Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing

E N Saridakis

Department of Physics, University of Athens, GR-15771 Athens, Greece
E-mail: msaridak@phys.uoa.gr

Received 20 December 2007
Accepted 20 March 2008
Published 15 April 2008

Online at stacks.iop.org/JCAP/2008/i=04/a=020
doi:10.1088/1475-7516/2008/04/020

Abstract. We apply the bulk holographic dark energy in general 5D two-brane models. We extract the Friedmann equation on the physical brane and we show that in the general moving-brane case the effective 4D holographic dark energy behaves as a quintom for a large parameter-space area of a simple solution subclass. We find that $w_\Lambda$ was larger than $-1$ in the past while its present value is $w_\Lambda_0 \approx -1.05$, and the phantom bound $w_\Lambda = -1$ was crossed at $z_p \approx 0.41$, a result in agreement with observations. Such a behavior arises naturally, without the inclusion of special fields or potential terms, but a fine-tuning between the 4D Planck mass and the brane tension has to be imposed.

Keywords: dark energy theory, cosmology with extra dimensions, cosmological applications of theories with extra dimensions

ArXiv ePrint: 0712.2672
Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing

## Contents

1. Introduction  
2. Formulation of holographic dark energy in a general bulk  
3. Holographic dark energy in 5D braneworld models with moving branes  
4. Discussion–conclusions  
   Acknowledgments  
   References

### 1. Introduction

Holographic dark energy [1]–[4] is a recently developed ingenious idea of explaining the observed Universe acceleration [5]. It arises from the cosmological application [6] of the more fundamental holographic principle [7, 8]. Although there are some objections about the applicability of holography to a cosmological framework [9], holographic dark energy has opened new research directions, revealing the dynamical nature of vacuum energy by relating it to cosmological volumes. The background on which it is based is black hole thermodynamics [10, 11] and the connection between the UV cutoff of a quantum-field theory, which is related to vacuum energy, and a suitable large distance of the theory [12]. This connection, which was also known from AdS/CFT correspondence, proves to be necessary for the applicability of quantum-field theory in large distances. The reason is that, while the entropy of a system is proportional to its volume, the black hole entropy is proportional to its area. Therefore, the total energy of a system should not exceed the mass of a black hole of the same size, since in this case the system would collapse to a black hole violating the second law of thermodynamics. In holographic statistical physics terms this is equivalent to the exclusion of those degrees of freedom that would collapse. When this approach is applied to the Universe, the resulting vacuum energy is identified as holographic dark energy.

Until now, almost all works on the subject have been formulated in the standard 4D framework. However, brane cosmology, according to which our Universe is a brane embedded in a higher-dimensional spacetime [13, 14], apart from being closer to a higher-dimensional fundamental theory of nature, has also great phenomenological successes [15]. In our recent works [16] we presented a generalized and restored holographic dark energy in the braneworld context. The basic argument was that, in a higher-dimensional spacetime, it is the bulk space which is the natural framework for the cosmological application (concerning dark energy) of the holographic principle, and not the lower-dimensional brane-Universe. This is obvious since it is the maximally dimensional subspace that determines the properties of quantum-field or gravitational theory (such as cutoff’s and vacuum energy), and this holds even if we consider brane cosmology as an intermediate limit of an even higher-dimensional fundamental theory of nature. To be more specific we recall that, in braneworld models, where the spacetime dimension is more than
four, black holes will in general be $D$-dimensional [10, 11], no matter what their 4D effective (mirage) effects could be. Therefore, although the holographic principle is itself applicable to arbitrary dimensions [7, 17] its cosmological application concerning dark energy should be considered in the maximal uncompactified space of the model, i.e. in the bulk. Subsequently, this bulk holographic dark energy gives rise to an effective 4D dark energy with ‘inherited’ holographic nature, and this one is present in the (also arisen from the full dynamics) Friedmann equation of the brane. One can in general acquire either different or exactly identical 4D behavior, compared to that obtained in the conventional 4D literature [1]–[4]. However, even in the second case, the physical interpretation is radically different.

In our previous works [16] we used, as a specific example, a general braneworld model with one brane. The arbitrary large extra dimension of this case imposes no restrictions on the application of bulk holographic dark energy; thus we recovered all the results of the 4D literature. In the present paper we are interested in investigating the case where the bulk is finite, and for this purpose we use the well-explored two-brane model, where the branes constitute the boundaries of the extra dimension [14], [18]–[20]. From the first moment it becomes clear that if the branes are steady, i.e. the extra dimension has a constant size, then the bulk dark energy loses its dynamical holographic nature. The reason is that, applying the holographic dark energy arguments, we cannot consider an arbitrary large bulk black hole in this case. Although we could still use non-spherical exotic solutions such as black rings and black ‘cigars’ [21], or highly rotational and/or charged black holes [22], if we desire to maintain the simplicity and universality which lies at the basis of holographic dark energy we must remain in the aforementioned framework, i.e. preventing the horizon of a spherical bulk black hole being larger than half the interbrane distance. It seems that holographic dark energy is in contradiction with the brane stabilization mechanism [23].

