FROM PRECISION COSMOLOGY TO ACCURATE COSMOLOGY

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This is the dawning of the age of precision cosmology, when all the important parameters will be established to one significant figure or better, within the cosmological model. In the age of accurate cosmology the model, which nowadays includes general relativity theory and the CDM model for structure formation, will be checked tightly enough to be established as a convincing approximation to reality. I comment on how we might make the transition. We already have some serious tests of gravity physics on the length and time scales of cosmology. The evidence for consistency with general relativity theory is still rough, but impressive, considering the enormous extrapolation from the empirical basis, and these probes of gravity physics will be considerably improved by work in progress on the cosmological tests. The CDM model has some impressive observational successes too, and some challenges, not least of which is that the model is based on a wonderfully optimistic view of the simplicity of physics in the dark sector. I present as a cautionary example a model for dark matter and dark energy that biases interpretations of cosmological observations that assume the CDM model. In short, cosmology has become an empirically rich subject with a well-motivated standard model, but it needs work to be established as generally accurate.

1 Introduction

Our colleagues in the more exact sciences distinguish the precision of a measurement, which is indicated by the number of significant figures, from the accuracy, which is what remains after due account of the interference by systematic errors. In cosmology we have to worry about systematic errors in the astronomy and, it is less commonly emphasized, in the physics. In the standard cosmology the latter includes general relativity and the rest of textbook physics, along with the cold dark matter model for structure formation. All this physics is a considerable extrapolation from the empirical basis. This means that, unlike the standard model for particle physics, in cosmology it is not a matter of measuring parameters in a reliably established theory: we have to check the physics too.

We have checks of the physics and the astronomy, from the growing network of cosmological tests. For example, the SNeIa redshift-magnitude measurements, combined with the CDM model interpretation of the anisotropy of the 3 K cosmic background radiation (the CBR), indicate the mean mass density, $\rho_m$, in low pressure matter is about one quarter of the critical Einstein-de Sitter value. A similar number follows from most dynamical analyses of galaxy peculiar velocities. These two approaches depend on very different astronomy, and they apply quite different aspects of the physics of the relativistic Friedmann-Lemaître cosmology. If the physics or the astronomy failed, the consistency of these estimates of $\rho_m$ would seem unlikely. Accidental coincidences do happen, of course, and we have to remember the natural human tendency to stop working so hard on an analysis when it approaches the wanted answer. Thus it is important
that similar estimates of $\rho_m$ follow from still other lines of evidence: weak gravitational lensing, the baryon mass fraction in clusters of galaxies, the abundance of clusters as a function of mass and redshift, and the power spectrum of the galaxy distribution. If this concordance survives further scrutiny that explains the remaining discrepant indications, for significantly larger and smaller values of $\rho_m$, it will eliminate the hypothesis of canceling errors.

What do we learn from this evidence for concordance? It certainly encourages the view that the mass density parameter, $\Omega_m = 8\pi G \rho_m / 3H_o^2$, is a physically meaningful number, and that we know its value to a factor of two or so. But it is useful to be more specific, by considering what aspects of gravity physics are probed by this concordance, and by all the other cosmological tests. I review four tests of gravity physics on the scales of cosmology in Sec. 2. All agree with GR so far. This is not surprising: we have no substantial reason within fundamental theory (apart maybe from brane worlds) to suspect GR fails on cosmological length scales. But positive empirical evidence is the thing.

Many of the cosmological tests assume the CDM model for structure formation. Is this model adequate for precision cosmology at the ten percent level? I discuss aspects of this issue in Sec. 3. The estimates of $\Omega_m$ based on CDM generally agree with independent indications from dynamics, and the successful fit to the 3 K CBR temperature anisotropy is also impressive. This is serious evidence that the CDM model is a useful approximation to reality. But the present precision of the evidence allows considerably more complicated physics in the dark sector, and more complicated physics may be indicated by the observational challenges from galaxy structure and formation. In short, significant adjustments to the CDM model would not be surprising, and a major shift not inconceivable. Until this is sorted out structure formation is a hazardous basis for cosmological tests.

These topics are discussed at length, with many references, in a paper with Bharat Ratra (in astro-ph/0207347). I refer the reader to this paper for details and references. Here I indulge in the luxury of a reference-free overview.

