Growing Self-organized Design of Efficient and Robust Complex Networks

Yukio Hayashi
School of Knowledge Science
Japan Advanced Institute of Science and Technology
Nomi, Ishikawa 923-1292
Email: yhayashi@jaist.ac.jp

Abstract—A self-organization of efficient and robust networks is important for a future design of communication or transportation systems, however both characteristics are incompatible in many real networks. Recently, it has been found that the robustness of onion-like structure with positive degree-degree correlations is optimal against intentional attacks. We show that, by biologically inspired copying, an onion-like network emerges in the incremental growth with functions of proxy access and reinforced connectivity on a space. The proposed network consists of the backbone of tree-like structure by copyings and the periphery by adding shortcut links between low degree nodes to enhance the connectivity. It has the fine properties of the statistically self-averaging unlike the conventional duplication-divergence model, exponential-like degree distribution without overloaded hubs, strong robustness against both malicious attacks and random failures, and the efficiency with short paths counted by the number of hops as mediators and by the Euclidean distances. The adaptivity to heal over and to recover the performance of networking is also discussed for a change of environment in such disasters or battlefields on a geographical map. These properties will be useful for a resilient and scalable infrastructure of network systems even in emergent situations or poor environments.

I. INTRODUCTION

Self-organizations with some outstanding properties of no central control, emerging structures, resulting complexity, and high scalability [1] appear in natural, social, and technological systems. To make a structure autonomously, movements or interactions of objects (materials or information) are necessary, therefore naturally induce their flows and form a network as the base. There are several fundamental mechanisms: preferential attachment [2], copying [6], [7], survival [9], [10], subdivision (fragmentation) [13], [14], [15], [16], or aggregation [17], for generating networks in the interdisciplinary research fields of physics, biology, sociology, and computer science. They are summarized in Table I. However, even the state-of-the-art science and technology is still far from fully understanding the potential for applying to a design of efficient and robust networks, while our daily life strongly depends on network infrastructures in energy supply, communication, transportation, economic, ecological, and biological (nervous or genetic) systems.

As one of the most elucidated evidences, it has been found [18], [19], [20], [21] that many real networks in social, biological, and information systems have a scale-free (SF) structure whose degree distribution follows a power-law; e.g. the networks of coauthors, movie actors, protein interactions, food webs, Internet, and WWW are included in this class [22]. The SF network consists of many low degree nodes and a few very large degree nodes. The key mechanism for network generation is the preferential attachment [2] called as rich-get-richer rule. In other words, people (= nodes) tend to do economic trade with richer persons (= linking to large degree nodes) to get more money in selfishness. The SF networks have the efficient small-world (SW) property [23] that the path length between any two nodes is short even for a large network size, however they have an extreme vulnerability against intentional attacks [24]. Thus, the efficiency and the robustness are incompatible in many real networks. In addition, the percolation analysis in statistical physics has recently found [25], [26], [27] that the vulnerability increases in interdependent networks: realistic modern systems consisting of mutually related vulnerable networks in power-grid, communication, transportation, economic-trading, and so on. Disasters or terrorism are no longer unusual, moreover the threat to induce a malfunction becomes more serious. Because the above current networks are extremely vulnerable in the interdependency, nevertheless to maintain the connectivity is the most fundamental function as network.

Such a fat-tail distribution as power-law is also emerged mathematically by other quite random and unselfish mechanisms, e.g. multiplicative process: a quantity \(X_t\) at time \(t\) is given by the product \(X_t = (\prod_{s=1}^{t} X_s)\ X_0\) [28], in which a temporal accumulation of random growth rate \(0 < r_s < 1\) contributes to make the inequality even for the uniform randomness at each time step. Because the distribution follows a log-normal from the central limit theorem for \(\log X_t = \sum_s \log r_s + \log X_0\). Similar distributions have been obtained in a simple model of random copying among individuals in the evolution for studying cultural change [3], [4], [5]. The distributions consist of many uncommon variants and a very few common variants, where cultural variants means first names, journal citations, decorative motifs on archaeological pottery, and patent citations in U.S. Thus, random copying may induce a key mechanism to self-organize a complex structure depending on a probabilistic selection history, however it is not a network model. On the other hand, the biologically inspired duplication process has been so far considered to be fundamental in a model of protein-protein interaction networks [6], [7]. It suggests that complex network structures can be generated by a simple random process.

