New Fermions at $e^+e^-$ Colliders:

II. Signals and Backgrounds.

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ABSTRACT

We discuss the production, at high–energy $e^+e^-$ linear colliders, of new heavy fermions predicted by extensions of the Standard Model. We analyze in great details the various signals and the corresponding backgrounds for both pair production and single production in association with ordinary fermions. Concentrating on new leptons, we use a model detector for $e^+e^-$ collisions at a center of mass energy of 500 GeV, to illustrate the discovery potential of the Next Linear Colliders.

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1. Introduction

Many theories beyond the Standard Model [SM] of the electroweak and strong interactions predict the existence of new fermions. In most of the cases, these new fermions have non–canonical SU(2)\textsubscript{L} × U(1)\textsubscript{Y} quantum numbers, e.g. the left–handed components are in weak isosinglets and the right–handed components in weak isodoublets. Examples of new fermions are the following (for a review, see for instance Ref. [1]):

i) Sequential fermions: this is the simplest extension of the SM, one simply has to add to the known fermionic spectrum with its three–fold replica a fourth family with the same quantum numbers. The existence of this fourth generation is still allowed by available experimental data, if the associated neutrino is heavy enough [3].

ii) Vector fermions: these occur for instance in the exceptional group E\textsubscript{6} [4], which is suggested as a low energy limit of superstring theories. In this group each generation of fermions lies in the representation of dimension 27, and in addition to the usual SM chiral fields, twelve new fields are needed to complete the representation. Among these, there will be two weak isodoublets of heavy leptons, a right–handed and a left–handed one.

iii) Mirror fermions: they have chiral properties which are opposite to those of ordinary fermions, i.e. the right–handed components are in weak isodoublets and the left–handed ones are in weak isosinglets; there is also a left–handed heavy neutrino [5]. These fermions appear in many extensions of the SM and provide a possible way to restore left–right symmetry at the scale of the electroweak symmetry breaking; they naturally occur in lattice gauge theories.

iv) Singlet fermions: these are the most discussed fermions in the literature, a prominent example being the SO(10) neutrino [6, 7]. Indeed, in this unifying group, which is one of the simplest and most economic extensions of the SM, the smallest anomaly–free fermion representation has dimension 16. It contains a right–handed neutrino in addition to the 15 Weyl fermions in one fermion generation; this neutrino is of the Majorana type. Singlet neutrinos, which can be either of Majorana or Dirac type, and quarks also occur in E\textsubscript{6} [4].

The search for these new fermions will be a major goal of the next generation of accelerators. In particular, high–energy e\textsuperscript{+}e\textsuperscript{−} colliders provide unique facilities to search for these fermions, and in case of discovery, to explore their basic properties.

In a preceding paper [8], hereafter referred to as (I), one of us discussed in a rather general context, the production mechanisms in e\textsuperscript{+}e\textsuperscript{−} collisions and the decay modes of the new fermions. Here, we will complete this discussion by analyzing the discovery potential of the next e\textsuperscript{+}e\textsuperscript{−} linear collider. Concentrating on new leptons, which are difficult to search for at hadron colliders [9], we study in great details their possible signatures and the corresponding backgrounds at a future high–energy e\textsuperscript{+}e\textsuperscript{−} collider with a center of mass energy of 500 GeV. To be as close as possible to the experimental conditions, we use a model detector for e\textsuperscript{+}e\textsuperscript{−} collisions at this energy, exploiting the various experimental studies which have been carried out on the potential of a 500 GeV e\textsuperscript{+}e\textsuperscript{−} machine [4].
The paper is organized as follows. In the next section, we first introduce the interactions of the new fermions, discuss their decay modes and summarize the existing constraints on their masses and couplings. In section 3, we study the pair production of neutral and charged heavy leptons. In section 4, we analyze in detail the signals and backgrounds for the single production of heavy leptons in association with their ordinary partners, paying a special attention to the first generation where additional production channels are present. Finally, section 5 contains our conclusions.

2. Physical Basis

In this section, we briefly summarize the couplings of the new fermions and their dominant decay modes, concentrating on those features which will be needed in the present analysis. A more general discussion can be found in (I).

2.1 Interactions

Except for singlet neutrinos which have no electromagnetic and weak charges, the new fermions couple to the photon and/or to the electroweak gauge bosons $W/Z$ with full strength. These couplings allow for the pair production in $e^+e^-$ annihilation of heavy leptons and quarks, if their masses are smaller than the beam energy. To calculate the production cross sections, one needs the vector and axial–vector couplings of the new fermions to the photon and $Z$ boson; in units of the proton charge, they are given by

$$v^F_\gamma = e^F, \quad a^F_\gamma = 0, \quad v^F_Z = v^F_F = \frac{2I^F_{3L} + 2I^F_{3R} - 4e^F s^2_W}{4s_W c_W}, \quad a^F_Z = a^F_F = \frac{2I^F_{3L} - 2I^F_{3R}}{4s_W c_W} (2.1)$$

where $e^F$ is the electric charge of the fermion $F$, $I^F_{3L}$, $I^F_{3R}$ the third components of the left–handed and right–handed weak isospin and $s^2_W = 1 - c^2_W \equiv \sin^2 \theta_W$. This expression clearly exhibits the facts that vector fermions have no axial–vector couplings, and that the axial–vector couplings of sequential and mirror fermions are of opposite signs.

If they have non–conventional quantum numbers, the new leptons and quarks will mix with the SM ordinary fermions which have the same $U(1)_Q$ and $SU(3)_C$ assignments. This mixing will give rise to new currents which determine to a large extent their decay properties and allow for a new production mechanism: single production in association with their light partners; see Fig. 1. The mixing pattern depends sensitively on the considered model and, in general, is rather complicated especially if one includes the mixing between different generations. However, this inter–generational mixing should be very small since it would induce at the tree level, flavor changing neutral currents which are severely constrained by existing data [4].

