Supersymmetric Domain Wall World from D=5 Simple Gauged Supergravity

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Abstract

We address a supersymmetric embedding of domain walls with asymptotically anti-deSitter (AdS) space-times in five-dimensional simple, N=2 U(1) gauged supergravity theory constructed by Gunaydin, Townsend and Sierra. These conformally flat solutions interpolate between supersymmetric AdS vacua, satisfy the Killing spinor (first order) differential equations, and the four-dimensional world on the domain wall is a flat world with N=1 supersymmetry. Regular solutions in this class have the energy density related to the cosmological constants of the supersymmetric AdS vacua. An analysis of such solutions is given for the example of one (real, neutral) vector supermultiplet with the most general form of the prepotential. There are at most two supersymmetric AdS vacua that are in general separated by a singularity in the potential. Nevertheless the supersymmetric domain wall solution exists with the scalar field interpolating continuously across the singular region.

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1 Introduction

The past few months have witnessed exciting progress in the study of domain walls in D=5 gravity theories. Such configurations are interesting from two, on a surface orthogonal perspectives: (i) in the context of AdS/CFT correspondence such conformally flat configurations provide new insights in the study of RGE flows [1, 2, 3, 4, 5, 6, 8, 9] and (ii) in the context of phenomenological implications, such configurations provide a framework [10, 11, 12, 13, 14, 15, 16, 17, 18] to address the physics implications of large dimensions for the four-dimensional world on the domain wall.

Within the first approach a number of conformally flat solutions were constructed and in particular the ones interpolating between supersymmetric anti-deSitter (AdS) vacua of N=8 D=5 gauged theory provide examples of static domain walls in D=5 with implications for the renormalization group flow and spectra in strongly coupled four-dimensional super Yang-Mills theories. One such example [2, 5] involves two scalar fields and thus was solved numerically and another most recent example with one scalar field can be solved explicitly [19].

Within the second approach infinitely thin, static, $Z_2$-symmetric domain wall solutions were constructed [10, 11] for pure AdS gravity theory. (Generalizations that incorporate effects of additional compactified dimensions were given in [12, 14, 15].) These solutions have to satisfy a specific relation between the domain wall tension $\sigma$ and the cosmological constant $\Lambda$ of the AdS vacua, thus implying a fine-tuning.

The purpose of this letter is few-fold. We shall address a supersymmetric embedding of domain walls with asymptotically AdS space-times in five-dimensions, in the simplest supergravity theory, i.e. the supergravity theory with least supersymmetry that allows for the explicit constructions of supersymmetric domain wall configurations. In order to demonstrate the existence of such domain walls, a supergravity theory necessarily has to have a potential for (gauge neutral) scalar fields, and the only known such examples are gauged supergravity theories and the matter fields responsible for the formation of the domain wall belongs to vector-supermultiplets. We thus choose to work within a framework of a five-dimensional N=2, U(1) gauged supergravity formulated by Gunaydin, Townsend and Sierra [20].

We derive the Killing spinor equations for domain wall solutions. For regular solutions in this class we also derive that the energy density $\sigma$ of the wall is related in a specific way to the cosmological constants of the supersymmetric vacua on each side of the wall. Namely, the relationship between the domain wall tension and the cosmological constant for regular supersymmetric domain walls is a consequence of the BPS nature of the solution, and not an artifact of fine-tuning. These configurations have four

$^3$Such a theory may have its origin as a compactification of Type IIB superstring theory on a specific Einstein-Sasaki-5-manifolds or as a compactification of M-theory on Calabi-Yau with non-trivial fluxes turned on. But the details are unknown so far and remain to be worked out.

$^4$These properties are very much parallel to those of supersymmetric domain walls of four-dimensional N=1 supergravity theory found in [22] and reviewed in [24].

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unbroken supercharges, or in other words break $\frac{1}{2}$ of N=2, D=5 supersymmetry, and thus the four-dimensional world on the domain wall has N=1 supersymmetry. A special example of infinitely thin supersymmetric domain walls (corresponding to the case of very large gauge coupling) with $\mathbb{Z}_2$ symmetry would provide a concrete supersymmetric realization of the static domain wall solution found by Randall and Sundrum [10].

