INTERNAL PROPER MOTIONS IN THE ESKIMO NEBULA

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ABSTRACT

We present measurements of internal proper motions at more than 500 positions of NGC 2392, the Eskimo Nebula, based on images acquired with WFPC2 on board the Hubble Space Telescope at two epochs separated by 7.695 yr. Comparisons of the two observations clearly show the expansion of the nebula. We measured the amplitude and direction of the motion of local structures in the nebula by determining their relative shift during that interval. In order to assess the potential uncertainties in the determination of proper motions in this object, in general, the measurements were performed using two different methods, used previously in the literature. We compare the results from the two methods, and to perform the scientific analysis of the results we choose one, the cross-correlation method, because it is more reliable. We go on to perform a “criss-cross” mapping analysis on the proper motion vectors, which helps in the interpretation of the velocity pattern. By combining our results of the proper motions with radial velocity measurements obtained from high resolution spectroscopic observations, and employing an existing 3D model, we estimate the distance to the nebula to be 1.3 kpc.

Key words: ISM: kinematics and dynamics – planetary nebulae: individual (NGC 2392) – techniques: spectroscopic

Supporting material: machine-readable table

1. INTRODUCTION

NGC 2392 (α = 07:29:10.76, δ = +20:54:42.47 [J2000.0]) is one of the most extensively studied high-ionization double-shell planetary nebulae (PNe; see García-Díaz et al. 2012, hereafter Paper I, and references therein) which is better known as the Eskimo nebula. The Eskimo nebula shows a very complex structure: a main inner shell with a filamentary shape surrounded by a ribbed structure (in Paper I “caps”), an outer shell, bright cometary knots, and collimated high-velocity bipolar outflows.

Comprehensive spectroscopic kinematic studies in Paper I have shown that the expansion velocity of the inner shell is $V_{\text{exp}} \approx \pm 120 \text{ km s}^{-1}$. This kinematic study also revealed that the knots are distributed in a disk very near the plane of the sky, expanding at velocities of $\approx 70 \text{ km s}^{-1}$. O’Dell et al. (1990) suggests that the outer shell is an oblate spheroid and the inner shell can be well described as a prolate spheroid oriented pole-on. In Paper I, we found that the inner shell is tilted by 9° with respect to the line of sight with a position angle, P.A., of 25° and, to a first approximation, has a width-to-length ratio of approximately 1.8.

The central star (CS) of the Eskimo has been studied by several authors. An important discussion about the temperature of the CS is given by Pottasch et al. (2008), in a paper about the abundances of the nebula using spectral data obtained in the mid-infrared with the Spitzer Space Telescope. The authors assume that [N II] was formed probably in the first dredge-up, that the abundance of carbon was produced during the third dredge-up, and that there is no evidence of the existence of a second dredge-up. From this analysis, the authors found that the CS must have evolved from a progenitor of 1.7 $M_\odot$. Several authors have calculated an effective temperature, $T_{\text{eff}}$, of the CS of around 40,000 K–45,000 K (Méndez et al. 2012; Pauldrach et al. 2004; Kudritzki et al. 1997). However, this temperature is not high enough to explain the high stages of ionization of some ions, such as O IV and Ne V, that have been observed in the nebula (Pottasch et al. 2008; Natta et al. 1980). Ciardullo et al. (1999) observed a weak companion of the CS which is undetected in V band. In an attempt to model the companion star, Danekar et al. (2012) used photoionization models and their result showed that the companion star must have a $T_{\text{eff}} = 250,000 \text{ K}$, which is much higher than that proposed by Pottasch et al. (2008). To date there is no decisive evidence of the nature of the putative companion star.

