The effect of shape and roughness on the maximum induction charge for small particles

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Abstract. Considerable analytical and numerical work has already been done on the charging characteristics of spherical and cylindrical particles. However, the majority of industrial processes involve irregular particles with rough surfaces. In this paper, the relationships between the magnitude of the induction charge and electric field on conductive particles in a uniform electric field as a function of the particle shape and roughness have been investigated. The COMSOL program based on the Finite Element Method was used in the numerical modelling. The results show that in evaluating the value of induction charge for a fixed applied electric field, as particle shape changes care must be taken to ensure that surface fields do not exceed breakdown. With this limitation it is shown that, for a given volume, a smooth sphere will gain more induction charge than either a fibrous or flake shaped particle. However for particles with rough surfaces for some levels of roughness it is possible to obtain a higher charge than an equivalent smooth sphere. These results suggest that the degree of surface roughness may be important in certain coating applications.

1. Introduction
Many industrial processes, such as electrostatic separation, precipitation, and powder coating, rely upon the electrical charging of particles. The charge may be achieved by various means depending upon whether the particles are conductive or insulating [1], [2]. Three common charging techniques are currently found in practical systems: triboelectrification, induction and corona charging. Each of these charging techniques has different advantages and applications.

The main emphasis here is on induction charging. The magnitude of induction charge and force acting on conducting particles in a uniform electric field for some regular shapes have been calculated analytically [2]-[4]. For a particle with a radius \( a \) in a uniform electric field \( E_0 \), the predicted charge is \( 3\pi\varepsilon_0 E_0 a^2 \) for a hemisphere, and \( 6.56\pi\varepsilon_0 E_0 a^2 \) for a sphere. Nevertheless, analytical methods have limitations in addressing the wide spectrum of irregular shapes of particles with rough surfaces encountered in actual electrostatic processes. Considerable work has already been done to numerically calculate the electric field, induction charge and force on particles of different shapes employed in industrial applications. Several electric field distributions produced by different arrangements in roll and plate-type electrostatic separators have been studied by using the Boundary Element Method (BEM) [5], [6]. Research work has been reported on the charging and behavior of spherical particles in electrostatic separators [7], [8] and in coating abrasives [9], [10]. Dascalescu [11] computed the
charge and force acting on conductive cylinders in contact with an electrode in a uniform field generated by parallel plane electrodes by using a charge simulation program. The computations showed that the charges and forces were functions of particle dimensions and particle-to-electrode distance, and that the field intensification caused by these particles was also a function of their charge. Dascalescu et al. [12]-[14] also numerically investigated the electric field, charge and force on conductive cylindrical particles in a non-uniform field generated by the roll-type corona-electrostatic separator, plate-type electrostatic separator and cylinder-plane system.

The aim of this paper is to determine numerically the relationships between the magnitude of induction charge and electric field on conductive particles in a uniform electric field as a function of the particle shape and surface roughness. The COMSOL program, based on the Finite Element Method, is used to solve the problem.

2. Computational Model

The induction charging model assumed that a conducting particle was sitting on a horizontal grounded plate subject to a uniform electric field $E_0$ produced by a parallel metal plate located 0.1m above the ground and connected to a DC voltage $V_0$. Since the configuration is axisymmetric, it was sufficient to simulate only the right half of the domain. The analysis of electric potential and electric field described by the Laplace equation was carried out with the COMSOL program and the resulting surface charge allowed the calculation of the induced charge $Q$ on the particle. The accuracy of the COMSOL model was tested by comparing results for several regular shapes with Felici's analytical formulae [3]. Excellent agreement was obtained. For example, agreement was within 0.3% for a sphere and 1.6% for a semi-ellipsoid having an aspect ratio of 10/1. Following this, the COMSOL program was extended to simulate fibrous and flaky particles modeled by conductive prolate and oblate ellipsoids respectively. In order to investigate the effect of surface roughness and sharpness of the particles, particles with cosine shaped perturbations for which the pitch and amplitude are variable were simulated. For all particles modeled, a fixed volume of 14.1 mm$^3$ was assumed (this is the equivalent volume of a spherical particle of $r = 1.5$ mm.)

2.1. Prolate and oblate ellipsoids

The fibrous and flaky particles were modeled by assuming conducting prolate and oblate ellipsoids with different ratios of long semi-axis $a$ and short semi-axis $b$ (Figure 1).

2.2. Spheres with cosine shaped perturbations on the surface

Rough particles were modeled by assuming the surface of a sphere covered with cosine shaped perturbations. By controlling the number and amplitude of the cosine curves distributed over the sphere the surface sharpness and roughness of the particles was modified correspondingly. This model is shown in Figure 2.
3. Computational results and discussion

3.1. Prolate and oblate ellipsoids

For fibrous particles modeled by prolate ellipsoids, first, the particles were assumed exposed to a uniform applied electric field of $E_0 = 0.5$ MV/m. However, it was found that when the particle becomes sharper, the maximum electric field at the tip can exceed the air breakdown value of approximately 3 MV/m. The calculations were then repeated with the electric field on the particle tip kept so it did exceed 3 MV/m to obtain a more accurate estimate of the maximum charge. The plots of the induction charge and maximum electric field for prolate ellipsoids both in the applied electric field of $E_0 = 0.5$ MV/m and with the constant electric field of $E_{\text{max}} = 3$ MV/m on the particle tip as a function of axes ratio $a/b$ are shown in Figures 3 and 4, respectively. For flaky particles modeled by oblate ellipsoids, the particles were assumed exposed to a higher uniform applied electric field of $E_0 = 0.71$ MV/m to raise the electric field at point $A$ shown in Figure 1(b) and the corresponding electric field at the tip of the sphere is 3 MV/m. The plots of induction charge and electric field at two different points shown in Figure 1(b) for flaky particles modeled by oblate ellipsoids as a function of the ratio $a/b$ are given in Figures 5 and 6, respectively.