In this paper, we consider a self-organized design of efficient and robust networks by biologically inspired copying. In particular, we focus on the robust network structure
Mechanism

preferential

survival

copying or
generation

subdivision

degradation

through the growth. In Sec. V, we discuss

networks for both the robustness and the efficiency does not

discussed. In Sec. VI, we summarize there results.

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for the robustness of connectivity and the emergent functions.

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to elucidate the universal properties and the generation rule

mechanism of network self-organization from a viewpoint in

we propose a basic model for understanding the fundamental

mechanism of network self-organization in complex network science [18], [19], [20], [21] which aims
to elucidate the universal properties and the generation rule

of networks. We show the important properties of our model

for the robustness of connectivity and the emergent functions.

In Sec. [VI] we consider an incrementally growing mechanism,

and investigate the strong robustness and the efficiency for path

lengths. We emphasis that the performance of our proposed

networks for both the robustness and the efficiency does not

degrade rather rises through the growth. In Sec. [VII] we discuss

the possibilities and the issues for applications especially in

communication or transportation systems. The adaptivity

for changing environment in disasters or battlefields is also
discussed. In Sec. [VII] we summarize there results.

II. RELATED WORKS IN COMPLEX NETWORK SCIENCE

It is a reasonable assumption that a biological network

grows according to a simple mechanism based on random

copying which differs from the preferential attachment [2]

by selecting large degree nodes in selfishness. Thus, in the
duplication-divergence (D-D) model [6], [7], a new node is

added at each time step, and duplicately creates links to con-
nected neighbor nodes of a randomly chosen node. Some links

in the duplication are deleted with a probability δ. Without

using degrees for the selection of linked nodes, a distribution

of power-law with exponential cutoff emerges in the D-D

dependence. However, there is a serious problem that the models

by pure duplication without deletion of δ = 0 and by D-D

with weak deletion of small δ < 1/2 have a singular property
called as non-self-averaging [7], [32]. In the pure duplication

[32], bipartite graphs \( K_{j,N-j} \) in any combinations of positive

integers \( j \) and \( N-j \) are generated with equiprobability, the
degrees of \( j \) and \( N-j \) distribute uniformly. The statistical

quantities such as the degree distribution and the total number

\( j(N-j) \) of links in a sample of the networks have no meaning
to characterize the topological structure, since the range

between the minimum \( N-1 \) at \( j = 1 \) and the maximum

(\( N/2 \))^2 at \( j = N/2 \) diverges for a large size \( N \). In addition,

the attack vulnerability remains in the D-D model because of

the SF-like network.

On the other hand, the optimal structure against the

targeted attacks to hub nodes has been shown numerically

[29], [30] and analytically [31] in a SF network. It is referred to

onion-like structure (see Fig. 1) from the character of

connectivity with positive degree-degree correlations, which

correspond to a natural tendency of homophily: nodes are more

likely likely connect to other nodes that are similar to them. Under

given degree distribution, an onion-like network with the

nearly optimal robustness can be generated by rewirings to

enhance the degree-degree correlations [33]. Thus, the onion-

like structure is applicable to a network with any degree

distribution. For example, after the rewirings, the attack vul-

nerability is decreased for a power-law degree distribution

in a SF network generated by the preferential attachment or

other methods. However, the entire rewiring process discards

already existing links, it is not effective utilization when a

network is growing in a realistic situation. Moreover, in spite

of the improvement of robustness, the positive degree-degree

correlations in a SF network tend to induce longer path lengths

[34], which are undesirable for the efficient communication or

transportation. We should not persist SF networks, and study

other candidates for future design of networks, especially in

considering self-organization mechanisms.

![Fig. 1. Illustration of onion-like network structure, in which nodes with a same degree are set on each concentric circle denoted by a dashed line. The solid line denotes a link between two nodes. The size of node is proportional to its degree.](image)

III. BASIC PROPERTIES

In this section, we consider a self-organization of onion-

like network, and show the tolerant and topological prop-

erties of robustness, degree distribution, and degree-degree

correlation. To study the connectivity in a network structure

independently of applications is important for clarifying the

basic property.

