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Multilayer-HySEA model validation for landslide generated tsunamis. Part II Granular slides

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Abstract

The overall objective of the present work is to benchmark the novel Multilayer-HySEA model using laboratory experiment data for landslide generated tsunamis. In particular, this second part of the work deals with granular slides, while the first part, in a companion paper, considers rigid slides. The experimental data used have been proposed by the US National Tsunami Hazard and Mitigation Program (NTHMP) and established for the NTHMP Landslide Benchmark Workshop, held in January 2017 at Galveston. Three of the seven benchmark problems proposed in that workshop dealt with tsunamis generated by rigid slides and are collected in the companion paper (Macías et al., 2020). Another three benchmarks considered tsunamis generated by granular slides. They are the subject of the present study. In order to reproduce the laboratory experiments dealing with granular slides, two models need to be coupled, one for the granular slide and a second one for the water dynamics. The coupled model used consists of a new and efficient hybrid finite volume/finite difference implementation on GPU architectures of a non-hydrostatic multilayer model coupled with a Savage-Hutter model. A brief description of model equations and the numerical scheme is included. The dispersive properties of the multilayer model can be found in the companion paper. Then, results for the three NTHMP benchmark problems dealing with tsunamis generated by granular slides are presented with

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1. Introduction

Following the introduction of the companion paper Macías et al. (2020), a landslide tsunami model benchmarking and validation workshop was held, January 9-11, 2017, in Galveston, TX. This workshop, which was organized on behalf of NOAA-NWS’s National Tsunami Hazard Mitigation Program (NTHMP) Mapping and Modeling Subcommittee (MMS), with the expected outcome being to develop: (i) a set of community accepted benchmark tests for validating models used for landslide tsunami generation and propagation in NTHMP inundation mapping work; (ii) workshop documentation and a web-based repository, for benchmark data, model results, and workshop documentation, results, and conclusions, and (iii) provide recommendations as a basis for developing best practice guidelines for landslide tsunami modeling in NTHMP work.

A set of seven benchmark tests were selected (Kirby et al., 2018). These benchmarks are based on a subset of available laboratory data sets for solid slide experiments and deformable slide experiments and include both submarine and subaerial slides. A benchmark based on a historic field event (Valdez, AK, 1964) closed the list of proposed benchmarks. The EDANYA group (www.uma.es/edanya) from the University of Málaga participated in the workshop with the Landslide-HySEA and the Multilayer-HySEA models and presented numerical results for six out of the seven benchmark problems proposed. Only BP6, which results are included here, was not performed at that time. The present work aims to show the numerical results obtained for Multilayer-HySEA in the framework of the validation effort described above for the case of granular slide generated tsunamis for the complete set of benchmark problems.

Fifteen years ago, at the beginning of the century, solid block landslide modeling challenged researchers and was undertaken by a number of authors (see...
companion paper Macías et al. (2020) for references) and laboratory experiments were developed for those cases and for tsunami model benchmarking. In contrast, some early models (e.g., Heinrich (1992); Harbitz et al. (1993); Rzadkiewicz et al. (1997); Fine et al. (1998)) and a number of more recent models have simulated tsunami generation by deformable slides, based either on depth-integrated two-layer model equations, or on solving more complete sets of equations in terms of featured physics (dispersive, non-hydrostatic, Navier-Stokes). Examples include solutions of 2D or 3D Navier-Stokes equations to simulate subaerial or submarine slides modeled as dense Newtonian or non-Newtonian fluids (Ataie-Ashtiani and Shobeyri, 2008; Weiss et al., 2009; Abadie et al., 2010, 2012; Horrillo et al., 2013), flows induced by sediment concentration (Ma et al., 2013), or fluid or granular flow layers penetrating or failing underneath a 3D water domain (for example, the two-layer models of Macías et al. (2015) or González-Vidal et al. (2019) where a fully coupled non-hydrostatic SW-Savege-Hutter is used or those of Ma et al. (2015); Kirby et al. (2016) in which the upper water layer is modeled with the non-hydrostatic σ-coordinate 3D model NHWAVE (Ma et al., 2012). For a more comprehensive review of recent modeling work, see Yavari-Ramshe and Ataie-Ashtiani (2016). A number of recent laboratory experiments have modeled tsunamis generated by subaerial landslides made of gravel (Fritz et al., 2004; Ataie-Ashtiani and Najafi-Jilani, 2008; Heller and Hager, 2010; Mohammed and Fritz, 2012) or glass beads (Viroulet et al., 2014). For deforming underwater landslides and related tsunami generation, 2D experiments were performed by Rzadkiewicz et al. (1997), who used sand, and Ataie-Ashtiani and Najafi-Jilani (2008), who used granular material. Well-controlled 2D glass bead experiments were reported and modeled by Grilli et al. (2017) using the model of Kirby et al. (2016).

