Damping the neutrino flavor pendulum by breaking homogeneity

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The most general case of self-induced neutrino flavor evolution is described by a set of kinetic equations for a dense neutrino gas evolving both in space and time. Solutions of these equations have been typically worked out assuming that either the time (in the core-collapse supernova environment) or space (in the early universe) homogeneity in the initial conditions is preserved through the evolution. In these cases one can gauge away the homogeneous variable and reduce the dimensionality of the problem. In this paper we investigate if small deviations from an initial postulated homogeneity can be amplified by the interacting neutrino gas, leading to a new flavor instability. To this end, we consider a simple two flavor isotropic neutrino gas evolving in time, and initially composed by only $\nu_e$ and $\bar{\nu}_e$ with equal densities. In the homogeneous case, this system shows a bimodal instability in the inverted mass hierarchy scheme, leading to the well studied flavor pendulum behavior. This would lead to periodic pair conversions $\nu_e \bar{\nu}_e \leftrightarrow \nu_\mu \bar{\nu}_\mu$. To break space homogeneity, we introduce small amplitude space-dependent perturbations in the matter potential. By Fourier transforming the equations of motion with respect to the space coordinate, we then numerically solve a set of coupled equations for the different Fourier modes. We find that even for arbitrarily tiny inhomogeneities, the system evolution runs away from the stable pendulum behavior: the different modes are excited and the space-averaged ensemble evolves towards flavor equilibrium. We finally comment on the role of a time decaying neutrino background density in weakening these results.

PACS numbers: 14.60.Pq, 97.60.Bw

I. INTRODUCTION

Neutrino–neutrino interactions in dense neutrino media are known to produce surprising flavor oscillation effects, in the form of self-induced conversions, when the typical neutrino self-interaction potential $\mu = \sqrt{2} G_F n_\nu$ is comparable with or greater than the vacuum oscillation frequency $\omega = \Delta m^2 / 2 E$ (see e.g. [1] for a recent review). This situation can be encountered in the early universe or in core-collapse supernovae (SN), where neutrino themselves form a background medium for their propagation. Differently from the usual Mykheyeev–Smirnov–Wolfenstein (MSW) effect [2], associated with the matter potential $\lambda = \sqrt{2} G_F n_e$, the self-induced effects do not change the flavor content of the neutrino ensemble. Yet, the flavor is exchanged between different momentum modes, leading to peculiar spectral features known as spectral swap and split [3].

The growth of these effects is associated with instabilities in the flavor space, which are amplified by the neutrino-neutrino interactions [4, 5]. An example is represented by the bimodal instability [6] of an isotropic and homogeneous dense gas of neutrinos and antineutrinos in equal amounts. They convert from one flavor to another in pair production processes $\nu_e \bar{\nu}_e \leftrightarrow \nu_\mu \bar{\nu}_\mu$, behaving as a flavor pendulum even if the mixing angle is very small [4, 5]. In this case, the vacuum mixing angle acts as a seed triggering the flavor instability. In non-isotropic neutrino gases, like the case of neutrinos streaming-off a SN core, the features of the self-induced effects are more involved, since the current-current nature of the low-energy weak interactions introduces an angle dependent term $(1 - v_\nu \cdot v_\nu)$, where $v_\nu$ are neutrino velocities [6, 10]. It has been shown that this term can lead to a multi-angle instability, which hinders the maintenance of the coherent oscillation behavior for different neutrino modes [10, 12]. In particular, in a symmetric gas of equal neutrino and antineutrino densities even a very small anisotropy is sufficient to trigger a run-away towards flavor equipartition [13]. An additional instability has been recently discovered in the SN context. Removing the assumption of axial symmetry in the $\nu$ propagation, a multi-azimuthal-angle instability emerges, even assuming a perfect spherically symmetric $\nu$ emission [14, 15].

