Squark flavor mixing and CP asymmetry of neutral $B$ mesons at LHCb

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Abstract

The CP violation of the neutral $B$ meson is the important phenomenon to search for the new physics. The like-sign dimuon charge asymmetry observed by the DØ Collaboration indicates the CP-violating new physics in the $B_s - \bar{B}_s$ mixing. On the other hand, LHCb observed the CP-violating asymmetry in $B^0_s \rightarrow J/\psi \phi$ and $B^0_s \rightarrow J/\psi f_0(980)$, which is consistent with the SM prediction. However, there is still room for new physics of the CP violation. The CKMfitter has presented the allowed region of the new physics parameters taking account of the LHCb data. Based on these results, we discuss the effect of the squark flavor mixing on the CP violation in the $B_d$ and $B_s$ mesons. We predict asymmetries in the non-leptonic decays $B^0_d \rightarrow \phi K_S$, $B^0_d \rightarrow \eta' K^0$, $B^0_s \rightarrow \phi \phi$ and $B^0_s \rightarrow \phi \eta'$. 

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1 Introduction

The CP violation in the $K$ and $B_d$ mesons has been successfully understood within the framework of the standard model (SM), so called Kobayashi-Maskawa (KM) model [1]. The source of the CP violation is the KM phase in the quark sector with three families. Until now, the KM phase has successfully described the experimental data of the CP violation of $K$ and $B_d$ mesons.

However, there could be new sources of the CP violation if the SM is extended to the supersymmetric (SUSY) models. The CP-violating phases appear in soft scalar mass matrices. These phases contribute to flavor changing neutral currents with the CP violation. Therefore, we should examine carefully CP-violating phenomena in the neutral mesons.

The Tevatron experiments have searched signals of the CP violation in the $B$ mesons. Recently, the DØ Collaboration reported the interesting result of the like-sign dimuon charge asymmetry $A_{b\text{sl}}(DØ) = -7.87 \pm 1.72 \pm 0.93 \times 10^{-3}$ [2]. This result is larger than the SM prediction $A_{b\text{sl}}(\text{SM}) = -2.3 \pm 0.5 \times 10^{-4}$ [2, 3] at the 3.9 $\sigma$ level, which indicates the CP-violating new physics in the $B_s$-$\bar{B}_s$ mixing [4, 5].

On the other hand, the LHCb [6, 7] and the CDF [8] observed the CP-violating phase $\phi_s$ in the non-leptonic decays of $B_s^0 \rightarrow J/\psi\phi$ and $B_s^0 \rightarrow J/\psi f_0(980)$. Those results are consistent with the SM prediction. However, there is still room for new physics on the CP violation of the $B$ meson. Actually, the CKMfitter has presented the allowed region of the new physics parameters taking into account of LHCb data [9, 10]. (See also the work in Ref. [11].)

The typical new physics is the gluino-squark mediated flavor changing process based on the SUSY model [12]-[21]. Relevant mass insertion parameters can explain the anomalous CP violation in the $B_s$ meson. In this paper, we discuss the effect of the squark flavor mixing on the CP violation in the non-leptonic decays of $B_d$ and $B_s$ taking account of the recent LHCb experimental data. Then, the CP-violating phases of the squark flavor mixing are constrained by the chromo electric dipole moment (cEDM) of strange quark [22, 23, 24].

The prediction of asymmetries in the penguin dominated decays is the crucial test of the squark flavor mixing. We predict the asymmetries of $B^0_d \rightarrow \phi K_S$, $B^0_d \rightarrow \eta' K^0$, $B^0_s \rightarrow \phi \phi$ and $B^0_s \rightarrow \phi \eta'$ decays.

In section 2, we summarize the recent experimental situation in the CP violation of the neutral $B$ mesons. In section 3, we discuss the contribution of the squark flavor mixing on the CP violation in the $B$ mesons. We also discuss the constraints from the $b \rightarrow s\gamma$ process and the cEDM of the strange quark. In section 4, we present the numerical result of the CP violation in the non-leptonic decays of $B$ mesons. Section 5 is devoted to the summary and discussion.

2 New physics of CP violation in $B_q$-$\bar{B}_q$ system

Let us discuss the possible contribution of the new physics on the $B_q$-$\bar{B}_q (q = d, s)$ system. The Tevatron experiment reported about the CP violation in like-sign dimuon charge asymmetry $A_{\text{sl}}^b$, which is defined as [2, 25]

$$A_{\text{sl}}^b \equiv \frac{N_{b}^{++} - N_{b}^{--}}{N_{b}^{++} + N_{b}^{--}} = (0.506 \pm 0.043)a_{sl}^d + (0.494 \pm 0.043)a_{sl}^s \ . \quad (1)$$
Here, $N_b^{±±}$ is the number of events of $b\bar{b} \to \mu^{±±}X$, and the "wrong-sign" charge asymmetry $a_{sl}^q$ of $B_q \to \mu^- X$ decay is defined as

$$a_{sl}^q \equiv \frac{\Gamma(B_q^0 \to \mu^+ X) - \Gamma(B_q^0 \to \mu^- X)}{\Gamma(B_q^0 \to \mu^+ X) + \Gamma(B_q^0 \to \mu^- X)} \sim \text{Im} \left( \frac{\Gamma_{12}^q}{M_{12}^q} \right),$$

(2)

where $M_{12}^q$ and $\Gamma_{12}^q$ are dispersive and absorptive parts in the effective Hamiltonian of the $B_q - \bar{B}_q$ system, respectively. The SM prediction of $A_{sl}^b$ is given as [2]

$$A_{sl}^b(\text{SM}) = (-2.3^{+0.5}_{-0.6}) \times 10^{-4},$$

(3)

which is calculated from [3]

$$a_{sl}^d(\text{SM}) = (-4.8^{+1.0}_{-1.2}) \times 10^{-4}, \quad a_{sl}^s(\text{SM}) = (2.06 \pm 0.57) \times 10^{-5}.$$  

(4)

The DØ Collaboration reported $A_{sl}^b$ with 9.0 fb$^{-1}$ data set as [2]

$$A_{sl}^b(\text{DØ}) = -(7.87 \pm 1.72 \pm 0.93) \times 10^{-3},$$

(5)

which shows 3.9 $\sigma$ deviation from the SM prediction of Eq. (3).

