Determination of the proton-to-helium ratio in cosmic rays at ultra-high energies from the tail of the $X_{\text{max}}$ distribution

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Abstract

We present a method to determine the proton-to-helium ratio in cosmic rays at ultra-high energies. It makes use of the exponential slope, $\Lambda$, of the tail of the $X_{\text{max}}$ distribution measured by an air shower experiment. The method is quite robust with respect to uncertainties from modeling hadronic interactions and to systematic errors on $X_{\text{max}}$ and energy, and to the possible presence of primary nuclei heavier than helium. Obtaining the proton-to-helium ratio with air shower experiments would be a remarkable achievement.

To quantify the applicability of a particular mass-sensitive variable for mass composition analysis despite hadronic uncertainties we introduce as a metric the ‘analysis indicator’ and find an improved performance of the $\Lambda$ method compared to other variables currently used in the literature. The fraction of events in the tail of the $X_{\text{max}}$ distribution can provide additional information on the presence of nuclei heavier than helium in the primary beam.

Keywords: depth of shower maximum, exponential slope, proton-to-helium ratio, mass composition

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1. Introduction

While the composition is known to be a key to understanding the origin of ultra-high energy cosmic rays its determination by air shower experiments is a challenge (for a review see e.g. [1]). Estimations of the primary mass based on observables such as the depth of shower maximum $X_{\text{max}}$, particle arrival times or muon content suffer from significant uncertainties in the description of high-energy hadronic interactions. In this paper we show that the exponential slope, $\Lambda$, of the tail of the $X_{\text{max}}$ distribution can be used for the determination of the proton-to-helium ratio, p/He, in the primary beam and that $\Lambda$ and the inferred p/He are only weakly dependent on details of hadronic interactions and experimental systematic uncertainties. Thus, even with the indirect observations performed by air shower experiments it becomes possible to obtain a robust measure of an important detail of the cosmic ray composition. Knowledge of p/He can be used to constrain astrophysical models of the origin of cosmic rays.

The exponential slope $\Lambda$ of the tail of the $X_{\text{max}}$ distribution was extensively studied and used with respect to the determination of the proton-air interaction cross-section in an energy region where the primary composition appears proton-dominated (see [2, 3, 4] and references therein). This is due to the fact that the tail of the $X_{\text{max}}$ distribution is intimately linked to the interaction cross-section of the first interaction. One possible choice by such measurements was to define the fit range so that it would contain 20% of the deepest events [2, 3]. The contamination of the primary beam with photons and helium nuclei was the main source of the systematic uncertainties on the proton-air cross-section. In the present work this sensitivity of $\Lambda$ to the primary mass composition is the very subject of the study. We define $\Lambda$ in a different way, using a pre-defined fit range satisfying the condition to be almost free from the contamination of nuclei heavier than helium. This makes $\Lambda$ sensitive predominantly to p/He in the primary beam and provides a possibility for estimating p/He in air shower experiments.

Current best constraints on p/He at ultra-high energies $\gtrsim 10^{18}$ eV can be deduced using the hadronic interaction models EPOS-LHC [5, 6] and QGSJet-II.04 [7] from mass composition fits of full $X_{\text{max}}$ distributions measured at the Pierre Auger Observatory [8]. Since the covariance matrices for the fitted nuclei fractions are not available an estimation of the upper limits on p/He from this data with account of systematic and statistical uncertainties can be done by taking the upper value of the proton fraction.
\( f_p + \Delta f_p (\text{stat + syst}) \) and dividing it by the lower value of the helium fraction \( f_{\text{He}} - \Delta f_{\text{He}} (\text{stat + syst}) \) (for the lower limits on \( p/\text{He} \) one has \( (f_p - \Delta f_p)/(f_{\text{He}} + \Delta f_{\text{He}}) \)). For the energy range \( \log(E/\text{eV}) = 17.8-18.4 \) where the fits indicate the presence of a large fraction of protons and in case of EPOS-LHC a very low fraction of helium, one gets \( p/\text{He}(\text{QGSJet-II.04}) = 1.8^{+6.1}_{-0.6} \) (stat + syst) and \( p/\text{He}(\text{EPOS-LHC}) = 22.1^{+142}_{-17.4} \) (stat + syst). Even when comparing just these two post-LHC event generators, the central values of \( p/\text{He} \) differ by around a factor 10. Taking into account the uncertainties it is possible to set only a lower limit \( p/\text{He} \gtrsim 1 \). For higher energies \( \log(E/\text{eV}) = 18.4-19.4 \) the corresponding values \( p/\text{He}(\text{QGSJet-II.04}) = 1.3^{+4.6}_{-0.9} \) (stat + syst) and \( p/\text{He}(\text{EPOS-LHC}) = 1.7^{+5.9}_{-1.1} \) (stat + syst) cover a range from helium dominance to helium being almost absent. These uncertainties may get even larger when other hadronic models will be used, like the new version of Sibyll \([9]\) producing showers with deeper \( X_{\text{max}} \) compared QGSJet-II.04 and EPOS-LHC. The \( \Lambda \) method proposed in this paper can considerably improve the ability to determine \( p/\text{He} \). The slope \( \Lambda \) is weakly sensitive to the variations of \( X_{\text{max}} \) distribution properties (mean \( \langle X_{\text{max}} \rangle \) and width \( \sigma(X_{\text{max}}) \)) which are influenced by shower development and fluctuations and drive the fits to the full \( X_{\text{max}} \) distributions, and thus to (i) variations of hadronic interaction parameters other than the cross-section and to (ii) the systematic uncertainties on \( X_{\text{max}} \) and energy, as will be discussed below. The use of \( \Lambda \) is hence expected to provide better and more robust constraints on \( p/\text{He} \) compared to fitting of the full \( X_{\text{max}} \) distributions.

