Quark Condensates in Nuclear Matter in the Global Color Symmetry Model of QCD

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Abstract

With the global color symmetry model being extended to finite chemical potential, we study the density dependence of the local and nonlocal scalar quark condensates in nuclear matter. The calculated results indicate that the quark condensates increase smoothly with the increasing of nuclear matter density before the critical value (about $12\rho_0$) is reached. It also manifests that the chiral symmetry is restored suddenly as the density of nuclear matter reaches its critical value. Meanwhile, the nonlocal quark condensate in nuclear matter changes nonmonotonously against the space-time distance among the quarks.

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1 Introduction

It is well known that quark condensates are the essential characteristics in the phase transition from the quark gluon plasma to hadrons of the early universe evolution. Meanwhile the condensates feature the nonperturbative structure of the QCD vacuum, and hence play essential roles in describing hadron structure, and further the properties of nuclear matter and finite nuclei [1, 2, 3, 4, 5, 6]. In the aspect of describing hadron structure in QCD, not only the local scalar quark condensate $<\bar{q}q>$, but also the nonlocal condensates $<\bar{q}(x)q(0)>$ are required. Especially, the later is more important to figure the nonpoint particle property of hadrons [4, 7, 8, 9, 10]. Then a large body of works on the quark condensates in both free space and hot and dense nuclear matter have existed (for a recent review, see for example Ref. [11]). Many of the approaches give a descent feature of the local scalar condensate in nuclear matter at low energy, which is believed to be the manner for the chiral symmetry to be restored. However an “upturn” emerges at higher density in the linear Walecka model [12], Dirac-Brueckner method [13] and Schwinger-Dyson formalism [14]. Recently, the Dyson-Schwinger equation (DSE) method of QCD [15, 16] and the global color symmetry model (GCM) of QCD [17, 18, 19, 20] have also been employed to evaluate the condensates in vacuum [21, 22, 23] and those at finite temperature [4] and finite density [24, 25]. The DSE method calculations [24, 25] show that the quark condensate increases with respect to the increase of chemical potential (or the nuclear matter density) before a critical value is reached. It means that one has not yet reached a common realization on how the quark condensate changes against nuclear matter density qualitatively. Since the DSE approach and the GCM have been shown to be quite successful in describing hadron properties both in free space and in nuclear matter [21, 22, 23, 24, 25, 26, 27, 28, 29, 30], with the global color symmetry model being extended to that at finite nucleon density $\rho$ (as usual, it is achieved by involving a finite chemical potential $\mu$), we will study the dependence of the local and nonlocal scalar quark condensates on nuclear matter density again in this paper.

The paper is organized as follows. In Sec. II we describe the formalism of the GCM at finite chemical potential $\mu$ and the relation between the chemical potential and the baryon density in nuclear matter. In Sec. III we represent the calculation and the obtained results of the local and non-local scalar quark condensates as functions of the nuclear matter density. In Section IV, a brief summary and some remarks are given.
2 Formalism

