Thermodynamics education for energy transformation: a Stirling Engine experiment

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Abstract

We present a thermodynamics experiment suitable for first year undergraduate students employing Stirling Engines to create a demonstration of energy transformation and to measure the mechanical efficiency of such engines. Using an inexpensive transparent chambered Stirling Engine, students can connect concepts such as the theoretical pressure-volume diagram with the physical movements of the engine’s pistons and the resultant useful output work of a spinning wheel. We found the majority of students successfully complete this experiment obtaining results similar to when performed by the authors. In addition to the core thermodynamics lesson, this experiment incorporates DC circuits, oscilloscopes, and data analysis so it can be integrated into a wider undergraduate physics course to combine the teaching of multiple subjects.

Keywords: Stirling Engine, heat engine, thermodynamics, energy transformation, thermodynamics education

1. Introduction

Thermodynamics is a key topic in a contemporary physics degree. Yet core concepts such as heat and work are often conflated by students [1, 2]. Particularly for process functions such as work done through pressure and volume changes of a gas, students may have more difficulty accurately describing the mathematics within a physics context [3]. Conversely a classical mechanics view of mechanical work being the product of force and distance is typically introduced to students before university. Novel mental models and theoretical justifications as methods for teaching thermodynamic concepts has been the subject of much recent research [4–6]. Providing an
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experiment for students to perform in addition to a theoretical justification enables the conceptualization of energy transformation to be ‘anchored’. In a review article, Mulop et al [7] highlighted the difficulty students have in visualising thermodynamic concepts as a barrier to learning. Stated difficulties included applying concepts of thermodynamic processes to a real life power plant operation and theories understood as abstractions that have no real life application. Further, students typically can hold varied conceptions of energy transformation that are applied in different situations: the energy transformation of a ball rolling down a slope is conceptually different to the energy transformation involved in the forming and breaking of chemical bonds [8]. Energy transformation has an associated efficiency. The thermal efficiency of a heat engine is the proportion of heat that is transformed into work. There has been previous research into the use of Stirling Engines as demonstration tools for physics education focusing on measuring the thermal efficiency [9]. Further, there has been other research that focuses specifically on investigating the thermal efficiencies of Stirling Engines [10, 11]. From a perspective of thermal efficiency, a heat engine serves as an abstract system given heat is somewhat difficult to see. However, in addition to the thermal efficiency of a heat engine, there is also the mechanical efficiency to consider which is the proportion of outputted work that is used to drive an applied load. Investigating the mechanical efficiency is a possible alleviation of the conceptual barrier as the the thermodynamic process and its resultant output can be ‘seen’.

The novelty of the present work from an educational perspective is the use of a spinning wheel to determine the mechanical efficiency of a Stirling Engine. Students calculate this mechanical efficiency through the spinning frequency of an attached wheel compared to an input temperature difference. The external wheel and internal pistons of the Stirling Engine are fixed together ensuring motion comes directly from the piston strokes. With this, the work done by the external wheel can be compared with the work done by the internal gas to calculate the mechanical efficiency. Finding the mechanical efficiency enables students to develop multiple competencies including use of electric circuits, signal detection with oscilloscopes and data analysis. Technically the outputted work from the spinning wheel can be used for finding the thermal efficiency of the Stirling Engines if an instantaneous kinetic energy of the external wheel is used. However, there are more effective methods to measure thermal efficiency such as the one used in [9]. Secondly, undergraduate students tend to be more familiar with a classical mechanical view of work thus tying a clearly mechanical spinning wheel with the work done by a gas facilitates the aforementioned visual demonstration of energy transformation. The following section will detail the thermodynamic principles of Stirling Engines. Section 3 will provide an overview of the experimental procedure and section 4 will present results and analysis to determine the engine’s efficiency.

2. Stirling Engine thermodynamic principles

Stirling Engines are heat engines that operate through the cycling of a work fluid through expansionary and contractionary states to drive pistons that produce useful work. The expansion and contraction of the gas is driven by exposure to hot and cold plates. Through measuring the mechanical work of the internal gas and the useful mechanical work produced, it is possible to calculate the mechanical efficiency of a Stirling Engine.

