A NEW METHOD TO MAP FLARES IN QUASARS

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ABSTRACT

Recently, Chartas et al. detected a rapid X-ray flare in the gravitationally lensed, multiple-image quasar RX J0911.4+0551. Dramatic events, such as rapid X-ray flares, are useful for providing high-precision measurements of the time delays between multiple images. In this paper we argue that there is a new possibility for measuring time delays between multiple images of gravitationally lensed quasars and constraining the locations of putative flares that give rise to the intrinsic rapid variabilities of quasars. The realization of these goals, however, cannot be achieved at present because of the limited accuracy of the current measurements. We predict that timing flares with accuracies of the order of a few seconds will be needed to probe the location of the flares. The method that we are proposing will work with better instruments in near future, such as the X-Ray Evolving Universe Spectroscopy Mission.

Subject headings: accretion, accretion disks — cosmological parameters — galaxies: starburst — gravitational lensing — quasars: general — supernovae: general

1. INTRODUCTION

It is well known that gravitationally lensed quasars with multiple images and their flux variabilities provide us opportunities for measuring $H_0$ (Refsdal 1964). Since the first detection of the lensed quasar Q0957+561 by Walsh, Carswell, & Weymann (1979), surveys in the optical and radio bands have uncovered over 70 gravitationally lensed quasars. For the purpose of measuring $H_0$, monitoring of several of these lensed objects has been performed and has resulted in measurements of time delays in about eight systems with accuracies of about 10%. The average value of $H_0$ based on these time-delay measurements clusters is around 70 km s$^{-1}$ Mpc$^{-1}$, with an uncertainty of $\sim 10\%$. Thus, the estimate of $H_0$ obtained with the gravitational lensing method is consistent with the value obtained by the Hubble Space Telescope Key Project (HSTKP) on the extragalactic distance scale.

Recently, the first-year results of the Wilkinson Microwave Anisotropy Probe (WMAP) (Spergel et al. 2003) have yielded fundamental numbers of universe, including $H_0$, with accuracies better than achieved by various independent techniques. Since the value of $H_0$ has also been measured with good accuracy, one may ask the whether there is a need to continue with measurements of the time delays in lensed systems. We point out that time-delay measurements can provide information in addition to $H_0$. As shown in Figure 1, the observed time delays depend not only $H_0$ but also on the cosmology, astrophysics, and astrometry. Even before the detection of the first gravitational lens system,Refsdal (1966) had noted that the cosmological parameters ($\Omega_m$ and $\Omega_\Lambda$) can be constrained from the time delays between multiple images of a gravitationally lensed source. Recently, the possibility of probing the mass profiles of the lens objects was discussed by several groups (e.g., Oguri et al. 2002), and a technique for constraining the physical properties of the source objects, i.e., quasars, was investigated by Yonehara (1999).

The realization of the above possibilities may depend on the accuracy of the time-delay measurements and the time-scale of flarelike flux variations in quasars. The former will improve with time, although the latter is somewhat unclear. Recently, Chartas et al. (2001, 2002) and Dai et al. (2003) detected with the Chandra and XMM-Newton X-ray observatories rapid X-ray flares with durations of $\sim 1$ ks in the gravitationally lensed quasars RX J0911.4+0551 ($z = 2.8$), PG 1115+080 ($z = 1.72$), and APM 08279+5255 ($z = 3.91$). The presence of such rapid flares is exciting and suggests that such rapid X-ray flares with the Kepler rotation period at the Schwarzschild radius do exist in quasars. We anticipate that in the near future the number of detected quasars that show such rapid flux variations will increase.

In this paper we describe a promising new application of gravitational lensing: we present a method of mapping the locations of flares in quasars employing an interferometric technique. We expect that this goal will become attainable in the near future as high-precision measurements of the time delay between multiple images of gravitationally lensed quasars become available. In § 2 we introduce a method for

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Fig. 1.—Schematic diagram describing the calculation of the time delay between images of a gravitationally lensed system.

2. METHODOLOGY FOR MAPPING THE LOCATION OF FLARES IN QUASARS

In the following section we outline a method that allows one to extract information about the location of flares in quasars from the observed time delay between images of a lensed quasar.

