Influence of a Tilt of Mirror Surface on the Measurement Accuracy of Laser Triangulation Rangefinder

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Abstract. Results of investigation of influence of tilt of mirror surface on the measurement accuracy of laser triangulation rangefinder are represented. Optical system of sensor satisfying to Scheimpflug’s principle is considered. The measuring errors produced by asymmetry of light distribution in photorecording plane at sensing a surface by anisotropic Gaussian light beam are analyzed. The example of measuring a melt level in the crucible during the Czochralski pulling is considered at which the tilt of a melt surface depends on angular velocity of crucible rotation.

1. Introduction
Laser triangulation is widely used in various applications to measure the distances up to the objects. A common triangulation principle is to project a light spot onto the object and extract the distance information from the reflected or scattered light. There are many works in which metrological characteristics of laser triangulation sensors have been investigated [1–4]. Usually, in these works the characteristics important only at sensing of scattering surface are observed. At the same time the laser triangulation is applied to sensing a smooth or mirror surface too. The examples of such devices are melt level measuring systems at crystal growing. Some peculiarities of mirror surface testing by laser triangulation, in particular, of measuring a melt level \( h \) at crystal growing, are considered in publications [5,6].

In this paper the results of investigations of influence of a tilt of mirror surface on the measurement accuracy of laser triangulation rangefinder are represented. We consider the optical system of sensor satisfying to Scheimpflug’s principle [1,6]. The measuring errors produced by asymmetry of light distribution in photorecording plane at sensing a surface by anisotropic Gaussian light beam are analyzed. The example of measuring a melt level in the crucible during the Czochralski pulling is considered at which the tilt of a melt surface depends on angular velocity of crucible rotation.

2. Principle of measurements
The principle of triangulation measurements is explained in figure 1. It shows a probing light beam \( a \) projected on a mirror surface \( F(x,y,z)=0 \) in point \( M \) at angle \( \alpha \) to the axis \( z \) of global coordinate system \( xyz \). The reflective ray \( b \) coincides with the optical axis of the sensor. The output (image) plane \( P_\theta(z_\theta,y_\theta) \) of the sensor is tilted to the optical axis under the angle \( \beta \) and optically conjugated with the plane \( P_\phi(z_\phi,y_\phi) \) passing through the probing beam according to the Scheimpflug condition: \( \tan(2\alpha) = M \tan(\beta) \), \( 1/d_1+1/d_2=1/F \), where \( F \) is the focal length of the lens and \( M=d_2/d_1 \) is the magnification of the optical system.
To calculate the light distribution at plane $P_\beta$ (or $P_c$) we define three additional coordinate systems $x_1y_1z_1$, $x_2y_2z_2$ and $x_\text{l}y_\text{l}z_\text{l}$, linked with incident $a$ and reflected $b$ light beams and also with the Scheimpflug’s plane $P_c$ (figure 2). If directions of coordinate axes $x_1$, $y_1$ are coincident with the directions of anisotropic axes $x_\text{l}$, $y_\text{l}$ of probing Gaussian light beam, the intensity distribution in light beam can be written in form:

$$I(x_1, y_1, z_1) = \left(\pi \sigma_x \sigma_y \right)^{-1} \exp\left(-\frac{x_1^2}{\sigma_x^2} - \frac{y_1^2}{\sigma_y^2}\right),$$

where the width of beam depends on coordinate $z_1$ along its axis:

$$\sigma_z = \sigma_z^0 \left[1 + \left(\frac{\lambda z_1}{\pi \sigma_z^2 z_0^2}\right)^2\right]^{1/2}$$

and $\lambda$ is the light wavelength. The vector of reflected light beam is determined by expression:

$$b = -a + 2n(a n),$$

where $n=(F_x', F_y', F_z')$ is a surface normal. Initial directions of an anisotropy of an incident light beam $x_1$ and $y_1$ are conversed to the corresponding directions of anisotropy of reflected beam $x_2$ and $y_2$ also according to expression (1). Relationship between the coordinates is set by the rotation matrix $R$:

$$(x_2, y_2, z_2)^T = R(x_c, y_c, z_c)^T.$$  (2)

Analyzeable light distribution $I_c(y_c, z_c)$ is set by cross-section of reflected light beam by Scheimpflug’s plane $P_c$:

$$I_c(y_c, z_c) = I(x_2(y_c, z_c), y_2(y_c, z_c), z_2(y_c, z_c)),$$  (3)
where coordinates relationships are determined by equation (2) at $x_c = 0$. The measuring error of sensor $\Delta z$ is defined by error $\Delta z_c$ of measuring the position of light spot in plane $P_c$: $\Delta z = \Delta z_c \cos(\alpha)$.

The analogous approach to calculation of light distribution was used when the incident light beam is situated in Scheimpflug's plane and is tilted to the plane $y = 0$ (beam direction $A_1$ in figure 2). The beam axis passes through the point $y_c = d$, $z_c = -z_0 / \cos(\alpha)$. Such measuring scheme can be used for multibeam triangulation measurements with various directions of beams $A$, $A_1$, etc. (figure 2) and allows to expand the range of admissible surface tilts at which measuring by laser triangulation is possible, in particular to increase the range of admissible angular velocities of melt rotation during the growing of a crystal [5].