A. Network model

We propose a modification of the D-D model [6], [7],

[32] without mutation. Mutations by connecting randomly

chosen two nodes are unnecessary for generating an onion-

like network. The basic processes of network construction by

using only local information are as follows (see Fig. 2).

| Model          | Mechanism               | Reference |
|----------------|-------------------------|-----------|
| network        | preferential attachment | [2]       |
| cultural change, network | copying or duplication | [3], [4], [5] |
| trail, network | survival                | [6], [7]  |
| cracking, network | fragmentation or subdivision | [8], [9], [10] |
| cluster of mass, network | aggregation       | [11], [12] |

TABLE I. TYPICAL SELF-ORGANIZATION MECHANISMS.
Step 0: Set an initial configuration (e.g. connected two nodes).

Step 1: At each time step \( t = 1, 2, \ldots \), a new node is added. The new node \( i \) connects to a randomly chosen node and to the neighbor nodes \( j \) with a probability \( (1 - \delta) \times p \) \[23\].

\[
p = \frac{1}{1 + a \left| k_i - k_j \right|},
\]

where \( \delta \) is a rate of link deletion, \( a \geq 0 \) is a parameter, and \( k_i \) and \( k_j \) denote the degrees of nodes \( i \) and \( j \).

Step 2: The above processes are repeated up to a given size \( N \) in prohibiting self-loops at a node and duplicate connections between two nodes.

In Step 1, \( k_i \) is temporary set as \( (1 - \delta) \times \) the degree of the chosen node, since the degree of new node is unknown in advance because of the stochastic process. When degrees \( k_i \) and \( k_j \) are close, the two nodes \( i \) and \( j \) are connected with high probability. The case of \( a = 0 \) without the correlation effect corresponds to the conventional D-D model except the mutual link between new node and chosen node. We set \( a = 3 \) as similar to Ref. \[33\].

Figure 3 shows that our proposed networks satisfy a good property of the self-averaging unlike the conventional D-D model: the statistical deviation \( \chi \equiv \sqrt{\langle M^2 \rangle - \langle M \rangle^2} / \langle M \rangle \) for the total number \( M \) of links converges to zero for a large size \( N \). Here, \( \langle \cdot \rangle \) denotes the statistical mean (expectation). We remark a reason of the self-averaging by that a sequence of complete graphs is generated in our case of pure duplication of \( \delta = 0 \) instead of bipartite graphs in the conventional D-D model \[32\]. Therefore, with a small change from the conventional D-D model by adding the mutual links, the tendency of dense connections (whose extreme case is a complete graph) induces a core of connected high degree nodes. In addition, since there is an effect of preferential attachment to the random neighbors \[35\], \[36\], large degree nodes \( i \) and \( j \) tend to be connected together when the chosen node has a large degree. The positive degree-degree correlations is suitable to improve the robustness, especially for the malicious attacks \[29\], \[30\]. However, such correlations between low degree nodes are weak in the tree-like structure as shown in the top of Fig. 4. Since a low degree node dangles from a higher degree node, the dangling part is easy to be disconnected by node removals. We remark that the majority is low degree nodes. Thus, we consider the addition of shortcut links \[23\] in Step 3.

Step 3 : After Step 2, some shortcut links between randomly chosen nodes \( i \) and \( j \) are added with the probability of Eq. \[1\]. The number of shortcut links are given by \( p_{sc} M \), where \( p_{sc} \) is an adding rate of shortcut and \( M \) is the total number of links in the original tree-like network generated for the size \( N \).

The bottom of Fig. 4 shows an onion-like structure by adding shortcut links after Step 2. In the next subsection, as a basic property, we will separately discuss the effects of copyings and adding shortcuts on the robustness in order to make them clearly.

We emphasize the two important functions. First, a new node act as a local proxy of the chosen node and becomes another access point for the neighbor nodes in an analogy of distributed computer communication systems. Second, the complementary added shortcuts improve the robustness. It has already been shown that adding some shortcut links between uniform randomly chosen nodes improves the robustness in the theory for the small-world model \[37\] and also in the numerical simulations for many networks \[10\], \[15\], \[16\], \[38\]. The robustness is further improved due to the positive degree-degree correlations in the onion-like network as shown later, however the case of \( \delta = 0 \) as the uniform random selections does not give the nearly optimal robustness.