Symmetries in the neutrino self-induced evolution are often assumed in order to reduce the complexity of the problem. Nevertheless, these recent findings question the validity of these assumptions, since they suggest that (unavoidable) small deviations from initial symmetries could be dramatically amplified by the interacting neutrinos during the evolution. In absence of collisions, the dynamics of the $\nu$ space-dependent occupation numbers or Wigner function $\rho_{p,x}(t)$ with momentum $p$ at position $x$ is ruled by the kinetic equations [20, 21]

\[
\partial_t \rho_{p,x} + v_p \cdot \nabla_x \rho_{p,x} + \dot{p} \cdot \nabla_p \rho_{p,x} = -i [\Omega_{p,x}, \rho_{p,x}] ,
\]

(1)
with the Liouville operator in the left-hand side. In particular, the first term accounts for an explicit time dependence, while the second is the drift term proportional to the neutrino velocity \( \nu_p \), due to particle free streaming. Finally, the third term is proportional to the force acting on neutrinos. On the right-hand-side the matrix \( \Omega_{p,x} \) is the full Hamiltonian containing the vacuum, matter and self-interaction terms. We remind the reader that the quantum-mechanical uncertainty between location and momentum implies that this formalism can be applied only for cases where spatial variations of the ensemble are weak on the microscopic length-scale defined by the typical particle wavelength.

In general, Eq. \( \text{(1)} \) describes a seven-dimensional problem that has never been solved in its complete form. For neutrino flavor conversions in the early universe one typically assumes initial space homogeneity, that allows one to reduce the dependence on space-time variables to time only. Conversely, for neutrinos in a SN environment, a spatial evolution under the assumption of a stationary neutrino emission is often considered. For a spherically symmetric neutrino emission with negligible variations in the transverse direction, the description further simplifies, the problem being reduced to a purely radial dynamics. However, small space inhomogeneities over the standard rotation and translation invariant background are expected in the early universe, with an initial spectrum in Fourier space very close to the scale invariant Harrison-Zeldovich one, the typical heritage of an inflationary expansion initial stage. On the other hand, in the SN environment one should account for deviations with respect to a stationary configuration, which are related to hydrodynamical instabilities. Both these deviations with respect to the assumed homogeneity conditions can act as seeds for instabilities.

In order to investigate this issue, rather than studying the behavior of the complex early universe or SN systems, we consider here a much more simple toy model, which however already illustrates the main point of this paper. Namely, unless spatial symmetry (or stationarity) is imposed by hand, the self-interacting neutrino dynamics is unstable with respect to even tiny ripples over a spatially constant (or time independent) background. In particular, we consider a neutrino ensemble in time and one spatial dimension, initially prepared with equal momentum implies that this formalism can be applied only for cases where spatial variations of the ensemble are weak on the microscopic length-scale defined by the typical particle wavelength.

The paper is organized as follows. In Section II we describe the equations of motion for the neutrino ensemble evolving in time in presence of inhomogeneities. By Fourier transforming the equations in the spatial coordinate, the problem is then reduced to ordinary differential equations in the time variable for the different Fourier modes, which are coupled each other. In Section III we numerically solve these equations for a constant background neutrino potential \( \mu > \omega \) and we show how the system exhibits a run–away from the flavor pendulum behavior, even for very small matter perturbations. The decoherence is indeed, associated with the growth of the different Fourier modes that destabilize the ordered pendulum solution. We also consider the effect of a time depending neutrino self-potential on this instability. If \( \mu \) is a decreasing function, as we expect to be the case in both the early universe (with respect to time) or SN scenarios (with respect to distance), the growth of higher wave number Fourier modes might be inhibited, for a sufficiently short decay time scale. Finally, in Section IV we summarize our results, we comment about the possible effects in realistic neutrino gases and we conclude.

II. EQUATIONS OF MOTION WITH INHOMOGENEITIES

A. The general formalism

We start from the equations of motion for a monoenergetic homogeneous and isotropic two-flavor \((\nu_e, \nu_x)\) relativistic neutrino gas, propagating in time. Expanding all quantities in Eq. \( \text{(1)} \) in terms of Pauli matrices, one gets the well-known pendulum equations \( \text{(2)} \)

\[
\begin{align*}
\partial_t \mathbf{P} &= [+\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{D}] \times \mathbf{P} , \\
\partial_t \mathbf{P} &= [-\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{D}] \times \mathbf{P} ,
\end{align*}
\]

