Letter

Evolution of quantum correlation undergoing decoherence using the information entropy inequality

Xia Liu, Huaxiin Lu, JiaQiang Zhao, Yang Yang, Yingde Li and LianZhen Cao

Department of Physics and Optoelectronic Engineering, Weifang University, Weifang, Shandong 261061, People’s Republic of China

E-mail: lianzhencao@wfu.edu.cn

Received 17 August 2018, revised 27 September 2018
Accepted for publication 27 September 2018
Published 7 November 2018

Abstract

Using the information-theoretic inequality proposed by Braunstein and Caves, we study the evolution of quantum correlation in the presence of different noisy channels. First, we experimentally prepare the high-fidelity two-bit entangled state and simulated the bit-flip and phase-shift noise using an all-optical experimental setup. Then, we study the quantum correlation evolution of a two-bit entangled system using the information difference of entropy under different noisy channels and noisy environments. It is found that the quantum correlation degradation depends on the type of noise and noisy channels. Our work provides a new essential tool for quantum information processing and measurement, and offers new insight into the evolution of quantum correlation in open systems.

Keywords: information entropy inequality, quantum correlation evolution, phase-shift noise, bit-flip noise

(Some figures may appear in colour only in the online journal)

1. Introduction

During recent decades, quantum correlations and the entanglement phenomenon in composite quantum systems, i.e. systems containing subsystems have been considered as a potential resource for technologies such as high-efficiency information processing, long-distance secure communications, ultra-sensitive metrology, and many others [1–3]. In information theory, the relation between two events can be quantified using the notion of conditional entropy. Quantum correlations for states of composite systems have been successfully described in terms of various information and entropic characteristics, including the von Neumann entropy and quantum mutual information, discord related measures, contextuality, causality, subadditivity, and new information and entropic inequalities with probability distributions [4–12].

Here, we study quantum correlation using the entropic inequality — Braunstein–Caves (BC) chained Bell inequalities [13]. It was shown that if local realism holds, then the joint Shannon entropies carried by the measurements on two distant systems must satisfy certain inequalities, which can be regarded as entropic Bell inequalities. This notion was successfully applied to study the relation between space-like separated measurements on different parts of an entangled quantum system [14, 15]. The advantage of this entropic
approach is that they do not rely on a labeling of measurement outcomes and that their form does not depend on a dimension of the system. This property is important for high-dimensional quantum systems. Furthermore, the BC chained Bell inequalities have some interesting applications in situations where the CHSH Bell inequality is inadequate. For instance, the use of a BC inequality with \( N = 3 \) solves a problem in Franson’s CHSH Bell experiment [16], and reduces the number of trials needed to rule out local realism in experiments with perfect detection efficiency [17]. Moreover, the use of BC inequalities with higher values of \( N \) improves the security of quantum key distribution protocols [18], and has also been used to investigate nonlocal theories [19, 20].

However, as most of the quantum composite systems are open systems, when a quantum system interacts with its environment it undergoes decoherence [21]. Decohering processes are one of the crucial problems for the observation of quantum correlations in practice. This in turn can decrease its capacity for quantum interference, which is essential for standard quantum information processing. An interesting study for exploring quantum devices is the experimental simulation of complex dynamics on controllable quantum systems of simple implementation [22]. These simulations allow for a better control and, therefore, understanding of the details leading to decoherence as well as the mechanisms underneath the system-environment exchange of excitation and/or information. Therefore, the characterization of quantum correlation evolution under the influence of decohering processes is required for any future realization of these quantum information applications.

In this paper, we aim to study quantum correlation in the presence of decoherence using the information-theoretic Bell inequalities. We experimentally prepared the high-fidelity two-bit entangled state and simulated the bit-flip and phase-shift noise using an all-optical experimental setup. Then, we study the quantum correlation evolution of the two-bit entangled system using the information difference of entropy under different noise channels and noise environments. The results show that the quantum correlation between a space-like separated entangled quantum system can be quantified using the quantum information entropy.

