Neutrinos With Seesaw Masses and Suppressed Interactions

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Mixing between light and heavy neutrino states has been proposed as an explanation (or partial explanation) for the 3-sigma NuTeV anomaly and the 2-sigma departure of the $Z^0$ invisible width from its expected value. I assume herein that neutrino masses and mixings result from the conventional seesaw mechanism involving six chiral neutrino states, the first three being members of weak doublets, the others weak singlets. A finely-tuned choice of both the (bare) Majorana masses and the (Higgs-induced) Dirac masses can fit solar and atmospheric neutrino data and also result in significant (but necessarily flavor-dependent) mixing between singlet and doublet states such as would yield detectable suppression of light neutrino interaction amplitudes. The possibility for this kind of suppressive mixing is constrained by the observed upper limit on radiative muon decay.
The three-sigma NuTeV anomaly \cite{1}, as well as the two-sigma anomaly \cite{2} in the invisible width of the $Z$, may require new physics for their resolution. Several proposed explanations invoke significant mixing of active neutrinos with heavy singlet states \cite{3}. The effect of this mixing is to reduce the couplings of $W$ and $Z$ bosons to the light neutrinos: for $W$ by the factors $1 - \epsilon_\ell/2$, and for $Z$ by the factors $1 - \epsilon_\ell$ (where $\ell = e, \mu, \tau$ is a flavor index).

Davidson et al. \cite{4} find that neutrino mixing can explain both anomalies, but only at the unacceptable cost of introducing additional departures from standard-model predictions. Recently, Takeuchi et al. \cite{5} \cite{6} revived the notion of suppressive neutrino mixing in a scheme also involving an unconventionally heavy Higgs boson. To carry out their analyses, they make the *ad hoc* hypothesis of flavor-independent ($\epsilon_\ell = \epsilon$) mixing between light and heavy neutrino states. Their fit requires $\epsilon = 0.003 \pm 0.001$, about which they remark that “a naive seesaw mechanism does not permit such a large mixing angle,” but that “the required pattern of mixings and masses can be arranged when there exists intergenerational mixing” \cite{6}.

This work asks whether neutrino mixing of the sort envisaged by Takeuchi et al. can result from a minimal seesaw mechanism: one involving exactly three weak-doublet neutrino states and exactly three singlet states, with Majorana masses for the singlets, Higgs-induced mass terms linking singlets and doublets, and no weak-triplet Majorana masses of the doublet states \cite{7}.

I find that the neutrino flavor eigenstates, in an appropriately fine-tuned model of this kind, *can* mix significantly with (inaccessible) heavy mass eigenstates, but this suppressive mixing *cannot* be flavor independent. Furthermore, I show how the current limit on the unobserved decay mode $\mu \to e + \gamma$ limits the nature and magnitude of the permitted flavor-dependent mixings.

The most general $6 \times 6$ seesaw neutrino mass matrix is

\[ \mathcal{M} \equiv \begin{pmatrix} 0 & \tilde{n} \\ \tilde{n}^T & M \end{pmatrix} \]

where $n$ is an arbitrary $3 \times 3$ matrix, $\tilde{n}$ is its transpose, and $M$ is an arbitrary $3 \times 3$ symmetric matrix. I shall assume there to be a basis in which these matrices take (or nearly take) the following special forms:

\[ n = \begin{pmatrix} 0 & 0 & 0 \\ 0 & n' & 0 \\ 0 & 0 & n \end{pmatrix}, \quad \text{and} \quad M = \begin{pmatrix} \Delta & 0 & M \\ 0 & M' & 0 \\ M & 0 & 0 \end{pmatrix}, \]

(2)
where \( M, M' \gg n, n' \gg \Delta > 0 \). This finely-tuned neutrino mass matrix is essentially the only one that can yield significant suppressive mixing in the minimal seesaw scheme. (The matrix \( n \) may always be chosen to be diagonal. However, the requirements that \( n_{11} \) and certain elements of \( M \) vanish can be relaxed, but only to the extent that our conclusions are unaffected.)

