Fractional-Wrapped Branes with Rotation, Linear Motion and Background Fields

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Abstract

We obtain two boundary states corresponding to the two folds of a fractional-wrapped Dp-brane, i.e. the twisted version under the orbifold \( \mathbb{C}^2/\mathbb{Z}_2 \) and the untwisted version. The brane has rotation and linear motion, in the presence of the following background fields: the Kalb-Ramond tensor, a \( U(1) \) internal gauge potential and a tachyon field. The rotation and linear motion are inside the volume of the brane. The brane lives in the \( d \)-dimensional spacetime, with the orbifold-toroidal structure \( T^n \times \mathbb{R}^{1,d-n-5} \times \mathbb{C}^2/\mathbb{Z}_2 \) in the twisted sector. Using these boundary states we calculate the interaction amplitude of two parallel fractional Dp-branes with the foregoing setup. Various properties of this amplitude such as the long-range behavior will be analyzed.

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1 Introduction

By using the boundary state formalism all properties of the D-branes can be extracted. In this formalism a D-brane can be completely represented in terms of all closed string states, internal fields, tension and dynamical variables of the brane. Hence, a D-brane appears as a source for emitting (absorbing) all closed string states. The D-branes interaction is obtained by overlap of two boundary states, associated with the branes, through the closed string propagator. Thus, this adequate formalism has been applied for various configurations of the D-branes [1]-[21].

Among the different configurations of branes the setups with fractional D-branes have some appealing behaviors [17]-[24]. The fractional branes appear in the various parts of string and M-theories. For example, they are useful tools for demonstrating the gauge/gravity correspondence [24], and the dynamical fractional branes prepare an explicit starting point for defining Matrix theory [25, 26]. On the other hand, we have the compactified D-branes which have a considerable application in string theory. Besides, there are D-branes with background fields which possess various interesting properties. For example, these fields drastically control the interactions of the branes [8]-[15], and they influence the emitted and absorbed closed strings by the branes. The fractional branes, wrapped branes and the background fields motivated us to study a configuration of the dynamical fractional-wrapped branes with background fields.

In this paper we use the method of boundary state to obtain the interaction amplitude between two parallel fractional-wrapped bosonic Dp-branes with background fields and dynamics. We introduce the background field $B_{\mu\nu}$, internal $U(1)$ gauge potentials and internal open string tachyon fields in the worldvolumes of the branes. In addition, the branes of our setup are dynamical, i.e. they rotate and move within their volumes. For the background spacetime in the twisted sector $\mathcal{T}$ we shall apply the following topological structure

$$\mathcal{T}^n \times \mathbb{R}^{1,d-n-5} \times \mathbb{C}^2 / \mathbb{Z}_2, \ n \in \{0,1,\ldots,d-5\}.$$

An arbitrary torus from the set $\{\mathcal{T}^n| n = 0,1,\ldots,d-5\}$ will be considered. Therefore, our configuration represents a generalized setup. We shall demonstrate that the twisted
sector does not contribute to the long-range force, i.e. the interaction of the distant branes completely comes from the untwisted sector $\mathcal{U}$.

This paper is organized as follows. In Sec. 2, we compute the boundary states corresponding to a rotating and moving fractional-wrapped $D_p$-brane with background and internal fields. In Sec. 3.1, the interaction amplitude for two parallel $D_p$-branes will be acquired. In Sec. 3.2, the contribution of the massless states of closed string to the interaction amplitude will be extracted. Section 4 is devoted to the conclusions.

2 The boundary states corresponding to a $D_p$-brane

We start by calculating the boundary states, associated with a fractional-wrapped $D_p$-brane. The $d$-dimensional background spacetime contains a toroidal compact part, and for the twisted sector includes a non-compact orbifold part $\mathbb{C}^2/\mathbb{Z}_2$. The $\mathbb{Z}_2$ group acts on the orbifold directions $\{x^a | a = d-4, d-3, d-2, d-1\}$. We begin with the string action

$$S = -\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \left( \sqrt{-g} g^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \right) + \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma \left( A_\alpha \partial_\tau X^\alpha + \omega_{\alpha\beta} J^\alpha_\tau + T^2( X^\alpha ) \right),$$

(2.1)

where $\alpha, \beta \in \{0, 1, \ldots, p\}$ represent the worldvolume directions of the brane, the metrics of the worldsheet and spacetime are $g_{ab}$ and $G_{\mu\nu}$, $\Sigma$ indicates the worldsheet of closed string and $\partial\Sigma$ is its boundary. Here we take the flat spacetime with the signature $G_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, 1, \ldots, 1)$ and a constant Kalb-Ramond field $B_{\mu\nu}$. The profile of the tachyon field is chosen as $T^2(X) = \frac{1}{2} U_{\alpha\beta} X^\alpha X^\beta$ with the constant symmetric matrix $U_{\alpha\beta}$ [27, 28]. For the internal gauge potential we chose the gauge $A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta$ with the constant field strength. The tachyon field and gauge potential belong to the spectrum of the open string theory, thus, they accurately appeared as the boundary terms. The antisymmetric constant angular velocity $\omega_{\alpha\beta}$ shows the rotation and linear motion of the brane, and $J^\alpha_\tau = X^\alpha \partial_\tau X^\beta - X^\beta \partial_\tau X^\alpha$ is the angular momentum density. Note that the rotation and linear motion of the brane are inside the volume of the brane. In fact, presence of the various internal fields indicates some preferred alignments in the brane, and hence the Lorentz symmetry in the brane worldvolume explicitly has been broken. We should
say that adding a tachyonic mode generally breaks the conformal invariance, however the conformal boundary state can still be considered at the fixed points of the orbifold. For string actions with tachyon fields e.g. see Ref. [23] and references therein, and also Refs. [27, 29, 30, 31, 32], in which some of them contain the resultant boundary states.

