Dynamics of interacting bubbles located in the center and vertices of regular polyhedra

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Abstract. The dynamics of spherical gas bubbles located in the center and vertices of regular polyhedra under a sharp pressure increase in liquid is studied. Before the pressure rise, the liquid and the bubbles are at rest, the liquid pressure is 1 bar, the radii of the bubbles are 0.25 mm. The liquid pressure rises by 3 bar. For comparison, the configurations with one bubble in the center of a sphere and the others stochastically distributed inside it are also considered. In all the cases, the initial minimum distance between the centers of the bubbles does not exceed 5 mm. A mathematical model is used, in which the dynamics of the bubbles is governed by second-order ordinary differential equations in the radii of the bubbles and the position-vectors of their centers. It is shown that in all the bubble configurations considered, the amplitude of the pressure oscillations in the central bubble decreases nonmonotonically. With a sufficiently large number of peripheral bubbles, the change in the amplitude begins with a phase of its significant increase. With a rise in the number of the peripheral bubbles, the oscillation frequency of all bubbles decreases.

1. Introduction

Until now the main attention in studying the dynamics of vapor and gas bubbles in liquid has focused on the consideration of single bubbles. At the same time, the hydrodynamic interaction between the bubbles [1] can have a significant effect on their dynamics, which is important for applications since in reality bubbles are rarely single. As a result of the interaction, the bubbles can be strongly deformed [2-4], they can perform translational motion [5-7], generate stable configurations (pairs, triples, streamers, clusters, etc.) [8, 9], the interaction can weaken or amplify radial pulsations of the bubbles [10-12], etc. The interaction between the bubbles is believed to enhance the potential of their destructive effect on the surface of closely spaced solids (i.e., it increases their damage and erosion [13, 14]). The authors of the experimental and theoretical studies of the phenomenon of neutron emission during acoustic cavitation of deuterated acetone claim that the presence of a cluster of bubbles is a necessary condition for the realization of this phenomenon [15, 16].

It is shown in [12] that, with an instantaneous increase in the liquid pressure, the maximum pressure in the central bubble of a cluster can far exceed the maximum pressure in a similar single bubble. Two types of clusters are considered in [12]. In the first case, the central bubble is surrounded by three equally spaced and equally oriented spherical layers of bubbles located at the vertices of three dodecahedra (a dodecahedron is a regular polyhedron with 20 vertices) concentric with the central bubble. In the second case, one of the bubbles is located in the center of a sphere, while the others are distributed stochastically inside it with a restriction on the minimum distance between the bubbles.
In the present work, the effect of the hydrodynamic interaction between bubbles in a cluster on the dynamics of the central bubble under a sharp pressure increase in liquid is studied in more detail. In particular, along with the cluster configurations considered in [12], other bubble structures are also investigated. In those structures, the bubbles distant from the central one are located at the vertices of regular polyhedra with a smaller number of the faces (i.e., icosahedra, hexahedra, etc.). When analyzing the clusters with a stochastic bubble distribution, averaging of the calculation results of ten variants of random distributions is used. Such a technique allows us to better clarify the trends in the influence of the random distribution. The first collapse of the bubbles is considered in more detail, which has enabled us to reveal a number of new features of the dynamics of the central bubble in a cluster resulted from its interaction with other bubbles.

2. Problem statement and computational technique

The hydrodynamic interaction of spherical gas (air) bubbles in a cluster consisting of the bubbles located in the center and vertices of regular polyhedra is considered. Before the interaction (up to the time \( t = 0 \)), the bubbles and the surrounding liquid (water-glycerin mixture) are at rest, the \( k \)-th bubble radius \( R_k \) (\( 1 \leq k \leq K \), \( K \) is the total number of bubbles in the cluster) is \( R_k = R_0 = 0.25 \) mm (i.e., all the bubbles are equal in size). At \( t = 0 \), the liquid pressure \( p_L \) instantly rises from 1 to 4 bar. As a result, the bubbles undergo a series of damped oscillations and go to a new equilibrium state corresponding to the liquid pressure \( p_L = 4 \) bar. The main attention is directed to studying the dependence of the dynamics of bubbles on their hydrodynamic interaction and position in the cluster. To this end, the dynamics of the central and peripheral bubbles in a cluster are compared among themselves and with the dynamics of a single bubble under similar conditions. The influence of the cluster structure is also analyzed. For this purpose, along with clusters consisting of bubbles located in the center and vertices of regular polyhedra, the bubble configurations with one of the bubbles in the center of a sphere and the others distributed stochastically inside it are also considered. In all the cases, the minimum distance between the centers of the bubbles does not exceed 5 mm.

