Thermal spectra of thin accretion disks of finite thickness around Kerr black holes

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ABSTRACT
The analysis of the thermal spectrum of geometrically thin and optically thick accretion disks of black holes, the so-called continuum-fitting method, is one of the leading techniques for measuring black hole spins. Current models normally approximate the disk as infinitesimally thin, while in reality the disk thickness is finite and increases as the black hole mass accretion rate increases. Here we present an XSPEC model to calculate the multi-temperature blackbody spectrum of a thin accretion disk of finite thickness around a Kerr black hole. We test our new model with an RXTE observation of the black hole binary GRS 1915+105. We find that the spin value inferred with the new model is slightly higher than the spin value obtained with a model with an infinitesimally thin disk, but the difference is small and the effect is currently subdominant with respect to other sources of uncertainties in the final spin measurement.

Key words: accretion, accretion discs – black hole physics

1 INTRODUCTION
Black holes are among the most exotic objects that can be found in the contemporary Universe (Bambi 2018, 2019). According to Einstein’s theory of general relativity, a black hole should be completely characterized by its mass $M$, spin angular momentum $J$, and electric charge $Q$ (Carter 1971; Robinson 1975), but the latter is thought to be completely negligible for astrophysical black holes. The mass $M$ is relatively easy to measure, by studying the orbital motion of individual stars orbiting the black hole (see, for instance, Casares & Jonker 2014; Ghez et al. 2008). The measurement of the spin is definitively more challenging. The spin of a rotating object has no gravitational effects in Newtonian gravity, so black hole spin measurements require the analysis of relativistic phenomena occurring in the strong gravity region of the black hole.

There are currently two leading techniques for measuring the spin of accreting black holes: the continuum-fitting method (Zhang et al. 1997; McClintock et al. 2011, 2014) and X-ray reflection spectroscopy (Brenneman & Reynolds 2006; Reynolds 2014). The continuum-fitting method is the analysis of the thermal spectrum of geometrically thin and optically thick accretion disks of black holes and is normally used for stellar-mass black holes only. Indeed, the temperature of a thin accretion disk scales as $M^{-0.25}$ and the disk mainly emits in the soft X-ray band for stellar-mass black holes and in the optical/UV bands for supermassive black holes. In the latter case, dust absorption limits the capability of an accurate measurement of the thermal spectrum and, in turn, the possibility of measuring the black hole spin. X-ray reflection spectroscopy refers to the analysis of the reflection spectrum of thin accretion disks and can measure the spin of black holes of any mass.

The standard framework to describe geometrically thin and optically thick accretion disks of black holes is the Novikov-Thorne model (Novikov & Thorne 1973; Page & Thorne 1974), which is normally thought to be a good approximation for accretion disks of sources in the thermal state and with an Eddington-scaled accretion luminosity between a few percent to about 30% (McClintock et al. 2006; Steiner et al. 2010; Penna et al. 2010; Kulkarni et al. 2011). Common models for the continuum-fitting method and X-ray reflection spectroscopy employ the Novikov-Thorne model and approximate the disk as infinitesimally thin, with the particles of the gas moving on nearly-geodesic, equatorial, circular orbits. However, in reality the disk has a finite thickness, which increases as the mass accretion rate increases.

In this paper, we present a model to calculate the thermal spectrum of a geometrically thin and optically thick accretion disk of finite thickness around Kerr black holes. We implement the disk model proposed in Taylor & Reynolds (2018) in the multi-temperature blackbody model \texttt{nkbb} (Zhou et al. 2019; Tripathi et al. 2020). A ray-tracing code calculates the transfer function of the spacetime for a disk with finite thickness (Cunningham 1975) and the transfer functions for a grid of black hole spins, mass accretion...
rates, and disk inclination angles are stored in a FITS file. The model \texttt{NKB} can be used in XSPEC (Arnaud 1996) and reads the FITS file of the transfer functions during the data analysis.

