Isospin eigenstates of the color singlet-singlet-type pentaquark states

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In this study, we constructed color singlet-singlet-type five-quark currents with isospins \((I, I_3) = (\frac{1}{2}, \frac{1}{2})\) and \((\frac{1}{2}, \frac{1}{2})\) unambiguously to explore the \(D\Sigma^-, D\Sigma^+, D^{*}\Sigma^-, D^{*}\Sigma^+\) pentaquark states via the quantum chromodynamics sum rules for the first time, where \(D, \Sigma, \cdots\) represent the color singlet clusters with the same quantum numbers as the corresponding physical mesons or baryons. The numerical results support assigning \(P_c(4312), P_c(4380), P_c(4440),\) and \(P_c(4457)\) as the \(D\Sigma^-, D\Sigma^+, D^{*}\Sigma^-, D^{*}\Sigma^+\) pentaquark states, respectively, with the isospin \(I = \frac{1}{2}\). The corresponding \(D\Sigma^-, D\Sigma^+, D^{*}\Sigma^-, D^{*}\Sigma^+\) pentaquark states with the isospin \(I = \frac{3}{2}\) have slightly larger masses. The observations of the high pentaquark candidates in the \(J/\psi K^- p\) invariant mass spectrum would shed light on the nature of the \(P_c\) states and contribute in distinguishing the scenarios of the color antitriplet-antitriplet-antitriplet-type and color singlet-singlet-type pentaquark states.

\textbf{pentaquark states, hadronic molecule, QCD sum rules}

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1 Introduction

In 1964, Gell-Mann [1] proposed that multiquark states could exist. Theoretically, the existence of exotic states has no prohibition, which cannot be embedded into the conventional charmonium spectrum. Since the observation of \(X(3872)\) by the Belle Collaboration [2] in 2003, many exotic \(X, Y,\) and \(Z\) particles have been observed at the Belle, BaBar, BESIII, and LHCb Collaborations [3]. The masses of some exotic states are close to the known two-particle thresholds and lead to possible hadronic molecule interpretations [4], namely, the bound states of the meson-meson, baryon-meson, or baryon-baryon. In 2015, the LHCb Collaboration [5] observed two pentaquark candidates \(P_c(4380)\) and \(P_c(4450)\) through the analysis of \(A_0^0 \rightarrow J/\psi K^- p\) decays. In 2019, the LHCb Collaboration [6] re-investigated the experimental data with an order of magnitude larger than that previously analyzed by the LHCb Collaboration and observed a narrow pentaquark candidate \(P_c(4312)\) in the \(J/\psi K^- p\) mass spectrum. It also proved that \(P_c(4450)\) consists of two narrow overlapping peaks \(P_c(4440)\) and \(P_c(4457)\). The measured Breit-Wigner masses and widths of the four exotic structures are as follows [5, 6]:

\[
P_c(4312) : M = 4311.9 \pm 0.7^{+6.5}_{-0.5} \text{ MeV}, \quad \Gamma = 9.8 \pm 2.7^{+2.7}_{-1.4} \text{ MeV},
\]

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\[ P_{c}(4380) : M = 4380 \pm 8 \pm 29 \text{ MeV}, \]
\[ \Gamma = 205 \pm 18 \pm 86 \text{ MeV}, \]
\[ P_{c}(4440) : M = 4440.3 \pm 1.3^{+4.1}_{-4.4} \text{ MeV}, \]
\[ \Gamma = 20.6 \pm 4.9^{+15}_{-10} \text{ MeV}, \]
\[ P_{c}(4457) : M = 4457.3 \pm 0.6^{+4.1}_{-1.7} \text{ MeV}, \]
\[ \Gamma = 6.4 \pm 2.0^{+5.7}_{-1.9} \text{ MeV}. \]

