Search for Periodicities in High Energy AGNs with a Time Domain Approach

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Abstract

This paper investigates a new methodology to search for periods in light curves of high-energy gamma-ray sources such as active galactic nuclei (AGNs). High-energy light curves have significant stochastic components, making period detection somewhat challenging. In our model, periodic terms, drifts of the light curves, and random walk with a correlation between flux points due to colored noise are taken into account independently. The parameters of the model are obtained directly from a Markov Chain Monte Carlo minimization. The time periods found are compared to the output of the publicly available Agatha program. The search method is applied to high-energy periodic AGN candidates from the Fermi-LAT catalog. The significance of periodic models over pure noise models is discussed. Finally, the variability of the period and amplitude of oscillating terms is studied on the most significant candidates.

Unified Astronomy Thesaurus concepts: Gamma-ray sources (633); BL Lacertae objects (158); Jets (870); Active galactic nuclei (16); Period search (1955); Time series analysis (1916)

1. Introduction

Systems of binary supermassive black holes (SMBH) are likely to arise in galaxy merging events. These binary black holes (BH) could be responsible for the apparent precession of radio jets (Begelman et al. 1980) and would show detectable periodic modulation of their fluxes. The typical Keplerian period is \(1.6 \text{ yr} \times 10^9 M_\odot \) BH located at \(10^{16} \text{ cm}\) from each other. The interest in binary SMBH has been renewed by the detection of gravitational waves from stellar-mass BH merger events by the LIGO and VIRGO collaboration and the prospect of observation of massive (up to \(10^9 M_\odot\)) BH merger events at upcoming instruments (Bailes et al. 2021). Black hole candidates are best identified through their radio or high-energy emission. One of the most straightforward ways to search for binary black hole candidates is by detecting periodicities on their light curves.

The Large Area Telescope embarked on the Fermi Gamma-ray Space Telescope (Fermi-LAT) monitors continuously the high-energy gamma-ray sky. Thousands of high-energy AGNs, mostly blazars, were detected by the Fermi-LAT in the 100 MeV–300 GeV passband. The Fermi-LAT data are publicly available and span a period of more than 12 yr (2008–2021). Periodic or “quasiperiodic” AGN light curves have been searched by various groups in the Fermi-LAT AGN sample (a few recent publications are Yang et al. 2021; Peñil et al. 2020; Ait Benkhali et al. 2020; Zhang et al. 2017a; Prokhorov & Moraghan 2017).

Methods for searching periods belong to two broad classes. The first class is spectral-domain searches, which involve building a power spectral densities from AGNs. Tarnopolski et al. (2020) use pure stochastic models and do not discuss the removal of long-term linear (or periodic) trends from their light curves.

This paper uses a time domain approach similar to the Gaussian process approach. The flux signal is analyzed as the sum of a periodic mean signal (with a possible linear trend) and a stochastic baseline. The baseline would be described by a Gaussian process for irregularly sampled light curves. In most time domain searches for periodicities (e.g., Covino et al. 2020; Zhang et al. 2021), the periodic component is modeled by including a periodic kernel on a stochastic light curve. While this method provides satisfactory fits to the data, we decided to have separate deterministic and stochastic components for two reasons. The first reason is methodological. Methods for stochastic time series used in this paper assume that they are stationary in time. Hence all trends and periodicities have to be fitted before analyzing the stochastic component. The second reason is related to our goal of searching for binary black holes. A periodic component of the light curve could be an “internal” damped oscillations (say from the disk emission) or an external forcing as in the case of binary black holes, which would be strictly periodic, or a combination of both. In our approach, the deterministic and stochastic components are clearly separated from the start and could thus describe both an external forcing and an internal oscillation.

In this paper, we further concentrate on regularly sampled light curves. The flux can then be modeled by the sum of a periodic flux, a linear trend, and an autoregressive part (an AR model). It would be more general to use ARMA models instead of AR models. But AR models have fewer parameters and they can be more easily related to physical properties of the AGN system such as correlation timescales or eigenfrequencies (see, e.g., Equations (2) and (6)). A very strong assumption we make on the stochastic component is that of stationarity. This assumption may be broken during an AGN outburst. The connection between our model and actual physical parameters such as the amplitude of the periodic signal is described in Section 2.1. Section 2 describes the integration of this model in a Markov Chain Monte Carlo (MCMC) minimization program. A sample of several periodic AGN candidates from the 4FGL...
catalog is analyzed with this program. Section 3 explains the production of light curves from these Fermi-LAT candidates. Results from the MCMC search are discussed in Section 4.

2. Methods

This section motivates the choice of the physical model employed for the description of the light curves. Following the standard practice in time series analysis, we separate the periodic and stochastic components of the light curve. The stochastic component is colored noise with a typical autocorrelation timescale of a few weeks. The basic idea of our method is to eliminate the correlation between flux measurements at consecutive times using an auxiliary variable (Equation (10)).

2.1. General Assumptions

The following considerations have to be taken into account to build a model of the AGN light curve.

1. The noise on AGN light curves is known to have a lognormal amplitude distribution (see, for instance, Ait Benkhali et al. 2020). In this paper, we work with the logarithm of the flux (designated below as “llux”).
2. Light-curve baselines are not stable, and exhibit drifts. In this paper, baseline drifts are taken as the linear drift of llux with time.
3. The noise on light curves could be additive, multiplicative, or a combination of both. Noise on a timescale of a few weeks is modeled as additive colored noise in this work.
4. Detected periods may not be stable in time. The period instability could be an artifact of a multiplicative source of the noise. In this paper, data are analyzed in small time intervals to take into account a possible drift of the llux period.

The model llux light curve $\phi(t)$ is the sum of a periodic term, a mean llux $\bar{\phi}$, a linear trend, and a stochastic component $\epsilon$. The periodic term can be written (including the mean llux) as

$$\phi_p(t) = \bar{\phi} + \sum_j (A_j \cos(\omega_j t) + B_j \sin(\omega_j t)).$$

The $A_j$, $B_j$, and $\omega_j$ are assumed to take constant values inside each analysis time interval.