![Figure 1. An example of a cluster of bubbles and some notations.](image)

A mathematical model is used, in which the dynamics of bubbles in a cluster is governed by second-order ordinary differential equations in the radii of the bubbles \( R_k \) and the position-vectors of their centers \( \mathbf{p}_k \) (figure 1). Those equations are a special case of the similar equations of [17] in that the bubbles are here assumed spherical. They are written as follows

\[
R_k \ddot{R}_k + \frac{3}{2} \dot{R}_k^2 - \frac{R_k^2}{4} - \frac{p_k - p_L}{\rho_L} + \frac{2\sigma}{\rho_L R_k} + \psi_{ok} + \Delta_k = \]

\[
= \sum_{j=1,j \neq k}^{K} \left[ \frac{\dot{B}_{0j}}{d_{kj}} - \frac{R_j^2 \dot{p}_{kj} \cdot (R_j \dot{p}_j + \dot{R}_j \mathbf{p}_j + 5\dot{R}_j \mathbf{p}_j)}{2d_{kj}^3} \right] + \sum_{l=1,l \neq j}^{K} \frac{3B_{jl}B_{0j} \mathbf{p}_{kj} \cdot \mathbf{p}_{lj}}{4d_{kj}^3d_{lj}^3} - \sum_{l=1,l \neq j}^{K} \frac{\left( R_j^3 B_{0j} \right) \mathbf{p}_{kj} \cdot \mathbf{p}_{lj}}{2d_{kj}^3d_{lj}^3}, \tag{1}
\]
Here the overdots and the primes mean differentiation with respect to time \( t \), \( B_{\text{ok}} = -\dot{R}_k^2 \dot{R}_k \), \( \mathbf{p}_k = x_k \mathbf{i} + y_k \mathbf{j} + z_k \mathbf{k} \), \( x_k, y_k, z_k \) are the Cartesian coordinates of the \( k \)-th bubble center with the unit vectors \( \mathbf{i}, \mathbf{j}, \mathbf{k} \), respectively, \( \mathbf{p}_{kj} = \mathbf{p}_k - \mathbf{p}_j \), \( d_{kj} = || \mathbf{p}_{kj} \| \) is the distance between the centers of the \( k \)-th and \( j \)-th bubbles, \( p_k \) is the pressure in the \( k \)-th bubble, \( \rho_L \) is the liquid density (\( \rho_L = 1156 \text{ kg/m}^3 \)), \( \sigma \) is the surface tension (\( \sigma = 0.07 \text{ N/m} \)), \( \psi_{\text{ok}}, \psi_{\text{ik}} \) are the corrections taking into account the influence of the liquid viscosity, \( \Delta_p \) is the correction for the influence of the liquid compressibility. Equations (1), (2) are valid for not too small distances between the bubbles, otherwise their deformations must be taken into account.

The pressure in the bubbles changes according to the adiabatic law

\[
P_k = \left( \rho_L + \frac{2\sigma}{R_0} \right) \left( \frac{R_k}{R_0} \right)^{3\gamma},
\]

where \( \gamma \) is the adiabatic exponent (\( \gamma = 1.4 \)).

The effects of the liquid viscosity and compressibility are assumed small and are described without taking into account the interaction between the bubbles, so that the corresponding corrections are determined by the following expressions

\[
\psi_{\text{ok}} = \frac{4\nu_L}{c_L} \frac{\dot{R}_k}{R_k}, \quad \psi_{\text{ik}} = \frac{18\nu_L}{c_L} \frac{\dot{\mathbf{p}}_k}{R_k},
\]

\[
\Delta_p = -\frac{\dot{R}_k}{c_L} \left( R_k \frac{\ddot{R}_k}{R_k} + \frac{\dot{R}_k^2}{2} + \frac{\Delta_p}{\rho_L} \right) - \frac{R_k}{c_L} \left( \frac{\dot{p}_k}{\rho_L} - \frac{4\nu_L}{c_L} \frac{\dot{R}_k}{R_k} \right),
\]

where \( \nu_L = \mu_L / \rho_L \) is the kinematic viscosity of the liquid (\( \mu_L = 0.011 \text{ Pa\cdot s} \)), \( c_L \) is the speed of sound in the liquid (\( c_L = 1500 \text{ m/s} \)).

Equations (1), (2) are solved numerically by the Runge-Kutta method with a variable integration step.

