Calculation of bulk viscosity of QCD at zero temperature and finite chemical potential

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In this letter, based on Kubo’s formula and the QCD low energy theorem, we propose a direct formula for calculating the bulk viscosity of QCD at finite chemical potential \(\mu\) and zero temperature. According to this formula, the bulk viscosity at finite \(\mu\) is totally determined by the dressed quark propagator at finite \(\mu\). We then use a dynamical, confining Dyson-Schwinger equation model of QCD to calculate the bulk viscosity at finite \(\mu\). It is found that no sharp peak behavior of the bulk viscosity at finite \(\mu\) is observed, which is quite different from that of the bulk viscosity at finite temperature.

Key-words: bulk viscosity, Dyson-Schwinger equations (DSEs), finite chemical potential, quantum chromodynamics (QCD).

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Recently transport coefficients of hot/dense quark matter attract lots of attentions. It is observed that the quark gluon plasma (QGP) created at RHIC (Relativistic Heavy Ion Collider) behaves like a nearly perfect fluid \(^1\)\(^2\) with the ratio of the shear viscosity \(\eta\) over entropy density \(s\) approaching the lower bound of 1/4\(\pi\) \(^3\) near the phase transition temperature \(T_c\). Such a behavior indicates that the created QGP is not a weakly coupled gas which is expected from perturbative quantum chromodynamics (pQCD), but a strongly coupled liquid. On the other hand, lattice QCD calculation shows that the ratio of the bulk viscosity \(\zeta\) over entropy density \(s\) has a sharp peak near the critical temperature \(^6\)\(^7\). Such a sharp peak behavior of \(\zeta\) has also been observed in many model calculations including linear sigma model \(^8\), real scalar model \(^9\), massless pion gas model \(^10\), quasiparticle model \(^11\), Z(2) and O(4) models \(^12\), and Dyson-Schwinger equations \(^13\). The bulk viscosity \(\zeta\) characterizes the response of the system to the conformal transformation and vanishes when the system has a conformal equation of state, therefore the sharp peak of the bulk viscosity would strongly affect the physics of the QCD matter near \(T_c\) and is very important for the study of QCD phase transition.

Because the phase structure of QCD at finite density is much richer than that at finite temperature (see, for example, Ref. \(^14\)), one may naturally ask whether the sharp peak behavior of \(\zeta\) exists at zero temperature and finite \(\mu\). As far as the present authors know, up to now there are only limited number of papers trying to answer the above question. For example, in Ref. \(^13\) the authors study the viscosity at finite \(\mu\) with Nambu-Jona-Lasinio (NJL) model and in Ref. \(^16\) the authors study the viscosity of strange quark matter at finite \(\mu\) with quasi particle model. Just as will be shown below, elucidating the bulk viscosity of QCD matter requires a knowledge of the equation of state (EOS) of strongly interacting matter, and the plausibility of the results of any given study rest on the EOS employed, which is difficult to judge a priori (Presently, it is not feasible to study the EOS at finite \(\mu\) rigorously in lattice simulation of QCD, due to the problems arising from the complex valued Euclidean fermion determinant at nonzero chemical potential). Consequently, the exploration of alternative equations of state and the identification of qualitatively consistent results is important. Here we would like to stress that the absence of confinement is a defect common to most existing model studies for the bulk viscosity of QCD matter \(^8\)\(^12\)\(^13\)\(^16\). A covariant, continuum approach incorporating both confinement and chiral symmetry in that direction is necessary.

It is well known that the Dyson-Schwinger equations (DSEs) provide a nonperturbative, Poincare invariant continuum framework for analyzing QCD, i.e., for the study of confinement and dynamical chiral symmetry breaking (DCSB), and hadron observables \(^17\)\(^22\). Herein we will employ a dynamical, confining Dyson-Schwinger equation model of QCD \(^22\) to calculate the bulk viscosity of QCD at finite chemical potential and zero temperature.