Setting the variation of this action to zero yields equation of motion of $X^\mu$ and the following equations for the boundary state

\[
\left( K_{\alpha\beta} \partial_{\tau} X^{\beta} + F_{\alpha\beta} \partial_{\sigma} X^{\beta} + B_{\alpha I} \partial_{\sigma} X^I + U_{\alpha\beta} X^{\beta} \right)_{\tau=0} |B_x\rangle = 0 , \\
\left( X^I - y^I \right)_{\tau=0} |B_x\rangle = 0 ,
\]

(2.2)

where $K_{\alpha\beta} = \eta_{\alpha\beta} + 4 \omega_{\alpha\beta}$, and the total field strength is $F_{\alpha\beta} = B_{\alpha\beta} - F_{\alpha\beta}$. The coordinates $\{x^I | I = p+1, \ldots, d-1\}$ show the directions which are perpendicular to the brane worldvolume, and the parameters $\{y^I | I = p+1, \ldots, d-1\}$ represent the location of the brane. Combination of Eqs. (2.2) eliminates the third term of the first equation. We observe that the background fields impose the mixed boundary conditions along the brane worldvolume.

The solution of the equation of motion for the non-orbifold directions has the form

\[
X^\lambda (\sigma, \tau) = x^\lambda + 2\alpha' p^\lambda \tau + 2L^\lambda \sigma + \frac{i}{2} \sqrt{2\alpha'} \sum_{m\neq 0} \frac{1}{m} \left( \alpha_m^\lambda e^{-2im(\tau-\sigma)} + \tilde{\alpha}_m^\lambda e^{-2im(\tau+\sigma)} \right) ,
\]

(2.3)

where $\lambda \in \{\alpha, I\}$ for the untwisted sector and $\lambda \in \{\alpha, i\}$ for the twisted one. In the twisted sector the set $\{x^i | i = p+1, \ldots, d-5\}$ represents the non-orbifold perpendicular directions to the brane worldvolume. In the solution (2.3) for the non-compact coordinates, like the time direction, the quantity $L^\lambda$ identically vanishes, while for the circular directions there are

\[
L^\lambda = N^\lambda R^\lambda , \quad N^\lambda \in \mathbb{Z} , \\
p^\lambda = \frac{M^\lambda}{R^\lambda} , \quad M^\lambda \in \mathbb{Z} ,
\]

(2.4)

where $N^\lambda$ is the winding number and $M^\lambda$ is momentum number of a closed string state, and $R^\lambda$ specifies the radius of compactification for the compact direction $x^\lambda$. Now look at the orbifold directions. The orbifold $\mathbb{C}^2/\mathbb{Z}_2$ is non-compact, thus, its fixed points
define a \((d-4)\)-dimensional hyperplane at \(x^a = 0\). As the Dp-brane has to sit on this hyperplane, and as the closed string is emitted (absorbed) at the brane position, the orbifold coordinates of the closed string possess the solution

\[
X^a(\sigma, \tau) = \frac{i}{2} \sqrt{2\alpha'} \sum_{r \in \mathbb{Z} + \frac{1}{2}} \frac{1}{r} \left( \alpha_r e^{-2\pi i (\tau - \sigma)} + \tilde{\alpha}_r e^{-2\pi i (\tau + \sigma)} \right).
\]  

(2.5)

In the twisted sector the solutions (2.3) and (2.5) decompose the second equation of (2.2) as in the following

\[
(X^i - y^j)_{\tau=0} |B\rangle^T = 0,
\]

\[
(X^a)_{\tau=0} |B\rangle^T = 0.
\]

(2.6)

By introducing Eqs. (2.3) and (2.5) into the boundary state equations we acquire the following equations

\[
\left[ \left( K_{\alpha\beta} - F_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta} \right) \alpha^\beta_m + \left( K_{\alpha\beta} + F_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta} \right) \tilde{\alpha}^\beta_{-m} \right] |B_{osc}\rangle^T U = 0,
\]

\[
(2\alpha' K_{\alpha\beta} p^\beta + 2F_{\alpha\beta} L^\beta + U_{\alpha\beta} x^\beta) |B\rangle^{(0)T} U = 0,
\]

\[
U_{\alpha\beta} L^\beta |B\rangle^{(0)T} U = 0,
\]

