Calculation of distribution of potential and electron concentration in the dust-electron thermal plasma with the axial geometry particles

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Abstract. We obtained the equation, which describes the distribution of the potential and the electron density in an equilibrium dust-electron plasma taking into account parameters of the electron gas inside the axial geometry dust particles. The inclusion of these parameters performed on the basis of the model of "solid-state plasma," considering the condensed particle system as the ion core and the free electron gas.

1. Introduction
The system which consists of dust particles, free electrons and neutral gas atoms at atmospheric or higher pressure at temperatures of 1000-3000 K called dust-electron thermal plasma [1-8]. Under these conditions, this system is an isothermal consequence of the heat of intense collisional, i.e. established the statistical equilibrium. Such a plasma is formed during the combustion of solid and liquid fuels, in the process of thermal spray coating, in the channels of the MHD generator, a plasma chemical reactors, the synthesis of nano- and micro-sized particles and magnetic fusion devices [9-11]. The characteristic size of the dust particles is in the range of 0.01 to 100 microns. Heated dust grains are a source of free electrons due to emission processes. As a consequence, the dust particles are positively charged. If such a plasma does not contain easily ionizable impurities of alkali metal, can be neglected by the presence of the ions.

One of the most important characteristics of dust-electron thermal plasma are potential and electron density distributions. To determine these characteristics of the data which are used in this paper the model of "jelly", which previously has been widely used in the description of the electrical properties of atomic clusters [12-14]. According to this model, the dust particle is represented as two-components system. The first component is ion core, which create uniform positive background in the whole volume of the dust particle. The second component is the electronic gas. The density of this gas distribute in the area from the condition of equilibrium of forces internal pressure of the gas and electrostatic forces. Thus, in this model, a dust particle can be regarded as the area of solid state plasma [15-17]. From the above it follows that the distribution of the potential and the electron density within and around the dust particles are dependent on the temperature, type of the substance, size and the concentration of particles.
2. Theoretical background and results
Consider the system of equations describing the potential distribution and the electron density in solid-state plasma and in the space around it in a state of statistical equilibrium [18] when the temperature is low and can be neglected of the ionization of the gas.

The distribution of the potential $\varphi$ of the electrostatic field is described by Poisson equation

$$\Delta \varphi = \frac{n_e - n_i}{\varepsilon_0 \varepsilon_1} q,$$

(1)

where $\varepsilon_1$ - relative permittivity, $\varepsilon_0$ - electric constant, $n_i$ - the concentration of positive charge of the ion core (the density of the positive background), $q$ - the absolute value of the electron charge.

From the Boltzmann formula

$$n_e = n_{e0} e^{\varphi / \theta},$$

(2)

where $n_{e0}$ - the electron density at the origin of the potential, i.e. when $\varphi = 0$, we obtain the Poisson-Boltzmann equation

$$\Delta \varphi = \frac{n_{e0} e^{\varphi / \theta} - n_i}{\varepsilon_0 \varepsilon_1} q.$$

(3)

The solution of equations (2) and (3) determines the distribution of the electron density. Consider a simple task when the dust particles are identical cylinders and these concentration will be constant in the space.

In the case of axial symmetry, of the equation (3) can be written as

$$\frac{\varepsilon_0 \varepsilon_1}{r} \frac{d}{dr} \left( r \cdot \frac{d \varphi}{dr} \right) = \left( n_{e0} e^{\varphi / \theta} - n_i \right) q.$$

(4)

As a result of the introduction of dimensionless quantities: $\frac{r}{R} = x$, $\frac{q \varphi}{kT} = \phi$, $\frac{n_i}{n_{e0}} = \bar{n}_i$, we obtain

$$\frac{1}{x} \frac{d}{dx} \left( x \cdot \frac{d \phi}{dx} \right) = b^2 (e^b - \bar{n}_i), \quad b^2 = \frac{q^2 R^2 n_{e0}}{kT \varepsilon_1 \varepsilon_0}.$$

(5)

Here $R$ – is a radius of the dust particle, $n_{e0}$ - the electron density in the center of the cylinder, $n_i = \text{const}$ in the field $0 \leq x \leq 1$ and $n_i = 0$ in the field of $x > 1$. Some of electrons leave from dust particles and therefore $n_i > n_{e0}$. The equation (11) will be solved under the conditions

$$\phi(0) = 0, \quad \phi'(0) = 0, \quad \phi'(\lambda) = 0,$$

where $\lambda = l/R$, $2l$ - the distance between the centers of two neighboring dust particles. The last boundary condition implies the vanishing of the total electric charge in the volume bounded by a sphere of radius $l$ and is used to determine the value of $n_i$.

Figure 1 shows plots of the electric potential distribution along the radius for different values of the absolute temperature. As can be seen in the dust particle with increasing $x$ potential first decreases slowly and then approaching the particle surface decreases rapidly. This is the due of the emergence of a large gradient of the electron density near the surface. Accordingly, there is a large electric field. With increasing temperature, the potential drop on the whole interval from 0 to 1 decreases owing to an increase in the electron yield of particles.
Fig. 1 Distribution of the potential at $R = 10^{-6}$ m, $n_i = 10^{19}$ m$^{-3}$, and at different temperatures (1000 K - red line, 1250 K - blue line, 1500 K - green line).

Since the potential and the electron density are is directly related to the Boltzmann formula, $x$ increases in $x \approx 1$ magnitude $n_e$ decreases sharply. With the removal of the particles decreases and the rate of change at the point of the condition $\frac{dn_e}{dx} = 0$ (see Fig. 2).

Fig. 2. Distribution of the electron density at $R = 10^{-6}$ m, $n_i = 10^{19}$ m$^{-3}$, and at different temperatures (1000 K - red line, 1250 K - blue line, 1500 K - green line).
At higher temperatures, with increasing $x$ in a particle density of electrons decreases rapidly. However, outside of the particle, the electron density is increases with increasing temperature.

In conclusion, we obtained the equation (4), which describes the distribution of the potential and the electron density in an equilibrium dust-electron plasma taking into account parameters of the electron gas inside the axial geometry dust particles. The inclusion of these parameters performed on the basis of the model of "solid-state plasma", considering the condensed particle system as the ion core and the free electron gas. Solving equation (4) we have been obtained the potential distributions and the electron density in thermal equilibrium dust-electron plasma. We have been found that near the particle surface there is a large gradient of the electric potential and as a result, the concentration of free electrons, i.e. electrons in the dust particle are located in a potential well. The obtained expressions allow to calculate the influence of several parameters such as temperature, concentration and dust particle size, type of the substance of particles on properties of the equilibrium dust-electron plasma.

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