Optimization design of gear reducer based on multi-attribute decision making and reliability sensitivity

Cao Tong\textsuperscript{1*}, Zhuang Tian\textsuperscript{2}

\textsuperscript{1} School of Mechatronics Engineering, Shenyang Aerospace University, Shenyang, Liaoning, 110136, China
\textsuperscript{2} School of Mechanical Engineering and Automation, Shenyang Institute of Technology, Fushun, Liaoning, 113122, China
*Corresponding author’s e-mail: tongcao19@163.com

Abstract. In order to solve the complexity and implicitness of lightweight optimization of the gear reducer, this paper constructs a multi-objective reliability optimization mathematical model of the gear reducer and obtains multi-objective Pareto solution, artificial intelligence algorithm is used to optimize structural parameters. Then, through repeatedly changing the structural parameters of the reducer, the experimental simulation analysis is carried out to obtain the maximum working stress. Based on the idea of the weight reduction and miniaturization, the stress state and reliability index values under different structural parameters are compared and analyzed. In order to determine the optimal solution of structural parameters, multi-attribute decision theory is used to determine the solution. Finally, in order to effectively reveal the inherent variation of attribute values and decision results and quantitatively reflect the impact of various structural parameters on system reliability, reliability sensitivity analysis of the gear reducer is carried out by the response surface-Monte Carlo and DPS methods. Thereby, an optimized structure that meets the safety reliability and the appropriate economy is obtained, and this study produces a balanced and improved optimized design that is coordinated with cost, quality, volume, safety and stability.

1. Introduction
In order to reduce the volume of the gear reducer and reduce the weight of the product, the modern method is to calculate and check the empirical formula of its strength and stiffness, so the cabinet design of the reduction gearbox tends to be conservative. With the development of virtual test technology, the virtual simulation technology is used to establish the full-body model of the gear reducer, and the stress distribution and deformation of the whole part are analyzed to optimize the overall structure of the product. The overall strength and rigidity of the reducer are guaranteed, and the weight of the reducer is optimized.

The optimization of gear reducer has been discussed in many literatures [1-4]. Tao [1] established the mathematical model of the optimization design of the gearbox of the tillage machine, which guarantees the given center distance range, transmission power, structure and installation. In the case of the case, the gear transmission parameters are reasonably selected to minimize the use of the gearbox. Zhu [2] taked the number of pinion gears, the modulus of the normal surface and the transmission ratio of the first two stages in the transmission of the spur gear reducer as the design variables, and establishes the third level with the minimum sum of the center distances of the gears of the reducer as the objective function. The mathematical model of the optimization design of the spur
gear reducer has achieved good results. Zhu [3] called Matlab to optimize the empirical mathematical formula in Visual Basic, and optimize the design of single-stage spur gear transmission. Although the accuracy is not high, the method is simple and easy. Row. Gai [4] comprehensively considered the factors such as displacement coefficient, tooth profile coefficient, equal strength and lubrication conditions, and establishes the optimization model with the minimum center distance as the target, and optimizes the transmission ratio of JS40-25 three-stage conical cylindrical gear reducer for coal mine. Distribution.

However, due to the complexity and implicit nature of multi-stage gear reducers, most studies have difficulty in considering their system reliability during the optimization process. In the past literature, although the previous literature has initially determined the optimal structural design parameters and basically met the specified reliability and light weight requirements, the previous methods are not the optimal solution and has some limitations because it does not consider the system reliability sensitivity factor. In order to get the optimal design solution more accurately, this paper combines each attribute with reliability sensitivity analysis. Based on the Pareto solution set, the multi-attribute decision theory is used to optimize the design of typical structural products.

2. Multi-attribute decision analysis theory

2.1. Decision matrix normalization method

In the multi-attribute decision problem, \( P = \{P_1, P_2, \ldots, P_m\} \) represents a set of \( m \) schemes, each scheme has \( n \) attributes, and \( I = \{I_1, I_2, \ldots, I_n\} \) represents a set of attributes. The \( j \)th attribute value of the \( i \)th scheme is represented by \( a_{ij} (i=1, 2, \ldots, m; j = 1, 2, \ldots, n) \), and then \( A = (a_{ij})_{m \times n} \) is the original decision matrix. The line number represents the scheme and the column number represents the attribute.