where \( \mathbf{P} (\mathbf{P}) \) are the neutrino (antineutrino) polarization vectors in flavor space. We define as usual \( \mathbf{D} = \mathbf{P} - \bar{\mathbf{P}} \). The vacuum oscillation frequency is \( \omega = \Delta m^2 / 2E \), \( \mathbf{L} = e_x \), and \( \lambda = \sqrt{2} G_F n_e \) is the effective potential due to forward scatterings with electrons. We remind the reader that a possibly large homogeneous and (in 3 dimensions) isotropic matter term only reduces the effective mixing angle, and can be rotated away from the equations of motion \( \text{(3)} \). The unit vector \( \mathbf{B} \) points in the mass eigenstate direction in flavor space, such that \( \mathbf{B} \cdot \mathbf{L} = -\cos \theta \), where \( \theta \) is the vacuum mixing angle. Finally, \( \mu \sim \sqrt{2} G_F n_e \) is the neutrino-neutrino interaction strength.

We now consider a non homogeneous background. In this case the evolution operator in the left-hand-side of Eq. \( \text{(2)} \) acquires also a space derivative. In the simplest case of neutrinos propagating in one spatial dimension only, the equation of motion for \( \nu \) becomes

\[
(\partial_t + \partial_x) \mathbf{P}(x,t) = [+\omega \mathbf{B} + \lambda(x,t) \mathbf{L} \\
+ \mu (x,t) \mathbf{D}(x,t)] \times \mathbf{P}(x,t),
\]

and analogously for \( \bar{\mathbf{P}} \). We notice that the third term in the left-hand side of Eq. \( \text{(1)} \) has been neglected since we
are considering a single momentum neutrino ensemble. We remark that in a multi-momentum scenario it easy to realize that it is of order \((\lambda_0 p)^{-1}\), with \(\lambda_0\) the length scale over which the background potential in the Hamiltonian is varying, and \(p\) is a typical neutrino momentum. As long as non homogeneities only contain Fourier modes with wavelengths much larger than the neutrino de Broglie wavelength, we have \((\lambda_0 p)^{-1} \ll 1\). Therefore, this term is expected to be smaller with respect to the other ones in the equations.

The partial differential equation \((6)\) can be transformed into a tower of ordinary differential equations for the Fourier modes

\[
P_k(t) = \int_{-\infty}^{+\infty} dx \ P(x, t) e^{-ikx}.
\]

We find

\[
\partial_t P_k = -ik P_k + \omega B \times P_k + \int_{-\infty}^{+\infty} \frac{dk'}{2\pi} \lambda_k L \times P_{k-k'} + \int_{-\infty}^{+\infty} \frac{dk'}{2\pi} \int_{-\infty}^{+\infty} \frac{dk''}{2\pi} \mu_{k-k'} \times P_{k''} \ D_{k''} \times P_{k''},
\]

where \(\lambda_k(t)\) and \(\mu_k(t)\) are the Fourier transform of \(\lambda(x, t)\) and \(\mu(x, t)\), respectively. An analogous set of coupled equations can be written for antineutrino polarization vector \(\bar{P}\).

**B. Monochromatic matter inhomogeneity**

We want to study the simplest model exhibiting an inhomogeneity feature. We assume that the background neutrino potential \(\mu\) is homogeneous, while we parametrize the space fluctuations of the matter background by a single wavelength oscillating term of amplitude \(\epsilon \ll \mu, \omega\) and wavenumber \(k_0\), i.e.

\[
\lambda = \epsilon \cos(k_0 x).
\]

Of course, this can be generalized to arbitrary fluctuation smooth profiles, which can be written as a linear superposition of several modes. The Fourier transform of Eq. \((6)\) is quite simple

\[
\lambda_k = \epsilon \pi \left[ \delta(k-k_0) + \delta(k+k_0) \right],
\]

and Eq. \((5)\) becomes

\[
\partial_t P_k = -ik P_k + \omega B \times P_k + \frac{\epsilon}{2} [P_{k-k_0}(t) + P_{k+k_0}(t)]
\]

\[
+ \mu \int_{-\infty}^{+\infty} \frac{dk'}{2\pi} \ D_{k-k'} \times P_{k'},
\]

where we have used the fact that since \(\mu\) is spatially constant, \(\mu_k = 2\pi \mu \delta(k)\).

