Can the scale factor be rippled?

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We address an issue: would the cosmological scale factor be a locally oscillating quantity? This problem is examined in the framework of two classical 1+1-dimensional models: the first one is a string against a curved background, and the second one is an inhomogeneous Bianchi I model. For the string model, it is shown that there exist the gauge and the initial condition providing an oscillation of scale factor against a slowly evolving background, which is not affected by such an oscillation “at the mean”. For the inhomogeneous Bianchi I model with the conformal time gauge, an initially homogeneous scale factor can become inhomogeneous and undergo the nonlinear oscillations. As is shown these nonlinear oscillations can be treated as a nonlinear gauge wave.

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I. INTRODUCTION

Cosmological constant problem is a subject of numerous investigations. The fact of the matter is that the enormous energy of zero point quantum fluctuations of matter fields as well as electromagnetic and gravitational waves have to produce an enormous universe expansion, which is not seen in reality. At the same time, a study of conformal fluctuations also has a long history and is still far from the completion owing to lack of a developed theory of quantum gravity.

A possible relation between these two problems has been considered in Refs. 12, 13. It has been suggested that, since the conformal fluctuations formally give a negative contribution to the Hamiltonian density, they can compensate an energy of the zero point fluctuations of matter fields, electromagnetic and gravitational waves. Nevertheless, as was shown in 14, it is not the case, and is still far from the completion owing to lack of a developed theory of quantum gravity.

In this paper we investigate the classical solutions of some simple models to determine a possibility of the scale factor modulation. This means that one explores some particular gauges and initial conditions, which result in an oscillating scale factor. This does not mean that the scale factor is an oscillating quantity under an arbitrary initial conditions. The aim of our work is to demonstrate, that the scale factor oscillations are not forbidden in general (e.g. owing to constraints).

Although, we investigate local oscillations of the scale factor here, the models admitting global oscillations are also of general interest.

II. STRING IN A BACKGROUND SPACE

The first model considered is a string against a curved background. For this model we have built a solution for which the scale factor is connected rigidly with the matter fields so that a mutual compensation of their fluctuations and, thereby, the solution of the cosmological constant problem look almost trivial.

The Lagrangian for a string against a background space is given as

\[ L = \int e^{2\alpha} \left( \frac{M_p^2}{2} (\partial_x \phi)^2 \left( N - \frac{N_1^2}{N} \right) + M_p^2 \partial_{\tau} \alpha \phi' \right) + \frac{1}{2} (\partial_x \phi)^2 \left( \frac{N_1^2}{N} - N \right) + \frac{\phi'^2}{2N} \right) dx, \]

where \( \phi(x, \tau) = \{ \phi_1(x, \tau), \phi_2(x, \tau), \ldots, \phi_N(x, \tau) \} \) are the scalar fields, \( \alpha(x, \tau) = \ln \alpha(x, \tau) \) is the logarithm of a “scale factor” \( \alpha(x) \), \( N \) and \( N_1 \) are the lapse and shift functions, respectively; \( M_p \) is a constant (an analog of the Plank mass); primes denote the derivatives over \( \tau \). This Lagrangian is not deduced directly from the general relativity action, however, it can be reduced to the form corresponding to a flat Friedman-Robertson-Walker (FRW) universe filled with the homogeneous scalar fields (i.e. if both \( \alpha \) and \( \phi \) do not depend on \( x \)). Thus Eq. (1) can be considered as a simplest inhomogeneous generalization of the FRW Lagrangian.
numerical values of parameters are $\sqrt{N}/M_p = 1.5$, $A = 1.3$, $k = 2$. Dashed line corresponds to $< \alpha >$, where the spatial averaging is implied.

Varying over $N$ and $N_1$ yields the Hamiltonian and momentum constraints, which, in the case of $N = 1$ and $N_1 = 0$, take the form:

$$\mathcal{H} = e^{2\alpha} \left( -M_p^2 \alpha'^2 - M_p(\partial_x \alpha)^2 + \phi^2 + (\partial_x \phi)^2 \right) = 0, \quad (2)$$

$$\mathcal{P} = e^{2\alpha} \left( -M_p^2 \alpha' \partial_x \alpha + \phi' \partial_x \phi \right) = 0. \quad (3)$$

Also, one can obtain the equations of motion $^{20}$

$$\phi'' - \partial_x \phi + 2\alpha' \phi' - 2\partial_x \alpha \partial_x \phi = 0, \quad (4)$$

$$M_p^2 \alpha'' - M_p^2 \partial_x \alpha + M_p^2 \alpha'^2 - M_p^2(\partial_x \alpha)^2$$

$$+ \phi^2 - (\partial_x \phi)^2 = 0. \quad (5)$$

The constraints evolve according to the equations

$$\mathcal{H}' = \partial_x \mathcal{P}, \quad \mathcal{P}' = \partial_x \mathcal{H}. \quad (6)$$

That is, if the constraints equal to zero initially, they remain to be zero during all the evolution.

