Cooperative Radar and Communications Signaling: The Estimation and Information Theory Odd Couple

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Abstract—We investigate cooperative radar and communications signaling. While each system typically considers the other system a source of interference, by considering the radar and communications operations to be a single joint system, the performance of both systems can, under certain conditions, be improved by the existence of the other. As an initial demonstration, we focus on the radar as relay scenario and present an approach denoted multiuser detection radar (MUDR). A novel joint estimation and information theoretic bound formulation is constructed for a receiver that observes communications and radar return in the same frequency allocation. The joint performance bound is presented in terms of the communication rate and the estimation rate of the system.

I. INTRODUCTION

Given the reality of the ever increasing strain on limited spectral resources, radar and communications systems are in some cases being forced into an uneasy coexistence. The typical assumption is that the existence of one type of system (either a radar or a communications system) will degrade the performance of the other system. Consequently, the systems are usually isolated temporally, spectrally, or spatially in most operations.

A. Background

During the last decade, cognitive radio technologies [1], [2] have been considered that implement opportunistic spectrum sharing as they are able to sense under-utilized spectrum and adaptively allocate it to other users [3]. A similar coexistence problem is currently faced by radars as their performance deteriorates due to coexisting wireless communications systems. Cognitive radars indicate initial attempts to adapt intelligently to complicated environments [4].

Current research on the spectral coexistence of radar and communications systems has mainly involved concepts similar to cooperative sensing [5]–[9]. Other methodologies that have been applied to the radar-communications coexistence problem include signal sharing [10], [11] and waveform shaping [12]–[15]. In other research and applied systems radars based on communication system waveforms have been considered. As an example, operating the radar passively or parasitically by using a broadcast communication system has been investigated (for example in [16] and references therein). Also, radios that communicate with radar systems by modulating the radar waveform have been considered [17].

B. Contributions

The principal contribution of the paper is that we develop a novel performance bound formulation to provide insight into the limits of coexisting radar and communications systems. For joint decoding and radar channel estimation (which we denote multiuser detection radar: MUDR), we allow the radar to demodulate and decode the communications signal jointly with estimating its radar channel. Rather than have radar and communications system performance degraded, the performance of both systems is potentially enhanced by the systems’ interactions. In its most general form, the coexisting radar and communications system becomes a large heterogenous multistatic radars or statistical multiple-input multiple-output (MIMO) radar [18]–[20] and simultaneously a heterogenous communication network [2]. These jointly cooperative systems are only possible under certain theoretical constraints that we begin to explore in this paper.

In this paper, as a preliminary exploration, we consider the limited scenario of a joint radar and communications relay. In this case, the node traditionally denoted “radar” is also a communications relay that jointly estimates the radar return and receives a communication signal. The radar waveform is then assumed to be a communications waveform. Because of the advantages of the radar power, the performance of the communications between two or more nodes is typically improved by using the radar as a relay compared to direct ground-to-ground communications. The principal constraint in performance of this system is in simultaneous reception of the radar return and communications signal, and is therefore the main thrust of this work.

II. JOINT ESTIMATION/COMMUNICATIONS BOUNDS

In general, much like network communications [2], exact bounds are challenging. However, in certain cases, such as the multiuser base station, bounds are tenable. We develop a generalization of the multiple-access receiver discussion for the joint radar channel estimation and communications reception.
A. Multiple-Access Communications Analogy

For reference, we review the multiple-access communications system performance bound [2], [21]. In the multiple-access channel that we discuss here, we assume that two independent transmitters are communicating with a single receiver. The channel-attenuation-power product for the two transmitters are given by \(a_1^2P_1\) and \(a_2^2P_2\), respectively. Their corresponding rates are denoted \(R_1\) and \(R_2\). Assuming that power is normalized so that the noise variance is unity, the fundamental limits on rate are given by

\[
R_1 \leq \log_2(1 + a_1^2P_1) \\
R_2 \leq \log_2(1 + a_2^2P_2) \\
R_1 + R_2 \leq \log_2(1 + a_1^2P_1 + a_2^2P_2) \tag{1}
\]