In addition, the proposed network has an efficiency without maintaining huge connections. Figure 5 shows that the degree distributions are approximated by exponential distributions. Therefore, huge degree nodes do not appear in the network, it is suitable for avoiding both the attack vulnerability \[24\] and the concentration of linking cost or traffic load \[39\]. Note that the proposed network does not belong to a SF network, since the degree distribution is not a power-law but an exponential.
the connections of randomly selected nodes $i$ and $j$ are repeated. Here, $s_i$ and $s_j$ denote the rank for the degrees of nodes $i$ and $j$. A remaining small part of unpaired stubs is remedied by the reshuffle procedure. In the stub-connection process, the initial configuration consists of all unpaired stubs: any node $i$ has $k_i$ free links (whose linked nodes are undetermined) assigned from a given degree distribution $P(k)$.

We have two measures for investigating the robustness and the degree-degree correlations. The robustness is measured by the following index

$$R = \frac{1}{N} \sum_{q=1/N}^{1} S(q),$$

where $S(q)$ denotes the number of nodes in the giant component (GC: largest connected cluster) after removing $qN$ nodes by the malicious attacks. $S(q)$ is monotonically decreased for increasing $q$ from an initial configuration of $S(0) = N$. The range of $R$ is $[0, 0.5]$, where $R = 0$ corresponds to a completely disconnected network consisting of isolated nodes, and $R = 0.5$ corresponds to the most robust network.

The degree-degree correlation is measured by the assortativity

$$r = \frac{S_1 S_e - S_2^2}{S_1 S_3 - S_2^2},$$

where $S_1 = \sum_i k_i$, $S_2 = \sum_i k_i^2$, $S_3 = \sum_i k_i^3$, $S_e = \sum_{ij} A_{ij} k_i k_j$, $A_{ij}$ denotes the $i$-$j$ element of the adjacency matrix. The right-hand side of Eq. (3) is a suitable scheme for the numerical calculation of $r$ than the original definition.

The range of $r$ is $[-1, 1]$ as the Pearson correlation coefficient of the degree. Nodes with similar degrees tend to be connected as $r > 0$ is larger, while nodes with different degrees tend to be connected as $r < 0$ is smaller (but $|r|$ is larger).

Figure 6 shows a scatter plot of robustness index $R$ versus assortativity $r$. The open marks correspond to the proposed networks, and the filled marks correspond to the rewired versions at the optimal.

We set $\delta = 0.5$ for the added versions with shortcut links for rates $p_{sc} = 0.2$ and 0.5. The values of $R$ become larger as $\delta$ is smaller in the tree-like networks with more links (denoted by open triangles and

![Fig. 4. Visualization examples for $N = 100$. (Top) a tree-like network by copies for $\delta = 0.5$. (Bottom) the extended onion-like network by adding shortcut links. The sizes of nodes are proportional to their degrees.](image)

![Fig. 5. Degree distributions over 100 realizations of our proposed network for $N = 5000$. The dotted magenta and chain cyan straight lines are the approximations by exponential distributions: $0.289 \exp(-0.209k)$ and $0.446 \exp(-0.309k)$, respectively. By adding shortcut links, the fraction of small degree nodes is slightly increased from the dotted blue line to the dashed green line.](image)
Fig. 6. Scatter plot of robustness index $R$ vs assortativity $r$ in sampling the networks for $N = 5000$.

IV. GROWING SELF-ORGANIZATION

We further consider an incrementally growing onion-like networks by simultaneous processes of copying and adding shortcuts. Because the tree-like network leaves its robustness weak in the growth except at the final stage by adding many shortcut links. Thus, we modify the shortcut process as follows in Step 2. Instead, Step 3 is omitted.

Step 2': In Step 2, at every interval $IT$, shortcut links are added between randomly chosen nodes $i$ and $j$ according to the probability of Eq. (1). This process is repeated up to $p_{sc}M(t)$ links. Here, $M(t)$ denotes the total number of links in the network at that time $t = IT, 2IT, 3IT, \ldots$.

In this section, we numerically show the strong robustness against both attacks and failures and the efficiency for path lengths in the incrementally growing onion-like networks. Not only the emergent structure in distributed manner but also the scalability without performance degradation are important for the self-organization, and don’t appear in the rewiring model [33].

