Global Mittag-Leffler Synchronization of Fractional-Order Neural Networks With Time-Varying Delay via Hybrid Sliding Mode Control

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ABSTRACT In this paper, the global Mittag-Leffler synchronization (GMLS) for fractional-order neural networks with time-varying delay (FNNTVD) is studied based on hybrid sliding mode control (HSMC). First, a class of sliding surface is designed to study the dynamic behavior of FNNTVD by using the integer order Lyapunov direct method, which resolves the problem that the activation function of FNNTVD cannot be applied to the construction of Lyapunov function in the previous literature. Then, a new HSMC with the fixed signal transmission time delay is developed to ensure GMLS of FNNTVD. Next, based on Lyapunov direct method and the designed HSMC, some criteria are put forward to achieve GMLS of FNNTVD. Finally, two examples are given to verify the effectiveness of the proposed synchronization criteria for FNNTVD.

INDEX TERMS Fractional order, neural networks, sliding mode control, Mittag-Leffler synchronization, time-varying delay.

I. INTRODUCTION

Fractional order system, as a more general dynamic system, has attracted increasing concerns [1]–[5] owing to their special properties: (1) the fractional order parameters can help analyze the dynamic behavior of systems by adding a degree of freedom [1]–[3]; (2) fractional-order systems have merits of memory and hereditary [4], [5]. Therefore, fractional-order systems have been intensively applied in many fields [6]–[9]. Idiou et al. proposed a novel time-domain identification method by using fractional calculus to deal with a more difficult identification problem [6]. Liu et al. designed a modified quantum bacterial foraging algorithm for parameters identification of the fractional-order systems [7]. In [8], a novel fractional-order chaotic financial system is presented by considering market confidence into a three-dimensional financial system. In [9], fractional-order chaotic systems with completely unknown dynamics and structure are proposed to study power systems. Recently, based on the unique advantages of fractional calculus, fractional-order neural networks (FNNs), as the popular and important kind of networks, are attracted increasing concerns [10]–[15]. Among these studies, stability and synchronization of FNNs are always major topics and many sufficient conditions have been proposed for the FNNs with or without time delay. For example, in [13], finite-time stability problem for fractional-order complex-valued neural networks with time delay is discussed. Asymptotic and finite-time cluster synchronization criteria of coupled FNNs with time delay are proposed in [14]. In [15], the existence and local Mittag-Leffler (ML) stability of multiple equilibria are studied for a class of FNNs with discontinuous and nonmonotonic activation functions.

Indeed, compared to the time delay, the time-varying delay is more popular phenomena in the real networks, which may bring about some complicated dynamic behaviors like chaos and instability. Moreover, considering the limited speed of signal transmission in a network, time-varying delay is unavoidable and should be considered into the model. It is well known that integer order neural networks with time-varying delay have been deeply studied in [16]–[20]. However, only few studies explore the fractional-order neural networks with time-varying delay (FNNTVD) [21]–[29]
because the studies of FNNTVD are more complex than those without one. In [21], [22], Growall-Bellman inequality is introduced to obtain synchronization criteria for FNNTVD. In [23]–[29], while Lyapunov functions of the primary and the quadratic functions (e.g., \( V(t) = \sum_{i=1}^{n} |x_i(t)|, V(t) = x^T(t)Qx(t), \)) and so on.) are adopted to render some sufficient criteria for FNNTVD, these functions don’t consider time-varying delay information in FNNTVD. Thereby, all these criteria are very strict, missing a lot of information on time-varying delay. Thus, how to obtain lower conservative stability or synchronization criteria for FNNTVD is still a big challenge for researchers.

As we know, a reasonable and efficient controller is critical to achieving the stability or synchronization of FNNTVD. From recent results in [30]–[38], we may find that various controlling approaches are put forward, such as adaptive feedback control [30]–[32], active pinning control [33], [34], adaptive impulsive control [35], periodical intermittent control [36], adaptive fuzzy observer-based cooperative control [37] and sliding mode control (SMC) in [38]. Compared with the controllers in [30]–[37], SMC can deal with uncertainties and external disturbances more effectively [39]. Thereby, many scholars adopt SMC to study the dynamic performance of linear and nonlinear system [40]–[42]. For instance, in [40], a non-fragile observer-based adaptive SMC is designed to achieve stability for fractional-order Markovian jump systems with time delay and input nonlinearity. Furthermore, fixed-time synchronization problem of fractional-order memristive MAM neural networks is discussed by using SMC in [41]. In [42], adaptive SMC for a class of fractional-order chaotic systems is proposed to ensure the fuzzy neural network-based chaos synchronization.

From the previous works [40]–[42], two important aspects of SMC have not been considered yet, which are the influence of input time delay and how to reduce over-consumption of network resources. In practical applications, especially when communication speed and channels are limited, the input time delay is an unavoidable phenomenon, which may affect the input control performance, leading to the local oscillation or even the instability of whole system. In addition, in the previous works [40]–[42], since all the control schemes are the continuous-time feedbacks, they easily lead to the over-consumption system’s resources and the channel congestion. In order to avoid these problems, recently sampled-data control, as a classical the discrete-time feedback control, is introduced [43]–[45]. Because the sampled-data controller only uses the sampled information collected by the system in a certain period of time, it can effectively relieve the communication pressure. Therefore, how to design a controller that includes the advantages from SMC and sampled-data control is another motivation for this article.

