Intermediate Mass Fragment Flow as a Probe to the
Nuclear Equation of State

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Abstract. The transverse flow of intermediate mass fragments (IMFs) has been investigated for the 35 MeV/u $^{70}$Zn$^{+}$70$^{70}$Zn, $^{64}$Zn$^{+}$64$^{64}$Zn, and $^{64}$Ni$^{+}$64$^{64}$Ni systems. A transition from the IMF transverse flow strongly depending on the mass of the system, in the most violent collisions, to a dependence on the charge of the system, for the peripheral reactions, is shown. This transition was shown to be sensitive to the density dependence of the symmetry energy using the antisymmetrized molecular dynamics model. The results present a new observable, the IMF transverse flow, that can be used to probe the nuclear Equation of State. Comparison with the simulation demonstrated a preference for a stiff density dependence of the symmetry energy.

1. Introduction

Heavy-ion collisions provide a unique opportunity to examine nuclear matter at temperatures, densities, and neutron-to-proton (N/Z) ratios away from that of ground state nuclei. A wide-range of observables from heavy-ion collisions have been used to study and constrain the nuclear Equation of State (EoS) \cite{1, 2}. While the EoS for symmetric nuclear matter (N=Z) is relatively well constrained [3–5], predictions for the density dependence of the symmetry energy, $E_{\text{sym}}(\rho)$, can still vary widely [2, 5]. Free neutron-proton ratios [6], isobar ratios [7], isoscaling [8], and isospin diffusion [9] measurements from heavy-ion collisions have provided evidence suggesting a stiff density dependence of the symmetry energy [10]. The collective transverse flow of light charged particles (LCPs) has been predicted to be a useful probe for applying constraints on the asymmetric part of the EoS at both high and low densities [2, 11].

The transverse collective flow has been shown to depend on both the mass and N/Z of the colliding system. The examination of the balance energy demonstrated that the transverse flow was strongly dependent on the mass, $A_{\text{sys}}$, of the colliding system \cite{12}. The balance
energy followed an $A_{sys}^{-1/3}$ power law which represents a balance between the attractive mean-field potential which scales with the surface, $A_{sys}^{2/3}$, and the repulsive nucleon-nucleon (NN) collisions which should scale with the interaction volume, $A$ [12, 13]. Pak et al. have shown that the transverse collective flow for light particles with $Z=1$-3, as well as the balance energy, increase with an increasing neutron to proton ratio of the system ($N/Z_{sys}$) [14, 15]. The isospin dependence of the transverse flow and balance energy was attributed to the isospin dependent potential and in-medium NN-cross sections [16, 17]. Along with the mass-dependent mechanisms (mean-field and NN-collisions), theoretical simulations have also demonstrated the importance of the Coulomb potential in describing the transverse flow [18–20].

While LCPs have typically been used in studying the transverse flow, we use IMFs to explore the dependences of the transverse flow on the mass and charge of the colliding systems. Additionally, the IMF flow is shown to be viable probe to the EoS, particularly the density dependence of the symmetry energy.

2. Experiment

The Superconducting K500 Cyclotron at the Texas A&M Cyclotron Institute was used to produce 35 MeV/u beams of $^{70}$Zn, $^{64}$Zn, and $^{64}$Ni which were impinging on $^{70}$Zn, $^{64}$Zn, and $^{64}$Ni self-supporting targets, respectively. The experimental data was collected using the 4$\pi$ NIMROD-ISiS array (Neutron Ion Multidetector for Reaction Oriented Dynamics with the Indiana Silicon Sphere) [21]. NIMROD-ISiS consisted of 14 concentric rings providing coverage from 3.6$^\circ$ to 167$^\circ$ in lab. The first 8 rings, ranging from 3.6$^\circ$ to 45.0$^\circ$, had the same geometry as the INDRA detector [22] and the final 6 rings were of the ISiS geometry [23]. Isotopic resolution was achieved, in the forward angles, for $Z=1$-17 particles and elemental identification was obtained up through the charge of the beam. In the backward angles detector thresholds allowed only for identification of $Z=1$-2 particles. The entire charged particle array was surrounded by the 4$\pi$ TAMU Neutron Ball which provided an average event by event neutron multiplicity.

