A New Method for Detecting Axion With Cylindrical Superconductor

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We propose a method for searching dark matter axion in axion-photon conversion. We consider a superconductor of cylindrical shape under strong magnetic field. The dark matter axion generates oscillating electric field which induces oscillating superconducting current in the surface of the superconductor. The current gives rise to dipole radiation with the frequency \( m_a/2\pi \) given by axion mass \( m_a \). We show that the radiation flux generated by the current is of the order of \( 10^{-18} \) W under magnetic field \( \sim 3T \) in the case of radius \( \sim 1cm \) and length \( \sim 10cm \) of the cylindrical superconductor. The large amount of the radiation flux arises because Cooper pairs with large number density ( \( \sim 10^{22}/cm^3 \) ) is present in the superconductor. We can simultaneously search wide bandwidth of the radio frequency with existing radio telescope.

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Axion has been introduced \([1]\) for solving strong CP problem in QCD. The axion is the Nambu-Goldstone boson\([1]\) associated with U(1) Peccei-Quinn symmetry. The symmetry is chiral and naturally solves the strong CP problem: The problem is why the CP violating term \( G_{\mu,\nu} \bar{G}^{\mu,\nu} \) is absent in QCD Lagrangian where \( G_{\mu,\nu} \) ( \( \bar{G}^{\mu,\nu} \) ) denotes (dual) fields strength of gluons. The axion is the phase of a complex scalar field carrying the U(1) charge. Although the axion is the Nambu-Goldstone boson, it acquires its mass \( m_a \) through chiral anomaly because the Peccei-Quinn symmetry is chiral. Thus, instantons in QCD give rise to the mass of the axion. The axion is called as QCD axion. In the paper we mainly consider the QCD axion. The axion is generated in early universe and becomes a dominant dark matter in the present universe when the Peccei-Quinn symmetry is broken after inflation.

In the present day, the axion is one of the most promising candidates for the dark matter. There proceed several projects for the detection of the axion. There are three types of the projects; Haloscope, Helioscope and others. The dark matter axion in our galaxy is searched with Haloscope, while axion produced in the Sun is searched with Helioscope. Haloscope projects are ADMX\([2]\), CARRACK\([3]\), HAYSTAC\([4]\), ABRACADABRA\([5]\), etc. Helioscope projects are CAST\([6]\), SUMICO\([7]\), XENON1T\([8]\), etc.. The others are LSW\([9]\) (Light Shining through a Wall), VMB\([10]\) (Vacuum Magnetic Birefringence) etc. Since the axion mass is unknown, the mass range in the search is very wide, \( 10^{-14}eV \sim 1eV \). But we assume QCD axion in this paper and expect that the appropriate mass range is of the order of \( 10^{-6}eV \sim 10^{-4}eV \).

In previous papers\([11]\) we have explained origin of fast radio bursts (FRB) assuming the QCD axion. The characteristic features of the FRBs\([12]\) are their frequency \( \sim 1GHz \) and very short duration \( \sim 10^{-3}s \). In our model the bursts arise in the collision of axion star\([13]\) and neutron star, or magnetized accretion disk around black hole. (The axion star causing the FRBs is gravitationally loosely bound state of the axions.) The radiations are photons converted from axions under strong magnetic fields of these astrophysical objects. We have obtained\([14]\) the axion mass \( \sim 7 \times 10^{-6}eV \) by examining spectral feature of the repeating FRB121102\([15]\). Our proposal for the detection of the dark matter axion is focused to such a mass range.

Our proposal is a type of Haloscope. The point in our proposal is that electric field induced by dark matter axion under magnetic field generates oscillating electric current in the surface of superconductor. The current is superconducting and produces the large amount of dipole radiations. This is because Cooper pairs with the large number density are present as carriers of the superconducting current. Our system is composed of a superconductor of cylindrical shape and a receiver for the detection of the radiations. We do not need to use resonant cavity. We need only sensitive radio receiver like radio telescope. We shall show in the paper that the larger amount of the radiation flux is generated by the cylindrical superconductor than those generated in resonant cavities of Haloscope experiments. We explain why such large amount of radiation flux is obtained in our apparatus, compared with resonant cavity experiments.

