System Life and Reliability Modeling of a Multiple Power Takeoffs Accessory Gearbox Transmission

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Abstract: A mathematical model for system life and reliability of a multiple power takeoffs aeroengine accessory gearbox transmission is presented. The geometry model of gear train is distributed into several subsystems by different transmitted powers. The lives of each component are combined to determine the units, subsystems and entire system lives sequentially according to a strict series probability model. The unit and subsystem interface models are defined to dispose the loads of common components. The algorithm verification is presented and a numerical example is given to illustrate the use of this program. The initial design could not fulfill the life requirement. A design modification shows that the gear train has a more balanced life distribution by strengthening the weak parts, and the overall life of entire system is increased above the design requirement. This program can help the designer to approach an optimal accessory gearbox transmission design efficiently.

Key words: aeroengine, accessory gearbox, power takeoff, gear train, life model, reliability, numerical program

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0 Introduction

Accessories (i.e., main fuel pump, lube pump, starter, etc.) are mounted on an accessory gearbox (AGB) located on the fan case or on the core of an aircraft engine. The AGB plays a vital role in the operation of an aircraft engine. General specification for aircraft turbojet and turbofan engines[1] requires specific life and reliability limits for the accessory gear transmission. It is imperative for the designer that accessory gear train skeleton should be created to meet the life and reliability requirements of specification in the preliminary design, which is an iterative process with manual intervention for moving designs across different design disciplines[2].

Mean time between overhauls (MTBO) or mean time between failures (MTBF) is an important property for the reliability of an AGB transmission module[3-4], since the engine maintenance cost is the major part of aircraft maintenance costs[5]. Tests of gear transmissions can be used to evaluate their reliabilities in selecting an optimal one, but it is time consuming and expensive to obtain the accurate results. Therefore, tests are usually conducted for performance verification and certification[6]. For a new AGB transmission, a high MTBO implies a long service life and a safe design. Selecting an extremely large MTBO or employing an over safety design leads to a heavy and overdesigned gear transmission, and it is not appropriate to the aero-engine application. In the preliminary design, engine and aircraft requirements are not specified. The designer needs to change the gear trains and to reevaluate these new designs. In cases such as this, there is a need for a tool that can quickly reevaluate a design based on new requirements, so the automated computer programs come to the aid of the designer.

The Lundberg-Palmgren fatigue life model is usually used for the life analysis of different types of bearings[7-8], and it is also used for the analysis of fatigue lives of spur, helical[9-10] and bevel gear pairs[11], assuming surface pitting is the eventual mode of failure. The reliability model is based on the reliabilities of the individual gears and bearings according to a two-parameter Weibull distribution, which is widely used in the numerical study of a single planetary transmission or a combinational reduction transmission with no power split[12-14]. This single power output transmission system is widely used in the helicopter transmission and wind turbine gearbox, and has received extensive attention and researches in these fields. Hu et al.[15] developed a reliability model based on the probabilistic design and reliability model, and studied the effects of load, tooth width and load sharing. Xue et al.[16] found that the contact fatigue life of gears for helicopter main
reducer is subject to two-parameter Weibull distribution. Li et al.\cite{17} established a model for the probabilistic life transformation based on the concept of minimal order statistics for planetary gear system. For the gear transmission system of wind turbine, Zhou and Xu\cite{18} built the dynamic and time-varying reliability model on the system level based on the stress-strength interference theory. An et al.\cite{19} established a reliability Copula model of a wind turbine gearbox based on failure correlation of each gear.

However, the life and reliability research on the AGB transmission system of the aeroengine is rarely reported, and the current models cannot be applied directly. For such multiple power takeoffs accessory gear transmission, motion and load of each component are more complicated. Therefore, how to establish the geometry model, how to reconstruct the system life model and how to achieve the algorithm implementation should be mainly focused on. Since some of their components may belong to multiple power flows simultaneously, the loads of the common components are combined in different angles. And model algorithm needs multiple iterations, while these are not included in the previous models.

In this paper, system life and reliability model of a multiple power takeoffs AGB transmission is developed. The system geometry model is distributed into subsystems by their transmitted powers. Each unit in a subsystem must be one of the three typical cases: spiral bevel gear unit, single mesh cylindrical gear unit and compound cylindrical gear unit. Several types of unit interfaces and subsystem interfaces are first introduced to constitute the subsystems and entire system separately, which can comprise most AGB gear trains of aeroengines in service. A case study is conducted for algorithm verification in comparison with that of the US National Aeronautics and Space Administration (NASA). Examples are also presented to illustrate the effect of this program. An initial design of the AGB transmission shows a low system life, and a refinement is performed to get a more satisfactory result. This program shows high efficiency to approach an optimal AGB transmission design.