Some particular solution can be easily found for the system $^{22}$, $^{23}$, $^{24}$, $^{25}$. Let us assume $\alpha(x, \tau) = \frac{\sqrt{N}}{M_p} \phi_1(x, \tau) = \frac{\sqrt{N}}{M_p} \phi_2(x, \tau) = ... \frac{\sqrt{N}}{M_p} \phi_N(x, \tau)$, then the equations of motion $^{24}$, $^{25}$ are reduced to the single equation

$$\alpha'' - \partial_x \alpha + \frac{2\sqrt{N}}{M_p} (\alpha'^2 - (\partial_x \alpha)^2) = 0. \quad (7)$$

Also, one can be sure that the constraints $^{22}$, $^{23}$ are satisfied, i.e. $\mathcal{H} = 0$, $\mathcal{P} = 0$.

It is interesting to note, that the analogous equation arises in the the Szekeres theory of colliding waves $^{21}$. However, the initial conditions for this equation in the Szekeres theory are restricted by the constraints, while in the present theory the constraints are already satisfied and the initial conditions for Eq. (7) are quite arbitrary.

FIG. 2: Evolution of $\alpha'(x, \tau)$ (a), $B'(x, \tau)$ (b) and $V'(x, \tau)$ (c) at $x = 0$ for the inhomogeneous Bianchi I model. Numerical values of the parameters are $\alpha_0 = -2$, $\pi V = 1/100$, $\pi B_0 = 2/100$, $B_0 = 0$, $V_0(x) = \cos x/100$. Dashed lines correspond to the mean values $< \alpha >$, $< B >$ and $< V >$.

The oscillations against a background can appear if the initial values meet the condition

$$|\partial_x \alpha(x, 0)| >> |\alpha'(x, 0)|. \quad (8)$$

In the opposite case, a monotonic evolution dominates.
The constraint evolution is given by (6) as it takes place
\[ \alpha \text{ the initial values of } \alpha \]
Here, the global evolution is defined by the value of \( k \) and the oscillations are defined by the amplitude of \( A \). The example of evolution at some fixed coordinate \( x \) is shown in Fig. 4.

From the point of view of the “energy balance” specified by the Hamiltonian constraint (2), the oscillations of scalar fields do not affect a slow “global” (i.e. “background”) evolution because they are compensated by the oscillations of scale factor \( \alpha \).

FIG. 3: Evolution of the dispersion \( \sqrt{< \alpha'^2 >} \). Dashed line corresponds to \( < \alpha' > \).

Solution of Eq. (7) can be written in the form of
\[ \alpha(x, \tau) = \frac{M_p}{2\sqrt{\mathcal{N}}} \ln \left( \frac{1}{2} e^{\frac{2\pi}{M_p}} \alpha_0(x+\tau) + \frac{1}{2} e^{\frac{2\pi}{M_p}} \alpha_0(x-\tau) \right) + \frac{\sqrt{\mathcal{N}}}{M_p} \int_{x-\tau}^{x+\tau} k(\xi) d\xi, \quad (9) \]
where the functions \( \alpha_0(\xi) \) and \( k(\xi) \) are connected with the initial values of \( \alpha \) and its derivative so that \( \alpha(x, 0) = \alpha_0(x), \alpha'(x, 0) = e^{-2\frac{2\pi}{M_p}} \alpha_0(x) k(x) \).

As an example, we take \( k = \text{const}, \alpha_0(\xi) = A \cos \xi \).

