Modeling of Human Development Index in Papua Province Using Spline Smoothing Estimator in Nonparametric Regression

D P Rahmawati¹, I N Budiantara¹,²*, D D Prastyo¹, and M A D Octavanny¹

¹Department of Statistics, Faculty of Mathematics, Computation and Data Science
Institut Teknologi Sepuluh Nopember, Surabaya, 60111, Indonesia

* e-mail: nyomanbudiantara65@gmail.com

Abstract. The development goal of a country must be focused on the quality of human life to achieve prosperity. One important indicator for measuring the success of a country's development is the Human Development Index (HDI). In 2018, Papua was the province with the lowest HDI in Indonesia. Special attention is needed to improve HDI in Papua Province, one of them is by paying attention to the variables that affect HDI such as population growth rate, percentage of poor population, and economic growth. The relationships between HDI and the predictor variables do not have a clear pattern and tend to change at certain subintervals. This case can be approached using Spline Smoothing in multivariable nonparametric regression. Spline Smoothing is a type of estimator in nonparametric regression that has an excellent ability to handle data that tend to change at certain subintervals. Therefore, the purposes of this study are to obtain the form of Spline Smoothing estimator function in multivariable nonparametric regression, estimate the function and apply it to the HDI in Papua Province. The empirical results of modeling HDI in Papua Province show that it can be adequately applied which gives GCV = 58.108, R² = 99.77% and RMSE = 0.0505.

Keywords: Spline Smoothing, Multivariable Nonparametric Regression, Penalized Least Square, Human Development Index.

1. Introduction
The development goal of a country must be focused on the quality of human life to achieve prosperity for its people. One indicator that has an essential role in measuring the success rate of a country's development is the Human Development Index (HDI) which was introduced by the United Nations Development Program (UNDP) in 1990. Human development has always been an essential issue in the design and strategy of sustainable development. At the world level, among the 17 SDGs goals, three of them are related to human development. Whereas at the national level, the human development agenda is contained in Nawacita at point five. In 2018, Papua Province was the province with the lowest HDI and became the only province with a low development status in Indonesia. The HDI in Papua Province is 60.06, and this value is much lower compared to the national HDI, which has reached 71.39 [1]. Therefore, special attention is needed to improve HDI in Papua Province, one of them is by paying
attention to the variables that can affect HDI such as population growth rate, percentage of poor population, and economic growth [2][3][4].

The statistical method that can be used to determine the relationship between response variables and predictor variables is regression analysis [5]. Regression analysis can be categorized into several types based on the assumption of the regression curve shape. In general, there are three approaches to estimating the regression curve, namely parametric regression, nonparametric regression, and semiparametric regression. In some cases, the relationships between response variables and predictor variables are sometimes unknown patterns, these cases are under nonparametric regression [6][7]. One of them is the case of HDI with several variables that influence it such as population growth rate, percentage of poor population, and economic growth. In this case the relationship between the response variable and the predictor variable has a pattern that tends to change at certain subintervals. So, the case is appropriate if approached with multivariable nonparametric regression with the Spline estimator. Spline is one of the nonparametric regression estimators that has excellent statistical and visual interpretation [8]. Spline Smoothing is one type of Spline estimator that is often used. The Spline Smoothing can handle data or functions that are smooth. Spline Smoothing also has an excellent ability to handle data that behavior changes at certain subintervals [9]. Therefore, this research will obtain the function form of the Spline Smoothing estimator in multivariable nonparametric regression, estimate the function and apply it to the HDI with several variables that influence it in Papua Province.

2. Materials and Methods

Regression analysis is a statistical method used to determine the functional relationship between response variables and predictor variables [5]. One type of regression that is appropriate for the pattern of relationships between response variables and predictors or assuming the shape of the regression curve is unknown is nonparametric regression [10]. In nonparametric regression the regression curve can be assumed smooth, this approach also has high flexibility where the data is expected to be able to obtain its regression curve estimation without the influence of subjectivity from the researcher [8].

