Serial and parallel reliability models for robot arm reliability analysis

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Abstract. For reliability evaluation of a robot arm designed for tunnel boring machine, a serial system reliability model for arm strength and a summation (cumulative deformation) system reliability model for the fingers of the robot arm are developed. To reflect the effect of load uncertainty, total probability theorem is applied to develop multi-layer, multi-variate reliability models. With the total probability theorem and the multi-layer configuration of the reliability models, such models reflect the effect of load intensity uncertainty, load action number uncertainty, load peak distribution uncertainty, and component strength uncertainty hierarchically.

1. Introduction
Robots are extensively applied in various of industry fields. The reliability of robot is an important quality index. Factors influencing product reliability are of variety. The conventional model to calculate system reliability[1], which was understood as the probability of survival of a serial system being equal to the product of the probabilities of survival of the components, is effective for systems of which component failures are independent of each other. Similarly, parallel system failure probability is calculated by the product of component failure probability.

For system reliability analysis, an important issue is the statistical dependence among component failures. Classical system reliability models were developed with the assumption of independent component failures, neglecting component failure dependence[2-4].

However, component failures are commonly dependent of each other, especially for mechanical systems. Traditional system reliability models fail to reflect the effect of component failure dependence, since those models were developed under the assumption of independent component failures[5-6].

For the reliability evaluation of variety of mechanical systems, the traditional series system reliability model is widely employed[7-9]. In fact, simplification was a common tactics for reliability analysis of complex systems[10-11], while inappropriate simplification may lead to unpredictable error.

Dedicated to robot arm reliability analysis, this paper presents serial and parallel system reliability models, in which the effect of component failure dependence on system reliability is incorporated inherently.

2. Robot arm structure and system type
The first paragraph after a heading is not indented (Bodytext style). A robot arm is composed of many structural parts and components. Fig.1 is a typical robot arm designed for TBM (Tunnel Boring Machine). The function of the robot arm is to change cut tool for a TBM. It should have enough
strength and motion accuracy. In more detail, all the arm components should have enough strength to resist the load, the hand composed of four fingers should not deform too much to hold the cut tool.

The stresses of the every structural parts and components can be calculated by finite element method (the FEM model is shown in Fig.2). The result is shown in Fig.3. From the point of view of strength reliability, the arm is a serial system composed of all the structural parts and components since any part or component failure means arm failure. The deformation of the every structural parts and components can also be calculated by FEM. The deformation of the arm is shown in Fig.4. To successfully hold the cutting tool, the total deformation should not be greater than a given value. For reliability analysis, it is a special system different from the conventional serial system or parallel system. The failure condition can be simply described as the sum of the deformations of the four fingers is equal or greater than a critical value. Therefore, it is called a summation system.

3. System reliability modelling

For a product operating in random load environments, reliability is affected by many random variables such as number of load actions in life time period and load amplitudes. By multi-level statistics analysis method, reliability model can be developed in a variable-by-variable extension way, depressing the individual factors hierarchically.

3.1. Serial system strength reliability model

Let $S$ stand for system strength with probability density function $f(S)$. For one time of load action, the reliability conditioning to a specified load $y$ is equal to

$$R(y) = \int_y^\infty f(S) dS$$  \hspace{1cm} (1)

For the situation of random load with pdf $h(y)$, the reliability equals

$$R = \int_0^\infty h(y)R(y)dy = \int_0^\infty h(y)(\int_y^\infty f(S) dS)dy$$  \hspace{1cm} (2)

During operation, a robot arm will be subjected to many times of load action during operation. In other words, it operates under a variable amplitude load history. Its reliability will decrease continuously with the increase of loading experience. For the situation of multiple random load action, the relationship between reliability and load action number $n$ is\(^{[12]}\)

$$R(n) = \int_0^\infty f(S)(\int_0^S g(s)ds)^n dS$$  \hspace{1cm} (3)

where, $f(S)$, as above, stands for the pdf of strength, $g(s)$ stands for the pdf of stress peaks inside the load history.
Alternatively, product reliability under multiple times of load action can be modelled by extreme load (load maximum order statistics) – strength interference relationship. That is, let \( g_n(s) \) be the pdf of the maximum order statistics of \( n \) random load observations, the reliability associated with \( n \) times of load actions can be modelled as
\[
R(n) = \int_0^\infty f(S) \left( \int_0^S g_{(n)}(s)ds \right) dS
\]
where, \( g_{(n)}(s) = n[G(s)]^{n-1}g(s) \), \( G(s) \) stands for the cumulative distribution function of stress \( s \).

