ABSTRACT. We introduce a route choice model that incorporates the notion of choice aversion in transportation networks. Formally, we propose a recursive logit model which incorporates a penalty term that accounts for the dimension of the choice set at each node of the network. We make three contributions. First, we show that our model overcomes the correlation problem between routes, a common pitfall of traditional logit models. In particular, our approach can be seen as an alternative to the class of models known as Path Size Logit (PSL). Second, we show how our model can generate violations of regularity in the path choice probabilities. In particular, we show that removing edges in the network can decrease the probability of some existing paths. Finally, we show that under the presence of choice aversion, adding edges to the network can increase the total cost of the system. In other words, a type of Braess’s paradox can emerge even in the case of uncongested networks. We show that these phenomena can be characterized in terms of a parameter that measures users’ degree of choice aversion.

Keywords: choice aversion, recursive logit, IIA, directed networks, transportation networks.

JEL classification: D001, C00, C51, C61
1. Introduction

Discrete choice models have been used extensively to understand the behavior of participants (who we will refer to as users) in transportation networks (McFadden (1978, 1981) and Ben-Akiva and Lerman (1985)). In this context, users choose a path that minimizes their total cost of traveling. A prominent model that has arisen from this literature is the Multinomial Logit Model (MNL), whose main advantage is its tractability and closed-form choice probabilities.

Despite its popularity, the MNL presents some drawbacks when applied to the case of transportation networks. In particular, the MNL can predict unrealistic choice probabilities for paths sharing common edges in the network. The root of this problem traces back to the Independence of Irrelevant Alternatives (IIA) axiom that is required to derive the MNL.

To explain the severity of the overlapping paths problem, consider the simple transportation network displayed in Figure 1.

![Figure 1. Paths (a1, a3) and (a1, a4) share edge a1.](image)

In this network we have three paths: \((a_1, a_3)\), \((a_1, a_4)\), and \((a_2)\). Assume the cost of each path is 1. At the edge level, assume that the cost of edge \(a_1\) is \(1 - \epsilon\), the cost of edges \(a_3\) and \(a_4\) is \(\epsilon\), and, finally, the cost of edge \(a_2\) is 1.

For the setting described above, the MNL will predict that each path is chosen with probability \(1/3\). However, these choice probabilities are unrealistic when paths lack distinctiveness or independence from another. In particular, an assignment in which path \((a_2)\) is chosen
with probability $\frac{1}{2}$ and paths $(a_1, a_3)$ and $(a_1, a_4)$ are chosen with probability $\frac{1}{4}$ is more sensible. More explicitly, this latter solution takes into account the fact that paths $(a_1, a_3)$ and $(a_1, a_4)$ share the common edge $a_1$ and in terms of total cost they are equivalent to each other as well as the cost of path $(a_2)$. The reason why the MNL model cannot accommodate situations where the path costs are correlated is because the MNL relies on the property of Independence of Irrelevant Alternatives (IIA). As the discussion above shows, the IIA property is hardly satisfied even in simple transportation networks.

Recognizing this pitfall, the transportation literature has proposed several corrections to the MNL. This class of extensions is known as Path Size Logit (PSL). The idea of this class of models is to correct the problem of overlapping paths by adding an extra correlation-penalizing term to path costs. Thus, when the choice probabilities are generated through a standard MNL, the correction will account for the degree of overlapping between different paths.

While its usefulness in correction is clear, the PSL class has two problems. First, the type of correction employed by the different models do not have a theoretical justification in terms of users’ behavior. In particular, the parameters describing the corrections do not have a direct interpretation from an economic viewpoint. Second, it is not clear how PSL models can be used to carry out welfare analysis in transportation networks. For instance, it is hard to interpret and predict the changes on welfare when edges are added to, or severed from, the network.

In this paper, we propose the use of a recursive logit model which incorporates the idea of choice aversion (choice overload) in users’ behavior. In doing so we follow Lorca and Melo (2020) who adapt the approach of Fudenberg and Strzalecki (2015) to the context of directed graphs. Simply put, the choice aversion hypothesis states that an increase in the number of alternatives to choose from may lead to adverse

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1 An in-depth discussion on path correlation and IIA can be found in McFadden (1974), which introduces the canonical red bus/blue bus problem.
consequences, such as lesser motivation to actually choose or lower satisfaction ex post (cf. Sheena and Lepper (2000) and Scheibehenne et al. (2010)).

Formally, we consider a transportation network with source node $s$ and designated sink node $t$. In this setting, we model users’ behavior as a sequential choice process: when assessing an edge $a$ at some node $i \neq t$, users evaluate both the flow cost and the appropriate continuation value associated to such an edge. Following Fudenberg and Strzalecki, we introduce a term that penalizes the size of each choice set that stems subsequently from every current edge under scrutiny. In particular, when considering an edge $a$ at node $i$, users will penalize the number of outgoing edges at $i$. In other words, when facing a set of alternatives in order to depart from a specific node, users incorporate the size of the ensuing choice set when they appraise the continuation value of each outgoing edge. Formally, at each node $i \neq t$, we consider the penalty $\kappa \log |A_i^+|$, where $|A_i^+|$ is the cardinality of the set of outgoing edges at node $i$ and $\kappa \geq 0$ is a parameter that captures users’ choice aversion degree.

We make three contributions. First, we show that a recursive logit with choice aversion can overcome the problem of overlapping in transportation networks. In particular, the parameter $\kappa$ plays a critical role in the form of the correction. We show that our model performs as well as the recent Adjusted PSL model introduced by Duncan et al. (2020). However, our correction has two main advantages. First, it is a simple correction based on users’ optimal behavior. Second, the parameter $\kappa$ has a clear interpretation in terms of users’ attitude with respect to size of choice sets.

In our second contribution, we show how our model captures violations of regularity (Luce and Suppes (1965)). Formally, we show that removing an edge in a particular node can decrease the choice probabilities of some paths in the network. To grasp how this result works, we note that removing an edge $a$ at node $i$ is equivalent to removing the set of all paths in which $a$ is a member. When removing an edge
in the traditional MNL (i.e., the choice aversion model with \( \kappa = 0 \)), the choice probability of the remaining paths increase proportionally. However, in our model, removing the edge \( a \) not only reduces the set of available paths (passing through node \( i \)) but also decreases the choice aversion costs associated to these paths. This latter reduction makes the set of paths passing through node \( i \) comparatively less expensive than those not using node \( i \). As a consequence, the path choice probabilities of paths passing through node \( i \) increase due to the reduction in the set of available paths and the choice aversion cost reduction.

We formalize the failure of regularity in terms of a precise relationship between the parameter \( \kappa \) and the choice probabilities of paths using the node where the edge is removed. As far as we know, this result is new to the literature on recursive models in transportation networks.

In our final contribution, we study how our model can capture a Braess’s-like paradox. In particular, we show how adding edges to the network can decrease users’ welfare. Similarly to regularity failure, we also characterize this result in terms of the degree of choice aversion \( \kappa \). Unlike existing models and extensions, the behavioral foundation of the choice aversion model and its formulation allows for a tradeoff between instantaneous route costs and choice aversion penalization such that decreases in welfare can be observed even in the absence of congestion. To our knowledge, this is also a novel result that can shed light on the design of transportation networks and its effects on welfare.

1.1. Related Literature. Recursive logit models have been studied by Baillon and Cominetti (2008), Fosgerau et al. (2013), and Mai et al. (2015) among others.\(^2\) Our paper differs from theirs in at least two dimensions. First, we extend their recursive approach to incorporate the notion of choice aversion. Second, we show how our model can handle the problem of path overlapping, violations of regularity, and Braess’s-like paradoxes. With respect to PSL models, the existing literature is

\(^2\)For an up to date survey on recursive models in traffic networks we refer the reader to Zimmermann and Frejinger (2020).
extensive (Ben-Akiva and Bierlarie (1999) and Freijinger and Bierlarie (2007)), and Duncan et al. (2020) present an up-to-date discussion of the PSL approach. In addition, they propose an alternative correction denominated as the Adapti ve Path Size Logit (APSL) model. Our results differ from theirs in the type of the behavioral foundation we use.

From a behavioral standpoint, a similar approach to this paper is found in Fosgerau and Jiang (2019) and Jiang et al. (2020), who incorporate a rational inattention model into the context of transportation networks. While choice aversion and rational inattention are interconnected in terms of information processing, we model a particular variant of costly decision-making in the form of aversion to increasing choices, rather than mutual information through observing a signal. This modeling choice allows us to generate clear path choice predictions, violations of regularity, and Braess’s-like paradox phenomena.

The rest of the paper is organized as follows. Section 2 introduces the recursive logit model with choice aversion. Section 3 explores the use of choice aversion in the path choice model and in comparison to existing Path Size Logit (PSL) models. Section 4 discusses the failure of regularity. Section 5 discusses a type of Braess’s paradox observed as a consequence of choice aversion. Section 6 concludes.

