BARGAINING IN A MULTI-ECHelon SUPPLY CHAIN WITH POWER STRUCTURE: KS SOLUTION VS. NASH SOLUTION

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Abstract. This paper studies a multi-echelon serial supply chain with negotiations over wholesale prices between successive echelons. Two types of bargaining systems with power structures are compared: one adopts the generalized Kalai-Smorodinsky (KS) solution and the other adopts the generalized Nash solution. Our analyses show that, for any KS bargaining system with a given bargaining power structure, there is a Nash bargaining system with another bargaining power structure, such that the two systems are the same. However, under the same power structure, the generalized KS solution results in lower wholesale price and higher total supply chain profit than the Nash solution does. Finally, we characterize the necessary and sufficient condition of the bargaining power structure under which the KS bargaining system Pareto dominates the Nash bargaining system, and the set characterized by such condition does not shrink to an empty set as the number of echelons increases to infinity.

1. Introduction. Bargaining commonly exists in supply chain member interactions (see, e.g., [6, 10, 7]). Hence, characterizing the bargaining outcomes in the supply chain is a key issue in supply chain management. The bargaining outcome varies with different fairness considerations, and different bargaining solutions have different axioms that represent different views of fairness. In this paper, two popularly adopted bargaining solution concepts are considered: the Nash solution (see, [17]) and the Kalai-Smorodinsky (KS) solution (see, [12]). The Nash solution is applicable to bargaining models with arbitrary number of players. However, the KS solution is mostly applied in negotiations with only two players. The extended KS solution (see, [23]) is applicable to multi-player bargaining models, but only weak Pareto-optimality is satisfied. That is, it is possible to find a result other

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than the extended KS solution, such that some players are strictly better whereas some players stay the same. We assume the supply chain members may have different bargaining powers, so the bargaining solutions are regarded as “generalized” bargaining solutions. In the generalized Nash solution (Nash solution for short), “Equality of Fear of Disagreement Relative to Bargaining Power” is a key characterization of fairness (see, [22]). Nash [17] assumes all bargainers have the same bargaining power, hence all bargainers’ fears of disagreements must be the same; while Svejnar [22] assumes the bargainers may have different bargaining powers, hence all bargainers’ fears of disagreements must be proportional to their relative bargaining powers. In contrast, the generalized KS solution (KS solution for short) emphasizes on bargainers’ aspirations, i.e., another axiom called “Individual Monotonicity (IM)” applies (see, [1]): If a bargainer’s alternatives are increased while all other bargainers’ best choices stay the same, then the bargainer with increased alternatives should become better off. In general, these two bargaining solutions may act differently due to different fairness considerations. However, most of the existing works in supply chain management adopt the Nash solution to characterize the bargaining outcome, while the KS solution is seldom used, and the comparison of these two bargaining solutions in supply chains is also missing in the literature. Our work tries to fill this gap by making use of the KS solution to characterize the negotiation outcome and comparing it with that of the Nash solution. In summary, this research contributes to the existing literature in the following manner: the first work to discuss the negotiation outcomes in a multi-echelon supply chain with respect to the fairness concerns characterized by KS solution; the first work that compares different fairness concerns in a multi-echelon supply chain negotiations.

We consider a multi-echelon serial supply chain in which there is only one firm at each echelon. For each pair of firms at neighboring echelons, they negotiate on the wholesale price between them, since wholesale price negotiations are necessary parts in supply chain communications (see, [2]). The bargaining processes start from the upstream to the downstream sequentially. Our study shows that, with an equivalent transform of the power structures, the KS bargaining system is identical to the Nash bargaining system. However, the two bargaining systems act differently with the same power structure. The resultant wholesale prices in the KS bargaining system are lower than that in the Nash bargaining system. Comparing supply chain members’ profits in these two systems, we obtain a necessary and sufficient condition under which the KS bargaining system Pareto dominates the Nash bargaining system. Additionally, we show that there exists a range (independent of the supply chain length), such that the KS bargaining system Pareto dominates the Nash bargaining system if the relative bargaining powers of all the bargainers are in that range.

Our work relates to the literature that studies supply chain models with bargaining processes. However, most of the existing works focuses on a two-echelon setting (see e.g., [10, 24]). For more detailed reviews on two-echelon supply chain negotiations, please refer to reviews, [3, 9, 15, 11]. There are a few works that consider supply chains with more than two echelons. Lovejoy [14] discusses a bargaining process in a multi-echelon supply chain with fixed demand, in which there are multiple competing members at each echelon. He mainly considers a static bargaining solution, i.e., all the bargaining outcomes between two neighboring echelons form a Nash equilibrium. Different to this work, we consider a serial supply chain where at each echelon there is only one firm, and we assume the demand to be
price sensitive and discuss a dynamic bargaining solution, i.e., all the bargaining outcomes between two neighboring echelons form a sequential equilibrium. Nguyen [18] also considers a bargaining process in a multi-echelon supply chain with one indivisible good. Similar to Lovejoy’s work, there are multiple competing members at each echelon, but Nguyen discusses a dynamic repeated bargaining process, i.e., the bargaining outcomes are achieved through a dynamic game. In specific, Nguyen studies the stationary equilibria, and characterizes the behavior of these stationary equilibria when the population of members at each echelon tends to infinity. Zhong et al. [26] consider a multi-echelon serial supply chain with price sensitive demand. They adopt Nash solution to characterize the bargaining outcomes, and mainly discuss the supply chain performance when the length of the supply chain tends to infinity. We consider a similar model as Zhong et al., but our work is different to theirs in two ways. On the one hand, we adopt the KS solution to characterize the bargaining outcomes. On the other hand, our work mainly focuses on comparing the KS bargaining system with the Nash bargaining system.

The reminder of the paper is organized as follows. Section 2 introduces the basic model and some assumptions. Section 3 presents model analyses including equilibrium pricing decisions and supply chain performance comparison between the KS bargaining system and the Nash bargaining system. Section 4 extends the main results in Section 3 to another type of demand functions. Finally, we conclude the paper in Section 5 with some future research directions.

2. Basic model and assumptions. In this section, we first introduce the concept of generalized KS solution to bargaining problems, and then presents the basic model and assumptions of the KS bargaining system for the multi-echelon supply chain.

2.1. The bargaining problem and the KS solution. For a two-player bargaining problem with relative bargaining powers \((\lambda, 1 - \lambda)\) in which \(\lambda \in [0, 1]\), let all feasible outcome utilities of both players be a convex compact set \(U \subset \mathbb{R}^2\), and denote by \((0, 0)\) the disagreement point without loss of generality. If there exists \(x = (x_1, x_2) \in U\) such that \(x_1 \geq 0\) and \(x_2 \geq 0\), then we define

\[
u_i(U) = \max_{x = (x_1, x_2) \in U, x \geq (0, 0)} x_i,
\]

and \(u(U) = (u_1(U), u_2(U))\) is called the utopia point. The generalized KS solution (see, [1]) is defined to be (see Figure 1)

\[
KS(U, \lambda) = (1 - \eta_0)(\lambda(u_1(U), 0) + (1 - \lambda)(0, u_2(U))) + \eta_0 u(U),
\]

in which

\[
\eta_0 = \max \{\eta \in [0, 1] | (1 - \eta)(\lambda(u_1(U), 0) + (1 - \lambda)(0, u_2(U))) + \eta u(U) \in U\}.
\]

2.2. The KS bargaining system. The configuration of our multi-echelon serial supply chain is similar to that in Li [13] and Zhong et al. [26] (Figure 2). There is only one member at each echelon, and the supply chain has \(n\) echelons in total. Denote by \(P_i\) the supply chain member at echelon \(i\), with \(P_1\) being the most downstream firm and \(P_n\) being the most upstream firm. The wholesale price contract is adopted between two neighboring supply chain members \(P_i\) and \(P_{i-1}\) \((i = n, n - 1, \ldots, 2)\). We assume that, the unit production cost of \(P_n\) is
Figure 1. Generalized KS solution for two players.

Figure 2. The multi-echelon supply chain.

