Baryogenesis from Primordial Blackholes after Electroweak Phase Transition

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Abstract

Incorporating a realistic model for accretion of ultra-relativistic particles by primordial blackholes (PBHs), we study the evolution of an Einstein-de Sitter universe consisting of PBHs embedded in a thermal bath from the epoch $\sim 10^{-33}$ sec to $\sim 5 \times 10^{-9}$ sec. In this paper we use Barrow et al’s ansatz to model blackhole evaporation in which the modified Hawking temperature goes to zero in the limit of the blackhole attaining a relic state with mass $\sim m_{pl}$. Both single mass PBH case as well as the case in which blackhole masses are distributed in the range $8 \times 10^2$ - $3 \times 10^5$ gm have been considered in our analysis. Blackholes with mass larger than $\sim 10^5$ gm appear to survive beyond the electroweak phase transition and, therefore, successfully manage to create baryon excess via $X - \bar{X}$ emissions, averting the baryon number wash-out due to sphalerons. In this scenario, we find that the contribution to the baryon-to-entropy ratio by PBHs of initial mass $m$ is given by $\sim \epsilon \zeta (m/1 \text{ gm})^{-1}$, where $\epsilon$ and $\zeta$ are the CP-violating parameter and the initial mass fraction of the PBHs, respectively. For $\epsilon$ larger than $\sim 10^{-4}$, the observed matter-antimatter asymmetry in the universe can be attributed to the evaporation of PBHs.

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1 Introduction

That the Milky Way is essentially made of matter is evident not only from the landings of space probes on Moon and other planets without any disastrous consequences but also from the absence of anti-nuclei in the observed cosmic rays, and from the observations of Faraday rotation. Observational support for absence of significant quantity of anti-matter beyond our Galaxy exists, but it is of indirect nature. Since visible mass in the universe is chiefly in the form of baryonic matter, the inferred matter-antimatter asymmetry essentially boils down to the problem of the origin of baryon asymmetry. The baryon asymmetry is characterized by the baryon-to-photon ratio $\eta = n_B/n_{\gamma}$, with $n_B$ and $n_{\gamma}$ being the number densities of net baryons and photons, respectively. According to standard big-bang nucleosynthesis calculations, the predicted abundances of light elements depend only on the free parameter $\eta$ and are in apparent agreement with the observed abundances provided $\eta$ lies in the range $(2.8 - 4.5) \times 10^{-10}$. Recently, Tytler et al. have estimated the baryon-to-photon ratio from the observations of deuterium abundance in a high red-shift quasar absorption system and according to their measurements, $\log \eta = -9.18 \pm 0.4 \pm 0.4 \pm 0.2$.

The esthetically appealing scenario of the universe consisting of equal amount of baryons and anti-baryons at the instant of creation is still compatible with a non-zero $\eta$ if one invokes Sakharov conditions, namely, of having $B$, $C$ and $CP$ violating interactions in out-of-thermodynamic equilibrium condition sometime in the early history of the universe. The Grand Unified Theories (GUTs) of fundamental forces incorporate baryon number violating interactions naturally while $CP$ violation can be introduced in such theories in many different ways (it is to be noted that $CP$-violation added theoretically in GUTs, in general, is not related to the observed $CP$-violation in the $K^0 - \bar{K}^0$ system) and therefore it is not surprising that GUTs provide a natural framework for the generation of baryon asymmetry through decay of $X - \bar{X}$ bosons. However, through the work of Kuzmin, Rubakov and Shaposhnikov, it came to be appreciated that baryon number violation can take place during electro-weak phase transition (EWPT) and that such processes could erase baryon-asymmetry generated prior to EWPT era.

Use of $B$-violation in electro-weak theories to produce excess baryons

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1 If neutrinos are massive then the gravitating mass may as well be dominated by leptons. However, there is hardly any direct measure of the lepton number of the universe.
has also been made in the literature\cite{2} but one of the major obstacles in this scenario is the requirement of low Higgs mass which is in direct conflict with the experimental lower limit of $m_H > 88$ GeV\cite{10}. It appears that in the minimal version of electro-weak theory, generating baryon asymmetry may not be possible at all\cite{11} and especially with the discovery of the top-quark with a mass around 175 GeV\cite{12} there is hardly any region left in the parameter space of the standard model to produce observed baryon-to-photon ratio\cite{13}.

The other major scenario of generating baryon asymmetry is to invoke Hawking evaporation of black-holes. The early sketchy ideas of Hawking and Zeldovich took proper shape with the advent of GUTs, giving rise to a picture of black-holes of small mass emitting $X$ and $\bar{X}$ bosons thermally which subsequently decay and in the process violate $B$, $C$ and $CP$, leading to a production of baryon excess\cite{14}. At the fundamental level, this scenario has an attractive feature in that it combines ideas of black-hole thermodynamics\cite{15}\cite{16} on one hand and GUT on the other, to explain the observed matter-antimatter asymmetry in the universe. One of the important ingredients of this picture is the occurrence of mini-black-holes having mass less than $10^{14}$ gm. It is obvious that such black-holes cannot emerge as end products of stellar evolution. However, Zeldovich and Novikov\cite{17} and Hawking\cite{18} argued that primordial black-holes (PBHs) of small mass can be generated from the space-time curvature, and subsequently, Carr\cite{19} showed the possibility of creating PBHs from density fluctuations in the early universe. In the context of inflation, several authors have discussed mechanisms to produce PBHs using the general idea that bubble wall collisions may trap pockets of false vacuum region that subsequently collapse to form black-holes\cite{20}. In a recent work, Nagatani\cite{21} has proposed an interesting blackhole-electroweak mechanism of baryogenesis that requires the presence of a blackhole to create a domain wall around it, leading to genesis of baryon excess without the need of a first order electroweak phase transition.

Previous paragraphs of this section indicate that although GUTs can naturally generate baryon asymmetry, any baryon excess generated prior to electro-weak era is erased due to sphaleron transitions, while at the same time, creation of baryon asymmetry solely due to electro-weak processes is fraught with uncertainties as well as the requirement of low Higgs mass, contrary to the experimental situation. Under the present circumstances, it is therefore natural to explore alternate means to explain matter-antimatter asymmetry. Since the existence of PBHs in the early universe is rather
generic, one ought to carefully re-examine the mechanism of generating baryon asymmetry through black-hole evaporation. In such a scenario, the crucial point to investigate is whether PBHs survive after the EWPT has taken place, so that the baryon asymmetry created due to their subsequent Hawking evaporation survives, leaving an imprint till the present epoch.