The resonances lie just a few MeV below the thresholds of the hidden-charm meson-baryon pairs \( D\Sigma_c, D\Sigma_c^*, D\Sigma_c^*, \) and \( D^*\Sigma_c^* \). Now, a typical interpretation of \( P_{c}(4312), P_{c}(4380), P_{c}(4440), \) and \( P_{c}(4457) \) is that they are the S-wave hidden-charm meson-baryon molecules and have a definite isospin \( I \), spin \( J \), and parity \( P \). For example, in ref. [7], it is proposed that \( P_{c}(4440) \) and \( P_{c}(4457) \) are the \( D^*\Sigma_c^* \)-bound states with \( I^P = \frac{1}{2}^{-} \) and \( \frac{3}{2}^{-} \), respectively, via the one-pion exchange potential between the heavy antimeson and heavy baryon. The result is consistent with the conclusion obtained in ref. [8] via the one-boson-exchange model. Interestingly, in ref. [9], isospins were considered via the one-pion/one-boson-exchange potential model, and a series of hidden-charm antimeson-baryon pentaquark molecules were predicted. In ref. [10], the \( D(\omega)\Sigma(\omega) \) molecular states were studied via a coupled-channel formalism with the scattering potential involving the one-pion exchange and short-range operators constrained by the heavy-quark spin symmetry. In ref. [11], \( P_{c}(4440) \) and \( P_{c}(4457) \) were interpreted as the \( D^*\Sigma_c^* \)-bound states with \( I^P = \frac{1}{2}^{-} \) and \( \frac{3}{2}^{-} \), respectively, via the quasipotential Bethe-Salpeter equation approach.

Among the popular theoretical tools, the quantum chromodynamics (QCD) sum rule approach is a powerful theoretical tool in studying exotic states. \( P_{c}(4312), P_{c}(4380), P_{c}(4440), \) and \( P_{c}(4457) \) have been studied with the QCD sum rules, irrespective of being assigned as pentaquark states [12] or pentaquark molecular states [13-19]. In the QCD sum rules, we choose the local five-quark currents. The pentaquark and molecular states are compact objects. Accordingly, we call the pentaquark molecular states as the color singlet-singlet-type pentaquark states. Thereafter, we will prefer the nomenclature “color singlet-singlet-type pentaquark states”.

If we prefer the interpretations of the color singlet-singlet-type pentaquark states and the theoretical approach of the QCD sum rules, we should distinguish their isospins and investigate their properties in an unambiguous way. However, in previous works, the isospins of the interpolating currents were not specified [13-19]. The currents couple potentially not only to the pentaquark states with the isospin \( I = \frac{1}{2} \) but also to the ones with the isospin \( I = \frac{1}{2} \), and there are unknown uncertainties. Because the \( P_{c} \) states were discovered in the \( J/\psi p \) invariant mass spectrum, their isospins should be \( I = \frac{1}{2} \) considering the conservation of the isospins in the strong interactions. Moreover, we should specify the isospins of the interpolating currents to make robust predictions. It is the key issue to solve the puzzle of the \( P_{c} \) states. In the present work, we explore the color singlet-singlet-type pentaquark states with \( I = \frac{1}{2} \) and \( \frac{3}{2} \) via the QCD sum rules in a systematic way.

This article is arranged as follows: We identify the QCD sum rules for the color singlet-singlet-type pentaquark states in sect. 2. We present the numerical results and discussions in sect. 3. We enumerate the conclusions in sect. 4.

2 QCD sum rules for the color singlet-singlet-type pentaquark states

The \( u \) and \( d \) quarks have the isospin \( I = \frac{1}{2} \). In detail, \( \bar{u}u = \frac{1}{2}u \) and \( \bar{d}d = \frac{1}{2}d \), where \( I \bar{I} \) is the isospin operator. Then, \( D^0, D^{*0}, D^-, D^{*-}, \Sigma^0, \Sigma^+, \Sigma^0, \) and \( \Sigma^{++} \) correspond to the eigenstates \( | \frac{1}{2}, \frac{1}{2} \rangle, | \frac{1}{2}, -\frac{1}{2} \rangle, | \frac{1}{2}, -\frac{1}{2} \rangle, | 1, 0 \rangle, | 1, 0 \rangle, | 1, 1 \rangle, \) and \( | 1, 1 \rangle \) in the isospin space \((I, I)\), respectively. Then, we can construct the following color singlet currents to interpolate them:

\[
J^{\rho\mu}(x) = \bar{c}(x)i\gamma_\mu u(x),
\]
\[
J^{\rho\mu}(x) = \bar{c}(x)i\gamma_\mu d(x),
\]
\[
J^{\rho\mu}(x) = \bar{c}(x)\gamma_\mu u(x),
\]
\[
J^{\rho\mu}(x) = \bar{c}(x)\gamma_\mu d(x),
\]
\[
F^{\bar{c}c}(x) = e^{ikx}u^T(x)C\gamma_\mu d(x)y^{\rho}y^{\mu}c(x),
\]
\[
F^{\bar{c}c}(x) = e^{ikx}u^T(x)C\gamma_\mu u(x)y^{\rho}y^{\mu}c(x),
\]
\[
F^{\bar{c}c}(x) = e^{ikx}T(x)C\gamma_\mu u(x)y^{\rho}y^{\mu}c(x),
\]
\[
F^{\bar{c}c}(x) = e^{ikx}T(x)C\gamma_\mu d(x)y^{\rho}y^{\mu}c(x).
\]