In the present analysis, we will neglect the inter–generational mixing and treat the few remaining mixing angles as phenomenological parameters. To describe our parametrization,
let us explicitly write down the interaction of the electron and its associated neutrino with exotic charged and neutral heavy leptons. Allowing for both left–handed and right–handed mixing, and assuming small angles so that one can write \( \sin \zeta_{L,R} \approx \zeta_{L,R} \), the Lagrangian describing the transitions between \( e, \nu_e \) and the heavy leptons \( N, E \) of the first generation is

\[
\mathcal{L} = g_W \left[ \zeta_{L}^{E} \bar{\nu}_e \gamma_\mu E_L + \zeta_{R}^{E} \bar{\nu}_e \gamma_\mu E_R \right] W^\mu + g_Z \left[ \zeta_{L}^{E} \bar{\nu}_\mu E_L + \zeta_{R}^{E} \bar{\nu}_\mu E_R \right] Z^\mu + \text{h.c.}
\]

with \( g_W = e/\sqrt{2} s_W \) and \( g_Z = e/2 s_W c_W \). The generalization to the other lepton families and to quarks is obvious. Note that in principle, the indices \( L,R \) refer to the handedness of the heavy lepton mixing with its light partners, but since the latter are almost massless they are also the chirality of the heavy leptons.

### 2.2 Decay Modes

The heavy leptons decay through mixing into massive gauge bosons plus their ordinary light partners; for masses larger than \( M_W/M_Z \) the vector bosons will be on–shell and decay into light quarks and leptons; Fig. 1c. Using the scaled masses \( v_{W,Z} = M^2_{W,Z}/m_F^2 \), the partial decay widths are

\[
\Gamma(F_{L,R} \to Z f) = \frac{\alpha}{32 s_W^2 c_W^2} \left( \zeta_{L,R}^{F} \right)^2 \frac{m_F^3}{M_Z^2} (1 - v_Z)^2 (1 + 2v_Z)
\]

\[
\Gamma(F_{L,R} \to W f') = \frac{\alpha}{16 s_W^2 c_W^2} \left( \zeta_{L,R}^{F} \right)^2 \frac{m_F^3}{M_Z^2} (1 - v_W)^2 (1 + 2v_W)
\]

For small mixing angles, the heavy fermions have very narrow widths: for \( \zeta_L/\zeta_R \sim 0.1 \) and masses around 100 GeV the partial decay widths are less than 10 MeV. The decay widths increase rapidly for increasing fermion masses, \( \Gamma \sim m_F^3 \), but for allowed values of the mixing angles [see below], do not exceed the 100 GeV range even for \( m_F \sim \mathcal{O}(1 \text{ TeV}) \). The charged current decay mode is always dominant and for fermion masses much larger than \( M_Z \), the branching ratios are 1/3 and 2/3 for the neutral and charged current decays, respectively. Note that for Majorana neutrinos, both the \( l^- W^+/\nu_l Z \) and \( l^+ W^-/\bar{\nu}_l Z \) are possible; this makes its total decay width twice as large as for Dirac neutrinos. However, for small mixing angles and/or moderate masses, the widths are too small to be resolved experimentally and no distinction between Dirac and Majorana neutrinos can be made at this level.

To fully reconstruct the heavy fermion from its final decay products one needs the branching ratio of its decay into visible particles. Neglecting the mixing between different generations, the branching ratios for the decays of the heavy leptons into charged ordinary leptons and gauge bosons which subsequently decay into jets take the asymptotic values [for large \( m_L \)]

\[
\text{Br}(E^- \to Z l^- \to jjl^-) \approx \frac{1}{3} \times 0.70 \approx 0.23
\]

\[
\text{Br}(N \to W^+l^- \to jjl^-) \approx \frac{2}{3} \times 0.66 \approx 0.43
\]
In the case of $E$, one can also include the cleaner $Z \to e^+e^- + \mu^+\mu^-$ decays, but the branching ratio is rather small: $\sim 6\%$ compared to $\sim 70\%$ for $Z \to$ hadrons.

For fermions which belong to the same isodoublets, the decay of the heaviest fermion into the lighter one plus a $W$ boson is in principle also possible. However, since the new fermions must be approximately degenerate not to induce large radiative corrections to electroweak parameters such as $M_W$ and $\sin^2 \theta_W$, the $W$ boson should be off-shell and these decays are strongly suppressed unless the mixing angles are prohibitively small. These cascade decays, the amplitudes of which can be found in (I), will not be considered here.

Finally, if the new fermions have masses smaller than $M_W$, the intermediate vector bosons will be off-shell and the decays are three-body decays. Complete expressions for partial widths, angular and energy distributions can be found in (I). In this paper, we will assume that the new fermions are always heavier than the $W$ boson; this will be justified below.

2.3. Constraints

Let us now briefly summarize the present experimental constraints on the masses of the new fermions and on their mixing with the ordinary fermions. The mixing will alter the couplings of the electroweak gauge bosons to light quarks and leptons from their SM values. Since the couplings of the latter to the $Z$ boson have been very accurately determined at LEP100 [through the measurement of total, partial and invisible decay widths as well as forward–backward and polarization asymmetries] and found to agree with the SM predictions up to the level of one percent, the mixing angles are constrained to be smaller than $\mathcal{O}(10^{-1})$ [10]. In the case of leptons, if the left and right–handed mixing angles have the same size, the precise measurement of the anomalous magnetic moments of the electron and muon leads to even more stringent constraints $\theta_{\text{mix}} < \mathcal{O}(10^{-2})$ [11]. Indeed, without the chiral protection $\zeta_L \text{ or } \zeta_R \sim 0$, the contribution of heavy lepton loops to $(g-2)_{e,\mu}$ will be proportional to $m_e/m_L$ [rather than to $m_{e,\mu}^2/m_L^2$] and very small $\theta_{\text{mix}}$ and/or extremely large $m_L$ are needed to protect the electron and muon from acquiring a too large magnetic moment.

From the negative search of new states and from the measurement of $Z$ decay widths at LEP100, one can infer a bound of the order of $M_Z/2$ on the masses of the new fermions [12] independently of their mixing, except for singlet neutrinos which have no full weak couplings to the $Z$ boson. Masses up to $m_F \sim M_W$ can be probed at LEP200.

In the case of heavy neutrinos, including the gauge singlets, an additional constraint is provided by the negative search [13, 14] of these states through single production in $Z$ decays: $Z \to \bar{\nu}_e + N \to \bar{\nu}_e + e^-W \to \bar{\nu}_e + e^-jj$. If the $\nu N$ mixing angle is of the order of $\sim 0.1$ or larger, the neutrino must be heavier than the $W$ boson [13]. A similar mass bound can be established for the charged lepton, which in $Z$ decays leads to the same final state, $Z \to e^+E^- \to e^+\nu_e W^- \to \nu_e e^-jj$, with approximately the same rate. Note that for mixing angles much smaller than $\mathcal{O}(10^{-2})$, no bound can be derived on the singlet neutrinos masses:
the production cross section is small and/or the heavy neutrino escapes detection because of its too large decay length.

