Spin-Wave Propagation in the Presence of Interfacial Dzyaloshinskii-Moriya Interaction

Jung-Hwan Moon¹, Soo-Man Seo¹, Kyung-Jin Lee¹,², Kyung-Whan Kim³, Jisu Ryu³, Hyun-Woo Lee³, R. D. McMichael⁵, and M. D. Stiles⁵
¹Department of Materials Science and Engineering, Korea University, Seoul 136-701, Korea
²KU-KIST Graduate School of Converging Science and Technology, Korea University, Seoul 136-713, Korea
³PCTP and Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Korea
⁴Basic Science and Research Institute, Pohang University of Science and Technology, Pohang 790-784, Korea
⁵Center for Nanoscale Science and Technology, National Institute of Standards and Technology, Gaithersburg, Maryland 20899, USA
(Dated: May 11, 2014)

In ferromagnetic thin films, broken inversion symmetry and spin-orbit coupling give rise to interfacial Dzyaloshinskii-Moriya interactions. Analytic expressions for spin-wave properties show that the interfacial Dzyaloshinskii-Moriya interaction leads to non-reciprocal spin-wave propagation, i.e. different properties for spin waves propagating in opposite directions. In favorable situations, it can increase the spin-wave attenuation length. Comparing measured spin wave properties in ferromagnet-normal metal bilayers and other artificial layered structures with these calculations can provide a useful characterization of the interfacial Dzyaloshinskii-Moriya interactions.

I. INTRODUCTION

Magnetic exchange is the root of magnetism. Intratatomic exchange stabilizes the magnetic moments and interatomic exchange tends to keep the magnetization spatially uniform. Interatomic exchange is usually symmetric in that the consequences of rotating the magnetization one way or the reverse are equivalent. It looses that symmetry when the system is subject to both spin-orbit coupling and broken inversion symmetry. The antisymmetric component of the exchange interaction, known as Dzyaloshinskii-Moriya (DM) interaction, can give chiral magnetic orders such as spin spirals and skyrmions. Understanding chiral magnetic order and its dynamics driven by magnetic fields or currents is currently of significant interest in the field of spintronics.

The DM interaction between two atomic spins \( S_i \) and \( S_j \) is

\[
\mathcal{H}_{\text{DM}} = -D_{ij} \cdot (S_i \times S_j) \tag{1}
\]

where \( D_{ij} \) is the Dzyaloshinskii-Moriya vector, which is perpendicular to both the asymmetry direction and the vector \( r_{ij} \) between the spins \( S_i \) and \( S_j \). The DM interaction can be classified into two classes depending on the type of inversion symmetry breaking, i.e. bulk and interfacial DM interactions corresponding to lack of inversion symmetry in lattices and at the interface, respectively. The bulk DM interaction has been studied mostly for B20 structures such as MnSi, FeCoSi, FeGe, etc. For the bulk DM interaction, \( D_{ij} \) is determined by the detailed symmetry of the lattice structure. On the other hand, the interfacial DM interaction, which is the main focus of this work, occurs at all magnetic interfaces. It can be particularly strong at the interface between a ferromagnet and a normal metal having strong spin-orbit coupling. The DM interaction can be modelled by a 3-site exchange between two atomic spins with a neighboring atom having a spin-orbit coupling. It has been investigated for epitaxial ferromagnet-heavy metal bilayers such as Mn/W, Fe/Ir, and Fe/W.

Recently Chen et al. reported that magnetic domain walls in epitaxial Fe/Ni/Cu(001) structures are Néel walls and the domain wall chirality is opposite to that of Ni/Fe/Cu(001) structures. Such behavior is expected for an interfacial DM interaction. The interfacial DM interaction in these structures may be not as large as that of structures having a heavy metal, but is still large enough to affect magnetic textures, which can in turn modify magnetization dynamics substantially. Furthermore, recent experiments on current-driven domain wall motion suggest that the interfacial DM interaction exists in sputtered Pt/CoFe/MgO and Pt/Co/Ni structures, and plays an important role in domain wall motion. Since sputtered thin films consist of small grains with different lattice orientation, the contributions from the bulk DM interactions tend to cancel and only the interfacial DM interaction contributions remain effective. In this respect, understanding the interfacial DM interaction in sputtered thin films is important not only for the fundamental understanding of topologically protected nanomagnetic structures but also to the development of spintronic devices based on domain walls.

