Investigation of spherical and cylindrical Luneburg lens antennas by the Green’s function method

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Abstract. Luneburg lens antenna radiation fields are calculated with Green’s functions of spherical and cylindrical layered structures. Electric field components of spherical and cylindrical Luneburg lenses excited by linear and circular polarized incident field are analysed. Dipole, slot and aperture antennas are described by electric and magnetic extraneous currents. Radiation patterns of cylindrical and spherical Luneburg lens are analysed. Co-polarized and cross-polarized field radiation patterns are shown. The proposed method significantly reduces the computing time for multi-layered lenses in comparison with the most commonly used in antenna design. The first step antenna structure optimization may be performed for a shorter time. The results may be used as the first approximation for Ansys HFSS and other software.

1. Introduction

Spherical and cylindrical Luneburg lenses are suitable for multiple-beam and scanning high-gain antenna systems design. The refraction index of Luneburg lens varies as follows [1]

\[ n(r) = \sqrt{\varepsilon'(r)} = \sqrt{2 - (r / a)^2}, \]

where \( \varepsilon' \) is the relative dielectric constant of lens material at the point \( r \), \( r \) is the radial coordinate in the spherical or cylindrical coordinate system, \( a \) is the outer lens radius in the corresponding coordinate system. Usually Luneburg lens is fabricated as a multilayer structure. The refractive index of the each layer is close enough to the law of Luneburg as in equation (1).

There are a lot of papers devoted to Luneburg lens antennas analysis with modern electromagnetic software as Ansys HFSS [2], CST Microwave Studio [3] and others. The authors have made a note of more processing time for simulation. The optimization of the complicated antenna system as the Luneburg lens with a feeder is a multivariate optimization problem. The initial solution is close to the optimum significantly accelerating calculations. Our goal is to offer a new method for the first step antenna structure approximation.

We suggest the well-known Green’s function method for the first step Luneburg lens antenna analysis and optimization. This method gives a chance for physical interpretation of obtained results but it is rather limited in the shape and structure of qualified objects. Fortunately, Luneburg lens may be well fitted by the spherical or cylindrical coordinate system. The antenna radiation field is defined by integrating of electric and/or magnetic extraneous currents with the Green’s functions suitable for the spherical or cylindrical coordinate system. As extraneous currents electric or magnetic dipoles are used. For example crossed electric and magnetic dipoles form the Huygens source. This kind of the...
source is used for aperture antennas design as a horn antenna. A linear waveguide with slots for cylindrical Luneburg lens excitation is described by magnetic dipoles.

In this paper a universal algorithm with the Green’s functions of layered spherical and cylindrical structures for Luneburg lens antenna radiation problems solving is described. The offered method demands at least one thousand less time for calculations in comparison with Ansys HFSS. Radiation patterns, antenna gain, directivity, radiation efficiency and polarization properties may be rather quickly analyzed.

2. Spectral-domain full-wave approach for multi-layered spherical and cylindrical structures

The spectral-domain full-wave approach suggested in [4],[5] uses the rigorous solution of the Maxwell equations for radially non-uniform structures with arbitrary electrodynamic properties and any distribution of impressed electrical and magnetic currents. This method with matrix approach is suitable for MATLAB. It is easy to simulate layered structures, impedance surfaces, and coating layers including metamaterials.

According to the suggested method the main electric and magnetic field component is chosen. Other components are defined with the main components. In the spherical coordinate system as the main radial electric \( E_r \) and magnetic \( H_z \) components are used. In cylindrical coordinates \( E_z \) and \( H_z \) are the main and the first one may be represented as

\[
E_z(r, \phi, z) = -\frac{1}{4\pi^2} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} E_m \cos(mp) j^n \frac{\exp(-ikr)}{kr} \left[ (\Phi_1^E + \Phi_2^E) \frac{\cos^2\theta}{k_0^2 \sin^2\theta} + \sin\theta \right], \tag{3}
\]

where functions \( \Phi_1^E, \Phi_2^E \) contain information about sources, cylindrical structure dimensions and electrodynamic material properties. These functions may be considered as the Green’s functions. In equation (2) all primed symbols are source points. The integration is carried out in relation to the source region \( V' \).

We can calculate electromagnetic field from any point of view. To obtain antenna radiation patterns the saddle-point method is used. For any point of the space the electric field components in the spherical coordinate system are defined as follows

\[
E_0(\theta, \phi, r) = \frac{1}{2\pi^2} \sum_{m=0}^{\infty} E_m \cos(mp) j^n \frac{\exp(-ikr)}{kr} \left( \Phi_1^E + \Phi_2^E \right) \frac{\cos^2\theta}{k_0^2 \sin^2\theta} + \sin\theta, \tag{4}
\]

where \( E_0 = 1, \ E_m = 2, \ m \neq 1 \).