Since for quarks and for the new leptons which have full couplings to the $Z$ boson, masses smaller than $M_W$ can be probed at LEP200 in pair production, and because the singlet neutrinos with such masses and with not too small mixing angles [which would prohibit their single production also at higher energies] are already ruled out, we will assume in our discussion at a 500 GeV collider, that the new fermions are heavier than the $W$ boson.

Finally, let us mention the constraints on the new gauge bosons which are also predicted by extended gauge models and which might have observable effects in the production and decay of the new fermions. Direct searches at present hadron colliders and indirect searches through the high–precision LEP measurements lead to bounds of the order of 200–400 GeV on the new vector boson mass, depending on the particular chosen model [3]; the accessible mass range can be pushed up to $\sim$ 500 GeV in the near future. Masses up to 3 TeV can be probed at the 500 GeV collider itself [13]. In our analysis, we will assume that these new gauge bosons are heavy enough not to affect the physics at the 500 GeV scale. The mixing between these new gauge bosons and the standard $Z$ boson would alter the couplings of the latter to ordinary and new fermions. However, since it is constrained to be smaller than $O(10^{-2})$ [3], this mixing will also be neglected in the present analysis.

3. Pair production

3.1 Cross sections and distributions

If their masses are smaller than the beam energy, the new fermions can be pair produced in $e^+e^-$ collisions; Fig. 1a. For charged fermions the process proceeds through $s$–channel $\gamma$ and $Z$ boson exchange, while for non–singlet heavy neutrinos only $Z$ boson exchange is present. There are also contributions from $t$–channel $W/Z$ exchange in the case of the first generation of heavy leptons, but they are quadratically suppressed by mixing angle factors and therefore totally negligible for experimentally allowed values of these angles.

The differential cross section $d\sigma/d\cos\theta$ [with $\theta$ specifying the direction of the fermion $F$ with respect to the incoming electron] for the process $e^+e^-\to FF$ has been given in (I) in the general case where several channels are present. In the case where only $\gamma$ and $Z$ $s$–channels exchanges contribute, the expression simplifies to

$$\frac{d\sigma}{d\cos\theta} = \frac{3}{8}\sigma_0 N_c \beta_F [\left\{ (1 + \cos^2 \theta)(\sigma_{VV} + \beta_F^2 \sigma_{AA}) + (1 - \beta^2) \sin^2 \theta \sigma_{VV} + 2\beta_F \cos \theta \sigma_{VA} \right\} (3.1)$$

where $N_c$ is a color factor, $\sigma_0 = 4\pi\alpha^2/3s$ the point–like QED cross section for muon pair production and $\beta_F = (1 - 4m_F^2/s)^{1/2}$ is the velocity of the fermion in the final state; in terms of the $FF\gamma$ and $FFZ$ couplings, the reduced cross sections read [neglecting the small decay width of the $Z$ boson]
\[
\sigma_{VV} = \frac{e^2 e_F^2 + 2e_v e_F v_F}{1 - M_Z^2/s} + \frac{(a_e^2 + v_e^2) v_F^2}{(1 - M_Z^2/s)^2}
\]
\[
\sigma_{AA} = \frac{(a_e^2 + v_e^2) a_F^2}{(1 - M_Z^2/s)^2}
\]
\[
\sigma_{VA} = \frac{2e_v e_F a_v a_F}{1 - M_Z^2/s} + \frac{4v_e a_v v_F a_F}{(1 - M_Z^2/s)^2}
\]

(3.2)

The total production cross sections are simply given by

\[
\sigma_F = \sigma_0 N_c \left[ \frac{1}{2} \beta_F (3 - \beta_F^2) \sigma_{VV} + \beta_F^2 \sigma_{AA} \right]
\]

(3.3)

The cross sections for the pair production of mirror and vector heavy leptons are displayed in Fig. 2 for a center of mass energy of 500 GeV. For charged leptons, they are of the order of 0.5 pb for masses not too close to the beam energy. With the expected integrated luminosity of 20 fb\(^{-1}\) for a 500 GeV \(e^+e^-\) collider, this corresponds to \(\sim 10,000\) events. Even for masses very close to the kinematical limit, \(\sim\) a few GeV, the cross sections exceed the 100 fb level, leading to a large number of events. Due to the absence of the photon exchange, the cross sections for the heavy neutrinos are smaller than for charged leptons. But the event rates are also very large, especially for vector neutrinos, allowing for the discovery of these leptons up to the kinematical limit. For instance, in the case of the mirror neutrino (which has the smallest cross section) with a mass of 247.5 GeV, one is still left with \(\sim 200\) events a year, assuming \(\int \mathcal{L} = 20\) fb\(^{-1}\).

The angular distributions are shown in Fig. 3 for lepton masses of 150 GeV. Because they have no axial–vector couplings, vector leptons have forward–backward symmetric angular distributions [the term proportional to \(\cos \theta\) is zero] contrary to mirror fermions for which \(d\sigma/d\cos \theta\) is larger in the backward–direction. The forward–backward asymmetries are given by

\[
A_F = \frac{3}{4} \beta_F \frac{\sigma_{VA}}{1/2(3 - \beta_F^2) \sigma_{VV} + \beta_F^2 \sigma_{AA}}
\]

(3.4)

For mirror leptons the asymmetries are sizeable [except near threshold where they vanish] and negative, contrary to the case of sequential heavy leptons for which they have the same magnitude but with the opposite sign. Of course, for vector leptons, the forward–backward asymmetries are zero.

The polarization four–vector \(P_\mu\) of the final state fermion \(F\) is defined by \(d\sigma^{\text{pol}}(\cos \theta) = d\sigma^{\text{unpol}}(\cos \theta) \times [1 + P_\mu n_\mu]\), with \(n_\mu\) the spin vector. In the \(F\) rest frame, the components are \((0, P_\perp, 0, P_\parallel)\) with \(P_\perp\) and \(P_\parallel\) being the transverse and longitudinal polarizations with respect to the flight direction; their expressions can be found in (I). The two components are shown in Fig. 4 for lepton masses of 150 GeV, as a function of the scattering angle.
For vector leptons, the magnitude of $P_\perp$ and $P_{||}$ is four times larger for $E$ than for $N$ but [due to the fact that their couplings to both the photon and the $Z$ are vectorial] the shape is exactly the same; $P_\perp$ is positive, forward–backward symmetric and maximal (minimal) for $\cos \theta = \pm 1(0)$ while $P_{||}$ is monotonically decreasing for increasing $\cos \theta$ and is zero for $\theta = \pi/2$. Mirror charged leptons have very small transverse and longitudinal polarizations contrary to their neutral partners.