To illustrate the impact of the disk thickness on the spin measurement, we analyze an RXTE observation of the black hole binary GRS 1915+105 with \texttt{NKB}, either assuming an infinitesimally thin accretion disk and employing the new version of the model with a disk of finite thickness. We find that the impact of the disk thickness on the estimate of the black hole spin inferred with the new model is slighter than the value found with the model assuming an infinitesimally thin disk. For the quality of the data analyzed, as well as considering the current typical uncertainties of black hole masses, distances, and inclination angles, the correction on the black hole spin measurement from the disk thickness can be ignored, but in the future, with more accurate and precise spin measurements, it may become necessary to take it into account for very high disk inclination angles.

The content of the paper is as follows. In Section 2, we review the accretion disk models with infinitesimally thin disk and with disk of finite thickness. In Section 3, we describe the construction of the model. In Section 4, we analyze an RXTE observation of the black hole binary GRS 1915+105 and we compare the black hole spin measurements obtained, respectively, with a model with infinitesimally thin disk and a model with a disk of finite thickness. We discuss our results in Section 5.

## 2 ACCRETION DISK MODELS

The Novikov-Thorne model is the standard framework for the description of geometrically thin and optically thick accretion disks around black holes (Novikov & Thorne 1973; Page & Thorne 1974). In this paper, we will assume that the spacetime metric is described by the Kerr solution (Kerr 1963), but the considerations and the expressions reported in this section hold for any stationary and axisymmetric black hole spacetime with a line element in spherical-like coordinates \((t, r, \theta, \phi)\) that can be written as

\[
ds^2 = \tilde{g}_{tt} dt^2 + 2 \tilde{g}_{t\phi} dt d\phi + \tilde{g}_{rr} dr^2 + \tilde{g}_{\theta\theta} \sin^2 \theta d\theta^2 + \tilde{g}_{\phi\phi} d\phi^2, \tag{1}
\]

where all the metric coefficients are independent of \(t\) and \(\phi\). Note that in this section we assume a metric with signature \((-+++\)) and employ units in which \(c = 1\). The details of the calculations can be found in Bambi (2017b, 2012).

If we approximate the disk as infinitesimally thin, the particles of the fluid move on nearly geodesic, circular \((r = \text{constant})\), equatorial \((\theta = \pi/2)\) orbits. We write the geodesic equations in the form

\[
\frac{d}{d\tau} \left( \tilde{g}_{\mu
u} \dot{x}^\nu \right) = \frac{1}{2} \left( \partial_{\mu} \tilde{g}_{\nu\rho} \right) \dot{x}^\nu \dot{x}^\rho. \tag{2}
\]

1. Note that this is not the most general line element for a stationary and axisymmetric spacetime; in general, \(\tilde{g}_{tr}\) may also be non-vanishing. Nevertheless, black hole solutions in general relativity and in many other theories of gravity have \(\tilde{g}_{tr} = 0\).

Since the particles of the fluid have \(\dot{r} = \dot{\theta} = 0\), for \(\mu = r\) we have

\[
(\partial_r \tilde{g}_{tt}) \dot{t}^2 + 2 \left( \partial_r \tilde{g}_{t\phi} \right) \dot{t} \dot{\phi} + \left( \partial_r \tilde{g}_{\phi\phi} \right) \dot{\phi}^2 = 0. \tag{3}
\]

The angular velocity of the fluid as measured by an observer at infinity is \(\Omega = \dot{\phi}/\dot{t}\). From Eq. (3) we find

\[
\Omega_k = - \frac{\left( \partial_r \tilde{g}_{t\phi} \right) \dot{\phi}}{\left( \partial_r \tilde{g}_{tt} \right) \dot{t}} \pm \sqrt{\left( \partial_r \tilde{g}_{t\phi} \right)^2 - \left( \partial_r \tilde{g}_{tt} \right) \left( \partial_r \tilde{g}_{\phi\phi} \right)}. \tag{4}
\]

where the upper (lower) sign refers to an accretion disk with angular momentum parallel (antiparallel) to the black hole spin.

