On the renormalization of higher derivative two dimensional gravity

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Abstract

The fourth derivative models for two dimensional gravity are shown to be equivalent to the special version of the nonlinear sigma models coupled to 2d quantum gravity. The reduction consists in the introduction of the auxiliary scalar fields and can be performed in an explicit way for both metric and general metric-dilaton cases. In view of this we can evaluate the structure of possible counterterms and show that they contain second derivative structures only. The calculations in the theory with an auxiliary fields require some special procedure to be applied. We perform the explicit calculations in a different gauges and explore the features of the auxiliary fields.

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1. Introduction. Recent developments in the field of two dimensional (2d) quantum gravity (see, for example, [1, 2, 3, 4, 5]) have inspired the interest to the link between 2d and 4d gravity theories. In fact, many people regard 2d case as the pattern (or, at least, as some toy model) for the more realistic four dimensional quantum gravity. However there is a very essential difference between these two theories. This difference is constituted, more or less completely, from two points. First of all in the 2d gravity there is no spin two states (see, for example, [6, 7] for the discussion of this in a harmonic gauge, which is most useful in 4d gravity) and the metric can be described by it’s conformal factor. Secondly, the loop integrals have better convergence properties in 2d quantum gravity. As a result the models of 2d gravity which are based on the usual action with second derivative terms only, are renormalizable. In 4d the renormalizability require the fourth derivative terms to be included to the action [8], that leads to the loss of unitarity within the standard perturbation scheme (see, for example, [9] for the introduction to higher derivative quantum gravity in 4d and references).

Last time there was some interest in the study of the higher derivative gravity in 2d [10, 11, 12, 13, 14, 15, 16]. The interest to the higher derivative dilaton gravity in 2d is partially caused by the analogy with the sigma model approach to the massive modes of string, where higher derivatives also appear. In particular, the general dilaton model with fourth derivatives has been formulated in Ref.[14]. This model can be regarded as some tool for investigation of massive string modes.

The study of the one loop renormalization of the higher derivative gravity in 2d has been started in [13, 14] (see also [15, 16]). The explicit one loop calculation [14] shows that the divergences which appear in the general fourth derivative model have the form of the second derivative metric - dilaton action. The same happens in a purely metric model [13]. In the last case the calculations has been performed in a classically equivalent second derivative model with an auxiliary fields. Such model has been regarded as the particular case of the known theory of the dilaton gravity, and thus the special calculations do not look necessary.

At the same time this way of study leads to some interesting puzzles. The expression for the one-loop divergences contains the singular dependence on the auxiliary field and thus it is not clear what is the link between this result and the starting $\Lambda + R^2$ model. On the other hand it turns out that the one-loop calculations in a harmonic gauge need some special procedure including the noncovariant terms to be introduced. Such terms exist in the dimension $d = 2 + \varepsilon$ and the contribution of this terms disappear when the regularization is removed and the parameter related with an extra terms tends to zero. The necessity of this noncovariant procedure leads to conclusion that the higher derivative $d = 2$ theory needs special renormalization procedure including the addition of some extra noncovariant counterterms [16]. All this is rather unnatural since there are no any difficulties of this sort in conformal gauge.

The goal of this letter is to solve the above puzzles. It is shown, how the covariant calculations can be performed without extra noncovariant terms. Next, we demonstrate that quantum theory with an auxiliary fields has to be modified as compared with the ordinary dilaton gravity. Then, in the framework of the modified theory the higher derivative gravity models are shown to be renormalizable in standard sence in both harmonic and conformal gauges. The use of conformal gauge enables one to show that only the second derivative counterterms arise at all loops. This concerns both $\Lambda + R^2$ model and general dilaton model.

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2The theories with an auxiliary fields have been studied in [17].
of [14]. The last is shown to be equivalent to the $D = 4$ second derivative sigma model where some of the sigma model coordinates are auxiliary scalars.

The paper is organized as follows. In section 2 we consider the pedagogical example of higher derivative $2d$ gravity without dilaton. In section 3 we deal with the general model of [14]. The last section is conclusion.

