Residue cross sections of $^{50}\text{Ti}$-induced fusion reactions based on the two-step model

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Abstract. $^{50}\text{Ti}$-induced fusion reactions to synthesize superheavy elements are studied systematically with the two-step model developed recently, where fusion process is divided into approaching phase and formation phase. Furthermore, the residue cross sections for different neutron evaporation channels are evaluated with the statistical evaporation model. In general, the calculated cross sections are much smaller than that of $^{48}\text{Ca}$-induced fusion reactions, but the results are within the detection capability of experimental facilities nowadays. The maximum calculated residue cross section for producing superheavy element $Z = 119$ is in the reaction $^{50}\text{Ti} + ^{247}\text{Bk}$ in $3n$ channels with $\sigma_{\text{res}}(3n) = 0.043$ pb at $E^* = 37.0$ MeV.

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1 Introduction

The synthesis of superheavy nuclei is a hot study field in nuclear physics, and it has obtained much progress experimentally and theoretically in recent years. Up to now, with the detecting of element $Z = 117$ in Dubna in 2010\cite{1}, the superheavy elements $Z = 110 – 118$ have been all synthesized\cite{2,3,4}. Theoretical supports for these very time-consuming and very high-expensive experiments are extremely vital for choosing the suitable target-projectile combinations and the optimum incident energy, and for the estimation of residue cross sections.

In synthesis of superheavy elements with proton number $Z = 114 – 118$, the so-called hot fusion reaction with $^{48}\text{Ca}$ as a projectile and actinide as a target is adopted. However, it comes increasingly difficult to synthesize heavier elements with projectile $^{48}\text{Ca}$. Maybe the last superheavy element which can be produced in the reaction with $^{48}\text{Ca}$ is the element with $Z = 118$ since the target heavier than Cf is too difficult to be obtained. Thus to produce heavier elements, heavier projectiles such as $^{50}\text{Ti}$, $^{54}\text{Cr}$, $^{58}\text{Fe}$, $^{64}\text{Ni}$ should be used.

Nevertheless, it is well known that in heavy ion induced reactions the deep-inelastic and quasi-fission processes are the dominant reaction channels because of the strong fusion hindrance, thus the fusion probability is much smaller than the light ion induced reactions. In the reactions with $^{48}\text{Ca}$ and actinide targets the probability of fusion relative to quasi-fission is less than 10%, and the ratio decreases for more symmetrical target-projectile combinations \cite{5}.

Mass asymmetry is one of the factors that influence quasi-fission and true fusion competition. Generally speaking, a decrease in the mass asymmetry in the reaction entrance channel leads to an increase in the quasi-fission and a decrease in the fusion contributions into the capture cross sections. It appears that fusion is strongly hindered as the size of the projectile relative to the target increases. Therefore, it is not at all surprising to see the failed attempt to make even heavier element 120 using the $^{58}\text{Fe} + ^{244}\text{Pu}$ reaction\cite{6}. Thus, a spherical neutron magic nucleus $^{50}\text{Ti}$ (only two protons greater than Ca) seems to be a promising candidate of projectile in the synthesis of superheavy element heavier than $Z = 118$. Furthermore, it is worth mentioning that the lower limit of residue cross section which can be detected experimentally is in the magnitude of 0.03 pb at present\cite{7}, so synthesis of superheavy elements with projectile $^{50}\text{Ti}$ is of great interest.

According to the theory of compound nucleus reactions, the whole process of synthesizing the superheavy nuclei is composed of fusion part and fission part. In the former part the projectile is captured by the target and an amalgamated system is formed that then evolves into a spherical compound nucleus, and then in latter part, besides being cracked into smaller fragments, few of the compound nucleus may cool down by evaporating particles and $\gamma$-rays and goes to its ground state. The evaporation residue cross section is usually expressed as a sum over all partial waves at a certain incident energy,

$$\sigma_{\text{res}}(E_{\text{c.m.}}) = \frac{\pi \hbar^2}{2\mu E_{\text{c.m.}}} \sum_J (2J+1)P_{\text{fus}}^J(E_{\text{c.m.}}) \cdot P_{\text{surv}}^J(E_{\text{c.m.}})$$  

(1)

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where $J$ is the total angular momentum quantum number, $E_{c.m.}$ the incident energy in the center of mass system; $P^J_{\text{fusion}}$ and $P^J_{\text{surv}}$ denote the fusion and the survival probabilities, respectively.

