From the $\gamma\gamma \rightarrow p\bar{p}$ reaction to the production of $p\bar{p}$ pairs in ultraperipheral ultrarelativistic heavy-ion collisions at the LHC

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Abstract

In this paper we consider the production of proton-antiproton pairs in two-photon interactions in electron-positron and heavy-ion collisions. We try to understand the dependence of the total cross section on the photon-photon c.m. energy as well as corresponding angular distributions measured by the Belle Collaboration for the $\gamma\gamma \rightarrow p\bar{p}$ process. To understand the Belle data we include the proton-exchange, the $f_2(1270)$ and $f_2(1950)$ s-channel exchanges, as well as the handbag mechanism. The helicity amplitudes for the $\gamma\gamma \rightarrow f_2 \rightarrow p\bar{p}$ process are written explicitly based on a Lagrangian approach. The parameters of vertex form factors are adjusted to the Belle data. Having described the angular distributions for the $\gamma\gamma \rightarrow p\bar{p}$ process we present first predictions for the ultraperipheral, ultrarelativistic, heavy-ion reaction $^{208}\text{Pb}^{208}\text{Pb} \rightarrow ^{208}\text{Pb}^{208}\text{Pb} p\bar{p}$. Both, the total cross section and several differential distributions for experimental cuts corresponding to the ALICE, ATLAS, CMS, and LHCb experiments are presented. We find the total cross section 100 $\mu$b for the ALICE cuts, 160 $\mu$b for the ATLAS cuts, 500 $\mu$b for the CMS cuts, and 104 $\mu$b taking into account the LHCb cuts. This opens a possibility to study the $\gamma\gamma \rightarrow p\bar{p}$ process at the LHC.

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I. INTRODUCTION

The baryon pair production via $\gamma\gamma$ fusion was measured at electron-positron colliders by various experimental groups: CLEO [1] at CESR, VENUS [2] at TRISTAN, OPAL [3] and L3 [4] at LEP, and Belle [5] at KEKB. In the latter experiment the $\gamma\gamma \rightarrow p\bar{p}$ cross sections were extracted from the $e^+e^- \rightarrow e^+e^- p\bar{p}$ reaction for the $\gamma\gamma$ center-of-mass (c.m.) energy range of $2.025 < W_{\gamma\gamma} < 4$ GeV and in the c.m. angular range of $|\cos\theta| < 0.6$.

QCD predictions for $\gamma\gamma \rightarrow p\bar{p}$ were first calculated in [6, 7] using the leading twist nucleon wave functions determined from QCD sum rules, see e.g. [8]. The calculated cross sections from the leading-twist QCD terms turned out to be about one order of magnitude smaller than the experimental data. To explain these experimental observations, various phenomenological approaches were suggested. For example, in the diquark model, which is a variant of the leading-twist approach, see e.g. [9] and references therein, the proton was considered to be a quark-diquark system and a diquark form factor was introduced. In the hand-bag approach, see e.g. [10], the $\gamma\gamma \rightarrow p\bar{p}$ amplitude was factorized into a hard $\gamma\gamma \rightarrow q\bar{q}$ subprocess and form factors describing a soft $q\bar{q} \rightarrow p\bar{p}$ transition. The transition form factors could not be calculated from first principles in QCD and were, therefore, determined phenomenologically. The pQCD-inspired phenomenological models have more chances to describe the absolute size of the cross section for $W_{\gamma\gamma} > 2.5$ GeV, however, they contain a number of free parameters that are fitted to data. Moreover, most data were taken at energies which are rather low for the kinematic requirements of large $s$, $|t|$, $|u|$ in the hand-bag approach.

The low center-of-mass energy region of $\gamma\gamma \rightarrow p\bar{p}$ may be dominated by $s$-channel resonance contributions. One of the effective approaches used for this region is the Veneziano model [11]. While a reasonable $\sigma(W_{\gamma\gamma})$ dependence was obtained without adjustable parameters, the agreement of the model with the angular distributions was only qualitative.

In a recent calculation [12] only the proton exchange contribution was considered. But we think that this calculation has some problems as we shall discuss below in Sec. II A.

In our approach we wish to include all important theory ingredients in order to achieve a quantitative description of the Belle data. Then we present our predictions for the production of $p\bar{p}$ pairs in the ultraperipheral, ultrarelativistic, heavy-ion collisions at the LHC. To describe the dynamics of the $\gamma\gamma \rightarrow p\bar{p}$ process we take into account not only the nonresonant proton exchange contribution but also the $s$-channel tensor meson exchange contributions and the hand-bag mechanism. A measurement of the $^{208}\text{Pb}^{208}\text{Pb} \rightarrow ^{208}\text{Pb}^{208}\text{Pb} p\bar{p}$ reaction will provide further information on the two-photon interactions involved and, thus, will allow further tests of existing theoretical approaches.

II. THE $\gamma\gamma \rightarrow p\bar{p}$ REACTION

We consider the reaction (see Fig. 1)

$$\gamma(k_1,\epsilon_1) + \gamma(k_2,\epsilon_2) \rightarrow p(p_3, s_3) + \bar{p}(p_4, s_4),$$

$$s_3, s_4 \in \{1/2, -1/2\},$$

(2.1)
$\mathcal{T}$-matrix element for the reaction (2.1),

$$
\langle p(p_3, s_3), \bar{p}(p_4, s_4)|\mathcal{T}|\gamma(k_1, \epsilon_1), \gamma(k_2, \epsilon_2)\rangle = M^{\mu\nu}(p_3, p_4; k_1, k_2) \epsilon_{1\mu} \epsilon_{2\nu}
$$

$$
\equiv M_{\gamma\gamma \rightarrow p\bar{p}}, \quad (2.2)
$$

for nonresonant proton exchange, exchange of spin 2 mesons in the s-channel, and for the hand-bag mechanism. We note that gauge invariance requires

$$
M^{\mu\nu}(p_3, p_4; k_1, k_2) k_1\mu = 0,
M^{\mu\nu}(p_3, p_4; k_1, k_2) k_2\nu = 0. \quad (2.3)
$$

Since the photons are bosons we must have

$$
M^{\mu\nu}(p_3, p_4; k_1, k_2) = M^{\nu\mu}(p_3, p_4; k_2, k_1). \quad (2.4)
$$

FIG. 1: Diagrams for the production of $p\bar{p}$ in $\gamma\gamma$ collisions. We consider the $t$- and $u$-channel proton exchange (diagrams (a) and (b), respectively), the exchange of $f_2$ meson in the $s$-channel (diagram (c)), and the hand-bag mechanism (diagram (d) plus the one with the photon vertices interchanged). Here $f_2$ stands generically for a $J^{PC} = 2^{++}$ meson.

The kinematical variables used in the present paper are (see Fig. 1)

\begin{align}
s &= (k_1 + k_2)^2 = (p_3 + p_4)^2 = W^2_{\gamma\gamma}, \\
t &= (k_1 - p_3)^2 = (k_2 - p_4)^2, \\
u &= (k_1 - p_4)^2 = (k_2 - p_3)^2, \\
s + t + u &= 2m_p^2; \\
ps &= k_1 + k_2 = p_3 + p_4, \\
p_t &= k_2 - p_4 = p_3 - k_1, \\
p_u &= k_1 - p_4 = p_3 - k_2; \quad (2.5) \\
p_s^2 &= s, \quad p_t^2 = t, \quad p_u^2 = u. \quad (2.6)
\end{align}

We shall work in the c.m. frame of the reaction (2.1); see Fig. 18 in Appendix A. For the incoming photons we use the polarization vectors (A27) and the helicity spinors for the
proton are as in (A2) – (A4) with \( \theta \to \theta, \phi \to 0 \). The helicity spinors for the antiproton are obtained from (A20) and (A10), (A11), with \( \theta \to \pi - \theta, \phi \to \pi \).

There are 16 helicity amplitudes

\[
\langle p(p_3, s_3), \bar{p}(p_4, s_4) | T | \gamma(k_1, m_1), \gamma(k_2, m_2) \rangle \equiv \langle 2s_3, 2s_4 | T | m_1, m_2 \rangle .
\]

Here \( s_3, s_4 \in \{1/2, -1/2\} \) and \( m_1, m_2 \in \{1, -1\} \) are the helicity labels of proton, antiproton and the photons, respectively. We have also introduced a convenient shorthand notation for the amplitudes. Using rotational, parity and charge-conjugation invariance one finds that only 6 of the 16 helicity amplitudes are independent which we denote by \( \psi_1(s, t), \ldots, \psi_6(s, t) \); see (A39) and Table V of Appendix A.

The unpolarized differential cross section for the reaction (2.1) is given by

\[
\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s} \left| \frac{p_3}{k_1} \right| \frac{1}{4} \sum_{\text{spins}} |M_{\gamma\gamma \to p\bar{p}}|^2 ,
\]

where \( s \) is the invariant mass squared of the \( \gamma\gamma \) system, \( \theta \) denotes the angle of the outgoing nucleon relative to the beam direction in the c.m. frame, see Fig. 18 in Appendix A and \( k_1 \) and \( p_3 \) are the c.m. 3-momenta of the initial photon and final nucleon, respectively; see (A26).

### A. Nonresonant proton exchange contribution

The amplitude for the proton exchange mechanism [see the diagrams (a) and (b) in Fig. 1] is written as

\[
M^{(p \text{ exchange})}_{\text{bare}} = (-i) \epsilon_{1\mu} \epsilon_{2\nu} \bar{u}(p_3) \left( i\Gamma^{(\gamma pp)}(p_3, p_4) \frac{i(p_4 + m_p)}{t - m_p^2 + i\epsilon} \right) v(p_4) \]

\[
+ i\Gamma^{(\gamma pp)}(p_3, p_4) \frac{i(p_4 + m_p)}{u - m_p^2 + i\epsilon} v(p_4) \right) v(p_4) .
\]

Here we use the free proton propagator for the internal proton lines and the photon-proton vertex function as for on-shell protons respectively antiprotons. This photon-proton vertex function is, with \( q = p' - p \), given by

\[
i\Gamma^{(\gamma pp)}(p', p) = -i e \left[ \gamma_\mu F_1(q^2) + \frac{i}{2m_p} \sigma_{\mu\nu} q^\nu F_2(q^2) \right] ;
\]

see e.g. (3.26) of [13]. In (2.11) \( \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \), \( F_1 \) and \( F_2 \) are the Dirac and Pauli form factors of the proton, respectively. For real photons we have \( F_1(0) = 1 \) and \( F_2(0) = \kappa_p = 1.7928 \), where \( \kappa_p \) is the anomalous magnetic moment of the proton. The amplitude (2.10) satisfies the gauge-invariance relations (2.3) and the Bose-symmetry relation (2.4).

Of course, the virtual protons in the diagrams of Fig. I (a) and (b) are off shell. Their propagators will, in general, not be the ones of free protons and the photon-proton vertex functions also will have an off-shell dependence. We take these off-shell dependences
into account via multiplication of the amplitude \(2.10\) by an extra form factor. We adopt here the scheme used in previous works \([14–17]\) and set
\[
F(t, u, s) = \left[ F(t) \right]^2 + \left[ F(u) \right]^2 + \left[ \tilde{F}(s) \right]^2,
\] (2.12)
with the exponential parametrizations
\[
F(t) = \exp \left( \frac{t - m_p^2}{\Lambda_p^2} \right),
\]
\[
F(u) = \exp \left( \frac{u - m_p^2}{\Lambda_p^2} \right),
\] (2.13)
\[
\tilde{F}(s) = \exp \left( \frac{-(s - 4m_p^2)}{\Lambda_p^2} \right).
\]
The parameter \(\Lambda_p\) should be fitted to the experimental data. Note that the form factor \(F(t)\) is normalized to unity for \(t = m_p^2\).

