Plane-symmetric inhomogeneous magnetized viscous fluid universe with a variable $\Lambda$

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Abstract
The behavior of magnetic field in plane symmetric inhomogeneous cosmological models for bulk viscous distribution is investigated. The coefficient of bulk viscosity is assumed to be a power function of mass density ($\xi = \xi_0 \rho^n$). The values of cosmological constant for these models are found to be small and positive which are supported by the results from recent supernovae Ia observations. Some physical and geometric aspects of the models are also discussed.

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1 Introduction
The standard Friedmann-Robertson-Walker (FRW) cosmological model prescribes a homogeneous and an isotropic distribution for its matter in the description of the present state of the universe. At the present state of evolution, the universe is spherically symmetric and the matter distribution in the universe is on the whole isotropic and homogeneous. But in early stages of evolution, it could have not had such a smoothed picture. Close to the big bang singularity, neither the assumption of spherical symmetry nor that of isotropy can be strictly valid. So we consider plane-symmetric, which is less restrictive than spherical symmetry and can provide an avenue to study inhomogeneities. Inhomogeneous cosmological models play an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts at the construction of such models have done by Tolman$^1$ and Bondi$^2$ who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub$^3$ $^4$ and later by Tomimura$^5$, Szekeres$^6$, Collins and Szafron$^7$, Szafron and Collins$^8$. Recently, Senovilla$^9$ obtained a new
class of exact solutions of Einstein’s equation without big bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere satisfying energy and causality conditions. Later, Ruis and Senovilla [10] have separated out a fairly large class of singularity free models through a comprehensive study of general cylindrically symmetric metric with separable function of $r$ and $t$ as metric coefficients. Dadhich et al. [11] have established a link between the FRW model and the singularity free family by deducing the latter through a natural and simple in-homogenization and anisotropization of the former. Recently, Patel et al. [12] presented a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid by assuming that the background space-time admits two space-like commuting killing vectors and has separable metric coefficients. Bali and Tyagi [13] obtained a plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electro-magnetic field. Recently, Pradhan et al. [14] have investigated a plane-symmetric inhomogeneous viscous fluid cosmological models with electro-magnetic field.

Models with a relic cosmological constant $\Lambda$ have received considerable attention recently among researchers for various reasons (see Refs. [15]−[19] and references therein). Some of the recent discussions on the cosmological constant “problem” and consequence on cosmology with a time-varying cosmological constant by Ratra and Peebles [20], Dolgov [21]−[23] and Sahni and Starobinsky [24] have pointed out that in the absence of any interaction with matter or radiation, the cosmological constant remains a “constant”. However, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying $\Lambda$ can be found. For these solutions, conservation of energy requires decrease in the energy density of the vacuum component to be compensated by a corresponding increase in the energy density of matter or radiation. Earlier researchers on this topic, are contained in Zeldovich [25], Weinberg [15] and Carroll, Press and Turner [26]. Recent observations by Perlmutter et al. [27] and Riess et al. [28] strongly favour a significant and positive value of $\Lambda$. Their finding arise from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \leq z \leq 0.83$ and these suggest Friedman models with negative pressure matter such as a cosmological constant ($\Lambda$), domain walls or cosmic strings (Vilenkin [29], Garnavich et al. [30]). Recently, Carmeli and Kuzmenko [31] have shown that the cosmological relativistic theory (Behar and Carmeli [32]) predicts the value for cosmological constant $\Lambda = 1.934 \times 10^{-35}\text{s}^{-2}$. This value of “$\Lambda$” is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al. [30]; Perlmutter et al. [27]; Riess et al. [28]; Schmidt et al. [31]). The main conclusion of these observations is that the expansion of the universe is accelerating.

Several ansatz have been proposed in which the $\Lambda$ term decays with time (see Refs. Gasperini [34], Berman [35], Freese et al. [14], Özer and Taha [19], Peebles and Ratra [37], Chen and Hu [38], Abdussattar and Viswakarma [39],

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Gariel and Le Denmat [10], Pradhan et al. [11]. Of the special interest is the ansatz \( \Lambda \propto S^{-2} \) (where \( S \) is the scale factor of the Robertson-Walker metric) by Chen and Wu [38], which has been considered/modified by several authors (Abdel-Rahaman [42], Carvalho et al. [19], Waga [43], Silveira and Waga [44], Vishwakarma [45]).