In order to understand the nature and origin of the Eskimo nebula, it is crucial to know its distance, which at present is not very well known. Several attempts have been made to find it using different statistical methods. Maciel (1981) estimated a distance of 1.1 kpc by using a mass–radius relation (Barker 1978). Hajian & Terzian (1995) measured the angular expansion of NGC 2392 at a radio frequency of 5 GHz with the Very Large Array with six years between epochs, employing the Doppler expansion velocity to calculate the distance to the nebula (assuming a spherical shape). The authors did not detect an angular expansion of the nebula, leading to a lower limit for the distance of 1.4 kpc. Stanghellini et al. (2008) revised the calibration of the PN distance scale from Cahn et al. (1992) using data for PNe in the Magellanic Cloud. This statistical method is based on a calibration of the relation between the ionized mass (assuming that all nebulae have the same ionized mass) of PNe and the optical thickness parameter. In this way, the authors obtained a distance for NGC 2392 of 1.259 pc. Liller & Liller (1968) reported a distance of 1.6 kpc with an uncertainty of 1.30 pc. Pottasch et al. (2011) reported 1.8 kpc for the distance, inferred from the core mass, the chemical nebular abundances, and the luminosity.

Another method for calculating distances is using proper motion measurements. A sample of a previous application of proper motion methods is found in Artigau et al. (2011), who used cross-correlation methods to measure the proper motions
of the knots and arcs of Eta Carinae using data from the Near-Infrared Coronagraphic Imager and NACO. Using the same method (cross-correlation), Ueta et al. (2006) calculated the proper motion patterns of the knots and arcs of Eta Carinae using data from the Hubble Space Telescope (HST, with a 5.5 yr interval). From these data, the authors determined the distance to the Egg nebula. Szyzska et al. (2011) used a different method on two epochs of HST imaging, separated by 9.43 yr in order to measure the expansion proper motions for NGC 6302’s bipolar lobes, calculating the $\chi^2$ of the difference image.

Li et al. (2002) observed the PN BD+30-3639 using the Wide Field Planetary Camera 2 (WFPC2) camera on board the HST. The data were obtained from two different epochs, separated by 5.663 yr. They used the $\chi^2$ method to measure the radial expansion of this PN. To derive the distance of the nebula, these authors combined the angular expansion with radial expansion velocities taken from the HST STIS Echelle spectrograph.

A recent paper about proper motions was published by O’Dell et al. (2013), where tangential velocities were calculated for NGC 6720 using the least-squares ($\chi^2$) method.

These studies all present the results of either only cross-correlation or the $\chi^2$ method for their determination of internal proper motion patterns. Up to now it is, however, unclear how these methods compare and how consistent the results are when applied to the same data set. In order to better assess the reliability of proper motion measurements in general and for this object, in particular, by these methods, we apply both of them (cross-correlation and $\chi^2$) to the same data sets.

In this paper, we report proper motion measurements for a large number of structures and arcs in the Eskimo nebula, using two [N II] 6584 Å images from the HST archive that were observed with a time interval of 7.695 yr. The results, along with the radial velocity measurements taken from long-slit, high-resolution, spectroscopic observations (Paper I) allow us to determine the distance.

The structure of the paper is as follows. In Section 2, we describe the observations and the data-reduction steps, including the alignment of the HST images. In Section 3, we describe in more detail the methods for finding the measurements of the proper motions in the nebula. In Section 3.1, we discuss the differences between the methods. In Section 4, we discuss the radial velocities. In Section 5, we explain the method for calculating the distance. In Section 6, we perform a criss-cross mapping analysis. Finally, we present a discussion of the results and our conclusions in Sections 7 and 8.

2. OBSERVATIONS AND DATA REDUCTION

2.1. High-resolution Spectroscopy

For the present work, we used the data from Paper I. They are high-resolution spectroscopic observations of the Eskimo nebula obtained at the Observatorio Astronómico Nacional San Pedro Mártir (OAN-SPM), Baja California, México, during the nights of 2002 January 7–10. The data were obtained with the Manchester Echelle Spectrometer (Meaburn et al. 2003) attached to the 2.1 m telescope in its f/7.5 configuration. For all positions, we used a 90 Å bandwidth filter to isolate the 87th order containing the H$\alpha$ and [N II] nebular emission lines. For the majority of the exposures, we used a 70 µm slit, while for some exposures a 150 µm slit was applied. The slit was oriented north–south, and exposures were taken at a series of 18 different parallel pointings and three pointings at other position angles, P.A. = $70^\circ$ (two positions) and P.A. = $110^\circ$ (one position). The positions and orientations of the slit are shown in Figure 1 of Paper I. All spectra were acquired using exposure times of 1800 s. Full details of the observations and data reduction process are given in Paper I.

