Consistent interactions in the Hamiltonian BRST formalism

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Abstract

A Hamiltonian BRST deformation procedure for obtaining consistent interactions among fields with gauge freedom is proposed. The general theory is exemplified on the three-dimensional Chern-Simons model.

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1 Introduction

The analysis of consistent interactions that can be introduced among fields with gauge freedom without changing the number of gauge symmetries [1–4] has been transposed lately at the level of the deformation of the master equation [5] from the antifield-BRST formalism [6–10]. This cohomological deformation technique has been applied, among others, to Chern-Simons models [5], Yang-Mills theories [11] and two-form gauge fields [12–13]. In this light, the antifield-BRST method was proved to be an elegant tool for investigating the problem of consistent interactions.

Recently, a Hamiltonian analysis of anomalies has been given [14]. Moreover, in a very interesting paper [15], there has been established the precise

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relation of the local BRST cohomologies in both Lagrangian and Hamiltonian formalisms (see Theorem 6 from this reference). The procedures developed within these papers strongly stimulate a Hamiltonian BRST approach to other interesting problems.

In this letter we analyze the problem of constructing consistent interactions among fields with gauge freedom in the framework of the Hamiltonian BRST formalism [10], [16]–[20]. Our strategy includes two main steps: (i) initially, we show that the problem of introducing consistent interactions among fields with gauge freedom can be reformulated as a problem of deforming the BRST charge and the BRST-invariant Hamiltonian with respect to a given “free” theory, and consequently we deduce the general equations that govern these two types of deformations; (ii) next, on behalf of the relationship between the Hamiltonian and antifield BRST formalisms for constrained systems we prove that the general equations possess solution. In the sequel, we reformulate the general equations in a manner that accounts for locality, and subsequently illustrate our general procedure in the case of three-dimensional Chern-Simons models. Finally, we remark that our method combined with the results in [15] may simplify the computation of local Lagrangian BRST cohomologies in some cases of interest.

2 General equations of the Hamiltonian deformation approach

We begin with a system described by the canonical variables $z^A$, subject to the first-class constraints

$$G_{a_0} \left( z^A \right) \approx 0, \ a_0 = 1, \ldots, M_0,$$

which are assumed to be $L$-stage reducible

$$G_{a_0} Z^{a_0}_{a_1} = 0, \ a_1 = 1, \ldots, M_1,$$

$$Z^{a_k}_{a_{k-1}} Z^{a_{k-1}}_{a_k} \approx 0, \ a_k = 1, \ldots, M_k, k = 2, \ldots, L,$$

and suppose that there are no second-class constraints in the theory. The Grassmann parities of the canonical variables and first-class constraints are
respectively denoted by $\varepsilon (z^A) = \varepsilon_A$ and $\varepsilon (G_{a_0}) = \varepsilon_{a_0}$. We denote the first-class Hamiltonian by $H_0$, such that the gauge algebra is expressed by

$$[G_{a_0}, G_{b_0}] = G_{c_0} C_{a_0 b_0 c_0}, \ [H_0, G_{a_0}] = G_{b_0} V_{a_0}^{b_0}. \tag{4}$$

It is known that a constrained Hamiltonian system can be described by the action

$$S_0 \left[ z^A, u^{a_0} \right] = \int_{t_1}^{t_2} dt \left( a_A(z) \dot{z}^A - H_0 - G_{a_0} u^{a_0} \right), \tag{5}$$

where the Grassmann parities of the Lagrange multipliers are given by $\varepsilon (u^{a_0}) = \varepsilon_{a_0}$. In (5), $a_A(z)$ is the one-form potential that gives the symplectic two-form $\omega_{AB} = (-)^{\varepsilon_A + 1} \frac{\partial a_A}{\partial z^B} - (-)^{\varepsilon_B (\varepsilon_A + 1)} \frac{\partial a_B}{\partial z^A}$, whose inverse, $\omega^{AB}$, corresponds to the fundamental Dirac brackets $[z^A, z^B] = \omega^{AB}$. Action (5) is invariant under the gauge transformations

$$\delta_{\xi} z^A = [z^A, G_{a_0}] \xi^{a_0}, \ \delta_{\xi} u^{a_0} = \dot{\xi}^{a_0} - V_{b_0}^{a_0} \xi^{b_0} - C_{b_0 c_0}^{a_0} \xi^{c_0} u^{b_0} - Z_{a_1}^{a_0} \xi^{a_1}. \tag{6}$$

