Interacting Fermi Gases in Disordered One-Dimensional Lattices

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Interacting two-component Fermi gases loaded in a one-dimensional (1D) lattice and subject to harmonic trapping exhibit intriguing compound phases in which fluid regions coexist with local Mott-insulator and/or band-insulator regions. Motivated by experiments on cold atoms inside disordered optical lattices, we present a theoretical study of the effects of a random potential on these ground-state phases. Within a density-functional scheme we show that disorder has two main effects: (i) it destroys the local insulating regions if it is sufficiently strong compared with the on-site atom-atom repulsion, and (ii) it induces an anomaly in the compressibility at low density from quenching of percolation.

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Introduction —The interplay between interactions and disorder in quantum many-body systems is an area of long-standing interest. For instance, both long-ranged Coulomb interactions and disorder from various mechanisms are believed to play an important role in the metal-insulator transition (MIT) in the two-dimensional (2D) electron liquid in semiconductor heterostructures. Disorder and interactions affect not only transport properties of the 2D liquid, but also thermodynamic quantities such as the compressibility and the spin susceptibility. “Dirty-boson” systems such as liquid 4He absorbed in aerogel, Vycor, or Geltech, or disordered granular superconductors, have also been extensively studied.

Cold atom gases are becoming important tools to understand the interplay between single-particle randomness and cooperative effects such as superfluidity and many-body effects induced by interactions. Atoms trapped in an optical lattice (OL) are particularly suitable candidates for such studies, especially because they allow one to reach the strongly-coupling regime through the depression of the kinetic energy associated with well-to-well tunneling. A 87Rb Bose-Einstein condensate (BEC) inside a disordered 1D OL has been used to study the interplay between repulsive interactions and disorder. In this work it has also been pointed out that thermodynamic quantities, such as the superfluid density in the case of a BEC, provide a better indicator of disorder-induced localization than time-of-flight absorption images. The present work has been motivated by the experiments in Refs. 7 and 8. We report a study of the interplay between interactions and randomness in a repulsive two-component Fermi gas trapped in a 1D OL. An added motive of interest is that the Landau Fermi-liquid paradigm does not apply. Two-component Fermi gases have recently been prepared in a quasi-1D geometry, thus opening the way to experimental studies of 1D phenomena such as spin-charge separation and atomic-density waves.

The ground state of an interacting Fermi gas moving under harmonic confinement in a 1D OL shows in the absence of disorder five qualitatively different phases (for a pictorial description see Fig. 1). How does disorder influence these phases and their thermodynamic properties? In the following we provide a quantitative answer to this question. In particular, we demonstrate that the incompressible Mott-insulating regions are very stable against disorder at strong coupling. We also show that the compressibility exhibits a disorder-induced low-density anomaly, similar in some respects to the one which has been found both experimentally and theoretically in the 2D electron liquid close to the onset of the MIT.

The 1D random Fermi-Hubbard model — We consider a two-component Fermi gas with N 1 atoms constrained to move under harmonic confinement of strength V 2 inside a disordered 1D OL with unit lattice constant and N s lattice sites i ∈ [1, N s]. The system is described by a single-band Hubbard Hamiltonian,

\[
\hat{H} = - \sum_{i,j} t_{ij} (\hat{c}^\dagger_i \hat{c}_{j\sigma} + \text{H.c.}) + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + V_2 \sum_i (i - N_s/2)^2 \hat{n}_i + \sum_i \epsilon_i \hat{n}_i. \tag{1}
\]

Here \(t_{ij} = t > 0\) if \(i, j\) are nearest sites and zero otherwise, \(\sigma = \uparrow, \downarrow\) is a pseudospin-1/2 label for two internal hyperfine states, \(\hat{n}_{i\sigma} = \hat{c}^\dagger_{i\sigma} \hat{c}_{i\sigma}\) is the pseudospin-resolved site occupation operator, and \(\hat{n}_i = \sum_{\sigma} \hat{n}_{i\sigma}\). The effect of disorder is simulated by the last term in Eq. (1), where \(\epsilon_i\) is randomly chosen at each site from a uniform distribution in the range \([-W/2, W/2]\).

