Spin Hall effect in diffusive Rashba two-dimensional electron systems with micrometer size

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Abstract. In an infinitely large diffusive n-type 2D semiconductor with Rashba spin-orbit coupling, the spin-Hall conductivity $\sigma_{SH}$ vanishes due to the cancellation of its intrinsic and impurity-related contributions, independently of temperature, of the spin-orbit coupling constant, of the impurity density, and of the specific form of the electron-impurity scattering potential. In this paper, we demonstrate that such a precise cancellation is associated with the continuity of electron momentum in infinitely large sample and hence may be removed when the size of the sample is reduced. Considering a square sample with side length, $L$, we find that depending on $L$ in the micrometer range, $\sigma_{SH}$ can be positive, zero or negative with a maximum absolute value reaching as large as $e/8\pi$ order of magnitude at low temperatures. As $L$ increases, $\sigma_{SH}$ oscillates with a decreasing amplitude. Such a size effect shows up only at low temperatures. When temperature increases the $\sigma_{SH}$ oscillations disappear and spin-Hall conductivity takes a small finite value before slowly approaching zero with further increasing the sample size to $L \leq 20$ micrometer.

The present study indicates that a nonvanishing spin-Hall conductivity may be obtained in a 2D Rashba electron systems of micrometer size, notwithstanding its disappearance in infinitely large samples. For it to appear, one has to accurately control the shape and size of the sample. In addition, the mobility of the sample should be high and the temperature should be low that both the collisional broadening of the electron energy level and the temperature smearing are smaller than the finite-size induced energy separation of the electron states around the Fermi surface.

1. Introduction
A dc electric field applied to system with spin-orbit (SO) coupling may cause a nonvanishing spin current along the transverse direction. This so-called spin-Hall effect has been extensively studied from both the theoretical[1, 2, 3, 4, 5] and experimental perspectives[6, 7] because of its potential application in spintronics.

In clean two-dimensional (2D) electron systems with spatial inversion asymmetry, the Rashba SO interaction may give rise to a spin-Hall conductivity, $\sigma_{SH}$, having a universal intrinsic value $e/8\pi$ at zero temperature[4]. However, in diffusive regime, the electron-impurity scattering can produce an additional contribution to spin-Hall current, leading to a complete suppression of spin-Hall current[8, 9, 10, 11, 12, 13, 14, 15, 16]. Furthermore, this cancellation of spin-Hall conductivity has been shown to be independent of the specific form of the electron-impurity scattering potential, of the impurity density, of the SO coupling constant, and of temperature[14, 15].
However, the elimination of spin-Hall current occurs only for infinitely large samples. Analyzing the coordinate dependence of spin-Hall current, Mischenko et al. demonstrated that the spin-Hall current remains finite near the boundaries of samples[11]. In the present paper, ignoring such spatial variation, we show that the spin-Hall conductivity \( \sigma_{SH} \) still may be nonvanishing in finite-size samples. We find that the cancellation of spin-Hall current is associated with the continuity of electron momentum in infinitely large samples. When the sample size is finite, the discretization of the energy levels may lead to a nonvanishing spin-Hall current in Rashba 2D semiconductors even in the quasiclassical regime. Performing a numerical calculation for Rashba 2D electron systems of square shape with size in micrometer regime, we find that depending on the system size, the spin-Hall conductivity can be positive, zero or negative, with a maximum absolute value up to the order of magnitude of \( e/8\pi \) at low temperatures. As a function of the sample size, \( \sigma_{SH} \) oscillates around zero with a decreasing amplitude when increasing sample size. Such a size effect can be observable only at low temperatures \( T \). When temperature increases that \( T \) becomes comparable with the finite-size induced energy separation of the electron states at the Fermi surface, \( \sigma_{SH} \) oscillation disappears and the spin-Hall conductivity reduces to a small nonvanishing value before it slowly approaches zero with further increasing sample size.