A. Robustness in the incrementally growing networks

In order to be $\langle k \rangle \approx 6.3$ at $N = 5000$ with $IT = 50$, we chose a combination of parameters: $\delta = 0.4$ & $p_{sc} = 0.003$, $\delta = 0.5$ & $p_{sc} = 0.006$, $\delta = 0.6$ & $p_{sc} = 0.010$, $\delta = 0.9$ & $p_{sc} = 0.018$, and the corresponding $\delta = 0.3$ in the tree-like networks. Since the robustness of connectivity depends on a degree distribution but increases as the average degree $\langle k \rangle$ is larger with more links in general, a same level of connection density must be set to compare the robustness in several networks. Note that the size is $N = t + N_0$ at time $t$, where $N_0$ denotes the initial size ($N_0 = 2$ when the initial configuration is a connected two nodes).

Figure 7(a) shows the relative size $S(q)/N$ of nodes belonging to the GC versus the fraction $q$ of removed nodes by the malicious attacks. From the tree-like (denoted by cyan dashed line) to the onion-like (denoted by other lines) networks, the robustness is significantly improved, as $\delta$ is larger. However the difference in $\delta = 0.4 \sim 0.9$ is small. For the breaking of the GC, the critical fraction $q_c$ is about $0.3 < q_c < 0.4$ for the onion-like networks but very small $\ll 0.1$ for the tree-like networks as shown in Fig. 7(b) and the inset. At the peak, the GC breaks off and is divided into small clusters. Note that the critical fraction $q_c$ is around 0.03 for the SF network models [33] and the Internet data at the level of autonomous system [24], [38], [42]. Figure 8(a) shows that the values of $S(q)/N$ is sustained without a considerable drop against random node removals, except the case of $\delta = 0.3$ (denoted by cyan dashed line). Note that the virtual line of angle $-45^\circ$ is the most robust case of $R = 0.5$ in the complete graph with the maximum $\langle k \rangle = N - 1$: wasteful links. The critical fraction $q_c$ is about 0.8 for the onion-like networks but $< 0.1$ for the tree-like networks as shown in Fig. 8(b). Remember that the robustness index $R$ defined in Eq. (2) indicates the area under a line of $S(q)/N$. Thus, higher robustness (values of $R$) in the growing onion-like networks is obtained than that in the tree-like networks for $\delta = 0.3$. Note that the value of $R$ include more information in order to compare the robustness, since different values of $R$ can exits even when some networks have a same value of $q_c$. The assortativity $r$ and robustness index $R$ are summarized in Table II. As the deletion rate $\delta$ is larger, $r$ is smaller, while $R$ is almost constant in both cases of the attacks and the failures.

| $\delta$ | $p_{sc}$ | $r$ | $R$ by attacks | $R$ by failures |
|---------|----------|----|----------------|----------------|
| 0.4     | 0.003    | 0.412 | 0.246          | 0.427          |
| 0.5     | 0.006    | 0.347 | 0.267          | 0.439          |
| 0.6     | 0.010    | 0.277 | 0.283          | 0.447          |
| 0.9     | 0.018    | 0.192 | 0.281          | 0.448          |
| 0.3     | -        | 0.442 | 0.127          | 0.356          |

Table II. Comparison of assortativity $r$ and robustness index $R$ in the networks for $N = 5000$ with $\langle k \rangle \approx 6.3$. 

and saturated around the nearly optimal for the rewired versions (denoted by filled marks). The dashed arrows show the improvement for the robustness, and the almost flat ones mean that the proposed networks with shortcuts have the nearly optimal robustness. Here, from ovals to dashed ovals, the decrease of $r$ is not strange for the rewired version to be onion-like structure with higher robustness. Because it has been pointed out that onion structure and assortativity are distinct property [29]: Not all assortative networks have onion structure but all onion networks are assortative [33].

