Contra generalized fuzzy precontinuous functions in fuzzy topological spaces

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Abstract: The concept of generalized preclosed (gp-closed) subsets and generalized preopen (gp-open) functions in topology were studied and some of their characterizations have been introduced. The aim of this paper is to introduce a new class of functions called fuzzy contra generalized precontinuous functions using generalized fuzzy preopen (gfp-fuzzy open) sets. We also studied their basic characterizations comparing with other types of fuzzy generalized open functions in fuzzy topological spaces.

1. Introduction
The purpose of present paper is to introduce and investigate fuzzy contra generalized preopen (contra gp-open), contra pre-generalized preopen (contra pgp-open) functions and their basic properties.

2. Literature Survey
Generalized closed sets in topological spaces were studied by N.Levin [1]. The definitions of generalized closed functions and the preservation theorem of normality and regularity were discussed by Malghan [2]. The concept of generalized preclosed sets in topology and the concept of generalized preopen functions and characterization of these functions with respect to the prenormal spaces were discussed by GovindappaNavalagi [3]. Some characterizations of generalized pre-irresolute and generalized pre-continuous maps between topological spaces were studied by I.Arokiarani, K.Balachandra and J.Dontchev[4]. Generalized preclosed functions were studied by T.Noir, H.Maki and J. Umehara [5]. Generalized preopen functions were studied by GovindappaNavalagiri and Mahesh Bhat [6]. Weakforms of preopen and preclosed functions were studied by Miguel Caldas and GovindappaNavalagiri [7,8].

3. Preliminaries
Throughout this paper (X, τ), (Y, σ) and (Z, γ) (or simply X, Y and Z) always means fuzzy topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space fuzzy topological space X. We denote the closure of A and the interior of A by Cl(A) and Int(A) respectively.
Definition 3.1: A subset $A$ of a fuzzy topological space $X$ is said to be preopen if $A \subseteq \text{Int } C_1(A)$. The complement of a preopen set of space fuzzy topological space $X$ is called preclosed. The family of all preopen (resp. preclosed) sets of a fuzzy topological space $X$ is denoted by $PO(X)$ (resp. $FP(X)$).

Definition 3.2: The intersection of all preclosed sets containing a subset $A$ is called the preclosure of $A$ and is denoted by $pCl(A)$ [9]; the union of all preopen sets contained in $A$ is called the preinterior of $A$ and is denoted by $pInt(a)$.

Definition 3.3: A subset $A$ of fuzzy topological space $(X, \tau)$ is called:

(i) Generalized fuzzy closed (in brief, fuzzy g-closed) set [10] if $\text{Cl}(A) \subseteq u$ whenever $A \subseteq U$ and $U$ is fuzzy open.

(ii) Generalized fuzzy preclosed (in brief, fuzzy gp-closed) set if $pCl(A) \subseteq U$ and $U$ is fuzzy open in $X$.

The complement of a gp-closed (resp. g-closed) set of a space fuzzy topological space $X$ is called generalized fuzzy preopen (in brief, fuzzy gp-open) (resp. g-open [4]) set of $X$.

Definition 3.4: A function $f : (X, \tau) \to (Y, \sigma)$ is said to be:

(i) Fuzzy precontinuous if inverse image of each fuzzy open set of $Y$ is fuzzy preopen in $X$,

(ii) Fuzzy Preopen if the image of each fuzzy open set $u$ of $X$, $f(u)$ is fuzzy preopen in $Y$,

(iii) Fuzzy preirresolute if the image of each fuzzy closed set $F$ of $X$, $f(F)$ is fuzzy preclosed in $Y$

(iv) Fuzzy pregeneralized preopen (briefly, pgp-open) if the image of each fuzzy closed set of $X$ is gp-closed in $Y$.

(v) Generalized fuzzy precontinuous (in brief, fuzzy gp-continuous) if the inverse image of each closed set of $Y$ is gp-closed in $X$.

(vi) Generalized fuzzy preirresolute (in brief, fuzzy gp-irresolute) if the inverse image of each fuzzy gp-closed set of $Y$ is fuzzy gp-closed in $X$.

