Are Leptons as elementary as Quarks?

Aharon Davidson

Physics Department, Ben-Gurion University of the Negev,
Beer-Sheva 84105, Israel
E-mail: davidson@bgu.ac.il

ABSTRACT: Nucleons and electrons were once considered elementary particles, a role nowadays taken by quarks and leptons. Here, mainly at the group theoretical level, we examine the unorthodox idea that nucleons and electrons share the same level of compositeness after all. We do it by first trading color $SU(3)_C$ for color/leptocolor $SU(3)_C \times SU(3)_{\ell C}$ confining gauge symmetry. Standard model leptons, equipped with inherited Yukawa couplings, make then their appearance at the intermediate (gauge protected) $SU(3)_C \times SU(3)_L \times SU(3)_R$ trinification stage as three pre-lepton composites. The addition of an exclusively prescribed (on anomaly free and Pauli exclusion grounds) spontaneously broken $SU(3)_F$ horizontal link to the unification chain, in the spirit of "One three to rule them all", strikingly preserves the anti-symmetric structure of the single family fermionic wave function. The threefold quark/lepton (spectators + composites) flavor chiral representation is then necessarily supplemented by a trinification singlet composite Majorana neutrino. The scheme serendipitously predicts a novel anomaly free lepto/dark portal.
1 Introduction

A long time ago, in what seems to be another era, nucleons (protons and neutrons) and electrons were considered indivisible elementary particles. Primarily owing to their discretely quantized electric charges (Millikan integer sequence), they govern atomic physics, and consequently the structure of matter at the Bohr scale and beyond. Quarks have entered the game through the group theoretical “eightfold way” path, and their strong local $SU(3)_C$ color interactions have been proposed and experimentally confirmed a bit later. The integer charged baryons and mesons have eventually become composites of fractionally charged confined quarks.

The leptons, on the other hand, have fully preserved their fundamental point-like perception during the quark revolution. Moreover, together with quarks they share the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ electro/nuclear model, and in particular, contribute their part to cancel the otherwise harmful ABJ anomalies. The latter feature takes us beyond the standard model, crossing the gauge desert all the way to grand unification. While staying essentially the same for $SU(5)$, anomaly cancelation becomes automatic for $SO(10)$ and $E(6)$.

By consistently embedding $SU(3)_C$ within a parent $SU(4)_{PS}$, the unity of quarks and leptons has been formally demonstrated in Pati-Salam’s ”Lepton number as the fourth color” [1]. Numerically speaking, $SU(4)_{PS}$ unitarity has been simply translated into the fundamental sum rule $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} - 1 = 0$. While such an imaginative idea has forcefully opened the door for the left-right symmetric grand unification variants $SO(10) \subset E(6)$, once the spontaneous symmetry breaking $SU(4)_{PS} \rightarrow SU(3)_C \times U(1)_{B-L}$ takes place, quarks and leptons are not treated on equal symmetric footing any more. After all, one can always build an integer out of thirds, but cannot vice versa build a third out of integers.

The idea of accompanying $SU(3)_C$ by extra $SU(3)$ factors (with or without an underlying discrete symmetry) is not new, with $E(6)$-embeddable [2–5] trinification [6–14]

$$G_{tr1} = SU(3)_C \times SU(3)_L \times SU(3)_R$$ \hspace{1cm} (1.1)
being the prototype example. Other examples include LR-symmetric chiral color $SU(3)_{CL} \times SU(3)_{CR}$ [15, 16], confining hypercolor (technicolor) $SU(3)_{HC}$ of various sorts (notably the Rishon model [17, 18] and some of its derivatives [19–21]), spontaneously broken flavor $SU(3)_F$ to account for the fermion threefold family replication [22, 23], and even leptocolor $SU(3)_{\ell C}$ aimed to enhance quark/lepton symmetry [24–29]. On symmetry grounds, with "lepton number as the fourth color" in mind, there is no group theoretical reason why not considering "baryon number as the fourth leptocolor" as well. This would bring nucleons and electrons, which anyhow share integer electric charges, back to the same level of elementariness (actually compositeness now), as used to be. Such an idea can be minimally realized within the framework of local $SU(3)_C \times SU(3)_{\ell C}$ symmetry. In fact, the latter symmetry has already been introduced in the literature [24–29]. However, the orthodox role of $SU(3)_{\ell C}$ as a spontaneously broken symmetry at heavy mass scale is herby replaced by a totally different scenario. Namely, an unbroken symmetry, characterized by a leptocolor scale substantially larger than the ordinary color scale $\Lambda_{\ell C} \gg \Lambda_C$ (with a gauge coupling constant $g_{\ell C} \gtrsim g_C$ at $\Lambda_{\ell C}$). Leptocolor is expected to enter its short range confining mode much earlier than $SU(3)_C$, at the stage where trinification prevails. As long as it stays unbroken, the latter local symmetry is capable of gauge protecting a flavor-chiral combination of spectator quarks and composite leptons.