(2.7)

for both twisted and untwisted sectors, and

\[
(\alpha^i_m - \tilde{\alpha}^i_{-m}) |B_{osc}\rangle^T = 0,
\]

\[
(\alpha^a_r - \tilde{\alpha}^a_{-r}) |B_{osc}\rangle^T = 0,
\]

\[
(x^i - y^j) |B\rangle^{(0)T} = 0,
\]

\[
L^i |B\rangle^{(0)T} = 0,
\]

(2.8)

for the twisted sector, and

\[
(\alpha^I_m - \tilde{\alpha}^I_{-m}) |B_{osc}\rangle^U = 0,
\]

\[
(x^I - y^J) |B\rangle^{(0)U} = 0,
\]

\[
L^I |B\rangle^{(0)U} = 0,
\]

(2.9)

for the untwisted sector, where we applied \(|B_x\rangle = |B\rangle^{(0)} \otimes |B_{osc}\rangle\). Since the fractional brane has stuck at the fixed points of the orbifold the state \(|B\rangle^{(0)}\) does not obtain any contribution from the orbifold directions.
According to the third equation of Eqs. (2.7) the tachyon field plays a crucial role for winding of closed strings around the compact directions of the brane. This equation implies that if the tachyon matrix is invertible we obviously receive the zero winding numbers \( \{ N^{\bar{\alpha}} = 0 | \bar{\alpha} = 1, 2, \ldots, p \} \), and hence closed strings cannot wrap around the circular directions of the brane. If the tachyon matrix possesses null determinant the vector \( \{ L^{\alpha} | \bar{\alpha} = 1, 2, \ldots, p \} \) can be nonzero, and therefore such wrapping of closed strings are allowable. If the perpendicular direction \( x^i \) (or \( x^I \)) is non-compact the last equation of Eqs. (2.8) (or (2.9)) becomes trivial, i.e. \( L^i \) (or \( L^I \)) identically vanishes, and if \( x^i \) (or \( x^I \)) is compact we observe that closed strings cannot wrap around it, that is \( N^i = 0 \) (or \( N^I = 0 \)).

The second equation of Eqs. (2.7) eventuates to the following valuable relation between the eigenvalues

\[
p^{\alpha} = -\frac{1}{2\alpha'} \left[ \left( K^{-1} U \right)^{\alpha}_{\beta} x^{\beta} + 2 \left( K^{-1} F \right)^{\alpha}_{\beta} \ell^{\beta} \right],
\]

(2.10)

where \( \ell^{\beta} \) is eigenvalue of the operator \( L^{\beta} \). We observe that any closed string state (wrapped or unwrapped) has a spacetime momentum along the worldvolume of the brane. This momentum includes two parts: continuous and discrete. The former is created by the tachyon while the latter originates from the Maxwell field and compactification. As we see this momentum is somewhat under the influence of the rotation and linear motion of the brane. This nonzero momentum extremely is unlike the conventional case in which the closed strings are radiated perpendicular to the brane worldvolume, for the conventional case e.g. see Refs. [7, 33, 34]. Thus, a peculiar potential, which is inspired by the background fields, the brane dynamics and compactification, acts on the center-of-mass positions of the emitted closed strings. If the brane directions are non-compact and or they are compact but the tachyon matrix is invertible Eq. (2.10) reduces to

\[
p^{\alpha} = -\frac{1}{2\alpha'} \left( K^{-1} U \right)^{\alpha}_{\beta} x^{\beta}.
\]

(2.11)

By the quantum mechanical technics, specially by using the commutation relations between \( x^{\alpha} \) and \( p^{\beta} \), and between \( x'^{\alpha} \) and \( L^{\beta} / \alpha' \), where \( x'^{\alpha} = x^{\alpha} - x^{\bar{\alpha}} \) and \( L^{\alpha} = \alpha' (p^{\alpha}_L - p^{\alpha}_R) \),

\[\text{...}\]

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the zero-mode part of the boundary state in the twisted sector finds the form

$$\left| B^{(0)} \right> = \frac{T_p}{2\sqrt{\det(U/2)}} \int_{-\infty}^{\infty} \exp \left[ i\alpha' \sum_{\alpha \neq \beta} (U^{-1}K + K^TU^{-1})_{\alpha\beta} p^\alpha p^\beta \right]$$

$$+ \frac{i}{2} \alpha' (U^{-1}K + K^TU^{-1})_{\alpha\alpha} (p^\alpha)^2 + 2i (U^{-1}F)_{\alpha\beta} \ell^\alpha p^\beta$$

$$\times \prod_{i=p+1}^{d-5} \left[ \delta(x^i - y^i) |p^i_L = 0\rangle \right] \prod_{\alpha=0}^{p} (|p^\alpha\rangle dp^\alpha) \cdot (2.12)$$