For the sake of concreteness we analyse the case with one physical (gauge neutral) vector superfield which allows for an explicit analysis of the possible domain wall configurations. For this simple model, kink solutions have been discussed some time ago [21]; the domain walls presented in this paper provide a concrete and explicit realisation of supersymmetric kink solutions. The upshot of the analysis is that this framework does provide examples of static domain wall solutions that satisfy the Killing spinor equations and are thus supersymmetric. The general one scalar case, however, has a potential that has at most two supersymmetric AdS vacua which are always separated by at least one (out of three) singularity points where the potential diverges. But nevertheless the scalar for the supersymmetric domain wall solutions interpolates across the singular point.

2  D=5 N=2 U(1) Gauged Supergravity

Supergravity in D=5 is very restrictive with respect to allowed potentials. The only allowed potentials come from gauging of isometries and especially interesting are potentials that have no “run-away” behavior (scalars become asymptotically constant) with non-trivial isolated extrema. This type of potential allows for the existence of domain walls with extrema corresponding to the AdS vacua on each side of the wall. The minimal gauged supergravity (N=2 gauged supergravity with $U(1)$ gauged R-symmetry), constructed in [21, 22], provides such a set-up. In this case one can consistently decouple the hyper-multiplets and the Lagrangian contains only the supergravity multiplet and the vector supermultiplets. (There are also domain wall solutions, that couple to non-trivial hypermultiplets [13], but they do not have asymptotic anti-de Sitter spaces.)

In this case the bosonic Lagrangian reads:

$$S_5 = \int \left[ \frac{1}{2} R + g^2 V - \frac{1}{4} G_{IJ} F^I \mu \nu F^{\mu \nu J} - \frac{1}{2} g_{AB} \partial_\mu \phi^A \partial^{\mu} \phi^B \right] + \frac{1}{48} \int C_{IJK} F^I \wedge F^J \wedge A^K$$

We chose the convention where the five-dimensional Newton’s constant is $\kappa = 1$ and $g$ is the gauge coupling. We work in the $(-, +, +, +, +)$ convention. The physical scalars $\phi^A$, which are real and neutral, correspond to the scalar components of the vector supermultiplets and define coordinates of the manifold defined by [20]

$$F = \frac{1}{6} C_{IJK} X^I X^J X^K = 1$$

with $C_{IJK}$ real, and the $X^I$ are the auxiliary real scalar fields. The metric(s) of the scalar manifold $g_{AB}$ (for physical scalars $\phi^A$) and $G_{IJ}$ (for auxiliary scalars $X^I$) are
defined by
\[ G_{IJ} = -\frac{1}{2} \left( \partial_I \partial_J \log F \right)_{F=1}, \quad g_{AB} = \left( \partial_A X^I \partial_B X^J G_{IJ} \right)_{F=1}, \] (3)
where \( \partial_A \equiv \frac{\partial}{\partial \phi^A} \). The auxially scalars \( X^I \) are accompanied by gauge field strengths \( F^I_{\mu\nu} \) entering the Lagrangian (1).

The gauging of a \( U(1) \) subgroup of the \( R \)-symmetry introduces a potential for the scalars \( h_I \)
\[ V = 6 h_I h_J \left( X^I X^J - \frac{3}{4} g^{AB} \partial_A X^I \partial_B X^J \right) \]
\[ = 6 \left( W^2 - \frac{3}{4} g^{AB} \partial_A W \partial_B W \right), \] (4)
where \( h_I \) are real constants, specifying the Fayet-Iliopoulos(FI) terms, and the superpotential \( W \) is defined as
\[ W = h_I X^I. \] (5)
Notice, \( W \) is subject to the constraint (2) which makes it non-linear in the physical scalars \( \phi^A \).

