Analysing the pseudoscalar hidden-charm tetraquark states with the QCD sum rules

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Abstract

In this work, we take all the color-antitriplet diquarks, such as the scalar, pseudoscalar, vector, axialvector and tensor diquarks, as the basic constituents to construct the local pseudoscalar four-quark currents without importing the explicit P-waves to implement the negative-parity, and investigate the mass spectroscopy of the pseudoscalar hidden-charm tetraquark states without strange, with strange and with hidden-strange in the framework of the QCD sum rules in a consistent and comprehensive way. We obtain the lowest mass 4.56 ± 0.08 GeV for the pseudoscalar tetraquark state with the symbolic quark constituents ccd, which is much larger than the experimental value 4.293 ± 0.045 MeV extracted by the LHCb collaboration.

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1 Introduction

In recent years, a number of exotic charmonium-like states have been reported by the BaBar, Belle, BESIII, CDF, CMS, D0, LHCb collaborations [1], such as the X(3860), X(3872), Zc(3900), X(3915), X(3940), Zcs(3985), etc, which have hidden-charm and therefore are charmonium-like, the traditional quark models are facing great challenges, we have to resort to additional quark and gluon constituents beyond the naïve quark-antiquark pairs to probe their properties, especially for those states with non-zero electric-charge, such as the Zc±(3900), Zc±(4020), Zc±(4430), etc. Several states lie slightly above or below the meson-antimeson or meson-meson thresholds provocatively, which maybe indicate that they are very good candidates for the hadronic molecules or threshold effects, but the others do not lie near the two-meson thresholds, why? Do they have absolutely different inner structures in nature? The theoretical physicists have tentatively suggested several possible assignments of those X, Y and Z states, such as the diquark-antidiquark type tetraquark states, hadronic molecular states (or color-singlet-color-singlet type tetraquark states), hadroquarkonium, dynamically generated resonances (another type molecular states), kinematical effects, cusp effects, virtual states, etc.

The attractive interactions inferred from one-gluon exchange support making diquark correlatings in the color-antitriplet 3c come into being [4, 5]. The diquark operators $\varepsilon^{ijk} q_j^T C T q_k^c$ in the $3c$ have five spinor structures, where the color indexes $i, j, k = 1, 2, 3$, the Dirac matrixes

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In this work, we extend our previous works \([10, 11, 12, 13, 14, 17, 18, 19]\) to explore the mass spectroscopy of the pseudoscalar hidden-charm tetraquark states and make great effects to obtain the best energy scales characterizing the QCD spectral properties of the pseudoscalar hidden-charm tetraquark states. The QCD sum rules method is a vigorous and powerful theoretical tool in probing the exotic X, Y and Z states, there have been several possible assignments of the X, Y and Z states, such as the diquark-antidiquark type tetraquark states, the tetraquark molecular states, the c\(\bar{c}\)-tetraquark mixing states, according to the analysis via the QCD sum rules \([6]\). The predictions depend on a great extent on the particular schemes in which the input parameters are accepted/adopted at the QCD side, for example, even in the same diquark-antidiquark type tetraquark scenario, the same quark currents can result in quite different predictions therefore quite different assignments of the Y states \([7, 8, 9]\). A comprehensive and consistent investigation of all the scalar, pseudoscalar, vector, axialvector and tensor hidden-charm tetraquark (molecular) states with the same input parameters and same treatments of the operator product expansion is necessary to avoid possible biased predictions.

In Refs.\([10, 11, 12]\) (\([13, 14]\)), we construct the diquark-antidiquark type four-quark currents without importing the explicit P-waves to investigate the mass spectroscopy of the ground state hidden-charm (doubly-charmed) tetraquark states with the \(J^{PC} = 0^{++}, 1^{++}, 1^{+-}, 1^{-+}, 1^{--}\) and \(2^{++}, 0^{++}, 1^{++}, 1^{+-}, 1^{-+}\) and \(2^{++}\) in the framework of the QCD sum rules in a comprehensive and consistent way, and revisit the assignments of all the observed X, Y, Z states in the scenario of tetraquark states, and make a series of predictions which can be compared with the experimental data in the future to illustrate (demonstrate) the nature of the tetraquark states and their inner quark-gluon structures. The predicted masses of the diquark-antidiquark type axialvector \(cc\bar{d}\bar{d}\) tetraquark states are \(3.90 \pm 0.09\) \(\text{GeV}\) \([13, 14]\), which are in very good agreement with the experimental value from the LHCb collaboration later \([15, 16]\). In Refs.\([17, 18]\), we import an explicit P-wave between the diquark and antidiquark building blocks to implement the negative-parity to construct the local four-quark currents to explore the mass spectroscopy of the ground state hidden-charm tetraquark states with the \(J^{PC} = 1^{--}\) in a systematic way, and obtain the lowest vector tetraquark states up to today.

In Ref.\([19]\), we construct the color-singlet-color-singlet type (or meson-meson type) local four-quark currents to investigate the mass spectroscopy of the hidden-charm tetraquark molecular states with the \(J^{PC} = 0^{++}, 1^{++}, 1^{+-}\) and \(2^{++}\) in a comprehensive and consistent way and make possible assignments of all the X, Y and Z states in a different scheme, and observe that the scenario of tetraquark molecule states can accommodate much less exotic states than that of the diquark-antidiquark type tetraquark states.