Our complete result for the nonresonant proton exchange contribution reads, therefore,
\[
\mathcal{M}(p\text{ exchange}) = \mathcal{M}_{\text{bare}}(p\text{ exchange}) F(t, u, s) .
\] (2.14)
The multiplication of the “bare” amplitude with a common form factor guarantees that the gauge-invariance relations (2.3) are satisfied for \(\mathcal{M}(p\text{ exchange})\). Also the Bose-symmetry relation (2.4) is satisfied \(^1\) by (2.14) since \(\mathcal{M}_{\text{bare}}(p\text{ exchange})\) satisfies (2.4) and the form factor \(F(t, u, s)\) is symmetric under the exchange \(t \leftrightarrow u\); see (2.10) and (2.12).

**B. \(f_2\) meson contributions**

In this section we discuss the contributions from the \(s\)-channel exchange of \(J^{PC} = 2^{++}\) mesons, generically denoted by \(f_2\) in diagram (c) of Fig. II. In the following we shall take into account the \(f_2(1270)\) and \(f_2(1950)\) resonances. That is, in the formulas \(f_2\) stands for any of these resonances. In the final calculations their contributions are summed.

The amplitude for the \(p\bar{p}\) production through the \(s\)-channel exchange of a tensor meson \(f_2\) [the corresponding diagram is shown in Fig. II(c)] is written as
\[
\mathcal{M}(f_2\text{ exchange}) = (-i) \bar{u}(p_3)i\Gamma^{(f_2 p\bar{p})\alpha\beta}(p_3, p_4) \sigma^\nu(p_4) i\Delta^{(f_2)}_{\alpha\beta\kappa\lambda}(p_s) i\Gamma^{(f_2\gamma\gamma)}_{\mu\nu\kappa\lambda}(k_1, k_2) \epsilon_1^\mu \epsilon_2^\nu .
\] (2.15)

\(^1\) The amplitude for \(\gamma\gamma \to p\bar{p}\) considered in Eqs. (8) - (10) of [12] does not satisfy the Bose-symmetry relation (2.4). Therefore, this amplitude and the corresponding cross section, (16) and (20) of [12], cannot correspond to reality.
The $f_2\gamma\gamma$ vertex is given as
\[ i\Gamma^{(f_2\gamma\gamma)}_{\mu\nu\kappa\lambda}(k_1, k_2) = i\left[2a_{f_2\gamma\gamma}F_a^{(f_2\gamma\gamma)}(p_s^2)\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k_1, k_2) - b_{f_2\gamma\gamma}F_b^{(f_2\gamma\gamma)}(p_s^2)\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k_1, k_2)\right], \]
(2.16)
with two rank-four tensor functions,
\[ \Gamma^{(0)}_{\mu\nu\kappa\lambda}(k_1, k_2) = \left[(k_1 \cdot k_2)g_{\mu\nu} - k_{2\mu}k_{1\nu}\right]\left[k_{1\kappa}k_{2\lambda} + k_{2\kappa}k_{1\lambda} - \frac{1}{2}(k_1 \cdot k_2)g_{\kappa\lambda}\right], \]
(2.17)
\[ \Gamma^{(2)}_{\mu\nu\kappa\lambda}(k_1, k_2) = \left[(k_1 \cdot k_2)(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa}) + g_{\mu\nu}(k_{1\kappa}k_{2\lambda} + k_{2\kappa}k_{1\lambda}) - k_{1\kappa}k_{2\lambda}g_{\mu\nu} - k_{1\nu}k_{2\mu}g_{\kappa\lambda} - k_{2\nu}k_{1\mu}g_{\kappa\lambda} - k_{2\kappa}k_{1\mu}g_{\nu\lambda}\right], \]
(2.18)
see (3.39) and (3.18) – (3.22) of [13]. In our case we have $k_1^2 = k_2^2 = 0$.

For the $f_2(1270)$ meson, the coupling constants $a_{f_2\gamma\gamma}$ and $b_{f_2\gamma\gamma}$ are estimated in Secs. 5.3 and 7.2 of [13]. In the case of the $f_2(1950)$ meson the numerical values of the $a$ and $b$ parameters will be obtained here from a fit to the Belle data [4]. In (2.16) we have introduced form factors $F^{(f_2\gamma\gamma)}(p_s^2)$ describing the $s$ dependence of the $f_2\gamma\gamma$ coupling. These form factors will be particularly important for the diagram Fig. 1(c) with $f_2(1270)$ exchange since in $p\bar{p}$ production this meson significantly contributes but only far off shell.

Let us now discuss in detail the $f_2p\bar{p}$ vertex. From the $l-S$ analysis, presented in Appendix B we know that there are two independent couplings corresponding to $(l, S) = (1, 1)$ and $(3, 1)$. In accord with this we choose two coupling Lagrangians, (2.19) and (2.20) below, which correspond to two linearly independent combinations of the two $(l, S)$ possibilities; see Appendix B. We set
\[ L^{(1)}_{f_2p\bar{p}}(x) = -\frac{g^{(1)}_{f_2p\bar{p}}}{M_0}f_{2\kappa\lambda}(x)\frac{i}{2}\bar{\psi}_p(x)\left[\gamma^\kappa \partial^\lambda + \gamma^\lambda \partial^\kappa - \frac{1}{2}g^{\kappa\lambda}\gamma^\rho \partial^\rho\right]\psi_p(x), \]
(2.19)
\[ L^{(2)}_{f_2p\bar{p}}(x) = \frac{g^{(2)}_{f_2p\bar{p}}}{M_0}f_{2\kappa\lambda}(x)\frac{i}{2}\bar{\psi}_p(x)\left[\gamma^\kappa \partial^\lambda - \frac{1}{4}g^{\kappa\lambda} \gamma^\rho \partial^\rho\right]\psi_p(x), \]
(2.20)
where $\psi_p(x)$ and $f_2(x)$ are the proton and $f_2$ meson field operators, respectively. The corresponding vertices, including form factors, are
\[ i\Gamma^{(f_2p\bar{p})^{(1)}}_{\kappa\lambda}(p_3, p_4) = -\frac{g^{(1)}_{f_2p\bar{p}}}{M_0} \left[\frac{1}{2}\gamma_\kappa(p_3 - p_4)\lambda + \frac{1}{2}\gamma_\lambda(p_3 - p_4)\kappa - \frac{1}{4}g_{\kappa\lambda}(p_3 - p_4)\right] \times F^{(f_2p\bar{p})^{(1)}}((p_3 + p_4)^2), \]
(2.21)
\[ i\Gamma^{(f_2p\bar{p})^{(2)}}_{\kappa\lambda}(p_3, p_4) = -\frac{g^{(2)}_{f_2p\bar{p}}}{M_0} \left[(p_3 - p_4)\kappa(p_3 - p_4)\lambda - \frac{1}{4}g_{\kappa\lambda}(p_3 - p_4)^2\right] \times F^{(f_2p\bar{p})^{(2)}}((p_3 + p_4)^2). \]
(2.22)
Here $g^{(j)}_{f_2p\bar{p}}$ ($j = 1, 2$) are dimensionless coupling constants and $M_0 \equiv 1$ GeV. The complete $f_2p\bar{p}$ vertex function is given by
\[ i\Gamma^{(f_2p\bar{p})}_{\kappa\lambda}(p_3, p_4) = \sum_{j=1,2} i\Gamma^{(f_2p\bar{p})^{(j)}}_{\kappa\lambda}(p_3, p_4). \]
(2.23)
For the $f_2$ propagator we use the simple formula

$$i\Delta^{(f_2)}_{\alpha\beta\lambda\kappa}(p_s) = ip^{(2)}_{\alpha\beta\lambda\kappa}(p_s) \Delta^{(2)}(p_s^2) = i\frac{1}{2} \left[ \hat{g}_{\alpha\lambda} \hat{g}_{\beta\kappa} + \hat{g}_{\alpha\kappa} \hat{g}_{\beta\lambda} - \frac{1}{3} \hat{g}_{\alpha\beta} \hat{g}_{\lambda\kappa} \right] \frac{1}{p_s^2 - m_{f_2}^2 + im_{f_2} \Gamma_{f_2}}, \quad (2.24)$$

where $\hat{g}_{\mu\nu} = -g_{\mu\nu} + p_{s\mu} p_{s\nu} / p_s^2$. $\Gamma_{f_2}$ is the total decay width of the $f_2$ resonance and $m_{f_2}$ its mass. For a more detailed analysis we should use a model for the $f_2$ propagator along the lines considered in [13]; see (3.6) – (3.8) and Appendix A of [13].

With the expressions from Appendix A we get the helicity amplitudes for the reaction $\gamma\gamma \to f_2 \to pp$, using the notation of (A36) and $\varepsilon = (\varepsilon_{s3})$ as defined in (A16), as follows

$$\langle 2s_3, 2s_4 | T | (+, +) \rangle = \langle 2s_3, 2s_4 | T | (-, -) \rangle$$

$$= \frac{1}{2} s^2 \sqrt{s - 4m_p^2} \Delta^{(2)}(s) a_{f_2\gamma\gamma} F_{a}^{(f_2\gamma\gamma)}(s)$$

$$\times \left\{ \frac{\hat{g}_{f_2pp}}{M_0} F_{a}^{(f_2\gamma\gamma)}(s) \left[ -2m_p \left( \cos^2 \theta - \frac{1}{3} \right) \delta_{s3s4} - \sqrt{s} \sin \theta \cos \theta \varepsilon_{s3s4} \right] + \frac{\hat{g}_{f_2pp}}{M_0} F_{a}^{(f_2\gamma\gamma)}(s) \left( s - 4m_p^2 \right) \left( \cos^2 \theta - \frac{1}{3} \right) \delta_{s3s4} \right\}, \quad (2.25)$$

$$\langle 2s_3, 2s_4 | T | (\pm, \mp) \rangle$$

$$= \frac{1}{2} s \sqrt{s - 4m_p^2} \Delta^{(2)}(s) b_{f_2\gamma\gamma} F_{b}^{(f_2\gamma\gamma)}(s)$$

$$\times \left\{ \frac{\hat{g}_{f_2pp}}{M_0} F_{b}^{(f_2\gamma\gamma)}(s) \left[ -2m_p \sin^2 \theta \delta_{s3s4} + \sqrt{s} \sin \theta \cos \theta \varepsilon_{s3s4} \pm \sqrt{s} \sin \theta \delta_{s3,-s4} \right] + \frac{\hat{g}_{f_2pp}}{M_0} F_{b}^{(f_2\gamma\gamma)}(s) \left( s - 4m_p^2 \right) \sin^2 \theta \delta_{s3s4} \right\}. \quad (2.26)$$

Note the different $s$ dependences in (2.25) and (2.26) that are due to the different dimensions of $a_{f_2\gamma\gamma}$ and $b_{f_2\gamma\gamma}$. Using different functional forms for the form factors $F_a$ and $F_b$ these $s$ dependences could be adjusted to experimental data.

In the calculation we assume the same form for $F_a$ and $F_b$

$$F_{a}^{(f_2\gamma\gamma)}(s) = F_{b}^{(f_2\gamma\gamma)}(s) = F^{(f_2\gamma\gamma)}(s). \quad (2.27)$$

A convenient ansatz for such a form factor is the exponential one (see (4.22) of [18])

$$F^{(f_2\gamma\gamma)}(s) = \exp \left( - \frac{(s - m_{f_2}^2)^2}{\Lambda_{f_2,exp}^4} \right) \quad (2.28)$$

with $\Lambda_{f_2}$ a parameter of the order 1 – 2 GeV. Alternatively, we can use

$$F^{(f_2\gamma\gamma)}(s) = \frac{\Lambda_{f_2,pow}^4}{\Lambda_{f_2,pow}^4 + (s - m_{f_2}^2)^2}. \quad (2.29)$$
The form factors $F^{(f_2\gamma\gamma)}(m_{f_2}^2)$ are normalized to $F^{(f_2\gamma\gamma)}(m_{f_2}^2) = 1$. For the $f_2p\bar{p}$ form factors we assume

$$F^{(f_2p\bar{p})}(1)(s) = F^{(f_2p\bar{p})}(2)(s) = F^{(f_2\gamma\gamma)}(s).$$

(2.30)

The numerical values of the form factor parameters will be adjusted to the Belle experimental data.

