I discuss how the fundamental theory of strong interactions given by QCD can be exploited to make accurate predictions for the solar fusion processes involving few-nucleon systems and to provide a novel description of nuclear matter as well as excitations near the chiral phase transition. The tool used is effective field theory combined with the modern technique of renormalization group flow. The notion of “BR scaling” is reinterpreted in terms of the vector manifestation à la Harada and Yamawaki. It is argued that the vector manifestation scenario predicts a QCD phase structure much richer than the standard scenario discussed up to date in the literature.

I. INTRODUCTION

There are broadly two ways the fundamental theory of strong interactions, QCD, can figure importantly in nuclear physics: The first is to legitimize the place that nuclear physics occupies within the framework of the Standard Model and the second is to enable one to make precise predictions for processes involving nuclei that play a crucial role in certain fundamental processes. In this talk I will not dwell on the first issue although it has attracted most of the recent efforts and simply refer to a recent review [1]. I shall instead focus on the second issue. It turns out that it is in the latter where several Korean theorists have made significant contributions. I am particularly proud of their contributions and feel honored to talk about them at this occasion of the 30th anniversary of Nuclear Physics Division of the Korean Physical Society.

I shall discuss two issues of fundamental nature here. One is the precise calculation of the weak processes that take place in the Sun that are relevant to the solar neutrino problem and the question of neutrino masses. Here we are dealing with light nuclei dilute in number density. The other is the property of hadronic matter under extreme conditions of high temperature and/or high density. The high temperature property is connected with the Early Universe and the high density with the interior of compact stars. QCD should describe what happens under such conditions, some which are currently accessible to the usual laboratory conditions and some not. In principle QCD should describe both – the dilute system and the dense system – within the same framework. However performing such a feat starting from QCD in its elegant form is impossible at the moment. Hence we are forced to resort to a certain number of strategies that rely on effective theories, in particular effective field theories (EFTs). Now in effectuating EFTs, the two cases I mentioned above require somewhat different strategies which in principle should be connected but how they are related is not well understood at the moment.

II. EFFECTIVE FIELD THEORIES (EFTs)

A. Setting up an EFT

In QCD, an effective field theory is defined below an arbitrary but suitably chosen scale \( \Lambda \) in terms of a set of effective degrees of freedom that delineate the EFT from a fundamental theory defined above \( \Lambda \) in terms of fundamental degrees of freedom. Below \( \Lambda \) the relevant degrees of freedom are hadrons with masses less than the cutoff scale and above \( \Lambda \) they are the quarks and gluons of QCD. In order for an EFT to accurately represent QCD, it has to be matched to QCD at the scale, say, \( \Lambda_M \). This can be done by matching physical observables, say, the current-current correlators, expressed in terms of the relevant degrees of freedom in both sectors. How this can be done has been developed and explained in detail by Harada and Yamawaki in [3] using hidden local symmetry Lagrangian. One of the important points that one learns from this development is that when the correlators are matched in medium at the scale \( \Lambda_M \), that is, at a finite temperature and/or finite density, the parameters of the effective Lagrangian are parametrically dependent on temperature and/or density as well as on the scale \( \Lambda_M \). In what precise form the dependence takes is not known within the framework of EFTs. They may be determined either on lattice or by experiments if available. We note that most of the EFT calculations that start with an effective Lagrangian defined in the matter-free vacuum do not have this “intrinsic dependence” whereas the approach based on Brown-Rho...
scaling does. This intrinsic dependence turns out to play a pivotal role for the arguments given below. The other important point is that the (Wilsonian) matching with QCD leads to the result that in the chiral limit (that is, when quark masses are set to zero) the system must flow to what Harada and Yamawaki call “vector manifestation (VM)” fixed point as the critical temperature or density is approached. At the VM, the mass of the vector mesons vanishes with the gauge coupling and the pion decay constant going to zero and the longitudinal components of the vector mesons (i.e., the de-Higgsed scalars) joining, for three flavors, the octet pseudoscalar Goldstone bosons, . The bottom line of this observation is that at least in the chiral limit, the chiral restoration phase transition is basically different from the standard sigma model one. There is no lattice QCD confirmation of this scenario but it is likely that it will be tested. In this talk, I will assume that the vector manifestation is a viable mechanism and discuss what comes out of the picture. It will turn out that one can make some interesting predictions ranging from light nuclear systems to dense stellar systems.