The superscripts \( i, j, k \) are color indices, and \( C \) represents the charge conjugation matrix. Accordingly, we can construct the color singlet-singlet-type five-quark currents to interpolate the \( D(\omega)\Sigma(\omega) \) pentaquark states, where \( D(\omega) \) and \( \Sigma(\omega) \) represent the color singlet clusters with the same quantum numbers as the physical states \( D(\omega) \) and \( \Sigma(\omega) \), respectively. We write down the two-point correlation functions:

\[
\Pi^{(0)}(p) = i \int d^4x e^{ipx} \langle 0|T \{ J(x)\bar{J}(0) \}|0 \rangle,
\]
\[
\Pi^{(0)}(p) = i \int d^4x e^{ipx} \langle 0|T \{ J_{\mu}(x)\bar{J}_{\mu}(0) \}|0 \rangle,
\]
\[
\Pi^{(0)}(p) = i \int d^4x e^{ipx} \langle 0|T \{ J_{\mu\nu}(x)\bar{J}_{\mu\nu}(0) \}|0 \rangle,
\]

where the currents

\[
J(x) = J^{D\Sigma}(x) + J^{D^*\Sigma^*}(x),
\]
\[
J_{\mu}(x) = J^{D\Sigma}_{\mu}(x) + J^{D^*\Sigma^*}_{\mu}(x),
\]
\[
J_{\mu\nu}(x) = J^{D\Sigma}_{\mu\nu}(x) + J^{D^*\Sigma^*}_{\mu\nu}(x).
\]
\( J_{\mu}(x) = \frac{1}{2} \gamma_{\nu}^{\nu}(x) \), \( J_{\mu}^{\nu}(x) \),

\[
J^{(\nu)}_{\frac{1}{2}}(x) = \frac{1}{\sqrt{3}} J^{\nu}(x), \quad J^{(\nu)}_{\frac{3}{2}}(x) = \frac{1}{\sqrt{3}} J^{\nu}(x),
\]

\[
J^{(\nu)}_{\frac{1}{2}}(x) = \frac{1}{\sqrt{3}} J^{\nu}(x), \quad J^{(\nu)}_{\frac{3}{2}}(x) = \frac{1}{\sqrt{3}} J^{\nu}(x),
\]

\[
J^{(\nu)}_{\frac{1}{2}}(x) = \frac{1}{\sqrt{3}} J^{\nu}(x), \quad J^{(\nu)}_{\frac{3}{2}}(x) = \frac{1}{\sqrt{3}} J^{\nu}(x),
\]

\[
J^{(\nu)}_{\frac{1}{2}}(x) = \frac{1}{\sqrt{3}} J^{\nu}(x), \quad J^{(\nu)}_{\frac{3}{2}}(x) = \frac{1}{\sqrt{3}} J^{\nu}(x),
\]

The subscripts \( \frac{1}{2} \) and \( \frac{3}{2} \) represent the isospins \( I \) [18]. The currents are the isospin eigenstates \( |I, I_3\rangle = |\frac{1}{2}, \frac{1}{2}\rangle \) or \( |\frac{1}{2}, \frac{3}{2}\rangle \).

The currents \( J(x) \), \( J_\mu(x) \), and \( J_{\mu}(x) \) couple potentially not only to the color singlet-singlet-type hidden-charm pentaquark states with negative parity but also to the ones with positive parity. We separate their ground-state contributions at the hadron side:

\[
\Pi(p) = \lambda \frac{2}{3} J_{\frac{1}{2}}(p^2) + M_5 - p^2 \gamma_\mu + \lambda \frac{2}{3} J_{\frac{3}{2}}(p^2) - p^2 \gamma_\mu + \cdots
\]

\[
\Pi_{\mu\nu}(p) = \lambda \frac{2}{3} J_{\frac{1}{2}}(p^2) - \frac{1}{2} \gamma_{\mu\nu} + \lambda \frac{2}{3} J_{\frac{3}{2}}(p^2) - \frac{1}{2} \gamma_{\mu\nu} + \cdots
\]

where the subscripts \( \frac{1}{2} \), \( \frac{3}{2} \), and \( \frac{5}{2} \) are the spins of the pentaquark states, and the subscript and superscript \( \pm \) denote the positive parity and negative parity, respectively. We have smeared the isospin indexes. The pole residues are defined by

