LOOKING BEYOND LAMBDA WITH THE UNION SUPERNOVA COMPILATION

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ABSTRACT

The recent robust and homogeneous analysis of the world’s supernova distance–redshift data, together with cosmic microwave background and baryon acoustic oscillation data—provides a powerful tool for constraining cosmological models. Here we examine particular classes of scalar field, modified gravity, and phenomenological models to assess whether they are consistent with observations even when their behavior deviates from the cosmological constant $\Lambda$. Some models have tension with the data, while others survive only by approaching the cosmological constant, and a couple are statistically favored over $\Lambda$ cold dark matter. Dark energy described by two equation-of-state parameters has considerable phase space to avoid $\Lambda$ and next-generation data will be required to constrain such physics, with the level of complementarity between probes varying with cosmology.

Key words: cosmology; observations – cosmology; theory – supernovae: general

Online-only material: color figures

1. INTRODUCTION

A decade after the discovery of the acceleration of the cosmic expansion (Perlmutter et al. 1999; Riess et al. 1998), we still understand little about the nature of the dark energy physics responsible. Improved data continue to show consistency with Einstein’s cosmological constant $\Lambda$, and in terms of a constant equation of state (EOS), or pressure-to-density, ratio $w$, the best fit to the data is $w = -0.969^{+0.050}_{-0.063}$ (statistical), $-0.05_{-0.06}$(systematic), where $\Lambda$ has $w = -1$ (Kowalski et al. 2008). However, the magnitude of $\Lambda$ required and the coincidence for it to be dominant so close to the present remain unexplained, and an abundance of motivated or unmotivated alternative models fills the literature. Using the latest, most robust data available, we examine the extent to which data really have settled on the cosmological constant.

The vast array of models proposed for dark energy makes comparison of every model in the literature with the data a Sisyphian task. Here we select some dozen models with properties such as well defined physical variables, simplicity, or features of particular physical interest. These embody a diversity of physics, including scalar fields, phase transitions, modified gravity, symmetries, and geometric relations. While far from exhaustive, they provide roadmarks for how well we can say that current data have zoomed in on $\Lambda$ as the solution.

For such comparisons, it is critical to employ robust data clearly interpretable within these “beyond $\Lambda$” cosmologies. Geometric probes from the Type Ia supernovae (SNe) distance–redshift relation, cosmic microwave background (CMB) acoustic peak scale shift parameter, and baryon acoustic oscillations (BAOs) angular scale serve this essential role. Equally important is confidence in the error estimates, incorporating systematics as well as statistical uncertainties. This has been studied in detail in the recent unified analysis of the world’s published heterogeneous supernova (SN) data sets—the Union08 compilation (Kowalski et al. 2008).

This SN compilation includes both the large data samples from the SNLS and ESSENCE survey, the compiled high redshift SNe observed with the Hubble Space Telescope (HST), a new sample of nearby SNe, and several other, small data sets. All SNe have been analyzed in a uniform manner and have passed a number of quality criteria (such as having data available in two bands to measure a color and sufficient light-curve points to make a meaningful fit). The samples have been carefully tested for inconsistencies under a blinded protocol before combining them into a single final data set comprising 307 SNe, the basis for this analysis. In this work, the SNe data will be combined with the constraints obtained from the BAO scale (Eisenstein et al. 2005) and from the 5 year data release of Wilkinson Microwave Anisotropy Probe (WMAP) and ground-based CMB measurements (Komatsu et al. 2009).

In Section 2, we describe the general method for cosmological parameter estimation and present a summary table of the various models considered and the $\chi^2$ statistics of the fit. Sections 3–12 then briefly describe the dark energy models, their parameters, and show the likelihood contours. The concluding discussion is given in Section 13.
2. CONSTRAINTING MODELS

Achieving informative constraints on the nature of dark energy requires restricting the degrees of freedom (dof) of the theory and the resulting degeneracies in the cosmological model being tested. One degree of freedom entering the model is the present matter density $\Omega_m$. For the case of the spatially flat cosmological constant $\Lambda$ model (or some of the other models considered below), this is the sole cosmological parameter determining the distances entering the SN magnitude-redshift, BAO scale, and CMB shift parameter relations.

Generally, further degrees of freedom to describe the nature of the dark energy, that is, its EOS, or pressure-to-density, ratio, are needed. In a few cases, the EOS is parameter free, as in the dark energy, that is, its EOS, or pressure-to-density, ratio, determining the distances entering the SN magnitude-redshift, BAO scale, and CMB shift parameter relations.

In general, combining SN with CMB or BAO data can deliver reasonable constraints on one-parameter descriptions of dark energy.

In addition to exploring the nature of dark energy through its EOS, one might also include another parameter for the dark energy density, that is, allow the possibility of nonzero spatial curvature. In this case, individual probes then generally do a poor job constraining the model with current data, although the combined data can sometimes still have leverage. Since cross-checks and testing consistency between probes are important (as particularly illustrated below in the DGP case), we consider spatial curvature only in the otherwise zero parameter cases of $\Lambda$ and DGP, and for the constant EOS dark energy model.

In the following sections, we investigate various one-parameter EOS models, discussing their physical motivation or lack thereof, and features of interest, and the observational constraints that can be placed upon them. In the last sections, we also investigate some two-parameter models of interest, with constrained physical behaviors and particular motivations. As a preview and summary of results, Table 1 lists the models, number of parameters, and goodness of fit for the present data.

The SN, CMB, and BAO data are combined by multiplying the likelihoods. Especially when testing models deviating from the cosmological constant, one must be careful to account for any shift of the CMB sound horizon arising from violation of high redshift matter domination on the CMB and BAO scales; details are given in Union08. Note that some doubt exists on the use of the BAO constraints for cosmologies other than $\Lambda CDM$ Models, as assumed in several places in the Eisenstein et al. (2005) analysis, for example, computation of the correlation function from redshift space, nonlinear density corrections, structure formation and the matter power spectrum, and color and luminosity function evolution. Properly, a systematic uncertainty should be assigned to BAO to account for these effects; however, this requires a complex analysis from the original data and we show only the statistical error. At the current level of precision, simplified estimates show that this does not strongly affect the results, but such systematics will need to be treated for future BAO data. All figures use the likelihood maximized over all relevant parameters besides those plotted, and contours are at the 68.3%, 95.4%, and 99.7% confidence levels.

