2D Image-based higher-order meshing for 3D modelling in MATLAB

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Abstract. We present a higher-order meshing technique to 3D modelling applications from 2D images. This method uses 2D images that are converted to 3D stereolithography files on which tetrahedral meshes of higher order are developed. The resulting 3D meshed structures find several applications in Computational fluid dynamics and finite element analysis problems. The generateMesh is the initial mesh used by the presented mesh generator, which is a MATLAB function from the PDE Toolbox. The required inputs for the mesh generator are stereolithography file and mesh edge length upper bound of the 3D structure. A higher-order tetrahedral mesh, connectivity matrix, node coordinates, and nodes lying on the border are the results we get from the presented work. For many finite element analysis problems, the outcomes obtained from the presented mesh generator can be used very efficiently to enhance the numerical results. In medical simulation, photogrammetric fields, material engineering, computational materials science, computational electromagnetics, etc., this method can be used.

Keywords- Mesh generation, Higher-order tetrahedral element, Image-based meshing, 3D modelling

1. Introduction

Three-dimension (3D) object modelling in the vision, graphic, medical simulations, and photogrammetric fields is an intense and long-lasting research objective. In many applications like object recognition, inspection, visualization, animation, and navigation these models are required. It has recently become quite a significant and fundamental initiative for the digital archiving of cultural heritage [1]. In certain fields, however, the exact definitions of geometry are not available a priori. For instance, in, the medical field, the primary source of geometric detail of images is from various dimensions - 2D, 3D, 4D, etc., such as computed tomography (CT), cryosection, magnetic resonance imaging (MRI), ultrasound imaging, etc [2]. These images do not, in general, provide explicit and obvious information of the intrinsic geometry and are therefore not readily accessible for meshing purposes. For many applications, therefore, the task of producing mesh from images is important [3]. Hence, there is a need for effective software to convert 2D images to higher dimensional meshed structures.
It is quite well known that in various medical science and engineering problems, higher-order (HO) approaches give remarkable accuracy and efficient simulation solutions [4-9]. Thus, these approaches could be used to design error threshold limit with less computing effort [10]. Fortunately, they should be integrated with effective HO mesh generators to be robust [10-14]. But, no 3D HO tetrahedral mesh generation software is available for MATLAB users which are obtained from 2D images. Therefore, we suggest an easy and efficient approach here for the conversion of 2D image to 3D higher-order meshes, especially for MATLAB users. Thus, the task of the generation of meshes from images is critical for many applications. To resolve many engineering and science problems, the presented approach can be effectively implemented. The approach described in this paper is to enhance the method exhibited in [13-14].

In this paper, we describe the approach of 2D image to 3D modelling using the HO technique of generating tetrahedral mesh and its mathematical formulations using MATLAB in section 2. We discuss the results and outcomes of the presented technique in section 3. In section 4, we conclude the article with suggestions for future studies.

2. 2D image to 3D modelling with the higher-order technique of generating tetrahedral mesh and its mathematical formulations using MATLAB

Meshes of 3D models is a basic pre-step in a wide range of applications, including Computational fluid dynamics (CFD) and finite element (FE) analysis, and developing a mesh from images is essential. Here, we provide the method for creating a basic and competent 3D HO tetrahedral mesh in MATLAB from the 2D image using the generateMesh mesh developer as the initial mesh from the PDE toolbox [15]. The outputs of the presented mesh generator can be used productively for finite element implementations as it provides outstanding HO meshes, guarantees the absence of redundant nodes, and also the border nodes lay precisely on the border with the consistent alignment of the numbering of finite element nodes as can be seen in Figures. 1 - 4.

As shown in Figures. 1-4, we position the nodes on the HO tetrahedral finite elements using a point mapping for linear to quartic order. We use the following point mapping as given in equation (1) for every HO tetrahedral element to describe the nodes on the border and inner part [13]:

\[
\begin{align*}
    r &= \sum_{j=1}^{6} \frac{(m+1)(m+2)(m+3)}{N_j^{(m)}(\xi, \eta, \zeta) r_j}, \\
    s &= \sum_{j=1}^{6} \frac{(m+1)(m+2)(m+3)}{N_j^{(m)}(\xi, \eta, \zeta) s_j}, \\
    t &= \sum_{j=1}^{6} \frac{(m+1)(m+2)(m+3)}{N_j^{(m)}(\xi, \eta, \zeta) t_j}. \quad (1)
\end{align*}
\]

where the Lagrange interpolation function \( N_j^{(m)}(\xi, \eta, \zeta) \) is defined in the natural coordinates for the \( m^{th} \) order tetrahedral element at node \( j \) in the natural coordinates \( (\xi, \eta, \zeta) \). After implementing the section formula for any HO tetrahedral element along each edge of the FE, the point mapping formula described by equation (1) reduces to:
For the 3D models from 2D images, we, therefore, established a HO mesh development code in MATLAB. The strategies that we used are as follows:

Step 1. Convert the 2D image to 3D stereolithography file using the Joint Photographic Experts Group to stereolithography file Converter [16].

Step 2. For the 3D model under analysis, insert the stereolithography file and an estimated mesh edge length upper bound thickness $h_{\text{max}}$.

Step 3. The stereolithography file is then imported from the PDE Toolbox into MATLAB with the function `importgeometry` [15].

Step 4. Produce the linear mesh using the PDE Toolbox in MATLAB from the `generateMesh` function [15].

Step 5. Produce the HO tetrahedral meshes with the desired HO finite element, which are shown in Figures 2 - 4, using the `HOMesh3d.m` code explained in [13].
The results of the presented HO meshes from the 2D images can be found in the following section. This mesh development techniques offer good quality meshes for FE analysis which can be used efficiently for various Engineering and Science problems by MATLAB users.

