Probing modified gravity theories and cosmology using gravitational-waves and associated electromagnetic counterparts

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The direct detection of gravitational waves by the LIGO-Virgo collaboration has opened a new window with which to measure cosmological parameters such as the Hubble constant \(H_0\), and also probe general relativity on large scales. In this paper we present a new phenomenological approach, together with its inferential implementation, for measuring deviations from general relativity (GR) on cosmological scales concurrently with a determination of \(H_0\). We consider gravitational waves (GWs) propagating in an expanding homogeneous and isotropic background, but with a modified friction term and dispersion relation relative to that of GR. We discuss the prospects for measuring such deviations from GR using GWs from a binary neutron star merger, its associated Gamma ray burst (GRB) emission and the identification of the host galaxy. We find that a single binary neutron star GW detection will poorly constrain the GW friction term. However, a joint analysis including the GW phase and GW-GRB detection delay could improve constraints on some GW dispersion relations provided the delay is measured with millisecond precision. We also show that by combining 100 binary neutron stars detections with observed electromagnetic counterparts and hosting galaxy identification, we will be able to constrain the Hubble constant, the GW damping term and the GW dispersion relation with 2%, 15% and 2% accuracy, respectively. Finally, our work also emphasises that these three parameters should be measured together in order to avoid biases.

I. INTRODUCTION

A fundamental building block of the Standard Cosmological model (ΛCDM) is Einstein’s theory of general relativity (GR). When supplemented with the assumption that on large scales the universe is homogeneous and isotropic (the Cosmological principle), together with the introduction of dark energy — in the form of a cosmological constant \(\Lambda\) — and dark matter, it is an excellent description of our observable universe, including its accelerated expansion today \([1]\). However, many theoretical questions remain open, most fundamental of which is perhaps the nature of dark energy and dark matter \([2]\). On the observational side, it is well known that the measurements of the Hubble constant through the Cosmic Microwave Background (CMB) \([3]\) and in the local Universe with Standard Candles \([4, 5]\) show a significant statistical discrepancy \([6]\). One of the possible solutions to these open problems is to consider that GR is modified on cosmological scales. There exists numerous models of modified gravity models which break different GR assumptions \([7]\), and in general in these models both scalar and tensor perturbations evolve differently from those of GR \([8–10]\). In this paper we focus on the tensor perturbations, namely Gravitational Waves (GWs).

Modifying gravity on cosmological scales generally results in a GW dispersion relation, i.e. GWs do not propagate at the speed of light; and also to a different GW friction term relative to that of GR. The parameters encoding these deviations are often denoted by \(\alpha_M\) (for the friction term) and \(\alpha_T\) (for the dispersion relation) \([11]\). For instance \(\alpha_M\) might arise from a running Planck mass, while a non-zero \(\alpha_T\) can occur in the case of a massive graviton \([7]\). GWs offers a unique opportunity to probe these parameters on cosmological scales. In fact, from a GW detection is possible to infer directly the luminosity distance of the source \([12–16]\) without the use of a cosmological ladder, thus giving the possibility to independently measure \(H_0\) with GWs \([17–18]\) even if an electromagnetic counterpart is not observed \([19]\). Moreover, the GW phase can be studied to probe for the presence of possible GW dispersion relations \([20–26]\). Finally, GWs can be also detected together with a gamma-ray burst (GRB), and any delay with respect to the GW can be used for probing \(\alpha_T\) \([27]\).

GWs have been used to measure \(H_0\) using the GW170817 hosting galaxy identification \([28]\) whereas for binary black holes, galaxy surveys have been employed \([29, 30]\). This type of study is indeed very promising for measuring independently \(H_0\), in fact with hundreds of GWs detection we will be able to measure \(H_0\) with 2% accuracy \([31]\). Recently, GWs have been used also to constrain \(\alpha_M\) and \(\alpha_T\) (or their equivalent quantities) independently of each other, often without considering the cross-correlations with the Hubble constant \(H_0\) (in the case of \(\alpha_T\)) and never (to the best of our knowledge) considering all of them at the same time. In \([9, 11, 32, 33]\) for instance, the authors present a methodology to measure jointly \(\alpha_M\) and \(H_0\) assuming that the redshift of the source is perfectly known from the identification of the host galaxy. In \([9, 26, 32]\) a more complete statistical analysis is presented using the binary neutron star merger (BNS) GW170817 \([34]\) for constraining the number of spacetime dimensions and a running Planck mass. Other works such as \([20, 21]\) focus on \(\alpha_T\) and the GW dispersion relation \([26]\) by fixing the cosmological parameters, and in particular \(H_0\), to the ones measured from CMB \([6]\). More recently there have been also some modelling effort for considering the effect of \(\alpha_T\) and \(\alpha_M\) together on the GW waveform \([35–38]\), however the cross-correlation
with Standard Cosmology are still not considered, the gamma-ray burst emission not taken into account and to the best of our knowledge without a complete inferential model.

In this paper we present a phenomenological method able to combine GW data, together with its associated GRB and hosting galaxy data to recover a joint estimation of the Hubble constant $H_0$ and the $\alpha_M$ and $\alpha_T$ parameters. The paper is organized as follow. In Sec. II we discuss the propagation equation for a GW traveling in modified gravity theories. We then link the GR deviation parameters $\alpha_M/T$ to three measurable quantities that we can infer from a GW event and for which we are usually provided with posterior samples: the luminosity distance, the GW phase evolution and the GW-GRB delay. In Sec. III we discuss the level of accuracy needed on these 3 observables to constraint the parameters $H_0, \alpha_M, \alpha_T$ using BNSs mergers observed with Advanced LIGO and Virgo. In Sec. IV we introduce a Bayesian inferential method which is able to provide a joint estimation of $H_0, \alpha_M, \alpha_T$ starting from the posteriors provided for the 3 observables and taking into account redshift uncertainties. In Sec. V we validate our method with simulated observations taking into account redshift uncertainties. Finally in Sec. VI we draw our conclusions and the prospects for this work.

II. PROPAGATION IN MODIFIED GRAVITY

We consider a flat Friedman-Lemaître-Robertson-Walker (FLRW) background space-time with line element

$$ds^2 = -c^2 dt^2 + a^2(t)dx^2$$

where $a(t)$ is the scale factor and conformal time $\eta$ is defined by

$$\eta = \int \frac{dt}{a(t)}.$$  \hspace{1cm} (2)

Though our focus is on modified gravity theories, we assume that the background evolution of the scale factor $a(t)$ is as in standard ΛCDM cosmology, so that only the dynamics of perturbations are modified relative to those of general relativity (a standard approximation on the literature.) Here we focus on tensor perturbations, namely gravitational waves. See for instance [39] for a discussion of scalar perturbations.