2. Recursive logit in Directed Networks

In this section we propose a recursive discrete choice model in directed networks. Formally, we model a set of users as solving a dynamic programming problem over a directed, acyclic graph. In a noticeable departure from previous literature, we adapt the choice aversion formulation of Fudenberg and Strzalecki (2015) into the context of directed graphs, and then we analyze the consequences on equilibria and welfare.\(^3\)

\(^3\)Throughout the text we use the term choice overload to refer to choice aversion.
2.1. Directed graphs. Consider a directed acyclic graph \( G = (N, A) \) where \( N \) is the set of nodes and \( A \) the set of edges, respectively. We denote the set of ingoing edges to node \( i \) by \( A_i^- \), and the set of outgoing edges from node \( i \) by \( A_i^+ \). We refer accordingly to the out-degree of node \( i \) as \( |A_i^+| \).

Without loss of generality, we assume that \( G \) has a single source-sink pair, where \( s \) and \( t \) stand for the source (origin) and sink (destination) nodes, respectively. Let \( j_a \) be the node \( j \) that has been reached through edge \( a \). We therefore define a path as a sequence of edges \((a_1, \ldots, a_K)\) with \( a_{k+1} \in A_{j_{a_k}}^+ \) for all \( k < K \).

The set of paths connecting nodes \( s \) and \( t \) is denoted by \( \mathcal{R} \). The set of paths connecting nodes \( s \) and \( i \neq t \) is denoted by \( \mathcal{R}_{si} \). Similarly, the set of all paths connecting nodes \( i \neq s \) and \( t \) is denoted as \( \mathcal{R}_{it} \). Let \( \mathcal{R}_i \) denote the set of paths passing through node \( i \). Finally, let \( \mathcal{R}_i^c \) denote the set of paths not passing through node \( i \).

A deterministic cost component \( c_a > 0 \) is associated with each edge \( a \in A_i^+ \) for all \( i \neq t \). Path costs are assumed to be edge additive, that is, for a path \( r = (a_1, \ldots, a_K) \in \mathcal{R} \) its associated cost is given by \( \sum_{k=1}^{K} c_{a_k} \).

We assume that at node \( s \) there is a unitary mass of network users who must choose a path from the set \( \mathcal{R} \). For the sake of exposition, the mass of users is summarized by the canonical vector \( e_s \), which has a 1 in the position of node \( s \) and zero elsewhere. The dimension of \( e_s \) is \( |N| - 1 \).

2.2. Choice aversion. We now develop a recursive logit choice model over \( G \) that incorporates choice overload by means of an specific kind of penalty on ensuing choice sets stemming from each edge appraisal. In particular, we adapt the choice aversion approach from Fudenberg and Strzalecki (2015) into the environment described by \( G \) as follows: for each \( a \in A_i^+ \) we associate a collection of i.i.d. random variables \( \{\epsilon_a\}_{a \in A_i^+} \) such that the recursive cost associated to edge \( a \) is defined...
as:

$$V_a = c_a + \mathbb{E} \left( \min_{a' \in A^+_{ja}} \left\{ V_a' + \epsilon_{a'} + \kappa \log |A^+_{ja}| \right\} \right)$$

for all $a \in A^+_i$, where $c_a$ denotes the instantaneous cost associated to edge $a$ and the term

$$\mathbb{E} \left( \min_{a' \in A^+_{ja}} \left\{ V_a' + \epsilon_{a'} + \kappa \log |A^+_{ja}| \right\} \right) = \mathbb{E} \left( \min_{a' \in A^+_{ja}} \left\{ V_a' + \epsilon_{a'} \right\} \right) + \kappa \log |A^+_{ja}|$$

is the adjusted continuation value associated to the selection of $a$. Notice that the latter term includes the factor $\kappa \log |A^+_{ja}|$, which is a penalty term that captures the size of the set $A^+_{ja}$, where $\kappa \geq 0$.

Following Fudenberg and Strzalecki (2015), we impose the following assumption on the random variables $\{\epsilon_a\}_{a \in A^+_i}$.

**Assumption 1** (Logit choice rule). At each node $i \neq t$ the collection of random variables $\{\epsilon_a\}_{a \in A^+_i}$ follows a Gumbel distribution with scale parameter $\mu = 1$.

Under this assumption, Eq. (1) can be expressed as:

$$V_a = c_a - \log \left( \sum_{a' \in A^+_{ja}} e^{-V_a'} \right) + \kappa \log |A^+_{ja}|,$$

where $- \log \left( \sum_{a' \in A^+_{ja}} e^{-V_a'} \right) + \kappa \log |A^+_{ja}|$ provides a closed-form expression for the adjusted continuation value.

Let us define $\varphi_{ja}(V) \triangleq - \log \left( \sum_{a' \in A^+_{ja}} e^{-V_a'} \right)$ for all $ja \neq t$. Accordingly Eq. (2) can be rewritten as:

$$V_a = c_a + \varphi_{ja}(V) + \kappa \log |A^+_{ja}|.$$

The previous expression deserves some remarks. First, the continuation value in Eq. (3) captures the complexity of the choice sets $A^+_{ja}$,

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4Fudenberg and Strzalecki (2015) study a recursive logit model in the context of intertemporal choice. In doing so, they consider a discount factor $\delta \in (0, 1)$. We focus on a digraph $G$ without discounting.

5See Train (2009), Chapter 3.
as measured by $\kappa \log |A_{j_a}^+|$, with $\kappa \geq 0$. Intuitively, $\kappa \log |A_{j_a}^+|$ penalizes the size of the choice sets at different nodes, where the parameter $\kappa \geq 0$ measures decision makers’ attitudes towards the size of $A_{j_a}^+$. In particular, $V_a$ is an increasing function of $\kappa$ and $\log |A_{j_a}^+|$.

Second, when $\kappa = 0$, Eq. (3) boils down to a traditional recursive logit model in which users are *choice-loving* in the sense that they always prefer to add additional items into the menu, as in the “preference for flexibility” of Kreps (1979). To see this, note that when $\kappa = 0$ the function $\varphi_{j_a}(V)$ is decreasing in $|A_{j_a}^+|$. As a consequence, the recursive cost $V_a$ is decreasing in the size of $A_{j_a}^+$. This latter feature implies that traditional recursive logit models in transportation networks (e.g. Baillon and Cominetti, 2008 and Fosgerau et al., 2013) can be associated with an intrinsic taste for plentiful options.

On the other hand, the case of $\kappa \in (0, 1)$ from an economic standpoint may be interpreted as a situation where the users prefer to include additional alternatives to the menu, provided the new options are not *too much worse* than the current average. Finally, the case $\kappa \geq 1$ is interpreted as a situation in which the users only wish to add alternatives that are perceived to be sufficiently better. In particular, the case $\kappa = 1$ captures a situation where the users want to remove choices that are worse than the average: they worry about choosing such additional alternatives by accident given appraisal costs—such as Ortoleva (2013)’s thinking aversion—that may offset the benefits of the corresponding random draw.

In sum, the parameter $\kappa$ encapsulates the scale of penalties on the set of ensuing actions arising from each nonterminal node, which unlocks keen consequences on users’ attitudes towards marginally increasing the set of edges. Following Fudenberg and Strzalecki (2015), we identify $\kappa$ as the users’ choice aversion parameter. We point out that all of our analysis extends to the case of node-specific choice aversion parameters $\kappa_i$ for all $i \in N$. We stress the relevance of allowing for heterogeneous $\{\kappa_i\}_{i \in N}$ in §3-5.
2.3. Flow allocation. Each user is looking for an optimal path connecting \( s \) and \( t \). Now, when they reach node \( i \neq t \), they observe the realization of the random costs \( V_a + \epsilon_a \) for all \( a \in A_i^+ \), and consequently choose the alternative \( a \in A_i^+ \) with the lowest cost.

This process is repeated at each subsequent node giving rise to a recursive discrete choice model, where the expected flow entering node \( i \neq t \) splits among the alternatives \( a \in A_i^+ \) according to the choice probability:

\[
\mathbb{P}(a | A_i^+) = \mathbb{P} \left( V_a + \epsilon_a \leq V_{a'} + \epsilon_{a'} \quad \forall a' \neq a \in A_i^+ \right) \quad \forall i \neq t.
\]

Due to Assumption 1, Eq. (4) can be rewritten as:

\[
\mathbb{P}(a | A_i^+) = \frac{e^{-(c_a + \varphi_{ja}(V) + \kappa \log |A_{ja}^+|)}}{\sum_{a' \in A_i^+} e^{-(c_{a'} + \varphi_{ja'}(V) + \kappa \log |A_{ja'}^+|)}} \quad \forall i \neq t.
\]

As \( \kappa \) increases, the edge choice probability \( \mathbb{P}(a | A_i^+) \) is increasingly penalized by the choice set \( |A_{ja}^+| \), reflecting the effect of choice overload onto a user’s edge cost from nodes with large choice sets. This is a fundamental difference with the traditional recursive logit model, which assumes \( \kappa = 0 \) as we mentioned before.

