New Index of CP Phase Effect and $\theta_{13}$ Screening in Long Baseline Neutrino Experiments

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Abstract

We introduce a new index of the leptonic CP phase dependence $I_{\text{CP}}$ and derive the maximal condition for this index in a simple and general form. $I_{\text{CP}} \simeq 100\%$ may be realized even in the JPARC experiment. In the case that the 1-3 mixing angle can be observed in the next generation reactor experiments, namely $\sin^2 2\theta_{13} > 0.01$, and nevertheless $\nu_e$ appearance signal cannot be observed in the JPARC experiment, we conclude that the CP phase $\delta$ becomes a value around 135° (45°) for $\Delta m^2_{31} > 0$ ($\Delta m^2_{31} < 0$) without depending the uncertainties of solar and atmospheric parameters.

1 Introduction

In future experiments, the determination of the leptonic CP phase $\delta$ is one of the most important aim in elementary particle physics. A lot of effort have been dedicated both from theoretical and experimental point of view in order to attain this aim, see [1, 2, 3, 4] and the references therein.

The CP asymmetry, $A_{\text{CP}} = (P_{\mu e} - \bar{P}_{\mu e})/(P_{\mu e} + \bar{P}_{\mu e})$, is widely used as the index of the CP phase dependence. Here, $P_{\mu e}$ and $\bar{P}_{\mu e}$ are the oscillation probabilities for the transition $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, respectively. However, this index has to be improved on the following three points. The first one is that the fake CP violation due to matter effect [5] cannot be separated clearly in $A_{\text{CP}}$. The second one is that only the effect originated from $\sin \delta$ is included in $A_{\text{CP}}$. The third one is that we need to observe the channels both in neutrino and anti-neutrino for calculating $A_{\text{CP}}$.

In this letter, we introduce a new index of the CP phase dependence improved on the above three points. In arbitrary matter profile, we derive the maximal condition of this index exactly for $\nu_\mu \rightarrow \nu_e$ transition. This index can be extended to the case for other channels and other parameters [6]. We can simply find the situation that the CP phase effect becomes large by using this index. As an example, we demonstrate the following interesting phenomena. It is commonly expected that a large $\nu_e$ appearance signal is observed in the JPARC experiment [7] if the 1-3 mixing angle $\theta_{13}$ is relatively large $\sin^2 2\theta_{13} > 0.01$ and is determined by the next generation reactor experiments like the Double Chooz experiment [8] and the KASKA experiment [9]. However, there is the possibility that $\nu_e$ appearance signal cannot be observed in certain mass squared differences and mixing angles even the case for large $\theta_{13}$. We call this “$\theta_{13}$ screening”. This occurs due to the almost complete cancellation of the large $\theta_{13}$ effect by the CP phase effect. If the background can be estimated precisely, we can obtain the information on the CP phase through the $\theta_{13}$ screening. This means that we cannot neglect the CP phase effect, which is actually neglected in many investigations as the first approximation.

2 General Formulation for Maximal CP Phase Effect

At first, we write the Hamiltonian in matter [10] as

$$H = O_{23} \Gamma H^\dagger O^T_{23}$$

(1)
by factoring out the 2-3 mixing angle and the CP phase, where $O_{23}$ is the rotation matrix in the 2-3 generations and $\Gamma$ is the phase matrix defined by $\Gamma = \text{diag}(1,1,e^{i\delta})$. The reduced Hamiltonian $H'$ is given by

$$H' = O_{13}O_{12}\text{diag}(0,\Delta_{21},\Delta_{31})O^T_{12}O^T_{13} + \text{diag}(a,0,0),$$

where $\Delta_{ij} = \Delta m^2_{ij}/(2E) = (m^2_i - m^2_j)/(2E)$, $a = \sqrt{2}G_FN_e$ is the Fermi constant, $N_e$ is the electron number density, $E$ is neutrino energy and $m_i$ is the mass of $\nu_i$. The oscillation probability for $\nu_\mu \to \nu_e$ is proportional to the $\cos \delta$ and $\sin \delta$ in arbitrary matter profile [11] and can be expressed as

$$P_{\mu e} = A \cos \delta + B \sin \delta + C = \sqrt{A^2 + B^2} \sin(\delta + \alpha) + C.$$  (3)

Here $A$, $B$ and $C$ are determined by parameters other than $\delta$ and are calculated by

$$A = 2\text{Re}[S'_{\mu e}S'_{\tau e}]c_{23}s_{23},$$

$$B = 2\text{Im}[S'_{\mu e}S'_{\tau e}]c_{23}s_{23},$$

$$C = |S'_{\mu e}|^2c_{23}^2 + |S'_{\tau e}|^2s_{23}^2,$$  (6)

where $S'_{\alpha\beta} = [\exp(-iH'L)]_{\alpha\beta}$, $\tan \alpha = A/B$ and $\sqrt{A^2 + B^2} \sin(\delta + \alpha)$ is the CP dependent term and $C$ is the CP independent term.