For fibrous particles modeled by prolate ellipsoids in the same applied electric field, the induction charge and maximum electric field all increase with the increase of the $a/b$ ratio. However, when the particle becomes sharper, the maximum electric field can exceed the air breakdown value. Therefore, the actual induction charge will be smaller than the values obtained from the simulation results for these cases. For the same fibrous particles modeled by prolate ellipsoids but constrained to have a maximum electric field of 3 MV/m at the particle tip, the induction charge decreases with the increase of the $a/b$ ratio. This indicates that under these conditions a sphere has the maximum charge. However, the actual maximum induction charge of prolate ellipsoids will be somewhat larger than these estimates since Peek’s formula predicts that the corona onset level increases higher than 3 MV/m as the tip radius decreases.

Figure 2. Model of a rough particle in the form of sphere with superimposed cosine curves.

Figure 3. Induction charge $Q$ versus axes ratio $a/b$ for a prolate ellipsoid.

Figure 4. Maximum electric field $E_{\text{max}}$ versus axes ratio $a/b$ for a prolate ellipsoid.
For flaky particles modeled by oblate ellipsoids in the same electric field, the induction charge first decreases then increases with the increase of the $a/b$ ratio; the electric field of point $A$ (Figure 1b) decreases when the $a/b$ ratio increases and finally approaches the value of applied electric field; when the $a/b$ ratio increases, the electric field of point $B$ (Figure 1b) first reduces because of the ground shielding effect then increases because the sharpness of the point increases. It can be seen for flaky particles modeled by oblate ellipsoids, the sphere is the optimum shape for maximum charge.

### 3.2. Spheres with cosine shaped perturbations on the surface

The particle with a rough surface was modeled by assuming the surface of the sphere was covered with cosine shaped perturbations. The amplitude of the cosine curve, $am$, represents the surface roughness of the particles, the number of perturbations, $f$, distributed on the large sphere represents the surface sharpness of the particles. For a single sphere, $f$ and $am$ both equal zero. For each case, the maximum electric field of the particles was kept at the corona onset level by changing the applied voltage and using Peek’s formula to calculate the corona onset level. For a single sphere with radius 1.5 mm, the maximum induction charge $Q$ equals 647.1 pC when the maximum electric field at the tip reaches the breakdown value of approximately 6.6 MV/m. For the four different values of the amplitudes of the cosine curve, the plot of the corona onset level as a function of the surface sharpness of the particles is given in Figure 7; the plot of the maximum induction charge as a function of surface area is shown in Figure 8; and the plot of the maximum induction charge as a function of the surface sharpness of the particles is given in Figure 9. For the four different numbers of perturbations distributed on the large sphere, the plot of the maximum induction charge as a function of the surface roughness of the particles is shown in Figure 10.
The corona onset level increases with the increase of surface sharpness of the particles, and for the same number of perturbations distributed on the large sphere, the corona onset level increases with the increase of amplitude of the cosine curve. Therefore, the induction charge for particles with larger surface sharpness can be increased by raising the applied voltage and for a mixture of the particles with varied surface sharpness, the lowest corona onset level for the particles should be considered to apply the optimum voltage. For the relatively large amplitudes of the cosine curve with the values of $0.5R$ and $0.1R$, the maximum induction charge decreases with the increase of surface area and surface sharpness of the particles starting from the value of sphere, while for the amplitudes of $0.05R$ and $0.01R$, the maximum induction charges first decrease then increase with the increase of surface area and surface sharpness. When the amplitude is the value of $0.01R$, the maximum induction charge surprisingly exceeds the value for spherical particle. It can also be seen that, for relatively small number of the perturbations ($4$ and $8$), the maximum induction charge decreases with the increase of surface roughness of the particles, while for the relatively large numbers of $16$ and $32$, the maximum induction charge first increases then decreases. Therefore, for particles with a certain degree of surface roughness, it is possible to attain a higher charge than on an equivalent smooth sphere.

4. Conclusion
Analytical formulae for computation of induction charge are available only for the particles with some regular geometries. The COMSOL program based on the Finite Element Method proved its ability in addressing a wide spectrum of irregular shapes of particles with rough surfaces encountered in actual electrostatic processes. For fibrous particles and the particles with rough surfaces of a fixed volume in the same applied electric field, when the particle becomes sharper, the maximum electric field can exceed the air breakdown value. Therefore, the actual induction charge will be smaller than the values obtained from the simulation results for these cases. Therefore, in evaluating the value of induction charge for a fixed applied electric field, as particle shape changes care must be taken to ensure that surface fields do not exceed the breakdown limit.

With this limitation it is shown that, for a given volume, a smooth sphere will gain more induction charge than either a fibrous or flake shaped particle. However for particles with rough surfaces for some levels of roughness it is possible to obtain a higher charge than an equivalent smooth sphere. These results suggest that surface roughness may be important in certain coating applications.

Acknowledgment
This work was partially funded by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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