Four-Alpha Linear-Chain States in $^{16}$O

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Abstract

We study 4$\alpha$ linear-chain states in $^{16}$O in comparison with 3$\alpha$ states in $^{12}$C by using the Generator Coordinate Method within a microscopic $N\alpha$-cluster model. It is shown to be very important to solve relative motion of $\alpha$-clusters by taking into account the orthogonality between the linear-chain states and other low-lying states including the ground state. For $^{12}$C, the 3$\alpha$ chain state disappears because of this orthogonality. The 4$\alpha$ chain state in $^{16}$O, on the other hand, is hardly affected by low-lying states and persists to remain above the 4$\alpha$ threshold. The calculated moment of inertia of the 4$\alpha$ linear-chain rotational band (88 keV) reproduces the experimentally suggested value qualitatively.
1 Introduction

A nuclear study of a large deformation is one of the frontier in nuclear spectroscopy. Since the first observation of the superdeformed band [1], various superdeformed nuclei (∼1:2 deformation) have been found, and nowadays extensive studies on these bands including their excited bands are in progress. Furthermore, there are several reports on the hyperdeformed band in which the deformation is around 1:3 [2]. These large deformed nuclei are mainly found in the medium-heavy mass number region where the liquid-drop energy and the shell-correction balances. Therefore, larger deformation than 1:3 is not expected to appear in this mass number region, since the shell-correction is not enough to make a barrier at larger deformations.

On the other hand, a large deformation above 1:3 may be possible in light nuclei due to the $\alpha$-cluster structure. For example, the first example of the “superdeformed” nuclei is nothing but $^8\text{Be}$ which has the $\alpha-\alpha$ structure, and there have been a lot of discussion on the nature of $0^+_2$ state of $^{12}\text{C}$ as the $3\alpha$ linear-chain state [3] [4]. Experimentally, it is more difficult to observe large-deformed states or linear-chain states of $\alpha$-clusters in light nuclei, since these states have large particle decay widths, and consequently a coincident detection of multi-$\alpha$ particles becomes necessary. From a theoretical point of view, on the other hand, it is required to estimate the couple-channel effects over the broad range of deformation or geometric structure, since a mean field picture may not be valid for the highly excited states in light nuclei. However, since it gives us the opportunity to explore the most deformed nuclei, the study of these $N\alpha$ linear-chain states [5] is valuable.

For example, there are several experimental candidates in light $4N$ nuclei for large deformed states above 1:3. The first example is the $4\alpha$ linear-chain band starting around the $4\alpha$ threshold energy region in $^{16}\text{O}$ suggested by Chevallier et al. [6] through the data analysis of the $^{12}\text{C}+\alpha \rightarrow ^8\text{Be}+^8\text{Be}$ reaction. This suggestion is supported by a theoretical work by Suzuki et al. [7]; they analyzed the decay widths from such a strongly coupled linear-chain state, and discussed that the excited states observed by Chevallier et al. are possibly characterized by a $4\alpha$ linear-chain structure. The second example is the $6\alpha$ linear-chain states, which is discussed as a candidate to explain a new molecular resonance state observed by Wuosmma et al. [8] in the excitation function of the inelastic reaction $^{12}\text{C}(^{12}\text{C}, ^{12}\text{C}(0^+_2))^{12}\text{C}(0^+_2)$ at $E_x=56.4 \text{ MeV}$ [9]. Although the clarification of the existence of a $6\alpha$ linear-chain state is very challenging, the coupling to $^{12}\text{C}^{*}-^{12}\text{C}^*$ and $^{16}\text{O}^{*-8}\text{Be}$ may be important [10] and makes it difficult to study a pure linear-chain state. Therefore, it is important and urgent to study the stability of linear-chain states in $^{12}\text{C}$ and $^{16}\text{O}$ at this stage.

The reliability of the linear-chain structure has been usually studied through kinematical analyses of decay widths. It has been also discussed on the analyses of small vibration around the equilibrium configuration by Ikeda [11]. In those previous studies, however, the linear-chain structure was assumed a priori. Namely, the coupling effects with other states have not been taken into account. These couplings, especially with lower excited states are expected to play an important role on the stability of linear-chain states, since they push-up the energy of linear-chain states and generally
increase the instability. Therefore, it is necessary to calculate in a wide wave function space, which covers not only the linear-chain configurations but also the lower excited states.

In this paper, we investigate the stability of \( 4\alpha \) linear-chain configurations in \( ^{16}\text{O} \) comparing with \( 3\alpha \) states of \( ^{12}\text{C} \) by applying a microscopic \( N\alpha \)-cluster model. In contrast to the previous works, we discuss the stability of the \( N\alpha \) linear-chain structure by solving dynamics of \( N\alpha \) systems. This is achieved by diagonalizing the Hamiltonian matrix in a wide space which also covers low-lying levels including the ground state. The framework used here is based on the Generator Coordinate Method (GCM) by which we can solve relative motion of \( \alpha \) clusters by taking account of the superposition of many different intrinsic configurations. We use the Constraint Cooling Method \([12, 13, 14, 15]\) proposed in the framework of AMD \([16]\) in order to generate GCM basis which describes low-lying levels. This method has been already shown to provide the suitable GCM basis states effectively \([17]\).

This paper is organized as follows; in section 2, we explain our model especially the way to construct the GCM basis states. In section 3, we show our results that the \( ^{12}\text{C} \) linear-chain state disappears because of the orthogonality to other states, however the \( ^{16}\text{O} \) linear-chain state exists above the \( 4\alpha \) threshold and shows the rotational band structure. In section 4 summary and conclusion are presented.

## 2 Method

In the framework of GCM, the energy spectra and the wave functions are solved by diagonalizing the Hamiltonian matrix through the Hill-Wheeler equation;

\[
\Psi^{JM} = \sum_j c_j \Phi_j^{JM} ,
\]

\[
\sum_k \left( \langle \Phi_j^{JM} | H | \Phi_k^{JM} \rangle - E \langle \Phi_j^{JM} | \Phi_k^{JM} \rangle \right) c_k = 0 ,
\]

where \( \Phi_j^{JM} \) is the \( j \)th GCM basis wave functions with the total angular momentum \( J \) and its \( z \)-component \( M \). Since the reliability of the GCM calculation largely depends on the adopted basis wave functions, we have adopted the GCM basis states which reasonably reproduce the low-lying excited states in addition to the linear-chain configurations described in the next section. The first set of GCM basis states is generated by applying the constraint cooling method \([13]\) to the Bloch-Brink wave function for an \( N\alpha \) system. The angular momentum projection from these wave packets is carried out numerically. A detailed explanation of this method is found in our previous paper \([17]\).