The Spline is one of the nonparametric regression estimators that has excellent statistical and visual interpretations [8]. In 1923, Spline was introduced by Whitaker as a data pattern approach that was initially used to approach continuous and differentiable functions with polynomial functions. Furthermore, Reinsch in 1967 developed Spline, which was used to solve optimization problems [11]. Since then, Spline estimators have developed rapidly and one type of Spline estimator that is often used is Spline Smoothing. Spline Smoothing can handle data characters or functions smoothly. Spline Smoothing also has an excellent ability to handle data whose behavior changes at certain subintervals [9].

If given data in pairs \((y_i, x_{i1}, x_{i2}, \ldots, x_{ip})\), which following the nonparametric regression model as follows:

\[
y_i = \sum_{j=1}^{p} g_j(x_{ij}) + \epsilon_i, \quad i = 1, 2, \ldots, n
\]

where \(\sum_{j=1}^{p} g_j(x_{ij})\) is approximated by the Spline Smoothing estimator with \(p\) predictor variables and \(\epsilon_i \sim N(0, \sigma^2)\). Equation (1) can be written in matrix form as follows:

\[
y = g + \epsilon
\]

In Spline Smoothing nonparametric regression, the regression curve \(g\) is unknown and is assumed to be smooth in the sense that it is contained in particular function space, especially in the Sobolev space \(W_2^m[a,b]\). Spline Smoothing can be obtained if the estimation \(\hat{g}\) in the regression model equation (2) is obtained by minimizing Penalized Least Square (PLS) which is an estimation method that combines the goodness of fit and penalty functions [8][12]:
\[
\min_{g \in W_T(a,b)} \left[ n^{-1} \left( y - g \right)^T \left( y - g \right) + \lambda \int_a^b \left| g''(x) \right|^2 \, dx \right]
\]
with \( \lambda \) is the smoothing parameter and \( 0 < \lambda < \infty \).

3. Result and Discussion

3.1. The Form of The Spline Smoothing Estimator in Multivariable Nonparametric Regression

Given there are \( p \) predictors, to obtain the Spline Smoothing estimator in nonparametric regression, the form of Spline Smoothing estimator function will be firstly described with one predictor. If \( g^* \) is a Spline Smoothing estimator with one predictor and the function is contained in a \( W \) space. The \( W \) space can be decomposed into a direct sum of two spaces \( W_0 \) and \( W_1 \) as \( W = W_0 \oplus W_1 \), dengan \( W_0 \perp W_1 \).

If \( \{ \theta_1, \theta_2, \ldots, \theta_m \} \) is the basis in \( W_0 \) and \( \{ \psi_1, \psi_2, \ldots, \psi_n \} \) is the basis in \( W_1 \), then for each function \( g^* \in W \) can be written as follows:

\[
g^* = u + v = \sum_{i=1}^m d_i \theta_i + \sum_{i=1}^n c_i \psi_i = \theta^T d + \psi^T c
\]

Equation (4) is a limited linear function in \( W \) space and \( g^* \in W \), then equation (4) can be presented as follows:

\[
\mathcal{L}_v g^* = \mathcal{L}_v (u + v) = g^* (x_i)
\]

Based on equation (4), equation (5) can be written as:

\[
g^* (x_i) = \langle \eta, g^* \rangle = \langle \eta, \theta^T d \rangle + \langle \eta, \psi^T c \rangle
\]

for \( i = 1 \), obtained:

\[
g^* (x_i) = \langle \eta, \theta^T d \rangle + \langle \eta, \psi^T c \rangle = d_1 \langle \eta, \theta_1 \rangle + d_2 \langle \eta, \theta_2 \rangle + \ldots + d_m \langle \eta, \theta_m \rangle + c_1 \langle \eta, \psi_1 \rangle + c_2 \langle \eta, \psi_2 \rangle + \ldots + c_n \langle \eta, \psi_n \rangle
\]