We convert heliocentric radial velocities to velocities relative to the Eskimo nebula by taking the systemic velocity, $V_{sys} = 70.5$ km s$^{-1}$, calculated in Paper I. The spectroscopic data are summarized in Table 1.

2.2. Hubble Space Telescope Data

The data used in this study to measure the proper motions of the Eskimo nebula consist of images which were retrieved from the HST archive. The observations were made in two separate observing runs: on 2000 January 1 (hereafter Epoch-1), as part of program 8499, with Andrew S. Fruchter as PI. During this run, three images were obtained with the WFPC2, with an exposure time of 350 s, using (among others) the F658N filter. The second-epoch data were obtained on 2007 September 21 (hereafter Epoch-2), from the science program 11122, with Bruce Balick, as PI, who employed the same configuration as in program 8499 to acquire three images with the F658N filter, using 400 s exposure times for all of them. For all of the images, we used the drizzled images (processed with Astrodizzle). Table 2 summarizes the imaging observations.

The time interval between observations is 7.695 yr with an angular resolution of $\approx0.1$. The mosaicking, geometric correction, and astrometry correction were done in the HST pipeline using Astrodizzle. The mosaics were median combined. In order to compare the two combined mosaics, we regridded them using the SWARP utility (Bertin et al. 2002), matching the X–Y directions with R.A.–decl. The correction of the very small residual
relative displacements between the two mosaics were performed recursively by visually analyzing the arithmetic difference, using the general structure that remains fixed as a reference. The shifting was carried out in 0.1 pixel steps.

We used the CS as a reference. The positions of the points where we calculated the proper motions in the nebula are given as offsets in R.A. and decl. referred to this point.

The resulting pictures are shown in Figure 1, which shows in the left and the center panels the two final images for each epoch (2000.03 and 2007.72, respectively) centered, taking the center of the Eskimo nebula and the difference image from the two epochs in the right panel as references. In general, we can see that the arcs of the inner bubble have outwards motion from the center of the nebula in the plane of the sky. The knots or arcs. We used two methods based on the assumption that the proper motion of any local structure in a nebula due to expansion can be measured by determining the translational displacements of the points. Generally, N = 15 was enough to measure the displacements of the points.

We denote the center of the box as p. This N × N pixel box is correlated with a box of the same size centered at p + δ in the other image. We then move δ so that −δmax ≤ δx, δy ≤ δmax and −δmax ≤ δx, δy ≤ δmax, obtaining a value for the correlation at each point, to form the function $C(p, \delta)$. The point with coordinates (δx, δy) where the function is maximum determines the displacement of the structure under study.

Calculating the cross-correlation for different displacements, δ, in one of the images, we can build the function $C(p, \delta) = C_{fg}(\delta_x, \delta_y)$. For the calculations, we used the task XREGISTER implemented in IRAF, where the cross-correlation function is computed in the following manner.

$$C_{fg}(\delta_x, \delta_y) = \frac{\sum_{x=1}^{M} \sum_{y=1}^{N} [f(x + \delta_x, y + \delta_y) - \bar{f}][g(x, y) - \bar{g}]}{\sigma_f \sigma_g}$$

where

$$\sigma_f = \left[ \sum_{x=1}^{M} \sum_{y=1}^{N} [f(x + \delta_x, y + \delta_y) - \bar{f}]^2 \right]^{1/2}$$

$$\sigma_g = \left[ \sum_{x=1}^{M} \sum_{y=1}^{N} [g(x, y) - \bar{g}]^2 \right]^{1/2}$$

$$\bar{f} = \frac{\sum_{x=1}^{M} \sum_{y=1}^{N} [f(x + \delta_x, y + \delta_y)]}{M \times N}$$

$$\bar{g} = \frac{\sum_{x=1}^{M} \sum_{y=1}^{N} [g(x, y)]}{M \times N}$$

Repeating the calculation in all of the 536 regions marked on the Epoch-1 image, we obtained the displacement map shown in Figure 2 (left panel), representing the internal motion of the nebula in the plane of the sky.

Second method: this method is based on the minimization of chi-squared ($\chi^2$). We used the same 536 different local regions identified above.

