Oscillations of offshore drilling platforms on layered anisotropic base

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Abstract. The article presents the results of the studies, including the elements of a theoretical calculation, factors of internal strength and deformation field during volumetric vibrations of offshore drilling platforms on a layered basis. The influence of factors such as seismic dynamic impacts at the regional scale and dynamics of the geomorphology of the sea floor have been studied. Structures with spatial resilient links connected rigidly, hingedly or otherwise are subjected to dynamic force action. Depending on the size of the external forces and the forces of inertia, the flexible elements of the structure of the offshore drilling platform are in a complex three-dimensional stress-strain state. In dynamic calculation of the characteristics of the systems, the finite element method was used by replacing the platform with a system with a finite number of nodal points interconnected by inertia less rods. The masses are concentrated at the nodal points.

1. Introduction

Offshore oil and gas facilities are continuously under the influence of wind, wave, currents and winter ice, which periodically assume intense importance. It is under extreme conditions that the suitability and durability of the structures adopted are determined. Thus, according to the literary data during the storm of 20–21 November 1957, in the area of the Oil Stones of Azerbaijan, almost 5% of the structures of the elevated type and dozens of island bases were destroyed, The sea was heavily polluted by oil products, hundreds of millions of dollars in losses and, tragically, dozens of people died [1,2]. This was a consequence of the incorrect accounting of the calculated values of force actions. Physical restrictions depending on the depth and fluctuations of sea level, storm surges, temperature amplitudes, and ice movements of a qualitative nature require further in-depth study and parameterization for specific zones of the considered part of the sea. It is necessary to detail the hydrometeorological data on winds (wind rose), waves (sea-wave degree roses), currents, displaced flows, ice drifts, etc.

The range of influencing factors should be significantly expanded from seismic-dynamic impacts at the regional scale to dynamic geomorphology of the sea floor. The long-standing impact of earthquakes on the Ashgabat-Spitak seismic belt in the Southern Caspian has so far been overlooked. The intensive and large-scale extraction of Tengiz oil on the north coast of the Caspian Sea triggers man-made earthquakes with marked fluctuations in soil on land and at the sea floor. In this and other cases, the impacts of transit seismic waves on maritime structures will have to be taken into account [3].

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Generally speaking, the seismic effects are preceded by the displacement of the roof rock caused by
the process of compacting the porous collector as oil is extracted. This process, in its weak
manifestation, leads to intermittent faults weakened by contact and, as a result, to a small, mild
earthquakes. At a time when the reservoirs are huge, the extent of rock slides and associated dynamic
events may be significant. Most importantly, the rocks can move up to the surface of the Earth, in the
shelf to the surface of the seabed. Such vertical displacements are called subsidence, sedimentation,
sagging, drawdown, etc. Mulds of displacement over mining workings have an analogy in the form of
changes in the geomorphology of areas (sometimes very extensive), where intensive and large-scale
development of oil and gas fields takes place [2,4].

2. Review and analysis of literature on the topic of research
Let us turn for credibility to the data of the world literature summarised in [1,2,4]. Illustrative
examples of sifting over oil and gas development include the following: sifting over the Wilmington
field (California, USA) forced a 9 m increase in the shoreline of the Port of Long Beach; horizontal
displacements formed on the surface of the sea floor as a result of the exploitation of the Inglewood
field (California, USA), caused the rupture of the Baldwin Hills dam; the creation of a hole deeper
than 2.5 m in diameter of 60 km led to the exploitation of oil and aquifers in the Houston-Galveston
zone (Texas, USA). In connection with the identification of this phenomenon, it would be useful to
draw attention to the geomorphological studies of Úzen, Zhetybay, Karazhanbas, Kalamkas and other
areas of relatively long-term development [5-7].

The purpose of the work is to develop methods for calculating non-linear deformation problems
of offshore drilling platforms using the Mathcad system.

To achieve the goal, the following objectives have been set and solved:

- based on the adopted mechanical and mathematical model of marine bottom sediments, design
  of marine structures and operational loads, development of calculation algorithms;
- compiling a suit of applications for calculating the class of static and dynamic problems of the
  stress and strain state of offshore structures;
- numerical solution of the problem and analysis of the elastic stress state of offshore structures
  on the bottom base from the action of external static forces;
- numerical integration of nonlinear equations using the Mathcad package [8,9].

3. Methods and models
Structures with spatial resilient links connected rigidly, hingedly or otherwise are subjected to
dynamic force actions. Depending on the magnitude of external forces and inertia forces, the elastic
structural elements of the offshore drilling platform are in a complex three-dimensional stress-strain
state [10-12].

