NEUTRINO-ANTINEUTRINO ASYMMETRY FROM THE SPACE-TIME NONCOMMUTATIVITY

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A new mechanism having as an origin the space-time non commutativity has been shown to generate anisotropy and axial like interaction giving rise to a leptonic asymmetry for fermionic particles propagating in a curved non commutative FRW universe. As a by product, for ultra relativistic particles like neutrinos, an analytical expression of this asymmetry is derived explicitly. Constraints and bounds from the cosmological parameters are also discussed.

Keywords: Noncommutative Geometry; Modified Theories of Gravity; Leptogenesis.

1. Introduction

Leptogenesis is a very important observation and can affect the present energy density and cosmic microwave background (CMB) etc...[1]. To explain the origin of this lepton number asymmetry, many mechanisms were proposed like the one of Affleck-Dine[2], fermion propagation in a curved space-time[3-4], Lorentz and CPT violating scenarios in the context of Riemannian-Cartan space-time[5-6] etc... On the other hand, the non commutative nature of the space-time has been a subject of a very active research[7-17]. The motivation was that the space-time non commutativity could be significant in the early universe where quantum gravity effects become important and very sensitive to search for signatures in the cosmological observations[18]. Moreover, the observed anisotropies of the CMB may be caused by the non commutativity of space-time geometry[19-20]. The main goal of this paper is to show that a dynamical leptonic (including neutrinos) asymmetry can be generated only by the non commutativity of a curved expanding space-time leading to a new mechanism explaining matter-antimatter asymmetry in the universe. In section 2 we present the mathematical formalism. In section 3 we derive the analytical expression of the neutrino-antineutrino asymmetry. Finally, in section 4 we discuss the numerical results and draw our conclusions.

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2. Mathematical Formalism

Following the general approach of ref. [21], we present a deformed FRW solution in a non commutative (NCG) gauge gravity where the structure of the space-time is affected by the commutation relations:

\[ [\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu} \]  

\( (\hbar = c = 1) \) (1)

where \( \Theta^{\mu\nu} \) are antisymmetric canonical parameters. The NCG gauge fields (spin connection) are denoted by \( \hat{\omega}^{AB}_{\mu} \) and can be expanded in power series of the \( \Theta^{\mu\nu} \) as:

\[ \hat{\omega}^{AB}_{\mu} = \omega^{AB}_{\mu} - i\Theta^{\nu\rho} \omega^{AB}_{\mu\rho} + \Theta^{\nu\rho} \Theta^{\lambda\tau} \omega^{AB}_{\mu\rho\lambda\tau} + ... \]  

where

\[ \omega^{AB}_{\mu\rho} = \frac{1}{4} \{ \omega_{\mu}, \partial_\rho \omega_{\mu} + R_{\rho\mu} \}^{AB} \]  

\( \omega^{AB}_{\mu\rho\lambda\tau} = \frac{1}{32} \left[ \{ \omega_{\lambda}, \partial_\tau \{ \omega_{\mu}, \partial_\rho \omega_{\mu} + R_{\rho\mu} \} \} + 2 \{ \omega_{\nu}, \{ R_{\tau\nu}, R_{\rho\mu} \} \} - \{ \omega_{\nu}, \partial_\rho \omega_{\mu} + R_{\rho\mu} \} , \partial_\tau \omega_{\mu} + R_{\tau\mu} \} + 2 \partial_\rho \omega_{\lambda}, \partial_\rho (\partial_\mu \omega_{\mu} + R_{\tau\mu}) \right]^{AB} \]  

and the covariant derivative \( D_\mu R^{AB}_{\rho\sigma} \) is such that:

\[ D_\mu R^{AB}_{\rho\sigma} = \partial_\mu R^{AB}_{\rho\sigma} + (\omega^{AC}_{\mu} R^{DB}_{\rho\sigma} + \omega^{BC}_{\mu} R^{DA}_{\rho\sigma}) \eta_{CD} \]  

here

\[ R^{AB}_{\mu\nu} = e^{A\rho} e^{B\sigma} R_{\mu\nu\rho\sigma} \]  

where \( e^{A\rho}, R_{\mu\nu\rho\sigma} \) and \( \omega^{AB}_{\mu} \) are the ordinary commutative inverses of the vierbein (tetrad), Riemannian tensor and spin connection respectively (Greek and Latin indices are for curved and flat space-time respectively).

Now, if we assume a vanishing commutative torsion, the components of the NCG tetrad fields \( \hat{e}^A_\mu \) read:

\[ \hat{e}^A_\mu = e^A_\mu - i\Theta^{\nu\rho} e^A_{\mu\rho} + \Theta^{\nu\rho} \Theta^{\lambda\tau} e^A_{\mu\rho\lambda\tau} + .... \]  

where

\[ e^{A\rho}_{\mu\nu} = \frac{1}{4} \left\{ \omega^{AC}_{\nu} \partial_\rho e^D_{\mu} + (\partial_\rho \omega^{AC}_{\mu} + R^{AC}_{\rho\mu}) e^D_{\nu} \right\} \eta_{CD} \]  

and
proper distances \( r \) between co-moving points increase proportionally to the scale factor, give the following non-zero components of the tetrad are in general complex (see Appendix Appendix B). Similarly, direct calculations using a Maple package, and writing \( \Theta_{\mu\nu} \) up to the \( \Theta^2 \) as it is shown in Appendix Appendix B.

