Fermi Gamma-Ray Pulsars: Understanding the High-energy Emission from Dissipative Magnetospheres

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Abstract

Based on the Fermi observational data, we reveal meaningful constraints for the dependence of the macroscopic conductivity ($\sigma$) of dissipative pulsar magnetosphere models on the corresponding spin-down rate, $\dot{E}$. Our models are refinements of the FIDO (Force-free Inside, Dissipative Outside) models, which have dissipative regions that are restricted on the equatorial current sheet outside of the the light-cylinder. Taking into account the observed cutoff energies of all of the Fermi pulsars and assuming that (a) the corresponding $\gamma$-ray pulsed emission is due to curvature radiation at the radiation-reaction-limit regime, and (b) this emission is produced at the equatorial current sheet near the light cylinder, we show that the Fermi data provide clear indications about the corresponding accelerating electric-field components. A direct comparison between the Fermi cutoff energies and the model ones reveals that $\sigma$ increases with $\dot{E}$ for high $\dot{E}$-values, while it saturates for low ones. This comparison indicates also that the corresponding gap width increases toward low $\dot{E}$-values. Assuming the Goldreich–Julian flux for the emitting particles, we calculate the total $\gamma$-ray luminosity ($L_\gamma$). A comparison between the dependence of the Fermi $L_\gamma$-values and the model ones on $\dot{E}$ indicates an increase of the emitting particle multiplicity with $\dot{E}$. Our modeling, guided by the Fermi data alone, enhances our understanding of the physical mechanisms behind the high-energy emission in pulsar magnetospheres.

Key words: gamma rays: stars – pulsars: general – stars: neutron

1. Introduction

Pulsars are among the most powerful and robust electromagnetic machines in the Universe. They operate in extreme physical conditions, producing low-frequency electromagnetic (EM) waves (<3 kHz) and particle radiation that covers the entire EM spectrum. Their fuel (energy) comes from their huge rotational kinetic energy ($\sim 10^{38}–10^{39}$ ergs), and their enormous surface magnetic field ($B_s \sim 10^8$ and $10^{13}$ G) mediates the conversion of this energy into the observed particle radiation.

Fermi has played a catalytic role in the current modeling of the high-energy emission in pulsar magnetospheres. Since its launch in 2008, the number of detected $\gamma$-ray pulsars has increased by a factor of 30. Thus, now more than 200 $\gamma$-ray pulsars have been detected (117 of them are compiled in the second pulsar catalog (2PC); Abdo et al. 2013). This has shifted the study of $\gamma$-ray pulsars from discovery to astronomy by establishing a number of trends and correlations.

Even though the general principles that govern the pulsar “machine” have been known for decades, the detailed physical mechanisms that provide a complete interpretation of the observations remain unknown. The numerical force-free (FF) and magnetohydrodynamical solutions that appeared in the literature over the past 18 years for the aligned (2.5D) rotator (Contopoulos et al. 1999; Gruzinov 2005; Komissarov 2006; McKinney 2006; Timokhin 2006; Parfrey et al. 2012; Cao et al. 2016) and for the oblique (3D) rotators (Spitkovsky 2006; Kalapotharakos & Contopoulos 2009; Petri 2012a; Tchekhovskoy et al. 2013) provided the impetus for the exploration of the field structure and the properties of more realistic configurations (compared to the analytic vacuum-retarded-dipole solution (VRD); Deutsch 1955).

Although the FF models are probably good indicators of the magnetic field structure, they say nothing about the necessary accelerating electric-field components $E_{\text{acc}}$ which are by definition zero ($E_{\text{acc}} = 0$). Kalapotharakos et al. (2012b) and Li et al. (2012) initiated the exploration of the properties of dissipative solutions that cover the entire spectrum of solutions between the VRD and FF ones. In this approach, each adopted prescription for the current density incorporates a conductivity $\sigma$ that regulates the $E_{\text{acc}}$. The FF (VRD) solutions correspond to the $\sigma \rightarrow \infty$ ($\sigma \rightarrow 0$) regimes.

