Gravitational and higher-order form factors of the pion in chiral quark models

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(Dated: 10 September 2008)

The gravitational form factor of the pion is evaluated in two chiral quark models and confronted to the recent full-QCD lattice data. We find good agreement for the case of the Spectral Quark Model, which builds in the vector-meson dominance for the charge form factor. We derive a simple relation between the gravitational and electromagnetic form factors, holding in the considered quark models in the chiral limit. The relation implies that the gravitational mean squared radius is half the electromagnetic one. We also analyze higher-order quark generalized form factors of the pion, related to higher moments in the symmetric Bjorken $X$-variable of the generalized parton distribution functions, and discuss their perturbative QCD evolution, needed to relate the quark-model predictions to the lattice data. The values of the higher-order quark form factors at $t = 0$, computed on the lattice, also agree with our quark model results within the statistical and method uncertainties.

PACS numbers: 12.38.Lg, 11.30, 12.38.-t
Keywords: pion gravitational form factor, generalized parton distributions, generalized form factors, structure of the pion, chiral quark models

I. INTRODUCTION

Form factors carry basic information on the extended structure of hadrons as they correspond to matrix elements of conserved currents between hadron states. There exist abundant experimental data concerning the charge form factor, related to the electromagnetic current and providing the distribution of charge in a hadron. The gravitational form factors, related to matrix elements of the energy-momentum tensor in a hadronic state and thus providing the distribution of matter within the hadron, are not experimentally known. This is because whereas the electromagnetic interactions are probed by structureless electrons and mediated by one-photon exchange, the parallel one-graviton exchange is extremely weak and impossible to measure. Recently, however, these objects were determined \textit{ab initio} and with sufficient accuracy in full-QCD lattice simulations by the QCDSF/UKQCD collaboration. In this paper we confront the predictions of chiral quark models for the pion gravitational form factors and higher-order generalized form factors to the lattice determination of Refs. 2, 3. We find that in the Spectral Quark Model (SQM), which is a variant of a chiral quark model with built-in vector meson dominance, the agreement with the lattice data is especially good. In particular, a slower fall-off with the large space-like momenta than for the case of the electromagnetic form factor is found. We also perform the calculations in the Nambu–Jona-Lasinio (NJL) model with the Pauli-Villars (PV) regularization, where the agreement is not as good as in SQM. In addition, we find that the higher-order generalized form factors (see below for a definition) at $t = 0$ provided by the full-QCD lattice simulations are properly reproduced in chiral quark models.

The form factors are related via sum rules to more general objects, the generalized parton distributions (GPDs) of the pion (for extensive reviews see e.g. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and references therein). Experimentally, the GPDs of the pion are elusive quantities, as they appear in exclusive processes which are difficult to measure, such as the deeply virtual Compton scattering or the hard electro-production of mesons. Recently, it has been suggested to study instead the deeply virtual Compton scattering on a virtual pion that is emitted by a proton under the operating conditions which will first be met after the energy upgrade at JLAB. This will eventually set important constraints on the pion GPDs.

Despite their fundamental and general character, GPDs are genuinely defined in the Minkowski space, thus hindering direct determinations on Euclidean lattices. The moments of GPDs in the $X$ variable (see Sect. 11) form polynomials in the $\xi$ variable, with coefficients depending on the $t$-variable only. The lowest moments yield the standard electromagnetic and the gravitational form factors, while higher moments are known as generalized form factors. These are useful quantities which may be computed directly on the lattice. Presently, apart from the values at $t = 0$, the generalized form factors are
not known even from lattice simulations due to insufficient statistics. In this work we make predictions for the first few generalized form factors. Since these objects do not correspond to conserved currents, they evolve with the QCD scale as they carry anomalous dimensions. We undertake this perturbative analysis at leading order and show how the generalized form factors evolve with the scale. We use the techniques described in detail in Ref. 15.

Chiral quark models have proved to properly describe the essential features of the pionic GPDs and related quantities. The special case of the parton distribution function (PDF) has been analyzed in the NJL model in Refs. 16, 17, 18. The diagonal GPD in the impact parameter space was obtained in 19. Other calculations of the pionic GPDs and PDFs were performed in instanton-inspired chiral quark models 20, 21, 22, 23, 24, 25, 26, 27. In the effective quark models the GPDs have been analyzed in 15, 21, 25, 28, 29, 30. Studies paying particular attention to polynomiality were carried out in 21, 31, 32, which proceeded via double distributions 33. The same technique was applied in Ref. 15, which allowed for a simple proof of polynomiality in chiral quark models. Also, other formal features, such as support, normalization, crossing, and the positivity constraints 34, 55, are all satisfied in the chiral quark model calculation of Ref. 15. Moreover, the obtained analytic expressions have a rather non-trivial form which is not factorizable in the $t$-variable. The parton distribution amplitude (PDA), related to the GPD via a low-energy theorem 36, was evaluated in Refs. 37, 38, 39, 40, 41, 42 (see Ref. 62 for a brief review of analyses of PDA). The authors of Ref. 14 provide standard electromagnetic and gravitational form factors based on a phenomenological model factorization assumption which is generally not satisfied by our dynamical field theoretical calculation 15. Finally, the related quantity, the pion-photon transition amplitude (PPA), 43, 44, 45, 46 has been obtained in Refs. 47, 48, 49, 50, 51.

In the present work we find that in the considered class of models a simple relation between the gravitational and charge form factors holds (see the Appendix). The essential element for the proof is the existence of the spectral representation of the quark model, which is the case both for SQM and NJL with the Pauli-Villars regularization. The relation implies that in the considered models and in the chiral limit the gravitational mean squared radius is half the electromagnetic one.

In Ref. 15 we have stressed the relevance of the QCD evolution for the phenomenological success of the chiral quark models in the description of the experimental and lattice data for the PDF and PDA of the pion. Indeed, the evolved results for the valence PDF compare very well to the Drell-Yan data from the E615 experiment 53 at the scale of 4 GeV, and to the transverse lattice results 53 at lower scales, $\sim 0.5$ GeV. Similarly, for the case of the PDA the QCD evolution leads to a fair description of the E791 dijet data 54 at the scale of 2 GeV, and of the transverse lattice data 53 at the lattice scale. The comparison is presented in Figs. 8-11 of Ref. 15.