In the evaporation process, though there is a certain margin of uncertainty in the estimations of evaporation residue cross sections [8], the statistical evaporation model is commonly accepted and used to calculate the evaporation probability $P^J_{\text{surv}}$. However in the fusion process, because of the complexity of heavy ion reactions, there is still no commonly accepted model to deal with this process. Several models are adopted to study the fusion reactions, such as fusion-by-diffusion model [9,10], DNS model [11,12], QMD-based model [13], etc. In this paper we adopt two-step model to study the $^{50}$Ti-induced fusion reactions leading to the synthesis of superheavy nuclei.

The paper is arranged as follow: Sec. 2 gives a brief description of the two-step model, and of the determination of parameters fitting the experimental capture cross sections; Sec. 3 shows the results of systematic calculations and discussions; Sec. 4 gives a summary.

2 Two-step model and determination of parameters

The two-step model was proposed to describe the fusion process in massive nuclear systems where fusion hindrance exists, as shown in Ref. [14,15,16]. In this model, the fusion process is divided into two stages: first, the sticking stage where projectile and target come to the touching point over the Coulomb barrier from infinite distance; and second, the formation stage where the touched projectile and target evolve to form a spherical compound nucleus. Therefore, the fusion probability gets the form,

$$P^J_{\text{fusion}}(E_{c.m.}) = P^J_{\text{stick}}(E_{c.m.}) \cdot P^J_{\text{form}}(E_{c.m.}).$$

The energy dissipations in sticking stage and in formation stage are substantially considered in the model. It is worth to emphasize that the two-step model provides a method for a connection between a two-body collision process and the subsequent one-body shape evolution. This is completely different from the adiabatic, or the diabatic connection, and should be called as a “statistical connection” [14].

In principle, both of the sticking probability and formation probability need to be calculated via fluctuation-dissipation model, as shown in Ref. [14]. However, for simplicity, we choose alternatively an empirical formula [17] to calculate the sticking probability, where the barrier height is supposed to be Gaussian-distributed around the Coulomb barrier to simulate the energy dissipation in the approaching phase. The $P_{\text{stick}}$ takes the form,

$$P^J_{\text{stick}}(E_{c.m.}) = \frac{1}{2} \left\{ 1 + \text{erf} \left[ \frac{1}{\sqrt{2H}}(E_{c.m.} - B_0) - \frac{\hbar^2 J(J+1)}{2\mu R_B^2} \right] \right\},$$

where $B_0$ is the barrier height of the Coulomb potential, $H$ the width of the Gaussian distribution of the barrier height, $\mu$ the reduced mass, and $R_B$ the distance between two centers of projectile and target at the Coulomb barrier. In a reasonable assumption $B_0/(\sqrt{2H})$ should be much greater than 1.

To calculate sticking probability, the parameter $C$ (a factor to calculate the width of the Gaussian distribution of the barrier height, see Ref. [15]) and the barrier height of the Coulomb potential $B_0 = B + \Delta B$ should be adjusted as adequate as possible for very heavy systems. The three systems to be fitted are $^{50}$Ti+$^{208}$Pb, $^{50}$Ti+$^{209}$Bi [18] and $^{50}$Ti+$^{244}$Pu [5]. The fitted results are shown in Fig. 1. With a constant value of $C = 0.095$ and a linear increase of the barrier height shift $\Delta B$ with proton number $Z$, the available experimental capture cross sections are well reproduced. The $\Delta B$ is extrapolated from the fitting formula $\Delta B = 1.225Z - 98.95$ for heavy targets having very close atomic numbers, namely Am, Cm, Bk, Cf, and Es, as is seen in Fig. 2.

With Eq. (3) and the re-fitted parameter $C$ and $\Delta B$, the sticking probability of $^{50}$Ti-induced fusion reactions is calculated with confidence. Further more, for hot fusion reactions, we are interested in the residue cross sections

![Fig. 1. Fitting (solid lines) the experimental capture cross sections $^{50}$Ti to get appropriate $C$ and $\Delta B$.](image1)

![Fig. 2. The extrapolation of the shift of Coulomb barrier for the nearby heavier targets. The solid circles correspond to the data fitting to experimental data in Fig. 1 while the open ones are extrapolations for Am, Cm, Bk, Cf, and Es.](image2)
around $E^* = 30 \sim 40$ MeV, and the experimental data are just located in the similar energy range, as shown in Fig. 1 the calculated $\sigma_{\text{res}}$ around the same energy range will not sensitively depends on the determination of $C$ and $\Delta B$.