The derivation of \( H_0 \) from measured time delays, in general, assumes that the source is a point source, which is a reasonable assumption for this derivation. However, astrophysical objects including quasar accretion disks have finite sizes, and the time delays due to gravitational lensing will depend on the location of the flares on the surface of the disks. Since flares may occur at various locations on the disk (Kawaguchi et al. 1998, 2000), we expect the observed time delay to vary depending on where the flare originated. This dependence of the time delay with the flare location is the basic idea of this paper and was initially investigated by Yonehara (1999). Even in the case when X-ray flares are originated from shocks within a relativistic jet (e.g., Rees 1978; Spada et al. 2001; Sikora et al. 2001), in principle, the proposed method is also applicable.

2.1. Time Delay and Time-Delay Difference

It is well known that the time delay (\( \tau_{ij} \)) between multiple images \( i \) and \( j \) produced by the gravitational lensing effect is given by the following equation (e.g., Schneider, Ehlers, & Falco 1992):

\[
\tau_{ij} = \frac{1}{2c} \frac{D_{ol} D_{os}}{D_{ls}} (1 + z_l) \times \left\{ \left( \theta_i - \beta \right)^2 - \left( \theta_j - \beta \right)^2 \right\} - 2 \delta \left( \psi(\theta_i) - \psi(\theta_j) \right),
\]

(1)

where \( c \) is the speed of light; \( D_{ol}, D_{os}, \) and \( D_{ls} \) are the angular diameter distances (e.g., Weinberg 1972) between observer and lens, observer and source, and lens and source, respectively; \( z_l \) is the redshift of the lens; \( \theta_i \) and \( \theta_j \) are the angular positions of the \( i \)th image and the source, respectively; and \( \psi(\theta) \) is the lens potential at \( \theta \). This lens potential may include external shear and/or contributions of neighboring galaxies, depending on the nature of the lens. A schematic diagram describing the calculation of the time delay is shown in Figure 1. For our purpose, we also derive the time delay between the same multiple images for different positions of the source, \( \tau'_{ij} \). Similar to equation (1), such a time delay is written as

\[
\tau'_{ij} = \frac{1}{2c} \frac{D_{ol} D_{os}}{D_{ls}} (1 + z_l) \left\{ \left( \theta_i + d\theta_i - \beta - d\beta \right)^2 - \left( \theta_j + d\theta_j - \beta - d\beta \right)^2 \right\} - 2 \delta \left( \psi(\theta_i + d\theta_i) - \psi(\theta_j + d\theta_j) \right),
\]

(2)

where \( d\beta \) is the difference of the source position from \( \beta \) in the equation (1) and \( d\theta_i \) is the corresponding deviation of the image position.

Given the source position and the lens potential in a gravitational lens system, one can obtain the ideal image positions by solving the lens equation. Thus, \( \theta_i \) and \( d\theta_i \) are determined by \( \beta \) and \( d\beta \) through the "lens equation." Subtracting equation (1) (\( \tau_{ij} \)) from equation (2) (\( \tau'_{ij} \)), we can obtain the time-delay difference (Fig. 2) between images.

\[ \delta\tau = \tau'_{ij} - \tau_{ij}, \]

as a function of the difference between the source positions, \( d\beta \).

If the difference in the source positions is small enough compared with the typical lens size, the latter time delay can be expressed in the form of a simple Taylor expansion. After

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If the difference in the source positions is small enough compared with the typical lens size, the latter time delay can be expressed in the form of a simple Taylor expansion. After
some algebra, we find that the time-delay difference ($\delta \tau$) can be expressed as

$$\delta \tau \simeq \frac{(1+z_l)}{c} \frac{D_{dl}D_{ol}}{D_{ls}} \left[ (\theta_i-\theta_j) \frac{d\beta}{r} \right],$$

(3)

where $d\beta$ represents the angular separation of two slightly different source positions. If the physical length that corresponds to this angular separation is $br$, then $d\beta$ can be rewritten as $br/D_{ol}$, and equation (3) can be rewritten in the form

$$\delta \tau \simeq \frac{(1+z_l)}{c} \frac{D_{ol}}{D_{ls}} \left[ (\theta_i-\theta_j) \frac{br}{r} \right].$$

