Boundary effects on energy dissipation in a cellular automaton model

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(Dated: April 25, 2009)

In this paper, we numerically study energy dissipation caused by traffic in the Nagel-Schreckenberg (NaSch) model with open boundary conditions (OBC). Numerical results show that there is a non-vanishing energy dissipation rate $E_d$, and no true free-flow phase exists in the deterministic and nondeterministic NaSch models with OBC. In the deterministic case, there is a critical value of the extinction rate $\beta_d$ below which $E_d$ increases with increasing $\beta$, but above which $E_d$ abruptly decreases in the case of the speed limit $v_{\text{max}} \geq 3$. However, when $v_{\text{max}} \leq 2$, no discontinuous change in $E_d$ occurs. In the nondeterministic case, the dissipated energy has two different contributions: one coming from the randomization, and one from the interactions, which is the only reason for dissipating energy in the deterministc case. The relative contributions of the two dissipation mechanisms are presented in the stochastic NaSch model with OBC. Energy dissipation rate $E_d$ is directly related to traffic phase. Theoretical analyses give an agreement with numerical results in three phases (low-density, high-density and maximum current phase) for the case $v_{\text{max}} = 1$.

PACS numbers: 05.65.+b, 45.70.Vn, 05.60.-k, 89.40.Bb

I. INTRODUCTION

In the last decades, traffic problems have attracted much attention of a community of physicists because of the observed nonequilibrium phase transitions and various nonlinear dynamical phenomena. A number of traffic models have been proposed to investigate the dynamical behavior of the traffic flow, including fluid dynamical models, gas-kinetic models, car-following models and cellular automata (CA) models [1, 2, 3]. These dynamical approaches represented complex physical phenomena of traffic flow among which are hysteresis, synchronization, wide moving jams, and phase transitions, etc. Among these models, the cellular automata approaches can be used very efficiently for computers to perform simulation [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. The Nagel-Schreckenberg (NaSch) model is a basic CA models describing one-lane traffic flow [5]. Based on the NaSch model, many CA models have succeeded in modeling a wide variety of properties of vehicular traffic [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

On the other hand, the problems of traffic jams, environmental pollution and energy dissipation caused by traffic have become more and more significant in modern society. Financial damage from traffic due to energy dissipation and environmental pollution is huge every year. In accordance with previously reported results in Refs. [15], more than 20% fuel consumption and air pollution is caused by impeded and "go and stop" traffic. Most recently, the problem of energy dissipation in traffic system has been investigated in the framework of car following model, city traffic model and NaSch model with periodic boundary conditions (PBC), respectively [16, 17, 18, 19, 20]. And analytical expressions for energy dissipation have also been provided in the nondeterministic NaSch model with PBC in the case of $v_{\text{max}} = 1$ and free-flow state [20]. However, the effects of boundary condition on energy dissipation have not been discussed yet.

The most significant difference between systems with open and periodic boundary conditions is the vehicle density $\rho$. In a periodic system, which has no maximum current phase, vehicle density is considered as an adjustable parameter. In systems with open boundary conditions (OBC), however, there are two adjustable parameters, namely the injection rate $\alpha$ and the extinction rate $\beta$, and the vehicle density $\rho$ is only a derived parameter. Compared with periodic systems, it implies that open systems show a different behaviour of quantities such as the global density, the current, the density profile and even the microscopic structure of traffic phase [21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. Therefore, energy dissipation in the CA model with OBC, which is relevant to many realistic situation in traffic, should be further investigated.

In this paper, we investigate the energy dissipation rate within the framework of the deterministic and nondeterministic NaSch model with OBC. The behaviours of the energy dissipation rate in different traffic phase are distinct. Theoretical analyses are presented in low-density, high-density and maximum current phase in the case $v_{\text{max}} = 1$. The influences of the speed limit $v_{\text{max}}$ on the energy dissipation are also investigated. Energy dissipation caused by braking is related not only to the velocity of vehicles, but also to the headway distribution. Thus, the behaviour of the energy dissipation rate is more complex than simpler quantities, and should be further inves-
tigated. The results of this article may lead to a profound understanding of some features of traffic system or may provide schemes for reducing energy dissipation of the existing traffic network.

