Rapid acceleration of protons upstream of earthward propagating dipolarization fronts

A. Y. Ukhorskiy,¹ M. I. Sitnov,¹ V. G. Merkin,¹ and A. V. Artemyev²

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[1] Transport and acceleration of ions in the magnetotail largely occurs in the form of discrete impulsive events associated with a steep increase of the tail magnetic field normal to the neutral plane ($B_z$), which are referred to as dipolarization fronts. The goal of this paper is to investigate how protons initially located upstream of earthward moving fronts are accelerated at their encounter. According to our analytical analysis and simplified two-dimensional test-particle simulations of equatorially mirroring particles, there are two regimes of proton acceleration: trapping and quasi-trapping, which are realized depending on whether the front is preceded by a negative depletion in $B_z$. We then use three-dimensional test-particle simulations to investigate how these acceleration processes operate in a realistic magnetotail geometry. For this purpose we construct an analytical model of the front which is superimposed onto the ambient field of the magnetotail. According to our numerical simulations, both trapping and quasi-trapping can produce rapid acceleration of protons by more than an order of magnitude. In the case of trapping, the acceleration levels depend on the amount of time particles stay in phase with the front which is controlled by the magnetic field curvature ahead of the front and the front width. Quasi-trapping does not cause particle scattering out of the equatorial plane. Energization levels in this case are limited by the number of encounters particles have with the front before they get magnetized behind it.

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1. Introduction

[2] Much of plasma heating in the terrestrial magnetosphere takes place in the magnetotail, where the magnetic energy brought with the solar wind is first accumulated and then suddenly released following magnetic reconnection [e.g., McPherron et al., 1973]. Magnetic energy is transferred to kinetic energy of particles in the process of magnetic field dipolarizations which bring energized particles from the tail to the inner magnetosphere [e.g., Lui et al., 1987; Brandt et al., 2001]. Subsequently, energized ions become the dominant source of plasma pressure in the inner magnetosphere. Hot plasma pressure drives large electric currents which determine global electrodynamics and coupling of the inner magnetosphere-ionosphere system [e.g., Roelof et al., 2004]. Pressure-driven currents can cause large distortions of magnetic field across the inner magnetosphere leading to rapid magnetopause loss of relativistic electrons from the outer radiation belts [e.g., Ukhorskiy et al., 2006]. Hot plasma also provides the energy source for multiple instabilities generating wave modes that contribute to acceleration and loss of energetic ions and relativistic electrons [e.g., Kennel and Petschek, 1966; Thorne, 2010].

[3] According to in situ satellite measurements, transport and energization of ions in Earth’s magnetotail largely occurs in the form of discrete activations such as substorms and bursty bulk flows [Baumjohann et al., 1990; Angelopoulos et al., 1992; Shiokawa et al., 1997, 1998; Fairfield et al., 1998, 1999]. Distinctive features of these impulsive events are high-speed plasma flow bursts with strong and steep increases of the tail magnetic field component ($B_z$) normal to the neutral plane. Such abrupt magnetic field changes, which make the stretched tail field more dipolar, are often referred to as dipolarization fronts [Nakamura et al., 2002; Runov et al., 2009, 2011, 2012].

[4] Dipolarization fronts are observed throughout the magnetotail and deep inside the inner magnetosphere (from $X = -3.5$ to $\geq 31 R_E$) [e.g., Ohtani et al., 2004, 2006; Nosé et al., 2010; Runov et al., 2011]. The analysis of 24 dipolarization fronts crossed by multiple THEMIS spacecraft between $X = -20$ and $-9 R_E$ [Runov et al., 2011] showed that the steep increase of $B_z$ at the fronts is preceded by a depletion with an amplitude of about a factor of three smaller than the increase and followed by a more gradual decrease of the field. A typical extent of
the entire structure along its propagation direction is from 2 to 3 $R_E$. The front thickness (i.e., the distance from the $B_z$ minimum to maximum) is between 500 and 1000 km, which is comparable to the gyroradius of thermal ions. The earthward propagation speed varies between 300 and 600 km/s. Dipolarization fronts are localized in the dawn-dusk direction. Studies based on comparisons between satellite and ground-based data [e.g., Angelopoulos et al., 1997] estimate the dawn-dusk extent to be 3–5 $R_E$, whereas the multipoint observation analysis gives an estimate of 2–3 $R_E$ [e.g., Nakamura et al., 2004]. Dipolarization fronts are also localized to the narrow region around the magnetic equator. With the use of Cluster measurements, Nakamura et al. [2004] estimated the north-south extent of the fronts to be no wider than 1.5–2 $R_E$.

[5] Dipolarization fronts are associated with dispersionless injections of hot tenuous plasmas [e.g., Runov et al., 2009]. Overall, macroscopic properties of dipolarization fronts are consistent with the concept of plasma bubbles, i.e., entropy depleted flux tubes which are propelled earthward due to interchange instability [e.g., Rosenbluth and Longmire, 1957; Pontius and Wolf, 1990; Chen and Wolf, 1993; Birn et al., 2011].

[6] A number of studies have been conducted to understand the mechanisms of plasma heating at dipolarization fronts at microscopic (i.e., individual particle) level as well as to investigate the sources of accelerated particles [e.g., Delcourt and Sauvaud, 1994; Sarris et al., 2002; Li et al., 2003; Zaharia et al., 2004; Elkington et al., 2005; Ashour-Abdalla et al., 2010; Zhou et al., 2011a]. In particular, Birn and Hesse [1994] and Birn et al., [1997, 2000, 2012] conducted a series of test-particle simulations of proton dynamics in three-dimensional time-dependent electric and magnetic fields of MHD simulations of dipolarizations triggered by magnetotail reconnection. Dipolarizations in the model have positive increase in $B_z$ similar to observed dipolarization fronts and therefore have strong positive $E_y$ due to their rapid earthward propagation. Protons were traced backward in time from selected locations in phase space till they either reach the simulation boundary or desired initial state. According to their results, some protons initially located in the dawn plasma sheet behind dipolarization site can intercept it and drift into its strong $E_y$, region from the dawn flank close to the magnetic equator. While inside the dipolarization region, protons exhibit gyration in the increased magnetic field and are accelerated by positive $E_y$ on the portion of their gyro-orbit along the $+y$ direction and deaccelerated on the portion of the orbit along the $−y$ direction. For some particles, the energy gain can significantly exceed the energy loss, and by the time they drift out of the dipolarization site, they can get accelerated by almost two orders of magnitude.