Therefore, we consider the new physics beyond the SM. The contribution of new physics to the dispersive part $M_{12}^q$ is parameterized as

$$M_{12}^q = M_{12}^{q,\text{SM}} + M_{12}^{q,\text{NP}} = M_{12}^{q,\text{SM}} \left( 1 + h_q e^{2i\sigma_q} \right), \quad (q = d, s)$$

(6)

where $M_{12}^{q,\text{NP}}$ are new physics contributions, and the SM contributions $M_{12}^{q,\text{SM}}$ are given as [26]

$$M_{12}^{q,\text{SM}} = \frac{G_F^2 M_{B_q}}{12\pi^2} M_{W}^2 (V_{tb} V_{tq}^*)^2 \hat{\eta}_B S_0(x_t) f^2_{B_q} Q_B.$$  

(7)

The SM contribution to the absorptive part $\Gamma_{12}^q$ is dominated by tree-level decay $b \to c\bar{c}s$, $\tau^+\tau^-$, and etc. Then, we assume $\Gamma_{12}^q = \Gamma_{12}^{q,\text{SM}}$. Numerical values of the new physics parameters $h_q$ and $\sigma_q$ have been obtained by the CKMfitter [9, 10].

Let us discuss the effect of the new physics in the non-leptonic decays of $B$ mesons. The time dependent CP asymmetry decaying into the final state $f$, which is defined as [27]

$$S_f = \frac{2\text{Im} \lambda_f}{|\lambda_f|^2 + 1},$$

(8)

where

$$\lambda_f = \frac{q}{p} \bar{\rho}, \quad \bar{\rho} \equiv \frac{A(B_q^0 \to f)}{A(B_q^0 \to \bar{f})}.$$  

(9)

In the decay of $B_d^0 \to J/\psi K_S$, we take

$$\lambda_{J/\psi K_S} = -e^{-i\phi_d}, \quad \phi_d = 2\beta_d + \text{arg}(1 + h_d e^{2i\sigma_d}),$$

(10)

by putting $|\bar{\rho}| = 1$ and $q/p \simeq \sqrt{M_{12}^{q,\text{SM}}/M_{12}^q}$, where the phase $\beta_d$ is given in the SM. The CKMfitter provided the allowed region of $h_d$ and $\sigma_d$, where the central value is [9, 10]

$$h_d \simeq 0.3, \quad \sigma_d \simeq 1.8 \text{ rad}.$$  

(11)
Since penguin processes are dominant in the case of \( f = \phi K_S, \eta' K^0 \), the loop induced new physics could contribute considerably on the CP violation of those decays. Then, those \( S_f \) is not any more same as \( S_{J/\psi K_S} \) due to \( |\rho| \neq 1 \). Those predictions provide us good tests for the new physics.

In the decay of \( B^0_s \to J/\psi\phi \), we have

\[
\lambda_{J/\psi\phi} = e^{-i\phi_s}, \quad \phi_s = -2\beta_s + \text{arg}(1 + h_s e^{2i\sigma_s}),
\]

where \( \beta_s \) is given in the SM.

Recently the LHCb [6] presented the observed CP-violating phase \( \phi_s \) in \( B^0_s \to J/\psi\phi \) and \( B^0 \to J/\psi f_0(980) \) decays using about 340 pb\(^{-1}\) of data. The combination of these results lead to

\[
\phi_s = 0.07 \pm 0.17 \pm 0.06 \text{ rad}.
\]

On the other hand, the SM prediction is [9]

\[
\phi_s^{J/\psi\phi,\text{SM}} = -2\beta_s = -0.0363 \pm 0.0017 \text{ rad}.
\]

Taking account of these data, the CKMfitter has presented the allowed values of \( h_s \) and \( \sigma_s \) [9, 10]. The allowed region is rather large including zero values. In order to investigate possible contribution of the new physics, we take the central values

\[
h_s = 0.1, \quad \sigma_s = 0.9 - 2.2 \text{ rad},
\]

as a typical parameter set in our work.

### 3 Squark flavor mixing

As the new physics contributing on the CP violation of the neutral \( B \) meson, we study the effect of the squark flavor mixing in SUSY. Let us consider the flavor structure of squarks, which gives the flavor changing neutral currents. When three families correspond to a triplet of a certain flavor symmetry, for example \( A_4 \) and \( S_4 \) [28], the squark mass matrix is diagonal with three degenerate masses in the supersymmetric limit. Then, the SUSY breaking induces soft SUSY breaking terms such as squark masses and scalar trilinear couplings, i.e. the so-called A-terms. The breaking of the flavor symmetry gives the small soft masses compared with the diagonal ones in the squark mass matrices. Therefore, in the super-CKM basis, we parametrize the soft scalar masses squared \( M_{d_{LL}}^2, M_{d_{RR}}^2, M_{d_{LR}}^2 \), and \( M_{d_{RL}}^2 \) for the down-type squarks as follows:

\[
M_{d_{LL}}^2 = m_q^2 \begin{pmatrix}
1 + (\delta_d^{LL})_{11} & (\delta_d^{LL})_{12} & (\delta_d^{LL})_{13} \\
(\delta_d^{LL})_{12} & 1 + (\delta_d^{LL})_{22} & (\delta_d^{LL})_{23} \\
(\delta_d^{LL})_{13} & (\delta_d^{LL})_{23} & 1 + (\delta_d^{LL})_{33}
\end{pmatrix},
\]

\[
M_{d_{RR}}^2 = m_q^2 \begin{pmatrix}
1 + (\delta_d^{RR})_{11} & (\delta_d^{RR})_{12} & (\delta_d^{RR})_{13} \\
(\delta_d^{RR})_{12} & 1 + (\delta_d^{RR})_{22} & (\delta_d^{RR})_{23} \\
(\delta_d^{RR})_{13} & (\delta_d^{RR})_{23} & 1 + (\delta_d^{RR})_{33}
\end{pmatrix},
\]

\[
M_{d_{LR}}^2 = (M_{d_{RL}}^2)^\dagger = m_q^2 \begin{pmatrix}
(\delta_d^{LR})_{11} & (\delta_d^{LR})_{12} & (\delta_d^{LR})_{13} \\
(\delta_d^{LR})_{12} & (\delta_d^{LR})_{22} & (\delta_d^{LR})_{23} \\
(\delta_d^{LR})_{13} & (\delta_d^{LR})_{23} & (\delta_d^{LR})_{33}
\end{pmatrix},
\]

\[
M_{d_{RL}}^2 = (M_{d_{LR}}^2)^\dagger = m_q^2 \begin{pmatrix}
(\delta_d^{LR})_{11} & (\delta_d^{LR})_{12} & (\delta_d^{LR})_{13} \\
(\delta_d^{LR})_{12} & (\delta_d^{LR})_{22} & (\delta_d^{LR})_{23} \\
(\delta_d^{LR})_{13} & (\delta_d^{LR})_{23} & (\delta_d^{LR})_{33}
\end{pmatrix},
\]

(16)
where \( m_\tilde{q} \) is the average squark mass, and \((\delta^L_{d})_{ij}, (\delta^R_{d})_{ij}, (\delta^R_{d})_{ij}, \text{ and } (\delta^R_{d})_{ij}\) are called as the mass insertion (MI) parameters. The MI parameters are supposed to be much smaller than 1.

The SUSY contribution by the gluino-squark box diagram to the dispersive part of the effective Hamiltonian for the \( B_q\bar{B}_q \) mixing are written as \[29, 30\]

\[
M^{q,\text{SUSY}}_{12} = A_1^q \left[ A_2 \left\{ (\delta^L_{d})^2_{ij} + (\delta^R_{d})^2_{ij} \right\} + A_3^q (\delta^L_{d})_{ij} (\delta^R_{d})_{ij} \right. \\
+ \left. A_4^q \left\{ (\delta^L_{d})^2_{ij} + (\delta^R_{d})^2_{ij} \right\} + A_5^q (\delta^L_{d})_{ij} (\delta^R_{d})_{ij} \right],
\]

where

\[
A_1^q = -\frac{\alpha_S^2}{216m_\tilde{g}^2} \frac{2}{3} M_{B_q} f_{B_q}, \quad A_2 = 24 x f_6(x) + 66 \tilde{f}_6(x),
\]

\[
A_3^q = \left\{ 384 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 + 72 \right\} x f_6(x) + \left\{ -24 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 + 36 \right\} \tilde{f}_6(x),
\]

\[
A_4^q = \left\{ -132 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 \right\} x f_6(x), \quad A_5^q = \left\{ -144 \left( \frac{M_{B_q}}{m_j + m_i} \right)^2 - 84 \right\} \tilde{f}_6(x).
\]

Here, we use \( x = m_\tilde{g}^2/m_\tilde{q}^2 \), where \( m_\tilde{g} \) is the gluino mass. For the cases of \( q = d \) and \( q = s \), we take \((i, j) = (1, 3)\) and \((i, j) = (2, 3)\), respectively, where \( m_1 = m_d, m_2 = m_s \) and \( m_3 = m_b \).

The loop functions \( f_6(x) \) and \( \tilde{f}_6(x) \) are given later in Eq.\([29]\).

Let us discuss the setup for the MI parameters in our analysis. For the case of \( x \approx 1 \), we estimate \( A_2 \approx -1, A_3^q \approx 30, A_4^q \approx -10 \) and \( A_5^q \approx 10 \). Therefore, we consider the case that \((\delta^L_{d})_{ij}\) and \((\delta^R_{d})_{ij}\) dominate \( M^{q,\text{SUSY}}_{12} \). Actually, magnitudes of \((\delta^L_{d})_{ij}\) and \((\delta^R_{d})_{ij}\) are constrained severely by the \( b \to s\gamma \) decay.

Including the double mass insertion, the transition amplitude of \( b \to s\gamma \) from the squark flavor mixing is given as \[31, 29, 30\]

\[
A^{\text{SUSY}} (b_L \to s_R \gamma) \propto m_b M_3(x)(\delta^L_{d})_{23} + m_\tilde{g} M_4(x)(\delta^R_{d})_{33}(\delta^L_{d})_{23} + m_\tilde{g} M_5(x)(\delta^R_{d})_{23},
\]

where functions \( M_1(x), M_3(x) \) and \( M_4(x) \) are given in Eq.\([29]\). At the electroweak scale, \((\delta^L_{d})_{33}\) is given in terms of \( \tan \beta \) and \( \mu \) as

\[
(\delta^L_{d})_{33} = m_b \frac{A_b - \mu \tan \beta}{m_\tilde{q}^2},
\]

where \( A_b \) is the A-term given at the high energy scale. In our numerical study, \( A_b \) is taken to be 0. Since \( m_\tilde{g} \gg m_q \), the magnitudes of \((\delta^L_{d})_{23}\) and \((\delta^R_{d})_{23}\) should be much smaller than \((\delta^L_{d})_{23}\) and \((\delta^R_{d})_{23}\).