2. Formalism

We consider a quasi-2D electron semiconductor in \( x - y \) plane, subjected to a Rashba SO interaction and a weak external dc electric field along the \( x \) axis, \( \mathbf{E} = (E, 0, 0) \). In this system, the motion of an electron with effective mass \( m \) and momentum \( \mathbf{p} \equiv (p_x, p_y) \equiv (p \cos \phi_p, p \sin \phi_p) \) can be described by Hamiltonian \( \hat{H} = \hat{H}_0 + \hat{H}_{imp} + \hat{H}_E \). \( \hat{H}_0 \) is the free electron Hamiltonian given by \( \hat{H} = p^2/(2m) + \alpha \mathbf{n} \cdot (\mathbf{r} \times \hat{\mathbf{r}}) \), with \( \alpha \) as the Rashba SO coupling constant, \( \hat{\mathbf{r}} \equiv (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z) \) as the Pauli matrices, and \( \mathbf{n} \) is the unit vector perpendicular to the 2D electron plane. By a local unitary transformation, \( \hat{U}_p = [u_1(p), u_2(p)] \), with \( u_\mu(p) = 1/(\sqrt{2}) [1, (-1)^\mu \exp(i\phi_p)]^T \) \((\mu = 1, 2)\). Hamiltonian \( \hat{H}_0 \) can be diagonalized as \( \hat{H}_0 \equiv \hat{U}_p^\dagger \hat{H}_0 \hat{U}_p = \text{diag}[\varepsilon_1(p), \varepsilon_2(p)] \) with \( \varepsilon_\mu(p) = p^2/(2m) + (1)^\mu \alpha p \) as dispersion relations of two spin-orbit-coupled bands[10]. \( \hat{H}_{imp} \) describes the electron-impurity scattering and takes the form, \( \hat{H}_{imp} = \sum_i V(|r - R_i|) (\mathbf{r} and R_i, respectively, are the electron and impurity coordinates, and \( V(r) \) is the electron-impurity scattering potential). \( \hat{H}_E \) describes the applied electric field, and, in the Coulomb gauge, it can be written as \( \hat{H}_E = -e\mathbf{E} \cdot \mathbf{r} \).

We are interested in the spin-Hall current polarized along the \( z \)-direction and flow along the \( y \) axis, \( J_y^z \). Its single-particle operator is defined in the spin basis as \( \hat{j}_y = (\hat{j}_y \hat{\sigma}_z + \hat{\sigma}_z \hat{j}_y)/\hbar e \) with \( \hat{j}_y \) as the electric current operator[17]. Applying the unitary transformation \( \hat{U}_p \), we find that \( \hat{j}_y \) reduces to an off-diagonal matrix in the helicity basis. Hence, the net spin-Hall current can be determined in the helicity basis via \( J_y^z = \sum_p \frac{p_x}{m} \int \frac{d\omega}{2\pi} \text{Im} \hat{G}_{12}\!^<_{\sigma}(p, \omega) \), with \( \hat{G}^<_{\sigma}(p, \omega) \) as the helicity-basis nonequilibrium lesser Green’s function.

It is evident that to determine the spin-Hall current, one has to consider the nonequilibrium lesser Green’s function \( \hat{G}^<_{\sigma}(p, \omega) \). Solving the kinetic equation for \( \hat{G}^<_{\sigma}(p, \omega) \), we find that the linear electric field part of \( \hat{G}^<_{\sigma}(p, \omega) \), \( \hat{G}^<_{\sigma}^I(p, \omega) \), can be expressed as a sum of two terms, \( (\hat{G}^<_{\sigma}^I)^I(p, \omega) \) and \( (\hat{G}^<_{\sigma}^I)^{I\dagger}(p, \omega) \), which, respectively, correspond to the diagonal and off-diagonal driving terms of the kinetic equation. In this way, the spin-Hall conductivity, \( \sigma_{SH} \), defined by \( \sigma_{SH} = J_y^z/E \), can be written as \( \sigma_{SH} = \sigma_{SH}^I + \sigma_{SH}^{II} \), with \( \sigma_{SH}^{I\dagger} \) coming from \( (\hat{G}^<_{\sigma}^I)^{I\dagger}(p, \omega) \) respectively, and taking the forms[14],

