Role of the Rheological Parameters in Debris Flow Modelling

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Abstract. Nowadays, the debris flow model has become an essential part of risk analysis and impact engineering. Coupled with field observations and historical records, these models have proven powerful tools to understand the behaviour of debris flow in complex terrain. However, their application poses several new challenges to scholars and engineers. A detailed understanding of the debris flow phenomena requires a sound knowledge of the shallow water equation and rheological model used to simulate the debris flow hazard. In this study, important rheological models used to analyse the debris flow process and their limitations have been highlighted. Furthermore, the suitability of the Voellmy-Salm rheological model has been studied for 2D pyroclastic flow taking different combinations of the coefficient of friction namely coulomb friction coefficient \( \mu \), and turbulent coefficient of friction \( \xi \) using IMEX SFLOW 2D dynamic continuum model. It was found that velocity and runout distances are significantly influenced by the variation of the coefficient of the turbulent friction \( \xi \) at a large scale. It is then concluded that the identification of a suitable rheological model is necessary to simulate the precise behaviour of complex and heterogeneous debris flow.

Keywords: Debris flow. Dynamic continuum model. Rheological model. Runout distance. Risk analysis. Shallow water equation

1. Introduction
Landslide in general terms can be defined as a whole range of mass and slope movements downward and outward in the form of rock, rubble, earth, or a combination of these materials under the action of gravity \([1]\). Based on the type of material (rock, stone, soil,) involved, and mode of the movement, landslide classified as rockfall, rockslide, debris flow, mudflow, earth spread, etc. Genetically and morphologically landslide areas have been divided into depletion and accumulation zone \([1]\). The accumulation zone is more critical in flow-like landslides as fluid moves on the rigid bed and covers large runout distances from the source area. Among the types of landslides, debris flow involves a water-sediment mixture and sort of mass wasting phenomenon that flowing along slope under gravitational influence \([2]\). Volcanic eruptions, earthquakes, and climate change due to global warming leads to heavy or uneven rainfall are the main cause of catastrophic debris flow in mountainous regions in certain parts of the world \([3,4]\). Further debris flow proves hazardous to the built environment and human life directly and indirectly. Structures like roads, pipelines,\([5]\) buildings \([6]\) underground structures\([7,8]\), and piles \([9]\) have been severely damaged due to soil movement and
mass flow reported in literature.

The dynamics of debris flow include a variety of the key physical factors; including strong and random interphase, particle-particle interactions, fluctuations of the fluid-solid motions, active sediment transport, substantial mass exchange with the bed, and the bottom boundary that usually undergoes evolution [10]. Generally, the debris flow model incorporates the shallow water equation with an appropriate rheological model for a particular type of debris flow. Hence, the rheological model is key to characterize the different types of flow that occur in nature from flood to Stoney debris flow. The various rheological model has been developed [11–13] till date to analyse the different type of mountainous flow.

1.1 Debris Flow Modelling

Analysis of landslide, debris flow, and other mountainous hazards extensively depends on the field observation, site investigation, interviewing the victims, and having a large accurate dataset of the affected area. Besides, mountainous hazards are also difficult to model in a laboratory from the challenges of scaling and proportioning point of view. However, the advancement of computational techniques, development of the open-access code, and demands of soft computing tools stressing the scientist to use the domain of numerical modelling for assessing the mountainous hazards. Further, it provides the window to researchers for simulating the properties of the debris flow and its impact quantitively as well as qualitatively on the infrastructure. Most of the existing debris flow model is the solution of the shallow water equation (based on the mass and momentum conservation), which is like the surface water equation and only differs in terms of the rheological model (description of friction), which is described in the subsequent section below.