3. Results
Figure 2 characterizes the effect of the interaction between bubbles in a cluster on their dynamics under an instant increase in the liquid pressure in the case of the cluster formed by many bubbles, one of which is in the center of regular polyhedra and the other are at their vertices located on a sphere with a radius of 5 mm. It can be seen that for all the bubbles presented in this figure, the bubble pressure changes with time in the form of oscillations. In the case of a single bubble (a), the amplitude of these oscillations decreases monotonically. In contrast, for almost all bubbles in all clusters (except for the peripheral bubbles in case (e)), the bubble pressure oscillation amplitude decreases nonmonotonically. Moreover, in cases (c)-(e), its change for the central bubble begins with a phase of its significant (by a factor of two or more) increase, and only then the phase of its nonmonotonic decrease sets in. The nonmonotonicity in the change of the pressure oscillation amplitude in the central bubbles of the clusters is explained by the influence of the peripheral bubbles, and the nonmonotonic variation in the pressure oscillation amplitude in the peripheral bubbles is due to the influence of the central ones. With an increase in the number of bubbles at the periphery of the cluster, their influence on the central bubble increases, whereas the impact of the central bubble on the peripheral ones decreases. As a result, the change in the pressure oscillation amplitude becomes more and more nonmonotonic in the central bubble, and less and less nonmonotonic in the peripheral ones. In case (e),
the number of the peripheral bubbles is so large that their oscillations are nearly independent from the oscillations of the central bubble. Therefore, the decrease in the peripheral bubble pressure oscillation amplitude is monotonic in this case, as it is in the case of a single bubble.

![Figure 2](image_url)

**Figure 2.** Change in the pressure inside a single bubble (a) and inside the central bubbles of the clusters with the peripheral bubbles located 5 mm away from the central ones at the vertices of a tetrahedron (b), a hexahedron (c), an icosahedron (d), and a dodecahedron (e). The upper and lower blue lines on (b)–(e) are the envelopes of the peripheral bubble pressure maxima and minima, respectively. Figure (f) shows the time interval corresponding to the first collapse of the single bubble (bold line), as well as the central (solid lines) and peripheral (dashed lines) bubbles in clusters (b)–(e) (lines 1–4 represent clusters (b)–(e), respectively). To better visualize the cluster structure, the thin lines connecting the peripheral bubbles of the clusters on (b)–(e) show the edges of the corresponding regular polyhedra.

Figure 2f illustrates the effect of interaction between the bubbles during the first collapse of the bubbles. It can be seen that the interaction between the bubbles slows down the collapse rate. With an increase in the number of the peripheral bubbles, the deceleration increases. This slowdown also manifests itself in decreasing the bubble pressure oscillation frequency with an increase in the number of the peripheral bubbles. As the number of the bubbles at the cluster periphery rises, the maximum pressure in the central bubbles noticeably grows, whereas in the peripheral bubbles it remains practically at the level achieved in the case of a single bubble.

![Figure 3](image_url)

**Figure 3.** Change in the pressure inside the central bubbles of the clusters with the peripheral bubbles located at the vertices of one (a), two (b) and three (c) dodecahedra equally oriented and concentric with the central bubble (the vertices of the dodecahedra are on spheres with a radius of 5, 10 and 15 mm). The upper and lower blue lines are the envelopes of the peripheral bubble pressure maxima and minima, respectively. The insets show the first collapse of a single bubble (bold lines), as well as the central (solid lines) and peripheral (dashed lines) bubbles in the clusters.
Figure 3 characterizes the dependence of the dynamics of bubbles on their interaction in the case of a cluster formed by bubbles with the peripheral ones located at the vertices of one, two and three dodecahedra identically oriented relative to the x, y, z axes and concentric with the central bubble. The vertices of the dodecahedra are on spheres with a radius of 5, 10 and 15 mm. It is seen that as the number of the spherical layers with bubbles around the central bubble increases, the maximum amplitude of the central bubble pressure oscillation increases. In the case of the three layers, its value becomes higher than 360 bar, which is more than 16 times the maximum pressure in the case of a single bubble. It follows from the insets in figure 3 that with an increase in the number of the spherical layers of bubbles around the central one, the first collapse of the central bubble becomes more and more delayed, whereas the period of the first collapse of the peripheral bubbles remains practically unchanged. Along with that, during the first collapse the maximum pressure increases in the central bubble and decreases in the peripheral ones.

Figure 4. Change in the pressure inside the central bubbles of clusters with the other 12, 24 and 36 bubbles respectively located at the vertices of one (a), two (b) and three (c) icosahedra concentric with the central bubble (12 vertices of the icosahedra are on the spheres with a radius of 6, 11 and 16 mm). The insets show the pressure change in the central bubbles of the clusters in which one bubble is in the center of a spherical region with a radius of 6 mm (a), 11 mm (b) and 16 mm (c), whereas the other 12, 24 and 36 bubbles, respectively, are stochastically distributed inside the region at distances between the centers of the bubbles not exceeding 5 mm.

Figure 4 illustrates the dependence of the dynamics of the central bubble on the stochastic distribution of the other bubbles in a cluster. The curves in the insets of this figure are obtained by averaging the calculation results of 10 variants of the stochastic distribution. In doing so, the time axis is also converted based on averaging the time instants of the local maxima and minima of the bubble pressure oscillations. It is seen in figure 4c that the central bubble pressure maximum in the case of stochastic distribution of bubbles surrounding the central one is much less than that in the case of the same number of the surrounding bubbles located at the vertices of the three corresponding icosahedra (concentric with the central bubble and identically oriented relative to the x, y, z axes).