It is easy to see that for a monochromatic matter perturbation \(\lambda\) as in Eq. \((6)\), only higher harmonics of the fundamental mode with \(k_n = nk_0\) are excited. Defining \(P_n = k_0 P_{kn}/(2\pi)\), Eq. \((8)\) reduces to a countable tower of coupled equations

\[
\partial_t P_n = -ik_n P_n + \omega B \times P_n + \frac{\epsilon}{2} L \times [P_{n-1} + P_{n+1}]
\]

\[
+ \mu \sum_{j=-\infty}^{+\infty} D_{n-j} \times P_j.
\]

Indeed, we can follow the evolution for positive modes only, i.e. \(n \geq 0\), since the polarization vector \(P(x, t)\) is a real function and therefore

\[
P_n* = P_{-n}.
\]

To have a clear feeling of how inhomogeneities propagate into the polarization vector, i.e. how \(P_n\) and \(P_0\) get excited, we explicitly write the two lowest-order equations for neutrinos, i.e.

\[
\partial_t P_0 = \omega B \times P_0 + \frac{\epsilon}{2} L \times [P_1 + P_1] + \mu D_0 \times P_0 + \mu \sum_{j=0}^{\infty} (D_j \times P_j + D_j \times P_j^*)
\]

\[
\partial_t P_1 = -ik_0 P_1 + \omega B \times P_1 + \frac{\epsilon}{2} L \times [P_0 + P_2] + \mu [D_1 \times P_0 + D_0 \times P_1 + D_2 \times P_1^* + \ldots].
\]

Suppose we start with a perfectly homogeneous initial condition, with only \(P_0 \neq 0\). The first Fourier mode \(P_1\) is then excited, since it is sourced by \(P_0\) in the inhomogeneous matter term. All other modes are then triggered in sequence in the same way. It is also worth noticing that the evolution of the fundamental mode \(P_0\) is perturbed by the presence of the other Fourier modes \(P_j\) in its equation. As we will see, this leads to a dephasing of the flavor pendulum with respect to the homogeneous evolution.

### III. NUMERICAL EXAMPLES

To illustrate the behavior of self–induced flavor conversions in presence of inhomogeneities we consider a neutrino gas initially composed by \(\nu_e\) and \(\bar{\nu}_e\), only, with equal densities. As initial condition we take \(P_{0,\bar{z}} = P_{0,\bar{z}} = 1\). These components represent the space-averaged flavor content in \(\nu\) and \(\bar{\nu}\). All other Fourier modes vanish, \(P_n(0) = P_n(0) = 0\) for \(n > 0\), so the system starts from a spatially constant configuration. Numerically, we fix the parameters in Eq. \((7)\) at \(\mu = 50\), \(\omega = 1\), \(\theta = 10^{-2}\) and we consider inverted mass hierarchy \(\Delta m^2 < 0\), for which the system is unstable in the homogeneous case \((\bar{\nu})\). In order to get stable numerical results we follow the evolution of the first \(N = 20\) Fourier modes. We also solved
the oscillation scale.

When the scale of the perturbation is of the order of the oscillation length scale, \( |\kappa| \) is of the order of \( \sqrt{2\omega\mu} \).

In Fig. 1 are also shown the evolution when we switch on an inhomogeneity in the matter potential with \( \epsilon = 10^{-7} \) (dashed curve) and \( \epsilon = 10^{-3} \) (continuous curve), respectively. In this case we have considered as wavenumber of the fluctuation \( k_0 = \kappa \). Notice that even for a very small inhomogeneity the coherent behavior of the pendulum is broken after some oscillation periods and the system decoheres towards flavor equilibrium. As expected, increasing the value of the inhomogeneity seed this flavor decoherence is reached earlier. This effect of flavor equilibrium is observed in \( P_{0,z} \), that represents the flavor content averaged over all the space. Indeed \( P_{0,z}(x, t) \) shows large space fluctuations, due to the interference of various contributions of higher harmonics \( k_n \). We expect that considering a more realistic multi-mode system, with the matter term seed containing several Fourier modes \( k_0 \), it would easily decohere also in the coordinate space.

Fig. 2 is a different way to see this phenomenon. We compare the evolution of the trajectory of the zero mode polarization vector \( P_0 \) in the \( x-z \) plane for the homogeneous case (left panel) with the case of an inhomogeneous seed with \( \epsilon = 10^{-3} \) (right panel). While in the first case the polarization vector performs stable pendular oscillations, keeping its modulus constant, in the inhomogeneous case after few periods, its length shrinks to zero, meaning that the flavor content averaged over the space coordinate is equal for two \( \nu \) species.

