Hard exclusive reactions and hadron structure

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The generalized Bjorken regime of exclusive reactions opens new ways to explore the hadron structure. We shortly review the present status of this domain where generalized parton distributions, generalized distribution amplitudes and transition distribution amplitudes describe various aspects of confinement physics.

1. Generalities

According to a now well established framework \cite{1}, the Bjorken limit of exclusive reactions with a hard probe allows factorization of the amplitudes into a perturbatively calculable subprocess at quark and gluon level on the one hand and hadronic matrix elements of light cone non local operators expressed through hadron (eventually generalized \cite{2}) distribution amplitudes and generalized parton distribution (GPD) (resp. a transition distribution amplitude \cite{3}) describing the transition from the baryon target to a baryon (resp. another hadron or photon) on the other hand.

As demonstrated in the presentations at this workshop, experimental data from DESY and JLab are now confirming this framework, and they seem to show that its applicability is quite precocious in terms of $Q^2$. Important in that respect is the analysis of spin asymmetries which are particularly sensitive to the interference between the dVCS and Bethe-Heitler processes \cite{4}.

2. Accessing transversity GPDs

Accessing chiral-odd hadronic matrix elements is notoriously difficult \cite{5}. We recently showed \cite{6} that the transversity GPD $H_T(x, \xi, t)$ contributes to a measurable electroproduction amplitude. We consider the process

\begin{equation}
\gamma^{(*)}N \rightarrow \rho_L \rho_T N'.
\end{equation}

For simplicity of discussion we specifically study the amplitude for the process

\begin{equation}
\gamma^{(*)}p \rightarrow \rho_L^0 \rho_T^+ n,
\end{equation}

\cite{UMR 7644 du CNRS}
Figure 1. The photoproduction process to access transversity GPDs.

that is, virtual or real photoproduction on a proton $p$, which leads via two-gluon exchange to the production of a vector meson $\rho^0$ separated by a large rapidity gap from another vector meson $\rho^+$ and the scattered neutron $n$. We consider the kinematical region where the rapidity gap between $\rho^+$ and $n$ is much smaller than the one between $\rho^0$ and $\rho^+$, that is the energy of the system ($\rho^+ - n$) is smaller than the energy of the system ($\rho^0 - \rho^+$) but, to justify our approach, still larger than baryonic resonance masses. In such kinematical circumstances the Born term for this process is calculable consistently within the collinear factorization method. The amplitude is represented as an integral (over the longitudinal momentum fractions of the quarks) of the product of two amplitudes: the first one describing the transition $\gamma^* \rightarrow \rho^0_L$ via two-gluon exchange and the second one describing the pomeron–proton subprocess $pp \rightarrow \rho^+ n$ which is closely related to the electroproduction process $\gamma^* N \rightarrow \rho N'$ where collinear factorization theorems allow separating the long distance dynamics expressed through the GPDs from a perturbatively calculable coefficient function. The hard scale appearing in the process is supplied by the relatively large momentum transfer $p^2$ in the two-gluon channel, i.e. by the virtuality of the pomeron.

Fig. 2 shows precitions for the differential cross section for $\gamma^*_L/T(Q) \rightarrow \rho^0_L, \rho^+_T n$ obtained within a model of the transversity distribution based on the one meson exchange in the t-channel. Our estimate is valid for the high energy limit. If one is willing to study the same processes at lower energy, one should include all polarization states of exchanged gluons.

An experimental determination of the transversity GPD $H_T$ seems feasible in photo- or electroproduction at high energies and we believe that the possible eRHIC machine may become the best place where the process discussed here could be measured, provided experimental setups allow a large angular coverage, ensuring a sufficient detection efficiency and a good control of exclusivity. The JLab CLAS-12 upgrade probably will have good enough detection efficiency for observing the two rho mesons, but only for relatively low $p_T$ of the order of 1–1.5 GeV. Moreover the smaller energy available prevents the theoretical framework used here from being adequate and one needs to supplement our studies
by adding contributions coming from other polarization states of exchanged gluons and the ones coming from quark exchanges.

Figure 2. The differential cross section for $\gamma^*_{L/T}(Q)p \rightarrow p^0_L p^+_T n$ for transverse virtual photon (left) and longitudinal virtual photon (right), plotted as a function of $p_T^2$ for $\xi = 0.3$ and $Q^2 = 0, 1, 5,$ and $10$ GeV$^2$.

3. Electroproduction of exotic hadrons

Hard production of hadrons outside the quark model has seemed to be a challenge, since it was generally believed that these exotic particles could not have a non-zero leading twist distribution amplitude (DA). We demonstrated recently \cite{7} that the non local nature of the quark correlators defining a DA was in fact allowing any $J^P_C$ values for the meson described by it. Moreover, a relation between the energy-momentum tensor and a moment of this correlator allows to estimate the magnitude of the leading twist DA of a $J^P_C = 1^{--}$ exotic vector meson. As seen in Fig$\text{3}$ electroproduction cross sections then turn out to be not small in comparison with those of usual mesons, and precise data at JLab should thus reveal the properties of these exotic mesons.