Then the solution of Eq. (9) takes the form
\[ \alpha(x, \tau) = \frac{M_p}{2\sqrt{\mathcal{N}}} \ln \left( \frac{1}{2} \left( e^{2A \cos(x+\tau)\sqrt{\mathcal{N}}/M_p} + e^{2A \cos(x-\tau)\sqrt{\mathcal{N}}/M_p} \right) + 2k\sqrt{\mathcal{N}}/M_p \right), \quad (10) \]

III. INHOMOGENEOUS BIANCHI I MODEL

As the next example, let analyse the inhomogeneous Bianchi I model. Hereinafter, we shall decompose the metric into the scale factor \( e^{2\alpha} \) and the components of conformal geometry, and use the conformal time \( \tau \). The argumentation for using the conformal time gauge is that it allows representing the equation of motion for a scale factor of universe in a form containing a difference of the potential and kinetic energies of the field oscillators \([22, 23]\).

Thus, the metric looks like
\[ ds^2 = e^{2\alpha} \left( d\tau^2 - e^{-4B} dx^2 - e^{2B+2\sqrt{3}V} dy^2 - e^{2B-2\sqrt{3}V} dz^2 \right), \quad (11) \]
where the functions \( \alpha, B, V \) depend on \( x \) and \( \tau \).

The Einstein equations allow obtaining the Hamiltonian and momentum constraints as well as the equations of motion:

\[ \mathcal{H} = \frac{1}{2} e^{2\alpha} \left( -\alpha'^2 + B'^2 + V'^2 \right) + e^{2\alpha+4B} \left( \frac{1}{6} (\partial_x \alpha)^2 + \frac{1}{3} \partial_{xx} \alpha + \frac{7}{6} (\partial_x B)^2 + \frac{1}{3} \partial_{xx} B + \frac{4}{3} \partial_x \alpha \partial_x B + \frac{1}{2} (\partial_x V)^2 \right), \quad (12) \]
\[ \mathcal{P} = e^{2\alpha} \left( -\frac{1}{3} \partial_x \alpha \alpha' + \partial_x BB' + \frac{2}{3} \partial_x \alpha B' + \partial_x VV' + \frac{1}{3} \partial_x B' + \frac{1}{3} \partial_x \alpha' \right), \quad (13) \]
\[ \alpha'' - e^{4B} \left( \partial_{xx} \alpha + (\partial_x \alpha)^2 + (\partial_x V)^2 + \frac{7}{3} (\partial_x B)^2 + \frac{2}{3} \partial_{xx} B + 4 \partial_x B \partial_x \alpha \right) + \alpha'^2 + V'^2 + B'^2 = 0, \quad (14) \]
\[ B'' + \frac{1}{3} e^{4B} (\partial_{xx} B + 2 \partial_x B)^2 + 6(\partial_x V)^2 - 2(\partial_x \alpha)^2 + 2 \partial_x \alpha \partial_x B + 2 \partial_x \alpha \partial_x B + 2 B' \alpha' = 0, \quad (15) \]
\[ V'' + 2V' \alpha' - e^{4B} (\partial_{xx} V + 2 \partial_x V \partial_x \alpha + 4 \partial_x B \partial_x \alpha) = 0. \quad (16) \]

It should be noted that the equations of motion can be also obtained from the Hamiltonian \( \mathcal{H} = \int \mathcal{H} dx \) directly. The constraint evolution is given by (6) as it takes place for a string.

Let ask: would the ripples of scale factor appear if it was initially homogeneous. Since there is no analytical solution, we shall investigate a particular solution of this model numerically.
The initial condition are taken in the form

\[ \begin{align*}
\alpha(x, 0) &= \alpha_0 = \text{const}, \\
B(x, 0) &= B_0 = \text{const}, \\
V(x, 0) &= V_0(x) = A \cos(x),
\end{align*} \tag{17} \]

\[ \begin{align*}
V'(x, 0) &= \exp(-2\alpha_0)\pi_V(x), \\
\alpha'(x, 0) &= \exp(-2\alpha_0)\pi_\alpha(x), \\
B'(x, 0) &= \exp(-2\alpha_0)\pi_B(x),
\end{align*} \tag{18} \]

where the velocities are expressed in the terms of initial momenta \( \pi_V(x), \pi_\alpha(x), \pi_B(x) \), which are chosen as

\[ \begin{align*}
\pi_V(x) &= \text{const}, \\
\pi_\alpha(x) &= \frac{1}{2}(\pi_{B0} - 3V_0(x)\pi_V) + \frac{\pi_V^2 + e^{4B_0+4\alpha_0}V_0'^2(x)}{2(\pi_{B0} - 3V_0(x)\pi_V)}, \\
\pi_B(x) &= -\pi_\alpha(x) - 3\pi_VV_0(x) + \pi_{B0},
\end{align*} \tag{19} \]

i.e. in such a way that the constraints (2), (3) are satisfied exactly.