First we show that the coherent axion induces an electric field under a magnetic field. It is well known that the axion \( a(\vec{x}, t) \) couples with both electric \( \vec{E} \) and magnetic fields \( \vec{B} \) in the following,
the oscillating current has the same spectrum as that of the axion. Typically, it is supposed to be given by
\[ p_{\text{osc}} \sim \frac{E_{\text{ax}} \cdot B}{a \pi} \]
with the decay constant \( f_a \) of the axion and the fine structure constant \( \alpha \approx 1/137 \), where the numerical constant \( k_a \) depends on axion models; typically it is of the order of one. The standard notation \( g_{a\gamma\gamma} \) is such that \( g_{a\gamma\gamma} = k_a \alpha / f_a \pi \approx 0.14(m_a / \text{GeV}^2) \) for DFSZ model\(^{[10]}\) and \( g_{a\gamma\gamma} \approx -0.39(m_a / \text{GeV}^2) \) for KSVZ model\(^{[17]}\). In other words, \( k_a \approx 0.37 \) for DFSZ and \( k_a \approx -0.96 \) for KSVZ. The axion decay constant \( f_a \) is related with the axion mass \( m_a \) in the QCD axion; \( m_a f_a \approx 6 \times 10^{-8} \text{eV} \times 10^{12} \text{GeV} \).

The interaction term in eq. (1) slightly modifies Maxwell equations,
\[ \partial \cdot \vec{E} + \frac{k_a \alpha \partial \cdot (a(x,t) \vec{B})}{f_a \pi} = 0, \quad \partial \times (\vec{B} - \frac{k_a \alpha a(x,t) \vec{E}}{f_a \pi}) - \partial_t (\vec{E} + \frac{k_a \alpha a(x,t) \vec{B}}{f_a \pi}) = 0, \]
\[ \partial \cdot \vec{B} = 0, \quad \partial \times \vec{E} + \partial_t \vec{B} = 0. \]

From the equations, we approximately obtain the electric field \( \vec{E} \) generated by the axion \( a \) under the background homogeneous magnetic field \( \vec{B} \),
\[ \vec{E}_a (r,t) = -k_a a \frac{a(x,t) \vec{B}}{f_a \pi} \]
by assuming the parameter \( k_a a / f_a \) extremely small and small momenta \( p_a \) of the dark matter axion; \( p_a \ll m_a \). Typically, it is supposed to be given by \( p_a \sim 10^{-3} m_a \). The energy density of the dark matter axion is given by
\[ \rho_a = \frac{1}{2} (\dot{a}^2 + (\dot{\alpha})^2 + m_a^2 a^2) \approx m_a^2 a^2 \]
where \( a(t) = a_0 \cos(t \sqrt{m_a^2 + p_a^2}) \approx a_0 \cos(m_a t) \). The local energy density in our galaxy is supposed such as \( \rho_a \sim 0.3 \text{GeV cm}^{-3} \approx 2.4 \times 10^{-42} \text{GeV}^4 \). This gives extremely small parameter \( a / f_a \approx \sqrt{\rho_a / (m_a f_a)} \approx 10^{-19} \). The energy density also gives the large number density of the axions \( \rho_a / m_a \sim 10^{15} \text{cm}^{-3} (10^{-8} \text{eV} / m_a) \), which causes their coherence. Thus we may treat the axions as the classical axion field \( a \).

The Cooper pair in a superconductor under the electric field \( E_a (t) \) oscillates with the frequency \( m_a / 2 \pi \) according to the equation of motion, \( m_e \ddot{v} = qE_a \), where \( m_e = 2 m_a \) ( \( q = 2 e \) denotes the mass (electric charge) of the Cooper pair with electron mass \( m_e \) and charge \( e \). Note that the motion of the Cooper pair is not disturbed by impurities in the superconductor, so there is no dissipative term in the equation of motion. Thus, the superconducting current density is given by \( J = qn_v = \frac{q^2 E_a}{m_a m_e} \) with the number density \( n \) of the Cooper pair. \( n \sim 10^{22} \text{cm}^{-3} \). The current oscillates with the frequency \( m_a / 2 \pi \). The frequency spectrum of the axions has the peak frequency \( m_a / 2 \pi \) with small bandwidth \( \Delta \omega \approx 10^{-6} \times m_a \), because of nonzero velocity \( v_a \sim 10^{-3} \) of the dark matter axion in our galaxy. Thus, the oscillating current has the same spectrum as that of the axion.

We impose the magnetic field \( B \) on a cylindrical superconductor, which is parallel to the direction along the length of the superconductor. We take the direction as \( z \) direction. The magnetic field is expelled from the superconductor. But the field penetrates into the superconductor to the depth \( \lambda = \sqrt{m_e / q^2 n} \) (London penetration depth). The penetration depth is assumed such that \( \lambda \approx 5 \times 10^{-6} \text{cm} \) corresponding to the number density \( n \approx 0.5 \times 10^{22} \text{cm}^{-3} \). Thus, the superconducting current flows in the surface to the depth \( \lambda \). Since it oscillates with the frequency \( m_a / 2 \pi \), the dipole radiation is emitted from the cylindrical superconductor.

