Addressing Catastrophic Forgetting in Few-Shot Problems

Pauching Yap¹ Hippolyt Ritter¹ David Barber¹²

Abstract
Neural networks are known to suffer from catastrophic forgetting when trained on sequential datasets. While there have been numerous attempts to solve this problem in large-scale supervised classification, little has been done to overcome catastrophic forgetting in few-shot classification problems. We demonstrate that the popular gradient-based model-agnostic meta-learning algorithm (MAML) indeed suffers from catastrophic forgetting and introduce a Bayesian online meta-learning framework that tackles this problem. Our framework utilises Bayesian online learning and meta-learning along with Laplace approximation and variational inference to overcome catastrophic forgetting in few-shot classification problems. The experimental evaluations demonstrate that our framework can effectively achieve this goal in comparison with various baselines. As an additional utility, we also demonstrate empirically that our framework is capable of meta-learning on sequentially arriving few-shot tasks from a stationary task distribution.

1. Introduction
Few-shot classification (Miller et al., 2000; Li et al., 2004; Lake et al., 2011) focuses on learning to adapt to unseen classes (known as novel classes) with very few labelled examples from each class. Recent works show that meta-learning provides promising approaches to few-shot classification problems (Santoro et al., 2016; Finn et al., 2017; Ravi & Larochelle, 2017). Meta-learning or learning-to-learn (Schmidhuber, 1987; Thrun & Pratt, 1998) takes the learning process a level deeper – instead of learning from the labelled examples in the training classes (known as base classes), meta-learning learns the example-learning process. The training process in meta-learning that utilises the base classes is called the meta-training stage, and the evaluation process that reports the few-shot performance on the novel classes is known as the meta-evaluation stage.

Despite being a promising solution to few-shot classification problems, meta-learning methods suffer from a limitation where a meta-learned model loses its few-shot classification ability on previous datasets as new ones arrive subsequently for meta-training. Some popular examples of the few-shot classification datasets are Omniglot (Lake et al., 2011), CIFAR-FS (Bertinetto et al., 2019) and miniImageNet (Vinyals et al., 2016). A meta-learned model is restricted to perform few-shot classification on a specific dataset, in the sense that the base and novel classes have to originate from the same dataset distribution. The current practice to few-shot classify the novel classes from different datasets is to meta-learn a model for each dataset separately (Snell et al., 2017; Vinyals et al., 2016; Bertinetto et al., 2019). This paper considers meta-learning a single model for few-shot classification on multiple datasets with evident distributional shift that arrive sequentially for meta-training. Figure 1 gives an example of the sequential few-shot classification setting of concern.

Figure 1. An example of the sequential few-shot classification problems with evident dataset distributional shift: Omniglot → CIFAR-FS → miniImageNet.

We introduce a recursive framework to train a model that is applicable to a broader scope of few-shot classification datasets by overcoming catastrophic forgetting. Bayesian online learning (BOL) (Opper, 1998) provides a principled framework for the posterior of the model parameters, while model-agnostic meta-learning (MAML) (Finn et al., 2017) finds a good model parameter initialisation (called meta-parameters) that can quickly few-shot adapt to novel classes. Our framework incorporates BOL and meta-learning to give a recursive formula for the posterior of the meta-parameters as new few-shot datasets arrive. Taking a MAP estimate in implementation leads to Laplace approx-
imation, whereas using a KL-divergence leads to variational inference. Our work builds on Ritter et al. (2018a) that combine BOL and Laplace approximation, and Nguyen et al. (2018) that use variational inference with BOL to prevent forgetting in large-scale supervised classification.

Advantage of our framework: An important reason to employ BOL over non-Bayesian approaches such as regret-based methods in an online setting is that BOL provides a grounded framework that suggests using the previous posterior as the prior recursively. BOL implicitly keeps a memory on previous knowledge via the posterior, in contrast to recent online meta-learning methods that explicitly accumulate previous data in a task buffer (Finn et al., 2019; Zhuang et al., 2019). Explicitly keeping a memory on previous data often triggers an important question: how should the carried-forward data be processed in future rounds, in order to accumulate knowledge? Finn et al. (2019) update the meta-parameters at each iteration using data sampled from the accumulated task buffer. This defeats the purpose of online learning, which by definition means to update the parameters each round using only the new data encountered.

Disadvantage of memorising past data: Having to retrain on previous data to avoid forgetting also increases the training time as the data accumulate (Finn et al., 2019; He et al., 2019). Certainly one can clamp the amount of data at some maximal limit and sample from the buffer, but the final performance of such an algorithm would be dependent on the samples being informative and of good quality which may vary across different seed runs. In contrast to memorising the datasets, having an implicit memory via the posterior automatically deals with the question on how to memorising the datasets, having an implicit memory via the posterior automatically deals with the question on how to accumulate previous data in a task buffer. This defeats the purpose of online learning, which by definition means to update the parameters each round using only the new data encountered.

Below are the contributions we make in this paper:

- We develop the Bayesian online meta-learning (BOML) framework for sequential few-shot classification problems. Under this framework we introduce the algorithms Bayesian online meta-learning with Laplace approximation (BOMLA) and Bayesian online meta-learning with variational inference (BOMVI).

- We propose an approximation to the Fisher corresponding to BOMLA that carries the desirable block-diagonal Kronecker-factored structure.

- We demonstrate that BOML can overcome catastrophic forgetting in the sequential few-shot datasets setting with apparent distributional shift in the datasets.

- We demonstrate empirically that BOML can also continually learn to few-shot classify the novel classes in the sequential meta-training few-shot tasks setting.

2. Meta-Learning

Most meta-learning algorithms comprise an inner loop for example-learning and an outer loop that learns the example-learning process. Such algorithms often require sampling a meta-batch of tasks at each iteration, where a task from a stationary task distribution \( p(T) \) is formed by sampling a subset of classes from the pool of base classes or novel classes during meta-training or meta-evaluation respectively. The \( N \)-way \( K \)-shot task, for instance, refers to sampling \( N \) classes and using \( K \) examples per class for few-shot quick adaptation.

An offline meta-learning algorithm learns a model only for a specific dataset \( \mathcal{D} \), which is divided into the set of base classes \( \mathcal{D}_b \) and novel classes \( \mathcal{D}_n \) for meta-training and meta-evaluation respectively. Upon completing meta-training on \( \mathcal{D} \), the goal is to perform well on an unseen task \( \mathcal{D}^* \) sampled from the novel set \( \mathcal{D}_n \) after a quick adaptation on a small subset \( \mathcal{D}^{\ast+}\mathcal{S} \) (known as the support set) of \( \mathcal{D}^* \). The performance of this unseen task is evaluated on the query set \( \mathcal{D}^{\ast+}\mathcal{Q} \), where \( \mathcal{D}^{\ast+}\mathcal{Q} = \mathcal{D}^* \setminus \mathcal{D}^{\ast+}\mathcal{S} \). Since \( \mathcal{D} \) is not accessible during meta-training, this support-query split is mimicked on the base set \( \mathcal{D} \) for meta-training.

Model-agnostic meta-learning: Each updating step of the well-known meta-learning algorithm MAML (Finn et al., 2017) aims to improve the ability of the meta-parameters to act as a good model initialisation for a quick adaptation on unseen tasks. Each iteration of the MAML algorithm samples \( M \) tasks from the base class set \( \mathcal{D} \) and runs a few steps of stochastic gradient descent (SGD) for an inner loop task-specific learning. The number of tasks sampled per iteration is known as the meta-batch size. For task \( m \), the inner loop outputs the task-specific parameters \( \hat{\theta}^m \) from a \( k \)-step SGD quick adaptation on the objective \( \mathcal{L}(\theta, \mathcal{D}^{m+}\mathcal{S}) \) with the support set \( \mathcal{D}^{m+}\mathcal{S} \) and initialised at \( \theta \):

\[
\hat{\theta}^m = \text{SGD}_k(\mathcal{L}(\theta, \mathcal{D}^{m+}\mathcal{S})),
\]

where \( m = 1, \ldots, M \). The outer loop gathers all task-specific adaptations to update the meta-parameters \( \theta \) using the loss \( \mathcal{L}(\hat{\theta}^m, \mathcal{D}^{m+}\mathcal{Q}) \) on the query set \( \mathcal{D}^{m+}\mathcal{Q} \).

The overall MAML optimisation objective is

\[
\arg\min_\theta \frac{1}{M} \sum_{m=1}^{M} \mathcal{L}(\text{SGD}_k(\mathcal{L}(\theta, \mathcal{D}^{m+}\mathcal{S})), \mathcal{D}^{m+}\mathcal{Q}).
\]

Like most offline meta-learning algorithms, MAML assumes a stationary task distribution during meta-training and meta-evaluation. Under this assumption, a meta-learned model is only applicable to a specific dataset distribution. When the model encounters a sequence of datasets with apparent distributional shift, it loses the few-shot classification ability on previous datasets as new ones arrive for
meta-training. Our work aims to meta-learn a single model for few-shot classification on multiple datasets that arrive sequentially for meta-training. We achieve this goal by incorporating meta-learning into the BOL framework to give the Bayesian online meta-learning (BOML) framework that considers the posterior of the meta-parameters.

3. Bayesian Online Meta-Learning Framework Overview

Our central contribution is to extend the benefits of meta-learning to the BOL scenario, thereby training models that can generalise across tasks whilst dealing with parameter uncertainty in the setting of sequentially arriving datasets.