The evolution in time of the stochastic component $\epsilon$ is governed by a stochastic equation. The Ornstein–Uhlenbeck model is a popular choice for this equation. In this model, $\epsilon$ satisfies

$$\dot{\epsilon} = -\frac{\epsilon}{\tau} + \frac{\sqrt{D}}{\tau} \zeta(t).$$

where $\tau$ is the correlation time of noise, $D$ is a constant, $\zeta$ is white noise, and the dot is the time derivation. The noise term can also be described by models involving higher-order derivatives such as CARMA models (Brockwell & Davis 1991).

Using as an example the Ornstein–Uhlenbeck noise model, the time evolution of the flux satisfies

$$\phi(t) = \bar{\phi} + \sum_j (A_j \cos(\omega_j t) + B_j \sin(\omega_j t)) + C t + \epsilon(t)$$

Equation (4) can be formally integrated to

$$\epsilon(t) = \exp((s - t)/\tau) \epsilon(s) + w(t - s), \ t \geq s$$

where $w(t - s) \sim N(0, D \times (1 - \exp(-2(t - s)/\tau))$ is a Gaussian random variable.

2.1.1. Regularly Sampled Light Curves

If the light curve is regularly sampled in time, then Equation (5) changes to

$$\epsilon(t_n) = \sum_{j=1}^{k} \beta_j \epsilon(t_{n-j}) + w(\delta t).$$

with $\delta t = t_1 - t_0$.

The time evolution of $\phi$ and $\epsilon$ is now described by:

$$\phi(t_n) = \bar{\phi} + \sum_j (A_j \cos(\omega_j t_n) + B_j \sin(\omega_j t_n)) + C t_n + \epsilon(t_n)$$

and $\epsilon$ can be eliminated from Equation (8) by using the linear combination

$$z = \phi(t_n) - \sum_{j=1}^{k} \beta_j \phi(t_{n-j}).$$

Note that a similar trick also works for irregularly sampled light curves, except that the $\beta_j$ coefficients would depend on time.

The evolution of $z$ is described by an equation similar to Equation (8), except for the $\epsilon$ term which is now replaced by a Gaussian random variable:

$$z = \bar{\phi} + \sum_j (A' j \cos(\omega_j t_n) + B' j \sin(\omega_j t_n)) + C' t_n + w(\delta t).$$

The physical $\bar{\phi}$, $A_j$, $B_j$, $C$ variables are obtained from the $\bar{\phi}'$, $A'_j$, $B'_j$, $C'$ variables by the transformation given in Appendix A.

The model of Equation (11) is closely related to ARIMA models with “exogenous covariates” (Feigelson et al. 2018) while its extension to irregular spacing would be a variant of space state models (Durbin & Koopman 2001). It is however not of common use in high-energy astrophysics. As emphasized in the introduction, it has the advantages of clearly separating the periodic and the stochastic part of the signal. Compared to CARMA-based approaches such as those from Yang et al. (2021), Goyal et al. (2018), it has the potential of identifying multiple periods such as harmonics, giving crucial clues to the underlying physical mechanism of the flux oscillation. Compared to the spectral methods such as Agatha (see Section 2.3), the amplitude of the periodic terms is
determined, allowing the study of their evolution with time. The implementation of our model in the MCMC minimization is described in further detail in Section 2.2. The formalism for the regular sampling is easier to implement in the MCMC search. We therefore focus on the study of regularly sampled light curves in this paper.

2.2. MCMC Implementation

The parameters of light-curve models are obtained by Bayesian inference.

The different stochastic models are composed of a mean value \( \bar{\phi} \), an AR term, and a white noise component \( N(0, \sigma) \). For each stochastic model, the deterministic components are added to perform six different MCMC fits: pure noise, linear, sinusoidal, harmonic, linear + sinusoidal, and linear + harmonic. This gives a total of 18 models computed. Finally, to account for observational uncertainties, each model includes a normal distribution \( N(0, \epsilon_{\text{obs}}) \) with standard deviation \( \epsilon_{\text{obs}} \) corresponding to the measurement of systematic 1\( \sigma \) errors in flux.

The following list indicates the parameters and the mathematical description of each stochastic model and deterministic component to fit.

Stochastic model:
1. White noise
   \[
   [\bar{\phi}, \sigma]: \phi(t_n) = \bar{\phi} + N(0, \sigma) \quad (12)
   \]
2. AR(1)
   \[
   [\bar{\phi}, \sigma, \beta_1]: \phi(t_n) = \bar{\phi} + \beta_1 \phi(t_{n-1}) + N(0, \sigma) \quad (13)
   \]
3. AR(2)
   \[
   [\bar{\phi}, \sigma, \beta_1, \beta_2]: \phi(t_n) = \bar{\phi} + \beta_1 \phi(t_{n-1}) + \beta_2 \phi(t_{n-2}) + N(0, \sigma). \quad (14)
   \]

Deterministic component:
1. Linear
   \[
   [C]: C t_n \quad (15)
   \]
2. Sinusoidal
   \[
   [A_j, B_j, \omega_j]: \sum_j (A_j \cos(\omega_j t_n) + B_j \sin(\omega_j t_n)) \quad (16)
   \]
3. Harmonic
   \[
   [A_j, B_j, A'_j, B'_j, \omega_j]: \sum_j (A_j \cos(\omega_j t_n) + B_j \sin(\omega_j t_n) + A'_j \cos(2\omega_j t_n) + B'_j \sin(2\omega_j t_n)). \quad (17)
   \]

There are between two (for a pure white noise model) and eleven or more parameters (for an AR(2) model with a linear term, a sinusoidal term, and its harmonics). Taking the example of an AR(2) model with a single period and no linear term, the conditional probability of obtaining flux \( \phi(t_n) \) is

\[
P(\phi(t_n)|\phi(t_{n-1}), \phi(t_{n-2}, \bar{\phi}, \beta_1, \beta_2, C, \omega_1, A_1, B_1, \sigma ...)
= N(\bar{\phi} + \beta_1 \phi(t_{n-1}) + \beta_2 \phi(t_{n-2}) + A \cos(\omega t_n) + B \sin(\omega t_n))
\]

The likelihood for the parameters \( \bar{\phi}, \beta_1, \beta_2, C, \omega, A_1, B_1, \sigma ... \) is (Robert 2007)

\[
L(\bar{\phi}, \beta_1, \beta_2, C, \omega, A_1, B_1, \sigma ...) = \frac{1}{\sigma_{\text{obs}}^{\text{tot}}} \prod_0^{\text{ntot}} \exp \left( \frac{1}{2 \sigma_{\text{obs}}^2} \right) 
\]

where notot is the number of measurements and \( \sigma_{\text{obs}}^2 = \sigma^2 + \epsilon(t_n)^2 \) takes into account the flux measurement error at time \( t_n \).