The above-mentioned success of the chiral quark models in describing properties related to the pionic GPD allows us to hope that also their other aspects, such as the form factors, including the generalized ones, can be reliably estimated in these models. Certainly, the electromagnetic form factor, as one of the most basic quantities, has been promptly computed in all chiral quark models of the pion. Note that these evaluations assume the large-$N_c$ limit, thus the effects of the pion loops, important at low momenta, are absent. We note that the NJL model leads to somewhat too small electromagnetic radius 41, while in SQM, which incorporates the vector meson dominance, one may fit the data accurately. Some predictions of the NJL model for the generalized form factors of the pion have been presented by one of us in Ref. 53, however, the data away from the physical pion mass have been used in that work.

The outline of the paper is as follows: In Sec. III we review the necessary formalism providing basic definitions and notation. The essential information on the two chiral quark models used in our work is provided in Sec. III while Sec. IV shows their predictions for the electromagnetic and gravitational form factors. In Sec. V containing our main results, we compare our predictions to the full-QCD lattice data from Refs. 2, 3. Then in Sec. VI we pass to the results for the higher-order form factors, where the QCD evolution effects are incorporated.

II. BASIC DEFINITIONS

In this Section we review the basic concepts necessary for our analysis and introduce the notation. Throughout this paper we work for simplicity in the strict chiral limit,

$$m_\pi = 0.$$ (1)

Since the lattice data of Refs. 2, 3 perform the extrapolation to the physical pion mass which is small, the assumption (1) is appropriate. Nevertheless, the extension to the physical pion mass in chiral quark models is straightforward. In any case, it is worth mentioning that although an attempt was made 2, 3 to incorporate chiral logarithms from Chiral Perturbation Theory ($\chi$PT) to the one-loop order 54, 55, 56 (for a review see e.g. Ref. 57), as described in Ref. 62, the data did not exhibit their presence when all other uncertainties were considered, suggesting instead a linear extrapolation.

In this paper we will deal with standard gravitational and vector form factors. The corresponding quark operators are

$$\Theta^{\mu\nu} = \sum_{q=u,d,...} \bar{q}(x) i \frac{1}{2} (\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) q(x),$$ (2)
and
\[ J_\nu^\phi = \sum_{q=u,d,...} \bar{q}(x) \frac{q_\nu}{2} \gamma^\nu q(x), \quad (3) \]
respectively. The gravitational quark form factors of the pion \([61]\), \(\Theta_1\) and \(\Theta_2\), are defined through the matrix element of the energy-momentum tensor in the one-pion state,
\[ \langle \pi^b(p') | T^{\mu\nu}(0) | \pi^a(p) \rangle = \frac{1}{2} \delta^{ab} \left[ (q^{\mu} q^2 - q^{\nu} q^2) \Theta_1(q^2) + 4P^\mu P^\nu \Theta_2(q^2) \right], \quad (4) \]
where \(P = \frac{1}{3}(p' + p)\), \(q = p' - p\), and \(a, b\) are the isospin indices. The gravitational form factors satisfy the low-energy theorem \(\Theta_1(0) - \Theta_2(0) = \mathcal{O}(m_\pi^2)\) \([61]\). In the low momentum and pion-mass limit one can establish contact with \(\chi PT\) in the presence of gravity \([61]\),
\[ \Theta_1(q^2) = 1 + \frac{2q^2}{f^2} (4L_{11} + L_{12}) - \frac{16m_\pi^2}{f^2} (L_{11} - L_{13}) + \ldots, \]
\[ \Theta_2(q^2) = 1 - \frac{2q^2}{f^2} L_{12} + \ldots, \quad (5) \]
where \(f = 86\) MeV is the pion weak decay constant in the chiral limit and \(L_{11,12,13}\) are the corresponding gravitational low energy constants. The calculation of the effective energy-momentum tensor as well as the low energy constants for chiral quark models has been carried out in Refs. \([62,63]\). The vector form factor is defined as
\[ \langle \pi^a(p') | J_{\mu}^V(0) | \pi^c(p) \rangle = \epsilon^{abc} (p'^\mu + p^\mu) F_V(q^2), \quad (6) \]
At low momentum one has
\[ F_V(q^2) = 1 + \frac{2q^2 L_9}{f^2} + \ldots \quad (7) \]
where \(L_9\) is a low energy constant of \(\chi PT\) \([58]\).

The generalized quark form factors of the pion (here we take \(\pi^+ \) for definiteness) are defined as matrix elements of quark bilinears accompanied by additional derivative operators,
\[ \langle \pi^+(p') | \Pi(0) \gamma^{\mu_1} i\nabla^{\mu_2} \ldots i\nabla^{\mu_{n-1}} u(0) | \pi^+(p) \rangle = 2P^{\mu_1} P^{\mu_2} \ldots P^{\mu_{n-1}} A_{n0}(t) + \]
\[ 2 \sum_{k=2}^{n} q^{\mu_1} q^{\mu_2} \ldots q^{\mu_{k-1}} P^{\mu_k} \ldots P^{\mu_{n-1}} 2^{-k} A_{nk}(t), \quad (8) \]
where \(A_{nk}(t)\) are the generalized form factors with \(n = 1, 2, \ldots\) and \(k = 0, 2, \ldots, n\). The symbol \(\nabla\) is the QCD covariant derivative, \(\nabla = \frac{1}{2} (\hat{\nabla} - \hat{\nabla})\), and \(\ldots\) denotes the symmetrization of indices and the subtraction of traces for each pair of indices. The factor of \(2^{-k}\) is conventional and makes our definition different than in Refs. \([2,3]\). For \(n = 1\) and \(2\) we have
\[ A_{10}(t) = F_V(t), \]
\[ A_{20}(t) = \frac{1}{2} \Theta_1(t), \quad A_{22}(t) = -\frac{1}{2} \Theta_2(t), \quad (9) \]
where \(F_V\) is the pion electromagnetic form factor, while \(\Theta_1^2\) denote the quark parts of the gravitational form factors of Eq. \([5]\).