\[
\langle 0 | J(0) | P_{\frac{1}{2}}(p) \rangle = \frac{1}{2} \gamma_{\mu} U^{-}(p, \epsilon),
\]

\[
\langle 0 | J(0) | P_{\frac{3}{2}}(p) \rangle = \frac{1}{2} \gamma_{\mu} U^{-}(p, \epsilon),
\]

\[
\langle 0 | J_{\mu}(0) | P_{\frac{1}{2}}(p) \rangle = \frac{1}{2} \gamma_{\mu} U^{-}(p, \epsilon),
\]

\[
\langle 0 | J_{\mu}(0) | P_{\frac{3}{2}}(p) \rangle = \frac{1}{2} \gamma_{\mu} U^{-}(p, \epsilon),
\]

\[
\langle 0 | J_{\mu}(0) | P_{\frac{3}{2}}(p) \rangle = \frac{1}{2} \gamma_{\mu} U^{-}(p, \epsilon),
\]

where \( U^{-}(p, \epsilon) \), \( U^{+}(p, \epsilon) \), and \( U^0(p, \epsilon) \) are the Dirac and Rarita-Schwinger spinors. For all the technical details, one can consult refs. [12, 17, 18].

In the present work, we choose the components associated with the structures \( \bar{q} \gamma_{\mu} \gamma_{\nu} \gamma_{\rho} \bar{q} \gamma_{\tau} \) and \( \bar{q} \gamma_{\mu} \gamma_{\nu} \gamma_{\rho} \gamma_{\tau} \bar{q} \gamma_{\rho} \gamma_{\tau} \) in the correlation functions \( \Pi(p) \), \( \Pi_{\mu\nu}(p) \), and \( \Pi_{\mu\nu\rho}(p) \), respectively, to investigate the color singlet-singlet-type pentaquark states with the spin parities \( J^P = \frac{1}{2} \), \( \frac{3}{2} \), and \( \frac{5}{2} \), respectively.

We conducted a complex operator product expansion and analyzed the contributions of all types of vacuum condensates. First, the contributions of the related vacuum condensates are small in the case of \( k \geq \frac{3}{2} \) for the counting rules in terms of the strong fine-structure constant \( O(\alpha_S) \)[20, 21]. Hence, it is accurate enough to calculate the terms for \( k \leq 1 \)[22]. Second, the highest dimension of the vacuum condensates is usually estimated from the leading-order Feynman diagrams. In the present work, the correlation functions contain two heavy-quark lines and three light quark lines. If each heavy-quark line emits a gluon and each light quark line contributes a quark-antiquark pair, we obtain the quark-gluon operator \( g_{\mu}G_{\rho\sigma}g_{\mu}G_{\rho\sigma} \) with the dimension 13. This operator can be factorized into the vacuum condensates \( \langle \bar{q} \gamma_{\rho} g_{\rho} \gamma_{\sigma} q \rangle \langle \bar{q} \gamma_{\sigma} g_{\rho} \gamma_{\sigma} q \rangle \), and \( \langle \bar{q} \gamma_{\rho} g_{\rho} \gamma_{\sigma} q \rangle \langle \bar{q} \gamma_{\sigma} g_{\rho} \gamma_{\sigma} q \rangle \). Third, the four-quark condensates \( \langle \bar{q} \gamma_{\rho} g_{\rho} \gamma_{\sigma} q \rangle \langle \bar{q} \gamma_{\sigma} g_{\rho} \gamma_{\sigma} q \rangle \) are neglected as they come from condensations between two heavy-quark lines through the equation of motion and play a tiny role [22]. Thus, in this work, we choose the terms \( \langle \bar{q} \gamma_{\rho} g_{\rho} \gamma_{\sigma} q \rangle \langle \bar{q} \gamma_{\sigma} g_{\rho} \gamma_{\sigma} q \rangle \), \( \langle \bar{q} \gamma_{\rho} g_{\rho} \gamma_{\sigma} q \rangle \langle \bar{q} \gamma_{\sigma} g_{\rho} \gamma_{\sigma} q \rangle \), and \( \langle \bar{q} \gamma_{\rho} g_{\rho} \gamma_{\sigma} q \rangle \langle \bar{q} \gamma_{\sigma} g_{\rho} \gamma_{\sigma} q \rangle \) in the operator product expansions for solid reasons.