Notice that contrary to the ordinary commutative case, \( \hat{\Theta}_{\mu\nu}^{AB} \) is in general a complex and not completely antisymmetric with respect to \( A \) and \( B \).
3. NCG Neutrino-Antineutrino Asymmetry

In an NCG curved space-time, the generalized Dirac Lagrangian density \( \mathcal{L} \) is assumed to have the form \([7], [14]\):

\[
\mathcal{L} = \sqrt{-\hat{g}} \Psi \, \left( -i \gamma^\mu \hat{D}_\mu - m \right) \Psi + c.c
\]

(14)

where the NCG covariant derivative \( \hat{D}_\mu \) is given by:

\[
\hat{D}_\mu = \partial_\mu - \frac{i}{4} \hat{e}^A_\mu \Sigma_{AB}
\]

(15)

with

\[
\gamma^\mu = \hat{e}^\mu_a \gamma^a, \quad \Sigma_{AB} = \frac{i}{2} [\gamma_A, \gamma_B], \quad \Sigma(AB) = \frac{1}{2} \{\gamma_A, \gamma_B\} = \eta_{AB}
\]

(16)

\((\gamma^i_s\) are the Dirac Gamma matrices in the flat Minkowski space-time and \( \hat{\Psi} \) the NCG 4-components Dirac spinor). Here “c.c” stands for complex conjugate and \( \hat{e}^\mu_a \) the inverse of NCG vierbein. Using the fact that

\[
\frac{1}{2} \left( \gamma_d \Sigma_{[AB]} + \Sigma_{[AB]} \gamma_d \right) = \varepsilon_{fabc} \gamma^c \gamma^b
\]

(17)

and

\[
\frac{i}{2} \left( \gamma_d \Sigma_{[AB]} - \Sigma_{[AB]} \gamma_d \right) = g_{db} \gamma_a - g_{da} \gamma_b
\]

(18)

as well as the NCG orthogonality relation

\[
\hat{e}^\mu_a \star \hat{e}_{\mu b} = \delta_{ab}
\]

(19)

where \( \varepsilon_{fabc} \) is the 4-rank totally antisymmetric tensor, one can show that the corresponding NCG Dirac equation takes the form:

\[
\left[ \gamma^f \left( i \partial_f + A_f \right) + \gamma^f \gamma^5 \bar{B}_f \right] \Psi = 0
\]

(20)

where

\[
\partial_f = \hat{e}^\mu_f \partial_\mu
\]

(21)

\[
A^f = 3(\hat{e}^af \sum_{a=1}^{4} \omega^{aa}_\mu) + \Re \left( \hat{e}^\mu_d \left( \hat{e}^{df}_\mu - \omega^{df}_\mu \right) \right) + O(\Theta^3)
\]

(22)

\[
B_f = \left[ 3 (\hat{e}^{ad} \Sigma_{\mu}^b) + \frac{1}{4} \Theta^{\rho\sigma} \Theta^{\alpha\beta} \left( \partial_\rho \partial_\alpha e^{\mu d} \right) \left( \partial_\sigma \partial_\beta \omega^{\mu b}_\mu \right) \right] \varepsilon_{fabc} + O(\Theta^3)
\]

(23)

one can show (as in ref. [27]) that the dispersion relations for the left and right chiral fields (particles \( X \) and antiparticles \( \overline{X} \)) read:

\[
E_X = \sqrt{(\overline{P} - \overline{\Lambda}_X)^2 + m^2 + \Lambda^X_4}
\]

(24)

and

\[
E_{\overline{X}} = \sqrt{(\overline{P} - \overline{\Lambda}_\overline{X})^2 + m^2 + \Lambda^{\overline{X}}_4}
\]

(25)
\[ \Lambda_X = -B + A, \quad \Lambda_{\overline{X}} = -B + A \]  
(26)

and

\[ A_X^4 = B_4 + A_4, \quad A_{\overline{X}}^4 = -B_4 + A_4 \]  
(27)

In fact, this result was expected because of \( CPT \) violation which affects the dispersion relations of particles and antiparticles leading to a difference between their energies. In our case, the violation of \( CPT \) can be induced by:

1) Curvature in a certain non trivial anisotropic \( NCG \) geometry. In fact, fermions in a curved space-time can interact via an axial vector current due to their spin with space-time curvature or torsion [27]. Therefore, the space-time curvature background has the effect of inducing an axial vector field especially in certain anisotropic space-time geometries like in \( NCG \).

2) \( NCG \) : since it violates Lorentz invariance leading to a non conservation of \( CPT \).

Thus, in our case the violation of \( CPT \) is induced by both the curvature and non commutativity of the space-time.