The paper is organized as follows. Section II is devoted to the description of the model and the definition of energy dissipation rate. In section III, the numerical studies are given, and the influences of the injection and extinction rate on energy dissipation rate are considered. And theoretical analyses are presented in the special case $v_{\text{max}} = 1$. Finally, the conclusions are given in section IV.

II. MODEL AND ENERGY DISSIPATION

Our investigations are based on a one dimensional cellular automaton model introduced by Nagel and Schreckenberg. The model is defined on a single lane road consisting of $L$ cells of equal size numbered by $i = 1, 2, \cdots, L$ and the time is discrete. Each site can be either empty or occupied by a car with the speed $v = 0, 1, 2, \cdots, v_{\text{max}}$, where $v_{\text{max}}$ is the speed limit. Let $x(i, t)$ and $v(i, t)$ denote the position and the velocity of the $i$th car at time $t$, respectively. The number of empty cells in front of the $i$th vehicle is denoted by $d(i, t) = x(i + 1, t) - x(i, t) - 1$. The following four steps for all cars update in parallel with periodic boundary.

1. Acceleration: $v(i, t + 1/3) \rightarrow \min[v(i, t) + 1, v_{\text{max}}]$;
2. Slowing down: $v(i, t + 2/3) \rightarrow \min[v(i, t + 1/3), d(i, t)]$;
3. Stochastic braking: $v(i, t + 1) \rightarrow \max[v(i, t + 2/3) - 1, 0]$ with the probability $p$;
4. Movement: $x(i, t + 1) \rightarrow x(i, t) + v(i, t + 1)$.

Open systems are characterized by the injection rate $\alpha$ and the extinction rate $\beta$, which means by the probability $\alpha$ and $\beta$ that a vehicle moves into and out of the system. In this paper, open boundary conditions are defined according to $28, 29$. At site $i = 0$ which means out of the system, a vehicle with speed $v = v_{\text{max}}$ is created with probability $\alpha$. The vehicle immediately moves forward in accordance with the NaSch rule. If the site $i = 1$ is occupied by a car, the injected vehicle at site $i = 0$ is deleted. At $i = L + 1$ a "block" occurs with probability $1 - \beta$ and causes a slowing down of the vehicles at the end of the system. Otherwise, a vehicle may leave freely from the end of the system.

It should be mentioned that for $v_{\text{max}} = 1$ the model above in the nondeterministic case is different from parallel updated asymmetric exclusion process (ASEP) with open boundary conditions $21, 22, 23$. In the ASEP model, if the site $L$ is occupied then the particle on that site exits with probability $\beta$, irrespective of their velocity. In the model above, however, the extinction of the vehicle at site $L$ depends not only on the probability $\beta$, but also on the braking probability $p$. Even if the vehicle with $v = 1$ is at site $L$ and there is no "block" at site $L + 1$, it may fail to exit because of being randomly delayed. This difference may influence the current and phase transition, which will be analysed in the following section. In the deterministic case, however, the model above with $v_{\text{max}} = 1$ is identical with parallel updated ASEP with OBC, for there is no stochastic delay.

The kinetic energy of the vehicle with the velocity $v = mv^2/2$, where $m$ is the mass of the vehicle. When braking the energy is lost. Let $E_d$ denotes energy dissipation rate per time step per vehicle. For simple, we neglect rolling and air drag dissipation and other dissipation such as the energy needed to keep the motor running while the vehicle is standing in our analysis, i.e., we only consider the energy lost caused by speed-down. The dissipated energy of $i$th vehicle from time $t - 1$ to $t$ is defined by $20$

$$e(i, t) = \begin{cases} \frac{m}{2} [v^2(i, t - 1) - v^2(i, t)] & \text{for } v(i, t) < v(i, t - 1) \\ 0 & \text{for } v(i, t) \geq v(i, t - 1). \end{cases}$$