### 3 Residue cross sections

In the excited compound nucleus, the de-excitation process includes usually light particle emissions, $\gamma$-ray emissions, and fission. However, because of existence of the Coulomb barrier for charged particle emissions, the probability for the emission of light charged particles is much smaller than the one for the neutron emission. Therefore, most of the superheavy nuclei are obtained through the consecutive neutron evaporations. In the calculation of survival probability, the HIVAP code, based on the statistical evaporation model, is adopted to evaluate the residue cross sections.

The very important parameter in the evaporation process is the fission barrier $B_f$. It is clear that for heavy nuclei, the more stable one, which means having larger shell correction energy $E_{\text{shell}}$, usually has a higher fission barrier. Thus, the classical way to calculate the fission barrier is $B_f = B_{\text{LD}} - E_{\text{shell}}$, as did in Ref. [19]. However the microscopic calculations does not prove such so simple relationship between $B_f$ and $E_{\text{shell}}$. Since the microscopic calculations do not give $B_f$ for so heavy compound nuclei discussed in the present paper, we thus used the classical form to calculate $B_f$, but with an arbitrary factor $f$ to the shell correction energy, i.e.,

$$B_f = B_{\text{LD}} - f \cdot E_{\text{shell}},$$

where $E_{\text{shell}}$ is taken from Ref. [20], and the factor $f$ is determined by fitting the experimental data. In the present case, the inclusion of the factor $f$ gives rise to reductions of the fission barrier, and then the reduction of residue cross sections, but does not change general feature of the excitation functions, i.e., peak positions etc., though decreasing slopes in higher energies are a little affected. The introduction of the factor $f$, thus, is appropriate for predictions of residue cross sections.

Up to now, there are no experimental residue cross sections for $^{50}$Ti-induced fusion reactions to synthesize superheavy nuclei with $Z \geq 110$. Fortunately, the residue cross sections of $^{48}$Ca+$^{249}$Bk had been measured by Dubna in 2010 [1], and together with the experimental residue cross section of $^{48}$Ca+$^{208}$Pb [2] and $^{50}$Ti+$^{208}$Pb [3], the corresponding factor $f$ for the three systems are determined to be 0.45 (which is the same as in Ref. [15]), 0.72 and 0.77, respectively, using HIVAP code. Then, the factor $f$ for reaction $^{50}$Ti+$^{249}$Bk can be approximately evaluated as 0.45 $\times$ (0.72/0.77) = 0.48. Since the target Bk of the reaction have only one or two protons more or less than the targets such as Am, Cm, Cf and Es, the factor $f$ for $^{50}$Ti+Bk should also work with these target in $^{50}$Ti-induced reactions.

Now, all the ingredients in our calculations do not leave any ambiguity. The target isotopes of Am, Cm, Bk, Cf, and Es with life times long enough for experiments are chosen, and then, the evaporation residue cross sections with $Z = 117 - 121$ in some selected reactions are calculated systematically using the two-step model and HIVAP with the purpose of searching the favorable reaction systems and collision energies for the synthesis of superheavy nuclei with even larger proton number. The corresponding results are shown in Figs. 3-7.

Fig. 3 shows very similar excitation functions of producing element $Z = 117$ in the $^{50}$Ti+$^{241,243}$Am reactions. It is reasonable since two targets differ with only two neu-
tron. The maximum residue cross sections for $^{50}$Ti + $^{243}$Am, is slightly larger with at in 3n channels.

The calculated residue cross sections for reactions $^{50}$Ti + $^{243-246}$Cm to produce elements $Z = 118$ are presented in Fig. 4. According to the results, the optimum reaction to synthesize $Z = 118$ are $^{50}$Ti + $^{244}$Cm with $\sigma_{\text{res}}(2n) = 0.053$ pb at $E^* = 28.0$ MeV and $^{50}$Ti + $^{246}$Cm with $\sigma_{\text{res}}(3n) = 0.040$ pb at $E^* = 37.1$ MeV. As one expects, the variation of the peak energies between 2n and 3n evaporation channels of the compound nuclei is around 8 MeV which is about one neutron separation energy. Furthermore, it can be seen from Fig. 4 that our results do not show strong isotope dependence of superheavy nucleus production. Generally speaking, the formation of the superheavy nucleus is a complex dynamical process and depends on many physical factors, such as Coulomb barrier, conditional saddle point, neutron separation energy, shell effect and so on. It needs more further explorations.