$c > 0$, and $P_1$ faces a deterministic price-sensitive demand $D(p_1) = (a + bp_1)^d$, in which either $d > 0$, $b < 0$, $a > -bc$; or $d < -1$, $b > 0$, $a \geq 0$ (see, [26, 21, 20]).

In the KS bargaining system, $P_i$ and $P_{i-1}$ adopt the generalized KS solution to negotiate the wholesale price $p_i$ with known $p_{i+1}$ (define $p_{n+1} = c$ for convenience). If the negotiation breaks down, all supply chain members gain nothing, hence we assume the disagreement point to be $(0, 0)$. Further, we assume the relative bargaining powers of $P_i$ and $P_{i-1}$ to be $\lambda_i$ and $1 - \lambda_i$, respectively ($\lambda_i \in [0,1]$, and define $\lambda_1 = 1$ for convenience). After the wholesale price $p_i$ is negotiated, $P_{i-1}$ determines his order quantities $q_{i-1}$. Similar to Zhong et al. [26] (in which the generalized Nash solution is adopted), we assume the negotiations take place sequentially from the upstream to the downstream, and finally $P_1$ determines $p_1$ and $q_1$ based on the negotiated wholesale price $p_2$. Further, we assume the production cost $c$, the demand function $D(p_1) = (a + bp_1)^d$ and the power structure $\{\lambda_i\}_{i=1,2,\ldots,n}$ are all common knowledge. As a consequence, each supply chain member can predict the final demand correctly, hence the order quantities satisfy $q_{n-1} = \ldots = q_1 = D(p_1)$.

In the next section, we analyze the KS bargaining system and compare it to the Nash bargaining system (see, [26]).

3. Model analyses. In this section, we first derive the negotiation outcomes of the KS bargaining system. Next, we compare the pricing decisions of the KS bargaining system and the Nash bargaining system. Then, we characterize the necessary and sufficient condition, under which the KS bargaining system Pareto dominates the Nash bargaining system. Finally, a simple numerical example is conduct to visually
show the areas, such that he KS bargaining system Pareto dominates the Nash bargaining system.

3.1. Pricing decisions: KS vs. Nash. The bargaining outcomes of the pricing decisions in the KS bargaining system are characterized in the following theorem.

(All proofs of theorems are presented in Appendices.)

**Theorem 3.1.** Given \( p_2 \) such that \( a + b p_2 > 0 \) and \( p_2 \geq c \), denote \( t_1 = d/(d+1) \). Then the optimal decision of \( P \) satisfies \( p_{i}^{KS}(p_2) \geq c \) and

\[
a + bp_{i}^{KS}(p_2) = t_1 \cdot (a + bp_2) > 0.
\]

For \( i = 2, 3, \ldots, n \), given \( p_{i+1} \) such that \( a + bp_{i+1} > 0 \) and \( p_{i+1} \geq c \), the KS solution \( p_{i}^{KS}(p_{i+1}) \geq c \), and satisfies

\[
a + bp_{i}^{KS}(p_{i+1}) = t_i \cdot (a + bp_{i+1}) > 0.
\]

Here \( t_i \in [\min\{d/(d+1), 1\}, \max\{d/(d+1), 1\}] \) is the unique solution of

\[
1 - \frac{\lambda_i}{\lambda_i} = \beta(t_i),
\]

in which

\[
\beta(t_i) = \frac{1 + t_i^{d-1} - 1)(d+1)}{1 - t_i^{d+1}}.
\]

Further, \( \beta \) is a strictly monotone function, and \( p_{i}^{KS}(p_{i+1}) \) is increasing in \( \lambda_i \).

**Remark:** 1. For \( \lambda_i = 0 \), we define \((1 - \lambda_i)/\lambda_i = \beta(1) = +\infty\), then the above equation (1) is well defined for all \( \lambda_i \in [0, 1] \).

2. In Zhong et al. [26], the Nash solution satisfies

\[
a + bp_{i}^{N}(p_{i+1}) = \left(1 - \frac{\lambda_i}{d+1}\right)(a + bp_{i+1}),
\]

in which the superscript \( N \) in \( p_{i}^{N} \) refers to the Nash solution.

The above theorem indicates that, a supply chain member with a larger power will obtain a more favorable wholesale price in negotiation.

The equations of \( p_i \) and \( p_{i+1} \) \((i = 1, \ldots, n)\) have a similar form in KS and Nash solutions. Hence, with a transformation of relative bargaining powers \( i.e., t_i \) corresponds to \( 1 - \lambda_i(d + 1)^{-1} \), \( i = 1, \ldots, n \), the KS solution and the Nash solution are the same in a multi-echelon supply chain. Therefore, almost all properties of the Nash bargaining system also hold in the KS bargaining system. In the following, we compare \( p_{i}^{KS}(p_{i+1}) \) and \( p_{i}^{N}(p_{i+1}) \) with the same relative bargaining powers \( (\lambda_i, 1 - \lambda_i) \).

**Theorem 3.2.** If \( d > 0, b < 0, a > -bc \), then \( t_i \geq 1 - \lambda_i(d + 1)^{-1} \); If \( d < -1, b > 0, a \geq 0 \), then \( t_i \leq 1 - \lambda_i(d + 1)^{-1} \). Furthermore, for any given \( p_{i+1} \) such that \( a + bp_{i+1} > 0 \) and \( p_{i+1} \geq c \), it holds \( p_{i}^{KS}(p_{i+1}) \leq p_{i}^{N}(p_{i+1}) \).

The above theorem indicates that, for a given wholesale price \( p_{i+1} \), the generalized KS solution will result in a lower negotiated wholesale price than the Nash solution does. Thus, the KS solution tends to allocate more profits to the retailer, who has a weak position in the negotiation in practice. This finding also coincides with the statement made by Bertsimas et al. [5], i.e., the KS solution are fairer
than the Nash solution. If we further calculate the negotiated wholesale prices of the whole supply chain system, we have

\[ p_{KS}^k = \left( \frac{a}{b} + c \right) \prod_{i=k}^n t_i - \frac{a}{b} \leq \left( \frac{a}{b} + c \right) \prod_{i=k}^n \left( 1 - \frac{\lambda_i}{d+1} \right) - \frac{a}{b} = p_N^k, \quad k = 1, ..., n. \]

Therefore, the wholesale prices of the KS bargaining system at all echelons are lower than that of the Nash bargaining system.

3.2. System performance: KS vs. Nash. The whole supply chain system’s profit is \( \Pi(p_1) = (p_1 - c)(a + b p_1)^d \), which is only determined by \( p_1 \). The following theorem compares the total supply chain profits of the KS bargaining system (\( \Pi^{KS} \)) and the Nash bargaining system (\( \Pi^{N} \)).

Theorem 3.3. The KS bargaining system always outperforms the Nash bargaining system, i.e., \( \Pi^{KS} \geq \Pi^{N} \).

The KS solution generates higher total supply chain profit for the following reasons. The KS solution is considered as a fairer solution than the Nash solution (see, [5]), which may allocate more profits to the weaker retailers. According to Zhong et al. [26], in a multi-echelon supply chain, powerful retailers may promote supply chain coordination. Hence, the KS solution generates higher total supply chain profit through allocating retailers with higher profits. This result differs from the findings of Bertsimas et al. [4], i.e., the minimum possible system efficiency of the Nash solution is higher than that of the KS solution. This difference occurs mainly because Bertsimas et al. [4] only consider a single Nash bargaining problem, whereas we consider a sequence of bilateral Nash bargaining problems in a multi-echelon supply chain setting. Since a supply chain with higher profitability (i.e., higher total profit) is considered to be more stable than one with lower profitability, this result reveals that in the negotiation it is more likely to build a long-term partnership when a firm seeks for a downstream and/or upstream partner with more fairness concerns (i.e., adopting KS solution) rather than more efficiency concerns (i.e., adopting Nash solution).