Kalapotharakos et al. (2012a) and Kalapotharakos et al. (2014, hereafter KHK) employed these dissipative magnetosphere models to generate model $\gamma$-ray light curves due to curvature radiation (CR). These studies revealed that the high-$\sigma$ (uniformly distributed) models place the emission at large distances near the equatorial current sheet (ECS), where the demand for current is high. Assuming that the radio emission originates near the stellar surface, KHK constrained their models using the observed dependence of the phase lags between the radio and $\gamma$-ray emission ($\delta$) on the $\gamma$-ray peak separation ($\Delta$). They found that a hybrid form of conductivity, specifically infinite conductivity interior to the light-cylinder (LC) and high-but-finite conductivity on the outside, provides a significant improvement in fitting the ($\delta - \Delta$)-data. In the so-called FIDO (FF Inside Dissipative Outside) models, the $\gamma$-ray emission is produced in regions near the ECS but is modulated by the local physical properties.

In Brambilla et al. (2015), we started an exploration of the spectral properties of the FIDO models. In our study, we used
FF geometry and approximate $E_{\text{acc}}$-values. We tried to find model parameters that fit eight bright pulsars with published phase-resolved spectra. The $\sigma$-values that best describe each of these pulsars showed an increase with the spin-down rate $\dot{E}$ and a decrease with the pulsar age.

In this paper, we demonstrate that the information needed to determine the $E_{\text{acc}}$-values (i.e., $\sigma$) is contained on the Fermi cutoff energies $\epsilon_{\text{cut}}$, and also reveals a dependence of $E_{\text{acc}}$ on $\dot{E}$. Moreover, we further specify the assumptions of the FIDO models. The comparison with the Fermi data exposes tight constraints on the $\sigma$-values, uncovering their dependence on $\dot{E}$. Finally, this comparison provides clear hints about the dependence of the corresponding gap widths and the multiplicity of the emitting particles on $\dot{E}$.

2. The FIDO Model Revisited

The FIDO model postulates that the magnetospheric plasma conductivity is finite only outside of the the LC. For solutions near the FF ones, the adopted approximated expressions used in KHK and Brambilla et al. (2015) produce significant $E_{\text{acc}}$-values only near the ECS. These studies also highlighted the necessity of low $\sigma$-values, even though the FF assumption implies only high $\sigma$. Nonetheless, we have found that the application of small $\sigma$-values everywhere outside of the the LC destroys the global FF-field structure (especially for low-inclination angles $\alpha$) the geometric properties of which are necessary for the successful reproduction of the $\epsilon \sim \Delta$ correlation. The only way to keep the field structure near the FF one is to apply the low-$\sigma$ in a narrow zone near the ECS outside of the LC (i.e., near the open field boundary). This actually implies that the conductivity is small in places where the requirement for current is high. However, this approach requires the detailed determination of the polar-cap rim at each time-step of the simulation, because the exact 3D locus of the ECS is not known a priori. Nonetheless, we have incorporated this into our code, which is now able to apply different $\sigma$-values (in the current density prescription shown in Equation (9) of KHK) along different magnetic field lines.

In Figure 1(a) we show schematically the dissipative region (i.e., finite-$\sigma$). In the light-orange region a finite-$\sigma$ has been applied, while all of the other regions are FF ($\sigma \rightarrow \infty$). Numerically, the FF condition is achieved by integrating Maxwell’s equations using a high-$\sigma$ ($\approx 10^2$; where $\Omega$ is the stellar angular-frequency) and nulling any remaining $E_{\text{acc}}$ (only inside of the FF region) at the end of each time-step; this ensures no parallel electric component ($E_B$) and $E < B$ (Spitkovsky 2006; Kalapotharakos & Contopoulos 2009). The dissipative region (finite-$\sigma$) is determined to be along the magnetic field lines (outside of the LC) that originate outside of a certain fraction $1 - w$ of the polar-cap rim radius (Figure 1(b)).

