On the Rotational Invariance and Non-Invariance of Lepton Angular Distributions in Drell-Yan and Quarkonium Production

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Abstract

Several rotational invariant quantities for the lepton angular distributions in Drell-Yan and quarkonium production were derived several years ago, allowing the comparison between different experiments adopting different reference frames. Using an intuitive picture for describing the lepton angular distribution in these processes, we show how the rotational invariance of these quantities can be readily obtained. This approach can also be used to determine the rotational invariance or non-invariance of various quantities specifying the amount of violation for the Lam-Tung relation. While the violation of the Lam-Tung relation is often expressed by frame-dependent quantities, we note that alternative frame-independent quantities are preferred.

Keywords: Drell-Yan Process, Quarkonium production, Lam-Tung relation, Rotational invariance

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The angular distributions of leptons produced in the Drell-Yan process\textsuperscript{1} and the quarkonium production in hadron-hadron collisions\textsuperscript{2,3} remain a subject of considerable interest. The polar and azimuthal angular distributions of leptons produced in unpolarized and polarized Drell-Yan process allow the extraction of various types of transverse-momentum dependent distributions\textsuperscript{4,5}. First (leading order) results on the extraction of the Boer-Mulders functions\textsuperscript{6,7} have been obtained from unpolarized Drell-Yan experiments using pion\textsuperscript{8,9} or proton\textsuperscript{10} beams, indicating that quarks are generally polarized inside unpolarized protons. A more precise determination of the amount of quark polarization requires inclusion of higher order perturbative corrections because gluon radiation can lead to the same nonzero angular distributions\textsuperscript{11,12,13,14}. Recent measurement of Drell-Yan angular distributions with a pion beam on a transversely polarized proton target provided the first information from Drell-Yan on the transverse momentum distribution of unpolarized quarks inside a polarized proton\textsuperscript{15}. For quarkonium production, the lepton angular distributions reveal sensitively the underlying partonic mechanisms, as various subprocesses could lead to distinct polarizations for the quarkonium\textsuperscript{3}.

The angular distributions of leptons produced in the Drell-Yan process or quarkonium production are described by the parameters\textsuperscript{16,17}

\begin{equation}
\lambda, \mu, \nu
\end{equation}

where \(\lambda\), \(\mu\), and \(\nu\) refer to the polar and azimuthal angles of \(l^- (e^- \text{ or } \mu^-)\) in the rest frame of the dilepton. While the polar angle dependence is specified by the parameter \(\lambda\), the azimuthal dependencies of the lepton angular distributions are described by the parameters \(\mu\) and \(\nu\). It is straightforward to show that the values of \(\lambda\), \(\mu\), and \(\nu\) depend on the choice of the coordinate system. While the Collins-Soper frame is chosen by most of the experiments for the data analysis, other reference frames are also utilized by some experiments. Going from one frame to another acts as a
nonlinear transformation on these three parameters, making it hard to connect the results in different frames intuitively. What is a large $\cos 2\phi$ angular coefficient in one frame need not correspond to a large $\cos 2\phi$ coefficient in another frame, for instance.

The frame-dependence of the angular distribution parameters could potentially lead to confusion when comparing results of lepton angular distributions or quarkonium polarizations measured in different experiments. In order to mitigate the confusion caused by the frame dependence of the parameters $\lambda$, $\mu$, and $\nu$, Faccioli et al. \cite{22, 23} pointed out that various quantities can be formed from $\lambda$, $\mu$, and $\nu$ with the property that they are invariant under the transformations among different reference frames. The comparison between measurements obtained with different reference frames could be readily performed, if the rotation invariant quantities are used rather than the individual $\lambda$, $\mu$, and $\nu$ parameters. Examples of such rotational invariant quantities include $F$, $\tilde{\lambda}$, and $G$.

\begin{equation}
F = \frac{1 + \lambda + \nu}{3 + \lambda},
\end{equation}

and

\begin{equation}
\tilde{\lambda} = \frac{2\lambda + 3\nu}{2 - \nu}.
\end{equation}

