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Numerical study of radiative non-Darcy nanofluid flow over a stretching sheet with a convective Nield conditions and energy activation

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Abstract: Numerous industrial processes such as continuous metal casting and polymer extrusion in metal spinning, include flow and heat transfer over a stretching surface. The theoretical investigation of magnetohydrodynamic thermally radiative non-Darcy Nanofluid flows through a stretching surface is presented considering also the influences of thermal conductivity and Arrhenius activation energy. Buongiorno’s two-phase Nanofluid model is deployed in order to generate Thermophoresis and Brownian motion effects [1]. By similarity transformation technique, the transport equations and the respective boundary conditions are normalized and the relevant variable and concerned similarity solutions are presented to summarize the transpiration parameter. An appropriate Matlab software (Bvp4c) is used to obtain the numerical solutions. The graphical influence of various thermal physical parameters are inspected for momentum, energy and nanoparticle volume fraction distributions. Tables containing the Nusselt number, skin friction and Sherwood number are also presented and well argued. The present results are compared with the previous studies and are found to be well correlated and are in good agreement. The existing modelling approach in the presence of nanoparticles enhances the performance of thermal energy thermoplastic devices.

Keywords: Buongiorno’s two-phase Nanofluid model, Arrhenius activation energy, non-Darcy, radiation, magnetohydrodynamics, velocity slip, Biot number

1 Introduction

Nanofluids are maneuvered by suspending nanoparticles with average size below 100 nm in traditional heat transfer fluids such as ethylene glycol, oil and water. An important role is simulated by thermal conductivity of water, oil and ethylene glycol with nanoparticles in the heat transfer between the heat and mass transfer medium and the heat and mass transfer surface. The rise in thermal conductivity is substantial in improving the heat transfer behavior of the fluids. High heat transfer performance is necessitated by several engineering applications. In several engineering and industrial applications, the thermal conductivity of nanofluid is not constant and it differs linearly with temperature. Nanofluid is the combination of nanoparticles with water. The addition of a surfactant to a base fluid improves thermal conductivity and convective heat transfer. Nanofluid technology has emerged as a modern heat transfer technique that is more effective. Nanofluids are used in solar energy applications such as heat exchanger design, medical applications such as cancer therapy and safer surgery by heat treatment applications. Nanofluid technology can be used to create better oil and lubricants for realistic applications [2]. Many scientists and engineers have been working on fluid forming for the past few decades in order to improve efficiency for various thermal applications. The term “nanofluid” has been proposed to address the new heat transfer problems by applying nanotechnology to heat transfer [3]. Many authors have predicted that nanofluids would have greater convective heat
transfer capabilities than base fluids. Heat and mass transfer of nanofluids have integrated many practical applications of thermal and solutal stratification. Patil et al. [4] investigated the unsteady mixed convection nanoliquid flows through an exponentially stretching surface in the presence of applied transverse magnetic field with realistic application to solar systems. Khan et al. [5] presented the entropy analysis and gyrotactic microorganisms of Buongiorno’s nanofluid between two stretchable rotating disks using homotopy analysis. Nainaru et al. [6] studied the effects of heat transfer characteristics on 3D MHD flows of nanofluid induced by stretching surface with thermal radiation. They observed a rise in the fluid temperature and velocity with a rise in variable thermal conductivity. Shiriny et al. [7] considered the forced convection flow of nanofluid in a horizontal microchannel with cross flow injection and slip velocity on microchannel walls. They observed that an increase in velocity slip and heat transfer rate by increasing the angle of injection to 94°. Suhail and Siddiqui [8] presented the numerical study the natural convection flows of nanofluid within a vertical annulus. They discussed the simulations for various nanoparticle concentrations for various heat flux values. Few recent studies on nanofluid includes [9–11].

Due of its universal and practical applications, the convective boundary layer plays an important role in the process of heat supply to the fluid via a particle surface with a restricted heat efficiency, especially in various manufacturing and advanced processes that include transpiration cooling, fabric cleaning, laser pulse heating and so on. Convective heat transfer is important in mechanisms like thermal insulation, gas turbines and nuclear power stations among others [11]. The analysis of electrically conducting fluids like sea water, antioxidants, plasma and metal oxides is known as magnetohydrodynamics (MHD). Alfven [12] was the one who invented the word MHD. The strength of magnetic induction affects MHD. The MHD fluid flows has a many engineering and industrial applications like crystal development, nuclear freezing, magnetic improved drug, MHD detectors, and energy production. The MHD fluid flows also has many applications in healthcare and biopharmaceutical fields such as hyperglycemia and emergency medicine, radiation therapy and many more. Khilap et al. [13] assessed the melting heat transfer and non-uniform heat source on magnetic nanofluid flows past a porous cylinder using Keller box technique. Jha and Malgwi [14] analyzed the mixed MHD convection flow of viscous fluid in a vertical microchannel considering the effects buoyancy forces, pressure gradient, Hall and ion-slip current. Haque [15] explored the effects of induced magnetic field and heat sink on the micropolar fluid flows past a semi-infinite vertical porous plate using the explicit finite difference technique. They observed that both heat sink and thermal diffusion have a growing effect of the species concentration and the lighter particles have a higher fluid concentration compared to heavier ones. Alam et al. [16] considered the principles of magnetohydrodynamics and ferrohydrodynamics to examine the biomagnetic flows of blood containing gold nanoparticles past a stretching surface in the presence of Biot number, suction and velocity slip using the bvp4c technique. Khan et al. [17] presented the entropy analysis of MHD radiative flows of Jeffrey fluid past an inclined surface using Keller-Box technique. Vedavati et al. [18] examined the MHD convection flows of nanofluid past an inverted cone considering the suction/injection effects and entropy analysis is also presented.

