The effects of pressure dependent constitutive model to simulate concrete structures failure under impact loads

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Abstract. The main objective of this paper is to explore the effect of confining pressure in the compression and tension zone by simulating the behaviour of reinforced concrete/mortar structures subjected to the impact load. The analysis comprises the numerical simulation of the influences of high mass low speed impact weight dropping on concrete structures, where the analyses are incorporated with meshless method namely as Smoothed Particle Hydrodynamics (SPH) method. The derivation of the plastic stiffness matrix of Drucker-Prager (DP) that extended from Von-Mises (VM) yield criteria to simulate the concrete behaviour were presented in this paper. In which, the displacements for concrete/mortar structures are assumed to be infinitesimal. Furthermore, the influence of the different material model of DP and VM that used numerically for concrete and mortar structures are also discussed. Validation upon existing experimental test results is carried out to investigate the effect of confining pressure, it is found that VM criterion causes unreal impact failure (flexural cracking) of concrete structures.

1. Introduction

A mathematical model to predict the behaviour of certain material that represent the relationships between stress and strain tensor in a material point of the body, is crucial for this purpose. The form of mathematical model is commonly named a constitutive equation or constitutive model. Generally, the constitutive equation are derived from the theories of elasticity and plasticity, where the linear elastic model based on the simple Hooke’s law principles. The Young’s modulus (E) and the Poisson’s ratio (ν) are obtained from the CHILE assumption (Continuous, Homogenous, Isotropic and Linear Elastic) as defined by [1], [2], [3]. With regard to that, more complicated models for isotropic matrix form can also be easily developed. To simulate the characteristic of materials in various states, failure criteria defined with stress invariants were derived, in which 1-parameter criterion and 2-parameter criterion are considered. Many researchers such as Bresler et al.[4], William et al. [5], Ottosen [6] or Hsieh et al. [7] have attempted to extend the application of plasticity model until 3- to 5-parameter criterion. However, this study only considered 1- and 2-parameter that are Von-Mises (VM) and Drucker-Prager (DP) yield criterion, respectively. These criterion are employed to describe deformational features of materials in the ultimate stress state. Meshless method namely as Smoothed Particle Hydrodynamics (SPH) were incorporated during the process of computation to solve the limitation in the mesh based technique when simulating impact case. One of their (mesh technique) limitation is the connecting nodes cannot be tracked accurately is use a fixed mesh. SPH has the ability to unravel this issue due to its connectivity.
of arbitrarily distributed particles without using any mesh [8]. The numerical calculation setting for SPH method can be found in the research paper of Mokhatar [8]. In order to explore the effect of pressure dependent criterion, this study involve numerical simulation of cementitious (concrete/mortar) materials under impact loads. Concrete material is chosen due to its brittle characteristic. In a two-parameter criteria the yielding of concrete material under hydrostatic pressure in compression region and fracture property in tension zone are combined, where the VM yield criterion is extended by including the effect of hydrostatic pressure on the shearing resistance of the material to form DP criterion. Details derivation of VM criterion can be found in [2] [3] and [8]. Figure 1 shows the graphic representation of general linear DP yield surfaces in the principal stress space. Based on the figure 1, it provides phenomenological explanations for the pressure dependent flow due to the internal friction, which is a typical feature of brittle materials.

2. Elasto-plastic constitutive equation of pressure dependent (Drucker-Prager criteria)

In a two-parameter criteria the yielding of concrete material under hydrostatic pressure in compression region and fracture property in tension zone are combined, where the VM yield criterion is extended by including the effect of hydrostatic pressure on the shearing resistance of the material to form DP criterion. Details derivation of VM criterion can be found in [2] [3] and [8]. Figure 1 shows the graphic representation of general linear DP yield surfaces in the principal stress space. Based on the figure 1, it provides phenomenological explanations for the pressure dependent flow due to the internal friction, which is a typical feature of brittle materials.

According to this criterion, the function of DP can be expressed as below:

\[ f_{DP}(J_{1}, I_{1}) = \sqrt{J_{2D}} + I_{1} \alpha - k < 0; \text{ Elastic} \]  
\[ f_{DP}(J_{1}, I_{1}) = \sqrt{J_{2D}} + I_{1} \alpha - k = 0; \text{ Plastic} \]  

When the stress point is on the yield surface, equation (1) is always satisfying, and hence the variation in \( f \) is zero. \( f_{DP} = 0 \). It follows that

\[ df_{DP} = \frac{\partial f_{DP}}{\partial \sigma_{ij}} d\sigma_{ij} = 0 \]  

