Abstract: In this study, an attempt was made to introduce the optimal values of effective parameters on the stress distribution around a circular/elliptical/quasi-square cutout in the perforated orthotropic plate under in-plane loadings. To achieve this goal, Lekhnitskii’s complex variable approach and Particle Swarm Optimization (PSO) method were used. This analytical method is based on using the complex variable method in the analysis of two-dimensional problems. The Tsai–Hill criterion and Stress Concentration Factor (SCF) are taken as objective functions and the fiber angle, bluntness, aspect ratio of cutout, the rotation angle of cutout, load angle, and material properties are considered as design variables. The results show that the PSO algorithm is able to predict the optimal value of each effective parameter. In addition, these parameters have significant effects on stress distribution around the cutouts and the load-bearing capacity of structures can be increased by appropriate selection of the effective design variables. The main innovation of this study is the use of PSO algorithm to determine the optimal design variables to increase the strength of the perforated plates. Finite element method (FEM) was employed to examine the results of the present analytical solution. The results obtained by the present solution are in accordance with numerical results.

Keywords: infinite orthotropic plates; quasi-square cutout; particle swarm algorithm; analytical solution; complex variable method

1. Introduction

Nowadays, the design of metal and composite plates with cutouts is of a great importance due to their extensive application in different industries [1,2]. It is well known that, due to geometric changes in different structures, highly localized stresses are created around discontinues areas, at which structural failure usually occurs [3]. Therefore, the analysis of this phenomenon, called stress concentration, has a significant importance for designers of engineering structures. The fracture strength of these structures depends strongly on the stress concentration caused by cutouts. Stress concentration and fracture criterions are very important in evaluating the reliability of engineering structures [4]. For instance, designing vehicles with the purpose of weight reduction in order to decrease fuel consumption and utilize engines with less power are some applications of these plates. In this study, according to the extensive usage of different types of cutouts and considering a long process of trial and error to find their optimum design, particle swarm optimization (PSO) algorithm (see, e.g., [5]) is employed for the
integrity of the search process in obtaining the optimum design. The main innovation of this paper is the use of PSO algorithm to determine the optimal design variables to increase the strength of the perforated plates.

2. Literature Review

Complex potential method established by G.V. Kolosov and N.I. Muskhelishvili (see, e.g., [6–8]) has been applied for anisotropic plates by Green and Zerna [7], Lekhnitskii [9], Sih et al. [10], Lekhnitskii [11], Bigoni and Movchan [12], Raci et al. [13], Craciun and Soós [14], Craciun and Barbu [15], and Chaleshtari and Jafari [16]. Tsutsumi et al. [17] investigated the solution of a semi-infinite plane with one circular hole. Their solution was based on repeatedly superposing the solution of an infinite plane with one circular hole and of a semi-infinite plane without holes to eliminate the stresses arising on both boundaries. Applying Lekhnitskii’s method [9,11], Rezaeezahand and Jafari [18] presented an analytical solution for the stress analysis of orthotropic plates with different cutouts and evaluated the stress distribution around a quasi-square cutout in orthotropic plates. They studied the effect of various parameters such as load angle, fiber angle, and cutout orientation for perforated orthotropic plates. Yang et al. [19] presented an analytical solution for the stress concentration problem of an infinite plate with a rectangular cutout under biaxial tensions. Rao et al. [20] found stress distribution around square and rectangular cutouts using Savin’s formulation [6]. Sharma [21] used Muskhelishvili’s complex variable approach [8] and presented the stress field around polygonal shaped cutouts in infinite isotropic plates. The effect of cutout shape, bluntness, load angle, and cutout orientation on the stress distribution was studied for triangular, square, pentagonal, hexagonal, heptagonal, and octagonal cutout shapes. Banerjee et al. [22] studied stress distribution around the circular cutout in isotropic and orthotropic plates under transverse loading using three-dimensional finite element models created in ANSYS. They investigated the effects of plate thickness, cutout diameter, and material on the amount of stress concentration in orthotropic plates. Marin et al. [23] studied the structural stability of an elastic body with voids and straight cracks in dipolar elastic bodies. Using the method of singular integral equations (see, e.g., [24]), Kazberuk et al. [25] presented the stress distribution in the quasi-orthotropic plane weakened by semi-infinite rounded V-notch.

Optimal structures with irregular geometry but with simple fields inside were investigated by Vigdergauz [26,27,28], Grabovsky and Kohn [29,30], Vigdergauz [31,32,33]. The related problem of an optimal shape of a cavity in an elastic plane was considered by Cherepanov [34], Banichuk [35], Banichuk and Karihaloo [36], Vigdergauz [28], Banichuk et al. [37], Vigdergauz and Cherkayev [38], Markenscoff [39], and Cherkayev et al. [40]. In addition, Cherepanov [34,41] proposed an effective exact solution of some inverse plane problems of the theory of elasticity concerning the determination of equally strong outlines of holes. Sivakumar et al. [42] studied the optimization of laminate composites containing an elliptical cutout by the genetic algorithm (GA) method (see, e.g., [43]). In this research, design variables were the stacking sequence of laminates, thickness of each layer, the relative size of cutout, cutout orientation, and ellipse diameters. The first and second natural frequencies were considered as a cost function. Cho and Rowlands [44] showed GA ability to minimize the tensile stress concentration in perforated composite laminates. Chen et al. [45] used a combination of PSO and finite element analysis to optimize composite structures based on reliability design optimization. Zhu et al. [46] considered the optimization of composite strut using the GA method and Tsai–Wu failure criterion [47]. They paid attention to minimizing the weight of the structure and the first buckling load. Fiber volume fraction and stacking sequence of laminates were considered as design variables. Artur and Daloğlu [48] used the GA to determine the optimum variable to achieve suitable steel frames. Moussavian and Jafari [49] calculated the optimal values of effective parameters on the stress distribution around a quasi-square cutout using different optimization algorithms such as Particle Swarm Optimization (PSO), GA, and Ant Colony Optimization (ACO) [50]. To achieve this goal, the analytical method based on Lekhnitskii’s method was employed to calculate the stress distribution around a square cutout in the symmetric laminated
composite. Jafari and Rohani [51] studied the optimization of perforated composite plates under tensile stress using GA method. The analytical solution was used to determine the stress distribution around different holes in perforated composite plates. Using GA, Jafari and Hoseyni [52] introduced the optimum parameters in order to achieve the minimum value of stress around different cutouts. Vosoughi and Gerist [53] proposed a hybrid finite element (FE), PSO, and conventional continuous GA (CGA) for damage detection of laminated composite beams. The finite element method (FEM) was employed to discretize the equations. Their design variables were damage ratios, the number of damaged elements, and the number of layers. Manjunath and Rangaswamy [54] optimized the stacking sequence of composite drive shafts made of different materials using PSO. The optimum results obtained by PSO are compared with results of GA and found that PSO yields better results than GA. Ghashochi Bargh and Sadr [55] used the PSO algorithm to the lay-up design of symmetrically laminated composite plates for maximization of the fundamental frequency. The design variables were the fiber orientation angles, edge conditions, and plate length/width ratios. Several algorithms are valid alternatives to PSO. Some of these alternatives are not heuristic algorithms but they have a strong theory behind them [56–58].