Consider a source at a fixed comoving position $r_{\text{com}} = |\vec{x}|$, which emits both a light (GRB) and a GW, see figure 1. Light rays travel along null geodesics and hence, assuming the observer being at the origin, $c(\eta^{EM}_{d} - \eta^{EM}_{s}) = r_{\text{com}}$, where $\eta^{EM}_{d}$ ($\eta^{EM}_{s}$) is the conformal time at which EM waves are detected (emitted). Light rays are redshifted in the usual way by the cosmological expansion;

$$f^{EM} = (1 + z)f^{EM}_d,$$  \hspace{1cm} (3)

where

$$1 + z = \frac{a(\eta^{EM}_{d})}{a(\eta^{EM}_{s})}. \hspace{1cm} (4)$$

As indicated in Fig. 1 in modified gravity models, GWs can travel with (possibly frequency dependent) speed $c_F \neq c$ and furthermore, as we discuss below, they are generally subjected to a modified friction term relative to that of general relativity (see [38] [40] [43] for some reviews and examples). Indeed here, as in many other papers in the literature, we only consider the effect of the modified gravity on the propagation of the GW signal [20] [21] [30] [37]. That is, the wave GW signal emitted by the binary source is assumed to be given by the standard GR expression. Hence we also restrict our analysis to the standard 2 degrees of freedom of GR.

![Figure 1](image)

FIG. 1. Schematic figure showing a binary neutron star merger at comoving distance $r_{\text{com}}$ emitting a GW and a GRB. The blue dots represent the two binary neutron stars. In modified gravity, GWs may propagate with a frequency dependent speed, and arrive with a relative time delay with respect to the electromagnetic counterpart. In this example, photons follow a time-like geodesic identified by an angle of 45 deg on the plot. Subluminal GWs follow a time-like geodesic identified by an angle lower than 45 deg (angle defined counterclockwise between $r - \eta$ axes).

\footnote{In many modified gravity models there are more than 2 propagating polarisations [43], which may possibly interact with each other. Here do not consider this case but focus on the effect of a modified propagation speed and friction term.}
Our starting point is a modified dispersion relation of the form [21]

\[ c^2 g_{\mu \nu} p^\mu p^\nu = -B_\alpha |p|^\alpha. \]  

(5)

where, for GWs emitted at \( r_{\text{com}} \) and propagating radially to the observer,

\[ p^\mu = (E/c, \hbar k/a^2, 0, 0) \]  

(6)

with \( k \) the (constant) comoving wave number, and \( |p| = (g_{ij} p^i p^j)^{1/2} = \hbar k/a^2 \). Thus the dispersion relation [5] is

\[ E^2 = c^2 h^2 k^2/a^2 + B_\alpha \left( c \frac{\hbar |k|}{a} \right)^\alpha, \]  

(7)

which depends on the physical momentum \( p_{ph} = k/a \). When the coefficients \( B_\alpha \) vanish, the dispersion relation Eq. [5] reduces to the standard one of a massless particle in general relativity \( \omega \equiv E/h = ck/a \). For \( B_0 \neq 0 \), Eq. [7] is the dispersion relation for the massive graviton \( B_0 = m_g^2 c^4 \) (in \([\text{eV}]^2\) ). Different theories give different predictions for the (generally \( \eta \)-dependent) \( B_\alpha \), see [21] for some examples. Here we aim to see what constraints GW observations can put on the \( B_\alpha \) without focusing on any particular theory.

Let us rewrite Eq. [7] as

\[ E^2 = \hbar^2 \omega^2 = c_T^2(\eta, k/a) \frac{\hbar^2 k^2}{a^2}, \]  

(8)

where

\[ c_T^2(\eta, k/a) \equiv c^2 \left[ 1 + B_\alpha \left( c \frac{\hbar |k|}{a} \right)^{-2} \right], \]  

(9)

Motivated by the very tight constraint on the speed of of gravitational waves [26, 45], we will assume that GWs are ultra-relativistic and that

\[ |B_\alpha| \left( c \frac{\hbar |k|}{a} \right)^{-2} \ll 1. \]  

(10)

Then from Eq. [8] it follows that

\[ \omega \simeq c |k|/a, \]  

(11)

so that the frequency of the emitted GW \( f^s_{GW} \) is related to that of the observed GW \( f^d_{GW} \) by the standard redshift relationship, namely

\[ a(t_d) f^d_{GW} \simeq a(t_s) f^s_{GW}. \]  

(12)

Hence we can identify the the GW redshift with the usual photon redshift \( z \), see Eq. [4]. With this approximation

\[ k \approx \frac{1}{c} \omega(\eta_d) a(\eta_d) = 2\pi f_d/c, \]  

(13)

since today \( a = 1 \). This allows us to write the phase velocity in Eq. [9] in terms of the detected GW frequency \( f_d \):

\[ c_T^2(f_d/a) = c^2 \left[ 1 + \hat{\alpha}_j \left( \frac{f_d}{a} \right)^2 \right], \]  

(14)

where we have defined

\[ \hat{\alpha}_j = B_{j+2}(2\pi \hbar)^j \]  

(15)

with \( j = \alpha - 2 \). Notice that the dimensions of \( [\alpha_j] = \text{Hz}^{-j} \). The radial propagation velocity of the waves is given by

\[ \frac{dr}{dt} = \frac{p^r}{p^t} = c_T^2 \frac{k}{a} \frac{1}{\alpha \omega} = \frac{v_g}{a} = \frac{1}{a} \frac{dr}{d\eta}, \]  

(16)

where the group velocity

\[ v_g \simeq c \left[ 1 - \frac{\hat{\alpha}_j}{2} \left( \frac{f_d}{a} \right)^2 \right], \]  

(17)

and we have used the approximation Eq. [10]. For massive gravitons, for example, \( c_T > c \), but the group velocity \( v_g \) is smaller than \( c \).

