Cassini-oval description of the energy balance at scission during $^{235}\text{U}(n_{th}, f)$

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Abstract

For a detailed description of the excitation energy of the fission fragments, that is used to evaporate neutrons and emit $\gamma$-rays, realistic nuclear shapes at scission are necessary. It is shown that the nuclear shapes around the scission point, along the main fission mode, are well described by Cassini ovals with only two parameters: $\alpha$ (elongation) and $\alpha_1$ (mass asymmetry). This shape parametrization is used in an attempt to solve, in the case of the low-energy fission of $^{236}\text{U}$, the puzzle of the energy balance at scission. The deformation energy liberated during the neck rupture ($\alpha_i=0.985 \rightarrow \alpha_f=1.001$) $\Delta E_{\text{dsc}}^{\text{def}}$ and the extra deformation energy of the fragments immediately after scission $\Delta E_{\text{iasc}}^{\text{def}}$ are calculated as a function of each-fragment mass $A$. This allows us to estimate the excitation energy for each fragment pair as well as its partition among the light and the heavy fragment and compare with data extracted from measured prompt neutron and $\gamma$-ray multiplicities.

The Coulomb repulsion of the fission fragments immediately after scission is also calculated as a function of their mass ratio and compared with experimental total fragment kinetic energies.

Keywords: nuclear fission, neutron multiplicity, total kinetic energy, total excitation energy

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1. Introduction

The amount of excitation energy accumulated in the primary fission fragments is extremely important. It is essential for all applications of the fission process since it represents the main input in the simulation of fragments’ de-excitation by neutron evaporation and $\gamma$-ray emission. Comparing existing experimental data with theoretical predictions one can also judge how correct is our understanding of nuclear fission.

The total excitation energy of the primary fission fragments $TXE = E_L^* + E_H^*$ as a function of their mass ratio $A_L/A_H$ is usually determined by the difference between the Q-value and the total kinetic energy TKE. The merit of this procedure is to involve only measured quantities and hence no modeling is needed.
The deficiency is to give no information on the sharing of TXE either between the two fragments or between different contributions (i.e., mainly between the extra-deformation energy and the intrinsic excitation energy at scission). Arbitrary assumptions regarding these sharings have been employed so far. In the present study, a method to calculate these individual contributions is proposed and applied to the low-energy fission of $^{236}U$.

The excitation energy of each primary fragment $E_{def}^* (L(H))$ has two main terms: the extra-deformation energy of the fragment immediately after scission $\Delta E_{def}^{isc} (L(H))$ and the excitation energy that the fragment already has at that moment $E_{def}^* (L(H))$.

The $\Delta E_{def}^{isc} (L(H))$ is the difference between the deformation energies of the fragment at scission and in the ground state, respectively. The calculation of the ground-state deformation energy is standard. One can use the microscopic-macroscopic method [1] and minimize with respect to the shape coordinates. To calculate the same quantity at scission one has to know the shape of the fissioning nucleus around this highly unstable configuration. Cassini ovals represent a realistic family of shapes for this purpose. With only two shape parameters, we can explain [2], for the thermal neutron fission of $^{235}U$, the most probable yield of the experimental mass distribution for the main fission mode $(A_L=95, A_H=141)$. Indeed, the variation of the deformation energy at scission with mass asymmetry has a minimum exactly at this value (95/141). Moreover in Sec. IV we show the Cassini ovals to be very close to the ‘optimal’ shapes obtained by a parameter free method proposed by Strutinsky [3, 4] and developed recently [5, 6].

In low-energy fission, $E_{def}^{isc} (L(H))$ is entirely due to the non-adiabatic coupling of the collective and intrinsic degrees of freedom during the descent from saddle to scission. This transition occurs in two phases: a relatively slow motion till the neck radius is too small ($\approx 1.5$ fm) to withhold the repulsive forces followed by a relatively fast neck rupture and absorption of the neck stubs by the fragments. Since the magnitude of the variation of the potential determines the amount of dissipation, it is reasonable to assume that the main contribution to this excitation energy is accumulated during scission and not before. In fact microscopic dynamical quantum calculations [7] in the frame of an independent particle model show that, if the saddle to scission time is longer than $5 \times 10^{-21}$ sec, the motion is adiabatic and no single-particle excitation occurs. The only information we have about this time is from heavy-ion induced fission. It is either extracted from experiment [8] or calculated [9]. In both cases classical approaches are used and saddle to scission times longer than $10^{-20}$ sec are reported. Even if at low energies, where quantum effects dominate, this time is expected to be shorter, $5 \times 10^{-21}$ sec can still be considered a lower limit. In the present evaluation we will consider only the amount of excitation energy induced in the fragments during the very last stage of the fission process. This energy was already estimated, using the sudden approximation [10], for each pair of fragments [11] and for each fragment separately [12].

