UV/IR Mixing in Noncommutative Field Theory
via Open String Loops

Youngjai Kiem and Sangmin Lee

School of Physics
Korea Institute for Advanced Study
Seoul 130-012 Korea

ABSTRACT

We explicitly evaluate one-loop (annulus) planar and nonplanar open string amplitudes in the presence of the background NS-NS two-form field. In the decoupling limit of Seiberg and Witten, we find that the nonplanar string amplitudes reproduce the UV/IR mixing of noncommutative field theories. In particular, the investigation of the UV regime of the open string amplitudes shows that certain IR closed string degrees of freedom survive the decoupling limit as previously predicted from the noncommutative field theory analysis. These degrees of freedom are responsible for the quadratic, linear and logarithmic IR singularities when the D-branes embedded in space-time have the codimension zero, one and two, respectively. The analysis is given for both bosonic and supersymmetric open strings.

*ykiem, sangmin@kias.re.kr
1 Introduction

Certain noncommutative field theories [1] can be systematically derived from open string theories in the presence of constant background NS-NS two-form field ($B$ field) [3]-[8]. The upshot of these developments is that noncommutative field theories are more stringy than what one might naively expect. For example, unlike the generic commutative field theories arising as decoupling limits of string theories, noncommutative field theories are T-duality invariant signaling its stringy nature [2]. Further considerations of loop effects in noncommutative field theories [9, 10, 11] add an intriguing new element in the analogy between open string theories and noncommutative field theories, namely, the UV/IR mixing. From the open string perspective, the simplest one-loop annulus diagram reveals a prominent stringy character, the open/closed string channel duality. The UV regime of the open string annulus amplitudes can naturally be interpreted as the IR closed string degrees of freedom. This behavior closely parallels the UV/IR mixing of Refs. [10, 11] in noncommutative field theories, where one can interpret certain UV divergences coming from nonplanar loops of high energy virtual particles as IR divergences.

In this paper, we embark upon the detailed study of one-loop annulus open string amplitudes in the presence of constant background $B$ field and recover many features found in Refs. [10, 11]. The one-loop open string amplitudes turn out to be of the same form as Ref. [10], except including the contributions from the massive string excitations. Upon taking the decoupling limit of [8], massive excitations decouple, while some UV degrees of freedom do not in nonplanar diagrams. Since our set-up is the string theory framework, via the standard open/closed string duality, we can unambiguously identify these extra degrees of freedom as IR closed string contributions. Their Wilsonian effective action decoded from the annulus amplitudes also turns out to be the same as the one proposed in Ref. [10] for the extra degrees of freedom responsible for the IR singularities of the one-loop noncommutative field theory amplitudes. In particular, for $(D - 1)$ D-branes (original critical open string theory), $(D - 2)$-branes and $(D - 3)$-branes, open string theory calculations reproduce the quadratic divergences [10], linear divergences and logarithmic divergences [11] caused by the extra degrees of freedom, respectively, where $D$ is the dimension of space-time. In short, noncommutative quantum field theories arising as limits of open string theories include closed string degrees of freedom, which survive the decoupling limit, couple linearly to the D-brane world-volume open string degrees.

\footnote{We note that our calculations have overlaps with the earlier literature on open string amplitudes, such as Refs. [12] and [13].}
of freedom and live in the bulk space-time.

This paper is organized as follows. In section 2, we compute the world-volume propagators on an annulus in the presence of the constant background $B$ field. In section 3, we evaluate the planar and nonplanar annulus diagrams in the bosonic open string theory. Via open/closed string duality, we identify the IR closed string degrees of freedom, which survive the decoupling limit, and study their properties. In section 4, we extend our analysis to open superstring amplitudes. We discuss further directions and implications suggested by our analysis in section 5.

## 2 World-sheet propagator on an annulus

One important ingredient in computing the one-loop string amplitude is the world-sheet propagator on an annulus. It was first obtained in [13] using a world sheet coordinate in which the two boundaries of the annulus are concentric circles. Here we present an equivalent but more concise form of the propagator following the notations of [14].