1 Motion Analyses and Component Load

1.1 Transmission Configurations

A typical AGB gear train of an aeroengine under consideration is shown in Fig. 1. The input shaft of inlet gearbox (IGB) is the high pressure spool of the aeroengine. Power of aeroengine turbine flows into IGB, transfer gearbox (TGB) and AGB in sequence, and ultimately flows into the accessories mounted on the AGB. The couplings between each gearbox are assumed to be splined, and only able to transmit torque loads. For
IGB and TGB, each consists of one spiral bevel gear unit. Figure 1 shows three accessories: A1, A2 and A3, on the AGB for this analysis. For A1 and A2, each is transmitted by a single mesh cylindrical gear unit from their input gear shaft AGB SH2 and AGB SH1 separately. A3 is transmitted by a compound cylindrical gear unit from its input gear shaft AGB SH1. It may contain more accessories in other applications, whose gear train skeleton could be constructed by adding additional gear units.

There are four power flows identified by different transmitted powers for AGB gear train shown in Fig. 1, which are named Subsystems 1, 2, 3 and 4, respectively. Gear train of Subsystem 1 (S1) includes IGB and TGB. Subsystem 2 (S2) includes gear shafts AGB SH1 and AGB SH2. Similarly, shafts AGB SH1, AGB SH4 and AGB SH5 comprise Subsystem 3 (S3); Shafts AGB SH2 and AGB SH3 comprise Subsystem 4 (S4). Each subsystem is composed of one of the three basic gear units or their combinations.

1.2 Basic Gear Unit Cases

Three basic gear unit cases comprise the transmission power flow subsystems. These unit cases are: spiral bevel gear unit, single mesh cylindrical gear unit and compound cylindrical gear unit, as shown schematically in Figs. 2—4, respectively. Here, $\delta_1$ and $\delta_2$ are respectively the pitch angles of bevel Gear 1 and Gear 2, and $b$ is the face width of cylindrical or bevel gears.

Each unit case has the input or output gear supported by two bearings: Bearing 1 and Bearing 2, from left to right. For the spiral bevel gear unit, the bearings are numbered in the direction from the apex to the back side of the gear. The location of the input or output gear is specified by the distances $A$ and $B$, where $A$ is measured from gear center to Bearing 1 center, and $B$ is measured from gear center to Bearing 2 center. Compound cylindrical gear unit contains an intermediate shaft including two intermediate gears and two bearings, which are named Bearing 1, Bearing 2, Gear 1 and Gear 2 in sequence from left to right. The
distances $C$, $D$, and $E$ locate these gears: $C$ is measured from Gear 1 center to Bearing 1 center; $D$ is measured from Gear 1 center to Gear 2 center; $E$ is measured from Gear 2 center to Bearing 2 center. For the spiral bevel gear unit, the shaft angle, $\Sigma$, is the angle between input gear axis and output gear axis. For the compound cylindrical gear unit, $\Sigma$ is measured from the input shaft and intermediate shaft center-line to the intermediate shaft and output shaft center-line, counterclockwise about the intermediate shaft looked at from the input shaft, as shown in Fig. 4.

For a spiral bevel gear unit, the gear forces include $F_t$, $F_r$, and $F_a$, as shown in Fig. 5. These forces are dependent on the direction of the spiral hand, and the direction of rotation whether it is a driver or a driven gear. These forces can be obtained:

\[
F_t = \frac{2T}{d_m}, \quad (1)
\]
\[
F_r = \frac{F_t}{\cos \beta_m} (\tan \alpha_n \cos \delta \pm \sin \beta_m \sin \delta), \quad (2)
\]
\[
F_a = \frac{F_t}{\cos \beta_m} (\tan \alpha_n \sin \delta \mp \sin \beta_m \cos \delta), \quad (3)
\]

where, $F_t$, $F_r$, and $F_a$ are the tangential, radial, and axial loads; $T$ is the torque; $d_m$ is the mean pitch diameter; $\beta_m$ is the spiral angle; $\alpha_n$ is the normal pressure angle at pitch surface; $\delta$ is the pitch angle.

The bearings supporting the gear shaft have reactions in three directions: axial, radial, and tangential, which act in the same plane of gear loads. The bearing axial reaction is equal to the axial gear load if it is the only bearing to stand axial load, or equals half of the axial gear load if two bearings share this load.

The tangential bearing reactions are

\[
F_{bt1} = F_t \frac{B}{A + B}, \quad (4)
\]
\[
F_{bt2} = F_t \frac{A}{A + B}, \quad (5)
\]

where $F_{bt1}$ and $F_{bt2}$ are the tangential reactions of Bearing 1 and Bearing 2.

