Development of a New Simulation Method of Mold Filling Based on a Body-fitted Coordinate System

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A new numerical model has been developed for the simulation of mold filling in curved-shape mold cavities. In the present method, the SIMPLE scheme was adopted to solve the momentum transport and the VOF (Volume of Fluid) method to trace the free surface in mold filling processes. In order to improve the solution accuracy in modeling mold filling with curved-shape and thin-walled mold cavities, the body-fitted-coordinate (BFC) concept, known as the most effective method to predict a flow field in a curved-shape cavity, was adopted. The governing equations for fluid flow were transformed based on the BFC concept. The non-staggered mesh and the momentum interpolation method were used which are essential for the BFC method. In addition, the standard VOF method was modified for the treatment of free surface in the BFC system. The standard DAFA (donor and acceptor flux approximation) method was also revised as a suitable form to the BFC system. In order to verify the present SIMPLE-BFC-VOF method, several examples on mold filling problems having curved-shape mold cavities were simulated, and the results were compared with the experimental results and other simulation methods. It is concluded that the present method can be used as an effective simulation method for the simulation of mold filling in thin-walled and curved-shape mold cavities.

KEY WORDS: mold filling; thin-walled casting; curved-shape; body fitted coordinates; volume of fluid; SIMPLE; numerical simulation.

1. Introduction

During the last decade, extensive efforts have been made to develop numerical models of mold filling in casting processes. The SMAC$^1$ and SOLA-VOF$^2$ techniques have been known as the effective methods for the simulation of mold filling with free surface. A number of studies using these methods have reported on the applications to mold filling in casting processes.$^3,4$ The algorithms mentioned above are solved using the finite volume or difference schemes, which are based on an orthogonal grid system, resulting in significant errors when curved-shaped or thin-walled castings are considered. Recently, Mampaey et al.$^5$ suggested a simulation method using an orthogonal-curvilinear-coordinate scheme, and Zhu et al.$^6$ reported a new scheme based on the direct finite difference method using regular and irregular mixed elements to preserve the advantages of orthogonal meshing. These methods may give relatively acceptable simulation results for treating curved shapes than the conventional ones. In spite of their efforts, these methods cannot be easily applied to the simulation of mold filling since they still have some difficulty in automatic grid generation and less user-friendly pre/post processors compared to the orthogonal meshing schemes. As an alternative scheme, the finite element method may be a useful method because it is easy to generate a grid for a complex geometry. However, it has been known to be unsuitable to predict the incompressible flow, and in addition the computational time and memory size will be much larger than those based on the finite difference method.$^7$

In the present study, a new coupled SIMPLE and VOF method based on the body-fitted coordinate (BFC) system was developed to model the mold filling process for curved-shape or thin-walled castings. A modified consideration on the volume of fluid (VOF) for treating the melt free surface was introduced. In order to verify the present SIMPLE-BFC-VOF method, several examples on mold filling problems having curved-shape mold cavities were simulated, and the results were compared with those reported in the literature.$^5$

2. Computational Method

2.1. Governing Equations

The governing equation for heat, fluid flow and mass transfer in mold filling processes can be expressed by the following generalized form of equation.

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x_j} (u \phi) = \frac{\partial}{\partial x_j} \left( \Gamma_\phi \frac{\partial \phi}{\partial x_j} \right) + S_\phi \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS
2.2. Transformation of the Governing Equations for Body-fitted Coordinates

In order to discretize Eq. (1) for the arbitrary geometry shown in Fig. 1(a), Eq. (1) must be transformed for the body-fitted coordinate (BFC) system. If the geometry is transformed based on the BFC (\( \xi \)), the grid cell is always kept to be rectangular, as shown in Fig. 1(b). Transformation can be made using the following equation.

\[
\frac{\partial \phi}{\partial x_j} = \frac{\partial \xi_i}{\partial x_j} \frac{\partial \phi}{\partial \xi_i} \quad (x_j: \text{Cartesian coordinate, } \xi_i: \text{BFC}) \ldots (2)
\]

Using the above relationship, the generalized equation, Eq. (1), can be rearranged for the BFC as follow.

\[
\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial \xi_j} \left( \rho U_j \phi \right) = \sum_{j=1}^{n} \left( D_j \frac{\partial \phi}{\partial \xi_j} \right) + S_{\phi} \quad \ldots (3)
\]

Here, \( D_j \) is a geometric coefficient and \( U_j = (\partial \xi_j / \partial x_i) u_i \) is a contravariant velocity at the cell face shown in Fig. 2.

