Parity-violating gravity and GW170817

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We consider gravitational waves (GWs) in generic parity-violating gravity including recently proposed ghost-free theories with parity violation as well as Chern-Simons modified gravity, and study the implications of observational constraints from GW170817/GRB 170817A. Whereas GWs propagate at the speed of light, c, in Chern-Simons gravity, we point out that this is specific to Chern-Simons gravity and the GW propagation speed deviates from c in general in parity-violating gravity. The constraint on the propagation speed of GWs can thus pin down the parity-violating sector, if any, to Chern-Simons gravity.

INTRODUCTION

The first detection of gravitational waves (GWs) from two merging black holes, GW150914 [1], has opened a new and intriguing arena for gravitational physics. Direct observation of GWs provides us a window into the regime of strong gravity and the propagating sector of gravity, enabling novel tests of general relativity. More recently, the nearly simultaneous detection of GWs and the gamma-ray burst from the merger of neutron stars, GW170817/GRB 170817A [2, 3], has a significant impact on constraining deviation from general relativity: the gamma-ray burst from the merger of neutron stars, GW150914 [1], has opened a new and intriguing arena for gravitational physics. Direct observation of GWs provides us a window into the regime of strong gravity and the propagating sector of gravity, enabling novel tests of general relativity.

Gravitational parity violation has been studied based mostly on one specific realization called Chern-Simons (CS) gravity [14] (see also [15]), whereas recent developments in modified gravity have revealed that in fact one can construct theories of parity-violating gravity other than CS gravity [16]. This leads us to consider a unifying framework to study the propagation of GWs in generic parity-violating gravity. We clarify how special CS gravity is among parity-violating theories, and show that the bound on the speed of GWs yields a stringent constraint on parity violation in theories other than CS gravity.

PARITY-VIOLATING GRAVITY

We consider parity-violating gravity whose action is of the form

\[ S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ R + \mathcal{L}_{PV} + \mathcal{L}_\phi \right], \]  

(1)

where \( R \) is the Ricci scalar, \( \mathcal{L}_{PV} \) is a parity-violating Lagrangian, and \( \mathcal{L}_\phi \) is the Lagrangian for a scalar field \( \phi \) which may be coupled non-minimally to gravity.

The most frequently studied example of parity-violating gravity is CS gravity for which \( \mathcal{L}_{PV} \) is given by

\[ \mathcal{L}_{PV} = \mathcal{L}_{CS} := f(\phi)P, \quad P := \varepsilon^{\mu\rho\sigma\tau} R_{\rho\sigma\tau\alpha} R^\alpha_{\mu\nu}, \]  

(2)

where \( \varepsilon^{\mu\rho\sigma\tau} \) is the Levi-Civita tensor defined as \( \varepsilon^{\mu\rho\sigma\tau} := \varepsilon^{\mu\rho\sigma\tau}/\sqrt{-g} \) with \( \varepsilon^{\mu\rho\sigma\tau} \) being the antisymmetric symbol. The Pontryagin term \( P \) is a topological invariant in four dimensions, and for this reason we need the dynamical scalar field \( \phi \) coupled to \( P \) via \( f(\phi) \) in order for this term to contribute to the field equations. The kinetic term for \( \phi \) is supposed to be included in \( \mathcal{L}_\phi \) and one of the simplest possibilities is \( \mathcal{L}_\phi = -(\partial \phi)^2/2 - V(\phi) \).

Since CS gravity has higher-derivative field equations as explicitly confirmed by varying the action with respect to the metric [14, 17], one expects that dangerous Ostrogradsky ghosts appear in this theory. This is indeed true, as can be seen directly, e.g., from a wrong sign kinetic term in the quadratic action for perturbations around a spherically symmetric background [18] (see also [19]). This conclusion is supported by the Hamiltonian analysis performed in [16]. The ghost degrees of freedom might not be problematic if the theory (1) is treated as a low-energy truncation of a fundamental theory, but they do cause instabilities if regarded as a complete theory.