The radial bearing reactions are

\[
F_{br1} = \frac{F_t B}{A + B} + \frac{F_t r A}{|A|(A + B)}, \quad (6)
\]
\[
F_{br2} = \frac{F_t A}{A + B} - \frac{F_t r A}{|A|(A + B)}, \quad (7)
\]

where, $F_{br1}$ and $F_{br2}$ are the radial reactions of Bearing 1 and Bearing 2; $r$ is the mean pitch radius.

The distance $A$ is taken as positive for a straddle mounting and negative for an overhung mounting. For the intermediate shaft or a shaft within more than one gear, the bearing reactions in each direction are the vector sums of the react components of each gear, and then sum to the total bearing reaction force. For example, the total reaction of Bearing 1 is

\[
F_{b1} = \sqrt{F_{bt1}^2 + F_{br1}^2}, \quad (8)
\]

where $F_{b1}$ is the total reaction of Bearing 1.

For the intermediate shaft with two gears, the analysis for the bearing loads is performed in the similar way, but the variables $C$, $D$, and $E$ are used to define the distances between gears and bearings instead of variables $A$ and $B$, as shown in Fig. 4.

1.3 Contact Line Length

The length of contact line is an important parameter for evaluating the gear tooth capacity. For spur and helical gears, it can be obtained by [20]

\[
Z = (r_{O1}^2 - r_{b1}^2)^{0.5} \pm \frac{(r_{O2}^2 - r_{b2}^2)^{0.5} - (r_1 + r_2) \sin \alpha_t}{(r_1 + r_2) \cos \alpha_t}, \quad (9)
\]

where, $Z$ is the length of contact line of cylindrical gear pair; $r_{O1}$, $r_{b1}$, $r_1$ and $r_{O2}$, $r_{b2}$, $r_2$ are the addendum radius, base and pitch radius of the pinion and the gear, respectively; $\alpha_t$ is the operating transverse pressure angle. Double signs are used in Eq. (9): the upper sign applies to external gears, and the lower sign applies to internal gears.

It is more complicated for the spiral bevel gear to get the value of contact line length. There are two
ways: by tooth contact analysis (TCA) and by formula equations. Here is the equation [21]:

\[ s = \left( bZ_N \eta \cos \beta_b \right) / \eta^2, \]  

(10)

where, \( s \) is the length of contact line for the spiral bevel gear pair; \( b \) is the net face width; \( Z_N \) is the length of action in mean normal section; \( \eta \) and \( \eta \) are intermediate variables; \( \beta_b \) is the mean base spiral angle.

### 1.4 Unit Transmission and Subsystem Transmission Interfaces

In a multiple power takeoffs AGB gear transmission, it contains several accessory power takeoff shafts and may have two or more power branches from the main power train on the interface shafts. The entire transmission system is composed of subsystems with certain transmitted powers, and the sum of each takeoff power is equal to the total system input power. Subsystems are composed of the basic gear units as described above.

For an arbitrary subsystem with more than one unit, the output shaft of the previous unit should be the same as the input shaft of the next unit, or they are two separate shafts in the same center line connected by couplings, which could be treated as a whole one. Similarly, it contains a common shaft between adjacent subsystems. Then the common shaft is defined as the interface shaft. In a subsystem, the structure of interface shaft between adjacent units can be sorted into three types, as shown in Fig. 6. The hollow arrow is the power output of the previous unit, and the solid arrow is the power input of the next unit. In Fig. 6(a), they act on the same gear. In Fig. 6(b), they act on two different gears. Both Figs. 6(a) and 6(b) contain a single shaft with two bearings. In Fig. 6(c), they act on two gears mounted on two shafts with their own supporting bearings.

![Fig. 6 Unit interfaces](image)

The first subsystem is on the input side of the whole system, which transmits the power equivalent to the entire system power input. If the whole system transmits a single power with no power split, then the first subsystem is the only one equivalent to the total system. One or more new subsystems should be created when a power branch joint or a power takeoff joint appears. The power branch joint is the interface shaft including two or more power outputs in different directions, which is named Subflow joint. The power takeoff joint is the interface shaft where one or two accessories are mounted on each end pad to deliver power, which is named Subpower joint. All of these joints comprise the subsystem interfaces. The Subpower joint configurations are the same as those in Figs. 6(a) and 6(b). The Subflow joint configurations are shown in Fig. 7. Similarly, the hollow arrow is the power output of the previous subsystem, and the solid arrow is the power input of the next subsystems. In a Subflow joint, there may be two or three flow branches, and the maximum number of branches herein is set to three. These cases in Fig. 7 can construct most accessory gear trains of current aeroengines. All these cases are allocated a specific integer as an array element in the input file of this program to identify the types of subsystems and their interfaces.