However, higher total supply chain profit does not necessarily mean that all supply chain members are better off. In the following, we aim to find conditions under which the KS bargaining system Pareto dominates the Nash bargaining system. Since

\[ p_{KS}^k = \left( \frac{a}{b} + c \right) \prod_{i=k}^n t_i - \frac{a}{b}, \]

we can calculate the profit of \( P_k \) (\( k = 1, 2, \ldots, n \)) in the KS bargaining system as

\[ \Pi_k^{KS} = \left( \frac{a + bc}{b} \right)^{d+1} (t_k - 1) \prod_{i=1}^n t_i^d \prod_{j=k+1}^n t_j. \] (2)

According to Zhong et al. [26], the profit of \( P_k \) (\( k = 1, 2, \ldots, n \)) in the Nash bargaining system is

\[ \Pi_k^{N} = \left( \frac{a + bc}{b} \right)^{d+1} \left( \frac{-\lambda_k}{1 + d} \right) \prod_{i=1}^n \left( 1 - \frac{\lambda_i}{1 + d} \right) \prod_{j=k+1}^n \left( 1 - \frac{\lambda_j}{1 + d} \right). \] (3)

Hence the following inequalities characterize the necessary and sufficient condition, under which the KS bargaining system Pareto dominates the Nash bargaining system:

\[ \left( \frac{a}{b} + c \right) \prod_{i=k}^n t_i - \frac{a}{b} \leq \left( \frac{a}{b} + c \right) \prod_{i=k}^n \left( 1 - \frac{\lambda_i}{d+1} \right) - \frac{a}{b} = \Pi_N^k, \quad k = 1, ..., n. \]
system,
\[
\frac{(a + bc)^{d+1}}{b} \left( \frac{-\lambda_k}{1 + d} \right) \prod_{i=1}^{n} \left( 1 - \frac{\lambda_i}{d + 1} \right)^d \prod_{j=k+1}^{n} \left( 1 - \frac{\lambda_j}{d + 1} \right) \]
\leq \frac{(a + bc)^{d+1}}{b} (t_k - 1) \prod_{i=1}^{n} t_i^d \prod_{j=k+1}^{n} t_j \quad \text{for} \quad k = 1, \ldots, n. \tag{4}
\]

The above inequalities are equivalent to (substituting \( \lambda_1 = 1, \ t_1 = d/(d + 1) \) and \( \lambda_i = (1 + \beta(t_i))^{-1}(i \geq 2) \) into (4))
\[
(d + 1)(\beta(t_k) + 1)(1 - t_k) \prod_{i=2}^{n} \left( \frac{(\beta(t_i) + 1)(d + 1)t_i^d}{(\beta(t_i) + 1)(d + 1) - 1} \right) \]
\cdot \prod_{j=k+1}^{n} \left( \frac{(\beta(t_j) + 1)(d + 1)t_j^d}{(\beta(t_j) + 1)(d + 1) - 1} \right) \geq 1 \quad \text{for} \quad k = 1, \ldots, n. \tag{5}
\]

The following theorem characterizes some properties of (5).

**Theorem 3.4.** For \( n = 2 \), we have \( \Pi_2^{KS} \leq \Pi_2^N \) and \( \Pi_1^{KS} \geq \Pi_1^N \). While for \( n \geq 3 \), there exists \( 0 \leq \underline{\lambda} < \lambda \leq 1 \) (that are independent of \( n \)), such that if \( \lambda_i \in [\underline{\lambda}, \lambda] \) for all \( i = 2, \ldots, n \), then (5) holds.

The above theorem indicates that, when there are only two echelons, the KS bargaining system can not strictly Pareto dominate the Nash bargaining system. However, strictly Pareto dominance is possible for a supply chain with three or more echelons. The main reason is that, more echelons means more dimensions of the bargaining power structures \( n - 1 \) dimensions in a \( n \)-echelon supply chain). More dimensions of the parameters provide a possibility for more flexible profit allocations, hence strictly Pareto dominance is achievable with more echelons. The theorem also shows that, the range determined by (5) does not shrink to an empty set as \( n \) goes to infinity. These results show that, although the KS bargaining system’s profit is higher than the Nash bargaining system’s profit, whether the profit surplus can benefit all supply chain members depends on the bargaining power structurer \( (\lambda_2, \ldots, \lambda_n) \) as well as the number of firms in the multi-echelon supply chain. A supply chain with more echelons (i.e., more bilateral negotiations) are more likely to reach Pareto improvement in the KS bargaining system. Because more negotiations allow profit surplus to be allocated with more flexibility, whereas less negotiations may cause firms to insist on their own profits in limited negotiations. This result also reveals that, the firms in the supply chain should focus more on fair allocations in the negotiations, since their fairness concerns may benefit not only the whole system but also themselves.

In the following, we conduct a numerical example to visually compare the KS bargaining system and the Nash bargaining system. Since the parameters \( a \) and \( b \) in demand function do not affect the comparing results of the KS and Nash bargaining systems, only the bargaining power structure \( (\lambda_2, \ldots, \lambda_n) \) and parameter \( d \) may affect the results. Therefore, in this numerical example, we mainly focus on the impact of bargaining power structurer. Noting that, the dimension of the bargaining power structure depends on the number of echelons, it is appropriate to consider a three-echelon supply chain in the numerical example (i.e., \( n = 3 \)). Further, the linear demand function \( D(p_1) = a + bp_1 \) \((b < 0)\) is popular in literature.
(see, [16]), hence we let $d = 1$ and $b < 0$. According to (4), the KS bargaining system Pareto dominates the Nash bargaining system if and only if
\[
(2 - \lambda_2)(2 - \lambda_3) \leq 4t_2t_3,
\]
\[
\lambda_2(2 - \lambda_2)(2 - \lambda_3)^2 \leq 16(1 - t_2)t_2t_3^2,
\]
\[
\lambda_3(2 - \lambda_2)(2 - \lambda_3) \leq 8(1 - t_3)t_2t_3,
\]
in which $(1 - \lambda_i)/\lambda_i = \beta(t_i) = (2t_i - 1)^2/(1 - t_i^2)$ for $i = 2, 3$. The first (second or third) inequality indicates that, $P_1$ ($P_2$ or $P_3$) prefers the KS bargaining system to the Nash bargaining system, i.e., $\Pi_1^{KS} \geq \Pi_1^N$ ($\Pi_2^{KS} \geq \Pi_2^N$ or $\Pi_3^{KS} \geq \Pi_3^N$). And the corresponding values of $(\lambda_2, \lambda_3)$ are shown in the blue region of Figure 3.1 (Figure 3.2 or Figure 3.3). The blue region in Figure 3.4 (which is the intersection of the blue regions in Figures 3.1, 3.2, and 3.3) characterizes the values of $(\lambda_2, \lambda_3)$, such that the KS bargaining system Pareto dominates the Nash bargaining system.

4. Extensions. In this section, we study another commonly used demand function $D(p_1) = ab^{-p_1}$ ($a > 0, b > 1$) as in Zhong et al. [26], and show the main results in Section 3 still hold.

**Theorem 4.1.** Given $p_2 \geq c$, $P_1$ will set $p_1^{KS}(p_2) = p_2 + s_1 \geq c$ with $s_1 = \ln b$. For $i = 2, \ldots, n$, given $p_{i+1} \geq c$, the KS solution of $p_i$ is $p_i^{KS}(p_{i+1}) = p_{i+1} + s_i \geq c$, in which $s_i \in [0, 1/\ln b]$ satisfies $\lambda_i = \gamma(s_i)$ and
\[
\gamma(s_i) = \frac{b^{s_i} - 1}{2bs_i - 1 - e \ln (bs_i)}.
\]

Further, $s_i$ is increasing in $\lambda_i$.

The above theorem indicates that a larger power in the negotiation means more favorable wholesale price for the bargainer. Such result coincides with the result in the previous section.
In the following, we compare the generalized KS solution \( p_i^{KS}(p_{i+1}) = p_{i+1} + \lambda_i / \ln b \) (see, [26]), and the result is shown in the following theorem.

