Influence of Ice Cover on the Motion of a Submerged Body

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Abstract. In this work studies the influence of the ice cover on the nature of the motion of a submerged body, depending on its length, cross-sectional area, speed, and submergence. Experimental studies of the submarine vessel models motion under the ice cover are carried out in the ice tank. To determine the influence of the cross-sectional area, relative elongation and displacement of the submarine on the parameters of the generated flexural-gravity waves and the nature of the load movement near the lower surface of the ice, nine models have been made in the form of drop-shaped rotation bodies in the scale $\lambda_l=1:120$. To conduct the experiments the necessary equipment and measuring system to register vibrations of the modelled ice are designed. By towing the vessel model its velocity and profiles of flexural-gravity waves are determined. The study showed that ice cover can have a significant impact on the nature of the model movement near its lower surface. An increase in the cross-sectional area of the model leads to an increase in values of ice deflections. When the length of the models are increased and the speeds of movement are the same, the efficiency of the ice destruction decreases.

1. Introduction

The problem of the flexural-gravity waves (FGW) generated by the motion of a solid body under the ice cover has been studied insufficiently. Among the first theoretical papers on the subject is a paper by Kheisin [1], who studied the plane steady-state problem of the motion of a vortex under a layer of broken ice. Kozin [2] performed model experiments which confirmed the possibility of breaking solid ice by a moving submarine. Bukatov and Zharkov [3] and Kozin and Pogorelova [4] studied the steady motion of a point source of mass under a floating elastic plate for the cases of a stratified and nonstratified fluids, respectively, analyzed the effect of the motion velocity, the depth of the source, and the thickness of the plate on its deflections. In Sturova’s work [5] the solution of the linear steady problem of the flow of an inviscid, incompressible and infinitely deep liquid around a sphere under an ice sheet, which is modelled by a thin elastic stressed plate of constant thickness is constructed. In Korobkin’s et al [6] deflections and stresses in an ice cover of a frozen channel is of rectangular cross section caused by a load moving with a constant speed along the channel are studied. The external load is modelled by a localized smooth pressure distribution moving along the central line of the channel. The effect of the channel walls on the ice response is studied. It is found that the presence of the vertical walls of the channel reduces the ice deflection but increases the elastic strains in the ice plate [7].

It is known that when a surface vessel moves in shallow water conditions, the draft of the vessel may change. In the bow of the vessel there is a maximum pressure, which is reduced to a minimum in the area of the midship frame, then rises again in the area of the aft end. The increase in pressure is
accompanying a decrease in the motion speed of water particles, and the pressure drop is accompanied by an increase in their motion speed and a decrease in supporting forces. The formation of a zone of increased pressure in the bow (much larger than in the stern) and a decrease in supporting forces in the midsection causes the fact that the ship ‘subsides’ and gets a trim on the stern. It is obvious that when moving a submerged body near the lower surface of the ice, a similar effect may occur.

The paper studies the influence of the ice cover on the nature of the motion of a submerged body (the occurrence of vertical displacement), depending on its length, cross-sectional area, speed, and submergence.

2. Equipment and technique for conducting experiments
Experimental studies of the submarine vessel model motion under the ice cover are carried out in the ice tank [8] of the Ice Technology Laboratory of Sholom-Alechem Priamursky State University (Birobidzhan, Russia). The dimensions of the ice tank are 14×3×1 m (Figure 1).

![Figure 1. Main view of the ice tank](image1)

![Figure 2. Schemes of models 1a, 1b, 1c](image2)
Figure 3. Schemes of models 2a, 2b, 2c

Figure 4. Schemes of models 3a, 3b, 3c

To determine the influence of the cross-sectional area, relative elongation and displacement of the submarine on the parameters of the generated FGW and the nature of the load movement near the lower surface of the ice, nine models have been made in the form of drop-shaped rotation bodies (Figures 2-4) in the scale $\lambda=1:120$. All dimensions are in millimeters. Schemes of models made in AutoCAD 2019. The models are made by means of the method of layer-by-layer creation of a physical object according to a digital 3D model; the Raise3D printer has been used for this purpose.

To visualize the process of moving a submerged body near the lower surface of the ice, a waterproof window is mounted in the side wall of the tank.

