The Via Lactea INCITE Simulation: Galactic Dark Matter Substructure at High Resolution

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Abstract. It is a clear unique prediction of the cold dark matter paradigm of cosmological structure formation that galaxies form hierarchically and are embedded in massive, extended dark halos teeming with self-bound substructure or "subhalos". The amount and spatial distribution of subhalos around their host provide unique information and clues on the galaxy assembly process and the nature of the dark matter. Here we present results from the \textit{Via Lactea} INCITE simulation, a one billion particle, one million cpu-hour simulation of the formation and evolution of a Galactic dark matter halo and its substructure population.

1. Introduction

According to the standard model of cosmology our Universe is made up mostly of mysterious dark energy (70\%) and cold dark matter (CDM, 25\%). Ordinary matter (baryons) makes up only 5\% of the mass density in the Universe today. The dark energy component is homogeneous and structure formation is dominated by the CDM component. In the last few years cosmological N-body simulations have successfully uncovered how the cold dark matter distribution evolves from almost homogeneous initial conditions near the Big Bang into the present highly clustered state \cite{1,2}. CDM halos are built up hierarchically in a long series of mergers of smaller halos. Early low resolution simulations and simple analytical models found that the end products are smooth, triaxial halos. Higher resolution cosmological N-body simulations later modified this simplistic picture. The merging of progenitors is not always complete: the cores of accreted halos often survive as gravitationally bound subhalos orbiting within a larger host system. CDM halos are not smooth, they exhibit a wealth of substructure on all resolved mass scales \cite{3}. Subhalos are now believed to host cluster galaxies and the satellite galaxies around the Milky Way.

In 2007 we received an Innovative and Novel Computational Impact on Theory and Experiment (INCITE) grant for one million cpu-hours on Oak Ridge National Laboratory’s \textit{Jaguar} Cray XT3 supercomputer, to run a one billion particle simulation of the formation and evolution of a Milky Way scale dark matter halo in an expanding cosmological volume. The resulting simulation, dubbed \textit{Via Lactea}\textsuperscript{\dagger}, has resolved the subhalo population at unprecedented detail and closer to the host halo’s center than ever before. In the following we describe some

\textsuperscript{\dagger}Actually this simulation is the second in a series of simulations of ever-increasing resolution and is referred to in the literature as \textit{Via Lactea II}.
of the technical details that lie behind this computational effort, and present some of the first scientific results that have emerged from this study.

2. Technical Details
Measurements of the cosmic microwave background by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [4] have given us an exquisite view of the state of the universe a mere \(\sim 100,000\) years after the big bang. At this time the universe was very simple: a complete statistical description of the primordial temperature fluctuations observed by WMAP requires only six parameters. This simple statistical description of the early universe allows us to generate an initial particle distribution at redshift \(\sim 100\) (only 15 million years after the Big Bang) with small linear density perturbations matching the WMAP measurements. Our simulation then follows the gravitational collapse of these fluctuations into the highly non-linear clustering regime today.

2.1. The N-body technique
The dark matter density field is sampled using a finite number \(N\) of Lagrangian tracer particles, whose evolution is governed by Newton’s equations of motion in coordinates co-moving with the expansion of the universe [5]:

\[
\frac{d\vec{x}}{dt} = \vec{v}
\]  
\[
\frac{d\vec{v}}{dt} + 2H(a)\vec{v} = -\frac{1}{a^2}\nabla \phi. 
\]  

Here \(a\) is the expansion scale factor, \(H(a) = \dot{a}/a\) the Hubble constant, and \(\phi\) a modified gravitational potential

\[
\phi = \Phi + \frac{1}{2}a\ddot{a}x^2, 
\]  

whose field equation is given by

\[
\nabla^2 \phi = 4\pi G\rho a^2 - \Lambda a^2 + 3a\ddot{a}, 
\]  

where we are allowing for the contribution of a vacuum energy of density \(\rho_v = \Lambda/(8\pi G)\). Using the 2\textsuperscript{nd} Friedman equation

\[
\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G\rho_b(t) + \frac{\Lambda}{3}, 
\]  

equation \(4\) reduces to

\[
\nabla^2 \phi = 4\pi G(\rho - \rho_b)a^2. 
\]  

Because the effective gravitational potential is determined by the density contrast \((\rho - \rho_b)\), the computational error on the gravitational potential is set by how accurately the density field is sampled. A larger \(N\) will thus result in a more faithful representation of the dark matter dynamics, and this has been the driving force leading to ever increasing problem sizes and the use of powerful supercomputers for their solution.