By noting that the second invariant of deviatoric stress, \( J_{2D} = \frac{1}{2} \sigma_{ij} \sigma_{ij} \) and \( I_{1} = \sigma_{ij} \delta_{ij} \), we can derive the function in equation (1) by;

\[ \frac{\partial f_{DP}}{\partial \sigma_{ij}} = \left( \frac{\partial f_{DP}}{\partial \sqrt{J_{2}}} \right) + \left( \frac{\partial f_{DP}}{\partial I_{1}} \right) = \left( 1 \sqrt{J_{2D}} ^{-1/2} \right) + (\alpha' \delta_{ij}) = \frac{\sigma_{ij}'}{2 \sqrt{J_{2D}}} + \alpha \delta_{ij} \]  

Generally, the incremental of plastic strain \( d\varepsilon_{pl} \) is calculated using flow rule associated with the yield criterion \( f \)[12]. When we substitute equation (4) into incremental plastic strain, it can be extended as;

\[ d\varepsilon_{ij} = \lambda_{dp} \frac{\partial f_{DP}}{\partial \sigma_{ij}} = \lambda_{dp} \left( \frac{\sigma_{ij}'}{2 \sqrt{J_{2D}}} + \alpha \delta_{ij} \right) \]  

Where \( \lambda_{dp} \) is plastic multiplier of DP model. Basically, the total strain increment \( d\varepsilon_{ij} \) can be separated into elastic and plastic components as shown in equation (6)
then, we can write the general incremental stress relationships as:

\[ d\sigma_{ij} = D_{ijkl}^e (d\varepsilon_{ij} - d\varepsilon_{ij}^p) \]  

and fourth-order tensor of elastic stiffness can be expressed by Lamè constant as:

\[ D_{ijkl}^e = \lambda (\delta_{ij} \delta_{kl}) + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \]  

For a yielding under a uniaxial state of stress, \( \sigma_y (\sigma_y > 0), f = 0 \) and hence parameter \( k \) can be expressed as in figure 2.

\[ \frac{\sqrt{J_{2D}}}{\sqrt{3}} \]}

\[ k = \left[ \alpha + \frac{1}{\sqrt{3}} \right] \sigma_y \]  

Substitute equation (9) into (1), we can define the effective stress \( \sigma_{eq} = \sigma_y \) as below;

\[ \sqrt{J_{2D}} + I_1 \alpha = \left[ \alpha + \frac{1}{\sqrt{3}} \right] \sigma_{eq} \]  

\[ \sigma_{eq} = \sqrt{J_{2D}} + I_1 \alpha \] where, \( \sigma_{eq} \left[ \alpha + \frac{1}{\sqrt{3}} \right] = \sqrt{J_{2D}} + I_1 \alpha \)  

Now calculate the effective plastic strain \( d\varepsilon_p \) by plastic work

\[ dW^p = \sigma_{ij} d\varepsilon_{ij}^p = \sigma_{eq} d\varepsilon_p \]  

Substitute equation (5) into (11) gives

\[ \sigma_{ij} \lambda_{dp} \left( \frac{\sigma_{ij}'}{2\sqrt{J_{2D}}} + \alpha \delta_{ij} \right) = \sigma_{eq} d\varepsilon_p \]  

Expand equation (12) and use notation used in the Drucker Prager criterion derivation [12] to yield,

\[ \lambda_{dp} \left( \frac{\sigma_{ij}'}{2\sqrt{J_{2D}}} \sigma_{ij} + \alpha \sigma_{ij} \delta_{ij} \right) = \sigma_{eq} d\varepsilon_p \]  

Substitute equation (10)b into (14) gives

\[ \lambda_{dp} \left( \frac{2J_{2D}}{2\sqrt{J_{2D}}} + I_1 \alpha \right) = \sigma_{eq} d\varepsilon_p \] or \( \lambda_{dp} \left( \sqrt{J_{2D}} + I_1 \alpha \right) = \sigma_{eq} d\varepsilon_p \)  

Substitute equation (10)b into (14) gives
\[
\lambda_{dp} \sigma_{eq} \left[ \alpha + \frac{1}{\sqrt{3}} \right] = \sigma_{eq} d\varepsilon_p
\]  

(15)

Therefore,

\[
d\varepsilon_p = \lambda_{dp} \left[ \alpha + \frac{1}{\sqrt{3}} \right]
\]  

(16)

In order to get the form of plastic multiplier \( \lambda_{dp} \), let consider equation (5), by squaring this equation, its yield;

\[
d\varepsilon^p_{ij} d\varepsilon^p_{ij} = \lambda_{dp}^2 \left( \frac{\sigma'_{ij} + \alpha \delta_{ij}}{2 \sqrt{2J_{2D}}} \right) \left( \frac{\sigma'_{ij} + \alpha \delta_{ij}}{2 \sqrt{2J_{2D}}} \right)
\]  