B. More effective effective field theory or MEEFT

The ultimate goal of an EFT would be to start with an effective Lagrangian as defined above and derive all properties of few-body as well as many-body systems in a systematic power counting scheme, without the necessity to resort to empirical data once a minimum (initial) set of parameters is fixed. Such a program would have to deal with not only the basic two-nucleon interactions but also many-body interactions, accounting for the formation of the Fermi sea which is in a sense a quantum critical phenomenon and arriving at a Fermi liquid structure at normal nuclear matter density. The ultimate objective would then be to arrive at the chiral phase transition expected in QCD in the chiral limit. Such an ambitious program does not yet exist and perhaps makes no sense just as it makes no sense to attempt to derive crystals from QED.

Now even in the framework of EFT, if one were to adhere strictly to the order-by-order power counting of the EFT series (one might call this “purist EFT”), one could not make much progress. All that has been accomplished up to now is to postdict things on two-nucleon systems and perhaps a little bit of three-nucleon systems but doing anything beyond in the mass number, unless one relaxes the strict rule of power counting, is at present out of reach. Even for the few-nucleon systems that can be handled, I do not see much more than just reproducing the phenomena well understood in the standard nuclear physics approach based on realistic potentials. It is not clear that this line of efforts would lead to anything new or even useful in deeper understanding of complex nuclear dynamics.

We take an alternative approach that takes full advantage of the tremendous accuracy achieved by the standard nuclear physics approach (SNPA). The sophisticated SNPA which has been unquestionably successful in correlating a large number of nuclear phenomena is based on “realistic” nuclear potentials – typically two- and three-nucleon interactions – that are fit to a large number of scattering as well as spectroscopic data over a wide range of nuclei. By realistic nuclear potentials, I will understand those potentials that have the long-range part constrained by chiral symmetry, that is, pionic properties and the short-range part determined by experiments. Since low-energy/momentum processes should not be sensitive to the detailed structure of the short-range part, the short-distance part of the interaction could not be uniquely determined by low-energy data. The issue is how to formulate the EFT such that transition matrix elements that sample also short-distance elements – though with much less importance – can be made accurate. The way to accomplish this is to incorporate the SNPA into the framework of effective field theories, while preserving the power counting of the EFT. This allows one to carry over the immense success of the SNPA and make the scheme as model-independent and unambiguous as possible. Note that the SNPA by itself does not lend itself to a systematic assessment of errors committed in the calculation. This is because there is no clear way to establish that what is obtained is not subject to uncontrolled corrections. The power of an EFT is precisely in providing the means to assess the error committed in making approximations and making it possible to systematically correct the errors. Let me call this version of EFT which combines the powers of the SNPA and EFT more effective effective field theory (MEEFT). What it means will be clearer later.

III. EFT PREDICTIONS FOR LIGHT NUCLEI

A. Solar fusion process

As a case where one can work with an effective Lagrangian constructed in the vacuum at zero temperature and density matched to QCD at the chiral scale near , consider the solar fusion processes

\begin{equation}
pp : \quad p + p \rightarrow d + e^+ + \nu_e, \quad (1)
\end{equation}

\begin{equation}
hep : \quad p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu_e. \quad (2)
\end{equation}

These two processes represent the lowest- and highest-energy neutrino production processes. The MEEFT enables one to treat these processes involving different numbers of nucleons on the same footing and further to make theoretical error estimates. The reactions (1) and (2) figure importantly in astrophysics; they have much bearing upon issues of great current interest such as, for example, the solar neutrino problem and non-standard physics in the neutrino sector. Since the thermal energy of the interior of the Sun is of the order of...
keV, and since no experimental data is available for such low-energy regimes, one must rely on theory for determining the astrophysical $S$-factors of the solar nuclear processes.