2.2. PKDGRAV, a hierarchical tree code
PKDGRAV [6], the code that we used for the Via Lactea simulation, is an example of a hierarchical tree code, in which the particles are grouped together hierarchically and the gravitational force on a particle from each group is approximated by a multipole expansion up to low order. The hierarchical grouping is realized in a tree structure, in which the whole computational domain is split up into ever smaller sub-regions containing successively smaller numbers of particles, down to the leaf nodes of the tree which contain only a small number of particles. The calculation
of the gravitational force felt by each particle proceeds by “walking” the tree, starting with the root node that covers the entire computational domain. A cell-opening criterion is applied to each node to determine whether to use its multipole approximation, ending the descent along this branch, or if the cells needs to be “opened” and its daughter nodes examined.

The use of such a hierarchical multipole expansion requires only $\log N$ interactions per particle, much less than the $N - 1$ separate force calculations required with direct-summation, and this allows much larger particle numbers to be used. The accuracy of the force calculation is determined by the order to which the multipole expansion is carried out and by how many nodes it is applied to. Increasing the order of the expansion allows greater accuracy, but comes at an additional computation cost. **PKDGRAV** extends the multipole expansion up to fourth order.

**PKDGRAV** employs a spatially balanced binary tree, a relative of the particle balanced k-d tree. Starting with the root node, every node is recursively split up into two rectangular pieces of equal volume by bisecting the tight bounding box containing all particles in the node. This splitting continues down to the leaf nodes, or “buckets”, which are allowed to contain at most 16 particles. Such a tree has been shown to be asymptotically superior to density based decompositions like the k-d tree [7].

The cell opening criterion applied by **PKDGRAV** is

$$r_{\text{open}} = \frac{2}{\sqrt{3}} \frac{b_{\text{max}}}{\theta},$$

where $b_{\text{max}}$ is the maximum distance from any point in the cell to the cell’s center of mass, and $\theta$ is a user-specified accuracy parameter typically set to $\theta = 0.7$ for $z < 2$ and $\theta = 0.5$ at higher redshifts. If during the walk a cell is deemed sufficiently far from bucket $B_i$ to allow its multipole approximation, then it is added to $B_i$’s list of particle-multipole interactions. Alternatively, if a particular branch is followed all the way down to its leaf node, then all the particles in that bucket are added to $B_i$’s particle-particle interaction list.

In order to avoid unphysical two-body collisions the potential is softened for small separations, $|\vec{x}_i - \vec{x}_j| < \epsilon$. This is accomplished by using a single particle density distribution that is given by a Dirac $\delta$-function convolved with a softening kernel, here taken to be a compensating $K_1$ kernel [8]. At separations greater than about $2\epsilon$ the potential becomes exactly Newtonian.

### 2.3. A new dynamical time step criterion

An accurate determination of the gravitational potential is a necessary, but not sufficient, requirement for a reliable solution to the equations of motion of dark matter. Of equal importance is the method used to integrate the equations and the choice of time step. Cosmological simulations typically exhibit a dynamic range of densities spanning many orders of magnitude. Denser particle distributions require shorter time steps for a given accuracy, but using a global time step set by the densest regions in the simulations becomes computationally impractical, since the majority of the particles reside in much lower density regions. A more efficient use of computational resources can be achieved by the use of individual time steps that adapt to the local density. **PKDGRAV** employs an adaptive leapfrog integrator, which allows such adjustable time steps. The time steps are quantized in powers of two of the basic time step $T_0$.

A widely used time step criterion is based on a rather ad-hoc combination of the force softening and the acceleration a given particle feels, $\Delta T \leq 0.2 \sqrt{\frac{|a|}{|\vec{a}|}}$. This criterion results in a very tight time step distribution and has been shown [9] to produce accurate results at all but the very highest densities. However, this criterion is not directly related to the local dynamical time, and it has been shown [10] that for very high dynamic ranges $(\gtrsim 10^5)$ it is not quite adaptive enough, resulting in overly conservative time steps in the outer low density regions and insufficiently short time steps in the very centers of dark matter halos. Instead Zemp et al. [10]
advocate a time step criterion that is directly set by the local dynamical time a particle feels,