![Fig. 7 Subflow joints](image)

The interface shaft is the common component for the connected subsystems or units. The bearings and gears on the common shaft are identical for the concerning subsystems or units. In a certain unit, the gears and bearings have steady loads, and they are analyzed as the components of a basic unit transmission. When the basic units assemble a subsystem or subsystems assemble the entire system, the loads of the gears and bearings should be vector superimposed for analyzing the lives and reliabilities of the subsystems or entire system transmission. An angle, \( \theta \), must be introduced to show the relationship of each force vector or power flow direction vector, as shown in Fig. 8. Here, \( \theta \) is measured counterclockwise from the output of the first unit gear load to the input of the second or the third unit gear load seen from Bearing 1 looking towards Bearing 2. Gear forces could be converted to a plane of the same baseline by \( \theta \), and then they are used for calculating bearing forces. The superimposed bearing forces on each bearing are given by

\[ F_{ba} = F_{bal} + F_{balI} + F_{balII}, \]  
\[ F_{bt} = F_{btl} - F_{btlI} \cos \theta_1 - F_{btlII} \cos \theta_2, \]  
\[ F_{br} = F_{brI} + F_{brII} \cos \theta_1 + F_{brIII} \cos \theta_2, \]  

(11)  
(12)  
(13)
where, $F_{ba}, F_{bt}$ and $F_{br}$ are the axial, tangential and radial forces of the bearing; $F_i$ and $F_i$ are the tangential and radial forces of the gear mesh, as shown in Fig. 8; $\theta_1$ and $\theta_2$ are the first and second output power flow direction angles relative to the input power flow direction on the interface shaft. In Fig. 8 and Eqs. (11)—(13), the subscripts I, II and III indicate the first, second and third units or subsystems, respectively.

![Fig. 8 Gear force superposition](image)

**2 Lives and Dynamic Capacities**

**2.1 Bearing Life and Capacity**

The Lundberg-Palmgren model has been applied to predict bearing life for a long time. A modified equation with adjustment factors is developed for the technology improvement in the area of material, processing, manufacturing and operating conditions. The modified equation is[22]

$$L_{10B} = a_1 a_2 a_3 \left( \frac{C_B}{P_B} \right)^{P_n},$$  \hspace{1cm} (14)

where, $L_{10B}$ is the adjusted 90% reliability life of the bearing in millions of load cycles; $a_1$ is a reliability factor; $a_2$ is a materials and processing factor; $a_3$ is an operating conditions factor (these factors can be obtained from relevant standards[7]); here it is assumed that $a_1 = 1$, $a_2 = 6$, and $a_3 = 1$ for typical values; $C_B$ is the basic dynamic load capacity; $P_B$ is the equivalent load; the power, $P_n$, is the load-life exponent, and the proposed value is 3 for ball bearings, and 10/3 for roller bearings[7].

**2.2 Gear Life and Capacity**

Based on the Lundberg-Palmgren theory, the spur gear life can be written as[22]

$$L_{10G} = \left( \frac{1}{z} \right)^{\frac{G}{p}} \frac{L_{10a}}{k},$$  \hspace{1cm} (15)

where, $L_{10G}$ and $L_{10a}$ are the 90% reliability lives of a gear and a single gear tooth, respectively, in millions of load cycles; $z$ is the number of teeth on the gear; $G$ is the Weibull slope for the gear and is taken as 2.5[13,22]; $k$ is the ratio from tooth load cycles to the system input shaft, which makes all components on the same counting base. Also, $k$ is a load count conversion factor especially useful for idler gears or intermediate gears, because their tooth will suffer more than one load cycle per gear rotation and the gear rotations will not always equal the input shaft rotations.

For helical and spiral bevel gears, the gear lives are modeled as equivalent spur gears in the normal plane[13]. The 90% reliability life of a single gear tooth in millions of load cycles is[22]

$$L_{10t} = a_2 a_3 \left( \frac{C_t}{P_t} \right)^{P_n},$$  \hspace{1cm} (16)

where, $a_2$ and $a_3$ are the life adjustment factors similar to those for rolling bearings; $C_t$ is the basic load capacity of the gear tooth; $P_n$ is the load-life exponent usually taken as 4.3 for AISI 9310 steel gears[22]; $P_t$ is the normal tooth load. For the intermediate gear on the interface shaft with multiple gear forces, $P_t$ is the equivalent normal tooth load determined as follows:

$$P_t = \left( \frac{P_{PG}^{(1)} + P_{PG}^{(2)} + \ldots + P_{PG}^{(n)}}{n} \right)^{\frac{1}{P_n}},$$  \hspace{1cm} (17)

where, $P_{11}, P_{12}, \ldots, P_{1n}$ are the normal tooth loads of gear meshes 1, 2, $\ldots$, $n$.