Since the momentum transfer involved is small, these processes are dominated by the Gamow-Teller matrix element given by the long-wavelength limit of the spatial component of the axial current operator. It turns out that the Gamow-Teller operator is particularly difficult to pin down if its matrix element is “accidentally” suppressed as in the case of the $hep$ process. One can understand this in terms of what is known as “chiral filter mechanism” [12] [13]. In an effective field theory in which chiral symmetry plays an essential role as in nuclei, corrections to the leading single-particle process in the transition matrix elements are strongly dominated by soft-pion exchanges whenever they are allowed by symmetry and kinematics. A consequence is that the subleading corrections remain small compared to the leading term and can be estimated reasonably well by going to one order higher: the total amplitude can be computed with a great deal of accuracy. The space part of the vector current [11] and the time part of the axial current [17] belong to this class of operators. However if the soft-pion exchanges are suppressed by symmetries independently of kinematic conditions as in the case of the time component of the vector current and the space component of the axial current, then leading corrections to the single-particle operators are sensitive to effects of short-distance nature that are not easily accessible by low-energy theorems and hence require chiral non-perturbative treatments. This chiral filter mechanism has been given a justification in terms of a chiral expansion at low orders of chiral perturbation [13] and also in a different and more general approach [13].

It is the lack of the chiral-filter aspect that has made the calculation of the $hep$ process notoriously difficult for a long time. The main operator for both $pp$ and $hep$ — which is the Gamow-Teller operator — is unprotected by the chiral filter. It turns out however that the leading GT single-particle matrix for $pp$ — which is unsuppressed by symmetry considerations — accounts for the bulk of the transition matrix element, so the short-distance corrections uncontrolled by chiral symmetry remain in the range of only a few percents at most. This is not the case for the $hep$ process. Because the leading single-particle matrix element involved here is highly suppressed “accidentally” by the symmetry mismatch between the initial and final wave functions, the corrections play a very important role, making orders of magnitude uncertainty in the $S$ factor depending on how they are computed as explained in detail in [13].

Let me quote the results here for the $pp$ and $hep$ for the sake of completeness as they are the values to be used by astrophysicists for solar neutrino issues and explain in the next subsection for nuclear and hadron physicists how the MEEFT proposed above can resolve this difficulty and allow the calculation of the $pp$ process with an unprecedented accuracy and that of the $hep$ process within \( \sim 15\% \) accuracy which represents a giant improvement over the past results [30]:

\[
S_{pp}(0) = 3.94 \times (1 \pm 0.004) \times 10^{-25} \text{MeV-b}, \quad (3)
\]

\[
S_{hep}(0) = (8.6 \pm 1.3) \times 10^{-20} \text{keV-b}. \quad (4)
\]

B. The strategy of MEEFT

Before I go into the details of this subsection, let me do away with other terms that enter mainly into the $hep$ calculation. In the weak matrix elements that contribute to both $pp$ and $hep$, the vector matrix elements are without any uncertainty: They are protected by low-energy theorems. However in the axial current sector, the time component of the axial current does make a significant contribution for the $hep$ whereas it is negligible for the $pp$. In most of the past works with the exception of the recent SNPA calculation of [14], the axial charge component has not been properly treated. This component, however, is protected by the chiral filter as mentioned above, so its calculation can be made fairly reliably. Thus I will focus on the Gamow-Teller operator which is the only difficult operator to handle for the processes involved.

