Application of Hyperelastic Materials in a Composite Hollow Brick for Assessing the Reduction of Dynamic Loads - Numerical Analysis

I Major, M Major
Częstochowa University of Technology, Faculty of Civil Engineering, Akademicka Street 3, 42-200 Częstochowa, Poland
E-mail: imajor@bud.pcz.czest.pl

Abstract. The article addresses the use of hyperelastic materials to evaluate the redistribution of stresses in a composite structural hollow brick by means of numerical analysis. Both rubber and concrete were modelled as hyperelastic materials with appropriate energy potential. The concrete was modelled on the Murnaghan potential, while the rubber was modelled on the Zahorski potential. The analysed prefabricated hollow brick was loaded with a force impulse. As a result, redistribution of effective stress was observed, which was important for the evaluation of the operation of the analysed prefabricated product. The presented numerical analysis was carried out using the ADINA program.

1. Introduction
The application of a force impulse, which is a point dynamic load, causes the disturbance propagation, which is of interest to many researchers. As a result of this interaction, a number of wave phenomena connected with the transport of energy inside the elastic medium, which can be modelled as a material continuum, can be observed. We assume that a material continuum can be modelled as a hyperelastic, compressible and non-compressible material, depending on the assumed elastic potential that determines the material under consideration. The description of the wave propagation in different types of materials includes a classical method based on mathematical analysis and a numerical method based on the finite element method. FEM is a commonly used method for the analysis of building materials [1, 2] and others used in the industry. The development of research methods has contributed to the development of many contemporary materials that reduce dynamic and acoustic loading (the so-called vibroacoustic loads); however, their mechanical properties exclude most of them for use in structural solutions in which significant compressive forces are transmitted.

The author's solution of a composite, concrete and rubber hollow brick, presented in this paper, allows to reduce dynamic loads and meet the requirements for structural materials intended for erecting masonry walls of enclosed buildings. This numerical analysis concerns attenuation of a mechanical wave, taking into account two non-linear hyperelastic materials describing the concrete with the use of Murnaghan elastic potential [3-5] and the rubber with the use of Zahorski potential. For the numerical analysis, the ADINA program was selected, which allows for the analysis of rubber and rubber-like materials, for which the Zahorski material was not originally implemented (see also [6, 7]). It was necessary to modify the material library using FORTRAN in order to obtain an appropriate elastic potential for the rubber filling the spaces in the developed structural hollow brick.
2. Material Models

The constitutive equations for non-linear hyperelastic materials are obtained from the mechanical energy balance equation, which describes the relationship between strain and stress or strain and energy. In the late 1930s Murnaghan defined a constitutive equation for a non-linear compressible elastic material which can be expressed by the following equation [3]:

\[
W(I_1, I_2, I_3) = \frac{I + 2m}{24} (I_1 - 3) + \frac{\lambda + 2\mu + 4m}{8} (I_1 - 3)^2 + \frac{8\mu + n}{8} (I_1 - 3)
\]

\[- \frac{m}{4} (I_1 - 3)(I_2 - 3) - \frac{4\mu + n}{8} (I_2 - 3) + \frac{n}{8} (I_3 - 1)\]

where: \( W \) is the elastic energy in the so-called reference configuration, \( I_1, I_2, I_3 \) are deformation tensor invariants, constants \( \lambda, \mu \) are Lamè constants, and the other constants \( l, m, n \) are second order elastic constants. The elastic constants describing concrete C20/25 are presented in Table 1.

