D-mesons in isospin asymmetric strange hadronic matter

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Abstract

We study the in-medium properties of $D$ and $\bar{D}$ mesons in isospin asymmetric hyperonic matter arising due to their interactions with the light hadrons. The interactions of $D$ and $\bar{D}$ mesons with these light hadrons are derived by generalizing the chiral SU(3) model used for the study of hyperonic matter to SU(4). The nucleons, the scalar isoscalar meson, $\sigma$ and the scalar-isovector meson, $\delta$ as modified in the strange hadronic matter, modify the masses of $D$ and $\bar{D}$ mesons. It is found that as compared to the $\bar{D}$ mesons, the $D$ meson properties are more sensitive to the isospin asymmetry at high densities. The effects of strangeness in the medium on the properties of $D$ and $\bar{D}$ mesons are studied in the present investigation. The $D$ mesons ($D^0, D^+$) are found to undergo larger medium modifications as compared to $\bar{D}$ mesons ($\bar{D}^0, D^-$) with the strangeness fraction, $f_s$ and these modifications are observed to be more appreciable at high densities. The present study of the in-medium properties of $D$ and $\bar{D}$ mesons will be of relevance for the experiments in the future Facility for Antiproton and Ion Research, GSI, where the baryonic matter at high densities will be produced. The isospin asymmetric effects in the doublet $D = (D^0, D^+)$ in the strange hadronic matter should show in observables like their production and flow in asymmetric heavy-ion collisions as well as in $J/\psi$ suppression.

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I. INTRODUCTION

Since long time the topic of study of the in-medium properties of hadrons has drawn considerable interest of particle physicists because of its importance in understanding the strong interaction physics and having relevance in heavy-ion collision experiments as well as in nuclear astrophysics. There have been extensive experimental efforts to study the in-medium properties of hadrons through experimental observables like production and propagation of the hadrons in hot and dense hadronic medium. The observed enhanced dilepton spectra [1–3] could be a signature of in medium vector meson mass reduction [4–8] and for the formation of thermalized quark gluon plasma [9]. The production of photons [10] arising from the heavy ion collision experiments is a promising observable which probes matter resulting from the high energy nuclear collisions. The hard photons production is a promising source that can provide information about the thermodynamical state of the plasma produced in heavy ion collision experiments [11]. Similarly kaons and antikaons properties have been studied experimentally by KaoS collaboration and the production of kaons and antikaons in the heavy-ion collisions and their collective flow are directly related to the medium modification of their spectral functions [5, 12–18]. At the Compressed baryonic matter (CBM) experiment at the future facility at GSI, one expects to produce matter at high densities [19] and the in-medium properties of strange and charm mesons are some of the important topics planned to be investigated extensively. Therefore, the topic of the study of charm mesons in dense hadronic matter has also gotten considerable interest in the recent past. The $D$ and $\bar{D}$ mesons are modified due to interaction of the light quark (u,d) or antiquark present in them with the nuclear medium. The experimental signatures for the mass modifications of $D$ and $\bar{D}$ mesons can be their production ratio as well as $J/\psi$ suppression in the hadronic medium [20–22]. In heavy-ion collision experiments of much higher energies, e.g., in RHIC or LHC, it is suggested that the $J/\psi$ suppression is because of the formation of quark-gluon plasma (QGP) [23, 24]. However, in Ref. [25–27] it is observed that the effect of hadron absorption of $J/\psi$ is not negligible. The $J/\psi$ suppression can also be due to comover interaction as suggested in Ref. [28]. An important difference between $J/\psi$ suppression pattern in comovers interaction model and in a deconfining scenario is that in the former case the anomalous suppression sets in smoothly from peripheral to central collisions rather than in
a sudden way when the deconfining threshold is reached \cite{28}. In Ref. \cite{29}, it was reported that the charmonium suppression observed in Pb + Pb collisions of NA50 experiment cannot be explained by nucleon absorption only, but need some additional density dependent suppression mechanism. It was suggested in these studies that the comover scattering can explain the additional suppression of charmonium. The $J/\psi$ suppression in nuclear collision at SPS energies has been studied in covariant transport approach HSD in Ref. \cite{30}. The calculations show that the absorption of $J/\psi$'s by both nucleons and produced mesons can explain reasonably not only the total $J/\psi$ cross section but also the transverse energy dependence of $J/\psi$ suppression measured in both proton-nucleus and nucleus-nucleus collisions. Due to the reduction in the masses of $D$ and $\bar{D}$ mesons in the medium it is a possibility that excited charmonium states can decay to $D\bar{D}$ pairs \cite{31} instead of decaying to lowest charmonium state $J/\psi$. Actually higher charmonium states are considered as the major source of $J/\psi$ \cite{32}. Even at certain higher densities it can become a possibility that $J/\psi$ itself will decay to $D\bar{D}$ pairs. So this can be an explanation of the observed $J/\psi$ suppression by NA50 collaboration at 158 GeV/nucleon in the Pb-Pb collisions \cite{23}. The excited states of charmonium also undergo mass drop in the nuclear medium \cite{33}. The modifications of the in-medium masses of $D$ mesons is larger than the $J/\psi$ mass modification \cite{34,35}. This is because $J/\psi$ is made of heavy quarks and these interact with the nuclear medium through gluon condensates. The change in gluon condensates with the nuclear density is very small and hence leads to only a small modification of the $J/\psi$ mass.