The benchmark problems performed in the present work are based on the laboratory experiments of Kimmoun and Dupont (see Grilli et al. (2017)) for BP4, Viroulet et al. (2014) for BP5, and Mohammed and Fritz (2012) for BP6. The basic reference for these three benchmarks, but also the three ones related to solid slides and the Alaska field case, all of them proposed by the NTHMP,
is Kirby et al. (2018). That is a key reference for readers interested in the benchmarking initiative in which the present work is based on.

2. The Multilayer-HySEA model for granular slides

First we consider the Landslide-HySEA model, applied in Macías et al. (2015) and González-Vida et al. (2019), which for the case of one-dimensional domains reads:

\[
\begin{align*}
\partial_t h + \partial_x (hu) &= 0, \\
\partial_t (hu) + \partial_x \left( hu^2 + \frac{1}{2} gh^2 \right) - gh \partial_x (H - z_s) &= \beta (u_s - u), \\
\partial_t z_s + \partial_x (z_s u_s) &= 0, \\
\partial_t (z_s u_s) + \partial_x \left( z_s u_s^2 + \frac{1}{2} g h (1 - r) z_s^2 \right) &= g z_s \partial_x ((1 - r) H - r\eta) \\
&\quad - n_a (u_s - u) + \tau_p,
\end{align*}
\]

(1)

where \( g \) is the gravity acceleration (\( g = 9.81 \text{ m/s}^2 \)); \( H(x) \) is the non-erodible bathymetry measured from a given reference level and unchanged in time; \( z_s(x, t) \) is the granular material depth at each point \( x \) at time \( t \); \( h(x, t) \) is the total water depth; \( \eta(x, t) \) is the free surface and is given by \( \eta = h + z_s - H \); \( u(x, t) \), \( u_s(x, t) \) the averaged horizontal velocity for the water and the granular material respectively; \( r = \rho_1 / \rho_2 \) is the ratio of densities between the ambient fluid and the granular material. \( n_a (u_s - u) \) defines the friction term between the fluid and the granular layers. Finally, here we will consider \( \tau_p(x, t) \) as the friction term given in Pouliquen and Forterre (2002) to be described more precisely in the next section.

System (1) presents the difficulty of considering the complete coupling between sediment and water, including the corresponding coupled pressure terms. That made its numerical approximation more complex. Moreover, it makes also difficult to consider its natural extension to non-hydrostatic flows.

Now, if \( \partial_x \eta \) is neglected in the momentum equation of the granular material, that is, the fluctuation of pressure due to the variations of the free-surface is
neglected in the momentum equation of the granular material, then the following weakly-coupled system could be obtained:

\[
\begin{align*}
\text{S-W system} & \quad \left\{ \begin{array}{l}
\partial_t h + \partial_x (hu) = 0, \\
\partial_t (hu) + \partial_x \left( hu^2 + \frac{1}{2} gh^2 \right) - gh \partial_x (H - z_s) = n_a (u_s - u), \\
\end{array} \right. \\
\text{S-H system} & \quad \left\{ \begin{array}{l}
\partial_t z_s + \partial_x (z_s u_s) = 0, \\
\partial_t (z_s u_s) + \partial_x \left( z_s u_s^2 + \frac{1}{2} g (1 - r) z_s^2 \right) - g (1 - r) z_s \partial_x H = -\beta (u_s - u) + \tau_P,
\end{array} \right.
\end{align*}
\]

(2)

(3)

where the first system is the standard one layer shallow-water system and the second one is the one layer reduced-gravity Savage-Hutter model (Savage and Hutter (1989)), that takes into account that the granular landslide is underwater. Note that the previous system could be also adapted to simulate subaerial/submarine landslides by a suitable treatment of the variation of the gravity terms. Under this formulation, it is now straightforward to improve the numerical model for the fluid phase by including non-hydrostatic effects.

3. Model Equations

The Multilayer-HySEA model implements a two-phase model in order to describe the interaction between the submarine/subaerial landslide and the fluid. In this work, a multi-layer non-hydrostatic shallow-water model is used for the evolution of the ambient water (see Fernández-Nieto et al. (2018)), and the Savage-Hutter model (3) is considered for the kinematics of the submarine/subaerial landslide. Both models are coupled through the boundary conditions at the sea-floor. The ratio of densities between the ambient fluid and the granular material is given by the parameter \( r \). Usually

\[
r = \frac{\rho_f}{\rho_b}, \quad \rho_b = (1 - \varphi) \rho_s + \varphi \rho_f,
\]

(4)
where $\rho_s$ is the typical density of the granular material, $\rho_f$ is the density of the fluid ($\rho_s > \rho_f$), and $\varphi$ is the porosity ($0 \leq \varphi < 1$). Here, we suppose that $\varphi$ is constant on space and time, and therefore $r$ is also constant. Note that $0 < r < 1$. Let us remark that even on a uniform material, $r$ is difficult to estimate as it depends on the porosity $\varphi$. Typical values for $r$ are in the interval $[0.3, 0.8]$.