Electric field components of a spherical Luneburg lens antenna in the far zone are simplified by saddle-point method in the spherical coordinate system. Electric field is defined as follows

\[
E(\theta, \phi, r) = E_0 \frac{\exp(-ikr)}{kr} \sum_{n=0}^{2n+1} \frac{2n+1}{n(n+1)} \left[ i \frac{\pi}{2} n + \phi \right] \left\{ \begin{array}{ll}
[a_\theta \tau_n(\theta) + ia_\phi \pi_n(\theta)](M_n + jM^*_n) & \\
[-a_\theta \pi_n(\theta) - ia_\phi \tau_n(\theta)](N_n + jN^*_n) & 
\end{array} \right\} \tag{5}
\]

where \( k_0 = 2\pi/\lambda \) is the wave number, \( a_\theta, a_\phi \) are unit vectors in the spherical coordinate system, \( \tau_n(\theta) \) and \( \pi_n(\theta) \) contain Legendre functions and derivatives of these function. Coefficients \( M_n, N_n \) in (5) depend on an excitation current and layered structure properties.

Luneburg lens with different kind of feed may be simulated at the corresponding choice of amplitude and phase of extraneous currents.

For radially layered structure, a model with equivalent lines is used [5]. Each layer is assigned with a part of the radial line. Transmission matrices of equivalent lines and boundaries define electromagnetic waves propagation in each layer and between layers. The method makes unnecessary field calculation inside the Luneburg lens when the far field and radiation patterns are defined. All these essentially reduce computing time.
3. Numerical results

Let us consider radiation properties of the cylindrical Luneburg lens excited by a longitudinal electric dipole. Dipole radiation is uniform in the azimuthal plane if there is no the lens. The lens increases antenna gain and radiation directivity as it is shown in figure 1. Two kinds of structures are under analysis. The first one has the equal thickness of the each layer. Dielectric constant is close enough to equation (1) as follows: $\varepsilon_1=1.93$, $\varepsilon_2=1.77$, $\varepsilon_3=1.61$, $\varepsilon_4=1.46$, $\varepsilon_5=1.31$, $\varepsilon_6=1.16$.

The second structure was optimized for maximum directivity. After optimization permittivity of the layers varies as follows: $\varepsilon_1=1.88$, $\varepsilon_2=1.72$, $\varepsilon_3=1.56$, $\varepsilon_4=1.40$, $\varepsilon_5=1.24$, $\varepsilon_6=1.08$. The radius of the $i$-th layer of the $3\lambda$ Luneburg lens is as follows: $r_1=0.6\lambda$, $r_2=1.35\lambda$, $r_3=1.8\lambda$, $r_4=2.16\lambda$, $r_5=2.49\lambda$, $r_6=2.76\lambda$, where $\lambda$ is the free space wavelength. The optimized lens structure forms more directed radiation. The similar effect is observed in the meridional plane (figure 2). Further this kind of the structure may be used as the first step approximation for Ansys HFSS or CST Microwave studio software.

![Figure 1](image1.png)

Figure 1. Radiation pattern in the azimuthal plane of the $3\lambda$ cylindrical Luneburg lens excited by the longitudinal electric dipole with equal-thickness (a) and optimized (b) structure.

![Figure 2](image2.png)

Figure 2. Radiation pattern in the meridional plane of the $3\lambda$ cylindrical Luneburg lens excited by the longitudinal electric dipole with equal-thickness (a) and optimized (b) structure.

Radiation patterns of the $4\lambda$ spherical Luneburg lens excited by the aperture antenna with circular polarization are shown in figure 3. Matching between the main polarization and cross-polarized radiation field levels may be simply carried out.

![Figure 3](image3.png)

Figure 3. Radiation pattern of the $4\lambda$ spherical Luneburg lens exited by the aperture antenna. Main polarization (solid line), Cross-polarization (dash line)
We tested our method in comparison with software Ansys HFSS. The three-layer spherical structure of Luneburg lens was designed. A circular polarized incident wave was excited by a circular horn antenna at the frequency of 25 GHz. The thickness and dielectric constant of each layer is shown in figure 4. Comparison of radiation patterns at the main polarization calculated by Ansys HFSS and our method are shown in figure 5. The main lobes are very similar. There is difference in sidelobes but it is compensated by a very short processing time (table 1).

![Image](image_url)

**Figure 4.** Three-layer spherical Luneburg lens and radiating field in the software Ansys HFSS

![Image](image_url)

**Figure 5.** Comparison of radiation patterns of the lens at the main polarization calculated by Ansys HFSS and the Green’s functions method

| 2Δθ^0.5 dB | 2Δθ^0.10 dB | t, sec |
|-------------|-------------|-------|
| a           | HFSS        | TGF   | HFSS  | TGF  | HFSS  | TGF  |
| λ           | 19.2        | 18.8  | 33.5  | 31.4 | 2.4×10^3| 1.8 |
| 2λ          | 11.1        | 10.0  | 17.5  | 16.6 | 3.7×10^3| 2.3 |
| 3λ          | 6.8         | 7.0   | 9.6   | 11.6 | 5.8×10^3| 3.1 |

The proposed method significantly reduces the computing time for multi-layered lenses analysis in comparison with the most commonly used in antenna design. The first step antenna structure optimization may be performed for a short time. The results may be used as the first approximation for Ansys HFSS, CST Microwave Studio and other software.

4. Conclusion
The efficiency of the universal method based on the Green’s functions of layered structures for the spherical and cylindrical Luneburg lenses radiation field calculations is shown.

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