Sliding Mode Output Feedback Controller Design of Discrete-Time Markov Jump System Based on Hidden Markov Model Approach

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ABSTRACT This paper investigates the problem of sliding mode controller design based on output feedback for discrete-time Markov jump systems. Considering the typical asynchronous phenomenon for jump systems, here we adopt the Hidden Markov Model (HMM) to quantify the asynchronous degree. Based on this, a static output feedback sliding surface is constructed. Sufficient conditions in terms of bilinear matrix inequalities are proposed ensuring the sliding motion dynamic to be stochastically stable and its $H_\infty$ performance for the considered discrete time Markov jump systems. Moreover, the reachability of the sliding mode surface is ensured by a predesigned control law. The whole design scheme is presented by using an iterative algorithm. Finally, simulation results are presented to illustrate the effectiveness of the design methodology.

INDEX TERMS Sliding mode control, HMM, output feedback, iterative algorithm.

I. INTRODUCTION

In the actual system, the change of working environment, the failure of parts and other factors often lead to the abrupt changes of system structure, which can be effectively described by the Markov jump system model. For decades, it has been widely used in economic system, flight control system and so on [1]–[6]. Early controller design for jump systems was carried out under the ideal assumption that all states of the target system could be obtained. However, it is not practical that all the modes of the target control system can be obtained due to technical limitations and other reasons. In this case, a large number of researchers have focused their attention on how to deal with a series of control or filtering problems when the modes of the system are constrained. In recent years, the HMM has been established to describe this kind of asynchronous phenomenon which is caused by incomplete mode information, and there have been many remarkable research results based on this model [7]–[10].

On the other hand, since it is often hard to get the actual state of the system in practical engineering applications, output feedback approach is a relatively easier way to conduct the control process. For many years output feedback control has been studied and has made great progress [9], [11], [12]. For example, The design of $H_\infty$ static output feedback control for discrete time Markov jump linear systems is discussed in [13]. In [14], a static output feedback controller has been developed for uncertain discrete time systems and the work in [9] and [16] have discussed the output feedback control method based on the HMM and presented a new LMI condition.

Sliding mode control as an effective robust control method has been widely used in electronic circuits, aerospace, robots and other engineering applications [16]–[20]. There are many methods of sliding mode control, including the use of singular system method. As we all know, the design of the sliding mode scheme is mainly divided into two stages. The first is to select a suitable sliding mode surface and construct the sliding motion dynamics. The second is to design a control law to ensure the reachability of the sliding surface within a limited time and keep the system state on the sliding mode.
surface to make the system achieve expected performance. In [21] sliding mode control has been successfully applied to the switching system with a space robot manipulator model. In addition, [22] and [23] discuss the problem of output feedback sliding control for uncertain linear systems and systems with state time-varying delay respectively. However, the above sliding mode control results are based on the condition that the controller is synchronized with the system. In actual situations, data loss, time delay and the instability of the network system and other factors often lead to the occurrence of asynchronous phenomenon. So keeping synchronization at all times is an ideal assumption in the actual system, which stimulates us to study the sliding mode control of the target system under the asynchronous case.

Up to now, the research on asynchronous sliding mode control of discrete-time Markov jump systems (MJSs) is incomplete according to the literature reviewed so we focus on enriching the sliding mode control theory of Markov jump nonlinear function that meets:

\[ \|\xi(x(k), k)\| \leq \rho_1\|x(k)\| + f(k, y) \] (2)

where \( \rho_1 \in [0, 1] \) and \( f(k, y) \) is a scalar valued function. The Markov chain \( \{\phi_k; k \geq 0\} \) take values in \( M = \{1, 2, \ldots, M\} \) and the related transition probability matrix \( \Theta = [\pi_{ij}] \), with \( \pi_{ij} \geq 0 \) and \( \sum_{i \in M} \pi_{ij} = 1 \), for all \( i, j \in M \), is given by

\[ \Pr(\phi_{k+1} = j|\phi_k = i) = \pi_{ij}. \]