Fig. 7(a) shows the relative size $S(q)/N$ of nodes belonging to the GC versus the fraction $q$ of removed nodes by the malicious attacks. From the tree-like (denoted by cyan dashed line) to the onion-like (denoted by other lines) networks, the robustness is significantly improved, as $\delta$ is larger. However the difference in $\delta = 0.4 \sim 0.9$ is small. For the breaking of the GC, the critical fraction $q_c$ is about $0.3 < q_c < 0.4$ for the onion-like networks but very small $\ll 0.1$ for the tree-like networks as shown in Fig. 7(b) and the inset. At the peak, the GC breaks off and is divided into small clusters. Note that the critical fraction $q_c$ is around 0.03 for the SF network models [33] and the Internet data at the level of autonomous system [24], [38], [42]. Figure 8(a) shows that the values of $S(q)/N$ is sustained without a considerable drop against random node removals, except the case of $\delta = 0.3$ (denoted by cyan dashed line). Note that the virtual line of angle $-45^\circ$ is the most robust case of $R = 0.5$ in the complete graph with the maximum $\langle k \rangle = N - 1$: wasteful links. The critical fraction $q_c$ is about 0.8 for the onion-like networks but $< 0.1$ for the tree-like networks as shown in Fig. 8(b). Remember that the robustness index $R$ defined in Eq. (2) indicates the area under a line of $S(q)/N$. Thus, higher robustness (values of $R$) in the growing onion-like networks is obtained than that in the tree-like networks for $\delta = 0.3$. Note that the value of $R$ include more information in order to compare the robustness, since different values of $R$ can exits even when some networks have a same value of $q_c$. The assortativity $r$ and robustness index $R$ are summarized in Table II. As the deletion rate $\delta$ is larger, $r$ is smaller, while $R$ is almost constant in both cases of the attacks and the failures.

We also study a spatially growing method. We consider the initial configuration of connected two nodes located at the center with the link length 20 in a 160 × 160 square. At each
time step in Step 1, from the center of a randomly chosen node, a new node is set on the radius of random number between $r_{\text{min}}$ and $r_{\text{max}}$ with any direction as similar to the basic idea of [43]. We set $r_{\text{min}} = 15$ and $r_{\text{max}} = 20$. As shown in the top of Fig. 9, the growing networks spread out diffusively on the space according to the time course. Figure 10 shows the relation of $\langle k \rangle$ and $R$ through the growth. In the incrementally growing onion-like networks for the values of $\delta$ and $p_{sc}$, the marks of green plus, red cross, blue open square, and magenta open circle on each line correspond to the size $N = 200, 400, 600, \ldots, 5000$ from left to right. Therefore, both $\langle k \rangle$ and $R$ increase with the time course. The robustness is reinforced with increasing $\langle k \rangle$. In other words, the network becomes more and more robust through the growth. However, $\langle k \rangle$ hardly vary and $R$ slightly decreases in the tree-like networks denoted by the marks of cyan triangle and black filled circle. The directions of lines are from top to bottom for $N = 200, 400, 600, \ldots, 5000$ in the time course. The long upper jump of a gray dashed line is due to become onion-like networks finally at $N = 5000$ by adding many shortcut links with the rate $p_{sc} = 0.5$ in Step 3. Through the growth, the assortativity $r$ is almost constant around $0.3 \sim 0.4$, except it increases (from $-0.1$ to $0.2$) in the case of $\delta = 0.9 \& p_{sc} = 0.018$.

**B. Efficiency for path lengths**

The efficiency for communication or transportation is measured by the average path lengths on the shortest paths over the network. The length is usually counted by the minimum number of hops between two nodes. Because a path which passes through as few mediator nodes as possible gives lower cost (energy consumption) and more tolerant (safety or stable) operation for time-delay or information loss. Short paths are also related to reduce traffic load defined by the betweenness centrality [44], the frequency of passing through a node or a link on the paths between all pairs of nodes. The frequency can be weighted by an inhomogeneously distributed (communication or transportation) requests, e.g. proportional to the products of populations around source and destination nodes.
degree nodes and the periphery of connected low degree nodes. The widths are narrower around 5 hops in the growing onion-like networks. Figure 10 shows that the distributions of path lengths in the networks for $N = 5000$ with $\langle k \rangle \approx 6.3$. The case for $\delta = 0.3$ is tree-like networks, and the other cases are onion-like networks.

Figure 11 shows that the distributions of path lengths in the networks at a same connection density level of $\langle k \rangle \approx 6.3$. The widths are narrower around 5 hops in the growing onion-like networks denoted by green dashed, red solid, blue dashed, and magenta dotted lines for $\delta = 0.4, 0.5, 0.6$, and 0.9. Thus, the growing onion-like networks are efficient with short paths.