The in-medium modifications of $D$ and $\bar{D}$ mesons have been studied in the literature using QCD sum rule approach, quark meson coupling (QMC) model and coupled channel approach. In the QCD sum rule approach, it is suggested that the light quark or antiquark of $D$ mesons interact with the light quark condensate leading to the medium modification of $D(\bar{D})$ meson masses \cite{36,37}. In the QMC model the scalar $\sigma$ meson couples to the confined light quark (u,d) in the nucleon thus giving a drop of the nucleon mass in the medium. The $D$-meson properties within the QMC model have been studied in Ref. \cite{38}. The drop in the mass of $D$ mesons arises due to the interaction with the nuclear medium and the mass drop of the $D$ mesons observed in the QMC model turns out to be similar to that calculated in the QCD sum rule approach. Charmed baryonic resonances have received a lot of attentions...
due to discovery of quite a few new states by CELO, Belle, and BABAR Collaborations \cite{39-44}. Whether these resonance states have $qqq$ structure or generated dynamically via meson baryon scattering processes is a matter of great interest \cite{45}. The study of the properties of $D$ mesons in the coupled channel approach using separable potential leads to the generation of resonance $\Lambda(2593)$ in $I = 0$ channel \cite{46} analogous to $\Lambda(1405)$ in the coupled channel approach for the $\bar{K}N$ interaction \cite{47}. The study of the spectral density of $D$ mesons at finite temperatures and densities, considering modifications of the nucleons in the medium, indicate a dominant increase in the width of the $D$-meson whereas there is a very small change in the $D$-meson mass in the medium \cite{48}. However, these calculations \cite{46, 48}, assume the interaction to be SU(3) symmetric in u,d,c quarks and ignore channels with charmed hadrons with strangeness. A coupled channel approach for the study of D-mesons has been developed based on SU(4) symmetry \cite{49} to construct the effective interaction between pseudoscalar mesons in a 16-plet with baryons in 20-plet representation through exchange of vector mesons and with KSFR condition \cite{50}. This model \cite{49} has been modified in aspects like regularization method and has been used to study DN interactions in Ref. \cite{51}. This reproduces the resonance $\Lambda_c(2593)$ in the $I=0$ channel and in addition generates another resonance in the $I=1$ channel at around 2770 MeV. These calculations have been generalized to finite temperatures \cite{52} accounting for the in-medium modifications of the nucleons in a Walecka type $\sigma - \omega$ model, to study the $D$ and $\bar{D}$ properties \cite{53} in the hot and dense hadronic matter. At the nuclear matter density and for zero temperature, these resonances ($\Lambda_c(2593)$ and $\Sigma_c(2770)$) are generated 45 MeV and 40 MeV below their free space positions. However at finite temperature, e.g., at $T = 100$ MeV resonance positions shift to 2579 MeV and 2767 MeV for $\Lambda_c (I = 0)$ and $\Sigma_c (I = 1)$ respectively. Thus at finite temperature resonances are seen to move closer to their free space values. This is because of the reduction of pauli blocking factor arising due to the fact that fermi surface is smeared out with temperature. For $\bar{D}$ mesons in coupled channel approach a small repulsive mass shift is obtained. This will rule out of any possibility of charmed mesic nuclei \cite{52} suggested in the QMC model \cite{38}. But as we shall see in our investigation, we obtain a small attractive mass shift for $\bar{D}$ mesons which can give rise to the possibility of the formation of charmed mesic nuclei. Using heavy quark symmetry, the $D^0$-nucleus bound states have been studied
in Ref. [54] using $D$ meson self-energy calculated in the nuclear medium [45]. In heavy quark symmetry, the pseudoscalar $D$ meson and the vector meson, $D^*$ are treated on equal footing and this leads to the generation of a broad spectrum of new resonant meson-baryon states in the charm one and strangeness zero [55] and the exotic charm minus one [56] sectors. The inclusion of vector mesons is the keystone for obtaining an attractive D-nucleus interaction that leads to the existence of $D^0$ nucleus bound states, as compared previous studies based on SU(4) flavor symmetry [54]. The study of $D$ meson self-energy in the nuclear matter is also helpful in understanding the properties of the charm and the hidden charm resonances in the nuclear matter [57]. In coupled channel approach the charmed resonance $D_s(2317)$ mainly couples to $DK$ system, while the $D_0(2400)$ couples to $D\pi$ and $D_s\bar{K}$. The hidden charm resonance couples mostly to $D\bar{D}$. Therefore any modification of $D$ meson properties in the nuclear medium will affect the properties of these resonances.

In the present investigation we study the in-medium modifications of the $D$ and $\bar{D}$ mesons in isospin asymmetric strange hadronic matter. The medium modifications of $D$ and $\bar{D}$ mesons are due to their interactions with the nucleons, the scalar isoscalar meson $\sigma$ and the scalar isovector meson $\delta$, which are modified in the hyperonic matter. The medium modifications of the light hadrons (nucleons and scalar mesons) are described by using a chiral SU(3) model [58]. The model has been used to study finite nuclei, the nuclear matter properties, the in-medium properties of the vector mesons [59, 60] as well as to investigate the optical potentials of kaons and antikaons in nuclear matter [61, 62] and in hyperonic matter in [63]. For the study of the properties of $D$ mesons in isospin-asymmetric strange hadronic matter, the chiral SU(3) model is generalized to SU(4) flavor symmetry to obtain the interactions of $D$ and $\bar{D}$ mesons with the light hadrons. Since the chiral symmetry is explicitly broken for the SU(4) case due to the large charm quark mass, we use the SU(4) symmetry here only to obtain the interactions of the $D$ and $\bar{D}$ mesons with the light hadron sector, but use the observed values of the heavy hadron masses and empirical values of the decay constants. This has been in line with the philosophy followed in Ref. [64] where charmonium absorption in nuclear matter was studied using the SU(4) model to obtain the relevant interactions. However, the values of the heavy hadron masses and the coupling constants in Ref. [64], were taken as the empirical values or as calculated from
other theoretical models. The coupling constants were derived by using the relations from SU(4) symmetry, if neither the empirical values nor values calculated from other theoretical models were available [64]. The $D$ meson properties in symmetric hot nuclear matter using the SU(4) model have been studied in Ref. [65] and for the case of asymmetric nuclear matter at zero temperature [66] and at finite temperature in Ref. [67]. The charmonium mass in hot asymmetric nuclear matter has been studied within present chiral model in Ref. [67, 68]. The mass modifications of charmonium states arise due to the interaction with the gluon condensates of QCD, simulated through the scalar dilaton field introduced to incorporate the broken scale invariance of QCD within effective chiral model Ref. [67].

Within the SU(4) model considered in the present investigation, the $D(\bar{D})$ energies are modified due to a vectorial Weinberg-Tomozawa, scalar exchange terms as well as range terms [62, 63]. The isospin asymmetric effects among $D^0$ and $D^+$ in the doublet, $D \equiv (D^0, D^+)$ as well as between $\bar{D}^0$ and $D^-$ in the doublet, $\bar{D} \equiv (\bar{D}^0, D^-)$ arise due to the scalar-isovector $\delta$ meson, due to asymmetric contributions in the Weinberg-Tomozawa term, as well as in the range term [62].

The outline of the paper is as follows: in section II, we give a brief introduction to the effective chiral $SU(3)$ model used to study the isospin asymmetric strange hadronic matter, and its extension to the $SU(4)$ model to derive the interactions of the charmed mesons with the light hadrons. In section III, we present the dispersion relations for the $D$ and $\bar{D}$ mesons which are solved to obtain their optical potentials in the dense strange hyperonic matter. The effects of isospin asymmetry, strangeness fraction of the hadronic medium on the optical potentials of the $D$ and $\bar{D}$ mesons are investigated. Section IV contains the results obtained for the medium modifications of the $D$ and $\bar{D}$ mesons in the hyperonic matter and finally, in section V, we summarize the findings of the present investigation and discuss possible outlook.

II. THE HADRONIC CHIRAL $SU(3) \times SU(3)$ MODEL

We use a chiral $SU(3)$ model for the study of the light hadrons in the present investigation [58]. The model is based on nonlinear realization of chiral symmetry [69, 71] and broken scale invariance [58, 60]. The effective hadronic chiral Lagrangian contains the following
terms
\[ \mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{\text{vec}} + \mathcal{L}_0 + \mathcal{L}_{SB} \]  
(1)

In Eq. (1), \( \mathcal{L}_{\text{kin}} \) is the kinetic energy term, \( \mathcal{L}_{BW} \) is the baryon-meson interaction term in which the baryons-spin-0 meson interaction term generates the baryon masses. \( \mathcal{L}_{\text{vec}} \) describes the dynamical mass generation of the vector mesons via couplings to the scalar mesons and contain additionally quartic self-interactions of the vector fields. \( \mathcal{L}_0 \) contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry as well as a scale invariance breaking logarithmic potential. \( \mathcal{L}_{SB} \) describes the explicit chiral symmetry breaking.

To study the hadron properties at finite densities in the present investigation, we use the mean field approximation, where all the meson fields are treated as classical fields. In this approximation, only the scalar and the vector fields contribute to the baryon-meson interaction, \( \mathcal{L}_{BW} \) since for all the other mesons, the expectation values are zero.