First, in the absence of the $B$ field, consider a rectangular torus whose modulus parameter $\tau = iT$ is purely imaginary. The world-sheet propagator is

$$
\langle X^\mu(z)X^\nu(w) \rangle = \frac{\alpha'}{2} \eta^{\mu\nu} G(z - w),
$$

(2.1)

where

$$
G(\nu) = -\log \left| \frac{\theta_1(\nu|iT)}{\theta'_1(0|iT)} \right|^2 + \frac{2\pi}{T} \left[ \text{Im}(\nu) \right]^2.
$$

(2.2)

Here, $\theta_1$ is the theta function defined as

$$
\theta_1(\nu|\tau) = -i \sum_{m=-\infty}^{\infty} (-1)^m q^{\frac{1}{2}(m+\frac{1}{2})^2} \omega^{m+\frac{1}{2}},
$$

where $q = e^{-\pi T}$. Figure 1: The world sheet coordinate for annulus.

![Image of annulus]

(a) Annulus as a ‘flattened’ torus

(b) Modular transformed annulus

Figure 1: The world sheet coordinate for annulus.
\[ q = \exp(2\pi i \tau), \quad \omega = \exp(2\pi i \nu), \]

The propagators are periodic under the two lattice transformations

\[ z \rightarrow z + 1, \quad z \rightarrow z + iT \]

and they satisfy the flux conversation; the integral of \( \partial^2 G(z) \) over the torus vanishes. To turn torus propagators to annulus propagators, we place a mirror charge at \( -\bar{w} \) (and at all their lattice translation points) for a source charge at \( w \). This operation imposes Neumann boundary conditions along the two boundaries at \( \text{Re}(z) = 0 \) and \( \text{Re}(z) = 1/2 \), while maintaining the periodicity in \( z \rightarrow z + iT \), thereby turning the original torus to an annulus. Written explicitly, the propagators look like

\[
\langle X^\mu(z)X^\nu(w) \rangle = \frac{\alpha'}{2} \eta^{\mu\nu} \{ G(z - w) + G(z + \bar{w}) \}. \tag{2.3}
\]

The propagators with Dirichlet boundary condition can also be straightforwardly written down. However, we will not need them for we will consider only open string vertex insertions.

Our next task is to find the explicit form of analogous expressions that are valid when \( B \neq 0 \). As noted in [15], one can bring the \( B \) field into a block-diagonal form and consider each \( 2 \times 2 \) block separately. Suppose for now that we turn on the \( B_{12} = B \) along \( X^1 = X \) and \( X^2 = Y \) directions parallel to the D-branes under consideration. The boundary conditions at \( \text{Re}(z) = 0 \) and \( \text{Re}(z) = 1/2 \) should be modified into

\[
\partial_n X + iB \partial_t Y = 0 \big|_{\text{Re}(z)=0,1/2}. \tag{2.4}
\]

The answer is:

\[
\frac{2}{\alpha'} \langle X(z)X(w) \rangle = G(z - w) + \frac{1 - B^2}{1 + B^2} G(z + \bar{w}) + \frac{B^2}{1 + B^2 T} [\text{Re}(z + \bar{w})]^2, \tag{2.5}
\]

\[
\frac{2}{\alpha'} \langle X(z)Y(w) \rangle = \frac{2B}{1 + B^2} \left[ \log \frac{\theta_1(z + \bar{w})}{\theta_1(z + w)} + \frac{4\pi i}{T} \text{Re}(z + \bar{w}) \text{Im}(z + \bar{w}) \right]. \tag{2.6}
\]

When \( |z|, |w| \ll 1, T \), the quadratic terms are negligible and \( \theta_1(z/iT) \) reduces to \( z \), so that we recover the propagators on a disk, which was obtained, for example, in Refs. [6, 8, 15]. The coefficients of the \( \theta \)-function terms are uniquely determined by comparison with the propagator

\[ \text{As in [13], it is possible to trade the quadratic terms of (2.5) with modified boundary conditions involving a constant term on the right hand sides of (2.4). Our choice in this paper is to keep the boundary conditions (2.4) intact.} \]
on the disk. The quadratic terms are required by the periodicity in $z \rightarrow z + iT$ and flux conservation.

