Monopoles and Spatial String Tension in the High Temperature Phase of SU(2) QCD

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Abstract

We studied a behavior of monopole currents in the high temperature (deconfinement) phase of abelian projected finite temperature SU(2) QCD in maximally abelian gauge. Wrapped monopole currents closed by periodic boundary play an important role for the spatial string tension which is a non-perturbative quantity in the deconfinement phase. The wrapped monopole current density seems to be non-vanishing in the continuum limit. These results may be related to Polyakov’s analysis of the confinement mechanism using monopole gas in 3-dimensional SU(2) gauge theory with Higgs fields.

I. INTRODUCTION

A characteristic feature of finite temperature QCD is the deconfinement phase transition. In the low temperature phase, the heavy quark potential give rise to a confining (linear) potential \( V(r) \sim \sigma r \), where \( \sigma \) is the string tension. The string tension vanishes at the critical temperature and the confining potential changes to a Debye screened potential in the high temperature phase. However, there are some non-perturbative quantities even in

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the high temperature (deconfinement) phase of QCD. The spatial string tension is known
to be such a non-perturbative quantity. This quantity is the string tension extracted from
space-like Wilson loops composed of only spatial link variables. The spatial string tension
is non-vanishing even in the high temperature phase.

This property in the high temperature phase may be understood by dimensional reduc-
tion [1]. 4-dimensional QCD at high temperature limit can be regarded as an effective
3-dimensional QCD with Higgs fields. The effective theory has confinement features. The
string tension in this effective theory may be the spatial string tension in 4-dimensional QCD.
The relation between the spatial string tension and that of the effective theory is confirmed
using Monte-Carlo simulations [2,3]. The spatial string tension shows a scaling behavior on
the lattice and is non-vanishing in the continuum limit. The behavior is reproduced by the
following obtained from the effective theory:

$$\sqrt{\sigma_s} \propto g^2(T) T,$$

where \( \sigma_s \) is the spatial string tension. \( g(T) \) is the 4-dimensional coupling constant.

On the other hand, the study of topological quantities (monopole, instanton, ...) may
be important in order to understand the non-perturbative phenomena. Many people believe
that the dual Meissner effect due to condensation of color magnetic monopoles is the color
confinement mechanism in QCD [4,5]. Polyakov showed that the confinement features of 3-
dimensional SU(2) gauge theory with Higgs fields in a Higgs phase is explained by monopole
gas (3-dimensional instanton) analytically [6].

Recently, we studied the contribution of the monopole to the string tension in abelian
projected SU(2) QCD in maximally abelian gauge and found the following results [7].

1. Both the physical and the spatial string tension can almost be explained by monopoles
alone. Although the physical string tension vanishes at the critical temperature, the
spatial string tension remain non-vanishing also in the case of the string tension cal-
culated by the monopole currents.
2. There exist a long monopole loop and some short loops in the confinement phase. The long monopole loop alone is important for the physical string tension.

3. The physical string tension and the long monopole loop disappear at the same temperature \( (T_c) \). In the high temperature phase, there are short monopole loops only and the total monopole number is small.

These results suggest that the important monopole loops are different in the cases of the spatial string tension and the physical one and the spatial string tension is produced by small number of monopoles.

The aim of this report is to find what kind of monopole is responsible for the spatial string tension in the high temperature phase of 4-dimensional QCD and to study the relation to the Polyakov’s monopole gas in the continuum theory.

II. WRAPPED MONOPOLE LOOP AND DIMENSIONAL REDUCTION

Polyakov [6] proved that monopole (instanton) gas can explain the string tension in 3-dimensional SU(2) gauge theory with Higgs fields. Hence it is expected that monopole gas may be important for the spatial string tension also in the dimensional reduced 3-dimensional effective theory. When we consider dimensional reduction in 4-dimensional SU(2) gauge theory, static monopole loops correspond to the monopole gas in the 3-dimensional effective theory. The static monopole loops are closed by periodic boundary in the time direction. We call these loops wrapped monopole loops.

Because the temperature is the inverse of the lattice size in the time direction, the monopole currents may become easier to wrap by the periodic boundary as the temperature becomes higher. It may be able to explain the scaling behavior of the spatial string tension. The spatial string tension becomes larger as the temperature rises. Since the only difference between time and space directions on the lattice is the lattice size, the asymmetry of the physical and spatial string tension must be reduced to the difference of the boundaries.
In order to know if the spatial string tension at high temperature is given by the monopole gas in the 3-dimensional theory, we have checked two points:

1. Do only the wrapped monopole loops produce the spatial string tension?

2. How is the temperature dependence of the wrapped monopole current density? Does this value remain non-vanishing in the continuum limit?

