The Entropy Production of a Nonequilibrium Open System

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Abstract

A nonequilibrium open system is studied in the projection operator formalism. The environment may linearly deviate from its initial state under the reaction from the open system. If the relevant statistical operator of the system is a generalized canonical one, the transport equation, the second kind of fluctuation-dissipation theorem and the entropy production rate of the open system can be derived and expressed in terms of correlation functions of fluctuations of random forces and interaction random forces.

Keywords: Non equilibrium open system; Projection operator; Entropy production rate

Introduction

In the study of nonequilibrium systems different projection operators are introduced to present a macroscopic description of the system in order to simplify the problem [1-6]. In this approach the macroscopic state of the system is determined by expectation values of a set of basis macro variables, and equations of motions for these expectation values, the transport equations, are derived in the projection operator formalism.

When studying a nonequilibrium open system, the influence of the environment upon the open system is one of the important topics in such studies. It has been shown [7] that the influence from the environment comes from two parts: one is the time-rate of the averaged projection operator formalism.

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Transport equations

Consider an open system S under the influence of its environment R. The total system S ⊗ R is characterized by Hamiltonian $H=H_S+H_R$ and statistical operator (so) $W(t)$. The open system $s$ is described by a reduced statistical operator $\rho(t)=tr_{\Sigma} W(t)$ satisfying $\partial \rho(t)/\partial t=-iL_\gamma\rho(t)+\eta(t)$ with

$$\eta(t)=-i\partial tr_{\Sigma} [I_{\Sigma} W(t)]$$

(2.1)

describing the influence of R upon S, where $L_{X}=i[HS,X], h=1$.

Suppose we are satisfied with the description of system S by the macroscopic level by expectation values (EVs) of a set of basis macrovariables $A_1,...,A_m$ of $S$, such macroscopic description can be realized by a relevant so $\rho(t)$ which is picked up by a time-dependent projection operator $\rho(t)$ from $\rho(t)$; $\rho(t)=\rho(t)\rho(t)$. We may choose the following projection operator as $\rho(t)$ [3]:

$$\rho(t)\rho(t) = \left[ \rho(t) - \sum_{\epsilon_{A(t)} < \Lambda(t)} \partial \rho(t) \right] + \sum_{\epsilon_{A(t)} > \Lambda(t)} \rho(t)$$

(2.2)

Introduce $q(t)=1-p(t), p(t)q(t)=0$, we have [6]

$$\rho(t)=\rho(t)+g(t)q(t)$$

(2.3)

with $P(t)\rho(t)=0, g(t)u=q(t)u$, is a time-ordered evolution operator satisfying

$$\partial g(t,u)/\partial u=i g(t,u)q(t)u$$

and $g(t,u)=1$.

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The transport equation for \( \langle A_j(t) \rangle = tr_j \rho(t) A_j \) takes the form [6]

\[
\frac{\partial}{\partial t} \langle A_j(t) \rangle = \frac{\partial}{\partial t} \langle A_j(t) \rangle + Y_j(t) \quad (j=1,2, \ldots, m).
\]

In Heisenberg picture,

\[
\frac{d}{dt} \langle \rho(t) A_j \rangle = \frac{\partial}{\partial t} \langle \rho(t) A_j \rangle + \int d\omega A_j \rho(\omega) G(\omega, t) \delta A_j.
\]

(2.5)

Here the first term gives the organized motion, the second term the initial condition and the third term the disorganized motion or the memory term [4] and

\[
Y_j(t) = \int_0^t dt \rho(t) \dot{A}_j.
\]

(2.6)

This is an additional term describing the external influence from the environment upon the open system; \( G(\omega, t) = T_\omega \exp \left[ \int dt \rho(\omega) G(u, t) \right] \) \((\omega > 0)\) is an anti-time-ordered evolution operator defined by

\[
\partial G(\omega, t)/\partial t = -i \omega G(\omega, t) \quad \text{and} \quad G(t, t) = \sum_{\omega} G(\omega, t) \rho(\omega) G(\omega, t) = 1.
\]

(3.6)

From its fluctuation and \( G(t, t)=1 \).

\[
\int \rho(t) \dot{A}_j = \int \langle \rho(t) A_j \rangle \partial_t \langle A_j(t) \rangle
\]

(3.1)

(3.2a)