The fluid model

The ambient fluid is supposed to be modeled by a multi-layer non-hydrostatic shallow-water system proposed in Fernández-Nieto et al. (2018) to account for dispersive water waves. The system, obtained by a process of depth-averaging, corresponds to a semi-discretization with respect to the vertical variable of the Euler equations. Total pressure is decomposed into a sum of a hydrostatic and a non-hydrostatic component. In this process, the horizontal and vertical velocities are assumed to have a constant vertical profile. The resulting multi-layer model admits an exact energy balance, and when the number of layers increases, the linear dispersion relation of the linear model converges to the same of Airy’s theory. The model proposed in Fernández-Nieto et al. (2018) can be written in compact form as

$$
\begin{align*}
\partial_t h + \partial_x (hu) &= 0, \\
\partial_t (hu_\alpha) + \partial_x (hu_\alpha^2 + \frac{1}{2}gh^2) - gh\partial_x (H - z_s) + u_{\alpha+1/2}\Gamma_{\alpha+1/2} + u_{\alpha-1/2}\Gamma_{\alpha-1/2} &= -h (\partial_x p_\alpha + \sigma_\alpha \partial_z p_\alpha) - \tau_\alpha, \\
\partial_t (hw_\alpha) + \partial_x (hu_\alpha w_\alpha) + w_{\alpha+1/2}\Gamma_{\alpha+1/2} - w_{\alpha-1/2}\Gamma_{\alpha-1/2} &= -h\partial_x p_\alpha, \\
\partial_x u_{\alpha-1/2} + \sigma_{\alpha-1/2}\partial_z u_{\alpha-1/2} + \partial_z w_{\alpha-1/2} &= 0,
\end{align*}
$$

for $\alpha \in \{1, 2, \ldots, L\}$, being $L$ the number of layers. In the previous system, we have used the following notation:

$$
f_{\alpha+1/2} = \frac{1}{2} (f_{\alpha+1} + f_{\alpha}), \quad \partial_z f_{\alpha+1/2} = \frac{1}{h\Delta_s} (f_{\alpha+1} - f_{\alpha}).$$
where \( \beta \) denotes one of the generic variables of the system, i.e., \( u \), \( w \) and \( p \), and, finally,

\[
\sigma_\alpha = \partial_x (H - z_s - h\Delta s(\alpha - 1/2)) \quad \text{and} \quad \sigma_{\alpha-1/2} = \partial_x (H - z_s - h\Delta s(\alpha - 1)).
\]

As depicted in Figure 1, the flow depth \( h \) is split along the vertical axis into \( L \geq 1 \) layers and \( \Delta s = 1/L \). \( u_\alpha \) and \( w_\alpha \) are the depth averaged velocities in the \( x \) and \( z \) directions respectively. The term \( p_{\alpha+1/2} \) is the non-hydrostatic pressure at the interface \( z_{\alpha+1/2} \). The free surface elevation measured from the still-water level is \( \eta = h - H + z_s \), where again \( H(x) \) is the unchanged non-erodible bathymetry measured from a given reference level. \( \tau_\alpha = 0 \), \( \alpha > 1 \) and \( \tau_1 \) is given by

\[
\tau_1 = \tau_b + n_{\alpha} (u_s - u_1),
\]

where \( \tau_b \) is a usual Manning-type friction formula for the bottom shear stress given by

\[
\tau_b = gh \frac{n_{\alpha}^2}{h^{4/3}} |u_1|,
\]

and \( n_{\alpha} (u_s - u_1) \) accounts for the friction between the fluid and the granular layers. Both are only present at the lowest layer \( (\alpha = 1) \). Finally, for \( \alpha = 1, \ldots, L - 1 \), \( \Gamma_{\alpha+1/2} \) account for the mass transfer across interfaces and are defined by

\[
\Gamma_{\alpha+1/2} = \sum_{\beta = \alpha+1}^L \partial_x (h\Delta s (u_\beta - \bar{u})) \quad \text{and} \quad \bar{u} = \sum_{\alpha = 1}^L \Delta s u_\alpha
\]
Here we suppose that $\Gamma_{1/2} = \Gamma_{L+1/2} = 0$, that is, there is no mass transfer through the bottom nor the free-surface. In order to close the system, the following boundary conditions are considered

$$p_{L+1/2} = 0, \ u_0 = 0, \ w_0 = -\partial_t (H - z_s).$$

Note that the last two conditions enter into the incompressibility condition for the lowest layer ($\alpha = 1$), given by

$$\partial_x u_{1/2} + \sigma_{1/2} \partial_z u_{1/2} + \partial_z w_{1/2} = 0.$$

Observe that both models are coupled through the unknown $z_s$, present in the equations and in the boundary condition ($w_0 = -\partial_t (H - z_s)$).

