Abstract

We calculate the effect of new $CP$ violating interactions parameterized by an anomalous $tbW$ coupling on $CP$-odd observables in $B$ decays. We find that couplings consistent with current bounds induce observable effects in some $CP$ asymmetries that will be measured in $B$-factories. The new effects are sufficiently large that they can actually test specific models that give rise to these $tbW$ interactions.
1 Introduction

The top quark is significantly heavier than the other five quarks. This has generated speculation that perhaps it plays a fundamental role in the breaking of electroweak symmetry \cite{1,2}. Models that incorporate this idea contain four-fermion operators that involve the third generation of quarks and perhaps exotic new fermions such as techniquarks.

At energy scales near the $W$ mass, this type of new theories gives rise to interactions between the $t$ and $b$ quarks and the electroweak gauge bosons $W$ and $Z$ that may deviate significantly from their standard model values. Such interactions are conveniently described by an effective Lagrangian \cite{3}.

In this note, we study the effects of the simplest $CP$-violating coupling in the effective Lagrangian to $CP$-odd observables in $B$ decays. The $CP$ conserving indirect effects of this coupling have been studied before in the literature \cite{4,5}. There are also studies of direct measurements of the $tbW$ coupling in future Tevatron experiments \cite{6}. $CP$ violating interactions beyond the standard model have also been studied in detail for $B$ decays \cite{7}. However, the specific scenario that we discuss here has not been studied previously.

2 High Energy Effective Lagrangian

We assume that whatever is responsible for generating the non-standard model top quark couplings occurs at a high energy scale $\Lambda$, perhaps a few $TeV$. We also assume that this physics is responsible for the breaking of electroweak symmetry and that there is no light Higgs boson. Therefore, we use a non-linear effective Lagrangian to describe the physics at the $W$ scale. Furthermore, in accordance with the prejudice that it is only the top-quark that plays a role in the new physics, we consider only the couplings of top and bottom quarks to $W$ and $Z$ gauge bosons.

To write the effective Lagrangian that describes the interactions of fermions to the electroweak gauge bosons we follow the formalism of Peccei and Zhang \cite{3}. We consider only the lowest order couplings that can violate $CP$, and we do not include dipole moment type couplings of the top to the $Z$ since these have been studied before in the literature \cite{8}. In unitary gauge we have:

$$\mathcal{L}_{\text{eff}} = \frac{g}{\sqrt{2}} V_{tb} \left[ (1 + \kappa_L e^{i\phi_L}) \bar{t}_L \gamma^\mu b_L + \kappa_R e^{i\phi_R} \bar{t}_R \gamma^\mu b_R \right] W_\mu^+ + h.c.$$  (1)

The new interaction effects are parameterized by the real couplings $\kappa_L$ and $\kappa_R$ and the new phases $\phi_{L,R}$. These phases contain the information on $CP$ violation that may exist in the new theory. For simplicity we assume that the form Eq. 1 occurs only in the $tbW$ coupling ignoring any possible effects on $tsW$ and $tdW$ due to CKM mixing. The existing bounds on $\kappa_{L,R}$ depend on naturalness assumptions, so they are not rigorous \cite{5}. To calibrate the sensitivity of the observables discussed in this paper, we will use the bounds $|\kappa_R| \lesssim 0.01$ from $b \rightarrow s \gamma$ and $|\kappa_L| \lesssim 0.2$; obtained
by setting any other anomalous coupling to zero in the results of Ref. [5]. There are no bounds at present on the phases $\phi_{L,R}$.

Eq. [4] contributes to observables in $B$ decays at one-loop order. However, we will not include in our calculation any additional effective operators that may be needed in a complete effective field theory to act as counter-terms at one-loop. Instead we resort to estimating the order of magnitude of the effects by keeping either the one-loop contribution from Eq. [1] when it is finite; or the leading non-analytic term when it is divergent [9]. Therefore, our results will depend on the naturalness assumption that contributions from different couplings do not cancel each other. This is similar in spirit to the bounds that are placed on new gauge boson self interactions from LEP observables [10].

3 Low Energy Effective Interactions

In this section we present the results of the one-loop order contributions of Eq. [4] to a low energy effective Lagrangian appropriate for the study of $B$ decays. Two types of terms are generated corresponding to effective $|\Delta b| = 1, 2$ transitions.