In Fig. 3 we fix \( \epsilon = 10^{-3} \) and we rather illustrate how changing the wavenumber of the matter perturbation affects the onset of the decoherence. With the definition \( k_0 = c\kappa \), the continuous dashed and dotted curves correspond to \( c = 1 \), \( c = 10^2 \) and \( c = 10^{-2} \), respectively. When the scale of the perturbation \( k_0 \) is of the order of the oscillation scale \( \kappa \), the flavor decoherence is approached earlier. Lowering \( c \) means considering a longer wavelength with respect to the oscillation length scale, and therefore the neutrino system needs more oscillation cycles to feel the inhomogeneities of the background and eventually decohere. On the other hand, increasing \( c \) the fluctuations tend to be averaged during an oscillation cycle, and this again tends to shift at larger time the onset of the decoherence with respect to the case \( c = 1 \).

The run away of the solution from the stable pendulum behavior is due to the growth of modes with \( n > 0 \), first triggered by the coupling of \( P_1 \) to \( P_0 \) and so on. This is shown in Fig. 4, where we show the evolution of the modulus of the first four modes \( |P_n| \), \( n = 1, \ldots, 4 \) for the case \( c = 1 \). After \( P_1 \) starts raising, the higher Fourier modes are also rapidly excited in sequence reaching \( |P_n| \sim 0.1 \).

**FIG. 1**: Evolution of the component \( P_{0,z} \) for \( k_0 = \kappa \). The continuous curve corresponds to a fluctuation seed \( \epsilon = 10^{-3} \), the dashed one to \( \epsilon = 10^{-7} \), while the dotted one is for the homogeneous case with \( \epsilon = 0 \).

**FIG. 2**: Trajectory of the zero mode polarization vector \( P_0 \) in the \( x-z \) plane for the homogeneous case (left panel) and for an inhomogeneity with \( \epsilon = 10^{-3} \) and \( k_0 = \kappa \) (right panel).

**FIG. 3**: Evolution of the component \( P_{0,z} \) for \( \epsilon = 10^{-3} \) and different values of \( k_0 = c\kappa \): \( c = 1 \) (continuous curve), \( c = 10^2 \) (dashed curve) and \( c = 10^{-2} \) (dotted curve).
FIG. 4: Evolution of the first four modes $|P_n|$ for the case $c = 1$ and $\epsilon = 10^{-3}$. The continuous curve corresponds to $n = 1$, the dashed one to $n = 2$, the dash-dotted one to $n = 3$ and the dotted to $n = 4$.

So far we have assumed that the background neutrino density is independent of time. This guarantees that the non linear term in the kinetic equations keeps always the same order of magnitude, so that eventually all $P_n$ are excited, after some sufficiently time laps. In more realistic scenarios, as in the early universe or in core-collapse SNe, neutrino density is rather expected to be diluted. In the first case, this is due to universe expansion, while for SN it is the effect of the radial matter profile. If $n_\nu$ decays too fast, the system is unable to develop the instability we have discussed so far, and the behavior of the system may be closer to the standard pendulum result.

To illustrate this point we have considered an exponential decaying term $\mu = \mu_0 \times \exp(-t/\tau)$, with different choices of the characteristic time $\tau$, where we take $\mu_0 = 50$. Our results are shown in Fig. 5, having set $\epsilon = 10^{-3}$, and $k_0 = \sqrt{2\mu_0\omega}$. In the left panels we show the evolution of $P_{0,z}$ and the modulus $|P_0|$, while in the right panels we report the evolution of the first Fourier modes $|P_1|$ (continuous curves) and $|P_2|$ (dashed curves). If the time evolution of $\mu$ is sufficiently slow (i.e. $\tau = 10^3$, upper panel) the neutrino ensemble quickly decoheres, basically as in the case of constant $\mu$. For a smaller $\tau = 10^2$ (middle panel) the system still shows this behavior, indeed $|P_0|$ drops (and the Fourier modes are excited). However the final value of $P_{0,z}$ is not zero, but the system tries to follow the slow decay of $\mu$, in a way closer to an homogeneous scenario with $\epsilon = 0$. Finally, for $\tau = 10$ (lower panel) $\mu$ declines too fast to allow the Fourier modes to develop. The system does not decohere, $|P_0|$ is fixed to unity, and the behavior is similar to what expected for an homogeneous system, i.e. an inversion of the polarization vector with respect to the initial value. Summarizing, we see that decoherence associated with inhomogeneities only grows for a adiabatic evolution of the neutrino background medium, with decay time scales larger than the typical oscillation frequency of the system.