In order to generate consistent interactions at the Hamiltonian level, we deform the action (5) by adding to it some interaction terms

$$S_0 \rightarrow \tilde{S}_0 = S_0 + g^{(1)} S_0 + g^{(2)} S_0 + \cdots, \tag{7}$$

and modify the gauge transformations (6) (to be denoted by $\tilde{\delta}_{\xi} z^A, \tilde{\delta}_{\xi} u^{a_0}$) in such a way that the deformed gauge transformations leave invariant the new action

$$\frac{\delta \tilde{S}_0}{\delta z^A} \tilde{\delta}_{\xi} z^A + \frac{\delta \tilde{S}_0}{\delta u^{a_0}} \tilde{\delta}_{\xi} u^{a_0} = 0. \tag{8}$$

Consequently, the deformation of the action (5) and of the gauge transformations (6) produces a deformation of the first-class constraints, first-class Hamiltonian and structure functions like

$$G_{a_0} \rightarrow \gamma_{a_0} = G_{a_0} + g^{(1)} \gamma_{a_0} + g^2 \gamma_{a_0} + \cdots, \tag{9}$$

$$H_0 \rightarrow H = H_0 + g^{(1)} H + g^2 H + \cdots, \tag{10}$$

$$V_{b_0}^{a_0} \rightarrow \tilde{V}_{b_0}^{a_0} = V_{b_0}^{a_0} + g \tilde{V}_{b_0}^{a_0} + g^2 \tilde{V}_{b_0}^{a_0} + \cdots. \tag{11}$$
\[ C^{a_0}_{b_0c_0} \to \tilde{C}^{a_0}_{b_0c_0} = C^{a_0}_{b_0c_0} + g C^{(1)a_0}_{b_0c_0} + g^2 C^{(2)a_0}_{b_0c_0} + \cdots, \]  

such that the deformed gauge algebra becomes

\[ [\gamma_{a_0}, \gamma_{b_0}] = C^{c_0}_{a_0b_0}, \quad [H, \gamma_{a_0}] = \tilde{V}^{b_0}_{a_0}. \]

In the meantime, we deform the reducibility relations, but we do not explicitly write down these relations.

As the BRST charge and BRST-invariant Hamiltonian contain all the information on the gauge structure of a given gauge theory, we can reformulate the problem of introducing consistent interactions within the Hamiltonian BRST context in terms of these two essential compounds. Indeed, if the interaction can be consistently constructed, then the BRST charge of the undeformed theory, \( \Omega^{(0)} \), can be deformed such as to be the BRST charge of the deformed theory, i.e.,

\[ \Omega^{(0)} \to \Omega = \Omega^{(0)} + g \Omega^{(1)} + g^2 \Omega^{(2)} + \cdots, \]

\[ [\Omega, \Omega] = 0. \]

At the same time, the deformation of the BRST charge induces the deformation of the BRST-invariant Hamiltonian of the undeformed theory, \( H_B^{(0)} \),

\[ H_B^{(0)} \to H_B = H_B^{(0)} + g H_B^{(1)} + g^2 H_B^{(2)} + \cdots, \]

in such a way that \( H_B \) is the BRST-invariant Hamiltonian of the interacting theory, i.e.

\[ [H_B, \Omega] = 0. \]