Vertices are found by jointly solving two bounds,

\[
R_2 = \log_2(1 + a_2^2P_2) \\
R_1 + R_2 - R_2 = \log_2(1 + a_1^2P_1 + a_2^2P_2) - \log_2(1 + a_2^2P_2) \\
R_1 = \log_2\left(\frac{1 + a_1^2P_1 + a_2^2P_2}{1 + a_2^2P_2}\right) \\
\{R_1, R_2\} = \left\{\log_2\left(1 + \frac{a_1^2P_1}{1 + a_2^2P_2}\right), \log_2(1 + a_2^2P_2)\right\} \tag{2}
\]

\[
\{R_1, R_2\} = \left\{\log_2(1 + a_1^2P_1), \log_2\left(1 + \frac{a_2^2P_2}{1 + a_1^2P_1}\right)\right\}. \tag{3}
\]

The region that satisfies these theoretical bounds is depicted in Figure 1

![Figure 1](image)

Fig. 1. Pentagon that contains communications multiple-access achievable rate region.

Unfortunately, this discussion serves only as a motivation because radar returns do not satisfy the fundamental communications assumption that they are drawn from a countable dictionary. Consequently, we do not expect that this form is directly applicable. However, by using a formalism similar to the communications multiple-access bound, we can gain insight into the simultaneous channel use by communications and radar.

B. Joint Radar-Communications Notation

Because there is a significant quantity of notation in discussing this topic, in Table I we present an overview of the important notation employed.

| Variable | Description |
|-----------------|-------------|
| \(\cdot\) | Expectation |
| \(\|\cdot\|\) | L2-norm or absolute value |
| \(B\) | Total system bandwidth |
| \(z(t)\) | Observed signal including radar and communications |
| \(\tilde{z}(t)\) | Observed signal with predicted radar return removed |
| \(s_{\text{radar}}(t)\) | Observed radar return |
| \(s_{\text{com}}(t)\) | Unit-variance transmitted radar signal |
| \(P_{\text{radar}}\) | Radar power |
| \(\tau_m\) | Time delay to \(m\)th target |
| \(\tau_m\) | \(k\)th observation of delay for \(m\)th target |
| \(\tau_m\) | Predicted time delay to \(m\)th target |
| \(a_m\) | Combined antenna gain, cross-section, and propagation |
| \(N\) | Number of targets |
| \(T\) | Radar pulse duration |
| \(N\) | Number of targets |
| \(T_{\text{pri}}\) | Pulse repetition interval |
| \(\delta\) | Radar duty factor |
| \(n(t)\) | Unit-variance transmitted communication signal |
| \(P_{\text{com}}\) | Total communications power |
| \(\eta\) | Communications propagation loss |
| \(b\) | Receiver thermal noise |
| \(\sigma^2_{\text{noise}}\) | Thermal noise power |
| \(k_B\) | Boltzmann constant |
| \(T_{\text{temp}}\) | Temperature |
| \(n_{\text{int}+n}\) | Interference plus noise for communications receiver |
| \(\gamma\) | Set of nonspecific system and target parameters |
| \(B_{\text{rms}}\) | Root-mean-squared radar bandwidth |
| \(\gamma\) | Radar spectral shape parameter |
| \(B_{\text{com}}\) | Communications-only subband |
| \(B_{\text{mix}}\) | Mixed radar and communications subband |
| \(\alpha\) | Fraction of bandwidth for communications only |
| \(\beta\) | Power fraction used by communications-only subband |
| \(\mu_{\text{com}}\) | Channel of communications-only subband |
| \(\mu_{\text{mix}}\) | Channel of mixed use subband |

