Latent heat of the large $N$ finite temperature phase transition

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Abstract

Reduced large $N$ gauge theories have a phase with unbroken center symmetry and phases in which that symmetry is broken for Polyakov loops in one or more lattice directions. The phase with unbroken symmetry is associated with the zero temperature, infinite volume, infinite $N$ theory while the phase in which the symmetry is broken in just one lattice direction has been conjectured to be the spatial reduction of the high temperature, infinite volume, infinite $N$ theory. Measurements of the scaling properties of the latent heat of the transition between these phases test that hypothesis. The results indicate a non-zero latent heat in the continuum limit. Substantial finite spacing effects remain, and finer lattices will be needed to confirm physical scaling.

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I. INTRODUCTION

Among the limited tools that can be used to investigate the nonperturbative aspects of gauge theories are lattice numerical calculations and large $N$ approximations. With continuing dramatic advances in computer hardware, it is possible to combine the two and make additional progress. Recent numerical results \[1\] \[2\] for $N$ up to 8 have confirmed that the finite $N$ corrections are surprisingly small and that $N = 3$ is remarkably close to $N = \infty$. However, since a straightforward large $N$ simulation is more expensive than the physical $N = 3$ case, there is little motivation to follow that indirect route to phenomenological results. The employment of reduction makes large $N$ numerical results more interesting.

Long ago, it was shown by Eguchi and Kawai \[3\] that certain infinite volume, $N = \infty$ quantities (such as the free energy) can be calculated in a reduced model where spacetime is reduced to one point—provided the center symmetry $Z(N)$ of $SU(N)$ gauge theory is unbroken. Unfortunately long before the coupling $\lambda = Ng^2$ is small enough to be near the continuum limit, the symmetry does break \[4\]. Quenching \[4\] and twisting \[5\] were developed as workarounds to this barrier. An alternative is to reduce not from infinite volume to a single site lattice but to a lattice of finite size $L^4$. For larger $L$, $\lambda$ can be pushed to smaller values while remaining in the phase with unbroken $Z(N)$. An investigation of this approach found evidence \[6\] that the transition to the phase with the symmetry broken for Polyakov loops in one lattice direction takes place at a physical value $L_c(\lambda) \sim (a\Lambda_{QCD})^{-1}$. Thus the infinite volume, $N = \infty$ theory can be simulated on a lattice of finite physical size.

However at each $L$, there is a critical coupling $\lambda_c$, which depends on $L$, and for which the center symmetry is broken when $\lambda < \lambda_c$. In \[6\], it was observed that there is a smaller coupling $\lambda_1$ defining a range $\lambda_1 < \lambda < \lambda_c$ in which the $Z(N)$ symmetry is broken only for the Polyakov loops in a single lattice direction. As $\lambda$ decreases further, the symmetry is broken in an increasing number of lattice directions. Does the phase with symmetry breaking in a single direction have physical significance? In \[6\], there is speculation that it is the large $N$ limit of the finite temperature phase with $T > T_c$. The value of $\lambda_c$ is roughly consistent with this claim, \textit{i.e.} $L_c \approx 1/T_c$.

There is numerical evidence that the finite temperature phase transition on large spatial lattices remains first order as $N$ increases \[1\] \[2\]. The latent heat $\Delta \epsilon$ appears to approach the form $N^2h$ with $h$ an $N$-independent physical energy density. In lattice units, $a^4h$ scales
as \((aA_{QCD})^4 \sim L_c^{-4}\). Thus the latent heat should be accessible in the reduced theory. The results in [6] indicated that the phase transition is also first order in the reduced theory at large \(N\).

If the phase of the reduced theory with the center symmetry broken in one direction is indeed physical, our expectation is that measurements of \(a^4 h\) will have a non-zero limit for \(N \to \infty\) and \(g_c^2 \to 0\) and will scale correctly with \(L_c\) as \(\lambda \to 0\). This report presents measurements of \(a^4 h\) on \(L^4\) reduced lattices with \(L = 5, N = 29; L = 6, N = 37; L = 7, N = 29;\) and \(L = 8, N = 29\). Since the large \(N\) corrections are \(O(1/N^2)\), the expectation is that they are negligible. Indeed some additional \(L = 6, N = 29\) data agree within statistical uncertainties with the \(L = 6, N = 37\) result shown below. The results indicate that the continuum limit \(\lambda \to 0\) of \(L_c^2 a^4 h\) is non-zero, but the asymptotic scaling region in which it would be independent of \(L_c\) has not been reached for these \(L_c\) values.

II. METHODS AND RESULTS

Numerical calculations follow those used in [6]. The Monte Carlo evolution uses the standard Wilson gauge action. Each update of the lattice consisted of a heatbath sweep followed by an overrelaxation sweep. In the heatbath sweep, the \(\text{SU}(N)\) group element on a link is modified by working with each of its \(N(N-1)/2\) \(\text{SU}(2)\) subgroups in turn. A typical run was for one or two thousand lattice updates.

The phase is determined by monitoring the four quantities

\[
P_\mu = \frac{1}{N^2} \left\langle \sum_{i,j} \sin^2 \left( \frac{1}{2} (\theta_i - \theta_j) \right) \right\rangle.
\]

The \(\theta's\) are the angles of the eigenvalues of a Polyakov loop in the \(\mu\) direction. The average is over sites in a plane perpendicular to that direction as described in [6]. For unbroken symmetry, \(P_\mu = 0.5\), and it decreases for symmetry breaking in the \(\mu\) direction.

