High Dynamic Range Image Quality Assessment Based on Frequency Disparity

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Abstract—In this paper, a novel and effective image quality assessment (IQA) algorithm based on frequency disparity for high dynamic range (HDR) images is proposed, termed as local-global frequency feature-based model (LGFM). Motivated by the assumption that the human visual system (HVS) is highly adapted for extracting structural information and partial frequencies when perceiving the visual scene, the Gabor and the Butterworth filters are applied to the luminance component of the HDR image to extract the local and global frequency features, respectively. The similarity measurement and feature pooling strategy are sequentially performed on the frequency features to obtain the predicted single quality score. The experiments evaluated on four widely used benchmarks demonstrate that the proposed LGFM can provide a higher consistency with the subjective perception compared with the state-of-the-art HDR IQA methods. Our code is available at: https://github.com/eezkni/LGFM.

Index Terms—Image quality assessment (IQA), high dynamic range (HDR), Gabor feature, Butterworth feature.

I. INTRODUCTION

With the rapid development of imaging technology and the growing demand for immersive experiences, the high dynamic range (HDR) images are increasingly indispensable due to the realistic experiences they can provide, which can significantly contribute to the development of TV and photography industry. Compared with the 8-bit low dynamic range (LDR) images, HDR images are linearly related to the physical luminance in the scene and can record more structural details by using 16-32 bit floating point values. Essentially speaking, there are two major differences between HDR and LDR images: 1) the data distribution of HDR images is much broader than that of LDR images; 2) more detailed structures can be preserved in HDR images. Therefore, most image quality assessment (IQA) methods designed for LDR images are not suitable for direct use in assessing the quality of HDR images, which makes it crucial for developing effective IQA models for HDR images.

The IQA models aim to objectively evaluate the image quality to align with the human visual system (HVS). According to the amount of available information of the reference perception, existing IQA models can be roughly divided into three categories, full-reference (FR) [1], [2], [3], reduced-reference (RR) [4], [5], and no-reference (NR) [6], [7], [8]. IQA models have been widely used to optimize the performance of various learning-based vision tasks and to improve image encoding capability by increasing compression ratio while preserving the original image quality [9], [10]. In this paper, we are committed to proposing a novel and effective FR IQA model for HDR images.

The most widely used objective IQA measure for HDR images is the HDR visual difference predictor (HDR-VDP) proposed by Mantik et al. [11], the latest release of which is HDR-VDP-3 [12]. They model the HVS by taking into account optical and retinal pathways in the human eye and evaluating visible differences between images. Currently, there is a metric specifically designed for the HDR video content, namely HDR video quality metric (HDR-VQM) proposed by Narwaria et al. [13]. The input HDR video sequences are first projected to the perceptual domain, then the Gabor filter is applied to extract the frequency features to obtain the final quality score. Nevertheless, both the HDR-VDP-3 and the HDR-VQM require additional viewing information such as display parameters and viewing distance.

Due to the lack of IQA measures specifically designed for HDR images, various objective LDR IQA models are also used to evaluate the quality of HDR images. However, the data distribution of the HDR image is quite different from that of the LDR image. Since traditional CRFs are designed for dim LDR displays, Aydin et al. [14] proposes the perceptual uniform (PU) space to map a wide range of luminance to a perceptual range that is consistent with the HVS. Therefore, the quality of encoded HDR images can be evaluated with LDR measures, such as peak signal-to-noise ratio (PSNR),...
Fig. 1. The flowchart of the proposed Local and Global Frequency feature-based Model (LGFM) for HDR IQA. Firstly, the reference and distorted HDR images are converted to the luminance perceptual space in the pre-processing stage. Subsequently, an odd log-Gabor filter is designed to extract local frequency features, with a spatial mask used to provide higher weights to the over-exposed region. Meanwhile, the Butterworth filter is used to simulate the contrast sensitivity function to highlight the sensitive frequency interval in HVS to generate the global frequency features of the reference and distorted HDR images. The local and global similarities are measured, followed by the feature pooling strategy to predict the final quality score.