3. Simulation results

For estimation of influence of surface tilt on metrological features of laser triangulation rangefinder the calculations of light distribution (3) and position of light spot was carried out at variations of parameters of laser beam and tilt of a surface, and also of geometry of the measuring scheme. Position of light spot was determined by two techniques: by measuring position of maximum intensity of light distribution ($\text{max}_z z_c$) and by measuring centre of light spot at some threshold level of intensity (center).

In figure 3 the results of calculations of coordinate $z_c$ of sensing light beam that determine a measuring error of rangefinder are presented, depending on surface tilt $\beta$ for two methods of beam position determination and for various distances $\text{dist}$ from a sensing point to beam waist plane. The presented results correspond to isotropic sensing beam with incidence plane $y = 0$. The tilt of surface changes symmetry of light distribution along the axis $z_c$ and causes the measuring errors of sensor. The measuring errors depend on triangulation angle $\alpha$, beam width $\sigma_b$, distance $\text{dist}$ between the sensing point and waist plane and quickly rise at negative angles of surface tilts. Error value also depends on technique of measuring the position of light spot: measuring the position of maximum intensity of light distribution results in greater error level in comparison with measuring the centre of light spot at some threshold level of intensity.
The results related to measuring of melt level during the crystal growth, are submitted in figures 4 and 5. The tilt of melt surface depends on coordinates of probing point and angular velocity of melt rotation \( \omega \). The shape of melt surface is described by expression [5]:

\[
F(x, y, z) = z - \omega^2 (x^2 + y^2 - r^2) / (2g) - h = 0,
\]

where \( h \) is the melt level, \( r_1 \) is the distance of point M (position of melt level measuring) with coordinates \((-htg(\alpha), y_0, h)\) from rotation axis of crucible, \( g \) is the acceleration of gravity. It is supposed, that condition \( z = h \) in equation (4) for any point of sensing is fulfilled by crucible control system.

In figure 4 the synthesized images of analyzed light distribution (3) are shown for various angular velocities of melt rotation \( N \) (rpm) and different values of melt level \( h \) (mm) for anisotropic sensing beam with parameters of anisotropy \( \sigma_{x0} = 0.02, \sigma_{y0} = 2 \) and distance of sensing point from centre of crucible \( y_0 = 150 \) mm. The triangulation angle \( \alpha \) in all cases is equal to 10 degree.

From figure 4 follows, that variations of angular velocity of melt rotation lead to declination of surface at point of sensing, according to equation (4), and cause rotation and asymmetry of light distribution along the analyzing direction \( z_c \). Variations of direction of probing beam \( (d \neq 0) \) lead to additional asymmetry of light distribution. Results of calculation of position and width of analyzed light spot versus angular velocity of melt rotation are shown in figure 5. It follows from calculations and figure 5 that the measured positions of analyzable light spot and measuring error of triangulation sensor depend on parameters of measuring scheme and can be considerable only for large values of angular velocity of melt rotation. Position of extremum of light distribution is most critical to change of angular velocity of melt rotation while the position of the center of light distribution, for chosen values of parameters, is weakly dependent on \( N \) and only for large values \( N \) can reach a few micrometers.
Figure 4. Synthesized images of light distribution in Scheimpflug’s plane.
4. Conclusion
From carried out investigations follows, that at use of laser triangulation range finder for precision measurements of distances up to objects with smooth (mirror) surface it is necessary to take into account the possibility of occurrence of measuring errors induced by variations of surface tilt in a point of probing. The tilt of surface causes rotation and asymmetry of registered light distribution. The distortions of light distribution depend on surface tilt, geometry of measuring scheme and parameters of laser beam: position of beam waist relative to surface and beam divergence. Distortions of light distribution produce the measurement errors of light beam position that lead to occurrence of measurement errors of a sensor.

References
[1] Baribeau R and Rioux M 1991 Influence of speckle on laser range finders J. Appl. Opt. 30 20 2873–78
[2] Hausler G and Herrmann J 1992 Physical limits of 3D-sensing Optics, Illumination, and Image Sensing for Machine Vision VII: Proc. SPIE ed D J Svetkoff 1822 150–158
[3] Dremel W, Hausler G and Maul M 1986 Triangulation with large dynamical range Optical Techniques for Industrial Inspection: Proc. SPIE ed P G Cielo 665 182–187
[4] Vertoprakhov V V 1995 Influence of the shape of an object and the orientation of its surface on the accuracy of laser triangulation measurements Optoelectronics, instrumentation and data processing (Avtometriya) 6 61–65
[5] Mikhlyaev S V 2003 Triangulation sensing of melt surface during crystal growth Optoelectronics, instrumentation and data processing (Avtometriya) 39 5 25–34
[6] Mikhlyaev S V 2005 Analysis of optical triangulation systems for measuring mirror surface profiles Optoelectronics, instrumentation and data processing (Avtometriya) 41 4 69–80