The sudden approximation is a simple prescription to calculate microscopically the amount of excitation induced in the fragments during an extremely fast scission process (neck rupture). We only need to know the neutron eigenstates ‘just before’ and ‘immediately after’ the scission.

In Sec. II the optimal shapes approach that defines the nuclear shapes along the fission path is presented and exemplified in the framework of the liquid drop model. In Sec. III the method used to calculate the shell and pairing corrections is described. In Sec. IV the Cassini ovals are introduced as a reliable nuclear shape parametrization at scission. The microscopic-macroscopic approach is applied to the calculation of the total deformation energy of $^{236}U$ around the scission line (Sec. V) and of the fission fragments in their ground state (Sec. VI) as well as of the fragments immediately after scission (Sec. VII). In Sec. VIII and IX the excitation energy of each fragment pair and of each primary fragment are calculated and compared with existing data. Among the experimental results, the dependence of the neutron multiplicity on the mass of the emitting fragment is most challenging to explain. Although the theoretical results present the saw-tooth structure, there is an obvious difficulty to reproduce its position. In Sec. X the Coulomb repulsion of the fission fragments immediately after scission is calculated. Pre-scission kinetic energies are inferred from a comparison with measured TKE values. Sec. XI contains the summary and the conclusions.
2. Optimal shapes of fissioning nuclei

The shape of an axially symmetric nucleus can be defined by rotation of some profile function \( \rho(z) \) around the \( z \)-axis. It was suggested [3] to define the profile function by looking for the minimum of the liquid-drop energy, \( E_{\text{LD}} = E_{\text{surf}} + E_{\text{Coul}} \) under the constraint that the volume \( V \) and the elongation \( R_{12} \) are fixed,

\[
\frac{\delta}{\delta \rho} (E_{\text{LD}} - \lambda_1 V - \lambda_2 R_{12}) = 0 ,
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the corresponding Lagrange multipliers. The elongation parameter \( R_{12} \) was chosen in [3] to be the distance between the centers of mass of the left and right parts of the nucleus,

\[
R_{12} = \frac{2\pi}{V} \int_{z_1}^{z_2} \rho^2(z) \left| z \right| dz , \quad V = \pi \int_{z_1}^{z_2} \rho^2(z) dz .
\]

Here, \( z_1 \) and \( z_2 \) are the left and right end of the nucleus, respectively. Note, that the spherical shape corresponds to \( R_{12} = 0.75 R_0 \).

The minimization of \( E_{\text{LD}} - \lambda_1 V - \lambda_2 R_{12} \) with respect to the profile function \( \rho(z) \) leads to an integro-differential equation for \( \rho(z) \)

\[
\rho \rho'' = 1 + (\rho')^2 - \rho [\lambda_1 + \lambda_2 \left| z \right| - 10 x_{\text{LD}} \Phi_S] [1 + (\rho')^2]^{\frac{3}{2}} .
\]

Here \( \Phi_S \equiv \Phi(z, \rho(z)) \) is the Coulomb potential at the nuclear surface, and \( x_{\text{LD}} \) is the fissility parameter of the liquid drop [13].

By solving Eq. (3) for given \( x_{\text{LD}} \) one obtains the profile function \( \rho(z) \).

The sequence of shapes (we call them optimal shapes) obtained for different values of the constraining parameter \( \lambda_2 \) is shown in Fig. 1. Varying the parameter \( \lambda_2 \) (\( \lambda_1 \) is fixed by the volume conservation condition) one obtains the shapes ranging from a very oblate shape (disk) up to two touching spheres.

