THE IMPACT OF COLD GAS ACCRETION ABOVE A MASS FLOOR ON GALAXY SCALING RELATIONS

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ABSTRACT

Using the cosmological baryonic accretion rate and normal star formation (SF) efficiencies, we present a very simple model for star-forming galaxies that accounts for the mass and redshift dependences of the star formation rate (SFR)–mass and Tully–Fisher (TF) relations from z ∼ 2 to the present. The time evolution follows from the fact that each modeled galaxy approaches a steady state where the SFR follows the (net) cold gas accretion rate. The key feature of the model is a halo mass floor M_{min} ≃ 10^{11} M_{⊙} below which accretion is quenched in order to simultaneously account for the observed slopes of the SFR–mass and TF relations. The same success cannot be achieved via an SF threshold (or delay) nor by varying the SF efficiency or the feedback efficiency. Combined with the mass ceiling for cold accretion due to virial shock heating, the mass floor M_{min} explains galaxy “downsizing,” where more massive galaxies formed earlier and over a shorter period of time. It turns out that the model also accounts for the observed galactic baryon and gas fractions as a function of mass and time, and the cosmic SFR density, which are all resulting from the mass floor M_{min}. The model helps us to understand that it is the cosmological decline of accretion rate that drives the decrease of cosmic SFR density between z ∼ 2 and z = 0. The model also predicts that the cosmic SFR density from z ∼ 6 to z ∼ 2 that allows us to put a constraint on our main parameter M_{min} ≃ 10^{11} M_{⊙}. Among the physical mechanisms that could be responsible for the mass floor, our view is that photoionization feedback (from first in situ hot stars) lowering the cooling efficiency is likely to play a large role.

Key words: cosmology; observations – galaxies: evolution – galaxies: high-redshift

Online-only material: color figures

1. INTRODUCTION

To a good first-order approximation, galaxies are either blue and active or red and passive, as indicated by the color modality. In the past few years, it has been realized that blue star-forming galaxies (SFGs) have a strong intimate connection between the growth of halos and the characteristic mass and time dependences of the SFR sequence, which correlates stellar mass M_{⋆} and mass index $q \equiv \log M_{\star}/M_{\odot}$ with a mass index $q \equiv (1+z)/(3.2)$, $\rho \equiv 0.8$, and $s \equiv 2.7$ in the redshift range $z \equiv 0$–2. Reproducing the characteristic mass and time dependences of the SFR sequence is a challenge for models of galaxy formation (e.g., Davé 2008; Damen et al. 2009).

Equation (1) is important as it is very reminiscent of the halo mean growth rate, which has been shown to be $M_{\text{h}} \propto M_{\text{h}}(1+z)^{2}$ with a mass index $s$ greater than unity $s \equiv 1.1$ (Neistein & Dekel 2008; Genel et al. 2008; McBride et al. 2009) that is set by the shape of the initial dark matter (DM) power spectrum (Neistein et al. 2006; Birnboim et al. 2007). The similarity motivates a closer investigation, and we will show that, indeed, there is a strong intimate connection between the growth of halos and the SFR sequence.

Another relevant scaling relation for (disky) SFGs is the well-known Tully–Fisher (TF; see, e.g., Tully & Fisher 1977) relation, which correlates stellar mass $M_{\star}$ and maximum circular velocity $V_{\text{max}}$. The TF relation appears to be already in place.

[The rest of the document follows with further details and equations.]
at high redshifts $z \geq 1$ (Kassin et al. 2007; Puech et al. 2008; Epinat et al. 2009; Cresci et al. 2009) and has evolved by a factor of 2.5 at a given mass (Cresci et al. 2009) from $z \simeq 2.2$ to the present. It is of the form:

$$M_* \propto V_{\text{max}}^m (1 + z)^n,$$

(2)

where $m \simeq 4$ to a 10% accuracy locally (e.g., Tully & Pierce 2000; McGaugh 2005; Meyer et al. 2008). As for the SFR sequence, Equation (2) is reminiscent of the virial relation for DM halos, namely, $M_h \propto V_h^3$. Thus, both the SFR sequence and the TF relation behave similarly to their DM counterparts, but have slightly different mass indices, i.e., they are tilted with respect to their DM counterparts.

Aside from our knowledge of these global properties, our detailed understanding of individual SFGs has also improved greatly thanks to spatially resolved kinematic studies of the ionized gas in $z \sim 2$ SFGs (e.g., SINS survey; Förster Schreiber et al. 2006, 2009; Genzel et al. 2006, 2008; Shapiro et al. 2008, 2009; Cresci et al. 2009). The SINS survey, consisting of 80 $z = 2$ SFGs, has revealed that 30%–50% of SFGs with $M_* \sim 10^{10.5}$–11.5 are gas-rich thick rotating disks (see also van Starkenburg et al. 2008; Wright et al. 2007), with the rest either dominated by dispersion velocities or showing the signature of recent mergers (e.g., Law et al. 2007, 2009; Förster Schreiber et al. 2009). The high-redshift SFG disks are different from local spirals in many ways. In particular, they are thick and correspondingly of high velocity dispersion (Förster Schreiber et al. 2006; Genzel et al. 2008; Cresci et al. 2009), and they are often made of giant SF “clumps” (Cowie et al. 1995; Genzel et al. 2006; Elmegreen et al. 2007; N. M. Förster Schreiber 2010, in preparation). This feature indicates wild gravitational instability, which is a likely outcome of a high gas density (Noguchi 1998; Immeli et al. 2004; Bournaud et al. 2007; Elmegreen et al. 2008; Dekel et al. 2009a). Indeed, direct evidence for high gas fractions at high redshifts is now mounting (Erb et al. 2006b; Daddi et al. 2008; Tacconi et al. 2010; Daddi et al. 2010a) thanks to rapid progress in the sensitivity of millimeter interferometers.

The global scaling relations and the recent surveys of galaxy kinematics raise several outstanding questions: (1) What causes the high SFRs at $z \gtrsim 2$? (2) What drives the evolution of the SFR sequence and TF relation from $z \sim 2$ to the present? (3) What drives the cosmological evolution of the average SFR density? (4) Why did more massive galaxies form their stars before less massive SFGs? (5) Why are $z = 2$ SFGs so gas rich?

In this paper, we address these questions in the context of the cosmological growth of DM halos. In particular, we construct a very simple model that ties the SFR sequence and TF relation to their DM counterparts. The important feature of the model is the suppression of the accretion below a mass floor at $M_{\text{min}} \sim 10^{11} \, M_\odot$, which, as we will show, has several other important consequences. We focus our attention at $z \gtrsim 2$, i.e., when the universe was just 3 Gyr old.

This paper is organized as follows. In Section 2, we present the model. In Section 3, we show that the SFGs reach a quasi-steady state and demonstrate the impact of the mass floor $M_{\text{min}}$ on the SFR sequence and the TF relation. In Section 4, we find that the mass floor naturally delays the star formation (SF) activity and leads to downsizing. The model also simultaneously accounts for the baryonic and gas fractions as a function of mass and time. In Section 5, we put direct constraints on the numerical value of $M_{\text{min}}$ from the cosmological history of SF density. In Section 6, we discuss the model limitations and present our conclusions. Finally, in Section 7, we discuss the possible origin of the mass floor. We use throughout the standard $\Lambda$CDM cosmology with the parameters $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, and $\sigma_8 = 0.8$.

2. THE RESERVOIR MODEL

In this section, we present our “reservoir” model and its two major ingredients (Section 2.1), namely, the accretion efficiency and the SF efficiency. We discuss the mass and redshift dependences of these two parameters in Section 2.2. The model ingredients are then summarized in Section 2.3. We will use this model as a learning tool to gain insights on the role played by several key physical parameters.

2.1. The Basic Equation

We consider a galaxy with its dark halo as a reservoir that is fed by a source and is emptied into a drain. The source represents the amount of newly accreted cold gas, and the drain is the gas consumption into stars as well as outflows. The basic equation of our model is the differential equation expressing the conservation of gas mass,

$$M_{\text{gas}} = M_{\text{gas, in}} - (1 - R) M_* - M_{\text{gas, out}}$$

(3)

where $M_{\text{gas, in}}$ is the gas accretion rate, $(1 - R) M_*$ is the net SFR corrected for the recycled fraction, and $M_{\text{gas, out}}$ is the mass outflow rate. For our purposes, $R$ is a kept time independent given its slow dependence on stellar-population age $T$ for $T > 10^9$ Gyr (Bruzual & Charlot 2003), i.e., at $z < 4$. We discuss the limitations of this assumption in the Appendix.

To be as general as possible, we included an additional drain in our model (Equation (3)) representing any outflowing gas. Observationally, the outflow rate $M_{\text{gas, out}}$ is observed to be roughly proportional to the SFR, $M_{\text{gas, out}} = a \times SFR$, with $a \simeq 1.0$ (Heckman et al. 2000; Martin 2005; Rupke et al. 2005; Erb 2008). We note, however, that some fraction of the supernova (SN) driven winds is likely to be recycled, especially in the most massive halos, where much of the outflowing gas falls back into the galaxy and can boost the stellar mass at $z = 0$ by 33% (simulations by Oppenheimer & Davé 2008). Hence, the net outflow rate $M_{\text{gas, out}}$ may be less than the SFR, i.e., $M_{\text{gas, out}} \lesssim SFR$.

Regardless of the numerical value of the proportionality constant $a$ between outflow rate and SFR, Equation (3) can be re-written as

$$M_{\text{gas}} = M_{\text{gas, in}} - a M_* ,$$

(4)

where $a$ includes the corrections for recycling and outflows. This equation expresses the trivial fact that the gas reservoir $M_{\text{gas}}$ will be filled up or get emptied depending on the relative power of the source and the drain terms. As discussed in Section 3.1, the system will self-regulate itself to a steady state ($M_{\text{gas}} \simeq 0$) where SFR is proportional to the cosmological accretion rate.