The disk partition function induces the normalization factor $1/\sqrt{\det(U/2)}$, \[35, 36\]. In the same sector, by using the coherent state method \[37\], we obtain the following boundary state for the closed string oscillators

$$\left| B_{osc} \right> = \prod_{n=1}^{\infty} [\det M_{(n)}]^{-1} \exp \left[ -\sum_{m=1}^{\infty} \left( \frac{1}{m} \alpha^\lambda \lambda \alpha^\lambda \lambda^\prime \tilde{\alpha}^\lambda \lambda^\prime \right) \right]$$

$$\times \exp \left[ -\sum_{r=1/2}^{\infty} \left( \frac{1}{r} \alpha^\lambda \lambda \alpha^\lambda \lambda^\prime \tilde{\alpha}^\lambda \lambda^\prime \right) \right] |0\rangle_{\alpha} |0\rangle_{\tilde{\alpha}} \cdot (2.13)$$

where $\lambda, \lambda' \in \{\alpha, i\}$, and the matrix $S_{(m)}$ is defined by

$$S_{(m)}^{\lambda\lambda'} = \left( Q_{(m)\alpha\beta} \equiv (M_{(m)}^{-1}N_{(m)})_{\alpha\beta}, -\delta_{ij} \right) ,$$

$$M_{(m)\alpha\beta} = K_{\alpha\beta} - F_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta} ,$$

$$N_{(m)\alpha\beta} = K_{\alpha\beta} + F_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta} \cdot (2.14)$$

Expansion of the exponential parts of Eq. (2.13) clarifies that the brane couples to the whole closed string spectrum in the twisted sector. The disk partition function gives the normalizing factor $\prod_{n=1}^{\infty} [\det M_{(n)}]^{-1}$ \[1, 16, 36\]. More precisely, the quadratic forms of the tachyon profile and rotating-moving term, accompanied by the gauge $A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta$, give a quadratic form to the boundary part of the action (2.1). Thus, there exists a Gaussian path integral, which induces the prefactors of Eqs. (2.12) and (2.13), and also the prefactors of the next Eqs. (2.15) and (2.16).

In a similar fashion, the untwisted sector $U$ has the following boundary states for the
zero-mode part and the oscillating part

\[ |B\rangle_{(0)\mu} = \frac{T_\mu}{2\sqrt{\det(U/2)}} \int_{-\infty}^{\infty} \exp \left[ i\alpha' \sum_{\alpha \neq \beta} (U^{-1}\mathcal{K} + \mathcal{K}^T U^{-1})_{\alpha\beta} p^\alpha p^\beta \right. \]

\[ + \left. \frac{i}{2} \alpha' \left( U^{-1}\mathcal{K} + \mathcal{K}^T U^{-1} \right)_{\alpha\alpha} (p^\alpha)^2 + 2i (U^{-1}\mathcal{F})_{\alpha\beta} \ell^\alpha p^\beta \right] \]

\[ \times \prod_{I=p+1}^{d-1} \left[ \delta (x' - y') \left| p_L = p_R = 0 \right\rangle \prod_{\alpha=0}^{p} (|p^\alpha\rangle dp^\alpha) \right) , \quad (2.15) \]

\[ |B_{osc}\rangle_{\mu} = \prod_{n=1}^{\infty} [\det M(n)]^{-1} \exp \left[ -\sum_{m=1}^{\infty} \left( \frac{1}{m} \alpha_{-m} S(m)_{\lambda\lambda'} \tilde{\alpha}_{-m}' \right) \right] |0\rangle_{\alpha} |0\rangle_{\tilde{\alpha}} , \quad (2.16) \]

where \( \lambda, \lambda' \in \{\alpha, I\} \), and \( S(m)_{\lambda\lambda'} = (Q(m)_{\alpha\beta}, -\delta_{IJ}) \).

For obtaining Eq. (2.15) we have used methods of quantum mechanics, specially the commutation relations between the position coordinates and their corresponding momenta, and for Eq. (2.16) we have applied the coherent state method. As expected, by setting all linear and angular velocities to zero the above boundary states reduce to the simple configurations of the D-branes, e.g. see Ref. [12]. Besides, by decompactifying the compact directions and quenching the background fields and velocities we receive the simpler boundary states, e.g. see Refs. [6, 21, 34, 38].

Look at the first equation of Eqs. (2.7). The coherent state method on the oscillators \( \{\alpha_m, \tilde{\alpha}_m | m = 1, 2, 3, \ldots \} \) introduces the matrix \( Q(m)_{\alpha\beta} \) in Eqs. (2.13) and (2.16), while this method on the set \( \{\tilde{\alpha}_m, \alpha_m | m = 1, 2, 3, \ldots \} \) recasts these boundary states with the matrix \( \left( [Q_{(-m)}]^{-1} \right)^T_{\alpha\beta} \). Equality of these matrices leads to the following conditions

\[ \eta U - U\eta + 4(\omega U + U\omega) = 0 , \]

\[ \eta F - F\eta + 4(\omega F + F\omega) = 0 . \quad (2.17) \]

These equations are independent of the mode numbers.