(17)

By noting that

\[
\sigma'_{ij} \delta_{ij} = \sigma_{ii} = 0, \delta_{ij} \delta_{ij} = 3
\]  

(18)

and consider notation used in the Drucker Prager criterion derivation [11]. Expand equation (17) and solve it

\[
d\varepsilon^p_{ij} d\varepsilon^p_{ij} = \lambda_{dp}^2 \left( \frac{\sigma'_{ij} \sigma'_{ij} + \alpha \sigma'_{ij} \delta_{ij} + \alpha \sigma'_{ij} \delta_{ij} + \alpha^2 \delta_{ij} \delta_{ij}}{2 \sqrt{2J_{2D}}} \right)
\]  

(19)

\[
= \lambda_{dp}^2 \left( \frac{2J_{2D}}{2J_{2D}} + \alpha^2 3 \right) = \lambda_{dp}^2 \left( \frac{1}{2} + 3 \alpha^2 \right)
\]  

(20)

Hence,

\[
\lambda_{dp} = \sqrt{\frac{d\varepsilon^p_{ij} d\varepsilon^p_{ij}}{\left( \frac{1}{2} + 3 \alpha^2 \right)}}
\]  

(21)

Substitute equation (21) into (16) in order to get \( d\varepsilon_p \)

\[
d\varepsilon_p = \lambda_{dp} \left[ \alpha + \frac{1}{\sqrt{3}} \right]
\]  

(22)

In plastic zone, both yield and subsequent stress states need to fulfil the condition \( f_{DP}(\sigma_{ij}, \varepsilon^p_{ij}, k(\varepsilon_p)) = 0 \), where a yield criterion of DP is a function of stress \( \sigma_{ij} \), plastic strain \( \varepsilon^p_{ij} \) and \( k(\varepsilon_p) \). Therefore, plastic flow is governed by the consistency condition, implying that,

\[
df_{DP} = \frac{\partial f_{DP}}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f_{DP}}{\partial \varepsilon^p_{ij}} d\varepsilon^p_{ij} + \frac{\partial f_{DP}}{\partial k(\varepsilon_p)} dk(\varepsilon_p) d\varepsilon_p = 0
\]  

(23)

Then, substitute equation (5), (7) and (22) into (23), we can get the consistency condition as;
During plastic loading, the general form of the DP function can be written as:

\[ f(J_{1D}^e, \varepsilon_p) = J_{2D} + I_1 \varepsilon_p - k = J_{2D} + I_1 \varepsilon_p - \left[ \alpha + \frac{1}{3} \right] \sigma_{eq}^2 \]  

(28)

if we derive the DP function in equation (28) to \( \sigma_{eq} \), we can get

\[ \frac{\partial f_{DP}}{\partial \sigma_{eq}} = - \left[ 2\alpha^2 + \frac{2}{3} \right] \sigma_{eq} \]

(29)
and the derivation term of DP function to the incremental plastic strain $d\varepsilon_p$ can be written as in equation (30) if we change hardening modulus $H$ from equation (27)

$$\frac{\partial f_{DP}}{\partial \varepsilon_p} = \frac{\partial f_{DP}}{\partial \sigma_{eq}} \left( \frac{\partial \sigma_{eq}}{\partial \varepsilon_p} \right) = \frac{\partial f_{DP}}{\partial \sigma_{eq}} (H)$$

(30)

Then, substitute equation (29) into (30), gives

$$\frac{\partial f_{DP}}{\partial \varepsilon_p} = -\left[ 2\alpha^2 + \frac{2}{3} \right] \sigma_{eq} (H)$$

(31)

In equation (1) and (28), the function does not depend on $\varepsilon_{ij}$. Therefore, when deriving the DP model, the term of $\partial f / \partial \varepsilon_{ij}$ can be eliminated in the general form of plastic multiplier as shown in equation (26)b. Let solve equation (26)b by substituting equation (4), (7) and (31) into it to obtain

$$\lambda_{dp} = \left( \frac{\sigma_{ij} + \alpha \delta_{ij}}{2\sqrt{J_{2D}}} \right) \left( \lambda (\delta_{ij} \delta_{kl}) + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \right) d\varepsilon_{kl}$$

(32)

