Estimate of the steel bridges fatigue life by application of the fracture mechanics

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Abstract. The bridge elements are subjected to the low-amplitude cyclic loading, causing the fatigue as the main problem that limits the carrying capacity and remaining working life of a structure. Identifying the parts prone to fatigue, inspection control, preventive maintenance, successful reinforcing and repair plans, secure the satisfying characteristics of a bridge during its working life.

For a new structure is important that elements subjected to fatigue are so designed to avoid the danger of the fatigue fracture. The fracture mechanics represents a useful tool for analyzing the fatigue crack growth in steel elements. The relevant parameters for the remaining working life estimate are the load level and frequency and the stress concentration.

A method, based on the Paris' law, for predicting the remaining fatigue working life of steel bridges, is proposed and verified on an actual bridge. The considered bridge was found to be safe against the fatigue fracture, both for the present and the future increased load, since it was verified that it has the reserve with respect to fatigue life. Based on obtained results, one can conclude that the fatigue working life of a bridge and its elements can be estimated based on application of the LEFM (linear elastic fracture mechanics) concept.

1. Introduction

Bridges are the very important part of the modern society infrastructure. In Europe, as well as in the rest of the developed world, the fast development of industry in the first half of the XIX century has caused construction of numerous roads and railways and bridges on them. Many of bridges constructed at those times are still in operation, despite the fact that their technical life has expired. Based on analysis of the age profile of the steel railway bridges in [1], it was established that about 70 % of those bridges are more than 50 years old, while about 30 % are more than 100 years old. Besides that, those bridges are now exposed to much higher requests related to traffic intensity and axle loading than in the times of their construction.

The main problem in steel bridges is fatigue, which limits the carrying capacity and the remaining working life of the existing structure. Recognizing the bridge parts prone to fatigue, inspection controls,
preventive maintenance, as well as successful reinforcing and repair plan, provide for the good characteristics of bridges during their working life. It is also very important for the new structures, that elements of structures, which have tendency to fatigue, to be properly designed to avoid the danger of appearance of fatigue cracks.

Haghani et al. [2] have presented more than 100 cases of damages, which were analyzed and grouped according to the mechanisms of appearance or the type of the structural element, where the fatigue crack was registered. In about 90% of cases the cause of damage was the crack due to deformation. This type of the fatigue damage is the most frequently the consequence of accidental and unpredictable restraints that appear due to mutual interactions of the various bridge elements. In addition, the weaker, not-reinforced elements of the structure, as well as the decrease in stiffness at points of individual bridge elements connections/extensions, also contribute to appearance of fatigue cracks. The calculation methods and rules of design usually do not provide sufficient data how to check and prevent this type of fatigue.

During their working life, the road and railway traffic produces large number of repeated load cycles to the bridge elements, which are, due to that, becoming sensitive to fatigue. The fatigue fracture appears as a consequence of a component's long-term exposure to the cyclic loading. There are three phases of the fatigue fracture: crack initiation, crack growth and fracture. The crack can appear due to surface defects of a component, caused by manual or machine processing, threading or due to appearance of the slip bands or dislocations, which are the result of the previous load cycles. In the crack growth phase, it continues to propagate due to loads that the component is subjected to. The fracture (failure) phase appears abruptly and it occurs when the component's undamaged portion without a crack cannot sustain the applied loads anymore. In Figure 1 is presented the schematic appearance of the cross-section of a component that has suffered the fatigue fracture, [3].

On the fatigue fracture surface in Figure 1, one can notice the initial surface defect, then mainly smooth surface, which is formed during the fatigue crack growth and the rough surface, which corresponds to the final fracture. Based on the surface appearance in the fatigue fracture, it is possible to discover the initial crack, since the lines in the fatigue fracture zone are bending curving backwards towards the fracture initiation point.