The above treatment ensures that the $E_{\text{acc}}$-values are consistent with the global solution. For low $\sigma$-values ($\sigma \lesssim 1\Omega$), $E_{\text{acc}}$ saturates locally to some $E_{\text{max}}$-value that depends on the assumed gap width (i.e., $w$). For high $\sigma$-values ($\sigma \gtrsim 10^2$), where the current density $J$ approaches the corresponding FF value, $E_{\text{acc}} \propto \sigma^{-1}$. The integration for high $\sigma$-values becomes cumbersome because of the stiff nature of the resistive term. Thus, for high $\sigma$-values ($\gtrsim 10^2$), we use the results for $\sigma_0 = 10^2$ and scale $E_{\text{acc}}$ according to

$$E_{\text{acc}} = E_{\text{acc}}^0 \frac{\sigma_0}{\sigma} \quad (\sigma > \sigma_0),$$  

where $E_{\text{acc}}^0$ corresponds to $\sigma_0$. We note that Equation (1) reproduces the correct $E_{\text{acc}}$ behavior for high $\sigma$ ($E_{\text{acc}} \rightarrow 0$ for $\sigma \rightarrow \infty$).

We use simulations with $w = 0.1$ (unless noted otherwise) that resolve the stellar radius $r_*$, and the LC radius $R_{\text{LC}}$ with 15 and 50 grid-points, respectively.

3. Guided by Fermi: Pulsar Cutoff Energies

2PC provides the total $\gamma$-ray luminosities ($L_\gamma$) and the phase-averaged $\epsilon_{\text{cut}}$ for most of the Fermi $\gamma$-ray pulsars. However, the $L_\gamma$-values depend on the assumed beaming-factor, $F_b$, and the estimated distances. The large spread in $L_\gamma$ with $\dot{E}$ indicates that other factors (i.e., $\alpha$-values, variability of $F_b$ with observer angle) play an important role in their determination. On the other hand, the range of $\epsilon_{\text{cut}}$ is more limited ($\sim 1$–6 GeV), does
pulsars, assuming CR at RRLR near the ECS at the LC. Fermi show the moving average values determination of the lines show the corresponding quadratic where \( m_{QC} \) is the radius of curvature, respectively. The \( \gamma_0 \) invariants (\( \alpha \) fundamentals), but also because it anticipates the Fermi because it is based entirely on simple/fundamental assumptions, but also because it anticipates the dependence of \( \sigma \) on \( E \).

4. Finding \( \sigma \)

Using the models described in Section 2, we integrate\(^6\) test particle trajectories assuming the Goldreich–Julian flux \( n_{GF} \) from the polar cap. Following an approach similar to what was used in KHK and Brambilla et al. (2015), we define particle trajectories considering that the velocity is everywhere determined by so-called Aristotelian electrodynamics (hereafter AE; Gruzinov 2012),

\[
v = \frac{E \times B + (eB_0 + E_0)}{B^2 + E_0^2},
\]

where the two signs correspond to the two different types of charge. We always choose the charge that is accelerated outwards. The quantities \( E_0 \) and \( B_0 \) are related to the Lorentz invariants (Gruzinov 2008; Li et al. 2012)

\[
E_0B_0 = E \cdot B, \quad E_0^2 - B_0^2 = E^2 - B^2.
\]

\( E_0 \) is the electric field in the frame, where \( E \) and \( B \) are parallel, and is the actual accelerating electric component which becomes zero only when \( E \cdot B = 0 \) and \( E < B \). Equation (4) describes accurately the asymptotic behavior of the particle velocities, and the corresponding trajectory determination is very close to the real one. Apparently, all of the velocities in AE are by definition equal to \( c \); i.e., the asymptotic value. This implies that Equation (4) can be used only for the determination of the particle shape and that no information about the particles’ dynamics/energetics can be derived by it. Thus, along each of these trajectories we compute \( \gamma_\ell \) by integrating Equation (2a), taking into account the local \( R_C \)-values that are calculated by the geometric shapes of the trajectories defined by Equation (4). The \( \gamma_\ell, R_C \)-values allow the derivation of the corresponding emission. By collecting all of the emitted photons, we can construct sky maps and compute spectra.

