We construct the $N = 2$, $D = 9$ supergravity theory up to the quartic fermionic terms and derive the supersymmetry transformation rules for the fields modulo cubic fermions. We consider a class of $p$-brane solutions of this theory, the stainless $p$-branes, which cannot be isotropically oxidized into higher dimensions. The new stainless elementary membrane and elementary particle solutions are found. It is explicitly verified that these solutions preserve half of the supersymmetry.
1 Introduction

The recent progress in the understanding of M-theory revived the interest in the supergravity theories in diverse dimensions. Much work has been done in constructing and classifying the \(p\)-brane solutions to these theories. However, the extended supergravity in nine dimensions has not yet been fully constructed and investigated. Purely bosonic \(N = 2, D = 9\) action in the context of type-II S- and T-duality symmetries has been discussed in [1, 2], but the full action of the theory and the supersymmetry transformation rules has not appeared in the literature. The goal of this paper is to fill this gap and present an explicit construction of the \(N = 2, D = 9\) supergravity up to the quartic fermionic terms and provide an exhaustive classification of stainless \(p\)-brane solutions of this theory.

With few exceptions, the lower dimensional supergravity theories can be obtained by dimensional reduction of the eleven dimensional Cremmer-Julia-Scherk (CJS) supergravity [3]. Some examples of such exceptions in \(D = 10\) are provided by type IIB and massive type IIA supergravity theories [4, 5]. Dimensional reduction of CJS action, apart from massless supergravities, can also give massive theories in \(D \leq 8\) [6, 7]. As one descends through the dimensions to obtain lower dimensional supergravities, a plethora of isotropic \(p\)-brane solutions arises [8, 9, 10]. Solutions of the dimensionally reduced theory are also solution of the higher-dimensional theory, however, in higher dimension these solutions may or may not exhibit isotropicity. The solutions which cannot be isotropically lifted to the higher dimension and, therefore, cannot be viewed as \(p\)-brane solutions in the higher dimension, are called stainless solutions [11]. The extended \(N = 2\) supergravity in nine dimensions can be truncated to \(N = 1\) supergravity whose stainless solutions consist of an elementary particle and a solitonic 5-brane [11]. It turns out that, whilst the elementary particle remains stainless in \(N = 2\) theory, the solitonic 5-brane becomes rusty and can be obtained from the type IIA \(D = 10\) solitonic 6-brane. We explain this phenomenon by employing a two-scalar 5-brane solution of \(N = 2\) supergravity.

The paper is organised as follows. In sections 2, using the ordinary Scherk-Schwarz dimensional reduction procedure [3], we obtain the bosonic Lagrangian of \(N = 2, D = 9\) supergravity theory by dimensionally reducing the eleven-dimensional CJS Lagrangian from eleven directly to nine dimensions. It is of interest to note that in \(D = 9\), unlike \(D = 8\) case [12], nontrivial group manifolds do not arise. In section 3, we perform an analogous dimensional reduction for fermions and obtain the fermionic part of the nine-dimensional supergravity up to quartic fermions. The supersymmetry transformation rules for the bosonic and fermionic fields, modulo trilinear fermions, are derived in section 4. In section 5, stainless solutions to the obtained \(N = 2, D = 9\) supergravity are analysed. New stainless elementary
particle and elementary membrane solutions are found, and it is shown that they preserve half of the supersymmetry. A stainless solitonic 6-brane is also discussed. It is noted that if one is to include type IIB chiral supergravity into consideration, the solitonic 6-brane and the elementary membrane solutions become rusty and can be treated as descendants of the type IIB solitonic 7-brane and self-dual 3-brane in $D = 10$.

2 Bosonic Sector

The bosonic part of the $N = 1$, $D = 11$ supergravity Lagrangian is given by

$$
\mathcal{L} = \frac{\hat{e}}{4\kappa^2} \hat{R}(\omega) - \frac{\hat{e}}{48} \hat{F}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \hat{F}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} + \frac{2\kappa}{(12)^4} \epsilon_{\hat{\mu}_1 \cdots \hat{\mu}_{11}} \hat{F}_{\hat{\mu}_1 \cdots \hat{\mu}_{11}} \hat{A} \cdots \hat{A}_{\hat{\mu}_{11}},
$$

(1)

where $\hat{e} = \text{det}(\hat{e}^{\hat{r}}_{\hat{\mu}})$, and $\hat{F}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$ is the field strength associated with the gauge field $\hat{A}_{\hat{\mu}\hat{\rho}\hat{\sigma}}$

$$
\hat{F}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} = 4\partial_{[\hat{\mu}} \hat{A}_{\hat{\rho}\hat{\sigma} \hat{\nu}]}.
$$

(2)

Here $\hat{\mu}, \hat{\nu}, \cdots = 0, 1, \cdots 10$ are the world indices and the square brackets represent the antisymmetrization with the unit strength. We take the metric to be mostly positive and perform the dimensional reduction in the space-like direction.

The Riemann tensor and the curvature scalar are as follows

$$
\hat{R}^{\hat{r}}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} = \partial_{\hat{\mu}} \hat{\omega}^{\hat{r}}_{\hat{\nu}\hat{\rho}\hat{\sigma}} + \hat{\omega}^{\hat{r}}_{\hat{\mu}\hat{t}} \hat{\omega}^{\hat{t}}_{\hat{\nu}\hat{\rho}\hat{\sigma}} - (\hat{\mu} \leftrightarrow \hat{\nu}),
$$

(3)

$$
\hat{R} = \hat{e}^{\hat{\mu}}_{\hat{\nu}} \hat{e}^{\hat{\rho}}_{\hat{\sigma}} \hat{R}^{\hat{r}}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \eta_{\hat{r}\hat{r}}.
$$

(4)

where $\hat{r}, \hat{s}, \cdots = 0, 1, \cdots 10$ denote the eleven-dimensional flat indices.

The spin connection is defined by

$$
\hat{\omega}_{\hat{r}\hat{s}}^{\hat{t}} = \frac{1}{2} (\hat{e}^{\hat{t}}_{\hat{\mu}} \hat{e}^{\hat{r}}_{\hat{s}} - \hat{e}^{\hat{t}}_{\hat{s}} \hat{e}^{\hat{r}}_{\hat{\mu}}) \partial_{\hat{\mu}} \hat{e}^{\hat{t}}_{\hat{\nu}}.
$$

(5)

We shall now perform the ordinary Scherk-Schwarz dimensional reduction of CJS Lagrangian (1) directly to $D = 9$ dimensions. We begin by dimensionally reducing the Einstein-Hilbert term in (1). It is convenient first to consider reduction from an arbitrary $D + d$ to $D$ dimensions, and then apply obtained formulas to our case where $D + d = 11$ and $D = 9$.

The Lorentz invariance of the supergravity theory in $D + d$ dimensions, enables one to cast the vielbein into the triangular form

$$
\hat{e}^{\hat{r}}_{\hat{\mu}} = \left( \begin{array}{cc} \hat{e}^{\mu}_{\hat{\mu}} & \hat{A}^{\alpha}_{\hat{\mu}} \\ 0 & \hat{e}^{\alpha}_{\alpha} \end{array} \right).
$$

(6)
Here $\mu, r = 0, 1, \ldots, D - 1$; $\alpha, i = 1, \ldots, d$, the hatted indices belong to $(D + d)$-dimensional space, and $\hat{A}_\mu^i$ are the vector gauge fields that give rise to 2-index field strengths in the dimensionally reduced theory.