The evolution of time derivatives at \( x = 0 \) is shown in Fig. 2. One can see, that \( \alpha'(x, t) \) evolves non-monotonically and can become negative at some moment of time and in some spatial point \( x \), i.e. the scale factor decreases here. Analogous oscillations appear for \( B' \). Figure 3 shows the evolution of the dispersion of \( \alpha' \), which finally becomes much greater than the mean value \( \langle \alpha' \rangle \).

**IV. LINEARIZED THEORY FOR THE INHOMOGENEOUS BIANCHI I MODEL**

The origin of the oscillations demonstrated above can be easily explained within the framework of a linearized theory. Let represent the variables as a sum of a spatially uniform part and a perturbation

\[ \begin{align*}
\alpha(x, \tau) &= \alpha_0(\tau) + \alpha_1(\tau)e^{ikx}, \\
B(x, \tau) &= B_0(\tau) + B_1(\tau)e^{ikx}, \\
V(x, \tau) &= V_0(\tau) + V_1(\tau)e^{ikx}.
\end{align*} \tag{20} \]

In the zero-order in perturbations, the equations of motion are

\[ \begin{align*}
V_0'' + 2V_0'\alpha_0' &= 0, \\
B_0'' + 2B_0'\alpha_0' &= 0, \\
\alpha_0'' + B_0'^2 + V_0'^2 + \alpha_0'^2 &= 0, \tag{21}
\end{align*} \]

and the only constraint is

\[ H_0 = \frac{1}{2}e^{2\alpha_0} \left( B_0'^2 + V_0'^2 - \alpha_0'^2 \right). \tag{22} \]

In the first-order in perturbations, one has

\[ \begin{align*}
V_1'' + k^2e^{4B_0}V_1 + 2V_0'\alpha_1' + 2V_1'\alpha_0' &= 0, \\
B_1'' + 2B_0'\alpha_1' + 2B_1'\alpha_0' - \frac{1}{3}k^2e^{4B_0}(B_1 + 2\alpha_1) &= 0, \\
\alpha_1'' + 2\alpha_0'\alpha_1' + 2B_0'B_1' + 2V_0'V_1' + \frac{1}{3}k^2e^{4B_0}(2B_1 + 3\alpha_1) &= 0, \\
H_1 &= -3B_0'B_1' - 3\alpha_1B_0'^2 + k^2e^{4B_0}B_1 + k^2e^{4B_0}\alpha_1 - 3V_0'V_1' - 3\alpha_1V_0'^2 \\
&\quad + 3\alpha_0'\alpha_1' + 3\alpha_1\alpha_0'^2 = 0, \\
\mathcal{P}_1 &= -3ikB_1B_0' - 2ik\alpha_1B_0' - ikB_1' - 3ikV_1V_0' + i\alpha_1\alpha_0' - i\alpha_1' = 0. \tag{23}
\end{align*} \]

However, only one of the two last constraints turns out to be independent. Thus one may exclude \( B_1 \) and after
V. RIPPLES FROM THE POINT OF VIEW OF THE ISAACSON’S THEORY

The Isaacson’s theory [24] considers an influence of the ripples on the evolution of background. An interpretation of the Isaacson’s theory is quite straightforward and does not depend on details of the averaging procedure. Let expound its main aspects. Let suppose, that there is a set of equations

\[ A(\xi(x, \tau)) = 0 \]  \hspace{1cm} (25)

for some quantities \( \xi \), which have a strongly inhomogeneous (oscillating) space-time behavior. Then, one may define an average quantity

\[ \xi_0(\tau) = \langle \xi(\tau, x) \rangle = \frac{1}{L} \int_0^L \xi(\tau, x) dx, \]  \hspace{1cm} (26)

where we shall consider only spatial averaging for the sake of simplicity.

As a result, the quantity \( \xi \) can be separated as

\[ \xi(x, \tau) = \xi_0(\tau) + \xi_1(x, \tau), \]  \hspace{1cm} (27)

where \( \xi_1(x, \tau) = \xi(x, \tau) - \xi_0(\tau) \). It is evident, that the mean value of \( \xi_1(x, \tau) \) equals to zero.