Based on the above discussion, in this paper, the global Mittag-Leffler synchronization (GMLS) for FNNTVD is studied by using the integer order Lyapunov direct method (IOLDM) and hybrid sliding mode control (HSMC). The main contributions of this paper can be summarized as follows: (1) by designing a class of sliding surface with fractional integrals, this paper uses IOLDM to analyze the stability of fractional-order systems, overcoming the problem that the activation function cannot be applied in the previously constructed Lyapunov function and extending the application scope of IOLDM. (2) A new HSMC with the fixed signal transmission time delay is designed based on sampled-data control technique, which improves the operating speed and decreases the communication bandwidth. (3) By employing IOLDM and the designed HSMC, delay-dependent conditions for GMLS of FNNTVD are obtained, which are more feasible and less conservative comparing with the previous results.

The framework of this article is organized as follows. In Section II, FNNTVD and some preliminaries of fractional calculus are introduced. Section III proposes a novel HSMC and the sufficient criteria for GMLS. After that, two numerical examples are provided to demonstrate the efficiency of the proposed criterions in Section IV and a summary of the study is presented in Section V.

Notations: \( \mathbb{R}^n \) and \( \mathbb{R}^{n \times n} \) are the n-dimensional Euclidean space and the set of n-order square matrices, respectively. \( C^m([0, +\infty), \mathbb{R}) \) denotes Banach space with all continuous and m-order differentiable functions. \( E \) represents the identity matrices with proper dimensions. \( \lambda_M(A) \) denotes the largest eigenvalue of matrix A. The symbol \( \text{diag}(\ldots) \) denotes a diagonal matrix. \( A > 0 \) represents that A is a real positive definite matrix. In a symmetric matrix, * is used to describe the symmetric terms. For any square matrix A, we define \( \text{Sym}(A) = A^T + A \). If \( a \) is a real number, then \( |a| \) is the absolute value of \( a \). \( \|x\| = \left( \sum_{k=1}^{n} |x_k|^2 \right)^{\frac{1}{2}} \) denotes the norm of \( x \in \mathbb{R}^n \).

II. PRELIMINARIES AND MODEL DESCRIPTION

In this section, some preliminaries of fractional calculus and the system discussed in this study are presented.

A. FRACTIONAL CALCULUS

**Definition 1 [46]:** The \( \alpha \)-order integration of function \( f(t) \) is defined as

\[ I_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t - \tau)^{\alpha - 1} f(\tau) d\tau, \]

where \( \Gamma(\cdot) \) is Gamma function.

**Definition 2 [46]:** The Riemann-Liouville derivative of \( \alpha \)-order of the continuously differentiable function \( f(t) \) is proposed by

\[ R_{t_0}^{\alpha} D_{t}^\alpha f(t) = \frac{1}{\Gamma(1 - \alpha)} \frac{d}{dt} \int_{t_0}^{t} (t - \tau)^{-\alpha} f(\tau) d\tau, \]

where \( 0 < \alpha < 1, f \in C([t_0, +\infty), \mathbb{R}) \).
**Definition 3 [46]:** The Caputo derivative of \( \alpha \)-order of the continuously differentiable function \( f(t) \) is proposed by

\[
D_0^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{0}^{t} (t-\tau)^{-\alpha} f'(\tau) d\tau,
\]

where \( 0 < \alpha < 1, f \in C([t_0, +\infty), \mathbb{R}) \).

**Definition 4 [46]:** \( E_{\mu,\nu}(z) \) denotes the ML function if and only if

\[
E_{\mu,\nu}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k \mu + \nu)},
\]

where \( \mu, \nu > 0 \) and \( z \in \mathbb{C} \) where \( \mathbb{C} \) is the complex set. In particular, \( E_{\mu,\nu}(z) = E_{\mu,\nu}(1) \).

In this paper, the notation \( D^\mu \) represents the \( \alpha \)-order Riemann-Liouville derivative without particular statement.

**B. SYSTEM DESCRIPTION**

In this paper, the following class of FNNTVD is discussed as the drive system.

\[
\begin{aligned}
D^\mu x(t) &= -Dx(t) + Af(x(t)) \\
&+ Bg(x(t) - \tau(t)) + \Delta(t) + u(t), \quad x(\theta) = \phi(\theta), \quad \theta \in [-\tau, 0],
\end{aligned}
\]

where \( 0 < \alpha < 1, x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \) is a state vector; \( D = \text{diag}(d_1, d_2, \ldots, d_n) \) is a diagonal positive definite matrix; \( A = (a_{ij})_{n \times n} \in \mathbb{R}^{n \times n} \) and \( B = (b_{ij})_{n \times n} \in \mathbb{R}^{n \times n} \) are the connection weight matrices between nodes; \( f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))) \in \mathbb{R}^n \) and \( g(x(t) - \tau(t))) = (g_1(x_1(t) - \tau(t)), g_2(x_2(t) - \tau(t)), \ldots, g_n(x_n(t) - \tau(t))) \in \mathbb{R}^n \) are the state activation functions; \( \tau(t) \) stands for time-varying delay satisfying \( 0 \leq \tau(t) \leq \tau, \tau(t) = \tau(t), \tau > 0 \) and \( \tau \in \mathbb{R} \); \( \Delta(t) = (\Delta f_1(t), \Delta f_2(t), \ldots, \Delta f_n(t))^T \in \mathbb{R}^n \) represents an unknown uncertainty information in the FNNTVD; \( I = (I_1, I_2, \ldots, I_n)^T \) denotes the external disturbance of the system; \( \phi(\theta) \) is a continuous initial function.

The corresponding response system can be described as

\[
\begin{aligned}
D^\mu y(t) &= -Dy(t) + Af(y(t)) \\
&+ Bg(y(t) - \tau(t))) + \Delta(t) + u(t), \\
y(\theta) &= \psi(\theta), \quad \theta \in [-\tau, 0],
\end{aligned}
\]

where \( y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \) is the state vector; \( \Delta g(t) = (\Delta g_1(t), \Delta g_2(t), \ldots, \Delta g_n(t))^T \in \mathbb{R}^n \) represents an unknown model uncertainty term; \( u(t) \in \mathbb{R}^n \) is a control input; \( \psi(\theta) \in \mathbb{R}^n \) is an initial function.