We stress that the feedback term will have little impact on the scaling relations, such as the SFR sequence, or the sSFR. Indeed, if half of the accreting gas is entrained and expelled by the SN-driven winds, the SFR will be lowered by a factor of 2. As a result, if the SFR is lower by a factor of 2, the stellar mass $M_* = \int dt SFR(t)$ will be lower by the same factor, and the SFR sequence will remain unchanged.

2.1.1. Halo Growth and Gas Accretion

The gas replenishment term $M_{\text{gas, in}}$ in Equation (3) is required observationally. For example, the G-dwarf problem (van den
Bergh 1962; Schmidt 1963) calls for a significant amount of newly accreted gas (e.g., Larson 1974). Furthermore, the gas depletion timescale in local massive galaxies is $\sim$ few Gyr (e.g., Wong & Blitz 2002; James et al. 2008), i.e., much shorter than the time required to build their stellar masses. Similarly, for $z \sim 2$ SFGs, the gas consumption timescale of less than 0.5 Gyr is shorter than their typical stellar ages of 1–2 Gyr (e.g., Erb 2008; Tacconi et al. 2010), thus requiring intense gas accretion.

Cold gas accretion is also required theoretically as it is a natural consequence of the “cold-accretion” regime (Birnboim & Dekel 2003; Kereš et al. 2005) when the cooling time is shorter than the dynamical time (White & Frenk 1991). Furthermore, since the cold gas is not shock-heated to the virial temperature, high gas accretion efficiency is a natural outcome of efficient penetration of cosmological cold streams into the inner galaxies at high redshift, as seen in hydrodynamical cosmological simulations (Ocvirk et al. 2008; Dekel et al. 2009b).

In this context, it is the halo growth rate of DM halos that is regulating the baryonic accretion. The halo growth rate is by now well understood based on $N$-body simulations and the extended Press–Schechter (EPS) analytic formalism (Efstathiou et al. 1985; Wechsler et al. 2002; van den Bosch 2002; Springel et al. 2006). An approximation for the average mass growth rate of DM halos of virial mass $M_h$ at redshift $z$ in cosmological $N$-body simulations (van den Bosch 2002; Genel et al. 2008; McBride et al. 2009), which is also understood using the EPS analytic formalism (Neistein & Dekel 2008), is obtained by the fitting function

$$M_h \simeq 510 M_{\odot} h^{-1} (1+z)^{0.2} M_{\odot} \text{yr}^{-1}.$$  
(5)

where $M_{h,12} \equiv M_h/10^{12} M_{\odot}$, $(1+z)^{3.2} \simeq (1+z)/3.2$, and $s \simeq 1.1$, with the estimates for $s$ ranging from 1.08 to 1.14 (Neistein & Dekel 2008; Genel et al. 2008; McBride et al. 2009).

Given the average halo growth rate, the corresponding average gas accretion rate is

$$M_{\text{gas,in}} = \epsilon_{\text{in}} f_b M_h$$

$$\simeq 90 \epsilon_{\text{in}} f_{b,0.18} M_{h,12}^{1.1} (1+z)^{2.2} M_{\odot} \text{yr}^{-1}.$$  
(6)

where $f_{b,18} \equiv f_b/0.18$ is the cosmic baryonic fraction, and $\epsilon_{\text{in}} \lesssim 1$ is the accretion efficiency expressing the effective fraction of the baryonic matter that is actually accreted as cold gas into the galaxy.

The accretion efficiency $\epsilon_{\text{in}}$ is a very important parameter and must be within the range 0.5–1.0 for several reasons. Theoretically, $\epsilon_{\text{in}}$ is high under the cold-accretion regime (Dekel et al. 2009b). Observationally, this efficiency is supported by the fact that the observed SFR in massive galaxies at $z \sim 2$ matches within a factor of 2 the maximum predicted gas accretion rate (Genel et al. 2008; Dekel et al. 2009b). In addition, Genel et al. (2008) showed that the halo major merger fraction in the mass range of interest log $M_h = 12$ is low and consistent with the low merger fractions in the SINS survey (Shapiro et al. 2008; Förster Schreiber et al. 2009), provided that the accretion efficiency $\epsilon_{\text{in}}$ is greater than 50% (see Figure 1 of Genel et al. 2008). Recently, Bauermeister et al. (2010) compared the inferred “external” gas accretion to the DM accretion rate and found this efficiency needs to be $> 70\%$. In light of all these results, we will adopt a fiducial value of $\epsilon_{\text{in}} = 0.7$.

As mentioned in Section 1, this baryonic growth rate (Equation (6)) is very reminiscent of the SFR–mass relationship (Equation (1)), given the similar near-linear increase of the accretion rate with mass. However, the mass index $s$ in the accretion rate is slightly larger than unity, while the mass index $p$ in the SFR sequence is somewhat smaller than unity. The clear implication from this difference is that the specific accretion rate increases with mass for the DM component, while the sSFR decreases with mass. Thus, if there is a connection between Equations (6) and (1), the difference has to be explained (see Section 3).

### 2.1.2. Star Formation Rate

SF, a very complex, local and inefficient process (giant molecular clouds (GMCs) typically turning 1%–2% of the gas mass into stars in a free-fall time) is the primary drain of the reservoir (in Equation (3)). On galaxy scales, the amount of gas consumed is well described by the empirical relation:

$$\Sigma_{\text{SFR}} = \epsilon_{\text{sfr}} \Sigma_{\text{gas}} / t_{\text{dyn}}, \quad \text{i.e.,}$$

$$\text{SFR} = \epsilon_{\text{sfr}} M_{\text{gas}} / (t_{\text{dyn}}).$$  
(7)

where $t_{\text{dyn}} = R_{1/2}/V_c$ is the galaxy dynamical time and $\epsilon_{\text{sfr}}$ is the SFR efficiency parameter. For a marginally unstable disk with Toomre $Q$ parameter $Q \sim 1$, it can be shown (Martin & Kennicutt 2001; Krumholz & Thompson 2007) that Equation (7) is equivalent to the traditional Kennicutt–Schmidt (KS) relation (Kennicutt 1979; Kennicutt 1998), which relates the surface densities of gas and SFR via $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^{1.5}$.

The locally inferred value by Kennicutt (1998) for the SFR efficiency is $\epsilon_{\text{sfr}} \sim 0.02$ and the most recent advancements in constraining the KS relation at $z \sim 2$ show no evidence for any evolution (Genzel et al. 2010; Daddi et al. 2010b). In addition, at $z \sim 2$, we know that massive disky SFGs, with $M_r > 10^{10} M_{\odot}$, extend to half-light radii of $R_{1/2} \sim 4$ kpc (Bouché et al. 2007b) and they rotate with circular velocities $V_c \sim 200$ km s$^{-1}$ (Förster Schreiber et al. 2006). Expressing the quantities in these units, the orbital time can be written as

$$t_{\text{dyn}} = 2 \times 10^7 \text{yr} \left( \frac{R_{1/2}}{4 \text{kpc}} \right) \left( \frac{V_c}{200 \text{ km s}^{-1}} \right)^{-1}.$$  
(8)

At $z > 2$, we assume that the orbital time scales with the galaxy dynamical time, $R_{1/2}/V_c$, which is a fixed fraction, about 18%, of the Hubble time at the given redshift, thus proportional to $(1+z)^{-3/2}$.

We note that our results on the scaling relations presented in Section 3.2 are completely independent of the SF efficiency (i.e., $\epsilon_{\text{sfr}}$ and $t_{\text{dyn}}$). This is because the SFR is going to be driven by the cosmological accretion rate, as explained in Section 3.1.

### 2.2. Mass Dependence

The two efficiency parameters $\epsilon_{\text{sfr}}$ and $\epsilon_{\text{in}}$ appearing in Equations (6)–(7) may in principle vary with mass and redshift. The SF efficiency $\epsilon_{\text{sfr}}$ does not appear to evolve with redshift. Indeed, the KS relation seems to be in place as early as $z = 2$, and shows no significant evolution between $z = 0$ and $z = 2$ (Bouché et al. 2007a).

On the other hand, the accretion efficiency $\epsilon_{\text{in}}$ must be a strong function of halo mass (e.g., White & Frenk 1991). At the massive end $M_h \gtrsim M_{\text{max}} \simeq 10^{12} M_{\odot}$, and at low redshifts, the cold flows are not expected to be in a form of narrow, dense streams and they therefore fail to penetrate through the shock-heated halo gas. This may play a significant role in quenching SF and causing the transition of blue galaxies onto the red sequence.
(Dekel & Birnboim 2006; Cattaneo et al. 2006). To model this effect, we set a ceiling for cold gas accretion at all redshifts,

\[ \epsilon_{\text{in}}(M_h) = 0 \text{ if } M_h > M_{\text{max}}. \] (9)

At the low-mass end, there is strong evidence for a drop in the efficiency of galaxy formation (e.g., Shankar et al. 2006; van den Bosch et al. 2007; Baldry et al. 2008; Kravtsov 2010; Moster et al. 2010; Guo et al. 2010). In the spirit of our simple toy model approach, we make the ansatz,

\[ \epsilon_{\text{in}}(M_h) = 0 \text{ if } M_h < M_{\text{min}}, \] (10)

i.e., we set a sharp mass floor for \( \epsilon_{\text{in}}(M_h) \) below \( M_{\text{min}} \). For any reasonable physical mechanism that may be responsible for the suppression of accretion and SFR in low-mass halos (see Section 7), modeling the effect as a sharp cutoff is clearly a very crude approximation, but it is useful as a first attempt in capturing the key features with a minimum number of parameters.