The interactions of the scalar mesons and vector mesons with the baryons are given as

\[ \mathcal{L}_{B\text{scal}} + \mathcal{L}_{B\text{vec}} = -\sum_i \bar{\psi}_i \left[ g_{\omega i} \gamma_0 \omega + g_{\phi i} \gamma_0 \phi + g_{\rho i} \gamma_0 \rho + m_i^* \right] \psi_i \]  
(2)

The interaction of the vector mesons, of the scalar fields and the interaction corresponding to the explicitly symmetry breaking in the mean field approximation are given as

\[ \mathcal{L}_{\text{vec}} = \frac{1}{2} \left( m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_\phi^2 \phi^2 \right) \frac{\chi^2}{\chi_0^2} + g_4 \left( \omega^4 + 6 \omega^2 \rho^2 + \rho^4 + 2 \phi^4 \right) \]  
(3)

\[ \mathcal{L}_0 = -\frac{1}{2} k_0 \chi^2 \left( \sigma^2 + \zeta^2 + \delta^2 \right) + k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right)^2 \]  

\[ + k_2 \left( \sigma^4 + \frac{\delta^4}{2} + 3 \sigma^2 \delta^2 + \zeta^4 \right) + k_3 \chi \left( \sigma^2 - \delta^2 \right) \zeta \]  

\[ - k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0^4} + \frac{d}{3} \chi^4 \ln \left( \frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \chi_0^2} \left( \frac{\chi}{\chi_0} \right)^3 \right) \]  
(4)

and

\[ \mathcal{L}_{SB} = -\left( \frac{\chi}{\chi_0} \right)^2 \left[ m_\pi^2 f_\pi \sigma + \left( \sqrt{2} m_k^2 f_k \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right] \]  
(5)
The effective mass of the baryon of species \( i \) is given as

\[
m_i^* = -g_{\sigma i} \sigma - g_{\zeta i} \zeta - g_{\delta i} \delta
\]  

(6)

The baryon-scalar meson interactions generate the baryon masses through the coupling of baryons to the nonstrange \( \sigma \), strange scalar mesons \( \zeta \) and also to scalar isovector meson \( \delta \). In analogy to the baryon-scalar meson coupling there exist two independent baryon-vector meson interaction terms corresponding to the F-type (antisymmetric) and D-type (symmetric) couplings. Here antisymmetric coupling is used because the universality principle \[72\] and vector meson dominance model suggest small symmetric coupling. Additionally, we choose the parameters \[58, 62\] so as to decouple the strange vector field \( \phi_{\mu} \sim \bar{s} \gamma_{\mu} s \) from the nucleon, corresponding to an ideal mixing between \( \omega \) and \( \phi \) mesons. A small deviation of the mixing angle from ideal mixing \[73–75\] has not been taken into account in the present investigation.

The concept of broken scale invariance leading to the trace anomaly in (massless) QCD, \( \theta_{\mu} = \frac{\beta_{QCD}}{2g} G_{\mu\nu}^a G^{a\mu\nu} \), where \( G_{\mu\nu}^a \) is the gluon field strength tensor of QCD, can be mimicked in an effective Lagrangian at tree level \[76\] through the introduction of the scale breaking potential (last two terms of Eq. (4))

\[
\mathcal{L}_{scalebreaking} = -\frac{1}{4} \chi^4 \ln \chi + \frac{d}{3} \chi^4 \ln \chi \left( \frac{(\sigma^2 - \delta^2)}{\sigma_0^2 \zeta_0} \right) \chi_0 \]  

(7)

The effect of these logarithmic terms \( \sim \chi^4 \ln \chi \) is to break the scale invariance, which leads to the trace of the energy momentum tensor as \[77\]

\[
\theta_{\mu} = -4\mathcal{L} + \chi \frac{\partial \mathcal{L}}{\partial \chi} = -(1 - d) \chi^4
\]  

(8)

Hence the scalar gluon condensate of QCD \( \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} \rangle \) is simulated by a scalar dilaton field in the present hadronic model. In the present investigation we use the frozen glue ball limit according to which the scalar dilaton field, \( \chi \) has very little dependence on the density of the medium and therefore its expectation value is taken to be constant (equal to the vacuum value, \( \chi_0 \)). The comparison of the trace of the energy momentum tensor arising from the trace anomaly of QCD with that of the present chiral model gives the relation of
the dilaton field to the scalar gluon condensate. We have, in the limit of massless quarks [78],
\[ \theta^\mu = \langle \frac{\beta_{QCD}}{2g} G^a_{\mu\nu} G^{a\mu\nu} \rangle \equiv -(1 - d) \chi^4 \quad (9) \]

The parameter \( d \) originates from the second logarithmic term of equation (7). To get an insight into the value of the parameter \( d \), we recall that the QCD \( \beta \) function at one loop level, for \( N_c \) colors and \( N_f \) flavors is given by
\[ \beta_{QCD} (g) = -\frac{11N_c g^3}{48\pi^2} \left( 1 - \frac{2N_f}{11N_c} \right) + O(g^5) \quad (10) \]

In the above equation, the first term in the parentheses arises from the (antiscreening) self-interaction of the gluons and the second term, proportional to \( N_f \), arises from the (screening) contribution of quark pairs. Equations (9) and (10) suggest the value of \( d \) to be 6/33 for three flavors and three colors, and for the case of three colors and two flavors, the value of \( d \) turns out to be 4/33, to be consistent with the one loop estimate of QCD \( \beta \) function. These values give the order of magnitude about which the parameter \( d \) can be taken [77], since one cannot rely on the one-loop estimate for \( \beta_{QCD}(g) \). This parameter, along with the other parameters corresponding to the scalar Lagrangian density, \( L_0 \) given by equation (4), are fitted so as to ensure extrema in the vacuum for the \( \sigma \) and \( \zeta \) field equations, to reproduce the vacuum masses of the \( \eta \) and \( \eta' \) mesons, the mass of the \( \sigma \) meson around 500 MeV, and pressure, \( p(\rho_0)=0 \), with \( \rho_0 \) as the nuclear matter saturation density [58, 66].

The coupled equations of motion for the non-strange scalar field \( \sigma \), strange scalar field \( \zeta \) and the scalar-isovector field \( \delta \) are derived from the Lagrangian densities given by equations (2) to (5) and are given as
\[ k_0 \chi^2 \sigma - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \sigma - 2k_2 \left( \sigma^3 + 3\sigma\delta^2 \right) - 2k_3 \chi \sigma \zeta \\
- \frac{d}{3} \chi \sigma^2 - \frac{2\sigma}{\sigma^2 - \delta^2} + \left( \frac{\chi}{\chi_0} \right)^2 \frac{m_\pi^2 f_\pi}{2} - \sum g_{\sigma_i} \rho_i^s = 0 \quad (11) \]

\[ k_0 \chi^2 \zeta - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \zeta - 4k_2 \chi^3 - k_3 \chi \left( \sigma^2 - \delta^2 \right) \\
- \frac{d}{3} \chi \zeta^2 + \left( \frac{\chi}{\chi_0} \right)^2 \left[ \sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right] - \sum g_{\zeta_i} \rho_i^s = 0 \quad (12) \]
\[ k_0 \chi^2 \delta - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \delta - 2k_2 \left( \delta^3 + 3\sigma^2 \delta \right) + k_3 \chi \delta \zeta + 2 \frac{d}{3} \chi^4 \left( \frac{\delta}{\sigma^2 - \delta^2} \right) - \sum g_{s_i} \rho_i^s = 0 \]  \hspace{1cm} (13)

In the above, \( \rho_i^s \) are the scalar densities for the baryons, which at zero temperature is given as

\[ \rho_i^s = \gamma_i \int_0^{k_{F_i}} \frac{d^3 k}{(2\pi)^3} \frac{m_i^*}{(k^2 + m_i^*)^{1/2}} \]  \hspace{1cm} (14)

where, \( k_{F_i} \) is the fermi momentum of the baryon of species \( i \) \( (i = p, n, \Lambda, \Sigma^{\pm, 0}, \Xi^{-, 0}) \) and \( \gamma_i = 2 \) is the spin degeneracy factor [62].