Table 1. Elastic constants for steel material required to describe Murnaghan elastic potential

| Material | Lamè constant [GPa] | Elastic second order constant [GPa] |
|----------|---------------------|------------------------------------|
| Concrete | 11.072              | 15.289                             |
|          | -3.091              | -2.310                             |
|          | -1.874              |                                    |

The first attempts to determine the constitutive equations describing the behaviour of rubber and rubber-like materials, [8, 9] were made in the 1940s and 1950s. In 1951, Rivlin and Saunders [10] presented a general form of the elastic energy function. This function depends on the first two strain tensor invariants and the additional two constants. Eight years later, in 1959, Zahorski gave a constitutive equation, i.e. the function of strain energy, for the non-compressible material described by the non-linear dependence on strain tensor invariants [11]. Then it became possible to describe the elastic behaviour of rubber and rubber-like materials subjected to large strain. This relationship is described by means of the equation [8]:

\[
W(I_1, I_2) = C_1(I_1 - 3) + C_2(I_2 - 3) + C_3(I_2^2 - 9)
\]

where \( C_1, C_2, C_3 \) are material constants and \( I_1, I_2 \) are strain tensor invariants. Zahorski's constitutive equation including non-linear member \( C_3(I_2^2 - 9) \) in comparison to the Mooney-Rivlin material model allows for a more comprehensive analysis and obtaining other quantities useful in the description of wave processes in terms of quality.

The constitutive Zahorski relationship reflects behaviour of rubber for the principal strain even for \( \lambda = 3 \), whereas for strain greater than \( \lambda > 2 \) Mooney-Rivlin material should not be used. Satisfactory results for the Mooney-Rivlin material are only obtained when \( \lambda \leq 1.4 \) [12].

The elastic constants for Zahorski’s material used in this article are presented in Table 2.

Table 2. Elastic constants corresponding to Zahorski material

| Constant | \( C_1 \) | \( C_2 \) | \( C_3 \) |
|----------|----------|----------|----------|
| Rubber   | 6.278·10^4 | 8.829·10^4 | 6.867·10^4 |

3. Numerical Models

The paper presents the analysis of a composite hollow brick made of C20/25 concrete with rubber inserts arranged transversely to the direction of the force impulse. It was assumed that each rubber insert passes through a 20 cm high hollow brick. In the external outline the hollow brick has dimensions of 25 cm by 47 cm (Figure 1).
In order to estimate the values of the reduction of dynamic loads induced by the force impulse, a numerical analysis was carried out with the help of ADINA. It was declared that the force impulse being the dynamic load is 1000 N and is applied perpendicularly to the transverse axis of the rubber inserts. It is applied to the lateral surface of the hollow brick in plane “XZ” - Figure 1. The force impulse is modelled as a point impact which reaches its maximum value for time \( t = 1 \times 10^{-5} \) s. After time \( t = 1.1 \times 10^{-5} \) s the force impulse decreases to 0 and the generated energy spreads inside the hollow brick under examination. It can be said that the assumed type of load generates interference wave propagation which can be described as mechanical wave propagation.

For the discrete model in question the following boundary conditions were assumed: the lower surface of the hollow brick stretched in the “XY” plane was blocked in the “Z” axis direction and the lateral surfaces defined by the “YZ” plane in the “X” axis direction and the “Y” axis were blocked.

The discrete model of the hollow brick under examination was carried out with the use of finite elements “3D-Solid” (tetrahedrons) for which a size of \( \sim 0.01 \) m is assumed. The model contained 29,717 nodes and 53,358 finite elements.

4. Results

The transfer of energy resulting from the propagation of mechanical waves in the analysed concrete-rubber hollow brick causes the formation of effective stresses in the horizontal section of the hollow brick. Figure 2 shows the values of these stresses for times \( t = 1.420 \times 10^{-5} \) s, \( t = 2.620 \times 10^{-5} \) s, \( t = 3.820 \times 10^{-5} \) s and \( t = 5.020 \times 10^{-5} \) s respectively.

Figure 2b for time \( t = 2.620 \times 10^{-5} \) s clearly shows the attenuation properties of the rubber inserts. Already the first row of rubber filling of the hollow brick results in a significant reduction of stress values. The ones reaching the second row of rubber filling are approx. 14 kPa lower than the first row values.
In subsequent time steps, i.e. for $t = 3.820 \times 10^{-5}$ s and $t = 5.020 \times 10^{-5}$ s, Figures 2c and 2d, the force impulse continues to propagate and is gradually attenuated. It is clearly visible that in the analysed composite hollow brick, at the height of the second row of rubber filling, the tested effective stresses are at a level of about 70 kPa, and at the height of the third row of rubber filling their value decreases to about 20 kPa.