In this work we examine braneworld models where the bulk is finite but with moving boundaries (branes). In this case holographic dark energy is applicable and the corresponding cosmological length is the interbrane distance. However, one still needs an additional fine-tuning assumption, arising from the time variation of the model parameters. We want to study the behavior of the effective 4D holographic dark energy, and especially its dependence on the metric scale factor. The rest of the paper is organized as follows. In section 2 we present the holographic dark energy in the bulk and in section 3 we apply it to a general two-brane model in $4 + 1$ dimensions. Finally, in section 4 we discuss the physical implications of the obtained results.

2. Formulation of holographic dark energy in a general bulk

In this section we display the basic results of the bulk holographic dark energy, formulated in [16]. The mass $M_{BH}$ of a spherical and uncharged $D$-dimensional black hole is related to its Schwarzschild radius $r_s$ through [11, 24]

$$M_{BH} = r_s^{D-3}(\sqrt{\pi}M_D)^{D-3}M_D\frac{D-2}{8\Gamma((D-1)/2)}, \quad (1)$$
where the $D$-dimensional Planck mass $M_D$ is related to the $D$-dimensional gravitational constant $G_D$ and the usual four-dimensional Planck mass $M_p$ through

\begin{align}
M_D &= G_D^{-1/(D-2)}, \\
M_p^2 &= M_D^{D-2}V_{D-4},
\end{align}

with $V_{D-4}$ the volume of the extra-dimensional space [11].

If $\rho_{AD}$ is the bulk vacuum energy, then application of holographic dark energy in the bulk gives

\begin{equation}
\rho_{AD} \text{Vol}(S^{D-2}) \leq r^{D-3}(\sqrt{\pi}M_D)^{D-3}M_D \frac{D - 2}{8\Gamma((D - 1)/2)},
\end{equation}

where $\text{Vol}(S^{D-2})$ is the volume of the maximal hypersphere in a $D$-dimensional spacetime, given by

\begin{equation}
\text{Vol}(S^{D-2}) = A_{D} r^{D-1},
\end{equation}

with

\begin{align}
A_D &= \frac{\pi^{((D-1)/2)}}{((D - 1)/2)!}, \\
A_D &= \frac{((D - 2)/2)!}{(D - 1)!} 2^{D-1} \pi^{(D-2)/2},
\end{align}

for $D - 1$ being even or odd, respectively. Therefore, by saturating inequality (3), introducing $L$ as a suitable large distance and $c^2$ as a numerical factor, the corresponding vacuum energy is, as usual, viewed as holographic dark energy:

\begin{equation}
\rho_{AD} = c^2(\sqrt{\pi}M_D)^{D-3}M_D A_{D}^{-1} \frac{D - 2}{8\Gamma((D - 1)/2)} L^{-2}.
\end{equation}

As was mentioned in [16], the ‘suitable large distance’ which was used in the definition of $L$ in (6) should be the Hubble radius, the particle horizon or the future event horizon [1,4,25], with the last ansatz being the most appropriate. However, in the case of a finite bulk with varying size it is obvious that $L$ should be just this bulk size, with the reason being again the foundations of holographic dark energy which prevent the use of a length larger than that. On the other hand, if the future event horizon is smaller than the bulk size, then one should use that instead of the bulk size. In this case our braneworld model does not ‘feel’ the other bulk boundary and the picture is equivalent to the single-brane model investigated in [16].

Finally, let us make a comment concerning the sign of bulk holographic dark energy. In the original Randall–Sundrum model [14] the bulk cosmological constant should be negative in order to acquire the correct localization of low-energy gravity on the brane. Such a negativity is not a fundamental requirement and is not necessary in more complex, non-static models, like the one of the application of the next section. However, as was mentioned in [16] and generally speaking, holographic dark energy is a simple idea of bounding the vacuum energy from above. It would be a pity if, despite this effort, one could still have a negative vacuum energy unbounded from below, because then holographic dark energy would lose its meaning. If holography is robust then one should reconsider the case of a negative bulk cosmological constant (although subspaces, such
Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing as branes, could still have negative tensions). Another possibility is to try to generalize holographic dark energy to negative values, in order to impose a negative bound. The subject is under investigation.

3. Holographic dark energy in 5D braneworld models with moving branes

We are interested in applying the bulk holographic dark energy in general 5D braneworld models where the bulk is bounded by two branes. The inclusion of a second brane is probably the best way of eliminating possible ‘naked’ metric singularities, thus allowing for more complex and realistic models [14], [18]–[20]. We consider an action of the form:

$$S = \int d^4x \sqrt{-g} (M_5^3 R - \rho_{A5}) + \sum_{i=1,2} \int_{br_i} d^4x \sqrt{-\gamma} (\mathcal{L}_{br_i}^{\text{mat}} - V_i).$$

(7)

In the first integral $M_5$ is the 5D Planck mass, $\rho_{A5}$ is the bulk cosmological constant, which is identified as the bulk holographic dark energy, and $R$ is the curvature scalar of the 5D bulk spacetime with metric $g_{AB}$. The second term corresponds to two $(3 + 1)$-dimensional branes, which constitute the boundary of the 5D space. $\gamma$ is the determinant of the induced 4D metric $\gamma_{\alpha\beta}$ on them, $V_i$ stand for the brane tensions and $\mathcal{L}_{br_i}^{\text{mat}}$ is an arbitrary brane matter content [26].