These structures should be located at slightly shifted positions in the Epoch-2 images due to the proper motion of the nebula. To quantify the displacements, we defined square image sections

![Figure 1. Left: Epoch-1 image [N ii] WFPC2-HST. Center: Epoch-2 image [N ii] WFPC2-HST. Right: difference of the two images taken 7.695 yr apart (2007–2000). In the difference image, the central bubble with proper motions shows up as “negative/positive” double ridge structures.](Image)
centered on each of these local structures in the Epoch-1 image. The size of these image sections has to be large enough so that the segmented structures can be uniquely identified. To analyze each section, we produced a custom IRAF task to shift the section of one epoch with respect to the corresponding section of the other epoch, to calculate the difference and to estimate the chi squared ($\chi^2$ ) on the difference. The script repeats this process spanning from $-\Delta x$ to $+\Delta x$, and from $-\Delta y$ to $+\Delta y$, where $\Delta x$ and $\Delta y$ were programmable, in increments of 0.1 pixel. Visually, we determined that the displacements are generally smaller than 1 pixel; thus, we carried out several trials putting $\Delta x = \Delta y = 2$ or 3 pixels. In Figure 3, we show a 3D plot of $\chi^2$ for 12 example cases, where we can see that the surface shows a minimum value of $\chi^2$ in these plots, which corresponds to the displacement for which the region of the second epoch is most similar to the same region on the first epoch. Then, measuring the position of this minimum gives the magnitude of the displacement vector, because the $x$, $y$ coordinates of this point are the relative positions given in 0.1 pixel steps with respect to the center of the region on the Epoch-1 image.

In general, the map obtained with this method (see Figure 2, right panel) is a good representation of the movements in the nebula. However, we found several cases where the minimum value is not so well defined, as we can see in Figure 4. Generally, in these cases the velocity vector was different from the movements observed in a visual approach. The reason for these discrepancies could be the uncertainties in the determination of the displacements, given by the center of the surface of the $\chi^2$ plot.

3.1. Differences between the Two Methods to Calculate the Proper Motions

We measured the proper motions of more than 500 regions of NGC 2392 by two methods: one using cross-correlation and another using the minimization of $\chi^2$. There are several points where the magnitude of the displacement obtained by minimizing the $\chi^2$ is larger than a full pixel, but visual inspection shows that the movements are smaller than that. We believe that these values, obtained by minimizing $\chi^2$, are generally oversized in the process of searching the minimum value, particularly in regions where the surface defined by the value of $\chi^2$ as a function of the displacements in $X$ and $Y$ is not as sharp and well behaved (see Figure 4) as those in Figure 3.

There are a couple of other points whose associated regions have very high ratios between the maximum and minimum values, or where this ratio is negative. In Figure 5, we show a plot of the ratio maximum/minimum of the pixel values in each subregion considered for each point in the $\chi^2$ method, taken as a measure of the contrast. We see those cases in Figure 5.

Discarding those points, we see that overall, the discrepancy between these two methods is really small. However, we preferred the values obtained with the cross-correlation method since we did not need to discard any values with that method. In addition, comparing the two images using a direct visual approach, we can see that the values obtained from the cross-correlation method are more representative of the motion in the nebula, since the global proper motions are more readily apparent in the cross-correlation method.

4. RADIAL VELOCITIES

The radial velocity of each local big structure or region (see Figure 1) was calculated using the [N ii] 6584 Å line profile for 20 individual slit positions originally used for Paper I. We identify each slit position over an HST [N ii] image relating each component of the emission profile with the corresponding region of the HST image. In the position–velocity ($P$–$V$) arrays shown in Figure 2 of Paper I, we found several distinct regions distributed along the slit, Figure 6 shows an example of the distribution of the regions in slits $i$–$l$.

These regions include the jets, but we did not use them because they are not visible in the HST images. We calculated the heliocentric velocity for each region by fitting a Gaussian to the 1D profile. The radial velocities are then calculated by subtracting the systemic velocity. The errors in the determination of the velocities are given by the $\sigma$ in each Gaussian fitted, according to the relation $FWHM = 2\sqrt{2\ln(2)}\sigma$. The median of those values of $\sigma$ is 10.5 km s$^{-1}$. 

