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MHD boundary layer flow of a power-law nanofluid with new mass flux condition

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An analysis is carried out to study the magnetohydrodynamic (MHD) boundary layer flow of power-law nanofluid over a non-linear stretching sheet. In the presence of a transverse magnetic field, the flow is generated due to non-linear stretching sheet. By using similarity transformations, the governing boundary layer equations are reduced into a system of ordinary differential equations. A recently proposed boundary condition requiring zero nanoparticle mass flux is employed in the flow analysis of power-law fluid. The reduced coupled differential equations are then solved numerically by the shooting method. The variations of dimensionless temperature and nanoparticle concentration with various parameters are graphed and discussed in detail. Numerical values of physical quantities such as the skin-friction coefficient and the reduced local Nusselt number are computed in tabular form.

I. INTRODUCTION

Recently, the study of nanotechnology based on nanofluids has received broad attention due to its wide-ranging applications in various engineering’s and technologies. Nanofluid comprises nano-sized particles (diameter less than 100 nanometers) which are suspended in the base fluid. Nanofluids have varied applications in the hybrid-powered engines, chemical catalytic reactors and so forth. The common fluids such as oil, water, ethylene glycol mixtures are in general poor heat transfer fluids because of their poor thermal conductivity. A very reliable technique to enhance thermal conductivity of such fluids is the usage of nanoparticles of relatively higher conductivities suspended in the base fluid. Improvement of the heat transfer in electronic cooling, heat exchangers, double plane windows etc is tremendously important topic from the energy saving point of view. There are a few studies regarding nanofluids and their applications.

The experimental as well as numerical studies on the nanofluids have increased in recent years for different models and configurations. Abu-Nada studied the natural convection in horizontal annuli using different types of water based nanofluids. Several researchers including Mahmoodi, Abu-Nada and Oztop, Aminossadati and Ghasemi, Abu-Nada et al. and Kanaafer et al. have showed that heat transfer improve with addition of nanoparticles for constant viscosity. Khan and Pop et al. studied the boundary layer flow of a nanofluid past a stretching sheet. Gorla and Chamkha investigated the natural convection flow past horizontal plate in a porous medium. The effects of natural convective flow of nanofluid over a convectively heated vertical plate were investigated by Aziz and Khan using Buongiorno’s model. Hamad et al. studied the effect of thermal radiation and convective surface boundary conditions. Further, the convection of liquid metal under the influence of a magnetic field has been studied extensively because of its application in medicine, physics and engineering. Khan and Pop investigated the free convection boundary layer flow past a horizontal flat plate embedded in a porous medium filled with a nanofluid. Khan and khan investigated the steady flow of Burgers nanofluid over a stretching surface with heat...
generation/absorption. Hayat et al.\textsuperscript{20} studied mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink. Hady et al.\textsuperscript{21} studied the effects of thermal radiation on the viscous flow of a nanofluid and heat transfer over a non-linear sheet. Kuznetsou and Nield\textsuperscript{22} provided the revised model of natural convective boundary-layer flow of nanofluid past a vertical plate subject to the new proposed boundary condition. Recently, Khan and Khan\textsuperscript{23} studied the forced convection analysis for generalized Burgers nanofluid flow over a stretching sheet. Many equipments such as MHD generators, pumps, bearings and boundary layer control are affected by the interaction between the electrically conducting fluid and a magnetic field. Additionally, the behavior of the flow strongly depends on the orientation and intensity of the applied magnetic field. The exerted magnetic field manipulates the suspended particles and rearranges their concentration in the fluid which strongly changes heat transfer characteristics of the flow. A magnetic nanofluid has both the liquid and magnetic characteristics. Such materials have fascinating applications like optical modulators, magneto-optical wavelength filters, nonlinear optical materials, optical switches, optical gratings etc. Magnetic particles have key role in the construction of loud speakers as sealing materials and in sink float separation. Magneto nanofluids are useful to guide the particles up the blood stream to a tumor with magnets. This is due to the fact that the magnetic nanoparticles are regarded more adhesive to tumor cells than non-malignant cells. Such particles absorb more power than microparticles in alternating current magnetic fields tolerable in humans i.e. for cancer therapy. Numerous applications involving nanofluids include drug delivery, hyperthermia, contrast enhancement in magnetic resonance imaging and magnetic cell separation. Motivated by all the aforementioned facts, various scientists and engineers are engaged in the discussion of MHD flows of nanofluids Aziz et al.\textsuperscript{24} studied MHD flow over an inclined radiating plate with temperature dependent thermal conductivity, variable reactive index and heat generation. Matin et al.\textsuperscript{25} presented the MHD mixed convective flow of a nanofluid over a stretching sheet. Zeeshan et al.\textsuperscript{26} examined the MHD flow of a third grade nanofluid between coaxial porous porous cylinders.