Now, in order to derive the neutrino-antineutrino asymmetry in the context of \( NCG \), one has to have a thermal equilibrium background coming after the neutrino decoupling. In fact, the synthetics of light elements depends strongly on the ratio of number of neutrinos / number of protons freezout abundance determined by the interplay between the weak interaction and expansion rate of the universe. Both are influenced by the neutrino decoupling temperature. The neutrino decoupling means that the neutrinos do not interact with baryonic matter and consequently do not influence the dynamics of the universe at early stage. The decoupling happens when the weak interaction rate of neutrinos is smaller than the expansion rate of the universe (\( kT_{\text{decoup}} \approx 1\text{MeV}, t_{\text{decoup}} \approx 1\text{seconde} \)). After decoupling, a thermal equilibrium background of relativistic neutrinos is expected with an effective temperature at late time \( T_{\nu} \approx 0.71T_{\gamma} \). Now, at this equilibrium temperature and using the Fermi-Dirac distribution together with the result:

\[ \int_0^\infty dx \frac{x^m}{1 + z^{-1}e^x} = z\Gamma(m + 1) \Phi(-z, m + 1, 1) \]  
(28)

where \( \Gamma(x) \) is the Euler Gamma function and \( \Phi(z, s, a) \) the Lerch transcendent function given by

\[ \Phi(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{(n + a)^s} \]  
(29)

the analytical expression of the difference between the neutrino and antineutrino \( NCG \) number density \( \Delta n_{NCG} \) is given by (in the system where the Boltzman con-
stant $k = 1$):

$$\Delta n_{NCG} \approx \frac{(T_\nu)^3}{2\pi^2} \sum_{i=1}^{\infty} \frac{(-1)^i}{i^3} \sinh \left( \frac{i\tilde{B}_4}{T_\nu} \right)$$

(30)

where $\tilde{B}_4$ has as an expression

$$\tilde{B}_4 = \Theta \frac{t_0^2}{\lambda} \left[ \frac{\hat{t} - \beta - 1}{2\sin \theta} - \frac{1}{4} \beta^2 \sin \theta \hat{t}^2 \beta^{-2} \right]$$

(31)

here $t_0$ stands for the decoupling time $t_{decomp}$.

Notice that since $\tilde{B}_4$ is proportional to $\Theta$, as a first approximation, eq. (30) reduces to

$$\Delta n_{NCG} \approx \frac{(T_\nu)^3}{2\pi^2} \frac{\tilde{B}_4}{\zeta(2)}$$

(32)

where $\zeta(2)$ is the Dirichlet zeta function. Now, using the CMB photon number density $n_\gamma$, the ratio $R_1 = \frac{\Delta n_{NCG}}{n_\gamma}$ reads:

$$R_1 = \frac{\pi^2}{12\zeta(3)} \left( \frac{\tilde{B}_4}{T_\nu} \right) \left( \frac{T_\nu}{T_\gamma} \right)^3$$

(33)

where $T_\gamma$ is the CMB photon temperature. Now, it is clear from eq. (33) that contrary to the commutative FRW universe (with isotropy, homogeneity and space-time spherical symmetry) where there is no geometrical contribution to the neutrino-antineutrino asymmetry $\Delta n$, the NCG space-time has generated anisotropy, non homogeneity and Broken spherical symmetry leading to a dynamical non vanishing net asymmetry $\Delta n_{NCG}$ which has a non commutative geometrical origin.

4. Discussions and Conclusions

We have found that if neutrinos are propagating in a curved NCG universe, where a space-time anisotropy as well as Lorentz violation invariance are generated, a net asymmetry between neutrinos and antineutrinos arises at the thermodynamical equilibrium. Thus, we have shown that a new source of a leptonic and antileptonic asymmetry which has as an origin the noncommutative geometrical structure of space-time can be generated leading to a new mechanism explaining matter-antimatter asymmetry. It is worth to mention that in the standard cosmology at present time, the ratio $R_2 = \frac{\Delta n}{n_\gamma}$ is given by $[1, 28]

$$R_2 = \frac{\pi^2}{12\zeta(3)} \left( \frac{T_\nu}{T_\gamma} \right)^3 \left( \frac{\mu_\nu}{T_\nu} \right)$$

(34)

$\mu_\nu$ is the neutrino chemical potential. At the decoupling time where $t_0 \approx 1s$, $\frac{T_\nu}{T_\gamma} \approx 0.71$ and $\left| \frac{\mu_\nu}{T_\nu} \right| \lesssim 0.04$, the ratio $R_2 \lesssim 9 \times 10^{-3}$. Now, if we consider the $\theta$-averaged
\[ \langle \hat{B}_4 \rangle_0 \left( -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right) \text{ at present time with the leading term } \sim \hat{t}^{2\beta - 2}, \text{ } R_1 \text{ takes the simple form:} \]

\[
R_1 \left|_{t=t_0} \approx \frac{\pi^2}{12\zeta(3)} \left[ -\frac{\bar{\Theta}}{4t_0^2} \beta^2 \frac{\hat{t}^{2\beta - 2}}{T_\nu} \right] \left( \frac{T_\nu}{T_\gamma} \right)^3 \right. , \tag{35}
\]

here \( \hat{t} = \frac{t_{ob}}{\bar{t}} \), using the fact that \( kT_\nu \approx 2 \times 10^{-23} J \) and \( t_{ob} \approx 0.3 \times 10^{18} s \), one gets:

\[
|R| = \left| \frac{R_1}{R_2} \right|_{t=t_{ob}} \approx \frac{\bar{\Theta} \beta^2}{12.8} \times 10^{-45} \left( 3 \times 10^{17} \right)^{2\beta - 2} J \tag{36}
\]

which means that if \( |R| \sim O(1) \), then \( \Lambda \sim \frac{\bar{\Theta} \beta^2}{20.5} \times 10^{-38} \left( 3 \times 10^{17} \right)^{2\beta - 2} \) TeV.