In this setting, meta-training occurs sequentially on the datasets $\mathcal{D}_1, \ldots, \mathcal{D}_T$. Each dataset $\mathcal{D}_i$ can be seen as a knowledge domain with an associated underlying task distribution $p(T_i)$. A newly-arrived $\mathcal{D}_{t+1}$ is separated into the base class set $\mathcal{D}_{t+1}^S$ and novel class set $\mathcal{D}_{t+1}^Q$ for meta-training and meta-evaluation respectively, where the tasks in these two stages are drawn from the task distribution $p(T_{t+1})$. Notationally, let $\mathcal{D}_{t+1}^S$ and $\mathcal{D}_{t+1}^Q$ denote the collection of support sets and query sets respectively from $\mathcal{D}_{t+1}$, so that $\mathcal{D}_{t+1} = \mathcal{D}_{t+1}^S \cup \mathcal{D}_{t+1}^Q$. Using Bayes’ rule on the posterior gives the recursive formula

$$p(\theta|\mathcal{D}_{t+1}) = \frac{p(\mathcal{D}_{t+1}^S|\theta, \mathcal{D}_{t+1}^Q)p(\theta|\mathcal{D}_{t+1})}{\int p(\mathcal{D}_{t+1}^Q|\tilde{\theta})p(\tilde{\theta}|\mathcal{D}_{t+1}^S)p(\mathcal{D}_{t+1}^Q|\theta)p(\theta|\mathcal{D}_{t+1})d\tilde{\theta}}$$

where Eq. (3) follows from the assumption that each dataset is independent given $\theta$. Figure 2 illustrates the BOML process flow for meta-training and meta-evaluation as datasets arrive sequentially.

From the meta-learning perspective, the parameters $\tilde{\theta}$ introduced in Eq. (5) can be viewed as the task-specific parameters in MAML. There are various choices for the distribution $p(\theta/\mathcal{D}_{t+1})$ in Eq. (5). In particular if we choose to set it as the deterministic function of taking several steps of SGD on loss $\mathcal{L}$ with the support set collection $\mathcal{D}_{t+1}^S$ and initialised at $\theta$, we have

$$p(\theta|\mathcal{D}_{t+1}) = \delta(\theta - SGD_k(\mathcal{L}(\theta, \mathcal{D}_{t+1}^S)))$$

where $\delta(\cdot)$ is the Dirac delta function.

The posterior in Eq. (5) is typically intractable for modern neural network architectures. This leads to the requirement for a good approximate posterior. This section demonstrates how we arrive at the algorithms Bayesian online meta-learning with Laplace approximation (BOMLA) and Bayesian online meta-learning with variational inference (BOMVI) by implementing Laplace approximation and variational continual learning (VCL) respectively to the BOML posterior in Eq. (5). We give a mini tutorial in Appendix A on BOL, VCL and Laplace approximation.

4. Implementation

The posterior in Eq. (5) is typically intractable for modern neural network architectures. This leads to the requirement for a good approximate posterior. This section demonstrates how we arrive at the algorithms Bayesian online meta-learning with Laplace approximation (BOMLA) and Bayesian online meta-learning with variational inference (BOMVI) by implementing Laplace approximation and variational continual learning (VCL) respectively to the BOML posterior in Eq. (5). We give a mini tutorial in Appendix A on BOL, VCL and Laplace approximation.

4.1. BomLA

As described in Appendix A.2, Laplace approximation justifies the use of a Gaussian approximate posterior by Taylor expanding the log-posterior around a mode up to the second order. The second order term corresponds to the log-probability of a Gaussian distribution. The BOML framework in Eq. (5) with a Gaussian approximate posterior $q$ of mean and precision $\phi_t = \{\mu_t, \Lambda_t\}$ from the Laplace approximation gives a MAP estimate:

$$\theta^* = \arg\max_{\theta} \{ \log \bar{p}_\theta + \log p(\mathcal{D}_{t+1}^S|\theta) + r_\theta \}$$

where

$$\bar{p}_\theta = \int p(\mathcal{D}_{t+1}^Q|\tilde{\theta})p(\tilde{\theta}|\theta, \mathcal{D}_{t+1}^S)d\tilde{\theta},$$

$$r_\theta = -\frac{1}{2}(\theta - \mu_t)^T\Lambda_t(\theta - \mu_t).$$
For an efficient optimisation, we use the deterministic \( \tilde{\theta} \) in Eq. (6) which leads to minimising the objective

\[
\tilde{f}_{t+1}^{\text{BOMLA}}(\theta, \mu_t, \Lambda_t) = \tilde{f}_\theta^{(1)} + \tilde{f}_\theta^{(2)} - \tau_\theta, \tag{8}
\]

where

\[
\tilde{f}_\theta^{(1)} = -\frac{1}{M} \sum_{m=1}^{M} \log p(D_t^{m,Q} | \tilde{\theta}^m),
\]

\[
\tilde{f}_\theta^{(2)} = -\frac{1}{M} \sum_{m=1}^{M} \log p(D_t^{m,S} | \theta),
\]

with \( \tilde{\theta}^m = SGD_k(\mathcal{L}(\theta, D_t^{m,S})) \) for \( m = 1, \ldots, M \) and \( M \) denotes the number of tasks sampled per iteration. The first term \( \tilde{f}_\theta^{(1)} \) in Eq. (8) corresponds to the MAML objective in Eq. (2) with a cross-entropy loss, the second term \( \tilde{f}_\theta^{(2)} \) can be viewed as the pre-adaptation loss on the support set and the last term \( \tau_\theta \) can be seen as a regulariser.

We discover that the Laplace approximation method provides a well-fitted meta-training framework for BOMLA in Eq. (5). Each updating step in the approximation procedure can be modified to correspond to the meta-parameters for few-shot classification, instead of the model parameters for large-scale supervised classification.

4.2. Hessian approximation

We calculate a block-diagonal Kronecker-factored Hessian approximation in order to update the precision \( \Lambda_t \), as explained in Appendix A.3.

The Hessian matrix corresponding to the first term of the BOMLA objective in Eq. (8) is

\[
\tilde{H}_{t+1}^{ij} = \frac{1}{M} \sum_{m=1}^{M} \left. \frac{\partial^2}{\partial \theta(i) \partial \theta(j)} \log p(D_t^{m,Q} | \tilde{\theta}^m) \right|_{\theta = \mu_{t+1}}. \tag{9}
\]

It is worth noting that the BOMLA Hessian deviates from the original Laplace approximation Hessian in Appendix A.2, and it is necessary to derive an adjusted approximation to the Hessian with some further assumptions.

The Hessian for a single data point can be approximated using the Fisher information matrix \( F \) to ensure its positive semi-definiteness (Martens & Grosse, 2015):

\[
F = \mathbb{E}_{x,y} \left[ \frac{d}{d\theta} \log p(y|x, \theta) \frac{d}{d\theta} \log p(y|x, \theta)^T \right]. \tag{10}
\]

Each \( (x, y) \) pair for the Fisher in BOMLA is associated to a task \( m \). The Fisher information matrix \( F \) corresponding to the BOMLA Hessian in Eq. (9) for a single data point is

\[
\tilde{F} = \frac{1}{M} \sum_{m=1}^{M} \mathbb{E}_{x,y} \left[ \left( \frac{\partial \tilde{\theta}^m}{\partial \theta} \right) \frac{d}{d\tilde{\theta}^m} \log p(y|x, \tilde{\theta}^m) \right. 
\]

\[
\times \left. \frac{d}{d\tilde{\theta}^m} \log p(y|x, \tilde{\theta}^m)^T \left( \frac{\partial \tilde{\theta}^m}{\partial \theta} \right)^T \right]. \tag{11}
\]

The additional Jacobian matrix \( \frac{\partial \tilde{\theta}^m}{\partial \theta} \) breaks the Kronecker-factored structure described by Martens & Grosse (2015) for the original Fisher in Eq. (10).

The results in Finn et al. (2017) show that the first step of the quick adaptation in \( \tilde{\theta}^m \) contributes the largest change to the meta-evaluation objective, and the remaining adaptation steps give a relatively small change to the objective. We can reasonably assume a one-step SGD quick adaptation \( \tilde{\theta}^m = \theta - \nabla_\theta \mathcal{L}(\theta, D_t^{m,S}) \) to approximate the Fisher, although in other parts of the implementation we use a few-step SGD. By imposing this assumption, the \((i, j)\)-th entry of the Jacobian term can be interpreted as

\[
\left( \frac{\partial \tilde{\theta}^m}{\partial \theta} \right)^{ij} = I^{ij} - \frac{\partial^2}{\partial \theta(i) \partial \theta(j)} \log p(D_t^{m,S} | \theta), \tag{12}
\]

where \( I \) is the corresponding identity matrix and the objective \( \mathcal{L} \) involved is the negative log-likelihood. The Hessian for a single data point in the second term of Eq. (12) can be approximated by \( F \) in Eq. (10) via the usual block-diagonal Kronecker-factored approximation. Putting the Jacobian back into Eq. (11) and expanding the factors give terms that multiply two or more Kronecker products together. The detailed derivation of \( F \) is explained in Appendix A.3.1. We introduce the posterior-regularising hyperparameter \( \lambda \) when updating the precision: \( \Lambda_{t+1} = \lambda H_{t+1} + \Lambda_t \) and the rationale for introducing \( \lambda \) is explained in Appendix A.3.2. The pseudo-code of the BOMLA algorithm can be found in Appendix B.1.

