Dynamic voltage stability of unbalanced distribution system with high penetration of single-phase PV units

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Abstract: Dynamic voltage instability (DVI) issues are the primary concern in low-voltage distribution network (DN) due to growing integration of low-inertia compressor motor loads such as air-conditioner and refrigerator. The concern of DVI is likely to increase owing to high penetration of rooftop type single-phase photovoltaic (PV) units in DN. On the other hand, DNs are inherently unbalanced as a result of load and line characteristics along with unbalanced PV penetration. This paper examines the impact of imbalance on the dynamic voltage stability (DVS) in DN and provides solutions to mitigate any adverse effects. Dynamic models of the single-phase PV units are developed and used in the paper. The degree of unbalanced is defined first, and then its impact on the DVS is investigated. From the investigation, it is observed that degree of instability is increased with the increment of imbalance. The paper has also proposed a mitigation strategy i.e. reactive power injection by PV inverter. Case studies are conducted on modified IEEE 4 bus system which represents a low-voltage DN. Results reveal that reactive power injection by PV inverter can improve the DVS by mitigating the impact of unbalance.

1 Introduction

The need for clean and affordable energy due to the concern of climate change and increasing energy demand has been leading the power system towards renewable. Among the renewable energy sources, PV systems are recognised as one of the most important energy sources due to their availability and low maintenance cost. Although the installation cost of PV units is still high, overall cost can be reduced through large-scale production. According to the recently published International Energy Authority (IEA) report, worldwide installations of PV systems have surpassed a landmark of 300 GW at the end of 2016 as illustrated in Fig. 1 [1]. It may be noted that a large portion of PV power are likely to be provided by single-phase rooftop type PV units, which are mainly being integrated into low-voltage DN [2]. With the increasing penetration of single-phase PV units, among many other concerns associated with DN, voltage instability issues are expected to be receiving of significant interest [3–5].

In twenty-first century, voltage instability has played a key role in some of the blackouts all over the world [6, 7]. According to the detailed ‘post-mortem’ of the very recent South Australian blackout incident, dynamic voltage instability is identified as the main reason for the collapse even though frequency instability was pointed out initially. There are several reasons behind the voltage instability problems. However, the root cause of DVI in DN is identified as the incapability of meeting increasing reactive power demand due to the induction motor (IM) stall following a contingency [6, 8]. Typically, IM loads draw 3–5 times of its rated current during stall [9].

Voltage instability can occur in both short and long term [10]. The time frame for long-term voltage stability analysis could be several minutes to hours, while it is few seconds for short-term study. Numerous researches have been accomplished to analyse the long-term voltage stability with high PV penetrations [5, 11, 12]. In contrast, very few studies have discussed the impact of high PV power on the short-term voltage stability (STVS), which is also known as DVS [13–17]. In [13], the authors have investigated the impact of LVRT by PV inverter on the STVS. It has been concluded that DVI could be ensued in the system following a contingency without LVRT capability of PV inverter. The same authors of [13] have proposed a new control strategy in [15] to improve the STVS. In [14, 17], STVS has been investigated for different control strategies of PV systems. Both papers [14, 17] have concluded that DVI is likely to occur with high PV penetration without LVRT capability, while it can be enhanced if PV units provide dynamic voltage support. However, the impact of unbalanced characteristics due to unequal PV penetrations and unbalanced loads has not been taken into account to study the DVS in all of the research presented in [13–17].

Fig. 1 Global evaluation of PV installation
As DN's are inherently unbalanced owing to its unbalanced load and line characteristics, it is important to investigate the impact of unbalanced characteristics on the dynamic voltage stability. Furthermore, because of numerous integration of single-phase PV system into DN, it is necessary to consider them to investigate the dynamic voltage stability of DN. Therefore, here, the impact of unbalanced characteristics of DN on the DVS is discussed first. It is observed that DVI could be impaired with the increase in imbalance. Therefore, to reduce the impact of imbalance, mitigation strategies are discussed and one of them, namely, reactive power injection by PV inverter is implemented. It has been shown that DVI problems due to increased imbalance can be mitigated by providing reactive power support from PV inverters.

The rest of the paper is organised as follows: the dynamic model of the single-phase PV system is developed in Section 2, followed by the definition of voltage unbalance and the methodology of DVS study in Sections 2 (C) and (D), respectively. Case studies are conducted in modified IEEE 4 bus systems, and the results are provided in Section 3. Finally, Section 4 accumulates the conclusion of this study.

2 Modelling and methodology of DVS

2.1 Modelling of the single-phase PV inverter

As the main motivation of this paper is to explore the impact of DVS on unbalanced DN with high PV penetration, an average model of the PV systems is developed by replacing the inverter switching, DC link, and PV with an ideal voltage source. The overall average model of the PV system is shown in Fig. 2, where $R$ and $L$ represent the grid impedance.