Thermal radiation is a key aspect in many engineering processes that take place at extreme temperatures. Many industrial applications involving extreme temperatures such as gas turbines, nuclear power stations and various combustion turbines for aircraft, rockets, spacecraft and aerospace engineering have stressed the effects of radiation on convection. Radiative heat transfer is crucial in oxidation, fossil fuel energy cycles, astrophysical flows, renewable energy engineering are the other applications. Mumtaz et al. [19] studied the mixed convective chemically radiative flows of tangent hyperbolic fluid with viscous dissipation in a doubly stratified medium using BVPh2 scheme. Yusuf et al. [20] investigated the entropy analysis on radiative MHD flows of Williamson nanofluid past an inclined porous plate considering the chemical reaction, gyrotactic microorganisms and convective heat transfer effects. Jawad et al. [21] presented the radiative MHD flows of second grade hybrid nanofluid past a stretching/shrinking surface using the homotopy analysis. They observed that an increase in the volume fraction of the nanoparticles increases the fluid’s thermal efficiency. Ge-jile et al. [22] discussed the radiative MHD flows of Jeffrey fluid past the horizontal walls in the porous medium and considering the viscous dissipation effects. Jawad et al. [23] considered the MHD mixed convection flows of Maxwell nanofluid past a stretching surface considering the influences of variable thermal conductivity, gyrotactic microorganisms, radiation, Dufour and Soret.

A surface will fluid-filled pores (voids) is termed as porous media. In industry, porous materials are commonly seen in reverse osmosis, steam turbines, heterogeneous catalysts, activated carbon columns, filtration centers and evaporative freezing. The analysis of fluid flow processes in a porous media has sparked researcher’s interest due to its various implications in scientific, bio-
logical and industrial producing goods such as porous bearings, atomic reservoirs, groundwater contamination, crude oil processing, casting and welding in production processes, porous bearing, hydro power, vapor movement in fibrous packaging, organic catalytic reactors, renewable energy and recycling devices. In 1856, Darcy formulated a principle that states that the volumetric flux of fluid across a medium has a direct relationship with the pressure gradient. Practically, the Darcy term [24] is commonly used in the problems relating to flow saturating porous space modeling and analysis. The Darcy’s law is only applicable when the velocity is low and the porosity is small. Inertia and boundary impacts are ignored by this rule. The customized version of classical Darcy’s principle results in the non-Darcian porous space which integrates inertial and boundary impacts. As a result, Forchheimer [25] took into account the inertia by using a square velocity term in the momentum term. Non-Darcian versions are extensions of the standard Darcy concept that includes inertial drag, vorticity dispersion and combinations of these impacts. Darcy law in the Darcian medium is criticized as failing at high velocity, high porosity medium and enormous Reynolds number. To take the responsibility of the inertia impacts of pressure drop, the Forchheimer expression is integrated into the square velocity term within the momentum equation during this method. Pop and Ingham [26] and Vafai [27] surveyed temperature change in heat and mass transfer flows of Darcian and non-Darcian porous media. Hayat et al. [28] presented the Darcy-Forchheimer 3D flows of nanofluid past a rotating surface considering the effects of activation energy and heat generation/absorption using NDsolve technique. Asma et al. [29] examined the convective heat transfer analysis of Darcy-Forchheimer 3D flows of nanofluid past a rotating disk in the presence of Arrhenius activation energy using shooting technique. Ramzan et al. [30] investigated the melting heat transfer effects of unsteady nanofluid flow between two parallel disks considering the Darcy-Forchheimer permeable media, Cattaneo-Christov heat flux ad homogeneous-heterogeneous reactions using bvψc technique. Sohail et al. [31] studied the MHD convection flows of hybrid nanofluid past a stretching porous surface with viscous dissipation impacts using successive over relaxation technique. Kareem and Abdulhadi [32] investigated the axisymmetric MHD Darcy-Forchheimer flows of third grade fluid past a stretching cylinder considering the Cattaneo-Christov effects using homotopy analysis technique. Jawad et al. [33] presented the entropy analysis of MHD radiative Darcy-Forchheimer 3D Casson nanofluid flows past a turning disk considering the Arrhenius activation energy using homotopy analysis scheme. Zhang et al. [34] MHD Darcy-Brinkman-Forchheimer flows of third-grade fluid between two parallel plates considering the influences of Joule heating and viscous dissipation.