The reason for considering these particular combinations is not just the rotational invariance, but also that they are measures for the deviation of the Lam-Tung relation $\tilde{\lambda} = 1 - \lambda = 2\nu$, that is satisfied in the Drell-Yan process at order $\alpha_s$ in case of collinear parton distributions. Its violation results from the acoplanarity of the partonic subprocess, as discussed in detail in Refs. \cite{12, 14}. This acoplanarity can arise from intrinsic transverse momentum of quarks inside the proton, but also from perturbative gluon radiation beyond order $\alpha_s$. They lead to a deviation of $F$ from $\frac{1}{2}$ and of $\tilde{\lambda}$ from 1. In contrast, the deviation of $1 - \lambda = 2\nu$ from zero often considered in experimental and theoretical studies is not a rotationally invariant quantity and hence a potential source of confusion when comparing its values obtained in different frames.

Another rotation-invariant quantity invoking all three parameters is $\tilde{\lambda}'$.

\begin{equation}
\tilde{\lambda}' = \frac{(\lambda - \nu/2)^2 + 4\mu^2}{(3 + \lambda)^2}.
\end{equation}

Although not immediately obvious from their definition in terms of $\lambda$, $\mu$, and $\nu$, the above three quantities, $F$, $\tilde{\lambda}$, $\tilde{\lambda}'$, are invariant only under rotations around the $y$ axis, which includes the transformations connecting the various references frames in the literature. On the other hand, the quantity $G$ is invariant under the rotation along the $x$ axis.

\begin{equation}
G = \frac{1 + \lambda - \nu}{3 + \lambda}.
\end{equation}

Finally, $\lambda$ is invariant under the rotation along the $z$-axis.

The rotational invariance of $F$, $\tilde{\lambda}$, $\tilde{\lambda}'$, and $G$ was obtained in Refs. \cite{22, 23} from the consideration of the covariance properties of angular momentum eigenstates of a vector meson. In a recent study \cite{12, 14}, it was shown that some salient features of the parameters $\lambda$, $\mu$, and $\nu$ in the Drell-Yan process and $Z$-boson production can be well described by a simple intuitive approach. In particular, the pronounced transverse-momentum dependence of $\lambda$ and $\nu$ for $Z$-boson production at the LHC \cite{27, 28}, as well as the clear violation of the Lam-Tung relation can be well understood. In this paper, we show how the rotational invariance properties of $F$, $\tilde{\lambda}$, $\tilde{\lambda}'$, and $G$ can be readily deduced using the approach of Refs. \cite{12, 14}. It is also clear from the analysis below that the rotational invariance or non-invariance of various quantities characterizing the violation of the Lam-Tung relation can be obtained.

In the dilepton rest frame, we first define three different planes, namely, the hadron plane, the quark plane, and the lepton plane, shown in Fig. 1. For dilepton with non-zero transverse momentum, $q_T$, the beam and target hadron momenta, $\vec{p}_B$ and $\vec{p}_T$, are not collinear in the rest frame of $\gamma^* / Z$, and they form the “hadron plane” shown in Fig. 1. Figure 1 also shows the “lepton plane”, formed by the momentum vector of $l^-$ and the $\hat{z}$ axis. In the rest frame of the dilepton, the $l^-$ and $l^+$ are clearly emitted back-to-back with equal momenta.