Averaged over the polar angle, the longitudinal polarization $\langle P_{||} \rangle$ is zero for vector leptons, while $P_\perp$ is small and positive. For mirror leptons, the two components have the same absolute values as for sequential leptons, only $P_{||}$ has the opposite sign.

Hence, the final polarization and the angular distributions of the produced leptons are very useful to discriminate between different types of particles: mirror, vector and also sequential fermions.

3.2. Signals and backgrounds

In this subsection, we discuss the signals and backgrounds for the pair production of the new fermions, concentrating on the heavy leptons which are more difficult to search for at hadron colliders [9].

The pair production of charged and neutral heavy leptons leads to final states with two ordinary leptons and two massive gauge bosons which subsequently decay into four massless fermions. To fully reconstruct the heavy leptons from their decay products, one needs the final states with charged ordinary leptons from the first vertex and jets from the decay of the gauge bosons [in the case of the $Z$ boson one can also include the $Z \to e^+e^- + \mu^+\mu^-$ decays, but the branching ratio is rather small, $\sim 6\%$ compared to $\sim 70\%$ for $Z \to$ hadrons]. Using the notation of the first generation, the branching fractions for these final states for large lepton masses are

\[
\begin{align*}
    e^+e^- &\to E^+E^- \to e^+e^-ZZ \to e^+e^- + 4 \text{ jets} \quad \text{B.R.} \approx 5.5\% \\
    e^+e^- &\to \bar{N}N \to e^+e^-WW \to e^+e^- + 4 \text{ jets} \quad \text{B.R.} \approx 20\% \quad (3.5)
\end{align*}
\]

The branching fraction for this specific final state is four times larger in the case of neutral leptons than for charged leptons but since the production cross sections for the latter are a factor 2.5 to 5 larger than for heavy neutrinos, the number of events for the two processes are of the same order. For instance, for a heavy lepton with a mass of 150 GeV, assuming a luminosity $\int \mathcal{L} = 20 \text{ fb}^{-1}$, one has 520 and 460 events for vector and mirror electrons respectively, while for vector and mirror neutrinos one has 820 and 320 events respectively.

Several conventional processes lead to the same final states, the most important ones being triple gauge boson production:

\[
\begin{align*}
    e^+e^- &\to ZZZ[\gamma] \to (e^+e^-) + (jj)(jj) \\
    e^+e^- &\to WWZ[\gamma] \to (e^+e^-) + (jj)(jj) \quad (3.6)
\end{align*}
\]
However, compared to the signals, these processes are of higher order \([\text{suppressed by one extra power of } \alpha]\) and the cross sections are rather small: if one neglects the Higgs induced contributions, one has \(\sigma(e^+e^- \rightarrow WWZ) \approx 40 \text{ fb}\) and \(\sigma(e^+e^- \rightarrow ZZ\gamma)\) is only \(\approx 1 \text{ fb}\). If in addition the \(Z\) boson is forced to decay into charged leptons, the cross sections drop by more than an order of magnitude. For a luminosity of \(20 \text{ fb}^{-1}\) one has only \(25 \, e^+e^- + 4\) jet events from the dominant \(WWZ\) process, to be compared with more than 300 events from the signals for \(m_L \approx 150 \text{ GeV}\). The cross sections for \(e^+e^- \rightarrow ZZ\gamma\) and \(WW\gamma\) are extremely small after forcing the virtual photon to have a mass of the order of \(M_Z\). Other background processes, lead to negligible cross sections at a c.m. energy of 500 GeV.

To be as close as possible to the experimental conditions, we will use in the following, a model detector for \(e^+e^-\) collisions at a c.m. energy of 500 GeV to simulate the efficiencies for reconstructing the signal events. For illustration, we will study the case of the pair production of charged leptons [the cross sections for mirror and vector leptons are nearly equal] with a mass of 200 GeV; before any cut is applied the cross section is 400 fb leading to 450 \(e^+e^- + 4\) jet events for an integrated luminosity of 20 \(\text{fb}^{-1}\). Since the cross sections for the production of the other types of leptons are approximately the same, the results for neutral heavy leptons should be similar. The background reactions will not be simulated since the cross sections are negligible, even before one requires that the invariant mass of one lepton and two jets combine to give the mass of the heavy lepton.

The process of pair production was simulated in PYTHIA5.6 \cite{16} by allowing for a fourth generation of fermions, which was then made to correspond to an excited copy of the first generation. Only the decay channel \(E \rightarrow Z\ell\) was turned on, with a weak matrix element similar to the one of the standard \(l \rightarrow W\nu\) process. This simulation should be representative, in terms of detector effects, of pair production of any lepton type.

In all cases, full hadronization was allowed to take place. Detector effects were taken into account by subjecting the particles to resolution smearing and acceptance cuts, as summarized in Table 1. The parameters for the model detector are similar to the “standard” set of Ref. \cite{17}, but with angular acceptance up to \(|\cos(\theta)| < 0.98\) for the electromagnetic calorimeter as well as for the charged particle tracker. This “standard” detector has similar properties to the ones of present high–energy colliders: in this respect we took a rather conservative approach since some improvements for the next generation detectors are expected.

All charged particles had their momentum “measured” by the central tracker, whereas the energy of photons and electrons was “reconstructed” with the resolution smearing appropriate for the electromagnetic calorimeter. In the case of electrons, the combined measurement of momentum and energy served to define the “reconstructed” \(4\)-vector. Muons were assumed to be identifiable by means of hypothetical muon chambers, or other subdetector type. Finally, other particles were assumed to be detected by the hadronic calorimeter.

As explained above, the simulation of detector effects imposed minimum requirements on
the particle directions and momenta. The loss in solid angle is small in this case [but not for processes with an angular distribution peaked along the beam line]. A further loss in efficiency resulted from the lepton selection criterion: in order to minimize backgrounds, only electrons with momentum higher than $p_{l}^{\text{min}} = 30 \text{ GeV}$ were considered. For pair production of not–too–heavy leptons, $p_{l}^{\text{min}}$ should be set lower.

In order to reconstruct the event consisting of four jets and two leptons, the following procedure was adopted:

(i) Accept only events having one and only one “detected” lepton pair.

