Intruders in the Dust: Air-Driven Granular Size Separation

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(Dated: June 5, 2022)

Using MRI and high-speed video we investigate the motion of a large intruder particle inside a vertically shaken bed of smaller particles. We find a pronounced, non-monotonic density dependence, with both light and heavy intruders moving faster than those whose density is approximately that of the granular bed. For light intruders, we furthermore observe either rising or sinking behavior, depending on intruder starting height, boundary condition and interstitial gas pressure. We map out the phase boundary delineating the rising and sinking regimes. A simple model can account for much of the observed behavior and show how the two regimes are connected by considering pressure gradients across the granular bed during a shaking cycle.

PACS numbers: 45.70.Mg, 64.75.+g, 83.80.Fg

Unlike thermal systems which favor mixing to increase entropy, granular systems tend to separate under an external driving mechanism such as vibrations. This is commonly known as the Brazil Nut Effect, in which a large particle, the “intruder”, rises to the top of a bed of smaller background particles. More recently, new behavior was discovered for the limit of very small bed particles (“dust”), in particular the sinking of light intruders, and a non-monotonic dependence of the rise time on density. A number of theory and experimental papers explored different aspects of this surprising behavior, but so far there has been no consensus about either the underlying mechanisms or the relative importance of various system parameters in driving the intruder motion.

Here we present results from a systematic investigation of both the intruder motion and the bed particle flow. Our central finding is that there is a phase diagram which delineates rising and sinking behavior of the intruder as a function of interstitial gas pressure, intruder density, and initial intruder height within the container. Our results lead to a physical model that provides a unifying framework to describe both rising and sinking regimes. In this way, the work presented here connects previously disjointed pieces of a puzzle that pointed to the importance of pressure gradients but approached the two regimes as separate phenomena. As a consequence, our findings directly contrast with the mechanisms proposed in Refs. that neglect interstitial gas flow.

We placed granular material inside an acrylic cylinder (inner diameter 8.2 cm) mounted on a shaker and used individual, well-spaced sine wave cycles (“taps”) of frequency $f$ and amplitude $A$ to vibrate the vessel vertically. The cell could be evacuated to a gas pressure, $P$. Both smooth and rough cells (created by gluing glass beads to the interior walls of an otherwise smooth cell) were used to study the effect of wall friction. A large intruder sphere of diameter $D$ was buried in the bed of background spheres (diameter $d$) at a height $h_s$ measured from the vessel bottom to the intruder top (See Fig. 1(a)). A range of diameter ratios $D/d$, shaking parameters, and background particle materials (glass, zirconium oxide, tapioca, and seeds [for MRI]) were investigated. The intruder density, $\rho$, could be tuned. We measured the number of taps required for the intruder to break through the upper free surface, $T_{\text{rise}}$, or to reach the bottom, $T_{\text{sink}}$. In addition, the intruder position could be recorded with high-speed video throughout the shaking process by attaching to the intruder a thin, vertical straw extending above the upper surface. We verified that the straw did not affect the intruder motion.

FIG. 1: MRI image of a large intruder sphere rising in a bed of smaller particles. Images are vertical cut through the center of the cylindric cell. (a) Layered bed and the starting position of the intruder before shaking. (b)-(d) Size separation in the presence of convection for intruders of three different densities at $\Gamma = 5$. $f = 13$ Hz: (b) $\rho/\overline{\rho}_m = 0.08$ after 23 taps; (c) $\rho/\overline{\rho}_m = 0.44 \approx \rho'/\overline{\rho}_m$ after 30 taps; and (d) $\rho/\overline{\rho}_m = 2.33$ after 11 taps. Here $\overline{\rho}_m = 1.10$ g/ml is the average density of the seeds used as bed particles.