Most cosmological models assume that the matter in the universe can be described by ‘dust’ (a pressure-less distribution) or at best a perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of expanding universe [46]–[48]. It has been shown that bulk viscosity leads to inflationary like solution [49], and acts like a negative energy field in an expanding universe [50]. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the Grand Unification Theories (GUT) phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [51] for a review on cosmological models with bulk viscosity). A number of authors have discussed cosmological solutions with bulk viscosity in various context [51]–[54].

The occurrence of magnetic fields on galactic scale is a well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out Zeldovich et al. [55]. Also Harrison [56] has suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [57]. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors [58]–[67]. Strong magnetic fields can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic fields give rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than if the pressure was isotropic [68] [69]. Such fields can be generated at the end of an inflationary epoch [70]–[74]. Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. The model studied by Murphy [88] possessed an interesting feature is that the big bang type of singularity of infinite spacetime curvature does not occur to a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. Several authors [75]–[81] have investigated cosmological models with a magnetic field in different context.

Recently Bali et al. [82] obtained some plane-symmetric inhomogeneous cosmological models for a perfect fluid distribution with electro-magnetic field. Motivated the situations discussed above, in this paper, we shall focus upon the
problem of establishing a formalism for studying the general relativistic evolution magnetic inhomogeneities in presence of bulk viscous in an expanding universe. We do this by extending the work of Bali et al.\[82\] by including an electrically neutral bulk viscous fluid as the source of matter in the energy-momentum tensor. This paper is organized as follows. The metric and the field equations are presented in section 2. In section 3 we deal with the solution of the field equations in presence of bulk viscous fluid. The sections 3.1, 3.2 and 3.3 contain the two different cases (i.e. for $n = 0$ and $n = 1$ ) and also contain some physical aspects of these models respectively. Section 4 describe some other generated models and their physical and geometric properties. Finally in section 5 concluding remarks have been given.

2 The metric and field equations

We consider the metric in the form

$$ds^2 = B^2(dx^2 - dt^2 + dy^2) + C^2dz^2,$$  

(1)

where the metric potential $B$ and $C$ are functions of $x$ and $t$.

The energy momentum tensor in the presence of bulk stress has the form

$$T^{ij} = (\rho + \bar{p})v^iv^j + \bar{p}g^{ij} + E^{ij},$$  

(2)

where $E^{ij}$ is the electro-magnetic field given by

$$E^{ij} = F_{i\alpha}F^{j\alpha} - \frac{1}{4}F^{\alpha\beta}F^{i\alpha\beta}g^{ij},$$  

(3)

and

$$\bar{p} = p - \xi v^i.$$  

(4)

Here $\rho, p, \bar{p}, F^{ij}$ and $\xi$ are the energy density, isotropic pressure, effective pressure, electromagnetic field tensor and bulk viscous coefficient respectively and $v^i$ is the flow vector satisfying the relation

$$g_{ij}v^iv^j = -1.$$  

(5)

Here the semicolon represents a covariant differentiation. The coordinates are considered to be comoving so that $v^1 = 0 = v^2 = v^3$ and $v^4 = 1\cB$. We consider the current to be flowing along the z-axis so that $F_{12}$ is the only non-vanishing component of $F_{ij}$.

The Einstein’s field equations ( in gravitational units $c = 1, G = 1$ ) read as

$$R^{ij} - \frac{1}{2}Rg^{ij} + \Lambda g^{ij} = -8\pi T^{ij},$$  

(6)

for the line element (1) has been set up as

$$8\pi B^2\left(\bar{p} + \frac{F_{12}^2}{2B^4}\right) = -\frac{B_{14}^4}{B} + \frac{B_{2}^2}{B^2} + \frac{B_{1}^2}{B^2} - \frac{C_{44}}{C} + \frac{2B_{1}C_{1}}{BC} - \Lambda B^2,$$  

(7)
\[8\pi B^2 \left( \bar{p} + \frac{F_{12}^2}{2B^4} \right) = -\frac{B_{44}}{B} + \frac{B_1^2}{B^2} + \frac{B_4}{B} - \frac{B_1^2}{B^2} - \frac{C_{11}}{C} + \Lambda B^2, \] 
\[8\pi B^2 \left( \bar{p} - \frac{F_{12}^2}{2B^4} \right) = -\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{2B_{11}}{B} - \frac{B_1^2}{B^2} - \Lambda B^2, \]
\[8\pi B^2 \left( \rho + \frac{F_{12}^2}{2B^4} \right) = -\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{B_1^2}{B^2} + \frac{2B_4}{B^2} + \frac{2B_4 C_4}{BC} + \Lambda B^2, \]
\[0 = \frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{2B_1 B_4}{B^2} - \frac{B_1 C_4}{BC} - \frac{B_4 C_4}{BC}, \]
where
\[\bar{p} = p - \xi \left( \frac{2B_4}{B} + \frac{C_4}{C} \right).\]

Here and in the following expressions the suffixes 1 and 4 at the symbols \(B\), \(C\), \(f\) and \(g\) denote differentiation with respect to \(x\) and \(t\) respectively.

