Few body Calculation of Neutrino Neutral Inelastic scattering on $^4$He

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The inelastic neutral reaction of neutrino on $^4$He is calculated using two modern nucleon–nucleon potentials. Full final state interaction among the four nucleons is considered, via the Lorentz integral transform (LIT) method. The effective interaction hyperspherical-harmonic (EIHH) approach is used to solve the resulting Schrödinger like equations. A detailed energy dependent calculation is given in the impulse approximation.

1. Introduction

Neutrino reactions with nuclear targets has led modern physics to numerous achievements and gave the first hint for deviations from the standard model. In astrophysics, for example, neutrino scattering on nuclei are the key ingredient in the supernova explosion of a massive star and the synthesis of elements within. The inelastic reactions of $^4$He with $\nu_x (\bar{\nu}_x)$ ($x = e, \mu, \tau$) have a part in these two phenomena. Core collapse supernovae are widely accepted to be a neutrino driven explosion of a massive star. When the iron core of a massive star becomes gravitationally unstable it collapses until short-range nuclear forces halt the collapse and drive an outgoing shock through the outer layers of the core and the inner envelope. However, the shock loses energy through dissociation of iron nuclei and neutrino radiation, and gradually stalls, it becomes an accretion shock. Meanwhile, the collapsed core (the proto-neutron star (PNS)) cools through neutrino radiation, originating in part by the deleptonization of the core, but mostly by thermally produced pairs of neutrinos and anti-neutrinos in all flavors. These neutrinos diffuse out of the PNS. Due to charge current interactions and electron scattering the electron–neutrinos decouple from matter in a bigger radius than the heavy-flavored neutrinos. An energy cascade is thus created, with 6-10 MeV temperature for $\nu_{\mu,\tau}$ ($\bar{\nu}_{\mu,\tau}$), 5-8 MeV for $\bar{\nu}_e$, and 3-5 MeV for $\nu_e$ [1].

It is believed that the shock is then revived, as these neutrinos deposit energy in the matter behind the shock to reverse the flow to an outgoing shock which explodes the star. This belief was qualitatively demonstrated in the detection of the neutrino signal from supernova 1987A [2]. However, quantitative proof in full hydro-reactive simulations is still missing [3,4]. The matter in the volume between the PNS and the shock is a hot dilute gas composed mainly of protons, neutrons, electrons, and $^4$He nuclei. In contrast to the fairly known cross-sections of neutrinos with electrons and nucleons, the interaction of neutrinos with $^4$He is not accurately known. Only recently a first microscopic calculation

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of the neutral cross-section has been published [5].
The role of $\alpha - \nu$ interaction in the supernova explosion mechanism is not yet completely understood (see however, [6]). The same interaction, however, has a most important role in neutrino induced nucleosynthesis [7]. The huge number of neutrinos are the seed of light element nucleosynthesis in the supernova environment, the so called $\nu$ process. In this process, an evaporation of a nucleon from $^4$He in the helium rich layer is followed by a fusion of the trinucleus with another $\alpha$ nucleus, resulting in a 7 body nucleus. This is an important source of $^7$Li, and for $^{11}$B and $^{19}$F through additional $\alpha$ capture reactions. A correct description of this process must contain an exact, energy dependent cross-section for the neutral inelastic $\alpha - \nu$ reaction, which initiates the process.
In this contribution we give the neutral cross-section in the impulse approximation, with two different modern NN potentials, Argonne V8' [8] and V18 [9].

2. Calculation And Results

The neutrino-nucleus scattering process is governed by the weak interaction model. In the limit of small momentum transfer (compared to the $Z$ particle rest mass), the effective Hamiltonian can be written as a current-current interaction:

$$\hat{H}_W = \frac{G}{\sqrt{2}} \int d^3x j_\mu(\vec{x}) J^\mu(\vec{x}),$$

where $G$ is the Fermi weak coupling constant, $j_\mu(\vec{x})$ is the leptonic current, and $J^\mu$ is the hadronic current. The matrix element of the leptonic current results only in kinematical factors to the cross-section. However, the nuclear current cannot be calculated in such a simple manner. The standard model dictates only the formal structure:

$$J_\mu^{\text{hadronic}} = (1 - 2 \cdot \sin^2 \theta_W) \frac{\tau_0}{2} J_\mu + \frac{\tau_0}{2} J_5^\mu - 2 \cdot \sin^2 \theta_W \frac{1}{2} J_\mu,$$  \hspace{1cm} (1)

here we denote axial currents by superscript 5, and no superscript denotes the vector currents. The latter are built of both isoscalar and isovector parts, whereas the axial currents are pure isovector operators. The nuclear current matrix elements consists of one body weak currents, but also many body corrections due to meson exchange. The many-body currents are a result of meson exchanged between the nucleons. The current work is done in the impulse approximation, thus taking into account only one-body terms. In order to estimate this approximation, we refer to studies of inclusive electron scattering off $^4$He [10], where it is shown that isovector electromagnetic two-body currents, which are proportional to the electroweak vector currents, produce a strong enhancement of the transverse response at low and intermediate energies. In the current calculation, the vector MEC are fully considered within the Siegert theorem, and the two-body axial currents are expected to give small contributions [11,12].

