The nuclear shell model toward the drip lines

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Received 28 September 2011
Accepted for publication 16 November 2011
Published 28 September 2012
Online at stacks.iop.org/PhysScr/T150/014030

Abstract

We describe the ‘islands of inversion’ that occur when approaching the neutron drip line around the magic numbers \(N = 20\), \(N = 28\) and \(N = 40\) in the framework of the interacting shell model in very large valence spaces. We explain these configuration inversions (and the associated shape transitions) as the result of the competition between the spherical mean field (monopole) that favors magicity and the correlations (multipole) that favor deformed intruder states. We also show that the \(N = 20\) and \(N = 28\) islands are in reality a single one, which for the magnesium isotopes is limited by \(N = 18\) and \(N = 32\).

PACS numbers: 21.10.–k, 21.60.Cs, 27.30.+t, 27.40.+z

(Some figures may appear in colour only in the online journal)

1. Monopole anomalies and multipole universality

The different facets of nuclear dynamics depend on the balance of the two main components of the nuclear Hamiltonian: the monopole which produces the effective spherical mean field and the multipole responsible for the correlations [1]. Large-scale shell model calculations have unveiled the monopole anomalies of two-body realistic interactions, namely that they tend to produce effective single-particle energies which are not compatible with the experimental data and which, if used without modifications, produce spectroscopic catastrophes. As early as the late 1970s, Pasquini and Zuker [2] showed that the Kuo and Brown [3] interaction could produce neither a magic \(^{48}\)Ca nor a magic \(^{56}\)Ni. In the latter case it made a nearly perfect rotor instead. A few monopole corrections (mainly \(T = 1\)) restored high-quality spectroscopy. Otsuka et al [4] have recently surmised that the monopole component of the three-body force may explain the monopole anomalies relevant for \(^{28}\)O and \(^{48}\)Ca. The multipole component of the realistic two-body interactions (dominated by \(L = 0\) pairings, quadrupole and octupole) does not seem to require any substantial modification and it is ‘universal’ in the sense that all the interactions produce equivalent multipole Hamiltonians. Magic numbers are associated with energy gaps in the spherical mean field. Therefore, to promote particles above the Fermi level costs energy. However, in some cases intruder configurations can compensate for their loss of monopole energy with their huge gain in correlation energy. Several examples of this phenomenon exist in stable magic nuclei in the form of coexisting spherical, deformed and
superdeformed states in a very narrow energy range, providing examples of nuclear allotropy. In the case of $^{40}$Ca they can be described in the spherical shell model framework [5].

2. The islands of inversion at $N = 20$ and $N = 28$ far from stability

The region around $^{31}$Na provides a beautiful example of intruder dominance in the ground states, known experimentally for a long time [6, 7]. Early shell model calculations (Poves and Retamosa [8] and Warburton et al [9]) unveiled the role of deformed intruder configurations, $2p-2h$ neutron excitations from the sd to the pf-shell, and started the study of the boundaries of the so-called ‘island of inversion’ and the properties of its inhabitants. Similar mechanisms produce the other known ‘islands of inversion’ centered in $^{11}$Li ($N = 8$), $^{42}$Si ($N = 28$) and $^{44}$Cr ($N = 40$). We propose now a unified description of the nuclei between oxygen and calcium, covering in many cases all the isotopes between the neutron and proton drip lines. The valence space comprises two major shells: the sd-shell ($0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$), and the pf-shell ($0f_{7/2}$, $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$) and the effective interaction is SDPF-U described in [10].

$N = 20$. Four protons away from doubly magic $^{40}$Ca, $^{34}$Si is a new doubly magic nucleus because the proton $Z = 14$ and the neutron $N = 20$ gaps reinforce each other. To go even more neutron rich, one needs to remove protons from the $0d_{5/2}$ orbit. This causes two effects: a reduction of the $N = 20$ neutron gap (see figure 1) and an increase of proton collectivity. Both conspire in the sudden appearance of an island of inversion in which deformed intruder states become ground states, as in $^{32}$Mg, $^{31}$Na and $^{30}$Ne.