C. Joint Radar-Communications Channel Model

In this section, we consider bounds for the multiple-access communications and radar return channel. We employ a number of simplifying assumptions for the sake of exposition; however, generalizations are possible. As an example, we estimate the range, but assume that the target cross-section is known. We assume that the targets are well separated and that return is modeled well by a Gaussian distribution before pulse compression. We assume that the range of any given target is predictable up to some Gaussian random process variation (not be confused with estimation error). We consider only the portion of time during which the radar return overlaps with the communications signal. We assume that temporal uncertainty of the random target process is within one over the bandwidth.
For $N$ targets, the observed radar return $z_{\text{radar}}(t)$ as a function of $t$ is given by
\[ z_{\text{radar}}(t) = \sum_{m=1}^{N} a_m s_{\text{radar}}(t - \tau_m) \sqrt{P_{\text{radar}}} + n(t). \] (4)

The zero-mean noise is drawn from the complex Gaussian with variance $\sigma^2_{\text{noise}},$
\[ \sigma^2_{\text{noise}} = k_B T_{\text{temp}} B, \] (5)
where $k_B$ is the Boltzmann constant, $T_{\text{temp}}$ is the absolute temperature, and $B$ is the full bandwidth. Range $r$ and delay $\tau$ are related by
\[ \tau = \frac{2r}{c}, \] (6)
where $c$ is the speed of light. The typical radar estimator attempts to estimate both the range and the amplitude. For the sake of discussion, we focus on range estimation. Similar developments can be found for amplitude estimation. A reasonable estimator (particularly if targets are well separated) under the assumption that Doppler shifts are unresolvable is given by
\[ \hat{\tau}_m = \arg\max_{\tau_m} \int dt z(t) s_{\text{radar}}^*(t - \tau_m). \] (7)

We assume that we are tracking the target, and we assume the optimistic model that we have some well understood expected value of the radar return (based upon prior observations); however, there is some range fluctuation in the return due to some underlying target process, so that the next observation is known up to some random Gaussian process variation $n_{\tau,\text{proc}},$
\[ \tau^{(k)}_m = \tau^{(k)}_{m,\text{pre}} + n_{\tau,\text{proc}} \] (8)
\[ \tau^{(k)}_{m,\text{pre}} = f(k; T_{\text{pri}}, \theta). \]

The function $f(k; T_{\text{pri}}, \theta)$ is a prediction function with parameters $T_{\text{pri}},$ which is the time between updates (pulse repetition interval), and $\theta$ which contains other parameters. The variance of the process is given by
\[ \sigma^2_{\tau,\text{proc}} = \left\langle \left\| \tau^{(k)}_m - f(k; T_{\text{pri}}, \theta) \right\|^2 \right\rangle. \] (9)

The observed signal at the receiver $z(t)$ at time $t$ in the presence of a communications signal and the radar return is given by
\[ z(t) = \sqrt{P_{\text{com}}} b s_{\text{com}}(t) \] (10)
\[ + \sqrt{P_{\text{radar}}} \sum_{m=1}^{N} a_m s_{\text{radar}}(t - \tau_m) + n(t) \]

D. Radar-Prediction-Suppressed Observed Signal

For the sake of the communications system, we can try to mitigate unnecessary interference by subtracting the predicted radar return at the receiver:\footnote{Note: this process would theoretically remove all clutter.}
\[ \hat{z}(t) = \sqrt{P_{\text{com}}} b s_{\text{com}}(t) + n(t) \] (11)
\[ + \sqrt{P_{\text{radar}}} \sum_{m=1}^{N} a_m [s_{\text{radar}}(t - \tau_m) - s_{\text{radar}}(t - \tau_{m,\text{pre}})], \]
where here we dropped the explicit indication of pulse index $(k).$ For small delay process variation, we can replace the difference between the waveforms at the correct and predicted delay with a derivative,
\[ s_{\text{radar}}(t - \tau_m) - s_{\text{radar}}(t - \tau_{m,\text{pre}}) \approx \frac{\partial s_{\text{radar}}(t - \tau_m)}{\partial t} n_{\tau,\text{proc}}. \] (12)
The observed signal is then given by
\[ \hat{z}(t) \approx \sqrt{P_{\text{com}}} b s_{\text{com}}(t) + n(t) \] (13)
\[ + \sqrt{P_{\text{radar}}} \sum_{m=1}^{N} a_m \frac{\partial s_{\text{radar}}(t - \tau_m)}{\partial t} n_{\tau,\text{proc}}. \]