The equations (17) and (17) split accordingly the deformation parameter as

\[ \left[ \Omega^{(0)}, \Omega^{(0)} \right] = 0, \quad \left[ \Omega^{(0)}, H_B^{(0)} \right] = 0, \]

\[ 2 \left[ \Omega^{(0)}, \Omega^{(1)} \right] = 0, \quad \left[ H_B^{(1)}, \Omega^{(1)} \right] + \left[ H_B^{(0)}, \Omega^{(1)} \right] = 0, \]

\[ 2 \left[ \Omega^{(2)}, \Omega^{(1)} \right] + \left[ \Omega^{(1)}, \Omega^{(1)} \right] = 0, \quad \left[ H_B^{(2)}, \Omega^{(1)} \right] + \left[ H_B^{(1)}, \Omega^{(1)} \right] + \left[ H_B^{(0)}, \Omega^{(1)} \right] = 0. \]
Equations (18–20) stand for the general equations of our deformation procedure. The equations (18) are checked by hypothesis. Then, it appears the natural question whether the next equations possess or not solution. This will be investigated in the next section.

3 Solution to the general equations

In order to prove that the equations (19–20), etc. possess solution, we use the link between the antifield and Hamiltonian BRST formalisms for constrained Hamiltonian systems [21]. First-class constrained Hamiltonian systems can be approached from the point of view of the BRST formalism in two different manners. One is based on the antibracket-antifield formulation [6]–[10], while the other relies on the standard Hamiltonian BRST treatment [10], [16]–[20]. The starting point of the antibracket-antifield formalism is represented by the invariance of the action (5) under the gauge transformations (6). In agreement with the general prescriptions of the antibracket-antifield procedure, we introduce the ghosts $(\eta_{a}^{k-1}, u_{a}^{k})$, with $k = 1, \ldots, L$ and 

$$
\varepsilon(\eta^{a_k}) = \varepsilon_{a_k} + k + 1 \mod 2, \quad gh(\eta^{a_k}) = k + 1, \quad k = 0, \ldots, L, \tag{21}
$$

$$
\varepsilon(u^{a_k}) = \varepsilon_{a_k} + k \mod 2, \quad gh(u^{a_k}) = k, \quad k = 1, \ldots, L, \tag{22}
$$

where $gh$ denotes the ghost number. The antifields associated with the fields $(z^{A}, u^{a_0}, \eta_{a}^{k-1}, u_{a}^{k})$ are denoted by $(\dot{z}^{A}, u^{a_0}, \dot{\eta}^{*}_{a_k-1}, u^{*}_{a_k})$ and display the properties $\varepsilon(antifield) = \varepsilon(field) + 1, \quad gh(antifield) = -gh(field) - 1$. Up to terms that are quadratic in the antifields, the solution to the master equation reads as

$$
S^{(0)} = \int_{t_1}^{t_2} dt \left( a_A(z) \dot{z}^{A} + \sum_{k=0}^{L} u^{*}_{a_k} \dot{\eta}^{a_k} - H_0 - G_{a_0} u^{a_0} + \dot{z}^{A} \left[ z^{A}, G_{a_0} \right] \eta^{a_0} - \right.

u^{*}_{a_0} \dot{V}_{a_0}^{b_0} \eta^{b_0} + (-)^{\varepsilon_{b_0} + 1} u^{*}_{a_0} C_{b_0 c_0}^{a_0} \eta^{c_0} \eta^{b_0} + \frac{1}{2} (-)^{\varepsilon_{b_0}} \eta^{*}_{a_0} C_{b_0 c_0}^{a_0} \eta^{c_0} \eta^{b_0} +