4.3. BOMVI

The VCL framework (Nguyen et al., 2018) is directly applicable to BOML. This section demonstrates how we arrive at the BOMVI algorithm by implementing VCL to the posterior of the BOML framework in Eq. (5).

As described in Appendix A.4, VCL approximates the posterior by minimising the KL-divergence over some pre-determined approximate posterior family \( \mathcal{Q} \). Fitting the BOML posterior in Eq. (5) into the VCL framework gives the approximate posterior:

\[
q(\theta | \phi_{t+1}) = \arg \min_{q \in \mathcal{Q}} D_{KL}(q(\theta | \phi) || \tilde{q}_\phi), \tag{13}
\]

where \( \tilde{q}_\phi = \int p(D_t^{Q} | \tilde{\theta}) p(\tilde{\theta} | \theta) D_t^{S} d\tilde{\theta} p(D_t^{S} | \theta) q(\theta | \phi_{t}) \). Similar to BOMLA, we use the deterministic \( \tilde{\theta} \) in Eq. (6).
This leads to minimising the objective
\[
\mathcal{L}_{t+1}^{\text{OML}}(\phi, \phi_t) = \hat{f}_{\phi}^{(1)} + \hat{f}_{\phi}^{(2)} + \hat{r}_{\phi},
\]
where
\[
\hat{f}_{\phi}^{(1)} = -\frac{1}{M} \sum_{m=1}^{M} \mathbb{E}_{q(\theta|\phi)} \left[ \log p(D_{t+1}^{m}) \right],
\]
\[
\hat{f}_{\phi}^{(2)} = -\frac{1}{M} \sum_{m=1}^{M} \mathbb{E}_{q(\theta|\phi)} \left[ \log p(D_{t+1}^{m}|\theta) \right],
\]
\[
\hat{r}_{\phi} = D_{KL}(q(\theta|\phi)||q(\theta|\phi_t)),
\]
with \( \hat{m} = \text{SGD}_{t}(L(\theta, D_{t+1}^{m})) \) for \( m = 1, \ldots, M \) and \( M \) denotes the number of tasks sampled per iteration. We use a Gaussian mean-field \( q(\theta|\phi) = \prod_{d=1}^{D} N(\mu_{t,d}, \sigma_{t,d}^2) \), where \( \phi_t = \{\mu_{t,d}, \sigma_{t,d}\}_{d=1}^{D}, D = \dim(\theta) \) and the objective in Eq. (14) is minimised over \( \phi \). The pseudo-code of the BOMVI algorithm can be found in Appendix B.1.

The term \( \hat{f}_{\phi}^{(1)} \) in Eq. (14) is rather cumbersome to estimate in optimisation. To compute its Monte Carlo estimate, we have to generate samples \( \theta_r \sim q \) for \( r = 1, \ldots, R \), and run a quick adaptation on each sampled meta-parameters \( \theta_r \) before evaluating their log-likelihoods. This is computationally intensive and gives an estimator with large variance. We propose a workaround by modifying the inner loop SGD quick adaptation, and the details can be found in Appendix B.2.

5. Related Work

5.1. Online Meta-Learning

**Regret minimisation:** The goal of this setting is to minimise the regret function, and the assumptions are made on the loss function rather than the task distribution. Recent works Finn et al. (2019) and Zhuang et al. (2019) belong to this category, where the aim is to compete with the best meta-learner and supersede it. These methods accumulate data as they arrive and meta-learn using all data acquired so far. Data accumulation is not desirable as the algorithmic complexity of training grows with the amount of data accumulated, and training time increases as new data arrive (Finn et al., 2019; He et al., 2019). The agent will eventually run out of memory for a long sequence of data. The BOML framework on the other hand is advantageous, as it only takes the current data and the posterior of the meta-parameters into consideration during optimisation. This gives a framework with an algorithmic complexity independent of the length of the dataset sequence.

**Same underlying task distribution:** Sequential tasks are assumed to originate from the same underlying task distribution \( p(T) \) in this setting. Denevi et al. (2019) introduce the online-within-online (OWO) and online-within-batch (OWB) settings, where OWO encounters tasks and examples within tasks sequentially while OWB encounters tasks sequentially but examples within tasks are in batch. Our work in the sequential datasets setting is novel in overcoming few-shot catastrophic forgetting, where the goal is to few-shot classify unseen tasks drawn from a sequence of distributions \( p(T_1), \ldots, p(T_T) \) as explained in Section 3. He et al. (2019), Harrison et al. (2019) and Jerfel et al. (2019) look into continual meta-learning for a non-stationary task distribution where the task boundaries are unknown to the model. Jerfel et al. (2019) consider a latent task structure to adapt to the non-stationary task distribution.

5.2. Offline Meta-Learning

Previous meta-learning works attempt to solve few-shot classification problems in an offline setting, under the assumption of having a stationary task distribution during meta-training and meta-evaluation. A single meta-learned model is aimed to few-shot classify one specific dataset with all base classes of the dataset readily available in a batch for meta-training. There are two general frameworks for the offline meta-learning setting:

**Probabilistic:** The MAML algorithm can be cast into a probabilistic inference problem (Finn et al., 2018) or with a hierarchical Bayesian structure (Grant et al., 2018; Yoon et al., 2018). Yoon et al. (2018) use Stein Variational Gradient Descent (SVGD) for task-specific learning. Gordon et al. (2019) implement probabilistic inference by considering the posterior predictive distribution with amortised networks. Grant et al. (2018) discuss the use of a Laplace approximation in the task-specific inner loop to improve MAML using the curvature information. Although at first sight our work seems similar to Grant et al. (2018) due to the use of Laplace approximation, our work is clearly distinct in terms of goal and context. Grant et al. (2018) use Laplace approximation at the task-specific level, whilst we use Laplace approximation at the meta-level for the meta-parameters approximate posterior. The formulation in Grant et al. (2018) does not accumulate past experience, whereas our work enables few-shot learning on unseen tasks from multiple knowledge domains sequentially.

**Non-probabilistic:** Gradient-based meta-learning (Finn et al., 2017; Nichol et al., 2018; Rusu et al., 2019) updates the meta-parameters by accumulating the gradients of a meta-batch of task-specific inner loop updates. The meta-parameters will be used as a model initialisation for a quick adaptation on the novel tasks. Metric-based meta-learning (Koch et al., 2015; Vinyals et al., 2016; Snell et al., 2017) utilises the metric distance between labelled examples. This method assumes that base and novel classes are from the
same dataset distribution, and the metric distance estimates can be generalised to the novel classes upon meta-learning the base classes.

5.3. Continual Learning

Modern continual learning works (Goodfellow et al., 2013; Lee et al., 2017; Zenke et al., 2017) focus primarily on large-scale supervised learning, in contrast to our work that looks into continual few-shot classification across sequential datasets with evident distributional shift. Wen et al. (2018) utilise few-shot learning to improve on overcoming catastrophic forgetting via logit matching on a small sample from the previous tasks. The online learning element in our work is closely related to recent works that overcome catastrophic forgetting in large-scale supervised classification (Kirkpatrick et al., 2017; Zenke et al., 2017; Ritter et al., 2018a; Nguyen et al., 2018). In particular, our work builds on the online Laplace approximation method (Ritter et al., 2018a). Our work extends this to the meta-learning scenario to avoid forgetting in few-shot classification problems. Nguyen et al. (2018) provide an alternative of using variational inference instead of Laplace approximation for approximating the posterior. Our work utilises this approach to adapt the variational method to approximate the posterior of the meta-parameters by adjusting the KL-divergence objective.

6. Experiments

6.1. N-athlon

We implement BOMLA and BOMVI\(^1\) to the 5-way 1-shot triathlon and pentathlon sequences. The experiment details and the datasets explanations are in Appendix C.1. We compare our algorithms to the following baselines:

**Train-On-Everything (TOE):** When a new dataset arrives for meta-training, we randomly re-initialise the meta-parameters and run MAML meta-training using all datasets encountered so far.

**Sequential MAML:** Upon the arrival of a new dataset, we run MAML to meta-train only on the newly-arrived dataset.

**Follow The Meta-Leader (FTML):** We introduce a slight modification to FTML (Finn et al., 2019) on its evaluation method, as FTML is not designed for few-shot learning on unseen tasks. In our experiments, we apply Update-Procedure in FTML to the data from unseen tasks, rather than the data from the same training task as in the original FTML.

---

\(^1\)Implementation code is available at [https://github.com/pauchingyap/boml](https://github.com/pauchingyap/boml)

6.1.1. TRIATHLON

This experiment considers the few-shot triathlon sequence as in Figure 3.

