2π-periodic Aharonov-Bohm effect
as a signature of nonlocal correlation of Majorana bound states

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We present the 2π-periodic Aharonov-Bohm (AB) effect as a verification scheme of the nonlocal correlation of Majorana zero modes (MZMs) without the restriction of fermion parity. We demonstrate the topological protection of the AB effect, where Andreev reflection mediated by MZMs strongly enhances the phase dependence. We investigate the influence of trivial bound states and show that a nonlocal index parameter enables a more explicit distinction between the trivial and topological bound states than local conductance measurements.

Introduction.— Owing to exotic properties such as nonlocality and topological protection, Majorana quasiparticles have been one of the central topics in condensed matter physics. The potential application for fault-tolerant quantum computation [1] has motivated the pursuit of Majorana zero modes (MZMs). One of the most promising platforms to host MZMs is topological superconductors. Several physical systems have been proposed to realize the topological bound states, including Rashba spin-orbit coupled nanowires with Zeeman energy in the proximity of s-wave superconductors [2–10]. This system is particularly favorable to experimentalists thanks to the emergence of the exotic property from the combination of the conventional ingredients.

Despite the substantial effort to prove the existence of MZMs, the consensus is yet to be established. The main focus of previous experimental studies has been local conductance measurements [11–18]. The topologically protected Andreev reflection mediated by MZMs causes quantized zero-bias conductance peaks (ZBCPs) [9, 10], which is one of the most well-known signatures of MZMs. For example, the experimentally observed near-quantized ZBCPs [19] had been interpreted as evidence of MZMs. However, theoretical studies have pointed out that the trivial bound states called partially-separated Andreev bound states (ps-ABSs) can mimic the signature [20–22]. The indistinguishability of the signatures emerging from MZMs and ps-ABSs questions the validity of the previous studies based on local conductance measurements, motivating experimental probes of the nonlocal nature peculiar to MZMs.

The methods to probe the nonlocality include the “teleportation” interferometry [23–27]. The process is the phase-coherent single electron transmission through MZMs, which reflects the nonlocal nature as the independence of the distance between MZMs. In this process, the phase of the transmission amplitude is proportional to the occupation number of the complex fermion composed of the MZMs. The interference measurement not only provides a smoking-gun signature of MZMs but also serves as readouts of topological qubits [28–29].

While the interferometry scheme attracts attention from both theoretical and experimental researchers [30–35], it requires parity conservation [27]. The condition demands the fabrication of mesoscopic systems with floating superconductors, which may lead to experimental intricacy. Therefore, it is desirable to develop a new approach for detecting the nonlocality of MZMs without the condition of parity restrictions.

In this paper, we present the 2π-periodic AB effect as an assessment of the nonlocal nature of Majorana bound states without the requirement of parity restrictions. We introduce two index parameters that characterize the AB effect to discuss their relations to the nonlocality and distinguish MZMs and ps-ABSs. We demonstrate that this effect is superior to local conductance measurements for differentiating the topological and trivial bound states.

2π-periodic AB effect.— Here we consider the AB effect in the setup expressed in Fig. 1(a) by introducing Peierls phase factors to the hopping of the metallic lead in the tight-binding model. Under the condition of parity restriction, MZMs at the ends of the wire mediate teleportation in the topological phase. The interference between the two paths of electron conduction, namely the metallic lead and the superconducting wire, yields the 4π-periodic AB effect [27]. In the trivial phase, the absence of transmission channels inside the energy gap eradicates the effect (see Fig. 1(b)).

This paper is devoted to the case without parity restriction, where the AB effect is 2π-periodic. The halved periodicity is attributed to the two processes with Andreev reflections at both ends of the SC wire (see Fig. 1). The Andreev reflections at both SN-junctions cause electrons and holes to gain the opposite-signed phase, resulting in the 2π periodicity. The halfed periodicity is universal whether the system is in the topological or trivial phase. However, the amplitude is more conspicuous by the orders of magnitude in the topological phase. The topological bound states mediate the perfect Andreev reflection [36–37], enhancing and topologically protecting the effect.

Model.— We model a topological superconductor as a Rashba spin-orbit coupled nanowire with Zeeman energy and s-wave superconducting proximity effect. The
increasing conductance reflects the gap closing behavior inside the energy gap. The trivial system (see Fig. 2(a)). The trivial system undergoes topological phase transition by changing magnetic field \( h \). Here the conductance of the SN junction system. The teleportation only occurs in the topological phase. (c) Interference process with grounded SCs. Electrons are doubly Andreev-reflected at the left and right junctions and interfere with the original electrons. (d) Another process with grounded SCs. Electrons are Andreev-reflected at the left or right junction. The holes created at each junction interfere with each other.

FIG. 1. (a) Setup for this research. The one-dimensional topological superconductor is placed parallel to the metallic lead. The lead and the superconducting wire are connected at both ends of the wire, forming the closed-loop system with SN junctions. (b) Interference process with floating SCs. The teleportation only occurs in the topological phase. (c) Interference process with grounded SCs. Electrons are Andreev-reflected at the left or right junction. The holes created at each junction interfere with each other.