This accretion floor means that the effective accretion of cold gas is suppressed in halos less massive than \( M_{\text{min}} \). Halos more massive than \( M_{\text{min}} \) accrete baryons that are embedded in other merging halos, which could themselves be more massive or less massive than \( M_{\text{min}} \), as well as smooth gas unbound to any halo (Stewart et al. 2008; Genel et al. 2010). The relative contribution of these different contributions leaves sufficient margins to meet an accretion efficiency of 0.7.

The accretion efficiency \( \epsilon_{\text{in}} \) at a given mass is likely to decline in time, as the overall fractions of stars and hot gas (\( \geq 10^4 \text{ K} \)) in the intergalactic medium (IGM) grow. We model this by incorporating a redshift dependence \( \epsilon_{\text{in}}(z) = \epsilon_{\text{in}} f(z) \), to allow for a decrease in cold-accretion efficiency from \( z = 2 \) to \( z = 0 \). For reasons of simplicity, we use a function linear in time with the boundary conditions \( f(z = 2.2) = 1 \) and \( f(z = 0) = 0.5 \). Note that our main results on the slope of the scaling relations are independent of this assumption since a change in \( \epsilon_{\text{in}} \) changes both SFR and \( M_h \) by the same amount. However, as we discuss in Section 3.4, this affects the evolution of the SFR sequence.

Figure 1 (left) shows the mass growth histories for our modeled halos and the model accretion efficiency \( \epsilon_{\text{in}}(M_h) \) with the upper and lower cutoffs at \( M_{\text{max}} \) and \( M_{\text{min}} \). Figure 1 (right) shows the maximum baryonic accretion rate \( f_h M_h \) available for each halo as a function of time. More massive halos reach the \( M_{\text{min}} \) threshold first and cross \( M_{\text{max}} \) over a shorter timescale, leading to downsizing (see Section 4.1).

2.3. Overview of Model Ingredients

At this stage, it is worth summarizing our model and its parameters. The model parameters are as follows:

1. The key assumption in our model is the halo mass floor for cold gas accretion \( M_{\text{min}} \), on the order of \( 10^{10}–10^{11} M_\odot \). As we will show in Sections 3.2 and 5, \( M_{\text{min}} \approx 10^{11} M_\odot \) will be required to match observations.

2. The halo mass floor at the mass \( M_{\text{max}} \) set at a value of \( 1.5 \times 10^{12} M_\odot \) to match the characteristic threshold for virial shock heating (Dekel & Birnboim 2006) and for best reproduction of the observed features of the galaxy bimodality (Cattaneo et al. 2006).

3. The cold gas accretion efficiency \( \epsilon_{\text{in}}(M_h, z) \). As illustrated in Figure 1, it vanishes below \( M_{\text{min}} \) and above \( M_{\text{max}} \), and is set to 0.7 at \( z > 2 \). Below \( z < 2 \), we use \( \epsilon_{\text{in}}(z) = f(z) \times 0.7 \), where \( f(z) \) is a function that is decreasing linearly with time with the boundary conditions \( f(2.2) = 1 \) and \( f(0) = 0.5 \).

4. The SFR efficiency \( \epsilon_{\text{sfr}} = 0.02 \) set from the observed KS relation, and the orbital timescale \( t_{\text{dyn}} = 2 \times 10^7 \text{ yr} (1 + z)^{1.5} \).

Our fiducial model described above is the “accretion floor” model, dubbed hereafter “accFloor,” where the accretion of cold gas and therefore SF are totally suppressed below \( M_{\text{min}} \). We emphasize that our terminology “accretion floor” means that the effective accretion of cold baryons is suppressed in halos with \( M_h(z) < M_{\text{min}} \). Either these baryons are prevented from cooling or they are prevented from entering and reaching the central object.

In Section 3, we will analyze two radically different models for comparison (listed in Table 1): (1) the other extreme “noMmin” model, the mass floor is not applied at all, i.e., we allow both gas accretion and SF to proceed unperturbed below \( M_{\text{min}} \); and (2) in the intermediate “sfrFloor” model, we do allow gas accretion below \( M_{\text{min}} \), but forbid SF there. In this case, the cold gas accumulates until the halo becomes more massive than \( M_{\text{min}} \), and it then turns into stars. We also investigate the impact of the outflow term \( M_{\text{out}} \) in Equation (3) with a model we term “accFloor+.” However, as described in Section 2.1, this term will have no impact on the scaling relations.

Figure 2 highlights the main differences between the three alternative models, “noMmin” (left), “accFloor” (middle), and “sfrFloor” (right). In the top panels, we show the assumed variation of \( \epsilon_{\text{in}} \) with redshift (thick solid lines, right axis) and the resultant evolution of the accretion rate (solid line), baryonic accretion rate (dashed line), and the SFR (dotted line) as a function of redshift. In the bottom panels, we show the resultant evolution of the DM halo mass (solid line), the maximum baryonic mass (dashed line), and the cold gas mass (dotted line). These three models differ significantly in their initial behavior. Compared to the “noMmin” model, the “accFloor” and “sfrFloor” models add a (mass-dependent) delay in the evolution of SFR. In contrast to the “accFloor” model, the “sfrFloor” model accumulates cold gas till the halo reaches \( M_{\text{min}} \). Both of these models lead to no SF below \( M_{\text{min}} \), but have rather distinct physical interpretations and consequences.

2.4. Numerical Approach

Using this simple reservoir setup (Equations (3) and (7)), we solve for its gas and stellar content at each redshift as follows. We first construct average mass growth histories \( M_h(M_0, z) \) for the main progenitors of halos of masses in the range \( M_0 = 10^{10.5–14} M_\odot \) today, as in Figure 1, following Neistein & Dekel (2008, see their Equation (8)). We then numerically integrate Equations (3)–(7) for \( M_{\text{gas}} \) and \( M_\star \) from \( z = 10 \) to

\[ \epsilon_{\text{in}}(M_h) = 0 \text{ if } M_h > M_{\text{max}}. \]
any $z$ until $z = 0$, with the initial conditions $M_{\text{gas}} = M_\ast = 0$ at $z = 10$. The stellar mass $M_\ast$ is updated at each step according to $M_\ast = (1 - R) \int \text{SFR} \, dt$, where $R = 0.52$ is the recycled gas fraction for a Chabrier initial mass function (IMF; Bruzual & Charlot 2003).8

3. RESULTS

3.1. Steady State

Before showing the numerical results, it is worth understanding the key behavior of the reservoir model from a qualitative inspection of Equations (3) and (7). When $M_{\text{gas}}$ in the galaxy is still low, the SFR is low by the KS law (Equation (7)), and it is smaller than the accretion rate dictated by the cosmological environment. The gas reservoir then gradually fills up until the SFR becomes comparable to the accretion rate (see Figure 3). The galaxy enters a quasi-steady state where the SFR is essentially set by the accretion rate, $\text{SFR} \simeq M_{\text{gas, in}}$. Should the gas reservoir be temporarily overfilled (for its current SFR), the SFR would then be larger than the accretion rate, and the galaxy will return to the quasi-steady state.

Figure 3 demonstrates the quasi-steady-state behavior. For our fiducial model (“accFloor”), the bottom panel shows the SFR (solid line) and the maximum accretion rate ($f_B M_h$) (dashed line). The reservoir model reaches rapidly a quasi-steady state in which the SFR scales with the accretion rate.

Because the accretion rate varies with redshift, the steady state can only be achieved when the timescale associated with SFR is comparable to or shorter than the accretion timescale. The SFR timescale, using Equation (8), is

$$t_{\text{sfr}} \equiv \frac{M_{\text{gas}}}{M_\ast} \simeq \left(t_{\text{dyn},7}/2\right) \epsilon_{\text{sfr},0.02}^{-1} (1 + z)^{-1.5} \text{Gyr} \,.$$  

where $t_{\text{dyn},7} = t_{\text{dyn}}/10^7$ yr, and $\epsilon_{\text{sfr},0.02} \equiv \epsilon_{\text{sfr}}/0.02$. From Equation (5), the accretion timescale is

$$t_{\text{acc}} \equiv M_h / \dot{M}_h \simeq 2 M_{h,12}^{-0.1} (1 + z)^{-2.2} \text{Gyr} \,.$$  

Therefore, the condition that the SFR timescale be shorter than the accretion timescale

$$t_{\text{sfr}} \leq t_{\text{acc}}$$  

is met at redshifts $z \lesssim 7$.

The steady-state solution $\text{SFR} \propto \dot{M}_{\text{gas, in}}$ has several important implications. First, it implies that the SFR($z$) is driven by the (net) accretion rate as illustrated on Figure 3. As a result, at around $z \sim 2$, SFR maintains a high and slowly varying value for a few Gyrs (Figure 1(b)), which means that the average SFR is expected to be comparable to the instantaneous SFR. This is indeed supported by the SINS sample at $z \sim 2$, where the birthrate parameter, defined as the ratio of instantaneous to past SFR, is estimated to be $b \simeq 1.2$ (Förster Schreiber et al. 2009).

Second, the resulting SFR, gas and stellar masses are thus independent of the initial conditions given the rapid growth of the gas mass (see Figure 2).

Lastly, when the steady-state condition (Equation (13)) is not met, the gas reservoir is being filled up at a rate faster than its consumption rate. In this case, SFR is time dependent, of the form $r^\alpha$, and sSFR is proportional to $1/t$.