The above coupled equations of motion are solved to obtain the density dependent values of the scalar fields \( (\sigma, \zeta \) and \( \delta \)), in the isospin asymmetric strange hadronic medium at zero temperature. The isospin asymmetry of the medium is defined through the parameter, \( \eta = (\rho_n - \rho_p) / (2\rho_B) \), where \( \rho_n \) and \( \rho_p \) are the number densities of the neutron and proton and \( \rho_B \) is the baryon density. The strangeness fraction, \( f_s \) of the medium is defined by \( \frac{\sum_i |s_i| \rho_i}{\rho_B} \), where \( s_i \) is the number of strange quarks of baryon \( i \). In this strange hadronic medium we have both nucleons and hyperons. The nucleons modified in the strange hadronic matter further interact with the \( D \) and \( \bar{D} \) mesons and modify their properties.

### III. \( D \) AND \( \bar{D} \) MESONS IN ISOPLAN ASYMMETRIC STRANGE HADRONIC MATTER

In this section we study the \( D \) and \( \bar{D} \) mesons properties in isospin asymmetric strange hadronic matter. As mentioned earlier, the medium modifications of the \( D \) and \( \bar{D} \) mesons arise due to their interactions with the nucleons, non strange scalar meson \( \sigma \) and the scalar isovector meson \( \delta \). The non strange scalar meson \( \sigma \) and the scalar isovector meson \( \delta \) are modified in the medium consisting of nucleons and hyperons. The nucleons modified in the strange hadronic medium interact with the \( D \) and \( \bar{D} \) mesons and modify their properties. The interaction Lagrangian density is given as

\[ \mathcal{L}_{DN} = -\frac{i}{8f_D^2} \left[ 3 \left( \bar{p} \gamma^\mu p + \bar{n} \gamma^\mu n \right) \left( D^0 (\partial_\mu \bar{D}^0) - (\partial_\mu D^0) \bar{D}^0 \right) + \left( D^+ (\partial_\mu D^-) - (\partial_\mu D^+) D^- \right) \right] \]
In Eq. (15), the first term is the vectorial Weinberg Tomozawa interaction term, obtained from the kinetic term of Eq. (11). The second term is obtained from the explicit symmetry breaking term and leads to the attractive interactions, as a function of density, for both the $D$ and $\bar{D}$ mesons in the medium. The next three terms of above Lagrangian density ($\sim (\partial_\mu \bar{D})(\partial^\mu D)$) are known as the range terms. The first range term (with coefficient ($-\frac{1}{f_D}$)) is obtained from the kinetic energy term of the pseudoscalar mesons. The second and third range terms $d_1$ and $d_2$ are written for the $DN$ interactions in analogy with those written for $KN$ interactions in [63]. It might be noted here that the interaction of the pseudoscalar mesons with the vector mesons, in addition to the pseudoscalar meson-nucleon vectorial interaction, leads to a double counting in the linear realization of chiral effective theories. Further, in the non-linear realization, such an interaction does not arise in the leading or subleading order, but only as a higher order contribution [79]. Hence the vector meson-pseudoscalar interactions will not be taken into account in the present investigation.

The dispersion relations for the $D$ and $\bar{D}$ mesons are obtained by the Fourier transformations of equations of motion. These are given as

$$ -\omega^2 + \vec{k}^2 + m_D^2 - \Pi(\omega, |\vec{k}|, \rho) = 0 $$

where, $m_D$ is the vacuum mass of the $D(\bar{D})$ meson and $\Pi(\omega, |\vec{k}|, \rho)$ denotes the self-energy of the $D(\bar{D})$ mesons in the medium. The self-energy $\Pi(\omega, |\vec{k}|, \rho)$ for the $D$ meson doublet ($D^0, D^+$) arising from the interaction of Eq. (15) is given as

$$ \Pi(\omega, |\vec{k}|, \rho) = \frac{1}{4f_D^2} [3(\rho_p + \rho_n) \pm (\rho_p - \rho_n)] \omega $$
\[ + \frac{m_D^2}{2 f_D} (\sigma' + \sqrt{2} \zeta_c' \pm \delta') \]
\[ + \left[ - \frac{1}{f_D} (\sigma' + \sqrt{2} \zeta_c' \pm \delta') + \frac{d_l}{2 f_D^2} (\rho_p^s + \rho_n^s) \right. \]
\[ + \frac{d_2}{4 f_D^2} \left( (\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s) \right) \]
\[ \left. (\omega^2 - \mathbf{k}^2) \right], \tag{17} \]

where the ± signs refer to the \( D^0 \) and \( D^+ \) mesons, respectively, and \( \sigma' (= \sigma - \sigma_0) \), \( \zeta_c' (= \zeta_c - \zeta_{c0}) \), and \( \delta' (= \delta - \delta_0) \) are the fluctuations of the scalar-isoscalar fields \( \sigma \), \( \zeta_c \) and the scalar-isoscalar field \( \delta \) from their vacuum expectation values in the strange hyperonic medium. The vacuum expectation value of \( \delta \) is zero \( (\delta_0 = 0) \), since a nonzero value for it will break the isospin-symmetry of the vacuum. (We neglect here the small isospin breaking effect arising from the mass and charge difference of the up and down quarks.) We might note here that the interaction of the scalar charm quark condensate \( \zeta_c \) (being made up of heavy charm quarks and charm antiquarks) leads to very small modifications of the masses \[80\]. So we will not consider the medium fluctuations of \( \zeta_c \). In Eq. \[17\], \( \rho_i \) and \( \rho_i^s \) with \( i = p, n \) are the number density and the scalar density of the baryon of type \( i \). The scalar density, \( \rho_i^s \), at zero temperature is already defined in Eq. \[14\], whereas the number density, \( \rho_i \) for the \( i \)-th baryon \( (i = p, n, \Lambda, \Sigma^{\pm, 0}, \Xi^{0, -}) \), is defined as
\[ \rho_i = \gamma_i \int_0^{k_{Fi}} \frac{d^3k}{(2\pi)^3}, \tag{18} \]
where \( \gamma_i = 2 \) is the spin degeneracy factor.

Similarly, for the \( \bar{D} \) meson doublet \((\bar{D}^0, D^-)\), the self-energy is calculated as
\[ \Pi(\omega, |\mathbf{k}|, \rho) = -\frac{1}{4 f_D^2} \left[ 3(\rho_p + \rho_n) \pm (\rho_p - \rho_n) \right] \omega \]
\[ + \frac{m_{\bar{D}}^2}{2 f_D} (\sigma' + \sqrt{2} \zeta_c' \pm \delta') \]
\[ + \left[ - \frac{1}{f_D} (\sigma' + \sqrt{2} \zeta_c' \pm \delta') + \frac{d_l}{2 f_D^2} (\rho_p^s + \rho_n^s) \right. \]
\[ + \frac{d_2}{4 f_D^2} \left( (\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s) \right) \]
\[ \left. (\omega^2 - \mathbf{k}^2) \right], \tag{19} \]

where the ± signs refer to the \( \bar{D}^0 \) and \( D^- \) mesons, respectively. The optical potentials of the \( D \) and \( \bar{D} \) mesons are obtained using the expression
\[ U(\omega, k) = \omega(k) - \sqrt{k^2 + m_{\bar{D}}^2}, \tag{20} \]
where $m_D$ is the vacuum mass for the $D(\bar{D})$ meson and $\omega(k)$ is the momentum-dependent energy of the $D(\bar{D})$ meson.