Chiral quark models work at the scale where the only explicit degrees of freedom are quarks, while the gluons are integrated out. Thus, the gluon form factors of the pion vanish,
\[ A_{nG}(t) = 0 \quad (\text{quark – model scale}). \quad (10) \]
When evolution to higher scales is carried out \([13]\), non-zero gluonic moments are generated. Note that the above mentioned soft-pion theorem \(\Theta_1(0) = \Theta_2(0)\) applies to the full trace of the energy momentum tensor, and consequently does not apply to their quark or gluonic contributions separately. The condition \([10]\) translates then into \(A_{20}(0) = -A_{22}(0)\) in the chiral limit at the quark model scale.

Now we pass to the definition of the GPDs, whose moments are related to the form factors listed above. The used conventions are the same as in Ref. \([15]\), where also all the details of the calculation of the GPDs can be found. The kinematics of the GPDs can be read off from Fig. \([1]\). The adopted notation is
\[ p^2 = m_\pi^2 = 0, \quad q^2 = -Q^2 = -2p \cdot q = t, \]
\[ n^2 = 0, \quad p \cdot n = 1, \quad q \cdot n = -\zeta, \quad (11) \]
with the null vector \(n\) defining the light cone. The two isospin projections of the quark GPDs of the pion, isosinglet (singlet) and isovector (non-singlet), are defined through the matrix elements of quark bilinears, with the quark fields displaced along the light cone,
\[ \delta_{ab} \mathcal{H}^{I=0}(x, \zeta, t) = \int \frac{dz}{4\pi} e^{ixp^+z^-} \langle \pi(0) | \bar{\psi}(0) \gamma \cdot n \psi(z) | \pi^a(p) \rangle |_{z^+ = 0, z^- = 0}, \quad (12) \]
\[ i\epsilon_{3ab} \mathcal{H}^{I=1}(x, \zeta, t) = \int \frac{dz}{4\pi} e^{ixp^+z^-} \langle \pi(0) | \bar{\psi}(0) \gamma \cdot n \psi(z) \tau_3 | \pi^a(p) \rangle |_{z^+ = 0, z^- = 0}. \quad (13) \]
The coordinate \(z\) lies on the light cone. At the quark-model scale the gluon GPD of the pion vanishes,
\[ \mathcal{H}^{G}(x, \zeta, t) = 0 \quad (\text{quark – model scale}). \quad (14) \]
When the QCD evolution to higher scales is performed, a non-vanishing gluonic contribution \(\mathcal{H}^{G}(x, \zeta, t)\) is generated \([13]\).

In the following analysis we use the symmetric notation for the GPDs,
\[ \xi = \frac{\zeta}{2 - \zeta}, \quad X = \frac{x - \zeta/2}{1 - \zeta/2}, \quad (15) \]
where \(0 \leq \xi \leq 1\) and the support is \(-1 \leq X \leq 1\). One introduces the corresponding GPDs
\[ \mathcal{H}^{I=0.1}(X, \zeta, t) = \mathcal{H}^{I=0.1} \left( \frac{\xi + X}{\xi + 1}, \frac{2\xi}{\xi + 1}, t \right). \quad (16) \]
The reflection about \( X = 0 \) yields the symmetry relations
\[
H^I(0, X, t) = -H^I(0, -X, t), \\
H^I(0, -X, t) = H^I(0, X, t).
\] (17)
At \( X \geq 0 \) the GPDs are related to the PDF, \( q(X) \),
\[
H^I(0, 0, 0) = H^I(0, 0, 0) = q(X).
\]

The \textit{polynomiality} conditions \cite{3, 6} follow from very basic field-theoretic assumptions, namely the Lorentz invariance, time reversal, and hermiticity, which yields the form \cite{4}. For the moments one finds
\[
\int dX X^{2j} H^I(0, X, t) = 2 \sum_{i=0}^{j} A_{2j+1,2i}(t) \xi^{2i},
\]
\[
\int dX X^{2j+1} H^I(0, X, t) = 2 \sum_{i=0}^{j+1} A_{2j+2,2i}(t) \xi^{2i},
\] (18)
with \( j = 0, 1, \ldots \). For the lowest moments one has
\[
\int dX H^I(0, X, t) = 2A_{10}(t) = 2F_V(t),
\]
\[
\int dX X H^I(0, X, t) = 2A_{20}(t) + 2A_{22}(t) \xi^2
\]
\[
= \Theta_2(t) - \Theta_1(t) \xi^2.
\] (20)
In the convention of Refs. \cite{2, 3} the equivalent expansion is in powers of \((2\xi)^2\) rather than \(\xi^2\), as in Eq. (18). In our approach, used in the following Sections polynomiality is explicitly manifest from the use of the double distributions \cite{15}. The equivalence of Eq. (8) and (18) is easily proven by contracting (8) with the null vectors \(n^{\mu_1} \ldots n^{\mu_n}\) and subsequently applying the definitions (11). We notice that for the isovector GPD only the even, and for the isoscalar GPD only the odd moments are non-vanishing. The gluon form factors are defined as the integrals
\[
\int dX X^{2j+1} H_g(X, \xi, t, Q^2) = 2 \sum_{i=0}^{j+1} A_{2j+2,2i}(t) \xi^{2i}.
\] (21)
Finally, we note that Eq. (13) for \( j = 0 \) expresses the electric charge conservation and the momentum sum rule operating in deep inelastic scattering.

III. CHIRAL QUARK MODELS

In chiral quark models at the leading-\( N_c \) level the calculation of the form factors and GPDs proceeds according to the one-loop diagrams of Fig. 1. Extensive details of the quark-model evaluation are given in \cite{15}. In this paper we carry out calculations in two chiral quark models: SQM \cite{4} and NJL with the Pauli-Villars regularization in the twice-subtracted version of Refs. \cite{41, 64, 65}. Variants of chiral quark models differ in the way of performing the necessary regularization of the quark loop diagrams.