Now a days, the flow analysis of non-Newtonian fluids has greatly fascinated the attention of scientists and engineers during the past few decades because of their numerous technological applications. Particularly such materials are involved in geophysics, oil reservoir engineering, bioengineering, chemical and nuclear industries, polymer solution, cosmetic processes, paper production etc. Additionally, all non-Newtonian materials on the basis of their behavior in shear are not predicted by one constitutive relationship. Thus different models of non-Newtonian fluids\textsuperscript{27,28} have been proposed for the discussion about their diverse characteristics. Amongst them the simplest model describing the most commonly existing nature of fluids i.e., shear thinning and shear thickening is the power-law model. Few studies dealing with flows of the power-law model can be mentioned by the Refs.\textsuperscript{29 and 30}.

Keeping in mind,\textsuperscript{28} in this paper our objective is to investigate the effects of Brownian motion and thermophoresis on the heat transfer of an electrically conducting power-law nanofluid over a non-linear stretching sheet under the influence of transverse magnetic field. By using similarity transformations system of partial differential equations is reduced to a system of non-linear ordinary differential equations. This system of non-linear ordinary differential equations is then solved numerically by the shooting method. The influence of the emerging parameters on the temperature and nanoparticles concentration is presented through several graphs and tables.

II. MATHEMATICAL FORMULATION

We investigate the steady two-dimensional flow of an incompressible power-law nanofluid over a non-linear stretching sheet. The flow is generated by action of two equal and opposite forces along the $x$-axis so that sheet is stretched with velocity $U = cx^s$ (where $c$ and $s$ are non-negative real numbers) by keeping the origin fixed. The stretching sheet is maintained at a constant temperature $T_w$ and a constant concentration $C_w$, and the ambient temperature and concentration far away from the surface of the sheet $T_\infty$ and $C_\infty$ are assumed to be uniform. A hot fluid with temperature $T_f$ is utilized to heat up or cool down the surface of the sheet by convective heat transfer mode, which provides the heat transfer coefficient $h_f$. In addition, a uniform magnetic field is applied in the $y$-direction.
with the magnetic field intensity \( B_0 \). For the steady two-dimensional flow due to stretching sheet we seek a velocity field of the form \( V = [u(x, y), v(x, y), 0] \), where \((x, y)\) denotes the Cartesian coordinates. Under the usual boundary layer approximations the governing equations (see Refs. 22 and 30) are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -K \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) - \frac{\sigma B^2_{0}}{\rho} u, \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} \right], \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}. \tag{4}
\]

The appropriate boundary conditions for the velocity, temperature and concentration field in the present problem are as follows:

\[
u (x, 0) = U = cx^2, \quad v(x, 0) = 0, -k \frac{\partial T}{\partial y} = h_f(T_f - T), \quad D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \text{ at } y = 0, \tag{5}
\]

\[
u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty. \tag{6}
\]

where \( u \) and \( v \) are the velocity components along \( x- \) and \( y- \) axes respectively, \( \rho \) the fluid density, \( K \) (\( > 0 \)) the rheological constant, \( (n > 0) \) the power-law index, \( \sigma \) the electrical conductivity of the fluid, \( T \) the temperature of the fluid, \( C \) the concentration of the fluid, \( \alpha \) the thermal diffusivity, \( \tau \) the ratio of effective heat capacity of the nanoparticle material to the heat capacity of the fluid, \( D_B \) the Brownian diffusion coefficient and \( D_T \) the thermophoresis diffusion coefficient.