This paper aims to introduce a suitable mapping function and optimal cutout geometry in the perforated orthotropic plate under uniaxial tensile loads, biaxial loads, and shear loads. The design variables are cutout orientation, the aspect ratio of the cutout, bluntness, load angle, and fiber angle. Minimizing normalized stress around the cutout the Tsai–Wu criterion is considered as a cost function of the particle swarm optimization algorithm. The normalized stress is the ratio of the maximum value of circumferential stress at the edge of the cutout to the nominal or applied stress, which is called stress concentration factor (SCF).

3. Theoretical Formulation

The problem studied in this paper is an infinite plate containing a quasi-square cutout. As shown in Figure 1, the plate is under biaxial loading at an angle \( \theta_1 \) (load angle) with respect to the \( x \)-axis. The square cutout has arbitrary orientations such that its major axis is directed at an angle \( \theta_3 \) (rotation angle) with respect to the \( x \)-axis and fiber angle is \( \theta_2 \) [59]. In this paper, the stress function is converted to an analytical expression with undetermined coefficients and displacements, and stresses can be calculated by stress function being determined. In this case, it is assumed that the plate has a linear elastic behavior. Because of the traction-free boundary conditions on the cutout edge, the stresses \( \sigma_x \) and \( \tau_{xy} \) at the cutout edge are zero and the circumferential stress \( \sigma_\theta \) is the only remaining stress. The equilibrium equations are satisfied by introducing \( F(x, y) \) as stress function [60–62] according to Equation (1)

\[
\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}.
\] (1)

The orthotropic stress–strain relation for plane problems in terms of the components of the reduced compliance matrix is as follows [62]:

\[
\begin{align*}
\varepsilon_x &= R_{11} \sigma_x + R_{12} \sigma_y \\
\varepsilon_y &= R_{12} \sigma_x + R_{22} \sigma_y \\
\tau_{xy} &= R_{66} \tau_{xy},
\end{align*}
\] (2)
The constitutive vector-matrix equation of an orthotropic material in the global coordinate system is as follows [62]:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{xz} \\
\tau_{xy}
\end{bmatrix}
\]

where \(S_{ij}\) are the components of the orthotropic compliance matrix. The inverted relation is [62]

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{xz} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix}
\]

with the components of the stiffness matrix \(C_{ij}\). The transform rules for \(S_{ij}\) into \(C_{ij}\) and vice versa are presented in [62].

![Figure 1](image-url) **Figure 1.** Infinite perforated orthotropic plate subject to biaxial loading (\(\theta_1\) is load angle, \(\theta_2\) is fiber angle, and \(\theta_3\) is cut out orientation).

For plane stress state, \(\sigma_z = \tau_{xz} = \tau_{yz} = 0\) is assumed, which means the last equation is degenerated to

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
0 \\
0 \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & C_{55}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yx} \\
\gamma_{xy}
\end{bmatrix}
\]

It is obvious that, if the shear stresses \(\tau_{xz} = \tau_{yz} = 0\), the conjugated strain must be zero if we have orthotropic material behavior. Finally, the remaining part of the last equation is

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
0 \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 \\
C_{12} & C_{22} & C_{23} & 0 \\
C_{13} & C_{23} & C_{33} & 0 \\
0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy}
\end{bmatrix}
\]
Thus, we have three constitutive equations and one constraint
\[ C_{13}\varepsilon_x + C_{23}\varepsilon_y + C_{33}\varepsilon_z = 0 \]
or
\[ \varepsilon_z = -\left( \frac{C_{13}}{C_{33}}\varepsilon_x + \frac{C_{23}}{C_{33}}\varepsilon_y \right). \]
The strain \( \varepsilon_z \) can now be substituted in the expressions for \( \sigma_x \)
\[ \sigma_x = C_{11}\varepsilon_x + C_{12}\varepsilon_y - C_{13}\left( \frac{C_{13}}{C_{33}}\varepsilon_x + \frac{C_{23}}{C_{33}}\varepsilon_y \right) \]
and finally we get
\[ \sigma_x = \frac{C_{11}C_{33} - C_{13}^2}{C_{33}}\varepsilon_x + \frac{C_{12}C_{33} - C_{13}C_{23}}{C_{33}}\varepsilon_y. \]
In a similar manner, \( \sigma_y \) can be expressed. The equations for both stresses can be solved with respect to the strains \( \varepsilon_x \) and \( \varepsilon_y \) and finally the reduced compliance components can be computed.

By replacing stress–strain relations in compatibility relation, we obtain
\[ \frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial y^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} \]
and rewriting the resultant equation in terms of stress function, the compatibility equation for orthotropic material yields:
\[ R_{11} \frac{\partial^4 F}{\partial y^4} + (2R_{12} + R_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} + R_{22} \frac{\partial^4 F}{\partial x^4} = 0. \] (3)
Lekhniskii [60] showed that this equation can be transferred to four linear operators of first order \( D_k \):
\[ D_1 D_2 D_3 D_4 F(x, y) = 0, \quad D_k = \frac{\partial}{\partial y} - \mu_k \frac{\partial}{\partial x}, \] (4)
and we obtain the characteristic equation as follows
\[ R_{11}\mu^4 + (2R_{12} + R_{66})\mu^2 + R_{22} = 0. \] (5)
It can be proved, in general, that Equation \( (5) \) has four complex conjugate roots \( \mu_1 = \mu_2 = \pm i, \overline{\mu_1} = \overline{\mu_2} = -\pm i \) and the general expression for the stress function is:
\[ F(x, y) = 2\Re[\varphi(z_1) + \psi(z_2)], \] (6)
where \( \Re[\ldots] \) indicates the real part of the expression inside the brackets and \( z_k = x + \mu_k y \) and \( \mu_k, k = 1, 2 \) are the roots of the characteristic equation of anisotropic materials.
Finally, the stress components in terms of two potential functions of \( \varphi(z_1) \) and \( \psi(z_2) \) are expressed [52]:
\[ \sigma_x = \sigma_x^\infty + 2\Re[\mu_1^2\varphi''(z_1) + \mu_1^2\psi''(z_2)], \]
\[ \sigma_y = \sigma_y^\infty + 2\Re[\varphi''(z_1) + \mu_2^2\psi''(z_2)], \]
\[ \tau_{xy} = \tau_{xy}^\infty - 2\Re[\mu_1\varphi''(z_1) + \mu_2\psi''(z_2)]. \] (7)
where