As usual the two branes are taken parallel, $y$ denotes the coordinate transverse to them and we assume isometry along three-dimensional $x$ slices including the branes. For the metric we choose the conformal gage [18, 19]:

$$ds^2 = e^{2B(t,y)} (-dt^2 + dy^2) + e^{2A(t,y)} dx^2.$$ 

(8)

This metric choice, along with the residual gage freedom $(t, y) \rightarrow (t', y')$ which preserves the 2D conformal form, allows us to ‘fix’ the positions of the branes. Without loss of generality we can locate them at $y = 0, 1$, having in mind that their physical distance is encoded in the metric component $B(t, y)$, and at a specific time it is given by [18, 19]

$$L_5(t) \equiv \int_0^1 dy \sqrt{g_{55}} = \int_0^1 dy \ e^{B(t,y)},$$

(9)

a quantity that is invariant under the residual gage freedom in our coordinates. The reason we prefer the metric (8), instead of the usual form in the literature, is that in the latter case the brane positions are, in general, time-dependent and the various boundary conditions are significantly more complicated. Thus, our coordinates are preferable for numerical calculations, despite the loss of simplicity in the definitions of some quantities. Eventually, the physical interpretation of the results is independent of the coordinate choice.

The non-trivial 5D Einstein equations consist of two dynamical:

$$\ddot{A} - A'' + 3 \dot{A}^2 - 3A^2 = \frac{2}{3M_5^3} e^{2B} \rho_{A5},$$

(10)

$$\ddot{B} - B'' - 3 \dot{A}^2 + 3A^2 = -\frac{1}{3M_5^3} e^{2B} \rho_{A5},$$

(11)
Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing

and two constraint equations:

\[-A'\dot{A} + B'\dot{A} + A'\dot{B} - \dot{A} = 0,\]  
\[2A'' - A'B' + A'' - A^2 - \dot{A}\dot{B} = -\frac{1}{3M_5^2} e^{2B} \rho_{\Lambda 5},\]

where primes and dots denote derivatives with respect to $y$ and $t$, respectively. It is easy to show that the constraints are preserved by the dynamical equations.

We consider a brane-Universe, which as usual is identified with the brane at $y = 0$, containing a perfect fluid with equation of state $p = w \rho$ (in the following we omit the index 0 for the physical brane quantities and we keep the index 1 for the ones on the brane at $y = 1$). For the hidden brane we consider for simplicity just a brane tension, although we could also consider some matter-field content \[27\]. The reason we use a second brane is to eliminate possible ‘naked’ metric singularities. Therefore, assuming $S^1/Z_2$ symmetry across each brane we restrict our interest only to the interbrane space.

Integrating on a small $y$ interval around the branes and using the boundary terms in the action we obtain the following junction (Israel) conditions:

\[ [A']_0 = -\frac{1}{3M_5^2} e^{B_0}(\rho + V), \]
\[ [B']_0 = \frac{1}{3M_5^2} e^{B_0}(2\rho + 3p - V) \]

for the physical brane and

\[ [A']_1 = \frac{1}{3M_5^2} e^{B_1} V_1, \]
\[ [B']_1 = \frac{1}{3M_5^2} e^{B_1} V_1 \]

for the hidden one, where $B_0 \equiv B_0(t)$ and $B_1 \equiv B_1(t)$ are the values of $B(y, t)$ at the branes at $y = 0, 1$, respectively. In the expressions above we use the following relations, resulting from $S^1/Z_2$ symmetry, for the jump of any function across the branes:

\[ [Q']_0 = 2Q'(0^+) \quad [Q']_1 = -2Q'(1^-). \]

Finally, the induced 4D metrics of the two (‘fixed’-position) branes in the conformal gage are simply given by

\[ ds^2 = -d\tau^2 + a^2(\tau) dx^2, \]

with

\[ d\tau_i = e^{B_i} dt, \]
\[ a_i = e^{A_i}, \]

the proper times and scale factors of the two branes ($i = 0, 1$). Thus, for the Hubble parameter on the branes we acquire

\[ H_i \equiv \frac{1}{a_i} \frac{da_i}{d\tau_i} = e^{-B_i} \dot{A}_i, \]
Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing which is invariant under residual gage transformations [18]. As we have mentioned, in the following we omit the index 0 for the physical brane quantities.

