Magnetically induced chessboard pattern in the conductance of a Kondo quantum dot

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We quantitatively describe the main features of the magnetically induced conductance modulation of a Kondo quantum dot – or chessboard pattern – in terms of a constant-interaction double quantum dot model. We show that the analogy with a double dot holds down to remarkably low magnetic fields. The analysis is extended by full 3D spin density functional calculations. Introducing an effective Kondo coupling parameter, the chessboard pattern is self-consistently computed as a function of magnetic field and electron number, which enables us to quantitatively explain our experimental data.

A quantum dot [1] with a finite net electron spin strongly coupled to its leads, enabling higher-order cotunneling processes, is able to exhibit the Kondo effect [2,3]. The Kondo effect in quantum dots manifests itself as an enhanced conductance in the Coulomb blockade regime and occurs for temperatures and source-drain voltages below an energy scale set by the Kondo temperature. The first experimental results on the Kondo effect in quantum dots [4–6] were described with help of the spin-1/2 Anderson impurity model. Assuming continuous filling of spin-degenerate single-particle levels, the dot is expected to either have total electron spin \(S = 0\) (for electron number \(N\) is even) or \(S = 1/2\) (for \(N\) is odd). The Kondo effect is therefore only expected for odd \(N\), hence giving rise to an “even-odd-effect” in the Coulomb valley conductance. A wide range of experiments, however, has shown a clear deviation from this picture [7–16]. Particularly striking is the observation of a “chessboard pattern” in the dot conductance as a function of magnetic field, \(B\), and gate voltage, \(V_g\) [11,12,14–16]. Characteristic for this pattern is the alternation of high and low valley conductance regions as a function of \(B\) within the same Coulomb valley, i.e. for constant \(N\). In addition, the conductance also alternates when \(N\) is changed by sweeping \(V_g\) at constant \(B\). The distinct regions in the \(V_g,B\) plane of either high or low conductance are associated with the fields of a chessboard due to the similar appearance when the conductance is plotted in color scale (see Fig. 1b). It has been experimentally shown that the enhanced conductance in certain Coulomb blockade regions can be ascribed to the Kondo effect, for both \(N\) odd and even [11,12,14–16].

In this paper, we start by presenting our experimental data on a single lateral quantum dot clearly exhibiting the chessboard pattern. Next, we calculate the skeleton of the chessboard pattern with a constant interaction (CI) model of two capacitively interacting dots, formed by the two lowest Landau levels (LLs) – an analogy (partly qualitatively) applied before at high magnetic fields [12,18–20] – and obtain the characteristic hexag...

![FIG. 1. (a) Scanning electron micrograph of the device. The metal gates are yellow and the dot is indicated by a red circle. The ungated 2DEG has a mobility of 2.3 × 10^6 cm^2/(Vs) and an electron density of 1.9 × 10^{15} m^{-2} at 4.2 K. The nominal dot size is 320 × 320 nm^2. (b) Color scale plot of the experimental linear conductance \(G\) through the dot as a function of \(B\) and \(V_g\) at 10 mK. The dotted hexagons highlight the shape of a few chessboard fields. The dotted hexagons indicate some regions where suppression of the Coulomb peak occurs. (c) Calculated self-consistent potential landscape of the device. The white lines denote the contours of the metal gates.](image-url)
nal, double dot (DD) stability diagram [17]. Interestingly, in this work we find that, due to small $N$ and the shallow potential, higher LLs only start to fill below a few tenths of a tesla. Thus the DD analogy applies down to remarkably low magnetic fields. We finally perform full, 3D spin density functional (SDF) calculations for our device and introduce an effective Kondo coupling, derived entirely from the self-consistent results. Hence we can simultaneously calculate the electronic states of the dot and an estimate of the Kondo coupling, exhibiting the chessboard pattern, where now $\text{LL}_0$ and $\text{LL}_1$, are occupied, the outline of the chessboard pattern, and quantitatively explain some of its subtler features.