Due to the lack of refrigerating machines, experimental studies are carried out in winter from 10 December 2018 to 31 January 2019. The ice cover model is made in the ice tank by freezing natural freshwater of the set thickness (0.002 m) without any impurity in natural conditions. The room where the tank is located is not heated for the whole period of the experiment. The experiments are carried out only at night from 9.00 p.m. to 5.00 a.m. At night there are negligible fluctuations in the room temperature, within the difference of 1°C to 2°C, although the room temperature varies from -10°C to -15°C, depending on the outside temperature, varying from -25°C to -30°C. Relatively uniform ice covers are, thus, obtained in 2-2.5 hours at night; that excludes patches of thaw on the ice. Due to the layer of heat-insulating material installed on the sides and under the bottom of the tank and compliance with the constant temperature regime, the ice thickness at the walls of the tank and in the middle of the ice field does not exceed 0.001 m. After carrying out each model test, broken ice pieces are removed and the depth of the model is readjusted, and then the ice cover is formed.

To conduct the experiments the necessary equipment and measuring system to register vibrations of the modelled ice are designed. By towing the vessel model its velocity and profiles of FGW are determined. To record the vibrations of the modelled ice we use LAS-Z infra-red non-contact laser movement sensors of Way Con Company (Germany). The sensor detects the ice deflection change depending on the time with the help of the Test viewer program 2.34.

The towing system makes it possible to test the model at speeds up to 2.4 m/s under steady motion conditions. Structurally, the system is made in the form of two frames installed on different sides of the tank and equipped with movable beams with guiding pulleys for the towing cable. A steel cable with a diameter of 0.003 m is used for towing the models; the strength of the cable provides the tension of the models necessary to exclude submergence changes in the course of motion that is determined experimentally by means of test starts on clear water before carrying out each series of the experiment. The changes in towing velocity of the submarine model are recorded with the help of the
SPS20 servo-driver of Servotechnik Company (Russia). The time of the model motion after the model begins working at a stationary regime does not exceed 3 s. The effect of wave reflection from the body of the vessel model is puttied and painted. The harnessing of the model in the towing system is made through carabiners connecting a tow cable with attachment points located at the ends of the body. The simulation scale is chosen based on the size of the ice tank, while the length of the tank provides access to the steady motion regime of the submarine model. The width is sufficient to eliminate the influence of the waves reflected from the channel sides on the main principle wave system [9].

Modeling of sea ice cover can be executed by using various ice models and for each of them there are corresponding conditions of similarity. The usual modeling is performed with partial satisfaction of conditions of similarity [2]:

\[ \lambda_E = \lambda_w = \lambda_h = \lambda_t \] (1)

Where \( \lambda_E \) is the model scale for Young’s modulus; \( \lambda_w \) is the model scale for deflections; \( \lambda_h \) is the model scale for ice thickness.

In this case, the requirements for similarity conditions with respect to Poisson’s ratio \( \mu \) and density \( \rho \) of the modelled ice are satisfied.

The vessel model must be geometrically similar to the full-scale one and their displacements must be in proportion to the cube of the geometrical scale-ratio:

\[ \frac{L_n}{L_m} = \lambda_l \; \frac{D_n}{D_m} = \lambda_l^3 \] (2)

Where \( L_n \) is the length of the full-scale vessel; \( D_n \) is the model vessel displacement. The model motion velocity \( u_m \) is determined by the condition of similarity

\[ \frac{u_n}{u_m} = \lambda_l^{1/2} \] (3)

Where \( u_n \) is the full-scale vessel velocity.

The parameters of model FGW are converted to full-scale ones in accordance with the geometrical similarity

\[ \frac{\lambda_n}{\lambda_m} = \frac{w_n}{w_m} = \lambda_l \] (4)

Where \( \lambda_n \) is the length of the full-scale vessel; \( \lambda_m \) is the length of model FGW; \( w_n \) is the full-scale FGW deflections; \( w_m \) is the model FGW deflections.

When the natural ice cover is used as a model one, the thickness of the modelled ice will be calculated when converted to a full-scale one in accordance with the following relations

\[ h_n = Hm \lambda_l^{4/3} \left( \frac{[\sigma_u]_n}{[\sigma_u]_m} \right)^{1/3} \] (5)

where \( h_n \) is the natural ice thickness; \( h_m \) is the modeled ice thickness; \( [\sigma_u]_n \) is the natural flexural stresses; \( [\sigma_u]_m \) is the modeled flexural stresses.