3. Results
This section presents the results obtained from the approach described in section 2 to a 2D image. The presented approach is implemented for the 2D image of a leaf successfully herein. In a similar approach, 2D image to 3D HO meshes can be created for any image and can be effectively used for many CFD and FE analysis. As the outcomes obtained from the approach may be productively utilised in the FEA of several problems as stated in section 1 [1-14]. Consequently, the FE approach can be used effectively to obtain accurate numerical results for video quality enhancement [17-19].

In the following example, we present the 2D to 3D HO mesh of the leaf. Figures 5-10 show the outcome for the structure from the presented method:

Example:
For analysis in many fields, FE method is becoming a useful research technique. It is an all-round tool for describing and testing complex structures [5-14]. The methodology suggested can therefore be useful to FE analysts. The Figures. 5 -10 shown below are the results from the presented method for the leaf image:

Figure. 5. 2D image of the leaf structure.

Figure. 6. The 3D stereolithography file of the leaf structure.
**Figure. 7.** Linear-order meshes of 3D of the leaf structure.

**Figure. 8.** Quadratic-order meshes of 3D of the leaf structure.
4. Conclusions
In this paper, we provided information on a novel approach of a 2D image conversion to 3D model meshing using HO tetrahedral elements in MATLAB, which can be used for different CFD and FE analysis. The mesh creator in the presented approach is established on the MATLAB `generateMesh` function and also it works fine with mesh refining. For FE analysts, it is indeed a powerful component in many application areas. In the Finite element approach, this mesh can also be used very effectively to solve any PDE in
several sciences and engineering problems. For curved geometries with curved finite elements, a similar
technique of meshing can be used.

5. References

[1] Remondino, Fabio, and Sabry El-Hakim, Image-based 3D modelling: a review, The photogrammetric record 21, no. 115, 269-291, 2006.
[2] Neittaanmäki, P., T. Rossi, K. Majava, O. Pironneau, O. Nevanlinna, and R. Rannacher. IMAGE-BASED UNSTRUCTURED 3D MESH GENERATION FOR MEDICAL APPLICATIONS, European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS), 2004.
[3] Cuadros-Vargas, Alex Jesús, Luis Gustavo Nonato, Rosane Minghim, and Tiago Etiene, Imesh: An image-based quality mesh generation technique, In XVIII Brazilian Symposium on Computer Graphics and Image Processing (SIBGRAPI'05), 341-348. IEEE, 2005.
[4] Shubhabrata Datta, J. Paulo Davim, Computational approaches to materials design: theoretical and practical aspects, IGI Global, Hershey, 2016.
[5] Wang, Lan, et al. "Analysis of the Osteogenic Effects of Biomaterials Using Numerical Simulation." BioMed research international 2017.
[6] El-Anwar, Mohamed I., and Mohamed M. El-Zawahry. "A three-dimensional finite element study on dental implant design." Journal of Genetic Engineering and Biotechnology 9(1) 77-82, 2011.
[7] Zadpoor, Amir Abbas. "Finite element method analysis of human hand arm vibrations." Int. J. Sci. Res 16 391-395, 2006.
[8] Mollica, F., and L. Ambrosio. "The Finite Element Method for the Design of Biomedical Devices." Biomaterials in Hand Surgery. Springer, Milano, 31-45, 2009.
[9] Boccaccio, A., et al. "Finite element method (FEM), mechanobiology and biomimetic scaffolds in bone tissue engineering." International journal of biological sciences 7(1) 112, 2011.
[10] Ims, Jeremy, and Z. J. Wang. Automated low-order to high-order mesh conversion, Engineering with Computers 1-13, 2018.
[11] M. Kardani, M. Nazem, J. P. Carter, A. J. Abbo, Efficiency of high-order elements in large-deformation problems of geomechanics, Int. J. Geomech. 15(6) 1-10, 2014.
[12] K. V. Nagaraja, V. Kesavulu Naidu, P. G. Siddheshwar, Optimal Subparametric Finite Elements for Elliptic Partial Differential Equations Using Higher-Order Curved Triangular Elements, International Journal for Computational Methods in Engineering Science and Mechanics 15(2) 83-100, 2014.
[13] Smitha, T. V., and K. V. Nagaraja. "Automated higher-order mesh generation in MATLAB on biomaterials." In IOP Conference Series: Materials Science and Engineering, vol. 577, 1, 012130. IOP Publishing, 2019.
[14] Smitha, T. V., and K. V. Nagaraja, “An efficient automated higher-order finite element computation technique using parabolic arcs for planar and multiply-connected energy problems”, Energy 183, 996-1011, 2019.
[15] MathWorks, Partial Differential Equation Toolbox: User's Guide (R2015a), <https://in.mathworks.com/help/pdf_doc/pde/doc/pde_m.pdf>, (2015).
[16] JPG to STL Converter, <https://anyconv.com/jpg-to-stl-converter/>, 2020.
[17] Madhura.S and Suresh K, “A new parallel DSP hardware compatible algorithm for noise reduction and contrast enhancement in video sequence using Zynq-7020”, International Journal of Computer Aided Engineering and Technology (IJCAET), vol-2, in press. Under Special Issue on: Computer-Aided Intelligent Systems, ISSN:978-91-27654-12-15
[18] Madhura.S and Suresh K, “Conversion of 2D image to 3D image using Histogram equalization function”, International Journal of Emerging Engineering Research and Technology vol-2, Issue 2, May 2014, pp 8-13
[19] Madhura.S, Suresh K, "A Novel Hardware Efficient Algorithm for Impulse Noise Filtering with Motion Estimation", *IEEE International Conference on Communication and Electronics Systems (ICCES 2016), part number-CFP16AWO-ART*, ISBN:978-1-5090-1066-0, 21-22 October 2016, Coimbatore, India.