Dynamics of structural - inhomogeneous coaxial-multi-layered systems "cylinder-shells"

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Abstract: A mathematical model and a technique for assessing the efficiency of the dissipative ability of structurally inhomogeneous mechanical systems consisting of multilayer cylinders bonded to a thin viscoelastic shell of finite length have been developed. A detailed analysis of the known works devoted to this problem is given. A model, methodology, and algorithm for studying the natural and forced oscillations of a system to assess the damping ability of structurally inhomogeneous elastic and viscoelastic mechanical systems, taking into account the influence of the geometric and physico-mechanical parameters of the shell and cylinder have been developed. In solving the problems considered, the method of divided variables, the method of the theory of potential functions, the Mueller method, the Gauss method and the orthogonal sweep method were used. The complex eigenfrequencies, amplitudes of forced oscillations are determined, and the largest dephasing abilities of the considered structurally inhomogeneous systems are estimated. It has been revealed that, the effect of the greatest damping ability in structurally heterogeneous systems is manifested when the real parts of complex natural frequencies come closer due to the interaction of close natural forms with each other.

Keywords: complex natural frequency, damping coefficient, inhomogeneous mechanical system, viscoelasticity, resonant amplitude.

1. INTRODUCTION

The intensive development of the national economy leads to the acceleration of scientific and technological progress to the creation of new technological systems, that should ensure the efficient operation of apparatuses and machines over a wide range of speeds, temperatures, pressures and loads. Moreover, they must be efficient for operation, comfortable for humans, environmentally friendly, noiseless, etc. Such technological systems include existing flying machines and various means of helicopters, airplanes, and others. Vibration suppression in such systems is, first of all, arising under the influence of dynamic loads in aircraft and vehicles.

One of the first studies of the problems of the dynamics of shell structures in contact with the cylinder, made of a different material property, was considered in [1]–[4], which is used today in various machines, devices and structures.

In [5], [6], the problems of oscillations of a thick-walled elastic cylinder (in a flat formulation) under
given unsteady loads acting on the external and internal surfaces were investigated. The solution to this problem is mainly based on the use of special functions of mathematical physics. In problems on the propagation of an axisymmetric elastic wave in a cylinder with an elastic filler[7], [8], the dispersion equation is obtained. Using special functions and Newton's method, the dispersion equation is obtained. The problems of unsteady radial vibrations of a thick-walled elastic cylinder bonded to a thin elastic shell are considered in [8]–[10], which is solved by the integral transformation method. Such problems are often found in solid fuel rocket engines, where the structural mechanical system consists of cylinders with various dissipative properties. Along with this, a large number of quasistatic and dynamic problems were solved for axisymmetric bodies under plane deformation. In solving such problems, the quasistatic linear theory of viscoelasticity was usually used. The quasistatic reaction of the system consisting of a cylinder and a housing to the action of a time-varying internal pressure was considered by Blend [9]. Williams et al. [10] also considered several problems of this type. A brief overview of problems for a cylinder with quasistatic viscoelasticity is given in the article by Rogers and Lee [11]. Moreover, the resulting dynamic effects were not taken into account in determining the viscoelastic behavior of the cylinder. However, the occurring vibrations in various elements of the aircraft have led to the need to study the oscillations of visco-elastic cylinders[12], [13]. Along with this, a class of problems related to the dynamics of shells in contact with an external elastic or viscoelastic medium of infinite length or limited volume has been considered[14], [15]. The problem of the action on the design of short-term pulses initiated, for example, by an explosive, a shock of a solid body, etc., in which the pressure distribution can be localized in the form of a spot of limited size, is considered[16], [17]. Experimental studies are limited mainly to recording the final surface parameters of the process, which do not allow us to trace how stress waves develop and interact in the material of structural elements[18]–[20]. Using various numerical methods [21]–[24] a large number of axisymmetric plane problems were solved in which mechanical systems consist of plates and shells. In[25]–[27], three-dimensional problems of the dynamics of cylindrical bodies were studied using equations of the theory of elasticity under the action of pulsed loads. It is assumed that pressure is applied to the outer surface. The desired components of the displacement vector are presented in the form of a double Fourier series along the longitudinal and angular coordinates.