II. PROPOSED LOCAL AND GLOBAL FREQUENCY FEATURE-BASED MODEL

The framework of the proposed LGFM is illustrated in Fig. 1, which consists of four processing stages. Firstly, the reference and distorted HDR image, $I_r$ and $I_d$, are converted to the perceptual space in the pre-processing stage. The corresponding luminance maps, $L_r(x, y)$ and $L_d(x, y)$, are obtained from the linear luminance space by the PU coding, where $(x, y)$ denotes the pixel coordinate in the image. In the second stage, the Gabor filter and Butterworth filter are used to extract the local frequency features ($G_r(x, y)$, $G_d(x, y)$) and global frequency features ($B_r(x, y)$, $B_d(x, y)$), respectively. In the third stage, the computed frequency features from the reference and distorted HDR images are compared separately to yield the local and global similarity maps. Finally, the two similarity maps are combined to generate the predicted quality score using the proposed feature pooling strategies.

A. Gabor Filter-Based Local Feature Extraction

The local frequency feature can be used to extract abundant structural and edge information. Ni et al. [22] use the odd-Gabor filter to model the edge information of the screen content images. The outstanding performance demonstrates that the Gabor filter is greatly consistent with the response of the HVS. Inspired by this, the local edge information is extracted using the odd log-Gabor filter, which can well represent the high-frequency component of nature images, $G(x, y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp\left\{ -\frac{1}{2} \left[ \log\left( \frac{x'}{\sigma_x} \right)^2 + \log\left( \frac{y'}{\sigma_y} \right)^2 \right] \right\} \times \sin(2\pi f x')$, (1)
where
\[ x' = x \cos \theta + y \sin \theta; \]
\[ y' = y \cos \theta - x \sin \theta, \tag{2} \]
where \( f \) and \( \theta \) denote the frequency and the rotation angle of \((x', y')\). The standard deviations of the Gaussian envelope in the two directions are represented as \( \sigma_x \) and \( \sigma_y \), respectively. More specifically, the rotation angle is set to 0 and \( \pi/2 \) to extract horizontal and vertical edge features, respectively, and the frequency \( f \) is empirically set to 2.5. The Gabor filter-based features of reference and distorted HDR images are denoted as \( G_r(x, y) \) and \( G_d(x, y) \), respectively. Furthermore, the HDR images can provide abundant texture information in the bright areas, which enables it to provide the better visual experience compared with the LDR image. Therefore, a spatial mask is applied on the extracted local features, focusing on the high luminance region of HDR images. The designed Gaussian function is applied to obtain the mask,
\[ M_g = 1 + \frac{1}{2\pi\sigma} \exp \left(-\frac{(L_r - \mu)^2}{2\sigma^2}\right), \tag{3} \]
where \( \sigma \) and \( \mu \) are empirically set to 0.2 and 250. Note that all feature extraction operations in this work are performed on the luminance of HDR images unless otherwise stated.

B. Butterworth Filter-Based Global Feature Extraction

Over the past few decades, frequency features have been widely adopted to extract the local texture or edge information of the nature images [1], [7]. However, the direct difference in the frequency domain between the reference and distorted images has not been extensively discussed. Furthermore, the contrast sensitivity of the human eye first increases and then decreases with the increase of spatial frequency [23], which implies that there is a frequency interval where HVS is highly sensitive. Motivated by this observation, this paper adopts the Butterworth filter to simulate the contrast sensitivity function (CSF) to directly extract the global feature from the frequency spectrum of the image. Given a 2D image, each pixel of its frequency representation is computed from all the pixels in its spatial domain. Therefore, the frequency representation of an image can be regarded as a global feature of the image. Taking the reference HDR image as an example, the corresponding 2D Discrete Fourier transform (DFT) is performed as:
\[ F_r(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} L_r(x,y) e^{-j2\pi(ux/M + vy/N)}, \tag{4} \]
where \( M \) and \( N \) are the size of the image, \((u, v)\) denotes the pixel coordinate in the frequency spectrum. After shifting the low frequencies to the middle of the frequency spectrum, the log operation is applied to compress the values. Subsequently, a bandpass Butterworth filter is designed to provide higher weights to the special frequency interval,
\[ M_b = \left(1 - \frac{1}{1 + (D_1/D)^{2n_1}}\right) \cdot \left(1 - \frac{1}{1 + (D_2/D)^{2n_2}}\right), \tag{5} \]
where \( D \) is the Euclidean distance of the 2D grid. In this work, the cut-off frequency and order value \( D_1, D_2, n_1, \) and \( n_2 \) are empirically set to 400, 100, 4 and 2, respectively. The masked frequency map is presented as,
\[ B_r^f(u,v) = \log(|F_r(u,v)| + 1) \cdot M_b. \tag{6} \]