**Assumption 1:** For any \( x, y \in \mathbb{R} \) and \( x \neq y \), the state activation functions \( g_i(x) \) and \( f_i(x) \) of model (1) are bounded and continuous, satisfying

\[
\left| \frac{g_i(x) - g_i(y)}{x - y} \right| \leq L^g_i, \quad \frac{f_i(x) - f_i(y)}{x - y} \leq L^f_i,
\]

where \( L^g_i, L^f_i \) and \( L^f_i \) are positive constants.

**Assumption 2:** Suppose the activation function \( f_i(x) \) is differentiable, then \( \hat{f}_i(x) \) is bounded, that is, \( |\hat{f}_i(x)| \leq \hat{L}^f_i \), where \( \hat{L}^f_i = \max(|L^f_i|, L^f_i) \). Denote \( \hat{L} = (\hat{L}^f_1, \hat{L}^f_2, \ldots, \hat{L}^f_n)^T \) and \( \hat{L}^f = \text{diag}(\hat{L}^f_1, \hat{L}^f_2, \ldots, \hat{L}^f_n) \).

Next, we consider GMLS between the drive system (1) and response system (2). Combining with drive-response systems (1) and (2), the error system can be described as follows:

\[
\begin{aligned}
D^\mu e(t) &= -De(t) + Af(e(t)) \\
&+ Bg(e(t) - \tau(t))) + \Delta(t) + u(t), \\
e(\theta) &= \psi(\theta), \quad \theta \in [-\tau, 0],
\end{aligned}
\]

where \( e(t) = y(t) - x(t), f(e(t)) \triangleq f(y(t)) - f(x(t)), g(e(t) - \tau(t))) \triangleq g(y(t) - x(t))), \Delta(t) = \Delta g(t) - \Delta f(t) \).

**Assumption 3:** Assume that the uncertainty term \( \Delta(t) \) satisfies the following inequality

\[
|\Delta(t)| \leq F_i, \quad F = \text{diag}(F_1, F_2, \ldots, F_n),
\]

where \( F_1 \) is a known nonnegative constant, \( i = 1, 2, \ldots, n \).

To obtain the main results, the following definition is proposed.

**Definition 5:** The ML synchronization for FNNTVD is achieved if there exist \( p, q > 0 \), such that

\[
\|e(t)\| \leq \frac{1}{2 \tau p - \alpha} E_{\alpha, \beta}(-q t),
\]

where \( e(t) \in \mathbb{R}^n, 0 < \alpha, \beta \leq 1, p \) and \( q \) are positive constants, respectively.

**III. GMLS OF FNNTVD BASED ON HYBRID SLIDING MODE CONTROLLER**

In this section, a novel HSMC scheme will be designed to guarantee the GMLS of the systems (1) and (2).

To investigate the GMLS of the error system (3), a simple sliding motion is proposed as

\[
s(t) = D^\alpha e(t).
\]

**Remark 1:** It is worth noting that the first-order differential of the sliding surface is equal to the equation of the error system. That is, by calculating the first-order derivative on both sides of (4), we can get

\[
\dot{s}(t) = D^\mu e(t) = -De(t) + Af(e(t)) \\
+ Bg(e(t) - \tau(t))) + \Delta(t) + u(t).
\]

To design the control scheme, the interval \( 0, +\infty \) is partitioned into \( \mathbb{T} = \{[t_k, t_{k+1}) \mid k \in \mathbb{N} \} \), where a sequence of times \( \{t_k \mid k \in \mathbb{N} \} \) satisfies \( 0 < t_{k+1} - t_k = h_k \leq h \) for all \( k \in \mathbb{N} \) and \( \lim_{k \to +\infty} t_k = +\infty \). We assume that the state vector \( x(t_k) \) of the FNNTVD at the sampling instant \( t_k \) is measurable.

Based on the SMC technique [40]–[42] and sampled-data feedback control approach [43]–[45], a hybrid or mixed SMC is designed to desire the GMLS for FNNTVD as follows

\[
u(t) = -(K_1 e(t_k) + K_2 e(t_k - \eta)) \\
+ Ws(t)) - Fsgn(s(t)), t \in [t_k, t_{k+1}).
\]
where $K_1, K_2$ and $W$ are designed matrices with appropriate dimension, $\eta$ is an invariable signal time delay, and $sgn(t)$ is defined by $|t| = t \times sgn(t)$ for all $t \in \mathbb{R}$.

**Remark 2:** Compared with the controller designed in [30]–[37], the designed HSMC not only has the properties of feedback SMC and sampled-data control, but also excludes $f(\cdot)$ and $g(\cdot)$ terms, keeping the property of the FNNTVD (3).

**Remark 3:** The designed controller (5) in this paper has two major advantages. First, SMC can effectively deal with external disturbances. Secondly, by designing the sliding surface (4) and HSMC (5), this paper uses IOLDM to the stability analysis of fractional-order systems, extending the application scope of the integer order Lyapunov method and overcoming the problem that the activation function cannot be used in the previously constructed Lyapunov function.

To obtain GMLS of the error system (3), we present the following theorems.