At present, there are many methods normalizing decision matrices. Linear proportional transformation method, range transformation method and gravity transformation method are commonly used [5]. Considering that an extreme value in an attribute is regarded as a constant, other attribute values are normalized. Therefore, the weighting method is used to normalize the original decision matrix.

When \( I_j \) is a benefit attribute,

\[
x_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}}
\]

(1)

When \( I_j \) is a cost-type attribute,

\[
x_{ij} = \frac{1/a_{ij}}{\sum_{i=1}^{m} 1/a_{ij}}
\]

(2)

And then Normalized decision matrix \( X = (x_{ij})_{m \times n} \) is obtained from (1) and (2).

2.2. Method for determining attribute weights

In multi-attribute decision making, we need to consider the mutual importance of the attributes. Usually, we use weights or weight coefficients to indicate the mutual importance of attributes, that is, weights are measures of the importance of attributes or indicators. Different weights will bring different evaluation results. Scientific and reasonable attribute weights may produce reliable and correct multi-attribute decision-making results. Therefore, weight determination or weight coefficient allocation is an important issue in the evaluation scheme selection process.
2.3. Multi-attribute decision-making evaluation method

Simple linear weighting is one of the most concise and multi-attribute evaluation methods that people often use. The attribute weight vector corresponding to the attribute set $I = \{I_1, I_2, \ldots, I_n\}$ is $W = \{w_1, w_2, \ldots, w_n\}$, where $\sum_{i=1}^{n} w_i = 1$, $0 \leq w_j \leq 1 (j = 1, 2, \ldots, n)$. Then the comprehensive evaluation value of the scheme $i$ is

$$y_i = \sum_{j=1}^{n} w_j x_{ij}, i = 1, 2, \ldots, m$$  \hspace{1cm} (3)

According to the size of $y_i$, you can get the result of the scheme sorting. If $y_1' > y_2' > \cdots > y_m'$, the order of the scheme is $P_1' > P_2' > \cdots > P_m'$.

3. Reliability sensitivity analysis method

3.1. Response surface method

The response surface method is an effective method for performing implicit functions of mechanical reliability analysis. The idea is to simulate a real limit state surface by fitting a response surface through a series of deterministic experiments. The response surface can be simulated with a mathematical equation, assuming the system response \(r\) and the random parameter vector affecting the structure $X = [X_1, X_2, \ldots, X_{NR}]$ can be described by a quadratic function with cross terms.

$$r = a_0 + \sum_{i=1}^{NR} a_i X_i + \sum_{i=1}^{NR} \sum_{j=i}^{NR} a_{ij} X_i X_j \hspace{1cm} (4)$$

Where $a_0, a_i, a_{ij}(i = 1 \ldots NR; j = i \ldots NR)$ are undetermined coefficients.

The NS sample points of the random parameters are numerically calculated to obtain NS output points $(z_1, z_2, \ldots, z_{NS})$, and the data are subjected to regression analysis by least squares method [5],

$$s = \sum_{i=1}^{NS} e_i^2 = \sum_{i=1}^{NS} \left[ z_i - \left( a_0 + \sum_{i=1}^{NR} a_i x_i + \sum_{i=1}^{NR} \sum_{j=i}^{NR} a_{ij} x_i x_j \right) \right]^2$$ \hspace{1cm} (5)

Where $e$ is the error term. The response surface function is used to replace the real response for the reliability analysis.

3.2. Sensitivity analysis based on response surface-Monte Carlo and PDS

To obtain the influence of random variables on structural reliability, the sensitivity of the limit state function to structural parameters (such as size, diameter, width, etc.) is characterized and explained, thus providing guidance and reference value for improving the structure.

Through the reliability sensitivity analysis module of ANSYS Probabilistic Design System (PDS), the influence of each structural variable on the limit state equation is obtained. The analysis file of the response surface function of the component is written by APDL, and the PDS module of ANSYS can be used effectively. Analyze the influence of various factors on the limit state equation. In the analysis process, each random parameter is described as an uncertainty variable obeying a certain probability density distribution, and the distribution characteristics, influence relationship and influence degree of each response parameter are analyzed through a large number of sampling points. Parameter sensitivity. Main steps is following as: select the experimental design scheme, virtual simulation test, establish response surface function, select analysis tool, specify reliability analysis file, cycle sampling, and post-processing result.