In our study, we have used a series of models for different combinations of \( \sigma \)-values, periods (\( P \)), stellar-surface magnetic-fields (\( B_\star \)), and \( \gamma_0 \)-values. The corresponding FF spin-down rate

\[
\eta_\gamma = \frac{3c}{2} \frac{\gamma_0^3}{R_C}. \tag{3}
\]

we can get an estimate of the corresponding \( \gamma_\ell \)-values. Applying this estimated value to Equations 2(a), (b) for \( \psi \approx c \), we get a final estimate of \( E_{\text{acc}} \). In Figure 2(b) we plot these \( E_{\text{acc}} \)-values (in the corresponding \( B_{1.0} \) units) versus \( E \) for all of the Fermi pulsars. The \( E_{\text{acc}} \) decreases for high \( E \) and saturates for low \( E \) around a value that is lower than \( B_{1.0} \).

The result depicted in Figure 2(b) is important not only because it is based entirely on Fermi data and simple/fundamental assumptions, but also because it anticipates the dependence of \( \sigma \) on \( E \).

\[
\eta_\gamma = \frac{3c}{2} \frac{\gamma_0^3}{R_C}. \tag{3}
\]

not suffer from geometry or distance uncertainties, and depends weakly only on the adopted-fit model.

In Figure 2(a), filled circles show the \( \epsilon_{\text{cut}} \) versus \( \dot{E} \) for both Fermi young pulsars (YP; green) and Fermi millisecond pulsars (MP; red). The open squares denote the moving averages values, and the solid lines are the corresponding log-log quadratic fits. We see that the \( \epsilon_{\text{cut}} \) of YPs increase with \( \dot{E} \) up to \( \sim 10^{36} \) erg s\(^{-1}\), and then they stabilize or even decrease. On the other hand, the \( \epsilon_{\text{cut}} \) of MPs shows a monotonic increase for the observed \( \dot{E} \)-values.

The Fermi \( \epsilon_{\text{cut}} \)-values provide a unique insight into the determination of the \( E_{\text{acc}} \), and through this for \( \sigma \). Assuming that the pulsar emission is due to CR at the radiation-reaction-limit regime (RRLR), we get

\[
\frac{d\gamma_\ell}{dt} = \frac{q_e E_{\text{acc}}}{m_e v^2} \gamma_\ell^4 - \frac{2q_e^2 \gamma_\ell^4}{3R_C^2 m_e c^3}
\]

\[
\hat{\gamma}_\ell = 0, \tag{2b}
\]

where \( m_e, q_e, c, \psi, \gamma_\ell, R_C \) are the electron mass, electron charge, speed of light, particle speed along \( E_{\text{acc}} \), Lorentz factor, and radius of curvature, respectively. The first term in Equation 2(a) describes the energy gain due to any \( E_{\text{acc}} \) that the particles encounter, while the second term describes the CR reaction losses. Assuming also that all of the radiative action is near the ECS close to the LC, we have an estimation for the \( R_C \approx R_{LC} \) (see KHK). Then, taking into account the Fermi \( \epsilon_{\text{cut}} \)-values (2PC) and the well-known expression

\[
\epsilon_{\text{cut}} = \frac{3c}{2} \frac{\gamma_0^3}{R_C}. \tag{3}
\]

\( \dot{E} \) is the electric field in the frame, where \( E \) and \( B \) are parallel, and is the actual accelerating electric component which becomes zero only when \( E \cdot B = 0 \) and \( E < B \). Equation (4) describes accurately the asymptotic behavior of the particle velocities, and the corresponding trajectory determination is very close to the real one. Apparently, all of the velocities in AE are by definition equal to \( c \); i.e., the asymptotic value. This implies that Equation (4) can be used only for the determination of the trajectory shape and that no information about the particles’ dynamics/energetics can be derived by it. Thus, along each of these trajectories we compute \( \gamma_\ell \) by integrating Equation 2(a), taking into account the local \( R_C \)-values that are calculated by the geometric shapes of the trajectories defined by Equation (4). The \( \gamma_\ell, R_C \)-values allow the derivation of the corresponding emission. By collecting all of the emitted photons, we can construct sky maps and compute spectra.