Our quantum dot is shown in Fig. 1a. Metal gates are deposited on top of a GaAs/AlGaAs heterostructure with a two-dimensional electron gas (2DEG) 100 nm below the surface [17]. By depleting the 2DEG below the gates, the quantum dot is defined. Current can flow from the source (S) to the drain (D) contact. The electron number is varied by sweeping the left gate voltage, $V_{gl}$. The SDF calculations show that our dot typically contains $\sim 20 - 40$ electrons.

Figure 1b shows a color scale plot of the linear conductance $G$ through the dot as a function of $B$ and $V_{gl}$. Red (blue) corresponds to large (small) $G$ (see scale in Fig. 1b). For the most negative values of $V_{gl}$ the coupling of the dot to the leads is weak. This results in relatively sharp Coulomb peaks (red lines) and low valley conductance (dark blue regions). However, if $V_{gl}$ is increased, the valley conductance reaches considerable values ($\sim e^2/h$) in certain regions of the $(B, V_{gl})$ plane. Most strikingly, the regions of low and high valley conductance alternate both along the $V_{gl}$ and the $B$ axis in a regular fashion, resulting in the aforementioned chessboard pattern. The $V_{gl}$ period ($\sim 10$ mV) is set by the energy required for adding an extra electron to the dot (addition energy), whereas the $B$ period ($\sim 0.1$ T) corresponds to adding a flux quantum to the effective dot area. Based on the temperature dependence of the high valley conductance regions (not shown here), we can ascribe the enhancement of $G$ to the Kondo effect. The transition from low to high valley conductance is associated with an abrupt jump of the $V_{gl}$ position of the Coulomb peaks. In some cases (see ellipses in Fig. 1b) the jump is accompanied by a suppression of the peak height. Below we present a quantitative description for the experimental features discussed above, using first an intuitive, though quantitative, approach followed by a fully self-consistent simulation of our device.

In the region where the two lowest LLs, labeled LL0 and LL1, are occupied, the outline of the chessboard pattern can be considered to be a DD (Fig. 2a) stability diagram, where now $B$ acts as one of the gates. $B$ couples to both “dots” (i.e. LLs) but with different “lever arms.” Similarly, $V_{gl}$ couples with different capacitances to the two LLs. We write an elementary CI functional for the total energy, $E$,

$$
E(N_0, N_1) = \sum_{nm} \varepsilon_{nm} + \frac{1}{2D} [C_{11}(N_0 e - C_{g0} V_{gl})^2 + C_{00}(N_1 e - C_{g1} V_{gl})^2]
- \frac{C_{01}}{D}(N_0 e - C_{g0} V_{gl})(N_1 e - C_{g1} V_{gl}),
$$

where $N_0(1)$ is the number of electrons in LL0(1), $\varepsilon_{nm} \equiv (2n + |m| + 1)\hbar \omega_c + m\hbar \omega_e$ are the Fock-Darwin (FD) energy levels [21], and the sum is over the lowest two LLs. A LL consists of all $n, m$ that satisfy LL index $\lambda = n + \frac{1}{2}(|m| + m) = \text{constant}$ (i.e. 0 or 1 for our specific case), and $\sigma$ is the spin index (since at the magnetic fields considered here the Zeeman splitting is much smaller than the other relevant energy scales, we assume spin-degenerate states). Here, $\omega_c^2 \equiv \omega_c^2 + \omega_e^2$ with $\omega_0$ the bare confining frequency and $\omega_c$ the cyclotron frequency. $C_{ij}$ are the capacitance matrix elements (off-diagonal elements are always negative [17,22]), $D \equiv C_{g0}C_{11} - C_{01}^2$. Here $V_{gl}$ is defined relative to the gate voltage that induces $N_{0(1)}^0$ electrons (i.e. $N_{0(1)}^0$ is the number of electrons in LL0(1) at $V_{gl}=0$).