Next, let us introduce a new index of the CP phase dependence $I_{\text{CP}}$. Suppose that $P_{\text{max}}$ and $P_{\text{min}}$ as the maximal and minimal values when $\delta$ changes from $0^\circ$ to $360^\circ$. Then, we define $I_{\text{CP}}$ as

$$I_{\text{CP}} = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}} = \frac{\sqrt{A^2 + B^2}}{C}.$$  (7)

Namely, the new index is expressed by the ratio of the coefficient of the CP dependent term to the CP independent term. $I_{\text{CP}}$ is a useful tool to explore where is the most effective region in parameter spaces to extract the CP effect from long baseline experiments although $I_{\text{CP}}$ is not an observable. $A_{\text{CP}}$ is also similar one and this is an observable. However $A_{\text{CP}}$ have to be expressed by $\delta$ though $\delta$ is still unknown parameter so that $A_{\text{CP}}$ seems not to be so good index to make the exploration. On the other hand, $I_{\text{CP}}$ is calculated without using $\delta$. This is the main difference between these two indices and it is more effective to use $I_{\text{CP}}$.

![Fig.1. Region with large $I_{\text{CP}}$. We write the lines for $I_{\text{CP}} = 95\%, 90\%, 85\%, 80\%$. $I_{\text{CP}}$ takes the large value in black region. Left and right panels are drawn with two different sets of parameters.](image)
In this letter, we show the region for taking the large value of $I_{CP}$ in the $E$-$L$ plane. In particular, we investigate how the region changes by the uncertainties of atmospheric and solar parameters if $\theta_{13}$ is determined by the future reactor experiment. In fig.1(left), we choose the parameters as $\Delta m^2_{21}=7.9 \times 10^{-5} eV^2$, $\sin^2 \theta_{12}=0.31$, $\Delta m^2_{31}=2.2 \times 10^{-3} eV^2$, $\sin^2 \theta_{23}=0.50$, which are the best-fit values in the present experiments [12]. We also use $\sin^2 2\theta_{13}=0.1$, which is near on the upper bound of the CHOOZ experiment [13]. On the other hand, in fig.1(right), we choose $\Delta m^2_{21}=8.9 \times 10^{-5} eV^2$, $\sin^2 \theta_{12}=0.40$, $\Delta m^2_{31}=1.4 \times 10^{-3} eV^2$, $\sin^2 \theta_{23}=0.34$, $\sin^2 2\theta_{13}=0.01$, which are within the 3-\sigma allowed region respectively [12]. We use the $\rho = 2.8 g/cm^3$ as the matter density. Fig.1(left) shows that $I_{CP}$ takes a large value along the line $L/E = const.$ in low energy region. In contrary, fig.1(right) shows that there is a wide region with almost $I_{CP} \simeq 100\%$. The region appears in the baseline shorter than 3000km and surprisingly it is independent of neutrino energy. So, what is the condition for realizing the maximal $I_{CP}$?

If we notice (4)-(6), we find that the relation between the denominator and the numerator of $I_{\ell}$ is a wide region with almost $100\%$ is realized when this condition is satisfied. Below, let us investigate the maximal condition $CP$-dependent term cannot be larger than the CP independent term” by using the approximate formula [14]. In this letter, we define $I_{CP}$ without depending on the unknown CP phase and derive the exact inequality in arbitrary matter profile for the first time. Furthermore, we consider the condition that both sides become equal in this inequality. This condition is given by

$$|S'_{\mu e}|c_{23} = |S'_{\tau e}|s_{23} \quad (\text{maximal condition}),$$

and $I_{CP} = 100\%$ is realized when this condition is satisfied. Below, let us investigate the maximal condition [9] in detail by using the approximate formula including the non-perturbative effect of small parameters $\theta_{13}$ and $\Delta m^2_{21}$, in constant matter profile [15]. The reduced amplitudes in the maximal condition are approximated by

$$S'_{\mu e} \simeq \lim_{s_{13} \to 0} \left[ \exp(-iH'L) \right]_{\mu e}, \quad (10)$$

$$S'_{\tau e} \simeq \lim_{\Delta_{21} \to 0} \left[ \exp(-iH'L) \right]_{\tau e}. \quad (11)$$