Composite model constructions are heavily restricted by 't Hooft’s anomaly matching equations [30–33]. General solutions are hard to find, and the more so solutions with integer coefficients (even in cases where the restrictive Apellquist-Carazzone decoupling condition [34] is not applicable). In fact, 't Hooft himself has pointed out that QCD with more than two flavors is not natural, and made the interesting remark that to construct tenable models with complete naturalness for elementary particles one may need more types of confining gauge theories besides QCD (an example is provided in [35]). At any rate, contrary to 't Hooft’s 3-quark model, where the associated $SU(3)_L \times SU(3)_R \times U(1)_V$ flavor symmetry is notably global, the latter symmetry is almost fully gauged in our case, except for the vector $U(1)_V$ remnant. Unfortunately, our model is not natural in 't Hooft sense, with the problematic matching equations being of course those involving $U(1)_V$. However, within the context of a flavor-chiral model (unlike QCD), it makes it a bit easier for us to assume that fermion masslessness is protected by anomaly free gauge invariance. While chiral symmetry suffices for this purpose, anomaly cancellation is stronger, and furthermore serves as a field theoretical consistency tool.

The threefold family structure cannot and does not stay out of the game. It integrates in naturally and inductively by means of adding another local $SU(3)_F$ link to the $SU(3)$ chain [28, 36–42]. And quite remarkably, it does it without upsetting the overall Pauli anti-symmetric structure of the lepton wave function, a phenomenon which we have already encountered at the single family level.
2 Composite leptons, spectator quarks

Let our $q \leftrightarrow \bar{\lambda}$ symmetric, anomaly free, flavor-chiral representation of left handed fermions transform as

$$\psi_L = C \ell C L R$$

under the local quartification gauge group

$$G_{quad} = [SU(3)_C \times SU(3)_{\ell C}] \times [SU(3)_L \times SU(3)_R] .$$

Note that $q \leftrightarrow \bar{\lambda}$ under $\{C \leftrightarrow \ell C, L \leftrightarrow R\}$, so that one cannot really tell quarks from pre-leptons at this stage. The representation is $C \ell C$ and $LR$ real, separately, yet overall complex, forming a moose chain. Conspicuous by its absence is the $(1, 1; 3^*, 3)$ leptonic piece which has been bifurcated into the $\lambda + \bar{\lambda}$ pair. Starting with $G_{quad}$, we end up with unbroken $SU(3)_C \times SU(3)_{\ell C} \times U(1)_Q$ once the dust settles down. The electric charge reads

$$Q = T_3L + T_3R + \frac{1}{2} (Y_L + Y_R) ,$$

so that $Q[q] + Q[\lambda] = 0$. At any rate, we keep the option of inductively enlarging $G_{quad}$, if needed, when proceeding beyond the scope of the single family level.

