Research on fractal dimension calculation method of machining surface based on wavelet transform

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Abstract. Combining the multi-scale analysis ability of wavelet with the multi-scale self-similar feature of fractal, multi-scale wavelet decomposition of fractal contour, based on logarithmic wavelet spectrum, and then evaluating the fractal features of contour and calculating its fractal dimension. The calculation method of fractal dimension logarithmic wavelet spectrum of machined surface contour is compared with the calculation error of power spectral density method and structure function method. Finally, it is applied to the calculation of fractal dimension of actual machined surface contour, which shows its practicability.

1. Introduction
From a microscopic point of view, the machined surface has protrusions and depressions of various sizes and complex arrangements, and the actual surface topography is rough and disordered. Surface topography is an important indicator for evaluating the quality of machined parts, directly affecting part performance, such as wear resistance, corrosion resistance, contact stiffness and mating properties[1]. Studies have shown that[2], mechanical rough surface contours have non-stationary, self-similarity and multi-scale characteristics, fractal theory just meets the above characteristics, and thus related scholars [3] introduced the fractal theory to the representation of rough surfaces. However, fractal theory is used to describe two characterization parameters of fractal dimension and scale factor when machining a machined surface. The fractal dimension reflects the complexity of the rough surface contour structure and qualitatively characterizes the proportion of high frequency components. Therefore, how to accurately obtain the fractal dimension of the rough surface is very important for the description of the rough surface.

Many scholars at home and abroad have carried out research on fractal dimension calculation [4, 5]. At present, commonly used fractal dimension calculation methods include box dimension method, size method, root mean square method, covariance method, power spectral density (PSD) method and structural function method. Li Chenggui [6, 7] et al. calculated the fractal dimension of the contour generated by the W-M function with a known fractal dimension of 1.6 using the power spectral density method and the structural function method. The calculated results are 1.65 and 1.63, respectively. Wang Jianjun [8] expounded the principles of the five methods of size method, box dimension method, R/S method, structure function method and power spectral density method and compared the theoretical
dimensions of the latter three methods to 1.2, 1.5. The results of the WM function of 1.8 show that the structural function method has the highest precision, the power spectral density method is the second, the R/S method has the lowest precision and the calculation amount is large, and its accuracy greatly decreases with the decrease of the theoretical fractal dimension. Ge Shirong[9] compared the calculation results of the WM function with fractal dimension 1.2, 1.5, and 1.8 by the method of size, box dimension, variance and structure function. The results show that the error of the size method is the biggest. The error of the variance method and the box dimension method is above 10%, and the error of the structure function method is small. The calculation results are 1.164, 1.455, 1.761. In addition, they also proposed the root mean square method [10, 11], and compared the root mean square method with the structure function method. The results show that both methods have better effects; for smaller fractal dimensions The calculation accuracy of the root mean square method is higher, and for the larger fractal dimension, the calculation accuracy of the structure function method is higher. In addition, the fractal parameters D and G can be solved by the system of the height standard deviation of the surface profile[12] and the standard deviation of the slope.

In this paper, the multi-scale analysis ability of wavelet is used for the representation of fractal multi-scale self-similar features. First, the multi-scale wavelet decomposition of fractal contour is carried out. Based on the logarithmic wavelet spectrum, the wavelet decomposition scale with fractal features is identified, and then the contour is evaluated. The fractal feature is calculated and its fractal dimension is calculated. The method of calculating the fractal dimension logarithmic wavelet spectrum of machined surface contour (hereinafter referred to as wavelet method) is proposed, and the power spectral density method, structural function method, root mean square method and equations The calculation accuracy of the method is compared, and finally it is applied to the calculation of the fractal dimension of the actual machined surface contour, which illustrates its practicability.

2. Wavelet transform

In 1984, French oil worker Morlet proposed the innovative concept of wavelet transform. Wavelet is called "mathematical microscope" and has good localization properties in both time and frequency domains. It has multi-scale and multi-resolution characteristics.

Wavelet signal decomposition principle [13]: Wavelet decomposes the original signal layer by layer. Each layer of signal can be regarded as the superposition of high frequency and low frequency components. The decomposition process is to extract the high frequency signal in the signal. Then, the lower layer signal is decomposed into the next layer. As the number of decomposition layers increases, the low-frequency signal can be well decomposed to achieve multi-scale, high-resolution interpretation of the original signal.

![Wavelet layering schematic](image)

**Figure 1.** Wavelet layering schematic.
The figure above shows the schematic diagram of wavelet three-layer decomposition. In the figure $\lambda_1(z), \lambda_2(z), \lambda_3(z)$ and $\gamma_1(z), \gamma_2(z), \gamma_3(z)$ are the approximation signal and detail signal under the decomposition layer number 1, 2, 3. $h(z), h(z^2), h(z^4)$ and $g(z), g(z^2), g(z^4)$ are the low-pass filter and high-pass filter under the decomposition layer $i = 1, 2, 3$. Among them, $x(z) = \lambda_1(z) + \gamma_1(z) = \lambda_2(z) + \gamma_2(z) + \gamma_1(z), \ x(z) = \lambda_3(z) + \gamma_3(z) + \gamma_2(z) + \gamma_1(z)$ according to the principle of three-layer decomposition, multi-scale decomposition of the analysis signal can be realized.