3.2. Effect of $M_{\text{min}}$ on Scaling Relations

Our goal is to better understand the global scaling relations of $z \simeq 2$ SFGs, and the arguments presented in this section are, however, applicable at $z = 0$, $z = 1$ as well. Thus, we take the naive point of view that the slopes of the scaling relations do not evolve with redshift and we stress that our aim is to demonstrate the key role played by the mass floor $M_{\text{min}}$ in connecting the baryonic scaling relations to their DM counterparts. Any evolution in the slopes of the SFR sequence or TF relation will lead to an evolution of this mass floor.

Figure 4 shows the SFR sequence predicted by the three fiducial models (“noMmin,” “accFloor,” and “sfrFloor”), in comparison with the observed SFR sequence at that redshift.

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8 For our purposes, $R$ is a constant given its slow dependence on stellar-population age $T$ for age $T > 10^9$ Gyr.
Figure 2. For a log $M_h = 11.6$ halo at $z = 2.2$, we show the behavior of each of our three models, namely, when $M_{\text{min}} = 0$ (“noMmin,” left panel), when $\epsilon_{\text{in}} = 0$ below the minimum mass floor (“accFloor,” middle panel), and when $\epsilon_{\text{sfr}} = 0$ below the SF floor $M_{\text{min}}$ (“sfrFloor,” right panel). The top panels show the redshift evolution of the dark matter accretion rate $\dot{M}_h$ (solid line), the baryonic accretion rates ($\dot{M}_h \times f_B$) (dashed line), and the SFR (dotted line). The bottom panels show the halo mass ($M_h$) (solid line), the maximum baryonic mass ($M_h \times f_B$) (dashed line), and the gas mass ($M_g$) (dotted line). The effect of $M_{\text{min}}$ is clearly apparent in each case. The “sfrFloor” model will lead to the same stellar mass $M_*$ as the “noMmin” model, given that the SF is simply delayed, and the amount of gas accreted remains the same.

(A color version of this figure is available in the online journal.)

from Pannella et al. (2009) and Daddi et al. (2007). The data show clearly that the sSFR is higher for lower mass galaxies (the mass index $p < 1$), whereas the expectation from the DM relation would predict an inverted trend since its mass index $s > 1$. The slope $s = 1.1$ associated with the predicted accretion rate as a function of halo mass is shown as the dashed line. Figure 5 compares the corresponding $z = 2$ TF relation with the observed relation from Cresci et al. (2009). Similarly, the data show a tilt from the slope 3 associated with the virial relation $M_h \propto V_h^3$ (dashed line).

These two figures highlight our first significant results: the “accFloor” model provides the best match to the two scaling relations. The mass floor $M_{\text{min}}$ affects the slopes of both the SFR sequence and the TF relation in the right direction. It tilts the SFR sequence away from the 1.1 dark accretion rate, and steepens the TF relation away from the 3 virial slope. This effect can easily be explained.

When a mass floor is not applied, once the system reaches the steady-state solution, SFR follows the halo accretion $\dot{M}_* \propto \dot{M}_h$ and thus scales with halo mass according to $\dot{M}_* \propto M_h^{1.1}$. Given that the halo mass is $M_h = \int_0^{t_{\text{min}}} M_h \, dt$, and that the stellar mass is $M_*(t) \propto \int_0^t M_* \, dt$, stellar mass and halo mass are proportional to each other: $M_* \propto M_h$. Hence, we expect in this case $M_* \propto M_h^{1.1}$, as seen in the left panel of Figure 4.

When a mass floor for accretion is imposed (model accFloor), accretion (hence SF) is suppressed until the time $t_{\text{min}}$ when the halo reaches $M_{\text{min}}$, which is a decreasing function of $M_h$, and thus of $M_*$. The stellar mass is now an integral from $t_{\text{min}}(M_h)$ to $t$: $M_*(t) \propto \int_{t_{\text{min}}(M_h)}^t M_* \, dt$. The less massive systems had less time to grow stars and $M_*$ is now smaller at a given halo mass, by
a larger factor for a smaller halo. This effect is responsible for the required tilt both in the SFR sequence and the TF relation as seen in the middle panels of Figures 4 and 5.

In the sfrFloor model, all the gas that has accumulated in the galaxy until \( t_{\text{min}} \) turns into stars after \( t_{\text{min}} \), so the total stellar mass after that time is expected to be the same as in model "noMmin," as seen in the right panels of Figures 4 and 5.

In summary, a mass-dependent delay in the gas accretion is found to be necessary to better reproduce the observed slopes of the scaling relations. In order for the time delay to have a significant effect, we find that \( M_{\text{min}} \) needs to be within 1 dex of \( M_{\text{max}} \), i.e., \( M_{\text{min}} \sim 10^{11} M_\odot \). In Section 5, we will put a stronger constraint on the numerical value of \( M_{\text{min}} \). The physical origin of the mass floor \( M_{\text{min}} \) is addressed in Section 7.

This mass-dependent delay can only be achieved in our model by the mass floor \( M_{\text{min}} \). Indeed, these results are completely independent of the SF efficiency \( \epsilon_{\text{sf}} \), and of the feedback term in Equation (3). A change in these two parameters will lead to a change in both \( M_\ast \) and SFR, i.e., leaving the SFR sequence intact. A change in the accretion efficiency \( \epsilon_{\text{in}} \) via its mass dependence is the only mechanism that we found to affect the slope of the scaling relations.

3.3. Connections Between the Two Relations

It is very interesting to realize that the TF relation and SFR sequence are in fact tied to each other. Indeed, the steady-state solution \( M_\ast \propto M_b^s \), together with the SFR-sequence \( M_\ast \propto M_b^p \) implies the TF relation \( (M_\ast \propto V_h^3) \). Indeed, using the virial relation \( M_b \propto V_h^3 \), we find

\[
m = 3s/p.
\]

With the indices being \( s \simeq 1.1 \) and \( p \simeq 0.8 \), the TF slope is indeed \( m \simeq 4 \) as observed (e.g., Meyer et al. 2008; McGaugh 2005). Note that this argument holds for various values of the indices \( s \) and \( p \) as long as \( s/p \simeq 4/3 \). As noted in Section 1, there are marginal indications that \( p \) varies from near 0.7 at \( z = 0–1 \) (Brinchmann et al. 2004; Noeske et al. 2007) to about 0.9 at \( z \sim 2 \) (Daddi et al. 2007; Santini et al. 2009; Pannella et al. 2009). These subtleties do not change our arguments given that all observational evidence shows that \( p < 1 \) (sSFR higher at lower mass), while the DM counterpart has a mass index \( s > 1 \) (specific accretion rate higher at higher mass). Should this redshift dependence of the mass index \( p \) be confirmed, it would point toward a redshift evolution of \( M_{\text{min}} \).

Given that the slopes of these two scaling relations are coupled together and that, as we will show, the accretion rate regulates the evolution of the SFR sequence, we expect the accretion rate to regulate the evolution of the TF relation as well. However, a more detailed analysis of the TF zero-point normalization requires knowledge of the explicit relation between the virial velocity of the halo, which we have crudely referred to so far as \( V_h \), and the observed maximum rotation velocity \( V_{\text{max}} \). Early models of galaxy formation assumed halo adiabatic contraction due to the infall of baryons into the halo center (Blumenthal et al. 1986; Mo et al. 1998), which typically gives \( V_{\text{max}} \simeq 1.6–1.8 V_h \) for a wide range of halo concentrations \( c = [5, 30] \) and disk mass fractions \( m_d = [0.02, 0.2] \). However, such models often fail to reproduce the observed TF zero point (e.g., Navarro & Steinmetz 2000; Dutton et al. 2007). Disk formation models can simultaneously fit the TF relation and the galaxy luminosity function if there was actually a slight halo expansion, where \( V_{\text{max}} \sim 1.2 V_h \) (Dutton et al. 2007). Evidence against adiabatic contraction is mounting also at \( z \sim 2 \) (Burkert et al. 2009). We find that a similar factor of about 1.2 between \( V_{\text{max}} \) and \( V_h \) provides a good match to the TF zero point at \( z \sim 2 \).

3.4. Redshift Dependence

According to Equation (1), the observed SFR-M\(_\ast\) relation (equivalently the sSFR (sSFR \( \equiv \text{SFR}/M_\ast \)) at a given stellar mass) is declining with time from \( z = 2 \) to the present by a factor \( \sim 20 \). Not only does our model reproduce this evolution, but it can help us understand its origin.

Figure 6(a) shows the model sSFR as a function of redshift for galaxies of different stellar masses, \( M_\ast = 10^{9.5}, 10^{10}, \) and \( 10^{10.5} M_\odot \), in comparison with observed sSFR for \( M_\ast \) in the range \( 10^{10}–10^{10.5} M_\odot \) by Daddi et al. (2009), Damen et al. (2009a), and Pannella et al. (2009). One sees that our toy model reproduces the variation of the sSFR with redshift relatively well to \( z = 2 \). We discuss the recent results at \( z = 4 \) to \( z = 7 \) in the Appendix.

The redshift dependence of the sSFR is driven by redshift evolution of the average accretion rate, Equation (5), which is \( \propto (1+z)^{2/3} \). Simply put, this factor is driven by the expansion of the universe. This can be clearly seen using the EPS formalism, where the factor \( (1+z)^{2/3} \) originates directly from the derivative of the linear growth factor \( D \) (Neistein & Dekel 2008; Birnboim et al. 2007). Indeed, under the EPS formalism, the specific accretion rate is \( M_b/M_\ast = -s(M_\ast) \dot{\omega} \), where \( s(M_\ast) \) is the self-invariant mass variable, and \( \dot{\omega}(z) = 1.69/D(z) \) is the self-invariant time variable. \( s(M_\ast) \) is a function of the variance of the initial density fluctuations on mass scale \( M_\ast \), \( \sigma(M_\ast) \) (see Equations (A5) and (A6) in Birnboim et al. 2007).