The determination of the flexural strength of the modelled ice is experimentally performed by testing beams afloat and for that purpose the modelled ice cover is prepared by building up the ice of the required thickness \( h_m = 0.002 \) m [10], \( [\sigma_u]_m = 1.53 \) MPa. The beams have a rectangular shape with the parameter of \( l \times b = 0.15 \times 0.45 \) m which are prepared in the modelled ice by cutting out according to the stamp by the rotating Dremel tool. The force required for beam destruction is determined by using the Mark-10 electronic dynamometer (USA). The value of the flexural strength of the freshwater ice \( [\sigma_u]_n = 0.7 \) MPa is chosen according to the work Petrov [11]. The thickness of the modelled ice cover after conversion to a natural one is \( h_n = 1.55 \) m.

It is known that when modelling sea ice by using fresh-water modelled ice, the elasticity modulus of natural ice should be less than the elasticity modulus of modelled ice \( E_n > E_m \), however the ratio \( E/\sigma_n \)
for sea ice is practically the same as that for fresh-water ice. To meet this condition in modelling ice is very important [12]. To enhance the visibility of ice fractures, the modelled ice surface is covered with a thin layer of snow, soon after the model field is formed.

The stages of ice destruction can be estimated using geometric characteristics. Laboratory studies [13] make it possible to determine the minimum angle of the tangent inclination to the deformed surface, sufficient for complete destruction of the ice during bending. The results show that the slope should not be less than 15°. Then, as a parameter for the complete destruction of the solid ice cover, we can take the value of the angle of the tangent inclination to the curved surface of the ice plate, characterizing the curvature of the FGW and equal to:

$$\alpha = \frac{2\pi w_m}{\lambda_m}$$

The dependence between the failure of the ice cover and the coefficient $\alpha$ has already been determined empirically in Kozin’s experimental work [2]. These model experiments on the failure of the natural ice cover of various thicknesses are conducted using of submarine models and hovercrafts. It is found, that for the load motion on the ice cover as well as for the submarine motion, when the maximum value of the slope of the ice surface exceeds the value of 0.04, it causes complete ice failure and crack opening.

Due to the complexity of the study, that is the duration of the preparation of the modelled ice field and the limited cold period (December-January), aiming to identify optional parameters of the experiment such as model displacement velocity, submergence and water depth a series of preliminary test runs have been conducted. The study shows that the most significant results are to be expected at submarine model speeds equal $u_m=1.4\div2.2$ m/s, the model submergence equal to $h_{svm}=0.16\div0.25$ m.

3. Results of the model experiments

3.1. Models 1a, 1b, 1c

We analyze the influence of the increase in the hull length of the submarine model on the parameters of generated flexural-gravity waves when the cross-sectional area $S$ is constant. Figure 5 shows the dependencies of ice cover deflections on the motion speed of models 1a, 1b, 1c $S_t=0.0106$ m$^2$ if $h_{svm}=0.16$ m. For model 1a the hull length is $L_m=0.933$ m.

![Figure 5. Dependence of the deflections of the model ice ($h_{m}=0.002$ m) on the motion speed of the load when $h_{svm}=0.16$ m; No. 1 is model 1a; No. 2 is model 1b; No. 3 is model 1c.](image-url)
Figure 6. Dependence of coefficient $\alpha$ on the motion speed of the load when $h_{svm}=0.16$ m; No.1 is model 1a; No.2 is model 1b; No.3 is model 1c.

Models 1b and 1c are lengthened by 23.7% and 42.9%, respectively. The graphs show that an increase in the length of the hull by 23.7% leads to a decrease in ice cover deflections by 10÷22%, and an increase in the length by 42.9% reduces deflections by 18÷47%. When motion speeds are about 1.85÷1.9 m/s the maximum values of ice cover deflections are observed.