(3.2b)

\[
\int \rho(t) \dot{A}_j = \int \langle \rho(t) A_j \rangle \partial_t \langle A_j(t) \rangle
\]

(3.3)

(3.4)

(3.5)

\[
\int \rho(t) \dot{A}_j = \int \langle \rho(t) A_j \rangle \partial_t \langle A_j(t) \rangle
\]

(3.6)

Therefore Eq.(2.5) takes the form

\[
\sum_{\pm} \sum_{i} \rho_{\pm} (i) \partial_t \langle A_j(t) \rangle = \sum_{\pm} \sum_{i} \rho_{\pm} (i) \partial_t \langle A_j(t) \rangle
\]

(3.7)

(3.8)

(3.9)

(3.10ab)

(3.10bc)

The meaning of (2.9) is clear and simple: The transport equation of an open system is the sum of transport equation of the corresponding closed system, the time-rate of the EV due to the interaction Hamiltonian \( H_a \), and the additional influence term \( Y_j(t) \).

The influence term (2.6) can be written as (j=1,2, ...,m) \((j=1,2, ...,m)\).

Where

\[
f_j(u,t) = Q(u)G(u,t) \dot{A}_j
\]

(2.11)

(2.12a)

(2.12b)

(2.12c)

being respectively the random force and interaction random force associated with the time rate of the basis variable \( A_j \) due to \( H_a \) and \( H_{SR} \), respectively. Since the average of the random force over given ensembles vanishes, so \( f_j(u,t) = \delta f_j(u,t) \) In the rest of the paper we will no longer distinguish \( f_j(u,t) \) from its fluctuation \( \delta f_j(u,t) \).

**Generalized canonical statistical operator In order to go steps further**

Let us assume \( \rho_j(t) \) to be a GCSO:

\[
\rho_j(t) = e^{-\sum_{\lambda} \lambda_j / z_j(t)} r_{\lambda}(t) = e^{-\sum_{\lambda} \lambda_j / z_j(t)}\]

(3.1)

Where \( \lambda_i(t) \) \((i=1, \ldots, m)\) are conjugate parameters of the basis macrovariables \( \{A_j\} \).

Making use of the Kubo identity \( \int d\omega e^{\omega t} [X,Y] e^{\omega(u-v)} \) we have

\[
\int d\omega \partial_{\omega} \rho_j(t) = \sum_{\lambda} \int d\omega e^{\omega t} (iL_{a\lambda} A_j) e^{-\sum_{\lambda} \lambda_j / z_j(t)} \rho_j(t) \lambda_j(t)
\]

(3.2a)

\[
\int d\omega \partial_{\omega} \rho_j(t) = \sum_{\lambda} \int d\omega e^{\omega t} (iL_{a\lambda}) e^{-\sum_{\lambda} \lambda_j / z_j(t)} \rho_j(t) \lambda_j(t)
\]

(3.2b)

Introducing the generalized quantum correlation function

\[
\langle X(t), Y(u) \rangle = \int d\omega \rho_j(t) e^{-\sum_{\lambda} \lambda_j / z_j(t)} \rho_j(u) \]

(3.3)

(3.4)

(3.5)

Where \( f_j(u,t) \) is given by (2.12b). Same argument will apply to similar cases later.

Therefore Eq.(2.5) takes the form

\[
\sum_{\pm} \sum_{i} \rho_{\pm} (i) \partial_t \langle A_j(t) \rangle = \sum_{\pm} \sum_{i} \rho_{\pm} (i) \partial_t \langle A_j(t) \rangle
\]

(3.7)

Where the memory term is expressed in terms of quantum correlation function of fluctuations of random forces. The influence term \( Y_j(t) \) will be further analysed in Section 5.

**Entropy production rate**

Now define the entropy of the nonequilibrium open system through its relevant statistical operator [1,15,16]

\[
S(t) = -k_b \sum_j r_j(t) \ln \rho_j(t)
\]

(4.1)

where \( k_b \) is the Boltzmann constant. The entropy production rate reads [7]

\[
\frac{\partial S(t)}{\partial t} = k_b \sum_j \frac{\partial}{\partial t} \langle A_j(t) \rangle \ln \rho_j(t)
\]