Due to our definition of the \( X_{\text{max}} \) fit range, the fraction of events in the fit range becomes a primary mass sensitive parameter itself. We show that this fraction can be useful for additionally constraining the abundances of nuclei heavier than helium. Having introduced more variables in addition to a number of existing ones based on \( X_{\text{max}} \), muon content and related observables (signal risetime, radius of curvature etc.), one would like to compare their performance for mass composition analysis in particular with respect to uncertainties from modeling hadronic interactions. We propose to use the ratio \( \Delta X(p-\text{He})/\Delta X(h) \) of the difference between proton and helium for some variable \( X \) to the uncertainties \( \Delta X(h) \) on this variable due to the limited knowledge of hadronic interaction properties. We call this ratio an ‘analysis indicator’. The analysis indicator for the exponential slope \( \Lambda \) turns out to be significantly larger than for other mass-sensitive variables such as \( \langle X_{\text{max}} \rangle \) or average number of muons.

All simulations are performed with CONEX \([10,11]\) for the hadronic

3
interaction models EPOS-LHC and QGSJet-II.04. For each primary particle
type (proton, helium, carbon, iron) and energy ($\log(E/eV) = 18.0$, 18.5 and
19.0) around $10^6$ showers are produced. To imitate the detector resolution
effect, each $X_{\text{max}}$ value is smeared by adding a Gaussian distributed random
variable with $\sigma = 20 \text{ g cm}^{-2}$, a typical value for current shower experiments
using the fluorescence technique \cite{12}.

In addition to showers with fixed energies, we simulated $10^5$ EPOS-LHC
proton showers from an energy spectrum with $\gamma = 3.23$ in the energy range
$\log(E/eV) = 17.8$–18.2 to confirm that a typical energy distribution does not
affect the determination of the value of $\Lambda$ due to its weak energy dependence
and that the simulations for fixed energies provide the necessary precision.

2. Method for the determination of $p/\text{He}$ in the primary beam

A main feature of our approach is the definition of the fit range, given
a measured $X_{\text{max}}$ distribution. In the cross-section analysis, where the fit
range is determined by requiring it to contain the deepest $\eta = 20\%$ of events,
for a simplified example of a pure carbon composition one will get $\Lambda_\eta$ corre-
sponding to the carbon-air cross-section. For mixed compositions $\Lambda_\eta$ takes
values from the spectrum of possibilities from proton to heavier nuclei and
intermediate values are degenerate, i.e. the same value of $\Lambda_\eta$ corresponds to
a number of different mixed compositions.

Instead, we define a lower limit of the $X_{\text{max}}$ fit range using the requirement
that only $\approx 0.5\%$ of the carbon-initiated showers survive. This way, a very
small contamination from nuclei heavier than helium can be achieved. In
this paper we use the lower limit value originating from QGSJet-II.04 which
is $5 - 7 \text{ g cm}^{-2}$ larger compared to the corresponding value for EPOS-LHC
(thus the fraction of the surviving carbon showers for EPOS-LHC is slightly
below 0.5\%). The lower limit of the fit range defined this way is $809 +
48(\log(E/eV) - 18) \text{ g cm}^{-2}$. The values of $\Lambda$ change within $\approx 1 - 2 \text{ g cm}^{-2}$
when the width of the fit range is varied within $60 - 150 \text{ g cm}^{-2}$, further we use
a width of $100 \text{ g cm}^{-2}$. For application to data the width could possibly be
further optimized depending on the characteristics of the specific experiment
and the available number of events.