We obtain the analytical spectral densities \( \rho_{\Pi_Q}(x) \) and \( \rho_{\Pi_Q}(x) \) at the quark-gluon level and take the quark-hadron duality below the continuum threshold \( x_0 \) and introduce the weight functions \( \sqrt{2} \exp\left(-\frac{\pi^2}{16}ight) \) and \( \exp\left(-\frac{\pi^2}{16}ight) \) to obtain the QCD sum rules:

\[
2M_{\lambda x^2} \exp\left(-\frac{M^2}{T^2}ight)
\]
\[
\int_{4m_c^2}^{s_0} \frac{dz}{z} \left[ \sqrt{s} \rho_{QCD}(s) + \rho_{QCD}^0(s) \right] \exp\left(-\frac{z}{T^2}\right).
\] (12)

As we are only interested in the pentaguard states with the negative parity, the explicit expressions of the spectral densities \(\rho_{QCD}(s)\) and \(\rho_{QCD}^0(s)\) at the quark level are neglected for simplicity.

We differentiate eq. (12) with respect to \(s = \frac{1}{T^2}\) and then eliminate the pole residues \(\lambda_j^*\) with \(j \neq \frac{1}{2}, \frac{3}{2}, \frac{5}{2}\) to obtain the QCD sum rules for the masses of the color singlet-singlet type pentaguard states:

\[
M_c^2 = \frac{\int_{4m_c^2}^{s_0} \frac{dz}{z} \left[ \sqrt{s} \rho_{QCD}(s) + \rho_{QCD}^0(s) \right] \exp\left(-\frac{z}{T^2}\right)}{\int_{4m_c^2}^{s_0} \frac{dz}{z} \left[ \sqrt{s} \rho_{QCD}^0(s) + \rho_{QCD}^0(s) \right] \exp\left(-\frac{z}{T^2}\right)},
\] (13)

where the spectral densities \(\rho_{QCD}(s) = \rho_{QCD}(s)\) and \(\rho_{QCD}^0(s) = \rho_{QCD}^0(s)\).

3 Numerical results and discussions

We apply the standard values of the vacuum condensates \(\langle \bar{q}q \rangle = -(0.24 \pm 0.01) \text{ GeV}^3\), \(\langle \bar{q}g_{i=1}^5 G q \rangle = m_c^6 \langle \bar{q}q \rangle \text{ GeV}^2\), \(m_c = (0.8 \pm 0.1) \text{ GeV}\), and \(\langle \bar{q}g_{j=1}^{10} G G \rangle = (0.33 \text{ GeV})^3\) at the energy scale \(\mu = 1 \text{ GeV}\) [23-26] and choose the \(M_5\) mass \(m_5(m_c) = (1.275 \pm 0.025) \text{ GeV}\) from the Particle Data Group [3]. We consider the energy-scale dependence of the following parameters:

\[
\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1 \text{ GeV}) \left[ \frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{13}{7\pi}},
\]

\[
\langle \bar{q}g_{i=1}^5 G q \rangle(\mu) = \langle \bar{q}g_{j=1}^{10} G q \rangle(1 \text{ GeV}) \left[ \frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{13}{7\pi}},
\]

\[
m_5(\mu) = m_5(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{13}{7\pi}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0} \left[ 1 - b_1 \log \frac{\mu^2}{b_0^2} + \frac{b_7}{3} \left( \log^2 \frac{\mu^2}{b_0^2} - 2 \log \frac{\mu^2}{b_0^2} + b_9 \right) \right],
\]

where \(t = \log \frac{\mu^2}{b_0^2}\), \(b_0 = \frac{33 - 2n_1}{12n_1}\), \(b_1 = \frac{135 - 19n_1}{24n_1}\), \(b_2 = \frac{2857 - 2n_1 + \frac{5n_1^2}{12n_1^2}}{72n_1^2}\), and \(\Lambda_{QCD} = 213, 296, 339 \text{ MeV}\) for the flavors \(n_1 = 5, 4, 3\), respectively [3, 27]. In this study, we choose the flavor number \(n_1 = 4\) for all the pentaguard states and apply the energy-scale formula to determine the best energy scales of the QCD spectral densities [12, 17, 18, 20, 21]:

\[
\mu = \sqrt{M_{c}^2 + \frac{4\Lambda_{QCD}^2}{I}}.
\] (14)

where \(M_c\) is the effective charm quark mass. We choose the updated value \(M_c = (1.85 \pm 0.01) \text{ GeV}\) [18].