The spectral quark model \cite{4} introduces the generalized spectral density \( \rho(\omega) \) in the quark mass \( \omega \), in the spirit of Ref. \cite{41}, supplied with chiral symmetry, gauge invariance, and vector meson dominance. The one-quark-loop action of SQM has the form
\[
\Gamma_{SQM} = -i N_c \int d\omega \rho(\omega) \text{Tr} \log (i\slashed{\partial} - m_\rho),
\] (22)
where \( \rho(\omega) \) is the quark generalized spectral function, and \( U^5 = \exp(i\gamma_5\tau \cdot \Phi/f) \), with \( \Phi \) denoting the pion field in the nonlinear realization. The vector part of the spectral function, needed in the present analysis, has the vector meson-dominance form \cite{4}
\[
\rho_V(\omega) = \frac{1}{2\pi i \omega} \frac{1}{1 - \omega^2/m_\rho^2} + \frac{1}{2\pi i \omega} \frac{1}{1 - \omega^2/m_\rho^2},
\] (23)
exhibiting the pole at the origin and cuts starting at \( \pm m_\rho/2 \), where \( m_\rho \sim 770 \text{ MeV} \) is the mass of the rho meson. The complex contour \( C \) for the integration in (22)
Villars regularization is introduced at the effective action

\[ M \text{ is the constituent quark mass. The Pauli-Villars } \]

regularization is introduced at the effective action level \[ 11, 64, 65, \] with the practical advantage that gauge

propriate vertex. For the electromagnetic form factor the

one-quark-loop action of the model is given in Ref. \[ 41 \]. SQM leads to conventional and suc-

cessful phenomenology for both the pion \[ 4, 13, 62, 67, \] the

nucleon \[ 68, \] and the photon parton distribution am-

plitude \[ 69. \]

We also study a more conventional chiral quark model, the NJL model with the Pauli-Villars regularization in

the twice-subtracted version proposed in Refs. \[ 41, 64, 65. \] The one-quark-loop action of the model is

\[ \Gamma_{\text{NJL}} = -iN_c \text{Tr} \log \left( i\theta - MU^2 \right), \] (24)

where \( M \) is the constituent quark mass. The Pauli-Villars regularization is introduced at the effective action

level \[ 11, 64, 65. \], with the practical advantage that gauge and relativistic symmetries as well as sum rules are man-

ifestly fulfilled \[ 70. \]. For the observables considered in

this paper the Pauli-Villars is implemented according to

the prescription, where instances of \( M^2 \) in an observable \( O \) are replaced with \( M^2 + \Lambda^2 \), and then the regularized observable is evaluated according to the prescription

\[ O_{\text{reg}} = O(0) - O(\Lambda^2) + \Lambda^2 \frac{dO(\Lambda^2)}{d\Lambda^2}. \] (25)

The Pauli-Villars regulator \( \Lambda \) is a free parameter of the model. In what follows we use

\[ M = 280 \text{ MeV}, \quad \Lambda = 871 \text{ MeV}, \] (26)

which yields \( f = 93.3 \text{ MeV} \) for the pion decay constant

\[ f^2 = -\frac{N_c M^2}{4\pi^2} \left[ \log(\Lambda^2 + M^2) \right]_{\text{reg}}. \] (27)

**IV. ELECTROMAGNETIC AND GRAVITATIONAL FORM FACTORS**

The form factors may be calculated in two different

ways. The first method uses the definition \[ 11, \] which leads to the evaluation of one-loop diagrams with an ap-

propriate vertex. For the electromagnetic form factor the

vertex is \( Q\gamma^\mu \), where \( Q \) is the electric charge of the quark.

For the gravitational form factor the vertex, correspond-

ing to the energy-momentum tensor, has the form

\[ \Theta^{\mu\nu}(k + q, k) = \frac{1}{4} [(2k + q)^\mu \gamma^\nu + (2k + q)^\nu \gamma^\mu] \]

\[ -\frac{1}{2} g^{\mu\nu} (2k + \gamma - \omega), \] (28)

with \( \omega \) denoting the quark mass. We illustrate the cal-

culation in the Appendix. The other method uses the

GPDs obtained earlier \[ 13, \] and evaluates their moments \[ 15. \]. The results are the same, which serves as a consistency test of the algebra. As mentioned in Sec. \[ III \] the equivalence is proven by contracting with the null vector. For instance in the case of the gravitational form factor we consider \( n_\rho \Theta^{\mu\nu} n_\nu. \) Then

\[ \left\langle n^\rho(p + q) \mid n_\rho \Theta^{\mu\nu}(0) n_\nu \mid n^\rho(p) \right\rangle = \]

\[ \delta^{ab} \frac{1}{2} \left[ \zeta^2 \Theta_1(q^2) + (2 - \zeta^2) \Theta_2(q^2) \right] \] (29)

and the vertex becomes

\[ n_\rho \Theta^{\mu\nu}(k + q, k) n_\nu = (x - \zeta/2) \gamma \cdot n. \] (30)

We recognize the same vertex as in the evaluation of the GPD’s multiplied by \( (x - \zeta/2) \). Upon passing to the symmetric notation Eq. \[ 20 \] follows.

Before showing the explicit results both for SQM and

NJL models in the specific realizations described in the previous section, we note some general results. Actually, in the considered quark models and in the chiral limit we have the following identity relating the gravitational and electromagnetic form factor,

\[ \frac{d}{dt} \left[ t \Theta_i(t) \right] = F_V(t), \quad (i = 1, 2), \] (31)

from which the identity between the two gravitational form factors \( \Theta_1(t) = \Theta_2(t) \equiv \Theta(t) \) follows. This remarkable quark model relations are proven explicitly in the Appendix. The essential ingredient of the proof is the existence of the spectral representation in both con-

sidered models. One consequence of relation \[ 31 \] is the expected consistency of normalizations at \( t = 0 \) for the charge and mass \( \Theta_i(0) = F_V(0), \) displaying the tight connection between the gauge and Poincare invariances. Furthermore, expanding in small \( t \) and using the fact that \( F(t) = F(0)[1 - \langle r^2 \rangle^2 / 6 + ...] \) with \( \langle r^2 \rangle \) denoting the mean squared radius, one gets

\[ 2\langle r^2 \rangle \Theta = \langle r^2 \rangle \nu, \] (32)

which means that in the considered models and in the chiral limit the gravitational mean squared radius is half the electromagnetic one.