Thus, the energy dissipation rate

$$E_d = \frac{1}{T N} \sum_{t=t_0+1}^{t_0+T} \sum_{1}^{N} e(i, t),$$

where $N$ is the number of vehicles in the system and $t_0$ is the relaxation time, taken as $t_0 = 10^3$. In this model, the particles are "self-driven" and the kinetic energy increases in the acceleration step. In the stationary state, the value of the increased energy while accelerating is equivalent to that of the dissipated energy caused by speed-down, and the kinetic energy is constant in the system. In the simulation, the system size $L = 1000$ is selected, and the results are obtained by averaging over $20$ initial configurations and $10^4$ time steps after discarding $10^3$ initial transient states.

III. NUMERICAL RESULTS

A. Effects of the boundary conditions on energy dissipation in the deterministic case

First, we investigate the influences of the boundary conditions on energy dissipation in the deterministic NaSch model with the maximum velocity $v_{\text{max}} = 5$. In the deterministic case, the stochastic braking is not considered, i.e., $p = 0$. Figure 1 shows the energy dissipation rate $E_d$ as a function of the extinction rate $\beta$ with different values of the injection rate $\alpha$. As shown in Fig. 1, there is a critical value of the extinction rate $\beta_d$ below which $E_d$ increases with the increase of the rate $\beta$, but above which $E_d$ abruptly decreases. The position of $\beta_d$ shifts towards a high value of $\beta$ with increasing the injection rate $\alpha$. The astonishing result is that there is
a nonvanishing energy dissipation, even though in low-density phase, which means that there is no "true" free-flow phase.

When \( \beta < \beta_{cd} \), the state of the system is high-density phase. As the extinction rate \( \beta \) increases, the kinetic energy possessed by vehicles increases because of the increase of the mean vehicle velocity, and the dissipated energy while braking increases. When \( \beta > \beta_{cd} \), the state of the system is low-density phase in which the distance-headways is larger and the interaction between vehicles is weaker than that in the high-density phase. The interaction is the only reason for dissipating energy in the deterministic case. Consequently, \( E_d \) decreases abruptly when the transition from high-density to low-density phase occurs. Because of boundary effects there are vehicular interactions even at low-density phase. Therefore, there is a nonvanishing energy dissipation rate \( E_d \), and no "true" free-flow phase exists in the deterministic NaSch model with open boundary conditions.

Energy dissipation rate \( E_d \) in the case of \( v_{\text{max}} = 3 \) and 4 show similar behavior to that for \( v_{\text{max}} = 5 \), and the position of \( \beta_{cd} \) shifts towards a low value of \( \beta \) with increasing the speed limit \( v_{\text{max}} \), as shown in Fig. 2. But in the case of \( v_{\text{max}} = 2 \) and 1, there is no critical value of the extinction rate \( \beta_{cd} \), i.e., no discontinuous change in \( E_d \) occurs. According to previously reported results in Refs. [19], in the case of \( v_{\text{max}} < 3 \), the state of a system with injection rate \( \alpha = 1 \) is the jamming phase, while in the case of \( v_{\text{max}} \geq 3 \), the jamming state exists in the region of low value of \( \beta \), and low-density lies in the region of high value of \( \beta \). Thus, there is no discontinuous change in \( E_d \) in the case of \( v_{\text{max}} = 2 \) and 1 for no phase transition occurs. Near \( \beta = 0 \), energy dissipation rate \( E_d \) is independent of the speed limit \( v_{\text{max}} \), and \( E_d \) increases linearly with the increase of \( \beta \). In the case of \( \alpha = 1 \) and \( \beta \rightarrow 0 \), the maximum velocity which vehicles can move is 1, so the speed limit has no influences on energy dissipation.

