Effect of the momentum dependence of nuclear symmetry potential on the transverse and elliptic flows

Lei Zhang\textsuperscript{1}, Yuan Gao\textsuperscript{1,2,}\textsuperscript{a}, Yun Du\textsuperscript{1}, Guang-Hua Zuo\textsuperscript{1}, and Gao-Chan Yong\textsuperscript{3}

\textsuperscript{1} School of Information Engineering, Hangzhou Dianzi University, Hangzhou 310018, China
\textsuperscript{2} School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China
\textsuperscript{3} Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

Received: 5 January 2012 / Revised: 2 February 2012
Published online: 12 March 2012 – © Società Italiana di Fisica / Springer-Verlag 2012
Communicated by Bo-Qiang Ma

Abstract. In the framework of the isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model, the effect of the momentum dependence of nuclear symmetry potential on nuclear transverse and elliptic flows in the neutron-rich reaction \textsuperscript{132}Sn+\textsuperscript{124}Sn at a beam energy of 400 MeV/nucleon is studied. We find that the momentum dependence of nuclear symmetry potential affects the rapidity distribution of the free neutron to proton ratio, the neutron and the proton transverse flows as a function of rapidity. The momentum dependence of nuclear symmetry potential affects the neutron-proton differential transverse flow more evidently than the difference of neutron and proton transverse flows as well as the difference of proton and neutron elliptic flows. It is thus better to probe the symmetry energy by using the difference of neutron and proton flows since the momentum dependence of nuclear symmetry potential is still an open question. And it is better to probe the momentum dependence of nuclear symmetry potential by using the neutron-proton differential transverse flow the rapidity distribution of the free neutron to proton ratio.

1 Introduction

Nowadays the equation of state (EOS) of isospin symmetric nuclear matter is now relatively well determined mainly by studying collective flows in heavy-ion collisions and nuclear giant monopole resonances \cite{1,2}. The major remaining uncertainty about the EOS of symmetric nuclear matter is due to our poor knowledge about the density dependence of the nuclear symmetry energy \cite{1,3–6}, which is crucial for understanding many interesting issues in both nuclear physics and astrophysics \cite{7–10}. And it is also crucial in connection with the structure of neutron stars and the dynamical evolution of proto-neutron stars \cite{11–13}. Considerable progress has been made recently in determining the density dependence of the nuclear symmetry energy around the normal nuclear matter density. However, much more work is still needed to probe the high-density behavior of the nuclear symmetry energy. Currently, to pin down the symmetry energy, the National Superconducting Cyclotron Laboratory (NSCL) at Michigan State University, the Gesellschaft für Schwerionenforschung (GSI) at Darmstadt, the Rikagaku Kenkyusho (RIKEN, the Institute of Physical and Chemical Research) of Japan, and the Cooler Storage Ring (CSR) in Lanzhou are planning to do related experiments to probe the symmetry energy.

The neutron-proton differential transverse flow and the difference of neutron and proton flows are both sensitive to the symmetry energy \cite{3,4}, but the used transport models always adopt different momentum-dependent interactions among nucleons. The importance of the momentum dependence of nuclear symmetry potential on the two kinds of nuclear flows was seldom mentioned. Considering the momentum dependence of nuclear symmetry potential is quite controversial \cite{14}, in the framework of the isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model, we find that, besides the ratio of $\pi^–/\pi^+$ \cite{15}, the momentum dependence of nuclear symmetry potential affects the neutron-proton differential transverse flow more evidently than the difference of neutron and proton transverse flows as well as the difference of proton and neutron elliptic flows. It is thus better to probe the symmetry energy by using the difference of nucleonic transverse (or elliptic) flows.

2 The IBUU04 transport model

The present study is based on the IBUU04 transport model \cite{3}. The initial neutron and proton density distributions of the projectile and target are obtained by us-
In the above equation, $\delta$ is the isospin asymmetry parameter, $\rho$ is the density, and $\rho_n, \rho_p$ are the neutron and proton densities, respectively. $\tau = 1/2 (-1/2)$ for neutron (proton) and $\tau \neq \tau'$, $\sigma = 4/3$, $f_\tau(r,p)$ is the phase-space distribution function at coordinate $r$ and momentum $p$. The parameters $A_\lambda(x), A_\lambda(x), B, C_{\tau,\tau'}, C_{\tau,\tau'}$ and $\Lambda$ were set by reproducing the momentum-dependent potential $U(\rho, \delta, p, \tau)$ predicted by the Gogny Hartree-Fock and/or the Brueckner-Hartree-Fock calculations, the saturation properties of symmetric nuclear matter and the symmetry energy of about 32 MeV at normal nuclear matter density $\rho_0 = 0.16 \text{ fm}^{-3}$. The propagations of nucleon are according to Hamilton's equations