![Figure 3. The triathlon 5-way 1-shot sequence in this experiment.](image)

The distributional shift from Omniglot to miniQuickDraw is less drastic, compared to the shift from miniQuickDraw to CIFAR-FS. The result in Figure 4 shows that BOMLA and BOMVI are able to prevent catastrophic forgetting in both dataset transitions. BOMLA, in particular, is able to proceed to the miniQuickDraw meta-training phase with almost no forgetting on Omniglot. In other words, the meta-level pattern of Omniglot is retained throughout the meta-training period of miniQuickDraw. There is a small trade-off in the performance of CIFAR-FS as BOMLA and BOMVI avoid catastrophically forgetting Omniglot and miniQuickDraw. Sequential MAML gives a noticeable drop in the performance of Omniglot and miniQuickDraw. TOE is able to retain the few-shot performance as it has access to all previous datasets, whilst FTML gives a mixed performance. We elaborate on the result interpretation, the BOMLA-BOMVI comparison and the choice of \(\lambda\) along with the next experiment pentathlon, which resembles the setting of this experiment except with a more challenging dataset sequence.

![Figure 4. Meta-evaluation accuracy across 3 seed runs on each dataset along meta-training. Higher accuracy values in the off-diagonals indicate less forgetting.](image)
6.1.2. **Pentathlon**

We implement BOMLA and BOMVI to the more challenging pentathlon sequence as in Figure 5.

![Figure 5. The pentathlon 5-way 1-shot sequence in this experiment.](image)

By implementing BOMLA and BOMVI to the pentathlon sequence, we can observe that they are capable of preventing catastrophic forgetting in the pentathlon dataset sequence. TOE is also able to retain the few-shot performance as it has access to all datasets encountered so far. Since TOE learns all datasets from random re-initialisation each time it encounters a new dataset, the meta-training time required to achieve a similarly good meta-evaluation performance is longer compared to other runs. Sequential MAML catastrophically forgets the previously learned datasets but has the best performance on new datasets compared to other runs. FTML gives a mixed performance on different datasets.

TOE and FTML can be memory-intensive as the dataset sequence becomes longer. They take the brute-force approach to prevent forgetting by memorising all datasets. Unlike TOE and FTML, our algorithms BOMLA and BOMVI only take the newly-arrived dataset and the posterior of the meta-parameters into consideration during optimisation. This gives a framework with an algorithmic complexity independent of the length of the dataset sequence.

![Figure 6. Meta-evaluation accuracy across 3 seed runs on each dataset along meta-training. Higher accuracy values indicate better results with less forgetting as we proceed to new datasets. BOMLA with $\lambda = 100$ gives good performance in the off-diagonal plots (retains performances on previously learned datasets), and has a minor performance trade-off in the diagonal plots (learns less well on new datasets). Sequential MAML gives better performance in the diagonal plots (learns well on new datasets) but worse performance in the off-diagonal plots (forgets previously learned datasets). BOMVI is also able to retain performance on previous datasets, although it may be unable to perform as good as BOMLA due to sampling and estimator variance.](image)
**BOMLA-BOMVI comparison:** As shown in Figure 6, BOMLA with appropriate $\lambda$ is superior to BOMVI in the performance. This is due to BOMLA having a better posterior approximation than BOMVI. Whilst BOMLA has a Gaussian approximate posterior with block-diagonal precision, BOMVI uses a Gaussian mean-field approximate posterior. Trippe & Turner (2017) compare the performances of variational inference with different covariance structures, and discover that variational inference with block-diagonal covariance performs worse than mean-field approximation. This is because the block-diagonal covariance in variational inference prohibits variance reduction methods such as local reparameterisation trick for Monte Carlo estimation. The variance of the Monte Carlo estimate has been proven problematic (Kingma et al., 2015; Trippe & Turner, 2017). We address this issue in Section 4.3 and Appendix B.2 specifically to the meta-learning setting by modifying the inner loop quick adaptation. We analyse the change in the approximate posterior covariance in Appendix C.2, as meta-training occurs sequentially on datasets from different knowledge domains.

**Choosing $\lambda$:** Tuning the posterior regulariser $\lambda$ mentioned in Section 4.2 and Appendix A.3.2 corresponds to balancing between a smaller performance trade-off on a new dataset and less forgetting on previous datasets. We compare BOMLA with different $\lambda$ values and BOMVI in Appendix C.3. A larger $\lambda = 1000$ results in a more concentrated Gaussian posterior and is therefore unable to learn new datasets well, but can better retain the performances on previous datasets. A smaller $\lambda = 1$ on the other hand gives a widespread Gaussian posterior and learns better on new datasets by sacrificing the performance on the previous datasets. In both triathlon and pentathlon experiments, the value $\lambda = 100$ gives the best balance between old and new datasets. Ideally we seek for a good performance on both old and new datasets, but in reality there is a trade-off between retaining performance on old datasets and learning well on new datasets due to posterior approximation errors.

### 6.2. Omniglot: Stationary Task Distribution

![Omniglot](image)

*Figure 7.* An example of the Omniglot task sequence for meta-training in this experiment.

In this experiment we demonstrate empirically that BOML can also continually learn to few-shot classify the novel classes in the sequential meta-training few-shot tasks setting, where all tasks originate from a stationary task distribution. This setting only involves one dataset $\mathcal{D}$ with an associated underlying task distribution $p(T)$, where $\mathcal{D}$ is separated into the base and novel class sets. In this setting, $\mathcal{D}_1, \ldots, \mathcal{D}_{t+1}$ denote the non-overlapping tasks formed from the base class set and they arrive sequentially for meta-training. We show the corresponding modifications of BOMLA and BOMVI under this setting in Appendix B.1 Algorithms 1 and 2.

We run the sequential tasks experiment on the Omniglot dataset. To increase the difficulty level, we split the dataset based on the alphabets (super-classes) instead of the characters (classes) as in Figure 7. The goal of this experiment is to classify the 5-way 5-shot novel tasks sampled from the meta-evaluation alphabets. The experimental details and the alphabet splits can be found in Appendix C.4. We compare our algorithms to the baselines TOE, Sequential MAML and FTML similar to the $N$-athlon experiments but in the sequential tasks setting.

As the tasks arrive sequentially for meta-training, Figure 8 shows that BOMLA and BOMVI can accumulate few-shot classification ability on the novel tasks over time. The knowledge acquired from previous meta-training tasks is carried forward in the form of a posterior, which is then used as a prior when a new task arrives for meta-training. Despite having access to all previous tasks, TOE shows no positive forward transfer in the meta-evaluation accuracy each time it encounters a new task. FTML and sequential MAML are inferior to BOMLA and BOMVI in the performance. BOMLA with $\lambda = 0.01$ gives the best performance in this experiment.

### 6.3. Discussion on Baselines

Finn et al. (2019) discover that TOE does not explicitly learn the structure across tasks, thus unable to fully utilise the data. The TOE performance in our Omniglot experiment is coherent with the TOE result in Finn et al. (2019). The result figures in Finn et al. (2019) show a TOE result similar to ours in the Omniglot experiment. In contrast, TOE in the $N$-athlon experiments performs well as it has access to drastically more data points than TOE in the Omniglot experiment, and samples numerous tasks from all previous datasets.

Sequential MAML in the $N$-athlon experiments suffers from catastrophic forgetting due to the apparent distributional shift in the datasets. The Omniglot experiment, on the other hand, has tasks originating from the same underlying distribution. As a result sequential MAML in this setting is able to accumulate few-shot ability, although it performs worse than BOMLA and BOMVI as shown in Figure 8 since there is only one task available at a time.
Since the original FTML is not aimed for unseen few-shot tasks and does not deal with sequential datasets setting as in the $N$-athlon experiments, we have to modify FTML as described in Section 6.1. Sampling from previous tasks in the buffer is a key feature of the FTML algorithm. Certainly one can sample many tasks from the buffer to achieve perfect memory in the $N$-athlon experiments, but such a baseline setup has been taken into consideration by TOE. Therefore we choose to retain the online characteristic of the original FTML in our modified implementation.

7. Conclusion

We introduced the Bayesian online meta-learning (BOML) framework with two algorithms: BOMLA and BOMVI. Our framework can overcome catastrophic forgetting in few-shot classification problems on datasets with evident distributional shift. BOML merged the BOL framework and meta-learning via Laplace approximation or variational continual learning. We proposed the necessary adjustments in the Hessian and Fisher approximation for BOMLA, as we optimise the meta-parameters for few-shot classification instead of the usual model parameters in large-scale supervised classification. The experiments show that BOMLA and BOMVI are able to retain the few-shot classification ability when trained on sequential datasets with apparent distributional shift, resulting in the ability to perform few-shot classification on multiple datasets with a single meta-learned model. BOMLA and BOMVI are also able to continually learn to few-shot classify the novel tasks, as the meta-training tasks from a stationary distribution arrive sequentially for learning.

Acknowledgements

We would like to thank the reviewers for their constructive comments, and Peter Hayes for the useful initial discussions.

References

Bertinetto, L., Henriques, J. F., Torr, P., and Vedaldi, A. Meta-Learning with Differentiable Closed-Form Solvers. In International Conference on Learning Representations, 2019.

Botev, A., Ritter, H., and Barber, D. Practical Gauss-Newton Optimisation for Deep Learning. In Proceedings of the 34th International Conference on Machine Learning, 2017.

Denevi, G., Stamos, D., Ciliberto, C., and Pontil, M. Online-Within-Online Meta-Learning. In Advances in Neural Information Processing Systems 32, 2019.

Denker, J. S. and LeCun, Y. Transforming Neural-Net Output Levels to Probability Distributions. In Advances in Neural Information Processing Systems 3, 1991.