The same way is applied for \( i = 2, 3, \ldots, n \). So, the vector \( g^* \) can be expressed as:

\[
g^* = \begin{pmatrix}
    d_1 \langle \eta, \theta_1 \rangle + \ldots + d_m \langle \eta, \theta_m \rangle + c_1 \langle \eta, \psi_1 \rangle + \ldots + c_n \langle \eta, \psi_n \rangle \\
    \vdots \\
    d_1 \langle \eta, \theta_1 \rangle + \ldots + d_m \langle \eta, \theta_m \rangle + c_1 \langle \eta, \psi_1 \rangle + \ldots + c_n \langle \eta, \psi_n \rangle \\
    \langle \eta, \theta_1 \rangle & \langle \eta, \theta_2 \rangle & \ldots & \langle \eta, \theta_m \rangle \\
    \vdots & \vdots & \ldots & \vdots \\
    \langle \eta, \theta_1 \rangle & \langle \eta, \theta_2 \rangle & \ldots & \langle \eta, \theta_m \rangle \\
    d_1 & d_2 & \ldots & d_m \\
    \vdots & \vdots & \ldots & \vdots \\
    d_1 & d_2 & \ldots & d_m \\
    \langle \eta, \psi_1 \rangle & \langle \eta, \psi_2 \rangle & \ldots & \langle \eta, \psi_n \rangle \\
    \vdots & \vdots & \ldots & \vdots \\
    \langle \eta, \psi_1 \rangle & \langle \eta, \psi_2 \rangle & \ldots & \langle \eta, \psi_n \rangle \\
    c_1 & c_2 & \ldots & c_n \\
\end{pmatrix} = U^T d + V^T c
\]

with \( \langle \eta, \theta_i \rangle = \frac{x_i^{s-1}}{(s-1)!}, i = 1, 2, \ldots, n; s = 1, 2, \ldots, m \),

\[
\langle \eta, \psi_i \rangle = \langle \psi_i, \psi_i \rangle; \langle \psi_i, \psi_i \rangle = \int_a^b \frac{(x_i - u)^{m-1} (x_i - u)^{m-1}}{(m-1)!} \, du, \quad i = 1, 2, \ldots, n; i' = 1, 2, \ldots, n.
\]

If the regression curve \( g \) in equation (2) is Spline Smoothing estimator with \( p \) predictors, then it can be stated as follows [13]:

\[
g = \sum_{j=1}^n g_j.
\]
\[ g = \sum_{j=1}^{p} (U_j d^j + V_j c^j) \]
\[ g = U d + V c \quad g = U d + V c \] (10)

where

\[ U = U_1 + U_2 + \ldots + U_p \]
\[ V = V_1 + V_2 + \ldots + V_p \]

\( U \) is a matrix sized \( n \times m \), \( d \) is a vector sized \( m \times 1 \), \( V \) is a matrix sized \( n \times n \), and \( c \) is a vector sized \( n \times 1 \).

### 3.2. Penalized Least Square Optimization for Spline Smoothing in Multivariable Nonparametric Regression

The Spline Smoothing estimator in the multivariable nonparametric regression model is estimated using Penalized Least Square method, therefore Penalized Least Square is formed as follows [11]:

\[ Q = n^{-1} \left\{ \sum_{i=1}^{n} \left( y_i - \sum_{j=1}^{p} g_j(x_{ij}) \right)^2 \right\} + \lambda \int_a^b \left[ g''(x) \right]^2 dx, 0 < \lambda < \infty \] (11)

with \[ \int_a^b \left[ g''(x) \right]^2 dx = c^T V c \] [14].