Turning now to the specific realization, by construc-

tion, the pion electromagnetic form factor in SQM has the monopole form

\[ F_{V}^{\text{SQM}}(t) = \frac{m_p^2}{m_p^2 - t}. \] (33)

A straightforward evaluation for the gravitational form factor yields

\[ \Theta_1^{\text{SQM}}(t) = \Theta_2^{\text{SQM}}(t) = \frac{m_q^2}{t} \log \left( \frac{m_q^2}{m_p^2 - t} \right) \equiv \Theta^{\text{SQM}}(t). \] (34)

These specific form factors fulfill trivially the general rela-

tions Eq. \[ 31 \] and Eq. \[ 32. \]

In the NJL model the pion electromagnetic form factor is equal to

\[ F_{V}^{\text{NJL}}(t) = 1 + \frac{N_c g_2^2}{8\pi^2} \left( \frac{2 \log(\frac{s + \sqrt{s^2 - t}}{\sqrt{s^2 - t}})}{\sqrt{s^2 - t}} \right) \] (35)

with \( g_2 = M/f \) denoting the quark-pion coupling con-

stant. We have introduced the short-hand notation \( s =
\[ \sqrt{4 (M^2 + \Lambda^2) - t}. \] The condition \( \lim_{t \to -\infty} F_{\text{NJL}}(t) = 0 \) is satisfied due to Eq. (27). For the gravitational form factor we obtain

\[ \Theta_{\text{NJL}}^{\text{NJL}}(t) = F_{\text{NJL}}^{\text{V}}(t) - \frac{N_c g_n^2}{8 \pi^2} \times \]

\[ \left( 4 (M^2 + \Lambda^2) t \right) \left[ \text{Li}_2 \left( \frac{2 \sqrt{-t}}{s + \sqrt{-t}} \right) + \text{Li}_2 \left( \frac{2 \sqrt{-t}}{\sqrt{-t} - s} \right) \right] \text{reg}, \]

where \( \text{Li}_2(z) \) is the polylogarithm function.

Of course, the explicit expressions (33,34) and (35,35) comply to the general relation (31).

Similar expressions, of growing complication, may be obtained for higher-order form factors in both SQM and NJL models.

V. COMPARISON TO THE LATTICE DATA

We are now ready to compare the results of the previous Section to the recent lattice data [2,3]. These data correspond to the scale of \( Q = 2 \text{ GeV} \) and give the quark part of the gravitational form factor and the electromagnetic form factor. On the other hand, the quark model calculation corresponds to a very low scale, \( Q_0 \sim 320 \text{ MeV} \) and in general the QCD evolution is needed to compare the model predictions to the data at a different scale \( Q \).

A detailed discussion of the evolution issue is presented in Ref. [15]. The low energy Chiral Quark Models contain quarks as the basic and explicit degrees of freedom and the considered twist-2 observables correspond to modeling QCD at a low renormalization point. Obviously, by the energy-momentum conservation in these models the quarks as the only degrees of freedom carry 100% of the total momentum in a hadron. On the other hand the momentum fraction carried by the valence quarks in the pion at the scale \( Q^2 = 4 \text{GeV}^2 \) is about 40%. The QCD evolution (LO, NLO, N2LO) tells us that this number grows as the scale is evolved to lower values. The quark model reference scale \( Q_0 \) is determined as the scale where the evolved QCD value yields exactly the inescapable 100% of the quark model. It would of course be highly desirable to include, e.g., explicit gluonic degrees of freedom as this would allow to stop the perturbative evolution at a higher scale. Nonetheless, the LO and NLO evolutions for the PDF’s are sufficiently close to each other [18] as to provide some confidence on the kind of calculations carried out in our work.
The issue of the QCD evolution of GPDs and the generalized form factors is addressed in detail in Sect. VI. However, the electromagnetic and gravitational form factors do not evolve with the scale. What changes is the ratio $R$ of the total momentum fraction carried by the quarks (valence and sea)

$$R = \frac{(x)_{q}(Q)}{(x)_{q}(Q_{0})} = \left( \frac{\alpha(Q)}{\alpha(Q_{0})} \right)^{\gamma_{1}^{(0)}/(2\beta_{0})},$$

where the anomalous dimension is given by $\gamma_{1}^{(0)}/(2\beta_{0}) = 32/81$ for $N_{F} = N_{c} = 3$ and

$$\alpha(Q^{2}) = \frac{4\pi}{\beta_{0} \log(Q^{2}/\Lambda_{QCD}^{2})},$$

$$\beta_{0} = \frac{11}{3}N_{c} - \frac{2}{3}N_{f},$$

where we take $\Lambda_{QCD} = 226$ MeV and $N_{c} = N_{f} = 3$. At the scale $Q_{0}$ we have $R = 1$, which then gradually decreases with the increasing scale. The quark part of the gravitational form factor is $\Theta^{G}(t) = R\Theta(t)$, the gluon part is $\Theta^{G}(t) = (1-R)\Theta(t)$, such that, of course, $\Theta^{G}(t) + \Theta^{G}(t) = \Theta(t)$. The value of $R$ depends on the evolution ratio $\alpha(Q^{2})/\alpha(Q_{0}^{2})$, which is not precisely known on the lattice. For that reason we shall treat $R$ as a free parameter when fitting the model results to the data for the gravitational form factor. In SQM the other parameter is the value of the rho meson mass, $m_{\rho}$. For SQM we fit jointly the electromagnetic and the quark part of the gravitational form factor. The $\chi^{2}$ method yields

$$R = 0.28 \pm 0.02, \quad m_{\rho} = 0.75 \pm 0.05 \text{ GeV},$$

with $\chi^{2}/\text{DOF} = 1.8$. The result of the fit is displayed in Fig. 2 with the solid line. The band corresponds to the uncertainties in the values of parameters in Eq. (39). We note an overall very good agreement for both form factors. Note a significantly slower fall-off for the gravitational form factor compared to the electromagnetic one. The optimum value of $m_{\rho}$ agrees within the error bars, which are substantial, with the physical mass of the rho meson.

For the case of the NJL model (dashed line in Fig. 2) the agreement is not as good as in the SQM model and it is not possible to improve it by changing the parameters $M$ and $\Lambda$. For that reason we have not carried out the $\chi^{2}$ fit in this case. The problems of the NJL model in reproducing the electromagnetic form factor, where the corresponding rms radius turns out to be too small, are well known, see e.g. the discussion in Ref. [11]. Similar discrepancy can be noted for the case of the gravitational form factor shown in the bottom panel of Fig. 2.