2 Metric model with fourth derivatives. As starting point, let us consider the higher derivative metric theory.

$$S_g = \int d^2x \sqrt{g} \left\{ \frac{1}{m^2} R^2 + \Lambda \right\}$$ (1)

Here $m^2, \Lambda$ are dimensional constants. We do not include the Einstein term because for the $2d$ space without boundaries it does not affect the renormalization. The action (1) may be supplemented by the action of $N$ scalar fields

$$S_m = \int d^2x \sqrt{g} \frac{1}{2} g^{\mu\nu} \partial_\mu \chi^i \partial_\nu \chi^i$$ (2)

where $i = 1...N$. Those fields give contribution to the Einstein type divergences [20]. Below we omit the terms, related with $S_m$ in all the intermediate expressions for the sake of brevity, and take them into account in the final expressions only.

The above theory has been studied on quantum level in [10, 11, 12, 14, 15, 16]. In particular, in the works [13, 15, 16] the one loop renormalization of the theory has been discussed. In [13, 14] the calculation of the one-loop divergences has been performed with the help of the auxiliary scalar field. Introducing the auxiliary scalar $\psi$ one can substitute (1) by the second derivative theory with the classical action

$$S = \int d^2x \sqrt{g} \left\{ -\frac{m^2}{4} \psi^2 + \psi R + \Lambda \right\}$$ (3)

On classical level the theories (1) and (3) are equivalent that can be seen if one take the value of $\psi$ from the corresponding equation of motion and substitute it back to the action (3) or to the dynamical equation for metric.

The theory (3) looks like a particular case of the general action for dilaton gravity

$$S_{gen} = \int d^2x \sqrt{g} \left\{ \frac{C_0}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + C_1 \phi R + V(\phi) \right\}$$ (4)

where $C_0 = 1, 0$ for different conformally equivalent versions of the theory and $V(\phi)$ is the potential function. The one loop renormalization in the theory with the action (4) has been investigated in [18, 19, 20, 21, 22]. In a harmonic type background gauge

$$S_{gf} = -\frac{C_1}{2} \int d^2x \sqrt{g} \frac{\phi}{\nu} \chi_\nu$$

$$\chi_\nu = \nabla_\nu h^\nu_\mu - \frac{1}{2} \nabla_\mu h - \frac{1}{2} \nabla_\mu \phi - X(\phi) \nabla_\mu \phi$$ (5)

the divergent part of the one-loop effective action has the form (it does not depend on the choice of $C_0$ as well as on the gauge parameter $X(\phi)$)

$$\Gamma_{div} = \frac{1}{\varepsilon} \int d^2x \sqrt{g} \left\{ A_1(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + A_2(\phi) R + A_3(\phi) \right\}$$ (6)
with

\[ A_1(\phi) = -\frac{1}{\varepsilon} \frac{\nu}{\phi^2}, \quad A_2 = \frac{1}{\varepsilon} \frac{24 - N}{12}, \quad A_3(\phi) = \frac{\nu VC_1}{\phi} + \frac{V'}{C_1} \]  

(7)

where \( \nu \) is an arbitrary gauge parameter and \( g_{\mu\nu} \) and \( \phi \) are background metric and scalar field. \( \varepsilon = (4\pi)^2 (d-2) \) is the parameter of dimensional regularization. Note that in the theory under discussion the propagator has smooth behaviour when \( \varepsilon \) tends to zero, and hence there is no additional effects like the oversubtraction problem \([1]\) which may come from dimensional regularization. Thus it seems possible to choose the special values of \( C_0 \) and \( V(\psi) \) corresponding to (3), substitute them into (6), and so obtain the divergences of the original theory (1) \([13, 15]\). However on this way one face some difficulties. The point is that \( \psi \) is the auxiliary field and one have to care about it and especially if the background field method is used. Let us consider this in details.