Finn, C., Abbeel, P., and Levine, S. Model-Agnostic Meta-Learning for Fast Adaptation of Deep Networks. In Proceedings of the 34th International Conference on Machine Learning, 2017.

Finn, C., Xu, K., and Levine, S. Probabilistic Model-Agnostic Meta-Learning. In Advances in Neural Information Processing Systems 31, 2018.

Finn, C., Rajeswaran, A., Kakade, S., and Levine, S. Online Meta-Learning. In Proceedings of the 36th International Conference on Machine Learning, 2019.

Goodfellow, I. J., Mirza, M., Xiao, D., Courville, A., and Bengio, Y. An Empirical Investigation of Catastrophic
Forgetting in Gradient-Based Neural Networks. arXiv preprint, arXiv:1312.6211, 2013.

Gordon, J., Bronskill, J., Bauer, M., Nowozin, S., and Turner, R. Meta-Learning Probabilistic Inference for Prediction. In International Conference on Learning Representations, 2019.

Grant, E., Finn, C., Levine, S., Darrell, T., and Griffiths, T. Recasting Gradient-Based Meta-Learning as Hierarchical Bayes. In International Conference on Learning Representations, 2018.

Grosse, R. and Martens, J. A Kronecker-Factored Approximate Fisher Matrix for Convolution Layers. In Proceedings of the 33rd International Conference on Machine Learning, 2016.

Ha, D. and Eck, D. A Neural Representation of Sketch Drawings. arXiv preprint, arXiv:1704.03477, 2017.

Harrison, J., Sharma, A., Finn, C., and Pavone, M. Continuous Meta-Learning without Tasks. arXiv preprint, arXiv:1912.08866, 2019.

He, X., Sygnowski, J., Galashov, A., Rusu, A. A., Teh, Y., and Pascanu, R. Task Agnostic Continual Learning via Meta Learning. arXiv preprint, arXiv:1906.05201, 2019.

Jerfel, G., Grant, E., Griffiths, T., and Heller, K. A. Reconciling Meta-Learning and Continual Learning with Online Mixtures of Tasks. In Advances in Neural Information Processing Systems 32, 2019.

Kingma, D. P. and Ba, J. Adam: A Method for Stochastic Optimization. In International Conference on Learning Representations, 2015.

Kingma, D. P., Salimans, T., and Welling, M. Variational Dropout and the Local Reparameterization Trick. In Advances in Neural Information Processing Systems 28, 2015.

Kirkpatrick, J., Pascanu, R., Rabinowitz, N., Veness, J., Desjardins, G., Rusu, A. A., Milan, K., Quan, J., Ramalho, T., Grabska-Barwinska, A., Hassabis, D., Clopath, C., Kumaran, D., and Hadsell, R. Overcoming Catastrophic Forgetting in Neural Networks. Proceedings of the National Academy of Sciences, 2017.

Koch, G., Zemel, R., and Salakhutdinov, R. Siamese Neural Networks for One-Shot Image Recognition. In 32th International Conference on Machine Learning Deep Learning Workshop, 2015.

Lake, B., Salakhutdinov, R., Gross, J., and Tenenbaum, J. One Shot Learning of Simple Visual Concepts. In Proceedings of the 33rd Annual Conference of the Cognitive Science Society, 2011.

Lee, S., Kim, J., Jun, J., Ha, J., and Zhang, B. Overcoming Catastrophic Forgetting by Incremental Moment Matching. In Advances in Neural Information Processing Systems 30, 2017.

Li, F., Fergus, R., and Perona, P. Learning Generative Visual Models from Few Training Examples: An Incremental Bayesian Approach Tested on 101 Object Categories. In Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition Workshops, 2004.

MacKay, D. J. C. A Practical Bayesian Framework for Backpropagation Networks. Neural Computation, 1992.

Maji, S., Rahtu, E., Kannala, J., Blaschko, M., and Vedaldi, A. Fine-Grained Visual Classification of Aircraft. arXiv preprint, arXiv:1306.5151, 2013.

Martens, J. and Grosse, R. Optimizing Neural Networks with Kronecker-Factored Approximate Curvature. In Proceedings of the 32nd International Conference on Machine Learning, 2015.

Miller, E. G., Matsakis, N. E., and Viola, P. A. Learning from One Example Through Shared Densities on Transforms. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 2000.

Nguyen, C. V., Li, Y., Bui, T. D., and Turner, R. E. Variational Continual Learning. In International Conference on Learning Representations, 2018.

Nichol, A., Achiam, J., and Schulman, J. On First-Order Meta-Learning Algorithms. arXiv preprint, arXiv:1803.02999, 2018.

Opper, M. A Bayesian Approach to Online Learning. Cambridge University Press, 1998.

Osawa, K., Tsuji, Y., Ueno, Y., Naruse, A., Foo, C., and Yokota, R. Scalable and Practical Natural Gradient for Large-Scale Deep Learning. IEEE Transactions on Pattern Analysis and Machine Intelligence, 2020.

Ravi, S. and Beatson, A. Amortized Bayesian Meta-Learning. In International Conference on Learning Representations, 2019.

Ravi, S. and Larochelle, H. Optimization as a Model for Few-Shot Learning. In International Conference on Learning Representations, 2017.
Ritter, H., Botev, A., and Barber, D. Online Structured Laplace Approximations for Overcoming Catastrophic Forgetting. In Advances in Neural Information Processing Systems 31, 2018a.

Ritter, H., Botev, A., and Barber, D. A Scalable Laplace Approximation for Neural Networks. In International Conference on Learning Representations, 2018b.

Robbins, H. and Monro, S. A Stochastic Approximation Method. The Annals of Mathematical Statistics, 1951.

Rusu, A. A., Rao, D., Sygnowski, J., Vinyals, O., Pascanu, R., Osindero, S., and Hadsell, R. Meta-Learning with Latent Embedding Optimization. In International Conference on Learning Representations, 2019.

Santoro, A., Bartunov, S., Botvinick, M., Wierstra, D., and Lillicrap, T. Meta-Learning with Memory-Augmented Neural Networks. In Proceedings of the 33rd International Conference on Machine Learning, 2016.

Schmidhuber, J. Evolutionary Principles in Self-Referential Learning. On Learning How to Learn: The Meta-Meta-Meta...-Hook. Diploma thesis, Institut für Informatik, Technische Universität München, 1987.

Snell, J., Swersky, K., and Zemel, R. Prototypical Networks for Few-Shot Learning. In Advances in Neural Information Processing Systems 30, 2017.

Thrun, S. and Pratt, L. Learning to Learn: Introduction and Overview. Springer, Boston, MA, 1998.

Trippe, B. L. and Turner, R. E. Overpruning in Variational Bayesian Neural Networks. In Advances in Neural Information Processing Systems 30 – Advances in Approximate Bayesian Inference Workshop, 2017.

Vinyals, O., Blundell, C., Lillicrap, T., Kavukcuoglu, K., and Wierstra, D. Matching Networks for One Shot Learning. In Advances in Neural Information Processing Systems 29, 2016.

Wen, J., Cao, Y., and Huang, R. Few-Shot Self Reminder to Overcome Catastrophic Forgetting. arXiv preprint, arXiv:1812.00543, 2018.

Yoon, J., Kim, T., Dia, O., Kim, S., Bengio, Y., and Ahn, S. Bayesian Model-Agnostic Meta-Learning. In Advances in Neural Information Processing Systems 31, 2018.

Zenke, F., Poole, B., and Ganguli, S. Continual Learning through Synaptic Intelligence. In Proceedings of the 34th International Conference on Machine Learning, 2017.

Zhuang, Z., Wang, Y., Yu, K., and Lu, S. No-Regret Non-Convex Online Meta-Learning. arXiv preprint, arXiv:1910.10196, 2019.
A. Background

This section provides a background explanation of using BOL to approximate the posterior of the model parameters and overcome catastrophic forgetting, commonly in large-scale supervised classification. We apply this approach to our recursion in Eq. (5).

The posterior is typically intractable due to the enormous size of the modern neural network architectures. This leads to the requirement for a good approximation of the posterior of the model parameters. A particularly suitable candidate for this purpose is the Laplace approximation (MacKay, 1992; Ritter et al., 2018b), as it simply adds a quadratic regulariser to the training objective. Variational continual learning (Nguyen et al., 2018) is another possible method to obtain an approximation for the posterior of the model parameters.

A.1. Bayesian Online Learning

Upon the arrival of the new dataset $D_{t+1}$, we consider the posterior $p(\theta|D_{1:t+1})$ of the parameters $\theta$ of a neural network. Using Bayes’ rule on the posterior gives the recursive formula

$$p(\theta|D_{1:t+1}) \propto p(D_{t+1}|\theta)p(\theta|D_{1:t})$$

where Eq. (15) follows from the assumption that each dataset is independent given $\theta$. As the normalised posterior $p(\theta|D_{1:t})$ is usually intractable, it can be approximated by a parametric distribution $q$ with parameter $\phi_t$. The BOL framework consists of the update step and the projection step (Opper, 1998). The update step uses the approximate posterior $q(\theta|\phi_t)$ obtained from the previous step for an update in the form of Eq. (15):

$$p(\theta|D_{1:t+1}, \phi_t) \propto p(D_{t+1}|\theta)q(\theta|\phi_t).$$

The new posterior $p(\theta|D_{1:t+1}, \phi_t)$ might not belong to the same parametric family as $q(\theta|\phi_t)$. In this case, the new posterior has to be projected into the same parametric family to obtain $q(\theta|\phi_{t+1})$. Opper (1998) performs this projection by minimising the KL-divergence between the new posterior and the parametric $q$, while Ritter et al. (2018a) use Laplace approximation and Nguyen et al. (2018) use variational inference.