2.2 Development of direct-quadrature (dq)-based control

Fig. 2 illustrates the overall control of the single-phase PV systems. Two levels of control (i) outer power control and (ii) inner current control are developed. In order to control the grid current, reference currents have been generated by power controllers. The instantaneous active power $P$ and reactive power $Q$ can be calculated from a set of direct-quadrature (dq) voltages and current as given in (1).

$$
\begin{bmatrix}
P \\
Q
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
V_d \\
V_q
\end{bmatrix} \begin{bmatrix}
I_d \\
-I_q
\end{bmatrix}
$$

(1)

where $V_d, V_q, I_d, I_q$ are the dq-components of grid voltage and current. For single-phase system, the $q$-axis component of grid voltage will be zero. Therefore, instantaneous active and reactive power for single-phase system would be:

$$
\begin{bmatrix}
P \\
Q
\end{bmatrix} = \frac{1}{2} V_m \begin{bmatrix}
I_d \\
-I_q
\end{bmatrix}
$$

(2)

where $V_m$ is the amplitude of the grid voltage. It can be seen from (2) that $P$ and $Q$ can be controlled through the dq-component of grid current. The reference current in $dq$-frame can be calculated by (3):

$$
\begin{bmatrix}
I_{dref} \\
I_{qref}
\end{bmatrix} = \frac{2}{V_m} \begin{bmatrix}
P_{ref} \\
-Q_{ref}
\end{bmatrix}
$$

(3)

In order to confirm precise controlling of grid current, two PI controllers are used. The transfer-function of the PI controllers can be written as shown in (4) and (5).

$$
G_p(s) = K_{pd} + \frac{1}{s}
$$

(4)

$$
G_q(s) = K_{pq} + K_{iq} \times \frac{1}{s}
$$

(5)

where $K_{pd}$ and $K_{pq}$ are the proportional gain, while $K_{id}$ and $K_{iq}$ are known as integral gain.

2.3 Definition of voltage unbalance

It is impractical to consider a residential DN to be fully balanced. However, due to the negative impact of voltage imbalance on the electrical equipment [18, 19], most of the international standards have defined the maximum acceptable voltage imbalance. It has been recommended by the International Electro-technical Commission (IEC) that the voltage imbalance of supply system should be limited within the maximum value of 2.0% [18]. On the other hand, maximum voltage imbalance should be limited to 3.0%, while measured at the utility revenue meter in no load condition suggested by another standard ANSI C84.1-1995 [18]. There are several methods in the literature to measure the voltage imbalance [18, 20]. The National Electrical Manufacturer Association (NEMA) has defined the percentage voltage imbalance as [18]:

$$
\% \text{Volatage unbalance} = \frac{\text{Maximum deviation from average}}{\text{Average of three phase – phase voltage}} \times 100
$$

(6)

In order to measure the degree of imbalance, another index called voltage unbalance factor (VUF) has been used in European standards. The VUF has been defined as the ratio of negative sequence to positive sequence voltage which is given in (7) [18].

$$
\% \text{VUF} = \frac{V_N}{V_P} \times 100
$$

(7)

where $V_P$ and $V_N$ are the positive and negative sequence voltages, respectively. Using the symmetrical components, $V_P$ and $V_N$ can easily be calculated as given in (8).

$$
\begin{bmatrix}
V_P \\
V_N
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 \\
a \\
a^2
\end{bmatrix} \begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
$$

(8)

where $a = 1 < 120$ and $a^2 = 1 < 240$. 

Fig. 2 Overall control structure of TL PV system
2.4 Method of dynamic voltage stability (DVS) study

The most popular method to investigate the DVS is time domain simulations (TDSs). Consequently, here, TDSs are conducted to study the impact of imbalance on the DVS as well as to explore the effectiveness of mitigation strategies. Furthermore, in order to quantitatively compare the DVS, an improvement of the index proposed in [21] called transient voltage severity index (TVSI) is developed and implemented. The TVSI which was originally proposed for a balanced system is extended here for unbalanced system. As the phase voltages of an unbalanced system are unequal, the TVSI for each phase are calculated according to (10) and the maximum value is taken to compare DVS. As higher TVSI value indicates weak DVS [21], to capture the poor situation in terms of DVS, maximum value of TVSI is taken. The modified TVSI can be defined as follows:

$$TVSI = \max(TVSI_A, TVSI_B, TVSI_C)$$  \hspace{1cm} (9)

where TVSI_A, TVSI_B, and TVSI_C are the TVSI of phase A, B, and C, respectively, which are given in (8).

