A late time acceleration of the universe with two scalar fields: many possibilities

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Abstract
In the present work, an attempt has been made to explain the recent cosmic acceleration of the universe with two mutually interacting scalar fields, one being the Brans-Dicke scalar field and the other a quintessence scalar field. Conditions have been derived for which the quintessence scalar field has an early oscillation and it grows during a later time to govern the dynamics of the universe.

1 Introduction
The present cosmic acceleration is now generally believed to be a certainty rather than a speculation. The recent data on supernovae of type Ia suggested this possibility quite strongly [1, 2, 3, 4] and the most trusted cosmological observations, namely that on Cosmic Microwave Background Radiation [5, 6, 7, 8, 9], appear to be quite compatible with an accelerated expansion of the present universe. The natural outcome of these observations is indeed a vigorous search for the form of matter which can give rise to such an expansion, as a normal matter distribution gives rise to an attractive gravity leading to a decelerated expansion. This particular form of matter, now popularly referred to as “dark energy”, is shown to account for as much as 70% of the present energy of the universe. This is also confirmed by the highly accurate Wilkinson Microwave Anisotropy Probe (WMAP) [10, 11, 12, 13, 14]. A large number of possible candidates suitable as a dark energy component have appeared in the literature. Excellent reviews on this topic are available [15, 16, 17]. Most of the dark energy candidates are constructed so as to generate an effective pressure which is sufficiently negative driving an accelerated expansion. The alleged acceleration can only be a very recent phenomenon and must have set in during the late stages of the matter dominated expansion of the universe. This requirement is crucial for the successful
nucleosynthesis in the radiation dominated era as well as for a perfect ambience for the formation of structure during the matter dominated era. Fortunately, the observational evidences are also strongly in favour of a scenario in which the expansion of the universe had been decelerated (the deceleration parameter $q > 0$) for high redshifts and becomes accelerated ($q < 0$) for low values of the redshift $z$ [18]. So the dark energy sector should have evolved in such a way that the consequent negative pressure has begun dictating terms only during a recent past.

This so easily reminds one about the inflationary universe models where an early accelerated expansion was invoked so as to wash away the horizon, fine tuning and some other associated problems of the standard big bang cosmology. The legendary problem, that the inflationary models themselves had, was that of a “graceful exit” - how the accelerated expansion gives way to the more sedate ($q < 0$) expansion so that the universe could look like as we see it now. For a very comprehensive review, we refer to Coles and Lucchin [19] or Kolb and Turner [20]. The “graceful exit” problem actually stems from the fact that for the potentials driving inflation, the phase transition to the true vacuum is never complete in a sizeable part of the actual volume of the universe. The attempts to get out of this problem involved the introduction of a scalar field which slowly rolls down its potential so that there is sufficient time available for the transition of phase throughout the actual volume of the universe. The current accelerated expansion thus poses the problem somewhat complementary to the graceful exit - that of a “graceful entry”.

The present work addresses this problem, and perhaps provides some clue regarding the solution of the problem. The basic motivation stems from existing literature on inflation. Mazenko, Wald and Unruh [21] showed that a classical slow roll is in fact invalid when the single scalar field driving inflation is self interacting. It was also shown that a slow roll with a single scalar field puts generic restrictions on the potentials driving inflation [22]. This kind of problems led to the belief that for a successful inflationary model, one needs to have two scalar fields [23]. The present work uses this idea of utilising two scalar fields, one of them being responsible for the present acceleration of the universe and is called the quintessence field. The second one interacts nonminimally with the former so that the quintessence field has an oscillatory behaviour at the early matter dominated epoch but indeed grows later to dominate the dynamics of the more recent stages of the evolution. If such a behaviour is achieved, some clue towards the resolution of the graceful entry problem or the coincidence problem may be obtained. There are quite a few quintessence potentials already in the literature [15] which drives a late time acceleration, but none of them really has an underlying physics explaining their genesis. As one is already hard pressed to find a proper physical background of the quintessence field, it will be even more embarassing to choose a second field without any physical motivation. Naturally, the best arena is provided by a scalar tensor theory, such as Brans-Dicke theory, where one scalar field is already there in the purview of the theory and is not put in by hand. It deserves mention that the Brans-Dicke scalar field was effectively used in “extended inflation” in order to get a sufficient slow roll of the scalar field [24, 25]. Later Brans-Dicke theory was used for finding a solution of the graceful exit problem with a large number of potentials [26], where the inflaton field evolves to an oscillatory phase during later stages.

