Optimal power flow studies in direct current grids: An application of the bio-inspired elephant swarm water search algorithm

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Abstract. Colombian power system is experienced important changes due to the large scale integration of renewable power generation based on solar and wind power; added to the fact that direct current networks have taken important attention, since they are efficient in terms of power loss and voltage profile at distribution or transmission levels. For addressing this problem, this paper presents the application of an emerging bio-inspired metaheuristic optimization technique known as elephant swarm water search algorithm to the optimal power flow problem in direct current networks. A master-slave hybrid optimization strategy for optimal power flow analysis is addressed in this paper by decoupling this problem in two optimizing issues. The first problem corresponds to the selection of the power generated by all non-voltage controlled distributed generators; While the second problem lies in the solution of the classical power flow equations in direct current networks. The solution of the master problem (first problem) is made by applying the elephant swarm water search algorithm, while the second problem (slave problem) is solved by a conventional Gauss-Seidel numerical method. The proposed hybrid methodology allows solving the power flow problem by using any basic programming language with minimum computational effort and well-precision when is compared with optimizing packages such as general algebraic modeling system/CONOPT solver and conventional metaheuristic techniques such as genetic algorithms.

1. Introduction
Direct-current (DC) networks have been widely incremented their presence into the modern electrical power grids under the microgrid’s paradigm [1]. In general terms, the electrical service in the modern power systems can be provided through classical alternating-current (AC) power grids as well as DC power grids [2]. The first case (AC grids) involves conventional power generation plants and integration of multiple distributed energy resources (DERs) via power electronic inverters (from DC to AC conversion), the objective of these grids is to support voltage and frequency to the end-users, in other words, these grids work as conventional AC distribution systems [3]. Second (DC networks) involves multiple integration of DERs via power electronic converters (some of them to transform AC energies into DC energies and some other to adequate DC signals into low/high DC voltage signals) to operate under a constant voltage output for supporting DC loads such as electric vehicles, batteries, home appliances or illumination systems, among others [4].
Optimal power flow studies correspond to a fundamental tool for optimizing electrical power grids where AC grids have been taken advantage when compared to DC power grids since the AC concept has predominated electrification of the countries continuously for more than 60 years, which have allowed studying detailed all AC power grids from planning to control. Nevertheless, the DC power grid paradigm has been underestimated until a few years ago, when DC power grids were not observed as a real and implementable power system [5]. However, this situation has changed significantly since DC power grids have been widely accepted as electrification alternative since they can integrate multiple DERs with low costs and efficient results in comparison to their AC counterpart [6].

Note that in the reviewing of the state-of-the-art, only two metaheuristic approaches were found (genetic algorithms and black hole optimization [7, 8]) were found as evidence about investigations related to the optimal power flow-direct current networks (OPF-DCN) via metaheuristic techniques, which clearly emerges an opportunity of research that this paper tries to fulfill. In this sense, we proposed a master-slave solution technique composed by a bio-inspired elephant swarm water search algorithm (ESWSA) in the master stage and the classical Gauss-Seidel numerical method (GSNM) in the slave stage. The main advantage of the proposed strategy lies in the possibility to solve the OPF-DCN problem efficiently without using any specialized software or optimizing packages since we offer an algorithmic solution easily implementable on any programming language. Additionally, the bio-inspired ESWSA corresponds to a modern metaheuristic technique recently developed [9], which allows efficiently solving nonlinear constrained optimization problems by using a population search method, in this sense, this method is mainly based on particle swarm optimization and it has been adapted to find the global solution on nonlinearly constrained optimization problems, which becomes into a promissory solution strategy for the OPF-DCN problem.

The remainder of this document is organized as follows: Section 2 presents the mathematical modeling of the OPF-DC problem; section 3 presents the bio-inspired ESWSA. Section 4 shows the classical formula of the Gauss-Seidel numerical method for solving power flow problems; test system, simulation cases, similar tools, and results are analyzed and discussed in section 5; while main conclusions and future works are presented in section 6.

2. Mathematical modeling

This section presents the general mathematical model of the optimal power flow problem in DC power grids through a nonlinear non-convex mathematical model [10] as well as its main characteristics and the classical Gauss-Seidel numerical method applied for power flow studies [1].