The dispersion relation in Eq. [8] can be obtained from the wave equation [4]

\[ \chi''(\eta, k) + k^2 c_T^2(\eta, k/a)^2 \chi(\eta, k) = 0 \]  

(18)

where \( ' = d/d\eta \) and \( \chi \) is the radial component of the propagating wave. The GW perturbation \( h \) (we drop the tensor indices for the moment) is related to \( \chi \) through (see e.g. [41])

\[ \chi = \tilde{a} h. \]  

(19)

Here \( \tilde{a} \) is an effective scale factor that encodes additional modifications to the GW friction term. We parameterize it as

\[ \frac{\tilde{a}'}{\tilde{a}} \equiv [1 + \alpha_M(\eta)] \frac{a'}{a}, \]  

(20)

where \( \alpha_M(\eta) \) is a deviation factor that can parameterize several theories such as scalar-tensor theories with a running Planck mass or theories with extra-dimensions. On subhorizon scales (that is, on scales smaller than \( \tilde{a}''/\tilde{a} \) [43]), Eq. [18] can be obtained from

\[ h'' + 2[1 + \alpha_M(\eta)] \frac{a'}{a} h' + k^2 c_T^2(\eta, k/a) h = 0, \]  

(21)

2 This assumes that \( a \) and \( B_\alpha \) varies on a cosmological time scale, which is much larger than any time-scale associated with the GW. Or in terms frequency (and in natural units), \( 1/k \ll r_{\text{com}} \ll H_0^{-1} \).
which is the wave equation of a GW propagating with a modified dispersion relation in the FRW universe. We can solve it using the WKB approximation following [34, 37], and obtain [38]

\[ h(\eta, k) = h_{\text{GR}}(\eta, k)C(\eta, \eta, k). \]  

where \( h_{\text{GR}}(\eta, k) \) is the solution in GR at the source at comoving distance \( r_{\text{com}} \), and \( C \) can be interpreted as the transfer function from the source to the detector for each GW mode \( k \). In terms of conformal time and detected GW frequency \( f_d \) (recall from Eq. (13) that \( k \approx 2\pi f_d/c \)) it is given by

\[
C(\eta, \eta, k) = \left[ \frac{c_T(\eta, f_d/a(\eta))}{c_T(\eta, f_d/a(\eta))} \right]^{1/2} \frac{\tilde{a}(\eta)}{a(\eta)} \times \exp[2\pi i (f_d/c) \int_{\eta}^{\eta'} c_T(\eta', f_d/a(\eta')) d\eta'] \\
\equiv |C(\eta, \eta, f_d)| e^{i\Psi(\eta, \eta, f_d)}. \]  

The modulus of \( C \) will contribute to the GW amplitude, that is to a modification of the luminosity distance. Its phase \( \Psi(\eta, \eta, f_d) \) leads to time delays and phase shifts, as we now discuss.

A. Observables

1. Luminosity distance

The first estimator that we define arises from the modulus of the transfer function. In GR, the amplitude of the GW scales as the comoving distance of the source. From Eq. (23), in modified gravity, the GW amplitude at the detector is now given by

\[
d_{\text{GW}}^{\text{GR}}(\eta_d, f_d) = r_{\text{com}} \frac{\tilde{a}(\eta_d)}{a(\eta_d)} \left[ \frac{c_T(\eta, f_d/a(\eta))}{c_T(\eta, f_d/a(\eta))} \right]^{1/2}. \]  

Since the results on GW dispersion relations are very tight \(|c - c_T| < 10^{-15} \) [25, 26], and measured errors on \( d_{\text{GW}}^{\text{GR}} \) are typically of at least a few percent, usually the effect of \( c_T \) on the distance is negligible. This is also consistent with the assumption in Eq. (11). The term \( \tilde{a} \) encodes the deviations in the GW friction and from Eq. (20), using redshift instead of conformal time, we obtain

\[
\tilde{a}(z) = a(z) \exp \left[ - \int_0^z \frac{\alpha_M(z)}{1+z} dz \right], \]  

where we have assumed that \( a(0) = \tilde{a}(0) = 1 \). In terms of the standard luminosity distance \( d_{\text{EM}}(z) = r_{\text{com}}/a(\eta_s) = r_{\text{com}}(1+z) \), we find that the GW luminosity distance in modified gravity is given by

\[
d_{\text{GW}}^{\text{GR}}(z) = d_{\text{EM}}(z) \exp \left[ \int_0^z \frac{\alpha_M(z)}{1+z} dz \right]. \]  

This equation is consistent with previous works [10, 43], which have shown the potential of the modified luminosity distance to be a good marker for testing possible deviations from GR on cosmological scales.

We now deviate from these references and use Eq. (26) to bound the parameter \( \alpha_M(z) \) such that the GW luminosity distance is a monotonically increasing function of the redshift. This condition is physically motivated, since if it were not satisfied one would detect an infinite number of GW sources at higher redshifts. In order to avoid this unphysical case, \( \alpha_M \) must satisfy

\[
\alpha_M(z) \geq - \frac{1}{z} \left[ \int_0^z \frac{dz'}{E(z')} \right]^{-1} - 1, \]  

where

\[
E(z) = \sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}. \]  

Since the right hand side of Eq. (27) is negative it follows that any positive values of \( \alpha_M \) (corresponding to a further GW), will satisfy this condition. Of course this is not valid for negative values of \( \alpha_M \) (GW might appear closer.) Fig. 2 shows the allowed values for GW friction parameter \( \alpha_M \) computed with Planck values of \( \Omega_M = 0.308 \) [3] and \( \Omega_\Lambda = 1 - \Omega_M \). Since at lower redshifts the \( \alpha_M \) contribution to the GW luminosity distance is small, this term is allowed to take very large values. However at higher redshifts, \( \alpha_M \) must be constrained to smaller values in order to satisfy the condition in Eq. (27).

![FIG. 2. The shaded area of on the plot shows the allowed value for the parameter \( \alpha_M \) with respect to the redshift. Any functional form of \( \alpha_M \) in the shaded area, will result in a monotonically increasing GW luminosity distance.](image)