[7] In a series of publications, Zhou et al. [2010, 2011b] analyzed interaction of dipolarization fronts with protons initially located earthward of the fronts. Perturbations in the magnetic and electric fields due to the fronts were represented by one-dimensional analytical functions $B_z(x,t)$ and $E_y(x,t)$, which were superimposed on equilibrium plasma sheet from the generalized Harris [1962] model. Their test particle experiments successfully reproduced characteristic features of energetic ion precursors observed by THEMIS spacecraft prior to dipolarization fronts. The predominantly earthward direction of the energized ion flows was interpreted as the indication that particles are energized by specular reflection from the fronts, similar to particle reflection from quasi-perpendicular shocks [e.g., Terasawa, 1979; Gosling et al., 1982]. At the encounter with a front, ions are turned around due to the sharp and strong increase in $B_z$ behind the front. Ions are reflected earthward at $\sim 2u−v_y$, where $v_x$ is the $x$ component of particle velocity prior to the reflection and $u$ is the earthward velocity of the front. For instance, particles initially with $v_x \simeq −u$ can gain a factor of 8 increase in energy.

[8] Recently, Artemyev et al. [2012] considered implications of negative magnetic field depletions ahead of dipolarization fronts, i.e., when the depletions are strong enough to cause magnetic reconnection. In this case, resonant protons, whose earthward velocities are close to the velocity of the front, can be stably trapped at the reconnection point. For as long as particles are trapped, they are linearly accelerated in the $−y$ direction by the electric field associated with the front motion. This mechanism similar to the surfatron acceleration [e.g., Sagdeev, 1966; Katsouleas and Dawson, 1983] has been actively explored in space plasmas in the context of particle acceleration by shock waves [e.g., Takeuchi, 2005]. With the use of analytical estimates and two-dimensional test-particle simulations, Artemyev et al. [2012] showed that protons trapped at dipolarization fronts can be rapidly accelerated by more than two orders of magnitude.

[9] All dipolarization events between $X = −20$ and $−9 R_E$ analyzed by Runov et al. [2011] produced ion heating. At the same time, when computed in the minimal variance coordinate system, the median value of the magnetic field depletions ahead of the fronts drops down to small values but remains positive. It is therefore important to understand whether the magnetic field depletion ahead of the front can contribute to ion energization in the absence of stable trapping enabled by magnetic reconnection. Moreover, due to the coupling of their gyromotion and bounce motion, energetic (> keV) protons in magnetotail exhibit complex quasi-adiabatic motion even in the absence of any transient structures such as dipolarization fronts [e.g., Büchner and Zelenyi, 1989; Chen, 1992]. Protons initially bouncing at the equator get eventually scattered to small pitch angles, which can lead to untrapping of particles moving with the front and limit possible energization. Therefore, while two-dimensional models are useful for elucidating physical mechanisms of particle acceleration at dipolarization fronts, a quantitative analysis of ion energization requires computation of three-dimensional particle orbits in realistic magnetotail configurations.

[10] In this paper we conduct a systematic analysis of proton energization at dipolarization fronts for the case when protons are initially located earthward of the fronts. We start with the analysis of equatorially mirroring protons. In the following section, we derive and discuss analytical solution of particle dynamics in the presence of an arbitrary soliton-like dipolarization front. Section 3 describes our analytical model of dipolarization fronts, which is used in all subsequent numerical simulations. In section 4, we discuss idealized test-particle simulations of equatorially mirroring particles interacting with earthward moving fronts. Section 5 followed by conclusions describes the results of full three-dimensional simulations of rapid proton energization at dipolarization fronts.
2. Exact Solution for Equatorial Particles

[11] Everywhere in this paper we assume that the fronts are solitary electromagnetic waves [e.g., Sagdeev, 1966]. The formation of soliton-like dipolarization fronts in the process of magnetic reconnection onset in the magnetotail has recently been shown in particle-in-cell simulations [Simov et al., 2013]. This assumption implies that the front shape does not considerably change on the time scales of particle interaction with the fronts. It means that the only electric field associated with the front (in the laboratory frame) is produced by the front motion, \( E = -u \times B_1/c \), where \( B_1 \) is the magnetic field of the front, and \( u = \mathbf{u} \) is its velocity, which we assume to be directed to Earth. If we also assume that neither the ambient magnetic field \( (B_0) \) nor the field of the front \( (B_1) \) depend on \( y \), the direction perpendicular to the front propagation, then the total magnetic field acting on equatorially mirroring particles can be written as follows:

\[
B_r(x, t) = B_0 + B_1(x - ut),
\]

The equations of motion of an equatorially mirroring non-relativistic proton are

\[
\begin{align*}
\dot{\xi} &= \vec{v}_x = \vec{v}_y, \\
\dot{\vec{v}}_x &= \frac{e}{m c} (B_0 + B_1(x - ut)) \vec{v}_y, \\
\dot{\vec{v}}_y &= -\frac{e}{m c} \left[ B_0 \vec{u} + (B_0 + B_1(x - ut)) \vec{v}_x \right],
\end{align*}
\]

where \( e \) is the proton charge, \( m \) is the mass, and \( c \) is the speed of light. The electric field of the front contributes only to the second equation of the system (first term on the right hand side).

[12] After making a transformation into the reference frame moving with the front,

\[
\xi = x - ut, \quad \vec{v}_\xi = \vec{v}_x - u,
\]

the equations of proton motion transform to the following:

\[
\begin{align*}
\dot{\xi} &= \frac{e}{m c} (B_0 + B_1(\xi)) \vec{v}_y, \\
\dot{v}_y &= -\frac{e}{m c} \left[ B_0 \vec{u} + (B_0 + B_1(\xi)) \vec{v}_x \right],
\end{align*}
\]

In the frame of the moving front, the electric field of the front itself is zero, whereas the first term on the right hand side of the second equation in system (4) is due to a constant electric field produced by the Lorentz transformation of the background magnetic field.