2 General Relativity Theory

General relativity passes searching tests on length scales ranging from the laboratory to the Taylor-Hulse pulsar ($\sim 10^{11}$ cm) and the Solar System ($\sim 10^{13}$ cm). But the extrapolation to cosmology, at Hubble length $c/H_o \sim 10^{28}$ cm is enormous, and to be checked.

I review four examples of the program of tests of gravity physics on the length and time scales of cosmology. Two concern the mean homogeneous expansion, under the assumption spacetime is described by a single metric tensor. The third test probes the inverse square law for the nonrelativistic dynamics of departures from homogeneity, the fourth gravitational lensing.

2.1 Active Gravitational Mass

In the homogeneous standard model the expansion parameter satisfies

$$\frac{\ddot{a}}{a} = \frac{4}{3} \pi G (\rho + 3p),$$

(1)

where the mean mass density $\rho$ and pressure $p$ satisfy the local energy conservation equation

$$\dot{\rho} = -3(\rho + p) \dot{a}/a.$$ 

(2)

The evidence that the universe is close to homogeneous and isotropic near the Hubble length is much more solid than for the issues under discussion here. In what follows, the mean mass density, $\rho$, and pressure, $p$, are the diagonal time and space parts of the stress-energy tensor, smoothed over scales much larger than the clustering length $\sim 10$ Mpc, in the frame of reference in which the smoothed stress-energy tensor is diagonal.
If we did not have GR, a naive Newtonian model might have led us to write
\[ \frac{\ddot{a}}{a} = -\frac{4}{3}\pi G\rho. \] (3)

Or we might have preferred to finesse the problem with the vacuum zero-point energy density. If the vacuum looks the same to any inertial observer, special relativity says the vacuum mass density and pressure satisfy \( p_v = -\rho_v \). They cancel from the right-hand side of the energy conservation Eq. 2, leaving \( \rho_v \) constant, as required of a velocity-independent vacuum. If the expansion equation were
\[ \frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + p), \] (4)

it would neatly remove the vacuum gravitational mass density. But to avoid confusion I emphasize that the merit I see in Eqs. 3 and 4 is their role as foils.

We have a test, from the standard model for the origin of deuterium and isotopes of helium and lithium, at \( z \sim 10^{10} \). At this redshift the pressure in the standard cosmology is close to \( p = \rho/3 \), so the expansion time satisfies
\[ \frac{1}{t^2} = \frac{16}{3}(1 + u)\pi G\rho. \] (5)

I have written the active gravitational mass density in a near homogeneous distribution as
\[ \rho_{\text{grav}} = \rho + 3up, \] (6)

with \( u = 1 \) in GR, \( u = 0 \) in the Newtonian model, and \( u = 1/3 \) in the model in Eq. 4 contrived to eliminate the gravity of a Lorentz-invariant vacuum zero-point energy density. The larger expansion times in these two foils lower the helium abundance, to \( Y \simeq 0.20 \) and \( Y \simeq 0.21 \). The experts assure me the former looks quite unpromising, and the latter is little better.

The conclusion is that the relativistic prediction \( u = 1 \) in Eq. 6 at redshift \( z \sim 10^{10} \) fits the observations significantly better than the foils in Eqs. 3 and 4. We should pause to admire a remarkable test of gravity physics.

2.2 The Effect of Space Curvature on the Expansion Rate

In the Friedmann-Lemaître model the expansion rate satisfies
\[ H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G \left(\rho + a^{-2}R_a^{-2}\right), \] (7)

and the line element may be written as
\[ ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 + R_b^{-2}r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right). \] (8)

Eq. 7 is the first integral of Eq. 1 with local energy conservation (Eq. 2), where \( R_a^{-2} \) is the constant of integration. The line element in Eq. 8, with the free constant \( R_b^{-2} \), follows from homogeneity and isotropy under the assumption spacetime is described by a single metric tensor. That is, Eqs. 7 and 8 are more general than GR. The GR prediction is \( R_a^{-2} = R_b^{-2} \).

The prediction has been probed, in discussions of a mass component with pressure \( p_c = -\rho_c/3 \), as in some models for cosmic strings. Local energy conservation says the mass density in this component varies as \( \rho_c \propto a(t)^{-2} \). This term produces the expansion time history of a universe with negative space curvature (positive \( R_a^{-2} \) in Eq. 7) in a Friedmann-Lemaître model with zero space curvature (\( R_b^{-2} = 0 \) in Eq. 8). I understand the SNeIa measurements tend to prefer \( R_a^{-2} = 0 \) and \( R_b^{-2} = 0 \), an encouraging start. It will be interesting to see the constraints on \( R_a^{-2} \) and \( R_b^{-2} \) treated as two independent parameters in the fit to the improving measurements.