Finally we shall use the following known boundary state, corresponding to the conformal ghost fields [16, 34],

\[ |B_{gh}\rangle = \exp \left[ \sum_{m=1}^{\infty} (c_{-m} \tilde{b}_{-m} - b_{-m} \tilde{c}_{-m}) \right] \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle |\tilde{q} = 1\rangle . \quad (2.18) \]
This state is independent of the orbifold projection, toroidal compactification, rotation and linear motion of the brane and the background fields. The total boundary state in the bosonic string theory, for each sector, is given by

\[ |B⟩_{\text{Total}}^{U} = |B_{\text{osc}}⟩^{U} \otimes |B⟩^{(0)}^{U} \otimes |B_{\text{gh}}⟩. \]

Compare the boundary states (2.12), (2.13), (2.15) and (2.16) with the boundary states of a bare brane, i.e. a stationary brane without any background and internal fields. This induces to define the following effective tension for the dressed brane

\[ T_p = \frac{T_p}{\sqrt{\text{det}(U/2)}} \prod_{n=1}^{\infty} \left| \left[ \text{det} M(n) \right]^{-1} \right|. \]

\[ (2.20) \]

### 3 Interaction between two Dp-branes

The interactions of the branes have appeared in many physical phenomena and in the main problems of physics. For example, in the brane-world scenario these interactions have been introduced as the origin of the inflation \[35, 39\]. Beside, interaction and collision of two D-branes create a Big-Bang \[40\]. In addition, in the early universe these interactions have been considered for describing the radiation-dominated era. Also there are Dp-branes that overlap with our D3-brane, hence, interact with it. Thus, these interactions induce the added gravity within our world \[41, 42\]. Furthermore, the branes interactions clarify some corners of the gauge/gravity correspondence \[24\]. Finally, the gravitational interaction between the branes describes creation of the dark matter \[43\]. There are many other satisfactory applications of such interactions, e.g. see the Refs. \[36, 44, 45, 46\].

The interaction between two D-branes can be described by the 1-loop graph of an open string worldsheet \[47-49\], or tree-level diagram of a closed string worldsheet \[1-21\]. In the second approach each brane couples to all closed string states through its corresponding boundary state. This is due to the fact that all properties of a D-brane are encoded into a boundary state. Thus, in the closed string channel closed string is radiated from one brane, then propagates toward the other brane, and finally is absorbed by the second brane. Therefore, for acquiring the interaction amplitude of two Dp-branes we should calculate the overlap of their corresponding boundary states via the closed string
propagator, i.e.,

\[ A = \langle B_1|D|B_2 \rangle, \quad (3.1) \]

where the total boundary states of the branes should be used. “D” is the closed string propagator, and is constructed from the closed string Hamiltonian. For the twisted sector the Hamiltonian is

\[
H^T = H_{\text{ghost}} + \alpha' p^\lambda p_\lambda + 2 \sum_{n=1}^{\infty} (\alpha^\lambda_n - \alpha^\lambda_{-n} + \tilde{\alpha}^\lambda_n - \tilde{\alpha}^\lambda_{-n}) + \sum_{r=1/2}^{\infty} (\alpha_n^{0} + \alpha_{ra}^{0} + \tilde{\alpha}_{-r}^{0} + \tilde{\alpha}^{0}_{-r}) - \frac{d - 6}{6}, \quad \lambda \in \{\alpha, i\}. \quad (3.2)
\]

For the untwisted sector there is

\[
H^U = H_{\text{ghost}} + \alpha' p^\lambda p_\lambda + 2 \sum_{n=1}^{\infty} (\alpha^\lambda_n - \alpha^\lambda_{-n} + \tilde{\alpha}^\lambda_n - \tilde{\alpha}^\lambda_{-n}) - d/6, \quad \lambda \in \{\alpha, I\}. \quad (3.3)
\]

The difference between the ground state energies of the two sectors is a consequence of the orbifold projection on the twisted sector. These ground state energies impose some significant effects in the branes interaction.

### 3.1 Interaction amplitude: arbitrary distance of the branes

According to the orbifold projection the total interaction amplitude has two parts: one part from the untwisted sector and the other part from the twisted sector

\[
A^{\text{Total}} = A^T + A^U. \quad (3.4)
\]

After a heavy calculation we receive the following amplitude for the twisted sector

\[
A^T = \frac{T^2 \alpha' V_{p+1}}{4(2\pi)^{d-p-5}} \prod_{n=1}^{\infty} \frac{[\det(M^i_{(n)})M(n)2]^{-1}}{\sqrt{\det(U_{1/2}) \det(U_{2/2})}} \int_0^\infty dt \left[ e^{(d-8)t/6} \left( \sqrt{\frac{\pi}{\alpha't}} \right)^{d_{in}} \right.
\]

\[
\times \exp \left( -\frac{1}{4\alpha't} \sum_{n} (y_1^n - y_2^n)^2 \right) \prod_{i} \Theta_3 \left( \frac{y_1^ic - y_2^ic}{2\pi R_{ic}} \right) \left( \frac{i\alpha't}{\pi R_{ic}^2} \right) \]

\[
\times \left[ \det Z(t) \right]^{-1/2} \sum_{\{N_{ac}\}} \exp (2W^\dagger Z(t)^{-1} W) \]

\[
\times \prod_{n=1}^{\infty} \left[ \det[1 - Q_{(n)1}^\dagger Q_{(n)2} e^{-4nt}]^{-1} \left( 1 - e^{-4nt} \right)^{p-d+7} \left( 1 - e^{-2(2n-1)t} \right)^{-4} \right]. \quad (3.5)
\]
where $V_{p+1}$ is the common worldvolume of the branes, and