To set up the strategy, one first has to pick the relevant degrees of freedom. Since the nucleons are the essential ingredient of the nuclear systems, they figure in the theory although the mass scale involved is of the same scale as the chiral scale \( \Lambda \sim 1 \text{ GeV} \). The pion is the lightest excitation whose mass is non-zero because of the explicit chiral symmetry breaking due to the quark masses. Its actual mass is \( \sim 140 \text{ MeV} \). Now if one is interested in very low-energy and low-momentum processes, say at most a few MeV, like $pp$ and $hep$, then the pion mass can be considered “high” and in this case, one might integrate out the pion as well $pp$. If the pion is integrated out with its effect incorporated into higher dimension terms in the effective Lagrangian, we lose however the power of the chiral filter consideration and thereby lose certain predictivity associated with current algebras. We shall therefore keep the pion in the theory. If the matter density is high, then as we shall see below, the vector degrees of freedom (such as the $\rho$ and $\omega$) cannot be ignored since their masses become small due to the “vector manifestation” of chiral symmetry [3]. However the systems that figure in $pp$ and $hep$ are dilute and hence the vector mesons remain massive, so can be integrated out. But this does not mean that they are not important, particularly if the process is not protected by the chiral filter. Nuclear processes generically experience a wide range of scales as nucleons in strong interactions sample all distances with varying degrees of importance and hence when the massive particles are integrated out their effects transferred to higher-dimension operators cannot be ignored a priori. How those effects are taken into account differentiate various approximations made in EFT calculations.
The computation of a transition matrix element between the initial $|i\rangle$ and final $|f\rangle$ states involves the full account of interactions between the nucleons and one insertion of an external field or current. In the Weinberg scheme \cite{15}, this amounts to computing the interaction potential and the current vertex in a series of “irreducible graphs” and summing to all orders the “reducible graphs” with the potential appearing as the kernel. The power counting goes into the irreducible graphs which are then computed to a certain order $N$. If one were to compute to all orders in the irreducible sector, then the procedure would give an exact answer \cite{31}. One is however limited to a finite $N$ because of the proliferation of high-order diagrams and of the number of consequent counter terms that cannot be fixed (a feature that is characteristic of the so-called non-renormalizable theory). This limitation is circumvented in MEEFT by resorting to the potentials that are phenomenologically fitted to an ensemble of experiments. Such a “realistic potential” must have the correct feature at long distance required by chiral symmetry. What is needed to get the correct matrix element that is independent of short-distance properties is involved in the potentials. This is the difficulty that the SNPA cannot control by itself.

The MEEFT resolves, albeit approximately, this difficulty as follows. We start with the wave functions for the initial and final nuclear states computed in many-body techniques \cite{1} using a potential that is fit to a large set of scattering and transition data. Such a potential typically possesses the long-range tail given by the pion exchange constrained by chiral symmetry – i.e., consistent with low-order chiral expansion – and the short-range parts determined by the fit to experiments. While the long-range part is unique, the short-range part may not be since a variety of short-distance components can give the same physical low-energy observables. The wave functions obtained with such different potentials would possess different short-distance properties, so the matrix element of an arbitrary operator computed to a finite order will differ depending upon how sensitive the operator is to the short-distance physics. What is needed to get the correct matrix element that is independent of short-distance ambiguities is the additional terms in the operator that compensate the short distance behavior in the wave functions. The way to do this is as follows. Since the current appears only once in the matrix element, one can focus on the irreducible graphs that in chiral perturbation theory contribute to a finite order $N$. In calculating the loop graphs, one puts a cutoff in the loop integrals, say, $\bar{\Lambda} \geq 300$ MeV, can however differ widely from one potential to another. It is also in these parts where off-shell ambiguities could figure strongly. This means that when one probes short-distance effects with such potentials one would obtain widely different results. It is not obvious then that when one uses these potentials in computing the transition matrix elements that involve low external energy/momentum, one is not getting different answers depending upon which short-distance properties are involved in the potentials. This is the difficulty that the SNPA cannot control by itself.

The price to pay in the MEEFT is that the strict power counting is sacrificed at the order $(N + 1)$ since the potential is most likely to contain not only the terms that are consistent with chiral symmetry up to order $N$ but also terms which go beyond. The counting error that shows up at $(N + 1)$-st order will then be in the regime of short-distance physics and the role of the MEEFT regularization is to largely, if not completely, compensate for this error.

What happens in the case at hand is very much like the situation with $V_{\text{twists}}$ discussed by Bogner et al \cite{16} that figures in the half on-shell $T$ matrix for low-momentum effective nuclear interactions: There it is found also to be remarkably independent of the short-distance uncertainty.