In the dilepton rest frame, a pair of collinear $q$ and $\bar{q}$ with equal momenta annihilate into a $\gamma^* / Z$ or a vector quarkonium, as illustrated in Fig. 1. We define the momentum unit vector of $q$ as $\hat{z}'$, and the “quark plane” is formed by the $\hat{z}'$ and $\hat{z}$ axes. The polar and azimuthal angles of the $\hat{z}'$ axis in the Collins-Soper frame are denoted as $\theta_1$ and $\phi_1$. The $q - \bar{q}$ axis, called the natural axis, has the important property that the $l^-$ angular distribution is azimuthally symmetric with respect to this axis, namely,\n
\begin{equation}
\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \lambda_0 \cos^2 \theta_0,
\end{equation}

where $\theta_0$ is the polar angle between the $l^-$ momentum vector and the $\hat{z}'$ axis (see Fig. 1), and $a$ is the forward-backward asymmetry originating from the parity-violating coupling, which is important only when the dilepton mass is close to the $Z$ boson mass. The parameter $\lambda_0$ depends on the reaction mechanism. For Drell-Yan process in which a virtual photon decays into a lepton pair, we have $\lambda_0 = 1$. This is a consequence of helicity conservation leading to a transversely polarized virtual photon with respect to the natural axis. For quarkonium production, the value of $\lambda_0$ for a given event depends on the specific mechanism. We note that $\lambda_0 = 0$ for an unpolarized quarkonium, while $\lambda_0 = -1$ for a longitudinally polarized quarkonium.

The angles $\theta$ and $\phi$ are experimental observables, and it is necessary to express $\theta_0$ in terms of $\theta$ and $\phi$. This can be accomplished using the following trigonometric relation:

\begin{equation}
\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos (\phi - \phi_1).
\end{equation}
The polar and azimuthal angles of \( \hat{z} \) in the Soper frame are collinearly with equal momenta to form \( \gamma \) beam (B) and target (T) hadrons. The \( \hat{P}_\text{hadron} \) plane is formed by the momentum vectors of the beam (B) and target (T) hadrons. The \( \hat{z} \) axis of the Collins-Soper frame both lie in the hadron plane with \( \hat{z} \) axis bisecting the \( \hat{P}_B \) and \( -\hat{P}_T \) vectors. The quark (\( q \)) and antiquark (\( \bar{q} \)) annihilate collinearly with equal momenta to form \( \gamma \) or a vector quarkonium, while the quark momentum vector \( \hat{z}' \) and the \( \hat{z} \) axis form the quark plane. The polar and azimuthal angles of \( \hat{z}' \) in the Collins-Soper frame are \( \theta_1 \) and \( \phi_1 \). The \( l^- \) and \( l^+ \) are emitted back-to-back with \( \theta \) and \( \phi \) as the polar and azimuthal angles for \( l^- \).

Figure 1: Definition of the Collins-Soper frame and various angles and planes in the rest frame of \( \gamma \).

Substituting Eq. (7) into Eq. (5), we obtain

\[
\frac{d\sigma}{dt} \propto (1 + \frac{1}{2} \lambda_0 \sin^2 \theta_1) + (\lambda_0 - \frac{3}{2} \lambda_0 \sin^2 \theta_1) \cos^2 \theta \\
+ \frac{1}{2} \lambda_0 \sin 2\theta_1 \cos \phi_1 \sin 2\theta \cos \phi \\
+ \frac{1}{2} \lambda_0 \sin^2 \theta_1 \cos 2\phi_1 \sin^2 \theta \cos 2\phi \\
+ (a \sin \theta_1 \cos \phi_1) \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta \\
+ \frac{1}{2} \lambda_0 \sin^2 \theta_1 \sin 2\phi_1 \sin^2 \theta \sin 2\phi \\
+ \frac{1}{2} \lambda_0 \sin 2\theta_1 \sin \phi_1 \sin 2\theta \sin \phi \\
+ (a \sin \theta_1 \sin \phi_1) \sin \theta \sin \phi.
\]

A comparison between Eq. (11) and Eq. (8) shows that \( \lambda \), \( \mu \), and \( \nu \) can be expressed as a function of \( \lambda_0 \), \( \theta_1 \) and \( \phi_1 \) (c.f. with [23]) for zero acoplanarity angle \( \phi_1 = 0 \):

\[
\lambda = \frac{2\lambda_0 - 3\lambda_0 \sin^2 \theta_1}{2 + \lambda_0 \sin^2 \theta_1}, \\
\mu = \frac{\lambda_0 \sin 2\theta_1 \cos \phi_1}{2 + \lambda_0 \sin^2 \theta_1}, \\
\nu = \frac{\lambda_0 \sin^2 \theta_1 \cos 2\phi_1}{2 + \lambda_0 \sin^2 \theta_1}.
\]