Hereafter, we briefly summarize our method. The Bloch-Brink wave function for an \( N\alpha \) system is the Slater determinant of Gaussian wave packets \([12, 16]\) with the assumption of the [4]-symmetry for the spin and iso-spin part,

\[
\Phi(\{r_i^{\alpha}; Z_i\}) = A \prod_{i=1}^{N} \prod_{\alpha=1}^{4} \phi_i^{\alpha}(r_i^{\alpha}; Z_i) ,
\]
\[
\phi_i^\nu(r^\alpha; Z_i) = \exp\{-\nu(r^\alpha - Z_i/\sqrt{\nu})^2 + Z_i^2/2\} \chi \quad (i = 1 \sim N),
\]
where \(\chi = \{p \uparrow, p \downarrow, n \uparrow, n \downarrow\}\) are the spin-isospin wave functions. In the spatial part, the complex parameters \(Z_i = \sqrt{\nu} (D_i + iK_i/2\nu h)\) represent phase space coordinates. In this work, the size parameter \(\nu = 0.27 \text{ fm}^{-2}\) is chosen to reproduce the energy and the r.m.s. radius of the \(\alpha\) particle.

In order to generate various intrinsic states, we employ the constraint cooling method \([12, 13, 14, 15]\), by which we can search for a set of suitable complex parameters \(\{Z_i\}\) to minimize the expectation value of the energy under some constraints. Following Ref. \([13]\), we obtain an optimum configuration state by solving the constraint cooling equation given by

\[
\frac{dZ_i}{d\beta} = -\frac{\partial\mathcal{H}}{\partial Z_i} - \sum_{l=1}^{\eta_l} \eta_l \frac{\partial \langle W_l \rangle}{\partial Z_i^*},
\]

where \(\mathcal{H}\) is the expectation value of the Hamiltonian operator. The expectation value of the constrained observable \(\langle W_l \rangle\) in Eq. (5) plays a role of the generator coordinate, and \(\eta_l\) is the Lagrange multiplier to keep the given constraint along the evolution. In this work, we constrain the expectation value of the total oscillator quantum numbers \(W_l = n_x, n_y, n_z\) in a rectangular coordinate system \([17]\). It is possible to constrain other observables, such as an angular momentum, however, cooling with an angular momentum constraint essentially results in giving the rotating nucleus with the similar intrinsic state when the angular momentum is not so high. Then these states are not very effective as GCM basis states.

Since the resultant cooled state is not the eigenstate of an angular momentum in general, it is necessary to carry out the angular momentum projection to construct GCM basis wave functions in Eq. (6).

\[
\Phi_{\bf J M}^{\nu}(\{r^\alpha_i; Z_i\}) = \frac{2J + 1}{8\pi^2} \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) \Phi(\{r^\alpha_i; Z_i\}),
\]

where \(\Omega\) is the Euler angle of \(\{r^\alpha_i\}\). In the numerical integration over the Euler angle \(\Omega(\alpha, \beta, \gamma)\), we have adopted \(25^3\) mesh points and integration is carried out with the Gauss-Legendre integral technique. The intrinsic angular momentum component \(K\) is chosen to be zero for simplicity.

### 3  \(N\alpha\) linear-chain states in \(^{12}\text{C}\) and \(^{16}\text{O}\)

#### 3.1 3\(\alpha\) states in \(^{12}\text{C}\)

In many previous analyses, it has been shown that a large part of low-lying excited states of \(^{12}\text{C}\) have clustering structures. Although Morinaga \([4]\) proposed the linear-chain structure for \(^{12}\text{C}\), the clustering states of \(0^+ (7.66 \text{ MeV})\) and \(2^+ (\sim 10 \text{ MeV})\) have been discussed to have a loosely-coupled 3\(\alpha\) configuration \([19]\) rather than the linear-chain structure. We here investigate the stability of the 3\(\alpha\)-linear-chain configuration in \(^{12}\text{C}\) from a viewpoint of orthogonality to low-lying states by applying a microscopic
3α model. In this calculation, we use the Volkov No.2 potential \[18\] with the Majorana value \( M = 0.592 \) as an effective nucleon-nucleon interaction. By using this interaction, the experimental binding energy of \(^{12}\text{C}\) (7.4 MeV) from the 3α threshold, which is calculated as \(-83.69\text{ MeV}\), is well reproduced.

Various intrinsic 3α configurations can be described by a parameter set \((l_1, l_2, \theta)\), where \(l_1\) and \(l_2\) are intervals between two α-clusters and \(\theta\) is a bending angle (See Fig.1a)). The linear-chain configuration corresponds to \(\theta = \pi\). Here, we treat \(l_1\) and \(\theta\) as generator coordinates and construct the basis states which have various sets of these coordinates. We employ \(l_1 = 1, 2, 3, 4, 5\text{ fm}\) and \(\theta = \pi \sim \pi/3\). We regard \(l_2\) as a variational parameter and apply a cooling method to search for the energy minimum. The 45 GCM basis states described by parameters \((l_1, l_2, \theta)\) are obtained, and their explicit values are presented in Table I.

After projecting the obtained pure linear-chain intrinsic states \(\theta = \pi\) to \(J^\pi = 0^+\), we obtain the energy \(-71.72\text{ MeV}\) at the minimum point \((l_1, l_2) = (4\text{ fm}, 2.89\text{ fm})\) (GCM basis No.4). This energy is much higher than the 3α threshold where the 0\(^+_2\) state is observed. To solve the longitudinal vibration of three α clusters, we employ 5 GCM basis states which are given by \((l_1, l_2) = (1\text{ fm}, 3.55\text{ fm}), (2\text{ fm}, 3.22\text{ fm}), (3\text{ fm}, 2.97\text{ fm}), (4\text{ fm}, 2.89\text{ fm}), \text{and} (5\text{ fm}, 2.92\text{ fm})\), keeping \(\theta = \pi\) (GCM basis of No.1 \sim No.5). After solving the GCM equation, we have the binding energy gain by 2.86 MeV \((-74.59\text{ MeV})\), but its energy is still too short for the energy of the experimentary observed 0\(^+_2\) state.