In our study, we have used a series of models for different combinations of \( \sigma \)-values, periods (\( P \)), stellar-surface magnetic-fields (\( B_\star \)), and \( \gamma_0 \)-values. The corresponding FF spin-down rate

\[
\eta_\gamma = \frac{3c}{2} \frac{\gamma_0^3}{R_C}. \tag{3}
\]
reads (Spitkovsky 2006)

\[
\dot{E} = \frac{4 \pi r_s^6 B_s^2}{c^2} (1 + \sin^2 \alpha),
\]

where \(r_s \approx 10^6\) cm is the stellar radius. Table 1 shows the \((P, B_s)\) combinations that produce the entire range of the observed \(\dot{E}\)-values for YPs and MPs.

Moreover, for each of these 30 \(\dot{E}_{\text{FP}}\)-values we have considered 4 conductivities \(\sigma = (1, 10, 10^2, 10^3) \Omega\) and 18 \(\alpha\)-values \((\alpha = 5^\circ, 10^\circ, 15^\circ, \ldots, 90^\circ, \text{every} 5^\circ)\). For \(\sigma = 10^3 \Omega\) and \(10^5 \Omega\) we use the simulation for \(\sigma = 10 \Omega\) and scale the accelerating electric field according to the relation (1). Thus, in total we have \(30 \times 4 \times 18 = 2160\) YP models and 2160 MP models.

We build the spectrum for each of these models, taking into account the emission from the entire magnetosphere (up to \(r = 2.5 R_C\)). The resulting spectral energy distributions are then fit with the model used in 2PC, namely

\[
\frac{dN}{d\epsilon} = A \epsilon^{-\Gamma} \exp\left(-\frac{\epsilon}{\epsilon_{\text{cut}}}\right),
\]

where \(\Gamma\) is the photon-index. For each \((B_s, P, \alpha)\) combination we compute \(\epsilon_{\text{cut}}\) for the considered four \(\sigma\)-values. A linear interpolation of these \((\log \sigma, \log \epsilon_{\text{cut}})\)-values is then used to find the optimum \(\sigma_{\text{opt}}\) value that reproduces the \(\epsilon_{\text{cut}}\) indicated by the fits shown in Figure 2(a) for the corresponding (through Equation (6)) \(\dot{E}\)-values.

In Figures 3(a), (b) we present these \(\sigma_{\text{opt}}\)-values for all the YP and MP models, respectively. The \(\sigma_{\text{opt}}\)-values of the points that are below the \(\sigma = \Omega\) gray-line have been determined by extrapolations of the linear-interpolations. We note that the dashed lines indicate the cumulative fraction of each Fermi pulsar group (YP, MP) that is observed below the corresponding \(\dot{E}\)-value. For each \(\dot{E}\)-value the \(\sigma_{\text{opt}}\) decreases with \(\alpha\), especially for high \(\alpha\)-values.

For YPs, \(\sigma_{\text{opt}} \propto \dot{E}\) for high \(\dot{E}\) and saturates toward lower \(\dot{E}\)-values. The \(\sigma_{\text{opt}}\) values below \(\dot{E} = \Omega\) imply the necessity of higher \(E_{\text{acc}}\)-values than those found in our models. Nonetheless, these \(\sigma\)-values are only slightly lower than \(\Omega\), while they appear close to the low end of the \(\dot{E}\)-values that Fermi observes in YPs. In our models, the \(E_{\text{acc}}\)-values do not depend only on the adopted \(\sigma\)-value, but also on the adopted gap width (i.e., \(w\)). In our modeling, \(w\) remains the same for all of the \(\dot{E}\)-values.

However, a smaller \(\sigma_{\text{opt}}\) indicates that the corresponding model struggles more to eliminate the \(E_{\text{acc}}\). This difficulty also implies wider gap widths (i.e., higher \(w\)-values). We tried a few models that have \(w = 0.2\) (instead of \(w = 0.1\)) and found an increase of \(\epsilon_{\text{cut}}\) that leads to an increase in the corresponding \(\sigma_{\text{opt}}\)-value by a factor of \(\sim 1.5\). This small increase is sufficient enough to restore most of the points that are below \(\dot{E}\) (Figure 3(a)) back to \(\sigma_{\text{opt}} > \Omega\).