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I mean the definition, “One that by contrast underscores or enhances the distinctive characteristics of another,” in the American Heritage Dictionary.
2.3 The Gravitational Inverse Square Law

An indirect but powerful test of the inverse square law for gravity is emerging from the theory and observations of large-scale dynamics.

In the standard cosmology the departures from homogeneity at sufficiently large length scales or redshifts are well described by linear perturbation theory. When nongravitational forces may be neglected, the mass density contrast, $\delta(x, t) = \delta\rho/\rho$, the peculiar velocity, $\mathbf{v}(x, t)$, and the peculiar gravitational acceleration, $\mathbf{g}(x, t)$, satisfy

$$\frac{\partial \delta}{\partial t} = -\frac{1}{a} \nabla \cdot \mathbf{v}, \quad \frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a} \mathbf{v} = \mathbf{g},$$

in linear theory and comoving coordinates. The first equation is local mass conservation. The second term in the second equation follows because a moving particle always is overtaking receding comoving observers. Both are more general than GR. In GR the peculiar gravitational acceleration satisfies Poisson’s equation,

$$\nabla \cdot \mathbf{g} = -4\pi G \rho_b \delta,$$

where the mean mass density is $\rho_b(t)$. These three equations yield

$$\frac{\partial^2 \delta}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} = 4\pi G \rho_b \delta.$$

We are interested in the growing solution,

$$\delta(x, t) = \delta(x, t_i) D(t)/D(t_i).$$

Eq. 12 has an observationally important property: the evolution of the mass density contrast $\delta(x, t)$ at fixed comoving position $x$ is independent of the density contrast everywhere else (in linear perturbation theory). This is because in linear theory the peculiar velocity $\mathbf{v}(x, t)$ in the growing mode is proportional to the peculiar gravitational acceleration $\mathbf{g}(x, t)$. The rate of change of $\delta(x, t)$ at fixed $x$ is set by the divergence of $\mathbf{v}(x, t)$, which is proportional to the divergence of $\mathbf{g}(x, t)$, which Poisson’s equation says is proportional to $\delta(x, t)$. Since the inverse square law for $\mathbf{g}$ follows from Poisson’s equation, a failure of the inverse square law would be reflected in a failure of the standard analysis of the evolution of large-scale structure.

For a foil let us consider what happens when Poisson’s equation is replaced with

$$\nabla^2 \phi/a^2 - \mu^2 \phi = 4\pi G \rho_b(t) \delta(x, t), \quad \mathbf{g} = -\nabla \phi/a,$$

where $\mu$ is constant. It is a good exercise for the student to check that Eq. 11 becomes

$$\frac{\partial^2 \delta_k}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta_k}{\partial t} = \frac{4\pi G \rho_b}{1 + (a\mu/k)^2} \delta_k,$$

for the Fourier component $\delta_k(t)$ with comoving wavenumber $k$. At short wavelengths, $a\mu/k \ll 1$, the Yukawa interaction is close to the inverse square law, and Eq. 14 is the Fourier transform of Eq. 11. On these scales the functional form of $\delta(x, t)$ is conserved while the amplitude grows, as in the standard model. At long wavelengths the foil Eq. 14 preserves the Fourier phases, but the amplitude $\delta_k$ stops growing at $a\mu/k \gg 1$.

In the CDM model the power spectrum of the primeval mass distribution usually is modeled as $P_k = |\delta_k|^2 = A k^n$, where $A$ and $n$ are constants. The fit to the observations requires the index is close to scale-invariant, $n \simeq 1$. In the foil this initial condition evolves to

$$P_k(t) \simeq A k^n a(t)^2 \quad \text{at} \quad k > \mu a(t),$$

$$\simeq A \mu^{-2} k^{n+2} \quad \text{at} \quad k < \mu a(t).$$

I am assuming conventional matter-dominated evolution, where the growing solution to Eq. 11 is $D \propto a \propto t^{2/3}$. It might be noted that if $n > 2$ small-scale nonlinear dynamics forces the large-scale tail of the power spectrum to decrease with increasing length no more rapidly than $P \propto k^4$. 
A conservative bound on the physical cutoff length in Eqs. (13 and 15) is
\[ \mu^{-1} \gtrsim 10h^{-1} \text{ Mpc}. \]  (16)

If the cutoff were a factor of ten smaller there would be serious problems with the power spectrum of the present galaxy distribution, which is well measured to \( k \sim 0.1h \text{ Mpc}^{-1} \), and with the interaction of matter and radiation at decoupling, since the comoving cutoff at decoupling would be less than the present Hubble length. On length scales larger than the cutoff we might look for some analog of the intermediate Sachs-Wolfe effect, but that depends on a more detailed model.