Furthermore, we take account of the bending motion \((\theta = 11\pi/12, 10\pi/12\text{ and } 9\pi/12)\) which corresponds to fluctuation around the linear-chain configuration.
solutions of the GCM equation with those basis states (No.1 ~ No.20) give the lowest energy $-80.40$ MeV. Although this energy is about 3 MeV higher than the energy of the $0^+_2$ state, it seems to give a good correspondence with that. The wave function of this solution has a dominant components of the linear-chain configuration, which are seen from the fact that overlap with the linear-chain GCM solution (described in the previous paragraph) is 0.65.

From the analyses described above, one may consider that the $0^+_2$ state can be interpreted as a $3\alpha$ linear-chain state with small bending vibrations. However, this solution may also have a large overlap with the ground state of the $3\alpha$ system, and it is necessary to solve the $0^+_2$ state by taking into account of the coupling with the ground state. In order to clarify this point, we performed the GCM calculation with larger basis states including those which describe low-lying levels. After diagonalizing the Hamiltonian with GCM basis states (No.1 ~ No.45 in table I), we obtain the ground state energy $-91.23$ MeV which well corresponds to the experimental one ($-92.2$ MeV).

In Fig.2, we show the conversion behavior of the energy spectrum. As the number of GCM basis states increases, each basis configuration generally becomes closer to equilateral triangle states. The ground $0^+$ state is reproduced around the basis states No.30 which corresponds to the configurations of $\theta = 6\pi/12$. By this increase of basis states, the second $0^+$ state converges energetically. This solution has the overlap with linear-chain ($\theta = \pi$) basis states of only 0.346 (absolute square), and cannot be interpreted as a linear-chain state.

| $l_1$ | $l_2$ | $\theta$ | $l_1$ | $l_2$ | $\theta$ | $l_1$ | $l_2$ | $\theta$ |
|-------|-------|---------|-------|-------|---------|-------|-------|---------|
| 1     | 1.0   | 3.5     | 16    | 1.0   | 4.7     | 9\pi/12| 31    | 1.0   | 1.4     | 6\pi/12|
| 2     | 2.0   | 3.2     | 17    | 2.0   | 2.9     | 9\pi/12| 32    | 2.0   | 1.9     | 6\pi/12|
| 3     | 3.0   | 3.0     | 18    | 3.0   | 2.9     | 9\pi/12| 33    | 3.0   | 2.4     | 6\pi/12|
| 4     | 4.0   | 2.9     | 19    | 4.0   | 2.9     | 9\pi/12| 34    | 4.0   | 2.7     | 6\pi/12|
| 5     | 5.0   | 2.9     | 20    | 5.0   | 2.9     | 9\pi/12| 35    | 5.0   | 2.9     | 6\pi/12|
| 6     | 1.0   | 3.5     | 21    | 1.0   | 4.5     | 8\pi/12| 36    | 1.0   | 1.5     | 5\pi/12|
| 7     | 2.0   | 3.2     | 22    | 2.0   | 2.5     | 8\pi/12| 37    | 2.0   | 2.0     | 5\pi/12|
| 8     | 3.0   | 3.0     | 23    | 3.0   | 2.7     | 8\pi/12| 38    | 3.0   | 2.4     | 5\pi/12|
| 9     | 4.0   | 2.9     | 24    | 4.0   | 2.9     | 8\pi/12| 39    | 4.0   | 2.7     | 5\pi/12|
| 10    | 5.0   | 2.9     | 25    | 5.0   | 2.9     | 8\pi/12| 40    | 5.0   | 2.9     | 5\pi/12|
| 11    | 1.0   | 3.3     | 26    | 1.0   | 4.9     | 7\pi/12| 41    | 1.0   | 1.3     | 4\pi/12|
| 12    | 2.0   | 3.1     | 27    | 2.0   | 2.2     | 7\pi/12| 42    | 2.0   | 2.1     | 4\pi/12|
| 13    | 3.0   | 2.9     | 28    | 3.0   | 2.5     | 7\pi/12| 43    | 3.0   | 2.6     | 4\pi/12|
| 14    | 4.0   | 2.9     | 29    | 4.0   | 2.8     | 7\pi/12| 44    | 4.0   | 2.8     | 4\pi/12|
| 15    | 5.0   | 2.9     | 30    | 5.0   | 2.9     | 7\pi/12| 45    | 5.0   | 2.9     | 4\pi/12|

Table 1: The GCM basis states for $^{12}$C described by $l_1$ (fm), $l_2$ (fm) and $\theta$ (radian) of Fig. 1 (a). $l_2$ is determined to minimize the energy at given $l_1$ and $\theta$. 


3.2 \textbf{4}\textalpha\textbf{ states in }^{16}\textbf{O}

In a similar way to the 3\textalpha\textbf{ system, we investigate the stability of the 4\textalpha\textbf{ linear-chain configuration in }^{16}\textbf{O}. We use the Volkov No.2 interaction \[18\] with the Majorana parameter \(M = 0.63\), in order to reproduce the ground state energy of \(^{16}\text{O} (-127.6\text{MeV}) \[17\]. However, this effective nuclear interaction has a shortcoming that the calculated binding energy of \(^{12}\text{C} (-82.0 \text{ MeV})\) does not reproduce the observed one (-92.2 MeV). This problem has been discussed \[17\] from both sides of model space and effective interactions. As one important conclusion, it has been shown that the clustering states are well reproduced around the corresponding threshold energy for various effective interactions.

We apply the microscopic 4\textalpha-cluster model and carry out the GCM calculation for stability examination of the 4\textalpha\textbf{ linear-chain configuration. The employed GCM basis states are listed in Table II. The linear-chain states and their bending motions are described by 45 GCM basis states (No.1 \sim No.45 in Table II) of the \(^{8}\text{Be}+^{8}\text{Be}\) configurations with length parameters \(l_{1}\) and \(l_{2}\) and an angle parameter \(\theta = \pi \sim \pi/2\) as shown in Fig.1 (b). The relative position of two \(^{8}\text{Be}\) clusters is determined by \(l_{2}\) \((l_{2} = 0.5, 1.0, 1.5, 2.0 \text{ and } 2.5 \text{ fm})\) and the distance \(l_{1}\) between the two \(\alpha\)-clusters of each \(^{8}\text{Be}\) is chosen to minimize the energy at given \(l_{2}\) and \(\theta\). In addition, we prepare 15 GCM basis states obtained by the Constraint Cooling Method for the description of other lower states.

