(g − 2)µ anomaly within the left-right symmetric model

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The muon anomalous magnetic moment is discussed within the model based on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$-gauge group. The contributions of the neutrinos having the dipole magnetic moment (DMM) are calculated. The cases of the Majorana and Dirac neutrino are considered. It is shown that to account for the (g − 2)µ anomaly DMMs connected with heavy neutrinos should be on the order of few × 10^{-8} \mu_B, that is at variance with the theoretically predicted values.

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1 Introduction

The muon anomalous magnetic moment (AMM) has been measured in a series of three experiments at CERN (1968-1977) and, most recently, in E-0821 experiment at the Brookhaven National Laboratory (BNL) on Alternating Gradient Synchrotron (in 1997-2001). The results of the CERN-III experiment were combined to yield a 7.3 ppm measurement in agreement with a theory. The measurements at BNL, using nearly equal samples of positive and negative muons, have been carried out with an impressive accuracy of 0.72 ppm and yielded the present world average \[a_{\mu}^{\exp} = 11 659 208.0(6.3) \times 10^{-10}\] (1)

with an accuracy of 0.54 ppm, which represents a 14-fold improvement compared to the previous measurements at CERN. However, contrary to expectations, the BNL data do not agree with the standard model (SM) prediction. For example, the result of Ref. [2] based on e^-e^+ data alone is as follows

\[a_{\mu}^{SM} = 11 659 178.5(6.1) \times 10^{-10}.\] (2)

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1 INTRODUCTION

Introducing the quantity
\[ \delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM}, \]
we obtain
\[ \delta a_\mu = (29.5 \pm 8.8) \times 10^{-10}. \] (3)
So, the SM evaluation displays 3.4 standard deviations below the experimental result. Analogous calculations obtained in Refs. [3],[4],[5] bring to slightly different results but all yield deviations of more than 3 \( \sigma \). In what follows we use the value \( a_\mu^{SM} \) given by Eq. (2). Then at 90% CL, \( \delta a_\mu \) must lie in the range
\[ 18.2 \times 10^{-10} \leq \delta a_\mu \leq 40.7 \times 10^{-10}. \] (4)

Since the BNL data have been perfectly collected and investigated for many years, it is highly improbable that this discrepancy could be due to a mere statistical fluctuation, as several earlier deviations from SM turned out to be. An extremely small variation of the muon AMM central value present in all the BNL results is also a serious argument in favor of the E-0821 experiment reliability. So, the \((g - 2)_\mu\) anomaly may be examined as the New Physics (NP) signal even at a weak scale. As this takes place, it is expected that NP contributions should be proportional to \( m_\mu^2/M_{NP}^2 \), where \( M_{NP} \) is the mass of a particle additional to the sector of the SM particles.

In this work we continue investigation of the \((g - 2)_\mu\) anomaly within the model based on the \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) gauge group (left-right model) [6]. The left-right model (LRM) contains the whole collection of particles, which are additional with respect to the SM. In case when the Higgs sector holds the bidoublet, the left- and right-handed triplets
\[ \Phi \left( \frac{1}{2}, \frac{1}{2}, 0 \right), \quad \Delta_L(1, 0, 2), \quad \Delta_R(0, 1, 2), \] (5)
where in brackets the quantum numbers of the \( SU(2)_L, SU(2)_R \) and \( U(1)_{B-L} \) groups are given, after a spontaneous symmetry breaking we have 14 physical Higgs bosons. In Ref. [7] it was shown that at the definite parameters values of the LRM the contributions from the physical Higgs bosons can explain the BNL results. The influence of the \( W_2 \) and \( Z_2 \) gauge bosons has been investigated earlier in Ref. [8], where it was shown that the \( Z_2 \) contribution is negative, while in order to explain the E-0821 result the \( W_2 \) gauge boson mass must lie around 100 Gev that is at variance with the experimental data. The problem is that, besides the Higgs and gauge bosons, the LRM contains another three additional particles: three heavy neutrinos \( N_e, N_\mu, N_\tau \) — representing a mixture of the mass eigenstates \( N_1, N_2 \) and \( N_3 \). As it was demonstrated in Ref. [9], the one of the \( N_{1,2,3} \) neutrinos could have the mass lying on the electroweak scale.