| Table 1 | "Beyond $\Lambda$" Dark Energy Models Considered in this Paper, Together with $\Lambda CDM$ Models |
|---------|-------------------------------------------------------|
| Model   | Motivation                                      | Parameters | $\chi^2$ (stat) | $\chi^2$ (sys) |
| $\Lambda CDM$ (flat) | Gravity, zeropoint                              | $\Omega_m$ | 313.1          | 309.9          |
| ACDM     | Gravity, zeropoint                              | $\Omega_m$, $\Omega_\Lambda$ | $-1.1$       | $-1.3$         |
| Constant $w$ (flat) | Simple extension                                | $\Omega_m$, $w$ | $-0.3$       | $-1.2$         |
| Constant $\mu$ (flat) | Simple extension                                | $\Omega_m$, $\mu$ | $-1.1$       | $-1.6$         |
| Brane worldview | Consistent gravity                             | $\Omega_m$, $\Omega_\Lambda$ | $5.0$       | $2.5$          |
| Doomsday | Simple extension                                | $\Omega_m$, $\Omega_\Lambda$ | $-0.1$       | $-0.7$         |
| Mirage   | CMB distance                                    | $\Omega_m$, $\Omega_\Lambda$ | $-0.2$       | $-0.1$         |
| Vacuum expansion | Induced gravity                               | $\Omega_m$, $\Omega_\Lambda$ | $0.0$       | $0.0$          |
| Geometric DE Rlow | Kinematics                                    | $r_0$, $r_1$, $\Omega_m$, $\Omega_\Lambda$ | $-0.1$       | $-1.1$         |
| Geometric DE Rhigh | Matter era deviation                       | $\Omega_m$, $w_m$, $\beta$ | $-1.9$       | $-2.2$         |
| PNGB     | Naturalness                                     | $\Omega_m$, $w_m$, $f$ | $-0.1$       | $-0.7$         |
| Algebraic thawing | Generic evolution                          | $\Omega_m$, $w_m$, $p$ | $-1.6$       | $-2.3$         |
| Early DE | Fine tuning problem                           | $\Omega_m$, $w_m$, $\Omega_e$ | $-0.3$       | $-1.2$         |
| Growing $\nu$-mass | Coincidence problem                      | $\Omega_m$, $\Omega_e$, $\nu^0$ | $-0.6$       | $-1.6$         |

Note. Models are listed in the order of discussion, and the cosmological fitting parameters shown. The $\chi^2$ of the matter plus cosmological constant case is given, and all other models list the $\Delta \chi^2$ from that model. The values refer to the best fit to the joint data of SN+CMB+BAO; in the last column, the SN systematics as analyzed in Union08 are included.

It is particularly important to note the treatment of systematic errors, included only for SN. We employ the prescription of Union08 for propagation of systematic errors. This introduces a new distance modulus $\mu_{\text{sys}} = \mu + M_{\Lambda} + \Delta M$, which is simply the usual distance modulus $\mu = 5 \log (H_0 d_1(z))$, where $d_1(z)$ is the luminosity distance and $H_0$ is the Hubble constant, shifted by a sample-dependent magnitude offset $M_\Lambda$ and a single-sample-independent magnitude offset $\Delta M$ added only for the higher redshift SNe (z > 0.2). The magnitude offsets $\Delta M_i$ reflect possible heterogeneity among the SNe samples while the $\Delta M$ step from SNe at $z < 0.2$ to $z > 0.2$ allows a possible common systematic error in the comparison of low versus high redshift SNe. Treating $\Delta M_i$ and $\Delta M$ as additional fit parameters, one defines $\chi^2_{\nu} = \chi^2 + \sum_i (\Delta M_i/\sigma_{\nu i})^2 + (\Delta M/\sigma_{\nu})^2$ to absorb the uncertainty in the nuisance parameters, $\sigma_{\nu i}$ and $\sigma_{\nu}$, and obtain constraints on the desired physical fit parameters that include systematic errors. This procedure of incorporating systematic errors provides robust quantification of whether or not a model is in conflict with the data and is essential for accurate physical interpretation. See Union08 for further, detailed discussion of robust treatment of systematics within the current world heterogeneous SN data.

3. CONSTANT EOS

Models with constant EOS $w$ within 20%, say, of the cosmological constant value $w = -1$, but not equal to $-1$, do not have much physical motivation. To achieve a constant EOS requires fine tuning of the kinetic and potential energies of a scalar field throughout its evolution. It is not clear that a constant $w \neq -1$ is a good approximation to any reasonable dynamical scalar field, where $w$ varies, and certainly does not capture the key physics. However, since current data cannot discern EOS variation on timescales less than or of order of the Hubble time, traditionally one phrases constraints in terms of a constant $w$. We reproduce this model from Union08 to serve as a point of comparison. Also see Union08 for models using the standard time-varying EOS $w(a) = w_0 + w_a (1 - a)$, where
\( a = 1/(1 + z) \) is the scale factor, and models with \( w(z) \) given in redshift bins.

In the constant \( w \) case, the Hubble expansion parameter \( H = \dot{a}/a \) is given by

\[
H^2/H_0^2 = \Omega_m(1+z)^3 + \Omega_w(1+z)^{3(1+w)} + \Omega_k(1+z)^2. \tag{1}
\]

where \( \Omega_m \) is the present matter density, \( \Omega_w \) the present dark energy density, and \( \Omega_k = 1 - \Omega_m - \Omega_w \) the effective energy density for spatial curvature.

Figure 1 shows the confidence contours in the \( w-\Omega_m \) plane, both without and with (minimized in the likelihood fit) spatial curvature. Note that allowing for spatial curvature does not strongly degrade the constraints. This is due to the strong complementarity of SN, CMB, and BAO data, combined with the restriction to a constant \( w \) model. As shown in Union08, the constraint on curvature in this model is \( \Omega_k = -0.010 \pm 0.012 \). See Union08 for more plots showing the individual probe constraints.

### 4. BRANEWORLD GRAVITY

Rather than from a new physical energy density, cosmic acceleration could be due to a modification of the Friedmann expansion equations arising from an extension of gravitational theory. In braneworld cosmology (Dvali et al. 2000; Deffayet et al. 2002), the acceleration is caused by a weakening of gravity over distances near the Hubble scale due to leaking into an extra dimensional bulk from our four-dimensional brane. Thus, a physical dark energy is replaced by an infrared modification of gravity. For DGP braneworld gravity, the Hubble expansion is given by

\[
H^2/H_0^2 = \left( \sqrt{\Omega_m(1+z)^3 + \Omega_{bw}} + \sqrt{\Omega_{bw}} \right)^2 + \Omega_k(1+z)^2 \tag{2}
\]

\[
\to \Omega_m(1+z)^3 + 2\Omega_{bw} + 2\sqrt{\Omega_{bw}/\Omega_m}(1+z)^3 + \Omega_{bw} \quad \text{ (flat).} \tag{3}
\]

Here, the present effective braneworld energy density is

\[
\Omega_{bw} = \frac{(1 - \Omega_m - \Omega_k)^2}{4(1 - \Omega_k)} \tag{4}
\]

\[
\to \frac{(1 - \Omega_m)^2}{4} \quad \text{(flat),} \tag{5}
\]

and is related to the five-dimensional crossover scale \( r_c = M_5^2/(2M_4^2) \) by \( \Omega_{bw} = 1/(4H_0^2 r_c^2) \). Note that the only cosmological parameters for this model are \( \Omega_m \) and \( \Omega_k \) (or \( \Omega_{bw} \)), so it has the same number of parameters as \( \Lambda \)CDM.