Quantum theory is conventionally constructed by means of generating functional in the form of the path integral. Since \( \psi \) is the auxiliary field one has to avoid the introduction of the external source for this field. The last can be interpreted as the simple fact that only the diagrams without external \( \psi \) lines are valid. Therefore the path integral representation of the theory (3) is different from the one of (1) just because some diagramms in the last must be removed. In the theory with auxiliary field \( \psi \) the propagator of this field works on the internal lines only. In the framework of the background field method the Green functions arise as a result of variation of the effective action with respect to the background fields. Hence if one need to abort the diagramms with the external \( \psi \) lines it is necessary to consider the theory without background \( \psi \) field. So we see that in the background field method (4) corresponds to the theory with purely quantum field \( \psi \). Let us notice that the same results from the formal analysis (one can find the details of the background field method in a numerous papers or in [3]).

From the above consideration it follows that it is not possible to obtain the correct expression for divergences in the theory (1) with the help of (6). Since both the gauge fixing term (5) and the divergences (6) contain \( 1/\psi \) factors they do not have smooth behaviour in the limit \( \psi \rightarrow 0 \) and thus this way does not provide us by correct result. Indeed all the singular terms vanish when the gauge parameter is choosen in a special way \( \nu = 0 \). However it is not clear whether the resulting expression has any link to the original theory (1).

In the rest of this section we calculate the one loop divergences in the theory (1) by three different ways, that is

i) In terms of original higher derivative theory (1) in a harmonic type background gauge,

ii) In terms of (3) in similar gauge,

iii) In terms of (4) in conformal gauge.

The last case is especially interesting since it enables one to establish the structure of renormalization at higher loops.

i) Calculation in original higher derivative theory. According to the background field method we divide the metric into background \( g_{\mu\nu} \) and quantum \( h_{\mu\nu} \) parts as

\[ g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} = \bar{h}_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \]  

(8)

\(^3\)In fact in [13] the independent calculation has been performed. Surprisingly the dependence of the gauge parameter \( X(\psi) \) has been found. This dependence contradict to the results of [19] and of third reference in [20]. Moreover the detailed analysis shows that such a dependence leads to the only shall gauge dependence of the one-loop divergences.
and introduce the gauge fixing term of the form

\[ S_{gf} = -\frac{1}{\alpha} \int d^2x \sqrt{g} (\nabla_{\tau} \bar{h}^\tau_\mu - \beta \nabla_\mu h) \left[ g^{\mu\nu} \Box + \gamma \nabla^\mu \nabla^\nu \right] (\nabla_{\lambda} \bar{h}^\lambda_\nu - \beta \nabla_\nu h) \]  

(9)

where \( \alpha, \beta, \gamma \) are the gauge fixing parameters. The one-loop effective action is given by the expression

\[ \Gamma_{\text{div}} = \frac{i}{2} \text{Tr} \ln \hat{H} - i \text{Tr} \ln \hat{H}_{gh} + \frac{i}{2} \text{Tr} \ln [g^{\mu\nu} \Box + \gamma \nabla^\mu \nabla^\nu] \]  

(10)

where \( \hat{H} \) is the bilinear form of the action \( S + S_{gf} \), \( \hat{H}_{gh} \) is the bilinear form of the ghost’s action and the last term is the contribution of the weight operator (8). The calculations are simplified considerably, if one choose the values of \( \beta \) and \( \gamma \) as \( \beta = \frac{\alpha}{2m^2} \left( \frac{\alpha}{m^2} - 1 \right) \) and \( \gamma = -\frac{\alpha}{m^2} \). Then, after some algebra, we arrive at the following bilinear part of \( S + S_{gf} \)

\[ S^{(2)} + S_{gf} = \int d^2x \sqrt{g} \left\{ \tilde{h}^{\mu\nu} \left[ -\frac{2\beta \gamma}{\alpha} - \frac{1}{2m^2} \right] R \nabla_\mu \nabla_\nu h + \right. \]

\[ \left. \tilde{h}^{\mu\nu} \left[ \frac{1}{\alpha} g_{\nu\beta} \nabla_\mu \nabla_\alpha \Box - \left( \frac{1}{2m^2} + \frac{1}{2\alpha} \right) R g_{\nu\beta} \nabla_\mu \nabla_\alpha - \frac{1}{2m^2} R^2 \delta_{\nu\beta,\alpha} - \frac{m^2}{4} \Lambda \delta_{\nu\beta,\alpha} \right] \tilde{h}^{\alpha\beta} + h \left[ \left( \frac{\beta^2 (m^2 - \alpha)}{4m^2} \right) \Box + \left( \frac{\beta^2}{2\alpha - 1} \right) R \right] h \right\} \]  