In the next section we write down the field equations in Brans-Dicke (B-D) theory with a quintessence field $\phi$, the potential $V(\phi)$ driving acceleration being modulated by the B-D scalar field $\psi$ as $V(\phi)\psi^{-\beta}$. With a slow roll approximation, the conditions for an initially oscillating $\phi$, which grows only during later stages, are found out for two examples, a power law expansion and an exponential expansion of the scale factor. The particular form of $V(\phi)$ is quite irrelevant in this context, the conditions only put some restrictions on the constants of the theory and the
parameters of the model. So the form of the potential is arbitrary to start with, only the conditions on the parameters and the ‘value’ of \( V(\phi) \) has to be satisfied and hence many possibilities are opened up to accommodate a physically viable potential as the driver of the late time acceleration. However, in some cases, this could restrict the form of \( V(\phi) \) as well. In the last section, we make some remarks on the results obtained.

2 A model with a Graceful Entry

The relevant action in Brans-Dicke theory is given by

\[
S = \int \left[ \frac{\psi R}{16\pi G_0} - \omega \frac{\psi_{,\mu} \psi_{,\mu}}{\psi} - \frac{1}{2} \phi_{,\mu} \phi^{,\mu} - U(\psi, \phi) + L_m \right] \sqrt{-g} \, d^4x
\]

where \( G_0 \) is the Newtonian constant of gravitation, \( \omega \) is the dimensionless Brans-Dicke parameter, \( R \) is the Ricci scalar, \( \psi \) and \( \phi \) are the Brans-Dicke scalar field and quintessence scalar fields respectively. If we now choose \( U(\psi, \phi) \) as \( V(\phi) \psi^\beta \) as explained, the field equations, in units where \( 8\pi G_0 = 1 \), can be written as

\[
3H^2 + 3H \frac{\dot{\psi}}{\psi} - \frac{\omega \dot{\psi}^2}{2 \psi^2} = V(\phi)\psi^{-(\beta+1)} + \frac{\rho}{\psi},
\]

\[
3H \dot{\phi} + V'(\phi)\psi^{-\beta} = 0,
\]

\[
(2\omega + 3)(\ddot{\psi} + 3H \dot{\psi}) = (\beta - 4)V(\phi)\psi^{-\beta} + \rho,
\]

where a dot represents differentiation with respect to time \( t \) and a prime represents differentiation with respect to the scalar field \( \phi \). As we require the potential \( U(\psi, \phi) \) to grow with time so that the effective negative pressure dominates at a later stage, \( \beta \) should be negative for a \( \psi \) growing with time or positive for a \( \psi \) decaying with time. The field equations are written in the slow roll approximation, i.e, where \( \dot{\phi}^2 \) and \( \ddot{\phi} \) are neglected in comparison to others. \( H = \frac{\dot{a}}{a} \) is obviously the Hubble parameter. As we dropped the field equation containing stresses, we can use the matter conservation equation as the fourth independent equation which yields on integration

\[
\rho = \frac{\rho_0}{a^3},
\]

\( \rho_0 \) being a constant. This is so as the fluid pressure is taken to be zero as we are interested in the matter dominated era.

Using the expression for \( V(\phi)\psi^{-\beta} \) from equation (2) in equation (4), we can write

\[
(2\omega + 3)\left[ \frac{\ddot{\psi}}{\psi} + 3H \frac{\dot{\psi}}{\psi} \right] = (\beta - 4)[3H^2 + 3H \frac{\dot{\psi}}{\psi} - \frac{\omega \dot{\psi}^2}{2 \psi^2}] - (\beta - 5)\frac{\rho_0}{a^3\psi}.
\]

If the scale factor \( a \) is known, this equation can be integrated to yield the Brans-Dicke field \( \psi \). Equations (3) and (4) yield

\[
\frac{V(\phi)}{V'(\phi)} = -\frac{(2\omega + 3)(\ddot{\psi} + 3H \dot{\psi}) - \rho}{3(\beta - 4)H \dot{\phi}}.
\]

Hence, if we define

\[
f(\phi) = \int \frac{V}{V'} d\phi,
\]
then
\[ f(\phi_0) - f(\phi_i) = \int_{t_i}^{t_0} F(t) dt , \] (9)
where
\[ F(t) = -\frac{(2\omega + 3)(\dot{\psi} + 3H\dot{\psi}) - \rho}{3(\beta - 4)H} \] (10)
and subscripts ‘o’ and ‘i’ stands for the present value and some initial value, such as the onset of the matter dominated phase of evolution.