E. Radar Estimation Information Rate

An essential tool of this paper is to consider the estimation information rate (estimating delay in this case). We develop this information rate by considering the entropy of a random parameter being estimated and the entropy of the estimation uncertainty of that parameter. As an observation, if the targets are well separated, then each target estimation can be considered an independent information channel.

1) Estimation Entropy: To find the estimation entropy, we find the delay estimation uncertainty for each target. For circularly symmetric Gaussian noise, we employ the complex Slepian-Bangs formulation of the Cramer-Rao bound \cite{2,22}. The variance of delay estimation for the $n^{th}$ target ignoring
inter-target interference) is given by
\[ \sigma^2_{\tau,est} = \text{Var}\{\hat{\tau}_m\} = \frac{1}{(2\pi)^2 B_{\text{rms}}^2} \text{SNR} \]
\[ = \frac{\sigma^2_{\text{noise}}}{(2\pi)^2 B_{\text{rms}}^2 TB a_{\text{in}}^2 P_{\text{radar}}} \]
\[ = \frac{\gamma^2 B (TB) a_{\text{in}}^2 P_{\text{radar}}}{k_B T_{\text{temp}}}, \quad (15) \]
where ISNR = \[TB a_{\text{in}}^2 P_{\text{radar}}/\sigma^2_{\text{noise}}\] indicates the integrated SNR, and the thermal noise is given by
\[ \sigma^2_{\text{noise}} = k_B T_{\text{temp}} B. \quad (16) \]

Under the assumption of Gaussian estimation error, the resulting entropy of the error is given by
\[ h_{\tau,est} = \log_2[\pi e \sigma^2_{\tau,est}] \]
\[ = \log_2 \left[ \pi e \frac{k_B T_{\text{temp}}}{\gamma^2 B (TB) a_{\text{in}}^2 P_{\text{radar}}} \right]. \quad (17) \]

2) Radar Random Process Entropy: The entropy of the process uncertainty plus estimation uncertainty under a Gaussian assumption for both is given by [2], [21]
\[ h_{\tau,rr} = \log_2 \left[ \pi e (\sigma^2_{\tau,\text{proc}} + \sigma^2_{\tau,\text{est}}) \right]. \quad (18) \]

3) Estimation Information Rate: Consequently, the mutual information rate in terms of bits per pulse repetition interval \( T_{\text{pri}} \), which is related to the integration period \( T \) by the duty factor \( T = \delta T_{\text{pri}} \), is approximately bounded by
\[ R_{\text{est}} = \sum_m \frac{h_{\tau,rr} - h_{\tau,est}}{T_{\text{pri}}} = \sum_m \frac{\delta}{T} \log_2 \left( 1 + \frac{\sigma^2_{\tau,\text{proc}}}{\sigma^2_{\tau,est}} \right) \]
\[ = \sum_m B \log_2 \left( 1 + \frac{\sigma^2_{\tau,\text{proc}} \gamma^2 B (TB) a_{\text{in}}^2 P_{\text{radar}}}{k_B T_{\text{temp}}} \right) \delta/(TB) \quad (19) \]

It is worth noting, that by employing this estimation entropy in the rate bound, it is assumed that the estimator achieves the Cramer-Rao performance. If the error variance is larger, then the rate bound is lowered.