2. Theoretical description of BC chained Bell inequalities

Braunstein and Caves (BC) proposed a two-party \( N \)-setting generalization of the CHSH Bell inequality [13], in which the first observer can choose one out of \( N \) alternative experiments \( A_1, A_2, \ldots, A_{2N-1} \), and the second observer one out of \( N \) alternative experiments \( B_2, B_3, \ldots, B_{2N} \), each of which is the measurable quantity of two widely separated systems, \( A \) and \( B \). The BC chained Bell inequalities can be described by the relation:

\[
H (A_1 | B_1) \leq H (A_1 | B_{2N}) + H (B_{2N} | A_1) \cdots + H (B_{2N} | A_{2N-1}) + H (A_{2N-1} | B_2).
\]

(1)

The quantum mechanical information \( H^{\text{QM}} (A_1 | B_1) = H^{\text{QM}} (B_1 | A_1) \equiv H^{\text{QM}} (\theta) \) can be taken from \( H^{\text{QM}} (\theta) = -\frac{1}{2\pi+1} \sum_{m_1, m_2} |D_{m_1-m_2} (R_n (\theta))|^2 \log |D_{m_1-m_2} (R_n (\theta))|^2 \), where the \( |D_{m_1-m_2} (R_n (\theta))|^2 = (2 \pm 1)P \), and the quantum number \( m \) is the possible value of observables. \( P \) is the joint probability for each measurable pair of quantities.

To prove the BC chained Bell inequalities by quantum mechanics, we consider a quantum spin-\( s \) system. Two spin-\( s \) particles are emitted by the decay of a system with zero angular momentum. In the simplest case \( S = \frac{1}{2} \), we use the operator \( S_z = (\hbar/2) \sigma_z \) with the eigenstates \( |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and \( |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). The state of two particles with zero total momentum is written as \( |\Phi\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle \right) \). This state can be experimentally realized using the polarization light field produced by pulsed type-II parametric down-conversion. When only considering the first-order term of parametric down-conversion, the quantum state \( |\Phi\rangle = \frac{1}{\sqrt{2}} \left( |H, V\rangle + |V, H\rangle \right) \) is prepared, where \( H \) and \( V \) are horizontal and vertical polarization of the photon, respectively. If we encoded the \( |0\rangle \) and \( |1\rangle \) with the photon’s \( H \) and \( V \) polarization, the quantum state is just the state \( |\Phi\rangle \). We now take the four unit vectors \( \vec{a}, \vec{a}', \vec{b}, \vec{b}' \). In the quantum mechanical description, the quantities \( A \) and \( A' \) are represented as the operators \( \vec{a} \cdot \hat{S} \) and ... The quantities \( B \) and \( B' \) are given in the same way. Following [10], we consider four coplanar vectors so that:

\[
\angle (\vec{a}, \vec{b}) = \angle (\vec{b}, \vec{a}') = \angle (\vec{a}', \vec{b}') = \theta/3, \quad \angle (\vec{a}, \vec{b}') = \theta.
\]

(2)

Using coplanar vectors is easy to realize and they are widely used. It can be seen that such a choice is illustrative for comparing different entropies in the noisy case.

3. Experimental preparation of the state and noise

The setup to generate the required entanglement state is shown in figure 1. Femtosecond laser pulse (\( \approx 100 \) fs, 80 MHz, 780 nm) passes through a frequency doubler LiB_3O_5 (LBO) crystal. The pump beam is converted into an ultraviolet (UV) light pulse with a central wavelength of 390 nm. Four dichroic beam splitters (DM; in the figure all DM are not shown) are placed behind the LBO to separate the mixed UV and infrared light components. Then, the pulses go through cross-polarized beam splitters, which divide the light into two paths. Different noise channels, where the noisy environment is simulated by a combination of birefringent quarter-wave plates (QWPs) and half-wave plates (HWP) in each arm. The HWP have switched the polarization by \( \theta \) and the QWPs are set at 0° with respect to the vertical direction. Polarization analysis were performed using further wave plates and polarizing beam splitters followed by silicon avalanche single-photon detectors.

The theoretical and experimental results indicate that if want to prove information-theoretic spin-1/2 inequality; the
fidelity of the entangled state must be high enough. Therefore, realizing a high-fidelity quantum state is the precondition for our experiment. In order to improve the fidelity of the entangled state, we have used the following methods: first, the femtosecond lasers are very vulnerable to the surrounding temperature, humidity and vibration environment. We take many measures to realize a constant temperature, constant humidity and constant mechanical vibration environment to reduce the pulse laser output power fluctuation. Second, through many experimental attempts, we get an optimal laser pump power of 40 mW for our experimental system. This power can guarantee enough coincidence counts, while ensuring a high-visibility quantum state. At the same time, by inserting and fine tuning the QWP + HWP + QWP combination (not shown here) we improve the fidelity of the entangled state. Finally, we get rid of the effect of dark count on the measurement data.

