Visibility, efficiency, and Bell violations in real Einstein-Podolsky-Rosen experiments.

W. A. Hofer

Department of Physics and Astronomy, University College, Gower Street, London WC1E 6BT, UK

The violation of Bell’s inequalities in Einstein-Podolsky-Rosen experiments has been demonstrated for photons and ions. In all experiments of this kind the relation between visibility, efficiency, and Bell violation is generally unknown. In this paper we show that simulations based on a local hidden variables models for entangled photons provide this information. It is established that these properties are closely related by the way, in which photons are detected after a polarizer beam splitter. On this basis we suggest controlled experiments which, for the first time, subject the superposition principle to experimental tests.

03.65.Ud,03.65.Ta

The Einstein-Podolsky-Rosen (EPR) problem has long occupied a central place in the understanding of quantum mechanics. Bell’s inequalities in conjunction with correlation measurements seemed to prove that reality in microphysics is manifestly nonlocal. Furthermore, the experimental evidence seems to contradict even the notion of an independent reality.

But as recently shown, the experimental data can be fully reproduced with a local and realistic model of correlation measurements. The model focussed on the electromagnetic fields of the propagating photons. The essential connection between the two points of measurement is the phase acquired at their common origin. The basic realistic model, Furry’s integral, was shown to be a wrong representation of the digital output of the polarizer beam splitters (PBS). The new model has three key advantages: (i) It is a hidden variables model, the hidden variable is the angle of polarization of a photon’s electromagnetic field; (ii) it is strictly local, all events can be traced from one point in space and time to the next; (iii) the main experimental limitations are included in the model. Decoherence, for example, is described as a random segment of a photon’s optical path. We also introduced a generic parameter, the threshold $\Delta s$, which describes the data selection at the critical polarization angle 45° of a PBS. In simulations of actual experiments it was shown that the Bell inequalities can be violated by a close to arbitrary amount, depending on the data suppressed around the critical angle.

This result points to a gap in our current understanding of these important experiments. The relation between visibility, decoherence, Bell violations, and data selection is in general not known. The problem is aggravated by the aim of experimenters for high visibility. Because this, in turn, may destroy the fair sampling of their experimental data. The most efficient method to gain a proper understanding of the relation between the important parameters in such an experiment are numerical simulations.

In this paper we simulate experiments over the whole range of experimental parameters. The parameter space is determined by decoherence and by the value of $\Delta s$. We simulate measurements from fully coherent to fully decoherent photon beams. The PBS parameters in one limit ($\Delta s = 0$) allow an arbitrarily precise resolution of angles at 45°. In the opposite limit ($\Delta s = 0.5$) not a single photon will be measured, because the limit is outside the range of possible values. The paper is structured as follows: first we shall briefly describe the setup of the experiment and explain, why the diagonal in a PBS is such a critical angle. Then we shall introduce the numerical model and explain its connection to experimental parameters. And finally, we shall present our simulations of EPR experiments under realistic conditions over the whole parameter space of the model.

The main idea which underlies the numerical model is a one to one correspondence between the model and the experiment for single events. It is, for example, conceivable that agreement between experiments and simulations could be obtained only after all results have been summed up. In this case the statistics enter the picture in the distribution of single events, which may be different for the model and the experiment. In fact, no successful model on this basis has ever been developed. On the contrary, the only local realistic model within this framework does not reproduce experiments. In this model the total probability $P(\alpha, \beta)$ of a coincidence between two polarizers set to $\alpha$ and $\beta$, respectively, is given by the sum over $\cos^2(\lambda - \alpha) \cos^2(\lambda - \beta)$, where $\lambda$ is the hidden angle of polarization. The reason for the disagreement is that in the experiments we do not sum up products of trigonometric functions, which in principle can be arbitrarily low, but we sum up integer coincidences on both measurement devices. In effect, we make the results digital on the level of single photons and thus introduce a cutoff in our trigonometric functions. This cutoff is due to the PBS.