Notice that the NCG scale \( \Lambda \) and \( \beta = \frac{2}{3(1+\omega)} \) are intimately related. Furthermore, for an accelerated expansion of the universe one has \( \beta > 1 \) and therefore: \( \Lambda \gtrsim \frac{\bar{\Theta}}{20.5} \times 10^{-38} \) TeV. This is a new lower bound of the non commutative scale \( \Lambda \) (\( \Theta < 1 \)).

Fig. 1 shows the ratio \( R \) as a function of the NCG parameter \( \frac{\Lambda}{\bar{\Theta}} \) (denoted by Lambda and the reduced time in a TeV unit), the reduced time \( \hat{t} \) (denoted by tilde, \( \hat{t} = 10^{17} \bar{t} \)) for a fixed \( \beta \approx 2.38 \) or equivalently the equation of state parameter \( (EOS) \) parameter \( \omega \approx -0.75 \). Notice that \( R \) is an increasing function of time for a fixed \( \frac{\Lambda}{\bar{\Theta}} \). Moreover, for smaller values of \( \frac{\Lambda}{\bar{\Theta}} \) of \( O(1 TeV) \), \( R \sim 10 \) when \( \hat{t} \sim 2t_{ob} \) (\( t_{ob} \) is the observed time). This shows that \( R \) becomes important and NCG effects dominate in comparison to the one of the standard cosmology. It is worth to notice that if the NCG effects become relevant \( (R > 1) \), one gets an upper limit for \( \frac{\Lambda}{\bar{\Theta}} \) of \( O(1 TeV) \).

Fig. 2 displays the ratio \( R \) as a function of the \( \beta \) parameter (denoted by beta) and reduced time \( \hat{t} \) for a fixed NCG parameter \( \frac{\Lambda}{\bar{\Theta}} \sim 1 \) TeV. Notice that the ratio \( R \) is very sensitive to the variations of \( \beta \); e.g. if \( \beta \sim 2.35 - 2.4 \), and \( \hat{t} \sim 1 - 2 \), the ratio \( R \sim 1 - 50 \).
Fig. 2. Ratio $R$ as a function of $\beta$ and $\tilde{t}$ for $\Lambda \approx 1$ TeV.

Fig. 3. Ratio $R$ as a function of $\Lambda \Theta$ and $\beta$ at present time.

Fig. 3 shows the ratio $R$ as a function of $\beta$ and $\Lambda \Theta$ at the present time. For $\Lambda \Theta \sim 4$ TeV and $\beta \sim 2$, the ratio $R$ reaches the value 1.8.

Fig. 4 displays the contour plots $(\Lambda(TeV), \beta)$ for a fixed $R$ at the present time. Notice that for a fixed $\beta$ or $\omega$ as $R$ increases; the value of the NCG parameter $\Lambda$ decreases and becomes relevant at a TeV scale. As an important and novel results, the NCG $\Lambda$ parameter is strongly related to the neutrino-antineutrino asymmetry which has as an origin the space-time NCG structure, and the EOS parameter $\omega$ (or equivalently $\beta$). Moreover, this asymmetry is dynamical in the sense that it is a time dependent quantity. It increases with time for an accelerated expansion of the universe. Finally, as a conclusion a new mechanism from the non commutativity of space-time generating the matter-antimatter asymmetry was found.

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Appendix A. Appendix

From Friedman’s equations:

\[2 \frac{\ddot{a}}{a} + \chi \left( P + \frac{1}{3} \rho \right) = 0 \quad (A.1)\]

and

\[2 \frac{\ddot{a}}{a} + \chi (P + \rho) - 2 \frac{a^2}{\dot{a}} = 0 \quad (A.2)\]

(\(\rho\) and \(P\) are the matter density and pressure respectively), one can get the following continuity equation (in what follows, we use the system where \(\chi = 1\) and “.” stands for time derivative)

\[\dot{\rho} + 3 \frac{\dot{a}}{a} (P + \rho) = 0 \quad (A.3)\]

leading to:

\[\frac{d}{dt} (\rho a^3) + \omega \rho \frac{da^3}{dt} = 0 \quad (A.4)\]

for a perfect fluid where \(P = \omega \rho\). One can show that the solution of eq. (A.4) is of the form \(\rho = D a^{-3(1 + \omega)}\). Moreover, from eqs. (A.1) and (A.2), one can obtain:

\[\dot{a}^2 = \rho \frac{a^2}{3} \quad (A.5)\]

consequently,

\[a(t) = \left( \frac{3}{2} \right) \left( 1 + \omega \right)^{\frac{-2}{1 + \omega}} \left( \frac{\chi D}{3} \right)^{\frac{1}{1 + \omega}} t^{\frac{2}{1 + \omega}} \quad (A.6)\]
Now, it is clear that:

$$\beta = \frac{2}{3(1 + \omega)}$$  \hspace{1cm} (A.7)

$$D = \rho_0 a_0^{3(1+\omega)}$$  \hspace{1cm} (A.8)

$$t_0 = \beta \left( \frac{3}{\chi \rho_0} \right)^{\frac{1}{2}}$$  \hspace{1cm} (A.9)