In order to avoid working with numbers in different orders of magnitude, which may result in lower efficiency for the MCMC sampling, data are standardized by being rescaled to their mean \( \bar{t} \) and std \( s_t \).

\[
\phi(t)_{\text{ad}} = \frac{\phi(t) - \bar{\phi}(t)}{s_{\phi(t)}} \quad (20)
\]

\[
t_{\text{ad}} = \frac{t - \bar{t}}{s_t}. \quad (21)
\]

By Bayes theorem, the probability distribution of the parameters is the product of the likelihood (Equation (19)) and priors on every parameter.

The prior distribution of the MCMC parameters is selected to be as vague and noninformative as possible for the data sample analyzed. This allows for minimizing the influence and bias on the parameters posterior inference.

The standardization of the data also helps to set the scale for the prior distributions. Thus, the priors of the offset (\( \bar{\phi} \)), the amplitude terms (\( A, B \), the slope (\( C \)), and the autoregressive terms (\( \beta_1, \beta_2 \) ) were chosen as a normal distribution around 0 with a standard deviation of 2.

For the period parameter, the posterior distribution is limited between a minimum and a maximum value. In the original scale, the lower value is set to 500 days to avoid the MCMC chains to be stuck in a possible artificial period of 1 yr and close values. The upper limit is set slightly above half of the data time span (~2200 days), so the period detection can be representative. Now, the prior distribution is a normal centered in the middle of the space drawn for this parameter (~1350 days), with a standard deviation of ~800 days.

Finally, the white noise parameter \( \sigma \) is uniformly distributed between \([1 \times 10^{-7}, 1 \times 10^3]\). After the completion of the MCMC, the parameter outputs are transformed back to the original scale.

The models have been implemented in the R version of JAGS (Plummer 2012). JAGS is an MCMC based on the Gibbs sampling algorithm. For each source, three independent chains were run, using 8000 iterations with a burning length of 4000 samples. The convergence of the chains was checked with the Gelman–Rubin diagnostic (Gelman & Rubin 1992). The output of the program is a set of posterior probability distributions, one for each parameter included in the model computed. Systematics of the output such as the prior dependence, the correlation between parameters, and the normality of residuals are described in the Appendix. Results from Tables 2 and 3 quote the mean values of the posterior distributions and 95% credible intervals around the mean.

\footnote{https://fermi.gsfc.nasa.gov/ssc/data/analysis/LAT_caveats_temporal.html}
As explained in Section 2.2.1, the fits also allow performing deviance comparisons between periodic and pure noise nonperiodic models.

2.2.1. Model Selection Through Information Theory

Information theory is introduced in model selection problems as a form of quantitative explanation of the best model’s goodness of fit. Akaike (1974) proposed a way to estimate divergence based on the maximized empirical log-likelihood estimator (MLE), the Akaike information criterion (AIC). AIC is used as a measure of the information lost when the fitted model is used to approximate the process that generates the empirical data:

\[
AIC = -2 \log(L(\theta | y)) + 2K
\]  

where \( \log(L(\theta | y)) \) is the log-likelihood of the model given the data \( y \), \( K \) is the number of model’s parameters (defined as \( \theta \)) and operates as a penalty to model complexity. Thus, AIC is an effective tool for selecting a simple model which describes and infers empirical data, avoiding both overfitting and underfitting. In our MCMC pipeline, the assessment is done using:

\[
AIC = D(\theta | y)_{\text{min}} + 2K
\]

where \( D(\theta | y)_{\text{min}} \) is the minimum deviance of the MCMC posterior sample.

For each light curve, within all the possible MCMC implementations, the one with the minimum AIC is selected. Now, for model statistical assessment, AIC does not carry much information as it is on a relative scale and it is dependent on sample size. What matters in model assessment, though, is \( \Delta AIC \), the difference between AIC values over multiple nested models. Given a full model \( F \) and a reduced model \( R \):

\[
\Delta AIC = AIC_F - AIC_R = -2 \log(L(\theta_0) / L(\theta)) - 2k
\]

where \( \Lambda \) is the likelihood ratio test statistic, \( L(\theta_0) \) and \( L(\theta) \) are the MLE under the null \( (R) \) and alternative \( (F) \) hypothesis, respectively, and \( k \) is the difference of parameters between models.

From this definition, a \( p_{\text{value}} \) can be computed, which shows the probability of obtaining the value \( \Lambda \) under the null hypothesis conditions. As stated in Efron & Hastie (2016, Chapter 13; see also Murtaugh 2014), the relationship between \( p_{\text{value}} \) and \( \Delta AIC \) can be drawn as:

\[
p_{\text{value}} = Pr(\chi^2_k > \Lambda) = Pr(\chi^2_k > \Delta AIC + 2k)
\]

where \( \Lambda = \Delta AIC + 2k \) follows a \( \chi^2 \) distribution with \( k \) degrees of freedom. In the following we therefore use this relation to assess a \( p_{\text{value}} \) when comparing nested models.

2.3. Spectral Method

In this paper, potential periods of light curves are searched by a time domain, MCMC-based method, introduced in Section 2.2. To validate our results, we found it useful to compare the periods found with those obtained with a different, spectral-based method. The public domain Agatha program (Feng et al. 2017) calculates Lomb–Scargle periodograms on astronomical time series and evaluates the significance of the periodic components found. Four different variants of the Lomb–Scargle algorithm are implemented and were studied with simulated light curves. The Bayes factor periodogram (BFP) was found to be efficient at finding periods in noisy light curves with trends. To assess the significance of the computed periodograms, the logarithm of the Bayes factor (InBF) is evaluated. InBF is related to the maximum likelihood ratio of the periodic and the noise model and can be seen as the significance of the given period. Agatha provides also a “moving periodogram” (periodogram calculated in different time windows) which is a useful tool for finding long-term changes in periods, studied in Section 4.2. The authors of Agatha recommend using their program in combination with an MCMC search to refine the results. As explained in Section 2.2, the priors of our MCMC search are not based on the Agatha results, which are only used as cross-checks.