$$dp_i/dt = -\nabla U(r_i) + q_i E, \quad dr_i/dt = p_i/\sqrt{m^2 + p_i^2} + \nabla U(r_i, p_i), \quad \text{(2)}$$

where $q_i$ is the charge of particle, $E$ is the Coulomb field of particle felt. The incompressibility of symmetric nuclear matter at normal density is set to be 211 MeV. According to essentially all microscopic model calculations, the EOS for isospin asymmetric nuclear matter can be expressed as

$$E(\rho, \delta) = E(\rho, 0) + E_{\text{sym}}(\rho) \delta^2 + O(\delta^4), \quad \text{(3)}$$

where $E(\rho, 0)$ is the energy per nucleon of symmetric nuclear matter, and $E_{\text{sym}}(\rho)$ is the nuclear symmetry energy. With the single-particle potential $U(\rho, \delta, p, \tau)$, for a given value $x$, one can readily calculate the symmetry energy $E_{\text{sym}}(\rho)$ as a function of density. Because the purpose of present studies is just to see how large the effect of momentum dependence of nuclear symmetry potential on the transverse and elliptic flows is, we let the variable $x$ be 1, since the IBUU04 model gives a super-soft symmetry energy at higher densities [24]. In fact, behavior of nuclear symmetry energy at supra-densities is still in controversy. The main characteristic of the present single particle is the momentum dependence of nuclear symmetry potential, which has evident effect on the energetic free $n/p$ ratio in heavy-ion collisions [25, 26]. In the present studies, to show the effects of the momentum dependence of symmetry potential, we kept the isoscalar potential fixed while changing the symmetry potential from the momentum-dependent symmetry potential to the momentum independent symmetry potential and keep the symmetry energy fixed [25, 26]. Figure 1 shows the momentum-dependent symmetry potential (MD) and the momentum-independent symmetry potential (MID) from the Gogny interaction. We can see that at higher densities the strength of the momentum-independent symmetry potential (MID) is always larger than that of the momentum-dependent symmetry potential. In the following, we give our results of the momentum dependence of nuclear symmetry potential on nuclear transverse and elliptic flows at higher densities (the density reached in the $^{124}\text{Sn}+^{124}\text{Sn}$ at a beam energy of 400 MeV is about 2 times that of saturation density [27]).
Fig. 2. (Color online) Rapidity distribution of free neutron to proton ratio $n/p$ in the reaction $^{132}\text{Sn}+^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 5 fm with and without momentum dependence of nuclear symmetry potential, signed with MD and MID, respectively.

Fig. 3. (Color online) Neutron and proton transverse flows analysis in the reaction $^{132}\text{Sn}+^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 5 fm with and without momentum dependence of nuclear symmetry potential, signed with MD and MID, respectively.

### 3 Results and discussions

We first study the neutron to proton ratio $n/p$ of free nucleons as a function of rapidity as shown in fig. 2. It is seen the case with momentum dependence of nuclear symmetry potential causes lower neutron to proton ratio, whereas the case without momentum dependence of nuclear symmetry potential causes higher neutron to proton ratio, especially for nucleons at large rapidities. Because the momentum dependence of nuclear symmetry potential decreases the strength of nuclear symmetry potential at high densities or high nucleonic momenta as shown in fig. 1. The small symmetry potential decreases the free neutron to proton ratio [25,26]. From this plot, we can see that while using the neutron to proton ratio $n/p$ of free nucleons to probe the symmetry energy, one should keep in mind that this observable is sensitive to the the momentum dependence of nuclear symmetry potential used, and thus may cause uncertainties while compared with the experimental data.

Figure 3 shows neutron and proton transverse flows analysis in the reaction $^{132}\text{Sn}+^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 5 fm with and without momentum dependence of nuclear symmetry potential. We can see that without (with) momentum dependence of nuclear symmetry potential, the strength of the neutron flow (especially neutron flow) increases (decreases). This is understandable since the momentum independence of nuclear symmetry potential overall increases the strength of nuclear symmetry potential as shown in fig. 1, thus neutrons are repelled more strongly than the case with the momentum-dependent symmetry potential. From fig. 3 we can also see that the effect of the momentum dependence of nuclear symmetry potential on the proton flow is less evident owing to the Coulomb potential added on protons. The Coulomb potential (always repulsive for protons) decreases the strength of symmetry potential (attractive for protons at high densities) added on protons.