All the QCD sum rules should satisfy the pole dominance and convergence of the operator product expansion, which are two basic criteria. Moreover, we should obtain Borel platforms to avoid additional uncertainties originating from the Borel parameters. The selections of the suitable energy scales, continuum threshold parameters, and Borel parameters are accomplished via trial and error: We tentatively choose an energy scale \(\mu\) and a continuum threshold parameter \(x_0\). Then, we obtain the numerical value of the pentaguard mass \(M_{c}\) from the QCD sum rules and evaluate whether or not the two basic criteria of the QCD sum rules (plus the constraint \(\sqrt{s_0} = M_{c} + 0.6-0.7 \text{ GeV}\) and the energy-scale formula \(\mu = \sqrt{M_{c}^2 + 4\Lambda_{QCD}^2}\) are satisfied. If not, then we choose another energy scale and continuum threshold parameter until we reach satisfactory results. In the calculations, we define the pole contributions (PCs) as:

\[
\text{PC} = \int_{4m_c^2}^{s_0} \frac{dz}{z} \left[ \sqrt{s} \rho_{QCD}(s) + \rho_{QCD}^0(s) \right] \exp\left(-\frac{z}{T^2}\right).
\] (15)

The convergence of the operator product expansion is quantified via the contributions of the vacuum condensates of the dimension \(n\):

\[
D(n) = \int_{4m_c^2}^{s_0} \frac{dz}{z} \left[ \sqrt{s} \rho_{QCD}(s) + \rho_{QCD}^0(s) \right] \exp\left(-\frac{z}{T^2}\right),
\] (16)

where \(\rho_{QCD}(s)\) and \(\rho_{QCD}^0(s)\) represent the spectral densities with the vacuum condensates of the dimension \(n\) selected from \(\rho_{QCD}(s)\) and \(\rho_{QCD}^0(s)\), respectively, and the total contributions are normalized to 1.

Lastly, we identify the best energy scales, ideal continuum threshold parameters, and Borel windows (see Table 1). The PCs for all the eight pentaguard states are approximately (or slightly larger than) 40%-60%. Thus, the pole dominance criterion for the QCD sum rules holds well.

The absolute values of the normalized contributions \(D(n)\) from the vacuum condensates are displayed in Figure 1, where the highest-dimensional condensate contributions \(|D(12)|\) and \(|D(13)|\) are approximately zero. The most important contributions are mainly from the lowest-order contributions \(\langle \bar{q}q \rangle\), \(\langle \bar{q}g_{i=1}^5 G q \rangle\), and \(\langle \bar{q}g_{j=1}^{10} G G \rangle\). The gluon condensate plays a less important role because \(|D(4)| < 5\%\) except for the \(D\Sigma_c\) pentaguard state with the isospin \(I = \frac{1}{2}\). All in all, the convergence of the operator expansions is very well satisfied.

We calculate the uncertainties of the masses and pole residues according to the standard error analysis formula. The numerical results of the masses and pole residues are shown in Table 1 (also Figure 2).

As shown in Table 1, the central value of the extracted mass of the \(D\Sigma_c\) pentaguard state with the quantum numbers \(I^P = \frac{1}{2}\) is 4.31 GeV. This value is only approximately
Table 1  Borel parameters, continuum threshold parameters, energy scales, PCs, masses, pole residues, and assignments for the eight color singlet-singlet-type pentaquark states, where the thresholds denote the corresponding thresholds of the meson-baryon scattering states

| $J^P$ | $T^2$ (GeV$^2$) | $\sqrt{s_0}$ (GeV) | $\mu$ (GeV) | PC | $M$ (GeV) | $\lambda$ (10$^{-3}$ GeV$^4$) | Assignments | Thresholds (MeV) |
|-------|----------------|-------------------|-------------|----|-----------|-----------------|-------------|-----------------|
| $\bar{D}_c$ | $\frac{3}{2}^-$ | 3.2-3.8 | 5.00 ± 0.10 | 2.2 | 42%-60% | 4.31 ± 0.07 | 3.25 ± 0.49 | $P_c$ (4312) | 4321 |
| $\bar{D}'_c$ | $\frac{3}{2}^-$ | 3.2-3.8 | 4.98 ± 0.10 | 2.2 | 44%-65% | 4.33 ± 0.09 | 1.97 ± 0.26 | resonance | 4321 |
| $D_c^*$ | $\frac{1}{2}^-$ | 3.3-3.9 | 5.06 ± 0.10 | 2.3 | 42%-60% | 4.38 ± 0.09 | 1.97 ± 0.24 | $P_c$ (4380) | 4385 |
| $D'_c^*$ | $\frac{3}{2}^-$ | 2.9-3.5 | 5.03 ± 0.10 | 2.4 | 44%-64% | 4.41 ± 0.08 | 1.24 ± 0.17 | resonance | 4385 |
| $D^*_c$ | $\frac{3}{2}^-$ | 3.0-3.6 | 5.10 ± 0.10 | 2.5 | 43%-61% | 4.47 ± 0.09 | 2.31 ± 0.31 | resonance | 4422 |
| $D^*_c$ | $\frac{3}{2}^-$ | 3.0-3.6 | 5.08 ± 0.10 | 2.5 | 43%-60% | 4.46 ± 0.08 | 4.05 ± 0.49 | $P_c$ (4457) | 4527 |
| $D^*_c$ | $\frac{3}{2}^-$ | 3.0-3.6 | 5.24 ± 0.10 | 2.8 | 42%-61% | 4.62 ± 0.09 | 2.40 ± 0.37 | resonance | 4527 |