In all the works \([10, 11, 12, 13, 14, 17, 18, 19]\), we resort to our unique benchmark, the energy scale formula \(\mu = \sqrt{M_{X/Y/Z}^2 - (2M_{c\bar{c}})^2}\) or its modifications with the effective c-quark mass \(M_{c\bar{c}}\), which has the universal value, to acquire the best energy scales characterizing the QCD spectral densities via trial and error. The energy scale formula plays an essential role in increasing the pole contributions and in making the convergent behaviors of the operator product expansion much better \([20]\), and it is a unique feature of our works.

In this work, we extend our previous works \([10, 11, 12, 13, 14, 17, 18, 19]\) to explore the mass spectroscopy of the pseudoscalar hidden-charm tetraquark states and make great effects to obtain
comprehensive investigations on the hidden-charm tetraquark states in a consistent way. We take the elementary constituents $C_\gamma$, $C$, $C_\gamma\gamma$, $C_\gamma\mu$, $C_\sigma\mu$ and $C_\sigma\mu\gamma$ (anti)diquark operators to construct the local four-quark pseudoscalar currents without resorting to the explicit P-wave. While in the dynamical (di)quark models and constituent (di)quark models, we always take only the scalar, pseudoscalar, vector, axialvector and tensor hidden-charm tetraquark states, and explicit P-waves between the diquark and antidiquark constituents are needed to acquire the pseudoscalar and vector tetraquark states [21, 22, 23]. The tetraquark spectroscopy obtained in Refs. [21, 22, 23] differ from the present work significantly. We investigate the mass spectroscopy of the pseudoscalar and vector tetraquark states [21, 22, 23] P-waves between the diquark and antidiquark constituents are needed to acquire the pseudoscalar in the dynamical (di)quark models and constituent (di)quark models, we always take only the scalar, pseudoscalar, vector, axialvector and tensor hidden-charm tetraquark states, and explicit P-waves between the diquark and antidiquark constituents are needed to acquire the pseudoscalar and vector tetraquark states [21, 22, 23].

The article is arranged as follows: we obtain the QCD sum rules for the masses and pole residues of the pseudoscalar hidden-charm tetraquark states in section 2; in section 3, we exhibit the numerical results and discussions; section 4 is reserved for our conclusion.

## 2 QCD sum rules for the pseudoscalar hidden-charm tetraquark states

According to the routine of the QCD sum rules, we write down the two-point correlation functions $\Pi(p)$ firstly,

$$
\Pi(p) = i \int d^4xe^{ipx} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle ,
$$

(1)

where the local four-quark currents,

$$
J^A(x) = J^A_{-T}(x), \ J^A_{AV}(x), \ J^A_{PS}(x), \ J^A_{T-}(x), \ J^A_{TT}(x),
$$

(2)

$$
J^A_{AV}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}} \left[ q_T^j(x) C_\gamma\mu c^k(x) q^m(x) \gamma_5 \gamma^\mu C \bar{c}^T n(x) - q_T^j(x) C_\gamma\mu \gamma_5 c^k(x) q^m(x) \gamma^\mu C \bar{c}^T n(x) \right],
$$

$$
J^A_{AV}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}} \left[ q_T^j(x) C_\gamma\mu c^k(x) q^m(x) \gamma_5 \gamma^\mu C \bar{c}^T n(x) + q_T^j(x) C_\gamma\mu \gamma_5 c^k(x) q^m(x) \gamma^\mu C \bar{c}^T n(x) \right],
$$

$$
J^A_{PS}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}} \left[ q_T^j(x) C\sigma\mu c^k(x) q^m(x) \gamma_5 C \bar{c}^T n(x) - q_T^j(x) C\sigma\mu \gamma_5 c^k(x) q^m(x) \gamma^\mu C \bar{c}^T n(x) \right],
$$

$$
J^A_{T-}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}} \left[ q_T^j(x) C\sigma\mu c^k(x) q^m(x) \gamma_5 \gamma^\mu C \bar{c}^T n(x) - q_T^j(x) C\sigma\mu \gamma_5 c^k(x) q^m(x) \gamma^\mu C \bar{c}^T n(x) \right],
$$

(3)

$$
J^A_{TT}(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}} \left[ q_T^j(x) C\sigma\mu c^k(x) q^m(x) \gamma_5 \gamma^\mu C \bar{c}^T n(x) + q_T^j(x) C\sigma\mu \gamma_5 c^k(x) q^m(x) \gamma^\mu C \bar{c}^T n(x) \right],
$$

with $q, q' = u, d, s$, the $i, j, k, m, n$ are color indexes, the $C$ is the charge-conjugation matrix, the superscripts $\pm$ symbolize the positive and negative charge-conjugation, respectively, the subscripts...
$P$, $S$, $V$, $A$ and $T$ stand for the pseudoscalar, scalar, vector, axialvector and tensor diquark (and antidiquark) operators, respectively. Under parity transform $\hat{P}$, the four-quark currents $J(x)$ have the property,

$$\hat{P}J(x)\hat{P}^{-1} = -J(\hat{x}),$$  \hspace{1cm} (4)

which warrants that the Lorentz scalar currents $J(x)$ have the negative parity, therefore they are pseudoscalar currents. Under charge-conjugation transform $\hat{C}$, the four-quark currents $J(x)$ have the properties,

$$\hat{C}J^\pm(x)\hat{C}^{-1} = \pm J^\pm(x)|_{q\leftrightarrow q'},$$  \hspace{1cm} (5)

which warrants that we can distinguish the positive and negative charge-conjugations unambiguously. By the way, we can prove that the current $J_{\overline{TT}}(x) = 0$ through performing the Fierz-transformation.