4. Transition distribution amplitudes

Let us now consider another class of exclusive processes, such as backward VCS $e p \rightarrow e p \gamma$, where the final photon flies in the direction of the initial proton, or its crossed version $\bar{p}p \rightarrow \gamma^* \gamma$, near the forward direction, which may be studied at GSI-FAIR \cite{8}. We propose \cite{3} that the amplitudes of such processes factorize in a quite similar way as for the dVCS reaction, but with a three quark exchange replacing the usual quark antiquark exchange characteristic of the handbag diagrams (see Fig. \text{4}). For instance, we write the $\bar{p}N \rightarrow \gamma^* \pi$ amplitude as

$$
\mathcal{M}(Q^2, \xi, t) = \int dx dy \phi(y_i, Q^2) T_H(x_i, y_i, Q^2) T(x_i, \xi, t, Q^2) ,
$$

\(3\)
where $\phi(y_i, Q^2)$ is the antiproton distribution amplitude, $T_H$ the hard scattering amplitude, calculated in the collinear approximation and $T(x_i, t, Q^2)$ the new TDAs.

To define the TDAs we introduce light-cone coordinates $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$ and transverse components $v^T = (v^1, v^2)$ for any four-vector $v$. The skewness variable $\xi = (p - p')^+(p + p')^+$ describes the loss of plus-momentum of the incident nucleon and is connected with $x_B$ by $\xi \approx x_B/(2 - x_B)$.

For instance, we define the leading twist TDAs for the $p \to \pi^0$ transition as:

$$
4\langle \pi^0(p')| \epsilon^{ijk} u^i(z_1 n) u^j(z_2 n) d^k(z_3 n) |p(p, s)\rangle
$$

$$
= - \frac{f_N}{2f_\pi} \left[ V^0_1(\hat{P}C)_{\alpha\beta}(B)_{\gamma} + A^0_1(\hat{P}\gamma^5 C)_{\alpha\beta}(\gamma^5 B)_{\gamma} - 3 T^0_1 (P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu B)_{\gamma} \right]
+ V^0_2(\hat{P}C)_{\alpha\beta}(\hat{\Delta} T B)_{\gamma} + A^0_2(\hat{P}\gamma^5 C)_{\alpha\beta}(\hat{\Delta} T^\gamma B)_{\gamma} + T^0_2 (\Delta^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(B)_{\gamma}
+ T^0_3 (P^\nu \sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\rho} \Delta^\rho T B)_{\gamma} + \frac{T^0_4}{M}(\Delta^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\hat{\Delta} T B)_{\gamma},
$$

where $\sigma^{\mu\nu} = i/2[\gamma^\mu, \gamma^\nu]$, $C$ is the charge conjugation matrix and $B$ the nucleon spinor. $\hat{P} = P^\mu \gamma_\mu$, the vector $\Delta = p' - p$ has - in the massless limit - the transverse components $\Delta^\nu = (g^{\mu\nu} - \frac{1}{P_n}(P^\mu n^\nu + P^\nu n^\mu))\Delta^\nu$.

$f_\pi$ is the pion decay constant ($f_\pi = 93$ MeV) and $f_N$ is the constant which determines the value of the nucleon wave function at the origin. The first three terms in (4) are the only ones surviving the forward limit $\Delta_T \to 0$. The constants in front of these three terms...
Figure 4. The factorization of the annihilation process $\bar{H} H \rightarrow \gamma^* \gamma$ into a hard subprocess (upper blob) and a transition distribution amplitude (lower blob) for the meson case and the baryon case.

have been chosen in reference to the soft pion limit results. With these conventions each function $V(z_i P \cdot n)$, $A(z_i P \cdot n)$, $T(z_i P \cdot n)$ is dimensionless.

The TDAs can then be Fourier transformed to get the usual representation in terms of the momentum fractions, through the relation

$$F(z_i P \cdot n) = \int_{-1+\xi}^{1+\xi} d^3x \delta(x_1 + x_2 + x_3 - 2\xi) e^{-i P n \sum x_i z_i} F(x_i, \xi)$$

(5)

where $F$ stands for $V_i, A_i, T_i$.

These TDAs are matrix elements of the same operator that appears in baryonic distribution amplitudes. They thus obey evolution equations which, as those of GPDs are of the ERBL type in some $x-$region, but are of a different nature in another $x-$region.

At fixed $\xi$ and $t$ the scaling behaviour of the amplitude is easily derived from its factorized expression, up to logarithmic corrections from the running of $\alpha_s$ and from the scale evolutions of the DA and of the TDA.

A partonic understanding of backward VCS is thus available for the first time, and precise data may be collected at JLab soon. This will be more discussed in J.Ph. Lansberg’s presentation [10].

5. Impact picture

One of the particularly nice feature of this class of exclusive reactions is its ability to uncover the deep transverse structure of hadrons [9]. GPDs, GDAs and TDAs contain information about the spatial structure of hadrons. For instance, the $p \rightarrow \pi$ TDA probes the partonic structure of the proton by requiring its wave function to overlap with the wave function of the configurations of the emerging meson, after it has been stripped from its valence quarks. Moreover, the Fourier transform of its dependence on $t$ tells us about the transverse position of these valence quarks in the proton. This may be phrased alternatively as detecting the transverse mean position of a pion inside the proton, when
the proton state is of the "next to leading Fock" order, namely $|qqq\pi>$. This is shown on Fig. 5.

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