Then the inverse vielbein is given by

$$\hat{e}_r^\mu = \begin{pmatrix} \hat{e}_r^\mu & -\hat{A}_r^\alpha \\ 0 & \hat{e}_i^\alpha \end{pmatrix},$$

(7)

where internal indices are raised and lowered by the metric

$$\hat{g}_{\alpha\beta} = \hat{e}_i^\alpha \hat{e}_j^\beta \delta_{ij}.$$  

(8)

The components of the spin-connection $\hat{\omega}_{rsi}$ are given by [6]

$$\hat{\omega}_{rsj} = \frac{1}{2} \hat{e}_j^a \hat{F}_{rs}^\alpha,$$

$$\hat{\omega}_{rij} = \frac{1}{2} \hat{e}_i^a \hat{e}_r^\mu \partial_\mu \hat{e}_j^a - (i \leftrightarrow j),$$

$$\hat{\omega}_{jrs} = -\hat{\omega}_{rsj},$$

$$\hat{\omega}_{irj} = -\frac{1}{2} \hat{e}_i^a \hat{e}_j^\beta \hat{e}_r^\mu \partial_\mu \hat{g}_{\alpha\beta},$$

(9)

where $\omega_{rst}$ is the torsion-free spin connection in $D$ dimensions and $\hat{F}_{rs}^\alpha = e_r^\nu e_s^\mu \hat{F}_{\mu\nu}^\alpha$ with

$$\hat{F}_{\mu\nu}^\alpha = \partial_\mu \hat{A}_\nu^\alpha - \partial_\nu \hat{A}_\mu^\alpha.$$  

(10)

Using (9), the following general formula for the dimensional reduction of the Einstein-Hilbert Lagrangian can be obtained [2, 6]:

$$\int d^{D+d}x \hat{e} R = \int d^Dx (det \hat{e}_r^\mu) \delta \left[ R - \frac{1}{4} \hat{g}_{\alpha\beta} \hat{F}_{\mu\nu}^\alpha \hat{F}_{\mu\nu}^\beta + \frac{1}{2} \hat{g}_{\mu\nu} \partial_\mu \hat{g}_{\alpha\beta} \partial_\nu \hat{g}_{\alpha\beta} + \hat{g}_{\mu\nu} \partial_\mu \ln \delta \partial_\nu \ln \delta \right],$$

(11)

(12)

where $\delta = det(\hat{e}_r^i)$ and $R$ is a Ricci scalar in $D$ dimensions.

We perform the following rescaling [6]

$$\hat{e}_r^\mu = \delta^\gamma e_r^\mu,$$

$$\hat{e}_i^\alpha = \delta^\frac{1}{2} \hat{L}_i^\alpha,$$

$$\hat{A}_\mu^\alpha = 2\kappa \hat{A}_\mu^\alpha,$$

(13)

where $\hat{L}_i^\alpha$ is a unimodular matrix $det\hat{L}_i^\alpha = 1$, and $\gamma$ is a free parameter that determines an exponential prefactor of the Einstein-Hilbert term in a lower dimensional theory.

As a result of rescaling, the vielbeins are brought to the following form

$$\hat{e}_r^{\hat{\mu}} = \begin{pmatrix} \delta^\gamma e_r^\mu & 2\kappa \delta^{\frac{1}{2}} \hat{L}_i^\alpha L_i^\alpha \\ 0 & \delta^{\frac{1}{2}} \hat{L}_i^\alpha \hat{L}_i^\alpha \end{pmatrix}, \quad \hat{e}_r^{\hat{\mu}} = \begin{pmatrix} \delta^{-\gamma} e_r^\mu & -2\kappa \delta^{-\gamma} \hat{A}_\mu^\alpha \\ 0 & \delta^{-\frac{1}{2}} \hat{L}_i^\alpha \hat{L}_i^\alpha \end{pmatrix}.$$  

(14)
Under the rescaling of the type (13), the metric in $D$ dimensions rescales as $\hat{g}_{\mu\nu} = \delta^{2\gamma} g_{\mu\nu}$, and a Ricci scalar changes as

$$R \rightarrow \delta^{-2\gamma} \left[ R - 2\gamma(D - 1)g^{\mu\nu} \nabla_\mu \nabla_\nu \ln \delta - \gamma^2(D - 1)(D - 2)g^{\mu\nu} \nabla_\mu \ln \delta \nabla_\nu \ln \delta \right].$$

Using (12) and (15), we obtain the expression for the dimensional reduction of the Einstein-Hilbert action from $D + d$ to $D$ dimensions (for $d > 1$) that generalises the result of ref. [6] for the arbitrary value of parameter $\gamma$

$$\int d^{D+d}x \hat{e} \hat{R} = \int d^Dx e^{\delta(D-2)+1} \left[ R - \kappa^2 \delta^{-2\gamma + \frac{2}{d}} g_{\alpha\beta} F^{\alpha\mu} F_{\mu\beta} + \beta(\gamma, D, d) g^{\mu\nu} \partial_\mu \ln \delta \partial_\nu \ln \delta + \frac{1}{2} g^{\mu\nu} \partial_\mu g_{\alpha\beta} \partial_\nu g^{\alpha\beta} \right],$$

where $\beta \equiv \gamma^2(D - 1)(D - 2) + 2\gamma(D - 1) + 1 - \frac{1}{d}$.

Using (13), the components of the spin connection are obtained [6]

$$\hat{\omega}_{rst} = \delta^{-\gamma} \left[ \omega_{rst} + \gamma \eta_{rs} \partial_t \ln \delta - \gamma \eta_{rt} \partial_s \ln \delta \right],$$
$$\hat{\omega}_{rsj} = \kappa \delta^{-2\gamma + \frac{2}{d}} F_{rsj},$$
$$\hat{\omega}_{rij} = \delta^{-\gamma} Q_{rij},$$
$$\hat{\omega}_{jrs} = -\hat{\omega}_{rsj},$$
$$\hat{\omega}_{ijr} = \delta^{-\gamma} (2P_{rij} + \frac{4}{\gamma} \delta_{ij} \partial_r \ln \delta),$$
$$\hat{\omega}_{ijk} = 0,$$ (18)

where $P_{rij}$ is symmetric and traceless and $Q_{rij}$ is antisymmetric and defined as

$$P_{rij} = \frac{1}{2} L^i_B \partial_r L_{aj} + (i \leftrightarrow j), \quad Q_{\mu ij} = \frac{1}{2} L^0_B \partial_\mu L_{0j} - (i \leftrightarrow j).$$ (19)

We now turn to CJS Lagrangian (1) and dimensionally reduce it to nine dimensions by applying the formulas (16)-(19) with $D + d = 11$ and $D = 9$. One can identify nine-dimensional gauge fields in terms of the eleven-dimensional gauge field as follows

$$A_{\mu\nu} = \hat{A}_{\mu\nu},$$
$$A_{\mu\alpha} = \hat{A}_{\mu\alpha},$$
$$A_{\mu\alpha\beta} = \hat{A}_{\mu\alpha\beta}.$$ (20)

It should be remarked here that there is an element of arbitrariness in defining the gauge field in $D = 9$. Let us recall that CJS Lagrangian is
invariant under the general coordinate transformation \([6]\). Upon compactification, this symmetry becomes
\[ D = 9 \]
general coordinate transformation and a set of the \([U(1)]^2\) reparametrization transformations. Denoting a parameter in \(D = 11\) by \(\xi^\mu\), one can show that under the \(\xi^\alpha\)-reparametrization transformation, the gauge fields defined in \([20]\) transform noncovariantly involving derivatives of the parameter \(\xi^\alpha\). Nevertheless, the supersymmetry transformation rules and the dimensional reduction procedure is somewhat simplified by this choice \([8, 11]\). In order to obtain results in terms of the covariant gauge fields, used in \([6]\), one has to change conventions by identifying the nine-dimensional gauge fields as
\[ B_{rst}^{A} = \delta^3_{\gamma} A_{rst}^{A}, \]
\[ B_{rsi}^{A} = \delta^2_{\gamma} A_{rsi}^{A}, \]
\[ B_{rij}^{A} = \delta^1_{\gamma} A_{rij}^{A}. \]