The dotted line in Figure 6(a) demonstrates that the time evolution of the sSFR (Equation (6)) is indeed roughly proportional to \( \dot{\omega} \), except that the predicted decline with time of the overall accretion rate is a bit slower than the decline in sSFR at \( z < 1 \) (Equation (1)). An improved fit to the evolution at low redshifts

![Figure 3](image-url)
Figure 4. $z = 2$ SFR sequence predicted from our three fiducial models with no mass floor $M_{\text{min}}$ (left), an accretion floor (middle), and an SF floor (right). The modeled points are shown as filled (open) squares, for halos below (above) the virial shock mass $M_{\text{max}}$. The observed SFR sequence from Pannella et al. (2009) and Daddi et al. (2007) is shown as gray shaded areas. The dotted line in each panel shows a line of slope 1.1 expected from the global accretion rate (Equation (5)). The “accFloor” model provides the best match to the observed SFR sequence with SFR $\propto M_\odot^{1.8}$.

(A color version of this figure is available in the online journal.)

is achieved by a small modification of the form $\dot{\omega} \times f(z)$ (long dashed line, defined in Section 2.3), attempting to correct for the gradual decline with time of the cold gas fraction in the overall baryonic accretion. This confirms that a gradual decrease in the accretion efficiency parameter, by a factor of $\sim 2$ between $z = 2$ and $z = 0$, helps in recovering the observed evolution of the sSFR.

Davé (2008) parameterized the evolution of the SFR sequence at a given $M_*$ in terms of an SF activity parameter, defined as $\alpha_{\text{sf}} = (M_*/M_\odot)/(t_H - 1 \text{ Gyr})$. He pointed out that most models predict that galaxies have a growth rate comparable to $t_H$, the Hubble time, i.e., with a constant $\alpha_{\text{sf}} \simeq 1$, whereas observations reveal a growth of $\alpha_{\text{sf}}$ by a factor of $\sim 3$ from $z = 2$ to $z = 0$ (see Figure 6). They used this apparent discrepancy to argue that the initial stellar mass function (IMF) must be evolving in time. Figure 6(b) shows the evolution of $\alpha_{\text{sf}}$ predicted by our fiducial “accFloor” model for log $M_* = 10.0$ and 10.7 $M_\odot$. An increase of $\alpha_{\text{sf}}$ by a factor of $\sim 3$ between $z = 2$ and $z = 0$, not far from the observational result, without any evolution in the IMF, is a general feature of the time dependence of the accretion efficiency.

The SFR sequence and sSFR($M_*$) at any given redshift are simply isochrones in their respective parameter plane. Our model gives us the opportunity to show the actual tracks of individual galaxies, which are shown in Figure 7 for the sSFR–$M_*$ (left) and the SFR–$M_*$ (right) relations.

4. ADDITIONAL ROLE OF $M_{\text{min}}$

Given that our toy model seems to reproduce the scaling relations and their evolution rather well, we now compare the predictions of the model to observations involving the “downsizing” effect (Section 4.1), the halo stellar fractions (Section 4.2), and the galaxy gas fractions (Section 4.3).

4.1. Downsizing and Tau Models

Stellar-population analyses of red galaxies in the local universe show that stars tend to have formed earlier and over a
shorter time span in the most massive ones (e.g., Heavens et al. 2004; Thomas et al. 2005; Jimenez et al. 2007; Thomas et al. 2010). This is sometimes termed as “archeological downsizing.” This age–mass correlation is also present in large samples (up to $3 \times 10^8$) of active galaxies in the Sloan Digital Sky Survey (SDSS; e.g., Gallazzi et al. 2005; Jimenez et al. 2007).

The SF history (SFH) tracks shown in Figure 8 demonstrate the origin of the age–mass anti-correlation. More massive halos reach the mass floor earlier, and spend less time between the mass floor $M_{\text{min}}$ and the mass ceiling $M_{\text{max}}$. This is a natural consequence of the halo tracks in Figure 1 combined with our key assumption of $M_{\text{min}}$.

Figure 9 shows the resulting age–mass relation at $z = 0$ (left) and $z = 2$ (right). The left panel shows the formation redshift $z_f$ as a function of stellar mass. Similarly to Noeske et al. (2007b), the formation redshift $z_f$ is defined here when SF began, i.e., when $M_h(z)$ reaches $M_{\text{min}}$. The mass index of the resulting age–mass relation is surprisingly close to the $(1 + z_f) \propto M^{0.3}$ inferred by Noeske et al. (2007b) shown as the dashed line (normalization arbitrary).

That the SFR sequence leads to the same mass-dependent SFH as in the archeological downsizing was already argued by Noeske et al. (2007b). Indeed, they fitted $\tau$-models to the $z \sim 1$ SFR–$M_*$ relation and concluded the sSFR relation required both $\tau$ and the formation redshift $z_f$ to be mass dependent ($\tau \propto M^{-1}$, $(1 + z_f) \propto M^{0.3}$). They call this mass-dependent SFH “Staged Galaxy Formation” where more massive galaxies are formed first on a shorter timescale. However, the mass dependency in our model did not have to agree with the results of Noeske et al. (2007a).

This age–mass relation may already be in place at $z \sim 2.2$ (Figure 9 (right)). Indeed, the large sample of the SINS survey (Förster Schreiber et al. 2009) shows a similar trend between spectral energy distribution (SED) derived ages and stellar masses. In Figure 9, we compare these observations (gray points) to the luminosity weighted ages in our model (solid squares), which are derived according to $\int f_{\text{obs}} \text{SFR}(t_{\text{obs}}) d_{\text{obs}}$ where $t_{\text{obs}}$ is the lookback time since $z = 2.2$.

Thus, our model reveals that the key player for downsizing is the accretion mass floor $M_{\text{min}}$. As was first realized in Neistein...
Figure 6. Left: evolution of specific star formation rate \( \text{sSFR} \equiv \frac{\text{SFR}}{M^*} \) at a fixed \( M^* \) from our fiducial model (“accFloor”). The dotted line shows the time dependence of the accretion rate through the function \( \dot{\omega} \propto (1 + z)^2 \) (see the text). The long-dashed line shows the slight modification of the introduced function \( f(z) \) (defined in Section 2.3). Right: Evolution of the star formation activity parameter \( \alpha_{sf} \equiv \left( \frac{M^*}{\dot{M}^*} \right) / (t_H - 1 \text{ Gyr}) \). In both panels, we show the observations by Daddi et al. (2009) (solid circles), Damen et al. (2009a) (open triangles), and Pannella et al. (2009) (solid squares) for galaxies with \( \log M^* = 10-10.5 \). The shaded area shows the hydro-simulation results of Davé (2008). The evolution of sSFR and \( \alpha_{sf} \) is mostly driven by the time dependence of the accretion.

(A color version of this figure is available in the online journal.)

Figure 7. Left: evolution of the sSFR–\( M^* \) relation. Right: evolution of the main sequence. In both panels, the solid lines show the isochrones of the scaling relations at \( z = 4, 2.2, 1.2, \) and \( z = 0 \). The dotted lines show the individual tracks for the simulated halos.

(A color version of this figure is available in the online journal.)

et al. (2006), the archeological downsizing effect is a direct consequence of a mass floor \( M_{\text{min}} \).

It has been noted that, for early type galaxies, the more massive ones reach the red sequence earlier, a phenomenon dubbed “downsizing in mass” (Cimatti et al. 2006). In our model, when galaxies reach the ceiling mass \( M_{\text{max}} \), their gas supply dries out and SF is quenched, i.e., the galaxy becomes red and passive. The same SFH tracks shown in Figure 8 lead to this other type of downsizing, in this case due to \( M_{\text{max}} \) (see also Cattaneo et al. 2008).

4.2. Stellar Fractions

Many have shown that the stellar fraction in galaxies is a strong function of the total halo mass (e.g., van den Bosch et al. 2003b; Eke et al. 2005; Shankar et al. 2006; van den Bosch et al. 2007; Kravtsov 2010; Moster et al. 2010; Guo et al. 2010) owing to the very different shapes of the halo mass function and the galaxy stellar mass function. We confront our toy model with this additional observational constraint.

Figure 10 compares the halo stellar fractions, defined as

\[
f_\star \equiv \frac{M_\star}{0.18 \times M_h},
\]

predicted from our model to relevant data at \( z = 0 \) (left) and \( z = 2.2 \) (right). The shaded area shows the results from the SDSS analysis of Guo et al. (2010), and the solid line shows the similar result from Moster et al. (2010). At \( z = 2.2 \), the observational estimates are from the disk-dominated galaxies
of the SINS survey (Förster Schreiber et al. 2009; Cresci et al. 2009), where the virial mass is estimated assuming that $V_h = V_{\text{max}}$.

We see that the models “noMmin” and “sfrFloor” overpredict the stellar fraction for halos below $10^{12} M_\odot$, while the “accFloor” model recovers the shape $f_\odot(M_h)$, including the drop toward lower masses occurring at the proper range of $M_h$. Once again, we learn that an effective halo mass floor for accretion at $M_{\min} \sim 10^{11} M_\odot$ is essential for a fit to the data.

However, the amplitude of the $f_\odot(M_h)$ function predicted in the simple accFloor model needs to be reduced roughly by a factor 2 for a match with the data. A better fit is obtained if we include in our model (Equation (3)) an outflow term $\dot{M}_{\text{out}} = 0.6 \dot{M}_\odot$.