In order to acquire the cosmological evolution on the physical brane we proceed as follows: equations (10)–(13) hold for the whole spacetime, including the branes. In the latter case we have to use the junction conditions (14) and (15) for the calculation of the first spatial derivatives. Therefore, eliminating $A^\mu$ from (10) and (13), and making use of (14) and the Hubble parameter relation (20), we finally obtain

$$
\frac{dH}{d\tau} + 2H^2 = \frac{1}{3M_5^3} \rho_{A5} + \frac{1}{36M_5^6} \left[ -\rho^2 + V \rho - 3p(\rho + V) + 2V^2 \right],
$$

where $H(\tau)$ is the Hubble parameter of the physical brane, with $\tau$ its proper time. The $\rho^2$ term on the right-hand side of (21) is the usual term present in braneworld cosmology. Taking the low-energy limit ($\rho \ll V$) and knowing that conventional Friedmann equations give

$$
\frac{dH}{d\tau} + 2H^2 = \frac{4\pi}{3M_p^2} (\rho - 3p) + \frac{16\pi}{3M_p^2} \rho_\Lambda,
$$

with $\rho_\Lambda \equiv \rho_{A4}$ the 4D dark energy, it is obvious that brane evolution coincides with that derived from conventional 4D cosmology if we identify

$$
V = 48\pi \frac{M_5^6}{M_p^2},
$$

and

$$
\rho_\Lambda = \frac{1}{16\pi M_5^6} \rho_{A5} + 24\pi \frac{M_5^6}{M_p^2}.
$$

Relation (24) provides the (effective in this higher-dimensional model) 4D dark energy in terms of the bulk holographic dark energy, which according to (6) is given by

$$
\rho_{A5} = c^2 3\frac{3}{4\pi^2} M_5^3 L_5^{-2}.
$$

The holographic nature of $\rho_{A5}$ is the cause of the holographic nature of $\rho_\Lambda$. As we have already mentioned, in the two-brane model examined in the present work, the cosmological length $L$ should be the interbrane distance $L_5$, which is given by (9). Furthermore, using (2) we eliminate the 5D Planck mass $M_5$, in terms of the standard 4D $M_p$, through $M_5^3 = M_p^2 / L_5$, since $L_5$ is the volume (size) of the extra dimension. Note that the varying behavior of the extra-dimension size will give rise to a varying $M_p$, i.e. varying 4D Newton constant $G_4$, and this is in general an inevitable consequence of moving-brane models [3,28]. A detailed discussion on this point is given at the end of this section. Thus, we finally acquire the following form for the effective 4D holographic dark energy:

$$
\rho_\Lambda = c^2 \left( \frac{3c^2}{64\pi^2} + 24\pi \right) M_p^2 L_5^{-2}.
$$

As we see, we have resulted in a simple holographic relation, despite the complicated nature of the model.
Our goal is to reveal the dependence of $\rho_\Lambda$ on the physical brane scale factor $a$. Since we have recovered the standard Friedmann equation (through identifications (23) and (24)) and using (9) for the calculation of $L_5$ we finally obtain

$$H^2 = \frac{8\pi}{3M_5^2} \rho + \frac{8\pi}{3} \left( \frac{3c^2}{64\pi^2} + 24\pi \right) \left( \int_0^1 dy e^{B(y)} \right)^{-2}. \quad (27)$$

Therefore, despite the simple form of relation (26), complexity has reappeared in the non-localized nature of the above equation. Such a behavior was expected and is a result of the ‘global’ properties of bulk holographic dark energy. This is a radical difference compared to the single-brane case of [16] where the arbitrary large bulk allowed for abrane-localized equation form. In the present case, the solution of the full 5D equations is indispensable. Namely, we have to solve (10)–(13) under boundary conditions (14) and (15), imposing the conventional time evolution for $\rho$ and $p$. Knowing $A(t, y)$ and $B(t, y)$ we calculate the interbrane distance $L_5(t)$ through (9) and then $\rho_\Lambda(t)$ through (26). We calculate the physical brane scale factor $a(t)$ using (19), and by eliminating $t$ we obtain the questioning relation for $\rho_\Lambda(a)$. Finally, as usual, we identify $\rho_\Lambda(a)$ with $\rho_\Lambda(a) \sim a^{-3(1+w_\Lambda)}$ and we extract the form of $w_\Lambda(z)$, with $z = a_0/a - 1$ and $a_0$ the value of $a$ at the present time.

The aforementioned procedure for the derivation of the $\rho_\Lambda(a)$ relation is impossible to be completed analytically (analytical solutions can be obtained only for stationary cases, i.e. with constant interbrane distance, which as we have mentioned are in contradiction with the holographic nature of dark energy). However, avoiding a full numerical approach, for the purpose of this work we examine the following solution class:

$$A(t_0, y) = B(t_0, y) = \ln \left[ \frac{-y + q_1}{q_2} \right], \quad (28)$$

$$\dot{A}(t_0, y) = \dot{B}(t_0, y) = q_3 \left[ y + q_1 \right], \quad (29)$$

which satisfy the constraint equations (12) and (13) at $t_0$ (and therefore at every $t$ since (12) and (13) are preserved by the equations of motion) provided that $(1 + 2q_1)^2 q_3^2 = c^2/(2\pi)$. Moreover, boundary conditions (14) and (15) are fulfilled imposing $V = 6M_5^2 q_2/q_1^2$ and $V_0 = -6M_5^2 q_2/(q_1 + 1)^2$.