Along with this study, the dynamics of various heterogeneous systems, taking into account their features and operating conditions, has been the subject of a number of works in which the natural vibrations and dynamic behavior of a structure under various influences are evaluated [16],[18],[36]–[42].

Here is a review, just some of the works that are devoted to assessing the dynamics of various structures, including axisymmetric systems. Therefore, the development of an effective methodology and algorithm, as well as the study of the dynamics of heterogeneous axisymmetric systems, is an urgent task. When solving such problems, it is more difficult to take into account the finite length of multilayer cylindrical bodies fastened in a thin elastic shell [28], [29].

This, in this article vibrational processes in spatial structurally inhomogeneous deformable mechanical systems consisting of multilayer cylinders of finite length, the materials of which have both elastic and viscoelastic properties is discussed. To assess the dissipative properties of such mechanical systems, mathematically developed models, methods and algorithms, and research results related to providing optimal damping capabilities of the system as a whole are given.

2. METHODS

2.1 Models and methods of solution

A viscoelastic multilayer cylinder of finite length of constant internal radius $a$ and outer radius of the outer surface $b$ (total radius) along which the cylinder is bonded to a thin elastic (or viscoelastic) shell we will consider (Figure 1). Some of the layers of mechanical systems (multilayer cylinder and shell) can
be massless. In the case under consideration, massless cylindrical bodies are characterized by operator stiffness coefficients, i.e.

$$\tilde{K}_0 f(t) = K_{0ij} \left[ f(t) - \int_0^t R_0(t - \tau) f(\tau) d\tau \right]. \quad (1)$$

where $K_{0ij}$ - moments of stiffness massless deformable elements, $R_0(t - \tau)$ - relaxation core, $f(t)$ - arbitrary function of time. The material of the multilayer cylinder is a highly filled polymer, the physico-mechanical characteristics of which are determined by the nature of the binder and the adhesion of the binder to the multilayer filler. The considered structurally inhomogeneous mechanical system “cylinder - shell (housing)” is subject to a uniform variable vibrational or pulsed pressure variable in time. The longitudinal section of the cylinder is shown in Figure 2. The relationship between stresses and strains, for structurally inhomogeneous bodies, can be represented as

$$\sigma_{ij} = \lambda_{\kappa} \left(1 - R_{\lambda_{\kappa}}\right) \Theta_{\kappa} \delta_{ij} + 2 \mu_{\kappa} \left(1 - R_{\mu_{\kappa}}^{\lambda_{\kappa}}\right) \varepsilon_{ij}. \quad (2)$$

Here $\lambda_{\kappa}, \mu_{\kappa}$ - constant’s Lame of the $\kappa$-th deformable element, $\varepsilon_{ij}$ - strain tensor component of the $\kappa$-th element, $\Theta$ - volumetric deformation,

$$R_{\lambda_{\kappa}} f(t) = \int_0^t R_{\lambda_{\kappa}} (t - \tau) f(\tau) d\tau;$$

$$R_{\mu_{\kappa}} f(t) = \int_0^t R_{\mu_{\kappa}} (t - \tau) f(\tau) d\tau$$

$$f(t)$$ - derivative of the function of time; $R_{\lambda_{\kappa}}(t - \tau), R_{\mu_{\kappa}}(t - \tau)$ - relaxation core; $\lambda_{0k}, \mu_{0k}$ - instantaneous modulus of elasticity ($k = 1, ..., N$) [9].
The equations of small vibrations of a multilayer (N-layer) cylinder have the form

\[ \ddot{u} = \dddot{u} + (\lambda + \mu) \nabla \nabla \nabla \dot{u} = \dot{\rho} \frac{\partial^2 \dddot{u}}{\partial t^2}, \quad (\kappa = 1, 2, 3, \ldots, N) \]  