Eqs. (25) can be expanded in powers of \( \xi_1(x, t) \):

\[ A(\xi(x, \tau)) = A_0(\xi_0(\tau)) + A_1(\xi_0(\tau), \xi_1(x, \tau)) + A_2(\xi_0(\tau), \xi_1(x, \tau)) + \cdots = 0. \]  \hspace{1cm} (28)

The first statement of the theory considered reads that the spatially inhomogeneous part obeys the linear equations \( A_1 = 0 \), and, in fact, is equivalent to the linearization considered above. This statement is not valid in our case, because the ripples of \( B \) and \( \alpha \) are strongly nonlinear.

The second statement reads that the evolution of background is determined by averaged terms of the second order in ripples:

\[ A_0(\xi_0(\tau)) = - \langle A_2(\xi_0(\tau), \xi_1(x, \tau)) \rangle. \]  \hspace{1cm} (29)

It is a consequence of the averaging of Eq. (28).
Let consider only single equation: the Hamiltonian constraint \( \{12\} \). We substitute
\[
\begin{align*}
\alpha(x, \tau) &= \alpha_0(\tau) + \alpha_1(\tau, x), \\
B(x, \tau) &= B_0(\tau) + B_1(\tau, x), \\
V(x, \tau) &= V_0(\tau) + V_1(\tau, x)
\end{align*}
\]
(30)

into the Hamiltonian constraint and obtain the second order contribution in the form of
\[
\mathcal{H}_2 = e^{2\alpha_0} \left( 2\alpha_1 \partial_x B_1 B_0' + e^{4B_0} \left( \frac{4}{3} \partial_x B_1 \partial_x \alpha_1 + \frac{2}{3} \alpha_1 \partial_x B_1 + \frac{4}{3} B_1 \partial_x B_1 + \frac{7}{6} (\partial_x B_1)^2 + \frac{4}{3} B_1 \partial_x \alpha_1 \right. \\
+ \frac{1}{2} (\partial_x V_1)^2 + \frac{1}{6} (\partial_x \alpha_1)^2 + \frac{2}{3} \alpha_1 \partial_x \alpha_1 \right) + \frac{1}{2} (\partial_x B_1)^2 + 2\alpha_1 \partial_x V_1 V_0' + \frac{1}{2} (V_1')^2 - 2\alpha_1 \alpha_0' \\
- \frac{1}{2} (\alpha_1')^2 + \alpha_1^2 \left( (B_0')^2 + (V_0')^2 - (\alpha_0')^2 \right)
\]
(31)

Using the results of the previous numerical calculations (Sec. \{III\}), one can see that the second statement of the Isaacson’s theory (i.e. \( \mathcal{H}_0 \approx - < \mathcal{H}_2 > \)) is satisfied with a high accuracy as it is shown in Fig. \(4 \) (a).

Let distinguish a “pure” gravitational wave contribution
\[
\mathcal{H}_{2V} = e^{2\alpha_0} \frac{1}{2} \left( (V_1')^2 + (\partial_x V_1)^2 \right)
\]
(32)
in \( \mathcal{H}_2 \). As one can see from Fig. \(5 \) (b), the plot of \( < \mathcal{H}_2 - \mathcal{H}_{2V} > \) is, in fact, symmetric relatively the time axis. This means that if one performs the time averaging in addition, then one can conclude that \( < \mathcal{H}_2 - \mathcal{H}_{2V} > \) does not contribute “at the mean” to the evolution of background, i.e. the evolution of background is determined by a “pure” gravitational wave \( < \mathcal{H}_{2V} > \).