\sum_{k=0}^{L} \eta^{*}_{a_k} Z_{a_{k+1}}^{a_k} \eta^{a_k} + \sum_{k=1}^{L-1} u^{*}_{a_{k-1}} Z_{a_{k-1}}^{a_k} u^{a_k} + \ldots \right). \tag{23}
$$
The Hamiltonian point of view is based on extending the phase-space through introducing the canonical pairs ghost-antighost \((\eta^a)_{k}, P^a_k\), with \([\eta^a)_{k}, P^a_k]\) = \(\delta^a_{b_k}\) and \(\varepsilon (P^a_k) = \varepsilon_a + k + 1, gh (P^a_k) = k + 1\). The BRST charge starts like

\[
(0) \Omega = G_a \eta^a + \frac{1}{2} (-\varepsilon b_0) \mathcal{P}_{a0} C^{b_0}_{c_0} \eta^c \eta^b_0 + \sum_{k=0}^{L-1} \mathcal{P}_{a_k} Z^{a_{k+1}} \eta^{a_{k+1}} + \cdots, \tag{24}
\]

such that \([\Omega, \Omega] = 0\). The BRST-invariant extension of \(H_0\)

\[
(0) H_B = H_0 + \mathcal{P}_{a0} V^a_0 \eta^b_0 + \cdots, \tag{25}
\]

satisfies the equation \([H_B, \Omega] = 0\). By employing the identifications

\[
u^*_a = \mathcal{P}_{a_k}, \ k = 0, \ldots, L, \tag{26}
\]

and extending the Dirac bracket such that \([\eta^a_k, u^*_a] = \delta^a_{b_k}\), we get that

\[
\frac{1}{2} \left( (S, S) \right) = \int_{t_1}^{t_2} dt \left( -\frac{d}{dt} (0) H_B, (0) \Omega \right) + \frac{1}{2} z^*_A \left( (0) \Omega, (0) \Omega \right) \right] + \frac{1}{2} \sum_{k=0}^{L} \eta^*_a \left( (0) \Omega, (0) \Omega \right) + \frac{1}{2} \sum_{k=0}^{L} \left( (0) \Omega, (0) \Omega \right) u^*_a u^a_k. \tag{27}
\]

The deformations \((14)\) and \((16)\) induce a deformation of the solution to the master equation

\[
(0) S \rightarrow S = (0) S + g^{(1)} S + g^2 S + \cdots, \tag{28}
\]

such that the equation \((27)\) for the deformed theory becomes

\[
\frac{1}{2} (S, S) = \int_{t_1}^{t_2} dt \left( -\frac{d}{dt} \Omega - [H_B, \Omega] + \frac{1}{2} z^*_A \left[ (0) \Omega, (0) \Omega \right] + \frac{1}{2} \sum_{k=0}^{L} \eta^*_a \left( (0) \Omega, (0) \Omega \right) + \frac{1}{2} \sum_{k=0}^{L} \left( (0) \Omega, (0) \Omega \right) u^*_a u^a_k. \tag{29}
\]
The equation (29) splits accordingly the deformation parameter as (27) and

\[
\begin{align*}
\left(\begin{array}{c}
(0) S \\
(1) S \\
\end{array}\right) = \int_{t_1}^{t_2} dt \left( -\frac{d}{dt} \Omega - \left[ (0) H_B, \Omega \right] - \left[ (1) H_B, (0) \Omega \right] + z^*_A \left[ z_A, \left(\begin{array}{c}
(0) \Omega \\
(1) \Omega \\
\end{array}\right) \right] \right) + \\
\sum_{k=0}^{L} \eta^*_a \left[ \eta^a, \left(\begin{array}{c}
(0) \Omega \\
(1) \Omega \\
\end{array}\right) \right] + \sum_{k=0}^{L} \left[ \left(\begin{array}{c}
(0) \Omega \\
(1) \Omega \\
\end{array}\right), u^*_a \right] u^a k, \\
\right)
\end{align*}
\]