(ii) Excluding these two lepton tracks, construct four jets out of the remaining particles, using the Durham algorithm [18].

(iii) Associate a pair of jets with each lepton in such a way that the following $\chi^2$ is minimized:

$$\chi^2 = \left( E_{\text{beam}} - E_{j1} - E_{j2} - E_{e+} \right)^2 \left( E_{\text{beam}} - E_{j3} - E_{j4} - E_{e-} \right)^2.$$

Fig. 5 shows the resulting mass histogram of one of the reconstructed $E^+$ for the case of a pair of charged heavy leptons of mass 200 GeV [an exactly analogous histogram can be made for $E^-$]. The tails of the distribution can be cleanly removed, as shown by the shaded area in the figure, with a loss of efficiency of 40%, by requiring that each of the invariant jet–jet masses be consistent with the mass of a $Z$ boson within 15 GeV. Note that only 6% of the events did not pass the cuts on the solid angle and the minimum lepton energy. The histogram is normalized for an integrated luminosity of 20 fb$^{-1}$ and in this case, one is left with 250 events [in the dashed area] after all cuts were applied.

As discussed above, similar numbers can be obtained for the other lepton types. Therefore, it is clear that the detection of pair–produced heavy leptons should be very easy at $e^+e^-$ colliders.

4. Single production

4.1. Cross sections and distributions

In $e^+e^-$ collisions one can also have access to the new fermions via single production in association with light fermions, if fermion mixing is not too small; Fig. 1b. The process proceeds through $s$–channel $Z$ exchange in the case of quarks and second/third generation leptons. For the first generation heavy leptons, one has to include additional $t$–channel gauge boson exchanges: $W$ exchange for $N$ and $Z$ exchange for $E$. Complete expressions for the angular distributions and the total cross sections in the general case can be found in (I). Neglecting the mass of the light fermion partner, the total cross sections for leptons of the second and third generations and for quarks take the simple form
\[
\sigma(F_{L,R}) = \sigma_0 N_c \frac{(e_{L,R}^f)^2}{128 c_W^4 s_W^4} (a_e^2 + v_e^2) \frac{(1 - \mu^2)^2 (1 + \mu^2 / 2)}{(1 - z)^2}
\]

where now \(a_e = -1\), \(v_e = -1 + 4 s_W^2\), \(\mu = m_h^2 / s\) and \(z = M_Z^2 / s\). The expressions of the cross section for the production of the first generation heavy leptons are more involved in the general case. However, for only \(Z\) and \(W\) exchange, once all the couplings are specified, they take the rather simple form

\[
\sigma(E_{L,R}) = \sigma_0 \frac{3 (e_{L,R}^E)^2}{256 c_W^4 s_W^4} \left\{ (v_e \pm a_e)^2 \left[ - \left( 2 - \mu^2 + 2 z + 2 (1 + z - \mu^2) \frac{1 + z}{1 - z} \right) L_Z \right. \\
+ \frac{1}{3} \frac{(1 - \mu^2)^2 (1 + \mu^2 / 2)}{(1 - z)^2} + \frac{(1 - \mu^2)(3 + 2 z - \mu^2)}{1 - z} + \frac{(1 - \mu^2)(1 + 2 z)}{z} \right] \\
+ (v_e \mp a_e)^2 \left[ - \left( (2 z - \mu^2)(1 - z) + 2 z (z - \mu^2) \right) \frac{1}{1 - z} L_Z \right. \\
\left. + \frac{1}{3} \frac{(1 - \mu^2)^2 (1 + \mu^2 / 2)}{(1 - z)^2} - \frac{(1 - \mu^2)(1 - 2 z + \mu^2)}{1 - z} + \frac{(1 - \mu^2)(1 + 2 z - 2 \mu^2)}{1 + z - \mu^2} \right] \right\}
\]

\[
\sigma(N_{L,R}) = \frac{3 \sigma_0}{16 s_W^4} \left\{ \frac{1}{24} \frac{(e_{L,R}^N)^2}{c_W^4} (v_e^2 + a_e^2) \frac{(1 - \mu^2)^2 (1 + \mu^2 / 2)}{(1 - z)^2} \\
- \frac{v_e + a_e}{4 c_W^2} \frac{s_W^2 N_{L,R}}{1 - z} \left[ (1 - \mu^2)(1 - 2 w + \mu^2) + 2 w (w - \mu^2) L_W \right] \\
+ (\tilde{\xi}_{L,R}^N)^2 \left[ \frac{(1 - \mu^2)(1 + 2 w - 2 \mu^2)}{1 + w - \mu^2} + (\mu^2 - 2 w) L_W \right] \right\}
\]

where for \(V = W, Z\) one has

\[
L_V = \log \frac{1 - \mu^2 + v}{v} \quad \text{with} \quad v = M_W^2 / s \quad \text{or} \quad M_Z^2 / s
\]

The cross sections are the same for the charge conjugate states.

The cross section for the single production of the heavy leptons in association with their light partners at a 500 GeV c.m. energy are displayed in Fig. 6 as a function of \(m_h\); for
the left and right-handed mixing angles, we have assumed values close to the experimental bounds $\zeta_{N_L,R}^{N_e} = \zeta_{N_L,R}^{N_\nu} = 0.1$. In the case of $N_L$ production, the cross section is of the order of 1 pb for masses of $\mathcal{O}(100 \text{ GeV})$, which corresponds to $2 \times 10^4$ events per year with a luminosity of 20 fb$^{-1}$. It drops to $\sim 0.1$ pb for $m_N \sim 450$ GeV. For smaller $\zeta_L$ values the cross section has to be scaled down; for $\zeta_L$ values ten times smaller, one is left with $\sim 1$ fb for $m_N \sim 450$ GeV. For this mass value and for $\zeta_R = 0.01$, one is left with only $\sim 10$ events for an integrated luminosity of 20 fb$^{-1}$.

The cross sections for the production of $\bar{E}e$ and $E\bar{e}$ pairs through right and left-handed mixing, which are approximately equal due to the fact that the vectorial coupling of the electron to the $Z$ is practically zero for $s_W^2 \simeq 1/4$, are one order of magnitude smaller than $\sigma(N_L)$ for the same value of the mixing angle. It is practically constant for most of the relevant range of $m_N$, being approximately 40 fb, and it starts to drop only for $m_N \sim 450$ GeV. For this mass value and for $\zeta_R = 0.01$, one is left with only $\sim 10$ events for an integrated luminosity of 20 fb$^{-1}$.