Applying the following non-dimensional variables

\[
u (x, y) = U f'(\eta), \quad v(x, y) = -URe^{\frac{-1}{\pi}} \left[ \{s(2n-1)+1\} f(\eta) + \{s(2n-1)-1\} \eta f'(\eta) \right],
\]

\[
u = \frac{U x}{Re^{\frac{1}{\pi}}} \psi, \quad \psi = U x e^{\frac{-1}{\pi}} f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_\infty}, \tag{7}
\]

where \( \psi \) is the Stokes stream function, the governing problem become

\[
\frac{n(-f''')f'''}{f'''} + \left( \frac{s(2n-1)+1}{n+1} \right) f f'' - s(f')^2 - M^2 f' = 0, \tag{8}
\]

\[
\frac{\theta'' + Pr \left( \frac{s(2n-1)+1}{n+1} \right) f \theta' + N_B \phi \theta' + N_t \theta'^2}{f'} = 0, \tag{9}
\]

\[
\frac{\phi'' + Pr \left( \frac{s(2n-1)+1}{n+1} \right) f \phi' + N_B \phi \theta'^2}{f'} = 0, \tag{10}
\]

\[
f(0) = 0, \quad f'(0) = 1, \quad \theta' = -\gamma(1 - \theta(0)), \quad N_B \phi'(0) + N_t \theta'(0) = 0, \tag{11}
\]

\[
f'(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0, \tag{12}
\]

where prime denotes differentiation with respect to \( \eta \), \( M \) is the magnetic parameter, \( Pr \) the generalized Prandtl number, \( Re \) the generalized Reynolds number, the generalized Biot number \( \gamma \), \( N_B \) the Brownian motion parameter, \( N_t \) the thermophoresis parameter and \( Le \) the Lewis number. These parameters are given by

\[
M^2 = \frac{\sigma B^2_0}{\rho U^2}, \quad Pr = \frac{xU}{\alpha} Re_{\tau}^{\frac{2}{\pi}}, \quad Re = \frac{\rho x^n U^{2-n}}{K}, \quad \gamma = \frac{x h_f}{k} Re_{\tau}^{\frac{1}{\pi}}, \tag{13}
\]

\[
N_B = \frac{\tau D_B (C_w - C_\infty)}{\alpha}, \quad N_t = \frac{\tau D_T (T_w - T_\infty)}{T_\infty \alpha}, \quad Le = \frac{\alpha}{D_B}.
\]
The skin-friction coefficient $C_f$ and the local Nusselt number $Nu_x$ can be defined as

$$C_f = \frac{\tau_{xy}}{\frac{1}{2} \rho U^2}, \quad Nu_x = \frac{x q_w}{K (T - T_\infty)},$$

(14)

where $\tau_{xy}$ and $q_w$ are the wall shear stress and heat flux, respectively, are given by

$$\tau_{xy} = \left( \frac{K}{\frac{1}{2}} \left( \frac{\partial u^{n-1}}{\partial y} \right) \right) \left. \frac{\partial u}{\partial y} \right|_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right) \bigg|_{y=0}$$

(15)

In terms of dimensionless quantities, we have

$$\frac{1}{2} \text{Re} \frac{\pi r}{\tau} C_f = \left[-f''(0)\right]^n, \quad \text{Re} \frac{\pi r}{\tau} Nu_x = -\theta'(0),$$

(16)

III. THE NUMERICAL METHOD

The similarity equations (8)-(10) are coupled and non-linear ordinary differential equations which possess no closed form solution. We solve these coupled non-linear differential equations by utilizing Runge-Kutta fourth order technique along with shooting method. The equations are firstly written as a system of first order ordinary differential equations and then the corresponding initial value problems are solved by the Runge-Kutta method. The initially guessed values are refined iteratively using the Newton’s method to satisfy boundary condition at infinity.

