Optimal short-term Scheduling of Industrial Packing Facilities

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Abstract

The simultaneous lot-sizing and production scheduling problem of a real-life large-scale industrial facility of packaged consumer goods is considered in this work. The problem under consideration is mainly focused on the packing stage which constitutes the major production bottleneck. Several packing lines, illustrating different design and operational characteristics, operate continuously in parallel. An efficient solution strategy is implemented to handle the high computational cost. A Mixed-Integer Linear Programming model (MILP) is used in parallel with a decomposition-based algorithm. Appropriate constraints, referring to the production/formulation stage of the plant, are also imposed in order to ensure the generation of feasible production schedules. The model relies on tight timing and sequencing constraints, as well as products’ allocation constraints. The main objective is the minimization of the makespan. A number of different case studies have been considered and detailed optimal production schedules have been generated for over 130 products scheduled weekly. The obtained results lead to nearly-optimal scheduling solutions in reasonable computation times. The proposed model can assist decision makers towards rigorous scheduling plans in a dynamic production environment under realistic uncertainty.

Keywords: production scheduling, consumer goods industry, mixed integer linear programming, decomposition technique

1. Introduction

Modern multi-product, multipurpose plants play a key role within the overall current climate of business globalization, aiming to produce highly diversified products that can address the needs and demands of customers spread over wide geographical areas. The inherent size and diversity of these processes gives rise to the need for planning and scheduling problem, of large-scale industrial production operations. Over the past 20 years the literature illustrates a large-number of scheduling models, which have been mostly applied on generic but relatively small problem instances (Méndez et al., 2006). However, current real-world industrial applications include hundreds of different final products, produced in flexible multi-purposes facilities, under several tight design and operating constraints (Harjunkoski et al., 2014). A few approaches have been used to solve large-scale industrial scheduling problems, utilizing advances in Mixed-Integer...
optimisation (Kopanos, Puigjaner and Georgiadis, 2011). Furthermore, hybrid methods for large scale industrial problems have been proposed. Kopanos et al. (2010), proposed a decomposition strategy for large-scale scheduling problems in multiproduct multistage pharmaceutical batch plants. Baumann and Trautmann (2014) proposed a hybrid method for large-scale short-term scheduling of make-and-pack production processes. In this work an MILP-based decomposition algorithm is proposed for large scale scheduling problems in a multiproduct continuous plant. The model focuses on the packing stage, taking also into account constraints referring to the production formulation stage in order to ensure the generation of feasible production schedules.

2. Problem Statement

This work considers the scheduling of packaged consumer goods in a real-life industry. More than 300 products can be produced continuously in parallel packing lines. The production process consists of the formulation/production and the packing stage. In the formulation stage the intermediate products are made. In most cases, more than one final product can be produced from the same intermediate product in the packing stage. Each packing line is connected to its own production/formulation unit. Sequence dependent changeovers take place in both stages. The changeover times differ among the various sequences, depending on the package size, the package color, the intermediate product etc. All changeovers, in the two stages, take place simultaneously and therefore in each sequence the most time-consuming changeover determines the total changeover time for a product sequence. In addition, in the formulation stage, due to technical plant restrictions, the total number of intermediate products’ changeovers should not exceed an upper limit. The short-term scheduling horizon of interest is one week, and both the packing and the formulation units are available 24 hours per day. Products’ due dates are taken into account along with the necessary planned maintenance activities. The main objective is the minimization of the makespan and the minimization of products’ changeover times.

3. MILP-based Decomposition Algorithm

For the aforementioned problem, an MILP-based decomposition algorithm has been developed. The large-scale initial problem is decomposed by scheduling, in an iterative mode, a subset of the involved product orders. The main model decisions are a) the products’ allocation to the packing lines, taking into account the technical constraints of unit’s restrictions, b) the relative sequence of products in the packing lines and c) the starting and the completion time of product orders in the packing stage. The proposed MILP-based decomposition algorithm consists of: a) the insertion policy, b) the MILP model and c) a decomposition technique. In case that a list of maintenance periods have to be taken into account, extra “maintenance-product” orders are scheduled, with processing time equal to the maintenance time. The ending times of these product orders are fixed to the ending times of these maintenance activities.