Despite the increase in the length of the hull and, accordingly, the decrease in the curvature of FGW, the efficiency of the ice cover destruction is high within the entire range of motion speeds for all models. Figure 6 shows the values of coefficient $\alpha$. The greatest destructive effect is achieved at motion speeds close to a critical value when $L_m=0.933$ m. The increase in the hull length by 23.7% leads to a reduction of the ice-breaking ability of FGW by 12÷20%, the increase by 42.9% reduces this ability by 17÷28%.

Figure 7. Fracture pattern of the model ice ($h_m=0.002$ m) by means of FGW caused by the motion of model 1a at the speed $u_m=1.42$ m/s, $h_{svm}=0.16$ m.

Figure 8. Fracture pattern of the model ice ($h_m=0.002$ m) by means of FGW caused by the motion of model 1a at the speed $u_m=1.72$ m/s, $h_{svm}=0.16$ m.
Figure 9. Fracture pattern of the model ice \( (h_m=0.002 \text{ m}) \) by means of FGW caused by the motion of model 1a at the speed \( u_m=2 \text{ m/s}, h_{sym}=0.16 \text{ m} \).

Figures 7-9 demonstrate the fracture patterns of the model ice caused by the motion of model 1a. When the motion speed of a model approaches to a critical value, the destruction area of the model ice increases (Figures 7-8). If the motion speed is supercritical, the efficiency of ice destruction decreases (Figure 9); the area of ice floes increases, due to the greater curvature of FGW.

When the length of the models are increased and the speeds of movement are the same, the efficiency of the ice destruction decreases, the size of ice floes becomes larger (Figure 10), main cracks are basically formed in the model field (Figure 11).

Figure 10. Fracture pattern of the model ice \( (h_m=0.002 \text{ m}) \) by means of FGW caused by the motion of model 1b at the speed \( u_m=1.61 \text{ m/s}, h_{sym}=0.16 \text{ m} \).

Figure 11. Fracture pattern of the model ice \( (h_m=0.002 \text{ m}) \) by means of FGW caused by the motion of model 1c at the speed \( u_m=1.61 \text{ m/s}, h_{sym}=0.16 \text{ m} \).

When the depth of the models increases, the amount of deflections begins to decrease significantly. So, if the submergence depth of model 1a is equal to \( h_{sym}=0.21 \text{ m} \), ice deflections decrease by 18-53\% in comparison with the submergence depth \( h_{sym}=0.16 \text{ m} \) (Figure 12). The ice breaking ability of FGW is reduced by 17-45\% (Figure 13). When the submergence depth is \( h_{sym}=0.25 \text{ m} \), the deflections decrease by 48-81\%, and the values of coefficient \( \alpha \) decrease by 43-78\%.
3.2. Models 2a, 2b, 2c

For models 2a, 2b, 2c compared to models 1a, 1b, 1c, the cross-sectional area is expanded by 38.7% and amounts to $S_c=0.0147 \text{ m}^2$. The length of the models is assumed to be equal to that of models 1a, 1b, 1c and is increased for models 2b and 2c by 23.7% and 42.9%, respectively, compared to model 2a. Figure 14 shows the dependences of ice cover deflections on the motion speed of models if the submergence depth is equal to $h_{svm}=0.16 \text{ m}$. Dependencies show that an increase in the cross-sectional area leads to an increase in deflections by 30÷39% regardless of the length of the models. An increase in the length of the hull by 23.7% leads to a decrease in the ice cover deflections by 6÷10% and an increase in the length by 42.9% leads to a decrease by 10÷16%. At speeds $u_m = 1.85÷1.9 \text{ m/s}$, maximum values of ice cover deflections are observed.

Despite the increase in the length of the hull and, accordingly, the decrease in the curvature of FGW, the efficiency of ice cover destruction is significant within the entire range of speeds for all models. The values of coefficient $\alpha$ are shown in Figure 15. The greatest destructive effect is achieved
at speeds close to the critical value when $L_m=0.933$ m. The increase in the length of the hull by 23.7\% leads to a reduction of the ice-breaking ability of FGW by 15\%-20\%, the increase by 42.7\% reduces this ability by 25\%-35\%.

Figure 1. Dependence of the deflections of the model ice ($h_m=0.002$ m) on the motion speed of the load when $h_{svm}=0.16$ m: No.1 is model 2a; No.2 is model 2b; No.3 is model 2c.