The effective dark energy EOS is given by the simple expression

\[
w(z) = -\frac{1 - \Omega_k(z)}{1 + \Omega_m(z) - \Omega_k(z)}, \tag{6}
\]

where \( \Omega_m(z) = \Omega_m(1+z)^3/(H^2/H_0^2) \) and \( \Omega_k(z) = \Omega_k(1+z)^2/(H^2/H_0^2) \). Thus, the dark energy EOS at present, \( w_0 \), is determined by \( \Omega_m \) and \( \Omega_k \); while time varying, it is not an independent parameter. So rather than plotting \( w_0 \) versus \( \Omega_m \) or showing constraints on the somewhat nonintuitive parameters \( r_c \) or \( \Omega_{bw} \) (but see the clear discussion and plots in Davis et al. 2007; Rydebeck et al. 2007, though without systematics), Figure 2 illustrates the confidence contours in the \( \Omega_k-\Omega_m \) plane. This makes it particularly easy to see how deviations from flatness pull the value of the matter density. In this and following figures, dotted contours show the BAO constraints, dashed show the CMB constraints, dot-dashed the SN with systematics, and solid contours give the joint constraints.

For a flat universe, in order for \( w \) to approach \(-1 \), the matter density is forced to small values. Alternately, pushing the curvature density \( \Omega_k \) negative, that is, introducing a positive spatial curvature \( k \), allows \( w \approx -1 \) with higher matter density. For a given \( w_0 \), the amount of curvature needed can be derived from Equation (6) to be approximately \( \Delta \Omega_k \approx -\Delta \Omega_m/\Omega_m \), so to move a flat, \( \Omega_m = 0.2 \) universe to \( \Omega_m = 0.3 \) requires \( \Omega_k = -0.5 \), in agreement with the SN contour (being most sensitive to \( w_0 \) of Figure 2).

Note that the curvature density cannot exceed \( 1 - \Omega_m \), corresponding to an infinite crossover scale \( r_c \), so the likelihood contours are cut off at this line and the region beyond is unphysical. However, this does not affect the joint contours. The BAO data contours do extend to the limit \( \Omega_k = 1 - \Omega_m \); here, \( \Omega_{bw} = 0 \), equivalent to the simple open, cold dark matter (OCDM) nonaccelerating universe.

Most importantly, the three probes do not reach concordance on a given cosmological model. The areas of intersection of any pair are distinct from other pairs, indicating that the full data disfavor the braneworld model, even with curvature. This is further quantified by the poor goodness of fit to the data, with \( \Delta \chi^2 = 2.7 \) relative to the flat \( \Lambda \)CDM model possessing one.
fewer parameter, or $\Delta \chi^2 = 4.0$ relative to $\Lambda$CDM allowing curvature. This indicates the crucial importance of crosschecking probes. Moreover, if we had used only the statistical estimates of uncertainties (see the “SN stat” 68% confidence level (cl) contour of Figure 2), we would have found that $\Delta \chi^2 = 15$ rather than 2.7, and possibly drawn exaggerated physical conclusions, considering the DGP model 2000 times less likely than it really is, as an illustration. Inclusion of systematics is essential for robust interpretation of results.

5. DOOMSDAY MODEL

Perhaps the simplest generalization of the cosmological constant is the linear potential model, pioneered by Linde (1987) and recently discussed by Weinberg (2008), motivated from high energy physics. Interestingly, while this gives a current accelerating epoch, in the future, the potential becomes negative and not only deceleration of the expansion but collapse of the universe ensues. Hence, this is why the name of a doomsday model is given.

The potential has two parameters: the amplitude and slope. The amplitude $V_0$ essentially gives the dark energy density, which is fixed by $\Omega_m$ in a flat universe (for the remainder of the paper, we assume a flat universe, for the reasons discussed in Section 2). The slope $V' = dV/d\phi$ can be translated into the present EOS $w_0$. Thus, this is a one-parameter model in our categorization. See Kelsoh et al. (2003) for discussion of the cosmological properties of the linear potential, Linde (1987) for a view of it as a perturbation about a zero cosmological constant, and Dimopoulos (2003) for links to the large kinetic term approach in particle physics. More recently, this has been considered as a textbook case by Weinberg (2008), so we will examine this model in some detail. Such dark energy is an example of a thawing scalar field (Caldwell & Linder 2005), starting with $w(z \gg 1) = -1$ and slowly rolling to attain less negative values of $w$, that is, it departs from $\Lambda$. If it has not evolved too far from $-1$, then its behavior is well described by $w_a = -1.5(1 + w_0)$, where $w(a) = w_0 + w_a(1-a)$. However, we solve the scalar field equation of motion exactly (numerically) for all results quoted here.

As the scalar field rolls to small values of the potential, the expansion stops accelerating, and when it reaches $V = 0$, then $w = 1$. However, it crosses through zero to negative values of the potential, further increasing $w$, and eventually the dark energy density itself becomes negative, causing $w$ to go to positive and then negative infinity. Thereafter, the negative dark energy density, now acting with an attractive gravitational force, not only causes deceleration but also forces the universe to start contracting. The rapid collapse of the universe ends in a big crunch or cosmic doomsday in a finite time.

In the notation used in Weinberg (2008), $V(\phi) = V_0 + (\phi - \phi_0) V_0'$, with $V_0$ being the potential energy during the initial frozen state (during high Hubble drag at high redshift) and $V_0'$ is the constant potential slope. Figure 3 shows the constraints in this high energy physics plane $V_0'$-$V_0$, Note the tight constraints on the initial potential energy $V_0$, given in units of the present critical density. The cosmological constant corresponds to the limit of $V_0' = 0$, but the slope must always be less than or of order $10^{-120}$ in Planck units, that is, unity when shown in terms of the present energy density, to match the data.

We can also translate these high energy physics parameters into the recent universe quantities of the matter density $\Omega_m$ and the present EOS $w_0$. Moreover, this is directly related to the doomsday time $t_{\text{doom}}$ or future time until collapse. A useful approximation (though we employ the exact solution) among $t_{\text{doom}}, w_0$, and the approximate time variation $w_a = -1.5(1+w_0)$ is

$$t_{\text{doom}} \approx 0.5H_0^{-1}(1+w_0)^{-0.8} \approx 0.6H_0^{-1}(-w_0)^{-0.8}. \quad (7)$$

Figure 4 shows the likelihood contours in the $t_{\text{doom}}$-$\Omega_m$ and $w_0$-$\Omega_m$ planes. The 95% confidence limit on $t_{\text{doom}}$ from present observations is $1.24H_0^{-1}$; that is, we are 95% likely to have at least 17 billion more years before doomsday.

6. MIRAGE MODEL

Given their limited sensitivity to the dynamics of dark energy, current data can appear to see a cosmological constant even in the presence of time variation. This is called the “mirage of $\Lambda$,” and we consider mirage models, with a form motivated by the
observations as discussed below, specifically to test whether the concordance cosmology truly narrows in on the cosmological constant as the dark energy.