**Theorem 1:** Under Assumptions 1-3 hold. The error system (3) with control scheme (5) is GMLS for any given scalars $0 < \beta < 1, 0 < \gamma, 0 \leq \mu$ and $0 < \eta$, if there exist matrices $Q_1 > 0, R_1 > 0, K_j (i = 1, 2)$ and $W$ with appropriate dimensional and positive definite diagonal matrices $P_1, P_2 \in \mathbb{R}^{n \times n}$, satisfying the following condition:

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix} < 0,$$

where

$$\Sigma_{11} = \begin{bmatrix} W_1 & -D & 0 \\ * & W_2 & 0 \\ * & * & W_4 \end{bmatrix},$$

$$\Sigma_{13} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{3e^{-\gamma \eta \mu -1}}{h} R_2 & 0 \end{bmatrix},$$

$$\Sigma_{12} = \begin{bmatrix} -K_1 & -K_2 & A \\ 0 & -Q_2 & W_8 \\ 0 & W_6 & 0 \end{bmatrix},$$

$$\Sigma_{23} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{3e^{-\gamma \eta \mu -1}}{h} R_2 & 0 \\ P_1 - \frac{3e^{-\gamma \eta \mu -1}}{h} R_1 & 0 & \frac{3e^{-\gamma \eta \mu -1}}{h} R_1 \end{bmatrix},$$

$$\Sigma_{32} = \begin{bmatrix} W_7 & 0 & 0 \\ 0 & \frac{3e^{-\gamma \eta \mu -1}}{h} R_2 & 0 \\ * & \frac{3e^{-\gamma \eta \mu -1}}{h} R_1 & \frac{3e^{-\gamma \eta \mu -1}}{h} R_1 \end{bmatrix},$$

$$W_1 = \gamma E + \frac{B^TB}{\beta} - Sym(W),$$

$$W_3 = -e^{-\gamma \eta} Q_2 - \frac{4}{h} e^{-\gamma \eta \mu -1} R_2,$$

$$W_4 = \beta L^2 - (1 - \tau_M)e^{-\gamma \eta} Q_1,$$

$$W_6 = (1 - e^{-\gamma \eta}) Q_2 - \frac{4}{h} e^{-\gamma \eta \mu -1} R_2,$$

$$W_8 = \frac{L^- + L^+}{2} P_1, \quad \dot{W}_8 = \frac{L^- + L^+}{2} P_2,$$

$$W_9 = e^{-\gamma \eta} Q_2 - \frac{4}{h} e^{-\gamma \eta \mu -1} R_2,$$

$$\begin{aligned}
e_i &= \begin{bmatrix} 0_{6 \times (i-1)n} \\ I_n \end{bmatrix} \begin{bmatrix} 0_{6 \times (10-i)n} \end{bmatrix}, & i = 1, 2, \ldots, 10.
\end{aligned}$$

**Proof:** First, denote

$$v_1(t) = \frac{2}{t-t_k} \int_{t_k-\eta}^{t} e(u)du,$$

$$v_2(t) = \frac{2}{t-t_k} \int_{t_k-\eta}^{t} f(e(u))du,$$

$$\xi(t) = [s^T(t), e^T(t), e^T(t-\eta), e^T(t-\tau(t)), e^T(t_k), e^T(t_k-\eta), f^T(e(t)), f^T(e(t_k)), v_1^T(t), v_2^T(t)]^T.$$ (6)

Consider the following Lyapunov function

$$V(t) = \sum_{i=1}^{4} V_i(t), \quad t \in [t_k, t_{k+1}).$$ (7)

where

$$V_1(t) = e^{\nu^T s^T(t)} s(t),$$

$$V_2(t) = \int_{t-\tau(t)}^{t} e^{\nu^T u^T(t)} Q_1 e(u)du,$$

$$V_3(t) = \int_{t-\eta}^{t} e^{\nu^T u^T(t)} Q_2 e(u)du,$$

$$V_4(t) = \int_{t_k}^{t} f(e(v)) R_1 \dot{f}(e(v))dvdu$$

$$+ \int_{t-k}^{t} e^{\nu^T u^T(t)} R_2 \dot{e}(v)dvdu,$$

here $e(t) = e(t) - e(t_k - \eta)$ and $r_k = \mu + \ln \left( \frac{V_4(t_k)}{V_4(t_k)} \right)$.

The time derivative of $V_1(t)$ can be calculated by

$$\frac{dV_1(t)}{dt} = e^{\nu^T [s^T(t) s(t) + 2 \Delta^T (t) \Delta u(t)]}$$

$$= \gamma e^{\nu^T s^T(t) s(t)} + 2 e^{\nu^T s^T(t) \Delta u(t)} + A f(e(t)) + B g(e(t) - \tau(t)) + \Delta(t) + u(t)]].$$ (8)

Based on Assumption 1, there exists $\beta \in (0, 1)$ such that

$$\frac{dV_1(t)}{dt} \leq e^{\nu^T [s^T(t) (\gamma E + \frac{1}{\beta} B^T B) s(t)]}$$

$$+ \beta e^{\nu^T (t - \tau(t)) L^2 e(t - \tau(t)) - 2 \Delta^T (t) \Delta u(t)}$$

$$+ 2 e^{\nu^T (t) A f(e(t))} + 2 e^{\nu^T (t) \Delta u(t)}.$$ (9)

Combining control law (5) into (9), we get

$$\frac{dV_1(t)}{dt} \leq e^{\nu^T [s^T(t) (\gamma E + \frac{1}{\beta} B^T B) s(t)]}$$

$$+ \beta e^{\nu^T (t - \tau(t)) L^2 e(t - \tau(t)) - 2 \Delta^T (t) \Delta u(t)}$$

$$+ 2 e^{\nu^T (t) A f(e(t))} + 2 e^{\nu^T (t) \Delta u(t)} + F sgn(s(t))$$

$$- 2 \Delta^T (t) (K_1 e(t_k) + K_2 e(t_k - \eta) + W s(t)).$$ (10)
From Assumption 3, we can have
\[ s(t) \Delta s(t) \leq F[s(t)]. \]  
(11)