### C. Hand-bag approach

The hand-bag contribution to $\gamma\gamma \rightarrow B\bar{B}$ processes was described in detail in [10]. The hand-bag amplitude can be written in terms of the hard scattering kernel for $\gamma\gamma \rightarrow q\bar{q}$ and a soft matrix element describing the $q\bar{q} \rightarrow p\bar{p}$ transition. Their c.m. helicity amplitudes, which we denote by $\bar{M}$, are written in terms of the light-cone helicity amplitudes $A$ (see Eq. (30) in [10]) as

$$\bar{M}_{s_3s_4,m_1m_2} = A_{s_3s_4,m_1m_2} + \frac{m_p}{\sqrt{s}} \left[ 2s_3A_{-s_3s_4,m_1m_2} + 2s_4A_{s_3-s_4,m_1m_2} \right] + O(m_p^2/s).$$

(2.31)

The light-cone helicity amplitudes, including terms suppressed only by $m_p/\sqrt{s}$, read [10]

$$A_{s_3s_4,+-} = -(1)^{s_3-s_4}A_{-s_3-s_4,--} =$$

$$4\pi\alpha em\frac{s}{\sqrt{tu}} \left\{ \delta_{s_3,s_4} \frac{t-u}{s} R_V(s) + 2s_3\delta_{s_3,-s_4} \left[ R_A(s) + R_P(s) \right] - \frac{\sqrt{s}}{2m_p} \delta_{s_3s_4} R_P(s) \right\}.$$

(2.32)

The authors of [10] argue that the amplitudes with identical photon helicities will be nonzero only at next-to-leading order in $\alpha$, in analogy to the photon helicity flip transitions in large-angle Compton scattering [19]. Note that for zero mass the light-cone helicity amplitudes (2.32) are identical with the helicity amplitudes (2.31), but not if the mass is finite. The $q\bar{q} \rightarrow p\bar{p}$ transition form factors $R_V(s)$, $R_A(s)$ and $R_P(s)$ were determined phenomenologically in [10]. In our calculation we neglect the term with $R_V(s)$ and assume $\frac{\sqrt{s}}{2m_p} \left| \frac{R_P(s)}{R_A(s)} \right| = 0.37$ (see formula (45) from [10]). In addition we take $R_A(s)$ and $R_P(s)$ as real and positive. We parametrize $R_A(s) = C_A/s$ (in parameter set A) with $C_A$ a parameter of dimension GeV$^2$ or $R_A(s) = \hat{C}_A/s^2$ (in parameter set B) with $\hat{C}_A$ a parameter of dimension GeV$^4$ which we shall determine from a fit to the Belle data in Sec. IV C. See Table II. Note that the $s$-dependence of $R_A$ with $C_A$ is different (less steep) than in [10], where only the hand-bag contribution was fitted to rather old experimental data. In [10] different phase conventions compared to ours are used. Taking this into account we find

$$\langle 2s_3,2s_4 | T | +, - \rangle_{hb} = 2s_4\bar{M}_{s_3s_4,+-},$$

$$\langle 2s_3,2s_4 | T | -, + \rangle_{hb} = 2s_4\bar{M}_{s_3s_4,--};$$

(2.33)

see Appendix C.

The hand-bag helicity amplitudes (2.33) must be added coherently within our approach (see previous subsections). At small momentum transfer $|t|$ or $|u|$ the hand-bag
and proton-exchange mechanisms compete and it would be a double counting to include both of them simultaneously. We emphasize, however, that in regions of small $|t|$ or $|u|$ the hand-bag approach has to be taken with a grain of salt. To avoid in addition double counting (we include explicitly the proton-exchange mechanism) we suggest to multiply the hand-bag amplitudes by a purely phenomenological factor:

$$ F_{\text{corr}}(t, u) = \left(1 - \exp\left(\frac{t}{\Lambda_{hb}}\right)\right) \left(1 - \exp\left(\frac{u}{\Lambda_{hb}}\right)\right) $$

with an extra free parameter $\Lambda_{hb}$. Its role is to cut off the region of small $|t|$ and $|u|$ where the hand-bag approach does not apply. As a consequence it also reduces the hand-bag contribution to the cross section at low $\sqrt{s}$ in the whole angular range.

### III. NUCLEAR REACTION

Now we will present theoretical formulas for the nuclear reaction

$$ ^{208}\text{Pb} + ^{208}\text{Pb} \rightarrow ^{208}\text{Pb} + ^{208}\text{Pb} + p + \bar{p} . $$

We focus on the processes for ultraperipheral collisions (UPC) of heavy ions, see the diagram shown in Fig. 1. The nuclear cross section is calculated in the equivalent photon approximation (EPA) in the impact parameter space. This approach allows to take into account the transverse distance between the colliding nuclei. The total (phase space integrated) cross section is expressed through the five-fold integral

$$ \sigma_{AA \rightarrow AAp\bar{p}} (\sqrt{s_{AA}}) = \int \sigma_{\gamma\gamma \rightarrow pp}(W_{\gamma\gamma}) N(\omega_1, b_1) N(\omega_2, b_2) S_{abs}^2(b) \times \frac{W_{\gamma\gamma}}{2} dW_{\gamma\gamma} dY_{p\bar{p}} d\bar{b}_x d\bar{b}_y 2\pi b \, db . $$

Above, $b = |b|$ is the impact parameter, i.e., the distance between colliding nuclei in the plane perpendicular to their direction of motion. $W_{\gamma\gamma} = \sqrt{4\omega_1\omega_2}$ is the invariant mass of the $\gamma\gamma$ system and $\omega_i, i = 1, 2,$ is the energy of the photon which is emitted...
from the first or second nucleus, respectively. \( Y_{\bar{p} p} = \frac{1}{2}(y_p + y_{\bar{p}}) \) is the rapidity of the \( p \bar{p} \) system. The quantities \( b_x = (b_{1x} + b_{2x})/2, b_y = (b_{1y} + b_{2y})/2 \) are given in terms of \( b_{1x}, b_{1y} \) which are the components of the \( b_1 \) and \( b_2 \) vectors which mark a point (distance from first and second nucleus) where photons collide and particles are produced. The diagram illustrating these quantities in the impact parameter space can be found in [20].

In Ref. [20] the dependence of the photon flux \( N(\omega_i, b_i) \) on the charge form factors of the colliding nuclei was shown explicitly. In our calculations we use the so-called realistic form factor which is the Fourier transform of the charge distribution in the nucleus. A more detailed discussion of this issue is given in [20].

The presence of the absorption factor \( S^2_{\text{abs}}(b) \) in (3.2) assures that we consider only peripheral collisions, when the nuclei do not undergo nuclear breakup. In the first approximation this geometrical factor can be expressed as

\[
S^2_{\text{abs}}(b) = \theta(|b| - (R_A + R_B)) = \theta(|b_1 - b_2| - (R_A + R_B)),
\]

(3.3)

where the sum of the radii of the two nuclei occurs.

In our present study we calculate also distributions in kinematical variables of each of the produced particles (for details how it is handled see [21]). Then one can impose easily experimental cuts on (pseudo)rapidities and transverse momenta.

**IV. RESULTS FOR THE \( \gamma\gamma \rightarrow p\bar{p} \) REACTION**

First we will show some features of the proton-exchange mechanism and the \( s \)-channel tensor meson exchanges. We will show the dependence of the cross section on the photon-photon energy and the angular distributions of individual helicity components. Then we will confront the model results with the experimental data and adjust the model parameters.

**A. Proton exchange mechanism**

In Fig. 3 we show that the proton exchange mechanism alone cannot describe the energy-dependence of the cross sections measured by Belle [5]. We show results for the Dirac- or Pauli-type couplings separately and when both couplings in the \( \gamma NN \) vertices are taken into account. We can see that the complete result indicates a large interference effect of Dirac and Pauli terms in the amplitudes. Clearly, the proton exchange contribution is not sufficient to describe the Belle data.

In Fig. 4 we show the unpolarized differential cross section \( d\sigma/d \cos \theta \) for three different \( \gamma\gamma \) c.m. energies. As one gets closer to \( \sqrt{s} = 2m_p \), the threshold energy, the angular distributions become flatter and flatter.

In Fig. 5 we present the helicity dependence of the differential cross section. We label the results for different helicity terms as \( (2s_3 2s_4 m_1 m_2) \) for \( (2s_3, 2s_4 | T | m_1, m_2) \) as defined in (A36). One can see the dominance of the \( (\pm \pm \pm \pm) \) and \( (\mp \mp \pm \mp) \) contributions over the \( (2s_3 2s_4 \pm \mp \mp) \) ones (see the red lines). In terms of the \( \psi_j (j = 1, ..., 6) \) from (A39) and Table V of Appendix A we find dominance of the amplitudes \( \psi_1 \) and \( \psi_2 \). Furthermore we see that the contributions of the amplitudes \( \psi_3, \psi_4, \psi_5 \) and \( \psi_6 \) are suppressed in the forward and backward directions, \( \cos \theta = \pm 1 \). This is clear from angular momentum.
FIG. 3: The $\gamma\gamma \rightarrow p\bar{p}$ cross section as a function of photon-photon energy $W_{\gamma\gamma} \equiv \sqrt{s}$. We present the results for the nonresonant contribution (see Sec. II A) for $\Lambda_p = 1.1$ GeV in (2.13). The solid line represents the complete result with both Dirac- and Pauli-type couplings included in the amplitude. Other combinations of electromagnetic couplings in the $\gamma NN$ vertices are also shown: only Dirac couplings, and only Pauli couplings at the two vertices in Figs. I(a) and (b). The Belle experimental data from [5] are shown for comparison.

FIG. 4: The angular distributions for $\sqrt{s} = 2.0$, 2.5 and 3.0 GeV for the nonresonant proton-exchange mechanism.

conservation. For $\psi_3$, $\psi_4$, and $\psi_5$, the state of the two photons has $J_z = \pm 2$. This cannot be reached by proton-antiproton produced in the forward or backward direction where we only get $J_z = 0$ or $\pm 1$. For $\psi_6$ the two-photon state has $J_z = 0$ and the two-baryon state in forward and backward direction has $J_z = +1$ and -1, respectively. We have again a mismatch. The contributions of four helicity states $(++--)$, $(+-++)$, $(++++)$, $(+-++-)$ vanish when only the Dirac-type coupling in the $\gamma NN$ vertices is included.
That is, the amplitude $\psi_6$ vanishes in this case.

\[ \gamma \gamma \rightarrow p\bar{p}, \text{ proton exchange, } \sqrt{s} = 2.0 \text{ GeV} \]

\[ \begin{align*}
(\pm \pm \pm), \psi_1 \\
(\mp \mp \mp), \psi_3 \\
(\pm \pm \pm), (\mp \mp \mp), \psi_5 \\
(\pm \pm \mp), (\pm \pm \pm), \psi_4 \\
(\mp \mp \mp), (\mp \mp \mp), \psi_7 \\
(\mp \mp \mp), (\mp \mp \mp), \psi_6
\end{align*} \]

\[ \begin{align*}
(\pm \pm \pm), \psi_1 \\
(\mp \mp \mp), \psi_3 \\
(\pm \pm \pm), (\mp \mp \mp), \psi_5 \\
(\pm \pm \mp), (\pm \pm \pm), \psi_4 \\
(\mp \mp \mp), (\mp \mp \mp), \psi_7 \\
(\mp \mp \mp), (\mp \mp \mp), \psi_6
\end{align*} \]

FIG. 5: The helicity components of $d\sigma/d\cos\theta$ as a function of $\cos\theta$ for the proton exchange mechanism for $\sqrt{s} = 2.0$ (the left panel) and 2.5 GeV (the right panel). Contributions of different helicities ($2s_3 2s_4 m_1 m_2$) of the photons and baryons are shown.