4. Reliability sensitivity analysis and optimization design of gear reducer

4.1. Multi-attribute decision making optimization based on Pareto solution set
After the multi-objective genetic algorithm is optimized, the Pareto solution set of the typical structural product gear reducer is obtained, which is a non-inferior solution that satisfies the reliability index and lightweight. It is used as the initial decision matrix of multi-attribute decision making, and the example optimization design and sensitivity analysis, table 1 shows the initial multiple solutions.

Table 1 Pareto optimal solution set

| $x_1$ (mm) | $x_2$ (mm) | $x_3$ (mm) | $x_4$ (mm) | $x_5$ (mm) | $x_6$ (mm) | $x_7$ (mm) | $x_8$ (mm) | $x_9$ (mm) | $f_1$ ($10^3$ mm$^3$) | $f_2$ |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------------|------|
| 34.0815    | 33.0442    | 31.5264    | 2.4626     | 28.2383    | 24.0321    | 27.0750    | 37.1806    | 8.3164     | 0.80759          | 0.99988 |
| 34.0835    | 32.0734    | 33.8462    | 1.7626     | 27.2383    | 24.0321    | 27.0750    | 37.1806    | 7.3486     | 0.80808          | 0.99926 |
| 32.0815    | 31.3642    | 31.7466    | 2.0128     | 25.0663    | 22.3033    | 25.9131    | 37.0002    | 7.6874     | 0.79808          | 0.99929 |
| 30.0112    | 31.7546    | 31.0086    | 1.8035     | 23.0638    | 23.3687    | 27.0642    | 34.9423    | 6.9468     | 0.80199          | 0.99943 |
| 32.0582    | 32.1346    | 33.8726    | 2.0121     | 24.7856    | 23.6472    | 27.0021    | 38.1648    | 7.3648     | 0.80057          | 0.99937 |
| 31.0086    | 32.9436    | 32.4367    | 2.0227     | 22.6984    | 23.3857    | 27.7841    | 36.8639    | 6.9431     | 0.79867          | 0.99955 |
| 33.0385    | 34.9647    | 32.4697    | 2.1689     | 23.5770    | 22.3756    | 28.067     | 37.0003    | 6.9738     | 0.79270          | 0.99935 |
| 32.4364    | 32.4673    | 32.2486    | 2.0735     | 22.8731    | 22.0479    | 27.9468    | 38.2487    | 7.4367     | 0.78892          | 0.99929 |

The reducer system volume $f_1$ is taken as the cost type attribute, and the system reliability $f_2$ is taken as the benefit attribute. The original decision matrix of the quality evaluation problem is

$$
A = \begin{bmatrix}
0.8076 \times 10^7, 0.8081 \times 10^7, 0.7981 \times 10^7, 0.8020 \times 10^7, 0.8006 \times 10^7, 0.7987 \times 10^7, 0.7927 \times 10^7, 0.7889 \times 10^7; \\
0.99988, 0.99926, 0.99929, 0.99943, 0.99937, 0.99955, 0.99935, 0.99929
\end{bmatrix}^T
$$

The normalized matrix obtained by normalizing the original decision-making annihilation using equations (1) and (2) is

$$
X = \begin{bmatrix}
1.0154, 1.0275, 1.0000, 1.0243, 1.0267, 1.0344, 1.0394; \\
0.12506, 0.12498, 0.12500, 0.12499, 0.12502, 0.12499, 0.12498
\end{bmatrix}^T
$$

The attribute vector given by experts according to experience is $W=[0.6, 0.4]$. The comprehensive evaluation value of each scheme calculated by equation (3) is $y_1=0.12556$, $y_2=0.12558$, $y_3=0.12464$, $y_4=0.1267$, $y_5=0.12488$, $y_6=0.12471$, $y_7=0.12414$, $y_8=0.12379$, so $y_4 > y_2 > y_1 > y_3 > y_6 > y_5 > y_7 > y_8$, the scheme is sorted to $P_1 > P_2 > P_3 > P_4 > P_5 > P_6 > P_7 > P_8$, the optimal scheme based on multi-attribute decision making is shown in Table 2.