Due to our definition, in case of a composition devoid of protons and
helium (such as the example of a pure carbon composition) there will be
too few events in the fit range to get a statistically reliable estimation of $\Lambda$
(the usage of the fraction of events in the fit range for composition studies is
inspected in Sec. 3). For mixed compositions, $\Lambda$ in our method is sensitive mainly to the relative abundances of protons and helium in the primary beam, and would take values between the $\Lambda$ values for pure proton and pure helium as will be shown below.

In Fig. 1 examples of proton, helium and carbon $X_{\text{max}}$ distributions are shown (left panel). There is a clear difference in the slopes between protons and helium. In the right panel of Fig. 1 fits of the function $\exp(-X_{\text{max}}/\Lambda)$ to the tail of proton EPOS-LHC $X_{\text{max}}$ distribution are given for energies of $\log(E/\text{eV}) = 18.0$ and 19.0. We use an unbinned maximum likelihood method, the binned histograms in the plots are given for visualization purpose.

The results for $\Lambda$ for all 3 primary energies and both interaction models are summarized in Fig. 2. The value of $\Lambda$ for proton is $\approx 20 - 25 \text{ g cm}^{-2}$ larger than that for helium (due to the significantly larger helium-air cross-section, see e.g. [13]). This difference is much larger than the difference of $\approx 2 - 3 \text{ g cm}^{-2}$ due to the hadronic model uncertainties, indicating an excellent suitability of the $\Lambda$ method for mass composition analysis with only a minor dependence on details of the hadronic interaction models.

In Fig. 3 we show the dependence of $\Lambda$ on $p/\text{He}$ for two-component $p$-$\text{He}$ mixtures. An accurate determination of $p/\text{He}$ is possible in a range $0.1 \lesssim p/\text{He} \lesssim 3$ where $\Lambda$ and $p/\text{He}$ are strongly correlated. Outside this range limits $p/\text{He} \lesssim 0.1$ or $p/\text{He} \gtrsim 3$ can be set.

Next, we investigate the impact of primary nuclei heavier than helium. We focus here on adding carbon ($A = 12$) and note that the effect of even heavier nuclei such as oxygen or iron is smaller than that of carbon. In Fig. 4 the values of $\Lambda$ for $p$-$\text{He}$ mixtures are compared to the values for three-component mixtures $p$-$\text{He}$-$C$ with carbon fractions of 25% and 50% in the primary beam. As expected from the definition of the $X_{\text{max}}$ fit range, the results on $\Lambda$ (and thus on primary $p/\text{He}$) are very robust. Specifically, a contamination with carbon of even 50% leads to a shift of $\Lambda$ of $2 - 3 \text{ g cm}^{-2}$ only. A determination of $\Lambda$ with an uncertainty of few $\text{g cm}^{-2}$ (e.g. for $p$-$\text{He}$ samples of 5000 events in total, the relative statistical uncertainty is $\Delta\Lambda/\Lambda \approx (8 - 10)\%$, c.f. also [3]) in several energy bins allows one to reconstruct the evolution of $p/\text{He}$ without a significant bias coming from variations in the fractions of heavier elements.

The value of $\Lambda$ is quite robust with respect to variations of the mean $\langle X_{\text{max}} \rangle$ and the width $\sigma(X_{\text{max}})$ of $X_{\text{max}}$ distributions, and thus to experimental systematic uncertainties on $X_{\text{max}}$ or to the variations of hadronic
interaction parameters other than the cross-section. For example, the smearing with a Gaussian distributed random variable with $\sigma = 20 \text{ g cm}^{-2}$ that we apply to the CONEX $X_{\text{max}}$ values, changes $\Lambda$ within 1.5 g cm$^{-2}$. The systematic uncertainty on $X_{\text{max}}$ reported by the Auger Observatory \cite{14} is smaller than 10 g cm$^{-2}$. Shifts of the simulated $X_{\text{max}}$ values by $\pm 10$ g cm$^{-2}$ produce changes in the values of $\Lambda$ within 1 g cm$^{-2}$. The systematic uncertainty on energy (measured with the fluorescence technique) of 14% corresponds to $\approx 3.5$ g cm$^{-2}$ uncertainty on $X_{\text{max}}$ and thus practically does not affect the values of $\Lambda$.