Now, we estimate the radiation flux emitted by the cylindrical superconductor under the magnetic field \( B \). We suppose that the superconductor has radius \( R \sim 1 \text{cm} \) and length \( l \sim 10 \text{cm} \). Then, the flux of the dipole radiation is given by
\[ S = \frac{m_a^2 (2 \pi R \lambda J)^2}{3} = \frac{m_a^2 (q^2 E_0 n)^2 (2 \pi R \lambda)^2}{3 m_e^2 m_e^2} = \frac{(q^2 E_0 n)^2 (2 \pi R \lambda)^2}{3 m_e^2 f_a^2 m_e^2} = \frac{(k_a a)^2 (2 \pi R \lambda)^2}{3 m_e^2 f_a^2 
\lambda^2} \]
with \( E_a = E_0 \cos(m_a t) \) ( \( E_0 = -k_a a_0 B / f_a \pi = -g_{a\gamma\gamma} a_0 B \), where we have used the formula of the London penetration depth \( \lambda = \sqrt{m_e / q^2 n} \). We note that the dipole current is \( I = 2 \pi R \lambda J \). Numerically, we estimate the flux \( \mathcal{S} \),
\[ S \simeq 4.1 \times 10^{-18} W \left( \frac{5 \times 10^{-6} \text{cm}}{\lambda} \right)^2 \left( \frac{B}{\text{ST}} \right)^2 \left( \frac{R}{1 \text{cm}} \right)^2 \left( \frac{l}{10 \text{cm}} \right)^2 \left( \frac{k_a}{1.0} \right)^2 \left( \frac{\rho_a}{0.3 \text{GeV/cm}^2} \right). \]  

(6)

where there is no dependence on the axion mass. (As an actual material used in superconducting magnet, the superconductor Nb\textsubscript{3}Sn shows the penetration depth \( \lambda \approx 8 \times 10^{-6} \text{cm} \). Then, the radiation flux similar to \( S \) in the above can be obtained by using \( B \sim \text{ST} \).

The spectrum of the radiation shows a sharp peak at the frequency \( m_a/2\pi \) with the bandwidth \( \Delta \omega \sim 10^{-6} m_a/2\pi \). We can make the flux \( S \) to increase more by enlarging the radius and length of the cylindrical superconductor as well as the strength of the magnetic field \( B \). The most significant contribution to the flux comes from the penetration depth \( \lambda \). Thus, by choosing sample of superconductor with smaller penetration depth than \( 5 \times 10^{-6} \text{cm} \), we can obtain much larger radiation flux than \( S \) in eq(6).

The flux is obtained by integrating a pointing vector over the sphere with radius \( r \gg 2/m_a \) around the superconductor; \( S = \int S(\theta, r)r^2 d\Omega = \int S(\theta, r)r^2 \sin \theta d\theta d\phi \), where

\[ S(\theta, r) = \frac{m_a^2 (2\pi R \alpha J)^2 (\sin \theta)^2}{8\pi r^2}, \]  

(7)

where we have taken the polar coordinate.

The dipole radiation is emitted mainly toward the direction ( \( \theta = \pi/2 \) ) perpendicular to the electric current \( I \) flowing in \( z \) direction. Thus, when we measure the radiation in the direction, we receive relatively strong flux density. For example, we observe the radiation using the radio telescope with diameter 32m by putting the cylindrical superconductor 100m away from the telescope (e.g. Yamaguchi 32-m radio telescope of National Astrophysical Observatory of Japan), the observed flux per frequency \( P \) is given by

\[ P = \int \frac{S(\theta, r)}{\Delta \omega} r^2 d\Omega = \int \frac{\pi^{\frac{\delta}{2}}}{\Delta \omega} S(\theta, r) r^2 \sin \theta d\theta d\phi \simeq \frac{m_a^2 (2\pi R \alpha J)^2 \delta^2}{8\Delta \omega} = \frac{35\delta^2}{8\Delta \omega} \simeq 0.4 \times 10^{-22} \text{W/Hz}, \]  

(8)

with \( \delta \approx 16 \text{m}/100 \text{m} = 0.16 \) and \( \Delta \omega = 10^9 \text{Hz} (m_a/(6 \times 10^{-6} \text{eV})) \). Thus, the antenna temperature is approximately \( T_a \equiv P \eta \simeq 1.5 \text{K} \) with the unit \( k_B = 1 \), assuming the antenna efficiency \( \eta \approx 0.6 \). Therefore, the radiation can be observed with the radio telescope with diameter such as 32m. We would like to stress that we can make the ratio of signal to noise \( S/N \propto \sqrt{\pi/\Delta \omega} \) being much larger than 1 by taking the observational time \( \tau \) long, different to the astrophysical observations. This is because we can perform the observation \( \tau \) even over one day.