The starting point of the global color symmetry model (GCM) is the action in Euclidean matric[17]
\[
S = \int d^4xd^4y \left[ \bar{q}(x) \left( \gamma \cdot \partial + m \right) \delta(x - y)q(y) - \frac{g^2}{2} j_\mu^a(x)D_{\mu\nu}(x - y)j_\nu^a(y) \right],
\]
where \( j_\mu^a = \bar{q}(x)\frac{i}{2} \gamma_\mu q(x) \) is the quark current, \( D_{\mu\nu}(x - y) \) is an effective two-point gluon propagator, \( m \) is the current quark mass, and \( g \) is the quark-gluon coupling constant. Taking the effective gluon propagator to be diagonal, i.e., \( D_{\mu\nu}(x - y) = \delta_{\mu\nu}D(x - y) \) and applying the Fierz reordering to the quark fields, one can rewrite the current-current term as
\[
\frac{g^2}{2} \int d^4xd^4y j_\mu^a(x)D(x - y)j_\nu^a(y) = -\frac{g^2}{2} \int d^4xd^4y j^\theta(x, y)D(x - y)j^\theta(y, x),
\]
where \( j^\theta(x, y) = \bar{q}(x)\Lambda^a q(y) \) with \( \Lambda^a \) being the direct products of Lorentz, flavor and color matrices which produce the scalar, vector, pseudoscalar and axial vector terms. More concretely, \( \Lambda^a = \frac{1}{2} K^a \otimes C^b \otimes F^c \) with \( \{ K^a \} = \{ I, i\gamma_5, \frac{i}{\sqrt{2}} \gamma_\mu, \frac{i}{\sqrt{2}} \gamma_\mu \gamma_5 \}, \{ C^b \} = \{ \frac{1}{2} I, \frac{i}{\sqrt{2}} \lambda \}, \{ F^c \} = \{ \frac{1}{\sqrt{2}} I, \frac{\sigma}{\sqrt{2}} \} \) where \( \{ \lambda^a \} (a = 1, 2, \ldots, 8), \{ \sigma \} (i = 1, 2, 3) \) are the generators of the groups \( SU_\text{C}(3) \) and \( SU_F(2) \), respectively. It is obvious that such a \( j^\theta(x, y) \) is a bilocal current. With two flavors \( u \) and \( d \) of quarks being taken into account, each \( \Lambda^a \) is either isoscalar or isovector. The color matrices involved in the Fierz transformation contain color-singlet and color-octet terms. Taking the bosonization procedure one can transfer the bilocal quark current structure into auxiliary Bose fields carrying the quantum number \( \theta \). The action of the GCM in free space (i.e., at the chemical potential \( \mu = 0 \)) for the zero-mass quark can then be rewritten[17] in the Euclidean space as
\[
S(B) = \int d^4xd^4y \bar{q}(x) \left[ \gamma \cdot \partial \delta(x - y) + \Lambda^a B^a(x, y) \right] q(y) + \int d^4xd^4y \frac{B^\theta(x, y)B^\theta(y, x)}{2g^2D(x - y)},
\]
where \( B^\theta(x, y) \) is the bilocal Bose field.

To extend the GCM to finite nuclear matter density (with finite chemical potential \( \mu \)), one should, in view of the statistical mechanics, take the partition function of the canonical (quark) ensemble into that of the grand ensemble with quarks and hadrons that are the solitons collecting quarks[18]. The quark field should be transformed under a constraint on the baryon number through the chemical potential \( \mu \)
\[
q(x) \rightarrow q'(x) = e^{ix_4}q(x).
\]
After some derivation, we have the action of the GCM in nuclear matter
\[
S(B^\theta, \mu) = \int d^4xd^4y \bar{q}(x) \left[ (\gamma \cdot \partial - \mu \gamma_4) \delta(x - y) + e^{ix_4} \Lambda^a B^a(x, y) e^{-ix_4} \right] q'(y)
+ \int d^4xd^4y \frac{B^\theta(x, y)B^\theta(y, x)}{2g^2D(x - y)},
\]
where \( B^\theta(x, y) \) is the bilocal Bose field.
and the generating functional is given as
\[ Z[\mu, \eta, \bar{\eta}] = \int D\bar{\eta} D\eta D\theta e^{-S(B^\theta, \mu) + \int d^4x (\bar{\eta} \gamma \gamma_4 \eta)} . \]  
(2)

where \( \eta \) and \( \bar{\eta} \) are the quark sources. After integrating the quark fields, we obtain
\[ S(B^\theta, \mu) = -\text{Tr} \ln \left[ (\gamma \cdot \partial - \mu \gamma_4)\delta(x-y) + e^{\mu x} \Lambda^\theta B^\theta e^{-\mu y} \right] + \int d^4x d^4y \frac{B^\theta(x,y)B^\theta(y,x)}{2g^2 D(x-y)} . \]  
(3)