A.2. Laplace Approximation

We consider finding a MAP estimate following from Eq. (15):

$$\theta^*_{t+1} = \arg \max_\theta p(\theta|D_{1:t+1}) = \arg \max_\theta \{ \log p(D_{t+1}|\theta) + \log p(\theta|D_{1:t}) \}. $$

Since the posterior $p(\theta|D_{1:t})$ of a neural network is intractable except for small architectures, the unnormalised posterior $\tilde{p}(\theta|D_{1:t})$ is considered instead. Performing Taylor expansion on the logarithm of the unnormalised posterior around a mode $\theta^*_t$ gives

$$\log \tilde{p}(\theta|D_{1:t}) \approx \log \tilde{p}(\theta|D_{1:t})|_{\theta=\theta^*_t} - \frac{1}{2}(\theta - \theta^*_t)^T \Lambda_t (\theta - \theta^*_t),$$

where $\Lambda_t$ denotes the Hessian matrix of the negative log-posterior evaluated at $\theta^*_t$. The expansion in Eq. (18) suggests using a Gaussian approximate posterior. Given the parameter $\phi_t = \{\mu_t, \Lambda_t\}$, a mean $\mu_{t+1}$ for step $t+1$ can be obtained by finding a mode of the approximate posterior as follows via a standard gradient-based optimisation:

$$\mu_{t+1} = \arg \max_\theta \left\{ \log p(D_{t+1}|\theta) - \frac{1}{2}(\theta - \mu_t)^T \Lambda_t (\theta - \mu_t) \right\}. $$

The precision matrix is updated as $\Lambda_{t+1} = H_{t+1} + \Lambda_t$, where $H_{t+1}$ is the Hessian matrix of the negative log-likelihood for $D_{t+1}$ evaluated at $\mu_{t+1}$ with entries

$$H_{t+1}^{ij} = -\frac{\partial^2}{\partial \theta^i \partial \theta^j} \log p(D_{t+1}|\theta) \bigg|_{\theta=\mu_{t+1}}.$$
A.3. Block-Diagonal Hessian Approximation

Since the full Hessian matrix in Eq. (20) is intractable for large neural networks, we seek for an efficient and relatively close approximation to the Hessian matrix. Diagonal approximations (Denker & LeCun, 1991; Kirkpatrick et al., 2017) are memory and computationally efficient, but sacrifice approximation accuracy as they ignore the interaction between parameters. Consider instead separating the Hessian matrix into blocks where different blocks are associated to different layers of a neural network. A particular diagonal block corresponds to the Hessian for a particular layer of the neural network. The block-diagonal Kronecker-factor approximation (Martens & Grosse, 2015; Grosse & Martens, 2016; Botev et al., 2017) utilises the fact that each diagonal block of the Hessian is Kronecker-factor for a single data point. This provides a better Hessian approximation as it takes the parameter interactions within a layer into consideration.

A.3.1. Kronecker-Factored Approximation

Consider a neural network with \(L\) layers and parameter \(\theta = [\text{vec}(W^1), \ldots, \text{vec}(W^L)]^T\) where \(W^\ell\) is the weight of layer \(\ell\) for \(\ell = \{1, \ldots, L\}\) and \(\text{vec}\) denotes stacking the columns of a matrix into a vector. We denote the input of the neural network as \(a_0 = x\) and the output of the neural network as \(h_L\). As the input passes through each layer of the neural network, we have the pre-activation for layer \(\ell\) as \(h_\ell = W_\ell a_{\ell-1}\) and the activation as \(a_\ell = f(h_\ell)\) where \(f\) is the activation function of layer \(\ell\). If a bias vector is applicable in calculating the pre-activation of a layer, we append the bias vector to the last column of the weight matrix and append a scalar one to the last element of the activation. The gradient \(g_\ell\) of loss \(L_\theta(x, y) = -\log p(y | x, \theta)\) with respect to \(h_\ell\) for an input-target pair \((x, y)\) is the pre-activation gradient for layer \(\ell\).

Martens & Grosse (2015) show that the \(\ell\)-th diagonal block \(F_\ell\) of the Fisher information matrix \(F\) can be approximated by the Kronecker product between the expectation of the outer product of the \((\ell - 1)\)-th layer activation and the \(\ell\)-th layer pre-activation gradient:

\[
F_\ell = \mathbb{E}_{x,y}[a_{\ell-1} a_{\ell-1}^T \otimes g_\ell g_\ell^T]
\]

\[
\approx \mathbb{E}_x[a_{\ell-1} a_{\ell-1}^T] \mathbb{E}_{y|x}[g_\ell g_\ell^T]
\]

\[
= A_{\ell-1} \otimes G_\ell, \tag{21}
\]

where \(A_{\ell-1} = \mathbb{E}_x[a_{\ell-1} a_{\ell-1}^T]\) and \(G_\ell = \mathbb{E}_{y|x}[g_\ell g_\ell^T]\). Grosse & Martens (2016) extend the block-diagonal Kronecker-factorised Fisher approximation for fully-connected layers to that for convolutional layers. For batch normalisation layers, we adopt the unit-wise approximation (Osawa et al., 2020). The Gaussian log-probability term can be calculated efficiently without expanding the Kronecker product using the identity

\[
(A_{\ell-1} \otimes G_\ell) \text{vec}(W_\ell - W_\ell^*) = \text{vec}(G_\ell(W_\ell - W_\ell^*) A_{\ell-1}^T).
\]

As we mentioned in Section 4.2, approximating the Hessian with the one-step SGD inner loop assumption results in having terms that multiply two or more Kronecker products together. The \(\ell\)-th diagonal block of \(\tilde{F}\) in Eq. (11) is

\[
\tilde{F}_\ell = \frac{1}{M} \sum_{m=1}^{M} (I - A_{\ell-1}^m \otimes G_\ell^m)(\tilde{A}_{\ell-1}^m \otimes \tilde{G}_\ell^m)(I - A_{\ell-1}^m \otimes G_\ell^m)^T,
\]

where \(\tilde{A}_{\ell-1}^m \otimes \tilde{G}_\ell^m\) is the Kronecker product corresponding to the non-Jacobian terms in Eq. (11) for task \(m\), and \(A_{\ell-1}^m \otimes G_\ell^m\) is the Kronecker product corresponding to the Hessian in Eq. (12). We expand \(\tilde{F}_\ell\) using the Kronecker product property:

\[
(A_{\ell-1}^m \otimes G_\ell^m)(\tilde{A}_{\ell-1}^m \otimes \tilde{G}_\ell^m) = A_{\ell-1}^m \tilde{A}_{\ell-1}^m \otimes G_\ell^m \tilde{G}_\ell^m.
\]

This gives

\[
\tilde{F}_\ell = \frac{1}{M} \sum_{m=1}^{M} \left\{ \tilde{A}_{\ell-1}^m \otimes \tilde{G}_\ell^m - A_{\ell-1}^m \tilde{A}_{\ell-1}^m \otimes G_\ell^m \tilde{G}_\ell^m - \tilde{G}_\ell^m (A_{\ell-1}^m)^T \otimes G_\ell^m (\tilde{G}_\ell^m)^T + A_{\ell-1}^m \tilde{A}_{\ell-1}^m (A_{\ell-1}^m)^T \otimes G_\ell^m \tilde{G}_\ell^m (G_\ell^m)^T \right\}.
\]

Finally, moving the meta-batch averaging into the Kronecker factors gives the approximation:

\[
\tilde{F}_\ell \approx \tilde{A}_{\ell-1} \otimes \tilde{G}_\ell - A_{\ell-1} \tilde{A}_{\ell-1} \otimes G_\ell \tilde{G}_\ell - \tilde{A}_{\ell-1} (A_{\ell-1})^T \otimes \tilde{G}_\ell (G_\ell)^T + A_{\ell-1} \tilde{A}_{\ell-1} (A_{\ell-1})^T \otimes G_\ell \tilde{G}_\ell (G_\ell)^T,
\]

where \(\tilde{A}_{\ell-1} = \frac{1}{M} \sum_m A_{\ell-1}^m, \tilde{G}_\ell = \frac{1}{M} \sum_m G_\ell^m, A_{\ell-1} \tilde{A}_{\ell-1} = \frac{1}{M} \sum_m A_{\ell-1}^m \tilde{A}_{\ell-1}^m\), and so on.
A.3.2. POSTERIOR REGULARISING HYPERPARAMETER FOR PRECISION UPDATE

Ritter et al. (2018a) use a hyperparameter $\lambda$ as a multiplier to the Hessian when updating the precision:

$$\Lambda_{t+1} = \lambda H_{t+1} + \Lambda_t.$$  \hspace{1cm} (29)

In the large-scale supervised classification setting, this hyperparameter has a regularising effect on the Gaussian posterior approximation for a balance between having a good performance on a new dataset and maintaining the performance on previous datasets (Ritter et al., 2018a). A large $\lambda$ results in a sharply peaked Gaussian posterior and is therefore unable to learn new datasets well, but can prevent forgetting previously learned datasets. A small $\lambda$ in contrast gives a dispersed Gaussian posterior and allows better performance on new datasets by sacrificing the performance on the previous datasets.