In the case of the processes \cite{1} and \cite{2}, it is possible to reduce the only two unknown two-body contact terms present in the theory into one combination in the Gamow-Teller operator effective for the two-nucleon, three-nucleon and four-nucleon processes in question (three-body and higher-body terms are suppressed compared to the terms calculated). The resulting single unknown constant can then be fixed once and for all by the accurately measured triton beta decay for given $\bar{\Lambda}$’s. This allows one to predict, totally parameter-free, the matrix elements for the $pp$ and $hep$ processes, i.e., the results \cite{2} and \cite{4}. The constant depends on $\bar{\Lambda}$ but the total Gamow-Teller matrix element is not. I find this result truly remarkable. It will be of help in the future work on solar neutrinos.
IV. EFT FOR DENSE MATTER

Let me turn now to dense hadronic matter. We will consider dense matter, ranging from nuclear matter all the way to that relevant to chiral restoration. For this purpose, we will need what I would call “double decimation” strategy which consists of first going to nuclear matter density and then to higher density near the chiral phase transition. Here the crucial role is played by the notion of “vector manifestation of chiral symmetry” introduced by Harada and Yamawaki [3, 5]. The significance of this notion is best appreciated if one recognizes that it in essence provides the mechanism by which the mass of the nucleons – which makes up 99% of the mass of ordinary matter [17] – can be made to “disappear.”

A. Vector manifestation

In going to nuclear matter and beyond, we must keep the vector-meson degrees of freedom *explicit* because of the vector manifestation (VM) mentioned above. To understand the VM, we consider the HLS Lagrangian [19] in which the pion and vector mesons are effective degrees of freedom. For the moment, we ignore fermions and heavier excitations as e.g., a1, glueballs etc. To make the discussion the most transparent, we consider three massless flavors. Masses and symmetry breaking can be introduced with attendant complications. The relevant fields are the (L,R)-handed chiral fields $\xi_{L,R} = e^{i\pi/F_\pi}e^{\pm i\pi/F_\pi}$ where $\pi$ is the pseudoscalar Goldstone bosons and $\sigma$ the Goldstone scalar absorbed into the HLS vector field $\rho_\mu$ coupled gauge invariantly with the gauge coupling constant $g$. If one matches this theory to QCD at a scale $\Lambda_M$ below the mass of the heavy mesons that are integrated out but above the vector $(\rho)$ meson mass; it comes out – when the quark condensate $\langle \bar q q \rangle$ vanishes as in the case of chiral restoration in the chiral limit – that

$$g(\bar g) \to 0, \quad a(\bar g) \equiv F_\pi/F_\sigma \to 1.$$  

(5)

Now the renormalization group analysis shows that $g = 0$ and $a = 1$ is the fixed point of the HLS theory and hence at the chiral transition, one approaches what is called the “vector manifestation” fixed point. The important point to note here is that *this fixed point is approached regardless of whether the chiral restoration is driven by temperature $T$ or density $n$ or a large number of flavors.* At the VM, the vector meson mass must go to zero in proportion to $g$, the transverse vectors decouple and the longitudinal components of the vectors join in a degenerate multiplet with the pions [18]. Specifically

$$m^*_\rho/m_\rho \approx g^*/g \approx \langle \bar q q \rangle^*/\langle \bar q q \rangle \to 0$$  

(6)

as the transition point $n = n_c$ is reached [20, 24]. This result can be understood as follows. Near the critical point the “intrinsic term” $g^*/F_\pi$ in the vector mass formula drops to zero faster than the dense loop term that goes as $\sim g^*H(n)$ where $H$ is a slowly (i.e., logarithmically) varying function of density. So the dense loop term controls the scaling. Now it seems to be a reasonable thing to do to assume that near the VM fixed point, we have the scaling

$$m^*_\rho/m_\rho \approx g^*/g \approx \langle \bar q q \rangle^*/\langle \bar q q \rangle.$$  

(7)

Our conjecture [21] is that this holds from nuclear matter density to the chiral restoration point.

B. Double-decimation conjecture

Let us now turn to the low-density regime, that is, density below nuclear matter density. At near zero density, one can apply chiral perturbation theory with a zero-density chiral Lagrangian. Using the HLS Lagrangian matched to QCD at a scale $\Lambda_M \sim \Lambda_\chi$, one can establish that

$$m^*_\rho/m_\rho \approx f^*_\rho/f_\rho \approx \sqrt{\langle \bar q q \rangle^*/\langle \bar q q \rangle}.$$  

(8)