### B. $f_2$ meson contributions

The Belle experimental angular distributions \[5\], at least at low energies, cannot be described solely with the proton-exchange mechanism discussed in Sec. II A. It seems that a mechanism is missing. A resonant $s$-channel contribution is a reasonable option for a second mechanism (see also \[16\] for the $\gamma\gamma \rightarrow \pi\pi$ reactions).

In Table \[I\] we have listed resonances that decay into $\gamma\gamma$ and $p\bar{p}$ and which, therefore, may contribute to the reaction (2.1). In principle, also subthreshold resonances, such as $f_2(1270)$, may play some, even an important, role. It is worth to mention that our knowledge about the $f_2(1950)$ resonance comes from the BES \[22\] and the CLEO \[23\] analyses for $\psi(2S) \rightarrow \gamma p\bar{p}$ radiative decays. In \[23\] the authors include also the $f_2(2150) \rightarrow p\bar{p}$ contribution in order to describe the $M_{p\bar{p}}$ and $M_{p\gamma}$ invariant mass distributions. For $\psi(2S) \rightarrow \gamma p\bar{p}$ a stringent upper limit for the threshold resonance $B(\psi(2S) \rightarrow \gamma R_{thr}) \times B(R_{thr} \rightarrow p\bar{p}) < 1.6 \times 10^{-5}$ at 90% confidence level was found \[23\].

In our paper we consider only the $f_2$ meson exchanges in the $s$-channel. In general also the $c\bar{c}$ mesons (e.g. $\eta_c(1S)$, $\chi_{c0}(1P)$) may contribute to the reaction (2.1). The charmonium states have rather small total widths (see Table \[I\]) thus they will appear in the invariant mass distribution as rather narrow peaks; see \[25\] for the $\gamma\gamma \rightarrow \gamma\gamma$ reaction. Even interference effects with other mechanisms may be important in this context. This goes, however, beyond the scope of the present paper and will be studied elsewhere.

Now we will discuss the helicity structure of $\gamma\gamma \rightarrow p\bar{p}$ from the contribution of the $s$-channel (below-threshold or above-threshold) $f_2$ resonances in our Lagrangian approach; see Sec. II B.
TABLE I: A list of resonances that may contribute to the $\gamma\gamma \rightarrow p\bar{p}$ reaction. Here we listed also the subthreshold $f_2(1270)$ resonance. The meson masses, their total widths $\Gamma$ and branching fractions are taken from PDG [24].

| Meson | $m$ (MeV) | $\Gamma$ (MeV) | $\Gamma_{pp}/\Gamma$ | $\Gamma_{\gamma\gamma}/\Gamma$ |
|-------|-----------|---------------|----------------------|-----------------------------|
| $f_2(1270)$ | 1275.5 ± 0.8 | 186.7$^{+2}_{-2}$ | | (1.42 ± 0.24) $\times 10^{-5}$ |
| $f_2(1950)$ | 1944 ± 12 | 472 ± 18 | seen | seen |
| $\eta_c(1S)$ | 2983 ± 0.5 | 31.8 ± 0.8 | (1.50 ± 0.16) $\times 10^{-3}$ | (1.59 ± 0.13) $\times 10^{-4}$ |
| $\chi_c(1P)$ | 3414.75 ± 0.31 | 10.5 ± 0.6 | (2.25 ± 0.09) $\times 10^{-4}$ | (2.23 ± 0.13) $\times 10^{-4}$ |
| $\chi_{c2}(1P)$ | 3556.20 ± 0.09 | 1.93 ± 0.11 | (7.5 ± 0.4) $\times 10^{-5}$ | (2.74 ± 0.14) $\times 10^{-4}$ |
| $\eta_c(2S)$ | 3639.2 ± 1.2 | 11.3$^{+3}_{-2}$ | < 2 $\times 10^{-3}$ | (1.9 ± 1.3) $\times 10^{-4}$ |

In Fig. 6, we show the contributions of different helicities for the two $\gamma\gamma \rightarrow f_2$ couplings in (2.16), $a_{f_2\gamma\gamma}$ (left panel) and $b_{f_2\gamma\gamma}$ (right panel). There are five independent helicity contributions since here the contributions of the amplitudes $\psi_1$ and $\psi_2$ turn out to be the same; see (2.25), (A39) and Table V of Appendix A. Only the distributions that are proportional to $(\cos^2 \theta - 1/3)^2$, see (2.25) and (2.26), (this corresponds to the solid line in the left panel) are favored by the Belle experimental data; see Figs. 8 and 9 below. Here the cutoff parameter of form factors ($\Lambda_{f_2,pow}$) and the products of coupling constants $(a_{f_2\gamma\gamma}^{(j)} g_{f_2pp}^{(j)}$ and $b_{f_2\gamma\gamma}^{(j)} g_{f_2pp}^{(j)}$) are fixed arbitrarily.

FIG. 6: The helicity components of the differential cross sections $d\sigma/d\cos\theta$ for the $\gamma\gamma \rightarrow f_2(1950) \rightarrow p\bar{p}$ reaction for $\sqrt{s} = 2.1$ GeV. Here, the coupling constants are fixed arbitrarily, for $j = 1, 2$: $a_{f_2\gamma\gamma}^{(j)} g_{f_2pp}^{(j)} = \frac{e^2}{4\pi} 1$ GeV$^{-3}$ (the left panel) and $b_{f_2\gamma\gamma}^{(j)} g_{f_2pp}^{(j)} = \frac{e^2}{4\pi} 1$ GeV$^{-1}$ (the right panel). The calculations have been done for $\Lambda_{f_2,pow} = 1.15$ GeV in (2.29).
C. Comparison with the Belle data

Here we wish to demonstrate that it is possible to describe the Belle data taking into account the $t$- and $u$-channel proton exchanges, the $s$-channel tensor meson exchanges, and the hand-bag mechanism discussed in Sec. III. In the following we shall take in our calculation a coherent sum of all the above amplitudes.

In Fig. 7 we show the energy dependence of the cross section for the $\gamma\gamma \rightarrow p\bar{p}$ reaction. In the panel (a) we present results for the proton exchange and the $f_2(1270)$ and $f_2(1950)$ channels exchanges together with the experimental data of the CLEO [1], VENUS [2], OPAL [3], L3 [4], and Belle [5] experiments. An agreement between the Belle experimental data [5] and the earlier measurements [1, 2, 4] with the exception of the OPAL experiment [3] in the low mass region $W_{\gamma\gamma} = M_{p\bar{p}} < 3$ GeV can be observed (within the quoted uncertainties); see also Fig. 11 below. For the $f_2(1270)$ contribution the coupling constants $a_{f_2\gamma\gamma}$ and $b_{f_2\gamma\gamma}$ are relatively well known and taken from [13]. We take into account only one $f_2(1270) p\bar{p}$ coupling ($\delta_{f_2(1270)p\bar{p}}^{(1)} = 11.04$) and neglect the term with $\delta_{f_2(1270)p\bar{p}}^{(2)}$. For the $f_2(1950)$ contribution we take only the term with $a_{f_2(1950)\gamma\gamma}$ and $b_{f_2\gamma\gamma}$ are relatively well known and taken from [13]. We take into account only one $f_2(1270) p\bar{p}$ coupling ($\delta_{f_2(1270)p\bar{p}}^{(1)} = 11.04$) and neglect the term with $\delta_{f_2(1270)p\bar{p}}^{(2)}$. In the vertices for the meson exchange contributions we assume the same type of the form factors (2.29) and $\Lambda_{f_2,pow} = 1.15$ GeV; see Eqs. (2.27) and (2.30). We take $\Lambda_p = 1.08$ GeV for the proton-exchange contribution; see (2.13). One can observe the dominance of the $f_2(1950)$ resonance term at low energies. We slightly underestimate the Belle data from $\sqrt{s} = 2.4$ to 2.9 GeV. The panels (b) and (c) show results including also the hand-bag contribution. The hand-bag contribution is important at $W_{\gamma\gamma} > 3$ GeV. To illustrate uncertainties of our model we take in the calculation two sets of parameters. For the convenience of the reader we collect in Table III the parameters of our model and their numerical values used here and in the following.

In Figs. 8 and 9 we show our fits to the Belle angular distributions. Here we use the same parametrization as in Fig. 7(a) (see set A of Table II). In Fig. 8 we present results for the $f_2(1270)$, $f_2(1950)$ and proton-exchange contributions separately, as well as their coherent sum. At large angles, $\cos \theta \approx 0$, the inclusion of the $f_2(1270)$ contribution lowers the cross section compared to the case when only the $f_2(1950)$ and proton-exchange are taken into account. In Fig. 9 we show results including the hand-bag contribution. The $C_A$ parameter obtained from the fit is $C_A = 0.14$ GeV$^2$. In Fig. 10 we use, as in Fig. 7(c), the parameter set B of Table II. The $C_A$ parameter obtained from the fit is $C_A = 2.5$ GeV$^4$. In Ref. 10 $C_A$ was estimated to be in the range $4.9 \div 8.0$ GeV$^4$ which is the same order of magnitude as we find.

Experimentally the angular distributions were averaged over rather large intervals of (sub)process energies. For a better comparison with the experimental data we use the formula, with $z \equiv \cos \theta$,

$$\left< \frac{d\sigma}{dz}(W_{\gamma\gamma}) \right>_{\Delta W_{\gamma\gamma}} = \frac{1}{\Delta W_{\gamma\gamma}} \int_{W_{\gamma\gamma} - \Delta W_{\gamma\gamma}}^{W_{\gamma\gamma} + \Delta W_{\gamma\gamma}} \frac{d\sigma}{dz}(W_{\gamma\gamma}) dW_{\gamma\gamma},$$

instead of $\frac{d\sigma}{dz}(W_{\gamma\gamma}) = \frac{W_{\gamma\gamma,max} + W_{\gamma\gamma,min}}{2}$.  