### 3 Solution of the field equations

From Equations (7), (8) and (9), we have
\[\frac{B_{11}}{B} + \frac{C_{11}}{C} - \frac{2B_1 C_1}{BC} - \frac{2B_1^2}{B^2} = 0, \] 
\[8\pi F_{12}^2 = \frac{B_{44}}{B} - \frac{B_{11}}{B} + \frac{C_{11}}{C} - \frac{C_{44}}{C}. \]

Equations (7)-(11) represent a system of five equations in seven unknowns \(B\), \(C\), \(\rho\), \(p\), \(F_{12}\), \(\Lambda\) and \(\xi\). The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations. To get a determinate solution, we need two extra conditions.

Let us consider that
\[B = f(x)g(t),\]
\[C = h(x)k(t). \]

Using Equation (14) in Equation (11), we obtain
\[\frac{f_1}{f} = \frac{k_4}{k} - \frac{g_4}{g} = K \ (constant, \ say), \]
which leads to
\[\frac{f_1}{f} = K \frac{h_1}{h}, \]
and
\[\frac{k_4}{k} - \frac{g_4}{g} = K \left( \frac{k_4}{k} + \frac{g_4}{g} \right). \]
Using Equation (14) in Equation (12) leads to

\[
\frac{f_{11}}{f} - \frac{2f_1}{f} \left( \frac{h_1}{h} + \frac{f_1}{f} \right) + \frac{h_{11}}{h} = 0.
\]  

(18)

Equations (10) and (18) give

\[(K + 1)hh_{11} - (3K + K^2)h_1^2 = 0,
\]  

(19)

which on integration leads to

\[
h = \left[ \frac{K + 1}{(1 - 2K - K^2)(\alpha x + \beta)} \right]^{(K+1)/(K+2K^2-1)},
\]  

(20)

where \(\alpha\) and \(\beta\) are constants of integration. Integrating Equations (10) and (17), we obtain

\[
f = ah^K,
\]  

(21)

and

\[
k = \left( \frac{g}{b} \right)^{(1+K)/(1-K)}.
\]  

(22)

respectively, where \(a\) and \(b\) are constants of integration. Hence from Equations (14), (20)-(22), we obtain

\[
B = ag \left[ \frac{K + 1}{(1 - 2K - K^2)(\alpha x + \beta)} \right]^{K(K+1)/(K+2K^2-1)}
\]  

(23)

and

\[
C = \left( \frac{g}{b} \right)^{(1+K)/(1-K)} \left[ \frac{K + 1}{(1 - 2K - K^2)(\alpha x + \beta)} \right]^{(K+1)/(K+2K^2-1)}.
\]  

(24)

After suitable transformation of coordinates and by taking \(\alpha\) as unity without any loss of generality the metric (11) reduces to the form

\[
ds^2 = M^2 \left( \frac{1}{X^2} \right)^{K(K+1)/(K+2K^2-1)} g^2(T) \left( dX^2 - dT^2 + dY^2 \right) +
\]

\[
\left( \frac{1}{X^2} \right)^{(K+1)/(K+2K^2-1)} \left\{ g^2(T) \right\}^{\frac{4+K}{4K}} dZ^2
\]  

(25)

where

\[
M = a \left[ \frac{K + 1}{(1 - 2K - K^2)} \right]^{K(K+1)/(K+2K^2-1)}.
\]  

(26)