In more detail for the emergence of SW property [23], it is important whether the average path length follows $O(\log N)$ for the size $N$ under a constant connection density. Inset of Fig. 12 shows the SW property in the tree-like networks denoted by cyan and orange chain lines for $\delta = 0.3$ and 0.5. Moreover, as shown in Fig. 12 the path lengths are very short around 5-6 hops in the growing onion-like networks denoted by green dashed, red solid, and blue dashed lines for $\delta = 0.4, 0.5, 0.6$, except with slightly higher values of $\langle L_{ij} \rangle$ (longer path lengths) in $N < 10^3$ denoted by magenta dotted line for $\delta = 0.9$ than the case of tree-like networks denoted by cyan and orange chain lines for $\delta = 0.3$ and 0.5. We should remark that the average degree $\langle k \rangle$ is not constant within a connection density but increasing in the growing onion-like networks. Moreover, $\langle k \rangle$ is smaller than that in the tree-like networks for $\delta = 0.3$ as shown in Fig. 10. Smaller $\langle k \rangle$ means fewer total links, and the network construction is less expensive in lower cost especially in an early stage of growth.

Another important measure is the path distance, which is defined by the sum of Euclidean distances for the links on a graph; the average path length $\langle L \rangle$ counted by the minimum number of hops) between two nodes. This measure is corresponding to the load of physical movements for (communication or transportation) flows on a space, instead of the load of routing process (for...
switching the transfer direction) defined by the number of mediator nodes on the path.

Figure 13 shows the distribution of path distances $D_{ij}$, in which the majorities are short around the size 160 of outer square. The case of $\delta = 0.3$ as the tree-like network denoted by cyan chain line has longer distances than other cases, since the distribution is shift to the right. We also confirm the SW property of $O(\log N)$ for the path distance as shown in Fig. 14. The green dashed, red solid, and blue dotted lines for $\delta = 0.4$, 0.5, and 0.6 in the onion-like networks are below the cyan and orange chain lines for $\delta = 0.3$ and 0.5 in the tree-like networks, except the magenta dotted line for $\delta = 0.9$. Thus, the onion-like networks has shorter path distances than the tree-like networks within a same connection density level, although the advantage in the Euclidean distances is weak in comparison with that in the number of hops (see Figs. 12 and 14). Note that we have similar results for the path length $L_{ij}$ and distance $D_{ij}$, when we chose the shortest distance path instead of the above shortest path defined by the minimum number of hops between two nodes. The difference of shortest (Euclidean) distance path and shortest (minimum hops) path is very small in these spatial networks, in other words, they give similar efficiency for the routing paths in these measures.

![Distribution of path distance](image)

V. DISCUSSIONS FOR APPLICATIONS

In this section, we will discuss the possibilities or issues in our proposed networks for applications of communication or transportation systems. There exist flows which represent movements of one entity relative to another on a network. Thus, we categorize the dynamics in networks into the following types.

Type 1. Dynamics of network configuration itself
Type 2. Dynamics of information flows, rumor spreading, opinion formation, synchronization, or logistics on a network

Through this paper, we study the Type 1 of dynamics in order to aim a fundamental mechanism for the self-organized design of efficient and robust networks. Temporal and/or fixed (corresponding to wireless and/or wired) connections are possible depending on the time-scale for changing the connection structure in a network. The quick change results in ad hoc networks, while the slow change is treated as an incremental modification of network. Both cases and the mixed one are not excluded, however we have assumed that each node or link is persisted once it is added unless removed by failures or attacks to simplify the discussion in Sections II-IV. The Type 2 of dynamics is significant for applications, e.g. wireless, sensor, mobile communication systems or autonomous transportation systems. In the Type 2 of dynamics, operation protocols for birth and death of communication or transportation request, routing, avoidance of congestion, task allocation, queuing, awareness of location, monitoring of system’s states or conditions, and so on, are necessary. There are many methods for the efficient operations that are independent (generally applicable) or dependent on a special network structure. The detail discussions are strongly related to device technologies, users, and situations of utilization, they are beyond our current scope.