[13] In the case of an arbitrary profile of the dipolarization \( B_1(\xi) \), the second equation of (4) can be integrated over \( t \). After inserting the result in the first equation of the system, it can be written as

\[
\frac{\xi}{t} = -\frac{\partial}{\partial \xi} U(\xi, \xi_0, t),
\]

where

\[
U(\xi, \xi_0, t) = \left( \frac{e^2}{2m} \right) (utB_0 F(\xi, \xi_0) + \frac{1}{2} F(\xi, \xi_0)^2),
\]

\[
F(\xi, \xi_0) = B_0(\xi - \xi_0) + A_1(\xi) - A_1(\xi_0).
\]

In the above expression, \( A_1(\xi) \) is the vector potential of the front: \( B_1 = dA_1/d\xi \).

[14] According to expressions (5)–(7), particle motion across the front is described by the Hamiltonian:

\[
H(\xi, \xi_0, t) = \frac{\xi^2}{2} + U(\xi, \xi_0, t)
\]

with the effective potential \( U \), which depends on time and particle initial position \( \xi_0 \).

[15] The derived expression is valid for any one-dimensional soliton-like front. However, the mechanisms of particle acceleration are different at the fronts with and without a negative \( B_z \) depletion. Before proceeding with a quantitative analysis of how the shape of the effective potential and particle acceleration differ in these two cases, let us elucidate key differences in particle dynamics. In the case when a front is preceded by a negative depletion in \( B_z \) (Figure 1a), the magnetic field goes through zero in two points across the front. In the vicinities of neutral (\( B_z = 0 \)) points, the last term in the second equation of (4) is negligible, and particles exhibit linear acceleration in the \(-y\) direction. The acceleration process is maintained as long as the particles stay at these points. Whether or not particles can be trapped at a neutral point is determined by the magnetic field derivative, \( dB_z/d\xi \), at this point. Particles are pushed out (by the \( \mathbf{v} \times \mathbf{B} \) force) from the neutral point closest to the front, preventing trapping, while being attracted to the second (magnetic reconnection) point, enabling trapping and constant acceleration in the \(-y\) direction.

[16] Stable trapping is not possible in the absence of magnetic reconnection (Figure 1b). Nonetheless, ahead of the front, the magnetic field is weak and the \(-uB_0\) term in the second equation of (4) dominates, which causes particle...
acceleration in the $-y$ direction. At the same time particles are pushed into the front by the increasing Lorentz force in the first equation. When particles penetrate behind the front where the $B_1$ is large, the magnetic term $B_1\xi$ becomes dominant in the second equation. Consequently, particles are turned around and are ejected back into the region of depleted magnetic field where they resume acceleration in the $-y$ direction. We refer to this acceleration regime as quasi-trapping.

### 3. Analytical Model of Dipolarization Fronts

[17] To understand the details of proton acceleration ahead of earthward moving dipolarization fronts, we set up controlled numerical experiments where the parameters of the background magnetic field and the electromagnetic field of the front can be adjusted to resemble the observed parameters of dipolarizations in the magnetotail. In the description of dipolarization fronts, we reproduce the following observational characteristics discussed in the introduction: (1) front width; (2) front thickness; (3) increase in $B$ preceded by a depletion; and (4) localization in the north-south direction. For this purpose we use a smooth function $X(\xi)$

$$X(\xi) = \frac{1}{\sqrt{\cosh(\xi L_x)}}.$$  (11)

with $L_x = 0.2 R_E$.

[19] To describe the dependence of the front shape on the direction of its propagation, we use a smooth function $X(\xi)$ composed of four pieces:

$$X = \begin{cases} 
B_1 (\cos(\alpha_1 (\xi - \xi_1)) - 1)/\alpha_1 & \text{if } \xi < \xi_1, \\
B_m (\cos(\alpha_m (\xi - \xi_m)) - 1)/\alpha_m & \text{if } \xi_1 \leq \xi < 0, \\
B_{m'} (\cos(\alpha_m (\xi - \xi_m)) - \sin(\alpha_m (\xi - \xi_m)))/\alpha_m & \text{if } 0 \leq \xi < \xi_2, \\
B_{m''} e^{-\alpha_m (\xi - \xi_m)} & \text{if } \xi \geq \xi_2.
\end{cases}$$  (12)

where at $\xi = 0$ the $z$ component of the front magnetic field goes through zero: $B_{1z} = X' = 0$. The maximum $B_{m'} > 0$ and the minimum $B_{m'} < 0$ field values are attained at $\xi_m < 0$ behind and $\xi_m > 0$ ahead of the front, which are related as $\xi_m = \xi_p B_p/B_m$ to ensure that $X'(0)$ is smooth. The reversed length scales are set as follows: $k_{m,\rho} = \pi/2|\xi_{m,\rho}|$.

$\xi_1 < \xi_{m,\rho}$ and $\xi_2 > \xi_{m,\rho}$ are chosen such that the derivatives $X'(\xi_{1,2})$ there are smooth:

$$\xi_{1,2} = \xi_{m,\rho} \mp \frac{1}{k_{m,\rho}} \arctan \frac{\alpha_{m,\rho}}{k_{m,\rho}}.$$  (13)

and equal to $B_{1,2} = \mp B_{m,\rho} \sin(\alpha_{m,\rho} \xi_{1,2})$. Constants $\alpha_{m,\rho}$ set the exponential decay of $B_{1,2}(\xi)$ behind and the recovery ahead of the front which determine the dipolarization extent along $\xi$.

