Self-organization of price fluctuation distribution in evolving markets

R. K. Pan and S. Sinha

The Institute of Mathematical Sciences, C.I.T. Campus - Taramani, Chennai, 600 113 India

received 21 June 2006; accepted in final form 16 January 2007
published online 21 February 2007

PACS 89.65.Gh – Economics; econophysics, financial markets, business and management
PACS 89.65.-s – Social and economic systems
PACS 05.65.+b – Self-organized systems

Abstract – Financial markets can be seen as complex systems in non-equilibrium steady state, one of whose most important properties is the distribution of price fluctuations. Recently, there have been assertions that this distribution is qualitatively different in emerging markets as compared to developed markets. Here we analyse both high-frequency tick-by-tick as well as daily closing price data to show that the price fluctuations in the Indian stock market, one of the largest emerging markets, have a distribution that is identical to that observed for developed markets (e.g., NYSE). In particular, the cumulative distribution has a long tail described by a power law with an exponent \( \alpha \approx 3 \). Also, we study the historical evolution of this distribution over the period of existence of the National Stock Exchange (NSE) of India, which coincided with the rapid transformation of the Indian economy due to liberalization, and show that this power law tail has been present almost throughout. We conclude that the “inverse cubic law” is a truly universal feature of a financial market, independent of its stage of development or the condition of the underlying economy.

Copyright © EPLA, 2007

Introduction. – Financial markets are paradigmatic examples of complex systems, comprising a large number of interacting components that are subject to a constant flow of external information [1,2]. Statistical physicists have studied simple interacting systems which self-organize into non-equilibrium steady states, often characterized by power law scaling [3]. Whether markets also show such behavior can be examined by looking for evidence of scaling functions which are invariant for different markets. The most prominent candidate for such a universal, scale-invariant property is the cumulative distribution of stock price fluctuations. The tails of this distribution have been reported to follow a power law, \( P_c(x) > x^{-\alpha} \), with the exponent \( \alpha \approx 3 \) [4]. This “inverse cubic law” had been reported initially for a small number of stocks from the S&P 100 list [5]. Later, it was established from statistical analysis of stock returns in the German stock exchange [6], as well as for three major US markets, including the New York Stock Exchange (NYSE) [7]. The distribution was shown to be quite robust, retaining the same functional form for time scales of up to several days. Similar behavior has also been seen in the London Stock Exchange [8]. An identical power law tail has also been observed for the fluctuation distribution of a number of market indices [9,10]. This apparent universality of the distribution may indicate that different markets self-organize to an almost identical non-equilibrium steady state. However, as almost all these observations are from developed markets, a question of obvious interest is whether the same distribution holds for developing or emerging financial markets. If the inverse cubic law is a true indicator of self-organization in markets, then observing the price fluctuation distribution as the market evolves will inform us about the process by which this complex system converges to the non-equilibrium steady state characterizing developed markets.

However, when it comes to empirical reports about such emerging markets there seems to be a lack of consensus. The market index fluctuations in Brazil [11] and Korea [10] have been reported to follow an exponential distribution, while, the distribution for an Indian market index was observed to be heavy tailed with exponent greater than 3 [12]. On the other hand, a comparative analysis of 27 indices from both mature and emerging markets found their tail behavior to be similar [13]. It is hard to
conclude about the nature of the fluctuation distribution for individual stock prices from the index data, as the latter is a weighted average of several stocks. Therefore, in principle, the index can show a distribution quite different from that of its constituent stocks if their price movements are not correlated.

Analysis of individual stock price returns for emerging markets have also not resulted in an unambiguous conclusion about whether such markets behave differently from developed markets. A study of the fluctuations in the daily price of the 49 largest stocks in an Indian stock exchange has claimed that the distribution has exponentially decaying tails [14]. This implies the presence of a characteristic scale, and the breakdown of universality of the power law tail for the price fluctuation distribution. On the other hand, it has been claimed that this distribution in emerging markets has even more extreme tails than developed markets, with an exponent α that can be less than 2 [15]. Recently, there has been a report of the “inverse cubic law” for the daily return distribution in the Chinese stock markets of Shanghai and Shenzhen [16]. These contradictory reports indicate that a careful analysis of the stock price return distribution for emerging markets is extremely necessary. This will help us to establish definitively whether the “inverse cubic law” is invariant with respect to the stage of economic development of a market.