$N = 28$. As we remove protons from doubly magic $^{48}$Ca, the $N = 28$ neutron gap slowly shrinks. In $^{46}$Ar the collectivity induced by the action of the four valence protons in the nearly degenerate quasi-spin doublet $1s_{1/2}-0d_{3/2}$ is not enough to beat the $N = 28$ closure. $^{46}$Ar is non-collective. In $^{44}$Si, the quadrupole collectivity sets in. The $N = 28$ closure blows out and prolate and non-collective states coexist. The ground state and the first excited $2^+$ form the germ of a prolate rotational band. In turn, $^{42}$Si is an oblate, well-deformed rotor with a first $2^+$ state at 770 keV [11], and $^{46}$Mg is predicted to be a very collective prolate rotor, with a $2^+$ at $\sim$720 keV. In addition, it could well develop a neutron halo because more than two neutrons are, on average, in p wave.

In the left panel of figure 2, we compare the experimental $2^+$ excitation energies of the even Mg isotopes with the shell model calculations with the SDPF-U interaction. Up to $N = 16$ the calculations are restricted to the sd-shell and therefore the results are the same as those produced by

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Figure 2. Comparison of the theoretical and experimental $2^+$ excitation energies of the even Mg isotopes (left panel) and $B(E2)$s (right panel). On the proton-rich side, some experimental energies are taken from their mirror nuclei.

Figure 3. Comparison of the theoretical and experimental $2^+$ excitation energies of the even Ne isotopes (left panel) and even Si isotopes (right panel).
the USD interaction [12]. Beyond $N = 16$ the calculations include up to 6p–6h excitations from the sd-shell to the full pf. The agreement is excellent and covers the entire range of isotopes from the proton to the neutron drip line. Note the disappearance of the semi-magic closures at $N = 20$ and $N = 28$ and the presence of a large region of deformation which connects the two islands of inversion, previously thought to be split apart. In the left panel, we compare the $B(E2)$s in the transition region with some very new experimental data from Riken. The agreement is very good as well.

The results for the neon isotopes (left panel of figure 3) are very similar to the magnesiums. In the right panel, we show the results for the silicon isotopes (note the very different energy scale). At odds with the magnesium case, we observe a majestic peak at $N = 20$, a fingerprint of the double magic nature of $^{34}$Si discussed above, and as in the Ne and Mg cases, no trace of the $N = 28$ shell closure is seen.

3. The island of deformation south of $^{68}$Ni

The situation at $N = 40$ is similar to that found at $N = 20$ except that $^{68}$Ni is not a ‘bona fide’ magic nucleus. Removing protons from the $0f_{5/2}$ orbit activates the quadrupole collectivity, which, in turn, favors the np–nh neutron configurations across $N = 40$, which take advantage of the quasi-SU3 coherence of the doublet $0g_{9/2}$–$1d_{5/2}$. Large-scale shell model (SM) calculations in the valence space of the full pf-shell for the protons and the $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$, $0g_{9/2}$ and $1d_{5/2}$ orbits for the neutrons predict a new region of deformation centered at $^{64}$Cr. In figure 4, we show our results for the $N = 40$ isotones; the inversion of configurations sets in very rapidly when we remove protons from $^{68}$Ni and persists all the way down to $^{66}$Ca even in the absence of deformation. This shows that the island of inversion and the island of deformation may not cover the same territory. More details of these calculations can be found in [13].

Acknowledgments

This work was partly supported by the Spanish Ministry of Ciencia e Innovación under grant no. FPA2009-13377, by the Comunidad de Madrid (Spain) project HEPHACOS S2009/ESP-1473 and by the IN2P3 (France)–CICyT (Spain) collaboration agreements.

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