Adiabatic Mach-Zehnder interferometer via an array of trapped ions

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We explore the possibility of implementing a Heisenberg-limited Mach-Zehnder interferometry via an array of trapped ions, which obey a quantum Ising model within a transverse field. Based upon adiabatic processes of increasing the Ising interaction and then decreasing the transverse field, we demonstrate a perfect transition from paramagnetic to ferromagnetic states, which can be used as the beam splitter for the multi-ion Mach-Zehnder interferometer. The achieved NOON state of the ions enables the Heisenberg-limited interferometry. Using currently available techniques for ultracold ions, we discuss the experimental feasibility of our scheme with global operations.

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Since the foundation of quantum theory, as a natural result of superposition principle, quantum interference has attracted continuous attentions in both theoretical and experimental studies. It has been widely used to implement high-precision measurement, quantum information processing and so on. A well-known scheme for performing quantum interferometry is the Mach-Zehnder (MZ) interferometer, which has a beam splitter for selecting the input states and another beam splitter for recombining the output states. Up to now, quantum MZ interferometry has been accomplished via photons \cite{1}, electrons \cite{2}, superconducting flux qubits \cite{3} and trapped ions \cite{4,5} etc.

Beyond the conventional quantum interferometry via unentangled states, it has been demonstrated that the measurement precision can be enhanced from the standard quantum limit (or the shot noise limit) to the Heisenberg limit by using multipartite entangled states \cite{6}. An excellent candidate is the NOON state \((\ket{N}_a\ket{0}_b+\ket{0}_a\ket{N}_b}/\sqrt{2}\), which is an equal-probability superposition of all \(N\) particles in one of two paths denoted by \(a\) and \(b\). The entangled ions for high-precision metrology have been proposed theoretically \cite{7} and demonstrated experimentally \cite{8,9}. However, these schemes are subject to limited numbers of ions or the requirement for individual addressing. For an ensemble of thousands of neutral atoms, the possibility of performing a Heisenberg-limited MZ interferometer has been demonstrated via a quantized Bose-Josephson junction \cite{10}.

In this article, based upon the adiabatic processes and global operations on an array of ultracold ions, we present a realizable scheme for performing a Heisenberg-limited MZ interferometry. The ion array is described by a quantum Ising model of ferromagnetic (FM) interaction \(J\) and transverse field \(B\), which has been used to simulate quantum magnetism \cite{11-13} and quantum phase transition (QPT) \cite{14-16} by virtue of spin-dependent optical dipole forces \cite{17}. In our scheme, if the system starts from a paramagnetic state dominated by \(B\), we adiabatically increase \(J\) and then decrease \(B\), and vice versa if the system starts from a FM state dominated by \(J\). In contrast to tuning either \(J\) or \(B\) in previous schemes \cite{11,16}, the adiabatic processes in our scheme perfectly connects the two limits solely controlled by one of \(B\) and \(J\). Therefore, theoretically, a pure NOON state can be prepared adiabatically from a SU(2) coherent state, which is the ground state for the system completely dominated by \(B\).

Our scheme requests only global operations, which is less challenging experimentally than individual addressing of the ions. By using the adiabatic process between paramagnetic and FM states as beam splitters, we are able to accomplish a Heisenberg-limited MZ interferometry via the NOON state of an array of trapped ions.

We consider \(N\) ultracold ions confined in a linear Paul trap. Because only two ionic hyperfine states are used for implementing the MZ interferometry, each ion can be regarded as a spin-1/2 particle of two spin states \(|\downarrow\rangle\) and \(|\uparrow\rangle\). By globally addressing all the ions with particular lasers, the system obeys a transverse-field quantum Ising Hamiltonian \cite{14-17},

\[
H = -\sum_{i<j} J_{ij}\sigma^i_x\sigma^j_x - B\sum_i \sigma^i_z, \tag{1}
\]

where \(\sigma^i_x, \sigma^i_z\) are Pauli operators for the \(i\)-th ion, \(B\) is the transverse field, and \(J_{ij} = J/|i-j|^3\) is the effective Ising interaction between ions \(i\) and \(j\) with \(J \geq 0\) denoting the nearest-neighboring interaction. For convenience, we consider the dimensionless model in units of \(\hbar = 1\) before the discussions of experimental possibility. This model has been experimentally realized by using either longitudinal \cite{11,17} or transverse modes \cite{12-16} of the ions.
Obviously, there are two extreme cases: \( J = 0 \) and \( B = 0 \).
If \( J = 0 \), the ground state is a paramagnetic state of all spins aligned with the magnetic field, i.e., \( |→→⋯→⟩ \) for \( B > 0 \) or \( |←←⋯←⟩ \) for \( B < 0 \). This ground state is a spin coherent state. If \( B = 0 \), there are two degenerate ground states of all spins in either the spin-down state \( |↓↓⋯↓⟩ \) or the spin-up state \( |↑↑⋯↑⟩ \). The equal-probability superposition of these two degenerate ground states is a NOON state.