Moreover, by separating the frequency spectrum into real and imaginary part, i.e., \( R_r = \text{Real}(F_r) \) and \( I_r = \text{Imag}(F_r) \), the phase map can be obtained as \( B_r^p = \tan^{-1}(I_r/R_r) \). In the same way, the frequency map \( B_d^f \) and phase map \( B_d^p \) of the distorted HDR image can be obtained. Therefore, the Butterworth filter-based global feature is composed of the frequency map and phase map, which is given by,
\[ B_r(x,y) = \{ B_r^f(x,y), B_r^p(x,y) \}; \]
\[ B_d(x,y) = \{ B_d^f(x,y), B_d^p(x,y) \}. \tag{7} \]

C. Feature Similarity Measurements

Since the generated local and global frequency features are represented in different domains, the feature similarity measurements are conducted on the spatial and frequency domains, respectively. The similarity map of the local frequency features is calculated as follows:
\[ S_L(x,y) = \frac{2G_r(x,y) \cdot G_d(x,y) + T_0}{G_r(x,y)^2 + G_d(x,y)^2 + T_0}, \tag{8} \]
where the \( T_0 \) is a positive constant to prevent the exception that the denominator equals to zero.

For the global feature, the similarity of the frequency map and phase map can be generated as,
\[ S_g^f(x,y) = \frac{2B_r^f(x,y) \cdot B_d^f(x,y) + T_1}{B_r^f(x,y)^2 + B_d^f(x,y)^2 + T_1}; \]
\[ S_g^p(x,y) = \frac{2B_r^p(x,y) \cdot B_d^p(x,y) + T_2}{B_r^p(x,y)^2 + B_d^p(x,y)^2 + T_2}, \tag{9} \]
where \( T_1 \) and \( T_2 \) are positive constants similar to \( T_0 \). The final similarity map of the global frequency feature is obtained as,
\[ S_G(x,y) = |S_g^f(x,y)|^\alpha \cdot |S_g^p(x,y)|^{(1-\alpha)}, \tag{10} \]
where \( \alpha \) is a positive constant used for weighting control of \( S_g^f(x,y) \) and \( S_g^p(x,y) \). In this work, \( T_0, T_1, T_2, \) and \( \alpha \) are empirically set as 0.014, 8, 1, and 0.5, respectively.

D. Feature Pooling

In feature maps generated from HDR images using the Gabor filter and Butterworth filter, the larger pixel value implies that the HVS is more sensitive and pays more attention to it. Therefore, the weighted maps for the local and global frequency feature maps can be generated as:
\[ W_G(x,y) = \max\{ |G_r(x,y)|, |G_d(x,y)| \}; \]
\[ W_L(x,y) = \max\{ |B_r^f(x,y)|, |B_d^f(x,y)| \}. \tag{11} \]

Therefore, the local frequency similarity score and global frequency similarity score can be calculated as the weighted
average over all the pixel locations \((x, y)\) on the corresponding similarity maps as:

\[
Q_G(x, y) = \sum_{(x,y)} W_G(x,y) \cdot S_G(x,y) / \sum_{(x,y)} W_G(x,y);
\]

\[
Q_L(x, y) = \sum_{(x,y)} W_L(x,y) \cdot S_L(x,y) / \sum_{(x,y)} W_L(x,y).
\]

The final quality score is obtained by combining the local similarity score and global frequency similarity score,

\[
Q_{\text{LGFM}} = Q_G(x, y) \cdot Q_L(x, y).
\]

### III. EXPERIMENTAL RESULTS

#### A. HDR Dataset and Evaluation Protocols

In this section, four publicly available datasets are used for performance evaluation [21], [24], [25], [26]. The first three datasets are constructed by subjective experiments, while the fourth dataset consists of two existing datasets using a specifically designed algorithm to align subjective scores [26], [27]. To determine the value of the parameters in LGFM, a subset of UIPIQ database chosen from the UIPIQ dataset is used, including 12 reference HDR images and the associated 152 distorted HDR images. Following the common practice suggested in [18], the parameter values that can lead to higher SROCC will be selected.