The liquid drop deformation energy \( E_{\text{def}} = E_{\text{LD}} - E_{\text{def}}^{(0)} \) (in units of the surface energy for spherical shape

\[
E_{\text{def}} / E_{\text{surf}}^{(0)} = B_{\text{surf}} - 1 + 2 x_{\text{LD}} (B_{\text{Coul}} - 1)
\]

calculated for the shapes shown in Fig. 1, is presented in Fig. 2, where \( B_{\text{Coul}} \equiv E_{\text{Coul}} / E_{\text{surf}}^{(0)} \) and \( B_{\text{surf}} \equiv E_{\text{surf}} / E_{\text{surf}}^{(0)} \). In (4) and everywhere below the index \( ^{(0)} \) refers to the spherical shape.
One can see from Fig. 2 that the elongation $R_{12}$ of the shapes shown in these figures is limited by some maximal value $R_{12}^{\text{max}}$. Above this deformation mono-nuclear shapes do not exist. With a good accuracy the maximal deformation is independent of the fissility parameter $x_{LD}$, $2.32R_0 \leq R_{12}^{\text{max}} \leq 2.35R_0$ for $0.4 \leq x_{LD} \leq 0.9$. This critical deformation was interpreted in [3] as the scission point. Note that, at scission the neck radius is still rather large: the neck radius at the critical deformation is approximately equal to $(0.25 - 0.30)R_0$ for a fissility parameter in the range $0.4 \leq x_{LD} \leq 0.9$ This value is in agreement with the minimum neck radius along the dynamical path calculated with one-body dissipation [14]. Therefore the nucleus breaks apart at a finite neck radius.

Another peculiarity of Fig. 2 is the upper branch of the deformation energy at large deformation. Along this branch the neck of the drop becomes smaller and smaller until it turns into two touching spheres. The nucleus breaks apart at a finite neck radius.

The above formalism can be generalized to mass-asymmetric shapes. One has to add one more constraint fixing the left-right asymmetry. Usually the mass asymmetry is the ratio $\delta \equiv (A_H - A_L)/(A_H + A_L)$, where $A_L$ and $A_H$ are the masses of the light and heavy parts of nucleus. For shapes with neck, it is the neck that divides the shape into the light and heavy parts. For pear-like shapes a neck does not exist and one has to introduce another quantity which would divide the shape into light and heavy parts.

For this purpose we analyse the local curvature of the nuclear surface $H(z)$,

$$H(z) = \frac{1}{2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right),$$

with $R_1$ and $R_2$ being the local principal radii of curvature. In the case of axially symmetric shapes the radii $R_1$ and $R_2$ are expressed in terms of the profile function,

$$R_1 = \rho(z)\sqrt{1 + (\rho')^2}, \quad R_2 = -[1 + (\rho')^2]^{\frac{3}{2}}/\rho''.$$

On any surface one can find a place, where the curvature is the largest. Let $z^*$ be that place. In case of left-right symmetric shapes $z^*$ would coincide with the center of mass, $z_{cm} = z^*$. For mass asymmetric shapes $z_{cm} \neq z^*$ and the difference $z_{cm} - z^*$ can be considered as a measure of the mass asymmetry. Formally $z_{cm} - z^*$ can be written as

$$z_{cm} - z^* = \frac{\pi}{V} \int_{z_1}^{z_2} (z - z^*)\rho^2(z)dz$$

which can be included as an additional constraint in the form $-\lambda_3(z_{cm} - z^*)$ into Eq. (1) leading to the following modification of equation (3)

$$\rho\rho'' = 1 + (\rho')^2 - \rho[\lambda_1 + \lambda_2|z - z^*| + \lambda_3(z - z^*) - 10x_{LD}\Phi S][1 + (\rho')^2]^{\frac{3}{2}}.$$

Few examples of mass asymmetric optimal shapes are shown in Fig. 3 for $x_{LD} = 0.75$.

Summarizing, we would like to stress that the optimal shape approach allows for the accurate definition of the scission shape without imposing a given shape parametrization.