(4)

Equation (4) provides a relation between the location of flares and the difference in time delays produced by these flares. A useful aspect of this equation is that it contains no dependence on the lens model or on $H_0$, and it depends only slightly on $\Omega_m$ and $\Omega_\Lambda$ through the ratio $D_{ol}/D_{ls}$ (or $D_{ol}/D_{ls}$). The value of $D_{ol}/D_{ls}$ is of order unity and has a weak dependence on the redshifts of the lens and the source (or lens systems), but we can calculate $D_{ol}/D_{ls}$ for any given redshift, as shown in Figure 3. This equation consists of two simple observables: the time-delay difference and image positions and the quantity that we intend to explore, the separation between flares. Application of equation (4) provides the locations of flares on the quasar accretion disk from two simple observables without any particular assumptions.

2.2. Gravitational Lens Interferometric Mapping Method

Next we describe in more detail a method for mapping accretion disk flares. A schematic diagram of the GLIMM is presented in Figure 4.

We first rewrite equation (4) in terms of typical values for the image separations and the expected difference in the time delays,

$$|\delta r| \sim 10r_s \left( \frac{M_{BH}}{10^9 M_\odot} \right)^{-1} \left( \frac{\delta \tau}{1 \text{ s}} \right)^{-1} \left( \frac{1+z_l}{2} \right)^{-1} \left( \frac{D_{ls}}{D_{ol}} \right) \times \left( \frac{\theta_i-\theta_j}{\cos \phi} \right) \left( \frac{10^{-\text{AU}}}{\text{AU}} \right),$$

(5)

or

$$|\delta r| \sim 200 \left( \frac{\delta \tau}{1 \text{ s}} \right)^{-1} \left( \frac{1+z_l}{2} \right)^{-1} \left( \frac{D_{ls}}{D_{ol}} \right) \times \left( \frac{\theta_i-\theta_j}{\cos \phi} \right)^{-1} \left( \frac{10^{-\text{AU}}}{\text{AU}} \right),$$

(6)

where $\phi$ is the angle measured from the direction between images $i$ and $j$ and $r_s$ is the Schwarzschild radius of the central black hole with mass $M_{BH}$. A simple application of these equations indicates that a measurement of the difference in the time delays of two flares to an accuracy of $\sim 1$ s results in a value for the separation of these flares that is accurate to about $10r_s$ for a quasar containing a $10^9 M_\odot$ black hole.

When equation (4) or (5) is applied to a gravitationally lensed quasar with more than three images, it provides tighter constraints on the separation of flares than when applied to gravitational lens systems with fewer images. For gravitational lens systems with only two images, the time-delay difference $\delta \tau$ is most sensitive to flares separated along the direction connecting the two images and is not sensitive to flares that are separated orthogonal to the image direction. For a gravitational lens system with more than three images, the time-delay difference $\delta \tau$ is sensitive to flares separated in any direction allowing for two-dimensional mapping of the locations of the flares. A flow diagram illustrating the use of GLIMM is presented in Figure 5.

3. A SPECIFIC EXAMPLE: CASE OF RX J0911.4+0551

Our earlier description of using GLIMM to infer two-dimensional distributions of flares in accretion disks was somewhat qualitative, and therefore, in the following

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4 This figure shows the contour of $D_{ls}/D_{ol}$. 

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subsection, we present numerical simulations to confirm our estimations and test the feasibility of our proposed mapping method. For our purpose we have chosen gravitational lens systems with more than three images, including the quadruple system RX J0911.4+0551, in which Chartas et al. (2001) detected a rapid X-ray flare.