The difference of proton and neutron transverse flows has been studied previously and was shown to be sensitive to the symmetry energy [4]. In order to show if the difference of proton and neutron transverse flows is sensitive to the momentum dependence of nuclear symmetry potential, we plot, in fig. 4, the difference of proton and neutron transverse flows analysis in the reaction $^{132}\text{Sn}+^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 5 fm with and without momentum dependence of nuclear symmetry potential. From this plot, we see that the difference of proton and neutron transverse flows is insensitive to the momentum dependence of nuclear symmetry potential. It is thus better to be used to probe the symmetry energy since the the momentum dependence of nuclear symmetry potential is still an open question [14].

The neutron-proton differential transverse flow also has been studied extensively and was shown to be sensitive to the symmetry energy [3,25,26]. To see if neutron proton differential transverse flow is sensitive to the momentum dependence of nuclear symmetry potential, we give, in fig. 5, the neutron proton differential transverse flow analysis in the reaction $^{132}\text{Sn}+^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 5 fm with and without momentum dependence of nuclear symmetry potential. From this plot, we see that the neutron proton differential transverse flow is sensitive to the momentum dependence of nuclear symmetry potential as shown in [25,26].

The neutron-proton differential transverse flow is defined as [27,28]

$$F_{n-p}^y(y) \equiv \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_i^y w_i,$$

$$= \frac{N_n(y)}{N(y)} \langle p_n^y(y) \rangle - \frac{N_p(y)}{N(y)} \langle p_p^y(y) \rangle,$$  \hspace{1cm} (4)
where \(N(y), N_n(y)\) and \(N_p(y)\) are the number of free nucleons, neutrons and protons, respectively, at rapidity \(y\); \(p^x_T(y)\) is the transverse momentum of the free nucleon at rapidity \(y\); \(w_i = 1 (-1)\) for neutrons (protons); and \(\langle p^x_n(y) \rangle\) and \(\langle p^x_p(y) \rangle\) are, respectively, the average transverse momenta of neutrons and protons at rapidity \(y\). One can see from eq. (4) that the constructed neutron-proton differential transverse flow depends not only on the proton flow and neutron flow but also on their relative multiplicities. Therefore the neutron-proton differential transverse flow is not simply the difference of the neutron and proton transverse flows, it in fact depends also on the isospin fractionation at the rapidity \(y\). If neutrons and protons have the same average transverse momentum in the reaction plane but different multiplicities in each rapidity bin, i.e., \(\langle p^x_n(y) \rangle = \langle p^x_p(y) \rangle = \langle p^x(y) \rangle\), and \(N_n(y) \neq N_p(y)\), then eq. (4) is reduced to

\[
F^x_{n-p}(y) = \frac{N_n(y) - N_p(y)}{N(y)} \langle p^x(y) \rangle = \delta(y) \cdot \langle p^x(y) \rangle, \tag{5}
\]

reflecting effects of the isospin fractionation. On the other hand, if neutrons and protons have the same multiplicity but different average transverse momenta, i.e., \(N_n(y) = N_p(y)\) but \(\langle p^x_n(y) \rangle \neq \langle p^x_p(y) \rangle\), then eq. (4) is reduced to

\[
F^x_{n-p}(y) = \frac{1}{2} (\langle p^x_n(y) \rangle - \langle p^x_p(y) \rangle). \tag{6}
\]

In this case, it reflects directly the difference of the neutron and proton transverse flows. Because the effect of the momentum dependence of nuclear symmetry potential for neutron to proton ratio is very large (shown in fig. 2) and the difference of the proton and neutron transverse flows has no such combination, we see a larger effect of the momentum dependence of nuclear symmetry potential on the neutron proton differential transverse flow than the difference of proton and neutron transverse flows. It is thus better to probe the symmetry energy by using the difference of neutron and proton flows since the momentum dependence of nuclear symmetry potential is still an open question [14].

The elliptic flow \(v_2(y, p_t)\), which is derived as the second coefficient from a Fourier expansion of the azimuthal distribution \(N(\phi, y, p_t) = v_0 (1 + v_1 \cos(\phi) + 2v_2 \cos(2\phi))\), can be expressed as

\[
v_2 = \left( \frac{p_x^2 - p_y^2}{p_T^2} \right), \tag{7}
\]

where \(p_t = \sqrt{p_x^2 + p_y^2}\) is the transverse momentum [29,30]. The difference of proton and neutron elliptic flows is also
shown to be sensitive to the symmetry energy [29,30]. It is thus also interesting to see if the difference of proton elliptic flow and neutron elliptic flow is sensitive to the momentum dependence of nuclear symmetry potential. Figure 6 shows the difference of proton elliptic flow and neutron elliptic flow analysis in the reaction $^{132}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 5 fm with and without momentum dependence of nuclear symmetry potential. Again, we see that the difference of proton elliptic flow and neutron elliptic flow is not sensitive to the symmetry energy.