The type of Stirling Engines of interest in this study are one chamber $\beta$ type engines with two dissimilar pistons shown in figure 1. Other types of Stirling Engines include $\alpha$ and $\gamma$, see [10] for details. Placing the Stirling Engine onto a cup of boiling water brings the bottom plate to temperature $T_H$ which heats up the air inside the Stirling Engine chamber causing it to expand. This expansion displaces the pistons and begins the cycle whereby the air cools and contracts when it is shuttled to the cold plate at temperature $T_C$. One piston serves as a displacer piston to shuttle the gas between the hot and cold plates and the other piston drives the rotation of the wheel.

2.1. Pressure—volume diagram of a Stirling Engine

The development of an analytical model is presented here and is suitable for an early
undergraduate physics course. To begin, we can express the thermodynamic cycle of a Stirling Engine in a Pressure–Volume ($P–V$) diagram. The four corners of the $P–V$ diagram represent the idealized four combinations of pressure and volume that define the borders between the thermodynamic processes involved in one cycle. In reality, the sharp corners of the $P–V$ diagram shown in figure 2 would be rounded but this idealized representation allows the stages to be understood more easily. The four stages of an idealized Stirling Engine are listed below.

(a) Isothermal expansion: The work fluid (air) expands at constant temperature, $T_H$, moving the work piston, the temperature remains constant and the pressure reduces.

(b) Isovolumetric cooling: The temperature of the enclosed air reduces to $T_C$ and the pressure drops further. The volume remains constant as the displacer piston moves the gas to the cold side of the engine.

(c) Isothermal compression: The volume of the gas reduces at constant temperature, $T_C$ and the pressure increases. Both pistons are pulled inward by the compressing gas displacing the gas to the hot side of the engine.

(d) Isovolumetric heating: The temperature of the gas increases from $T_C$ to $T_H$ whilst the volume remains constant.

The Stirling Engine shown in figure 1 can provide an effective way to visualise the stages of the $P–V$ diagram. As the chamber is transparent, students can identify the position of the displacer piston together with the work piston as the engine moves through multiple cycles. In practice however guidance from teachers is typically required for students to fully identify the specific positions of the pistons.
2.2. Theory

We are interested in determining the mechanical efficiency, \( \eta \), of the Stirling Engine; how much useful work (W), is performed by the engine through spinning the wheel compared to the net work done by the gas. Thus, \( \eta \) can be expressed as

\[
\eta = \frac{W_{\text{wheel}}}{W_{\text{gas}}}.
\]

The mechanical work done by the Stirling Engine wheel is defined as its rotational kinetic energy:

\[
W_{\text{wheel}} = KE_{\text{wheel}} = \frac{1}{2} I \omega^2.
\]

where I is the rotational inertia of the wheel and \( \omega \) is the angular velocity of the wheel. We have approximated the wheel as a disk of radius \( r \) and mass \( M \) with an angular velocity \( \omega = 2\pi f \). Equation (2) can thus be written as:

\[
W_{\text{wheel}} = \frac{1}{2} \left( \frac{1}{2} Mr^2 \right) \omega^2 = \frac{1}{2} \left( \frac{1}{2} Mr^2 \right) (2\pi f)^2 = Mr^2 \pi^2 f^2.
\]

Next, as depicted in figure 2, we can express \( W_{\text{gas}} \) as the heat entering the cycle in steps 1 \( \rightarrow \) 2 minus the heat leaving the cycle in steps 3 \( \rightarrow \) 4. We will assume that the gas equilibrates with the temperatures \( T_H \) and \( T_C \) of the bottom and top plates that are shown in figure 1. The work done by the gas is expressed through \( \int P \cdot dV \). Hence \( W_{\text{gas}} \) can be expressed as:

\[
W_{\text{gas}} = W_{1\rightarrow2} + W_{2\rightarrow3} + W_{3\rightarrow4} + W_{4\rightarrow1}
= \int P \cdot dV.
\]

However, in steps 4 \( \rightarrow \) 1 and 2 \( \rightarrow \) 3 the gas changes isochorically (\( \Delta V = 0 \)) thus the work is equal to zero:

\[
W_{4\rightarrow1} = W_{2\rightarrow3} = \int P \cdot dV = 0.
\]