![Figure 2](image-url) Figure 2. Left panel: map of proper motions obtained with the cross-correlation method. Right panel: map of proper motions obtained with the $\chi^2$ method.
Figure 3. Some of the 3D plots of $\chi^2$ as a function of the displacements in $X$ and $Y$. The scale of the $Z$ axis is in arbitrary units. In this figures, we can see the minimum very well defined.

In some cases, we had problems with the identification of the velocities of some of the regions, because we found two or three different velocities in the same position of the HST image. In those cases, we took the velocity of the brightest node in the $P-V$ array.

The radial velocities with respect to the CS are listed in Table 3.

5. DISTANCE

We calculate the distance to the Eskimo nebula using the proper motion vectors calculated from the cross-correlation method and taking only the vectors of the inner shell, given that we know its geometry. Statistically, the components of proper motions are $\langle pm_x \rangle \approx \langle pm_y \rangle$. If we consider a spherical distribution, the root mean square (rms) of the radial velocity would be: $\langle rv \rangle \approx \langle pm_x \rangle \approx \langle pm_y \rangle$. Assuming the shape of the inner shell as modeled in Paper I (see Figure 7), where the ratio between the major axis ($b$) and minor axis ($a$) is $b/a = 1.8$,

$$\langle rv \rangle = (b/a) \langle pm_x \rangle = (b/a) \langle pm_y \rangle,$$

or, in terms of the rms of the proper motions ($\langle pm \rangle = \sqrt{pm_x^2 + pm_y^2}$)

$$\langle rv \rangle = (b/a) \frac{\langle pm \rangle}{\sqrt{2}}$$

(7)

If we express the time interval between the two epochs of observation by $\delta t$ (in years), the rms of the measured transverse displacements (on the images) by $\langle \delta r \rangle$ (in arcsec), and the distance $D$ (in parsecs) to the Eskimo nebula, the rms of proper motions can be expressed by (see Appendix, Equation (A3)),

$$\langle pm \rangle = 4.74 \frac{D \langle \delta r \rangle}{\delta t}$$

(8)

Using the Equation (8), we can compute $D$ by,

$$D = 0.211 \frac{\delta t}{\langle \delta r \rangle} \frac{\sqrt{2} \langle rv \rangle}{(b/a)}$$

(9)

and from our measurements, we calculate a distance of 1.3 kpc. This value is similar to other published values: 1259 kpc from Stanghellini et al. (2008), 1.4 kpc from Hajian & Terzian (1995), 1.6 kpc from Liller & Liller (1968), and 1.8 kpc from Pottasch et al. (2011).

Applying our newly computed distance, Table 3 lists the proper motion values converted from angular displacements to velocity in km s$^{-1}$, where we have used the scale factor to convert arcsec into km s$^{-1}$ from Equation (9) (801.3 km s$^{-1}$ arcsec$^{-1}$ calculated in the Appendix, Equation (A7)).
Figure 4. Same as Figure 4, but it can be seen that in this case the function around the minimum is more difficult to be defined. In several of these cases, the calculated velocity vector was different than the movement observed using a visual approach.

Figure 5. Plot of the ratio maximum/minimum value for the pixels for each of the 536 regions selected in the $\chi^2$ method, having discarded those for which the magnitude of the displacement is larger than a full pixel. We can see that in some cases this ratio is very large or negative. The larger and the minimum values correspond in this case to regions near the central star, where we found saturated pixels.
Figure 6. Example of regions on slits $i$ where we measured the radial velocity. Details about these observations are in Paper I.