In order to make this letter self-contained, let us first recall the general approach of calculating bulk viscosity in existing literature. According to Kubo’s formula the bulk viscosity \(\zeta\) can be expressed as \(^6\)

\[
\zeta = -\frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G^R(\omega, \vec{0}),
\]

where the retarded Green’s function \(G^R\) is defined as:

\[
G^R(x) = \int \frac{d\omega d^3k}{(2\pi)^4} G^R(\omega, \vec{k}) e^{-i(\omega t - \vec{k} \cdot \vec{x})}
= -i\theta(t) \langle [T^\mu(x), T^\mu(0)] \rangle
\]
with $T_{\mu\nu}$ the stress tensor of the system. In the small frequency region one can assume the spectral density $\rho(\omega, \vec{p})$ which is defined as

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \text{Im} G^R(\omega, \vec{p})$$

has the following form

$$\rho(\omega, \vec{p}) = \frac{9\zeta}{\pi} \frac{\omega^3}{\omega_0^3 + \omega^2},$$

where the parameter $\omega_0$ identifies the scale at which the perturbation theory becomes valid \cite{3}. With such ansatz the bulk viscosity can be expressed as

$$9\zeta \omega_0 = 2\int_0^\infty \frac{\rho(u, \vec{0})}{u} du = \int d^4x \langle T_t \{ T_\mu^\nu(x) T_\mu^\nu(0) \} \rangle,$$

For QCD, the trace of stress tensor reads

$$T_\mu^\nu = m_q \bar{q}q + \frac{\beta(g)}{2g} F_\mu^\nu F^\nu_\mu \equiv T_F + T_G,$$

where $g$ is the strong coupling constant, $T_F$ and $T_G$ are the contribution of quark fields and of gluon field, respectively, and $\beta(g)$ is the QCD $\beta$-function which determines the running behavior of $g$. In Eq. (6) $q$ are quark fields with two flavors (in this letter we will limit ourselves in two flavor case and set the current quark mass $m_u = m_d = m$). From the QCD low-energy theorems at finite temperature $T$ and $\mu$ \cite{23}, one can find

$$(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - d) \langle \hat{O} \rangle_T = \int d^4x \langle T_t \{ T_G(x) \hat{O}(0) \} \rangle,$$

where $d$ is the canonical dimension of the operator $\hat{O}$. Using the above equation, one has

$$(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4) \langle T_G \rangle_T = \int d^4x \langle T_t \{ T_G(x) T_G(0) \} \rangle,$$

$$(T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 3) \langle T_F \rangle_T = \int d^4x \langle T_t \{ T_G(x) T_F(0) \} \rangle.$$  

From the above two relations one obtains

$$9\zeta \omega_0 = \int d^4x \langle T_t \{ T_\mu^\nu(x) T_\mu^\nu(0) \} \rangle$$

$$= (T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4) \langle T_G \rangle_T + 2 \left( T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 3 \right) \langle T_F \rangle_T$$

$$+ \int d^4x \langle T_t \{ T_F(x) T_F(0) \} \rangle$$

$$\approx \left( T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4 \right) \langle T_\mu^\nu \rangle_T + \left( T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 2 \right) \langle T_\mu^\nu \rangle_T$$

$$= (T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 4) (E - 3P)$$

$$+ \left( T \frac{\partial}{\partial T} + \mu \frac{\partial}{\partial \mu} - 2 \right) \langle T_F \rangle_T,$$

where $E$ is the energy density and $P$ is the pressure density of QCD. Here, because the current quark mass $m$ of u and d quark is very small, in deriving Eq. (10) we have neglected the term proportional to $m^2$.

Now we take the $T \to 0$ limit of the above expression. In this limit the terms involving $T \partial/\partial T$ vanish. Therefore one would obtain the following expression of $\zeta$ at zero $T$ and finite $\mu$:

$$9\zeta \omega_0 = -4\rho_q(\mu) + 16P(\mu) + 2m_\mu \frac{\partial \langle \bar{q}q \rangle}{\partial \mu} - 4m \langle \bar{q}q \rangle \langle 11 \rangle,$$

where $\rho_q(\mu)$ is the quark number density which can be written as (see, for example, Ref. \cite{24})

$$\rho_q(\mu) = \frac{\partial P(\mu)}{\partial \mu} = -N_c N_f \int \frac{d^4p}{(2\pi)^4} \text{tr} \{ G_q[\mu](p) \gamma_4 \},$$

and $\langle \bar{q}q \rangle$ is the quark condensate at zero $T$ and finite $\mu$ which can be defined as

$$\langle \bar{q}q \rangle = -N_c N_f \int \frac{d^4q}{(2\pi)^4} \text{tr} \{ G_q[\mu](p) \},$$

In Eqs. (12) and (13) $G_q[\mu](p)$ is the dressed quark propagator at finite $\mu$. $N_c$ and $N_f$ denote the number of colors and of flavors, respectively, and the trace operation tr is over Dirac indices (we will not write the renormalization constants explicitly because one would find in the final result the renormalization constants cancel each other).