This result follows from an in-medium GMOR relation for the pion with the assumption that at low density the pion mass does not scale (as indicated experimentally [22]), that the vector meson mass is dominantly given by the “intrinsic term” $\sqrt{aF_\pi g}$ with small loop corrections that can be ignored and that the gauge coupling constant does not get modified at low density (as indicated by chiral models and also empirically). We will now simply assume that this relation holds from zero density to nuclear matter density. An important ingredient to interject here is that normal nuclear matter is a many-body fixed point, known as Fermi-liquid fixed point, so we are essentially summarizing the phase structure by two fixed points, namely, the Fermi-liquid fixed point and the vector-manifestation fixed point. This suggests a double-decimation approximation. It may be too simplistic to approximate the complex phase structure by only two decimations but we find that we can go a long distance with this approximation.

V. PREDICTIONS

A variety of evidences that lend support to the scaling behavior given above are discussed in [23]. Here I would like to discuss a few recent developments.

A. Parametric density dependence

The one feature that distinguishes the HLS/VM theory from other EFTs is the parametric dependence on the background of the “vacuum” – density and/or temperature – which controls the fixed point structure of the VM. At low density, this dependence is relatively weak but in precision experiments, it should be visible.
Consider a chiral Lagrangian in which only the nucleon and pion fields are kept explicit with the vectors and other heavy hadron degrees of freedom integrated out (say, from the HLS/VM Lagrangian). The relevant parameters of the Lagrangian are the “bare” nucleon mass $M(M, n)$, the “bare” pion mass $m_{\pi}(M, n)$, the “bare” pion decay constant $F_{\pi}(M, n)$, the “bare” axial-vector coupling $g_{A}(M, n)$, and so on. The structure of this Lagrangian is the same as the familiar one apart from the intrinsic dependence of the parameters on $n$. In the usual approach, the scale $\Lambda_{M}$ is fixed at the chiral scale and the dependence on $n$ is absent. Suppose that one does a chiral perturbation theory with the HLS/VM Lagrangian. The power counting will be the same as in the conventional approach. Now if the density involved in the system is low enough, say, no greater than nuclear matter density, then one could work to leading order in nuclear matter density, then one could work to leading order in $\rho$. To this order, the parameters of the Lagrangian are the “bare” nucleon mass $m_{\pi}(\Lambda_{M}, n)$, the “bare” pion fields are kept explicit with the vectors and other heavy hadron degrees of freedom integrated out (say, from the HLS/VM Lagrangian). The relevant parameters of the Lagrangian can be identified with physical quantities. For instance, the bare pion decay constant $F_{\pi}$ can be identified with the physical constant $\pi^{*}$ etc. Now in the framework at hand, the only dependence in the constants on density will then be the intrinsic one determined by the matching to QCD living in the background of density $n$.

If we apply the above argument to the recent measurement of deeply bound pionic atom systems $22$, we will find that the measurement gives information on the ratio $f_{\pi}^{*}/f_{\pi}$ at a density $n \lesssim n_{0}$. We have from $f_{\pi}^{*}/f_{\pi}$

$$\Phi(n) \equiv f_{\pi}^{*}/f_{\pi} \approx \sqrt{<qq^*/<qq>(\approx m_{\rho}/m_{\rho})}.$$ \hspace{1cm} (9)

The scaling quantity $\Phi$ has been obtained from nuclear gyromagnetic ratio in $23, 24$. At nuclear matter density, it comes out to be $\Phi(n_{0}) = 0.78$ with a small error bar. Thus it is predicted that

$$(f_{\pi}^{*}/f_{\pi})^{2}(n_{0}) \approx 0.61.$$ \hspace{1cm} (10)

This agrees with the value extracted from the pionic atom data of $22$. $(f_{\pi}^{*}/f_{\pi})^{2}(n_{0}) = 0.65 \pm 0.05$.

It is perhaps important to stress that this “agreement” can be taken as an evidence neither for the validity of the tree approximation nor for the signal for “partial chiral restoration.” If one were to go to higher orders in chiral expansion, the parametric pion decay constant cannot be directly identified with the physical pion decay constant since the latter should contain two important corrections, i.e., quantum corrections governed by the renormalization group equation as the scale is lowered from $\Lambda_{M}$ to the physical scale and dense loop corrections generated by the flow. At the chiral restoration, it is this latter that signals the phase transition: The parametric pion decay constant with the scale fixed at the matching scale does not go to zero even at the chiral restoration point $4$. Thus when one does higher-order chiral perturbation calculation of the same quantity, one has to be careful which quantity one is dealing with.

