D-Branes and Dual Gauge Theories in Type 0 Strings

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Abstract

We consider the type 0 theories, obtained from the closed NSR string by a diagonal GSO projection which excludes space-time fermions, and study the D-branes in these theories. The low-energy dynamics of $N$ coincident D-branes is governed by a $U(N)$ gauge theory coupled to adjoint scalar fields. It is tempting to look for the type 0 string duals of such bosonic gauge theories in the background of the R-R charged $p$-brane classical solutions. This results in a picture analogous to the one recently proposed by Polyakov (hep-th/9809057). One of the serious problems that needs to be resolved is the closed string tachyon mode which couples to the D-branes and appears to cause an instability. We study the tachyon terms in the type 0 effective action and argue that the background R-R flux provides a positive shift of the (mass)$^2$ of the tachyon. Thus, for sufficiently large flux, the tachyonic instability may be cured, removing the most basic obstacle to constructing the type 0 duals of non-supersymmetric gauge theories. We further find that the tachyon acquires an expectation value in presence of the R-R flux. This effect is crucial for breaking the conformal invariance in the dual description of the 3 + 1 dimensional non-supersymmetric gauge theory.

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1. Introduction

The idea that supergravity may be used to obtain exact results in strongly coupled $\mathcal{N} = 4$ SYM theory with a large number of colors has been explored extensively in recent literature. Of course, a more ambitious goal is to extend this success to a stringy description of non-supersymmetric Yang-Mills theory in $d$ dimensions. One fruitful approach to this problem, proposed in [7], is to start with a maximally supersymmetric gauge theory in $d + 1$ dimensions and heat it up, so that the temperature breaks all supersymmetry. A supergravity description of such thermal strongly coupled gauge theories is indeed possible in terms of near-extremal brane solutions, whose near-horizon geometry corresponds to a black hole in AdS space. The new insights into non-supersymmetric gauge theories obtained in [7], and further developed in [13], include a new conceptual approach to the quark confinement and to the discreteness of the glueball spectrum. Subsequent work has revealed an interesting structure of the glueball spectra calculated in a limit akin to the lattice strong coupling limit. However, extrapolation of these calculations to the continuum (weak coupling) limit poses a very difficult problem because it corresponds to the large $\alpha'$ limit of string theory.

A rather different approach to non-supersymmetric gauge theories was suggested in [1], building on earlier results in [18]. It is based on a non-supersymmetric string theory obtained via a diagonal (non-chiral) GSO projection which is usually referred to as type 0 (A or B) theory. In $D = 10$ the spectra of these theories are:

- type 0A: $(\text{NS}^-,\text{NS}^-) \oplus (\text{NS}^+,\text{NS}^+) \oplus (R^+,R^-) \oplus (R^-,R^+)$,
- type 0B: $(\text{NS}^-,\text{NS}^-) \oplus (\text{NS}^+,\text{NS}^+) \oplus (R^+,R+) \oplus (R^-,R^-)$.

Both of these theories have no fermions in their spectra but produce modular invariant partition functions. The massless bosonic fields are as in the corresponding type

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1. In [21,22] it was also suggested that strings with world sheet supersymmetry may be related to non-supersymmetric gauge theories.
2. We adopt the notation of [23]: for example, $\text{NS}^-$ and $\text{NS}^+$ refer to odd or even chiral world sheet fermion number in the NS sector.
3. One may ask, for example, why we cannot have a consistent theory where only the $(\text{NS}^+,\text{NS}^+) \oplus (R^+,R^+)$ part of the full type IIB spectrum (which is $(\text{NS}^+,\text{NS}^+) \oplus (R^+,R^+) \oplus (\text{NS}^+,R^+) \oplus (R^+,\text{NS}^+)$) is kept. The answer is that such a theory would not be modular invariant.
II theory (A or B), but with the doubled set of the Ramond-Ramond (R-R) fields. One of the main claims of [1] is that, although the spectrum of type 0 theory contains a tachyon from the \((NS-, NS-)\) sector, this theory can be made to describe a tachyon-free gauge theory.

In this paper we further study possible relations between the type 0 strings and non-supersymmetric gauge theories (for now we restrict ourselves to the critical dimension \(D = 10\), postponing the non-critical case discussed in [1] for the future). We follow a route analogous to the one that has proven successful in the context of type II theory and construct gauge theories by stacking large numbers of coincident D-branes [25,26]. D-branes certainly can be introduced into the type 0 string theory, and the only question that needs to be addressed is: what is the appropriate GSO projection for open strings attached to the D-branes?

Since the type 0 theory has no space-time fermions in the bulk, it is fairly obvious that there are none localized on a single D-brane either, i.e. the Ramond sector of the open strings has to be removed. In the Neveu-Schwarz open string sector we adopt the same projection as for the bulk NS-NS states and keep only the vertex operators of even fermion number in the 0-picture. This way the tachyon of the NS sector is projected out, and the low-energy degrees of freedom on a D\(_p\)-brane consist of the massless gauge fields and scalars only. We will see that this conclusion continues to hold for parallel like-charged D-branes where the gauge theory naturally generalizes to \(U(N)\). Thus, there is no tachyon on parallel D-branes in type 0 theory. This is in accord with the conclusion reached in [1] that the dual gauge theory has no tachyonic states. The resulting D-brane action in the quadratic approximation may be obtained by dimensionally reducing 9 + 1 dimensional pure glue theory to \(p + 1\) dimensions. This purely bosonic \(U(N)\) gauge theory may therefore be built by stacking \(N\) coincident D\(_p\)-branes in the type 0 (0A for \(p\) even, 0B for \(p\) odd) theory, analogously to the construction of maximally supersymmetric gauge theories by stacking the D-branes of the type II theory. By analogy, we anticipate that the dual (closed string) description of the gauge theory on a large number \(N\) of D\(_p\)-branes

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\[\text{4 Let us note that type 0B theory has open string descendants which were originally constructed by orientifold projection in [23]. They, in general, have open-string tachyons in their spectra. A non-tachyonic model (which is anomaly-free and contains chiral fermions in its open-string spectrum) was found by a special Klein-bottle projection of the 0B theory in [24].}\]

\[\text{5 Our discussion of the D-branes has significant overlap with section 3.1 and Appendix C of an earlier paper [27], which we were unfortunately unaware of at the time of writing, where the D-branes of type 0B theory and their boundary states were discussed. We thank O. Bergman for pointing out this omission from the original version of this article.}\]
involves the classical R-R-charged $p$-brane background. Here, however, we encounter a puzzle because the theory in the bulk appears to suffer from a tachyonic instability. Since no such instability is seen in the gauge theory on the D-branes, the conjectured duality to a closed string background suggests that there the tachyonic instability disappears due to some non-perturbative mechanism.

With this idea in mind, we have examined the structure of the low-energy effective action of type 0 theory, with the emphasis on terms involving the tachyon field $T$. We find that their structure in the type 0 theory is far more constrained than in the conventional bosonic string. For example, due to the effects of the world sheet fermions in the vertex operators, it is immediately clear that the graviton-dilaton-tachyon part of the effective action has no odd powers of the tachyon field. This implies that the tachyon effective potential has no $T^3$ term, so that a positive $T^4$ term may cure the instability. While this raises the possibility of the tachyon condensation in the absence of the R-R backgrounds, we find even more intriguing results for the terms that couple the R-R $n$-form gauge field strength, $F_{\mu_1...\mu_n}$, to the tachyon: we demonstrate the presence of $F^2T^2$ terms which shift the effective $(mass)^2$ of the tachyon field. Moreover, the sign of such terms is such that the effective $(mass)^2$ can become positive in the presence of R-R flux. Since the conjectured type 0 duals of the gauge theories necessarily include the R-R flux, this gives an appealing mechanism for stabilizing such backgrounds.

While our results amount to a scenario for generalizing the AdS/CFT duality to type 0 string backgrounds, we should caution the reader that this scenario is hard to establish unambiguously based on on-shell amplitude calculations, which is the only tool we have used so far. Thus, for instance, the $F^2T^2$ term in the effective action is difficult to distinguish from a derivative coupling $F^2T\nabla^2T$. Nevertheless, our analysis of the effective action does make a rather convincing case for the stabilization of the tachyon in the presence of R-R charges. We emphasize that the mechanism for stabilization is inherent to the type 0 string, which is based on supersymmetric world sheet theory, and would not be available in the conventional bosonic string. This is related to the fact that in the type 0 string, but not in the conventional bosonic string, the D-branes carry R-R charges. In this respect the type 0 theory is much more similar to the type II theory than to the bosonic string.

The structure of the paper is as follows. In section 2 we introduce the D-branes of type 0 theory and study their interactions and world volume effective actions. In section 3 we calculate various three- and four-point functions, and deduce the corresponding terms in the bulk effective action. In section 4 we show that the near-horizon region of the type 0 threebrane solution is not of the form $AdS_5 \times S^5$, in agreement with the fact that the dual non-supersymmetric gauge theory is not conformal. In Appendix A we further discuss the bulk effective action by calculating the two graviton – two tachyon amplitude. In Appendix B we present the general form of the classical equations for the electric threebrane background of type 0 theory.
2. Forces between parallel D-branes

In this section we calculate the interaction energy of two parallel D-branes in the type 0 theory. This will provide a useful check of our prescription for the GSO projection in the open string sector. The calculation of the cylinder amplitude is analogous to the corresponding calculation in type II theory [25,28]. In fact, the only difference is that now we omit the contribution of the open string $R$ sector. Thus, we find

$$A = V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2 \alpha' t)^{(p+1)/2} e^{-\frac{q^2}{2\pi\alpha'}} \left( \left[ \frac{f_3(q)}{f_1(q)} \right]^8 - \left[ \frac{f_4(q)}{f_1(q)} \right]^8 \right),$$

where $q = e^{-\pi t}$ and we adopt the notation of [29]. The first term in parenthesis comes from the NS spin structure, while the second from the NS($-1)^F$ spin structure. Just as in the type II case, the negative power of $q$ corresponding to the open string tachyon cancels between these two terms.

In order to factorize the cylinder in the closed string channel, it is necessary to expand the integrand for small $t$. For the NS-NS closed string exchange, which corresponds to the NS open string spin structure, we have

$$\left[ \frac{f_3(q)}{f_1(q)} \right]^8 = t^4 (e^{\pi/t} + 8 + O(e^{-\pi/t})).$$

The first term corresponds to the NS-NS tachyon with $m^2 = -2/\alpha'$, while the second to the attractive massless exchange. This indicates that a D-brane serves as a source for the tachyon, as well as for the graviton and dilaton.

For the R-R closed string exchange, which corresponds to the NS($-1)^F$ open string spin structure, we have

$$- \left[ \frac{f_4(q)}{f_1(q)} \right]^8 = t^4 (-16 + O(e^{-2\pi/t})).$$

Here, as expected, we do not find the tachyon.

Note that, while in the type II case the R-R repulsion exactly balanced the NS-NS attraction, here the R-R massless repulsion is twice as strong as the NS-NS massless attraction. We believe that this is due to the doubling of the number of R-R fields in the type 0 theory. For instance, while in the type IIB case we have only the $(R+,R+)$ states coming from the positive chirality spinors, in the type 0B theory we also have the $(R-,R-)$ states coming from the negative chirality spinors. The spectrum of the 0B theory thus contains two R-R scalars, two R-R 2-forms, and also a 4-form without the constraint that its 5-form field strength is selfdual.
A question that comes to mind immediately is whether the two parallel D-branes will actually fly apart when the tachyon is removed from the spectrum (this is not the conclusion that we want to reach, since we want to build a gauge theory by stacking the D-branes on top of each other). A possible way to evade this conclusion is to note that the tachyon will work to attract the D-branes to a sufficiently small distance. Only when a large number of D-branes have coalesced to form a bound state with a macroscopic R-R field around it, will the tachyon (mass)\(^2\) become shifted to remove the instability. Unfortunately, we cannot make this argument quantitative at present.

On the basis of the cylinder calculation and the low-lying spectra of the open and closed strings, it is not hard to write down the low-energy action for a D-brane. It has the Born-Infeld form similar to the analogous action in type II theory except that now the R-R spectrum is doubled, and there is also a tachyon in the NS-NS sector. Thus, we find

\[
S_p = -T_p \int d^{p+1}\sigma \ e^{-\Phi(X)} \sqrt{-\hat{G}} \left[ 1 + \frac{1}{4} \bar{q} q \ T(X) + \frac{1}{4} F_{\alpha\beta}^2 + \frac{1}{2} \sum_{i=1}^{9-p} (\partial_\alpha X^i)^2 + \ldots \right]
\]

\[
+ \mu_p \int d^{p+1}\sigma \left( q C_{p+1} + \bar{q} \bar{C}_{p+1} \right),
\]

where \(T\) and \(\Phi\) are the background (bulk) tachyon and dilaton fields, \(\hat{G}_{\alpha\beta}\) is the metric induced on the D-brane, and \(C_{p+1}\) and \(\bar{C}_{p+1}\) are the projections of the R-R fields.\(^6\)

The discrete charges \(q = \pm 1\) and \(\bar{q} = \pm 1\) distinguish the branes with respect to \(C_{p+1}\) and \(\bar{C}_{p+1}\) from the anti-branes.\(^6\) The cylinder amplitude in (2.1) gives the potential between the like-charged branes: \(q_1 = q_2, \ \bar{q}_1 = \bar{q}_2\). In order to study the case \(q_1 = -q_2, \ \bar{q}_1 = -\bar{q}_2\) we need to change the sign of the NS(\(-1\))^\(F\) term which corresponds to the R-R interaction. Then, just as in the type IIB case \(28,30\), the open string tachyon is no longer projected out.

Finally, we should be able to construct the cylinder amplitude for \(q_1 = -q_2, \ \bar{q}_1 = \bar{q}_2\) or \(q_1 = q_2, \ \bar{q}_1 = -\bar{q}_2\). Then the R-R contribution to the interaction potential cancels between the \(C_{p+1}\) and \(\bar{C}_{p+1}\) exchanges. The NS-NS couplings of the two D-branes are also different

\(^6\) Here we use the normalization of \(C_{p+1}\) (and \(\bar{C}_{p+1}\)) as in \(29\) (i.e. \(L = \frac{1}{2\kappa^2} R + \frac{1}{2} |F_{p+2}|^2\), \(|F_{p+2}|^2 = \frac{1}{(p+2)!} F_{p+2} F_{p+2}\)). The tachyon is normalized so that \(L(T) = \frac{1}{8\kappa^2} (\partial^n T \partial^n T + m^2 T^2)\), \(m^2 = -\frac{2}{\alpha'}\). We used the notation \(F_{\alpha\beta} = 2\pi \alpha' F_{\alpha\beta} + B_{\alpha\beta}\) where \(F_{\alpha\beta}\) is the world-volume vector field strength and absorbed \(2\pi\) into the collective coordinates \(X^i\). We also omitted other possible \(F \wedge \ldots \wedge F \wedge (C_k + \bar{C}_k)\) terms.

\(^7\) The presence of the factor \(q\bar{q}\) in the coupling of the D-brane to the tachyon is related to the effective action R-R couplings \(F \bar{F} T\) whose existence we demonstrate in section 3.1.
now due to the factor $q\bar{q}$ in the source term for the tachyon. The correct projection in the open string channel is to retain the R sector only [27], so that the cylinder amplitude becomes

$$A = -V_{p+1} \int_0^\infty \frac{dt}{2t} (8\pi^2\alpha')^{(p+1)/2} e^{-\frac{\mu^2}{2\pi\alpha'}} \left[ \frac{f_2(q)}{f_1(q)} \right]^8.$$  

Expanding for small $t$, we have

$$- \left[ \frac{f_2(q)}{f_1(q)} \right]^8 = t^4 (-e^{\pi/t} + 8 + O(e^{-\pi/t})) ,$$

so that the sign of the closed-string tachyon contribution is indeed reversed compared to (2.1). The open string tachyon does not appear in this loop diagram, but, somewhat unexpectedly, the fermions do. Thus, the worldvolume theory of two D-branes with $q_1 = -q_2$, $\bar{q}_1 = \bar{q}_2$ or $q_1 = q_2$, $\bar{q}_1 = -\bar{q}_2$ contains fermions in addition to bosons. Since the total charge $q$ or $\bar{q}$ vanishes for such a composite D-brane, it decouples from the bulk tachyon.

Altogether we find 4 different types of ‘elementary’ D-branes depending on the signs of $q$ and $\bar{q}$ (i.e. there are 2 kinds of D-branes, and each one has a corresponding antibrane). An alternative way of labeling the D-branes and the R-R fields is to form

$$q^\pm = \frac{1}{2} (q \pm \bar{q}) , \quad (C_{p+1})^\pm = \frac{1}{\sqrt{2}} (C_{p+1} \pm \bar{C}_{p+1}) .$$

The D-branes with $q^- = 0$ are sources of $C_+$ only, while the D-branes with $q^+ = 0$ are sources of $C_-$ only. Note that for $p = 3$ the cases $q^+ = 1, q^- = 0$ ($q^+ = 0, q^- = 1$) correspond to electrically (magnetically) charged D3-branes. The presence of the two types of 3-branes is related to the fact that in type 0B theory the 5-form field strength is unconstrained (in type IIB theory it satisfies the selfduality constraint which forces the 3-branes to be dyonic). For example, for an electric 3-brane the last term of (2.4) is replaced by

$$\mu_5 \int d^4\sigma \, F_5 ,$$

where $F_5 = dC_4$ is the unconstrained 5-form field strength. To construct the self-dual 3-branes of type 0 theory we need to bring together equal numbers of the elementary electric and magnetic 3-branes. In this paper we will mainly restrict ourselves to discussing purely electric or purely magnetic configurations.

In the type II case the D-brane tensions and charge densities were normalized by Polchinski on the basis of the cylinder calculation, and we will adopt the same method. Since in (2.1) we have discarded the contribution of the open string R-sector, the graviton-dilaton attraction comes from the NS sector alone and turns out to be 1/2 of what it was in the type II case. This implies that the tension is reduced by a factor of $\sqrt{2}$, i.e.
\( T_p = T^{\text{II}}_p / \sqrt{2} \). However, the relation between tension \( T_p \) and charge density \( \mu_p \) remains as it was in type II theory [29]

\[
T_p = \frac{\mu_p}{\sqrt{2} \kappa},
\]

where \( \kappa \) is the gravitational constant. Thus

\[
\mu_p = \frac{\mu_p^{\text{II}}}{\sqrt{2}} = \sqrt{\pi} \left( 2\pi \sqrt{\alpha'} \right)^{3-p}.
\]

This assignment ensures that the massless R-R exchange term is also normalized correctly: both the \( C_{p+1} \) and the \( \bar{C}_{p+1} \) contributions work out to be \( 1/2 \) of the type II result, so that the sum is equal to the type II result. Indeed, the R-R exchange term, which comes from the \( \text{NS(}-1\text{)}^F \) sector, is identical in the type 0 and type II theories.

A similar argument shows that the Dirac-Nepomechie-Teitelboim charge quantization condition is satisfied by the type 0 D-branes. The net phase \( \theta \) accumulated in circling a \( p \)-brane around the \( (6-p) \)-brane magnetically dual to it, is doubled due to the presence of both the \( C_{p+1} \) and \( \bar{C}_{p+1} \) fields:

\[
\theta = 2\mu_p \mu_{6-p} = 2\pi.
\]

This shows that, just as in the type II theory, D-branes carry minimal charges consistent with the quantization condition.

### 3. Tree-level amplitudes and the effective action

In this section we perform a number of on-shell amplitude calculations in the type 0 theory (we shall concentrate on the type 0B theory which has D3-branes, but a similar discussion applies to the type 0A theory). Our goal is to construct the leading terms in the tree-level effective action \( S \) for the lowest-level states in all the four sectors: \((\text{NS}^-, \text{NS}^-), (\text{NS}^+, \text{NS}^+), (R^+, R^+), \) and \((R^-, R^-)\). The novel features of the type 0 theory as compared to type II are the presence of the \((\text{NS}^-, \text{NS}^-)\) sector which contains the tachyon (but no massless states), and the doubling of the R-R sector. \( S \) will thus involve the tachyon \( T \), the massless NS-NS fields \((G_{mn}, B_{mn}, \Phi)\), the R-R fields \((C, C_{mn}, C_{mnklt}^{(+)})\) and their ‘doubles’ \((\bar{C}, \bar{C}_{mn}, \bar{C}_{mnklt}^{(-)})\) (we shall combine \( C_{mnklt}^{(+)}) \) and \( \bar{C}_{mnklt}^{(-)} \) with selfdual and anti-selfdual field strengths into a single field \( C_{mnklt} \) with unconstrained field strength).

The effective action has a number of interesting properties. It is easy to see that all the tree amplitudes which involve only the fields from the \((\text{NS}^+, \text{NS}^+)\) and \((R^+, R^+)\) (or \((\text{NS}^+, \text{NS}^+)\) and \((R^-, R^-)\)) sectors are identical to those in the type IIB theory (in
particular, they factorize only on states from these sectors). Thus, in spite of the absence of space-time supersymmetry, the world-sheet supersymmetry implies the same restrictions on the tree-level string effective action as in the type II theory (e.g., the absence of \( \alpha' \) corrections to 3-point functions). The leading \( \alpha' \) correction to the second-derivative terms for the \( (G_{mn}, B_{mn}, \Phi) \) fields is again of the form \( \alpha'^3 \zeta(3)RRRR + ... \), i.e.

\[
S = -2 \int d^D x \sqrt{G} e^{-2\Phi} \left[ c_0 + R + 4(\partial_n \Phi)^2 - \frac{1}{12} H^2_{mnk} + O(\alpha'^3) \right].
\]  

(3.1)

In this section we set \( e^{\Phi_0} = 1, \kappa = \frac{1}{2} \), so that the graviton term in the Einstein frame has the canonical normalization. While we shall consider only the case of \( D = 10 \), for generality we have included the central charge term \( c_0 = -\frac{D-10}{\alpha'} \). Most of the calculations in this section can be repeated for general \( D \) and, in particular, the coefficients of the terms in the string-frame effective action below (apart from tachyon mass \( m^2 = -\frac{D-2}{4\alpha'} \)) are independent of \( D \), as they should be for consistency with the world-sheet sigma-model (conformal invariance conditions) approach.

The world-sheet supersymmetry also constrains the tachyonic sector of the action. Since \( T \) is the only field from the \( (NS-\bar{NS}, NS-\bar{NS}) \) sector which we include in \( S \), it is not hard to see that all the NS-NS amplitudes involving odd powers of \( T \), e.g., \( \langle TTT \rangle \), vanish. It is usually assumed that the presence of the tachyon renders the notion of a low-energy action ill-defined. One may argue that there is no unambiguous derivative expansion because, for instance, \( \nabla^2 T \) may be replaced by \( m^2 T \) on shell, i.e. the structure of the tachyon potential is ambiguous (see, for instance, \([31,32]\) for discussions). It seems natural, however, to look for an off-shell definition of the effective action which satisfies the following conditions: (i) it reproduces the string S-matrix when expanded near the usual tachyonic vacuum, and (ii) it has other stable (non-tachyonic) stationary points not visible in standard string perturbation theory. In the absence of some (yet unknown) string-theoretic principle which unambiguously favors that particular action over others which agree with the string S-matrix and have only tachyonic vacua, our main criterion is the self-consistency of this approach: the ability to satisfy conditions (i) and (ii) simultaneously is far from trivial.\(^8\)

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\(^8\) One may try to use the world-sheet sigma-model approach as a guide in defining the off-shell theory. The discussions \([31,32]\) of bosonic theory may not necessarily apply to the type 0 string due to the presence of world-sheet supersymmetry. While it may look as if the tachyonic coupling in the sigma model \( \int d^2 \sigma d^2 \theta T(X) \to \int d^2 \sigma \psi^m \bar{\psi}^n \partial_m \partial_n T \) is built in terms of derivatives of \( T \) so that the constant mode of the tachyon has no physical meaning, the fact that the tachyon operator may come in different (not only in the \((0,0)\) but also in the \((-1,-1)\)) pictures complicates the situation.
Our strategy will be to adopt the parametrization of the tachyonic terms in the effective action that appears most natural to us (for instance, has the smallest number of derivatives), and to check its consistency.

The absence of the $T^3$ interaction is an important hint that the type 0 theory is more stable than the usual bosonic string: the $T^3$ is present in the dual models and means that that the leading correction to the tachyon potential does not stabilize the theory. On the other hand, if the leading correction is of the positive $T^4$ form (as discussed in section 3.2), then it is possible that the instability disappears in one of the broken symmetry vacua.

Furthermore, we shall see below that the tachyon is coupled to the R-R fields (sections 3.1 and 3.3). Thus, the curved backgrounds with non-zero R-R charges may lead to corrections to the effective tachyon potential and, hopefully, to its stabilization.

Below we shall first discuss the three-point amplitudes and then consider the four-tachyon amplitude and the two R-R – two tachyon amplitude. The two graviton – two tachyon amplitude is discussed in Appendix A.

3.1. Three-point amplitudes

Since the $T^3$ amplitude vanishes, and the amplitudes involving two tachyons and a graviton or a dilaton are easily shown to correspond to the standard covariant kinetic term in the effective action, we have

$$\int d^D x \sqrt{G} \ e^{-2\Phi}\left(\frac{1}{2} G^{mn} \partial_m T \partial_n T + \frac{1}{2} m^2 T^2\right), \quad (3.2)$$

the most interesting calculation involves a tachyon and two R-R massless particles. In type 0B theory the R-R fields come from the $(R+, R+)$ and the $(R-, R-)$ sectors. The $(R+, R+)$ vertex operators have the form

$$e^{-\frac{1}{2} \phi(z) - \frac{1}{2} \tilde{\phi}(\bar{z})} \Theta(z) C \Gamma^{m_1 \ldots m_n} (1 + \Gamma_{11}) \tilde{\Theta}(\bar{z}) \ F_{m_1 \ldots m_n}(k) \ e^{ik \cdot x(z, \bar{z})}, \quad (3.3)$$

where the spin operators $\Theta$ and $\tilde{\Theta}$ are both $D = 10$ Majorana spinors. Similarly, the $(R-, R-)$ vertex operators have the form

$$e^{-\frac{1}{2} \phi(z) - \frac{1}{2} \tilde{\phi}(\bar{z})} \Theta(z) C \Gamma^{m_1 \ldots m_n} (1 - \Gamma_{11}) \tilde{\Theta}(\bar{z}) \ \bar{F}_{m_1 \ldots m_n}(k) \ e^{ik \cdot x(z, \bar{z})}. \quad (3.4)$$

---

9 As already mentioned above, the tachyon-graviton-graviton amplitude vanishes in type 0 theory (in the bosonic string it does not vanish, indicating the presence of a $TR_{m n k l} R^{m n k l}$ term in the effective action [32]).

10 Here we follow the notation of [20] and will use equations from section 12.4 there. $\phi$ is the bosonized ghost and $C$ is the charge conjugation matrix which satisfies $C^T = -C$ and anticommutes with $\Gamma_{11}$. Note that, in general, the R-R vertex operator in the sigma-model action on a sphere has a dilaton prefactor $e^{\Phi(x)}$. 

9
For \( n = 1, 3 \) the \( n \)-form field strengths are related to the potentials \( C_{n-1} \) and \( \bar{C}_{n-1} \) through

\[
F_{m_1...m_n} = n\partial[m_1, C_{m_2...m_n}], \quad \bar{F}_{m_1...m_n} = n\partial[m_1, \bar{C}_{m_2...m_n}].
\]

The 5-form case is special: here there is only one unconstrained field strength \( F_{m_1...m_5} \) which enters both the \((R^+, R^+)\) and the \((R^-, R^-)\) vertex operators,

\[
e^{-\frac{1}{2}\phi(z) - \frac{1}{2}\tilde{\phi}(\bar{z})} \Theta(z) \bar{\Theta}(\bar{z}) \Gamma^{m_1...m_5} (1 \pm \Gamma_{11}) F_{m_1...m_5}(k) e^{ik \cdot x(z, \bar{z})}. \tag{3.5}
\]

The positive sign in the projector picks out the selfdual part of the field strength present in the \((R^+, R^+)\) sector, while the negative sign picks out the anti-selfdual part present in the \((R^-, R^-)\) sector.

We will also need the detailed form of the tachyon vertex operator. In the \((0,0)\) picture it is

\[
k_m \psi^n (z) k_n \tilde{\psi}^n (\bar{z}) e^{ik \cdot x(z, \bar{z})}. \tag{3.6}
\]

However, to saturate the ghost number conservation on the sphere in the tachyon-(R-R)-(R-R) correlator, with the R-R operators in the \((-1/2, -1/2)\) picture, the tachyon should be taken in the \((-1, -1)\) picture

\[
e^{-\phi(z) - \tilde{\phi}(\bar{z})} e^{ik \cdot x(z, \bar{z})}. \tag{3.7}
\]

Since

\[
\langle \Theta_\alpha(z_1) \Theta_{\alpha'}(z_2) \rangle = z_{12}^{-5/4} C_{\alpha\alpha'} \tag{3.8}
\]

is non-zero only for spinors of the opposite chirality \((1 + \Gamma_{11})C (1 - \Gamma_{11}) = C\) we conclude that \( \langle FFT \rangle = \langle \bar{F} \bar{F} T \rangle = 0 \), i.e. the tachyon does not couple separately to the fields of the \((R^+, R^+)\) or \((R^-, R^-)\) sectors (this fact is, of course, necessary for consistency of the type IIB theory). At the same time, the \( \langle \bar{F} \bar{F} T \rangle \) amplitude is non-vanishing and leads to the \( F_n \bar{F}_n T \) terms in the effective action. For the 5-form field strength, the tachyon coupling is of the form \( F_5^{(+)} \bar{F}_5^{(-)} T \), where \( F_5^{(\pm)} = \frac{1}{2} (F_5 \pm \bar{F}_5^{\ast}) \) are the selfdual and anti-selfdual parts.

The graviton-(R-R)-(R-R) amplitude is identical to the corresponding type II amplitude. Here the graviton vertex operator is to be taken in the \((-1, -1)\) picture,

\[
e^{-\phi(z) - \tilde{\phi}(\bar{z})} \psi^m \bar{\psi}^n \zeta_{mn} (k) e^{ik \cdot x(z, \bar{z})}. \tag{3.9}
\]

Using the basic holomorphic correlators (see, e.g., [24])

\[
\langle e^{-\frac{1}{2}\phi(z_1)} e^{-\frac{1}{2}\phi(z_2)} e^{-\phi(z_3)} \rangle = z_{12}^{-1/4} (z_{13} z_{23})^{-1/2}, \tag{3.10}
\]

\( z_{ij} = z_i z_j \).
\[
(\Theta_\alpha(z_1)\Theta_{\alpha'}(z_2)\psi^m(z_4)) = 2^{-1/2}z_{12}^{-3/2}(z_{14}z_{24})^{-1}(C\Gamma^m)_{\alpha\alpha'}, \quad (3.11)
\]
we get the 3-point function corresponding to the standard kinetic R-R terms coupled to gravity (and to the dilaton, if we switch to the Einstein frame). As a result, the corresponding leading R-R terms in the action are
\[
\int d^Dx \sqrt{G} \left[ \frac{1}{2}|F_1|^2 + \frac{1}{2}|\bar{F}_1|^2 + \frac{1}{2}|F_3|^2 + \frac{1}{2}|\bar{F}_3|^2 + \frac{1}{2}|F_5|^2 + |F_1\bar{F}_1|^2 + |F_3\bar{F}_3|^2 + \frac{1}{2}|F_5|^2 \right],
\]
where \(|F_n\bar{F}_n| \equiv \frac{1}{\sqrt{n}}F_{m_1...m_n}\bar{F}_{m_1...m_n}|. In the last term we used
\[
|F_5^{(+)}F_5^{(-)}| = \frac{1}{2}|F_5|^2
\]
because \(\epsilon_{10}F_5F_5 = 0\) and \(\epsilon_{10}^2 = -10!\).

The \(F_1, F_3\) part of the action (3.12) may be diagonalized,
\[
\int d^Dx \sqrt{G} \left[ \frac{1}{2}(1+T)|F_{1+}|^2 + \frac{1}{2}(1+T)|F_{3+}|^2 + \frac{1}{2}(1-T)|F_{1-}|^2 + \frac{1}{2}(1-T)|F_{3-}|^2 \right], \quad (3.13)
\]
where we introduced, as in (2.7), \(F_{n\pm} = \frac{1}{\sqrt{2}}(F_n \pm \bar{F}_n)\) (for \(n = 1, 3\)). The unconstrained field \(F_5\) has both ‘electric’ and ‘magnetic’ components which are the 5-form counterparts of the degrees of freedom in \(F_{1\pm}\) and \(F_{3\pm}\).

3.2. The four-tachyon amplitude

In this section we attempt to find the leading \(T^4\) non-linear correction to the tachyon potential. The starting point is the 4-tachyon amplitude in type 0 theory. Taking two tachyons in the \((0,0)\) picture and two in the \((-1, -1)\) picture one finds that, up to normalization, the amplitude is given by
\[
A_4 = (k_1 \cdot k_2)^2 \int d^2z |z|^{-2-t}|1-z|^{-2-u} = 2\pi(1 + \frac{1}{2}s)^2 \frac{\Gamma(-1-\frac{1}{2}s)\Gamma(-\frac{1}{2}t)\Gamma(-\frac{1}{2}u)}{\Gamma(2 + \frac{1}{2}s)\Gamma(1 + \frac{1}{2}t)\Gamma(1 + \frac{1}{2}u)}
\]
\[
= -2\pi \frac{\Gamma(-\frac{1}{2}s)\Gamma(-\frac{1}{2}t)\Gamma(-\frac{1}{2}u)}{\Gamma(1 + \frac{1}{2}s)\Gamma(1 + \frac{1}{2}t)\Gamma(1 + \frac{1}{2}u)}, \quad (3.14)
\]
where (setting \(\alpha' = 2\))
\[
s = -(k_1 + k_2)^2, \quad t = -(k_1 + k_4)^2, \quad u = -(k_1 + k_3)^2, \quad (3.15)
\]
\[
s + t + u = 4m^2 = -4, \quad k_1 + k_2 + k_3 + k_4 = 0, \quad k_i^2 = -m^2 = 1. \quad (3.16)
\]
As expected, the amplitude is completely symmetric in $s, t, u$. It has only massless and $m_n^2 > 0$ massive poles (in agreement with the vanishing of the 3-tachyon amplitude). The massless pole is found by expressing $t$ and $u$ in terms of $s, u$ and $t, s$ and sending $s$ to zero

$$\frac{\Gamma(-\frac{1}{2}s)\Gamma(2 + \frac{1}{2}u + \frac{1}{2}s)\Gamma(2 + \frac{1}{2}t + \frac{1}{2}s)}{\Gamma(1 + \frac{1}{2}s)\Gamma(1 + \frac{1}{2}t)\Gamma(1 + \frac{1}{2}u)} \to -\frac{2}{s}(1 + \frac{1}{2}t)(1 + \frac{1}{2}u) + O(s^0) \quad (3.17)$$

The residue is proportional to $(k_1 \cdot k_3)^2$, making it clear that this pole is due to the graviton/dilaton exchange between the two $TT$-vertices in (3.2). Indeed, in the Einstein frame ($G_{mn} = e^{\frac{4}{D-2}\Phi}g_{mn}$) the sum of (3.1) and (3.2) becomes

$$S = \int d^Dx \sqrt{g} \left[ -2R + \frac{8}{D-2}(\partial_n \Phi)^2 + \frac{1}{2}(\partial_n T)^2 + \frac{1}{2}m^2 e^{\frac{4}{D-2}\Phi}T^2 + ... \right]$$

$$= \int d^Dx \left[ \frac{1}{2}h_{mn}(\Delta + ...)h_{mn} + \frac{1}{2}\Phi'\Delta \Phi' + \frac{1}{2}T(\Delta + m^2)T - T_{mn}^{(T)}h_{mn} + T\Phi' + ... \right], \quad (3.18)$$

where

$$\Delta = -\partial^2, \quad g_{mn} = \delta_{mn} + h_{mn}, \quad \Phi' \equiv \frac{4}{\sqrt{D-2}}\Phi,$$

and the tachyon contributions to the sources are

$$T_{mn}^{(T)} = \frac{1}{2}[\partial_m T\partial_n T - \frac{1}{2}\delta_{mn}(\partial_k T\partial_k T + m^2 T^2)], \quad T = \frac{1}{2}\frac{1}{\sqrt{D-2}}m^2 T^2. \quad (3.19)$$

Integrating the graviton and the dilaton out to the leading order we get the $T^4$ exchange amplitude which may be written as a contribution to the S-matrix generating functional $S(T^{(in)})$

$$S(T) = \int d^Dx \left[ \frac{1}{2}T(\Delta + m^2)T + W(T) \right], \quad (3.20)$$

$$W = -\frac{1}{2}T_{mn}^{(T)}\Delta^{-1}_{mn,kl}T_k^{(T)} - \frac{1}{2}T\Delta^{-1}T,$$

where $\Delta^{-1} = -\partial^2$ and $\Delta^{-1}_{mn,kl} = (\delta_{m(k}\delta_{\ell)n} - \frac{1}{D-2}\delta_{mn}\delta_{kl})\Delta^{-1}$ is the graviton propagator in the harmonic gauge. Evaluating the contractions in the effective $T^4$ term in the coordinate

\footnote{Note that, if written in the form

$$\frac{\Gamma(1 + m^2 - \frac{1}{2}s)\Gamma(1 + m^2 - \frac{1}{2}t)\Gamma(1 + m^2 - \frac{1}{2}u)}{\Gamma(-m^2 + \frac{1}{2}s)\Gamma(-m^2 + \frac{1}{2}t)\Gamma(-m^2 + \frac{1}{2}u)},$$

this expression gives the 4-tachyon amplitudes in both the bosonic ($m^2 = -\frac{4}{\alpha'}$) and the type 0 ($m^2 = -\frac{2}{\alpha'}$) string theories.


space we find that all $D$-dependence disappears and we are left with the following simple result

\[ W = \frac{1}{4} \left[ (\partial_m T \partial_n T) \Delta^{-1}(\partial_m T \partial_n T) - \frac{1}{2} (\partial_n T \partial_n T + m^2 T^2) \Delta^{-1}(\partial_k T \partial_k T + m^2 T^2) \right] \]

\[ = \frac{1}{4} \left[ (\partial_m T \partial_n T) \Delta^{-1}(\partial_m T \partial_n T) + \frac{1}{8} T^2 \partial^2 T^2 \right] , \quad (3.21) \]

where we have used the on-shell condition $\partial^2 T = m^2 T$.\(^{12}\) The corresponding expression in the momentum space (symmetric in $k_1, k_2, k_3, k_4$) is

\[ W = \int \prod_{i=1}^4 \frac{d^D k_i}{(2\pi)^D} e^{ik_i \cdot x} W(k_1, ..., k_4) T(k_1) T(k_2) T(k_3) T(k_4) , \quad (3.22) \]

\[ W_{\text{exch}} = -\frac{1}{12} \left[ \frac{(1 + \frac{1}{2} u)(1 + \frac{1}{2} t)}{s} + \frac{(1 + \frac{1}{2} s)(1 + \frac{1}{2} t)}{u} + \frac{(1 + \frac{1}{2} u)(1 + \frac{1}{2} s)}{t} \right] . \quad (3.23) \]

This is the same massless pole combination as found in the string amplitude (3.17). Subtracting the massless exchanges from the string amplitude we find that the correctly normalized contribution to the ‘contact’ (massive exchange) $T^4$ term in the effective action is given by

\[ W_{\text{subtr}} = \frac{1}{24} \left\{ \frac{\Gamma(-\frac{1}{2} s)\Gamma(-\frac{1}{2} t)\Gamma(-\frac{1}{2} u)}{\Gamma(1 + \frac{1}{2} s)\Gamma(1 + \frac{1}{2} t)\Gamma(1 + \frac{1}{2} u)} + \left[ \frac{2(1 + \frac{1}{2} u)(1 + \frac{1}{2} t)}{s} + (s, t, u \text{ cycle}) \right] \right\} . \quad (3.24) \]

To determine the leading contact term in this expression one may first extend it off shell and then expand in powers of momenta. In contrast to the case when all external legs are massless there seems to be no unique way of doing this, but we suspect that the situation in type 0 theory, where the 4-tachyon amplitude does not have the tachyonic poles, is better defined than in the bosonic string. There are at least two expansion procedures that preserve symmetry between $s, t, u$: (i) we may evaluate (3.24) at the symmetric point $s = t = u = -\frac{4}{3}$ getting a $T^4$ term in the effective action,

\[ S = \int d^D x \left[ \frac{1}{2} T(\Delta + m^2) T + c_1 T^4 + \ldots \right] , \quad (3.25) \]

\(^{12}\) The same expression can be found in the string frame by solving the ‘beta-function’ equations

\[ R_{mn} + 2\nabla_m \nabla_n \Phi - \frac{1}{4} \partial_m T \partial_n T = 0 , \quad c_0 + 2\nabla^2 \Phi - 4(\partial_n \Phi)^2 - \frac{1}{4} m^2 T^2 = 0 , \]

for the graviton and dilaton in terms of $T$ and substituting the result back into the action.
or (ii) we may set \( s = -2 - 2k_1 \cdot k_2, \ t = -2 - 2k_1 \cdot k_3, \ s = -2 - 2k_1 \cdot k_4 \) and expand in powers of scalar products of momenta, \( k_i \cdot k_j \), treating them as independent; that leads to the derivative-dependent, e.g., \( T \partial_m \partial_n T \partial_m T \partial_n T \), terms in the effective action.

While it is not clear if the \( T^4 \) correction may provide a contribution to \( S(T) \) to remove the tachyon, we will see below that in type 0 theory there is a more viable mechanism for tachyon stabilization based on the fact that the tachyon couples to the R-R fields. These R-R contributions to the tachyon effective potential dominate over the possible \( T^4 \) terms in the limit of strong R-R fields.

### 3.3. The two R-R field – two tachyon amplitude

Our aim below will be to determine the \( FFTT \) and \( FFTT \) terms in the effective action from the corresponding 4-point amplitudes, and also to check the coefficients of the cubic terms in the action (3.12) via factorization. For simplicity, we will first consider the R-R scalars of type 0B theory. Then the generalization to other R-R particles will be quite obvious. If the two R-R vertices are in the \((-1/2, -1/2)\) picture, then one of the two tachyon vertex operators is to be taken in the \((-1, 0)\) picture (3.6), and the other in the \((0,0)\) picture (3.7). Ordering the two R-R vertices as 1,2 and the two tachyon vertices as 3,4 and using the basic correlators (3.10),(3.11) we find the following expression for the string amplitude (as usual, we fix the Möbius gauge as \( z_{1,2,3} = 0, 1, \infty \)):

\[
J = \int d^2z|z|^{-2-t}|1-z|^{-2-u} = 2\pi \frac{\Gamma(-\frac{1}{2}s)\Gamma(-\frac{1}{2}t)\Gamma(-\frac{1}{2}u)}{\Gamma(1+\frac{1}{2}s)\Gamma(1+\frac{1}{2}t)\Gamma(1+\frac{1}{2}u)} \tag{3.27}
\]

is the same ratio of \( \Gamma \)-functions as in the final expression for the 4-tachyon amplitude in (3.14), except for the fact that now

\[
s + t + u = -2, \quad k_1^2 = k_2^2 = 0, \quad k_3^2 = k_4^2 = 1. \tag{3.28}
\]

In the kinematic factor (which is non-vanishing only if the two R-R vertices are from the same sector) we have used the fact that each R-R scalar vertex contains the field strength \( F_m = \partial_m C \) and thus an extra factor of momentum. Since \( C \Gamma^m = -(\Gamma^m)^T C \), \( C^2 = 1 \), one finds that the kinematic factor reduces to

\[
\text{Tr}(\Gamma_m \Gamma_p \Gamma_n \Gamma_q)k_4^m k_4^n k_2^p k_2^q = 32(2k_1 \cdot k_4 k_2 \cdot k_4 - k_1 \cdot k_2 k_4 \cdot k_4) = 16(-1 + tu). \tag{3.29}
\]

The string amplitude has massless poles in \( s,t,u \) channels. The \( s \)-channel pole \( (\sim \frac{tu^{-1}}{s}) \) is due to the graviton/dilaton exchange between the R-R and the tachyon ‘stress
tensor’ vertices, while the $t, u$ poles are due to the R-R field exchange between the two $F_1 F_1 T$ vertices in ($3.12$). The normalisation can be fixed by reproducing the $s$-channel pole from field theory. We start with the effective action parametrized by two constants $a, b$ ($C$ and $\bar{C}$ are the R-R scalars)

\[ S = \int d^D x \sqrt{G} \left[ e^{-2\Phi} (-2R - 8\partial^m \Phi \partial_m \Phi + \frac{1}{2} \partial^m T \partial_m T + \frac{1}{2} m^2 T^2) 
\]

\[
\quad + \frac{1}{2} (\partial^m C \partial_m C + \partial^m \bar{C} \partial_m \bar{C})(1 + b T^2 + ...) + a \partial^m C \partial_m \bar{C} T + ... \right], \quad (3.30)
\]

or, in the Einstein frame,

\[ S = \int d^D x \sqrt{g} \left[ -2R + \frac{1}{2} \partial^m \Phi' \partial_m \Phi' + \frac{1}{2} \partial^m T \partial_m T + \frac{1}{2} e^{\frac{1}{\sqrt{D-2}} \Phi'} m^2 T^2 
\]

\[
\quad + \frac{1}{2} e^{\frac{1}{\sqrt{D-2}} \Phi'} (\partial^m C \partial_m C + \partial^m \bar{C} \partial_m \bar{C})(1 + b T^2 + ...) + a e^{\frac{1}{\sqrt{D-2}} \Phi'} C \partial_m C \partial_m \bar{C} T + ... \right]. \quad (3.31)
\]

Calculating the graviton and dilaton exchange $CC - TT$ amplitude we find that, as in $TT - TT$ case ($3.21$), all $D$-dependence cancels out and (after use of the on-shell conditions for the legs) we are left with the following simple result for $W$ in ($3.20$)

\[ W_{grav} = -\frac{1}{4} \Delta^{-1} (\partial_m T \partial_n T) \]

\[ = -\frac{1}{4} \left[ (\partial_m C \partial_n C) \Delta^{-1} (\partial_m T \partial_n T) + \frac{1}{4} C^2 T \partial_l \partial_l T \right]. \quad (3.32)
\]

In the momentum space (symmetrizing over $t, u$) we get for the analogue of $W$ in ($3.22$)

\[ W^{(s)}_{field} = W_{grav} = \frac{1}{32} \left[ (1 + u)^2 + (1 + t)^2 \right] - (2 + s) \]

\[ = \frac{1}{16} \frac{1 - tu}{s}. \quad (3.33)
\]

This is the same $s$-channel pole we got in the string amplitude. This fixes the overall coefficient in the string amplitude which thus corresponds to the following $CCTT$ term in the generating functional for the S-matrix (the kernel below is to be multiplied by $C(k_1)C(k_2)T(k_3)T(k_4)$ and integrated over momenta as in ($3.22$))

\[ W_{string} = \frac{1}{32} (-1 + tu) \frac{\Gamma(-\frac{1}{2}s)\Gamma(-\frac{1}{2}t)\Gamma(-\frac{1}{2}u)}{\Gamma(1 + \frac{1}{2}s)\Gamma(1 + \frac{1}{2}t)\Gamma(1 + \frac{1}{2}u)}. \quad (3.34)
\]

The $\bar{C}$ exchange in field theory ($3.31$) gives the following contribution to the $CCTT$ amplitude

\[ W_{form} = -\frac{1}{2} a^2 C \partial_m C \partial_m T \Delta^{-1} \partial_n C \partial_n T \rightarrow W_{form} = \frac{1}{16} a^2 \left[ \frac{(1 + u)^2}{u} + \frac{(1 + t)^2}{t} \right]. \quad (3.35)
\]
The leading $\frac{1}{t}$ and $\frac{1}{u}$ poles are as in the string amplitude (3.34) if
\[ a^2 = 1 , \] (3.36)
which is the same value of $a$ as in (3.12). The contact $\partial C \partial CT$ vertex in (3.31) combined with the $\bar{C}$-exchange part (3.33) gives the following contribution to the field-theoretic $CCTT$ amplitude $\mathcal{W}$ (which is to be added to the $s$-channel exchange (3.33))
\[ \mathcal{W}^{(t,u)}_{\text{field}} = \mathcal{W}_{\text{form}} + \mathcal{W}_{\text{cont}} = \frac{1}{16} \left[ \frac{(1 + u)^2}{u} + \frac{(1 + t)^2}{t} + 4bs \right] . \] (3.37)
To fix the constant $b$ let us compare (3.37) with the string-theory expression for the sum of the $t$ and $u$ channel poles. Rewriting (3.34) as
\[ \mathcal{W}_{\text{string}} = \frac{1}{16} \frac{(1 - tu)(t + u)}{tu} P(t, u) , \quad P \equiv \frac{\Gamma(1 + \frac{1}{2}t + \frac{1}{2}u)\Gamma(1 - \frac{1}{2}t)\Gamma(1 - \frac{1}{2}u)}{\Gamma(1 - \frac{1}{2}u - \frac{1}{2}t)\Gamma(1 + \frac{1}{2}t)\Gamma(1 + \frac{1}{2}u)} , \] (3.38)
and taking the limit $t, u \to 0$, we get $P = 1 + c_1 tu(t + u) + O(t^5, u^5)$, where $c_1 = [\frac{d^3}{dz^3} \log \Gamma(z)]_{z=1}$. Thus
\[ \mathcal{W}_{\text{string}}(t, u \to 0) = \frac{1}{16}(1 - tu)(\frac{1}{t} + \frac{1}{u}) + O(t^2, u^2) , \] (3.39)
and comparison with the field-theory result (3.37) gives
\[ b = \frac{1}{2} . \] (3.40)
It is worth emphasizing the difference between the $TTTT$ amplitude discussed above and the present case: there the field-theory exchanges were reproducing the leading terms in the expansion of the string-theory amplitude; here in the $t, u$ channels this happens only if the extra contact $CCTT$ and $\bar{C}\bar{C}TT$ terms are added to the effective action.

Similarly, one finds that the effective action (3.12) contains the terms $\frac{1}{4}(|F_3|^2 + |\bar{F}_3|^2 + |F_5|^2)T^2$. We will be particularly interested in the $F_5$ background which is created by the threebrane solution. The $F_5$-dependent terms in the action that we have determined are (cf. (3.13))
\[ \int d^Dx \sqrt{G} \frac{1}{2}(1 + T + \frac{1}{2}T^2)|F_5|^2 . \] (3.41)
It is instructive to sketch an explicit derivation of these terms. Let us start with the field theory (3.30) and replace the $C, \bar{C}$ part by $\frac{1}{2}(1 + aT + bT^2)|F_5|^2$. The sum of the leading-order graviton and $C_4$ exchanges, and the contact contributions to the $F_5F_5TT$ part of the field-theory generating functional is found to be (cf. (3.32), (3.35))
\[ W_{\text{field}} = W_{\text{grav}} + W_{\text{form}} + W_{\text{cont}} \]
\[
= \frac{1}{8\cdot 5!} (10F_{mklpq}F_{nklpq} - \delta_{mn}F_{sklpq}F_{sklpq}) \partial^{-2}(\partial_m T \partial_n T) \\
+ \frac{1}{2\cdot 4!} a^2 (F_{mklpq}\partial_m T) \partial^{-2}(F_{nklpq}\partial_n T) + \frac{1}{2\cdot 5!} b F_{mklpq}F_{mklpq} TT. \quad (3.42)
\]

The string amplitude computed using the non-chiral R-R vertex operator (3.3), i.e.
\[e^{-\frac{1}{2}\phi(z)}-\frac{1}{2}\bar{\phi}({\bar{z}})\Theta C\Gamma_{m_1...m_5}\bar{\Theta} F_{m_1...m_5}(k) e^{ik\cdot x},\]
contains instead of (3.29) the following kinematic factor (here it is useful to keep the external field strength as a single object without separating the momentum factor in it):
\[\text{Tr}(\Gamma_m \Gamma_{p_1...p_5} \Gamma_n \Gamma_{q_1...q_5}) F_{p_1...p_5}(k_1) F_{q_1...q_5}(k_2) k_4^m k_4^n. \quad (3.43)\]

The total amplitude is similar to the one in (3.34),
\[W_{\text{string}} = \frac{1}{16\cdot 5!} \left[ 10F_{mklpq}(k_1) F_{nklpq}(k_2) k_4^m k_4^n - F_{mklpq}(k_1) F_{mklpq}(k_2) k_4 \cdot k_4 \right] \]
\[\times \frac{\Gamma(-\frac{1}{2}s)\Gamma(-\frac{1}{2}t)\Gamma(-\frac{1}{2}u)}{\Gamma(1+\frac{1}{2}s)\Gamma(1+\frac{1}{2}t)\Gamma(1+\frac{1}{2}u)}. \quad (3.44)\]

Extending this kinematic factor off shell in the most obvious way gives the following expression for the sum of the s-channel and the t,u channel poles of the amplitude (3.44) (in the coordinate space representation)
\[W_{\text{string}} = -\frac{1}{8\cdot 5!} \left[ 10(F_{mklpq} F_{nklpq}) \partial^{-2}(T \partial_m \partial_n T) - (F_{nklpq} F_{nklpq}) \partial^{-2}(T \partial^2 T) \right] \\
- \frac{1}{4\cdot 5!} \left[ 10(F_{mklpq}T) \partial^{-2}(F_{nklpq} \partial_m \partial_n T) - (F_{nklpq}T) \partial^{-2}(F_{nklpq} \partial^2 T) \right]. \quad (3.45)\]

Now we integrate by parts and use the $F_5$ equation of motion, $\partial_m F_{mklpq} = 0$, and the Bianchi identity, $F_{mklpq}(\partial_n F_{mklpq} - 5\partial_m F_{nklpq}) = 0$. As in the R-R scalar case, we then find that the s-channel pole in (3.44) is in agreement with the graviton exchange in (3.42), and the sum of the t, u poles is in agreement with the sum of the $C_4$ exchange and contact term in (3.42), iff $a^2 = 1$, $b = \frac{1}{2}$. This establishes explicitly that the $F_5$-dependent terms in the effective action are given by (3.41).

While it may seem that, since the string kinematic factor in (3.44),(3.45) involves only derivatives of the tachyon field, the effective action should also involve $\partial T$, it is worth stressing that the contact $F_5 F_5 TT$ term with no derivatives on $T$ comes out of the expression (3.43) upon rearranging it in the form (3.42). We believe, therefore, that our off-shell definition of the action as given in (3.41) is a natural one.

In the next section we will discuss some implications of the action (3.41) for threebrane solutions in type 0B theory.
4. The threebrane near-horizon solution

In this section we discuss the threebrane solution. More specifically, we focus on its near-horizon limit, but it is precisely this region that is expected to be dual to the gauge theory \[1\]. In the type IIB case, this throat region has the geometry of $AdS_5 \times S^5$ with a constant selfdual 5-form flux. The fact that the dilaton is constant indicates the scale-independence of the gauge coupling, which is in accord with the vanishing of the beta function in the dual $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory.

The $SU(N)$ gauge theory that we are considering is non-supersymmetric: it is related to the $\mathcal{N} = 4$ theory through removing all the fermionic partners. This theory is asymptotically free in the ultraviolet and is expected to be confining in the infrared. In fact, it may be in the same universality class as the pure glue theory, since the 6 adjoint scalars $X^i$ may become massive in the absence of supersymmetry. In the gravity (effective action) approximation we can study this theory reliably only for large bare coupling. While it is hard to predict precisely what should happen in this limit, it would be strange indeed if we found that its gravity dual is still $AdS_5 \times S^5$ \[13\] This would correspond to a line of fixed points that we can essentially rule out on the gauge theory side. Luckily, we are able to demonstrate that $AdS_5 \times S^5$ is not a solution of the type 0B theory with an electric 5-form flux. The mechanism for breaking of conformal invariance is quite subtle and involves tachyon condensation.

One lesson that we learned from the analysis of elementary type 0B D3-branes is that they can carry two types of charges, electric and magnetic. If we wish to construct a non-supersymmetric gauge theory on coincident D3-branes, we may stack D3-branes of one type: say, the positively charged electric D3-branes. Thus, the dual gravity description will carry $N$ units of electric 5-form flux. For such a 5-form background $|F_5|^2$ does not vanish, and from the action in (3.41) we find a shift of the tachyon (mass)\(^2\), and also an effective linear term for the tachyon. If the 5-form field were taken to be selfdual, then $|F_5|^2$ would vanish and none of the interesting effects that we are discussing would occur.\[14\] Thus, the doubling of the R-R sector in type 0B theory appears to be crucial for shifting the tachyon.

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13 In fact, we will argue that for large bare coupling the gravitational background has a tachyonic instability.

14 This is the field that surrounds a self-dual 3-brane, and it is quite easy to see that in the type 0B theory there exists an $AdS_5 \times S^5$ background with self-dual 5-form flux and the vanishing tachyon. The bulk tachyon simply decouples from the type 0B selfdual 3-brane.
The string frame field equations following from (3.1), (3.41) can be put into the form (cf. footnote 8; \( \kappa_{10} = \frac{1}{4} \))

\[
c_0 + 2\nabla^2 \Phi - 4\nabla^n \Phi \nabla_n \Phi - \frac{1}{4} m^2 T^2 = 0 , \tag{4.1}
\]

\[
R_{mn} + 2\nabla_m \nabla_n \Phi - \frac{1}{4} \nabla_m T \nabla_n T
- \frac{1}{4!} e^{2\Phi} f(T)(F_{mklpq} F_{n}^{klpq} - \frac{1}{10} G_{mn} F_{sklpq} F_{sklpq}) = 0 , \tag{4.2}
\]

\[
(-\nabla^2 + 2\nabla^n \Phi \nabla_n + m^2)T + \frac{1}{2!} e^{2\Phi} f'(T) F_{sklpq} F^{sklpq} = 0 , \tag{4.3}
\]

\[
\nabla_m [f(T) F^{mnkpq}] = 0 , \tag{4.4}
\]

where

\[
f(T) \equiv 1 + T + \frac{1}{2} T^2 . \tag{4.5}
\]

The central charge deficit term \( c_0 = \frac{10-D}{\alpha'} \) vanishes in the present case. Note that (4.1) does not depend on the 5-form terms in the action. This is a consequence of the Weyl-invariance of the \(|F_5|^2\) term in 10 dimensions, i.e. the fact that it drops out of the trace of eq. (1.2).

Equation (4.1) can be rewritten in the following useful form:

\[
- \nabla^2 e^{-2\Phi} + M^2 e^{-2\Phi} = 0 , \quad M^2 \equiv -\frac{1}{4} m^2 T^2 = \frac{1}{2\alpha'} T^2 . \tag{4.6}
\]

Remarkably, \( e^{-2\Phi} \) is not tachyonic precisely because \( T\) is. This equation can always be ‘integrated once’: indeed, the equation for \( \Phi \) can always be written in the first-order form as follows from eliminating \( \nabla^2 \Phi \) from (1.1) using the trace of (1.2):

\[
4 \nabla^n \Phi \nabla_n \Phi = -R + \frac{1}{4} \nabla^n T \nabla_n T - \frac{1}{4} m^2 T^2 . \tag{4.7}
\]

The first-order character of this equation with respect to the dilaton seems consistent with the interpretation of the radial \( e^\Phi \) equation as the RG evolution equation for \( g_{YM}^2 \).

It is quite easy to analyze the case where \(|F_5|^2\) is so large that it dominates over the quadratic action (3.2). Assuming \( e^{2\Phi} |F_5|^2 \gg |m^2|\), a particular solution of the tachyon equation is

\[
T = T_{vac} = \text{const} , \quad f'(T_{vac}) = 0 . \tag{4.7}
\]

\[\text{If there are corrections to the tachyon potential in the absence of R-R fields, then } \frac{1}{2} m^2 T^2 \text{ in } M^2 \text{ should be replaced by } V(T) = \frac{1}{2} m^2 T^2 + c_1 T^4 + \ldots , \text{ i.e. } M^2(T) = -\frac{1}{2} V(T). \text{ This will not seriously affect the R-R stabilization mechanism of the tachyon.}\]
Then $M^2$ in the equation (4.3) for $\Phi$ becomes constant, and also (4.6) reduces to
\[ |\nabla_n \Phi|^2 = \frac{1}{4} (M^2 - R) . \]  
(4.8)

In the background where the tachyon field acquires vacuum expectation value $T_{\text{vac}} = -1$ the dilaton thus has a source term coming from the quadratic tachyon action (the $|F_5|^2$ terms are Weyl invariant and do not generate a dilaton source). Thus, in the Einstein frame the dilaton equation is
\[ \nabla^2 \Phi = -\frac{1}{4\alpha'} e^{\frac{\Phi}{2}} T_{\text{vac}}^2 . \]  
(4.9)

This seems to imply that the dilaton decreases toward larger distance from the brane, i.e. the coupling indeed appears to decrease in the UV region, as expected from the asymptotic freedom of the gauge theory. The running of the dilaton means that the conformal invariance is lost, and $AdS_5 \times S^5$ is not a solution. The fact that the tachyon condensation is the mechanism for the breaking of 4-d conformal invariance provides support for our scenario.

It is interesting to note that, under the assumption that an approximate solution has $T = \text{const}$, the equations (4.1),(4.2),(4.4),(4.5) become formally the same as in the non-critical string theory without a tachyon condensate, but with the ‘effective’ central charge deficit $c_{\text{eff}} = M^2 = \frac{1}{2\alpha'} T^2$. The non-zero value of $M^2$ in (4.5) sets a scale for the gradients of the fields and, in the Einstein frame (4.9), plays the role of the coefficient of the exponential dilaton potential (making it hard to find the solution analytically, as we explain below).

Let us look for the electrically charged 3-brane background using the following ansatz for the metric and the R-R field
\[ ds^2 = d\tau^2 + e^{2\lambda(\tau)} (-dt^2 + dx_idx_i) + e^{2\nu(\tau)} d\Omega_5^2 , \]  
(4.10)
\[ C_{0123} = A(\tau) , \quad F_{0123\tau} = A'(\tau) , \]  
(4.11)
where $\tau$ is related to the radial direction transverse to the 3-brane ($i = 1, 2, 3$). All the fields including the tachyon are, in general, functions of $\tau$. Substituting the solution of the equation for $C_4$,
\[ A' = 2Qe^{4\lambda - 5\nu} f^{-1}(T) , \quad Q = \text{const} , \]  
(4.12)
into the remaining equations (4.1),(4.2),(4.3), we find that, as usual, they can be derived from an effective action $S(\lambda, \nu, \Phi, T)$ for a mechanical system with a potential.\footnote{One gets the wrong sign of the ‘electric’ $Q^2$-term in $S$ if one substitutes (4.12) directly into the action – when varying the action over $\lambda, \nu$ one is not taking into account the variation of $A$.}

The ‘mechanical’ equations are to be supplemented by the ‘zero-energy’ constraint following from $G_{\tau\tau}$ variation of the original action.
The resulting action and equations in the general case of $T \neq \text{const}$ are given in Appendix B, while here we shall simply assume, as suggested above, that for large 3-brane charge $Q$ one may ignore the tachyonic mass term in the equation for $T$ so that there should exist a self-consistent solution with $T = \text{const}$, $f'(T) = 0$. This should be true for large $Q$ in the near-horizon region provided

$$Q^2 e^{2\Phi - 10\nu} f^{-1}(T) \gg \frac{1}{8} |m^2| T^2 .$$

(4.13)

Under the assumption that $T = T_{\text{vac}} = -1$ is an approximate solution we get $f(T_{\text{vac}}) = \frac{1}{2}$, $M^2 = \frac{1}{2\alpha'} T_{\text{vac}}^2 = \frac{1}{2\alpha'}$. Introducing the new ‘time’ parameter $\rho$ and the fields $\xi, \eta$

$$d\rho = e^{2\Phi - 4\lambda - 5\nu} d\tau , \quad \xi = \Phi - 4\lambda , \quad \eta = \Phi - 2\lambda - 2\nu ,$$

(4.14)

we find that our problem is described by the following Toda-like mechanical system (dot denotes $\rho$-derivative)

$$S = \int d\rho \left[ \frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} \dot{\xi}^2 - 5\dot{\eta}^2 - V(\Phi, \xi, \eta) \right] ,$$

(4.15)

$$V = M^2 e^{\frac{\Phi}{2} + \frac{\Phi + \frac{1}{2}\xi - 5\eta}{2}} + 20 e^{-4\eta} - 2Q^2 e^{-2\xi} .$$

(4.16)

The three terms in the potential have a clear interpretation: the first originates from the ‘effective central charge’ or tachyon mass term, the second represents the curvature of $S^5$, and the third is produced by the electric R-R charge. The zero-energy constraint that supplements the second-order equations following from (4.15),

$$\frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} \dot{\xi}^2 - 5\dot{\eta}^2 + V(\Phi, \xi, \eta) = 0 ,$$

(4.17)

is equivalent to (4.18) (with the second derivatives of the metric components in $R$ eliminated using their equations of motion). When $M^2 = 0$ the dilaton has no potential and $\xi$ and $\eta$ are described by decoupled Liouville-type actions. As a result, one finds the electric analogue of the standard R-R 3-brane solution [34]:

$$\Phi = \Phi_0 , \quad e^\xi = e^{\Phi_0} + 2Q\rho \equiv e^{\Phi_0} H , \quad e^\eta = 2\sqrt{\rho} , \quad \rho = \frac{e^{2\Phi_0}}{4r^4} ,$$

(4.18)

which implies

$$d\tau^2 = H^{1/2} dr^2 , \quad e^{2\lambda} = H^{-1/2} , \quad e^{2\nu} = H^{1/2} r^2 , \quad H = 1 + \frac{e^{\Phi_0} Q}{2r^4} .$$
When \( M^2 \neq 0 \) all the three fields are coupled and the exact analytic solution does not seem possible. This increase in complexity is a general feature of the black hole solutions in the presence of a dilaton potential \([33,35]\). It should be sufficient, however, to analyze the solution qualitatively in the near-horizon region \( r \to 0 \) (or, equivalently, the asymptotic ‘long-time’ region \( \rho \to \infty \)). We shall not attempt to do this here (for some potentially relevant discussions see \([35]\)).

Although the complete near-horizon solution is not known, we would like to estimate the range of parameters for which the tachyon instability is removed. For this purpose we will assume that, just as for \( AdS_5 \times S^5 \), the overall scale of the metric is \( \sqrt{N g_s} \). The condition for the effective (mass)\(^2\) of \( T \) to be positive (which is, essentially, equivalent to (4.13)) is

\[
\frac{1}{4} g_s^2 |F_5|^2 > \frac{1}{\alpha'}.
\]

(4.19)

Since \( F_5 \sim N \), while the inverse metric is of order \( 1/\sqrt{N g_s} \), (4.19) implies

\[
\frac{1}{\sqrt{N g_s}} > O(1).
\]

(4.20)

In the dual field theory \( N g_s \) is the bare ‘t Hooft coupling, which we need to send to zero in order to achieve the continuum limit. It is satisfying that for small \( N g_s \) the type 0B background indeed appears to be stable. However, if the stability criterion (4.20) is satisfied, then there are significant corrections to the gravity approximation. Thus, in order to study such RR-charged backgrounds reliably, we really need to find the corresponding conformal \( \sigma \)-models.

### 5. Discussion

In this paper, following up on the idea of \([1]\), we studied possible connections between type 0 strings and non-supersymmetric gauge theories. It turns out that non-supersymmetric, yet tachyon-free, field theories are naturally realized as the world volume theories on coincident D-branes of the type 0 theory. By analogy with the recently discovered AdS/CFT duality, we attempt to give dual descriptions of these large \( N \) gauge theories.

\(^{17}\) To get a feeling for the complexity of this system let us make a simplifying assumption that for large \( Q \) the curvature of \( S^5 \) is supposed to be small so that the second term in \( V \) (4.16) may be ignored. Then one finds that \( \Phi = \eta + q \tau + p \), \( q, p = \text{const} \) and \( \tilde{\xi} = \frac{1}{2} M^2 e^{\frac{1}{2} \Phi + \frac{1}{2} \xi - 5 \eta} + 4 Q^2 e^{-2 \xi}, \)

\( \eta = -\frac{1}{2} M^2 e^{\frac{1}{2} \Phi + \frac{1}{2} \xi - 5 \eta} \) plus the constraint (4.17). Even this simplified system of equations does not appear to be exactly solvable.
in terms of the R-R-charged \( p \)-brane classical backgrounds. An immediate problem is the presence of a tachyon in the closed string spectrum. However, based on our analysis of the tachyon effective action in the presence of R-R field strength, we argue that the tachyon may condense, while its effective (mass)\(^2\) may become shifted to a positive value. This provides an appealing mechanism for curing the tachyonic instability. Our analysis of the near-horizon threebrane geometry shows that the tachyon instability is indeed removed, while the condensation breaks the conformal invariance, in agreement with the fact that the dual gauge theory is not conformal.

While we feel that we have constructed an appealing scenario for a new generalization of the AdS/CFT duality, much work remains to be done in checking its consistency. The only truly convincing check would be the construction of a conformal field theory with the R-R and tachyon backgrounds, but unfortunately we are still far from this goal.

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Appendix A. The two graviton – two tachyon amplitude

Let us first make some general comments on which terms we should expect in the graviton-tachyon sector of the effective Lagrangian. On general grounds, one may have (in the Einstein frame, up to factors of dilaton and terms with derivatives of dilaton)

\[
L = a_1 \alpha' R^{mn} \nabla_m T \nabla_n T + a_2 \alpha' R \nabla_k T \nabla_k T + a_3 R T^2 + TTTT \text{ terms } ,
\]

where \( TTTT \) stands for \( T^4 \) as well as terms with derivatives of the tachyon field. Using equations of motion

\[
R_{mn} = \frac{1}{2} \left( \mathcal{T}^{(T)}_{mn} - \frac{1}{D-2} g_{mn} \mathcal{T}^{(T)}_{kk} \right) , \quad R = - \frac{1}{D-2} \mathcal{T}^{(T)}_{kk} , \quad \nabla^m \mathcal{T}^{(T)}_{mn} = 0 ,
\]

where \( \mathcal{T}^{(T)}_{mn} \) was defined in (3.19) and we have taken into account \( \nabla^2 T = m^2 T \), we can thus transform the \( R T T \) terms in (A.1) into the \( TTTT \) terms. That means that such \( R T T \)
terms can be redefined away and give a vanishing contribution to the \( hhTT \) amplitude in particular. Their contribution to the 4-tachyon amplitude is equivalent to that of the contact \( TTTT \) terms in the action. Since the coefficients of such \( RTT \) terms are thus ambiguous, it is natural to define the effective action by setting them equal to zero. At the same time, it is natural to keep the \( FFTT, \bar{F}FTT \) and \( TTTT \) terms in the action.

Our aim will be to compare the string-theory and field-theory expressions for the \( hhTT \) amplitude. Let us first compute the field-theory amplitude. We shall consider only at the \( h_{mn}h_{mn} \) part of the amplitude where the polarization tensors of the two gravitons are contracted onto each other (not on momenta). The relevant \( h^2 \) and \( h^3 \) terms in the expansion of the Einstein term in the action near flat space are  \( (g_{mn} = \delta_{mn} + h_{mn}) \)

\[
-2\sqrt{g}R = \frac{1}{2} \partial_k h \partial_k h - \frac{1}{2} h_{kl} \partial_k h_{mn} \partial_l h_{mn} \\
+ \frac{1}{4} h_{tt} \partial_k h_{mn} \partial_l h_{mn} - h_{mn} \partial_k h_{mn} \partial_l h_{kl} + h_{mn} \partial_k h_{mn} \partial_l h_{tt} - h_{kl} \partial_n h_{mk} \partial_l h_{ml} . \tag{A.3}
\]

We have included the last ‘cyclic contraction’ term \( h_{kl} \partial_n h_{mk} \partial_l h_{ml} \) since it also contributes to the \( h_{mn}h_{mn}TT \) amplitude. After integration by parts and dropping terms where \( \partial^2 \) acts on one of the contracted (external) \( h_{mn} \) legs we get

\[
-2\sqrt{g}R = \frac{1}{2}(\partial_k h_{mn} \partial_l h_{mn} + \ldots) - h_{kl} T^{(h)}_{kl} - h_{kl} \partial_n h_{mk} \partial_l h_{ml} , \tag{A.4}
\]

\[
T^{(h)}_{kl} \equiv h_{mn} \partial_k h_{mn} + \frac{1}{2} \partial_k h_{mn} \partial_l h_{mn} - \frac{3}{4} \delta_{kl} \partial_k h_{mn} \partial_l h_{mn} , \quad \partial^k T^{(h)}_{kl} = 0 .
\]

The relevant terms in the tachyonic action are (we ignore the dilaton dependence as the dilaton exchanges do not contribute to the \( hhTT \) amplitude)

\[
\frac{1}{2}(\partial_k T \partial_k T + m^2 T^2)(1 - \frac{1}{4} h_{mn} h_{mn}) - h_{mn} T^{(T)}_{mn} , \tag{A.5}
\]

where the stress tensor \( T^{(T)}_{mn} \) is the same as in (B.19). Integrating out the graviton connecting the \( TTh \) and \( hhh \) vertices and adding also the contact \( hhTT \) term we find for the \( hhTT \) term in the resulting generating functional (see (B.20))

\[
W = W_{cont} + W_{exch} , \quad W_{cont} = -\frac{1}{8} \partial_k h_{mn} \partial_l h_{mn} TT , \tag{A.6}
\]

\[
W_{exch} = -(T^{(h)}_{mn} - \partial_p h_{qm} \partial_h h_{qn}) \Delta_{mn,kl}^{-1} T^{(T)}_{kl} . \tag{A.7}
\]

\(^{18}\) Note that while the \( RTT \) terms do not contribute to \( hTT \) 3-point function, they could still contribute to the 4-point amplitude \( hhTT \) (e.g., \( R = -\frac{1}{3} \partial_k h_{mn} \partial_l h_{mn} + \ldots \)). However, the contribution of such contact vertices cancels against that of the exchange diagram obtained by pairing \( hTT \) vertex from the \( RTT \) with \( hhh \) vertex from \( R \) term.
where the last term is the contribution of the ‘cyclic contraction’ term. The total result is very simple

\[ W = -\frac{1}{4}(h_{mn}\partial_k \partial_l h_{mn})\partial^{-2}(\partial_k T\partial_l T), \quad (A.8) \]

were we have used the mass shell condition for the tachyon and graviton legs to transform \( h_{mn}h_{mn}(\partial_k T\partial_l T + m^2T^2) \) into \( \partial_k h_{mn}\partial_k h_{mn}T^2 \). In the momentum space we get (this should be multiplied by the product of the graviton and tachyon factors \( \zeta_{mn}(k_1)\zeta_{mn}(k_2)T(k_3)T(k_4) \) as in (3.22))

\[ W = -\frac{1}{16} \frac{(1+t)(1+u)}{s}, \quad (A.9) \]

where we used that

\[ k_{1,2}^2 = 0, \quad k_{3,4}^2 = -m^2 = 1, \quad s + t + u = -2. \]

The string \( hhTT \) amplitude is easily computed in type 0 theory, but for simplicity we shall again consider only its part where the two graviton polarization tensors are contracted onto each other. The amplitude is found by taking the two gravitons (particles 1 and 2) in the \((-1,-1)\) picture and the two tachyons (particles 3 and 4) in the \((0,0)\) picture. As in (3.14) the momentum factor \((k_3 \cdot k_4)^2 = (1 + \frac{1}{2}s)^2\) comes from the contraction of fermions in the two tachyonic vertices, and the rest of the amplitude is given by \( \int d^2z|z|^{-1-t}|1-z|^{-1-u} \), i.e., up to normalization,

\[ A_4 = (1 + \frac{1}{2}s)^2 \frac{\Gamma(-1 - \frac{1}{2}s)\Gamma(\frac{1}{2} - \frac{1}{2}t)\Gamma(\frac{1}{2} - \frac{1}{2}u)}{\Gamma(2 + \frac{1}{2}s)\Gamma(\frac{1}{2} + \frac{1}{2}t)\Gamma(\frac{1}{2} + \frac{1}{2}u)} = \frac{2}{s} \frac{\Gamma(1 - \frac{1}{2}s)\Gamma(\frac{1}{2} - \frac{1}{2}t)\Gamma(\frac{1}{2} - \frac{1}{2}u)}{\Gamma(1 + \frac{1}{2}s)\Gamma(\frac{1}{2} + \frac{1}{2}t)\Gamma(\frac{1}{2} + \frac{1}{2}u)} \quad (A.10) \]

The amplitude is \( t - u \) symmetric, has massless pole in the \( s \)-channel and no tachyonic poles. The pole \( A_4(s \to 0) \to \frac{1}{2} \frac{(1+u)(1+t)}{s} + O(s^0) \) is in perfect agreement with the field

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19 We omit other terms with different index contraction patterns coming from the ‘cyclic’ term.

Note also that the relative signs of \( T(h) \) and the ‘cyclic’ term are different in (A.4) and in (A.7): there are 3 possible contractions of \( h_{pq} \) in the ‘cyclic’ term, with two having opposite sign to that of the third one.

20 Though the two \( \partial T \partial T h \) vertices could be, in principle, connected by the tachyon propagator, this does not produce the graviton polarization tensor factor we are considering.
theory result (A.9), implying that the normalized expression for the string amplitude with massless exchange subtracted is given by (cf. (3.24))

\[ \mathcal{W}_{\text{subtr}} = -\frac{1}{16} \frac{(1 + u)(1 + t)}{s} \left[ \frac{\Gamma(1 - \frac{1}{2}s)\Gamma(\frac{1}{2} + \frac{1}{2}t + \frac{1}{2}s)\Gamma(\frac{1}{2} + \frac{1}{2}u + \frac{1}{2}s)}{\Gamma(1 + \frac{1}{2}s)\Gamma(\frac{1}{2} + \frac{1}{2}t)\Gamma(\frac{1}{2} + \frac{1}{2}u)} - 1 \right]. \quad (A.11) \]

Appendix B. Classical equations for electric p-brane backgrounds

Here we shall discuss the general system of equations corresponding to the ansatz (4.10), (4.11) and \( T = T(\tau) \). For generality we will replace the 5-dimensional sphere in (4.10) by \( k \)-dimensional one, the number 3 of brane spatial dimensions by \( p \) and the rank of the R-R potential by \( n = p + 1 \). Thus, the case discussed in Section 4 is \( k = 5, n = 4 \). The parametrisation (4.10) is similar to the one used in the study of cosmological solutions with a non-trivial dilaton [36], with \( \tau \) playing the role of (Euclidean) time. The corresponding effective action for the fields \( \lambda, \nu, A, T \) and the redefined dilaton

\[ \varphi \equiv 2\Phi - n\lambda - k\nu \]

which follows from (3.1), (3.41) is

\[
S = -\int d\tau \left( e^{-\varphi} \left[ c_0 + k(k - 1)e^{-2\nu} - n\lambda'^2 - k\nu'^2 + \varphi'^2 - \frac{1}{4}T'^2 - \frac{1}{4}m^2T^2 \right] \\
+ \frac{1}{4}e^{-n\lambda + k\nu} f(T) A'^2 \right). \quad (B.2)
\]

Substituting the solution of the equation for the R-R potential

\[ A' = 2Qe^{n\lambda - k\nu} f^{-1}(T), \quad Q = \text{const} \quad (B.3) \]

into the remaining equations (4.1), (4.2), (4.3) we find that they become (prime denotes the \( \tau \)-derivative)

\[ n\lambda'^2 + k\nu'^2 - \varphi'^2 + \frac{1}{4}T'^2 + V = 0, \quad (B.4) \]

\[ \lambda'' - \varphi'\lambda' = -\frac{1}{2n} \frac{\partial V}{\partial \lambda}, \quad \nu'' - \varphi'\nu' = -\frac{1}{2k} \frac{\partial V}{\partial \nu}, \quad (B.5) \]

\[ \varphi'' - n\lambda'^2 - k\nu'^2 = \frac{1}{2} \frac{\partial V}{\partial \varphi}, \quad (B.6) \]

\[ T'' - \varphi'T' = -2 \frac{\partial V}{\partial T}. \quad (B.7) \]
These equations can be derived from the following action:

\[
S = \int d\tau \ e^{-\varphi} \left[ n\lambda'^2 + k\nu'^2 - \varphi'^2 + \frac{1}{4}T'^2 - V(\lambda, \nu, \varphi, T) \right],
\]

(B.8)

\[
V \equiv c_0 + k(k - 1)e^{-2\nu} - \frac{1}{4}m^2T^2 - Q^2e^{\varphi + n\lambda - k\nu}f^{-1}(T),
\]

(B.9)

provided one adds also the zero-energy constraint (B.4).

The action (B.8) may be transformed into a simpler form by making redefinitions similar to (4.14). Specifying to the case of our interest, \( n = 4, \ k = 5 \), we thus find the following action which generalizes (4.15), (4.16) to the case of non-constant tachyon:

\[
S = \int d\rho \left[ \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}\dot{\xi}^2 - 5\dot{\eta}^2 + \frac{1}{4}T'^2 - V(\Phi, \xi, \eta, T) \right],
\]

(B.10)

\[
V = e^{-2\varphi}V = (c_0 - \frac{1}{4}m^2T^2)e^{\frac{1}{2}\Phi + \frac{1}{2}\xi - 5\eta} + 20e^{-4\eta} - Q^2e^{-2\xi}f^{-1}(T).
\]

(B.11)

The 3-brane solution corresponding to this action will, in general, involve \( T \) changing with \( \rho \). As we discussed in section 4, for large \( Q \) it is natural to expect that for large \( \rho \) (or near the horizon) the tachyon will asymptotically approach a constant value. It would be interesting to try to confirm this in a more rigorous way.
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