**Theorem 4.2.** It holds that \( s_i \leq \lambda_i / \ln b \), i.e., \( p_i^{KS}(p_{i+1}) \leq p_i^{N}(p_{i+1}) \).

Similar to Theorem 3.2, the above theorem shows that, for a given wholesale price \( p_{i+1} \), the generalized KS solution leads to a lower wholesale price than the Nash solution does. We can also calculate the wholesale prices of the whole supply chain system as

\[
p_{KS}^k = c + \sum_{i=k}^{n} s_k \leq c + \sum_{i=k}^{n} \frac{\lambda_k}{\ln b} = p_{N}^k.
\]

Therefore, the wholesale prices at all echelons are lower in the KS bargaining system than that in the Nash bargaining system. In the following theorem, we show that the total supply chain profit of the KS bargaining system is higher than that of the Nash bargaining system.

**Theorem 4.3.** The KS bargaining system always outperforms the Nash bargaining system, i.e., \( \Pi^{KS} \geq \Pi^{N} \).

Finally, we find the conditions on the bargaining power structure, such that the KS bargaining system Pareto dominates the Nash bargaining system. Since

\[
p_{KS}^k = c + \sum_{i=k}^{n} s_k,
\]
we can calculate the profit of \( P_k \) \( (k = 1, 2, \ldots, n) \) in the KS bargaining system as

\[
\Pi^{KS}_k = ab - c b - \sum_{i=1}^{n} s_i s_k.
\]

According to Zhong et al. [26], the profit of \( P_k \) \( (k = 1, 2, \ldots, n) \) in the Nash bargaining system is

\[
\Pi^{N}_k = ab - c e - \sum_{i=1}^{n} \lambda_i \frac{\lambda_k}{\ln b}.
\]

Hence the following inequalities characterize the necessary and sufficient condition, under which the KS bargaining system Pareto dominates the Nash bargaining system:

\[
\frac{\lambda_k}{\ln b} \cdot e^{-\sum_{i=1}^{n} \lambda_i} \leq s_k \cdot b^{-\sum_{i=1}^{n} s_i} \quad \text{for} \quad k = 1, \ldots, n.
\]

Inequalities in (9) are equivalent to (substituting \( \lambda_1 = 1, s_1 = 1 / \ln b \) and \( \lambda_i = \gamma(s_i) (i \geq 2) \) into (9))

\[
\frac{s_k \ln b}{\gamma(s_k)} \cdot e^{\sum_{i=2}^{n} (\gamma(s_i) - s_i \ln b)} \geq 1 \quad \text{for} \quad k = 1, \ldots, n.
\]

The following theorem characterizes some properties of (10).

**Theorem 4.4.** For \( n = 2 \), we have \( \Pi^{KS}_2 \leq \Pi^{N}_2 \) and \( \Pi^{KS}_1 \geq \Pi^{N}_1 \). While for \( n \geq 3 \), there exists \( 0 \leq \lambda < \lambda \leq 1 \) (that are independent of \( n \)), such that if \( \lambda_i \in [\lambda, \lambda] \) for all \( i = 2, \ldots, n \), then (10) holds.
5. Concluding remarks. In this paper, we study a multi-echelon supply chain that faces a price-sensitive demand with sequential bargaining over the wholesale prices. When the generalized KS solution is applied as the bargaining outcome, we derive the equilibrium pricing decisions. We first show that the KS bargaining system and the Nash bargaining system generate the same bargaining outcome under a one to one mapping between bargaining power structures. That is, for any given bargaining power structure of the KS bargaining system, there is a Nash bargaining system with another bargaining power structure such that all wholesale price decision of the two systems are the same.

Comparing with the Nash bargaining system under the same power structure, the KS bargaining system leads to lower wholesale prices at all echelons. In terms of the total supply chain profit, the KS bargaining system always outperforms the Nash bargaining system. Further, we show that the KS bargaining system Pareto dominates the Nash bargaining system under certain conditions. Our analytical results reveal that, applying fairer KS solution (rather than the Nash solution) in a multi-echelon supply chain does not necessarily reduce each firm’s profit. It is possible that both fair and efficiency can be achieved when multiple bilateral negotiations exist. That is, fair negotiation results may finally benefit all players when multiple bilateral negotiations exist. And our research suggests that, supply chain firms in their bilateral negotiations should make fairer joint decisions, so that all the firms in the supply chain may earn more profits. Nevertheless, there are still some limitations of our research. The bargaining problem and the sequential decision model are built based on rational assumptions of all decision makers. However, in practice it is likely that the decision makers may take irrational actions, and thus our results may not fit the situation in practice perfectly. Also, the bilateral negotiation between two firms may involve multiple stages, and not only the relative bargaining power may affect negotiation outcomes, but also some random events outside the system (e.g., COVID-19) may have impacts on the outcomes. In our model, we apply the KS solution to statically characterize the negotiation outcomes, which is a simplification of the real situation.

Our paper opens up some possibilities for future research. For example, there are other widely studied bargaining solutions in game theory (e.g., the Perles-Maschler (PM) solution, see, [19]), and it might provide new insights to supply chain managers to evaluate their performances under the multi-echelon supply chain. Besides, we assume the demand is a deterministic function of the retail price with certain formula in our work, and it might be more interesting to consider other types of demand functions, or assume that the demand is stochastic other than being deterministic. Further, it is interesting to consider information asymmetry and apply the bargaining models with incomplete information (see, [8, 25]) in multi-echelon supply chains.

Appendix.

**Proof of Theorem 3.1.** Given $p_2$, the profit of $P_1$ is $\Pi_1^{KS}(p_1) = (p_1 - p_2) \cdot (a + bp_1)^d$, and

$$\frac{\partial \Pi_1^{KS}}{\partial p_1} = (a + b(1 + d)p_1 - bdp_2) \cdot (a + bp_1)^{d-1}.$$ 

Noticing that $b(1 + d) < 0$, we know $\frac{\partial \Pi_1^{KS}}{\partial p_1} > (>)0$ when $p_1 < (>)(a - bdp_2)/[-b(1 + d)]$. Thus $P_1$ will choose $p_1^{KS}(p_2) = (a - bdp_2)/[-b(1 + d)] \geq p_2 \geq c$
to maximize his profit, i.e.,
\[ a + bp^K_1 p_2 = \frac{d}{d+1}(a + bp_2) = t_1(a + bp_2) > 0. \]

We proceed to prove the theorem by induction. Assuming that the theorem holds for \( i \in \{1, 2, \ldots, k-1\} \), then for given \( p_k \), we know from the assumption of induction that \( p_i \) satisfies
\[ a + bp_i^K = t_i \cdot (a + bp^K_i) = \cdots = (a + bp_k) \prod_{i=1}^{k-1} t_i. \]

Now we consider the case of \( i = k \). For given \( p_{k+1} \), the profits of \( P_k \) and \( P_{k-1} \) are
\[ \Pi_k^K(p_k) = (a + bp_k)^d p_{k+1} \prod_{i=1}^{k-1} t_i, \]
\[ \Pi_{k-1}^K(p_k) = (a + bp_k)^d (p_{k+1}^K p_k) \prod_{i=1}^{k-1} t_i \]
\[ = (a + bp_k)^d (t_k - 1) \frac{b}{d} \prod_{i=1}^{k-1} t_i. \]

Denoting the bargaining set by \( U_k \), we can calculate the utopia point
\[ u_k(U_k) = \max_{p_k \geq p_{k+1}} \Pi_k^K(p_k) = \frac{-1}{b(d+1)} \left( \frac{d}{d+1} \right)^d (a + bp_{k+1}) \prod_{i=1}^{k-1} t_i. \]
\[ u_{k-1}(U_k) = \max_{p_k \geq p_{k+1}} \Pi_{k-1}^K(p_k) = (a + bp_{k+1}) \frac{(t_k - 1) + 1}{b} \prod_{i=1}^{k-1} t_i. \]

By the similar arguments to \( \Pi_i^K \), we know \( \Pi_i^K(p_k) \) is maximized when \( a + bp_k = (a + bp_{k+1}) d/(d+1) \), i.e.,
\[ p_k = \frac{d}{d+1} p_{k+1} - \frac{a}{b(d+1)}. \]