\[ \sigma_\infty^x = \frac{\sigma}{2} [ (\lambda + 1) + (\lambda - 1) \cos 2\theta_1 ], \]
\[ \sigma_\infty^y = \frac{\sigma}{2} [ (\lambda + 1) - (\lambda - 1) \cos 2\theta_1 ], \]
\[ \tau_{\infty}^{xy} = \frac{\sigma}{2} [ (\lambda - 1) \sin 2\theta_1 ] \]

with \( \sigma \) as applied load (see Figure 1). In the above-presented equations, by taking appropriate values of \( \lambda \) describing the type of loading and \( \theta_1 \) for stress applied at infinity \((\sigma_\infty^x, \sigma_\infty^y, \tau_{\infty}^{xy})\), uniaxial loading, equibiaxial loading, and shear loading can be considered. The following values of \( \lambda \) and \( \theta_1 \) may be taken into Equation (8) to obtain various cases of in-plane loading:

- inclined uniaxial tension: \( \lambda = 0 \) and \( \theta_1 \neq 0 \);
- equibiaxial tension: \( \lambda = 1 \) and \( \theta_1 = 0 \); and
- shear loading: \( \lambda = -1 \) and \( \theta_1 = \frac{\pi}{4}, \frac{3\pi}{4} \).

We denote by \( \varphi''(z_1), \psi''(z_2) \) the derivatives of the functions \( \varphi(z_1) \) and \( \psi(z_2) \) with respect to \( z_1 \) and \( z_2 \). These analytic functions can be determined by applying the boundary conditions. To calculate the stress components in the polar coordinates system, we use the following equations

\[ \sigma_\theta + \sigma_\rho = \sigma_y + \sigma_x \]  
(9)
\[ \sigma_\theta - \sigma_\rho + 2i\tau_{\theta\rho} = (\sigma_y + 2i\tau_{xy})e^{2i\Omega}. \]  
(10)

In these equations, \( \Omega \) is the angle between the positive \( x \)-axis and the direction \( \rho \) (Figure 2).

4. Conformal Mapping

To apply the Lekhnitskii’s method to quasi-square cut out, establishing a relation between the cutout and a circular cutout is necessary [63]. A conformal mapping can be used to map the external area of a quasi-square cutout in \( z \)-plane into the area outside the unit circle in \( \zeta \)-plane (Figure 3). Such a mapping function is represented thus:

\[ z = \omega(\xi) = x + \mu_k y, \]  
(11)

where \( x \) and \( y \) are obtained as follows:

\[ x = (\cos \theta + w \cos n\theta), \]  
(12)
\[ y = -c(\sin \theta - w \sin n\theta). \]  
(13)
The parameter \( w \) determines the bluntness factor and changes the radius of curvature at the corner of the cut out (Figure 4).

As can be concluded from Equations (12) and (13), \( w = 0 \) presents the circular cutout. Integer \( n \) in the mapping function represents the shape of the cutout. The cutout sides are given by \( n + 1 \).

Bluntness \( w \) and cutout orientation \( \theta_3 \) are important parameters that influence the stress distribution around the different cutouts. Parameter \( c \) is the aspect ratio of cutout (length/width ratio) and Figure 5 shows the good effects of these parameters on the cutout geometry. With increasing of \( c \) at a constant value of \( w \), the cutout is elongated in one direction. For circular and elliptical cutout, \( c = 1 \) and \( c \neq 1 \), respectively, and, for both cases, \( w \) is equal to zero. For an elliptical cutout, \( c \) is the ratio of diameters the ellipse \( (c = b/a) \), where \( a \) and \( b \) are semi-major and semi-minor axis of the ellipse, respectively.

5. Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a population-based stochastic search optimization algorithm [64,65]. This algorithm starts to work with a number of initial answers which are determined randomly, and it looks to find an optimum answer by moving these answers through consecutive iterations. In
each iteration, the position of each particle in the search space is determined based on the best position obtained by itself and the best position obtained by the whole particles during the searching process. In each iteration, the particles, velocities and particle position are updated according to Equations (14) and (15), respectively [66],

\[
V_i(t + 1) = \omega V_i(t) + r_1 c_1 [P_i(t) - X_i(t)] + r_2 c_2 [p_i^\ast(t) - X_i(t)],
\]

\[
X_i(t + 1) = X_i(t) + V_i(t + 1),
\]

where \(V_i(t)\) and \(X_i(t)\) are the current velocity and position of the particle respectively. Let \(X_i(t) = \{x_1(t), \ldots, x_{N_{\text{var}}}(t)\}\) be the position of particle in a \(N_{\text{var}}\)-dimensional search space at iteration \(t\). We denote by \(X_i(t + 1)\) and \(V_i(t + 1)\) the updated velocity and position, respectively, and by \(\omega\) the inertia weight coefficient that controls the exploration and exploitation of the search space. \(c_1\) and \(c_2\) are two positive constants called the cognitive and social coefficients, respectively. A high inertia weight causes the available particles in the algorithm to search newer areas and perform a global search. On the contrary, the low inertia weight leads the particles to stay in a limited area. When the value of \(c_1\) increases, the particles tend to move toward the best individual experience and their motion toward the best group’s experience decreases, whereas, by increasing the \(c_2\), the particles move toward the best group’s experience, thus their motion toward the best individual experience decreases. Let \(r_1, r_2 \in [0, 1]\) be two random numbers, and \(P_i(t)\) and \(p_i^\ast(t)\) are the best individual and group’s experiences position, respectively. Choosing the appropriate values for \(c_1, c_2,\) and \(\omega\) results in an acceleration in convergence and leads to find the absolute optimum and prevents premature convergence in a local optimum. Here, \(c_1\) and \(c_2\) parameters update as in Equation (16) where \(c_{1,f}, c_{2,f}, c_{1,i}\) and \(c_{2,i}\) are constant values. In addition, Equation (17) is considered for \(\omega\) operator where \(\omega_i\) and \(\omega_f\) are initial and final values of weight factor, respectively; \(I\) is the number of particle’s current iteration; and \(I_{\text{max}}\) is the number of the greatest iteration [67].

\[
c_1 = (\omega_1 - \omega_f) \frac{I}{I_{\text{max}}} + c_{1,i}, \quad c_2 = (c_{2,f} - c_{2,i}) \frac{I}{I_{\text{max}}} + c_{2,i},
\]

\[
\omega = (\omega_1 - \omega_f) \frac{I_{\text{max}} - I}{I_{\text{max}}} + \omega_f.
\]

In a \(N_{\text{var}}\)-dimensional problem, a particle includes a row vector with \(N_{\text{var}}\) elements. This arrangement is defined as

\[P = [p_1, p_2, \ldots, p_{N_{\text{var}}}].\]

To begin the algorithm, a number of these particles (as the number of the primary particle algorithm) must be created.

