Entanglement as a complexity measure for quantum state preparation

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Abstract

State preparation algorithms are routines used in many quantum computing applications, from solving a linear system of equations to quantum machine learning. In this paper, we present an exact state preparation algorithm that considers entanglement to reduce the complexity of the generated quantum circuit. The proposed method allows approximation by entanglement reduction. The approximation of an entangled state by two separable states allows a quadratic decrease in the number of CNOT gates in exchange for an approximation error. Through experimental analysis with 7-qubits devices, the approximation error observed through measurements is smaller than the error due to the noise generated by the circuit to prepare the original state.

1. Introduction

Quantum devices can execute information processing tasks that classical computers cannot perform efficiently \[1\]. The initialization of an \(n\)-qubit quantum state (state preparation) is an important step in quantum information processing and requires quantum circuits with \(O(2^n)\) controlled-NOT (CNOT) gates. Several works focus on the development of algorithms that supposes data-efficient initialization. For instance, this supposition leads to exponential advantages in the solution of systems of linear equations \[2\] and principal component analysis \[3\].

There are several quantum state preparation algorithms \[4, 5, 6, 7\] with a lower bound of \(O(2^n)\) CNOT gates to prepare an arbitrary quantum state with \(n\) qubits. Attempts to prepare quantum states more efficiently include a divide-and-conquer strategy that exchanges circuit depth by circuit width \[7, 8\], probabilistic approaches \[9, 10\], and strategies to initialize approximated quantum states \[11\]. Most recently there are works that focus on specific classes of quantum states. For instance, how to prepare uniform \[12\] or \[6\] sparse states. However, there is no clear understanding of what are the class of quantum states that we can create more efficiently.

Entanglement is one of the quantum resources that allow the development of more efficient algorithms and protocols. However, the circuit complexity of the current algorithms to create a quantum state does not consider the amount of entanglement. The main goal of this work is to define the complexity of state preparation as a function of an entanglement measure instead of only the number of qubits. We investigate a concrete algorithm that explores the level of entanglement of the input vector and also define how to prepare an \(r\)-rank approximation of a quantum state. The proposed algorithm has the potential to speed-up quantum algorithms where the use of low-rank state approximation is possible or that requires the initialization of low-rank states. The method is based on the state preparation algorithm proposed in \[5\].

2. State preparation

The quantum state preparation proposed in \[5\] is based on the Schmidt decomposition. Given a quantum state \(|\psi\rangle = \sum_j \lambda_j |j\rangle\), the first step of the algorithm runs a Schmidt decomposition \(|\psi\rangle = \sum_i \sigma_i |\alpha_i\rangle \otimes |\beta_i\rangle \in\)
\[ H = H_A \otimes H_B \] where the state is factored in two quantum subsystems \( H_A \) and \( H_B \), values \( \sigma_i \) are the Schmidt coefficients, \( \{ |\alpha_i\rangle\} \in H_A \) and \( \{ |\beta_i\rangle\} \in H_B \) are orthonormal basis, and \( 1 \leq i \leq \min(\dim(H_A), \dim(H_B)) \). The second step performs the initialization of the quantum state \( \sum_i \sigma_i |i\rangle |0\rangle \) in the first quantum register. The objective of this step is to initialize a state with the Schmidt coefficients. The third step applies \( \lceil n/2 \rceil \) CNOTs to create the state \( \sum_i \sigma_i |i\rangle |i\rangle \). Let \( U \) and \( V \) be unitaries, where \( U |\alpha_i\rangle = |\beta_i\rangle \) and \( V |\beta_i\rangle = |\gamma_i\rangle \). The last step of the algorithm applies \( U \) in the first register and \( V \) in the second register.

With an even number of qubits, the total number of CNOTs of Plesch’s state preparation algorithm is bounded by \( \frac{2n^2}{3} \) and with an odd number of qubits, the number of CNOTs is bounded by \( \frac{11n^2}{8} + 2^n \). In the case with an odd number of qubits the quantum state preparation can be improved using an isometry \([13]\). The Schmidt decomposition reduces the depth of the quantum circuit used to perform the state preparation. The idea in this work is to change the behavior of this state preparation algorithm when there are Schmidt coefficients equal to zero. This also leads to an approximated state preparation algorithm that removes the smallest Schmidt coefficients. This reduction uses a low-rank matrix approximation \([14]\).