Equation (3) is discretized by the finite volume method based on the non-staggered grid system as shown in Fig. 2. In the non-staggered grid system, the velocity and the pressure are calculated at the center of a control volume, and the contravariant velocities at the cell faces are evaluated by the momentum interpolation method. This interpolation method prevents the zig-zag problem known as a serious trouble in the non-staggered grid.

As a result, the final discretized equation is given by

\[
A_p \phi_p = A_{E} \phi_E + A_{W} \phi_W + A_{S} \phi_S + A_{N} \phi_N + A_{T} \phi_T + A_{B} \phi_B + b \ldots (4)
\]

where

\[
b = b_{S} + b_{SS} \quad \ldots \ldots \ldots \ldots \ldots (5)
\]

\( J \) is the volume of a control volume and \( b_{SS} \) is an extra source term called as the cross derivative term appeared due to the BFC transformation. The coefficient \( A_{nb} \) (\( nb = P, W, E, N, S, T, B \)) is determined by the convective scheme.

2.3. Volume of Fluid (VOF) Method in BFC

The standard VOF method has been known as more effective than the marker and cell method in case of tracing the free surface in mold filling because it is based on the mass conservation equation and uses less memory size. However, the standard VOF method cannot be directly applied to the BFC system since it has been developed for an orthogonal grid system. The standard VOF method was modified in order to use it for the BFC method as follows.

In the VOF method, the fluid function \( F \) is calculated by Eq. (6).

\[
\frac{\partial F}{\partial t} + u_j \frac{\partial F}{\partial x_j} = 0 \quad \ldots \ldots \ldots \ldots \ldots (6)
\]

Similar to the case for the transformation of Eq. (3), Eq. (6) can also be transformed for the BFC system as follow.

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Table 1. Flow variables, diffusion coefficients and source terms for the governing equations.

| \( \theta \) | \( D_p \) | \( S_{\phi} \) |
|---|---|---|
| 1 | 0 | 0 |
| \( u \) | \( \mu \) | \( \frac{\partial p}{\partial x} \) |
| \( v \) | \( \mu \) | \( \frac{\partial p}{\partial y} \) |
| \( w \) | \( \mu \) | \( \frac{\partial p}{\partial z} \) |

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Fig. 1. Grid configurations and velocity components: (a) \( x-y \) coordinate system and (b) body-fitted coordinate system. \( u \) and \( v \) are the velocity components in \( x-y \) coordinate, \( U \) and \( \phi \) are the contravariant velocities.

Fig. 2. Notations of the calculation points, the cell-face points, the contravariant velocities, and the cell-center velocities.
The discretized equation of Eq. (6) is given by
\[ F^{n+1} = F^n - \Delta t \cdot \frac{\partial (U_n F)}{\partial \xi_j} \] \hspace{1cm} (8)

In order to preserve the sharp definition of free surface, a special care must be taken in computing the cell flux, \(D U_j F\). The Donor and Acceptor Flux Approximation (DAFA), which is usually used in the SOLA-VOF method,\(^2\) is adopted in the present method. However, the standard DAFA, which is proposed for an orthogonal grid system, cannot be used directly. The modified DAFA for the BFC method is given as follows.
\[ \Delta U_j F \cdot \Delta t = \text{sgn}(U_j^n) \min \left[ F_{AD} \left| U_j^n \cdot \Delta t + CF, F_{BD} L_\xi \right| \right] \] \hspace{1cm} (9)

Here,
\[ CF = \max \left\{ (F - F_{AD}) \cdot U_j^n \cdot \Delta t + (F - F_{BD}) L_\xi, 0, 0 \right\} \] \hspace{1cm} (10)
and
\[ (F) = \max \left\{ F_{D}, F_{DM}, 0.1 \right\} \] \hspace{1cm} (11)

The selection of the subscripts, D, DM and AD is similar to the previous DAFA.\(^2\) The distance and \(L_\xi\) the contravariant velocity are shown in Fig. 3.

