Degeneracy and colorings of squares of planar graphs without 4-cycles. (English)

Summary: We prove several results on coloring squares of planar graphs without 4-cycles. First, we show that if \( G \) is such a graph, then \( G^2 \) is \((\Delta(G) + 72)\)-degenerate. This implies an upper bound of \( \Delta(G) + 73 \) on the chromatic number of \( G^2 \) as well as on several variants of the chromatic number such as the list-chromatic number, paint number, Alon-Tarsi number, and correspondence chromatic number. We also show that if \( \Delta(G) \) is sufficiently large, then the upper bounds on each of these parameters of \( G^2 \) can all be lowered to \( \Delta(G) + 2 \) (which is best possible). To complement these results, we show that 4-cycles are unique in having this property. Specifically, let \( S \) be a finite list of positive integers, with \( 4 \notin S \). For each constant \( C \), we construct a planar graph \( G_{S,C} \) with no cycle with length in \( S \), but for which \( \chi(G_{S,C}^2) > \Delta(G_{S,C}) + C \).

MSC: 05C15 Coloring of graphs and hypergraphs

Keywords: 4-cycles; planar graphs; colorings; degeneracy

References:

[1] Álón, N.; Tarsi, M., Colorings and orientations of graphs, Combinatorial, 12, 125-134 (1992) · Zbl 0756.05049 · doi:10.1007/BF01204715
[2] Amini, O.; Esperet, L.; van den Heuvel, J., A unified approach to distance-two colouring of graphs on surfaces, Combinatorial, 33, 253-296 (2013) · Zbl 1324.05052 · doi:10.1007/s00493-013-2573-2
[3] Bonamy, M.; Cranston, D. W.; Puekle, L., Planar graphs of girth at least five are \((\Delta + 2)\)-choosable, J. Combin. Theory Ser. B, 134, 218-238 (2019) · Zbl 1402.05043 · doi:10.1016/j.jctb.2018.06.005
[4] Borodin, O. V.; Glebov, A. N.; Ivanova, A. O.; Neustroeva, T. K.; Tashkinov, V. A., Sufficient conditions for planar graphs to be \( 2 \)-distance (\( \Delta + 1 \))-colorable, Sib. Elektron. Mat. Izv., 1, 129-141 (2004) · Zbl 1076.05032
[5] Cranston, D. W.; Jaeger, B., List-coloring the squares of planar graphs without 4-cycles and 5-cycles, J. Graph Theory, 85, 721-737 (2017) · Zbl 1368.05047 · doi:10.1002/jgt.22101
[6] Cranston, D. W.; West, D. B., An introduction to the discharging method via graph coloring, Discrete Math., 340, 766-793 (2017) · Zbl 1355.05014 · doi:10.1016/j.disc.2016.11.022
[7] Dong, W.; Xu, B., 2-distance coloring of planar graphs without 4-cycles and 5-cycles, SIAM J. Discrete Math., 33, 1297-1312 (2019) · Zbl 1426.05041 · doi:10.1137/17M1157313
[8] Dvořák, Z.; Král, D.; Nejedlý, P.; Škrekovski, R., Coloring squares of planar graphs with girth six, European J. Combin., 29, 838-849 (2008) · Zbl 1143.05027 · doi:10.1016/j.ejc.2007.11.005
[9] F. Havet, J. van den Heuvel, C. McDiarmid and B. Reed: List colouring squares of planar graphs, July 2008, preprint available at https://arxiv.org/abs/0807.3233. · Zbl 1341.05073
[10] Kimball Jonas, T., Graph coloring analogues with a condition at distance two: \( L(2, 1) \)-labellings and list lambda-labellings (1993), Ann Arbor, MI: ProQuest LLC, Ann Arbor, MI
[11] Molloy, M.; Salavatipour, M. R., A bound on the chromatic number of the square of a planar graph, J. Combin. Theory Ser. B, 94, 189-213 (2005) · Zbl 1071.05036 · doi:10.1016/j.jctb.2004.12.005
[12] Schauz, U., Flexible color lists in Alon and Tarsi’s theorem, and time scheduling with unreliable participants, Electronic J. Combin., 17, 13 (2010) · Zbl 1192.91045 · doi:10.37236/285
[13] Wang, W-F; Lih, K-W, Labeling planar graphs with conditions on girth and distance two, SIAM J. Discrete Math., 17, 264-275 (2003) · Zbl 1041.05068 · doi:10.1137/S089548010390448

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.