The currents $J_{AV}^+(x)$, $J_{PS}^+(x)$ and $J_{TT}^+(x)$ have the same quantum numbers $J^{PC} = 0^{-+}$, while the currents $J_{\overline{AV}}(x)$ and $J_{PS}(x)$ have the same quantum numbers $J^{PC} = 0^{--}$. The currents having the same quantum numbers could mix with each other under re-normalization, we have to import mixing matrixes (or transformation matrixes) $\mathcal{M}$ to obtain diagonal currents $\tilde{J}(x)$ under re-normalization,

$$\tilde{J} = \mathcal{M}J,$$
$$\gamma_{\tilde{J}} = \mathcal{M}\gamma_J\mathcal{M}^{-1},$$  \hspace{1cm} (6)

where the $\gamma_J$ are the anomalous dimension matrixes of the current operators $J(x)$, and the $\gamma_{\tilde{J}}$ are the diagonal anomalous dimension matrixes. In the present case, the matrices $\gamma_J$ are $3 \times 3$ or $2 \times 2$ matrixes in the case of $J(x) = J_{\overline{AV}}(x)$, $J_{PS}^+(x)$, $J_{TT}^+(x)$ or $J(x) = J_{AV}(x)$, $J_{PS}(x)$, respectively.

In general, the matrices $\gamma_J$ can be expanded in terms of the strong fine structure constant $\alpha_s = \frac{g^2}{4\pi}$,

$$\gamma_J = C_{\gamma_{J,1}}\frac{\alpha_s}{4\pi} + C_{\gamma_{J,2}}\left(\frac{\alpha_s}{4\pi}\right)^2 + \cdots,$$  \hspace{1cm} (7)

where the $C_{\gamma_{J,1}}$ and $C_{\gamma_{J,2}}$ are the coefficients corresponding to the next-to-leading-order and next-to-next-to-leading-order radiative corrections, respectively. If we choose the diagonal currents $\tilde{J}(x)$, then the current operators $\tilde{J}(x)$ do not mix under re-normalization, and are expected to couple potentially to the physical pseudoscalar tetraquark states, as the physical masses are invariant under re-normalization, they are determined by experimental detections.

Generally speaking, a physical pseudoscalar tetraquark state, just like other hadron states, maybe have several Fock components, we can choose any current with the same quark structure as one of the Fock components to interpolate this tetraquark state due to the non-vanishing current-hadron coupling constant. Under re-normalization, there are new components induced in this special current operator, accordingly, we have to import new Fock components of the orders $\frac{\alpha_s}{4\pi}$, $(\frac{\alpha_s}{4\pi})^2$, etc to match with the updated current operator. In this aspect, the diagonalized current operators $\tilde{J}(x)$ are preferred, however, at the present time, we cannot acquire the mixing matrixes $\mathcal{M}$ without calculating the anomalous dimension matrixes $\gamma_J$, this maybe our next work.

Now go back to Eq. (1), at the hadron side, we insert a complete set of intermediate hadronic states, such as the tetraquark states, two-meson scattering states, continuum states, etc, having the same quantum numbers as the current operators $J(x)$ into the correlation functions $\Pi(p)$ to obtain the hadronic spectral representation [25, 26, 27], and distinguish the contributions of the lowest pseudoscalar hidden-charm tetraquark states without strange, with strange and with hidden-strange, respectively,

$$\Pi(p) = \frac{\lambda_2^2}{M_2^2 - p^2} + \cdots,$$  \hspace{1cm} (8)
where the pole residues \( \lambda_Z \) are defined by \( \langle 0 | J(0) | Z_\chi(p) \rangle = \lambda_Z \). In the isospin limit \( m_u = m_d \), the four-quark currents with the symbolic quark structures,

\[
\bar{c} c \bar{d} u, \quad \bar{c} c \bar{u} d, \quad \bar{c} c \bar{u} + \bar{d} d, \quad \frac{\bar{c} c u - \bar{d} d}{\sqrt{2}}, \quad \frac{\bar{c} c u + \bar{d} d}{\sqrt{2}},
\]

couple potentially to the pseudoscalar tetraquark states with degenerated masses, and they result in the same QCD sum rules as a matter of fact. On the other hand, the four-quark currents with the symbolic quark structures,

\[
\bar{c} c \bar{u} s, \quad \bar{c} c \bar{s} d, \quad \bar{c} c \bar{s} u, \quad \bar{c} c \bar{d} s,
\]

couple also potentially to the pseudoscalar tetraquark states with degenerated masses according to the isospin symmetry. Therefore, we will not distinguish the \( u \) and \( d \) quarks.

