Ascertaining the cosmological constant using galactic superclusters in $f(R,T)$ gravity

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In this work we estimated the cosmological constant in a pioneering approach by using galactic superclusters in the layout of $f(R,T)$ gravity. We set $f(R,T) = R + 2\lambda T$ where $\lambda$ is the model parameter. We report that appropriate values of $\lambda$ generate cosmological constant ($\Lambda$) values in harmony with observational value of $1.1056 \times 10^{-52} m^{-2}$. We also delineate that for $\lambda = 0$ which corresponds to GR, yields physically unacceptable results.

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I. INTRODUCTION

The cosmological constant ($\Lambda$) problem is one of the major unsolved mysteries concerned with the dissimilarity between the tiny observed value of the cosmological constant and the extremely large value of zero point energy. Based on Planck energy cutoff along with other factors, the disaccord is as high as 120 orders of magnitude [1], a predicament often quoted as [3] “the worst theoretical prediction in the history of physics”. After the discovery of the expansion of the universe by E. Hubble in 1929 [2], it was expected that the rate of expansion must be slowing down owing to the attractive nature of the gravity. Nonetheless, measurement of the intrinsic brightness of distant Type Ia supernovae [4, 5] showed that the expansion is in fact accelerating. This mysterious component which fuels the expansion at an ever increasing rate account for nearly 70% of the energy budget of the universe and is termed Dark Energy (DE).

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The paper is organized as follows: In Section II we provide a summary of the cosmological constant. Section III contains a brief summary and conclusions.

II. OVERVIEW OF $f(R,T)$ GRAVITY

The action in $f(R,T)$ gravity is given by

$$S = \frac{1}{16\pi G} \int \sqrt{-g} \left[ f(R,T) + \mathcal{L}_m \right] dv^4$$ (1)
where $\mathcal{L}_m$ denote matter Lagrangian. Stress-energy-momentum tensor of matter fields reads

$$T_{\mu\nu} = -2 \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\sqrt{-g}}$$

(2)

varying the action (1) with respect to the metric yields

$$\Pi_{\mu\nu} f_{,R}(R, T) + f_{,R}(R, T) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f(R, T) = \Theta_{\mu\nu} + \kappa^2 T_{\mu\nu} - f_{,T}(R, T) (T_{\mu\nu})$$

(3)

where

$$\Pi_{\mu\nu} = \epsilon_{\mu\nu} \Box - \nabla_\mu \nabla_\nu$$

(4)

$$\Theta_{\mu\nu} \equiv \epsilon_{\alpha\beta} \delta T_{\alpha\beta} \delta g_{\mu\nu}$$

(5)

and $f_{,X} = \frac{df}{dx^X}$. The field equations (3) reduces to standard GR form when $f(R, T) \equiv R$.

Contracting equation (3) with inverse metric $g^{\mu\nu}$, one obtain the trace of the field equations as

$$3 \Box f_{,R}(R, T) + f_{,R}(R, T) R - 2f(R, T) = \Theta_{\mu\nu} + \kappa^2 T_{\mu\nu} - f_{,T}(R, T) (T_{\mu\nu})$$

(6)

Considering a spatially flat FLRW metric as

$$ds^2 = dt^2 - a(t)^2 [dx^2 + dy^2 + dz^2]$$

(7)

where $a(t)$ represents the scale factor. Assuming the universe to be dominated by a perfect fluid and hence matter Lagrangian density can be assumed $\mathcal{L}_m = -p$. Applying this to equations (3) and (6) we obtain

$$\frac{\kappa^2 + f_{,T}(R, T)}{f_{,R}(R, T)} \rho + \frac{1}{2} f^{1}_{,R}(R, T) \left[ p f^{1}_{,T}(R, T) - 3 \dot{R} H f^{2}_{,R}(R, T) + \frac{1}{2} \left( f(R, T) - R f^{1}_{,R}(R, T) \right) \right] = 3 H^2$$

(8)

$$\frac{\kappa^2 + f_{,T}(R, T)}{f_{,R}(R, T)} p + \frac{1}{2} f^{1}_{,R}(R, T) \left[ \dot{R} f^{2}_{,R}(R, T) + \dot{R}^2 f^{3}_{,R}(R, T) - \frac{1}{2} \left( f(R, T) - R f^{1}_{,R}(R, T) \right) \right] - p f^{1}_{,T}(R, T) + 2H \dot{R} f^{1}_{,R}(R, T)$$

$$= -3 H^2 - 2 \dot{H}$$

(9)

where dots represent time derivative and $H$ is the Hubble parameter, $\rho$ represents density and $p$ represents pressure with $T = \rho - 3p$.

### III. DETERMINING THE COSMOLOGICAL CONSTANT

For this work we assume the $f(R, T)$ functional form to be

$$f(R, T) = R + 2\lambda T$$

(10)

Substituting (10) in (8) we obtain the field equation as

$$H^2 = \frac{8\pi G}{3} \left( 8\pi + 3\lambda \right) \rho$$

(11)

where $p = 0$ for galaxies and their clusters in the present universe \[24\]. Friedmann equation in standard GR with cosmological constant $\Lambda$ is given by \[21\ \[22\]

$$H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda c^2}{3}$$

(12)

Since a void-dominated phase of cosmic fluid can be regarded as a quasi-vacuum dominated state \[24\], therefore we can equate (11) and (12) to obtain

$$\Lambda \approx \frac{75 G \rho}{c^2} (8 + \lambda)$$

(13)
By plugging $G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$ & $c = 3 \times 10^8 m s^{-1}$ we finally obtain

$$\Lambda \approx 5.58 \times 10^{-26} \rho (8 + \lambda) m^{-2} \quad (14)$$

Table I contain data of masses and sizes of 8 cosmic superclusters from which we calculate average densities by assuming a spherical symmetry. After inserting the densities in (14) we obtain the value of the cosmological constant.