Since cosmological distances involve an integral over the energy density of components, which in turn are integrals over the EOS as a function of redshift, there exists a chain of dependences between these quantities. Fixing a distance, such as \( d_{\text{ls}} \) to the CMB last scattering surface, can generally lead to an “attractor” behavior in the EOS to a common averaged value or the value at a particular redshift. Specifically, Linder (2007) pointed out that if CMB data for \( d_{\text{ls}} \) are well fitted by the \( \Lambda \)CDM model, then this forces \( w(z \approx 0.4) \approx -1 \) for quite general monotonic EOS. So even dark energy models with substantial time variation could thus appear to behave like the cosmological constant at \( z \approx 0.4 \), near the pivot redshift of current data.

Since current experiments insensitive to time variation inherently interpret the data in terms of a constant \( w \) given by the EOS value at the pivot redshift, this in turn thus leads to the “mirage of \( \Lambda \)”: thinking that \( w = -1 \) everywhere, despite models very different from \( \Lambda \) being good fits. See Section 5.2 of Linder (2008b) for further discussion. Also note that attempting to constrain the EOS by combining the CMB \( d_{\text{ls}} \) with a precision determination of the Hubble constant \( H_0 \) only tightens the uncertainty on the pivot EOS value (already taken to be nearly \( -1 \)) and so similarly does not reveal the true nature of dark energy.

We test this with a family of “mirages” models motivated by the reduced distance to CMB last scattering \( d_{\text{ls}} \). These correspond to the one-parameter subset of the two-parameter EOS model \( w(a) = w_0 + w_a (1 - a) \) with \( w_0 \approx -3.63 (1 + w_0) \), as shown in Linder (2007). They are not exactly equivalent to imposing a CMB prior since \( d_{\text{ls}} \) will still change with \( \Omega_m \), that is, they essentially test the uniqueness of the current concordance model for cosmology: \( \Lambda \)CDM with \( \Omega_m = 0.28 \).

For any model well approximated by a relation \( w_a = -A (1 + w_0) \), as this model (and the previous one) is, the Hubble parameter is given by

\[
H^2/H_0^2 = \Omega_m (1 + z)^3 + (1 - \Omega_m) \times (1 + z)^{(1+w_0+w_a)} e^{-3 w_a z/(1+z)} \approx \Omega_m (1 + z)^3 + (1 - \Omega_m) \times (1 + z)^{(1+w_0)(1-A)} e^{3 A (1+w_0) z/(1+z)}. \tag{8}
\]

\[
\frac{H^2}{H_0^2} = \Omega_m (1 + z)^3 + (1 - \Omega_m) \times (1 + z)^{(1+w_0+w_a)} e^{-3 w_a z/(1+z)} \approx \Omega_m (1 + z)^3 + (1 - \Omega_m) \times (1 + z)^{(1+w_0)(1-A)} e^{3 A (1+w_0) z/(1+z)}. \tag{9}
\]

We have

\[
H(z) = H_0 z \sqrt{\Omega_m (1 + z)^3 + (1 - \Omega_m) (1 + z)^{(1+w_0+w_a)} e^{-3 w_a z/(1+z)}},
\]

where \( H(z) \) is the Hubble parameter at \( z \) and \( H_0 \) is the Hubble constant.

Figure 4. Future expansion history in the linear potential model has a collapse, or cosmic doomsday, at a finite time in the future. The left panel shows the confidence contours for the time remaining until collapse; the likelihood contours extend to infinity, with \( t_{\text{doom}} = \infty \) corresponding to the \( \Lambda \) model. The contours can also be viewed in the equivalent \( w_0 - \Omega_m \) plane (right panel). Current data constraints indicate cosmic doomsday will occur no sooner than \( \sim 1.24 \) Hubble times from now at 95% confidence.

(A color version of this figure is available in the online journal.)

Figure 5. Mirage subclass of time varying dark energy looks like \( \Lambda \) in an averaged sense. Note that CMB contours are almost vertical, indicating both that the mirage holds, preserving the \( \Lambda \)CDM distance to last scattering, and yet imposes little constraint on \( w_0 \) and hence \( w_a \). Thus the appearance of \( \Lambda \) does not actually exclude time variation. The mirage is broken when the EOS at high redshift exceeds the matter domination value of zero; this causes the wall in the likelihood at \( w_0 = A/(1 - A) \approx -1.4 \) (see Equation (9)).

(A color version of this figure is available in the online journal.)

Figure 5 shows constraints in the \( w_0 - \Omega_m \) plane. It is important to note that \( w \) is not constant in this model. A significant range of \( w_0 \) (and hence a larger range of \( w_a \) too, roughly \(+0.55 \) to \(-1.1 \) at 68% cl) is allowed by the data, even though all these models look like a cosmological constant in an averaged sense. Thus, experiments sensitive to the time variation \( w_a \) (e.g., \( \sigma(w_a) < 0.36 \) to know that \( w(z) \) is really, not just apparently, within 10% of \(-1 \)) are required to determine whether the mirage is reality or not.

7. VACUUM METAMORPHOSIS

An interesting model where the cosmic acceleration is due to a change in the behavior of physical laws, rather than to a new physical energy density, is the vacuum metamorphosis model (Parker & Raval 2000; Caldwell et al. 2006). As in Sakharov’s induced gravity (Sakharov 1968, 2000), quantum fluctuations of a massive scalar field give rise to a phase transition in

\[
H^2 = \frac{3m \phi^2}{2} + \frac{\Lambda}{3},
\]

where \( m \) is the mass of the scalar field and \( \phi \) is its field value.

\[
H = \sqrt{\frac{3\Lambda}{8\pi G} + \frac{m^2 \phi^2}{8\pi G}},
\]

which is the Hubble parameter.

\[
H(z) = \frac{H_0}{z} \sqrt{\Omega_m (1 + z)^3 + (1 - \Omega_m) (1 + z)^{(1+w_0+w_a)} e^{-3 w_a z/(1+z)}},
\]

where \( H(z) \) is the Hubble parameter at \( z \) and \( H_0 \) is the Hubble constant.

\[
H(z) = \frac{H_0}{z} \sqrt{\Omega_m (1 + z)^3 + (1 - \Omega_m) (1 + z)^{(1+w_0+w_a)} e^{-3 w_a z/(1+z)}},
\]

which is the Hubble parameter.

\[
H(z) = \frac{H_0}{z} \sqrt{\Omega_m (1 + z)^3 + (1 - \Omega_m) (1 + z)^{(1+w_0+w_a)} e^{-3 w_a z/(1+z)}},
\]

where \( H(z) \) is the Hubble parameter at \( z \) and \( H_0 \) is the Hubble constant.
gravity when the Ricci scalar curvature $R$ becomes of order the mass squared of the field and freezes $R$ there. This model is interesting in terms of its physical origin and nearly first principle derivation, and further because it is an example of a well behaved phantom field, with $w < -1$.