(vii) Generalized fuzzy preclosed (in brief, fuzzy gp-closed) if the image of each fuzzy closed set of $X$ is fuzzy gp-closed in $Y$.

(viii) Generalized fuzzy preopen (in brief, pgp-open) if the image of each fuzzy open set of $X$ is fuzzy gp-open in $Y$.

(ix) Fuzzy pregeneralized preopen (in brief, pgp-open) if the image of each preopen set of $X$ is gp-open in $Y$.

4. Fuzzy Contra gp-open Functions
In this section, we define the fuzzy contra generalized form of preopen functions in the following.

Definition 4.1: A function $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy contra generalized preopen (briefly, contra gp-open) if for each fuzzy open set $U$ of $X$, $f(u)$ is gp-fuzzy closed in $Y$.

Definition 4.2: A function $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy contra pre-generalized preopen (briefly, contrapgp-open) if for each fuzzy preopen set $U$ of $X$, $f(u)$ is pgp-fuzzy closed in $Y$.

Clearly, every contra fuzzy preopen function is contra fuzzy gp-open.

We, characterize fuzzy contra gp-open and contra fuzzy pgp-open functions in the following.

Theorem 4.3: A surjective function $f : (X, \tau) \to (Y, \sigma)$ is fuzzy contra gp-open (resp. contra pgp-open) if and only if for each subset $B$ of $Y$ and each closed (resp. preclosed) $F$ of $X$ containing $f^{-1}(B)$, there exists a fuzzy gp-open set $H$ of $Y$ such that $B \subseteq H$ and $f^{-1}(H) \subseteq F$.

Proof: Necessity: Suppose $f$ is contra fuzzy gp-open (resp. contra fuzzy pgp-open). Let $B$ be any fuzzy subset of $Y$ and $F$ is fuzzy closed (resp. preclosed) set of $X$ containing $f^{-1}(B)$. Then $H$ is fuzzy gp-open in $Y$, $B \subseteq H$ and $f^{-1}(H) \subseteq u$. 

Proof: Sufficiency: Suppose $f$ is fuzzy contra gp-open (resp. contra pgp-open). Then $B \subseteq H$ and $f^{-1}(H) \subseteq u$. 

2
Definition 4.8: Now define the following. for the fuzzy gp-open set \{c\} in X, \(f^{-1}(\{c\}) = \{c\}\) is not fuzzy open in Y. We, recall the following.

Theorem 4.10: Let \(u\) be any fuzzy open (resp. preopen) set in X. Put \(B = Y- u\), then we have \(f^{-1}(B) \subseteq X- u\) and \(X - U\) is closed (resp. preclosed) set of in X. There exists fuzzy gp-open set \(H\) of \(Y\) such that \(B = Y- f(u) \subseteq H\) and \(f^{-1}(H) \subseteq X- u\), therefore, we obtain \(f(u) = Y-H\) and hence \(f(u)\) is fuzzy gp-closed in Y. This shows that \(f\) is fuzzy contra gp-open (resp. contra pgp-open).

Proof: Let \(F\) be any fuzzy closed set of \(Y\) contained in \(f(A)\). Then \(A \supseteq f^{-1}(F)\) and \(f^{-1}(F)\) is fuzzy closed in \(X\) since \(f\) is a continuous function. Since \(A\) is fuzzy gp-open in \(X\), \(f^{-1}(F) \supseteq pInt(A)\) and hence \(F \subseteq f(pInt(A)) \subseteq f(A)\). Since \(f\) is fuzzy pgp-open and \(pInt(A)\) is fuzzy preopen in \(X\), \(f(pInt(A))\) is fuzzy open in \(Y\) and hence \(F \subseteq pInt(f(pInt(A))) \supseteq pInt(f(A))\). This shows that \(f(A)\) is fuzzy gp-open in \(Y\).

\textbf{Theorem 4.11:} If \(f: X \rightarrow Y\) is fuzzy continuous, fuzzy pgp-open and \(A\) is gp- fuzzy open set in \(X\) then \(f(A)\) is fuzzy gp-open in \(Y\).