For application to real data, it is worth noting that special care has to be taken to achieve an unbiased data set and, thus, an unbiased extraction of $\Lambda$. This may involve a dedicated event selection based on shower geometries such as performed, for instance, in \cite{2, 3}.

![Figure 1: Left: $X_{\text{max}}$ distributions for proton, helium and carbon for EPOS-LHC and QGSJet-II.04 at $\log(E/eV) = 18.0$. The vertical line indicates the lower limit of the fit range. Right: examples of $\exp(-X_{\text{max}}/\Lambda)$ fits of the tails of the $X_{\text{max}}$ distributions for EPOS-LHC proton at $\log(E/eV) = 18.0$ and 19.0. $\Lambda$ values are in units of g cm$^{-2}$.](image)

3. Fraction of events in the fit range

For the proposed $\Lambda$ definition the fraction of events in the fit range becomes itself a mass-sensitive variable. For both models the fractions of the carbon $X_{\text{max}}$ distribution in the fit range are below 0.5%. The fractions of the distributions for protons and helium nuclei are shown in Fig. 5, ranging from 11.5% – 15.5% for protons and 3.2% – 4.3% for helium depending on energy and interaction model. Thus, one can see a good separation between protons and helium also in this variable though differences between the hadronic interaction models are larger compared to the case of $\Lambda$ (cf. Fig. 2). While
Figure 2: Energy dependence of the exponential slopes $\Lambda$ of the tails of proton and helium $X_{\text{max}}$ distributions for two hadronic interaction models.

Figure 3: Dependence of $\Lambda$ on $p/\text{He}$ for EPOS-LHC (left) and QGSJet-II.04 (right) in two-component $p$-He mixtures at $\log(E/eV) = 18.0$ and $\log(E/eV) = 19.0$.

with $\Lambda$ one can extract the primary $p/\text{He}$, the fraction of events can give additional information about the presence of heavier nuclei in the primary beam. Therefore one can combine the information from both variables in a single plot, as shown in Fig. 3. One can see that for proton-carbon mixtures $\Lambda$ decreases only slightly ($< 2 \text{ g cm}^{-2}$) for 50% fractions of carbon (c.f. Fig. 4). In case the observed $\Lambda$ is compatible with the expectation for protons, the fraction of events can be used for constraining the absolute proton abundance. For example, a fraction of events $< 6\%$ corresponds to a proton fraction $\lesssim 50\%$ for both interaction models and all compositions. In case both $\Lambda$ and the fraction of events are compatible with the values for proton-
helium mixtures, one can conclude that the admixture of heavier elements is small. Dilution of the primary beam with heavier elements would be seen as a horizontal shift of the data point to the left from the proton-helium graph, towards smaller values of the fraction of events in the fit range. Any data points in the area above the proton-only value of Λ or to the right of the proton-helium line would serve as an indication of shortcomings in the particular interaction model.

Compared to the fraction of events in the fit range, Λ is a more robust variable with respect to the uncertainties in the interaction models, being sensitive essentially to the cross-section only. The fraction of events in the fit range is influenced both by the position of the shower maximum and by the width of the $X_{\text{max}}$ distributions. Specifically, shifting $X_{\text{max}}$ values in accordance with systematic uncertainties of the Auger Observatory by ±10 g cm$^{-2}$ one gets the following variations in the fraction of events in the fit range at log($E$/eV) = 18.5: 14.4$^{+2.9}_{-2.4}$% (protons) and 4.1$^{+1.6}_{-1.2}$% (helium) for EPOS-LHC; and 11.5$^{+2.3}_{-1.9}$% (protons) and 3.2$^{+1.2}_{-0.8}$% (helium) for QGSJet-II.04.

A metric to quantify the impact of the hadronic interaction uncertainties for variables such as Λ and fraction of events is introduced in the next section.