Figure 11 compares the predicted $f_g(M_\ast)$ from our model “accFloor+” to those observations at $z = 0$ (left) and $z = 2$

4.3. Gas Fractions

Another interesting constraint is provided by the gas fraction in galaxies, defined as

$$ f_g \equiv \frac{M_g}{M_g + M_\ast}. $$

At low redshift, we learn from all-sky HI surveys (e.g., Zwaan et al. 2003; Rosenberg & Schneider 2003; Meyer et al. 2004; Giovanelli et al. 2005; Wong et al. 2006; Catinella et al. 2010) that the gas fractions in SFGs vary systematically with stellar mass (e.g., Baldry et al. 2008, and references therein). These data are presented in the left panel of Figure 11. At $z \sim 0$, thanks to the rapid progress in the sensitivity of millimeter interferometers (such as the IRAM Plateau de Bure interferometer, PdBI), it is now possible to study CO rotational emission lines of normal high-redshift SFGs at $z > 1$ and therefore constrain their gas masses. The observed gas fraction for $z \sim 1–2.5$ SFGs with $M_\ast > 10^{10.6} M_\odot$ ranges from 0.3 to 0.6 (Daddi et al. 2010a; Tacconi et al. 2010) with an average of about 0.44 for the large sample of 19 SFGs of Tacconi et al. (2010).

Figure 11 compares the predicted $f_g(M_\ast)$ from our model “accFloor+” to those observations at $z = 0$ (left) and $z = 2$. This constraint is very consistent with the observations of Heckman et al. (2000) and Martin (2005) who found that $M_{\text{out}} \approx \text{SFR}$. Indeed, the corrections involved in deriving the outflow rates in galaxies (for ionization, metallicity, and depletion) are usually of several orders of magnitude. Moreover, the instantaneous outflow rate needs not be equal to our net outflow $M_{\text{out}}$ term, which only describes the baryons that leave the halo forever. We note that this constraint on the outflow rate is degenerate with the accretion efficiency $\epsilon_{\in}$, i.e., $M_{\text{out}} = (0.4–0.8) M_\ast$ for $\epsilon_{\in}$ ranging from 0.5 to 1.0.

It is important to realize that the results presented thus far are independent of this outflow term. The SFR sequence is not sensitive to feedback as lowering the SFR by a factor of 2 leads to a stellar mass $M_\ast$ lower by the same factor. Dutton et al. (2010) demonstrate this in a similar work.

Figure 9. Archeological downsizing. Left: formation redshift $z_f$ (defined when $M_h(z)$ reaches $M_{\min}$) as a function of stellar mass for our modeled galaxies (solid squares). The dashed line shows the $M_h \sim z_f$ relation inferred by Noeske et al. (2007b) from the $z \sim 1$ SFR sequence, namely, $(1 + z_f) \propto M_\ast^{-0.3}$ (normalization is arbitrary). Right: the modeled $z_f \sim M_\ast$ relation is shown with the solid squares. Modeled ages are the luminosity weighted ages (see the text). The gray points show the ages (inferred from SED fitting) for the large sample of $z = 2$ SFGs available in the SINS survey (Förster Schreiber et al. 2009). At all epochs, the age–mass dependence is a direct consequence of the mass floor $M_{\min}$.

(A color version of this figure is available in the online journal.)
Figure 10. Stellar fractions \( f_\star \equiv M_\star / (0.18 \times M_h) \) at \( z = 0 \) (left) and \( z = 2 \) (right) predicted from our fiducial model \( \text{"accFloor+"} \). In each panel, the alternative models, namely, with no mass floor \( M_{\text{min}} \) (dotted line), with an accretion floor (long dashed line), and with an SF floor (short dashed line) are indicated. The \( z = 0 \) stellar fractions from Moster et al. (2010) and Guo et al. (2010) are shown as the shaded area and thick line, respectively. The stellar fractions in \( z = 2 \) SFGs are shown as gray circles (Cresci et al. 2009).

(A color version of this figure is available in the online journal.)

Figure 11. Solid squares show the gas fractions as a function of \( M_\star \) predicted from our fiducial model \( \text{"accFloor+"} \) at \( z = 0 \) (left) and \( z = 2 \) (right). The observed local gas fractions from Baldry et al. (2008) and Zhang et al. (2009) are shown as the solid circles and shaded area, respectively. The new \( z \sim 2 \) gas fraction is represented by the shaded box for dozen SF galaxies with \( \log M_\star > 10.6 \) (see Tacconi et al. 2010). The upturn of \( f_g \) at the low-mass end is again a result of the mass floor \( M_{\text{min}} \).

(A color version of this figure is available in the online journal.)

We see that the model is consistent with the data both at \( z = 0 \) and at \( z \sim 2 \). Locally, the upturn of \( f_g \) at the low-mass end is again a result of the mass floor \( M_{\text{min}} \).

Contrary to the other results presented so far, the gas fractions are sensitive to SF efficiency \( \epsilon_{\text{sfr}} \), given that the SFR is set by the steady-state solution (or by the accretion rate). A lower SF efficiency leads to higher gas fractions and vice versa. We used a universal \( \epsilon_{\text{sfr}} \) set by the KS relation and the redshift evolution of \( f_g \) in our model appears to be very consistent with observations. Indeed, the model predicts \( f_g = 0.4 - 0.55 \) at \( z = 2 \) and \( f_g = 0.3 - 0.45 \) at \( z = 1 \), which is in good agreement with the average gas fractions estimated from the CO observations, 0.44 and 0.34, respectively (Tacconi et al. 2010; see also Daddi et al. 2010a).

Our simplistic toy model, which was only tuned (with \( M_{\text{min}} \)) to reproduce the scaling relations, helps us also to understand the trends of gas fraction with mass and redshift for SFGs.

5. STAR FORMATION HISTORY: CONSTRAINING THE MASS FLOOR

Thus, based on the two scaling relations, the stellar and gas fractions, our model shows that a mass floor \( M_{\text{min}} \) must play an important role in galaxy formation and we argued that \( M_{\text{min}} \) should be \( \sim 10^{11} \, M_\odot \). We are about to show that the evolution of the average density of SF, \( \rho_s(z) \), will give us a quantitative constraint on \( M_{\text{min}} \).
dropping by an order of magnitude between $z > 2$ observations (Schiminovich et al. 2005; Dahlen et al. 2007; Mobasher et al. 2009) with the $z > 2$ data (Hopkins & Beacom 2006; Verma et al. 2007; Wilkins et al. 2008) all converted to a Chabrier IMF following Hopkins & Beacom (2008).

(A color version of this figure is available in the online journal.)

Figure 12 shows a compilation of data for the cosmic SFH, derived from observations under the assumption of a Chabrier IMF. The average density of SF, $\rho_\star$, is observed to be rising from $z \sim 6$ to $z \sim 2$, reaching a plateau at about $z \sim 2$, and then dropping by an order of magnitude between $z \sim 1$ and $z = 0$ (e.g., Lilly et al. 1996; Madau et al. 1996; Hopkins & Beacom 2006, and references therein).

Our “accFloor+” model convolved with the Sheth–Tormen model for the halo mass function (Sheth et al. 2001) is shown for four different values of $M_{\text{min}}$, ranging from $10^9$ to $3 \times 10^{11} M_\odot$. We see that the value of $M_{\text{min}}$ indeed determines the growth of $\rho_\star$ with time at high redshift. Best agreement to the growth rate observed at $z > 2$ is obtained with $M_{\text{min}} = 10^{11} M_\odot$. Thus, we conclude that the rise in the cosmic SFH from $z = 6$ to $z \sim 2$ confirms the necessity of a mass floor and provides the strongest constraint on the value of $M_{\text{min}}$.

Our model can also help us to understand the origin of the decline at late times. Possible reasons for this sharp decline could in principle be the acceleration of the universe at recent cosmological epochs, gas exhaustion into stars and outflows, the quenching of SFR above the mass ceiling, or simply the generic decline in the accretion rate into DM halos due to the expansion of the universe. The sharp decline at $z < 2$ is predominantly driven by the decreasing accretion rate as a direct result of the expansion of the universe as it can be seen from Figure 1 (right) and discussed in Section 3.4. All the other potential sources for the decline, including the acceleration of the universe and the introduction of $M_{\text{max}}$ or gas consumption, have only minor effects on this decline.

Many other groups (e.g., Springel & Hernquist 2003; Nagamine et al. 2006; Schaye et al. 2010; Choi & Nagamine 2009) have used cosmological hydrosimulations to investigate the cosmic SFH and to identify the physical processes that drive its evolution. Similarly to our model, Hernquist & Springel (2003) used an analytical approach and reached similar conclusions. They found that, at high-$z$, the rise in $\dot{\rho}_\star(t)$ originates from the gravitationally driven growth of halos, while at low-$z$, the decline is due to the expansion of the universe, through the inhibition of cooling. While in our model the SFR at a given halo mass is driven by the halo growth rate, their model assumes an ad hoc form for the SFR that is constant as a function of mass for all halos of virial temperature above $10^5$ K. Similarly to our $M_{\text{min}}$ assumption, in their model stars do not form in halos below $T_{\text{vir}} = 10^4$ K. Their assumption also leads to a peak in the SFH, albeit at a higher redshift $z \sim 5$ or 6. In spite of the differences between their results and our results, this again illustrates the impact of a mass floor.

6. DISCUSSION AND CONCLUSIONS

The purpose of this paper is to seek a basic understanding of the relations between global properties of SFGs and their time evolution between $z \sim 2$ and today. This includes the origin of the SFR sequence and the TF relation. We achieved this via an idealized and remarkably simple “toy” model, based on the cosmological evolution of accretion rate into DM halos, the efficient deep penetration of cold gas streams into the central galaxies, and the standard efficiency of SFR in galactic disks.

The model should be interpreted as a learning tool for gaining insight into the role played by several key physical processes.