Investigating the low-energy (late-time) evolution of the aforementioned solution class, i.e. omitting $\rho$ and $p$ in boundary conditions which make them significantly simpler, we obtain interesting results. In particular, for a large area of the parameter space we find a reasonable $w_\Lambda(z)$ form, with the basic requirement (necessary but not always efficient) being the decreasing of interbrane distance. Fortunately, a decreasing interbrane distance seems to have larger probability than an increasing one (since in the latter case the system is often unstable or ‘naked’ singularities do appear between the branes). Furthermore, there are many parameters present in the model and in solution class (28) and (29). These characteristics make it substantially easier to acquire a reasonable $w_\Lambda(z)$ form in the 5D framework described in this work than in conventional 4D holographic dark energy [1]–[4]. In addition, we do not have to fine tune the constant $c$ in the definition of $\rho_{55}$ in relation (25). In figure 1 we depict $w_\Lambda(z)$ for $q_1 = -1.4$, $q_2 \approx 11$, $q_3 \approx 0.22$ and $c = 1$, and using the unit $M_5 = 1$ (the specific scale does not affect the $w_\Lambda(z)$ form). We observe that the effective 4D holographic dark energy behaves like a ‘quintom’ [29], that is $w_\Lambda$ was larger than $-1$ in the past while its present value is $w_{\Lambda_0} \approx -1.05$, and the phantom bound
$w = -1$ was crossed at $z_p \approx 0.41$. This result is in agreement with late acceleration of the Universe and dark energy constraints imposed by observations [30,31]. Note that quintom behavior arises naturally in our higher-dimensional brane model, without the inclusion of extra fields or specific potential terms by hand. Moreover, as we mentioned above, we do not have to fine tune the solution parameters, since a large area of the parameter space leads to similar behavior (the case of figure 1 just corresponds to a good representative in comparison with observations). Finally, other parameter areas in the aforementioned solution subclass, as well as a general numerical investigation beyond this ansatz (a hard task due to various instabilities [18,19]) reveal interesting but likely unphysical $w_\Lambda$ behavior, such as chaotic oscillations with respect to $z$. However, this could still be the case in our Universe for larger $z$. The subject is under investigation.

Let us finish this section with some comments on the dynamical nature of some quantities. As we have mentioned in the introduction, the main goal of the present work is the study of bulk holographic dark energy in a finite braneworld model. From the first moment we deduce that, if the bulk boundaries (the two branes) are steady, then holographic dark energy becomes a constant, losing its dynamical nature. In other words, the concept of holographic dark energy is in contradiction to the finite-bulk framework, unless we consider moving-brane models as a necessary outlet. Unfortunately, this choice leads to some undesirable consequences. Indeed, with a varying interbrane distance it is obvious from (2) that either the 5D Planck mass $M_5$ or its 4D counterpart $M_p$, or even both, should change with time. Since $M_5$ is a fundamental quantity of our model we desire to maintain it as a constant. Thus, $M_p$ and therefore the 4D Newton’s gravitational constant $G_4$ are the ones that reflect the dynamical nature of the bulk, and this is a common feature of moving-brane models [3,28]. However, in order to acquire the correct effective 4D cosmological evolution we need to make the identification (23), a usual approach of braneworld cosmology. This relation ‘matches’ the 4D Planck mass (which is determined by dimensional reduction of the 5D action) with the brane tension (which should be given by the vacuum energy predicted by the effective 4D QFT) and
Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing

Figure 2. Evolution of the 4D Newton’s constant $G_4$, divided by its present value, versus $z$ for solution class (28) and (29), using $q_1 = -1.4$, $q_2 \approx 11$, $q_3 \approx 0.22$, $c = 1$ and the unit $M_5 = 1$. The minimum value is obtained at $z_p \approx 0.41$, which corresponds to the phantom-divide crossing of $w_\Lambda(z)$.

brane-cosmologists hope to acquire its justification by a fundamental theory of nature, unknown up to now. Unfortunately, in the present model of moving branes this fine-tuning must hold at all times and this is definitely an additional assumption, independent of the rest formulation. Thus, the extension of holographic dark energy to finite-bulk models is still obscure, since one expects from a fundamental theory to justify the aforementioned eternal fine-tuning.

In order to provide a clear picture of these features, in figure 2 we depict the evolution of the 4D Newton’s constant $G_4$ divided by its present value, in terms of $z$, for the same parameter values of figure 1. We observe that $G_4$ acquires its minimum value at $z_p \approx 0.41$, which corresponds to the phantom-divide crossing of $w_\Lambda(z)$. This behavior of $G_4$ is consistent with cosmological constraints which restrict its deviation between $\pm 5\%$ [32]. However, we mention here that not all numerical solutions satisfy these limits. Indeed, our numerical elaboration reveals that, out of $10^3$ solutions that present an acceptable (quintom) form for $w_\Lambda(z)$ (similar to that depicted in figure 1), only $\approx 20\%$ correspond to an acceptable $G_4$-behavior, too.