In experiments with entangled photons a pair of photons is emitted from a common source. After the polarization of one photon is altered by an angle $\alpha$ both photons are analyzed at their respective PBS. In our model we simulate this feature by a switch at the critical angle of the PBS of $|\lambda - \alpha| = 45°$. The polarizer beam splitter projects a given field vector of the electromagnetic field.
onto two orthogonal directions, say $\alpha_1$ and $\alpha_2$. The output of one channel is the projection onto one axis ($+$), the intensity is $\cos^2(\phi_1 - \alpha_1)$, where $\phi_1$ is the polarization angle of the electromagnetic field. The projected intensity onto the other axis ($-$) is $\sin^2(\phi_1 - \alpha_1)$, since $\alpha_2$ is perpendicular to $\alpha_1$. If we set $\alpha_1 = 0$ and $\alpha_2 = \pi/2$, then if $\phi_1 = \pi/4$ we get the same intensity on both outputs ($+$) and ($-$). This case must be excluded, since it is not compatible with the desired results in quantum mechanics, where we have either ($+$) or ($-$), but never both. For this reason we include a threshold to account for the elimination of undesired results. The threshold $\Delta s$ simulates the way, the combination of a PBS and a detector operates. Statistics enter our model due to an $\Delta$ for the elimination of undesired results. The threshold $\Delta s$ is not compatible with the desired results in quantum mechanics, because they entail incompatible with quantum mechanics, because they entail

It should be noted that the probability $P(\alpha, \beta)$ of the ideal measurement can also be described as an integral of two factored functions $\bar{A}(\phi_1, \alpha)$ and $\bar{B}(\phi_2, \alpha)$. In the ideal case these functions for photons of perpendicular polarization ($\phi_2 = \phi_1 + \pi/2$) are given by:

$$\bar{A}(\phi_1, \alpha) = \frac{1}{2} \left[ \text{sign} \left[ \cos^2(\phi_1 - \alpha) - \frac{1}{2} \right] + 1 \right]$$ (1)

$$\bar{B}(\phi_2, \beta) = \frac{1}{2} \left[ \text{sign} \left[ \sin^2(\phi_1 - \beta) - \frac{1}{2} \right] + 1 \right]$$ (2)

$$P(\alpha, \beta) = \frac{1}{2\pi} \int_{0}^{2\pi} d\phi_1 \bar{A}(\phi_1, \alpha) \bar{B}(\phi_2, \beta)$$ (3)

The model is deterministic in the sense that it is uniquely determined by the angles $\alpha$ and $\beta$, and that it is a unique integral over the angle of polarization $\phi_1$ (the local hidden variable). In this formulation we encounter no longer any element of randomness. But the model is also compatible with the fundamental assumption in Bell’s inequalities. The inequalities, however, are not compatible with quantum mechanics, because they entail infinitely precise measurements even for non-commuting observables. Since operators of different settings do not commute, the spin states at different settings cannot be simultaneously eigenstates of the system. Thus it is impossible to obtain the limit of the standard inequalities within the framework of quantum mechanics.