Some dispersive properties of the system (5) were originally studied in Fernández-Nieto et al. (2018). Moreover, for a better-detailed study on the dispersion relation such as 'phase velocity', 'group velocity', and 'linear shoaling', the reader is referred to the companion paper Macías et al. (2020).

Finally, let us mention that in the derivation of this two-phase model, we have supposed a rigid-lid assumption concerning the free surface of the ambient fluid: that is, the pressure variations induced by the fluctuation on the free surface of the ambient fluid over the landslide are neglected.

The Landslide model

The 1D Savage-Hutter model that we consider in this work is given by the system (3). The Pouliquen-Folterre friction law $\tau_P$ is given by

$$\tau_P = -g (1 - r) \mu z_s \frac{u_s^2}{|u_s|},$$

where $\mu$ corresponds to a constant friction coefficient that is quite relevant, as it controls the motion of the landslide. Usually $\mu$ is given by the Coulomb friction law as the simpler that can be used in landslide models. However, it is well-known that a constant friction coefficient does not allow to reproduce steady uniform flows over rough beds observed in the laboratory for a range of inclination angles. To reproduce these flows, in Pouliquen and Forterre (2002)
authors introduced an empirical friction coefficient $\mu$ that depends on the norm of the mean velocity $u_s$, on the thickness $z_s$ of the granular layer and on the Froude number $Fr = \frac{u_s}{\sqrt{g z_s}}$. The friction law is given by:

- If $Fr \geq \beta$:
  \[
  \mu(z_s, u_s) = \tan(\delta_1) + (\tan(\delta_2) - \tan(\delta_1)) \exp \left( -\frac{\beta z_s}{d_s Fr} \right),
  \]

- If $Fr = 0$:
  \[
  \mu(z_s) = \mu_{\text{start}}(z_s) = \tan(\delta_3) + (\tan(\delta_2) - \tan(\delta_1)) \exp \left( -\frac{z_s}{d_s} \right),
  \]

- If $0 \leq Fr \leq \beta$:
  \[
  \mu(z_s, u_s) = \mu_{\text{start}}(z_s) + \left( \frac{Fr}{\beta} \right)^\gamma \left( \mu_{\text{stop}}(z_s) - \mu_{\text{start}}(z_s) \right),
  \]

where in this expressions $d_s$ represents the mean size of grains. $\beta = 0.136$ and $\gamma = 10^{-3}$ are empirical parameters. $\tan(\delta_1)$, $\tan(\delta_2)$ are the characteristic angles of the material, and $\tan(\delta_3)$ is other friction angle related to the behavior when starting from rest. This law has been widely used in the literature (see e.g. [Brunet et al. (2017)]).

Note that the two-phase system can also be adapted to simulate subaerial landslides. The presence of the term $(1 - r)$ in the definition of the Poulilquen-Folterre friction law is due to the buoyancy effects, which must be taken into account only in the case that the granular material layer is submerged in the fluid. Otherwise, this term must be replaced by 1.

4. Numerical Solution Method

System (3) could be written in the following compact way:

\[
\partial_t U_s + \partial_x F_s(U_s) = G_s(U_s) \partial_x H - S_s(U_s),
\]
\[ U_s = \begin{bmatrix} z_s \\ u_s z_s \end{bmatrix}, \quad F_s(U_s) = \begin{bmatrix} z_s u_s \\ z_s u_s^2 + \frac{1}{2} g (1 - r) z_s^2 \end{bmatrix}, \]
\[ G_s(U_s) = \begin{bmatrix} 0 \\ g (1 - r) z_s \end{bmatrix}, \quad S_s(U_s) = \begin{bmatrix} 0 \\ -n_a (u_s - u) + \tau_p \end{bmatrix}. \]