The terms proportional to \( \sin 2\phi \) and \( \sin \phi \) do not appear due to Lorentz invariance, provided there are no vectors (like transverse polarization) normal to the hadron plane. Such terms integral to zero due to the acoplanarity angle average. Unless one considers polarized leptons, parity or time-reversal violation, Eq. (8) reduces to Eq. (11).

First, we consider the quantity \( F \) in Eq. (2). From Eq. (9), it is straightforward to obtain

\[
F = \frac{1 + \lambda_0 - 2\lambda_0 \sin^2 \theta_1 \sin^2 \phi_1}{3 + \lambda_0} = \frac{1 + \lambda_0 - 2\lambda_0 y_1^2}{3 + \lambda_0},
\]

where \( y_1 = \sin \theta_1 \sin \phi_1 \) is the component of the unit vector \( \hat{z}' \) along the \( y \)-axis in the dilepton rest frame. The invariance of \( F \) with respect to a rotation along the \( y \)-axis is clearly shown in Eq. (10), since \( \lambda_0 \) and \( y_1 \) are both invariant under such a rotation. It is interesting to note that for the Drell-Yan process, where \( \lambda_0 = 1 \), \( F \) becomes \((1 - y_1^2)/2 \). As pointed out in Refs. [12, 14], \( y_1 \), or the non-coplanarity angle \( \phi_1 \) between the hadron and the quark planes in Fig. 1, is in general not equal to zero. For the special case of \( \phi_1 = 0 \) (or \( y_1 = 0 \)), \( F = 1/2 \) and \( F \) is invariant under any arbitrary rotation in the dilepton’s rest frame. As discussed in Refs. [12, 14], the Lam-Tung relation in the Drell-Yan process is satisfied when the angle \( \phi_1 \) vanishes. This is readily verified from Eq. (9), when the values of \( \lambda_0 \) and \( \phi_1 \) are set at 1 and 0, respectively.

We next consider the quantity \( \lambda \). Using Eq. (11), Eq. (8) becomes

\[
\lambda = \frac{\lambda_0 + 3\lambda_0 \sin^2 \theta_1 \sin^2 \phi_1}{1 + \lambda_0 \sin^2 \theta_1 \sin^2 \phi_1} = \frac{\lambda_0 + 3\lambda_0 y_1^2}{1 + \lambda_0 y_1^2}.
\]

Again, it is evident that \( \lambda \) must be invariant under a rotation along the \( y \)-axis, since \( \lambda_0 \) and \( y_1 \) are both invariant under such rotation. In the special case of coplanarity between the hadron plane and the quark plane, we have \( y_1 = 0 \), and Eq. (11) becomes \( \lambda = \lambda_0 \). This reflects the nomenclature for \( \lambda \), and it also implies that in that case \( \lambda \) is invariant under rotation along any axis. However, \( \lambda \) is in general not the same as \( \lambda_0 \), and \( \lambda \) is in general not invariant under an arbitrary rotation.

We turn our attention next to the quantity \( \lambda' \) in Eq. (11). All three parameters, \( \lambda, \mu, \) and \( \nu \) are involved in \( \lambda' \). Using Eq. (9), we obtain

\[
\lambda' = \frac{\lambda_0^2 (z_1^2 + x_1^2)^2}{3 + \lambda_0 (z_1^2 + x_1^2)}.
\]

where \( z_1 \) is the component of the unit vector \( \hat{z}' \) along the \( z \)-axis and the identity \( x_1^2 + y_1^2 + z_1^2 = 1 \) is used. It is evident that \( \lambda' \) is invariant under a rotation along the \( y \)-axis. For the coplanar case, \( y_1 = 0 \) and \( \lambda' \) is invariant under rotation along any axis.