3. Fermi Data

This section is dedicated to explaining the selection and analysis of the sample of AGNs light curves from the Fermi-LAT data.

3.1. Object Selection

The sources selected in this work compose a subsample of the AGN population of the Fermi-LAT 8-year Source Catalog (4FGL; Abdollahi et al. 2020). The 4FGL represents a daily full-sky survey in the 50 MeV–1 TeV energy range. Of the total of 5064 sources in the catalog, 3207 are tagged as AGNs of which 3137 are blazars, 42 are radio galaxies, and 28 are other kinds of AGNs. Our AGN selection was motivated by previous studies on period detection in gamma-ray literature. Information about the sources’ properties and periodicity literature results is shown in Table 1.

3.2. Data Analysis

The analysis of each source was performed using Enrico, a community-developed Python package to conduct Fermi-LAT analysis (Sanchez & Deil 2013), which consists of a simplified full analysis chain based on the FermiTools. The version of the FermiTools is 2.0.8 via the Conda repository.

The data are obtained from Fermi-LAT Data Server3 introducing Astroquery (Ginsburg et al. 2019) in our pipeline. More than 12 yr of Pass 8 LAT data (Atwood et al. 2013) are used with evclass = 128 and evtype = 3 for photon-like events in point sources analysis. Events with a zenith angle greater than 100° are rejected to reduce the gamma-ray contamination of the Earth limb. Good time intervals with high quality data are selected using (DATA_QUAL>0) & & (LAT_CONV = = 1). On sky model generation, the background emission is modeled adopting the Galactic diffuse emission file gll_iem_v07.fits and the extragalactic isotropic diffuse emission file iso_P8R3_SOURCE_V2_v1.txt.

The analysis software computes a binned likelihood analysis to find the best-fit model parameters and the light curves are obtained by running the entire chain into time bins. The light curves between 1 and 300 GeV were computed in 145 time bins from the start of the mission (239557418 MET) until the end of March 2021 (63876517 MET), using an ROI of 10 degrees. Within each time bin, the pipeline generates a light-
Table 1
List of Fermi-LAT AGN Sample with 4FGL

| 4FGL Name      | Common Name | Type      | Period (Days) | Reference |
|----------------|-------------|-----------|---------------|-----------|
| J0043.8+3425   | GB6 J0043+3426 | FSRQ     | 657           | 3         |
| J0102.9+5825   | TXS 0105+581  | FSRQ     | 767           | 3         |
| J0158.5+0133   | 4C +01.28    | BL Lac    | 445           | 4         |
| J0210.7-5101   | PKS 0210-512  | FSRQ     | 949           | 3         |
| J0211.2+1051   | CGRab J0211+1051 | BL Lac | 621           | 3         |
| J0252.8-2218   | PKS 0250-225  | FSRQ     | 438           | 3         |
| J0303.4-2407   | PKS 0301-243  | BL Lac    | 730, 766 ± 109 | 3, 9     |
| J0428.6-3756   | QSO B0426-380 | FSRQ     | 1241, 1223 ± 248 | 3, 8     |
| J0449.4-4350   | PKS 0447-439  | BL Lac    | 913           | 3         |
| J0457.0-2324   | QSO J0457-2324 | FSRQ | 949           | 3         |
| J0501.2-0158   | PKS 0458-039  | FSRQ     | 621           | 3         |
| J0521.7+2112   | RX J0521.7+2112 | BL Lac | 1022          | 3         |
| J0691.9+7120   | PKS 0716+71  | BL Lac    | 1022, 346     | 3, 4      |
| J0808.2-0751   | QSO B0805-077 | FSRQ     | 658           | 4         |
| J0811.4+0146   | QSO B0808+019 | BL Lac    | 1570          | 3         |
| J0818.2+0422   | QSO B0814+42  | BL Lac    | 803           | 3         |
| J1146.9+3958   | B2 1144+40   | FSRQ     | 1205          | 3         |
| J1248.3-5820   | QSO B1246-586 | BL Lac    | 803           | 3         |
| J1303.0+2434   | MG2 J1303+2434 | BL Lac | 730           | 3         |
| J1454.4+5124   | TXS 1452+516  | BL Lac    | 767           | 3         |
| J1555.7+1111   | PG 1553+113  | BL Lac    | 790, 803, 798, 780 ± 63, 803 | 2, 3, 4, 5, 6 |
| J1649.4+5235   | 87 GB 164812.2+52403 | BL Lac | 986          | 3         |
| J1903.2+5540   | 1RXS J1903.15+55403 | BL Lac | 1387          | 3         |
| J2056.2-4714   | PMN J2056-4714 | FSRQ     | 620, 637      | 3, 4      |
| J2158.8-3013   | PKS 2155-304  | BL Lac    | 685, 610, 621, 644, 620 ± 41, 635 ± 47 | 1, 2, 3, 4, 5, 7 |
| J2202.7+4216   | BL Lac       |           | 698, 680 ± 35 | 4, 5      |
| J2238.1-2759   | VSOP J2258-2758 | FSRQ     | 475           | 3         |

References. (1) Chevalier et al. (2019) (2) Covino et al. (2020) (3) Peñil et al. (2020) (4) Prokhorov & Moraghan (2017) (5) Sandrinelli et al. (2018) (6) Tavani et al. (2018) (7) Zhang et al. (2017a) (8) Zhang et al. (2017b) (9) Zhang et al. (2017c).

curve point unless the TS is under 9, in which case an upper limit is derived.

Finally, as justified in Section 2, the logarithm of the flux is applied. For the study on periodicity, only light curves without important flux gaps are considered. Thus, the final AGN sample is limited to the sources included in Section 4.

4. Results

4.1. Full Light Curve

The light curves obtained after the analysis chain explained in 3.2 are analyzed following the MCMC procedures in 2.2. The results are shown in Table 2.