Figure 15. Dependence of coefficient $\alpha$ on the motion speed of the load when $h_{svm}=0.16$ m: No.1 is model 2a; No.2 is model 2b; No.3 is model 2c.

Figures 16-18 show the fracture patterns of the model ice caused by the motion of model 2a. Experiments demonstrate that if the displacement is sufficiently large and the elongation is relatively small (for model 2a $L_m/B_m=6.8$) an intensive destruction of ice takes place at speeds much lower than the critical value (Figure 16). As the motion speed of the model approaches to the critical value, the destruction area of the model ice reaches its maximum (Figure 17). When the motion speed approaches to its critical value, the destruction area of the model ice sharply decreases, the size of ice floes increases, and the destruction area decreases (Figure 18), due to the greater curvature of FGW.
Figure 16. Fracture pattern of the model ice ($h_m=0.002$ m) by means of FGW caused by the motion of model 2a at the speed $u_m=1.42$ m/s, $h_{svm}=0.16$ m.

Figure 17. Fracture pattern of the model ice ($h_m=0.002$ m) by means of FGW caused by the motion of model 2a at the speed $u_m=1.78$ m/s, $h_{svm}=0.16$ m.

Figure 18. Fracture pattern of the model ice ($h_m=0.002$ m) by means of FGW caused by the motion of model 2a at the speed $u_m=2.08$ m/s, $h_{svm}=0.16$ m.

Figure 19. Dependence of the deflections of the model ice ($h_m=0.002$ m) on the motion speed of model 2b at such submergences as No. 1 is $h_{svm}=0.16$ m; No. 2 is $h_{svm}=0.21$ m; No. 3 is $h_{svm}=0.25$ m.
The study of the influence of the submergence depth of the submarine on the parameters of generated FGW is conducted for model 2b. The results of experiments are shown in Figures 19-20. When the submergence of the model increases, the deflection value begins decreasing. If the submergence depth of model 2b is equal to $h_{svm}=0.21$ m, ice deflections are reduced by 24÷53% compared to the submergence depth of $h_{svm}=0.16$ m (Figure 19). The ice-breaking ability of FGW is reduced by 20÷56% (Figure 20). If there is a submergence $h_{svm}=0.25$ m, the deflections decrease by 47÷68%, and the values of coefficient $\alpha$ are reduced by 46÷65%.

The fracture pattern of the model ice at a critical speed is shown in Figures 21-22. Despite the submergence increase a dense network of cracks is formed in the ice. The size of ice floes significantly grows as submergence depth increases.

3.3. Models 3a, 3b, 3c

For models 3a, 3b, 3c, the cross-sectional area is reduced by 30% and amounts to $S_3=0.0077$ m$^2$. The length of the models is assumed to be equal to models 1a, 1b, 1c and is increased for models 3b and 3c by 23.7% and 42.9%, respectively, compared to model 3a. Figure 23 shows the dependences of ice cover deflections on the motion speed of models at the submergence $h_m=0.16$ m.

Dependencies show that a decrease in the cross-sectional area by 30% leads to a decrease in deflections by 27÷84%. An increase in the length of the hull by 23.7% leads to a decrease in the
deflection of the ice cover by \(33\% \div 40\%\) and an increase in the length by \(42.9\%\) leads to a decrease by \(38\% \div 58\%\). At speeds \(u_m=1.85\% \div 1.9\ m/s\), maximum values of the ice cover deflections are observed.

**Figure 23.** Dependence of the deflections of the model ice \((h_m=0.002\ m)\) on the motion speed of the load when \(h_{swm}=0.16\ m;\ No.1\ is\ model\ 3a;\ No.2\ is\ model\ 3b;\ No.3\ is\ model\ 3c.\)

The ice-breaking ability of FGW is also reduced significantly (Figure 24). An increase in the length of the hull by \(23.7\%\) leads to a reduction of the ice-breaking ability of FGW by \(22\% \div 32\%\), the increase by \(42.9\%\) reduces this ability by \(27\% \div 43\%\). The greatest destructive effect is achieved at speeds close to the critical value when \(L_m=0.933\ m\).