$$\Delta T \leq \eta \frac{1}{\sqrt{G\rho_{\text{enc}}(r)}},$$

(8)

where $\rho_{\text{enc}}$ is the enclosed density from the particle’s location to the dominant dynamical structure, and $\eta$ is typically set to $0.02 - 0.03$. The challenge is to quickly determine the dominant structure governing the orbit of a given particle, and Zemp et al. [10] have achieved this by cleverly making use of the hierarchical tree structure used for the force calculation. Figure 1 (taken from [10]) shows a comparison of the time step distribution using the old and new time step criteria, and reveals that the new time step is more adaptive, with smaller time steps close to the center and larger one in the outer regions. The most recent version of PKDGRAV contains an implementation of this new algorithm, and it allowed our Via Lactea simulation to push to unprecedented central densities at a manageable computational cost.

An unfortunate side effect of the use of individual time steps is that it destroys the symplectic nature of the integrator. This would be highly undesirable for any long term integration of planetary orbits, for example. In cosmological simulations, however, dark matter particles typically complete a comparatively much smaller number of orbits over a Hubble time, and so any secular error is not likely to grow significantly during the simulation. In this case the computational advantage of using individual and adaptive time steps outweighs the negative side-effects.
Table 1. Simulation Parameters

| $L_{\text{box}}$ (Mpc) | $\epsilon$ (pc) | $z_i$ | $N_{\text{hires}}$ | $M_{\text{hires}}$ ($\text{M}_\odot$) | $r_{200}$ (kpc) | $M_{200}$ ($\text{M}_\odot$) | $V_{\text{max}}$ (km s$^{-1}$) | $r_{V_{\text{max}}}$ (kpc) | $N_{\text{sub}}$ |
|------------------------|----------------|------|------------------|--------------------------|--------------|------------------|-----------------|-----------------|---------|
| 40.0                   | 40.0           | 104  | $1.05 \times 10^9$ | $4.10 \times 10^3$       | 402          | $1.93 \times 10^{12}$ | 201             | 60.0            | 53,653  |

Box size $L_{\text{box}}$, (spline) softening length $\epsilon$, initial redshift $z_i$, number $N_{\text{hires}}$ and mass $M_{\text{hires}}$ of high resolution dark matter particles, host halo $r_{200}$ (radius enclosing 200 times the mean density), $M_{200}$ (mass within $r_{200}$), $V_{\text{max}}$ (the peak of the circular velocity curve, $v_{\text{circ}} = \sqrt{GM(< r)/r}$) and $r_{V_{\text{max}}}$ (the radius at which $V_{\text{max}}$ occurs), and number of subhalos within $r_{200}$ $N_{\text{sub}}$ at $z = 0$. Force softening lengths $\epsilon$ are constant in physical units back to $z = 9$ and constant in co-moving units before.

2.4. Multi-mass initial conditions

The Via Lactea simulation makes use of multi-mass initial conditions [11, 9], which allow the halo of interest to be resolved with very high resolution, while at the same time allowing for the large scale cosmological tidal field to be represented at much lower resolution. The overall computational volume is $(40 \text{ Mpc})^3$, but at the end of the simulation 95% of all high resolution particles are found within the inner $(2 \text{ Mpc})^3$. In total the Via Lactea simulation consists of a hierarchy of three particle masses: $(16,689,261, 28,645,888, 1,048,772,608)$ particles with masses $(32,768, 64, 1)$ times the high resolution particle mass of $M_{\text{hires}} = 4.10 \times 10^3 \text{ M}_\odot$. The remaining parameters of the Via Lactea simulation are summarized in Table 1.

3. Results

The main scientific driver for our Via Lactea simulation is to better characterize the dark matter subhalo population of a Milky-Way-scale halo. A visual impression of the enormous abundance of subhalos is given by Figure 2, which shows a density squared projection of the central $(566 \text{kpc})^3$ cube. Projecting density squared enhances the contrast between individual subhalos and the smooth background. The two insets show progressive zooms into a region of interest. The deepest zoom is a projection of a $(40 \text{kpc})^3$ cube centered on subhalo with $V_{\text{max}} = 21.4 \text{ km s}^{-1}$ and a tidal radius of 46.8 kpc. The high resolution of Via Lactea is able to resolve the second level of the subhalo hierarchy – subhalos of a subhalo.