**Appendix B. Appendix**

Starting from eqs. (2), (8) and (11) and using maple 18 package, one obtains the following complex NCG vierbeins, the metric components and the spin connection components respectively; (We set : $\hbar = \frac{\rho_0}{t_0}$ and $\tilde{\Theta} = \frac{\Theta}{t_0}$), in what follow, the upper (respectively lower) indices stand for flat (respectively curved) space-time,

$$\hat{e}_1 = \hat{e}_4 = \frac{\bar{\beta}}{128} \left[ 128 + \frac{\tilde{\Theta}_0^2}{12} \hat{\tau}^{2\hat{\tau}^3} \beta^2 (\beta + 1) \hat{\tau}^{2\hat{\tau}^3} h^2 \right]$$  \hspace{1cm} (B.1)

$$\hat{e}_2 = -\frac{3}{256} \beta^2 \hat{r} \hat{t} \beta \left( 4 - \hat{t}^2 \beta - 2 \beta^2 h^2 \right)$$  \hspace{1cm} (B.2)

$$\hat{e}_3 = -\frac{i}{2} \hat{r} \hat{t} \beta$$  \hspace{1cm} (B.3)

$$\hat{e}_4 = -\frac{1}{8} \Theta^2 \beta \left( 1 - 1 \right) \hat{r} \hat{t} \beta^3$$  \hspace{1cm} (B.4)

$$\hat{e}_1 = -\frac{7}{64} \beta^2 \hat{r} \hat{t} \beta^2 (\beta + 1) \hat{r} \hat{t} \beta^3 h^3$$  \hspace{1cm} (B.5)

$$\hat{e}_2 = -\frac{1}{128} \hat{r} \hat{t} \beta \left\{ -128 + \tilde{\Theta}_0^2 \beta^2 \left[ \frac{\beta^2 \hat{r} \hat{t} \beta^2 - 1 h^2}{-17 \beta^2 \hat{t} \beta^3 - 4 h^2} \sin^2 \theta - \tilde{\beta} \left( 2 \sin^2 \theta + 1 \right) \right] \right\}$$  \hspace{1cm} (B.6)

$$\hat{e}_3 = -\frac{i}{4} \hat{r} \hat{t} \beta \left[ \cos 2\theta - \beta^2 \hat{t} \beta^2 h^2 \sin^2 \theta \right]$$  \hspace{1cm} (B.7)

$$\hat{e}_4 = -\frac{7}{64} \tilde{\Theta}^2 \beta^2 (\beta - 1) \hat{t} \beta^3 h^3 \sin 2\theta$$  \hspace{1cm} (B.8)

$$\hat{e}_1 = \hat{e}_4 = 0$$  \hspace{1cm} (B.9)

$$\hat{e}_2 = -\frac{i}{4} \hat{r} \beta \hat{r} \beta \hat{t} \beta^3 h^2 \sin \theta$$  \hspace{1cm} (B.10)
\[
\bar{c}_3 = \frac{1}{128} r_0 \hat{\Theta}^3 \sin \theta \left\{ 128 + \hat{\Theta}^2 \left[ -\beta^2 \hat{r}^2 \hat{t}^2 \hat{\beta}^3 - 2 \hat{h}^2 (5 + 16 \cos^2 \theta) \right] + 17 \beta^4 \hat{r}^4 \hat{t}^4 \hat{\beta}^3 - 4 \hat{h}^4 \sin^2 \theta + 2 \right\} \] (B.11)

\[
\bar{c}_4 = \frac{1}{128} \hat{\Theta}^2 \beta \hat{r}^2 \hat{t}^2 \hat{\beta} \hat{h} \left[ 5 + \sin^2 \theta + 22 \beta^2 \hat{r}^2 \hat{t}^2 \hat{\beta}^3 - 2 \hat{h}^2 \right] \] (B.12)

\[
\bar{c}_2 = -\frac{3}{256} \beta \hat{\Theta}^2 \hat{r}^2 r_0 \hat{h} \hat{t}^2 \hat{\beta} - 1 \left[ -4 + \beta^2 \hat{r}^2 \hat{t}^2 \hat{\beta}^3 - 2 \hat{h}^2 \right] \] (B.13)

\[
\bar{c}_4 = \frac{1}{4} \hat{\Theta}^2 r_0 \beta \hat{h} \hat{t}^2 \hat{\beta} - 1 \sin 2 \theta \] (B.14)

\[
\bar{c}_4 = -\frac{1}{64} \left\{ -64 + \hat{\Theta}^2 \left[ -18 \beta^3 (\beta - 1) \hat{t}^4 \hat{\beta}^3 - 4 \hat{h}^4 \sin^2 \theta \right] \right\} \] (B.15)