### 3. Microscopic - macroscopic approach

Our goal is to calculate the potential energy of deformation for the fissioning nucleus $^{236}U$ around the scission line as well as for the individual fission fragments (both immediately after scission and in their ground state). For this we recourse to the widely used Strutinsky’s approach [1] in which the deformation energy

$$E_{\text{def}} = E_{\text{def}}^{LD} + E_{\text{shell}}$$

was used.
Figure 3: Examples of solutions of (8) obtained for different values of parameter \( \lambda_2 \) for the fissility parameter \( x_{LD} = 0.75 \).

with

\[
E_{def}^{LD} = 15.869248A^{2/3} \left[ 2 \left( \frac{Z^2/A}{45} \right) (B_{Coul} - 1) + 1.13037 \left( 1 - 1.78 \left( \frac{N - Z}{A^2} \right)^2 \right) (B_{surf} - 1) \right]
\]  

and the shell correction \( E_{shell} = \delta E + \delta P. \) \( \delta E \) is the difference between the sum of single particle energies of occupied states \( E_{ipm} \) and the corresponding averaged quantity,

\[
E_{ipm} = \sum_{occ.} \epsilon_k, \quad \delta E = E_{ipm} - \int_{-\infty}^{\tilde{\mu}} e \tilde{g}(\epsilon) \, d\epsilon, \quad \int_{-\infty}^{\tilde{\mu}} \tilde{g}(\epsilon) \, d\epsilon = N, \quad \tilde{g}(\epsilon) = \frac{1}{\gamma} \sum_k f \left( \frac{\epsilon_k - \tilde{\mu}}{\gamma} \right)
\]

where \( f(x) \) is the smoothing function

\[
f(x) = \frac{e^{-x^2}}{\sqrt{\pi}} P_M(x), \quad P_M(x) = \sum_{n=0,2,\ldots}^M a_n H_n(x), \quad a_0 = 1, \quad a_{n+2} = -a_n/(n+2).
\]

\( H_n(x) \) are the Hermite polynomials. The order \( M \) of the correcting polynomial \( P_M(x) \) and the smoothing width \( \gamma \) are the parameters of the averaging procedure. They are fixed by the "plateau condition".

We consider the pairing energy \( E_{pair} \) in BCS approximation, \( E_{pair} = E_{BCS} - E_{ipm}. \)

\[
E_{pair} = \sum_{k=k_1}^{k_2} (2v_k^2 - n_k)(\epsilon_k - \lambda) - \frac{\Delta^2}{G}.
\]

\( n_k = 2 \) for filled states, \( n_k = 1 \) for half-filled states. The \( \lambda \) and \( \Delta \) are fixed by the particle number conservation and the gap equation

\[
\sum_{k=k_1}^{k_2} 2v_k^2 = N - k_1 + 1, \quad \frac{1}{G} = \sum_{k=k_1}^{k_2} \frac{\Delta}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}
\]

The shell correction to the pairing energy is then the difference,

\[
\delta P = E_{pair} - \tilde{E}_{pair}.
\]
The averaged part $\tilde{E}_{\text{pair}}$ is often calculated by replacing the sum in (13) by the integral and assuming that the density of state is constant over the pairing window \[1\]

$$
\tilde{E}_{\text{pair}} = \tilde{g}(\lambda) \int_{\lambda-\hbar\Omega}^{\lambda+\hbar\Omega} (e - \lambda) \left( 1 - \frac{e - \lambda}{\sqrt{(e - \lambda)^2 + \Delta^2}} \right) de - \frac{\Delta^2}{G} - 2\tilde{g}(\lambda) \int_{\lambda-\hbar\Omega}^{0} (e - \lambda)d(e - \lambda) \quad (16)
$$

$$= \tilde{g}(\lambda)\hbar\Omega^2 [1 - \sqrt{1 + \Delta^2/(\hbar\Omega)^2}] \approx -\frac{1}{2} \tilde{g}(\lambda) \Delta^2$$

The pairing strength $G$ was removed from (16) by the gap equation. Solving the gap equation in the same approximation one gets a relation between $G$ and $\Delta$,

$$
\frac{1}{G} = -\tilde{g}(\lambda) \ln \left[ \sqrt{1 + (\hbar\Omega)^2/\Delta^2} - \hbar\Omega/\Delta \right] \quad (17)
$$

Following \[18\] we used the following approximation for the average pairing gap $\tilde{\Delta}$

$$\tilde{\Delta} = \begin{cases} 
  r e^{-tI^2+sI/Z^{1/3}}, & \text{for protons,} \\
  r e^{-tI^2-sI/N^{1/3}}, & \text{for neutrons,}
\end{cases} \quad (18)
$$

with $r = 5.72 MeV, s = 0.118, t = 8.12, I \equiv (N - Z)/A$.