As the first step in research of the 4\textalpha-linear-chain state, we search the 0\textsuperscript{+} \textbf{optimal linear-chain configuration with \((l_{1} = 3.29 \text{ fm}, l_{2} = 1.50 \text{ fm, } \theta = 0)\) at the energy \(E = 34.79 \text{ MeV}\) from the 4\textalpha\textbf{ threshold within a single Slater determinant. As the next
Table 2: The GCM basis states for $^{16}$O. Basis state configurations configuration No.1 ~ No.45 are described by $l_1$ (fm) $l_2$ (fm) and $\theta$ (radian) of Fig. 1 (b). The value of $l_2$ is determined to minimize the total energy. The GCM basis states No.46 ~ No.60 are obtained by the constraint cooling method, and classified with the constraint principal quantum numbers ($n_x$, $n_y$ and $n_z$).

step, we examine its stability for mixing with configurations other than the linear-chain by solving the GCM equation with increasing the number of the basis states.

The $0^+$-state energy convergence is shown in Fig.3, and the ground state energy converges to $-129.1$ MeV. The energy of the $0^+$ state obtained at $-91.68$ MeV (finally corresponds to $0^+_6$) converges with a very small number of GCM basis states ($\leq$ No.15), which means this state contains large linear-chain components with small bending ($\leq 10\pi/12$). As an evidence, this state has an overlap (absolute square) with the GCM basis states Nos.1-10 ($\theta = \pi$ and $11\pi/12$) of 0.461 and with the GCM basis states Nos.1-15 ($\theta = \pi, 11\pi/12$ and $10\pi/12$) of 0.613. Even when a larger number of GCM basis states are adopted, the state of $-91.68$ MeV is hardly affected by other lower states and approximately keeps its own energy value. However this energy is higher about 20 MeV compared with the $4\alpha$ threshold energy ($-111.6$ MeV).

There may be at least two reasons for the too-high-energy solution of $0^+_6$: 1) The $^{12}$C+$\alpha$ threshold is not correctly reproduced due to the present $NN$ interaction, and is calculated at the energy lower than experiment by about 10 MeV. As was discussed in the previous paper, it is very difficult problem to reproduce both threshold energies of $^{12}$C+$\alpha$ and $4\alpha$ by an effective $NN$ interaction. Because of this shortcoming, $4\alpha$ clustered configurations are calculated at rather high excited energies. 2) The GCM basis states describing the $0^+_6$ may be insufficient. We have employed the configurations of bending motion deviating from the linear-chain, and of the three-dimensional $4\alpha$ structure though they contribute little to this $0^+_6$ state. Then the energy of this state
would be lower if we solve other small vibrations around the linear-chain configuration such as $Z$ letter-type mode or three-dimensional twinst mode.

However, an improved calculation based on the above reason for the fault would only increase the quantitative precision, and at least we can suggest two things from the resent result. First, the state of $-91.68$ MeV has a large overlap with the linear-chain configuration which shows the $\theta$ stability. Second, this state is hardly affected by other lower energy states and there is a large possibility to correspond to remain as resonance state.

In Fig.4, the linear-chain states with different angular momenta are presented. From the $2^+$ state, the rotational band is shown to appear with a very small slope of 88 keV because of its large inertia moment. This moment of inertia is similar to the experimentally suggested value (about 60 keV).

4 Summary and Conclusion

In this paper, we have microscopically studied the existence and the stability of $N\alpha$ linear-chain state in $^{12}C$ and $^{16}O$. We have emphasized the importance of the orthogonality between the linear-chain state and other lower states. The linear-chain state of $^{12}C$ is hardly considered to be a stable state as we increase the number of the GCM basis states. In other words, their components fragment to the levels over broad range of excitation energy, especially to the $0^+_2$ state, and these couplings make a pure linear-chain state unstable. On the other hand, the linear-chain state of $^{16}O$ is hardly affected by other states, and the rotational band structure is shown to appear, although
Figure 4: The rotational band of $^{16}$O linear-chain state. The numbers show the order of excited state.

The absolute energy of the band head is higher by about 20 MeV compared with the $4\alpha$-threshold.

These conclusions are consistent with the previous work by Ikeda [11], where the stability against small vibrations around the equilibrium configuration is considered. However, we have shown in this work that the linear-chain state of $^{16}$O may survive even after the couplings to the lower states are taken into account.

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References

[1] B. M. Nyakó et al., Phys. Rev. Lett. 52 (1984) 507.
[2] A. Galindo-Uribarri et al., Phys. Rev. Lett. 71 (1993) 231.
[3] H. Morinaga, Phys. Rev. 101 (1956) 254.
[4] H. Morinaga, Phys. Letters 21 (1966) 78.

[5] K. Ikeda, N. Takigawa and H. Horiuchi, Prog. Theor. Phys. Suppl. Extra Number (1968) 464;

[6] P. Chevallier and F. Scheibling, Phys. Rev. 160 (1967) 160.

[7] Y. Suzuki, H. Horiuchi and K. Ikeda, Prog. Theor. Phys. 47 (1972) 1517.

[8] Wuosmuma et al., Phys. Rev. Lett. 68 (1992) 1295.

[9] W.D.M. Rea, A.C. Merchant and B. Buck, Phys. Rev. Lett 69 (1992) 3709.

[10] Y. Hirabayashi, Y. Sakuragi and Y. Abe, Phys. Rev. Lett 74 (1995) 4141.

[11] H. Horiuchi, K. Ikeda and Y. Suzuki, Prog. Theor. Phys. Suppl. 52 (1972) 89.

[12] H. Horiuchi, M. Maruyama, A. Ohnishi and S. Yamaguchi, Preprint KUNS 1028.

[13] Y. Kanada-En’yo and H. Horiuchi, Prog. Theor. Phys. 93 (1995) 115.