The maximal condition is rewritten as

$$\frac{\Delta_{23} \sin 2\theta_{12}}{\Delta_{\ell}} c_{23} \sin \left( \frac{\Delta_{\ell} L}{2} \right) = \left| \frac{\Delta_{31} \sin 2\theta_{13}}{\Delta_{h}} s_{23} \sin \left( \frac{\Delta_{h} L}{2} \right) \right| \quad (12)$$

in this approximation, where $\Delta_{h} = \Delta m^2_{h}/(2E)$, $\Delta_{\ell} = \Delta m^2_{\ell}/(2E)$ and $\Delta m^2_\ell$ ($\Delta m^2_{h}$) stands for the effective mass squared differences corresponding to high (low) energy MSW effect. The concrete expression for $\Delta_{h}$ is given by

$$\Delta_{h} = \sqrt{(\Delta_{31} \cos 2\theta_{13} - a)^2 + \Delta_{31}^2 \sin^2 2\theta_{13}}. \quad (13)$$

We obtain $\Delta_{\ell}$ by the replacements $\Delta_{h} \rightarrow \Delta_{\ell}$, $\Delta_{31} \rightarrow \Delta_{21}$, $\theta_{13} \rightarrow \theta_{12}$. The energy dependence of $\Delta m^2_{h}$ and $\Delta m^2_{\ell}$ is mild and roughly speaking we regard these as constant. At this time, (12) becomes the equation for $L/E$ and the region for large $I_{CP}$ appears along the line $L/E = const.$ in fig.1(left). On the other hand, there appears no $L/E$ dependence at short baseline in fig.1(right). This is interpreted as follows. In the case of small $x$, the approximation $\sin x \simeq x$ becomes good and the $E$ and $L$ dependencies of both sides vanish and the maximal condition can be simplified as

$$\Delta m^2_{31} \sin 2\theta_{12} c_{23} = |\Delta m^2_{31}| \sin 2\theta_{13} s_{23} \quad (14)$$
The inequality for $L$ is obtained by $\Delta L/2 \ll 1$ as

$$ L \ll \frac{2}{a} = \frac{2}{\sqrt{2}G_F \rho Y_e} \simeq 3500 \text{[km]}, \quad (15) $$

where we use $\rho = 2.8 \text{g/cm}^3$ as the matter density and $Y_e = 0.494$ as the electron fraction. The inequality for $E$ is also obtained by $\Delta h L/2 \ll 1$ as

$$ E \gg \frac{\Delta m^2_{31} L}{4} \simeq \frac{L}{500} \text{[GeV]}, \quad (16) $$

where the baseline length $L$ is measured in the unit of km. The region for satisfying these conditions coincides with that for taking large $I_{CP}$ in fig.1(right).

Next, let us investigate the condition (14). That is rewritten as

$$ \sin 2\theta_{13} = 0.036 \times \frac{\Delta m^2_{21}}{8 \times 10^{-5}} \sin 2\theta_{12} \frac{1.0}{\tan \theta_{23}} \frac{2 \times 10^{-3}}{|\Delta m^2_{31}|}. \quad (17) $$

In the case that the parameters except $\theta_{13}$ have their best-fit values in the present experiments, the value of $\theta_{13}$ satisfying (17) becomes small compared with the bound $\sin 2\theta_{13} > 0.1$ of next generation reactor experiments. However, it is possible to satisfy Eq. (14) if the parameters possess values, which slightly deviate from those of the best-fit. We chose the parameters for satisfying Eq. (14) in fig.1(right). The readers may think that such a situation with large $I_{CP}$ independent of $E$ and $L$ is extraordinary and it is not likely realized. However, if we write the ratio of both sides of (14) as

$$ r = \frac{\Delta m^2_{21} c_{23} \sin 2\theta_{12}}{|\Delta m^2_{31}| s_{23} \sin 2\theta_{13}}, \quad (18) $$

$I_{CP}$ is calculated by using $r$ as

$$ I_{CP} = \frac{2r}{1 + r^2}, \quad (19) $$

and we found that the large CP phase effect is realized as $I_{CP} = 97.5\%(88\%)$ with $r = 0.8(0.6)$ for example. Thus, the decrease of $I_{CP}$ according to the difference from Eq. (14) is comparatively mild and the region with large $I_{CP}$ becomes wider than expected. If we describe for the reference, $r = 0.10(0.87)$ and $I_{CP} \approx 20\%(99\%)$ in fig.1(left) (fig.1(right)). The discovery of such a situation, where the maximal condition is satisfied independently of $E$ and $L$ is one of our main results in this letter. This may be realized in the JPARC experiment under the condition of certain parameter combinations, and is very important to analyze the experimental result.

