A Simulation Based Study of Maximum Development of Extensive Air Showers in Highest Energies

P. Davoudifar\(^1\), K. Rowshan Tabari\(^1\)
Research Institute for Astronomy and Astrophysics of Maragha, P.O.B 55177-36698, Vali Asr Town, Eastern Azarbayjan, Iran
dfpantea@riaam.ac.ir ; p_davoudifar@yahoo.com

Abstract. The maximum development of an Extensive Air Shower (EAS) is an important parameter which is possible to be studied with EAS simulation programs. This parameter is also related to hadronic interaction models in high and low energy ranges. A detailed study has been considered to investigate the effects of zenith angle and interaction models as well as thinning parameters on the maximum development of an EAS and its mean values. Diagrams of Maximum shower development vs. energy being used to recognize the masses of the primary particles initiated those EASs. An empirical formula presented to remove the effects of zenith angle.

1. Introduction
The question of cosmic rays type has been essential to many fields of physics. Knowing the composition of very high energy particles (EeV range and above) been researched alongside the development of cosmic ray large area detectors. For primaries with energies of ~ \(10^{15}\) eV and above, due to their low rate of hitting the upper atmosphere (1 \(m^2\) \(yr^{-1}\) to 1 \(km^2\) \(yr^{-1}\) and lower), their direct observation is practically impossible and they can only be investigated through their interactions with the atmosphere, where they produce Extensive Air Showers (EASs).

Till now, many experimental techniques were used to detect the features of these EASs amongst them is to measure the “elongation rate” [1, 2, 3] which is made essentially by detection of Cherenkov light [4]. Hence, the mean values of “maximum longitudinal shower development”, \(<X_{\text{max}}>\), is possible to be estimated using “elongation rates”. But the random nature of shower development in the atmosphere, causes uncertainties arise against the nature of those primaries which are declared through experimental techniques.

Development of simulation programs, like as CORSIKA (Cosmic Ray Simulation for Kascade)[5, 6, 7], made it possible to explore the random features of EASs and it appeared that to compare the results of a simulation based study and experimental values, high statistic is essential.

2. CORSIKA simulation
We started to study the mass composition of higher energy particles using CORSIKA 6.98[5], were different primary particles (gamma, proton, CNO group, and Iron) were used. As the maximum

\(^{1}\) This work has been supported by Research Institute for Astronomy & Astrophysics of Maragha (RIAAM) under research project No. 1/2522.
development of an EAS, $X_{\text{max}}$, is used in mass composition studies we considered a detailed analysis of the zenith angle dependence of $X_{\text{max}}$.

Due to this study using CORSIKA simulation, it was seen that there are considerable fluctuations in $<X_{\text{max}}>$ with respect to its mean values when zenith angles were changed up to 60 degrees. The fluctuations seemed to be related in primary type as there were lower for Iron primaries. For all primary types the fluctuations were higher in lower zenith angles.

Then we considered our simulation for QGSJETII [8, 9] as high and FLUKA 2011[10, 11] as low energy interaction models, thinning parameter of $10^{-4}$[6, 7], for Yakutsk array (observation level of 105 gr/cm$^2$, 62° North latitude and 129° East longitude [12]).

To calculate mean values of $<X_{\text{max}}>$, the zenith angles were ranged in 30 ranges by separations of 2 degrees (i.e. 0-2, 2-4, 4-6, 6-8… and 58-60), 100 showers in each range and total of 3000 showers for each energy were produced. Energy bins were ranged from EeV to $10^3$ EeV (i.e. 1EeV, 2EeV, …, 10 EeV, 20 EeV, …, 100 EeV, 200 EeV, …, and 1000 EeV), total of 28 energy bins and 252000 showers were simulated.

In a given energy, the distribution of the shower maximum development in different zenith angles is shown in figure 1:

**Distribution of Maximum Depth of Shower Longitudinal Development**

![Distribution of Maximum Depth of Shower Longitudinal Development](image)

**Figure 1.** Distribution of shower development maxima in different zenith angles for a 100 EeV proton (this simulation).

in which, it is seen that not only the frequency, but the values of $X_{\text{max}}$ differs when the zenith angle changes.
Due to later result, we considered a path length distribution function to estimate the average values in each zenith angle range. The path length distribution function considered to be in the form of:

\[
f(x) = \begin{cases} 
\text{polynomial}(2), & \text{lower limit} < x < x_{\text{max}} \\
p[2], & x = x_{\text{max}} \\
\text{polynomial}(4), & x_{\text{max}} < x < \text{upper limit}
\end{cases}
\]

in which lower and upper limits been considered as the lower and upper ranges of distribution of shower development maxima, where no more event were recorded. The common form of a function of this type is shown in figure 2.

![Distribution of Shower Maxima](image1)

**Figure 2.** Common form of the function of formula 1.

These functions were fitted to the frequencies and then normalized to get the probability functions:

\[
P(x) = \frac{f(x)}{\int_{\text{lower limit}}^{\text{upper limit}} f(x) \, dx}
\]

The comparison of these probabilities shows that in lower zenith angles (i.e. less than 10 deg.), the shape of probability function does not change with zenith angle increase, but this change is obvious in larger zenith angles up to about 30 deg. (i.e. the width of probability function increases) and then the width of probability function slowly decreases without drastic changes in its maxima.