Mathematically, the recursive process just described induces a Markov chain over the graph \( G \), where the transition probabilities are given by Eq. (5). Let \( x_i \) be the expected flow entering at node \( i \) towards sink node \( t \). Then the flow received by edge \( a \) is given by:

\[
f_a = x_i \mathbb{P}(a | A_i^+) \quad \forall a \in A_i^+,
\]

with \( f = (f_a)_{a \in A} \) denoting the expected flow vector.

In addition, let \( \hat{\mathbb{P}} = (\hat{P}_{ij})_{i,j \neq t} \) denote the restriction to the set of nodes \( N \setminus \{t\} \). Then the expected demand vector \( x = (x_i)_{i \neq t} \) may be expressed as \( x = e_s + \hat{\mathbb{P}}^T x \) which generates the following stochastic conservation flow equations

\[
x_i = \sum_{a \in A_i^+} f_a \quad \text{for all } i \neq t.
\]
A flow vector \( f \) satisfying (7) is called feasible. It is worth mentioning that there exists a unique flow vector \( x^* \) satisfying the flow constraints (7). In fact, using Baillon and Cominetti (2008, Lemma 1) it is possible to show that \( [I - \mathbb{P}^T]^{-1} \) is well defined. Then \( x^* \) is the unique vector that satisfies \( x^* = [I - \mathbb{P}^T]^{-1}e_s \) and \( f_a = x_i^*\mathbb{P}(a|A_i^+) \) for all \( a \in A_i^+, i \neq t \).

2.4. Path choice and choice aversion. In this section we note that the solution of our recursive choice model can be equivalently written in terms of path choice probabilities. In doing so, assume that for each path \( r \in \mathcal{R} \) the cost associated to it is a random variable defined as

\[
\tilde{C}_r = C_r + \epsilon_r \quad \forall r \in \mathcal{R},
\]

where \( C_r = \sum_{a \in r} (c_a + \kappa \log |A_{ja}|) = \sum_{a \in r} c_a + \kappa \sum_{a \in r} \log |A_{ja}| \) and \( \{\epsilon_r\}_{r \in \mathcal{R}} \) is a collection of absolutely continuous random variables satisfying Assumption 1.

Under these conditions, the probability of choosing path \( r \) is defined as:

\[
\mathbb{P}_r \triangleq \mathbb{P}
\left(r = \arg\min_{r' \in \mathcal{R}} \{C_{r'} + \epsilon_{r'}\}\right) \quad \forall r \in \mathcal{R}.
\]

Equations (8) and (9) jointly define a path choice model over \( \mathcal{R} \), where we again refer to the Gumbel assumption to obtain:

\[
\mathbb{P}_r = \frac{e^{-C_r}}{\sum_{r' \in \mathcal{R}} e^{-C_{r'}}} \quad \forall r \in \mathcal{R}.
\]

However, it is well known that the path choice probability \( \mathbb{P}_r \) can be decomposed in terms of the edge probabilities (e.g., Fosgerau et al. (2013) and Lorca and Melo (2020)). Formally, we have that for each path \( r = (a_1, \ldots, a_K) \in \mathcal{R} \) with \( K \geq 2 \), the following equality holds

\[
\mathbb{P}_r = \prod_{k=1}^{K} \mathbb{P}(a_k|A^+_k)\mathbb{P}(a_{k+1}|A^+_{ja_{nk}}).
\]

The previous characterization will play a key role in next sections. Intuitively, Eq. (11) establishes that the \( \mathbb{P}_r \) can be decomposed in terms of the recursive choice probabilities. This equivalence allows us
to highlight the role and effect of the terms $\kappa \log |A_{ja}^+|$ in the path choice probabilities $P_r$.

2.4.1. Heterogenous $\kappa$. As we mentioned earlier, holding $\kappa$ fixed across nodes is not a necessity in our setup. In fact, to model user behavior accurately, it is sensible to reevaluate the assumption of a homogeneous choice aversion parameter when considering a variety of transportation network contexts. In many cases, a user might be more sensitive to choice overload at one particular node relative to another. For instance, in a context where nodes might represent locations along routes where a user must choose which direction to turn, nodes may differ across characteristics like visibility, intersection type, or lane width. If we allow the choice aversion parameter to be node-specific, then we update Eq. (8) such that $C_r = \sum_{a \in r} (c_a + \kappa_{ja} \log |A_{ja}^+|) = \sum_{a \in r} c_a + \sum_{a \in r} \kappa_{ja} \log |A_{ja}^+|$. As a result, the path choice probability $P_r$ for each path $r \in R$ takes the form

$$P_r = \frac{e^{-c_r - \rho_r}}{\sum_{r' \in R} e^{-c_{r'} - \rho_{r'}}} \quad \text{for all } r \in R,$$

where $c_r = \sum_{a \in r} c_a$ and $\rho_r \triangleq \sum_{a \in r} \kappa_{ja} \log |A_{ja}^+|$. From here, we note that the path choice probability above simplifies to Eq. (10) in the special case of $\kappa_{ja} = \kappa \geq 0$ for all $j_a \neq t$.

3. Choice aversion and the IIA property

In the context of path choice models, it is well known that the traditional MNL model is restricted by the IIA property, which does not hold in the context of route choice due to the overlapping paths problem. The main implication of overlapping paths is that the traditional
MNL produces unrealistic path choice probabilities (Ben-Akiva and Ramming and Ben-Akiva and Bierlarie (1999)).

In order to solve this problem, the route choice literature has proposed the Path Size Logit (PSL) approach. In simple terms, PSL models extend the MNL by adding a correction term to path costs which account for the degree of overlapping among paths. For instance, Ben-Akiva and Lerman, 1985; Ben-Akiva and Bierlarie, 1999, Frejinger and Bierlarie, 2007, and recently Duncan et al. (2020) propose different corrections to the MNL in order to solve the overlapping problem. From a behavioral point of view, PSL models try to correct the fact that the IIA property should be relaxed in contexts where paths are not distinct or independent.

In this section, we show how choice aversion can be seen as a natural mechanism that overcomes the problem of overlapping paths in transportation networks.

Assuming $\kappa_i = \kappa$ for all $i \neq s, t$, let us rewrite Eq. (11) in §2 as follows:

\begin{equation}
Pr_r = \frac{e^{-c_r - \kappa \gamma_r}}{\sum_{r' \in R} e^{-c_{r'} - \kappa \gamma_{r'}}} \quad \text{for all } r \in R,
\end{equation}

where $c_r = \sum_{a \in r} c_a$, $\gamma_r \triangleq \sum_{a \in r} \log |A_{ja}^+|$, and $\kappa \geq 0$ is the (homogeneous) choice aversion parameter.\footnote{Note that equation (12) simplifies to (13) when the choice aversion parameters are fixed across nodes. In this case, $\rho_r = \kappa \gamma_r$.}

It is easy to see that for $\kappa > 0$, the term $\kappa \gamma_r$ can be seen as a penalty term that accounts for the size of the choice set at each of the nodes accessed along path $r$.\footnote{Note that paths passing through a common node $i$ would share the penalty $\kappa \log |A_i^+|$.}

Formally, the term $\kappa \gamma_r$ accounts for the degree of overlapping among different paths. In particular, the presence of $\kappa \gamma_r$ accounts for the fact that, in our recursive model, the IIA property does not hold.
To see how our model works, consider two paths \( r \) and \( r' \) with associated probabilities \( P_r \) and \( P_{r'} \) respectively. Computing the probability ratio between \( r \) and \( r' \) we get:

\[
\frac{P_r}{P_{r'}} = \frac{e^{-c_r}}{e^{-c_{r'}}} \times \frac{e^{-\kappa \gamma_r}}{e^{-\kappa \gamma_{r'}}}
\]

(14)

Note that expression (14) shows that the ratio \( \frac{P_r}{P_{r'}} \) depends on the ratio between the costs associated to paths \( r \) and \( r' \) times the ratio between \( \kappa \gamma_r \) and \( \kappa \gamma_{r'} \). This latter term incorporates information about \( r \) and \( r' \) regarding choice sets at each node crossed by these paths. This information is captured by the terms \( \kappa \gamma_r \) and \( \kappa \gamma_{r'} \).

From (14) it follows that adding or deleting links in a particular node contained (crossed) in \( r \) (or \( r' \)) will affect the ratio \( \frac{P_r}{P_{r'}} \), and as a consequence \( \frac{P_r}{P_{r'}} \) will be modified. In other words, \( \frac{P_r}{P_{r'}} \) depends not only on \( r \) and \( r' \), but also on other paths passing by the same nodes as \( r \) and \( r' \) do. Note that when \( \kappa = 0 \), then \( \frac{P_r}{P_{r'}} = \frac{e^{-c_r}}{e^{-c_{r'}}} \) and we recover the IIA property in the MNL model. Similarly, if paths \( r \) and \( r' \) pass through the same nodes, we get \( \kappa \gamma_r = \kappa \gamma_{r'} \), so that \( \frac{P_r}{P_{r'}} = \frac{e^{-c_r}}{e^{-c_{r'}}} \). Thus, the factor \( \frac{e^{-\kappa \gamma_r}}{e^{-\kappa \gamma_{r'}}} \) captures the degree of overlapping between different paths.