The cross sections for the production of $\bar{E}e$ and $E\bar{e}$ pairs through right and left-handed mixing, which are approximately equal due to the fact that the vectorial coupling of the electron to the $Z$ is practically zero for $s_W^2 \simeq 1/4$, are one order of magnitude smaller than $\sigma(N_L)$ for the same value of the mixing angle. It is practically constant for most of the relevant range of $m_N$, being approximately 40 fb, and it starts to drop only for $m_N \sim 450$ GeV. For this mass value and for $\zeta_R = 0.01$, one is left with only $\sim 10$ events for an integrated luminosity of 20 fb$^{-1}$.

The differential cross sections are shown in Fig. 7. In the case of Dirac $N_L$ production, $d\sigma/dc\cos\theta$ is peaked in the forward direction, while for $N_R$ it is practically flat except near $c\cos\theta=1$. For the production of Majorana particles, the angular distribution is forward–backward symmetric and in the case of $N_L$, it is peaked in both directions $[\text{I}]$. Exploiting the different behavior of $d\sigma/dc\cos\theta$, it should be rather easy to distinguish between left–handed and right–handed mixings and between Dirac and Majorana neutrinos. As in the case of $N_L$, the angular distributions for $E$ production $d\sigma/dc\cos\theta$ are peaked in the forward direction due the $t$–channel $Z$ exchange. Since they are practically the same for $E_L$ and $E_R$ production, it is very difficult to distinguish between left and right–handed mixing. In this respect, the polarization of the final heavy state can be very useful.

The longitudinal and transverse components of the polarization vector of heavy fermions produced in association with light partners have been given in (I). In Fig. 8, the two components of the polarization vectors of $E$ and $N$ are shown as a function of the scattering angle and for a lepton mass of 400 GeV. In the case of $N$, the transverse components are positive and similar in shape for left–handed and right–handed mixing; the longitudinal polarizations are rather different for $N_L$ and $N_R$. In the case of the charged leptons, both longitudinal
and transverse components are completely different for left and right–handed mixings, in particular, they are of opposite signs for a given value of $\theta$. This important feature will be useful to distinguish $E$ particles which couple to the $Z$ boson with left or right–handed currents, in particular as the distinction is rather difficult using the angular distributions.

4.2. Signals and backgrounds

The single production of the heavy leptons leads to less complicated final states than for pair production: just two ordinary leptons, one coming from the production process and the other from the decay of the heavy lepton, and a gauge boson which subsequently decays into two massless fermions. As previously discussed for the case of pair production, to fully reconstruct the heavy leptons from their decay products one has to concentrate on the following final states

\[
e^+e^- \rightarrow E^+e^- \rightarrow e^+e^\mp Z \rightarrow e^+e^- + 2 \text{ jets} \quad \text{B.R.} \approx 23\%
\]
\[
e^+e^- \rightarrow \bar{\nu}N/\nu\bar{N} \rightarrow \nu_e e^\mp W^\mp \rightarrow \nu_e e^\mp + 2 \text{ jets} \quad \text{B.R.} \approx 43\% \quad (4.6)
\]

where in the case of $N$, $\nu_e$ stands for the electronic neutrino and its conjugate state. The branching fractions are for large lepton masses compared to $M_W$ and for $E$ one can also use the decays $Z \rightarrow e^+e^-/\mu^+\mu^-$. \(13\)

In the following we will simulate the single production of the heavy leptons which leads to the final states shown above, as well as the corresponding background reactions. We will concentrate on the case of the first generation neutral and charged leptons and assume a left–handed mixing with $\theta_{\text{mix}}$ taken to be $\theta_{\text{mix}} = 0.025$ for $N$ and $0.05$ for $E$.

The processes for the production of the charged and neutral heavy leptons $E$ and $N$ were simulated by incorporating in the PYTHIA \[16\] generator the matrix elements for the three body reactions: $e^+e^- \rightarrow N\nu_e/E^\pm e^\mp \rightarrow \nu_e e^\mp W^\mp/e^\pm e^\mp Z$ and forcing the gauge bosons to decay hadronically; full hadronization was allowed to take place. All the resulting particles were then subjected to detector resolution and acceptance effects. The parameters for the detector are shown in Table 1. In the case of the neutral heavy lepton, the missing momentum vector was calculated and subsequently assumed to be the reconstructed neutrino momentum. The background processes were simulated using existing parameter options in PYTHIA.

In the case of the charged lepton $E$, the signal consists of an $e^+e^-$ pair and two jets. Other processes likely to produce such a configuration are:

(i) $e^+e^- \rightarrow e^+e^-Z$, with the $Z$ decaying hadronically; the cross section is 3800 fb.

(ii) $e^+e^- \rightarrow ZZ$, where the cross section is 615 fb after forcing one of the $Z$ bosons to decay hadronically and the other into electrons.

(iii) $e^+e^- \rightarrow t\bar{t}$, followed by $t \rightarrow bW$ and leading to two electrons and 2 jets but with missing momentum.
(iv) $\gamma\gamma \rightarrow e^+e^-q\bar{q}$ which has a large cross section but the jets have small invariant masses and the resulting events have the primary electrons going mostly along the beam pipe.

In the case of the neutral lepton $N$, the signal consists of an electron, a pair of jets and missing momentum. The backgrounds that one has to consider are:

(i) $e^+e^- \rightarrow e\nu W$ with a cross section of $5800$ fb when the $W$ boson decays hadronically.

(ii) $e^+e^- \rightarrow WW$ where one of the $W$'s decays hadronically and the other to an $e\nu$ pair and the cross section in this case is $1140$ fb;

(iii) $\gamma\gamma \rightarrow e(e)q\bar{q}$ where one of the electrons escapes observation; the cross section is large but the jets have small invariant masses.

The analysis proceeded by selecting among all “reconstructed” final state particles the electrons and/or positrons with a momentum greater than $30$ GeV. All particles within an angle $10^\circ$ along the beam direction were rejected. Since for the signal, we have incorporated the correlations between all the particles involved in the process, which have been worked out in detail in (I), we have full control on the kinematics of the process. This allowed us to select efficiently the cuts which suppress considerably the background processes without affecting significantly the signals.

For the charged lepton, the following cuts were found to be very efficient against the background without affecting drastically the signals:

(1) Require one and only one $e^+e^-$ pair; this cut makes that one has not to consider the huge background from single $W$ production, and eliminates a very large part of the $e^+e^-Z$ background in which one of the electrons goes along the beam pipe and therefore escapes detection.