Eq. (11) indicates that the bulk viscosity of QCD at zero $T$ and finite $\mu$ is determined by $\rho_q(\mu)$, the pressure density $P(\mu)$ and the quark condensate at finite $\mu$. From Eqs. (12) and (13) it can be seen that the quark number density and quark condensate at finite $\mu$ is totally determined by the dressed quark propagator at finite $\mu$. As will be shown below, the pressure density is also totally determined by the dressed quark propagator at finite $\mu$. This means that the bulk viscosity at finite $\mu$ is totally determined by the dressed quark propagator at finite $\mu$. This is quite different from the case of finite $T$ in which the term involving the speed of sound contributes to the peak behavior of $\zeta/s$ around $T_c$, \cite{4}.

From Ref. \cite{24} one can obtain the following:

$$P(\mu) = P(\mu = 0) + \int_0^\mu \rho_q(\mu') d\mu' = P(\mu = 0)$$

$$- N_c N_f \int_0^\mu d\mu' \int \frac{d^4p}{(2\pi)^4} \text{tr} \{ G_q[\mu'](p) \gamma_4 \}. \quad (14)$$

Based on the above equation it can be seen that the pressure density $P(\mu)$ is the sum of two terms: the first term $P(\mu = 0)$ (the pressure density of vacuum) is only a $\mu$-independent term; the second term, which is totally determined by $G_q[\mu](p)$, contains all the nontrivial $\mu$-dependence. Just as was shown in Ref. \cite{24}, the constant term $P(\mu = 0)$ can be calculated through Cornwall-Jackiw-Tomboulis effective action \cite{23}. However, in the
present paper we are only interested in the $\mu$-dependence of $\zeta$, so we ignore this term.

According to Eqs. (8-10) and Eq. (14), once the dressed quark propagator at finite $\mu$ is known, one can obtain the bulk viscosity of QCD at finite $\mu$. However, at present it is very difficult to calculate $G_q(\mu)(p)$ from first principles of QCD and one has to resort to various QCD models. Over the past few years, considerable progress has been made in the framework of the rainbow-ladder approximation of the DSEs approach [17–21], which provides a successful description of various nonperturbative aspects of strong interaction physics. In the rest of this paper we will use DSEs approach to calculate $\zeta$.

To obtain the dressed quark propagator at finite $\mu$ let us first recall the following general result given in Refs. [20–30]: If one adopts the rainbow approximation of the DSEs (rainbow approximation means one uses the bare vertex to approximate the full quark-gluon vertex in the quark DSE) and ignores the $\mu$ dependence of the dressed gluon propagator (one often makes this approximation in the study of finite density QCD because it is usually believed that the effect of chemical potential on the gluon propagator arising from quark-loop insertions is small in comparison with that on the quark propagator; this is a commonly used approximation in studying the dressed quark propagator at finite $\mu$ [18, 24–35]), then one can obtain the dressed quark propagator at finite $\mu$ from the one at $\mu = 0$ by the following substitution (more details can be found in Refs. [20 30]):

$$G_q(\mu)(p) = G_q(\mu = 0)(\hat{p}), \quad (15)$$

where $\hat{p} = (\vec{p}, p_\perp + i\mu)$. The validity of this approximation has been discussed in detail in Refs. [28 30 36] and has been applied to the study of properties of quark stars [37, 38]. Therefore, once the dressed quark propagator at $\mu = 0$ is known, the dressed quark propagator at finite $\mu$ and $T = 0$ can be obtained by means of Eq. (15).

Now one needs to specify the form of the dressed quark propagator at $\mu = T = 0$. In Ref. [22], guided by the solution of the coupled set of DSEs for the ghost, gluon and quark propagator in the Landau gauge, the following meromorphic form of the dressed quark propagator is proposed:

$$G_q(p) = \sum_{j=1}^{n_p} \left( \frac{r_j}{ip + a_j + ib_j} + \frac{r_j}{ip + a_j - ib_j} \right). \quad (16)$$