All AGNs analyzed at present a correlated colored noise, depicted from the autoregressive AR(N) terms. The inclusion of these components over white noise is fundamental in terms of significance, regarding the model selection procedure presented in Section 2.2.1. The $\Delta AIC_w = AIC_{white} - AIC_{colored}$ are between 10 and 60 for all sources, which is more than sufficient to reject a white noise model over a colored noise one. Nonetheless, it is important to remark that using an autoregressive noise model decreases the significance of periodic signals compared to using a white noise model. This means that, in most sources analyzed, $\Delta AIC_w = AIC_{white noise} - AIC_{white periodic}$ is greater than $\Delta AIC_c = AIC_{colored noise} - AIC_{colored periodic}$.

For all sources, a periodic model is prefered and the periodic signal is assessed through the computation of the $p_{value}$ by its relationship with $\Delta AIC$ (Equation (25)).

The most significant periodic signal is that for PG 1553+113, with a $p_{value}$ of $1.2 \times 10^{-7}$ denoting very strong evidence of the source’s periodic behavior. The period found of 774 days ($\sim 2.1$ yr) is compatible with previous periodicity studies on the source. Besides, the light curve shows a clear linear trend and a small noise/autoregressive behavior. The period found of 774 days is not compatible with previous literature periodic analysis (Peñil et al. 2020) where a period of $\sim 3.8$ yr is found in the low significance level ($\sim 2.5\sigma$). On the other hand, for PKS 2155-304, the fitted periodic signal of 614 days ($\sim 1.7$ yr) is compatible with the literature. Both sources show also a linear trend and autoregressive noise behavior. The aforementioned descriptions for the three most significant periodic sources MCMC fits are shown in Figure 1.

Two sources, PKS 0301-243 and RX J0521.7+2112, are below a $p_{value}$ of 0.01, with a period of 821 days ($\sim 2.3$ yr) and 1136 days ($\sim 3.1$ yr), respectively. Both values are higher and not compatible with those in the literature, where periods of $\sim 2$ and $\sim 2.8$ yr are found in the high significance level ($\sim 3\sigma$), respectively. RX J0521.7+2112 does not show a linear trend and it is dominated by an autoregressive component of second order with $\beta_1 = 0.38$ and $\beta_2 = 0.24$. PKS 0301-243 shows a special sinusoidal behavior, a principal periodic component of 821 days ($\sim 2.3$ yr) with a second harmonic. The result of this source is shown in Figure 2. For CGRaBS J0211+1051 a
harmonic oscillation of 1398 days (~3.8 yr) is also found with lower significance. The appearance of a harmonic component in the source’s light curve will be discussed in Section 5.

The periodicity significance of the remaining AGNs in Table 2 is low ($p_{\text{value}} > 0.01$) and the MCMC performance is not as good as for the high significance cases. As a result, the standard deviation of the period parameter increases (>150 days) as well as the highest density interval (HDI), which, for some of the sources, is stuck either in the lower or the upper posterior limit. Also, the colored noise terms are much higher, being all above 0.5. For the sources marked with $^*$, the period values are taken from a good posterior distribution obtained by the use of specific priors different from the general prior in Section 2.2. This is described in Appendix B.1. For the sources marked with $^{**}$, the MCMC chains convergence for the period parameter is poorer, resulting in posterior distributions with shapes different from an expected symmetric Gaussian. An example is also shown in Appendix B.1.

After the completion of the time series analysis through the MCMC fits, a spectral analysis, presented in Section 2.3, is performed as a cross-check. For each source, the BFP is performed using the same AR noise model as the one retrieved in the MCMC fit. The results of the Agatha analysis can be found in the right part of Table 2. As an example, the BFPs of the sources represented in Figure 1 are shown in Figure 3.

Of the 14 periodic light curves analyzed, the Agatha periods for all sources but for 4FGL J0818.2+4222 and 4FGL J0303.4-2407 are compatible with the MCMC results. Nonetheless, the two most significant signals of 4FGL J0818.2+4222 are 1337 days and 805 days, with very close lnBF values of 2.1 and 1.6 respectively. The second most significant signal at 805 days is compatible with the MCMC result at 867 days. For 4FGL J0303.4-2407, an lnBF = −1.2 means that a noise model is favored over the periodic model at this given period. This might be due to the presence of a harmonic signal in the MCMC fit, which is not reproducible with Agatha. 4FGL J1903.2+5540 and 4FGL J2202.7+421 have a lnBF = 0.6 and lnBF = 0.1 respectively, meaning that the periodic model is poorly favored over the noise model. All the remaining sources present compatible periods with an important level of significance, i.e., lnBF ≥ 3.

As will be discussed in Section 5, the time stability of the period and the variation of the amplitude of the periodic modulation as a function of the period are two important clues of the physical mechanism causing the oscillations. The local period and amplitudes can be obtained by building a spectrogram with the time windows of the light curve.

### 4.2. Time Windows

For this analysis, several time windows are selected to search for periodicity change in the light curve time span. Each time window has a width of 0.4 of the full light curve time span, this is, 0.4 × 4620 days ~ 1848 days (~5 yr). Thus, only the most significant periodic sources with periods smaller than 900 days are studied. A total of five time windows are computed, each of them centered in 920, 1607, 2294, 2981, and 3669 days from the start, respectively. The MCMC fits are applied at each time window for every source included. Then, an Agatha moving periodogram is performed as a cross-check as explained in Section 2.3. The results are shown in Table 3 and Figure 4. Figure 5 shows the light curves analyzed with the fitted MCMC components at each time window.

For PG 1553+113, a periodic component is found in every time window with a $p$-value below $10^{-2}$. Windows 1, 4, and 5 present a period below the entire light-curve period of 774 days. For windows 2 and 3, the period is ~840 days and with larger error bars. There is a total difference of 105 days between the minimum and the maximum period found. To quantify this dispersion, a constant fit is performed with a $\chi^2$ test, considering the standard deviations, that indicates the probability that the different time window periods come from a constant distribution. The fit estimate of the constant value is 756 ± 16 days, with a $p$-value equal to 0.64. The Agatha values agree with the MCMC results with a significance above 3.8 for every time window except the last one with a value of 1.5.