In step 1 \( \rightarrow \) 2 (3 \( \rightarrow \) 4), the pressure and volume change simultaneously during isothermal expansion (compression). We can use the ideal gas law, \( PV = nRT \), to derive the work done by the gas. For 1 \( \rightarrow \) 2, equation (5) becomes

\[
W_{1\rightarrow2} = \int_{V_1}^{V_2} P \cdot dV = \int_{V_1}^{V_2} \frac{nRT_H}{V} \cdot dV
= nRT_H \cdot \ln \frac{V_2}{V_1}
\]

and in step 3 \( \rightarrow \) 4, equation (5) becomes

\[
W_{3\rightarrow4} = \int_{V_3}^{V_4} P \cdot dV = \int_{V_3}^{V_4} \frac{nRT_C}{V} \cdot dV
= nRT_C \cdot \ln \frac{V_4}{V_3}.
\]

The gas inside of the Stirling Engine that performs the work is termed the work fluid. The ratio of the maximum volume of work fluid, \( V_2 \), to the minimum volume of the work fluid, \( V_1 \), is called the compression ratio, \( C_R \). In the case of the Stirling Engine we used, shown in figure 1, this is the ratio of the maximum volume of air between the plunger and hot plate during the cycle divided by the maximum volume of air between the plunger and cold plate during the cycle. Using equations (6) and (7), equation (4) can be written as:

\[
W_{\text{gas}} = W_{1\rightarrow2} + W_{3\rightarrow4}
= nRT_H \cdot \ln \frac{V_2}{V_1} + nRT_C \cdot \ln \frac{V_1}{V_2}
= nRT_H \cdot \ln C_R + nRT_C \cdot \ln \frac{V_1}{V_2}
= nR(T_H - T_C) \cdot \ln C_R.
\]

The thermal efficiency can be determined using equation (8) as described in appendix A. However, for this student investigation, we are interested in the mechanical efficiency. As the rotation of the wheel is mechanically fixed to the cycling of the cylinders where one cycle of the cylinders corresponds to one on the wheels and vice-versa. This can be derived by substituting equations (3) and (8) into equation (1):

\[
\eta = \frac{Mr^2 \pi^2 f^2}{nR(T_H - T_C) \cdot \ln C_R}.
\]
A final form relevant for the experiment can be written in terms of measured variables of $f$ and $\Delta T = T_H - T_C$.

$$f^2 = \frac{nR \cdot \ln C_R}{Mr^2 \cdot \pi^2} \cdot \eta \Delta T.$$  \hspace{1cm} (10)

Hence, equation (9) can be compared to a linear function $y = mx + c$ in completing a regression analysis to determine the efficiency, $\eta$. This is described further in section 4.

3. Experimental procedure

This experiment requires the following equipment: a Stirling Engine; a light gate, to measure the engine’s rotation; a breadboard with approximately ten wires and two 100 Ω resistors; a benchtop power supply; an oscilloscope; two thermocouple probes with tape attached to them; a cup and a kettle to boil water. As shown in figure 3, the Stirling Engine is mounted on a cup of boiling water that serves as a hot source to create a temperature difference across the chamber of the Stirling Engine. Thermocouples are placed on the top and bottom of the Stirling Engine to measure this temperature difference. A Light Gate is mounted across the wheel to detect its rotation. The light gate is powered by a simple DC circuit shown in figure 4.

For our student’s experiment, the Stirling Engines were purchased from a supplier on Amazon.com for £30 each. The wheel of the engine weighed 65.20 ± 0.01 g and its radius was 4.35 ± 0.01 cm. Through deconstructing our Stirling Engine and measuring the internal dimensions of the chamber and plunger, we calculated the volume of the internal air and used the density of air to calculate its weight to be 0.1 g. Taking the molar mass of air to be 28.97 g mol$^{-1}$ we found $n$ in equation (9) to be 3.45 mmol of gas. Students determine a value of $C_R$ by measuring the relevant maximum distances of the displacement pistons relative to the hot and cold plates. These values describing the engine are summarised in table 1.

The experiment is performed through initially filling a cup with boiling water and then immediately placing a room temperature Stirling Engine on top of the cup. Within 90 s the Stirling Engine wheel begins to turn, accelerating rapidly as a temperature differential between the hot and cold plates is now present. Sometimes, a slight push on the wheel is required to overcome its static inertia. The light gate is placed to allow the wheel spokes of the engine to cut across it. When $V_{out}$ is measured via an oscilloscope, the passage of the spokes through the Light Gate will appear as distinct steps in the waveform. While digital oscilloscopes can measure the frequency of this waveform automatically, students should be encouraged to perform manual measurements of $f$ to build confidence in the data. For example, students can measure the time between six peaks to determine the rotational frequency. As its role is to measure the frequency, the oscilloscope in this experiment could be replaced with a computer with an appropriate I/O device. The separate digital thermometer could be replaced in a similar manner.