The present paper is an attempt to critically examine the evolution of the masses of a collection of PBHs created after the end of inflation, taking into account both the accretion of background matter by the black-holes as well as the mass loss due to Hawking emission. The paper has been organized in the following manner. In Section 2 we discuss processes responsible for the change in a black-hole’s mass, and thereafter, we develop a formalism to describe accretion of relativistic matter by mini-black-holes. The subject of black-hole mass spectrum and its evolution is tackled next, in section 3, along with a discussion on the cosmological evolution of a mixture of PBHs and relativistic matter. Section 4 deals with the study of evolution equations numerically as well as a detailed analysis of the numerical solutions pertaining to the survival of PBHs past the EWPT. In section 5, we calculate baryon excess resulting from the decay of $X - \bar{X}$ bosons emitted by the PBHs during their final stages of Hawking evaporation, and then discuss the implications of these results to the question of matter-antimatter asymmetry. Finally, we end with a brief discussion of the above scenario in section 6.

2 Evolution of the mass of a black-hole

2.1 Mass loss due to evaporation

Bekenstein’s conjecture\cite{15} that the area of the event horizon of a black-hole being a measure of its entropy was vindicated by the classic work of Hawking in the early seventies who showed that when quantum effects around a black-hole are included, the black-hole emits particles with a thermal distribution corresponding to a temperature $T_{BH}$ that is proportional to the surface gravity at the event horizon\cite{15} \cite{16}, and is given by the relation,

$$T_{BH} = \frac{m_{pl}^2}{8\pi m c^2}$$  \hspace{1cm} (1)

where $m$ and $m_{pl}$ are the mass of the black-hole and the Planck mass, respectively. According to eq. (1), PBHs created in the early universe with a
mass $\approx 10^{14}$ gm would be decaying today in a burst of high energy radiation, and there exists in the literature, upper bounds on the abundance of such PBHs from the observed level of cosmic $\gamma$-ray flux. As pointed out by Zeldovich and others, the expression in eq. (1) for the Hawking temperature can only be an approximation and is amenable to modifications at Planck scale because of the effects of quantum gravity. In fact, particle physicists have shown from various angles that Hawking evaporation may cease when the black-hole reaches the Planck mass scale leading to a massive relic. In this context, an interesting toy model inspired by superstring theories has been considered by Barrow et al\cite{22} in which the expression for the black-hole temperature has been modified by including correction terms that contain powers of black-hole mass in units of Planck mass. Following Barrow et al’s ansatz, one can therefore express the black-hole temperature as

$$T_{BH} = \frac{m_{pl}}{8\pi} \left[ \frac{m_{pl}}{m} - \kappa \left( \frac{m_{pl}}{m} \right)^n \right] \frac{c^2}{k} \tag{2}$$

where $\kappa$ is a non-negative constant. For $n > 2$ and $\kappa \approx O(1)$, it is clear that for holes of mass $m \gg m_{pl}$, eq. (1) is a limiting case of eq. (2). According to eq. (2), as the hole mass decreases due to evaporation, initially there is a rise in the hole’s temperature but as $m$ approaches $m_{pl}$ the temperature starts falling and becomes zero when the mass of the hole reaches the value $m_{rel} = \kappa^{1/(n-1)} m_{pl}$. Therefore, Barrow et al’s ansatz implies stable black-hole relics of mass $m_{rel} \approx m_{pl}$. To estimate the rate of mass-loss from eq. (2), we may work in the frame-work of radiative transfer, assuming that the hole’s event horizon acts like a perfect black-body surface. In such a case, it is easy to show that the energy flux $F$ is related to the energy density $\varepsilon$ at the surface of interest in the following manner\cite{23}

$$F = \frac{c}{4} \varepsilon \tag{3}$$

The effective energy-density of ultra-relativistic particles due to Hawking evaporation in the vicinity of the event horizon is related to the temperature of the black-hole by

$$\varepsilon = \frac{\pi^2 g_z^{BH}}{30} \frac{k^4}{(hc)^3} T_{BH}^4 \tag{4}$$

where $g_z^{BH} = g_b^{BH} + (7/8)g_f^{BH}$ is the effective number of degrees of freedom at the temperature $T_{BH}$, and $g_b^{BH}$ and $g_f^{BH}$ are the corresponding degrees of
freedom for bosons and fermions, respectively. Therefore, the rate of mass-loss from the event-horizon is given by

\[
\frac{dm}{dt} = -\frac{1}{c^2} F \cdot 4\pi R_S^2
\]

\[
= -\alpha_2 m^2 \left[ \frac{m_{pl}}{m} - \kappa \left( \frac{m_{pl}}{m} \right)^n \right] \tag{5}
\]

\[
\frac{dm}{dt} = -\alpha_2 m^2 \left[ \frac{m_{pl}}{m} - \kappa \left( \frac{m_{pl}}{m} \right)^n \right] \tag{6}
\]

where \( \alpha_2 = g_{BH}^2 c^2 / (30720 \pi \hbar) \) and \( \kappa \approx 1 \). In arriving at eq. (6), we have made use of eqs. (2)-(4) as well as the standard result for the Schwarzschild radius \( R_S = 2Gm/c^2 \). The calculations that led to eq.(6) were based on modeling the black-hole event-horizon to be the surface of a black-body of radius \( R_S \) at a thermodynamic temperature \( T_{BH} \). It is, therefore, interesting to compare our result with that of Don Page\[24\] which is based on rigorous numerical computations for black-holes of mass \( m > 10^{17} \) gm. According to his calculations, the mass-loss rate for such holes is

\[
\frac{dm}{dt} = -2.011 \times 10^{-4} \frac{\hbar c^4}{G^2 m^2} \tag{7}
\]

If we assume that eq.(7) is valid also for \( 10^{-2} \) gm < \( m < 10^{17} \)gm, then comparing (6) and (7) one obtains \( g_{BH} \approx 20 \), which is not too unreasonable since for holes of mass \( 10^{-2} \)gm one expects \( g_{BH} \) to be as high as \( \approx 100 \) (in most GUTs).

2.2 Accretion of relativistic matter by a mini black-hole

The temperature of the universe is expected to be extremely high just after the end of inflation, and therefore matter during that period will be in the form of ultra-relativistic particles. For particles with de Broglie wavelength \( \lambda \ll R_S \), the capture cross-section corresponding to a Schwarzschild black-hole is \( \sim \pi r_c^2 \) where \( r_c = (3\sqrt{3}/2)R_S \)[25]. When the de Broglie wavelength of a particle is larger than \( R_S \), the capture cross-section is likely to be negligible as the blackhole sees an incident wave rather than a point particle. For high energy particles with \( \lambda \ll R_S \), we will make use of the geometric optics approximation in which any such ultra-relativistic particle hitting a fictitious sphere of radius \( r_c \) around the hole will be absorbed.