2. Time delay

We now compute the time delay at the detector between two monochromatic GWs which were emitted at different times from the source at fixed comoving distance \( r_{\text{com}} \), see Fig. [1]. Consider a GW emitted at \( \eta_s^A \) and received at \( \eta_d^A \), with detected frequency \( f_{d,A} \). From
Eq. (17) it follows that
\[ r_{\text{com}} = \int_{\eta^A}^{\eta^B} \frac{c}{1 - \frac{1}{2} \hat{\alpha}_j} df_{j,A} \, d\eta. \] (29)
Similarly for second GW labelled by B we have
\[ r_{\text{com}} = \int_{\eta^B}^{\eta^A} \frac{c}{1 - \frac{1}{2} \hat{\alpha}_j} df_{j,B} \, d\eta. \] (30)
Hence the conformal time delay at the detector between the two GWs A and B is given by
\[ \Delta \eta_d^{AB} = \Delta \eta_s^{AB} + \frac{f_j^{j,A} - f_j^{j,B}}{2} \eta_j. \] (31)
To a good approximation the two GWs are emitted and detected on timescales which are smaller than the cosmological timescale, and thus
\[ \Delta t_s^{AB} = (1 + z_s) \Delta t_s^{AB} + \frac{f_j^{j,A} - f_j^{j,B}}{2} \eta_j \] (32)
where
\[ \eta_j = \int_0^{z_s} d\eta' \hat{\alpha}_j(z) \frac{(1 + z')^3}{H_0 E(z')} \] (33)
and \( z_s \) is the fixed source redshift.
In GR \( \hat{\alpha}_j = 0 = T_j \) and Eq. (32) reduces to the standard time dilation due to cosmological expansion. Although time delays between different GW modes are not usually measured (see below for a discussion of the measured GW phase), we can exploit Eq. (32) to obtain another observable quantity, namely the GW-photon time delay. To do so we set the non-GR delay contribution of \( B \) to zero, so that the time delay between the GW and its associated electromagnetic counterpart (a GRB) is
\[ \Delta t_d^{GW-EM} = (1 + z_s) \Delta t_s^{GW-EM} + \frac{f_j^{j,R,d}}{2} \eta_j, \] (34)
where \( f_j^{j,R,d} \) is understood to be the GW reference frequency used to compute the time delay [For instance for GW170817, \( f_j^{j,R,d} \) was the merger frequency [27]] and \( \Delta t_d^{GW-EM} \) is the prompt time delay of the GW and its EM counterpart at the source.

3. GW phase shift

Usually at the detector we do not measure the delay between different GW frequencies but rather the phase of the GW \( \psi(f_d) \). This is well approximated by [15]
\[ \psi(f_d) = 2\pi \int_{f_{R,d}}^{f_d} (t_d - t_{R,d}) df_d' + 2\pi f_d t_{R,d} - \phi_{R,d} - \pi/4, \] (35)
where \( t_{R,d} \) is the reference time at the detector at which the GW had frequency \( f_{R,d} \). Substituting \((t - t_{R,d})\) with the time-delay term computed in Eq. (32) and integrating gives
\[ \psi(f_d) = \psi_{\text{GR}}(f_d) + \pi T_j f_j^{j+1} \] when \( j \neq -1 \) (36)
\[ \psi(f_d) = \psi_{\text{GR}}(f_d) + \pi T_j \ln f_d, \] when \( j = -1 \) (37)
where
\[ \psi_{\text{GR}}(f_d) = 2\pi f_d t_{R,d} - \phi_{R,d} + 2\pi \int_{f_{R,d}}^{f_d} (1 + z_s)(t_s - t_{R,s}) df_d'. \]
(Here the primes take into account the redefinition of the reference time and phase after the integration on the non-GR delay term [21].) Note that Eq. (36) predicts a constant phase shift for the non-GR model with \( j = 0 \). This is normal since for \( j = 0 \) the GW group velocity in Eq. (17) is frequency independent.

Usually \( \psi_{\text{GR}}(f_d) \) is measured in terms of Post-Newtonian (PN) coefficients \( \beta_n^{PN}, \beta_{n,ln}^{PN} \)
\[ \psi_{\text{GR}}(f_d) = \sum_n \psi_n^{GR}(f_d) \]
\[ = \sum_n \left[ \beta_n^{PN} + \beta_{n,ln}^{PN} \ln f_d \right] f_d^{(n-5)/3}. \] (38)
These may be written in the form [49]
\[ \beta_n^{PN} = \frac{3}{128\nu} (\pi M)^{n-\nu} g_n(\nu, S_1, S_2), \] (39)
where \( M = m_1 + m_2 \) is total mass of the binary in seconds, \( \nu = (m_1 m_2)/M^2 \) is the symmetric mass ratio, \( S_1, S_2 \) the reduced spins and \( g_n(\nu, S_1, S_2) \) are numerical functions provided in [49]. By comparing the frequency dependency of Eqs. (39) and (38), we therefore see that a modified dispersion relation with power \( j \) will appear as a GR PN parameter of order
\[ n = 3j + 8. \] (40)
GW waveforms are known up to the PN orders \( n=0,...,7 \) thus meaning we can probe dispersion relations with eight different powers \( j \) given by \( j = -8/3, -7/3, \ldots, -1/3 \). Amongst these cases is massive gravity for which \( j = -2 \).

With current GW posteriors, there exists two methodologies for probing GW dispersion relations. The first one provides posteriors for GW dispersion relations which are not directly linked to the PN parameters [21]. This method fixes \( H_0 \) to Planck’s cosmology [3] and use the entire GW signal. The other one, is more agnostic and provides posteriors on the fractional phase deviations in the GW phase that would correspond to a given PN order [50, 51] from the inspiral part of the merger. In this paper, we start from the agnostic posterior samples on the PN parameters in [50, 51], since we would like to analyse
GW dispersion relation connection to the PN coefficients $\beta_{\text{PN}}$.

$$\frac{\psi_{3j+8}(f_d) - \psi_{3j+8,\text{GR}}(f_d)}{\psi_{3j+8,\text{GR}}(f_d)} = \pi \frac{T_j}{\beta_{3j+8}'(j+1)},$$

when \( j \neq -1 \) \( (41) \)

$$\frac{\psi_{5}(f_d) - \psi_{5,\text{GR}}(f_d)}{\psi_{5,\text{GR}}(f_d)} = \pi \frac{T_{-1}}{\beta_{5,\text{in}}},$$

when \( j = -1 \). \( (42) \)

III. IMPACT OF GR DEVIATIONS ON THE THREE OBSERVABLES

In this section we study the accuracy to which measurements of our three observables — luminosity distance, time delay and phase shift — are required in order to make constraints on deviations from GR. These deviations are encoded in the $\alpha_M$ and $\hat{\alpha}_j$ parameters; these two parameters, together with the Hubble constant $H_0$ (which is a third parameter) will affect the three GW related observables. From now on, in this paper, we will use the 0th order Taylor’s expansion of $\alpha_M(z) \approx \alpha_M(0), \hat{\alpha}_j(z) \approx \hat{\alpha}_j(0)$ since we are looking at events at very low redshift.

As a first step, in this section we discuss the possibility of measuring $\alpha_M$ and $\hat{\alpha}_j$ from the three observables independently when fixing $H_0$. This will help us anticipate the results on the inference on $\alpha_M, \hat{\alpha}_j$ and $H_0$ which we will show for our statistical method (introduced in Sec. V) applied to software simulated signals in Sec. V.