Recently, ghost-free parity-violating theories of gravity have been explored in [16]. At least in the unitary gauge in which \( \phi \) is homogeneous on \( t = \text{const} \) hypersurfaces, it is found that one can indeed construct Ostrogradsky-stable theories. One of the theories proposed in [16] is given by the following Lagrangian:

\[ \mathcal{L}_{PV1} = \sum_{A=1}^{4} a_A(\phi, \phi_\mu \phi^\mu) L_A, \]  

(3)
with

\[ L_1 := \varepsilon^{\mu
u\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\nu,\lambda} \phi^\rho \phi^\sigma \phi^\lambda, \]

\[ L_2 := \varepsilon^{\mu
u\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\rho,\lambda} \phi^\nu \phi^\lambda, \]

\[ L_3 := \varepsilon^{\mu
u\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\nu,\rho,\lambda} \phi^\mu \phi^\lambda, \]

\[ L_4 := \phi \phi^\lambda \phi^\lambda, \quad (4) \]

\[ (5) \]

\[ (6) \]

\[ (7) \]

where \( \phi_\mu := \nabla_\mu \phi \). In order to remove the Ostrogradsky modes, it is required that \( 4a_1 + 2a_2 + a_3 + 6a_4 = 0 \). Similarly, another ghost-free, parity-violating theory found in [16] contains second derivatives of the scalar field, \( \phi^\mu := \nabla_\mu \phi \), and is described by the Lagrangian of the form

\[ \mathcal{L}_{PV2} = b_1(\phi, \phi_\lambda \phi^\lambda) \varepsilon^{\mu
u\alpha\beta} R_{\alpha\beta\rho\sigma} \phi^\rho \phi^\sigma \phi^\nu + \cdots, \quad (8) \]

though its explicit expression is not important in this paper.

The purpose of the present paper is to study the propagation of GWs in such parity-violating theories of gravity. Let us consider GWs propagating on a homogeneous and isotropic background. The spatial metric is written as \( g_{ij} = a^2(t) [\delta_{ij} + h_{ij}(t, \vec{x})] \) with the scale factor \( a(t) \). To derive the evolution equation for \( h_{ij} \), we substitute the perturbed metric to Eq. (1) and expand it to second order in \( h_{ij} \). Let us focus on \( \mathcal{L}_{PV1} \) for the moment. After some manipulation, we find

\[ \mathcal{L}_{PV1}^{(2)} \supset \varepsilon^{ijk} h_{ii} \partial_j h_{kl}, \quad \varepsilon^{ijk} \partial^2 h_{ii} \partial_j h_{kl}, \quad (9) \]

where \( \varepsilon^{ijk} \) is the antisymmetric symbol and the coefficients of these terms depends on time in general. In the language of the Arnowitt-Deser-Misner (ADM) formalism, the two terms come respectively from \( \varepsilon^{ijk} K_{il} D_j K^l_k \) and \( \varepsilon^{ijk} R_{il}^{(3)} D_j K^l_k \), where \( K_{il} \) and \( R_{il}^{(3)} \) are the extrinsic and intrinsic curvature tensors of the spatial hypersurfaces and \( D_i \) is the three-dimensional covariant derivative. The second term in Eq. (9) can be recast in \( \varepsilon^{ijk} \partial^2 h_{ii} \partial_j h_{kl} \) by performing integration by parts. Therefore, the final form of the quadratic action for \( h_{ij} \) is of the form

\[ S^{(2)} = \frac{1}{16\pi G} \int dtd^3 x \, a^3 \left[ \mathcal{L}_{GR}^{(2)} + \mathcal{L}_{PV}^{(2)} \right], \quad (10) \]

where

\[ \mathcal{L}_{GR}^{(2)} = \frac{1}{4} \left[ \dot{h}_{ij}^2 - a^{-2} (\partial_t h_{ij})^2 \right], \quad (11) \]

is the standard Lagrangian obtained from the Einstein-Hilbert term \( R \) and

\[ \mathcal{L}_{PV}^{(2)} = \frac{1}{4} \left[ \frac{\alpha(t)}{a^2 \Lambda} \varepsilon^{ijk} h_{ii} \partial_j h_{kl} + \frac{\beta(t)}{a^2 \Lambda} \varepsilon^{ijk} \partial^2 h_{ii} \partial_j h_{kl} \right] \quad (12) \]

is the Lagrangian signaling parity violation. Here \( \alpha \) and \( \beta \) are dimensionless functions of time and \( \Lambda \) is some energy scale. Note that \( \alpha \) and \( \beta \) are independent in general. At least either of \( \alpha \) and \( \beta \) is taken to be an \( O(1) \) quantity by rescaling \( \Lambda \). We obtain only the first term in Eq. (9) if we start from \( \mathcal{L}_{PV2} \). Therefore, the Lagrangian (12) contains \( \mathcal{L}_{PV2} \) as the special case with \( \beta(t) = 0 \). By expanding \( \mathcal{L}_{CS} \) one sees that CS gravity corresponds to the special case of the above general action satisfying

\[ \alpha(t) = \beta(t). \quad (13) \]