For the $|\Delta b| = 2$ transition we compute the usual box diagrams but with the $tbW$ coupling modified as indicated in Eq. [4]. In this case our result is divergent because the new interaction explicitly violates GIM. We keep the leading non-analytic term:

$$H_{\text{eff}} = G_F^2 M_W^2 \frac{(V_{tb} V_{tb}^*)^2}{2 \pi^2} \left\{ \left( H(x_t) - \kappa_L e^{i \phi_L} \log \left( \frac{\mu}{M_W} \right) \right) \overline{D}\gamma_\alpha P_L b D\gamma_\alpha P_L b \\
- 2 \kappa_R^2 e^{2 i \phi_R} \log \left( \frac{\mu}{M_W} \right) \overline{D}P_R b \overline{D}P_R b \right\}$$

We use $D$ to denote either a strange or down quark and, for comparison, we have included the standard model result as the first term. With $x_t = m_t^2/m_W^2$ we have [11]:

$$H(x_t) = \frac{\alpha_S}{4\pi} \left( F_L(x_t) + \left( F_L(x_t) + \frac{1}{9} \right) \kappa_L e^{i \phi_L} \right) \overline{D}\gamma_\alpha \gamma_\alpha P_L b \sum_q \overline{q} \gamma_\alpha \gamma_\alpha q + g_S 8 \pi^2 F_R(x_t) \kappa_R e^{i \phi_R} m_t \overline{D}\sigma^{\mu\nu} \gamma_\alpha P_R b G_{\mu\nu}^\alpha$$

For the $|\Delta b| = 1$ transition we compute the standard gluonic penguin but use the complete interaction of Eq. [4] for the $tbW$ coupling. After we include the factors corresponding to wave function renormalization for the external fermions, we obtain the finite result:

$$H_{\text{eff}} = G_F \sqrt{2} V_{tb} V_{tb} \left\{ \frac{\alpha_S}{4\pi} \left[ F_L(x_t) + \left( F_L(x_t) + \frac{1}{9} \right) \kappa_L e^{i \phi_L} \right] \overline{D}\gamma_\alpha \gamma_\alpha P_L b \sum_q \overline{q} \gamma_\alpha \gamma_\alpha q \right\} + h. c.$$  

where, again, $D$ stands for either a strange or down quark, and the form factors are:

$$F_L(x_t) = - \log(x_t) \frac{9 x_t^2 - 16 x_t + 4}{6(1 - x_t)^4} + x_t \frac{18 - 11 x_t - x_t^2}{12(1 - x_t)^3}$$
\[ F_R(x_t) = 3 \left[ \log(x_t) \frac{x_t}{(1-x_t)^3} + \frac{(1+x_t)}{2(1-x_t)^2} + \frac{1}{6} \right] \]  

(5)

The first term, independent of \( \kappa_L \), corresponds to the standard model result and, not surprisingly, has the same form factor as the new left-handed coupling. The additional constant term, 1/9, is present because the new interaction does not have a GIM mechanism. For the same reason there is the term 1/6 in \( F_R \). It is interesting to note the potential for very large effects from the \( \kappa_R \) term, where there is an enhancement factor of \( m_t/m_b \sim 35 \) compared to the corresponding operator in the standard model. This large factor can compensate for the smallness of \( \kappa_R \) in the same way as it does in \( b \to s\gamma \) transitions \[4\].

4 CP Violation in neutral B decays

With the results of the previous section it is straightforward to estimate the effects of the new phases in the CP asymmetries that will be studied in the B-factories. The general analysis of these CP asymmetries has been reviewed in Ref. \[12\].

The simplest way to estimate the potential size of the effect due to the new phases is to look at processes with mixing induced CP violation that are dominated by a tree-level amplitude. In this case the quantity of interest is (in the standard notation of \[12\]):

\[ \left( \frac{q}{p} \right)_{B_D} = \frac{V_{tb}^* V_{tD}}{V_{tb} V_{tD}} \left( \frac{F^* F}{F} \right)^{\frac{1}{2}} \]  

(6)

where the first factor corresponds to the standard model value and the second factor is the modification due to the new phases:

\[ F = H(x_t) - \kappa_L e^{i\phi_L} \log(\frac{\mu}{M_W}) - 2\kappa_R^2 e^{2i\phi_R} \log(\frac{\mu}{M_W}) \frac{5}{8} \left( \frac{M_B}{m_b} \right)^2 \]  

(7)