\footnote{The cross section $d\sigma/d|z|$, $z = \cos \theta$, was calculated for the Belle angular range of $-0.6 < z < 0.6$, but plotted for $0 < z < 0.6$ after multiplication by a factor 2.}
TABLE II: Model parameters and their numerical values used. The second column indicates the equation numbers where the parameter is defined.

| parameter for $p\bar{p}$ | eq. | value (set A) | value (set B) |
|---------------------------|-----|---------------|---------------|
| $\kappa_p$                | (2.11) et seq. | 1.7928        | 1.7928        |
| $\Lambda_p$               | (2.12), (2.13) | 1.08 GeV      | 1.07 GeV      |

| $f_2(1270)$                | | | |
| $a_{f_2\gamma\gamma}$     | (2.16); (3.40) of [13] | $\frac{s^2}{4\pi}$ 1.45 GeV$^{-3}$ | $\frac{s^2}{4\pi}$ 1.45 GeV$^{-3}$ |
| $b_{f_2\gamma\gamma}$     | (2.16); (3.40) of [13] | $\frac{s^2}{4\pi}$ 2.49 GeV$^{-1}$ | $\frac{s^2}{4\pi}$ 2.49 GeV$^{-1}$ |
| $M_0$                      | (2.19) et seq. | 1 GeV | 1 GeV |
| $g_{f_2pp}^{(1)}$          | (2.19), (2.21) | 11.04 | 11.04 |
| $g_{f_2pp}^{(2)}$          | (2.20), (2.22) | 0 | 0 |
| $\Lambda_{f_2,\text{pow}}$| (2.29) | 1.15 GeV | 1 GeV |

| $f_2(1950)$                | | | |
| $a_{f_2\gamma\gamma}g_{f_2pp}^{(2)}$ | (2.16), (2.20), (2.22) | $\frac{s^2}{4\pi}$ 13.05 GeV$^{-3}$ | $\frac{s^2}{4\pi}$ 12 GeV$^{-3}$ |
| $b_{f_2\gamma\gamma}$     | (2.16) | 0 | 0 |
| $g_{f_2pp}^{(1)}$          | (2.19), (2.21) | 0 | 0 |
| $\Lambda_{f_2,\text{pow}}$| (2.29) | 1.15 GeV | 1.15 GeV |

| hand-bag contribution      | | | |
| $C_A$                      | $R_A(s) = C_A/s$ | 0.14 GeV$^2$ | 2.5 GeV$^4$ |
| $\tilde{C}_A$              | $R_A(s) = \tilde{C}_A/s^2$ | 0.85 GeV | 0.85 GeV |

In Fig. 11 we compare the Belle data [5] and the earlier OPAL and L3 data [3, 4] with our model results. Due to the large error bars of the OPAL and L3 data only the comparison of the model results with the Belle data gives significant information.

Having shown that the results of our approach, including three mechanisms, describe the Belle experimental data reasonably well we shall present our predictions for the nuclear reaction (3.1) in the next section.
FIG. 7: Energy dependence of the total cross section for $\gamma\gamma \to p\bar{p}$ for $|\cos \theta| < 0.6$. The experimental data are from the CLEO [1], VENUS [2], OPAL [3], L3 [4], and Belle [5] experiments. In the panel (a) we show the results for the tensor meson exchanges and the proton-exchange contributions, and their coherent sum (see the red solid line). In the panels (b) and (c) we show the results including, in addition, the hand-bag contribution. In the panels (a) and (b) we used the parameter set A while in the panel (c) we used the parameter set B; see Table II.
FIG. 8: Differential cross sections for the $\gamma\gamma \rightarrow p\bar{p}$ reaction as a function of $|\cos \theta|$ for different $W_{\gamma\gamma}$ ranges. For the Belle data [5] both statistical and systematic uncertainties are included. Calculations were done with $\Lambda_{f_2,\text{pow}} = 1.15$ GeV in (2.29), and $\Lambda_p = 1.08$ GeV in (2.13). The hand-bag model contribution is not included here. Here we used the parameter set A from Table [I].
FIG. 9: The same as in Fig. 8 but here the hand-bag contribution is included. The green dotted line shows the contribution of the hand-bag mechanism. Here we used the parameter set A from Table III.
FIG. 10: The same as in Fig. 9 but here we used the parameter set B from Table II.
FIG. 11: Differential cross sections for the $\gamma\gamma \rightarrow p\bar{p}$ reaction as a function of $|\cos \theta|$ for different $W_{\gamma\gamma}$ ranges. We compare our total model results (including the hand-bag contribution) with the Belle data [5], the L3 data [4], and the OPAL data [3]; see the black solid line, the red long-dashed line, and the blue short-dashed line, respectively. Here we used the parameter set A from Table II.
V. PREDICTIONS FOR THE NUCLEAR ULTRAPERIPHERAL COLLISIONS

Having described the Belle angular distributions we go to the predictions for the nuclear collisions. In this section we show the integrated cross sections and several differential distributions for the nuclear process (3.1) calculated as described in Sec. III including three mechanisms discussed in Secs. III and IV. In the calculations below we used the parameter set A from Table II.

FIG. 12: The distribution in $z = \cos \theta$, integrating over $2m_p < W_{\gamma\gamma} < 4$ GeV, for the $PbPb \rightarrow PbPbpp$ reaction at the $PbPb$ collision energy $\sqrt{s_{NN}} = 5.02$ TeV.

FIG. 13: Distribution in $(z, W_{\gamma\gamma})$ for the $PbPb \rightarrow PbPbpp$ reaction (3.1) at the LHC energy $\sqrt{s_{NN}} = 5.02$ TeV.

In Fig. 12 we present the angular distribution $d\sigma/dz (z = \cos \theta$ in the $\gamma\gamma$ c.m. system) at the $PbPb$ collision energy $\sqrt{s_{NN}} = 5.02$ TeV. Here we show the nuclear results when
the hand-bag mechanism is included (solid line) and excluded (dotted line). One can conclude that the hand-bag contribution does not play an important role in the $p\bar{p}$ angular distribution. We wish to emphasize that the enhancements at $z = \pm 1$ are the consequence of our model presented in Sec. II. One can better visualize this behavior with the help of the two dimensional distribution $d^2\sigma/dz dW_{\gamma\gamma}$. From Fig. 13 we clearly see that the result for the nuclear reaction corresponds to that for elementary $\gamma\gamma \rightarrow p\bar{p}$ reaction discussed in the previous section. The $f_2(1950)$ contribution dominates at smaller $W_{\gamma\gamma}$ and at $z \approx 0$ and $z \approx \pm 1$. This coincides with the result which was presented in Fig. 6 (left panel, solid line). In contrast to the resonant contribution, the proton-exchange one is concentrated mostly at larger invariant masses and around $z = 1$.

In Fig. 14 we present the nuclear differential cross sections for two ranges of $z$: the red lines are for $|z| < 0.6$, as in the Belle measurement, the black lines are for $|z| \leq 1$ (full
Panel (a) shows the distribution in proton-antiproton invariant mass ($M_{p\bar{p}} \equiv W_{\gamma\gamma}$). The $M_{p\bar{p}}$ distribution for the full $z$-range extends to much larger invariant masses while for the Belle $z$-range it falls steeply down. Similar as for the elementary cross section (Fig. 7), the hand-bag mechanism contributes significantly at $M_{p\bar{p}} > 3$ GeV. Simultaneously, the difference between the results with (solid lines) and without (dotted lines) hand-bag contribution appears more pronounced for the case when the angular phase space is narrowed. In the present calculations we integrate for $2m_p < W_{\gamma\gamma} < 4$ GeV. The transverse momentum distributions of protons and antiprotons shown in panel (b) are identical. Therefore we label them by $p_t$. For large $p_t$ the distributions fall steeply.

The limitation on the phase space ($|z| < 0.6$) has a significant impact for smaller values of $p_t$ and has no influence for $p_t > 1.4$ GeV. In the panel (c) we show distributions in rapidity of the proton or antiproton (which are identical). Here we see only a difference in the normalization, and not in the shape for the two different ranges of $z$. Finally, in the panel (d) we show the distribution in rapidity distance between proton and antiproton $y_{\text{diff}} = y_p - y_{\bar{p}}$. The larger the range of phase space the broader is the distribution in $y_{\text{diff}}$. There are three maxima when no extra cuts are imposed. The broad peak at $y_{\text{diff}} \approx 0$ corresponds to the region $|z| < 0.6$. It seems that observation of the broader $y_{\text{diff}}$ distribution, in particular identification of the outer maxima, could be a good test of our model. As we see from Fig. 12 the cross section decreases quickly with $W_{\gamma\gamma} = M_{p\bar{p}}$ for $|z| < 0.6$, but stays large for $|z| \rightarrow 1$. Thus, extending the integration to $W_{\gamma\gamma} > 4$ GeV should not change the distributions of Fig. 14 (b) - (d) for $|z| < 0.6$ but could have a sizeable influence on those for $|z| \leq 1$.

FIG. 15: The two-dimensional distributions in proton and antiproton rapidities for the reaction (3.1) at $\sqrt{s_{NN}} = 5.02$ TeV for two different $z$-ranges of outgoing nucleons. The results include the hand-bag contribution. The results are integrated for $2m_p < W_{\gamma\gamma} < 4$ GeV.

In Fig. 15 we show the two-dimensional distributions in $(y_p, y_{\bar{p}})$ again for two ranges of $z$ (left panel relates to the Belle angle limitation and right panel is for full phase space). The cross section is concentrated along the diagonal $y_p \approx y_{\bar{p}}$.

The ALICE Collaboration can measure $p\bar{p}$ in $Pb-Pb$ collisions for $|y| < 0.9$; see [26]
where the $J/\psi \rightarrow p\bar{p}$ decay was observed. We predict 46 events for $|y| < 0.9$ and $p_t > 1$ GeV for our $\gamma\gamma \rightarrow p\bar{p}$ contribution, including three mechanisms, for ALICE integrated luminosity $L_{int} = 95 \mu b^{-1}$.[26] On the other hand the coherent $J/\psi$ photoproduction [27] in the $p\bar{p}$ channel gives 583 events assuming approximately isotropic decay of $J/\psi \rightarrow p\bar{p}$. This strongly suggests dominance of the coherent photoproduction mechanism of $J/\psi$ over the $\gamma\gamma$ contribution. With such a transverse momentum cut as for the ALICE preliminary result a lot of the $\gamma\gamma \rightarrow p\bar{p}$ contribution is lost (with respect to the full phase space) but considerably less of coherent $J/\psi \rightarrow p\bar{p}$ contribution, where the maximum of the $p\bar{p}$ emission occurs at $p_t = \frac{m_{J/\psi}}{2} \approx 1.5$ GeV (sharp Jacobian peak associated with the fact that transverse momentum of the coherent $J/\psi$ is very small). Generally, the range covered by the ATLAS and CMS detectors for $p\bar{p}$ pairs in UPC is somewhat larger, $|y| < 2.5$. The LHCb Collaboration can measure $p\bar{p}$ production in nuclear collisions for $2 < \eta < 4.5$ and $p_t > 0.2$ GeV.[28]

In Fig. 16 we present distributions in $W_{\gamma\gamma} \equiv M_{p\bar{p}}$ (the left panel) and $y_{diff} = y_p - y_{\bar{p}}$ (the right panel) for the $PbPb \rightarrow PbPb p\bar{p}$ reaction (3.1). The results for different experimental cuts are presented.

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experiments.

For completeness, we give the cross sections for the \( \text{PbPb} \to \text{PbPb} p \bar{p} \) reaction for the \( \gamma\gamma \) contribution for various experimental cuts on proton and antiproton (pseudo)rapidities and transverse momenta at \( \sqrt{s_{NN}} = 5.02 \) TeV. We find the cross section of 100 \( \mu b \) taking into account the ALICE cuts (\(|y| < 0.9, p_t > 0.2 \) GeV), 160 \( \mu b \) for the ATLAS cuts (\(|y| < 2.5, p_t > 0.5 \) GeV), 500 \( \mu b \) for the CMS cuts (\(|y| < 2.5, p_t > 0.2 \) GeV), and 104 \( \mu b \) for the LHCb cuts (\(2 < \eta < 4.5, p_t > 0.2 \) GeV).

VI. CONCLUSIONS

We have discussed in detail the production of proton-antiproton pairs in photon-photon collisions. Previous theoretical papers on the subject tried to pick up only one simple mechanism out of many in principle possible ones. In our work we have tried to incorporate the known mechanisms, such as proton exchange, \( s \)-channel resonance exchange and the hand-bag contribution.

In our calculation of the nonresonant proton exchange we have included both Dirac- and Pauli-type couplings of the photon to the nucleon and form factors for the exchanged off-shell protons. We have found that the Pauli-type coupling is very important, enhances the cross section considerably, and cannot therefore be neglected.