(4.2)

which is the sum of products of transport equations and the conjugate parameters. If assume that the initial state of the system is a GCSO: \( \rho(0)=\rho(0) \) then the initial term in (3.6) vanishes. Combining (4.2) with (2.4) given by (3.6) and (2.11), we obtain

\[
\frac{\partial S_e(t)}{\partial t} = \frac{\partial S(t)}{\partial t} + \frac{\partial S(t)}{\partial t} + \frac{\partial S(t)}{\partial t}
\]

(4.3)

the first term resulting from the organized motion in (3.6) reads [7]
\[
\frac{dS}{dt} = \int \left[ \rho(u) f(u, t) \right] \lambda(u) dt
\]  
(4.4)

The second term resulting from the disorganized motion in (3.6) takes the form
\[
\frac{dS}{dt} = k_s \int \left[ \rho(u) f(u, t) \right] \lambda(u) dt
\]  
(4.5)

because of (3.5); and the third term resulting from the influence term (2.11) is
\[
\frac{dS}{dt} = k_s \int \left[ \rho(u) f(u, t) \right] \lambda(u) dt
\]  
(4.6)

These expressions represent the contributions of each term in the transport equation to the entropy production, respectively. Besides, Eq.(4.4) does not involve \( dL \), indicating that in the organized motion term \( H \), contributes nothing to the rate.

**Non-reservoir environment**

Now we further analyze the contribution of the influence term \( Y(t) \). Suppose that the environment \( R \) is not a reservoir and may linearly deviate from its initial state under the reaction from \( S \). For simplicity, we assume \( H_{SR} = \sum \gamma_j A_j \), and \( A_j \) and \( B_j \) respectively pertain to \( S \) and \( R \), and they are initially independent:
\[
W(0) = \rho(0) R(0) \text{ where } R(t) = tr W(t).
\]

By \( W(0) = \rho(0) R(0) \), we have \( W(u) = \rho(u) R(u) \), here \( \rho(u) = e^{\lambda_{\omega(t)+iu} \lambda(t)} \rho(0) \) and \( R(u) = e^{\lambda_{\omega(t)+iu} \lambda(t)} R(0) \). By (2.1), \( \eta(u) \) may be written as
\[
\eta(u) = -i\lambda - L_{SR}(u) > \omega, \rho(u) = -i\lambda [[H_{SR}(u) > \omega, \rho(t)]]
\]  
(5.1)

\[
< H_{SR}(u) > \omega = \sum \gamma_j A_j = B_k(u) > \omega;
\]  
(5.2)

\[
< B_k(u) > \omega \text{ determines the evolution of the EV of macrovariable } B_k.
\]

Since \( \lambda(t) = \lambda_{\omega(t)+iu} \lambda(t) \), thus \( B_k(t) = e^{\lambda_{\omega(t)+iu} \lambda(t)} B_k(0) \). For weak interaction, keeping only the linear term in \( \lambda \), we obtain
\[
B_k(t) = e^{\lambda \lambda_{\omega(t)+iu} \lambda(t)} B_k(0) + \lambda \lambda_{\omega(t)+iu} \lambda(t) L_{SR}(u) = e^{\lambda \lambda_{\omega(t)+iu} \lambda(t)} L_{SR}(u)
\]  
(5.3a)

\[
< B_k(u) > \omega = \sum \gamma_j A_j = B_k(u) > \omega;
\]  
(5.3b)

\[
< B_k(u) > \omega = \sum \gamma_j A_j = B_k(u) > \omega
\]  
(5.3c)

\[
\text{being the zeroth and first order terms of the EV of } B_k \text{, when } R \text{ linearly deviates from its initial state under the weak reaction from } S. \text{ By (5.1) and (5.2), the integrand in (2.11) takes the form}
\]
\[
\eta(u) f(u, t) = -i\lambda \sum \gamma_j \lambda_j f(u, t) \left[ A_j, \rho(u) \right] < B_k(u) > \omega
\]  
(5.4a)

\[
\text{in Schrodinger and Heisenberg pictures, respectively, where}
\]
\[
F_j(u, t) = e^{(\lambda_{\omega(t)+iu} \lambda(t))} f_j(u, t) \text{ and } A_j(u) = e^{(\lambda_{\omega(t)+iu} \lambda(t))} A_j.
\]

\[
\text{Hence we have the influence term}
\]
\[
Y_j(t) = -i\lambda \sum \gamma_j \int \lambda_j f_j(u, t) \left[ A_j, \rho(u) \right] < B_k(u) > \omega
\]  
(5.5a)

\[
= -i\lambda \sum \gamma_j \int \lambda_j f_j(u, t) \left[ A_j, \rho(u) \right] < B_k(u) > \omega
\]  
(5.5b)

\[
\text{and its contribution to the entropy production in Schrodinger and Heisenberg pictures:}
\]
\[
\frac{dS}{dt} = -i\lambda \sum \gamma_j \int \lambda_j f_j(u, t) \left[ A_j, \rho(u) \right] < B_k(u) > \omega
\]  
(5.6a)

\[
= -i\lambda \sum \gamma_j \int \lambda_j f_j(u, t) \left[ A_j, \rho(u) \right] < B_k(u) > \omega
\]  
(5.6b)