IV. RESULTS AND DISCUSSIONS

In this section we present the results and discussions of our investigation of the in-medium properties of $D$ and $\bar{D}$ mesons in isospin asymmetric strange hadronic matter. We have generalized the chiral $SU(3)$ model to $SU(4)$ to include the interactions of the charmed mesons. The present calculations use the following model parameters: $k_0 = 2.54, k_1 = 1.35, k_2 = -4.78, k_3 = -2.77, k_4 = -0.22$ and $d = 0.064$, which are the parameters occurring in the scalar meson interactions defined in equation (4). The vacuum values of the scalar isoscalar fields, $\sigma$ and $\zeta$ and the dilaton field $\chi$ are $-93.3$ MeV, $-106.6$ MeV and $409.8$ MeV respectively. The values, $g_{\sigma N} = 10.6$ and $g_{\zeta N} = -0.47$ are determined by fitting vacuum baryon masses. The other parameters fitted to the asymmetric nuclear matter saturation properties in the mean-field approximation are: $g_{\omega N} = 13.3$, $g_{\rho p} = 5.5$, $g_4 = 79.7$, $g_{\delta p} = 2.5$, $m_\zeta = 1024.5$ MeV, $m_\sigma = 466.5$ MeV and $m_\delta = 899.5$ MeV. The coefficients $d_1$ and $d_2$, calculated from the empirical values of the $K N$ scattering lengths for $I = 0$ and $I = 1$ channels, are $2.56/m_K$ and $0.73/m_K$, respectively.

The $D$ and $\bar{D}$ mesons properties in isospin asymmetric strange hyperonic matter are modified due to their interactions with nucleons, the scalar meson $\sigma$ and scalar isovector meson $\delta$. As discussed earlier the non-strange scalar meson $\sigma$ and scalar isovector meson $\delta$ are modified in the strange hadronic medium consisting of nucleons and hyperons. The nucleons modified in the strange hadronic matter, interact with the $D$ and $\bar{D}$ mesons. In our present investigation, the Weinberg Tomozawa term introduces the isospin asymmetry in the medium through the isospin asymmetry in proton and neutron number densities. The scalar exchange term and first range term (term with coefficient $(-1/f_D)$) also contribute to the isospin asymmetry of the medium through the scalar isovector meson $\delta$. The $d_2$-term in the interaction Lagrangian density given by Eq. (15) also introduces the isospin asymmetry in the $D(D^0, D^+)$ and $\bar{D}(D^0, \bar{D}^-)$ meson doublets.

First, we study the effect of strangeness fraction, $f_s$ of the medium on the energies of $D$ and $\bar{D}$ mesons arising due to the various terms of the Lagrangian density given by
equation (15). In figures 1 and 2 we show the variation of the energies of $D$ and $\bar{D}$ mesons at zero momentum, with density in the isospin symmetric medium ($\eta = 0$). The results are shown for values of the strangeness fraction, $f_s = 0$ and 0.5. For a given value of isospin asymmetry parameter, $\eta$ and the strangeness fraction, $f_s$, the masses of $D^+$ and $D^0$ mesons are seen to decrease with increase in the density. For a given value of density, $\rho_B$ and isospin asymmetry parameter, $\eta$, as we move from nuclear medium ($f_s=0$) to the hyperonic matter, the attractive contribution to the in-medium energies of $D$ mesons from the Weinberg-Tomozawa term becomes smaller and the repulsive contribution arising from this Weinberg-Tomozawa term becomes smaller for the $\bar{D}$ mesons. To have an understanding of this behavior of $D$ and $\bar{D}$ mesons with the strangeness fraction, $f_s$, let us consider the term corresponding to the contribution of Weinberg-Tomozawa term to the self energy of the $D$ mesons at low densities. The $D$ meson self-energy arising from the Weinberg-Tomozawa interaction, $\Pi_{WT}(\omega, |\vec{k}|)$, is given by the first term of Eq. (17), and, at low densities, this turns out to be much smaller than $(k^2 + m_D^2)$. One can then, as a first approximation, replace $\Pi_{WT}(\omega, |\vec{k}|)$ by $\Pi_{WT}(m_D, |\vec{k}|)$ and solve for the dispersion relation given by Eq. (16). Confining our attention to the Weinberg Tomozawa interaction only, one finds that the energies of $D$ mesons($D^0$ and $D^+$) are given as

$$\omega(|\vec{k}|) \simeq \left(|\vec{k}|^2 + m_D^2\right)^{1/2} - \frac{1}{8 f_D^2} \frac{m_D}{\sqrt{k^2 + m_D^2}} \left[3 (\rho_p + \rho_n) \pm (\rho_p - \rho_n)\right].$$

(21)

Due to the presence of hyperons in the strange medium, for a fixed value of baryon density, $\rho_B$, the values of the proton and neutron densities, $\rho_p$ and $\rho_n$ are smaller in the hyperonic matter (nonzero $f_s$) as compared to the non-strange ($f_s = 0$) medium. This causes an increase in the energy of the $D$ mesons with the strangeness of the medium, as can be seen from Eq. (21). For $\bar{D}$ meson the sign of the Weinberg-Tomozawa term is opposite as compared to that of the $D$ meson, as can be seen from Eq. (19), and this leads to a smaller repulsive contribution for the the energy of $\bar{D}$ mesons arising from the Weinberg-Tomozawa interaction with strangeness fraction, for a fixed value of density and isospin asymmetry of the medium.

For a given value of isospin asymmetry parameter, $\eta$ and the strangeness fraction, $f_s$, the scalar meson exchange term has attractive contributions to the energies of both $D$ and $\bar{D}$ mesons, as can be seen from the second terms of the equations (17) and (19). From
FIG. 1: (Color online) The various contributions to $D$ meson energies at zero momentum in isospin symmetric strange hadronic medium ($\eta = 0$) for (a) $D^+$ and for (b) $D^0$ in MeV plotted as functions of baryon density in units of nuclear matter saturation density, $\rho_B/\rho_0$, shown for the strangeness fraction $f_s = 0.5$ and compared with the case of $f_s = 0$ (dotted line).
FIG. 2: (Color online) The various contributions to $\bar{D}$ meson energies at zero momentum in isospin symmetric strange hadronic medium ($\eta = 0$) for (a) $D^-$ and for (b) $\bar{D}^0$ in MeV plotted as functions of baryon density in units of nuclear matter saturation density, $\rho_B/\rho_0$, shown for the strangeness fraction $f_s = 0.5$ and compared with the case of $f_s = 0$ (dotted line).
figures [1] and [2], we observe that in isospin symmetric medium, $\eta = 0$, the energies of $D$ and $\bar{D}$ mesons, due to the scalar exchange term, increase as we move from the value of the strangeness fraction, $f_s = 0$ to 0.5. This can be understood from the following relation of the energy of $D$ and $\bar{D}$ mesons in the symmetric strange hadronic medium at zero momentum due to scalar meson exchange term only

$$\omega(|\vec{k}| = 0) = m_D \left[ 1 - \frac{\sigma'}{2f_D} \right]^{1/2} \quad (22)$$

Note that we have neglected the small fluctuations in the $\zeta_c$ field. As we move from the non-strange medium to strange medium, the magnitude of the $\sigma$ field increases and hence, the fluctuation, $\sigma' = \sigma - \sigma_0$ becomes smaller in the strange medium as compared to its value in the nuclear medium. This leads to an increase in the energy of $D$ and $\bar{D}$ mesons due to scalar meson exchange term as we move from symmetric non-strange hadronic medium ($f_s = 0$) to symmetric strange hadronic medium (nonzero $f_s$).