We have shown that the Belle data \cite{5} for low photon-photon energies can be nicely described by including in addition to the proton exchange the \( s \)-channel exchange of the \( f_2(1950) \) resonance, which was observed to decay into the \( \gamma\gamma \) and \( pp \) channels \cite{24}. We include in the calculation also the \( s \)-channel \( f_2(1270) \) meson exchange contribution. These two tensor mesons were also needed to describe the Belle data for the \( \gamma\gamma \to \pi^+\pi^- \) and \( \gamma\gamma \to \pi^0\pi^0 \) processes \cite{16,28}. Our simple model has a few parameters; see Table II. Adjusting the parameters of the vertex form factors for the proton exchange, of the tensor meson \( s \)-channel exchanges, and of the form factor \( (2.34) \) in the hand-bag contribution we have managed to describe both total cross section and differential angular distributions of the Belle Collaboration with significantly better agreement with the data than in all previous trials.

Having described the Belle data we have used the \( \gamma\gamma \to p\bar{p} \) cross section to calculate the integrated cross section and differential distributions for production of \( p\bar{p} \) pairs in ultraperipheral, ultrarelativistic, collisions (UPC) of heavy ions at \( \sqrt{s_{NN}} = 5.02 \) TeV. We have presented distributions in rapidity and transverse momentum of protons and antiprotons, invariant mass of the \( p\bar{p} \) system as well as in the difference of rapidities for protons and antiprotons. We have presented results for the full angular range of \( z = \cos \theta \) as well as for the Belle range \(|z| < 0.6\). The integrated cross section for the full phase space is by a factor 5 larger than the one corresponding to the Belle angular coverage. The larger the range of phase space the broader is the distribution in \( y_{\text{diff}} \), the rapidity difference between proton and antiproton.

We have also made predictions for \( \text{Pb-Pb} \) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV and experimental cuts for the ALICE, ATLAS, CMS, and LHCb experiments. Corresponding total cross sections and differential distributions have been presented. The UPC of heavy ions may provide new information compared to the presently available data from \( e^+e^- \) collisions, in particular, if the structures of the \( y_{\text{diff}} \) distributions shown in Figs. 14 and 16 can be observed.
Acknowledgments

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Appendix A: Helicity states for protons and antiprotons and helicity amplitudes

The general theory of helicity amplitudes for collisions of particles with spin was developed in [29]. To make our article self-contained and to fix the phases of our states we discuss in the following the construction of helicity states for protons and antiprotons as we found convenient for our purposes. These states are then used to determine the independent helicity amplitudes for the reaction $\gamma\gamma \rightarrow p\bar{p}$ (2.1).

We consider protons and antiprotons in a fixed reference frame; see Fig. 17. Let $p$ be the 3-momentum of the proton and

$$\hat{p} = \frac{p}{|p|} = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix},$$

$$0 \leq \theta \leq \pi, \quad 0 \leq \phi < 2\pi. \tag{A1}$$

We use throughout our paper a boldface notation for 3-vectors, $p$, $e_z$ etc.

For $\hat{p} \cdot e_z \neq -1 (\theta \neq \pi)$ we define the spinors of definite helicity of type $a$ as

$$u^{(h,a)}_s(p) = \sqrt{p^0 + m_p} \begin{pmatrix} \chi^{(a)}_s(\hat{p}) \\ 2s \frac{|p|}{p^0 + m_p} \chi^{(a)}_s(\hat{p}) \end{pmatrix},$$

$$s \in \{+1/2, -1/2\}. \tag{A2}$$

![FIG. 17: Coordinate system and momentum vector $p$.](image-url)
where
\[
\chi_s^{(a)}(\hat{p}) = \frac{1 + 2s (\sigma \cdot \hat{p})}{\sqrt{2(1 + \hat{p} \cdot e_z)}} \chi_s^{(1)} ,
\]
\[
\chi^{(1)}_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \quad \chi^{(1)}_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .
\] (A3)

This gives
\[
\chi^{(a)}_{1/2}(\hat{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1 + \hat{p}_z} \\ \hat{p}_x + i \hat{p}_y \sqrt{1 + \hat{p}_z} \end{pmatrix} = \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix} ,
\]
\[
\chi^{(a)}_{-1/2}(\hat{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} -\hat{p}_x - i \hat{p}_y \sqrt{1 + \hat{p}_z} \\ \sqrt{1 + \hat{p}_z} \end{pmatrix} = \begin{pmatrix} -\sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix} ,
\] (A4)

\[
\bar{u}_{(h,a)}(p) u_{s(0,a)}(p) = 2m_p \delta_{rs} .
\] (A5)

Let us denote the usual spinors with spin in \pm z direction as
\[
u_r(p) = \sqrt{p^0 + m_p} \begin{pmatrix} \chi^{(1)}_r \\ \sigma \cdot p / (p^0 + m_p) \chi^{(1)}_r \end{pmatrix} ,
\] (A6)

\[ r \in \{+1/2, -1/2\} ; \]

see for instance [30].

We get then
\[
\left( \bar{u}_{(h,a)}(p) \right) u_{s(0,a)}(p) = 2m_p \left( B^{(a)}_{rs}(\hat{p}) \right) ,
\]
\[
B^{(a)}(\hat{p}) = \left( B^{(a)}_{rs}(\hat{p}) \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1 + \hat{p}_z} \\ \hat{p}_x + i \hat{p}_y \sqrt{1 + \hat{p}_z} \end{pmatrix} \begin{pmatrix} -\hat{p}_x - i \hat{p}_y \sqrt{1 + \hat{p}_z} \\ \sqrt{1 + \hat{p}_z} \end{pmatrix} = \begin{pmatrix} \cos \frac{\theta}{2} \\ -\sin \frac{\theta}{2} e^{-i\phi} \end{pmatrix} \begin{pmatrix} \sin \frac{\theta}{2} \cos \phi \\ \cos \frac{\theta}{2} \end{pmatrix} .
\] (A7)

Furthermore we define the creation operators for a proton in the helicity state \( s \) of type \( a \) by
\[
a^{(a)}_{h,a}(p, s) = a^{(a)}_r(p) B^{(a)}_{rs}(\hat{p}) ,
\] (A8)

where \( a^{(a)}_r(p) \) are the usual creation operators corresponding to the spinors (A6). We have then
\[
\bar{u}_{s(0,a)}(p) u_{s(0,a)}(p) = \bar{p} + m_p ,
\]
\[
a^{(a)}_{h,a}(p, s) \bar{u}_{s(0,a)}(p) = a^{(a)}_r(p) \bar{u}_r(p) .
\] (A9)
For $\hat{p} \cdot e_z \neq 1 (\theta \neq 0)$ we can define helicity spinors of type $b$ as follows

$$u_s^{(h,b)}(p) = \sqrt{p^0 + m_p} \left( \begin{array}{c} \chi_s^{(b)}(\hat{p}) \\ 2s |p| p^0 + m_p \chi_s^{(b)}(\hat{p}) \end{array} \right),$$

$$\chi_s^{(b)}(\hat{p}) = \frac{1 + 2s (\sigma \cdot \hat{p})}{\sqrt{2(1 - \hat{p} \cdot e_z)}} \chi_s^{(1)},$$

$s \in \{+1/2, -1/2\}$.

This gives

$$\chi_{1/2}^{(b)}(\hat{p}) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \hat{p}_x - i \hat{p}_y \\ \sqrt{1 - \hat{p}_z} \end{array} \right) = \left( \begin{array}{c} \cos \frac{\theta}{2} e^{-i\phi} \\ \sin \frac{\theta}{2} \end{array} \right),$$

$$\chi_{-1/2}^{(b)}(\hat{p}) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{1 - \hat{p}_z} \\ -\hat{p}_x + i \hat{p}_y \sqrt{1 - \hat{p}_z} \end{array} \right) = \left( \begin{array}{c} \sin \frac{\theta}{2} \\ -\cos \frac{\theta}{2} e^{i\phi} \end{array} \right).$$

Comparing with (A4) we find for $0 < \theta < \pi$

$$\chi_{1/2}^{(b)}(\hat{p}) = e^{-i\phi} \chi_{1/2}^{(a)}(\hat{p}),$$

$$\chi_{-1/2}^{(b)}(\hat{p}) = -e^{i\phi} \chi_{-1/2}^{(a)}(\hat{p}).$$

With $u_r(p)$ from (A6) we find

$$\left( \bar{u}_r(p) u_s^{(h,b)}(p) \right) = 2m_p \left( B_s^{(b)}(\hat{p}) \right),$$

$$B^{(b)}(\hat{p}) = \left( B_s^{(b)}(\hat{p}) \right) = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} \hat{p}_x - i \hat{p}_y & \sqrt{1 - \hat{p}_z} \\ \sqrt{1 - \hat{p}_z} & -\hat{p}_x + i \hat{p}_y \sqrt{1 - \hat{p}_z} \end{array} \right)$$

$$= \left( \begin{array}{cc} \cos \frac{\theta}{2} e^{-i\phi} & \sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & -\cos \frac{\theta}{2} e^{i\phi} \end{array} \right).$$

Defining creation operators analogous to (A8)

$$a_{h,b}^{\dagger}(p, s) = a_r^{\dagger}(p) B_s^{(b)}(\hat{p}),$$

we get

$$u_s^{(h,b)}(p) \bar{u}_s^{(h,b)}(p) = \hat{p} + m_p,$$

$$a_{h,b}^{\dagger}(p, s) \bar{u}_s^{(h,b)}(p) = a_r^{\dagger}(p) \bar{u}_r(p).$$
Now we go to antiprotons. For this we use the charge-conjugation matrix

\[ S(C) = i \gamma^2 \gamma^0 = -i \gamma_2 \gamma_0 = \begin{pmatrix} 0 & -\varepsilon \\ -\varepsilon & 0 \end{pmatrix}, \]

\[ \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \]  \hspace{1cm} \text{(A16)}

see for instance chapter 4 of [30]. We have

\[ S(C) = S(C) = -S^{-1}(C) = -S(C)^\dagger = -S(C)^T, \]

\[ S^{-1}(C) \gamma^\mu S(C) = -\gamma^\mu. \]  \hspace{1cm} \text{(A17)}

We define the antiproton spinors as

\[ \varphi_r(p) = u^T_r(p) S(C) = -\sqrt{p^0 + m_p} \left( -\chi^{(1)}_r \varepsilon \frac{\sigma \cdot p}{p^0 + m_p}, \chi^{(1)}_r \varepsilon \right), \]  \hspace{1cm} \text{(A18)}

\[ \varphi_s^{(h,a)}(p) = u^T_s^{(h,a)}(p) S(C) = -\sqrt{p^0 + m_p} \left( \chi^{(a)}_s (\hat{p}) \varepsilon 2s \frac{|p|}{p^0 + m_p}, \chi^{(a)}_s (\hat{p}) \varepsilon \right), \]  \hspace{1cm} \text{(A19)}

\[ \varphi_s^{(h,b)}(p) = u^T_s^{(h,b)}(p) S(C) = -\sqrt{p^0 + m_p} \left( \chi^{(b)}_s (\hat{p}) \varepsilon 2s \frac{|p|}{p^0 + m_p}, \chi^{(b)}_s (\hat{p}) \varepsilon \right); \]  \hspace{1cm} \text{(A20)}

see (A6), (A2), and (A10).

The creation operators for antiprotons are in the standard basis

\[ b^\dagger_r(p) = U(C) a^\dagger_r(p) U^{-1}(C), \]  \hspace{1cm} \text{(A21)}

where \( U(C) \) is the charge-conjugation operator. Analogously we define the creation operators for antiprotons of definite helicity

\[ b^\dagger_{h,a}(p, s) = U(C) a^\dagger_{h,a}(p, s) U^{-1}(C) = b^\dagger_r(p) B_{rs}^{(a)}(\hat{p}), \]  \hspace{1cm} \text{(A22)}

\[ b^\dagger_{h,b}(p, s) = U(C) a^\dagger_{h,b}(p, s) U^{-1}(C) = b^\dagger_r(p) B_{rs}^{(b)}(\hat{p}), \]  \hspace{1cm} \text{(A23)}

where we used (A8) and (A14).

