Transverse spin waves in isotropic ferromagnets

V. P. Mineev
Commissariat à l’Energie Atomique, DSM/DRFMC/SPSMS 38054 Grenoble, France and Landau Institute for Theoretical Physics, Chernogolovka, Russia

(Dated: July 9, 2005)

The comparison of transverse spin wave spectra and its attenuation in Heisenberg ferromagnet and in ferromagnetic Fermi liquid as well in polarized Fermi liquid is undertaken. The transverse spin waves frequency in polarized paramagnetic Fermi liquid as well in a Fermi liquid with spontaneous magnetization is found to be proportional to $k^2$ with complex diffusion coefficient such that the damping has a finite value proportional to the scattering rate of quasiparticles at $T = 0$.

This behavior of polarized Fermi liquid contrasts with the behavior of Heisenberg ferromagnet in hydrodynamic regime where the transverse spin wave attenuation appears in terms proportional $k^4$.

The reactive part of diffusion coefficient in paramagnetic state at $T = 0$ proves to be inversely proportional to magnetization whereas in ferromagnetic state it is directly proportional to magnetization. The dissipative part of diffusion coefficient at $T = 0$ in paramagnetic state is polarization independent, whereas in ferromagnetic state it is proportional to square of magnetization. Moreover, the spin wave spectrum in ferromagnetic Fermi liquid proves to be unstable that demonstrates the difficulty of the Fermi liquid description of itinerant ferromagnetism.

I. INTRODUCTION

The significant role in physics belong to the simplest models taking in consideration the essential features of physical phenomena. The particular examples of such kind models in the magnetism theory are the models of isotropic ferromagnets. There are two type of isotropic ferromagnets. First, it is the system of localized moments occupying the sites of some crystal lattice with Heisenberg exchange interaction between them

$$\hat{H}_{Heis} = - \sum_{\langle ij \rangle} J_{ij} S_i S_j. \tag{1}$$

Second, it is Stoner-Hubbard ferromagnet formed of fermi-particles moving over a crystal lattice with local repulsive interaction between the particles with opposite spins

$$\hat{H}_{S-H} = \sum_{\langle ij \rangle, \sigma} t_{ij} a_{i \sigma} ^\dagger a_{j \sigma} + U \sum_i n_i \sum_{\sigma} n_{i \sigma} = \sum_{\langle ij \rangle, \sigma} t_{ij} a_{i \sigma} ^\dagger a_{j \sigma} + \frac{NU}{2} - \frac{2U}{3} \sum_i S_i^2. \tag{2}$$

The latter system can be treated on more phenomenological level as Landau Fermi liquid with short range interaction between quasiparticles with opposite spins. The polarization in the Fermi liquid can be either spontaneous, this is the case due to Pomeranchuk type instability at $1 + F^{a_0} < 0$, or caused by external field or pumping in a paramagnetic state. We shall discuss below both of these possibilities.

It is obvious that the systems described by the Hamiltonians (1) and (2) are isotropic that means the invariance of the Hamiltonians under simultaneous homogenious rotation of all the spins. Unlike this common feature we shall see that the systems of localized and itinerant spins posess quite different properties. A comparison of them it is the goal of this paper. We shall be particularly interested in the transverse spin dynamics and present here results of its phenomenological and microscopic theory approach considerations.

II. TRANSVERSE SPIN WAVES IN HEISENBERG FERROMAGNET

The treatment of transverse spin waves hydrodynamics has been undertaken by several authors (see eqs[1,2]). We rederive it here in somewhat different manner. With this purpose instead of Hamiltonian (1) we shall use a phenomenological expression for the free energy

$$F = F(M) - MH_0 + \frac{a}{2} (\nabla_i M_\alpha)^2. \tag{3}$$
where the minimum of \( F(M) \) gives the equilibrium value of magnetization \( M \) in ferromagnetic state and \( H_0 \) is an external magnetic field. Being interested in the dispersion law of small transverse vibrations of magnetization \( \delta M_\alpha = \epsilon_{\alpha\beta\gamma} \Theta_\beta M_\gamma \), where \( \Theta \) is a vector of infinitesimal rotation lying in the plane perpendicular to \( M \) we rewrite the free energy in terms of these angles

\[
F = F(M) - MH_0 + \frac{a}{2} M^2 (\nabla_i \Theta_\alpha)^2.
\]

Then, by introducing the magnetization current as

\[
J_{i\alpha} = -\frac{\delta F}{\delta \nabla_i \Theta_\alpha} = -a(M \times \nabla_i M)_\alpha, \tag{5}
\]

we obtain the equation of motion of magnetization or the equation of spin density conservation

\[
\frac{\partial M}{\partial t} + \frac{\partial J_i}{\partial x_i} - M \times \gamma H_0 = 0 \tag{6}
\]

known as Landau-Lifshits equation. The simple derivation from eqns (4), (5) results in dispersion law of linear transverse spin waves

\[
\omega = \omega_L + aMk^2. \tag{7}
\]

Here \( \omega_L = \gamma H_0 \) is the Larmor frequency.

