Dynamic analysis of vibrating screener system

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Abstract. Transversal vibrations of a screen for a fine granular material are studied using both analytical and numerical methods. In the analytical approach the motion of the screen is described by partial differential equations. The general solution of the screen free vibrations is derived from variables separation method. In the numerical computations the finite element method is applied. The screen compound geometry and the mass of the granular material are considered. Screen natural frequencies and natural mode shapes are determined. Numerical results are compared with the analytical solution. The screen geometry simplifications are proposed and validated by benchmark tests. The presented research allows determining the impact of the fine–granular material mass on the vibrating screener. The influence of the granular material on the screen natural frequencies is also investigated. It is important to note that the results of this research provide practical hints for vibrating screens designers.

1. Introduction
The large number of cut-outs and their distribution allows effective fine granular material sieving. A theoretical solution for a body of such geometry does not exist. Both static and mode shapes solutions exist only for a plate without openings. Many questions arise considering this model of sophisticated geometry. Is it possible to replace the screen geometry by a solid plate without the loss of accuracy in prediction of natural frequencies and natural mode shapes? What is the sufficient number and necessary concentration of openings? How to modify material data - Young’s modulus and density - in order to obtain comparable results for the screen plate and the solid plate?

The answers for the above questions allow developing simplified vibrating screen model which has the same properties as considered screen. Numerical results for the solid plate can be compared with the ones derived from analytical methods. This way, the reliability of proposed simplified model can be validated.

2. Free vibrations of rectangular plate - theoretical solution
A vibrating system for fine-granular material is considered (figure 1). The vibrating screen is a rectangular plate 0.85 x 1.5 m one millimeter thick with the set of rectangular openings (figure 2). The solution of free vibrations of the rectangular plate simply supported (pinned) on all edges is commonly known. Here, the analytical solution of natural frequencies and mode shapes for the rectangular plate in which two opposite edges are pinned while the other two are free is presented. The considered plate in the Cartesian coordinate system is shown in figure 3.

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The differential equation for plate natural vibrations is:

\[ \frac{\partial^2 w}{\partial t^2} + \frac{D}{\rho h} \nabla^4 w = 0 \]  \hspace{1cm} (1)

where \( \rho \) is the density, \( h \) is the plate thickness, \( D \) is the plate bending stiffness

\[ D = \frac{Eh^3}{12(1-\nu^2)} \]  \hspace{1cm} (2)

where: \( E \) - Young’s modulus, \( \nu \) - Poisson ratio
The boundary conditions for the plate are:

\[ w(0, y, t) = w(a, y, t) = 0; \quad \frac{\partial^2 w(x, y, t)}{\partial x^2} \Bigg|_{x=0} = \frac{\partial^2 w(x, y, t)}{\partial x^2} \Bigg|_{x=a} = 0 \]  

(3)

and

\[ \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \Bigg|_{y=0} = 0; \quad \frac{\partial}{\partial y} \left( \frac{\partial^2 w}{\partial y^2} + (2 - \nu) \frac{\partial^2 w}{\partial x^2} \right) \Bigg|_{y=b} = 0 \]  

(4)

In the variables separation method the solution \( w(x, y, t) \) is assumed to be a product of spatial and time functions:

\[ w(x, y, t) = W(x, y)T(t) \]  

(5)

After substituting (5) into (1), (3) and (4) one obtains:

\[ \ddot{T}(t) + \omega^2 T(t) = 0 \]

\[ \nabla^4 W(x, y) - p^4 W(x, y) = 0 \]  

(6)

where: \( \omega \) is the natural frequency. After introducing parameter \( p \):

\[ p^4 = \frac{\rho h \omega^2}{D} \]  

(7)