(30)

\[
\begin{align*}
\left(\begin{array}{c}
(0) S \\
(2) S \\
\end{array}\right) + \frac{1}{2} \left(\begin{array}{c}
(1) S \\
(1) S \\
\end{array}\right) = \int_{t_1}^{t_2} dt \left( -\frac{d}{dt} \Omega - \left[ (0) H_B, \Omega \right] - \left[ (1) H_B, (1) \Omega \right] - \\
\left[ (2) H_B, (0) \Omega \right] + z^*_A \left[ z_A, \left(\begin{array}{c}
(0) \Omega \\
(1) \Omega \\
\end{array}\right) \right] + \frac{1}{2} \left[ (1) \Omega, (1) \Omega \right] \right) + \\
\sum_{k=0}^{L} \eta^*_a \left[ \eta^a, \left(\begin{array}{c}
(0) \Omega \\
(2) \Omega \\
\end{array}\right) \right] + \frac{1}{2} \left[ (1) \Omega, (1) \Omega \right] + \\
\sum_{k=0}^{L} \left[ \left(\begin{array}{c}
(0) \Omega \\
(2) \Omega \\
\end{array}\right), u^*_a \right] u^a k, \\
\right)
\end{align*}
\]

(31)

The last equations emphasize that the existence of \((1) S\) guarantees the existence of \((1) \Omega\) and \((1) H_B\), the existence of \((2) S\) guarantees the existence of \((2) \Omega\) and \((2) H_B\), and so on. Moreover, the equations (19–20), etc. are equivalent to the equations \((0) S, S) = 0, (0) S, (2) S + \frac{1}{2} (1) S, (1) S = 0, etc. modulo imposing some appropriate boundary conditions for \(\Omega\) [20]. On the other hand, the last equations possess solution. The existence of such solutions was proved in [4] on behalf of the triviality of the antibracket in the cohomology. Thus, the existence of the solutions in the antibracket proves the existence of the solutions to (19–20), etc. In conclusion, we can construct consistent interactions by means of the equations (19–20), etc.

In practical applications, as commonly required, the deformation should be local, i.e., \((1) \Omega, (2) \Omega, (1) H_B, (2) H_B\), etc. should be local functionals. Let \(F_1 = \int d^{D-1} x f_1\) and \(F_2 = \int d^{D-1} x f_2\) be two local functionals. Then, \([F_1, F_2] = \int d^{D-1} x \) is
local, namely, there exists a local \([f_1, f_2]\) (but defined up to a \((D - 1)\)-dimensional divergence), such that \([F_1, F_2] = \int d^{D-1}x [f_1, f_2]\). Thus, the equations (19–20), etc. can be written as

\[
2 \left( S (0)^{(1)} \omega = \partial^k \frac{(1)}{j_k} \left( S h_B + \left( \left( \frac{(0)}{h_B} \frac{(1)}{\omega} \right) \right) \right. \right. = \partial^k \left( \frac{(1)}{m_k} \right), \quad (32)
\]

\[
2 \left( S (0)^{(2)} \omega + \left( \frac{(1)}{\omega} \left( \frac{(1)}{\omega} \right) \right) \right. = \partial^k \left( \frac{(2)}{j_k} \left( S h_B + \left( \left( \frac{(0)}{h_B} \frac{(1)}{\omega} \right) \right) \right. \right. = \partial^k \left( \frac{(2)}{m_k} \right), \quad (33)
\]

in terms of the integrands \(h_B\) and \(\omega\). In the above, \(\frac{(0)}{s}\) stands for the undeformed BRST symmetry. The formalism developed so far does not guarantee locality. For instance, even if \(\left( \frac{(1)}{\Omega} \frac{(1)}{\Omega} \right)\) is \(\frac{(0)}{s}\)-exact, it is not granted that it is the BRST variation of a local functional. Such locality problems appear also in the Lagrangian deformation procedure \([3]\). However, in the case of most important applications \([3], [11]–[13]\), the Lagrangian BRST deformation procedure leads to local interactions. Thus, we expect that the Hamiltonian BRST deformation treatment also outputs local vertices in practical applications.