The goal of this work is to study the influence of the LRM neutrino sector on the muon AMM value. We assume that neutrinos possess a dipole magnetic moment (DMM) and consider the cases when the neutrinos have both the Majorana and Dirac nature. In the next section we estimate the contributions of the diagrams with the neutrino exchange into the \((g - 2)_\mu\) anomaly. Comparing the results obtained, we find the constraints on the neutrino DMM. And in Sec. III our conclusions are summarized.
2 Neutrino corrections to the muon AMM

The conventional choice of the LRM Higgs sector is given by Eq. (5). Such a choice ensures the Majorana nature of the neutrinos. Let us start our investigation with this very case. Then, if we are dealing with diagonal elements of the multipole moments, the only nonvanishing multipole moment is an anapole moment. However this is not true for nondiagonal elements. For example, nondiagonal elements of the dipole magnetic moment only nonvanishing multipole moment is an anapole moment. Then, if we are dealing with diagonal elements of the multipole moments, the ensures the Majorana nature of the neutrinos. Let us start our investigation with this

\[ H_{\text{add}} = \sum_{i \neq j} \left[ \mu_{\nu_i \nu_j} |i(p)\sigma_{\lambda \tau} q_{\tau}(p') + \mu_{N_i N_j} \overline{N}_i(p)\sigma_{\lambda \tau} q_{\tau} N_j(p') + \right. \\
\left. + \mu_{\nu_i N_j} \overline{N}_i(p)\sigma_{\lambda \tau} q_{\tau} N_j(p') + \mu_{N_i \nu_j} \overline{N}_i(p)\sigma_{\lambda \tau} q_{\tau} \nu_j(p') \right] A_\lambda(q), \]  

(6)

where

\[ \sigma_{\lambda \tau} = \frac{i}{2} (\gamma_\tau \gamma_\lambda - \gamma_\lambda \gamma_\tau), \quad q_{\tau} = (p' - p)_{\tau}. \]

For the sake of simplicity, we suppose that mixing takes place between the muon and tau neutrino only, namely:

\[
\begin{pmatrix} \nu_{\mu L} \\ \nu_{\tau L} \\ N_{\mu R} \\ N_{\tau R} \end{pmatrix} = \mathcal{M} \begin{pmatrix} \nu_2 \\ \nu_3 \\ N_2 \\ N_3 \end{pmatrix},
\]

(7)

where

\[
\mathcal{M} = \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} \begin{pmatrix} c_{\phi_{\mu}} & 0 & s_{\phi_{\mu}} & 0 \\ 0 & c_{\phi_{\tau}} & 0 & s_{\phi_{\tau}} \\ -s_{\phi_{\mu}} & 0 & c_{\phi_{\mu}} & 0 \\ 0 & -s_{\phi_{\tau}} & 0 & c_{\phi_{\tau}} \end{pmatrix},
\]

(8)

\[ \varphi_{l} \] is the mixing angle between light and heavy neutrinos in the l-generation, \( \theta_{\nu} (\theta_{N}) \) is the mixing angle between light (heavy) neutrinos belonging to different generations, \( \beta_{\nu} (\beta_{N}) \) is the \( CP \)-violating phase for light (heavy) neutrinos, \( s_{\phi_{\mu}} = \sin \varphi_{\mu} \), \( c_{\phi_{\mu}} = \cos \varphi_{\mu} \), and so on.

In the LRM the charged current Lagrangian is of the form

\[ \mathcal{L}_{CC}^{L} = \frac{g_L}{2\sqrt{2}} \overline{\tau}(x)\gamma_\lambda (1 + \gamma_5) \nu_l(x) W_{L\lambda}(x) + \frac{g_R}{2\sqrt{2}} \overline{\tau}(x)\gamma_\lambda (1 - \gamma_5) N_l(x) W_{R\lambda}(x), \]

(9)

where

\[ W_{L\lambda} = W_{1\lambda} c_\xi + W_{2\lambda} e^{-i\phi} s_\xi, \quad W_{R\lambda} = -W_{1\lambda} e^{i\phi} s_\xi + W_{2\lambda} c_\xi \]

\( \xi \) is the mixing angle in the sector of the charged gauge bosons and \( \phi \) is the phase to be responsible for the \( CP \) violation.
Figure 1: One-loop diagrams contributing to the muon AMM due to the light and heavy neutrinos.