Therefore, we consider the contribution from \((\delta^L_{d})_{ij}\) and \((\delta^R_{d})_{ij}\) in \( M^{q}_{12} \). In order to estimate the larger contribution of squark flavor mixing on \( M^{q}_{12} \) with keeping smaller magnitudes of MI parameters, we take \(|(\delta^L_{d})_{ij}| = |(\delta^R_{d})_{ij}|\). This condition is derived from that the coefficient \( A_3^q \) is much larger than \( A_2 \). On the other hand, we take phases of these MI
parameters $\theta_{ij}^{LL}$ and $\theta_{ij}^{RR}$ to be different each other. Therefore, we can parametrize the MI parameters as follows:

$$
(\delta_{d}^{LL})_{ij} = r_{ij}e^{2i\theta_{ij}^{LL}}, \quad (\delta_{d}^{RR})_{ij} = r_{ij}e^{2i\theta_{ij}^{RR}}.
$$

(21)

Since magnitudes of $(\delta_{d}^{LR})_{23}$ and $(\delta_{d}^{RL})_{23}$ are expected to be tiny from $b \to s\gamma$, we neglect them in our following calculations. Then, $r_{ij}$, $\theta_{ij}^{LL}$, and $\theta_{ij}^{RR}$ are related with the new physics contribution $h_q$ and $\sigma_q$. Inserting Eq.(17) with Eq.(21) into the following ratio

$$
\frac{M_{12}^{q,SUSY}}{M_{12}^{q,SM}} = h_q e^{2i\sigma_q},
$$

(22)

we obtain two equations as follows:

$$
r_{ij} = \sqrt{\frac{h_q |M_{12}^{q,SM}|}{|A_1^q (2A_2 \cos 2(\theta_{ij}^{LL} - \theta_{ij}^{RR}) + A_3^q)|}},
$$

$$
\theta_{ij}^{LL} + \theta_{ij}^{RR} = \sigma_q + \phi_q^{SM} + \frac{n\pi}{2}, \quad (n = 0, \pm1, \pm2, \cdots),
$$

(23)

where $(\delta_{d}^{LR})_{ij} = (\delta_{d}^{RL})_{ij} = 0$ is taken. Here, we use the definition $2\phi_q^{SM} = \text{arg}(M_{12}^{q,SM})$ in the CKM basis. The numerical study of these parameters are presented in the next section.

There is another constraint for MI parameters from the cEDM of the strange quark. The T violation is expected to be observed in the electric dipole moment of the neutron. The experimental upper bound of the electric dipole moment of the neutron provides us the upper-bound of the cEDM of the strange quark [22, 23, 24]. The cEDM of the strange quark was discussed to constrain the MI parameters $(\delta_{d}^{LL})_{23}$ and $(\delta_{d}^{RR})_{23}$ [14, 22, 23, 32].

The cEDM of the strange quark is given by

$$
d_s^C = c \frac{\alpha_s m_q}{4\pi} \left( -\frac{1}{3} N_1(x) - 3N_2(x) \right) \text{Im} \left[ (\delta_{d}^{LL})_{23}(\delta_{d}^{LR})_{33}(\delta_{d}^{RR})_{23} \right],
$$

(24)

where $c$ is the QCD correction, and $c = 0.9$ is taken. The $N_1(x)$ and $N_2(x)$ are given in Eq.(20). By using Eq.(20) and Eq.(21) with $A_h = 0$, $d_s^C$ is rewritten as

$$
d_s^C = c \frac{\alpha_s m_q m_b \mu \tan \beta}{4\pi m_q^4} \left( \frac{1}{3} N_1(x) + 3N_2(x) \right) r_{23}^2 \sin 2(\theta_{23}^{LL} - \theta_{23}^{RR}).
$$

(25)

Thus, the phase difference $(\theta_{23}^{LL} - \theta_{23}^{RR})$ is constrained from the experimental upper bound $c|d_s^C| < 1 \times 10^{-25}\text{ecm}$ [14, 22, 23, 32].

The squark flavor mixing can be tested in the CP-violating asymmetries in the neutral B meson decays. Since the $B_s^0 \to J/\psi K_S$ process occurs at the tree level of SM, the CP-violating asymmetry originates from $M_{12}^d$. Although the $B_d^0 \to \phi K_S$ and $B_d^0 \to \eta' K^0$ decays are penguin dominant ones, their asymmetries also come from $M_{12}^d$ in SM. Then, asymmetries of $B_s^0 \to J/\psi K_S$, $B_s^0 \to \phi K_S$ and $B_s^0 \to \eta' K^0$ are expected to be same magnitude. On the other hand, if the squark flavor mixing contributes to the decay at the one-loop level, its magnitude could be comparable to the SM penguin one in $B_d^0 \to \phi K_S$ and $B_d^0 \to \eta' K^0$, but
it is tiny in $B_d^0 \to J/\psi K_S$. Therefore, it is important to study carefully these asymmetries [33].