IV. NUMERICAL RESULTS AND DISCUSSION

The main objective here is to study the influence of emerging physical parameters on temperature and concentration profile, respectively. The variation of physical parameters such as $s$ and $M$ on the temperature profile are discussed in figures 1-2. Figures 1(a) – 1(b) present the temperature profile $\theta(\eta)$ for different values of stretching parameter $s$. We can see that with the increase in stretching parameter the temperature profile decreases for $n = 1$ and $n = 1.5$. Furthermore, the boundary layer thickness, with the increase in $s$ decreases for $n = 1$ and $n = 1.5$.

Figures 2(a) – 2(b) show the effects of the magnetic parameter $M$ on temperature profile $\theta(\eta)$ It is clear from these figures that temperature profile increases with the increase of $M$. Moreover, it is also observed that fluids for $n = 1$ decay more slowly when compared to fluid for $n = 1.5$.

Figures 3-8 show the variations of $\phi(\eta)$ for different values of flow parameters $s$, $M$, $\gamma$, $Pr$, $N_b$, and $N_t$. Figures 3(a) – 3(b) and 4(a) – 4(b) suggests that nanoparticle concentration decreases with the increase of the stretching parameter $s$ and increases with the increase of magnetic parameter $M$. However, we can observe that stretching parameter $s$ has very small effect on concentration profile.

FIG. 1. Variation of temperature profiles $\theta(\eta)$ for different values of the stretching parameter $s$. 
FIG. 2. Variation of temperature profiles $\theta(\eta)$ for different values of the magnetic parameter $M$.

FIG. 3. Variation of concentration profiles $\phi(\eta)$ for different values of the stretching parameter $s$.

FIG. 4. Variation of concentration profiles $\phi(\eta)$ for different values of the magnetic parameter $M$.

The impact of the generalized Biot number $\gamma$ on the nanoparticle concentration distribution $\phi(\eta)$ is shown by 5(a) - 5(b) for the power-law index $n = 1$ and $n = 1.5$. It is concluded that concentration distribution as well as concentration boundary layer thickness increase for higher values of the generalized Biot number $\gamma$.

Figures 6(a) and 6(b) illustrate the impact of the generalized Prandtl number $Pr$ on the concentration distributions for the power-law index $n = 1$ and $n = 1.5$, respectively. These figures show a diminishing behavior of concentration distribution and concentration boundary layer thickness for larger the generalized Prandtl number $Pr$ for both the cases. Additionally, we can observe that power index $n$ plays significant role. The increase of power index $n$ results in thinning the thermal boundary layer.

Figures 7(a) and 7(b) are depicted for the variation of the Brownian motion parameter $N_b$ to concentration distribution for the power-law index $n = 1, 1.5$. It is straightforwardly appeared from
these figures that the fluid concentration distribution diminishes for stronger the Brownian motion parameter $N_b$. This happens because of the way that the Brownian movement of particles is simply the result of all the impulses of the fluid molecules on the surface of the particles. Moreover, away from the surface the larger values of $N_b$ stifle the diffusion of the nanoparticles in the fluid regime which reduces the concentration distribution.

The variation in non-dimensional concentration distribution with the increment for a few sets of values of thermophoresis parameter $N_t$ is illustrated in figures 8(a) and 8(b) for the power-law index $n = 1, 1.5$. It is anticipated by these figures that the concentration distribution increases as the thermophoresis parameter $N_t$ increases. Physically, when there is a temperature gradient in the flow domain of the particulate system, small particles tend to disperse faster in hotter regions and slower in colder regions. The collective effect of the differential dispersion of the particles is their apparent
FIG. 8. Variation of concentration profiles $\phi(\eta)$ for different values of the thermophoresis parameter $N_t$.