We experimentally simulate the bit-flip and phase-shift noise models. To simulate bit-flip noise, all the qubits must be rotated along the Y-axis at a random angle. For simplicity, we chose the angle to be ±θ with equal probability. Note that, for a single qubit, this is nothing but a bit-flip noise with probability \( p = \sin^2 2\theta \). Experimentally, the noise channels can be simulated by setting the angles of the HWPs to be ±θ with equal probability, and each HWP is sandwiched by two QWPs at 0° with respect to the vertical direction. The phase-shift noise is simulated by adding two HWPs with each setting at 45° before and after the wave plates.

4. Experimental results of the evolution of information entropy inequality

To characterize the prepared two-qubit state, we have extracted its density matrix by the method of over-complete state tomography. We collect the experimental data for each of 16 combinations of measurement basis \(|H\rangle, |V\rangle, |D\rangle, |R\rangle, \text{ and } |L\rangle\). \( H \), \( V \) and \( D \) are respectively horizontal, vertical and diagonal linear
polarization, while $R$ and $L$ are right- and left-circular polarization. With these data, the maximum-likelihood technique is used to construct the density matrix of the state. The density matrix of the two-qubit entangled state is shown in figure 2. From the estimated density matrix, we calculate the fidelity characterizing the quality of the state as $\rho = 0.993 \pm 0.004$. Similar to the method in [23, 24], the error bar of the fidelity is calculated by performing a 100 run Monte Carlo simulation of the whole state tomography analysis, with Poissonian noise added to each experimental data in each run.

The information-theoretic metric is a useful tool to investigate nonclassical correlations [25, 26]. The information-theoretic Bell inequalities can be written in terms of the mean entropic theoretic prediction, we first experimentally give the relation of quantum mechanical information conditional entropic theoretic prediction, we first experimentally give the relation of quantum mechanical information conditional entropy $H_{QM}(A|B) = H_{QM}(B)$ dependent on angle $\theta$ between $\vec{a}$ and $\vec{b}$. The experimental result is shown in figure 3. It can be seen that the location of maximum theoretic information deficit is at angle $\theta = 52.31^\circ$, corresponding to the concavity of entropy [28]. The experimental results agree well with the theoretical calculation by Braunstein and Caves [13].

We first study the evolution of the information difference under a single bit-flip noise channel when the rotation angle is fixed at 52.31°. The variation of noise intensity can be realized by adjusting the angles of the middle HWP. The experimental results are shown in figure 4. The general behavior shows that the quantum correlation between the entangled system decreases as the noise intensity increases. When the rotation angle of the middle HWP reaches 5°, the maximum information difference tends to zero-information difference value. This means that the quantum correlation will be destroyed at this noise intensity of the single bit-flip noise channel. It should be pointed out that the difference between the theoretical and experimental value is bigger at the small-angle zone, which can be explained by the quantum Zeno paradox [13].

In figure 5, we give the change of information difference depending on the rotation angle of the HWP under the collective bit-flip and phase-shift noise environment. In order to reflect the objective physical law, we studied the two different kinds of rotation angles 48° and 60°, respectively. It can be seen that the decreasing rates of information difference are obviously different when the photons pass through different noisy channels — bit-flip or phase-shift. Comparing the experimental results in the presence of different noisy channels, it can be seen that the quantum correction is more robust.
under the phase-shift noisy environment than the bit-flip noisy environment [29].

At the same time, the degradation trend of the two curves corresponding to rotation angles 48° and 60° are becoming consistent for identical noisy channels. For the collective phase-shift noise, the information difference is still negative when the rotation angle of the middle HWP is 3.5°, corresponding to rotation angles 60°. While for the rotation angles 48°, the upper bound of the rotating angles to destroy the quantum correlation property is 3.3°. It can be seen that the upper bounds of the rotating angles to destroy the quantum correlation property are 2.5° and 2.4°, corresponding to rotation angles 60° and 48°, respectively, for the collective bit-flip noisy environment [30, 31].

The aim of the quantum decoherence research is to understand the impact of quantum coherence on quantum information processing, and then to propose a protected quantum decoherence scheme to ensure the successful implementation of quantum computation and quantum information processing. Using quantum weak measurement and the quantum weak measurement reversal technology can effectively protect entanglement bits or qutrit entanglement properties in the decoherence environment, especially to avoid the entanglement sudden death phenomenon [32, 33]. Therefore, we
are carrying out the experimental research of entanglement system coherent protection and restoration based on the weak and the weak measurement technology.