\textbf{Proof:} Let \(F\) be any fuzzy closed set of \(Y\) contained in \(f(A)\). Then \(A \supseteq f^{-1}(F)\) and \(f^{-1}(F)\) is fuzzy closed in \(X\) since \(f\) is a continuous function. Since \(A\) is fuzzy gp-open in \(X\), \(f^{-1}(F) \supseteq pInt(A)\) and hence \(F \subseteq f(pInt(A)) \subseteq f(A)\). Since \(f\) is fuzzy pgp-open and \(pInt(A)\) is fuzzy preopen in \(X\), \(f(pInt(A))\) is fuzzy open in \(Y\) and hence \(F \subseteq pInt(f(pInt(A))) \supseteq pInt(f(A))\). This shows that \(f(A)\) is fuzzy gp-open in \(Y\).

We, recall the following.

\textbf{Definition 4.5:} A function \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called strongly fuzzy M- preopen if the image of each preopen set of \(X\) is open in \(Y\).

\textbf{Definition 4.6:} A function \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called strongly fuzzy gp-open if the image of each gp-open set in \(X\) open in \(Y\).

Clearly every strongly fuzzy gp-open function is strongly M-preopen but not conversely. For,

\textbf{Example 4.7:} Let \(X = \{a, b, c\}\) and \(\tau = \{\emptyset, \{a\}, \{a, c\}\}\) and \(\sigma = \{Y, \{a\}, \{a, b\}, \{a, c\}\}\). Then identity function \(f: X \rightarrow Y\) is strongly fuzzy M- fuzzy preopen but not strongly fuzzy gp-open since for the fuzzy gp-open set \(\{c\}\) in \(X\), \(f(\{c\}) = \{c\}\) is not fuzzy open in \(Y\).

Now define the following.

\textbf{Definition 4.8:} A function \(f: X \rightarrow Y\) is said to be contra fuzzy strongly gp-open if the image of each fuzzy gp-open set of \(X\) is closed in \(Y\).

\textbf{Definition 4.9:} A function \(f: X \rightarrow Y\) is said to be contra fuzzy strongly M-preopen if the image of each fuzzy preopen set of \(X\) is closed in \(Y\).

We prove some of the decompositions of contra fuzzy gp-open and contra fuzzy pgp-open functions in the following.

\textbf{Theorem 4.10:} Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) and \(g: (Y, \sigma) \rightarrow (Z, \gamma)\) be two functions. Then, if \(f\) is fuzzy preopen and \(g\) is contra fuzzy pgp-open function, the composition \(g \circ f\) is contra fuzzy gp-open.

\textbf{Proof:} Let \(u\) be any open set in \(X\). Since \(f\) is preopen. Then \(f(u)\) is preopen set in \(Y\). Hence \(g(f(u))\) is closed in \(Z\). Because \(g\) is contra fuzzy pgp-open function. But \(g(f(u)) = gof(u)\). This shows that \(gof\) is contra fuzzy gp-open.

\textbf{Theorem 4.11:} Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) and \(g: (Y, \sigma) \rightarrow (Z, \gamma)\) be two functions. If for a M-preopen function \(f\) and contra fuzzy pgp-open function \(g\), then their composition \(gof\) is fuzzy contra pgp-open.

\textbf{Proof:} Let \(u\) be any preopen set in \(X\). Since \(f\) is fuzzy M-preopen, then \(f(u)\) is fuzzy preopen set in \(Y\). As \(g\) is contra fuzzy pgp-open function, \(g(f(u)) = gof(u)\) fuzzy gp-closed set in \(Z\). This shows that \(gof\) is fuzzy contra gp-open.

\textbf{Theorem 4.12:} Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) and \(g: (Y, \sigma) \rightarrow (Z, \gamma)\) be two fuzzy functions. If \(f\) is fuzzy preopen and \(g\) is fuzzy pgp-open, then the composition \(gof\) is fuzzy pgp-open.