3. Entanglement measures

The Schmidt decomposition describes the entanglement correlations for pure bipartite states \([13, 16, 17, 18]\). A numerical measure of entanglement between the bipartite state subsystems can be based on squares of the Schmidt coefficients \( |\sigma_i|^2 \) since unitary operations performed locally on the subsystems do not change these quantities \([19]\).

Purity \( P(\rho) = \text{Tr}(\rho^2) \) \([18, 19]\) can be used to quantify the degree of entanglement operating on the reduced density matrices for the two subsystem \( H_A \) and \( H_B \):

\[
\rho_A = \sum_i |\sigma_i|^2 |\alpha_i\rangle \langle \alpha_i| \quad \rho_B = \sum_i |\sigma_i|^2 |\beta_i\rangle \langle \beta_i| \quad P(\rho_A) = P(\rho_B) = \sum_i |\sigma_i|^4
\]

Since the quantum state of the total system is pure and the purity of both subsystems is equal, the conclusion is that the purity of the reduced state represents a quantifier of entanglement between \( H_A \) and \( H_B \) \([19]\).

Related to the purity of a state regarding a measure of entanglement has been proposed by Meyer-Wallach \([20]\) with the current interpretation of the average complement of purity given by Brennen \([21]\)

\[
Q(|\psi\rangle) = 2 \left(1 - \frac{1}{n} \sum_{k=1}^{n} \text{Tr}[\rho_k^2] \right)
\]

where \( \rho_k \) are the \( n = \text{dim}(H) \) single-qubit bipartitions that exist in a \( n \) qubit state \( |\psi\rangle \).

Another simple alternative to quantify entanglement uses the Schmidt rank (the number of non-zero \( \sigma_i \) amplitudes). The Schmidt measure of entanglement \([19]\) is defined in units of e-bits by

\[
E_S(|\psi\rangle) = \log_2(\text{rank})
\]

The Bell states have one e-bit (the amount of entanglement contained in a maximally entangled two-qubit state) and define e-bit as the unit of bipartite entanglement. The rank of a Bell state is two, so \( E_s(|\text{bell}\rangle) = 1 \) e-bit.

4. Low-rank state preparation

Based on the state preparation proposed by Plesch \([5]\) (Fig. 1d), one can observe how entanglement affects the complexity of the circuit. Such a circuit is a direct application of the Schmidt decomposition, and it is modified as follows to achieve low-rank state preparation (LRSP). Let \( |\psi\rangle \) be an \( n \)-qubit quantum state with Schmidt decomposition \( |\psi\rangle = \sum_{i=0}^{k} \sigma_i |i_A\rangle |i_B\rangle \), where subsystem \( H_A \) has \( \lceil n/2 \rceil \) qubits, subsystem \( H_B \) has \( \lfloor n/2 \rfloor \) qubits and \( k \) is the Schmidt rank (also known as Schmidt number). If the Schmidt measure \( m = \lfloor \log_2(k) \rfloor < \lceil n/2 \rceil \), operator \( S \) (Fig. 1a) initializes a state with \( m \) qubits in phase 1 (Fig. 1b) and Fig. 1c),
Instead of $\lfloor n/2 \rfloor$ qubits (Fig. 2). The second phase requires $m$ CNOT gates, as they are not needed where the control qubit is $|0\rangle$, reducing entanglement. In this circuit configuration, $m$ is the Schmidt measure and quantifies the entanglement between subsystems $\mathcal{H}_A$ and $\mathcal{H}_B$. Finally, in phases 3 and 4, LRSP is made possible by replacing the unitary decomposition with isometry decomposition \cite{13} (Fig. 1). The number of columns in each isometry is equal to the Schmidt rank.