2.4 Treatment of a Surface Cell

2.4.1. Momentum Equation

In order to calculate the momentum on a surface cell in Fig. 4, the following tangential and normal stress conditions are applied.

The normal stress condition is given by
\[ \sigma_{\xi \xi} |_{n} = 0 \] \hspace{1cm} (12)

And the tangential stress condition is
\[ \tau_{\xi \eta} |_{n} = 0 \] \hspace{1cm} (13)

In order to explain the procedure how to apply these conditions, the momentum equation for a two-dimensional problem is represented with respect to the normal and tangential stresses.
\[ \rho \frac{D u}{D t} = \frac{\partial \sigma_{\xi \xi}}{\partial \xi} + \frac{\partial \tau_{\xi \eta}}{\partial \eta} + \rho \cdot f_{bx} \] \hspace{1cm} (14)
\[ \rho \frac{D v}{D t} = \frac{\partial \tau_{\xi \eta}}{\partial \xi} + \frac{\partial \sigma_{\eta \eta}}{\partial \eta} + \rho \cdot f_{by} \] \hspace{1cm} (15)

The above equations are transformed as follows.
\[ \rho \frac{D u}{D t} = \frac{\partial}{\partial \xi} \left[ \sigma_{\xi \xi} + \tau_{\xi \eta} \right] + \frac{\partial}{\partial \eta} \left[ \tau_{\xi \eta} + \sigma_{\eta \eta} \right] + \rho \cdot f_{bx} \] \hspace{1cm} (16)
\[ \rho \frac{D v}{D t} = \frac{\partial}{\partial \xi} \left[ \tau_{\xi \eta} + \sigma_{\eta \eta} \right] + \frac{\partial}{\partial \eta} \left[ \tau_{\xi \eta} + \sigma_{\eta \eta} \right] + \rho \cdot f_{by} \] \hspace{1cm} (17)

From the rule of transformation, the stresses in BFC can be written as
\[ \sigma_{\xi \xi} = \sigma_{\xi \xi} + \tau_{\xi \eta}, \quad \sigma_{\eta \eta} = \sigma_{\eta \eta} + \tau_{\xi \eta}, \quad \tau_{\xi \eta} = \tau_{\xi \eta} + \tau_{\xi \eta} \] \hspace{1cm} (18)

By substituting Eq. (18) into Eq. (16), the discretized form is obtained as follow.
\[ \rho \frac{D u}{D t} + \left[ \sigma_{\xi \xi}(U \cdot u) + (V \cdot u) \right] - \left[ \sigma_{\xi \xi}(V \cdot v) + (V \cdot v) \right] = \rho \cdot f_{bx} \] \hspace{1cm} (19)

If the stress condition is applied to Eq. (19) on a surface cell in Fig. 4, the equation can be reduced as follow.
\[ \rho \frac{D u}{D t} + \left[ \sigma_{\xi \xi}(U \cdot u) + (V \cdot u) \right] - \left[ \sigma_{\xi \xi}(V \cdot v) + (V \cdot v) \right] = \rho \cdot f_{bx} \] \hspace{1cm} (20)

And the equation for the v component is similarly given by
\[ \rho \frac{D v}{D t} + \left[ \tau_{\xi \eta}(U \cdot u) + \tau_{\tau \eta}(V \cdot v) \right] - \left[ \tau_{\xi \eta}(V \cdot v) + \tau_{\tau \eta}(V \cdot v) \right] = \rho \cdot f_{by} \] \hspace{1cm} (21)
2.4.2. Pressure at a Surface Cell

In the SOLA-VOF method, the pressure of a surface cell in Fig. 5 is evaluated by interpolating from its neighbor cell. The interpolation function is given by

\[ P_s = \left(1 - \frac{d}{d_c}\right) P_n + \frac{d}{d_c} P_{\text{surface}} \quad \ldots \quad (22) \]

The nomenclatures of Eq. (22) are shown in Fig. 5. The pressure is rapidly changed from a negative to a positive value during a small time interval because it depends on the distance \( d \) between the neighbor cell and the free surface, resulting in an unstable result and an increase in computational time. In order to solve this problem, the distance \( d \) in Fig. 5 is redefined as a distance from the neighbor cell to the across surface of the control volume of a surface cell, as shown in Fig. 6.