4. Analysis indicator

Denoting for some variable $X$ the uncertainty due to the hadronic (h) interaction models as $\Delta X(h)$ and the difference between values of $X$ for protons and helium as $\Delta X(p - He)$ one can define the following quantity:

Figure 4: Dependence of Λ on p/He for EPOS-LHC (left) and QGSJet-II.04 (right) in two-component p-He mixtures (0% C) compared to the dependence in three-component p-He-C mixtures with carbon fractions of 25% and 50% for log($E$/eV) = 18.5.
We propose to name it ‘analysis indicator’: values $I^A_h(X) \gg 1$ (that is, $\Delta X(p-\text{He}) \gg \Delta X(h)$) indicate that $X$ is excellent for mass composition (label ‘A’ in $I^A_h(X)$) analysis, while for $I^A_h(X) \ll 1$ the variable is good for constraining interaction models (label ‘h’). Thus $I^A_h$ provides a metric to judge on the reliability of a variable to determine the composition or hadronic parameters. For many variables it may be close to 1, indicating that they are of limited use as an isolated parameter for distinguishing changes in the mass composition from changes or uncertainties in hadronic interactions. Let us consider several examples.

The depth of the shower maximum is known as a good shower observable for estimating the mass composition. A main variable extracted from a set of measured $X_{\text{max}}$ values is the mean $\langle X_{\text{max}} \rangle$. At energies around 10 EeV the difference between protons and helium in $\langle X_{\text{max}} \rangle$ is around 35 g cm$^{-2}$ and the uncertainty due to interaction models is around 20 g cm$^{-2}$ (see e.g. [14]) giving a quite large value of the analysis indicator $I^A_h(\langle X_{\text{max}} \rangle) \approx 35$ [g cm$^{-2}$]/20 [g cm$^{-2}$] $\approx 1.7$.

An opposite example can be given by variables related to the muon shower content. For the muon production depth [15] the analysis indicator

$$I^A_h(X) = \frac{\Delta X(p-\text{He})}{\Delta X(h)}.$$
Figure 6: Exponential slope vs. fraction of events in the fit range for EPOS-LHC (left) and QGSJet-II.04 (right) for p-He, p-C and p-Fe mixtures at log\((E/eV) = 18.5\). The relative fraction of primary particles changes with a step of 0.05. Numbers 0.1, 0.5, 1.0 near the graphs indicate the proton fraction in the samples. For pure carbon and iron samples there are too few events in the fit range to obtain a reliable estimation of \(\Lambda\), thus there are no corresponding points in the plots.

\[ I_A^h(\langle X_{max}^\mu \rangle) \approx 25 \, [\text{g cm}^{-2}] / 50 \, [\text{g cm}^{-2}] = 0.5. \]

Similarly for the number of muons in very inclined air showers one gets
\[ I_A^h(\langle \ln R_\mu \rangle) \approx 0.1 / 0.2 = 0.5. \]

Uncertainties on these variables from the description of hadronic interactions are limiting their usage for composition analysis. Results on \(\langle \ln A \rangle\) leading to masses heavier than iron\(^{15,16}\) may be used to restrict interaction models.

For \(\Lambda\) the uncertainties due to interaction models are small \(\approx 2 - 3 \, \text{g cm}^{-2}\) and the difference between protons and helium is around 20 \(\text{g cm}^{-2}\) (Fig. 2), thus
\[ I_A^h(\Lambda) \approx 7 - 10. \]

Allowing for an uncertainty (statistical and systematic) of approximately 10% on the proton-air cross-section\(^2\) which corresponds to an uncertainty of \(\approx 5 - 6 \, \text{g cm}^{-2}\) on \(\Lambda\), the analysis indicator is still large
\[ I_A^h(\Lambda) \approx 20 \, [\text{g cm}^{-2}] / 6 \, [\text{g cm}^{-2}] \approx 3.3. \]

This indicates that the \(\Lambda\) method is suitable for differentiating between primaries even as close as proton and helium.

For the fraction of events in the fit range the analysis indicator is
\[ I_A^h(\text{fraction}) \approx 7\% / 3\% \approx 2.3 \text{ (Fig. 3)}\]

which is clearly smaller compared to \(I_A^h(\Lambda)\) but still indicates a good applicability for composition studies.

\(^1\Delta \langle X_{max}^\mu \rangle(p-\text{He})\) is obtained using \(\Delta \langle X_{max}^\mu \rangle(p-\text{Fe}) \times \ln 4 / \ln 56\)

\(^2R_\mu\) is the ratio of the total number of muons in simulations or data to a number of muons in a typical proton shower at 10 EeV produced with QGSJet-II.03
5. Discussion

We introduced a method to determine p/He in primary cosmic rays at ultra-high-energies from the slope of the exponential fit of the tail of the $X_{\text{max}}$ distribution. The method is robust with regard to a possible contamination of the primary beam with nuclei heavier than helium, to experimental systematic uncertainties on $X_{\text{max}}$ and energy, and - importantly - to uncertainties in hadronic models being sensitive mainly to the interaction cross-section. A solid determination of p/He with air shower experiments will be a remarkable achievement. The fraction of events in the fit range is proposed as an additional mass sensitive variable that can be used to constrain the abundance of nuclei heavier than helium in the primary beam.