[20] The numerical values of independent parameters, which were used in expressions (11)–(13) to define the front are listed in Table 1. To describe the ambient magnetic field of the magnetotail, we used the T96 magnetic field model [Tsyganenko, 1995, 2002] at $P_{\text{dyn}} = 4$ nPa, $B_z = 5$ nT, $B_r = 0$, $D_{st} = 0$, and zero tilt angle such that the $z = 0$ plane corresponds to the magnetic equator. The T96 model underestimates the magnetic field curvature in the tail, which is compensated by the parameter $\Delta X$ in expression (12). The radius of curvature is calculated as $\rho = 1/\kappa$, where $\kappa = b \cdot \nabla b$ and $b$ is the unit vector along the magnetic field. The $\Delta X$ value is chosen such that $\rho$ estimated at $x = -16 R_E$ ahead of the front is equal to 0.25 $R_E$ which falls into the statistical range of the current sheet curvature values 500–10,000 km as reported by Runov et al. [2005]. The contour plots of the magnetic field components of the described dipolarization front are shown in Figure 2.

### 4. 2-D Analysis

[21] Depending on the value of the ambient magnetic field, the $B_z$ depletion of a dipolarization front may or may not cause reconnection ahead of the front. Both cases are illustrated in Figure 3 showing the superposition of the front described in previous section with the T96 magnetic field at different locations in the tail. Figure 3a corresponds to the front at $x = -16 R_E$. The ambient magnetic field there is $B_0 = 3.35$ nT and the front is therefore preceded by a region of negative $B_z$ bounded by two neutral points. The bottom part of Figure 3a shows magnetic field lines in the meridional plane. The reconnecting field line is marked in red. Similarly, Figure 3b shows the front at $x = -12 R_E$ where the ambient magnetic field of $B_0 = 6.28$ nT exceeds the magnitude of

### Table 1. Parameters of the Dipolarization Front Model

| $B_{1z}$, nT | $B_{m'}$, nT | $\xi_{m,\rho}$, $R_E$ | $\alpha_{m,\rho}$ | $L_x$, $R_E$ | $\Delta X$, nT $\cdot R_E$ |
|--------------|--------------|----------------------|------------------|-------------|-------------------------|
| 20.0         | -6.0         | -0.1                 | 2.5              | 1.0         | 0.2                     |

Figure 2. Longitudinal distributions of the magnetic field components in our dipolarization front model.
Figure 3. Equatorial profiles of total magnetic field and field lines of model dipolarization front at two locations in the magnetotail where the ambient magnetic field is (a) weaker and (b) stronger than the negative depletion of the field associated with the front. The reconnecting line in Figure 3a is shown in red.

the negative dip ahead of the front. Consequently, the front in this case depletes the field but does not cause reconnection. Before conducting full three-dimensional simulations, we analyze proton dynamics in these two cases in two dimensions restricting our consideration to the equatorially mirroring particles.

[22] The effective potential of particle motion across the front (expressions (6) and (7)) computed at three different moments of time for both cases is shown in Figure 4. Figure 4a corresponds to the case with a negative $B_z$ ahead of the front. The potential in this case was calculated for the initial position $\xi_0 = 0.61 \, R_E$ corresponding to the reconnection point ahead of the front. Red vertical lines indicate neutral points. The potential in this case has a local maximum at the first neutral point and a local minimum at the reconnection point forming a potential well which narrows with time. It can therefore be expected that the protons can be stably trapped ahead of the front exhibiting oscillations in $\xi$ with diminishing amplitude.

[23] The effective potential in the case without a negative $B_z$ ahead of the front is illustrated in Figure 4b for the initial position $\xi_0 = 0.1 \, R_E$. The potential in this case does not have a local maximum at the front. Consequently, particles cannot be trapped ahead of the front. After penetrating behind the front, particles are turned around at the left wall of the potential and return to the region of depleted magnetic field ahead of the front. With time, the left wall of the effective potential shifts toward negative values of $\xi$, suggesting that with each oscillation in $\xi$, particles fall father behind the front. It has to be noted though that when particles separate sufficiently far from the front, their trajectory may no longer be bounded by the walls of the effective potential (6) derived in the reference frame moving with the front. Therefore, in the case of quasi-trapping, the time-dependent effective potential (6) provides a useful insight into particle motion only at initial stages when particles are confined to the front.

[24] Figure 5 shows phase portraits of proton motion for both cases computed by numerically integrating equation (4) with the earthward front speed of $u = 600 \, \text{km/s}$. Initial positions are marked with red dots. Particles had the initial energy of 2 keV. Proton energy along the orbits is marked with color. In the first case of a negative $B_z$ depletion ahead of the front (Figure 5a), the particle was launched earthward ($v_x \approx 619 \, \text{km/s}$) from the reconnection point $\xi_0 = 0.61 \, R_E$, i.e., very close to the resonance with the front. As was suggested in the above consideration of the effective potential functions, the particle was stably trapped ahead of the front. It was localized within $\Delta \xi \approx 0.02 \, R_E$ of the front and exhibited small oscillation in velocity $\Delta \xi \approx 70 \, \text{km/s}$. Over...
500 s of the simulation process particle energy increased up to almost $5 \times 10^4$ keV in accordance with linear acceleration in the $-y$ direction by the $-B_0 u_t$ term in the second equation of system (4) attributed to the Lorentz transformation of the ambient magnetic field. The unrealistically high energy gain is attributed to the idealized character of the simulation setup, i.e., an infinitely wide dipolarization front and a homogeneous background magnetic field.

As was mentioned in the introduction, the surfatron acceleration was extensively discussed in the context of ion acceleration at quasi-perpendicular shocks. In the case of kinetic shocks, the difference in electron and ion inertia results in an ambipolar electrostatic field across the shock front [e.g., Hoshino, 2001; Shapiro and Ücer, 2003]. The localized electrostatic potential well enables stable trapping of ions ahead of the front and subsequent shock surfing acceleration similar to the surfatron acceleration of stably trapped ions at dipolarization fronts preceded by negative $B_z$. However, unlike the acceleration at dipolarization fronts, which (under idealized conditions) is limited only by the system size, the maximum energy of ions in shock surfing is determined by the trapping limit defined by balance between the electric and the magnetic components of the Lorenz force across the front: $v_y,\text{max} \sim cE_x/B_z$.