Now we come back in a greater detail to the issue of the quark to gluon momentum ratio $R$. It evolves with the scale from the value $R = 1$ at the quark model scale to $R \rightarrow 0$ at $Q^{2} \rightarrow \infty$. We use the standard LO DGLAP evolution for the single channel. We know the final scale, $Q = 2$ GeV, but the quark model scale is a priori unknown. We thus adjust $Q_{0}$ in order to reach the value of $R$ from Eq. (39) at the known scale $Q$. The result is

$$Q_{0} = 0.31 \pm 0.03 \text{ GeV}$$

with $\Lambda_{QCD} = 226$ MeV. This value agrees with the earlier independent determinations of the quark-model scale, see Ref. [13] for a detailed discussion on these issues. We note that at such low scales as in Eq. (40) the perturbative expansion parameter in the DGLAP equations is large, $\alpha(Q_{0}^{2})/(2\pi) = 0.34$, which makes the evolution very fast for the scales in the vicinity of $Q_{0}$.

One should note that the large value of $\alpha(Q_{0}^{2})$ calls for the use of improved QCD evolution at low scales. The presented analysis could be extended in several ways, for instance incorporating the next-to-leading (NLO) effects, or by modifying the dependence of $\alpha$ on $Q^{2}$ at low scales incorporating the “infra-red protection” [73]. The NLO corrections to the meson PDF’s in chiral quark models were studied in Ref. [13], where, somewhat surprisingly, it was found that these effects are small. Modifications in the evolution of $\alpha(Q^{2})$ could be incorporated along the lines of Refs. [76, 77, 78] where they were used for the pion form factor, but this calculation is outside of the scope of the present paper.

We stress that the chiral quark model explanation of the lattice data used in this work as well as the physical and lattice data explored in Ref. [13] requires “strong” evolution, with a large value of the evolution ratio $\alpha(Q_{0}^{2})/\alpha(Q^{2})$. Our simple-minded analysis based on the LO evolution may probably be viewed as an approximation to a more elaborate scheme, nevertheless it is quite remarkable that various observables (PDF, PDA, their moments) lead to the same evolution ratio, giving in the LO approximation compatible quark-model scales [13]. Moreover, the electromagnetic and the gravitational form factors are independent of the evolution, hence the issue does not arise for these observables.

In Fig. 3 we show the form factor multiplied by $-t$, which is a popular way to present the results at large Euclidean momenta. In the case of the vector (electromagnetic) form factor we also display the experimental TJLAB data [71, 72, 73] and the earlier Cornell data [74] and the approximately constant value for $-tF_{V}(t)$ is clearly seen. In the gravitational case, however, the lattice data [2, 3] show an increasing trend which is well mimicked by our SQM form factor, $-t\Theta(t) \sim \log(-t)$.

In the large-$N_{c}$ limit in the single resonance approximation (SRA) [61, 79] one would have a monopole form factor

$$\Theta^{SRA}(t) = \frac{AM_{f}^{2}}{M_{f}^{2} - t},$$

which agrees with our quark model at low $t$ if $M_{f} = \sqrt{2}M_{V} \sim 1100$ MeV in agreement with the result of Eq. (32). The difference between the SRA form factor
and our Eq. (34) is less than 10% for momentum values up to \( t = -1 \text{GeV}^2 \). The SRA monopole fits well the data extrapolated to the physical pion mass, yielding \( A = 0.261(5) \) and \( M_f = 1320(60) \text{MeV} \).

To conclude this Section, several general remarks are in order. One has to bear in mind that the full-QCD lattice calculations are linearly extrapolated to the physical pion mass. Our quark-model analysis incorporates only the leading-\( N_c \) contributions, while the full-QCD simulations include all orders in that expansion. Nevertheless, despite these caveats, we note a quite remarkable agreement, in particular for the case of SQM. This is noteworthy, as our study is a genuine dynamical field-theoretical calculation, and the parameters providing the optimal fit are highly compatible with calculations of other processes within the same model and scheme.

Admittedly, at very large values of \(-t\) perturbative QCD results for the form factors should be reached. At very low values of \( t \) chiral corrections are important. Since the data we use are at intermediate values of Euclidean momenta, we may neglect both above-mentioned effects and use the chiral quark models to explain the data.

We stress that our calculation conforms to the low-energy theorem \( \Theta_1(0) - \Theta_2(0) = \mathcal{O}(m_\pi^2) \) which is dictated by chiral symmetry. The quark model predicts, in addition, \( \Theta_1(t) = \Theta_2(t) \), and the purely multiplicative character of the QCD evolution yields in our conventions

\[
A_{22}(t) = -A_{20}(t). \tag{42}
\]

Probably due to insufficient statistics, this formula is not quite seen in the data of Ref. 3 (note that with conventions adopted in that reference one should have instead \( A_{22}(t) = -\frac{1}{t} A_{20}(t) \)). For that reason we have not used the data for \( A_{22} \) in our numerical analysis.

VI. HIGHER-ORDER FORM FACTORS

Analogous calculations as in the previous section can be performed for the higher-order generalized form factors. Here we only give the results for the case of SQM, as the results in NJL are qualitatively similar, while also SQM works better for those quantities which can be confronted to the data. In Fig. 4(a) we show \( A_{3,2i} \) and \( A_{4,2i} \) obtained at the quark model scale. In the chiral quark models in the chiral limit one has at \( t = 0 \) very simple expressions \( 15, 21, 22 \)

\[
H^I = (X, \xi, 0) = \theta \left( 1 - X^2 \right) - \theta \left( (1 - X)(X - \xi) \right) - \theta \left( (X + 1)(-\xi - X) \right),
\]

where \( \theta(x) \) is the Heavyside step function. It follows from the definition \( 15 \) that at the quark-model scale the following relations hold \( 55 \),

\[
A_{2j+1,2i}(0) = \begin{cases} \frac{1}{2j+1} & \text{for } i = 0 \\ 0 & \text{otherwise} \end{cases}
\]

\[
A_{2j+2,2i}(0) = \begin{cases} \frac{1}{2j+2} & \text{for } i = 0 \\ -\frac{1}{2j+2} & \text{for } i = j + 1 \\ 0 & \text{otherwise} \end{cases}
\]