At the QCD side, we compute the vacuum condensates \( \langle \bar{q} q \rangle, \langle \bar{q} g_s \sigma G q \rangle, \langle \bar{q} q \rangle^2, \langle \bar{q} q \rangle \langle \bar{q} g_s \sigma G q \rangle, \langle \bar{q} g_s \sigma G q \rangle^2 \) and \( \langle \bar{q} q \rangle^2 \langle \bar{q} g_s \sigma G q \rangle \) with \( q = u, d \) or \( s \), which are vacuum expectations of the quark-gluon operators of the order \( \mathcal{O}(\alpha_s^k) \) with \( k \leq 1 \) [20, 28, 29], furthermore, we take the light flavor \( SU(3) \) mass-breaking effects into consideration by computing the terms of the order \( \mathcal{O}(m_s) \). In calculations, we adopt vacuum saturation for the higher dimensional vacuum condensates, for detailed discussions of this subject, one can consult Ref. [30]. Then we acquire the spectral representation of the correlation functions \( \Pi(p) \) through dispersion relation. At the end, we match the hadron side with the QCD side of the \( \Pi(p) \) below the continuum thresholds \( s_0 \), and accomplish the Borel transform by taking the large squared Euclidean momentum \( P^2 = -p^2 \) as the variable to get the QCD sum rules:

\[
\lambda_Z^2 \exp \left( -\frac{M_Z^2}{T^2} \right) = \int_{4m_s^2}^{s_0} ds \rho_{QCD}(s) \exp \left( -\frac{8}{T^2} \right),
\]

where we ignore the cumbersome analytical expressions of the QCD spectral densities \( \rho_{QCD}(s) \) to save the layout of printed sheets.

We differentiate Eq. (11) with respect to the inversed Borel parameter \( \tau = \frac{1}{T^2} \), and get the QCD sum rules for the masses of the pseudoscalar hidden-charm tetraquark states without strange, with strange and with hidden-strange through a fraction,

\[
M_Z^2 = -\frac{\int_{4m_s^2}^{s_0} ds \frac{d}{d\tau} \rho_{QCD}(s) \exp (-\tau s) }{\int_{4m_s^2}^{s_0} ds \rho_{QCD}(s) \exp (-\tau s) }.
\]

3 Numerical results and discussions

At first, we write down the energy-scale dependence of the quark masses and vacuum condensates,

\[
\langle \bar{q} q \rangle(\mu) = \langle \bar{q} q \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}},
\]

\[
\langle \bar{q} g_s \sigma G q \rangle(\mu) = \langle \bar{q} g_s \sigma G q \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33-2n_f}},
\]

\[
m_q(\mu) = m_q(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{12}{33-2n_f}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1}{b_0^2} \log t + \frac{b_2^2}{b_0^4} \left( \log^2 t - \log t - 1 \right) + b_0 b_2 t \right],
\]

1
from the renormalization group equation, where \( q = u, d, s, t = \log \frac{\mu^2}{\Lambda_{QCD}} \), \( b_0 = \frac{33 - 2n_f}{12\pi} \), \( b_1 = \frac{153 - 19n_f}{24\pi} \), \( b_2 = \frac{2857 - 5033n_f + 325n_f^2}{192\pi^2} \), \( \Lambda_{QCD} = 210 \text{ MeV} \), 292 MeV and 332 MeV for the flavors \( n_f = 5 \), 4 and 3, respectively \([1, 31]\), then try to get the ideal energy scales.

In this work, we explore the properties of the hidden-charm tetraquark states without strange, with strange and with hidden-strange, it is better to adopt the flavor number \( n_f = 4 \).

At the initial points, we take the standard values of the vacuum condensates \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3 \), \( \langle ss \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle \), \( \langle ar{q}g_s \sigma G \bar{q} \rangle = m_0^2\langle \bar{q}q \rangle \), \( \langle s \bar{s} G \bar{s} \rangle = m_0^2\langle ss \rangle \), \( m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2 \), \( \langle s \bar{s} GG\rangle = (0.012 \pm 0.004) \text{ GeV}^4 \) at the energy scale \( \mu = 1 \text{ GeV} \) \([25, 26, 27, 32]\), where \( q = u, d \), and take the \( \overline{MS} \) quark masses \( m_c(m_c) = (1.275 \pm 0.025) \text{ GeV} \) and \( m_s(\mu = 2 \text{ GeV}) = (0.095 \pm 0.005) \text{ GeV} \) from the Particle Data Group \([1]\). In numerical computations, we neglect the small masses of the \( u \) and \( d \) quarks.

In this work, we resort to our unique benchmark, the energy scale formula \( \mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2} \) with the effective \( c \)-quark mass \( M_c = 1.82 \text{ GeV} \) to get the best energy scales characterizing the QCD spectral densities \([20, 33]\). For detailed discussions about the energy scale formula, one can consult Refs. \([29, 34, 35, 36]\). We can rewrite the energy scale formula in the following form,

\[
M_{X/Y/Z}^2 = \mu^2 + C,
\]

where the constants \( C \) have the universal value \( 4M_c^2 \) and are fitted numerically via the QCD sum rules, the tetraquark masses and the best energy scales characterizing the QCD spectral densities have a Regge-trajectory-like relation.