2.1. Optimal power flow modeling for direct-current power grids

The optimal power flow problem in DC power grids can be modeled as a set of nonlinear non-convex optimization problem [10], which is composed by single-objective function, classically defined in specialized literature as a power losses minimization [11]; and a set of constraints related to the power balance as well as maximum and minimum voltages bounds and maximum and minimum distributed generation capabilities, respectively [12]. A complete detailed mathematical of the OPF-DCN is presented in the objective function, Equation (1).

\[
\min p_{loss} = v^T \mathcal{G}_L v; \quad (1)
\]

where \( \mathcal{G}_L \) represents the component of the conductance matrix \( \mathcal{G} \) related to the contribution of the resistive line effects, and \( v \) is a column vector that contains all voltage variables [13].

The complete interpretation of the mathematical model presented by Equation (1) to Equation (4) is presented as follows: Equation (1) is the objective function of the problem.
and it evaluates the total power loss in all the branches of the distribution network; Equation (2) are set a nonlinear non-convex constraints (multiple quadratic equality constraints) [14] that define the energy balance in all the nodes of the network. Equation (3) defines the upper and lower voltage constraints for all the voltage profiles in the network, and Equation (4) defines power capabilities in all the distributed generators connected to the DC grid.

\[ p_g - p_d + p_{dg} = D(v)[G_L + G_N]v; \]  

where \( D(v) \) is a square matrix that contains the voltage variables in its main diagonal, \( G_N \) represents the component of the conductance matrix \( G \) related to the contribution of the resistive loads (constant resistances connected to the nodes). \( p_g \) represents the vector that contains all power generation in the voltage controlled nodes, \( p_d \) corresponds to the vector associated to the all constant power load consumptions, \( p_{dg} \) is the vector that contains all power generation in the distributed generators (non-controlled voltage nodes).

\[ v^\text{min} \leq v \leq v^\text{max}; \]  

where \( v^\text{min} \) and \( v^\text{max} \) contain information about the maximum and minimum allowed voltage profiles.

\[ P^\text{min}_{dg} \leq p_{dg} \leq P^\text{max}_{dg}; \]  

where \( p^\text{min}_{dg} \) and \( p^\text{max}_{dg} \) contain information about the maximum and minimum power generation capabilities associated to the distributed generators.

3. Elephant swarm water search algorithm
ESWSA is a bio-inspired optimization technique for solving continuous nonlinear constrained optimization problems [9]. Algorithm 1 resumes the application of the ESWSA for minimization problems. The interpretation of all the variables and parameters that intervene in the implementation of the ESWSA can be consulted in [9].

4. Gauss-Seidel method for power flow analysis
The GSNM for power flow studies corresponds to an adaptation of the Gauss-Seidel method for solving linear equations [1]. A recursive formula for power analysis in direct current networks is presented in Equation (5),

\[ v_{i}^{t+1} = \frac{1}{G_{ii}} \left( \frac{p_{i}^{dg} - p_{i}^{d}}{v_{i}^{b}} + \sum_{j<i} |G_{ij}| v_{j}^{t+1} + \sum_{j>i} |G_{ij}| v_{j}^{t} \right) \{ \forall i \in \{1, 2, ..., n\} \}, \]

where \( p_{i}^{dg} \) and \( p_{i}^{d} \) correspond to the total power generated and consumed at the \( i^{th} \) node, while \( G_{ii} \) and \( G_{ij} \) represent the self-conductance and mutual-conductance values, respectively, (these values are obtained from the conductance matrix \( G \)). Note that \( t \) is the iterative counter, which is linearly incremented until the maximum error between voltages of two consecutive iterations be lower than the convergence error.
Algorithm 1. Proposed pseudo-code for the hybrid GSNM–ESWSA for solving the optimal DC power flow problem.