The approximate eigenvalues of the real symmetric matrix \( M \) (the neutrino masses) are easily seen to be \( \pm M \) and \( M' \) (for the heavy and predominantly weak singlet states) and \( 0, n^2/M' \) and \( n^2\Delta/M^2 \) (for the light and predominantly weak singlet states). Both \( M \) and \( M' \) must probably exceed \( M_Z \) because no heavy neutrino state has been detected. I have not as yet specified the basis states of \( M \) in terms of flavor eigenstates, other than by identifying the first three components as (active) doublet states and the last three components as (sterile) singlet states. The three light mass eigenstates of \( M \) are easily found. Properly normalized, they are:

- \( \nu_a = (1, 0, 0; 0, 0, 0) \): This neutrino state is purely doublet and massless (\( m_a = 0 \)).
- \( \nu_b = (0, \cos \chi', 0; 0, \sin \chi', 0) \): This state, with mass \( m_b \simeq n'^2/M' \), mixes negligibly with the singlet states. (From the plausible constraints, \( m_b < 10 \text{ eV} \) and \( M' > 100 \text{ GeV} \), I find \( n' < 1 \text{ MeV} \) and \( \chi' \simeq n'/M' < 10^{-5} \).) Little error is incurred by setting \( \nu_b \simeq (0, 1, 0; 0, 0, 0) \).
- \( \nu_c \simeq (0, 0, \cos \chi; 0, 0, \sin \chi) \): This is the only light neutrino eigenstate which may mix significantly with singlet neutrinos. Its mass is \( m_c \simeq n^2\Delta/M^2 \), whilst its mixing parameter is \( \chi \simeq n/M \). (For example, we might set \( M = 1 \text{ TeV} \), \( n = 100 \text{ GeV} \) and \( \Delta = 10 \text{ eV} \) to obtain a neutrino with a mass of 0.1 eV, which would then suffer 10% suppressive mixing with singlet states.) We define the parameter \( \epsilon \) in terms of which the suppression parameters \( \epsilon_\ell \) corresponding to the various flavor eigenstates will be expressed:

\[
\epsilon \equiv \sin^2 \chi \simeq (n/M)^2. \quad (3)
\]

The basis in which the light mass eigenstates \( \nu_{a,b,c} \) are exhibited above is one from which inter-family mixing is removed. The flavor eigenstates are 6-component vectors spanning the space of doublet neutrinos. They are orthonormal vectors whose last three components vanish. These states, for \( \epsilon \neq 0 \), involve a possibly significant admixture of inaccessible heavy neutrino states. The ‘reduced’ flavor eigenstates, denoted by \( \hat{\nu}_{e,\mu,\tau} \) (the hats indicating 3-component vectors), are given by a unitary matrix—the usual analog to the Kobayashi-Maskawa matrix—acting on \( \hat{\nu}_{a,b,c} \), the projections of \( \nu_{a,b,c} \) on the space
of doublet states. In other words, $\nu_{a,b,c}$ are simply the states $\nu_{a,b,c}$ with their last three components deleted. In the following equation, the reduced flavor eigenstates are expressed in terms of a permutation of the states $\nu_{a,b,c}$:

$$
\begin{pmatrix}
\hat{\nu}_e \\
\hat{\nu}_\mu \\
\hat{\nu}_\tau
\end{pmatrix} =
\begin{pmatrix}
c_2c_3 & c_2s_3 & s_2e^{-i\delta} \\
-c_1s_3 - s_1s_2c_3 e^{i\delta} & c_1c_3 - s_1s_2s_3 e^{i\delta} & s_1c_2 \\
+s_1s_3 - c_1s_2c_3 e^{i\delta} & -s_1c_3 - c_1s_2s_3 e^{i\delta} & c_1c_2
\end{pmatrix}
\begin{pmatrix}
\hat{\nu}_{a'} \\
\hat{\nu}_{b'} \\
\hat{\nu}_{c'}
\end{pmatrix},
$$

(4)

where $s_i, c_i \equiv \sin \theta_i, \cos \theta_i$, and $(a', b', c')$ is a permutation of $(a, b, c)$. Because $|\hat{\nu}_c|^2 = \cos^2 \chi < 1$, the reduced flavor eigenstates, $\hat{\nu}_e$, $\hat{\nu}_\mu$ and $\hat{\nu}_\tau$, are neither normalized nor are they orthogonal to one another.