By application of Levy’s method for considered boundary conditions, the natural frequencies can be obtained from the following equation:

\[ \sin(\delta_1 b) \sinh(\delta_1 b) \delta_1^2 \left( p^2 - \alpha_m^2 (1 - \nu) \right)^2 - \delta_1^2 \left( p^2 + \alpha_m^2 (1 - \nu) \right)^2 \]

\[ - 2 \left( \cosh(\delta_1 b) \cos(\delta_1 b) - 1 \right) \delta_1^2 \left[ p^4 - \alpha_m^4 (1 - \nu)^2 \right] = 0 \]  

(8)

where:

\[ \alpha_m = \frac{m \pi}{a}, \quad m = 1, 2, \ldots, \quad \delta_1^2 = p^2 - \alpha_m^2, \quad \delta_2^2 = p^2 + \alpha_m^2. \]  

(9)

Equation (8) represents highly non-linear and extremely ill-conditioned problem. Very small variations of \( p \) parameter can cause rapid jumps on the left-hand-side of (8) e.g. if \( f(p) \) is the left-hand-side of (8) for the considered plate \( f(5.21) = 3.24e9 \) while \( f(5.22) = -1.69e9 \). Moreover, if \( p < \alpha_m \), then the function (8) returns a complex number.

After setting \( m \) the \( n \) values the \( p \) parameter can be found i.e. \( p = p_{mn} (m,n = 1, 2, \ldots). \) When \( p \) is known the natural frequencies can be obtained from (7)

\[ \omega_{mn} = p_{mn} \sqrt{\frac{D}{\rho h}} \]  

(10)

The mode shapes for the considered plate are [1]:
\[
W_{mn}(x, y) = \left[ 1 - (\cosh(\delta_1 b) - \cos(\delta_1 b)) \right] \left[ p_{mn}^2 - \alpha_m^2(1 - \nu)^2 \right] \left[ \sinh(\theta_1 y) \right] + \left[ p_{mn}^2 - \alpha_m^2(1 - \nu) \right] \left[ \sin(\delta_1 b) \right] - \left[ p_{mn}^2 - \alpha_m^2(1 - \nu) \right] \left[ \sinh(\delta_1 b) \right] - \left[ p_{mn}^2 + \alpha_m^2(1 - \nu) \right] \left[ \sin(\delta_1 y) \right] \left[ \cos(\delta_1 y) \right] \left[ \frac{mn\pi}{a} \right].
\]

(11)

For the considered plate the magnitude of natural frequencies found analytically are presented in table 1.

| Parameter | Frequency (s\(^{-1}\)) |
|-----------|------------------------|
| m=1, n=1  | 3.3516                 |
| m=1, n=2  | 4.1974                 |
| m=1, n=3  | 6.7790                 |
| m=1, n=4  | 11.1764                |
| m=2, n=1  | 13.4999                |

3. Free vibrations of rectangular plate - numerical solution

The solution for free vibrations of the rectangular plate described in the previous chapter is also found numerically using the finite element method [2]. Two commercial FEM programs are used independently: ANSYS and ABAQUS. In both programs the rectangular plate is modeled as a mesh of shell elements. The mesh consisting of higher-order rectangular elements is generated in both programs. An example of the solution obtained in ANSYS is presented in figure 4.

The magnitudes of natural frequencies of the rectangular plate computed by ANSYS and ABAQUS programs are summarised in table 2. Results presented in table 2 show excellent accuracy and convergence to the theoretical results for the first two-three natural frequencies. For higher natural frequencies the precision is still acceptable but the approximation errors are slightly larger. Fortunately, these frequencies are not important from the point of view of presented research.

|          | ANSYS | ABAQUS |
|----------|-------|--------|
| 3.2719   | 3.3514|
| 4.0972   | 4.1959|
| 6.6171   | 6.7751|
| 10.9110  | 11.1710|
| 13.1930  | 13.4990|

4. Free vibrations of rectangular plate with cut-outs

Following numerical simulations for the solid rectangular plate, similar computations are made for the rectangular screen. This problem cannot be solved analytically. In the numerical computations very dense finite elements meshes are used. In the best-fitted model each gap visible in figure 2 is covered by three layers of shell elements. The size of the problem and computation time is much larger now than those in previous example. The mesh generation is a very laborious task because of sophisticated
geometry which includes thousands of openings. An exemplar solution obtained by ANSYS program is presented in figure 5 (a mesh of holes is clearly visible).

