Tube bundle buckling analysis with support effects on tubesheet in heat exchangers

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Abstract. Tubesheet strength calculation is important work in design of heat exchangers. The thickness of the tubesheet depends on the two major factors: loads and structure. The former includes fluid pressure and metal thermal gradient. The later involves tubes bundle located at inner perforated area of tubesheet and constraints applied by flanges or shells at the outer ring of tubesheet. As an elastic foundation, tubes bundle is one of critical factors in calculation of tubesheet system. The effect and current problems of tubes bundle support is reviewed in this paper. Analysis and experiments show that current single tube buckling criterion used in the strength design of the tubesheet may lead to an inaccurate calculation of the tubesheet thickness since this criterion is conservative for a long tube or aggressive for a short tube. A more reasonable and safe criterion for considering the tube bundle support is proposed in this paper which could make the tubesheet thinner and thus, decrease the cost of the heat exchangers. In addition, the idea proposed in this paper may be helpful to develop a new standard for the strength design of the tubesheet or even the tubesheet system.

1. Introduction
Heat exchangers (HTX) are widely used in chemical or petroleum plants. Optimization of HTX plays an import role in economic investment because the HTXs takes about 30% of the whole weight in a plant. Tubesheet of HTX is the most important part in HTX though its calculation is a complex procedure. The common practice is design by analytical formula that are developed from empirical formula as TEMA [1] codes to complicated shell plate theory method as ASME [2], GB/T151 [3] and CODAP [4]. There are three typical mathematical calculation models corresponding to three different structures, namely fixed tubesheet HTX, floating heat HTX and U-tube type HTX. The first two are more complex than the last one. In the U-tube type HTX, the tube bundles support will not be considered and thus, will not be discussed in this paper.

Tube bundle support is evaluated by rules. If the tube is in tensile, the tube stress should be less than the allowable stress of the material and if the tube is in compression, the tube stress should be less than the critical buckling stress of the tube or allowable stress of the material. In engineering, floating head HTX sometimes fail in buckling of tube bundle, so it is necessary to perform tube bundle buckling analysis with the support effects on the tubesheet.

Figure 1 shows a basic mathematical calculation model of tubesheet for floating head HTX. There are about 100 relevant parameters in the calculation based on shell plate theory method in current
standards. To facilitate the design, some complex calculation procedures are simplified or presented in curves in standards or codes such as ASME and GB/T 151.

From previous research [5,6,7], there are two key parameters that affect the deformation of tubesheet. One parameter is K for bundle support and another is m for constraints along the tubesheet. In general design, the engineers adjust the constraints that include flanges, shell of tube side, shell of shell side, expansion joint etc.

In the model of Figure 1, the tube bundle supports the tubesheet like a spring and is called elastic foundation. The tube must keep validation under the pressure and reaction from the tubesheet or the elastic foundation will collapse and the elastic hypothesis is invalid.

![Figure 1](image)

**Figure 1.** Typical calculation model of floating head HTX.

In shell plate theory, the tube with maximum force can be calculated. The classic Euler formula for single beam under end compressive forces is used to evaluate buckling failure. If a single tube buckles, the whole bundle is considered to be collapsed. The single tube failure criterion in standards will be introduced in next section.

2. **Current single tube bulking failure criterion in standards**
Tubes are the foundation of tubesheets. Buckling mode and critical load of tubes in HTX are the import work in calculation of tubesheet system. Some new phenomenon different from the classic hypothesis of HTX is found and discussed below.

2.1. **Basic theory**
As an elastic support, tubes in HTX must be kept in the state of stability, which is the calculation hypothesis of the tubesheet. In standards, the buckling criterion for the tubes is based on the theory of Euler formula.

Factors that cause deviation from the pure Euler tube behavior include imperfections in geometry of the tubes and the plasticity/non-linear stress strain behavior of the tube’s material. Researcher Koiter[5] have proved that the if the defect reaches the 1/5 thickness of shell, the critical buckling load will decrease to 50%.

Consequently, a number of empirical formula have been developed that depend on test data, all of which embody the slenderness ratio.

2.1.1. **Euler formula.** The basic theory of the tube buckling was investigated in 1757 by mathematician Leonhard Euler. It is called Euler formula that gives the maximum axial load that a long, slender, ideal column can carry without buckling [8]. The formula is shown in formula (1).
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\[ F = \frac{\pi^2 EI}{(KL)^2} \]  

(1)

Where, \( F \) = critical force for buckling. \( E \) = modulus of elasticity of the tube. \( I \) = area moment of inertia of the tube. \( L \) = unsupported length of the tube. \( K \) = tube effective length factor, whose value depends on the conditions of end support of the tube. For both ends hinged, (free to rotate, tube length between two baffles), \( K = 1.0 \). For both ends fixed (tube length between two tubesheets without baffles), \( K = 0.50 \). For one end fixed and the other end hinged, \( K = 0.707 \). For one end fixed and the other end free to move laterally, (no application for HTX), \( K = 2.0 \).