For PKS 2155-304, the periodic components are also found with $p$-values below $10^{-2}$, Windows 1 and 2 present a period higher than the entire light-curve period of 613 days. Windows 3 and 4 present a period below this value. For window 5, the best-fitted model corresponds to an AR(1) stochastic component, a linear trend, and a sinusoidal with a second harmonic

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**Table 2**

MCMC Fit Results and Agatha Cross-check Comparison of the AGN Fermi-LAT Sample

| 4FGL Name     | Best Model | AIC     | Period  | Period HDI95% | ΔAIC | $p_{\text{value}}$ | PeriodAGATHA | lnBF |
|---------------|------------|---------|---------|--------------|------|-------------------|--------------|------|
| J1555.7+1111  | AR1 + lin + sin | 298.03 | 774 ± 10 | 755-793 | 28.98 | $1.2 \times 10^{-7}$ | 771 ± 29     | 10.9 |
| J2158.8-3013  | AR1 + lin + sin | 340.07 | 614 ± 16 | 589-642 | 9.85  | $1.2 \times 10^{-3}$ | 615 ± 26     | 6.3  |
| J1903.2+5540  | AR1 + lin + sin | 381.92 | 1120 ± 95 | 1040-1230 | 8.7  | $2.1 \times 10^{-3}$ | 1163 ± 55    | 0.6  |
| J0303.4-2407  | AR1 + lin + harm | 323.83 | 821 ± 40 | 761-870 | 5.58  | $8.2 \times 10^{-3}$ | 773 ± 26     | -1.2 |
| J0521.7+2112  | AR2 + sin  | 309.11 | 1136 ± 128 | 990-1280 | 5.52  | $9.2 \times 10^{-3}$ | 1139 ± 73    | 11   |
| J1248.3+5820  | AR2 + sin  | 383.73 | 2048 ± 169 | 1800-2350 | 4.29  | $1.6 \times 10^{-2}$ | 2039 ± 133   | 3    |
| J0211.2+1051  | AR1 + harm | 301.31 | 1398 ± 122 | 1190-1630 | 3.62  | $1.8 \times 10^{-2}$ | 1446 ± 59    | 3    |
| J0449.4-3350  | AR1 + lin + sin | 296.29 | 746 ± 229 | 505-1030 | 3.6   | $2.2 \times 10^{-2}$ | 669 ± 14     | 7.2  |
| J0457.0-2324  | AR1 + sin  | 293.61 | 1300 ± 153 | 975-1350 | 2.11  | $4.4 \times 10^{-2}$ | 1330 ± 59    | 7.9  |
| J2202.7+4216  | AR1 + lin + sin | 261.19 | 1799 ± 219 | 1430-2250 | 2.01  | $4.6 \times 10^{-2}$ | 1763 ± 89    | 0.1  |
| J0721.9+7120  | AR1 + sin  | 321.08 | 987 ± 220 | 574-1520 | 1.94  | $4.7 \times 10^{-2}$ | 1011 ± 96    | 7.3  |
| J0818.2+4222  | AR2 + sin  | 360.61 | 955 ± 356 | 501-1790 | 1.68  | $5.3 \times 10^{-2}$ | 1333 ± 21    | 2.1  |
| J0428.6-3756  | AR1 + lin + sin | 288.65 | 1310 ± 175 | 889-1650 | 0.94  | $7.4 \times 10^{-2}$ | 1262 ± 114   | 13.2 |
| J0210.7-5101  | AR2 + lin + sin | 210.93 | 1080 ± 351 | 502-1640 | 0.68  | $8.3 \times 10^{-2}$ | 1025 ± 13    | 3.9  |

Note. For each source, the list indicates: the best model in terms of AIC; the AIC value; the period mean and standard deviation in days; the period 95% highest density intervals (HDI) in days; the ΔAIC between the periodic and the noise model; the $p$-value computed from ΔAIC; the Agatha period mean and standard deviation; the Agatha lnBF. $^*$ indicates a specific prior assumption and $^{**}$ indicates inferior posterior convergence for the period (see Appendix B.1). The results are sorted by MCMC period detection significance.
Figure 1. MCMC fits for the three sources with the most significant periodic signals. Left panel: separated fitted components of the model. The green line indicates the linear trend. The blue line indicates the sinusoidal term. The red line indicates the stochastic term. Right panel: general fit and white noise $\sigma$ component.

Figure 2. MCMC fit for PKS 0301-243. Left panel: separated fitted components of the model. The green line indicates the linear trend. The blue line indicates the sinusoidal harmonic term. The red line indicates the stochastic term. Right panel: general fit and white noise $\sigma$ component.
component. Again, the $\chi^2$ test is computed, resulting in a constant value of $618 \pm 25$ days with a $p$-value of 0.02.

As explained in Appendix A, the transformed amplitude $Z'$ of the periodic modulations is obtained from the MCMC posterior values of parameters $A$ and $B$. Then, the physical amplitude $Z$ is derived from $Z'$ through $U$. Physical amplitudes as a function of the period is shown in Figure 6. The $\chi^2$ test is computed to quantify the amplitude variation with the period. This gives a $p$-value of 0.87 and 0.95 for PG 1553+113 and PKS 2155-304 respectively.

5. Discussion and Conclusions

Models explaining the periodicity of high-energy emission of quasars involve a variety of mechanisms from jet precession (Caproni et al. 2013) to periodic changes in the disk accretion flow (Gracia et al. 2003) somehow transmitted to the jet.

An important class of models is geometric models (Rieger 2004). In geometric models, the periodicity in the observed emission is due to a change in the viewing angle of the jet components. The lighthouse model of Camenzind & Krockenberger (1992) explains QPOs with periods of a few hundred days by the rotation of plasma bubbles around the central axis of the jet. In that model, the observed period increases with time and the amplitude of the emission is also time dependent (see Figures 2 and 3 of Camenzind & Krockenberger 1992). Another possible mechanism is based on the instability of the boundary (at transition radius $r_{tr}$) between an outer thin disk (Gracia et al. 2003) or a torus (Zanotti et al. 2003) and an inner radiatively inefficient flow (ADAF). In the model of Gracia et al. (2003), the boundary is slowly moving outward and the period of oscillations increases with time.