The multi-layer non-hydrostatic shallow-water system (5) could also be expressed in a similar way:

\[
\begin{aligned}
\partial_t U_f + \partial_x F_f(U_f) + B_f(U_f) \partial_x U_f &= G_f(U) \partial_x (H - z_s) + T_{NH} - S_f(U_f), \\
B(U_f, (U_f)_x, H, (H)_x, z_s, (z_s)_x) &= 0,
\end{aligned}
\]

where

\[
U_f = \begin{bmatrix} h \\ hu_1 \\ \vdots \\ hu_L \\ hw_1 \\ \vdots \\ hw_L \end{bmatrix}, \quad F_f(U_f) = \begin{bmatrix} h\bar{u} \\ hu_1^2 + \frac{1}{2} gh^2 \\ \vdots \\ hu_L^2 + \frac{1}{2} gh^2 \\ hu_1 w_1 \\ \vdots \\ hu_L w_L \end{bmatrix}, \quad G_f(U_f) = \begin{bmatrix} 0 \\ gh \\ \vdots \\ gh \\ 0 \\ \vdots \\ 0 \end{bmatrix}.
\]
$B_f(U_f)\partial_x(U_f)$ contains the non-conservative products involving the momentum transfer across the interfaces, and $S_f(U_f)$ the friction terms:

$$B_f(U_f)\partial_x(U_f) = \begin{bmatrix}
0 \\
\frac{u_3}{2} \Gamma_{3/2} \\
\frac{u_5}{3} \Gamma_{5/2} - \frac{u_3}{2} \Gamma_{3/2} \\
\vdots \\
-\frac{u_{L-1}}{2} \Gamma_{L-1/2} \\
w_{3/2} \Gamma_{3/2} \\
w_{5/3} \Gamma_{5/2} - w_{3/2} \Gamma_{3/2} \\
\vdots \\
-\frac{w_{L-1}}{2} \Gamma_{L-1/2}
\end{bmatrix}, \quad S_f(U_f) = \begin{bmatrix}
0 \\
\tau_b + n_\alpha (u_s - u_1) \\
\vdots \\
0
\end{bmatrix}. $$

The non-hydrostatic corrections in the momentum equations are given by

$$T_{NH} = T_N H(h, h, x, H, H, z_s, (z_s)_x, p, p_x) = - \begin{bmatrix}
0 \\
h(\partial_x p_1 + \sigma_1 \partial_z p_1) \\
\vdots \\
h(\partial_x p_L + \sigma_L \partial_z p_L) \\
h \partial_z p_1 \\
\vdots \\
h \partial_z p_L
\end{bmatrix},$$

and finally, the operator related with the incompressibility condition at each layer is given by:

$$B(U_f, (U_f)_x, H, H, x, z_s, (z_s)_x) = \begin{bmatrix}
\partial_x u_{1/2} + \sigma_{1/2} \partial_z u_{1/2} + \partial_z w_{1/2} \\
\vdots \\
\partial_x u_{L-1/2} + \sigma_{L-1/2} \partial_z u_{L-1/2} + \partial_z w_{L-1/2}
\end{bmatrix}. $$

The discretization of systems (6) and (7) becomes difficult. In this article, we have considered the natural extension of the numerical schemes proposed in Escalante et al. (2018b,a), where a splitting technique has been described. Firstly,
the systems (6) and (7) can be expressed as the following non-conservative hyperbolic system:

\[
\begin{align*}
\partial_t U_s + \partial_x F_s(U_s) &= G_s(U_s) \partial_x H, \\
\partial_t U_f + \partial_x F_f(U_f) + B_f(U_f) \partial_x (U_f) &= G_f(U_f) \partial_x (H - z_s).
\end{align*}
\]

Both equations are solved simultaneously using the same time step, by means of a second order HLL, positivity-preserving and well-balanced, path-conservative finite volume scheme (see Castro and Fernández-Nieto (2012)). The synchronization of time steps is done taking into account the CFL condition of the complete system (8). A first order estimation of the maximum of the wave speed for system (8) is the following:

\[
\lambda_{\text{max}} = \max(|u_s| + \sqrt{g(1 - r)z_s}, |\bar{u}| + \sqrt{gh}).
\]

Next, the non-hydrostatic pressure corrections \( p_{1/2}, \ldots, p_{L-1/2} \) at the vertical interfaces are computed from

\[
\begin{align*}
\partial_t U_f &= \mathcal{T}_{NH}(h, h_x, H, H_x, z_s, (z_s)_x, p, p_x), \\
B(U_f, (U_f)_x, H, H_x, z_s, (z_s)_x) &= 0.
\end{align*}
\]

That requires the discretization of an elliptic operator using standard second-order central finite differences. The resulting linear system is solved using an iterative Scheduled Jacobi method (see Adsuara et al. (2016)). Finally, the horizontal and vertical momentum equations at each layer are updated using the computed non-hydrostatic corrections. At this stage, the frictions \( S_s(U_s) \) and \( S_f(U_f) \) are also discretized (see Escalante et al. (2018b,a)). We refer the reader to Fernández-Nieto et al. (2008) for the discretization of the Coulomb friction term.