We always refer to the experimental data on the energy gaps between the ground states (1S) and first radial excited states (2S) as references to get the continuum threshold parameters. Considering for the possible quantum numbers, decay modes and energy gaps, if we prefer the assignments in terms of compact tetraquark states in stead of other assignments, we can tentatively assign the \( Z_c(3900) \), \( X(3915) \), \( Z_c(4020) \), \( X(4140) \), \( X(4500) \), \( Z_c(4430) \), \( Z_c(4600) \) and \( X(4685) \) as the hidden-charm tetraquark states in perfect union, see Table 1. From the Table, we obtain the energy gaps 0.57 \( \sim \) 0.59 GeV between the 1S and 2S hidden-charm tetraquark states, therefore we set the continuum threshold parameters to be \( \sqrt{s_0} = M_Z + 0.4 \sim 0.6 \text{ GeV} \).

The pole dominance and convergence of the operator product expansion are two elementary criteria, we should satisfy them to reach reliable QCD sum rules. Now we define the pole contributions (PC),

\[
\text{PC} = \frac{\int_{4m_c^2}^{s_0} ds \rho_{QCD}(s) \exp \left( -\frac{s}{\mu^2} \right)}{\int_{4m_c^2}^{s_0} ds \rho_{QCD}(s) \exp \left( -\frac{s}{\mu^2} \right)},
\]

and the contributions of the terms involving the vacuum condensates of dimension \( n \),

\[
D(n) = \frac{\int_{4m_c^2}^{s_0} ds \rho_{QCD,n}(s) \exp \left( -\frac{s}{\mu^2} \right)}{\int_{4m_c^2}^{s_0} ds \rho_{QCD}(s) \exp \left( -\frac{s}{\mu^2} \right)},
\]

Table 1: The mass gaps between the 1S and 2S hidden-charm tetraquark states with the possible assignments.
where the total contributions are normalized to be 1.

We search for the best Borel parameters and continuum threshold parameters in the framework of trial and error following the routine in our previous works [10, 11, 12, 13, 14, 17, 18, 19]. At last, we reach the satisfactory destinations, such as the Borel windows, continuum threshold parameters, energy scales of the QCD spectral densities and pole contributions, which are shown distinctly in Table [2]. From the Table, we can see distinctly that the pole contributions are about (40 – 60)% at the hadron side, just like in our previous works investigating the hidden-charm (doubly-charmed) tetraquark states with the $J^{PC} = 0^{++}, 1^{++}, 1^{+-}, 1^{--}, 1^{++}$ and $2^{++}$ $(0^{++}, 1^{++}, 1^{+-}$ and $2^{++})$ [10, 11, 12, 13, 14, 19], while the central values are larger than 50%, the pole dominance criterion is matched with very good. As an example, in Fig. 1 we plot the contributions of the vacuum condensates $D(n)$ under the condition of the central values of all the input parameters for the [uc]$_{A} [dc]_{V} - [uc]_{V} [dc]_{A}$, [uc]$_{P} [dc]_{S} + [uc]_{S} [dc]_{P}$ and [uc]$_{T} [dc]_{T} + [uc]_{T} [dc]_{T}$ tetraquark states with the $J^{PC} = 0^{--}$. The figure displays that the dominant contributions come from the perturbative terms, compared with the lower vacuum condensates, the higher vacuum condensates play a minor (or tiny) important role (or have very little effects), especially $|D(10)| \ll 1\%$.

We take all the uncertainties of the input parameters into consideration and acquire the masses and pole residues of the pseudoscalar hidden-charm tetraquark states without strange, with strange and with hidden-strange having the quantum numbers $J^{PC} = 0^{--}$ and $0^{--}$, and we also present them distinctly in Table [2]. From Table [2] we can see distinctly that the modified energy scale formula $\mu = \sqrt{M_{X/Y/Z}^{2} - (2M_{c})^{2}} - k m_{s}(\mu)$ with $k = 0, 1$ or $2$ is strikingly satisfied, where we subtract the small $s$-quark mass approximately to account for the small light flavor $SU(3)$ mass-breaking effects. In calculations, we have added an uncertainty $\delta \mu = \pm 0.1$ GeV to the energy scales $\mu$ to account for the possible uncertainty in determining the effective $c$-quark mass $M_{c}$. In Fig. 2 we plot the predicted masses of the $[uc]_{A} [dc]_{V} - [uc]_{V} [dc]_{A}$ and $[uc]_{A} [dc]_{V} + [uc]_{V} [dc]_{A}$ tetraquark states with the $J^{PC} = 0^{--}$ and $0^{--}$ respectively via the variations of the Borel parameters at much larger ranges than the Borel widows as a typical example. From Fig. 2 and Table [2] we can see distinctly that the values of the tetraquark masses emerge smoothly flat platforms in the Borel windows, which can lead to reliable predictions.

As can be seen distinctly from Table [3] that the lowest mass of the pseudoscalar hidden-charm tetraquark state with the symbolic quark constituents $c\bar{c}u\bar{d}$ is about 4.56 ± 0.08 GeV, which is much larger than the value $4239 \pm 18^{+15}_{-10}$ MeV from the LHCb collaboration [2].