(See, e.g., [20].) Thus, the quadratic action (10) with (11) and (12) offers us a unifying framework to study the propagation of GWs in parity violating theories of gravity described above. The concrete forms of \( \alpha(t) \) and \( \beta(t) \) depend on the background cosmological evolution of \( a(t) \) and \( \phi(t) \) as well as the theory under consideration.

Note that \( \alpha(t) = \beta(t) \) could in principle occur even in the \( \mathcal{L}_{PV1} \) theory. However, an extreme fine-tuning of the time-dependent functions is required in the \( \mathcal{L}_{PV1} \) theory, while Eq. (13) is automatically satisfied in CS gravity.

The Lagrangian for CS gravity is sometimes expressed using some length scale \( \ell_{CS} \) and the dimensionless scalar field \( \vartheta \) as \( \mathcal{L}_{CS} \sim (\ell_{CS}^2 \vartheta / 4) P \), and \( \ell_{CS} \) is often denoted as \( \ell_{CS}^1 / 4 \). This notation can be converted to ours as \( \alpha / \Lambda = \beta / \Lambda \sim \ell_{CS}^1 / 4 \) (ignoring the cosmic expansion).

Before proceeding, let us give two comments on possible further generalization of the above framework. The first comment is that one can generalize the standard piece \( \mathcal{L}_{GR}^{(2)} \) to

\[ \mathcal{L}_{H}^{(2)} = \frac{1}{4} \left[ A(t) \dot{h}_{ij}^2 - a^{-2} B(t) (\partial_k h_{ij})^2 \right] \]

by considering, e.g., the Horndeski/generalized Galileon Lagrangian as \( \mathcal{L}_\varphi \) [21–23]. However, for the moment we assume the standard Lagrangian (11) for the parity-preserving part.

The second comment is that Eq. (12) is not only the unifying description of the known parity-violating terms \( \mathcal{L}_{CS}, \mathcal{L}_{PV1}, \) and \( \mathcal{L}_{PV2} \), but also the low-energy effective description of generic parity-violating GWs. Indeed, the same quadratic action was derived from the viewpoint of the effective field theory in [24]. In light of this viewpoint, one may, for example, further add terms like \( \Lambda^{-5} \varepsilon^{ijk} h_{ii} \partial^2 \partial_j h_{kl} \), \( \Lambda^{-5} \varepsilon^{ijk} \partial^2 h_{ii} \partial^2 \partial_j h_{kl} \), \cdots, which are suppressed by powers of \( 1 / \Lambda \). The latter was studied in the context of Hořava gravity [25].
PROPAGATION OF PARITY-VIOLATING GWS

Varying the action (10) with respect to $h_{ij}$, we obtain the equation of motion for GWs,

\[ h''_{ij} + 2\mathcal{H} h'_{ij} - \partial^2 h_{ij} + \frac{1}{a^2} \epsilon_{ijk \ell} [a h''_{jk} + (\mathcal{H} a + \alpha') h'_{jk} - \beta \partial^2 h_{jk}] = 0, \]

(15)

where the prime denotes differentiation with respect to the conformal time defined by $d\eta = a^{-1} dt$, and $\mathcal{H} := \alpha'/a$.

We decompose $h_{ij}$ into the circular polarization basis defined by the following linear combination of the standard $+$ and $\times$ polarization basis:

\[ e^R_{ij} := \frac{1}{\sqrt{2}} (e^+_{ij} + ie^\times_{ij}), \quad e^L_{ij} := \frac{1}{\sqrt{2}} (e^+_{ij} - ie^\times_{ij}). \]

(16)

This choice of the polarization basis is convenient for parity-violating GWs because the equations for the left and right circular polarizations are decoupled even though the parity-violating terms mix the $+$ and $\times$ polarizations. Performing a Fourier decomposition, we write

\[ h_{ij}(\eta, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \sum_{A=R, L} \int d^3k \sqrt{\hat{A}^2(\eta)} e^{i\hat{A}_{ij}} e^{i\vec{k} \cdot \vec{x}}. \]