The criticality condition

$$R = 6(\dot{H} + 2H^2) = m^2$$  \hspace{1cm} (10)

after the phase transition at redshift $z_t$ leads to a Hubble parameter

$$H^2 / H_0^2 = \left(1 - \frac{m^2}{12}\right)(1 + z)^4 + \frac{m^2}{12}, \quad z < z_t, \quad (11)$$

$$H^2 / H_0^2 = \Omega_m (1 + z)^3 + \frac{m^2}{3} \frac{1 - \Omega_s}{4 - 3\Omega_m}, \quad z > z_t. \quad (12)$$

There is one parameter, $\Omega_s = \Omega_m(z_t)$, in addition to the present matter density $\Omega_m$, where $1 - \Omega_s$ is proportional to the cosmological constant. The variables $z_t$ and $m$ are given in terms of $\Omega_m$ and $\Omega_s$ by $z_t = (m^2 \Omega_s / [3\Omega_m(4 - 3\Omega_m)])^{1/3} - 1$ and $m^2 = 3\Omega_m[(4 - 3\Omega_m)/\Omega_s]^{1/3}[(4/m^2) - (1/3)]^{-3/4}$, respectively. The original version of the model had fixed $\Omega_s = 1$, that is, no cosmological constant, but if the scalar field has a nonzero expectation value (which is not required for the induced gravity phase transition), then there will be a cosmological constant, and $\Omega_s$ deviates from unity.

Figure 6 shows the confidence contours in the $\Omega_s - \Omega_m$ plane. To consider constraints on the original vacuum metamorphosis model, without an extra cosmological constant, slice across the likelihood contours at the $\Omega_s = 1$ line. We see that the three probes are inconsistent with each other in this case, with disjoint contours (indeed $\Delta \chi^2 = 28.5$ relative to flat $\Lambda$CDM). Allowing for a cosmological constant, that is, $\Omega_s \neq 1$, brings the probes into concordance, and the best joint fit approaches the lower bound of the region $\Omega_s \geq \Omega_m$. The condition $\Omega_s = \Omega_m$ corresponds to the standard cosmological constant case, with $\Omega_s = 1 - \Omega_m$, since the phase transition then only occurs at $z_t = 0$. Thus, the data do not favor any vacuum phase transition. Although this model comprises very different physics, and allows phantom behavior, the data are still consistent with the cosmological constant.

8. GEOMETRIC DARK ENERGY

According to the equivalence principle, acceleration is manifest in the curvature of spacetime, so it is interesting to consider geometric dark energy, the idea that the acceleration arises from some property of the spacetime geometry. One example of this involves the holographic principle of quantum field theory as applied to cosmology. This limits the number of modes available to the vacuum energy and so could have an impact on the cosmological constant problem (Bousso 2002). The basic idea is that there is a spacelike, two-dimensional surface on which all the field information is holographically encoded, and the covariant entropy bound relates the area of this surface to the maximum mode energy allowed (UV cutoff). The vacuum energy density resulting from summing over modes ends up being proportional to the area or inverse square of the characteristic length scale. However, choosing what is perhaps the natural surface (see Bousso 2002), the causal event horizon, does not lead to an energy density with accelerating properties.

Many of the attempts in the literature to overcome this have grown increasingly distant from the original concept of holography, though they often retain the name. It is important to realize that, dimensionally, any energy density, including the vacuum energy density, has $\rho \sim L^{-2}$, so merely choosing some length $L$ does not imply any connection to quantum holography. We, therefore, do not consider these models but instead turn to the spacetime curvature.

8.1. Ricci Dark Energy $R_{\text{low}}$

A different approach involves the spacetime curvature directly, as measured through the Ricci scalar. This is similar in motivation to the vacuum metamorphosis model of Section 7. Here we consider it purely geometrically, with the key physical quantity being the reduced scalar spacetime curvature, in terms of the Ricci scalar and Hubble parameter, as in the model of Linder (2004):

$$R \equiv \frac{R}{12H^2} = r_0 + r_1(1 - a).$$  \hspace{1cm} (13)

Through the equivalence principle, this quantity directly involves the acceleration. Moreover, we can treat it purely kinematically, as in the last equality above, assuming no field equations or dynamics. Of course, any functional form contains an implicit dynamics (see, e.g., Linder 2008b), but we have effectively chosen a Taylor expansion in the scale factor $a$, valid for any dynamics for small deviations $1 - a$ from the present, that is, the low redshift or low scalar curvature regime.

At high redshift, as $1 - a$ is no longer small, we match it to an asymptotic matter-dominated behavior for $a < a_t = 1 - (1 - 4r_0)/(4r_1)$. Solving for the Hubble parameter, we have

$$H^2 / H_0^2 = a^{4(r_0 + r_1)} e^{4r_1(1 - a)}, \quad a > a_t \quad \hspace{1cm} (14)$$

$$H^2 / H_0^2 = \Omega_m a^{-3}, \quad a < a_t. \quad \hspace{1cm} (15)$$

The matching condition determines

$$\Omega_m = \left(\frac{4r_0 + 4r_1 - 1}{4r_1}\right)^{4r_0 + 4r_1 - 1} e^{1 - 4r_0}, \quad \hspace{1cm} (16)$$
so there is only one parameter independent of the matter density.

Note also that we can define an effective dark energy as that part of the Hubble parameter deviating from the usual matter behavior, with EOS generally given by

\[ w(a) = \frac{1 - 4R}{3} \left[ 1 - \Omega_m e^{-\frac{2\delta H^2}{R^2}} \right]^{-1}. \]

For the particular form of Equation (13), we have

\[ w_0 = \frac{1 - 4r_0}{3(1 - \Omega_m)}. \]

This model has one EOS parameter in addition to the matter density. We can, therefore, explore constraints either in the general kinematic plane \( r_0 - r_1 \) or view them in the \( \Omega_m - w_0 \) plane. Figure 7 shows both.

Figure 7. Geometric dark energy in the \( R_{low} \) model describes the acceleration directly through the reduced Ricci scalar or spacetime curvature. This can be viewed in a kinematic sense, in the \( r_0 - r_1 \) plane, or in a dark energy sense, in the \( \Omega_m - w_0 \) plane. The data favor \( w_0 = -1 \) but this is not \( \Lambda \), instead representing distinct physics. For \( r_0 + r_1 > 1/4 \), above the diagonal line, early matter domination is violated, and the CMB and BAO likelihoods avoid this region, as seen in the left panel; the matter density also cannot then be uniquely defined so the equivalent region is excluded from the right panel.

(A color version of this figure is available in the online journal.)

early universe and ask how the data favor acceleration coming about. In this second geometric dark energy model (call it \( R_{high} \) for high redshift or large values of scalar curvature), the value of \( \mathcal{R} \) evolves from 1/4 at high redshift. From the definition of \( \mathcal{R} \), it must asymptotically behave as

\[ \mathcal{R} = \frac{1}{4} \left[ 1 - 3w_\infty \frac{\delta H^2}{H^2} \right] \approx \frac{1}{4} [1 + 4\alpha a^{-3w_\infty}], \]

where \( \delta H^2 = (H^2 / H_0^2) - \Omega_m(1 + z)^3 \) is the deviation from matter-dominated behavior and \( w_\infty \) is the associated, effective EOS at high redshift, approximated as asymptotically constant.