3.1. Lens Model and Time-Delay Contour

We model the potential of the lens in the gravitational lens system RX J0911.4+0551 with a pseudoisothermal elliptic mass distribution (PIEMD) lens model (e.g., Kassiola & Kovner 1993) plus external shear. The observable constraints are the positions of the lensed images and lensing galaxy. Observed flux ratios were not considered in our lens model since these may be affected by many factors, such as differential extinction and microlensing. By fitting our lens model using a $\chi^2$ minimization method (e.g., Kayser et al. 1990; Kochanek 1991), we obtain the best-fit parameters, which are presented in Tables 1 and 2. The reduced $\chi^2$ is less than 1, and the best-fit lens model reproduces the observed positions of the images accurately. The resulting caustic, critical curve, source position, and image positions are depicted in the upper panel of Figure 6. Using these best-fit lens parameters, we can calculate the time-delay difference between images. Contours of the time-delay difference are presented in the lower panel of Figure 6. As expected from equation (4), the contour lines are perpendicular to the direction vector connecting image pairs, which are represented by arrows in the upper panel of Figure 6. It is also apparent from equation (4) that the density of the contour levels of the time-delay differences is proportional to the image separation. To convert from apparent angular distances to actual distances, we have assumed $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.3$, and $\Omega_L = 0.7$. Adopting this cosmology and considering an accretion disk with a size of $1000 \text{ r}_g$ extending around a $10^9 \text{ M}_\odot$ black hole, the expected time-delay difference between images A and D varies from $-200$ to $+200$ s for source positions spreading across the assumed extent of the accretion disk. This result supports our analytic estimate in the previous subsection, since $z_{os} = 0.269$, $z_{os} = 2.8$, and $D_L/D_{os} \sim 0.5$ in the case of RX J0911.4+0551 (see Fig. 3), and $\theta_A - \theta_D \sim 3^\prime$, the expected variation of the time-delay difference between images A and D for the assumed extent of the accretion disk ($\sim10^{17} \text{ cm}$) becomes $\sim\pm200$ s from equation (5). These simulations also indicate that the spatial resolution of mapping flares using GLIMM may reach several $\text{ r}_g$ when observing techniques

![Fig. 4.—Schematic figures of GLIMM compared with a radio interferometric observation](image)

**TABLE 1**

| Position (arcsec) | Image A          | Image B          | Image C          | Image D          | Galaxy          |
|-------------------|------------------|------------------|------------------|------------------|-----------------|
| $x$               | $0.000 \pm 0.004$| $-0.259 \pm 0.007$| $+0.013 \pm 0.008$| $+2.935 \pm 0.002$| $+0.709 \pm 0.026$ |
| $y$               | $0.000 \pm 0.008$| $+0.402 \pm 0.006$| $-0.946 \pm 0.008$| $+0.785 \pm 0.003$| $+0.507 \pm 0.046$ |

Note.—Positions of images and lensing galaxy relative to image A (the brightest image), were taken from Burud et al. 1998; $x$ and $y$ positions are defined a positive toward the north and the west.

5 It will have a radius of $\sim2 \times 10^{17} \text{ cm}$. 
improve to the level of constraining time delays to an accuracy of $\sim 1\text{ s}$.

### 3.2. Simulated Light Curves

To simulate light curves of flares in gravitational lensed quasars, we take the following steps: 

1. We start the simulation by creating a flare with an exponential growth and decay profile. The flare is produced randomly in time and space on the surface of the accretion disk. The intensity of the flare is also chosen to be random. 

2. We numerically calculate the time delay and fluxes of the flare for each image. 

3. Steps 1 and 2 are repeated a number of times to produce a light curve for each image. In Figure 7a we show the simulated location of five flares on the surface of the disk. In Figure 7b we show the contours of the time-delay difference. The simulated light curves of the flares in the four images are shown in Figure 7c. We shift the light curves of images B, C, and D by the median time delay between these images.
three images and image A. We then apply GLIMM to the simulated light curves to determine the locations of the flares. The inferred locations of the flares are overplotted with crosses in Figure 7b.

It is possible to identify flares of common origin in the light curves of images since each flare has a unique shape and the duration of a flare is not affected by the gravitational lens effect. Variations of the flux ratios of flares between images are expected to occur because of the different locations of the flares. For our simulation, the variations in flux ratios were estimated to be less than 1%, and such a tiny change of flux ratios does not complicate the identification of the flares. Therefore, if we take into account flux ratios between images, we will successfully identify the corresponding flares between images.