Many scientists and engineers have considered the activation energy, which was originally proposed by Svante Arrhenius in 1889 and is defined as the minimum amount of energy needed to carry out a reaction phase. The activation energy is the energy given to the reactants to transform them into products in different chemical reactions. The kinetic and potential energy involved with the molecules are of deemed importance to break bonds or stretch and twist bonds. It is observed that molecules rebound with each other without completion of reaction if their movement is detected slowly with low kinetic energy or they smash improperly. However, due to high momentum energy, a chemical reaction is initiated for which minimum activation energy is required. The activation energy concept is more significant in suspension of oil, hydrodynamics, oil storage industries and in geothermal. Owing to such interesting applications, this phenomenon is studied by many researchers. For instance, Umair et al. [35] explored the impacts of Soret and Dufour on chemically radiative magnetohydrodynamics flows of Cross liquid through a shrinking/stretching wedge also considering the effects of activation energy. Aldabesh et al. [36] dealt with the unsteady flow of Williamson nanofluid with gyrotactic microorganisms through a rotating cylinder considering the impacts of activation energy, chemical reaction and variable thermal conductivity. Mehbob et al. [37] investigated the MHD thermally radiative 3D flows of Cross fluid considering the effects convective heat transfer, stratification phenomena, heat source/sink and activation energy using bvψc technique. Sami et al. [38] explored effects of velocity slip and Arrhenius activation energy on MHD radiative 3D rheology of Eyring-Powell nanofluid past a stretching surface using shooting technique. Muhammad et al. [39] studied the effects of Brinkman number, magnetic parameter, diffusion parameter, Weissenberg number and activation energy on the entropy analysis of Carreau-Yasuda fluid past a stretching surface using the homotopy analysis method. Few recent studies on activation energy include [40–42].

By keeping the above studies in mind, the main objective of the current analysis is to study the effects of Arrhenius activation energy of Darcy-Forchheimer Nanofluid past a stretching surface in the presence of thermal radiation and magnetohydrodynamics. In addition, the influences of thermal conductivity, velocity slip, Biot number and Nield boundary condition are also considered. An appropriate bvψc along with 3-stage Lobatto IIIa method from MATLAB software is maneuvered to solve the two-
point boundary value problem using the similarity variables. The influences of several thermo physical parameters on velocity, temperature, nanoparticle spices concentration fields, skin friction, Nusselt number and Sherwood numbers are presented graphically and numerically. The results of the present study are compared with those of Wang [42], Gorla and Sidawi [44] and Khan and Pop [45] and found to be a good agreement.

Under these assumptions along with the Boussinesq and boundary layer approximations, the governing equations for Nanofluid [9–11] are:

\[
\frac{\partial u}{\partial y} + \frac{v}{\partial x} = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + g ((1 - C_{\infty}) \rho_f \beta_f (T - T_{\infty})
- (\rho_p - \rho_f) \beta_c (C - C_{\infty}) - \frac{\partial B_0^2}{\partial y} u - \frac{v}{\sqrt{K_s}} u^2 - \frac{C_b}{\sqrt{K_s}} u^2 \tag{2}
\]

\[
\frac{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}}{\rho c_p} + \frac{(\rho c_p)_p}{(\rho c)_f} \left( \frac{D_B \frac{\partial C}{\partial y} + D_T \frac{\partial T}{T_{\infty}}}{\frac{\partial y}{\partial y}} \right)^2 \tag{3}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{D_B \frac{\partial C}{\partial y} + D_T \frac{\partial T}{T_{\infty}}}{\frac{\partial y}{\partial y}} - K_c (C - C_{\infty}) \left( \frac{T}{T_{\infty}} \right)^n \cdot e^{-\frac{T}{T_{\infty}}} \tag{4}
\]

Using Rosseland approximation, \(q_r\) can be expressed in non-linear form as:

\[
q_r = \frac{4 \sigma \alpha \delta T^4}{3K_s^4} \tag{5}
\]

By assuming that the temperature differences within the flow are sufficiently small, using Taylor’s series expansion, can be expressed as,

\[
T^4 \approxeq 4T^3_{\infty} - 3T^4_{\infty} \tag{6}
\]

Using Eqs. (5) & (6) in (3), we get,

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k \frac{\partial^2 T}{\partial y^2} - \frac{16 \sigma \alpha T^3_{\infty}}{3K_s^4 \rho c_p} \frac{\partial^2 T}{\partial y^2}}{\rho c_p} + \frac{(\rho c_p)_p}{(\rho c)_f} \left( \frac{D_B \frac{\partial C}{\partial y} + D_T \frac{\partial T}{T_{\infty}}}{\frac{\partial y}{\partial y}} \right)^2 \tag{7}
\]

The Navier’s slip condition, convective condition and Nield boundary conditions are:

\[
u = U(x) + L \frac{dy}{dx}, \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h_w (T_w - T), \quad D_B \frac{\partial C}{\partial y} + D_T \frac{\partial T}{T_{\infty}} = 0, \quad At \ y = 0 \quad u \rightarrow 0, \quad T \rightarrow T_{\infty}, \quad C \rightarrow C_{\infty}, \quad As \ y \rightarrow \infty \tag{8}
\]

Introducing the similarity variables:

\[
\eta = \sqrt{\frac{u}{v}}, \quad \psi = \sqrt{\frac{u}{v}} \psi_{xf}(\eta), \quad \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad \phi = \frac{C - C_{\infty}}{C_w - C_{\infty}} \tag{9}
\]

The continuity equation is satisfied by considering the stream function \(\psi(x, y)\) as follows:

\[
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{10}
\]
Using Eq. (6) in Eqs. (2), (7) and (4), we get,
\[ f''' + f'' - (M + K_1) f' - (1 + Fr) f' + 1 \theta + \lambda (\theta - N \phi) = 0 \]  
(10)

\[ \frac{1}{Fr} (1 + R) \theta'' + \phi' + Nt \phi' + Nt (\phi')^2 = 0 \]  
(11)

\[ \phi'' + Sc \phi' + \frac{Nt}{Nb} \phi'' - Sc \sigma_R (1 + \delta \theta)^2 e^{-\left( \frac{r}{r_m} \right)} = 0 \]  
(12)