\[ \bar{g}_{\mu \nu} \text{ components:} \]

\[
\bar{g}_{11} = \frac{\hat{\Theta}^2}{64} \left\{ 64 + \hat{\Theta}^2 \hat{t}^2 \hat{r}^2 \hat{h}^2 [14 (\beta^2 \hat{t}^2 \hat{r}^2 \hat{h}^2 - 1) \sin^2 \theta - 3] \right\} \] (B.16)

\[
\bar{g}_{12} = -\frac{\hat{\Theta}^2}{256} r_0 \hat{t}^2 \hat{r} \sin 2 \theta [12 + 25 \beta^2 \hat{t}^2 \hat{r}^2 \hat{h}^2] \] (B.17)

\[
\bar{g}_{13} = \bar{g}_{23} = \bar{g}_{31} = \bar{g}_{32} = \bar{g}_{34} = \bar{g}_{43} = 0 \] (B.18)

\[
\bar{g}_{14} = \frac{2}{257} \hat{\Theta}^2 \beta \hat{r}^2 \hat{t}^2 \hat{\beta} - 1 \hat{h} [2 \beta (19 \beta + 8) \hat{t}^2 \hat{r}^2 \hat{h}^2 \sin^2 \theta - 10 \cos 2 \theta] \] (B.19)

\[
\bar{g}_{21} = -\frac{\hat{\Theta}^2}{64} r_0 \hat{t}^2 \hat{r} \sin 2 \theta [12 + 25 \beta^2 \hat{t}^2 \hat{r}^2 \hat{h}^2] \] (B.20)

\[
\bar{g}_{22} = \frac{1}{64} \hat{r}^2 r_0 \hat{t}^2 \hat{r} \left\{ 64 + \hat{\Theta}^2 \left[ 12 \sin^2 \theta + 6 + 21 \beta^4 \hat{r}^4 \hat{t}^4 \hat{\beta}^3 - 4 \hat{h}^4 \left( 7 \sin^2 \theta - 18 \cos^2 \theta \right) \beta^2 \hat{r}^2 \hat{t}^2 \hat{\beta}^2 - 2 \right] \right\} \] (B.21)

\[
\bar{g}_{24} = -\frac{1}{64} \beta^2 r_0 h^3 \hat{r}^4 \hat{t}^4 \hat{\beta}^3 - 3 \sin^2 \theta [25 \beta - 28] \] (B.22)

\[
\bar{g}_{33} = \hat{r}^2 r_0 \hat{t}^2 \hat{r} (1 + \hat{\Theta}^2 \sin^2 \theta) \] (B.23)

\[
\bar{g}_{41} = -\frac{1}{64} \hat{\Theta}^2 \beta \hat{r}^2 \hat{t}^2 \hat{\beta} - 1 \hat{h} \left[ (19 \beta \sin^2 \theta - 8) \hat{r}^2 \hat{t}^2 \hat{r}^2 \beta \hat{r}^2 - 8 \sin^2 \theta \right] \] (B.24)
Spin connection components at order of $\Theta$:

\[
\begin{align*}
\hat{\omega}_i &= \frac{\Theta^2}{64} \beta^2 r_0^4 \beta^3 h^3 [25\beta - 28] \\
\hat{\omega}_{ij} &= \frac{1}{32} \left\{-32 + \Theta^2 h^2 \beta^2 \gamma^2 \beta^2 r^2 \left[ \frac{-18\beta^2 (\beta - 1) \gamma^2 - 2h^2 r^2 \sin^2 \theta}{(\beta - 1)(6\cos^2 \theta - 7)} \right] \right\}
\end{align*}
\]

Spin connection components at order of $\Theta$:

\[
\begin{align*}
\hat{\omega}_1^{12} &= \hat{\omega}_1^{13} = \hat{\omega}_1^{42} = \hat{\omega}_1^{43} = \hat{\omega}_1^{31} = \hat{\omega}_1^{42} = \hat{\omega}_4^{42} = 0 & (B.27) \\
\hat{\omega}_1^{11} &\sim \hat{\omega}_2^{21} \sim \hat{\omega}_1^{22} \sim \hat{\omega}_1^{24} \sim \hat{\omega}_1^{33} \sim \hat{\omega}_1^{44} \sim \hat{\omega}_2^{11} \sim \hat{\omega}_2^{22} \sim \hat{\omega}_2^{33} \sim \hat{\omega}_2^{44} & (B.28) \\
&\sim \hat{\omega}_4^{14} \sim \hat{\omega}_4^{11} \sim \hat{\omega}_4^{21} \sim \hat{\omega}_4^{22} \sim \hat{\omega}_4^{34} \sim \hat{\omega}_4^{31} \sim \hat{\omega}_4^{41} \sim \hat{\omega}_4^{44} \sim O \left( \Theta^2 \right) & (B.29)
\end{align*}
\]
\[ \hat{\omega}^{43}_2 = -\frac{i\widehat{\Theta}}{4} \beta hr \tilde{r}^{\beta-1} (1 + \tilde{r}^{2\beta-2} \beta^2 h^2 r^2) + O \left( \widehat{\Theta}^3 \right) \] (B.39)