TABLE I. Numerical values of the skin friction coefficient and reduced Nusselt numbers for different values of physical parameters.

| $n$ | $M$ | $s$ | $\gamma$ | $Pr$ | $N_b$ | $N_t$ | Le | $-\theta'(0)$ | $-\theta'(0)$ |
|-----|-----|-----|-------|-----|------|------|----|-----------|-----------|
| 0.5 | 2.0 | 1.5 | 0.6   | 2.0 | 0.5  | 0.2  | 2.0 | 1.981438   | 0.242519   |
| 1.1 |     |     |       |     |      |      |     | 2.327012   | 0.33611    |
| 1.5 |     |     |       |     |      |      |     | 2.520191   | 0.36938    |
| 0.5 |     |     |       |     | 0.2  | 1.0  |     | 1.305714   | 0.391159   |
| 1.0 |     |     |       |     |      |      |     | 1.594711   | 0.385919   |
| 1.5 |     |     |       |     |      |      |     | 2.01419    | 0.378325   |
| 0.5 |     |     |       |     | 0.2  | 1.0  |     | 2.249116   | 0.295778   |
| 1.0 |     |     |       |     |      |      |     | 2.387183   | 0.340422   |
| 1.5 |     |     |       |     |      |      |     | 2.520191   | 0.36938    |
| 0.2 |     |     |       |     | 0.2  |      |     | 2.520191   | 0.165797   |
| 0.4 |     |     |       |     |      |      |     | 2.520191   | 0.282753   |
| 0.6 |     |     |       |     |      |      |     | 2.520191   | 0.36938    |
| 0.9 |     |     |       |     |      |      |     | 2.520189   | 0.275217   |
| 1.0 |     |     |       |     |      |      |     | 2.520189   | 0.320642   |
| 1.3 |     |     |       |     |      |      |     | 2.520180   | 0.351723   |
| 1.7 |     |     |       |     |      |      |     | 2.520189   | 0.36938    |
| 0.1 |     |     |       |     | 0.1  |      |     | 2.520191   | 0.36938    |
| 0.6 |     |     |       |     |      |      |     | 2.520191   | 0.36938    |
| 0.9 |     |     |       |     |      |      |     | 2.520191   | 0.36938    |
| 0.1 |     |     |       |     | 0.1  |      |     | 2.520191   | 0.37051    |
| 0.6 |     |     |       |     |      |      |     | 2.520191   | 0.364689   |
| 0.9 |     |     |       |     |      |      |     | 2.520191   | 0.360979   |
| 0.9 |     |     |       |     |      |      |     | 2.52019    | 0.370125   |
| 1.3 |     |     |       |     |      |      |     | 2.520189   | 0.369789   |
| 1.7 |     |     |       |     |      |      |     | 2.520189   | 0.369536   |

migration from hotter to colder regions. The result of the migration is the accumulation of particles and higher particle concentrations in the colder regions of the particulate mixture. This is due to fact that the thermophoresis parameter $N_t$ is directly proportional to the heat transfer coefficient associated with the fluid. Furthermore, for the larger values of $N_t$ the thermophoretic forces are produced. These forces have the tendency to migrate the nanoparticles in the reverse direction of temperature gradient which causes a non-uniform nanoparticle distribution.

Table I is presented for the numerical values of the skin friction coefficient $\frac{1}{Re} \pi \tau C_f$ and local Nusselt number $Re^{-\pi \tau} Nu_{x}$ for different values of $n$, $\gamma$, $M$, $s$, $Pr$, $N_b$, $N_t$. From table it is noticed that magnitude of skin friction coefficient for large values of $M$ and $s$. Also, it can be seen from Table I that local heat and mas fluxes increases for large values of power index $n$ and $s$ respectively.
V. CONCLUSIONS

The effect of two dimensional MHD free convective boundary layer flow of power-law nanofluid over nonlinear stretching sheet was investigated. The governing partial differential equations were converted into nonlinear ordinary differential equations by using suitable similarity transformation. The influence of various parameters on the dimensionless velocity, temperature, nanoparticle volume fraction can be summarized as follows:

- It is observed that velocity profile as well as boundary layer thickness decreases for increasing stretching parameter s.
- Temperature and concentration profiles decreases for the increasing values of stretching parameter s.
- Temperature distribution increases with in increase of parameters \( N_p \), \( N_r \).
- Behavior of \( N_p \) and \( N_r \) on the concentration profile is quite opposite.

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