In the special case of \( v_{\text{max}} = 1 \), energy dissipation rate \( E_d \) is proportional to the mean density of ”go and stop” vehicles per time step. The mean density of ”go and stop” vehicles is defined in the following way:

\[
\rho_{gs} = \frac{1}{N_T} \sum_{i=1}^{t_0+T} \sum_{i=1}^{N} n(i,t) [1 - n(i,t+1)],
\]

(3)

where \( n(i,t) = 0 \) for stopped cars and \( n(i,t) = 1 \) for moving cars at time \( t \), and \( t_0 \) is the relaxation time as mentioned in the section II. And energy dissipation rate \( E_d \) in the case of \( v_{\text{max}} = 1 \) can be written as

\[
E_d = \frac{1}{2} m \rho_{gs}.
\]

(4)

For open boundary conditions, the mean density of ”go and stop” vehicles per time step reads

\[
\rho_{gs} = n_0 - n_0^2 = \beta(1 - \beta),
\]

(5)

where \( n_0 = N_0/N \) is the fraction of the stopped vehicles, and \( N_0 \) is the number of stopped vehicles on the road.
As a consequence, energy dissipation rate $E_d$ in the case of $v_{\text{max}} = 1$ can be obtained as

$$E_d = \frac{m}{2}(\beta - \beta^2). \quad (6)$$

In formula (6), energy dissipation rate $E_d$ is directly related to the probability for a vehicle moving out of the system or the probability for a vehicle occupying the last site of the system. As shown in figure 3, theoretical analysis is in good agreement with numerical results.

For $v_{\text{max}} = 1$ and $\alpha < 1$, a discontinuous change in $E_d$ also occurs at a critical point $\beta_{cd}$, and the position of $\beta_{cd}$ shifts towards a high value of $\beta$ with increasing $\alpha$, which is similar to that in the case of $v_{\text{max}} = 5$.

B. Effects of the boundary conditions on energy dissipation in the nondeterministic case

Next, we investigate the rate of energy dissipation $E_d$ when the stochastic braking behaviours of drivers are considered, i.e., $p \neq 0$. In the nondeterministic case, the dissipated energy has two different contributions: one coming from the stochastic noise and one from the interactions between vehicles (this in fact is the only reason for dissipating energy in the deterministic case). Let $E_{di}$ and $E_{dr}$ denote the rate of energy dissipation caused by the interactions and randomizations, respectively. And energy dissipation rate $E_d = E_{di} + E_{dr}$. Figure 4 and 5 show the relation of the energy dissipation rate $E_{di}$ and $E_{dr}$ to the extinction rate $\beta$ with different values of the injection rate $\alpha$, respectively, in the case of $v_{\text{max}} = 5$ and $p = 0.5$. As shown in figure 4, there is a critical value of the extinction rate $\beta_c$ below which $E_{di}$ increases with increasing $\beta$, but above which $E_{di}$ abruptly decreases, except for the case of $\alpha = 1$. Different from $E_{di}$, energy dissipation rate $E_{dr}$ sharp increases at the critical point $\beta_c$, above which $E_{dr}$ shows the approximate plateau and is independent of $\beta$, except for $\alpha = 1$, as shown in figure 5. And the position of $\beta_c$ shifts towards a high value of $\beta$ with the increase of the injection rate $\alpha$. For $\alpha = 1$, there is no discontinuous change in $E_{di}$ and $E_{dr}$. Compared figure 4 with 5, it turns out that energy dissipation is mainly caused by the randomization in the low-density phase, and by the interactions in the high-density phase. When $\alpha > 0.35$, the values of the rate of energy dissipation $E_{di}$ and $E_{dr}$ for various value of $\alpha$ collapse into a single curve (not shown).

In fact, in the case of $p = 0.5$ and $v_{\text{max}} = 5$, there are the transitions from the high-density to low-density phase for $\alpha \leq 0.35$ and from high-density to maximum current phase for $\alpha > 0.35$. In the high-density phase, with the increase of the extinction rate $\beta$, the mean velocity increases and the dissipated energy increases. In the low-density and maximum current phase, with increasing the rate $\beta$, the distance headways increases and the interactions lowers; thus energy dissipation rate $E_{di}$ decreases. Energy dissipation rate $E_{dr}$ reaches a constant value in the low-density phase, but continuatively increases with the increase of $\beta$ in the maximum current phase.