Table 3
Proper Motions of NGC 2392

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|-----|-----|-----|-----|-----|-----|-----|-----|
| X (arcsec) | Y (arcsec) | $\text{pm}_x$ (pixel) | $\text{pm}_y$ (pixel) | $\text{pm}_x$ (km s$^{-1}$) | $\text{pm}_y$ (km s$^{-1}$) | Proper Motion (km s$^{-1}$) | Radial Velocity (km s$^{-1}$) |
| 25.94 | −12.02 | 0.4210 | 0.3641 | 33.76 | 29.20 | 47.74 | −5.50 |
| 24.87 | −12.82 | 0.3913 | 0.1027 | 31.38 | 8.24 | 44.38 | −2.80 |
| 24.87 | −11.92 | 0.2686 | 0.4216 | 21.54 | 33.81 | 30.46 | −2.80 |
| 24.44 | −14.12 | 0.4829 | 0.1922 | 38.72 | 15.41 | 54.76 | −2.80 |
| 23.37 | 12.18 | 0.4522 | −0.0030 | 36.26 | −0.24 | 51.28 | −5.10 |
| 23.05 | −18.62 | 0.1641 | 0.2169 | 13.16 | 17.39 | 18.61 | 0.20 |
| 22.62 | −17.42 | 0.1062 | 0.0486 | 8.52 | 3.90 | 12.04 | 0.20 |
| 22.08 | −2.52 | 0.4363 | −0.0529 | 34.99 | −4.24 | 49.48 | −5.50 |
| 20.91 | −8.22 | 0.2596 | 0.0559 | 20.82 | 4.48 | 29.44 | ... |
| 20.59 | −10.92 | 0.2903 | −0.1512 | 23.28 | −12.12 | 32.92 | ... |
| 20.59 | −2.72 | 0.4351 | 0.2505 | 34.89 | 20.09 | 49.34 | ... |
| 20.37 | −16.82 | −0.0935 | 0.1247 | −7.50 | 10.00 | 10.60 | ... |
| 20.16 | −17.82 | 0.1391 | 0.1053 | 11.15 | 8.44 | 15.78 | ... |
| 20.05 | 11.08 | 0.1588 | −0.1636 | 12.73 | −13.12 | 18.01 | ... |
| 20.05 | 11.88 | 0.1974 | −0.2830 | 15.83 | −22.69 | 22.39 | ... |
| 19.94 | −1.42 | 0.3877 | 0.1159 | 31.09 | 9.29 | 43.97 | ... |
| 19.94 | 15.08 | 0.2191 | −0.2212 | 17.57 | −17.74 | 24.85 | −3.80 |
| 19.84 | −7.62 | 0.1277 | 0.2136 | 10.24 | 17.13 | 14.48 | −5.30 |
| 19.73 | −9.42 | 0.0458 | 0.2229 | 3.67 | 17.87 | 5.19 | ... |
| 19.62 | −10.32 | 0.0679 | 0.0536 | 5.45 | 4.30 | 7.70 | ... |

Notes. Columns 1 and 2 give the position of each region, with respect to the central star. Columns 3 and 4 give the proper motion in pixels of each knot. Columns 5 and 6 contain the proper motions in km s$^{-1}$. Column 7 gives the magnitude of the proper motion vector in km s$^{-1}$. Column 8 gives the radial velocity in km s$^{-1}$ calculated from high resolution spectra.

(This table is available in its entirety in machine-readable form. A portion is shown here for guidance regarding its form and content.)

6. CRISS-CROSS MAPPING

Criss-cross mapping was recently developed by Steffen & Koning (2011) as an analysis tool to identify patterns in proper motion measurements, in particular, systematic deviations from homologous expansion. The basic idea is to find regions where the projected velocity vectors converge or diverge. For the mapping, the vectors are extended over the full image range independently of the direction. All lines have a fixed brightness value (e.g., one) and are then added together in an image. The resulting image, which may be convolved with a Gaussian smoothing function, will show enhanced values where the line crossing points cluster, thereby revealing regions on which the motion converges or from where it diverges.

As can be appreciated by close inspection of Figure 2, the proper motion pattern obtained by applying the cross-correlation and the $\chi^2$ methods are somewhat different from each other and clearly not consistent with the radial pattern expected...
from homologous expansion, which produces a central point-like concentration (Steffen & Koning 2011). In order to assess whether the deviations from homologous expansion are purely intrinsic to the methods or might contain information about deviations of the 3D velocity field from homologous expansion, we perform a criss-cross mapping analysis on the data and on a correction to the model of homologous expansion. The model is based on the following hypothesis.