Translating the DM interaction in Eq. (1) to a continuum model with magnetization direction \( \hat{m} \), and symmetry breaking in the \( \hat{y} \) direction, the DM energy density is given by

\[
E_{\text{DM}} = -D \left( \hat{x} \times \hat{y} \right) \cdot \left( \hat{m} \times \frac{\partial \hat{m}}{\partial x} \right) + (\hat{z} \times \hat{y}) \cdot \left( \hat{m} \times \frac{\partial \hat{m}}{\partial z} \right) \tag{2}
\]

In this paper, we consider the case where the equilibrium
magnetization lies along the $\hat{z}$ axis, in the plane of the film. We also restrict our attention to the case of spin waves propagating in the $x$-direction so that $\mathbf{m}$ varies only in the $x$-direction, and the second term in Eq. (2) is zero (see Fig. 1).

A net DM interaction is present in any trilayer structure when the first nonmagnetic layer supplies a spin-orbit coupling, the middle layer is a ferromagnet, and the third layer is nonmagnetic, but different from the first layer to break symmetry. Since the observation of efficient domain wall motion in bilayers and trilayers is nonmagnetic, but different from the first layer to break symmetry. Since the observation of efficient domain wall motion in bilayers and trilayers is correlated with the conditions for a strong DM interaction, it is useful to study various artificial structures to find large interfacial DM interactions. The spin wave properties we present below provide a useful probe of the DM interactions in these systems.

In this work, we compute analytical expressions for asymmetric spin-wave propagation induced by the interfacial DM interaction. There have been several related studies on specific systems. Udvari and Szunyogh predicted that the DM interaction gives rise to asymmetric spin-wave dispersion depending on the sign of wavevector, based on first principles calculations for Fe/W(110).

Costa et al. predicted that the spin-wave frequency, amplitude, and lifetime differ depending on the sign of wavevector, based on a multiband Hubbard model for Fe/W(110). Zakeri et al. reported a series of spin-wave experiments based on the spin-polarized electron-loss spectroscopy for a single crystalline Fe/W(110) consistent with these predictions. Cortés-Ortuño and Landeros developed a spin-wave theory for bulk DM interaction where they demonstrated that the spin-wave dispersion is asymmetric with respect to wavevector inversion.

We focus on the influence of the interfacial DM interaction on spin-wave properties and pay attention to asymmetric spin-wave attenuation and excitation amplitude with respect to wavevector inversion. We provide analytic expressions for asymmetric dispersion, attenuation length, and amplitude of interfacial DM spin-waves. In Sec. II, we present spin-wave theory in the presence of the interfacial DM interaction. Section III gives comparisons between analytic expressions and micromagnetic simulations. We summarize our work in Sec. IV.

II. SPIN-WAVE THEORY

We begin with a quantum spin-wave theory to find the contribution of the interfacial DM interaction to the dispersion. Quantum spin-wave theory for the symmetric exchange interaction is well established and shows that the exchange interaction results in $k^2$-dependence of the dispersion for small wavevector $k$. Here we focus on the interfacial DM interaction in a one-dimensional spin system. The interfacial DM interaction Hamiltonian is given as

$$\mathcal{H}_{\text{DM}} = -2D_0 \sum_{j,\delta} \hat{z} \cdot (S_j \times S_{j+\delta}) \quad (3)$$

$$= \frac{D_0}{\hbar^2} \sum_{j,\delta} (S_j^+ S_{j+\delta}^- - S_j^- S_{j+\delta}^+),$$