It is impressive that structure formation gives a quite direct probe of the inverse square law for nonrelativistic motion on scales \( \sim 10^{25} \text{ cm} \), some twelve orders of magnitude larger than the standard tests. This does assume the adiabatic scale-invariant initial condition of ΛCDM, which has been accepted into the standard cosmology because it gives a consistent fit to the measurements of the power spectra of the galaxy and CBR distributions, under standard gravity physics. We don’t know for sure whether a modified gravitational force law could account for the observations under isocurvature initial conditions.

The length \( \mu^{-1} \) (which can be taken to be comoving or physical) in the Yukawa force law seems awkward from a phenomenological point of view. Another maybe more interesting foil, in which the force law at large separations is (made by hand to be) a power law with index different from two, is under discussion.

### 2.4 The Gravitational Deflection of Light

Also under discussion are tests of the large-scale relativistic gravitational dynamics relevant to the anisotropy of the CBR and gravitational lensing. Here I comment on an easy parametrization of the latter.

The relativistic factor of two difference from the Newtonian gravitational deflection of light figures in the luminous arcs produced by mass concentrations in clusters of galaxies, the rate of lensing of quasars by the masses in foreground galaxies, and the mass estimates from weak lensing. It can be checked. For example, we have estimates of the mass distributions in clusters from measurements of galaxy redshifts, X-ray observations of the intracluster plasma, and measurements of the inverse Compton-Thomson scattering of the CBR by the plasma. Their interpretation depends on nonrelativistic gravitational dynamics, at scales less than about a megaparsec, along with standard local physics. The results may be compared to what is needed to account for luminous arcs under models for the gravitational deflection of light and the angular size distances. It would be interesting to know whether analyses of luminous arcs and the other lensing phenomena, along with the relevant mass estimates, have become precise enough to distinguish the factor of two difference between the Newtonian and GR models.

### 3 The Dark Sector

In ΛCDM the three dominant contributions to the present mass of the universe are dark energy — the modern variant of Einstein’s cosmological constant — nonbaryonic dark matter, and baryonic matter, with density parameters
\[ \Omega_A \simeq 0.75, \quad \Omega_{DM} \simeq 0.2, \quad \Omega_{\text{baryon}} \simeq 0.05. \]  (17)

The hypothetical dark sector, with density parameter \( \Omega_A + \Omega_{DM} \simeq 0.95 \), interacts with gravity, but extremely weakly if at all with ordinary matter and radiation.

We have pretty good evidence the dark sector exists, at about the parameters in Eq. (17), but little empirical guidance to the physics. We accordingly adopt the simplest physics we can get away with, which is good strategy, but need not be the way it is.
3.1 The Empirical Situation

Observational problems with the ΛCDM picture for galaxy structure and formation are widely discussed, but there are considerable divisions of opinion on which are serious. My list is headed by the predicted formation of elliptical galaxies by mergers at modest redshifts, which seems out of synch with the observation of quasars at $z \sim 6$; the prediction of appreciable debris in the voids defined by $L_*$ galaxies, which seems contrary to the observation that dwarf, irregular, and $L_*$ galaxies have quite similar distributions; and the prediction of cusp-like dark matter cores in low surface brightness galaxies, which are not observed. The warm and collisional dark matter variants of the standard model may remedy the last problem with little effect on the cosmological tests. The first two seem less easily resolved, and their significance for the cosmological tests that depend on the structure formation model is an open issue.

We do have a serious case for the existence of the dark sector. Nonbaryonic matter at about the mass density in Eq. [17] is indicated by two independent lines of evidence. First, this nonbaryonic matter allows us to reconcile the baryon density parameter $\Omega_b \simeq 0.05$ derived from the standard model for the light elements with the evidence that the net mass density in matter capable of clustering is $\Omega_m \gtrsim 0.15$. Second, nonbaryonic matter is an essential ingredient in the standard and so far largely successful model for the large-scale distributions of galaxies and the CBR: the absence of radiation drag on the nonbaryonic component finesses dissipation of primeval adiabatic density fluctuations, allowing the observed hierarchical “bottom up” growth of structure, as opposed to the pancake “top-down” growth to be expected from adiabatic initial conditions in a baryonic dark matter model. Perhaps primeval isocurvature initial conditions can produce a viable baryonic dark matter model, but I have not seen an example. My conclusion is that the case for nonbaryonic matter is substantial, though not yet as compelling as the abundance of evidence that the matter density parameter is $\Omega_m = 0.25 \pm 0.1$.