We have in this limit \( 15 \)

\[
H^I = \frac{3}{2\xi} \left( 1 - \frac{X^2}{\xi^2} \right) F_V(t) \tag{47}
\]

\[
H^I = (1 - \xi^2) \frac{15}{4\xi^2} \frac{N_f}{4C_F + N_f} X \left( 1 - \frac{X^2}{\xi^2} \right) \Theta(t) \tag{48}
\]

\[
XH^G = (1 - \xi^2) \frac{15}{4\xi^2} \frac{C_F}{4C_F + N_f} \left( 1 - \frac{X^2}{\xi^2} \right)^2 \Theta(t), \quad (Q^2 \rightarrow \infty)
\]

where \( C_F = (N_c^2 - 1)/(2N_c) \) and \( N_f = 3 \). The proportionality factors follow from the normalization at the ini-
Evaluation of moments in Eq. (47) yields immediately
\[
A_{2j+1,2i}(t) = \begin{cases} 
\frac{3}{4}(j+2)+3F_V(t) & \text{for } i = j \\
0 & \text{otherwise}
\end{cases}
\]
\[
A_{2j+2,2i}(t) = \begin{cases} 
\frac{N_f}{4C_F+N_f}2^{(4j+3)+15}\Theta(t) & \text{for } i = j \\
\frac{15}{2^{(4j+1)+15}}A_{2j+2,2i}(t) & \text{for } i = j + 1 \\
0 & \text{otherwise}
\end{cases}
\]
\[
A_{2j+2,2i}^G(t) = \frac{4C_f}{N_f} \frac{1}{2j+1} A_{2j+2,2i}(t),
\]
\[\langle Q^2 \to \infty \rangle\] (49)

We note a qualitative difference of the asymptotic form factors compared to the form factors at the quark model scale shown in Fig. [1]. For the isovector case \((n = 2j+1)\) only the highest form factor, with \(i = j\), does not vanish, while for the isoscalar case \((n = 2j+2)\) only the highest moments, with \(i = j + 1\) and \(i = j\) are non-zero. The remaining moments tend to zero. This result is a prompt conclusion from the asymptotic forms (47). Note that in contrast to this behavior, at the quark-model scale all generalized form factors are non-zero. As mentioned previously, the form factors \(A_{10}, A_{20}\), and \(A_{22}\) are invariants of the evolution. The asymptotic gluon form factors in Eq. (49) are related to the isoscalar quark form factors in a simple manner. Asymptotically, all generalized form factors become proportional to \(F_V(t)\) or \(\Theta(t)\) in the isovector and isoscalar channels, respectively.

Finally, we compare our values of the higher-order form factors at \(t = 0\) to the lattice data provided in Sec. 7 of Ref. [3]. With the notation for the moments at \(t = 0\),
\[
\langle x^n \rangle = A_{n+1,0}(0),
\]
one finds at the lattice scale of \(Q = 2\) GeV
\[
\langle x \rangle = 0.271 \pm 0.016,
\]
\[
\langle x^2 \rangle = 0.128 \pm 0.018,
\]
\[
\langle x^3 \rangle = 0.074 \pm 0.027.
\]
(lattice)

while in both chiral quark models we obtain after the LO DGLAP evolution to the lattice scale
\[
\langle x \rangle = 0.28 \pm 0.02,
\]
\[
\langle x^2 \rangle = 0.10 \pm 0.02,
\]
\[
\langle x^3 \rangle = 0.06 \pm 0.01.
\]
(chiral quark models)

where the error bars come from the uncertainty of the scale \(Q_0\) in Eq. (10). The two sets of numbers overlap within the error bars. This result is quite remarkable, as it shows that the hierarchy of the form factors at \(t = 0\) found in full-QCD lattice calculations is properly reproduced in chiral quark models. The above form factors are simply the moments of the PDF of the pion. We recall that the PDF itself in the chiral quark models reproduces very well the experimental [52] and transverse lattice data of Ref. [53], see Figs. 8 and 9 of Ref. [13].

FIG. 4: (Color online) Generalized form factors \(A_{2i}\), and \(A_{2i}\) of the pion in SQM at the quark-model scale \(Q_0\) (a), at the lattice scale \(Q = 2\) GeV (b), and the gluon form factors \(A_{2i}^G\) at \(Q = 2\) GeV(c).
VII. CONCLUSIONS

Here are our main points:

1. The gravitational form factor obtained from the spectral quark model in the chiral limit agrees very well with the lattice data of Refs. [2, 3]. We have performed a global fit to the electromagnetic and the quark part of the gravitational form factors and obtained very reasonable values for the evolution ratio and the vector-meson mass. The longer range of the gravitational form factor follows from a different analytic expression compared to the electromagnetic form factor.

2. The NJL model does not provide such an excellent agreement, although the qualitative features are very similar to SQM.

3. We provide analytic expressions for the lowest-order form factors in both considered models. For the considered models in the chiral limit we find an explicit relation between the gravitational and vector form factors. In particular, the relation shows that in our case both gravitational form factors are equal, and that the mean squared electromagnetic form factors for the considered models in the chiral limit we find an excellent agreement, although the qualitative features are very similar to SQM.

4. The electromagnetic and gravitational form factors do not evolve with the scale, but the higher-order generalized form factors do. We have performed the leading-order ERBL DGLAP QCD evolution of the pion GPDs and obtained via moments the generalized form factors at the scale of the lattice measurements.

5. The generalized form factors at $t = 0$ found in full-QCD lattice simulations are reproduced in chiral quark models within the error bars corresponding to statistical and model uncertainties.

6. Our predictions can be further tested with future lattice simulations for higher-order form factors. The behavior is non-trivial, with form factors having different signs, magnitude, and asymptotic fall-off.

7. Lattice simulations for the gluon form factors would provide a very useful independent information, which could be used to verify the model predictions.

Acknowledgments

The authors are grateful to Dirk Brömmel for helpful e-mail exchanges and for providing the lattice data of Refs. [2, 3] for our figures, as well as cordially thank Krzysztof Golec-Biernat for making available his computer code for solving the QCD evolution equations for the GPDs.

