ESTIMATES WITH AN EFFECTIVE CHIRAL LAGRANGIAN FOR HEAVY MESONS

R. Casalbuoni
Dipartimento di Fisica, Univ. di Firenze
I.N.F.N., Sezione di Firenze

A. Deandrea, N. Di Bartolomeo and R. Gatto
Département de Physique Théorique, Univ. de Genève

F. Feruglio
Dipartimento di Fisica, Univ. di Padova
I.N.F.N., Sezione di Padova

G. Nardulli
Dipartimento di Fisica, Univ. di Bari
I.N.F.N., Sezione di Bari

UGVA-DPT 1992/07-779
BARI-TH/92-117
Revised version, September 1992

* Partially supported by the Swiss National Foundation
ABSTRACT

On the basis of an effective lagrangian incorporating approximate chiral symmetry and heavy-quark spin and flavor symmetries, and by use of information on leptonic decays, we estimate the effective $D^*D\pi$ coupling.
1 Introduction

Much work has been recently devoted to formulate a heavy-quark effective theory, for hadrons containing a heavy quark, incorporating the heavy-quark flavour symmetry and the heavy-quark spin symmetry that are expected to hold for infinite heavy-quark mass, and estimate deviations from such a limit [1]. We recall that another long-standing attack to the complexity of the QCD dynamics has been the effective chiral approach, based on the approximate chiral invariance of the light QCD sector for small masses of the light quarks. It is important to look for possibilities of using at the same time both systems of symmetries (heavy quark and chiral) in appropriate kinematical situations where they both, separately and acting on the different fields, may be expected to have some approximate validity. The quantitative analysis of the involved approximations will undoubtedly require efforts and hard calculations of corrections terms. We shall attempt here some first estimates, openly based on a certain optimism, considering to come back to more accurate evaluations of the approximations involved when some theoretical issues will be solved and possibly more data accumulated.

In a recent paper [2] (see also [3], [4]) an effective chiral symmetric lagrangian describing low momentum interactions of heavy mesons with the pseudo Goldstone bosons of the $0^-$ octet has been constructed. The lagrangian is as follows

\[
L = \frac{f^2}{8} Tr \left[ \partial^\mu \Sigma \partial_\mu \Sigma^\dagger \right] + \lambda_0 Tr [m_q \Sigma + m_q \Sigma^\dagger] \\
- i Tr [\bar{H}_a v_\mu \partial^\mu H_a] + \frac{i}{2} Tr [\bar{H}_a v^\mu H_b (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)_{ba}] \\
+ \frac{g}{2} Tr [\bar{H}_a H_b \gamma^\mu \gamma_5 (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)_{ba}] \\
+ \lambda_1 Tr [H_a H_b (\xi m_q \Sigma + \xi^\dagger m_q \Sigma^\dagger)_{ba}] \\
+ \lambda_1' Tr [\bar{H}_a H_a (m_q \Sigma + m_q \Sigma^\dagger)_{bb}] \\
+ \frac{\lambda_2}{m_Q} Tr [H_a \sigma_{\mu\nu} H_a \sigma^{\mu\nu}] + \cdots
\]

(1.1)

Here the fields $H_a$ describe the mesons $\bar{q}_a Q$ made up by the heavy quark $Q$ and the light anti-quark $\bar{q}_a$ ($a = 1, 2, 3$). The lagrangian is assumed to describe the limit $m_Q \rightarrow \infty$, where, as shown in refs. [5], [6], QCD has an additional spin flavour symmetry and a heavy quark effective theory (HQET) can be developed [7]. In eq. (1.1) $f = 132 \, MeV$, $m_q$ is the diagonal light quark mass matrix; $H_a$ is a $4 \times 4$ matrix that contains the pseudoscalar and vector meson fields $P_a$ and $P_a^\mu$

\[
H_a = \frac{(1 + \gamma^5)}{2} [P_a^* \gamma^\mu - P_a \gamma_5] \\
\bar{H}_a = \gamma_0 H_a^\dagger \gamma_0
\]

(1.2)

with $P_a^\mu v_\mu = 0$. These fields have dimension $3/2$ since factors of $\sqrt{m_P}$ have been absorbed in their definition. We also note that the meson fields depend on the meson four-velocity $v_\mu$; each term in eq. (1.1) which is bilinear in $H_a$ is diagonal in $v$, according to the velocity superselection rule [8] and a sum over velocities is understood. Finally $H_a$ also depends on the heavy flavour $Q$. 