![Figure 1. Appearance of the fatigue fracture surface.](image)

2. **Application of the fracture mechanics to estimate the fatigue working life**

The fatigue working life of a bridge structural element can be predicted by application of the fracture mechanics, based on the fact that the stress field at the crack tip can be described by a single parameter, the stress intensity factor, \( K \), or in the case of the cyclic loading, the change of the stress intensity factor \( \Delta K \), i.e. the difference between values of the stress intensity factor at maximum and minimum load. The latter parameter is defined as:

\[
\Delta K = \Delta \sigma \cdot Y \cdot \sqrt{\pi a}
\]

where \( \Delta \sigma \) – is the range of the applied load, \( a \) – is the crack length and \( Y \) – is the dimensionless parameter, which depends on the sample geometry and applied load.
Change of the stress intensity factor $\Delta K$, based on the Paris' law, determines the number of cycles of the applied load until the appearance of fracture. The Paris' law is, for the case of the stable crack growth, defined by the following expression, [4]:

$$\frac{da}{dN} = C(\Delta K)^m,$$  \hspace{1cm} (2)

where: $da/dN$ – is the crack growth rate, $da$ – is the crack length change that is within the range from the initial length to the critical length, which causes the fracture, $N$ – is the number of the load cycles, $C$ and $m$ – are the material constants. The remaining working life is obtained by integration of equation (2):

$$N = \int_{a_i}^{a_c} \frac{da}{C(\Delta K)^m},$$  \hspace{1cm} (3)

where: $a_i$ – is the initial crack length and $a_c$ is the critical crack length.

The unstable crack growth appears when the stress intensity factor $K_I$ becomes greater than the experimentally determined material property – the fracture toughness $K_{IC}$, namely when the crack reaches the critical length $a = a_c$. Based on equation (1), the critical crack length can be determined as:

$$a_c = \frac{1}{\pi} \left( \frac{K_{IC}}{\sigma_c \cdot Y_c} \right),$$  \hspace{1cm} (4)

where: $Y_c$ – is the geometrical factor for $a_c$ and $\sigma_c$ – is the highest load level. The fracture toughness for the structural steels does not vary strongly during the fatigue working life and can be adopted as about $K_{IC} = 3000$ Nmm$^{-3/2}$, [5]. The geometrical factor for the plate with the edge crack, subjected to the bending moment is, [6]:

$$Y = \sqrt{\frac{2 \cdot w}{\pi \cdot a}} \left( \frac{\pi \cdot a}{2 \cdot w} \right) \cos \left( \frac{\pi \cdot a}{2 \cdot w} \right) \left( 0.923 + 0.199 \cdot \left( 1 - \sin \left( \frac{\pi \cdot a}{2 \cdot w} \right) \right)^3 \right),$$  \hspace{1cm} (5)

where $w$ – is the connecting plate thickness.

The stress intensity factor $K_I$ governs the crack growth rate and the size of the plastic zone around the crack tip. If the plastic zone size or the applied load are of the order of magnitude of the crack length or the yield stress, respectively, the assumptions of the linear elastic fracture mechanics (LEFM) would not be valid, due to the extreme plasticity at the crack tip. However, the plastic zone size is in the cyclic loading usually smaller than in monotonous loading, so the application of the LEFM concept for analysis of the fatigue crack growth is justified, [7].

The fatigue working life up to fracture, of a structural element, can be divided into three phases, as shown in Figure 2, [8]. As can be seen from Figure 2, the dependence $da/dN$ on $\Delta K$ in the log-log coordinate system is presented by the S-shaped curve (the sigmoidal curve). The curve is asymptotically approaching the limits of the crack growth $\Delta K_{th}$ and $\Delta K_C$. The stress intensity factor range threshold value $\Delta K_{th}$ is the limit value below which the fatigue crack does not grow. At the $\Delta K_C$ value the fracture occurs.

The S curve spreads over three regions. In the first region (I), the dominant influence on the crack growth rate is exhibited by the microstructure, average stress and environment. The plastic zone size at the crack tip is of the order of the grain size and the deformation is crystallographic. In the second region (II), the plastic zone size is larger than the grain size. Influences of the microstructure and the sample thickness are small in this phase, while the influences of stress, environment and cyclic loading frequency are strong. This is the phase of the stable crack growth and equation (2) of the fatigue crack growth is valid.
In the third region (III) – close to the phase of the unstable crack growth, the strong influence is exhibited by the microstructure, average stress and the sample thickness, while the influence of environment is small.