Under the condition that \( \text{rank}(C_iB_i) = n_w \), there exists a state transformation \( z(k) = T_i x(k) \), where \( T_i \) is a non-singular matrix [22]. The system (1) could be written as the following form:

\[
\begin{align*}
\dot{z}(k + 1) &= \tilde{A}_1 z(k) + \tilde{D}_1 w(k) + \begin{bmatrix} 0 \\ B_{21} \end{bmatrix} (u(k) + \xi(x(k), k)) \\
y(k) &= [0 \ C_{21}] z(k)
\end{align*}
\] (3)

where \( B_{21} \in \mathbb{R}^{n_w} \) and \( C_{21} \in \mathbb{R}^{n_x \times n_w} \) is invertible, and

\[
\tilde{A}_1 = T_i A_i T_i^{-1}, \quad \tilde{D}_1 = \begin{bmatrix} \tilde{D}_{11} \\ \tilde{D}_{21} \end{bmatrix}.
\]

The first equation of (3) can be written as

\[
\begin{align*}
z_1(k + 1) &= \tilde{A}_{11} z_1(k) + \tilde{A}_{12} z_2(k) + \tilde{D}_1 w(k) \\
z_2(k + 1) &= \tilde{A}_{21} z_1(k) + \tilde{A}_{22} z_2(k) + \tilde{D}_2 w(k) + B_{21} (u(k) + \xi(x(k), k))
\end{align*}
\] (4)

In this paper, we consider the asynchronous problem between the modes of the target system and the modes of the controller as is depicted in Fig. 1.

![Sliding mode controller](image)

**FIGURE 1.** Configuration of the control systems.

The parameter \( \{\sigma_k, k \geq 0\} \) represents the mode of controller and take values in \( N = \{1, 2, \ldots, N\} \). The HMM \( \psi(\phi_k, \sigma_k) \) satisfies the following conditional probability relations:

\[ \Pr(\sigma_k = l|\phi_k = i) = \alpha_{il}. \]

which characterizes the severity of asynchronous phenomenon and the random process based on this model will
be sent to the controller. Similar to $a_{ij}$, the conditional probability $a_{ij}$ is between 0 and 1, and $\sum_{i \in \mathcal{N}} a_{ij} = 1$ for all $i \in \mathcal{M}$.

A static output based asynchronous sliding mode surface can be synthesized as

$$s(k) = \left[ -K_f I_n \right] C_2^{-1} y(k)$$  

(5)

$K_f$ represents the control parameter to be solved. Then on the sliding surface, we have $s(k) = 0$, i.e.,

$$s(k) = \left[ -K_f I_n \right] \begin{bmatrix} C_1 & 0 & 0 & I_n \end{bmatrix} z(k) = 0$$  

(6)

with $C_1 = \begin{bmatrix} 0 & I_n & -n_a & 0 \end{bmatrix}$, which gives $z_2(k) = K_f C_1 z_1(k)$. Further we can obtain the following sliding motion dynamic:

$$z_{1}(k+1) = \hat{A} z_1(k) + \hat{D}_1 w(k)$$  

(7)

where $\hat{A} = \hat{A}_{11} + \hat{A}_{12} K_f C_1$.

In order to analyze the $\mathcal{H}_\infty$ performance of system (7), we take the measured output $z_m(k)$ as

$$z_{m}(k) = L z_1(k) + E_i w(k).$$  

(8)

Recall the following definitions and lemmas which will be needed for further analysis.

**Definition 1 ([25]):** For the sliding motion dynamic (7) with measured output equation (8) is SS with $\mathcal{H}_\infty$ performance index, i.e. under zero initial condition, $\|z(k)\|_2 < \gamma \|w(k)\|_2$ holds for all $w(k) \in \mathcal{L}_2[0, +\infty)$.

**Definition 2 ([26]):** For a given constant $\gamma > 0$, the sliding motion dynamic (7) with measured output equation (8) is SS with $\mathcal{H}_\infty$ performance index, i.e. under zero initial condition, $\|z(k)\|_2 < \gamma \|w(k)\|_2$ holds for all $w(k) \in \mathcal{L}_2[0, +\infty)$.

**Lemma 1 ([27]):** For given matrices $U$ and $V$, symmetric matrix $\Gamma$ with appropriate dimensions, the following statements are equivalent:

1. $U^T \Gamma U > 0$ where $U^T \tilde{U} = 0$.
2. $\Gamma + \text{Her}(U \Xi) > 0$ for some matrix $\Xi$.

