The Multi-Role Hidden Tree Model for Scale-free Networks

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Abstract. Based on the spatial factor other than the temporal accumulation, the hidden tree model was built up to model scale-free networks. This paper further assumed that a node has multiple roles in different hidden trees, and explored the multi-role hidden tree model. The experimental results showed that multi-role hidden tree model can also produce scale-free networks. This conclusion indicates that the hidden tree model is robust with multiple roles.

1. Introduction
Modelling scale-free networks is an important issue in the research of complex networks [1, 2, 3, 4, 5], because some important systems such as the Internet and the WWW were found to be scale-free [1, 2, 6, 7, 8]. To understand the real-world networks and to illustrate the origins and mechanisms of the scale-free property, hundreds of models are proposed in the last decades [9, 10]. Among all of the models, the BA model [1] proposed by the pioneer scientists A.-L. Barabási et al. is the most popular. The BA model indicates that the scale-free property is based on the temporal accumulation. Based on the BA model, a lot of researches were carried out.

In another side, scale-free networks has an important feature: the hierarchy [11, 12, 13, 14, 15]. Therefore, some researchers tried to integrate the hierarchy into the models to explore the mechanisms of the scale-free property [16]. Thus, Zheng et al. further proposed that the hierarchy is the key to the scale-free property and thus proposed the hidden tree model [12, 17, 18].

Zheng et al.’s hidden tree model suggested that the hierarchy is a kind of hidden order [17, 19, 18] that is represented by a tree structure. In the hidden tree model, all the nodes are organized into a tree, and the offspring nodes are controlled in cascade by the parents nodes; every node has a probability to trade with a certain random node; when the transaction happens, the node rising the transaction would link to every node which control the transaction in cascade. With this method, scale-free networks can be generated. Different to the temporal accumulation of the BA model, this model are based on the spatial factor. This model has been used to explain many phenomena in the real world.

However, the individuals in the real world often have multiple roles, i.e., they may be organized into different hidden trees. Thus, whether the overall relationship network is still scale-free or not? To answer this problem, we explored the multi-role hidden tree model in this paper.
2. The Model

Assume that the whole system has \( N \) nodes, and these nodes are organized into \( R \) hidden trees, i.e., every node has \( R \) roles. For simplicity, every hidden tree is assumed as the \( n \)-tree. Moreover, the roles are assigned randomly, i.e., the places of every node in different hidden trees are random. Similar to the original hidden tree, every node has a probability to do transactions with the others randomly; because every node has multiple roles, a hidden tree is chosen randomly to reflect the cascade controlling; when a node has a transaction to another, the source node would link to all nodes in the route between the source node and destination node in the selected hidden tree. That is, when node \( a \) has a transaction to node \( b \), the proposed model chooses a hidden tree randomly, and finds a route from node \( a \) to node \( b \) in the chosen hidden tree, and finally node \( a \) links itself to all the nodes in the route.

This model can be depicted as Fig. 1.

![Figure 1. The hidden trees](image)

As shown in Fig. 1, the identical node with the same number would have different places in the left hidden tree and in the right hidden tree. Assume that node 1 arises a transaction with node 11, if the left hidden tree is chosen, then node 1 will link to nodes 2, 5 and 11; if the right hidden tree is chosen, then node 1 will link to nodes 5, 7, 14, 6 and 11.

3. Experimental Results

The proposed model has four parameters, i.e., the number of hidden trees \( T \), the number of nodes \( N \), the number of sibling nodes \( n \) and the probability of arising a transaction \( A \). We focus on the simulation results under \( T = 2 \) and \( T = 3 \).

First, we studied the circumstance when the number of roles is 2, i.e., \( T = 2 \). To study the effect of the parameter \( N \), i.e., the number of node, we set the other parameters \( n = 2 \) and \( A = 0.4 \), and set \( N = 1000, 2000, 5000, 10000 \) and \( 20000 \). To study the effect of the parameter \( n \), we set the other parameters \( N = 1000 \) and \( A = 0.4 \), and set \( n = 2, 3, 4, 5 \) and \( 6 \). To study the effect of the parameter \( A \), we set the other parameters \( N = 1000 \) and \( n = 2 \), and set \( A = 0.08, 0.16, 0.32, 0.64 \) and \( 1.00 \). Moreover, to assure the robustness of the model, we carried out every parameters setting 10 times. The simulation results were very robust. Therefore, we chose the generated scale-free networks in the first run and demonstrated their cumulative degree distributions as Fig. 2.

From Fig. 2, we can see that the degree distributions of the generated networks are approximately linear in the log-log coordination, indicating that the generated networks are scale-free. Moreover, the curves in Fig. 2 are similar to previous studies [12, 17, 18]. According
to the theoretical analysis, previous studies have proved that the curves demonstrated the scale-free property. Therefore, the degree distributions in Fig. 2 are theoretically approximately power-law.

Second, we studied the circumstance when the number of roles is 3, i.e., $T = 3$. In order to compare with the results under $T = 2$, we used the same parameters settings as those under $T = 2$. The experimental results are listed as Fig. 3.

From Fig. 3, we can see that the experimental results are very similar to the results under $T = 2$. The experimental results indicate that when all nodes have multiple roles, the hierarchy or the hidden order can still produce the scale-free property.

4. Conclusions and Discussion
The experimental results have demonstrated that the proposed model can generate scale-free networks. The proposed model did not employ the time factor, i.e., did not rely on the time accumulation, but was based on the (topological) spatial factor. The spatial factor, i.e., the hidden tree, can be explained as the hidden order. On one side, because the proposed model was independent to the time factor, it would be full parallelism. On the other side, because the proposed model was based on the spatial factor, the hierarchy, i.e., the hidden order, is the key factor to the scaling phenomenon.
Moreover, the proposed model explored the circumstances that all nodes have multiple roles, which is more general in the real world. When every node has a few roles, the generated networks are still scale-free. This conclusion indicates that the hidden tree model is quite robust with multiple roles.

The conclusions in this paper can also explain the invulnerability of scale-free networks [20, 21, 22, 23, 24, 25, 26, 27]. The hidden order is like the crystal in the matters, therefore, it means the minimal energy status. When the hidden order is destroyed, the additional energy should be used. That is, the networks with the hidden order, i.e., scale-free networks, would be more stable under the attacks. In another words, destroying scale-free networks is more difficult than destroying random networks in most cases[22, 23, 24, 28].

Furthermore, because the hidden order can be reflected as the minimal energy status, the multi-role hidden order model can be regarded as the optimization process. Therefore, the proposed model is compatible to the optimization explanation of scale-free networks [29, 30].

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