(11)

Let us now use the two - dimensional identity

\[ \tilde{h}^{\mu\nu} X \left[ g_{\nu\beta} \nabla_\mu \nabla_\alpha \right] \tilde{h}^{\alpha\beta} = \frac{1}{2} \tilde{h}^{\mu\nu} X \left[ \delta_{\nu\beta,\alpha} \Box - \delta_{\nu\beta,\alpha} R \right] \tilde{h}^{\alpha\beta} \]  

(12)

advocated in [3, 4]. Here \( X \) is an arbitrary scalar quantity. Since the prove of (12) use only the tracelessness of \( \tilde{h}^{\alpha\beta \gamma} \) it is not affected by the action of \( X = \Box \). Then, after simple rescaling of the fields \( \tilde{h}^{\alpha\beta} \), \( h \), the bilinear form \( \hat{H} \) has the form of the minimal higher derivative operator

\[ \hat{H}' = \hat{1} \Box^{\alpha} + \hat{L}^{\alpha\beta \nu} \nabla_\alpha \nabla_\beta \nabla_\nu + \hat{V}^{\alpha\beta} \nabla_\alpha \nabla_\beta + \hat{N}_\alpha \nabla_\alpha + \hat{U} \]  

(13)

with \( \hat{L}^{\alpha\beta \nu} = 0 \). The divergences of \( Tr \ln \hat{H}' \) has been derived in [19].

\[ Tr \ln \hat{H}'_{\text{div}} = -\frac{2}{\varepsilon} tr \left( \frac{1}{16} \hat{V}_\alpha^\alpha - \frac{1}{16} \hat{L}^{\alpha\beta \nu} \hat{L}_{\alpha\beta \nu} - \frac{3}{32} \hat{L}_\alpha^\alpha \hat{L}_\alpha \right) \]  

(14)

If one use (14) and take into account the contributions of ghosts and of the weight operator, the dependence on the gauge parameter \( \alpha \) is dropped out and the divergences of the theory (3) are found to be

\[ \Gamma_{\text{div}} = \frac{1}{\varepsilon} \int d^2x \sqrt{g} \left( \frac{24 - N}{12} \right) R \]  

(15)

Thus the only one counterterm which appears in higher derivative theory (3) is (13).

Some note is in order. From the above consideration we learn that in two dimensional gravity any second or fourth order differential operator in \( \bar{h} \bar{h} \) sector is minimal due to identity (12). As a consequence the calculation in harmonic type gauge in a higher derivative \( d = 2 \) gravity is possible without introduction of the extra noncovariant term like \( \xi \bar{h}^{\mu\nu} \Box (R_{\mu\nu} - \)
\( \frac{1}{2} R g_{\mu \nu} \) that has been done in [14, 16]. From what follows that both the loop calculations and renormalization procedure can be performed in two dimensions and there is no any need to introduce an extra noncovariant structures in \( d = 2 + \varepsilon \) [16].

ii) Now we shall look how all this works in the classically equivalent theory (3) when the auxiliary field \( \psi \) is introduced. Since the auxiliary field is purely quantum we must introduce the background metric only, as a result the calculations are tiny. Then, performing the splitting of the metric as in (8) we choose the gauge fixing term in the form

\[
S_{gf} = \frac{1}{\alpha} \int d^2 x \sqrt{g} \left( \nabla_\nu \bar{h}_\mu + \frac{\alpha}{2} \nabla_\mu h \right) \left( \nabla_\lambda \bar{h}^{\mu \lambda} + \frac{\alpha}{2} \nabla^{\mu} h \right)
\]

(16)