There is a lot of known solutions to the Einstein field equations but Equation (25) is indeed a new one. The effective pressure \(\bar{p}\) and density \(\rho\) for the model (25) are given by

\[
8\pi\bar{p} = -\Lambda + \frac{1}{g^2M^2X^{(1-2K-K^2)/2}}
\]
\[
8\pi \rho = \Lambda + \frac{1}{g^2 M^2 X^{\frac{2K(K+1)}{(1-2K-K^2)}}} \times \left[ \frac{2K^2(K+1)(K+2)}{(1-2K-K^2)^2 X^2} - \frac{(2-K) g_{44}}{(1-K) g} + \frac{(1-3K) g_4^2}{(1-K)^2 g^2} \right],
\]
(27)
\[
8\pi\rho = \Lambda + \frac{1}{g^2 M^2 X^{\frac{2K(K+1)}{(1-2K-K^2)}}} \times \left[ \frac{2K(K+1)(K+2)}{(1-2K-K^2)^2 X^2} - \frac{g_{44}}{(1-K) g} + \frac{(4-5K-K^2) g_4^2}{(1-K)^2 g^2} \right].
\]
(28)

For the specification of \( \xi \), we assume that the fluid obeys an equation of state of the form
\[
p = \gamma \rho,
\]
(29)

where \( 0 \leq \gamma \leq 1 \) is a constant.

Thus, given \( \xi(t) \) we can solve for the cosmological parameters. In most of the investigations involving bulk viscosity is assumed to be a simple power function of the energy density\[83\,–\,85\].

\[
\xi(t) = \xi_0 \rho^n,
\]
(30)

where \( \xi_0 \) and \( n \) are constants. If \( n = 1 \), Equation (26) may correspond to a radiative fluid\[86\]. However, more realistic models\[87\] are based on \( n \) lying in the regime \( 0 < n < \frac{1}{2} \).

On using (30) in (27), we obtain
\[
8\pi \rho - \xi_0 \rho^n \theta = -\Lambda + \frac{1}{g^2 M^2 X^{\frac{2K(K+1)}{(1-2K-K^2)}}} \times \left[ \frac{2K^2(K+1)(K+2)}{(1-2K-K^2)^2 X^2} - \frac{(2-K) g_{44}}{(1-K) g} + \frac{(1-3K) g_4^2}{(1-K)^2 g^2} \right],
\]
(31)

where \( \theta \) is the scalar of expansion calculated for the flow vector \( \nu^i \) and is given by
\[
\theta = \frac{g_4}{g^2 \sqrt{M X^{\frac{K(K+1)}{(1-2K-K^2)}}}} \left( \frac{K-3}{K-1} \right).
\]
(32)

3.1 Model I: \( (\xi = \xi_0) \)

When \( n = 0 \), Equation (30) reduces to \( \xi = \xi_0 \). With the use of Equations (28), (29) and (31), Equation (31) reduces to
\[
8\pi(1+\gamma)\rho = \frac{8\pi \xi_0 g_4}{g^2 \sqrt{M X^{\frac{K(K+1)}{(1-2K-K^2)}}}} \left( \frac{K-3}{K-1} \right) + \frac{1}{g^2 M^2 X^{\frac{2K(K^2-1)(K+2)}{(1-2K-K^2)^2 X^2}}} \left[ \frac{2K(K^2-1)(K+2)}{(1-2K-K^2)^2 X^2} - \frac{(3-K) g_{44}}{(1-K) g} + \frac{(5-8K-K^2) g_4^2}{(1-K)^2 g^2} \right],
\]
(33)
Eliminating $\rho(t)$ between (28) and (33), we get

$$(1 + \gamma)\Lambda = \frac{8\pi\xi_0g_4}{g^2\sqrt{MX}^{\frac{K(K+1)}{2K(K+2)}}\left(\frac{K - 3}{K - 1}\right) + \frac{1}{g^2M^2X^{\frac{2K(K+1)}{2K(K+2)}}}} \times$$

$$\left[\frac{2K(K + 1)(K + 2)(K + \gamma)}{(1 - 2K - K^2)^2X^2} - \frac{(2 - K - \gamma)g_{44}}{g} + \frac{(1 - 3K - (4 - 5K + K^2)\gamma)g_4^2}{g^2}\right].$$

(34)

3.2 Model II: ($\xi = \xi_0\rho$)

When $n = 1$, Equation (30) reduces to $\xi = \xi_0\rho$. With the use of (28), (29) and (32), Equation (31) reduces to

$$8\pi \left[1 + \gamma - \frac{\xi_0g_4}{g^2\sqrt{MX}^{\frac{K(K+1)}{2K(K+2)}}\left(\frac{K - 3}{K - 1}\right)}\right] \rho =$$

$$\frac{1}{g^2M^2X^{\frac{2K(K+1)}{2K(K+2)}}} \left[\frac{2K(K^2 - 1)(K + 2)}{(1 - 2K - K^2)^2X^2} \frac{(3 - K)g_{44}}{g} + \frac{(5 - 8K - K^2)g_4^2}{g^2}\right].$$