In addition, we also know that the Pareto frontier of the bargaining set is the curve
\[ (\Pi_k^K(p_k), \Pi_{k-1}^K(p_k))_{p_k \leq p_{k+1} \leq \frac{d}{d+1} p_{k+1} - \frac{a}{b(d+1)}}. \]

Therefore, the generalized KS solution satisfies \( p_k^K \geq p_{k+1} \geq c \), and \( (\Pi_k^K(p_k^K), \Pi_{k-1}^K(p_k^K)) \) is on the segment determined by \( (u_k(U_k), u_{k-1}(U_k)) \) and \((\lambda_k u_k(U_k), (1 - \lambda_k) u_{k-1}(U_k))\), i.e.,
\[ [(1 - \lambda_k) u_k(U_k)] \times [u_{k-1}(U_k) - \Pi_{k-1}^K(p_k^K)] \]
\[ = [u_k(U_k) - \Pi_k^K(p_k^K)] \times [\lambda_k u_{k-1}(U_k)]. \]

This is equivalent to
\[ a + bp_k^K = t_k(a + bp_{k+1}) > 0, \]
in which \( t_k \in [\min\{d/(d+1), 1\}, \max\{d/(d+1), 1\}] \) satisfies the following equation:
\[ (1 - \lambda_k) (1 - t_k^{d+1}) = \lambda_k \left[ 1 + t_k^d(t_k - 1)(d+1) \left( \frac{d+1}{d} \right)^d \right]. \]
Noting that \( t_k = 1 \) when \( \lambda_k = 0 \), together with the definition that \( 1/0 = \beta(1) = +\infty \), we know \( t_k \) satisfies
\[
1 - \frac{\lambda_k}{\lambda_k} = \frac{1 + t_k^d(t_k - 1)(d + 1)\left(\frac{d + 1}{d}\right)^d}{1 - t_k^{d+1}} = \beta(t_k).
\]

When \( d > 0, b < 0, a > -bc \) we know that \( t_k \in [d/(d + 1), 1] \). Furthermore, we know that \( 1 - t_k^d(1 - t_k)(d + 1)^{d+1}d^{-d} \geq 0 \) is strictly increasing in \( t_k \) and \( 1 - t_k^{d+1} \geq 0 \) is strictly decreasing in \( t_k \), i.e., \( \beta \) is a strictly increasing function in the interval \([d/(d + 1), 1]\). Thus, \( t_k \) increases from \( d/(d + 1) \) to 1 as \( \lambda_k \) decreases from 1 to 0, which further indicates that \( p_k^{KS} \) is increasing in \( \lambda_k \).

When \( d < -1, b > 0, a \geq 0 \), we know that \( t_k \in [1, d/(d + 1)] \). Furthermore, we know that \( 1 + t_k^d(t_i - 1)(d + 1)[(d + 1)/d]^d \geq 0 \) is strictly decreasing in \( t_k \) and \( 1 - t_k^{d+1} \geq 0 \) is strictly increasing in \( t_k \). i.e., \( \beta \) is a strictly decreasing function in the interval \([1, d/(d + 1)]\). Thus, \( t_k \) decreases from \( d/(d + 1) \) to 1 as \( \lambda_k \) decreases from 1 to 0, which further indicates that \( p_k^{KS} \) is increasing in \( \lambda_k \).

In summary, \( \beta \) is a strictly monotone function (which implies Equation (1) has a unique solution), and \( p_k^{KS} \) is increasing in \( \lambda_k \). Hence, we have proved the theorem. \( \square \)

**Proof of Theorem 3.2.** We first give a lemma that will be used later.

**Lemma 1** If \( x \geq 0 \) and \( y \geq 1 \), then \((1 + x)^y - 1 - xy, \) then \((d/dx)\kappa = y(1 + x)^{y-1} - y \geq 0 \), hence \( \kappa(x) \geq \kappa(0) = 0 \).

**Proof.** Define \( \kappa(x) = (1 + x)^y - 1 - xy \), then \((d/dx)\kappa = y(1 + x)^{y-1} - y \geq 0 \), hence \( \kappa(x) \geq \kappa(0) = 0 \).

When \( \lambda_i = 0 \), we have \( t_i = 1 \), which means \( t_i = 1 - \lambda_i(d+1)^{-1} \). In the following, we only consider \( \lambda_i \in (0, 1) \) (which indicates \( t_i \neq 1 \)).

If \( d > 0, b < 0, a > -bc \), we know that \( \beta \) is a strictly increasing function in the interval \([d/(d + 1), 1]\). Therefore \( t_i \geq 1 - \lambda_i(d + 1)^{-1} \) is equivalent to \( \beta(t_i) \geq \beta(1 - \lambda_i(d + 1)^{-1}) \), i.e., we only need to prove
\[
\frac{1 - \lambda_i}{\lambda_i} \geq \frac{1 - \lambda_i}{\lambda_i} \left(\frac{d+1-\lambda_i}{d+1}\right)^{d+1}
\]
\[
\iff 1 - 2\lambda_i - (1 - \lambda_i) \left(\frac{d+1-\lambda_i}{d+1}\right)^{d+1} + \lambda_i^2 \left(\frac{d+1-\lambda_i}{d}\right)^d \geq 0.
\]

Define the following function
\[
W_1(x) = 1 - 2x - (1 - x) \left(\frac{d+1-x}{d+1}\right)^{d+1} + x^2 \left(\frac{d+1-x}{d}\right)^d.
\]

Then
\[
W_1'(x) = -2 + \left(\frac{d+1-x}{d+1}\right)^{d+1} + (1 - x) \left(\frac{d+1-x}{d+1}\right)^d
\]
\[
+ 2x \left(\frac{d+1-x}{d}\right)^d - x^2 \left(\frac{d+1-x}{d}\right)^{d-1},
\]
\[
\left(\frac{d}{d+1-x}\right)^{d-2} W_1''(x) = 2 \left(\frac{d+1-x}{d}\right)^2 - \frac{4x(d+1-x)}{d} + \frac{d-1}{d} x^2
\]
Together with 

\[ W(0) = 0 \text{ and negative later, i.e., } W(1) = (d/(d+1))^{d+1} - 1 < 0. \]

Therefore, as \( x \) increases from 0 to 1, \( W''(x) \) is a convex quadratic function. Note that, 

\[
\left( \frac{d}{d+1-x} \right)^{d-2} W_1''(x) = \left( \frac{d}{d+1} \right)^{d-2} \left\{ 2 \left( 1 + \frac{1}{d} \right)^d - 2 - \frac{d}{d+1} \right\} 
\]

\[
\geq \left( \frac{d}{d+1} \right)^{d-2} \left\{ 2 \left( 1 + \frac{d+1}{d} \right) \frac{d}{d+1} - 2 - \frac{d}{d+1} \right\} 
\]

\[
= \left( \frac{d}{d+1} \right)^{d-2} \frac{d}{d+1} > 0,
\]

\[
\left( \frac{d}{d+1-x} \right)^{d-2} W_1''(x) \bigg|_{x=1} = -1 - \frac{1}{d} - 2 \left( \frac{d}{d+1} \right)^d < 0.
\]

The above calculation indicates that, as \( x \) increases from 0 to 1, \( W''(x) \) is positive at first and negative later, i.e., \( W'(x) \) is increasing first but decreasing later. Note that \( W_1(0) = 0 \) and \( W_1(1) = (d/(d+1))^{d+1} - 1 < 0. \) Therefore, as \( x \) increases from 0 to 1, \( W_1(x) \) starts from 0 and increases to positive, then decreases below 0, i.e., \( W_1(x) \) is increasing first but decreasing later. Note that, \( W_1(0) = W_1(1) = 0, \) hence \( W_1(x) \geq 0 \) for all \( x \in [0, 1]. \) Therefore, \( W_1(\lambda_i) \geq 0, \) i.e., \( t_i \geq 1 - \lambda_i(d+1)^{-1}, \) which indicates \( p_{KS}^i(p_{i+1}) \leq p_N^i(p_{i+1}) \) when \( a + bp_{i+1} \geq 0 \) and \( p_{i+1} \geq c. \)