In ΛCDM the second hypothetical component, dark energy, is present in an amount sufficient to make space sections flat. The SNeIa and CBR measurements agree with this: within the CDM cosmology they indicate $\Omega_\Lambda \simeq 0.7$ and $\Omega_m \simeq 0.3$. We have a check, from the many other lines of evidence for a similar value of $\Omega_m$. This is very encouraging, but we have to bear in mind the hazards of astronomy. How do we know the supernovae observed at redshift $z \sim 1$ are statistically similar to those seen at low redshift? How do we know we can trust a structure formation model, ΛCDM, that is somewhat beclouded?

Before considering one of the clouds — physics in the dark sector — we should pause to note a related issue. Well checked physics says the zero-point energies of particles and fields at laboratory scales are as real as any other, and contribute to gravity like any other energy. But the known fields make absurdly large positive and negative contributions to the vacuum energy density. This has been known since the discovery of quantum physics. The usual prescription — just ignore the vacuum part — is observationally successful but certainly not an acceptable theory. We do not understand the role of the material content of the space between the galaxies, and we do not know whether this ignorance is hazardous to the cosmological tests.

3.2 A Cautionary Example

The point of this example is that the dark sector could be complicated. Consider the Lagrangian

$$L = \frac{\phi_i \phi^i}{2} + \frac{\psi_i \psi^i}{2} - K \phi^{-\alpha} - (m^2 + \lambda \phi^2) \psi^2 / 2,$$

(18)

where $\alpha$, $\lambda$, $m$, and $K$ are positive constants, the first two dimensionless. This is to be added to the Ricci scalar and the terms for ordinary matter and radiation that interact with $\psi$ and $\phi$ only by gravity.

When $\lambda = 0$ this is a familiar model for dark matter, represented by the scalar field $\psi$ with mass $m$, and dark energy, represented by the scalar $\phi$. The mass $m$ is supposed to be
much larger than Hubble’s constant $H_o \sim t_o^{-1}$ (where the subscripts here and below mean the present values), so at $\lambda = 0$ the field $\psi$ oscillates with frequency $m$ and amplitude proportional to $a(t)^{-3/2}$ (assuming the departures from a homogeneous spatial distribution of the $\psi$ energy are nonrelativistic). The observations say the present value of the dark matter mass density, $\rho_o$, is not far from the Einstein-de Sitter value, so $\rho_o = \psi^2/2 + m^2\psi^2/2 \sim m_{\text{Pl}}^2 t_o^{-2} (a_o/a)^3$, where $m_{\text{Pl}} = G^{-1/2}$ is the Planck mass. When $\rho_\psi$ is the dominant mass density the attractor power law solution for the near homogeneous dark energy field is $\phi \propto t^{2/(\alpha + 2)}$, which means the ratio of energy densities varies as $\rho_\phi/\rho_\psi \propto t^{4/(\alpha + 2)}$. The dark energy in $\phi$ thus eventually dominates and the universe starts to act as if it had an appreciable cosmological constant. The notorious coincidence is that this seems to have happened just as we flourish, that is, $\rho_\phi^0 \sim \rho_\psi^0$. This would mean the present value of the dark energy field is $\phi_o \sim m_{\text{Pl}}$.

When $\lambda \neq 0$ the dark matter field amplitude varies with time as $a(t)^{-3/2}(m^2 + \lambda\phi(t)^2)^{-1/4}$, and the dark matter mass density is

$$\rho_\psi = \frac{1}{2} \psi^2 + \frac{1}{2}(m^2 + \lambda\phi^2)\psi^2 \sim \frac{m_{\text{Pl}}^2}{t_o^2} (\frac{a_o}{a(t)})^3 \left( \frac{m^2 + \lambda\phi^2}{m^2 + \lambda\phi_o^2} \right)^{1/2}.$$ (19)