In order to see how our model overcomes the overlapping problem, we study a concrete case. Let us reintroduce the network in Figure 2 where the set of paths is given by \( \mathcal{R} = \{r_1, r_2, r_3\} \) with \( r_1 = (a_1, a_3) \), \( r_2 = (a_1, a_4) \), and \( r_3 = (a_2) \). For this example, we assume \( c_{a_1} = 1.9 \), \( c_{a_3} = c_{a_4} = 0.1 \), and \( c_{a_2} = 2 \).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{logit_path_choice}
\caption{Logit path choice.}
\end{figure}
As we have previously discussed for this network structure, paths \(r_1\) and \(r_2\) overlap, sharing the common edge \(a_1\). Under this parameterization of instantaneous cost, coupled with \(\kappa = 0\), it follows that \(c_{r_1} = c_{r_2} = c_{r_3} = 2\), and, consequently, the logit choice rule \((13)\) assigns one third of flow to each path. In other words, with \(\kappa = 0\), we get \(P_{r_1} = P_{r_2} = P_{r_3} = \frac{1}{3}\). However, since paths \(r_1\) and \(r_2\) are identical, the assignment \(P_{r_1} = P_{r_2} = \frac{1}{3}\) and \(P_{r_3} = \frac{1}{2}\) is a more appropriate allocation.

However, as \(\kappa \to 1\), the logit model with choice aversion predicts a flow allocation approaching \((\frac{1}{4}, \frac{1}{4}, \frac{1}{2})\). To understand this, we look at the probability ratio between paths \(r_1\), \(r_2\), and \(r_3\):

\[
\frac{P_{r_1}}{P_{r_2}} = 1 \quad \text{and} \quad \frac{P_{r_1}}{P_{r_3}} = \frac{P_{r_2}}{P_{r_3}} = e^{-\kappa \log 2}.
\]

From the previous expression, it follows that the value of \(\kappa\) will affect the ratios \(\frac{P_{r_1}}{P_{r_3}}\) and \(\frac{P_{r_2}}{P_{r_3}}\), but not \(\frac{P_{r_1}}{P_{r_2}}\). In particular, Eq. \((15)\) shows that the ratios \(\frac{P_{r_1}}{P_{r_3}}\) and \(\frac{P_{r_2}}{P_{r_3}}\) are decreasing in \(\kappa\). In other words, as the degree of choice aversion increases, the probabilities \(P_{r_1}\) and \(P_{r_2}\) decrease while the probability associated to \(r_3\) increases.

Figure 3 shows how the route choice probabilities in Figure 2 respond to \(\kappa \in [0, 2.5]\) under choice aversion.

Now assume that a new edge \(\hat{a}\) is added at node \(i_1\). This implies that the new set of paths is \(R = R \cup \{\hat{a}\}\). In terms of Eq. \((15)\), adding \(\hat{a}\) implies that:

\[
\frac{P_{r_1}}{P_{r_2}} = 1 \quad \text{and} \quad \frac{P_{r_1}}{P_{r_3}} = \frac{P_{r_2}}{P_{r_3}} = e^{-\kappa \log 3}.
\]

This latter expression makes explicit the fact that changes in \(R\) will change the probability ratio between different paths.

3.1. **Choice aversion compared to PSL models.** What the analysis just laid out shows—which applies to the general case of directed networks—is that from the vantage point of path selection, choice aversion is a robust way to derive path choice probabilities, even in the case of overlapping of different routes. This robustness feature makes our
approach similar to the class of PSL models, which is widely used in applied work (e.g. Duncan et al. (2020)).

In this section we compare how our approach compares with some of the best well-known PSL models. Figure 4 displays a network topology originally featured in Fosgerau et al. (2013). We test the performance of the choice aversion model on this network in calculating path choice probabilities. We assume that this is a directed acyclical graph, where the set of paths is given by \( \mathcal{R} = \{r_1, r_2, r_3, r_4\} \) with 
\[
\begin{align*}
 r_1 &= (12, 23, 35), \\
 r_2 &= (12, 23, 34, 45), \\
 r_3 &= (12, 24, 45), \\
 r_4 &= (15).
\end{align*}
\]

This graph represents a more complex uncongested network topology where the cost of all routes \( r_i \in \mathcal{R} \) are equal. Thus, the only difference in routes 1 through 4 are the choice sets at each node along the path. The MNL model (equivalent to \( \kappa = 0 \) in the choice aversion model) predicts equal path choice probabilities, i.e., \( P_{r_i} = \frac{1}{4} \) for \( i = 1, 2, 3, 4 \).

Figure 5(A) shows choice probabilities as \( \kappa \) increases from 0 to 10. As the choice aversion penalization grows larger, \( P_{r_4} \) approaches 1, since it is the only route with no choice set for the user after node 1. On

\footnote{For ease of exposition we provide the details of the PSL models discussed in this section in Appendix A.}
the other hand, while $r_1$ and $r_2$ have equivalent choice aversion terms, $r_3$ has the advantage of lacking an additional downstream choice in comparison, allowing $p_{r_3} > p_{r_1} = p_{r_2}$ for $\kappa > 0$ until the choice aversion parameter grows so large that the respective route choice probabilities converge and approach zero.

The prediction in Figure 5(A) differs from route choice probabilities generated by many PSL models and extensions, including the models discussed in Fosgerau et al. (2013) and the Adaptive PSL model proposed by Duncan et al. (2020). This latter model is shown in Figure 5(B). For these models, path correction cost is based on correlation of routes through link-path incidence rather than a penalization for size.
of choice sets along the path. As a result, we see that for most PSL models, including the Adaptive PSL model, $P_{r_1} = P_{r_3}$ for all values of $\beta$, not $P_{r_1} = P_{r_2}$ as in the choice aversion model.

The behavioral nature of the choice aversion model allows for a different type of correction than most PSL models. The choice aversion model and other PSL models work in a similar way in the sense of overcoming the overlapping path problem. However, the choice aversion model has a simple and clear behavioral interpretation. This feature sets the choice aversion model apart from other PSL models both in terms of interpretation and performance.

3.2. RNL and IIA. In this section we compare our approach with the Recursive Nested logit (RNL) model (Mai et al. (2015)). Similar to our approach, the RNL does not impose the IIA property. In particular, in order to allow for correlation among paths, Mai et al. (2015) extend the recursive logit model by allowing the scale parameter $\mu_a$ of the Gumbel-distributed random variables $\{\epsilon_a\}_{a \in A}$ to be link-specific (in contrast with our Assumption 1). Under this more general assumption, Mai et al. (2015) show that the RNL allows for situations where the IIA property does not hold. They show that the RNL generates more realistic path choice probabilities.

Formally, the main difference between the recursive logit and the RNL model is that the continuation values $\hat{\varphi}$ are defined as

$$\hat{\varphi}_{jb}(V) = \mathbb{E}\left(\min_{a \in A^b_j} \{c_a + \hat{\varphi}_{ja} + \mu_{jb} \epsilon_a\}\right) \quad \forall b \in A.$$  \hfill (16)

From (16) it is easy to see that the RNL allows for heterogeneous scale parameters $\mu_{jb}$ that are edge-specific. This modification allows for correlation between alternatives and payoffs. They show that the IIA property holds for paths within the same nest but not for paths in different nests. More importantly, Mai et al. (2015) show that the RNL can be seen as a solution to the overlapping paths problem.

Note that given the structure of the NRL, we can extend our choice aversion model to this context. To see this note that Eq. (16) can be
rewritten as:

\[
\hat{\varphi}_{jb}(V) = \mathbb{E}\left( \min_{a \in A_{jb}^+} \{c_a + \hat{\varphi}_{ja} + \mu_{jb}\epsilon_a\} \right) + \kappa \log |A_{jb}^+| \quad \forall b \in A.
\]

Thus choice aversion can be combined with the RNL approach in a simple way.

However, there are at least two important differences between the choice aversion and the RNL models. First, in our approach, the behavioral mechanism that allows for situations where IIA does not hold is the idea of choice aversion (or choice overload). Second, as we shall see in §4, our model allows for the failure of the regularity property in the path choice probabilities. Thereby, our approach can be seen as more flexible than the RNL model.

4. Choice aversion and the failure of regularity

In the standard MNL (\(\kappa = 0\)), adding an additional alternative to the choice set cannot increase the probability that an existing action is selected (and vice versa). This is known as the regularity property in discrete choice models (Luce and Suppes (1965, Def. 26)).

In this section, we show that there exists a critical value of \(\kappa\), which allows us to understand how varying the network \(G\) can generate violations of regularity. In order to gain some intuition, we discuss a simple network that allows us to show how regularity may break down. In particular, we study one of the nested network structures considered in Mai et al. (2015). Figure 6 replicates their Figure 3. This network has four nodes \{A, B, C, D\} and eight links between the source node A and sink node D. There are six possible paths from \(o\) to \(d\): \((o, a, a_1, d), (o, a, a_2, d), (o, a, a_3, d), (o, b, b_1, d), (o, b, b_2, d), \) and \((o, b, b_3, d)\). We denote these path by \(r_1, r_2, r_3, r_4, r_5, \) and \(r_6\), respectively.