If \( I_\nu \) represents the specific intensity of such particles corresponding to energy \( h\nu \) and if \( dA \) is an area element on this fictitious sphere then the rate
at which energy is accreted by the hole per unit range of $\nu$ per unit area is given by

$$\frac{dE_\nu}{dt d\omega dA} = \int d\Omega \cos \theta I_\nu = \pi I_\nu$$

(8)

Since the effective area of capture is $4\pi r_c^2$, the rate at which energy is accreted in the frequency range $[\nu, \nu + d\nu]$ is given by

$$\frac{dE_\nu}{dt} = (2\pi r_c)^2 I_\nu d\nu$$

(9)

To obtain the total rate of accretion of energy we integrate eq. (9) over frequency keeping in mind that geometric optics approximation requires the lower limit of integration $\nu_{\text{min}}$ to be a few times $c/r_c$. For ultra-relativistic particles, momentum is $p \approx \hbar \nu/c$ so that the number density of particles of species A in the frequency-range $(\nu, \nu + d\nu)$ takes the form

$$n_A(\nu)d\nu = \frac{4\pi g_A}{c^3} \frac{\nu^2 d\nu}{e^{\hbar \nu/kT} \pm 1}$$

(10)

where $g_A$ is the spin-degeneracy factor for the $A^{th}$ species and the $+(-)$ sign refers to fermions (bosons). In eq. (10) $T$ is the temperature of the universe. Therefore, the specific intensity $I_{\nu A}$ corresponding to the species $A$ is given by

$$I_{\nu A} = \frac{\hbar \nu^3}{4\pi c^2} e^{\hbar \nu/kT} \pm 1$$

(11)

Making use of eqs. (9) and (11), we can express the net rate of energy accretion by a hole in the following manner

$$\frac{dE}{dt} = \left( \frac{2\pi r_c}{c} \right)^2 \left[ g_b^{\text{uni}} \int_{\nu_{\text{min}}}^{\infty} \frac{\hbar \nu^3}{e^{\hbar \nu/kT} - 1} d\nu + g_f^{\text{uni}} \int_{\nu_{\text{min}}}^{\infty} \frac{\hbar \nu^3}{e^{\hbar \nu/kT} + 1} d\nu \right]$$

(12)

where $\nu_{\text{min}} = \alpha_1 c/r_c$ is the lower frequency cut-off, $\alpha_1$ being a number of the order of 10 (this takes care of the fact that only particles with $\lambda \ll R_S$ are considered to have been captured by the blackhole). In eq. (12), $g_b^{\text{uni}}$ and $g_f^{\text{uni}}$ are the total bosonic and fermionic degrees of freedom, respectively, for the cosmic soup. These are to be distinguished from $g_b^{BH}$ and $g_f^{BH}$ introduced in section (2.1).
The rate at which the hole’s mass grows as a result of accretion is

\[
\frac{dm}{dt} = \frac{405}{\pi^3 c^5} \varepsilon_R G^2 m^2 \left[ \frac{g_b^{uni}}{g_*^{uni}} \int_{x_{min}}^{\infty} \frac{x^3}{e^x - 1} \, dx + \frac{g_l^{uni}}{g_*^{uni}} \int_{x_{min}}^{\infty} \frac{x^3}{e^x + 1} \, dx \right]
\]

where \( x_{min} = \frac{\hbar \nu_{min}}{kT} \).

In obtaining the above equation, we have made use of a change of variable in eq.(12) along with \( r_c = \frac{3\sqrt{3}/2}{R_S} \). We note that \( \varepsilon_R \) appearing in eq.(13) is the energy-density of the background relativistic particles, \( \varepsilon_R = \pi^2 g_*^{uni}(kT)^4/(30h^3 c^3) \), \( g_*^{uni} \) being the temperature-dependent effective spin-degeneracy factor and is equal to \( g_b^{uni} + 7/8g_f^{uni} \). From eq.(13) it is evident that accretion plays an important role for massive PBHs at early epochs when the temperature of the universe is very high so that energy density \( \varepsilon_R \) of the relativistic particles is large while \( x_{min} \) is small. This is easy to understand from a physical point of view in the sense that only when the temperature is large that there are sufficient number of particles with de Broglie wavelength much less than the Schwarzschild radius of the PBHs, ready to be accreted. By the same token, when the hole-mass reaches a size of the order of \( m_{pl} \), neither accretion nor quantum evaporation is significant.

### 3 Blackhole mass spectrum and evolution of the universe

It is evident that the mass distribution of PBHs is intimately linked to the mechanism of their production. Several authors [19, 20, 26, 27] in the literature have discussed blackhole mass spectrum from diverse angles. Since the mass spectrum is sensitive to production mechanisms and, since so far no particular model of PBH creation has been singled out, we adopt a very general procedure in this paper to analyse the evolution of blackhole mass spectrum.

We consider a distribution function \( N(m, t) \) such that \( N(m, t)dm \) represents number-density of PBHs with mass in the range \( (m, m + dm) \) at the cosmic epoch \( t \). We assume that the creation of PBHs stopped after a cosmic epoch \( t_{pbh} \) so that at later times in a given comoving volume the number of holes remain the same while their masses change due to a combination of Hawking radiation and accretion of background matter. Note that we are working under the assumption that the ultimate state of a PBH along the
course of its evolution is a stable relic of mass $\approx m_{pl}$, i.e. a hole does not disappear completely as the original Hawking radiation mechanism would demand. Also, since the mass $m$ of a hole changes with time, the mass distribution function at time $t$ and at time $t + dt$ are related as

$$a^3(t)N(m, t)dm = a^3(t + dt)N(m', t + dt)dm'$$  \hspace{1cm} (14)$$

where $m'$ is related to $m$ through $m' = m + \dot{m}dt$ and $a(t)$ is the FRW scale-factor at cosmic epoch $t$. Making a Taylor expansion of quantities at the RHS of eq.(14), and using the relation

$$dm' = dm \left(1 + \frac{\dot{m}}{\dot{m}} dt \right)$$  \hspace{1cm} (15)$$

we obtain

$$\frac{\partial N}{\partial t} + 3 \frac{\dot{a}}{a} N + \frac{\partial}{\partial m} (N\dot{m}) = 0$$  \hspace{1cm} (16)$$

With the help of the mass distribution function $N(m, t)$, we can also obtain an expression for the mass-density associated with PBHs as

$$\rho_{BH}(t) = \int_{m_{rel}}^{\infty} mN(m, t)dm$$  \hspace{1cm} (17)$$

It is useful to express the black-hole mass-distribution as

$$N(m, t) = N_0(t)f(m, t)$$  \hspace{1cm} (18)$$

where $N_0 \propto a^{-3}$ so that $\int_{m_{rel}}^{\infty} f(m, t)dm$ is independent of time. With the help of eq.(18) it can be easily shown that eq.(16) reduces to