(3)

here \( \ddot{u} (u_r, u_\varphi, u_z) \) - displacement vector, \( \dot{\rho} \) - density of the \( \kappa \)-th material. In addition to (3), we write out the equations of oscillations of the fastened viscoelastic cylindrical shell (Figure 1). The equation of motion of a thin viscoelastic shell in the space of displacements takes the following form [30]

\[ L(1-R^2)\ddot{u} = \left(1 - \frac{\rho \partial^2}{E_o n_o}\right) \dddot{u} + \rho_o \frac{\partial^2 \dddot{u}}{\partial t^2}, \]  

(4)

where \( L \) - differential operator, which, in the framework of the Kirchhoff–Love hypotheses, takes, in matrix form, the form

\[ L = \begin{bmatrix}
\frac{\rho \partial^2}{2R^2 \partial \theta^2}
& \frac{1}{R \partial \zeta}
& \frac{\rho \partial^2}{R \partial \theta}

\frac{1}{R \partial \theta}
& \frac{1}{R \partial \theta} - a(2-v)\frac{\partial^3}{\partial \zeta^3 \partial \theta} - \frac{a}{R^2 \partial \theta}
& \frac{1}{R^2} + a\left(\frac{\partial^2}{\partial \zeta^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2}\right)

\end{bmatrix}.
\]

The integral terms in (2) are assumed to be small. Then the function \( f(t) \) satisfies the following equality: \( f(t) = \psi(t) e^{-i\omega_k t} \). This corresponds to linear oscillations of the mechanical system. Here \( \omega_k \) - slowly changing function of time, \( \omega_k \) - actual value. Then (2) are replaced by the following approximate relations

\[ R_{\lambda \kappa} f(t) = \lambda_{\kappa} \right[ \Gamma_{\lambda \kappa}^S (\omega_k) + i \Gamma_{\lambda \kappa}^S (\omega_k) \right] f(t), \]  

\[ R_{\mu \kappa} f(t) = \mu_{\kappa} \left[ \Gamma_{\mu \kappa}^S (\omega_k) + i \Gamma_{\mu \kappa}^S (\omega_k) \right] f(t), \]

where

\[ \Gamma_{\lambda \kappa}^C (\omega_k) = \int_0^\infty R_{\lambda \kappa} (\tau) \cos \omega_k \tau d\tau, \quad \Gamma_{\lambda \kappa}^S (\omega_k) = \int_0^\infty R_{\lambda \kappa} (\tau) \sin \omega_k \tau d\tau, \]

\[ \Gamma_{\mu \kappa}^C (\omega_k) = \int_0^\infty R_{\mu \kappa} (\tau) \cos \omega_k \tau d\tau, \quad \Gamma_{\mu \kappa}^S (\omega_k) = \int_0^\infty R_{\mu \kappa} (\tau) \sin \omega_k \tau d\tau, \]

- respectively, the cosine and sine images of the Fourier kernel \( R(t) \), \( a = h^2/12R^2 \).

The boundary conditions at the contact of the layers (equality of displacements and stresses are satisfied) have the form

\[ \sigma^{i'}_r = \sigma^{i+1'}_r; \quad u^{i'}_r = u^{i+1'}_r; \]

\[ \tau^{i'}_{rz} = \tau^{i+1'}_{rz}; \quad u^{i'}_{\varphi r} = u^{i+1'}_{\varphi r}; \]

\[ \tau^{i'}_{r\varphi} = \tau^{i+1'}_{r\varphi}; \quad u^{i'}_z = u^{i+1'}_z; \]  

(5)
Where is the index \( i \) corresponds to the layer number \( i = 1, 2, 3, \ldots, N \); \( r = r_j \). Layers are numbered starting from the outside. If between \( i \)-th and \( (i+1) \)-th layer is the massless cylinder, then the boundary conditions take the form (for \( r = r_j \)):

\[
\begin{align*}
\sigma_{rr_i} &= k_r (u_{ri} - u_{r(i+1)}) , \\
\sigma_{r\theta_i} &= k_\theta (u_{\theta i} - u_{\theta(i+1)}) , \\
\sigma_{r zi} &= k_z (u_{zi} - u_{z(i+1)})
\end{align*}
\]

where \( k_r, k_\theta, k_z \) - interaction factors.