VI. THE YORK CURVATURE FOR THE NONUNIFORM BIANCHI MODEL

Why is the \( V \)-variable considered as that corresponding to a “pure” gravitational wave? To answer this question, one may use the description of a conformal three-geometry in the terms of the York curvature \( \{12\} \). We have for the York curvature \( \{33\} \),
\[
Y^{ab} = g^{1/3} e^{\alpha f} (R^b_f - \frac{1}{4} \delta^b_R R)_{,c},
\]
(33)
where \( g_{ab} \) denotes the spatial part of the metric \( \{11\} \), \( R_{ab} \) is the Ricci tensor of the three-metric \( g_{ab} \), \( e^{\alpha f} \) is the fully antisymmetric symbol and the :c means the covariant derivative over the metric \( g_{ab} \). Direct calculation gives the following result
\[
Y^{23} = \sqrt{5} e^{4B} \left( 9\partial_x B \partial_x V + 3\partial_x V (\partial_x B + 6(\partial_x B)^2 \\
-2(\partial_x V)^2) + \partial_x x V \right),
\]
(34)
which is exactly the metric \( \{11\} \) if the conformal time \( dt = e^{2\alpha_0} dt \) is used instead of \( t \).

VII. DISCUSSION AND CONCLUSIONS

Let compare the model considered with the well-known Gowdy \( \{26 \} \) model described by the metric:
\[
ds^2 = e^{-T + \lambda/2} (e^{4T} dt^2 - dx^2) - e^{2T}(e^{2\beta} dy^2 + e^{-2\beta} dz^2),
\]
(35)
where \( T, \lambda, \beta \) are the functions of \( t, x \) only. Denoting
\[
\begin{align*}
\beta &= \sqrt{3} V, \\
T &= B + \alpha, \\
\lambda &= 6(B - \alpha)
\end{align*}
\]
(36)
we come to the metric
\[
ds^2 = e^{2\alpha} ((e^{2\alpha} dt)^2 - e^{-4B} dx^2 - e^{2B+2\sqrt{3} V} dy^2 \\
- e^{2B-2\sqrt{3} V} dz^2),
\]
(37)
which is exactly the metric \( \{11\} \) if the conformal time \( dt = e^{2\alpha_0} dt \) is used instead of \( t \).

For the Gowdy model, it is shown that the function \( T \) can be chosen as the function of the time \( t \) only \( \{26 \} \), and the function \( \lambda \) ceases to depend on \( x \) at \( t \to \infty \). Thus, in principle, the ripples of \( \alpha = \frac{1}{2}(T - \frac{\lambda}{\beta}) \) found above can be eliminated by some coordinate transformation.

It should be noted, that, if the gauge invariance turns out to be violated in quantum theory, the ripples of the
scale factor in the conformal time can appear. Since, as it has been demonstrated above (Figs. 3-5), the ripples consist of the sharp spikes, their spectrum will be broad and “noise-like”. It should be taken into account, that such spikes have appeared when the Bianchi I model was considered. As has been demonstrated in Sec. [14] the unstable rising modes exist for this model. In contrast, concerning a contribution of the ripples to the energy balance, one may conclude that such ripples occur in both string and Bianchi I models. But, in the string model the balance, one may conclude that such ripples occur in both space-time, the ripples could exist only in a strongly non-linear regime and this issue needs a further analysis.

Concerning a contribution of the ripples to the energy balance, one may conclude that such ripples occur in both string and Bianchi I models. But, in the string model the ripples of the scale factor and the oscillations of the matter fields compensate mutually each other and, thereby, the background evolution is not affected by them. For the Bianchi I model, the results of Sec. [14] demonstrate that the ripples of the scale factor compensate only the oscillations of the $B$-field and the background evolution “at the mean” becomes to be determined by the “pure” gravitational wave $V$ in agreement with the results of [14]. Nevertheless, such a conclusion can become invalid in the quantum gravity case, since a strong gauge wave, consisting of the scale factor $\alpha$ and the $B$-field could mixed up the gravitational waves and contribute to the background evolution, as well.

It should be noted, that both models (i.e. the string and Bianchi I ones) have been considered on the classical level that cannot shed a light on the solution of the vacuum energy problem. This results from the fact that a compensation of only zero-point fluctuations of matter fields is required in the quantum case. But in the classics, it is impossible to distinguish the fluctuations in the ground state from those in an excited state.