It is also to be noted that entrapped air in a closed loop is one of the principal factors affecting the evolution of free surface in mold filling. Entrapped air may float due to the buoyancy force or deform because of its back pressure, affecting the geometry of free surface. However, the effects of buoyancy force and a rise in pressure of entrapped air or air bubbles are not considered in the present numerical model.

2.4.3. Contravariant Velocity on a Surface Cell

The contravariant velocity on the surface of a control volume can be calculated by the momentum interpolation method\(^9\) when two neighbor cells are not empty cells. Therefore, if an empty cell exists as in Fig. 4, the contravariant velocity is obtained not by the momentum interpolation method, but by the mass conservation equation suggested in SMAC.\(^1\)

For a two dimensional problem, the mass conservation equation is given by

\[ \dot{U}_e - U_n + V_n - V_s = 0 \quad \ldots \quad (23) \]

The contravariant velocity \( U_e \) in Fig. 4 is calculated by Eq. (23) using the known velocities, \( U_n, V_n \), and \( V_s \). According to the scheme used in SMAC,\(^1\) 16 cases for a two dimension and 64 cases for a three dimension must be considered.

In addition, when an empty cell becomes a surface cell, the initial velocity \( u_e \) at the center point of a control volume can be estimated by averaging the values of its neighbor cells.

2.5. Determination of the Direction Normal to a Free Surface

It is important to find the direction normal to a free surface since it is used for calculating the volume of fluid and the pressure. In the SOLA-VOF method,\(^2\) the slope of a free surface is firstly calculated and then the direction normal to a free surface is selected. This method is very simple to use for two dimensional mold filling problems, but not easy for three-dimensional problems since the definition of the slope for three dimensional surfaces is very complicated. In the present study, a new approach has been developed to determine the normal direction to a free surface. The procedure is as follows.

1. Check neighbor cells and find the locations of full fluid cells.
2. If there is no full fluid cell in the first step, check flow fluxes through all surfaces of a control volume and find the direction of the maximum flux.
3. If there is no flux through the cell surface, check the value of fluid function (\( F \)) of neighbor cells and find the location of the cell having the maximum value of \( F \).

The direction of the normal vector at a free surface is the same with the above direction. This approach can be easily applied to three-dimensional problems.

3. Solution Scheme

In the original SIMPLE algorithm, the discretized equations for a full calculation domain are implicitly solved by TDMA (Tri-Diagonal Matrix Algorithm). In mold filling analyses, the discretized momentum equations are to be solved for surface and full fluid cells. Therefore, the solution procedure using TDMA is considered not to be efficient during the initial stage of mold filling since the empty cells are not necessary to be solved. In order to reduce the computational time, all the cells except empty cells are solved by the Jacobi Iteration method\(^10\) instead of TDMA.

4. Results and Discussion

The present SIMPLE-BFC-VOF method has been applied to simulate several mold filling problems, and compared with the simulation and experimental results reported in the literature.\(^8\)
4.1. Mold Filling Simulation and Comparison with a Standard VOF Method