Finally, models with binary supermassive black holes (BSMBH) are natural candidates to explain an observed periodicity in the emission. Sobacchi et al. (2017) have modeled the high-energy light curve of PG 1553+113 with a geometrical model based on a BSMBH system in which one of the black holes has a precessing jet. In this model, the periodicity is due to the orbital motion of the BSMBH and is not expected to change significantly with time while the flux amplitude is related to the geometrical properties of the jet, which can change smoothly with time. Tavani et al. (2018) have interpreted the 2.2 yr periodicity of the high energy light curve of PG 1553+113 with a model where blazar jets of a BSMBH are periodically perturbed by magnetogravitational stresses (Cavaliere et al. 2017). In this model, the smaller black hole stresses periodically the jet launched by the heavier one, triggering synchrotron emission and inverse Compton scattering in the GeV energy range. The observed emission could come either from a single jet or from the 2 jets of the BSMBH system. In the latter case, Tavani et al. (2018) predict a stable period with smooth amplitude changes from cycle to cycle, while in the former case the amplitude changes are erratic.
In this paper, a number of high-energy periodic source candidates from the Fermi-LAT 4FGL catalog have been searched for periodicity by a novel method that separates clearly the stochastic and the periodic component in the light-curve fitting. Adding a periodic component to three sources, 4FGL J1555.7+1111, 4FGL J2158.8-3013, and 4FGL J1903.2+5540 improves significantly the fit to the data compared to an ARMA (autoregressive moving average) noise only model. A study of period and amplitude as a function of time was attempted by dividing the light curve into different time windows for light curves with periods less than 900 days. This excludes 4FGL J1903.2+5540 from the time window study.

We now discuss the results of the time window study for 4FGL J1555.7+1111 and 4FGL J2158.8-3013. The measured period of the best candidate 4FGL J1555.7+1111 does not significantly change with time. The amplitude of the periodic term depends only weakly on the period. The almost constant amplitude and period are in agreement with a BSMBH model such as the 2-jet model of Tavani et al. (2018). An evolution of both the period and the amplitude is expected in models based on the lighthouse effect (Camenzind & Krockenberger 1992) such as the model of Mohan & Mangalam (2015). This model aims at explaining short-term variability (≤1 yr), but could perhaps be extended to longer timescales (Ait Benkhali et al. 2020).
In the case of 4FGL J2158.8-3013, there is a marginally significant drift of the period with time. The amplitude of the oscillating component does not significantly change with time. The lack of correlation between amplitude and period disfavors again models based on the lighthouse effect. A harmonic of the period is detected in one of the time windows. This harmonic is similar to the value obtained by Liu et al. (2006). The value of $k_{\text{trans}}$ for the torus model is similar to the value obtained by Liu et al. (2007) for a disk or a torus to an ADAF occurs at the radius $r_{\text{trans}} = K_{\text{trans}} \left( \frac{10^8 M_\odot}{M_{\text{BL}}} \right) \left( \frac{P_{\text{true}}}{1 \text{ yr}} \right)^{2/3}$ (26)

with $K \approx 524$ for the transition radius to a disk (Gracia et al. 2003) and $K \approx 2100$ for the transition to a torus (Liu et al. 2006).

The discussion on 4FGL J1555.7+1111 and 4FGL J2158.8-3013 has shown the advantages of our time domain approach: the possibility of separating the periodic signal from the stochastic noise, accounting for harmonics, and studying the evolution in time of periods and amplitudes. This paper dealt exclusively with regularly sampled light curves. In the next step, we will extend the study to the whole Fermi-LAT data set, including light curves with holes in the observations.

**Appendix A**

**Derivation of the MCMC Model for Regularly Spaced Data**

Subtracting $k$ times Equation (8), one gets

$$z = \phi(t_n) - \sum_{j=1}^{k} \beta_j \phi(t_{n-j})$$

$$= \left(1 - \sum_{j=1}^{k} \beta_j + \sum_{j=1}^{k} \beta_j \delta t \right) \bar{\phi}$$

$$+ C t_n \left(1 - \sum_{j=1}^{k} \beta_j \right) + w(\delta t) + S_n$$

$$S_n = \sum_{j} \left(A_j \cos(\omega_j t_n) - \sum_{i=1}^{k} \beta_i \cos(\omega_j t_{n-i}) \right)$$

$$+ B_j \left(\sin(\omega_j t_n) - \sum_{i=1}^{k} \beta_i \sin(\omega_j t_{n-i}) \right)$$

with $\delta t = t_1 - t_0$.

$S_n$ can be simplified by defining

$$U_j \exp i\psi_j = 1 - \sum_{l=1}^{k} \beta_l \exp(-i\omega_l \delta t)$$

(A1)

with $U_j$ real.

Then

$$S_n = \sum_{j} \left(A_j U_j \cos(\omega_j t_n + \psi_j) + B_j U_j \sin(\omega_j t_n + \psi_j) \right)$$

Finally, $z$ can be written as

$$z = \bar{\phi}' + \sum_{j} \left(A'_j \cos(\omega_j t_n) + B'_j \sin(\omega_j t_n) \right) + C' t_n + w(\delta t)$$

(A2)

with

$$\bar{\phi}' = \left(1 - \sum_{j=1}^{k} \beta_j \right) \bar{\phi}$$

(A3)

$$C' = C(1 - \sum_{j=1}^{k} \beta_j)$$

(A4)

$$A'_j = U_j (A_j \cos(\psi_j) + B_j \sin(\psi_j))$$

(A5)

$$B'_j = U_j (-A_j \sin(\psi_j) + B_j \cos(\psi_j))$$

(A6)

The time-averaged square amplitude of each oscillating term is

$$Z_j^2 = 1/2(A_j^2 + B_j^2)$$

(A7)

$$= 1/2(U_j)^2 (A_j^2 + B_j^2) = (U_j)^2 Z_j^2.$$ 

(A8)

The transformed amplitude $Z'$ picks up an additional period dependence compared to the physical amplitude $Z$. Specializing to the case of an AR(1) model with a single period $T = 2\pi$, the ratio of the transformed amplitude to the physical amplitude is

$$\frac{Z'}{Z} = U = \sqrt{1 + \beta^2 - 2\beta \cos \left(\frac{2\pi \delta t}{T}\right)}.$$ 

(A9)

If $\beta > 0$ as in the Ornstein–Uhlenbeck model (Equation (5)), $Z'/Z$ is a decreasing function of $T$ in the limit of small sampling times ($\delta t/T \ll 1$).

