Scalar mesons: in search of the lightest glueball

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According to the QCD expectations the lightest glueball should be a scalar particle ($J^{PC} = 0^{++}$). Different scenarios have been considered for a classification of these states but – despite considerable progress in recent years – the experimental basis for various parameters is still rather weak. We present a new analysis of the elastic and charge exchange $\pi\pi$ scattering between 1000 and 1800 MeV. A unique solution is selected which shows clear evidence for $f_0(1500)$ and a broad state ($\sigma$ or “$f_0(1000)$”), but there is no evidence for $f_0(1370)$ at a level of $\gtrsim 10\%$ branching ratio to $\pi\pi$. Arguments in favour of the broad state to be a glueball are summarized.

1. EXPECTATION FOR GLUEBALLS IN QCD

The existence of glueballs has been considered from the very beginning as characteristic prediction of QCD because of the self-interaction of gluons; early investigations suggested different scenarios\cite{1}. Today, quantitative results are available from

\begin{itemize}
\item[(1)] \textit{Lattice QCD}
\item[(2)] \textit{QCD sum rules}
\end{itemize}

Detailed results on glueballs together with a scenario for the $q\bar{q}$ sector have been obtained by Narison\cite{3}. The glueball around 1500 MeV is reproduced, in addition a lighter gluonic state around 1000 MeV is required with strong decay into $\pi\pi$ and a large width mixing with the nearby $q\bar{q}$ state. Other analyses for the gluonic sector find similar results with two states\cite{4} or only one state in the region around 1250 MeV\cite{5}.

2. INTERPRETATIONS OF THE OBSERVED SPECTRUM

From these computations one would expect the lightest glueball in the scalar sector in the mass range up to about 1800 MeV. In this search it is necessary to identify at the same time the $q\bar{q}$ (or possibly the $qq\bar{q}\bar{q}$) nonets and a possible mixing of states. The Particle Data Group lists the following scalar particles\cite{6}:

$I=0$: $f_0(600)$ ($\sigma$), $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$…

$I=\frac{1}{2}$: $K_0^*(800)$ ($\kappa$ (?), $K_0^*(1430)$, $K^*(1950)$ (?))…

$I=1$: $a_0(980)$, $a_0(1450)$…

There are different scenarios for the interpretation of this spectrum and we describe shortly two variants, for more details and references, see, for example, Ref.\cite{7}.

\textit{Scenario A:}

A light nonet is formed with the states $\sigma(600)$, $\kappa(800)$, $a_0(980)$, $f_0(980)$, either with $q\bar{q}$ or $qq\bar{q}\bar{q}$ composition. Then, a heavier nonet can be built with $a_0(1450)$, $K_0^*(1430)$ and two isoscalars which mix with the bare glueball into the observed $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ (for an early Ref., see\cite{8}).

\textit{Scenario B:}

The lightest members of the nonet are $f_0(980)$ and $a_0(980)$ (or $a_0(1450)$), furthermore $K_0^*(1430)$ and $f_0(1500)$ (or $f_0(1370)$). The glueball is considered a very broad object with mass in the region 1000 MeV\cite{9} - 1500 MeV\cite{11} and width of the order of mass.

The scheme for light scalars proposed by us\cite{9} follows Scenario B. The isoscalar partners $f_0(980)$ and $f_0(1500)$ are “non-ideal” mixed near an
Figure 1. Argand diagrams for $\pi\pi S_I$ waves: $S_0$ solutions $-+-$ and $---$ from elastic $\pi^+\pi^-$ (CM-II), corrected for new $I=2$ results, and $S_{00}$ amplitude for $\pi^+\pi^-\rightarrow\pi^0\pi^0$ (GAMS) and component $S_0$ thereof. Also shown on the left is the amplitude $S_2$.