\textbf{Theorem 4.13:} Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) and \(g: (Y, \sigma) \rightarrow (Z, \gamma)\) be two fuzzy functions. If \(f\) is strongly fuzzy M-preopen and \(g\) is fuzzy gp-open, then the composition \(gof\) is fuzzy pgp-open.
Proofs of the above two theorems are easy and hence omitted.

Hence we give the following, 

**Theorem 4.14:** Let \( f : (X, \tau) \to (Y, \sigma) \) and \( g : (Y, \sigma) \to (Z, \gamma) \) be two function. Then,

(i) If \( f \) is fuzzy continuous and surjective, then \( g \) is fuzzy contra gp-open.

(ii) If \( g \) is fuzzy gp-irresolute and injective, then \( f \) is fuzzy gp-open.

**Proof:**

(i) Let \( V \) be and arbitrary open set in \( Y \). Then, \( f^{-1}(V) \) is fuzzy open is \( X \) since \( f \) is continuous. Since \( g \) is fuzzy contra gp-open and \( f \) is surjective, \( (gof)(f^{-1}(V)) = g(V) \) is fuzzy gp-closed in \( Z \). This shows that \( g \) is fuzzy contra gp-open.

(iii) As, we have \( f(A) = g^{-1}(gof(A)) \) this is true for every subset \( A \) of \( X \) since \( g \) is given injective function. Let \( u \) be an arbitrary fuzzy open subset of \( X \), then \( gof(u) \) is fuzzy gp-closed in \( Z \). But \( g \) is also fuzzy gp-irresolute and hence we obtain that, \( g^{-1}(gof(A)) = f(u) \) is fuzzy gp-closed set in \( Y \).

This shows that \( f \) is a fuzzy contra gp-open function.

Similar to the above Theorem – 3.14, we prove the following.

**Theorem 4.15:** Let \( f : (X, \tau) \to (Y, \sigma) \) and \( g : (Y, \sigma) \to (Z, \gamma) \) be two fuzzy functions and \( gop \) is contra fuzzy pgp-open function. Then,

(iv) If \( f \) is fuzzy preirresolute and surjective, then \( g \) is fuzzy contra pgp-open.

(v) If \( g \) is fuzzy gp-irresolute and injective, then \( f \) is fuzzy gp-open.

**Proof:**

(i) Let \( V \) be an arbitrary fuzzy preopen set in \( Y \). Then, \( f^{-1}(V) \) is preopen is \( X \) since \( f \) is continuous. Since \( g \) is fuzzy contra pgp-open and \( f \) is surjective, \( (gof)(f^{-1}(V)) = g(V) \) is fuzzy gp-closed in \( Z \). This shows that \( g \) is fuzzy contra gp-open.

(iii) As, we have \( f(A) = g^{-1}(gof(A)) \) this is true for every subset \( A \) of \( X \) since \( g \) is given fuzzy injective function. Let \( u \) be an arbitrary preopen subset of \( X \), then \( gof(u) \) is fuzzy gp-closed in \( Z \) since \( gof \) is contra fuzzy pgp-open. But \( g \) is also fuzzy gp-irresolute and hence we obtain that, \( g^{-1}(gof(u)) = f(u) \) is fuzzy gp-closed set in \( Y \). This shows that \( f \) is a contra fuzzy pgp-open function.

We, recall the following.

**Definition 4.16:** A function \( f : (X, \tau) \to (Y, \sigma) \) is said to be always fuzzy generalized preopen (briefly, always gp-open) if for each fuzzy gp-open set \( U \) of \( X \), \( f(U) \) is fuzzy gp-open in \( Y \).

**Definition 4.17:** A function \( f : (X, \tau) \to (Y, \sigma) \) is said to be always fuzzy generalized preclosed (briefly, always gp-closed) if for each fuzzy gp-closed set \( F \) of \( X \), \( f(F) \) is fuzzy gp-closed in \( Y \).

**Remark 4.18:** A bijective function is always fuzzy go-open iff it is always fuzzy go-closed.

We, give the following.