The factor \( 5/8(M_B/m_b)^2 \) for the right handed coupling takes into account the difference in the hadronic matrix elements using factorization and vacuum insertion. Numerically, we can use \( m_t = 175 \) GeV, and \( \mu = 1 \) TeV to find:

\[ \left( \frac{F^* F}{F} \right)^{\frac{1}{2}} \approx 1 + i[5\kappa_L \sin \phi_L + 8\kappa_R^2 \sin(2\phi_R)] \equiv 1 + i\phi_{Box} \]  

(8)

Experimental constraints on \( \kappa_R \) from \( b \to s\gamma \) \[4\] make its contribution much smaller than that of \( \kappa_L \) in Eq. 8 so we will drop it.

Since we assumed that the only new interaction is of the form shown in Eq. 1, there is no corresponding modification for mixing in the \( K \) or \( D \) systems. Strictly speaking, \( \phi_{Box} \), may have a different value for \( B_d \) and \( B_s \) because, in principle, the two amplitudes may have different counter-terms. In our approximation we are ignoring the counter-terms and keeping only the leading logarithm so we end up with the same \( \phi_{Box} \) for \( B_d \) and \( B_s \).
This is sufficient to study decay modes with tree-level dominated amplitudes (or after penguin effects are disentangled with an isospin analysis [13]). For example, the modes \( B_d \rightarrow \Psi K_s \) and \( B_d \rightarrow \pi^+\pi^- \) could be used to measure the angles \( \beta \) and \( \alpha [12] \). With our new phases they would really measure the combinations:

\[
\begin{align*}
\lambda(B_d \rightarrow \Psi K_s) &= e^{-i(2\beta - \phi_{Box})} \\
\lambda(B_d \rightarrow \pi^+\pi^-) &= e^{i(2\alpha + \phi_{Box})}
\end{align*}
\] (9)

Assuming that the phases \( \alpha \) and \( \beta \) are known with small theoretical uncertainties, for example from better measurements of \( V_{ub} \) and \( K^+ \rightarrow \pi^+\nu\bar{\nu} \), one can use the \( B \)-factory measurements of \( \alpha \) and \( \beta \) to look for physics beyond the standard model. The BaBar technical design report quotes achievable errors on the measurements of \( \alpha \) and \( \beta \) of 8.5% and 5.9% respectively for \( 30 fb^{-1} [15] \).

The mode \( B_d \rightarrow \Psi K_s \) is free of hadronic uncertainties both in the standard model [12], and in our model. For hadronic uncertainties to appear in this mode, we would have to enhance penguin amplitudes by factors of at least 20. This is impossible for values of \( \kappa_L \) and \( \kappa_R \) that make sense in the context that we are discussing. Therefore, in the scenario in which \( \beta \) is known and the \( B \)-factory measures \( (\beta - \phi_{Box}/2) \) with a 5.9% accuracy we can place the bound:

\[ \kappa_L \sin \phi_L \lesssim 0.03 \] (10)

This is a very significant constraint since current \( CP \) conserving data allows \( \kappa_L \) to be as large as \( \kappa_L \sim 0.2 [3] \).

The mode \( B_d \rightarrow \pi^+\pi^- \) has hadronic uncertainties that may be resolved experimentally by carrying out an isospin analysis [13]. Assuming that this isospin analysis is possible and that the standard model is correct, this mode will give a measurement of \( \alpha \). Our model for new physics does not introduce new \( \Delta I = 3/2 \) transitions, so the same isospin analysis would isolate the phase \( (\alpha + \phi_{Box}/2) \). Since the experimental accuracy in this mode is worse than that of the previous mode, it will not lead to better bounds on \( \kappa_L \sin \phi_L \). However, in our model the deviations in \( \alpha \) and \( \beta \) are related as in Eq. (9) and, thus, the value of \( (\alpha + \phi_{Box}/2) \) is a prediction that could be tested.

We should point out that a value of \( \kappa_L \) as large as 0.2 would change the \( B - B \) mixing amplitude with respect to its standard model value by about 50%. This is well within the current theoretical uncertainty in the standard model calculation due to the hadronic form factors \( f_{B^0} B_B [10] \).