Let us present the framework of these calculations. The effective Hamiltonian for $\Delta B = 1$ process is defined as

$$H_{eff} = \frac{4G_F}{\sqrt{2}} \left[ \sum_{q'=u,c} V_{qb} V_{q'b}^{*} \sum_{i=1,2} C_i O_i^{(q')} - V_{tb} V_{ts}^{*} \sum_{i=3-7,8G} \left( C_i O_i + \tilde{C}_i \tilde{O}_i \right) \right], \quad (26)$$

where the local operators are given as

$$O_1^{(q')} = (\bar{s}_i \gamma_\mu P_L q'_j)(\bar{q}'_j \gamma^\mu P_L b_i), \quad O_2^{(q')} = (\bar{s}_i \gamma_\mu P_L q'_j)(\bar{q}'_j \gamma^\mu P_L b_j),$$

$$O_3 = (\bar{s}_i \gamma_\mu P_L b_i) \sum_q (\bar{q}_j \gamma^\mu P_L q_j), \quad O_4 = (\bar{s}_i \gamma_\mu P_L b_j) \sum_q (\bar{q}_j \gamma^\mu P_L q_i),$$

$$O_5 = (\bar{s}_i \gamma_\mu P_L b_i) \sum_q (\bar{q}_j \gamma^\mu P_R q_j), \quad O_6 = (\bar{s}_i \gamma_\mu P_L b_j) \sum_q (\bar{q}_j \gamma^\mu P_R q_i),$$

$$O_{7\gamma} = \frac{e}{16\pi^2} m_b \bar{s}_i \gamma_\mu \gamma^\nu P_R b_i F_{\mu\nu}, \quad O_{8G} = \frac{g_s}{16\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} P_R T_i b_j G_{\mu\nu}, \quad (27)$$

where $P_R = (1 + \gamma_5)/2$, $P_L = (1 - \gamma_5)/2$, and $i$ and $j$ are color indices, and $q$ is taken to be $u, d, s, c$. Here, $C_i$'s are the Wilson coefficients, and $\tilde{C}_i$'s are the operators by replacing $L(R)$ with $R(L)$ in $O_i$. In our work, $C_i$ includes both SM contribution and gluino one, such as $C_i = C_i^{SM} + C_i^{\tilde{g}}$, where $C_i^{SM}$ is given in Ref. [34] and $C_i^{\tilde{g}}$ is presented as follows [35]:

$$C_3^{\tilde{g}} \approx \frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{ts}^{*} m_q^2} \left( \delta_{dL}^{LL} \right)_{23} \left[ - \frac{1}{9} B_1(x) - \frac{5}{9} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right],$$

$$C_4^{\tilde{g}} \approx \frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{ts}^{*} m_q^2} \left( \delta_{dL}^{LL} \right)_{23} \left[ - \frac{7}{3} B_1(x) + \frac{1}{3} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right],$$

$$C_5^{\tilde{g}} \approx \frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{ts}^{*} m_q^2} \left( \delta_{dL}^{LL} \right)_{23} \left[ \frac{10}{9} B_1(x) + \frac{1}{3} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right],$$

$$C_6^{\tilde{g}} \approx \frac{\sqrt{2} \alpha_s^2}{4 G_F V_{tb} V_{ts}^{*} m_q^2} \left( \delta_{dL}^{LL} \right)_{23} \left[ - \frac{2}{3} B_1(x) + \frac{7}{6} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right],$$

$$C_7^{\tilde{g}_\gamma} \approx - \frac{\sqrt{2} \alpha_s \pi}{6 G_F V_{tb} V_{ts}^{*} m_q^2} \left( \delta_{dL}^{LL} \right)_{23} \left[ \frac{8}{3} M_3(x) - \mu \tan \beta \frac{m_{\tilde{g}}}{m_{\tilde{q}}} 8 M_4(x) \right] + \left( \delta_{dL}^{LR} \right)_{23} \frac{m_{\tilde{g}}}{m_q} \frac{8}{3} M_1(x),$$

$$C_8^{\tilde{g}_G} \approx - \frac{\sqrt{2} \alpha_s \pi}{2 G_F V_{tb} V_{ts}^{*} m_q^2} \left( \delta_{dL}^{LL} \right)_{23} \left\{ \left( \frac{1}{3} M_3(x) + 3 M_4(x) \right) \right.$$  

$$- \mu \tan \beta \frac{m_{\tilde{g}}}{m_q} \left( \frac{1}{3} M_3(x) + 3 M_4(x) \right) \right\} + \left( \delta_{dL}^{LR} \right)_{23} \frac{m_{\tilde{g}}}{m_q} \left( \frac{1}{3} M_1(x) + 3 M_2(x) \right). \quad (28)$$

The Wilson coefficients $\tilde{C}_i^{\tilde{g}}$'s are obtained by replacing $L(R)$ with $R(L)$ in $C_i^{\tilde{g}}$'s. The loop
functions, which we use in our calculations, are summarized as

\[
\begin{align*}
  f_6(x) &= \frac{6(1 + 3x) \log x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5}, \\
  \bar{f}_6(x) &= \frac{6x(1 + x) \log x - x^3 - 9x^2 + 9x + 1}{3(x - 1)^5}, \\
  N_1(x) &= \frac{3 + 44x - 36x^2 - 12x^3 + x^4 + 12x(2 + 3x) \log x}{6(1 - x)^6}, \\
  N_2(x) &= \frac{-10 + 9x - 18x^2 - x^3 + 3(1 + 6x + 3x^2) \log x}{3(1 - x)^6}, \\
  B_1(x) &= \frac{1 + 4x - 5x^2 + 4x \log x + 2x^2 \log x}{8(1 - x)^4}, \\
  B_2(x) &= \frac{x - 5x - 2 \log x + 4x \log x}{2(1 - x)^4}, \\
  P_1(x) &= \frac{1 - 6x + 18x^2 - 10x^3 - 3x^4 + 12x^3 \log x}{18(x - 1)^5}, \\
  P_2(x) &= \frac{7 - 18x + 9x^2 + 2x^3 + 3 \log x - 9x^2 \log x}{9(x - 1)^5}, \\
  M_1(x) &= 4B_1(x), \\
  M_2(x) &= -xB_2(x), \\
  M_3(x) &= \frac{-1 + 9x + 9x^2 - 17x^3 + 18x^2 \log x + 6x^3 \log x}{12(x - 1)^5}, \\
  M_4(x) &= \frac{-1 - 9x + 9x^2 + x^3 - 6x \log x + 6x^2 \log x}{6(x - 1)^5}, \\
  M_a(x) &= \frac{1 + 9x - 9x^2 - x^3 + (6x + 6x^2) \log x}{2(x - 1)^5}, \\
  M_b(x) &= \frac{-3 - 3x^2 + (1 + 4x + x^2) \log x}{(x - 1)^5}.
\end{align*}
\]