**Theorem 4.19:** A surjective function \( f : (X, \tau) \to (Y, \sigma) \) is always fuzzy gp-open (resp. always gp-closed) if and only if for each fuzzy subset \( B \) of \( Y \) and each fuzzy gp-closed (resp. gp-open) \( F \) of \( X \) containing \( f^{-1}(B) \), there exists fuzzy gp-closed (resp. gp-open) set \( H \) of \( Y \) such that \( B \subseteq H \) and \( f^{-1}(H) \subseteq F \).
Proof: Necessity: Suppose \( f \) is always fuzzy gp-open (resp. always fuzzy gp-closed) and let \( B \) be any fuzzy subset of \( Y \) and \( F \) is fuzzy gp-closed (resp. gp-fuzzy open) set of \( X \) containing \( f^{-1}(B) \). Put \( H = Y - f(X - F) \). Then \( H \) is fuzzy gp-closed (resp. gp-open) in \( Y \), \( B \subset H \) and \( f^{-1}(H) \subset F \).

Sufficiency: Let \( u \) be any fuzzy gp-open (resp. gp-closed) set in \( X \). Put \( B = Y - f(u) \), then we have \( f^{-1}(B) \subset X - u \) and \( X - u \) is fuzzy gp-closed (resp. gp-open) set in \( X \). There exists fuzzy gp-closed (resp. gp-open) set \( H \) of \( Y \) such that \( B = Y - f(u) \subset H \) and \( f^{-1}(H) \subset X - u \). Therefore, we obtain \( f(u) = Y - H \) and hence \( f(u) \) is fuzzy gp-open (resp. gp-closed) in \( Y \). This shows that \( f \) is always fuzzy gp-open (resp. always gp-closed).

We, define the following.

Definition 4.20: A function \( f : (X, \tau) \to (Y, \sigma) \) is said to be always contra fuzzy generalized preopen (briefly, always contra gp-open) if for each fuzzy gp-open set \( u \) of \( X \), \( f(u) \) is fuzzy gp-closed in \( Y \).

Definition 4.21: A function \( f : (X, \tau) \to (Y, \sigma) \) is said to be always contra fuzzy generalized preclosed (briefly, always contra gp-closed) if for each gp-closed set \( F \) of \( X \), \( f(F) \) is fuzzy gp-open in \( Y \).

We, give the following.

Theorem 4.22: Let \( f : (X, \tau) \to (Y, \sigma) \) and \( g : (Y, \sigma) \to (Z, \tau') \) be two fuzzy functions and \( g \circ f \) is always contra fuzzy gp-open function. Then,

(i) If \( g \) is fuzzy gp-irresolute and surjective, then \( g \) is always fuzzy contra gp-open.

(ii) If \( g \) is fuzzy gp-irresolute and injection, then \( g \) is always fuzzy contra gp-open.

Proof:

(i) Let \( V \) be an arbitrary fuzzy gp-open set in \( Y \). Then, \( f^{-1}(V) \) is fuzzy gp-open in \( X \) since \( f \) is fuzzy gp-irresolute. Since \( g \circ f \) is always fuzzy contra gp-open and \( f \) is surjective, \( (g \circ f)(f^{-1}(V)) = g(V) \) is fuzzy gp-closed in \( Z \). This shows that \( g \) is always contra fuzzy gp-open.

(ii) As, we have \( f(A) = g^{-1}(g(f(A))) \) this is true for every subset \( A \) of \( X \) since \( g \) is given injective function. Let \( u \) be an arbitrary fuzzy gp-open subset of \( X \), then \( g(f(u)) \) is fuzzy gp-closed in \( Z \) since \( g \circ f \) is always fuzzy contra gp-open. But \( g \) is also fuzzy gp-irresolute and hence we obtain that \( g^{-1}(g(f(u))) = f(u) \) is fuzzy gp-closed set in \( Y \). This shows that \( f \) is always contra fuzzy gp-open function.

5. Conclusion

Thus in this paper we have introduced a new class of functions called fuzzy contra generalized preopen (contra gp-open) functions using generalized fuzzy preopen (gp-fuzzy open) sets and we have also studied their basic characterizations comparing it with other types of fuzzy generalized open functions.

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