So, (10) can be rewritten as follows
\[
dV(t) \leq \exp^{\gamma T}(t)(\gamma E + \frac{1}{\beta}B^T B - 2W)s(t)
\]
\[
+ 2\exp^{T}(t)A(\xi(t)) + \beta^2 e(t) - \tau(t))L_2 e(t - t(\tau(t))]
\]
\[
+ 2\exp^{T}(t)K_1 e(t) - 2\exp^{T}(t)K_2 e(t - \eta(t))
\]
\[
eq \exp^{\gamma T}(t)[e_1(t)(\gamma E + \frac{1}{\beta}B^T B - 2W)e_1 + \beta e_1^T L_2 e_4
\]
\[
- \text{Sym}[e_1^T (De_2 - Ae_7 + K_1 e_5 + K_2 e_6)]\xi(t). \]  
(12)

Calculating the time-derivative of $V_2(t)$, $V_3(t)$ and $V_4(t)$, we can obtain
\[
\frac{dV_2(t)}{dt} \leq \exp^{\gamma t}(t)[e_1^T Q_1 e_2 - \frac{1}{\tau}Q_1 e_4]\xi(t), \]  
(13)
\[
\frac{dV_3(t)}{dt} \leq \exp^{\gamma t}(t)[Q_2 e(t) - \exp^{\gamma \tau}(t)\xi(t)]\xi(t). \]  
(14)
\[
\frac{dV_4(t)}{dt} \leq -e^{\gamma t - \tau} \int_{\tau - \eta}^{\tau} \hat{f}(e(v))R_2\hat{f}(e(v))dv \]  
(15)

We have the following inequalities according to Corollary 5 in [47]
\[
-\int_{\tau - \eta}^{\tau} \hat{f}(e(v))R_2\hat{f}(e(v))dv \leq \frac{3}{h}X_1(t)R_1 \chi_1(t)
\]
\[
-\frac{1}{h}(f(e(t)) - f(e(t_\eta)))^T R_1(f(e(t)) - f(e(t_\eta))), \]  
(16)
\[
-\int_{\tau - \eta}^{\tau} \hat{f}(e(v))R_2\hat{f}(e(v))dv \leq \frac{3}{h}X_2(t)R_2 \chi_2(t)
\]
\[
-\frac{1}{h}(e(t - \eta) - e(t_\eta))^T R_2(e(t - \eta) - e(t_\eta)), \]  
(17)

where $\chi_1(t) = (e_7 + e_8 - e_10)\xi(t)$, $\chi_2(t) = (e_3 + e_6 - e_9)\xi(t)$, namely
\[
\frac{dV_1(t)}{dt} \leq \exp^{\gamma t - \tau} \exp^{\gamma t}(t) \left[ -\frac{1}{h}(e_7 - e_8)^T R_1(e_7 - e_8)
\right.
\]
\[
-\frac{3}{h}(e_7 + e_8 - e_10)^T R_1(e_7 + e_8 - e_10)
\]
\[
-\frac{1}{h}e^{-\gamma \eta}(e_3 - e_6)^T R_2(e_3 - e_6)
\]
\[
-\frac{3}{h}e^{-\gamma \eta}(e_3 + e_6 - e_9)^T R_2(e_3 + e_6 - e_9) \right] \xi(t). \]  
(18)

Under Assumption 1, there exist diagonal matrices $P_1 > 0$ and $P_2 > 0$ such that
\[
\exp^{\gamma t}(t)[e_7 - L^+ e_7]^T P_1[L^+ e_7 - e_7]\xi(t) \geq 0, \]  
(19)
\[
\exp^{\gamma t}(t)[e_8 - L^+ e_8]^T P_2[L^+ e_8 - e_8]\xi(t) \geq 0. \]  
(20)

From (12)-(14) and (18)-(20), we obtain the following estimate the derivation of (7) for all $t \in [t_k, t_{k+1})$
\[
\dot{V}(t) \leq \exp^{\gamma t}(t)\xi(t) \leq \lambda_M(\Sigma)\exp^{\gamma t}(t)\xi(t). \]  
(21)

Therefore, the following inequality holds for $t \in [t_k, t_{k+1})$
\[
V(t) \leq V(t_k) \leq \cdots \leq V(0). \]  
(22)

Then, according to (7), we can obtain
\[
V(0) = s(0) + \int_{s(t_k)}^{0} \exp^{\gamma \tau}(\xi(t))Q_1 e(\tau)d\tau
\]
\[
+ \int_{0}^{\tau} \exp^{\gamma \tau}(\xi(t))Q_2 e(\tau)d\tau = \frac{1}{\Gamma(1 - \alpha)} \int_{t_k}^{t} (t - \tau)^{-\alpha} e(\tau)d\tau \]  
(24)

Therefore, $s(0) = 0$ and the following inequality holds
\[
V(0) \leq \lambda_M(\Sigma) \sup_{-\tau \leq \theta \leq 0} \{ ||e(\theta)||^2 \}
\]
\[
+ 4\lambda_M(\Sigma) \sup_{-\tau \leq \theta \leq 0} \{ ||e(\theta)||^2 \}
\]
\[
+ \frac{(\gamma \eta - 1 + e^{-\gamma \eta})\lambda_M(R_1)}{\gamma^2} \sup_{-\tau \leq \theta \leq 0} \{ ||e(\theta)||^2 \}
\]
\[
+ \frac{\gamma^2 \lambda_M(R_2)}{\gamma^2} \sup_{-\tau \leq \theta \leq 0} \{ ||e(\theta)||^2 \}], \]  
(25)

where $\lambda^2 = \lambda_M(\Sigma) + 4\lambda_M(\Sigma) + \frac{(\gamma \eta - 1 + e^{-\gamma \eta})\lambda_M(R_1) + \gamma^2 \lambda_M(R_2)}{\gamma^2} \lambda_M(R_2)$. Then we can get
\[
\exp^{\gamma \tau}(t)||s(t)||^2 \leq \lambda^2 \left( \sup_{-\tau \leq \theta \leq 0} \{ ||e(\theta)||^2, ||\dot{L}||, ||\dot{e}(\theta)||^2 \} \right). \]  
(26)

