Possible Suppression of Neutron EDM

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ABSTRACT

Employing generalized Schiff’s transformation on electric dipole moments (EDM) in quantum field theory, we show that the chromoelectric EDM lagrangian density is transformed into the electric EDM term with a new coefficient. Under the new constraint on the EDM operators, the neutron EDM can be described by a unique combination of electric EDM $d_f$ and chromoelectric EDM $\tilde{d}_f$ of quarks. If the special relation of $d_f = \frac{e_f}{2g_s} \tilde{d}_f$ holds, then the neutron EDM is suppressed significantly.

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1. Introduction

Recent measurements of electric dipole moments (EDM) in neutrons and atoms have presented tight upper bounds [1-5] which are rather close to theoretical predictions of the EDM by supersymmetric theory calculations [6-10]. Even though there are some free parameters in the calculations, it is expected that the finite EDM values may well be observed soon. In particular, the recent measurement of the muon $g - 2$ [23] should give severe constraints on the theoretical EDM values.

Recently, there have been many theoretical investigations of the neutron EDM [11-18]. In particular, Pospelov and Ritz [19] suggest that the contributions to the neutron EDM which come from the chromoelectric interactions may well be comparable to the electric EDM estimation. Therefore, careful studies of the EDM operator arising from the chromoelectric fields must be quite important for the reliable estimation of the neutron EDM.

In this Letter, we study a constraint on the chromoelectric EDM operators due to the unitary transformation which is analogous to Schiff’s theorem in nonrelativistic quantum mechanics. Here, we show that the chromoelectric EDM lagrangian density $-i\frac{d_f}{2}\bar{\psi}\sigma_{\mu\nu}\gamma_5 t^a\psi G_{\mu\nu}^a$ is transformed into a new term $i\frac{ef}{4g_s}\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi F_{\mu\nu}$ which is just the same as the electric EDM lagrangian density. Thus, the coefficient of the EDM lagrangian density is modified from $\frac{d_f}{2}$ to $\frac{1}{2}\left(d_f - \frac{ef}{2g_s}\tilde{d}_f\right)$. Therefore, any neutron EDM calculations should be described with the combinations of $\left(d_f - \frac{ef}{2g_s}\tilde{d}_f\right)$. This should present some constraints which should be satisfied by any calculations of the neutron EDM.

2. Schiff’s theorem in QED

In 1963, Schiff proved [20] that the effect of the EDM interaction cannot be measured in the nonrelativistic quantum mechanics with electromagnetic interactions. This can be easily seen since the hamiltonian with the EDM interaction

$$H = \frac{P^2}{2m} + eA_0(r) - \mathbf{d} \cdot \mathbf{E}$$

(1)
can be rewritten with the replacement of $r \rightarrow r - \frac{d}{e}$ as

$$H = \frac{p^2}{2m} + eA_0(r) + d \cdot \nabla A_0(r) = \frac{p^2}{2m} + eA_0(r) + O(d^2)$$

(2)

where $E(r) = -\nabla A_0(r)$, and we have made use of the fact that the momentum $p$ does not change under the replacement of $r \rightarrow r - \frac{d}{e}$. This means that the spectrum of the original Hamiltonian with the EDM term becomes just the same as that of the Hamiltonian without the EDM term since the $O(d^2)$ term is negligibly small. This is Schiff’s theorem, and the EDM effect cannot be observed in the nonrelativistic quantum mechanics with electromagnetic interactions.

Next, we treat the field theory version of Schiff’s theorem and consider the unitary transformation in QED [21]. The Lagrangian density of QED with the electromagnetic EDM terms can be written as

$$\mathcal{L} = \bar{\psi}(i\partial_\mu + ieA_\mu)\gamma^\mu \psi - m_0 \bar{\psi}\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i \frac{d_f}{2} \bar{\psi} \sigma_{\mu\nu} \gamma^5 \psi F^{\mu\nu}$$

(3)

where $F_{\mu\nu}$ denotes the field tensor given as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$  

(4)