III. INNER RATE BOUNDS

It would be surprising if the performance bound displayed for the communications multiple-access scenario in Figure [1] achieved the performance bounds of the joint estimation and communications problem. Here, we search for a good achievable (inner) bounds. The fundamental system performance limit lies between these achievable bounds and the outer bounds found above. To find these inner bounds, we hypothesize an idealized receiver and determine the bounding rates. To simplify the discussion, we consider only a single target with delay \( \tau \) and gain-propagation-cross-section product \( a^2 \), and drop the explicit index to the target. For example \( \sigma^2_{\tau,\text{proc}} \rightarrow \sigma^2_{\text{proc}} \).

If \( R_{\text{est}} \approx 0 \) is sufficiently low, then the communications operates according to the bound determined by the isolated communications system,
\[ R_{\text{com}} \leq B \log_2 \left( 1 + \frac{b^2 P_{\text{com}}}{\sigma^2_{\text{noise}}} \right) \]
\[ = B \log_2 \left( 1 + \frac{b^2 P_{\text{com}}}{k_B T_{\text{temp}} B} \right). \quad (20) \]

If \( R_{\text{com}} \) is sufficiently low for a given transmit power then the communications signal can be decoded and subtracted completely from the underlying signal, so that the radar parameters can be estimated without contamination,
\[ R_{\text{com}} \leq B \log_2 \left[ 1 + \frac{b^2 P_{\text{com}}}{\sigma^2_{\text{int+n}}} \right] \]
\[ = B \log_2 \left[ 1 + \frac{b^2 P_{\text{com}}}{a^2 P_{\text{radar}} \gamma^2 B^2 \sigma^2_{\text{proc}} + k_B T_{\text{temp}} B} \right], \quad (21) \]
where we used Equation (14). In this regime, the corresponding estimation rate bound \( R_{\text{est}} \) is given by Equation (19).

These two vertices correspond to the points 2 (associated with Equation (20)) and 4 (associated with Equations (21) and (19) in Figure 1) if \( R_1 \) is interpreted as the estimation rate, and \( R_2 \) is interpreted as the communications rate. An achievable rate lies within the triangle constructed by connecting a straight line between these points.

A. Water-filling

We hypothesize that we can construct tighter (larger) inner bounds than we constructed in the previous section. In this section, we consider a water-filling approach that splits the total bandwidth into two sub-bands and we water fill the communications power between these bands. Water filling optimizes the power and rate allocation between multiple channels [2], [21]. For this application, we separate the band into two frequency channels. One channel has only communications, and the other channel is mixed-use and operates at the SIC rate vertex define by Equations (19) and (21).

Given some \( \alpha \), that defines the bandwidth separation,
\[ B = B_{\text{com}} + B_{\text{mix}} \quad (22) \]
\[ B_{\text{com}} = \alpha B \]
\[ B_{\text{mix}} = (1 - \alpha) B, \]
then we optimize \( \beta \) that defines the power utilization,
\[ P_{\text{com}} = P_{\text{com,com}} + P_{\text{com,mix}} \quad (23) \]
\[ P_{\text{com,com}} = \beta P_{\text{com}} \]
\[ P_{\text{com,mix}} = (1 - \beta) P_{\text{com}}. \]

There are two effective channels
\[ P_{\text{com}} = \frac{\beta^2}{k_B T_{\text{temp}} \alpha B} \]
\[ = \frac{b^2}{k_B T_{\text{temp}} B} \quad (24) \]
for the channel with only communications signal, and the mixed-use channel that includes the interference to the communications system from the radar

\[
\mu_{\text{mix}} = \frac{b^2}{\sigma_{\text{int+n}}^2}
\]

\[
= \frac{b^2}{a^2 P_{\text{radar}}(1-\alpha)^2 \gamma^2 B^2 \sigma_{\text{proc}}^2 + k_B T_{\text{temp}}(1-\alpha) B}.
\]

The communications power is split between the two channels \[2,21\].

\[
P_{\text{com}} = P_{\text{com,com}} + P_{\text{com,mix}}
\]

\[
= \left(\alpha \nu - \frac{1}{\mu_{\text{com}}}ight)^+ + \left((1-\alpha) \nu - \frac{1}{\mu_{\text{mix}}}ight)^+.
\]