As the cup of boiled water cools down, the temperature difference across the chamber of the Stirling Engine reduces and the speed of rotation slows. This creates a decrease in the independent variable: the temperature difference $\Delta T$. The student’s data acquisition will be recording the corresponding rotational frequency, $f$, for a range of $\Delta T$. The subsequent analysis of this data together with equation (10) can be used to calculate the engine efficiency $\eta$.

4. Measurements and analysis

As described in section 3, the variation of rotational frequency with temperature difference comprise the main data for this experiment. As the hot source gradually cools, students can record this data, $f(\Delta T)$, at regular intervals until $\Delta T$ is sufficiently low whereupon the engine stops. Students are encouraged to repeat the experiment a number of times to enable a statistical analysis to be performed. Given the likely imperfections of these engines, together with slight variations in method, this repeat data will serve to demonstrate statistical variation to students.

Recording repeat data sets can involve two approaches. The first approach is to record $f(\Delta T)$, at distinct values of $\Delta T$. Here students would need to be attentive to record $f$ at the same values of $\Delta T$ during each repeat as the hot source cools, for example at $\Delta T = 60.0 K, 50.0 K, 40.0 K$ etc. The difficulty here is trusting that the system
will conveniently return to these precise values in subsequent experiments. The second approach is to simply measure \( f \) at arbitrary values of \( \Delta T \) and to do at more regular intervals for example at \( \Delta T = 60.20\,\text{K}, 59.31\,\text{K}, 58.10\,\text{K} \) etc. Here the measurements could be sampled at a constant rate every few seconds. Each subsequent repeat of the experiment would then follow a similar sample rate without having to match the same \( \Delta T \) values. This approach naturally results in larger data sets and the analysis to determine the statistical averages will be more involved.

| Table 1. Parameters of Stirling Engine used in this experiment. |
|---------------------------------------------------------------|
| **Parameter** | **Symbol** | **Value** |
| Mass of wheel | \( M \) | \( 6.52 \pm 0.01 \times 10^{-2} \,\text{kg} \) |
| Radius of wheel | \( r \) | \( 4.35 \pm 0.0110 \times 10^{-2} \,\text{m} \) |
| Number of moles of gas | \( n \) | \( 3.45 \pm 0.0110^{-3} \,\text{mol} \) |
| Gas constant | \( R \) | \( 8.31 \,\text{J mol}^{-1} \,\text{K}^{-1} \) |
| Compression ratio | \( C_R \) | \( 1.3 \pm 0.3 \) |

### 4.1. Binned statistics

The combining of repeat data at arbitrary values of \( \Delta T \) presents an opportunity to teach a useful analysis method called binned statistics. First the repeat data is merged into a single set, this is then combined further into bins by a specific algorithm. Statistics can then be calculated for the contents for each of these \( N \) bins. Fortunately, the algorithm for performing this is available to students via the Python programming language. An implementation of this is presented in appendix B.

An example result is shown in figure 5. Here the student recorded raw data for three repeats of the experiment. The data was combined into \( N = 8 \) bins across the range of \( \Delta T \). The average
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Figure 5. Example student data of rotational frequency, $f$, of the Stirling Engine as a function of temperature difference $\Delta T$. Here the raw data comprises of three repeat experiments. The average of the data grouped into eight bins is also shown together with error bars representing the error on the mean in both variables. Note the clear divergence of measurements of frequency at higher values of $\Delta T$ in the student’s data is indicative of a systematic error between repeats.

4.2. Regression analysis

Once the repeat data sets have been combined, students next carry out a regression analysis to determine the engine’s efficiency. The theoretical model presented to the students has a dependence of $f^2(\Delta T)$ as described by equation (9). The simplest regression method for students to use here is the least squares approach. Use of this algorithm is standard in most physics courses and its implementation is available in all analysis software. To perform the analysis, students equate equation (9) to the linear function $y = mx + c$. The algorithm calculates the values of the gradient, $m$, and the intercept, $c$, which best fit the experimental data. The final step is then to use the gradient value

\[ m = \frac{\eta m R \cdot \ln C_R}{\pi \cdot r^2 M} \]  

(11)

together with basic error propagation to determine the engine efficiency $\eta \pm \Delta \eta$.