### 3 $\theta_{13}$ Screening in JPARC Experiment

We found that the screening condition may be realized in the JPARC experiment. Next, we consider the case such that $I_{CP}$ takes a large value and the CP phase effect contributes to the probability destructively. It is commonly expected that a large $\nu_e$ appearance signal is observed in the JPARC experiment, if $\theta_{13}$ is large and will be confirmed in next generation reactor experiments. We point out here that the probability can be zero over the entire region in the JPARC experiment due to the cancellation of $\theta_{13}$ effect by the CP phase effect. We use the same parameters as in fig.1(right), where Eq. (14) holds. Fig.2 shows the oscillation probabilities with $\delta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$ in the energy range $0.4$-1.2 GeV of the JPARC experiment. In fig.2, one can see that the CP dependent term has the same sign as the CP independent
term when $\delta = 315^\circ$. They interfere constructively with each other and generate the large probability. On the other hand, they have opposite sign and almost completely cancel each other when $\delta = 135^\circ$. As a result, the probability for $\nu_\mu \to \nu_e$ transition is strongly suppressed and we call this phenomena "$\theta_{13}$ screening".

Fig. 2. CP dependence of $P_{\mu e}$ under the maximal condition. Four lines stand for the oscillation probabilities with $\delta = 45^\circ, 135^\circ, 225^\circ$ and $315^\circ$, respectively.

Let us calculate the value of $\delta$ for the $\theta_{13}$ screening. At first, the value of $\alpha$ is determined by $\sin \alpha = A/\sqrt{A^2 + B^2}$, $\cos \alpha = B/\sqrt{A^2 + B^2}$. This leads to

$$\tan \alpha = \frac{A}{B} \simeq \frac{-1}{\tan \left( \frac{\Delta m^2_{12} L}{4E} \right)} = \tan \left( \frac{\pi}{2} + \frac{\Delta m^2_{32} L}{4E} \right).$$

Substituting $\Delta m^2_{12} = 1.4 \times 10^{-3}\text{eV}^2$, $L = 295\text{km}$ and $E = 0.7\text{GeV}$ into this relation, we obtain $\alpha \simeq 135^\circ$. If $\delta \simeq 135^\circ$, we obtain $\sin(\delta + \alpha) = \sin 270^\circ = -1$ and as a result $C$ and $\sqrt{A^2 + B^2}$ cancel each other from eq. (20). As seen from (20), the value of $\delta$ for the $\theta_{13}$ screening changes with $E$ and $L$ in general. However, $\sin(\delta + \alpha)$ takes a local minimum around $\delta + \alpha = 270^\circ$ and the magnitude of the CP dependent term changes at most 10% even if $\alpha$ changes $30^\circ$ around this minimum. This is the reason for $P_{\mu e} \simeq 0$ in a wide energy region.

Here, we discuss the relation between the problems of parameter degeneracy and the $\theta_{13}$ screening. In this letter, we investigated the case for large $\theta_{13}$, which will be confirmed by next generation reactor experiments. Namely, we have considered the case of $\theta_{13}-\delta$ ambiguity free. Next, let us consider the $\theta_{23}$ ambiguity. In order for the $\theta_{13}$ screening to be realized, the parameters should satisfy the relation (14). Under this condition, only a small $\theta_{23}$ near the present lower bound is permitted, namely $\theta_{23} \simeq 35^\circ$, which corresponds to $\sin^2 \theta_{23} = 0.34$. Therefore, if the $\theta_{13}$ screening is observed, $\theta_{23}$ degeneracy is solved. Finally, let us consider the $\Delta m^2_{31}$ sign ambiguity. In the case for also $\Delta m^2_{31} < 0$, the maximal condition is almost independent of $E$ and $L$ at the baseline of the JPARC experiment $L = 295\text{ km}$. The sign of $A$ changes and becomes negative according to the replacement of the sign of $\Delta m^2_{31}$. On the other hand, the sign of $B$ does not change and is negative. This leads to $\alpha = 225^\circ$ and the $\theta_{13}$ screening occurs around $\delta = 45^\circ$ for $\Delta m^2_{31} < 0$.