For the computation of the open string amplitudes, we insert open string vertex operators along the boundaries at $\text{Re}(z) = 0$ and $\text{Re}(z) = 1/2$. For later convenience, we note that for the planar insertions, the propagators become

\begin{align}
\langle X(0)X(iy) \rangle &= \alpha' \frac{1}{1 + B^2} G(iy), \\
\langle X(0)Y(iy) \rangle &= i(\pi \alpha') \frac{B}{1 + B^2} \varepsilon(y),
\end{align}

where we introduced the Heaviside step function $\varepsilon(y) = y/|y|$. For nonplanar insertions, we have

\begin{align}
\langle X(0)X(1/2 + iy) \rangle &= \alpha' \frac{1}{1 + B^2} G(1/2 + iy) + \frac{\pi \alpha'}{2T} \frac{B^2}{1 + B^2}, \\
\langle X(0)Y(1/2 + iy) \rangle &= i(2\pi \alpha') \frac{B}{1 + B^2} \frac{y}{T}.
\end{align}

The quadratic term in (2.5),

\[ \frac{2B^2}{1 + B^2} \frac{2\pi}{T} \left[ \text{Re}(z + \bar{w}) \right]^2, \]

(2.11)
deserves further comments. We note that (2.11) distinguishes between the planar and nonplanar vertex insertions. In particular, it vanishes for planar insertions along $\text{Re}(z) = 0$ but gives a non-vanishing contribution for nonplanar insertions where one vertex is separated from another. We will find that it will give finite contributions to the amplitudes under the decoupling limit of Ref. [8] thus surviving in the effective noncommutative field theory.

### 3 One-loop bosonic open string amplitudes

Utilizing the world-sheet propagators of section 2, we explicitly evaluate the annulus amplitudes, inserting two open string vertex operators along the world-sheet boundaries in the presence of D-branes. In the following, we will ignore all numerical constants in the overall normalization of the amplitudes, but the dimensions and dependence on coupling constant will be unambiguous.

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Footnote:

The periodicity in this context means the periodicity of the physical objects, such as $\langle \partial X Y \rangle$. We note that $\langle X Y \rangle$ itself is not periodic, but this does not give an ambiguity when computing physical amplitudes.
3.1 Planar and nonplanar bosonic open string amplitudes on an annulus

The one-loop amplitude in the presence of an \((n - 1)\)-brane is given by

\[
A = \int_0^\infty \frac{dT}{T} Z(T) \int_0^T dy_1 \int_0^T dy_2 (V_1(p, y_1) V_2(-p, y_2))_T \\
= g^2 \alpha' \int_0^\infty \frac{dT}{T} (2\pi \alpha'T)^{-n/2} f_1(q)^{-24T} \int_0^T dy_1 I(p; y, T) .
\] (3.1)

The “partition function” part is computed in the same way as in [16];

\[
Z(T) = \int \frac{d^n k}{(2\pi)^n} \sum_k e^{-2\pi \alpha' T(k^2 + M_k^2)},
\] (3.3)

\[
f_1(q) = q^{1/24} \prod_{m=1}^{\infty} (1 - q^m), \quad q = e^{-2\pi T}.
\] (3.4)

The vertex operators for a tachyon and a gauge boson are, respectively,

\[
V_T = g \sqrt{\alpha'} e^{ikX}, \; V_A = g \epsilon \cdot \partial X e^{ikX},
\] (3.5)

where the coupling constant \(g\) is the one that appears in the low energy effective action of open strings. Schematically,

\[
S = - \int d^n x \{(\partial \phi)^2 + m^2 \phi^2 + g^2 \phi^4 + \cdots \}
\] (3.6)

A simple dimensional analysis shows that \(g\) is related to string coupling by \(g^2 = (\alpha')^{n/2-2} g_{st}\).