It should be noted that energy dissipation rate $E_d$ for $\alpha = 1$ is minimum, i.e., energy dissipation in the maximum current phase is lower than that in the low-density phase (not shown). Traffic flow, however, in the maximum current phase is maximal.

The relationship of $E_d$ to the extinction rate $\beta$ for different values of $v_{\text{max}}$ in the case of $\alpha = 1$ is shown in figure 6. Similar to the deterministic case, when the value of $\beta$ is very small, the rate of energy dissipation shows a scaling relation and is independent of the speed limit, as shown in figure 6. In the region of high values of $\beta$, the scaling relations of energy dissipation $E_d$ to the speed limit cannot be observed; and $E_d$ increases with the increase of $v_{\text{max}}$. When $v_{\text{max}} > 3$, energy dissipation rate $E_d$ increases with increasing the rate $\beta$ in the region of high values of $\beta$. However, when $v_{\text{max}} \leq 3$, the value of $E_d$ tends to be invariable. Though the states of the system for different speed limit are maximum current phase in the region of high values of $\beta$, with the increase of $\beta$, the value of $E_d$ continues to increase.
the mean velocity increases for $v_{\text{max}} > 3$ but does not vary in the case of $v_{\text{max}} \leq 3$. Consequently, there is a plateau for the case $v_{\text{max}} \leq 3$ in the region of high values of $\beta$, which is different from that in the case of $v_{\text{max}} > 3$.

In the system with $v_{\text{max}} = 1$, some vehicles can be stopped due to the stochastic braking, therefore the rate of energy dissipation $E_d$ is proportional to the mean "go and stop" density $\rho_{gs}$, which demonstrates that the probability for "go and stop" vehicle to appear per time step in the system.

In the low-density phase A ($\alpha < \beta$, $\alpha < \alpha_c = 1 - \sqrt{p}$), the fraction of the stopped vehicles reads

$$n_0 = 1 - q - \alpha \over 1 - \alpha.$$  \hfill (7)

where $q = 1 - p$. And the mean "go and stop" density $\rho_{gs}$ can be obtain as

$$\rho_{gs} = n_0 - n_0^2 = q - \alpha \over 1 - \alpha \cdot \left(1 - q - \alpha \over 1 - \alpha \right).$$ \hfill (8)

Substituting formula (8) into (4), we can obtain energy dissipation rate $E_d$ in the low-density phase A

$$E_d^A = m q - \alpha(1 - q) \over 2(1 - \alpha^2).$$ \hfill (9)

Figure 7 shows the relation between the rate of energy dissipation $E_d$ and the injection rate $\alpha$ with various values of $p$, in the case of $\beta = 1$. As shown in Fig. 7 in the region of low values of $\alpha$, formula (9) gives good agreement with the simulation data.

From $\text{d}E_d^A \over \text{d} \alpha = 0$, we obtain the critical value $\alpha_{mc} = 1 - 2p$ in which the curve described by formula (9) reaches the maximum. Compared with the critical rate $\alpha_c = 1 - \sqrt{p}$, the critical value of the stochastic braking probability $p_c^L = \frac{1}{4}$ can be obtained. When $p < p_c^L$, energy dissipation rate $E_d^A$ increases with increasing the injection rate $\alpha$. When $p \geq 0.5$, however, with the increase of the rate $\alpha$, energy dissipation rate $E_d^A$ decreases. In the interval $0.5 > p \geq p_c^L$, with increasing $\alpha$, the rate of energy dissipation $E_d^A$ increases first and decreases after a maximum value is reached.

In the high-density phase B ($\beta < \alpha$, $\beta < \beta_c = \frac{1}{1 + \sqrt{p}}$), the model of this paper is different from the ASEP with parallel update and the mean velocity is determined not only by the extinction rate $\beta$, but also the stochastic braking probability $p$. The fraction of the stopped vehicles reads

$$n_0 = 1 - q\beta.$$ \hfill (10)

And the mean "go and stop" density $\rho_{gs}$ can be written as

$$\rho_{gs} = n_0 - n_0^2 = q\beta(1 - q\beta).$$ \hfill (11)
Substituting formula (11) into (4), we can obtain the rate of energy dissipation $E_d$ in the high-density phase B

$$E_d^B = \frac{m}{2}(q\beta - q^2\beta^2). \quad (12)$$

Figure 8 shows the relation the energy dissipation rate $E_d$ as a function of the extinction rate $\beta$ with different values of the stochastic braking probability $p$, in the case of $\alpha = 1$. As shown in Fig. 8, the agreement can be obtained in the case of low value of the extinction rate $\beta$.