To prepare the NOON state for the multi-ion MZ interferometry under global operations, one may use a quantum adiabatic evolution connecting the initial spin coherent state and the NOON state. Obviously, the initial state is chosen as \( |→→⋯→⟩ \), which can be easily prepared. The system first evolves with \( B = 1 \) and \( J = t/\tau \) from \( t = 0 \) to \( t = \tau \) and then evolves with \( J = 1 \) and \( B = 2 - t/\tau \) if \( t \in [\tau, 2\tau] \). (d) Population evolutions for \( N = 8 \) under different sweeping schemes. In (a), (b) and (c), the insets stand for the temporal modulations of \( B \) and/or \( J \).

The first scheme, \( J \) is adiabatically increased from 0 to \( B/J ≪ 1 \) (with constant \( B \)) [11]. In the second scheme, \( B \) is adiabatically decreased from \( B/J \gg 1 \) to 1 (with constant \( J \)) [16]. Starting from \( |→→⋯→⟩ \), we show the dynamical populations in the two FM states \( |↓↓⋯↓⟩ \) and \( |↑↑⋯↑⟩ \), see Fig. 1(a) and (b). The two FM states always have the same populations due to the absence of longitudinal fields and the fidelity to the desired NOON state is associated with the sum of these two populations. The final populations depend on the sweeping rate and the system size. But even if the time-evolution is perfectly adiabatic, for a finite-size system, the final fidelity to the NOON state can only approach unity but not exactly unity. This is because that the final ground state of the first scheme [11] is not exactly the FM states and the initial state is not the ground state for the second scheme [16].

To improve the fidelity to the desired NOON state, different from the single-step schemes of sweeping \( B \) or \( J \), we adopt in the present work a two-step scheme exactly connecting two extreme cases, i.e., \( (J = 0, B > 0) \) and \( (J > 0, B = 0) \), in which \( J \) and \( B \) are alternately changed in one of the two steps. Thus, the ground state for the initial Hamiltonian \( H_i = -B\sum_i \sigma_i^z \) is exactly the spin coherent state along \( x \)-axis, and the NOON state is exactly a ground state for the final Hamiltonian \( H_f = -\sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z \). In the first step, the ratio \( J/B \) increases from 0 to 1 with \( B \) remaining unchanged. While in the second step, the ratio \( B/J \) decreases from 1 to 0 with \( J \) remaining unchanged. In our calculations, the initial state is chosen as \( |→→⋯→⟩ \), which can be easily prepared. The system first evolves with \( B = 1 \) and \( J = t/\tau \) from \( t = 0 \) to \( t = \tau \) and then evolves with \( J = 1 \) and \( B = 2 - t/\tau \) from \( t = \tau \) to \( t = 2\tau \). Therefore, for a sufficiently large \( \tau \), the system will adiabatically evolve into the desired NOON state \( |Ψ⟩ = (|↓↓⋯↓⟩ + |↑↑⋯↑⟩) / √2 \) with very high fidelities, see Fig. 1(c) and Fig. 2. Besides, Fig. 1(d) shows that the NOON state achieved by our two-step scheme is much better than the ones achieved by the single-step schemes.
Heisenberg-limited MZ interferometry by using $|↑↑\cdots↑⟩$ and $|↓↓\cdots↓⟩$ as two paths and by global operations for all the ions. The schematic diagram for our adiabatic MZ interferometry is shown in Fig. 3, in which a free evolution is sandwiched by two beam splitters achieved by adiabatic processes connecting paramagnetic and FM states. The first beam splitter (BS1) is the adiabatic preparation of the NOON state $|Ψ⟩$ discussed above. Then the state $|Ψ⟩$ evolves into $|Ψ⟩ = (e^{-iNφ/2} |↑↑\cdots↑⟩ + e^{iNφ/2} |↓↓\cdots↓⟩)/\sqrt{2}$ in the following free evolution of time duration $T$ under the government of $H_0 = ω_0 ∑_i σ_i^z$, where $ω_0$ is the transition frequency between $|↑⟩$ and $|↓⟩$ and the accumulated phase $φ = ω_0 T$. Lastly, the second beam splitter (BS2) is accomplished by the inverse process of the BS1.