To track the evolution of both the intruder and the surrounding bed material we used magnetic resonance imaging (MRI). Fig. 1 shows for the rough cell how the motion of the intruder is coupled to that of the background convection. Horizontal layers of poppy seeds (MRI active) were alternated with layers of rajagara seeds (MRI in-
active), with the intruder initially placed on the lowest black layer (Fig. 1(a)). Figures 1(b)-(d) show the situation, for intruders with progressively higher densities, after several taps when the intruder had risen slightly more than its own diameter. In both Fig. 1(b) and 1(d), the intruders move faster relative to the dark layer on which they were originally placed. In Fig. 1(c), the intermediate weight intruder with density close to the effect density of the bed remains at the same relative position to that layer. Thus, its rise time corresponds to the convection speed of the bed particles immediately below it. Above the intruder in Figs. 1(b) and 1(c), the alternating light and dark layers are no longer horizontal, because convection speeds up near the free surface and convection rolls churn the material into a swirl pattern near the side walls. In Fig. 1(d), the dense intruder rises so rapidly that these convection rolls have not had a chance to that layer. Thus, its rise time corresponds to the convection speed of the bed particles immediately below it. However, when either bed permeability or wall-driven friction is sufficiently reduced, the rolls in the bed (5, 18). However, when either bed permeability or wall-driven friction is sufficiently reduced, the convection becomes asymmetric so that intruders are driven towards the cell wall. We find the onset of asymmetric flow for \( d \leq 0.5 \) mm. This is precisely also the size regime in which it becomes very difficult to excite axi-symmetric bed flow patterns (18). In general, wall-driven convection sets up axi-symmetric convection rolls in the bed (5, 18). However, when either bed permeability or wall-driven friction is sufficiently reduced, the convection becomes asymmetric so that intruders are driven towards the cell wall. We find the onset of asymmetric flow for \( d \leq 0.5 \) mm. In the following, we focus on representative results obtained with \( d = 0.5 \) mm glass beads (\( \rho_m = 2.5 \) g/ml), \( D = 25 \) mm intruders, bed fill height 85 mm, and dimensionless shaking acceleration \( \Gamma = A(2\pi f)^2/g = 5 \) at \( f = 13 \) Hz. The two cell types then allow us to investigate how the overall bed flow affects the intruder motion.

Despite the clearly different flow patterns in the two cells, we find a qualitatively similar dependence of the intruder rise time, \( T_{\text{rise}} \), on the ratio of intruder density, \( \rho \), to the bed material density, \( \rho_m \) (Fig. 2). For both boundary conditions, at ambient pressure, \( T_{\text{rise}} \) has a pronounced peak at \( \rho^* \sim 0.5\rho_m \). On either side of this peak, there is a large increase of the intruder velocity (i.e., a decrease in \( T_{\text{rise}} \)). When \( P \) is decreased, the peak amplitude shrinks and \( \rho^* \) moves to smaller values as shown for both cells in the inset to Fig. 2(a). From MRI measurements (Fig. 1), we know that at the peak \( \rho^* \) the intruder rises with the convection, while intruders on either side of the peak rise faster. This result is in contrast with previous experiments (5, 10), where convection is measured without the intruder. This indicates the presence of the intruder slows down the convection. When \( P \) is lowered, the convection speeds up so that the amplitude of the rise-time peak shrinks as shown in Fig. 2. For the rough cell, \( \rho^* \) decreases more strongly than in the smooth cell and at low \( P \) it is indistinguishable from \( \rho^* = 0 \), having reached our lowest measurable density. At sufficiently low pressure (\( P \leq 0.13 \) kPa), the non-monotonic behavior in \( T_{\text{rise}} \) disappears and the curve is featureless. The slow increase in \( T_{\text{rise}} \) as \( \rho \) increases we believe is due to the heavy intruder burrowing back into the bed at the end of each cycle. Aside from this there is essentially no density dependence to the rise time. For \( \rho < \rho^* \) and \( P \sim 2.7 \) kPa a dramatic change in behavior can be observed, seen most clearly in the smooth cell. As shown in the inset to Fig. 2(b), at \( P = 2.7 \) kPa, instead of speeding up again as \( \rho \) is lowered below \( \rho^* \), the intruder stops rising and begins to sink. In this case, the peak in \( T_{\text{rise}} \) at \( \rho^* \) turns into a discontinuity as shown in the inset of Fig. 2(b). We find similar behavior in the rough cell but over a much smaller region of parameter space.