Three phenomenologically distinct permutations are compatible with the observed mass hierarchy of solar and atmospheric neutrino oscillations. (The remaining permutations, with $a$ and $b$ interchanged, are accounted for by the variation of $\theta_3$ from 0 to $\pi$.)

I. $(a', b', c') = (a, b, c)$: where the parameters in (2) are chosen so that $m_c^2 \gg m_b^2$.

For this case, using the above expressions for $\nu_{a,b,c}$ and Eq.(4), I find the following suppression factors applicable to the several flavor eigenstates:

$$
\epsilon_e = \epsilon s_2^2, \quad \epsilon_\mu = \epsilon s_1^2 c_2^2, \quad \epsilon_\tau = \epsilon c_1^2 c_2^2.
$$

(5)

II. $(a', b', c') = (a, c, b)$: where the parameters are chosen so that $m_b^2 \gg m_c^2$. For this case, I may neglect the small subdominant angle $\theta_2$ in the expressions for these suppression factors:

$$
\epsilon_e \simeq \epsilon s_3^2, \quad \epsilon_\mu \simeq \epsilon c_1^2 c_3^2, \quad \epsilon_\tau \simeq \epsilon s_1^2 c_3^2.
$$

(6)

III. $(a', b', c') = (b, c, a)$: where the parameters are chosen so that $m_c^2 \gg |m_c^2 - m_b^2|$. For this instance of an inverted mass hierarchy, the suppression factors are identical to those of Case II. (In all three Cases, $\epsilon_e + \epsilon_\mu + \epsilon_\tau = 1$.)

The departures of the reduced flavor eigenstates from orthonormality,

$$
|\hat{\nu}_\ell^\dagger \cdot \hat{\nu}_{\ell'}| = \epsilon_\ell \epsilon_{\ell'}, \quad \text{where} \quad \ell, \ell' = e, \mu, \tau,
$$

(7)

have phenomenological consequences other than their relevance to the analyses of Takeuchi et al. [5][6]. In the seesaw models we have considered, the simple one-loop diagrams

$\dagger$ For the angles usually designated by $\theta_{23}, \theta_{13}, \theta_{12}$, we use the simpler names $\theta_1, \theta_2, \theta_3$, resp.
involving the emission and absorption of a $W$ boson lead to a nonvanishing contribution to radiative decays such as $\mu \to e + \gamma$. This occurs because both the heavy and light components of the neutrino flavor eigenstates are involved in the virtual process. It follows from Eq.(7) and Ref.[8] that the branching ratio $B$ for the radiative decay mode $\mu \to e + \gamma$ is:

$$B = \frac{\alpha}{24\pi} \epsilon_\mu \epsilon_e N(M),$$

where the monotone function $N(M)$ equals $1/2$ at $M = M_Z$, increasing to 1 as $M \to \infty$. This result is to be compared with the experimental upper bound [9]:

$$B < 1.2 \times 10^{-11}.$$  

Eqs.(8) and (9) yield the inequalities:

$$10^{-6} > \begin{cases} \epsilon^2 s_1^2 \sin^2 2\theta_2, & \text{for Case I;} \\ \epsilon^2 c_1^2 \sin^2 2\theta_3, & \text{for Cases II and III.} \end{cases}$$

With no further ado, we may set $s_1^2 \simeq c_1^2 \simeq 1/2$ in Eq.(10) from the Super-Kamiokande atmospheric neutrino data [10].

For Case I, Eq.(11) cannot provide a constraint on $\epsilon$ until an experimental lower bound is set on the subdominant angle $|\theta_2|$. In the event that $\sin^2 2\theta_2 \simeq 0.1$, its largest CHOOZ-allowed [11] value, the constraint $\epsilon < 4.5 \times 10^{-3}$ would be obtained. For Cases II or III, solar-neutrino data [12] determine $\sin^2 \theta_3 > 0.68$ with 95% confidence. Here the stronger constraint $\epsilon < 1.7 \times 10^{-3}$ results.