The failure criterion and SCF are taken as a cost function (C.F.) for orthotropic and isotropic plate, respectively. It should be mentioned that, in [41,68], an alternative approach is presented. SCF is defined as the ratio of the von Mises stress, which is the maximum value of circumferential stress at the edge of the cutout \((\sigma_y)\), to the nominal or applied stress. In the case of a composite lamina, the strength is calculated by using the Tsai–Wu criterion:

\[
\text{Cost Function C.F.} = \text{SCF} = \min \sigma_i^2 = \left( \frac{c_1}{\sigma} \right)^2 \frac{1}{F_1^2} + \left( \frac{c_2}{\sigma} \right)^2 \frac{1}{F_2^2} + \left( \frac{\sigma_6}{\sigma} \right)^2 \frac{1}{F_6^2} - \frac{c_1 c_2 \frac{1}{F_1^2}}{\sigma^2},
\]

where \(c_i\) is the failure stress following from the Tsai–Wu criterion and \(c_1, c_2, \sigma_6\) are the transformed stress components in material principle coordinate [62], which are calculated using \(\sigma_x, \sigma_y, \tau_{xy}\) obtained in Equation (7). We denote by \(F_1\) and \(F_2\) the longitudinal and transverse strength in tension, respectively, and by \(F_6\) the shear strength. In this case, the simplified Tsai–Wu criterion is used (no linear terms, orthotropic
material behavior, or plane stress state), as suggested by Tsai and Wu [47]. The Tsai–Wu criterion is a degenerated Gol’denblat–Kopnov (tensor-polynomial) criterion [69], which is an extension of the anisotropic von Mises [70] or orthotropic Hill [71] criterion.

For isotropic materials, the cost function is defined as follows:

Cost Function \( C.F. = SCF = \min \frac{\sigma_{\text{von Mises}}}{\sigma} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2}{\sigma}} \) \hspace{1cm} (20)

with \( \sigma_{\text{von Mises}} \) as failure stress following from the von Mises criterion. By evaluating the C.F. for variables \( p_1, p_2, p_3, \ldots, p_{N_{\text{var}}} \), the cost of each particle is obtained:

\[ C.F._i = f(p_1, p_2, p_3, \ldots, p_{N_{\text{var}}}). \] \hspace{1cm} (21)

Moreover, the value range of design variables is defined as follows:

\[
\begin{align*}
0^\circ < \theta_i < 90^\circ, & \quad i = 1, 2 \\
0^\circ < \theta_3 < 180^\circ \\
0 < w < 1/3 \\
1 < c < 2.
\end{align*}
\] \hspace{1cm} (22)

Finally, each particle based on the best performance of his relationship has to be updated with the condition:

\[
P_i(t+1) = \begin{cases} 
  P_i(t), & \text{if } f(X_i(t+1)) > f(P_i(t)) \\
  X_i(t+1), & \text{otherwise.}
\end{cases} \] \hspace{1cm} (23)

The velocity and position of a particle on the basis of the best position among the particles are updated according to condition:

\[
\text{if } f(X_i(t+1)) < f(P_i^*(t)), \text{ then } P_i^*(t+1) = X_i(t+1). \] \hspace{1cm} (24)

The values of effective parameters for the PSO algorithm are listed in Table 1.

### Table 1. The value of effective parameter for PSO algorithm.

| PSO Parameters     | Value     |
|--------------------|-----------|
| Population Size    | 40        |
| Maximum of Iteration | 50        |
| Cognitive Component | \( c_1 = (c_{1,f} - c_{1,i}) \frac{I}{I_{\text{max}}} + c_{1,i} \) |
| Social Component   | \( c_2 = (c_{2,f} - c_{2,i}) \frac{I}{I_{\text{max}}} + c_{2,i} \) |
| Inertia Weight     | \( \omega = (\omega_1 - \omega_1) \frac{I}{I_{\text{max}} - I} + \omega_1 \) |

The convergence diagrams for the SCF and fracture criterion (Tsai–Wu) with quasi-square cutout \( c = 1 \) and in the case of uniaxial loading are shown in Figures 6 and 7, respectively.
31
16
11
26
,
s provided. Good agreements between the results obtained from the two methods indicates the accuracy of the present analytical solution.

Comparison of the present results in a special case to the results obtained by the present solution and FEM show the accuracy and precision of the present analytical solution.

According to this, in an isotropic plate under shear loading, Figures 8 and 9 show the optimum stress distribution modeled in ABAQUS and MATLAB, respectively ($\theta_1 = 90^\circ$, $\theta_2 = 90^\circ$, and $\theta_3 = 135.5^\circ$).

6. Solution Verification

To examine results obtained from the present analytical method, FEM (ABAQUS software) was employed. For this purpose, firstly, using PSO program code, optimum parameters with quasi-square cutout were determined. Then, the cutout geometry was modeled in accordance with optimum parameters obtained from program execution in ABAQUS software. To achieve optimum mesh number and increased accuracy in the results obtained from finite element numerical solution, meshing was finer around the cutout than external boundaries of the plate.

According to this, in an isotropic plate under shear loading, Figures 8 and 9 show the optimum stress distribution modeled in ABAQUS and MATLAB, respectively ($\theta_3 = 90^\circ$, $w = 0.078$). The values obtained from analytical solutions and FEM are compared in Figure 10. Angle $\theta$ indicates the points on the boundary cutout relative to the horizontal axis. In isotropic plates, because of the symmetry of stress distribution around the cutout, results to $\theta = 180^\circ$ are provided. Good agreements between the results obtained by the present solution and FEM show the accuracy and precision of the present analytical solution.
Comparison of the present results in a special case ($\theta_3 = 0^\circ$) and for shear loading with Pilkeys’ results [72] for an elliptical cutout in the isotropic plate is shown in Figure 11. As shown in this figure, the investigation was conducted based on changing the aspect ratio of cutout. The conformity of results obtained from the two methods indicates the accuracy of the present analytical solution.