\[
\sigma_{SH}^I = -\sum_p \frac{e p_x}{4m \alpha p} \frac{\partial}{\partial p_x} \left[ f_1(p) - f_2(p) \right],
\]
Figure 1. Sample-size dependence of $\sigma_{SH}$ in Rashba 2D GaAs-based semiconductors at different temperatures: (a) $T = 0.5$ K, (b) $T = 0.8$ K, and (c) $T = 1$ K. The electron density and broadening parameter are $n_e = 5 \times 10^{10}$ cm$^{-2}$ and $\gamma_0 = 0.1$ meV, respectively. The SO coupling constant is $\alpha = 1$ meV$\cdot$nm.

and

$$\sigma^{II}_{SH} = \frac{-e}{4\pi a_0} \sum_p \frac{p_y^2}{p^3} [f_1(p) - f_2(p)].$$

Here, $f_\mu(p)$ ($\mu = 1, 2$) are the unperturbed distribution functions.

In an infinitely large Rashba 2D semiconductor, the spin-Hall conductivity vanishes, independently of temperature, of the spin-orbit coupling constant, of the impurity density, and of the specific form of the electron-impurity scattering potential. This can be seen from Eqs. (1) and (2) by considering the fact that, for an infinitely large system, the electron momentum is continuous and the summation over the electron momentum can be replaced by a momentum integral. Performing the integration in Eq. (1) by parts, we find that $\sigma^{I}_{SH} = -\sigma^{II}_{SH}$, i.e. the spin-Hall conductivity vanishes.

However, when sample size is reduced to be comparable with $2\pi/k_F$ [$k_F \equiv (k_{1F} + k_{2F})/2$ and $k_{\mu F}$ is the Fermi momentum of electrons in spin-orbit-coupled (helicity) band $\mu$], the discretization of the electron momentum can no longer be ignored. Since $\sigma^{I}_{SH}$ is associated mainly with the states near the Fermi surface while $\sigma^{II}_{SH}$ is related to all electron states in the Fermi sea (this can be seen from the fact that $\sigma^{I}_{SH}$ and $\sigma^{II}_{SH}$ depend on $f_\mu(p)$ through its derivative and itself, respectively), the discretization of the electron momentum has an effect on $\sigma^{I}_{SH}$ stronger than that on $\sigma^{II}_{SH}$. As a result, the spin-Hall conductivity may be finite in finite size samples.

3. Numerical results

Further, we perform a numerical calculation to investigate the spin-Hall conductivity in finite Rashba GaAs-based heterojunctions within the quasiclassical and diffusive regime. For definiteness, we consider a square Rashba 2D electron system with length $L$ in both $x$ and $y$ directions. The possible values of the electron momentum are $p_x = 2\pi n_x/L$ and $p_y = 2\pi n_y/L$.
with integers $n_x$ and $n_y$. In calculation, we also take account of the collisional broadening of the electron states, which is described by a constant parameter $\gamma_0$ and leads to distribution function $f_{\mu}(p)$ taking the form, $f_{\mu}(p) = \text{Im}[\Psi(1/2 + C_{\mu}(p))] / \pi + \gamma_0$, with $\Psi(x)$ as the Digamma function and $C_{\mu}(p) = (\gamma_0 - i(\varepsilon_{\mu}(p) - \mu_c)) / 2\pi T$. The results obtained from Eqs. (1) and (2) are plotted in Figs. 1, 2 and 3.