Next we will focus on solving the parameter $K_f$ of the asynchronous output sliding surface and the design of a reaching motion controller.

**III. MAIN RESULTS**

In this section, we give the conditions that guarantee the asymptotically stability of the system (7) with measured output equation (8) and satisfy the $\mathcal{H}_\infty$ performance. An asynchronous sliding mode surface based on output knowledge is designed and its reachability analysis will be performed under a predefined controller.

**A. ASYNCHRONOUS OUTPUT-FEEDBACK SLIDING MODE SURFACE DESIGN**

**Theorem 1:** For a given scalar $\gamma > 0$, the sliding motion dynamic system (7) with measured output equation (8) is SS and with a predefined $\mathcal{H}_\infty$ attenuation index $\gamma$ if there exist symmetric matrices $P_i > 0$, and matrices $Q_{il}^3 > 0$, $\star \in \{1, 2, 3\}$, $H_{il}, X_i, Y_i, F_i$ such that for all $i \in \mathcal{M}, l \in \mathcal{N}$ the following inequalities hold

$$\begin{bmatrix} P_i & 0 & 0 \\ 0 & \gamma^2 I & 0 \\ 0 & 0 & \gamma^2 I \end{bmatrix} > \sum_{i \in \mathcal{N}} \alpha_{il} \begin{bmatrix} Q_{il}^1 & * & * \\ * & Q_{il}^2 & * \\ * & * & Q_{il}^3 \end{bmatrix},$$

$$H_{il}^\top \hat{A} H_{il} D_{1l} + \text{Her}(H_{il}) - \zeta_i(P) * \* *$$

$$\begin{bmatrix} L_i & E_i & 0 & I & * \\ \hat{F} & 0 & \hat{A}_{12}^T H_{il}^\top & 0 & \text{Her}(X_i) \end{bmatrix} > 0$$  

(10)

where $\zeta_i(P) = \sum_{j=1}^M \pi_j P_j$ and

$$\hat{A} = \hat{A}_{11l} + \hat{A}_{12l} F_i$$

$$\hat{F} = X_i F_l - Y_i C_1.$$  

Moreover, if the above inequalities have feasible solution, the asynchronous sliding surface can be given by

$$s(k) = \left[ -X_{il}^{-1} Y_i I_n \right] C_2^{-1} y(k)$$  

(11)

**Proof:** Choose the Lyapunov function as follows:

$$V(k, z_1(k)) = z_{1}^T(k) P_i z_1(k)$$

With $w(k) = 0$, we can obtain that

$$\Delta V(k, z_1(k)) = E [V(k + 1, z_1(k + 1))] - V(k, z_1(k))$$

$$= z_{1}^T(k) \sum_{i \in \mathcal{N}} \alpha_{il} (\hat{A}^T \zeta_i(P) \hat{A} - P_i) z_1(k)$$

Hence $\Delta V(k, z_1(k)) < 0$ can be ensured by

$$\hat{A}^T \zeta_i(P) \hat{A} - P_i < 0$$  

(12)

Further, under the zero initial condition, we can have the following index function

$$\mathcal{J}_\gamma = E \left\{ \sum_{k=0}^{\infty} z_{m}^T(k) z_m(k) - \gamma^2 w^T(k) w(k) \right\}$$

$$\leq \sum_{k=0}^{\infty} \eta^T(k) \sum_{i \in \mathcal{N}} \alpha_{il} \Theta_{il} \eta(k)$$

where $\eta(k) = \left[ z_{1}^T(k) \, w^T(k) \right]^T$ and

$$\Theta_{il} = \begin{bmatrix} \hat{A}^T \zeta_i(P) \hat{A} + L_i^T L_i - P_i \\ \hat{D}^T_{1l} \zeta_i(X) \hat{D}_{1l} + E_i^T E_i - \gamma^2 I \end{bmatrix}$$

which shows that if $\alpha_{il} \Theta_{il} < 0$, $\mathcal{J}_\gamma < 0$ and (12) hold, further guaranteeing the stability of the sliding motion dynamic with a predefined $\mathcal{H}_\infty$ performance.