The basic load capacity of the gear tooth is

$$C_t = B_m b^{0.907} \rho^{-1.165} l^{-0.093},$$  \hspace{1cm} (18)

$$\rho = \left( \frac{1}{\rho_1^*} + \frac{1}{\rho_2^*} \right) \frac{1}{\sin \alpha_n},$$  \hspace{1cm} (19)

where, $B_m$ is a material constant taken as 139 MPa[22]; $l$ is the length of stressed track (for spur and helical gears, its value equals $Z$ in Eq. (9); for spiral bevel gear, its value equals $s$ in Eq. (10)); $b$ is the net face width for spur gear, and is the effective face width for helical and spiral bevel gears; $\rho$ is the curvature sum at the start of single-tooth contact; $\rho_1$ and $\rho_2$ are the radii of curvatures of the pinion tooth surface and gear tooth surface, respectively, at the contact point.

**2.3 System Life and Capacity**

The reliabilities of the individual bearing and gear are assumed to be a strict series probability model. The lives of each component are combined to determine the system life using the two-parameter Weibull distribution function[9], as follows:

$$\ln \frac{1}{R_S} = \ln \frac{1}{0.9} \sum_{i=1}^{n} \left( \frac{L_{10,i}}{L_{10,i}} \right)^{e_i},$$  \hspace{1cm} (20)

where, $R_S$, $L_S$ and $e_i$ are the reliability and the life of the entire system; $e_i$ and $L_{10,i}$ are the Weibull slope and the 90% reliability life of the component number $i$; $n$ is the number of components.
Components such as gears and different types of bearings have different Weibull slopes. Equation (20) can be solved at a reliability range, and the values of reliability versus life are plotted on Weibull paper. Using a least-squares fit, the Weibull slope of the entire system can be determined, and then the two-parameter Weibull distribution with the system reliability can be written as

\[
\ln \frac{1}{R_S} = \ln \frac{1}{0.9} \left( \frac{L_S}{L_{10,S}} \right)^{e_S},
\]

(21)

where \(e_S\) and \(L_{10,S}\) are the Weibull slope and the 90% reliability life of the entire system.

The system mean life is the average of the mean lives of individual components. The failure rate is defined as the reciprocal of the mean life. Therefore, the transmission system life is given as

\[
\frac{1}{L_{av,S}} = \sum_{i=1}^{n} \frac{1}{L_{av,i}},
\]

(22)

where \(L_{av,S}\) and \(L_{av,i}\) are the mean lives of the entire system and component number \(i\).

Using the Palmgren load-life model in terms of component torques and lives, Eq. (20) becomes

\[
1 = \sum_{i=1}^{n} \left( \frac{D_S}{D_i} \right)^{e_i p_i},
\]

(23)

where, \(D_S\) and \(D_i\) are the conversion dynamic capacity of the entire system and component number \(i\); \(p_i\) is the load-life exponent of component number \(i\).

A series of 90% reliability lives of the entire system are calculated at input torques between 10% and 100% of the dynamic capacity. The negative reciprocal of the load-life exponent of the entire system can be found using a linear regression which is similar to the calculation of Weibull slope. The conversion dynamic capacity of the entire system is given by

\[
L_{10,S} = \left( \frac{D_S}{T_1} \right)^{p_S},
\]

(24)

where, \(p_S\) is the load-life exponent of the entire system; \(T_1\) is the input torque of the overall transmission.

### 3 Program Structure and Algorithm Verification

#### 3.1 Program Structure

The flow chart of the program is shown in Fig. 9. AGB gear train as a whole transmission system contains a series of connected subsystems and each subsystem contains one or more units. Each unit should be one of the three basic unit cases, which are the basic input modules in the input file. The input file contains all unit array parameters, including configuration of subsystems (i.e., number of units, input power, speed, direction of input shaft rotation, units, type and serial number of subsystem, type of interface to the next connected subsystem, number of flow branches, and serial number of the next connected subsystem), configuration of units (i.e., type of unit and type of interface to the next connected unit), properties and configurations of all gears and bearings, and also the shaft geometry and properties. Each unit is described

![Fig. 9 Program flow chart](image-url)
in a fixed-field format of a two-dimensional array with special name, i.e., sys1_unit2, which implies the second unit in the first subsystem. Then the input array data can be passed to a global three-dimensional array used in the whole program calculation, whose third dimension corresponds to a specific subsystem. The output gear train layout, subsystem knot diagram and orthographic projection can be used to check the transmission configuration. The life and capacity of each component in units are determined, and then the lives and capacities of units can be obtained by iteration. When the calculations of all units in a subsystem are completed, the life and capacity of the subsystem can be obtained by iteration. Ultimately, the results of each subsystem can be used to obtain the life and capacity results of the entire system iteratively.