Thus obtained:

\[ \lambda \int_a^b \left[ g''(x) \right]^2 dx = \lambda c^T V c \] (13)

Equation (11) can be written in matrix notation as follows:

\[ Q(c, d) = n^{-1} \left( y - (U d + V c) \right)^T \left( y - (U d + V c) \right) + \lambda c^T V c \]

Next is completing Penalized Least Square optimization as follows:

\[ \text{Min}_{c \in \mathbb{R}^n} Q(c, d) = \text{Min}_{c \in \mathbb{R}^n} \left( n^{-1} \left( y - (U d + V c) \right)^T \left( y - (U d + V c) \right) + \lambda c^T V c \right) \] (14)

the solution for optimization above can be obtained by performing the partial derivative \( Q(c, d) \) to \( c \) and \( d \), then the result is equal to zero. The partial derivative results are obtained as follows:

\[ \frac{\partial Q(c, d)}{\partial c} = -2V^T y + 2V^T Ud + 2V^T V \hat{c} + 2n \lambda V \hat{c} = 0 \]

with \( R = V + n \lambda \), thus obtained:

\[ \hat{c} = R^{-1} (y - Ud) \] (15)

Next is the partial derivative of \( Q(c, d) \) with \( d \) as follows:

\[ \frac{\partial Q(c, d)}{\partial d} = -2U^T \hat{y} + 2U^T Ud + 2U^T V \hat{c} = 0 \]

\[ -2U^T \hat{y} + 2U^T Ud + 2U^T V \left[ R^{-1} (y - Ud) \right] = 0 \]

Considering that \( R = V + n \lambda \), then

\[ VR^{-1} = (R - n \lambda) R^{-1} = I - n \lambda R^{-1} \]

Thus obtained:

\[ \hat{d} = (U^T R^{-1} U)^{-1} U^T R^{-1} \hat{y} \] (16)

by substituting Equation (16) to the equation (15), thus:

\[ \hat{c} = R^{-1} \left( I - U \left( U^T R^{-1} U \right)^{-1} U^T R^{-1} \right) \hat{y} \] (17)
\( \hat{c} \) and \( \hat{d} \) are substituted into the Spline Smoothing estimator form in (10), then the following Spline Smoothing in the multivariable nonparametric regression model is obtained:

\[
\hat{g}_i = U\hat{d} + V\hat{c}
\]

\[
= \left( U \left( U^T R^{-1} U \right)^{-1} U^T R^{-1} + VR^{-1} \left( I - U \left( U^T R^{-1} U \right)^{-1} U^T R^{-1} \right) \right) y
\]

\[
= A(\lambda) y
\]

with \( A(\lambda) = U \left( U^T R^{-1} U \right)^{-1} U^T R^{-1} + VR^{-1} \left( I - U \left( U^T R^{-1} U \right)^{-1} U^T R^{-1} \right) \).

(18)

3.3. Application on HDI in Papua Province

Multivariable nonparametric regression models with Spline Smoothing is applied to the HDI \( y \) and the factors that influence it, such as population growth rate \( x_1 \), percentage of poor population \( x_2 \), and economic growth \( x_3 \) in Papua province. The data was data in 2017, which was obtained from BPS (Badan Pusat Statistik) of Papua Province. The observation units of this study were 29 regencies/cities in Papua Province.

![Scatterplot of HDI vs x1,x2] Figure 1. The plot between HDI and each predictor variable.

Figure 1. Illustrates a partial relationship between HDI and each predictor variable. Based on the plot, the data appear to have no clear pattern and tend to change in certain subintervals. For this reason, multivariable nonparametric regression with the Spline Smoothing estimator is applied to the data. The model is a Spline Smoothing model with order \( m = 2 \). The results of the model show that the best model is model with smoothing parameter \( \lambda = 1 \) with the minimum GCV value of 58,108. The model gives the coefficient of determination of 99.77% and RMSE of 0.05053. Because the model is model with order \( m = 2 \), then the equation (8) and (9) can be described as follows:

for \( s = 1, 2 \), \( \langle \eta_s, \theta \rangle = \frac{x^{s-1}}{(1-1)!} = 1 \) and \( \langle \eta_s, \theta \rangle = \frac{x^{s-1}}{(2-1)!} = x_i \)