In Fig. 1, the spin-Hall conductivity $\sigma_{SH} \equiv \sigma_{SH}^I + \sigma_{SH}^{II}$ is shown as a function of the sample size at different temperatures. We see that sensitively depending on the size of micrometer samples, the spin-Hall conductivity can be positive, zero and negative. At a given temperature, $\sigma_{SH}$ oscillates with a decreasing amplitude when increasing the sample size from $L = 1 \mu m$. The period of the oscillation is approximately equal to $2\pi / k_F$. When temperature increases, the oscillation amplitude decreases. Note that at low temperature, the maximum value of the amplitude can be as large as $e / 8\pi$.

In Fig. 2, we plot the temperature dependence of spin-Hall conductivity for different sample sizes. At high temperature, $\sigma_{SH}$ approaches a small (nonzero) constant value. When temperature goes down from 1 K, $\sigma_{SH}$ spreads and approaches different low-temperature values between $-e / 8\pi$ and $1.4e / 8\pi$ for different sample sizes. It should be noted that such a finite-size effect of $\sigma_{SH}$ originates from the change of the electron distribution around the Fermi surface and relates to three energy scales: (i) the finite-size induced energy separation of the electron states around the Fermi surface, $\Delta_F = 2\pi v_F / L$, (ii) the collisional broadening of the energy level described by temperature-independent parameter $\gamma_0 \approx 0.1 \text{meV}$, and (iii) temperature $T$. When $\gamma_0$ and $T$ are smaller than $\Delta_F$, the finite size effect leads to a nonvanishing $\sigma_{SH}$ of order of $e / 8\pi$. When temperature $T$ increases from zero to $\gamma_0$, the collisional broadening dominates the smearing of the electron distribution and $\sigma_{SH}$ exhibits a plateau due to the temperature independence of $\gamma_0$. As $T$ further increases to the range of $T > \gamma_0$, the temperature smearing dominates and $|\sigma_{SH}|$ shrinks with increasing $T$. When temperature becomes larger than $\Delta_F$, $T > \Delta_F$, the effect of the energy level separation is washed out and $\sigma_{SH}$ approaches a small nonzero constant.

Note that the strong sample-size and temperature dependencies of $\sigma_{SH}$ discussed above, come almost entirely from the change of $\sigma_{SH}^I$ with variation of $L$ and $T$. Actually, there exists finite size effect associated with the change of $\sigma_{SH}^{II}$ with sample size. Such an effect becomes important in the larger $L$ scale, where the finite size effect on $\sigma_{SH}^{II}$ is washed out. In Fig. 3, we plot the spin-Hall conductivity of finite Rashba 2D semiconductors having size from $L = 2 \mu m$ to $20 \mu m$ at temperature $T = 1 \text{K}$. We see that, though at this temperature the $\sigma_{SH}$ oscillation disappears for $L > 2 \mu m$, the spin-Hall conductivity remains to have a small finite value. Only when sample size

**Figure 2.** Temperature dependence of $\sigma_{SH}$ in Rashba 2D semiconductors of different sample sizes, $L = 1.47$, 1.5, 1.52, and 1.54 $\mu m$. Other parameters are the same as in Fig. 1.
size increases to $L \geq 20 \mu m$, can $\sigma_{SH}$ close to zero, the result of an infinitely large sample.

4. Conclusions
We have investigated the finite-size effect on spin-Hall current in Rashba two-dimensional electron systems. A numerical calculation for Rashba 2D electron systems of square shape indicates that, as a function of the sample size in micrometer regime, $\sigma_{SH}$ oscillates around zero. The oscillation amplitude decreases when increasing sample size and its maximum absolute value may reach up to the order of magnitude of $e/8\pi$ at low temperatures. Such a size effect disappears when temperature increases that $T$ becomes comparable with energy separation of the electron states induced by finite sample size at the Fermi surface. With further increasing sample size, $\sigma_{SH}$ takes a small finite value before it slowly approaches zero.

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