In the unconfined limit \((V_2 = 0)\) the Hamiltonian \(\hat{H}\) reduces to the Anderson localization problem for \(U = 0\) and to the exactly-solvable Lieb-Wu model for \(W = 0\). The Lieb-Wu model describes a Luttinger liquid away from half and full filling, a Mott insulator at half filling, and a band insulator at full filling. In the unconfined limit the 2D version of \(\hat{H}\) has been studied in connection with the 2D MIT. In the clean limit the ground-state of \(\hat{H}\) has been studied by several authors, finding five different phases as already shown in Fig. 1.

A particular set \(\epsilon_i(\alpha)\) of randomly chosen values of
compressibility can be obtained from the stiffness $S$ against further increases of

that the density profiles that we report below are sta-

the strength of disorder increases and we have checked

such large numbers of realizations becomes necessary as

— The functional
terms, external potential

occupation, in the sense that it does not depend on the

parameters

leads to the site occupation

be derived

exchange-correlation energy of the Lieb-Wu model

estimated through an LDA based on the exactly known

many-body effects beyond mean field. This is approx-

imated through an LDA based on the noninter-

potential

i.e.

N = 70 instead (bottom panel of Fig. 2), the Mott plateau at

center of the trap is unstable against the formation of a fluid phase with $N_i > 1$, and the Mott phase can

survive for weak disorder only in an intermediate region

between the edge and the center of the trap. In fact,

$N = 70$ is the critical number of atoms at which the

phase transition $B \rightarrow C$ occurs in the clean limit: a weak

disorder potential can shift the transition and induce a fluid region embedded in the Mott plateau (see the plot

corresponding to $W/t = 1$ in the bottom panel of Fig. 2).

Eventually, the scenario depicted above for $N_i = 60$ is

re-established when $W$ is strong enough.

In the top panel of Fig. 3 we show the site occupa-

tion for a disordered gas with $u = 8$, which is in phase

$D$ for $W = 0$. Clearly, the band-insulating region can

only be corrupted from below. We note that at $W/t = 3$

the Mott-insulating regions still exist, while the band-

insulating region has been destroyed. This confirms the

expectation that a Mott-insulating region, having its ori-

gin in exchange-correlation effects, is more stable against

disorder than a band-insulating region. These behaviors

are examined in detail in the bottom panel of Fig. 3.

We turn in Fig. 4 to illustrate the effect of disorder on the

stiffness of the Fermi gas. In the top panel we show $S_p$ as a function of $N_i$ at different values of $u$ in the ab-

sence of disorder. At $u \geq 4$ this quantity exhibits three

non-analyticity points associated with the three phase

transitions: $A \rightarrow B$, $B \rightarrow C$ and $C \rightarrow D$. In the bot-

tom panel of Fig. 4 we show the same disorder-averaged

quantity for a disordered Fermi gas with $u = 8$. We

see that the disorder has two main effects. It not only

leads to smoothing of the non-analytic behaviors found

in the clean limit, but also induces a strong stiffening at

low density. The latter is an “anomalous” behavior com-

pared to that found in the clean limit. In fact, for finite

$W$ the stiffening appears to grow unbounded at very low

density (see bottom panel of Fig. 4), following the power

law $S_p \propto (N_i)^{-\nu}$ with an exponent $\nu \approx 0.6$. The value

of the exponent is essentially independent of the param-

eters $u$ and $W/t$, but depends on the confinement: for

example, we find $\nu \approx 0.4$ for an open lattice with $V_2 = 0$.

At high density $S_p$ appears instead to be essentially un-

affected by disorder.

The low-density behavior of $S_p$ is reminiscent of what

has been found in Refs. 2 and 3 for a 2D electron liquid.
Following Ref. 3 we explain the origin of the anomaly using the concept of density percolation. As \( N_f \) decreases, the high-density regions tend to become disconnected, since the atoms tend to occupy just the deepest valleys in the disorder landscape. At given \( u \) and low \( N_f \), the system thus stiffens as the disorder grows (see the bottom panel in Fig. 4). For a given realization of disorder, the number \( N_g \) of essentially empty sites increases with \( W/t \), as it is shown in the inset.