From the conservation of the rest-mass of the particles of the fluid \(\rho_{\mu\nu} u^\mu u^\nu = -1\) and the conditions \(\dot{t} = \dot{\theta} = 0\) on the fluid motion, we derive \(i\)

\[
i = \frac{1}{\sqrt{-\tilde{g}_{tt} - 2\Omega_k \tilde{g}_{t\phi} - \Omega^2 \tilde{g}_{\phi\phi}}}. \tag{5}
\]

As the disk is infinitesimally thin, its surface is on the equatorial plane, and the 4-velocity of the particles on the surface of the disk is \(u^\mu = (1, 0, 0, \Omega)\), with the expressions of \(i\) and \(\Omega\) are, respectively, in Eq. (5) and Eq. (4), and in both cases all quantities are evaluated on the equatorial plane \((\theta = \pi/2)\).

Taylor & Reynolds (2018) have recently proposed a simple framework to take the thickness of the disk into account. The mid-plane of the accretion disk is still on the equatorial plane \(\theta = \pi/2\). For a radiatively dominated, optically thick disk, the pressure scale height is (Shakura & Sunyaev 1973)

\[
H = \frac{3}{2} \frac{1}{\eta} \left( \frac{M}{M_{\text{ISCO}}} \right) \left( i - \frac{R_{\text{ISCO}}}{\rho} \right), \tag{6}
\]

where \(\eta = 1 - E_{\text{ISCO}}\) the radiative efficiency of the Novikov-Thorne accretion disk, \(R_{\text{ISCO}}\) is the specific energy of a test-particle at the radius of the innermost stable circular orbit (ISCO) on the equatorial plane, \(M/M_{\text{ISCO}}\) is the Eddington-scaled mass accretion rate, \(R_{\text{ISCO}}\) is the ISCO radius, and \(\rho = r \sin \theta\) is the pseudo-cylindrical radius. The surface of the disk is set at \(z(\rho) = 2H(\rho)\)

All the particles of the fluid with the same pseudo-cylindrical radius \(\rho\) are supposed to rotate with the angular velocity of a test-particle on a geodesic, equatorial, circular orbit at that value of \(\rho\). Since \(\eta \gg R_{\text{ISCO}}/M\) depend on the black hole spin parameter \(a_\ast\) in the Kerr spacetime, black holes with the same Eddington-scaled mass accretion rate can have disks with different thickness according to the value of their spin parameter. Fig. 1 shows the disk profiles for a central black hole with \(a_\ast = 0.0, 0.8, \) and 0.998, and an Eddington-scaled mass accretion \(M/M_{\text{ISCO}} = 0.1, 0.2, \) and 0.3. For a given \(a_\ast\), \(M/M_{\text{ISCO}}\) is the parameter regulating the thickness of the disk.

In the model proposed in Taylor & Reynolds (2018), the surface of the disk is determined by Eq. (7). The 4-velocity of the particles of the fluid on the surface of the disk is still \(u^\mu = (1, 0, 0, \Omega)\), with \(\Omega\) still evaluated on the equatorial plane \((i = \rho = \pi/2)\) and \(i\) is evaluated from Eq. (5) with \(\Omega\) evaluated on the equatorial plane and the metric coefficients evaluated at the exact point on the surface of the disk.
3 THERMAL SPECTRA OF ACCRETION DISKS OF FINITE THICKNESS

The finite thickness disk geometry can be implemented in the relativistic thermal model NKBB (Zhou et al. 2019; Tripathi et al. 2020). While NKBB is specifically designed for testing the Kerr metric (Bambi 2017a), in this work we will ignore such a possibility and we will only consider the Kerr background, either with an infinitesimally thin disk or a disk of finite thickness. The extension to non-Kerr backgrounds is not so straightforward, because the FITS file is now for \( (\alpha_i, M/M_{\text{Edd}}) \), where \( M/M_{\text{Edd}} \) replaces the deformation parameter of the spacetime of the normal version of NKBB and a FITS file with 4 parameters would become too heavy.