The non-orthogonality of $\nu_\mu$ and $\nu_\tau$ has another interesting consequence. ‘Young’ $\nu_\mu$’s (prior to the onset of oscillations) can act as if they were $\nu_\tau$’s (thereby producing $\tau$’s via charged-current interactions) with probability $\epsilon_\mu \epsilon_\tau$. Thus the unsuccessful NOMAD search [13] for this process yields the inequalities:

$$6.8 \times 10^{-4} > \begin{cases} \epsilon^2 c_4^2 \sin^2 2\theta_1, & \text{for Case I;} \\ \epsilon^2 c_4^2 \sin^2 2\theta_1, & \text{for Cases II and III.} \end{cases}$$

For Case I, Eq.(11) establishes the firm but weak upper limit $\epsilon < 2.6 \times 10^{-2}$. For Cases II and III, the bound on $\epsilon$ set by the NOMAD result is less stringent than our earlier result.
To summarize, I have shown that current experimental data establishes the bounds:

\[
\epsilon < \begin{cases} 
2.6 \times 10^{-2}, & \text{for Case I;} \\
1.7 \times 10^{-3}, & \text{for Cases II and III.}
\end{cases}
\] (12)

To illustrate the significance of these results, note that the currently reported value \cite{2} for the number of light neutrinos is \(N_\nu = 2.984 \pm 0.008\). Its explanation in the present context (should a mere 2-sigma discrepancy need explanation) would indicate \(\epsilon = 4 \pm 2 \times 10^{-3}\). A similar remark probably applies to a discussion of the NuTeV anomaly. Thus, Case I is likely to provide the only feasible route to effective suppressive mixing in the minimal seesaw scheme.*

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* Other schemes of suppressive neutrino mixing, including those with flavor-independent suppression, can emerge from more elaborate seesaw frameworks, such as one involving six heavy singlet neutrino states rather than three.
References

[1] K.S. McFarland et al., hep-ex/0205081; G.P. Zeller et al., hep-ex/0207037.

[2] The LEP Collaborations, the LEP Electroweak Working Group, and the SLD Heavy Flavor Group, hep-ex/0212036.

[3] E.g., J. Bernardéu et al., Phys. Lett B187(1987)303; A. de Gouvêa et al., Nucl. Phys. B623(2002)395, hep-ph/0107156; K.S. Babu and J.C. Pati, hep-ph/0203029.

[4] S. Davidson et al., JHEP 0202(2002)037, hep-ph/0112302.

[5] T. Takeuchi, hep-ph/0209103; T. Takeuchi et al., 2002 International Workshop at Nagoya, www.eken.phys.nagoya-u.ac.jp/SCGT02/.

[6] W. Loinaz, N. Okamura, T. Takeuchi, and L.C.R. Wijewardhana, et al., hep-th/0210193.

[7] S.L. Glashow, in Quarks and Leptons, Cargèse 1979, eds. M. Lévy, et al., (Plenum 1980 New York), p. 707; M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, eds. D. Freedman et al., (North Holland 1980 Amsterdam); Y. Yanagida in Proc. Workshop on Unified Theories &c., eds. O. Sawada and A. Sugamoto (Tsukuba, 1979); R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44(1980)912.

[8] G. Feinberg, Phys. Rev. 110(1958)1482.

[9] M.L. Brooks, Phys. Rev. Lett. 83(1999)521.

[10] Y. Fukuda et al. [Super-K Collaboration], Phys. Rev. Lett. 81(1998)1562.

[11] M. Apollonio et al., Phys. Lett. B466(1999)415.

[12] E.g., J.N. Bahcall, M.C. Gonzalez-Garcia and C. Peña, hep-ph/0212147.

[13] P.Astier et al. [NOMAD Collaboration], Nucl. Phys. B611(2001)3.