In Plesch’s original algorithm \cite{5}, the number of CNOT gates is $O(2^n)$ and considers only the amount of qubits $n$. The LRSP algorithm (Alg. 1) complexity is defined considering the number of qubits $n$ and the Schmidt measure of entanglement $m$ in units of e-bits. The unitaries $U$ and $V^T$ are isometries $2^m$-to-$2^\lfloor n/2 \rfloor$ and $2^n$-to-$2^\lfloor n/2 \rfloor$ which require $O(2^{m+\lfloor n/2 \rfloor})$ CNOTs. The number of CNOTs of the complete LRSP circuit is $O(2^{m+\lfloor n/2 \rfloor})$ because the cost of the isometry dominates the cost of the algorithm. The best-case occurs when the bipartition is not entangled $|\psi\rangle = |\psi_A\rangle |\psi_B\rangle$. Therefore, if rank $= 1$ ($m = 0$ e-bits) there is no operations in phase 1 (to encode singular values), no entanglement between the two subsystems in phase 2 (since there is 0 e-bits between the subsystems), and there are one 1-to-2 $\lfloor n/2 \rfloor$ and one 1-to-2 $\lfloor n/2 \rfloor$ isometries in phases 3 and 4 (Fig. 1b) (which is equivalent to two parallel sub-state preparations in $\lfloor n/2 \rfloor$ and $\lfloor n/2 \rfloor$ qubits). If all qubits of the state are separable, applying recursively the same algorithm to prepare the sub-states in phases 3 and 4 generates a circuit without CNOT gates. If the state is initially separable but

\footnote{The difference between even and odd $n$ is discussed in Section 4.1}
**Algorithm 1: Low-rank state preparation algorithm**

**Input:** A state vector.
**Output:** Encoding quantum circuit.

1. Decompose the state vector using SVD (store unitaries $u$ and $v^T$, and vector $s$).
2. Set rank equal to the number of non-zero elements of $s$ ($\text{rank} = \text{count}(s \neq 0)$).
3. Set Schmidt measure $m = \lfloor \log_2(\text{rank}) \rfloor$.
4. If $m < \lfloor n/2 \rfloor$, set the number of columns of $u$, $v^T$, and $s$ to $2^m$, otherwise continue.
5. Create a quantum circuit with $n$ qubits ($n = \log_2(\text{length(state vector)})$).
6. If $m > 0$, encode the normalized vector $s$ on qubits $q_0$ ($0 \leq q < \lfloor n/2 \rfloor$) using an amplitude encoding state preparation algorithm. This step is named Phase 1 (Fig. 1a).
7. Perform $m$ CNOT gates between control qubit $q_j$ and target $q_j + \lfloor n/2 \rfloor$ ($0 \leq j < m$). This step is named Phase 2.
8. Encode $u$ on qubits $q_0$ ($0 \leq q < \lfloor n/2 \rfloor$) using an algorithm according to its dimension (vector, isometry, and unitary). This step is named Phase 3.
9. Encode $v^T$ on qubits $q_j + \lfloor n/2 \rfloor$ ($0 \leq j < \lfloor n/2 \rfloor$) using an algorithm according to its dimension (vector, isometry or unitary). This step is named Phase 4.
10. Output the encoding quantum circuit.

Not all sub-states of the recurrence (some qubits are not separable), the cost of the state preparation is $O(2^{n_e})$, where $n_e$ is the number of qubits of the higher entangled subsystem. The worst case occurs when $m = \lfloor n/2 \rfloor$, recovering the $O(2^n)$ complexity of the original Plesch circuit (Fig. 1d). The number of CNOT gates is discussed in Section 4.1.

The LRSP algorithm also allows a low-rank approximation in exchange for an error. Limiting the Schmidt rank imposes an entanglement reduction, as this action reduces both purity and e-bits. Fidelity-loss can be used to quantify the loss by the approximation, as presented in Section 4.2. To achieve low-rank approximation, the corresponding input parameter $r$ is introduced to Algorithm 1 and can assume values between 1 and $2\lfloor n/2 \rfloor$. The following conditional statement is introduced between lines 2 and 3: If $r < \text{rank}$ then $\text{rank} = r$. Setting $r = 1$ completely disentangles subsystems $\mathcal{H}_A$ and $\mathcal{H}_B$, which implies a quadratic reduction in the number of CNOT gates in the circuit to prepare a state. Additionally, the circuit depth is also reduced.

### 4.1. Complexity

When acting on $k$ qubits, a quantum state preparation requires $2^k - k - 1$ CNOTs [4], a unitary operator $\frac{23}{48}2^{2k} - \frac{3}{2}2^k + \frac{4}{3}$ [22] and an isometry $2^{m+k} - \frac{1}{24}2^k + O(k^2)2^m$ [13]. Considering the complete low-rank state preparation (LRSP) circuit and summing up contributions from all four phases, the overall number of CNOT gates is represented by:

- $m < k$ and $n$ even:
  \[
  2^m - m - 1 + m + 2\left(2^{m+k} - \frac{1}{24}2^k\right)
  \]
- $m < k$ and $n$ odd:
  \[
  2^m - m - 1 + m + 2^{m+k} - \frac{1}{24}2^k + 2^{m+k+1} - \frac{1}{24}2^{k+1}
  \]
- $m = k$ and $n$ even:
  \[
  2^k - k - 1 + k + 2\left(\frac{23}{48}2^{2k} - \frac{3}{2}2^k + \frac{4}{3}\right)
  \]
where \( k = n/2 \) for \( n \) even and \( k = (n - 1)/2 \) for \( n \) odd. The phases brackets indicate the contribution from each phase of the LRSP procedure to the number of CNOTs. Phase 1 is a state preparation, phase 2 a sequence of CNOT gates, phases 3 and 4 are isometry decompositions when \( m < k \) or unitary decompositions when \( m = k \). The numbers are bounded by the results presented in Table 1.

Table 1: Upper bound on the number of CNOT gates required by LRSP for large \( n \). When \( m < k \) the upper bound achieves a quadratic reduction in the number of qubits. It is possible to reduce the upper bound for \( m = k \) and \( n \) odd by using an efficient isometry \( k \)-to-\((k + 1)\), such as cosine-sine decomposition, in place of the unitary decomposition.

4.2. Bounded Approximation Algorithm

The above idea can be generalized by a recursive algorithm that we will describe here. It provides an approximation up to a given error bound – if possible – and thus saves the use of CNOT gates while providing a guarantee of proximity to the original state. It is a bounded approximation error state preparation algorithm (BAA) that potentially has a classical exponential run-time with respect to the number of qubits of the state. As it is a branch-and-bound algorithm using breadth-first search the complexity usually converges a lot quicker.

The algorithm is as follows. Any (pure) \( n \)-qubit quantum state \(|\psi\rangle\) will allow for a total of

\[
b(n) = \sum_{k=1}^{\lfloor n/2 \rfloor} \binom{n}{k}
\]

bipartitions, with each the number of Schmidt coefficients \( k = k(b) = \min(\dim(H_A), \dim(H_B)) \) \((b \in \{1, \ldots, b(n)\})\), which we also call branches. Given the low-rank parameter \( r \), the main idea of the Algorithm 1 is to truncate the coefficients for \( 1 \leq r \leq k \), i.e. \( \sigma_j = 0 \), \( r < j \leq k \). Then a renormalization is necessary, which means that

\[
Z = \sqrt{\sum_{j=1}^{r} |\sigma_j|^2} = \sqrt{1 - \sum_{j=r+1}^{k} |\sigma_j|^2}, \quad \sigma'_i = \frac{\sigma_i}{Z}, \quad i = 1, \ldots, r
\]

and the state that emerges from this description is written as \(|\psi'\rangle = \sum_{i=1}^{r} \sigma'_i |\alpha_i \rangle \otimes |\beta_i \rangle\). The state overlap with the original one is then

\[
|\langle \psi', \psi\rangle|^2 = \frac{1}{Z^2} \left( \sum_{i=1}^{r} |\sigma_i|^2 \right)^2 = \frac{Z^4}{2Z^2} = Z^2.
\]

Thus, recursively, we will start from \(|\psi\rangle\) into the \( b(n) \) possible bipartitions / branches and then continue from the resulting set of states of the bipartition. The quantity

\[
l(b, |\psi\rangle) := 1 - |\langle \psi, \psi'\rangle|^2 = 1 - Z^2 = \sum_{i=r+1}^{k} |\sigma_i|^2
\]

5
is called the fidelity loss of the state $|\psi\rangle$ on the bipartition / branch $b \in \{1, \ldots, b(n)\}$. Taking from now on the low-rank parameter $r = 1$ ($m = 0$, recall $1 \leq r \leq 2^m$), we aim to disentangle the state to the product state that has the maximal overlap with the original state $|\psi\rangle$. To that end, a tree is recursively spanned by an adaptation of the Algorithm 1 with $r = 1$. A tree of an example three-qubit state is shown in Fig. 2. The algorithm ends when a product state is hit. To calculate the total loss over several steps in the tree, one needs to calculate each step’s left fidelity of $(1 - l_i)$ and multiply that with the left fidelity of another step, $(1 - l_j)$. Therefore, $l = 1 - \prod_i (1 - l_i)$.