(35)

Eliminating $\rho(t)$ between (28) and (35), we get

$$\left[1 + \gamma - \frac{\xi_0g_4}{g^2\sqrt{MX}^{\frac{K(K+1)}{2K(K+2)}}\left(\frac{K - 3}{K - 1}\right)}\right] \Lambda = \frac{1}{g^2M^2X^{\frac{2K(K+1)}{2K(K+2)}}} \times$$

$$\left[\frac{2K(K + 1)(K + 2)(K + \gamma)}{(1 - 2K - K^2)^2X^2} - \frac{1 - \gamma(2 - K)g_{44}}{g} + \frac{(4 - 5K - K^2) - (1 - 3K)\gamma g_4^2}{g^2}\right]$$

$$+ \frac{\xi_0g_4}{g^4M^2X^{\frac{3K(K+1)}{2K(K+2)}}\left(\frac{K - 3}{K - 1}\right)} \times$$

$$\left[\frac{2K(K + 1)(K + 2)}{(1 - 2K - K^2)^2X^2} - \frac{(2 - K)g_{44}}{g} + \frac{(1 - 3K)g_4^2}{(1 - K)^2 g^2}\right].$$

(36)

In spite of homogeneity at large scale our universe is inhomogeneous at small scales, so physical quantities having position dependent are more natural in our observable universe if we do not go to super high scale. This result shows this kind of physical importance. In recent times the $\Lambda$-term has interested theoreticians and observers for various reasons. The nontrivial role of the vacuum in the early universe generate a $\Lambda$-term that leads to inflationary phase. Observationally this term provides an additional parameter to accommodate conflicting data on the values of the Hubble constant, the deceleration parameter, the density parameter and the age of the universe (for example, see the references [39, 31]). Assuming that $\Lambda$ owes its origin to vacuum interactions, as suggested in particular by Sakharov [91] it follows that it would in general be a function
of space and time coordinates, rather than a strict constant. In a homogeneous universe $\Lambda$ will be at most time dependent \[92\]. In our case this approach can generate $\Lambda$ that varies both with space and time. In considering the nature of local massive objects, however, the space dependence of $\Lambda$ cannot be ignored. For details discussion, the readers are advised to see the references (Narlikar, Pecker and Vigier \[93\], Ray et al. \[94\]).

The effect of bulk viscosity is to introduce a change in the perfect fluid model. We also observe here that the condition of Murphy \[88\] about the absence of a big bang type of singularity in the finite past in models with bulk viscous fluid is, in general, not true. We have freedom of choosing the function $g(T)$ so that to give a physical behaviour of above parameters. As a matter of fact, there are multiple choices, for example, $g(T) = c^2 + d^2 t^2, c^2 + e^{-d} t^2, c^2 + d^2 \cos \omega t, c^2 > d^2$, where $c$ and $d$ are some real constants. From Equations \[34\] and \[36\], we observe that the cosmological constant is a decreasing function of time and it approaches a small positive value at late times under some suitable conditions which explains the small value of $\Lambda$ at present.

### 3.3 Some physical aspects of the models

We shall now give the expressions for kinematical quantities and the components of conformal curvature tensor. With regard to the kinematical properties of the velocity vector $v^i$ in the metric \[25\], a straightforward calculation leads to the following expression for the shear of the fluid:

$$\sigma^2 = \frac{4K^2 g_4^2}{3\sqrt{M} g^4 (1 - K)^2 X^2 \left(1 - 2K - K^2\right)}.$$  \( (37) \)