Among the three range terms, the first range term (term with coefficient $(-\frac{1}{f_D})$) has repulsive contributions to the energies of $D$ and $\bar{D}$ mesons. However, as we move from the nuclear medium ($f_s=0$) to hyperonic matter (nonzero $f_s$), the magnitude of the first range term decreases. This is due to the fact that there is an increase in the magnitude of the $\sigma$ field in moving from non-strange hadronic medium to strange hadronic medium, which leads to a smaller value of $\sigma'$, the fluctuation of $\sigma$. The second ($d_1$ term) and third ($d_2$ term) range terms have attractive contributions to the energies of $D$ and $\bar{D}$ mesons. However, the attractive contributions are lessened when we go from nuclear matter ($f_s=0$) to hyperonic matter, due to smaller values of the proton and neutron scalar densities $\rho^p_s$ and $\rho^n_s$ in the strange medium as compared to the non strange medium, for a given value of baryon density. The combined effect of all the range terms is observed to give an increase in the energies of the $D$ and $\bar{D}$ mesons with increase in strangeness of the hadronic medium, as can be seen from figures [1] and [2].

In figure [3], we show the variation of energies of D-mesons ($D^+, D^0$) at zero momentum, with density, for different values of isospin asymmetry parameters ($\eta = 0$, 0.3 and 0.5). At each value of isospin asymmetry parameter, $\eta$, the results are shown for strangeness fractions, $f_s = 0, 0.1, 0.3$ and 0.5. The $D$ and $\bar{D}$ mesons properties in isospin asymmetric nuclear matter at zero temperature have been studied in Ref. [66]. For a given value of
density, as we move from isospin symmetric nuclear medium ($\eta = 0$) to isospin asymmetric nuclear medium (finite $\eta$), the $D^+$ mesons feel a drop in the mass whereas the mass of $D^0$ mesons increases. For example, at density $\rho_B = \rho_0$, $f_s = 0$, the mass of $D^+$ drops by 18 MeV whereas the mass of $D^0$ meson increases by 22 MeV as we move from $\eta = 0$ to $\eta = 0.5$. At baryon density $\rho_B = 4\rho_0$, $f_s = 0$, the energy of $D^+$ meson decreases by 36 MeV whereas the energy of $D^0$ mesons increases by 92 MeV in going from $\eta = 0$ to $\eta = 0.5$. At strangeness fraction $f_s = 0.5$, as we move from $\eta = 0$ to $\eta = 0.5$, the energy of $D^+$ mesons decreases by 21 MeV at density $\rho_B = \rho_0$ whereas at density $4\rho_0$ it increases by 3 MeV.

We observe that, at baryon density, $\rho_B = \rho_0$, as we move from isospin symmetric medium ($\eta = 0$) to isospin asymmetric medium ($\eta = 0.5$), there is a larger drop in the energy of $D^+$ meson at strangeness fraction $f_s = 0.5$ as compared to $f_s = 0$. This is due to the larger drop in the energy of $D^+$ meson, given by scalar meson exchange term, as a function of isospin asymmetry of the medium at strangeness fraction, $f_s = 0.5$ as compared to $f_s = 0$. However, the increase in the energy of $D^+$ mesons as a function of isospin asymmetry of the medium at baryon density, $\rho_B = 4\rho_0$ and strangeness fraction, $f_s = 0.5$, is because of larger increase in the energy of $D^+$ mesons, given by the first range term (term with coefficient $\frac{1}{f_D}$) at $f_s = 0.5$ as compared to $f_s = 0$. For $D^0$ mesons, at strangeness fraction, $f_s = 0.5$, as we move from $\eta = 0$ to $\eta = 0.5$ the energy increases by 0.14 MeV and 35 MeV at a baryon density of $\rho_0$ and $4\rho_0$ respectively.

For a given value of density and isospin asymmetry parameter, $\eta$, the energies of $D^+$ and $D^0$ mesons increase with increase in the strangeness fraction, $f_s$, of the medium. For example, at nuclear saturation density $\rho_0$, isospin asymmetry parameter $\eta = 0$, as we move from $f_s = 0$ to 0.5, the energy of both $D^+$ and $D^0$ mesons increases by 47 MeV. At $\rho_B = 4\rho_0$, $\eta = 0$, as we move from $f_s = 0$ to 0.5, the energies of both $D^+$ and $D^0$ mesons increase by about 128 MeV. In isospin asymmetric medium ($\eta = 0.5$), the energy of $D^+$ increases by 43 MeV at $\rho_0$ and by 167 MeV at $4\rho_0$ whereas the energy of $D^0$ meson increases by 25 and 71 MeV at $\rho_0$ and $4\rho_0$ respectively. It may be noted that as a function of strangeness fraction the increase in the energy of $D^+$ mesons is larger as compared to $D^0$ mesons.

For a given value of isospin asymmetry and strangeness fraction, the energy of $D$ mesons ($D^0, D^+$) is found to decrease with increase in the density of the medium. In isospin sym-
FIG. 3: (Color online) The energies of $D^+$ mesons ((a),(c) and (e)) and of $D^0$ mesons ((b),(d) and (f)), at momentum $k = 0$, versus the baryon density (in units of nuclear saturation density), $\rho_B/\rho_0$, for different values of the strangeness fractions ($f_s = 0, 0.1, 0.3, 0.5$) and for a given value of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5).
metric nuclear medium ($\eta = 0$), the energy of $D^+(D^0)$ meson at zero momentum decreases by 80 (79.5) MeV and 360 (359.5) MeV at $\rho_0$ and $4\rho_0$ respectively from its vacuum value. In isospin symmetric nuclear medium ($\eta = 0$), at the value of the strangeness fraction $f_s = 0.5$, the energy of $D^+(D^0)$ meson at $|\vec{k}| = 0$ decreases by 33(32.5) and 232(231.5) MeV at $\rho_0$ and $4\rho_0$ respectively from its vacuum value. In isospin asymmetric nuclear medium with $\eta = 0.5$, the energy of $D^+(D^0)$ meson decreases by 98 (57) MeV and 397 (268) MeV at densities of $\rho_0$ and $4\rho_0$ respectively. At the same value of $\eta$, but with $f_s = 0.5$, the energy of $D^+(D^0)$ is observed to decrease by 54 (33) MeV and 229 (196) MeV at densities of $\rho_0$ and $4\rho_0$ respectively from its vacuum value. The smaller drop in the masses of $D$ mesons in the hyperonic medium is because of an increase in the mass of $D$ mesons with strangeness fraction of the medium, as has already been discussed.

In figure 4, we show the variation of the energies of $\bar{D}$ mesons ($D^-, \bar{D}^0$) at zero momentum, with density, for different values of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5). For each value of the isospin asymmetry parameter $\eta$, the results are shown for different values of the strangeness fraction, $f_s$. For a given value of density and isospin asymmetry of the medium, similar to the $D$ mesons, the energies of $\bar{D}$ mesons also increase with increase in the strangeness fraction $f_s$. In isospin symmetric medium ($\eta = 0$), for a given value of density, as we move from $f_s = 0$ to $f_s = 0.5$, the energy of $D^-$ meson increases by 32 MeV at $\rho_0$ and 86 MeV at $4\rho_0$. At isospin asymmetry parameter $\eta = 0.5$, the energy of $D^-$ increases by 28 and 62 MeV at density $\rho_0$ and $4\rho_0$ respectively. In isospin symmetric medium ($\eta = 0$), as we move from $f_s = 0$ to 0.5, the energy of $\bar{D}^0$ meson increases by 32 MeV and 84 MeV at $\rho_0$ and $4\rho_0$ respectively. At $\eta = 0.5$, the energy of $\bar{D}^0$ meson increases by 17 MeV and 45 MeV at density $\rho_0$ and $4\rho_0$ respectively. The increase in the energy of $\bar{D}$ mesons as a function of the strangeness fraction of the medium is because the range term, which is repulsive, dominates over the Weinberg-Tomozawa and scalar exchange terms of the Lagrangian density. The $D^-$ meson is observed to have a larger increase in the mass with increase in the strangeness fraction. This is because the total range term has larger repulsive contributions to the energy of $D^-$ mesons as compared to $\bar{D}^0$ mesons as we increase the strangeness fraction of the hadronic medium.