The propagator of this form has $n_p$ pairs of complex conjugate poles located at $a_j \pm ib_j$. When some $b_j$ is set to zero, the pair of complex conjugate poles degenerates to a real pole. The residues $r_j$ are real (note that a similar meromorphic form of the quark propagator was previously proposed in Ref. [39], in which the residues in the two additive terms are complex conjugate of each other). The values of parameters in Eq. (16) are listed in Table I [22] where 2CC means two pairs of complex conjugate poles, 1R1CC means one real pole and one pair of complex conjugate poles and 3R means three real poles. The parameters of these meromorphic forms in Table I are fitted by requiring that these meromorphic forms of propagator reproduce some physical observables (the two-quark condensate and pion decay constant) and lattice data (the zero momentum values of the mass function and wave-function renormalization, $M_0$ and $Z_0^q$, and an approximate width of the region of large dynamical mass generation, $\omega_L$) (for more details, please see [22]). Here we should stress that the real dressed quark propagator has much more complicated analytic structures than that given in Eq. (16). However, despite its simplicity, this kind of parametrization of the dressed quark propagator has several appealing features: it can simultaneously describe DCSB and quark confinement which are very important for low energy QCD; at large momenta it reduces to the free fermion propagator, as required by asymptotic freedom. Phenomenologically, these parameterizations of the dressed quark propagator can give a quite good description of low energy QCD [22].

| Parameterization | $r_1$ | $a_1$ (GeV) | $b_1$ (GeV) | $r_2$ | $a_2$ (GeV) | $b_2$ (GeV) | $r_3$ | $a_3$ (GeV) |
|------------------|-------|-------------|-------------|-------|-------------|-------------|-------|-------------|
| 2CC              | 0.360 | 0.351       | 0.08        | 0.140 | -0.899      | 0.463       | -     | -           |
| 1R1CC            | 0.354 | 0.377       | -           | 0.146 | -0.91       | 0.45        | -     | -           |
| 3R               | 0.365 | 0.341       | -           | 1.2   | -1.31       | -           | -1.06 | -1.40       |

With such propagator the quark number density and the $\mu$-dependent term of $P(\mu)$ in Eq. (14) can be calculated as [22]

$$\rho_q(\mu) = \frac{2N_cN_f}{3\pi^2} \sum_{j=1}^{n_p} r_j \theta(\mu - |a_j|)$$

$$\times \left( \mu^2 - \frac{a_j^2b_j^2}{\mu^2} - a_j^2 + b_j^2 \right)^{\frac{1}{2}}, \quad (17)$$

$$\int_0^\mu \rho_q(\mu')d\mu' = \frac{2N_cN_f}{3\pi^2} \sum_{j=1}^{n_p} r_j \theta(\mu - |a_j|)I_j, \quad (18)$$

Table I. The parameters for the quark propagator. These parameters are taken directly from Table II of Ref. [22].
where $I_j$ reads:

$$I_j = \frac{3(a_j^2 + b_j^2 - 6a_j^2b_j^2)}{16} \ln \left( \frac{\mu^2 + b_j^2 + \sqrt{\mu^2 - a_j^2}}{\mu^2 + b_j^2 - \sqrt{\mu^2 - a_j^2}} \right)$$

$$+ \frac{3(a_j^2 - b_j^2) |a_j b_j|}{4} \arctan \left( \frac{b_j^2(\mu^2 - a_j^2)}{a_j^2(\mu^2 + b_j^2)} \right)$$

$$+ \frac{\mu^2 - 5}{8} (a_j^2 - b_j^2) \sqrt{\mu^2 - a_j^2}(\mu^2 + b_j^2)$$

$$+ \frac{a_j^2 b_j^2}{2 \mu^2} \sqrt{\mu^2 - a_j^2}(\mu^2 + b_j^2). \quad (19)$$

From Eq. (17) it can be seen that when $\mu$ is below a critical value $\mu_c = \min \{ |a_j| \}$ ($\mu_c = 351$ MeV, 377 MeV and 341 MeV for the 2CC, 1R1CC and 3R parametrization, respectively), the quark number density vanishes identically. Namely, $\mu = \mu_c$ is a singularity which separates two regions with different quark number densities. This result agrees qualitatively with the general conclusion of Ref. [10]. In that reference, based on a universal argument, it is pointed out that the existence of some singularity at the point $\mu = \mu_c$ and $T = 0$ is a robust and model-independent prediction. The numerical value of the critical chemical potential in pure QCD (i.e., with electromagnetic interaction being switched off) is estimated to be 307 MeV. The value of $\mu_c$ obtained here is of the same order of magnitude as the estimates in Ref. [40].

