Non-traditional approaches to the problems of forming the structure of the vibration field of technological machines

A V Eliseev\textsuperscript{1,2,*}, N K Kuznetsov\textsuperscript{1}, S V Eliseev\textsuperscript{2} and Q T Vuong\textsuperscript{2}

\textsuperscript{1}Irkutsk National Research Technical University, 83, Lermontov St., Irkutsk, 664074, Russia
\textsuperscript{2}Irkutsk State Transport University, 15, Chernyshevskiy St., Irkutsk, 664074, Russia

* eavsh@ya.ru

Abstract. A technology for evaluating the dynamic properties of mechanical vibration systems within the framework of structural and mathematical modeling methods with the use of dynamic characteristics of the structure of vibration fields of technological machines has been developed. We consider the problem of developing a mathematical model that allows us to correct the dynamic state depending on the system parameters based on local estimates of the properties of the vibration field. The ratio of reactions is used to evaluate the dynamic state of the vibration field. The coupling coefficient of external harmonic perturbations and the mass-inertia characteristic of a device for converting motion are considered as parameters of a mechanical oscillatory system. Based on a model with two degrees of freedom and two external influences, a detailed method for constructing mathematical models for evaluating local features of the vibration fields of technological machines is proposed. The obtained analytical expressions allow us to construct diagrams of the characteristics of mechanical oscillating systems depending on the parameters of the adjustment tools.

1. Introduction

The widespread use of vibration in various fields of technology and, above all, when creating vibration machines, equipment and technological processes, on the one hand, and the development of methods and means of vibration protection for modern machines, communication devices and equipment with increasing speeds and loads on the drives, on the other hand, actualizes the importance of the development of research in the field of mechanics of oscillatory systems, providing the possibility of a rational choice of tuning and regulation [1-4].

In recent years, theoretical and applied research in the field of regulation of vibration fields has been developed in the direction of the search and development of technical means using new mechanisms, in particular, devices for converting motion [5].

The structural and technical complexity of the forms of vibrating technological machines determines the application of the analytical apparatus of circuit theory, automatic control theory and mechatronics.

In works reflecting the current level of development of methods of structural and mathematical modeling, technology for developing a methodological basis for constructing mathematical models is aimed at detailing the possibilities of varying the characteristics of mechanical oscillatory systems using various types of mechanisms, which include devices for converting motion [6].
In this article, we consider the problem of constructing structural-mathematical models of objects with two degrees of freedom, in which the dynamic local features of the vibration field are provided by introducing and using a device for converting motion that takes into account a setting parameter that reflects the functional relationship of two external disturbances.

2. Problem statement

The technical object is a mechanical system formed by two elements \( m_1 \) and \( m_2 \), elastic elements \( k_1, k_2, k_3 \), a device for converting motion \( L \) (figure 1). Inertia element \( m_1 \) is connected to the support surface I through an elastic element \( k_3 \) and to the support surface II through a quasi-spring formed, by elements \( k_1, k_2 \) and \( L \).

![Figure 1. Schematic diagram of a mechanical system with a device for converting motion \( L \)](image)

The law of motion of supporting surfaces I and II is given by the harmonic functions \( z_1(t) \) and \( z_2(t) \) one frequency. An essential feature of the considered mechanical system is a functional tie

\[
\ddot{z}_2 = \alpha \ddot{z}_1, \tag{1}
\]

where \( \alpha \) is the constant coefficient of connectivity of external disturbances.

The displacement of the inertia element \( m_1 \) relative to the position of static equilibrium is determined by the coordinate \( y_1 \), and the mass \( m_1 \) by the coordinate \( y_1 \). It is assumed that elements \( m_1 \) and \( m_2 \) make small fluctuations relative to the position of static equilibrium.

To build a mathematical model, we will make expressions for kinetic and potential energies

\[
T = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 + \frac{1}{2} L (\dot{y}_2 - \dot{y}_1)^2 + \frac{1}{2} m_2 \dot{y}_2^2, \tag{2}
\]

\[
\Pi = \frac{1}{2} k_1 (y_1 - z_1)^2 + \frac{1}{2} k_2 (y_2 - y_1)^2 + \frac{1}{2} k_3 (y_2 - z_2)^2. \tag{3}
\]

Using the Lagrange equation of the second kind, and then the Laplace transform under zero initial conditions, we write the system of differential equations in operator form [33, 41]:

\[
\ddot{y}_1 (m_1 + L) p^2 + \ddot{y}_1 (k_1 + k_2) - \ddot{y}_2 (L p^2 + k_2) = k_1 \ddot{z}_1, \tag{4}
\]
\[ \ddot{y}_2 (m_2 + L) p^2 + \ddot{y}_2 (k_2 + k_3) - \ddot{y}_1 (Lp^2 + k_2) = k_3 \ddot{z}_2. \]  

(5)

A structural mathematical model or block diagram of an automatic control system is shown in figure 2.