Now, we consider the following unitary transformation with $p_\mu = i \partial_\mu$,

$$\psi' = \exp \left( i \frac{d_f}{e} \gamma_5 p_\mu \gamma^\mu \right) \psi.$$  

(5)

Under this transformation, the total Lagrangian density becomes, up to the order of $d_f$

$$\mathcal{L}' = \bar{\psi}'(i\partial_\mu + ieA_\mu)\gamma^\mu \psi - m_0 \bar{\psi}\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - 2i \frac{d_f}{e} \bar{\psi} \gamma_5 (p_\mu p^\mu - eA_\mu p^\mu) \psi.$$  

(6)

This is the new Lagrangian density in QED and states that the new EDM term does not couple to the external electric field while the original EDM term has the coupling with the electric field $E$ in the first order as seen in eq.(3). This is just the same as Schiff’s statement that the EDM interaction does not have the first order perturbation energy with the external electric field any more. Here, it should be noted that, even though eq.(6) contains the $A_\mu p^\mu$ term, the Hamiltonian constructed from eq.(6) contains only the
three dimensional vector potential $\mathbf{A} \cdot \mathbf{p}$ term which does not contribute to the EDM. This is wellknown in the EDM evaluation in atomic physics [1,21,25].

Therefore, the EDM of a composite system can be generated in the second order perturbation as

$$d_n = 2 \sum_{N^*} \frac{<N|\sum_i e_i z_i|N^*><N^*|H_{edm}|N>}{E_N - E_{N^*}}$$

(7)

where the sum should be taken over excited states in the composite system and $H_{edm}$ corresponds to the EDM lagrangian term in eq.(6). $N$ and $N^*$ denote the ground and excited states of the composite system.

At this point, we make a comment on the gauge symmmetry violation of the unitary transformation of eq.(5). This transformation leads to the violation of the gauge symmetry, but this is justified as long as one is interested in evaluating the low energy state property of the system up to the order of $g$.

This is the same as Weinberg-Salam treatment of spontaneous symmetry breaking where the gauge symmetry is broken when evaluating the ground state property of the system. Even though Weinberg-Salam’s model is not exact, their treatment is economical. Clearly, one can obtain the same result with the spontaneous symmetry breaking procedure as the result that is solved exactly for the low energy property. But obviously the exact calculation of the Weinberg-Salam’s model is far from economical.

3. Unitary transformation in QCD plus QED

Recently, there have been several papers which treat the chromoelectric EDM terms [15-19]. The neutron EDM which comes from the chromoelectric EDM of quarks seems to depend on the model calculations. The calculations with QCD sum rules in ref.[16] suggest a suppression of the neutron EDM from the chromoelectric EDM of quarks while the chiral loop estimate gives a sizable contribution to the neutron EDM [18]. Further, Pespelov and Ritz [19] show that the chromoelectric EDM term gives a significant contribution to the neutron EDM when they employ QCD sum rules.
In this section, we show that the dominant contributions from the chromoelectric EDM can be transformed into the identical shape to the electric EDM term with some modified coefficients.

We start from QCD plus QED lagrangian density with electric and chromoelectric EDM terms where the charge of the $f$ flavor quark is denoted by $e_f$,

$$\mathcal{L} = \bar{\psi} i(\partial_\mu + ig_s A_\mu^a t^a + ie_f A_\mu) \gamma^\mu \psi - m_0 \bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}$$

$$-i \frac{d_f}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 t^a \psi G^{\mu\nu,a} - i \frac{d_f}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu} \quad (8)$$

where

$$G^a_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s C_{abc} A_\mu^b A_\nu^c. \quad (9)$$

Now, we consider the following unitary transformation

$$\psi' = \exp \left( i \frac{d_f}{2 g_s} \gamma_5 (p_\mu - g_s A_\mu^a t^a) \gamma^\mu \right) \psi. \quad (10)$$