IV. CONCLUSIONS

The role of symmetries in reducing the complexity of the dynamics of self-interacting neutrino systems has been widely exploited. The general seven-dimensional differential problem can be reduced to more treatable models, and numerically solved with less demanding computation powers. However, simplifying the scenarios to the only time or radial evolution of neutrino density matrix, neglecting space inhomogeneities or non stationary features, provides useful results only if the dynamics is stable against perturbations. If this is not the case, the behavior of neutrino medium can be very different from what is found when a particular symmetry is imposed by hand.

In this context we have investigated the emergence of a new kind of instability in the flavor evolution of a dense neutrino gas, when space homogeneity is slightly perturbed. In order to illustrate this effect we have considered the time evolution of a simple system based on an isotropic neutrino ensemble, initially composed by only
$\nu_e$ and $\bar{\nu}_e$ with equal densities. We have introduced a small amplitude position-dependent perturbation. In order to follow the simultaneous temporal and spatial flavor evolution we have Fourier transformed the equations of motion, obtaining a tower of equations for the different modes associated with the space variable. These modes are coupled because of the effect of the inhomogeneities on the neutrino-neutrino interaction term. We found that in inverted mass hierarchy, where an homogeneous neutrino gas would have evolved according to the flavor pendulum solution [2], the presence of the inhomogeneous term destroys this coherent behavior and leads to flavor decoherence. This is due to the growing of modes with non zero wavenumber, excited by their coupling to the neutrino homogeneous zero mode. The system is instead stable in the normal mass hierarchy case.

The instability discussed here complements the findings of [13], where for the same $\nu - \bar{\nu}$ symmetric case, breaking the isotropy of the neutrino propagation leads to a quick decoherence in both mass hierarchies. In that case the equations of motion were expanded in multipoles through the Legendre functions and the decoherence was associated to the excitation of the higher multipoles. Breaking the homogeneity we are observing a similar phenomenology.

Despite the simplicity of our model – one space dimension and a monochromatic matter potential disturbance – we think that features similar to those described in this paper would be also present in more realistic cases. There are indeed, two main frameworks where inhomogeneities (in time or space) could play a role in self-induced flavor conversions: neutrinos evolution in the early universe or in their streaming off core-collapse supernovae. In the first case we know that lepton and neutrino number densities keep the imprint of small space perturbations over the homogeneous background configuration. These are likely produced during the inflationary phase with an almost scale invariant spectrum and initial amplitudes of order $10^{-5}$. In the second example, deviations from a stationary evolution are expected to be triggered by hydrodynamical instabilities.

In both these ensembles the neutrino density decreases with respect to the evolution variable. In order to mimic this effect, we have considered a declining matter potential $\mu$. We found that this could inhibit the growth of the different Fourier modes, since the evolution is not enough adiabatic to let them to develop. The decoherence effect could be thus, suppressed in realistic scenarios. However, it is not guaranteed that the simultaneous breaking of isotropy and homogeneity may not lead to novel phenomena. At this regard the impact of multi-angle effects requires a detailed investigation that we leave for a future work.

Finally, we would like to stress that the formalism we have developed here to treat the simultaneous time and space flavor evolution could be applied to study other deviations from a stationary SN neutrino flavor evolution, as those which could be induced by the small backward flux caused by residual neutrino scattering that causes significant refraction [24, 25]. In any case it is intriguing that even the simplest neutrino flavor pendulum is still a source of new instabilities that were not appreciated before.

Acknowledgements

A.M. acknowledges Günter Sigl for inspiring discussions. We thank Georg Raffelt for useful comments on the draft. G.M. acknowledges support by the Istituto Nazionale di Fisica Nucleare I.S. FA51. The work of A.M. was supported by the German Science Foundation (DFG) within the Collaborative Research Center 676 “Particles, Strings and the Early Universe”. N.S. acknowledges support from the European Union FP7 ITN INVISIBLES (Marie Curie Actions, PITN- GA-2011- 289442).

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