Generally, the bilocal field \( B^\theta(x, y) \) can be written as\[^{20}\]
\[ B^\theta(x, y) = B^\theta_0(x, y) + \sum_i \Gamma_i^\theta(x, y) \phi_i^\theta \left( \frac{x + y}{2} \right) , \]  
(4)

where \( B^\theta_0(x, y) = B^\theta_0(x - y) = B^\theta(x - y) \) is the vacuum configuration of the bilocal field. \( \Sigma_i \Gamma_i^\theta(x, y) \phi_i^\theta \left( \frac{x + y}{2} \right) \) correspond to the fields which can be interpreted as effective meson fields. In the lowest order approximation with only the Goldstone bosons being taken into account, the \( \phi_i^\theta \) includes \( \pi \) and \( \sigma \) mesons. The corresponding width of the fluctuations are approximately the same as the vacuum configuration\[^{20}\], i.e., \( \Gamma_i^\theta = B^\theta_0 \). Since the bilocal field arises from the bilocal current of quarks, the internal \( \bar{\eta} q \) structure of the mesons can be well described in the GCM. The vacuum configuration can be determined by the saddle-point condition \( \frac{\partial S}{\partial B^\theta_0} = 0 \). Then an equation of the translation invariant quark self-energy \( \Sigma(q, \mu) \) is obtained as a truncated Dyson-Schwinger equation
\[ \Sigma(p, \mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{1}{2} \gamma^\nu \frac{1}{i \gamma^\nu \gamma_4 + \Sigma(q, \mu) \gamma_4} \gamma^\alpha \frac{1}{2} . \]  
(5)

Considering the fact that the inclusion of the chemical potential breaks the O(4) symmetry in the four-dimensionanl space, one should rewrite the decomposition of the self-energy \( \Sigma \) as
\[ \Sigma(p, \mu) = i[A(p, \mu) - 1] \gamma^\nu \gamma_4 + i[C(p, \mu) - 1] \gamma_4 (p_4 + i\mu) + B(p, \mu) , \]  
(6)

where \( B(p, \mu) \) is the counterpart of the vacuum configuration in the momentum space, i.e.,
\[ B(p, \mu) = \int \frac{1}{(2\pi)^4} B^\theta_0(z) e^{i\bar{p}z} dz . \]

It requires then that the vacuum configuration of \( \sigma \) and \( \bar{\sigma} \) should satisfy the restriction \( \sigma^2 + \bar{\sigma}^2 = 1 \). With \( \bar{q}_\nu = (\bar{q}, q_4 + i\mu) \) being introduced, and combining Eqs. (5) and (6), one can obtain the equations to determine the \( A(\bar{p}) \), \( B(\bar{p}) \) and \( C(\bar{p}) \) as follows
\[ [A(\bar{p}) - 1] \bar{p}^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{A(\bar{q}) \bar{q} \cdot \bar{p}}{A^2(\bar{q}) \bar{q}^2 + C^2(\bar{q}) \bar{q}_4^2 + \bar{B}^2(\bar{q})} , \]  
(7a)

\[ [C(\bar{p}) - 1] \bar{p}_4^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{C(\bar{q}) \bar{q}_4 \bar{p}_4}{A^2(\bar{q}) \bar{q}^2 + C^2(\bar{q}) \bar{q}_4^2 + \bar{B}^2(\bar{q})} , \]  
(7b)

\[ B(\bar{p}) = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{B(\bar{q})}{A^2(\bar{q}) \bar{q}^2 + C^2(\bar{q}) \bar{q}_4^2 + \bar{B}^2(\bar{q})} . \]  
(7c)
Basing on the solution of the Dyson-Schwinger equation or the gap equations, one can determine the bilocal field, and fix further the GCM action \( S \) and the quark propagator \( G \) with

\[
G^{-1} = iA(\tilde{p})\vec{\gamma} \cdot \vec{p} + iC(\tilde{p})\gamma_4\tilde{p}_4 + \Lambda^\theta B^\theta(\tilde{p}).
\]  

(8)