The prescribed two-point boundary conditions are redesigned as:
\[ f = 0, \quad f' = 1 + S f''', \quad \theta = 1 + \frac{\phi'}{M}, \quad Nt \phi' + Nb \phi' = 0 \quad \text{at} \quad \eta = 0 \]
\[ f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty \]  
(13)

Where \( K_c (C - C_\infty) \left( \frac{T - T_\infty}{T} \right)^n e^{-\left( \frac{E}{RT} \right)} \) is an Arrhenius expression, \( Fr = \sqrt{\frac{T \sigma a}{\pi \kappa}} \), \( K_1 = \frac{k}{\pi \kappa} \), \( M = \frac{\sigma a}{\pi \kappa} \), \( Pr = \frac{k}{\pi \kappa} \), \( \lambda = \frac{(1 + C - C_\infty) \beta (T_a - T_\infty)}{x a^2} \), \( N_t = \frac{(\rho - \rho_f) \beta (C - C_\infty)}{1 + C - C_\infty} \), \( R = \frac{k}{\pi \kappa} \), \( S = \sqrt{\frac{T}{\gamma}} \), \( Sc = \frac{v}{\beta} \), \( Nb = \frac{E(1 - C - C_\infty)}{v} \), \( Nt = \frac{E(1 - C - C_\infty)}{v} \), \( E = \frac{E}{k} \), \( Bi = \frac{h \beta}{k} \), \( \sigma_R = \frac{\kappa}{a} \), \( \delta = \frac{T_a - T_\infty}{T} \).

To calculate the heat and concentration transfer rates, the physical quintiles shear stress rate \( (C_f) \), local Nusselt number \( (Nu_x) \) and local Sherwood number \( (Sh_x) \) are defined as:
\[ C_f Re_x^{1/2} = f'' (0) \]  
(14)

\[ Nu_x Re_x^{1/2} = - (1 + R) \theta' (0) \]  
(15)

\[ Sh_x Re_x^{1/2} = - \phi' (0) \]  
(16)

Here \( Re_x = \frac{ax}{v} \) is the local Reynolds number.

### 3 Numerical procedure

Mathematically, the system of coupled dimensionless Eqs. (7) – (9) subject to boundary conditions Eq. (10) is strongly non-linear and are indeed very difficult to solve analytically. Hence the bvp4c technique from matlab is used to solve this system of equations numerically.

The Matlab BVP solver bvp4c from matlab, a finite difference code which implements the 3stage Lobatto IIIa formula is used to obtain the numerical solutions. In this technique the Eqs. (7) - (9) are first transformed into a set of coupled first-order equations as follows:

\[ y = \begin{bmatrix} f \\ f' \\ \theta \\ \phi \\ \phi' \\ \phi'' \\ \phi''' \\ \theta' \\ \phi' \\ \phi'' \\ \phi''' \\ \theta'' \\ \phi' \\ \phi'' \\ \phi''' \\ \theta'' \\ \phi' \\ \phi'' \\ \phi''' \end{bmatrix} \]  
(17)

Therefore, Eqs. (7) – (8) can be written as:

\[ \frac{d}{d \eta} \begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \\ y(6) \\ y(7) \end{bmatrix} = \begin{bmatrix} y(2) \\ y(3) \\ -y(1)y(3) + (M + K_1)y(2) + (1 + Fr)y(2) + (\lambda y(4) - N \gamma y(6)) \\ y(5) \\ -Pr \left[ y(1)y(5) + Nby(5)y(7) + Nty(5)^2 \right] / (1 + R) \\ y(6) \\ y(7) \end{bmatrix} \]  
(18)

And then this is set up as a boundary value problem (bvp) and use the bvp solver in matlab to solve the system along with the specified boundary conditions numerically using the RK method of order 4(i.e., bvp4c). The iterative process will be terminated when the error involved is \(< 10^{-6} \).

For further information about the algorithm of bvp4c the readers can refer to Shampine et al. [46] and Ibrahim [47].
4 Results and discussion

The system of Eqs. (7) – (10) are solved using the bvp4c technique and the results of velocity, temperature, concentration, shear stress rate, Nusselt number and Sherwood number for distinct values of various dimensionless thermo-physical parameter namely, Brownian motion parameter \((N_b)\), thermophoresis parameter \((N_t)\), buoyancy ratio parameter \((N_r)\), Darcy number \((K_1)\), Energy activation number \((E)\), Forchheimer number \((Fr)\), Velocity slip parameter \((S)\), Biot number \((Bi)\), magnetic parameter \((M)\), radiation parameter \((R)\), Prandtl number \((Pr)\) and Schmidt number \((Sc)\) are presented graphically and numerically. The results of the present numerical code for reduced Nusselt number (local heat transfer rate, \(Nu_x\)) are compared with the solutions of Wang [43], Gorla & Sidawi [44] and Khan & Pop [45] for different values of \(Pr\) and are displayed in Table 1. Exceptionally excellent agreement is attained. To study the influence of the involved variables, other parameters are fixed as \(Bi = 0.3, R = Nr = K_1 = Fr = S = A = E = 0.1, Pr = 3, Sc = M = \sigma = 0.2, \delta = n = Nb = Nt = 0.5\). The results of shear stress rate \((C_f)\), reduced local heat transfer coefficient (i.e., local Nusselt number, \(Nu_x\)) and reduced mass transfer coefficient (i.e., local Sherwood number, \(Sh_x\)) are presented in Table 2 for distinct values of \(Pr, M, Nr, R, A, K_1, E, Fr, Sc, S, Bi, Nb, Nt\). A rise in \(Pr\) is noted to increase \(C_f\), \(Nu_x\) and \(Sh_x\). Hence the heat transfer to the surface is raised with lower thermal conductivity (higher Prandtl number) and the nanoparticle diffusion (mass transfer) is raised. Further, it is seen that \(C_f\) is enhanced with a rise in \(M\) values whereas both \(Nu_x\) and \(Sh_x\) are reduced. Fluid energizes with the impact of stronger magnetic field and the heat is expanded from the boundary that leads to further decrease in heat transfer to the wall. This process affects adversely on the nanoparticles diffusion to the wall. The magnetic quantities leads the energy to the fluid and improves the heat transportation through the sheet. Hence, the magnetic parameter could be used as a method for plotting the features of flow and heat transport. It is further seen that increasing \(Nr\) values reduces \(C_f\), \(Nu_x\) and \(Sh_x\) since radiation stimulates the nanofluid. However, increment of \(Nr\) with high radiation implies elevation of nanoparticles diffusion to the boundary layer and diminishes nanoparticles concentration at the boundary layer. Also, an increase in radiation parameter is seen to decrease \(C_f\) and \(Sh_x\) but \(Nu_x\) is boosted. Increasing mixed convection parameter (Richardson number) is seen to largely reduce \(C_f\) however, a significant rise is observed in the case of both \(Nu_x\) and \(Sh_x\). An increase in Darcy number is seen to enhance \(C_f\) greatly but both \(Nu_x\) and \(Sh_x\) are seen to decrease.