In the dynamical quark model, the lowest masses of the pseudoscalar hidden-charm tetraquark states with the symbolic quark constituents $c\bar{c}u\bar{d}$ are about 4.2 ∼ 4.3 GeV [21], while in the constituent quark models, the lowest masses of the pseudoscalar hidden-charm tetraquark states are about 4.25 GeV [22], 4.55 ∼ 4.60 GeV [23]. In Refs. [21, 22, 23], the authors prefer the explicit P-waves, which lie between the diquark and antidiquark constituents. The present predictions are compatible with the calculations in Ref. [23], but we should bear in mind that the P-waves are implicitly embodied in the negative parity of the diquarks (or antidiquarks) themselves in the present work, which differ from the quark structures in Refs. [21, 22, 23] remarkably.

In Ref. [24], Chen and Zhu study the hidden-charm tetraquark states with the symbolic quark constituents $c\bar{c}u\bar{d}$ in the framework of the QCD sum rules, and obtain the ground state masses 4.55 ± 0.11 GeV for the tetraquark states with the $J^{PC} = 0^{--}$, the masses 4.55 ± 0.11 GeV, 4.67 ± 0.10 GeV, 4.72 ± 0.10 GeV for the tetraquark states with the $J^{PC} = 0^{--}$. The present predictions are consistent with their calculations, again, we should bear in mind that their interpolating currents and schemes in treating the operator product expansion and input parameters at the QCD side differ from the present work remarkably. Any current operator with the same quantum numbers and same quark structure as a Fock state in a hadron couples potentially to this hadron, so we can construct several current operators to interpolate a hadron, or construct a current operator to interpolate several hadrons. The compare between the present work and Ref. [24] is not entirely vague at all.

From Table [3] we can see distinctly that the central values of the masses of the $J^{PC} = 0^{--}$
tetraquark states with the symbolic quark constituents $ucd\bar{c}, ucs\bar{c}, scs\bar{c}$ are about 4.56 ~ 4.58 GeV, 4.61 ~ 4.62 GeV and 4.66 ~ 4.67 GeV, respectively, the central values of the masses of the $J^{PC} = 0^{--}$ tetraquark states with the symbolic quark constituents $ucd\bar{c}, ucs\bar{c}$ and $scs\bar{c}$ are about 4.58 GeV, 4.63 GeV and 4.67 GeV, respectively. We can obtain the conclusion tentatively that the currents $J^{+}_{AV}(x)$, $J^{+}_{PS}(x)$ and $J^{+}_{TT}(x)$ couple potentially to three different pseudoscalar tetraquark states with almost degenerated masses, or to one pseudoscalar tetraquark state with three different Fock components; the currents $J^{+}_{AV}(x)$ and $J^{+}_{PS}(x)$ couple potentially to two different pseudoscalar tetraquark states with almost degenerated masses, or to one pseudoscalar tetraquark state with two different Fock components. As the currents with the same quantum numbers couple potentially to the pseudoscalar tetraquark states with almost degenerated masses, the mixing effects cannot improve the predictions remarkably if only the tetraquark masses are concerned. All in all, we obtain reasonable predictions for the masses of the pseudoscalar tetraquark states without strange, with strange and with hidden-strange, the central values are about 4.56 ~ 4.58 GeV, 4.61 ~ 4.63 GeV and 4.66 ~ 4.67 GeV, respectively.

The following two-body strong decays of the pseudoscalar hidden-charm tetraquark states,

$$
Z_{c}(0^{--}) \rightarrow \chi_{c1}\rho, \eta_{c}\rho, J/\psi a_{1}(1260), J/\psi \pi, D\bar{D}_{0} + h.c., D^{*}\bar{D}_{1} + h.c., D^{*}\bar{D} + h.c.,
$$

$$
Z_{c}(0^{++}) \rightarrow \chi_{c0}\pi, \eta_{c}\eta, J/\psi f_{0}(500), J/\psi \rho, D\bar{D}_{0} + h.c., D^{*}\bar{D}_{1} + h.c., D^{*}\bar{D} + h.c.,
$$

$$
Z_{cs}(0^{--}) \rightarrow \chi_{c1}K^{*}, \eta_{c}K^{*}, J/\psi K, J/\psi K, D_{s}\bar{D}_{0} + h.c., D\bar{D}_{s0} + h.c., D_{s}^{*}\bar{D}_{1} + h.c.,
$$

$$
D^{*}\bar{D}_{1} + h.c., D^{*}\bar{D} + h.c., \bar{D}^{*}\bar{D}_{1} + h.c., \bar{D}^{*}\bar{D} + h.c.,
$$

$$
Z_{cs}(0^{++}) \rightarrow \chi_{c0}K, \eta_{c}K_{0}^{*}(700), J/\psi K^{*}, D_{s}\bar{D}_{0} + h.c., D\bar{D}_{s0} + h.c., D_{s}^{*}\bar{D}_{1} + h.c.,
$$

$$
D^{*}\bar{D}_{1} + h.c., D^{*}\bar{D} + h.c., \bar{D}^{*}\bar{D}_{1} + h.c., \bar{D}^{*}\bar{D} + h.c.,
$$

$$
Z_{c\bar{s}}(0^{--}) \rightarrow \chi_{c1}\phi, \eta_{c}\phi, J/\psi f_{1}, J/\psi \eta, D_{s}\bar{D}_{0} + h.c., D^{*}_{s}\bar{D}_{1} + h.c., D^{*}_{s}\bar{D} + h.c.,
$$

$$
Z_{c\bar{s}}(0^{++}) \rightarrow \chi_{c0}\eta, \eta_{c}f_{0}(980), J/\psi \phi, D_{s}\bar{D}_{0} + h.c., D^{*}_{s}\bar{D}_{1} + h.c., D^{*}_{s}\bar{D} + h.c.,
$$

(17)

can take place through the Okubo-Zweig-Iizuka super-allowed fall-apart mechanism. We can probe those pseudoscalar hidden-charm tetraquark states at the BESIII, LHCb, Belle II, CEPC, FCC, ILC in the future, and confront the present predictions with the experimental data to illustrate the nature of the exotic $X, Y$ and $Z$ states.