We first give the answers for the planar amplitudes. When \(B = 0\), for the tachyon-tachyon insertion, we have

\[
I = \left| \frac{\theta_1(iy|iT)}{\theta_1'(0|iT)} \right|^{-2} \exp \left( \frac{2\pi T y^2}{T} \right),
\] (3.7)

while for gauge boson-gauge boson insertion, we have

\[
I = \left\{ \epsilon_1 \cdot \epsilon_2 J'' + \alpha'(\epsilon_1 \cdot p)(\epsilon_2 \cdot p)(J')^2 \right\} e^{\alpha' p^2 J} \\
= -\alpha' \left\{ (\epsilon_1 \cdot \epsilon_2) p^2 - (\epsilon_1 \cdot p)(\epsilon_2 \cdot p) \right\} (J')^2 e^{\alpha' p^2 J} + (\text{total derivative in } y),
\] (3.8)

where we define \(J(y) = G(iy)\). When we turn on the \(B\) field \((B \neq 0)\), the answer is exactly the same as the one for \(B = 0\), except that the external momentum squared is evaluated with respect to the open string metric \(G^{\mu \nu}\) defined in Ref. [8] as

\[
G_{\mu \nu} \equiv \eta_{\mu \nu} - (B \eta^{-1} B)_{\mu \nu}.
\] (3.9)

We now consider the nonplanar insertions where \(B \neq 0\) effect is conspicuous. For the tachyon-tachyon insertions, we get

\[
I = \left[ \frac{\theta_2(iy|iT)}{\theta_2'(0|iT)} \right]^{-2} \exp \left( \frac{2\pi T y^2}{T} \right) \exp \left( -\frac{p \cdot p}{2\pi \alpha' T} \right),
\] (3.10)
and the gauge boson-gauge boson insertions yield
\[ I = -\left[ \alpha' \left\{ (\varepsilon_1 \cdot p_2)^2 - (\varepsilon_1 \cdot p)(\varepsilon_2 \cdot p) \right\} (K')^2 + \frac{(\varepsilon_1 \times p_2)(\varepsilon_2 \times p)}{\alpha' T^2} \right] \times \exp\left( \alpha' p_2^2 K - \frac{p_1 p_2}{2\pi\alpha'} \right), \]
where we define \( K(y) = G(1/2 + iy) \). We introduced \( \circ \)-product and \( \times \)-product following Ref. [10] as
\[ p \circ p \equiv -\frac{1}{4} p_\mu (\Theta G \Theta)^{\mu\nu} p_\nu, \quad \varepsilon \times p = \varepsilon_\mu \Theta^{\mu\nu} p_\nu, \]
where \( \Theta \) is the noncommutativity parameter defined in [8],
\[ \Theta^{\mu\nu} \equiv -2\pi \alpha' \{(\eta + B)^{-1} B(\eta - B)^{-1}\}^{\mu\nu}. \]

The sign in the definition of \( p \circ p \) is introduced to make it nonnegative.

The \( B \) dependence in the amplitudes come from two combinations \( p \circ p \) and \( \varepsilon \times p \). Due to the prefactor \( 1/T \) in front of \( p \circ p \), the effect of noncommutativity becomes stronger as we approach the UV corner of the moduli integral. For the higher spin world-volume fields, there are polarization dependences, as exemplified in \( \varepsilon \times p \) for the gauge boson amplitudes.

### 3.2 Open/Closed string duality and the Decoupling limit

For the rest of this section, we only consider the tachyon amplitudes in detail for simplicity. We first review the well-known world-sheet duality for \( B = 0 \) to contrast it with the \( B \neq 0 \) situation. The relation between the nonplanar tachyon-tachyon amplitude
\[ A = \int_0^\infty \frac{dT}{T} (2\pi\alpha'T)^{-n/2} T \int_0^T dy \left| \frac{\theta_2(iy|iT)}{\theta_1(0|iT)} \right|^{-2} \exp\left( \frac{2\pi}{T} y^2 \right), \]
and the one-loop amplitudes in the field theory of the type (3.6) is manifest in the region $T \gg 1$, where we can expand (3.14) in $e^{-2\pi T}$. For example, when $y \ll T$, the open string diagram looks very much like the field theory diagram in Fig. 2(a). This intuitive picture is confirmed by an explicit calculation which shows that

$$A = g^2 \alpha' \sum_I a_I \int \frac{dT}{T} (2\pi \alpha'T)^{-n/2} e^{-2\pi \alpha'T M_I^2} = \sum_I \int d^n k \frac{a_I g^2}{k^2 + M_I^2}, \quad (3.15)$$

where $a_I$ are some numerical coefficients.