By Schur complement, $\alpha_{il} \Theta_{il} < 0$ is equivalent to

$$\begin{bmatrix} P_i & 0 & 0 \\ 0 & \gamma^2 I & 0 \\ 0 & 0 & \gamma^2 I \end{bmatrix} > \sum_{i \in \mathcal{N}} \alpha_{il} \hat{A} \hat{D}_{1l} \zeta_i(P)^{-1} * \* *$$

(13)

where $\hat{A} = \hat{A} + \hat{A}_{12l} R_{il}$ with $R_{il} = K_i C_1 - F_i$.  

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Adopt the change of variables $X_iK_i = Y_i$, then we can rewrite (10) in the following form:

$$\Phi_i + \text{Her}(UX_iV^T) > 0$$  \quad (14)

where $U = [0 \ 0 \ 0 \ I]^T$, $V^T = [-R_{ii} \ 0 \ 0 \ I]$ and

$$\Phi_i = \begin{bmatrix} Q_{l_i}^1 & * & * & * \\ Q_{l_i}^2 & Q_{l_i}^3 & * & * \\ H_{il}A H_{il}D_{il} I_{l_i} \text{Her}(H_{il}) - \zeta_i(P) & * & * \\ L_i & E_i & 0 & I \end{bmatrix}$$

Now we define

$$\tilde{N}(R_{il}) = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \\ R_{il} & 0 & 0 & 0 \end{bmatrix}$$

and we can verify that $V^T \tilde{N}(R_{il}) = 0$. By using Lemma 1, it can be found that (10) is equivalent to

$$\tilde{N}(R_{il})^T \Phi_i \tilde{N}(R_{il}) > 0$$  \quad (15)

i.e.,

$$\begin{bmatrix} Q_{l_i}^1 & * & * & * \\ Q_{l_i}^2 & Q_{l_i}^3 & * & * \\ H_{il}A H_{il}D_{il} I_{l_i} \text{Her}(H_{il}) - \zeta_i(P) & * & * \\ L_i & E_i & 0 & I \end{bmatrix} > 0$$  \quad (16)

where $\hat{A} = \hat{A} + \hat{A}_{12}R_{il}$. Note that we always has $\text{Her}(H_{il}) - \zeta_i(P) < H_{il}\zeta_i(P)^{-1}H_{il}^T$ which indicates

$$\begin{bmatrix} Q_{l_i}^1 & * & * & * \\ Q_{l_i}^2 & Q_{l_i}^3 & * & * \\ H_{il}A H_{il}D_{il} I_{l_i} \zeta_i(P)^{-1} & * & * \\ L_i & E_i & 0 & I \end{bmatrix} > 0$$  \quad (17)

By making congruence transformation with diag$[I, I, H_{il}^{-1}, I]$, we get

$$\begin{bmatrix} Q_{l_i}^1 & * & * & * \\ Q_{l_i}^2 & Q_{l_i}^3 & * & * \\ \hat{A} \hat{D}_{il} \zeta_i(P)^{-1} & * & * \\ L_i & E_i & 0 & I \end{bmatrix} > 0$$  \quad (18)

Then it follows from (10) that (13) holds, which shows that Theorem 1 holds, thus the proof is completed.

Remark 1: It should be noted that Theorem 1 cannot be solved directly since the conditions are not linear. It is a bilinear problem as discussed in [28] and [29]. Following a similar line with the above proof, we can obtain that

$$U^T \tilde{N}(0) = 0,$$

with $U^T \tilde{N}(0) = 0$, i.e.,

$$\begin{bmatrix} P_i & * & * & * \\ 0 & Y & * & * \\ \sum_{l \in N} \alpha_{il}(\hat{A}_{11i} + \hat{A}_{12}F_l) \hat{D}_{il} \zeta_i(P)^{-1} & * & * \\ L_i & E_i & 0 & I \end{bmatrix} > 0$$  \quad (19)

which shows that the parameter $F_l$ stochastically stabilizes the matrix pair $(\hat{A}_{11i}, \hat{A}_{12}F_l)$ and can be seen a state feedback gain of the sliding motion dynamic (7). Therefore with a feasible state feedback gain $F_l$, the conditions can be solved by using a coordinate descent algorithm which will be presented later.