[14] Y. Kanada-En’yo, H. Horiuchi and A. Ono, Phys. Rev. C52 (1995), 628.

[15] Y. Kanada-En’yo and H. Horiuchi Phys. Rev. C52 (1995), 647.

[16] A. Ono, H. Horiuchi, T. Maruyama and A. Ohnishi, Prog. Theor. Phys. 87 (1992) 1185; Phys. Rev. Lett. 68 (1992) 2898.

[17] N. Itagaki, A. Ohnishi and K. Katō Prog. Theor. Phys. 94 (1995), 1019.

[18] A. B. Volkov Nucl. Phys. 74 (1965) 33, C47 (1992), 210.

[19] Y. Fujiwara et al., Prog. Theor. Phys. Suppl. No. 68 (1980), 29.
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1 Introduction

A nuclear study of a large deformation is one of the frontier in nuclear spectroscopy. Since the first observation of the superdeformed band [1], various superdeformed nuclei ($\sim 1:2$ deformation) have been found, and nowadays extensive studies on these bands including their excited bands are in progress. Furthermore, there are several reports on the hyperdeformed band in which the deformation is around 1:3 [2]. These large deformed nuclei are mainly found in the medium-heavy mass number region where the liquid-drop energy and the shell-correction balances. Therefore, larger deformation than 1:3 is not expected to appear in this mass number region, since the shell-correction is not enough to make a barrier at larger deformations.

On the other hand, a large deformation above 1:3 may be possible in light nuclei due to the $\alpha$-cluster structure. For example, the first example of the “superdeformed” nuclei is nothing but $^8$Be which has the $\alpha - \alpha$ structure, and there have been a lot of discussion on the nature of $0^+_2$ state of $^{12}$C as the $3\alpha$ linear-chain state [3, 4]. Experimentally, it is more difficult to observe large-deformed states or linear-chain states of $\alpha$-clusters in light nuclei, since these states have large particle decay widths, and consequently a coincident detection of multi-$\alpha$ particles becomes necessary. From a theoretical point of view, on the other hand, it is required to estimate the couple-channel effects over the broad range of deformation or geometric structure, since a mean field picture may not be valid for the highly excited states in light nuclei. However, since it gives us the opportunity to explore the most deformed nuclei, the study of these $N\alpha$ linear-chain states [5] is valuable.

For example, there are several experimental candidates in light $4N$ nuclei for large deformed states above 1:3. The first example is the $4\alpha$ linear-chain band starting around the $4\alpha$ threshold energy region in $^{16}$O suggested by Chevallier et al. [6] through the data analysis of the $^{12}$C+$\alpha \rightarrow ^{8}$Be+$^8$Be reaction. This suggestion is supported by a theoretical work by Suzuki et al. [7]; they analyzed the decay widths from such a strongly coupled linear-chain state, and discussed that the excited states observed by Chevallier et al. are possibly characterized by a $4\alpha$ linear-chain structure. The second example is the $6\alpha$ linear-chain states, which is discussed as a candidate to explain a new molecular resonance state observed by Wuosmaa et al. [8] in the excitation function of the inelastic reaction $^{12}$C($^{12}$C, $^{12}$C($0^+_2$))$^{12}$C($0^+_2$) at $E_x=56.4$ MeV [9]. Although the clarification of the existence of a $6\alpha$ linear-chain state is very challenging, the coupling to $^{12}$C$^*$$-^{12}$C$^*$ and $^{16}$O$^*$$-^8$Be may be important [10] and makes it difficult to study a pure linear-chain state. Therefore, it is important and urgent to study the stability of linear-chain states in $^{12}$C and $^{16}$O at this stage.

The reliability of the linear-chain structure has been usually studied through kinematical analyses of decay widths. It has been also discussed on the analyses of small vibration around the equilibrium configuration by Ikeda [11]. In those previous studies, however, the linear-chain structure was assumed a priori. Namely, the coupling effects with other states have not been taken into account. These couplings, especially with lower excited states are expected to play an important role on the stability of linear-chain states, since they push-up the energy of linear-chain states and generally
increase the instability. Therefore, it is necessary to calculate in a wide wave function space, which covers not only the linear-chain configurations but also the lower excited states.

In this paper, we investigate the stability of $4\alpha$ linear-chain configurations in $^{16}\text{O}$ comparing with $3\alpha$ states of $^{12}\text{C}$ by applying a microscopic $N\alpha$-cluster model. In contrast to the previous works, we discuss the stability of the $N\alpha$ linear-chain structure by solving dynamics of $N\alpha$ systems. This is achieved by diagonalizing the Hamiltonian matrix in a wide space which also covers low-lying levels including the ground state. The framework used here is based on the Generator Coordinate Method (GCM) by which we can solve relative motion of $\alpha$ clusters by taking account of the superposition of many different intrinsic configurations. We use the Constraint Cooling Method [12, 13, 14, 15] proposed in the framework of AMD [16] in order to generate GCM basis which describes low-lying levels. This method has been already shown to provide the suitable GCM basis states effectively [17].

This paper is organized as follows; in section 2, we explain our model especially the way to construct the GCM basis states. In section 3, we show our results that the $^{12}\text{C}$ linear-chain state disappears because of the orthogonality to other states, however the $^{16}\text{O}$ linear-chain state exists above the $4\alpha$ threshold and shows the rotational band structure. In section 4 summary and conclusion are presented.

2 Method

In the framework of GCM, the energy spectra and the wave functions are solved by diagonalizing the Hamiltonian matrix through the Hill-Wheeler equation;

$$\Psi^J = \sum J c_J \Phi^J ,$$  \hspace{1cm}  (1)

$$\sum_k (\langle \Phi^J | H | \Phi^J \rangle \phi^J_k ) c_k = 0 ,$$  \hspace{1cm}  (2)

where $\Phi^J$ is the $j$th GCM basis wave functions with the total angular momentum $J$ and its $z$-component $M$. Since the reliability of the GCM calculation largely depends on the adopted basis wave functions, we have adopted the GCM basis states which reasonably reproduce the low-lying excited states in addition to the linear-chain configurations described in the next section. The first set of GCM basis states is generated by applying the constraint cooling method [13] to the Bloch-Brink wave function for an $N\alpha$ system. The angular momentum projection from these wave packets is carried out numerically. A detailed explanation of this method is found in our previous paper [17].