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial m}(\dot{m}f) = 0$$  \hspace{1cm} (19)$$

Essentially, $f(m, t)dm$ represents the number of black-holes with mass in the interval $(m, m + dm)$ in a unit coordinate volume at the cosmic epoch $t$, while the dilution of black-hole number density due to the expansion of the universe is taken care of by the factor $N_0(t) = A/a^3(t)$. Differentiating eq.(17) with respect to $t$ and then making use of eqs.(18) and (19) it can be shown that

$$\frac{d\rho_{BH}}{dt} + \frac{3\dot{a}}{a}\rho_{BH} = N_0 \int_{m_{rel}}^{\infty} \dot{m}f(m, t)dm$$  \hspace{1cm} (20)$$
Since the total energy-momentum tensor is divergence free, we also have the equation:

\[ c^2 \frac{d}{dt} \left[ (\rho_R + \rho_{BH})a^3 \right] + 3p_R a^2 \ddot{a} = 0 \tag{21} \]

where \( \rho_R = \frac{\varepsilon_R}{c^2} \) is the mass density of radiation. Here we have assumed that the black-holes possess negligible peculiar speeds so that their contribution to pressure is insignificant. Using \( p_R = c^2 \rho_R / 3 \) and eq.(21) in eq.(20) we obtain:

\[ \frac{d\rho_R}{dt} + 4 \frac{\dot{a}}{a} \rho_R = -N_0 \int_{m_{min}}^{\infty} \dot{m} f(m, t) dm \tag{22} \]

Eq.(22) just reflects, as is to be expected, the fact that an effective black-hole mass loss (or gain) would imply \( \rho_R \propto a^{-4-\alpha} \) where \( \alpha(t) \) is negative(positive) because of black-holes acting as source (sink) of radiation.

In any mechanism of PBH production, the actual masses of the black-holes will be distributed in a discrete fashion, and therefore without loss of generality the distribution function can be expressed as

\[ f(m, t) = \sum_{i=1}^{K} \beta_i \delta(m - m_i(t)) \tag{23} \]

where \( \beta_i \) are constant weights corresponding to \( m_i \), and \( K \) is the number of distinct black-hole masses. It can easily be ascertained that the distribution function in eq.(23) indeed is a solution of eq.(19), since

\[ \frac{\partial}{\partial t} \delta(m - m_i(t)) = \frac{\dot{m}_i}{m - m_i(t)} \delta(m - m_i(t)) \tag{24} \]

and

\[ \frac{\partial}{\partial m} [\dot{m} f(m, t)] = -\sum_{i=1}^{K} \beta_i \frac{\dot{m}_i}{m - m_i(t)} \delta(m - m_i(t)) \tag{25} \]

Consequently, we have:

\[ \frac{d\rho_R}{dt} + 4 \rho_R \frac{\dot{a}}{a} = -N_0(t) \sum_{i=1}^{K} \beta_i \dot{m}_i \tag{26} \]

The manner in which the individual mass \( m_i \) of a PBH changes with time depends on the combination of Hawking evaporation rate and the accretion of background relativistic matter as discussed in section 4. Therefore, making
use of eq (8) and eq (13) in the context of a PBH with mass $m_i$, we get the following result:

$$\frac{dm_i}{dt} = \frac{405}{\pi^3 c^3} \rho R G^2 m_i^2 \left[ \frac{g^{uni}_i}{g^{uni}_G} \int_{x_{min}}^{\infty} \frac{x^3}{e^x - 1} + \frac{g^{uni}_i}{g^{uni}_G} \int_{x_{min}}^{\infty} \frac{x^3}{e^x + 1} \right]$$

(27)

$$-\alpha_2 m_i^2 \left[ \frac{m_i}{m} - \kappa \left( \frac{m_i}{m} \right) \right]^4$$

From eqs. (17), (18) and (23), the mass density $\rho_{BH}$ associated with the PBHs can be written as:

$$\rho_{BH}(t) = N_0(t) \sum_{i=1}^{K} \beta_i m_i(t)$$

(28)

The evolution of the scale-factor $a(t)$ then follows from the flat FRW Einstein equation:

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left[ \rho_R + N_0(t) \sum_{i=1}^{K} \beta_i m_i(t) \right]$$

(29)

In writing down the above equation, we have adopted the inflationary paradigm according to which universe in the post-inflationary phase is described essentially by a flat FRW model. In this paper, the evolution of the universe is determined by three coupled differential equations (26), (27) and (29) along with the fact that $N_0(t) \propto a^{-3}(t)$.

4 Numerical Evolution

In this section, we solve $2 + K$ coupled non-linear, first order differential equations (26), (27) and (29), set up in the preceding section, numerically using Hemming’s fourth-order, double precision predictor-corrector method. To begin with, we fix $N_0(t)$ by demanding that $\beta_i m_i(t_0) N_0(t_0)$ represents the initial fraction $\zeta_i$ of total mass density $\rho(t_0)$ that lies in blackholes having initial mass $m_i(t_0)$ so that,

$$\beta_i m_i(t_0) N_0(t_0) = \zeta_i \rho(t_0)$$

(30)

As $N_0(t) \propto a^{-3}(t)$, we have, from eq (30),

$$N_0(t) = \frac{a^3(t_0)}{a^3(t)} \frac{\zeta_i \rho(t_0)}{\beta_i m_i(t_0)}$$

(31)
Since \( N_0(t) \) is independent of \( i \), we obtain the following relation between \( \zeta_i \) and \( \beta_i \),

\[
\zeta_i \propto \beta_i m_i(t_0)
\]  

(32)

where the constant of proportionality in eq(32) can be determined from the following identity,

\[
\rho_R(t_0) = \rho(t_0) - \sum_{i=1}^{K} \zeta_i \rho(t_0)
\]  

(33)

leading to the following expression,

\[
\text{proportionality constant} = \left( \sum_{i=1}^{K} \beta_i m_i(t_0) \right)^{-1} \left( 1 - \frac{\rho_R(t_0)}{\rho(t_0)} \right)
\]  

(34)

Substituting eq(31) in eqs (26) and (29), we obtain,

\[
\frac{d\rho_R}{dt} + 4 \rho_R \frac{\dot{a}}{a} = -\frac{a^3(t_0)}{a^3(t)} \rho(t_0) \sum_{i=1}^{K} \zeta_i \frac{\dot{m}_i}{m_i(t_0)}
\]  

(35)

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left[ \rho_R + \frac{a^3(t_0)}{a^3(t)} \rho(t_0) \sum_{i=1}^{K} \zeta_i \frac{m_i(t)}{m_i(t_0)} \right]
\]  

(36)

respectively.