Now consider the case that the initial state of \( S \) is given by a GCSO:
\[
\rho(0) = e^{-\lambda_{\omega(t)+iu} \lambda(t)}
\]  
(5.7)

By the Kubo identity and the initial condition \( \rho(0) = \rho(0) \) we have
\[
\left[ A_j(u), \rho(0) \right] = -\lambda \lambda_{\omega(t)+iu} \lambda(t)
\]  
(5.8)

and its contribution to the rate.

\[
\text{Thus (5.4) may be written as}
\]
\[
\eta(u) f(u, t) = -i\lambda \sum \gamma_j \int \lambda_j f_j(u, t) \left[ A_j, \rho(u) \right] < B_k(u) > \omega
\]  
(5.9)

\[
\text{and}
\]
\[
\text{the memory term in (3.6) becomes}
\]
\[
\lambda \int \left[ f(u, t), f^*(u, t) \right] \rho(u) dt
\]  
(5.10)

\[
\text{If the open system is initially in an equilibrium state}
\]
\[
\rho(0) = \rho_0 = e^{-\lambda_{\omega(t)+iu} \lambda(t)}
\]  
(5.11)

\[
\beta_0 = 1/k_B T, \text{the initial inverse temperature of the system,}
\]
\[
\text{then } \rho_0 \text{ is a special case of (5.7) in which } A_0 = H_\beta, \lambda(0) = \beta_0
\]

\[
\text{and } A_0 = A(0) \text{ for } t > 0. \text{ Since } A_0 \text{ is the only basic variable, so}
\]
\[
\rho(0) = e^{-\lambda_{\omega(t)+iu} \lambda(t)}
\]  
(5.12)

the memory term in (3.6) becomes
\[
\lambda \int \left[ f(u, t), f^*(u, t) \right] \rho(u) dt
\]  
(5.13)
Besides, as a special case of (5.9) and (5.10), we have

\[
Y_{f}(t) = \lambda^{2} \beta_{f}^{0} \int_{0}^{\infty} dt \left\{ F_{f}^{SR}(u,t), f_{f}^{0}(u,t) \right\}
\]

(5.14)

\[
\frac{dS_{f}(t)}{dt} = \frac{\lambda^{2}}{T_{0}} \int_{0}^{\infty} dt \left\{ F_{f}^{SR}(u,t), f_{f}^{0}(u,t) \right\} \beta(t)
\]

(5.15)

Here \((X(t), Y(u)) = \int_{0}^{\infty} d\sigma \varphi_{ \lambda}(t) e^{-\eta_{ \lambda}^{0} T_{0} \rho_{ \lambda}} (u)\).

Finally we obtain the transport equation for the only basis variable \(u_{eq}^{0}\):

\[
\frac{dS_{e}(t)}{dt} = \frac{\lambda}{T_{0}} \varphi(t) \left\{ \rho_{ \lambda}(t) \mid u_{eq}^{0}, H_{ \lambda} \right\}
\]

(5.16)

Eqs. (5.9), (5.10) and (5.14), (5.15) involve the averaged interaction random force (5.8) which has incorporated the linear deviation of the environment from its initial state.