The resulting numerical scheme is well-balanced for the water at rest solution and is linearly \( L^\infty \)-stable under the usual CFL condition related to the hydrostatic system. It is also worth mentioning that the numerical scheme is positive preserving and can deal with emerging topographies. Finally, its extension to 2D is straightforward. In this case, the computational domain is decomposed
into subsets with a simple geometry, called cells or finite volumes. The numerical algorithm adapts well to GPU architectures, as is shown in Castro et al. (2011). Moreover, the compactness of the numerical stencil and the easy and massively parallelization of the Jacobi method makes that the second step can also be implemented on GPUs (see Escalante et al. (2018b,a)). That results in much shorter computational times.

5. Benchmark Problem Comparisons

In this section, we show the numerical results obtained with the Multilayer-HySEA model and the comparison with the measured lab data for water waves generated by the interaction of a granular slide. In particular, BP4 deals with a 2D submarine granular slide, BP5 with a 2D subaerial slide, and BP6 with a 3D subaerial slide. The description of all these benchmarks can be found at LTMBW (2017) and Kirby et al. (2018). In the following numerical simulations, unless otherwise indicated, the quantities of the parameters are expressed in units of measure of the International System of Units.

5.1. Benchmark Problem 4: Two-dimensional submarine granular slide

This benchmark problem is based on 2D laboratory experiments of tsunami generation by underwater slides made of glass beads that were performed at the Ecole Centrale de Marseille (see Grilli et al. (2017) for a description of the experiment). A set of 58 (29 with their corresponding replicate) experiments were performed at the IRPHE (Institut de Recherche sur les Phénomènes Hors Équilibre) precision tank. The experiments were performed using a triangular submarine cavity filled with glass beads that were released by lifting a sluice gate and then moving down a plane slope, everything underwater. Figure 2 shows a schematic picture of the experiment set-up. The one-dimensional domain \([0, 5.47]\) is discretized with \(\Delta x = 0.005\ m\) and wall boundary conditions were imposed. The final time is 10 s. We set the \(CFL = 0.5\) and the parameters for the model have been set as follows

\[
g = 9.81, \quad r = 0.78, \quad n_s = 0, \quad n_m = 0.2 \cdot 10^{-3},
\]
Figure 2: BP4 sketch showing the longitudinal cross section of the IRPHE’s precision tank. Location of the sluice gate and the 4 gages (WG1, WG2, WG3 WG4) is marked.

\[ d_s = 7 \cdot 10^{-3}, \quad \delta_1 = 6^\circ, \quad \delta_2 = 24^\circ, \quad \delta_3 = 12^\circ, \quad \beta = 0.136, \quad \gamma = 10^{-3}. \]

Figure 3 shows the comparison of model results with lab data for the four wave gauges considered. Figure 4 depicts the water free surface and the grain location at several times during the numerical simulation.

The number of layers was set up to 5, although similar results can be obtained with fewer layers, for example, 3. In this case, the number of layers was increased to obtain a similar agreement with measured data as in previous benchmark problems, but the numerical results remained similar for a larger number of layers. That may indicate that improving the numerical results it is not a question of the dispersive properties of the model (that improve with the number of layers) but is more likely due to some missing physics.
Figure 3: Comparison of data time series (red) and numerical (blue) at wave gauges (A) WG1, (B) WG2, (C) WG3, and (D) WG4.
Figure 4: Numerical profiles of the water free surface elevation and the grain layer location at times $t = 0, 0.3, 0.6, 0.9$ s.
5.2. Benchmark Problem 5: Two-dimensional subaerial granular slide

This benchmark is based on a series of 2D laboratory experiments performed by Viroulet et al. (2014) in a small tank at École Centrale de Marseille, France. The simplified picture of the set-up for these experiments can be found in Figure 5. The granular material was confined in triangular subaerial cavities and composed of dry glass beads of diameter \( d_s \) (that was varied) and density \( \rho_s = 2,500 \text{ km/m}^3 \). This was located on a plane 45° slope just on top of the water surface. Then the slide was released by lifting a sluice gate and entering right away in contact with water. The experimental set-up used by Viroulet et al. (2014) consisted in a wave tank, 2.2 m long, 0.4 m high, and 0.2 m wide.