Since the inner bubble of the Eskimo nebula contains hot X-ray emitting gas (Guerrero et al. 2005; Ruiz et al. 2013), it may well be that its expansion is dominated by thermal pressure. Therefore, instead of expanding homologously, the direction of the velocity field might be perpendicular at every surface point of the bubble. In Figure 7 (left panel) we therefore show the expected proper motion pattern for an axisymmetric ellipsoid with an axis ratio of 1.8 at an inclination angle of 20°, velocity vectors that are perpendicular to the local surface.

The predicted proper motion pattern is not unlike the observed pattern in that it contains strongly deviating vector directions and magnitudes very close to each other. This is because in this type of model the vectors from the front and back portions of the nebula may in fact have different magnitudes and directions at the same projected positions. In a homologous expansion the projections of all vectors onto the sky are radial, no matter what the orientation or whether they come from the front or the back. For a more detailed qualitative analysis, we apply the criss-cross mapping technique to the observations and this model (Figure 8).

The criss-cross map for the non-homologous ellipsoidal model shows a very characteristic pattern of an approximately straight line with uniform brightness along the direction of inclination (Figure 8, left). The criss-cross maps for the observations (Figure 8, middle and right) show a more complex structure centered to the north of the CS. The overall structure is, however, much less elongated in the north–south direction compared to the ellipsoidal model and are more consistent with an off-center noisy point structure. The criss-cross mapping is, therefore, not consistent with an expansion perpendicular to the nebular surface, but rather with radial expansion. The center of expansion is, however, off-centered from the CS.

7. DISCUSSION

In this article, we have produced proper motion velocity maps for the Eskimo nebula, using images from HST taken at two different epochs separated by 7.695 yr. One map was generated using a method based on χ^2 minimization and the other was obtained by calculating the cross-correlation. Both methods used the same (>500) subregions of the images. We find that the cross-correlation method provides a slightly more continuous pattern of proper motion vectors than the χ^2 method.

The X-ray emission, constrained by the outline of the inner bubble, suggests the possibility that the expansion of the bubble is dominated by the thermal pressure of the hot gas rather than by the inertia of the bubble, resulting in an expansion perpendicular to its surface, rather than radially outwards. A comparison with a simple ellipsoidal model with velocity vectors perpendicular to the surface shows that such a model indeed reproduces the overall local deviations of neighboring proper motion vectors.
Criss-cross mapping analysis is incompatible with the pressure driven overall ellipsoidal model and with the velocity perpendicular to the surface.

We also present the radial velocities calculated at the different points where we measured the proper motions. From those, we infer a distance to the nebula of 1.3 kpc, a value well within the range of the published values (1.1–1.8 kpc) for the distance to this nebula.

8. CONCLUSIONS

A key result of this study is that the application of two different methods for the determination of internal proper motion in the Eskimo Nebula (NGC 2392), based on the same observational data, has shown that the results are quite similar with minor deviations between the two methods.

Criss-cross mapping of the proper motion vector field yields no evidence in favor of an expansion perpendicular to the inner bubble, which in turn is an indication that the hot gas inside the bubble is not driving the expansion, rather than inertia of the dense shell of NGC 2392.

Last, but not least, based on our data, we determined a distance to the Eskimo Nebula of approximately 1.3 kpc.

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APPENDIX

Using our proper motion measurements, we are able to calculate the distance to the Eskimo nebula. Considering any distance subtended by an angle $\delta r$ (in this case, $\delta r$ is the measured transverse displacements in arcsec), as $s$. Then the distance $D$ to the Eskimo nebula is calculated as

$$s({\text{km}}) = D({\text{km}})\delta r({\text{rad}}) = 3.0856 \times 10^{13}({\text{km pc}^{-1}}) \times D({\text{pc}})\frac{\delta r({\text{arcsec}})}{206264.8(\text{arcsec/rad})}. \quad (A1)$$

Expressing the time interval between observations by $\delta t$, the rms of the measured proper motions is given by

$$\langle pm (\text{km s}^{-1}) \rangle = \frac{S(\text{km})}{\delta \tau(s)} = \frac{3.0856 \times 10^{13} D(\text{pc}) \delta r(\text{arcsec})/206264.8}{\delta \tau(\text{yr}) \times 365.24 \times 24 \times 3600}. \quad (A2)$$

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