In our original formulation (see section 3), the blackhole initial mass-spectrum was completely specified by the set of numbers \{\beta_i, m_i(t_0); i = 1, \ldots K\}. Equivalently, since \( \beta_i \) and \( \zeta_i \) are related by equation (32), we may as well specify the spectrum by the set \{\zeta_i, m_i(t_0); i = 1, \ldots K\}.

For the purpose of numerical evolution, it is convenient to cast equations (27) , (29) and (36) in terms of dimensionless quantities defined below

\[
\tau = t \sqrt{G \rho_0}
\]  

(37)

\[
\alpha(\tau) = \frac{a(t)}{a_0}
\]  

(38)

\[
R(\tau) = \frac{\rho_R(t)}{\rho_0} \alpha^4
\]  

(39)

\[
M_i(\tau) = \frac{m_i(t)}{m_i(t_0)}
\]  

(40)
where $\rho_0 \equiv \rho(t_0)$ and $a_0 \equiv a(t_0)$. In terms of the above quantities, the system of differential equations assumes the following form:

\[
\begin{align*}
\alpha' &= \frac{1}{\alpha} \sqrt{\frac{8\pi}{3}(R + \alpha \sum_{i=1}^{K} \zeta_i M_i)} \quad (41) \\
R' &= -\alpha \sum_{i=1}^{K} \zeta_i M_i' \\
M_i' &= m_i(t_0) M_i^2 (G\rho_0)^{-1/2} \left( \frac{405}{\pi^3 c^3} G^2 \rho_0 R \alpha^{-4} J_i - \alpha_2 \left[ \left( \frac{m_{pl}}{m_i(t_0)} \right) \frac{1}{M_i} \right] - \kappa \left( \frac{m_{pl}}{m_i(t_0)} \right)^{n/4} \right) \quad (43)
\end{align*}
\]

where prime denotes differentiation with respect to $\tau$ and for convenience we have introduced

\[
J_i \equiv J(x_0_i, T) = \frac{g_{b}^{uni}}{g_{*}^{uni}} \int_{x_0_i}^{\infty} \frac{x^3 \, dx}{e^x - 1} + \frac{g_{f}^{uni}}{g_{*}^{uni}} \int_{x_0_i}^{\infty} \frac{x^3 \, dx}{e^x + 1}
\]

with

\[
x_0_i = \frac{h}{kT} \left[ \frac{\alpha_1 c}{r_{ci}} \right]
\]

and

\[
r_{ci} = \frac{3\sqrt{3} G m_i(t)}{c^2}
\]

We choose $t_0$ to be the cosmic-epoch when inflation ends $\approx 10^{-33}$ sec., and set $\rho_0 = 10^{56}$ GeV$^4$, which is the density expected at GUT scale. In our numerical evolution program, the actual values used for the following parameters are listed below:

\[
\begin{align*}
\alpha_1 &= 10 \\
\kappa &= 0.1 \\
n &= 3 \\
g_b^{BH} &= g_f^{BH} = g_{b}^{uni} = g_{f}^{uni} = 50
\end{align*}
\]
First we consider the case when $K = 1$, i.e. at the end of inflation a fraction $\zeta$ of matter lies in blackholes, all with initial mass $m_0 = m_0(t_0)$. We study different models by varying $\zeta$ in the range $10^{-3}$ to $10^{-1}$ while $m_0$ runs through the range $10^3$ to $5 \times 10^5$ gm. In fig (1) we plot $a(t)$ for a typical choice of $\zeta$ and $m_0$. The plots of $a(t)$ for a radiation-dominated (RD) FRW universe ($a \sim t^{1/2}$) and a matter-dominated (MD) FRW universe ($a \sim t^{2/3}$) are also given in the same figure. The initial behaviour of the system is that of a RD universe but soon the evolution of the scale-factor $a$ becomes similar to that in an MD universe, and subsequently, as the PBHs evaporate, the dynamics becomes RD again. This is because initially energy density of relativistic matter gets depleted owing to accretion by PBHs, resulting in its decrease faster than the kinematic rate $a^{-4}$ (see fig (1)) so that the dominant contribution from ‘dust’ like blackholes drives a faster expansion rate. We have also compared our results with approximate estimates obtained by assuming that $a(t) \sim t^{2/3}$ from $t = t_0$ (end of inflation) to $t = t_{EWPT}$ (epoch of EWPT) and that $a(t) \sim t^{1/2}$ afterwards. For the range of parameters $10^3 < m_0 < 10^5$ (gm) and $0.001 < \zeta < 0.1$, the estimates agree with our numerical results to within an order of magnitude.

In fig (2) we plot a typical mass $m$ as a function of time. It is evident from the figure that growth of blackhole mass due to accretion takes place only in the initial period when the temperature and density of the universe is very high. This is anyway expected since the de Broglie wavelength $\lambda$ of a typical particle just after the end of inflation is $\sim 10^{-28}$ cm, while the $R_S$ for a blackhole of mass as low as $\sim 100$ gm is $\sim 10^{-26}$ cm leading to a substantial accretion because of $\lambda < R_S$ criteria. At intermediate times the curve flattens out reflecting a balance between accretion and Hawking evaporation. During this phase, the dynamics is essentially MD since radiation loses out in the competition because of the expansion of the universe as well as its attenuation due to accretion by PBHs.

Towards the end, Hawking evaporation begins to dominate the evolution of PBH mass as the accretion automatically gets switched off due to the decrease in temperature and density of background radiation. In fig (3) we plot, for a typical choice of parameters $\zeta = 0.01$ and $m_0 = 2.5 \times 10^5$, the ambient temperature of the universe $T$ as well as the Hawking temperature $T_{BH}$ of the black-hole. The straight line portion of the curve has slope equal to $-2/3$. Thus $T$ falls, at intermediate times, as if the dynamics of the universe was akin to that of a MD universe. At later times, when evaporation becomes the dominant process in the evolution of the holes, the universe at first starts

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cooling at a slower rate, but eventually reheats due to rapid evaporation of the blackholes (the reheat portion is not included in the figure). The point at which EWPT occurs is marked by an arrow in the figure, (i.e. \( T \approx 100 \text{ GeV} \) at this instant of time) corresponding to a value of \( \sim 2 \times 10^{-13} \text{ sec} \). We note that the epoch of EWPT is considerably lower than the standard value of \( \sim 10^{-10} \text{ sec} \) obtained from the time-temperature relation in big-bang models. The reason for this is not hard to understand, as depletion of radiation by the accreting PBHs leads to a MD phase causing the \( T \) to decline faster than the usual \( t^{-1/2} \) fall.