$SU(3)$ flavour singlet and octet respectively, just like $\eta'$ and $\eta$ but with opposite mass ordering as also suggested in [11]. This proposal is consistent with various observed decay rates; especially, the relative phases between the $s\bar{s}$ components in $f_0(980)$, $f_0(1500)$ follow from interference effects [9,12]. In addition, this multiplet fulfils the Gell-Mann Okubo mass formula. Ultimately, we hope that the correct choice of the $q\bar{q}$ multiplet will be established by the verification of flavour symmetry relations, such as those suggested for charmless $B$-decays [7].

We don’t discuss these arguments here any further but try to elucidate the properties of $f_0(1370)$ – which has an important impact on the classification schemes – and the broad object, our glueball candidate. Phenomenologically, one observes in the elastic $S$ wave $\pi\pi$ scattering cross section a broad bump modified by dips at the masses of $f_0(980)$ and $f_0(1500)$ (the “red dragon”). All resonance fits to this phenomenon, first observed by the CERN Munich collaboration (CM-I [13]), revealed a pole in the $T$ matrix corresponding to a state with $M \gtrsim 1000$ MeV and comparable width (also called $f_0(1000)$ [14]). This object may be identical with the broad $\sigma$ particle.

3. NEW ANALYSIS OF $\pi\pi$ SCATTERING FOR $M_{\pi\pi} \gtrsim 1000$ MEV.

The first analysis of CM-I data [13] on $\pi^+\pi^- \rightarrow \pi^+\pi^-$ in the range $600 \leq M_{\pi\pi} \leq 1800$ MeV was based on an energy dependent $K$-matrix fit followed by a bin-by-bin energy-independent analysis. Ambiguities expected on general grounds have not been excluded above 1000 MeV. Subsequently, they have been classified into 4 types by Estabrooks and Martin [15] and in CM-II [16] according to the “Barrelet zero” systematics. Additional information has been added by the CERN-Krakow-Munich (CKM) experiment with polarised target [17,18].

3.1. Selection of the physical solution

A unique solution for $M_{\pi\pi} < 1400$ MeV has been suggested recently by Kamiński, Peláez and Ynduráin [21] who found good agreement for the $S$ wave phases from the three above mentioned determinations up to 1400 MeV: CM-I, CM-II (Sols. $---$ or, equivalently here $---$, or Sols. A,C [15]), and CKM, but not Sols. $+++$ and $+-+-$ (B,D).

We can obtain a further selection among the remaining ambiguities above 1400 MeV by comparing with the reaction $\pi^+\pi^- \rightarrow \pi^0\pi^0$ with its different ambiguity structure. Such data have been obtained from $\pi^-p$ collisions at 100 GeV/c.
(GAMS [19]) and at 18.3 GeV (E852 [20]). For our study we use the GAMS data which extend to masses $M_{\pi\pi} > 1400$ MeV with good accuracy.

We also take into account the $I=2$ amplitude $S_2$. For the mass range $1000 < M_{\pi\pi} < 1800$ MeV we represent the data [22,23] on phases and inelasticities by the parametrizations $\delta_2^0 = -25^\circ + 25^\circ(M_{\pi\pi} - 1.3)^2$ and $\eta_2^0 = \text{Min}[1.0 - (M_{\pi\pi} - 1.2) + 0.5(M_{\pi\pi} - 1.2)^2, 1.0]$ (masses in GeV). The errors on the inelasticity at the higher energies are rather large $\eta_2^0 \approx 0.5 \pm 0.2$.

For further analysis of elastic $\pi^+\pi^-$ scattering we take the CM-II results which use the full correlation matrix in the determination of the amplitudes and therefore have small statistical errors, but the results in Ref. [15] are qualitatively similar. As CM-II used an older purely elastic $S_2$ wave we recalculated $S_0$ from the measured $\pi^+\pi^-$ amplitude assuming $S_{+} = (S_0 + (1/2)S_2)_{\text{old}} = (S_0 + (1/2)S_2)_{\text{new}}$. The remaining two $I=0$ amplitudes $S_0$ ($-++$ and $--$) corrected in this way are shown in in Fig. 1 together with the $I=2$ wave.