B. ASYNCHRONOUS SLIDING MODE CONTROLLER DESIGN

In this subsection, we design the control law to guarantee that the states of system (4) can be driven onto the sliding surface. To this end, we first define a sliding patch:

$$\Omega = \{ z(k) \in \mathbb{R} : \|z(k)\| \leq \sigma \}$$  \quad (20)

then we can obtain the following theorem.

Theorem 2: The reachability of the asynchronous switching surface in finite time locally for $z(k) \in \Omega$ by the following sliding mode controller law:

$$u(k) = -B_{z1}^{-1} \left\{ vs(k) + (\beta_i + \|B_{2l}\| f(k, y)) \frac{s(k)}{\|s(k)\|} \right\}$$  \quad (21)

where

$$\beta_i \\
\geq \sigma \left( \sum_{d=1}^{N} \alpha_{id} \left\{ \sum_{j=1}^{M} \pi_{ij}\|[-K_dC_1 I] \hat{A}_i\| + \sum_{j=1}^{M} \pi_{ij}\rho_{ij}\|B_{2l}\|\|T_{1j}^{-1}\| \right\} \right) \\
+ \sum_{j=1}^{M} \pi_{ij}\|[-K_dC_1 I] D_{il}\| \bar{w})$$

and $v$ is a positive scalar.

Proof: Define the Lyapunov function for the switching surface $s(k)$:

$$V_1(k) = \frac{1}{2} s^T(k)s(k)$$

It can be obtained that

$$\Delta V_1(k) = E[V_1(k + 1)] - V_1(k)$$

$$= E\left\{ \frac{1}{2} s^T(k+1)s(k+1) - \frac{1}{2} s^T(k)s(k) \right\}$$

$$= s^T(k)\Delta s(k) + \frac{1}{2} \Delta s^T(k)\Delta s(k)$$

where $\Delta s(k) = E[s(k + 1)] - s(k)$, and

$$E[\Delta s(k)] = E[s(k + 1)] - s(k)$$

$$= \sum_{d=1}^{N} \alpha_{id} \left\{ \sum_{j=1}^{M} \pi_{ij}[{-K_dC_1 I}]z(k + 1) \right\} - s(k)$$

It follows from sliding mode control law (21) that

$$\Delta V_1(k) = s^T(k)\{E[\Delta s(k)] - s(k)\} + \frac{1}{2} \Delta s^T(k)\Delta s(k)$$
\[ s^T(k) \left\{ \sum_{d=1}^{N} \alpha_{id} \left[ \sum_{j=1}^{M} \pi_{ij} [ -K_d C_1 I ] \tilde{A}_i \right] \right. \\
+ \tilde{D}_i w(k) \left] \right. + \sum_{j=1}^{M} \pi_{ij} [ [ -K_d C_1 I ] \tilde{A}_i ] \right. \\
- s(k) \right) + \frac{1}{2} \Delta s^T(k) \Delta s(k) \\
\leq \sum_{d=1}^{N} \alpha_{id} \left( \sum_{j=1}^{M} \pi_{ij} \| [ -K_d C_1 I ] \tilde{A}_i \| \\
+ \sum_{j=1}^{M} \pi_{ij} \| B_{2j} \| \| T_{ij}^{-1} \| \| \sigma \| \| \tilde{D}_i w(k) \| \\
- v \| s(k) \|^2 - \beta_i \| s(k) \| \right) - \| s(k) \|^2 \\
+ \frac{1}{2} \Delta s^T(k) \Delta s(k) \\
\leq - \sum_{d=1}^{N} \alpha_{id} (v I + I) \| s(k) \|^2 + \frac{1}{2} \Delta s^T(k) \Delta s(k). \]

According to the condition mentioned in Theorem 2 we can obtain that \( \Delta V_1(k) \leq - \sum_{d=1}^{N} \alpha_{id} (v I + I) \| s(k) \|^2 + \frac{1}{2} \Delta s^T(k) \Delta s(k) \). By adjusting the parameter \( v \) according to the actual situation and \( \Delta s(k) \) is reasonable norm-bounded, it can be guaranteed that \( \Delta V_1(k) \) is a negative scalar. Therefore, under the action of the control law (21), the state trajectory of the system (7) can be forced to the asynchronous sliding surface within a limited time, and it can be locally reachable in the predetermined sliding patch (20). This proof is completed.