We find that the transition between the two regimes of

FIG. 2: (a) Intruder rise time \( T_{\text{rise}} \) versus \( \rho/\rho_m \) at different pressures \( P \). (a) Rough cell: (■) 101 kPa, (●) 47 kPa, (▼) 13 kPa, (▲) 6.7 kPa, (○) 0.13 kPa. Inset: rise time peak/divergence position \( \rho^*/\rho_m \) as a function of \( P \) in the rough (●) and smooth (■) cell. (b) Smooth cell: (■) 101 kPa, (○) 27 kPa, (●) 0.13 kPa. Inset: The sinking regime at 2.7 kPa. In both cells, \( h_s = 5.5 \) cm.
intruder rising and sinking is controlled by at least five main parameters: \( \rho/\rho_m, P, d, h_s \) and cell wall roughness. This allows us to construct phase diagrams for different wall roughness as shown in Fig. 3, where we plot the maximum starting height for sinking, \( h_c \), as function of relative density. Results for different \( d \) will be presented elsewhere [19]. If the intruder is placed close enough to the top surface, it will invariably rise. The regions where the intruder sinks depend on pressure, \( P \), and, when they occur, happen at low relative densities, \( \rho/\rho_m \), and low heights, \( h_s \). Interestingly, the sinking regime is largest for intermediate pressures around 1−30 kPa. At lower pressure, this regime disappears completely and all particles rise at all depths as they are swept along via the background convection rolls. We find the sinking regime only for \( \rho < \rho^* \).

In order to determine how the intruder interacts with the background, we measured its motion during flight using high-speed video. In Fig. 4 we plot the vertical displacement \( \Delta h \) (measured in the moving frame of the cell) versus time. The inset of Fig. 4(a) shows intruder trajectories in the lab frame. In both rough and smooth cell, the bed lifts off the cell bottom at \( t = 0 \), the particles near the walls land at 0.05 s and the condensation front of the bed reaches the intruder at \( \sim 0.07 \) s. Between liftoff and landing of the bed, the heavy intruder in the rough cell follows a parabolic trajectory with a total downward acceleration only slightly greater than \( g \) (Fig. 4 inset) whereas, as is evident from the larger effective downward acceleration, the lighter intruder experiences larger drag due to the air flow. Before landing, the light particles show a sharp change in behavior. This is seen as a kink in the curves occurring near 0.045 s (apparent in the lab frame). This kink, which slows down the landing of the light particles, appears to be responsible for the increase in upward velocity for \( \rho < \rho^* \). In the smooth cell, the kink is not as dramatic as in the rough cell but the departure from the downward parabolic profile is still evident and the intruder clearly moves down slower than the bed. When a light intruder is in the sinking region of the phase diagram, its trajectory, as seen in Fig. 4(b), is pulled down with respect to the bed in the first part of the shaking period.

These results clearly identify the interstitial gas as the cause of the non-monotonic behavior in \( T_{\text{rise}} \) and of the reversal from rising to sinking. They contradict several recent models that ignore air effects or treat the rising and sinking regimes as unrelated phenomena [10, 14, 16]. However, we can explain the rise/sink cross-over in the phase diagram and predict quantitatively the peak at

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**FIG. 3:** Phase diagrams delineating the rising and sinking regimes for \( d = 0.5 \) mm beds at various pressures \( P \). Above each phase boundary, intruders rise. Shaded area shows the sinking regime at ambient pressure. Main panel: (■) 101 kPa, (×) 27 kPa, (△) 0.67 kPa in the smooth cell. Inset: (■) 101 kPa, (○) 27 kPa in the rough cell. In both cells, all intruders rise from the bottom \( (h_s = D) \) when \( P < 0.13 \) kPa. The dashed lines indicate \( h_s \) of Fig. 2.

**FIG. 4:** High-speed video intruder displacement in the moving frame. The vertical dashed lines delineate the part I and part II of the period (see text). (a) Three different densities with \( h_s = 7.0 \) cm in the rough cell. (■) \( \rho/\rho_m = 0.043, \) (□) \( \rho/\rho_m = 0.52 \simeq \rho^*/\rho_m, \) (▲) \( \rho/\rho_m = 3.3 \). Inset: Same trajectories in the lab frame. The “kink” for the lightest intruder is highlighted by the dashed circle. (b) Light intruder \( (\rho/\rho_m = 0.043) \) in smooth cell rising at \( h_s = 6.5 \) cm \( > h_c \) (■) and sinking at \( h_s = 3.5 \) cm \( < h_c \) (○). The third curve (△) tracks the gap underneath the bed.
$T_{\text{rise}} (\rho^* / \rho_m)$ with a model that treats the bed as a porous piston with permeability $k$. We consider two parts of the free-flight portion during each shaking cycle (vertical lines in Fig. 4): In part I, gas flows down into the gap opened up at the cell bottom and produces a drag that adds to the gravitational acceleration $g$. In part II, the gap closes, so that the gas pushed upward through the bed creates a drag force opposing the inertial force.