Next, we extend this behavior to a form that takes the reduced scalar curvature to a constant in the far future (as it must if the EOS of the dominant component goes to an asymptotic value):

\[ \mathcal{R} = \frac{1}{4} + \frac{\alpha a^{-3w_\infty}}{1 + \beta a^{-3w_\infty}}. \]

So today, \( \mathcal{R} = 1/4 + \alpha/(1 + \beta) \) and in the future, \( \mathcal{R} = 1/4 + \alpha/\beta \). By requiring the correct form for the high redshift Hubble expansion, one can relate the parameters \( \alpha \) and \( \beta \) by

\[ \alpha = (3\beta w_\infty / 4)[\ln \Omega_m / \ln(1 + \beta)]. \]

and finally

\[ H^2 / H_0^2 = \Omega_m a^{-3(1 + \beta a^{-3w_\infty}) - \ln \Omega_m / \ln(1 + \beta)}. \]

The \( R_{high} \) geometric dark energy model has two parameters \( \beta \) and \( w_\infty \), in addition to the matter density \( \Omega_m \). This is the first such model we consider, and all remaining models also have two EOS parameters. Current data cannot in general satisfactorily constrain two parameters, so for all remaining models, we do not show individual probe constraints; if the EOS phase-space behavior of the model is sufficiently restrictive, then reasonable joint constraints may result.

Figure 8 shows the joint likelihoods in the \( \Omega_m - w_\infty \) and \( \Omega_m - \beta \) planes, with the third parameter minimized over (see the caption for discussion of the individual probe likelihoods). We see that the data were consistent with the cosmological constant behavior.

8.2. Ricci Dark Energy \( R_{high} \)

Rather than expanding the spacetime curvature around the present value, we can also consider the deviation from a high redshift matter-dominated era. That is, we start with a standard

\[ \Omega_m, \]

rather than expanding the spacetime curvature around the present value, we can also consider the deviation from a high redshift matter-dominated era. That is, we start with a standard
$w_\infty = -1$ in the past (this is only a necessary, and not sufficient, condition for $\Lambda$CDM), and indeed constrain the asymptotic high redshift behavior reasonably well, in particular to negative values of $w_\infty$. This indicates that the Ricci scalar curvature definitely prefers a nearly-standard early matter-dominated era, that is, the deviations faded away into the past. This has important implications as well for scalar–tensor theories that can be considered strongly physically motivated (perhaps even more so than $\Lambda$). See Frieman (1995) for an early cosmological analysis of PNGB as dark energy and more recent work by Dutta & Sorbo (2007) and Abrahamse et al. (2008).

The potential for the PNGB model is

$$V(\phi) = V_\star [1 + \cos(\phi/f)],$$

(24)

with $V_\star$, setting the magnitude, $f$ the symmetry energy scale or steepness of the potential, and $\phi_\star$ the initial value of the field when it thaws from the high redshift, high Hubble drag, frozen state. These three parameters determine, and can be thought of as roughly analogous to, the dark energy density, the time variation of the EOS, and the value of the EOS. The dynamics of this class of models is sometimes approximated by the simple form

$$w(a) = -1 + (1 + w_0) a^F,$$

(25)

with $F$ being roughly inversely related to the symmetry energy scale $f$, but we employ the exact numerical solutions of the field evolution equation.

PNGB models are an example of thawing dark energy, where the field has recently departed from its high redshift cosmological constant behavior, evolving toward a less negative EOS. Since the EOS only recently deviates from $w = -1$, the precision in measuring $w_0$ is more important than the precision in measuring an averaged or pivot EOS value. SN data provide the tightest constraint on $w_0$. In the future, the field oscillates around its minimum with zero potential and ceases to accelerate the expansion, acting instead like nonrelativistic matter.

Figure 9 illustrates the constraints in both the particle physics and the cosmological parameters. The symmetry energy scale could provide a key clue for revealing the fundamental physics.

9. PSEUDO-NAMBU GOLDSTONE BOSON MODEL

Returning to high energy physics models for dark energy, one of the key puzzles is how to prevent quantum corrections from adding a Planck energy scale cosmological constant or affecting the shape of the potential. This is referred to as the issue of technical naturalness. Pseudo-Nambu Goldstone boson (PNGB) models are technically natural, due to a shift symmetry, and so could be considered strongly physically motivated (perhaps even more so than $\Lambda$). See Frieman (1995) for an early cosmological analysis of PNGB as dark energy and more recent work by Dutta & Sorbo (2007) and Abrahamse et al. (2008).
behind dark energy, and it is interesting to note that these astrophysical observations essentially probe the Planck scale. For values of $f$ below unity (the reduced Planck scale), the potential is steeper, causing greater evolution away from the cosmological constant state. However, the field may be frozen until recently and then quickly proceed down the steep slope, allowing values of $w_0$ far from $-1$ but looking in an average or constant $w$ sense like $\langle w \rangle \approx -1$. Small values of $\phi_i/f$ have the field set initially near the top of the potential; starting from such a flat region, the field rolls very little and $w$ stays near $-1$ even today. In the limit $\phi_i/f = 0$, the field stays at the maximum, looking exactly like a cosmological constant. The two effects of the steepness and initial position mean that the cosmological parameter likelihood can accommodate both $w_0 \approx -1$ and $w_0$ approaching 0 as consistent with current data. However, to agree with data and $1 + w_0 \sim 1$ requires $f \ll 1$ and fine tuning; for example, for $f = 0.1$, one must balance the field to within one part in a thousand of the top. Thus, in the left panel, there exists an invisibly narrow tail extending along the $y$-axis to $f = 0$. In the right panel, we show how taking more natural values $f \gtrsim 0.5$ removes the more extreme values of $w_0$ caused by the unnatural fine tuning.

10. ALGEBRAIC THAWING MODEL

While PNGB models involve a pseudoscalar thawing field, we can also consider scalar fields with thawing behavior. Any such fields that are neither fine tuned nor have overly steep potentials must initially depart from the cosmological constant behavior along a specific track in the EOS phase space, characterized by a form of slow roll behavior in the matter-dominated era (see Caldwell & Linder 2005; Linder 2006; Scherrer & Sen 2008; Cahn et al. 2008). Here, we adopt the algebraic thawing model of Linder (2008a), specifically designed to incorporate this physical behavior:

$$1 + w = (1 + w_0) a^p \left( \frac{1 + b}{1 + ba^{-3}} \right)^{1-p/3}$$  \hspace{1cm} (26)

where $\alpha = (1 + w_0)/a^p - 1$ and $b = 0.3$ is a fixed constant and not a parameter. The two parameters are $w_0$ and $p$, and this form follows the scalar field dynamics not only to leading order but also to next-to-leading order (see Cahn et al. 2008).