A simulation run starts by an initialization of the random number generator. The generator is initialized only once, at the beginning of a simulation cycle. The random number is mapped onto the initial phase from 0 to $2\pi$ of the photon pair. Simulations are generally made with a phase difference of $\pi/2$ between the angles of polarization of photon one and photon two. We introduce decoherence as a random segment of the optical path of a photon. 100 % decoherence, for example, means that half a wavelength of the optical path is random. After covering the distance to polarizer one and two the photons are measured. We assumed, without lack of generality, that both distances are integer multiples of the wavelength. After a single pair has been measured, we record the coincidences ($++,+-,-+,–$). The procedure is repeated for all pairs, then the polarization angle of device one is changed by $\pi/100$. A run ends, when all pairs at the final position of polarizer one have been measured ($\pi$). In the simulation we recorded the correlations of 2000 pairs of photons. We vary decoherence from 0% to 100%, and the threshold of the polarizer from 0.0 to 0.5. The results displayed, the correlation function, the visibility, and efficiency plots are all derived from $N_{++}$. One key requirement in EPR experiments is the space like separation of the two measurements. In general this involves an optical waveguide between the source of the photon pair and the polarizers. Unless all components in such an experiment are cooled to temperatures near 0K and decoupled from their environment, we expect the components of the optical system to oscillate. This oscillation introduces randomness into the correlation measurements, which has the same effect as decoherence of the electromagnetic field of the photon. Any realistic theoretical model must include a random optical path of at least 5% or about 10 - 20nm, if the laser operates with visible light. This translates into about 10% decoherence. Depending on the setup and the experimental precision much higher decoherence seems possible. In Fig. 1 we show the correlation function for the whole range of decoherence from 0% (fully coherent fields) to 100% (fully random polarization angles). As expected, the correlation function becomes a straight line in the limit of full decoherence. In the intermediate region (10 - 50 % decoherence) it is sinusoidal, while the ideal correlation is a sawtooth. It should be noted that the phase is a hidden variable in all measurements, also of linear polarization. Because the polarization measurements at a device which combines polarizer beam splitters and photodetectors is an energy measuring device. And energy, from a field theoretical point of view, is intensity, which depends on the phase at the point of measurement.

In all experiments efficiency is less than 10% and efficiency decreases due to increased polarizer thresholds. This decrease is close to linear in the high efficiency range. Efficiency in terms of the polarizer threshold is plotted in Fig. 1. The curve shows the statistical noise due to the random variables in our simulation. Given the values we obtain, the main reason for the lack of efficiency in the measurements should not be the polarizer threshold. For realistic values around 0.1, values which can be deduced from the visibility in a measurement, the efficiency is still around 80%. We thus have to conclude that the experimental settings are not the main reason for lacking efficiency. However, if the efficiency decreases with a change of experimental parameters, one possible reason is the increase of the polarizer threshold. And this, in turn, changes the most

2
important values measured in such an experiment.

One of these values is the visibility of the correlation function. It is defined as \((\max - \min)/(\max + \min)\), where \(\max\) (\(\min\)) denote the maximum (minimum) number of coincidences over all polarizer settings. Essentially, the visibility can be increased by reducing the minimum count to a value close to zero. The increase of the polarizer threshold has exactly this effect. In Fig. 3 we show the result of our simulation. The visibility is color coded and emphasized by contour lines. Even in case of decoherence of more than 20% it is possible to obtain 95% visibility in the experiment if the polarizer threshold is raised to 0.13. The experimental values of Weihs et al. [4], for example, point to a decoherence of 10 – 12% and a threshold of 0.1 – 0.12. It seems quite unexpected that the parameter range with a visibility of more than 99% covers roughly one fourth of the whole parameter space. The requirement of high visibility therefore does not decisively limit the parameter space of real measurements. Visibility seems thus unsuitable as a measure of experimental precision. As a technical point we remark that the statistical spread from one value to the next shows a high fluctuation. This is due to a multiplication of statistical deviations in the computation of the visibility.

Experimentally, one seeks to obtain high visibility in an experiment before embarking on the actual run, where valid data are measured. In this case it is to be expected that practically all the data in these experiments are obtained in the high visibility range. Incidentally, this is the same range where the Bell inequalities are maximally violated. For the calculation of the Bell violations we fixed polarizer one and two at the angles of maximum violation \((0\degree, 22.5\degree, 45\degree, 67.5\degree)\), and simulated the measurement of 10000 pairs of photons for every datapoint. We computed the Bell violation from the coincidence counts using the formulation of Clauser et al. [11]. From this value we have subtracted 2.0 and show only the values above zero. The simulation is shown in Fig. 4. It can be seen that every threshold allows for a wide range of Bell violations. For a threshold value > 0.2, for example, the inequalities can be violated by 0.0 – 2.0, depending on the decoherence of the photon beams. The contour lines describe the violation. Comparing with the previous plot it can be seen that the condition of high visibility alone guarantees that the Bell inequalities are maximally violated. If a violation exceeding 0.82 (the value in quantum mechanics) has so far not been observed, we refer this to the low coherence in the measurements and the requirement of high efficiency. The former generally reduces the violation, while the latter makes a high threshold undesirable.