Figure 2. Resonance fit Eq. (1) in comparison with data (CM-I/II): Argand diagram for corrected $S_0$ wave, phase $\delta_0^0$ and inelasticity $\eta_0^0$.

Rightmost in Fig. 1 is shown the amplitude $S_{00}$ for the process $\pi^+\pi^- \rightarrow \pi^0\pi^0$ which we extracted from the magnitude $|S_{00}|^2$, the phase $|\Phi_S - \Phi_{D_0}|$ and the Breit Wigner phase of $f_2(1270)$ in the $D$ wave as determined from GAMS data [12]. The $I=0$ amplitude $S_0$ is obtained from $S_{00} = S_0 - S_2$ and is shown in the same diagram. A second solution from the unresolved sign of $\Phi_S - \Phi_{D_0}$ is rejected by the unitarity requirement.

The amplitude $S_0$ so obtained shows a circular motion from 1000 to 1740 MeV with a smaller circle superimposed above 1400 MeV. This behaviour is only consistent at the qualitative level with the $\pi^+\pi^-$ solution $++$ which we therefore select as the physical solution. There are some remaining discrepancies between the results on $S_0$ from both processes. These we attach to systematic errors in the determination of the overall phase from leading resonances, from truncation of higher partial waves and the uncertainty in the $I=2$ amplitude. These uncertainties over large mass scales should not affect the nature of local resonance phenomena like $f_0(1500)$ or $f_0(1370)$.

It is nevertheless satisfactory that agreement within these limitations is obtained. It shows the production of $f_0(1500)$, the small circle, with parameters roughly according to PDG ($M=1500$ MeV, $\Gamma=109$ MeV) and with branching ratio into $\pi\pi$ of $x_{\pi\pi} \sim 0.349$, whereas we estimate from the depth of the circle $x_{\pi\pi} \sim 0.25$. The polarised target CKM data on $|S|^2$ [14] which are determined in a model independent way agree better with CM-II for $M \lesssim 1400$ MeV and better with GAMS for $M \gtrsim 1400$ MeV with its higher inelasticity.
3.2. Resonance fit to the $I = 0$ $S$ wave

Next we look for solutions in the cases $S_{0}^{+} (\pi \pi)$ of CM-II (corrected). We represent the $S$ matrix for the $3$ channels $(\pi \pi, K \bar{K}, 4\pi)$ as product of $3$ matrices for resonances $S_{R} = 1 + 2i T_{R}$

$$S = S_{f_{0}(980)} S_{f_{0}(1500)} S_{\text{broad}}$$

$$T_{R} = \left[ M_{0}^{2} - M_{\pi \pi}^{2} - i (\rho_{1} g_{1}^{2} + \rho_{2} g_{2}^{2} + \rho_{3} g_{3}^{2}) \right]^{-1}$$

$$\times \rho_{1}^{2} T_{i} (g_{1}, g_{2}, g_{3}) \rho_{1}^{2}$$  \hspace{1cm} (2)

where $\rho_{i} = 2k_{i}/\sqrt{3}$. We fit these formulae to the data from CM-II for $M_{\pi \pi} > 1000$ MeV and the phases from CM-I in $600 < M_{\pi \pi} < 1000$ MeV for definiteness. The details will be given elsewhere. As can be seen in Fig. 2 the three resonances give a good overall description of the data.