In an analogous fashion, we can readily show the invariance of \( \lambda' \) and \( \lambda \) under the rotation along the \( x \) and \( z \) axes, respectively. Using Eq. (9), Eq. (11) becomes

\[
\lambda' = \frac{1 + \lambda_0 - 2\lambda_0 \sin^2 \theta_1 \cos^2 \phi_1}{3 + \lambda_0} = \frac{1 + \lambda_0 - 2\lambda_0 y_1^2}{3 + \lambda_0}.
\]
where \( x_1 = \sin \theta_1 \cos \phi_1 \) is the component of the unit vector \( \hat{z}' \) along the \( x \) axis in the dilepton rest frame. Similarly, from Eq. (9), the parameter \( \lambda \) can be written as

\[
\lambda = \frac{-\lambda_0 + 3\lambda_0 \cos^2 \theta_1}{2 + \lambda_0 - \lambda_0 \cos^2 \theta_1} = \frac{-\lambda_0 + 3\lambda_0 \gamma^2}{2 + \lambda_0 - \lambda_0 \gamma^2},
\]

(14)

where \( z_1 = \cos \theta_1 \) is the component of the unit vector \( \hat{z}' \) along the \( z \) axis in the dilepton rest frame. From Eq. (13) and Eq. (14) we note that \( \hat{c} \) and \( \lambda \) are invariant under the rotation along the \( x \) and \( z \) axis, respectively.

Using the above results one can see that despite the nonlinear transformation of \( \lambda, \mu \) and \( \nu \) under rotations, the linear combination \( 1 - \lambda - 2\nu \) remains zero in all other rotated frames if it is zero in one particular frame, as was observed for specific rotations in [10]. If the combination is nonzero however, then its value will change under rotations, even around the \( y \) axis. From Eq. (9), it is indeed straightforward to see that the quantity \( 1 - \lambda - 2\nu \) is not invariant under rotations along the \( y \) axis. On the other hand, the quantity, \( (1 - \lambda - 2\nu)/(3 + \lambda) \), is invariant under such rotations, namely

\[
\frac{1 - \lambda - 2\nu}{3 + \lambda} = 1 - 2F = \frac{1 - \lambda_0 + 4\lambda_0 \gamma^2}{3 + \lambda_0}.
\]

(15)

Therefore, to examine the amount of the violation of the Lam-Tung relation, the quantity, \( (1 - \lambda - 2\nu)/(3 + \lambda) \), is preferred.

Often in the literature for the Drell-Yan process, another set of angular coefficients are considered: \( A_0, A_1, A_2 \), where

\[
\frac{d\sigma}{d\Omega} \propto \left(1 + \cos^2 \theta\right) + \frac{A_0}{2} \left(1 - 3 \cos^2 \theta\right) + A_1 \sin 2\theta \cos \phi
\]

\[+ \quad \frac{A_2}{2} \sin^2 \theta \cos 2\phi.
\]

(16)

The Lam-Tung relation is then expressed as \( A_0 = A_2 \). The violation of the Lam-Tung relation, \( A_0 - A_2 = 2(1 - 2F) \), is rotationally invariant around the \( y \) axis. On the other hand, the quantity \( \Delta_{LT} = 1 - A_2/A_0 \) of \( [30] \) is not.

In conclusion, we have presented an intuitive derivation for rotation-invariant quantities for lepton angular distributions in Drell-Yan and vector quarkonium production. By expressing these quantities in terms of the \( \lambda_0 \) and the \( x, y \) and \( z \) components of the unit vector of the quark momentum in the dilepton rest frame, the invariant properties of these quantities become transparent. This approach also offers a useful insight regarding the roles of \( \lambda_0 \) and the acoplanarity of the partonic subprocesses in determining the applicability and values of these invariant quantities. We also noted that, while the violation of the Lam-Tung relation is often expressed by a frame-dependent quantity, an alternative quantity (Eq. (15)), which is invariant under the rotation along the \( y \) axis, is preferred. This approach could also be extended to other hard processes, such as hadron pair production in \( e^+e^- \) annihilation, which is closely connected to the Drell-Yan and vector quarkonium production.

