The Temp Secretary Problem *

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Abstract. We consider a generalization of the secretary problem where contracts are temporary, and for a fixed duration $\gamma$. This models online hiring of temporary employees, or online auctions for re-usable resources. The problem is related to the question of finding a large independent set in a random unit interval graph.

1 Introduction

This paper deals with a variant of the secretary model, where contracts are temporary. E.g., employees are hired for short-term contracts, or re-usable resources are rented out repeatedly, etc. If an item is chosen, it “exists” for a fixed length of time and then disappears.

Motivation for this problem are web sites such as Airbnb and oDesk. Airbnb offers short term rentals in competition with classic hotels. A homeowner posts a rental price and customers either accept it or not. oDesk is a venture capitalizing on freelance employees. A firm seeking short term freelance employees offers a salary and performs interviews of such employees before choosing one of them.

We consider an online setting where items have values determined by an adversary, (“no information” as in the standard model [15]), combined with stochastic arrival times that come from a prior known distribution (in contrast to the random permutation assumption and as done in [21,7,16]). Unlike much of the previous work on online auctions with stochastic arrival/departure timing ([18]), we do not consider the issue of incentive compatibility with respect to timing, and assume that arrival time cannot be misrepresented.

The temp secretary problem can be viewed

1. As a problem related to hiring temporary workers of varying quality subject to workplace capacity constraints. There is some known prior $F(x) = \int_0^x f(z)dz$ on the arrival times of job seekers, some maximal capacity, $d$, on the number of such workers that can be employed simultaneously, and a bound $k$ on the total number than can be hired over time. If hired, workers cannot be fired before their contract is up.

2. Alternately, one can view the temp secretary problem as dealing with social welfare maximization in the context of rentals. Customers arrive according

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to some distribution. A firm with capacity $d$ can rent out up to $d$ boats simultaneously, possibly constrained to no more than $k$ rentals overall. The firm publishes a rental price, which may change over time after a customer is serviced. A customer will choose to rent if her value for the service is at least the current posted price. Such a mechanism is inherently dominant strategy truthful, with the caveat that we make the common assumption that customers reveal their true values in any case.

We give two algorithms, both of which are quite simple and offer posted prices for rental that vary over time. Assuming that the time of arrival cannot be manipulated, this means that our algorithms are dominant strategy incentive compatible.

For rental duration $\gamma$, capacity $d = 1$, no budget restrictions, and arrival times from an arbitrary prior, the time-slice algorithm gives a $\frac{1}{2e}$ competitive ratio. For arbitrary $d$ the competitive ratio of the time-slice algorithm is at least $(1/2) \cdot (1 - 5/\sqrt{d})$. This can be generalized to more complex settings. The time slice algorithm divides time into slices of length $\gamma$. It randomly decides if to work on even or odd slices. Within each slice it uses a variant of some other secretary problem (E.g., [26], [2], [24]) except that it keeps track of the cumulative distribution function rather than the number of secretaries.

The more technically challenging Charter algorithm is strongly motivated by the $k$-secretary algorithm of [24]. For capacity $d$, employment period $\gamma$, and budget $d \leq k \leq d/\gamma$ (the only relevant values), the Charter algorithm does the following:

- Recursively run the algorithm with parameters $\gamma, \lfloor k/2 \rfloor$ on all bids that arrive during the period $[0, 1/2)$.
- Take the bid of rank $\lceil k/2 \rceil$ that appeared during the period $[0, 1/2)$, if such rank exists and set a threshold $T$ to be it’s value. If no such rank exists set the threshold $T$ to be zero.
- Greedily accept all items that appear during the period $[1/2, 1)$ that have value at least $T$ — subject to not exceeding capacity ($d$) or budget ($k$) constraints.

For $d = 1$ the competitive ratio of the Charter algorithm is at least

$$\frac{1}{1 + k\gamma} \left( 1 - \frac{5}{\sqrt{k}} - 7.4\sqrt{\gamma \ln(1/\gamma)} \right).$$

Two special cases of interest are $k = 1/\gamma$ (no budget restriction), in which case the expression above is at least $\frac{1}{2} \left( 1 - 12.4\sqrt{\gamma \ln(1/\gamma)} \right)$. We also show an upper bound of $1/2 + \gamma/2$ for $\gamma > 0$. As $\gamma$ approaches zero the two bounds converge to 1/2. Another case of interest is when $k$ is fixed and $\gamma$ approaches zero in which this becomes the guarantee given by Kleinberg’s $k$-secretary algorithm.

For arbitrary $d$ the competitive ratio of the Charter algorithm is at least

$$1 - \Theta\left( \frac{\sqrt{\ln d}}{\sqrt{d}} \right) - \Theta(\gamma \log (1/\gamma)).$$