3.1. Insertion policy

In order to solve the initial problem iteratively, the number and the sequencing of the inserted products have to be decided. Two insertion criteria are adopted in order to ensure the avoidance of infeasible schedules. The products with the earliest due dates are inserted
first. Furthermore, according to the technical plant restrictions, the products with the same recipe (intermediate product) are inserted first, in order to minimize the total number of intermediate products’ changeovers. It was shown, by several test runs, that a 5-by-5 product insertion policy is the optimal one, as by increasing the number of products the solution is not improved and the computational cost is increased as well. In case that maintenance periods have to be taken into account these “maintenance-product” orders are inserted first.

3.2. MILP model

The MILP model applied in this case study is an extension of a general-immediate precedence framework as developed by Kopanos, Mendez and Puigjaner (2010). A brief description of the model is presented below:

\[
\sum_{j \in \mathcal{I}^i} Y_{i,j} = 1 \quad \forall i
\]

\[
X_{i',j,i} + X_{i',j,j} + 1 \geq Y_{i',j} + Y_{i,j} \quad \forall i,i' > i,j \in (i_{\cap}i')
\]

\[
C_{i'} \geq C_i + T_{i'} + XX_{i,i'} \text{ changeover}_{i,i'} - M(1 - X_{i,i'})
\]

\[
\forall i, i' \neq i, j \in (i_{\cap}i')
\]

\[
Z(X_{i',j,i} + X_{i',j,j}) \leq Y_{i',j} + Y_{i,j} \quad \forall i,i' > i,j \in (i_{\cap}i')
\]

\[
Z_{i,i'} + XX_{i,i'} \geq X_{i,i'} \quad \forall i,i' \in (i_{\cap}i')
\]

\[
Z_{i,i'} = \sum_{i'' \neq i} \sum_{j \in (i_{\cap}i')} \left(XX_{i',i''} + XX_{i'',i'}\right) + M(1 - XX_{i,i'})
\]

\[
\forall i, i', j \in (i_{\cap}i')
\]

\[
\sum_{i} \sum_{j \neq i}XX_{i,j} \leq \text{Limit}_j \quad \forall j
\]

\[
\min C_{\text{max}} \geq C_i \quad \forall i, j
\]

Constraint (1) forces that every product order \( i \) goes through one packing line \( j \) via the allocation binary variable \( Y_{i,j} \). Constraint (2), (3) and (4) give the relative sequencing of product orders. The big-M constraint (3) determines the completion time \( C_{i'} \) of a product order \( i' \) to be greater that the completion time and the processing time \( T_{i'} \) of whichever product \( i \) is produced beforehand at the same unit, and greater than the changeover time, \( \text{changeover}_{i,i'} \), only if the binary variable \( X_{i,i'} \) is active. The binary variable \( X_{i,i'} \) is active only if product \( i' \) is produced after product \( i \). The constraints (2) and (4) state that when two products are produced at the same unit, only one global sequencing binary
variable has to be active and when one of the binary variable $X_{i',i,j}$ is active at least one of the $Y_{i,j}$ and $Y_{i',j}$ has to be active as well. The variable $Z_{i,i',j}$ signifies the position difference among two products produced in the same packing line. When $Z_{i,i',j}$ is equal to 0, the product i is produced exactly before the $i'$. The variable $Z_{i,i',j}$ is calculated in equation (6). As a result, from equation (5) the immediate precedence binary variable $XX_{i,i',j}$ takes the value 1 only when the variable $Z_{i,i',j}$ is equal to 0. The binary variable $XX_{i,i',j}$ takes the value 1 when the product $i'$ is produced exactly after the product i. The constraint (7) ensures that the number of sequences between products with different recipe ($formula_{i'}$) does not exceed an upper limit ($Limit_j$) which is determined by the plants’ technical restrictions. In order this constraint to be included in the model both the immediate and the general precedence binary variables are used. Finally, the objective function of the model is expressed by constraints (8), which is the minimization of the total production makespan, $C_{max}$, hence it also considers the minimization of changeovers and unnecessary idle times.