The density effects on the in-medium masses of $\bar{D}$ mesons are observed to be quite
FIG. 4: (Color online) The energies of $D^-$ mesons ((a),(c) and (e)) and of $\bar{D}^0$ mesons ((b),(d) and (f)), at momentum $k = 0$, versus the baryon density (in units of nuclear saturation density), $\rho_B/\rho_0$, for different values of the strangeness fractions ($f_s = 0, 0.1, 0.3, 0.5$) and for a given value of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5).
appreciable. In isospin symmetric nuclear medium ($\eta = 0$, $f_s = 0$), the energy of $D^- (\bar{D}^0)$ meson is seen to decrease by 29 (30.5) and 183 (181.5) MeV from its vacuum value at densities of $\rho_0$ and $4\rho_0$ respectively. As already mentioned, the in-medium energies of $\bar{D}$ mesons increase with increase in the strangeness fraction of the medium. At a value of the strangeness fraction $f_s = 0.5$, we observe a small increase in the in-medium energies of $\bar{D}$ mesons with density at small baryon densities. For example, in isospin symmetric nuclear medium, $\eta = 0$, at strangeness fraction $f_s = 0.5$, the energy of $D^-$ and $\bar{D}^0$ mesons increase up to a density of about $0.4\rho_0$, above which they start decreasing with further increase in the density of the medium. This behaviour of $\bar{D}$ mesons is because of the fact that the scalar meson exchange term and the attractive range terms ($d_1$ and $d_2$ terms) become less attractive with increase in the strangeness fraction of the medium. Because of this, at small densities the in-medium masses of the $\bar{D}$ mesons become larger than their vacuum values, since the repulsive vectorial interaction dominates over the attractive scalar meson and attractive $d_1$ and $d_2$ terms. However, with further increase in the baryon density the masses of the $D$ mesons decrease with density although the decrease is less for non-zero values of the strangeness fraction, $f_s$, as compared to the nuclear matter case ($f_s = 0$), as can be seen from figure [4]. In isospin symmetric strange hadronic medium, with $f_s=0.5$, at the nuclear saturation density, $\rho_B = \rho_0$, the in-medium energy of the $D^-$ and $\bar{D}^0$ mesons are observed to increase by 3 MeV and 2.5 MeV respectively from their vacuum values, whereas these masses are seen to drop by about 30.4 MeV and 34.4 MeV, for the nuclear medium ($f_s=0$). At baryon density $\rho_B = 4\rho_0$, for isospin symmetric hyperonic matter with $f_s=0.5$, the energy of the $D^- (\bar{D}^0)$ meson is observed to drop by 97.5(97) MeV, whereas the mass drop is 183 (182) MeV for $f_s=0$.

In the present investigation, we observe that the effect of strangeness fraction of the medium is to increase the energies of the $D$ and $\bar{D}$ mesons and the effect is seen to be more appreciable for the $D$ mesons as compared to the $\bar{D}$ mesons. The reason for this behaviour can be understood in the following way. The magnitude of the Weinberg-Tomozawa term, which is attractive (repulsive) for the in-medium masses of the $D(\bar{D})$ mesons is observed to be lessened when we increase the strangeness fraction in the hadronic medium, due to the presence of hyperons in the medium, which gives smaller values for $\rho_p$ and $\rho_n$, for a given
baryon density, and hence the magnitude of the contribution to the Weinberg-Tomozawa term is smaller. On the other hand, in the magnitude of the range terms, there is seen to be an increase for both the $D$ and $\bar{D}$ mesons, as can be observed from figures 1 and 2. This is due to smaller values of $\rho_s^p$ and $\rho_s^n$ in the presence of hyperons, for a given density, leading to smaller attractive contributions from the $d_1$ and $d_2$ terms of the range term. The scalar exchange term is seen to decrease in magnitude for both $D$ and $\bar{D}$ mesons, though the change is observed to be small. The difference in the effect of increasing strangeness in the medium on the masses of the $D(\bar{D})$ mesons is due to the positive (negative) contribution arising from the Weinberg-Tomozawa term, leading to a larger increase of the $D$ meson masses as compared to the $\bar{D}$ meson masses, with strangeness of the medium.

The mass modification of the $D$-meson at finite density has been studied in the QCD sum rule approach. Borel-transformed QCD sum rules were used to describe the interactions of $D$ mesons with nucleons by taking into account all the lowest dimension-4 operators in the operator product expansion (OPE) and the mass shift at the nuclear matter saturation density was found to be around $-50$ MeV [36]. In the quark meson coupling (QMC) model, where the $D(\bar{D})$ mesons are assumed to be bound states of a light quark (antiquark) and a charm antiquark (quark), interacting through exchange of scalar and vector mesons, the mass shift of the $D$ meson was calculated to be around $-60$ MeV [38]. In the present calculations, we observe a drop in the masses of the $D$ as well as $\bar{D}$ mesons. However, the mass drop is seen to be less in the presence of hyperons in the medium, as can be observed from figures 3 and 4. The drop in the masses of the $D$ mesons is also supported by the recent calculations in the coupled channel approach based on heavy quark symmetry [54], which may lead to the possibility of the formation of D-mesic nuclei. In figures 5 and 6 we show the optical potentials for the $D$ and $\bar{D}$ mesons as functions of momentum, at the nuclear matter saturation density, and in figures 7, 8, 9 and 10 the optical potentials for densities of $2\rho_0$ and $4\rho_0$, are plotted. These are illustrated for different values of the isospin asymmetry parameter. The effects of isospin asymmetry and strangeness fraction of the medium on the in-medium masses of $D$ and $\bar{D}$ mesons are in turn reflected in their optical potentials. From the plots we observe that the optical potentials of $D$ and $\bar{D}$ mesons do not depend significantly on the momentum of the mesons at low densities, but as we move to
high densities e.g. to $\rho_B = 4\rho_0$, there does seem to be appreciable momentum dependence. It is observed that for a given baryon density, with increase of strangeness fraction, $f_s$, of the medium, there is an increase in the magnitude of the optical potential of $D^+$ mesons as compared to the case of the $D^0$ mesons, in isospin asymmetric hyperonic matter. This is a reflection of the fact that the mass of $D^+$ meson has a larger drop with increase in the strangeness fraction of the medium, as compared to the mass drop of $D^0$ meson. For a given density and strangeness fraction, the drop in the optical potential of $D^+$ meson is increased, whereas, there is seen to be a decrease in the drop of the $D^0$ meson as we increase the isospin asymmetry in the medium. This is due to the interaction with the scalar isovector $\delta$ meson, which has opposite signs for the $D^+$ and $D^0$ mesons, in the isospin asymmetric hadronic matter, as can been seen from equation (17). The isospin asymmetric contribution of the Weinberg-Tomozawa term also leads to an increase (decrease) in the mass drop of the $D^+(D^0)$ meson in the asymmetric hadronic matter. On the other hand, the dependence of $D^-$ and $\bar{D}^0$ meson masses on the isospin asymmetry is seen to be larger for the $D^-$ mesons as compared to the $\bar{D}^0$ meson masses, as can be seen from figure 4. These are in turn reflected in their optical potentials, plotted in figures 6, 8 and 10 for densities, $\rho_B = \rho_0, 2\rho_0$ and $4\rho_0$ respectively. The effect of strangeness is also observed to be more appreciable for the $D^-$ mesons as compared to the $\bar{D}^0$ mesons. At high densities, the effects of strangeness as well as isospin asymmetry are observed to be quite dominant. The ratios $D^+/D^-$ and $D^0/\bar{D}^0$ could be promising experimental tools to verify the effect of strangeness fraction of the isospin asymmetric medium on the properties of $D$ and $\bar{D}$ mesons in compressed baryon matter asymmetric heavy ion collision experiments at the FAIR project in the future facility at GSI.