That’s to say,
\[
||s(t)|| \leq \lambda \left( \sup_{-\tau \leq \theta \leq 0} \{ ||e(\theta)||^2, ||\dot{L}||, ||\dot{e}(\theta)||^2 \} \right)^{1/2}. \]  
(27)

Combining (4) with (27), we can acquire that
\[
\exp^{\gamma \tau - 1} e(t) \leq \frac{\lambda}{\exp^{\gamma \tau}} \left( \sup_{-\tau \leq \theta \leq 0} \{ ||e(\theta)||^2, ||\dot{L}||, ||\dot{e}(\theta)||^2 \} \right)^{1/2}. \]  
(28)
For the $ith$ component of vector $e(t)$, we have

$$\lambda \sup_{-\tau \leq \theta \leq 0} \{ \| e(\theta) \|, \| \hat{L} \|, \| \hat{\dot{e}}(\theta) \| \} e^{-\frac{\lambda}{2} t}$$

Taking Laplace transform for both sides of (29), then the following inequality holds

$$\lambda \sup_{-s+\frac{\lambda}{2} \leq \theta \leq 0} \{ \| e(\theta) \|, \| \hat{L} \|, \| \hat{\dot{e}}(\theta) \| \} \leq \lambda \sup_{-s+\frac{\lambda}{2} \leq \theta \leq 0} \{ \| e(\theta) \|, \| \hat{L} \|, \| \hat{\dot{e}}(\theta) \| \}$$

Consequently, based on Laplace inverse transformation, we have the following inequality

$$\| e(t) \| \leq \sup_{-\tau \leq \theta \leq 0} \{ \| e(\theta) \|, \| \hat{L} \|, \| \hat{\dot{e}}(\theta) \| \} \frac{\lambda}{\Gamma(1-\alpha)} E_{1,\alpha}(-\frac{\lambda}{2} s).$$

From the inequalities (27) and (32), the sliding mode surface $s(t)$ and the system state trajectory $e(t)$ converge exponentially to zero. Therefore, the system (3) is GMLS. This finishes the proof.

In the proof process of Theorem 1, we find that the system (3) is still GMLS under Assumption 1 and Assumption 3.

**Theorem 2:** Suppose that Assumptions 1 and 3 hold. The system (3) with HSMC (5) is GMLS for given scalars $0 < \beta < 1$, $0 < \gamma$, $0 \leq \mu$ and $0 < \eta$, if there exist matrices $Q_i > 0$, $R_i > 0$, $K_i$ ($i = 1, 2$) and $W$ with appropriate dimension, and positive definite diagonal matrices $P_j \in \mathbb{R}^{n \times n}$ ($j = 1, 2, 3$) satisfying the following condition:

$$\tilde{\Sigma} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix} < 0,$$

where

$$\Sigma_{12} = \begin{bmatrix} -K_1 -K_2 A \\ 0 & -Q_1 \\ 0 & 0 \end{bmatrix},$$

$$\Sigma_{22} = \begin{bmatrix} W_5 0 0 \\ 0 W_6 0 \\ 0 0 -P_1 \end{bmatrix},$$

$$\Sigma_{23} = \begin{bmatrix} \hat{W}_8 0 0 \\ 0 \hat{W}_8 0 \\ 0 0 \frac{3e^{-\gamma \eta - \mu - 1}}{n} R_2 \end{bmatrix},$$

$$\tilde{\Sigma}_{33} = \begin{bmatrix} -P_2 0 0 \\ -P_3 0 \end{bmatrix} + \frac{3e^{-\gamma \eta - \mu - 1}}{n} R_2,$$

$$\hat{W}_6 = (1 - e^{-\gamma \eta})Q_2 - \frac{4}{h} e^{-\gamma \eta - \mu - 1} R_2 - L - P_3 L^+, \quad \hat{W}_7 = \frac{Q_2}{e^{\gamma \eta}} \frac{2}{h} e^{-\gamma \eta - \mu - 1} R_2, \quad \hat{W}_8 = \frac{L + L^+}{2} P_3,$$

and other notations are the same as Theorem 1.

**Proof:** Let $\xi(t) = [\xi^T(t), e^T(t), e^T(t - \tau(t)), e^T(t_k), e^T(t_k - \eta), f^T(e(t)), f^T(e(t_k)), f^T(e(t_k - \eta)), v^T(t)]^T$. Consider the following Lyapunov function

$$V(t) = \sum_{i=1}^{n} V_i(t) + \hat{V}_d(t), \quad t \in [t_k, t_{k+1}],$$

where $\hat{V}_d(t) = \int_{t_{k-\eta}}^{t_k} e^{\gamma \eta - \tau(t')} e^T(t') R_2 e(t') dvdu$. By Assumption 1, there exist diagonal positive definite matrices $P_i$, $i = 1, 2, 3$, such that

$$\xi^T(t) [e_1 - L^{-} e_1] P_1 [L^+ e_2 - e_1] \xi(t) \geq 0, \quad \xi^T(t) [e_2 - L^{-} e_2] P_2 [L^+ e_3 - e_2] \xi(t) \geq 0, \quad \xi^T(t) [e_3 - L^{-} e_3] P_3 [L^+ e_4 - e_3] \xi(t) \geq 0.$$

From (12)-(14), (18) and (34)-(36), we can obtain

$$\dot{V}(t) \leq e^{\gamma t} \xi^T(t) \tilde{\Sigma} \xi(t) \leq \lambda_M(\tilde{\Sigma}) e^{\gamma t} \xi^T(t) \xi(t).$$

The proof is similar to Theorem 1, thus it is omitted here for the sake of simplicity.