APPENDIX A: RELATION BETWEEN GRAVITATIONAL AND ELECTROMAGNETIC FORM FACTOR

In this Appendix we show an explicit calculation of form factors in SQM and prove as a by-product the result of Eq. (21). The proof also applies to the NJL model with the PV regularization, as that model can also be written in terms of the spectral representation (see below).

The SQM is defined by the generalized Lehman representation

$$S(p) = \int d\omega \frac{\rho(\omega)}{p - \omega}. \quad (A1)$$

The method exploited in Ref. [1] was mainly based in writing down Ward identities for the corresponding vertex functions at the quark level, and then closing the quark line to make one loop calculations. As discussed in later works (see e.g. [2, 3]) this is fully equivalent to proceeding directly through the effective action which we sketch now. Using the action of Eq. (22) one can compute the energy momentum tensor as a functional derivative of the action with respect to an external space-time-dependent metric, $g_{\mu\nu}(x)$, around the flat space-time metric $\eta_{\mu\nu}$ (we take the signature $(+ - -)$)

$$\frac{1}{2} \Theta^{\mu\nu}(x) = \frac{\delta T}{\delta g_{\mu\nu}(x)} \bigg|_{g_{\mu\nu} = \eta_{\mu\nu}} = -i \frac{N_c}{2} \int d\omega \rho(\omega) \langle x | \{ O^{\mu\nu}, (i\not\partial - \omega U^5)^{-1} \} | x \rangle,$$

where

$$O^{\mu\nu} = \frac{i}{2} (\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) - g^{\mu\nu} (i\not\partial - \omega), \quad (A3)$$

and $U^5 = \exp(i\gamma_5 \tau \cdot \phi / f)$ with $\phi$ the pion field in the nonlinear realization. For the calculation of the gravitational form factor, we expand up to second order in the pion field corresponding to evaluating the diagrams of Fig. 4. In the cartesian isospin basis one has

$$\langle \pi^b | \Theta_{\mu\nu}(\pi^a) = -N_c \int d\omega \rho(\omega) \int \frac{d^4k}{(2\pi)^4} \text{Tr} \{ \Theta_{\mu\nu}(k + q, k) \times \left[ \frac{i}{k - \omega} \left( -\tau_a \gamma_5 \omega \right) \frac{i}{k - \not{p} - \omega} \left( -\tau_5 \gamma_5 \omega \right) \frac{i}{k + \not{q} - \omega} \right. \right.$$

$$\left. + \left. \frac{i}{k - \omega} \frac{i}{2f} \frac{i}{k + \not{q} - \omega} + \text{crossed} \right] \right}, \quad (A4)$$

where the quark gravitational vertex is given by

$$\Theta_{\mu\nu}(k + q, k) = \frac{1}{4} \left[ (2k_\mu + q_\mu)\gamma_\nu + (2k_\nu + q_\nu)\gamma_\mu \right]$$

$$- \frac{1}{2} g_{\mu\nu}(2k + \not{q} - \omega). \quad (A5)$$

After computing the traces and using the Feynman trick
in the integrals, in the chiral limit the result is

\[
\Theta_1(q^2) = \frac{N_c}{4\pi^2 f^2} \int d\omega \rho(\omega) \omega^2 \int_0^1 \frac{dx}{x(1-x)q^2} \times \left[ -\omega^2 \log \omega^2 + x(1-x)q^2 \right. \\
+ \left. (\omega^2 - x(1-x)q^2) \log(\omega^2 - x(1-x)q^2) \right].
\]

\[
\Theta_2(q^2) = \Theta_1(q^2).
\]

(A6)

In the low-momentum limit we may use Eq. (5) as deduced from \( \chi \)PT in the presence of gravity [61] to get

\[
L_{11} = \frac{N_c}{192\pi^2},
\]

\[
L_{12} = -\frac{N_c}{96\pi^2},
\]

\[
L_{13} = -\frac{N_c}{(4\pi^2)12B_0} = \frac{f^2}{24M_S^2},
\]

(A7)

(A8)

(A9)

in agreement with the derivative expansion of SQM [62].

Proceeding similarly with the vector form factor we get

\[
\langle \pi^a | J_V^{\mu,\nu} | p^c \rangle = -N_c \int d\omega \rho(\omega) \int \frac{d^4k}{(2\pi)^4} Tr \left[ \gamma_\mu \gamma_\nu \frac{f}{f - \omega - M^2} \right]
\]

\[
\times \left[ \frac{i}{f} \left( \frac{\tau_\nu \gamma_\mu \omega}{f} \right) \frac{i}{f} \left( \frac{\tau_\nu \gamma_\mu \omega}{f} \right) \frac{i}{f} \left( \frac{\tau_\nu \gamma_\mu \omega}{f} \right) \frac{i}{f} \left( \frac{\tau_\nu \gamma_\mu \omega}{f} \right) \right].
\]

(A10)

For on-shell massless pions the electromagnetic form factor reads

\[
F_V(q^2) = \frac{N_c}{4\pi^2 f^2} \int d\omega \rho(\omega) \omega^2 \\
\times \int_0^1 dx \log \left[ \omega^2 - x(1-x)q^2 \right].
\]

(A11)

Charge normalization, \( F_V(0) = 1 \), and energy-momentum normalization \( \Theta_2(0) = 1 \) implies

\[
1 = F_V(0) = \Theta_2(0) = -\frac{N_c}{4\pi^2 f^2} \int d\omega \rho(\omega) \omega^2 \\
= \frac{N_c m^2}{24\pi^2 f^2}.
\]

(A12)

where in the second line the vector meson realization, Eq. (23), has been used. The value agrees with the value of the pion weak decay constant obtained from the corresponding axial matrix element [4].

With the representations of Eq. (A6) and Eq. (A11) the result in Eq. (31) can be readily derived, without any reference to the specific realization given by Eq. (23).

The above proof also holds for the NJL model in the PV regularization. This class of models can be cast explicitly in the spectral-representation form, using \( \rho(\omega) = \delta(\omega - M) + \sum c_i \delta(\omega - \sqrt{M^2 + \Lambda_i^2}) \), where \( c_i \) and \( \Lambda_i \) are the PV constants. Then all above algebraic steps carry over and the result (31) holds for the NJL model with PV regularization as well.
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