If \( d < -1, b > 0, a \geq 0, \) we know that \( \beta \) is a decreasing function in the interval \((1, d/(d+1)]. \) Therefore \( t_i \leq 1 - \lambda_i(d+1)^{-1} \) equivalents to \( \beta(t_i) \geq \beta(1 - \lambda_i(d+1)^{-1}), \) i.e., we only need to prove 

\[
\frac{1 - \lambda_i}{\lambda_i} \geq \frac{1 - \lambda_i}{\left( \frac{d+1 - \lambda_i}{d+1} \right)^{d+1}} \iff W_1(x) \geq 0.
\]

Define \( d_0 = -1 - d > 0, \) then

\[
\left( \frac{d}{d+1-x} \right)^{d-2} W_1''(x) \bigg|_{x=0} = \left( \frac{d_0}{d_0+1} \right)^{d_0+3} \left\{ 2 \left( 1 + \frac{1}{d_0} \right)^{d_0+1} - 2 - \frac{d_0 + 1}{d_0} \right\} 
\]

\[
\geq \left( \frac{d_0}{d_0+1} \right)^{d_0+3} \left\{ 2 \left( 1 + \frac{d_0 + 1}{d_0} \right) - 2 - \frac{d_0 + 1}{d_0} \right\} > 0,
\]

\[
\left( \frac{d}{d+1-x} \right)^{d-2} W_1''(x) \bigg|_{x=1} = -1 - \frac{1}{d} - 2 \left( \frac{d}{d+1} \right)^d < 0.
\]

Together with \([d/(d+1-x)]^{d-2}W_1''(x)\) being a quadratic function, we know that, as \( x \) increases from 0 to 1, \( W''(x) \) is positive at first and negative later i.e., \( W'(x) \) is increasing first but decreasing later. Note that \( W_1'(0) = 0 \) and \( W_1'(1) = (d/(d+1))^{d+1} - 1 < 0. \) Therefore, as \( x \) increases from 0 to 1, \( W_1'(x) \) starts from 0 and increases to positive, then decreases below 0, i.e., \( W_1'(x) \) is increasing first but
decreasing later. Note that \( W_1(0) = W_1(1) = 0 \), hence \( W_1(x) \geq 0 \) for all \( x \in [0,1] \). Therefore, \( W_1(\lambda_i) \geq 0 \), i.e., \( t_i \leq 1 - \lambda_i(d+1)^{-1} \), which further indicates 
\[ p_{i}^{KS}(p_{i+1}) \leq p_{i}^{N}(p_{i+1}) \text{ when } a + b_{p_{i+1}} \geq 0 \text{ and } p_{i+1} \geq 0. \]

**Proof of Theorem 3.3.** Since \( b(d+1) < 0 \) and
\[ \frac{\partial \Pi}{\partial p_1} = [b(d+1)p_1 + a - bdc](a + b_{p_1})^{d-1}, \]
it holds \( \partial \Pi/\partial p_1 \leq 0 \) when \( p_1 \geq (a - bdc)/[b(d+1)] \), hence \( \Pi(p_1) \) is a decreasing function when
\[ p_1 \geq \left( \frac{a}{b} + c \right) \frac{d}{d+1} - \frac{a}{b}. \]

According to Theorem 3.1, we know that \( t_i \in [\min\{1,d/(d+1)\}, \max\{1,d/(d+1)\}] \). Thus \( t_i \geq 1 \) when \( d < -1, b > 0, a \geq 0 \); and \( t_i \leq 1 \) when \( d > 0, b < 0, a > -bc \). Therefore
\[
p^{N}_1 \geq p^{KS}_{1} = \left( \frac{a}{b} + c \right) \prod_{i=1}^{n} t_i - \frac{a}{b} \geq \left( \frac{a}{b} + c \right) \frac{d}{d+1} \prod_{i=2}^{n} t_i - \frac{a}{b} \geq \left( \frac{a}{b} + c \right) \frac{d}{d+1} - \frac{a}{b}.
\]
Hence
\[ \Pi^{KS} = \Pi(p^{KS}) \geq \Pi(p^{N}) = \Pi^{N}. \]

**Proof of Theorem 3.4.** For \( n = 2 \), according to Equations (2) and (3), we have
\[
\Pi^{KS}_{1} = (a + bc)^{d+1} \frac{-1}{b(d+1)} \left( \frac{d}{d+1} \right)^{d+1},
\]
\[
\Pi^{N}_{1} = (a + bc)^{d+1} \frac{-1}{b(d+1)} \left( \frac{d}{d+1} \right)^{d} \left( 1 - \frac{\lambda_2}{1+d} \right)^{d+1},
\]
\[
\Pi^{KS}_{2} = (a + bc)^{d+1} \left( \frac{d}{d+1} \right)^{d} \frac{t_2 t_2 - 1}{b},
\]
\[
\Pi^{N}_{2} = (a + bc)^{d+1} \left( \frac{d}{d+1} \right)^{d} \left( 1 - \frac{\lambda_2}{1+d} \right)^{d} \frac{-\lambda_2}{b(d+1)}.
\]

Define a function \( W_2(x) = x^d(x-1)/b \) with domain \( x \in (0, +\infty) \), then
\[ W'_2(x) = \frac{d+1}{b} \left( x - \frac{d}{d+1} \right) x^{d-1}. \]
Hence \( W_2(x) \) is increasing if \( x \in (0, d/(d+1)) \) and decreasing if \( x \in [d/(d+1), +\infty) \).

If \( d > 0, b < 0, a > -bc \), we know that \( t_2 \geq 1 - \lambda_2 (d+1)^{-1} \geq d/(d+1) > 0 \), hence \( t_2^{d+1} \geq (1 - \lambda_2 (d+1)^{-1})^{d+1} \) and \( W_2(t_2) \geq W_2(1 - \lambda_2 (d+1)^{-1}) \), i.e., \( \Pi^{KS}_{2} \geq \Pi^{N}_{2} \) and \( \Pi^{KS}_{2} \leq \Pi^{N}_{2} \).

If \( d < -1, b > 0, a \geq 0 \), we know that \( 1 \leq t_2 \leq 1 - \lambda_2 (d+1)^{-1} \leq d/(d+1) \), hence \( t_2^{d+1} \geq (1 - \lambda_2 (d+1)^{-1})^{d+1} \) and \( W_2(t_2) \geq W_2(1 - \lambda_2 (d+1)^{-1}) \), i.e., \( \Pi^{KS}_{2} \geq \Pi^{N}_{2} \) and \( \Pi^{KS}_{2} \leq \Pi^{N}_{2} \).
For $n \geq 3$, consider the following three functions:

$$W_3(x) = \left(\frac{(\beta(x) + 1)(d + 1)x}{(\beta(x) + 1)(d + 1) - 1}\right)^{d+1},$$

$$W_4(x) = (d + 1)(\beta(x) + 1)(1 - x) \left(\frac{(\beta(x) + 1)(d + 1)x}{(\beta(x) + 1)(d + 1) - 1}\right)^{2d},$$

$$W_5(x) = (d + 1)(\beta(x) + 1)(1 - x) \left(\frac{(\beta(x) + 1)(d + 1)x}{(\beta(x) + 1)(d + 1) - 1}\right)^{2d+1}.$$  

By the definition of $\beta(x)$ in Equation (1), we can easily see that

$$\beta\left(\frac{d}{d+1}\right) = \beta'\left(\frac{d}{d+1}\right) = 0.$$

Therefore, we have

$$W_3\left(\frac{d}{d+1}\right) = W_4\left(\frac{d}{d+1}\right) = W_5\left(\frac{d}{d+1}\right) = 1,$$

and

$$W_3'\left(\frac{d}{d+1}\right) = W_4'\left(\frac{d}{d+1}\right) = \frac{(d + 1)^2}{d}, W_5'\left(\frac{d}{d+1}\right) = d + 1.$$