In analogy with the color singlet baryons which are made of three quarks, we now attempt to construct leptocolor singlet composites using

$$\lambda_L(1, 3^*; 3, 1) , \quad \lambda_R(1, 3^*; 1, 3)$$

as our building blocks. Here, invoking the traditional basis, $f_R$ denotes the right handed Weyl partner of the left handed $\bar{f}_L$ (they transform into each other under charge conjugation). In turn, $\lambda_L$ and $\lambda_R$ transform alike, that is $\sim 3^*$ under $S(3)_{\ell C}$. We are after a set of left handed composites which would compensate for the $SU(3)_{L,R}$ triangle anomalies contributed by the quark spectators. The fact that $\lambda_R \lambda_R \lambda_R$ and $\lambda_L \lambda_L \lambda_R$ are right-handed leaves us with the two left-handed candidates:

(i) $\lambda_L \lambda_L \lambda_L$: From $SU(2)_L \times SU(2)_R^1$ Lorentzian point of view, we can have $(2+2+4, 1)$, spinors nicely included (in fact, there are two of them). But this turns out irrelevant given that under local $SU(3)_L \times SU(3)_R$, the resulting configuration $(1 + 8 + 8 + 10, 1)$ does not contain triplets nor anti-triplets. Put it in other words, no standard model leptonic candidates are available. Note in passing that, unlike complex $(10, 1)$, the real representations $(1 + 8 + 8, 1)$ cannot anyhow be protected by gauge invariance. The $(1, 1)$ piece will play a role in a later stage.

(ii) $\lambda_L \lambda_R \lambda_R$: The Lorentz representation $(2, 1 + 3)$ also contains the mandatory $(2, 1)$ piece, but unlike in the previous case, one faces now $(3, 3^* + 6)$ under $SU(3)_L \times SU(3)_R$,
including the mandatory \((3, 3^*)\) representation. With this in hands, we can now complete our composite level lepton identification, namely

\[
\ell_L + \bar{\ell}_L = (1, 1; 3, 3^*) \ .
\]  

(2.5)

Together with the quarks and anti quarks, hereby treated as spectators, the leptocolor singlets close upon

\[
\psi_{\text{tri}}^L = \begin{array}{c|c|c|c}
\hline
& C & L & R \\
\hline
q_L & 3 & 3^* & 1 \\
\hline
\bar{q}_L & 3^* & 1 & 3 \\
\hline
\ell_L + \bar{\ell}_L & 1 & 3 & 3^* \\
\hline
\end{array}
\]  

(2.6)

forming a complex single family anomaly free, that is \(3 \times (-1) + 9 \times 0 + 3 \times 1 = 0\), trinification representation. It is important to note that the \(SU(3)_R\) triangle anomaly \(A(6) = -7A(3^*)\) makes the \((1, 1; 3, 6)\) representation irrelevant for our anomaly cancelation purposes. Had we incorporated \((1, 1; 3, 6)\), then owing to \(A_{3L} + A_{3R} = 3(2 + 7) \neq 0\), there would be no way to simultaneously cancel both \(SU(3)_L^3\) anomalies \(A_{3L}, A_{3R}\), respectively. As long as \(G_{\text{tri}}\) stays unbroken, this complex set is protected and remains massless. Other composite states, such as (say) \(\lambda_L \lambda_L \lambda_L\) which belongs to \(SU(3)_L/Z_3\), are not protected from picking up the large \(\Lambda_{\text{IC}}\) mass scale.

The Yukawa interactions, introduced at the fundamental Lagrangian level, forcefully reappear at the intermediate trinification stage. As depicted in Fig.(1), spectator quarks and composite leptons share the one and the same Yukawa coupling diagrams. As far as the leptons are concerned, only one pre-lepton is involved in the coupling, while its two companions stay inert. A similar situation occurs in the leading order of beta decay Fig.(2).

\[\text{Figure 1.} \quad \text{Yukawa at trinification: Spectator quarks and composite leptons share similar type Yukawa diagrams, involving the one and the same Higgs scalar. Only one pre-lepton is actively involved, while two others stay inertly out of the game.}\]
Figure 2. Beta decay as depicted at the fundamental quark/pre-lepton level: The weak gauge boson mediators $W_{L}^{\pm}$ are members of the $SU(3)_{L}$ adjoint representation.

3 Threefold family structure

Once the single quark/lepton family model has been established, the obvious challenge is to incorporate the underlying threefold family structure, and do it naturally without any superfluous replication. With the underlying motto of "One three to rule all flavors", we are driven to the quite traditional step of introducing an extra local $SU(3)_{F}$ group factor ($F$ stands for Family). One has to be careful though, especially when dealing with composite models, to ensure the overall anti-symmetry of the lepton wave function under the interchange of the two identical $\lambda_{L}$s. This is however not automatic and should be regarded as a consistency test for the model under construction.