By using the notation in equation (18), we can solve equation (32) as;

$$\lambda_{dp} = \left[ \frac{\mu \sigma_{ij}}{J_{2D}} + \alpha (3\lambda + 2\mu) \delta_{ij} \right] - \frac{3\alpha^2 (3\lambda + 2\mu) + \mu + \left[ 2\alpha^2 + \frac{2}{3} \right] \sigma_{eq} (H) \left[ \alpha + \frac{1}{\sqrt{3}} \right]}{3\alpha^2 (3\lambda + 2\mu) + \mu + \left[ 2\alpha^2 + \frac{2}{3} \right] \sigma_{eq} (H) \left[ \alpha + \frac{1}{\sqrt{3}} \right]} d\varepsilon_{ij}$$

(33)

Then, recall equation (7) and substitute (5) into it.

$$d\sigma_{ij} = D_{ijkl} (d\varepsilon_{kl} - d\varepsilon_{ij}) = D_{ijkl} d\varepsilon_{kl} - D_{ijkl} \lambda_{dp} \frac{\partial f_{DP}}{\partial \sigma_{ij}}$$

(34)

Substitute again the derivation form in equation (32) into (34), yield to

$$\frac{\mu \sigma_{ij}}{J_{2D}} + \alpha (3\lambda + 2\mu) \delta_{ij} \left[ \frac{\sigma_{ij} + \alpha \delta_{ij}}{2\sqrt{J_{2D}}} \right] d\varepsilon_{ij}$$

(35)

Substitute $D_{ijkl}$ into equation (8). Then, expand $[D]^\nu$ in equation (35) and modify the index notation,

$$[D]^\nu = \left[ \lambda (\delta_{ij} \delta_{kl}) + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \right] \left( \frac{\sigma_{ij} + \alpha \delta_{ij}}{2\sqrt{J_{2D}}} \right) \left( \frac{\mu \sigma_{mn}}{J_{2D}} + \alpha (3\lambda + 2\mu) \delta_{mn} \right) d\varepsilon_{mn}$$

(36)

Finally, the constitutive equation of DP can be expressed as;
\[
\sigma_{ij} = D_{ijkl}^{e} \delta_{kl} \]  

(37)

and the plastic stiffness matrix form of \([D]^p\) for DP model can be expressed as below

\[
[D]^p = \begin{bmatrix}
[H_{ij}^p]
\end{bmatrix}
\]

(38)

3. Numerical example

The effect of mean stress on shear failure criteria is studied in this sub-topic. Figure 4 shows the difference of failure envelopes of pressure dependent (DP) criteria and pressure independent (VM) criterion model.

\[ I_{1, \text{tension}} \]

\[ I_{1, \text{comp}} \]

\[ \sigma_{c} \]

\[ \sigma_{t} \]

\[ \sqrt{3} \]

\[ \sqrt{3} \]

\[ f_{VM} \]

\[ f_{DP} \]

\[ J_{2D} \]

Red: DP envelope

Blue: VM envelope

**Figure 4. Envelope for DP and VM models**

In this figure, x-axis represents the first invariant of stress tensor as shown in equation (39). Meanwhile, second invariant of deviatoric stress tensor becomes the y-axis.

\[
I_i = \sigma_{11} + \sigma_{22} + \sigma_{33} = \sigma_{ij} \delta_{ij} 
\]

(39)

where the two-parameter of this model, slope of failure line, \(\alpha\) and intercept of the failure line, \(k\) can be defined as:

\[
\alpha = \frac{(\sigma_{c} - \sigma_{t})}{\sqrt{3}(\sigma_{c} + \sigma_{t})} \quad \text{and} \quad k = \left[ \alpha + \frac{1}{\sqrt{3}} \right] \sigma_{t} 
\]

(40)
4. Comparisons between numerical and experimental tests - First case

To further investigate the effects of pressure dependent model, comparisons between numerical simulation and experimental tests for cementitious material (reinforced concrete beam) is conducted, from which the practical tests were carried out at Kyushu University, Japan concerning the high mass-low velocity impact loads. The details of the experimental setup, structural specifications of testing beams and its results can be referred in [13] and its material properties as shown below,

| Material  | Young Modulus (N/m$^2$) | Poisson’s ratio, $\nu$ | Density (kg/m$^3$) | Yield Stress, $\sigma_t$ (N/m$^2$) | Yield stress, $\sigma_c$ (N/m$^2$) |
|-----------|-------------------------|------------------------|---------------------|-----------------------------------|---------------------------------|
| Concrete  | 16600 x 10$^6$          | 0.22                   | 2400                | 23.78 x 10$^6$                    | 2.37 x 10$^6$                   |
| Steel     | 206000 x 10$^6$         | 0.3                    | 7800                | 300 x 10$^6$                      | 300 x 10$^6$                    |