Then, after use of (12) we find that the bilinear (with respect to quantum fields \( h_{\mu \nu}, \psi \)) part of the expression \( S + S_{gf} \) has the form

\[
S^{(2)} + S_{gf} = - \int d^2 x \sqrt{g} \left\{ \frac{1}{2 \alpha} \bar{h}^{\mu \nu} (\Box - R + \frac{\alpha}{2} \Lambda) \bar{h}_{\mu \nu} + \frac{1}{2} \psi \Box h + \frac{\alpha}{4} \psi \Box \psi + \frac{m^2}{4} \psi^2 \right\}
\]

(17)

The above expression enables us to use standard algorithm for the minimal second order operators. Taking into account the contribution of gauge ghosts we finally obtain the divergences of the theory (3) with an auxiliary field \( \psi \)

\[
\Gamma_{\text{div}} = \frac{1}{\varepsilon} \int d^2 x \sqrt{g} \left( \frac{24 - N}{12} R - \frac{\alpha}{2} \Lambda \right)
\]

(18)

The last expression differs from (15) by the cosmological term. The source of this deviation is the different gauge fixing conditions. In fact (18) are the divergences of the theory (3) with zero background \( \psi \). For zero \( \psi \) the equation of motion for the theory (3) reads \( \Lambda = 0 \), and two expressions coincide on this ”mass shell”. The problem is that \( \Lambda = 0 \) does not correspond to equations of motion of the original theory (1). Thus the defect of the divergences (18) is caused by the calculational scheme, and we conclude that in the harmonic gauge two theories (1) and (3) are not completely equivalent on quantum level even if we treat auxiliary fields in a correct way.

iii) The story is much more simple if we apply the conformal gauge. Let us write

\[
g_{\mu \nu} = \bar{g}_{\mu \nu} e^{2\sigma}
\]

(19)

where \( \sigma \) is quantum field and \( \bar{g}_{\mu \nu} \) is the background metric. The scalar curvature is transformed as

\[
R = e^{-2\sigma} [\bar{R} - 2 \Box \sigma]
\]

(20)

and thus (14) is reduced to the ordinary sigma-model action (see also the paper of Russo and Tseytlin [20] where similar representation has been used in second derivative 2d dilaton gravity.)

\[
S = \int d^2 x \sqrt{\bar{g}} \{ \bar{g}^{\mu \nu} G_{ab} \partial_\mu X^a \partial_\nu X^b + B(X) \bar{R} + T(X) \}
\]

(21)

where

\[
X^a = (\psi, \sigma), \quad G_{ab} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B(X) = \psi + \alpha, \quad T(X) = \Lambda - \frac{m^2}{4} \psi^2 e^{2\sigma}
\]

(22)
Thus in conformal gauge the theory \((\text{3})\) becomes the linear sigma model which is known to have the divergences of the tachyon type, and also the anomalous one of the Einstein type. This is the divergences structure not only at one, but also at higher loops. Since in the case of the theory \((\text{1})\) only metric is the dynamical field, one can expect for the divergences of Einstein and cosmological form only. Let us notice that even if we consider the theory with auxiliary field, and some diagramms have to be omitted, the superficial index of the remaining diagramms is in an accord with the power counting of the general theory \((\text{22})\) (or \((\text{4})\)), and thus our consideration here is correct.

It is important that the linear sigma model calculations can be performed without mixing of conformal factor \(\sigma\) with the auxiliary field \(\psi\) and that the result is not singular when we put \(\psi\) equal to zero. Since in the case the "tachyon" term is constant for the zero background \(\psi\), such a divergences are lacking as well. The explicit one-loop calculations show that the divergences of \((\text{3})\) have the form \((\text{15})\) just as in original formulation of the theory \((\text{1})\).