**Supersymmetry Transformations and BPS-Saturated Domain Walls**

We are searching for supersymmetric (BPS-saturated) domain wall solutions: those are solutions that preserve part of the supersymmetry, and thus satisfy the Killing spinor equations, which are first order differential equations what ensure that the supersymmetry transformations in this domain wall background are preserved.

We choose these domain wall solutions to be neutral, and thus they are supported only by (gauge neutral) scalars with the gauge fields turned off. Thus, the supersymmetry transformations for these backgounds read [20]:
\[ \delta \lambda_A = \left( -i \frac{1}{2} g_{AB} \Gamma_{ab} \partial_a \Phi^B + i \frac{3}{2} g \partial_A W \right) \epsilon, \]
\[ \delta \psi_\mu = \left( \partial_\mu + \frac{1}{4} \omega^a_{\mu a} \Gamma_{ab} + \frac{1}{2} \partial A \Gamma_{\mu W} \right) \epsilon. \] (6)

The vacuum is given by the asymptotic space, where the scalars are constant and thus supersymmetry requires \( \partial_A W = 0 \). The form of the potential \( V \) (4) implies that supersymmetric vacua are always extrema of the potential.

The domain wall Ansatz for the metric is of the form:
\[ ds^2 = A(z) \left[ -dt^2 + dx^2 + dx^2 + dx^2 + dz^2 \right] + dz^2, \] (7)

5Note the parallels with the potential in D=4 N=1 supergravity where: \( V = e^K \left( g^{AB} D_A W D_B W - 3 |W|^2 \right) \) where \( W \) and \( K \) are the superpotential and Kähler potential for the chiral superfields.
and the scalars have the form \( \phi^A = \phi^A(z) \), where \( z = \{-\infty, +\infty\} \) is a direction transverse to the wall.

Then the Killing spinor equations \( \delta \psi_\mu = 0 \) and \( \delta \lambda_A = 0 \) are solved by \( 6 \):

\[
\partial_z \log A = 2gW ,
\]

and

\[
\partial_z \phi^A = -3gg^{AB}\partial_B W ,
\]

where the four component spinor satisfies the constraint: \( \Gamma_z \epsilon = -\epsilon \). (Killing spinor equations for domain walls of D=5, N=8 supergravity can be cast in a similar form \([3]\).) Note that as long as the domain of physical fields contain two isolated supersymmetric vacua, this set of solutions specify the BPS domain wall. The physical domain of such solutions requires that the scalar metric \( g_{AB} \) remains positive definite. (In D=4 N=1 supergravity, the Killing spinor equations are similar \([22]\): \( \partial_z \log A \sim e^{2\Phi}W \), \( \partial_z \phi^A \sim e^\Phi g^{AB}D_B W \).)

The domain wall tension can be determined by applying Nester’s procedure which relates the wall tension \( \sigma \) to the central charge of the supersymmetry algebra; the central charge is determined by the values of the superpotential at each asymptotically supersymmetric vacuum. (For D=4 N=1 domain wall solutions, see Appendix A of \([22]\).) More concretely, one considers the integral over the spatial boundary

\[
\int_{\partial \Sigma} N^{\mu\nu} d\Sigma_{\mu\nu} = \int_{\Sigma} \nabla_\mu N^{\mu\nu} d\Sigma_\nu = \int_{\Sigma} \nabla_\mu N^{\mu0} d\Sigma_0 .
\]

\( \Sigma_{\mu\nu} \) is a space-like hypersurface and thus \( d\Sigma_0 \sim dzd\vec{x} \). The Nester tensor reads \( N^{\mu\nu} = \epsilon^\nu_\rho \partial_\mu \delta \psi_\lambda \) where \( \delta \psi_\lambda \) is the gravitino variation. In \([10]\) we used the Stokes theorem, and thus assumed that the Nester tensor is non-singular. In order to determine the energy density, we can factor out the integral over the domain wall coordinates \( (d\vec{x}) \) and the integration over the transverse direction \( (z) \) yields in \([10]\) the contributions far away from the wall. Inserting the gravitino variation in \([10]\), one obtains two contributions, the first one represents the domain wall tension \( \sigma \) (energy density) and the second one corresponds to the central charge \( \mathcal{C} \). The latter one is a topological term that corresponds to the difference of the boundary values of the superpotential. For the supersymmetric configuration the gravitino variation is zero, and thus \([10]\) is zero which implies:

\[
\sigma = \mathcal{C} \equiv -\frac{1}{2} \left( \epsilon \sqrt{\det g} \right) gW \bigg|_{-\infty}^{+\infty} ,
\]

where we used the projector \( \Gamma_z \epsilon = -\epsilon \). Again, in the derivation of \([11]\) it was assumed that the Stokes theorem can be applied and thus the \( \partial_z W \) is non-singular inside the wall. In the one-scalar example this is not the case (see later).

\(^6\)We would like to thank S. Gubser for pointing out the sign error in eq. \((8)\) in the original version of the manuscript. As a consequence the explicit solution for the metric coefficient \( A(z) \), discussed in Section 3 of the original version, becomes \( A(z)^{-1} \) of the revised version.
Normalizing the Killing spinor as \( (\bar{\epsilon} \Gamma^0 \sqrt{A} \epsilon) = 1 \), yields the result:

\[
\sigma_{BPS} = -\frac{g}{2}(W_+ - W_-) = -\frac{1}{2\sqrt{6}}\left(\text{sign}[W_\infty] \sqrt{-\Lambda_+} - \text{sign}[W_-] \sqrt{-\Lambda_-}\right),
\]

(12)

where \( W_{\pm\infty} \equiv W(\phi^A|z=\pm\infty) \). In the second part of (12) we have used the relationship between the cosmological constant \( \Lambda \) and the value of the superpotential \( W \) for supersymmetric vacua. Thus, the domain wall tension is specified by the values of the cosmological constants of the asymptotic AdS vacua.

According to the asymptotic behavior of the Killing spinor equation (8) there are the following types of BPS-saturated domain walls (very much parallel to the analysis of the types of BPS-saturated domain walls in D=4 [25], their global space-time structure [26] and their relationship to non-supersymmetric configurations [27]):

- **Type I** domain walls interpolate supersymmetric Minkowski space-time (\( \Lambda_{-\infty} = 0 \)) and the AdS space-time (\( \Lambda_{+\infty} \equiv \Lambda \neq 0 \)). On the asymptotic AdS side the metric coefficient takes the form \( A(z) = e^{-\sqrt{-\frac{2}{3}\Lambda_+}|z|} \); on the AdS side of the wall \( z \to \infty \) limit corresponds to the AdS horizon. The geodesic extension of these space-times could either be pure AdS or new regions that involve an infinite tiling with the “mirror” domain walls. Regular solutions have \( \sigma_{BPS} = \frac{1}{2\sqrt{6}} \sqrt{-\Lambda} \). These walls saturate an analog of the Coleman-De Luccia bound [28] in five dimensions.

- **Type II** domain walls interpolate between supersymmetric AdS vacua where \( \text{sign}[W_{-\infty}] = -\text{sign}[W_{+\infty}] \) and the metric behaves as \( A(z) = e^{\sqrt{-\frac{2}{3}\Lambda_{\pm\infty}}|z|} \) for \( z \to \pm\infty \), i.e. one approaches the AdS horizons and thus the geodesic extensions could be either pure AdS or new regions that involve an infinite tiling with the mirror domain walls. For regular solutions \( \sigma_{BPS} = \frac{1}{2\sqrt{6}} \left(\sqrt{-\Lambda_{+\infty}} + \sqrt{-\Lambda_{-\infty}}\right) \). These domain walls can be viewed as “stable”; non-supersymmetric generalizations are expanding bubbles on either side of the AdS vacua [24]. A special case of a \( Z_2 \) symmetric solution \( (W_{+\infty} = -W_{-\infty}) \) satisfies the constraint: \( \sigma_{BPS} = \frac{1}{2\sqrt{6}} \sqrt{\Lambda} \) which is a relationship found in [10].