With a rise in activation energy number, a very slight variation is observed in \(C_f\), \(Nu_x\) and \(Sh_x\). A rise in velocity slip parameter is seen to reduce \(C_f\), \(Nu_x\) and \(Sh_x\). A rise in biot number is seen to enhance \(Nu_x\) and \(Sh_x\). A rise in Schmidt number is seen to reduce \(Sh_x\). A rise in Brownian motion parameter is seen to reduce \(Sh_x\). A rise in Thermophoresis parameter is seen to enhance \(Sh_x\).

![Figure 2: Variations of \(f'\) for various \(M\) Values](image)

![Figure 3: Variations of \(\theta\) for various \(M\)](image)

Figures 2 - 4 depicts the effect of Hartmann number, \(M\) on velocity, temperature and concentration distributions. A diminishing trend for velocity is noted as \(M\) increases. A strong Lorentz force substantially reduces the velocity with greater \(M\) values. As the Lorentz force is resistive, the velocity distribution decreases resulting in a reduction in particles motion. The temperature profiles are found...
Table 1: Comparison values of $Nu_x$ for various values of $Pr$.

| $Pr$ | Wang [43] | Gorla & Sidawi [44] | Khan & Pop [45] | Present |
|------|------------|----------------------|-----------------|---------|
| 0.07 | 0.0656     | 0.0656               | 0.0663          | 0.0660  |
| 0.2  | 0.1691     | 0.1691               | 0.1691          | 0.1689  |
| 0.3  | 0.4539     | 0.5349               | 0.4539          | 0.4539  |
| 2    | 0.9114     | 0.9114               | 0.9113          | 0.9112  |
| 7    | 1.8954     | 1.8905               | 1.8954          | 1.8950  |
| 20   | 3.3539     | 3.3539               | 3.3539          | 3.3535  |
| 70   | 6.4622     | 6.4622               | 6.4621          | 6.4620  |