3. Event Characterization, Reaction Plane Reconstruction, and Flow Quantification

An estimation of the impact parameter, for the experimental data, was completed using the minimum bias 2-D distributions of the raw neutron multiplicity plotted against the charged particle multiplicity for each system. Five bins (0-4) were created from the 2-D distributions such that each bin would represent a $b/b_{max}$, or $b_{red}$, width of 0.2 if one assumes a corresponding triangular impact parameter distribution. Molecular dynamics simulations [24, 25], filtered using a software representation of the NIMROD-ISiS array, showed that the 80% (20%) of the events in the most peripheral (central) bin were correctly identified. Thus, Bin 0 does not necessarily contain the most central events but rather the most violent events, while the bins representing the peripheral collisions do provide a relatively accurate impact parameter estimation.

In order to calculate the in-plane transverse momentum for the fragments, the reaction plane for each event was reconstructed using the azimuthal correlation method [26]. The azimuthal correlation method does not differentiate the forward, quasi-projectile, and backward, quasi-target, sides of the flow. Therefore, the forward flow side of the reaction plane was determined using the transverse momentum analysis method [27]. The particle of interest (POI) was removed from the calculation of the reaction plane in order to avoid autocorrelations [26–28]. Thus, the reaction plane was calculated for each particle in an event rather than once for the whole event. In order to ensure that only quasi-complete events were used in the analysis an event criterion was imposed such that the total detected charge for an event must be greater than 40% of the total charge in the colliding system.

The transverse flow is often quantified as the slope of the average in-plane momentum, $\langle P_x \rangle$, at mid-rapidity. However, in the NIMROD-ISiS array thresholds produced incomplete detection...
of intermediate mass fragments (IMFs) at negative reduced rapidities ($Y_r=Y_{cm}/Y_{cm,proj}$). Therefore, the transverse flow was quantified by calculating the average in-plane transverse momentum from $0.0 \leq Y_r \leq 0.45$ [29–31]. The flow is extracted only from the positive rapidity fragments and is designated as $\langle P_x \rangle$. In calculating the $\langle P_x \rangle$ any lack of momentum conservation, due to the reaction plane reconstruction [26, 29], was manually corrected by adjusting the $\langle P_x \rangle$ versus $Y_r$ plot such that it passed through the origin, (0,0).

In examining the transverse flow of the IMFs, an anti-correlation between the reaction plane and the IMF’s azimuthal angle was observed. The anti-correlation is due to the removal of the POI from the reaction plane calculation. For the most violent collisions, there was a significant probability that the removal of the heavy POI would result in a reaction plane oriented $\sim 180^\circ$ from the reaction plane calculated with the heavy POI, producing negative flow values. This anti-correlation for heavy fragments has also been observed by the INDRA collaboration [28]. Therefore, if the difference between the reaction plane calculated without the POI and the reaction plane calculated using the entire event was greater than $90^\circ$ the POI removed reaction plane was re-oriented (rotated $180^\circ$). The molecular dynamics simulations showed that magnitude of the flow could be overestimated for some IMFs by up to 43% in the most central collisions. However, the strength of the method is that the trend and correct sign of the IMF flow is reproduced allowing for system to system comparisons to be studied.

4. Results and Discussion
The transverse flow, $\langle P_x \rangle$, for $Z=1$-9 particles is shown in Figure 1 for the five centrality bins, ranging from Bin 0 (most violent collisions) to Bin 4 (most peripheral collisions). The expected increase in the transverse flow with the increasing charge of the fragments is clear [14, 32, 33]. However, a decrease in the magnitude of the flow from the central to peripheral bins is observed rather than the typical maximum flow in the mid-peripheral collisions (Bin 2) [14, 32]. While the un-filtered AMD-Gemini results showed a maximum flow for the mid-peripheral collisions, the filtered results demonstrated the same trend shown in the experimental data (Figure 1). This was attributed to the reaction plane re-orientation method, which overestimates the magnitude of the flow in the most violent collisions (Bins 0-1) with respect to the mid-peripheral selection.