So, the reactive part of spin waves dispersion proves to be directly proportional to the magnetization value. This general property of Landau-Lifshits equation is sometimes formulated as resulting of finite domain wall rigidity.

The dissipation can be also taken into consideration. It is only necessary to generalize the spin current expression

\[
J_{i\alpha} = -\frac{\delta F}{\delta \nabla_i \Theta_\alpha} + J_{i\alpha}^{\text{diss}}, \quad J_{i\alpha}^{\text{diss}} = -\frac{\delta R}{\delta \nabla_i \Theta_\alpha}, \tag{8}
\]

where

\[
R = b\epsilon_{\alpha\beta\gamma} M_\alpha \nabla_i \nabla_j \Theta_\beta \nabla_i \nabla_j \Theta_\gamma
\]

is the dissipation function which according to general rules is chosen being quadratic on gradients of spin velocity \( \nabla_i \Theta_\alpha \) (it is variable conjugated to the spin current) and such that dissipative part of the spin current is an odd function in respect of time inversion

\[
J_{i\alpha}^{\text{diss}}(-t) = -J_{i\alpha}^{\text{diss}}(t). \tag{10}
\]

Taking into account the dissipative spin current we obtain from Landau-Lifshits equation the transverse spin waves dispersion law with dissipation

\[
\omega = \omega_L + aMk^2 - ibk^4. \tag{11}
\]

The microscopic calculation gives the value of coefficient \( b \propto (\ln T/k^2)/|M|^3 \) meaning the nonanalytic wave vector dependence of dispersion law.

In conclusion of this Section we stress that all the results found here are valid in hydrodynamic or local equilibrium regime that is under the following condition

\[
aMk^2\tau \ll 1 \tag{12}
\]

### III. TRANSVERSE SPIN WAVES IN POLARIZED FERMI LIQUID

In spin polarized Fermi liquid the equation of spin density conservation is still valid

\[
\frac{\partial M}{\partial t} + \frac{\partial J_i}{\partial x_i} - M \times \gamma H_0 = 0 \tag{13}
\]

but the spin current density has the following form

\[
J_i = -D'\nabla_i M + D'' \hat{m} \times \nabla_i M, \tag{14}
\]
where $\hat{m} = M/M$. Here the second term has the same structure as the reactive part of spin current in Heisenberg ferromagnet [3]. It is time reversal invariant while the first term describes the dissipative and odd in respect of time reversal current. Such a term is absent in Heisenberg ferromagnet but it is always present in spin polarized Fermi liquid even at absolute zero temperature as diffusion current in the solution of two liquids with up and down spins.

For the weakly polarized paramagnetic Fermi liquid the equations [13, 14] has been derived from semiclassical Landau-Silin kinetic equation [7] by A.Leggett [8]. Then the exact form of reactive and dissipative part of diffusion constant has been found in frame of the same approach [9] with general form of two particle collision integral in weakly polarized Fermi liquid with arbitrary relationship between temperature $T$ and polarization $\gamma H$. Finally the expressions applicable both for the description of spin dynamics in paramagnetic Fermi liquid with finite polarization and in a ferromagnetic Fermi liquid with spontaneous polarization was found [10, 11]. These results are confirmed by derivation of transverse spin wave dispersion law in frame of field theoretical methods from the integral equation for the vortex function [10]. It is shown that similar derivation taking into consideration the divergency of static transverse susceptibility also leads to the same attenuating spin wave spectrum.

The dispersion law of the transversal spin waves following from equations (13), (14) is

$$\omega = \omega_L + (D'' - iD')k^2,$$

(15)

where $\omega_L = \gamma H_0$ is the Larmor frequency,

$$D' = \frac{w^2\tau}{3(1 + (C\tau)^2)} \approx \frac{w^2}{3C^2\tau},$$

(16)

is the dissipative part of diffusion coefficient and

$$D'' = C\tau D' \approx \frac{w^2}{3C},$$

(17)

is its reactive part. Here the second approximative values of $D'$ and $D''$ correspond to the limit $C\tau \gg 1$.

The parameter $C$ expresses through the Fermi liquid constants and integral of the shifted in respect each other spin-up and spin-down distributions

$$C = \frac{\hat{m}}{N_0}(F_0^a - \frac{F_1^a}{3}) \int d\tau \Delta n_0(\varepsilon).$$

(18)

The value of relative shift $\gamma H$ in paramagnetic Fermi liquid is determined by the external field, Landau molecular field and in general nonequilibrium case it is also can be created by pumping. In the ferromagnetic Fermi liquid the relative shift exist even in the absence of external field and it is determined by Fermi liquid (exchange) interaction. The current relaxation time is

$$\frac{1}{\tau} = \frac{m^*3}{6(2\pi)^{5/2}}(W_1 + W_2) \left[(2\pi T)^2 + (\gamma H)^2\right].$$

(19)