Tables 1 and 2 display route choice probabilities when links \(a_1, a_2, b_1,\) and \(b_2\) are removed from the nested network in Figure 6. Table 1 shows the results when choice parameters are homogeneous (i.e., \(\kappa_B = \)
\[ \kappa_C = \kappa = 1 \), and Table 2 shows the results when \( \kappa_C = 2 \), all else held equal.

\[
\begin{array}{ccc}
\text{Route} & \text{Homogen.} & \text{Route Choice Probabilities when Edge is Removed} \\
\hline
r_1=(o, a_1, d) & 0.4485 & a_1 \\
r_2=(o, a_2, d) & 0.1650 & 0.6174 (38\%) b_1 \\
r_3=(o, a_3, d) & 0.0607 & 0.3726 (126\%) b_2 \\
r_4=(o, b_1, d) & 0.0607 & 0.1371 (126\%) b_2 \\
r_5=(o, b_2, d) & 0.1001 & 0.0607 (51\%) b_2 \\
r_6=(o, b_3, d) & 0.1650 & 0.2484 (51\%) b_2 \\
\end{array}
\]

From Table 1, we note that when link \( a_1 \) is removed, the choice probabilities of remaining paths \( \{r_2, \ldots, r_6\} \) increase. Note that the probabilities of paths \( r_2 \) and \( r_3 \) increase in the same proportion (126%). Similarly, the probabilities of paths \( r_4, r_5, \) and \( r_6 \) also increase in the same proportion (51%). Each increase is even more pronounced in Table 2 where \( \kappa_C = 2 \). However, in both tables, the increase is not
proportional across paths crossing different nodes. This feature comes from the IIA property, which holds within nodes (nests) but not across them.

Now, consider the case of removing edge $a_2$. In Table 1, the probabilities of paths $r_1$ and $r_3$ increase proportionally (38%). However, the probability of paths $r_4$, $r_5$, and $r_6$ actually decrease. This counterintuitive result is a consequence of the fact that removing edge $a_2$ not only reduces the number of available paths but also decreases the cost associated to paths $r_1$ and $r_3$. In other words, this effect can be decomposed into two parts. First, the IIA property implies that the probabilities of $r_1$, $r_3$, $r_4$, $r_5$, and $r_6$ will increase upon removing edge $a_2$. The second force behind this counterintuitive result is that removing edge $a_2$ reduces the choice set when taking paths $r_1$ and $r_3$. For a choice averse user, this latter effect implies that $\kappa \log 3$ reduces to $\kappa \log 2$, which makes paths $r_1$ and $r_3$ relatively more attractive than $r_4$, $r_5$, and $r_6$, such that the probabilities of $r_4$, $r_5$, and $r_6$ decrease. When this second effect dominates, we will observe the failure of regularity associated with removing edge $a_2$.

It may be tempting to think that the effect in path choices described above may be driven by the assumption that $\kappa$ is homogeneous. However, Table 2 shows that a similar pattern occurs when we consider $\kappa_B = 1$ and $\kappa_C = 2$. In this case, the failure of regularity is even more pronounced through relatively larger changes to remaining path probabilities.

We formalize the previous intuition in Proposition 1 below. In doing so, recall that $\mathcal{R}_i$ is the set of paths passing through node $i$. Similarly the set of paths not passing through node $i$ is defined as $\mathcal{R}_i^c$. In addition, define $\mathcal{R}_{ia}$ as the set of paths passing through node $i$ after removing edge $a$ at node $i$. We note that $\mathcal{R}_{ia} \subseteq \mathcal{R}_i$. Note that before and after removing and edge the set $\mathcal{R}_i^c$ is the same.
We remark that after the edge $a$ at node $i$ is removed, the cost of paths in $\mathcal{R}_{ia}$ can be expressed as

$$\bar{C}_r = C_r + \Delta_i \quad \forall r \in \mathcal{R}_{ia}$$

where $\Delta_i \triangleq \kappa_i \left( \log |A_i^+| - 1 | - \log |A_i^+| \right)$. Note that $\Delta_i$ is constant across all paths in $\mathcal{R}_{ia}$. Let $\mathbb{P}(\mathcal{R}_i) = \sum_{r \in \mathcal{R}_i} P_r$ and $\mathbb{P}(\mathcal{R}_{ia}) = \sum_{r \in \mathcal{R}_{ia}} P_r$.

**Proposition 1.** The probability of choosing a path $r \in \mathcal{R}_i^c$ decreases after removing an edge $a \in A_i^+$ if

$$(17) \quad \kappa_i > \frac{\log \left( \frac{\mathbb{P}(\mathcal{R}_i)}{\mathbb{P}(\mathcal{R}_{ia})} \right)}{\log \left( \frac{|A_i^+|}{|A_i^+| - 1} \right)}.$$ 

**Proof.** Without loss of generality, fix a path $r \in \mathcal{R}_i^c$. Let $\tilde{P}_r$ and $\bar{P}_r$ be the probability of choosing path $r$ before and after removing edge $a$ at node $i$, respectively. We want to show under what conditions we have $\tilde{P}_r - P_r < 0$ for any path $r \in \mathcal{R}_i^c$.

Note that $\tilde{P}_r$ can be written as:

$$\tilde{P}_r = \frac{e^{-C_r}}{\sum_{l \in \mathcal{R}_{ia}} e^{-C_l - \Delta_i} + \sum_{k \in \mathcal{R}_i^c} P_k e^{-C_k}} \quad \forall r \in \mathcal{R}_i^c$$

Dividing the numerator and denominator by $\sum_{l \in \mathcal{R}_i} e^{-C_l}$, we find that:

$$\bar{P}_r = \frac{P_r}{\sum_{l \in \mathcal{R}_{ia}} P_l e^{-\Delta_i} + \sum_{k \in \mathcal{R}_i^c} P_k} \quad \forall r \in \mathcal{R}_i^c$$

From the previous expression, it follows that $\tilde{P}_r - P_r$ can be expressed as:

$$\tilde{P}_r - P_r = P_r \left( \frac{1}{\sum_{l \in \mathcal{R}_{ia}} P_l e^{-\Delta_i} + \sum_{k \in \mathcal{R}_i^c} P_k} - 1 \right)$$

Since $P_r > 0$, it follows that $\tilde{P}_r - P_r$ is negative iff

$$\left( \frac{1}{\sum_{l \in \mathcal{R}_{ia}} P_l e^{-\Delta_i} + \sum_{k \in \mathcal{R}_i^c} P_k} - 1 \right) < 0.$$ 

Rearranging this expression, we get

$$1 - \sum_{k \in \mathcal{R}_i^c} P_k < \sum_{l \in \mathcal{R}_{ia}} P_l e^{-\Delta_i}. $$
Using the fact that $P(R_i) = \sum_{l \in R} P_l = 1 - \sum_{k \in R'} P_k$, we get

$$\frac{P(R_i)}{P(R_{ia})} < e^{-\Delta_i}.$$ 

Noting that $e^{-\Delta_i} = e^{\log(|A_i^+|/|A_i^+ - 1|)\kappa_i} = \left(\frac{|A_i^+|}{|A_i^+| - 1}\right)^{\kappa_i}$, then we conclude:

$$\kappa_i > \frac{\log \left( \frac{P(R_i)}{P(R_{ia})} \right)}{\log \left( \frac{|A_i^+|}{|A_i^+| - 1} \right)}.$$ 

Some remarks are in order. First, Proposition 1 provides a simple condition to know when removing an edge at node $i$ can decrease the probability of paths not crossing node $i$. Condition (17) captures the fact that removing an edge at node $i$ will not only modify the set of available paths but also the choice aversion cost. Concretely, condition (17) establishes a lower bound on the parameter $\kappa_i$ in terms of the path choice probabilities $P(R_i)$ and $P(R_{ia})$ and the magnitude of $|A_i^+|$ and $|A_i^+ - 1|$. To the best of our knowledge this result is new to the literature on recursive discrete choice models in transportation networks.

Second, we note from a practical point of view that in order to test whether condition (17) is satisfied or not, we only need the information contained in the original network. For instance, we can use the information contained in the estimation of the probabilities $P_r$ for each $r \in R_i$ (before removing edge $a$) to understand how users react to changes on the topology.

Third, Proposition 1 predicts that removing edge $a \in A_i^+$ can decrease the probability of paths passing through nodes different from $i$. We have identified this phenomenon as a failure of regularity in the sense of Luce and Suppes (1965). However, behind this result is the factor that reducing the cardinality of $A_i^+$ reduces the cost associated to choice aversion. This cost reduction can overweight the impact of reducing the number of paths available. In the context of rational inattention, Matějka and McKay (2015) have shown that regularity may fail in the logit model. Our result is different in two aspects. First,
we study a recursive logit model with choice aversion in the context of transportation networks. Second, our result highlights the role of choice aversion by providing a specific condition on $\kappa_i$. Matějka and McKay (2015) use the idea of information acquisition in order to derive their result.\(^\text{10}\)

Finally, we mention that a particular case of Proposition 1 is when $\kappa_i = \kappa$ for all $i \in A$.