The critical point (the transition between using one or both channels for communications) occurs when

\[
(1-\alpha) \nu - \frac{1}{\mu_{\text{mix}}} = 0
\]

\[
P_{\text{com}} = \alpha \nu - \frac{1}{\mu_{\text{com}}},
\]

so both channels are used if

\[
P_{\text{com}} \geq \frac{\alpha}{(1-\alpha) \mu_{\text{mix}}} - \frac{1}{\mu_{\text{com}}}.
\]

If the communications-only channel is used exclusively for communications, then \(P_{\text{com}} = P_{\text{com,com}}\). If both channels are employed for communications then

\[
P_{\text{com,com}} = \alpha \nu - \frac{1}{\mu_{\text{com}}},
\]

\[
P_{\text{com,mix}} = (1-\alpha) \nu - \frac{1}{\mu_{\text{mix}}},
\]

and thus when Equation \(28\) is satisfied

\[
P_{\text{com}} = \alpha \nu - \frac{1}{\mu_{\text{com}}} + (1-\alpha) \nu - \frac{1}{\mu_{\text{mix}}}
\]

\[
\nu = \left(P_{\text{com}} + \frac{1}{\mu_{\text{com}}} + \frac{1}{\mu_{\text{mix}}}ight).
\]

The value of power fraction \(\beta\) is then given by

\[
\beta = \frac{P_{\text{com,com}}}{P_{\text{com}}}
\]

\[
= \frac{\alpha \nu - \frac{1}{\mu_{\text{com}}}}{P_{\text{com}}}
\]

\[
= \frac{\alpha}{P_{\text{com}}} \left(P_{\text{com}} + \frac{1}{\mu_{\text{com}}} + \frac{1}{\mu_{\text{mix}}}ight) - \frac{1}{\mu_{\text{com}}}
\]

\[
= \alpha + \frac{1}{P_{\text{com}}} \left(\frac{\alpha - 1}{\mu_{\text{com}}} + \frac{1}{\mu_{\text{mix}}}ight);
\]

\[
\text{when } P_{\text{com}} \geq \frac{\alpha}{(1-\alpha) \mu_{\text{mix}}} - \frac{1}{\mu_{\text{com}}}.
\]

The resulting communications rate bound in the communications-only subband is given by

\[
R_{\text{com,com}} \leq B_{\text{com}} \log_2 \left[1 + \frac{P_{\text{com,com}} b^2}{k_B T_{\text{temp}} B_{\text{com}}}\right]
\]

\[
\leq \alpha B \log_2 \left[1 + \frac{\beta P_{\text{com}} b^2}{k_B T_{\text{temp}} \alpha B}\right].
\]

If \(P_{\text{com}} < 1/\mu_{\text{com}} - 1/\mu_{\text{mix}}\) then \(R_{\text{com,mix}} = 0\) because no communications power is allocated to the “mixed” use channel, otherwise the mixed use communications rate inner bound is given by

\[
R_{\text{com,mix}} \leq B_{\text{mix}} \log_2 \left[1 + \frac{b^2 P_{\text{mix}}}{\sigma_{\text{int+n}}^2}\right]
\]

\[
= (1-\alpha) B \log_2 \left[1 + \frac{b^2 (1-\beta) P_{\text{com}}}{\sigma_{\text{int+n}}^2}\right]
\]

\[
= (1-\alpha) B \log_2 \left[1 + \frac{b^2 (1-\beta) P_{\text{com}}}{\sigma_{\text{int+n}}^2}\right] + \left(\frac{\gamma^2 \sigma_{\text{proc}}^2 + k_B T_{\text{temp}}(1-\alpha) B}{\sigma_{\text{int+n}}^2}\right)
\]

\[
\sigma_{\text{int+n}}^2 = a^2 P_{\text{radar}}(1-\alpha)^2 \gamma^2 B^2 \sigma_{\text{proc}}^2 + k_B T_{\text{temp}}(1-\alpha) B
\]