Decays in which penguin amplitudes are dominant offer the possibility to place bounds on \( \kappa_R \sin \phi_R \). For penguin dominated modes one would have (using the same values of \( m_t \) and \( \mu \) as before and in the notation of [12]):

\[ \frac{\mathcal{A}}{\mathcal{A}} \approx \left( \frac{\mathcal{A}}{\mathcal{A}} \right)_{SM} \left( 1 + i[3\kappa_L \sin \phi_L + 90\kappa_R \sin \phi_R] \right) \] (11)

\footnote{Alternatively one could use the modes \( B \rightarrow \pi\rho \) to measure \( \alpha [14] \).}
The large numerical factor in front of $\kappa_R$ is due to the $m_t/m_b$ enhancement discussed earlier. To actually calculate this number one would have to be able to compute the hadronic matrix elements of the two operators in Eq. 4 and this is impossible at present. The number 90 follows from a simple dimensional analysis in which we compare the two operators, imagine the gluon splitting into a quark-anti quark pair and replace any quark or gluon momentum with a factor of $M_B$. In this case the $\kappa_L$ term is less important so we drop it.

The general analysis of $CP$ asymmetries in $B$ decays with new physics in the decay amplitudes has been carried out in Ref. [17]. For a case like ours, it is convenient to compare the asymmetry in $B_d \to \Psi K_s$ with that in $B_d \to \phi K_s$. In models that only have new $CP$ violating phases in the mixing, the asymmetries in these two modes measure the same phase $(\beta + \delta_{Md})$ in the notation of Ref. [17], (in our case $\delta_{Md} = -\phi_{Box}/2 \approx -2.5\kappa_L \sin \phi_L$). With additional new phases in the decay amplitudes, the phase measured in $B_d \to \Psi K_s$ remains the same, whereas the one measured in $B_d \to \phi K_s$ (or any other $b \to s\bar{s}s$ mode) becomes $(\beta + \delta_{Md} + \delta \phi_A)$ (again in the notation of Ref. [17]). From Eq. 11 we can read off the value $\delta \phi_A \approx 3\kappa_L \sin \phi_L + 90\kappa_R \sin \phi_R$.

Assuming that the difference in the phases of $b \to c\bar{s}s$ and $b \to s\bar{s}s$ modes can be measured to 10% one could place the bound:

$$\kappa_R \sin \phi_R \lesssim 0.001$$

(12)

Even with the stringent bounds (of order a few percent) that $b \to s\gamma$ places on $\kappa_R$ [1], this term can produce measurable corrections to $CP$ asymmetries in penguin dominated modes. The bound in Eq. 12 is extremely good, and would place constraints on models like one of Appelquist and Wu [20] where $\kappa_R \sin \phi_R$ can be as large as 0.01. The characteristic pattern of asymmetries induced by our anomalous couplings is the same as that of models with an enhanced chromomagnetic dipole operator of Ref. [18] that are discussed in Ref. [17] [1].

5 Conclusions

In conclusion we have found that $CP$ asymmetries in $B$ decays are in principle sensitive to new $CP$ violating phases in the $tbW$ interaction. This is theoretically interesting because models of electroweak symmetry breaking in which the top-quark plays a special role may give rise to this type of interactions. Appelquist and Wu estimate in a technicolor model that $\kappa_R \sin \phi_R$ could be as large as 0.01 [20], which we have seen

2Unless penguins are enhanced by factors of 20 or more, but we argue this does not happen in our model.

3 It is difficult to be quantitative because we do not know how to calculate the necessary matrix elements, and because we don’t yet know what will be the ultimate experimental accuracy. If we take $\kappa_R \sim 0.01$, our new interactions respect the CLEO bounds for $b \to s\gamma$ and for certain values of $\phi_R$ they can produce direct $CP$ violating asymmetries of order 10% in modes where the standard model does not induce a sizable asymmetry such as $B \to \phi K$. We thank A. Kagan who computed these numbers for us using his factorization model for the hadronic matrix elements [13].
is large enough to be seen at the $B$-factory. We have also seen that the $B$-factory can place bounds on $\kappa_L \sin \phi_L$ of order a few percent and that these bounds are meaningful in light of the current bounds on $\kappa_L$. Our result is phenomenologically interesting because the new $CP$ violating effects could be large enough to distinguish them from standard model $CP$ violation in future experiments at a $B$-factory. Finally, it is important to point out that the $B$-factory effects that we have discussed are likely to be the only place where one can search for new $CP$ violating phases of the type in Eq. 1. This is because the top couples so dominantly to bottom, that in direct top production and decay the phases cancel out as they enter both the production and decay vertices.

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