In order to see how Proposition 1 applies in the concrete case of the network in Figure 6, Table 3 summarizes the information after removing edges $a_1, a_2, b_1$, and $b_2$, respectively. The main message from this table is the simplicity in checking Proposition 1.

| Edge Removed | Condition |
|--------------|-----------|
| $a_1$        | $\kappa > 2.699$ |
| $a_2$        | $\kappa > 0.692$ |
| $b_1$        | $\kappa > 0.508$ |
| $b_2$        | $\kappa > 0.905$ |

Table 3. Conditions for regularity failure in Figure 6.

5. Welfare Analysis and Braess’s Paradox

In previous sections we have shown how the recursive choice aversion model corrects the problem of predicting routing behavior when there are overlapping paths. Similarly, we have shown how this model may generate violations of regularity when some edge at the network is removed.

In this section, we show how choice aversion can capture changes to users welfare when the network topology is modified. Formally, we make two contributions. First, we show that, under the presence of choice aversion, adding edges to the network can decrease users’ welfare.

\(^{10}\)We note that Matějka and McKay (2015)’s analysis has been extended to the general class of additive random utility models by Fosgerau et al.
In particular, we show how a type of Braess’s paradox (Braess (1968) and Braess et al. (2005)) can emerge even in the case of uncongested networks. Second, we compare our approach with the APSL model in terms of its ability to capture changes on welfare.

5.1. Welfare. Following McFadden (1981, Ch. 5), we define the users’ welfare as follows:

\[
C(\kappa) = \mathbb{E} \left( \min_{r \in \mathcal{R}} \{ C_r + \epsilon_r \} \right) = -\log \left( \sum_{r \in \mathcal{R}} e^{-C_r} \right),
\]

where the last equality follows from Assumption 1. Notice that this definition exploits the equivalence in Eq. (11) and makes explicit the dependence of user welfare on the choice aversion parameter \( \kappa \).

Following the literature on discrete choice models, expression (18) can be interpreted as the inclusive value of paths in \( \mathcal{R} \), which is equivalent to say that \( C(\kappa) \) measures the inclusive value of the source node \( s \). In particular, \( C(\kappa) \) represents the expected cost faced by the network users.

It is easy to show that \( C(\kappa) \) is decreasing on \( \kappa \). Similarly, it can be shown that \( C(\kappa) \) is increasing on \( c_a \) for \( a \in A \). In particular, Lorca and Melo (2020, Prop. 4) show that \( \frac{dC(\kappa)}{dc_a} = x_i \mathbb{P}(a|A_i^+) \).

The goal of this section is to use \( C(\kappa) \) to quantify changes on welfare in response to adding or deleting edges to the network \( G \). To that end, we now connect \( \kappa \) with changes on \( C(\kappa) \) when the network \( G \) is modified. Formally, we have the following:

**Proposition 2.** Fix a node \( i \neq s, t \). Suppose that a new link \( a' \) is added to node \( i \). Then \( C(\kappa) \) decreases if and only the following condition holds:

\[
\kappa < \frac{\log(1 - \mathbb{P}(a'|A_i^+ \cup \{a'\}))}{\log \left( \frac{|A_i^+|}{|A_i^+|+1} \right)}.
\]

The previous result is a restatement of Lorca and Melo (2020, Thm. 1). Its relevance comes from the fact that we have a clear way to understand how adding an edge to the network is welfare-improving as
a function of the value of $\kappa$. In particular, for a given network, Eq. (19) can be easily checked.

Subsequently, the choice aversion model predicts that there exists a range of values for $\kappa$ where the addition of costless edges can lead to a decrease in welfare, even if the cost of newly created routes is lower than existing routes. Thus, Braess’s Paradox-like phenomena may emerge even in the case of uncongested transportation networks.

From an empirical point of view, Eq. (19) can be estimated providing a simple test to understand when modifications to the network are welfare improving or not.

5.1.1. Welfare and PSL models. In addition to being able to compute changes on welfare using the choice aversion model, we also compute changes in welfare using several PSL models. In particular, for this class of models the welfare is defined as follows:

\begin{equation}
\hat{C}(\theta) \triangleq E \left( \min_{r \in R} \{ \hat{C}_r + \epsilon_r \} \right) = -\log \left( \sum_{r \in R} e^{-\hat{C}_r} \right),
\end{equation}

where $\theta$ is a parameter vector describing the specific PSL model and $\hat{C}_r$ represents the adjusted cost after applying the respective correction.

It is worth pointing out that the PSL models are not designed to capture changes on welfare. So, Eq. (20) should be interpreted as an adapted welfare measure. The reason to consider these measures is to compare how traditional PSL models might be used to compute welfare changes with the choice aversion model.

5.2. Braess’s Paradox. Traditionally, Braess’s paradox is studied as the result of a congestion game in a transportation network. In this context, Braess’s paradox predicts that introducing additional edges with zero cost can actually contribute to a greater total network cost, and, therefore, a decrease in welfare, than without the additional edges. This counterintuitive result relies in the fact that users in the transportation network are selfish (Roughgarden (2016)). As a consequence,
adding edges to the network can make everybody worse off in the system.

However, the choice aversion model reveals that welfare decreases can arise naturally even when networks are uncongested through increasing choice set cardinality. To see this more explicitly, consider the parameterization of Figure 7(A) where $c_{a_1} = c_{a_4} = x$ with $x \in [0, 3]$ and $c_{a_2} = c_{a_3} = 1$. This figure depicts the case of a simple parallel serial link network where the set of paths is given by $\mathcal{R}_A = \{r_1, r_2\}$ with $r_1 = (a_1, a_2)$ and $r_2 = (a_3, a_4)$.

Figure 7(B) shows the network with edge $a_5$ added to the directed acyclical graph, connecting $i_1$ to $i_2$. For the purpose of observing Braess’s Paradox, we set $c_{a_5} = 0$. The set of paths is given by $\mathcal{R}_B = \mathcal{R}_A \cup \{r_3\}$, where $r_3 = (a_1, a_5, a_4)$. We calculate the welfare according to (18) for 7(A) and 7(B) and compare the difference.

According to Proposition 2, we can compute the threshold for $\kappa$ that determines when adding links is welfare improving. In particular, we

\footnote{Note that in the case of uncongested networks, users do not get involve in strategic interaction.}
have that adding a link is welfare-improving when

\[
\kappa < \frac{\log (1 - e^{-x}/(e^{-1} + e^{-x}))}{\log(1/2)}.
\]

After some algebra, we can express a relationship between \( \kappa \) and \( x \) as follows:

\[
(21)
\kappa < -\log \left( \frac{e^{x-1}}{e^{x-1} + 1} \right) / \log 2.
\]

From (21) it is easy to see that when \( x = 0 \) adding an edge is welfare improving when \( \kappa < 1.89 \), approximately. Similarly, when \( x = 1 \), the welfare increases when \( \kappa < 1 \). In particular, Eq. (21) shows that there is an inverse relationship between \( \kappa \) and \( x \).

Based on this observation, we show how \( C(\kappa) \) changes when \( x \) varies for a constant \( \kappa \). Figure 8 shows the welfare change for the choice aversion model with \( \kappa = \{1, 2\} \) as well as the MNL model and other PSL models and extensions mentioned in Section 4.

The choice aversion model where \( \kappa = 1 \) matches this characterization, reflecting a welfare upgrade for \( x < 1 \) and a welfare downgrade when \( x > 1 \).

![Figure 8. Welfare change from network Figure 7(A) to 7(B) for various logit models.](image)
In contrast, the MNL model only reflects a nonnegative change in welfare from adding edge $a_5$, a symptom of a model where additional gains in welfare are realized from any additional route added to the choice set of the agent. From a theoretical standpoint, this feature of the MNL model captures the preference for flexibility in users’ preferences (Kreps (1979)).

Interestingly, most PSL models and extensions shown in Figure 5 reflect a welfare change similar to each other: a initial welfare gain which decreases in $x$, ultimately leading to a decrease in welfare.

However, Duncan et al. (2020)’s Adaptive PSL model breaks from the welfare change pattern observed by other models. For this model, the difference in welfare is positive and decreasing until $x = 1$, but for $x > 1$, the change in welfare is zero. This is a compelling result which speaks to the strengths of this PSL extension.\(^\text{12}\)

As pointed out before, an important property of the choice aversion model is its microfoundation based in the behavioral concept of choice overload, which provides justification for the penalization term in the cost function as well as the observed outcomes. While there is a similar welfare difference observed among the choice aversion model with $\kappa = 1$ and other PSL models and extensions, this microfoundation sets the choice aversion model apart from other models.