The $\pi \pi$ component of $S_{\text{broad}}$ in (1) is shown separately in Fig. 3. It corresponds to a mass parameter in (2) of $M_{0} = 1100$ MeV and a total width of similar size. The parametrization (2) with constant $g_{i}$ has not the threshold behaviour expected from chiral theory. The Adler zero can be enforced by multiplication of the couplings with $(s - m_{A}^{2}/2)/(s - s_{A})$. We found that this modification has little effect in the mass range considered here ($M_{\pi \pi} > 600$) MeV, but one may expect that the pole position in the amplitude has shifted from $1100$ MeV towards a lower value. Therefore the $f_{0}(1000)$ and the $\sigma(600)$ effects are presumably the same phenomena. The dependence of the pole position on the actual line shape can be seen in a simple unitary model amplitude with Adler zero, communicated to us by H. Zheng $^{[21]}$ ($m_{\pi} \rightarrow 0$): $T = \rho \sigma s^{2} / (M^{2} - s - i \rho \sigma s^{2})$ with pole at $s_{\pi} \sim M^{2} / (1 + i g^{2})$. In the extreme cases, for small coupling the pole is near $s_{\pi} = M^{2} (\sim 1000$ MeV), whereas for very strong coupling $g^{2} \geq 1$ Re $s_{\sigma} \rightarrow 0$, Im $s_{\sigma} \gg$ Re $s_{\sigma}$.

We note that there is no evidence for $f_{0}(1370)$ from our fitting. This state would show up as a second circle in the Argand diagram and as another dip in $\eta_{0}^{2}$ at the corresponding mass. If we allow for such a state in the fit the mass tends towards $1500$ MeV and the width to decrease such that the additional circle fits into the first one. We therefore exclude an additional state with branching ratio $x_{\pi \pi} \equiv B(f_{0}(1370) \rightarrow \pi \pi) \geq 0.1$ near $1370$ MeV.

Results are also available on the inelastic processes $\pi \pi \rightarrow \eta \eta, K \bar{K}$ where the reconstructed Argand diagrams $^{[6]}$ show one extra circle above background. Explicit fits $^{[29,30]}$ have denied a second narrow resonant state beyond $f_{0}(1500)$, contrary to the recent study $^{[31]}$, so this situation has to be clarified. The main evidence for $f_{0}(1370)$ comes from fits to $3$-body final states in low energy $pp$ annihilation and $J/\psi$ decays, also central $pp$ production of the decays $f_{0} \rightarrow \pi \pi, K \bar{K}$. None of these studies have presented a full $2$-dim energy-independent phase shift analysis showing the existence of a second circle. Such studies are essential in processes with large background from a broad resonance and considerable $I = 2$ components.

4. Glueball interpretation of “$f_{0}(1000)$”

We see the following arguments in favour of the broad state to be a glueball.

1. In our interpretation of the spectrum the broad state, presumably identical to $\sigma(600)$, does not belong to a nonet. In particular, we would not add this effect centered around $1$ GeV, at least not entirely, to a light nonet of any kind, so it is

Figure 3. Argand diagram for the broad component $(f_{0}(1100)/\sigma)$, the hypothetical glueball, in our resonance fit of Fig. 2, to CM-II data. GAMS data would suggest larger inelasticity at higher masses.
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“left over” in the $q\bar{q}$ systematics.

2. The state is produced in most processes considered as “gluon rich”. In radiative $J/\psi$ decays a broad or constant contribution has been seen in $KK\gamma$ \[23\], in $\sigma\gamma$ \[20\], but not yet in $\gamma\pi\pi$ which may be difficult; furthermore in central production, like $pp \rightarrow p\pi\pi p$ and in annihilation, like $p\bar{p} \rightarrow 3\pi$ and also in $B \rightarrow K + X$ \[24\] with the gluonic contribution $b \rightarrow s g$.

3. The broad state appears together with $f_0(1500)$ in channels $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$ and $B \rightarrow K(\pi\pi), K(K\bar{K})$. The signs of interference in these channels are consistent with $f_0(1500)$ being a flavour octet (as in models \[9\],[11\]) and $f_0(1000)$ being near a flavour singlet \[7\].

4. Another expectation is the suppression of glueball production in $\gamma\gamma$ processes. Based on the earlier result \[27\] on $f_0(400 - 1200)$ we noted \[28\] a relative suppression in comparison with $f_2(1270)$. A better study in the 1000 MeV region is important.

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