In the following we will assume a GW detection horizon (this means that every GW emitted within this range is detected with probability 1) of 333 Mpc and the values of $(\alpha_M, \hat{\alpha}_j)$ are limited through the condition given in Eq. \( (27) \). The GW horizon we consider roughly corresponds to the BNS range for a 3 detector network with same sensitivity during the observing run 4 (O4) scenario \( ([22]) \).

A. Measuring $\alpha_M$ from the GW luminosity distance

As seen from Eq. \( (26) \), the parameter $\alpha_M$ appears in the GW luminosity distance in Eq. \( (26) \). We define the fractional change introduced by the parameter $\alpha_M$ on the GW luminosity distance as

$$\epsilon_d \equiv \frac{d_{\text{GW}} - d_{\text{EM}}}{d_{\text{EM}}} = \exp \left[ \int_0^{z'} \frac{\alpha_M(z')dz'}{1 + z'} \right] - 1. \quad (43)$$

In principle, in order to measure the contribution of $\alpha_M$, one would need an accuracy on the GW luminosity distance measurement of the same order of the deviation introduced by the GR deviation parameter (assuming we perfectly know $H_0$ and the source redshift).

In Fig. \( 3 \) we show the absolute value of the GW luminosity distance fractional change, $\epsilon_d$, for different source redshifts and varying values of $\alpha_M$. In this case study, we fix the value of the Hubble constant to $H_0 = 70 \text{ km Mpc}^{-1} \text{s}^{-1}$.

![BNS Horizon = 333 Mpc](image)

**FIG. 3.** Absolute value of the fractional change (colorbar) on the GW luminosity distance introduced by $\alpha_M$ at different redshifts for a BNS detection during O4 observing scenario. We assume a value for the Hubble constant of $H_0 = 70 \text{ km Mpc}^{-1} \text{s}^{-1}$. The hatched region corresponds to BNS that cannot be detected since they are above the GW horizon. The dotted region is excluded from Eq. \( (27) \).

We see that a precision on the GW luminosity distance between 10\% and 1 \% is needed in order to constrain $\alpha_M$ between -5 and 5 for sources detected between redshifts 0.01 and 0.06. Unfortunately, this accuracy is not achievable with current GW detections; indeed, most of the error budget for the luminosity distance inferred from GW data comes from the well-known distance-binary inclination degeneracy \( ([53] [54]) \), and as a result, typical values of the luminosity distance uncertainty are of the order of 20\%-40\% \( ([54]) \) depending on the detected signal-to-noise ratio. This means at the best we expect to constrain the $\alpha_M$ parameter between values of order of -20 and 20 for BNSs between redshift 0.02 and 0.04. For BNSs detected at redshifts around 0.01 (like GW170817), the constraint will be of the order of -40 and 40. This prediction is consistent with the value of $\alpha_M$ found in \( ([9]) \) from GW170817. As we will also see later in Sec. V a better accuracy will be reached combining the results from many GW detections.

B. Measuring $\hat{\alpha}_j$ from the GW phase and the GRB delay

We again consider the O4 BNS observing scenario and a value of $H_0 = 70 \text{ km Mpc}^{-1} \text{s}^{-1}$. Furthermore, in this subsection, in order to gain intuition on the parameter $\hat{\alpha}_j$, we fix $\alpha_M = 0$.

Let us define the fractional phase shift change on the GW phase introduced by $\hat{\alpha}_j$ by

$$\epsilon_\psi \equiv \frac{\psi_{3j+8}(f_d) - \psi_{3j+8,\text{GR}}(f_d)}{\psi_{3j+8,\text{GR}}(f_d)}.$$  

(See Eqs. \( (41) \) - \( (42) \).) In addition, for this study, we consider the additional constraint on the speed of gravity...
FIG. 4. Fractional deviation in the PN coefficients introduced by the GW dispersion parameter $\hat{\alpha}_j$ with respect to the source redshift for a BNS O4 observing scenario. The Hubble constant has been fixed to $H_0 = 70 \, \text{km} \, \text{Mpc}^{-1} \, \text{s}^{-1}$. The hatched area corresponds to sources above the O4 GW horizon for detectability, and in each case the range of the $y$-axis satisfies (45).

FIG. 5. Time delay in seconds (colorbar) introduced by the GW dispersion parameter $\hat{\alpha}_j$ with respect to the source redshift for a BNS O4 observing scenario. The Hubble constant has been fixed to $H_0 = 70 \, \text{km} \, \text{Mpc}^{-1} \, \text{s}^{-1}$. The hatched area corresponds to sources above the O4 GW horizon for detectability, and in each case the range of the $y$-axis satisfies (45).
set from GW170817 and GRB170817A which can be converted into a constraint on \( \hat{\alpha}_j \), i.e.

\[
\left| \frac{\hat{\alpha}_j}{2} f_{R,d} \right| < 10^{-15},
\]

where \( f_{R,d} \) is the GW170817 merger frequency, which is of the order of \( f_s = 2000 \text{ Hz} \).

Fig. 4 shows the fractional phase shift introduced by the \( \hat{\alpha}_j \) for each dispersion relation (alongside the PN order to which they contribute). In each case the range of the \( y \)-axis satisfies (45), and one observes that the fractional GW phaseshift introduced by \( \hat{\alpha}_j \) varies over many orders of magnitude. In particular, for the largest values of \( \hat{\alpha}_j \), allowable by (45), the fractional GW phaseshifts are very large, of order \( 10^4 \). In general the measurements of GW phaseshifts are more accurate, and hence one expects the PN measurements to give better constraints than the GW-GRB time delay. For instance in the case of massive gravity (panel with \( j = -2 \) in Fig. 4), one can see that a mass of the graviton of the order of \( 2 \times 10^{-20} \text{ eV/c}^2 \), emitted by a source at redshift \( \sim 0.01 \), would modify the corresponding PN coefficient of \( 10^4 \) times its GR value. For GW170817, the constraint on the GR value of this PN coefficient is of the order of 10% \[26\], which from Figure 5 would lead to an upper limit on the mass of the graviton of order a few times \( 10^{-22} \text{ eV/c}^2 \).