B. Effective degrees of freedom at the chiral phase transition

Let me go near the chiral transition point and ask what new physics is introduced by the VM. I will do this for high temperature. There are lots of interesting phenomena triggered by high density in the present framework but this will be a topic for next publication.

Let me start by recalling the standard scenario of the chiral phase transition driven by temperature. The standard scenario assumes that in the heat bath, there are no other low-energy degrees of freedom than pions to consider, so all other mesons and all baryons can be integrated out with their imprints lodged in the coefficients of higher-dimension operators in the Lagrangian. In making this assumption, one is disregarding the possible relevance of the VM fixed point. Consider now the vector susceptibility (VSUS) and axial-vector susceptibility (ASUS) defined in terms of Euclidean QCD current correlators as

$$\delta^{ab} \chi_{V} = \int_{0}^{1/T} d\tau \int d^{3}\vec{x} V_{0}^{a}(\tau, \vec{x}) V_{0}^{b}(0, \vec{0})|_{\beta},$$ \hspace{1cm} (11)

$$\delta^{ab} \chi_{A} = \int_{0}^{1/T} d\tau \int d^{3}\vec{x} A_{0}^{a}(\tau, \vec{x}) A_{0}^{b}(0, \vec{0})|_{\beta}$$ \hspace{1cm} (12)

where $<|\beta>$ denotes thermal average and

$$V_{0}^{a} \equiv \bar{\psi} \gamma^{0} \tau^{a} \psi, \quad A_{0}^{a} \equiv \bar{\psi} \gamma^{0} \gamma^{5} \tau^{a} \bar{\psi}$$ \hspace{1cm} (13)

with the quark field $\psi$ and the $\tau^{a}$ Pauli matrix the generator of the flavor $SU(2)$. These SUS’s can be readily computed via chiral Lagrangians defined in a heat bath with the sole assumption that the effective Lagrangian $L_{eff}$ can be written in terms of local fields and their derivatives only -- an assumption which is standard in EFTs. Indeed, Son and Stephanov $25$ computed the ASUS, $\chi_{A}$, and found at the critical temperature and in zero density that

$$\chi_{A} = -\frac{\partial^{2}}{\partial \mu^{2}} L_{eff} |_{\mu = 0} = f_{\pi}^{2}$$ \hspace{1cm} (14)

where $f_{\pi}$ is the time component of the pion decay constant. The principal point to note here is that as long as the effective action is given by local terms, this is the entire story. There are no other contributions to the ASUS than the temporal component of the pion decay constant. Now at the chiral phase transition, the vector SUS and the axial-vector SUS must be equal to each other, $\chi_{V} = \chi_{A}$ and the lattice data on $\chi_{V}$ $26$ clearly indicates that

$$\chi_{V}|_{T=T_{c}} \neq 0,$$ \hspace{1cm} (15)

which leads to the conclusion $26$ that

$$f_{\pi}^{2}|_{T=T_{c}} \neq 0.$$ \hspace{1cm} (16)
On the other hand, it is expected and verified by lattice simulations that the space component of the pion decay constant $f_\pi^2$ should vanish at $T = T_c$. One therefore arrives at

$$v_\pi^2 \sim f_\pi^2 / f_\pi^4 \to 0, \ T \to T_c$$  \hspace{1cm} (17)$$

where $v_\pi$ is the pion velocity. This is the main conclusion of the pion-only theory.

This elegant result is unfortunately marred by the caveat that the same formalism applied to the VSUS does not work. In fact, one can see easily from the effective Lagrangian that the same theory predicts $\chi_V = 0$ for all temperature which of course cannot be correct and is ruled out by the lattice result. Son and Stephanov attribute this failure to the possibility that a diffusive mode in hydrodynamic language, which is not described by the chiral Lagrangian, contributes to the VSUS. In the standard picture, this is not surprising since there are no other degrees of freedom than the pions and it must be nonlocal effects involving pions that must intervene. This situation is entirely different when the vectors are present as relevant degrees of freedom as in the VM.