Figure 6. Results from linear regression analysis on experimental data gathered by two students and a teacher. Each includes statistical error bars representing the error on the mean of the repeat measurements. Representative examples of this regression analysis are shown in figure 6. Here, data sets originally recorded by students has been used as a comparison to data recorded by teaching staff. All three sets of measurements were recorded using individual Stirling Engines of the type described in section 1. The student repeat data sets were binned using the approach described in section 4.1 while the teacher data was recorded at discrete values of $\Delta T$ and averaged using several repeat experiments.

Excluding the last value for Student 2 on figure 6, the statistical errors are relatively constant for both the binned and discrete value approaches. Given the same sets of equipment are used in all three data sets, the random errors associated with the precision of the equipment can also be assumed to be roughly constants. Therefore the remaining discrepancies between the data sets are due to systematic errors. The chief systematic difference is the rotational friction of the individual Stirling Engine wheels. This manifests in either different minimum temperatures to start rotation or different gradients of $\Delta T$ against $f^2$. Different wheels have different static inertia’s and thus different threshold temperatures; this is the systematic error causing the differences in
the $x$-axis offset on figure 6. Conversely, wheel misalignment or other similar differences leads to the rotational friction each Stirling Engine wheel experiences to be slightly different causing the value of $f^2$ for a $\Delta T$ (and thus the gradients) to be different between data sets. These two systematic errors are the main causes of the differences between the data sets.

4.3. Analysis results

The results of the regression analysis are summarised in table 2. Values of the gradient range from 0.6–0.7 Hz$^2$ K$^{-1}$ with relative errors ranging from 5% to 14%. The y-intercept values can be used to determine the x-intercept (the minimum value of $\Delta T$ for rotation) which range from 20 to 40 K. The goodness of fit, $r^2$, for both Student 1 and Teacher are $>0.9$ but for Student 2, $r^2 < 0.9$ which is due to the outlier at high $\Delta T$. In the absence of this outlier, the goodness of fit for Student 2 is $r^2 = 0.9672$ and the corresponding slope is $0.58 \pm 0.05$ Hz$^2$ K$^{-1}$.

Finally, the efficiency of the Stirling Engines can be calculated. From equation (10), we can note that the efficiency will be proportional to the gradient determined from the regression analysis as well as the square of the wheel size and the mass of the wheel. The resulting efficiency values are all approximately 10%. Given the combination of the uncertainty on the gradient and the compression ratio, the overall relative error on $\eta$ is 20%–25%. These efficiency results are also summarised in table 2.

For comparison, an ideal machine has a mechanical efficiency of 100% and a typical internal combustion engine has a mechanical efficiency of around 95% [12]. Thus a 10% mechanical efficiency is very low and this means around 90% of the potential power output is lost to the surroundings. Whilst this efficiency is low, it is understood as valid as the wheels on the Stirling Engines used typically fishtail and squeak as they rotate indicating losses. Secondly, this is in comparison to a commercial engine thus a lower value is somewhat expected. Students should also be encouraged to make the connection between this mechanical efficiency result and the thermal efficiency. A typical internal combustion engine found in a car could have a thermal efficiency of 35% [13] which using the aforementioned mechanical efficiency gives an overall efficiency of 33.25%. For a $\beta$ type Stirling Engine the thermal efficiency would be around 7.5% [10]. However, this can be generously estimated as 18% by applying a Carnot model as described in appendix A. Hence, a maximum of 18% of the initial heat energy can be converted into work done by the gas for moving the wheel. Just 10% of this amount is actually converted to kinetic energy of the rotating wheel thus the overall efficiency is 1.8%. Given a thermal efficiency of 7.5% may be more apt the final overall efficiency could be as low as 0.75%. This very low number can be used to illustrate just how little of the energy from the hot source goes into spinning the Stirling Engine wheel.