MPs show similar behavior, even though they extend over a smaller \(\dot{E}\) range of rather low \(\dot{E}\)-values (Figure 3(b)). The rising part is less steep (\(\sigma \propto \dot{E}^{1/3}\)) than that of YPs for the high \(\dot{E}\)-values. As mentioned in the previous section, the \(\epsilon_{\text{cut}}\) of YPs stops increasing for \(\dot{E} \lesssim 10^{34} \text{ erg s}^{-1}\), while the \(\epsilon_{\text{cut}}\) of MPs seems to increase for all of the observed \(\dot{E}\)-values. Thus, a faster increase of \(\sigma_{\text{opt}}\) of YPs is required to reduce the corresponding \(E_{\text{acc}}\) more efficiently.

Similarly to YPs, a wider gap is implied for the low \(\dot{E}\)-values of MPs. Wider gaps mean larger emission domains in the magnetosphere, which is totally consistent with the observations (for many Fermi MPs and the low-\(\dot{E}\) Fermi YPs) that show wider \(\gamma\)-ray pulses and, in general, more complex \(\gamma\)-ray light curves (N. Renault-Tinacci et al. 2017, in preparation).

Our analysis also provides a possible explanation for why YPs and MPs are not observed for \(\dot{E} \lesssim 10^{34} \text{ erg s}^{-1}\) and \(\dot{E} \lesssim 10^{33} \text{ erg s}^{-1}\), respectively. We have already seen in Figure 2(a) that the \(\epsilon_{\text{cut}}\) of Fermi YPs and MPs decreases toward low \(\dot{E}\)-values. In Figure 3(c) we plot the \(\epsilon_{\text{cut}}\) versus \(\dot{E}\) for all of the models for \(\alpha = 45^\circ, \sigma = \Omega\), and \(w = 0.2\). Below some \(\dot{E}\) the corresponding \(\epsilon_{\text{cut}}\) becomes small, approaching the Fermi threshold (\(\sim 0.1 \text{ GeV}\)), which, in combination with lower luminosities, apparently makes their detection more difficult.

In the Appendix (see also Gruzinov 2013) we show that, assuming emission due to CR at RRLR (for the same \(\sigma, w\)), we get

\[
\epsilon_{\text{cut}} \propto \dot{E}^{3/8}.
\]

The slope 3/8 is followed very well by the model data points in Figure 3(c). We also show that the \(\epsilon_{\text{cut}}\)-values for pulsars of the same \(\dot{E}\), but of different \((B_s, P)\) combinations, read

\[
\epsilon_{\text{cut}} \propto B_s^{-1/8}.
\]
Applying the previous expression to Fermi MPs and YPs, taking into account that $B_{\text{MP}} \approx 10^{-4} B_{\text{YP}}$, we get that $\epsilon_{\text{cut,MP}} \approx 3 \epsilon_{\text{cut,YP}}$, which is followed exactly by the models (Figure 3(c)). This rule also explains why Fermi MPs have, on average, higher $\epsilon_{\text{cut}}$-values than YPs for the same $E_\gamma$. The actual average $\epsilon_{\text{cut}}$ ratios, as can be derived by the data shown in Figure 2(a), vary by a factor $\sim 1.5–2$ ($<3$) mainly because of the slightly different values of $\sigma$ and $w$.

Finally, for completeness, in Figures 4(a), (b) we present the model $L_\gamma$ versus $E_\gamma$ corresponding to the $\sigma_{\text{opt}}$-values ($w = 0.1$) for YPs and MPs, respectively. We note that, for the cases that $\sigma_{\text{opt}} < 1 \Omega$ (Figures 3(a), (b)), we plot the extrapolated $L_\gamma$-values of the linear interpolation of $(\log \sigma, \log L_\gamma)$ that we have for $\sigma > 1 \Omega$. Figure 4 shows that $L_\gamma$ decreases with $\alpha$ with the dependence on $\alpha$ becoming stronger at high $\alpha$-values mainly because of the lower particle-fluxes $n_B \propto B \cdot \Omega \propto \cos(\alpha)$, which implies higher relative multiplicities for higher $\alpha$.

