Dipole and quadrupole collectivity in atomic nuclei

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Abstract. The interrelation of the dipole, i.e. cluster and quadrupole collectivity of atomic nuclei is studied in the examples of the superdeformed and hyperdeformed states of \( N = Z \) nuclei. Our method is largely based on symmetry-considerations.

1. Introduction

The atomic nuclei are typical examples of mesoscopic systems, in which the number of building blocks is not too small, and not too large. It is a longstanding problem how to find the relevant degrees of freedom, which are few enough such that a correct theoretical description can be based on them, yet the important features of the system are involved. Actually this task is as old as the nuclear research, and the methods which were developed for its solution are exported to other fields of research as well.

Our present understanding is that nuclear collectivity serves as a useful guide in this complex problem. There are different types of collective motion in nuclei, and they can be most easily characterized by their multipole nature.

The dipole collectivity is related to the clusterization. In this picture a nucleus is considered to be built up from two (or more) smaller nuclei. The relative motion of these clusters is characterized by a dipole vector. This kind of molecule-like (or in simple cases dumbbell-like) structure-model is a very old one. It was invented by G. Gamow [1] (as an alpha-particle model), even before the discovery of the neutron, i.e. before realizing that the nucleus consists of protons and neutrons.

Another important type of collectivity is the quadrupole one, which describes the deformation of the nuclear shape from spherical to prolate or oblate. When dealing with the quadrupole collectivity the nucleus is considered as a microscopic liquid drop, which can vibrate and rotate. This behaviour turned out to be very general, almost all nuclei show this feature.

The relation of the different structure models, or that of the different collective phenomena, like. e.g. clusterization and quadrupole deformation is of utmost interest. Without exploring this connection we do not have a coherent theoretical understanding.

In this contribution we investigate the interrelation of the quadrupole deformation and clusterization in some examples of superdeformed and hyperdeformed states of \( N = Z \) nuclei.
2. Methods of investigation
The superdeformed (SD) and hyperdeformed (HD) states of nuclei are defined by their quadrupole shape. They correspond to ellipsoids with ratios of main axes of 2:1:1, and 3:1:1, respectively. The investigation of their clusterization is important from two aspects. First it contributes to our understanding of the nuclear structure. Second it gives valuable information on the nuclear reactions, which can populate the state in question. In particular, a binary cluster configuration with the clusters in their ground intrinsic states is uniquely related to a reaction channel, like target and projectile in their ground states.

Both in our determination of the shape isomers, and in the investigation of their possible clusterizations symmetry considerations play an important role. In particular, the quasidynamical (or effective) $U(3)$ symmetry is used [2]. It is a generalization of the concept of the real $U(3)$ symmetry, known to be approximately valid in light nuclei [3]. The quasi-dynamical symmetry is more general in the following sense. The Hamiltonian breaks the symmetry in such a way that the $U(3)$ quantum numbers are not valid for its eigenvectors either (contrary to the case of the real $U(3)$ dynamical symmetry). In other words neither the operator is symmetric (i.e. it is not a $U(3)$ scalar), nor its eigenvectors (i.e. they do not transform according to a single irreducible representation) [4]. Yet, the symmetry is present in some sense.

An asymptotic Nilsson-state serves as an intrinsic state for the quasi-dynamical $SU(3)$ representation. Thus the effective quantum numbers are determined by the Nilsson states in the regime of large deformation [5]. When the deformation is not large enough, then we can expand the Nilsson-states in the asymptotic basis, and calculate the effective quantum numbers based on this expansion [6].

The $SU(3)$ quantum numbers uniquely determine the quadrupole shape of the nucleus [7], thus we obtain the shape isomers from them. In particular, a self-consistency calculation is performed with respect to the quadrupole shape of nucleus. It is based on the application of the quasidynamical $U(3)$ quantum numbers [8], and in those cases when a detailed comparison can be made with the more traditional energy-minimum calculations, the results are very similar [8, 9, 10].