Obviously, the identical flux \( P \) can be obtained even when we use a radio telescope with small diameter 32cm, by putting the superconductor 1m away from the telescope, as long as the distance 1m much larger than the wave length of the radiation; \( 1 \text{m} \gg 2/m_a \). The merit of our proposal is that we can simultaneously search wide bandwidth of the radio frequency without tuning the shape of the superconductor. In this way, we can observe the radiation from the dark matter axion.

Here, we explain why we obtain much larger flux in our apparatus than flux obtained in resonant cavity. For comparison, we write down the flux \( S_{cav} \) of TM mode in the resonant cavity;

\[ S_{cav} = 4 \left( \frac{k_a \alpha B}{m_a} \right)^2 \frac{\rho_a V C}{m_a} \times \min(Q_l, Q_a) \approx 4 \left( \frac{k_a \alpha B}{m_a} \right)^2 \frac{V \rho_a}{\pi^2 m_a^2 f_a^2 m_a R_c^2} \times \min(Q_l, Q_a), \]  

(9)

with \( C = 4/(m_a R_c)^2 \), the volume \( V = \pi R_c^2 l_c \) of the cavity and \( \min(Q_l, Q_a) = Q_l/(Q_a) \) for \( Q_l < Q_a (Q_a < Q_l) \), where \( R_c (l_c) \) denotes the radius (length) of the cavity. \( Q_a \simeq 3 \times 10^6 \) and \( Q_l = R_c/\delta_c \) with the skin depth \( \delta_c \) of the cavity; e.g. \( \delta_c \approx 2 \times 10^{-4} \text{cm} \) for the radio frequency \( m_a/2\pi = 1 \text{GHz} \) in copper. We note that the resonance condition of the cavity gives \( m_a R_c \sim 2.4 \) in eq(4), i.e. \( C \simeq 0.7 \). Then, \( R_c \simeq 11.5 \text{cm} \) for \( m_a/2\pi = 1 \text{GHz} \). On the other hand, the flux received by the telescope in the above is given by

\[ S_{tel} = \frac{35\delta^2}{8} = \frac{3(\delta C)^2 (2\pi R_l)^2 \rho_a}{24\pi^2 m_a^2 f_a^2 \lambda^2} \times \delta^2 = \frac{(k_a \alpha B)^2 V l R}{2\pi^2 m_a^2 f_a^2 R_l \lambda^2} \times \delta^2 \]  

(10)

with \( \delta \approx 0.16 \) and the volume \( V_l \equiv \pi R_l^2 l \) of the cylindrical superconductor. The small factor \( 3\delta^2/8 \) is the solid angle per 4\( \pi \) when looking at the telescope from the superconductor. Thus, the ratio of \( S_{tel} \) to \( S_{cav} \) is
can be explored. As we have pointed out, a Cooper pair oscillates harmonically under the electric field in a microscopic point of view. Then, we will find the applicability of the formula to what the extent the axion mass much small such as $\lambda_{300cm} (6 \times lR/\lambda)$ contribution comes from the factor $R$ with $wN_{\pi/m}$. The wave length 2\(\pi/m\) when we explore the axion mass such as $\lambda_{30cm}$ (10cm/m) the number of the region are incoherent.

Thus, we find the ratio $S_{tel}/S_{cav} \sim 360$. The large flux $S_{tel}$ mainly comes from the small penetration depth $\lambda \sim 5\times 10^{-6}cm$ of the magnetic field, in other words, the large number density of Cooper pairs. (The main contribution comes from the factor $IR/\lambda^2 \simeq 4 \times 10^{11}$.) That is, the difference in the fluxes arises from the difference between the penetration depth $\lambda$ and the skin depth $\delta_t$. Therefore, our apparatus for the detection of the axion works more efficiently than resonant cavities do.