4. Discussion–conclusions

In this work we apply the bulk holographic dark energy in a general braneworld model with moving branes. Such a generalized bulk version of holographic dark energy is necessary if we desire to match the successes of brane cosmology at both the theoretical and phenomenological–observational level with the successful, simple, and inspired by first principles, notion of holographic dark energy in conventional 4D cosmology. In particular, as we showed in [16], the bulk space is the natural framework for the cosmological application, concerning dark energy, of the holographic principle, since it is the maximally dimensional subspace that determines the properties of quantum-field and gravitational theory, and black hole formation. Subsequently, this bulk holographic dark energy will
Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing give rise to an effective 4D dark energy with ‘inherited’ holographic nature, and this one will be present in the effective Friedmann equation of the brane.

Applying the bulk holographic dark energy in the well-investigated two-brane model, and using the interbrane distance (bulk size) as the cosmological length in its definition, we deduce that the branes have to move in order for dark energy to preserve its holographic-dynamical nature. In this case we extract the effective Friedmann equation on the physical brane, which has a non-localized (on the brane) form since the interbrane distance must be calculated from the full 5D dynamics. This complexity is a result of the ‘global’ characteristics of bulk holographic dark energy. Numerical investigation on a simple solution subclass reveals a quintom behavior \cite{29} for $w_\Lambda(z)$. In particular, $w_\Lambda$ was larger than $-1$ in the past, it crossed the phantom divide $w_\Lambda = -1$ at $z_p \approx 0.41$ and its present value is $w_\Lambda_0 \approx -1.05$. This behavior is in notable agreement with observations \cite{30,31} which give $w_\Lambda_0 = -1.02^{+0.13}_{-0.19}$ and $z_p = 0.46 \pm 0.13$. However, although we have not included any special fields or specific potentials, an additional assumption has to be imposed in order for this holographic dark energy application to be valid, namely, the fine-tuning between the brane tension and the 4D Planck mass. Furthermore, Newton’s constant acquires a dynamical nature too and one has to be careful in order to be consistent with cosmological constraints \cite{32}. In conclusion, the investigation of the extension of holographic dark energy in finite-bulk models reveals that a reasonable $w_\Lambda(z)$ behavior is acquired relatively easily, only under a fundamental fine-tuning. Definitely, the combination of holographic dark energy with brane cosmology is an interesting subject which needs further investigation.

Acknowledgments

The author is grateful to G Kofinas, K Tamvakis, N Tetradis, F Belgiorno, B Brown, S Cacciatori, M Cadoni, R Casadio, G Felder, A Frolov, B Harms, N Mohammed, M Setare, Y Shtanov and to an anonymous referee for useful discussions and suggestions.

References

[1] Li M, 2004 Phys. Lett. B 603 1 [SPIRES] [hep-th/0403127]
[2] Huang Q-G and Li M, 2004 J. Cosmol. Astropart. Phys. JCAP08(2004)013 [SPIRES] [astro-ph/0404229]
Ke K and Li M, 2005 Phys. Lett. B 606 173 [SPIRES] [hep-th/0407056]
Gong Y, Wang B and Zhang Y-Z, 2005 Phys. Rev. D 72 043510 [SPIRES] [hep-th/0412218]
Myung Y S, 2005 Phys. Lett. B 610 18 [SPIRES] [hep-th/0412224]
Elizalde E, Nojiri S, Odintsov S D and Wang P, 2005 Phys. Rev. D 71 103504 [SPIRES] [hep-th/0502082]
Pavon D and Zimdahl W, 2005 Phys. Lett. B 628 206 [SPIRES] [gr-qc/0505020]
Nojiri S and Odintsov S D, 2006 Gen. Rel. Grav. 38 1285 [SPIRES] [hep-th/0506212]
Zhang X, 2007 Phys. Lett. B 648 1 [SPIRES] [astro-ph/0604454]
Setare M R, 2006 Phys. Lett. B 642 1 [SPIRES] [hep-th/0609069]
Setare M R, 2006 Phys. Lett. B 642 421 [SPIRES] [hep-th/0609104]
Zhang X, 2006 Phys. Rev. D 74 103505 [SPIRES] [astro-ph/0609699]
Simpson F, 2007 J. Cosmol. Astropart. Phys. JCAP03(2007)016 [SPIRES] [astro-ph/0609755]
Setare M R, Zhang J and Zhang X, 2007 J. Cosmol. Astropart. Phys. JCAP03(2007)007 [SPIRES]
[gr-qc/0611084]
Lee J-W, Lee J and Kim H-C, 2007 J. Cosmol. Astropart. Phys. JCAP08(2007)005 [SPIRES]
[hep-th/0701119]
Myung Y S, 2007 Phys. Lett. B 649 247 [SPIRES] [gr-qc/0702032]
Zhang J, Zheng W, Zhu Z-H and He S, 2007 Preprint 0705.4409 [astro-ph]
Zhang J, Zhang X and Liu H, 2007 Preprint 0708.3121 [hep-th]
Setare M R, 2007 Preprint 0708.3284 [hep-th]
Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing

Feng C-J, 2007 Preprint [hep-th]
Horvat R, 2007 Preprint [gr-qc]
Gao C, Chen X and Shen Y-G, 2007 Preprint [astro-ph]

[3] Guberina B, Horvat R and Nikolic H, 2005 Phys. Rev. D 72 125011 [SPIRES] [astro-ph/0507666]

