Thermodynamics of strong coupling 2-color QCD

Yusuke NISHIDA

Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

2-color QCD at finite temperature \( T \) and chemical potential \( \mu \) is revisited in the strong coupling limit on the lattice. The phase structure in the space of \( T, \mu \) and the quark mass \( m \) is elucidated with the use of the mean field approximation and the large dimensional expansion. We put special emphasis on the interplay among the chiral condensate, the diquark condensate and the quark density in the \( T-\mu-m \) space. Qualitative comparison is made between our results and those of recent Monte-Carlo simulations in 2-color QCD.

§1. Introduction

Physics of matter under high baryon density is one of the most challenging problems in quantum chromodynamics (QCD) both from technical and physical point of view. Unfortunately, analysing the finite baryon chemical potential region of 3-color QCD from the first principle lattice simulations is retarded due to the complex fermion determinant. However, the situation is different for 2-color QCD in which the fermion determinant can be made real and the Monte-Carlo simulations are attainable. Therefore, 2-color QCD provides an unique opportunity to compare various ideas at finite chemical potential with the results from lattice simulations.

One of the major differences between 2-color QCD and 3-color QCD is that the color-singlet baryon is boson (diquark) in the former. This implies that the ground state of the 2-color system at finite baryon density in the color-confined phase is an interacting boson system, i.e. a Bose liquid, although the quark Fermi liquid may be realized at high baryon density in the color-deconfined phase. How this Bose liquid changes its character as a function of the temperature \( T \), the quark chemical potential \( \mu \) and the quark mass \( m \) is an interesting question by itself and may also give us a hint to understand physics of the color superconductivity in 3-color QCD in which the crossover from the Bose-Einstein condensate of tightly bound quark pairs to the BCS type condensate of loosely bound Cooper pairs may take place.

In this article, we revisit the thermodynamics of the strong coupling limit of 2-color lattice QCD with chiral and diquark condensates which was originally studied in Ref.\(^2\). Our main purpose is to analyse its phase structure and the interplay among the chiral condensate \( \langle \bar{q}q \rangle \), the diquark condensate \( \langle qq \rangle \) and the baryon density \( \langle q^\dagger q \rangle \) as functions of \( T, \mu \) and \( m \). This would give us a physical insight and useful guide for the actively pursued lattice QCD simulations of the same system in the weak coupling.

§2. Formulation

We derive an effective free energy for meson and diquark fields starting from the lattice action with the single component staggered fermion. In the strong cou-
pling limit $g \to \infty$, the gluonic part of the action vanishes because it is inversely proportional to $g^2$. Then the action on the lattice is given by only fermionic part;

$$S = m \sum_x \bar{\chi}(x)\chi(x) + \frac{1}{2} \sum_x \sum_{j=1}^{d} \eta_j(x) \left\{ \bar{\chi}(x)U_j(x)\chi(x+j) - \bar{\chi}(x+j)U_j^\dagger(x)\chi(x) \right\}$$

$$+ \frac{1}{2} \sum_x \eta_0(x) \left\{ \bar{\chi}(x)e^{\mu}U_0(x)\chi(x+\hat{0}) - \bar{\chi}(x+\hat{0})U_0^\dagger(x)e^{-\mu}\chi(x) \right\},$$

(2.1)

with

$$\eta_0(x) = 1, \quad \eta_j(x) = (-1)^{\sum_{i=1}^{d-1} x_i - 1}. \quad (2.2)$$

$\chi$ stands for the quark field in the fundamental representation of the color SU(2) group and $U_\mu$ is the SU(2) valued gauge link variable. $d$ represents the number of spatial directions.

This action has a global $U_V(1) \times U_A(1)$ symmetry at $\mu \neq 0$ in the chiral limit $m = 0$. $U_V(1)$ corresponds to the baryon number conservation, which is spontaneously broken by the diquark condensation. On the other hand, the action has a larger symmetry $U(2)$ for $m = \mu = 0$ due to the Pauli-Gürsey’s fermion—anti-fermion symmetry. This is the special feature of 2-color QCD.

Here we summarise how to derive the effective free energy for the meson and diquark system from the original action Eq. (2.1).

1. Large dimensional $(1/d)$ expansion is employed in the spatial directions in order to facilitate the integration over the spatial link variable $U_j$.
2. Bosonization is performed by introducing the auxiliary fields $\sigma$ for $\bar{\chi}\chi$ and $\Delta$ for $\chi\chi$. Then the mean field approximation is adopted for the auxiliary fields.
3. Integration with respect to $\chi$, $\bar{\chi}$ and $U_0$ are accomplished exactly.

After performing these procedure[3], we can write down the effective free energy for the chiral condensate ($\sigma$) and the diquark condensate ($\Delta$) as follows;

$$F_{\text{eff}}[\sigma, \Delta] = \frac{d}{2}\sigma^2 + \frac{d}{2}|\Delta|^2 - T \log \{1 + 4 \cosh (E_+/T) \cdot \cosh (E_-/T)\},$$

(2.3)

where $E_{\pm}$ is the excitation energy of (anti) quasi-quarks,

$$E_{\pm} = \arccosh \left( \sqrt{(1 + M^2) \cosh^2 \mu + (d/2)^2|\Delta|^2} \pm M \sinh \mu \right).$$

(2.4)

with dynamical quark mass $M = m + \sigma d/2$. In the chiral limit $m = 0$ with zero chemical potential $\mu = 0$, this is a function only in terms of $\sigma^2 + |\Delta|^2$. As a result, the effective free energy is invariant under the transformation mixing the chiral condensate with the diquark condensate. This symmetry comes from the Pauli-Gürsey symmetry of the original action at $m = \mu = 0$. The computational details to derive the effective free energy and its analytical properties can be found in Ref. [4].