Thereafter we can evaluate the quark condensates from the above saddle-point expansion. It is straightforward that, under the mean-field approximation, for any quark operator \( O \)

\[
O_n = (\bar{q}_{j_1}(1)\Gamma_{j_1}^{(1)}q_{i_1})(\bar{q}_{j_2}(2)\Gamma_{j_2}^{(2)}q_{i_2})\cdots(\bar{q}_{j_n}(n)\Gamma_{j_n}^{(n)}q_{i_n}),
\]

one can get the condensate

\[
\langle : O_n : \rangle = (-1)^n \sum_P \text{Tr}(-)^P \Gamma_{j_1}^{(1)}\Gamma_{j_2}^{(2)}\cdots\Gamma_{j_n}^{(n)}(G)_{j_1,j_2} P(1)(G)_{j_2,j_3} P(2)\cdots(G)_{j_n,j_1} P(n),
\]

where \( G \) is the quark propagator and \( P \) denotes the permutation of the \( n \) indices. We obtain then the local scalar quark condensate as

\[
\langle : \bar{q}q : \rangle = -\text{Tr}_C G(x,x)
\]

\[
= -8\sqrt{2} \int \frac{d^4p}{(2\pi)^4} \frac{B(\tilde{p}^2)}{\tilde{p}^2 A^2(\tilde{p}^2) + \tilde{p}_4^2 C^2(\tilde{p}^2) + B^2(\tilde{p}^2)},
\]

(9)

Meanwhile, the nonlocal quark condensate is given as

\[
\langle : \bar{q}(x)q(0) : \rangle = -\text{Tr}_C [G(x,0)]
\]

\[
= -8\sqrt{2} \int \frac{d^4p}{(2\pi)^4} \frac{B(\tilde{p}^2)}{\tilde{p}^2 A^2(\tilde{p}^2) + \tilde{p}_4^2 C^2(\tilde{p}^2) + B^2(\tilde{p}^2)} e^{ipx}
\]

(10)

It is apparent that, with the solutions of the gap equations (7a)-(7c) being taken as the input for Eqs. (9) and (10), the variation of the quark condensates against the chemical potential can be evaluated. In practical calculation, since the knowledge about the exact behavior of \( g^2 \) and \( D(p - q) \) in low energy region is still lacking, one has to take some approximations or phenomenological form to solve the gap equations. For simplicity, we adopt the infrared dominant form\(^{31, 17}\)

\[
g^2 D(p - q) = \frac{3\pi^2 \eta^2}{q^2} \delta(p - q),
\]

(11)

where \( \eta \) is an energy-scale parameter and can be fixed by experimental data of hadrons. Although this form does not include the contribution from the ultraviolet energy region, it maintains the main property of QCD in low energy region. With Eqs. (7a)-(7c) and (11), one has the Nanbu-Goldstone solution

\[
A(\tilde{p}) = C(\tilde{p}) = 2, \quad B(\tilde{p}) = (\eta^2 - 4\tilde{p}^2)^{1/2}, \quad \text{for Re}(\tilde{p}^2) < \frac{\eta^2}{4},
\]

(12a)

\[
A(\tilde{p}) = C(\tilde{p}) = \frac{1}{2} \left[ 1 + \left( 1 + \frac{2\eta^2}{\tilde{p}^2} \right)^{1/2} \right], \quad B(\tilde{p}) = 0, \quad \text{for Re}(\tilde{p}^2) > \frac{\eta^2}{4}.
\]

(12b)
which describes the phase where chiral symmetry is spontaneously broken and the dressed-quarks are confined. Meanwhile one has also the Wigner solution

\[ A(\tilde{p}) = C(\tilde{p}) = \frac{1}{2} \left[ 1 + \left( 1 + \frac{2\eta^2}{\tilde{p}^2} \right)^{1/2} \right], \quad B(\tilde{p}) = 0, \]  

(13)

which characterizes a phase in which chiral symmetry is not broken and the dressed-quarks are not confined.