Hereafter, we briefly summarize our method. The Bloch-Brink wave function for an $N\alpha$ system is the Slater determinant of Gaussian wave packets [12, 16] with the assumption of the $[4]$-symmetry for the spin and iso-spin part,

$$\Phi(\{r_\alpha^a; Z_i\}) = A \prod_{i=1,N} A \prod_{\alpha=1,4} \phi^a_i (r_\alpha^a; Z_i) ,$$  \hspace{1cm}  (3)
\[
\phi_i^\alpha(r^\alpha; Z_i) = \exp\{-\nu(r^\alpha - Z_i/\sqrt{\nu})^2 + Z_i^2/2\} \chi_\alpha \quad (i = 1 \sim N),
\]
where \( \chi_\alpha = \{ p \uparrow, p \downarrow, n \uparrow, n \downarrow \} \) are the spin-isospin wave functions. In the spatial part, the complex parameters \( Z_i = \sqrt{\nu}(D_i + i K_i/2\nu \hat{h}) \) represent phase space coordinates. In this work, the size parameter \( \nu = 0.27 \text{ fm}^{-2} \) is chosen to reproduce the energy and the r.m.s. radius of the \( \alpha \) particle.

In order to generate various intrinsic states, we employ the constraint cooling method [12, 13, 14, 15], by which we can search for a set of suitable complex parameters \( \{Z_i\} \) to minimize the expectation value of the energy under some constraints. Following Ref. [13], we obtain an optimum configuration state by solving the constraint cooling equation given by

\[
\frac{dZ_i}{d\beta} = -\frac{\partial \mathcal{H}}{\partial Z_i^*} - \sum_{i=1}^{n} \eta_i \frac{\partial \langle W_i \rangle}{\partial Z_i^*},
\]
where \( \mathcal{H} \) is the expectation value of the Hamiltonian operator. The expectation value of the constrained observable \( \langle W_i \rangle \) in Eq. (5) plays a role of the generator coordinate, and \( \eta_i \) is the Lagrange multiplier to keep the given constraint along the evolution. In this work, we constrain the expectation value of the total oscillator quantum numbers \( W_i = n_x, n_y \) and \( n_z \) in a rectangular coordinate system [17]. It is possible to constrain other observables, such as an angular momentum, however, cooling with an angular momentum constraint essentially results in giving the rotating nucleus with the similar intrinsic state when the angular momentum is not so high. Then these states are not very effective as GCM basis states.

Since the resultant cooled state is not the eigenstate of an angular momentum in general, it is necessary to carry out the angular momentum projection to construct GCM basis wave functions in Eq. (1).

\[
\Phi^{JM}(\{ r^\alpha_i; Z_i \}) = \frac{2J + 1}{8\pi^2} \int d\Omega D_{MK}^{J*}(\Omega) \tilde{R}(\Omega) \Phi(\{ r^\alpha_i; Z_i \}),
\]
where \( \Omega \) is the Euler angle of \( \{ r^\alpha_i \} \). In the numerical integration over the Euler angle \( \Omega(\alpha, \beta, \gamma) \), we have adopted 25\(^3\) mesh points and integration is carried out with the Gauss-Legendre integral technique. The intrinsic angular momentum component \( K \) is chosen to be zero for simplicity.

3 \( \alpha \) linear-chain states in \(^{12}\text{C}\) and \(^{16}\text{O}\)

3.1 \( 3\alpha \) states in \(^{12}\text{C}\)

In many previous analyses, it has been shown that a large part of low-lying excited states of \(^{12}\text{C}\) have clustering structures. Although Morinaga [4] proposed the linear-chain structure for \(^{12}\text{C}\), the clustering states of \( 0^+_2(7.66 \text{ MeV}) \) and \( 2^+_2(\sim 10 \text{ MeV}) \) have been discussed to have a loosely-coupled \( 3\alpha \) configuration [19] rather than the linear-chain structure. We here investigate the stability of the \( 3\alpha \)-linear-chain configuration in \(^{12}\text{C}\) from a viewpoint of orthogonality to low-lying states by applying a microscopic
3α model. In this calculation, we use the Volkov No.2 potential [18] with the Majorana value $M = 0.592$ as an effective nucleon-nucleon interaction. By using this interaction, the experimental binding energy of $^{12}$C (7.4 MeV) from the 3α threshold, which is calculated as $-83.69$ MeV, is well reproduced.

Various intrinsic 3α configurations can be described by a parameter set $(l_1, l_2, \theta)$, where $l_1$ and $l_2$ are intervals between two α-clusters and $\theta$ is a bending angle (See Fig. 1 a)). The linear-chain configuration corresponds to $\theta = \pi$. Here, we treat $l_1$ and $\theta$ as generator coordinates and construct the basis states which have various sets of these coordinates. We employ $l_1 = 1, 2, 3, 4, 5$ fm and $\theta = \pi \sim \pi/3$. We regard $l_2$ as a variational parameter and apply a cooling method to search for the energy minimum. The 45 GCM basis states described by parameters $(l_1, l_2, \theta)$ are obtained, and their explicit values are presented in Table I.

After projecting the obtained pure linear-chain intrinsic states ($\theta = \pi$) to $J^\pi = 0^+$, we obtain the energy $-71.72$ MeV at the minimum point $(l_1, l_2) = (4$ fm, $2.89$ fm) (GCM basis No.4). This energy is much higher than the 3α threshold where the $0^+_2$ state is observed. To solve the longitudinal vibration of three α-clusters, we employ 5 GCM basis states which are given by $(l_1, l_2) = (1$ fm, $3.55$ fm), (2 fm, $3.22$ fm), (3 fm, $2.97$ fm), (4 fm, $2.89$ fm), and (5 fm, $2.92$ fm), keeping $\theta = \pi$ (GCM basis of No.1 $\sim$ No.5). After solving the GCM equation, we have the binding energy gain by $2.86$ MeV ($-74.59$ MeV), but its energy is still too short for the energy of the experimental observed $0^+_2$ state.