In the previous section, we discussed the uncertainties of the observed Fermi $L_\gamma$-values. From the model perspective, the main uncertainty is the number of particles that accelerate at every point of the magnetosphere and contribute to the high-energy emission. Assuming a GJ flux for all of the models, we see that the intermediate and low $\alpha$ of both YPs and MPs are able to reproduce the observed $L_\gamma$-values at low-$E_\gamma$ even though they never reach close to the 100% efficiency (yellow dashed line). For higher $E_\gamma$, the model $L_\gamma$-values are lower than the observed ones with the effect being more prominent for the YPs. The Fermi $L_\gamma$-values of YPs increase slower with $E_\gamma$ at high $E_\gamma$, implying lower $\gamma$-ray efficiency. The YP models show a similar behavior, although their $L_\gamma$-values increase much more slowly and seem even to decrease at very high $E_\gamma$. This discrepancy can be reconciled by assuming an increased particle multiplicity for $E_\gamma \gtrsim 10^{38} \text{erg s}^{-1}$; such an assumption is in agreement with the increase in $\sigma$ (which is supposedly attributed to higher particle multiplicities) above this $E_\gamma$-value shown in Figure 3(a). We note also that the $L_\gamma$ inconsistency at the relatively higher $E_\gamma$-values for MPs (Figure 4(b)) appears milder because the corresponding $\sigma_{\text{opt}}$ increase is smaller (Figure 3(b)).

**Figure 3.** (a), (b) The YP and MP model $\sigma_{\text{opt}}$-values for $w = 0.1$ that reproduce the Fermi $\epsilon_{\text{cut}}$. The values below the gray lines have been derived by extrapolation (see the text for more details) and indicate larger $w$-values. The dashed lines show the cumulative fraction of the corresponding Fermi group (right vertical axes). (c) The model $\epsilon_{\text{cut}}$-values for the indicated $(\alpha, \sigma, w)$-values.

**Figure 4.** The model $L_\gamma$-values (color points) together with the corresponding Fermi values as indicated in the figure. The yellow dashed lines denote 100% efficiency. The comparison between models and observations indicates higher multiplicities for the emitting particles at high $E_\gamma$. 
5. Conclusions

In this paper, by expanding our previous studies, we interpret the Fermi pulsar γ-ray phenomenology within the framework of sophisticated dissipative pulsar magnetosphere models.

We refine our FIDO models by restricting the dissipative regions to near the ECS. For this, we run simulations that have magnetic-field-line-dependent conductivity. This approach allows the exploration of low σ-values and provides $E_{\text{acc}}$ that is consistent with the global structure. Although these solutions are dissipative, the corresponding field structure remains close to the FF one, which is necessary for the reproduction of the nice $\delta - \Delta$ correlation (2PC; KHK).

Moreover, based on very basic assumptions that the observed γ-ray emission

(a) is due to CR at RRLR, and

(b) is produced at the ECS, near the LC,

we show that the Fermi $\epsilon_{\text{cut}}$-values reveal the required $E_{\text{acc}}$-values (in $B_{\text{LC}}$ units) which decrease with $\E$, at high $\E$, while they stabilize at low $\E$, below $B_{\text{LC}}$.

Motivated by the previous result and taking into account the Fermi $\epsilon_{\text{cut}}$ variation with $\E$, we derive, for two series of models that cover the entire range of the observed $\E$ of YPs and MPs, the different $\sigma_{\text{opt}}$-values that reproduce the corresponding $\epsilon_{\text{cut}}$. We find that the $\sigma_{\text{opt}}$ increases with $\E$, at high $\E$. For the low $\E$ the models struggle to produce the observed $\epsilon_{\text{cut}}$-values, indicating the need for larger dissipative regions that can provide the slightly higher $E_{\text{acc}}$ needed in these cases.