![Figure 2. Block diagram of a system with a device for converting motion](image)

The transfer functions of the system under kinematic disturbance from the base side are defined by the expressions

\[ W_1 (p) = \frac{\ddot{y}_1}{\ddot{z}_1} = \frac{k_1 [(m_2 + L) p^2 + k_2 + k_3]}{A(p)}, \]  

(6)

\[ W_2 (p) = \frac{\ddot{y}_2}{\ddot{z}_1} = \frac{k_4 (k_2 + Lp^2)}{A(p)}, \]  

(7)

\[ W_3 (p) = \frac{\ddot{y}_1}{\ddot{z}_2} = \frac{k_3 (k_2 + Lp^2)}{A(p)}, \]  

(8)

\[ W_4 (p) = \frac{\ddot{y}_2}{\ddot{z}_2} = \frac{k_3 [(m_1 + L) p^2 + k_1 + k_3]}{A(p)}, \]  

(9)

where

\[ A(p) = [(m_1 + L) p^2 + k_1 + k_2] [(m_2 + L) p^2 + k_2 + k_3] - (Lp^2 + k_2)^2 \]  

(10)

– is the characteristic frequency equation of the system.

The task of the study is to evaluate the dynamic features introduced into the mechanical oscillatory system by a device for converting motion, as well as the introduction of an intermediate mass – intermediate inertia element \(m_1\), which creates two cascades in the system, sequentially connected to each other.

3. Evaluating the dynamic properties of the system when adding motion conversion devices

Usually \(p\). (B1), in which three typical elements with transfer functions \(k_1\), \(k_2\) and \(Lp^2\) simultaneously contact, is not considered as a characteristic point. The authors interpret such a point as a nodal point; its position is determined by the coordinate \(\ddot{y}_1\). The limiting case can be assumed to be the situation when \(m_1\) tends to 0, which determines the approach to estimating the dynamic stiffness of a two-stage sequentially connected system elements, the value of \(m_1\) affects the distribution of dynamic coupling reactions in pp. (B), (B1) and (B2).

Using the block diagram in figure 2, write down the transfer functions, taking into account the connectivity of external influences, defined by the expression (1)

\[ W_1^1 (p) = \frac{\ddot{y}_1}{\ddot{z}_1} = \frac{k_1 [(m_2 + L) p^2 + k_2 + k_3] + \alpha k_3 (Lp^2 + k_2)}{A(p)}, \]  

(11)

\[ W_2^1 (p) = \frac{\ddot{y}_2}{\ddot{z}_1} = \frac{\alpha k_1 [(m_1 + L) p^2 + k_1 + k_3] + k_1 (Lp^2 + k_2)}{A(p)}. \]  

(12)
In figures 3, a, and b, the transformed structural schemes are shown with the allocation of elements \( m_1 \) and \( m_2 \), covered by the corresponding negative feedbacks. In a physical sense, the transfer functions of such connections correspond to (or determine) the dynamic rigidity of structural formations.

Dynamic reactions at the points of contact with the support surface are defined by the expressions

\[
\begin{align*}
\vec{R}_{\text{sup}} &= \vec{R}_A + \vec{R}_B = k_3 \cdot \vec{z}_i \cdot W_i'(p) + k_1 \cdot \vec{z}_i \cdot W_i'(p) = \\
&= k_3 \cdot \vec{z}_i \{\alpha k_3 ((m_1 + L) p^2 + k_1 + k_2) + k_1 (L p^2 + k_2)\} + \\
&+ k_1 \cdot \vec{z}_i \{k_1 ((m_2 + L) p^2 + k_2 + k_3) + \alpha k_3 (L p^2 + k_2)\}. \tag{13}
\end{align*}
\]

or

\[
\vec{z}_i \{\alpha k_3 ((m_1 + L) + k_2^2 (m_2 + L) + k_1 k_3 \alpha (1 + \alpha)) p^2 + \\
+ \alpha k_3^2 (k_1 + k_2) + k_1^2 (k_2 + k_3) + k_1 k_2 k_3 (1 + \alpha)\}.
\tag{14}
\]

The dynamic response \( \vec{R}_{\text{sup}} \) is reset to zero at the frequency of external influence

\[
\alpha^2 = \frac{\alpha k_3^2 (k_1 + k_2) + k_1^2 (k_2 + k_3) + k_1 k_2 k_3 (1 + \alpha)}{\alpha k_3^2 (m_1 + L) + k_1^2 (m_1 + L) + k_1 k_2 k_3 (1 + \alpha)}.
\tag{15}
\]

The dynamic response on an «object» \( m_2 \) is defined in terms of dynamic stiffness, which can be found using structural transformation rules

\[
\tilde{k}_{\text{red}}' = \frac{(L p^2 + k_2)((m_1 + L) p^2 + k_1 + k_2) - (L p^2 + k_2)^2}{(m_1 + L) p^2 + k_1 + k_2} = \frac{(L p^2 + k_2)(m_1 p^2 + k_1)}{(m_1 + L) p^2 + k_1 + k_2}
\tag{16}
\]