[4] Gong Y-G, 2004 Phys. Rev. D 70 064020 [SPIRES] [hep-th/0404030]

Sadjadi H M, 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)026 [SPIRES] [gr-qc/0701074]

[5] Bahcall N A, Ostriker J P, Perlmutter S and Steinhardt P J, 1999 Science 284 1481

Balbi A et al, 2000 Astrophys. J. 545 L1 [SPIRES] (MAXIMA-1)

[6] Fischler W and Susskind L, 1998 Preprint [hep-th/9806039]
Bak D and Rey S-J, 2000 Class. Quantum Grav. 17 L83 [SPIRES] [hep-th/9902173]
Verlinde H L, 2000 Nucl. Phys. B 580 264 [SPIRES] [hep-th/9906182]
Horava P and Minic D, 2000 Phys. Rev. Lett. 85 1610 [SPIRES] [hep-th/0001145]
Verlinde E P, 2000 Preprint [hep-th/0008140]
Gregory J P and Padilla A, 2003 Class. Quantum Grav. 20 4221 [SPIRES] [hep-th/0304250]

Kiritsis E, 2005 J. Cosmol. Astropart. Phys. JCAP10(2005)014 [SPIRES] [astro-ph/0504219]

[7] ‘t Hooft G, 1993 Salamfest pp 284–96 [gr-qc/9310026]

[8] Susskind L, 1995 J. Math. Phys. 36 6377 [SPIRES] [hep-th/9409089]
Witten E, 1998 Adv. Theor. Math. Phys. 2 253 [SPIRES] [hep-th/9802150]
Bousso R, 2002 Rev. Mod. Phys. 74 825 [SPIRES] [hep-th/0203101]

[9] Easther R and Lowe D A, 1999 Phys. Rev. Lett. 82 4967 [SPIRES] [hep-th/9902088]
Kaloper N and Linde A D, 1999 Phys. Rev. D 60 103509 [SPIRES] [hep-th/9904120]

[10] Tavakol R K and Ellis G, 1999 Phys. Lett. B 469 37 [SPIRES] [hep-th/9908093]

[11] Myers R C and Perry M J, 1986 Ann. Phys., NY 172 304 [SPIRES]

[12] Argyres P C, Dimopoulos S and Russell J M., 1998 Phys. Lett. B 441 96 [SPIRES] [hep-th/9808138]
Casadio R and Harns B, 2000 Phys. Lett. B 487 209 [SPIRES] [hep-th/0004004]
Kanti P and Tamvakis K, 2002 Phys. Rev. D 65 084010 [SPIRES] [hep-th/0110298]
Tanaka T, 2003 Prog. Theor. Phys. Suppl. 148 307 [SPIRES] [gr-qc/0203082]

[13] Kofinas G, Papantonopoulos E and Zamarias V, 2002 Phys. Rev. D 66 104028 [SPIRES] [hep-th/0208207]
Kanti P and Tamvakis K, 2003 Phys. Rev. D 68 024014 [SPIRES] [hep-th/0303073]

[14] Cavaglia M, 2003 Int. J. Mod. Phys. A 18 1843 [SPIRES] [hep-ph/0210296]

[15] Cohen A G, Kaplan D B and Nelson A E, 1999 Phys. Rev. Lett. 82 4971 [SPIRES] [hep-ph/9803132]
Susskind L and Witten E, 1998 Preprint [hep-th/9805114]

[16] Padmanabhan T, 2005 Class. Quantum Grav. 22 L107 [SPIRES] [hep-th/0406060]

[17] Padmanabhan T, 2005 Class. Quantum Grav. 22 L107 [SPIRES] [hep-th/0406060]

[18] Rubakov V A and Shaposhnikov M E, 1983 Phys. Lett. B 125 136 [SPIRES]
Antoniadis I, Bachas C, Lewellen D and Tomaras T, 1988 Phys. Lett. B 207 441 [SPIRES]
Horava P and Witten E, 1996 Nucl. Phys. B 460 506 [SPIRES]
Horava P and Witten E, 1996 Nucl. Phys. B 475 94 [SPIRES]

[19] Lukas A, Ovrut B A, Stelle K S and Waldram D, 1999 Phys. Rev. D 59 086001 [SPIRES] [hep-th/9803235]
-Hamed N A, Dimopoulos S and Dvali G, 1998 Phys. Lett. B 429 263 [SPIRES] [hep-ph/9803315]
Binetruy P, Deffayet C and Langlois D, 2000 Nucl. Phys. B 565 269 [SPIRES] [hep-th/9905012]

[20] Randall L and Sundrum R, 1999 Phys. Rev. Lett. 83 4690 [SPIRES] [hep-th/9906064]

[21] Randall L and Sundrum R, 1999 Phys. Rev. Lett. 83 3370 [SPIRES] [hep-ph/9905221]

[22] Ida D, 2000 J. High Energy Phys. JHEP09(2000)014 [SPIRES] [gr-qc/9912002]
Brax P and de Bruck C V, 2003 Class. Quantum Grav. 20 R201 [SPIRES] [hep-th/0303095]