where $D_0$ is the DM energy, $S_j^+ (= S_{jx} + i S_{jy})$ and $S_j^- (= S_{jx} - i S_{jy})$ are the spin raising and lowering operators. We treat the case where the equilibrium magnetization direction is along $\mathbf{D}_0$ because this configuration exhibits the strongest spin-wave asymmetry. Based on the Holstein-Primakoff transformation and assuming that the total number of flipped spins in the system is small compared to the total number of spins, $S_j^+ (S_j^-)$ can be approximated as $\hbar \sqrt{2s} a_j^+ (\hbar \sqrt{2s} a_j^-)$ where $s$ is the total spin on the site, and $a_j (a_j^+)$ is magnon annihilation (creation) operator. Substituting these approximations into Eq. (3) gives

$$\mathcal{H}_{\text{DM}} = \frac{2sD_0}{\hbar} \sum_{j,\delta} (a_j a_{j+\delta}^+ - a_j^+ a_{j+\delta}). \quad (4)$$

Introducing the operators $a_k^+$ and $a_k$, which are the Fourier transforms of the $a_j$'s, and summing over $j$, Eq. (4) becomes

$$\mathcal{H}_{\text{DM}} = \frac{2sD_0}{\hbar} \sum_{\delta, k} (e^{-ik\delta} a_k a_k^+ - e^{ik\delta} a_k^+ a_k). \quad (5)$$

The contribution to the magnon energy in Eq. (5) is

$$\mathcal{H}_{\text{DM}}^{\text{magnon}} = -4sD_0 \sum_k \sin(ka) a_k^+ a_k = \sum_k \hbar \omega_k^{\text{DM}} n_k, \quad (6)$$

where $\hat{n}_k = a_k^+ a_k$ is the number operator for magnons with wavevector $k$, $a$ is the lattice constant, and the DM interaction contribution to the dispersion is given by

$$\hbar \omega_k^{\text{DM}} = -4sD_0 \sin(ka). \quad (7)$$

For small $k$, Eq. (7) reduces to

$$\hbar \omega_k^{\text{DM}} = -4sD_0 ka, \quad (8)$$
a contribution to the dispersion that is linear in $k$. This antisymmetric contribution to the energy leads to asymmetric spin-wave propagation, i.e. dependent on the direction of $k$.

A similar contribution arises in a classical theory of spin-waves in thin films with an interfacial DM interaction. We consider small amplitude spin-waves propagating along the $x$-axis in the perturbative limit, where the equilibrium magnetization is in the $z$-direction perpendicular to both the film thickness direction and the spin-wave propagation direction,

$$\dot{m} = p\dot{z} + m_0 \exp[i(kx - \omega t)] \exp[-x/\Lambda],$$

where $m_0 = (m_x, m_y, 0)$, $|m_0| \ll 1$, $p = \pm 1$, and $\Lambda$ is the spin-wave attenuation length. The spin-wave dynamics is described by the Landau-Lifshitz-Gilbert (LLG) equation,

$$\frac{\partial \dot{m}}{\partial t} = -\gamma \dot{m} \times \mu_0 H_{\text{eff}} + \alpha \dot{m} \times \frac{\partial \dot{m}}{\partial t},$$

where $\gamma$ is the gyromagnetic ratio and $\alpha$ is the damping constant. The effective field $H_{\text{eff}}$ is given as

$$H_{\text{eff}} = pHz + J\nabla^2 \dot{m} - D^* \left( \dot{z} \times \frac{\partial \dot{m}}{\partial x} \right) + H_{\text{dipole}},$$

where $H$ is the external field, $J$ is $2A/\mu_0 M_s$, $D^*$ is $2D/\mu_0 M_s$, $A$ is the exchange stiffness constant, $M_s$ is the saturation magnetization, $H_{\text{dipole}} = -M_s/(1 - e^{-2|k|d})m_x \hat{x} - M_s(1 - (1 - e^{-2|k|d})/4)m_y \hat{y}$, and $d$ is the film thickness. We note that $H_{\text{dipole}}$ consists of local contribution (independent of $d$ and $k$) and nonlocal contribution (dependent on $d$ and $k$). Inserting Eqs. (9) and (11) into Eq. (10), and neglecting small terms proportional to $1/(k\Lambda)^2$, $\alpha^2$, and $\alpha/(k\Lambda)$, gives