4. Generalized Cassini ovals

As shown in the previous section, to calculate the total potential energy along the fission path one needs, besides the liquid drop energy, also the shell correction. For this one has to solve the eigenvalue problem of the corresponding Hamiltonian. Ideally, one should use a mean field generated by the optimal shapes.

Unfortunately, at present, such codes do not exist. Instead, we approximate the optimal shape by a distorted Cassini ovaloid \[15, 16\].

In this shape parametrization the lemniscate coordinate system \{R, x\} is used. The coordinates \{R, x\} are related to some cylindrical coordinates \{\rho, z\} by the equations

$$
\rho = \frac{1}{\sqrt{2}} \sqrt{p(x) - R^2(2x^2 - 1) - s}, \quad z = \frac{\text{sign}(x)}{\sqrt{2}} \sqrt{p(x) + R^2(2x^2 - 1) + s},
$$

$$p(x) \equiv (R^4 + 2sR^2(2x^2 - 1) + s^2), \quad 0 \leq R \leq \infty, \quad -1 \leq x \leq 1.
$$

The coordinate surfaces of the lemniscate system $R(x) = R_0$ are the Cassini ovaloids with $s \equiv \varepsilon R_0^2$, where $s$ is the squared distance between the focus of Cassinian ovals and the origin of coordinates. The spherical shape of the nucleus corresponds to $\varepsilon = 0$. For $0 < \varepsilon < 0.4$ the Cassinian ovals are very close to ellipses with the ratio of half-axes equal to $(1 - 2\varepsilon/3)/(1 + \varepsilon/3)$. At larger $\varepsilon$ values a neck appears and at $\varepsilon = 1$ the neck becomes zero (see Fig. 4).

The deviation of the nuclear surface from Cassini ovaloids is defined by expansion of $R(x)$ in series in Legendre polynomials $P_n(x)$,

$$
R(x) = R_0[1 + \sum_n \alpha_n P_n(x)], \quad (19)
$$

where $R_0$ is the radius of the spherical nucleus with the same volume. The volume conservation condition is satisfied by the scaling of the cylindrical coordinates, namely, \{\rho, z\}

$$
\rho \rightarrow \rho \equiv \rho/c, \quad z \rightarrow z \equiv (z - z_{cm})/c, \quad c = (V/V_0)^{1/3}, \quad (20)
$$

where $V$ and $V_0$ are the volumes of the deformed and spherical nuclei respectively and $z_{cm}$ is the $z$-coordinate of the center of mass of Cassini ovaloid.
The parameters $\varepsilon$ and $\alpha_n$ are considered as the deformation parameters. Instead of $\varepsilon$, it is convenient to introduce another parameter, $\alpha$, which is defined so that at $\alpha = 1$ the neck radius turns into zero for any value of all other deformation parameters $\alpha_n$,

$$
\varepsilon = \frac{\alpha - 1}{4} \left[ 1 + \sum_n \alpha_n \right] + \left[ 1 + \sum_n (-1)^n \alpha_n \right] + \frac{\alpha + 1}{2} \left[ 1 + \sum_n (-1)^n \alpha_{2n} (2n - 1)!! / (2n)! \right]^2.
$$

As one can see in Fig. 5 the maximal-deformation optimal shapes (i.e., the scission shapes), obtained in Sec. II, are well reproduced, in the experimental mass-asymmetry range, by Cassini ovals with $\alpha = 0.985$. This is another argument, beside the one mentioned in the Introduction, to safely use two-parameter Cassini ovals to describe the shape of the fissioning nucleus around scission, along the main fission mode.

5. Energy liberated during scission

Scission is a fast process that begins when the neck starts to break and ends when it is completely reabsorbed by the fragments. Based on Fig. 5 we fix $\alpha_i = 0.985$ for the just before scission configuration. For immediately after scission we fix $\alpha_f = 1.001$ by symmetry, i.e., by looking for two separated fragments that resemble the nascent fragments at $\alpha_i$. The calculated deformation energies (9) just before scission and immediately after scission are shown in Fig. 6, for $^{236}U$, as a function of the light fragment mass number $A_L$. In the limit of extremely diabatic coupling during scission the difference between these two curves, $\Delta E_{\text{def}}^\text{disc}$, is transformed into primary-fragments’ excitation. Under this assumption is calculated TXE in Sec. VIII.