Figure 7(a) indicates the geometry of a mold cavity used for the simulation. The domain size is \(30 \times 30 \times 4\) (cm) with a semicircular core of the radius of 10 cm. The inlet velocity through the gate is 50 cm/s. The working fluid is a cast iron \((\rho = 7.254 \text{ g/cm}^3, \mu = 0.0696 \text{ g/cm\cdot s})\). The simulated filling pattern by the present SIMPLE-BFC-VOF method was compared with a standard SIMPLE-VOF method in which a rectangular coordinate system was used. The predicted filling sequences are shown in Fig. 8: (a) the present SIMPLE-BFC-VOF method with \(20 \times 20 \times 4\) grids, (b) the standard SIMPLE-VOF method with \(20 \times 20 \times 4\) grids, and (c) the standard SIMPLE-VOF method with \(80 \times 80 \times 12\) grids, respectively. It can be seen from the figure that the present SIMPLE-BFC-VOF method can describe the filling pattern and sequences more quantitatively, especially around the semicircular core, as shown in Fig. 8(a). However, the standard SIMPLE-VOF method using a rectangular grid system cannot predict the filling pattern around the semicircular core exactly. In case of a small number of grids used as in Fig. 8(b), it predicts a second rotational flow occurred around the semicircular core during filling, which is different from the simulation result by the present method. As the number of grids increases as shown in Fig. 8(c), the filling sequence around the semicircular core becomes similar to that by the SIMPLE-BFC-VOF method. It can be said that the present method can exactly predict the filling pattern and sequences in cases of curved-
shape mold cavities, and that the computational time can be reduced a lot since a small number of grids is needed for the simulation as shown in the figure.

4.2. Comparison with Other Simulation Scheme

It has been known that the standard VOF method using an orthogonal grid system is not suitable for the simulation of mold filling with curved gating systems. Xu and Mampaey\(^8\) reported an orthogonal-curvilinear-coordinate method based on the VOF scheme, employing a structured non-orthogonal mesh, and comparison with some experimental results were also made on mold filling with a curved gating system. Their simulation and experimental models were adopted in the present study in order to verify the present model. Two models were simulated: one is with a stair-like gating system and the other with a curved gating system, and the numbers of grids for these two models were 6636 and 5736, respectively, as shown in Figs. 7(b) and 7(c)\(^8\).

Figure 9 indicates the filling sequences in a mold cavity with a stair-like gating system: (a) and (b) the experimental and simulation results by Xu and Mampaey\(^8\); and (c) the simulation results by the present SIMPLE-BFC-VOF method. As another example of application, the filling pattern and

Fig. 9. Mold filling sequences in a die cavity having a stair-like gating system: (a) and (b) the experimental and simulation results by Mampaey et al.\(^8\); and (c) the simulation results by the present SIMPLE-BFC-VOF method.

Fig. 10. Mold filling sequences in a die cavity having a curved gating system: (a) and (b) the experimental and simulation results by Xu and Mampaey\(^8\); and (c) the simulation results by the present SIMPLE-BFC-VOF method.
sequences through a curved gating system shown in Fig.7 (c) were investigated. It is well known that the standard VOF method with a rectangular grid system can hardly obtain the exact filling pattern and sequences in case of mold filling with thin-walled and curved-shape mold cavities. **Figure 10** indicates the filling sequences in a mold cavity through a curved gating system: (a) and (b) the experimental and simulation results by Xu and Mampaey (c) the simulation result by the present SIMPLE-BFC-VOF method. As shown in Fig. 10(a), the experimental observation shows that the melt through a curved gate is ejected into the mold cavity with a slight slope against the vertical axis and reaches up to the top plane of the mold cavity until the end of filling. It is found from Fig. 10(c) that the simulation by the present method can exactly predict the filling pattern and sequences for this model. However, the simulation by Mampaey _et al._ predicts that the molten melt enters into the mold cavity vertically, which is different from the experimental observation. It can also be said that the present SIMPLE-BFC-VOF method needs a small number of grids even for the simulation of the filling pattern and sequences in a curved-shape mold cavity.

5. **Concluding Remarks**

A new simulation method, based on the coupling of the body-fitted-coordinate (BFC) concept and the VOF (Volume of Fluid) method has been developed for the simulation of mold filling in curved-shape mold cavities. In the present method, the SIMPLE scheme was adopted to solve the momentum transport in mold filling processes. The present SIMPLE-BFC-VOF method was verified by simulating the filling pattern and sequences of several mold filling problems having curved-shape mold cavities, and the simulated results were compared with the experimental results and other simulation methods reported in the literature. It can be concluded that the present SIMPLE-BFC-VOF method can be used as an effective simulation method for the simulation of mold filling in thin-walled and curved-shape mold cavities. In addition, in cases of curved-shape and thin-walled castings, the SIMPLE-BFC-VOF method has a significant advantage compared to the standard VOF method since it needs only a small number of grids, leading to a decrease in computational time.

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