$$\frac{\omega}{\gamma \mu_0} = \sqrt{(H + M_s/4 + Jk^2)(H + 3M_s/4 + Jk^2) - \frac{e^{-4|k|d}M_s^2}{16}(1 + 2e^{2|k|d}) + pD^*k},$$

and

$$\Lambda_\pm = \frac{1}{\alpha \omega} \left( 2\gamma \mu_0 J|k_\pm| + \frac{\gamma \mu_0 M_s^2 d e^{-4|k_\pm|d}(1 + e^{2|k_\pm|d})/8 \pm pD^*(\omega \mp \gamma \mu_0 pD^*|k_\pm|)}{H + M_s/2 + Jk_\pm^2} \right),$$

where the upper (lower) sign corresponds to the case $k > 0$ ($k < 0$). The dispersion (Eq. (12)) is the sum of the terms in the square root, which is the dispersion in the absence of the DM interaction, and a term linear in $k$. Therefore, the interfacial DM interaction generates a term linear in $k$ in the dispersion (Eq. (12)) as in the quantum spin-wave theory (Eq. [5]). As a result, the wavevectors are different for propagation in different directions at a fixed frequency $\omega$. The spin-wave attenuation length also depends on the sign of $k$ when $D \neq 0$ (Eq. (13)).

In the large $k$ limit (i.e., exchange-DM spin-waves), one may neglect the nonlocal magnetostatic contribution so that Eqs. (12) and (13) reduce to

$$\frac{\omega}{\gamma \mu_0} = \sqrt{(H + Jk^2)(H + M_s + Jk^2) + pD^*k},$$

and

$$\Lambda_\pm = \frac{1}{\alpha \omega} \left( 2\gamma \mu_0 J|k_\pm| + \frac{pD^*(\omega \mp \gamma \mu_0 pD^*|k_\pm|)}{H + M_s/2 + Jk_\pm^2} \right).$$

On the other hand, in the small $k$ limit (i.e., magnetostatic-DM spin-waves) that is more relevant to experimental conditions, one may neglect the exchange contribution and assume $|k_\pm|d \ll 1$ so that Eqs. (12) and (13) reduce to

$$\frac{\omega}{\gamma \mu_0} = \sqrt{H(H + M_s) + \frac{M_s^2 |k|d}{4\sqrt{H(H + M_s)}} + pD^*k},$$

and

$$\Lambda_\pm = \frac{1}{\alpha \omega} \left( \frac{\gamma \mu_0 M_s^2 d/4 \pm pD^*(\omega \mp \gamma \mu_0 pD^*|k_\pm|)}{H + M_s/2} \right).$$

In this small $k$ limit, one finds from Eq. (16) that not only the interfacial DM interaction but also the dipolar coupling generates a term linear in $k$. However, there is an important difference. The interfacial DM interaction contribution changes its sign with respect to the inversion of the wavevector $k$ or the magnetization direction $p$, whereas the dipolar contribution does not. Due to this feature, one can distinguish the interfacial DM interaction contribution from the dipolar contribution unambiguously.

Not only are the wave vectors, Eq. (12), and the decay lengths, Eq. (13) asymmetric, the amplitudes of the spin wave are different when symmetrically excited. We approximate the ratio of spin-wave amplitudes $\kappa$
by neglecting contributions from nonlocal dipolar coupling, where the plus (minus) sign corresponds to \( k > 0 \) (\( k < 0 \)). From the susceptibility, one finds

\[
m_y = \sqrt{\frac{H_e}{H_y}} \int_0^\infty dk \frac{h_k H_y}{2\pi} \frac{1}{2\omega_0 d\omega + i}\frac{1}{v_g}.
\]

(18)

where \( H_e = H + Jk^2 \), \( H_y = H + M_s + Jk^2 \), \( h_k \) is the Fourier component of the driving field, \( \omega_0 = \gamma \mu_0 \sqrt{H_e H_y} \), \( d\omega \) describes the frequency difference from the resonance frequency, \( \Gamma \) describes the damping term, and \( v_g \) is the group velocity. Thus, the spin-wave amplitude ratio \( \kappa \) is

\[
\kappa = \frac{-pD^* + Jk - \sqrt{2H + M_s + 2Jk^2}}{\sqrt{(H + Jk^2)(H + M_s + Jk^2)}} + pD^* + Jk + \sqrt{2H + M_s + 2Jk^2}\frac{\sqrt{H + M_s + Jk^2}}{(H + Jk^2)(H + M_s + Jk^2)}.
\]

(19)

This equation shows that the interfacial DM interaction makes the spin-wave amplitude asymmetric depending on the sign of \( k \) or \( p \).

