Polarization and angular distribution of the radiation emitted in laser-assisted recombination

S. Bivona, G. Bonanno, R. Burlon, and C. Leone
Dipartimento di Fisica e Tecnologie Relative, Università degli Studi di Palermo - Italy and C.N.R. - C.N.I.S.M.
Viale delle Scienze - Ed. 18 - I-90128 Palermo Italy
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The effect of an intense external linear polarized radiation field on the angular distributions and polarization states of the photons emitted during the radiative recombination is investigated. It is predicted, on symmetry grounds, and corroborated by numerical calculations of approximate recombination rates, that emission of elliptically polarized photons occurs when the momentum of the electron beam is not aligned to the direction of the oscillating field. Moreover, strong modifications to the angular distributions of the emitted photons are induced by the external radiation field.

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The capture of a free electron by a positive ion is one of the fundamental processes occurring in the electromagnetic interaction of charged particles. The recombination of an electron with a bare ion is made possible by the spontaneous emission of a photon, in order to fulfill both energy and momentum conservation.

In view of its application to the analysis of astrophysical and laboratory plasmas as one of the principal means to obtain information about their physical conditions through the observation of the emitted radiation, this process has been extensively studied both theoretically and experimentally. In the last years much work has been devoted to radiative recombination (RR) in the presence of laser field \[1, 2\].

The availability of intense laser sources has stimulated theoretical investigation of laser assisted radiative recombination (LARR) aimed at exploring the spectrum of the radiation emitted during the recombination event. In the presence of intense monochromatic electromagnetic fields, the electron can exchange a large number of photons of the assisting radiation field and may recombine in a bound state \[3, 4, 5\] emitting x-rays whose frequency depends on the number of the exchanged laser photons, according to the energy conservation law:

\[\hbar \omega_X(n) = \frac{p^2}{2m} + I_f + \Delta + n \hbar \omega_L\]  \hspace{1cm} (1)

In Eq. (1) \(\omega_X(n)\) is the frequency of the emitted photon, \(p^2/2m\) is the incoming electron energy, \(I_f\) is the ionization energy of the bound state in which the electron is captured, \(\Delta = e^2 E_0^2/4m\omega_L^2\) is the ponderomotive shift, \(n\) denotes the number of exchanged photons, \(\omega_L\) and \(E_0\) the frequency and the amplitude of the oscillating laser field, \(e\) and \(m\) the electron charge and the mass respectively.

From Eq. (1), it may be easily seen that the spectrum of the emitted x-ray results in a series of frequencies evenly separated by the assisted field frequency.

The main features of the spectra of the emitted radiation have been found by several authors by treating the process in the framework of a Keldysh-type approach, where the interaction of the incoming electron with the ion in the initial continuum state is neglected \[6\]. Slight modifications of this scheme have been considered in order to take into account, though approximately, the electron-ion interaction. In all the proposed treatments, the calculated emission spectra show a large plateau due to absorption of a large number of photons, followed by an abrupt cutoff \[4, 5, 6\].

As it is well known, in the field-free process, when the electron recombines with a bare ion in the ground state, the matrix element of the electron dipole moment is directed along the direction of the incoming electron momentum, and the emitted photon turns out to be linearly polarized. In the dipole approximation, the photon polarization vector lies in the plane formed by the direction of the electron initial asymptotic momentum \(q\) and the one of the wavevector of the emitted photon \(k_f\). Due to the cylindrical symmetry about the axis passing through the nucleus, directed along \(q\), the intensity of the emitted radiation depends only on the angle between \(q\) and \(k_f\). Moreover, the linear polarization degree of the emitted radiation is equal to one.