4. Conclusion
The dependence of the dynamics of spherical gas bubbles located in the center and vertices of regular polyhedra on their hydrodynamic interaction under a sharp liquid pressure increase is studied. The dynamics of the bubbles is governed by second-order ordinary differential equations in the radii of the bubbles and the position-vectors of their centers. In terms of the ratio of the radii of the bubbles to the distance between their centers, these equations are of the forth order of accuracy.

It is shown that under the considered conditions, the bubble pressure oscillation amplitude in the case of bubble clusters of the indicated structure decreases nonmonotonically. Moreover, with a sufficiently large number of the peripheral bubbles, the change in the amplitude of the central bubble pressure oscillations begins with a phase of its significant growth. With an increase in the number of the peripheral bubbles, the nonmonotonicity of the change in the pressure oscillation amplitude inside the central bubble increases whereas the nonmonotonicity inside the peripheral bubbles decreases. With increasing the number of the peripheral bubbles, the oscillation frequency decreases. It is found
that in the case of a cluster formed by bubbles with the central one surrounded by those located at the vertices of one, two and three dodecahedra (concentric with the central bubble and identically oriented relative to spatial coordinates), the maximum amplitude of the central bubble pressure oscillation increases with increasing the number of the spherical layers of bubbles around the central one.

The dependence of the dynamics of the central bubble in a cluster on the stochastic distribution of the other bubbles has also been considered. It is shown that the central bubble pressure maximum in the case of a stochastic distribution of the other bubbles is much less than it is in the case where the surrounding bubbles are located at the vertices of three icosahedra (concentric with the central bubble and identically oriented relative to spatial coordinates).

5. References
[1] Bjerknes V F K 1906 Field of Force (New York: Columbia Univ. Press)
[2] Aganin A A and Davletshin A I 2009 Simulation of interaction of gas bubbles in a liquid with allowing for their small asphericity Mathematical Models and Computer Simulations 21 6 pp 89–102
[3] Aganin A A, Davletshin A I and Toporkov D Yu 2014 Dynamics of a line of cavitation bubbles in an intense acoustic wave Computational Technologies 19 1 pp 3–19
[4] Aganin A A and Davletshin A I 2018 Influence of spatial position of gas bubbles in liquid on their joint dynamics Journal of Physics: Conference Series 1058 012067
[5] Kuznetsov G N and Shchekin I E 1973 Interaction of pulsating bubbles in a viscous liquid Ultrasonics 18 pp 466–9
[6] Aganin A A and Davletshin A I 2016 A refined model of spatial interaction of spherical gas bubbles Izvestia Ufimskogo Nauchnogo Tsentra RAN (Proceedings of the RAS Ufa Scientific Centre) 4 pp 9–13
[7] Doinikov A A 2004 Mathematical model for collective bubble dynamics in strong ultrasound fields Journal of the Acoustical Society of America 116 2 pp 821–7
[8] Aganin A A and Davletshin A I 2013 Interaction of spherical bubbles with centers located on the same line Mathematical Models and Computer Simulations 25 12 pp 3–18
[9] Doinikov A A 2001 Translational motion of two interacting bubbles in a strong acoustic field Phys. Rev. E 64 2 026301
[10] Gubaidullin A A and Gubkin A S 2013 Investigation of bubble cluster dynamics Tyumen State University Herald 7 pp 81–7
[11] Gubaidullin A A and Gubkin A S 2013 Behavior of bubbles in a cluster under acoustic influence Modern Science: Researches, Ideas, Results, Technologies 1 pp 363–7
[12] Gubaidullin A A and Gubkin A S 2015 Peculiarities of the dynamic behavior of bubbles in a cluster caused by their hydrodynamic interaction Thermophysics and Aeromechanics 22 4 pp 453–62
[13] Pearsall I S 1972 Cavitation (London: Mills and Boon Limited)
[14] Philipp A and Lauterborn W 1998 Cavitation erosion by single laser-produced bubbles J. Fluid Mech. 361 pp 75–116
[15] Taleyarkhan R P, West C D, Cho J S, Lahey R T (Jr), Nigmatulin R I and Block R C 2002 Evidence for nuclear emissions during acoustic cavitation Science 295 pp 1868–73
[16] Taleyarkhan R P, West C D, Lahey R T (Jr), Nigmatulin R I, Block R C and Xu Y 2006 Nuclear emissions during self-nucleated acoustic cavitation Phys. Rev. Let. 96 034301
[17] Aganin A A and Davletshin A I 2018 Equations of interaction of weakly non-spherical gas bubbles in liquid Lobachevskii Journal of Mathematics 39 8 pp 1047–52