Once the shape isomers have been found, the next question is how they are related to cluster configurations. To find their connection we use the Harvey prescription [11] and the $U(3)$ selection rule [10]. They can incorporate the effects of the exclusion principle, only in an approximate way, of course. But it is a well-defined way, and its validity can be checked by making a comparison with the results of the fully microscopic description, where they are available. In geometrical terms the $U(3)$ selection rule expresses the similarity of the quadrupole-deformation of the cluster configuration and the shell-model (or collective model) state.

Energetic preference represents a complementary viewpoint for the selection of clusterization. We incorporate it in two different ways: i) by applying simple binding energy arguments [12], and ii) by performing double-folding calculations, according to the dinuclear system model [13, 14].

3. Case studies
In this section we discuss some cases, which are in the focus of recent theoretical and experimental investigations.

3.1. $^{36}$Ar
The SD band of the $^{36}$Ar nucleus was reported in [15]. Following the experimental observation a considerable theoretical effort has been concentrated on this band. In [16] e.g. the possible binary clusterizations of this state was studied systematically. Similar studies have been done also for the ground, and the hyperdeformed bands. The latter one had been predicted from alpha-cluster model calculations [17]. One of the interesting conclusions of this work [16], was
that the HD state of the $^{36}$Ar nucleus could be populated in the $^{24}$Mg+$^{12}$C and $^{20}$Ne+$^{16}$O reactions.

A recent analysis of the $^{24}$Mg+$^{12}$C elastic scattering [18] revealed the fact that the cross section can be described only by supposing resonances on top of the potential scattering. This very careful analysis incorporated phase-shift study, as well as Regge-pole and energy-dependent resonance calculations. The existence of five resonances have been proved, which have angular momenta 2, 4, 6, 7, 8. These states together with the resonances from the $^{20}$Ne+$^{16}$O reactions seem to establish a rotational band [18]. Its moment of inertia is in a very good agreement with that of the HD shape predicted from alpha-cluster model [17]. The similarity of the (predicted and observed) moments of inertia, and the fact that the resonances were seen in exactly those reactions, which define the preferred cluster-configurations of the HD shape suggest that the recently observed band is a good candidate for the hyperdeformed shape isomer of the $^{36}$Ar nucleus.

Since a candidate for the HD state showed up, the exciting question arises if such a shape can be seen in shell model calculation as well. In [8] we have carried out a Nilsson-model + quasi-dynamical $SU(3)$ calculation in order to find the answer. This calculation gives a HD state which has exactly the same symmetry as that from the alpha-cluster model. The moment of inertia from these models agrees well with the one indicated by the experiment.

### 3.2. $^{56}$Ni

The stable elongated shapes of the $^{56}$Ni nucleus, which are relevant for clusterization, have been determined from a similar Nilsson-model + quasi-dynamical $U(3)$ symmetry calculation in [19], and are shown in Figure 1. In this figure it is not the minima, rather the horizontal plateaus, which correspond to the stable shapes. (They are insensitive to the small changes of the input parameter. Furthermore, these deformations fulfill the self-consistency argument between the input and output deformation-parameters to some approximation.)

![Figure 1. Quadrupole deformation of the $^{56}$Ni nucleus from the Nilsson model with effective U(3) quantum numbers.](image)

The triaxial ground-state (for which the experimental deformation is $\beta_2 = 0.173$) is followed by a prolate-like deformed state of $0h\omega$ excitation. The next region of stability corresponds to the superdeformed shape. This state represents 4 nucleon excitation, being very much in