(2) The invariant mass of the two jets should reconstruct to the $Z$ boson mass, $85 < M_{jj} < 105$ GeV; the lower bound was set intentionally high so as to avoid a possibility of confusing a $Z$ with a $W$ boson. This cut suppresses the $ZZ$ and $e^+e^-Z$ backgrounds by approximately a factor of two.

(3) In the case of $E^-$ production, the momentum of the positron should be $|p_{l^+}| > (E_{\text{beam}} - M_E^2/4E_{\text{beam}}) - 40$ GeV and the momentum of the electron $|p_{l^-}| > \frac{2}{3}M_E - 133$ GeV, with $M_E$ the reconstructed mass of the heavy lepton; these kinematic constraints ensure energy-momentum conservation, with some tolerance for detector resolution effects. For $E^+$ production, one has to interchange the cuts on the $e^+$ and $e^-$. 

(4) The invariant mass of the $e^+e^-$ pair should be different from $M_Z$: $|M_{l^+l^-} - M_Z| > 12$ GeV; this cut is effective against the $ZZ$ background since the latter process is suppressed by a factor of two.
(5) Cuts on the angle between the $Z$ boson and the initial electron $\cos \theta_Z > -(M_E + 440)/720$ and $\cos \theta_Z < (2100 - M_E)/2000$, which are necessary to reduce the background from $e^+e^- \rightarrow e^+e^-Z$; this cut applies for $E^-$ production, similar ones apply for $E^+$. 

(6) An angular cut $\cos \theta_{ll} < 0.5$ since in the signal, the two leptons are mostly back–to–back. 

In the case of neutral leptons, similar cuts were applied:

(1) One and only one electron or positron; this cut is very effective against the background from single $W$ production where the electron goes along the beam pipe and therefore is undetected.

(2) The invariant mass of the two jets should reconstruct to the $W$ boson mass within 10 GeV, $70 < M_{jj} < 90$ GeV.

(3) The missing momentum is constrained to be $|p_\nu| > (E_{\text{beam}} - M_N^2/4E_{\text{beam}}) - 40$ and the momentum of the charged lepton to be $|p_l| > \frac{2}{3}M_N - 133$.

(4) The invariant mass of the two leptons should be large: $M_{ll} > 120$ GeV; this cut is effective against the $WW$ background since in this case the invariant mass peaks at $M_W$.

(5) The angular cut $\cos \theta_W < 2.58 - M_N/240$ reduces the background from single $W$ production $e^+e^- \rightarrow e^-\bar{\nu}W$.

(6) An angular cut $\cos \theta_{l\nu} < 0.5$; here also the two leptons from the signal are mostly back–to–back.

The effects of the previous selection criteria on the main background processes and on the signals for single production of $N$ and $E$ with masses of 350 GeV and mixing angles of 0.025 and 0.05 respectively are shown in Table 2 and 3. After these cuts, no events from the $t\bar{t}$ background survived. In the case of the $\gamma\gamma$ background, because it was not possible to generate a sufficient number of events, only an estimate of < 10 surviving background events/5 GeV can be given for the mass region $M < 250$ GeV. The background events from vector boson pair production have been suppressed to a very low level; the background events from single $W$ and $Z$ production are relatively higher. Note that in the tables, the event number for the signal and the latter background are of the same order: this is simply due to the fact that we have taken a large bin [50 GeV] for the reconstructed lepton–jets invariant mass. Note also that although we have tried to optimize all the above cuts, we believe that further improvements are still possible.

Fig. 9 shows the reconstructed invariant mass histograms for heavy leptons with masses of 250, 350 and 450 GeV and with mixing angles $\theta_{\text{mix}} = 0.05$ in the case of the charged lepton and $\theta_{\text{mix}} = 0.025$ for the heavy neutrino. For these values, one can see that the signal peaks stand out clearly from the background events, especially for heavy lepton masses not too
close to the total c.m. energy of the collider. Only the background events from single gauge boson production are relatively important.

For smaller mixing angles, the signal cross sections have to be scaled down correspondingly. For lepton masses of the order of 450 GeV, only slightly smaller $\theta_{\text{mix}}$ values can be probed, while for masses around 350 GeV one can go down by at least a factor of two. The situation is much more favorable for heavy neutrinos than for charged leptons, the cross section being one order of magnitude larger. For instance, assuming a mass of 350 GeV and requiring that the ratio of the signal events to the square root of the background events be larger than unity, one can probe mixing angles down to $\theta_{\text{mix}} \sim 0.005$ for neutral leptons and $\theta_{\text{mix}} \sim 0.03$ for charged leptons.

5. Summary

In this paper we have studied the production, at future high-energy $e^+e^-$ colliders, of new heavy fermions predicted by several extensions of the Standard Model. Exploiting the various studies done for a 500 GeV $e^+e^-$ collider, we have used a model detector to analyze the discovery potential of an $e^+e^-$ linear collider operating at this center of mass energy.

If the new fermions have non-zero electromagnetic and weak charges, they can be pair produced when their masses are smaller than the beam energy of the $e^+e^-$ collider. At a 500 GeV collider, the cross sections are fairly large being, up to phase space suppression factors, of the order of the point like QED cross section for muon pair production. This leads to samples of several thousand events per year for a luminosity of $\mathcal{L} = 10^{33}$ cm$^{-2}$s$^{-1}$. The backgrounds are rather small and the signals very clean, the large number of events allows to probe masses up to practically the kinematical limit of 250 GeV.

The mixing between the new fermions and the ordinary fermions of the Standard Model allows an additional production mechanism: the single production in association with their light ordinary partners. The cross sections are suppressed by mixing angle factors and are very small for quarks and for second and third families of leptons for which the process is mediated only by $s$-channel $Z$ boson exchange. In the case of the first generation of new leptons, the reactions are mediated by additional $t$-channel exchanges: $W$ exchange for heavy neutrinos and $Z$ exchange for charged leptons. This increases the cross sections by several orders of magnitude, and leads to large numbers of events for not too small mixing angles. Because of the purity of the environment of $e^+e^-$ colliders, the signals are clear and rather easy to separate from backgrounds due to more conventional processes. Lepton masses close to the total center of mass energy of the collider can be probed this way.
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Table Captions

Tab. 1 Assumed parameters of a standard detector for $e^+e^-$ collisions at a c.m. energy of 500 GeV.