Additionally, it can be noted that in the analysed composite hollow brick, the disturbance propagation caused by the force impulse does not significantly affect the values of stresses in the applied filling, which oscillate in the range from 0 to 10 kPa.

5. Conclusions
The article presents two hyperelastic materials described by the Murnaghan potential and the Zahorski potential, respectively. For the structural composite hollow brick made of concrete modelled as Murnaghan’s material and the rubber modelled as Zahorski’s material, a numerical analysis of the phenomenon of attenuation of mechanical wave propagation was carried out. The designed composite hollow brick, because it is made of recyclable materials, can be considered environmentally friendly. Such a solution may also be considered innovative due to a lack of research for this type of solution.

Production of the composite hollow brick presented in the analysis is connected with making only a special mould, which in the first phase should be filled with concrete. Once the concrete blocks reach 80% of their compressive strength, the mould should be removed and the voids filled with rubber material by injection or insertion.

The analysis confirmed the preliminary assumptions that in composite hollow bricks prepared in such a way, the mechanical wave can be significantly attenuated with the use of rubber inserts. On the basis of the above analysis, it can be assumed that the designed concrete and rubber hollow bricks can be used in structures exposed to various sources of vibration, and can also be an additional barrier for acoustic waves. In addition, the use of rubber inserts does not reduce the compressive strength of the entire element so that it meets structural requirements.
The paper also shows that typical construction materials used in construction can be treated as hyperelastic materials and modelled using an appropriate elastic potential, taking into account non-linear effects in the transport of strain energy induced by a force impulse, i.e. the dynamic load.

The effectiveness of the use of hyperelastic materials for numerical modelling of the energy transport phenomena is the subject of many scientific studies. The authors of this publication successfully conduct such research for building materials [6, 7].

The original material and structural solution based on a composite hollow brick modified with rubber inserts, presented in this article, is an effective solution that can be implemented in the EU.

6. References

[1] Krejsa M, Brozovsky J, Mikolasek D, Koubova L, Parenica P and Materna A 2017 Numerical modeling of fillet and butt welds in steel structural elements with verification using experiment Procedia Engineering 190 pp 318-325

[2] Kotrasova K, Grajciar I and Kormanikova E 2015 A case study on the seismic behavior of tanks considering soil-structure-fluid interaction Journal of Vibration Engineering & Technologies 3 3 pp 315-330

[3] Murnaghan F D 1937 Finite deformations of an elastic solid American Journal of Mathematics 59 pp 235-260

[4] Marasco A and Romano A 2009 On the acceleration waves in second-order elastic, isotropic, compressible, and homogeneous materials Mathematical and Computer Modelling 49 7–8 pp 1504-18

[5] Jemioł S and Franus 2018 A Numerical implementation of the Murnaghan material model in ABAQUS/Standard MATEC Web of Conferences 196 p 8

[6] Major I and Major M 2016 Wave phenomena in composite rubber-sandstone structures using ADINA program Engineering Mechanics 2016 pp 387-390

[7] Major M, Kuliński K and Major I 2017 Dynamic analysis of an impact load applied to the composite wall structure MATEC Web of Conferences 107 pp 6

[8] Mooney M 1940 A theory of large deformations Journal of Applied Physics 11 pp 582-592

[9] Rivlin R S 1948 Large elastic deformations of isotropic materials I Fundamental concepts, Phil. Trans. Roy. Soc. Lond. A 240, pp 459-490

[10] Rivlin R S and Saunders D W 1951 Large elastic deformations of isotropic materials. VII Experiments of the deformation of rubber Phil. Trans. Roy. Soc. Lond. 243 pp 251-288

[11] Zahorski S 1959 A form of elastic potential for rubber-like materials Archives of Mechanics 5 pp 613-617

[12] Kosiński S 2007 Fale sprężyste w gumopodobnych kompozytach warstwowych Wydawnictwo Politechniki Łódzkiej, Łódź (in polish)