The model employs the formalism of the transfer function proposed by Cunningham (Cunningham 1975; Speith et al. 1995). We consider a static observer at spatial infinity. The flux of the accretion disk as measured by the distant observer can be written as

\[
F_0(v_e) = \int I_o(v_e) \, dX \, dY = \int g^3 I_e(v_e) \, dX \, dY, \tag{8}
\]

where \( v_o \) and \( v_e \) are the photon frequencies in the rest-frame of the distant observer and of the gas, respectively, \( X \) and \( Y \) are the Cartesian coordinates of the plane of the distant observer, \( I_o \) and \( I_e \) are the specific intensities of the radiation in the rest-frame of the distant observer and of the gas, respectively, \( I_e = g^3 I_o \) follows from Liouville’s theorem (Lindquist 1966), and \( g \) is the redshift factor

\[
g = \frac{v_o}{v_e} = \frac{(k_B) \omega^R \omega^\mu}{(k_B) \omega^e \omega^\mu}, \tag{9}
\]

where \( \omega^\mu = (1, 0, 0, 0) \) is the 4-velocity of the distant observer, \( \omega^\mu = (1, 0, 0, \Omega) \) is the 4-velocity of the gas on the surface of the accretion disk (which changes if we assume infinitesimally thin disk or finite thickness disk), \( k^\mu \) is the 4-momentum of the photon, which is evaluated, respectively, at the detection point in the numerator and at the emission point in the denominator.

Introducing the transfer function \( f \), the flux of the accretion disk can be written as

\[
F_0(v_e) = \int_{R_{\text{out}}}^{R_{\text{in}}} \int_0^1 \frac{\pi r^2 g^* f(g^*, r_e, i)}{\sqrt{g^*(1 - g^*)}} \, I_e \, dg^* \, dr_e \tag{10}
\]

where \( R_{\text{in}} \) and \( R_{\text{out}} \) are the inner and the outer edge of the accretion disk, respectively, and \( g^* \) is the relative redshift factor defined by

\[
g^* = \frac{g - g_{\text{min}}}{g_{\text{max}} - g_{\text{min}}}, \tag{11}
\]

where \( g_{\text{min}} = g_{\text{min}}(r_e, i) \) and \( g_{\text{max}} = g_{\text{max}}(r_e, i) \) are, respectively, the minimum and the maximum values of the redshift factor \( g \) for the photons emitted from the radial coordinate \( r_e \) and for an inclination angle of the disk \( i \) (i.e., the angle between the black hole spin and the line of sight of the distant observer). \( f(g^*, r_e, i) \) is the transfer function

\[
f(g^*, r_e, i) = \frac{g \sqrt{g^*(1 - g^*)}}{\pi r_e} \frac{\partial (X, Y)}{\partial (g^*, r_e)}, \tag{12}
\]

where \( |\partial (X, Y)/\partial (g^*, r_e)| \) is the Jacobian between the Cartesian coordinates of the screen of the distant observer and the disk variables \( g^* \) and \( r_e \).

The specific intensity of the radiation at the emission point is

\[
I_e(v_e) = \frac{2h \nu^3}{f_{\text{col}}^3 \exp \left( \frac{h \nu_e}{k_B T_{\text{col}}} \right) - 1}, \tag{13}
\]

where \( h \) is Planck’s constant, \( k_B \) is the Boltzmann constant, \( T \) is a possible parameter of order 1 that depends on the angle between the normal to the disk and the propagation direction of the photon (but in what follows we will assume \( T = 1 \), corresponding to isotropic emission), and \( f_{\text{col}} \) is the color factor (a phenomenological parameter to take non-thermal effects into account, mainly the electron scattering...
in the disk atmosphere). \( T_{\text{col}} = f_{\text{col}} T_{\text{eff}} \) is the color temperature, and \( T_{\text{eff}} \) is the effective temperature of the accretion disk, which is obtained assuming \( F = \sigma T_{\text{eff}}^4 \), where \( \sigma \) is the Stefan-Boltzmann constant and \( F \) the time-averaged energy flux emitted from the disk surface (Bambi 2017b, 2012)

\[
F(r) = \frac{M}{4\pi M_\odot^2} F(r).
\]  

(14)

\( F(r) \) is a dimensionless function that depends on the spacetime geometry only. For an infinitesimally thin disk, the mass accretion rate enters only via Eq. (14). The temperature profile is the same for infinitesimally thin disk and finite thickness disk.