The dynamic system equation is based on the Hamilton variation principle. If all kinetic energy of
the system is denoted through \( T \), all potential energy of internal and external forces is - \( \Pi \), and the
work of non-conservative forces of the system, including damping forces, is \( W_{ne} \), then the variational
principle of Hamilton is represented by the following expression

\[
\int_{t_1}^{t_2} \delta(T - \Pi)dt + \int_{\eta}^{T} \delta W_{ne}dT = 0
\] (1)

Respectively

where \( L = (T - \Pi) \times W_{ne} \) - is the Lagrange functional \( (2) \) [13-15]:
\[ \delta II = O, \quad (3) \]

which constitutes the starting point for the FEM (Finite Element Method) variation formulation. Suppose the following functional dependencies exist:

\[ \delta II = O, \quad II = (u_i) : \quad \delta W_w = Q, \delta u_i : i = 1, 2, \ldots, N \quad (4) \]

Applying integration by parts of expression (5) and taking into account that \( \delta u(t_1) = \delta u(t_2) = 0 \) on the basis of expression (1), we obtain the expression (6)

\[ \int \left\{ \sum_i \left[ \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{u}_i} \right) + \frac{\partial T}{\partial u_i} - \frac{\partial II}{\partial \dot{u}_i} + Q \right] \delta u_i \right\} = 0 \quad (5) \]

Since \( \delta u(i = 1, 2, \ldots, N) \) are arbitrary numbers that are generally different from zero, then equation (6) implies that

\[ \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{u}_i} \right) + \frac{\partial T}{\partial u_i} = Q_i \quad (6) \]

the expression (7) is the Lagrange motion equation, which serves as the basis for the formulation of the equations of motion in FEM. The kinetic energy of the finite volume element is written in matrix form [16].

\[ T = \frac{1}{2} \int \rho u^T u dV \quad (7) \]

where \( \rho \) is the density of the material, \( u = du/dt \) is the velocity vector.

Potential energy of an element

\[ II = \frac{1}{2} \int \epsilon^T D \epsilon dV - \int \int \epsilon^T F dV - \int u^T P dS \quad (8) \]

in which the first member represents the internal energy of the deformation, the second is the potential energy of the volumetric forces, and the third is the potential energy of the surface forces.

Using the form functions, the displacement field and the deformation field in the finite element are written according to the displacement parameters in the nodes [17].

\[ \{U\} = [N]\{u\}, \quad \{\epsilon\} = [B]\{\epsilon\} \quad (9) \]

In the dynamic calculation of displacement in nodes, as well as displacement and deformations in an element, are time functions, while the functions of the forms depend only on the geometric characteristics of the element. By time differentiation of the first equation in expression (10), we establish a link between the speed of the element and the generalized speeds of the element nodes [18].

\[ \{\dot{U}\} = [N]\{\dot{u}\} \quad (10) \]

by substituting the expressions (10) and (11) in equations (8) and (9), we get expressions for the kinetic and potential energy of the element:

\[ T = \frac{1}{2} u^T \left( \int \int \rho N dV \right) u, \quad II = \frac{1}{2} u^T \left( \int \int B^T D B \epsilon dV \right) u - \int \int u^T F dV - \int u^T P dS \quad (11) \]
in which the generalized displacements and the generalized speeds in the nodes of the element are unknown. The non-conservative forces in the element function as a damping force, assumed to be proportional to the velocities, i.e. with the direction of action opposite to the direction of speed \[19\].

\[
\{F_r\} = -[C]\{\ddot{u}\} = -[C][N]\{\ddot{u}\}
\]  

(12)

If the damping forces are taken as the forces distributed in a unit of volume, the corresponding generalized forces in the nodes of the element are obtained according to the expression (13)

\[
Q_p = \int \mathbf{N}^T F_p dV \left( -\int \mathbf{N}^T C \mathbf{N} dV \right) \ddot{u}
\]  

(13)

Substituting the expression (2) and (13) into (6), considering that \(\frac{\partial T}{\partial u_i} = 0\), we get

\[
\left( \int \mathbf{N}^T c \mathbf{N} dV \right) \ddot{u} + \left( \int B^T DBdV \right) u = \int \mathbf{N}^T FdV + \int N^T PdS
\]  

(14)

respectively

\[
m \ddot{u} + c \dot{u} + ku = Q_e
\]  

Where \(m = \int \mathbf{N}^T \rho \mathbf{N} dV\); \(c = \int \mathbf{N}^T c \mathbf{N} dV\); \(k = \int B^T DBdV\);

\[
Q_e = \int \mathbf{N}^T FdV + \int N^T PdS
\]  

(15)

The expression (15) is the matrix formulation of the finite element’s equations of motion. Based on the equations of motion for one finite element, we construct the equation of motion for the finite element system as (16):

\[
[M]\dddot{U} + [C]\ddot{U} + [K]U = \{Q(t)\}
\]  

(16)

where \([M],[C],[K]\) - respectively, the mass matrix, the structural damping of the rigidity of the system; \(\{U\}, \{\dot{U}\}, \{\ddot{U}\}\) – accordingly, vectors of displacement, paces and accelerations in nodes.

The mass matrix of an element is determined by expression (16). If the elements in this matrix are computed using the same form functions as the field approximation of the element, then the matrix thus obtained is called the matrix of distributed masses. It is a symmetric and positively distributed matrix of \(mm\) order, where \(mm\) is the number of degrees of freedom of the element. The structure of the matrix \([M]\) is the same as the rigidity matrix \([K]\) and consists of blocks.
Similarly to the elements of the stiffness matrix, elements of the mass matrix can be given physical meaning. The element of the mass matrix is a generalized force at node \(i\) in the direction of \(k\), due to a unit generalized acceleration at node \(I\) in the direction of \(L\). Still the remaining generalized accelerations are equal to zero [19,20].

