Comparison of the atomic clock orientational error in on-board equipment of Galileo, GPS and BeiDou satellite systems

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Abstract. An assessment of the influence of the orientational frequency shift of an optically pumped quantum standard with a gas cell in the total error of the navigation systems Galileo, GPS and BeiDou satellites ephemeris is presented. A comparison of the calculated and experimental data of the time dependencies of the orientation errors for different working magnetic field values and directions is shown. For a numerical estimate of the orientational error influence on the ephemeris of different satellites, the values of the correlation coefficients between the calculated and the experimental dependencies on time are presented.

1. Introduction
In [1-10], one noted the importance of the accurate satellites positioning for modern global navigation systems. The authors of the [11-17] noticed that the main destabilizing factor of quantum optically pumped frequency standards is the so-called light shifts, due to the influence of the dynamic Stark effect on the atoms of the working substance in the absorption cell. In this case, the dependence of the light shift on the angle between the direction of the working field and the optical axis of the device leads to significant orientation errors of the measuring module, which can manifest themselves in the variant of its use in the on-board equipment of communication satellites.

However, the manifestation of the dynamic Stark effect is most significant with a small internal magnetic field. When this magnetic field becomes larger, the quadratic dependence of the microwave resonance frequency of the working transition makes a significant contribution to the error:

\[ f_r = f_0 + aH^2, \]

where \( f_r \) is the microwave resonance frequency of the working transition, \( f_0 \) is the microwave resonance frequency of the rubidium atoms undisturbed working transition, \( a \) is the constant, \( H \) is the magnetic field.

The variable \( H \) in formula (1) is determined by the internal working magnetic field, which is created inside the device, and by the weakened magnetic screen, which is used as part of the optically pumped quantum frequency standard, external magnetic field — the geomagnetic field.

This paper presents the results of calculations of the measurement error of the parameters of a communication satellite orbit for two magnetic field values: 0.1 μT и 7 μT.

2. Comparison of experimental with calculated data
In the authors’ [1], the calculations result of the orientational error of Galileo and GPS satellites ephemeris for the working magnetic field value 5 μT were presented. These calculations were carried...
out using a specially written program in C++. In this work, an improved program algorithm was used, that is responsible for the atomic clock destabilizing factor due to light shifts. Improved algorithm led to an accuracy increase of calculation and, consequently, to better calculation results of the dependence of satellite navigation system orientation error on time. In addition, a new algorithm was added to the program, which takes into account formula (1) at increased values magnetic fields.

The calculated and experimental coordinates deviations of the Galileo satellite from time are presented in figures 1 and 3, and for the GPS satellite - in figures 2 and 4. Obtaining dependencies for Galileo and GPS satellites were carried out with the assumption that the quantum frequency standards on gas cell were used as onboard equipment of satellite systems. Also, the following Galileo satellite orbital parameters were used:

- the period of the satellite 14 hours;
- the coefficient of dynamic shielding magnetic screen: 100;
- the average magnetic field intensity in the satellite orbit: 0.1 μT and 7 μT;
- the angle of inclination of the orbital plane to the equatorial plane: 56°;
- the average height of the satellite above the Earth's surface: 23200 km.

GPS satellite orbital parameters were used:

- the period of the satellite 12 hours;
- the coefficient of dynamic shielding magnetic screen: 100;
- the average magnetic field intensity in the satellite orbit: 0.1 μT and 7 μT;
- the angle of inclination of the orbital plane to the equatorial plane: 55°;
- the average height of the satellite above the Earth's surface: 20200 km.

The dynamic shielding coefficient presented above was not chosen randomly, its value is confirmed by a direct experiment with a commercial frequency standard.

![Figure 1](image-url)
Figure 2. Experimental (solid line) and calculated with a working magnetic field of 0.1 μT (dotted line) dependencies GPS satellite orientation error on the time.

Figure 3. Experimental (solid line) and calculated with a working magnetic field of 7 μT (dotted line) dependencies Galileo satellite orientation error on the time.

Figure 4. Experimental (solid line) and calculated with a working magnetic field of 7 μT (dotted line) dependencies GPS satellite orientation error on the time.
The numerical assessment of the orientation errors effect on the ephemeris of the Galileo and GPS navigation satellites is represented by the values of the correlation coefficients between the calculated (it taking into account only orientational shifts in atomic hours) and experimental time dependencies. The obtained these coefficients values for satellites with a working magnetic field of 0.1 μT are shown in table 1, and for satellites with a working magnetic field of 7 μT - in table 2. In both tables, the results are given for different relative shift variants of the start time Δt, expressed in units of the satellite rotation period.