We have used the standard formula for the flux $S$ of the dipole radiation. Here we would like to derive the formula in a microscopic point of view. Then, we will find the applicability of the formula to what the extent the axion mass can be explored. As we have pointed out, a Cooper pair oscillates harmonically under the electric field $E_a$ according to the equation of motion, $m_e \ddot{v} = qE_a(t)$. The amplitude $qE_0/(m_e m_a^2)$ ($E_a = E_0 \cos m_a t$) of the oscillation is much small such as $\sim 10^{-18}eV/cm(B/3T)(m_a/10^{-6}eV)^2$. The Cooper pair emits a dipole radiation with the wave length 30cm (6 $\times 10^{-6}eV/m_a$) and flux $\dot{S} = q^2 E_0^2/3m_c^2$. When the number density of the Cooper pair is $n$, the number of the Cooper pairs emitting the radiation is given by $N(l) = 2\pi RLln$. The point is that their emissions are coherent when the wave length $2\pi/m_a$ is larger than the length $l$ of the cylindrical superconductor. Thus, we find that the total flux $\dot{S} N^2$ is just equal to $S$ shown in eq(3),

$$S = \dot{S} N(l)^2 = \frac{q^4 E_0^2 N(l)^2}{3m_c^2} = \frac{m_c^2 (2\pi R L J)^2}{3}$$ (13)

with $J = \frac{q^2 E_0}{m_a m_c}$. When the wave length $2\pi/m_a$ is shorter than the length $l$, the formula need to change. For example, when we explore the axion mass such as $m_a = 10^{-6}eV$, the coherent radiations arise from the partial region with length $2\pi/m_a \simeq 1.3cm$ in the superconductor. Thus, the total flux is given such that

$$S = \dot{S} N(2\pi/m_a = 1.3cm)^2 \left( \frac{l = 10cm}{2\pi/m_a = 1.3cm} \right) \sim 0.5 \times 10^{-18}W$$ (14)

because there are (10cm/1.3cm) regions in each of which the raditions are coherent, but the radiations from different regions are incoherent.

We make a few comments. First, normal, not superconducting currents also flow in the surface. The amount of the normal current is proportional to the number density of normal electrons. It is strongly suppressed in much lower temperature than critical temperature separating normal and superconducting states. They would not contribute the power $P$ detected with the receiver so much. Furthermore, the cylindrical superconductor should be Type 2 because the superconductivity must hold even under the strong magnetic fields 3T. The magnetic field penetrates inside of the type 2 superconductor and the magnetic vortices are formed. The electric fields $E_a$ induce oscillating electric currents even in the vortices, but the currents are normal, not superconducting ones. As we have mentioned, such normal currents would not contribute the power $F_{cavity}$ in much lower temperature than the critical one.

Secondly, the strong magnetic field $B$ parallel to the direction along the length of the cylindrical superconductor is produced by coils surrounding it. They should have open space for the dipole radiations to escape outside the coils and reach the telescope. In particular, the open space should be present in the $\theta = \pi/2 \pm \delta$ directions ( $\delta \simeq 16cm$ ) perpendicular to the cylindrical superconductor ( magnetic field ). That is, the coils are composed of two parts; one covers the upper side of the superconductor and the other one covers the lower side. Then, there is an open space in the coils through which the raditions can escape from the coils. Although there is the open space in the coils, the magnetic field must be parallel to the cylindrical superconductor.
Finally, the axion mass for our main interest is about $4 \times 10^{-6}$eV $\sim 10^{-4}$eV. The corresponding frequencies (wave lengths) of the radiations are 1GHz$\sim$ 24GHz (30cm$\sim$ 1.3cm). The interest comes from our previous prediction $\simeq 7 \times 10^{-6}$eV$^{[4]}$ for the axion mass, which has been obtained by the analysis of repeating fast radio bursts. But our proposal for the detection of the dark matter axion is not restricted to the mass range. It can be applied to larger axion masses. In the standard resonant cavity experiments for the axion detection, it would be necessary to almost change the setup in order to detect the axion mass larger than $10^{-5}$eV. In our proposal, all radiations emitted by the superconductor of the cylindrical shape can be searched without any essential modification. We only need to diminish the radius and length of the superconductor for the dipole approximation to be valid. In this way, we can simultaneously search wide bandwidth of the radio frequency without tuning the shape of the apparatus to achieve the resonance condition.

In summary, we have proposed a new method for the detection of the dark matter axion. It is to use a superconductor of the cylindrical shape in which the oscillating superconducting current flows under strong magnetic field. We have shown that the superconductor can emit sufficiently large amount of the dipole radiation so as for existing radio telescopes to easily detect them.

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