The model naturally leads to the observed scaling relations, $\dot{M}_\star \propto M_\star^{0.8}$ and $\dot{M}_\star \propto V_{\text{max}}^4$, and to the archeological “downsizing” phenomenon, provided that there is a mass floor $M_{\text{min}} \sim 10^{10} M_\odot$ below which the accretion of cold gas is substantially suppressed. An analysis of the SFH allowed us to put an additional strong constraint on the mass floor: $M_{\text{min}} \sim 10^{11} M_\odot$, corresponding to $V_{\text{vir}} \sim 100 M_\odot h^{-1/3}$ for the $10^{11} M_\odot h^{-1/3}$ km s$^{-1}$, best accounts for the observed scaling relations.

Recently, Dutton et al. (2010) have used a full semianalytic approach to study the origin of the SFR sequence. While we agree with the conclusions of Dutton et al. (2010) with respect to the steady state, the impact of feedback, and to the evolution of the SFR sequence, our model differs from theirs on one significant aspect: the lack of a mass floor. Cattaneo et al. (2010) used a full semianalytical model with an effective efficiency $\epsilon_{\text{in}}$ similar to ours (compare their Figure 4 to our Figure 1). Their effective mass floor is due to the combined effect of SN feedback and photoheating due to reionization. These authors, however, used a reionization threshold at $V_{\text{re}} = 40$ km s$^{-1}$ in order to fit the luminosity function, which is higher than expected ($V_{\text{re}} \sim 10$ km s$^{-1}$) from pure reionization (e.g., Barkana & Loeb 1999; Iliev et al. 2005) by an order of magnitude in halo mass.

6.1. Caveats and Limitations

Our model, which relies mainly on the cosmological accretion rate, is based on very few parameters ($\epsilon_{\text{in}}, M_{\text{max}},$ and $M_{\text{min}}$) compared to semianalytic models (e.g., Somerville et al. 2008). Our main free parameter is $M_{\text{min}}$ (affecting the cooling/accretion efficiency $\epsilon_{\text{in}}(M)$), as we use a normal SF efficiency $\epsilon_{\text{sfr}}$ and set $M_{\text{max}}$ following Cattaneo et al. (e.g., 2006, 2008).

The remarkably few parameters in our model are clearly an advantage (see also Neistein & Weinmann 2010), but this inevitably leads to oversimplifications. For instance, modeling the lower and upper thresholds for cold gas accretion as sharp

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9 We note that the threshold could have been defined in terms of a constant virial velocity $V_{\text{vir}}$, and this would have led to similar results.
of the scaling relations at the massive end, $M_{\ast} \sim 10^{11} M_{\odot}$, in Figures 4 and 5.

The toy model presented in this paper does not allow us to address other observational constraints such as the mass–metallicity relation because this is rather sensitive to the exact treatment of the metal content of the stellar, SN outflows combined with the unknown fraction of ISM that is entrained in the outflow.

We also cannot address the scatter in the scaling relations as the model deals only with the mean global properties. Variations in the mass assembly history are to be expected (e.g., Genel et al. 2008) and will likely contribute to the scatter in the scaling relations (e.g., Dutton et al. 2010).

Another limitation of our simplified model is that we do not include bulge growth, which can be an additional drain for SF disks in our model.

6.2. Conclusions

In spite of its simplistic assumptions, our model captures many of the observed properties of SF galaxies and their evolution. This approach helps us gain insight into the following questions:

1. Why are the $z = 2$ SFRs so high? What sets the SFR-sequence? Once the timescale for SF becomes shorter than the accretion timescale, galaxies settle in a quasi-steady state where the SFR is driven by the accretion rate (Figure 3). As a consequence, the high SFRs for $z = 2$ SFGs are driven by the high accretion rate of cold baryons. The steady state also implies that the instantaneous SFR is comparable to the average SFR ($\langle SFR \rangle$) for a few Gyr from $z \sim 4$ to $z \sim 1.5$ (see Figure 1(b)). This is consistent with observations from the SINS survey (Förster Schreiber et al. 2009) where the ratio $\langle SFR \rangle/SFR$ is found to be $\simeq 1.0$ in $z = 2$ SFGs.

2. What is the origin of the scaling relations of SFGs? We find that a single parameter, namely, the mass floor $M_{\rm min} \sim 10^{11} M_{\odot}$ for accretion, is key in generating the correct slopes of both the SFR sequence and the TF relation (Figures 4 and 5). Thus, the two scaling relations are not independent relations, as we show in Section 3.3. We emphasize that the SFR sequence and its evolution are insensitive to the values of the efficiency parameters $\epsilon_{\rm in}$ and $\epsilon_{\rm sfr}$, because changes in either of these efficiencies affect both $M_t$ and $M_\ast$, and shift galaxies along the SFR sequence. The same argument holds for the impact of feedback on the SFR sequence (see Dutton et al. 2010). This is the reason for the failure of any model that attempts to reproduce the tilts of the scaling relation by varying either the SF efficiency or the feedback efficiency (see Davé 2008; Damen et al. 2009a, and references therein).

3. What sets the evolution of the SFR–$M_\ast$ relation? The redshift evolution of the SFR sequence (or sSFR) is driven predominantly by the redshift evolution of the cosmological DM accretion rate. Both the SFR sequence and the sSFR at a fixed mass follow the cosmological decline of average accretion rate onto halos as a result of the expansion of the universe (Figure 6). A slight improved match with the observations is obtained when we force the accretion efficiency to decrease with time at $z < 2$.

4. What is the origin of archeological downsizing? Why did massive galaxies form their stars earlier and over a shorter duration (e.g., Heavens et al. 2004; Thomas et al. 2005; Gallazzi et al. 2005; Jimenez et al. 2007)? An inspection of the halo trajectories in (Figure 1) reveals that more massive halos reach the mass floor earlier, and spend less time between the mass floor and the mass ceiling (see also Figure 8). It is the mass floor $M_{\rm min}$ that naturally produces the age–mass relation (Figure 9) as Neistein et al. (2006) already pointed out.

5. Is the observed low stellar fraction compatible with the high accretion efficiency of cold gas? Having a high accretion efficiency ($\epsilon_{\rm in} > 50\%$) is entirely compatible with the observed stellar mass fractions $f_{\ast}$ that are low (<50%), i.e., $f_{\ast} \sim 0.20$ (Figure 10), provided a net outflow rate $M_{\rm out} \propto 0.6 M_{\ast}$ is included. This is consistent with observations $M_{\rm out} \simeq 1.0 M_{\ast}$ given that a significant fraction of the outflowing gas may be returned into the interstellar medium by $z = 0$ (e.g., Oppenheimer & Davé 2008).

6. Why are the gas fractions large at $z > 1$? The observed high gas fractions at $z > 1$ arise naturally from the high, mass-dependent accretion efficiency and the low SF efficiency. The SFR is set by the accretion rate in the steady-state solution, while the gas fraction is determined by the SFR efficiency $\epsilon_{\rm sfr}$. The model predicts mean gas fractions of $30\%$–45% at $z = 1.2$ and $40\%$–55% at $z = 2.2$ (Figure 11) in good agreement with the recent observations of Tacconi et al. (2010) and Daddi et al. (2010a).

7. What is the origin of the shape of the SFH? The decline in the SFH from $z = 2$ to the present is primordially due to the decline in accretion rate associated with the cosmological expansion. The rise of $\rho_\ast$ from $z = 6$ to $z = 2$ necessitates a mass floor near $M_{\rm min} \sim 10^{11} M_{\odot}$ (Figure 12).

The successes of our model are evidence for the central role played by the key ingredient of our reservoir model, namely, the halo growth rate in the standard cosmology. While our model reveals the importance of a mass floor $M_{\rm min}$ for accretion, it does not address its physical origin.

7. ON THE ORIGIN OF THE MASS FLOOR

The exact mechanism that suppresses or holds accretion below $M_{\rm min}$ may be due either to photoheating associated with the fresh UV background after reionization (e.g., Thoul & Weinberg 1996; Quinn et al. 1996; Barkana & Loeb 1999; Gnedin 2000; Dijkstra et al. 2004), or to SN feedback (e.g., Dekel & Silk 1986; Dutton & van den Bosch 2009; Oppenheimer & Davé 2008).\footnote{Note, in the context of momentum driven winds (Oppenheimer & Davé 2008), our mass floor corresponds to a threshold in the mass loading factor $\eta$ where $\eta \rightarrow \infty$ below $M_{\rm min}$.} Photoheating associated with reionization increases the mean temperature of the IGM to $\sim 10^4$ K. This reduces the cooling rate of the hot gas and prevents the assembly of new low-mass galaxies with $V_{\rm esc} < 10$ km s$^{-1}$ (or $M_t \sim 10^8 M_{\odot}$), and those that have already collapsed are relatively quickly photoevaporated (e.g., Barkana & Loeb 1999; Iliev et al. 2005). SN explosions, like photoheating from reionization, provide a negative feedback on SF. If present...
simultaneously, photoheating and SN feedback effects can mutually amplify each other’s ability to suppress the SFR (Pieri et al. 2007; Pawlik & Schaye 2009). Lastly, the formation of cold ($T \sim 10-30$ K) molecular gas (necessary for SF) requires specific physical conditions in addition to high gas densities, such as high gas pressure (Blitz & Rosolowsky 2006) and pre-enrichment. Whether these conditions are met in low-mass halos is an open question. Furthermore, radiative feedback effects from massive stars during intense SF episodes could play a crucial role in disrupting the GMCs as proposed by Murray et al. (2010).