To fix the energy-scale parameter \( \eta \) so that the above mentioned calculations can be accomplished, we take the way described in the original work of the GCM\(^{[17]} \). Extending the relation between the GCM and the bag models given in Ref.\(^{[17]} \) to that in nuclear matter, we have the relation among the nucleon radius \( R \), nucleon mass \( M \) and the bag constant \( B \) of a nucleon as

\[ R = \left( \frac{a}{4\pi B} \right)^{1/4}, \]  

(14)

\[ M = \frac{4a}{3} \left( \frac{4\pi B}{a} \right)^{1/4}, \]  

(15)

where \( a = 3(\omega_0 - \mu) - Z_0 = 6.12 - 3\mu - Z_0 \), with \( Z_0 \) being a parameter to include the corrections of the motion of center-of-mass, zero-point energy, and other effects of the three quarks in a nucleon. Meanwhile, in advantage of the Friedberg-Lee nontopological soliton ansatz\(^{[32]} \), the bag constant comes from the difference of the action for hadrons and that for the vacuum. We have then

\[ B = S(\text{hadron}) - S(\text{vacuum}). \]

In the GCM, the interaction is transferred by the Goldstone bosons \( \pi \) and the scalar meson \( \sigma \). From the restriction \( \pi^2 + \sigma^2 = 1 \), one can take a simple approximation \( \pi = 0, \sigma = 1 \) for the vacuum. Referring the configuration of the mesons in a hadron as \( (\sigma, \pi) \), we have

\[ B = S(\sigma, \pi) - S(1,0). \]

In principle, the configuration of \( \sigma \) and \( \pi \) should be determined by solving the coupled equations of motion of the mesons and the quarks. As an approximation\(^{[17]} \), they can be taken as \( \sigma = 0, \pi = 0 \). After some derivation we obtain

\[ B = S(0,0,) - S(1,0) = 12 \int \frac{d^4\tilde{p}}{(2\pi)^4} \left\{ \ln \left[ \frac{A^2(\tilde{p}^2)\tilde{p}^2 + C^2(\tilde{p}^2)\tilde{p}^2_1 + B^2(\tilde{p}^2)}{A^2(\tilde{p}^2)\tilde{p}^2 + C^2(\tilde{p}^2)\tilde{p}^2_1 + B^2(\tilde{p}^2)} \right] \right\}, \]

(16)
In order to investigate the dependence of the quark condensate on the nuclear matter density $\rho$ explicitly, we must transfer the above $\mu$-dependence to the $\rho$-dependence. We should then determine the relation between the nuclear matter density $\rho$ and the chemical potential $\mu$ in the GCM. It has been known that the baryon number in nuclear matter can be related with the generating functional of the system as

$$n = \frac{\partial}{\partial \mu} \ln Z(\mu, B^0(x-y)) \approx \frac{\partial}{\partial \mu} Tr \ln G^{-1}(\mu, B^0(x-y)) \, ,$$

(17)

where $G^{-1}(\mu, B^0(x-y))$ is the inverse of the quark propagator in the medium. Taking some algorithm, we obtain finally

$$n \approx 2 \frac{\partial}{\partial \mu} \int \frac{d^4x}{(2\pi)^4} \int d^4p \frac{[B^0(\vec{p}, p_4 + i\mu)]^2}{p^2} \, .$$

The baryon number density can thus be given as

$$\rho_n = \frac{n}{\int d^4x} \approx \frac{2}{(2\pi)^4} \frac{\partial}{\partial \mu} \int d^4p \frac{[B^0(\vec{p}, p_4 + i\mu)]^2}{p^2} \, .$$

(18)

3 Numerical Result and Discussion

Taking the energy scale by fitting the properties of $\pi$ and $\rho$ mesons $\eta = 1.37$ GeV and calibrating the nucleon mass $M_0 = 939$ MeV in free space in the GCM, we obtain the center-of-mass motion correction parameter $Z_0 = 3.707$ and the nucleon radius $R_0 = 0.69$ fm. With the fitted parameters and the above formulae, we get at first, in the Nanbu-Goldstone phase, the quark condensate in free space $<$ $\bar{q}q$ $>$ $0$ $= -(148$ MeV)$^3$. Meanwhile, in the Wigner phase, the quark condensate vanishes, i.e., $<$ $\bar{q}q$ $>$ $\equiv 0$. It is evident that the presently obtained $<$ $\bar{q}q$ $>$ in the Nanbu-Goldstone phase is very close to the results given with a fully dressed gluon propagator ($- <$ $\bar{q}q$ $>^{1/3}$ can be 150-180 MeV) even though it is smaller than those given in QCD sum rules and some other approaches. From Eqs.(7) and (10) one can know that the value of the scalar quark condensate in vacuum depends on the integration interval determined by the parameter $\eta$. With the increase of $\eta$, the absolute value of the condensate will increase. In the present calculation, the $\eta$ is fixed by fitting the nucleon properties consistently but not freely. What we are now interested in is the changing characteristic of the condensate in nuclear matter against the nuclear matter density, which can be identified as the ratio of the condensates in nuclear matter to that in vacuum. Since the condensate in nuclear matter depends also on the parameter $\eta$ with the same relation, the smaller absolute value in vacuum will not affect the changing feature.