The damping matrix of an element according to expression (16) is determined in the same way and using the form functions as the mass matrix, only the damping coefficient \(C\) is introduced instead of the density. According to expression (17), the damping forces are proportional to the velocities, and the direction of their actions is opposite to the velocity action. The FEM cannot form a system \([C]\) damping matrix using individual element damping matrices analogous to mass and rigidity matrices because it is impossible to obtain damping matrices for individual elements. In this regard, the damping matrix \([C]\) is determined for the entire system based on the total dissipative energy of the system during the load. In determining the damping \([C]\), it is assumed that the total attenuation of the system is equal to their sum, which correspond to the individual characteristic modes of vibration of the system. Thus, the decomposition method of principal mode produces a proportionality equation.

\[
\{\Phi_I\}^T [C] \{\Phi_I\} = 2\alpha_i \xi_i \delta_y
\]  

(18)

the relative attenuation coefficient for individual waveforms based on the Rayleigh hypothesis is depicted as a linear superposition of mass and stiffness matrices

\[
F_i(t) = \sum_{j} U_j Q_j(t) \frac{B_j}{B_i}
\]  

(19)

where \(\alpha, \beta\) - are the corresponding constants determined by the attenuation coefficients for two independent vibrational modes, substituting the expression in (19) into equation (20), we obtain

\[
\{\Phi_I\}^T \left(\alpha [M] + \beta [K]\right) \{\Phi_I\} = 2\alpha_i \xi_i
\]  

(20)

When the interaction of the structure with water is taken into account, the mass matrix includes, in addition to the platform weight, the water mass attached, and the damping matrix includes both structural and hydrodynamic damping. The solution of the equation is to be found as

\[
\{u\} = [U]\{\xi\} = \sum_{j} \xi_j \{U\}
\]  

(21)

where \(M\) is the number of principal mode. Substituting (21) into the general equation and multiplying the result by \([U]^T\), we finally obtain

\[
[M^*] \{\ddot{\xi}\} + [C^*] \{\dot{\xi}\} + [K^*] \{\xi\} = \{Q'(t)\}
\]  

(22)
where the mass and stiffness matrices are $[M']=[U]'[M][U]$, $[K']=[U]'[K][U]$, which are diagonal in view of the orthogonality of the mass matrices and rigidity to the principal mode of the system, $[C']=[U]'[C][U]$. In the general case, it is not diagonal. By separating the equation for dynamic equilibrium equation components can not satisfy the condition Rayleigh, which is formulated as a possible representation of the damping matrix in the form of matrices combination of stiffness and mass. If the condition is satisfied, the matrix $[C']$ will be diagonal, since equation (23) is separated. In the general case, when the Rayleigh condition is not satisfied, the matrix $[C']$ leads to the diagonal form of $[C']$ by minimizing the error vector $\{E\}$, where

$$\{E\} = [C']\{\ddot{\xi}\} - [C^0]\{\ddot{\xi}\}$$  \hspace{1cm} (23)

The average deviation is minimized according to [7]

$$\left\langle \frac{\partial\{E\}}{\partial[C^0]} \right\rangle = \left\langle \left( \sum [C^0]_i \{\ddot{\xi}\} - [C^0]^T\{\ddot{\xi}\} \right) \right\rangle = 0$$

From here we get the expression for elements with $c^0_i$ of matrix $[C^0]$

$$c^0_i = c_i^* + \sum_j c^*_j \left\langle \frac{\ddot{\xi}_j}{\ddot{\xi}_i} \right\rangle$$  \hspace{1cm} (24)

and in a first approximation we put $C^0_u = C^1_u$. After that, the equilibrium equation splits into $N$ of unrelated equations of the form

$$\ddot{\xi}_i + 2\varsigma_i \dot{\xi}_i + \Omega^2 \xi_i = F_i$$  \hspace{1cm} (25)

where $\ddot{\xi}_i, \varsigma_i, F_i$ - respectively, the generalized coordinate, the generalized damping, the generalized load along the $i$ oscillation, $\Omega^2$ - is the natural frequency of the system. The expression for generalized loads has the form of

$$F_i(t) = \sum_{j=1}^{N} U_j Q_j(t) / B_i$$  \hspace{1cm} (26)

where $B_i = \sum_{j=1}^{N} \sum_{k=1}^{N} Q_j m_j U_j U_{ik}$, $N$ - is the number of degrees of freedom of the system.

4. Conclusions

A substantiated statement of the problem of calculating the stress-strain state of an offshore drilling platform on anisotropic bases has been completed. Such a statement of the problem reflects the specifics of dynamic calculations of the stress-strain state of offshore structures.

In the case of dynamic problems, the proposed calculation method makes it possible to take into account the structural features of the marine structure on the one hand and the heterogeneous structure of the material on the other hand, The peculiarities of the interaction of individual links make it possible to determine the internal efforts in them of the action of external force fields over time.

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