The rotation $\omega$ and acceleration are identically zero. The expansion scalar $\theta$ has already been given by \[32\]. The non-vanishing physical components of the conformal curvature tensor are given by

$$C_{(1212)} = -C_{(3434)} = \frac{1}{6M g^2 X^2 \left(1 - 2K - K^2\right)} \times$$

$$\left[ \begin{array}{c} 2K(K + 1)(K - 1) \\ (1 - 2K - K^2)^2 X^2 \end{array} \right] - \frac{4K}{(1 - K)} g_{44} + \frac{4K(1 - 3K)}{(1 - K)^2} g_4^2,$$  \( (38) \)

$$C_{(1313)} = -C_{(2424)} = \frac{1}{6M g^2 X^2 \left(1 - 2K - K^2\right)} \times$$

$$\left[ \begin{array}{c} 4K(K + 1)(1 - K) \\ (1 - 2K - K^2)^2 X^2 \end{array} \right] + \frac{2K}{(1 - K)} g_{44} + \frac{2K(3K - 1)}{(1 - K)^2} g_4^2,$$  \( (39) \)

$$C_{(1224)} = -C_{(1334)} = \frac{K(K + 1)}{M(1 - 2K - K^2) X^2 \left(1 - 2K - K^2\right)} g_4.$$  \( (40) \)
\[ C_{(2323)} = -C_{(1414)} = \frac{1}{3Mg^2X^{\frac{2K(K+1)}{1-2K-K^2}}} \left( \frac{K(K+1)^2(1-K)}{(1-2K-K^2)^2X^2} - \frac{2K}{(1-K)g^2} \right). \]  

(41)

The non-vanishing component \( F_{12} \) of the electromagnetic field tensor and \( J^2 \), the component of charge current density, are given by

\[ F_{12} = \frac{1}{2\sqrt{\pi}} \left[ \frac{K}{(1-K)g} - \frac{K(1+K)g_4^2}{(1-K)^2g^2} - \frac{K(K+1)(K-1)(K+2)}{(1-2K-K^2)X^2} \right]^{\frac{3}{2}} B, \]

(42)

\[ J^2 = \frac{1}{M^2g^4X^{\frac{(1+K)(1+3K)}{(1-2K-K^2)}}} \times \left[ \sqrt{\zeta(1-K^2)}X^{\frac{2K}{1-2K-K^2}} + \frac{2K(K+1)(K-1)(K+2)\sqrt{\zeta}}{(1-2K-K^2)^2X^2} \right], \]

(43)

where

\[ \zeta = \left[ -\frac{2K}{(1-K)g} - \frac{2K(1+K)g_4^2}{(1-K)^2g^2} - \frac{2K(K+1)(K-1)(K+2)}{(1-2K-K^2)^2X^2} \right]. \]

(44)

The models represent shearing, non-rotating and Petrov type I non-degenerate in general, in which the flow is geodetic. The expansion in the model stops when \( K = 3 \) but it will continue indefinitely when \( K > 3 \). Since \( \lim_{T \to 0} \frac{\Theta}{\hat{\Theta}} \neq 0 \), hence the models do not approach isotropy for large values of \( T \).

### 4 Other generated model

In the metric (25), the function \( g(T) \) is indeterminate. To get the determinate value of \( g \), we assume in Equation (15) as

\[ \frac{k_4}{k} - \frac{g_4}{g} = r, \]

(45)

\[ \frac{k_4}{k} + \frac{g_4}{g} = s, \]

(46)

where \( r \) and \( s \) are constants. From Equations (15) and (19), we have derived

\[ k = le^{\frac{(r+s)}{2}T}, \]

(47)

\[ g = me^{\frac{(r-s)}{2}T}, \]

(48)

where \( l \) and \( m \) are integrating constants. In this case the geometry of the universe (25) reduces to the form

\[ ds^2 = N^2 \left( \frac{1}{X^2} \right)^{\frac{a(n+1)}{(c^2+2K-1)}} e^{\frac{(r-s)}{2}T}(dX^2 - dT^2 + dY^2) \]

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\[ + \left( \frac{1}{X^2} \right) \left( \frac{(\kappa + 1)}{(\kappa^2 + 2\kappa - 1)} \right)^{\frac{(\kappa + 1)}{(\kappa^2 + 2\kappa - 1)}} e^{\frac{(r-s)}{2}T}dZ^2, \tag{49} \]

where
\[
N = \frac{am}{(1 - 2\kappa - \kappa^2)} \left( \frac{\kappa + 1}{(\kappa^2 + 2\kappa - 1)} \right) \text{ and } \kappa = \frac{r}{s}. \tag{50} \]