Recalling the property of the quark confinement phase and deconfinement phase, one can know easily that, in the confinement phase, the motion of a quark is restricted in a
hadron. In a phenomenological word, the motion of the quark is limited in a bag. The 
bag constant provides the constraint to limit the motion of the quark. It is apparent that, 
if the bag constant $B$ becomes zero, the quark can move freely. In view of the variation 
of the bag constant, $B = 0$ can then be taken as the critical point of the confinement 
phase and the deconfinement phase. Consequently, the gap equations (7a)-(7c) take the 
Nanbu-Goldstone solution or Wigner solution. Accomplishing the integral in Eq. (16) 
with various values of the chemical potential $\mu$, we get the chemical potential dependence 
of the bag constant. With the relation given in Eq. (18), we obtain the nuclear matter 
density dependence of the bag constant. The result is illustrated in Fig. 1. The figure 
shows evidently that the bag constant decreases with respect to the increasing of the 
nuclear matter density, and a critical nuclear matter density $\rho \approx 11.8 \rho_0$ (corresponding to 
a critical chemical potential $\mu_c \approx 0.316$ GeV), where $\rho_0$ is the normal density of nuclear 
matter, exists for the quark deconfinement to happen.

By varying the chemical potential $\mu$ and accomplishing the calculation in Eqs (9), 
(10) and (18) at several space-time distances $x$, we obtain the variation behavior of the 
ratios of local and non-local scalar quark condensates in nuclear matter to the local 
condensate in vacuum. The results with several fixed space-time distances are illustrated 
in Fig. 2 [simply denoted as $R_2(\rho)$]. The figure shows clearly that, in the Nanbu-Goldstone 
phase, the local scalar quark condensate and the nonlocal condensates with each fixed 
space-time distance between the quarks increase smoothly with respect to the increase of 
nuclear matter density. As the density of nuclear matter reaches its critical value $\mu_c$, the 
condensates vanish suddenly. In the Wigner phase, the condensates maintain zero since 
$B(\tilde{p}) \equiv 0$. This result provides a clue that the chiral symmetry is broken more strongly as 
the nuclear matter density increases gradually. When the nuclear matter density reaches 
the critical value, the chiral symmetry is restored suddenly. Such a varying behavior 
is consistent with the result of the DSE calculations[24, 25] and the sudden changing 
characteristic is similar to that given in the composite-operator formalism of QCD[35]. 
However, it is evident that such a changing behavior of the scalar quark condensates in 
nuclear matter is different from most of the previous results. It has been conventionally 
known that the nucleon mass is a monotonous function of the scalar quark condensate, and 
the mass of a nucleon in nuclear matter decreases with respect to the increase of nuclear 
matter density. Then the absolute value of the scalar quark condensate should decrease as 
the nuclear matter density increases. In fact, the condensate depends also on the number 
of nucleons in nuclear matter which increases rapidly against the increase of nuclear matter 
density. Combining these two aspects together, one can know that it is reasonable for 
the condensates to increase with respect to the increase of nuclear matter density. On 
the other hand, according to the well known Gell-Mann–Oakes–Renner relation[36], the 
mass of a pion in nuclear matter will increase with respect to the increase of nuclear matter 
density. Consequently, the threshold of the pion production in ultrarelativistic heavy ion
collisions will increase. One of the reason for the most recently observed relatively small amount of pion production to proton production in the RHIC experiment may be such an increase of the pion production threshold.