We now consider the fractional deviation introduced by the GW dispersion relation \( \hat{\alpha}_j \) on the GW-EM time delay in Eq. (34). We define the time delay introduced by \( \hat{\alpha}_j \) as

\[
\epsilon_t \equiv - \frac{f_{R,d}}{2} T_j(z, H_0, \hat{\alpha}_j).
\]

The above quantity is measured in seconds since we have chosen for this definition \( \Delta t_{GW-EM} = 0 \). Figs. 5 shows the values of \( \epsilon_t \) for the 8 dispersion relations corresponding to the PN orders that we can study from the GW phase. Note that almost all the dispersion relations predicts the same maximum value of \( \epsilon_t \sim 100\text{s} \). The reason is that we are computing \( \epsilon_t \) from the maximum value given in Eq. (15). As an example, consider the case of massive gravity, given by the dispersion relation with \( j = -2 \). From Fig. 6 we see that one would need a massive graviton with mass of about \( 10^{-21} \text{ eV/c}^2 \) for a GW170817 like-source (redshift \( \sim 0.01 \)) in order to explain the 1.74 second delay observed (assuming that the GW and GRB were prompt at the same time). This limit is higher than those set in \[25\], which make use of the BBHs detections, and use the phase of study of the PN orders of the GW phase. We conclude that, with the current accuracy, the GW-EM time delay cannot constrain massive gravity better than the GW phase study.

For each GW dispersion relation model, one can predict which observable — the GW phase or GW-GRB delay — will best constrain the GW dispersion relation. The ratio \( \epsilon_{\psi}/\epsilon_t \) (see Eqs. (15) and (46)) quantifies which of the two observables is more modified by the introduction of \( \hat{\alpha}_j \). Assuming that our GW waveform model is well approximated by the PN expansion in Eq. (30) and the reference GW frequency for computing the GRB time delay is the last stable orbit \( f_{R,d} = (63/2\pi M)^{-1} \), we ob-
In this section we present a statistical method able to take all these effects into account. Let us discuss which are the statistical variables of the model. We assume a FLRW background specified by some parameters Λ (in the particular case we consider here Λ is simply the parameter $H_0$), a deviation in the GW friction term $\alpha_M$ and dispersion relation encoded in $\hat{\alpha}_j$. These are the population parameters on which we would like to have posterior distributions. Furthermore, there are other parameters which are intrinsic to each source but not measured or provided as posteriors: in particular, the initial prompt-delay between the GW and its EM counterpart $\Delta t_s$ and the merger frequency at the detector $f_d$. These, together with $H_0$ and the source redshift $z$ will determine values for the three observables: the GW luminosity distance $d_{GW}$, the GW-EM time delay $\Delta t_d$ and the GW phase shift $\delta \psi$. These observables are measured from several observed datasets. The GW luminosity distance and phase shift are measured from GW data $x_{GW}$, the redshift is observed from the hosting galaxy observations $x_z$ and the GW-GRB time delay is observed from GRB data $x_{GRB}$. Let us refer to these three datasets as $\hat{x}$.

\[
\begin{align*}
\epsilon_\psi = \frac{\pi T_j}{(j + 1)\beta_{3j+8}^{PN} f_{R,d}^{\psi}} \frac{2}{3(j + 1)M g_3 j + 8 (\nu, S_1, S_2)} & = \frac{256 \nu^j}{\beta_{3j+8}^{PN} f_{R,d}^{\psi}} \\
\epsilon_t = \frac{\pi T_{-1}}{\beta_{3j+8}^{PN} f_{R,d}^{\psi} f_{-1}^{\psi}} \frac{2}{3M g_3 j + 8 (\nu, S_1, S_2)} & = \frac{256 \nu^{j-2}}{3M g_3 j + 8 (\nu, S_1, S_2)}
\end{align*}
\]
each other. However, this is not true if we want to measure also the Hubble constant.

In our analysis we are interested in the likelihood of observing the three datasets \( x_{GW} \), \( x_t \) and \( x_M \) given some values of the FLRW background parameters and the deviation parameters \( \alpha_M \) and \( \tilde{\alpha}_j \). We would like to sample the posterior probability \( p(\lambda, \alpha_M, \tilde{\alpha}_j|\vec{x}) \). This is given by

\[
p(\Lambda, \alpha_M, \tilde{\alpha}_j|\vec{x}) = \frac{p(\Lambda, \alpha_M, \tilde{\alpha}_j, \vec{x})}{p(\vec{x})}.
\] (49)

Let us focus on the numerator in the above Eq., this can be factorized following the Fig. 7

\[
p(\vec{x}, \Lambda, \alpha_M, \tilde{\alpha}_j) = \int p(\alpha_M)p(\Lambda)p(\tilde{\alpha}_j)p(\Delta t_s)p(f_d)p(z|\Lambda) \cdot p(d_{GW}|\alpha_M, \Lambda, z)p(\delta \psi|\tilde{\alpha}_j, \Lambda, z)p(\Delta t_d|\tilde{\alpha}_j, \Lambda, \Delta t_s, f_d) \cdot p(x_{GW}|d_{GW}, \delta \psi)p(x_d|z)p(x_{EM}|\Delta t_d)
\]

In Eq. (50), the marginalization is carried out on all the variables except for \( \Lambda, \alpha_M, \tilde{\alpha}_j \). In Eq. (50), the terms \( p(\alpha_M), p(\Lambda), p(\tilde{\alpha}_j) \) represent the priors probabilities. The terms \( p(\Delta t_s), p(f_d) \) are prior distributions. The probability \( p(z|\Lambda) \) is a prior probability for the source to be located at a redshift \( z \). We can chose for example a uniform in comoving volume prior, which in the local universe scales as \( p(z|H_0) \propto z^2/H_0^3 \). The probabilities \( p(d_{GW}|\alpha_M, \Lambda, z), p(\delta \psi|\tilde{\alpha}_j, \Lambda, z), p(\Delta t_d|\tilde{\alpha}_j, \Lambda, \Delta t_s, f_d) \) represents the probability of having a value for one of the three observables given some values for the FLRW background parameters and GR deviation parameters. We assume these probabilities to be given by Dirac delta functions, since the observables can be computed deterministically from the relations that we defined in Sec. II.