Under this transformation, we obtain the following new lagrangian density,

$$\mathcal{L}' = \bar{\psi} i(\partial_\mu + ig_s A_\mu^a t^a + ie_f A_\mu) \gamma^\mu \psi - m_0 \bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}$$

$$-i \left( d_f - \frac{e_f}{2 g_s} d_f \right) \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu} - i \frac{d_f}{g_s} \bar{\psi} \gamma_5 \left\{ (p_\mu - g_s A_\mu^a t^a)^2 - e_f A_\mu^a (p_\mu - g_s A_\mu^a t^a) \right\} \psi. \quad (11)$$

Eq.(11) shows that the chromoelectric EDM term disappears, and instead the coefficient of the electric EDM term now changes into a new form. Further, the last term which is similar to the QED case of eq.(6) is generated. This term gives rise to the neutron EDM in the second order perturbation theory in the same way as eq.(7).

Here, it should be noted that the two representations of the EDM operators are indeed equivalent when we estimate their expectation values with exact wave functions. However, when we consider the EDM of any composite systems like neutrons, the difference between the two operators becomes important since we cannot normally obtain the exact wave functions of neutrons. Employing the new EDM operators, we can calculate the EDM of neutrons in a transparent and economical fashion since it can be evaluated in the first order perturbation theory.
4. Neutron EDM

Now, we want to estimate the neutron EDM based on the lagrangian density obtained in the previous section. The EDM hamiltonian which corresponds to the new EDM lagrangian can be written as

\[ H_{edm} = -\sum_f \left\{ \left( d_f - \frac{e_f}{2g_s} \tilde{d}_f \right) \gamma_0 \Sigma \cdot E - \frac{i\tilde{d}_f}{g_s} \gamma_0 \gamma_5 \left( p_\mu^2 - 2g_s A^a \cdot \not{t}^a + g_s^2 (A^{a\mu} t^a)^2 \right) \right\} \tag{12} \]

where we dropped the term which is proportional to the electromagnetic vector potential \( A_\mu \) since the neutron state does not contain any photon states in the ground state. Also, the magnetic term is not included in eq.(12).

Now, we estimate the first term of eq.(12) in the first order perturbation. Using the SU(6) quark model, we obtain for the neutron EDM,

\[ d_n^{(1)} = \left[ \frac{4}{3}(d_d - \frac{e_d}{2g_s} \tilde{d}_d) - \frac{1}{3}(d_u - \frac{e_u}{2g_s} \tilde{d}_u) \right] N_R \tag{13} \]

where \( N_R \) denotes the overlapping integral, and \( N_R = 1 \) for nonrelativistic quark models and \( N_R \simeq 0.5 \) for the MIT bag model [24].

Next, we evaluate the contribution to the neutron EDM from eq.(12) in the second order perturbation as given in eq.(7). Since there is no \( \frac{1}{2}^- \) state nearby in the neutron excited states, there is no enhancement, contrary to the atomic cases [21].

Here, we carry out a naive estimation of eq.(7) with the MIT bag model [24], and therefore, we employ the closure approximation. In this case, eq.(7) becomes

\[ d_n^{(2)} = \frac{2}{\Delta E} < N | \sum_{i=d,d,u} e_i z_i \sum_{j=d,d,u} \left\{ \left( d_j - \frac{e_j}{2g_s} \tilde{d}_j \right) \gamma_0^{(j)} \Sigma^{(j)} \cdot \not{E}^{(j)} \right. \]

\[ - \left. \frac{i\tilde{d}_j}{g_s} \gamma_5^{(j)} \left( p_\mu^{(j)} \right)^2 - 2g_s A^a \cdot \not{t}^a_j + g_s^2 (A^{a\mu} t^a_j)^2 \right\} | N > \tag{14} \]

where \( \Delta E \) is an average excitation energy of neutron \( \frac{1}{2}^- \) states which should be of the order of nucleon mass \( M_N \). \( \not{E}^{(j)} \) denotes the electric field which the \( j \)-th quark feels inside the bag. After some numerical calculations, we obtain