In Bin 0 the $\langle P_x \rangle$ of the IMFs from the $^{64}$Ni and $^{64}$Zn systems are nearly equivalent and larger than the $^{70}$Zn system. This can be understood through the mass dependence of the transverse
flow which is related to the balance energy relationship derived by Westfall et al. [12]. Thus, one would expect the $^{70}$Zn system to exhibit a decreased flow in comparison to the $A_{sys}=128$ systems since it has a lower balance energy due to the increased repulsive NN-collisions relative to the attractive mean-field potential.

In the peripheral reactions, Bins 3 and 4, the $\langle P_x \rangle$ of the IMFs from the Zn systems become similar and decreased with respect to the $^{64}$Ni system. This represents a clear dependence of the charge of the system on the IMF flow. The larger repulsive Coulomb force in the Zn (Z=30) systems causes a decreased flow in comparison to the $^{64}$Ni (Z=28) system. The increased effect of the charge-dependent forces, relative to the mass-dependent forces, in the peripheral collisions may be due to the decreased interaction volume. For example, the number of NN-collisions would be greatly diminished in the peripheral reactions.

A separation of the IMF flow between all 3 systems occurs in the mid-peripheral reactions, Bin 2, in which the IMFs $\langle P_x \rangle$ from the $^{64}$Zn system is less than that from the $^{64}$Ni system yet larger than the $^{70}$Zn flow, exhibiting a behavior between the extremes of the mass (Bin 0) and charge (Bin 4) dependent flow. The difference between the IMF flow in the $^{64}$Ni and $^{64}$Zn systems is similar to the $(N/Z)_{sys}$ dependence observed by Pak et al. for LCPs in $A_{sys}=116$ systems [14]. However, in context with the results from the IMF flow of the $^{70}$Zn system, which has a similar $(N/Z)_{sys}$ to the $^{64}$Ni system, the difference between the $A_{sys}=128$ IMF flow appears to be due to a balancing between the mass and charge dependent mechanisms.

In order to examine this trend more quantitatively the ratio

$$R_{Flow} = \frac{\langle P_x / A \rangle_{^{64}Zn} - \langle P_x / A \rangle_{^{70}Zn}}{\langle P_x / A \rangle_{^{64}Ni} - \langle P_x / A \rangle_{^{70}Zn}} \tag{1}$$

can be used to define the magnitude of the flow from the $^{64}$Zn system in comparison to the $^{64}$Ni and $^{70}$Zn systems. Thus, when $R_{Flow}=1$ the IMF flow of the $^{64}$Zn system equals that of the $^{64}$Ni system and when $R_{Flow}=0$ the $^{64}$Zn and $^{70}$Zn systems have equivalent values of flow. In Figure 2, the individual $R_{Flow}$ values of the Z=6-9 fragments and the average $R_{Flow}$ value of Z=4-9 fragments are plotted as a function of the centrality bin number. The ratio values exhibit a systematic trend from $R_{Flow} \approx 1$ for the most violent collisions to $R_{Flow} \approx 0$ for the most peripheral reactions. This trend, observed in Figures 1 and 2, shows a transition from the IMF’s $\langle P_x \rangle$ being strongly dependent on the mass of the system to a dependence on the charge of the system.

![Figure 2](image.png)

The observed mass to charge dependence of the IMF transverse flow should be sensitive to the density dependence of the symmetry energy since there is a mean-field component to the flow.
Scalone et al. predicted that the difference in the transverse flow of LCPs from two systems with the same mass and differing \((N/Z)_{sys}\) would be sensitive to \(E_{sym}(\rho)\) \cite{34}. Therefore, changing the isospin-dependent part of the mean-field should affect the balance between the mass and charge dependent forces.