As suggested in the VQEG HDTV test [28], [29], a logistic regression function is applied to map the predicted objective scores to a common scale,

\[
S_i = \gamma_1 \left( \frac{1}{1 + e^{-\gamma_2(q_i - \gamma_3)}} \right) + \gamma_4 q_i + \gamma_5,
\]

where \(q_i\) and \(S_i\) denote the generated quality score of the \(i\)-th image from the IQA model and the corresponding mapped score. \(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \) and \(\gamma_5\) are the regression parameters determined by minimizing the sum of squared differences between the predicted objective score \(S_i\) and corresponding ground truth score (i.e., MOS/DMOS). Subsequently, the Spearman rank order correlations coefficient (SROCC), Kendall rank order correlation coefficient, and root mean square error (RMSE) are adopted to evaluate the performance of various HDR IQA metrics. Note that higher values of SROCC and KROCC represent a stronger correlation, while lower values of RMES indicate smaller differences.

#### B. Performance Comparison

To illustrate the superiority of the proposed LGFM, several classic and state-of-the-art IQA metrics are adopted for comparison, including HDR-VDP-3 [12], HDR-VQM [13], PU-PieAPP [21], PSNR, SSIM [15], FSIM [18], VIF [17], GMSD [16], ESIM [30], GFM [22], and MSSIM [31], where the first three algorithms are specifically designed for HDR images, while the other metrics are for LDR images. More specifically, we apply the display-referred normalization method to calculate the HDR-VDP-3 and HDR-VQM score as described in [32]. Since the parameters in part of the IQA models (i.e., display and distance parameters in HDR-VDP-3 and HDR-VQM) would affect the final results, we use the default settings during the experiments. Moreover, the HDR images are converted into perceptual space by PU or PQ coding before applying the LDR IQA metrics.

Table I presents the performance comparison of various IQA models on four datasets, where the highlighted in bold with red, blue, and black represent the first, second, and third-ranked performance of the measurement criterions. Compared with the state-of-the-art IQA metrics, our proposed LGFM model yields the best overall performance in terms of SROCC, KROCC, and RMSE on four datasets. Besides,
the HDR-VDP-3, PU-ESIM, and PU-GFM also achieved relatively promising results. Moreover, there is an interesting observation that most of the highlighted values are from PU coding, which indicates that PU encoding performs higher consistency with the HVS compared with the PQ coding.

### C. Ablation Study

This subsection verifies the effectiveness of each component in the proposed LGFM model, including the Gabor filter-based local feature, Butterworth filter-based global feature, as well as the proposed two masks \( M_g \) and \( M_b \). More specifically, four LGFM variants are conducted as follows: 1) L: only Gabor filter-based local frequency feature is used for evaluation. 2) L w/o \( M_g \): removing the weighting mask \( M_g \) in L. 3) G: only Butterworth filter-based global frequency feature is used for evaluation. 4) G w/o \( M_b \): removing the weighting mask \( M_b \) in G. 5) LGFM w/o \( B \): removing the phase map in LGFM. As shown in Table II, both the \( M_g \) and \( M_b \) contribute to feature extraction in L and G, respectively, which illustrate the effectiveness of the over-exposed regions and the frequency interval. Furthermore, the combination of local and global frequency features outperforms using either one alone, indicating that the proposed two feature maps achieve complementarity between local and global frequency features.

### IV. Conclusion

This paper proposes the LGFM model, a novel image quality assessment algorithm for HDR images. The key contribution of our model lies in the combination of the local and global frequency features. More specifically, the local feature map is extracted by the Gabor filter to measure the structure similarity, while the global feature map is obtained by simulating the contrast sensitivity function with the Butterworth filter to detect the frequency interval similarity. Subsequently, the feature pooling strategy is adopted to generate the quality scores based on the local and global similarity maps, leading to the final quality score by combining them. Extensive experiments demonstrate that each component in the proposed LGFM contributes to the final results. Moreover, the proposed model provides higher consistency with the HVS and outperforms other state-of-the-art IQA models.

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