These results only hold when the DM interaction is not strong enough to change the ground state of the magnetic configuration. Setting Eq. (12) to be zero and neglecting nonlocal dipolar coupling, the threshold \( D^\text{th} \) is

\[
D^\text{th} = \sqrt{(2H + M_s + 2\sqrt{H(H + M_s)}).}
\]

(20)

When \( D^* > D^\text{th} \), the ground state is not a single domain but rather a chiral magnetic texture and our results may not apply. However, to study a particular interface, one can reduce the thickness-averaged effective \( D^* \) below \( D^\text{th} \) by simply increasing the thickness of ferromagnet because the DM interaction in sputtered thin films is an interface effect. By doing so, one can study the DM interaction associated with a particular interface in an appropriate layered structure.

III. NUMERICAL RESULTS AND DISCUSSION

To test the equations derived above, we perform micromagnetic simulations in the large \( k \) and small \( k \) limits for a semi-one dimensional system (i.e., the system is discretized along the length direction, but assumed to be uniform along the width and thin enough that there is no variation along the thickness). In simulations for the large \( k \) limit, we neglect the nonlocal dipolar coupling but keep the local demagnetization field to mimic a thin film geometry. On the other hand, we keep all micromagnetic interactions in simulations for the small \( k \) limit. We use the damping constant \( \alpha = 0.01 \), the saturation magnetization \( M_s = 800 \text{ kA/m} \), the exchange constant \( A = 1.3 \times 10^{-11} \text{ J/m} \), the film thickness \( d = 1 \text{ nm} \), and the film width of 20 \( \mu \text{m} \). We use the external uniform field \( \mu_0 H = 0.1 \text{ T} \) (0.01 T) and a unit cell size of 2 nm (5 nm) in simulations for the large (small) \( k \) limit. To excite spin-waves, we apply an ac field \((0.1 \text{ mT}) \times \cos(2\pi ft)\) to two unit cells at \( x = 0 \). Therefore, the wavevector \( k \) of spin-waves for \( x > 0 \) is positive whereas \( k \) for \( x < 0 \) is negative. For a legitimate comparison between theoretical and numerical results, we include absorbing boundary conditions at the system edges to suppress spin-wave reflection.

Figure 2 shows snap-shot images of the spin-wave distribution along the \( x \)-axis for (a) \( D = 0 \text{ mJ/m}^2 \) and (b) \( D = 1.5 \text{ mJ/m}^2 \). The spins-wave frequency \( f \) is 11 GHz.

\[ \text{FIG. 2: (color online) Snap-shot images of spin-wave distribution along the } x \text{-axis for (a) } D = 0 \text{ mJ/m}^2 \text{ and (b) } D = 1.5 \text{ mJ/m}^2. \text{ The spin-wave frequency } f \text{ is } 11 \text{ GHz.} \]
In the large $k$ limit (Eqs. (16) and (17)). In Fig. 4(b), one finds a dispersion and attenuation length agree with analytic expression of the frequency $f$. Symbols and lines correspond to numerical and analytic results, respectively.

FIG. 3: (color online) Asymmetric spin-wave propagation induced by an interfacial DM interaction in the large $k$ limit: (a) Dispersion relation. (b) Attenuation length $\Lambda$ as a function of the frequency $f$. (c) Amplitude ratio $\kappa$ as a function of $f$. Symbols and lines correspond to numerical and analytic results, respectively.