### 4 Example

Let us exemplify the prior procedure in the case of abelian Chern-Simons model in three dimensions. We start with the Lagrangian action

\[
S_0 \left[ A^a_{\mu} \right] = \frac{1}{2} \int d^3 x \varepsilon^{\mu\nu\rho} k_{ab} A^a_{\mu} F^{b}_{\nu\rho}, \quad (34)
\]

where \(k_{ab}\) is a non-degenerate symmetric and constant matrix, while \(F^{b}_{\nu\rho} = \partial_{\nu} A^b_{\rho} - \partial_{\rho} A^b_{\nu} \equiv \partial_{[\nu} A^b_{\rho]}\). Performing the canonical analysis and eliminating the second-class constraints (the independent variables are \(A^a_{\mu}, \pi^a_0\) and \(A^a_{k}\)), we infer the first-class constraints \(G_{1a} \equiv \pi^a_0 \approx 0, \; G_{2a} \equiv -\frac{1}{2} \varepsilon^{0ik} k_{ab} F^{b}_{ik} \approx 0\) and the first-class Hamiltonian \(H_0 = -2 \int d^2 x A^a_0 G_{2a}\). The non-vanishing
fundamental Dirac brackets read as $[A^a_0, \pi^b_0] = \delta^a_b$, $[A^a_k, A^b_j] = \frac{1}{2} \varepsilon^{0ik} k^a b^b$, hence the BRST charge takes the simple form

$$\Omega^{(0)} = \int d^2 x \left( \pi^0_a \eta^a_1 - \frac{1}{2} \varepsilon^{0ik} k^a b^b F^b_{ik} \eta^a_2 \right),$$

(35)

where $k^{ab}$ is the inverse of $k_{ab}$, and $(\eta^a_1, \eta^a_2)$ stand for the fermionic ghost number one ghosts. Thus, the BRST operator $s^{(0)}$ splits as $s = \delta + \gamma$, where $\delta$ is the Koszul-Tate differential and $\gamma$ represents the longitudinal exterior derivative along the gauge orbits. Then, we have

$$\delta A^a_0 = 0, ~ \delta \pi^0_a = 0, ~ \delta A^a_k = 0, ~ \delta \eta^a_1 = \delta \eta^a_2 = 0,$$

(36)

$$\delta P_{1a} = -\pi^0_a, ~ \delta P_{2a} = \frac{1}{2} \varepsilon^{0ik} k^a b^b F^b_{ik},$$

(37)

$$\gamma A^a_0 = \eta^a_1, ~ \gamma \pi^0_a = 0, ~ \gamma A^a_k = \frac{1}{2} \partial_k \eta^a_2, ~ \gamma \eta^a_1 = \gamma \eta^a_2 = 0,$$

(38)

$$\gamma P_{1a} = \gamma P_{2a} = 0.$$

(39)

Now, we solve the former equation from (32). In view of this, we develop $\omega^{(1)}$ accordingly the antighost number

$$\omega^{(1)} = \omega^{(1)}_0 + \omega^{(1)}_1 + \cdots + \omega^{(1)}_{\Delta}, ~ \text{antigh} \left( \omega^{(1)}_{\Delta} \right) = \Delta, ~ \text{gh} \left( \omega^{(1)}_{\Delta} \right) = 1,$$

(40)

where the last term in (40) can be assumed to be annihilated by $\gamma$. As $\text{pgh} \left( \omega^{(1)}_{\Delta} \right) = \Delta + 1$, we can represent $\omega^{(1)}_{\Delta}$ under the form

$$\omega^{(1)}_{\Delta} = \mu_{a_1 \cdots a_{\Delta+1}} \eta^a_1 \cdots \eta^a_{\Delta+1}.$$

(41)

With this choice, it results that the $\gamma$-invariant coefficient $\mu_{a_1 \cdots a_{\Delta+1}}$ belongs to $H_{\Delta} (\delta | \tilde{d})$, i.e., is solution to the equation

$$\delta \mu_{a_1 \cdots a_{\Delta+1}} + \partial_k b^k_{a_1 \cdots a_{\Delta+1}} = 0,$$