For the subsequent analysis concerning the photon polarization state emitted in the presence of the radiation field it may result more suitable to look at the above result in terms of the symmetries of the physical system. In fact, upon reflection in the plane containing \(q\) and \(k_f\), the system does not change, while the photon helicity reverse its sign. Therefore the emission probability of photons characterized by opposite helicities and equal \(k_f\) is the same, and the unitary value of the degree of linear polarization of the photon emitted in the radiative recombination follows from the property that a linearly polarized state may be considered as a superposition of two circularly polarized states with opposite helicities and equal amplitudes.

It is the aim of this paper to study the modifications of both the polarization states and the angular distribution of the emitted photons when the recombination occurs...
in the presence of a linearly polarized laser. Due to the circumstance that the highest coupling between the recombining electron and the assisting radiation takes place when the oscillating electric field is directed along the incoming electron momentum $\mathbf{q}$, all the previous study, to the best of our knowledge, has been carried out mainly in this geometry. As the cylindrical symmetry, in this configuration, is still maintained, for each emission channel characterized by the number of photons that the assisting field exchanges with the recombining electron, the angular distribution of the emitted radiation depends on the intensity of the external field, while the photon polarization properties remain unaltered. By changing the angle between the direction of the laser field polarization and $\mathbf{q}$, the above symmetry breaks down, affecting both the polarization states and the angular distribution of the emitted photons.

In order to make quantitative estimations of the external field effects we have to calculate the recombination transition amplitude of the electron with a bare ion of charge $Ze$ in presence of an intense, spatially and temporally homogeneous, radiation field that will be taken in dipole approximation and treated classically. During the recombination emission of x-ray photon occurs with simultaneous exchange of laser photons. The x-ray radiation, characterized by the wavevector $\mathbf{k}$, and the polarization vector $\hat{\epsilon}_{\mathbf{k},\lambda}$, will be treated in second quantization and in dipole approximation. Accordingly, the electric field operator associated to a single mode x-ray radiation is taken, in Gaussian units, as

$$\hat{E}_X(t) = \sum_{\lambda=1,2} \frac{i}{V} 2\pi \hbar \omega_X \hat{e}_{\mathbf{k},\lambda}(\hat{a}_{\mathbf{k},\lambda} e^{-i\omega_X t} - \hat{a}_{\mathbf{k},\lambda}^+ e^{i\omega_X t})$$

with $V$ the quantization volume of the radiation, $\hat{a}_{\mathbf{k},\lambda}$ and $\hat{a}_{\mathbf{k},\lambda}^+$ the annihilation and creation operator, respectively, for a photon in the state characterized by $\mathbf{k}$ and $\lambda$. The nonrelativistic hamiltonian of the atomic system interacting with the radiation field reads

$$\hat{H} = \hat{H}_{at} - e\hat{\mathbf{E}}_L \cdot \mathbf{r} - e\hat{E}_X \cdot \mathbf{r}$$

where $\hat{H}_{at}$ is the field-free atomic hamiltonian $\hat{H}_{at} = \hat{p}^2/(2m) + Ze^2/r$, $\mathbf{r}$ is the electron coordinate and $\mathbf{E}_L(t)$ the external oscillating electric field assumed linearly polarized and directed along the unitary vector $\hat{\epsilon}_L$, $E_L = \hat{\epsilon}_L E_0 \cos(\omega_L t)$.

The transition amplitude of emitting one x-ray photon characterized by $\hat{\epsilon}_{\mathbf{k},\lambda}$ during the recombination of an electron with asymptotic average momentum $\mathbf{q}$ from a continuum state $\psi_q^+(\mathbf{r}, t)$ into a bound state $\psi_0(\mathbf{r}, t)$, treating the electron-x-ray photon interaction at the first order of the time-dependent perturbation theory, is given by (hereafter atomic units will be used)

$$T_{l,f} = -\sqrt{\frac{2\pi \hbar \omega_X}{V}} \int_{-\infty}^{\infty} dt \langle \psi_0(\mathbf{r}, t) | \hat{e}_{\mathbf{k},\lambda} \cdot \mathbf{r} e^{i\omega_X t} | \psi_q^+(\mathbf{r}, t) \rangle$$

(4)

By assuming the oscillating laser field amplitude $E_0$ to be much less than the interatomic electric field experienced by the electron in the ground state of the hydrogenic ions, $\psi_0(\mathbf{r}, t)$ may be approximated by the field-free ground state of the hydrogenic ion with charge $Z - 1$ and energy $Z^2 I_0$, where $I_0$ denotes the hydrogen ground state energy.