In the opposite end $T \ll 1$, the usual channel duality allows us to rewrite (3.14) from the point of view of the closed strings. In particular, using the modular transformation of the theta functions, we find

$$A = \int_0^\infty \frac{dS}{S} S^{n/2 - 12} f_1(\bar{q})^{-18} \int_0^1 dx |\theta_4(x|S)|^{-2}, \quad (3.16)$$

where $S = 1/T$ and $\bar{q} = \exp(-2\pi S)$. The picture now is a closed string connecting the open string states as in Fig. 2(b). For the case of the space-time filling 25-brane, (3.16) can be expanded to give

$$A = \sum_J b_J \int dSe^{-2\pi S \alpha'(p^2 + M_J^2)/4} \sim \sum_J \frac{b_J^2 \kappa^2}{p^2 + M_J^2}, \quad (3.17)$$

for some numerical constants $b_J$. The coupling constant $\kappa$ appears in the low energy effective action of the form

$$S = \int d^n x \kappa \chi \phi, \quad (3.18)$$

where $\chi$ is a closed string field.

The noncommutative field theory arises in the decoupling limit $\alpha' \to 0$ while keeping $G^{\mu\nu}$ and $\Theta^{\mu\nu}$ fixed [8]. We note that in the bosonic string theory, the mass spectrum is known to be $\alpha' M_I^2 = N_I - 1$ (open) and $\alpha' M_J^2 = 4(N_J - 1)$ (closed) for nonnegative integers $N_I$ and $N_J$. In this limit, therefore, if we ignore the tachyons, all but the contribution from massless intermediate states disappear, as can be seen from (3.17). When $B = 0$, the massless intermediate degrees of freedom give a trivial IR divergence that should be cancelled with other divergences via Fischler-Susskind type mechanism. When $B \neq 0$, their contribution is non-trivial as we will see shortly; we need to take the Wilsonian point of view regarding the cutoff and the effective degrees of freedom.
3.3 What survives the decoupling limit

The key issue is to identify the contributions from the $T \ll 1$ UV regime to the open string moduli integral when $B \neq 0$. For this purpose, in the spirit of string field theory [17, 18], we explicitly introduce a short distance UV regulator $1/\Lambda^2$ in the open string description; the regulated open string contribution comes from the region of the moduli space where $2\pi\alpha'T > 1/\Lambda^2$. Then, as depicted in Fig. 3, the possible extra UV degrees of freedom originating from the extreme UV open string loops should come from the corner of moduli space where $0 < 2\pi\alpha'T < 1/\Lambda^2$. When $2\pi\alpha'T$ goes below the UV cutoff, we have a factorization channel where the original nonplanar annulus diagram becomes two string states connected by a long closed string tube. Via open/closed string channel duality, it is natural to investigate $0 < 2\pi\alpha'T < 1/\Lambda^2$ corner of the open string moduli space in terms of the closed string picture. We thus resort to the nonplanar amplitude expression in the closed string channel, Eq. (3.16). In this channel, the open string UV cutoff $1/\Lambda^2$ transforms to the closed string IR cutoff $\Lambda^2$. As shown in Fig. 3, the open string UV regime gets mapped to the closed string IR regime $S/2\pi\alpha' > \Lambda^2$.

The contribution to the nonplanar amplitude from these IR closed string degrees of freedom can be computed as

$$A_{IR} = A(\infty) - A(\Lambda) \simeq \int_0^{\infty} \frac{dS}{S} S^{n/2-12} \left( e^{-p \cdot p S/2\pi\alpha'} - e^{-(p \cdot p + 1/\Lambda^2) S/2\pi\alpha'} \right),$$

(3.19)

where the IR regulated closed string amplitude $A(\Lambda)$ is defined as

$$A(\Lambda) \simeq \int_0^{\infty} \frac{dS}{S} S^{n/2-12} e^{-(p \cdot p + 1/\Lambda^2) S/2\pi\alpha'},$$

(3.20)

4 When computing perturbative string amplitudes, the external momenta are always put on-shell. Here we are assuming that the final expression holds for off-shell amplitudes as well.