**Table 1.** Correlation coefficients values to dependencies in figure 1 and figure 2 (working magnetic field is 0.1 μT) at the different starting times.

| Δt   | -6T | -4T | -2T | 0   | 2T  | 4T  | 6T  |
|------|-----|-----|-----|-----|-----|-----|-----|
| $R_{Galileo}$ | 0.559 | 0.587 | 0.706 | 0.712 | 0.635 | 0.541 | 0.559 |
| $R_{GPS}$    | 0.477 | 0.477 | 0.477 | 0.477 | 0.477 | 0.477 | 0.477 |

**Table 2.** Correlation coefficients values to dependencies in figure 3 and figure 4 (working magnetic field is 7 μT) at the different starting times.

| Δt   | -6T | -4T | -2T | 0   | 2T  | 4T  | 6T  |
|------|-----|-----|-----|-----|-----|-----|-----|
| $R_{Galileo}$ | 0.940 | 0.871 | 0.780 | 0.764 | 0.830 | 0.915 | 0.940 |
| $R_{GPS}$    | 0.559 | 0.559 | 0.559 | 0.559 | 0.559 | 0.559 | 0.559 |

In contradistinction to the satellites mentioned above, BeiDou is geostationary, and, therefore, the orientation error does not contribute. However, with the help of a geostationary satellite, it is possible to estimate the effect of other error sources, for example, the ionosphere, that will allow to take into account this influence on the total error of the ephemeris of different navigation satellites. The calculated (taking into account only the orientational shift) and experimental dependencies of the BeiDou satellite coordinates deviation on time are presented in figure 5.

![Figure 5](image-url)

**Figure 5.** Experimental (solid line) and calculated (dotted line), which takes into account only the orientational error, dependencies coordinates deviation of the BeiDou satellite on time.

**3. Conclusion**

A comparison of the data in the table confirms the connection between the total error of the communication satellites ephemeris and the frequency orientational shift, which makes a greater contribution with an increased value of the working magnetic field. Also, the calculations showed that the correlation coefficients for both working magnetic field values for the Galileo satellite prevail over the correlation coefficients values for the GPS satellite. This can be explained by the difference in the
magnitudes of the working magnetic field, optical pump powers, magnetic screen and satellites orbits parameters.

References
[1] Lozov R K, Baranov A A, Ermak S V and Semenov V V 2019 Comparison of orientational error of an optically pumped quantum sensor in on-board equipment of Galileo and GPS satellite systems J. Phys. Conf. Ser. 1236 012077
[2] Rachitskaya A P and Tsikin I A 2018 Gnss integrity monitoring in case of a priori uncertainty about user’s coordinates Proceedings of the 2018 IEEE International Conference on Electrical Engineering and Photonics, EExPolytech 2018 83–87
[3] Melikhova A P and Tsikin I A 2018 Optimum Array Processing with Unknown Attitude Parameters for GNSS Anti-Spoofing Integrity Monitoring 41st International Conference on Telecommunications and Signal Processing 2018 8441358
[4] Tsikin I A and Shcherbinina E A 2018 Algorithms of GNSS signal processing based on the generalized maximum likelihood criterion for attitude determination 25th Saint Petersburg International Conference on Integrated Navigation Systems, ICINS 2018 - Proceeding 1–4
[5] Melikhova A P and Tsikin I A Decision-making algorithms based on generalized likelihood ratio test for angle-of-arrival GNSS integrity monitoring 2018 25th Saint Petersburg International Conference on Integrated Navigation Systems, ICINS 2018 - Proceedings 1–5
[6] Tsikin I and Melikhova A 2016 Angle-of-arrival GPS integrity monitoring insensitive to satellite constellation geometry Lect. Notes Comput. Sci. (including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics) 9870 LNCS 584–92
[7] Tsikin I A and Melikhova A P 2017 Direct signal processing for GNSS integrity monitoring Lect. Notes Comput. Sci. (including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics) 10531 LNCS 635–43
[8] Melikhova A and Tsikin I 2016 Antenna array with a small number of elements for angle-of-Arriving gnss integrity monitoring 2016 39th Int. Conf. Telecommun. Signal Process, TSP 2016 190–3
[9] Tsikin I and Shcherbinina E 2016 GNSS attitude determination based on antenna array space-time signal processing Lect. Notes Comput. Sci. (including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics) 9870 LNCS 573–83
[10] Tsikin I and Melikhova A 2016 Angle-of-arrival GPS integrity monitoring insensitive to satellite constellation geometry Lect. Notes Comput. Sci. (including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics) 9870 LNCS 584–92
[11] Lozov R K, Baranov A A, Ermak S V and Semenov V V 2018 The influence of the orientation frequency shift of the quantum sensor with optical pumping on the measurement of the orbit parameters of the satellites of navigation systems Russia J. Radioengineering 5-12
[12] Sagitov E A, Ermak S V, Petrenko M V and Semenov V V 2016 Quantum magnetometers as a base for atomic clock J. Phys. Conf. Ser. 769
[13] Fedorov M I, Ermak S V, Petrenko M V, Pyatychev E N and Semenov V V 2016 Investigation of coherent population trapping signals in 87Rb cells with buffer gas J. Phys. Conf. Ser. 769
[14] Baranov A, Ermak S, Smolin R and Semenov V 2016 Long-term stability dependence of the quantum magnetometers dual scheme on the correlation of their double resonance signals 2016 IEEE Int. Freq. Control Symp. IFCS 2016 - Proc.
[15] Baranov A A, Ermak S V, Sagitov E A, Smolin R V and Semenov V V 2016 Signal correlation in the tandem of a spin oscillator and microwave frequency discriminator with laser-pumped alkali atoms Tech. Phys. Lett. 42 186–90
[16] Ermak S V, Petrenko M V and Semenov V V 2016 Coherent population trapping on 87Rb atoms in small-size absorption cells with buffer gas Tech. Phys. Lett. 42 127–30