(17)

Using the identity

\[ \epsilon_{ijk \ell} \epsilon^{R, L}_{jk} = i\lambda_{R, L} e^{R, L}_{ij}, \]

(18)

where $\lambda_R = +1$, $\lambda_L = -1$, and $n_{l}$ is a unit vector pointing to the direction of propagation, we obtain

\[
\left( 1 - \lambda_A \hat{k} \alpha \right) (h^A_{k})'' + \left[ 2 - \lambda_A \hat{k} \left( \alpha + \alpha' \mathcal{H}^{-1} \right) \right] \mathcal{H} (h^A_{k})' + \left( 1 - \lambda_A \hat{k} \beta \right) k^2 h^A_{k} = 0, \]

(19)

for $A = R$ and $L$. Here we defined the dimensionless wavenumber $\hat{k} := k/(a\Lambda)$, which controls the magnitude of the corrections to general relativity. This parameter depends on the frequency of GWs, indicating that the parity-violating effect is more efficient at higher frequencies such as LIGO’s observation band than at lower frequencies corresponding, e.g., to CMB scales. In order to compare the propagation equation derived above with the general framework of GW propagation [26], we rewrite Eq. (19) in a physically more transparent form as

\[
(h^A_{k})'' + (2 + \nu_A) \mathcal{H} (h^A_{k})' + (c^A_T)^2 k^2 h^A_{k} = 0, \]

(20)

with the additional amplitude damping and the GW propagation speed squared,

\[
\nu_A = \frac{\lambda_A \hat{k} (\alpha - \alpha' \mathcal{H}^{-1})}{1 - \lambda_A \hat{k} \alpha}, \quad (c^A_T)^2 = \frac{1 - \lambda_A \hat{k} \beta}{1 - \lambda_A \hat{k} \alpha}. \]

(21)

If one takes $\alpha = \beta$, CS gravity is recovered, in which case we exactly have $c^A_T = 1$ and only the amplitude is modified through $\nu_A$. This agrees with the previous argument [27]. In general parity-violating gravity, however, the propagation speed is also modified. Note that since the sign of $\nu_A$ is determined by $\lambda_A, \eta$, and $\nu_l$ always have opposite signs. That is, if the amplitude of one polarization mode is enhanced, the other is suppressed. This is also true for $(c^A_T)^2 - 1$: If one polarization mode is superluminal, then the other is subluminal.

OBSERVATIONAL CONSTRAINTS

Assuming that the parity-violating effect is a small correction to general relativity, namely, $\hat{k} \ll 1$, the observables given in Eq. (21) are

\[
\nu_A = \lambda_A \hat{k} (\alpha - \alpha' \mathcal{H}^{-1}) + O(\hat{k}^2), \]

(22)

\[
(c^A_T)^2 = 1 + \lambda_A \hat{k} (\alpha - \beta) + O(\hat{k}^2). \]

(23)

The GW speed has already been measured from the coincident detections of GW170817/GRB 170817A [2, 3] and it is constrained so tightly in the range $-7 \times 10^{-16} < -1 - c_T < 3 \times 10^{-15}$. From this and Eq. (23), we have the constraint on parity-violating gravity, $\hat{k} |\alpha - \beta| \lesssim 10^{-15}$. Since the LIGO constraint is for the GW speed at a frequency of $k/a \sim 100$ Hz, this can also be written as

\[
\Lambda^{-1} |\alpha - \beta| \lesssim 10^{-11} \text{ km}. \]

(24)

This implies that either of the following statements holds in the low-redshift Universe: (i) the parity-violating sector is given by CS gravity, which is plagued with ghost instabilities (if considered as a complete theory); (ii) the parity-violating sector is given by a linear combination of $\mathcal{L}_{PV1}$ and $\mathcal{L}_{PV2}$ with $\alpha - \beta = O(1)$, and the stringent constraint is obtained as $\Lambda^{-1} \lesssim 10^{-11}$km; (iii) the parity-violating sector is given by a linear combination of $\mathcal{L}_{PV1}$ and $\mathcal{L}_{PV2}$, and the two time-dependent functions are extremely fine-tuned. Note that in the third case at least $\mathcal{L}_{PV1}$ is necessary because $\mathcal{L}_{PV2}$ generates only the $\alpha$ term. Note also that of course one has the freedom to add $\mathcal{L}_{CS}$ in the second and third cases.