In Ref. [43], we assign the $Z^+_c(3900)$ as the diquark-antidiquark type tetraquark state with the quantum numbers $J^{PC} = 1^{+-}$, and investigate the hadronic coupling constants in its two-body strong decays with the QCD sum rules based on the novel analysis, i.e. rigorous current-hadron duality, and obtain satisfactory total width to match to the experimental data. The novel analysis has been successfully applied to study the decay widths of the $X(4140), X(4274), Y(4660)$, $P_c(4312)$, etc [37, 40, 47, 48]. We can extend our previous works to explore the two-body strong decays shown in Eq. (17) with the three-point QCD sum rules based on the rigorous current-hadron duality, and get the branching fractions, which can be confronted with the experimental data in the future to identify the pseudoscalar hidden-charm tetraquark states in more reasonable foundations. We prefer to accomplish the complex and arduous calculations in an independent work.

## 4 Conclusion

In the present work, we take all the color-antitriplet diquark operators, such as the scalar, pseudoscalar, vector, axialvector and tensor diquark operators, as the elementary building blocks to construct the four-quark currents without importing the explicit P-waves to implement the negative-parity, and take account of all the light flavor $SU(3)$ breaking effects, such as the vacuum condensates and quark masses, to investigate the mass spectroscopy of the pseudoscalar hidden-charm tetraquark states without strange, with strange and with hidden-strange in the framework of the QCD sum rules comprehensively as a further extension of our previous works. We obtain the lowest mass $4.56 \pm 0.08$ GeV for the tetraquark state with the symbolic quark constituents $c\bar{c}u\bar{d}$,
Figure 1: The contributions of the vacuum condensates with the central values of the input parameters for the tetraquark states with the $J^{PC} = 0^{-+}$, where the $A-V$, $P-S$ and $T-T$ denote the $[uc]_A[\bar{d}\bar{c}]_V - [uc]_V[\bar{d}\bar{c}]_A$, $[uc]_P[\bar{d}\bar{c}]_S + [uc]_S[\bar{d}\bar{c}]_P$ and $[uc]_T[\bar{d}\bar{c}]_T + [uc]_T[\bar{d}\bar{c}]_T$ tetraquark states, respectively.

Figure 2: The masses of the pseudoscalar tetraquark states $[uc]_A[\bar{d}\bar{c}]_V - [uc]_V[\bar{d}\bar{c}]_A (C = +)$ and $[uc]_A[\bar{d}\bar{c}]_V + [uc]_V[\bar{d}\bar{c}]_A (C = -)$ with variations of the Borel parameters.
| $Z_c$ | $J^{PC}$ | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | $\mu$(GeV) | pole |
|------|--------|-------------|-----------------|------------|------|
| $uc[A|d\bar{c}]_{\bar{V}} - uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{-+}$ | 3.7 - 4.1 | 5.10 ± 0.10 | 2.7 | (42 - 60)% |
| $uc[A|d\bar{c}]_{\bar{V}} - uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{-}$ | 3.7 - 4.1 | 5.10 ± 0.10 | 2.8 | (42 - 60)% |
| $uc[A|d\bar{c}]_{\bar{V}} - uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{+}$ | 3.7 - 4.1 | 5.15 ± 0.10 | 2.7 | (43 - 61)% |
| $uc[A|d\bar{c}]_{\bar{V}} + uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{-}$ | 3.7 - 4.1 | 5.15 ± 0.10 | 2.8 | (43 - 61)% |
| $sc[A|\bar{d}\bar{c}]_{\bar{V}} - sc[V|\bar{d}\bar{c}]_{\bar{A}}$ | 0$^{+}$ | 3.8 - 4.2 | 5.20 ± 0.10 | 2.7 | (42 - 60)% |
| $sc[A|\bar{d}\bar{c}]_{\bar{V}} + sc[V|\bar{d}\bar{c}]_{\bar{A}}$ | 0$^{-}$ | 3.8 - 4.2 | 5.20 ± 0.10 | 2.8 | (43 - 60)% |
| $uc[p|d\bar{c}]_{\bar{S}} + uc[S|d\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 3.7 - 4.1 | 5.10 ± 0.10 | 2.8 | (42 - 60)% |
| $uc[p|d\bar{c}]_{\bar{S}} - uc[S|d\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 3.7 - 4.1 | 5.10 ± 0.10 | 2.8 | (42 - 60)% |
| $uc[p|d\bar{c}]_{\bar{S}} + uc[S|d\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 3.7 - 4.1 | 5.15 ± 0.10 | 2.8 | (43 - 61)% |
| $uc[p|d\bar{c}]_{\bar{S}} - uc[S|d\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 3.7 - 4.1 | 5.15 ± 0.10 | 2.8 | (43 - 61)% |
| $sc[p|\bar{d}\bar{c}]_{\bar{S}} + sc[S|\bar{d}\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 3.8 - 4.2 | 5.20 ± 0.10 | 2.8 | (43 - 61)% |
| $sc[p|\bar{d}\bar{c}]_{\bar{S}} - sc[S|\bar{d}\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 3.8 - 4.2 | 5.20 ± 0.10 | 2.8 | (43 - 61)% |
| $uc[T|d\bar{c}]_{\bar{T}} + uc[T|d\bar{c}]_{\bar{T}}$ | 0$^{-}$ | 3.7 - 4.1 | 5.10 ± 0.10 | 2.7 | (41 - 60)% |
| $sc[T|\bar{d}\bar{c}]_{\bar{T}} + sc[T|\bar{d}\bar{c}]_{\bar{T}}$ | 0$^{-}$ | 3.8 - 4.2 | 5.20 ± 0.10 | 2.7 | (43 - 61)% |