From $\frac{dE_d^B}{d\beta} = 0$, we can obtain the critical value $\beta_{cr} = \frac{1}{2(1-p)}$ in which the curve corresponding to formula (12) reaches the maximum. Compared with the critical rate $\beta_c = \frac{1}{1+\sqrt{p}}$, the critical value of the stochastic braking probability $p^h_c = \frac{1}{\sqrt{p}}$ can be obtained. When $p \geq p^h_c$, energy dissipation rate $E_d^B$ increases with the increase of the extinction rate $\beta$. When $p < p^h_c$, however, with increasing the rate $\beta$, energy dissipation rate $E_d^B$ increases first and decreases after a maximum value is reached.

In the maximum current phase $C (\alpha > \alpha_c, \beta > \beta_c)$, the fraction of the stopped vehicles reads

$$n_0 = \sqrt{p}. \quad (13)$$

And the mean ”go and stop” density $\rho_{gs}$ can be obtain as

$$\rho_{gs} = n_0 - n_0^2 = \sqrt{p}(1 - \sqrt{p}). \quad (14)$$

Substituting formula (14) into (4), we can obtain energy dissipation rate $E_d$ in the maximum current phase $C$

$$E_d^C = \frac{m}{2}(\sqrt{p} - p). \quad (15)$$

Equation (15) demonstrates that energy dissipation in the maximum current phase is independent of the rate $\alpha$ and $\beta$, and is only determined by the stochastic braking probability. Figure 7 and 8 give a comparison between number results and Eq. 15. As shown in the right region of figure 7 and 8, formula (15) gives good agreement with the simulation data.

IV. SUMMARY

In this paper, we investigate the rate of energy dissipation caused by braking in the NaSch model with open boundary conditions. Different from periodic systems, open systems in which the vehicle density is only a derived parameter are controlled by the injection and extinction rate. In fact, real traffic systems are usually open, hence it is highly desirable to investigate energy dissipation in traffic systems both numerically and theoretically.

Numerical results show that in the deterministic case there is a critical value of the extinction rate $\beta_{cr}$ above which $E_d$ decreases abruptly for $v_{max} \geq 3$, however, no discontinuous change in $E_d$ occurs when $v_{max} < 3$. The rate $\beta_{cr}$ is related not only to the injection rate $\alpha$, but also to the maximum velocity of vehicles. In the non-deterministic case, there is also a critical value of the extinction rate $\beta_{cr}$ below which $E_d$ and $E_{dr}$ increase with increasing $\beta$, above which $E_{di}$ abruptly decreases but $E_{dr}$ sharp increases and shows the approximate plateau with further increase of $\beta$, when the transition from the high-density to low-density phase occurs. However, when the transition from the high-density to the maximum current phase occurs, the values of energy dissipation rate $E_{di}$ and $E_{dr}$ for various value of $\alpha$ collapse into a single curve, and no discontinuous change occurs. Moreover, there is a nonvanishing energy dissipation rate, and no ”true” free-flow phase exists in the deterministic and non-deterministic NaSch models with open boundary conditions.

Energy dissipation rate $E_d$ is directly related to traffic phase. Energy dissipation in maximum current phase is smaller than that in the low-density phase. A phenomenological mean-field theory is presented to describe the energy dissipation rate $E_d$ in three phases (low-density, high-density and maximum current phase) in the case of $v_{max} = 1$. Theoretical analyses give an excellent agreement with numerical results. But in the case of $v_{max} > 1$, explicit expressions about the energy dissipation rate $E_d$ do not be obtained because of effects of long length of time space correlations, and deserve further investigate.

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