The CP-violating asymmetries \( S_f \) in Eq. (5) are calculated by using \( \lambda_f \), which is given for \( B_d^0 \to \phi K_S \) and \( B_d^0 \to \eta' K^0 \) as follows:

\[
\lambda_{\phi K_S, \eta' K^0} = -e^{-i\phi_d} \sum_{i=3-6,7,8G} \left( C_i^{\text{SM}} \langle O_i \rangle + C_i^{q} \langle O_i \rangle + \tilde{C}_i^{q} \langle \tilde{O}_i \rangle \right) \sum_{i=3-6,7,8G} \left( C_i^{\text{SM}^*} \langle O_i \rangle + C_i^{q^*} \langle O_i \rangle + \tilde{C}_i^{q^*} \langle \tilde{O}_i \rangle \right),
\]

(30)

where \( \langle O_i \rangle \) is the abbreviation of \( \langle f|O_i|B_q^0 \rangle \). It is noticed that \( \langle \phi K_S|O_i|B_d^0 \rangle = \langle \phi K_S|\tilde{O}_i|B_d^0 \rangle \) and \( \langle \eta' K^0|O_i|B_d^0 \rangle = -\langle \eta' K^0|\tilde{O}_i|B_d^0 \rangle \) because of the parity of the final state. We have also \( \lambda_f \)
for $B^0_s \to \phi \phi$ and $B^0_s \to \phi \eta'$ as follows:

$$
\chi_{\phi \phi, \phi \eta'} = e^{-i\phi_s} \sum_{i=3-6,7,8G} C^{SM}_i \langle O_i \rangle + C^{g}_i \langle O_i \rangle + C^{g\phi}_i \langle O_i \rangle,
$$

with $\langle \phi \phi | O_i | B^0_s \rangle = -\langle \phi \phi | \tilde{O}_i | B^0_s \rangle$ and $\langle \phi \eta' | O_i | B^0_s \rangle = \langle \phi \eta' | \tilde{O}_i | B^0_s \rangle$.

Although the $C^3_{8G} \langle O_{8G} \rangle$ dominates these decay amplitude, we take account of other terms in our calculations. Therefore, we estimate each hadronic matrix elements by using the factorization relations in Ref. [36].

We remark numerical input of phases $\phi_d$ and $\phi_s$. The phase $\phi_d$ is derived from the observed value $S_f = 0.671 \pm 0.023$ in $B^0_d \to J/\psi K_S$ [37] because we have $\lambda_f = -e^{-i\phi_d}$ for $f = J/\psi K_S$. On the other hand, we use the SM value of $\beta_s$ and the values of the new physics parameters, $h_s$ and $\sigma_s$ in Eq. (15) to estimate $\phi_s = -2\beta_s + \arg(1 + h_s e^{2i\sigma_s})$. We do not use the observed value of $\phi_s$ in $B^0_s \to J/\psi \phi$ due to the large experimental error in Eq. (13).

In our framework, we have taken the assumption $|\langle \delta_d^{LL} \rangle_{ij}| = |\langle \delta_d^{RR} \rangle_{ij}|$. Let us compare our numerical results with the ones from another assumption, in which $\delta_d^{RR} = 0$ is taken. Then, the MI parameters come from only left-handed soft scalar masses and phase is only one. Now, the SUSY contribution by gluino-squark box diagram to the dispersive part of the effective Hamiltonian for the $B_q \tilde{B}_q$ mixing is simply written as

$$
M_{12}^{SUSY} = A_1^d A_2 (\delta_d^{LL})_{ij}^2.
$$

Then, the magnitude of the MI parameters and the phase are given as

$$
r_{ij} = \sqrt{\frac{h_q |M_{12}^{SM}|}{|A_1^d A_2|}},
$$

$$
\theta_{ij}^{LL} = \frac{1}{2} \sigma_q + \frac{1}{2} \phi_q^{SM} + \frac{n\pi}{4}, \quad (n = 0, \pm 1, \pm 2, \cdots),
$$

instead of Eq. (33). The numerical discussion are presented in the next section.

4 Numerical analysis

Let us show numerical results. The magnitude of the MI parameter $r_{23}$ is calculated from Eq. (23) or Eq. (33), where $M_{12}^{SM}$ is fixed by putting relevant parameters shown in Table 1. The phases $\theta_{23}^{LL}$ and $\theta_{23}^{RR}$ are constrained as seen in Eq. (23) or Eq. (33). On the other hand, the cEDM of the strange quark constrains the phase difference $\theta_{23}^{LL} - \theta_{23}^{RR}$ in the case of $|\langle \delta_d^{LL} \rangle_{23}| = |\langle \delta_d^{RR} \rangle_{23}|$ as seen in Eq. (23). Especially, the constraint of the cEDM of the strange quark becomes severe in the case of larger $\mu \tan \beta$.