The transfer function encodes the details of the spacetime metric and of the disk geometry. The transfer function is calculated with a ray-tracing code, firing photons from the screen of the distant observer backward in time to the accretion disk. The numerical scheme has been already discussed in Bambi et al. (2017), Abdikamalov et al. (2019), and Zhou et al. (2019). The transfer functions are tabulated and stored into a FITS file. The main grid of the FITS file is 3-dimensional, for the black hole spin parameter \( a_s \), the Eddington-scaled mass accretion rate \( \dot{M}/\dot{M}_{\text{Edd}} \), and the inclination angle of the disk \( i \). The grid is \( 30 \times 30 \times 22 \), namely we have 30 values for \( a_s \) and \( \dot{M}/\dot{M}_{\text{Edd}} \) and 22 values for \( i \). \( a_s \) and \( i \) have the same spacing as in our previous version of NKBK (Zhou et al. 2019). The values of \( \dot{M}/\dot{M}_{\text{Edd}} \) are evenly distributed over the range 0 to 0.3. Fig. 2 shows the grid points of the FITS file on the plane spin parameter \( a_s \) vs mass accretion rate \( \dot{M}/\dot{M}_{\text{Edd}} \). For every set of \((a_s, \dot{M}/\dot{M}_{\text{Edd}}, i)\), the transfer function is evaluated at 100 emission radii \( r_e \), from the ISCO to \( 10^6 M \). For every emission radius, the transfer function is evaluated at 40 values of \( g^+ \).

Once the transfer function is stored in the FITS file for some specific accretion disk model, we can calculated the thermal spectrum of the accretion disk using Eq. (10). Fig. 3 shows the output of our model for three different spin parameters \((a_s = 0, 0.8, \text{and } 0.998, \text{from top to bottom})\), three different inclination angles of the disk \((i = 10^\circ, 45^\circ, \text{and } 80^\circ, \text{from left to right})\), and three different mass accretion rates \((\dot{M}/\dot{M}_{\text{Edd}} = 0.1, 0.2, \text{and } 0.3, \text{respectively black, blue, and red curves})\). The dotted curves are for infinitesimally thin accretion disks and the solid curves are for finite thickness disks. In some cases, the difference between the spectra of infinitesimally thin disks and finite thickness disks is hard to
see. The general trend is that the discrepancy between the two spectra increases for higher values of the spin parameter and the inclination angle.

4 IMPACT OF THE DISK THICKNESS ON THE SPIN MEASUREMENT: GRS 1915+105

In order to evaluate the impact of the thickness of the disk on the estimate of black hole spins, we analyze an X-ray observation of a black hole and we measure its spin parameter with NKB, either with the standard assumption of an infinitesimally thin disk (model 1 in this and next section) and with the disk model with finite thickness (model 2 in this and next section). The difference between the two spin measurements provides an estimate of the systematic uncertainty due to the disk thickness in the models with infinitesimally thin disks. From Fig. 3 we expect that the difference between the spectra of the two disk models is larger when the source has its spin parameter $a_i$ close to 1 and the inclination angle of the accretion disk $i$ is high. We thus decide to test the new model with the black hole binary GRS 1915+105.

GRS 1915+105 is a low-mass X-ray binary. The distance of the source is $D = 8.6^{+2.0}_{-1.6}$ kpc and the mass of its black hole is $M = 12.4^{+2.0}_{-1.8} M_\odot$ (Reid et al. 2014). Employing models with infinitesimally thin accretion disks, the black hole spin parameter has been estimated by several authors and different observations, always finding a value close to 1. McClintock et al. (2006) find $a_i > 0.98$ with the continuum-fitting method and the analysis of RXTE and ASCA data. Blum et al. (2009) and Miller et al. (2013) analyze, respectively, a Suzaku and a NuSTAR observation of GRS 1915+105; both studies find $a_i = 0.98 \pm 0.01$ (1-$\sigma$ statistical error) from the analysis of the reflection spectrum of the source. Zhang et al. (2019) and Abdikamalov et al. (2020) reanalyze the Suzaku data of Blum et al. (2009) without assuming the Kerr metric and find that the black hole spin parameter is very close to 1 even in the presence of possible deviations from the Kerr geometry. The inclination angle of the accretion disk is also thought to be high, even if its exact value is a bit controversial. Assuming that the jet of the source is parallel to the black hole spin and orthogonal to the accretion disk, the inclination angle is $i = 66^\circ \pm 2^\circ$ (Fender et al. 1999). From the analysis of the reflection spectrum of the accretion disk, one can find an independent estimate of the inclination angle of the inner part of the disk, with $i$ ranging from $16^\circ$ up to $80^\circ$ (Blum et al. 2009; Miller et al. 2013; Zhang et al. 2019; Abdikamalov et al. 2020).