It is possible that no single mechanism can produce such a mass floor. For instance, since simple arguments on SN feedback give $f_* \propto M_*/M_h \propto M_h^{\beta}$ with $\beta = 2/3$ (Dekel & Silk 1986), an additional physical process must be at play. The combination of SN feedback and a (very high) reionization threshold can produce a steeper $f_*$ as shown in Cattaneo et al. (2010). Similarly, van den Bosch et al. (2003a) and Mo et al. (2005) showed that artificially induced re-heating of the IGM to $T \sim 10^5$ K directly leads to a mass-dependent efficiency $f_*$. However, this effect alone also cannot account for the steep mass dependence of $f_*$ (e.g., Lu & Mo 2007).

Lastly, Cantalupo (2010) argued that the missing ingredient could be the effect of photoionization by local sources on the incoming cooling gas. He showed that soft X-rays generated by SF efficiently alter the ionization state of atoms, such as O. This effectively removes the main coolants and increases the cooling times by orders of magnitude, essentially stopping accretion, preferentially in low-mass halos. While this is an attractive mechanism, further work is needed in order to understand quantitatively whether photoionization from local sources are able to regulate the cosmological gas accretion rates.

One way to gain insight into which of these possible mechanisms are at play is to constrain the transition in accretion efficiency near $M_{\text{min}} (\epsilon_m \propto M_h^{\eta})$, which is related to the slope $\beta$ of the relation $f_* \propto M_*/M_h \propto M_h^{\beta}$ under the quasi-steady-state solution with $\eta \sim \beta$. Our modeling indicates that $\eta$ should be steeper than unity. Observationally, various groups have determined that $\beta \geq 2$ (Shankar et al. 2006; Baldry et al. 2008; Kravtsov 2010; Moster et al. 2010; Guo et al. 2010).

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APPENDIX

SPECIFIC SFR IN THE EARLY UNIVERSE

Upon completion of this work, several groups have used deep Hubble Space Telescope data to their limit and published constraints on the SFR sequence beyond $z = 4$ (Stark et al. 2009), at $z = 5$ (Yabe et al. 2009), and $z = 7$ (Gonzalez et al. 2010). As shown in Figure 13, some groups (Stark et al. 2009; Gonzalez et al. 2010) find that the SFR sequence no longer evolves, i.e., the sSFR remains constant from $z \sim 4$ to $z \sim 7$, while Schaerer & de Barros (2010) find moderate and Yabe et al. (2009) strong evolution (see also Sawicki et al. 2007). The analysis of Schaerer & de Barros (2010) includes the sample of Gonzalez et al. (2010) with the added WFC3 data from Oesch et al. 2010. Given that it is a challenge to derive accurate (dust-corrected) SFR and stellar masses at those redshifts in a self-consistent way, it may be premature to view the (non-)evolution of Gonzalez et al. (2010) beyond $z = 2$ as definitely understood.

A.1. Assumptions in the Model

Should the non-evolution of SFR be real claimed by Gonzalez et al. (2010), could the discrepant results of our model be due to some of our assumptions? Our model fails to indicate the turnover of Gonzalez et al. (2010) in the evolution of sSFR($z$) beyond $z = 2$ as shown in Figure 13. We turn to some of our assumptions in order to examine the robustness of this failed prediction. Could our model be modified in such a way so as to account for this apparent non-evolution of SFR? Our main assumption, the accretion mass floor $M_{\text{min}}$, only alters the mass index of the SFR sequence, but does not alter its redshift evolution. An assumption certain to break down at $z > 4$ is our assumed constant recycling fraction $R$. Indeed, $R$ is a strong function of stellar ages (Bruzual & Charlot 2003) for ages <1 Gyr. In order to understand how this affects our calculations in the first Gyr, we note that $R$ affects the SF efficiency. The recycled fraction $R$ is low at early times, meaning that the gas consumption is more efficient via the $(1-R)$SFR factor in Equation (3). Since the SF efficiency is increased, the steady-state solution is reached faster, i.e., sSFR($z$) is going to be set by the redshift dependence.
of the accretion rate sooner. Therefore, a more realistic \( R(t) \) will not work in the right direction.

At those early times, the main assumption to break down in our model is related to the steady-state solution. As detailed in Section 3.1, galaxies in our model are out of the steady-state solution beyond \( z > 5 \). Therefore, the assumption made for the SF timescale (or efficiency), namely, that \( t_{\text{bb}} \), scales with the halo dynamical time \( t_{\text{dyn}} \) may break down: gas (or fuel for SF) is pouring at a rate faster than it can be consumed implying that the SF efficiency may be far from the local value. A more efficient gas consumption (shorter SF timescale) at early times means that the steady-state solution is reached more rapidly. This will only exacerbate the discrepancy between the \( z = 4–8 \) data and the model.

A less efficient gas consumption (longer SF timescale) at early times means that the SFR is increasing to some power of time \( t \) or redshift \((1 + z)\). Thus, the sSFR is always going to be proportional to \( \text{SFR}(t) \propto 1/t \).

Therefore, upon examining our assumptions, we find that the non-evolution of sSFR beyond \( z = 2 \) cannot be achieved easily. In other words, the evolution of sSFR beyond \( z = 4 \) is giving strong constraints on early galaxy formation. Taken at face value, the Gonzalez et al. (2010) results are going to be extremely challenging for any theoretical model.

A.2. Assumptions in the Observations

In the observations, in order to fit the observed SEDs, one has to assume (1) an SFH and (2) treat the nebular emission lines consistently (Schaerer & de Barros 2009). These two can affect the sSFR significantly. For instance, at high SFRs and young ages, the nebular emission lines (e.g., \([\text{O} \text{II}], \text{H}\beta\)) can affect the color significantly (Schaerer & de Barros 2009; Yabe et al. 2009; Finlator et al. 2010). When this is omitted, the resulting Balmer break is overestimated, and hence the stellar mass, leading to an underestimated sSFR.

Similarly, the SFH can have a strong impact on the derived sSFR. At \( z = 2 \), the often assumed “constant SFH” generally works well (e.g., Förster Schreiber et al. 2009) and this assumption is in good agreement with the growth predicted (see Figure 6 and Section 3.1). At \( z > 5 \), different groups make different assumptions. For example, Gonzalez et al. (2010) prefer a constant SFH, while Stark et al. (2009) and Schaerer & de Barros (2010) use an exponentially declining SFH with a range of ages. Unfortunately, the SFH is difficult to constrain from the data themselves, but has a significant impact on the sSFR. Indeed, if \( \text{SFR}(t) \) is constant, then sSFR goes as \( 1/t \). If \( \text{SFR}(t) \) is declining exponentially, then \( M_*(t) \) is dominated by the initial SFR \( t = t_0 \), \( M_*(t) \approx \text{exp}(-t_0/t) \), and sSFR is dominated by the age \( \Delta t = t - t_0 \), with \( \text{sSFR} \propto \text{exp}(-\Delta t/\tau) \). If \( \text{SFR}(t) \) is increasing exponentially, then \( M_*(t) \) is proportional to \( \text{exp}(+t/\tau) \), and the current SFR is also \( \propto \text{exp}(+t/\tau) \). Hence, the sSFR is constant in this case.

Given the short times between \( z = 5, z = 6, \) and \( z = 7 \), the various populations observed at these epochs are likely descendants and progenitors of one another and the inferred involution of sSFR(\( z \)) ought to be consistent with the assumed SFH. Currently, it appears that this self-consistency in the observational results is not present.

A.3. Star Formation Histories

At \( z > 5 \), galaxy formation is surely more uncertain. Protogalaxies are more metal poor and the universe was just reionized, two facts that will impact molecular gas formation, hence the KS relation itself. Hence, several of our assumptions are likely to fail at those early epochs. However, a robust and generic feature of any model at those epochs is the rapid growth rate \( M_* \), which implies that the SFHs are very far from exponentially declining. In order to guide interested readers with SFHs appropriate in this regime of steady growth (at \( z > 4 \)), we note that in the redshift range \( z = 5 \) to \( z = 7 \), our SFHs are approximate increasing exponential, \( \text{SFR}(t) \propto \text{exp}(+t/\tau) \), with a timescale \( \tau \) close to \( \sim \)0.5 Gyr as shown in Figure 14 with the dotted line. Using \( \tau = 0.5 \) Gyr, one can approximate the SFR(\( t \)) with

\[
\text{SFR}(t) \propto e^{(e^{t/\tau})} \quad \text{for times } t > t_f, \tag{A1}
\]

where the formation time \( t_f \) is given by \( (1+z_f) \) \( \approx 0.4 \times M_*/13 \). As Figure 14 shows that \( \tau \) varies from 0.3 Gyr to 0.6 Gyr between \( z = 5 \) and \( z = 7 \), a physically motivated form (dashed line) is given by

\[
\text{SFR}(t) = 100 e^{(e^{(t-t_f)/\tau(z)})} \quad \text{with } \tau(z) = 0.5 \text{ Gyr} \left(1 + \frac{z}{1+z_0}\right)^{-2.2}, \tag{A2}
\]

where \( t_f \) and \( z_0 \) are the fitted parameters. Table 2 lists the fitted parameters. A more accurate fit is obtained using polynomials of the type \( a_0 + a_1 U^3 + a_2 U^2 \), where \( U = t/(10^9 \text{ yr}) \), and shown as the red curves in Figure 14.

![Figure 14](https://example.com/figure14.png)  
**Figure 14.** At \( z > 5 \), the star formation histories are close to being *increasing* exponential, \( \text{SFR}(t) \propto e^{t/\tau(t)} \) with \( \tau = 0.5 \) Gyr (dotted line). Motivated by the halo growth, the dashed line shows an evolving \( \tau(z) \approx 0.5 \) Gyr/(1+\( z \))^2. The red curves show second-order polynomial fits. (A color version of this figure is available in the online journal.)