It is important to note the contrast in welfare change between $\kappa = 1$ and $\kappa = 2$ for the choice aversion model. For $\kappa = 2$, Figure 8 clearly

\(^\text{12}\)To understand why the APSL differs from traditional PSL models, we note that $c_{r_3} \leq c_{r_1} = c_{r_2}$ when $x \leq 1$, with the equality holding only for $x = 1$. Here, it is reasonable that we would observe an increase in welfare when $a_5$ is added to the network: for an uncongested network, we are adding a cheaper route choice for users, and it is intuitive that this would improve welfare. Indeed, this result is consistent with most models as shown in Figure 8. However, for $x > 1$, where $c_{r_3} > c_{r_1} = c_{r_2}$, the APSL model reflects no decrease in welfare. Again, this is intuitive: when $x > 1$, adding costless edge $a_5$ does not provide users with a cheaper route choice. Since the users would incur cheaper costs from choosing routes $r_1$ or $r_2$, the APSL model considers $r_3$ to be an irrelevant addition to the choice set and thus would not contribute to, nor detract from, welfare.
shows that there is no value of \( x > 0 \) such that a positive welfare change will be observed. Again, the decrease in welfare stems from a dominating effect on route cost by the choice aversion term, amplified by the choice aversion parameter \( \kappa \). This contrast is important and draws our attention to how welfare changes as \( \kappa \) varies for a fixed \( x \).

Figure 9(A) shows how the difference in welfare calculated by the choice aversion model responds to an increase in \( \kappa \) from 0 to 10 for fixed \( x = \{0, 0.5, 1, 1.5, 2, 2.5, 3\} \). For each value of \( x \) displayed, we first see a positive difference in welfare for low values of \( \kappa \). As \( \kappa \) increases, the difference in welfare continues to decrease until the welfare change is negative. In other words, we see that the threshold \( \kappa \) (i.e., the \( \kappa \) where the welfare change from adding edge \( a_5 \) is no longer positive) decreases in \( x \). This occurrence stems from the fact that for low values of \( \kappa \), the instantaneous cost of each route dominates the choice aversion term in the users’ cost functions. Thus, for low values of \( x \), the welfare change is initially positive, but as \( \kappa \) increases, the choice aversion term is updated with increasing weight until the user’s aversion to choice dominates any reduction in cost incurred by an additional route created by \( a_5 \).

While the choice aversion model predicts this welfare decrease as a result of its behavioral motivation, other logit models may not provide the same outcome. For example, the APSL model does not show a
welfare decrease for any value of \( x \) as shown by Figure 9(B). Rather, this model seems to predict that the welfare change is nonnegative for all values of \( \beta \in [0, 10] \) and only positive for \( \beta < 1 \).\(^{13}\) While this may be sensible for \( x > 1 \), we would expect that a model looking to explain decision-making behavior accurately would display a continuous trade-off between route costs and aversion to increasing choice set cardinality. However, we see that the threshold \( \beta \), where the welfare change is no longer positive, is approximately 1 for all values of \( x \) featured and most notably for \( x < 1 \). The APSL model does not seem to be capable of taking into account the tradeoff mentioned above, perhaps as a result of its corrective nature and intentional design. While the APSL model and other PSL models may have a strength in producing more intuitive route choice probabilities, there appears to be a weakness in predicting reasonable welfare changes from a behavioral point of view.

To summarize, while PSL models and extensions are developed to correct the outcome of a standard logit model and provide what may be deemed as more reasonable choice probabilities for a given network topology, what they often lack is the behavioral foundation for any penalties or adjustments in the cost function of the network user, which would justify the outcomes they provide. Their designs also limit their capability to model changes in welfare with respect to internal parameters. This is not to say that the existing PSL models are inferior; indeed, these models are powerful and useful in various applications at providing reasonable predictions in network flow allocations.\(^{14}\) We simply wish to speak to the strengths of the choice aversion model in the context of applications in transportation networks where choice overload is a factor in users’ decision-making, as well as encourage a convergence of the PSL literature with choice aversion models in transportation network applications.

\(^{13}\)Despite the difference in behavioral motivation, \( \kappa \) in the choice aversion model and \( \beta \) in most PSL models, including the APSL model, serve a similar purpose and can be compared equivalently.

\(^{14}\)For instance, Duncan et al. (2020) analyze an example similar to Figure 2. Their conclusions, while being quantitatively different, agree with the predictions made by the choice aversion model.
5.3. Node-Specific Choice Aversion Parameter. We can also modify our welfare analysis by incorporating node-specific choice aversion parameters as previously defined in §2 such that \( \kappa_{ja} \geq 0 \) for all \( a \in A^+_ja \) and all \( j_a \neq t \).

Accordingly, we update users’ welfare according to Eq. (18) under node-specific choice aversion parameters such that

\[
C(\kappa) = -\log \left( \sum_{r \in R} e^{-\kappa_r} \right) = -\log \left( \sum_{r \in R_c} e^{-c_r - \rho_r} \right)
\]

where \( \kappa = \{\kappa_i\}_{i \in N} \), and \( c_r \) and \( \rho_r \) follow Eq. (12). With updated rules for choice probability and welfare, we can examine how heterogeneous choice aversion parameters may be used in practice.

In particular, the impact of changes on a specific \( \kappa_i \) on \( C(\kappa) \) can be formalized as follows:

**Proposition 3.** Let \( R_i \) be the set of all paths passing through node \( i \). Similarly, let \( R_i^c \) be the set of all paths passing through nodes other than \( i \). Then:

i) \( C(\kappa) \) is increasing in \( \kappa_i \). In particular,

\[
\frac{dC(\kappa)}{d\kappa_i} = \sum_{r \in R_i} P_r \log |A^+_i| > 0.
\]

ii) Fix a node \( i \neq s, t \) and suppose that a new edge \( a' \) is added to node \( i \). Then \( C(\kappa) \) decreases if and only the following condition holds:

\[
(23) \quad \kappa_i < \frac{\log(1 - P(a' | A^+_i \cup \{a'\}))}{\log \left( \frac{|A^+_i|}{|A^+_i + 1|} \right)} \quad \forall i \neq s, t.
\]

**Proof.** i) Using the definitions of \( R_i \) and \( R_i^c \), \( C(\kappa) \) can be written as:

\[
C(\kappa) = -\log \left( \sum_{r \in R_i} e^{-c_r - \rho_r} + \sum_{r' \in R_i^c} e^{-c_{r'} - \rho_{r'}} \right)
\]
Then taking the differential, we find:

\[
\frac{dC(\kappa)}{d\kappa_i} = \frac{1}{\sum_{r \in R} e^{-c_r - \rho_r}} \left( \sum_{r \in R_i} e^{-c_r - \rho_r} \right) \log |A_i^+|.
\]

Noting that \(\sum_{r \in R} e^{-c_r - \rho_r} \left( \sum_{r \in R_i} e^{-c_r - \rho_r} \right) = \sum_{r \in R_i} \mathbb{P}_r\), we conclude that

\[
\frac{dC(\kappa)}{d\kappa_i} = \sum_{r \in R_i} \mathbb{P}_r \log |A_i^+| > 0.
\]

ii) Follows the same argument used in proving Proposition 2. \(\square\)

For an example of application, we borrow the complex network example from Figure 4 and initialize \(\kappa_i = 1\) for all \(i \in N\). To see how node-specific choice aversion parameterization can change outcomes to path choice probability and welfare, we let \(\kappa_2\) and \(\kappa_3\) (that is, each respective choice aversion parameter \(\kappa_i\) for node \(i \in \{2, 3\}\)) vary from 0 to 5 independently. Figure 10 displays the responses to choice probabilities and welfare.

\[\text{Figure 10. Choice probability and welfare responses as } \kappa_2/\kappa_3 \text{ varies across nodes for Figure 4 (example from Fosgerau et al. (2013)).}\]

In this example, a user will reach node 2 when taking \(r_1, r_2,\) or \(r_3\). However, node 3, the only remaining node in this example with a
choice set cardinality greater than 1, is reached only by taking routes $r_1$ or $r_2$. When $\kappa_2$ is small and $\kappa_3 = 1$, Figure 10(A) illustrates that $P_{r_1} = P_{r_2} < P_{r_3} \leq P_{r_4}$, with the equality holding at $\kappa_2 = 0$. As $\kappa_2$ increases, $P_{r_3}$ declines further away from $P_{r_4}$ and converges to $P_{r_1}$ and $P_{r_2}$ as the common $\kappa_2$ parameter dominates the cost functions of the three routes.

Conversely, when $\kappa_2 = 1$ and $\kappa_3$ is small, shown in Figure 10(C), the path choice probabilities for $r_1$, $r_2$, and $r_3$ are close in value, but as $\kappa_3$ increases, $P_{r_3}$ increases until $P_{r_4}$ and $P_{r_2}$ are sufficiently close to zero. However, $P_{r_3} < P_{r_4}$ for all $\kappa_3 \in [0, 5]$ since the user is averse to the choice set at node 2 with a fixed $\kappa_2 = 1$.

Figures 10(B) and 10(D) show the changes to welfare as $\kappa_2$ and $\kappa_3$ increase from 0 to 5, respectively. There is a marked difference in the level of welfare variation as $\kappa_2$ increases and as $\kappa_3$ increases. Because node 2 is included in all routes in the network except for $r_4$, an increase or a decrease in $\kappa_2$ has a larger effect on welfare than a change to $\kappa_3$, which is only accounted for along routes $r_1$ and $r_2$.