In our following numerical calculations, we fix the squark mass and the gluino mass as

$$
m_{\tilde{q}} = 1000 \text{ GeV}, \quad m_{\tilde{g}} = 1000 \text{ GeV}.
$$
The parameters of new physics, $h_s$ and $\sigma_s$ are given in Eq.(15). Phase parameters $\theta_{23}^{LL}$ and $\theta_{23}^{RR}$ are taken in the region $[0, \pi]$. It is noticed that the squark mass $m_\tilde{q}$ is a variable for only Figure 1. In Fig. 1(a), we show $r_{23}$ versus the squark mass value for the case of $|(\delta_{d}^{LL})_{23}| = |(\delta_{d}^{RR})_{23}|$ with $\mu \tan \beta = 5000$ GeV. The region between the upper curve and lower one is excluded by the constraint of phases $\theta_{23}^{LL}$ and $\theta_{23}^{RR}$ from the cEDM of the strange quark $d_s^C$. The value of $r_{23}$ is around 0.02 at $m_\tilde{q} = 1000$ GeV. Its value is almost same for larger $\mu \tan \beta$ such as 20000 GeV.

In Fig. 1(b), we show $r_{23}$ for the case of $(\delta_{d}^{RR})_{23} = 0$. There is no constraint from $d_s^C$ because of $(\delta_{d}^{RR})_{23} = 0$. The value of $r_{23}$ is around 0.13 at the $m_\tilde{q} = 1000$ GeV. Thus, the obtained $r_{23}$ is six times larger compared with the one for $|(\delta_{d}^{LL})_{23}| = |(\delta_{d}^{RR})_{23}|$.

The phases $\theta_{23}^{LL}$ and $\theta_{23}^{RR}$ are constrained by the CP or T violating experimental data. The cEDM of the strange quark in Eq.(24) constrains the phase difference $\theta_{23}^{LL} - \theta_{23}^{RR}$. Let us show the severe constraint from the cEDM of the strange quark. In Figs. 2(a) and 2(b), the predicted values of $d_s^C$ are presented versus the phase difference $\theta_{23}^{LL} - \theta_{23}^{RR}$ at $\mu \tan \beta = 5000$ GeV and 20000 GeV, respectively, where the red horizontal line denotes the experimental upper bound. It is noted that considerable tuning of the phase difference around $n\pi/2(n = 0, \pm 1, \cdots)$ is required for $\mu \tan \beta = 20000$ GeV. These constraints affect the CP-violating
Figure 2: The predicted cEDM of the strange quark versus the phase difference $\theta_{23}^{LL} - \theta_{23}^{RR}$ at (a) $\mu \tan \beta = 5000$ GeV and (b) $\mu \tan \beta = 20000$ GeV. The experimental upper bound is denoted by the red horizontal line.

asymmetries in the non-leptonic $B$ meson decays. On the other hand, for the case of $(\delta_{RR}^{\tau})_{23} = 0$, there is no constraint from the cEDM of the strange quark.

Figure 3: Predicted CP-violating asymmetries of $B_d^0$ non-leptonic decays in the case of $|(\delta_{d}^{LL})_{23}| = |(\delta_{d}^{RR})_{23}|$ at (a) $\mu \tan \beta = 5000$ GeV and (b) $\mu \tan \beta = 20000$ GeV. The SM prediction $S_{J/\psi K_S} = S_{\phi K_S} = S_{\eta' K^0}$ is plotted by the slant dashed lines. The experimental data with error bar is plotted by the red solid lines at 1 $\sigma$ level.

By using the constrained MI parameters, we predict the allowed region of the CP-violating asymmetries for the non-leptonic decays of the neutral $B$ mesons. Let us discuss $S_f$, which is the measure of the CP-violating asymmetry, for $B_d^0 \rightarrow J/\psi K_S$, $\phi K_S$, $\eta' K^0$. If there is no new physics, these $S_f$’s are predicted to be same ones. On the other hand, if the squark flavor mixing contributes to the decay process at the one-loop level, its magnitude is comparable to the SM penguin one in $B_d^0 \rightarrow \phi K_S$ and $B_d^0 \rightarrow \eta' K^0$, but it is negligible small in $B_d^0 \rightarrow J/\psi K_S$. Therefore, we expect different $S_f$’s for these decays from Eq.(30).

In Figs. 3(a) and 3(b), we show our predictions on the plane $S_{\phi K_S}$ and $S_{\eta' K^0}$ at $\mu \tan \beta = 5000$ GeV and $\mu \tan \beta = 20000$ GeV, respectively. The blue regions denote predicted ones.
Figure 4: Predicted CP-violating asymmetries of $B_d^0$ non-leptonic decays in the case of $(\delta_d^{RR})_{23} = 0$ at (a) $\mu \tan \beta = 5000$ GeV and (b) $\mu \tan \beta = 20000$ GeV. The SM prediction denoted by the slant dashed line is on the predicted line.

Figure 5: Predicted CP-violating asymmetries of $B_s^0$ non-leptonic decays in the case of $|(\delta_d^{LL})_{23}| = |(\delta_d^{RR})_{23}|$ at (a) $\mu \tan \beta = 5000$ GeV and (b) $\mu \tan \beta = 20000$ GeV. The central value of the SM prediction is plotted at ($-0.036, -0.036$).
Figure 6: Predicted CP-violating asymmetries of $B^0_s$ non-leptonic decays in the case of $(\delta^{RR}_{d})_{23} = 0$ at (a) $\mu \tan \beta = 5000$ GeV and (b) $\mu \tan \beta = 20000$ GeV. The central value of the SM prediction is plotted at ($-0.036, -0.036$).