There is, however, an important conceptual difference between the present Fermi gas and the 2D electron liquid. In the latter the density is also an inverse measure of the coupling strength: the stiffness anomaly at low density occurs in the strongly correlated regime. In the present case the atom number \( N_f \) and the interaction parameter \( U \) are instead independent parameters. The anomaly that we observe occurs also in the noninteracting limit (see the bottom panel of Fig. 4), demonstrating the crucial role of the quenching of percolation in originating the anomaly. The interatomic repulsions enhance the stiffness at low density in the disordered case just as they do in the clean case, in accord with the intuitive expectation that a repulsive system is less compressible.

In summary, we have shown how disorder affects the quantum phases of interacting Fermi gases moving under harmonic confinement in 1D lattices. In particular we have seen that Mott insulating regions are quite stable against uniformly distributed uncorrelated disorder and that the disorder induces an anomalous increase of the stiffness at low density from quenching of percolation.

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FIG. 1: (color online) Sketch of the ground-state site occupation $n_i$ of an interacting Fermi gas in a harmonic trap and a clean 1D lattice. Phase $\mathcal{A}$ is a fluid (known as the metallic phase in the case of electrons) with $0 < n_i < 2$. In phase $\mathcal{B}$ an incompressible Mott insulator occupies the bulk of the trap with $n_i$ locally locked to 1. In phase $\mathcal{C}$ a fluid with $1 < n_i < 2$ is embedded in the Mott plateau. In phase $\mathcal{D}$ a band insulator with $n_i$ locally locked to 2 is surrounded by fluid edges and embedded in the Mott plateau. Finally, in phase $\mathcal{E}$ a band insulator in the bulk of the trap coexists with fluid edges.
FIG. 2: (color online) Site occupation $N_i$ as a function of site position $i$ for $u = 4$ and $V_2/t = 2.5 \times 10^{-3}$ in a lattice with $N_s = 200$ sites. The number of atoms is $N_f = 60$ in the top panel and $N_f = 70$ in the bottom panel.
FIG. 3: (color online) Top panel: Site occupation $N_i$ as a function of $i$ for $N_f = 200$ atoms in a lattice with $N_s = 200$ sites, in the case $u = 8$ and $V_2/t = 2.5 \times 10^{-3}$. Bottom panel: Number of consecutive sites $N_{\text{Mott}}$ ($N_{\text{Band}}$) at which $|N_i - 1| \leq 10^{-5}$ ($|N_i - 2| \leq 10^{-5}$), as a function of $W/t$ for the system shown in the top panel. The steps show that each insulating region is stable over a finite range of $W/t$. $N_{\text{Mott}}$ can be fitted with a linear function over the whole range of $W/t$ (solid line through the squares, showing that $N_{\text{Mott}} = 0$ at $(W/t)_{c1} \approx 5$). $N_{\text{Band}}$ has a linear behavior up to $W/t \approx 1.8$ and beyond can only be fitted by a nonlinear function (solid and dashed lines through the dots, showing that $N_{\text{band}} = 0$ at $(W/t)_{c2} \approx 3 < (W/t)_{c1}$).
FIG. 4: (color online) Thermodynamic stiffness $S_\rho$ (in units of $t$) as a function of $N_f$ for $V_2/t = 2.5 \times 10^{-3}$ and $N_s = 200$ lattice sites. Top panel: results for a clean system at various values of $u$ (in the noninteracting case only the phase transition $A \rightarrow E$ can occur). Bottom panel: results for a disordered system at $u = 8$ and for $0 \leq W/t \leq 10$. The black triangles report the low-density stiffness of a noninteracting system at $W/t = 10$. The inset shows the number $N_0$ of sites at which $n_i(\alpha) \leq 10^{-5}$ in a particular realization of disorder, as a function of $N_f$ for $u = 8$ and $0 \leq W/t \leq 10$. 