Now consider the case without a given initial condition. By (3.1) and the Kubo identity, we have

\[
i_{<L_{ \lambda}(u) >_{\lambda}, \rho_{ \lambda}(u)} = \sum_{ \lambda} \alpha_{ \lambda}(u) \int_{0}^{\infty} d\sigma \sum_{ \lambda} \rho_{ \lambda}(u) \int_{0}^{\infty} d\eta \int_{0}^{\infty} d\xi
\]

\[
= \sum_{ \lambda} \alpha_{ \lambda}(u) \int_{0}^{\infty} d\sigma \sum_{ \lambda} \rho_{ \lambda}(u) \int_{0}^{\infty} d\eta \int_{0}^{\infty} d\xi
\]

(5.17)

With (5.1) and (2.3),

\[
\tau_{ \lambda} \left\{ \eta(u), f_{f}(u,t) \right\} = -\lambda \int_{0}^{\infty} d\sigma \int_{0}^{\infty} d\eta \sum_{ \lambda} \alpha_{ \lambda}(u) \int_{0}^{\infty} d\xi
\]

(5.18)

J_{1} = \lambda^{2} \beta_{f}^{0} \int_{0}^{\infty} dt \left\{ F_{f}^{SR}(u,t), f_{f}^{0}(u,t) \right\}

(5.19)

J_{1} = \lambda^{2} \beta_{f}^{0} \int_{0}^{\infty} dt \left\{ F_{f}^{SR}(u,t), f_{f}^{0}(u,t) \right\}

(5.20)

J_{2} = \lambda \sum_{ \lambda} \alpha_{ \lambda}(u) \int_{0}^{\infty} dt \left\{ K_{f}(u,t), f_{f}^{0}(u,t) \right\}

(5.21)

K_{i}(u,t) = i L_{f} Q(u) G(u,t)

(5.22)

J_{3} = \lambda \sum_{ \lambda} \alpha_{ \lambda}(u) \int_{0}^{\infty} dt \left\{ K_{f}(u,t), f_{f}^{0}(u,t) \right\}

(5.23)

K(t) = i L_{f} Q(u) G(u,t)

(5.24)

Substituting (2.3) into (5.23) and repeating the above arguments, we have

\[
\tau_{ \lambda} \left\{ \eta(u), f_{f}(u,t) \right\} = \lambda \sum_{ \lambda} \alpha_{ \lambda}(u) \int_{0}^{\infty} dt \left\{ K_{f}(u,t), f_{f}^{0}(u,t) \right\}
\]

(5.25)

which may be written as

\[
\tau_{ \lambda} \left\{ \eta(u), f_{f}(u,t) \right\} = \lambda \sum_{ \lambda} \alpha_{ \lambda}(u) \int_{0}^{\infty} dt \left\{ K_{f}(u,t), f_{f}^{0}(u,t) \right\}
\]

(5.26)

M(t) = \sum_{ i} \lambda_{i} \int_{0}^{\infty} dt \left\{ \lambda_{i}(u), f_{f}^{0}(u,t) \right\}

(5.27)

here we have had argument similar to that leads (3.4) to (3.5). Thus we obtain the influence term and its contribution to the entropy production:

\[
Y_{f}(t) = \lambda \sum_{ i} \lambda_{i} \int_{0}^{\infty} dt \left\{ \lambda_{i}(u), f_{f}^{0}(u,t) \right\} + M_{i}(u,t)
\]

(5.28)

\[
\frac{dS_{f}(t)}{dt} = \lambda k_{i} \sum_{ i} \int_{0}^{\infty} dt \left\{ \lambda_{i}(u), f_{f}^{0}(u,t) \right\} + M_{i}(u,t)
\]

(5.29)

If we are satisfied with keeping the linear term of \(\lambda\) in \(M_{i}(u,t)\), then

\[
M_{i}(u,t) = \sum_{ i} \int_{0}^{\infty} dt \left\{ \lambda_{i}(u), f_{f}^{0}(u,t) \right\} + o(\lambda^{2})
\]

(5.30)

\[
\tau_{ \lambda} \left\{ \eta(u), f_{f}(u,t) \right\} = \lambda \sum_{ \lambda} \int_{0}^{\infty} dt \left\{ \phi(u), f_{f}^{0}(u,t) \right\}
\]

(5.31)

Here we have the approximate expressions

\[
Y_{f}(t) = \lambda \sum_{ i} \int_{0}^{\infty} dt \left\{ \phi(u), f_{f}^{0}(u,t) \right\}
\]

(5.32)

\[
\frac{dS_{f}(t)}{dt} = \lambda k_{i} \sum_{ i} \int_{0}^{\infty} dt \left\{ \phi(u), f_{f}^{0}(u,t) \right\}
\]

(5.33)

they are up to \(\lambda^{2}\) by (2.12a). Comparing (5.34) with (4.5), we see clearly that \(\phi(u)\) plays the role of \(\lambda_{i}(u)\) in the case of corresponding isolate system.