All the previous studies of price fluctuations in emerging markets have been done on low-frequency daily data. For the first time, we report analysis done on high-frequency tick-by-tick data, which are corroborated by analysis of daily data over much longer periods. The data set that we have chosen for this purpose is from the National Stock Exchange (NSE) of India, the largest among the 23 exchanges in India, with more than 85% of the total value of transactions for securities in all market segments of the entire Indian financial market in recent times [17]. This data set is of unique importance, as we have access to daily data right from the time the market commenced operations in the equities market in November 1994, up to the present when it has become the world’s third largest stock exchange (after NASDAQ and NYSE) in terms of transactions [18]. Over this period, the market has grown rapidly, with the number of transactions having increased by more than three orders of magnitude. Therefore, if markets do show discernible transition in the return distribution during their evolution, the Indian market data is best placed to spot evidence for it, not least because of the rapid transformation of the Indian economy in the liberalized environment since the 1990s.

In this paper, we focus on two important questions: i) Does an emerging market exhibit a different price fluctuation distribution compared to developed markets, and ii) if the market is indeed following the inverse cubic law at present, whether this has been converged at starting from an initially different distribution when the market had just begun operation. Both of these questions are answered in the negative in the following analysis.

Data description. – We have looked at two data sets having different temporal resolutions: i) The high-frequency tick-by-tick data contains information about all transactions carried out in the NSE between January 2003 and March 2004. This information includes the date and time of trade, the price of the stock during transaction and the volume of shares traded. This database is available in the form of CDs published by NSE. For calculating the price return, we have focused on 489 stocks that were part of the BSE 500 index (a comprehensive indicator for the Indian financial market) during this period. The number of transactions for each company in this set is $\sim 10^6$, on the average. The total number of transactions for the 489 stocks is of the order of $5 \times 10^8$ during the period under study. ii) The daily closing price of all the stocks listed in NSE during its period of existence between November 1994 and May 2006. This was obtained from the NSE website [19] and manually corrected for stock splitting. For comparison with US markets, in particular the NYSE, we have considered the 500 stocks listed in S&P 500 during the period November 1994–May 2006, the daily data being obtained from Yahoo! Finance [20].

Results. – To measure the price fluctuations such that the result is independent of the scale of measurement, we calculate the logarithmic return of price. If $P_i(t)$ is the stock price of the $i$-th stock at time $t$, then the (logarithmic) price return is defined as

$$R_i(t, \Delta t) \equiv \ln P_i(t + \Delta t) - \ln P_i(t).$$

However, the distribution of price returns of different stocks may have different widths, owing to differences in their volatility, defined (for the $i$-th stock) as $\sigma_i^2 \equiv \langle R_i^2 \rangle - \langle R_i \rangle^2$. To compare the distribution of different stocks, we normalize the returns by dividing them with their volatility $\sigma_i(t)$ as in ref. [2]. The resulting normalized price return\(^\dagger\) is given by

$$r_i(t, \Delta t) \equiv \frac{R_i(t) - \langle R_i(t) \rangle}{\sigma_i(t)} = \frac{P_i(t + \Delta t) - P_i(t)}{\sigma_i(t)}.$$

\(\dagger\)The normalization of return $R_i(t)$ is performed by removing its own contribution from the volatility, i.e., $\sigma_i(t) = \sqrt{\frac{1}{N} \sum_{t'=t}^{t+\Delta t} (R_i(t') - \langle R_i(t) \rangle)}$.

58004-p2
for all the 489 stocks that we have considered is shown in fig. 1 (right).