As shown by observational (e.g., Giveon et al. 1999; Uttley, McHardy, & Papadakis 2002) and theoretical (e.g., Kawaguchi et al. 1998, 2000) studies for flux variabilities of quasar (or active galactic nuclei), flares with small amplitude occur randomly and frequently. Thus, unfortunately, flares crowd each other, and we may not be able to apply GLIMM to such flares. In contrast, the same studies also show that flares with large amplitude do not occur so frequently, and we can separate individual flares. Moreover, the shape, amplitude, and duration of flares with large amplitude are known from previous observations to show stochastic natures. Taking into account such properties, an individual flare should have a unique shape, and it could be quite rare or unrealistic for flares that are almost identical but with different origins to occur. Thus, we can successfully identify the corresponding flares between multiple images, at least in the case of large flares.

In Table 3 we list the time-delay difference of common-origin flares between images. The median time delays between images have been subtracted from the time-delay differences presented in Table 3. These time-delay differences are expressed with respect to the times of the flares that are detected in image A. Measured time-delay differences can be used to constrain the locations of the flares using GLIMM. We apply GLIMM to the two pairs of images in our simulated case of a quadruple-lens system. The nonparallel directions of these image pairs allows us to produce a two-dimensional map of the flare. We apply GLIMM to the remaining image pair to cross-check our results with those obtained from the application of GLIMM to the other two image pairs.

In the case of flares with small amplitude, we cannot say much about the nature of their shapes because of the limited time resolution or effective area of observational instruments and the limited resolution for numerical simulations.

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**Table 2: Lens Model Parameters for RX J0911.4+0551**

| Parameter       | Value   |
|-----------------|---------|
| $\gamma$ (deg)  | 0.3179  |
| $\phi_0$ (deg)  | 102.30  |
| $E_p$ (arcsec)  | 1.1095  |
| $\epsilon$ (arcsec) | 0.0948 |
| $\omega$ (arcsec) | 0 (fix) |
| $\phi_{\text{ext}}$ (deg) | 140.33 |
| $\beta_1$ (arcsec) | 0.4388 |
| $\beta_2$ (arcsec) | 0.0013 |

**Note.**—Best-fit parameters for RX J0911.4+0551. Specifically, we present the external shear, $\gamma$, the PIEMD parameters, $E_p$, $\epsilon$, and $\omega$, and the source positions $\beta_1$ and $\beta_2$. The notation of the PIEMD parameters is similar to that used in Kassiola & Kovner 1993. Position angles of the external shear, $\phi_0$, and of the ellipsoid, $\phi_{\text{ext}}$, are measured from north to east. Because of the absence of a fifth image in this lens system, we have set the core radius, $\omega$, to zero. The reduced $\chi^2$ of this lens model is 0.65.

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**Fig. 6.**—In (a) we show the caustic (inner thin and cuspy curve), critical curve (outer thick and smooth curve), source position (open circle), and image positions (filled circles) resulting from a fit of a lens model to the gravitational lens system RX J0911.4+0551. We model the potential of the lens with a pseudoisothermal elliptic mass distribution (PIEMD) lens model plus shear. The direction vectors connecting image pairs are represented by arrows. These directions correspond to mapping directions of GLIMM. (b) The contours of time-delay differences between images A and B (dashed lines), images A and C (dotted lines), and images A and D (solid lines). Along the direction of arrows shown in the upper panel (or from the thick to the thin lines), values of time-delay difference are $+100$, $+50$, $+0$, $−50$, and $−100$ s. The plus (minus) sign here means that the time delay on this location is longer (shorter) than that on the center. In the case of the difference between image A and B (dashed lines), $+100$ and $−100$ s contours are out of range of this figure.

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6 In the case of flares with small amplitude, we cannot say much about the nature of their shapes because of the limited time resolution or effective area of observational instruments and the limited resolution for numerical simulations.
4. DISCUSSION

In this paper we have presented a new application of gravitational lensing that may be employed in the near future; we have described how measurements of the time delays of accretion disk flares with accuracies of a few seconds may provide two-dimensional mapping of the flare positions. At present, time-delay accuracies of order 1 s seem to be somewhat unrealistic. However, we anticipate that such accuracies will become attainable with future satellites.