As explained in the introduction, for the determination of p/He the use of the full $X_{\text{max}}$ distribution to fit the cosmic ray composition is not superior or equivalent to the $\Lambda$ method presented here which exploits only the tail of the distribution. This can be understood because the absolute $X_{\text{max}}$ values (and related $\langle X_{\text{max}} \rangle$) do not have such a direct relation to the primary p/He as has $\Lambda$. Moreover, the interpretation of the absolute $X_{\text{max}}$ values is much stronger affected by experimental systematics e.g. in $X_{\text{max}}$, by the (uncertain) presence of heavier nuclei, and, in particular, by hadronic model uncertainties ($I_{h}^{A}(\Lambda) > I_{h}^{A}(X_{\text{max}})$). Since, however, the absolute $X_{\text{max}}$ values are a main driver when fitting the full $X_{\text{max}}$ distribution, the primary p/He from elemental fractions derived this way suffers from larger uncertainty compared to exploiting the direct correlation between $\Lambda$ and primary p/He. While the $\Lambda$ method provides only partial but not full information on composition such as abundances of all elements or mass groups (as fitting the full $X_{\text{max}}$ distribution does), it focuses on that part of the data which allows us to draw robust conclusions about the mass composition, even if the price to pay is to determine only one of its specific aspects, namely in this case p/He.

In the paper on proton-air cross-section measurement by the Pierre Auger Observatory [2] the fit of $\eta = 20\%$ of the deepest events with energies log$(E/eV) = 18.0 - 18.5$ gave $\Lambda_{\eta} = 55.8 \pm 2.3 \, (\text{stat}) \pm 1.6 \, (\text{syst}) \, \text{g cm}^{-2}$. A naive, straightforward comparison of this value with the values found in the current paper (Fig.3) indicates that protons are indeed existing and are more abundant than helium nuclei at these energies, in agreement with the estimations of [2]. However, such a comparison should be taken with great care. The fit range in the cross-section analysis (both the starting depth and the width) differs from the definition in the current work, optimized for p/He.
determination. Such a difference will result in a different data sample for the p/He analysis compared to the data sample used for the cross-section analysis. As a result the value of $\Lambda_\eta$ might differ as well both due to an earlier start of the fit (closer to the maximum of the $X_{\text{max}}$ distribution) and due to a possible larger contamination with nuclei heavier than helium. Thus for a quantitative interpretation of data a proper accounting for detector acceptance, event selection criteria and definition of the fit range is needed.

In the method proposed in the current paper, an important characteristic we exploit is the quasi-discreteness of light nuclei ($A < 12$) in the cosmic ray beam with protons and helium being the dominating light primary particles. Other light nuclei are not expected to significantly impact the relation between $\Lambda$ and p/He. The primary abundance of the other light nuclei is expected to be very small due to their relative instability compared to protons and helium. As confirmed by simulations of cosmic ray propagation with CR-Propa [17], their primary abundance is suppressed by factors $> 10$ compared to the abundance of $(p + \text{He})$ [17, 18]. Furthermore, elements heavier than helium are additionally suppressed by our definition of the $X_{\text{max}}$ fit range. For instance, suppression factors due to the fit range relative to protons (to helium) are about 8 (2.5) for $^7\text{Li}$ and 20 (6) for $^{10}\text{B}$.

Finally, we also introduced a new quantity called ‘analysis indicator’ for the characterization of the performance of the mass-sensitive variables for mass composition analysis. The value of the analysis indicator for $\Lambda$ is at least twice as large as that for $\langle X_{\text{max}} \rangle$. Compared to $\langle X_{\text{max}} \rangle$ or to an analysis making use of absolute $X_{\text{max}}$ values, $\Lambda$ is more robust with regard to experimental systematic uncertainties and to uncertainties in the simulation of hadronic interactions since it depends essentially only on the cross-section. Though the fraction of events in the fit range is more sensitive to hadronic interaction parameters compared to $\Lambda$, it still has a value of the analysis indicator comparable to that for $\langle X_{\text{max}} \rangle$. The analysis indicator can provide a common base for comparing performances of the different mass-sensitive variables for mass composition analysis or for constraining hadronic interactions.

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