- **Type III** domain walls are those between two supersymmetric AdS vacua where \( \text{sign}[W_{-\infty}] = +\text{sign}[W_{+\infty}] \). The metric coefficient grows exponentially on one side of the wall: \( A(z) = e^{\pm\sqrt{-\frac{2}{3}\Lambda_{\pm\infty}}|z|} \), and thus on this side, \( |z| \to \infty \) limit corresponds to the boundary of the AdS space! For regular solutions \( \sigma_{BPS} = \frac{1}{2\sqrt{6}} \left(\sqrt{-\Lambda_{+\infty}} - \sqrt{-\Lambda_{-\infty}}\right) \); those are the “unstable” domain wall solutions whose non-supersymmetric generalizations corresponds to false vacuum decay bubbles, only [24].

- **Type IV** domain walls correspond to a class of solutions where \( \text{sign}[W_{-\infty}] = -\text{sign}[W_{+\infty}] \) (just like Type II walls), but now the metric behaves asymptotically as \( A(z) = e^{\sqrt{-\frac{2}{3}\Lambda_{\pm\infty}}|z|} \), i.e. for \( |z| \to \infty \) one approaches the boundary of the AdS space instead the horizon. They have negative surface density. A special limit in this class (Type V) would correspond to the case where one, say, \( \Lambda_{-\infty} = 0 \).
3 BPS Domain Walls with One Vector Supermultiplet

For the sake of being explicit we will address the case of a single vector multiplet. Defining the physical scalar as \( \phi = X^1/X^0 \) the constraint (2) takes the form:

\[
F = (X^0)^3 (A + B \phi + C \phi^2 + D \phi^3) = 1,
\]

and the superpotential (3) becomes:

\[
W = X^0 (h_0 + h_1 \phi).
\]

where \( X^0 \) is the auxiliary field eliminated by eq. (13). The metric \( g_{\phi\phi} \), and the derivative of the potential \( \partial \phi W \) take the form:

\[
g_{\phi\phi} = \frac{1}{3} \left( C^2 - 3BD \right) \phi^2 + \frac{1}{3} \left( BC - 9AD \right) \phi + \frac{1}{3} \left( B^2 - 3AC \right),
\]

\[
\partial \phi W = \frac{1}{3} \left( h_1 C - h_0 D \right) \phi^2 + \frac{1}{3} \left( h_1 B - h_0 C \right) \phi + h_1 A - \frac{1}{3} h_0 B,
\]

and the potential reads:

\[
V = 6 \left[ W^2 - \frac{3}{4g_{\phi\phi}} (\partial \phi W)^2 \right].
\]

One can make the following general observations about the nature of supersymmetric vacua. (i) The superpotential (14) allows for at most two extrema, where \( \partial \phi W = 0 \). (ii) Expanding \( W \) around a given extremum yields: \( \partial^2 \phi W |_{extr} = \frac{2}{3} g_{\phi\phi} W |_{extr} \) (see e.g., [6]). This relationship implies that for physical solutions with \( g_{\phi\phi} > 0 \) the two extrema of \( W \) cannot be connected, there is at least one pole between them. Thus, the supersymmetric domain wall solution necessarily involves a “jump” over a region where the superpotential (as well as the scalar metric and the potential) has a pole.

Note, these lines of arguments hold for the one-scalar case, only. If \( W \) depends on more than one scalar, it may allow for two minima, which can be smoothly connected.

In order to discuss the solution in more detail we choose, without loss of generality, the following parameterization:

\[
g = 1, \quad A = 0, \quad B = D = h_0 = 1, \quad C = \sqrt{3} \chi, \quad h_1 = \sqrt{3} \xi.
\]