In Fig. 1 we represent Feynman diagrams with light and heavy neutrinos making a contribution into the muon AMM. The vertex function of the third order $\Lambda_\beta(p, p')$ that corresponds to the diagrams with the $W_1^(-)$-boson exchange has the form

$$\Lambda_\beta(p, p') = \frac{eg^2c^2}{(4\pi)^4m_e} \sum_{i,j} \mu_{ij}' \int \gamma_\alpha (1 + \gamma_5) \left \{ [i(p' - k) - m_i] \sigma_\beta q_\tau [i(p - k) - m_j] M^\dagger_{i\mu} M_{\mu i} \times \right.$$  

$$ \times M^\dagger_{j\mu} M_{\mu j} + C \lambda_{j\odot} m_i \sigma_\beta q_\tau C \lambda_{j\odot} m_j M_{i\mu} M_{\mu i} M^\dagger_{j\mu} M^\dagger_{\mu j} \} \gamma_\nu (1 + \gamma_5) \times \right.$$  

$$ \left. \frac{[\delta_{\sigma\nu} + k_\sigma k_\nu/m^2_{W_1}]}{[k^2 + m^2_{W_1}][(p' - k)^2 + m^2_2][(p - k)^2 + m^2_2]} \right \},$$

(10)

where $\mu_{ij}' = \mu_{ij}/\mu_B$, $\mu_B$ is the Bohr magneton, $\lambda_{j\odot}$ is the creation phase factor of $\nu_j$-neutrino ($|\lambda_{j\odot}|^2 = 1$), $C$ is the charge conjugation matrix, $i, j = \nu_2, \nu_3, N_2, N_3$. Besides, it is taken into account that for the Majorana field the relations

$$< 0|T(\nu_\alpha(x)\overline{\nu}_\beta(y))|0 > = -i S^c_{\alpha\beta}(x - y), \quad < 0|T(\nu_\alpha(x)\nu_\beta(y))|0 > = -i \lambda_{\odot} m C_{\alpha\beta} \Delta^c(x - y)$$

(11)

are valid. It is easy to verify that the second term in the integrand vanishes. As a result, we arrive at the same expression as in the case of the Dirac neutrino.

Of course, since the spontaneously broken gauge theories are renormalizable, the vertex function $\Lambda_\beta(p, p')$ should be finite. In fact, the total bare Lagrangian of the LRM includes no interactions of the form given by Eq.(6). In the unitary gauge the propagator of the $W_{1,2}^\pm$-bosons contains $k_\sigma k_\nu/m^2_{W_{1,2}}$ term, resulting in a linearly divergent piece. Although these divergent terms are canceled, the finite parts depend on the routing of the momenta through the graphs. Such an ambiguity could be resolved in several ways.

For example, one could employ the method presented in [10], where the $W_{1,2}^\pm$-boson lines carry no external momenta, with the use of the $\xi$-limiting procedure. The second way is to calculate the matrix elements in the $R_\xi$ gauge. Here we choose more simple way,
proportional to \( \mu \) masses are not independent. In the two-flavor approximation the \( y \) are connected with the \( f \) where
\[
\sum_j m_j = m_{\nu_2} + m_{\nu_3} + m_{N_2} + m_{N_3} = (f_{\mu \mu} + f_{\mu \tau})(v_L + v_R),
\]
where \( f_{\mu \tau} \) and \( f_{\mu \mu} \) are the triplet Yukawa coupling constants, \( v_{L,R} \) is the vacuum expectation values (VEV) for the left- and right-handed triplets of the Higgs fields.