To extract the accumulated relative phase between the two paths, one has to transform the phase information into the amplitude information of the final state itself or the expectation information of a particular observable for the final state. For systems of few trapped ions, this could be done by a controlled-NOT gate with the first ion being the control qubit and the rest being the target ones followed by a Hadamard operation on the first ion [7]. However, the individual addressing used in this procedure is very difficult to be accomplished in systems of large numbers of particles. In our scheme with full global operations, we may accomplish this procedure in another adiabatic process, which is the inverse process of the BS1, i.e., by first slowly increasing $B/J$ from 0 to 1 (with $J$ fixed) and then slowly decreasing $J/B$ from 1 to 0 (with $B$ fixed). As a result, for extracting the phase shift from our multi-ion MZ interferometry, we have to measure populations of the ground state $|0⟩$ and the first excited state $|1⟩$ of the Hamiltonian $H_1$ [18]. Under a global Hadamard operation, $|0⟩$ turns to be spin-up with all the ions, and $|1⟩$ becomes an entangled state with one spin-down and others spin-up. By applying a probe laser for coupling $|↓⟩$ to an excited level, it is possible to distinguish $|1⟩$ from $|0⟩$ by detecting spin-dependent fluorescence signals [16]. Therefore, we could obtain the population of the first excited state,

$$P_1 = \sin^2(Nφ/2),$$

and the corresponding phase sensitivity is

$$Δφ = Δ\hat{P}_1/(\partial \langle\hat{P}_1⟩/\partial φ) = 1/N,$$

with the population operator $\hat{P}_1 = |1⟩⟨1|$ and its variance $Δ\hat{P}_1$. This is the Heisenberg limit, where the relative phase $φ$ is measured more precisely than the disentanglement case by $\sqrt{N}$ times. Since $φ = ω_0 T$ and $ω_0$ depends on the atomic configuration and the local magnetic field, our Heisenberg-limited MZ interferometry is of practical applications in understanding the atomic configuration and measuring the local magnetic field.

Experimentally, the transverse-field quantum Ising model could be realized by an array of ultracold ions which are globally illuminated with off-resonant lasers and resonant Raman beams [12–17]. Because of the off-resonant lasers, the inter-ion Ising interactions are induced by the spin-dependent forces [19] with the assistance of phonon modes. The effective transverse fields are generated by the resonant Raman beams, which resonantly couple the two spin states. In the rotating frame, the ion array under the Lamb-Dicke limit is equivalent to a quantum Ising system within a transverse magnetic field. With the control of the frequencies and intensities of the off-resonant lasers, the sign and strength of the Ising interaction can be dynamically adjusted [15]. By tuning the strength of the resonant Raman beams, we can dynamically adjust the strength of the transverse field [13, 14, 16].

For an array of $^{171}$Yb$^+$ ions, two clock states $^2S_{1/2}$ $|F = 0, m_F = 0⟩$ and $^2S_{1/2}$ $|F = 1, m_F = 0⟩$ are employed as the two spin states $|↓⟩$ and $|↑⟩$ for a spin-1/2 particle, respectively. Based upon an array of ultracold $^{171}$Yb$^+$ ions in a linear trap, the QPT in the transverse-field quantum Ising model has been simulated [16], in which both $B$ and $J$ may be larger than the order of kHz. Therefore, we assume the maximum value for $B$ and $J$ is $B_0 = J_0 = 50$ kHz for our two-step sweeping scheme shown in the inset of Fig. 1(c). To compare with the dimensionless model, we transform the original Hamiltonian $H$ into $H/J_0$, which means that the energy is in units of $J_0$ and the time is in units of $1/J_0$. In the first step, we fix $B = 50$ kHz and slowly vary $J$ according to $J = J_0 \times t/τ$ from $t = 0$ to $t = τ$, where $τ = 100μs$. Then we fix $J = 50$ kHz and slowly sweep $B$ following $B = B_0 \times (2 − t/τ)$ from $t = τ$ to $t = 2τ$. Besides, to maintain the coherence of the NOON state, the free evolution time $T$ should be much shorter than $τ$. Therefore the required operations from BS1 to BS2 in our proposed MZ interferometry take about $400 \mu$s, which is shorter than the gating time (longer than $500 \mu$s) in Ref. [16] and should be feasible with currently available techniques for finite spin size.

Our scheme could also be realized by using other types of ions. For an array of $^{40}$Ca$^+$ ions in a linear trap, $|↓⟩$ and $|↑⟩$ are encoded by two hyperfine states $|S_{1/2}, m_S = −1/2⟩$ and $|S_{1/2}, m_S = 1/2⟩$ or $|S_{1/2}, m_S = −1/2⟩$ and the metastable state $|D_{5/2}, m_S = −1/2⟩$, respectively.