$$TVSI_{A,B,C} = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=T_c}^{T} TVDI_{A,B,C}^{i,t}$$  \hspace{1cm} (10)

where $N$ is the total number of buses, $T$ and $T_c$ are the considered transient time-frame and fault clearing time, respectively, and $TVDI_{A,B,C}^{i,t}$ is the transient voltage deviation index for phases $a$, $b$, and $c$. The $TVDI_{A,B,C}^{i,t}$ can be defined in (9).

$$TVDI_{A,B,C}^{i,t} = \begin{cases} \frac{V_{A,B,C}^{i,t} - V_{A,B,C}^0}{V_{A,B,C}^0} & \text{if } \frac{V_{A,B,C}^{i,t} - V_{A,B,C}^0}{V_{A,B,C}^0} \geq \gamma \forall t \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (11)

$$\in [T_c, T]$$

where $V_{A,B,C}^0$ are the phase voltages magnitude of bus $i$ at time $t$, $V_{A,B,C}^{i,t}$ are the pre-fault phase voltages of bus $i$, and $\gamma$ is the threshold voltage.

3 Results and discussion

In order to examine the DVS in unbalanced DN, several case studies are accomplished for different scenarios in modified IEEE 4 bus system as shown in Fig. 3. Modifications have been accomplished by accommodating PV systems and IM loads at bus 4 through a transformer (Tr2). As a result, a new bus (bus 5) is created. As the PV systems are connected at bus 5, it is called as point of common coupling (PCC). The per unit (pu) parameters of the new transformer is considered to be same as transformers 1 (Tr1). Characteristically, a residential DN possess as high as $70\%$ of IM load during a hot summer day [9]. As IEEE 4 bus system represent a residential DN, the loads of the modified IEEE 4 bus system are modelled as the combination of $30\%$ constant impedance, constant current, and constant power (ZIP-type) and $70\%$ IM-type. The modified loads of IEEE 4 bus system and the per unit parameters of IM loads are accessible in Tables 1 and 2. The short-circuit capacity (SCC) and the reactive to resistive (X/R) ratio of the system are designed to be $115$ MVA and $6.0$. Percentage of PV penetration is calculated according to (12). It can be noted that $50\%$ PV penetrations are reflected in all the scenarios here.

$$PV\text{ Penetration Level} = \frac{\text{Injected PV Power}}{\text{Total Load}} \times 100$$  \hspace{1cm} (12)

3.1 Impact of imbalance on DVS

TDS are performed for the cases presented in Table 3 to study the impact of imbalance on DVS. Cases 1 to 5 are based on percentage of VUF, which increases from $0.0\%$ in case 1 to $5.0\%$ in case 5. As the VUF is $0\%$ in case 1, it is considered as base case with balanced load. Voltage unbalance is created by increasing and decreasing the reference voltages of phases $C$ and $B$, respectively, at infinite bus, while voltage of phase $A$ is retained at $1.0$ pu. For the case study, PV systems are developed with LVRT capability and a three-phase to ground fault is created at bus 1 for the duration of $0.2$ s.

The dynamic voltage profiles at PCC for all three phases are shown in Fig. 4. It can be observed from Figs. 4a and b that DVS can be impaired with the increasing imbalance. In contrast, Fig. 4c indicates that DVS of phase $C$ is enhanced. This is due to improved voltage profile at phase $C$ when VUF increases in other cases compared to case 1. Although the voltage profile of phase $A$ is kept steady, the voltage profile of phase $B$ will deteriorate as the percentage of VUF increases.

![Fig. 3 Modified IEEE 4 bus system](image-url)
constant at 1.0 pu for all cases, post-contingency voltage recovery
is decelerated when imbalance among phases increases as seen in
Fig. 4a.

The TVSI values are calculated according to (9) for all cases
and listed in Table 4. Throughout the calculation of TVSI, the
parameters are set as follows: $N = 5$, $T = 2$ s, $T_c = 1.2$ s, step time =
20 μs, and $\gamma = 20\%$. It can be seen that TVSI values are gradually
increased for cases 1 to 5. To be precise, TVSI increases 34.3%
when VUF rises 0 to 5%, clearly indicates weaken DVS with high
imbalance.

3.2 Mitigation strategies

There are several strategies to mitigate the DVI problems, for
example installation of D-STATCOM and/or BESS, partial load
shedding, reactive power support from PV systems etc. However,
installation of BESS or D-STATCOM might not be a cost-effective
solution because of its higher price. The reported cost is roughly
20,000 Australian dollar (AUS) for 5 kVA–20 kWh BESS, while it
is US$ 50–55/kVAR for a D-STATCOM [9]. In contrast, the cost for
reactive power support by PV inverter includes implementation of
fast and dynamic control which is very negligible as the PV
systems are already installed in the network. Therefore, reactive
power support by PV systems could be more effective solutions in
terms of cost compared to BESS and D-STATCOM. Last but not
least, rearranging the single-phase PV systems along with loads
among phases could be another solution to mitigate the impact of
imbalance on DVS. This solution might be less costly compared to
BESS and PV reactive power support.