Now we rewrite the results obtained above in the form of special dependent. For simplicity, we focus on the simpler expression (5.34). The entropy production of the open system reads

\[
\frac{dS(x,t)}{dt} = \frac{dS_{f}(x,t)}{dt} + \frac{dS_{s}(x,t)}{dt} + \frac{dS_{s}(x,t)}{dt}
\]

(5.35)

Where

\[
\frac{dS_{f}(x,t)}{dt} = \lambda k_{i} \sum_{ i} \int_{0}^{\infty} dt \left\{ \rho_{i}(x), [L_{f} A_{i}] \right\}
\]

(5.36a)

results from the organized motion in the transport equation due to \(H_{f}\).
$$\frac{d\delta S}{dt}(x,t) = k_s \sum_{j=1}^{m} \int du \left( f_j(u,t), f_j^*(u,u) \right) \lambda_j(x,u) \lambda_j(x,t)$$

(5.36b)

from the disorganized motion and

$$\frac{d\delta S}{dt}(x,t) = k_h \sum_{j=1}^{m} \int du \left( \phi_j(x,u), f_j(u,u) \right) \lambda_j(x,u) \lambda_j(x,t)$$

(5.36c)

from the influence term, respectively.

**Comparison**

In this section, we compare the results obtained in the proceeding sections with the well-known approaches in the linear nonequilibrium thermodynamics and statistical mechanics. The time rate of the entropy density $\delta S(x,t)$ of a nonequilibrium system takes the form [8,18,19]:

$$\frac{d\delta S}{dt}(x,t) = \sigma(x,t) - \nabla \cdot J(x,t) \tag{6.1}$$

where $\sigma(x,t) = \sum X_i(x,t) J_i(x,t)$ is the entropy production density occurring inside the system which is given in terms of the sum of products of thermodynamic fluxes $J_i(x,t)$ and the conjugate thermodynamic forces $X_i(x,t)$: $J_i(x,t)$ is the density of entropy flux through the border into the system.

Onsager proposed a linear relationship between the fluxes and forces

$$J_i(x,t) = \sum X_i(x,t) = J_{si} \tag{6.3}$$

with reciprocity relations

$$K_{ik} = L_{ki} \tag{6.4}$$

Thus we have

$$\frac{d\delta S}{dt}(x,t) = \sum_{i,k} L_{ik} X_i(x,t) X_k(x,t) - \nabla \cdot J(x,t) \tag{6.5}$$

For the special case considered in Sect.5, the interaction between open system S and its environment R takes the form $H_{SR} = \sum \gamma_i A_i B_i$, for example, S and R are composed of different kinds of harmonic oscillator [7]. Such interaction implies no obvious border separating S and R, leading to absence of the divergence term on the right hand side of (6.5). Thus the variation of entropy density results from inner entropy production $\sigma(x,t)$ only:

$$\frac{d\delta S}{dt}(x,t) = \sum_{i,k} L_{ik} X_i(x,t) X_k(x,t) - \nabla \cdot J(x,t) \tag{6.6}$$

Besides, in the Green-Kubo formalism, the transport coefficients $L_{ik}$ can be expressed in terms of time correlation functions of the time rate of corresponding variables [19,20].

$$L_{ik} \sim \int_0^\infty dt \left\{ \dot{A}_i(t) \right\} \dot{A}_k(0) \tag{6.7}$$

where the average is taken over an equilibrium ensemble and the Markovian effect is taken into account.

In this paper, we study a nonequilibrium open system whose transport equations (2.9)-(2.11) are nonlinear differential-integral ones. Now let us compare (5.36b) with (6.6). We notice that (5.36b) share the same structure as (6.6) because of the facts: (1) parameters $\lambda_i(x,t)$ \ ($i=1,\ldots,m$) play the role Thermodynamic forces since they may involve spacial gradients of, e.g., temperature, velocity, chemical potential or electric, magnetic fields, etc; (2) the random forces (2.12b) involve the time rates of variables because of using projection operator technique and (3) the average is taken over GCSO (3.1) instead of an equilibrium ensemble. As for (5.36c), the contribution of $Y_j (t)$ to the entropy production, in which the free term $\phi_j (x,u)$ in the Volterra equation is indeed a generalization of $\lambda_j(x,u)$ in (5.26b), hence (5.36c) possesses the same structure as (6.6) also. Accordingly, we see clearly that the entropy production rate (5.35) is a natural generalization of (6.6) where the non-linearity and the non-Markovian effect have been taken into consideration. In addition to the entropy production, the entropy generation is another useful tool in the study of nonequilibrium systems [8] and especially useful in the analysis of a process occurring in the system during a period of time. It is worth noticing the major differences between the two: the entropy production needs the hypothesis of local equilibrium but the entropy generation does not; the former does not consider the time but the latter introduces the lifetime of the process. The two different approaches are closely related and complementary one to another.