Figure 4. Free vibrations of rectangular plate - mode 3.

Figure 5. Free vibrations of rectangular plate - numerical solution, mode 3.

The natural frequencies of the rectangular screen plate computed by ANSYS and ABAQUS programs are presented in table 3. Comparison of the natural frequencies of solid and screen rectangular plates shows that for the first three frequencies obtained results are comparable and they are similar to the analytical solution of the solid plate. Thus, the question arises if the solid model of rectangular plate can be used instead of screen one? It is very important from the point of view of the total cost of numerical computations [3]. The answer for the above question is positive if material data is properly modified. Two material constants influence natural frequencies radically i.e. Young’s modulus (representing stiffness) and the density (representing mass). If the ratio of $E/\rho$ is preserved, the solution obtained for the same geometry and the same boundary conditions is the same. In this case the geometries of solid and screen plates are different.

Table 3. Natural frequencies (s$^{-1}$) - numerical solution.

|          | ANSYS | ABAQUS |
|----------|-------|--------|
| 3.0745   | 3.1440|
| 4.0239   | 4.0568|
| 6.2825   | 6.2399|
In the numerical tests considering replacing the model of the screen plate with the solid one the mass of the plate should remain constant. Thus, the density of the solid plate is corrected. The magnitude of the Young’s modulus is also modified. This modification is based on the accuracy of the first natural frequency which is the main frequency of interest. During the series of computations the optimal Young’s modulus was found. The magnitude $E = 1.06 \times 10^{11}$ Pa is selected in the analysis of solid plate model used instead of the screen plate model. Of course, if other natural frequencies were also important, the choice of Young’s modulus may have taken them into account and the least squares based-methods could have been applied.

5. Forced vibrations of rectangular plate

Another problem solved within presented research is the prediction of rectangular plate forced vibrations. In this case the plate is loaded by the mass of stones. The layer of stones covering the plate is shown in figure 6.

The contact between the plate (modeled as a shell) and stones (modeled as a solid body) is assumed to be of type “bonded” (ANSYS program). It means that transverse displacements of both entities are common. The presented analysis is the first attempt to solve the screen forced vibration problem [4]. The assumed numerical model is as simple as possible. Instead of the screen model the solid plate model is used. Material data is modified as described in the previous chapter. Unlike in the reality, the thickness of stones layer is constant here. Of course, this model does not allow individual stones to pass through the screen.

The main goal of this test is the investigation of the influence of stones mass on the natural frequencies of the screen. The results of analysis are summarised in table 4. As expected, the natural frequencies have decreased because of the larger vibrating mass. Although presented model is relatively simple, the results of numerical simulations are promising. They should be verified experimentally. Appropriate research is planned in the future.

| Freq. number | ANSYS       |
|--------------|-------------|
| 1            | 1.3264      |
| 2            | 1.7369      |
| 3            | 2.6806      |
| 4            | 3.9253      |
| 5            | 5.3594      |
6. Conclusions

Numerical investigations presented in this paper are a part of a larger project considering development of effective vibrating screen system. Frequencies of vibration inductors applied in this system should be far away from the resonance i.e. far from the natural and forced frequencies of the vibrating screen. The set of free and forced natural frequencies can be found by the numerical analysis. The most important are the first few frequencies.

In this research the finite element method is used in order to provide free/forced frequencies and appropriate mode shapes. Results of numerical analyses are compared, if possible, with the theoretical solution. The simplified model of the solid plate based on the modified material data is proposed. In the future research the experimental investigations will be undertaken to help validate and improve proposed numerical models.

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