Figure 3(c) shows numerical results of the amplitude ratio $\kappa$ as a function of the frequency $f$, in agreement with the analytic expression (Eq. (19)). We note that an asymmetry of spin-wave amplitude has been observed when the spin-waves are excited by a magnetic field generated by microwave antennas, and has been called non-reciprocity of spin-waves.\cite{note1,note2} In this case, the amplitude asymmetry results from a non-reciprocal antenna-spin-wave coupling, caused by the spatially non-uniform distribution of the antenna field. However, we use reciprocal coupling in deriving the analytic expressions and performing the numerical simulations, so that the amplitude asymmetry shown in Fig. 3(c) is purely due to the interfacial DM interaction. This interfacial DM interaction-induced amplitude asymmetry may find use in spin-wave logic devices as proposed by Zakeri et al.\cite{note3} In addition, it suggests a reexamination of the interpretation of experiments reporting non-reciprocal antenna-spin-wave coupling. These experiments have been done on relatively thin structures, in which interfacial DM interaction may also be an important source of asymmetry.

Figure 4 summarizes numerical results obtained in the small $k$ limit. Numerical results for both the spin-wave dispersion and attenuation length agree with analytic expressions (Eqs. 19 and 21). In Fig. 4(b), one finds a difference in the attenuation length $\Lambda$ from Fig. 3(b). In the large $k$ limit, $\Lambda$ with $D \neq 0$ is larger than $\Lambda$ with $D = 0$ regardless of the sign of $k$. In contrast, in the small $k$ limit with $pD > 0$, $\Lambda$ with $D \neq 0$ is larger than $\Lambda$ with $D = 0$ for $k > 0$, whereas it is smaller for $k < 0$. This result is again related to the group velocity. From Eq. (21), one finds $v_g^\pm = v_0^\pm \pm \gamma \mu_0 p D^* \pm \frac{\gamma \mu_0 M_0^2}{2} \sqrt{H(H+M_s)}$ is the group velocity with $D = 0$ in the small $k$ limit. Therefore, for a sign of $k$, $\Lambda$ with $D \neq 0$ is larger than $\Lambda$ with $D = 0$ whereas for the other sign of $k$, it is smaller.

Since the analytic expression of the dispersion (Eq. 12) is valid regardless of $k$, the strength of interfacial DM interaction $D$ can be estimated experimentally by measuring the frequency shift $\Delta f \approx |f_{k, \pm p} - f_{-k, \pm p}|$, given as

$$\Delta f = \gamma \mu_0 D^* |k| / \pi.$$  \hspace{1cm} \text{(21)}

With the parameters $M_s = 800$ kA/m, $\gamma = 1.76 \times 10^{11}$ T$^{-1}$s$^{-1}$, and $2\pi/k = 1$ mm, $\Delta f$ is about 880 MHz for $D = 1$ mJ/m$^2$ that is smaller than the threshold value ($\approx 3.1$ mJ/m$^2$ for the parameters used in simulations, see Eq. 20). We note that propagating spin-wave spectroscopy\cite{note4,note5} can resolve $\Delta f$ smaller than 20 MHz.

Another interesting consequence of the interfacial DM interaction is that the dispersion is asymmetric depending not only on the wavevector direction but also on the magnetization direction (i.e., the sign of $p$). This $k$- and magnetization-direction-dependent asymmetry in the dispersion is similar to the electron dispersion in a ferromagnet subject to Rashba spin-orbit coupling.\cite{note6} We note that this is not an accident because the interfacial DM interaction is directly connected with the Rashba spin-orbit coupling at magnetic interfaces\cite{note7}.

IV. SUMMARY

We theoretically study asymmetric spin-wave propagation induced by interfacial DM interactions. We derive analytic expressions of dispersion, attenuation length, and amplitude of interfacial DM spin-waves and compare...
them with numerical results. The frequency shifts induced by the interfacial DM interaction range from MHz to GHz, which should be large enough to be resolved by state-of-the-art experimental tools such as propagating spin-wave spectroscopy. Assuming that the damping does not change with the interfacial DM interaction, the spin-wave attenuation length can increase with increasing the interfacial DM interaction. The spin-wave amplitude may be useful to investigate interfacial magnetic properties.