Table 2 the optimized structural parameters

| Type           | $x_1$ (mm) | $x_2$ (mm) | $x_3$ (mm) | $x_4$ (mm) | $x_5$ (mm) | $x_6$ (mm) | $x_7$ (mm) | $x_8$ (mm) | $x_9$ (mm) | $f_1$ ($10^3$ mm$^3$) | $f_2$ |
|----------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------------|------|
| Multiple attribute decision | 30.0       | 31.7       | 31.0       | 1.8        | 23.0       | 23.3       | 27.0       | 34.9       | 6.9        | 0.80199           | 0.99943 |
|                | 112        | 546        | 086        | 035        | 638        | 687        | 642        | 423        | 468        |                   |      |

Due to the Pareto solution set based on multi-objective genetic optimization algorithm, the structural parameters are all decimals. In order to facilitate the convenience of the processing technology, the improved optimized structural parameters are shown in Table 3.

Table 3 the rounding optimized structural parameters

| Type           | $x_1$ (mm) | $x_2$ (mm) | $x_3$ (mm) | $x_4$ (mm) | $x_5$ (mm) | $x_6$ (mm) | $x_7$ (mm) | $x_8$ (mm) | $x_9$ (mm) | $f_1$ ($10^3$ mm$^3$) | $f_2$ |
|----------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------------|------|
| Multiple attribute decision | 30         | 32         | 31         | 1.8        | 23         | 23         | 27         | 35         | 7          | 0.81066           | 0.99939 |

Compared with the virtual simulation results, it is found that except for the comparison between the $x_3$ parameters and the virtual simulation results, the $x_2$, $x_4$ and $x_5$ parameters have slight differences, and the remaining parameters are within a reasonable range.

4.2. Establishment of response surface function

The reducer is the most important power and motion transmission equipment, and the gear transmission is the main part of the reducer. It is known from the FMECA failure analysis, the gear failure is the main failure mode of the system, and the system response surface function is established
by the gear force characteristics. From the analysis results of the virtual reliability test, the reliability of the secondary transmission gear is the lowest one, which is the main failure mode of the typical structural product.

The finite element calculation of the pinion gear is carried out by ANSYS/LS-DYNA. The calculation result is shown in Fig. 1. The maximum contact stress of the gear tooth surface is 396.743 MPa. The values of the first 10 sample points (55 in total) calculated.

Table 4 Latin hypercube sampling sample points and response values

| Sample point | x1 (mm)  | x2 (mm)  | x3 (mm)  | x4 (mm)  | x5 (mm)  | x6 (mm)  | x7 (mm)  | x8 (mm)  | x9 (mm)  | r (Mpa) |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|---------|
| 1            | 32.2798  | 33.4707  | 32.5310  | 2.1386   | 27.3180  | 24.9273  | 27.2937  | 36.9625  | 8.3741   | 399.1719|
| 2            | 34.7682  | 31.4077  | 33.7937  | 1.7073   | 26.7761  | 24.6343  | 27.9916  | 37.1524  | 8.0908   | 399.1608|
| 3            | 33.1683  | 32.6973  | 31.6474  | 2.0719   | 24.8727  | 22.9054  | 27.4393  | 37.4772  | 8.6963   | 392.9833|
| 4            | 31.0792  | 31.2857  | 33.6953  | 2.4009   | 22.4355  | 24.2992  | 25.7169  | 36.3087  | 6.3664   | 406.5987|
| 5            | 31.4895  | 32.6653  | 32.9997  | 1.8356   | 28.5827  | 23.7147  | 26.1053  | 36.8197  | 7.3591   | 392.2891|
| 6            | 33.6839  | 31.2073  | 31.3155  | 2.3650   | 28.8820  | 24.0189  | 27.1204  | 38.5628  | 8.8849   | 395.5237|
| 7            | 32.7056  | 31.5278  | 32.4862  | 2.2462   | 27.6757  | 23.2670  | 28.8590  | 37.2840  | 7.4801   | 393.1715|
| 8            | 33.3324  | 33.8571  | 33.1030  | 2.1530   | 22.8733  | 22.6144  | 28.7572  | 36.7840  | 7.0974   | 391.4969|
| 9            | 33.7812  | 32.4385  | 33.5762  | 2.3455   | 24.9844  | 23.8279  | 26.5286  | 38.2749  | 8.9942   | 398.4493|
| 10           | 31.5623  | 31.3419  | 32.2004  | 1.7349   | 25.2649  | 24.2481  | 28.9484  | 36.5426  | 6.4174   | 387.8587|