3.3. Decomposition Algorithm

The large industrial scheduling problem is decomposed into smaller problems in an iterative fashion. In each iteration a subset of the product orders is scheduled. In this way the MILP subproblems are solved much easier and the computational time is significantly decreased as well. A number of inserted products are scheduled in each iteration until all product orders are scheduled. After the resolution of the MILP model in each iteration, the global sequencing variables as well as the allocation variables of the inserted products are fixed. However, the timing variables (ending time) and the immediate precedence binary variables remain free. When all products are inserted, the final schedule is constructed. Each iteration, aims to find a 0% integrality gap solution. However, the industrial necessities impose the usage of an upper bound in the total computational time. Therefore, a time limit of 10 minutes, has been set for each subproblem. In the Figure 1. the decomposition technique is briefly described.

![Figure 1: Decomposition-based solution algorithm](image)

4. Results

A representative industrial case study of 178 final products and 6 packing lines is used to illustrate the applicability and efficiency of the proposed solution strategy. All the data have been provided from a real-life, large-scale consumer goods industry. Significant packing lines restrictions do not allow full flexibility of the products’ allocation. The first 80 products can be produced only in the first 3 packing lines, as the rest 98 products can be produced only in the next 3 packing lines. Utilizing the aforementioned decomposition algorithm 5 products are scheduled in each iteration, except for the last one, where the
remaining 3 products are scheduled. The decomposition technique, was implemented in GAMS and solved using the CPLEX 12.0 solver. A 0% optimality gap has been reached in all iterations. Figure 2. illustrates the final schedule of the study under consideration. Maintenance activities have also been taken into account.

Figure 2: Gantt chart of the packing lines referring to the examined case study

Table 1: Comparison between the exact MILP model and the decomposition approach applying different insertion policies

|                  | Exact MILP Approach | Decomposition Approach |
|------------------|---------------------|------------------------|
|                  |                     | Insertion policy 1-by-1 | Insertion policy 5-by-5 | Insertion policy 10-by-10 |
| Makespan (hrs)   | 118.9               | 122.6                  | 120.2                  | 120.8                  |
| Computational CPU Time (minutes) | 15        | 1.6                    | 2.46                   | 31.4                   |

Another medium-scale problem instance, including 80 products and 3 packing lines, is also demonstrated, in order to assess the solutions of the proposed decomposition technique. The case study is solved using both the decomposition algorithm and the exact MILP model, described above. The results are illustrated in the Table 1. It is observed that the decomposition algorithm leads to nearly optimal solutions. In addition, the computation time is significantly decreased using the decomposition algorithm, when the products are inserted 1-by-1, or 5-by-5. The application of the 10-by-10 insertion policy, increases the complexity of the subproblems and the zero integrality gap cannot be achieved in many iterations without exceeding the defined execution time limit. As a result, the computational time is increased and higher makespan values are obtained. The
The aforementioned exact MILP model can only be applied to medium problem instances. For larger problem instances the computational cost becomes prohibitively high and not even a feasible solution can be returned.

5. Conclusions
The main contribution of this work is the application of an MILP-based decomposition technique in large scale packing facilities. A continuous, real-life, large-scale, industrial facility is considered. More specifically, the packing stage of a consumer goods facility is scheduled taking also into account restrictions related to the formulation/production stage. The proposed decomposition-based approach leads to nearly optimal solutions of large-scale problem instances, while on the contrary, the exact MILP model can only be applied to medium problem instances. The solution strategy can assist decision makers towards rigorous scheduling plans in a dynamic production environment under realistic uncertainty. As future step, an additional improvement algorithm step will be considered.

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