The continuum-fitting method requires thermal dominant spectral data, which are defined by three conditions in Remillard & McClintock (2006): i) the flux of the thermal component accounts for more than 75% of the total 2-20 keV unabsorbed flux, ii) the root mean square (RMS) variability in the power density spectrum in the 0.1-10 Hz range is lower than 0.075, and iii) quasi-periodic oscillations (QPOs) are absent or very weak. Imposing these conditions, in the RXTE archive McClintock et al. (2006) find 20 observations of GRS 1915+105, which become 5 observations after requiring that the Eddington-scaled luminosity is less than 30%. Since our goal here is only to illustrate the impact of the disk thickness, and not to repeat a detailed measurement of the black hole spin of GRS 1915+105, we consider only one of these observations, corresponding to observation number 20 in McClintock et al. (2006). The observation was on 24 November 2003 and the exposure time was around 4.2 ks.

For the data reduction, we strictly follow that in McClintock et al. (2006). We only use the pulse-height spectra of PCU-2 because it is the best calibrated PCU and is the most operational one. The only difference with McClintock et al. (2006) is in the calibration correction and systematic error. In our case, data are corrected for calibration using the python script pcacorr (García et al. 2014) and we add a systematic error of 0.1% to all the PCA energy channels. The data are fitted with the XSPEC model (McClintock et al. 2006)

\[ \text{TBABS}\times\text{SMEDGE}\times\text{NKBB + POWERLAW}. \]

TBABS describes the Galactic absorption (Wilms et al. 2000) and we freeze the hydrogen column density to $N_H = 8 \times 10^{22}$ cm$^{-2}$ (Abdikamalov et al. 2020); however, its exact value does not appreciably affect the fit because the RXTE data do not cover low energies. SMEDGE describes a broad iron absorption edge (Ebisawa et al. 1994) and GABS is a Gaussian absorption line around 7 keV. NKBB describes the thermal spectrum of the accretion disk; we first consider the case of an infinitesimally thin disk (model 1) and then the case of a disk with finite thickness (model 2). We freeze the black hole mass to $M = 12.4 M_\odot$ and the black hole distance to $D = 8.6$ kpc (Reid et al. 2014). The inclination angle of the disk is frozen to $i = 73^\circ$, which is the measurement obtained from X-ray reflection spectroscopy in Abdikamalov et al. (2020); however, even in this case the exact value does not have a significant impact on the fit. We ignore the uncertainties of $M$, $D$, and $i$ as an accurate spin measurement of the black hole in GRS 1915+105 is beyond the scope of our study, which is only focused on a preliminary estimate of the impact of the disk thickness on spin measurements. POWERLAW describes a power law component.

The best-fit values of the two models are reported in Tab. 1, where all parameter uncertainties are at the 90% of confidence level for one relevant parameter. As we can see, the best-fit values of the two models are all consistent. While we find quite high best-fit values for the photon index $\Gamma$, our measurements are consistent with that reported in McClintock et al. (2006). The spin measurements of the two models are

\[ 0.9859 < a_i < 0.9903 \quad (\text{model 1}) \]
\[ 0.9899 < a_i < 0.9962 \quad (\text{model 2}) \]

Note that here we are only considering the statistical uncertainty of the fit. We are ignoring all systematic uncertainties of the model and the contribution from the uncertainties on $M$, $D$, and $i$, three quantities that are usually poorly constrained and tend to dominate the final error on the black hole spin parameter (Kulkarni et al. 2011; McClintock et al. 2014).