Dimensionally reducing the field strength, we find
\[ \hat{F}_{rstu} = \delta^{-4\gamma} \left( F_{rstu} - 2k F_{rsti} A_{i}^{A} + 4k^2 F_{rsji} A_{j}^{A} A_{i}^{A} \right) \equiv \delta^{-4\gamma} F'_{rstu} , \]
\[ \hat{F}_{rsti} = \delta^{-3\gamma-\frac{1}{2}} \left( F_{rsti} - 4k F_{rsji} A_{j}^{A} \right) \equiv \delta^{-3\gamma-\frac{1}{2}} F'_{rsti} , \]
\[ \hat{F}_{rsij} = \delta^{-2\gamma-1} F_{rsij} , \]
\[ \hat{F}_{rijk} = 0 , \]
\[ \hat{F}_{ijkl} = 0 . \]

Above rules reflect the fact that the field strengths in \(D = 9\) do not transform covariantly under the \(U(1)\) transformations arising from eleven-dimensional general coordinate transformation. This \(U(1)\) reparametrization invariance should not be confused with another \(U(1)\) symmetry which is a gauge symmetry of the antisymmetric field in \(D = 9\).

In \(D = 11\), the \(U(1)\) gauge transformation is given by \([8]\)
\[ \delta_{\lambda} A_{\mu\nu\rho}^{\phi} = 3\partial_{[\mu} A_{\nu\rho]}^{\phi} , \]
which upon reduction gives the following \(U(1)\) gauge transformations in \(D = 9\):
\[ \delta_{\lambda} A_{\mu\nu}^{\rho} = 3\partial_{[\mu} A_{\nu\rho]}^{\rho} , \]
\[ \delta_{\lambda} A_{\mu\nu}^{\alpha} = 2\partial_{[\mu} A_{\nu\alpha]}^{\alpha} , \]
\[ \delta_{\lambda} A_{\mu\alpha\beta}^{\phi} = \partial_{[\mu} A_{\alpha\beta]}^{\phi} . \]

Notice that since we are performing the ordinary dimensional reduction, none of the fields and parameters in \(D = 9\) depend on extra compactification coordinates, in other words, all \(\partial_{\alpha}\) derivatives are identically zero.
Turning to the kinetic term for the antisymmetric tensor field in (1) and performing the straightforward reduction of the field strengths using (22), we obtain

\[-\frac{2\kappa}{(12)^4\varepsilon^{\mu_1\cdots\mu_9}g^{\alpha\beta}} \left( 3F_{\mu_1\mu_2\mu_3\mu_4}F_{\mu_5\mu_6\mu_7\mu_8}A_{\alpha\mu_9\beta} + 24F_{\mu_1\mu_2\mu_3\mu_4}F_{\mu_5\mu_6\gamma\alpha}A_{\mu_7\mu_8\beta} - 16F_{\mu_1\mu_2\mu_3\mu_4}F_{\mu_5\mu_6\beta}A_{\alpha\mu_7\mu_8\beta} + 12F_{\mu_1\mu_2\mu_3\mu_4}F_{\mu_5\mu_6\alpha\beta}A_{\mu_7\mu_8\alpha} \right) \right].

(25)

The interaction term in the bosonic Lagrangian can also be dimensionally reduced by decomposing summation in the hatted indices

\[\varepsilon^{\mu_1\cdots\mu_9}\varepsilon^{\alpha\beta} = \varepsilon^{\mu_1\cdots\mu_9\alpha\beta}.\]

In order to have the canonical normalization of a scalar field kinetic term in (16), we introduce a dilaton in D dimensions

\[\delta = e^{\sqrt{-\kappa/7}\phi}.\]

(27)

and choose \(\gamma = -\frac{1}{D-2}\) to bring the Einstein-Hilbert term of the reduced theory to the standard form. A generalized method of dimensional reduction for this particular choice of parameters was discussed in [6].

Combining (16), (25), (26), and choosing \(\gamma = -\frac{1}{D-2} = -\frac{1}{7}\) and \(\delta = e^{\sqrt{-\kappa/7}\phi}\), we get the bosonic action of the \(N = 2, D = 9\) supergravity

\[S = \int d^9x e \left[ \frac{1}{12}\kappa R - \frac{1}{4\sqrt{\kappa}} g_{\alpha\beta}F^{\mu\nu\alpha\beta}F_{\mu\nu} - \frac{1}{2\kappa} (\partial_\mu \phi)^2 - \frac{1}{2\kappa} P_{\mu\nu} P^{\mu\nu} - \frac{1}{2\kappa} e^{-\sqrt{\kappa/7}\phi} F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} - \frac{1}{2\kappa} e^{-\sqrt{\kappa/7}\phi} g_{\alpha\beta} F_{\mu\nu\rho}^\alpha F^{\mu\nu\rho\beta} - \frac{1}{2\kappa} e^{-\sqrt{\kappa/7}\phi} F_{\mu\nu\alpha\beta}F^{\mu\nu\alpha\beta} + \mathcal{L}_{F_{\alpha\beta}} \right],\]

(28)

where \(\mathcal{L}_{F_{\alpha\beta}}\) is given by (27).

The bosonic action (28) is invariant under the Abelian \(U(1)\) gauge transformations (24). We shall consider the supersymmetry and Lorentz invariance of the full \(N = 2, D = 9\) action in section 4 where the supersymmetry transformation rules for the fields will be derived.

Using the general expression for dimensional reduction of the Einstein-Hilbert term (14), one can rewrite the action (28) in the \(p\)-brane metric which appears naturally in the \(p\)-brane \(\sigma\)-models and is related to the canonical gravitational metric in \(D\) dimensions as \(g_{\mu\nu} (p - \text{brane}) = e^{a/(p+1)} g_{\mu\nu}\), with \(\Delta\)

\[a^2 = \Delta = \frac{2(p+1)d}{D-2},\]

(29)
where \( \tilde{d} = D - p - 3 \) and \( \Delta \) in maximal supergravity theories is equal to 4. Then for the \( p \)-brane metric, the parameter \( \gamma \) is defined by the equation:

\[
2\kappa(p + 1)(7\gamma + 1)(-2/\beta)^{1/2} = -(D - 2)a. \tag{30}
\]

In the following, we use the canonical value of \( \gamma \), which in \( D = 9 \) is \(-1/7\).

## 3 Fermionic sector

We shall now compactify the fermionic part of the eleven-dimensional supergravity Lagrangian which reads \([3]\)

\[
\mathcal{L}_F = \mathcal{L}_F^{(1)} + \mathcal{L}_F^{(2)} + \text{quartic fermions}, \tag{31}
\]

\[
\mathcal{L}_F^{(1)} = \frac{\kappa}{2} \bar{\psi}_i \hat{\Gamma}^{\hat{r}\hat{s}\hat{t}} \hat{D}_{\hat{s}}(\hat{\omega}) \hat{\psi}_i, \tag{32}
\]

\[
\mathcal{L}_F^{(2)} = \frac{\kappa \hat{e}}{98} \left( \bar{\psi}_i \hat{\Gamma}^{\hat{r}\hat{s}\hat{t}\hat{u}\hat{v}} \hat{\psi}_i + 12 \bar{\psi}_i \hat{\Gamma}^{\hat{r}\hat{u}\hat{v}} \hat{\psi}_i \hat{F}_{\hat{t}\hat{u}\hat{v}} \right), \tag{33}
\]

where the covariant derivative is given by

\[
\hat{D}_{\hat{s}} \hat{\psi}_i = \partial_{\hat{s}} \hat{\psi}_i + \frac{1}{4} \hat{\omega}_{\hat{s}\hat{u}\hat{v}} \hat{\Gamma}^{\hat{u}\hat{v}} \hat{\psi}_i + \hat{\omega}_{\hat{s}\hat{u}} \hat{\Gamma}^{\hat{u}\hat{v}} \hat{\psi}_i. \tag{34}
\]

The covariant derivative (34) commutes with the \( \Gamma \)-matrices: \([\hat{D}_s, \hat{\Gamma}_r] = 0\), which obey the algebra

\[
[\hat{\Gamma}_r, \hat{\Gamma}_s] = 2 \eta_{rs}, \tag{35}
\]

where \( \eta_{rs} = \text{diag}(- + + \cdots +) \).