A plethora of numerical models have been suggested such as DAMBRK [14] is one-dimensional hydrodynamic flood routing software, FLO-2D [15], DAN3D [16], DMM [17], Debris2D [18,19], TRENT2D [20], Mass-Mov2D [21] among others for the assessment of runout distance and areas affected by debris flows [10,21–23]. These methods mainly depend on topography, debris-flow volume, and rheological characteristics of the material involved which are derived and calibrated based on the data obtained from field measurements and laboratory experiments. However, they cannot provide enough and adequate information to quantify the debris flow evolution, initiation and stop mechanism, and flow on the three-dimensional terrain. To predict the debris flow runout distances, flow velocities, and impact pressures in natural three-dimensional terrain Christian et al. [24] have developed the (RAMMS), while Vitturi et al. [25] developed (IMEX SFLOW 2D 1.0) dynamic continuum model using the principle of conservation of mass and momentum equation. The depth-averaged equation used by Vitturi et al. [25] to develop the IMEX SFLOW 2D was based on the following assumption.

- Flow in the vertical direction was neglected and flow depth \( h(x,y,t) \) changes with space and time only.
- Horizontal velocities \( u(x,y,t) \) and \( v(x,y,t) \) in \( x \)- and \( y \)- direction over a topography \( B(x,y) \) was integrated using a shallow water equation.
- Topography \( B(x,y) \) is assumed to be not changed with time \( t \).
- Hydrostatic pressure distribution was considered throughout the model and the density of the fluid was also constant.

By using the above assumption, and without considering the frictional terms, the simplified depth-averaged differential (shallow water) equation can be written in the following way:

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hu)}{\partial y} = 0
\]  

\[
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2)}{\partial x} + \frac{\partial (huv)}{\partial y} + gh \frac{\partial (h+B)}{\partial x} = 0
\]
\[
\frac{\partial (hv)}{\partial t} + \frac{\partial (huv)}{\partial x} + \frac{\partial (hv^2)}{\partial y} + gh \frac{\partial (h+B)}{\partial y} = 0
\]  
Eq. (3)

Eq. 1 represents the conservation of mass, while Eqs. 2 and 3 represent the conservation of the momentum in x- and y-direction, respectively. The parameter \( g \) in Equation 2 and 3 represents the gravity-induced force resulting from hydrostatic pressure on bathymetry. The advantage of IMEX SFLOW 2D is that it includes the topography that can be defined by the ASCII file or importing the DEM file of a particular region.

1.2. Debris flow rheology

Rheology deals with the relationship between the frictional parameter and shear stress of the fluid moving on the slope or base. The Identification of appropriate rheology has long been seen as significant to successful interpretation, modeling, and prediction of debris-flow behavior [26]. Due to its complexity and wide range of sediment involved in it, debates about the most suitable rheological formula have been continued for several decades. At the same time, evidence from the field observations and video recordings of torrential flows, mud slurry has shown that there is no precise rheology of flow to explain the variety of mechanical behaviors exhibited by poorly sorted, water-saturated debris flow. The most crucial part of the debris flow model is to conceptualize the rheological parameter associated with a particular type of flow and to estimate the shear stress developed during the flow involves, which depends on the following five different components[13]

\[
\tau = \tau_d + \tau_t + \tau_v + \tau_{mc} + \tau_c
\]  
Where, \( \tau_d \) is the dispersive term, \( \tau_t \) is the turbulent term, \( \tau_v \) is the viscous term, \( \tau_{mc} \) is the mohr-coulomb terms, and \( \tau_c \) is the cohesive term. The rheological model developed to date considers the combination of the various terms among these five components. The important and widely accepted rheological models have been discussed below.

1.2.1. Bingham model. Bingham plastic model is used to simulate the viscoelastic behaviour of the mud slurry and is widely accepted in mud and debris flow analysis [27]. This model has been characterized by a two-parameter model that includes yield stress and plastic viscosity term of fluid. Bingham model possesses a certain viscosity discontinuity by which it changes the viscosity value from finite to infinity that causes the instabilities in the application of the Bingham model in debris flow analysis at a high shear rate. In simple shear, the basic equation of the Bingham fluid which relates the stress and shear rate is:

\[
\frac{\partial u}{\partial y} = \begin{cases} 
0, & \tau < \tau_0 \\
\frac{\tau - \tau_0}{\mu_{so}}, & \tau \geq \tau_0 
\end{cases}
\]  
Where, \( \frac{\partial u}{\partial y} \) is shear rate, \( \tau \) is the elastic shear stress, \( \tau_0 \) is the critical shear stress, and \( \mu \) is the coulomb viscous force. The disadvantage of the Bingham model is that the shearing thickening and thinning phenomenon of the experiment cannot be simulated [28]. Another shortcoming of the Bingham model is the linear relationship between the shear stress tensor and strain rate tensor under high strain rate cannot predict the precise (non-linear) behavior of the debris flow simulation.