So far we have assumed that the parity-preserving part is described by general relativity. If one generalizes this part to the Horndeski-type Lagrangian (14), $\nu_A$ and $c^A_T$ are also affected by this modification. However, the two effects cannot be canceled out because the parity-violating modification depends on $\lambda_A$ and the wave number, while the Horndeski-type modification does not. The two ways of modifying gravity are thus distinct, and therefore the above statements are robust.

The above constraint should be taken with a caution when one compares it with the existing constraints for the typical length scale $\Lambda^{-1}$ in literature. The analysis is made basically for CS gravity only, and the constraints are based on the assumptions that the scalar
sector is given simply by \( L_\phi = - (\partial \phi)^2 / 2 - V(\phi) \) and that \( \phi \) has a specelike gradient, or \( \phi \) is considered to be a non-dynamical field, or \( \phi \) is considered to be a non-dynamical field, or \( \phi \) is considered to be a non-dynamical field, or \( \phi \) is considered to be a non-dynamical field, or \( \phi \) is considered to be non-dynamical field. In the latter case, the bound on the typical length scale is given for example by \( \Lambda^{-1} \lesssim 10^{-1} \text{km} \) [28]. However, this would depend on the form of \( L_\phi \) as well as the parity-violating sector of gravity. It should be emphasized that our constraint has been derived without assuming any specific form of the scalar-field Lagrangian.

The observational bound (24) constrains the combination \( \Lambda^{-1} |\alpha - \beta| \) and we cannot distinguish the two possibilities (ii) and (iii) above. That is, if \( \alpha \) and \( \beta \) are extremely fine-tuned, the constraint tells nothing about the energy scale \( \Lambda \) of parity violation. Let us remark that another constraint can possibly come from the measurement of the gravitational constant at a binary pulsar. In modified gravity, the effective gravitational constant for the tensor modes, \( G_{\text{GW}} \), can be different from that of Newtonian gravity, \( G_N \). However, the binary-pulsar constraint from PSR B1913+16 leads to \( 0.995 \lesssim G_{\text{GW}} / G_N \lesssim 1.00 \) [29]. This strongly limits for instance viable scalar-tensor theories satisfying \( c_{\text{GW}} = 1 \) [30]. In our cases, from the Lagrangians (11) and (12) with \( \tilde{k} \alpha \) and \( \tilde{k} \beta \) now being the same at the level of \( 10^{-15} \), the effective gravitational constant for the tensor modes is defined by including the additional contribution from the parity-violating terms as

\[
G_{\text{GW}}^4 = G \left( 1 - \lambda A \right)^{-1} .
\]

Even in the presence of the parity-violating terms, spherically symmetric solutions remain the same as in general relativity [14] if \( \phi \) is minimally coupled to matter and gravity (except for the gravitational parity-violating part). In this case, we may set \( G_N = G \). Then, the binary-pulsar constraint is translated to \( |k \alpha| \lesssim 5 \times 10^{-3} \), or

\[
\Lambda^{-1} |\alpha| \lesssim 10^6 \text{km} ,
\]

using the GW frequency of \( k/a \sim 4 \times 10^{-4} \text{Hz} \) [31]. This indicates that the deviation from general relativity due to the parity violation effect must be smaller than 0.5% in the Lagrangian (10).

**CONCLUSIONS**

We have studied the propagation of gravitational waves (GWs) in Chern-Simons (CS) modified gravity and recently proposed ghost-free theories of parity-violating gravity. Along with this latter extension of gravity, we have found that the propagation speed of GWs is modified in general, together with the amplitude damping, which is modified in CS gravity as well. From the measurement of the GW speed with GW170817/GRB 170817A, we conclude that the possible parity-violating extension of gravity at low redshifts has already been tightly restricted to fine-tuned models without ghosts or CS gravity with ghosts. We have not considered any specific Lagrangian for the scalar degree of freedom, \( L_\phi \). Our result thus relies only on the propagation of GWs and the assumption that the scalar field \( \phi \) has a timelike gradient, and hence is robust irrespective of this scalar sector (provided that such a scalar-field configuration is allowed).

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