It is evident that the main contributions to \( \delta a_\mu \) may be due to the terms which are proportional to \( \mu^L_{N_2N_3} \) and \( \mu^L_{N_a\nu_b} \) (\( a, b = 2, 3 \)). First, we consider the contribution coming
Figure 2: The curves constraining the allowed region of the $\mu'_{N_2N_3}$ and $m_{N_2}$ values (the values of $\mu'_{N_2N_3}$ is taken in the Bohr magneton units).

from the terms associated with $\mu'_{N_2N_3}$ only. In our numerical calculation we use the following parameter values:

$$\theta_{\nu} = \theta_N = 50^0, \quad \xi = 5 \times 10^{-2}, \quad m_{W_2} = 800 \text{ GeV}. \quad (16)$$

For the case $\sum m_j = 1500 \text{ GeV}$ and $\varphi_{\mu} = 0$ in the $\mu'_{N_2N_3}$ vs $m_{N_2}$ parameter space, Fig. 2 presents two curves corresponding to the values of $\delta a_{\mu}$ equal to $407 \times 10^{-11}$ (upper curve) and $182 \times 10^{-11}$ (lower curve). The range of the parameters for the heavy neutrino sector allowed by the BNL data lies between these curves. It is seen that, in order to account for the observed muon AMM, the values of $\mu_{N_2N_3}$ must be as large as $10^{-7} \mu_B$.

Further we allow for the contributions made by $\mu_{N_2\nu_b}$. In so doing, for the sake of simplicity, we assume that

$$\mu_{N_2\nu_b} = \mu_{N_2N_3} = \mu_{hn},$$

and take for $\varphi_{\mu}$ its upper bound $10^{-2}$ [13]. In that case we obtain that the $(g - 2)_\mu$ anomaly finds its explanation under fulfilment of the condition

$$\mu_{hn} \geq 3 \times 10^{-8} \mu_B.$$

Let us examine, whether these values are at variance with the literature data. Note that, up to now, there have been no experimental limitations on the values of the heavy
neutrino DMMs (only the laboratory constraints for light neutrino DDMs have been obtained). Therefore we should appeal to the values the theory predicts.

Within the LRM, the transition DMMs have been computed in Refs. [14]. In doing so the contributions of the diagrams with the virtual charged gauge bosons were considered only. The diagrams produced by the Lagrangians describing the interactions of the singly charged Higgs bosons \( h^\pm \), \( \tilde{\eta}^\pm \) with leptons and gauge bosons were disregarded. The authors of Refs. [14] have neglected the mixing between the light and heavy neutrinos too. The transition moments to be of interest here are defined as follows:

\[
\mu_{N_a N_b} \approx -\frac{3eg_L^2 (m_{N_a} + m_{N_b})}{27\pi^2} \left( \frac{s_\tau^2 m_\tau^2}{m_W^4} + \frac{c_\tau^2 m_\tau^2}{m_W^4} \right) \text{Im} [V_{\tau a}^* V_{\tau b}], \tag{17}
\]

\[
\mu_{N_a \nu_b} \approx \frac{eg_1^2}{8\pi^2 s_\xi c_\xi} \frac{m_\tau}{m_W} \text{Im} [e^{-i\phi} V_{\tau a}^* U_{\tau b}]. \tag{18}
\]

First of all, we note that in order to give positive contributions to \( \delta a_\mu \), the mixing matrices \( U, V \), and CP-violating phase \( \phi \) must satisfy the conditions

\[
\text{Im} [V_{\tau a}^* V_{\tau b}] < 0, \quad \text{Im} [e^{-i\phi} V_{\tau a}^* U_{\tau b}] > 0. \tag{19}
\]

In the case when the heavy neutrino mass is on the order of a few TeV, from Eqs. (17) and (18) it follows that

\[
\mu_{N_a N_b} \leq 10^{-13} \mu_B, \quad \mu_{N_a \nu_b} \leq 7 \times 10^{-11} \mu_B, \tag{20}
\]

where we have made use of (16). So, for the case of the Majorana neutrino the transition moment values required for the explanation of the \((g - 2)_\mu\) anomaly happen to be much larger than those predicted by the theory.

As long as the values of \( J_{W_i}^{L^R} \) as well as of the neutrino DMMs are proportional to the neutrino mass, one might hope to obtain the \((g - 2)_\mu\) explanation at larger values of \( m_{N_2,3} \) (for example, at \( m_{N_2} + m_{N_3} > \text{few} \times 10 \text{ TeV} \)). But, an increase in the heavy neutrino mass is mainly caused by increasing size of \( v_R \). On the other hand, the quantity \( v_R \) enters the definitions both of the \( W_2 \) gauge boson mass and mixing angle \( \xi \)

\[
m_{W_2}^2 = \frac{g_2^2}{2} \left[ (k_1^2 + k_2^2 + v_L^2 + v_R^2) - \sqrt{(v_R^2 - v_L^2)^2 + k_1^2 k_2^2} \right],
\]

\[
\tan 2\xi = \frac{2k_1 k_2}{v_R^2 - v_L^2},
\]

where \( k_1 \) and \( k_2 \) are VEV of neutral components of the Higgs bidoublet. We recall that

\[
v_L \ll \max(k_1, k_2) \ll v_R.
\]