5 DISCUSSION

In the traditional framework of the continuum-fitting method, the model depends on 5 parameters: the black hole mass $M$, the black hole spin parameter $a_i$, the black hole distance $D$, the mass accretion rate $M$, and the inclination angle...
of the disk $i$. However, the resulting spectrum is simply a multi-temperature blackbody-like spectrum without particular features and it is not possible to infer the values of all the free parameters from the fit. It is thus necessary to have independent estimates of the black hole mass, black hole distance, and inclination angle, often obtained from optical observations, and then one can fit the thermal component of the source to measure the black hole spin parameter and the mass accretion rate (Zhang et al. 1997; McClintock et al. 2011, 2014).

The model commonly used for the continuum-fitting method is KERRBB (Li et al. 2005), and its extension KERRBB. There are now about 15 stellar-mass black holes with an estimate of the spin parameter via the continuum-fitting method. The model employs Novikov-Thorne disks and assumes that the disks are infinitesimally thin. The impact of the theoretical model of KERRBB on the estimate of black hole spins has been investigated in Penna et al. (2010) and Kulkarni et al. (2011): the authors ran GRMHD simulations of thin accretion disks for different values of the black hole spin parameter, calculated the spectra emitted by their simulated disks, evaluated the differences with the spectra calculated from the Novikov-Thorne model with an infinitesimally thin disk and no emission inside the ISCO, and eventually estimated the errors on the spin measurements obtained with an infinitesimally thin disk model. The conclusion of Kulkarni et al. (2011) is that the uncertainties on current spin measurements with the continuum-fitting method are dominated by observational uncertainties, while systematic uncertainties due to the theoretical model are negligible. Their conclusion is consistent with our results, where the estimates of the black hole spin with models 1 and 2 overlap at 90% of confidence level and we have not included the larger uncertainty contributions from the errors on $M$, $D$, and $i$.

An extension of KERRBB/KERRBB was presented in Straub et al. (2011) and called SLIMBB, as capable of describing the thermal spectra of thin and slim accretion disks (Sadowski et al. 2011), so valid up to Eddington-scaled luminosities ~ 0.7. SLIMBB not only takes the thickness of the disk into account, but includes also the radial advection of heat, which changes the emission profile, and deviations of the inner edge of the disk from the ISCO radius, both effects important at high mass accretion rates. Straub et al. (2011) analyze a large number of RXTE data of the black hole binary LMC X-3, which is thought to be a black hole with a moderate value of the spin parameter ($a_*$ < 0.7) and a high disk inclination angle ($i$ ~ 70°). Straub et al. (2011) find no discrepancy between the black hole spin measurements obtained with KERRBB/KERRBB and SLIMBB when the Eddington-scaled accretion luminosity of the source is below 30%.

The impact of the disk structure has been investigated even for spin measurements obtained from the analysis of the reflection spectrum of the disk. X-ray reflection spectroscopy can potentially provide more precise spin measurements, because the latter do not require independent estimate of the black hole mass, distance, and disk inclination angle, three quantities that are often difficult to measure and are affected by large systematic uncertainties. Moreover, the reflection spectrum has more features than the thermal component, and this also helps constraining the model parameters. However, the model itself is more complicated and the theoretical uncertainties in the model can have a larger impact on the final spin measurement.

Reynolds & Fabian (2008) simulate geometrically thin accretion disks in a pseudo-Newtonian potential. They find that spin measurements employing the standard infinitesimally thin disk model lead to overestimate the black hole spin, which is the result contrary to ours, but the main contribution to the final spin measurement is not determined by the disk thickness but by the radiation emitted from the plunging region inside the ISCO.

In our work, we have implemented the disk model proposed in Taylor & Reynolds (2018), where the authors study the impact of the disk thickness on X-ray reflection spectroscopy spin measurements. Taylor & Reynolds (2018) find that the analysis with an infinitesimally thin disk model leads to underestimate the black hole spin, like in our analysis for the continuum-fitting method, but it should be noted that the coronal geometry and the corresponding intensity profile play quite an important role on the actual impact of the disk geometry, so a direct comparison is not straightforward. The systematic uncertainty on the final spin measurement from an infinitesimally thin disk is not negligible in