Our method is the following. We perform usual Monte-Carlo simulations of SU(2) gauge theory using the Wilson action. An abelian theory is extracted by abelian projection \(^8,9\). The partial gauge fixing is done in the maximally abelian gauge in which the quantity

\[
R = \sum_{s, \mu} \text{Tr}\left(\sigma_3 U(s, \mu) \sigma_3 U^\dagger(s, \mu)\right)
\]

is maximized as much as possible by gauge transformation \[^9\]. An abelian link field \(u(s, \mu)\) is decomposed from the gauge fixed SU(2) link variable \(U(s, \mu)\) as follows:

\[
U(s, \mu) = c(s, \mu) u(s, \mu),
\]

\[
c(s, \mu) = \begin{pmatrix}
\sqrt{1 - |c_\mu(s)|^2} & -c_\mu^*(s) \\
c_\mu(s) & \sqrt{1 - |c_\mu(s)|^2}
\end{pmatrix}, \quad u(s, \mu) = \begin{pmatrix}
e^{i\theta_\mu(s)} & 0 \\
0 & e^{-i\theta_\mu(s)}
\end{pmatrix},
\]

where \(\theta_\mu(s)\) is the abelian gauge field. \(c_\mu(s)\) is a complex matter field. The monopole current \(k_\mu(s)\) is defined as

\[
k_\mu(s) = (1/4\pi) \epsilon_{\alpha\beta\gamma} \partial_\alpha \Theta_{\beta\gamma}(s)
\]

\[
\Theta_{\mu\nu}(s) = \partial_\mu \theta_\nu(s) - \partial_\nu \theta_\mu(s)
\]

\[
= \bar{\Theta}_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s)
\]

\((-\pi < \bar{\Theta}_{\mu\nu} \leq \pi, \quad n_{\mu\nu} : \text{integer}\)

following Degrand-Toussaint \[^{10}\].

We define wrapping number of every cluster of connected monopole currents as follows:

\[
\text{(wrapping number)} = \frac{1}{N_t} \sum_{\{\text{cluster}\}} k_4(s),
\]

(6)
where $\sum_{\text{cluster}}$ means summing up in a cluster, $N_t$ is the lattice size in the time direction. Non-wrapped monopole loops closed without using periodic boundary condition give the vanishing wrapping number, since there are the same number of monopole currents taking $k_4 = \pm 1$. On the other hand, if the monopole currents are wrapped by the periodic boundary, $\sum_{\text{cluster}} k_4(s)$ can take the value of $N_t$ times integer. When a monopole current belongs to a cluster which has non-zero wrapping number, we regarded it as a wrapped monopole current.

In the confinement phase, the difference between the wrapped monopole currents and the non-wrapped monopole currents is unclear since about half of the monopole currents are connected into one long monopole loop. However, in the deconfinement phase, there are short loops only and we can discriminate both monopole currents.

III. WRAPPED MONOPOLE CONTRIBUTION TO THE SPATIAL STRING TENSION

We have studied the wrapped monopole contribution to the spatial string tension in the high temperature phase. We have considered the monopole contribution to the Wilson loop as discussed in [11,12].

First, we extract abelian component by performing abelian projection in the maximally abelian gauge. In this gauge, the spatial string tension can be reproduced by residual abelian link variables. Next, we decompose the abelian Wilson loop $W$, which is the Wilson loop composed of abelian link variables, into two parts $W_1$ (photon part) and $W_2$ (monopole part) as follows:

$$W = \exp\{i \sum \theta_\mu(s) J_\mu(s)\}$$

$$= W_1 \cdot W_2$$

$$W_1 = \exp\{-i \sum \partial_\mu \Theta_{\mu\nu}(s) D(s - s') J_\nu(s')\}$$

$$W_2 = \exp\{2\pi i \sum k_\beta(s) D(s - s') \frac{1}{2} \epsilon_{\alpha\beta\rho\sigma} \partial_\alpha M_{\rho\sigma}(s)\},$$

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where $D(s-s')$ is the lattice Coulomb propagator. $J_\mu(s)$ is an external current corresponding to the Wilson loop and $M_{\mu\nu}(s)$ is an antisymmetric variable taking $\pm 1$ on a surface with the Wilson loop boundary as $J_\nu(s) = \partial_\mu^s M_{\mu\nu}(s)$. The string tension is obtained from these Wilson loops by the least square fit. We have assumed that the static quark anti-quark potential is given by linear + Coulomb + constant terms. The Creutz ratio of the size $R \times S$ on the lattice is

$$\chi(R, S) = \chi_0 + \chi_1 \left( \frac{1}{R(R-1)} + \frac{1}{S(S-1)} \right) + \chi_2 \left( \frac{1}{R(R-1)S(S-1)} \right),$$

(11)

where $\chi_0$ is the spatial string tension in the lattice unit. We have adopted this fitting function. The spatial string tension $\sigma_s(T)$ can be expressed in units of the critical temperature ($T_c$) using the relation between the lattice spacing and the temperature [2]:

$$\sqrt{\frac{\sigma_s(T)}{T_c}} = \sqrt{\chi_0(T)} N_{tc},$$

(12)

where $N_{tc}$ is the critical lattice size at each $\beta$.