Table 2: Values of $C_f$, $Nu_x$, and $Sh_x$

| $Pr$ | $M$ | $Nr$ | $R$ | $\lambda$ | $K_t$ | $E$ | $Fr$ | $Sc$ | $S$ | $Bi$ | $Nb$ | $Nt$ | $C_f$ | $Nu_x$ | $Sh_x$ |
|------|-----|------|-----|------------|-------|-----|------|------|-----|------|------|------|-------|--------|--------|
| 3    | 0.2 | 0.1  | 0.1 | 0.1        | 0.1   | 0.1 | 0.2  | 0.1  | 0.3 | 0.5  | 0.5  | 0.9424 | 0.2552 | 0.2320 |
| 4    |     |      |     |            |       |     |      |      |     |      |      | 0.9442 | 0.2648 | 0.2407 |
| 5    |     |      |     |            |       |     |      |      |     |      |      | 0.9453 | 0.2714 | 0.2467 |
| 6    |     |      |     |            |       |     |      |      |     |      |      | 0.9461 | 0.2763 | 0.2512 |
| 0.5  |     |      |     |            |       |     |      |      |     |      |      | 1.0172 | 0.2538 | 0.2307 |
| 1.0  |     |      |     |            |       |     |      |      |     |      |      | 1.2394 | 0.2490 | 0.2264 |
| 1.5  |     |      |     |            |       |     |      |      |     |      |      | 1.5196 | 0.2420 | 0.2200 |
| 2.0  |     |      |     |            |       |     |      |      |     |      |      | 1.8150 | 0.2333 | 0.2121 |
| 1    |     |      |     |            |       |     |      |      |     |      |      | 0.9411 | 0.2552 | 0.2320 |
| 5    |     |      |     |            |       |     |      |      |     |      |      | 0.9366 | 0.2549 | 0.2317 |
| 7    |     |      |     |            |       |     |      |      |     |      |      | 0.9357 | 0.2547 | 0.2315 |
| 0.5  |     |      |     |            |       |     |      |      |     |      |      | 0.9398 | 0.3321 | 0.2214 |
| 1.0  |     |      |     |            |       |     |      |      |     |      |      | 0.9368 | 0.4205 | 0.2103 |
| 1.5  |     |      |     |            |       |     |      |      |     |      |      | 0.9340 | 0.5020 | 0.2008 |
| 1    |     |      |     |            |       |     |      |      |     |      |      | 0.8731 | 0.2562 | 0.2329 |
| 5    |     |      |     |            |       |     |      |      |     |      |      | 0.6038 | 0.2594 | 0.2359 |
| 10   |     |      |     |            |       |     |      |      |     |      |      | 0.3191 | 0.2621 | 0.2383 |
| 15   |     |      |     |            |       |     |      |      |     |      |      | 0.0686 | 0.2640 | 0.2400 |
| 1.0  |     |      |     |            |       |     |      |      |     |      |      | 1.2235 | 0.2494 | 0.2267 |
| 2.0  |     |      |     |            |       |     |      |      |     |      |      | 1.4573 | 0.2436 | 0.2215 |
| 3.0  |     |      |     |            |       |     |      |      |     |      |      | 1.6460 | 0.2384 | 0.2167 |
| 0.5  |     |      |     |            |       |     |      |      |     |      |      | 0.9424 | 0.2553 | 0.2320 |
| 1.5  |     |      |     |            |       |     |      |      |     |      |      | 0.9425 | 0.2553 | 0.2321 |
| 3.0  |     |      |     |            |       |     |      |      |     |      |      | 0.9425 | 0.2553 | 0.2321 |
| 5.0  |     |      |     |            |       |     |      |      |     |      |      | 1.6126 | 0.2445 | 0.2222 |
| 10   |     |      |     |            |       |     |      |      |     |      |      | 1.9917 | 0.2372 | 0.2156 |
| 15   |     |      |     |            |       |     |      |      |     |      |      | 2.2622 | 0.2314 | 0.2104 |
| 0.5  |     |      |     |            |       |     |      |      |     |      |      | 0.6280 | 0.2466 | 0.2242 |
| 1.0  |     |      |     |            |       |     |      |      |     |      |      | 0.4518 | 0.2392 | 0.2175 |
| 1.5  |     |      |     |            |       |     |      |      |     |      |      | 0.3556 | 0.2337 | 0.2124 |
| 0.5  |     |      |     |            |       |     |      |      |     |      |      | 0.3694 | 0.3358 |        |
| 0.8  |     |      |     |            |       |     |      |      |     |      |      | 0.4933 | 0.4485 |        |
| 1.2  |     |      |     |            |       |     |      |      |     |      |      | 0.6062 | 0.5511 |        |
| 0.5  |     |      |     |            |       |     |      |      |     |      |      | 0.2317 |        |        |
| 1.0  |     |      |     |            |       |     |      |      |     |      |      | 0.2312 |        |        |
| 1.5  |     |      |     |            |       |     |      |      |     |      |      | 0.2308 |        |        |
| 0.1  |     |      |     |            |       |     |      |      |     |      |      | 1.1600 |        |        |
| 0.2  |     |      |     |            |       |     |      |      |     |      |      | 0.5800 |        |        |
| 0.3  |     |      |     |            |       |     |      |      |     |      |      | 0.3867 |        |        |
| 0.1  |     |      |     |            |       |     |      |      |     |      |      | 0.0465 |        |        |
| 0.2  |     |      |     |            |       |     |      |      |     |      |      | 0.0929 |        |        |
| 0.3  |     |      |     |            |       |     |      |      |     |      |      | 0.1393 |        |        |
to increase as $M$ increases. The greater variance in $M$ includes Lorentz forces resisting the motion of the fluid particles and increases the temperature profiles. When the fluid flow decreases the energy is released as heat, resulting in a thickening of the thermal boundary layer. A slight decrease in the concentration profiles is observed with a rise in $M$ values.

Figures 5 - 7 display the effects of Richardson number, $\lambda$ on the velocity, temperature and concentration distributions. The velocity increases with a rise in $\lambda$ values. Physically, the mixed convection parameter compares buoyancy and viscous forces. The viscous forces are reduced when the mixed convection parameter is increased, which improves the velocity shift. A very slight decrease in temperature is noted with a rise in $\lambda$ values. The fluid density varies with temperature variation due to thermal expandability and the flow can be influenced by the buoyancy force. As a consequence, the more we go from pure force convection ($\lambda = 0$) to natural convection under defined Reynolds, the more cooler the interface becomes. The nanoparticle concentration is noted to increase with an increase in $\lambda$.

Figures 8 – 10 display the effect of Darcy number, $K_1$, on the velocity, temperature and concentration distributions. An increase in $K_1$ values leads the velocity profiles to decay. The existence of permeable space increases the fluid stream safety by lowering fluid velocity and the energy layer associated with it. Physically, the presence of porosity causes resistance to fluid motion, resulting in a reduction in fluid velocity. The temperature profiles are enhanced with a rise in $K_1$ values. Porousness, in general, increases fluid flow resistance, resulting in increased temper-
Figure 8: Variations of $f'$ for various $K_1$

Figure 9: Variations of $\theta$ for various $K_1$

Figure 10: Variations of $\phi$ for various $K_1$

Temperature profiles and thermal boundary layer thickness. The concentration profiles are seen to decrease with a rise in $K_1$ values. It is noted that the presence of porous media increases the flow resistance, resulting in improved temperature distribution.