[1] S. Weinberg. The cosmological constant problem. Review of Modern Physics 61, 1 (1989).
[2] S.M. Carroll, Living Rev. Relativity 4, 1 (2001).
[3] T. Padmanabhan, Cosmological Constant - the Weight of the Vacuum. Phys. Rep. 380, 235 (2003).
[4] J. R. Ellis. Dark matter and dark energy: summary and future directions. Phil. Trans. Roy. Soc. Lond. A 361, 2607 (2003).
[5] P. J. Steinhardt. A quintessential introduction to dark energy. Phil. Trans. Roy. Soc. Lond. A 361, 2497 (2003).
[6] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt. Essentials of k-essence. Phys. Rev. D 63, 103510 (2001).
[7] E.J. Copeland, M. Sami and S. Tsujikawa. Dynamics of dark energy. Int.J.Mod.Phys.D 15, 1753 (2006).
[8] T. Padmanabhan. Planck length is the lower bound to all physical length scales. Gen. Rel. Grav. 17, 215 (1985).
[9] I. Antoniadis, P. O. Mazur and E. Mottola. Four-dimensional quantum gravity in the conformal sector. Phys. Rev. D 45, 2013 (1992).
[10] I. Antoniadis, P. O. Mazur and E. Mottola. Physical States of the Quantum Conformal Factor. Phys. Rev. D 55, 4770 (1997).
[11] C. Cunliff. Conformal fluctuations do not establish a minimum length. arXiv:1201.2247.
[12] C. H.-T. Wang, R. Bingham and J. T. Mendonca. Quantum gravitational decoherence of matter waves. Class. Quant. Grav. 23, L59 (2006).
[13] S. V. Anischenko, V. L. Kalashnikov and S.L. Cherkas. To the question about vacuum energy in cosmology. Proc. 2nd Congress of Physicists of Belarus (Minsk, November 3-5) (2008) [in Russian].
[14] P. M. Bonifacio. Spacetime Conformal Fluctuations and Quantum Dephasing. PhD thesis (University of Aberdeen). arXiv:0906.0463.
[15] J.W. Jr. York. Gravitational Degrees of Freedom and the Initial-Value Problem. Phys. Rev. Lett. 26, 1656 (1971).
[16] J. D. Barrow and J. Levin. Chaos in the Einstein-Yang-Mills Equations. Phys.Rev.Lett. 80, 656 (1998), arXiv:gr-qc/9706065.
[17] J. D. Barrow, D. Kimberly and J. Magueijo. Bouncing Universes with Varying Constants. Class. Quant. Grav. 21, 4289 (2004), arXiv:astro-ph/0406369.
[18] A. Davidson. Rippled Cosmological Dark Matter from Damped Oscillating Newton Constant. Class. Quant. Grav. 22, 1119 (2005), arXiv:gr-qc/0409059.
[19] J. D. Barrow and C. G. Tsagas, On the stability of static ghost cosmologies. Class. Quant. Grav. 26, 195003 (2009), arXiv:0904.1340.
[20] S. L. Cherkas and V. L. Kalashnikov. An inhomogeneous toy-model of the quantum gravity with explicitly evolvable observables. arXiv:1107.2224.
[21] P. Szekeres. Colliding plane gravitational waves. J. Math. Phys. 13, 286 (1972).
[22] S.L. Cherkas and V.L. Kalashnikov, JCAP 0701, 028 (2007).
[23] S.L. Cherkas, V.L. Kalashnikov. Universe driven by the vacuum of the scalar field: VFD model. Proc. Int. Conf. "Problems of Practical Cosmology", held at Russian Geographical Society, 23-27 June 2008, Saint Petersburg. Ed. Yu. V. Baryshev, J.N. Taganov, P. Teerikorpi, 2, 135 (2008), arXiv: astro-ph/0611795.
[24] R.A. Isaacson. Gravitational Radiation in the Limit of High Frequency. I. The Linear Approximation and Geometrical Optics. Phys. Rev. 166, 1263 (1968).
[25] C.W. Misner, K.S. Torn and J.A. Wheeler. Gravitation. (W. H. Freeman & Company, 1973) Vol. 2.
[26] R.H. Gowdy. Vacuum Spacetimes and Compact Invariant Hyperspaces: Topologies and Boundary Conditions. Ann. Phys. (N.Y.) 83, 203 (1974).
[27] C.W. Misner. A minisuperspace Example: The Gowdy T3 Cosmology. Phys. Rev. 8, 3271 (1973).
[28] B.K. Berger. Quantum Graviton Creation in a Model Universe. Ann. Phys. 83, 458 (1974).
[29] B.K. Berger. Quantum cosmology: Exact solution for the Gowdy T3 model. Phys.Rev. D 11, 2770 (1975).
[30] L.D. Landau and E.M. Lifshits. Field Theory (Oxford, Pergamon Press) (1982).