**Standard GR case ($\lambda = 0$):**

For $\lambda = 0$ we obtain

$$\Lambda \approx 44.64 \times 10^{-26} \rho m^{-2} \quad (15)$$

Putting average density of Draco supercluster from Table I ($\rho_{Draco} = 0.6 \times 10^{-26} kg m^{-3}$, yields $\Lambda \approx 2.6 \times 10^{-51} m^{-2}$, which is unacceptable. Similar unacceptable results can be obtained for all other superclusters. For Saraswati supercluster the discrepancy is as high as 4 orders of magnitude.

**TABLE I: Data of Cosmic Superclusters**

| Supercluster        | Redshift $z$ | Binding Mass $k_g$ | Mean Radius $m$ | Mean Density ($\rho$) $10^{-26} kg m^{-3}$ | References |
|---------------------|--------------|--------------------|-----------------|-------------------------------------------|------------|
| Laniakea            | 0.000        | $\simeq 1.0 \times 10^{27}$ | $\simeq 2.40 \times 10^{-24}$ | $\simeq 0.173$                          | 24, 25     |
| Draco               | -            | $\simeq 2.0 \times 10^{47}$ | $\simeq 2.00 \times 10^{24}$ | $\simeq 0.600$                          | 26         |
| Virgo               | 0.000        | $\simeq 3.0 \times 10^{55}$ | $\simeq 5.00 \times 10^{23}$ | $\simeq 0.570$                          | 27         |
| Saraswati           | 0.280        | $\simeq 4.0 \times 10^{48}$ | $\simeq 3.08 \times 10^{22}$ | $\simeq 328.0$                          | 28         |
| Horologium          | 0.063        | $\simeq 2.0 \times 10^{47}$ | $\simeq 4.50 \times 10^{22}$ | $\simeq 52.50$                          | 29, 30     |
| Corona Borealis     | 0.070        | $\simeq 2.0 \times 10^{46}$ | $\simeq 1.54 \times 10^{24}$ | $\simeq 0.129$                          | 31, 32     |
| Caelum              | 0.126        | $\simeq 4.0 \times 10^{47}$ | $\simeq 4.30 \times 10^{24}$ | $\simeq 0.119$                          | 33         |
| Hyperion-Proto      | 2.450        | $\simeq 9.5 \times 10^{45}$ | $\simeq 1.50 \times 10^{24}$ | $\simeq 6.700$                          | 34         |

Planck satellite data reported 24 $\Omega_\Lambda = 0.6889 \pm 0.0056$ & $H_0 = 67.66 \pm 0.42 (km/s)/Mpc$, from which we obtain

$$\Lambda = 3 \left( \frac{H_0}{c} \right)^2 \Omega_\Lambda \approx 1.1056 \times 10^{-52} m^{-2} \quad (16)$$

In Figure 1 we show the dependency of $\Lambda$ on the model parameter ($\lambda$). We see that to obtain observationally acceptable values of $\Lambda$, $\lambda$ must be non-zero, negative and greater than -8. It can also be positive, however for this to happen density would have to be very low ($\sim 10^{-28} kg m^{-3}$ and lesser).

In Table I we show some viable values of $\lambda$ which generate acceptable values of $\Lambda$. It is also clear that for $\lambda = 0$ which corresponds to standard GR would fail to produce feasible values of cosmological constant using our approach. Hence, our work can serve as an observational verification of $f(R, T)$ gravity models.

In a recent study by 24, the cosmological constant was calculated by assuming voids acting as a source of negative pressure, or in other words, as dark energy. But their model could only work for voids of sizes $\sim 3.1 \times 10^{24} m$. Since voids located in low density environments are bigger than the ones located in high density spaces 35, their model fails when one uses voids of larger sizes like The KBC Void 25 ($\sim 2 \times 10^{28} m$) or The Giant Void 27 ($\sim 1.23 \times 10^{29} m$) as they produce $\Lambda$ an order of magnitude less than the observational value.
TABLE II: Derived values of the cosmological constant

| Supercluster  | Model Parameter ($\lambda$) | cosmological constant (\(\Lambda\)) (10^{-52} m^{-2}) |
|--------------|-----------------------------|------------------------------------------------|---|
| Laniakea     | -6.9000                     | 1.0560                                       |
| Draco        | -7.7000                     | 1.0044                                       |
| Virgo        | -7.6000                     | 1.2722                                       |
| Saraswati    | -7.9989                     | 2.0132                                       |
| Horologium   | -7.2000                     | 2.1240                                       |
| Corona Borealis | -6.3000                 | 1.2240                                       |
| Caelum       | -6.4000                     | 0.9600                                       |
| Hyperion-Proto | -7.9680                  | 1.1936                                       |

IV. CONCLUSIONS

In this work we calculated the cosmological constant in a novel approach by using observational data of 8 cosmic superclusters in the framework of \(f(R, T)\) gravity. We assumed \(f(R, T) = R + 2\lambda T\) as our model where \(\lambda\) plays the role of model parameter. We showed that in all cases suitable values of \(\lambda\) generate \(\Lambda\) values compatible with observations. We also showed that for \(\lambda = 0\) which corresponds to GR, yields physically unacceptable results. Hence our work directly confirms the viability of minimal \(f(R, T)\) gravity model of the form \(f(R, T) = R + 2\lambda T\).

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