As all the individual stocks follow very similar distributions, we can merge the data for different stocks to obtain a single distribution for normalized returns. The aggregated return data set with $\Delta t = 5$ mins has $6.5 \times 10^6$ data points. The corresponding cumulative distribution is shown in fig. 2 (left), with the exponents for the positive and negative tails estimated as

$$\alpha = \begin{cases} 
2.87 \pm 0.08 \text{ (positive tail),} \\
2.52 \pm 0.04 \text{ (negative tail).}
\end{cases} \quad (3)$$

From fig. 2 we confirm that the distribution does indeed follow a power law decay, albeit with different exponents for the positive and negative return tails. Such a difference between the positive and negative tails has also been observed in the case of stocks in the NYSE [7]. To further verify that the tails are indeed consistent with a power law form, we perform an alternative measurement of $\alpha$ using the Hill estimator [21,22]. We arrange the returns in decreasing order such that $r_1 > \cdots > r_n$ and obtain the Hill estimator (based on the largest $k+1$ values) as $H_{k,n} = \frac{1}{k} \sum_{i=1}^{k} \log \frac{r_i}{r_{k+1}}$, for $k = 1, \cdots, n-1$. The estimator $H_{k,n} \rightarrow \alpha^{-1}$ when $k \rightarrow \infty$ and $k/n \rightarrow 0$. For our data, this procedure gives $\alpha = 2.86$ and 2.56 for the positive and the negative tail, respectively (when $k = 20000$), which are consistent with (3).

Next, we extend this analysis for longer time scales, to observe how the nature of the distribution changes with increasing $\Delta t$. As has been previously reported for US markets, the distribution is found to decay faster as $\Delta t$ becomes large. However, upto $\Delta t = 1$ day, i.e., the daily
closing returns, the distribution clearly shows a power-law tail (fig. 3, left). The deviation is because of the decreasing size of the data set with increase in $\Delta t$. Note that, while for $\Delta t < 1$ day we have used the high-frequency data, for $\Delta t = 1$ day we have considered the longer data set of closing price returns for all stocks traded in NSE between November 1994 to May 2006. In fig. 3 (right) we have also shown the distributions of the power law exponents for the individual stocks, for $10 \text{ min} \leq \Delta t \leq 60 \text{ min}$. We observe that the bulk of the exponents falls between 2 and 4, consistent with the results from the merged data sets.

To compare the distribution of returns in this emerging market with that observed in mature markets, we have considered the daily return data for the 500 stocks from NYSE listed in S&P 500 over the same period. As seen in fig. 4, the distributions for NSE and NYSE are almost identical, implying that the price fluctuation distribution of emerging markets cannot be distinguished from that of developed markets, contrary to what has been claimed recently [14].

We now turn to the second question, and check whether it is possible to see any discernible change in the price fluctuation distribution as the stock market evolved over time. For this we focus on the daily return distribution for all stocks that were traded during the entire period of existence of NSE. This period is divided into four intervals (a) 1994–1996 (□), (b) 1997–1999 (▽), (c) 2000–2002 (○) and (d) 2003–2006 (♦), each corresponding to increase in the number of transactions by an order of magnitude. Figure 5 shows that the return distribution at all four periods is similar, the negative tail even more so than the positive one. While the numerical value of the tail exponent may

\[\text{Total number of stocks traded in these four intervals were 1460, 1560, 1321 and 1160, respectively.}\]
appear to have changed somewhat over the period that
the NSE has operated, the power law nature of the tail
is apparent at even the earliest period of its existence.
We therefore conclude that the convergence of the return
distribution to a power law functional form is extremely
rapid, indicating that a market is effectively always at
the non-equilibrium steady state characterized by the inverse
cubic law.

We have also verified that stocks in the Bombay Stock
Exchange (BSE), the second largest in India after NSE,
follow a similar distribution [23]. Moreover, the return
distribution of several Indian market indices (e.g., the
NSE Nifty) also exhibits power law decay, with exponents
very close to 3 [24]. As the index is a composite of
several stocks, this behavior can be understood as a
consequence of the power law decay for the tails of
individual stock price returns, provided the movements
of these stocks are correlated [7,23]. Even though the Indian
market microstructure has been refined and modernized
significantly in the period under study as a result of
the reforms and initiatives taken by the government, the
nature of the return distribution has remained invariant,
indicating that the nature of price fluctuations in financial
markets is most probably independent of the level of
economic development.