![FIG. 2.](image-url) (a) Schematic of the dot in terms of two Landau levels (LLs). (b) Results of numerical minimization of Eq. 1. Color stripes are regions of constant $N = N_0 + N_1$. Numbers in parentheses show $N_0, N_1$. Black lines bound regions of constant $N_0$. Dotted lines show “$N_1$ boundaries”. The $N_1$ boundary lengths alternate (compare to Fig. 1b). Capacitances and dot parabohility estimated for $N_0^0 = 25$, $N_1^0 = 10$, $B_0 = 0.6 T$: $C_{00} = 3.3$, $C_{11} = 2.2$, $C_{01} = -2.1$, $C_{g0} = -0.10$ and $C_{g1} = -0.056$ (all in aF), $\omega_0 = 0.4$ meV. The dashed line is the approximate 3rd LL filling boundary. (c) Schematic honeycomb structure in the $B, V_{gl}$ plane resulting from the double dot model. Typical occupancy configurations (energy vs. position) of LL1 (gray arrows) and LL0 (black arrows) at the dot boundary are shown for different honeycomb cells.
First, we estimate capacitances $C_{ij}$ and bare confining potential $\omega_0$ from the full SDF calculations for $(N_0^a, N_0^b)$ electrons and for a “central” magnetic field $B_0$ (see caption, Fig. 2) [23]. Then, we numerically minimize Eq. 1 with respect to $N_0$ and $N_1$, keeping $C_{ij}$ and $\omega_0$ constant (even though in principle these change slightly over our $(B,V_{gl})$ range). We find a “honeycomb” structure characteristic for DDs [17]. We compute self-consistent eigenfunctions $\psi_{p,\sigma}$, occupancies $n_{p,\sigma}$ and tunneling coefficients $\gamma_{p,\sigma}$, with $p$ the orbital and $\sigma$ the spin indices, as well as the total interacting energy $F$ of the dot-gate-leads system, all as a function of $N$, $V_{gl}$ and $B$. However, we cannot compute the coherent Kondo-assisted conductance of the dot from the ground state properties provided by the SDF calculation. Instead, we introduce an effective Kondo parameter, which enables us to reproduce the conductance modulation characteristic for the chessboard pattern.

In Ref. [12] it was qualitatively argued that the alternating Kondo conductance with varying $B$ at fixed $N$ observed in their experiments in the strong edge state regime resulted from Coulomb regulated redistribution of electrons, one at a time, from LL1 to LL0. Since LL0 was assumed much more strongly coupled to the leads, the Kondo effect was argued to occur only when $N_0$ is odd. Here, the computed electronic structure allows us to quantitatively exhibit this depopolation process, to show how it varies in phase from one Coulomb valley to the next and to show how the $B$-dependent tunneling coefficients affect the structure of individual chessboard fields. The model’s assumption of spin-degenerate states is accounted for at the end of the paper.

In the standard derivation of the s-d model from the Anderson model [26], a Schrieffer-Wolff transformation is used to define an effective coupling between spin-degenerate states of the dot (or impurity) via virtual electron exchange with a neighboring Fermi sea. These co-tunneling processes then lead to coherent hybridization of the degenerate states, which results in a logarithmic increase of the conductance at low temperature, a feature of the Kondo effect in quantum dots. In general, a dot can contain many singly occupied electron states, for which spin-flip can occur via co-tunneling contributing to the Kondo coupling. In order to estimate the contribution of all singly-occupied states to the Kondo effect, we define a parameter $K$, which we call Kondo parameter, as the sum of all co-tunneling amplitudes that leave the ground state of the system unchanged except for the flip of a single spin

$$K \equiv \frac{1}{\sqrt{\gamma_{p,\sigma} \gamma_{p,\sigma^*}}} \sum_{p,\sigma} n_{p,\sigma} (1 - n_{p,\sigma}) \left( \frac{1}{F(N + 1,V_{gl} B)} - \frac{1}{F(N,V_{gl} B)} \right)$$

where $\sigma$ is the spin opposite to $\sigma$. We conclude from the experimental data that the spin polarization of the leads is negligible, in contrast to Ref. [24].