Hamiltonian is expressed as

\[ H = H_{\text{wire}} + H_{\text{lead}} + H_{\text{hop-LW}}, \]  

(1)

\[ H_{\text{wire}} = \sum_{j, \sigma} \left[ -t_{\text{wire}} c_{j+1, \sigma}^\dagger c_{j, \sigma} + \text{h.c.} \right] - \mu_{\text{wire}} \sum_{j, \sigma} c_{j, \sigma}^\dagger c_{j, \sigma} + h \sum_{j} \left[ c_{j, \uparrow}^\dagger c_{j, \downarrow}^\dagger - c_{j, \downarrow} c_{j, \uparrow} \right] \]

\[ + \sum_{j} \left[ -\lambda c_{j-1, \downarrow}^\dagger c_{j, \uparrow} + \lambda c_{j+1, \uparrow}^\dagger c_{j, \downarrow} + \text{h.c.} \right] + \sum_{j} \left[ \Delta c_{j, \downarrow}^\dagger c_{j, \uparrow} + \text{h.c.} \right], \]

(2)

\[ H_{\text{lead}} = \sum_{j, \sigma} \left[ -t_{\text{lead}} \psi_{j+1, \sigma}^\dagger \psi_{j, \sigma} + \text{h.c.} \right] - \mu_{\text{lead}} \sum_{j, \sigma} \psi_{j, \sigma}^\dagger \psi_{j, \sigma}, \]

(3)

\[ H_{\text{hop-LW}} = \sum_{\sigma} \left[ -t_{\text{LW}} c_{1, \sigma}^\dagger \psi_{1, \sigma} - t_{\text{LW}} c_{N, \sigma}^\dagger \psi_{N, \sigma} + \text{h.c.} \right]. \]

(4)

Throughout this paper, we set parameters \( \lambda = 0.3t_{\text{wire}}, \Delta = 0.1t_{\text{wire}}, t_{\text{lead}} = t_{\text{wire}}, \mu_{\text{lead}} = 0 \). The nonlocal conductance \( G = dI_R/dV_L \) (see Fig. 1(a)) is calculated as,

\[ G = \frac{e^2}{\hbar} \int_{-\infty}^{\infty} dE \left( -\frac{\partial f(E - eV)}{\partial E} \right) [T_{ee}(E) - T_{eh}(E)]. \]

(5)

Here \( T_{ee} \) (\( T_{eh} \)) is the transmission rate from electron sectors of the left lead to electron (hole) sectors of the right lead. The transmission rates are calculated with the help of Kwant, which is the Python package for the scattering problem in tight-binding models.

Results.— Figure 2(b) shows the magnetic field dependence of the conductance of the SN junction system. The system undergoes topological phase transition by changing magnetic field \( h \), with the critical value of \( h_c \) (see Fig. 2(a)). The trivial system (\( h < h_c \)) yields a nonzero and constant conductance inside the energy gap. The conductance reflects the gap closing behavior with increasing \( h \). After the phase transition and the emergence of MZMs, the conductance shows the valley structure at zero energy. The emergence of the structure can be interpreted as a signature of the topological phase.

Figure 3 illustrates the AB effect. In the trivial phase, the conductance shows almost the constant value inside the gap, with the slight Peierls phase dependence of 2\( \pi \)-periodicity. In the case of the topological phase, the periodicity is identical to the trivial one. However, the amplitude is more prominent by a few orders of magnitude. The amplification demonstrates the topologically protected 2\( \pi \)-periodic AB effect.

Trivial bound states.— Now let us consider the effects of low-energy trivial states referred to as ps-ABSs, which appear at SN junctions with smooth gate potentials. In the previous section, we assumed a clean superconductor, where MZMs mediate Andreev reflection and amplify the 2\( \pi \)-periodic AB effect. In this case, the absence of MZMs in the trivial phase strongly suppresses the amplitude. Meanwhile, ps-ABSs, a possible cause of the experimentally observed ZBCPs \[19\], can also contribute
The trivial and topological bound states show contrasting spatial distributions. First we introduce the gate potential at the left end (see Fig. 3). By diagonalizing the superconducting wire Hamiltonian, we find the eigenfunctions $\phi_{\varepsilon,\sigma}(x)$ with eigenenergy $\varepsilon$, labeled by electron-hole degrees of freedom $\tau$ and spin $\sigma$. Figure 4(c, d) show the distributions of the squared wave functions of ps-ABSs and MZMs in the trivial and topological regime, respectively. The distributions are calculated by

$$|\psi_{\pm}(x)|^2 = \sum_{\tau=e,h,\sigma=\uparrow,\downarrow} \left| \frac{\phi_{\varepsilon,\sigma}(x)}{\sqrt{2}} \right|^2$$

These are consistent with the previous research [22].