**Appendix B**

**Systematics of the MCMC Search**

**B.1. Period Prior Dependence**

On the high significance sources, changes in the prior distribution have no remarkable influence on the MCMC sampling and vague priors are suitable for the MCMC fit. The posterior distributions are approximately symmetric Gaussians from where the parameters are obtained as the mean value with standard deviation, as shown for example in the left panel of Figure 7 for PG 1553+113.

The right panel of Figure 7 shows an example of a poorer MCMC sampling where the chains are not converging properly and the posterior spreads wider to lower and higher values around the peak. Even so, the peak of the distribution is close to its mean value. This is the case for low significant sources marked with ** in Table 2 where a change in the prior distribution does not change the output significantly.

For low significant sources marked with *, different priors might result in better posterior results. As can be seen in the
Figure 7. Examples of posterior distributions for the period parameter. Left panel: 4FGL J1555.7+1111. Right panel: 4FGL J0721.9+7120.

Figure 8. Examples of 4FGL J0457.0-2324 posterior distributions for the period parameter using (left panel) the general prior and (right panel) the specific prior.

Figure 9. Example of a corner plot for 4FGL J1555.7+1111. The diagonal plots show the posterior distributions for every standardized parameter. The 2D maps below the diagonal show the density of the posterior distribution with one parameter on each axis. The values above the diagonal show the Pearson correlation coefficients between parameters.
example in Figure 8, the use of a general prior leads to a result of bad chain convergence. Thus, the HDI and standard deviation are larger and the mean of the distribution is not close to the peak value. Centering the prior distribution in the highest value and reducing its standard deviation results in a posterior distribution closer to a symmetric Gaussian. This shows a strong dependence between the posterior and the choice of the prior distribution. Thus, the acceptance of these results is lower and can be correlated with the inferior significance.

B.2. Correlations between Parameters

For every source analyzed, the correlation between MCMC parameters is not as important as to affect the efficiency of the sampling chains. There are some correlations between periodic parameters (period, A and B) and between AR parameters ($\beta_1$, $\beta_2$) with the others in the model. All correlation values are below $\sim$0.6. In Figure 9 the corner plot for PG 1553+113 is shown, useful to visualize the pairwise correlations between model parameters.

B.3. Tests for Normality

As suggested by Feigelson et al. (2018) the results of the light-curve fits have been tested for normality by performing an Anderson–Darling test on the residuals. Also, a Gaussian distribution $N(\text{mean}, \sigma)$ is fitted. The results are shown in Table 4. An example is shown in Figure 10.

The $p$-value from the Anderson–Darling test rejects the hypothesis of normality if its value is lower or equal to 0.05. For all sources but 4FGL 1903.2+5540 the $p$-value is above 0.05. As can be seen in Figure 10 for 4FGL 1903.2+5540, some outliers are found on the left side of the residuals distribution and the test for normality fails.

![Figure 10](https://example.com/figure10.png)

**Figure 10.** Residuals histogram for the three sources with the most significant periodic signals. The black line shows a Gaussian distribution fit with $N_{\text{mean}}$ and $N_{\sigma}$ parameters given in Table 4. Left panel: 4FGL J1555.7+1111. Center panel: 4FGL 2158.8-3013. Right panel: 4FGL J1903.2+5540.

![Figure 11](https://example.com/figure11.png)

**Figure 11.** Examples of posterior distributions for the sigma parameter for the three sources with the most significant periodic signals. Left panel: 4FGL J1555.7+1111. Center panel: 4FGL 2158.8-3013. Right panel: 4FGL J1903.2+5540.

### Table 4

| 4FGL Name   | $P_{\text{valueAD}}$ | $N_{\text{mean}}$ | $N_{\sigma}$ | $\sigma_{\text{MCMC}}$ |
|-------------|----------------------|--------------------|--------------|------------------------|
| J1555.7+1111| 0.89                 | $-1 \times 10^{-4}$| 0.2          | 0.2                    |
| J2158.8-3013| 0.36                 | $9 \times 10^{-4}$ | 0.33         | 0.34                   |
| J1903.2+5540| $9 \times 10^{-4}$   | $-1.8 \times 10^{-3}$ | 0.33         | 0.34                   |
| J0303.4-2407| 0.38                 | $7 \times 10^{-3}$  | 0.45         | 0.47                   |
| J0521.7+2112| 0.66                 | $9.7 \times 10^{-3}$ | 0.41         | 0.43                   |
| J1248.3+5820| 0.43                 | $5 \times 10^{-4}$  | 0.3          | 0.31                   |
| J0211.2+1051| 0.21                 | $7.3 \times 10^{-3}$ | 0.46         | 0.46                   |
| J0449.4-4350| 0.29                 | $1.4 \times 10^{-3}$ | 0.33         | 0.34                   |
| J2202.7+4216| 0.29                 | $-2.6 \times 10^{-3}$ | 0.53         | 0.55                   |
| J0818.2+4222| 0.324                | $8.7 \times 10^{-2}$ | 0.38         | 0.38                   |
| J0721.9+7120| 0.4                  | $7 \times 10^{-4}$  | 0.52         | 0.53                   |
| J0457.0-2324| 0.3                  | $1 \times 10^{-2}$  | 0.55         | 0.56                   |
| J0428.6-3756| 0.06                 | $4 \times 10^{-3}$  | 0.51         | 0.52                   |
| J0210.7-5101| 0.86                 | $7.8 \times 10^{-2}$ | 0.59         | 0.61                   |

**Note.** For each source, the list indicates: the $p$-value of the Anderson–Darling test; Gaussian distribution fit mean $N_{\text{mean}}$ and standard deviation $N_{\sigma}$; the $\sigma$ of white noise term in the MCMC fit.

From the Gaussian distribution fit some conclusions can be drawn. As expected from a proper data fit, the mean value of the residuals is centered close to 0. Furthermore, the standard deviation values $N_{\sigma}$ are equal to those obtained from the MCMC fit white noise term $\sigma_{\text{MCMC}}$ and all posterior distributions are well sampled as approximately symmetric Gaussians. An example is shown in Figure 11.

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