To each edge, which has a fidelity loss, also the number of saved CNOT gates can be calculated. Through a branch-and-bound with a breadth-first search type of algorithm, given a maximal acceptable fidelity loss of $l_{\text{max}}$, it is possible to maximize the number of saved CNOT gates. In our example (Fig. 2), if an acceptable fidelity loss of $l_{\text{max}} = 0.1$ is given, only one edge from the root is possible, with a loss of $l \approx 0.0583$, a saving of two CNOT gates can be achieved. Given, that a three-qubit state preparation only needs four CNOT gates, this is a squared reduction.

![Figure 2: An example of a three-qubit state that is disentangled into product states (leafs). Its Meyer-Wallach measure is $\approx 0.60954$. It starts with one vector in the root and then all $b(3) = 3$ bipartitions are branched off, creating each two states. From there, the bigger subspace has two qubits, thus it branches $b(2) = 2$ branches. Each edge has a fidelity-loss and each node has the number of CNOT gates saved using the proposed algorithm. The purple nodes show the bipartitions. Adding the weights from root to any leaf gives the total fidelity-loss of $\approx 0.3085$, meaning that the reduced product state to its original state have an overlap of about $\approx 0.6915$, but with a saving of 4 CNOT gates. The adaptive approximation algorithm goes to ask if we can save some CNOT gates with an acceptable fidelity-loss. There is the bipartition 2 that introduces only a fidelity-loss of $\approx 0.0583$ by simultaneously saving two CNOT gates.](image)

The example in Fig. 2 is a simple one and given more complicated examples, say a 5 qubit random state, for as much as a fidelity loss of $l \approx 0.033309$ one can attain 17 CNOT gates saved when the full state preparation would need 26. Indeed, one can hypothesize that a low-entanglement state should be representable by a product state with a small fidelity-loss $l$.

Through an empirical evaluation, we verified that the fidelity loss to create a disentangled approximation of a state is independent of which path one moves along in the tree, thus it is a general value associated with a given state. Furthermore, it is within the interval [0, 1] and can be shown that it is invariant under
single-qubit unitary operations. It is thus an alternative measure for entanglement that is connected to the complexity to create the state.

4.3. Experiments

Two sets of experiments were performed to evaluate the low-rank state preparation algorithm. The first set evaluates the required number of CNOTs to initialize a quantum state with the number of qubits \( n \) from 4 to 14 and Schmidt measure \( m = \lceil \log_2(\text{rank}) \rceil \) from 0 to \( \lfloor n/2 \rfloor \). The quantum states are initialized randomly from a uniform distribution. After the Schmidt decomposition, only the first 2\(^m\) Schmidt coefficients are considered. Table 2 and Figure 3 show the reduction in circuit depth and number of CNOTs with smaller values of \( m \). The results show that the number of CNOTs and the circuit depth are an exponential function concerning \( m \), not the number of qubits \( n \).

In Figure 3b and Figure 3d, the number of CNOTs when \( m = n-1 \) is higher than the number of CNOTs when \( m = n \). This increase in the number of CNOTs occurs because the implemented isometry is not efficient when \( m = n-1 \). The cosine-sine decomposition of the unitary proposed in [22] is used for \( m = n \).

![Figure 3: Circuit depth and number of CNOTs to load a 2\(^n\)-dimensional complex vector into a quantum computer by adjusting parameter \( m \).](image)

The decreasing number of CNOTs and lower depths is the consequence of exchanging computational complexity for an approximated quantum state. It is achieved by reducing the entanglement between the qubits.

| \( m \) | \( n = 4 \) CNOTs | \( n = 6 \) CNOTs | \( n = 8 \) CNOTs | \( n = 10 \) CNOTs | \( n = 12 \) CNOTs | \( n = 14 \) CNOTs |
|---|---|---|---|---|---|---|
| 0 | 2 3 | 8 9 | 15 11 | 52 53 | 108 57 | 240 241 |
| 1 | 7 9 | 21 23 | 51 51 | 113 111 | 239 233 | 493 481 |
| 2 | 9 11 | 51 51 | 117 115 | 247 243 | 505 497 | 1019 1005 |
| 3 | 9 11 | 54 57 | 250 242 | 528 518 | 1062 1048 | 2108 2088 |
| 4 | 25 31 | 253 247 | 1104 1074 | 2226 2191 | 4373 4333 | 8798 8764 |
| 5 | 1087 1081 | 1087 1081 | 4631 4588 | 4631 4588 | 9085 9031 | 9085 9031 |
| 6 | 4474 4345 | 4474 4345 | 18798 18642 | 18798 18642 | 18798 18642 | 18798 18642 |
| 7 | 18175 17781 | 18175 17781 | 70798 69442 | 70798 69442 | 70798 69442 | 70798 69442 |

Table 2: Circuit depth and number of CNOTs by adjusting the parameter \( m = \log_2(\text{rank}) \). \( m \) can be interpreted as a hyperparameter to fine-tune the encoding circuit to hardware characteristics such as relaxation time, dephasing time, and CNOT gate error.