The comparison between the model $L$, with the observed values becomes difficult because of the existing uncertainties in both the 2PC data (i.e., pulsar distances, unknown beaming factors) and the models (i.e., multiplicity of the emitting particles). However, by comparing the trends of the $L_{\gamma}$ dependence on $\E$, it becomes clear that relatively higher emitting particle multiplicities are needed for high $\E$ models and the very high $\sigma$-values.

We emphasize that the seemingly unbiased initial choice of the model parameters (i.e., same size of the dissipative region, same emitting particle multiplicity, independent of the $\E$, led to some problems, the solutions of which are consistent with the underlying theoretical view. Thus, the emerging necessities of larger dissipative regions toward low $\E$ and of higher emitting particle multiplicities toward high $\E$ are consistent with the lower (higher) $\sigma_{\text{opt}}$ at low (high) $\E$ and the associated lower (higher) pair production efficiency, respectively. We also note that even though our simple consideration of only two regimes of conductivity (finite $\sigma$ near the ECS and infinite everywhere else) is successful in interpreting the observations, in reality the situation is expected to be more complex. Thus, a possible generalization might be a gradual variation of the conductivity with the polar-cap radius (i.e., polar angle from the magnetic pole) and the spherical radius.

Guided by observations, our models provide a complete macroscopic picture with meaningful constraints that deepens our understanding about the pulsar γ-ray emission mechanisms and shows that CR can provide the observed Fermi pulsar emission, in contrast to models that advocate synchrotron emission at GeV energies (Pétrí 2012b; Cerutti et al. 2016). However, they are not self-consistent in the sense that they cannot provide unambiguous information about the microscopic properties of the magnetospheric plasma, such as pair creation and particle distribution function. This kind of study requires the use of kinetic particle-in-cell simulations (Chen & Beloborodov 2014; Philippov & Spitkovsky 2014; Belyaev 2015; Philippov et al. 2015; Cerutti et al. 2016) and are expected to reveal the dependence of the macroscopic parameters found in the present study on the microphysical processes of pulsar magnetospheres. We have started exploring this research path and we will present our results in forthcoming papers.

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Appendix

We assume emission at the LC, near the ECS, due to CR at RRLR for models of specific $\sigma$ and $\E$. Then $E_{\text{acc}} \propto B_{\text{LC}} \propto B_{\text{LC}} \propto B_{\text{LC}} \propto P$

$$E_{\text{acc}} \propto B_{\text{LC}} \propto B_{\text{LC}} \propto B_{\text{LC}} \propto P$$

and from $E \propto B_{\text{LC}}^2 P^{-4}$ (Equation (6))

$$E_{\text{acc}} \propto B_{\text{LC}}^2 P^{-4}$$

Equation (2) gives $\gamma_{1L} \propto E_{\text{acc}}^{1/4} R_{\text{LC}}^{1/2}$ and because $R_{\text{C}} \propto R_{\text{LC}} \propto P$

$$\gamma_{1L} \propto E_{\text{acc}}^{1/4} R_{\text{LC}}^{1/2}$$

and using Equation (11)

$$\gamma_{1L} \propto E_{\text{acc}}^{1/8} P^{-1/4}.$$

From Equation (3) we have also

$$\epsilon_{\text{cut}} \propto \gamma_{1L}^3 P^{-1}$$

and using Equation (13)

$$\epsilon_{\text{cut}} \propto \gamma_{1L}^{3/8} P^{-1/4}.$$

Taking into account the fact that for each pulsar group (YP, MP) the range of the observed $\E$-values is much broader than that of $P$-values, it becomes clear that for pulsars $\epsilon_{\text{cut}} \propto \E^{3/8}$, while the weak dependence on $P$ produces just a small spread of the $\epsilon_{\text{cut}}$-values.

Moreover, for pulsars of the same $\E$, $P \propto B_{\text{LC}}^{1/2}$ (Equation (6)) and thus, from Equations (10), (12), and (14)

$$\epsilon_{\text{cut}} \propto B_{\text{LC}}^{-1/8}.$$
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