Without losing generality, let $\lambda_{L} \sim 3^{*}$ under $SU(3)_{F}$, so that

$$\lambda_{L}\lambda_{L}\lambda_{L} \sim 3^{*} \times 3^{*} \times 3^{*} = 1 + 8 + 8 + 10^{*}.$$  \hspace{1cm} (3.1)

There are still two possibilities regarding the assignments of $\bar{\lambda}_{L}$ (or equivalently of $\lambda_{R}$). To choose between these two, one needs to first check the associated transformation laws:

(i) $\bar{\lambda}_{L} \sim 3$ ($\lambda_{R} \sim 3^{*}$) implies

$$\lambda_{L}\lambda_{R}\lambda_{R} \sim 3^{*} \times 3^{*} \times 3^{*} = 1 + 8 + 8 + 10^{*}.$$  \hspace{1cm} (3.2)

(ii) $\bar{\lambda}_{L} \sim 3^{*}$ ($\lambda_{R} \sim 3$) implies

$$\lambda_{L}\lambda_{R}\lambda_{R} \sim 3^{*} \times 3 \times 3 = 3 + 3 + 6^{*} + 15.$$  \hspace{1cm} (3.3)

And since the left handed composites must include a 3 (or a 3*) to consistently enter the family picture, it is only the second option which can deliver. The main bonus is fixing the inner structure of the tenable composite fermion, namely choosing $\lambda_{L}\lambda_{R}\lambda_{R}$ rather than $\lambda_{L}\lambda_{L}\lambda_{L}$. It is by no means trivial that the very same conclusion was drawn previously,
where only $\lambda_L \lambda_R \lambda_R$ was shown to transform as required, that is via $(3, 3^*)$, under the electro/weak $SU(3)_L \times SU(3)_R$. Still, see eq.(3.3), there are two $\lambda_L \lambda_R \lambda_R$ triplet candidates. Using $SU(3)_F$ spinor indices, they are

\begin{equation}
\text{anti-symmetric: } \epsilon^{iqr} \epsilon_{pqk} \lambda_{Lq} \lambda^l R \lambda^k R ,
\end{equation}

\begin{equation}
\text{symmetric: } \lambda_{Lp} (\lambda^i R \lambda^p R + \lambda^p R \lambda^i R) .
\end{equation}

The choice between these two candidates will be eventually made on symmetry (or anti-symmetry) grounds when the full picture will be in front of us.

A word of caution is now in order. Note that $\lambda_L \lambda_R \lambda_R$ is expected to transform according to the $(\frac{1}{2}, 0)$ representation under Lorentz group $SU(3)_L \times SU(3)_R \dagger$. This means that $\lambda_R \lambda_R$ must be treated as a Lorentz scalar, that is $\epsilon^{ij} \lambda^l R \lambda^j R$ (here $i, j$ are $SU(3)_R$ spinor indices). To sharpen this point, with the focus on its underlying antisymmetric Lorentz structure, a somewhat better notation for the composite is perhaps $\lambda_L [\lambda_R \lambda_R]$. The same should hold of course for the left handed composite fermions $\lambda_L \lambda_L \lambda_L$ as well, whose inner Lorentz structure is better manifested via $\lambda_L [\lambda_L \lambda_L]$. At any rate, recalling that $\lambda_L$ and $\lambda_L$ are both $SU(3)_F$ anti-triplets, there seems to be just one way to cancel the potentially newly induced ABJ anomalies. Quarks and anti-quarks are called to the rescue, and they do it quite naturally by means of

\begin{equation}
\lambda_L, \lambda_L \sim 3^* \implies q_L, \bar{q}_L \sim 3 ,
\end{equation}

to be regarded a fundamental feature of the quark/lepton symmetry.