[23] Saridakis E N, 2008 Phys. Lett. B 660 138 [SPIRES] [0712.2228] [hep-th]
Saridakis E N, 2008 Phys. Lett. B 661 335 [SPIRES] [0712.3806] [gr-qc]

[24] KaBousso R, 1999 J. High Energy Phys. JHEP06(1999)028 [SPIRES] [hep-th/9906022]
Savonije I and Verlinde E P, 2001 Phys. Lett. B 507 305 [SPIRES] [hep-th/0102042]

[25] Thomas S D, 2002 Phys. Rev. Lett. 89 081301 [SPIRES]
Cai R-G and Myung Y S, 2003 Phys. Lett. B 559 60 [SPIRES] [hep-th/0210300]
Iwasaka Y, Kobayashi T, Shiromizu T and Yoshino H, 2006 Phys. Rev. D 74 064027 [SPIRES] [hep-th/0606027]

[26] Martin J, Felder G N, Frolov A V, Peloso M and Kofman L, 2004 Phys. Rev. D 69 084017 [SPIRES] [hep-th/0309001]

[27] Diakonos F K and Saridakis E N, 2007 Preprint 0708.3143 [hep-th]
Saridakis E N, 2007 Preprint 0710.3269 [hep-th]

[28] Kobayashi S and Koyama K, 2002 J. High Energy Phys. JHEP12(2002)056 [SPIRES] [hep-th/0210029]
Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing

Shtanov Y and Viznyuk A, 2005 *Class. Quantum Grav.* **22** 987 [SPIRES] [hep-th/0312261]

Schwindt J-M and Wetterich C, 2005 *Nucl. Phys. B* **726** 75 [SPIRES] [hep-th/0501049]

Chamblin A, Hawking S W and Reall H S, 2000 *Phys. Rev. D* **61** 065007 [SPIRES] [hep-th/9909205]

Emparan R and Reall H S, 2002 *Phys. Rev. Lett.* **88** 101101 [SPIRES] [hep-th/0110260]

Lemos J P S, 1995 *Phys. Lett. B* **353** 46 [SPIRES] [gr-qc/9404041]

Cai M and Galloway G J, 2001 *Class. Quantum Grav.* **18** 2707 [SPIRES] [hep-th/0102149]

Goldberger W D and Wise M B, 1999 *Phys. Rev. Lett.* **83** 4922 [SPIRES] [hep-ph/9907447]

[21] Chamblin A, Hawking S W and Reall H S, 2000 *Phys. Rev. D* **61** 065007 [SPIRES] [hep-th/9909205]

[22] Lemos J P S, 1995 *Phys. Lett. B* **353** 46 [SPIRES] [gr-qc/9404041]

Shapiro I L, Sola J and Stefancic H, 2005 *J. Cosmol. Astropart. Phys.* JCAP01(2005)012 [SPIRES] [astro-ph/0410095]

Zhao W, 2007 *Phys. Lett. B* **655** 97 [SPIRES] [0706.2211] [astro-ph]

Apostolopoulos P S, Brouzakis N, Saridakis E N and Tetradis N, 2005 *Phys. Rev. D* **72** 064013 [SPIRES] [hep-th/0502115]

[26] Apostolopoulos P S and Tetradis N, 2004 *Phys. Lett. B* **594** 13 [SPIRES] [hep-th/0403052]

[28] Apostolopoulos P S and Tetradis N, 2005 *Phys. Rev. D* **72** 064013 [SPIRES] [hep-th/0502115]

[29] Sahni V and Shtanov Y, 2003 *J. Cosmol. Astropart. Phys.* JCAP11(2003)014 [SPIRES] [astro-ph/0202346]

[32] D’Innocenti S, Fiorentini G, Raffelt G G, Ricci B and Weiss A, 1996 *Astron. Astrophys.* **312** 345 [SPIRES] [astro-ph/9509090]

[30] Riess A G *et al* (Supernova Search Team), 2004 *Astrophys. J.* **607** 665 [SPIRES] [astro-ph/0402512]

[31] Huang Q-G and Gong Y-G, 2004 *J. Cosmol. Astropart. Phys.* JCAP04(2004)006 [SPIRES] [astro-ph/0403590]

[33] Umezu K, Ichiki K and Yahiro M, 2005 *Phys. Rev. D* **72** 043524 [SPIRES] [astro-ph/0506310]

[27] Diakonos F K, Saridakis E N and Tetradis N, 2005 *Phys. Lett. B* **605** 1 [SPIRES] [hep-th/0409025]

[25] Nojiri S, Obregon O, Odintsov S D and Ogushi S, 2000 *Phys. Rev. D* **62** 064017 [SPIRES] [hep-th/0003148]

[34] Guo Z-K, Piao Y-S, Zhang X and Zhang Y-Z, 2006 *Phys. Rev. D* **74** 127304 [SPIRES] [astro-ph/0608165]

[24] Goldberger W D and Wise M B, 1999 *Phys. Rev. Lett.* **83** 4922 [SPIRES] [hep-ph/9907447]

[23] Dimopoulos S and Landsberg G L, 2001 *Phys. Rev. Lett.* **87** 161602 [SPIRES] [hep-th/0403052]