For orthotropic plate containing an elliptical cutout with aspect ratio $c = 2$, the amount of failure strength based on the Tsai–Wu failure criterion was compared with the results obtained by Ukadgaonker and Rao [73]. For this case, fiber and rotation angles were considered $60^\circ$ and $0^\circ$, respectively, and the perforated plate was subjected to biaxial tensile. Table 2 shows the conformity of the present solution method with Ukadgaonker and Rao [73].
Figure 11. Variation of SCF with aspect ratio of elliptical cut out by different methods for isotropic plate ($\theta_3 = 0^\circ$).

Table 2. Comparison of Tsai–Wu failure strength (MPa) obtained by present solution (p.s.) as well as those of Ukadgaonker and Rao [73] (U&R).

| $\theta$ | p.s. | U&R | $\theta$ | p.s. | U&R |
|---------|------|-----|---------|------|-----|
| 0       | 32.55| 32.6| 95      | 35.73| 35.7|
| 5       | 40.64| 40.6| 100     | 33091| 33.1|
| 10      | 58.045| 58  | 105     | 30.756| 30.8|
| 15      | 185.28| 185.3| 110     | 28.676| 28.7|
| 20      | 166.18| 166.2| 115     | 26.813| 26.8|
| 25      | 181.79| 181.8| 120     | 25.14 | 25.1|
| 30      | 198.51| 198.5| 125     | 23.64 | 23.6|
| 35      | 181.2 | 181.2| 130     | 22.32 | 22.3|
| 40      | 150.93| 150.9| 135     | 22.32 | 22.3|
| 45      | 123.42| 123.4| 140     | 20.23 | 20.2|
| 50      | 102.07| 102.1| 145     | 19.52 | 19.5|
| 55      | 86.027| 86  | 150     | 19.12 | 19.1|
| 60      | 73.9 | 73.9 | 155     | 19.108| 19.1|
| 65      | 64.55| 64.50| 160     | 19.66 | 19.6|
| 70      | 57.177| 57.2 | 165     | 20.93 | 20.9|
| 75      | 51.234| 51.2 | 170     | 23.27 | 23.3|
| 80      | 46.343| 46.3 | 175     | 27    | 27  |
| 85      | 42.244| 42.2 | 180     | 35.7  | 180 |
| 90      | 38.751| 38.8 |         |       |     |

7. Results and Discussions

Mechanical properties of the used materials are given in Table 3. The normalized stress and the Tsai–Wu criterion are considered as a cost function for the PSO algorithm.
Table 3. Material properties of the plate by Daniel and Ishai [74].

| Material                        | \( F_1 \) (MPa) | \( F_2 \) (MPa) | \( F_6 \) (MPa) | \( E_1 \) (MPa) | \( E_2 \) (MPa) | \( G_{12} \) (MPa) | \( \nu_{12} \) |
|--------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------------------|------------|
| Steel                          | -               | -               | -               | 207             | 207             | 79.3              | 0.3        |
| Graphite/Epoxy (T300/5208)     | 1500            | 40              | 68              | 181             | 10.3            | 7.17              | 0.28       |
| S-glass/Epoxy                  | 1280            | 49              | 69              | 43              | 8.9             | 4.5               | 0.27       |
| Woven-glass/Epoxy (7781/5245C) | 367             | 367             | 97.1            | 29.7            | 29.7            | 5.3               | 0.17       |
| E-glass/epoxy                  | 1080            | 39              | 89              | 39              | 8.6             | 3.8               | 0.28       |
| Carbon/Epoxy (IM6/SC1081)      | 2860            | 49              | 83              | 177             | 10.8            | 7.6               | 0.27       |
| Boron/Epoxy (B5.6/5505)        | 1380            | 56              | 62              | 201             | 21.7            | 5.4               | 0.17       |
| Glass/Epoxy                    | 1062            | 31              | 72              | 38.6            | 8.27            | 4.14              | 0.26       |

7.1. Isotropic Plates

For isotropic plate with quasi-square cutout, a variation of optimal SCF with bluntness parameters for different in-plane loadings is shown in Figure 12. According to this figure, the results of uniaxial and biaxial loadings are different from the shear loading. For biaxial loading, by increasing the value of \( w \), the C.F. rises and minimum C.F. occurs at \( w = 0 \). For uniaxial and shear loadings, minimum C.F. happens at \( w = 0.052 \) and \( w = 0.078 \), respectively. \( w = 0 \) indicates a circular cutout. In other words, for an isotropic plate with quasi-square cutout and under uniaxial and shear loadings with \( w = 0.052 \) and \( w = 0.078 \), respectively, minimum SCF will be less than SCF related to a circular cutout. By changing the value of \( c \), the aspect ratio of the cutout can be controlled. According to Equations (12) and (13), because the aspect ratio parameter \( c \) is in the \( y \)-direction of the mapping function, the shape of the cutout is stretched in the \( y \)-direction. To study the effect of \( c \), the value of \( c \) is considered between 1 and 2 (1 < \( c \) < 2).

Figure 12. Variations of the C.F. in terms of \( w \) in different loading (\( c = 1 \)).

Figure 13 shows the effect of aspect ratio (\( c \)) for various in-plane loadings on C.F. in optimal values of load angle and rotation angle and \( w = 0.05 \).
Figure 13. Variations of the C.F. in terms of $c$ for rectangular cutout ($w = 0.05$).

According to this figure, the C.F. varies linearly with $c$. Except for equibiaxial loading, with increasing value of $c$, C.F. is reduced. The values of the cost function in an optimal state for circular and elliptical cutout ($w = 0$) are shown in Table 4 and for rectangular cutout in different values of $w$ are shown in Table 5. Figure 14 shows the change of normalized von Mises stress (cost function) around cutouts in an optimal condition for the isotropic plate.

![Stress distribution](image)

(a) (b) (c) (d) (e)

Figure 14. Stress distribution around different cutouts in an optimal condition for isotropic plates: (a) uniaxial tensile loading ($w = 0$, C.F. = 1.998), elliptical ($c = 2$, $\theta_1 = 45^\circ$, $\theta_3 = 135^\circ$); (b) equibiaxial loading ($w = 0$, C.F. = 3.994), elliptical ($c = 2$, $\theta_3 = 135^\circ$); (c) shear loading ($w = 0$, C.F. = 2.998), elliptical ($c = 2$, $\theta_3 = 44.5^\circ$); (d) uniaxial tensile loading ($w = 0.05$, C.F. = 2.208), rectangular ($c = 2$, $\theta_1 = 70.13^\circ$, $\theta_3 = 0^\circ$); and (e) shear loading ($w = 0.05$, C.F. = 3.214), rectangular ($c = 2$, $\theta_3 = 26.9^\circ$).