$$W_\alpha = (U_1^{-1} \mathcal{F}_1)_{\beta, \alpha} \ell^\beta + (U_2^{-1} \mathcal{F}_2)_{\beta, \alpha} \ell^\beta;$$

$$Z(t)_{\alpha \beta} = \begin{cases} 
2t\alpha' \delta_{\alpha \beta} + \alpha'[(U_1^{-1} \mathcal{K}_1 + \mathcal{K}_1^T U_1^{-1}) - (U_2^{-1} \mathcal{K}_2 + \mathcal{K}_2^T U_2^{-1})]_{\alpha \beta}, & \text{if } \alpha = \beta \\
2t\alpha'[(U_1^{-1} \mathcal{K}_1 + \mathcal{K}_1^T U_1^{-1}) - (U_2^{-1} \mathcal{K}_2 + \mathcal{K}_2^T U_2^{-1})]_{\alpha \beta}, & \text{if } \alpha \neq \beta.
\end{cases}$$

Besides, we decomposed each set of the directions into the compact and non-compact subsets, i.e.

$$\{i = p + 1, \ldots, d - 5\} = \{i_n\} \cup \{i_c\}, \quad \{\alpha = 0, \ldots, p\} = \{\alpha_n\} \cup \{\alpha_c\},$$

where the index “c” (“n”) represents the word “compact” (“non-compact”). Thus, $d_{i_n}$ is the dimension of the directions $\{x^{i_n}\}$. The factor $\prod_{n=1}^{\infty}(1 - e^{-4nt})^{p-d+7}$ originates from the oscillators of the non-orbifoldy perpendicular directions and the conformal ghosts, and the last factor of the last line is contribution of the orbifold directions.

The interaction amplitude in the untwisted sector is given by

$$\mathcal{A}^t = \frac{T^2}{4(2\pi)^{d-p-1}} \prod_{n=1}^{\infty} \left( \frac{\det(\mathcal{M}_{(n)}^1 \mathcal{M}_{(n)}^2)}{\det(U_1/2) \det(U_2/2)} \right) \int_0^\infty dt \left[ e^{(d-2)t/6} \left( \sqrt{\frac{\pi}{\alpha' t}} \right)^{d_{I_n}} \right. \times \exp \left( -\frac{1}{4\alpha' t} \sum_{I_n} (y_1^{I_n} - y_2^{I_n})^2 \right) \prod_{I_c} \Theta_3 \left( \frac{y_1^{I_c} - y_2^{I_c}}{2\pi R_{I_c}} \right) \right]^{1/2} \left[ \prod_{\{N_{I_n}\}} \right] \exp \left( 2W^+ Z(t)^{-1} W \right),$$

$$\times \prod_{n=1}^{\infty} \left( \det[1 - Q_{(n)}^1 Q_{(n)}^2 e^{-4nt}]^{-1} (1 - e^{-4nt})^{p-d+3} \right),$$

where $\{I = p + 1, \ldots, d - 1\} = \{I_n\} \cup \{I_c\}$, and $d_{I_n} = \dim \{x^{I_n}\}$. The factor $\prod_{n=1}^{\infty}(1 - e^{-4nt})^{p-d+3}$ originates from the oscillators of the perpendicular directions and the conformal ghosts.

For computing the amplitudes (3.5) and (3.7) we receive the factor $\prod_{\alpha=0}^{p} \langle p_1^\alpha | p_2^\alpha \rangle$. This implies that a nonzero interaction requires the equation

$$p_1^\alpha - p_2^\alpha = 0, \quad \alpha = 0, 1, \ldots, p.$$

According to Eq. (2.10) this equation eventuates to the following conditions

$$\det(K_1^{-1} U_1 - K_2^{-1} U_2) = 0,$$

$$\det(K_1^{-1} F_1 - K_2^{-1} F_2) = 0.$$
The conditions (2.17) and (3.8) reduce \( n + (p + 1)(3p + 2)/2 \) parameters of the theory to \( n - 2 + p(p + 1)/2 \), where “\( n \)” is the dimension of the asymmetric torus \( T^n \).

The second lines of the amplitudes (3.5) and (3.7) imply that the interaction is exponentially damped by the square distance of the branes. In the last lines of these equations the determinants come from the oscillators of the string coordinates \( \{X^\alpha\} \). The overall factors in front of the integrals, which include the parameters of the system, partially specify the strength of the interaction.

The variety of the parameters in the setup, i.e., the matrix elements of: the Kalb-Ramond tensor and field strengths and tachyon matrices, the linear and angular speeds of the branes, the dimensions of the spacetime and the branes, the closed string winding and momentum numbers, the coordinates of the branes location, and the radii of the circular directions, specifies a general interaction amplitude \( \mathcal{A}^{\text{Total}} = \mathcal{A}^T + \mathcal{A}^U \).