Figure 1  (Color online) Contributions of the vacuum condensates of the dimension $n$, where A, B, C, D, E, F, G, and H denote the pentaquarks $\bar{D}_c$ with $l = \frac{3}{2}$, $D_0c$ with $l = \frac{1}{2}$, $D_0c^*$ with $l = \frac{1}{2}$, $D_0c^*$ with $l = \frac{1}{2}$, $D_0c^*$ with $l = \frac{3}{2}$, $D_0c^*$ with $l = \frac{1}{2}$, $D_0c^*$ with $l = \frac{3}{2}$, and $D_0c^*$ with $l = \frac{3}{2}$, respectively.

10 MeV below the $D_0c^*$ threshold, so we can naturally assign this state as the $P_c$ (4312). For the $D_0c^*$ pentaquark state with the quantum numbers $J^P = \frac{3}{2}^-$, the central value of the mass is 4.33 GeV. This value is approximately 10 MeV above the $D_0c^*$ threshold. Hence, we can assign this one as the $D_0c^*$ resonance state, the isospin cousin of the $P_c$ (4312).

In a similar way, considering the numerical results of the extracted masses, we can assign $P_c$ (4380), $P_c$ (4440), and $P_c$ (4457) as the $\bar{D}_c^*$, $D_0^*c$, and $D_0^*c^*$ pentaquark states, respectively. For the color singlet-singlet-type pentaquark states (resonances) $\bar{D}_c^*$ with $J^P = \frac{1}{2}^-$, $D_0^*c$ with $J^P = \frac{3}{2}^-$, and $D_0^*c^*$ with $J^P = \frac{3}{2}^-$, the central values of the extracted masses are approximately 20, 10, and 90 MeV above the corresponding meson-baryon thresholds, respectively.

If we choose the same input parameters, then the color singlet-singlet-type pentaquark states with the isospin $I = \frac{3}{2}$ have slightly larger masses than the corresponding pentaquarks with the isospin $I = \frac{1}{2}$. In the calculations, the masses and pole residues monotonously increase with the increase in the continuum threshold parameters. Hence, we determine the continuum threshold parameter $s_0$ by adopting the uniform constraints, such as the continuum thresholds $\sqrt{s_0} = M_c = 0.65 \pm 0.1$ GeV, PCs 40%-65%, and intervals $T^2_{max} - T^2_{min} = 0.6$ GeV$^2$, to acquire reliable predictions. $T^2_{max}$ and $T^2_{min}$ stand for the maximum and minimum values of the Borel parameters, respectively.

Clearly, $P_c$ (4312), $P_c$ (4380), $P_c$ (4440), and $P_c$ (4457) can be assigned as the $\bar{D}_c^*$, $\bar{D}_c^*$, $D_0^*c$, and $D_0^*c^*$ pentaquark states with the isospin $I = \frac{1}{2}$, respectively, because the two-body strong decays $P_c \rightarrow J/\psi p$ conserve isospin. If the assignments are robust, then four slightly high pentaquark states, namely, $\bar{D}_c^*$, $\bar{D}_c^*$, $D_0^*c$, and $D_0^*c^*$, exist with the isospin $I = \frac{1}{2}$. We can search for the four resonances in the $J/\psi \Delta$ invariant mass spectrum, as the two-body strong decays $P_c \rightarrow J/\psi \Delta$ also conserve isospin. $J/\psi$, $p$, and $\Delta$ have the isospins $I = 0$, $\frac{1}{2}$ and $\frac{3}{2}$, respectively. If the four resonances are observed one day, then we can obtain additional proofs for the color singlet-singlet-type pentaquark assignments and shed light on the nature of the $P_c$ states and dynamics of the low-energy QCD.