The overall picture is reminiscent of the “turtles all the way down” image. In fact, in our simulation the clumpiness extends over six orders of magnitude in mass down to our subhalo resolution limit of $\sim 10^5 \text{ M}_\odot$. The theoretical expectation is that this clumpiness continues even further, all the way down to the cut-off in the density fluctuation power spectrum set by the finite velocity dispersion (or temperature) of the cold dark matter particle [12, 13], some 10 to 15 orders of magnitude below the scales that we currently resolve. An important caveat here is that our simulation does not include the effects of baryons. Interactions with stars or giant molecular clouds in the disk of the Milky Way could destroy some fraction of the subhalos passing through the center. Such baryonic affects are unlikely to affect the subhalo population as whole, however, since most of the subhalos are not on orbits that carry them through the central disk.

3.1. The dark matter subhalo population

The density profiles of isolated dark matter halos are known to follow a common radial dependence, falling off as $r^{-3}$ in the outer regions, becoming shallower towards the center, and

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3One megaparsec (Mpc) is approximately 3 million light years.

* $V_{\text{max}}$ is the peak of the circular velocity curve ($v_{\text{circ}} = \sqrt{GM(< r)/r}$) and is used as a proxy for mass.
Finally approaching a central cusp of $r^{-\gamma}$, with the exact value of $\gamma$ being somewhat uncertain, but less than 1.5 \cite{14, 15, 16}. The very large number of particles in the *Via Lactea* simulation and the extremely accurate integration of their orbits has allowed a precise determination of the density profile within the inner kpc of the Galactic halo and its satellite halos, see the left panel of Figure 3. The host halo density profile is best fit with a central cusp of slope $\gamma = 1.24$, and we find that the inner profiles of subhalos are also consistent with cusps, with slopes scattered around $\gamma = 1.2$. At larger radii the subhalo density profiles generally fall off faster than the host halo (see inset). The explanation is simply that subhalo density profiles are modified by tidal mass loss, which strips material from the outside in, but does not affect the inner cusp structure.
Overall *Via Lactea* predicts a remarkably self-similar pattern of dark matter clustering properties. The right panel of Figure 3 shows that subhalos appear to be scaled down versions of their host halo not just in the inner mass distribution, but also in terms of their relative substructure abundance. The overall subhalo abundance as a function of mass, as measured by the cumulative $V_{\text{max}}$ function, is well described by a single power law

$$N(> V_{\text{max}}) \propto (V_{\text{max}}/V_{\text{max, host}})^{-3},$$

independently of distance, within 50, 100, or 402 kpc of the center. Furthermore, for the first time it has become possible to determine the abundance of subhalos at the second level of the substructure hierarchy, and the sub-subhalo $V_{\text{max}}$-functions (see inset) appear to be quite similar to the scaled down version of the host halo’s subhalo distribution.

### 3.2. The “missing satellites problem”

It has been known for many years that our host galaxy, the Milky Way, is orbited by a number of so-called dwarf galaxy satellites. The original census of about 11 dwarfs [19] has recently doubled in size by the discovery of $\sim 12$ additional ultra-faint dwarfs in the Sloan Digital Sky Survey (SDSS) [20 21 22 23 24 25 26 27]. Many, if not all, of these systems are strongly dark matter...
dominated in their central regions \cite{28, 29}, and are thus thought to be the luminous counterparts to the massive end of the dark matter subhalo population. The apparent discrepancy between the number of observed dwarf galaxies (around 50, after correcting for the incomplete sky coverage of the SDSS) and the vastly larger number of dark matter subhalos predicted by numerical simulations (\( \sim 50,000 \) in \textit{Via Lactea}) is well known as the “missing satellites problem”.

The solution to this problem could lie in a modification of the standard cold dark matter paradigm of structure formation. For example, \textit{warm} dark matter, with its higher intrinsic velocity dispersion, would lead to a suppression of power at small scales and greatly reduce the number of low mass subhalos seen in CDM simulations. On the other hand, the standard CDM picture might be fine, if complicated “gastrophysics”, such as radiative or thermal feedback from young galaxies, quasars, or supernovae, prevents star formation in the vast majority of dark matter subhalos. In this case the Galactic halo would be populated by an enormous number of invisible dark matter clumps. The resolution of the “missing satellites problem” depends on both progress from the observational side: deeper surveys, more and improved measurements of stellar kinematics in the centers of dwarfs, etc.; and from the theoretical side: better characterization of the properties of dark matter subhalos from simulations, a better understanding of the efficacy of feedback in preventing star formation, etc. Our \textit{Via Lactea} simulations are one step in this direction \cite{30}.