By varying the coupling at fixed \(L\), one can identify the narrow region of metastability where both the symmetric phase and the phase with the symmetry broken for one lattice direction can exist for substantial periods of Monte Carlo time. In terms of \(b = 1/(Ng^2) = 1/\lambda\), the critical values are at about 0.347, 0.352, 0.356, and 0.3595 for \(L = 5, 6, 7,\) and 8 respectively. The metastable regions are of width about 0.0005 in \(b\), i.e. about a part in a thousand. At each \(L\), a value for the jump in the average plaquette \(\Delta s\) is obtained at a \(b\)
where one phase is stable and the other metastable. For $L = 8$, this could be done at one $b$ value, while for the other $L$s, two $b$s separated by 0.0005 were possible.

Table I shows the results for $L^4 c \Delta s$. The normalization of the average plaquette is such that it approaches one as the coupling goes to zero. In the cases where there are two $b$ values at a single $L$, there is also a line that shows the average of the two jumps. The uncertainty in the jump at one $b$ value is based on the statistics of several runs of one or two thousand sweeps at that $b$ value. The differences in the jumps for two $b$ values at the same $L$ are larger than the uncertainties in each jump. Thus there is a systematic uncertainty associated with the choice of $b$. The uncertainty in an average jump is estimated as the difference between the jump values at the two $b$ values.

The jump in the average plaquette $\Delta s$ is proportional to the discontinuity in $\epsilon - 3P$. Since the pressure $P$ is continuous at a transition, the jump in the plaquette also gives the jump in the energy density $\epsilon$ and thus $a^4 h$.

Table I uses the data from Table I at definite $b$ values and plots $L^4 c \Delta s$ as a function of $L_c$. This would be a constant in the continuum limit, but there are substantial variations from that limiting case. Most likely these are a finite spacing effect, $i.e.$ the coupling is not sufficiently small or $L_c$ sufficiently large to see continuum scaling. For comparison, consider the results [2] or [9] in non-reduced calculations. At $N_t$ values that are the same as the $L_c$
values used here, the scaling violations are also substantial and comparable to results shown here.

If the dominant scaling violations at these lattice spacings are an $a^2$ correction, then $L^4\Delta s$ vs. $L^{-2}$ would be a line. Figure 2 shows the average data plotted in that way. A straight line fit to the data has a $L^4\Delta s$ intercept of 0.30. On the other hand, if the $L = 8$ point were run in assuming a perturbative $\beta$ function, the intercept would be considerably higher.

Thus it appears that these calculations are not at $L_c$ values that are sufficiently large ($\lambda$ sufficiently small) to confirm weak coupling continuum scaling for a physical value of the latent heat. However, since the results for the jump in the plaquette are close in value to the non-reduced results and show a similar scaling violation, the hypothesis that the phase of the reduced model with the center symmetry broken in one direction is the reduced $T > T_c$ phase remains viable. Pushing the calculations to larger $L$ would be fairly expensive.

To assess the strength of the transition, it is conventional to compare the latent heat per unit three volume in lattice units

$$a^4\Delta \epsilon = N^2a^4h = -12N^2a \frac{1}{\partial a} \frac{1}{\lambda} \Delta s$$

with the blackbody energy density per massless vector particle

$$\epsilon_{SB} = 4\sigma T^4$$

with

$$\sigma = \frac{\pi^2}{60}$$
FIG. 2: The average values for $L_c^4 \Delta s$ from Table I are plotted along with the line that gives the best fit.

the dimensionless Stephan-Boltzmann constant. In the continuum limit,

$$-\frac{\partial}{\partial a} \frac{1}{\lambda} = \frac{11}{24\pi^2}. \quad (5)$$

$$\frac{\Delta \epsilon}{N^2 \epsilon_{SB}} = \frac{165}{2\pi^4} L_c^4 \Delta s \quad (6)$$

For an $L_c^4 \Delta s$ intercept of 0.30, this is 0.26.

If the phase with intact center symmetry is the reduced zero temperature theory, and the phase with center symmetry broken in one direction is the reduced $T > T_c$ phase, one may wonder what has happened to the phase with $0 < T < T_c$. In fact, the finite temperature effects in this region are nonleading in $N$ and are therefore invisible in the reduced theory which captures only the leading $N^2$ term correctly [10] [11] [12].

III. CONCLUSION

Reduced large $N$ gauge theories have a transition between phases with unbroken and broken center symmetry. If the phase with the center symmetry broken in one direction is indeed physical, our expectation is that $a^4 h$ will have a non-zero limit for $N \to \infty$ and $g_c^2 \to 0$ at fixed $\lambda = Ng^2$ and will scale correctly with $L_c$ as $\lambda \to 0$. The results show that $a^4 h$ is non-zero in the limit $N \to \infty, g^2 \to 0$ with $\lambda$ fixed and also indicate that $L_c^4 a^4 h$ has a non-zero limit as $\lambda \to 0$. However, substantial finite spacing effects remain, and finer lattices will be needed to confirm asymptotic physical scaling. This means that $h$ has a finite, non-zero value in physical units, but the proper functional form for its approach to the continuum limit remains to be confirmed.
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