The quark condensate at finite $\mu$ with dressed quark propagator given in Eq. (16) has been obtained in Ref. [11]:

$$\langle \bar{q} q \rangle = \langle \bar{q} q \rangle_0 + \frac{2N_c N_f}{\pi^2} \sum_{j=1}^{n_F} r_j a_j \theta(\mu - |a_j|) f_j(\mu), \quad (20)$$

where $\langle \bar{q} q \rangle_0$ is the quark condensate at $\mu = 0$ and $f_j$ reads

$$f_j = \frac{1}{2} \left( 1 - \frac{b_j^2}{\mu^2} \right) \ln \left( \frac{\sqrt{\mu^2 - a_j^2}(\mu^2 + b_j^2)}{\mu^2 + b_j^2 - \sqrt{\mu^2 - a_j^2}} \right)$$

$$+ \frac{3b_j^2 - a_j^2}{4} \ln \left( \frac{\sqrt{\mu^2 + b_j^2} + \sqrt{\mu^2 - a_j^2}}{\mu^2 + b_j^2 - \sqrt{\mu^2 - a_j^2}} \right)$$

$$+ \frac{b_j^2 - 3a_j^2}{2} \frac{b_j}{a_j} \arctan \left( \frac{b_j^2(\mu^2 - a_j^2)}{a_j^2(\mu^2 + b_j^2)} \right). \quad (21)$$

From Eq. (20) it can also be seen that when $\mu < \mu_c$, the quark condensate equals its vacuum value.

Substituting Eqs. (17), (18) and (20) into Eq. (11) one can calculate the bulk viscosity at zero $T$ and finite $\mu$. The numerical result is shown in Fig. 1. From Fig. 1 one can find the bulk viscosity is kept unchanged from the value at $\mu = 0$ when $\mu$ is smaller than $\mu_c$, and $\mu_c$ is the point when quarks get populated (like the liquid-gas point, if there were a self-bound nuclear or constituent-quark groundstate). This is understandable because as was analyzed in Ref. [24], when $\mu < \mu_c$, the ground state at finite $\mu$ remains the vacuum and the physical quantities should be kept unchanged. When $\mu > \mu_c$, it can be seen from Fig. 1 that the bulk viscosity decreases monotonously as $\mu$ increases and no sharp peak is observed at finite $\mu$. This behavior of bulk viscosity at finite $\mu$ does not show a first-order chiral phase transition (or a rapid cross-over), which is quite different from that at finite temperature. The result of such a behavior is qualitatively consistent with the result obtained in Ref. [13]. Here, we would like to point out that the NJL model [42, 43] is the most widely adopted nonperturbative QCD model in the study of finite density QCD at present. The NJL model can phenomenologically incorporate DCSB but does not have quark confinement, whereas the DSEs approach can simultaneously incorporate DCSB and quark confinement. Over the past few years considerable progress has been made in the framework of the rainbow-ladder approximation of the DSEs approach at finite temperature and finite density (see, e.g., the review article Ref. [18]). Due to this we naturally expect that the DSEs approach can be applied to the study of bulk viscosity of QCD. This is our main motivation to use the DSEs approach to calculate the bulk viscosity of QCD at finite density in this work. Finally, it should be noted that in obtaining the result of bulk viscosity in this paper, we have made use of Eq. (15). However, as was pointed out in Refs. [20, 30], Eq. (15) is obtained under the approximation that one neglects the $\mu$ dependence of the dressed gluon propagator. This approximation is valid only in small $\mu$ region. Therefore, if one applies Eq. (15) to the case of large $\mu$, one should be cautious.

To summarize, in this letter, based on the Kubo’s formula and the QCD low energy theorem, we have derived a direct formula for calculating the bulk viscosity of QCD at finite chemical potential $\mu$ and zero temperature. According to this formula, the bulk viscosity at finite $\mu$ is totally determined by the dressed quark propagator at finite $\mu$. Then, under the rainbow approximation of DSEs and ignoring the $\mu$-dependence of the dressed gluon prop-
agator, we use the meromorphic quark propagator proposed in Ref. [22] to calculate the bulk viscosity at finite \( \mu \). It is found that when \( \mu \) is below a critical value \( \mu_c \), the bulk viscosity equals its vacuum value. When \( \mu > \mu_c \), the bulk viscosity decreases monotonously as \( \mu \) increases, and no sharp peak is observed at finite \( \mu \). This means that in our model calculation in the framework of DESs approach the bulk viscosity at finite \( \mu \) does not show a first-order chiral phase transition (or a rapid cross-over), which is quite different from that at finite temperature.

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