The unhatted fermions and \( \Gamma \)-matrices are defined as follows

\[
\psi_r = \hat{\psi}_r, \quad \psi_i = \hat{\psi}_i, \quad \Gamma_r = \hat{\Gamma}_r, \quad \Gamma_i = \hat{\Gamma}_i. \tag{36}
\]

The fermions and the \( \Gamma \)-matrices with the world indices are defined by \( \hat{\Gamma}_\mu = \hat{e}_\mu{}^\hat{r} \hat{\Gamma}_\hat{r} \) and \( \hat{\psi}_\mu = \hat{e}_\mu{}^\hat{r} \hat{\psi}_\hat{r} \), or using (14) and (36),

\[
\hat{\Gamma}_\mu = e^{-2\kappa \gamma_{\mu\phi}} \Gamma_\mu + 2\kappa e^{\frac{1}{36} \gamma_{\mu\phi}} A^i \Gamma_i, \tag{37}
\]

\[
\hat{\psi}_\mu = e^{-2\kappa \gamma_{\mu\phi}} \psi_\mu + 2\kappa e^{\frac{1}{36} \gamma_{\mu\phi}} A^i \psi_i. \tag{38}
\]

In order to bring the reduced Lagrangian to the canonical form, we have to redefine the fermionic fields

\[
\psi_r \rightarrow \left( \psi_r - \frac{1}{7} \Gamma_7 \chi_i \right) e^{\frac{1}{36} \gamma_{\phi\phi}}, \quad \psi_i \rightarrow \chi_i e^{\frac{1}{36} \gamma_{\phi\phi}}. \tag{39}
\]
In the kinetic term for the fermions, for example, the exponential in (39) cancels against the corresponding factors coming from the determinant $\hat{\epsilon}$ and the covariant derivative $\hat{D}_s$, and the shift of the fermionic field ensures that the Lagrangian is diagonalised. In the arbitrary dimension $D$, one has to make the following redefinitions

$$\psi_\rho \longrightarrow \left( \psi_\rho - \frac{1}{D-2} \Gamma_\rho \Gamma^i \chi_i \right) \delta^{-\frac{1}{2}(\gamma(D-1)+1)}, \quad (40)$$

$$\psi_i \longrightarrow \chi_i \delta^{-\frac{1}{2}(\gamma(D-1)+1)}. \quad (41)$$

Substituting (34), (36), (39) into (32) and (33), and using the identity

$$\kappa e \psi^i \Gamma^j \psi^\rho = - \frac{1}{2} \Gamma^j \Gamma^\rho \Gamma^i \psi^\sigma + \psi^\rho \Gamma^\mu \Gamma_\sigma \left( \frac{1}{7} \Gamma^j \Gamma^k + \delta^{jk} \right) \chi_k + \frac{1}{49} \chi_i \Gamma^\mu \left( -23 \Gamma^k \delta^{ij} + 59 \Gamma^j \delta^{ik} \right) \chi_k \quad (42)$$

we derive the fermionic part of the $N = 2, D = 9$ supergravity Lagrangian

$$L_F^{(1)} = \frac{\kappa e}{2} \bar{\psi}_\mu \Gamma^{\rho \sigma} \mathcal{D}_\nu \psi_\rho + \frac{\kappa e}{2} \bar{\chi}_i \Gamma^\mu \left( \frac{1}{7} \Gamma^i \Gamma^j + \delta^{ij} \right) \mathcal{D}_\mu \chi_j + \frac{\kappa e}{24} e^{\frac{1}{2} \kappa \phi} \mathcal{F}_{\mu \nu} \left[ - \frac{1}{2} \bar{\psi}^\rho \Gamma_{[\rho} \Gamma^\sigma \Gamma_{\sigma]} \psi^\sigma + \bar{\psi}^\rho \Gamma^\mu \Gamma^\nu \Gamma^\lambda \left( \delta^{ij} - \frac{2}{7} \Gamma^i \Gamma^j \right) \chi_k \right], \quad (43)$$

$$L_F^{(2)} = \frac{\kappa e}{96} e^{\frac{1}{2} \kappa \phi} \mathcal{F}_{\mu \nu \rho \sigma} \left[ \bar{\psi}^\lambda \Gamma_{[\rho} \Gamma^{\mu \nu \rho \sigma} \Gamma_{\sigma]} \psi^\tau + \frac{6}{7} \bar{\psi}^\lambda \Gamma^{\mu \nu \rho \sigma} \Gamma^i \chi_i \right. \left. + \bar{\chi}_i \Gamma^{\mu \nu \rho \sigma} \left( \frac{11}{49} \Gamma^i \Gamma^j - \delta^{ij} \right) \chi_j \right]$$

$$+ \frac{\kappa e}{24} e^{\frac{1}{2} \kappa \phi} \mathcal{F}_{\mu \nu \rho \sigma} \left[ - \bar{\psi}^\lambda \Gamma_{[\rho} \Gamma^{\mu \nu \rho \sigma} \Gamma_{\sigma]} \psi^\sigma + 2 \bar{\psi}^\lambda \Gamma^{\mu \nu \rho \sigma} \Gamma_\lambda \left( \delta^{ij} - \frac{2}{7} \Gamma^i \Gamma^j \right) \chi_k \right]$$

$$+ \frac{\kappa e}{16} e^{\frac{1}{2} \kappa \phi} \mathcal{F}_{\mu \nu \rho \sigma} \left[ \bar{\psi}^\lambda \Gamma_{[\rho} \Gamma^{\mu \nu \rho \sigma} \Gamma_{\sigma]} \psi^\sigma - \frac{24}{7} \bar{\psi}^\lambda \Gamma^{\mu \nu \rho \sigma} \Gamma^i \chi_i \right] + 2 \bar{\chi}_i \Gamma^{\mu \nu} \left( \frac{16}{49} \Gamma^i \Gamma^j + \delta^{ij} \right) \chi_j \quad (44)$$

where the covariant derivatives $\mathcal{D}_\mu \psi_\nu = e_\mu \epsilon_\nu \xi \mathcal{D}_s \psi_\tau$ and $\mathcal{D}_\mu \chi_i = e_\mu \epsilon_\nu \xi \mathcal{D}_s \chi_i$ are defined as

$$\mathcal{D}_s \psi_\tau = \partial_\tau \psi_\tau + \frac{1}{4} \omega_{uv} \Gamma^{uv} \psi_\tau + \omega_{st} \psi_u + \frac{1}{4} q_{sij} \Gamma^{ij} \psi_t$$

$$\mathcal{D}_s \chi_j = \partial_\tau \chi_j + \frac{1}{4} \omega_{uv} \Gamma^{uv} \chi_j + \frac{1}{4} q_{sik} \Gamma^{ik} \chi_j + q_{sij} \chi_k. \quad (45)$$

In deriving (43) and (44), we have used the following flipping property of Majorana spinors in nine dimensions

$$\bar{\psi}^\Gamma_{\tau^1 \cdots \tau^n} \psi = (-1)^n \bar{\eta}^\Gamma_{\tau^1 \cdots \tau^n} \psi. \quad (47)$$
4 Supersymmetry transformations

In this section, we obtain the supersymmetry transformation laws in nine dimensions. In order to preserve the triangular form of the vielbein \( \hat{e}_\alpha^r = 0 \), one has to consider combined Lorentz and supersymmetry transformation laws. As we shall see, the requirement of the off-diagonal part of the vielbein be zero, imposes an additional constrain on the Lorentz group parameters. This, in its turn, affects the supersymmetry transformation laws of the fields.