4.3. System Reliability Sensitivity Analysis

The input coefficient and the correlation coefficient between the input variables is selected and defined according to the response surface function. The maximum contact stress of the gear tooth surface is regarded as a random output variable, and the distribution type, distribution function and its parameters is specified for each input variable. According to equation (4), the limit state function is established as G. If G<0, it is invalid. In the probabilistic design process, the 2-axis pinion analysis module is sampled for 50,000 cycles to obtain the reliability and the sensitivity value of each random parameter. Fig. 2 shows the limit state variable G sample history curve, which reproduces the value of the limit state variable G in 50,000 samples.
For those sensitive factors, do not change the size of the structure design as much as possible. In processing and production, it is necessary to strictly control the precision. For those parameters that are less sensitive, in order to improve the economy, the requirements can be appropriately widened. No strict control is required. The above numerical calculation results can provide a basis for the engineering designer to accurately design the reducer gear. The above calculation results are completely consistent with the usual qualitative analysis results, which further demonstrates the comprehensiveness and correctness of the sensitivity matrix to the analysis of various factors of the reducer gear.

Fig. 3(a) shows the sensitivity by the bar graph. Fig. 3 (b) is the sensitivity by the pie chart. As can be seen from the figure 3, the sensitivity of the gear modulus $x_4$ is 1.5216, and the direct $x_5$, $x_6$, $x_7$ and $x_8$ of each axis are 0.22204, 0.37733, 0.34559, and 0.19341, respectively. The sensitivity values of the other parameters are very small. It indicates that these parameters have less impact on system reliability. The sensitivity of the tooth width is negative, indicating that the system reliability will decrease as the tooth width of the secondary gear pair increases. This is probably due to the mutual influence of the bending and torsion deformation of each shafting system, resulting in the tooth surface deviation. Load, and thus local stress is too large, resulting in a reduction in system reliability, so this is consistent with the gear analysis we know. The sensitivity of all other parameters is positive, indicating that the reliability of the gear reducer system is increased by increasing the parameters. For the parameters of the gear modulus and the shaft diameter, the larger the value, the higher the bearing capacity, so the reliability is also The higher the accuracy is, therefore, the theoretical analysis is performed to verify the correctness of the results of the reliability sensitivity analysis.

From the previous reliability sensitivity analysis, the change of gear modulus has the greatest influence on the reliability of the system. Therefore, in the optimization design and evaluation of the gear, the change of the gear modulus parameter should be strictly controlled and obtained by the multi-objective genetic algorithm. In the optimal Pareto solution set, only the modulus of scheme 2 and the current optimal scheme 4 are small. Therefore, if the factors of the structural parameters of the gear reducer are changed due to human factors or factors of expert weighting in multi-attribute decision analysis, scheme 2 is a better choice.
5. Conclusion
Aiming at solving the complexity and implicit problems in the optimization design of gear reducer, this paper proposes an optimization design method based on the multi-attribute decision-making and reliability sensitivity. Through analysis of the reducer's example, the optimal structure with the lightweight and high reliability are obtained. The design scheme provides an important theoretical basis for the application of the industrial gear reducers.

Acknowledgments
This research was financially supported by the Young Doctor Scientific Research Foundation of College (Grant No. 19YB27) and Liaoning Provincial Natural Science Foundation (Grant No. 2018010334-301). Their financial supports are gratefully acknowledged.

References
[1] Tao, D.C., Sun S.L., Lu Y.E. (2001) Optimization design of gearbox of cultivator. Transactions of the Chinese Society for Agricultural Machinery, 32(2): 118-20.
[2] Zhu, L.L., Wang G.X. (2005) Optimization design of parameters of three-stage cylindrical gear reducer. Journal of Dalian Jiaotong University, 26(2): 20-24.
[3] Zhu Z.B., Pan D.L. (2005) Optimization Design of Cylindrical Gear for Mine ZGB Car Based on Matlab. Machinery Design & Manufacture, (12): 72-73.
[4] Gai M.M., Ren J., Wei B.Y., Guo Q. (2015) Optimization Design of JS40 Three-stage Conical Cylindrical Gear Reducer Drive System. Mining & Processing Equipment, 43(02):105-109.
[5] Hwang C.L., Yoon K. (1981) Multiple attribute decision making: methods and applications . Berlin: Springer-Verlag.
[6] Yan M., Sun Z.L., Yang Q. (2007) Reliability sensitivity method based on response surface method. Journal of Mechanical Engineering, 43(10): 67-71.