(20)

for \( u \in [0,1] \), \( \langle \nu \rangle = \frac{1}{(m-1)!} \int_0^1 (x_i - u)^{m-1} (x_i - u)^{m-1} du = x_i, x_i, -\frac{1}{2} (x_i + x_i) + \frac{1}{3} \)

(21)

so, the model of Spline Smoothing in multivariable nonparametric regression for HDI in Papua Province can be defined as follows:
\[
\hat{y}_i = \sum_{j=1}^{k} \left( d_j + \hat{d}_j x_{ji} + \sum_{r=1}^{2} \hat{c}_r \left( x_{jr} x_{ji} - \frac{1}{2} (x_{jr} + x_{ji}) + \frac{1}{3} \right) \right), \quad i = 1, 2, \ldots, 29
\]

with the estimation results of the model parameters \((\hat{d}_j, \hat{c}_i)\) are presented in Table 1.

### Table 1. The results of the model parameters estimation on the HDI in Papua.

| Parameters | Parameters Estimate |
|------------|---------------------|
| \(\hat{d}_s\), \(s = 1, 2\) | \(\hat{d}_1 = 21.682\); \(\hat{d}_2 = 0.306\). |
| \(\hat{c}_i\), \(i = 1, 2, \ldots, 29\) | \(\hat{c}_1 = -4.076 \times 10^{-3}\); \(\hat{c}_2 = -7.289 \times 10^{-4}\); \(\hat{c}_3 = 3.711 \times 10^{-3}\); \(\hat{c}_4 = -4.172 \times 10^{-4}\); |
|  | \(\hat{c}_5 = 9.355 \times 10^{-6}\); \(\hat{c}_6 = 8.751 \times 10^{-4}\); \(\hat{c}_7 = -9.188 \times 10^{-4}\); \(\hat{c}_8 = -6.067 \times 10^{-4}\); |
|  | \(\hat{c}_9 = -4.213 \times 10^{-3}\); \(\hat{c}_{10} = -3.342 \times 10^{-3}\); \(\hat{c}_{11} = 6.296 \times 10^{-4}\); \(\hat{c}_{12} = -1.153 \times 10^{-3}\); |
|  | \(\hat{c}_{13} = 8.436 \times 10^{-4}\); \(\hat{c}_{14} = -1.199 \times 10^{-3}\); \(\hat{c}_{15} = 6.258 \times 10^{-4}\); \(\hat{c}_{16} = -2.361 \times 10^{-3}\); |
|  | \(\hat{c}_{17} = 3.201 \times 10^{-3}\); \(\hat{c}_{18} = 8.856 \times 10^{-4}\); \(\hat{c}_{19} = -2.146 \times 10^{-5}\); \(\hat{c}_{20} = -1.539 \times 10^{-3}\); |
|  | \(\hat{c}_{21} = -8.087 \times 10^{-4}\); \(\hat{c}_{22} = 1.076 \times 10^{-3}\); \(\hat{c}_{23} = -2.244 \times 10^{-4}\); \(\hat{c}_{24} = -4.907 \times 10^{-5}\); |
|  | \(\hat{c}_{25} = -1.61 \times 10^{-4}\); \(\hat{c}_{26} = 5.803 \times 10^{-4}\); \(\hat{c}_{27} = 5.181 \times 10^{-3}\); \(\hat{c}_{28} = 1.604 \times 10^{-4}\); |
|  | \(\hat{c}_{29} = 7.444 \times 10^{-4}\). |

The HDI values and the estimated HDI values using Spline Smoothing in multivariable nonparametric regression model in each district/city are presented in Table 2. Based on Table 2, it is known that between real HDI values and HDI estimated values have small difference. This model can provide accurate prediction results for cases HDI in Papua Province. Furthermore, Figure 2. Shows the plot of the HDI estimation results using Spline Smoothing in multivariable nonparametric regression. The plot shows that the predictive ability of the model is very well, where all of the HDI data points can be approached with the prediction curve. This evidence is also shown by the several 3-dimensional scatterplots in Figure 3. which show the estimation results of Spline Smoothing in multivariable nonparametric regression model give the estimated fit values close to the actual HDI in Papua Province.