**Data:** Define DC grid, ESWSA and Gauss-Seidel parameters.

for \( t = 1 : t_{\text{max}} \) do
  if \( t == 1 \) then
    \( \Delta = \infty \);
    for \( k = 1 : n_g \) do
      Generate \( X_t^k \) and \( V_t^k \), and assign \( Y_t^k = X_t^k \);
      if \( z_t^k (Y_t^k) < \Delta \) then
        \( Z_t^k = Y_t^k \) and \( \Delta = z_t^k (Y_t^k) \);
      end
    end
  else
    Assign the weight factor \( \omega_t \);
    for \( k = 1 : n_g \) do
      \( a = \text{rand} \), and update \( V_t^k \) and \( X_t^k \);
      if \( z_t^k (X_t^k) < z_t^k (Y_t^k) \) then
        Actualize the best local solution, and \( Y_t^k = X_t^k \);
      end
      if \( z_t^k (Y_t^k) < z_t^k (Z_t^k) \) then
        Actualize the best global solution;
        \( Z_t^k = Y_t^k \) and \( z^* = z_t^k (Z_t^k) \);
      end
    end
  end
  \( X^* = Z_t^k \) Verification of the stopping criteria;
end

**Result:** Return \( X^* \) and \( z_t (X^*) \).

5. Computational validation

The test system employed for the numerical validation corresponds to the 21-node test feeder reported in [12] for analyzing the convergence of Newton’s method for the power flow problem in DC power grids. The detailed information of this test feeder, i.e., branch parameters, nodal consumption and topology can be consulted in [12]. In order to evaluate the OPF-DC problem, we select three nodes arbitrarily to locate distributed generators. These nodes are 7, 12 and 15, respectively; and the power capabilities of these generators are contained from 0 p.u. to 1.5 p.u., respectively.

The capability and efficiency of the proposed hybrid GSNM-ESWSA optimization algorithm for solving the OPF-DC problem is verified by employing the general algebraic modeling system (GAMS) and the nonlinear optimization solver named CONOPT which is based on the generalized reduced gradient algorithm as well as a hybrid optimization algorithm based on a Gauss-Seidel numerical method and a continuous version of a genetic algorithm (GSNM-GA) [7]. All implementations are made in MATLAB and GAMS software by using a desk-computer INTEL(R) Core(TM) i5-3550, 3.50 GHz, 8 GB RAM with 64 bits Windows 7 Professional.

Table 1 presents the power generation for each distributed generator as well as the objective function for each comparison method. Additionally, this table and the errors given by the GSNM-GA and GSNM-ESWSA when these are compared to GAMS/CONOPT.
Table 1. Power generation and active power losses (kW).

| Method          | $p_{g9}^{gd}$ | $p_{13}^{gd}$ | $p_{21}^{gd}$ | $P_{loss}$ | error [%] |
|-----------------|---------------|---------------|---------------|------------|-----------|
| GAMS/CONOPT    | 143.689       | 124.815       | 122.217       | 6.620      | 0.00      |
| GSNM-GA        | 145.173       | 128.260       | 120.213       | 6.623      | $5.436 \times 10^{-4}$ |
| GSNM-ESWSA     | 143.509       | 126.097       | 118.882       | 6.623      | $4.963 \times 10^{-4}$ |

Considering that the power losses at the base case (system without distributed generation) correspond to 27.603 kW, in general terms the total power reduction for the comparison methods as well as the proposed method is around of 76.01%. Besides, the GSNM-ESWSA evidences the best approximation to the objective function when compared to the GSNM-GA, since it has a lower estimation error.

On the other hand, when the total power generated by the distributed generators are compared, it is possible to observe that: the GAMS/CONOPT solver injects 390.721 kW, the GSNM-GA injects 393.646 kW, and the GSNM-ESWSA injects 388.488 kW; which implies that the proposed method reduces around 2.233 kW and 5.158 kW when compared to the GAMS/CONOPT and GSNM-GA solution strategies.

6. Conclusions and future works
This paper addressed the OPF problem in DC power grids via an emerging metaheuristic optimization technique named elephant swarm water search algorithm, which corresponds to a variant of the conventional particle swarm optimization methodology. This optimization technique was successfully combined with a classical Gauss-Seidel numerical method through a master-slave optimization strategy. Moreover, this combination allowed solving the OPF-DC problem algorithmically, without recurring to specialized software or programming languages, since it can be intuitively implementing on any basic programming software.

As future works, the proposed hybrid method can be extended for optimal power flow analysis in alternating current power grids as well as for solving economic dispatch problems including energy storage devices and high penetration of distributed generators. Besides, it can be embedded into large scale optimization problems with combines binary and continuous variables, such as distribution system planning problems.

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