![FIG. 3: (Color online) The schematic diagram for the adiabatic MZ interferometry via trapped ions. Here, BS1 prepares the NOON state via adiabatic processes connecting paramagnetic and FM states, then two paths accumulate a relative phase in a following free evolution, and BS2 recombines the two paths for interference.](Image)
It has demonstrated the entanglement of fourteen $^{40}$Ca$^+$ ions in a linear trap [20]. More recently, the universal digital quantum simulation based upon a transverse-field quantum Ising model has been implemented by using $^{40}$Ca$^+$ [21]. For a system of fourteen $^{40}$Ca$^+$ ions, our interferometry scheme could enhance the measurement precision by nearly four times in comparison to the standard quantum limit.

Besides unpredictable imperfection in operations, the main errors in realistic experiments are caused by spontaneous emission and intensity/frequency fluctuations of the Raman beams, which lead to decoherence and suppress the populations of FM states [16]. The spontaneous emission in our scheme can be minimized by enlarging the Raman detuning from the excited state, provided that we increase the laser power accordingly to maintain the original Ising interaction and the transverse field strength. The intensity fluctuations of the Raman beams only yield AC Stark shift with 2% rms error [12], which is not serious in our scheme. Alternatively, we may employ $^{40}$Ca$^+$ ions irradiated by a 729 nm laser to couple the ground state $|S_{1/2}, m_S = -1/2\rangle$ with the metastable state $|D_{5/2}, m_S = -1/2\rangle$, which completely remove the necessity of Raman beams in manipulation [22]. For the operational imperfection, we simply consider a typical one in the non-resonant carrier transition when we tune the parameter $B$, which yields unwanted longitudinal bias field $\delta \sum_i \sigma_i^z$ and may cause a deviation from the expected NOON state. This imperfection is shown in Fig. 4, and it could be suppressed by carefully adjusting the direction and the polarization of the laser beams and/or by using additional pulses [11] to compensate the deviation.

In our scheme, the two beam splitters form a loop-like adiabatic operation, which may generate a geometric phase [23]. Fortunately, the generated geometric phase is a constant relevant to the loop area, but do not depend on the free evolution time $T$ (i.e. the accumulated phase $\omega_0 T$) [24]. Therefore, by measuring the relative phase for different free evolution time, we may eliminate the influence of the geometric phase by reducing a common shift from measurement results and the measurement precision of $\omega_0$ is thereby independent from the geometric phase accumulated in the adiabatic operation.

In conclusion, we have proposed a simple and practical scheme to carry out a Heisenberg-limited MZ interferometer with an array of trapped ions, which obey a transverse-field quantum Ising model. The multi-ion MZ interferometry is implemented by only global operations, which favors scalability. However, more challenges would appear if more ions are involved, such as less homogenous laser irradiation on the ions, smaller energy gap in the Ising model and weaker laser-ion coupling. Nevertheless, for finite ions, the global operation is less difficult experimentally than individual addressing. Provided the nearly perfect adiabaticity, our scheme includes a QPT in each beam splitter. By suppressing unpredictable errors in realistic experiments and elaborately modifying the results, our scheme for quantum metrology is in principle able to reach the ultimate precision limit beyond the standard quantum limit [25].

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[18] For N ions, the ground state \( |0\rangle = \frac{|↑↑\cdots↑⟩ + |↓↓\cdots↓⟩}{\sqrt{2}} \) and the first excited state \( |1\rangle \) consists of N degenerate sub-states. For a system of N = 3, its three degenerate sub-states are \( |e_1\rangle = (|↑↑↑⟩ - |↓↓↓⟩ + |↓↑↑⟩ - |↑↓↓⟩ + |↑↑↓⟩ - |↓↑↓⟩ + |↓↓↑⟩ - |↑↓↑⟩ - |↑↑↓⟩)/2, \)
\( |e_2\rangle = (|↑↑↓⟩ - |↓↓↑⟩ + |↓↑↓⟩ - |↑↓↑⟩ + |↑↑↑⟩ - |↓↓↓⟩ + |↓↑↑⟩ - |↑↓↓⟩ + |↑↑↓⟩ - |↓↓↑⟩)/2. \) Under a global Hadamard gate, \( |0\rangle \) turns to be \( |↑\rangle \) and each sub-state of \( |1\rangle \) is an entanglement of two spin-up and one spin-down. We may accomplish the readout by detecting the spin-dependent spontaneously emitting photons.

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