For a given \( a = a(t) \), therefore, equation (6) can be used to find \( \psi \), which in turn, with equations (8) and (9) determines \( f(\phi) \). Now, these equations can be used to put bounds on the values of derivatives of the potential, which would ensure that the dark energy has an oscillating phase in the early stages.

The complete wave equation for the quintessence field \( \phi \) is
\[ \ddot{\phi} + 3H\dot{\phi} + V'(\phi)\psi - \beta = 0. \]
The condition for a small oscillation of \( \phi \) about a mean value is \( V'(\phi) = 0 \). This provides a kind of plateau for the potential which hardly grows with evolution and hence the dynamics of the universe is practically governed by the B-D field \( \psi \) and the matter density \( \rho \). We find the condition for such an oscillation of \( \phi \) at some initial epoch by choosing \( V'(\phi) \approx 0 \) which yields
\[ \left. \frac{\ddot{\phi}}{3H\dot{\phi}} \right| \approx -1 . \]
During later stages, the universe evolves according to the equations (2) - (4) where \( V'(\phi) \neq 0 \), and the scalar field slowly rolls along the potential so that the quintessence field takes an active role in the dynamics and gives an accelerated expansion of the universe.

Two examples, one for power law and the other exponential expansion, will be discussed in the present work.

I. Power law expansion :-

If \( a = a_0 t^n \) where \( n > 1 \), the universe expands with a steady acceleration, i.e, with a constant negative deceleration parameter \( q = -\frac{(n-1)}{n} \).

With this,
\[ H = \frac{\dot{a}}{a} = \frac{n}{t} , \] (11)
and equation (6) has the form
\[ c_1 \frac{\ddot{\psi}}{\psi} + c_2 \frac{\dot{\psi}}{\psi} \frac{1}{t} + c_3 \frac{\dot{\psi}^2}{\psi^2} + c_4 \frac{1}{t^2} + c_5 \frac{1}{t^{3n}\psi} = 0 , \] (12)
ci’s being constants given by,
\[ c_1 = (2\omega + 3), \quad c_2 = 3n(2\omega - \beta + 7), \quad c_3 = \frac{\omega}{2}(\beta - 4), \quad c_4 = -3n^2(\beta - 4), \quad c_5 = (\beta - 5)\frac{\rho_0}{a_0^3} . \] (13)
The simplest solution for \( \psi \) in equation (12) is
\[ \psi = \psi_0 t^{2-3n} , \] (14)
\( \psi_0 \) being a constant.
The consistency condition for this is,
\[
(2\omega+3)(2-3n)(1-3n)+3n(2\omega-\beta+7)(2-3n)+\frac{\omega}{2}(\beta-4)(2-3n)^2-3n^2(\beta-4)+ (\beta-5) \frac{\rho_0}{a_0^3\psi_0} = 0 .
\]
From equations (9) and (10),
\[
f(\phi_0) - f(\phi_i) = D \left( t_i^{2-3n} - t_0^{2-3n} \right), \tag{15}
\]
where \( D \) is a constant involving \( c_i \)'s, i.e, \( n, \omega, \psi_0, \rho_0 \) etc. given by
\[
D = \frac{(2\omega+3)\psi_0(2-3n) - \frac{\rho_0}{a_0^3}}{3n(\beta-4)(2-3n)} .
\]
From the form of potential \( V = V(\phi) \), \( f(\phi) \) can be found from the relation (8). So, the different constants will be related by equation (15).

If the quintessence field oscillates with small amplitude about the equilibrium at the beginning, i.e, at \( t = t_i \) and grows during the later stages of evolution, then \( \left. \frac{\ddot{\phi}}{3H\phi} \right|_i \approx 1 \), which puts more constraints amongst different parameters of the theory.

Equations (7) and (14) yield
\[
\dot{\phi} = A t^{(1-3n)} (\ln V)' ,
\]
where \( A \) is a constant given by
\[
A = -\frac{(2\omega+3)\psi_0(2-3n) - \frac{\rho_0}{a_0^3}}{3n(\beta-4)} = -D(2-3n) .
\]
So,
\[
\frac{\ddot{\phi}}{3H\phi} = -1 + \frac{1}{3n} + \frac{A}{3n} t^{2-3n}(\ln V)'' .
\]
The condition for the small oscillation of \( \phi \) close to \( t = t_i \) is
\[
\left. \frac{\ddot{\phi}}{3H\phi} \right|_i \approx -1 ,
\]
which gives the condition on \((\ln V)'\) as
\[
(\ln V)'_i \approx -\frac{1}{A} t_i^{3n-2} . \tag{16}
\]