**Remark 4:** It is obvious that the above two theorems are proposed for all $\alpha \in (0, 1]$. The results, of course, are still valid for integer-order chaotic systems with system uncertainties and time-varying delays, which had been discussed in [16]–[20].

**Remark 5:** When $\tau(t)$ is a constant function, FNNTVD (3) turns into FNN with time delay, which were discussed in [13], [14]. Therefore, GMLS criteria of this paper are more general than the previous works.

**Remark 6:** From [23]–[26], synchronization criteria for FNNTVD has been proposed. However, it is different from that delay-dependent synchronization criteria are proposed by considering the effect of input time delay on FNNTVD in this paper. Thus, compared to the previous works [23]–[26], delay-dependent synchronization criteria are less conservative.

**Remark 7:** In light of Theorem 2, the parameters $K_1, K_2$ $W$ for HSMC can be designed. To facilitate implementation in practical application, the computational complexity of approach to achieve GMLS of FNNTVD (3) is considered. From [48], the computational complexity of approach Theorem 2 is solved as $D^3 L$, where the number of rows $L$ and decision variables $D$ in the linear matrix inequalities are $L = 10n$ and $D = 5n(n + 1)$.

**Remark 8:** According to the definitions of Riemann-Liouville derivative and Caputo fractional derivative, we have $\frac{D^\alpha f(t)}{D_t^\alpha} = \frac{f(\alpha t)}{\Gamma(\alpha)} \frac{D^\alpha f(t)}{D_t^\alpha} + \frac{f(t) - f(0)}{\Gamma(1-\alpha) \Gamma(\alpha)}$. Then,
Riemann-Liouville derivative and Caputo fractional derivative can be converted to each other. In order to further explore synchronization for FONNTVD based on Caputo fractional derivative, HSMC (5) is improved as follows for all \( t \in (t_\kappa, t_{\kappa+1}) \)

\[
u(t) = -K_1 e(t_\kappa) - K_2 e(t_\kappa - \eta) - F sgn(s(t)) - W s(t) - \frac{t^{-\alpha} e(0)}{\Gamma(1 - \alpha)} sgn(s(t)).
\]  
(38)

Thus, the proposed synchronization criteria in Theorems 1 and 2 are valid to achieve GMLS for FONNTVD based on Caputo fractional derivative.

Finally, we consider the specific system as follows

\[
\begin{align*}
\mathcal{D}^\alpha e(t) &= -De(t) + Af(e(t)) + Bf(e(t - \tau(t))) + \Delta(t) + u(t), \\
e(t) &= \phi(t) - \phi(t), \quad \theta \in [-\tau, 0].
\end{align*}
\]  
(39)

According to Theorem 1, we can directly obtain the criteria of GMLS for the system (39).

**Corollary 1**: Suppose that Assumptions 1-3 hold. The system (39) with control strategy (5) is GMLS for given scalars \( 0 < \beta < 1, 0 < \gamma, 0 \leq \mu \) and \( 0 < \eta, \) if there exist matrices \( Q_i > 0, K_i (i = 1, 2), R_2 > 0 \) and \( W \) with appropriate dimension, and positive definite diagonal matrices \( P_1, P_2 \in \mathbb{R}^{n\times n} \) satisfying the following condition:

\[
\Sigma = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\
* & \Sigma_{22} & \Sigma_{23} \\
* & * & \Sigma_{33}
\end{bmatrix} < 0,
\]

where

\[
\Sigma_{11} = \begin{bmatrix}
W_1 & -D & 0 & 0 \\
* & W_2 & 0 & 0 \\
* & * & W_3 & 0 \\
* & * & * & W_4
\end{bmatrix},
\]

\( W_4 = \beta L^2 - (1 - \tau_M) e^{-\gamma \tau} Q_1 \) and other notations are defined in Theorem 1.

**Proof**: The proof is similar to the Theorem 1, therefore it is omitted here.

**IV. NUMERICAL SIMULATIONS**

In this section, the validity and feasibility of the proposed criteria are demonstrated by two examples. The numerical simulations are performed by MATLAB software via the Adams-Bashforth-Moulton algorithm.

**A. EXAMPLE 1**

Here, we consider the three-dimensional FNNs in [49].

\[
\mathcal{D}^\alpha x(t) = -Dx(t) + Af(x(t)) + I,
\]  
(40)

where \( I = 0, g(s) = \tanh(s) + 0.05 \text{sign}(s), f(x(t)) = (g(x_1(t)), g(x_2(t)), g(x_3(t)))^T, x(0) = (2.5, -1.5, 0.8)^T \) and

\[
D = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad A = \begin{bmatrix}
2 & -1.2 & 0 \\
1.8 & 1.71 & 1.15 \\
-4.75 & 0 & 1.1
\end{bmatrix}.
\]

Consider the following response systems

\[
\mathcal{D}^\alpha y(t) = -Dy(t) + Af(y(t)) + I + u(t),
\]  
(41)

where \( u(t) = -K_1 e(t_\kappa) - K_2 e(t_\kappa - \eta) - W s(t) + Fs(t) + (2.5, -1.5, 0.8)^T \) and \( y(0) = (-2.2, 2.5, -3)^T \). System parameter are given as \( t_{\kappa+1} - t_\kappa = 0.05, \eta = 0.5, K_1 = K_2 = \text{diag}(0.1, 0.1, 0.1) \) and \( W = \text{diag}(17, 17, 17) \).