\( KL \) can be considered as the effective length of the tube.

2.1.2 Thin shell buckling load formula. Euler formula gives the buckling mode of the long tubes. However, FEA analysis shows that there is a different buckling when the tube is short as shown in Figure 2. This buckling mode is of a characteristic of shell instability.

The buckling stress formula [9] for thin shell is

\[ P_{cr}^s = \frac{k\pi^2E}{12(1-v^2)} \left( \frac{h}{L} \right)^2 \]  

(2)

where, \( P_{cr}^s \) =Critical buckling stress.

\( h \) =Thickness of shell.

\( a \) =Inner radius of shell.

\( k \) =Parameter concerns to \( L^2/ah \).

2.1.3 Comparison between Euler formula, thin shell formula and FEA analysis with experiments. To compare the critical forces obtained by three methods with experiments, let’s examine tubes under end compressive forces. The outer diameter of tubes is 12mm and material is S30408 with the elastic module of 124.5x10^3 MPa.

| Tube length (mm) | Euler formula (KN) | Shell Formula (KN) | FEA Linear (KN) | FEA Nonlinear^1 (KN) | Compression experiment (KN) |
|-----------------|---------------------|-------------------|----------------|---------------------|-----------------------------|
| 50              | 218                 | 187.1             | 164 (60^2)     | 13.5^1              | 7.1(16.447)                 |
| 100             | 54.5                | 46.76             | 42.5           | 9.4^2               | 6.6(9.038)                  |
| 200             | 13.6                | 11.69             | 13.38          | 12.5^1              | 6.576                       |
| 300             | 6.06                | 5.196             | 6.0            | 5.8^1               | 6.217                       |
| 400             | 3.41                | 2.9               | 3.38           | 3.2^1               | 5.275                       |
| 500             | 2.18                | 1.87              | 2.17           | 2.1^1               | 4.393                       |

^1 \( P_e \) as in Figure 3 by Arc-Length-Type Method; ^2 Shell buckling mode like Figure 4 (a).
The finite element analysis and experiment results are listed in Table 1. The compression experiment curves of tubes with different length are shown in Figure 5.

The data in Table 1 indicate that the Euler formula gives the trend forces of elastic buckling failure but it does not fully agree with compression experiment results. When the tube is short, the critical buckling load is higher than experimental data. When the tube is longer, the critical buckling load is lower than experimental result.

Figure 5. Experimental curves of tubes with compress forces

Nonlinear FEA on Arc-Length-Type Method in column 5 of Table 1 is widely used to evaluate the buckling load. The value is lower than that in column 6 of the Table 1. It means that the Nonlinear FEA yields a safe result to evaluate the tube buckling when the tube is longer than 200mm.

When the tube is short, both Euler Formula and Nonlinear FEA are aggressive compared with experiments. Though the collapse failure depends on the yield limit of material that is lower than buckling critical limit generally, more attention should be paid to this change. Figure 4 (c) shows a radial bulking failure in engineering plant.

2.2. Use of Euler Formula in engineering standards

The Euler formula is adopted in major HTX standards such as USA code ASME VIII and Chinese code GB/T 151 as listed in Table 2 for the relevant contents where the nomenclatures are the same as in the corresponding codes.
Table 2. Applications of Euler formula in heat exchangers standards.

|                  | ASME VIII-2 | ASME III | Chinese Code GB/T 151 |
|------------------|-------------|----------|-----------------------|
| $S_n$            | $\min \left\{ \left[ \frac{\pi^2 E}{F_s F_c} \right], S \right\}$ | $C_i \leq F_i$ | For $0 \leq \lambda \leq 1$ |
|                  | $S_n = \min \left\{ \left[ \frac{S_{1,1} F_c \left( 1 - \frac{F_c}{2C} \right)}{F_c} \right], S \right\}$ | $C_i > F_i$ | $P = \frac{1.11 + 0.50 \lambda + 0.17 \lambda^2 - 0.28 \lambda^3}{1 - \lambda^2 / 4}$ |
| When $Pe \neq 0$ | $F_s = \min \left\{ \max \left\{ C1.25, 2.0 \right\} \right\}$ | $[\sigma_{cr}] = \frac{\pi^2 E}{n(I_{cr}/i)^3} = \frac{\pi^2 E}{1.5(I_{cr}/i)^3}$ |
| $C = 3.25 - 0.25(3Zd + QZw)Xa^4$ | For $\lambda > \sqrt{2}$ | |
| When $Pe = 0$, $F_s = 1.25$ | $P = \frac{2}{3} (1 - \lambda^2 / 4)$ | $P = \frac{2}{3 \lambda^2}$ |

The main differences in the above codes is safety factors. The safety factor is $F_i = 1.25$–2, 1.5 and 1.5 respectively in ASME VIII-2, GB/T 151 and ASME III. Figure 6 shows the critical loads changing with compressive end forces for different calculation approaches.