Combining the supersymmetry and the Lorentz transformation laws in eleven dimensions, we have [3]

\[
\delta \hat{e}_{\hat{\mu} \hat{\nu}} = -\bar{\eta} \Gamma_{\hat{\mu} \hat{\nu}} \psi^{\hat{\mu}}, \\
\delta \hat{A}_{\hat{\mu} \hat{\nu}} = -\frac{1}{2} \bar{\eta} \Gamma_{\hat{\mu} \hat{\nu}} \psi^{\hat{\mu}}, \\
\delta \hat{\psi}_{\hat{\mu}} = \hat{D}_{\hat{\mu}} \eta - \frac{1}{144} \left( \Gamma_{\hat{\mu} \hat{\nu}} \psi_{\hat{\mu}} + 8 \Gamma_{\hat{\mu} \hat{\nu}} \delta_{\hat{\mu}}^{\hat{\nu}} \right) \hat{F}_{\hat{\mu} \hat{\nu}} \eta + \frac{1}{4} \Lambda_{\hat{\mu} \hat{\nu} \hat{\rho} \hat{\sigma}} (8 \Gamma_{\hat{\mu} \hat{\nu}} \delta_{\hat{\mu}}^{\hat{\nu}}),
\]

where \( \eta \) is a supersymmetry parameter, \( \hat{D}_{\hat{\mu}} \eta \) is a torsion free covariant derivative, and \( \Lambda_{\hat{\mu} \hat{\nu} \hat{\rho} \hat{\sigma}} = -\Lambda_{\hat{\rho} \hat{\sigma} \hat{\mu} \hat{\nu}} \) is the Lorentz group parameter.

Redefining the fermionic fields according to (39) and taking different projections of (48)-(50), we obtain the corresponding transformations in nine dimensions. The off-diagonal (\( r\alpha \)) projection of (48) fixes the (\( ri \)) component of the Lorentz parameter in terms of the supersymmetry parameter as follows

\[
\Lambda_{ri} = \bar{\varepsilon} \Gamma_r \chi_i,
\]

where \( \varepsilon = \eta e^{\frac{1}{\sqrt{7}} \kappa \phi} \). Eq.(51) is the necessary condition for the triangular gauge \( \hat{e}_\alpha^r = 0 \) to be preserved. Naively, one might expect that the \( \xi^\mu \)-reparametrization invariance alone would be sufficient to maintain the triangular gauge of the vielbein. However, the explicit calculation shows that the reparametrization transformation drops out of the (\( r\alpha \)) projection of (48) altogether and, therefore, cannot modify constrain (51) on the Lorentz parameter. Substituting (51) into the (\( r\mu \)) projection of (48), we obtain the vielbein transformation law

\[
\delta e_{\mu r} = \frac{2}{3} \sqrt{7} \kappa \delta \phi - \frac{1}{2} \bar{\varepsilon} \Gamma_{\mu} \psi_{\mu} + \frac{1}{4} \bar{\varepsilon} \Gamma_{\mu} \chi_{\mu} \delta_{\mu r} + \Lambda'_{\mu r} ,
\]

where symmetrization is performed with the unit strength and \( \Lambda'_{\mu r} \) is the redefined local \( SO(1,8) \) Lorentz transformation parameter

\[
\Lambda'_{\mu r} = \Lambda_{\mu r} - \varepsilon \Gamma_{[\psi_{\mu r]} + \frac{1}{4} \bar{\varepsilon} \Gamma_{\mu} \chi_{\mu} \delta_{\mu r} .
\]

Remaining two projections of (48) give the transformation rules for the vector field \( \hat{A}_{\mu}^i \) and the internal vielbein \( \hat{L}_{\alpha i}^\mu \):

\[
\delta \hat{A}_{\mu}^i = -\frac{1}{2} \bar{\varepsilon} \kappa \hat{A}_{\mu}^i \delta \phi - \frac{1}{2} \bar{\varepsilon} e^{-\frac{3}{2} \bar{\varepsilon}} \left( \Gamma_i \psi_{\mu} + \Gamma_{\mu} (\delta_{ij} + \frac{1}{4} \Gamma^j \chi_j) \right)
\]
where the redefined \( SO(2) \) Lorentz parameter is

\[
\Lambda'_{ij} = \Lambda_{ij} - \bar{\varepsilon}\Gamma[i\chi_j].
\]

Using (34), (36) and (39), and performing the straightforward reduction of (53), one finds the transformation law for \( \delta\phi \), which upon substitution into (52), puts the vielbein transformation law into canonical form. Suppressing the \( \Lambda'_{rs} \) and \( \Lambda'_{ij} \) transformations, we obtain the supersymmetry transformation rules for the bosonic fields:

\[
\delta e^r_{\mu} = -\bar{\varepsilon}\Gamma^r\psi_{\mu},
\]

\[
\delta\phi = -\frac{1}{2\kappa}\bar{\varepsilon}\Gamma^i\chi_i,
\]

\[
L_j^a\delta L_{ai} = -\bar{\varepsilon}\left(\Gamma(i\chi_j) - \frac{i}{2}\delta_{ij}\Gamma_k\chi_k\right),
\]

\[
\delta A_i^a = A_i^a \left(L_j^a\delta L_{ai} - \frac{\bar{\varepsilon}}{2\kappa}\varepsilon\Gamma^{i\phi} \left(\Gamma_i^j\psi_{\mu} + \Gamma_{\mu}(\delta_{ij} + \frac{i}{2}\Gamma_j^j)\chi_j\right)\right),
\]

\[
\delta A_{\mu\alpha\beta} = -\frac{1}{2}\varepsilon\varepsilon^\mu\varepsilon^\kappa\varepsilon^\gamma (\Gamma_\alpha\Gamma_\gamma A_{i\mu}^a + 3\kappa\varepsilon^\mu\varepsilon^\alpha\varepsilon^\gamma A_{i\mu}^a \Gamma_\gamma^a),
\]

\[
\delta A_{\mu\nu\rho} = -\frac{3}{2}\varepsilon\varepsilon^\mu\varepsilon^\nu\varepsilon^\rho (\Gamma_{[\mu\nu]}) + \frac{3}{4}\varepsilon\varepsilon^\mu\varepsilon^\nu\varepsilon^\rho (\Gamma_\alpha\Gamma_\gamma A_{i\mu}^a \Gamma_\gamma^a),
\]