Looking over Fig. 2, one may also easily realize that, at a small space-time distance, the condensate ratios at a definite density of nuclear matter are larger than that of the local condensate. However, at large space-time distance, they are smaller. It indicates that the quark condensate in nuclear matter does not change monotonously with respect to the space-time distance. To show this point more explicitly, we display the changing feature of the ratio of scalar quark condensate in nuclear matter to the corresponding values in free space against the space-time distance in Fig. 3 [denoted as $G_2(x^2)$]. The figure shows evidently that the condensate in free space ($\rho = 0$) decreases gradually with the increase of the space-time distance among the quarks definitely. However, the condensate in nuclear matter ($\rho \neq 0$) increases if the space-time distance is very small. As the distance is about 0.2-0.24 fm (about one fourth of the nucleon radius), the condensate ratio reaches its maximum if the nuclear matter density changes from $3\rho_0$ to $11\rho_0$, and such a distance increases with the increase of nuclear matter density. If the space-time distance gets larger, the condensate ratio decreases with the increasing of space-time distance and the decreasing rate increases with respect to the increase of nuclear matter density. It manifests that the chiral symmetry can be restored more rapidly with the increase of nuclear matter density if the space-time distance among quarks is approximately equal to or larger than the one forth of the nucleon radius. Furthermore, such a nonmonotonous changing characteristic also indicates that there exists a repulsive interaction at very small space-time distance between the quarks. Since increasing the density of nuclear matter results generally in a decrease of the distance among quarks, the decrease of condensate ratios as the quarks are located farther away beyond the repulsive core is just the other manifestation of the fact that the absolute values of the scalar quark condensates increase against increasing nuclear matter density.

4 Summary and Remarks

In summary we have investigated the nuclear matter density dependence of the local and nonlocal scalar quark condensates in nuclear matter in an effective field theory model of QCD, namely the global color symmetry model (GCM). The calculated results indicate that the quark condensates increase smoothly with the increase of the density of nuclear matter and vanish suddenly as the nuclear matter density reaches its critical value (about $11.8\rho_0$). Meanwhile the nonlocal condensates change nonmonotonously as the quarks are separated far away from each other. This result manifests that the chiral symmetry is broken more seriously as the density of nuclear matter increases and can be restored suddenly at the critical point. However, if the quarks are separated farther away from
each other, the chiral symmetry restoration process can be enhanced with the increase of nuclear matter density.

Since only a very simple form of the $g^2$ and $D(p - q)$ is taken into account in the present calculation, detailed effects of the running couple constant $g^2$ (or $\alpha$), the gluon propagator $D(p - q)$ and the other degrees of freedom have not yet been included. Besides, the dependence of the quark, meson states on the nuclear matter density (or chemical potential) has not yet been taken into account self-consistently. Moreover, even though the presently obtained result is coincident with that obtained in the DSE formalism\cite{24, 25} and the sudden change feature is consistent with that given in the composite-operator formalism of QCD\cite{35}, the changing behavior in the continuous region is not consistent with most of the previous results (e.g., those given in Ref.\cite{11}). Then more sophisticated investigation is necessary and under progress.

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References

[1] E. G. Drukarev and E. M. Levin, Nucl. Phys. A 511, 679 (1990); Prog. Part. Nucl. Phys. 27, 77 (1991).

[2] E. M. Henley and J. Pasupathy, Nucl. Phys. A 556, 467 (1993).

[3] X. Jin, Phys. Rev. C 51, 2260 (1995); C 52, 3344 (1995).

[4] M. B. Johnson and L. S. Kisslinger, Phys. Rev. D 61, 074014 (2000).

[5] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979); B 147, 448 (1979).

[6] L. L. Reinders, H. Rubenstei, and S. Yazaki, Phys. Rep. 127, 1 (1985).

[7] H. Jung and L. S. Kisslinger, Nucl. Phys. A 586, 682 (1995).

[8] A. P. Bakulev and A. V. Radyushkin, Phys. Lett. B 271, 223 (1991).

[9] S. V. Mikhailov and A. V. Radyushkin, Phys. Rev. D 45, 1754 (1992).

[10] Z. G. Wang, S. L. Wan, and K. L. Wang, Phys. Lett. B 498, 195 (2001).