The result for primaries with larger mass numbers is different as for example, for an Iron primary there are fewer changes in all zenith angle ranges and the value of maxima decreases with zenith angle very smoothly. In this stage no more could be down. In each angle range, the mean value of \(X_{\text{max}}\) is then equal to:

\[
\langle X_{\text{max}} \rangle = \int_{\text{lower limit}}^{\text{upper limit}} x \cdot P(x) \, dx
\]

As the width of this probability function is more at lower zenith angles, this treatment will affect
the calculated \( <X_{\text{max}} > \) at lower angles. The results are shown in table 1.

To remove the effect of zenith angle, a function of type:

\[
\Theta(\theta) = a \cdot \cos \theta + b
\]

(4)

been used. Coefficients were estimated to be \( a=753.1315 \) and \( b=11.5003 \). Eliminating the effect of this function:

| \( \theta \) (deg) | 1   | 3   | 5   | 7   | 9   | 11  | 13  | 15  | 17  | 19  |
|-------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( <X_{\text{max}} > \) | 773 | 721 | 789 | 751 | 782 | 776 | 738 | 742 | 711 | 723 |
| \( \theta \) (deg) | 21  | 23  | 25  | 27  | 29  | 31  | 33  | 35  | 37  | 39  |
| \( <X_{\text{max}} > \) | 735 | 744 | 700 | 630 | 651 | 630 | 642 | 639 | 622 | 579 |
| \( \theta \) (deg) | 41  | 43  | 45  | 47  | 49  | 51  | 53  | 55  | 57  | 59  |
| \( <X_{\text{max}} > \) | 583 | 559 | 543 | 529 | 517 | 481 | 479 | 439 | 426 | 397 |

the value of \( <X_{\text{max}} > \) for a primary proton of 100 EeV is estimated:

\[
\langle X_{\text{max}} \rangle = 764.7
\]

(5)

Figure 4. Mean value of maximum depth of shower development for Iron, Proton and Gamma primaries as resulted in this simulation.
To produce a diagram of $<X_{\text{max}}>$ v.s. energy (figure 4), this procedure been repeated for Proton, Iron and Gamma primaries in different energies (i.e. 10 EeV up to 1000 EeV).

A function of the logarithmic form (figure 3), were adopted to consider the effect of first interaction height:

$$ H(x) = c + d \cdot \ln x $$  \hspace{1cm} (6)

The function were fitted to our results, in which $x$ denoted the atmospheric depth in gr/cm$^2$. For a primary proton of 100 EeV, these parameters were estimated to be $c= 2394.2031$ and $d= -451.8835$. From equation 7:

$$ \langle H \rangle = \frac{\int_{\text{lower limit}}^{\text{upper limit}} x(c + d \cdot \ln x) \, dx}{\int_{\text{lower limit}}^{\text{upper limit}} (c + d \cdot \ln x) \, dx} $$  \hspace{1cm} (7)

the average value of H were obtained ~50. More studies showed that this average value for a proton primary remains unchanged (or have small changes) when the interaction models are not varied.

3. Results

These results are collected in figure 3, in which a set of resent experimental data of Yakutsk array are also shown [4]. Though the most of these primaries at higher energies seems to be protons but the effect of other interaction models should be considered in detail.

The effect of zenith angle on the depth of shower maximum was studied. It is shown that in lower zenith angles, the values of shower maximum development, $X_{\text{max}}$, have drastic changes and a statistical study of $<X_{\text{max}}>$ values always should be considered to remove these effects. We introduced a procedure to remove the effect of zenith angle. As a result it shown that in a simulation, for practical means, it is essential to consider this effect when use the data from an arrays with lower resolution. For higher resolution arrays the accuracy of zenith angle estimation is a few degrees and this resolution determines the zenith angles ranges should be considered in the simulation.

4. References

[1] Linsley J 1977 Proceedings of 15th International Conference on Cosmic Rays 12 89-96
[2] Linsley J and Watson A A 1981 Physical Review Letters 46 459-463
[3] Badea A F, et al. 2001 Astroparticle Physics, 15 19-28
[4] Knurenko S P and Sabourova A 2011 Astrophysics and Space Science Transactions 7 251-255
[5] Heck D and Pierog T 2011 Extensive Air Shower Simulation with CORSIKA: A User’s Guide (Version 6.980)
[6] Heck D and et al. 1998 FZKA 6019
[7] Knapp J, Heck D and Schatz G 1996 FZKA 5828
[8] Kalmykov N N, Ostapchenko S S, and Pavlov A I 1994 Bulletin of Russian Academy of Science (Physics) 58 1966
[9] Ostapchenko S S 2006 Nuclear Physics B (Proceedings Supplements) 151 143, 147; 2006 Physical Review D 74 014026
[10] Fasso A, Ferrari A, Ranft J, and Sala P R 2005 Report CERN-2005-1
[11] http://www.fluka.org/

1988 “Catalogue of Highest Energy Cosmic Rays, No. 3, Yakutsk” World Data Center C2 for Cosmic Rays Institute of Physical and Chemical Research