For a weakly polarized fluid $C = (F_0^a - F_1^a/3)\gamma H$. The expression for $w^2$ depends of state of liquid. One can find it analytically in the case of weak polarization [10]. In a paramagnetic Fermi liquid it is

$$w^2 = v_F^2(1 + F_0^a)(1 + \frac{F_1^a}{3}),$$

(20)

where $v_F$ is the Fermi velocity in unpolarized liquid. In a ferromagnetic Fermi liquid (if an external field is smaller than spontaneous) it is

$$w^2 = -v_F^2(1 + \frac{F_1^a}{3}) \left(\frac{\gamma H}{4\mu}\right)^2.$$

(21)

Thus, the reactive part of diffusion coefficient in paramagnetic state at $T = 0$ proves to be inversely proportional to magnetization

$$D'' = \frac{v_F^2(1 + F_0^a)(1 + F_1^a/3)}{3(F_0^a - F_1^a/3)\gamma H},$$

(22)

whereas in ferromagnetic state it is directly proportional to magnetization

$$D'' = \frac{v_F^2\gamma H}{3(4\varepsilon_F)^2}.$$

(23)
The latter is in exact correspondence with known result obtained in frame of Stoner-Hubbard model\textsuperscript{11}.

The dissipative part of diffusion coefficient given by eqn (16) at \( T = 0 \) in paramagnetic state is polarization independent, whereas in ferromagnetic state it is proportional to the square of magnetization. More important, however, that imaginary part of dispersion law in Fermi liquid with spontaneous magnetization proves to be positive. This means the instability of Fermi liquid with spontaneous magnetization. This conclusion is obtained in frame of linear approximation, another words, for the infinitesimally small transversal deviations of magnetization. Does the instability disappear in nonlinear theory or it is principal lack of itinerant ferromagnetism description in frame of polarized Fermi liquid theory? This is an open problem.

So, the transverse spin waves frequency in polarized paramagnetic Fermi liquid as well in a Fermi liquid with spontaneous magnetization is found to be proportional to \( k^2 \) with complex diffusion coefficient such that the damping at \( C\tau \gg 1 \) has a finite value proportional to the scattering rate of quasiparticles at \( T = 0 \). As it was pointed out in\textsuperscript{8} the latter is in formal analogy with ultrasound attenuation in collisionless regime. It is worth noting, however, that in neglect of processes of longitudinal relaxation the parameter \( \gamma H \tau \) has no relation to the local equilibrium establishment.

The results (15)-(23) are valid both in hydrodynamic \( Dk^2\tau \ll 1 \) and in collisionless regime \( Dk^2\tau \gg 1 \) so long

\[ Dk^2 \ll \gamma H \]  

(24)

that is the condition of two moment approximation for the solution of the kinetic equation\textsuperscript{8}. This behavior of polarized Fermi liquid contrasts with the behavior of Heisenberg ferromagnet in hydrodynamic regime where the transverse spin wave attenuation appears in terms proportional \( k^4 \).

\section*{IV. CONCLUSION}

We discussed the transverse spin waves dispersion in two types of isotropic ferromagnetic systems: Heisenberg localized ferromagnet and itinerant polarized Fermi liquid. In contrast with the Heisenberg ferromagnet, where spin wave attenuation appears in terms proportional to the wave vector in the fourth power, the spin polarized Fermi liquid has attenuation already in quadratic in the wave vector terms. At the phenomenological level this difference originates from the diffusive current which exists in the mixture of two spin-up and spin-down Fermi liquids even at zero temperature.

Unlike paramagnetic polarized Fermi liquid the spectrum of transverse spin waves in ferromagnetic Fermi liquid demonstrates the inherent instability pointing out on troubles of pure Fermi liquid description of itinerant ferromagnetism.

\begin{thebibliography}{9}
\bibitem{1} B.I.Halperin and P.C.Hohenberg, Phys.Rev. \textbf{188}, 898 (1969).
\bibitem{2} T.C.Lubensky, Ann. Phys \textbf{64}, 424 (1971).
\bibitem{3} D.Forster, \textit{Hydrodynamic fluctuations, Broken Symmetry, and Correlation functions}, W.A. Benjamin, Inc., London-Tokyo, 1975.
\bibitem{4} L.Landau and E.Lifshits, Physik. Z. Sowjetunion \textbf{8}, 153 (1935).
\bibitem{5} V.N.Kashcheev and M.A.Krivoglaz, Fiz.Tverd.Tela \textbf{3}, 1541 (1961) [Soviet Phys.-Solid State \textbf{3}, 1117 (1961)].
\bibitem{6} A.B.Harris, Phys.Rev. \textbf{175}, 674 (1968).
\bibitem{7} V.P.Silin, Zh. Eksp.Teor.Fiz. \textbf{33}, 1227 (1957) [Sov. Phys.JETP \textbf{6}, 945 (1958)]
\bibitem{8} A.J.Leggett, J.Phys.C \textbf{3}, 448 (1970).
\bibitem{9} V.P.Mineev, Phys. Rev. B \textbf{69}, 144429 (2004).
\bibitem{10} V.P.Mineev, cond-mat/0507675, Phys. Rev. B to be published (2005).
\bibitem{11} T. Moriya “Spin fluctuations in itinerant electron magnetism”, Springer-Verlag, Berlin, 1985.
\end{thebibliography}