1.2.2. Herschel-Bulkley model. The Herschel–Bulkley (HB) model of non-Newtonian fluid in which the strain experienced by the fluid has been related to the stress in a complicated non-linear way. Yield stresses, viscosity, and turbulent dispersive stresses are the three different terms collectively defined the total frictional stresses in the HB model that are functions of the sediment concentration in the mixture. In any case, it is appropriate to simulate the flow stoppage with the hypothesis of the Binghamic nature of the fluid [29]. This non-Newtonian fluid model was introduced by Winslow Herschel and Ronald Bulkley in 1926. The constitutive equation of the Herschel-Bulkley model is commonly written as.

\[
\tau = \tau_0 + k\gamma^n
\]  
(6)
Where, \(\tau\) is the shear stress, \(\gamma\) is the shear rate \(\frac{\partial u}{\partial y}\), \(k\) is the consistency index, and \(n\) is the flow index. The limitation of the Herschel-Bulkley (HB) model is that it does incorporate the effective viscosity at the low shear rate in the numerical divergence of the shallow water equations.

1.2.3. Voellmy-Salm rheology model. It is a non-linear model, which simulates the mixture motion as a homogeneous mass flow. Voellmy–Salm friction model [30] is the theory that is commonly used for analyzing the frictional terms of non-Newtonian flows, and explain the motion of snow avalanches, pyroclastic flow, and granular debris flow [13] in a wide variety of the frictional term. The constitutive equation of the model can be written as:

\[
\tau = \mu \rho g h + \rho g U v \xi^2 / h \xi^2
\]

In above equation first term defined the Coulomb friction stress \((\tau_\mu)\) which linearly depend on the flow depth \((h)\) and dry coulomb friction \((\mu)\). The second term explains the turbulent friction stress \((\tau_\xi)\) which directly depends on the square of the velocity \((v)\) and inversely proportional to the turbulent frictional coefficient \((\xi)\) [31]. The ranges of these coefficients largely depend upon the type of sediment involve in the flow. Snow avalanches, pyroclastic flow, granular debris flow, and rock avalanches can easily be differentiated based on these parameters and the density of the fluid. Previous literature indicated that the friction coefficient \((\mu)\) is in the range of 0.1 to 0.3 and the turbulence coefficient \((\xi)\) from 1000 to 3000 ms\(^{-2}\) [32] for pyroclastic or volcanic origin debris flow. Similarly, for snow avalanches, the values \((\mu)\) from 0.1 to 0.6 and \((\xi)\) ranges 50 to 4000 ms\(^{-2}\). Numerical computation of debris flow equation largely depends on the techniques used to solve the shallow water equation and rheological model to simulate the initiation and stopping mechanism of the debris flow.

In this study, the importance of the rheological model for non-Newtonian debris flow has been highlighted by considering the critical forces during the flow. Besides, the effect of the turbulent coefficient friction on the velocity of debris flow and runout distances on arbitrary slope (by expanding case 4 created by Vitturi et. al [25]) using IMEX SFLOW 2D has also been discussed. The avalanches of finite granular mass flowing down on an inclined plane merging continuously into a horizontal plane has been simulated. The initiation and topographical condition have been taken from the article by Wang et al. [26]. The computational domain was a rectangular domain of [0,30] x [-7,7] as shown in Figure 2 on which the hemispherical shell containing granular material is released on the inclined plane of 35\(^\circ\).