With the above considerations Eq. (50) reduces to

\[
p(\vec{x}, \Lambda, \alpha_M, \tilde{\alpha}_j) = p(\alpha_M)p(\Lambda)p(\tilde{\alpha}_j) \int p(\Delta t_s)p(f_d)p(z|\Lambda) \cdot p(x_{GW}|d_{GW}, \delta \psi)p(x_d|z) \cdot p(x_{EM}|\Delta t_d|\tilde{\alpha}_j, \Lambda, \Delta t_s, f_d, z) dz df_d d\Delta t_s.
\] (50)

The remaining terms in Eq. (50) are the likelihoods, which are computed from the different datasets. Often we will not have access to the likelihood values but to the posterior distributions. For instance, we will have access to the joint posterior for the GW luminosity distance and PN parameters. The posteriors can be used in our method using the Bayes theorem

\[
p(x_{GW}|d_{GW}, \delta \psi) = \frac{p(d_{GW}, \delta \psi|x_{GW})p(x_{GW})}{\pi(d_{GW}, \delta \psi)},
\] (51)

where \( \pi(\cdot) \) is the prior used to generate the posterior samples. Therefore, we can compute the posterior \( p(\Lambda, \alpha_M, \tilde{\alpha}_j|\vec{x}) \) using Eq. (51) and the Bayes theorem as

\[
p(\vec{x}, \Lambda, \alpha_M, \tilde{\alpha}_j) = p(\alpha_M)p(\Lambda)p(\tilde{\alpha}_j) \int p(\Delta t_s)p(f_d)p(z|\Lambda) \cdot p(x_{GW}|d_{GW}, \delta \psi)p(x_d|z) \cdot p(x_{EM}|\Delta t_d|\tilde{\alpha}_j, \Lambda, \Delta t_s, f_d, z) dz df_d d\Delta t_s.
\] (52)

Finally, we can plug the joint probability in Eq. (52) in Eq. (49) to obtain

\[
p(\Lambda, \alpha_M, \tilde{\alpha}_j|\vec{x}) = p(\alpha_M)p(\Lambda)p(\tilde{\alpha}_j) \int p(\Delta t_s)p(f_d)p(z|\Lambda) \cdot \frac{p(d_{GW}|\alpha_M, \Lambda, z)\delta(\tilde{\alpha}_j, \Lambda, z)p(x_{GW}|\Lambda, z)}{\pi(z)} \cdot \frac{p(x_{GW}|\alpha_M, \Lambda, \tilde{\alpha}_j, \Lambda, \Delta t_s, f_d, z)p(x_d|z)}{\pi(z)} \cdot \frac{p(x_{EM}|\Lambda, \Delta t_s, f_d, z)}{\pi(\Delta t_d|\tilde{\alpha}_j, \Lambda, \Delta t_s, f_d, z)} dz df_d d\Delta t_s.
\] (53)

The posterior distribution in Eq. (52) does not consider yet selection effects [55, 59], which takes into account that for some values of \( \Lambda, \alpha_M, \tilde{\alpha}_j \) some binaries are more probable to be detected. When combining multiple sources for inferring the population parameters \( \Lambda, \alpha_M, \tilde{\alpha}_j \) this is a crucial term to include in order to recover a measure which is unbiased by the selection bias. Selection effects can be included dividing Eq. (53) by the selection probability

\[
\beta(\Lambda, \alpha_M, \tilde{\alpha}_j) = \int P_{GW}^{GR}(z, \Lambda, \alpha_M, \tilde{\alpha}_j)P_{GW}^{GR}(z, \Lambda, \alpha_M, \tilde{\alpha}_j) dz d\Delta t_s d\Delta t_s.
\] (54)

In the above Eq., \( P_{GW}^{GR} \) are the detection probabilities of the different datasets given the parameters of the binary. It is usually assumed that the detection probabilities will be dominated by the GW detection probability [31] as it will go to zero faster with respect to the other detection probabilities. In principle the GW detection probability will also depend on the phase shift of the GW, but at small redshift and small values of \( \tilde{\alpha}_j \) it is usually assumed that the GWs modified by a non-GR dispersion relation can be always detected [20].

V. SIMULATIONS

In this section we implement the statistical framework discussed previously showing that the two deviation parameters \( \tilde{\alpha}_j \) and \( \alpha_M \) will be in general correlated if we assume that we do not know \( H_0 \). We consider a Universe with \( H_0 = 70 \text{ km Mpc}^{-1} \text{s}^{-1} \), \( \alpha_M = 5 \) and \( \tilde{\alpha}_j = 9.23 \cdot 10^{-14} \text{Hz}^2 \) (which corresponds to a massive graviton of \( m_g = 2 \cdot 10^{-22} \text{eV} / c^2 \)). In order to compute the selection function in Eq. (50), we consider that all the BNS within a horizon of 100 Mpc are detected with probability 1. We take a GW170817-like scenario, with \( \rho = 33 \) at 40 Mpc. This corresponds to a signal-to-noise ratio for detection of about 12 at 100 Mpc. We sample for \( H_0, \alpha_M, \tilde{\alpha}_j, \Delta t_s | x_{GW}, x_t, x_{EM} \). This means that in the likelihood Eq. (52) we do not integrate in \( \Delta t_s \). We use a Monte Carlo Markov Chain (MCMC). In particular we use the implementation of the Parallel tempering Ensemble MCMC [60] in [61]. As we will show in the following sections, GW170817-like simulated signals bring to the same results and uncertainties observed in [9]. This further validates the analysis.
A. Single source

The first source that we consider is a BNS merger at redshift \( z = 0.010 \), this value corresponds in our simulated Universe to a GW luminosity distance of \( d_{GW} = 48 \) Mpc (computed using Eq. (26)) and to a detected SNR of \( \rho = 27.5 \). We assume the redshift to be perfectly known:

\[
p(x_s | z) = \delta(z_{\text{inj}} - z).
\]  

(55)

We also assume that we have observed a time delay between the GW and its associated GRB. We compute the GW-GRB delay observed using Eq. (54) and we assume the prompt delay between the GW and the GRB at the source to be \( \Delta t_s^{GW-GRB} = -1.78 \) s. We also assume the GW reference frequency for computing the time delay to be \( f_{R,d} = 2000 \) Hz.

Hence we simulate posterior samples for \( p(\Delta t_s | x_{GRB}) \) as a normal distribution centered at the injected GW-GRB time delay with standard deviation of 0.05s (which is a value similar to GW170817). We simulate the likelihood samples for the GW luminosity distance and phase shift \( p(x_{GW} | d_{GW}, \delta \psi) \). We assume the posterior \( p(\delta \psi, d_{GW} | x_{GW}) \) as a multi-variate gaussian distribution centered around the signal values. Following [31], the standard deviation of the luminosity distance is assumed to be the \( \sigma_d = (1.8/\rho)d_{GW} \) of the injected distance value. Regarding the phase shift, we assume a standard deviation of the 10% its injected value. In our first run, we fix the value of \( H_0 \) and \( \delta t_s \) to the injected ones. We run the MCMC and sample for \( \alpha_M \) and \( \alpha_{-2} \). We recover the values of \( \alpha_M = -8.20^{+6.10}_{-6.84} \), \( \alpha_{-2} = 9.68^{+2.40}_{-2.77} \times 10^{-14} \) Hz

(56)

(median with 68% confidence level). Fig. [8] shows the joint and marginal posterior distributions for \( \alpha_M \) and \( \alpha_j \). The two deviation parameters for GR are not correlated. This is due to the fact that the value of \( H_0 \) and \( z \) are fixed.