6. Ground-state fragment deformation energy

The ground-state energy of each fragment $E_{\text{def}}^{\text{gs}}(L)$ or $E_{\text{def}}^{\text{gs}}(H)$ is calculated as the sum of the liquid drop energy plus the shell correction (including the correction to the pairing energy). The ground state shape was parametrised in terms of distorted Cassini ovaloids. The values of deformation parameters $\alpha_2 - \alpha_6$ ($\alpha = \alpha_1 = 0$) were found by the minimization of the energy (9) with respect to the variation of these parameters.

The dependence of the ground-state energy on the fragment mass is seen in the left part of Fig. 7. As expected there is a sharp minimum of the energy around the double magic nucleus $Z = 50, N = 82$. In this region the shape is close to spherical and the liquid drop part is small. There is another minimum on the light-fragment side around the single magic $Z = 32, N = 50$. The heaviest fragments ($A > 150$)
are stabilized by deformed shells. In the right side of Fig. 7 the ground-state deformation energy for each fragment pair is plotted. It represents the origin from which the energy release per fission event is estimated. It has a pronounced minimum around $A_L = 104$ (the partner of $A_H = 132$) and a shallow minimum around $A_L = 82$.

The staggering of the energy is the odd-even effect. The point is that the shell correction to the pairing energy reduces the total shell correction. If the number of neutrons (protons) is odd, then the contribution to $\delta E$ from the last filled level is not diminished by the $\delta P$ and the sum $\delta E + \delta P$ is somewhat larger (in absolute value) as compared with the neighbouring nucleus with even number of neutrons (protons).

In short, Fig. 7 confirms our knowledge about nuclear stability and can be used with confidence in Secs. VIII and IX to calculate the excitation energy of the fission fragments.

7. Fragment deformation energy immediately after scission

As discussed in Sec. V, we assume that the shape immediately after scission is given by a Cassinian ovaloid with $\alpha = 1.001$. A second shape parameter $\alpha_1$ defines the mass asymmetry. Two such shapes, one for symmetric and one for asymmetric mass divisions are shown in Fig. 8 by solid lines.
To obtain the shape of each individual fragment, the shapes of the light and heavy fragments are fitted separately by the expansion (19) with 10 deformation parameters \( \alpha_n \), \( 1 \leq n \leq 10 \), \( \alpha_0 = 0 \). The value of these parameters \( \alpha_n \) are defined by minimizing the deviation of the shape of fragment from the one given by expansion (19). The result is shown by dash lines in Fig. 8. One can see that the fit is very accurate.

For each shape (for each set of \( \alpha_n \) parameters) the deformation energy \( E_{\text{iasc} \text{def}} \) (9) is calculated. Its dependence on the fragment mass number is shown in Fig. 9. It has a pronounced minimum around \( A \approx 142 \) and a shallower one around \( A \approx 82 \). The difference between Fig. 7 (left) and Fig. 9 is due to the proximity of the partner fragment. It mainly shifts the most probable \( A \) from 132 to 142 and widened the minima. To understand why this happens, we plot in Fig. 10 the deformation energy immediately after scission for each fragment pair. There is now only one minimum around \( A_L = 95 \) that coincides with the most probable experimental mass division 95/141. Therefore \( A_H = 141 \) is the result of the competition between \( A_H = 154 \) (the partner of \( A_L = 82 \)) and \( A_H = 132 \).

8. Total excitation energy

For each fragment pair (L,H) the total excitation energy is:

\[
TXE = \Delta E_{\text{def}}^{\text{iasc}} + \Delta E_{\text{def}}^{\text{iasc}}(L) + \Delta E_{\text{def}}^{\text{iasc}}(H)
\]

where

\[
\Delta E_{\text{def}}^{\text{iasc}}(L) = E_{\text{def}}^{\text{iasc}}(L) - E_{\text{def}}^{\text{gs}}(L), \quad \Delta E_{\text{def}}^{\text{iasc}}(H) = E_{\text{def}}^{\text{iasc}}(H) - E_{\text{def}}^{\text{gs}}(H).
\]
The comparison of the calculated and measured dependence of the total excitation energy on $A_L$ are shown in Fig. 11. The theoretical and experimental quantities are of the same order of magnitude but their values and their dependence on the fragment mass differ considerably. Surprisingly, the experimental values of $TXE$ do not show the well established shell effects which are seen in the calculated quantities. Instead, they are close to the liquid drop estimate.