Further, different robot arm will experience different load histories in service. To consider the uncertainty of variable amplitude load history, the uncertainty of stochastic load environment needs to be characterized at both macro level (over load histories) and the micro level (inside one load history), while described by \( g(s) \) is only the micro level uncertainty.

The uncertainty of load environment firstly manifests as the variation of the intensity of load histories on the whole. In this paper, an overall intensity parameter \( L \) is used to characterize a load history at the macro level, and the pdf \( h(L) \) describes the macro statistical property of the load environment. The parameter \( L \) can be the maximum or the RMS (root mean square) of the peak values in a load history.

By means of the total probability theorem to reflect the effect of the macro uncertainty of service load, the serial system reliability model can be written as
\[
R(n) = \int_0^\infty h(L) f(S) \left( \int_0^S g_{(n)}(s)ds \right) dSdL
\]
Or
\[
R(n) = \int_0^\infty h(L) \left( \int_0^S f(S) \left( \int_0^S g_{(n)}(s)ds \right) dS \right) dL
\]

For a serial system, denoting the pdf of the ith component by \( f_i(S) \), the cumulative distribution function of the ith component by \( F_i(S) \), then the cumulative distribution function of the serial system is
\[
F(S) = 1 - \prod_{i=1}^n [1 - F_i(S)]
\]

And Eq.6 can be written as
\[
R(n) = \int_0^\infty h(L) \left( \int_0^S f(S) \left( \int_0^S g_{(n)}(s)(1-F(s))ds \right) dS \right) dL
\]

3.2. Summation (cumulative deformation) system reliability model
For the four fingers to hold cut tool steadily, the total deformation should not be greater than a critical value. It can be described as that the entire stiffness of the four fingers is high enough. Denoting finger stiffness as \( x_i \) \((i=1,2,\ldots,4)\), the entire stiffness can be expressed as the sum of the finger stiffness, i.e.
\[
X = \sum_{i=1}^n X_i
\]

For the robot arm, the stiffness of the every finger can be taken as independently identical random variables, thus Eq.10 becomes
\[
X = nX_f
\]
where, \( X_f \) stands for the stiffness random variable of a finger of the robot arm.

Provide that the finger stiffness follows Normal distribution, e.g. \( X_f \sim N(\mu_f, \sigma_f^2) \), the entire stiffness will also follow the Normal distribution \( X \sim N(n\mu_f, (n\sigma_f)^2) \).

The reliability of the holding system is
\[
R_{stiff} = \int_{X_{stiff}}^\infty f_{stiff}(x)dx
\]
And the reliability of the robot arm is
\[ R_{\text{robot-arm}} = R(n)R_{\text{stiff}} \] (13)

4. Conclusion
To evaluate the reliability of a robot arm, a serial system reliability model and a summation system reliability model are developed for robot arm strength failure and robot finger stiffness failure, respectively. The serial system reliability model associated with robot arm strength failure incorporates the factors of load level, load action number, component strength, and the statistical dependence among component failures. Such a model is constructed by means of the total probability theorem. The uncertainty of variable amplitude load history is decomposed into the uncertainty over load histories and the uncertainty of the load peaks inside one load history. The summation system reliability model is a simple load-strength interference type of reliability model, calculated is the probability that the total stiffness of the four fingers is greater than a critical value, or the probability that the total deformation of the four fingers is less than a critical deformation. Finally, the robot arm reliability is modelled as the product of robot arm strength reliability and finger stiffness reliability.

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