Figure 11: Variations of $f'$ for various $Fr$

Figure 12: Variations of $\theta$ for various $Fr$

Figures 11 – 13 portrays the effect of Forchheimer parameter, $Fr$, on the velocity, temperature and concentration distributions. The velocity profiles are lowered with greater values of $Fr$ since the inertia coefficient is directly proportional to the drag coefficient. Therefore, the drag coefficient increases as $Fr$ increases. As a consequence, the fluid’s resistance force increases and hence the velocity decreases. An increase in $Fr$ values leads to thickening the thermal boundary layer and doesn’t allow the fluid to pass easily. The temperature profiles are strongly in-
increased with the raising values of $Fr$. However, the concentration profiles are lowered with increasing $Fr$ values.

Figures 14 - 16 display the effect of velocity slip parameter, $S$, on the velocity, temperature and concentration distributions. A decay in velocity profile is seen with an increase in $S$ values. As the $S$ values raise, the stretching impacts are partly pass through the fluid and hence the velocity decreases. An increase in $S$ values is noted to increase both temperature and concentration profiles. If $S = 0$, the fluid holds to the boundary and the fluid slides with no resistance as $S \to \infty$. As the $S$ values increase, the motion of the fluid particles declines and hence the temperature and concentration increases.

Figures 17 - 18 display the effect of Prandtl number, $Pr$ on the temperature and concentration distributions. Generally, the temperature profiles decline with a rise in $Pr$ values. The parameter $Pr$ plays a significant role in engineering and manufacturing processes. The Prandtl num-
Figure 18: Variations of $\phi$ for various $Pr$

Figure 19: Variations of $\theta$ for various $R$

Figure 20: Variations of $\phi$ for various $R$

Figure 21: Variations of $\theta$ for various $Bi$

Figure 22: Variations of $\phi$ for various $Bi$

Prandtl number is defined as the ratio of force diffusivity to warm diffusivity. Clearly, $Pr$ is inversely related to thermal diffusivity. Hence for greater $Pr$ values the thermal diffusivity is very low. High values of $Pr$ lead the fluid become more viscous and small values of $Pr$ lead the fluid become less viscous. Therefore, increasing $Pr$ causes the fluid temperature to decrease. A significant reduction in concentration profiles is observed with a rise in $Pr$ values.

Figure 19 - 20 display the effect of thermal radiation, $R$, on the temperature and concentration distributions. A strong elevation in temperature and concentration profiles is noted with a rise in $R$ values. This is due to the fact that certain amount of heat energy is emitted during the radiation cycle.
Figures 21 – 22 display the effect of Biot number, Bi on the temperature and concentration distributions. A strong increase in both temperature and concentration profiles is seen with a rise in Bi values. The buoyancy force increases as the Biot number rises and the fluid carries the heat energy at a faster rate. Due to thermal expandability, the fluid density varies with temperature and the flow is influenced by the buoyancy force. Biot number is defined as the relationship between solid conduction and surface convection. Physically, as the Biot number rises, the surface’s thermal resistance decreases dramatically. Convection is increased, resulting in a higher surface temperature.

![Figure 23: Variations of $\phi$ for various $Nt$](image)

Figure 23 display the effect of thermophoresis parameter, $Nt$, on the concentration distributions. The process of moving the particles so that they contract temperature due to the temperature gradient force is called thermophoresis. The parameter $Nt$, plays a crucial role in nanoparticle volume fraction. A significant enhancement is observed in the concentration profile with an increase in $Nt$ values. As $Nt$ increases, the heat transfer in the boundary layer rises and at the same time exacerbates particle deposition away from the fluid region, thereby raising the nanoparticles volume fraction.

![Figure 24: Variations of $\phi$ for various $Nb$](image)

Figure 24 display the effect of Brownian motion parameter, $Nb$, on the concentration distributions. The nanoparticle concentration is substantially decreased with increasing $Nb$ values. The nanoparticles reorganize to create a new structure because of the spontaneous diffusivity. Thus the thermal conductivity of the nanofluid is improved. The Brownian motion warms the fluid in the boundary layer and rages the particles away from the fluid system thereby reducing the concentration distributions.

![Figure 25: Variations of $\phi$ for various $Sc$](image)

Figure 25 display the effect of Schmidt number, $Sc$, on the concentration distributions. A significant decrease in concentration profiles and the corresponding boundary layer thickness is noted with a rise $Sc$ values. By def., $Sc$ represents the diffusion ratio of momentum to mass. Hence, mass diffusivity is the reason for the decrease in the concentration.

Figure 26 display the effect of Reaction rate, $\sigma_R$, on the concentration distributions. A significant reduction in concentration profiles is noted with rising $\sigma$ values. An increase in $\sigma$ values results in an increase in the Arrhenius expression which eventually damages the chemical reaction. Hence the concentration profiles decay.
Figure 26: Variations of $\phi$ for various $\sigma$

Figure 27: Variations of $\phi$ for various $E$

Figure 28: Variations of $C_f$ for various $M$ values

Figure 29: Variations of $N_u$ for various $M$ values

Figure 27 illustrates the influence of Energy activation number, $E$, on the concentration profiles. It is found that the concentration profiles increase with an increase in $E$ values. Usage of activation energy is more efficient in enhancing the reaction process and hence increasing the concentration. The Arrhenius expression declines with an increase in $E$ values, resulting in the development of the relational chemical reaction leading to an increase in the concentration profiles. Due to the phenomenon of low temperature and greater activation energy results in a lower reaction rate that slows down the chemical reaction. This way the concentration increases.