Now, the amount of reheating should be such that the temperature of the universe does not rise above the EWPT temperature (\( \sim 100 \text{ GeV} \)), because, otherwise the sphaleron processes will be re-ignited, leading once again to a washing out of BAU generated. This, in effect, constrains our parameters \( \zeta \) and \( m_0 \). In fig (4) we plot the combination of \( \zeta \) and \( m_0 \) for which the re-heat temperature is 100 GeV, and these points are empirically fitted with a curve. From the numerical evolution, we find that the region lying below to the right of the curve consists of those values of \( (\zeta, m_0) \) for which reheat temperature remains below 100 GeV. While the region lying left of the curve consists of those combinations for which PBHs evaporate away, reaching the relic state before EWPT, and therefore are not of any use as far as baryogenesis is concerned. From fig (4) it is evident that for \( \zeta \) lying in the interval \( (10^{-5}, 0.1) \) a PBH with initial mass less than \( \sim 2 \times 10^5 \text{ gm} \) converges to the relic state before EWPT, and hence does not contribute to generation of baryons. Therefore, we find that if the initial PBH mass spectrum is a delta-function peaking at the mass \( m_0 \), baryogenesis through blackhole evaporation is viable only when the initial mass of the PBHs exceeds \( \sim 2 \times 10^5 \text{ gm} \) for reasonably low values of \( \zeta \).

Next, we consider the case in which blackhole masses at time \( t_0 \) are distributed in a pseudo-Maxwellian manner as shown in fig (5). The blackhole masses fall in a range from \( 8 \times 10^2 \text{ gm} \) to \( 3 \times 10^5 \text{ gm} \), with 22 distinct mass values contributing to a total fraction \( \sum_{i=1}^{22} \zeta_i \approx 0.09 \) of the mass density of the universe just after the end of inflation. From the plot fig (6) it is apparent that blackholes with larger initial mass accrete background hot matter at higher rates than those with smaller initial mass, as expected from the fact that higher mass PBHs have larger cross-section for absorbing matter. We find that those PBHs with initial mass greater than \( 1.5 \times 10^5 \text{ gm} \) reach \( X - \bar{X} \) emitting phase after the epoch \( 1.9 \times 10^{-11} \text{ sec} \), the instant at which EWPT takes place for this spectrum of masses. Once again we find that EWPT oc-
curs sooner than that in the standard model. There are 7 of such blackhole masses which finally contribute to the production of baryon excess. Because of the wide distribution of blackhole masses, the instants at which the PBHs reach the relic mass are staggered, hence no sharp re-heating takes place in our analysis, rather the temperature of the universe falls at a slower rate till the largest size blackhole (with initial mass = $3 \times 10^5$ gm) evaporates, leaving behind a relic mass around the epoch $\sim 3 \times 10^{-9}$ sec, when the temperature of the universe is $\sim 9$ GeV. The decline of temperature with time is shown in fig (7).

Even in the case of blackhole mass distribution, $K$ being larger than 1, in principle, one can constrain the parameter space ($\zeta, m_i(t_0)$) from the requirement of re-heating less than 100 GeV (as undertaken when $K = 1$, see fig (4)), however the exercise is enormously time consuming, and is beyond the scope of the present paper.

5 Baryogenesis

We saw in the previous section that for $\zeta \approx 0.01$, blackholes created with mass less than $\approx 2 \times 10^5$ gm evaporate and reach the relic state before the EWPT and hence their contribution to baryon asymmetry is doubtful due to the expected $B$-violation induced by sphalerons. However, blackholes with initial mass larger than $\approx 2.5 \times 10^5$ gm certainly ought to be considered as sources of baryogenesis since they reach the $X - \bar{X}$ emission phase well past the EWPT. In this section, we proceed to estimate the quantity of excess baryons resulting from blackholes whose Hawking temperature reaches GUT scale after EWPT.

Representing the specific intensity of $X$-bosons radiated with energy $h\nu$ from a blackhole by $I_\nu^X$, we have the relation (e.g., see [23])

$$I_\nu^X = \frac{u_\nu^X(\Omega)}{v}$$

(44)

where $u_\nu^X(\Omega)$ is the specific energy density and $v$ is the speed of the emanating $X$-bosons.

With

$$v = c \left[1 - \left(\frac{mc^2}{h\nu}\right)^2\right]^{1/2}$$

(45)
and
\[ u^X_\nu(\Omega) = \frac{\hbar \nu^3}{c^4} \frac{vgX}{e^{\hbar \nu/kT_{BH}(m)} - 1} \] (46)
we may express \( I^X_\nu \) as
\[ I^X_\nu = \frac{\hbar \nu^3}{c^2} \left[ 1 - \left( \frac{mc^2}{\hbar \nu} \right)^2 \right] \frac{gX}{e^{\hbar \nu/kT_{BH}(m)} - 1} \] (47)

It is to be noted that \( gX \) and \( T_{BH}(m) \) are the spin degeneracy factor of \( X \)-bosons and Hawking temperature of a blackhole of mass \( m \), respectively.

The flux-density of \( X \)-bosons at a distance \( r \) from the blackhole is given by
\[ F^X_\nu = \pi I^X_\nu r^2 \] (48)

Therefore, from equations (47) and (48) the rate of emission of \( X \)-bosons from a blackhole of mass \( m \) is derived to be
\[ \frac{dN^X(m)}{dt} = \int_{mXc^2/\hbar}^{\infty} \frac{F^X_\nu \cdot 4\pi r^2 \, d\nu}{\hbar \nu} \] (49)
\[ = \frac{4\pi R^2_\Sigma c^4 gX m^3_X}{\hbar^3} I(y_i) \] (50)

where \( m_X \) is the mass of the \( X \)-boson and
\[ I(y_i) = \int_1^{\infty} \frac{y^2 - 1}{e^{y/y_i} - 1} \, dy \] (51)

while
\[ y_i(t) \equiv \frac{kT_{BH}(m_i(t))}{m_Xc^2} \] (52)

Since, at any given cosmic epoch \( t \), the number density of blackholes with mass lying in the interval \((m, m + dm)\) is \( N_0(t) f(m, t) \) (see eq (18)), the rate at which \( X \) and \( \bar{X} \) bosons are generated in a unit proper volume is given by
\[ \frac{dn^X_{\bar{X}}}{dt} = 2N_0(t) \int \frac{dN^X(m)}{dt} f(m, t) \, dm \] (53)

In eq (53) the factor 2 arises because we have included production of \( \bar{X} \)-bosons as well. Making use of the form given in eq (23) we can express eq (53) as
\[ \frac{dn^{X_{\bar{X}}}}{dt} = 2N_0(t) \sum_{i=1}^{K} \beta_i \frac{dN^X(m_i)}{dt} \] (54)

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The lifetime of a $X$-boson $\tau_X = \Gamma_X^{-1}$ turns out to be $\approx 10^{-36}$ sec when $m_X \approx 10^{14}$ GeV \[29\] which is negligible in comparison with the time scales over which blackhole mass changes or the universe expands appreciably. Hence, the rate of increase of net baryon number in a unit proper volume is

$$\approx \epsilon \frac{dn_{XX}}{dt}$$

with

$$\epsilon \equiv \frac{\Gamma(X \rightarrow ql) - \Gamma(\bar{X} \rightarrow \bar{q}l)}{\Gamma_{tot}}$$

being the net baryon number generated by the decay of a pair of $X$ and $\bar{X}$ \[30\].