3. Metric - dilaton model.

Let us now consider a general version of quantum dilaton gravity which has been recently formulated in \([\text{14}]\). The action of the model has the form

\[
S = \int d^2x \sqrt{g} \left\{ a_1(\varphi)R^2 + a_2(\varphi)R\Box \varphi + a_3(\varphi)R(\nabla \varphi)^2 + a_4(\varphi)(\Box \varphi)^2 + a_5(\varphi)(\Box \varphi)(\nabla \varphi)^2 + +a_6(\varphi)(\nabla \varphi)^4 + a_7(\varphi)R + a_8(\varphi)(\nabla \varphi)^2 + a_9(\varphi) \right\}.
\]

Here \(a_i(\varphi)\) are an arbitrary functions of the dimensionless scalar field \(\varphi\) and \((\nabla \varphi)^2 = (\nabla_\mu \varphi)(\nabla^\mu \varphi)\).

The one-loop calculations in the model \((\text{23})\) have been performed in \([\text{14}]\) for both general case and for \(a_2 = a_4 = 0\). Here we show how the general model \((\text{23})\) can be reduced to the second derivative theory, that enables one to establish the structure of divergences at higher loops. Moreover we shall derive the one loop divergences in this second derivative theory and check the correspondence between two formulations on quantum level.

Despite the action \((\text{23})\) looks rather combersome it is possible to introduce the auxiliary fields and to reduce \((\text{23})\) to the lower derivative action. To show this one can rewrite \((\text{23})\) in the following form

\[
S_{dg} = \int d^2x \sqrt{g} \left\{ a_7(\varphi)R + a_8(\varphi)(\nabla \varphi)^2 + a_9(\varphi) + +a(\varphi) \left[ R + y(\varphi)(\Box \varphi) + z(\varphi)(\nabla \varphi)^2 \right]^2 + b(\varphi) \left[ (\Box \varphi) + u(\varphi)R + v(\varphi)(\nabla \varphi)^2 \right]^2 \right\}.
\]

Here \(a(\varphi), b(\varphi), y(\varphi), z(\varphi), u(\varphi), v(\varphi)\) are some functions which are related with \(a_1, \ldots, a_6(\varphi)\). Now we are able to introduce the auxiliary fields \(\psi\) and \(\xi\) and to rewrite \((\text{23})\) in the form

\[
S_{dg} = \int d^2x \sqrt{g} \left\{ \frac{1}{4a(\varphi)}\psi^2 - \frac{1}{4b(\varphi)}\xi^2 + \psi \left[ R + y(\varphi)(\Box \varphi) + z(\varphi)(\nabla \varphi)^2 \right] + +\xi \left[ (\Box \varphi) + u(\varphi)R + v(\varphi)(\nabla \varphi)^2 \right] + a_7(\varphi)R + a_8(\varphi)(\nabla \varphi)^2 + a_9(\varphi) \right\}.
\]

The last action can be regarded as the special case of the nonlinear sigma model coupled to quantum metric. The one loop counterterms for the general nonlinear sigma model coupled
to quantum 2d gravity have been recently calculated in [6]. Unfortunately the result of [7] can not be applied directly to the model (25) because the auxiliary fields $\psi, \xi$ need special care. Moreover the metric of the sigma model (without quantum gravity contributions) is degenerate in the last case. However the method of [7] is applicable, and thus we can use the statement of equivalence between covariant and conformal gauges which was proved there at one-loop level. Therefore it is sufficient to consider the conformal gauge only. In the conformal gauge the action (25) becomes (the argument $\phi$ and bars are omitted below)

$$S_{dg} = \int d^2x \sqrt{g} \left\{ g^{\mu\nu} G_{ab} \partial_\mu X^a \partial_\nu X^b + B(X^a) R + T(X^a) \right\}$$

(26)

where

$$X^a = (\psi, \xi, \phi, \sigma), \quad G_{ab} = \begin{pmatrix} 0 & 0 & -y/2 & 1 \\ 0 & 0 & -1/2 & u \\ -y/2 & -1/2 & (v\xi + z\psi + a_8 - y'\psi) & u'\xi \\ 1 & u & u'\xi & 0 \end{pmatrix}, \quad B(X) = \psi + u\xi + a_7, \quad T(X) = \left( a_9 - \frac{1}{4a} \psi^2 - \frac{1}{4b} \xi^2 \right) e^{2\sigma}$$