The physical behavior of a minimally coupled scalar field evolving from a matter-dominated era would tend to have $p \in [0, 3]$. Since we want to test whether the data point to such a thawing model, we consider values of $p$ outside this range. Results are shown in Figure 10.

For $p < 0$, the field has already evolved to its least negative value of $w$ and returned toward the cosmological constant. The more negative $p$ is, the less negative (closer to 0) the extreme value of $w$ is, so these models can be more tightly constrained as $p$ gets more strongly negative. As $p$ gets more positive, the field takes longer to thaw, increasing its similarity to the cosmological constant until recently, when it rapidly evolves to $w_0$. Such models will be very difficult to distinguish from $\Lambda$. If we restrict consideration to the physically expected range $p \in [0, 3]$, this implies $w_0 < -0.57$ at 95% confidence in these thawing models, so considerable dynamics remains allowed under current data. This estimation is consistent with the two specific thawing models already treated, the doomsday and PNGB cases.

The goodness of fit to the data is the best of all models considered here ($\Delta \chi^2 = 2.3$), even taking into account the addition of two fit parameters. This may indicate that we should be sure to include a cosmological probe sensitive to $w_0$ (not necessarily the pivot EOS $w_p$) and to recent time variation $w_a$, such as SN, in our quest to understand the nature of dark energy.

11. EARLY DARK ENERGY

The other major class of dark energy behavior is that of freezing models, which start out dynamical and approach the
cosmological constant in their evolution. The tracking subclass is interesting again from the point of view of fundamental physics motivation: they can ameliorate the fine-tuning problem for the amplitude of the dark energy density by having an attractor behavior in their dynamics, drawing from a large basin of attraction in initial conditions (Zlatev et al. 1999). Such models generically can have nontrivial amounts of dark energy at high redshift; particularly interesting are scaling models, or tracers, where the dark energy has a fixed fraction of the energy density of the dominant component. These can be motivated by dilatation symmetry in particle physics and string theory (Wetterich 1988).

As a specific model of such early dark energy, we adopt that of Doran & Robbers (2006), with

$$\Omega_{\text{DE}}(a) = \frac{1 - \Omega_m - \Omega_\nu (1 - a^{-3w_0})}{1 - \Omega_m + \Omega_m a^{3w_0}} + \Omega_e (1 - a^{-3w_0}) \tag{28}$$

for the dark energy density as a function of the scale factor $a = 1/(1+z)$. Here, $\Omega_{\text{DE}} = 1 - \Omega_m$ is the present dark energy density, $\Omega_m$ is the asymptotic early dark energy density, and $\Omega_\nu$ is the present dark energy EOS. In addition to the matter density, the two parameters are $\Omega_e$ and $w_0$, respectively.

The Hubble parameter is given by $H^2/H_0^2 = \Omega_m a^{-3} / [1 - \Omega_{\text{DE}}(a)]$. The standard formula for the EOS, $w = -1/(3[1 - \Omega_{\text{DE}}(a)]) d\ln \Omega_{\text{DE}}(a)/d\ln a$, does not particularly simplify in this model. Note that the dark energy density does not act to accelerate expansion at early times, and in fact $w \to 0$. However, although the energy density scales like matter at high redshift, it does not appreciably clump and so slows growth of matter density perturbations. We will see that this effect is crucial in constraining early dark energy.

Figure 11 shows the constraints in the $\Omega_m - \Omega_e$ and $\Omega_e - w_0$ planes. Considerable early dark energy density appears to be allowed, but this is only because we used purely geometric information, that is, distances and the acoustic peak scale. The high redshift Hubble parameter for a scaling solution is multiplied by a factor $1/\sqrt{1 - \Omega_e}$ relative to the case without early dark energy (see Doran et al. 2007a). This means that the sound horizon is shifted according to $s \sim \sqrt{1 - \Omega_e}$, but a geometric degeneracy exists whereby the acoustic peak angular scale can be preserved by changing the value of the matter density $\Omega_m$ (see Linder & Robbers 2008 for a detailed treatment). This degeneracy is clear in the left panel.

However, as mentioned, the growth of perturbations is strongly affected by the unclustered early dark energy. This suppresses growth at early times, leading to a lower mass amplitude $\sigma_8$ today. To explore the influence of growth constraints, we investigate adding a growth prior of 10% to the data, that is, we require the total linear growth (or $\sigma_8$) to lie within 10% of the concordance model. The innermost, white contour of the left panel of Figure 11 shows the constraint with the growth prior. In the right panel, we zoom in and show $\Omega_e$ versus $w_0$, seeing that the degeneracy is effectively broken. The amount of early dark energy is limited to $\Omega_e < 0.038$ at 95% cl. Similar conclusions were found in a detailed treatment by Doran et al. (2007b).

We find that a convenient theoretical fitting formula is that for an early dark energy model, the total linear growth to the present, $\Omega_e$, is suppressed by

$$\frac{\Delta \sigma_0}{\sigma_0} \approx \left( \frac{\Omega_e}{0.01} \right) \times 5.1\%, \tag{29}$$

relative to a model with $\Omega_e = 0$ but all other parameters fixed. Thus, appreciable amounts of early dark energy have significant effects on matter perturbations, and we might expect nonlinear growth to be even more sensitive (e.g., see Bartelmann et al. 2006).

12. GROWING NEUTRINO MODEL

While freezing or scaling models, such as the early dark energy model just considered, are interesting from the physics perspective, they generally have difficulty in naturally evolving to sufficiently negative EOS by the present. The growing neutrino model of Amendola et al. (2007) and Wetterich (2007) solves this by coupling the scalar field to massive neutrinos, forcing the scalar field to a near cosmological constant behavior when the neutrinos go nonrelativistic. This is an intriguing model that solves the coincidence problem through cosmological selection (the time when neutrinos become nonrelativistic) rather than tuning the Lagrangian.

The combined dark sector (cosmon scalar field plus mass-running neutrinos) energy density is

$$\Omega_{\text{ch}}(a) = \frac{\Omega_{\text{ch}} a^3 + 2\Omega_{\nu} (a^{3/2} - a^3)}{1 - \Omega_{\text{ch}}(1 - a^3) + 2\Omega_{\nu} (a^{3/2} - a^3)}, \quad a > a_t, \tag{30}$$

$$\Omega_{\text{ch}}(a) = \Omega_\nu, \quad a < a_t, \tag{31}$$

where $\Omega_{\text{ch}} = 1 - \Omega_m$ is the present dark sector energy density. The Hubble parameter can be found by $H^2/H_0^2 = \Omega_m a^{-3} / [1 - \Omega_{\text{ch}}(a)]$ as usual. The two free dark parameters are the neutrino mass or density $\Omega_{\nu} = m_\nu(z = 0)/(30.8 h^2 \text{eV})$ and the early dark energy density $\Omega_e$. The transition scale factor $a_t$ is determined by intersection of the two behaviors given for $\Omega_{\text{ch}}(a)$.