Thus, with increase in a heavy neutrino mass, the \( W_2 \) gauge boson mass is growing, whereas the mixing angle \( \xi \) decreases. As a result, the transition moment size for the heavy neutrino and also its contribution to the muon AMM is diminishing.
3 CONCLUSIONS

In LRM neutrino could be of the Dirac nature as well. Then, the theory predicts that the diagonal elements of the neutrino DMMs are nonzero and could be much greater than the nondiagonal elements. When the heavy neutrino masses lie on the TeV scale, the heavy neutrino DMM is defined by the expression [14]

\[
\mu_{N_a} \approx \frac{3eg_L^2 m_{N_a}}{64\pi^2} \left[ \frac{s^2}{m_{W_1}^2} + \frac{c^2}{m_{W_2}^2} \right].
\]

Setting, for example, \( \xi = 5 \times 10^{-2}, m_{W_2} = 800 \text{ GeV} \) and \( m_{N_a} = 1400 \text{ GeV} \), we obtain

\[
\mu_{N_a} \approx 4 \times 10^{-9} \mu_B.
\]

So it may be hoped that big values of \( \mu_{N_a} \) ensure the \((g - 2)\mu\) anomaly explanation.

Taking into consideration the contribution to be made by the diagrams with the heavy neutrino exchange, we arrive at

\[
\delta a_\mu \approx \frac{G_F m_{W_1}^2 c^2_{\theta_N} m_{\mu}^2}{16\sqrt{2}\pi^2 m_e} \left\{ \int_0^1 s^2 \xi \left( \frac{x(1 - x)^2 - x^3 m_{N_2}^2 m_{W_1}^2}{(x m_{\mu}^2 - m_{W_1}^2)(x - 1) + x m_{N_2}^2} \right) + m_{W_1}^2 x(1 - 3x) \ln \left| \frac{m_{W_1}^2 (1 - x) + m_{N_2}^2 x}{(m_{W_1}^2 - x m_{\mu}^2)(1 - x) + m_{N_2}^2 x} \right| + m_{N_2}^2 x^2 \right\} + \left( \theta_N \to \theta_N + \frac{\pi}{2}, \mu'_{N_3} \to \mu'_{N_3}, m_{N_2} \to m_{N_3} \right). \tag{22}
\]

However, the integrals in Eq. (22) prove to be too small and, despite possible big values of \( \mu_{N_a} \), the value of the muon AMM is inexplicable by the presence of the neutrino DMM.

3 Conclusions

The \((g - 2)_\mu\) anomaly within the LRM has been discussed. We have considered the contributions coming from neutrinos possessing the DMMs. As shown, in the Majorana case, in order to explain the BNL result, the neutrino DMM must be as large as \( \times 10^{-7} \mu_B \). The obtained value proves to be three orders of magnitude higher than that calculated in Ref. [13]. Such a situation is maintained as a neutrino has the Dirac nature. Then, taking into account the results of Refs. [7, 8, 9] we come to the conclusion: among the additional particles of the LRM only the Higgs bosons could give explanation for the \((g - 2)_\mu\) anomaly.

At present, a new BNL muon experiment is under discussion. It is scheduled to improve the results by at least a factor of 2. We note that the up-to-date theoretical error is somewhat greater than the experimental one. It is fully dominated by the uncertainty in the hadronic low energy cross section data defining the hadronic vacuum polarization and, in part, by the uncertainty in the hadronic light-by-light scattering contribution. Therefore, the theoretical results associated with calculations of the hadronic corrections...
should be improved as well. Progress in this area is awaited owing to the lattice QCD techniques. Comparison of the theoretical and experimental results will give a more precise \((g - 2)_{\mu}\) value that, in turn, will enable finding of more trustworthy constraints on the Higgs sector parameters for the LRM.
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