The effective pressure \( \bar{p} \) and density \( \rho \) for the model (49) are given by
\[
8\pi \bar{p} = \frac{e^{\frac{(r-s)}{2}T}}{N^2 X \frac{2\kappa + 1}{(1 - 2\kappa - \kappa^2)}} \left[ \frac{2\kappa^2(\kappa + 1)(\kappa + 2)}{(1 - 2\kappa - \kappa^2)X^2} - \frac{1}{16}(r^2 + s^2) \right] - \Lambda, \tag{51} \]
\[
8\pi \rho = \frac{e^{\frac{(r-s)}{2}T}}{N^2 X \frac{2\kappa + 1}{(1 - 2\kappa - \kappa^2)}} \left[ \frac{-2\kappa(\kappa + 1)(\kappa + 2)}{(1 - 2\kappa - \kappa^2)X^2} + \frac{1}{8}(s^2 - r^2) \right] + \Lambda. \tag{52} \]

On using Equations (4) and (30) in (51), we obtain
\[
8\pi (p - \xi_0^a \theta) = \frac{e^{\frac{(r-s)}{2}T}}{N^2 X \frac{2\kappa + 1}{(1 - 2\kappa - \kappa^2)}} \left[ \frac{2\kappa^2(\kappa + 1)(\kappa + 2)}{(1 - 2\kappa - \kappa^2)X^2} - \frac{1}{16}(r^2 + s^2) \right] - \Lambda, \tag{53} \]

where \( \theta \) is the scalar expansion calculated for the flow vector \( v^i \) and is given by
\[
\theta = \frac{(r + 3s)e^{\frac{(r-s)}{2}T}}{4\sqrt{N} X \frac{2\kappa + 1}{(1 - 2\kappa - \kappa^2)}}. \tag{54} \]

### 4.1 Model I: Solution for \( (\xi = \xi_0) \)

In this case, using Eqs. (32), (29) and (54) in Eq. (33), we obtain
\[
8\pi (1 + \gamma) = \frac{8\pi \xi_0 (r + 3s)e^{\frac{(r-s)}{2}T}}{N^2 X \frac{2\kappa + 1}{(1 - 2\kappa - \kappa^2)}} + \frac{e^{\frac{(r-s)}{2}T}}{N^2 X \frac{2\kappa + 1}{(1 - 2\kappa - \kappa^2)}} \left[ \frac{2\kappa(\kappa + 1)(\kappa + 2)}{(1 - 2\kappa - \kappa^2)X^2} - \frac{1}{16}(r^2 + 2rs - 3s^2) \right]. \tag{55} \]

Eliminating \( \rho(t) \) between (32) and (55), we get
\[
(1 + \gamma)\Lambda = \frac{8\pi \xi_0 (r + 3s)e^{\frac{(r-s)}{2}T}}{N^2 X \frac{2\kappa + 1}{(1 - 2\kappa - \kappa^2)}} + \frac{e^{\frac{(r-s)}{2}T}}{N^2 X \frac{2\kappa + 1}{(1 - 2\kappa - \kappa^2)}} \left[ \frac{2\kappa(\kappa + 1)(\kappa + 2)(\kappa + \gamma)}{(1 - 2\kappa - \kappa^2)X^2} - \frac{1}{16}(r^2 + 2rs(2 + \gamma) - s^2(7 + 4\gamma)) \right]. \tag{56} \]
4.2 Model II: Solution for \((\xi = \xi_0 \rho)\)

In this case with the use of Eqs. (52), (29) and (54) in Eq. (53) reduces to
\[
8\pi \left[ 1 + \gamma \frac{\xi_0(r + 3s)e^{\frac{(r-s)T}{2}}}{4\sqrt{NX} (1-2\kappa - \kappa^2)} \right] \rho =
\frac{e^{\frac{(r-s)T}{2}}}{N^2X \frac{2s(k+1)}{(1-2\kappa - \kappa^2)}} \left[ \frac{2\kappa(k+1)(\kappa-1)(\kappa+2)}{(1-2\kappa - \kappa^2)X^2} - \frac{1}{16} \{r^2 + 2rs - 3s^2\} \right].
\]

Eliminating \(\rho(t)\) between (52) and (57) we get
\[
\frac{1}{16}(r^2 + s^2) + \left\{ \frac{2\kappa(k+1)(\kappa-1)(\kappa+2)}{(1-2\kappa - \kappa^2)X^2} + \frac{1}{8}(r - 2s)s \right\} \times \left\{ \gamma - \frac{(r + 3s)\xi_0 e^{\frac{(r-s)T}{2}}}{4\sqrt{NX} \frac{2s(k+1)}{(1-2\kappa - \kappa^2)}} \right\}.
\]

From Equations (50) and (58), we observe that cosmological constant \(\Lambda\) may be positive or negative under specific conditions. A negative cosmological constant adds to the attractive gravity of matter, therefore, universe with a negative cosmological constant are invariably doomed to recollapse. A positive cosmological constant resists the attractive gravity of matter due to its negative pressure. For most universe, the positive cosmological constant eventually dominates over the attraction of matter and drives the universe to expands exponentially.