For simplicity, we assume that the edges of the cylinder and the shell can rotate without deformation in the longitudinal direction (Figure 2). This is equivalent to the fact that the ends of the cylinder are fastened with a flexible membrane, absolutely rigid in its plane:

\[
\mathcal{G} = w = -\frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial z^2} = 0, u_r = u_\phi = u_z = O \quad (z = 0, l).
\]

The inner surface is not loaded, i.e.

\[
\sigma_r = \tau_{r\phi} = \tau_{rz} = O \quad (r = a) \quad .
\]

The outer surface may be loaded

\[
\sigma_r = -P(\phi, z, t); \quad \tau_{r\phi} = \tau_{rz} = O \quad (r = b) \quad .
\]

In the construction (Figure 1), the left end is rigidly pinched \((z = 0)\), and the right end is free \((z = l)\) from loads. In the study of forced oscillations, it is assumed that a local load is applied to the external surface (7).

Now the general problem under consideration can be formulated as follows: it is necessary to find the function \( u(u_r, u_\phi, u_z) \) satisfying equations (3) - (4) under boundary conditions (5-6) under the action of a load (7).

2.2 Methods for solving the problem

In a linear formulation, a solution to the equations of a multilayer cylinder is sought in the form of a Green – Lame expansion [31]

\[
\begin{align*}
u_{1k} &= \frac{1}{r} \frac{\partial \Phi_{1k}}{\partial r} + \frac{1}{r} \frac{\partial \Phi_{2k}}{\partial \phi} + \frac{1}{v_{p1}} \frac{\partial^2 \Phi_{3k}}{\partial z \partial r} \\
u_{2k} &= \frac{1}{r} \frac{\partial \Phi_{1k}}{\partial \phi} - \frac{1}{r} \frac{\partial \Phi_{2k}}{\partial r} + \frac{1}{v_{p1}} \frac{\partial^2 \Phi_{3k}}{\partial z \partial \phi} \\
u_{3k} &= \frac{1}{v_{p1}} \frac{\partial^2 \Phi_{2k}}{\partial z^2} + v_{p1} \Phi_{3k}
\end{align*}
\]

Where are the functions \( \Phi_i \) – the essence of solving scalar wave equations.
\[ \nabla^2 \Phi_{ik} = \frac{1}{a_{ik}^2} \frac{\partial^2 \Phi_{ik}}{\partial t^2}, \]
\[ \nabla^2 (\Phi_{2k}, \Phi_{3k}) = \frac{1}{a_{2k}^2} \frac{\partial^2 \Phi_{1k}}{\partial t^2}. \] 

Here \( \alpha_{ik}^2 = (\alpha_t + 2\mu_t) R_{ik} / R_k, \) \( \rho_k \alpha_{2k}^2 = \mu_k R_k / R_k, \)
\[ R_{\alpha,\mu k} = (2\mu_0 + \mu_0 k) \Gamma^c_{\alpha k} (\omega_R) + 2\mu_0 k \Gamma_{\mu k} (\omega_R) + i(\mu_0 k \Gamma^c_{\alpha k} (\omega_R) + 2\mu_0 k \Gamma_{\mu k} (\omega_R)) \]
\[ R_{\mu k} = \mu_0 k \Gamma_{\mu k} (\omega_R) + i\mu_0 k \Gamma^c_{\mu k} (\omega_R) \]