The corresponding radar estimation rate inner bound is then given by

\[
R_{\text{est}} \leq B_{\text{mix}} \log_2 \left[1 + \frac{\sigma_{\text{proc}}^2 (T_p B_{\text{mix}}) a^2 P_{\text{radar}}}{k_B T_{\text{temp}} \alpha B}\right]^{\delta/(T B_{\text{mix}})}
\]

\[
= (1-\alpha) B \log_2 \left(1 + \text{SNR}_{\text{radar}}\right)^{\delta/(1-\alpha)TB}
\]

\[
\text{SNR}_{\text{radar}} = \frac{\sigma_{\text{proc}}^2 (1-\alpha) B (1-\alpha) TB a^2 P_{\text{radar}}}{k_B T_{\text{temp}}}
\]

We assume that \(1-\alpha) TB\), which is the waveform integration, is held constant as \(\alpha\) is varied so \(R_{\text{est}}\) is given by

\[
R_{\text{est}} \leq (1-\alpha) B \log_2 \left(1 + \frac{\sigma_{\text{proc}}^2 (1-\alpha) B \kappa a^2 P_{\text{radar}}}{k_B T_{\text{temp}}}\right)\]

\[
= \left(\frac{\gamma^2 \sigma_{\text{proc}}^2 + k_B T_{\text{temp}}(1-\alpha) B}{\sigma_{\text{int+n}}^2}\right)
\]

\[
\text{where waveform integration is denoted } \kappa = (1-\alpha) TB.\]

\[
\text{For some very large value of } \kappa, \text{ corresponding to a very small radar subband, the problem is no longer self consistent because } T > T_{\text{Rpi}}.
\]

**B. Examples**

In Figure 2, we display an example of inner bounds on performance. The parameters used in the example are displayed in Table 1. It is assumed that the communications system is received through an antenna sidelobe, so that the radar and communications receive gain are not identical. In the figure, we indicate a outer bound in red. We indicate in green, the bound on successive interference cancellation (SIC), presented in Equation (21). The best case system performance given SIC is at the vertex (at the intersection of the green and red lines), which is determined by the joint solution of Equations (21) and (19). The inner bound that linearly interpolates between this vertex and the radar-free communications bound in Equation (20) is indicated by the gray dashed line. The water-filling bound is indicated by the blue line. The water-filling bound
is not guaranteed to be convex. The water-filling bound is not guaranteed to be greater than the linearly interpolated bound. In general, the inner bound is produced by the convex hull of all contributing inner bounds. In the example, we see that the water-filling bound exceeds the linearly interpolated bound.

### Table II

| Parameter                        | Value   |
|----------------------------------|---------|
| Bandwidth                        | 5 MHz   |
| Center Frequency                 | 3 GHz   |
| Temperature                      | 1000 K  |
| Communications Range             | 10 km   |
| Communications Power             | 20 dBm  |
| Communications Antenna Gain      | 0 dB    |
| Radar Target Range               | 100 km  |
| Radar Antenna Gain               | 30 dB   |
| Radar Power                      | 1 kW    |
| Target Cross Section             | 10 m²   |
| Target Process Standard Deviation| 100 m   |
| Time-Bandwidth Product           | 100     |

![Graph showing data rate and estimation rate bounds](image)

Fig. 2. Data rate and estimation rate bounds. Outer bounds on communications and radar are indicated by the red lines. Successive interference cancellation (SIC) bound for the communications rate is indicated by the green dashed line. The linear interpolation between SIC vertex and the radar-free data rate bound is indicated by the gray dashed line. The water-filling inner bound is indicated by the blue line.

### IV. Conclusion

In this paper, we provide a novel approach for producing joint radar and communications performance bounds. An achievable inner bound based on a water-filling approach is developed and an example is presented. This is an initial investigation. There are a range of potentially significant improvements to the inner bounds, and potentially interesting extensions to the scenarios to which these bounds may be applied.

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