![Figure 2. Phases of the fatigue crack growth, [8].](image)

Determination of the stress intensity range threshold, needed for the beginning of the fatigue crack growth $\Delta K_{th}$ is a time-consuming and expensive process, since it is considered that the threshold is reached when the crack at sufficient number of cycles still does not grow. The $\Delta K_{th}$ values are usually smaller than $10 \text{ MPa} \cdot \text{m}^{1/2}$, [9].

3. An example of the method application and discussion of results

The method defined in the previous section was checked on an example of a bridge on the railroad between Žilina and Košice in Slovakia, Figure 3. The bridge carries one railroad track and both passenger and freight trains are passing over it.

The bridge was commissioned in 1941 and it consists of three spans: the two side spans of 29.4 m by the left and the right bank and the middle span of 57.4 m, Figure 4. The bridge is built of steel of the tensile strength $\sigma_M = 360 \text{ N/mm}^2$, with the elasticity modulus of $E = 210000 \text{ N/mm}^2$ and the yield stress of $\sigma_T = 235 \text{ N/mm}^2$.

The trains with axial load of 250 kN represent the worst possible case - trains of type 11, Table D.2: – Heavy traffic mix with 25 t (250 kN) axles), [10]. It is assumed that 5824 trains cross the bridge annually (16 trains per day $\times$ 7 working days per week $\times$ 52 weeks a year). The critical cross-section for analysis is the cross-section 3’ (Figure 4 (b)). The upper flange is made of the 13 mm thick plate. It is connected by the rivets to the wall of dimensions $2800 \times 15 \text{ mm}$ and the pair of angles of dimensions $L160 \times 160 \times 15 \text{ mm}$. The lower flange is made of angles only. The parameters used for analysis of the fatigue crack growth are $C = 9.14 \times 10^{-12}$, $m = 3$, $K_{th} = 8 \text{ MPa} \cdot \text{m}^{1/2}$ and $K_{Ic} = 120 \text{ MPa} \cdot \text{m}^{1/2}$, [11].
Figure 3. The bridge on the Žilina - Košice railway in Slovakia over river Váh in Strečno.

Figure 4. Geometry of the bridge [mm]: (a) Elevation and (b) Cross section.
The diagram of the crack growth rate $da/dN$ [m/cycle] dependence on the relative crack length, $a/W$, presented in Figure 5, is obtained by application of the programming routine Mathematica®.

![Graph showing crack growth rate versus relative crack length](image)

**Figure 5.** Crack growth rate in terms of the relative crack length.

From diagram in Figure 5 can be seen that the crack growth rate increases with the relative crack length increase up to 50 % and then it decreases, what corresponds to the beginning of fracture. It can also be noticed that the fatigue crack starts to grow when the initial crack reaches 7 % of the plate thickness.

In Figure 6 is shown the number of cycles needed for the crack growth in terms of the relative crack length. The diagram is obtained according to equation (3) and by application of the programming package Mathematica®.

![Graph showing number of cycles versus relative crack length](image)

**Figure 6.** Number of load cycles needed for the fatigue crack growth vs. the relative crack length.
From Figure 6 can be seen again that the fatigue crack starts to grow when the initial crack length reaches 7% of the plate thickness. In addition, one can read-off the value of the fatigue life of the plate, from the moment of the fatigue crack appearance. That value for the considered bridge is about 3 million cycles. For the assumed traffic level that amounts to 515 years.

4. Conclusion
The paper presented a method for calculation of the remaining working life of bridge components, based on application of the linear elastic fracture mechanics. The conditions for the LEFM application are that the cyclic plastic zone around the fatigue crack is small and that the crack would propagate only if the difference between values of the stress intensity factor at the maximum and the minimum load is greater than the stress intensity factor value needed for the fatigue crack growth. The number of load cycles that the component can still sustain was calculated based on the Paris' law.

The method was applied for calculating the remaining working life of a component of a real bridge on the railway between Žilina and Košice in Slovakia. It was shown that the component of a bridge, commissioned in 1941, with the present cyclic load due to traffic, can still be operational for over 3 million cycles, i.e. 515 years. This, of course, is assumed if the adequate maintenance, repair plans and operations of the bridge structure were regularly executed.

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