Tab. 2 Cross sections for the single production of a heavy charged lepton detected in the process $e^+e^- \rightarrow e^\mp E^\pm \rightarrow e^\mp e^-jj$ and for the principal background processes from $e^+e^- \rightarrow ZZ$ and $e^+e^- \rightarrow e^+e^-Z$, after successive application of selection criteria, as described in the text. For the heavy charged lepton, a mass of 350 GeV and a mixing angle of 0.05 are assumed.

Tab. 3 Cross sections for the single production of a heavy neutral lepton detected in the process $e^+e^- \rightarrow \nu_eN \rightarrow \nu_ejj$ and for the principal background processes $e^+e^- \rightarrow WW$ and $e^+e^- \rightarrow \nu_eW$, after successive application of selection criteria, as described in the text. For the heavy neutrino, a mass of 350 GeV and a mixing angle of 0.025 are assumed.
Figure Captions

Fig. 1 Feynman diagrams for the pair production of heavy leptons (a), for their single production in association with their light partners (b) and for their decay into a light lepton and a gauge boson which subsequently decays into two fermions.

Fig. 2 Total cross sections for the pair production of vector and mirror charged and neutral heavy leptons at a c.m. energy of $\sqrt{s} = 500$ GeV, as a function of the mass.

Fig. 3 Differential cross sections in the pair production of charged and neutral leptons as a function the scattering angle. The c.m. energy is $\sqrt{s} = 500$ GeV and the mass of the lepton is fixed to 150 GeV.

Fig. 4 Longitudinal and transverse components of the polarization vectors of the pair produced charged and neutral leptons as a function of the scattering angle. The lepton masses are fixed to 150 GeV.

Fig. 5 Reconstructed invariant mass histograms for pair produced heavy charged lepton with masses of 200 GeV in the decays $E^- \rightarrow e^- Z \rightarrow e^- jj$. For the various cuts and experimental resolutions see the main text.

Fig. 6 Cross sections for the single production of charged and neutral heavy leptons in association with their light partners at $\sqrt{s} = 500$ GeV as a function of the lepton masses. The left and right–handed mixing angles are taken to be $\zeta_L, \zeta_R = 0.1$.

Fig. 7 Differential cross sections in the single production of charged and neutral leptons as a function the scattering angle. The c.m. energy is $\sqrt{s} = 500$ GeV, the mass of the lepton is fixed to 400 GeV and the mixing angles are taken to be $\zeta_L, \zeta_R = 0.1$.

Fig. 8 Longitudinal and transverse components of the polarization vectors of the singly produced charged and neutral heavy leptons as a function of the scattering angle. The lepton masses are fixed to 400 GeV.

Fig. 9 Reconstructed masses of the singly produced heavy leptons in the processes $e^+ e^- \rightarrow e^+ E^- \rightarrow e^+ e^- Z \rightarrow e^+ e^- jj$ (a) and $e^+ e^- \rightarrow \nu_e N \rightarrow \nu_e e^- W \rightarrow \nu_e e^- jj$ (b) for three lepton masses 250, 350 and 450 GeV and for the main backgrounds. The mixing angles have been fixed to $\theta_{\text{mix}} = 0.05$ for the charged lepton and $\theta_{\text{mix}} = 0.025$ for the neutral lepton. For the various cuts and experimental resolutions see the main text.
|                          | \( \Delta p; A \) | \( \Delta p; B \) | \( \cos \theta_{cut} \) | \( \Delta \theta \) (mr) | \( \Delta \phi \) (mr) | \( p_T^{\min} \) (GeV) |
|--------------------------|------------------|------------------|------------------|------------------|------------------|-------------------|
| Charged Particle Tracker | 0.001            | 0.005            | 0.98             | 2                | 0.5              | 0.1               |
| Electromagnetic Calorimeter | \( \Delta E; A \) | \( \Delta E; B \) | \( \cos \theta_{cut} \) | \( \Delta \theta \) (mr) | \( \Delta \phi \) (mr) |
|                          | 0.08             | 0.02             | 0.98             | 2                | 2                |
| Hadronic Calorimeter     | \( \Delta E; A \) | \( \Delta E; B \) | \( \Delta \theta \) (mr) | \( \Delta \phi \) (mr) |
|                          | 0.60             | 0.02             | 10               | 10               |

Momentum resolution \( \Delta p/p = \sqrt{(Ap)^2 + B^2} \) (\( p \) in GeV)

Energy resolution \( \Delta E/E = \sqrt{A^2/E + B^2} \) (\( E \) in GeV)

Table 1:


| Process | $E^+e^- + E^-e^+$ | $e^+e^-Z$ | $ZZ$ |
|---------|-------------------|-------------|------|
| $\sigma$ [fb] | 9.5 | 4960 | 615 |
| $\times$ B.R. | 2.19 | 3470 | 28.8 |
| one $e^+e^-$ pair | 1.74 | 93.0 | 23.0 |
| $330 < M_E < 370$ GeV | 1.56 | 11.7 | 5.30 |
| $85 < M_Z < 105$ GeV | 1.41 | 5.84 | 2.87 |
| $|M_{ll} - M_Z| > 12$ GeV | 1.39 | 5.18 | 1.02 |
| $\cos \theta_{ll} < 0.5$ | 1.33 | 4.32 | 0.56 |
| $f(M_E, \cos \theta_Z)$ | 1.30 | 1.90 | 0.43 |
| kinem. cuts | 1.30 | 1.55 | 0.39 |

Table 2:
| Process                  | $\bar{N}\nu + \bar{N}$ | $e\nu W$ | $WW$ |
|-------------------------|-------------------------|----------|------|
| $\sigma$ [fb]           | 490                     | 8610     | 2600 |
| $\times$ B.R.           | 13.7                    | 5823     | 1140 |
| one $\nu$               | 13.2                    | 198      | 883  |
| $330 < M_N < 370$ GeV   | 12.5                    | 11.9     | 100  |
| $70 < M_W < 90$ GeV     | 12.3                    | 10.3     | 70.3 |
| $M_{l\nu} > 120$ GeV    | 11.8                    | 10.0     | 7.93 |
| $\cos\theta_{l\nu} < 0.5$ | 11.7                   | 10.0     | 7.80 |
| $f(M_N, \cos\theta_Z)$ | 11.7                    | 10.0     | 7.80 |
| kinem. cuts             | 11.7                    | 10.0     | 4.13 |

Table 3: