Large capacity multi-float configurations for the wave energy converter M4 using a time-domain linear diffraction model

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A B S T R A C T

The moored three-float line absorber WEC M4 has been developed to optimise power capture through experiments and linear diffraction modelling. With the progression down wave from small to medium to large floats, the device heads naturally into the wave direction. The bow and mid floats are rigidly connected by a beam and a beam from the stern float is connected to the hinge point above the mid float for power take off (PTO). Increasing the bow to mid float spacing to be more than 50% greater than the mid to stern float spacing has been found to improve power capture. To increase power capture further and potentially reduce electricity generation cost the number of mid floats and stern floats is increased while maintaining a single bow float for mooring connection. The bow and mid floats still form a rigid body while the stern floats may respond independently. A time domain linear diffraction model based on Cummins method has been applied to configurations of 121, 123, 132, 133, and 134 floats where the numbers indicate the number of floats: bow, mid, stern. This shows how power capture is increased while response remains similar. We only consider uni-directional (long-crested) waves with narrow band width typical of swell. By considering scatter diagrams for various offshore sites capacities may range from 3.7 MW to 17.3 MW for the eight float system with a capacity factor of 1/3 while the cost of electricity assuming capital cost to be a fixed multiple of steel cost is reduced from that for the three-float system.

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1. Introduction

Many devices have been considered for the conversion of ocean wave motion into electricity, e.g. [1]. The wave resource is greatest offshore in deeper water and here we consider developments of the moored three-float line absorber with high capture width known as M4 [2]. The Cockerel raft in the 1980s was probably the first of this kind of system, also referred to as an attenuator, followed by Pelamis with connected cylindrical tubes [3] and Sea Power with box-type floats [4]. With M4 the floats are circular in cross section with hemi-spherical or rounded bases giving minimal drag losses and increase in diameter and volume from bow to stern giving a range of natural periods in heave and pitch, providing a broad band response; significant power generation across the range of wave periods occurring at a typical site may be obtained. The distance between floats is about half a typical wavelength so that forces and adjacent float motions are predominantly in antiphase. With the progression down wave from small to medium to large floats, the device naturally heads into the wave direction. The bow and mid floats are rigidly connected by a beam and a beam from the stern float is connected to the hinge point above the mid float for power take off (PTO). The machine has been investigated by laboratory experiments at two scales differing by a factor of 5 showing the accuracy of Froude scaling [2]. Frequency domain linear diffraction models have been shown to give good prediction of relative pitch rotation and slightly overestimate power absorption by Sun et al. [5]. Interestingly the predictions became more accurate as wave complexity increased from regular to irregular to multi-directional probably because any effect of reflections in a confined (albeit large) wave basin will be minimal when frequency averaged [5].

Time domain linear diffraction modelling showed that reducing the diameter of the mid and stern floats, from 0.3 m to 0.25 m and 0.4 m to 0.35 m at laboratory scale, was found not to reduce power capture while reducing float mass significantly; also increasing the bow to mid float spacing to be more than 50% greater than the mid to stern float spacing was found to improve power capture [6]. This configuration has been tested in the laboratory and a diagram is shown in Fig. 1. The time domain linear diffraction model has been set up for irregular waves based on Cummins’ method with convolution integrals defined by impulse response functions, summarised in Stansby et al. [6]. The added mass, radia-

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tion damping and excitation force coefficients are determined from WAMIT. In the experiments a pneumatic damper at the hinge provided almost linear damping. The option of nonlinear PTOs is not considered here although there was preliminary investigation in [6] of Coulomb damping and clutch rectifying power take off (PTO). While these studies were for operational conditions some focussed wave experiments were also undertaken in large (non-breaking) waves without the PTO engaged, giving effectively zero damping; these showed that the response remained remarkably linear until float deck submergence (dunking) occurred further supporting the use of linear modelling [7]. Extreme analysis indicated that the M4 response limit will not be exceeded in most extreme conditions even in the open North Atlantic west of Orkney. Mooring forces have not been analysed; they are a predominantly second-order effect and a light mooring line was used to prevent drift in the experiments.

Results of laboratory experiments are shown here and compared with time domain linear diffraction modelling. This provides some model validation before generalisation to multi-float configurations with 121, 122, 131, 132, 133 and 134 floats where the numbers indicate the number of bow, mid, stern floats. For all configurations a single bow float forms a rigid body with the mid floats while the stern floats may respond independently. Only narrow band, uni-directional (long-crested) waves typical of swell are considered at this stage. The aim is to determine how power capture is increased and whether cost of electricity is likely to be reduced by considering scatter diagrams for several locations assuming capital cost to be a fixed multiple of steel cost.

Following this introduction the next section provides a full description of the body dynamics and hydrodynamic modelling. The following section describes the experiments and model comparisons. Results for the multi-float configurations are then presented. These are then discussed with implications for energy capture and cost of electricity. Finally some conclusions are drawn.

2. Mathematical formulation and solution

Although we are concerned with irregular waves we first give the mathematical formulation for a single frequency before generalising for the input of a wave spectrum.

Fig. 2 shows a representative multi-float configuration and notation. Angular rotation \( \Theta \) is clockwise positive, \( h \) is horizontal distance from O to a float positive in stern direction, \( v \) is vertical distance from O to a float positive below O. \( H \) and \( V \) are total hydrodynamic forces (due to diffraction, added mass, radiation damping) in conventional \( x, z \) directions (shown in Fig. 1) and \( M \) is moment about O.

Although there are multiple floats (\( n \)) these may be considered as two bodies \( A (n_A \text{ floats}) \) and \( B (n_B \text{ floats}) \) as shown for the six float case in Fig. 2.

For body \( A \) taking moments about \( O \) accounting for all masses (floats, ballast, beams)

\[
\sum_{i=1}^{n_A} m_i v_i \dot{x}_i + \sum_{i=1}^{n_A} m_i h_i z_i + I_0 \dot{\Theta}_0 = M_{\text{mech}} + \sum_{i=1}^{n_A} M_i - \sum_{i=1}^{n_A} h V_i - \sum_{i=1}^{n_A} v H_i
\]

(1)

\( M_{\text{mech}} \) is due to mechanical damping or PTO at \( O \).

For body \( B \) for each float \( i = 1 + n_A \), \( n \) taking moments about \( O \) as each float may respond individually

\[
-m_i v_i \dot{x}_i - m_i h_i \dot{z}_i + I_0 \ddot{\Theta}_0 = -M_{\text{mech}} + M_i - h V_i - v H_i
\]

(2)

![Fig. 1. Diagram of laboratory scale three-float M4.](image)
where \( M_{\text{mech}} = -B_{\text{mech}} \dot{\theta}_i + M_{\text{mech}} = \sum_{i=1}^{n} m_i z_i \) and \( \theta_{ni} = \theta_A - \theta_i \).

For the whole system there is no net force or moment on the hinge.

In the horizontal direction

\[
\sum_{i=1}^{n} m_i \dot{x}_i = \sum_{i=1}^{n} H_i \tag{3}
\]

and in the vertical direction

\[
\sum_{i=1}^{n} m_i \dot{z}_i = \sum_{i=1}^{n} V_i \tag{4}
\]

The positions of the centres of gravity of each float \( x_i, z_i \) in relation to \( O \), linearised for small angles, are defined by:

For body A floats, \( i = 1, n_A \)

\[
x_i = x_0 - v_i \dot{\theta}_A \tag{5a}
\]

\[
z_i = z_0 - h_i \dot{\theta}_A \tag{5b}
\]

For body B, \( i = 1 + n_A, n \)

\[
x_i = x_0 - v_i \dot{\theta}_B \tag{5c}
\]

\[
z_i = z_0 - h_i \dot{\theta}_B \tag{5d}
\]

We thus have 4 equations for 4 unknowns \( x_0, z_0, \theta_A, \dot{\theta}_A ; i = 1 + n_A \).

For A

\[
- \sum_{i=1}^{n_A} m_i v_i (\ddot{x}_0 - v_i \ddot{\theta}_A) - \sum_{i=1}^{n_A} m_i h_i (\ddot{z}_0 - h_i \ddot{\theta}_A) + I_A \ddot{\theta}_A = M_{\text{mech}}
\]

\[
+ \sum_{i=1}^{n_A} M_i - \sum_{i=1}^{n_A} h_i V_i - \sum_{i=1}^{n_A} v_i H_i \tag{6}
\]

giving

\[
\ddot{\theta}_A \left( \sum_{i=1}^{n_A} m_i v_i 2 + \sum_{i=1}^{n_A} m_i h_i 2 + I_A \right) = \sum_{i=1}^{n_A} m_i v_i \ddot{x}_0 + \sum_{i=1}^{n_A} m_i h_i \ddot{z}_0 + M_{\text{mech}}
\]

\[
+ \sum_{i=1}^{n_A} M_i - \sum_{i=1}^{n_A} h_i V_i - \sum_{i=1}^{n_A} v_i H_i \tag{7}
\]

And for body B float \( i = 1 + n_A, n \)

\[
-m_i v_i (\ddot{x}_0 - v_i \ddot{\theta}_B) - m_i h_i (\ddot{z}_0 - h_i \ddot{\theta}_B) + I_i \ddot{\theta}_i = -M_{\text{mech}} i
\]

\[
+ M_i - h_i V_i - v_i H_i \tag{8}
\]

\[
\ddot{\theta}_i (m_i v_i^2 + m_i h_i^2 + I_i) = m_i v_i \ddot{x}_0 + m_i h_i \ddot{z}_0 - M_{\text{mech}} i
\]

\[
+ M_i - h_i V_i - v_i H_i \tag{9}
\]

And for the whole system

In horizontal

\[
\sum_{i=1}^{n} m_i (\ddot{x}_0 - v_i \ddot{\theta}_i) = \sum_{i=1}^{n} H_i \tag{10}
\]

giving

\[
\ddot{x}_0 \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} H_i + \sum_{i=1}^{n} m_i v_i \ddot{\theta}_i \tag{11}
\]

and in vertical

\[
\sum_{i=1}^{n} m_i (\ddot{z}_0 - h_i \ddot{\theta}_i) = \sum_{i=1}^{n} V_i \tag{12}
\]

giving

\[
\ddot{z}_0 \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} V_i + \sum_{i=1}^{n} m_i h_i \ddot{\theta}_i \tag{13}
\]

We thus have equations for \( \ddot{\theta}_A, \ddot{\theta}_B, \ddot{x}_0, \ddot{z}_0 \) which are further complicated by \( H_i, V_i, M_i \) defined below also being a function of \( \ddot{\theta}_A, \ddot{\theta}_B, \ddot{x}_0, \ddot{z}_0 \) and hydrodynamic (WAMIT) coefficients. In the above form iterating to a solution by assuming values of \( \ddot{x}_0, \ddot{z}_0 \) and substituting to give \( \ddot{\theta}_A, \ddot{\theta}_B \) then using updated values could be unstable. The equations were advanced in time using Beeman’s method with time step \( \Delta t \)

\[
\dot{\theta}_A^{n+1} = \theta_A^n + \dot{\theta}_A^n \Delta t + \ddot{\theta}_A^n \Delta t^2 / 2 \tag{14a}
\]

\[
\dot{\theta}_B^{n+1} = \theta_B^n + \dot{\theta}_B^n \Delta t + \ddot{\theta}_B^n \Delta t^2 / 2 \tag{14b}
\]

\[
\dot{x}_0^{n+1} = x_0^n + \dot{x}_0^n \Delta t + x_0^n \Delta t^2 / 2 \tag{14c}
\]

\[
\dot{z}_0^{n+1} = z_0^n + \dot{z}_0^n \Delta t + z_0^n \Delta t^2 / 2 \tag{14d}
\]

\[
\dot{\theta}_A^{n+1} = \theta_A^n + \dot{\theta}_A^n \Delta t \tag{14e}
\]

\[
\dot{\theta}_B^{n+1} = \theta_B^n + \dot{\theta}_B^n \Delta t \tag{14f}
\]

\[
\dot{x}_0^{n+1} = x_0^n + \dot{x}_0^n \Delta t \tag{14g}
\]

\[
\dot{z}_0^{n+1} = z_0^n + \dot{z}_0^n \Delta t \tag{14h}
\]
\[ \frac{dz_{O}^{n+1}}{dO} = \frac{dz_{O}^{n}}{dO} + \frac{dz_{O}}{dO} \Delta t \]  

(14h)

As previously stated, the device is restrained by the light mooring line in the experiments giving horizontal motion less than about 1 cm in these tests and for convenience the end of a time step \( t_0 \) is returned to zero in the model.

The WAMIT coefficients are for all cross coupled terms between floats as well as for the directly coupled (diagonal) terms which have greatest magnitude. There are thus \((3n)^2\) non-zero coefficients for \( n \) floats with 3 modes (heave, surge and pitch) for radiation damping and added mass and with \( 3n \) coefficients for diffraction forces. Forming a direct formulation for each of \( \hat{\theta}_A, \hat{\theta}_B, x_O, z_O \) with all cross coupled terms would be tedious to generalise if in fact possible. However, the dominant diagonal terms in added mass for each of \( \hat{\theta}_A, \hat{\theta}_B, x_O, z_O \) may be removed from each of \( H_i, V_i, M_i \) and added to the LHS of Eqs. (7), (9), (11) and (13). An iteration is still required with updated values of \( \hat{\theta}_A, \hat{\theta}_B, x_O, z_O \) for terms on the RHS but this should converge with less than 10 iterations (default value). The radiation damping and diffraction force terms were not modified in the iteration. A time step size of \( \Delta t \) was sufficiently small to give converged results (to plotting accuracy).

Hydrodynamic moments and forces are defined using WAMIT notation as shown in Table 1.

With \( \theta_A = \theta_i, i = 1, n_A \) for body A, for each float \( i = 1, n \) moments are defined by:

\[
M_i = M_{01}(\theta_{ij}) = \sum_{j=1}^{n} A_{ij}(\theta_{ij}) \cdot \hat{\theta}_j - \sum_{j=1}^{n} B_{ij}(\theta_{ij}) \cdot \dot{\theta}_j
\]

\[
- \sum_{j=1}^{n} A_{ij}(\theta_{ij}) \cdot \hat{\theta}_j - \sum_{j=1}^{n} B_{ij}(\theta_{ij}) \cdot \dot{\theta}_j - \sum_{j=1}^{n} A_{ij}(\theta_{ij}) \cdot \hat{x}_j - \sum_{j=1}^{n} B_{ij}(\theta_{ij}) \cdot \dot{x}_j
\]

(15)

The restoring heave force and pitch moment for a single float are given by: \( V_{rest} = -\rho g \pi r^2 z \) and \( M_{rest} = -\rho g \pi r^3 \theta \) where \( r \) is float radius \( (H_{rest} = 0) \).

And total power \( P = \sum_{i=1}^{n} B_{mech} \hat{\theta}_i^2 \)  

(18)

The formulation above is for a single frequency and here we input a wave spectrum of JONSWAP form and added mass and radiation damping are defined using Cummins method. For an example variable \( x \) we have

\[
\sum_{j=1}^{n} \left( (m_{ij} + A_{ij}^{\infty}) \chi_j(t) + \int_{-\infty}^{t} L_j(t-\tau) \chi_j(\tau) d\tau \right) = f_j(t)
\]

(19)

where \( f \) includes all forces: excitation, restoring and PTO; \( i \) and \( j \) denote mode with all combinations for surge, heave and pitch for the floats, \( A_{ij}^{\infty} \) is added mass for infinite frequency replacing the added mass terms in Eqs. (15)–(17) and the impulse response function for radiation damping is given by

\[
L_j(t) = \frac{2}{\pi} \int_{0}^{\infty} B_j(\omega) \cos(\omega t) d\omega
\]

(20)

where \( B_j \) is the radiation damping for angular wave frequency \( \omega \). The impulse response function \( L_j \) is precomputed and stored. The lower time limit of \(-\infty \) in Eq.(19) was set at \(-4T_p \) or \(-2T_p \) making negligible difference. The convolution term (2nd term on LHS in Eq.(19)) replaces the damping terms for each mode in Eqs. (15)–(17).

The equation set with numerical solution is thus complete and proved stable and convergent.

3. Experimental comparison

Experiments were undertaken in the COAST (coastal, ocean and sediment transport) laboratory of Plymouth University. The flume is 15.5 m wide, 35 m long and 1.0 m deep for these studies and has been used for previous versions of M4: Stansby et al. [2], Sun et al. [5], Santo et al. [7]. The new device dimensions were shown in Fig. 1 and a sketch in the basin is shown in Fig. 3. The wave probes measure surface elevation with accuracy of about 1 mm.

The standard pneumatic actuator or damper (Norgren Type RM/8816/M/100) shown in Fig. 1 was almost linear, although the damping factor varied from one wave case to another. The force in the actuator was measured with a load cell (Omega LDME-10N with perfect linear calibration within measurement accuracy) and converted to a moment \( M_b \) about hinge O (equivalent to \( M_{mech} \)) by multiplying by the lever arm. The relative angle between column and beam \( \theta \) was measured using an incremental shaft encoder (Wachendorff 10000 PPR TTL) with a resolution of 2000 points per revolution (0.0031 rad). The sampling frequency for surface elevation was 128 Hz, while for the other signals 200 Hz. The damping factor was determined by post processing the damping moment assumed to be of the form \( M_{eq} = B_0 + B_1 \theta + B_2 \dot{\theta} \). The least squares goodness-of-fit \( R^2 \) was always greater than 0.9 and generally around 0.95. An example of the time variation of moment at the hinge \( M_b \) is shown in Fig. 4. The inertial component with \( B_0 \) was very small in relation to that due to body masses and the small mean \( B_0 \) does not contribute to damping; both are ignored in the numerical model and moment is simplified as \( M_b = B_0 \theta + B_2 \dot{\theta} \) and an example fit is shown in Fig. 4.

Due to limitations of regular wave testing in flumes only irregular waves were input, of JONSWAP form with spectral peakedness factor \( \gamma = 1 \). It will be noticed in Fig. 5 that there is a small mooring buoy (red), 10 cm in diameter, to which the device is attached by a light string which restricts the longitudinal position to within about
1 cm, as already stated. The mass and inertia for each component are shown in Table 2.

Results were obtained for $H_s$ in the range 0.03 m to 0.08 m as listed in Table 3. Examples of standard JONSWAP spectra have been compared in Fig. 5 with the spectra of the measured data at Probe 1 shown in Fig. 3. There is a wave generator frequency cut off at 2 Hz and agreement is very close for $T_p = 1.2$ and 1.6 s but not for 0.8 s which is associated with the frequency cutoff; the measured spectra are input into the linear diffraction model. Figs. 6 and 7 show variations of CWR and rms $\theta_r$ with $T_p$ respectively. CWR is capture width ratio defined as $P_w/P_m/L_c$ where $P_m$ is the average total power, $P_w$ is wave power per metre width of wave crest (obtained from the measured spectra) and $L_c$ is the wavelength associated with the energy period $T_c$. In a practical power take off system, power would be smoothed but this is not considered here; we basically assume this does not incur energy loss. The modelled CWR curves would be independent of $H_s$ for constant $B_{mech}$ but $B_{mech}$ varies somewhat as shown in Table 3 and the curves show some variation. Note CWR is sometimes normalised by body width but this does not generalise performance enabling device comparison, e.g. the theoretical maximum CWR for a point absorber in heave and pitch and/or surge is $3/2\pi$ [8] and for a slender two-raft wave energy converter is $4/3\pi$ [9], providing useful baselines. It can be seen that measured rms $\theta_r$ is in close agreement with linear diffraction modelling for $H_s \approx 0.035$ m and 0.05 m, although overestimating for $H_s \approx 0.07$ m, and linear diffraction modelling generally slightly overestimates CWR, consistent with the frequency domain results which did not cover the larger $H_s$ [5]. However intermittent, sometimes substantial, roll could occur to a greater degree than for previous configurations which is clearly undesirable, although it should be mentioned that roll does not affect power capture or response within a linear diffraction context. To eliminate roll outrigger buoys were added to the stern float as shown in Fig. 8 in blue. Both mooring buoy and outrigger buoys were of 10 cm diam-
eter. In still water their bases just touch the surface and in motion move in and out of the water. These tests were made in the smaller Manchester 5 m wide flume of 0.45 m depth giving close to the shallow depth wave conditions rather than the intermediate conditions intended for this application; these results showed that the model now slightly underestimates experimental CWR. The influence of the outriggers in generating a buoyancy force was included in the model but only had a very small effect and their influence is certainly more complex. Further study was not undertaken here where we are concerned with power absorption by a large number of floats. These results however show that the linear diffraction modelling is approximately accurate.

4. Multi-float results

Results with an increased number of floats were obtained from linear diffraction modelling. Nine additional configurations have been analysed as shown in Fig. 9. Note with two or more mid floats roll will not occur. For ease of comparison $B_{\text{mech}}$ is fixed at 6 Nm (giving CWR close to maxima); depth is fixed at 1 m and $H_s = 0.04$ m. It can be seen that CWR and average power generally increases as the number of floats increases in Figs. 10 and 11 respectively while relative angular motion shows little consistent trend with float configuration, generally varying by ±20% for a given $T_p$, shown in Fig. 12. That increasing the number of floats from 3 to 8 increases average power by a factor of about 3 suggests that larger systems will be more cost effective for electricity generation. This may be
assessed approximately by determining energy capture through scatter diagrams for various sites and estimating the cost of the system. This has previously been undertaken for the three-float system for eight sites [10]. Here we choose 4 sites on western Atlantic coasts (Wavehub in SW England, Belmullet in W Ireland, Death Coast in N Spain and Leixões in Portugal) where swell is likely to be prominent [11] and we assume waves are long-crested with $\gamma \approx 3.3$. Scatter diagrams are provided for Wavehub in Van Nieuwkoop et al. [12], for Belmullet in Dalton et al. [13], for Death Coast in Iglesias and Carballo [14] and for Leixões in Silva et al. [15]. A capacity factor of 1/3 is assumed for each sea state in a scatter diagram so average power does not exceed three times the annual average; this is a typical value for offshore wind enabling comparison and in fact it has insignificant effect on annual energy capture [16]. To estimate the cost of electricity, capital cost (CAPEX) is assumed to be a fixed multiple of the cost of steel. The CAPEX involves all subsystems or components of the wave energy converter: the structure and prime mover, the PTO, foundations, moorings, installation and grid connection. To estimate structural mass we assume steel mass is proportional to float surface area with a constant steel thickness of 10 mm. The baseline is 2.2 t designed for a device 5 times the laboratory scale referred to here; this includes steel beams and columns. This device was designed more for ease of manufacture than economy, e.g. a steel thickness of 5–6 mm was used to facilitate welding.
but it gives a useful baseline from engineering drawings. A cost of £2000/tonne is assumed based on 2016 UK construction rates for fabricated steel to give structural cost which is then assumed to be 38% of the total (CAPEX) following Carbon Trust guidelines [17]. This does of course depend on country of construction and time of construction as prices fluctuate but the intention is to give representative estimates.

To determine the levelised cost of electricity (LCoE) the widely used discounting approach (e.g. [18]) is used assuming a 20 years project life and a discount rate of 15%. Expenditure also includes expenses incurred after commissioning due to operation and maintenance (OPEX). The LCoE is obtained from the ratio of the present value for all expenditure over the project lifetime and the electricity generated over the same period. As wave energy is a renewable (free) energy source for power conversion, the initial cost is the most significant expenditure. The economic assessment is based on the CAPEX with annual operation and maintenance costs (OPEX) assumed to be a percentage of the CAPEX and 5% is assumed as a representative value. For energy production an efficiency of 85% and availability of 95% are assumed to be representative values. For a design life N years and a discount rate r, LCoE is given by

\[
LCoE = \frac{CAPEX(1 + 0.05 Ann)}{(Ann \times annual\ energy\ yield)}
\]

where Annuity \( Ann = \frac{(1 + r)^N - 1}{r(1 + r)^N} \) and annual energy yield is the output from the machine. These assumptions may be considered to give conservative cost estimates. Table 4 shows values of the length scale factor (LSF), a multiple of the laboratory dimension giving the size of machine for minimum electricity cost, with corresponding annual energy yield, annual average power, rated power set equal to 3 times annual average power obtained through iteration (typical of offshore wind to enable comparison), cost per rated MW, total cost and levelised cost of electricity (LCoE) for each of the four sites. The sites of Belmullet and Death Coast have relatively long period waves as shown by the rms occurrence in Fig. 13 supporting the swell assumption. The scale relative to laboratory size for optimum energy cost is quite large at 70 and 120 respectively. For WaveHub the periods are shortest and the optimum scale of 50 suggests that the swell assumption is weakest here. The tables contain this information for various sites. The range of capacities from 3 to 8 floats is a factor of 3 for each site. For the 8 float device the capacity ranges from 3.7 MW for Wavehub to 17.3 MW for Death Coast. The LCoE is smallest for the 6 float configuration with the lowest value at Belmullet at 9 p/kWh and largest at Wavehub at 15 p/kWh.

These tables are useful to give relative rather than absolute values while they are intended to be realistic. With the larger number of floats, rated powers are greater than wind turbines in the case of Death Coast, Belmullet and Leixões with horizontal device lengths similar to the tip heights of wind turbines above water. Costs would be reduced by multiple production but other costs will fluctuate. Overall the energy costs appear worthy of consideration.

5. Discussion

This study applies linear diffraction modelling to the hydrodynamic interactions within the multi-float device M4. It is important to suppress roll which is achieved with outriggers on the 111 configuration and multiple mid floats. With rounded or hemi-spherical bases the floats are assumed to have negligible drag and this has been supported by CFD for heave response giving drag coefficients of less than about 0.3 [19]. The internal interactions are defined by relative angular response between bodies and power absorbed from a damper at the hinge point. It is assumed that there are no mechanical friction losses which have previously been shown to be negligible. Mooring forces would require 2nd order or possibly fully nonlinear modelling and here it was observed that horizontal motion was small (order 1% of a wavelength and assumed zero in the model) in these operational conditions. Breaking waves in extreme conditions would further require modelling which handles free surface overturning and preferably aeration and impact forces, such as SPH (smoothed particle hydrodynamics), e.g. [20].

For the earlier larger diameter floats frequency linear diffraction modelling showed similar trends in relation to measured rms \( \theta_2 \) and CWR. The advantage of the time domain modelling is that nonlinear PTO may be included, for example Coulomb damping or rectifying clutch for supply to an induction generator [6]. It is the intention to investigate control for optimising energy capture through various levels of MPC (model predictive control).

The advantage of such an efficient model is that more complex configurations may be readily assessed. The code is written in Fortran 77 and even with 8 floats computer time including computation of IRFs is less than 5 mins on a laptop. The generation of hydrodynamic coefficients, here using WAMIT, can however take several hours. The configurations considered here are a natural pro-
Fig. 9. Configurations of floats with notation defining each case with dimensions for laboratory scale. Bow diameter is 0.2 m, mid diameters 0.25 m, stern diameters 0.35 m. The solid bar shows a hinge, normal to plane of rotation.

gression from three in line floats. The basic principle of a single point mooring and float size increasing in diameter and volume from bow to stern is maintained. It is not certain in practice whether more complex configurations would align precisely with the wave direction; ‘vaning’ may occur. Experiments with three floats indicated that small misalignment had negligible influence on power
output [5]. With these more complex arrays alignment may be controlled through side thrusters requiring some small power input. With several stern floats each would have a separate PTO although coupling through a common shaft is also possible if desirable for PTO design. Importantly all components remain above deck for access and maintenance. While we have covered several configurations it is probable that others may have better LCoE characteristics and indeed the ‘best’ configuration may well be site dependent. The intention here is to establish methodology and potential. In extreme waves the three float configuration performed well in lab-
6. Conclusions

A linear diffraction time domain model has been shown to approximately predict power and response of the three-float line absorber M4, consistent with previous frequency domain modelling. The model is extended to multi-flots, up to 8, while maintaining a rigid triangular configuration with the bow and two or three mid floats; stern floats each with a connecting beam hinged above the mid floats enable power extraction from dampers at each hinge. Increasing the floats from 3 to 8 markedly increases capacity. By estimating energy capture through scatter diagrams for four sites capacity is estimated to be between 3.7 and 17.3 MW for the 8 float configuration assuming a capacity factor of 1/3. The minimum LCoE was however obtained with the 132 6-float system at between 15–9 p/kWh based on cost estimates as a multiple of the cost of steel. While this assumed long crested swell waves it is indicative of device performance.

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