Now we introduce the gate potentials to both ends of the wire and calculate the conductance, as illustrated in Fig. 5. The MZMs and the corresponding zero energy structure are observed in the topological regime of $h > h_c$. However, unlike the clean case considered in the previous section, the structure extends into the trivial regime, reflecting the presence of ps-ABSs (see Fig. 5). Figure 6 demonstrates the AB effect in each case with topological and trivial bound states. The identical periodicity of $2\pi$ and comparable amplitude in the trivial phase are attributed to the Andreev reflection mediated by ps-ABSs.

Robustness. Here we investigate the stability of the $2\pi$-periodic AB effect with variations of the potential structure at the ends of the superconducting wire. We consider two index parameters to characterize the effect: “dip” and “amplitude.” The former is the depth of valley structures of conductance $(2 - \bar{G}_{\text{ave}})$, and the latter is the width of the $2\pi$-periodic oscillation $(\bar{G}_{\text{max}} - \bar{G}_{\text{min}})$.
FIG. 5. (a) Spatial variation of the potential. (b) Magnetic field dependence of the energy eigenvalues of the isolated superconductor. (c) Magnetic field dependence of the conductance in the unit of $e^2/h$ at zero temperature.

FIG. 6. (a, c) Magnetic flux dependence of the conductance at the specified bias voltage. The voltage dependence is plotted in (b) and (d) with varying the flux (Peierls phase). The top and bottom rows correspond to the trivial ($h/h_c = 0.75$) and topological ($h/h_c = 1.05$) cases, respectively. $\Phi_0$ is the superconducting magnetic flux quantum $h/2e$.

FIG. 7. The two parameters, (a) for “dip” and (b) for “amplitude”, are plotted with the varying hoppings at the left and right junctions ($t_{LW-left}$ and $t_{LW-right}$, respectively). The superconductor is in the topological phase ($h/h_c = 2$).

\[
\tilde{G}_{\text{ave}} = \frac{1}{2\Phi_0} \int_{0}^{2\Phi_0} d\Phi \frac{G(eV = 0, \Phi)}{(e^2/h)}, \quad (9)
\]

\[
\tilde{G}_{\text{max}} = \max_{0 \leq \Phi < 2\Phi_0} G(eV = 0, \Phi)/(e^2/h), \quad (10)
\]

\[
\tilde{G}_{\text{min}} = \min_{0 \leq \Phi < 2\Phi_0} G(eV = 0, \Phi)/(e^2/h). \quad (11)
\]

Let us discuss the relationship between the parameters and nonlocality. Figure 7 shows contrasting behavior of “dip” and “amplitude.” The first index “dip” remains finite even if either of the SN-junctions is switched off. The unnecessity of the hopping at both ends corresponds to signatures of local conductance, which only depends on states at either end. On the other hand, Fig. 7(b) shows that “amplitude” survives only when both hoppings are finite. This dependence arises because the parameter relies on sequential Andreev reflection at both junctions. Therefore, “amplitude” depends on the condition of the spatially separated ends of the wire, potentially encoding the nonlocal information.

In Fig. 8, we compare the robustness of the two parameters with the topological or trivial bound states. In the case of MZMs, both parameters are robust against variations in potential, as expected from the topological protection. The trivial bound states can also exhibit similar behavior in certain parameter regimes for the local index “dip”, posing potential difficulty in differentiating ps-ABSs from MZMs. These regimes correspond to the case where both junctions independently satisfy the optimal condition for Andreev reflection. The independence suggests an absence of nonlocal information in the index. On the other hand, “amplitude” of ps-ABSs mimics that of MZMs only in a limited region compared to “dip.” Even in the parameter regime where the distinction between trivial and topological bound states by the local parameter “dip” is difficult, “amplitude” for ps-ABSs is highly sensitive to variations of the potential profile. This result reflects the nonlocal nature of “amplitude” since the index requires sequential Andreev reflection at both ends of the superconductor.
Conclusion.— In this paper, we discussed the $2\pi$-periodic AB effect in the context of the nonlocal correlation. This effect is attributed to Andreev reflection at multiple SN-junctions and does not require parity restriction. Due to the spatial separation of the junctions, the phenomenon reflects the nonlocal nature peculiar to Majorana bound states. We demonstrated the topological protection of the effect, where Andreev reflection mediated by MZMs strongly enhances the phase dependence. By introducing the two characteristic index parameters, we investigated the influence of trivial bound states. The trivial and topological bound states exhibit comparable phase dependence in the case with the parameters set ideally for ps-ABSs to mimic MZMs. However, the robustness against variation of potential profile demonstrated striking contrast between the trivial and topological bound states. Even in a parameter regime where the local index parameter for ps-ABSs is indistinguishable from the topological one, their nonlocal index parameter strongly depends on junction profile, making a sharp contrast to MZMs. The difference enables a more explicit distinction between the trivial and topological bound states than local conductance measurements. Therefore, the observation of the effect provides a clear signature that reflects the nonlocality of MZMs, which will advance the exploration of their existence.

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