The next superheavy element to be synthesized in experiment may be $Z = 119$ or 120, therefore, the investigations of the synthesis of $Z > 118$, especially of 119 and 120, are interesting and useful. It shows in Fig. 5 that in reactions $^{50}$Ti + $^{247-249}$Bk the maximum residue cross sections for producing superheavy element $Z = 119$ are both in 3n channels and are, respectively, 0.043 pb and 0.033 pb, which are almost one order of magnitude smaller than that of $^{48}$Ca + $^{247,249}$Bk in reference [15]. This could be explained by the fact that for the same actinide target, the Coulomb potential for $^{50}$Ti-induced reaction is roughly 10% larger than that for $^{48}$Ca-induced reactions, and the fusion hindrance for the former one is also stronger than the latter case [22]. Moreover, the results for $^{50}$Ti + $^{249}$Bk from Ref. [9] with fusion-by-diffusion model and from Ref. [23] with dinuclear system model are 0.57 pb and 0.11 pb respectively, while our present calculation value is very close to 0.035 pb calculated with an analytical expression for the description of the fusion probability [21] and 0.05 pb in Ref. [25]. The different results can be attributed to the dependence of model and parameter. However, although the cross sections are relatively small and are more than two orders of magnitudes lower than pico-barn, they are in the detection capability of the present experimental facilities.

Fig. 4 and Fig. 7 gives the results for producing $Z = 120$ and 121 in reactions $^{50}$Ti + $^{249-252}$Cf and $^{50}$Ti + $^{252,254}$Es, respectively. It can be seen from the figures that the residue cross sections are too small, approaching to the order of femto-barn. As was noted in the figures, the maximum residue cross sections in the reactions $^{50}$Ti + $^{250,251,252}$Cf are, 9.7 fb, 7.5 fb, and 12.2 fb, respectively, which are obviously larger than those of the reaction $^{50}$Ti + $^{249}$Cf (4.6 fb). Recently, Siwek-Wilczynska et al. predict the cross section of $^{50}$Ti + $^{249}$Cf to synthesize the element $Z = 120$ to be only 6 fb [10] which is more consistent with our present result. In contrast, our result is smaller than other several predictions with $\sigma_{\text{res}} \approx 20 \sim 200$ fb [10, 24, 25]. In addition, it is worth arguing that the residue cross section with target $^{252}$Cf is several times larger than those of the targets $^{249-251}$Cf and hence is theoretically the most favorable one for the synthesis of element 120. However, $^{252}$Cf may be difficult to be target because its spontaneous fission would bring about serious background in the experiment. Therefore, to produce element $Z = 120$, $^{50}$Ti + $^{249-251}$Cf could be considered in the future experiments. As a further extension, we evaluate the residue cross sections of element 121 with targets $^{252,254}$Es though Einsteinium is rather exotic and may be hardly prepared presently. It shows that the maximum residue cross sections for nucleus $Z = 121$ have comparable value of about 3 fb and are far below the present experimental limit of registration (30 fb). Thus, the synthesis of elements with $Z > 120$ is rather problematic in the near future due to extremely low cross sections and short half-lives of these elements.

To illustrate the results clearly, the relatively larger residue cross sections for different reactions are listed in Table 1. It should be mentioned that our reduction factor $f$ of $E_{\text{shell}}$ of the compound system influence only on the
absolute values of residue cross sections, not on the shapes of the residue excitation functions.

4 Summary

In summary, $^{50}$Ti-induced fusion reactions to synthesize superheavy nucleus with $Z = 117 \sim 121$ are studied with the two-step model and statistical evaporation model, where fusion process is divided into two consecutive phases, i.e., approaching phase and formation phase. The results show that the reactions $^{50}$Ti + $^{241,241}$Am, $^{50}$Ti + $^{243,244}$Cm, $^{50}$Ti + $^{247,248}$Bk to synthesize superheavy nucleus with $Z = 117, 118$ and $119$ have smaller residue cross sections than $^{48}$Ca-induced ones with nearly one order of magnitude. However, the calculated residue cross sections are still within the detection capability of experiment nowadays. Whereas, $^{50}$Ti-induced fusion reactions with a target $^{249,250}$Cf, and $^{252,254}$Es, to synthesize superheavy elements with $Z = 120$ and $121$ respectively, have so small residue cross sections that the experiments can be performed only when the experimental facilities are developed in the future. Of course, for planning the experiments on the synthesis of superheavy nuclei of up to $Z = 122$, new mechanism and more precise data obtained in the processes of fusion-fission and quasi-fission of these nuclei are required.

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