3.2 Algorithm Verification

It is impractical for laboratory test of a statistically significant number of aeroengine gearboxes to determine their life and reliability\cite{22}, due to the fact that the small variations in operating profile can result in significant changes in life, and also for the huge cost and time expenditure. Fortunately, Zeretsky\cite{10} verified the Lundberg-Palmgren model by collecting field data of the commercial turboprop gearbox, which is also used in this study to predict gearbox life. The field data results follow the similar trend as the simulated predictions, but the $L_{10}$ life is a bit underpredicted. Even so, from the view of transmission engineering design application, the system level reliability quick optimization under uniform standard is still a valuable and meaningful job.

Here, we use the same Lundberg-Palmgren model to predict gearbox life and reliability, but the algorithm in this program is defined for a multiple power takeoffs accessory gear train with a completely different program structure. It is also applied to a single component, unit or subsystem. For checking the validity of the algorithm, the bearing life, gear life and unit life calculations made by this program are conducted to compare with the results of NASA\cite{13}. A more complicated calculation case of a multiple power takeoffs accessory gear train will be presented in the next section. Based on the same input data file, as shown in Table 2 of Ref. [13], the loads, output dynamic capacities and mean lives of components and the unit are compared with those in Tables 3 and 4 of Ref. [13]. The loads of components are the same and are no longer listed here. The capacities and mean lives are listed in Table 1 with the results of Ref. [13] in parenthesis, and the life of system presented in the Weibull plot is shown in Fig. 10, while Ref. [13] only shows tabular results. The major different data are those of gears, as shown in Table 1 below. This is mainly caused by the update calculation methods of the curvature sum of tooth face and the contact line length in obtaining of basic load capacity of gear tooth. However, the greatest relative error between them is less than 5%, and the impact to the system properties is very trivial. So the results of the algorithm are considered reasonable and acceptable.

| Component          | Conversion dynamic capacity/(N·m) | $L_{10}$ life in million output rotations | Mean life/h |
|--------------------|----------------------------------|------------------------------------------|-------------|
| Input pinion       | 2 786.88 (2 772.36)              | 4 604 783.75 (4 393 803.00)             | 86 031 332.63 (82 089 460.00) |
| Input roller bearing| 1 752.48 (1 752.48)               | 62.73 (62.73)                            | 3 429.65 (3 429.32) |
| Input ball bearing  | 1 629.36 (1 629.36)               | 34.61 (34.60)                            | 1 939.60 (1 939.40) |
| Output gear        | 2 889.32 (2 904.53)               | 6 339 971.61 (6 659 760.00)             | 118 449 789.02 (124 424 400.00) |
| Output roller bearing | 2 227.92 (2 227.92)             | 138.52 (138.51)                           | 7 572.88 (7 572.15) |
| Output ball bearing | 3 853.48 (3 853.48)              | 457.81 (457.77)                           | 25 657.98 (25 655.27) |
| Transmission       | 1 683.67 (1 313.67)              | 20.66 (20.66)                            | 1 146.23 (1 146.13) |

Mean component life: 1 022.24 (1 022.19)

4 Applications

4.1 Numerical Example

A multiple power takeoffs AGB transmission used in this study consists of IGB, TGB and AGB, as shown in Fig. 1. The input power is 150 kW (equivalent input torque of 95.49 N·m) at the speed of 15 000 r/min in the counterclockwise direction looking towards the flying orientation. Transmitted power for each accessory is 50 kW for A1, 75 kW for A2 and 25 kW for A3. Gears are initially designed to satisfy the requirement of bending strength and pitting resistance. Bearings are selected from the bearing catalogue or from the past experience in similar applications. The basic gear and bearing parameters are listed in Tables 2 and 3, respectively, where the values in parenthesis are used for the redesign (Design 2) later.
### Table 2 Basic gear parameters with redesigned values in parenthesis