3.3. \textit{Indirect detection of dark matter substructure?}

An intriguing, if somewhat speculative, possibility for learning more about the dark matter subhalos of our Milky Way arises if the dark matter particle is a WIMP, a weakly interacting massive particle, like the neutralino in the supersymmetric extension to the standard model of particle physics. In that case the dark matter particle would be its own anti-particle, and in regions of sufficiently high density colliding dark matter particles would pair-annihilate. The energetic standard model particles that are produced in such annihilations would decay in a particle cascade, resulting in a potentially observable signal of electron-positron pairs, neutrinos, and gamma-rays. Space based experiments (like PAMELA and the upcoming AMS-02, \cite{31}) could detect anti-particles produced in dark matter annihilations within about one kpc, and huge neutrino telescopes built deep into Antarctic ice (e.g. AMANDA-II and its successor IceCube \cite{32}) or in the Mediterranean sea (ANTARES \cite{32}) have also been pursuing such an indirect detection. The strength of such a signal depends on the local dark matter density in the solar neighborhood, and can be boosted by the clumpiness of the dark matter distribution. Extrapolating from the measured subhalo abundance in our simulation, we have estimated a local boost (within 1 kpc) of about 40\% \cite{33}.

The hope for the gamma-ray signal rests on ground-based atmospheric Cerenkov telescopes (like H.E.S.S., VERITAS, MAGIC, and STACEE, \cite{34}) and on NASA’s recently launched Gamma-ray Large Area Space Telescope (GLAST, \cite{35}). We have used the \textit{Via Lactea} simulation to make quantitative predictions for the possible dark matter annihilation signal that GLAST may detect \cite{36}. By placing fiducial observers at 8 kpc (the distance from the Sun to the Galactic center) from the host halo center and summing up the annihilation fluxes from all particles in our simulation, we produced allsky maps of the gamma-ray flux from dark matter annihilations within the Galaxy. An example of such an allsky map is shown in Figure 4. Converting these flux maps into expected number counts of gamma-ray photons requires assumptions about the nature of the dark matter particle, namely its mass, its annihilation cross section, and the gamma-ray spectrum resulting from a single annihilation event. Particle physics theory today does not uniquely predict these quantities, and so we have sampled a region of the theoretically motivated parameter space. Using detailed models of the known astrophysical gamma-ray backgrounds that the dark matter annihilation signal from Galactic substructure has to contend with, we determined the expected detection significance for every subhalo in our simulation.
Although the vast majority of the subhalos are too faint to be detectable, we found that for reasonable choices of the particle physics parameters, GLAST should be able to detect a handful of individual subhalos at more than five sigma significance.

4. Summary
The Via Lactea simulation is a one billion particle simulation of the formation and evolution of a Milky Way scale dark matter halo in an expanding cosmological volume. At a cost of about 1 million cpu hours on ORNL’s Cray XT3 supercomputer Jaguar, it provides an unprecedented view of the subhalo population in a dark matter halo like the one harboring our own Milky Way.

PKDGRAV, the code we used for our simulation, is an example of a hierarchical tree code that uses a multipole expansion of the gravitational potential to solve the N-body problem in a fast and accurate manner. In Via Lactea we employed for the first time a new dynamical time step criterion that ensures a highly accurate orbit integration even for particles in very dense environments, while at the same time allowing longer time steps for particles in the outer regions of the halo. This allowed us to follow the central density profile and resolve the subhalo population closer to the host halo center than ever before.

We find that the dark matter clustering properties are remarkably self-similar: isolated halos and subhalos contain about the same relative amount of substructure and both have cuspy inner density profiles. The resolution of the “missing satellites problem”, a well known discrepancy between the number of luminous dwarf galaxies orbiting our Galaxy and the predicted number of dark matter subhalos, will require either a modification of the standard cold dark matter paradigm of structure formation or baryonic physics that suppresses star formation in all but a small fraction of subhalos. In the future it may be possible to indirectly detect dark matter substructure, through the measurement of electron-positron pairs, neutrinos, or gamma-rays produced in pair-annihilations of dark matter particles in high density regions such as the centers of subhalos.
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