| Model | 1 | 2 |
|-------|---|---|
| TRABS |   |   |
| $n_H$ [10$^{22}$ cm$^{-2}$] | 8$^*$ | 8$^*$ |
| SEDGE |   |   |
| $E_\text{in}$ [keV] | 7.62$^{+0.06}_{-0.06}$ | 7.62$^{+0.09}_{-0.08}$ |
| $\tau_e$ | 0.95$^{+0.24}_{-0.16}$ | 0.95$^{+0.16}_{-0.17}$ |
| GABS |   |   |
| $E_{\text{line}}$ [keV] | 7.07$^{+0.05}_{-0.05}$ | 7.07$^{+0.07}_{-0.06}$ |
| $\sigma$ [keV] | 0.5$^*$ | 0.5$^*$ |
| NKB |   |   |
| $M$ [$M_\odot$] | 12.4$^*$ | 12.4$^*$ |
| $D$ [kpc] | 8.6$^*$ | 8.6$^*$ |
| $i$ [deg] | 73$^*$ | 73$^*$ |
| $a_*$ | 0.988$^{+0.002}_{-0.002}$ | 0.9926$^{+0.0036}_{-0.0027}$ |
| $M$ [$M_\odot$] | 0.183$^{+0.003}_{-0.005}$ | 0.1834$^{+0.0017}_{-0.0011}$ |
| $f_{\text{bol}}$ | 1.7$^*$ | 1.7$^*$ |
| POWERLAW |   |   |
| $\Gamma$ | 3.78$^{+0.01}_{-0.01}$ | 3.78$^{+0.05}_{-0.05}$ |
| norm | 62$^{+4}_{-3}$ | 63$^{+5}_{-5}$ |
| $\chi^2/\nu$ | 54.97/39 = 1.410 | 55.18/39 = 1.415 |

Table 1. Summary of the best-fit values for model 1 (infinitesimally thin disk) and model 2 (disk with finite thickness). The reported uncertainties correspond to the 90% of the confidence level for one relevant parameter. $^*$ indicates that the parameter is frozen in the fit.

2 The color factor $f_{\text{bol}}$ can be calculated with a model for the disk atmosphere and mainly depends on the mass accretion rate $M$, so it is not a free parameter (McClintock et al. 2014).
Suzaku than those in the increases. For higher inclination angles and/or thicker disks of the thickness of the disk increases as the inclination angle be included in the theoretical model. Moreover, the impact the thickness of thin disks may become a new ingredient to et al. (2011). However, future observational measurements by the theoretical model, as already pointed out in Kulkarni et al. (2011). However, future observational measurements will be likely more precise and accurate, and in such a case the thickness of thin disks may become a new ingredient to be included in the theoretical model. Moreover, the impact of the thickness of the disk increases as the inclination angle increases. For higher inclination angles and/or thicker disks than those in the Suzaku observation of GRS 1915+105, a more significant part of the very innermost region of the accretion disk may be obscured by the disk itself (Taylor & Reynolds 2018), and this could increase the difference in the best-fit values from the models with infinitesimally thin disk and disk with finite thickness.

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REFERENCES

Abdikamalov, A. B., Ayzenberg, D., Bambi, C., et al. 2019, ApJ, 878, 91
Abdikamalov, A. B., Ayzenberg, D., Bambi, C., et al. 2020, arXiv e-prints, arXiv:2003.09663
Arnaud, K. A. 1996, Astronomical Data Analysis Software and Systems V, 101, 17
Bambi, C. 2012, ApJ, 761, 174
Bambi, C. 2017a, Reviews of Modern Physics, 89, 025001
Bambi, C. 2017b, Black Holes: A Laboratory for Testing Strong Gravity
Bambi, C. 2018, Annalen der Physik, 530, 1700430
Bambi, C. 2019, arXiv e-prints, arXiv:1906.03871
Bambi, C., Cárdenas-Avendaño, A., Dauser, T., et al. 2017, ApJ, 842, 76
Blum, J. L., Miller, J. M., Fabian, A. C., et al. 2009, ApJ, 706, 60
Brenneman, L. W., & Reynolds, C. S. 2006, ApJ, 652, 1028
Carter, B. 1971, Phys. Rev. Lett., 26, 331
Casares, J., & Jonker, P. G. 2014, Space Sci. Rev., 183, 223
Cunningham, C. T. 1975, ApJ, 202, 788

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