\[ \hat{\omega}^{11}_3 = \frac{i}{8} \widehat{\Theta} \sin 2\theta + O \left( \widehat{\Theta}^3 \right) \] (B.40)

\[ \hat{\omega}^{12}_3 = \frac{i\widehat{\Theta}}{4} (\cos^2 \theta - \tilde{r}^{2\beta-2} \beta^2 h^2 r^2 \sin^2 \theta) + O \left( \widehat{\Theta}^3 \right) \] (B.41)

\[ \hat{\omega}^{13}_3 = -\hat{\omega}^{31}_3 = - \sin \theta + O \left( \widehat{\Theta}^2 \right) \] (B.42)

\[ \hat{\omega}^{14}_3 = \hat{\omega}^{41}_3 = -\frac{i}{8} \widehat{\Theta} \beta hr \tilde{r}^{\beta-1} \sin 2\theta + O \left( \widehat{\Theta}^3 \right) \] (B.43)

\[ \hat{\omega}^{21}_3 = -\frac{i\widehat{\Theta}}{4} (1 + 2\tilde{r}^{2\beta-2} \beta^2 h^2 r^2)^2 + O \left( \widehat{\Theta}^3 \right) \] (B.44)

\[ \hat{\omega}^{22}_3 = -\frac{i\widehat{\Theta}}{8} (1 + 3\tilde{r}^{2\beta-2} \beta^2 h^2 r^2)^2 + O \left( \widehat{\Theta}^3 \right) \] (B.45)

\[ \hat{\omega}^{23}_3 = -\hat{\omega}^{32}_3 = - \cos \theta + O \left( \widehat{\Theta}^2 \right) \] (B.46)

\[ \hat{\omega}^{24}_3 = \frac{i\widehat{\Theta}}{4} \beta hr \tilde{r}^{\beta-1} (1 + 2\tilde{r}^{2\beta-2} \beta^2 h^2 r^2)^2 + O \left( \widehat{\Theta}^3 \right) \] (B.47)

\[ \hat{\omega}^{33}_3 = \frac{i\widehat{\Theta}}{2} \beta^2 h^2 r^2 \tilde{r}^{2\beta-2} + O \left( \widehat{\Theta}^3 \right) \] (B.48)

\[ \hat{\omega}^{34}_3 = -\hat{\omega}^{43}_3 = -\beta hr \tilde{r}^{\beta-1} \sin \theta + O \left( \widehat{\Theta}^2 \right) \] (B.49)

\[ \hat{\omega}^{42}_3 = -\frac{i\widehat{\Theta}}{4} \beta hr \tilde{r}^{\beta-1} \left[ \cos^2 \theta - \tilde{r}^{2\beta-2} \beta^2 h^2 r^2 \sin^2 \theta \right] + O \left( \widehat{\Theta}^3 \right) \] (B.50)

\[ \hat{\omega}^{44}_3 = \frac{i\widehat{\Theta}}{8} \beta^2 h^2 r^2 \tilde{r}^{2\beta-2} + O \left( \widehat{\Theta}^3 \right) \] (B.51)

\[ \hat{\omega}^{33}_4 = -\hat{\omega}^{32}_4 = -\frac{i}{2} \beta^2 (\beta - 1) \frac{\tilde{r}^2}{r_0} \beta^2 h^2 \tilde{r}^{2\beta-3} \sin \theta + O \left( \widehat{\Theta}^3 \right) \] (B.52)
\[ \hat{\omega}_{43}^{34} = -\frac{i}{2} \beta (\beta - 1) \frac{\tilde{p}}{t_0} \hbar \beta^{\beta - 2} \cos \theta + O(\tilde{\Theta}^3) \quad (B.53) \]

\[ \hat{\omega}_{4}^{43} = 0. \quad (B.54) \]

References

1. J. Legourgues and S. Pastor, Cosmological implications of a relic neutrino asymmetry, *Phys. Rev. D* 60, 103521 (1999); S. Hannestad, New constraints on neutrino physics from BOOMERANG data, *Phys. Rev. Lett.* 85, 4203-4206 (2000); J. Kneller, R. J. Sherrer, G. Steigman and T. P. Walker, How does the cosmic microwave background plus big bang nucleosynthesis constrain new physics?, *Phys. Rev. D* 64, 123506 (2001); S. H. Hansen, G. Mangano, A. Melchiorri, G. Miele and O. Pisanti, Constraining neutrino physics with big bang nucleosynthesis and cosmic microwave background radiation, *Phys. Rev. D* 65, 023511 (2001); A. D. Dolgov, Neutrinos in cosmology, *Phys. Rep.* 370, 333-535 (2002).

2. I. Affleck and M. Dine, A new mechanism for baryogenesis, *Nucl. Phys. B* 249, 361-380 (1985).

3. S. Chandrasekhar, The solution of Dirac’s equation in Kerr geometry, *Proc. Roy. Soc. Lond. A* 349, 571-575 (1976); D. Chouldhary, N. D. Hari Dass, and M. V. N. Murthy, Gravitational helicity flip for massive neutrinos and SN 1987 A, *Class. Quantum Grav.* 6, L167-L171 (1989).