4 Conclusions

Based on the IBUU04 transport model, the effect of the momentum dependence of nuclear symmetry potential on nuclear transverse and elliptic flows in the neutron-rich reaction $^{132}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon is studied. It is found that the momentum dependence of nuclear symmetry potential affects the rapidity distribution of the free neutron to proton ratio, neutron flow and proton flow as a function of rapidity. The momentum dependence of nuclear symmetry potential affects neutron-proton differential transverse flow more evidently than the difference of neutron and proton transverse flows as well as the difference of proton elliptic flow and neutron elliptic flow. Therefore it is better to probe the symmetry energy by using the difference of neutron flow and proton flow since the momentum dependence of nuclear symmetry potential is still an open question. And it is better to probe the momentum dependence of nuclear symmetry potential by using the neutron-proton differential transverse flow and the rapidity distribution of the free neutron to proton ratio.

One of the authors (YG) thanks Prof. Bao-An Li for providing the code and useful guidance while he stayed at the Institute of Modern Physics, Chinese Academy of Sciences and thanks Prof. Wei Zuo for helpful discussions. The work is supported by the National Natural Science Foundation of China (10975064, 11175074, 11175219 and 11105035) and the Zhejiang Provincial Natural Science Foundation of China (Y6110644).

References

1. P. Danielewicz, R. Lacey, W.G. Lynch, Science 298, 1592 (2002).
2. D.H. Youngblood et al., Phys. Rev. Lett. 82, 691 (1999).
3. B.A. Li, L.W. Chen, C.M. Ko, Phys. Rep. 464, 113 (2008).
4. V. Baran, M. Colonna, V. Greco, M. Di Toro, Phys. Rep. 410, 335 (2005).
5. J. Piekarewicz, Phys. Rev. C 69, 041301 (2004).
6. G. Cio, N. Van Giai, J. Meyer, K. Bennaceur, P. Bonche, Phys. Rev. C 70, 024307 (2004).
7. B.A. Brown, Phys. Rev. Lett. 85, 5296 (2000).
8. K. Sumiyoshi, H. Toki, Astrophys. J. 422, 700 (1994).
9. J.M. Lattimer, M. Prakash, Science 304, 536 (2004).
10. A.W. Steiner, M. Prakash, J.M. Lattimer, P.J. Ellis, Phys. Rep. 411, 325 (2005).
11. M. Kutscher, Phys. Lett. B 340, 1 (1994).
12. M. Prakash et al., Phys. Rep. 280, 1 (1997).
13. J.M. Lattimer, M. Prakash, Astrophys J. 550, 426 (2001).
14. Rong Chen, Bao-Jun Cai, Lie-Wen Chen, Bao-An Li, Xiao-Hua Li, Chang Xu, arXiv:1112.2936 (2011).
15. Y. Gao, L. Zhang, H.F. Zhang, X.M. Chen, G.C. Yong, Phys. Rev. C 83, 047602 (2011).
16. G.C. Yong, Y. Gao, W. Zuo, X.C. Zhang, Phys. Rev. C 84, 034609 (2011).
17. B.A. Li, L.W. Chen, Phys. Rev. C 72, 064611 (2005).
18. G.C. Yong, Eur. Phys. J. A 46, 399 (2010).
19. G.Q. Li, R. Machleidt, Phys. Rev. C 48, 1702 (1993).
20. C. Fuchs, A. Faessler, M. El-Shabbahi, Phys. Rev. C 64, 024003 (2001).
21. H.F. Zhang, Z.H. Li, U. Lombardo, P.Y. Luo, F. Sammarra, W. Zuo, Phys. Rev. C 76, 054601 (2007).
22. H.F. Zhang, U. Lombardo, W. Zuo, Phys. Rev. C 82, 015805 (2010).
23. C.B. Das et al., Phys. Rev. C 67, 034611 (2003).
24. Z.G. Xiao, B.A. Li, L.W. Chen, G.C. Yong, M. Zhang, Phys. Rev. Lett. 102, 062502 (2009).
25. B.A. Li, C.B. Das, S. Das Gupta, C. Gale, Nucl. Phys. A 735, 563 (2004).
26. B.A. Li, C.B. Das, S. Das Gupta, C. Gale, Phys. Rev. C 69, 064602 (2004).
27. G.C. Yong, B.A. Li, L.W. Chen, Phys. Rev. C 74, 064617 (2006).
28. G.C. Yong, B.A. Li, L.W. Chen, Phys. Rev. C 80, 044608 (2009).
29. V. Greco, V. Baran, M. Colonna, M. Di Toro, T. Gaitanos, H.H. Wolter, Phys. Lett. B 562, 215 (2003).
30. V. Giordano, M. Colonna, M. Di Toro, V. Greco, J. Rizzo, Phys. Rev. C 81, 044611 (2010).