Heterogeneous specification of choice aversion at different nodes provides a more precise way of modeling and predicting changes to user welfare. While some transportation contexts might warrant similar degrees of choice aversion across nodes, we anticipate that other applications will benefit from the use of node-specific parameterization and the more nuanced welfare predictions that follow.

6. Final Remarks

The recursive choice model with choice aversion is a highly tractable extension of the standard MNL that can be used to predict reasonable route choice probabilities and provide welfare interpretations in transportation networks. Upon testing our approach against existing PSL models, our model exhibits the power to provide reasonable corrections and predictions with the benefit of a microfoundation in choice overload.
In addition, we explore how the choice aversion model allows for a break in the regularity condition typically preserved by path choice probabilities in logit models and extensions. In doing so, we show that, conditional on the degree of choice aversion, removing edges in the network can lead to a decrease in choice probability of certain existing paths.

We also simulate the welfare implications of the choice aversion model and find a novel prediction: even in uncongested networks, a decrease in welfare akin to Braess’s Paradox can arise when costless edges are added. Here, we also provide a simple characterization for welfare changes conditional on choice aversion which is testable in empirical settings.

It is worth remarking that given its simplicity, our model can be estimated following the methodology proposed by Fosgerau et al. (2013). Exploiting these techniques allows one to test the hypothesis $\kappa_i = \kappa$ for all nodes $i \neq t$.

An important extension of this work is to consider the role of choice aversion in the context of congested traffic networks and the respective modeling approach. One way to study this question is to extend the model in Baillon and Cominetti (2008) by introducing choice aversion in their recursive approach.

Finally, we remark that there is much work to be done regarding the empirical support of choice aversion in transportation networks. Future work regarding experiments on behavior of participants in transportation networks would help establish a better understanding of the significance of choice overload in making routing choices.

Appendix A. Path size logit models

PSL models include correction terms to penalize routes that share links with other routes, so that the deterministic cost of route $r \in R$ is $C_r = c_r + \mu_r$, where $\mu_r \geq 0$ is a correction term for route $r \in R$. The
probability that a user chooses path \( r \) is given by:

\[
\hat{p}_r = \frac{e^{-c_r + \mu_r}}{\sum_{r' \in \mathcal{R}} e^{-c_{r'} + \mu_{r'}}} \quad \forall r \in \mathcal{R}
\]

Following Ben-Akiva and Bierlarie (1999), Path Size Logit (PSL) models adopt the form \( \mu_i = \beta \ln (\gamma_r) \), where \( \beta \geq 0 \) is the path size scaling parameter, and \( \gamma_r \in (0, 1] \) is the path size term for route \( r \in \mathcal{R} \).

A distinct route with no shared links has a path size term equal to 1, resulting in no penalization. Less distinct routes have smaller path size terms and incur greater penalization. The probability that a user chooses route \( r \in \mathcal{R} \) is:

\[
\hat{p}_r = \frac{\left( \gamma_r \right)^\beta e^{-\theta c_r}}{\sum_{r' \in \mathcal{R}} \left( \gamma_{r'} \right)^\beta e^{-\theta c_{r'}}} = \frac{1}{\sum_{r' \in \mathcal{R}} \left( \frac{\gamma_{r'}}{\gamma_r} \right)^\beta e^{-\theta (c_r - c_{r'})}}
\]

The Path Size Logit (PSL) model was first proposed by Ben-Akiva and Ramming, and states that the PSL path size term for route \( r \in \mathcal{R} \), \( \gamma_{rPS}^r \), is defined as follows:

\[
\gamma_{rPS}^r = \sum_{a \in r} \frac{c_a}{c_r} \frac{1}{\delta_{ar'}}
\]

where \( \delta_{ar'} = 1 \) if edge \( a \) belongs to path \( r' \) and \( \delta_{ar'} = 0 \) otherwise.

In Eq. (25) each link \( a \) in route \( r \) is penalized (in terms of decreasing the path size term and increasing the cost of the path) according to the number of paths in the choice set that also use that link \( \left( \sum_{r' \in \mathcal{R}} \delta_{ar'} \right) \), and the significance of the penalization is weighted according to how prominent edge \( a \) is in route \( r \), i.e. the cost of edge \( a \) in relation to the total cost of path \( r \), \( \left( \frac{c_a}{c_r} \right) \).

A.1. Generalized Path Size Logit (GPSL). Ben-Akiva and Bierlarie (1999) formulate an alternative PSL model (PSL') that attempts to reduce the contributions of excessively expensive routes to the path size terms of more realistic routes in the choice set. The GPSL model states that the PSL path size term for route \( r \in \mathcal{R} \), \( \gamma_{rPSL'}^r \), is defined as
follows:

\[
\gamma_{PSL}^{r'} = \sum_{a \in r} \frac{c_a}{c_r} \sum_{r' \in \mathcal{R}} \left( \min_{c_{r''} \in \mathcal{R}} \frac{c_{r''}}{c_{r'}} \right) \delta_{ar'}
\]

where \(\delta_{ar'} = 1\) if edge is in path \(r'\) and \(\delta_{ar'} = 0\) otherwise.

In Eq. (26) the contribution of route \(r\) to path size terms is weighted according to the ratio of route \(r\) and the cheapest route in the choice set \(\left( \min_{c_{r''} \in \mathcal{R}} \frac{c_{r''}}{c_{r'}} \right)\), and hence contributions of high costing routes compared to the cheapest alternative are reduced.

As Ramming describes, however, when a route is completely distinct its path size term is not always equal to 1 which results in an undesired penalization upon the utility of that route. To combat this, Ramming proposes the Generalized Path Size Logit (GPSL) model. The GPSL model states that the GPSL path size term for route \(r \in \mathcal{R}, \gamma_{GPSL}^{r}\), is defined as follows:

\[
\gamma_{GPSL}^{r} = \sum_{a \in r} \frac{c_a}{c_r} \sum_{r' \in \mathcal{R}} \left( \frac{c_a}{c_{r'}} \right)^\lambda \delta_{ar'}
\]

where \(\delta_{ar'} = 1\) if edge \(a\) is in path \(r'\) and \(\delta_{ar'} = 0\) otherwise and \(\lambda \geq 0\). It is easy to see that the GPSL model is equivalent to the PSL model when \(\lambda = 0\). In Eq. (27) the contribution of route \(r'\) to the path size term of route \(r\) (the path size contribution factor) is weighted according to the cost ratio between the routes, \(\left( \frac{c_a}{c_{r'}} \right)^\lambda\), and hence the contributions of high costing routes to the path size terms of low costing routes is reduced. \(\lambda \geq 0\) is the path size contribution scaling parameter to be estimated.

A.2. Adaptive Path Size Logit Model (APSL). In an attempt to improve on existing PSL models and extensions, Duncan et al. (2020) propose an internally consistent PSL model where all components assess the feasibility of routes according to its relative attractiveness due to travel cost and distinctiveness. Formally, their correction can be defined as follows:
Definition 1. The APSL choice probabilities, $\mathbb{P}^*$, for a choice set of size $\mathcal{R}$ are a solution to the fixed-point problem $\mathbb{P}^* = G\left(g\left(\gamma_{\text{APSL}}(\mathbb{P}^*)\right)\right)$ where:

\begin{align}
G_r \left(g_r\left(\gamma_{\text{APSL}}(\mathbb{P}^*)\right)\right) &= \tau + (1 - N\tau) \cdot g_r\left(\gamma_{\text{APSL}}(\mathbb{P}^*)\right) \\
g_r\left(\gamma_{\text{APSL}}(\mathbb{P}^*)\right) &= \frac{\left(\gamma_{r,\text{APSL}}(\mathbb{P}^*)\right)^\beta e^{-\theta c_r}}{\sum_{r' \in \mathcal{R}} \left(\gamma_{r',\text{APSL}}(\mathbb{P}^*)\right)^\beta e^{-\theta c_{r'}}} \\
\gamma_{r,\text{APSL}}(\mathbb{P}^*) &= \sum_{a \in r} \frac{c_a}{c_r} \sum_{r'' \in \mathcal{R}} \left(\frac{P_{r''}}{P_r}\right) \delta_{ar''}
\end{align}

$\forall r \in \mathcal{R}, \forall \mathbb{P}^* \in D^{(r)}, \theta > 0, \beta \geq 0, 0 < \tau \leq \frac{1}{R}$

$D^{(r)} = \{\mathbb{P}^* \in \mathbb{R}_0^{\mathcal{R}} : \tau \leq \mathbb{P}_r^* \leq (1 - (\mathcal{R} - 1)\tau), \forall r \in \mathcal{R}, \sum_{r' \in \mathcal{R}} \mathbb{P}_{r'} = 1\}$

Despite the fact that there is no closed-form representation of the choice probabilities for the APSL model, the APSL model corrects for many of the internal consistency issues in route cost and distinctiveness that trouble other PSL models, which makes its recent introduction in the literature particularly useful in working to predict route choice probabilities more appropriately.

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