9. Saw-tooth structure of neutron multiplicity

The excitation energy $E^*_L(H)$ of each primary fission fragment is estimated by

$$ E^*_L(H) = B_n + E^*_{\text{iasc}}(L(H)) + \Delta E^*_{\text{.sys}}(L(H)) $$

where $B_n$ is the binding energy of the incident neutron and $E^*_{\text{iasc}}$ is the excitation energy immediately after scission taken from [12]. To obtain the excitation energy used to evaporate prompt neutrons, we subtract the neutron separation energy $S_n$ from $E^*_L(H)$ since, once this level is attained, only $\gamma$-rays can be further emitted. The result is shown in Fig. 12. For a zero-order comparison we show also the experimental values of the neutron multiplicity [19, 20] multiplied by the average neutron separation energy ($\approx 6.5$ MeV).

It is interesting to note that the primary excitation energy has a value compatible with the experimental data and shows fluctuations very similar to the "saw tooth" structure. However the minima and maxima are shifted by approx. 20 units to higher mass numbers as compared with the measured $\bar{\nu}(A)$.

The position of the calculated maximum of the primary excitation energy is given by the maximal ground state shell correction of the nuclei around $Z = 50, N = 82$. This is an established fact. It is difficult to imagine how to shift the maximum of the primary excitation energy either to the left or to the right of this double-magic nucleus.

In the present work we have considered only one (mass asymmetric) fission mode (StII). The experimental results on the mass distribution of fission fragments [21] indicate the existence of other two modes: one asymmetric (STI) and one symmetric (SL). It means that, to complete our study, we should investigate also the other two scission configurations. However, even with two more scission modes, the highest number of neutrons will still be emitted from the region around $Z = 50, N = 82$. 

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Figure 11: The liquid drop part and the total excitation energy as a function of the mass number $A_L$ of light fragment. The solid line is the experimental total excitation energy taken from [19].

Figure 12: The dependence of the primary fragment excitation energy on the fragment mass number $A$, calculated (empty symbols) and deduced from experiment (full symbols).
10. Coulomb repulsion and TKE

To complete this study, it remains to be seen if the Cassini ovals used are compatible with the measured total kinetic energies of the fission fragments. In Fig. 13 the Coulomb repulsion of the fragments immediately after scission ($\alpha_f=1.001$) is plotted as a function of the heavy fragment mass. On the same figure the experimental data [22, 23, 24] together with their analyses in terms of fission modes [25] are shown. One can see that the calculated points lie below the values corresponding to the main fission mode (StII). The differences (around 7 MeV at the peak of the mass distribution) are interpreted here as pre-scission kinetic energies. Note that, our just-before scission point lies only 10 MeV below the saddle point.

11. Summary and Conclusions

Generalized Cassini ovaloids are used to describe the shape of the fissioning nucleus $^{236}\text{U}$ just before and immediately after scission as a function of the fission-fragment mass-asymmetry. The total deformation energy is calculated as the liquid drop energy plus the Strutinsky’s shell correction for each fragment pair as well as for each fragment separately. This allows us to estimate the excitation of the primary fragments available for prompt neutron evaporation and compare with the same quantity extracted approximately from measured neutron multiplicities during $^{235}\text{U}(n_{th},f)$.

The well-known "saw-tooth" shape of $\bar{\nu}(A)$ is reproduced by the calculation without any normalization. However its position is shifted, relative to the experimental data, towards heavier fragments by about 20 mass units, so that, as expected, the double-magic $^{132}\text{Sn}$ evaporates the largest number of neutrons. Since the data do not show this, difficult to refute, prediction we conclude that something important is missing in our understanding of nuclear fission.

If one extracts $TXE$ from experiment (mainly from $\bar{\nu}(A_L) + \bar{\nu}(A_H)$) the structure disappears. An almost flat curve is obtained that is very close to our pure liquid-drop estimate.

Finally, the Coulomb repulsion of the primary fragments immediately after scission is compatible with a pre-scission kinetic energy of about 7 MeV for the most probable mass yields. This is in agreement with the weak damping assumed here during the saddle to just-before scission descent.
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