5. Discussion and conclusion

The inexpensive Stirling Engines used in this study have been shown to work successfully in an experiment that can be incorporated into a first year undergraduate degree. Even for non-scientists, the sight of the engine beginning to quickly cycle after placement on top of a cup of hot water, then slowing as the water cools, provides a direct visual representation of energy transformation. For students of thermodynamics, the work done by the gas causing the wheel to spin ties the theoretical $P$–$V$ changes with an intuitive
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classical mechanics idea of work. Student’s conceptual framework of the work done by the gas can be further explored by asking them to identify the processes listed in section 2.1 with the individual strokes of the Stirling Engine’s pistons during one full cycle.

This experiment can also be integrated into wider experimental skills development. For example, prior sessions can be devoted to developing competence with DC circuits, motion sensing and signal measurement via oscilloscopes. The subsequent engine experiment can then incorporate these individually developed skills to build and carry out an experimental investigation. Further skills development including merging repeat data and performing statistical data analysis can follow on from the experimental session. This multi-session approach was the case for the student cohorts who performed this experiment at the author’s institution.

In teaching this experiment with two first year undergraduate cohorts, the vast majority complete the experiment within three hours. However, there are some regular issues students encountered with the experiment. A typical problem was the wheel not moving as the plates heat up. This is solved simply by giving the wheel a slight push to overcome the static inertia. Students may also struggle wiring up the Light Gate circuit shown in figure 4. We found that explicitly showing the wire colors on the figure used in the lab handout and emphasizing to students to beware of inadvertent contact between circuit components helped with this.

Occasionally, after some use, the Stirling Engines would seize up and either not cycle or cycle slowly despite an obviously large temperature difference. This can be solved by taking the wheel off and pulling the smaller piston completely out its containment tube. After this, upon reassembly the Stirling Engine performs as normal. We speculate this is due to air gradually being forced out of the central engine chamber as the pistons cycle which alters the pressure inside the chamber. Another cause of slow cycling occurs when the Stirling Engine wheel spins slightly off-axis creating a fishtail motion. The solution here is to alter the axis alignment by wiggling the wheel and bending the support arms inwards to secure the wheel better.

To improve performance of the Stirling Engines, optional modifications can be added. Lubricant can be applied to the pin bearings and piston rods to ease their movement and thermal insulation can be added to achieve a higher $\Delta T$ across the top and bottom plates. Looking closely at the top plate in the photo on the left of figure 1 reveals the thermal insulation modification. Whilst this is a large amount of work for multiple engines, this modification provides a strong performance improvement with peak $\Delta T > 65$ K and a slower reduction in temperature difference compared to without the modification.

Due to $\Delta T$ being a temperature difference, it is possible to drive the Stirling Engine through a colder $T_C$ rather than a hotter $T_H$. This can be achieved though setting the Stirling Engine on a Petri dish full of dry ice to cool the bottom plate to around $-70 \, ^{\circ}C$. Hence with the top plate at room temperature, a higher maximum $\Delta T \approx 90$ K can be achieved. However, thermal conduction will also gradually cool the top plate reducing the $\Delta T$ until the wheel slows to a stop. Interestingly, placing the dry ice on the top plate of the Stirling Engine will cause the wheel to spin in the opposite direction as the $T_C$ and $T_H$ plates are ‘flipped’. However, the advantage of placing the Stirling Engines on top of dry ice is that the teacher can prepare this more easily so the students do not have to handle the dry ice.

To conclude, the thermodynamics experiment described in this paper is a valuable addition to an undergraduate first year physics course. The experiment provides students the opportunity to connect thermodynamic process functions with physical movements of a wheel creating a conceptual framework for work. Students are exposed to oscilloscopes, DC circuits and light gates creating a thorough experience in physics experimentation. Further, the regression analysis and use of binned statistics provides students with a gentle introduction into data analysis.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.
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Appendix A. Thermal efficiency
The student experiment described in the main paper aims to determine the mechanical efficiency of the Stirling Engine—the useful work divided by the outputted work. However, to determine the thermal efficiency the work done on the system must be evaluated. This can be found through transforming equation (5), and therefore equation (8), into

\[ W_{\text{output}} = -\int P \cdot dV = -nR(T_H - T_C) \cdot \ln C_R. \] (A.1)

The heat inputted into the system comes from the hot plate \( Q_H = T_H \Delta S \). Using the first law of thermodynamics and equation (6) we find \( Q_H \) to be

\[ Q_H = -nRT_H \cdot \ln C_R. \] (A.2)