Using (34), (36) and (39), and performing the straightforward reduction of (50), one obtains the transformation laws for the fermionic fields:

\[
\delta\psi_{\mu} = \mathcal{D}_{\mu\varepsilon} - \frac{1}{4\kappa}\varepsilon^\mu\varepsilon^\nu\varepsilon^\phi F_{\nu\rho}^\phi \left(\Gamma_{\mu\nu\rho} + 12\Gamma_{\mu\rho}\delta^\nu_{\rho}\right)\Gamma^i\varepsilon
\]

\[
-\frac{1}{4\kappa}\varepsilon^\mu\varepsilon^\nu\varepsilon^\rho F_{\nu\rho\sigma\lambda} \left(\frac{9\Gamma_{\nu\rho\sigma\lambda}}{4} + \frac{43}{4}\Gamma_{\nu\sigma}\delta^\rho_{\lambda}\right)\varepsilon
\]

\[
+\frac{1}{4\kappa}\varepsilon^\mu\varepsilon^\nu\varepsilon^\rho F_{\nu\rho\sigma} \left(\Gamma_{\mu\nu\sigma} + \frac{i}{2}\Gamma_{\mu\sigma}\delta^\nu_{\rho}\right)\Gamma^j\varepsilon
\]

\[
-\frac{1}{4\kappa}\varepsilon^\mu\varepsilon^\nu\varepsilon^\rho F_{\nu\rho\sigma\lambda} \left(\frac{3\Gamma_{\nu\rho}}{4} + 3\Gamma_{\nu\rho}\delta^\rho_{\lambda}\right)\Gamma^{ij}\varepsilon + \text{cubics},
\]
\[
\delta \chi_i = -\frac{i}{4} \kappa e^{\frac{3}{4} \sqrt{7} \kappa \phi} F^i_{\mu \nu} \Gamma^{\mu \nu \varepsilon} + (P_{\mu ij} + \frac{2}{\sqrt{7} \kappa \delta_{ij} \partial_{\mu} \phi}) \Gamma^\mu \Gamma^j \\
- \frac{1}{144} e^{\frac{3}{4} \sqrt{7} \kappa \phi} F'_{\mu \nu \rho \sigma} \Gamma^{\mu \nu \rho \sigma} \Gamma_i \varepsilon + \frac{1}{36} e^{- \frac{1}{4} \sqrt{7} \kappa \phi} F'_{\mu \nu \rho j} \Gamma^{\mu \nu \rho} (\Gamma_{ij} - 2 \delta_{ij}) \varepsilon \\
- \frac{1}{4} e^{- \frac{1}{4} \sqrt{7} \kappa \phi} F'_{\mu \nu j} \Gamma^\mu \Gamma^j \varepsilon + \text{cubics},
\]

where
\[
D_{\mu} \varepsilon = \partial_{\mu} \varepsilon + \frac{i}{4} \omega_{\mu st} \Gamma^{st} \varepsilon + \frac{1}{4} Q_{\mu ij} \Gamma^{ij} \varepsilon.
\]

5 Stainless p-brane solutions

Most supergravity theories in \( D < 11 \) dimensions can be obtained by dimensionally reducing \( D = 11 \) supergravity theory. Therefore, solutions of the dimensionally reduced theory are also solutions of the higher-dimensional theory. However, in higher dimension these solutions may or may not exhibit isotropicity. The solutions which cannot be isotropically lifted to the higher dimension and, therefore, cannot be viewed as p-brane solutions in the higher dimensions, are called stainless solutions \([11]\). In other words, stainless p-branes are genuinely new solutions of the supergravity theory in the given dimension and should not be treated on the same footing as solutions which are descendants of the higher dimensional p-branes.

We first briefly review some of the main results on p-brane solutions \([8, 10, 11]\) and then apply general results to constructing and classifying stainless p-branes in \( N = 2, D = 9 \) theory. The p-brane solutions in general involve the metric tensor \( g_{\mu \nu} \), a dilaton \( \phi \) and an \( n \)-index antisymmetric tensor \( F_{M_1 M_2 \cdots M_n} \). The Lagrangian for these fields takes the form
\[
\mathcal{L} = e^{-1} R - \frac{1}{4} (\partial \phi)^2 - \frac{1}{4} e^{-a \phi} F_n^2,
\]
where \( a \) is a constant given by \([23, 8, 11]\). In \( D = 11 \), the absence of a dilaton implies that \( \Delta = 4 \). The value of \( \Delta \) is preserved under the dimensional reduction procedure and, hence, all antisymmetric tensors in maximal supergravity theories have \( \Delta = 4 \). However, if an antisymmetric tensor used in a particular p-brane solution is formed from a linear combination of the original field strengths, then it will have \( \Delta < 4 \). An example of this is a solitonic 5-brane in \( N = 1, D = 10 \) supergravity considered below.

We shall be looking for isotropic p-brane solutions for which the metric ansatz is given by \([8, 11]\)
\[
ds^2 = e^{2A} dx^\mu dx^\nu \eta_{\mu \nu} + e^{2B} dy^m dy^m,
\]
where \( x^\mu (\mu = 0, \cdots, d - 1) \) are the coordinates of the \((d - 1)\)-brane world volume, and \( y^m \) are the coordinates of the \((9 - d)\)-dimensional transverse space. The functions \( A \) and \( B \), also the dilaton \( \phi \), depend only on \( r = \sqrt{y^m y^m} \).
This ansatz for the metric preserves an $SO(1, d - 1) \times SO(9 - d)$ subgroup of the original $SO(1, 8)$ Lorentz group.

For the elementary $p$-brane solutions, the ansatz for the field strength is given by \cite{8, 11}
\[
F_{m\mu_1\cdots \mu_{n-1}} = \varepsilon_{\mu_1\cdots \mu_{n-1}} \partial_m e^C, \tag{68}
\]
where $\varepsilon_{\mu_1\cdots \mu_{n-1}} \equiv g_{\mu_1\nu_1} \cdots \varepsilon^{\nu_1\cdots}$ with $\varepsilon^{012\cdots} = 1$, and $C$ is a function of $r$ only. The dimension of the brane world volume is $d = n - 1$.

For the solitonic $p$-brane solution, the ansatz for the antisymmetric tensor is given by \cite{8, 11}
\[
F_{m_1\cdots m_n} = \lambda \varepsilon_{m_1\cdots m_n} \frac{y^p}{y^{n+1}}, \tag{69}
\]
where $\lambda$ is a constant and the dimension of the world volume is $d = 9 - n - 1$.

The solutions to the equations of motion obtained from the Lagrangian (66) are given by
\[
A = -\frac{2\tilde{d}}{\Delta (D - 2)} \ln \left(1 + \frac{k}{y^{\tilde{d}}} \right), \quad B = -\frac{d}{d} A, \quad \phi = \frac{7\epsilon a}{d} A, \tag{70}
\]
where
\[
k = \frac{\epsilon \lambda}{2d} \sqrt{\Delta}, \tag{71}
\]
$d = D - d - 2$ and $\epsilon = 1$ ($\epsilon = -1$) for the elementary (solitonic) ansatz. In the solitonic case, the equation of motion for the field strength is automatically satisfied, whilst in the elementary case the function $C$ is given by
\[
e^C = \frac{2}{\sqrt{\Delta}} \left(1 + \frac{k}{y^{\tilde{d}}} \right)^{-1}. \tag{72}
\]

The solutions (70)-(72) are valid for an $n$-index field strength with $n > 1$. When $n = 1$, i.e. $d = 0$, there only exists a solitonic solution described by (74) with $kr^{-\tilde{d}} \to k\log r$ and $\tilde{d} \to 0$.