Acknowledgements

This work was supported by the NRF (2010-0023798, 2011-028163, NRF-2013R1A2A2A01013188) and KU-KIST School Joint Research Program. (*) Corresponding email: kj_lee@korea.ac.kr

1 I. E. Dzyaloshinskii, Sov. Phys. JETP 5, 1259 (1957).
2 T. Moriya, Phys. Rev. 120, 91 (1960).
3 U. K. Rößler, A. N. Bogdanov, and C. Pfleiderer, Nature (London) 442, 797 (2006).
4 M. Uchida, Y. Onose, Y. Matsui, and Y. Tokura, Science 311, 359 (2006).
5 S. Mül Bauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and F. Böni, Science 323, 915 (2009).
6 S. D. Yi, S. Onoda, N. Nagaosa, and J. H. Han, Phys. Rev. B 80, 054416 (2009).
7 A. B. Butenko, A. A. Leonov, U. K. Rößler, and A. N. Bogdanov, Phys. Rev. B 82, 052403 (2010).
8 X. Z. Yu, Y. Onose, N. Kanazawa, J. H. park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, Nature (London) 465, 901 (2010).
9 X. Z. Yu, N. Kanazawa, Y. Onose, K. Kimoto, W. Z. Zhang, S. Ishiwata, Y. Matsui, and Y. Tokura, Nat. Mater. 10, 106 (2011).
10 S. Heinze, K. von Bergmann, M. Menzel, J. Brede, A. Kubetzka, R. Wiesendanger, G. Bihlmayer, and S. Blügel, Nat. Phys. 7 713 (2011).
11 S. X. Huang and C. L. Chien, Phys. Rev. Lett. 108, 267201 (2012).
12 F. Jonietz, S. Mül Bauer, C. Pfleiderer, A. Neubauer, W. Münzer, A. Bauer, T. Adams, R. Georgii, P. Böni, R. A. Dunie, K. Everschor, M. Garst, and A. Rosch, Science 330, 1648 (2010).
13 Y. Tchoe and J. H. Han, Phys. Rev. B 85 174416 (2012).
14 X. Z. Yu, N. Kanazawa, W. Z. Zhang, T. Nagai, T. Hara, K. Kimoto, Y. Matsui, Y. Onose, and Y. Tokura, Nature Commun. 3, 988 (2012).
15 A. Thiaville, S. Rohart, É. Jué, V. Cros, and A. Fert, Europhys. Lett. 100 57002 (2012).
16 J. Iwasaki, M. Mochizuki, and N. Nagaosa, Nat. Commun. 4, 1463 (2013).
17 A. Fert, V. Cros, and J. Sampaio, Nat. Nanotech. 8, 152 (2013).
18 A. Fert and P. M. Levy, Phys. Rev. Lett. 44, 1538 (1980).
19 M. Bode, M. Heide, K. von Bergmann, P. Ferriani, S. Heinze, G. Bihlmayer, A. Kubetzka, O. Pietzsch, S. Blügel, and R. Wiesendanger, Nature (London) 447, 190 (2007).
20 P. Ferriani, K. von Bergmann, E. Y. Vedmedenko, S. Heinze, M. Bode, M. Heide, G. Bihlmayer, S. Blügel, and R. Wiesendanger, Phys. Rev. Lett. 101, 027201 (2008).
21 M. Heide, G. Bihlmayer, and S. Blügel, Phys. Rev. B 78, 140403(R) (2008).
22 L. Udvardi and L. Szunyogh, Phys. Rev. Lett. 102, 207204 (2009).
23 Kh. Zakeri, Y. Zhang, J. Prokot, T. H. Chuang, N. Sakr, W. X. Tang, and J. Kirschner, Phys. Rev. Lett. 104, 137203 (2010).
24 A. T. Costa, R. B. Muniz, S. Lounis, A. B. Klautau, and D. L. Mills, Phys. Rev. B 82 014428 (2010).
25 Kh. Zakeri, Y. Zhang, T.-H. Chuang, and J. Kirschner, Phys. Rev. Lett. 108, 197205 (2012).
26 G. Chen, J. Zhu, A. Quesada, T. P. Ma, Y. Chen, H. Y. Kwon, Z. Q. Qin, A. K. Schmid, and Y. Z. Wu, Phys. Rev. Lett. 110, 177204 (2013).
27 S. Emori, U. Bauer, S.-M. Ahn, E. Martinez, and G. S. D. Beach, Nat. Mater. 12, 611 (2013).
28 K.-S. Ryu, L. Thomas, S.-H. Yang, and S. S. P. Parkin, Nat. Nanotech. 8, 527 (2013).
29 R. Skomski, Z. Li, R. Zhang, R. D. Kirby, A. Enders, D. Schmidt, T. Hofmann, E. Schubert, and D. J. Sellmyer, J. Appl. Phys. 111, 07E116 (2012).
30 A. Yamaguchi, T. Ono, S. Nasu, K. Miyake, K. Mibu, and T. Shinjo, Phys. Rev. Lett. 92, 077205 (2004).
31 M. Yamanouchi, D. Chiba, F. Matsukura, and H. Ohno, Nature (London) 428, 539 (2004).
32 M. Kläui, C. A. F. Vaz, J. A. C. Bland, W. Wernsdorfer, G. Faini, E. Cambril, L. J. Heyderman, F. Nolting, and U. Rudiger, Phys. Rev. Lett. 94, 106601 (2005).
33 D. Cortés-Ortúñ o and P. Landeros, J. Phys.: Condens. Matter 25, 156001 (2013).
34 D. D. Stancil and A. Prabhakar, Spin Waves - Theory and Applications (Springer, 2009).
35 T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).
36 R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids 19, 308 (1961).
37 R. Arias and D. L. Mills, Phys. Rev. B 60, 7395 (1999).
38 D. V. Berkov, and N. L. Gorn, J. Appl. Phys. 99, 08Q701 (2006).
39 S.-M. Seo, K.-J. Lee, H. Yang, and T. Ono, Phys. Rev. Lett. 102, 147202 (2009).
40 Y. Tserkovnyak, A. Brataas, G.E.W. Bauer, Phys. Rev. Lett. 88, 117601 (2002).
41 K.-W. Kim, J.-H. Moon, K.-J. Lee, and H.-W. Lee, Phys. Rev. Lett. 108, 217202 (2012).
42 G. Tatara, N. Nakabayashi, and K.-J. Lee, Phys. Rev. B 87, 054403 (2013).
43 M. Bailleul, D. Olligs, and C. Fermon, Appl. Phys. Lett. 83 927 (2003).
44 P. K. Amiri, B. Rejaei, M. Vroubel, and Y. Zhuang, Appl.
Phys. Lett. 91 927 (2007).