Through combining equations (A.1) and (A.2) the Carnot efficiency can thus be recovered

\[ \eta = \frac{W}{Q_H} = \frac{-nR(T_H - T_C) \cdot \ln C_R}{-nRT_H \cdot \ln C_R} = 1 - \frac{T_C}{T_H}. \] (A.3)

The maximum Carnot efficiency, using a \( \Delta T = 65 \) K and \( T_H = 85 ^\circ C \) is around 18%. However our simplified view does not take into account the additional terms included in \( Q_H \) such as regenerative heat loss

\[ Q_r = MC_v(1 - \epsilon_r)(T_H - T_C) \] (A.4)

where \( M \) is the molar mass of the work fluid, \( C_v \) the molar specific heat capacity at constant volume of the work fluid and \( \epsilon_r \) is the regenerator effectiveness. Equation (A.4) would thus have to be included in the denominator in equation (A.3) meaning we will not reach the Carnot efficiency. For a more detailed breakdown, see [11].

Appendix B. Binning algorithm
Below is the Python script for binning irregular repeat data used in figure 5:

```python
import numpy as np
from scipy.stats import binned_statistic

def average(x, y, nbins):
    
    """
    Combine data into bins and calculate statistics
    x: independent variable
    y: dependent variable, same size as x
    nbins: number of bins to use
    return:
    average values for y and x within each bin
    std: standard deviation
    N: number of values used per bin
    """

    # Calculate the mean of the binned data in y
    y_bins, bin_edges, misc = binned_statistic(x, y, statistic = "mean", bins = nbins)

    # Calculate the bin centres in x
    x_bins = (bin_edges[:-1] + bin_edges[1:]) / 2

    # Determine std of binned data
    y_std, bin_edges, misc = binned_statistic(x, y, statistic = "std", bins = nbins)
    x_std, bin_edges, misc = binned_statistic(x, x, statistic = "std", bins = nbins)

    # Determine how many data points N per bin
    y_std, bin_edges, misc = binned_statistic(x, y, statistic = "count", bins = nbins)

    return x_bins, y_bins, x_std, y_std, N
```
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References

[1] Meltzer D E 2004 Investigation of students reasoning regarding heat, work and the first law of thermodynamics in an introductory calculus-based general physics course Am. J. Phys. 72 1432–46
[2] van Roon P H, van Sprang H F and Verdonk A H 1994 Work and heat: on a road towards thermodynamics Int. J. Sci. Educ. 16 131–44
[3] Pollock E B 2007 Thompson J R and Mountcastle D B Student understanding of the physics and mathematics of process variables in P-V diagrams AIP Conf. Proc. 951 168–71
[4] Kim J H and Nam J 2021 Thermodynamic identities with sunray diagrams Eur. J. Phys. 42 035101
[5] Lipscombe T C and Mungan C E 2020 Breathtaking physics: human respiration as a heat engine Phys. Teach. 58 150–1
[6] Wu G and Wu A Y 2019 A new perspective of how to understand entropy in thermodynamics Phys. Educ. 55 015005
[7] Mulop N, Yusof K M and Tasir Z 2012 A review on enhancing the teaching and learning of thermodynamics Proc. Soc. Behav. Sci. 56 703–12 Int. Conf. on Teaching and Learning in Higher Education in conjunction with Conf. on Engineering Education and Research in Higher Education
[8] Macrie-Shuck M and Talanquer V 2020 Exploring students explanations of energy transfer and transformation J. Chem. Educ. 97 4225–34
[9] Deacon C G, Goulding R, Haridass C and de Young B 1994 Demonstration experiments with a Stirling engine Phys. Educ. 29 180–3

[10] Abuelyamen A and Ben-Mansour R 2018 Energy efficiency comparison of Stirling engine types (α, β and γ) using detailed CFD modeling Int. J. Therm. Sci. 132 411–23
[11] Ahmadi M H, Ahmadi M A and Mehrpooya M 2016 Investigation of the effect of design parameters on power output and thermal efficiency of a Stirling engine by thermodynamic analysis Int. J. Low-Carbon Technol. 11 141–56
[12] ElBahloul M A, Aziz E S and Chassapis C 2019 Mechanical efficiency prediction methodology of the hypocycloid gear mechanism for internal combustion engine application Int. J. Interact. Des. Manuf. 13 221-233
[13] Caton J A 2018 Maximum efficiencies for internal combustion engines: thermodynamic limitations Int. J. Engine Res. 19 1005–23

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