Since the LHCb observed the $B^0_s \rightarrow J/\psi \phi$ decay, we can now discuss the effect of the squark flavor mixing on other CP-violating asymmetries such as the ones in $B^0_s \rightarrow \phi \phi$ and $B^0_s \rightarrow \phi \eta'$ decays. In Figs. 5(a) and 5(b), we predict the CP-violating asymmetries of $S_{\phi \phi}$ and $S_{\phi \eta'}$ decays at $\mu \tan \beta = 5000$ GeV and 20000 GeV, respectively, for the case of $|\delta^{LL}_{d}| = |\delta^{RR}_{d}|$. The blue region denotes the predicted region, and the central value of the SM prediction is plotted at ($-0.036, -0.036$), which is given in Eq. (14). As seen in Fig. 5(b), the allowed region on the $S_{\phi \phi} - S_{\phi \eta'}$ plane is complicated at $\mu \tan \beta = 20000$ GeV due to the severe phase constraint from the cEDM of the strange quark as seen in Fig. 2(a).

We also show the result of the CP-violating asymmetry for the case of $(\delta^{RR}_{d})_{23} = 0$. In Figs. 6(a) and 6(b), we predict the CP-violating asymmetries at $\mu \tan \beta = 5000$ GeV and 20000 GeV. In this case, there is no constraint from the cEDM of the strange quark. These asymmetries are expected to be observed at LHCb, and then, new physics of squark flavor mixing will be testable.

Finally, we discuss the constraint from the $b \rightarrow s \gamma$ decay, in which the transition amplitude from the squark flavor mixing is given in Eq. (19). The observed $b \rightarrow s \gamma$ branching ratio is $(3.60 \pm 0.23) \times 10^{-4}$ [37], on the other hand the SM prediction is given as $(3.15 \pm 0.23) \times 10^{-4}$ at $O(\alpha_s^2)$ [38, 39]. Therefore, the contribution of our new physics should be suppressed compared with the experimental data. For $|\delta^{LL}_{d}| = |\delta^{RR}_{d}|$, with $(\delta^{L}_{d})_{23} = \delta^{LL}_{d})_{23} = 0$, we show the branching ratio including the contribution of the SM and the squark flavor mixing versus $\mu \tan \beta$ in Figure 7(a), where we neglect the error for the SM contribution. Due to the phases $\theta^{LL}_{d}$ and $\theta^{RR}_{d}$, the predicted region is extended. As seen in Fig. 7(a), the contribution of the squark flavor mixing becomes seizable as $|\mu \tan \beta|$ increases larger than $O(5000)$ GeV. It is found that the contribution of the squark flavor mixing is consistent with the experimental data when we take account of the error for the SM prediction $(3.15 \pm 0.23) \times 10^{-4}$.

For the case of $(\delta^{RR}_{d})_{23} = 0$, the contribution of the squark flavor mixing is larger than the one in the case of $|\delta^{LL}_{d}| = |\delta^{RR}_{d}|$ as seen in Figure 7(b). The phase $\theta^{LL}_{d}$ is somewhat constrained to be consistent with the experimental data for the large $|\mu \tan \beta|$.

In conclusion, the $b \rightarrow s \gamma$ decay ratio hardly affects our predictions of the CP-violating asymmetries.
Figure 7: The $b \to s\gamma$ branching ratio versus $\mu \tan \beta$ for (a) $|\delta_d^{LL}| = |\delta_d^{RR}|$ and (b) $(\delta_d^{RR})_{23} = 0$. The region between horizontal lines is allowed by the experimental data at $3\sigma$.

5 Summary and Discussion

We have discussed the contribution of the squark flavor mixing on the CP violation in the non-leptonic decays of $B_d^0$ and $B_s^0$ mesons based on the recent LHCb data. In our predictions, we take account of the constraint from the cEDM of the strange quark, which is severe for larger $\mu \tan \beta$ such as 20000 GeV. CP-violating asymmetries of penguin dominated decays are the crucial test for the squark flavor mixing. We predict that the CP-violating asymmetries $S_f$ of $B_d^0 \to \phi K_S$ and $B_d^0 \to \eta' K^0$ could deviate considerably from the one of $B_d^0 \to \phi K_S$ if $\mu \tan \beta \simeq 20000$ GeV. Although these observed values seem to be different from the predictions of SM, more precise data are required in order to conclude the effect of the new physics. Since $B_s^0 \to J/\psi \phi$ was observed at LHCb, we have also predicted the asymmetries of $B_s^0 \to \phi \phi$ and $B_s^0 \to \phi \eta'$.

Since the global fit results of the CKMfitter do not guarantee the Tevatron anomaly, we should discuss our input parameters of NP, $h_d$, $h_s$, $\sigma_d$ and $\sigma_s$ in Eq.(11) and Eq.(15) in respect of the like-sign dimuon charge asymmetry data at the DØ Collaboration. Our parameters predict $A_{sl} = -(0.75 \sim 1.0) \times 10^{-3}$, which is significantly deviated from the SM prediction. However, the experimental value of the DØ Collaboration $-(7.87 \pm 1.72 \pm 0.93) \times 10^{-3}$ still show $3.5\sigma$ deviation from our predicted value. In conclusion, it is difficult to explain the Tevatron anomaly in our framework of the squark flavor mixing.

The magnitudes of MI parameters may be important to build a flavor model such as the flavor symmetry. In our work, we obtained $|\delta_d^{LL}| = |\delta_d^{RR}| \simeq 0.02$. Putting the central values of CKMfitter, $(h_d \sim 0.3, \sigma_d \sim 1.8 \text{ rad})$, we obtain $|\delta_d^{LL}| = |\delta_d^{RR}| \simeq 0.008$. The CP violation of the neutral $K$ meson also gives us $|\delta_d^{LL}|_{12} = |\delta_d^{RR}|_{12} \leq 10^{-6}$. Thus, we have the hierarchy of MI parameters $|\delta_d^{LL}|_{23} \geq |\delta_d^{LL}|_{13} \gg |\delta_d^{LL}|_{12}$. Such flavor structure of the squark mass matrix gives us a clue of the flavor symmetry. We will discuss the flavor symmetry in the further coming paper.

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