| Gear description | Number of teeth | Module/mm | Pressure angle/° | Face width/mm | Spiral angle/° | Hand of spiral |
|------------------|-----------------|-----------|------------------|---------------|----------------|----------------|
| IGB input gear   | 37              | 4.2       | 20               | 25            | 35             | Left           |
| IGB output gear  | 35              | 4.2       | 20               | 25            | 35             | Right          |
| TGB input gear   | 29              | 4.2       | 20               | 25            | 35             | Right          |
| TGB output gear  | 31              | 4.2       | 20               | 25            | 35             | Left           |
| AGB SH1 gear     | 37 (33)         | 2 (2.5)   | 25               | 12 (18)       | 0              | —              |
| AGB SH2 gear     | 67 (59)         | 2 (2.5)   | 25               | 12 (18)       | 0              | —              |
| AGB SH3 gear     | 63 (55)         | 2 (2.5)   | 25               | 6 (10)        | 0              | —              |
| AGB SH4 gear 1   | 63 (56)         | 2 (2.5)   | 25               | 6 (12)        | 0              | —              |
| AGB SH4 gear 2   | 37 (33)         | 2 (2.5)   | 25               | 8 (12)        | 0              | —              |
| AGB SH5 gear     | 73 (65)         | 2 (2.5)   | 25               | 8 (12)        | 0              | —              |

### Table 3 Basic bearing parameters with reselected values in parenthesis

| Bearing description | Bore diameter/mm | Outside diameter/mm | Width/mm | Basic dynamic capacity/kN | Static capacity of ball bearing/kN |
|---------------------|------------------|---------------------|----------|---------------------------|-----------------------------------|
| IGB input bearing 1 | 100              | 180                 | 30       | 130                       | 135                               |
| IGB input bearing 2 | 100              | 150                 | 24       | 145                       | —                                 |
| IGB output bearing 1| 40               | 70                  | 20       | 45                        | —                                 |
| TGB output bearing 2| 40               | 68                  | 15       | 30                        | 20                                |
| All bearings in AGB except AGB SH5 bearing 2 | 30 | 62 (55) | 16 (13) | 20.3 (13.8) | 11.2 (8.3) |
| AGB SH5 bearing 2   | 25               | 52 (47)             | 15 (8)   | 14.8 (8.06)               | 7.8 (4.75)                        |

Fig. 10 Weibull plot of predicted life for spiral bevel gear unit compared with Ref. [13]

![Weibull plot](image)

4.2 Discussion of Results

The graphical user interface (GUI) of the program is shown in Fig. 11. The decomposed gear train skeleton, subsystem knot diagram and orthographic projection are shown in the upside of Fig. 11. They are applied to check whether the analyzing gear train is consistent with the required layout, and to check whether the input parameters in the input file are correct. There are four subsystems in this gear train. For the initial Design 1 as shown in Tables 2 and 3, the lives of the entire system and each subsystem presented in the Weibull plot are shown in Fig. 12. It can be found that Subsystem 1 has the largest life against the other three subsystems. The life data of Subsystem 2 is the most close to the entire system, and it is the weakest part of the complete transmission. By further examination, Fig. 13 shows...
the mean lives and dynamic capacities of the entire system and each component. These results show that the gears in AGB have the lowest lives below the design requirement of 10000 h except the gears of AGB SH4 and SH5. AGB SH1 gear has a special low life of 892 h, which dominates the life and dynamic capacity of the entire transmission. For the entire system, the mean life is only 879 h, and the value of dynamic capacity is 251 N·m in units of input torque. It can also be found that the lives of each component in this gear train are not balanceable, where the values of bearings are much higher than those of gears. In order to improve this transmission, some changes of gears are performed and
all AGB bearings are reselected from the Svenska Kullager Fabriken (SKF) catalogue in Design 2 as listed in parenthesis within Tables 2 and 3, respectively.

For Design 2, the system Weibull plots are shown in Fig. 14, and the lives and dynamic capacities of components and the entire system are shown in Fig. 15. The lives of Subsystems 2, 3 and 4 increase significantly, since the AGB gears are modified by changing module, face width and number of teeth, but the transmission ratio of each accessory changes only slightly. The lives of the AGB gears are increased by increasing their dynamic capacities. The lives and dynamic capacities of AGB bearings in Design 2 are lower compared with the results of Design 1, since these bearings are reselected from those of smaller size and lower basic dynamic capacities, but they are still much higher than the values of gears. For the entire system, the dynamic capacity equals 467 N·m, and the mean life is 10639h which
is above the design requirement. This indicates that the weakest components of the gear train are the AGB gears. The lives and dynamic capacities of gears can be improved significantly by increasing their modules and face widths. When these components are strengthened, the entire system will get a much higher MTBO. However, the Design 2 of this AGB gear train may not be the best design and can also be improved by a further optimization.

5 Conclusion

(1) Based on the three basic gear unit cases and the interface models, the system geometry model of a multiple power takeoffs AGB is developed. Then based on the Lundberg-Palmgren model, the two-parameter Weibull distribution model and a strict series probability model, the lives of each component are combined to obtain the unit, subsystem and entire system lives sequentially.