As for light mesons, the matrix $\Sigma$ contains the pseudo Goldstone bosons in the form
\[
\Sigma = \xi^2 = \exp \left(2iMf\right)
\]
where $M$ is the usual $3 \times 3$ matrix of the $0^-$ pseudoscalar octet.

Besides Lorentz invariance and discrete symmetries, the strong interaction lagrangian (1.1) possesses, in the limit $m_Q \to \infty$, $m_q \to 0$, a $SU(2)$ spin symmetry, a heavy flavour symmetry and a $SU(3)_L \otimes SU(3)_R$ symmetry. Under the chiral symmetry the fields transform as follows
\[
\begin{align*}
\Sigma & \to L\Sigma R^\dagger \\
\xi & \to L\xi U^\dagger = U\xi R^\dagger \\
H & \to HU^\dagger
\end{align*}
\]
where $U$ is, in general, a non-linear function of the fields and the matrices $L \in SU(3)_L$ and $R \in SU(3)_R$. Under the $SU(2)$ spin symmetry
\[
H_a \to SH_a
\]
\[\text{(1.5)}\]
with $S \in SU(2)$.

Finally in eq. (1.1) terms with additional derivatives are omitted, since one aims at describing only the low momentum interactions of the light mesons; the ellipsis in (1.1) denotes these terms as well as higher order mass corrections ($1/m_Q$, etc.).

### 2 Semileptonic decays

Weak interactions between light and heavy mesons can be described by the weak current
\[
L^\mu_a = i\frac{\alpha}{2} Tr[\gamma^\mu(1-\gamma_5)H_b\xi^\dagger_b] + \cdots
\]
where again the ellipsis denotes terms vanishing in the limit $m_q \to 0$, $m_Q \to \infty$ or terms with derivatives. This current has the same transformation properties of the quark weak current $\bar{q}_a\gamma^\mu(1-\gamma_5)Q$ under the chiral group, i.e. $(3_L, 1_R)$. The constant $\alpha$ can be obtained by considering the matrix element of $L^\mu_a$ between the meson state and the vacuum:
\[
\langle 0|\bar{q}\gamma^\mu\gamma_5Q|P\rangle = if_Pm_Pv^\mu
\]
\[\text{(2.2)}\]
In the limit $m_Q \to \infty$ one has the result:
\[
\alpha = f_P\sqrt{m_P}
\]
\[\text{(2.3)}\]
and $\alpha$ has a logarithmic dependence on the heavy quark masses. The scaling law $m_P^{-1/2}$ for $f_P$ predicted by (2.3) has relevant $O(1/m_Q)$ corrections at the charm mass, as shown by QCD sum rules [8] and lattice QCD calculations [9]. We take into account these $O(1/m_Q)$ terms since they represent a well defined set of mass corrections and we assume for $f_D$ and $f_B$ the central values obtained by the QCD sum rule analyses [8] (recent lattice QCD analyses favour values of $f_B$ slightly larger [9]):
\[
f_D \approx 200 \text{ MeV}
\]
\[\text{(2.4)}\]
In this letter we will present an estimate of the constant \( g \) in (1.1). We shall see that this can be done by using the experimental data \( \Gamma(D^0 \to \pi^-e^+\nu_e) = (3.9_{-1.2}^{+2.3}) \times 10^{-3}/\tau_{D^0} \) \[10\], together with the results (2.4) and (2.5) and reasonable assumptions on semileptonic form factors.