If $n_B(t)$ represents net baryon number density at the cosmic epoch $t$ then

$$\frac{d}{dt} (a^3(t)n_B(t)) = \epsilon a^3(t) \frac{dn_{XX}}{dt}$$

Employing eq \[31\] and \[54\] in \[56\] and then integrating the latter, we obtain

$$a^3(t)n_B(t) - a^3(t_{EWPT})n_B(t_{EWPT}) = 2\epsilon(\rho_0a_0^3) \sum_{i=1}^{K} \frac{\zeta_i}{m_i(t_0)} \int_{t_{EWPT}}^{t} \frac{dN_X(m_i)}{dt'} dt'$$

Assuming that prior to blackhole baryogenesis, the net baryon number in the universe is zero (i.e. $n_B(t_{EWPT}) = 0$) and making use of eq \[31\] in eq \[57\], we get the following expression for the net baryon number density at any time,

$$n_B(t) = \frac{\rho_0a_0^3}{a^3(t)} \left( \frac{4G^2}{\pi\hbar^2} \right) \cdot \epsilon g_X m_X^3 \cdot \sum_{i=1}^{K} \frac{\zeta_i}{m_i(t_0)} \int_{t_{EWPT}}^{t} m_i^2(t') I(y_i(t')) dt'$$

After EWPT has taken place, the evolution of PBH mass is totally dominated by eq \[\Box\] since the de Broglie wavelength $\lambda$ of a typical particle is larger than $\sim 10^{-15}$ cm, while the $R_S$ corresponding to a blackhole of mass as high as $\sim 10^7$ gm is only $\sim 10^{-21}$ cm. Hence, using eq \[\Box\] we can change the variable of integration in eq\[58\] from $t'$ to $m_i(t')$ so that,

$$\int_{t_{EWPT}}^{t} m_i^2(t') I(y_i(t')) dt' = \frac{2m_{pl}^4}{\alpha_2(8\pi m_X)^4} H(m_i(lower), m_i(t_{EWPT}))$$

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where $H$ is defined to be,

$$H(m_i(\text{lower}), m_i(t_{EWPT})) \equiv \int_{m_i(\text{lower})}^{m_i(t_{EWPT})} \frac{1}{y_i^2} \left[ y_i \sum_{k=1}^{\infty} \frac{e^{-k/y_i}}{k^3} + \sum_{k=1}^{\infty} \frac{e^{-k/y_i}}{k^2} \right] dm_i$$  \hspace{1cm} (60)

In obtaining eqs (59) and (60) we have used the series equivalent of the integral given in eq(51). The value of $m_i(\text{lower})$ is set by requiring $y_i$ to be $10^{-3}$ since the series given in eq(60) is negligibly small for smaller values of $y_i$. This automatically takes into account the fact that only those PBHs matter for BAU that are capable of emitting $X - \bar{X}$ after EWPT. For PBH masses larger than $10^5 m_{pl}$, the value of $H$ is $2.7 \times 10^{-2}$ and becomes insensitive to the exact value of $m_i(t_{EWPT})$ thereafter. Therefore, for the 7 PBHs that survive the EWPT, we have,

$$\sum_{i=16}^{22} \frac{\zeta_i}{m_i(t_0)} H(m_i(\text{lower}), m_i(t_{EWPT})) = 1.97 \times 10^{-9}$$  \hspace{1cm} (61)

The entropy density of the universe at any epoch $t$ is given by,

$$s = \frac{2\pi^2}{45} g_{\text{uni}}^* \frac{k^4}{(\hbar c)^3} T^3(t)$$  \hspace{1cm} (62)

We estimate the baryon-to-entropy ratio at $t \sim 3 \times 10^{-9} \text{ sec}$, when all the PBHs settle on to the relic state, by making use of eqs(58),(59),(60),(61) and (62),

$$\frac{n_B(t)}{s(t)} = 7.5 \times 10^{-8} g_X \left( \frac{g_{BH}^*}{100} \right)^{-1} \left( \frac{g_{\text{uni}}^*}{100} \right)^{-1}$$  \hspace{1cm} (63)

The contribution to baryon-to-entropy ratio by PBHs with initial mass $m_0$ and initial mass fraction $\zeta$ goes roughly as,

$$\frac{n_B}{s} \approx \epsilon \zeta g_X \left( \frac{m_0}{1 \text{ gm}} \right)^{-1} \left( \frac{g_{BH}^*}{100} \right)^{-1} \left( \frac{g_{\text{uni}}^*}{100} \right)^{-1}$$  \hspace{1cm} (64)

Hence, in the case of a delta-function mass spectrum with $\zeta \approx 0.01$ and $m_0 \approx 2.5 \times 10^5 \text{ gm}$, one obtains a baryon-to-entropy ratio of $\approx 4 \times 10^{-8}\epsilon$, with $g_X = 1$. Thus one may use eq(64) along with the value of $n_B/s \approx 10^{-11}$, that follows from observations, to put a constraint on $\epsilon \zeta/m_0$. This implies that one requires the CP-violating parameter $\epsilon$ to be around $\sim 10^{-4}$ to generate excess baryons from evaporating PBHs.
6 Discussions

To study the evolution of PBHs, in the early universe, that undergo accretion along with steady mass loss due to Hawking evaporation, we have laid down a formalism which can handle any blackhole mass spectrum that can be decomposed as a sum of weighted $\delta$-functions. Accretion of ambient hot matter by a blackhole has been modeled in the limit of geometric approximation, so that only those particles with de Broglie wavelength less than about a tenth of Schwarzschild radius are considered for absorption by the blackhole. The evolution of a flat FRW universe and the PBHs has been studied numerically to find conditions under which blackholes survive past the electroweak phase transition in order that their subsequent evaporation leads to baryogenesis.