Discussion and conclusion. – Most of the previous
studies on emerging markets had focussed on either stock
indices or a small number of stocks. In addition, all these
studies were done with low-frequency daily data. Thus,
the number of data points used for calculating the return
distribution were orders of magnitude smaller compared to
ours. Indeed, the paucity of data can result in missing the
long tail of a power law distribution and falsely identifying
it to be an exponential distribution. Matia et al. [14]
claimed that differences in the daily return distribution for
Indian and US markets were apparent even if one looks at
only 49 stocks from each market. However, we found
that this statement is critically dependent upon the choice
of stocks. Indeed, when we made an arbitrary choice of
50 stocks in both Indian and US markets, and compared
their distributions, we found them to be indistinguishable.
Therefore, the results of analysis done on such small data
sets can hardly be considered stable, with the conclusions
depending on the particular sample of stocks.

In this study, we have shown conclusively that the
inverse cubic law for price fluctuations holds even in
emerging markets. It is indeed surprising that the nature of
price fluctuations is invariant with respect to large changes
in the number of stocks, trading volume and number of
transactions that have all increased significantly at NSE
during the period under study. The robustness of the
distribution implies that it should be possible to explain
it independently of the particular features of different
markets, or the various economic factors underlying them.

***

We are grateful to M. Krishna for assistance in obtaining
and analyzing the high-frequency NSE data. We thank A.
Chatterjee, J. D. Farmer and the referees for helpful comments.

REFERENCES

[1] Mantegna R. N. and Stanley H. E., An Introduction to
Econophysics (Cambridge University Press, Cambridge)
1999.
[2] Bouchaud J. P. and Potters M., Theory of Financial
Risk and Derivative Pricing (Cambridge University Press,
Cambridge) 2003.
[3] Privman V. (Editor), Nonequilibrium Statistical Mechanics
in One Dimension (Cambridge University Press,
Cambridge) 1997.
[4] Gopikrishnan P., Meyer M., Amaral L. A. N. and
Stanley H. E., Eur. Phys. J. B, 3 (1998) 139.
[5] Jansen D. W. and de Vries C. G., Rev. Econ. Stat., 73
(1991) 18.
[6] Lux T., Appl. Finan. Econ., 6 (1999) 463.
[7] Plerou V., Gopikrishnan P., Amaral L. A. N.,
Meyer M. and Stanley H. E., Phys. Rev. E, 60 (1999)
6519.
[8] Farmer J. D., Gillemot L., Lillo F., Mike S. and
Sen A., Quant. Finance, 4 (2004) 383.
[9] Gopikrishnan P., Plerou V., Amaral L. A. N.,
Meyer M. and Stanley H. E., Phys. Rev. E, 60 (1999)
5305.
[10] Oh G., Um Cheol-Jun and Kim S., preprint (2006)
physics/0601126.
[11] Couto Miranda L. and Riera R., Physica A, 297
(2001) 509.
[12] Sarma M., Eurorandom Report, 2005-003 (2005).
[13] Jondeau E. and Rockinger M., J. Emp. Finance, 10
(2003) 559.
[14] Matia K., Pal M., Salunkay H. and Stanley H. E.,
Europhys. Lett., 66 (2004) 909.
[15] Bouchaud J. P., Chaos, 15 (2005) 026104.
[16] Gu G.-F. and Zhou W.-X., preprint (2006) physics/
0603147.
[17] Indian Securities Market, A Review (ISMR)
(National Stock Exchange of India) 2004.
[18] Annual Report and Statistics 2005 (World Federation of
Exchanges) 2006, p. 77.
[19] http://www.nseindia.com/.
[20] http://finance.yahoo.com/.
[21] Hill B. M., Ann. Stat., 3 (1975) 1163.
[22] Drees H., de Haan L. and Resnick S., Ann. Stat., 28
(2000) 254.
[23] Sinha S. and Pan R. K., Econophysics of Stock and Other
Markets, edited by Chatterjee A. and Chakrabarti
B. K. (Springer, Milan) 2006 (also at physics/0605247).
[24] Pan R. K. and Sinha S., preprint (2006) physics/
0607014.