The trajectory in Figure 5b is an example of quasi-trapping in the case without a negative depletion in $B_z$ ahead of the front. The particle was launched from $\xi_0 = 0.1 R_E$ in earthward direction, i.e., toward the front. Consequently, the particle penetrates behind the front where it is turned around by the strong $B_z$ and is accelerated by the electric field associated with the front motion. After the particle is ejected back into the region of weak magnetic field ahead of the front, it turns into $-y$ direction and is further accelerated by the electric field ahead of the front until it encounters the front. The particle does not overtake the front for the second time. It falls farther and farther behind the front with each gyration experiencing the diminishing influence of the front electric field $E_y = uB_1/c > 0$. The electric field amplitude on $-y$ leg of its gyro-orbit is larger than on the $+y$ leg. Consequently, by the time the particle is completely separated from the front, it looses more than 50% of the energy it acquired during quasi-trapping.

Quasi-trapping can be best illustrated by approximating the dipolarization front with two-step functions with $B_1(\xi < 0) = B_1 \gg B_0$ behind the front and $B_2(\xi > 0) = -B_2 < 0$ ahead of the front such that $B_2 \lesssim B_0$. Particle trajectory at the quasi-trapping stage is schematically illustrated in Figures 6a–6c in three different reference frames along with...
the electric and magnetic fields acting on the particle in these frames. Figure 6b corresponds to the reference frame moving with the front. The presence of constant electric field, \( E_y = -uB_0/c \), in this frame makes it problematic to apply the concept of specular reflection for estimating the change in particle velocity (energy) after its escape out of the front.

In the laboratory frame (Figure 6a), the electric field is constant both behind and ahead of the front. It is therefore possible to consider particle motion behind and ahead of the front separately in two different reference frames where the electric field is zero and particle orbits are circular (Figure 6c). Behind the front \( E = 0 \) in the frame, \( x' = x - u_E t \), where \( u_E = uB_1/(B_0+B_1) \) (Figure 6d). The front in this frame is moving in the \( +x' \) direction with velocity \( u - u_E \). Consequently, not all particles that encounter the front will be able to escape out of it. Indeed, proton gyration behind the front is given by: \( (x', y) = \rho(\sin \varphi', \cos \varphi') \), where \( \rho' \) is the Larmor radius. If a proton encounters the front at \( \varphi' \), then it will overtake it and escape out, only if the time it takes for the proton to complete its gyro-orbit, \( \Delta \varphi'/\omega' \), is less than the time it takes the front to pass the distance \( \Delta = \rho'(1 - \cos \varphi') \) (see Figure 6d). Thus, we obtain the condition for particle escape out of the front:

\[
\frac{\varphi'}{u} \left( 1 + \frac{B_0}{B_1} \right) > \frac{\varphi'}{1 - \cos \varphi'},
\]

where \( \varphi' = [(v_x - u_E)^2 + v_y^2]^{1/2} \) is the magnitude of particle velocity in the moving frame, and \( \varphi' = \arctan(v_y/(v_x - u_E)) + \pi/2. \)

Taking into account that the proton escape point will lie on its gyro-orbit somewhere between points 1 and 2 indicated in Figure 6d, which correspond to \( \varphi'' = 0 \) (the last point before the particle gets trapped) and \( \varphi'' = 2\pi - \varphi' \) (the limit when the front does not move), we obtain the following estimate for the change in proton energy at the escape point:

\[
\Delta K_1 = x \frac{mv^2}{2} \left( \frac{B_1}{B_1+B_0} \right)^2 \left( 1 - \frac{B_1+B_0 v_x}{B_1 u} \right),
\]

where the factor \( x \in (2,4) \). According to estimate (15), the gain in kinetic energy at the escape point is generally less than what is expected from specular reflection, which corresponds to the limit \( x = 4 \) and \( B_1 \gg B_0 \).

Ahead of the front \( E = 0 \) in the frame: \( x'' = x + u_E t \), where \( u_E = uB_2/(B_0-B_2) \). Since the front velocity \( u + u_E \) in this frame is in the \( +x'' \) direction and all protons ahead of the front are eventually turned in \( -x'' \) direction by the depleting magnetic field \( B_0-B_2 > 0 \), any proton initially ahead of the front will encounter it at some point of its gyro-orbit. At given values of energy and pitch angle, the exact location of the encounter point can be found only numerically. To get an estimate of a characteristic proton energy change, we assume the encounter to be at the point where the particle turns toward the front: \( v''_x = 0, v''_y = v'' \), where \( v'' \) is the magnitude of the initial velocity ahead of the front. As a result, the change in proton kinetic energy ahead of the front can be estimated as:

\[
\Delta K_2 \sim x m u^2 \left( \frac{B_2}{B_0-B_2} \right)^2 \left( 1 - \frac{B_0-B_2 v_y}{B_2 u} \right).
\]

After taking into account that \( B_0-B_2 \) is small, we obtain \( \Delta K_2 \sim m u^2 B_2/(B_0-B_2)^2 \gg m u^2/2 \).

According to the above estimates, proton energy gain ahead of the front can surpass the energy gain behind the front. Also, with the use of expression (14), it can be shown numerically that for realistic parameters of the front and initial energies, protons cannot exhibit more than two encounters with the front. After the second encounter, particles are overtaken by the front and lose most of the kinetic energy gained during quasi-trapping.

5. 3-D Analysis

To determine whether the additional complexity introduced by three-dimensional particle motion in the magnetotail modifies the properties of trapping and quasi-trapping acceleration of equatorially mirroring protons, we conducted two numerical simulations. In both simulations we used our analytical model of the front with the constant
trapped particles.

The upper limit of proton acceleration ahead of the fronts is determined by the magnetic field curvature. After protons leave the equatorial plane, they can pass through the front on the course of their gradient-bounce motion and either lose or pick-up additional energy, which causes the spread of the effective electric field values in Figure 7b as well as the existence of large spread in final energies of protons with close initial gyrophase values in Figure 7a.