II. Exponential expansion :-

Similarly for an exponential expansion at the present epoch, the constraints can be derived. For such an expansion,
\[
a = a_0 e^{\alpha t} ,
\]
where \( a_0, \alpha \) are all positive constants.
Then,
\[
H = \frac{\dot{a}}{a} = \alpha \quad \text{and} \quad \rho = \frac{\rho_0}{a_0^3} e^{-3\alpha t} .
\]
Equation (6) with this solution has the form

\[ b_1 \dddot{\psi} + b_2 \ddot{\psi} + b_3 \dot{\psi}^2 + b_4 + b_5 \frac{1}{e^{3\alpha t} \psi} = 0 , \]  

where \( b_i \)'s are constants given by

\[ b_1 = (2\omega + 3), \quad b_2 = 3\alpha(2\omega - \beta + 7), \quad b_3 = \frac{\omega}{2}(\beta - 4), \quad b_4 = -3\alpha^2(\beta - 4), \quad b_5 = (\beta - 5)\frac{\rho_0}{a_0^3} . \]  

A simple solution for \( \psi \) is

\[ \psi = \psi_0 e^{-3\alpha t} , \]  

\( \psi_0 \) being a constant. The consistency condition for this is,

\[ 9\alpha^2(\beta + \frac{\omega \beta}{2} - 2\omega - 4) - 3\alpha^2(\beta - 4) + (\beta - 5)\frac{\rho_0}{a_0^3} \psi_0 = 0 . \]  

Equations (9) and (10) will put restrictions on the parameters of potential by

\[ f(\phi_0) - f(\phi_i) = \frac{\rho_0}{9\alpha^2 a_0^3(\beta - 4)} [e^{-3\alpha t_i} - e^{-3\alpha t_0}] . \]  

The condition \( \frac{\ddot{\phi}}{3H^2} \approx -1 \) for the small oscillation of the scalar field at \( t = t_i \), with the help of equations (7) and (19), leads to the interesting result

\[ (\ln V)' \big|_i \approx 0 , \]  

which indicates that for an exponentially expanding present stage of evolution, the quintessence potential behaves exponentially at least at the beginning.

Thus, if \( V = V(\phi) \) is given, then equations (15) and (21) will help finding out the bounds on the values of the scalar field \( \phi \) and the constants appearing in \( V(\phi) \) in terms of the initial and final epochs.

The conditions (15) or (21) put bounds on the potential so that the quintessence field \( \phi \) has an oscillatory behaviour in the beginning. As the solutions in the examples considered has accelerated expansion, the Q-field \( \psi \) has a steady growth at later stages. For a power law expansion, the growth has some arbitrariness as \( V(\phi) \) is not specified. For the exponential expansion, however, the growth of \( \phi \) is governed by an exponential potential.

### 3 Conclusion

It deserves mention that non-minimally coupled scalar fields were utilised by Salopek et al[27] and Spokoiny [28, 29] in the context of an early inflation, where the field had an oscillation or a plateau at the end of the inflation. In the present case, we need just the reverse for the Q-field, and that is aided by the non-minimally coupled field \( \psi \).

We see that for a wide range of choice of \( V(\phi) \), a power law acceleration is on cards, only the value of \( V(\phi) \) at some initial stage is restricted by equation (16). For an exponential expansion of the scale factor, however, the potential \( V(\phi) \) has to be an exponential function of \( \phi \).
of equation (22). This investigation can be extended for more complicated kinds of accelerated expansion.

It is true that general relativity is by far the best theory of gravity and the present calculations are worked out in Brans-Dicke (B-D) theory, but this should give some idea about how a second scalar field may be conveniently used to get some desired results. Although B-D theory lost a part of its appeal as the most natural generalization of general relativity (GR) as the merger of B-D theory with GR for large $\omega$ limit is shown to be somewhat restricted [30], it still provides useful limits to the solution of the cosmological problems [24]. Another feature of the present work is that the numerical value of $\omega$ required is not much restricted. For power law expansion, $\omega$ is restricted by equation (16) which clearly shows that it can be adjusted by properly choosing values of some other quantities, whereas for exponential expansion, equation (22) shows that $\omega$ is arbitrary. This is encouraging as it might be possible to get an acceleration even with a high value of $\omega$, compatible with local astronomical observations [31]. B-D theory had been shown to generate acceleration by itself [32], although it had problems with early universe dynamics. B-D theory with quintessence or some modifications of the theory [33, 34, 35, 36, 37] were shown to explain the present cosmic acceleration, but all these models, unlike the present work, required a very low value of $\omega$, contrary to the local observations.

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