As mentioned above, the three-setting information entropy inequality has special significance in quantum information processing. For instance, the use of inequality with $N=3$ solves a problem in Franson’s CHSH Bell experiment, and reduces the number of trials needed to rule out local realism in experiments with perfect detection efficiency, and improves the security of quantum key distribution protocols [16–18]. Of course, in order to study the inequality, the qutrit states must be realized experimentally. Therefore, our next work is to realize a high-quality spin-1 quantum state based on photons, and to study the experimental verification and application of high-dimensional information entropy inequality [11, 12].

5. Conclusion

In summary, we give the experimental rules of quantum correlation variation of the high-fidelity two-bit entangled system using the information difference of entropic criteria under bit-flip and phase-shift noisy environment. It is found that the quantum correlation degradation depends on the type of noise and noisy channels. The decreasing rate follows the law of exponential form. The results improve our understanding of decoherence and will provide new strategies to control it. Our analysis offers new insight into the dynamics of entanglement in open systems.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant No.11404246), the Natural Science Foundation of Shandong Province (Grant No. ZR2018LA014) and the Higher School Science and Technology Program of Shandong Province (Grant No. J17KA188).

References

[1] Giovannetti V, Lloyd S and Maccone L 2004 Science 306 1330–6
[2] Wang X L, Cai X D, Su Z N, Chen M C, Wu D, Li L, Liu N L, Lu C Y and Pan J W 2015 Nature 518 516–9
[3] Unden T et al 2016 Phys. Rev. Lett. 116 230502
[4] Ringbauer M, Wood C J, Modi K, Gilchrist A, White A G and Fedrizzi A 2015 Phys. Rev. Lett. 114 090402
[5] Chen P F, Sun W Y, Ming F, Huang A A J, Wang D and Ye L 2018 Laser Phys. Lett. 15 015206
[6] Modi K, Brodutch A, Cable H, Paterek T and Vedral V 2012 Rev. Mod. Phys. 84 1655–707
[7] Wehner S and Winter A 2009 New J. Phys. 12 65–85
[8] Strakhov A A and Man’ko V I 2013 J. Russ. Laser Res. 34 267–77
[9] Kiktenko E O and Korotaev S M 2012 Phys. Lett. A 376 820–3
[10] Chernega V N, Man’ko V O and Man’ko V I 2017 J. Russ. Laser Res. 38 324–33
[11] Chernega V N, Man’ko V O and Man’ko V I 2017 J. Russ. Laser Res. 38 416–25
[12] Chernega V N, Man’ko V O, Man’ko V I and Seiloz Z 2017 Theor. Math. Phys. 193 1715–24
[13] Braunstein S L and Caves C M 1988 Phys. Rev. Lett. 61 662–5
[14] Cao H and Ma W P 2018 Laser Phys. Lett. 15 095201
[15] Branciard C, Gisin N and Pironio S 2010 Phys. Rev. Lett. 104 170401–4
[16] Aerts S, Kwiat P, Larsson J Å and Zukowski M 1999 Phys. Rev. Lett. 83 2872–5
[17] Peres A 2000 Fortschr. Phys. 48 531–5
[18] Barrett J, Hardy L and Kent A 2005 Phys. Rev. Lett. 95 010503
[19] Barrett J, Kent A and Pironio S 2006 Phys. Rev. Lett. 97 170409
[20] Colbeck R and Renner R 2008 Phys. Rev. Lett. 101 050403
[21] Zurek W H 2003 Rev. Mod. Phys. 75 715
[22] Georgescu I M, Ashhab S and Nori F 2014 Rev. Mod. Phys. 86 153–85
[23] Lu H X, Zhao J Q, Cao L Z and Wang X Q 2011 Phys. Rev. A 84 044101
[24] Lu H X, Cao L Z, Zhao J Q, Li Y D and Wang X Q 2014 Sci. Rep. 4 4476
[25] Kurzynski P and Kaszlikowski D 2014 Phys. Rev. A 89 012103
[26] Fritz T and Chaves R 2013 IEEE Trans. Inf. Theory 59 803–17
[27] Mukohyama S 1998 Phys. Rev. D 58 104023
[28] Rastegin A E 2015 Ann. Phys. 355 241–57
[29] Shi J D, Wu T, Song X K and Ye L 2014 Quantum Inf. Process. 13 1045–56
[30] Hosten O, Engelsen N J, Krishnakumar N and Kasevich M A 2016 Nature 529 505–8
[31] Cao Z, Yin Z Q and Han Z F 2016 Phys. Rev. A 93 022310
[32] Pramanik T and Majumdar A S 2013 Phys. Lett. A 377 3209–15
[33] Xu S, He J, Song X K, Shi J D and Ye L 2015 Quantum Inf. Process. 14 755–64