The basic picture which emerges is the following. In the case of a blackhole mass spectrum that peaks sharply at a single mass value $m_0$, when $\zeta$ (the initial mass fraction of PBHs) is of the order of 1%, PBHs with initial mass $m_0$ less than about $2.3 \times 10^5$ gm evaporate before EWPT. Therefore, only PBHs with $m_0$ greater than this critical value need be considered for generation of BAU. Here, we wish to point out that the model of accretion which one considers can make an immense difference in the final result of the analysis. If one uses a simple spherical model of accretion in which the capture-cross section is just $\pi R_0^2$ and with no de Broglie wavelength based cutoff then blackholes of initial mass $m_0 \approx 10^3$ gm can successfully live past the EWPT, and eventually contribute to the BAU (see Majumdar et al. in [14]). While on using the same set of parameters with a wavelength based cutoff model of accretion, we find that PBHs of such small initial mass do not survive beyond the EWPT.

For reasonable choice of parameters, we find that in the case of PBHs with a distribution of mass ranging from $8 \times 10^2$ - $3 \times 10^5$ gm, blackholes with initial mass larger than about $\sim 10^5$ gm reach the relic state much after EWPT. Because of the presence of blackholes with mass less than $10^5$ gm that evaporate at a faster rate, pumping in energetic particles into the surrounding medium, the ambient temperature in this case declines at a slower rate, and hence EWPT takes place later than in the case when all PBHs had the same mass of $2.5 \times 10^5$ gm. As described in sections (3) and (4), the evolution of mass spectrum is totally determined by the manner in which individual blackhole masses change with time, $\beta_i$ or equivalently $\zeta_i$ remaining fixed for all times. As an illustration, we have shown the evolution of mass spectrum in fig (6) for a particular set of $\zeta_i$. 

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We wish to point out that accretion is important only during the initial stages just after the end of inflation when the temperature of the universe is \( \sim 10^{13} \) GeV, causing an increase in the mass of a blackhole by a factor of \( \sim 4 \). There are two factors responsible for a blackhole of initial mass of \( \sim 10^5 \) gm to live after the EWPT. One being the increase in the mass due to accretion, while the other is the occurrence of EWPT sooner than that in a model in which there is no depletion of radiation due to PBHs acting as sinks. For blackholes with mass less than \( \sim 10^5 \) gm, accretion is less due to the reduction in capture cross-section because of which the rate of depletion of radiation is not large leading to a delayed occurrence of EWPT, after the blackholes have reached the final relic state.

Barrow et al’s ansatz\[22\] which has been used in this paper to take into account expected modification of Hawking emission becomes important only when the mass of evaporating blackholes fall below \( \sim 10 m_{pl} \). Our numerical results are not sensitive to the exact form of modified blackhole temperature. For baryogenesis, significant quantity of \( X - \bar{X} \) are emitted only during the phase when blackhole temperature is \( \sim T_{GUT} \), because of which the integral \( H \) is not sensitive to the upper limit \( m(t_{EWPT}) \) so long as the latter is larger than \( 10^5 m_{pl} \). Therefore, the final expression for baryon-to-entropy ratio turns out to be rather simple (see eq (64)), implying that if at the end of inflation 1\% of total matter goes into creating PBHs with initial mass \( 2.5 \times 10^5 \) gm then this scenario can successfully lead to BAU provided the CP-violating parameter \( \epsilon \) is over \( 10^{-4} \). Thus production of baryon excess through blackhole evaporation is a viable alternative to GUTs or electroweak baryogenesis, although there is no denying that because of the presence of parameters like \( \zeta_i \) and \( m_i(t_0) \) whose values \textit{a priori} are uncertain, this scenario cannot provide meaningful constraint on the value of \( \epsilon \).

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Figure Captions

1. Figure 1: The evolution of the scale-factor $a(t)$ for $\zeta = 0.01$, $m_0 = 2.5 \times 10^5$ gm. The plots $a \sim t^{1/2}$ and $a \sim t^{2/3}$ are provided for comparison.

2. Figure 2: The evolution of the mass $m(t)$ of the PBHs for a typical choice $\zeta = 0.01$, $m_0 = 2.5 \times 10^5$ gm.

3. Figure 3: The temperature $T$ of the background thermal bath and the Hawking temperature $T_{BH}$ for a typical choice $\zeta = 0.01$ and $m_0 = 2.5 \times 10^5$ gm. The instant of EWPT is marked by an arrow.

4. Figure 4: The combinations $\zeta$ and $m_0$ for which the reheat temperature $= T_{EWPT} = 100$ GeV. The region with acceptable reheat temperatures $< 100$ GeV is indicated in the figure. The analytical fit with dotted line is purely empirical.

5. Figure 5: Black-hole mass spectrum: plot of $\zeta$ against $m_i(t_0)$.

6. Figure 6: The evolution of the masses $m_i(t)$ of a collection of PBH masses distributed according to the spectrum shown in fig (5).

7. Figure 7: The cooling of the universe for the case where PBH masses are distributed according to the spectrum displayed in fig (5). The epoch of EWPT is marked by an arrow, and it takes place at $1.9 \times 10^{-11}$ sec.
Figure 1: The evolution of the scale-factor $a(t)$ for $\zeta = 0.01$, $m_0 = 2.5 \times 10^5$ gm. The plots $a \sim t^{1/2}$ and $a \sim t^{2/3}$ are provided for comparison.
Figure 2: The evolution of the mass $m(t)$ of the PBHs for a typical choice $\zeta = 0.01$, $m_0 = 2.5 \times 10^5 \text{ gm}$. 
Figure 3: The temperature $T$ of the background thermal bath and the Hawking temperature $T_{BH}$ for a typical choice $\zeta = 0.01$ and $m_0 = 2.5 \times 10^5$ gm. The instant of EWPT is marked by an arrow.
The region with reheat temperatures $\leq 100$ GeV is indicated in the figure. The analytical fit with dotted line is purely empirical. The combinations $\zeta$ and $m_0$ for which the reheat temperature $T_{\text{EWPT}} = \frac{m_0}{m'}$ is $\leq 100$ GeV are shown.
Figure 5: Black-hole mass spectrum: plot of $\zeta$ against $m_i(t_0)$. 

![Graph showing black-hole mass spectrum](image-url)
Figure 6: The evolution of the masses $m_i(t)$ of a collection of PBH masses distributed according to the spectrum shown in fig (3).
Figure 7: The cooling of the universe for the case where PBH masses are distributed according to the spectrum displayed in fig (3). The epoch of EWPT is marked by an arrow, and it takes place at $1.9 \times 10^{-11}$ sec.