Table 6 shows the results of the cost function and one of the optimal modes for quasi-square cutout when all effective parameters such as rotation angle, load angle, and bluntness are considered as design variables. The last column of this table represents the percent difference between the optimal C.F. of quasi-square cutout and the corresponding value related to a circular cutout (P.D.).

Table 4. Optimal results for circular/elliptical cutout in isotropic plates.

| Optimal Values | Uni-Axial Tensile Loading | Equi-Biaxial Loading | Shear Loading |
|----------------|--------------------------|----------------------|--------------|
| $w = 0, c = b/a$ | $\theta_1$ | $\theta_3$ | $|\theta_1 - \theta_3|$ | C.F. | $\theta_3$ | C.F. | $\theta_3$ | C.F. |
| 1 (circular) | 45 | - | - | 2.996 | - | 2.002 | - | 3.995 |
| $c = 1.5$ (elliptical) | 45 | 134.55 | 89.55 | 2.331 | 45-135 | 2.996 | 44.33 | 3.330 |
| $c = 2$ (elliptical) | 45 | 135 | 90 | 1.998 | 45-135 | 3.994 | 44.5 | 2.998 |
Table 5. Optimal results for rectangular cutout in isotropic plates.

| Optimal Values | Uni-Axial Tensile Loading | Equi-Biaxial Loading | Shear Loading |
|----------------|---------------------------|----------------------|--------------|
| $w$ | $c$ | $\theta_1$ | $\theta_3$ | $|\theta_1 - \theta_3|$ | C.F. | $\theta_3$ | C.F. | $\theta_3$ | C.F. |
| 1 | 39.61 | 174.59 | 134.98 | 2.548 | 45-135 | 2.702 | 0-90-180 | 3.476 |
| 0.05 | 1.5 | 58.85 | 117.55 | 58.7 | 2.428 | 45-135 | 3.877 | 77 | 3.429 |
| 2 | 70.13 | 0.00 | 70.13 | 2.208 | 45-135 | 5.052 | 26.9 | 3.214 |

| 1 | 61.77 | 16.81 | 44.96 | 2.857 | 45-135 | 3.709 | 180 | 3.389 |
| 0.1 | 1.5 | 15.6 | 74 | 58.4 | 2.560 | 45-135 | 5.136 | 80.5 | 3.261 |
| 2 | 17.82 | 131.32 | 135.5 | 2.932 | 45-135 | 5.136 | 15.8 | 3.141 |

| 1 | 38.15 | 173.1 | 134.95 | 3.683 | 45-135 | 5.265 | 0-90-180 | 3.928 |
| 0.15 | 1.5 | 47 | 167.5 | 120.5 | 3.005 | 45-135 | 7.081 | 79.8 | 3.488 |
| 2 | 39.15 | 106.9 | 67.75 | 2.566 | 45-135 | 8.897 | 16.35 | 3.240 |

| 1 | 8.6 | 53.6 | 45 | 5.256 | 45-135 | 7.989 | 0-90-180 | 5.149 |
| 0.2 | 1.5 | 17.4 | 135.33 | 117.93 | 3.723 | 45-135 | 10.486 | 78 | 4.063 |
| 2 | 87.6 | 17.3 | 70.3 | 2.988 | 45-135 | 12.983 | 18.23 | 3.564 |

Table 6. All optimal values of design parameters for quasi-square cutout in isotropic plate ($c = 1$).

| All Optimal Values for Minimum SCF | $w$ | $\theta_1$ | $\theta_3$ | $|\theta_1 - \theta_3|$ | C.F. | P.D. |
|-----------------------------------|------|-------------|-------------|----------------|------|------|
| Uniaxial tensile loading          | 0.052 | 90 | 135 | 45 | 2.547 | 15% |
| Equibiaxial loading               | 0.00 | - | - | - | 2.002 | 0.00% |
| Shear loading                      | 0.078 | - | 0-90-180 | - | 3.328 | 16.7% |

Stress distribution around square cutout in an optimal condition in different values of $w$ and for uniaxial and biaxial tensile loading is shown in Figures 15 and 16.

Figure 15. Distribution of the cost function around square cutout in an optimal state (uniaxial tensile loading).
7.2. Orthotropic Plates

For orthotropic material, the ratio of the maximum stress created around the cutout to applied stress is called a SCF. Variation of SCF for Graphite/Epoxy (T300/5208) plate with quasi-square cutout under different in-plane loadings with bluntness parameter \( w \) is illustrated in Figure 17. According to this figure, the minimum values of C.F. for all three types of loadings occurs in non-zero values for \( w \). Minimum C.F. happens at \( w = 0.035, w = 0.020, \) and \( w = 0.045 \) for uniaxial, biaxial, and shear loadings, respectively. \( w = 0 \) is equivalent to a circular cutout. This means square cutout leads to less SCF than circular cutout. Figure 18 shows the effect of aspect ratio of cutout at different types of loadings on SCF. In this case, for \( w = 0.05 \), the optimal results have been achieved for optimal values of load angle, fiber angle, and rotation angle. According to this figure, there is nearly a linear relation between SCF and \( c \).
Figure 18. Variations of the SCF with $c$ in different types of loadings ($w = 0.05$).

The values of the cost function in an optimal mode for circular, elliptical cutouts in different values of $c$ and for rectangular cutout for different values of bluntness parameters $w$ are tabulated in Tables 7 and 8, respectively. Figure 19 shows the stress distribution around quasi-square and elliptical cutouts for graphite/epoxy plate in an optimal condition.

Figures 20 and 21 show the variations of cost function obtained based on Tsai–Wu failure criterion with the bluntness parameter $w$. The results of Figure 20 are for a square cutout ($c = 1$) and biaxial and shear loadings, whereas Figure 21 shows strength variations of the graphite/epoxy with $w$ in different values of $c$; unexpectedly, the optimal value of $w$ is not zero. This means that, by selecting the appropriate values of bluntness parameter, the strength of graphite/epoxy plate with rectangular cutout based on the Tsai–Wu criterion is more than those of a circular cutout. For different values of bluntness ($w$) and aspect ratio of cutout ($c$), the optimal values of the effective parameters are listed in Table 9. In addition, similar results are presented in Table 10 for triangular cutout. For all values of $w$, strength increases with increasing $c$. In this paper, we try to present the results of a square cutout in more detail while the other cutouts only the final results are presented.