Solutions of equations (9) can be found depending on whether the wave numbers are positive, zero or negative[32]. At \( R_{\alpha,\mu k} = R_{\mu k} = 0 \) solutions of equation (9) are known [33]. Thus, using (8), we determine the displacements of the cylinder points:

\[ u_{\alpha k} = \sum_{n=0}^{\infty} \frac{\gamma_{\alpha k}}{\alpha \nu_{\alpha k}^2} \left[ A_n J_{\alpha} (\nu_{\alpha k} r) + A_m Y_{\alpha} (\nu_{\alpha k} r) \right] \sin(\alpha z + \epsilon) \cos n \rho e^{-it}, \]
\[ u_{\mu k} = \sum_{n=0}^{\infty} \frac{\gamma_{\mu k}}{\alpha \nu_{\mu k}^2} \left[ A_n J_{\mu} (\nu_{\mu k} r) + A_m Y_{\mu} (\nu_{\mu k} r) \right] \sin(\alpha z + \epsilon) \sin n \rho e^{-it}, \]
\[ u_{\kappa k} = -\sum_{n=0}^{\infty} \frac{\gamma_{\kappa k}}{\alpha \nu_{\kappa k}^2} \left[ A_n J_{\kappa} (\nu_{\kappa k} r) + A_m Y_{\kappa} (\nu_{\kappa k} r) \right] \cos(\alpha z + \epsilon) \cos n \rho e^{-it}. \] 

Here the prime means differentiation with respect to \( \gamma_{\alpha k}^R \) or \( \gamma_{2k}^R \), \( J_n (z), Y_n (z) \) - Bessel functions complex argument. The following notation is introduced:

\[ \gamma_{\alpha k}^2 = \nu_{\alpha k}^2 - \alpha_{\mu k}^2 = \omega^2 / a_{\mu k}^2 - \alpha_{\mu k}^2, \]
\[ \gamma_{2k}^2 = \nu_{2k}^2 - \alpha_{\mu k}^2 = \omega^2 / a_{2k}^2 - \alpha_{2k}^2, \alpha_m = m \pi / l (m = 1, 2, ...), \]

Knowing the displacements (10) of various points of structurally inhomogeneous mechanical systems, according to Hooke's law, one can find the components of the stress tensor [34]. Satisfying the boundary and contact conditions (5) - (7), we obtain a system of homogeneous algebraic systems of equations with complex coefficients with nine unknowns

\[ \sum_{j=1}^{9} A_j a_{ij} = 0 \quad (i = 1, 2, 3 \ldots 9) \]

where

\[ a_{11} = \alpha H_k J_n (\gamma_{\alpha k}^R b), a_{12} = \alpha H_k Y_n (\gamma_{\alpha k}^R b), \]
\[ a_{13} = \alpha n H_k J_n (\gamma_{2k}^R b) / (2 b \gamma_{1k}), a_{14} = \alpha n H_k Y_n (\gamma_{2k}^R b) / (2 b \gamma_{1k}), \]
\[ a_{15} = -\alpha^2 H_k J_n (1 - v_{2k}^2 / (2 \alpha_2) \gamma_{2k}^R b) / v_{2k} \gamma_{1k}, \]
\[ a_{16} = -\alpha^2 H_k Y_n (1 - v_{2k}^2 / (2 \alpha_2) \gamma_{2k}^R b) / v_{2k} \gamma_{1k}, \]
\[ a_{17} = \beta_{km}^2 + B_{212} n^2 - \tilde{g}_{\omega}^2, a_{18} = -B_{112} + B_{112} \beta_{km} n, a_{19} = -B_{112} \beta_{km}, \]
\[ a_{21} = H_k X_{1k} (\gamma_{1k}^R b) / b, a_{22} = H_k Z_{1k} (\gamma_{1k}^R b) / b, \]
\[ a_{23} = H_k \gamma_{2k}^R X_{3k} (\gamma_{2k}^R b) / (b \gamma_{1k}), a_{24} = H_k \gamma_{2k}^R Z_{3k} (\gamma_{2k}^R b) / (b \gamma_{1k}), \]
\[ a_{25} = -\alpha H_k \gamma_{2k}^R X_{1k} (\gamma_{2k}^R b) / (b \gamma_{1k} \nu_{2k}), a_{26} = -\alpha H_k \gamma_{2k}^R Z_{1k} (\gamma_{2k}^R b) / (b \gamma_{1k} \nu_{2k}). \]
In order for a system of homogeneous algebraic equations with complex coefficients to have solutions, the determinant of this system must be equal to zero. This will give a frequency equ...
Where $\omega = \omega_R + i\omega_I$ - complex frequency, $l$ - geometric and physico-mechanical parameters of the mechanical system. The non-trivial solutions (or roots) of complex unified equations (12) are sought numerically by the Mueller method[43].