With this we get

\[ \varphi_s^{(h,a)}(p) \varphi_s^{(h,a)}(p) = \hat{p} - m_p, \]  \hspace{1cm} \text{(A24)}

\[ \varphi_s^{(h,b)}(p) \varphi_s^{(h,b)}(p) = \hat{p} - m_p, \]

\[ \varphi_s^{(h,a)}(p) b^\dagger_r(p, s) = v_r(p) b^\dagger_r(p), \]

\[ \varphi_s^{(h,b)}(p) b^\dagger_r(p, s) = v_r(p) b^\dagger_r(p). \]  \hspace{1cm} \text{(A25)}

Now we come to the reaction (2.1). We consider (2.1) in the c.m. system with the \( x-z \) plane giving the reaction plane; see Fig. [18]. The usual kinematic variables are given...
FIG. 18: The reaction $\gamma \gamma \rightarrow pp$ in the c.m. system.

by (2.5). Let $e_x, e_y, e_z$ be the Cartesian unit vectors in the reference system of Fig. 18. Then $k_1^0 = k_2^0 = p_3^0 = p_4^0 = \frac{1}{2} \sqrt{s}$ and the momenta of the particles are

$$k_1 = -k_2 = |k_1| e_z,$$
$$p_3 = -p_4 = |p_3| (\sin \theta e_x + \cos \theta e_z),$$
$$|k_1| = \frac{1}{2} \sqrt{s},$$
$$|p_3| = \frac{1}{2} \sqrt{s - 4m_p^2}. \quad (A26)$$

As polarization vectors for the incoming photons of definite helicity we choose

$$\epsilon_1^{(\pm)} = \mp \frac{1}{\sqrt{2}} (e_x \pm i e_y),$$
$$\epsilon_2^{(\pm)} = \mp \frac{1}{\sqrt{2}} (e_x \pm i e_y). \quad (A27)$$

The corresponding photon creation operators are

$$a^\dagger(k_j, m) = \epsilon_j^{(m)} a^\dagger(k_j),$$
$$j = 1, 2, \quad m = \pm 1. \quad (A28)$$

For the proton we choose the helicity basis $a$, for the antiproton the basis $b$. From (A8) and (A23) we have for the corresponding creation operators

$$a_{h,a}^\dagger(p_3, s) = a_{r}^\dagger(p_3) B_{rs}^{(a)} (\hat{p}_3),$$
$$b_{h,b}^\dagger(p_4, s) = b_{r}^\dagger(p_4) B_{rs}^{(b)} (\hat{p}_4). \quad (A29)$$

Note that in calculating $B_{rs}^{(a)} (\hat{p}_3)$ from (A27) we have to make the replacements $\theta \rightarrow \theta$, $\phi \rightarrow 0$. Calculating $B_{rs}^{(b)} (\hat{p}_4)$ from (A13) we have to make the replacements $\theta \rightarrow \pi - \theta$, $\phi \rightarrow \pi$.

The symmetries of the reaction (2.1) are the following. The parity ($P$) transformation followed by a rotation by $\pi$ around the positive $y$-axis:

$$U_2(\pi)U(P). \quad (A31)$$
The charge-conjugation (C) transformation followed by a rotation by \( \pi \) around the positive \( y \)-axis:

\[ U_2(\pi)U(C). \]  \hfill (A32)

From the transformation laws of the standard creation operators (see e.g. \[30\]) and from the relations (see (A7), (A13))

\[ \epsilon^T B^{(a)}(\hat{\psi}_3) \epsilon = B^{(a)}(\hat{\psi}_3), \]
\[ \epsilon^T B^{(b)}(\hat{\psi}_4) \epsilon = -B^{(b)}(\hat{\psi}_4), \]  \hfill (A33)
\[ \epsilon B^{(a)}(\hat{\psi}_3) = -B^{(b)}(\hat{\psi}_4) \sigma_3, \]
\[ \epsilon B^{(b)}(\hat{\psi}_4) = B^{(a)}(\hat{\psi}_3) \sigma_3, \]  \hfill (A34)

we get the transformation laws for the helicity creation operators shown in Table III.

| \( A^\dagger \) | \( U_2(\pi)U(P)A^\dagger U_2^{-1}(\pi) \) | \( U_2(\pi)U(C)A^\dagger U_2^{-1}(\pi) \) |
|---|---|---|
| \( a^\dagger_3(p_3) \) | \(-a^\dagger_r(p_3) \epsilon_{rs} \) | \(-b^\dagger_f(p_4) \epsilon_{rs} \) |
| \( b^\dagger_3(p_4) \) | \(b^\dagger_f(p_4) \epsilon_{rs} \) | \(-a^\dagger_f(p_3) \epsilon_{rs} \) |
| \( a^\dagger_{h,a}(p_3, s) \) | \(-a^\dagger_{h,a}(p_3, r) \epsilon_{rs} \) | \( b^\dagger_{h,b}(p_4, r) (\sigma_3)_{rs} \) |
| \( b^\dagger_{h,b}(p_4, s) \) | \(-b^\dagger_{h,b}(p_4, r) \epsilon_{rs} \) | \(-a^\dagger_{h,a}(p_3, r) (\sigma_3)_{rs} \) |
| \( a^\dagger(k_1, m) \) | \(-a^\dagger(k_1, -m) \) | \(-a^\dagger(k_2, m) \) |
| \( a^\dagger(k_2, m) \) | \(-a^\dagger(k_2, -m) \) | \(-a^\dagger(k_1, m) \) |

TABLE III: Transformation properties of creation operators for protons, antiprotons, and photons under the transformations (A31) and (A32).

We define now the helicity states for the reaction (2.1) using (A28), (A29) and (A30) as

\[ |\gamma(k_1, m_1), \gamma(k_2, m_2)\rangle = a^\dagger(k_1, m_1)a^\dagger(k_2, m_2)|0\rangle, \]
\[ m_1, m_2 \in \{+1, -1\}, \]
\[ |p(p_3, s_3), \bar{p}(p_4, s_4)\rangle = a^\dagger_{h,a}(p_3, s_3)b^\dagger_{h,b}(p_4, s_4)|0\rangle, \]
\[ s_3, s_4 \in \{+1/2, -1/2\}. \]  \hfill (A35)

The transformation laws of these states are shown in Table IV.

| \( \ ) \ | \( U_2(\pi)U(P) \) | \( U_2(\pi)U(C) \) |
|---|---|---|
| \( |p(p_3, s_3), \bar{p}(p_4, s_4)\rangle \) | \( |p(p_3, r_3), \bar{p}(p_4, r_4)\rangle (c_3)_{rs,s} (\epsilon_3)_{rs,s} \) | \( |p(p_3, r_3), \bar{p}(p_4, r_4)\rangle (c_3)_{rs,s} (\epsilon_3)_{rs,s} \) |
| \( |\gamma(k_1, m_1), \gamma(k_2, m_2)\rangle \) | \( |\gamma(k_1, -m_1), \gamma(k_2, -m_2)\rangle \) | \( |\gamma(k_1, m_2), \gamma(k_2, m_1)\rangle \) |

TABLE IV: Transformation laws of the states (A35) under the transformations (A31) and (A32).

Finally we come to the helicity amplitudes for the reaction (2.1)

\[ \langle p(p_3, s_3), \bar{p}(p_4, s_4)|\mathcal{T}|\gamma(k_1, m_1), \gamma(k_2, m_2)\rangle \equiv \langle 2s_3, 2s_4|\mathcal{T}|m_1, m_2 \rangle, \]
\[ 2s_3, 2s_4, m_1, m_2 \in \{+1, -1\}, \]  \hfill (A36)
where we use the convenient shorthand notation of (2.8). There are 16 helicity amplitudes. The symmetry $U_2(\pi)U(P)$ (A31) gives the relation, using Table IV.

$$
\langle s_3, s_4 | T | m_1, m_2 \rangle = \langle r_3, r_4 | T | -m_1, -m_2 \rangle \epsilon_{r_3 s_3} \epsilon_{r_4 s_4}.
$$

(A37)

From the symmetry of $U_2(\pi)U(C)$ (A32) we get

$$
\langle s_3, s_4 | T | m_1, m_2 \rangle = \langle r_3, r_4 | T | m_2, m_1 \rangle (\sigma_3)_{r_3 s_4} (\sigma_3)_{r_4 s_5}.
$$

(A38)

The relations (A37) and (A38) are written explicitly for the helicity amplitudes in Table V. From this we find that there are only 6 independent helicity amplitudes for (2.1) which we choose as follows:

\begin{align*}
\psi_1(s,t) &= \langle + + | T | + + \rangle, \\
\psi_2(s,t) &= \langle + + | T | - - \rangle, \\
\psi_3(s,t) &= \langle + - | T | + - \rangle, \\
\psi_4(s,t) &= \langle + - | T | - + \rangle, \\
\psi_5(s,t) &= \langle + + | T | + - \rangle, \\
\psi_6(s,t) &= \langle + - | T | + + \rangle.
\end{align*}

(A39)

| $U_2(\pi)U(P)$ | $U_2(\pi)U(C)$ | $\psi$ |
|----------------|----------------|-------|
| $\langle + + | T | + + \rangle$ | $\langle + + | T | + + \rangle$ | $\psi_1$ |
| $\langle + - | T | + + \rangle$ | $\langle + - | T | + + \rangle$ | $\psi_6$ |
| $\langle - + | T | + + \rangle$ | $\langle - + | T | + + \rangle$ | $-\psi_6$ |
| $\langle - - | T | + + \rangle$ | $\langle - - | T | + + \rangle$ | $-\psi_2$ |
| $\langle + + | T | + - \rangle$ | $\langle + + | T | + - \rangle$ | $\psi_2$ |
| $\langle + - | T | + - \rangle$ | $\langle + - | T | + - \rangle$ | $\psi_5$ |
| $\langle - + | T | + - \rangle$ | $\langle - + | T | + - \rangle$ | $-\psi_3$ |
| $\langle - - | T | + - \rangle$ | $\langle - - | T | + - \rangle$ | $-\psi_5$ |
| $\langle + + | T | - + \rangle$ | $\langle + + | T | - + \rangle$ | $\psi_4$ |
| $\langle + - | T | - + \rangle$ | $\langle + - | T | - + \rangle$ | $\psi_6$ |
| $\langle - + | T | - + \rangle$ | $\langle - + | T | - + \rangle$ | $-\psi_1$ |
| $\langle - - | T | - + \rangle$ | $\langle - - | T | - + \rangle$ | $-\psi_3$ |

TABLE V: Helicity amplitudes for $\gamma\gamma \rightarrow p\bar{p}$ (2.1) and their symmetry relations.

With this we have obtained a complete overview of the general constraints of the helicity amplitudes of $\gamma\gamma \rightarrow p\bar{p}$ following from rotational, parity, and charge-conjugation invariance of strong and electromagnetic interactions.

Finally we note that the same analysis applies to any reaction

$$
\gamma + \gamma \rightarrow B + \bar{B}.
$$

(A40)
where $B$ stands for a spin 1/2 baryon. We only have to replace in all our formulas $m_p$ by $m_B$. Interesting examples may be $B = \Lambda, \Sigma^+, \Lambda^c$. The polarization of these baryons can be obtained from their decay distributions.