Hence, when $d > 0$, there exists $x_d \in (d/(d+1), 1)$ such that $W_3(x_d) > 1$, $W_4(x_d) > 1$ and $W_5(x_d) > 1$; when $d < -1$, there also exists $x_d \in (1, d/(d+1))$ such that $W_3(x_d) > 1$, $W_4(x_d) > 1$ and $W_5(x_d) > 1$. Define the following functions:

$$\Phi_1(x, y) = (d + 1)(\beta(x) + 1)(1 - x) \left(\frac{(\beta(x) + 1)(d + 1)x}{(\beta(x) + 1)(d + 1) - 1}\right)^{d} \cdot \left(\frac{(\beta(y) + 1)(d + 1)y}{(\beta(y) + 1)(d + 1) - 1}\right)^{d},$$

$$\Phi_2(x, y) = (d + 1)(\beta(x) + 1)(1 - x) \left(\frac{(\beta(x) + 1)(d + 1)x}{(\beta(x) + 1)(d + 1) - 1}\right)^{d} \cdot \left(\frac{(\beta(y) + 1)(d + 1)y}{(\beta(y) + 1)(d + 1) - 1}\right)^{d+1}.$$  

Then $\Phi_1(x_d, x_d) = W_4(x_d) > 1$ and $\Phi_2(x_d, x_d) = W_5(x_d) > 1$. Thus, there exists $t$ and $\bar{t}$ near $x_d$ such that, for all $x_0, x_1, x_2 \in [\min, \max]$ it holds $W_3(x_0) > 1$, $\Phi_1(x_1, x_2) > 1$ and $\Phi_2(x_1, x_2) > 1$.

Note that, $t(x)$ is a monotone function (increasing when $d > 0$; decreasing when $d < -1$). Hence there exists $\underline{x}$ and $\overline{x}$ in $(0, 1)$ such that $t(x) \in [\underline{x}, \overline{x}]$ for all $x \in [\underline{x}, \overline{x}]$.

In the following, we show that, if $\lambda_i \in [\underline{x}, \overline{x}]$ for $i = 2, \ldots, n$, then (5) holds. (It is easy to see that $\lambda$ and $\overline{\lambda}$ are independent of $n$.)

For $\lambda_i \in [\underline{x}, \overline{x}]$ for $i = 2, \ldots, n$, we know that $t_i = t(\lambda_i) \in [\underline{t}, \overline{t}]$ for $i = 2, \ldots, n$. When $k = 1$, we have

$$\prod_{i=2}^{n} \left(\frac{(\beta(t_i) + 1)(d + 1)t_i}{(\beta(t_i) + 1)(d + 1) - 1}\right)^{d+1} = \prod_{i=2}^{n} W_3(t_i) \geq 1.$$
When $k=2$, we have
\[
(d+1)(\beta(t_2)+1)(1-t_2) \left( \frac{(\beta(t_2)+1)(d+1)t_2}{(\beta(t_2)+1)(d+1)-1} \right)^d \\
\cdot \left( \frac{(\beta(t_3)+1)(d+1)t_3}{(\beta(t_3)+1)(d+1)-1} \right)^{d+1} \\
\prod_{i=4}^{n} \left( \frac{(\beta(t_i)+1)(d+1)t_i}{(\beta(t_i)+1)(d+1)-1} \right)^{d+1} \\
= \Phi_2(t_2,t_3) \prod_{i=4}^{n} W_3(t_i) \geq 1.
\]

When $k \geq 3$, we have
\[
(d+1)(\beta(t_k)+1)(1-t_k) \left( \frac{(\beta(t_k)+1)(d+1)t_k}{(\beta(t_k)+1)(d+1)-1} \right)^d \\
\cdot \left( \frac{(\beta(t_2)+1)(d+1)t_2}{(\beta(t_2)+1)(d+1)-1} \right)^{d+1} \\
\prod_{i=4}^{n} \left( \frac{(\beta(t_i)+1)(d+1)t_i}{(\beta(t_i)+1)(d+1)-1} \right)^{d+1} \\
= \Phi_2(t_k,t_2) \prod_{i=4}^{n} [W_3(t_i)]^{\frac{d}{d+1}} \cdot \prod_{i=k+1}^{n} W_3(t_i) \geq 1.
\]

Hence, Equation (5) holds, and the theorem is proved. \qed

**Proof of Theorem 4.1.** The idea of this proof is similar to the proof of Theorem 3.1. For given $p_2$, the profit of $P_1$ is $\Pi_1^{KS}(p_1) = ab^{-p_1} \cdot (p_1 - p_2)$. It is easy to see that
\[
p_1^{KS}(p_2) = p_2 + \frac{\lambda_1}{\ln b} = p_2 + \frac{1}{\ln b} = p_2 + s_1.
\]

Assuming that the theorem holds for $i \in \{1, 2, \ldots, k-1\}$, then for given $p_k$, we know from the assumption of induction that $p_1$ will be
\[
p_1^{KS} = p_2^{KS} + s_1 = \cdots = p_k^{KS} + s_1 = p_{k-1}^{KS} + \sum_{i=1}^{k-1} s_i.
\]

Now we consider the case of $i = k$. The profits of $P_k$ and $P_{k-1}$ are
\[
\Pi_k^{KS}(p_k) = a(p_k - p_{k+1})b^{-p_k - \sum_{i=1}^{k-1} s_i},
\]
\[
\Pi_{k-1}^{KS}(p_k) = a(p_{k-1}^{KS}(p_k) - p_k)b^{-p_k - \sum_{i=1}^{k-1} s_i} = as_{k-1}b^{-p_k - \sum_{i=1}^{k-1} s_i}.
\]

Hence we can calculate the utopia point,
\[
u_k(U_k) = \max_{p_k \geq p_{k+1}} \Pi_k^{KS}(p_k) = \Pi_k^{KS} \left( p_{k+1} + \frac{1}{\ln b} \right) = \frac{a}{\ln b} b^{-p_{k+1} - \frac{1}{\ln b} - \sum_{i=1}^{k-1} s_i},
\]
\[
u_{k-1}(U_k) = \max_{p_k \geq p_{k+1}} \Pi_{k-1}^{KS}(p_k) = \Pi_{k-1}^{KS}(p_{k+1}) = \Pi_{k-1}^{KS}(p_{k+1}) = as_{k-1}b^{-p_{k+1} - \sum_{i=1}^{k-1} s_i}.
\]

The Pareto frontier is the curve
\[
(\Pi_k^{KS}(p_k), \Pi_{k-1}^{KS}(p_k))_{p_k \leq p_k \leq p_{k+1} + \frac{1}{\ln b}}.
\]
Thus, the generalized KS solution \((\Pi^K_{p_k}^{KS}(p_k), \Pi^{KS}_{k-1}(p_k))\) lies on the segment determined by \((u_k(U_k), u_{k-1}(U_k))\) and \((\lambda_k u_k(U_k), (1 - \lambda_k) u_{k-1}(U_k))\), i.e.,

\[
[(1 - \lambda_k) u_k(U_k)] \times [u_{k-1}(U_k) - \Pi^{KS}_{k-1}(p_k^{KS})] = [u_k(U_k) - \Pi^{KS}_{k}(p_k^{KS})] \times [\lambda_k u_{k-1}(U_k)]
\]  

(11)

Equality (11) is equivalent to

\[p_k^{KS} = p_{k+1} + s_k,\]

in which \(s_k \in [0, 1/\ln b]\) satisfies the following equation:

\[\lambda_k = \frac{b^{s_k} - 1}{2b^{s_k} - 1 - es_k \ln b} = \gamma(s_k).\]  

(12)

In the following, we show that \(\gamma\) is strictly increasing in \(s_k\). Since

\[
\frac{\partial}{\partial s_k} (eb^{-s_k} + es_k \ln b) = (1 - b^{-s_k}) e \ln b \geq 0,
\]

and \(s_k \in [0, 1/\ln b]\), we have

\[
\frac{\partial \gamma}{\partial s_k} = \left(1 + e - eb^{-s_k} - es_k \ln b\right) \frac{b^{s_k} \ln b}{(2b^{s_k} - 1 - e \ln (b^{s_k}))^2}
\]

\[
\geq \left\{ (1 + e - eb^{-s_k} - es_k \ln b) \big|_{s_k = 1/\ln b} \right\} \frac{b^{s_k} \ln b}{(2b^{s_k} - 1 - e \ln (b^{s_k}))^2} = 0.
\]

The above inequality becomes equality if and only if \(s_k = 1/\ln b\), hence \(\gamma\) is a strictly increasing function, and the unique solution \(s_k\) of Equation (12) is increasing in \(\lambda_k\).