4.1. Another Possible Application of GLIMM

Flares may also originate from supernovae in the host galaxies of quasars or starburst galaxies. If the total extent of detected flares is much larger than expected in quasar accretion disks, flare mapping can be realized by coordinating current observational instruments such as world-wide monitoring networks (e.g., GLITP)\(^7\) in the proper way. Thus, even with the currently available time resolution, GLIMM will be able to constrain the location of supernovae in the host galaxy of quasars or starburst galaxies.

Currently, there is considerable effort being devoted to the investigation of a possible physical connection between quasars and starburst activity (e.g., Boyle & Terlevich 1998; Ohsuga et al. 1999). We expect that the host galaxies of several lensed quasars may exhibit starburst activity and that supernovae explosions may occur frequently in these galaxies. The total extent of a starburst region is expected to be \(\geq 100\) pc. Using equations (4) or (6), the expected time-delay difference between different supernovae is predicted to be of the order of days because of the large spatial separation between supernovae in the host galaxy. Recently, Goicoechea (2002) reported the presence of multiple time delays in the double quasar Q0957+561 and attributed this to the large extent of the variable emission region. They suggested that this variability is partly caused by some explosive phenomena of stellar objects in the host galaxy. Other proposed mechanisms, based on the statistical properties of the flares (e.g., Kawaguchi et al. 1998) of quasar variability, are instabilities of the accretion disk. If it is shown that quasar variability is partly produced by supernova-type explosions, we can apply GLIMM in these cases to map their spatial distribution and thus map the starburst region and/or locations of supernova explosions in distant galaxies. Such information may provide insight to the star

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\(^7\) See http://www.iac.es/proyect/gravs_lens/GLITP/index.html.
formation history or evolution process of (starburst) galaxies.

We also note that if the conclusions of Goicoechea (2002) are correct, we can plan a detailed spectroscopic observation of a supernova by triggering on an event detected in the leading image and waiting by the known time delay to make a detailed spectroscopic observation of the delayed event in another lensed image. This will enable us to obtain a spectrum of the very early stage of a supernova. We conclude that GLIMM can be a powerful tool not only for understanding accretion disk physics, but also for stellar and/or supernova physics.

4.2. Practical Difficulties

Here we discuss practical difficulties that may arise when applying GLIMM to real data. To achieve our goal of mapping flares in quasar accretion disks, satellites with collecting areas larger than those of present X-ray telescopes will be required. As indicated in § 2, mapping of accretion disk flares via GLIMM requires measuring time delays of flares to accuracies of a few seconds. Recently, a rapid X-ray flare with a duration of about ~2000 s was detected in a Chandra observation of the gravitationally lensed quasar RX J0991.4+0551 (Chartas et al. 2001). X-ray flares have also been detected with Chandra in several other gravitational lens systems, such as PG 1115+080 and APM 08279+0552 (Dai et al. 2003; Chartas et al. 2002). If these flares are intrinsic, i.e., originate from the quasar accretion disk, GLIMM can, in principle, be applied to map the spatial structure of the quasar accretion disk. For example, the flare that Chartas et al. (2001) detected in RX J0991.4+0551 shows 30 counts in 2 ks, and the relatively low effective area of present X-ray telescopes precludes time-delay measurements with the specific accuracies needed to successfully apply GLIMM. However, future X-ray missions with large effective areas such as the X-Ray Evolving Universe Spectroscopy Mission (XEUS) may be capable of measuring the arrival times of flares to accuracies of a few seconds. In the case of XEUS, effective areas will be increased by 3 orders of magnitude and expected photon counts will be 30 × 10^3 = 3 × 10^4 in 2 ks or 15 counts s^{-1} for the flare that Chartas et al. (2001) detected. Assuming a low background, a signal-to-noise ratio within a 1 s bin becomes ~4, and this value is sufficient for a 3 σ detection.