We approximate both the packing fraction $f$ of bed particles and the pressure gradient to be constant throughout. A more detailed discussion will be presented elsewhere [10]. By Darcy’s law, incompressible gas of viscosity $\mu$ and velocity $u$ flowing through the bed produces a pressure gradient $\partial P / \partial z = \mu u / k$. This gradient leads to a drag force $F_d = (\mu u / k)V$ on a bed volume $V$. Similarly, an intruder of volume $V_i$ experiences a force $\int_P (\partial P / \partial z) dS = (\mu u / k)V_i$. Consider the free flight motion of an intruder of mass $m_i = \rho V_i$ relative to a neighboring, identical volume $V_i$ comprised only of bed material. Depending on whether the sign of the mass difference $\Delta m = (\rho - f \rho_m)V_i$ between the two volumes is positive (negative), the intruder during part I will push against the material above (below) it. We calculate the net force on the compound object consisting of the intruder and the vertical column (diameter $D$) of bed material either above or below it. We find that, during part I, intruders with $\rho / \rho_m < f$ will sink, while those with $\rho / \rho_m > f$ will rise relative to the neighboring bed.

During part II of the free flight the situation changes. Now the heavy (light) intruder accelerates together with the column of material below (above) it. Because inertial and drag forces oppose each other, light intruders experience smaller acceleration magnitudes and fall more slowly than the surrounding bed. In particular, near the top surface light intruders can reach their terminal velocity before the bed collides with the base. We see evidence of this in the inset of Fig. 4(a) as the parabolic trajectory changes to a linear, constant velocity segment in the lab frame. Finally, we assume that any gap around the intruder changes to a linear, constant velocity segment in the presence of convection, this divergence should be cut off by the convective rise time. Second, the peak in $T_{\text{rise}}$ should occur at a packing fraction value corresponding to a loosely packed “in-flight” bed configuration. Thus, we expect similar values for $\rho^* / \rho$ around $f \sim 0.5$ using different bed particle sizes and shaking parameters [8, 9, 10]. Third, for light intruders the amount of sinking during part I and rising during part II should depend on their vertical position in the bed. Thus, for $\rho / \rho_m < f$ there should be a critical initial height, $h_c$, separating rising and sinking behavior. Because convection will produce a bias towards rising, it will reduce $h_c$ in the rough cell. Qualitatively, this explains the key features of the phase diagrams in Fig. 3. Finally, the model implies that the sinking found by us and others [6, 7] depends on a pressure gradient and should vanish in vacuum, where only the convection-driven, density-independent rising effect survives [8]. This is confirmed by the data for $P < 0.13 \text{kPa}$ in Fig. 2.

While the above model provides a mechanism for the demise of the density dependence with decreasing pressure it neglects a number of aspects that might be important: variation in $\partial P / \partial z$, compressibility of the bed and associated variations of $f$ with position, and pressure dependence of convection. Thus, the model cannot predict the detailed shape of the phase diagram (Fig. 3, nor the inset of Fig. 2a). From earlier results [17] by the Duke group, we would have expected a significant pressure dependence only once $P$ approached values close to 2 kPa. However, our results clearly demonstrate changes in the behavior at much larger $P$.

By presenting the phase diagram as a function of various system parameters, we showed that the previous perplexing results obtained by different groups [3, 4, 5, 6, 10] belong to different regimes of the same phenomenon. Using MRI and high-speed video, we found the underlying mechanisms of the density-dependent behavior of intruders. Moreover, a simple model can give a qualitative description of the key experimental results.

We thank N. Mueggenburg and E. Corwin for fruitful discussions and J. Rivers for help with the MRI. This work was supported by the NSF under Grant CTS-0009490 and under the MRSEC program, DMR-0213745. MEM acknowledges a Burroughs-Wellcome Fellowship.

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