Furthermore, we take account of the bending motion ($\theta = 11\pi/12$, $10\pi/12$ and $9\pi/12$) which corresponds to fluctuation around the linear-chain configuration. The solutions of the GCM equation with those basis states (No.1 $\sim$ No.20) give the lowest
Table 1: The GCM basis states for $^{12}$C described by $l_1$ (fm), $l_2$ (fm) and $\theta$ (radian) of Fig. 1 (a). $l_2$ is determined to minimize the energy at given $l_1$ and $\theta$.

| $l_1$ | $l_2$ | $\theta$ | $l_1$ | $l_2$ | $\theta$ | $l_1$ | $l_2$ | $\theta$ |
|-------|-------|---------|-------|-------|---------|-------|-------|---------|
| 1     | 1.0   | 3.5     | 16    | 1.0   | 4.7     | 31    | 1.0   | 1.4     | 6\pi/12 |
| 2     | 2.0   | 3.2     | 17    | 2.0   | 2.9     | 32    | 2.0   | 1.9     | 6\pi/12 |
| 3     | 3.0   | 3.0     | 18    | 3.0   | 2.9     | 33    | 3.0   | 2.4     | 6\pi/12 |
| 4     | 4.0   | 2.9     | 19    | 4.0   | 2.9     | 34    | 4.0   | 2.7     | 6\pi/12 |
| 5     | 5.0   | 2.9     | 20    | 5.0   | 2.9     | 35    | 5.0   | 2.9     | 6\pi/12 |
| 6     | 1.0   | 3.5 11\pi/12 | 21    | 1.0   | 4.5     | 36    | 1.0   | 1.5     | 5\pi/12 |
| 7     | 2.0   | 3.2 11\pi/12 | 22    | 2.0   | 2.5     | 37    | 2.0   | 2.0     | 5\pi/12 |
| 8     | 3.0   | 3.0 11\pi/12 | 23    | 3.0   | 2.7     | 38    | 3.0   | 2.4     | 5\pi/12 |
| 9     | 4.0   | 2.9 11\pi/12 | 24    | 4.0   | 2.9     | 39    | 4.0   | 2.7     | 5\pi/12 |
| 10    | 5.0   | 2.9 11\pi/12 | 25    | 5.0   | 2.9     | 40    | 5.0   | 2.9     | 5\pi/12 |
| 11    | 1.0   | 3.3 10\pi/12 | 26    | 1.0   | 4.9     | 41    | 1.0   | 1.3     | 4\pi/12 |
| 12    | 2.0   | 3.1 10\pi/12 | 27    | 2.0   | 2.2     | 42    | 2.0   | 2.1     | 4\pi/12 |
| 13    | 3.0   | 2.9 10\pi/12 | 28    | 3.0   | 2.5     | 43    | 3.0   | 2.6     | 4\pi/12 |
| 14    | 4.0   | 2.9 10\pi/12 | 29    | 4.0   | 2.8     | 44    | 4.0   | 2.8     | 4\pi/12 |
| 15    | 5.0   | 2.9 10\pi/12 | 30    | 5.0   | 2.9     | 45    | 5.0   | 2.9     | 4\pi/12 |

energy $-80.40$ MeV. Although this energy is about 3 MeV higher than the energy of the $0^+_1$ state, it seems to give a good correspondence with that. The wave function of this solution has a dominant component of the linear-chain configuration, which are seen from the fact that overlap with the linear-chain GCM solution (described in the previous paragraph) is 0.65.

From the analyses described above, one may consider that the $0^+_1$ state can be interpreted as a $3\alpha$ linear-chain state with small bending vibrations. However, this solution may also have a large overlap with the ground state of the $3\alpha$ system, and it is necessary to solve the $0^+_1$ state by taking into account of the coupling with the ground state. In order to clarify this point, we performed the GCM calculation with larger basis states including those which describe low-lying levels. After diagonalizing the Hamiltonian with GCM basis states (No.1 $\sim$ No.45 in table 1), we obtain the ground state energy $-91.23$ MeV which well corresponds to the experimental one ($-92.2$ MeV).

In Fig.2, we show the conversion behavior of the energy spectrum. As the number of GCM basis states increases, each basis configuration generally becomes closer to equilateral triangle states. The ground $0^+$ state is reproduced around the basis states No.30 which corresponds to the configurations of $\theta = 6\pi/12$. By this increase of basis states, the second $0^+$ state converges energetically. This solution has the overlap with linear-chain ($\theta = \pi$) basis states of only 0.346 (absolute square), and cannot be interpreted as a linear-chain state.
3.2 4α states in $^{16}$O

In a similar way to the 3α system, we investigate the stability of the 4α linear-chain configuration in $^{16}$O. We use the Volkov No.2 interaction [18] with the Majorana parameter $M = 0.63$, in order to reproduce the ground state energy of $^{16}$O ($-127.6\text{MeV}$) [17]. However, this effective nuclear interaction has a shortcoming that the calculated binding energy of $^{12}$C ($-82.0 \text{MeV}$) does not reproduce the observed one ($-92.2 \text{MeV}$). This problem has been discussed [17] from both sides of model spaces and effective interactions. As one important conclusion, it has been shown that the clustering states are well reproduced around the corresponding threshold energy for various effective interactions.

We apply the microscopic 4α-cluster model and carry out the GCM calculation for stability examination of the 4α linear-chain configuration. The employed GCM basis states are listed in Table II. The linear-chain states and their bending motions are described by 45 GCM basis states (No.1 ~ No.45 in Table II) of the $^8\text{Be}+^8\text{Be}$ configurations with length parameters ($l_1$ and $l_2$) and an angle parameter ($\theta = \pi \sim \pi/2$) as shown in Fig.1 (b). The relative position of two $^8\text{Be}$ clusters is determined by $l_2$ ($l_2 = 0.5, 1.0, 1.5, 2.0$ and $2.5 \text{fm}$) and the distance ($l_1$) between the two α-clusters of each $^8\text{Be}$ is chosen to minimize the energy at given $l_2$ and $\theta$. In addition, we prepare 15 GCM basis states obtained by the Constraint Cooling Method for the description of other lower states.