(27)

and the prime stands for the derivative with respect to $\phi$. According to our analysis in the previous section, on quantum level the theory (25) corresponds to the path integral with the external sources for the metric and $\phi$ only, but not to the auxiliary fields $\psi, \xi$. In the framework of the background field method it means that we have both these fields as purely quantum. However the aborting of the diagrams with an external $\psi$ and $\xi$ lines does not change the power counting. Therefore the general form of divergences in the theory (25) [1] is given by the expression (4). Indeed the functions $A_{1,2,3}(\phi)$ can depend on $a(\varphi), b(\varphi), y(\varphi), z(\varphi), u(\varphi), v(\varphi)$. Thus the introduction of higher derivative terms is not caused by the renormalizability, in contrast with the four dimensional gravity. And so we observe that the result of direct calculations of Ref.[14] is just that it has to be.

The divergences of the sigma model (26) are well known [23, 24]. However the general result of [23, 24] is not applicable in our case since the method of calculation is assumed to preserve the covariance with respect to the target space reparametrizations and the corresponding calculational method can mix the auxiliary fields with $\phi$ and $\sigma$. As a result one can not simply put the background auxiliary fields equal to zero, and all the consideration is essentially more complicated.

Since we are interested in the application of the auxiliary fields to the one-loop calculations, let us consider the example of (25) and compare the one loop divergences with the ones of [14]. It is reasonable to apply conformal gauge which has been used in [14]. Expanding the conformal factor and $\phi$ as

$$\sigma \rightarrow \bar{\sigma} + \sigma \quad \phi \rightarrow \phi + \tau$$

(28)

we shall regard $\phi$ and $g_{\mu\nu} = \bar{g}_{\mu\nu} e^{2\bar{\sigma}}$ as the background quantities and $\sigma, \tau$ as quantum ones. After small algebra we obtain the bilinear part of the action (26) in the form

$$S^{(0)} = \int d^2x \sqrt{g} \left\{ G_{ab} \nabla_\mu X^a \nabla^\mu X^b + A_{ab}(\nabla^\mu \phi) X^a \nabla_\mu X^b + B_{ab} \right\}$$

(29)

Perhaps with one exception, that will be discussed below.
where $G_{ab}$ is given by (27) with $\xi = \psi = 0$ and

$$A_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2a''_7 & 2a_s & -y' & 0 \\ 0 & 2z - 2y' & 0 & 0 \\ 2u' & 2v & 0 & 0 \end{pmatrix}. $$

$$B_{ab} = \begin{pmatrix} 2a_9 \\ 2a_9 \\ 0 \\ 0 \end{pmatrix}/(2a''_9 + a''_9 R + a''_8 (\nabla \phi)^2) \begin{pmatrix} (z - y')(\nabla \phi)^2 & u' R + v'(\nabla \phi)^2 \\ -1/4a \\ 0 \\ -1/4b \end{pmatrix}$$

(30)

The above expression leads to the standard bilinear form of the action. The one loop divergences can be easily estimated within the Schwinger-DeWitt method and finally are given by expression

$$\Gamma_{div} = \frac{1}{\varepsilon} \int d^2 x \sqrt{g} \, tr \left\{ G^{-1} M - \frac{1}{4} G^{-1} L G^{-1} L + \frac{1}{6} R \right\}$$

$$L(\nabla_\mu \phi) = (\nabla_\mu G) + \frac{1}{2} (A^T - A)(\nabla_\mu \phi) \quad M = -\frac{1}{2} (B + B^T) + \frac{1}{2} A^T (\Box \phi) + \frac{1}{2} (\nabla_\mu A^T)(\nabla_\mu \phi)$$

(31)

Substituting (30) into (31) we obtain

$$\Gamma_{div} = \frac{1}{\varepsilon} \int d^2 x \sqrt{g} \left\{ \frac{2u'}{1 - uy} R - \frac{u(a'_7 + u a_s)}{a(1 - uy)^2} - \frac{a_s + ya'_7}{(1 - uy)^2} \right. + \frac{2uz - 3uy' - u'y - 2v}{1 - uy} (\Box \phi) +