The last expression is the spatial mean value. The time dependence is easy to understand: the oscillation of $\psi$ is adiabatic, so the particle number is conserved, the mean number density varies as $n_\psi \propto a(t)^{-3}$, and the mean mass density varies as $\rho_\psi \propto m_{\text{eff}} a^{-3}$, where the particle mass is

$$m_{\text{eff}} = (m^2 + \lambda\phi^2)^{1/2}.$$ (20)

I shall comment on the simple weak coupling case,

$$0 < \lambda m_{\text{Pl}}^2 \lesssim m^2.$$ (21)

When $\rho_\psi$ is dominant the interaction term in the wave equation for $\phi$ varies approximately as $\phi/t^2$, so the attractor solution is $\phi \sim t^{2/(\alpha + 2)}$, and as before $\rho_\phi$ eventually dominates.

The coupling of the dark matter and energy fields causes the particle mass $m_{\text{eff}}$ to increase as the dark energy field $\phi$ increases (Eq. [21]), and it produces a long-range fifth force: lumps of dark matter, with masses $M_1$ and $M_2$ at separation $r \ll H_o^{-1}$, interact by the potential

$$U = -\kappa G \frac{M_1 M_2}{r}, \quad \kappa = \frac{\lambda^2 \phi_o^2 m_{\text{Pl}}^2}{2\pi m_{\text{eff}}^4} \sim \frac{\lambda^2 m_{\text{Pl}}^4}{m^4}.$$ (22)

In the visible sector we have tight constraints on fifth forces and variable masses (in units where $\hbar$ and $c$ are fixed). Here are examples of the much looser constraints in the dark sector.

The mass of an isolated dark matter halo varies as $M \propto m_{\text{eff}}$. This adiabatic evolution conserves $M \sigma r$, causing the halo radius and velocity dispersion to vary as

$$\sigma \propto m_{\text{eff}}(t)^2,$$

when the halo is dominated by dark matter. The fifth force biases the apparent density parameter derived from the nonlinear dynamics of relative motions of dark matter halos from the true value, $\Omega_m$, to

$$\Omega_{\text{apparent}} = (1 + \kappa)\Omega_m.$$ (24)

In linear perturbation theory the evolution of the dark matter density contrast is adjusted from Eq. [11] to

$$\frac{\partial^2 \delta}{\partial t^2} + \left( \frac{\dot{a}}{a} + \frac{\dot{m}_{\text{eff}}}{m_{\text{eff}}} \right) \frac{\partial \delta}{\partial t} = 4\pi G \rho_0 (1 + \kappa)\delta.$$ (25)
And we have to take account of the exchange of energy between $\phi$ and $\psi$ in the computation of the expansion rate as a function of time.

It would be a curious coincidence if the value of the parameter $\lambda$ happened to be just such as to make one or more of these departures from CDM observationally acceptable and significant. But there are many such coincidences in physical science. In this case, the coincidence would bias cosmological tests that assume the CDM model.

4 Concluding Remarks

It is standard and efficient practice to stick with the theory that has brought us this far until it fails. Experience reenforces the strategy; GR is a good example. Einstein’s modest empirical basis came from laboratory physics: Maxwell’s equations, that contain special relativity, and the evidence for the equivalence principle. Beginning with Einstein’s calculation of the perihelion of Mercury, GR has been shown to pass searching tests out to the much larger scales of the Solar System. We are now seeing that the theory passes nontrivial tests on the enormous scales of cosmology. One might argue that this is to be expected, from the compelling physical logic of GR. I respect the logic, but am much more impressed by the prospect of actually weighing the physics of GR on the observational scales of cosmology.

The physics of the CDM model for structure formation is not as logically compelling as GR, as witness the broad interest in the warm and self-interacting variants. Alternatives that upset the cosmological tests are less widely discussed, but certainly will be useful, maybe as foils to help establish the CDM model, maybe as leads to better physics in the dark sector.

We may be lucky enough to get a laboratory detection and exploration of the properties of dark matter, but most of the physics in the dark sector and the rest of cosmology will have to be established in quite indirect ways, like much of physical science these days. One way to organize this follows the PPN approach to tests of GR: assign parameters to the aspects of gravity physics that are of interest to cosmology, as discussed in Sec. 2, other parameters for such physics in the visible sector as rolling coupling constants, more parameters for physics in the dark sector, and still more for initial conditions. Overconstraining them all will be quite a challenge, but Nature has provided opportunities for lots of observations, the pursuit of which we may hope will show us when we have arrived at the dawning of the age of accurate cosmology.

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