5 In [19], the same amplitude was computed using the open-closed string field theory for $B = 0$. Division of the moduli space into two connected parts is inherent in their formalism.
explicitly introducing the cut off $\Lambda^2$. The $A(\Lambda)$ defined in (3.20) is the IR regulated amplitude where we restrict the closed string moduli integral to the distance up to the IR cutoff scale $\Lambda^2$. In (3.19), we wrote down only the part of the amplitude that survives the decoupling limit and we deleted the tachyonic intermediate contribution. Reinstating the tachyonic contribution would produce the negative eigenvalue for the quadratic effective action for the small value of $p \cdot p$ by shifting $p \cdot p$ into $p \cdot p + M_{\text{tachyon}}^2$ in (3.19), indicating the tachyonic instability [10]. For the space-time filling 25-brane ($n = 26$), we have
\[
A_{IR} = \kappa^2 (\alpha')^2 \left[ \frac{1}{p \cdot p} - \frac{1}{p \cdot p + 1/\Lambda^2} \right] = \frac{\kappa^2 (\alpha')^2}{p \cdot p + \Lambda^2 (p \cdot p)^2}
\]
from (3.19). From the ‘long tube’ IR closed string picture of Fig. 2, we find that (3.21) is nothing but the propagator of the extra degree of freedom multiplied by the coupling constant. From the low energy effective description point of view, the extra degree of freedom (denoted as $\chi$ field) then has the effective Lagrangian of the form
\[
\int dx^{26} \left[ \partial \chi \cdot \partial \chi + \Lambda^2 (\partial \cdot \partial \chi)^2 \right] + \int d^n x \kappa \chi \phi,
\]
where $\phi$ is a generic world-volume open string scalar field. From our derivation, it is clear that $\chi$ field with the effective action (3.22) gives the effective description of the ‘long tube’ IR closed strings at low energies. The effective action (3.22) is identical to the one found in the noncommutative field theory one-loop analysis [10].

For the codimension one 24-brane, (3.19) yields
\[
A_{IR} = \left[ \frac{\kappa^2 \alpha'}{\sqrt{p \cdot p}} - \frac{\kappa^2 \alpha'}{\sqrt{p \cdot p + 1/\Lambda^2}} \right],
\]
while for the codimension two 23-brane, we get
\[
A_{IR} = \kappa^2 \left[ \log(p \cdot p) - \log(p \cdot p + 1/\Lambda^2) \right].
\]
We note that (3.23) and (3.24) are the same as the 1PI amplitudes found in Ref. [11] for the extra low energy degrees of freedom. From our derivation, it is clear that they also represent the IR closed string degrees of freedom; they live in the bulk space-time while the open string degrees of freedom are confined on a codimension one and two D-brane, respectively. The extra dimensions found in Ref. [11] are indeed space-time dimensions transversal to the brane, at least when the noncommutative field theory under consideration derives from the decoupling limit of open string theory.
4 One-loop open superstring amplitudes

In this section, we repeat the calculations of the previous section for the case of open superstrings. The main finding that the space-time supersymmetry makes the two point amplitudes vanish regardless of the value of $B$ is consistent with Ref. [20]. In Ref. [20], it was argued that for the supersymmetric gauge theories with sixteen supercharges, the noncommutative IR singularities do not show up. In our present context with parallel D-branes, we clearly have sixteen supercharges.