(2) Algorithm verification of the program is performed by comparison with the results of US NASA for the same unit sample. Similar results are obtained with the maximum error less than 5%, which mainly occurs in the gears due to the different calculation methods of the curvature sum of tooth face and the contact line length.

(3) A numerical example of a multiple power takeoffs AGB gear train is presented. It shows that the weakest component dominates the life and dynamic capacity of the entire transmission. The minimum design requirement of MTBO for the system is obtained by strengthening the weak gears with higher modules and face widths.

(4) The program acting as a uniform standard for system level reliability optimization can help the designer to approach an optimal AGB transmission design efficiently.

References

[1] Engine, aircraft, turbojet and turbofan, general specification: MIL-E-5007E [S]. Washington: United States Department of Defense, 1983.
[2] CHAUDHARI V, MONIZ T, NAIKTARI C, et al. Integrated preliminary design approach for turbomachinery design [C]// ASME 2011 Turbo Expo. Vancouver: ASME, 2012: 171-176.
[3] Joint service specification guide: Engines, aircraft, turbine: JSSG-2007A [S]. Washington: United States Department of Defense, 2004.
[4] INGISTOV S. Failures of the high energy, high pitch velocity gear boxes [C]// ASME Turbo Expo 2004. Vienna: ASME, 2008: 783-794.
[5] DAVIS J D. An appraisal of cost-effectiveness models used in the air force and navy aircraft engine component improvement programs [D]. Monterey: Naval Postgraduate School, 1991.
[6] KUZNAR R. GE Aircraft Engines and Fiat Avio high electrical output accessory drive system [C]// AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit. Cleveland, OH: AIAA, 1998: 3584.
[7] Rolling bearings: Dynamic load ratings and rating life: ISO 281 [S]. Geneva: ISO, 2007.
[8] HARRIS T, RUMBARGER J H, BUTTERFIELD C P. Wind turbine design guideline DG03: Yaw and pitch rolling bearing life [R]. Golden: National Renewable Energy Laboratory, 2009.
[9] ZARETSKY E V. Design of oil-lubricated machine components for life and reliability [C]// Seventh International Symposium on Tribology. Cracow: NASA, 2006: 1-10.
[10] ZARETSKY E V. Design of oil-lubricated machine components for life and reliability [C]// Twenty-Eighth Joint Propulsion Conference and Exhibit. Nashville, TN: AIAA, 1992: 1-15.
[11] ZARETSKY E V, LEWICKI D G, SAVAGE M, et al. Spur, helical, and spiral bevel transmission life modeling [J]. Journal of Propulsion and Power, 1996, 12(2): 283-288.
[12] SAVAGE M, PRASANNA M, COE H. Maximum life spiral bevel reduction design [C]// 28th Joint Propulsion Conference and Exhibit. Nashville, TN: AIAA, 1992: 1-15.
[13] ZARETSKY E V, LEWICKI D G, SAVAGE M, et al. Determination of turboprop reduction gearbox system fatigue life and reliability modeling [R]. Adelphi: Army Research Laboratory, 1994.
[14] HU Q C, DUAN F H, WU S S. Research on reliability of closed planetary transmission systems [J]. China Mechanical Engineering, 2007, 18(2): 146-149 (in Chinese).
[15] XUE X Z, LI Y X, WANG S M. Method of lifetime and reliability of some helicopter’s main reducer [J]. Journal of Aerospace Power, 2011, 26(3): 635-641 (in Chinese).
[16] LI M, XIE L Y, LI H Y, et al. Life distribution transformation model of planetary gear system [J]. Chinese Journal of Mechanical Engineering, 2018, 31: 24.
[17] ZHOU Z G, XU F. Dynamic reliability analysis of gear transmission system of wind turbine considering strength degradation and dependent failure [J]. Journal of Mechanical Engineering, 2016, 52: 80-87.
[18] AN Z W, ZHANG Y, WANG Z L. Reliability Copula model for wind turbine gearbox based on failure correlation [J]. Journal of Shanghai Jiaotong University (Science), 2015, 20(3): 312-316.
[19] Geometry factors for determining the pitting resistance and bending strength of spur, helical and herringbone gear teeth: ANSI/AGMA 908-B89 [S]. Alexandria: AGMA, 1989.
[20] Geometry factors for determining the pitting resistance and bending strength of generated straight bevel, zero bevel and spiral bevel gear teeth: ANSI/AGMA 2003-B97 [S]. Alexandria: AGMA, 1997.
[21] Geometry factors for determining the pitting resistance and bending strength of generated straight bevel, zero bevel and spiral bevel gear teeth: ISO 6336-2007 [S]. Geneva: ISO, 2007.
[22] ZARETSKY E V. Rolling bearing life prediction, theory, and application [R]. Cleveland: Glenn Research Center, 2013.