**Conclusion**

In the present paper we have studied a nonequilibrium open system in interaction with its environment which may linearly deviate from its initial state under the reaction of the open system. We have shown that if the relevant statistical operator of the system is of the form of GCSO, then the transport equation is given by (3.6) and (5.28) or (5.33). The memory term in (3.6) and the influence term (5.28) or (5.33) can be expressed in terms of quantum correlation functions of fluctuations of random forces and interaction random forces, giving the second kind of fluctuation-dissipation theorem for this nonequilibrium open system. We have also shown that the entropy production rate is given by the sum of products of transport equations and the corresponding parameters. In the organized motion term, $H_s$ contributes nothing to the rate, but $H_{SR}$ does: the contributions of the memory and influence terms are expressed in terms of quantum correlation functions of fluctuations of random forces and interaction random forces. The total entropy production rate is given by the sum of contributions resulting from each term in the transport equation, given respectively by (4.4), (4.5) and (5.29) or (5.34). They are natural generalizations of those for a linear nonequilibrium closed system to a nonlinear open system.

**References**

1. Robertson B (1996) Equations of Motion in Nonequilibrium Statistical Mechanics. Phys Rev 144: 151.
2. Li KH (1986) Physics of open systems. Phys Rep 134: 1-85.
3. Kawasaki K, Gunton JD (1973) Phys Rev A 8: 2048.
4. Grabert H (1978) Projection Operator Techniques in Nonequilibrium Statistical Mechanics. Phys Rev 144: 151.
5. Oppenheim I, Levin RD (1979) Nonlinear Transport Processes: Hydrodynamics. Physica A 99: 383-402.
6. Yu MB (1996) Physica A 137.
7. Yu MB (2008) Influence of environment and entropy production of a nonequilibrium open system. Phys Left A 372: 2572-2577.
8. Lucia U (2013) Stationary open systems: A brief review on contemporary theories on irreversibility. Physica A 392: 1051-1062.
9. Hatsopoulos GN, Gyftopoulos EP (1976) A Unified Quantum Theory of Mechanics and Thermodynamics. Part I. Postulates. Foundations of Phys 6: 15.
10. Hatsopoulos GN, Gyftopoulos EP (1976) A Unified Quantum Theory of...
11. Hatsopoulos GN, Gyftopoulos EP (1976) A Unified Quantum Theory of Mechanics and Thermodynamics. Part IIb. Stable Equilibrium States. Foundations of Phys 6: 439.

12. Hatsopoulos GN, Gyftopoulos EP (1976) A Unified Quantum Theory of Mechanics and Thermodynamics. Part III Irreducible Quantal Dispersions. Foundations of Phys 6: 561.

13. Beretta GP (2010) Maximum entropy production rate in quantum thermodynamics. J Phys: Conference Series 237: 1-32.

14. Sciacovelli A, Smith CE, Vonspakovsky MR, Verda V (2010) Quantum Thermodynamics: Non-equilibrium 3D Description of and Unbounded System at an Atomistic Level. Int J of Thermodynamics 13: 23-33.

15. Nettleton RE (2001) Positive definiteness of entropy production in the nonlinear Robertson formalism. J Chem Phys 14: 6007-6013.

16. Jarvis JB (2005) Time-dependent entropy evolution in microscopic and macroscopic electromagnetic relaxation. Phys Rev E 72.

17. MB Yu (1990) Phys Lett A 150: 1-10.

18. Groot SD, Mazur P (1982) Non-Equilibrium Thermodynamics, North-Holland.

19. Martyushev LM, Seleznev VD (2006) The restrictions of the Maximum Entropy Production Principle. Phys Rept 426: 1-45.

20. Kubo R (1957) The Glass Transition: Relaxation Dynamics in Liquids and Disordered Materials. J Phys Soc Jpn 12: 570-586.