Stainlesness of a $p$-brane solution crucially depends on a degree of the antisymmetric tensor involved in a solution, and the value of constant $a$ occurring in the exponential prefactor. There are two different situations when a stainless $p$-brane solution may arise in a given dimension. In the first scenario, no $(D + 1)$-dimensional theory contains the necessary field strength for a brane solution. In particular, if in $D$ dimensions the solution is elementary, the $(D + 1)$-dimensional theory must have a field strength of degree one higher than that in $D$ dimensional theory. If it is a solitonic solution, the $(D + 1)$-dimensional theory must contain a field strength of the same degree as in $D$ dimensional theory. In the second case, the required field strength
exists in the \( (D + 1) \)-dimensional theory, but a \( p \)-brane is stainless only if the constant \( \hat{a} \) of a corresponding antisymmetric tensor in \( (D + 1) \) dimensions is not related to the constant \( a \) of the \( D \)-dimensional theory as \[ \hat{a}^2 = a^2 - \frac{2d^2}{(D - 1)(D - 2)} \] (73)

It should be noted that in determining whether or not a particular \( p \)-brane solution is stainless, we restrict our attention only to the supergravity theories which can be obtained from \( D = 11 \) supergravity. This, for example, will lead us to conclude that a solitonic 6-brane and an elementary membrane in the nine-dimensional theory are stainless. However, if we include type-IIB theory into consideration, we will see that these \( p \)-branes are no longer stainless and can be isotropically oxidized to the solitonic 7-brane and the self-dual 3-brane of type IIB supergravity.

In order to consider solutions to the obtained \( N = 2, D = 9 \) supergravity theory, we need to parametrize tensor \( P_{\mu ij} \) in the Lagrangian (28). Since after separating out the determinant, the internal vielbein has only two degrees of freedom left, we introduce two scalar fields, \( \varphi \) and \( E \), and parametrize the vielbeins as follows:

\[
L^\alpha_i = \begin{pmatrix} e^{\kappa \varphi} & \kappa E e^{-\kappa \varphi} \\ 0 & e^{-\kappa \varphi} \end{pmatrix}, \quad L^\alpha_i = \begin{pmatrix} e^{-\kappa \varphi} & -\kappa E e^{-\kappa \varphi} \\ 0 & e^{\kappa \varphi} \end{pmatrix}.
\] (74)

The metric \( g_{\alpha \beta} \) is given by

\[
g_{\alpha \beta} = \begin{pmatrix} e^{2\kappa \varphi} + \frac{1}{4} E^2 e^{-2\kappa \varphi} & \frac{1}{2} e^{\kappa \varphi} \\ \frac{1}{2} e^{-\kappa \varphi} & e^{-2\kappa \varphi} \end{pmatrix}.
\] (75)

The internal metric (73) is not diagonal, therefore, the terms in the Lagrangian containing \( g_{\alpha \beta} \) will not be diagonal as well. To diagonalize the Lagrangian, the following redefinitions have to be made:

\[
F_{MN}^2 + EF_{MN}^1 \to F_{MN}^{(2)}, \quad F_{MN}^1 \to F_{MN}^{(1)},
\]

\[
F_{MNP}^{(2)} + EF_{MNP}^{(1)} \to F_{MNP}^{(2)}, \quad F_{MNP}^{(1)} \to F_{MNP}^{(1)}.
\] (76)

Here and throughout this subsection \( M, N, P = 0, \ldots, 8 \) denote the curved nine-dimensional world volume indices, whilst \( R, S, T = 0, \ldots, 8 \) denote the flat indices.

Then the Lagrangian (28) can be written as

\[
e^{-1} \mathcal{L} = R - \frac{i}{2}(\partial \varphi)^2 - \frac{i}{2}(\partial \varphi)^2 - \frac{1}{4} e^{\frac{3}{4} \varphi} (F_2^{(1)})^2 - \frac{1}{4} e^{\frac{3}{4} \varphi} (F_2^{(2)})^2
\]

\[
- 4 e^{2 \varphi} (H_1)^2 + 4 e^{-\frac{3}{4} \varphi} (F_2^2) - 4 e^{-\frac{3}{4} \varphi} (F_3^{(1)})^2
\]

\[
+ \frac{1}{12} e^{-\frac{3}{4} \varphi} (F_3^{(2)})^2 - \frac{1}{48} e^{\varphi} (F_4')^2 + \mathcal{L}_{FFA}.
\] (77)
where
\[ F^2_n \equiv F_{M_1 M_2 \cdots M_n} F^{M_1 M_2 \cdots M_n} \]
denotes the square of an \( n \)-index field strength, \( H_M = \partial_M E \) and the parameter \( \kappa \) is set to \( 1/2 \).

If in (77) one retains only one field strength and a corresponding dilaton, which for \( F^{(i)}_2 \) and \( F^{(i)} \) is a linear combination of \( \phi \) and \( \varphi \), one arrives at the Lagrangian of the form (77). Then general results described above can be applied to constructing single \( p \)-brane solutions in the given supergravity theory.

It should be noted that for the purposes of finding a purely elementary or a purely solitonic \( p \)-brane solution, the \( FFA \)-term in the Lagrangian and the Chern-Simons modifications of the field strengths can be disregarded due to the fact that the constraints implied by these terms are automatically satisfied in \( D = 9 \). However, in general, for certain \( p \)-brane solutions, the \( L_{FFA} \) term and the Chern-Simons modifications to the field strengths give rise to nontrivial equations in some dimensions (77). Good examples illustrating this point are dyonic \( p \)-branes in \( D = 4 \) and \( D = 6 \) dimensions.

Since our principal interest lies in the stainless \( p \)-brane solutions, we have first to determine which of the \( p \)-branes of \( N = 2, D = 9 \) supergravity are stainless. For this, one recalls that the \( N = 2, D = 10 \) supergravity contains a 2-index field strength, a 3-index field strength and a 4-index field strength with the \( \hat{a}^2 \) values \( 1, \frac{1}{4}, \frac{9}{4} \) respectively. Using the criteria for stainlessness of a \( p \)-brane and the equation (73), we find that the stainless solutions of \( N = 2, D = 9 \) supergravity theory are an elementary particle, an elementary membrane and a solitonic 6-brane. Applying (74), we obtain the metrics for these solutions

\[
\text{particle : } \quad ds^2 = \left( 1 + \frac{k}{r^6} \right)^{-6/7} dt^2 + \left( 1 + \frac{k}{r^6} \right)^{2/7} dy^m dy^m , \quad (78)
\]
\[
\text{membrane : } \quad ds^2 = \left( 1 + \frac{k}{r^4} \right)^{-4/7} dx^\mu dx^\nu \eta_{\mu\nu} + \left( 1 + \frac{k}{r^4} \right)^{3/7} dy^m dy^m , \quad (79)
\]
\[
\text{6-brane : } \quad ds^2 = dx^\mu dx^\nu \eta_{\mu\nu} + \left( 1 + klog r \right)^{3/7} dy^m dy^m . \quad (80)
\]

It is of interest to note that a solitonic 5-brane, which is stainless as a solution to \( N = 1, D = 9 \) supergravity theory, does not remain stainless in \( N = 2, D = 9 \) supergravity. This seeming paradox can be resolved if we consider details of the truncation of \( N = 2 \) to \( N = 1 \) supergravity, which contains a dilaton, a 2-index field strength and a 3-index field strength. One cannot consistently truncate out either two 2-index antisymmetric tensors or a scalar field. Nonetheless, it is possible to make a consistent truncation if we first rotate the scalar fields:

\[
\varphi = \sqrt{\frac{2}{5}} \phi_1 - \sqrt{\frac{3}{5}} \phi_2 , \quad \phi = \sqrt{\frac{1}{5}} \phi_1 + \sqrt{\frac{4}{5}} \phi_2 \quad (81)
\]
and then set $\phi_2 = F_4 = H_1 = F_3^{(1)} = F_2^{(1)} = 0$ which now is consistent with the equations of motion. Defining $\tilde{F}_2 \equiv \sqrt{2} F_2 = \sqrt{2} F_2^{(2)}$, we get the Lagrangian for the bosonic sector of $N = 1, D = 9$ supergravity [11, 14]:

$$\mathcal{L} = eR - \frac{1}{2} e (\partial \phi_1)^2 - \frac{1}{4} e e^{-\sqrt{2} \phi_1} (F_3^{(2)})^2 - \frac{1}{4} e e^{-\sqrt{2} \phi_1} \tilde{F}_2^2. \quad (82)$$