Below, the continuum electron state $\psi_q^+$ will be approximated by the Coulomb-Volkov ansatz $\hat{\psi}_q^+(\mathbf{r}, t) = \chi_{\mathbf{q}}(\mathbf{r}, t) u_q^+(\mathbf{r})$ with

$$\chi_{\mathbf{q}} = \exp\{i[\hat{k}_L(t) \cdot \mathbf{r} - \frac{1}{2} \int dt' |\mathbf{q} + \hat{k}_L(t)|^2] \} \quad (5)$$

$$\hat{k}_L(t)$$ the quiver momentum imparted to the electrons by the intense field, $\hat{k}_L(t) = \hat{\epsilon}_L(E_0/\omega_L) \sin \omega_L t$, and

$$u_q^+ = \exp(1/2\pi \nu) \frac{\Gamma(1-\nu)}{(2\pi)^2} F_1(i\nu, 1, i|\mathbf{q} \cdot \mathbf{r} - qr) \exp(i\mathbf{q} \cdot \mathbf{r})$$

(6)

the field-free outgoing Coulomb wave ($\nu = Z/q$).

Proceeding in the usual way, by using the Fermi golden rule, the differential recombination probability per unit time is obtained as a sum of probabilities of single events in which the emission of an x-ray photon occurs simultaneously to exchange of $n$ laser photons

$$\frac{dP}{d\Omega} = \sum_{n;i=1,2} P_n(\hat{\epsilon}_{\mathbf{k},\lambda})$$

(7)

In Eq. (7) $d\Omega$ is the element of solid angle surrounding the vector $\mathbf{k}$, and $P_n(\hat{\epsilon}_{\mathbf{k},\lambda})$ is the single-channel differential rate of recombination with exchange of $n$ laser photons and emission of an x-ray photon with energy $\omega_X(n)$ and polarization vector $\hat{\epsilon}_{\mathbf{k},\lambda}$ with $\omega_X(n)$ given by Eq. 11

$$P_n(\hat{\epsilon}_{\mathbf{k},\lambda}) = \int_0^{\infty} d\omega \frac{\omega^3}{2\pi c^3} |T_n(\hat{\epsilon}_{\mathbf{k},\lambda})|^2 \delta[\omega - \omega_X(n)]$$

(8)

with $T_n(\hat{\epsilon}_{\mathbf{k},\lambda})$ the corresponding transition amplitude given by
The coordinate system used to calculate the differential recombination rate. The asymptotic electron momentum is directed along $z$; the emitted photon wavevector $k_e$ points into the direction $(\theta, \phi)$; the oscillating electric field, lying in the plane $xz$ makes an angle of $\chi$ with the $z$ axes.

As shown in Fig. 1, the emission direction of the x-ray photon is specified by the angles $\theta$ and $\phi$, while the angle between $\hat{e}_L$ and $\hat{z}$, the unit vector indicating the incoming electron momentum direction ($\hat{q} = \hat{z}q$), is denoted by $\chi$. Without loosing generality, let us denote by $\hat{e}_1(0) \equiv \hat{e}_{k_e, -1}$ the linear polarization vector lying in the plane containing $k_e$ and $\hat{z}$, and by $\hat{e}_2(0) \equiv \hat{e}_{k_e, 2}$ the one lying in the $xy$ plane, so that $(\hat{e}_1(0), \hat{e}_2(0), k_e, [k_e, \hat{z}])$ form a right-handed set of mutually orthogonal unit vectors.