Inspired by the work in [28], we conclude the asynchronous sliding mode controller design scheme as follows based on the discussions in Theorem 1 and 2:

Remark 2: For solving the parameter \( K_1 \), it is transformed into a static output feedback problem. Compared with the work in [22] where the slack variable is chosen in a special form. However, an iterative method is adopted here to solve the parameter in the sliding mode surface. Compared to [22], the method adopted here is more direct to find the feasible solution and reduce the conservatism to a certain extent.

### IV. NUMERICAL EXAMPLES

In this section, we present an illustrative examples to demonstrate the obtained results in this paper.

**Example:** Consider a Markov jump system of the form (1) with its parameters given by the following:

\[
A_1 = \begin{bmatrix} 0.9997 & 0.001 & 0 \\ -0.0171 & 0.9998 & -0.0122 \\ 0 & 0 & 0.9934 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9866 & 0.172 & 0 \\ -0.0171 & 0.9910 & -0.0122 \\ 0 & 0 & 0.9952 \end{bmatrix}
\]

\[
B_1 = \begin{bmatrix} 0 \\ 0 \\ 0.0071 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.0066 \end{bmatrix}
\]

\[
L_1 = L_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
\]

\[
E_1 = E_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T, \quad w(k) = 0.5e^{-k},
\]

\[
D_1 = D_2 = \begin{bmatrix} 0.2 & 0.2 & 0.2 \end{bmatrix}^T.
\]
The system contains two modes and the transition matrix is set as
\[
[\pi_{ij}] = \begin{bmatrix}
0.6 & 0.4 \\
0.3 & 0.7
\end{bmatrix}
\]
Considering the asynchronous scheme, the conditional probability is set as
\[
[\alpha_{il}] = \begin{bmatrix}
0.9 & 0.1 \\
0.2 & 0.8
\end{bmatrix}
\]
For the purpose of simple calculation, set the function of the transformation matrix \(T_1 = T_2 = I_{3 \times 3}\), we can get
\[
A_{111} = \begin{bmatrix}
0.9997 & 0.0010 \\
-0.0171 & 0.9998
\end{bmatrix}, \quad A_{121} = \begin{bmatrix}
0 \\
-0.0122
\end{bmatrix}
\]
\[
B_{21} = 0.0071
\]
\[
A_{112} = \begin{bmatrix}
0.9866 & 0.1720 \\
-0.0171 & 0.9910
\end{bmatrix}, \quad A_{122} = \begin{bmatrix}
0 \\
-0.0122
\end{bmatrix}
\]
\[
B_{22} = 0.0066
\]
Set the positive scalar \(v = 0.5\) in (21) and take the initial iteration value as
\[
F_1 = \begin{bmatrix}
95.1373 & 94.4750
\end{bmatrix}, \quad F_2 = \begin{bmatrix}
99.4366 & 99.6295
\end{bmatrix}
\]
through the iterative algorithm we can find the parameter
\[
K_1 = 0.0567, \quad K_2 = 0.0579
\]
and the sliding surface is
\[
s(k) = \begin{cases}
-0.0567 y(k), & l = 1; \\
-0.0579 y(k), & l = 2,
\end{cases}
\]
which gives a guaranteed \(H_\infty\) performance index \(\gamma_{\min} = 1.0173\) for the system (7). Fig. 2 shows asynchronous phenomenon between the system and the sliding mode controller. In addition, we can see that the unforced system is not stable from Fig. 3. When the asynchronous sliding mode controller has been implemented, we can obtain the trajectories of the states presented in Fig. 4. The Fig. 5 shows the trajectories of the measured output \(z_{m1}\) and \(z_{m2}\). It can be seen from the simulation results that the original unforced system gradually tends to stable under the action of asynchronous sliding mode controller, which also proves the effectiveness of the proposed method.
V. CONCLUSION

This paper mainly studies the output feedback sliding control problem of discrete-time MJSs system based on HMM. Use the form of conditional probability to process the controller design when the mode information of the system is constrained. Then an asynchronous sliding surface and a reaching controller that depends on the output information are constructed. An iterative algorithm is adopted to solve the bilinear conditions. Finally, a numerical example is used to verify the validity of the obtained results. It is worth noting that one of the future research work is to extend the obtained results to the nonsynchronous fuzzy sliding mode control of the fuzzy jump systems.

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