The answer for the amplitude is given by

$$A = \sum_{a=2}^{4} (-1)^a \int_0^\infty \frac{dT}{T} (2\pi\alpha'T)^{-n/2} \left\{ \frac{f_a(q)}{f_1(q)} \right\}^8 T \int_0^T dy (V_1(p, y)V_2(-p, 0)),$$

where the index $a$ labels spin structures. For the definitions of $f_a(q)$, see Ref. [16]. The vertex operators for massless gauge bosons are

$$V(p, y) = \varepsilon_\mu(\partial X^\mu + ip \cdot \psi \bar{\psi}^\mu)e^{ipX}.$$  \hfill (4.1)

In addition to the terms in Eqs. (3.8) and (3.11), we have the contraction of four world-sheet fermions. After the summation over spin structures, however, the two point function completely vanishes due to the Jacobi’s fundamental formulae [21]. Naturally, one attributes this property to the space-time supersymmetry. In fact, only the terms with eight or more world-sheet fermions give nonzero contribution.

5 Discussions

The term (2.11) should be present in the world-sheet propagators as a consequence of the boundary conditions (2.4). Its contribution to nonplanar amplitudes is the crucial exponential factor $\exp(-(p \circ p)/2\pi\alpha'T)$ necessary for the emergence of IR closed string degrees of freedom. Its existence, however, might seem rather puzzling from the open string theory point of view; the direction $\text{Re}(z)$ is the spatial direction along which the open string lies. Therefore, it implies that there is a term in the mode expansion of $X$, which is linearly proportional to $\text{Re}(z)$. For closed strings, this would usually signal the presence of a non-trivial winding state, while we are apparently considering open strings. This behavior, however, is consistent with Ref. [22] where the thermodynamic evidence for the ‘winding states’ in noncommutative field theories is given. As was shown from the calculations in section 3, its contribution to amplitudes is from
the corner of the open string moduli space where the dual closed string interpretation is appropriate. From this point of view, the ‘winding states’ of Ref. [22] represent nothing but closed string states. We note that the authors of Ref. [22] do not find any wrapping states, which would correspond to higher extended objects than strings.

For the commutative field theories, the amplitude \( A_{\text{IR}} \) becomes a simple divergence involving the cutoff \( \Lambda^2 \) without the momentum dependence. In realistic models, via the Fischler-Susskind mechanism, the cutoff dependence in open string channel gets cancelled by closed string sigma model divergence, ultimately resulting the vanishing beta function. In fact, the dual supergravity background geometries of commutative field theories have the asymptotic isometry group isomorphic to the conformal group, a familiar AdS/CFT correspondence [23]; in this context, we consider the perturbations around a conformal fixed point. On the other hand, the conjectured dual supergravity backgrounds of the noncommutative field theories do not have the asymptotic isometry group isomorphic to the conformal group [24, 25]. The emergence of the non-trivial IR closed strings in noncommutative field theories appears to be related to the nonconformality of the world-volume theory. Naturally, the detailed investigation of the Fischler-Susskind mechanism in the noncommutative context along the line of Ref. [26] should reveal interesting physics.

The noncommutative field theory calculations mimic the open string calculations to a remarkable degree, as shown from the analysis in this paper. By turning the viewpoint around, one might consider using the noncommutative field theory as a useful guide that provides us with a systematic organization tool for the open/closed string loop diagrams. In this spirit, the disentangling of higher loop diagrams in noncommutative field theories via open string perturbation theory should be an exciting venue, especially in relation to string field theory [8, 17].

**Note added**

After the completion of the first version of this paper, Ref. [28] appeared. After reading [28], we found some calculational errors in section 3.1 for the gauge boson amplitudes in the original version of our paper. The corrected calculation revealed the \((\epsilon_1 \times p)(\epsilon_2 \times p)\) part in Eq. (3.11). As noted in [28], some on-shell string calculations can be extended to off-shell in field theory limits. Reinstalling the part that vanishes in on-shell, the \([(\epsilon_1 \cdot p)^2 - (\epsilon_1 \cdot p)(\epsilon_2 \cdot p)]\) part, in Eqs. (3.8) and (3.11), we find that our gauge boson amplitudes are identical to the ones given in Ref. [28]. We emphasize that our boundary propagator expressions, Eqs. (2.7)-(2.10), are
identical to Eqs. (2.42)-(2.45) in [28] obtained by using the boundary state formalism. To explicitly see this, we need variable changes \(2\pi y = \log \left| \frac{\rho}{\rho'} \right|\) and \(2\pi T = -\log k\).

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