Notice that we can see that a time-like monopole current ($k_4$) such as the static monopole current does not affect the physical string tension and it contributes only to the spatial one as seen from equation (10). When we measure the physical string tension, we use the Wilson loop having the antisymmetric variable $M_{i4}$ or $M_{4i}$, where $i = 1, 2, 3$. Because of the antisymmetric $\epsilon$ tensor, the physical string tension is evaluated by the space-like monopole currents only.

The monopole contribution in the deconfinement phase is a little lower than the full one, but it almost reproduce the behavior of the full one in the maximally abelian gauge [7].

Here, we calculate the wrapped monopole contribution and the non-wrapped monopole contribution to the spatial string tension separately in the high temperature phase. We have performed Monte-Carlo simulations on $24^3 \times N_t$ lattices with the periodic boundary condition, $N_t = \{2, 4, 6, 8\}$, at $\beta = \{2.30, 2.51, 2.74\}$ which are the critical $\beta$ for $N_t = \{4, 8, 16\}$ respectively. Measurements have been done every 50 sweeps after a thermalization of 2000 sweeps. We have taken 50 configurations for measurements. The data are plotted
in Fig. [1]. These data show that the spatial string tension from the wrapped monopole is almost the same as that from total monopole loops and show that the non-wrapped loops do not contribute to the spatial string tension.

IV. SCALING BEHAVIOR OF WRAPPED MONOPOLE DENSITY

The spatial string tension looks non-vanishing in the continuum limit. If the wrapped monopole produces the spatial string tension, the wrapped monopole density must remain non-vanishing in the continuum limit. We have investigated the monopole density on a lattice of the size $N_s^3 \times N_t$. The monopole density $\rho(T)$ is defined in the physical unit as follows:

$$\rho(T) = \frac{\sum |k_\mu(s)|a}{(N_s a)^3(N_t a)}.$$  

(13)

Here $a$ is the lattice spacing. Considering the relation between the temperature and the lattice size $N_t a = 1/T$, we can rewrite the monopole density as follows:

$$\rho(T) = \frac{\sum |k_\mu(s)|N_{tc}^3}{N_s^3 N_t} T_c^3,$$

(14)

where $T_c$ is the critical temperature and $N_{tc}$ is the critical lattice size at each $\beta$.

We have measured the temperature dependence and the $\beta$ dependence both of the total and the wrapped monopole densities varying both $\beta$ and $N_t$ on $24^3 \times N_t$ lattices. ($\beta = \{2.30, 2.51, 2.74\}, \ N_t = \{2, 4, 6, 8, 12\}$)

The data of the total and the wrapped monopole densities are shown in Fig. [2] and in Fig. [3]. The total monopole density does not show good scaling behavior. It depends not only on $T$ but also on $\beta$. On the other hand, the wrapped monopole density in the high temperature phase is independent of $\beta$, and looks remaining in the continuum limit. The wrapped monopole density seems to be proportional to $T^3$ at high temperature. This scaling behavior is similar to that of the spatial string tension.
V. CONCLUSIONS AND DISCUSSION

We have found the following results by the numerical studies of Monte-Carlo simulations. The spatial string tension can almost be reproduced by the wrapped monopole loops closed by the periodic boundary in the time direction in the high temperature phase. The wrapped monopole density is independent of $\beta$ and appears non-vanishing in the continuum limit. Moreover the scaling behavior of the wrapped monopole density is similar to that of the spatial string tension. These results suggest that the spatial string tension at high temperature is produced by the monopole gas in the effective 3-dimensional theory.

In Polyakov's analytical calculation, it is essential that the Higgs field has non-zero vacuum expectation value. On the other hand, the study of the scaling behavior of the spatial string tension suggested that the Higgs-sector in dimensionally reduced QCD does not contribute significantly to the spatial string tension. Furthermore, It was reported that the Higgs field does not have a non-zero vacuum expectation value. We expect that the monopole gas can be discussed without the expectation value of the Higgs field as we are discussing originally the monopole currents in 4-dimensional QCD without a Higgs field. If one wants to study the confinement mechanism using the classical solution of the monopole gas, the non-zero expectation value of the Higgs field is necessary. However, in the scheme of the abelian projection, the monopoles are produced dynamically and such monopoles are expected to play an important role for the confinement mechanism. Our results suggest that the spatial string tension has a close relation to the monopole gas.

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FIG. 1. The full spatial string tension (circle), the total monopole contribution (cross), the wrapped monopole contribution (square) and the non-wrapped monopole contribution (triangle). The full one is cited from [2].
FIG. 2. The temperature dependence of the total monopole density at $\beta = 2.30$ (circle), $\beta = 2.51$ (square) and $\beta = 2.74$ (triangle) on $N_t = 2, 4, 6, 8$ and 12 lattices respectively.
FIG. 3. The temperature dependence of the wrapped monopole density at $\beta = 2.30$ (circle), $\beta = 2.51$ (square) and $\beta = 2.74$ (triangle) on $N_t = 2, 4, 6, 8$ and 12 lattices respectively.