Next, in fig. 3, we numerically estimate $\nu_e$ appearance signal, namely the total number of events distinct from the background noise, obtained in the JPARC-SK experiment within the energy range $E = 0.4\text{ to }1.2\text{GeV}$ when the CP phase $\delta$ changes from $0^\circ$ to $360^\circ$. In top and down figures, we use $\Delta m^2_{31} = 1.4 \times 10^{-3}\text{eV}^2$ and $3.0 \times 10^{-3}\text{eV}^2$ respectively. In left and right figures, we use $\sin^2 \theta_{23} = 0.34$ and 0.66. Other parameters are taken as in fig.1 (right). We assume here only the neutrino beam data as realized in the JPARC-SK for five years. We also give the statistical error within the 1-$\sigma$ level in fig.3. We use the globes software to perform the numerical calculation.
As expected from the oscillation probability in fig. 2, $\nu_\mu$ appearance signal will become almost zero around $\delta = 135^\circ$ even during the five years data acquisition in the SK experiment in the top-left of fig. 3. Note that this occurs only when the maximal condition $[\ref{eq:14}]$ is satisfied, namely $I_{CP} \simeq 100\%$. Other panels in fig. 3 show that the minimal value of $\nu_\mu$ appearance signal rise and is a little different from zero because $[\ref{eq:14}]$ is not satisfied so precisely. We obtain the similar results in the case that $\Delta m_{31}^2$ or $\sin^2 \theta_{23}$ changes within the allowed region obtained from solar and the KamLAND experiments. Let us here illustrate how the value of $\delta$ is constrained by the experiment. Below, suppose that the atmospheric parameters have some uncertainties as $\Delta m_{31}^2 = 1.4 \times 10^{-3} \text{eV}^2$ and $\sin^2 \theta_{23} = 0.34-0.66$, while the solar parameters $\Delta m_{21}^2$ and $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ are fixed for simplicity. For example, if 15 appearance signals are observed in the experiment, we obtain the allowed region of $\delta$ as $0^\circ$-$50^\circ$ or $130^\circ$-$220^\circ$ or $240^\circ$-$360^\circ$ from four figures in fig. 3. Namely, combined range of all allowed region is totally $260^\circ$. Next, we consider the case that no appearance signal is obtained. This gives the allowed region of $\delta$ as $110^\circ$-$160^\circ$. Namely, combined range of all allowed region is totally $50^\circ$. Thus, we found from above rough estimation that the stronger constraint is obtained in the case of $\theta_{13}$ screening even if the uncertainties of parameters except for $\delta$ are considered. In other words, we can also obtain the information on the atmospheric and solar parameters. Although the precise estimation of the background is a difficult problem, it is interesting to have strong constraint for not only the value of the CP phase but also other parameters like $\Delta m_{31}^2$ and $\sin^2 \theta_{23}$ when the $\nu_\mu$ appearance signal is not observed and the 1-3 mixing angle has a comparatively large value $\sin^2 2 \theta_{13} > 0.01$.

4 Summary and Discussion

In summary, we introduced a new index of the CP phase dependence $I_{CP}$ and derived their maximal condition in a simple and general form. In particular, we showed that $I_{CP} \simeq 100\%$ is realized in a rather wide region in the $E$-$L$ plane at certain values of parameters. In the case that $\theta_{13}$ has a comparatively large value $\sin^2 2 \theta_{13} = 0.01$, (namely $\theta_{13}$ will be observed in next generation reactor experiments) nevertheless we cannot observe $\nu_\mu$ appearance signal in the JPARC experiment, we obtain the information on the CP phase as $\delta \simeq 135^\circ$ ($\delta \simeq 45^\circ$) for $\Delta m_{31}^2 > 0$ ($\Delta m_{31}^2 < 0$) without depending on the uncertainties of other
parameters. Also for $\sin^2 2\theta_{13} < 0.01$, there is a possibility that the $\theta_{13}$ screening will occur. In this case, we need to consider the reason for the absence of $\nu_e$ appearance signal in the JPARC experiment more carefully, in order to understand whether $\theta_{13}$ is small or the $\theta_{13}$ effect is canceled out by the CP phase effect. We also note that the $\theta_{13}$ screening may be realized for not only $\nu_\mu \rightarrow \nu_e$ oscillation in super-beam experiments but also $\nu_e \rightarrow \nu_\tau$ oscillation in neutrino factory experiments. We can also use the zero probability in the $\theta_{13}$ screening to explore new physics like non-standard interaction.

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