As the first step in research of the 4α-linear-chain state, we search the $0^+$ optimal linear-chain configuration with ($l_1 = 3.29 \text{fm}, l_2 = 1.50 \text{fm}, \theta = 0$) at the energy $E = 34.79 \text{MeV}$ from the 4α threshold within a single Slater determinant. As the next
Table 2: The GCM basis states for $^{16}$O. Basis state configurations configuration No.1 ~ No.45 are described by $l_1$ (fm) $l_2$ (fm) and $\theta$ (radian) of Fig. 1 (b). The value of $l_2$ is determined to minimize the total energy. The GCM basis states No.46 ~ No.60 are obtained by the constraint cooling method, and classified with the constraint principal quantum numbers ($n_x$, $n_y$ and $n_z$).

step, we examine its stability for mixing with configurations other than the linear-chain by solving the GCM equation with increasing the number of the basis states.

The $0^+$-state energy convergence is shown in Fig.3, and the ground state energy converges to $-129.1$ MeV. The energy of the $0^+$ state obtained at $-91.68$ MeV (finally corresponds to $0^+_0$) converges with a very small number of GCM basis states ($\simeq$ No.15), which means this state contains large linear-chain components with small bending ($\leq 10\pi/12$). As an evidence, this state has an overlap (absolute square) with the GCM basis states Nos.1-10 ($\theta = \pi$ and 11$\pi/12$) of 0.461 and with the GCM basis states Nos.1-15 ($\theta = \pi, 11\pi/12$ and 10$\pi/12$) of 0.613. Even when a larger number of GCM basis states are adopted, the state of $-91.68$ MeV is hardly affected by other lower states and approximately keeps its own energy value. However this energy is higher about 20 MeV compared with the 4$\alpha$ threshold energy ($-111.6$ MeV).

There may be at least two reasons for the too-high-energy solution of $0^+_0$: 1) The $^{12}$C+$\alpha$ threshold is not correctly reproduced due to the present $NN$ interaction, and is calculated at the energy lower than experiment by about 10 MeV. As was discussed in the previous paper, it is very difficult problem to reproduce both threshold energies of $^{12}$C+$\alpha$ and 4$\alpha$ by an effective $NN$ interaction. Because of this shortcoming, 4$\alpha$ clusterized configurations are calculated at rather high excited energies. 2) The GCM basis states describing the $0^+_0$ may be insufficient. We have employed the configurations of bending motion deviating from the linear-chain, and of the three-dimensional 4$\alpha$ structure though they contribute little to this $0^+_0$ state. Then the energy of this state
would be lower if we solve other small vibrations around the linear-chain configuration such as $Z$ letter-type mode or three-dimensional twin mode.

However, an improved calculation based on the above reason for the fault would only increase the quantitative precision, and at least we can suggest two things from the recent result. First, the state of $-91.68$ MeV has a large overlap with the linear-chain configuration which shows the $\theta$ stability. Second, this state is hardly affected by other lower energy states and there is a large possibility to correspond to remain as resonance state.

In Fig.4, the linear-chain states with different angular momenta are presented. From the $2^+$ state, the rotational band is shown to appear with a very small slope of 88 keV because of its large inertia moment. This moment of inertia is similar to the experimentally suggested value (about 60 keV).

4 Summary and Conclusion

In this paper, we have microscopically studied the existence and the stability of $N\alpha$ linear-chain state in $^{12}\text{C}$ and $^{16}\text{O}$. We have emphasized the importance of the orthogonality between the linear-chain state and other lower states. The linear-chain state of $^{12}\text{C}$ is hardly considered to be a stable state as we increase the number of the GCM basis states. In other words, their components fragment to the levels over broad range of excitation energy, especially to the $0^+_2$ state, and these couplings make a pure linear-chain state unstable. On the other hand, the linear-chain state of $^{16}\text{O}$ is hardly affected by other states, and the rotational band structure is shown to appear, although
Figure 4: The rotational band of $^{16}$O linear-chain state. The numbers show the order of excited state.

the absolute energy of the band head is higher by about 20 MeV compared with the 4α-threshold.

These conclusions are consistent with the previous work by Ikeda [11], where the stability against small vibrations around the equilibrium configuration is considered. However, we have shown in this work that the linear-chain state of $^{16}$O may survive even after the couplings to the lower states are taken into account.

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References

[1] B. M. Nyakó et al., Phys. Rev. Lett. 52 (1984) 507.
[2] A. Galindo-Uribarri et al., Phys. Rev. Lett. 71 (1993) 231.
[3] H. Morinaga, Phys. Rev. 101 (1956) 254.
[4] H. Morinaga, Phys. Letters 21 (1966) 78.

[5] K. Ikeda, N. Takigawa and H. Horiuchi, Prog. Theor. Phys. Suppl. Extra Number (1968) 464.

[6] P. Chevallier and F. Scheibling, Phys. Rev. 160 (1967) 160.

[7] Y. Suzuki, H. Horiuchi and K. Ikeda, Prog. Theor. Phys. 47 (1972) 1517.

[8] Wuosma et al., Phys. Rev. Lett. 68 (1992) 1295.

[9] W.D.M. Rea, A.C. Merchant and B. Buck, Phys. Rev. Lett 69 (1992) 3709.

[10] Y. Hirabayashi, Y. Sakuragi and Y. Abe, Phys. Rev. Lett 74 (1995) 4141.

[11] H. Horiuchi, K. Ikeda and Y. Suzuki, Prog. Theor. Phys. Suppl. 52 (1972) 89.

[12] H. Horiuchi, M. Maruyama, A. Ohnishi and S. Yamaguchi, Preprint KUNS 1028.

[13] Y. Kanada-En'yo and H. Horiuchi, Prog. Theor. Phys. 93 (1995) 115.

[14] Y. Kanada-En'yo, H. Horiuchi and A. Ono, Phys. Rev. C52 (1995), 628.

[15] Y. Kanada-En'yo and H. Horiuchi Phys. Rev. C52 (1995), 647.

[16] A. Ono, H. Horiuchi, T. Maruyama and A. Ohnishi, Prog. Theor. Phys. 87 (1992) 1185; Phys. Rev. Lett. 68 (1992) 2898.

[17] N. Itagaki, A. Ohnishi and K. Katō Prog. Theor. Phys. 94 (1995), 1019.

[18] A.B. Volkov Nucl. Phys. 74 (1965) 33, C47 (1992), 210.

[19] Y. Fujiwara et al., Prog. Theor. Phys. Suppl. No.68 (1980), 29.