Table 7. Optimal values of design variables in different aspect ratios of cutout ($w = 0$).

| $c$          | Uni-Axial Tensile Loading | Equi-Biaxial Loading | Shear Loading |
|--------------|---------------------------|----------------------|---------------|
|              | $\theta_1$ | $\theta_2$ | $\theta_3$ | $|\theta_1 - \theta_3|$ | SCF | $\theta_2$ | $\theta_3$ | SCF | $\theta_2$ | $\theta_3$ | SCF |
| 1 (circular) | 90 | 0 | - | - | 2.371 | 49 | - | 2.551 | 45.45 | 2.152 |
| 1.5 (elliptical) | 90 | 1.5 | 180 | 90 | 1.914 | 66.15 | 180 | 2.704 | 45.45 | 2.152 |
| 2 (elliptical) | 88.6 | 0 | 178.6 | 90 | 1.685 | 0 | 69.3 | 3.397 | 4 | 45.3 | 1.924 |
Table 8. The optimal values of design variables in different values of bluntness $w$ and aspect ratio of rectangular cutout $c$.

| $w$  | $c$  | $\theta_1$ | $\theta_2$ | $\theta_3$ | $|\theta_1 - \theta_3|$ | SCF $\theta_2$ | SCF $\theta_3$ | SCF $\theta_2$ | SCF $\theta_3$ |
|------|------|------------|------------|------------|----------------|-------------|-------------|-------------|-------------|
| 0    | 0.05 | 90         | 0          | 45         | 45            | 2.232       | 84.5        | 129.5       | 2.548       | 45          | 0           | 2.338       |
| 0.1  | 1.5  | 90         | 135        | 0          | 45            | 2.688       | 5.25        | 50.25       | 3.842       | 45          | 180         | 2.586       |
| 2    | 77.5 | 0         | 180        | 102.5      | 1.826         | 16          | 84.7        | 4.126       | 43          | 26.4        | 2.073       |

Table 9. The optimal values of design variables in different values of bluntness $w$ and aspect ratio of rectangular cutout $c$.

| $w$  | $c$  | $\theta_1$ | $\theta_2$ | $\theta_3$ | T.W. (MPa) | $\theta_2$ | T.W. (MPa) | $\theta_3$ | T.W. (MPa) |
|------|------|------------|------------|------------|------------|-------------|------------|-------------|------------|
| 0    | 0.05 | 48.2       | 48.2       | 48.2       | 96.7702    | 61          | 18.7526    | 0           | 23.6921    |
| 0.1  | 1.5  | 90         | 32         | 31.7       | 126.4      | 135.8846    | 47          | 23.8670     | 90          | 25.8111    |
| 2    | 85.8 | 15         | 6.4        | 79.4       | 2.1317     | 1009.6      | 41.8        | 23.53       | 2.136      |

| $w$  | $c$  | $\theta_1$ | $\theta_2$ | $\theta_3$ | T.W. (MPa) | $\theta_2$ | T.W. (MPa) | $\theta_3$ | T.W. (MPa) |
|------|------|------------|------------|------------|------------|-------------|------------|-------------|------------|
| 0    | 0.05 | 43.8       | 42.65      | 121.8      | 49         | 30          | 25.1931    | 90          | 26.3309    |
| 0.1  | 1.5  | 44.65      | 47.9       | 87.3       | 96.8535    | 77.2        | 122.2      | 18.5250     | 90          | 22.2486    |
| 2    | 18.6 | 14.8       | 88.8       | 143.0703   | 90         | 107.5      | 23.3656    | 90          | 28.5282    |

| $w$  | $c$  | $\theta_1$ | $\theta_2$ | $\theta_3$ | T.W. (MPa) | $\theta_2$ | T.W. (MPa) | $\theta_3$ | T.W. (MPa) |
|------|------|------------|------------|------------|------------|-------------|------------|-------------|------------|
| 0    | 0.05 | 26.6       | 20         | 166.5      | 69.2347    | 45          | 14.3278    | 90          | 17.7315    |
| 0.1  | 1.5  | 37.2       | 32         | 100.7      | 118.3234   | 41.5        | 70.5       | 20.8137     | 90          | 30.2795    |
| 2    | 71   | 75.3       | 180        | 120.9234   | 82.3       | 87         | 20.4352    | 90          | 27.8044    |

| $w$  | $c$  | $\theta_1$ | $\theta_2$ | $\theta_3$ | T.W. (MPa) | $\theta_2$ | T.W. (MPa) | $\theta_3$ | T.W. (MPa) |
|------|------|------------|------------|------------|------------|-------------|------------|-------------|------------|
| 0    | 0.05 | 54         | 54.3       | 98.8       | 47.8697    | 64          | 19         | 10.0770     | 90          | 13.6550    |
| 0.1  | 1.5  | 63.2       | 67.8       | 178        | 74.4147    | 68.7        | 46         | 14.0185     | 90          | 23.7177    |
| 2    | 79.8 | 83.8       | 7.2        | 99.6860    | 55         | 51.7       | 17.2145    | 90          | 23.8164    |
Table 10. The optimal values of design variables in different values of bluntness \( w \) and aspect ratio of quasi-triangular cutout \( c \).

| \( w \) | \( c \) | \( \theta_1 \) | \( \theta_2 \) | \( \theta_3 \) | Uni-Axial Tensile Loading (MPa) | Equi-Biaxial Loading (MPa) | Shear Loading (MPa) |
|---|---|---|---|---|---|---|---|
| 1 | 61.2 | 59.6 | 127.7 | 88.3026 | 90 | 180 | 18.0113 | 90 | 0 | 22.6820 |
| 0.05 | 1.5 | 18.2 | 18.14 | 105.56 | 119.7700 | 17.23 | 17.67 | 21.7493 | 0 | 90 | 24.1125 |
| 2 | 90 | 90 | 180 | 154.5169 | 0 | 0 | 25.7136 | 12.42 | 7.34 | 24.7409 |
| 1 | 88.36 | 81.58 | 150.41 | 76.2248 | 90 | 180 | 18.0113 | 90 | 0 | 22.6820 |
| 0.1 | 1.5 | 46.91 | 47.95 | 107.7110 | 150.59 | 42.33 | 26.53 | 20.1418 | 81.63 | 14 | 22.5973 |
| 2 | 90 | 90 | 180 | 138.6020 | 0 | 180 | 23.8684 | 75 | 20.23 | 23.2315 |
| 1 | 121.5 | 59.6 | 127.7 | 88.3026 | 90 | 180 | 18.0113 | 90 | 0 | 22.6820 |
| 0.15 | 1.5 | 46.91 | 47.95 | 107.7110 | 150.59 | 42.33 | 26.53 | 20.1418 | 81.63 | 14 | 22.5973 |
| 2 | 90 | 90 | 180 | 138.6020 | 0 | 180 | 23.8684 | 75 | 20.23 | 23.2315 |