The bending failure mechanism is the first main point of this example to investigate the influence of pressure dependent (DP) and pressure independent (VM) criteria under the 0.20 m and 0.62 m drop height of steel weight (approximately 2.0 ~ 3.5 m/s velocity). As shown in figure 5 (c), the 900 mm concrete beam indicates the flexure crack occurs at the centre of the tension region. This phenomenon has been calculated numerically by using VM and DP [see figure 4 (a) and (b)]. In both analysis cases, the cracking is considered when the maximum principal strain $\varepsilon_{\text{mm}}^{\max}$ more than 300|$\mu$ and 500|$\mu$. Based on these figures, the bending crack in the maximum deformation state as shown in grey and red colour using DP model well corresponds to the experimental result. Meanwhile, VM results give to overestimate bending crack pattern. It is due to the VM yield criteria requiring small yield stress in the positive confining pressure domain. On the contrary, DP model has comparatively large yield stress due to the pressure dependency. In this analysis, the anisotropic constitutive equation is employed to decrease the stiffness in the tensile zone due to the crack growth.

![Comparison results between numerical and experimental](image)

Figure 5: Comparison results between numerical and experimental

The displacement-time relationships of the numerical and experimental results are shown in Fig. 4-6. Based on this graph, it is indicated that the maximum mid-span displacement by VM criterion is larger than that of DP criterion. Furthermore, the DP criterion curve attenuates earlier than the VM criterion curve. This result indicates that the VM criterion gives larger degradation of stiffness than the DP criterion under this impact load condition. Although the mid-span displacement of experimental data shows very large rebound motion due to the lack of rebound prevention jig, the maximum mid-span displacement is comparatively the same as the numerical results. Therefore, it is confirmed that both analyses by VM and DP criterion give an appropriate displacement response of the beam specimens.
However, the VM yield criterion cannot consider the accurate crack growth because of the pressure independent assumption. In DP model, though the maximum displacement under 2.0 m/s velocities is smaller than VM and experimental result, maximum displacement and crack pattern are in reasonable agreement with the test result.

\[ \text{Figure 4.6. Displacement-time histories} \]

5. Comparisons between numerical and experimental tests - Second case
The impact velocity is increased for the same length and dimension of the concrete beam until 3.5 m/s in order to investigate the change of failure mode of this structural element. Again, an original VM and DP model have been compared in this analysis. Based on the figure 6 (a) and (b), the significant differences between VM and DP yield criteria can be also investigated under high stress state conditions. The VM yield function shows an inappropriate crack distribution as compared to DP model and experimental results [refer figure 6 (a), (b) and (c)]. In which, the failure is considered when the maximum principal strain $\varepsilon_{\text{mm}}^{\text{max}}$ more than 3000$\mu$ and 5000$\mu$. The bending crack patterns predicted using DP yield function essentially matched the test results. However, under the highly compressed state, the compression failure cannot be estimated using the DP model because no limitation in the compression zone. Generally, the original DP yield surface gives large shear strength with large compressive confining pressure. Based on these two numerical examples, it can be summarized that the DP yield criteria is more appropriate than the VM yield criteria and further improvement is expected in order to simulate an accurate compression failure of RC beam under impact loads.

\[ \text{Figure 6. Comparison results between numerical and experimental} \]
Figure 7 shows the mid-span displacement-time history using original VM and DP criterion as well as the experimental results. It is confirmed that the maximum displacement of two numerical examples by DP model gives ten percent small response compared with the experimental. As for the oscillation period, the analysis result with fixed vertical support is different from the test result due to the lack of a rebound prevention jig in the experiment.

In this analysis, we adopted straight envelop for DP plane and it might cause the discrepancies of displacement between analysis and experiments. In general, easy improvement of yield surface requires the increase of parameters, and thus we should further consider the optimum criterion from the viewpoint of few parameters and appropriate prediction ability. However, an application of the DP model in this study gives affordable results of displacement response and cracked pattern compared with experimental results.

6. Conclusions
As the results of displacement time histories for first and second case using DP criterion are smaller than that of the experimental results, we should revise the effect of dependence on confining pressure. Though VM criterion gives a reliable displacement response, it cannot predict adequate flexural cracking and compressive damage. Therefore, DP criterion is more suitable than VM criterion. It is due to the compression forces mainly affect the crushing phenomenon. In addition, inaccurate stress evaluation by VM criterion (because of the coupling between hydrostatic stress and deviatoric stress) causes unreal flexural cracking. Thus, two-parameter (like DP) models with cap surface modification correspond to the above-mentioned feature is necessary. The development of the cap surface associated with compression softening can be obtained in [8].

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