The EOS is

$$w = -1 + \frac{\Omega_{\text{ch}} a^{-3/2}}{\Omega_{\text{ch}} + 2\Omega_{\nu} (a^{-3/2} - 1)}, \quad a > a_t, \tag{32}$$

with $w = 0$ before the transition, that is, a return to the standard early dark energy model. One can, therefore, translate $\Omega_e$ or $m_\nu(z = 0)$ into $w_0 = -1 + \Omega_e/\Omega_{\text{ch}} = -1 + \Omega_\nu/(1 - \Omega_m)$. Figure 12 shows the constraints in the $m_\nu(z = 0) - \Omega_e$ plane. As in the previous early dark energy model, the geometric degeneracy is clear. Again, when we add growth information
in the form of a 10% prior on the total linear growth (or the mass variance $\sigma_8$), the constraints tighten considerably, as shown in the right panel. Note that the neutrinos themselves cluster and large-scale observations may be able to provide future constraints on model parameters (Mota et al. 2008). The 95% confidence level limit on the neutrino mass from the current cosmological data (plus growth) is $2.1 (h/0.7)^2$ eV (1.2 if only statistical uncertainties are taken into account). These limits are comparable to astrophysical constraints from similar types of data applied to standard, constant mass neutrinos (Goobar et al. 2006; Tegmark et al. 2006). Note that because the neutrino mass grows due to the coupling, the value today can actually be larger than that at, say, $z \approx 3$ where Lyman alpha forest constraints apply (Seljak et al. 2006).

13. CONCLUSION

We have considered a wide variety of dark energy physics quite different from the cosmological constant. These include a diversity of physical origins for the acceleration of the expansion: from dynamical scalar fields to dark energy that will eventually cause deceleration and collapse, to gravitational modifications arising from extra dimensions or from quantum phase transitions, to geometric or kinematic parametrization of the acceleration, to dark energy that may have influenced the early universe and that may have its magnitude set by the neutrino mass. The comparison to $\Lambda$CDM and constant $w$ cases covers 5 one-parameter and 5 two-parameter dark energy EOS models. (Linder & Huterer 2005 had given in detail how even next-generation data will not generically be able to tightly constrain more than two such parameters.)

Two key results to emphasize are that current data (1) are consistent with $\Lambda$ and (2) are also consistent with a diversity of other models and theories, even when we restrict consideration to those with at least modest physical motivation or justification. As explicitly shown by the mirage model, any inclination toward declaring $\Lambda$ the answer based on consideration of a constant $w$ has an overly restricted view. The need for next-generation observations with far greater accuracy, and the development of precision growth probes, such as weak gravitational lensing, is
clear. All major classes of physics to explain the nature of dark energy are still in play.

However, there are already quite hopeful signs of imminent progress in understanding the nature of dark energy. For example, for the braneworld model, tight control of systematics would decrease the goodness of fit to $\Delta \chi^2 = +15$, even allowing for spatial curvature, diminishing its likelihood by a factor of 2000 naively, effectively ruling out the model. For the doomsday model, improving errors by 30% extends our “safety margin” against cosmic collapse by 10 billion years—a non-negligible amount. Every improvement in uncertainties pushes the limits on the neutrino mass within the growing neutrino model closer toward other astrophysical constraints plus this model essentially guarantees a deviation from $w = -1$ of 0.1 ($m_\nu/eV$), which is excitingly tractable. Terrestrial neutrino oscillation bounds already provide within this model that $1 + w > 0.005$.

As points of interest, we note that the model with noticeably positive $\Delta \chi^2$ relative to $\Lambda$, and hence disfavored, is completely distinct from the cosmological constant, that is, the braneworld model has no limit within its parameter spaces equivalent to $\Lambda$. This does not say that no such model could fit the data—the $R_{hew}$ model is also distinct from $\Lambda$ but fits as well as many models. Certainly, many successful models under current data do look in some averaged sense like a vacuum energy but this does not necessarily point to static dark energy. Two serious motivations to continue looking for deviations are that physicists have failed for 90 years to explain the magnitude required for a cosmological constant and that the previous known occurrence of cosmic acceleration— inflation—evidently involved a dynamical field and not a cosmological constant.

To guide further exploration of the possible physics, we highlight those models that do better than $\Lambda$: the geometric dark energy and algebraic thawing approaches. One of the sole models where adding a degree of freedom is justified (albeit modestly) by the resulting reduction in $\chi^2$ is the $R_{hew}$ model directly studying deviations of the spacetime curvature from the matter-dominated behavior. This has one more parameter than the constant $w$ EOS approach, but improves in $\chi^2$ by 1. In addition, it has a built-in test for the asymptotic de Sitter fate of the future expansion. We recommend that this model be considered a model of interest for future fits. The other model improving by at least one unit of $\chi^2$ is the algebraic thawing model, performing better than the other thawing models, with a general parametrization explicitly incorporating the physical conditions imposed by matter domination on the scalar field dynamics.

The diversity of models also illustrates some properties of the cosmological probes beyond the familiar territory of vanilla $\Lambda$CDM. For example, for the algebraic thawing and other such evolutionary models, the premium is on precision of $w_0$ and $w_a$ much more than the averaged or pivot EOS value $w_p$. Not all models possess the wonderful three-fold complementarity of the probes seen in the constant $w$ case; for many of the examples, BAO and CMB carry much the same information as each other. However, we clearly see that for every model, SN plays a valuable role, complementary to CMB/BAO, and often carry the most important physical information, such as on the doomsday time or the de Sitter fate of the universe or the Planck scale nature of the PNGB symmetry breaking.

The diversity of physical motivations and interpretations of acceptable models highlights the issue of assumptions, or priors, on how the dark energy should behave. For example, in the $R_{hew}$ model, should priors be flat in $r_0$, $r_1$, or $\Omega_m$, $w_0$; in the PNGB model should they be flat in $f$, $\phi/f$ or in $\Omega_m$, $w_0$, etc.? Lacking clear physical understanding of the appropriate priors restricts the physical meaning of any Bayesian evidence one might calculate to employ model selection; the $\chi^2$ goodness of fit used here does not run into these complications that can obscure physical interpretation.

We can use our diversity of models for an important consistency test of our understanding of the data. If there would be systematic trends in the data that do not directly project into the $\Lambda$CDM parameter space (i.e., look like a shift in those parameters), then one might expect that one of the dozen models considered might exhibit a significantly better fit. The fact that we do not observe this can be viewed as evidence that the data considered here are not flawed by significant hidden systematic uncertainties. The data utilize the Union08 compilation of uniformly analyzed and crosscalibrated Type Ia SNe data, constituting the world’s published set, with systematics treated and characterized through blinded controls. The data are publicly available at http://supernova.lbl.gov/Union, and will be supplemented as further SN data sets become published; the site contains high resolution figures for this paper as well.

However, to distinguish deeply among the possible physics behind dark energy requires major advances in several cosmological probes, enabling strong sensitivity to the time variation of the EOS. This is especially true for those models that are now or were in the past close to the cosmological constant behavior.

We get our first glimpses looking beyond $\Lambda$, but await keen improvements in vision before we can say that we understand the new physics governing our universe.

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