4.3 Some physical aspects of the models

The coefficient of shear \(\sigma\), non-vanishing physical components of conformal curvature tensor \(C_{ijkl}\), non-vanishing component of electromagnetic field tensor \(F_{ij}\) and component of charge density \(J^2\) for the model (40) are given as:
\[
\sigma^2 = \frac{r^2 e^{\frac{(r-s)T}{2}}}{8NX \frac{2s(k+1)}{(1-2\kappa - \kappa^2)}},
\]
\[
C_{(1212)} = -C_{(3434)} = \frac{e^{\frac{(r-s)T}{2}}}{6NX \frac{2s(k+1)}{(1-2\kappa - \kappa^2)} } \left[ \frac{2\kappa(k+1)(\kappa-1)}{(1-2\kappa - \kappa^2)X^2} + \frac{r^2}{2} \right],
\]
\[
C_{(1313)} = -C_{(2424)} = \frac{e^{\frac{(r-s)T}{2}}}{6NX \frac{2s(k+1)}{(1-2\kappa - \kappa^2)} } \left[ -\frac{4\kappa(k+1)(\kappa-1)}{(1-2\kappa - \kappa^2)X^2} + \frac{r^2}{4} \right],
\]
\[
C_{(1224)} = -C_{(1334)} = \frac{r e^{\frac{(r-s)T}{2}}}{4NX \frac{2s(k+1)}{(1-2\kappa - \kappa^2)}}.
\]
\[ C_{(2323)} = -C_{(1414)} = e^{\frac{(r-s)T}{2N^{2}(1-2\kappa-\kappa^{2})}} \frac{1}{3N^{2}(1-2\kappa-\kappa^{2})} \left[ \frac{\kappa(\kappa + 1)(\kappa + 2)(\kappa - 1)}{(1-2\kappa-\kappa^{2})X^{2}} + \frac{1}{8}(r-s) \right], \]  
\[ F_{12} = \frac{1}{\sqrt{8\pi}} \left[ \frac{2\kappa(\kappa + 1)(1-\kappa)(\kappa + 2) - rs}{(1-2\kappa-\kappa^{2})X^{2}} \right]^{\frac{3}{2}} B, \]  
\[ J^{2} = -\frac{e^{\frac{(r-s)T}{2N^{2}(1-2\kappa-\kappa^{2})}} \times}{N^{2}X^{(2\kappa + 3\kappa - 1)}} \left[ \frac{(1-\kappa^{2})}{(1-2\kappa-\kappa^{2})} \psi \frac{2n}{X^{(1-2\kappa-\kappa^{2})}} + \frac{2\kappa(\kappa + 1)(\kappa - 1)(\kappa + 2)}{(1-2\kappa-\kappa^{2})^{2}} \psi^{-\frac{1}{2}} X^{\frac{2(\kappa^{2} + 3\kappa - 1)}{(1-2\kappa-\kappa^{2})}}} \right], \]  
where
\[ \psi = \frac{2\kappa(\kappa + 1)(1-\kappa)(\kappa + 2) - rs}{(1-2\kappa-\kappa^{2})X^{2}} - \frac{r-s}{4}. \]

The rotation \( \omega \) is identically zero. The models start expanding at \( T = 0 \) and continue till \( T = \infty \) for \( N > 0 \). The expansion stops when \( T = -\infty \) or \( r = -3s \). The models represent shearing, non-rotating and Petrov type I non-degenerate in general.

## 5 Conclusions

We have obtained a new class of plane-symmetric inhomogeneous cosmological models of electromagnetic bulk viscous fluid as the source of matter. Generally the models represent expanding, shearing, non-rotating and Petrov type I non-degenerate universe in which the flow vector is geodetic. In all these models, we observe that they do not approach isotropy for large values of time.

The cosmological constants in all models given in Sections 3 are decreasing functions of time and they all approach a small value at late times. The values of cosmological “constant” for these models are found to be small and positive which are supported by the results from recent supernova Ia observations recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al. [30]; Perlmutter et al. [27]; Riess et al. [28]; Schmidt et al. [33]).

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