The granular is initially retained by a vertical gate on the dry slope. The gate is suddenly lowered, and in the numerical experiments, it should be assumed that the gate release velocity is large enough to neglect the time it takes the gate to withdraw. The front face of the granular slide touches the water surface at \( t = 0 \). The initial slide shape has a triangular cross-section over the width of the tank, with down-tank length \( L \), and front face height \( B = L \) as the slope angle is 45°.

For the present benchmark, two cases are considered. Case 1 defined by the following set-up: \( d_s = 1.5 \text{ mm} \), \( H = 14.8 \text{ cm} \) and \( L = 11 \text{ cm} \) and Case 2 given by \( d_s = 10 \text{ mm} \), \( H = 15 \text{ cm} \) and \( L = 13.5 \text{ cm} \). The benchmark problem proposed consists in simulating the time series of free surface elevations at the

Figure 5: BP5 sketch of the set-up for the laboratory experiments.
four gauges WG1 to WG4 for the two test cases described above.

The same model configuration as in the previous benchmark problem is used here. The number of layers was set up to 3. The one-dimensional domain $[0, 2.2]$ is discretized with $\Delta x = 0.003 \, m$ and wall boundary conditions were imposed. The simulation time is 2.5 s. We set the $CFL = 0.9$ and we choose the parameters given by

$$
\begin{align*}
  g &= 9.81, \quad r = 0.6, \quad n_a = 0, \quad n_m = 0.1 \cdot 10^{-3}, \\
  \delta_1 &= 6^\circ, \quad \delta_2 = 24^\circ, \quad \delta_3 = 12^\circ, \quad \beta = 0.136, \quad \gamma = 10^{-3}.
\end{align*}
$$

Finally $d_s$ was set to $1.5 \cdot 10^{-3}$ and $10 \cdot 10^{-3}$ depending on the test case. Figure 6 shows the comparison for Case 1. In this case, model agreement with measured data is extremely good and most of all for the two leading waves. Figure 7 shows the comparison for Case 2. In this case, the agreement is good, but larger differences appear between model and lab measurements. Figure 8 depicts the free surface elevation and the granular layer spatial distribution at several times for Case 1. It can be observed that the agreement with lab data is much better for Case 1 than for Case 2 and also that this agreement is also better for gauges located further from the slide. This latter behavior can be explained as a consequence of the fact that the hydrodynamic component is much better resolved and simulated than the morphodynamic component (the movement of the slide material), obviously much more difficult to reproduce.
Figure 6: Comparison of data time series (red) and numerical (blue). Case 1. (A) G1, (B) G2, (C) G3, and (D) G4
Figure 7: Comparison of data time series (red) and numerical (blue). Case 2. (A) G1, (B) G2, (C) G3, and (D) G4
Figure 8: Numerical profiles of the water free surface elevation and the grain layer at times $t = 0, 0.2, 0.4, 0.8 \text{ s}$ for the Case 1.
5.3. Benchmark Problem 6: Three-dimensional subaerial granular slide

This benchmark problem is based on the 3D laboratory experiment of Mohammed and Fritz (2012); Mohammed (2010). Benchmark 6 simulates the rapid entry slide into a 3D water body. The landslide tsunami experiments were conducted in the tsunami wave basin at Oregon State University in Corvallis. The landslides are deployed off a plane 27.1° slope, as shown in Figure 9. The landslide material is deployed in a box measuring 2.1 m × 1.2 m × 0.3 m, with a volume of 0.756 $m^3$ and weight of approximately 1360 $kg$. The case we choose for this study has a still water depth of $H = 0.6 m$. The computational domain $[0, 48] \times [-14, 14]$ is discretized with $\Delta x = \Delta y = 0.06 m$ and wall boundary conditions were imposed. The simulation time is 20 s and we set the $CFL = 0.5$. According to Mohammed and Fritz (2012), Mohammed (2010), the

\[ \text{Figure 9: The computational domain. Upper panel: plan view of the domain. Lower panel: a cross-section at } y = 0 \text{ m. The red dots show an sketch of the wave gauge positions used to measure the water surface elevation.} \]
three-dimensional granular landslide parameters were set to

\[ g = 9.81, \quad r = 0.55, \quad n_a = 0.1, \quad n_m = 0.4 \times 10^{-3}, \]

\[ d_s = 13.7 \times 10^{-3}, \quad \delta_1 = 6^\circ, \quad \delta_2 = 24^\circ, \quad \delta_3 = 12^\circ, \quad \beta = 0.136, \quad \gamma = 10^{-3}. \]

Three vertical layers were employed to simulate the upper-layer water motion, although it was observed similar results with 2 layers.