While $\lambda_L, \lambda_L \sim 3^*$, notice that the emerging composite lepton state $\lambda_L \lambda_R \lambda_R$, previously identified as $\ell_L + \bar{\ell}_L$, transforms under $SU(3)_F$ precisely like $q_L, \bar{q}_L$, that is $\lambda_L \lambda_R \lambda_R \sim 3$. In turn, with the addition of $SU(3)_F$, the attractive trinification scheme, dynamically encountered at the single family level following leptoquark confinement, is apparently in jeopardy. The accumulated anomaly of twenty seven $SU(3)_F$ triplets must be taken care of in order to gauge protect the masslessness of the three $\{q_L, \bar{q}_L, \ell_L + \bar{\ell}_L\}$ flavor-chiral standard trinification families. One is thus after a leptocolor singlet, $SU(3)_F$ non-trivial composite fermion which, on self consistency grounds, must further transform as a total multi-singlet $(1, 1, 1)$ under $SU(3)_C \times SU(3)_L \times SU(3)_R$.

4 Lepto/Dark portal

It comes with no surprise that precisely such a composite fermion does exist in the model and is at our service (see anomaly details below). We refer of course to $\Lambda_L \equiv \lambda_L \lambda_L \lambda_L \sim 10^*$ which has already made its appearance in Eq.(3.1). The fact that $\Lambda_L$ is totally symmetric in its $SU(3)_F$ spinor indices is crucial. By the same symmetry token, it chooses for us the symmetric $\lambda_L \lambda_R \lambda_R$ triplet candidate Eq.(3.4b). The anti-symmetric structure of the lepton wave function under identical pre-lepton interchange, encountered already at the single family model, is preserved (it could not have been done differently) in the modified threefold family version.
Figure 3. Lepto/Dark portal: $\phi\Lambda$ Yukawa coupling involves the same Higgs scalar which couples (and generates masses upon developing VEV) to standard model fermions. $\phi_S \sim 6$ under $SU(3)_F$ couples symmetrically in family space.

In 4-dim, the $SU(n)$ triangle anomaly table $[43–45]$ offers the relation

$$A(\Box\Box\Box) = \frac{1}{2}(n + 6)(n + 3)A(\Box) ,$$

(4.1)

where the box stands for the fundamental representation, and in general we have $A(r^*) = -A(r)$. In particular for $n = 3$, the above formula reads

$$A(10^*) + 27A(3) = 0 ,$$

(4.2)

thereby elevating $\Lambda_L = \lambda_L\lambda_L\lambda_L$ to the level of a mandatory supplement to the threefold quark/lepton family (spectators + composites) flavor chiral representation. Altogether, the latter is given explicitly by

$$\psi_L^{tri} = \begin{array}{c|c|c|c|c}
\hline
\text{ } & C & L & R & F \\
\hline
q_L & 3 & 3^* & 1 & 3 \\
\hline
\bar{q}_L & 3^* & 1 & 3 & 3 \\
\hline
\ell_L + \bar{\ell}_L \equiv \lambda_L\lambda_R\lambda_R & 1 & 3 & 3^* & 3 \\
\Lambda_L \equiv \lambda_L\lambda_L\lambda_L & 1 & 1 & 1 & 10^* \\
\hline
\end{array}$$

(4.3)

The overall anomaly is now $9 \times 1 + 9 \times 1 + 9 \times 1 - 1 \times 27 = 0$. Had we started from $SU(n)_F$, thereby facing the anomaly equation $3 \times 9 \times 1 - 1 \times \frac{1}{2}(n + 6)(n + 3) = 0$, the special case of $n = 3$ would have been singled out on anomaly free and Pauli exclusion grounds. This establishes a novel group theoretical interplay between the single fermionic family and its threefold replication.