Upon replacement, in Eq. (9), of $\hat{e}_{k_e, \lambda}$ with $\hat{e}_{\pm 1}$ defined by $\hat{e}_{\pm 1} = \mp \hat{e}_1(0) \pm i \hat{e}_2(0)/\sqrt{2}$, the emission rate of an x-ray photon with helicity equal, respectively, to $+1$ or $-1$ is obtained. Below we characterize the polarization state of the emitted photon through the so called Stokes parameter $S_0, S_1, S_2, S_3$. These are defined in terms of four independent set of measurements giving information about the total intensity of the x-ray radiation and the intensity transmitted by a polarizer oriented at particular angles that will be specified below.

The parameters $S_0 \equiv I_r \propto (P_n[\hat{e}_1(0)] + P_n[\hat{e}_2(0)]) = P_n$ is given by the total intensity of the radiation of a given frequency emitted in a particular direction. Usually, the following normalized parameters $\eta_i = S_i/S_0$ $(i = 1, 2, 3)$ are used. $\eta_1 = [P_n[\hat{e}_1(0)] - P_n[\hat{e}_2(0)]]/P_n$ gives the degree of linear polarization with respect to the axes along $\hat{e}_1(0)$ and $\hat{e}_2(0)$. It is obtained by measuring the radiation intensity $I[\hat{e}_1(0)] \propto P_n[\hat{e}_1(0)]$ transmitted by a polarization filter oriented along the direction of $\hat{e}_1(0)$ and $\hat{e}_2(0)$; $\eta_2 = [P_n[\hat{e}_1(\pi/4)] - P_n[\hat{e}_2(\pi/4)]]/P_n$ is the degree of linear polarization with respect to two orthogonal axes rotated by $\pi/4$ with respect to the above ones. The degree of circular polarization is given by $\eta_3 = (I_+ - I_-)/(I_+ + I_-) = \{P_n[\hat{e}_+] - P_n[\hat{e}_-]\}/P_n$ with $I_+ \propto P_n[\hat{e}_+]$ and $I_- \propto P_n[\hat{e}_-]$ the intensities transmitted by polarization filters that transmit photons with positive or negative helicity, respectively. We remark that for the case under study, the radiation is completely polarized and, hence, $\eta_1^2 + \eta_2^2 + \eta_3^2 = 1$.

By using Eq. (9), calculation of energy spectra, angular distributions and degrees of polarization of the emitted photons have been carried out at moderate laser intensities, $I_L = 3 \cdot 10^{13} W/cm^2$, for such incoming electron energies and nuclear charges that the requirements of radiation dipole approximation and non relativistic treatments are satisfied.

In Fig. 2 the effect of the disalignment between $\hat{e}_L$ and $\hat{q}$ on the angular distribution of the emitted x-ray for selected channels characterized by the number of the exchanged assisting-field photons is illustrated by plotting, as a function of $\theta$, the differential recombination rate $P_n[\hat{e}_1(0)]$ evaluated for three different values of the angle $\chi$, keeping fixed at $45^\circ$ the value of the angle $\phi$. The incoming electron energy has been taken equal to 60 eV. The unusual behavior of the differential emission rate as a function of the angle $\theta$, when $\theta = 0$, is remarkable.

FIG. 1: The coordinate system used to calculate the differential recombination rate. The asymptotic electron momentum is directed along $z$; the emitted photon wavevector $k_e$ points into the direction $(\theta, \phi)$; the oscillating electric field, lying in the plane $xz$ makes an angle of $\chi$ with the $z$ axes.

FIG. 2: Differential recombination rate $P_n[\hat{e}_1(0)]$, with exchange of $n$ laser photons, evaluated at three different values of $\chi$, as a function of the angle $\theta$ between $k_e$ and $\hat{q}$. $k_e, \hat{q}$ points in the direction $(\theta, \phi)$ with $\chi$ kept at $45^\circ$. Full line $n = 25$; dashed line $n = 28$.