Table 2: The Borel parameters, continuum threshold parameters, energy scales of the QCD spectral densities and pole contributions for the pseudoscalar hidden-charm tetraquark states.

| $Z_c$ | $J^{PC}$ | $M_Z$(GeV) | $\lambda_Z$(GeV$^2$) |
|------|--------|------------|----------------|
| $uc[A|d\bar{c}]_{\bar{V}} - uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{-+}$ | 4.56 ± 0.08 | (1.33 ± 0.18) × 10$^{-1}$ |
| $uc[A|d\bar{c}]_{\bar{V}} - uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{-}$ | 4.58 ± 0.07 | (1.37 ± 0.17) × 10$^{-1}$ |
| $uc[A|d\bar{c}]_{\bar{V}} - uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{+}$ | 4.61 ± 0.08 | (1.41 ± 0.19) × 10$^{-1}$ |
| $uc[A|d\bar{c}]_{\bar{V}} + uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{-}$ | 4.63 ± 0.08 | (1.45 ± 0.19) × 10$^{-1}$ |
| $uc[A|d\bar{c}]_{\bar{V}} + uc[V|d\bar{c}]_{\bar{A}}$ | 0$^{+}$ | 4.66 ± 0.08 | (1.50 ± 0.20) × 10$^{-1}$ |
| $uc[p|d\bar{c}]_{\bar{S}} + uc[S|d\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 4.67 ± 0.08 | (1.53 ± 0.20) × 10$^{-1}$ |
| $uc[p|d\bar{c}]_{\bar{S}} + uc[S|d\bar{c}]_{\bar{P}}$ | 0$^{+}$ | 4.68 ± 0.07 | (6.92 ± 0.86) × 10$^{-2}$ |
| $uc[p|d\bar{c}]_{\bar{S}} + uc[S|d\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 4.68 ± 0.07 | (6.91 ± 0.86) × 10$^{-2}$ |
| $uc[p|d\bar{c}]_{\bar{S}} + uc[S|d\bar{c}]_{\bar{P}}$ | 0$^{+}$ | 4.68 ± 0.07 | (7.30 ± 0.90) × 10$^{-2}$ |
| $sc[p|\bar{d}\bar{c}]_{\bar{S}} + sc[S|\bar{d}\bar{c}]_{\bar{P}}$ | 0$^{-}$ | 4.67 ± 0.08 | (7.73 ± 0.97) × 10$^{-2}$ |
| $sc[p|\bar{d}\bar{c}]_{\bar{S}} + sc[S|\bar{d}\bar{c}]_{\bar{P}}$ | 0$^{+}$ | 4.67 ± 0.08 | (7.73 ± 0.96) × 10$^{-2}$ |
| $uc[T|d\bar{c}]_{\bar{T}} + uc[T|d\bar{c}]_{\bar{T}}$ | 0$^{-}$ | 4.57 ± 0.08 | (4.62 ± 0.61) × 10$^{-1}$ |
| $sc[T|\bar{d}\bar{c}]_{\bar{T}} + sc[T|\bar{d}\bar{c}]_{\bar{T}}$ | 0$^{-}$ | 4.62 ± 0.08 | (4.89 ± 0.63) × 10$^{-1}$ |
| $sc[T|\bar{d}\bar{c}]_{\bar{T}} + sc[T|\bar{d}\bar{c}]_{\bar{T}}$ | 0$^{+}$ | 4.67 ± 0.08 | (5.19 ± 0.67) × 10$^{-1}$ |

Table 3: The masses and pole residues for the ground state pseudoscalar hidden-charm tetraquark states.
which is much larger than the experimental value $4239 \pm 18^{+45}_{-10} \text{MeV}$ from the LHCb collaboration, and the discrepancy does not support assigning the $Z_c(4240)$, which still needs confirmation, to be the pseudoscalar hidden-charm tetraquark state with the symbolic quark constituents $c\bar{c}u\bar{d}$. We can search for those pseudoscalar hidden-charm tetraquark states at the Okubo-Zweig-Iizuka super-allowed two-body strong decays at the BESIII, LHCb, Belle II, CEPC, FCC, ILC in the future, and confront the present predictions with the experimental data to examine reliability of the calculations.

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