The wavelet analysis operation of the signal is to decompose the signal to be processed into a linear superposition of a family of wavelet functions $\psi_k^m(t) \in \mathbb{Z}^2$ obtained by scale expansion and translation of the wavelet mother function. The Fourier transform is used to decompose the signal into a series of sines of different frequencies. The superposition of functions. Wavelet analysis is to spread the signal in a set of orthogonal bases $\psi_k^m(t) \in \mathbb{Z}^2$. When the scale changes in binary, the scale expansion and translation of the mother wavelet can be expressed as

$$\psi_k^m(t) = 2^{-m/2} \psi(2^{-m} t - k)$$ (1)

In the formula, $2^{-m}$ is the scale factor, $k$ is the translation factor.

The discrete wavelet series of an arbitrary signal $f(t) \in L^2(\mathbb{R})$ (square integrable function space) is expressed as:

$$f(t) \in \sum_{m \in \mathbb{Z}} \sum_{k \in \mathbb{Z}} d_k^m \psi_k^m(t)$$ (2)

The wavelet decomposition coefficient can be expressed as:

$$d_k^m = \langle f(t), \psi_k^m(t) \rangle = \int_{-\infty}^{\infty} f(t) \psi_k^m(t) dt$$ (3)

In the formula, $\langle \cdot, \cdot \rangle$ denotes the inner product symbol, $m$ is the scale number, $d_k^m$ is the $k$ decomposition coefficient of the layer decomposition.

Wavelet transform is performed on the contour curve to obtain the wavelet coefficient, that is $d_k^m$. For a random signal with a contour curve, the wavelet coefficients of a set of $m$ are calculated by wavelet transform, and the linear regression analysis $\text{Var}(d_k^m)$ with $m$ is performed in double logarithmic coordinates to obtain the slope of the line $\alpha$. Then the fractal dimension is:

$$D = 2 - \alpha$$ (4)

3. Wavelet calculation method of fractal dimension

3.1. First construct the M-B function

The rough surface contour curve is characterized by the M-B function, and its expression is

$$z(x) = G^{\frac{1}{D-1}} \sum_{n=0}^{\infty} \nu^{-2(D-1)n} \cos(2\pi \nu^n x), \quad 1 < D < 2, \quad \nu > 1$$ (5)

In the formula, $z(x)$ is the height of the random surface contour, $G$ is the feature scale factor, $\nu$ is a constant, $D$ is the fractal dimension.
The M-B function is used to numerically simulate the fractal contour of the machined surface. According to the measurement of the actual machined surface in [14], the contour parameter is: sampling interval \( l = 0.204 \mu m \), Fractal feature \( G = 2.86 \times 10^{-10} m \), sampling interval length \( L = 2^{13}l = 1671 \mu m \). The fractal dimension \( D \) is \( 1.1, 1.2, \ldots, 1.8, 1.9 \). To satisfy the sampling theorem, take the highest frequency index \( n_{\text{max}} = \log_{1/2} \frac{1}{l} \). The figure below shows the M-B function topography with the scale factor \( G=1.5 \).

\[ M = \text{floor} \left[ \frac{\ln (l_s / (l_s - 1))}{\ln 2} \right] = \text{floor} \left[ \ln (l_s / (l_s - 1)) \right] \]

In the formula, \( \text{floor}(g) \) is a round-down function; \( l_s \) is the signal length, \( l_s \) is the filter length, which is related to the wavelet type. For the wavelet \( l_w = 2N \), \( N \) is the vanishing moment of the wavelet function.

**Figure 2.** M-B function topography with scale factor \( G=1.5 \).

**Figure 3.** Wavelet method example.
The structure function method and the wavelet method of this paper respectively solve the W-M function under different fractal dimensions, and the obtained results and errors are shown in the following table:

| Theoretical fractal dimension | 1.1     | 1.2     | 1.3     | 1.4     | 1.5     | 1.6     | 1.7     | 1.8     | 1.9     |
|------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Structural function method   | Calculating dimensions | 1.2965  | 1.3413  | 1.4215  | 1.4956  | 1.5798  | 1.6721  | 1.7528  | 1.8401  | 1.9321  |
| Error%                       |         | 17.8636 | 11.7750 | 9.3461  | 6.8286  | 0.7533  | 4.5063  | 3.1059  | 2.2278  | 1.6894  |
| Wavelet method               | Calculating dimensions | 1.0999  | 1.2008  | 1.3013  | 1.4017  | 1.5020  | 1.6023  | 1.7025  | 1.8026  | 1.9026  |
| Error%                       | -0.0    | 0.0633  | 0.1009  | 0.1231  | 0.1361  | 0.1428  | 0.1447  | 0.1421  | 0.1351  |         |

4. Conclusion

A logarithmic wavelet spectrum calculation method for the fractal dimension of machined surface contours and the concept of effective decomposition scale are proposed. The key to calculate the fractal dimension of the contour is to choose the wavelet basis function and the decomposition scale. However, the maximum decomposition scale can be determined by the surface contour sequence and the wavelet order, which can be decomposed according to the maximum decomposition scale. Logarithmic wavelet spectroscopy can deal with the multi-scale features of fractals well. For the fractal contour simulated by MB function, when the sym4 wavelet base is used, the logarithmic wavelet spectrum method has a calculation error of less than 0.15% Accuracy.

Acknowledgments

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