The real part ($\omega_R$) integrated natural frequency $\omega = \omega_R + i\omega_I$ means the oscillation frequency of the system in question, and the imaginary ($\omega_I$) determines the rate of damping of oscillations and the damping coefficient makes sense. When exposed to a system of external vibrational loads, equations (11) take the following form

$$\sum_{j=1}^{9} A_j a_{ij} = P_j \quad (i = 1, 2, 3...9).$$

As a result, we obtain a system of inhomogeneous algebraic equations with complex coefficients (13). Here $P_j$ - the amplitude of the vibration effect. This problem is solved by the Gauss methods [42],[44] with the selection of the main element.

3. RESULTS AND DISCUSSIONS

3.1. Natural oscillations.

A structurally heterogeneous system consisting of a viscoelastic multilayer cylinder with a circular cavity bonded to a thin elastic (or viscoelastic) shell is considered. The relaxation core for deformable viscoelastic cylinders was chosen as the Rzhanitsyn – Koltunov core [9,28], i.e.:

$$R(t) = Ae^{-\beta t}t^{a-1},$$

where $A, \alpha, \beta$ - kernel parameters. The viscous properties of a multilayer dissipatively inhomogeneous cylinder are adopted so that its creep deformation during the quasistatic process amounts to a small fraction (~ 12%) of the total deformation. The procedure for determining the parameters of the relaxation core (14) for various materials is described in [28, [35]. In [36], on the basis of the described methodology, parameters are given for some specific materials. In the case under consideration, in specific calculations, the kernel parameters were taken as follows:

$$A = 0.01, \alpha = 0.1, \beta = 0.05, a = 30sm, b = 100sm, h_0 = 0.5sm, \quad h_1 = h_2 = 0.5sm.$$  
$$E_0 = 2 \cdot 10^5 MPa, \quad E_1 / E_0 = 5 \cdot 10^{-2}, E_2 / E_0 = 5 \cdot 10^{-3}, \quad \nu_0 = 0.25, \quad \nu_1 = \nu_2 = 0.35,$$

$$\rho_0 = 1.8 \cdot 10^{-3} kg / sm^3, \quad \rho_1 = \rho_2 = \rho_0 = 0.5.$$  

The real relationship was investigated here. ($\omega_R$) and many ($\omega_I$) parts of the first few complex natural frequencies $\omega = \omega_R + i\omega_I$ on the value of the instantaneous stiffness of a massless elite (multilayer cylinder). In calculating the instantaneous stiffness varied within $10^4$ before $10^2$.  