\[ d_n^{(2)} \simeq \frac{2}{3R_0 \Delta E} \left[ 2e_d^2 \left( d_d - \frac{e_d}{2g_s} \tilde{d}_d \right) + e_u^2 \left( d_u - \frac{e_u}{2g_s} \tilde{d}_u \right) \right] + \frac{4R_0}{9g_s \Delta E} \left( 2e_d m_d^2 \tilde{d}_d + e_u m_u^2 \tilde{d}_u \right) \tag{15} \]
where $m_d$ ($m_u$) denotes the mass of the $d$ ($u$) quarks. Now, we take the $\Delta E \sim M_N$ and $R_0 \sim \frac{1}{m_\pi}$ with $m_\pi$ the pion mass, and therefore, the total neutron EDM can be written as

$$d_n = \left[ \frac{4}{3} \left( d_d - \frac{e_d}{2g_s} \tilde{d}_d \right) - \frac{1}{3} \left( d_u - \frac{e_u}{2g_s} \tilde{d}_u \right) \right] N_R$$

$$+ \frac{2m_\pi}{3M_N} \left[ 2e_d^2 \left( d_d - \frac{e_d}{2g_s} \tilde{d}_d \right) + e_u^2 \left( d_u - \frac{e_u}{2g_s} \tilde{d}_u \right) \right] + \frac{4}{9g_s m_\pi M_N} \left( 2e_d m_d^2 \tilde{d}_d + e_u m_u^2 \tilde{d}_u \right).$$

(16)

Since the second line of eq.(16) is much smaller than the first line contribution, we can practically neglect the second order perturbation contribution to the neutron EDM as long as the first two terms (the first line) survive.

5. Possible suppression of neutron EDM

As can be seen from eq.(16), the neutron EDM must be significantly suppressed if the following relation holds for $f$ flavor quark,

$$d_f = \frac{e_f}{2g_s} \tilde{d}_f.$$  \hspace{1cm} (17)

Recently, Chang, Keung and Pilaftsis [22] presented the two-loop calculation to the EDM in supersymmetric theories. They show that the electric EDM $d_f$ and the chromoelectric EDM $\tilde{d}_f$ of a light fermion $f$ can be given as

$$d_f = \frac{e_f 3\alpha_{em} R_f m_f}{64\pi^3 M_a^2} \sum_{q=t,b} Q_q^2 H_q$$ \hspace{1cm} (18a)

$$\tilde{d}_f = g_s \frac{\alpha_s}{128\pi^3} \frac{R_f m_f}{M_a^2} \sum_{q=t,b} H_q$$ \hspace{1cm} (18b)

where $\alpha_{em}$ and $\alpha_s$ are the electromagnetic and the chromomagnetic fine structure constants, respectively. Also, $R_f$ is given as $R_f = \cot \beta$, and $M_a$ denotes the tree level mass of the CP-odd Higgs. $H_q$ is defined as

$$H_q = \xi_q \left[ F \left( \frac{M_q^2}{M_a^2} \right) - F \left( \frac{M_{q_2}^2}{M_a^2} \right) \right]$$ \hspace{1cm} (19)

where $F(z)$ denotes a two loop function as given in ref.[22],

$$F(z) = \int_0^1 dx \frac{x(1-x)}{z-x(1-x)} \ln \left[ \frac{x(1-x)}{z} \right].$$
Also, $\xi_q$’s are the CP violating couplings which are given in the MSSM scheme \[7\].

Now, we want to see the relation between $d_f$ and $\tilde{d}_f$. From eqs.(18), we find

$$d_f = \frac{e_f}{2g_s} \tilde{d}_f \left\{ \left( \frac{12\alpha_{em}}{\alpha_s} \right) \frac{\sum_{q=t,b} Q_q^2 H_q}{\sum_{q=t,b} H_q} \right\}. \tag{20}$$

Since the value of $\frac{12\alpha_{em}}{\alpha_s}$ is close to unity at the electroweak scale, the parameters in $H_q$ will control the magnitude of the curly bracket of eq.(20). If the value of the curly bracket is unity, then the relation of eq.(17) holds.

At the present stage, we do not know whether the parameters that appear in eqs.(18) can be estimated reliably or not in the supersymmetric theories. If these parameters can be determined reliably, it would be very interesting to examine the validity of the relation.

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