The ice-breaking ability of FGW is also reduced significantly. An increase in the length of the hull by \(23.7\%\) leads to a reduction of the ice-breaking ability of FGW by \(22\% \div 32\%\), the increase by \(42.9\%\) reduces this ability by \(27\% \div 43\%\). The greatest destructive effect is achieved at speeds close to the critical value when \(L_m=0.933\ m\) (Figure 25). We have not recorded an effective destruction of ice and a loss of a load-bearing capacity for models 3a, 3b, 3c. As the hull is lengthened, the size of ice floes substantially increases (Figure 26).
Figure 25. Fracture pattern of the model ice ($h_m=0.002$ m) by means of FGW caused by the motion of model 3a at the speed $u_m=1.85$ m/s, $h_{svm}=0.16$ m.

Figure 26. Fracture pattern of the model ice ($h_m=0.002$ m) by means of FGW caused by the motion of model 3c at the speed $u_m=1.92$ m/s, $h_{svm}=0.16$ m.

Figure 27. Dependence of the deflections of the model ice ($h_m=0.002$ m) on the motion speed of model 3c at such submergences as No.1 is $h_{svm}=0.16$ m; No.2 is $h_{svm}=0.21$ m; No.3 is $h_{svm}=0.25$ m.

Figure 28. Dependence of coefficient $\alpha$ on the motion speed of model 3c at such submergences as No.1 is $h_{svm}=0.16$ m; No.2 is $h_{svm}=0.21$ m; No.3 is $h_{svm}=0.25$ m.
The study of the influence of the submergence depth of the submarine on the parameters of generated FGW is conducted for model 3c. The results of experiments are shown in Figures 27-28. When the submergence of the model increases, the deflection value begins decreasing significantly. If the submergence depth of model 3c is equal to \( h_{svm} = 0.21 \) m, ice deflections are reduced by 10÷26% compared to the submergence depth of \( h_{svm} = 0.16 \) m (Figure 27). The ice-breaking ability of FGW is reduced by 27÷37% (Figure 28). If there is a submergence \( h_{svm} = 0.25 \) m, the deflections decrease by 52÷68%, and the values of coefficient \( \alpha \) are reduced by 48÷58%.

While doing experiments, it has been recorded that when models 2 move near the lower surface of the ice sheet, regardless of their lengths, there is a change in the depth of the submergence. The vertical displacement is not recorded during movement for models 1, 3 that have a smaller cross-sectional area. It is found that the value of the displacement depends on the motion speed of models and the submergence. The greatest deviations from the initial value of the submergence depth are recorded at motion speeds equal to 1.47÷1.68 m/s. The submergence decreases by 30% when \( h_{svm} = 0.16 \) m (Figure 29).

![Figure 29. Dependence of the relative vertical displacement of model 3 on the motion speed at such submergences as No.1 is \( h_{svm} = 0.16 \) m; No.2 is \( h_{svm} = 0.21 \) m; No.3 is \( h_{svm} = 0.25 \) m.](image)

At speeds greater than 2 m/s, the vertical displacement of the models is not fixed. As the submergence depth increases, the vertical displacement becomes less and for \( h_{svm} = 0.25 \) m it is 12% at the speed \( u_m = 1.47 \) m/s. The vertical displacement of models takes place because in the bow of the model there is a zone of increased pressure that is much larger than in the stern. The supporting forces in the area of the midship frame decreases, which leads to a vertical displacement. We should notice that the models also have a trim on the stern, which is about 2 degrees. It is obvious that when the cross-sectional area increases and its shape changes, the value of the vertical displacement can increase.

4. Conclusion
The study concludes
- the ice cover can have a significant impact on the nature of the load movement near its lower surface;
- an occurrence of a vertical displacement depends on the cross-sectional area of the load, the motion speed and the submergence depth;
- an increase in the cross-sectional area of the load leads to an increase in values of ice deflections and, accordingly, to an increase in the ice breaking ability of FGW;
- when the length of the models are increased and the speeds of movement are the same, the efficiency of the ice destruction decreases, the size of ice floes becomes larger, main cracks are basically formed in the model field because of an increase in the length of FGW;
- a significant decrease in the ice breaking ability of FGW is observed when the value of the load depth increases.

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Acknowledgments
The reported study was funded by RSF (Russian Science Foundation) according to the research project No. 16-19-10097