However, a lot of investigations and operational planning are
required to realise the effectiveness of this solution. Here,
effectiveness of reactive power support by PV inverter to augment
the DVS in unbalanced DN is investigated.

3.2.1 Reactive power support from PV inverters: Recently,
most of the international regulations have been updated including
the requirement of reactive power controlling capability by
distributed generators (DGs) to support the grid following a
disturbance [23–26]. According to the German grid code (E.ON),
DG units are required to inject 100% reactive power when the grid

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**Fig. 4** Dynamic voltage profile at PCC for different cases
(a) Phase A, (b) Phase B and, (c) Phase C

**Table 4** TVSI for different cases presented in Table 3

| Cases  | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|--------|--------|--------|--------|--------|--------|
| TVSI   | 1288   | 1485   | 1682   | 1871   | 2084   |
voltage goes below 0.5 pu [25]. Here, the reactive power injections have been accomplished by setting the reactive current reference value as follows:

\[
I_q = \mu (1 - V_{g})I_n \quad \text{when} \quad 1 - \frac{1}{\mu} \leq V_g \leq 0.95
\]

\[
I_q = I_n \quad \text{when} \quad V_g < 1 - \frac{1}{\mu}
\]

(13)

where \(V_g\) is the instantaneous grid voltage in pu, \(I_n\) and \(I_q\) are the inverter rated current and injected reactive current, respectively, and \(\mu\) is the reactive current scaling factor which can be defined in (14).

\[
\mu = \frac{I_q - I_{qo}}{I_n (1 - V_{g})}
\]

(14)

where \(I_{qo}\) is the initial reactive current before the fault. It is assumed that the PV systems operate at unity power factor during normal operation (0.95 \(\leq V_g \leq 1.05\)). Therefore, \(I_{qo}\) is set to be zero. According to (13), percentage of reactive current injection is calculated for different values of \(\mu\) in all condition of grid voltage sag and plotted in Fig. 5. It can be seen that reactive power injection is reduced while the value of \(\mu\) is dropped. Here, the value of \(\mu\) is selected to be 3.0 to realise high reactive power injection with comparatively low grid voltage sag.

In order to examine the effectiveness of the prior discussed mitigation strategy, TDS are carried out for the cases given in Table 5. Cases 1, 3, and 5 are similar to the cases presented in Table 1, while cases 2, 4, and 6 are formed by providing reactive power support from PV systems.

Fig. 6 depicts all the three phases’ dynamic voltage profile at PCC for the cases presented in Table 5. It can be seen that post-contingency voltage for all cases has been recovered, realising fastest recovery for cases 2, 4, and 6 in all phases. This is due to the reactive power support by PV systems in those cases. Furthermore, it can be observed from Figs. 6a and b that the voltage recovery process in cases 2, 4, and 6 are almost similar though it is different without reactive power support in cases 1, 3, and 5. This clearly indicates that the impact of imbalance on the DVS is mitigated. Therefore, the DVI problems with high imbalance which has also been observed in the previous Section could be avoided if the PV systems are capable to provide sufficient reactive power support.

The TVSI are calculated for all the cases in Table 5 and narrated in Table 6. In Table 6, high TVSI is found in cases 1, 3, and 5, while it has become less and very close with reactive power support in cases 2, 4 and 6 which in result improved DVS. Precisely, TVSI is reduced \(\sim 84.31, 83.16,\) and \(81.81\%\) for Scenarios 1, 2, and 3, respectively, by providing reactive power support.

4 Conclusions

Here, the impact of imbalance on DVS in residential DN with high PV penetration is investigated through time domain simulations and an index. Time domain simulations are performed by developing dynamic model of the elements involved in the network. Case studies are conducted in modified IEEE 4 bus system for different scenarios. Simulation results reveal that DVS is likely to be impaired with the increase in imbalance. In order to compare, TVSI is calculated for different scenarios. It has been shown that if VUF rises 0–5%, TVSI increases 34.3% which is the clear indication of weaken DVS with high imbalance. However, to alleviate the adverse effect of imbalance, mitigation strategies are proposed. One of the strategies is the reactive power injection by PV inverter which shows supreme performance in mitigating the impact of imbalance on the DVS. Furthermore, the DVS is improved compared with no reactive power support.
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