Phase averaging method for the modeling of the multiprobe and cutaneous cryosurgery

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Abstract. In this paper we consider the problem of planning and optimization of the cutaneous and multiprobe cryosurgery operations. An explicit scheme based on the finite volume approximation of phase averaged Pennes bioheat transfer model is applied. The flux relaxation method is used for the stability improvement of scheme. Skin tissue is considered as strongly inhomogeneous media. Computerized planning tool is tested on model cryotip-based and cutaneous cryosurgery problems. For the case of cutaneous cryosurgery the method of an additional freezing element mounting is studied as an approach to optimize the cellular necrosis front propagation.

1. Introduction

During the last decades the cryosurgery methods has been considerably developed and has become a competitive approach for human cancer treatment. The prediction of the cryosurgery results is one of the actual problems of the computational medicine. The cryosurgical treatment is known to be based on the tissue necrosis effect.

For the cryotip based cryosurgery necrosis effect is caused by the ice ball propagation from the tips of cryoneedles in the tissue to treat, in the case of cutaneous cryosurgery the ice ball propagates for the contact region between the cutaneous surface and liquid nitrogen being sprayed on the skin. For the cutaneous cryosurgery cancer tissue, which is located under the skin surface, often have a quite complicated shape, so the total destruction of the tumor may lead to the significant damage to close-fitting healthy tissue. On the other hand, incomplete destruction of the tumor tissue lead to the disastrous increasing of the tumor growth speed.

This paper is concerned with numerical study of the possibilities of the cutaneous cryosurgery based on the combined effect from liquid nitrogen spraying and introducing of cryotips in order to optimize the tissue necrosis front propagation to the shape of tumor tissue. In order to take into account the peculiarities of physical processes in the bio-tissues which take place under the cryogenic treatment on the cell scale the phase averaged Pennes bioheat transfer model is used to describe macroscopical evolution of the thermal regime in target region. The explicit finite volume scheme for indexed convex grids with flux relaxation is applied for the numerical computation of the problem under study.
2. Phase averaging procedure

According to the phase averaging method the cryogenic heat transfer processes in biotissue are described by 2-parametrically modified Pennes model [1, 2]:

$$C(T; T_\alpha, T^*) \frac{\partial T}{\partial t} = \text{div} \left( \bar{k}(T; T_\alpha, T^*) \nabla T \right) + \rho_\omega \omega_b \bar{C}_b(T_b - T) + q_{\text{met}},$$

$t > 0, \mathbf{r} \in D \subset \mathbb{R}^3,$ \hspace{1cm} (1)

Heat conductivity coefficient is linearly interpolated on the interval $(T_\alpha, T^*)$.

After introducing $\delta = (T^* - T_\alpha)/(T_c - T_\alpha)$ phase averaged heat capacity of the biotissue can be approximated as follows:

- $C(T; T_\alpha, \delta) = C_1(T)$ if $T \leq T_\alpha$;
- $C(T; T_\alpha, \delta) = \frac{T - T_\alpha}{T_c - T_\alpha} C_1(T_\alpha) + (1 - \delta) C_2$ if $T_\alpha < T < T_c$ and
- $C(T; T_\alpha, \delta) = C_2$ otherwise.

The indices 1 and 2 relate to frozen and unfrozen states correspondingly.

In this way the parameters of the phase averaging are the lower temperature of the phase change $T_\alpha$ and $T^*$, which has sense of the normalized phase change temperature in terms of the Stefan problem. The upper phase change temperature is assumed to be known value $T_c = 273K$.

For the cryoprobe based cryosurgery problem the optimal phase averaging parameter values are $T_\alpha = 265K, \delta = 0.6$.

In case of cutaneous cryosurgery the piecewise constant fields $T_\alpha$ and $\delta$ are considered: $(T_\alpha, \delta) = (260, 0.7)$ in 2mm range from cryo-active surfaces and $(T_\alpha, \delta) = (265, 0.5)$ otherwise.

3. The results of multiprobe cryosurgery modeling

An explicit scheme based on the finite volumes method is used for the numerical modeling of cryosurgery [3, 4, 5]. The flux relaxation approach [6] is involved for the stability improvement of explicit finite difference scheme. The last one leads to a decreasing of the computations time by taking a higher time steps.

The geometry of the considered tumor tissue is shown on Figure 1 (left). The resulting localization of the tissue necrosis front and Cooper’s isosurface of the temperature when the necrosis starts compared to the tissue shape are shown on the Figure 1 (right). The temperature
Figure 2. Critical temperature isosurfaces propagation

Figure 3. Cutaneous cryosurgery modeling. Left: Computation area. Center: Resulting temperature distribution. Right: Tumor/cryodamaged tissue regions.

of the working surface of 3 cryotips was taken as $\Phi(t) = \max(300 - At, 100)K$ with freezing speed $A = 80K/min$. Assuming the geometry of the tissue the sliced pyramid (5 and 2.5 cm foundations, 5 cm height) was taken as the computational area with $10^6$ cells discretization.

The evolution of the critical temperature zones with 30 sec time steps is shown on the Figure 2
4. Numerical modeling of the cutaneous cryosurgery with correcting cryotip
In current work we assume the skin tissue to consist of 3 layers: epidermis (0.5mm), dermis (3.5mm), subcutaneous fat (5mm). Tumor tissue bio-physical parameters are assumed to have the same with the dermis tissue. We consider the introducing of correcting cryotip in tumor tissue for the optimization of the propagation of tissue necrosis front from the skin surface under the LN₂ spray. The values of thermophysical parameters for the skin tissue was taken from [7]. Correcting cryotip’s working surface is assumed to have a sphere shape (radius is 2mm). Center slice of the computation area geometry is shown on the Figure 3(Left).

Numerical experiments shown that due to flux relaxation mechanism the stability condition may be reduced from $\tau \sim h^2$, which is peculiar to explicit schemes for parabolic equations, to $\tau \sim h^3$.

The temperature of the working surface of the correcting cryotip was taken as $\Phi(t) = \max(300 - At; 100) K$ with freezing speed $A = 80 K/min$. Quasi-gradient method was applied to find an optimal position of the cryotip which would decrease the damage to the healthy tissue.

The resulting temperature distribution in the target region is shown be means of the center slice of the computation area on the Figure 3(Center). The resulting tissue necrosis region (yellow) and tumor tissue (blue) are shown of Figure 3(Right).

5. Conclusion
The phase averaged macroscopic model with scalar averaging fields for the cryogenic bioheat transfer is proposed as the tool for predicting the results of cryosurgery with respect to peculiarities of phase change processes in biological solutes. Introducing of the additional cryotip into the skin tissue during the cutaneous cryosurgery is considered. Numerical experiments on the model problems showed that the developed planning tool can cause a valuable increasing of effectiveness and safety of cryosurgery operations performing.

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