4. B. Mukhopadhyay, Behaviour of spin-1/2 particle around a charged black hole, *Class. Quantum Grav.* 17, 2017-2026 (2000); P. Singh and B. Mukhopadhyay, Gravitationally induced neutrino asymmetry, *Mod. Phys. Lett. A* 18, 779-785 (2003); B. Mukhopadhyay, Neutrino asymmetry around black holes: neutrinos interact with gravity, *Mod. Phys. Lett. A* 20, 2145-2155 (2005).

5. V. A. Kostelecký, Lorentz and CPT violation in the neutrino sector, *Phys. Rev. D* 69, 105009 (2004).

6. V. A. Kostelecký, and M. Mewes, *Phys. Rev. D* 70, 031902 (2004).

7. N. Mebarki, L. Khodja and S. Zaim, On the noncommutative space-time Bianchi I universe and particles pair creation process *Electron. J. Theor. Phys.* 7, 181-196 (2010).

8. N. Mebarki, "Dark Energy, Induced Cosmological Constant and Matter Asymmetry from Non Commutative Geometry" in Third International Meeting on Frontiers of Physics, edited by Suee-Ping Chia et al, AIP Conference Proceedings 1150, American Institute of Physics, Kuala Lumpur, Malaysia , pp. 38-42, (2009).

9. N. Mebarki, "Dark Dynamical Cosmology in a Noncommutative Geometry" In. 4th International Workshop On The Dark Side Of Universe, edited by S. Khalil, AIP Conference Proceedings 1115, American Institute of Physics, Cairo, Egypt, pp. 248-253, (2008).

10. N. Mebarki, F. Khelili and J. Mimouni, Extended non symmetric gravitation theory with a scalar field in non commutative geometry, *Electron. J. Theor. Phys.* 3, 55-69 (2006).

11. N. Mebarki, F. Khelili and J. Mimouni, Discrete groups approach to non symmetric gravitation theory, *Electron. J. Theor. Phys.* 4, 51-60 (2007).

12. N. Mebarki, F. Khelili, Noncommutative geometry and modified gravity, *Electron. J. Theor. Phys.* 5, 65-68 (2008).
13. N. Mebarki, S. Zaim, L. Khodja and H. Aissaoui, Gauge gravity in noncommutative de Sitter space and pair creation, Phys. Scripta 78, 045101 (2008).
14. F. Khelili, J. Mimouni and N. Mebarki, Nonsymmetric gravitation theory in noncommutative geometry, J. Math. Phys. 42, 3615-3627 (2001).
15. N. Mebarki, F. Khelili, S. Kalli and M. Haouchine, Matter-Noncommutative Space-Time Induced Gravity and a Possible Solution to the Cosmological Constant Problem, Chin. J. Phys. 44, 180-188 (2006).
16. N. Mebarki, F. Khelili, H. Bouhalouf and O. Mebarki, New Seiberg-Witten fields maps through Weyl symmetrization and the pure geometric extension of the standard model, Electron. J. Theor. Phys. 6, 193-210 (2009).
17. N. Seiberg and E. Witten, String theory and noncommutative geometry, JHEP 9909:032 (1999).
18. T. Eguchi, P. B. Gilkey and A. Hanson, Gravitation, gauge theories and differential geometry, Phys. Report. 66, 213-393 (1980).
19. F. W. Hehl, J. D. McCrea, E. W. Mielke and Y. Ne’eman, Metric-affine gauge theory of gravity: field equations, Noether identities, world spinors, and breaking of dilation invariance, Phys. Report. 258, 1-171 (1995).
20. S. Tsujikawa, R. Maartens and R. Brandenberger, Non-commutative inflation and the CMB, Phys. Lett. B 574, 141-148 (2003).
21. A. H. Chamseddine, Deforming Einstein’s gravity, Phys. Lett. B 504, 33-37 (2001).
22. M. Chaichian, M. R. Setar, A. Tureanu, G. Zet, On black holes and cosmological constant in noncommutative gauge theory of gravity, JHEP 0804, 064 (2008).
23. M. Chaichian, A. Tureanu, G. Zet, Corrections to Schwarzschild solution in noncommutative gauge theory of gravity, Phys. Lett. B 660, 573-578 (2008).
24. J. Gomis, T. Mehen, Space–time noncommutative field theories and unitarity, Nucl. Phys. B 591, 265-276 (2000).
25. C. Rim, J. Hyung, Yee, Unitarity in space–time noncommutative field theories, Phys. Lett. B 574, 111-120 (2003).
26. D. Bahns. S. Doplicher, K. Fredenhagen, G. Piacitelli, On the unitarity problem in space-time noncommutative theories, Phys. Lett. B 533,178-181 (2002).
27. U. Debnath, B. Mukhopadhyay and N. Dadhich, Spacetime curvature coupling of spinors in early universe: neutrino asymmetry and a possible source of baryogenesis, Mod. Phys. Lett. A 21, 399-408 (2006).
28. J. Lesgourgues, S. Pastor, Massive neutrinos and cosmology, Phys. Rept. 429, 307-379 (2006).