The effects of the toroidal compactification have been gathered in \( i_n, d_i, I_n, d_I \), the Jacobi theta function \( \Theta_3 \) and the worldvolume vector \( W_\alpha \). Thus, for obtaining the interaction amplitudes in the non-compact spacetime it is sufficient to exert the following replacements: \( i_n \to i, d_i \to d = d - p - 5, \Theta_3 \to 1 \) and \( W \to 0 \) in Eq. (3.5); and \( I_n \to I, d_I \to d = d - p - 1, \Theta_3 \to 1 \) and \( W \to 0 \) in Eq. (3.7).

### 3.2 Interaction amplitude: large distance of the branes

Behavior of the total interaction amplitude for large distances of the branes is very important. This prominently defines the long-range force of the theory, which is determined by

\[
\mathcal{A}_{\text{long-range}}^{\text{Total}} = \mathcal{A}_{\text{long-range}}^T + \mathcal{A}_{\text{long-range}}^U. \tag{3.9}
\]

In fact, this picks out the contributions of the closed string tachyon and massless states to the interaction. For this purpose, since the states of the graviton, Kalb-Ramond tensor and dilaton have zero winding and zero momentum numbers we shall impose \( \ell_\beta = 0 \) for every \( \beta_c \). Besides, in the critical string theory, i.e. for the dimension \( d = 26 \), we impose the limit \( t \to \infty \) on the oscillating parts of the amplitudes (3.5) and (3.7). Since the nature of an emitted (absorbed) closed string is independent of the locations of the
interacting branes the position factors in Eqs. (3.5) and (3.7) do not change. In this limit the contribution of all massive states, except the tachyon state, vanish.

For the twisted sector the limit is

$$
\lim_{t \to \infty} e^{3t} \prod_{n=1}^{\infty} \left( \det[1 - Q_{(n)}^1 Q_{(n)}^2 e^{-4nt}]^{-1} (1 - e^{-4nt})^{p-d+7} (1 - e^{-2(2n-1)t})^{-4} \right) \\
\to e^{3t} + \left[ 21 - p + \text{Tr} \left( Q_{(n=1)}^1 Q_{(n=1)}^2 \right) \right] e^{-t}.
$$

Thus, the interaction amplitude of the distant branes, in the twisted sector, has the following form

$$
\mathcal{A}_{\text{long-range}}^T = \frac{T_p^2 \alpha' V_{p+1} \prod_{n=1}^{\infty} \left[ \det (M_{(n)}^1 M_{(n)}^2) \right]^{-1}}{4(2\pi)^{21-p} \sqrt{\det(U_1/2) \det(U_2/2)}} \int_0^\infty dt \left\{ \left( \sqrt{\frac{\pi}{\alpha' t}} \right)^{d_{in}} \right\} \\
\times \left[ \det Z(t) \right]^{-1/2} \exp \left( -\frac{1}{4\alpha' t} \sum_i \left( y_{1i} - y_{2i} \right)^2 \right) \prod_{ic} \Theta_3 \left( \frac{y_{1ic} - y_{2ic}}{2\pi R_{ic}} \right) \frac{i\alpha' t}{\pi R_{ic}^2} \\
\times \lim_{t \to \infty} e^{3t} + \left[ 21 - p + \text{Tr} \left( Q_{(n=1)}^1 Q_{(n=1)}^2 \right) \right] e^{-t}.
$$

According to the negative mass squared of the tachyon, the divergent part in the last line exhibits exchange of the tachyonic state. The last bracket in Eq. (3.11) clarifies that in the twisted sector the $\mathbb{Z}_2$ projection extremely damps the long-range force. This is due to the fact that this projection modified the zero-point energy of the Hamiltonian of this sector.

In fact, the twisted spectrum of closed string does not have any massless state, but contains the tachyonic state with a modified imaginary mass. Therefore, the vanishing long-range force in this sector is an expected result. However, we calculated this force to find the damping form of it and the divergence form for the tachyon exchange.

We should also calculate the long-time behavior of the interaction amplitude in the untwisted sector. By considering the following limit in the 26-dimensional spacetime

$$
\lim_{t \to \infty} e^{4t} \prod_{n=1}^{\infty} \left( \det[1 - Q_{(n)}^1 Q_{(n)}^2 e^{-4nt}]^{-1} (1 - e^{-4nt})^{p-23} \right) \\
\to e^{4t} + 23 - p + \text{Tr} \left( Q_{(n=1)}^1 Q_{(n=1)}^2 \right),
$$

(3.12)
the long-range force of the untwisted sector takes the form

\[ A_{\text{long-range}}^U = \frac{T_2^2 \alpha' V_{p+1}}{4(2\pi)^{25-p}} \frac{\prod_{n=1}^{\infty} [\det(M_{(n)1}^{\dagger} M_{(n)2})]^{-1}}{\sqrt{\det(U_{1/2}) \det(U_{2/2})}} \int_0^\infty \frac{dt}{\alpha'} \left\{ \left( \frac{\sqrt{\pi}}{\alpha'} t \right)^{d_{1n}} \right. \]

\[ \times \left[ \det Z(t) \right]^{-1} \exp \left( -\frac{1}{4\alpha' t} \sum_i \left( y_{1n}^i - y_{2n}^i \right)^2 \right) \prod_{I_c} \Theta_3 \left( \frac{y_{1c}^I - y_{2c}^I}{2\pi R_{I_c}} \left| \frac{i\alpha' t}{\pi R_{I_c}^2} \right| \right) \]

\[ \times \left( \lim_{t \to \infty} e^{4t} + 23 - p + \text{Tr} \left( Q_{(n=1)1}^\dagger Q_{(n=1)2} \right) \right) \].