(42)

for some $b^k_{a_1 \cdots a_{\Delta+1}}$, where $\tilde{d} = dx^i \partial_i$. Using the result from [22] adapted to the Hamiltonian context, it follows that $H_{\Delta} (\delta | \tilde{d})$ vanish for $\Delta \geq 2$, hence

9
\[
\begin{aligned}
\omega^1 &= \omega_0^1 + \omega_1^1, \quad \text{with} \quad \omega_1^1 = \frac{1}{2} \mu_{ab} \eta_2^a \eta_2^b, \quad \text{where} \quad \mu_{ab} \text{ from } H_1(\delta | \partial). \\
\text{A general representative of } H_1(\delta | \partial) \text{ is of the type } \mu_{ab} = C^c_{ab} \mathcal{P}_{2c}, \quad \text{where } C^c_{ab} \text{ are some constants, antisymmetric in the lower indices, } C^c_{ab} = -C^c_{ba}. \quad \text{The necessity for } C^c_{ab} \text{ to be constant results from the equation that must be satisfied by } \mu_{ab}, \quad \text{namely, } \delta \mu_{ab} = \partial_k \left( C^c_{ab} \varepsilon^{0kj} k_{cd} A_j^d \right).
\end{aligned}
\]

In this way, we obtained that \( \omega_1^1 = \frac{1}{2} C^c_{ab} \mathcal{P}_{2c} \eta_2^a \eta_2^b \). The former equation from [32] at antighost number zero reads as \( \delta (\omega_1^1 + \gamma \omega_0^1) = \partial_k m^k \), which further yields \( \omega_0^1 = C^c_{ad} k_{cb} \varepsilon^{0kj} A_k^a A_j^d \). In this manner, we inferred \( \omega^1 = C^c_{ab} \left( \frac{1}{2} \mathcal{P}_{2c} \eta_2^a \eta_2^b + k_{cd} \varepsilon^{0kj} A_k^a A_j^d \right) \). Simple computation leads to

\[
\left[ (\omega^1, \Omega^1) \right] = \int d^2x \left( -\frac{1}{3} C^m_{[nc} C^c_{ab]} \mathcal{P}_{2m} \eta_2^a \eta_2^b \eta_2^n \right. - \left. \varepsilon^{0ij} k_{ad} C^e_{[bc} \mathcal{P}_{2e} \eta_2^a \eta_2^b \eta_2^e \right). \tag{43}
\]

The last relation shows that \( \left[ (\omega^1, \Omega^1) \right] \) cannot be written like a \((0)^{st}\) -exact modulo \( \partial \partial \) local functional. For this reason it is necessary to have \( \left[ (\omega^1, \Omega^1) \right] = 0 \). This condition takes place if and only if \( C^m_{[nc} C^c_{ab]} = 0 \), so if and only if the constants verify the Jacobi identity. This further implies \( \Omega^1 = 0, \kappa \geq 2 \). The deformed BRST charge takes the final form

\[
\Omega = \int d^2x \left( \pi_a^0 \eta_1^a - \varepsilon^{0ik} k_{ca} \left( \frac{1}{2} F^c_{ik} - g C^c_{bd} A_k^b A_d^c \right) \eta_2^a + \frac{1}{2} g C^c_{ab} \mathcal{P}_{2c} \eta_2^a \eta_2^b \right), \tag{44}
\]

so it is clearly a local functional.