Thus

\[
\begin{align*}
\vec{R}_{m_2} &= \vec{R}_A + \vec{R}_B = \tilde{k}_{\text{red}}' \cdot W_i(p) \cdot \vec{z}_i = \\
&= \frac{k_3 \{(m_1 + L) p^2 + k_2 + k_3\} + k_1 (L p^2 + k_2)\} \times \\
&\times \{k_1 ((m_2 + L) p^2 + k_2 + k_3) + \alpha k_3 (L p^2 + k_2)\} \frac{A(p)}{\alpha k_3^2 (m_1 + L) + k_1^2 (m_1 + L) + k_1 k_2 k_3 (1 + \alpha)}.
\tag{17}
\end{align*}
\]

We will find the relation of dynamic reactions \( \vec{R}_{m_1} \) and \( \vec{R}_{\text{sup}} \).
In turn, in the first frequency range \(\omega \rightarrow \infty\) the ratio \(\frac{R_{m1}}{R_{sup}}\) will tend to a certain limit.

When \(L = 0, m_1 = 100 \text{ kg}\), we obtain an ordinary chain with two degrees of freedom, as shown in figure 4.

![Figure 4. Schematic diagram of a mechanical chain oscillatory system with two degrees of freedom](image)

If the setting parameter is \(m_1\), then it is not very convenient – the material consumption for requirements of large \(m_1\). If we consider the possibilities, assuming that \(L \neq 0\) \((L\) is the reduced mass), we can obtain large values of \(m_1 + Lp^2\) due to the effects of the reduced parameters, thereby providing savings in material consumption (reducing \(m_1\) to zero values).

The amplitude-frequency characteristics of \(\frac{\bar{Y}_1}{\bar{Z}_1}\) the system and \(\frac{\bar{Y}_2}{\bar{Z}_2}\) at different values of \(\alpha\) are given in figure 5.

In figure 5(a) note that the graphs \(\frac{\bar{Y}_1}{\bar{Z}_1}\) intersect the x-axis (p(1), p(2), p(3)) in the second frequency range \(\omega_{1}\text{nat} < \omega < \omega_{2}\text{nat}\); increasing values of \(\alpha\) the point of intersection is to the right by \(\omega_{2}\text{nat}\). Point of intersection shows the frequency of dynamic oscillation damping of the y-coordinate \(\bar{Y}_1\). In turn, in figure 5(b), the points of \(\frac{\bar{Y}_2}{\bar{Z}_2}\) intersection of the dependence graphs with the abscissa axis (p . (1'), p . (2'), p.(3')) are in the third frequency range. Figure 5(c) p .(1) shows the intersection \(\frac{\bar{Y}_1}{\bar{Z}_1}\) point
with the abscissa axis, which is located in the second frequency range, respectively, for the value $\alpha = 0$ (figure 5(a)); and $p \cdot (2'')$, respectively, is located in the first range for the value $\alpha = -0.5$; with the value $\alpha = -2$, there will be no frequency of dynamic damping of vibrations (there is no intersection point).

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{The amplitude-frequency characteristics of the system (a)– in coordinate $\tilde{y}_1$ with $\alpha = 0, 0.5, 2$; (b) – coordinate $\tilde{y}_2$ with $\alpha = 0, 0.5, 2$; (c) – in coordinate $\tilde{y}_1$ with $\alpha = 0, -0.5, -2$; (d) – coordinate $\tilde{y}_2$ with $\alpha = 0, -0.5, -2$}
\end{figure}

In figure 5(d), we see that at $\alpha = -0.5, -2$, the intersection points of the dependency graphs $\frac{\tilde{y}_2}{\tilde{y}_1}(\omega)$ ($p \cdot (2'')$, $p \cdot (3'')$) placed in the second frequency range (compared to pp. (2'), (3') in figure 5(b)).

\section{Conclusion}

The proposed technology within the framework of structural mathematical modeling based on the use of transfer functions provides the possibility of rational regulation of local dynamic characteristics of the vibration field of technological equipment. The method used for the analysis of dynamic systems is based on the representation of a system of differential equations in the form of a block diagram of an automatic control system.

A detailed representation of the method for constructing mathematical models of technological vibration machines is proposed, taking into account the simultaneous action of two external perturbing factors that have a functional relationship. In particular, the concept of amplitude-frequency response is introduced, taking into account the connectivity coefficient - the characteristic of the functional dependence between external harmonic perturbations of the same frequency.

Regulation of the vibration field and evaluation of its dynamic properties can be performed using such dynamic characteristics as the ratio of coupling reactions occurring on the support surfaces and
on the inertial elements of the system. In this sense, the dynamic state of the vibration field can be adjusted depending on the coupling coefficient of external influences and the parameter of the device for converting motion.

The proposed approach can be used, in particular, to determine the parameters of vibration processing equipment used in industrial production.

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