Let us first consider the semileptonic \( D \to \pi \) decays. With the usual definition

\[
\langle \pi^-(p_\pi)|i\bar{d}\gamma^\mu(1-\gamma_5)|D^0(p_D)\rangle = f_+(q^2)(p_D + p_\pi)^\mu + f_-(q^2)(p_D - p_\pi)^\mu \tag{2.6}
\]

it follows from (1.1) and (2.1) that the form factors \( f_+ \) and \( f_- \) have the following contributions at \( q^2 = q^2_{\text{max}} = (m_D - m_\pi)^2 \):

\[
f_+(q^2_{\text{max}}) = -\frac{f_D}{2f} \left( 1 + g \frac{m_D - m_\pi}{\Delta + m_\pi} \right) \tag{2.7}
\]
\[
f_-(q^2_{\text{max}}) = -\frac{f_D}{2f} \left( 1 - g \frac{m_D + m_\pi}{\Delta + m_\pi} \right) \tag{2.8}
\]

where \( \Delta = m_{D^*} - m_D = 145 \text{ MeV} \) is the mass difference between the 1\(^-\) and 0\(^-\) low-lying charmed mesons. The value \( f_+(0) \) can be tentatively obtained, by assuming for the form factor the \( q^2 \) dependence suggested by single pole dominated dispersion relations:

\[
f_+(q^2) = m_1^2 \frac{f_+(0)}{m_1^2 - q^2} \tag{2.9}
\]

where \( m_1 \) is the mass of the vector meson state with the same quantum numbers of the current \( \bar{q}_a\gamma^\mu Q \) (the \( D^*(2010) \) state in the present case). From this it follows

\[
f_+(0) = -\frac{f_D}{2f} \left( 1 + g \frac{m_D - m_\pi}{\Delta + m_\pi} \right) \frac{(\Delta + m_\pi)(2m_D + \Delta - m_\pi)}{m_D^2} \tag{2.10}
\]

From experimental data on \( D^0 \to \pi^-e^+\nu_e \) decay \[10\] and eqs. (2.4) and (2.5), we get two possible values for \( g \) according to the sign of \( f_+(0) \):

\[
g = \begin{cases} 0.48 \pm 0.15 & (f_+(0) < 0) \\ -0.80 \pm 0.15 & (f_+(0) > 0) \end{cases} \tag{2.11}
\]

We observe that both solutions are smaller than the value \( g \approx 1 \) obtained by PCAC \[11\] or in a non relativistic quark model (the first reference in \[3\]). They are also within the present experimental bound from \( D^* \to D\pi \) decay: \( |g| \leq 1.7 \). In fact from (1.1) one gets \( \Gamma(D^{*+} \to D^0\pi^+) = g^2 p_\pi^3/(6\pi f^2) = 0.20g^2 \text{ MeV} \) and experimentally \[10\] one has \( \Gamma(D^{*+} \to D^0\pi^+) < 0.6 \text{ MeV} \).

We can now use the result (2.11) to get information on the coupling \( f_+(0) \) for \( \bar{B}^0 \to \pi^+e^-\bar{\nu}_e \); we get the values

\[
\begin{align*}
f^\pi_{\bar{B} \to \pi} &= -0.67 \pm 0.20 & (g = 0.48 \pm 0.15) \\
f^\pi_{\bar{B} \to \pi} &= 1.08 \pm 0.20 & (g = -0.80 \pm 0.15) \tag{2.12}
\end{align*}
\]

for the form factors at \( q^2 = 0 \); using again (2.9) for the \( q^2 \) dependence (with \( m_1 = m_{B^*} = 5325 \text{ MeV} \)) we obtain the following predictions for the branching ratio (we use

\[
f_B \approx 180 \text{ MeV} \tag{2.5}
\]
\[ \tau_B = 1.24 \text{ psec, eq. (2.10) with the parameters } \Delta = 46 \text{ MeV, } f_B = 180 \text{ MeV and the substitutions } m_D \rightarrow m_B, m_{D*} \rightarrow m_{B*}: \]

\[ BR(\bar{B}^0 \rightarrow \pi^+ e^- \bar{\nu}_e) = 6 \times 10^{-4} \left( \frac{V_{ub}}{0.004} \right)^2 \quad (g = 0.48 \pm 0.15) \quad (2.13) \]

\[ BR(\bar{B}^0 \rightarrow \pi^- e^+ \bar{\nu}_e) = 15.6 \times 10^{-4} \left( \frac{V_{ub}}{0.004} \right)^2 \quad (g = -0.80 \pm 0.15) \quad (2.14) \]