A.4. Variational Continual Learning

The variational continual learning method (Nguyen et al., 2018) also provides a suitable meta-training framework for BOML in Eq. (5). Consider approximating the posterior $q$ by minimising the KL-divergence between the parametric $q$ and the new posterior as in the projection step in Eq. (16), where $q$ belongs to some pre-determined approximate posterior family $Q$ with parameters $\phi$:

$$q(\theta|\phi_{t+1}) = \arg \min_{q \in Q} D_{KL}(q(\theta|\phi)\|p(D_{t+1}|\theta)q(\theta|\phi_t))$$  \hspace{1cm} (30)

$$= \arg \min_{q \in Q} \left\{ -\mathbb{E}_{q(\theta|\phi)}[\log p(D_{t+1}|\theta)] + D_{KL}(q(\theta|\phi)\|q(\theta|\phi_t)) \right\}. \hspace{1cm} (31)$$

The optimisation in Eq. (31) leads to the objective

$$\phi_{t+1} = \arg \min_{\phi} \left\{ -\mathbb{E}_{q(\theta|\phi)}[\log p(D_{t+1}|\theta)] + D_{KL}(q(\theta|\phi)\|q(\theta|\phi_t)) \right\}. \hspace{1cm} (32)$$

One can use a Gaussian mean-field approximate posterior $q(\theta|\phi_t) = \prod_{d=1}^D N(\mu_{t,d}; \sigma_{t,d}^2)$, where $\phi_t = \{\mu_{t,d}, \sigma_{t,d}\}_{d=1}^D$ and $D = \dim(\theta)$. The first term in Eq. (32) can be estimated via simple Monte Carlo with local reparameterisation trick (Kingma et al., 2015), and the second KL-divergence term has a closed form for Gaussian distributions.
Algorithm 1  Bayesian online meta-learning with Laplace approximation (BOMLA)

1: Require: sequential base sets (or tasks) \( D_1, \ldots, D_T \), learning rate \( \alpha \), posterior regulariser \( \lambda \), number of meta-training iterations (or epochs) \( J \), meta-batch size (or number of mini-batches) \( M \)
2: Initialise: \( \mu_0, \Lambda_0, \theta \)
3: for \( t = 1 \) to \( T \) do
4: for \( i = 1, \ldots, J \) do \( \triangleright \) meta-training on base set (or task) \( D_t \)
5: for \( m = 1 \) to \( M \) do
6: Sample task (or split the batch) \( D^m_t = D^m_{t,S} \cup D^m_{t,Q} \)
7: Inner update \( \tilde{\theta}^m = SGD_k(\mathcal{L}(\theta, D^m_{t,S})) \)
8: end for
9: Evaluate loss \( f^\text{BOMLA}_t(\theta, \mu_{t-1}, \Lambda_{t-1}) \) in Eq. (8)
10: Outer update \( \theta \leftarrow \theta - \alpha \nabla_\theta f^\text{BOMLA}_t(\theta, \mu_{t-1}, \Lambda_{t-1}) \)
11: end for
12: Update mean \( \mu_t \leftarrow \theta \) \( \triangleright \) update posterior mean
13: For sequential datasets, sample a number of tasks for Hessian approximation
14: Run inner update in line 7 for each sampled task (or for each batch)
15: Approximate \( \tilde{H}_t \) with block-diagonal Kronecker-factored approximation to \( \tilde{F} \) in Eq. (11)
16: Update precision \( \Lambda_t \leftarrow \lambda \tilde{H}_t + \Lambda_{t-1} \) \( \triangleright \) update posterior precision
17: end for

Algorithm 2  Bayesian online meta-learning with variational inference (BOMVI)

1: Require: sequential base sets (or tasks) \( D_1, \ldots, D_T \), learning rate \( \alpha \), number of meta-training iterations (or epochs) \( J \), meta-batch size (or number of mini-batches) \( M \)
2: Initialise: \( \phi_0 = \{\mu_0, \sigma_0\} \)
3: for \( t = 1 \) to \( T \) do
4: for \( i = 1, 2, \ldots, J \) do \( \triangleright \) meta-training on base set (or task) \( D_t \)
5: for \( m = 1 \) to \( M \) do
6: Sample task (or split the batch) \( D^m_t = D^m_{t,S} \cup D^m_{t,Q} \)
7: Inner update \( \tilde{\theta}^m = SGD_k(\mathcal{L}(\theta, D^m_{t,S})) \)
8: end for
9: Evaluate loss \( f^\text{BOMVI}_t(\phi, \phi_{t-1}) \) in Eq. (14)
10: Outer update \( \mu \leftarrow \mu - \alpha \nabla_\mu f^\text{BOMVI}_t(\phi, \phi_{t-1}) \), and \( \sigma \leftarrow \sigma - \alpha \nabla_\sigma f^\text{BOMVI}_t(\phi, \phi_{t-1}) \)
11: end for
12: Update \( \mu_t \leftarrow \mu \) and \( \sigma_t \leftarrow \sigma \) \( \triangleright \) update posterior parameters
13: end for

B. Algorithms

B.1. BOMLA and BOMVI

Algorithm 1 gives the pseudo-code of the BOMLA algorithm, with the corresponding variation for the Section 6.2 Omniglot sequential tasks setting in brackets. The algorithm is formed of three main elements: meta-training on a specific base set or task (line 4 – 11), updating the Gaussian mean (line 12) and updating the Gaussian precision (line 13 – 16). For the precision update, we approximate the Hessian using block-diagonal Kronecker-factored approximation.

Algorithm 2 gives the pseudo-code of the BOMVI algorithm, with the corresponding variation for the Section 6.2 Omniglot sequential tasks setting in brackets. The algorithm is formed of two main elements: meta-training on a specific base set or task (line 4 – 11) and updating the parameters of the Gaussian mean-field approximate posterior (line 12).
B.2. BOMVI Monte Carlo Estimator

Recall that the BOMVI objective is:

$$f_{t+1}^{\text{BOMVI}}(\phi, \phi_t) = -\frac{1}{M} \sum_{m=1}^{M} \mathbb{E}_{q(\theta|\phi)} \left[ \log p(D_{t+1}^{m,Q} | \hat{\theta}^m) \right] - \frac{1}{M} \sum_{m=1}^{M} \mathbb{E}_{q(\theta|\phi)} \left[ \log p(D_{t+1}^{m,S} | \theta) \right] + D_{\text{KL}}(q(\theta|\phi) || q(\theta|\phi_t)),$$

where $\hat{\theta}^m = \text{SGD}_k(L(\theta, D_{t+1}^{m,S}))$ for $m = 1, \ldots, M$. The Monte Carlo estimator for the first term of the BOMVI objective is difficult to compute, as every sampled meta-parameters $\theta_r$ for $r = 1, \ldots, R$ has to undergo a few-shot quick adaptation prior to the log-likelihood evaluation. As a consequence the estimator is prone to a large variance. Moreover, every quickly-adapted sample $\theta_r$ contributes to the meta-learning gradients of the posterior mean and covariance, resulting in a high computational cost when taking the meta-gradients.

To solve these impediments, we introduce a slight modification to the SGD quick adaptation $\tilde{\theta}^m$. Instead of taking the gradients with respect to the sampled meta-parameters, we consider the gradients with respect to the posterior mean. A one-step SGD quick adaptation, for instance, becomes:

$$\tilde{\theta}^m = \theta - \alpha \nabla_{\mu_t} L(\mu_t, D_{t+1}^{m,S}).$$

This gives $\tilde{\theta}^m \sim N(\tilde{\mu}_t, \text{diag}(\sigma_{\tilde{t}}^2))$ where

$$\tilde{\mu}_t = \mu_t - \alpha \nabla_{\mu_t} L(\mu_t, D_{t+1}^{m,S}),$$

since $\theta \sim N(\mu_t, \text{diag}(\sigma_{t}^2))$. A quick adaptation with more steps works in a similar fashion. With this modification, we can calculate the Monte Carlo estimate for the first term using the local reparameterisation trick as usual.
C. Experiments

C.1. Triathlon and Pentathlon

In these experiments, we use the model architecture proposed by Vinyals et al. (2016) that takes 4 modules with 64 filters of size $3 \times 3$, followed by a batch normalisation, a ReLU activation and a $2 \times 2$ max-pooling. A fully-connected layer is appended to the final module before getting the class probabilities with softmax. Tables 1 and 2 are the hyperparameters used in these experiments.

Omniglot: Omniglot (Lake et al., 2011) comprises 1623 characters from 50 alphabets and each character has 20 instances. We use 1100 characters for meta-training, 100 characters for validation and the remaining for meta-evaluation. New classes with rotations in the multiples of $90^\circ$ are formed after splitting the characters as mentioned.

miniQuickDraw: QuickDraw (Ha & Eck, 2017) comprises 345 categories of drawings collected from the players in the game “Quick, Draw!”’. We generate miniQuickDraw by randomly sampling 1000 instances in each class of QuickDraw.