(One can show that \( g = D = h_0 = 1 \) corresponds to an overall rescaling of the potential, \( A = 0 \) can be obtained by shifts \( \phi \rightarrow \phi - \phi_0 \) and \( h_0 \rightarrow h_0 + h_1 \phi_0 \), and \( B = 1 \) corresponds to a rescaling of \( \phi \).) In this case the metric, superpotential and its derivate can be written in the following form:

\[
g_{\phi\phi} = \frac{3(\chi^2 - 1) \phi^2 + \sqrt{3} \chi \phi + 1}{3 \phi^2 (1 + \sqrt{3} \chi \phi + \phi^2)^2},
\]
\[ W = \frac{1 + \sqrt{3} \xi \phi}{[\phi(1 + \sqrt{3} \chi \phi + \phi^2)]^3}, \quad (20) \]

\[
\partial_{\phi}W = \frac{3(\chi \xi - 1)\phi^2 - 2\sqrt{3}(\chi - \xi)\phi - 1}{3[\phi(1 + \sqrt{3} \chi \phi + \phi^2)]^4}. \quad (21)
\]

The corresponding Killing spinor equations for the metric coefficient \(A(z)\) is given in (8) and that for the scalar field (9) takes the form:

\[
g_{\phi\phi} \partial_{\phi} \phi = -3 \partial_{\phi} W. \quad (22)
\]

Let us mention that in a proper coordinate system, these first order differential equations are solved by an algebraic constraint [6], which says that the normal vector on the scalar manifold, defined by (13), has to behave monotonically and becomes parallel to \(h_I\) in the asymptotic AdS vacuum.

**Features of the Solutions and an Example**

The first useful observation is that in the region where the metric \(g_{\phi\phi}\) has real poles, \(g_{\phi\phi}\) has no real zeroes. Namely, the poles and zeros are at the following values of \(\phi\):

- Poles of \(g_{\phi\phi}\):
  \[
  \{ \frac{1}{2}(-\sqrt{3} \chi \pm \sqrt{3} \chi^2 - 4), 0 \}.
  \quad (23)
  \]

- Zeros for \(g_{\phi\phi}\):
  \[
  -\chi \pm \sqrt{-\chi^2 + 4} \over 2\sqrt{3}(\chi^2 - 1)\].
  \quad (24)

Thus for \(\chi^2 > 4/3\) there are no zeroes of the metric, but there are poles for the values of \(\phi\) specified by (23).

Supersymmetric vacua are determined by zeroes of \(\partial_{\phi} W\) (21). As discussed at the beginning of this section, there are at most two, where \(\phi\) takes the value:

\[
\text{Zeros for } \partial_{\phi} W : \quad \frac{(\chi - \xi) \pm \sqrt{\chi^2 - \xi \chi + \xi^2 - 1}}{\sqrt{3}(\xi \chi - 1)}. \quad (25)
\]

Note also that the poles of \(W, \partial_{\phi} W\) and \(g_{\phi\phi}\) are identical.

For the parameter range \(\chi^2 < \frac{4}{3}\), \(W\) has no poles and thus, due to the relationship \(\partial_{\phi}^2 W|_{extr} = \frac{2}{3} g_{\phi\phi} W|_{extr}\), one extremum has to be a maximum and the other one a minimum. Therefore, the scalar metric \(g_{\phi\phi}\) is negative for one value of (23) and corresponds to a non-physical vacuum (\(\phi\) becomes tachyonic).

Thus the only physical region for the domain wall solutions corresponds to \(\chi^2 > \frac{4}{3}\). Now the scalar metric \(g_{\phi\phi}\) is always positive definite and \(W\) has two real extrema which are necessarily separated by at least one pole. Thus these domain walls interpolate between supersymmetric extrema with the kink (\(\phi\)) solution transversing a region where the potential (the superpotential and the metric) have a pole. Near the pole the Killing
spinor equations (8) and (9) can be solved approximately: instead of a typical kink behaviour \( \phi - \phi_{\text{pole}} \sim (z - z_{\text{pole}}) \) (in the case of a finite potential) now the kink “slows-down” and behaves near the pole as \( \phi - \phi_{\text{pole}} \sim (z - z_{\text{pole}})^3 \) and the metric coefficient behaves as \( A(z) \sim (z - z_{\text{pole}})^{2c} \) where \( c \) depends on the value of \( \phi \) at the pole. For \( c \neq 0 \) the curvature blows-up \(^7\). \((c < 0 \text{ for } \xi > \frac{1}{2}(\chi \pm \sqrt{\chi^2 - 4/3}) \text{ for poles at } \phi = \frac{1}{2}(-\sqrt{3}\chi \pm \sqrt{3\chi^2 - 4}).)\)