A synopsis of all data obtained in the simulations reveals that the experimental results can be changed arbitrarily by changing the parameters in the experiments. Generally speaking, EPR experiments if analyzed with our model exhibit a much larger span of possible parameters and results than presently assumed. From our simulations we conclude that there will be not one value for the Bell violation, which can be compared with theoretical predictions, but there should be many, depending entirely on the parameters in the experiment. In this sense the simulation allows, for the first time, to check quantum mechanics in controlled EPR experiments. If the Bell violation decreases from an initial value as experimental conditions become more ideal, then the predictions in quantum mechanics must be wrong. Increasing coherence can be obtained e.g. by cooling down the experimental components to very low temperature and by reducing their distances to a minimum. If, however, increasing coherence and efficiency leads to a higher violation of the Bell inequalities, then the present local model of EPR measurements must be inadequate. The actual point in the parameter space of such an experiment can be obtained from experimental values. Increasing visibility and efficiency is related to to lower threshold values and lower decoherence, in short, to more ideal experimental conditions. With these experiments also the superposition principle can be tested experimentally. The principle is fundamental for all current standard frameworks, it is based on linearity of fields and wavefunctions between two points of measurement. The principle is violated in our model, because the two measurements have been factored. We can therefore use the predictions of our theoretical model for a first experimental test of superposition. In case the local hidden variables model is correct, which can be checked by the above procedure, the superposition principle is not generally applicable: a consequence which may require major adjustment in current theories.

In summary we have shown that the whole parameter space of EPR experiments can be analyzed by numerical simulations within a local hidden variables model. We pointed out that the Bell inequalities are maximally violated in all cases, where an experiment possesses high visibility. And we suggested controlled EPR experiments to detect a possible deviation of quantum mechanical predictions from the results of measurements.

Acknowledgements. - The EPR problem has been widely discussed with colleagues over the last year. In particular I am indebted to G. Adenier, D. Bowler, A. Fisher, J. Gavartin, J. Gittings, and R. Stadler. Computing facilities at the HiPerSPACE center were funded by the Higher Education Funding Council for England.

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935)
[2] J.S. Bell, Physics 1, 195 (1964)
[3] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 61, 91 (1982)
[4] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998)
[5] A. Aspect, Nature 398, 189 (1999)
[6] M. A. Rowe et al. *Nature* **409**, 791 (2001)
[7] B. d’Espagnat, *Conceptual Foundations of Quantum Mechanics*, Addison Wesley, New York (1989)
[8] W.A. Hofer, Simulation of Einstein-Podolsky-Rosen experiments in a local hidden variables model with limited efficiency and coherence, *Phys. Rev. Lett.*, submitted (2001)
[9] W.H. Furry, *Phys. Rev.* **49**, 393 (1936)
[10] W.H. Press, B.P.Flannery, S.A. Teukolsy, W.T. Vetterling, *Numerical Recipes*, Cambridge Univ. Press, Cambridge (1987)
[11] J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, *Phys. Rev. Lett.* **23**, 880 (1969)

**FIG. 1.** Variation of the correlation function with decoherence. The value of the correlation function is color coded. For fully coherent field we obtain a sawtooth, while fully decoherent photon beams will show only statistical noise.

**FIG. 2.** Efficiency of an EPR experiment for varying polarizer threshold. The efficiency decreases as the threshold is increased.

**FIG. 3.** Visibility in an EPR experiment. Even in experiments, where the photon beams are not fully coherent, we can obtain close to 100 % visibility by increasing the polarizer threshold. The visibility is color coded.

**FIG. 4.** Violation of the Bell inequalities depending on decoherence and polarizer threshold. The violation is color coded. The Bell inequalities are violated in all realistic experimental setups. The violation depends crucially on the experimental parameters and can reach a value of up to 2.0.