It is only when $SU(3)_F$ gets broken down, and leaves no residual gauge symmetry to protect $\Lambda_L$ from acquiring the heavy mass scale involved, that one finally faces the familiar $E(6)$-unifiable trinification scheme (threefold ‘superfluous’ replication included). Be reminded that Lorentz invariance allows for two types of mass terms. Dirac mass term $f_{1L}^\dagger f_{2R}$ involves two opposite helicity Weyl fermions $f_{1L}$ and $f_{2R}$ which must then carry the same conserved charges $Q(f_{1L}) - Q(f_{2R}) = 0$. Majorana mass term, say $f_{1L}f_{2L}$,
involves two (may be the same) same helicity Weyl fermions, with \( Q(f_{1L}) + Q(f_{2L}) = 0 \). By acquiring a heavy Majorana mass \( m_{\Lambda} \Lambda_L \Lambda_L \) at this level (no \( \Lambda_R \) at our disposal to produce a Dirac mass as well), \( \Lambda_L \) plays here a role similar to that of the \( \nu_R \) in the standard model. In fact, appreciating its electro/nuclear neutrality, it may trigger the very same neutrino seesaw mechanism provided it can couple to standard model particles. A possible (there may be others) tree level mechanism by which \( \Lambda_L \) can acquire its Majorana mass is suggested in Fig.(4). Owing to the \( SU(3)_F \) multiplication feature \( 10 \times 10 \subset 6 \times 6 \times 6 \), it involves exactly three Yukawa couplings. The relevant inner fermion lines are associated with heavy Majorana neutrinos (there are two per trinification family) which live outside minimal \( G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \). Note that \( \Lambda_L \) Yukawa mix with both heavy LR components \( (2_1, 2_{-1}) \) and \( (1_{-2}, 1_{2}) \) of \( \ell_L + \bar{\ell}_L \). Two of the three scalar lines end with a \( \langle 2_{-1}, 2_1 \rangle \) VEV, while the third one ends with a \( \langle (1_2, 1_{-2}) \rangle \) VEV, respectively.

![Figure 4](image)

**Figure 4.** A possible dark mass origin: Depicted is a tree level diagram, involving exactly three Yukawa couplings, by which the trinification singlet fermion \( \Lambda_L \) acquires its Majorana mass.

To refresh the Higgs mechanism scheme in the presence of the newly introduced \( SU(3)_F \) group, we first observe that the Yukawa interactions are now governed by \( \phi(1, 1; 3^*, 3; 3^* + 6) \). They couple symmetrically (\( \phi_S \sim 6 \)) and/or anti-symmetrically (\( \phi_A \sim 3^* \)) in the family space. Given the odd number of families, \( \phi_A \) is not allowed to Yukawa couple by itself, as otherwise one family stays massless and the other two degenerate. This makes the symmetric Yukawa \( \phi_S \) couplings essential. The novel ingredient is that the ordinary Yukawa couplings, depicted in Fig.(1), are now supplemented, as is demonstrated in Fig.(3), by \( \phi_S(\ell_R + \bar{\ell}_R)\Lambda_L \). This can be interpreted as a lepto/dark portal. After all, \( \Lambda_L \) is a Majorana heavy trinification singlet, striped from all electro/nuclear interactions, capable of seesaw mixing (\( \theta \sim m_\ell/m_\Lambda \)) with standard model fermions.

5 Epilogue

While quarks and leptons are equal rights members of the standard electro/nuclear theory, they are not really treated on symmetrical footing. Lepton number can serve as the fourth "color", but it is harder to imagine baryon number serving as the fourth "leptocolor". The fact that an integer can be built out of thirds, but not vice versa, naively suggests that leptons (integer charges) are made out of three pre-leptons (fractional charges), such that one would not be able to tell quarks from pre-leptons at the fundamental level. A group theoretical realization of this idea is provided in this paper, with \( E(6) \)-unifiable
trinification emerging in the intermediate stage, and with the threefold family structure inductively incorporated. Thus, to answer the question in the title in a challenging (not to say provocative) way, leptons need not be, at least theoretically, as elementary as quarks.

A few comments are finally in order:

(i) In analogy with LR symmetry, one may anticipate $C\ell C$ symmetry to be spontaneously violated, such that leptocolor becomes the first interaction to confine. Until such a symmetry breaking mechanism is introduced, one has to explicitly assume $gE \gtrsim gC$. (ii) We can say nothing compelling at this stage about the quark/lepton mass spectrum and mixings. While we rely on the dynamical emergence of the intermediate trinification phase, we have not incorporated any particular trinification model (from existing ones [6–14] or beyond) for probing fine details.

(iii) It is not clear what ingredients can be added to composite models in general, and to our model in particular, in order to achieve (or at least tenably avoid) the restrictive 't Hooft naturalness requirement.

(iv) In addition, we have not investigated at this stage what could be the benefits, if at all, of incorporating supersymmetry. We leave it for a future investigation.

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