![Figure 2. The superdeformed state of the $^{56}$Ni nucleus (central part), and its allowed alpha-like binary cluster configurations. The energetics prefer $^{28}$Si+$^{28}$Si and $^{40}$Ca+$^{16}$O configurations to $^{32}$S+$^{24}$Mg and $^{36}$Ar+$^{20}$Ne.](image)
We have found that the ground state of $^{56}$Ni prefers asymmetric cluster configurations, from among the alpha-like clusterization only $^4\text{He}+^{52}\text{Fe}$ is allowed. The deformed, superdeformed and largely deformed triaxial states match with several clusterizations. The allowed alpha-like binary configurations of the SD state are shown in Fig. 2. Structure considerations suggest that the correlated $^{28}\text{Si}+^{28}\text{Si}$ and $^{40}\text{Ca}+^{16}\text{O}$ resonances correspond to the superdeformed state of $^{56}\text{Ni}$, but not to the hyperdeformed one. In the latter case the $^{40}\text{Ca}+^{16}\text{O}$ configuration has a strong structural forbiddenness [19]. The $^{24}\text{Mg}+^{32}\text{S}$ cluster configuration on the other hand, which is determined by the entrance channel of the ternary fission experiment [22] matches both with the SD and HD states, and with the largely deformed triaxial state in between.

The triaxial state is of special interest, because it is thought to be related to the molecular resonances of two ground-state-like (oblate) $^{28}\text{Si}$ clusters in their equator-to-equator configuration. If the equator-to-equator configuration is not exactly parallel, then other alpha-like binary clusterizations, like e.g. $^{24}\text{Mg}+^{32}\text{S}$, are also possible.

The energetic preference of the cluster configurations were obtained from binding-energy arguments [12], and from calculations based on the Dinuclear System Model [13]. The latter ones were performed both for the pole-to-pole configurations and for the ones derived from the microscopic considerations. The $^4\text{He}+\text{core}$ configuration turned out to be the most preferred one, followed by the $^8\text{Be}+\text{core}$ one. Then a group of the $^{12}\text{C}$, $^{28}\text{Si}$, and $^{16}\text{O}$ clusters come, with close-lying values, but in different order from different calculations. The $^{24}\text{Mg}$ and $^{20}\text{Ne}$ turned out to be the least-preferred alpha-like clusters.

More detailed investigations on this nucleus is presented in [19]. In this work the results of the energetic calculation with three different methods are compared: in addition to the binding energy, and double folding, mentioned above, there the extended liquid drop model [23] was also applied.

3.3. $^{28}\text{Si}$

Concerning the superdeformed state of the $^{28}\text{Si}$ nucleus, there has been an experimental candidate from [24]. Recent AMD calculations [25], as well as our Nilsson-model studies [26] predict a moment of inertia, which is different from that of the experimental data of [24]. The alpha-cluster model also gave a SD state [27] in line with these recent theoretical studies. Both AMD and our binary-cluster studies indicate the importance of the $^{24}\text{Mg}+^4\text{He}$ and $^{16}\text{O}+^{12}\text{C}$ clusterizations.

These circumstances initiated a critical overview of the available experimental information, and their extension [26]. It turned out that some states were populated in $(\alpha, \gamma)$ reactions, but not in $(p, \gamma)$ [28]. These states represent another candidate for the rotational band with superdeformed shape. The analysis of a recent in-beam gamma-ray measurement [26] gives further support for this idea. The new candidate for the SD state has a moment of inertia very close to the theoretical predictions.

4. Summary and conclusions

In this paper we have considered the connection between the dipole and quadrupole collectivity of some states in $N=Z$ nuclei. In particular elongated shape isomers, e.g. superdeformed and hyperdeformed states and their possible binary clusterizations have been investigated. Both in finding the stable shapes and in determining their relations to cluster configurations symmetry considerations were applied extensively.

We have determined the shape isomers from the quasidynamical $U(3)$ symmetry, obtained from Nilsson-calculations. In searching for the possible binary clusterizations of the shape
isomers we have taken into account both natural laws which govern the building up of a nucleus from smaller constituents: the exclusion principle and the energy-minimum principle.

This kind of investigations contribute to the understanding of the structure of the specific states, and give information on the possible reaction channels, which can populate them. Furthermore, they deepen our understanding on the connection between the different collective modes of the atomic nucleus.

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