In the field-free recombination, or when the oscillating assisting electric field is directed along the direction of the incoming electron momentum, the emission of photon along $\hat{q}$ is strictly forbidden. In fact, the electron states involved in the transition are characterized by the same magnetic quantum number $m = 0$, and photon emission along $\hat{q}$ would violate the angular momentum conserva-
tion. Instead, if \( \mathbf{q} \) and \( \mathbf{e}_L \) are not aligned, \( m \) ceases to be a good quantum number, and the above restriction on the emission direction no longer applies.

Moreover, as already mentioned, the lack of cylindrical symmetry affects the polarization states of the emitted photons. This is shown in Fig. 3 where together with the differential emission rate of x-ray photons into different channels characterized by \( n \), for a fixed emission direction, are displayed the degrees of polarization \( \eta_1, \eta_2 \) and \( \eta_3 \) as a function of \( \chi \). We note that, as \( \eta_1^2 + \eta_2^2 + \eta_3^2 = 1 \), the highest degree of circular polarization occurs in proximity of values of \( \chi \) at which both the linear polarization degrees \( \eta_1 \) and \( \eta_2 \) reach their minimum absolute value. The circular polarization degree may be written, by using Eqs. (8,9), as

\[
\eta_3 = \frac{4 \cdot \text{Im} \{ T_n[\epsilon_1(0)] \cdot T_n^*[\epsilon_2(0)] \}}{|T_n[\epsilon_1(0)]|^2 + |T_n^*[\epsilon_2(0)]|^2} = \sin 2\psi
\]  

(10)

with \( \psi \) the ellipticity angle of the emitted radiation.

When \( \mathbf{e}_L \) is parallel to \( \mathbf{q} \), \( T_n[\epsilon_2(0)] \) is zero because the dipole moment transition matrix between states with the same magnetic quantum number has no component along the direction perpendicular to the quantization axis, and \( \eta_2, \eta_3 \) turn out to be zero. Once \( \mathbf{e}_L \) is no longer aligned to \( \mathbf{q} \), a time-dependent dipole moment with components along \( \epsilon_1(0) \) and \( \epsilon_2(0) \) is induced by the driving field. Interference between the components of the transition dipole moment along these two perpendicular directions allows emission of photons whose circular polarization degree depends on \( \chi \) and their propagation direction.

For \( \chi = \pi/2 \), \( \eta_3 \) vanishes. This result may be easily explained by resolving the periodic part of the dipole moment matrix element into terms of its Fourier component. From the invariance of the Hamiltonian under simultaneous reflection in the plane \( xy \) and time translation of \( \pi/\omega_L \), it may be shown that \( T_n[\epsilon_2(0)] \) is zero for processes involving exchanges of even numbers of the laser field assisting photons, while for odd \( n \), the phase difference between \( T_n[\epsilon_1(0)] \) and \( T_n[\epsilon_2(0)] \) is zero. Therefore, according to Eq. (10), \( \eta_3 \) is found to be zero for both odd and even \( n \).

Finally, we observe that by increasing \( \chi \), the coupling of the electron with the assisting radiation field weakens, leading to a decreasing of the number of the laser photons exchanged during the recombination process. Therefore the disalignment of the oscillating field with the incoming electron momentum causes the narrowing of the emission spectra width. For the channels shown in Fig. 3 when \( \chi \) is close to \( \pi/2 \) the differential emission rates become vanishing small, therefore results for \( \chi > 80^\circ \) have not been shown.

By concluding, we remark that, although numerical calculations of the recombination rates have required use of approximate wavefunctions, the major goal of our work has been to show, on symmetry grounds, that in the presence of a linear polarized laser, during the recombination event, elliptically polarized photons are emitted, the ellipticity depending on the disalignment between the electron beam momentum and the oscillating electric field direction. Moreover, due to the presence of the external field, strong modifications to the angular distribution of the emitted photons have been found.

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