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References

[1] S.D. Drell and T.M. Yan, Phys. Rev. Lett. 25, 316 (1970); Ann. Phys. (NY) 66, 578 (1971).
[2] C. Biino et al., Phys. Rev. Lett. 58, 2523 (1987).
[3] E. Braaten and J. Russ, Annu. Rev. Nucl. Part. Sci. 64, 221 (2014).
[4] V. Barone, A. Drago, and P.G. Ratcliffe, Phys. Rept. 359, 1 (2002).
[5] J.C. Peng and J.W. Qiu, Prog. Part. Nucl. Phys. 76, 43 (2014).
[6] D. Boer and P.J. Mulders, Phys. Rev. D57, 5780 (1998).
[7] D. Boer, Phys. Rev. D60, 014012 (1999).
[8] NA10 Collaboration, S. Falciano et al., Z. Phys. C31, 513 (1986); M. Guanziroli et al., Z. Phys. C37, 545 (1988).
[9] E615 Collaboration, J.S. Conway et al., Phys. Rev. D39, 92 (1989); J.G. Heinrich et al., Phys. Rev. D44, 1909 (1991).
[10] Fermilab E866 Collaboration, L.Y. Zhu et al., Phys. Rev. Lett. 99, 082301 (2007); Phys. Rev. Lett. 102, 182001 (2009).
[11] A. Brandenburg, O. Nachtmann and E. Mirkes, Z. Phys. C 60, 697 (1993).
[12] J.C. Peng, W.C. Chang, R.E. McClellan, and O.V. Teryaev, Phys. Lett. B 758, 384 (2016).
[13] M. Lamberts and W. Vogelsang, Phys. Rev. D 93, 114013 (2016).
[14] W.C. Chang, R.E. McClellan, J.C. Peng, and O.V. Teryaev, Phys. Rev. D 96, 054020 (2017).
[15] COMPASS Collaboration, M. Aghasyan et al., Phys. Rev. Lett. 119, 112002 (2017).
[16] J.C. Collins and D.E. Soper, Phys. Rev. D10, 2219 (1977).
[17] K. Gottfried and J.D. Jackson, Nuovo Cimento 33, 309 (1964).
[18] C.S. Lam and W.K. Tung, Phys. Rev. D18, 2447 (1978).
[19] D. Boer and W. Vogelsang, Phys. Rev. D 74, 014004 (2006).
[20] P. Faccioli, C. Lourenco, J. Seixas, and H. Wohri, Phys. Rev. Lett. 102, 151802 (2009).
[21] P. Faccioli, C. Lourenco, and J. Seixas, Phys. Rev. D81, 111502 (2010).
[22] P. Faccioli, C. Lourenco, and J. Seixas, Phys. Rev. Lett. 105, 061601 (2010).
[23] P. Faccioli, C. Lourenco, J. Seixas, and H. Wohri, Phys. Rev. D83, 056008 (2011).
[24] C.S. Lam and W.K. Tung, Phys. Rev. D21, 2712 (1980).
[25] S. Palestini, Phys. Rev. D83, 031503 (2011).
[26] Y. Q. Ma, J. W. Qiu and H. Zhang, arXiv:1703.04752.
[27] CMS Collaboration, V. Khachatryan et al., Phys. Lett. B750, 154 (2015).
[28] ATLAS Collaboration, G. Aad et al., JHEP 08 (2016) 159.
[29] O. Teryaev, Proceedings of XI Advanced Research Workshop on High Energy Spin Physics, Dubna, 2005, pp. 171-175; Nucl. Phys. Proc. Suppl. 214, 118 (2011). doi:10.1016/j.nuclphysbps.2011.03.070
[30] R. Gauld, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and A. Huss, JHEP 1711, 003 (2017).