Figure 19. Stress distribution around cutout for graphite/epoxy plates in an optimal condition: (a) uniaxial tensile loading \( (w = 0, \text{C.F.} = 1.685) \), elliptical \( (c = 2, \theta_1 = 88.6^\circ, \theta_2 = 0^\circ, \theta_3 = 178.6^\circ) \); (b) equibiaxial loading \( (w = 0, \text{C.F.} = 3.397) \), elliptical \( (c = 2, \theta_2 = 0^\circ, \theta_3 = 69.3^\circ) \); (c) shear loading \( (w = 0, \text{C.F.} = 1.924) \), elliptical \( (c = 2, \theta_2 = 45^\circ, \theta_3 = 45.3^\circ) \); (d) uniaxial tensile loading \( (w = 0.05, \text{C.F.} = 1.826) \), rectangular \( (c = 2, \theta_1 = 77.5^\circ, \theta_2 = 0^\circ, \theta_3 = 180^\circ) \); and (e) shear loading \( (w = 0.05, \text{C.F.} = 1.826) \), rectangular \( (c = 2, \theta_2 = 43^\circ, \theta_3 = 26.4^\circ) \).

Figure 20. Strength variations of the graphite/epoxy with \( w (c = 1) \).
Figure 21. Strength variations of the graphite/epoxy with \( w \) in different values of \( c \) (Uniaxial tensile loading).

Table 11 gives the optimal values of all design variables for quasi-square \((c = 1)\) cutout to achieve the greatest fracture strength. As shown in this table, the maximum value of Tsai–Wu strength occurs at \( w \neq 0 \). P.D. in this table refers to the percent difference between the optimal C.F. of rectangular cutout and the corresponding value related to a circular cutout.

**Table 11. Optimal values of all design variables for square cutout \((c = 1)\).**

| All Optimal Values | All Optimal Values for minimum SCF | All Optimal Values for maximum Tsai–Wu |
|--------------------|-----------------------------------|---------------------------------------|
| \( w \) \( \theta_1 \) \( \theta_2 \) \( \theta_3 \) SCF P.D. | \( w \) \( \theta_1 \) \( \theta_2 \) \( \theta_3 \) T.W. (MPa) | P.D. |
| Uniaxial 0.035 0 90 135.5 2.175 8% | 0.052 86.7 88.4 39.4 | 110.4345 14% |
| Equibiaxial 0.020 - 55 10 2.031 20% | 0.045 - 73.6 118.6 | 21.4945 14% |
| Shear 0.045 - 44.5 90.5 2.336 10% | 0.039 - 90 0 | 25.3407 7% |

Finally, the optimal values of the design variables for other cutouts are listed in Table 12. As shown in this table, for cutout with an odd number of sides, the highest strength for all in-plane loads occurs at \( w = 0 \). This behavior is not always seen for cutout with an even number of sides.

**Table 12. All optimal values of design parameters for another cutout \((c = 1)\).**

| Uni-Axial Tensile Loading | Equi-Biaxial Loading | Shear Loading |
|---------------------------|----------------------|--------------|
| \( w \) \( \theta_1 \) \( \theta_2 \) \( \theta_3 \) T.W. (MPa) | \( w \) \( \theta_1 \) \( \theta_2 \) \( \theta_3 \) T.W. (MPa) | \( w \) \( \theta_1 \) \( \theta_3 \) T.W. (MPa) |
| Pentagonal 0.00 90 90 180 96.3992 0.00 80.5 180 18.7526 0.00 90 39.5 | 23.6940 |
| Hexagonal 0.013 0.00 0.00 180 99.2559 0.013 34 4 19.9764 0.00 90 15.5 | 23.6940 |
| Heptagonal 0.00 90 90 121.5 96.4769 0.00 90 141 18.7526 0.00 90 15.5 | 23.6940 |
| Octagonal 0.00 90 90 104.5 96.4769 0.005 88 20.5 19.4578 0.00 90 145.5 | 23.6940 |

For perforated composite plates made of different materials, the optimal values of all design variables \((c = 1)\) are listed in Table 13. The results are provided using PSO algorithm. The perforated plate is subjected to uniaxial loading. The results show that for all materials, the optimal values of bluntness parameter \( w \) are not zero. Namely, for the case of \( c = 1 \), square cutout with a certain value of \( w \) leads to higher failure strength than a circular cutout. The percent difference between failure strength of plate with the square cutout and circular cutout is shown in this table. Optimal cost function (Tsai–Wu strength) is highly dependent on the mechanical properties of the materials. The highest percentage difference is related to Boron/Epoxy and the lowest is related to E-glass/Epoxy. The value of bluntness parameters \( w \) is different for various materials.
Table 13. Optimal values of the design parameters for different materials \((c = 1)\) with respect to Tsai–Wu.

| Material                  | Optimum Failure Strength Subjected to Uniaxial Tensile Loading | Optimal Tsai–Wu P.D. | Optimal Tsai–Wu (circular) |
|---------------------------|---------------------------------------------------------------|----------------------|---------------------------|
| Graphite/Epoxy \((T300/5208)\) | 0.052, 0.047, 0.072                                           | 14%                  | 96.770 (86.548)           |
| S-Glass/Epoxy             | 0.032, 0.047                                                 | 5%                   | 98.072 (98.054)           |
| Woven-Glass/Epoxy         | 0.0047, 0.072                                                | 10%                  | 66.105 (66.105)           |
| Graphite/Epoxy            | 0.050, 0.089                                                 | 2.5%                 | 117.967 (117.967)         |
| Carbon/Epoxy              | 0.089, 0.056                                                 | 13%                  | 84.206 (84.206)           |
| E-Glass/Epoxy             | 0.056, 0.089                                                 | 1.5%                 | 114.795 (114.795)         |
| Boron/Epoxy \((B5.6/5505)\)| 0.056, 0.089                                                 | 19.5%                | 96.110 (96.110)           |

8. Conclusions

In this study, the PSO algorithm was used to determine the optimal values of effective parameters on stress distribution around different cutouts in orthotropic/iso-tropic infinite plates under in-plane loading. The failure strength obtained from Tsai–Wu criterion was considered as cost function of the PSO algorithm. The analytical solution based on Lekhnitskii method was used to calculate the stress components around the cutout. The results show that the bluntness \((w)\) and aspect ratio of cutout \((c)\) and fiber angle \((\theta_2)\), load angle \((\theta_1)\), and the cutout orientation \((\theta_3)\) have significant effects in reducing the amount of the cost function and by appropriate selection of these parameters the higher failure strength can be achieved. In addition, the effect of material properties of perforated plates on the values of optimal design variables was studied. Optimal values of design variables depend strongly on the mechanical properties of the perforated plate.

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