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Figure 3. Frequency dependence $\omega_k$ and damping factors $\theta_k$ shock absorber stiffness $C_0$
for mechanical systems considered

For each value of the instantaneous stiffness of the massless cylinder, the corresponding natural frequencies are determined ($\omega_R$) and damping factors ($\theta_k$). The characteristic values of the instantaneous stiffness $C$ are determined, since at $C = C_0$ there is a change in the forms of vibrations. Figure 3 shows the dependence of the first five frequencies $\omega_k$ ($\kappa = 1, 2, 3, 4, 5$) and corresponding damping factors $\theta_k$ from the instantaneous stiffness of the massless cylinder $C$. It follows from the analysis of the graphs that the damping properties of the vibrations of the mechanical system under consideration are determined not only by the viscoelastic (or rheological) properties of its elements, but also substantially depend on the interaction of various eigenmodes. The detecting effect is expressed in the fact that under certain conditions, for and up to a value of the rigidity of the massless cylinder, the dissipative abilities of the system under consideration sharply increase and the damping of the oscillation is enhanced due to the interaction of close intrinsic ones (in this case, the second, third, fourth, fifth) forms. Then, starting with a value of the instantaneous stiffness of the shock absorber $C_0$ (in this case $C_0 = 104 \cdot 10^{-4}$) the process of energy dissipation by own forms is normalized and proceeds according to the energy hierarchy of forms. A clear illustration of the detected effect is the intersection of the graphs of the imaginary part of the complex frequency (damping coefficients) $\omega_k^i, \theta_k^i$ at $C = C_0$. At this point (at the intersection of the graphs), the difference in the attenuation rates of the two vibrational forms of the structure changes sign if the cylinder system is structurally inhomogeneous (i.e., the mechanical system consists of elastic and viscoelastic elements).

Such effects do not occur in structurally homogeneous mechanical systems. This statement is proved by introducing normal generalized coordinates. Then a structurally homogeneous mechanical system (all elements of the mechanical system are viscoelastic and with the same rheological properties) is characterized by the fact that in the form of normal coordinates $\theta_n$ the problem under consideration has equations of the form [36]:

$$\ddot{\theta}_n + \Omega_n^2 (1 - \Gamma_k - \Pi_n) \theta_n = \Psi_n,$$

(15)

where $\Omega_n^2$ - $\kappa$- natural frequency of the elastic system; $\Psi_n$ - generalized force corresponding to the $k$-
th normal coordinate. It can be seen from system (15) that each form of oscillation of the system of
equations is independent of each other, i.e., they oscillate separately from each other. If system (15)
is written in matrix form, then a diagonal matrix is obtained (or, without special transformations of
generalized coordinates, they are reduced to a diagonal matrix). This means that the structurally
homogeneous mechanical system under consideration is, as it were, a collection of independent partial
systems with one degree of freedom. Each equation (15) is solved independently of each other. The
difference between a structurally homogeneous system and an elastic mechanical system is only that
the natural normal vibrations of a structurally homogeneous system have a damped character.

In structurally inhomogeneous systems (in this case, with elastic and viscoelastic elements) and with a
noticeable approximation of the real parts \( \omega_k^r, \omega_k^e, \omega_k^s \) complex natural frequencies, and imaginary parts
\( \omega_k^i \) complex frequencies \( \omega = \omega_k^r + i\omega_k^i \) sharply increase or decrease. Thus, in order to use in practice
the discovered effect of enhancing the dissipative properties of the system in structurally
inhomogeneous mechanical systems, the above conditions must be satisfied. Let us now find out the
reason for the origin of the interaction effect of various eigenmodes for structurally inhomogeneous
viscoelastic mechanical systems by introducing normal generalized coordinates.

![Figure 4. The dependence of the frequencies and damping coefficients on the ratio of the thicknesses
of the first and second cylinder](image)