In the second set of experiments, the Mean Absolute Error (MAE) of the square amplitudes of the desired state and measurements of the state generated in noisy quantum devices are evaluated. A set of 10 random complex states with 7 qubits following a uniform distribution were initialized using an exact algorithm and the low-rank approximation on IBM Lagos, Casablanca, and Jakarta devices. For each rank and device, the 10 random states were initialized with 8192 shots each. Table 3 shows the MAE with values of \( m \) from 0 to 3. On noisy devices, the smaller MAE occurs when \( m = 0 \). This indicates that in noisy devices it is better to use smaller ranks because the loss of fidelity introduced by the approximation is smaller than the loss caused by the noise of more complex circuits.

![Fig. 4](image)

Fig. 4 shows experimental results with the initialization of a 4 qubits state for \( m = 0 \), 1 and 2. In this case, the low-depth quantum state preparation does not present advantages. One probable reason is that

```python
In
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the original circuit has a complexity (depth and number of CNOT gates) small enough to be handled by
the noisy devices and is therefore not suitable for the approximation.

5. Conclusion

Previous state preparation algorithms require $O(2^n)$ CNOTs to initialize a quantum state. However,
these algorithms do not consider the amount of entanglement. This work shows that the complexity of state
preparation is related to entanglement through numerical measures of pure state entanglement based on the
square of Schmidt coefficients. These measures are directly related to the number of CNOTs required for
state initialization. The relation between entanglement and state preparation gives a better understanding
of the initialization complexity of classes of states.

The second result of this work is the low-rank approximation for the initialization of quantum states.
Experiments with seven qubits quantum devices show that measurements of the state generated by the
low-rank approximation circuits are closer to the expected results than the exact initialization algorithm
case. It occurs because of the reduction in the complexity of the circuits, which reduces the number of noisy
operations. A low-rank approximation of the state can make the final circuit feasible on noisy devices for
algorithms that rely on loading data into an arbitrary quantum state.

A variant of this approach permits a bounded approximation error state preparation algorithm which is
useful if a certain approximation error must be observed. Low entanglement states can usually be created
by product states with a low approximation error, while high-entanglement state remain hard to create,
but still for medium entanglement ($\approx 0.5$ Meyer-Wallach measure) there are considerable improvements in
terms of saved CNOT gates.

This work leaves some open questions. How do quantum machine learning, solving systems of linear
equations, principal component analysis, and systems of differential equations perform with low-rank state
preparation? As a future investigation, one could verify if the quality of a quantum algorithm reduces when
it operates on an approximated quantum state and if this could allow an advantage in solving problems that
have state preparation as a bottleneck. Another very promising application is enhancing quantum adiabatic
state preparation [23, 24, 25, 26] by preparing a low-entanglement bounded approximation error state and
find its Hamiltonian for which it is an eigenvalue and then apply the adiabatic theorem. The goal would
be to potentially skip spectral gap bottlenecks and therefore make either the state preparation possible or
reduce the adiabatic time significantly. In fact, a possible way forward could be to reach high-entanglement
states by this approach. It remains unclear how this could actually be achieved, but the applications would
be obviously beneficial.

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policy or position of IBM or the IBM Quantum team.
Figure 4: Experimental results with a 16-dimensional complex input vector on three IBM quantum devices. Blue and orange bars indicate respectively the ideal results and the full-rank results. Green and red bars indicate the low-rank approximation results. Error bars are the standard deviation. State singular values: 0.9033, 0.3256, 0.2348, 0.1509. State purity: 0.6807. Fidelity loss for ranks 1 and 2: 0.077931 and 0.022792.

Competing interests

The authors declare no competing interests.

Data availability

The site https://www.cin.ufpe.br/~ajsilva/qclib contains all the data and software generated during the current study.

Additional Information

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