**Appendix B: The $lS$ coupling scheme and helicity amplitudes for the reaction $\gamma\gamma \to f_2 \to p\bar{p}$**

In this Appendix we discuss the relation of $lS$ couplings to the helicity amplitudes for the reaction $\gamma\gamma \to f_2 \to p\bar{p}$. Here $l$ stands for the orbital angular momentum and $S$ for the total spin of the $p\bar{p}$ system.

Let us see in how many ways one can construct a $p\bar{p}$ state with $J^{PC} = 2^{++}$. The partial-wave analysis (which is perfectly relativistic) says that we can combine the spins of $p$ and $\bar{p}$ to give the total spin $S = 0, 1$. Now we must combine this with the orbital angular momentum $l$ to the total angular momentum $J = 2$. This gives the four possibilities listed in Table VI. In general we have the parity of $p\bar{p}$ state $P = (-1)^{l+1}$ ($p$ and $\bar{p}$ have opposite intrinsic parity) and charge-conjugation $C = (-1)^{l+S}$. There are, thus, two possible $(l, S)$ couplings for $f_2(2^{++}) \to p\bar{p}$: $(1, 1)$ and $(3, 1)$.

| $l$ | $S$ | $J^{PC}$ |
|-----|-----|----------|
| 2   | 0   | 2$^-$   |
| 1   | 1   | 2$^{++}$|
| 2   | 1   | 2$^-$   |
| 3   | 1   | 2$^{++}$|

| TABLE VI: The $l$ and $S$ values leading to $p\bar{p}$ states with $J = 2$. |

We shall now analyze the $lS$ content of the $f_2 p\bar{p}$ couplings (2.19) and (2.20).

Let $u_r(p)$, $v_r(p)$ be the usual Dirac spinors with spin in $\pm z$ direction for $r = \pm 1/2$; see (A6) and (A18). For these we find in the c.m. system of reaction (2.1) the matrix elements of the vertex functions $\Gamma_{k^l}(f_2 p\bar{p})^{(j)}(j = 1, 2)$ (see (2.21), (2.22)) with $P^{(2)}_{k, l, k', l'}$ the spin 2 projector (the term in square brackets in (2.24)) as follows. For $j = 1$ we get

$$p^{(2)}_{k, l, k', l'}(p_3 + p_4)u_{r_3}(p_3)\Gamma_{k^l}(f_2 p\bar{p})^{(j)}(p_3, p_4) v_{r_4}(p_4) = 0$$

unless $k = k$, $l = l$, $k, l \in \{1, 2, 3\}$,

$$p^{(2)}_{k, l, k', l'}(p_3 + p_4)u_{r_3}(p_3)\Gamma_{k^l}(f_2 p\bar{p})^{(1)}(p_3, p_4) v_{r_4}(p_4)$$

$$= -\frac{4\delta^{(1)}_{f_2 p\bar{p}}}{M_0}F(f_2 p\bar{p})^{(1)}[(p_3 + p_4)^2]$$

$$\times \xi_{r_3} \{ -p_3^0 \left[ \frac{1}{2}p_3^k \sigma^l + \frac{1}{2}p_3^l \sigma^k - \frac{1}{3} \delta^{kl}(p_3 \cdot \sigma) \right]$$

$$+ \left[ \frac{1}{2}p_3^k p_3^l + \frac{1}{2}p_3^l p_3^k - \frac{1}{3} \delta^{kl} |p_3|^2 \right] \frac{1}{p_3^0 + m_p} (p_3 \cdot \sigma) \} \epsilon_{r_4}.$$  

5 The $\Lambda$ baryon has a magnetic moment $\mu_\Lambda = -0.613 \pm 0.004 \mu_N$. Thus, the reaction $\gamma\gamma \to \Lambda\bar{\Lambda}$ can proceed through the analogue of the diagrams of Fig.1(a) and (b).
Here and in the following we set \( \chi_r \equiv \chi_r^{(1)} \); see (A6) and (A18). For \( j = 2 \) we get

\[
p^{(2)\kappa\lambda\kappa'\lambda'}(p_3 + p_4) \bar{u}_{r_3}(p_3) \Gamma_{\kappa'\lambda'}^{(f_2p\bar{p})}(p_3, p_4) v_{r_4}(p_4) = 0
\]

for \( \kappa = 0 \), \( \lambda \) arbitrary and \( \kappa \) arbitrary, \( \lambda = 0 \);

\[
p^{(2)\kappa l\kappa'\lambda'}(p_3 + p_4) \bar{u}_{r_3}(p_3) \Gamma_{\kappa'\lambda'}^{(f_2p\bar{p})}(p_3, p_4) v_{r_4}(p_4)
\]

\[
= -\frac{8\hat{g}_{f_2p\bar{p}}^{(2)}}{M_0} F^{(f_2p\bar{p})}(2)[(p_3 + p_4)^2] \left[ p_3^k p_3^l - \frac{1}{3} \delta^{kl} |p_3|^2 \right] p_3 \cdot \chi^+_r \sigma \varepsilon \chi^*_r .
\]

The \( l\)-\( S \) amplitudes are as follows. For \( l = 1, S = 1 \) we have

\[
A_{(1,1)}^{kl} = \chi^+_r \left[ \frac{1}{2} p_3^k \sigma^l + \frac{1}{2} p_3^l \sigma^k - \frac{1}{3} \delta^{kl} (p_3 \cdot \sigma) \right] \varepsilon \chi^*_r .
\]

The traceless symmetric \( (l = 3) \) tensor is

\[
T_{3}^{klm} = p_3^k p_3^l p_3^m - \frac{1}{5} |p_3|^2 \left( \delta^{kl} p_3^m + \delta^{km} p_3^l + \delta^{lm} p_3^k \right) .
\]

This gives, for instance, with \( \theta \) as defined in Fig. 18 and \( P_3 \) the Legendre polynomial

\[
T_{3}^{klm} e_2^k e_2^l e_2^m = |p_3|^2 \left( \frac{5}{2} \cos^3 \theta - \frac{3}{2} \cos \theta \right) = |p_3|^2 \frac{2}{5} P_3(\cos \theta) .
\]

The \( l = 3, S = 1 \) the amplitude is

\[
T_{3}^{klm} \chi^+_r \sigma^m \varepsilon \chi^*_r = A_{(3,1)}^{kl}
\]

\[
= \left[ p_3^k p_3^l - \frac{1}{3} \delta^{kl} |p_3|^2 \right] \chi^+_r (p_3 \cdot \sigma) \varepsilon \chi^*_r
\]

\[
- \frac{2}{5} |p_3|^2 \chi^+_r \left( \frac{1}{2} p_3^k \sigma^l + \frac{1}{2} p_3^l \sigma^k - \frac{1}{3} \delta^{kl} (p_3 \cdot \sigma) \right) \varepsilon \chi^*_r .
\]

From (B2), (B4), (B5) and (B8) we get the \( l\)-\( S \) decomposition of our couplings \( j = 1 \) and 2 as follows:

\[
p^{(2)\kappa l\kappa'\lambda'}(p_3 + p_4) \bar{u}_{r_3}(p_3) \Gamma_{\kappa'\lambda'}^{(f_2p\bar{p})}(p_3, p_4) v_{r_4}(p_4)
\]

\[
= -\frac{4\hat{g}_{f_2p\bar{p}}^{(1)}}{M_0} F^{(f_2p\bar{p})}(1)[(p_3 + p_4)^2]
\]

\[
\times \left\{ -\left( \frac{3}{5} p_3^0 + \frac{2}{5} m_p \right) A_{(1,1)}^{kl} + \frac{1}{p_3^0 + m_p} A_{(3,1)}^{kl} \right\} ,
\]

\[
p^{(2)\kappa l\kappa'\lambda'}(p_3 + p_4) \bar{u}_{r_3}(p_3) \Gamma_{\kappa'\lambda'}^{(f_2p\bar{p})}(p_3, p_4) v_{r_4}(p_4)
\]

\[
= -\frac{8\hat{g}_{f_2p\bar{p}}^{(2)}}{M_0^2} F^{(f_2p\bar{p})}(2)[(p_3 + p_4)^2]
\]

\[
\times \left\{ \frac{2}{5} \left( (p_3^0)^2 - m_p^2 \right) A_{(1,1)}^{kl} + A_{(3,1)}^{kl} \right\} .
\]
Note that – for $\chi_{r3}$ and $\chi_{r4}$ not depending on $\theta - A_{(1,1)}^{kl}$ clearly has only $l = 1$ and $A_{(3,1)}^{kl}$ clearly has only $l = 3$; see (B5) and (B8), respectively.

But now we can go to the helicity amplitudes. All we have to do is to replace the two-component spinors as follows

$$\chi_{r3} \rightarrow \chi_{s3}^{(a)}(\hat{p}_3) \text{ from (A4)} \text{ with the replacements } \theta \rightarrow \theta, \phi \rightarrow 0,$$

$$\chi_{r4} \rightarrow \chi_{s4}^{(b)}(-\hat{p}_3) \text{ from (A11) with the replacements } \theta \rightarrow \pi - \theta, \phi \rightarrow \pi.$$

Note that these spinors depend on $\theta$.

We get

$$\begin{align*}
&\left(\chi_{s3}^{(a)\dagger} \varepsilon \chi_{s4}^{(b)*}\right) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = (\sigma_{ss}^3), \\
&\left(\chi_{s3}^{(a)\dagger} \sigma_1 \varepsilon \chi_{s4}^{(b)*}\right) = \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix}, \\
&\left(\chi_{s3}^{(a)\dagger} \sigma_2 \varepsilon \chi_{s4}^{(b)*}\right) = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \\
&\left(\chi_{s3}^{(a)\dagger} \sigma_3 \varepsilon \chi_{s4}^{(b)*}\right) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \\
&\left(\chi_{s3}^{(a)\dagger} \hat{p}_3 \cdot \sigma \varepsilon \chi_{s4}^{(b)*}\right) = \delta_{ss}, \\
&\hat{p}_3 = p_3 / |p_3|.
\end{align*}$$

(B12)

Inserting these expressions in (B2) and (B4) we see that the $p_3$ dependence, that is, the $\theta$ dependence of the amplitudes will in general be changed. Take, for instance, (B4) which is a combination of $l = 3$ plus $l = 1$; see (B5) and (B8). With the replacements (B11) we get from (B12)

$$\begin{align*}
&\left[p_3^k p_3^l - \frac{1}{3} \delta^{kl} |p_3|^2 \chi_{s3}^{(a)\dagger} p_3 \cdot \sigma \varepsilon \chi_{s4}^{(b)*}\right] = \left[p_3^k p_3^l - \frac{1}{3} \delta^{kl} |p_3|^2 \right] p_3 | \delta_{ss}. 
\end{align*}$$

(B13)

From $l = 3$ plus $l = 1$ we go, effectively, to $l = 2$.

The replacements (B11) lead from (B2) and (B4), using the expression for the diagram for $\gamma\gamma \rightarrow f_2 \rightarrow p\bar{p}$ [see Fig. 18(c)], to the helicity amplitudes (2.25) and (2.26).

**Appendix C: Phase conventions**

For the hand-bag contribution, Sec. II C we must take into account different phase conventions used in [10] relative to ours, as explained in Appendix A. In [10] the orientation of the particle momenta corresponds to a rotation by $\frac{\pi}{2} - \theta$ relative to the momenta in Fig. 18. Considering this we find that their spinors for proton and antiproton correspond to our $u_{s3}^{(h,a)}(p_3)$ and $-2s_4 v_{s4}^{(h,b)}(p_4)$, respectively. The phase conventions for the photons are not stated explicitly in [10]. From a comparison of the calculations (22)
and (23) of [10] with the corresponding ones with our conventions we conclude that the \( |\gamma(k_1, \pm), \gamma(k_2, \mp)\rangle \) states of [10] have an extra minus sign compared to ours. Taking everything together we obtain (2.33) for the amplitudes.

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