CIFAR-FS: CIFAR-FS (Bertinetto et al., 2019) has 100 classes of objects and each class comprises 600 images. We use the same split as Bertinetto et al. (2019): 64 classes for meta-training, 16 classes for validation and 20 classes for meta-evaluation.

miniImageNet: miniImageNet (Vinyals et al., 2016) takes 100 classes and 600 instances in each class from the ImageNet dataset. We use the same split as Ravi & Larochelle (2017): 64 classes for meta-training, 16 classes for validation and 20 classes for meta-evaluation.

VGG-Flowers: VGG-Flowers (Nilsback & Zisserman, 2008) comprises 102 different types of flowers as the classes. We randomly split 66 classes for meta-training, 16 classes for validation and 20 classes for meta-evaluation.

Aircraft: Aircraft (Maji et al., 2013) is a fine-grained dataset consisting of 100 aircraft models as the classes and each class has 100 images. We randomly split 64 classes for meta-training, 16 classes for validation and 20 classes for meta-evaluation.

Table 1. Hyperparameters for the triathlon and pentathlon experiments (same value for all datasets)

| Hyperparameter                  | BOMLA | BOMVI |
|---------------------------------|-------|-------|
| Posterior regulariser $\lambda$ | (various values) | -     |
| Precision initialisation values | $10^{-4} \sim 10^{-2}$ | -     |
| Number of tasks sampled for Hessian approx. | 5000 | -     |
| Covariance initialisation values | - | $\exp(-5)$ |
| Number of Monte Carlo samples | - | 20     |
| Meta-batch size $M$             | 32    | 32    |
| Number of query samples per class | 15   | 15    |
| Number of iterations per dataset | 5000 | 5000  |
| Outer loop optimiser            | Adam  | Adam  |
| Outer loop learning rate         | 0.001 | 0.001 |
| Number of tasks sampled for meta-evaluation | 100  | 100   |

Table 2. Hyperparameters for the triathlon and pentathlon experiments (individual datasets)

| Hyperparameter                  | Omniglot | miniQuickDraw | CIFAR-FS | miniImageNet | VGG-Flowers | Aircraft |
|---------------------------------|----------|---------------|----------|--------------|-------------|----------|
| Number of inner SGD steps in meta-training ($k$) | 1        | 3             | 5        | 5            | 5           | 5        |
| Inner SGD learning rate ($\alpha$) | 0.4      | 0.2           | 0.1      | 0.1          | 0.1         | 0.1      |
| Outer learning rate decay schedule (none for BOMVI) | - | $\times0.1$ halfway | $\times0.1$ halfway | $\times0.1$ halfway | $\times0.1$ every 1000 iterations | $\times0.1$ halfway |
| Number of inner SGD steps in meta-evaluation | 3        | 5             | 10       | 10           | 10          | 10       |
C.2. Pentathlon: Analysing the Change in Approximate Posterior Covariance

We visualise the covariance of the meta-parameters approximate posterior from BOMVI to better understand how the uncertainty in the algorithm prevents catastrophic forgetting in few-shot classification problems. We follow the pentathlon sequence going from left to right of the figure: Omniglot → CIFAR-FS → miniImageNet → VGG-Flowers → Aircraft. The Gaussian mean-field approximate posterior becomes increasingly concentrated in general as it learns on more datasets. This is especially true for the earlier layers (Conv 1 and Conv 2), meaning that the posterior progressively becomes very confident on the meta-parameters of the raw-level filters. The covariance for the layer closest to the classifier (Conv 4) remains large in general, although there are some filters with decreasing covariance. As the convolutional layer gets closer to the classifying layer, a larger fine-tuning in the meta-parameters is needed (Ravi & Beatson, 2019) to cope with few-shot tasks from different knowledge domains. The approximate posterior covariance from BOMLA is too large for visualisation as it is block-diagonal. The BOMLA covariance for each convolutional layer has dimension $D \times D$ where $D$ is the number of parameters in a convolutional layer. In theory, the BOMLA covariance should also follow the same pattern as the BOMVI covariance.

![Figure 9. The change in the approximate posterior covariance after meta-training is completed on each dataset. Going from left to right are the pentathlon sequence of datasets. Going from top to bottom are the convolutional layers of the neural network which gets closer to the classifying layer.](image)

---

**Addressing Catastrophic Forgetting in Few-Shot Problems**

---
C.3. Pentathlon: the Comparison between BOMVI and BOMLA with Different Values of $\lambda$

![Meta-evaluation accuracy across 3 seed runs on each dataset along meta-training. Higher accuracy values indicate better results with less forgetting as we proceed to new datasets. BOMLA with a large $\lambda = 1000$ gives better performance in the off-diagonal plots (retains performances on previously learned datasets) but worse performance in the diagonal plots (does not learn well on new datasets). A small $\lambda = 1$ gives better performance in the diagonal plots (learns well on new datasets) but worse performance in the off-diagonal plots (forgets previously learned datasets). BOMVI is also able to retain performance on previous datasets, although it may be unable to perform as good as BOMLA due to sampling and estimator variance.](image)

Figure 10. Meta-evaluation accuracy across 3 seed runs on each dataset along meta-training. Higher accuracy values indicate better results with less forgetting as we proceed to new datasets. BOMLA with a large $\lambda = 1000$ gives better performance in the off-diagonal plots (retains performances on previously learned datasets) but worse performance in the diagonal plots (does not learn well on new datasets). A small $\lambda = 1$ gives better performance in the diagonal plots (learns well on new datasets) but worse performance in the off-diagonal plots (forgets previously learned datasets). BOMVI is also able to retain performance on previous datasets, although it may be unable to perform as good as BOMLA due to sampling and estimator variance.

Tuning the posterior regulariser $\lambda$ mentioned in Section 4.2 and Appendix A.3.2 corresponds to balancing between a smaller performance trade-off on a new dataset and less forgetting on previous datasets. As shown in the figure above, a larger $\lambda = 1000$ results in a more concentrated Gaussian posterior and is therefore unable to learn new datasets well, but can better retain the performances on previous datasets. A smaller $\lambda = 1$ on the other hand gives a widespread Gaussian posterior and learns better on new datasets by sacrificing the performance on the previous datasets. In this experiment, the value $\lambda = 100$ gives the best balance between old and new datasets. Ideally we seek for a good performance on both old and new datasets, but in reality there is a trade-off between retaining performance on old datasets and learning well on new datasets due to posterior approximation errors.
C.4. Omniglot: Sequential Tasks from a Stationary Task Distribution

In this experiment, we use the model architecture proposed by Vinyals et al. (2016) that takes 4 modules with 64 filters of size $3 \times 3$, followed by a batch normalisation, a ReLU activation and a $2 \times 2$ max-pooling. A fully-connected layer is appended to the final module before getting the class probabilities with softmax. Table 3 shows the hyperparameters used in this experiment.

The Omniglot dataset comprises 50 alphabets (super-classes). Each alphabet has numerous characters (classes) and each character has 20 instances. As the meta-training alphabets arrive sequentially, we form non-overlapping sequential tasks from each arriving alphabet, and the tasks also do not overlap in the characters. We use 35 alphabets for meta-training, 7 alphabets for validation and 8 alphabets for meta-evaluation. The alphabet splits are as follows:

35 alphabets for meta-training:

Alphabet_of_the_Magi, Angelic, Armenian, Asomtavruli_(Georgian), Atlantean, Aurek-Besh, Avesta, Balinese, Bengali, Braille, Burmese_(Myanmar), Early_Aramaic, Grantha, Gujarati, Gurmukhi, Hebrew, Inuktitut_(Canadian_Aboriginal_Syllabics), Japanese_(hiragana), Japanese_(katakana), Kannada, Keble, Korean, Latin, Malayalam, Malay_(Jawi_-_Arabic), Manipuri, Mongolian, Ojibwe_(Canadian_Aboriginal_Syllabics), Old_Church_Slavonic_(Cyrillic), Oriya, Sanskrit, Sylheti, Tengwar, Tifinagh, ULOG

7 alphabets for validation:

Anglo-Saxon_Futhorc, Arcadian, Blackfoot_(Canadian_Aboriginal_Syllabics), Cyrillic, Ge_ez, Glagolitic, N_Ko

8 alphabets for meta-evaluation:

Atemayar_Qelisayer, Futurama, Greek, Mkhedruli_(Georgian), Syriac_(Estrangelo), Syriac_(Serto), Tagalog, Tibetan

| Hyperparameter                               | BOMLA | BOMVI |
|---------------------------------------------|-------|-------|
| Posterior regulariser $\lambda$              | 0.01  | -     |
| Precision initialisation values              | $10^{-4} \sim 10^{-2}$ | -     |
| Covariance initialisation values             | -     | $\exp(-10)$ |
| Number of Monte Carlo samples                | -     | 5     |
| Number of mini-batches $M$                   | 1     | 1     |
| Number of query samples per class (meta-evaluation) | 15     | 15     |
| Number of epochs per task                   | 50    | 50    |
| Number of inner SGD steps in meta-training ($k$) | 5     | 5     |
| Inner SGD learning rate ($\alpha$)           | 0.1   | 0.1   |
| Outer loop optimiser                         | Adam  | Adam  |
| Outer loop learning rate                     | 0.001 | 0.001 |
| Number of tasks sampled for meta-evaluation  | 100   | 100   |
| Number of inner SGD steps in meta-evaluation ($k$) | 10     | 10     |

Table 3. Hyperparameters for the Omniglot sequential tasks experiment.