The one-body currents connect the $^4$He ground state and final state wave functions. An explicit calculation of the nuclear response functions demands an accurate description of all the excited states of the nucleus. For $^4$He this is currently out of reach, as the nucleus has no excited bound states, and no accurate continuum wave functions exist for all channels of system with more than 3 particles. In order to avoid this complexity we use the Lorentz integral transforms [13]. As a result, the problem reduces to a bound state like problem. A most successful method for solving these kind of problems is the EIHH [14] method. The combination of the EIHH and LIT methods brings to a rapid
Table 1
Flavor and temperature averaged inclusive inelastic cross-section

| T [MeV] | $\langle \sigma \rangle_T$ $[10^{-42} cm^2]$ | AV8’ | AV18 | Ref. [7] |
|---------|---------------------------------------------|-------|-------|---------|
| 4       | 2.09(-3)                                    | 2.31(-3) | -     |
| 6       | 3.84(-2)                                    | 4.30(-2) | 3.87(-2) |
| 8       | 2.25(-1)                                    | 2.52(-1) | 2.14(-1) |
| 10      | 7.85(-1)                                    | 8.81(-1) | 6.78(-1) |
| 12      | 2.05                                        | 2.29    | 1.63  |
| 14      | 4.45                                        | 4.53    | -     |
| 16      | 8.52                                        | 9.48    | -     |

convergence in the response functions [5,16].

It is of interest to note that since $^4$He is almost pure isoscalar and approximately spin zero, the leading contributions to the cross-section are proportional to $q r$, where $q$ is the momentum transfer and $r$ is the coordinate of the nucleon. The contribution of the isoscalar current to the neutral cross-section is of higher order in this small parameter, thus negligible.

It is assumed that the neutrinos are in thermal equilibrium, thus their spectrum can be approximated by the Fermi-Dirac distribution with characteristic temperature $T$. As a result, the most interesting physical quantities are the temperature averaged cross-section,

$$\frac{d\langle \sigma \rangle_T}{d\omega} = \int dk f(T, k) \frac{d\omega}{dk}$$

and energy transfer cross-section,

$$\frac{d\langle \sigma \omega \rangle_T}{d\omega} = \omega \frac{d\langle \sigma \rangle_T}{d\omega}$$

where $f(T, k)$ is normalized Fermi-Dirac spectrum with zero chemical potential, temperature $T$, and energy $k$. In Table 1 we present the calculated total temperature averaged cross-section, $\langle \sigma \rangle_T = \frac{1}{2 \pi} (\langle \sigma_\nu + \sigma_\nu \rangle)_T$, as a function of the temperature of the neutrinos. Also presented are earlier results reported by Woosley et. al. [7,15]. It can be seen that the current work predicts a substantial enhancement in the cross-section.

The energy transfer cross-section was fitted by Haxton to a convenient formula [15],

$$\langle \sigma \omega \rangle_T = \alpha \left( \frac{T - T_0}{10 \text{MeV}} \right)^\beta$$

with the parameters $\alpha = 0.62 \cdot 10^{-40} \text{cm}^2 \text{MeV}$, $T_0 = 2.54 \text{MeV}$, $\beta = 3.82$. A similar fit to our results yields for AV8': $\alpha = 0.64 \cdot 10^{-40} \text{cm}^2 \text{MeV}$, $T_0 = 2.05 \text{MeV}$, $\beta = 4.46$, and for AV18: $\alpha = 0.72 \cdot 10^{-40} \text{cm}^2 \text{MeV}$, $T_0 = 2.12 \text{MeV}$, $\beta = 4.42$. It can be seen that the current work predicts a stronger temperature dependence of the cross sections. For example, a $15 - 30\%$ difference between these calculations at $T = 10 \text{ Mev}$, grows to a $50 - 70\%$ difference at $T = 16 \text{ MeV}$.

3. Conclusions

A detailed realistic calculation of the inelastic neutrino-$^4$He neutral scattering cross-section is given. The calculation was done in the impulse approximation with numerical accuracy of about $1\%$. The different approximations used here should result in about $10\%$ error, mainly due to many-body currents and three nucleon force (3NF), which were
not considered in the current work. Axial vector part contributes more than 90% of the cross-section given. As these currents are not protected by current conservation, they should be included explicitly in the calculation. However, comparing to the size of these effects in electron scattering processes, one should expect only a few percents effect.

The nuclear hamiltonian used in the current work consists only of nucleon-nucleon potential, and neglects 3NF. Lately, a detailed description of the photoabsorption on $^4$He has shown that including 3NF results in a reduction of 10% of the cross-section [16]. Thus, one should also check this effect on neutrino-$^4$He cross-section.

The influence of the current calculation on the supernova explosion mechanism should be checked through hydrodynamic simulations, of various progenitors. Nonetheless, it is clear that our results facilitate a stronger neutrino-matter coupling in the supernova environment. First, our calculations predict an enhanced cross section with respect to previous estimates. Second, we obtained steeper dependence of the energy transfer cross-section on the neutrino’s temperature. Thus, supporting the observation that the core temperature is a critical parameter in the explosion process. It is important to notice that the energy-transfer due to inelastic reactions are $1 - 2$ orders of magnitude larger than the elastic reactions, ergo the inelastic cross-section are important to an accurate description of the Helium shell temperature.

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