Example trajectories of protons from both groups are shown in Figure 8. Proton initial conditions are marked with a purple dot. The proton which gets magnetized behind the front \( \mathbf{E} \times \mathbf{B} \) drifts earthward for about 2 \( R_E \) lagging behind the front until it gets scattered off the equator and resumes its quasi-adiabatic westward motion. The proton trapped ahead of the front is displaced by about 6 \( R_E \) in the \( -\gamma \) direction being accelerated by almost 100 keV. After it is scattered off the equator, its trajectory turns westward in the direction of its gradient-curvature drift and passes behind the front where the proton picks up additional energy due to its interaction with the positive \( E_y \) field behind the front.

In the second simulation, protons were launched at \( x_0 = -12 R_E \) where the ambient field is larger than the magnitude of the negative \( B_z \) depletion of the front, which was started at \( x = -12.1 R_E \). Figure 9 shows one trajectory from this simulation with the initial conditions same as in the two-dimensional simulation of the quasi-trapping shown in Figure 5b. Figure 9a shows a three-dimensional view of the particle trajectory in the laboratory reference frame, while Figure 9b shows the equatorial projection in the reference frame moving with the front. Particle kinetic energy (in the laboratory frame) is marked with color along the trajectories. This example is in good agreement with the simplified two-dimensional analysis of the previous section. During the first \( \sim 8 \) s, the proton was quasi-trapped at the front. After the encounter with the front, it was turned around by the strong \( B_z \) behind the front. In about 3 s, the proton was ejected out of the front being accelerated by \( \sim 10 \) keV. Over the next \( \sim 4 \) s, it was accelerated by the electric field ahead of the front in the \( -\gamma \) direction being simultaneously turned back toward the front by the depleted magnetic field. By the second encounter with the front, the proton gained additional \( \sim 40 \) keV. At this point, it was overtaken by the front. Over the following \( 20 \) s, the proton kept falling behind the front losing its kinetic energy due to the diminishing influence of the electric field associated with the front motion. By the time the proton was scattered off the equatorial plane its energy dropped down to about 20 keV and it was separated from the front by \( 1.5 R_E \) at which point the influence of the front electric field was negligible and particle energy was conserved through the end of the simulation. During most of the interaction process, the proton was magnetized behind the front due to its strong \( B_z \), which kept it localized to the equatorial plane. Therefore, despite the fact that the proton was eventually scattered off the equator in accordance with

earthward propagation speed of 600 km/s superimposed onto the T96 magnetic field model. Ensembles of \( 3 \times 10^4 \) protons evenly distributed in gyrophase between 0 and \( 2\pi \) were launched at two different radial distances in the midnight meridian of the equatorial plane (\( y_0 = z_0 = 0 \)) with the initial energy of 2 keV and 90° pitch angle.

[33] In the first simulation, protons were initiated at \( x_0 = -16 R_E \) and the front was launched from \( x = -16.61 R_E \), such that particles were initially located at the reconnection point ahead of the front, where according to our two-dimensional analysis they can be stably trapped. Figure 8a shows distribution of final proton energy computed as a function of the initial gyrophase (\( \psi_0 \)). There are two groups of particles in the distribution. Particles with the initial gyrophase around \( \pi \) (i.e., those initially moving in the direction opposite to the front) penetrate into the region of the increased magnetic field behind the front deeper than their gyroradius. These particles get magnetized behind the front and \( \mathbf{E} \times \mathbf{B} \) drift toward Earth. They fall behind the front due to the difference between the front velocity \( u \) and the velocity of their \( \mathbf{E} \times \mathbf{B} \) motion, \( u_B \approx u_B/(B_0 + B_1) \), until the electric field at their location diminishes such that the gradient-curvature drift becomes prevalent at which point they separate from the front. These particles exhibit no or weak acceleration.

[34] The second larger group of particles (63%) exhibit substantial acceleration by more than 40 keV. This acceleration is associated with trapping. To verify this, we computed the effective electric field accelerating particles in the \( -\gamma \) direction using the minimal values of particle displacement \( \Delta y \) (the span of the acceleration region along the front) and energy: \( E_y^\Delta = \Delta K/e\Delta y \). The distribution of the effective field values as a function of \( \Delta y \) is shown in Figure 7b. The average value \( \langle E_y^\Delta \rangle = -2.10 \) mV/m of the effective field is very close to \( E_y = -uB_0/c = 2.01 \) mV/m, the electric field in the frame moving with the front, which accelerates trapped particles.

[35] As follows from Figure 7b, protons energized by 40 keV and more are displaced in the \( -\gamma \) direction across the front by at least 3 \( R_E \). However, despite the fact that the front width in this simulation is not restricted, the displacement in \( -\gamma \) has the upper boundary of \( -6 R_E \). This is attributed to the fact that particle orbits in the vicinity of the equatorial plane become unstable when particle gyroradius exceeds the radius of the magnetic field curvature [e.g., Chen and Palmadesso, 1986]. After protons are accelerated to a certain energy, they are pitch-angle scattered off the equator and are quickly separated from the front. The larger is the magnetic field curvature, the faster particles are detrapped. Thus, for wide dipolarizations preceded by negative depletions in \( B_z \), the upper limit of proton acceleration ahead of the fronts is determined by the magnetic field curvature. After protons leave the equatorial plane, they can pass through the front on the course of their gradient-bounce motion and either lose or pick-up additional energy, which causes the spread of the effective electric field values in Figure 7b as well as the existence of large spread in final energies of protons with close initial gyrophase values in Figure 7a.
the quasi-adiabatic motion of energetic particles in the tail, it did not impact its interaction with the front.