Now, we derive the deformed BRST-invariant Hamiltonian. The BRST-invariant Hamiltonian for the free theory is given by \( H_B = H_0 + 2 \int d^2x \eta_1^a \mathcal{P}_{2a} \). Consequently, we find

\[
\left[ (H_B, \omega^1) \right] = -2 C^c_{ab} k_{cd} \varepsilon^{0ij} A_j^b \left( \eta_1^a A_i^a + \eta_2^d \partial_i A_0^a \right) - 2 C^c_{ab} \mathcal{P}_{2c} \eta_2^a \eta_1^b. \tag{45}
\]
Then, the solution of the latter equation in (32) reads as
\[ h_B^{(1)} = 2C_{ab}^c \epsilon^{0ij} A_0^i A_0^j + A_0^b \mathcal{P}_{2c} \eta_2^a. \tag{46} \]

Straightforward computation leads to \[ [H_B, \Omega] = 0, \] hence the latter equation from (33) is satisfied with \( h_B^{(2)} = 0. \) Therefore, the higher-order deformation equations for the BRST-invariant Hamiltonian are verified with \( H_B^{(3)} = H_B^{(4)} = \cdots = 0. \) Thus, the complete deformed BRST invariant Hamiltonian reads as
\[ H_B = 2 \int d^2 x \left( -A_0^a \epsilon^{0ik} k_{ca} \left( \frac{1}{2} F_{ik}^c - g C_{bd}^c A_i^b A_k^d \right) + ( \eta_1^a - g C_{ab}^c A_0^b \eta_2^c ) \mathcal{P}_{2a} \right), \tag{47} \]
and is a local functional, too. With the help of (44) and (47) we identify the new gauge theory. From the antighost-independent terms in (44) we observe that the deformation of the BRST charge implies the deformed first-class constraints
\[ \gamma_2^a \equiv -\epsilon^{0ik} k_{ca} \left( \frac{1}{2} F_{ik}^c - g C_{bd}^c A_i^b A_k^d \right) \approx 0, \tag{48} \]
the remaining constraints being undeformed. The term \( \frac{1}{2} g C_{ab}^c A_0^b \mathcal{P}_{2c} \eta_2^a \eta_2^b \) shows that the new constraint functions form a Lie algebra, i.e.,
\[ [\gamma_2^a, \gamma_2^b] = C_{ab}^c \gamma_2^c. \tag{49} \]
On the other hand, the antighost-independent piece in (47)
\[ H = -2 \int d^2 x A_0^a \epsilon^{0ik} k_{ca} \left( \frac{1}{2} F_{ik}^c - g C_{bd}^c A_i^b A_k^d \right), \tag{50} \]
is precisely the first-class Hamiltonian of the deformed theory. The components linear in the antighosts from (47) indicate that the Dirac brackets among the new first-class Hamiltonian and the new constraint functions are modified as \( [H, \gamma_2^a] = -C_{ab}^c A_0^b \gamma_2^c. \) Thus, the resulting first-class theory is nothing but the nonabelian version of the Chern-Simons model in three dimensions, described by the local Lagrangian action
\[ S_0[A_µ] = \int d^3 x \epsilon_\mu^{\nu\rho} A_\mu \left( \frac{1}{2} k_{ab} F_{\nu\rho}^b - \frac{2}{3} g C_{abc} A_\nu^b A_\rho^c \right). \tag{51} \]
where $C_{abc} = C^d [_{[b} k_{c]}d]$. As the first-class constraints generate gauge transformations, from the deformations (48) and (49) we can conclude that the added interactions involved with (50) modify both the gauge transformations and their gauge algebra.

5 Conclusion

To conclude with, in this letter we have presented a Hamiltonian BRST approach to the construction of consistent interactions among fields with gauge freedom. Our procedure reformulates the problem of constructing Hamiltonian consistent interactions as a deformation problem of the BRST charge and BRST-invariant Hamiltonian of a given “free” theory. We have derived the general equations from the Hamiltonian BRST deformation method, and proved that they possess solution. Next, we have written down the local version of these equations and discussed on the locality of their solutions. Finally, the general theory was exemplified in the case of the Chern-Simons model in three dimensions. We think that our approach together with the general results in [13] might be successfully applied to computing local BRST cohomologies for those theories whose Lagrangian version is more intricate than the Hamiltonian one.

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