One practical difficulty of applying GLIMM is the identification of flares of common origin. If the actual flare is isolated, such an identification will be simple because the durations of flares are not affected by the gravitational lens effect and flux ratios are almost identical over the quasar accretion disk. On the other hand, in a worst case scenario, a flare could be superposed on another flare, and the identification of a flare may not be an easy task in this situation. If the typical time separations between flares are comparable or smaller than the typical timescales of flares, a superposition of flares may occur. In such a case, the fading and/or brightening part of a flare shape will be strongly distorted, and its original shape will be altered. However, the identification of the peaks of flares may not be so difficult to determine. After we determine the peaks of superposed flares, we can subtract individual shapes of flares from observed light curves by extrapolating the light curves from the peak, assuming exponential growth and decay of the light curves. If there is a particularly active region, unfortunately, the situation will become worse. The reason is that such regions could possibly produce multiple flares of very similar types, and identification of the corresponding flares can be difficult. To overcome this problem, some statistical arguments will be required, but there is no guarantee that scientific results can be extracted clearly from such multiple flares.

Another practical difficulty in using GLIMM is that other confusing phenomena may be superposed on the light curves of the quasar, e.g., quasar microlensing and additional gravitational lens effects due to the small-scale structures in the lens galaxy. Flux variability induced by quasar microlensing is distinguishable from intrinsic variability of quasars because quasar microlensing events do not lead to time delayed variations. Furthermore, typical timescales of quasar microlensing events are long (e.g., Wambsganss, Paczynski, & Schneider 1990) and can be effectively removed prior to applying GLIMM. Additional gravitational lens effects due to small-scale structures in the lens galaxy may change the time-delay difference. However, even if cold dark matter (CDM) substructures are present, such events will lead to changes of the time delay of order of a few percent (X. Yonehara, H. Susa, & M. Umemura, in preparation 2003). Moreover, even if CDM substructure is present along the line of sight toward an image, we can calibrate the remaining image pairs in multiple-image quasars with four or more images. This calibration may allow us to probe the existence of small-scale structures in the lens galaxy.

Finally, we briefly mention gravitationally lensed quasars with less than three images. In § 2 we showed that applying GLIMM to gravitational lens systems with three or more images may provide two-dimensional mapping of accretion disk flares. We note that GLIMM can also be applied to double-image quasars, resulting in one-dimensional mapping of the spatial distribution of flares on the accretion disk.

Current X-ray telescopes do not have the sensitivity to allow implementation of GLIMM, even in the case of the most luminous gravitationally lensed quasars. But this technique will work with better instruments developed in near future. Until now, the lensing method has been used primarily to obtain a direct value of $H_0$. We hope that in the near future the lensing method will allow us to map the spatial distribution of flares on the accretion disks of distant quasars.

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REFERENCES

Boyle, B. J., & Terlevich, R. J. 1998, MNRAS, 293, L49
Burud, I., et al. 1998, ApJ, 501, L5
Chartas, G., et al. 2001, ApJ, 558, 119
———. 2002, ApJ, 579, 169
Dai, X., Chartas, G., Agol, E., Bautz, M. W., & Garmire, G. P. 2003, ApJ, 589, 100
Giveon, U., et al. 1999, MNRAS, 306, 637
Goicoechea, L. J. 2002, MNRAS, 334, 905
Kassiola, A., & Kovner, I. 1993, ApJ, 417, 450
Kawaguchi, T., et al. 1998, ApJ, 504, 671
———. 2000, PASJ, 52, L1
Kayser, R., et al. 1990, ApJ, 364, 15
Kochanek, C. S. 1991, ApJ, 373, 354
Oguri, M., et al. 2002, ApJ, 568, 488

Ohsuga, K., et al. 1999, PASJ, 51, 345
Rees, M. 1978, MNRAS, 184, 61
Refsdal, S. 1964, MNRAS, 128, 295
———. 1966, MNRAS, 132, 101
Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses (2d ed.; New York: Springer)
Sikora, M., et al. 2001, ApJ, 561, 1154
Spada, M., et al. 2001, MNRAS, 325, 1559
Spergel, D. N., et al. 2003, ApJS, 148, 175
Utley, P., McHardy, I. M., & Papadakis, I. E. 2002, MNRAS, 332, 231
Walsh, D., Carswell, R. F., & Weymann, R. J. 1979, Nature, 279, 381
Wambsganss, J., Paczyński, B., & Schneider, P. 1990, ApJ, 358, L33
Weinberg, S. 1972, Gravitaion and Cosmology (New York: Wiley)
Yonehara, A. 1999, ApJ, 519, L31