![Critical stress changing with compressive end forces.](image)

From Figure 6 it is found that the critical load in ASME III is higher than that in other codes. When the tube is short, the buckling mode is something like Figure 2 or Figure 4 (a). In this case, buckling evaluation based on Euler formula is not accurate enough and need to be adjusted.

2.3. summary of this sections

From Table 1 and Figure 5, we can find these phenomena:

(a) Tubes under end compressive forces for different lengths have mixed buckling modes.
(b) Many analytic solutions such as Euler formula can’t cover all buckling modes especially for short tubes.
(c) Figure 5 shows that when the tubes under end compressive forces reached the first bifurcation point of instability, the tubes have still potentiality to afford the forces but the elastic module decreased. In another word, the elastic foundation of tubesheet become different from that in original elastic tubes phase.
(d) Standards have given safety factor 1.25–2. Sometimes the factor is conservative and sometimes the factor is aggressive. The factor depends on slender of tubes as per Table1. The designer should pay more attention to this point.

Different safety factor in standards will affect the tubesheet thickness based on the shell plate theory and our study which will be presented in next section.
3. Effects of buckling criteria on thickness of the tubesheet

Reasonable tube buckling evaluation and proper safety factor are both important to thickness calculation for the tubesheet. Based on analytical method, the tubesheet deformation or stress is relevant to key parameters K and m. K is a measurement of tube bundle foundation, m is measurement of constraint of outer edge of tubesheet [10].

As an example, a floating head HTX is considered the parameters of which are listed in table 3 and the shell side pressure Ps of 4.0 MPa is applied as shown in Figure 7. One is the average stress from all tubes in a bundle, which is balanced by the pressure Ps. Another stress is maximum stress among all tubes from center to edge of the tubesheet, which much depends on the thickness of the tubesheet. Figure 7 shows the compressive stress ratio which is defined as ratio of the second stress divided by the first stress. In Figure 7, the x-axial is the thickness of the tubesheet and the y-axial is the tube compressive stress ratio. It shows that the tube support affects the thickness of tubesheet. The larger, the critical load of stability of the tube bundle, the less the thickness of the tubesheet is.

| Tubesheet Items          | Parameters | Tube Items          | Parameters |
|--------------------------|------------|---------------------|------------|
| Thickness, mm            | 54         | Tube length, mm     | 720        |
| Material                 | S30408     | Tube OD, mm         | 12         |
| Outer diameter, mm       | 316.3      | Tube thickness, mm  | 0.8        |
| modulus of elasticity, MPa| 1.95E5     | Tube number         | 223        |
|                         |            | modulus of elasticity, MPa | 1.245E5 |

For accurate design of the tubesheet, reasonable buckling load criteria and proper safety factors should be given in standards. In fact, all the buckling criteria mentioned above are based on a single tube. The tubes in bundle are interrelated and interact on each other through the tubesheet. To illustrate their difference, finite element analysis is carried on to compute the single tube buckling criterion and bundle buckling criterion.

![Figure 7](image7.png)

**Figure 7** Relation between tube compressive load and thickness of tubesheet.

![Figure 8](image8.png)

**Figure 8.** Tube bundle buckling in FEA.
Based on shell plate theory, the maximum tube stress is 144.531 MPa, the minimum thickness of tubesheet is 54 mm under 6.4 MPa to guarantee that the tube with maximum compressive stress is not buckling. But from FEA, as shown in Figure 8. The maximum axial stress before buckling collapse is 319 MPa. The maximum tube compressive critical stress corresponding to the whole bundle critical load is 319 MPa, much larger than 144.531 MPa corresponding to the single tube critical load. The stress ratio is 319/144.531 = 2.2. In fact, if one tube is compressed approaching to its critical buckling load, the surrounding tubes can still undertake compressive load because the tube bundle is connected by the tubesheet. As a result, the whole bundle critical buckling load is larger than that for a single tube buckling.

So it is too conservative to evaluate the tube buckling as per single critical tube stress criterion in current standards. Using the bundle buckling criterion other than current single tube buckling criterion is more reasonable and could decrease the thickness of the tubesheet according to Figure 7.

4. Conclusions
(a) Current buckling criterion for heat transfer tubes in heat exchanger based on Euler formula can’t cover all buckling modes, especially for short tubes. In standards, the bulking criterion is based on the buckling failure of a single tube, which is conservative for a long tube but aggressive for a short tube.
(b) Buckling failure criterion affects the thickness of tubesheets. The larger the critical buckling load of tubes, the less thickness of the tubesheet is.
(c) Finite element analysis for tube bundle buckling shows that the whole tube bundle presents larger critical buckling load than that of a single tube. Bundle buckling criterion is more reasonable to represent the tube elastic foundation than current single tube buckling criterion.

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