In the beginning, the slide box is driven using four pneumatic pistons. Here we provide comparisons for the case of pressure in the pneumatic pistons of the landslide tsunami generator of \( P = 0.4 \) MPa (\( P = 58 \) PSI). In Mohammed (2010), it is shown that for this test case, the landslide box velocity reached a velocity of \( v_b = 2.3 \times \sqrt{g} \cdot 0.6 = 5.58 \) m/2 that serve us as a constant initial condition for the \( x \)-component of \( u_s \) wherever \( z_s > 0 \).

The benchmark problem proposed consist in simulating the time series of free surface elevations at some wave-gauges. Here we show the data time series for the 9 wave gauges displayed in Figure 9 as red dots. The wave-gauge in coordinates \((r, \theta)\) are given more precisely in Table 1. We first compare the simulated landslide velocity at impact with the measurement. In the experiment, the landslide impact velocity is 5.72 m/s at time \( t = 0.44 \) s. The computed landslide impact velocity by the model is slightly underestimated with a value of 5 m/s at the time \( t = 0.4 \) s as it can be seen in the upper panel of Figure 10. The final deposition is located at the plane bottom close to the transition of the slope as shown in the lower panel of Figure 10 which is a bit more offshore than that observed in the experiments Mohammed and Fritz (2012). The offshore run-out of the slide is probably due to the present models neglect of the additional friction due to the curvature change at the transition of the slope.

In Ma et al. (2015) a similar result and discussion can be found. In Figure 11
we show the comparisons of simulated and measured tsunami waves at 9 wave gauges. Generally, the model simulates the tsunami waves well, although the wave heights are overestimated at some stations, specially those closer to the shoreline (for example, the station with $\theta = 30^\circ$ and $r = 3.9$). This effect has been also observed and discussed in Ma et al. (2015). By the end of some of the time series the small free-surface oscillations are not captured by the model partially due to the relatively coarse horizontal grids used in the simulation. These observations can be also stated in Figure 11 for the comparisons of simulated and measured tsunami waves at some wave gauges situated at the shoreline ($x = 7.53$). Table 2 shows the execution times on a NVIDIA Tesla P100 GPU. In can be observed that including non-hydrostatic terms in the SWE-SH system results in an increase of the computational time in 2.9 times. If a richer
Table 2: Execution times in seconds for SWE-SH and non-hydrostatic GPU implementations. Ratios compared with SWE-SH.

|        | Runtime (s) | Ratio |
|--------|-------------|-------|
| SWE-SH | 186.55      | 1     |
| 1L NH-SH | 541.11    | 2.9   |
| 2L NH-SH | 649.19    | 3.48  |
| 3L NH-SH | 869.32    | 4.66  |

vertical structure is considered, then larger computational times are required. As examples for the two and three-layer systems, 3.48 and 4.66 times increase in the computational effort.
Figure 11: The comparisons of simulated (solid lines) and measured (dashed lines) impulse waves.
Figure 12: The comparisons of simulated (solid lines) and measured (dashed lines) impulse waves.
6. Concluding Remarks

Numerical models need to be validated previous to their use as predictive tools. This requirement becomes even more necessary when these models are going to be used for risk assessment in natural hazards where human lives are involved. The present work aims to benchmark the novel Multilayer-HySEA model for landslide generated tsunamis produced by granular slides.

Multilayer-HySEA implements a two-phase model to describe the interaction between submarine/subaerial landslides and water. The upper phase describes the hydrodynamic component. That is done using a stratified vertical structure that includes non-hydrostatic terms to incorporate dispersive effects in the propagation of simulated waves. The motion of the landslide is then taken into account by the lower phase consisting of a Savage-Hutter model. To reproduce these flows, the friction model given in Pouliquen and Forterre (2002) is considered here. Both models are weakly-coupled through the boundary condition at the sea-floor.

The numerical scheme employed combines a finite volume path-conservative scheme for the underlying hyperbolic system and finite differences for the discretization of non-hydrostatic terms. The numerical model is implemented to be run in GPU architectures. From a computational point of view, the two-layer non-hydrostatic code coupled with the Savage-Hutter presents good computational times with respect to the one-layer SWE/Savage-Hutter GPU code. For the numerical simulations performed here, the wall-clock times for the non-hydrostatic model are always below 4.66 times the times for the SWE model for a number of layers less than three. We can conclude that the numerical scheme presented here is robust, extremely efficient, and can model dispersive effects generated by submarine/subaerial landslides with at a low computational cost considering that dispersive effects and a vertical multilayer structure are included in the model. Model results show a good agreement with the experimental data for the three benchmark problems considered. In particular, for BP5, but this also occurs for the other two benchmark problems. In general, it
is shown a better agreement for the hydrodynamic component, compare with their morphodynamic counterpart, which is more challenging to reproduce.

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