(3.13)

Again the divergent part represents the exchange of the tachyon state, and the remainder indicates the long-range force.

The amplitudes (3.11) and (3.13) demonstrate that the orbifold projection does not deform the total long-range force. In addition, this projection imposed the divergence \( e^{3t} \) as the contribution of the tachyon exchange in the twisted sector. Besides, these amplitudes reveal that the compactification of the branes directions does not have any role in the long-range force.

According to Eqs. (2.14) the matrices \( Q_{(n)1} \) and \( Q_{(n)2} \) contain \( 2(p+1)(2p+1) \) parameters

\[ \{ \omega_{(l)\alpha\beta}, F_{(l)\alpha\beta}, B_{(l)\alpha\beta}, U_{(l)\alpha\beta} | \alpha, \beta = 0, 1, \ldots, p \}, \]

with \( l = 1, 2 \) for the first and second interacting branes. By adjusting these parameters we can receive

\[ 23 - p + \text{Tr} \left( Q_{(n=1)1}^\dagger Q_{(n=1)2} \right) = 0, \]

(3.14)

and hence, we acquire a vanishing total long-range force. In fact, for the two D0-branes there are only two parameters \( U_{(1)00} \) and \( U_{(2)00} \), thus this equation is not satisfied. However, for the systems with \( p \geq 1 \) there are enough parameters for satisfying this equation.

For example, consider two parallel D1-branes. For simplification let \( \omega_{(1)01} = \omega_{(2)01} = 0, \)
therefore Eq. (3.14) is decomposed to the following equations
\[ U_{(1)11}U_{(2)00} - U_{(1)00}U_{(1)11}U_{(2)11} + U_{(1)00}U_{(1)11}U_{(2)00} 
- U_{(1)00}U_{(2)00}U_{(2)11} + 4U_{(1)11} - 4U_{(2)11} + 4U_{(2)00} - 4U_{(1)00} 
-4U_{(1)11}F^2_{(2)01} + 4U_{(2)11}F^2_{(1)01} - 4U_{(2)00}F^2_{(1)01} + 4U_{(1)00}F^2_{(2)01} 
- U_{(1)11}U^2_{(2)01} + U_{(2)11}U^2_{(1)01} - U_{(2)00}U^2_{(1)01} + U_{(1)00}U^2_{(2)01} = 0 , \]
\[ -12U_{(1)11}U_{(2)11} - 48F^2_{(1)01}F^2_{(2)01} - 4U_{(1)01}U_{(2)01} - 16F^2_{(1)01}F_{(2)01} 
-12U^2_{(1)01}F^2_{(2)01} - 12U^2_{(2)01}F^2_{(1)01} - 12U_{(1)00}U_{(2)00} + 10U_{(1)11}U_{(2)00} 
+10U_{(1)00}U_{(2)11} + 3U^2_{(1)01}U_{(2)00}U_{(2)11} + 3U^2_{(2)01}U_{(1)00}U_{(1)11} + 10U^2_{(2)01} 
+10U^2_{(1)01} + 40F^2_{(2)01} + 40F^2_{(1)01} - 10U_{(1)00}U_{(1)11} - 10U_{(2)00}U_{(2)11} 
-3U^2_{(1)01}U^2_{(2)01} + 12F^2_{(1)01}U_{(2)00}U_{(2)11} + 12F^2_{(2)01}U_{(1)00}U_{(1)11} 
-3U_{(1)00}U_{(1)11}U_{(2)00}U_{(2)11} - 48 = 0 . \]

(3.15)

4 Conclusions

We constructed the boundary states, associated with a non-stationary fractional-wrapped Dp-brane, in the presence of the Kalb-Ramond background field, an internal $U(1)$ gauge potential and an internal open string tachyon field in the twisted and untwisted sectors of the orbifold projection.

We observed that the emitted closed strings cannot wrap around the compact directions which are perpendicular to the brane. In addition, wrapping of them around the compact directions of the brane is controlled by the tachyon matrix. Besides, each emitted closed string possesses a momentum along the worldvolume of the brane. This momentum depends on the position of the closed string center-of-mass, its winding numbers, and the parameters of the setup. This noticeable result clarifies that the background fields, accompanied by the toroidal compactification and linear and angular velocities of the brane, induce a marvelous potential on the emitted closed string.

For both twisted and untwisted sectors the interaction amplitudes of two dynamical fractional-wrapped Dp-branes, in the above-mentioned setup, were obtained. The multi-
plicity of the parameters designed a generalized amplitude. The strength of the interaction accurately is adjustable via these parameters to any desirable value.

From the total interaction amplitude the total long-range force was extracted. The long-range force only originates from the untwisted sector. That is, the orbifold directions quenches the contribution of the massless states to this interaction. By a specific adjustment of the parameters we can eliminate the long-range force.

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