[38] Figure 10 shows distributions of proton kinetic energy versus their initial gyrophase in the simulation at \( x_0 = -12 \, R_E \) computed at three different instances of the simulation process. Figure 10a corresponds to \( t = 4 \, s \) when particles that initially encounter the front are ejected to the depleted magnetic field region ahead of the front. Figure 10b shows the distribution at \( t = 9 \, s \) when the particles that were ejected out of the front encounter it for the second time. Particle distribution at the end of the simulation is shown in Figure 10c. It follows from the figure that the energization is most efficient for \( \psi_0 \sim 2\text{--}3 \), which corresponds to the particles that are accelerated both behind and ahead of the front (i.e., encounter the front twice) similar to the example shown in Figure 10. Particles that are first energized ahead of the front and then encounter it at an angle such that they are overtaken by the front are considerably less energized. In accordance with our two-dimensional analysis and the example shown in Figure 10, particle energy is the highest at the point when they encounter the front for the second time. From the comparison of distributions in Figure 10a and Figure 10b it follows that protons are energized by a factor of 2 more ahead of the front than behind the front. By the end of the simulation (Figure 10c) particles lose more than 50% of the energy acquired during quasi-trapping. As opposed to the distribution from the previous simulation of trapped particles shown in Figure 7a, there is no dispersion in the energy gain for particles with similar initial gyrophase. This is attributed to the fact that in the case of quasi-trapping, equatorial particles are not scattered off the equator during their interaction with the front. Particle trajectories leave the equatorial plane only after they are disengaged from the front and their energy is conserved.

Figure 9. An example of a quasi-trapped orbit from 3-D test-particle simulation of proton dynamics at \( x_0 = -12 \, R_E \) where the front does not have a negative \( B_z \) depletion. (a) A 3-D view of particle trajectory in the laboratory reference frame. (b) Orbit projection onto the equatorial plane in the reference frame moving with the front. Particle kinetic energy (as calculated in the laboratory frame) along the trajectories is marked with color.

Figure 10. The distribution of proton energy as function if their initial gyrophase computed at different times of the 3-D test-particle simulation of proton dynamics at \( x_0 = -12 \, R_E \) where the front does not have a negative \( B_z \) depletion.
6. Conclusions

This paper investigated acceleration of protons at the encounter of earthward propagating dipolarization fronts. We derived a general analytical solution for equatorially mirroring particles interacting with a soliton-like front. Particle motion in the direction parallel to the front in this case is described by a Hamiltonian function with a time-dependent effective potential. From the analysis of the potential function as well as simplified two-dimensional simulations, it follows that there are two different regimes of particle acceleration: trapping and quasi-trapping. Similar results were recently obtained by Takeuchi [2012] in the context of particle acceleration at interstellar plasma shocks and by Hoshino [2001] for magnetic solitary waves in astrophysical plasma.

In the case when the front is preceded by a negative depletion in $B_z$, particles can be stably trapped in the vicinity of the reconnection point. Trapped particles are accelerated in the $-y$ direction by the electric field produced by the Lorentz transformation of the background magnetic field into the reference frame moving with the front as was previously suggested by Artemyev et al. [2012].

In the absence of a negative $B_z$ depletion ahead of the front, particles can be quasi-trapped. Particles encounter and penetrate behind the front where they are accelerated by the electric field and turned around by strong magnetic field. Consequently, particles are ejected back into the region of depleted magnetic field where they are accelerated until they encounter the front for the second time. Quasi-trapped particles gain most of their energy ahead of the front.

To understand how these acceleration regimes operate in a realistic magnetotail geometry, we conducted three-dimensional test-particle simulations. The electromagnetic field of earthward moving fronts was described with a simple analytical model, which reproduces their key observational features. It was superimposed onto a constant ambient magnetic field of the T96 models. Ensembles of equatorially mirroring protons evenly distributed in the gyrophase were initiated ahead of the front. We conducted two series of simulations representative of particle acceleration at the fronts with and without a negative depletion in $B_z$. While the simulations show that protons can be rapidly accelerated by up to two orders of magnitude, understanding the global effect of dipolarization fronts on ion energization in the tail requires expanding our analysis onto the full pitch-angle and energy spectra, which we defer to future studies.

In the first simulation, test particles were initialized sufficiently far in the tail where the dipolarization front is preceded by a negative depletion in $B_z$ enabling stable trapping ahead of the front. The level of particle energization, in this case, depends on how long they can be trapped, i.e., stay in phase with the earthward motion of the front. Trapping time is limited by the width of the front and the curvature of the ambient field ahead of the front. The higher is the curvature, the faster particles get scattered out of the equatorial plane. In our simulations with a front unbounded in the dawn-dusk direction, trapped protons were accelerated by up to $\sim 100$ keV before being scattered off the equatorial plane due to high magnetic field curvature. In the realistic conditions, the maximum energy of accelerated protons will depend on the dawn-dusk extent of the front.

Closer to Earth where the background magnetic field is high enough to prevent reconnection ahead of the front, particles can be quasi-trapped. For the field parameters chosen in the simulations, protons were accelerated by $> 40$ keV. During most of their interaction with the front, particles stay magnetized behind the front which keeps them localized in the equatorial plane. Consequently, the acceleration is not limited by three-dimensional aspects of particle orbits in the tail as in the case of trapping. However, it was found that quasi-trapped particles cannot exhibit more than two encounters with the front, which sets the upper limit for particle acceleration. At the second encounter, protons are overtaken by the front and lose more than 50% of their energy gained during quasi-trapping.

It has to be noted that in this paper the dipolarization fronts were treated as MHD structures neglecting kinetic effects such as the differences in electron and ion motion, which produce electrostatic field across the fronts reported from in situ spacecraft measurements [e.g., Fu et al., 2012]. As was shown in the analysis of shock surfing acceleration [e.g., Hoshino, 2001; Shapiro and Ücer, 2003], such electrostatic fields can enable stable trapping of ion ahead of the fronts and enhance energization even in the absence of negative depletion in $B_z$. It is therefore important to address in subsequent publications what role the electrostatic fields play in ion acceleration at dipolarization fronts. It also has to be noted that dipolarization fronts are often accompanied by the magnetic field oscillations at frequencies close to local gyrofrequency of energetic ions which can contribute to local particle acceleration [e.g., Ono et al., 2009].

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References

Angelopoulos, V., W. Baumjohann, C. F. Kennel, F. V. Coroniti, M. G. Kivelson, R. Pellat, R. J. Walker, H. Luhr, and G. Paschmann (1992), Bursty bulk flows in the inner central plasma sheet, J. Geophys. Res., 97, 4027.