The type of the domain wall is specified by the relative signs of \( W \) on each side, as well as by the asymptotic behavior of the metric coefficient. Typically one encounters Type IV domain wall solutions, i.e. \( \text{sign}[W_{+\infty}] = -\text{sign}[W_{-\infty}] \), \( A(z) \sim e^{\sqrt{-\frac{4A_{\pm\infty}}{3}|z|}} \) as \(|z| \to \infty\), and at least one pole in-between. (Typical Randall-Sundrum scenario would correspond to Type II domain walls.) This is a consequence of the fact that the supersymmetric extrema are minima of the potential \( V \) \(^3\). Details will be given elsewhere \([24]\). For the sake of concreteness we exhibit a numerical solution for \( \chi = 1.4, \xi = -0.6 \) with the two supersymmetric minima \([25]\) at \( \{-1.0887, -0.1664\} \) sandwiched between the pole in the middle (poles \([23]\) are at \( \{-1.8980, -0.5269, 0\}\)). This solution is close to a \( Z_2 \) symmetric solution; \( W|_{-\infty} = 2.6944 \) and \( W|_{+\infty} = -2.4953 \). Notice the “slow-down” of the kink solution \( \phi(z) \) and a power-law “spike” of the metric \( A(z)^{-1} \) in the middle of the wall. Moreover, the smoothness of the scalar is due to the definition as ratio, \( X^0 \) as well as \( X^1 \) are singular at the pole (recall \( \phi = X^1/X^0 \)).

4 Conclusions

The specific realization of supersymmetric domain walls with asymptotically AdS space-times, in the simplest five-dimensional supergravity, demonstrates a number of interesting features. The superpotential \( W \) as a function of a single scalar can have at most two extrema, but there is no smooth flow possible while demanding that the scalar metric remains positive \( (g_{\phi\phi} > 0) \). Two AdS vacua with positive scalar metric have to be separated by a pole in the superpotential and the corresponding domain wall represents a supergravity solution that interpolates between the two branches. Despite this singularity, a stable kink solution exists with the scalar field “slowing-down” mildly in the region crossing the pole.

Within a more general framework of five-dimensional \( N=2 \) \( U(1) \) gauged supergravity we derived the Killing spinor equations for static domain wall solutions with asymptotic AdS space-times. For regular solutions in this class the wall tension and the cosmological constants of the supersymmetric AdS vacua are related. Such a relationship is therefore a consequence of the supersymmetry and not an artifact of fine-tuning. As another by-product we see that the domain wall world is flat and supersymmetric, i.e. along with the massless graviton there is an accompanying gravitino. The hypermulti-

\(^7\)We thank R. Myers for pointing that to us.
Figure 1: The domain wall solution for the scalar field $\phi(z)$ and the metric coefficient $A(z)^{-1}$ is depicted for the choice of parameters $\chi = 1.4$ and $\xi = -0.6$. Note the “slow-down” of the kink and a “spike” of the metric coefficient $A(z)^{-1}$ in the region in the middle of the wall.

...plets of D=5 gauged supergravity could potentially play a role of matter on the domain wall, a subject of further investigations. The break-down of supersymmetry (by either of the vacua) would ensure that the non-extreme walls would become expanding bubbles (see the analysis given in for non-supersymmetric domain walls in D=4 in [27] and for a somewhat related analysis in D=5 in [18, 29, 30].)

Acknowledgments

We would like to thank S. Gubser and R. Myers for helpful comments, P. Townsend for correspondence and T. Banks, H. Lü, A. Tseytlin, S. Gukov and E. Witten for useful discussions. The work is supported by a DFG grant, in part by the Department of Energy under grant number DE-FG03-92-ER 40701 (K.B.), DOE-FG02-95ER40893 (M.C.) and the University of Pennsylvania Research Foundation (M.C).

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