The medium modifications of the masses of $D$ and $\bar{D}$ mesons can lead to an explanation of $J/\psi$ suppression observed by NA50 collaboration at 158 GeV/nucleon in the Pb-Pb collisions [23]. Due to the drop in the mass of the $D\bar{D}$ pair in the nuclear medium, it can become a possibility that the excited states of charmonium ($\psi', \chi_{c2}, \chi_{c1}, \chi_{c0}$) can decay to $D\bar{D}$ pairs [66] and hence the production of $J/\psi$ from the decay of these excited states can be suppressed. Even at some densities it can become a possibility that the $J/\psi$ itself decays to $D\bar{D}$ pairs. Thus the medium modifications of the $D$ mesons can change the decay widths
FIG. 5: (Color online) The optical potential of $D^+$ meson (a,c and e) and of $D^0$ meson (b,d and f), are plotted as functions of momentum for $\rho_B = \rho_0$, for different values of the strangeness fractions ($f_s = 0, 0.1, 0.3, 0.5$) and for a given value of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5).

The decay of the charmonium states have been studied in Ref. [31, 35]. It is seen to depend sensitively on the relative momentum in the final state. These excited states might become narrow [35] though the $D$ meson mass is decreased appreciably at high densities.
FIG. 6: (Color online) The optical potential of $D^-$ meson (a,c and e) and of $\bar{D}^0$ meson (b,d and f), are plotted as functions of momentum for $\rho_B = \rho_0$, for different values of the strangeness fractions ($f_s = 0, 0.1, 0.3, 0.5$) and for a given value of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5).

It may even vanish at certain momentum corresponding to nodes in the wave function $35$. Though the decay widths for these excited states can be modified by their wave functions, the partial decay width of $\chi_{c2}$, owing to absence of any nodes, can increase monotonically with the drop of the $D^+D^-$ pair mass in the medium. This can give rise to depletion in the
FIG. 7: (Color online) The optical potential of $D^+$ meson (a,c and e) and of $D^0$ meson (b,d and f), are plotted as functions of momentum for $\rho_B = 2\rho_0$, for different values of the strangeness fractions ($f_s = 0, 0.1, 0.3, 0.5$) and for a given value of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5).

$J/\psi$ yield in heavy-ion collisions. The dissociation of the quarkonium states ($\Psi', \chi_c$, $J/\psi$) into $D\bar{D}$ pairs has also been studied [81, 82] by comparing their binding energies with the lattice results on the temperature dependence of the heavy-quark effective potential [83].
FIG. 8: (Color online) The optical potential of $D^-$ meson (a,c and e) and of $\bar{D}^0$ meson (b,d and f), are plotted as functions of momentum for $\rho_B = 2\rho_0$, for different values of the strangeness fractions ($f_s = 0, 0.1, 0.3, 0.5$) and for a given value of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5).

V. SUMMARY

We have investigated in the present work, the in-medium masses of the $D$, $\bar{D}$ mesons in isospin asymmetric strange hadronic matter. The properties of the light hadrons (the nu-
FIG. 9: (Color online) The optical potential of $D^+$ meson (a,c and e) and of $D^0$ meson (b,d and f), are plotted as functions of momentum for $\rho_B = 4\rho_0$, for different values of the strangeness fractions ($f_s = 0, 0.1, 0.3, 0.5$) and for a given value of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5).

cleons, scalar $\sigma$ meson and the scalar-isovector $\delta$ meson) in the hadronic medium containing the nucleons as well as the hyperons – as studied in $SU(3)$ chiral model – modify the $D(D)$ meson properties in the strange hadronic matter. The chiral $SU(3)$ model, with parameters fixed from the properties of the hadron masses in vacuum and low-energy KN scattering
FIG. 10: (Color online) The optical potential of $D^-$ meson (a,c and e) and of $\bar{D}^0$ meson (b,d and f), are plotted as functions of momentum for $\rho_B = 4\rho_0$, for different values of the strangeness fractions ($f_s = 0, 0.1, 0.3, 0.5$) and for a given value of isospin asymmetry parameter ($\eta = 0, 0.3$ and 0.5).
data, is extended to SU(4) to derive the interactions of \( D(\bar{D}) \) mesons with the light hadron sector. The mass modifications of \( D^+ \) and \( D^0 \) mesons are observed to be more strongly dependent on the isospin-asymmetry of the hadronic medium as compared to the medium modifications of the masses of \( D^- \) and \( \bar{D}^0 \) mesons. For a given value of density and isospin asymmetry, the strangeness of the medium is seen to lead to an increase in the masses of \( D \) and \( \bar{D} \) mesons. The effect of strangeness fraction on the in-medium masses of \( D \) mesons is observed to be larger as compared to those of the \( \bar{D} \) mesons. The mass modification for the \( D \) mesons are seen to be similar to the earlier finite density calculations of \( D \) mesons using QCD sum rule approach [37, 84] as well as to the results obtained using the quark-meson coupling model [38]. This is contrary to the small mass modifications observed in the coupled channel approach [48, 52]. In the present calculations, we obtain attractive mass shifts for the \( D \) as well as the \( \bar{D} \) mesons, contrary to the repulsive mass shifts for the \( \bar{D} \) mesons within the coupled channel approach [52]. The observed attractive mass shifts for the \( D \) mesons is also supported by the recent calculations in coupled channel approach based on heavy quark symmetry [54], which can give rise to the possibility of the \( D \)-mesic nuclei.

The ratios \( D^+/D^0 \) and \( D^-/\bar{D}^0 \) could be promising observables to study the effect of strangeness fraction of the medium on the properties of \( D \) and \( \bar{D} \) mesons. The isospin dependence of \( D^+ \) and \( D^0 \) masses is seen to be a dominant medium effect at high densities, which might show in their production \((D^+/D^0)\), whereas, for the \( D^- \) and \( \bar{D}^0 \), one sees that, even though these have a strong density dependence, their in-medium masses remain similar at a given value for the isospin-asymmetry parameter \( \eta \). The strong density, strangeness and isospin dependence of the \( D(\bar{D}) \) meson optical potentials in asymmetric strange hadronic matter can be tested in the asymmetric heavy-ion collision experiments at future GSI facility [19]. The study of the in-medium modifications of \( D \) mesons in strange hadronic matter at finite temperatures within the \( SU(4) \) model will be a possible extension of the present investigation. The medium modifications of the charmonium states as well as strange charmed mesons in the strange hadronic matter are also planned to be investigated.
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