Figures 28 – 29 depicts the influences of magnetic parameter $M$ on skin friction and heat transfer rate. It has been seen that as the values of $M$ increases, the skin friction increases near the wall and as we move away from the wall it decreases and a quite opposite trend is observed in the case of heat transfer rate. This confirms the earlier observations that the flow is slowed by the magnetic field but the fluid is heated. Higher heat transfer rates and a decrease in fluid temperature are associated with heat transfer from the fluid to the surface.

Figures 30 – 31 depicts the influences of radiation parameter $R$ on heat and mass transfer rates. A greater increase in heat transfer rate is observed with a rise in radiation parameter whereas the mass transfer rate is decreased. The flow is accelerated by the strong radiation, but the heat transfer to the surface is reduced. Species diffusion to the surface is hampered by the greater contribution of thermal conduction heat transfer.

Figure 32 – 33 depicts the influences of velocity slip $S$ and Darcy number $K_1$ on skin friction. A significant decrease in skin friction is seen with an increase in velocity slip. With an increase in Darcy number the skin friction is
Figure 30: Variations of $N_{ux}$ for various $R$ values

Figure 31: Variations of $S_{hx}$ for various $R$ values

Figure 32: Variations of $Cf_x$ for various $S$ values

Figure 33: Variations of $Cf_x$ for various $K_1$ values

Figure 34: Variations of $N_{ux}$ for various $Fr$ values

Figure 35: Variations of $S_{hx}$ for various $K_1$ values

increased slightly near to the wall and as we move away from the wall the skin friction is decreased.

Figure 34 presents the influences of Forchheimer number $Fr$ on heat transfer rate. With an increase in $Fr$, the skin friction is slightly decreased near the wall and as we move away from the wall it is increased significantly.

Figure 35 illustrates the influences of Schmidt number $Sc$ and reaction rate, $\sigma_R$ on mass transfer rate. The mass transfer rate is increased with an increase in $Sc$ values. While the mass transfer rate is decreased with an increase in $\sigma_R$ values. An increase in $Sc$ results in a decline in species mass diffusivity and hence increases mass transfer rate.
The fluid velocity is decreased with increasing values of $M$ whereas the fluid temperature and concentration are increased.

2. Increasing mixed convection parameter elevates the fluid velocity and nanoparticle concentration slightly whereas temperature is slightly increased.

3. Increasing Darcy number is noted to reduce the fluid velocity and nanoparticle concentration whereas temperature is enhanced. A similar behavior is observed with an increase in Forchheimer number and velocity slip parameter.

4. Increasing $Pr$ reduces the temperature and nanoparticles concentration. Conversely, increasing Biot number enhances both temperature and nanoparticles concentration.

5. Increasing radiation parameter enhances the fluid temperature and nanoparticle concentration.

6. Increasing thermophoresis parameter strongly accelerates the nanoparticle concentration whereas increasing Brownian motion parameter and Schmidt number reduces nanoparticle concentration.

7. The nanoparticles concentration is decreased with increasing reaction rate parameter whereas increased slightly with increasing activation energy parameter.

**Nomenclature:**

- $a$: Positive constant associated with linear stretching
- $Bi$: Thermal Biot number
- $B_0$: Constant imposed magnetic field
- $C$: Concentration of the fluid
- $C_b$: Forchheimer inertial Drag coefficient
- $c_p$: Specific heat parameter
- $D_T$: Thermophoretic diffusion constraint.
- $D_B$: Brownian diffusivity
- $E_a$: Activation energy
- $E$: Energy activation number
- $f$: Non-dimensional steam function
- $Fr$: Inertia-coefficient (Forchheimer number)
- $g$: Acceleration due to gravity
- $h_w$: Convective heat transfer coefficient
- $k^*$: Rosseland’s mean absorption coefficient
- $k$: Thermal conductivity
- $K^*$: Permeability
- $K_1$: Darcy parameter
- $K_c$: Reaction rate
- $L$: The parameter related to velocity slip
- $n$: Motile density
- $M$: Hartmann number
- $Nr$: Buoyancy parameter
\( \frac{N.b}{N.b} \) Brownian motion Constraint
\( N.t \) Thermophoresis parameter
\( \text{Pr} \) Prandtl number
\( q.\tau \) Radiative heat flux
\( R \) Radiation parameter
\( S \) Velocity slip parameter
\( Sc \) Schmidt number
\( T \) Temperature of Nanofluid
\( T.w \) convective fluid temperature
\( u, v \) The velocity components along (x, y) directions

Greek symbols

\( \beta_T \) Thermal expansion coefficient
\( \beta_c \) Concentration expansion coefficient
\( \lambda \) The buoyancy or mixed convective parameter
\( \rho_f \) Base fluid density
\( (\rho c)_p \) Effective nano particles heat capacity
\( \rho_p \) Nanoparticle density
\( \nu \) Kinematics viscosity
\( \sigma \) Electric conductivity of the fluid
\( (\rho c)_f \) Fluid heat capacity
\( \sigma_f \) Thermal diffusivity
\( \sigma_R \) Reaction rate constant
\( \delta \) Temperature difference parameter
\( \sigma^* \) Stefan-Boltzmann constant
\( \theta \) Non-dimensional temperature
\( \varphi \) Non-dimensional concentration
\( \psi \) Dimensionless stream function
\( \eta \) The dimensionless radial coordinate
\( \mu \) Dynamic viscosity

Subscripts

\( \infty \) Free stream condition
\( w \) wall condition

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