\frac{1}{(1 - uy)^2} (\nabla \phi)^2 \left[ (uy'' + uy')(1 - uy) +

\left. + u^2 (y'')^2 + y^2 (u'')^2 + 2(z - y')(z u'' - u') + 2v (u'y + uy' - 2uv + z) \right] \right\}$$

(32)

Now, substituting the relations between $a_i(\phi)$ (23) and $b(\phi), y(\phi), z(\phi), u(\phi), v(\phi)$ (24) into the corresponding expression from [14] we find that both results are in a good relation with each other. One can regard this as an additional check of our treating of the auxiliary fields. Note that applying the standard result [23, 24] for the sigma model (26) we arrive at different expression. This can be easily seen from the "tachyon" sector already. Therefore the starting model (24) is equivalent to the sigma model (26) only on classical level. Already at one loop the auxiliary fields need special care. At the same time one can use representation (26) to analyze the general structure of divergences and also to classify the different versions of the general model (24), (26) into several sets. It is easy to see that for some versions of (23) there is only one significant big bracket in (24). For this particular cases we miss one auxiliary field in (25), and thus obtain one less sigma model coordinate in (26), (27). The analysys of propagator of the starting higher derivative model (4) shows that these degenerate models correspond to [14]

$$\Delta = det \begin{pmatrix} 4a_1(\phi) & -a_2(\phi) \\ -a_2(\phi) & a_4(\phi) \end{pmatrix} = (1 - uv)^2 = 0$$

(33)

If the condition (33) holds and the rank of the matrix in (33) is nonzero then the above scheme has to be modified because of less amount of the auxiliary fields, but doesn’t fail.
The result is similar to that we have observed in the previous section. So we see that the higher derivative terms with $(\nabla \varphi)^2$ are of less importance because they do not give rise to an additional auxiliary degree of freedom. It can lead to some difficulty, if we start, for example, with the theory (14) with $a_1(\phi) = a_2(\phi) = a_4(\phi) = 0$ when $a_3(\phi)$ or $a_5(\phi)$ is nonzero. In such theory the target space metric in the space of scalar field $\phi$, auxiliary field and conformal factor will be degenerate, and the above scheme does not work. In the framework of the higher derivative model (23) the picture looks as follows. The inverse propagator of the theory contains only the second derivative terms, and there are four derivative vertices. As a result the theory has worst structure of divergences and can be nonrenormalizable because of possible higher derivative counterterms of $a_1, a_2, a_4$ type. In terms of the original model (23) the one-loop divergences are not related with the functional determinant of the minimal higher derivative operator (13) but with some complicated nonminimal second order operator. So this version of the higher derivative model (23) strongly differs from the others.

**Conclusion.** We have discussed the renormalization of a higher derivative dilaton quantum gravity in two dimensional space, and solved some puzzles which have been found in this field. In particular it was shown that the loop calculations in a harmonic gauge can be performed in a completely covariant way. Then we learned to deal with an auxiliary fields.

It turns out that the general higher derivative model (23) can be reduced to $D = 4$ nonlinear sigma model with the second derivatives only. In this approach the sigma model coordinates are dilaton scalar field, conformal factor of the metric and two auxiliary fields, which correspond to higher derivatives in the original formulation. On quantum level the auxiliary fields need some special procedure to be applied, and then the results of calculations are the same as in original higher derivative model.

Our consideration may be relevant for the study of massive modes in string theory and also for the higher derivative 4$d$ quantum gravity. In this case (as well as in any dimension different from 2) the curvature tensor is not defined completely by scalar curvature. In fact it reflects the existence of spin two states. That is why in four dimensional theory one can not remove highed derivative fields introducing auxiliary scalars. However it is quite possible to develop the ”second order formalism” for the fourth derivative gravity, introducing auxiliary tensors of second rank. If these auxiliary fields are treated in a correct way (that doesn’t look difficult) then we can obtain new frame for the study of the general higher derivative 4$d$ gravity. Perhaps such description of the theory will be useful for the better understanding of the important problem of unitarity.

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