[11] E. G. Drukarev, M. G. Ryskin, and V. A. Sadovnikova, Prog. Part. Nucl. Phys. 47, 73 (2001); and references therein.

[12] M. Malheiro, M. Dey, A. Delfino, and J. Dey, Phys. Rev. C 55, 521 (1997).

[13] G. Q. Li and C. M. Ko, Phys. Lett. B 338, 118 (1994).

[14] T. Mitsumori, N. Noda, H. Kouno, A. Hasegawa, and M. Nakano, Phys. Rev. C 55, 1577 (1997).

[15] C. Itzyson and J-B. Zuber, Quantum Field Theory (McGraw-Hill, New York, 1985).

[16] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. 33, 477 (1994), and references therein.

[17] R. T. Cahill and C. D. Roberts, Phys. Rev. D 32, 2419 (1985); R. T. Cahill, C. D. Roberts, and J. Praschifka, Ann. Phys. (N.Y.) 188, 20 (1988); C. D. Roberts, R. T. Cahill, M. E. Sevior, and N. Iannella, Phys. Rev. D 49, 125 (1994).

[18] M. R. Frank, P. C. Tandy, and G. Fai, Phys. Rev. C 43, 2808 (1991); M. R. Frank and P. C. Tandy, Phys. Rev. C 46, 338 (1992); C. W. Johnson and G. Fai, Phys. Rev. C 56, 3353 (1997).
[19] P. C. Tandy, Prog. Part. Nucl. Phys. 39, 177 (1997), and references therein.

[20] X. F. Lü, Y. X. Liu, H. S. Zong, and E. G. Zhao, Phys. Rev. C 58, 1195 (1998).

[21] T. Meissner, Phys. Lett. B 405, 8 (1997).

[22] L. S. Kisslinger and T. Meissner, Phys. Rev. C 57, 1528 (1998).

[23] H. S. Zong, X. F. Lü, J. Z. Gu, C. H. Chang, and E. G. Zhao, Phys. Rev. C 60, 055208 (1999).

[24] P. Maris, C. D. Roberts, and S. Schmidt, Phys. Rev. C 57, R2821 (1998).

[25] A. Bender, G. I. Poulies, C. D. Roberts, S. Schmidt, and A. W. Thomas, Phys. Lett. B 431, 263 (1998).

[26] A. Bender, D. Blaschke, Yu L. Kalinovsky, and C. D. Roberts, Phys. Rev. Lett. 77, 3724 (1996); A. Bender, C. D. Roberts, and L. V. Smekal, Phys. Lett. B 380, 7 (1996).

[27] D. Blaschke, C. D. Roberts, and S. Schmidt, Phys. Lett. B 425, 232 (1998).

[28] D. Blaschke, H. Grigorian, G. Poghosyan, C. D. Roberts, and S. Schmidt, Phys. Lett. B 450, 207 (1999).

[29] M. Harada and A. Shibata, Phys. Rev. D 59, 014010 (1999).

[30] Y. X. Liu, D. F. Gao, and H. Guo, Nucl. Phys. A 695, 353 (2001).

[31] H. J. Munczek and A. M. Nemirovsky, Phys. Rev. D 28, 181 (1983).

[32] R. Friedberg and T. D. Lee, Phys. Rev. D 15, 1694 (1977); D 16, 1096 (1977); D 18, 2623 (1978).

[33] J. I. Kapusta, Finite-Temperature Field Theory (Cambridge University Press, Cambridge, 1989)

[34] L.F. Yang and X. F. Lü, Commun. Theor. Phys. 37, 589 (2002).

[35] A. Barducci, R. Casalbuoni, S. De Curtis, R. Gatto, and G. Pettini, Phys. Rev. D 41, 1610 (1990).

[36] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[37] C. Seife, Science 298, 718 (2002).
Figure 1: The variation of the bag constant with respect to the nuclear matter density
Figure 2: Calculated ratio of the two-quark condensate in nuclear matter to that in free space at several fixed space-time distances.
Figure 3: Calculated ratio of the nonlocal two-quark condensate in nuclear matter to that of the local one