Here, as illustrated in Figs. 1 and 2, we consider \( \alpha = 0.75 \). Therefore, we can clearly know from Theorem 2 that the drive-response systems (40) and (41) are GMLS. Obviously, it is clear that the control input is effective and feasible in practice. Fig. 1 shows the state trajectories \( x(t) \) and \( y(t) \) of the drive-response systems (40) and (41) with the initial conditions \( x(t_0) = (2.5, -1.5, 0.8)^T \) and \( y(0) = (-2.2, 2.5, -3)^T \). In Fig. 2, the state error \( e(t) \) with initial values \( x(t_0) \) and \( y(t_0) \) converges to zero, which show that drive-response systems (40) and (41) are GMLS with \( \alpha = 0.75 \). By comparing the controller designed in [49], it is easy to find that the designed HSCM in this paper can not only reduce the controller’s information capacity, but also achieve GMLS of the drive-response systems (40) and (41) more effectively. Fig. 3 shows it takes less time for the system to achieve
synchronization under the proposed controller in this paper than [49].

### B. EXAMPLE 2

In this example, we consider the following more general three-dimensional FNNTVD.

\[
\begin{align*}
D^\alpha x(t) &= -Dx(t) + Af(x(t)) \\
+ &Bg(x(t - \tau(t)) + I + \Delta f(t),
\end{align*}
\]  
\( x(\theta) = \phi(\theta), \quad \theta \in [-\tau, 0], \)

where \( \alpha = 0.98, \tau(t) = 0.5 - 0.2\sin(t), I = 0, \\
\( x(t) = (x_1(t), x_2(t), x_3(t))^T, \Delta f(t) = B\phi(x(t)), \psi(s) = -0.9\cos(s) - 0.1\sin(s), \varphi(x(t)) = (\psi(x_1(t)), \psi(x_2(t)), \psi(x_3(t))), \phi(\theta) = (-2\tanh(0.5\theta), -4\cos(0.2\theta), 4\sin(\theta))^T, \)
\( g(x(t)) = (\cos(x_1(t)), \cos(x_2(t)), \cos(x_3(t)))^T, f(x(t)) = (\tanh(x_1(t)), \tanh(x_2(t)), \tanh(x_3(t)))^T, D = \text{diag}(1.25, 1.15, 1.22), \)
\( A = \begin{bmatrix} 2 & -1.2 & 0 \\
1.8 & 1.71 & 1.15 \\
-4.75 & 0 & 1.1 \end{bmatrix}, B = \begin{bmatrix} 0.3 & -0.24 & 0.1 \\
-0.26 & -0.3 & 0.23 \\
-0.15 & -0.4 & 0.2 \end{bmatrix} \)

By applying these parameters, Fig. 4 is obtained to show the dynamic trajectory of system (42). Then, the corresponding response system is given by

\[
\begin{align*}
D^\alpha y(t) &= -Dy(t) + Af(y(t)) \\
+ &Bg(y(t - \tau(t)) + I + u(t) + \Delta f(t),
\end{align*}
\]  
\( y(\theta) = \psi(\theta), \quad \theta \in [-\tau, 0], \)

where \( \psi(\theta) = (-2\sin(0.5\theta), -4\tanh(0.2\theta), 4\cos(\theta))^T. \)

Denote \( e(t) = y(t) - x(t), \) then, the controller is designed as

\[ u(t) = -K_1e(t_k) - K_2e(t_k - \eta) - Ws(t) - Fs(\eta)(s(t)), \]

where the time sequence and decay satisfy \( t_k - t_{k-1} = 0.1 \) and \( \eta = 0.5, \) respectively. By solving the LMIs of Theorem 1, the parameter matrices of the corresponding HSCM gains are \( K_1 = \text{diag}(0.0204, 0.0192, 0.02), \)
\( K_2 = \text{diag}(0.02, 0.02, 0.02) \) and \( W = \text{diag}(6.9225, 7.6530, 4.9455). \)

According to Theorem 1, we can obtain that the drive-response systems (42) and (43) are GMLS based on HSMC. From Figs. 5 and 6, we can clearly see that state vectors of the systems (42) and (43) show the synchronization behaviors. Obviously, it is clear that the control input is effective and feasible in practical applications. From Fig. 7, the error vectors of the drive-response systems (42) and (43) converge to zero, and we can conclude that the synchronous performance of the systems (42) and (43) is not affected by fractional order \( \alpha. \) From the discussion in Example 2, we can obtain that the proposed GMLS criteria in this paper are reasonable and feasible in practice.
FIGURE 7. Phase synchronization maximal error state $\max_{1 \leq i \leq 3} |e_i(t)|$ of the drive-response systems with different $\alpha$.

V. CONCLUSION

In this paper, GMLS has been discussed for the FNNTVD, which was more complex and challenging than the asymptotically complete synchronization of FNNs without time-varying delay. In order to deal with time-varying delay items more efficiently, IOLDM was applied to design sliding surface with fractional-order integral. Meanwhile, to guarantee GMLS for the FNNTVD, an adaptive HSMC with a fixed signal transmission time delay was designed. By constructing a class of Lyapunov functions and applying IOLDM, some GMLS criteria of FNNTVD were proposed. Finally, two numerical examples were presented to demonstrate the validity and feasibility of the proposed GMLS criteria. In future work, to further explore the proposed method, we will consider the more general fractional-order systems such as fractional-order complex-valued neural networks with time delay, fractional-order complex dynamical networks. From the perspective of cost saving, we will consider event-triggered control that can effectively save system resources in future work.

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