Angelopoulos, V., et al. (1997), Magnetotail flow bursts: Association to global magnetospheric circulation, relationship to ionospheric activity and direct evidence for localization, Geophys. Res. Lett., 24, 2271.

Artemyev, A. V., V. N. Lutsenko, and A. A. Petrukovich (2012), Ion resonance acceleration by dipolarization fronts: Analytic theory and spacecraft observation, Ann. Geophys., 30, 316, doi:10.5194/angeo-30-317-2012.

Ashour-Abdalla, M., M. El-Alaoui, M. L. Goldstein, M. Zhou, D. Schriver, R. Richard, R. Walker, M. G. Kivelson, and K.-J. Hwang (2010), Observations and simulations of non-local acceleration of electrons in magnetotail magnetic reconnection events, Nature Phys., 7, 360.

Baumjohann, W., G. Paschmann, and H. Lühr (1990), Characteristics of high-speed ion flows in the plasma sheet, J. Geophys. Res., 95, 3801.

Birn, J., and M. Hesse (1994), Particle acceleration in the dynamic magnetotail: Orbits in self-consistent three-dimensional MHD fields, J. Geophys. Res., 99, 109.

Birn, J., M. F. Thomsen, J. E. Borovsky, G. D. R. D. J. McComas, R. D. Bellan, and M. Hesse (1997), Substorm ion injections: Geosynchronous observations and test particle orbits in three-dimensional dynamic MHD fields, J. Geophys. Res., 102, 2325.

Birn, J., M. F. Thomsen, J. E. Borovsky, and G. D. Reeves (2000), Particle acceleration in the dynamic magnetotail, Physics of Plasmas, 7, 2149.

Birn, J., R. Nakamura, E. V. Panov, and M. Hesse (2011), Bursty bulk flows and dipolarization in MHD simulations of magnetotail reconnection, J. Geophys. Res., 116, A01210, doi:10.1029/2010JA016083.

Birn, J., A. V. Artemyev, D. N. Baker, M. Echim, M. Hoshino, and L. M. Zelenyi (2012), Particle acceleration in the magnetotail and aurora, Space Sci. Rev., 173, 49–102, doi:10.1007/s11214-012-9874-4.
Brandt, P. C., D. G. Mitchell, E. C. Roclof, and J. L. Burch (2001), Basilea Day storm: Global response of the terrestrial ring current, Solar Phys., 204, 377.

Blüchener, J., and L. M. Zelenyi (1989), Regular and chaotic charged particle motion in magnetotaillike field reversals: I. Basic theory of trapped motion, J. Geophys. Res., 94, 11,821.

Chen, C. X., and R. A. Wolf (1993), Interpretation of high-speed flows in the plasma sheet, J. Geophys. Res., 98, 21,409.

Chen, J. (1992), Nonlinear dynamics of charged particles in the magnetotail, J. Geophys. Res., 97, 15,011.

Chen, J., and P. J. Palmedasso (1986), Chaos and nonlinear dynamics of single particle orbits in a magnetotaillike magnetic field, J. Geophys. Res., 91, 14,499.

Delcourt, D. C., and J. A. Sauvaud (1994), Plasma sheet ion energization during dipolarization events, J. Geophys. Res., 99, 97.

Elkington, S. R., D. N. Baker, and M. Wiltberger (2005), Injection of energetic ions during the March 0630 substorm, in The Inner Magnetosphere: Physics and Modeling, edited by T. Pulkkinen et al., p. 565, American Geophysical Union, Washington, D.C.

Fairfield, D. H., et al. (1998), Geotail observations of substorm onset in the inner magnetotail, J. Geophys. Res., 103, 122.

Fairfield, D. H., M. B. T. Mukai, G. D. Reeves, S. Kokubun, G. K. Parks, T. Nagai, H. Matsumoto, K. Hashimoto, D. A. Gurnett, and T. Yamamoto (1999), Earthward flow bursts in the inner magnetotail and their relation to auroral brightenings, AKR intensifications, geosynchronous particle injection, and substorm onset, J. Geophys. Res., 104, 355.

Fu, H. S., Y. V. Khotyaintsev, A. Vaivads, M. André, and S. Y. Huang (2012), Electric structure of dipolarization front at sub-proton scale, Geophys. Res. Lett., 39, L06105, doi:10.1029/2012GL051724.

Gosling, J. T., M. F. Thomsen, S. J. Bame, W. C. Feldman, G. Paschmann, and N. Schopke (1982), Evidence for specularly reflected ions upstream from the quasi-parallel bow shock, Geophys. Res. Lett., 9, 1333.

Harris, E. G. (1962), On a plasma sheath separating regions of oppositely directed magnetic field, Nuovo Cimento, 113, 23.

Hoshino, M. (2001), Nonthermal particle acceleration in shock front, J. Geophys. Res., 106, 15,011.

Katsouleas, T., and J. Dawson (1983), Unlimited electron acceleration in directed magnetic field, Ann. Geophys., 11, 259.

Li, X., T. E. Sarris, D. N. Baker, and W. K. Peterson (2003), Observations and two-fluid simulations, Geotail and GOES coordinated study, plasma sheet flow on the geosynchronous magnetic configuration: I. Basic motion, Adv. Space. Res., 33, 747.

Nosé, M., H. Koshiishi, H. Matsumoto, P. C. Brandt, K. K. Koga, T. Goka, and T. Obara (2010), Magnetic field dipolarization in the deep inner magnetosphere and its role in development of O+-rich ring current, J. Geophys. Res., 115, A01204, doi:10.1029/2009JA015321.

Ono, Y., M. Nöse, S. P. Christon, and A. T. Y. Lui (2009), The role of magnetic field fluctuations in nonadiabatic acceleration of ions during dipolarization, J. Geophys. Res., 114, A05209, doi:10.1029/2009JA013918.

Pontius, D. H., and R. A. Wolf (1990), Transient flux tubes in the terrestrial magnetosphere, Geophys. Res. Lett., 17, 49.