T. Schneider, A. A. Serga, T. Neumann, B. Hillebrands, and M. P. Kostylev, Phys. Rev. B 77 214411 (2008).

V. E. Demidov, M. P. Kostylev, K. Rott, P. Krzysteczko, G. Reiss, and S. O. Demokritov, Appl. Phys. Lett. 95 112509 (2009).

K. Sekiguchi, K. Yamada, S. M. Seo, K. J. Lee, D. Chiba, K. Kobayashi, and T. Ono, Appl. Phys. Lett. 97 022508 (2010).

V. Vlaminck and M. Bailleul, Science 322, 410 (2008).

M. Zhu, C. L. Dennis, and R. D. McMichael, Phys. Rev. B 81, 140407(R) (2010).

M. Zhu, B. D. Soe, R. D. McMichael, M. J. Carey, S. Maat, and J. R. Childress, Appl. Phys. Lett. 98, 072510 (2011).

K. Sekiguchi, K. Yamada, S.-M. Seo, K.-J. Lee, D. Chiba, K. Kobayashi, and T. Ono, Phys. Rev. Lett. 108, 017203 (2012).

J.-H. Park, C. H. Kim, H.-W. Lee, and J. H. Han, Phys. Rev. B 87, 041301(R) (2013).

K.-W. Kim, H.-W. Lee, K.-J. Lee, and M. D. Stiles, arXiv:1308.1198.