§3. Numerical Results and Discussions

We determine the chiral condensate $\sigma$ and the diquark condensate $\Delta$ numerically by minimising the effective free energy in Eq. (2.3). The quark density $\rho = -\partial F_{\text{eff}}/\partial \mu$
First we consider the chiral and diquark condensates as functions of chemical potential (the left panel of Fig. 1). There exist two critical chemical potentials, the lower one $\mu_c^{\text{low}}$ and the upper one $\mu_c^{\text{up}}$. We can understand the behavior of the condensates as the manifestation of two different mechanisms: One is a continuous “rotation” from the chiral condensation to the diquark condensation above $\mu = \mu_c^{\text{low}}$ with the total condensate $\sqrt{\sigma^2 + \Delta^2}$ varying smoothly. The other is the “saturation effect”; quark density forces the diquark condensate to decrease and disappear for large $\mu$.

The “rotation” can be understood as follows: As we have discussed, the effective free energy at vanishing $m$ and $\mu$ has a symmetry between the chiral condensate and the diquark condensate. The effect of $m$ ($\mu$) is to break this symmetry in the direction of the chiral (diquark) condensation favored. Therefore at finite $m$, a relatively large chiral condensate predominantly appears for small $\mu$ region. Once $\mu$ exceeds a threshold value $\mu_c^{\text{low}}$, the chiral condensate decreases while the diquark condensate increases, because the effect of $\mu$ surpasses that of $m$.

As $\mu$ becomes larger, the diquark condensate begins to decrease in turn by the effect of the “saturation” and eventually disappears when $\mu$ exceeds the upper critical value $\mu_c^{\text{up}}$ (order of unity for $T = 0$). On the other hand, the quark density $\rho$ increases until the saturation point ($\rho = 2$) where quarks occupy the maximally allowed configurations by the Fermi statistics. Those behaviors of $\Delta$ and $\rho$ are also observed in the recent Monte-Carlo simulations of 2-color QCD.

Next we consider the chiral and diquark condensates as functions of $T$ (the right panel of Fig. 1). At low $T$, both the chiral and diquark condensates have finite values for $\mu = 0.2$. The diquark condensate decreases monotonously as $T$ increases and shows a second order transition. On the other hand, the chiral condensate increases as the diquark condensate decreases so that the total condensate $\sqrt{\sigma^2 + \Delta^2}$ is a smoothly varying function of $T$. The understanding based on the approximate
symmetry between chiral and diquark condensates is thus valid. An interesting observation is that the chiral condensate has a cusp shape associated with the phase transition of the diquark condensate.

Now we show the phase diagram of the strong coupling 2-color QCD in the $\mu$-$m$ plane at $T = 0$ in Fig. 2. The lower right of the figure corresponds to the vacuum with no baryon number present, $\rho = 0$. On the other hand, the upper left of the figure corresponds to the saturated system, $\rho = 2$, in which every lattice site is occupied by two quarks. There is a diquark superfluid phase with $\Delta \neq 0$ and $0 < \rho < 2$ bounded by the above two limiting cases. It is worth mentioning here that we can see the corresponding system in the context of condensed matter physics; the hardcore boson Hubbard model has a similar phase diagram in which superfluid phase is sandwiched by Mott-insulating phases with zero or full density.\[5)

Finally, the phase structure in the three dimensional $T$-$\mu$-$m$ space is shown in Fig. 3. The diquark condensate has a none-vanishing value inside the critical surface and the phase transition is of second order everywhere on this critical surface.

Acknowledgements

This article is based on a work Ref. [4] in collaboration with K. Fukushima and T. Hatsuda to whom the author is grateful.

References

1) H. Abuki, T. Hatsuda and K. Itakura, Phys. Rev. D 65 (2002), 074014.
K. Itakura, Nucl. Phys. A 715 (2003), 859.
2) E. Dagotto, F. Karsch and A. Moreo, Phys. Lett. B 169 (1986), 421.
E. Dagotto, A. Moreo and U. Wolff, Phys. Lett. B 186 (1987), 395.
J.-U. Klaetke and K.-H. Mütter, Nucl. Phys. B 342 (1990), 764.
3) J.B. Kogut, D. Toublan and D.K. Sinclair, Phys. Lett. B 514 (2001), 77; Nucl. Phys. B 642 (2002), 477; Phys. Rev. D 68 (2003), 054507.
4) Y. Nishida, K. Fukushima and T. Hatsuda, hep-ph/0306066 to appear in Phys. Rep. (2003).
5) G. Schmid, S. Todo, M. Troyer and A. Dorneich, cond-mat/0110024

Fig. 2. Phase diagram of strong coupling 2-color QCD in the $\mu$-$m$ plane at $T = 0$.

Fig. 3. Phase structure of strong coupling 2-color QCD in the $T$-$\mu$-$m$ space. The surface separates the region where $\Delta = 0$ (outside) from the region where $\Delta \neq 0$ (inside).