In this work, we construct local color singlet-singlet-type
five-quark currents with definite isospins, which couple potentially to the color singlet-singlet-type hidden-charm pentaquark states rather than to the meson-baryon scattering states or thresholds. The thresholds in Table 1 are taken from the PDG [3], as the traditional charmed mesons and baryons are spatial extended objects and have average spatial sizes $\sqrt{\langle r^2 \rangle} \approx 0.5$ fm and 0.5-0.8 fm, respectively [18, 28]. Therefore, the loosely bound molecular states, meson-baryon scat-
tering states, or thresholds have spatial extensions larger than 1 fm, which is too large to be interpolated by the local currents. In the local limit $r \to 0$, in such small spatial separations, the $\varepsilon g$ meson and $\varepsilon q \bar{q}$ baryon lose themselves and merge into color singlet-singlet-type pentaquark states. The scenario of the color singlet-singlet-type pentaquark states in the QCD sum rules is quite different from other theoretical methods. In the QCD sum rules, there are two color singlet clusters, which have the same quantum numbers as the physical states $D^{(*)}$ and $\Sigma^{(*)}$, respectively, but they are not the physical states. Then, we performed the operator product expansion at the quark-gluon level at the QCD side and distinguished the short-distance and long-distance contributions, and no hadronic degrees of freedoms are needed. In fact, we can abandon the conception of molecular states in the QCD sum rules. Hence, we just investigate the color singlet-singlet-type pentaquark states, which have masses near the meson-baryon thresholds.

Meanwhile, in the one-pion exchange potential model [29] and heavy-quark spin symmetry model [30], there are physical charmed meson and baryons states. In the one-pion exchange potential model, the short-range interaction by the coupling to the 5-quark-core states plays a major role in determining the ordering of the multiplet states, whereas the long-range force of the pion tensor force affects the production of the decay widths [29]. In the heavy-quark spin symmetry model, the pentaquark-like resonances can be naturally accommodated in a contact-range effective field theory description that incorporates the heavy-quark spin symmetry [30].

The $P_c(4380)$ observed in the six-dimensional amplitude analysis is obsolete in the updated analysis [6], which weakens the previously reported evidence for the $P_c(4380)$. However, it does not contradict its existence as the one-dimensional analysis is not sensitive to wide $P_c$ states. Whether or not there exist $P_c(4380)$-like wide pentaquark candidates, a six-dimensional amplitude analysis of the $\Lambda_c^0 \to J/\psi p K^- \bar{K}^0$ decays in the future could answer the question. Our calculations just indicate that a color singlet-singlet-type pentaquark candidate exists with the mass of approximately 4.38 GeV, and it is not necessary to be $P_c(4380)$.

In ref. [31], we assign $Z^0(3900)$ as the diquark-antidiquark-type tetraquark state with the quantum numbers $J^{PC}=1^{--}$ and study the hadronic coupling constants in its two-body strong decays with the QCD sum rules based on the rigorous current-hadron duality. Then, we obtain a satisfactory total width to match the experimental data. We can explore the two-body strong decays of the color singlet-singlet-type pentaquark states based on the rigorous current-hadron duality and determine the branching fractions. This finding can be confronted with the experimental data in the future to assign the color singlet-singlet-type pentaquark states in reasonable foundations.

4 Conclusions

In the present work, we distinguish the isospins of the color singlet-singlet-type pentaquark states and construct the color singlet-singlet-type five-quark currents with the isospins $(I, I_z) = (\frac{1}{2}, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$ unambiguously to explore their properties with the QCD sum rules for the first time. To obtain accurate numerical results, we consider the vacuum condensates up to dimension 13 to be constant. Based on the extracted pentaquark masses from the Borel windows, we assign the $D_{\Sigma^c}$, $D_{\Sigma_c^-}$, $D_{\Sigma_c^0}$, and $D_{\Sigma_c^+}$ pentaquark states with the isospin $I = \frac{1}{2}$ to be $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, and $P_c(4457)$, respectively (see Table 1). Furthermore, the present calculations indicate that there exist four slightly higher pentaquark states $D_{\Sigma_c^-}$, $D_{\Sigma_c^0}$, $D_{\Sigma_c^+}$ with the isospin $I = \frac{1}{2}$, which lie slightly above the thresholds of the corresponding meson-baryon pairs $D_{\Sigma_c^-}$, $D_{\Sigma_c^0}$, $D_{\Sigma_c^+}$ respectively. We can search for the four resonances in the $J/\psi \Delta$ invariant mass spectrum, which can lead to additional proofs for the color singlet-singlet-type pentaquark assignments and shed light on the nature of the $P_c$ states and dynamics of the low-energy QCD.

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