![Figure 2](image-url)
Tabel 2. The HDI values and the estimated HDI values in each District/City.

| District/City | HDI Value | HDI Estimation Value | District/City | HDI Value | HDI Estimation Value |
|---------------|-----------|----------------------|---------------|-----------|----------------------|
| Merauke       | 68.64     | 68.75819             | Sarmi         | 62.31     | 62.37848             |
| Jayawijaya    | 55.99     | 56.01114             | Keerom        | 64.99     | 64.89718             |
| Jayapura      | 70.97     | 70.86238             | Waropen       | 64.08     | 64.05432             |
| Nabire        | 67.11     | 67.1221              | Supiori       | 61.23     | 61.23062             |
| Kepulauan Yapen | 66.07   | 66.06973             | Mamberamo Raya | 50.25     | 50.29463             |
| Biak Numfor   | 71.56     | 71.53462             | Nduga         | 27.87     | 27.89345             |
| Paniai        | 54.91     | 54.93665             | Lanny Jaya    | 46.49     | 46.45879             |
| Puncak Jaya   | 46.57     | 46.58759             | Mamberamo Tengah | 45.5      | 45.50651             |
| Mimika        | 72.42     | 72.29782             | Yalimo        | 46.19     | 46.19142             |
| Boven Digoel  | 60.14     | 60.23693             | Puncak        | 41.06     | 41.06467             |
| Mappi         | 57.1      | 57.08174             | Dogiyai       | 54.04     | 54.02317             |
| Asmat         | 48.49     | 48.52345             | Intan Jaya    | 45.68     | 45.6785              |
| Yahukimo      | 47.95     | 47.92554             | Deiyai        | 49.07     | 49.06535             |
| Pegunungan Bintang | 43.24 | 43.27477             | Kota Jayapura | 79.23     | 79.20841             |
| Tolikara      | 47.89     | 47.87185             |               |           |                      |

Figure 3. (a) 3-D scatterplot $x_1$, $x_2$ and $y$, (b) 3-D scatterplot $x_1$, $x_3$ and $y$, (c) 3-D scatterplot $x_2$, $x_3$ and $y$. 
4. Conclusion

Based on the results of this study, the main conclusions are:

1. The form of Spline Smoothing estimator function from the multivariable nonparametric regression

\[ y_i = \sum_{j=1}^{p} g_j(x_{ij}) + \varepsilon_i \]  
\[ g(x) = Ud + Vc. \]

2. Spline Smoothing in multivariable nonparametric regression is estimated using Penalized Least Square (PLS) method by minimizing the following PLS functions:

\[ \min_{\lambda \in \mathbb{R}} \left\{ n^{-\frac{1}{2}} \left( y - (Ud + Vc) \right)^T \left( y - (Ud + Vc) \right) + \lambda c^T Vc \right\} \]

so, the estimator is \[ \hat{g}_x = A(\lambda) \] with \[ A(\lambda) = U(U^T R^{-1} U)^{-1} U^T R^{-1} \] 

3. The model of Spline Smoothing in multivariable nonparametric regression for HDI in Papua Province can be defined as follows:

\[ \hat{y}_i = \sum_{j=1}^{p} \hat{d}_j + \hat{d}_x x_{ij} + \sum_{j=1}^{29} \hat{c}_j \left( x_{ij} - \frac{1}{2} \left( x_{ij} + x_{ij} - \frac{1}{3} \right) \right), \]

the model can be appropriately applied, which gives GCV = 58.108, R^2 = 99.77% and RMSE = 0.0505.

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