The antisymmetrized molecular dynamics with wave packet Diffusion and Shrinking (AMD-DS) model \cite{25} was used to investigate the sensitivity of the IMF flow to \(E_{sym}(\rho)\). The dynamics of the reaction were simulated up to a time of 300 fm/c, after which the GEMINI code \cite{35} was used to statistically de-excite the hot fragments. The momentum dependent Gogny and Gogny-AS effective interactions provided an incompressibility of symmetric nuclear matter of \(K=228\) MeV while allowing for the density dependence of the symmetry energy to be varied \cite{25}. The Gogny and Gogny-AS interactions produce a soft and stiff density dependence of the symmetry energy, respectively \cite{36}.

In Figure 3 the average \(R_{Flow}\) value for \(Z=4-9\) fragments is shown as a function of \(b_{\text{red}}\) from the AMD-Gemini simulations in comparison to the experimental data. The experimental results are equivalent to those presented in Figure 2 except that \(R_{Flow}\) is shown as a function of the average \(b_{\text{red}}\) of each centrality bin. The average \(b_{\text{red}}\) was determined using the filtered molecular dynamics simulations to provide an estimate of the impact parameter range selected in each centrality bin. The impact parameter for each event of the AMD-Gemini simulation was known and, therefore, the average \(b_{\text{red}}\) values shown in Figure 3 are exact. While the same experimental procedure discussed above was used to extract the IMF flow, the AMD-Gemini results (Figure 3) were not filtered due to statistical limitations.

![Figure 3](image-url)

**Figure 3.** Average \(R_{Flow}\) for \(Z=4-9\) fragments \((\langle R_{Flow} \rangle_{Z=4-9})\) is plotted as a function of the reduced impact parameter, \(b_{\text{red}}\), for the experimental data (yellow area) and AMD-Gemini simulation with both a stiff (red squares) and soft (blue open squares) \(E_{sym}(\rho)\).

The results of Figure 3 demonstrate that the differences in the IMF flow between systems have a strong sensitivity to the density dependence of the symmetry energy. The Gogny-AS interaction, or stiff \(E_{sym}(\rho)\), clearly demonstrates the best agreement with the experimental data, showing a decreasing \(\langle R_{Flow} \rangle_{Z=4-9}\) value with increasing \(b_{\text{red}}\). In comparison, the soft symmetry energy parameterization, or Gogny interaction, is unable to reproduce the experimental trend. In the Gogny calculation the \(^{64}\text{Zn}\) flow increases relative to the \(^{64}\text{Ni}\) flow, eventually becoming larger (\(R_{Flow} > 1\)). This is related to the larger symmetry energy at low-density for the Gogny interaction, which is more repulsive for the more neutron-rich \(^{64}\text{Ni}\) system relative to the \(^{64}\text{Zn}\) system. The isospin-dependent part of the Gogny-AS interaction is less repulsive at low-density and therefore, the \(^{64}\text{Ni}\) flow remains larger than the \(^{64}\text{Zn}\) flow producing agreement with the experimental data. It is clear that the isospin-dependent part of the interaction is an important component in describing the observed transition from a mass to charge dependence of the IMF transverse flow.
5. Conclusions
The transverse flow of the IMFs has been shown to be sensitive to both the mass and charge of the colliding system. The strong dependence of the IMF flow on the charge of the system in the peripheral collisions provides new insight into the mechanisms responsible for the transverse flow. The AMD-Gemini simulation demonstrated that the differences in the IMF flow between the systems was sensitive to the isospin-dependent part of the nucleon-nucleon interaction. Comparison between the experiment and AMD-Gemini provided strong evidence supporting a stiff $E_{sym}(\rho)$. Future research examining the IMF flow should allow for additional constraints on the nuclear Equation of State.

6. Acknowledgements
We would like to thank the staff members of the Texas A&M Cyclotron Institute for the excellent beam quality. This work was supported in part by the Robert A. Welch Foundation through grant No. A-1266, and the Department of Energy through grant No. DE-FG03-93ER40773. We would further like to thank Dr. A. Ono for use of the AMD code and Dr. R. J. Charity for use of the Gemini code.

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