The present experimental bound \[ BR(B^+ \rightarrow \pi^0 e^+ \nu_e) < 2.2 \times 10^{-3} \] does not allow for a choice between (2.13) and (2.14) and we should wait for more precise data. We can nevertheless get some hints from the decay \( D^0 \rightarrow K^- e^+ \nu_e \). In this case we would get from (2.10) (with \( m_\pi \rightarrow m_K, m_{D*} \rightarrow m_{D*} = 2110 \text{ MeV} \) and \( \Delta = 245 \text{ MeV} \)):

\[ f_+(0) = -0.83 \pm 0.12 \quad (g = 0.48 \pm 0.15) \]

\[ f_+(0) = 0.21 \pm 0.12 \quad (g = -0.80 \pm 0.15) \quad (2.15) \]

to be compared to the value obtained from the semileptonic width \[ |f_+(0)| = 0.74 \pm 0.03 \quad (2.16) \]

Eqs. (2.15) and (2.16) point towards a smaller value of \( g \), i.e. \( g \approx 0.48 \), but it should be noted that for \( D \rightarrow K \) decays the role of terms with higher derivatives is expected to be not negligible.

### 3 Conclusions

The calculations we have performed neglect a number of corrections (in \( 1/m_Q \), in higher derivatives, non-polar contributions, etc.) so that one would have to be careful before taking them with an absolute faith. Nevertheless it seems that the chiral perturbation theory for mesons containing a heavy quark, supplemented by information on the leptonic decay constant from QCD sum rules, can be used to get two possible values for the \( D^*D\pi \) coupling constant from semileptonic \( D \rightarrow \pi \) decay. A (more debatable) application to \( D \rightarrow \bar{K} \) semileptonic decay would favour the smaller value for \( g \) (\( g \approx 0.48 \)) in the lagrangian of eq. (1.1).

**ACKNOWLEDGMENT** We would like to thank S. De Curtis and D. Dominici for many interesting discussions.
References

[1] The subject is reviewed in M.Wise, CALT preprint 68-1721 (1991); H.Georgi, HUTP preprint 91-A039 (1991), B.Grinstein, SSCL preprint 34 (1992) (to appear in Annual Review of Nucleus and Particle Sciences); N.Isgur and M.B.Wise, CEBAF preprint TH-92-10 (1992). We refer to these reviews for a full bibliography of the work done on the subject.

[2] M.B.Wise, Phys. Rev. D45 (1992) R2188

[3] T.-M.Yan, H.-Y.Cheng, C.-Y.Cheung, G.-L.Lin, Y.C.Lin and H.-L.Yu, preprint CLNS 92/1138, IP-ASTP-03-92 (1992); G.Burdman and J.F.Donoghue, UMHEP-365 (1992)

[4] C.L.Y.Lee, M.Lu and M.B.Wise, preprint CALT-68 1771 (1992)

[5] N.Isgur and M.B.Wise, Phys. Lett. B232 (1989) 113 and B237 (1990) 527; E.Eichten and B.Hill, Phys. Lett. B240 (1990) 447; B.Grinstein, Nucl. Phys. B339 (1990) 253; J.D.Bjorken, SLAC report: SLAC-PUB 5278 (1990) unpublished; F.Hussain, J.G.Körner and R.Migneron, Phys. Lett. B248 (1990) 299

[6] M.B.Voloshin and M.A.Shifman, Yad. Fiz. 45 (1987) 463 (Sov. J. Nucl. Phys. 45 (1987) 292) and Yad. Fiz. 47 (1988) 801 (Sov. J. Nucl. Phys. 47 (1988) 511); H.D.Politzer and M.B.Wise, Phys. Lett. B206 (1988) 681 and B208 (1988) 504

[7] H.Georgi, Phys. Lett. B240 (1990) 447; A.F.Falk, H.Georgi, B.Grinstein and M.B.Wise, Nucl. Phys. B343 (1990) 1

[8] C.A.Domínguez and N.Paver, Phys. Lett. B197 (1987) 427; L.J.Renders and S.Yazaki, Phys. Lett. B212 (1988) 245; P.Colangelo, G.Nardulli, A.A.Ovchinnikov and N.Paver, Phys. Lett. B269 (1991) 201

[9] C.R.Allton et al., Nucl. Phys. B349 (1991) 598; C.Alexandrou et al. Phys. Lett. B256 (1991) 60

[10] Particle Data Group, Review of Particle Physics, Phys. Rev. D45 (1992) 51

[11] S.Nussinov and W.Wetzel, Phys. Rev. D36 (1987) 130