![Fig. 8](image8.png)

FIG. 8. Marginal and joint posterior distributions for \( \alpha_M, \alpha_{-2} \) for a software injection with SNR 27 in modified cosmology.

![Fig. 9](image9.png)

FIG. 9. Marginal and joint posterior distributions for the 4 parameters \( H_0, \alpha_M, \alpha_{-2}, \Delta t_s \) for a software injection with SNR 27.5 in modified cosmology.

![Fig. 10](image10.png)

FIG. 10. Marginal and joint posterior distributions for the 4 parameters \( H_0, \alpha_M, \alpha_{-2}, \Delta t_s \) for a software injection with SNR 27.5 in modified cosmology. In this simulation no selection effects have been considered.

In our second run, we also allow \( H_0 \) and \( \Delta t_s \) to
vary, and we sample the 4 dimensional parameters space $(H_0, \Delta t_s, \alpha_M, \hat{\alpha}_j)$. We obtained the values for the parameters $H_0 = 75.9^{+11.5}_{-9.4} \text{ km Mpc}^{-1} \text{s}^{-1}$, $\alpha_M = 11.5^{+6.61}_{-7.74}$, $\hat{\alpha}_2 = 9.88^{+2.79}_{-2.57} \cdot 10^{-14} \text{ Hz}^2$, $\Delta t_s = -1.78^{+0.48}_{-0.48} \text{s}$. Fig. [11] shows the joint and marginal posterior distributions for these 4 parameters. Let us first note that we obtain a marginal distribution for initial prompt delay $\Delta t_s$ centered around the injected value of $-1.78 \text{s}$ and with variance 0.05s. This means that the GW-GRB time delay at the detector is only decided by their initial prompt delay and the GW dispersion is being constrained by the GW phase. This is consistent with our discussion of Sec. [III] for Figs. [4] and [5]. For massive gravity, the parameter $\hat{\alpha}_{-2}$ strongly affects the GW phase shift but not the GW-GRB delay. From Fig. [2] we can also observe that the two GR deviation parameters show a weak positive correlation. This correlation arises from the fact that we are allowing $H_0$ to vary.

Finally let us discuss the correlation between $H_0$ and $\alpha_M$. From Eq. [20], these two parameters are degenerate each other at fixed redshift, that is the same value of the GW luminosity distance can be obtained with different pairs of $(H_0, \alpha_M)$. The strong positive correlation seen in Fig. [6] is due to this fact. However, a preference for a given pair $(H_0, \alpha_M)$ comes from the selection function in Eq. [22]: indeed higher values of $\alpha_M$ are preferred since the GW detection horizon is smaller. As expected, this preference for smaller $\alpha_M$ disappears if we do not include the selection effect: in Fig. [10] we show the corner plots for the same injection but with no selection effect considered. As this figure shows, the MCMC prefers smaller values of $H_0$. This is explained by the fact that we are using a uniform in comoving volume prior which scales as $1/H_0^2$. The value of the parameters that we recover are $H_0 = 73.2^{+10.6}_{-9.9} \text{ km Mpc}^{-1} \text{s}^{-1}$, $\alpha_M = 8.95^{+7.17}_{-7.08}$, $\hat{\alpha}_2 = 9.71^{+2.93}_{-2.74} \cdot 10^{-14} \text{ Hz}^2$, $\Delta t_s = -1.78^{+0.05}_{-0.05} \text{s}$.

B. Population

We now use our inferential method to combine the results from 100 BNSs detections. We simulate BNSs mergers in a Universe with $H_0 = 70 \text{ km Mpc}^{-1} \text{s}^{-1}$, $\alpha_M = 5$ and $\hat{\alpha}_2 = 9.23 \cdot 10^{-14} \text{Hz}^2$ with a uniform in comoving volume redshift prior

$$p(z|H_0) \propto \frac{1}{E(z)H_0^2} \left[ \int_0^z \frac{dz'}{E(z')} \right]^2. \quad (56)$$

Again, we consider that GWs with a luminosity distance < 100 Mpc are always detected. We fix the GW SNR threshold at 12 at 100 Mpc as in previous the case. For each event we generate the recovered posterior samples for the GW luminosity distance, phase shift and GW-GRB delay following the same assumptions of the previous section. The initial prompt delay between the GW and the GRB, $\Delta t_s$, is generated uniformly between 0 and -10 seconds, following the model assumptions in [27].

VI. CONCLUSIONS

In this paper we have presented a new method for probing deviations from GR at cosmological scales. In Sec. [III] we have discussed how in theories beyond GR, the GW propagation is modified in an expanding universe, and in particular we have shown how to relate the GW friction to a modified GW luminosity distance, and how to relate the GW speed (or dispersion relation) to the possible GW-GRB delay and GW phase evolution. These 3 observables, which depend on the Hubble constant $H_0$, can
contribute to measuring GR deviations on cosmological scales.

In Sec. [III] we discussed the required level of accuracy that we would need on the 3 observables to accurately measure GR deviations at a given redshift. Regarding the GW friction term \( \alpha_M \), we have shown that the GW luminosity distance uncertainty is too high to allow for the accurate measurement of \( \alpha_M \). We have also discussed the possibility to constrain the GW dispersion parameter \( \hat{\alpha}_j \) from the GW-GRB time delay and the GW phase independently. We have shown that the GW phase already provides one of the most stringent upperlimits on the GW dispersion relation.

In Sec. [IV] we have presented a Bayesian statistical framework able to combine GW, GRB and galaxy redshift measurements to jointly constrain the Hubble constant, the GW friction and the GW dispersion relation. The statistical method can be used starting from the posterior distributions of the redshift, GW parameters and GW-GRB time delay. Using this statistical framework, we have shown that our lack of knowledge on the value of the Hubble constant \( H_0 \) is a crucial variable to include if we are trying to measure the GW friction and dispersion relation. We have shown that by combining 100 BNSs events with accurate redshift estimations, \( H_0 \) will be constrained to an accuracy of 2% and the GW friction and dispersion relation (for the case of massive gravity) to an accuracy of 15% and 2% respectively. We argue that with the combination of 100 BNSs events, the error budget on \( \alpha_M \) and \( \hat{\alpha}_j \) is impacted by the \( H_0 \) determination and fixing an \( H_0 \) value would lead to a biased measurement of GR deviations. The method presented in this paper will be applied in a separate work to the BNS merger GW170817 using the event galaxy information and the GRB observation delay [62].

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