The situation changes in the case of a structurally inhomogeneous mechanical system. Here the
dissipative properties of the elements are different. In this case, the generalized stiffness matrix
consists of the sum of two matrices: real and complex. In this case, we obtain three square symmetric
dissimilar matrices: the matrix of generalized masses, the real and imaginary parts of the matrix of
generalized stiffnesses. Then equations (15) for structurally inhomogeneous mechanical systems take
the following form
\[
\ddot{\varphi}_k + \Omega_k^2 \varphi_k - \Omega_k^2 \sum_{j=1}^{n} (\varphi_j^r + i\varphi_j^i) \varphi_k = \Psi_k.
\]

where \( \Psi_k \) - generalized power, \( \varphi_j^r \) and \( \varphi_j^i \) - positive definite real matrices. The system of differential
equations (16) consists of \( n \) related differential equations. Physically, this connection (16) means that
the normal coordinates for a structurally inhomogeneous mechanical system are interconnected. Each
movement of elements of a structurally inhomogeneous mechanical system is a superposition
of interacting vibrations of several normal coordinates.

Figure 5. Change in resonant amplitude depending on the ratio of thicknesses of the first and second cylinder

Consequently, with free damped vibrations of structurally inhomogeneous mechanical systems, an exchange of energy occurs between the forms of vibrations. This phenomenon is strongly manifested if the vibration modes of a structurally inhomogeneous mechanical system have close natural frequencies. Then at the point of approximation of the graphs of natural frequencies $\omega_1$, $\omega_2$, $\omega_3$ (at the point $C = C_0$) imaginary parts of complex multiple frequencies $\omega_1$, $\omega_2$ and $\omega_3$ intersect, which provides energy loss in two forms due to the interactions of the forms with each other. After the intersection of the curves of complex frequencies, the nature of the interaction of the corresponding forms decreases and their dissipative properties take on a normal character. Frequency dependence $\omega_R$ and damping factors $\omega_I$ from the ratios of the thicknesses of the first and second cylinders are presented in Figure 4.

3.2. Forced vibrations

The steady-state forced vibrations of a structurally inhomogeneous mechanical system (Figure 1) are considered with a uniform load applied on the external surface of the form

$$P_n(t) = P_n^0 e^{-\omega t}$$

where $\Omega$ — known frequency of external exposure, and $P_n^0$ — amplitude of external loads. The vector of amplitudes of this load (force) is equal to a unit vector. The parameters of its relaxation core were taken as: $A=0.078$, $a=0.9$, $\beta=0.05$. A load uniformly distributed over the surface of the shell of the mechanical system. The vector of amplitudes of these loads (forces) has unit components. It is required to evaluate (for the case of forced oscillations) the damping (dissipative) properties of an inhomogeneous viscoelastic mechanical system depending on the instantaneous stiffness of a massless cylinder.

Calculation results, i.e. changes in the maximum resonance amplitudes are shown in Figure 5. Amplitudes were plotted for the center points of the inner cylinder ($a = 1$, $\theta = \pi / 2$). In Figure 5 shows some values of the resonant amplitude (peaks) depending on the instantaneous stiffness of the massless cylinder. From Figure 5 shows that the behavior of the maximum values of the resonance amplitudes (some characteristic points of the mechanical system) depends on the global
damping coefficient (GDC).

Thus, the effect of the interaction of natural forms, discovered during natural vibrations of structurally inhomogeneous mechanical systems, is confirmed once again in the problem of forced vibrations. Differences in the optimal values of the instantaneous stiffnesses of a massless cylinder for forced and natural vibrations ($C_o = 104 \cdot 10^{-4}$), due to the difference in viscosities of elements

5. CONCLUSIONS

1. A mathematical model and methodology has been developed for evaluating the efficiency of dissipative abilities of structurally inhomogeneous mechanical systems consisting of multilayer cylinders bonded to a thin viscoelastic shell of finite dimensions under various dynamic influences.

2. The natural and forced oscillations of structurally homogeneous and inhomogeneous mechanical systems are investigated, the complex natural frequencies and resonant amplitudes of forced oscillations are determined for various values of the instantaneous stiffness of the shock absorber under periodic effects.

3. The greatest dissipative abilities of structurally heterogeneous mechanical systems were revealed when the real parts of complex eigen frequencies came closer due to the interaction of close eigen modes, which is not observed in homogeneous systems.

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