We now see that to obtain a 5-brane solution described by $\tilde{F}_2$ in $N = 1$ theory, one has to start with a multi-brane solution [9] in $N = 2$ supergravity, namely, a two-scalar 5-brane described by $F_2$ and $F_2^{(2)}$. The metric for this solution, which preserves quarter of the supersymmetry of $N = 2$ supergravity, is given by

$$ds^2 = \left(1 + \frac{k_1}{r}\right)^{-1/7} \left(1 + \frac{k_2}{r}\right)^{-1/7} dx^m dx^n \eta_{mn} + \left(1 + \frac{k_1}{r}\right)^{6/7} \left(1 + \frac{k_2}{r}\right)^{6/7} dy^m dy^m. \quad (83)$$

For $k_1 = k_2$, the two-scalar 5-brane can also be viewed as a single solitonic 5-brane of $N = 1$ supergravity preserving half of the supersymmetry of this theory. It should also be remarked that the $\Delta$ value for the $N = 1$ 5-brane is 2, as opposed to $\Delta = 4$ for the $N = 2$ 5-brane, which is yet another indication of a multi-brane origin of this solution.

Thus we see that an interesting phenomenon of supersymmetry enhancement may occur when a single-brane solution of truncated theory can be viewed as a particular limit of a multi-brane solution of extended supergravity. Notice that not only does one obtain a solution preserving more supersymmetries, but one may also find a new stainless $p$-brane in the truncated supergravity.

We now begin to examine in detail the stainless solutions (78)-(80) and verify that they preserve half of the supersymmetry. To consider the supersymmetry of the elementary particle solution (78), we make a $1 + 8$ split of the gamma matrices:

$$\Gamma^0 = \gamma_9, \quad \Gamma^m = \gamma_m, \quad (84)$$

where $\gamma_9 = \gamma_1 \gamma_2 \cdots \gamma_8$ and $\gamma_m$ are numerical matrices with flat indices. The transformation rules for the fermions become

$$\delta \chi = -i \sqrt{7} e^{-B} \partial_m \phi \Gamma_i \gamma_m \varepsilon - \frac{1}{4} e^{-A-B+C} \sqrt{\phi} \partial_m C \gamma_m \gamma_9 \varepsilon_{ij} \Gamma^j \varepsilon, \quad \delta \psi_0 = \frac{1}{4} e^{A-B} \partial_m A \gamma_9 \gamma_m \varepsilon - \frac{1}{4} e^{-B+C} \sqrt{\phi} \partial_m C \gamma_9 \varepsilon_{ij} \Gamma^j \varepsilon, \quad (85)$$

$$\delta \psi_m = \partial_m \varepsilon + \frac{1}{4} \partial_n B \gamma_m \gamma_n \varepsilon + \frac{1}{4} e^{-A+C} \sqrt{\phi} \partial_m C \gamma_m \gamma_9 \varepsilon_{ij} \Gamma^j \varepsilon + \frac{3}{16} e^{-A-C} \sqrt{\phi} \partial_m C \gamma_9 \varepsilon_{ij} \Gamma^j \varepsilon.$$

Substituting the solution (78) into the equations above, we find that the variations of all fermionic fields vanish provided that

$$\varepsilon = e^{-A} \varepsilon_0, \quad \gamma_9 \varepsilon_0 = \varepsilon_0, \quad \varepsilon_{ij} \Gamma^j \varepsilon_0 = \varepsilon_0, \quad (86)$$
where \( \varepsilon_0 \) is a constant spinor. Thus the elementary particle solution preserves half of the supersymmetry.

To verify that the elementary membrane solution also preserves half of the supersymmetry, we make a \( 3 + 9 \) split of the gamma matrices:

\[
\Gamma^\mu = \gamma^\mu \otimes \gamma_7, \quad \Gamma^m = 1 \otimes \gamma^m, \quad (87)
\]

where \( \gamma_7 = \gamma_0 \gamma_1 ... \gamma_5 \) in the transverse space and \( \gamma_1 \gamma_2 \gamma_3 = 1 \) on the world volume. The transformation rules for the fermions become

\[
\delta \chi_i = -\frac{\sqrt{7}}{6} e^{-B} \partial_m \phi \gamma_7 \otimes \gamma_m \Gamma_i \varepsilon - \frac{1}{6} e^{-B - 3A + C + \frac{1}{\sqrt{7}} \phi} \partial_m C \otimes \gamma_m \Gamma_i \varepsilon, \\
\delta \psi_\mu = \partial_m A e^{A - B} \gamma_\mu \otimes \gamma_7 (\gamma_m \varepsilon + \frac{1}{6} e^{-B - 3A + C + \frac{1}{\sqrt{7}} \phi} \gamma_\mu \otimes \gamma_7 \gamma_m \varepsilon), \quad (88)
\]

\[
\delta \psi_\mu = \partial_m \varepsilon + \frac{1}{6} \partial_B e^{-A} \varepsilon + \frac{1}{6} e^{-B - 3A + C + \frac{1}{\sqrt{7}} \phi} \partial_m C \otimes \gamma_m \varepsilon .
\]

This solution preserves half of the supersymmetry provided that

\[
\varepsilon = e^{-A} \varepsilon_0, \quad \gamma_7 \otimes 1 \varepsilon_0 = \varepsilon_0. \quad (89)
\]

It can be shown that a solitonic 6-brane solution \((80)\) also preserves half of the supersymmetry \([11]\). This solution, as was discussed above, is stainless in \( N = 2, D = 9 \) supergravity, the reason being the absence of the required one-index field strength in type IIA \( D = 10 \) supergravity \([11]\). However, in \( D = 10 \) there also exists the chiral type IIB supergravity theory which, unlike type IIA version, cannot be obtained by compactification of the eleven-dimensional \( N = 1 \) supergravity, and which contains the necessary one-index field strength. Recalling that the coefficient \( a \) in the exponential prefactor of this field strength is 2, and that \( \tilde{d} = 0 \), one concludes, based on \((73)\), that the solitonic 6-brane of \( N = 2, D = 9 \) supergravity can be isotropically oxidized to the type IIB solitonic 7-brane. A similar situation is encountered with the elementary membrane solution which turns out to be a descendant of the type IIB self-dual elementary 3-brane described by a five-index self-dual field strength with \( a = 0 \). Thus, if besides eleven-dimensional supergravity one is to include type IIB supergravity in the classification of \( p \)-brane solutions, one necessarily arrives at a conclusion that the elementary particle is the only stainless solution in \( N = 2, D = 9 \) supergravity theory.

### 6 Conclusions

In this paper, we constructed and studied the extended \( N = 2 \) supergravity theory in \( D = 9 \). The full action was obtained by compactifying \( D = 11 \) supergravity directly to \( D = 9 \). The method employed was the ordinary
Scherk-Schwarz dimensional reduction procedure which gives an advantage of constructing the lower dimensional theory in one step, as opposed to the standard Kaluza-Klein step by step dimensional reduction, and also enables one to consider compactification on a non-trivial group manifolds \[12\]. We explored the stainless $p$-brane solutions to the obtained $N = 2$ supergravity in $D = 9$. Having derived the supersymmetry transformation laws for the fields, we were in the position to examine the supersymmetry of the found stainless $p$-branes. Discussing the relation of the $N = 2$ solutions to the $N = 1$ stainless solutions, it was observed that the stainless solutions of truncated theory may or may not remain stainless in the extended supergravity. The notion of stainlessness was discussed in the case when, along with $D = 11$ supergravity, type IIB supergravity was taken into consideration.

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