**Proof of Theorem 4.2.** It is easy to see that \(p_i^{KS}(p_{i+1}) \leq p_i^{N}(p_{i+1})\) is equivalent to \(s_i \leq \lambda_i/\ln b\). Note that \(\gamma\) is an increasing function, thus we only need to prove \(\gamma(s_i) \leq \gamma(\lambda_i/\ln b)\), i.e.,

\[\lambda_i \leq \frac{e^\lambda_i - 1}{2e^\lambda_i - 1 - e\lambda_i}.\]

Note that \(\partial^2(2e^x - 1 - ex)/\partial x^2 = 2e^x > 0\), hence \(2e^x - 1 - ex\) reaches the minimum when \(\partial(2e^x - 1 - ex)/\partial x = 2e^x - e = 0\), i.e., \(x = 1 - \ln 2\). Therefore, \(2e^x - 1 - ex \geq e - 1 - e(1 - \ln 2) = \ln 2 - 1 > 0\), which indicates

\[\lambda_i \leq \frac{e^\lambda_i - 1}{2e^\lambda_i - 1 - e\lambda_i} \iff (2\lambda_i - 1)e^\lambda_i - e\lambda_i^2 - \lambda_i - 1 \leq 0.
\]

Define a function \(W_0(x) = (2x - 1)e^x - ex^2 - x + 1\), then

\[
W'_0(x) = (2x + 1)e^x - 2ex - 1,\]

\[
W''_0(x) = (2x + 3)e^x - 2e,\]

\[
W'''_0(x) = (2x + 5)e^x > 0.
\]

Hence, \(W'_0(x)\) is a convex function. Note that \(W'_0(0) = 0, W'_0(1) = e - 1 > 0\) and \(W''_0(0) = 3 - 2e < 0\), thus as \(x\) increases from 0 to 1, \(W'_0(x)\) starts at 0, and decreases to negative, then increases to positive, and finally reaches \(e - 1\). Therefore, \(W_0(x)\) is decreasing at first and increasing later as \(x\) increases from 0 to 1. Note that \(W_0(0) = W_0(1) = 0\), thus \(W_0(x) \leq 0\) for \(x \in [0, 1]\). This result indicates that \(W_0(\lambda_i) \geq 0\), thus \(s_i \leq \lambda_i/\ln b\), i.e., \(p_i^{KS}(p_{i+1}) \leq p_i^{N}(p_{i+1})\).
Proof of Theorem 4.3. The system profit $\Pi = ab^{-p_1(p_1 - c)}$ is only determined by $p_1$. Note that
\[
\frac{\partial \Pi}{\partial p_1} = ab^{-p_1}(1 - (p_1 - c) \ln b),
\]
hence $\Pi$ is decreasing in $p_1$ if $p_1 \in [c + 1/\ln b, +\infty)$. Since
\[
c + \frac{1}{\ln b} \leq c + \frac{1}{\ln b} + \sum_{k=2}^{n} s_k = p_1^{KS} \leq p_1^N,
\]
it holds $\Pi^{KS} = \Pi(p_1^{KS}) \geq \Pi(p_1^N) = \Pi^N$. \qed

Proof of Theorem 4.4. For $n = 2$, according to Equations (7) and (8), we have
\[
\Pi_1^{KS} = \frac{ab^{-c}}{e \ln b} b^{-s_2} \geq \frac{ab^{-c}}{e \ln b} b^{-\frac{\lambda_2}{\gamma}} = \Pi_1^N,
\]
\[
\Pi_2^{KS} = \frac{ab^{-c}}{e} s_2 b^{-s_2},
\]
\[
\Pi_2^N = \frac{ab^{-c}}{e} \frac{\lambda_2}{\gamma} b^{-\frac{\lambda_2}{\gamma}}.
\]
Define $W_7(x) = xb^{-x}$, then
\[
W_7^'(x) = (1 - x \ln b)b^{-x} \geq 0, \quad \forall x \leq \frac{1}{\ln b}.
\]
Note that $s_2 \leq \lambda_2/\ln b \leq 1/\ln b$, hence
\[
\Pi_2^{KS} = \frac{ab^{-c}}{e} W_7(s_2) \leq \frac{ab^{-c}}{e} W_7\left(\frac{\lambda_2}{\ln b}\right) = \Pi_2^N.
\]

For $n \geq 3$, we consider the following function:
\[
W_8(x) = \frac{x \ln b}{\gamma(x)} \cdot e^{2(\gamma(x) - x \ln b)}.
\]
Since $\gamma(1/\ln b) = 1$ and
\[
\left. \frac{\partial \gamma}{\partial x} \right|_{x=1/\ln b} = (1 + e - eb^{-x} - ex \ln b) b^x \ln b \bigg|_{x=1/\ln b} = 0,
\]
we have
\[
\left. \frac{\partial W_8(x)}{\partial x} \right|_{x=1/\ln b} = \frac{\ln b}{\gamma(x)} \cdot (1 - 2x \ln b) \cdot e^{2(\gamma(x) - x \ln b)} \bigg|_{x=1/\ln b} = -\ln b < 0.
\]
Note that $W_8(1/\ln b) = 1$, hence there exists $0 < x_0 < 1/\ln b$ such that $W_8(x_0) > 1$.
Define
\[
\Phi_3(x, y) = \frac{x \ln b}{\gamma(x)} \cdot e^{\gamma(x) - x \ln b} \cdot e^{\gamma(y) - y \ln b},
\]
then $\Phi_3(x_0, x_0) = W_8(x_0) > 1$. Therefore, there exists $0 < x \leq y < 1/\ln b$ such that $\Phi_3(x_1, x_2) > 1$ for all $x_1, x_2 \in [s, \overline{s}]$. Let $\lambda = \gamma(\overline{s}) \in [0, 1]$ and $\Lambda = \gamma(\underline{s}) \in [0, 1]$, then $\lambda_i \in [\lambda, \overline{\Lambda}]$ indicates that $\gamma(s_i) \in [\gamma(\overline{s}), \gamma(\underline{s})]$, i.e., $s_i \in [\underline{s}, \overline{s}]$ ($i = 2, \ldots, n$). In the following, we show that Equation (10) holds if $\lambda_i \in [\lambda, \overline{\Lambda}]$ for all $i = 2, \ldots, n$. When $k = 1$, since $\lambda_i = \gamma(s_i) \geq s_i \ln b$ (Theorem 4.2), we have
\[
e^{\sum_{i=2}^{n} (\gamma(s_i) - s_i \ln b)} \geq 1.$
When \( k = 2 \),
\[
\frac{s_2 \ln b}{\gamma(s_2)} \cdot e^{\sum_{i=2}^{n}(\gamma(s_i) - s_i \ln b)} \geq \frac{s_2 \ln b}{\gamma(s_2)} \cdot e^{\gamma(s_2) - s_2 \ln b} \cdot e^{\gamma(s_3) - s_3 \ln b} = \Phi_3(s_2, s_3) \geq 1.
\]
When \( k \geq 3 \),
\[
\frac{s_k \ln b}{\gamma(s_k)} \cdot e^{\sum_{i=2}^{n}(\gamma(s_i) - s_i \ln b)} \geq \frac{s_k \ln b}{\gamma(s_k)} \cdot e^{\gamma(s_k) - s_k \ln b} \cdot e^{\gamma(s_2) - s_2 \ln b} = \Phi_3(s_k, s_2) \geq 1.
\]
Therefore, the theorem is proved.

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