In the VM scenario, the vector mesons with a dropping mass must enter near the phase transition. In addition, one can conceive of quasiquarks with a similarly dropping mass also contributing. It is rather straightforward to compute the VSUS and ASUS at one-loop order. The results obtained by Harada, Kim, Rho and Sasaki [27] are

$$f_\pi^2 |_{T=T_c} = f_\pi^4 |_{T=T_c} = 0, \ v_\pi |_{T=T_c} = 1$$  \hspace{1cm} (18)$$

and

$$\chi_A |_{T=T_c} = \chi_V |_{T=T_c} = 2N_f \left[ \frac{N_f}{12} T_c^2 + \frac{N_c}{6} T_c^2 \right].$$  \hspace{1cm} (19)$$

These results are not difficult to understand. Both the time part and space part of the pion decay constant go to zero with the ratio going to 1. Thus the pion velocity approaches the speed of light, which is completely opposite to the Son-Stephanov result of vanishing velocity.

The vector mesons and the quasiquarks contribute to the vector and axial-vector SUS’s, the latter bigger than the former by a factor of 2. It is easy to see how the two SUS’s come out equal here. Both are given by quasiparticles, the vector mesons and constituent quarks. There is no need to invoke diffusive modes.

It is interesting that both pion decay constants go to zero simultaneously giving the pion velocity equal to the light velocity. It appears to restore Lorentz symmetry broken by the medium. It is however counterintuitive: one could think of the pion as an analog to the sound in condensed matter, the velocity of which is known to go to zero on critical surface. The VM result looks elegant and simple but is puzzling at the moment.

There is also a caveat here. The result obtained for the pion velocity is in one-loop approximation. One might wonder what happens at higher-loop orders. We expect the $f_\pi^4$ to remain zero at the critical point to all orders. This is because it is the order parameter for the chiral restoration. The question is what happens to the time part, $f_\pi^2$, at higher orders. It is possible that the VM fixed point protects it so that it remains zero but this has not been proven. If however there is an additional contribution at higher order that does not vanish, then we will go over to the Son-Stephanov result of a vanishing pion velocity. More work needs to be done to clarify this issue.

On the other hand, if (18) does turn out to be correct, then the implication will be that the phase structure at the chiral transition is entirely different in the HLS/VM from that of the standard picture. In fact it indicates that there can be a much richer structure in the phase diagram than the standard one. In particular, it indicates that a certain number of degrees of freedom hitherto ignored must be taken into account and the transition will be a lot smoother than imagined in the standard scenario since the number of degrees of freedom below the critical point will be considerably increased. I expect that future lattice and experimental developments will validate or invalidate this beautiful VM scenario.

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[28] For two flavors, the situation is a bit more subtle. We will ignore this subtlety here.

[29] I must mention here as a side remark that this terminology is entirely mine, merely meant to tickle, and not to “anger,” the aficionados of the “purist EFT.” My coworkers like to call it EFT*.

[30] I would like to reiterate my challenge to the aficionados of the “purist EFT” to come up with a parameter-free prediction for the $\text{hep S-factor}$. The first person who succeeds in doing so within the coming year will be offered a bottle of a great French wine.

[31] There could be a problem of consistency in power counting if this program is executed to a finite $N$ as discussed in [1] but if summed to all orders, this problem would be avoided by fiat.

[32] How to do this at higher loop order while preserving chiral symmetry is a tricky matter when one uses the cut-off procedure. To avoid this problem, one might use the dimensional regularization suitably generalized for the problem at hand, e.g., power divergence subtraction.

[33] One might wonder whether the decoupled transverse vectors require chiral partners to be consistent with the Wigner mode of chiral symmetry. If so, then, there would be a difficulty as there are no massless axial vectors that belong to the same multiplet. The answer is that there is no need for them. The transverse vector fields must transform invariantly under chiral transformation. How this can happen is suggested in [18]. I am grateful for discussions with M. Harada, T. Kugo and K. Yamawaki on the status of the transverse vectors.

[34] We will specialize in density. Qualitatively the same argument holds also in temperature but the details can be different away from the chiral restoration point.