Even when we focus on the Type 1 of dynamics, there are two kinds of interactions among individuals (nodes) and with environment I for the operation and control in a self-organized system. In particular, we consider changes of environment and the adaptivity in a network. Here, adaptivity is defined as the capability to work in different or changing environment without intervention and configuration by an administrator II. The levels of adaptation are distinguished as shown in Table III.

At the Level 1, a compensatory growth for removed parts by attacks or failures is related to an adaptivity with healing function in the incrementally growing onion-like networks because of the potential with strong robustness. The function
of proxy at a new node locally contributes to make different access paths through the copying process as mentioned in subsection II A. On the other hand, node mobility may give rise to other problems of disconnections in a limited transmission range of wireless beam, handover for high-speed environment, or loss of data and navigational routes. When a communication network is often disconnected but resilient due to node mobility, limited radio power, node or link failure, it is known as a Delay/Disruption Tolerant Network (DTN) where a mobile device or software agent temporary stores and carries local information for forwarding messages until an end-to-end route is re-established or re-generated. It is used in disasters, battlefields, and vehicular communications. There are many protocols in the concept of DTN routing [47], [48]. As a base structure for DTN, it is obviously better to have many short paths and to maintain tolerant path connectivity for temporal changes of nodes or links. Our growing onion-like network becomes one of the candidates of DTN.

At the Level 2, in our growing network, it will be useful to regulate the parameters of $\delta$, $p_{sc}$, and $IT$ in order to repair the damaged parts and to recover the performance for communication or transportation according to a limited resource and the state of network system monitored at a time. The scheduling is a further issue, and it is also important to detect the damaged parts or malfunctions caused from overload beyond a capacity in a realistic network. At the Level 3, a change of the intrinsic mechanism for generating a network is required. Since the required change depends on the scale or the speed of damages or evolutions, we should develop a proper measure to evaluate them in many complex situations.

Apart from the basic property, only random failures and malicious attacks by removing nodes in decreasing order of their current degrees are insufficient to investigate the robustness in a realistic network. Natural disasters such as earthquake or flood and unremitting armed conflicts with aerial bombing give rise to spatially spreading damages. They probably make complex shapes of removed part in the network according to a geographical map. Although we treat a simple spatially growing method in Section IV we need more consideration. In addition, how to locate a node on a space should be discussed more carefully e.g. for the geographical allocation of mobile base stations for wireless communication in an emergent case of disaster or poor infrastructural environment. In any cases, incremental repair and reinforced construction will be powerful approaches.

## VI. Conclusion

We have proposed a self-organized design method for generating efficient and robust networks. It is based on biologically inspired copyings and the complementary adding shortcut links to enhance the robustness. In particular, taking into account positive degree-degree correlations, we have considered incrementally growing onion-like networks, which emerges from the backbone of tree-like structure generated by copyings and the periphery of shortcut links between low degree nodes. The obtained properties are fine as follows. We emphasize that these properties are not trivially predicted from a combination of copyings and adding shortcuts.

- non-singular self-averaging property unlike the conventional D-D model [6], [7], [32] to ensure statistically meaningful convergence for a large size $N$ in samples
- growing with the compensatory function of local proxy as another access point
- exponential degree distribution without huge hubs to avoid the concentration of linking cost and traffic load at a few nodes [39]
- strong tolerance of connectivity against failures and attacks equivalent to the nearly optimal in the rewired version [53] under a same degree distribution
- efficiency with short path lengths and distances (counted by the number of hops and the Euclidean distances, respectively, on the shortest path) for communication or transportation, in addition the path length is superior to the $O(log N)$ SW property [23] in a sense

Moreover, we have discussed the possibilities and issues for applications in wireless and/or wired communication or transportation systems, and particularly mentioned the adaptivity to heal over and to recover the performance of networking for a change of environment in such disasters or battlefields on a geographical map. In this paper, we have considered a self-configuration of efficient and robust network in an incremental and distributed manner. The proposed method does not require the entirely rewiring processes on a network [53]. Thus, the obtained results will be useful for temporal evolution of a resilient network system. Other capabilities and realistic strategies for self-diagnosis, self-protection, self-healing, self-repair, and self-optimization [1] are further issues. We will study effectively applicable methods to self-organize networks which keep high robustness and efficient paths for communication or transportation in both normal and emergent situations.

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