A color scale plot of $K$ in the $(B,V_{gl})$ plane is shown in Fig. 3, for $N = 32$ to 39. Here, the $V_{gl}$ dependence has been approximated as follows. The two denominators in Eq. 2 are additions energies, which are, to a good approximation, linear in $V_{gl}$ and vanish at the Coulomb oscillations. Thus, $F(N + 1,V_{gl} B) - F(N,V_{gl} B) \approx (e^2/2C) + e\alpha(V_{gl} - V_{gl}^{N,min}) + \varepsilon_{N+1}$, where $(e^2/2C) \equiv (\partial^2 F/\partial N^2)$ defines $C$, and $\partial F(N,V_{gl}^{N,min},B)/\partial N = 0$ defines the valley center gate voltage, $V_{gl}^{N,min}$, and where $\alpha = C_{dot-gate}/C$ is the lever arm (a similar analysis holds for the second energy denominator). By calculating the full electronic structure only near the valley centers, we can determine the CI parameters $V_{gl}^{N,min}$, $C$ and $\alpha$ and thereby show the $V_{gl}$ dependence of $K$ due to the
addition energies, Fig. 3. The full calculation of Eq. 2 on a mesh of $B, V_{gl}$ values, which is numerically taxing, is shown for a small region in the inset to Fig. 3.

The alternating pattern of Kondo “zones” in successive Coulomb valleys is evident. Even in this low magnetic field regime, the coupling of the LL0 states to the leads typically exceeds that of the LL1 states by two orders of magnitude. Therefore, although the parameter $K$ is a sum of all possible co-tunneling amplitudes, the amplitudes of the LL0 states dominate. Hence, even when $N$ is odd, $K$ is negligible as long as $N_0$ is even.

Within a Kondo zone an abrupt increase of $K$ is followed by a gradual decrease. This results from the contraction, with $B$, of the half-filled orbit at the dot edge, and the resulting decrease of its tunnel coefficient. When another electron depopulates from LL1 to that orbit, that spin flip process is no longer available, and $K$ collapses. Depopulation of LL1 coincides with a drop of the dot’s Fermi level [28] relative to the leads (Fig. 3, top panel). The denominators in Eq. 2 are responsible for the increase of $K$ away from the valley centers; a feature which is clearly observed in the experiment Fig. 1b.

The approximation used in Fig. 3 includes the $V_{gl}$ dependence of the charging energies, but not that of the $n_{p,σ}$. The latter dependence, specifically that of the transfer from LL1 to LL0, controls the slant of the chessboard fields. This slant only emerges in the calculation of $K$ without approximation, as shown for a small region of $V_{gl}$ and $B$ in the inset to Fig. 3.

Finally, we comment on our assumption of spin-degenerate states. For $N$ odd, this only requires that the Zeeman splitting ($\sim 10$ μeV) is negligible, which is the case in the $B$ range considered. However, for $N$ even, a singly filled orbital in LL0 implies another one in LL1 and these electrons have, in general, an exchange interaction. In Ref. [27] it was found that this exchange interaction diminishes rapidly with $N$. For our dot, we explicitly determine the singlet-triplet splitting for various (even) $N$ and $B$ by computing separate ground states constrained to $S = 0$ and $S = 1$. We find that the splitting is typically tens of μeV. The experimental signature of a split ground state is a split Kondo resonance (in source-drain voltage). Our analysis suggests that this splitting, which has been observed [9,15], would be characteristic of $N$ even in regimes where Zeeman energy is small.

We acknowledge financial support from the DARPA grant number DAAD19-01-1-0659 of the QuIST program.

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