2. Methodology
Most of the debris flow dynamic simulation models are based on the solution of the mass and momentum conservation equations, which simulate the single and multiphase debris flow behaviour as describe in section 1.2. In this study numerical modelling of debris flow has been achieved using the depth average dynamic model (IMEX SFLOW 2D). The effect of the coefficient of turbulent friction \((\xi)\) on the runout distances and velocity, a combination of frictional parameters has been made in the range (100 to 300 m/s\(^2\)) as shown in Table 1. Each simulation was run for a constant time of 30s and a small volume of 13.26 m\(^3\) for comparison purposes. A brief insight of the IMEX SFLOW 2D has been discussed in the subsequent section below:

2.1 Depth averaged dynamic model (IMEX SFLOW 2D)
IMEX-SFLOW2D 1.0 [25] is a dynamic continuum model, which utilizes the Eqs.1 to 3 for defining the velocity in \(x\)-and \(y\)-directions, in contrast, the average depth in flow direction as in Eq.7 explains the rheological characteristics of the flow. These equations were collectively solved with the above assumption using a finite-volume central-upwind scheme in space and an implicit-explicit Runge-Kutta scheme in time. This code aims to run simulations on co-located grids derived from DEMs.
ESRI, ASCII format standard input files defining the topography used in which square sized equally spaced grid pixels are arranged in rows and columns. The method to define the elevation values at the face centers and cell centers of the computational grid is represented in Figure 1. For details, interested readers are advised to refer to the work of Vitturi et.al [25].

![Computational grids of the IMEX SFLOW 2D code](image)

**Figure 1.** Computational grids of the IMEX SFLOW 2D code in which colored pixels reflect the elevation values of the original DEM, and lines define the edges of the IMEX-Sflow2D computational cells [7].

2.2 *Features, limitations, and input parameter of the model*

The model is formulated to simulate the wide range of the hazard from rock avalanches, snow avalanches, and hyper-concentrated flow by defining the appropriate rheological model and density of the mixture. The model simulates the single-phase fluid on the geographical coordinate system using the elevation defined in ASCII and ESRI format which can easily be handled by any GIS platform. Debris flow intensities can be obtained using the IMEX SFLOW 2D depth-averaged model by compiling the different files in FORTRAN code along with a Python script used to obtain different curves associated with the debris flow analysis. It has three input parameters including the *digital elevation model* (DEM), *release volume*, and *friction parameters*. The digital elevation model plays a valuable role in run out simulation since it determines the extent of hazard in its direction of flow and also provides the visualization of the model. The friction parameter consists of viscous turbulent friction ($\xi$), and dry coulomb friction ($\mu$), which describe the rheology of the fluid. In the result runout distance, velocity of flow, average depth contours will be the output of the model. The adopted range of the frictional parameter at constant volume and its output has been shown in Table 1.

| Volume (m$^3$) | Friction parameter ($\mu$, $\xi$) | Time (s) | Maximum runout velocity (m/s) | Maximum Runout distance (m) | Remark |
|----------------|----------------------------------|----------|------------------------------|-----------------------------|--------|
| 13.26          | 0.3, 150                         | 30       | 2.5                          | 30                          | All the analysis has been run on a 200*134 grid taking the constant volume of granular material |
| 13.26          | 0.3, 200                         | 30       | 2.7                          | 30.5                        |        |
| 13.26          | 0.3, 250                         | 30       | 3.0                          | 30.7                        |        |
| 13.26          | 0.3, 300                         | 30       | 3.2                          | 30.9                        |        |
3. Result and discussion

In Figure 2, it is well observed, that the granular material is released from the initial position starts flowing under the influence of gravity and turbulent frictional stresses till it attains the maximum velocity near the transition phase. From the transition phase coulomb frictional stresses ($\mu$) dominate and opposes the flow and finally stop it. The numerical output at four different times (1, 10, 20, and 30 s) showing on the arbitrary topography has presented in Figure 2 with depth average contour.

![Figure 2](image)

**Figure 2.** Thickness contour of the flowing material at a different time interval and in the bottom right panel flow was stopped after 30s by merging continuously in the horizontal plane.

As shown by the plots in Figure 2 average thickness of the flow has been presented by the contours at different time intervals which justifies the suitability of the model without the need for any artificial numerical diffusion. Motion of the material has been controlled by the volume of flow and slope of the terrain and the rheological model defined for flow.
It has been noticed that the turbulent frictional parameter ($\xi$) with constant Coulomb friction coefficient ($\mu$) has a marginal influence on the runout velocity in granular debris flow (see Figure 3) due to the small range (i.e., 100 to 300 ms$^{-2}$) of the parameter selected and also due to the small computational domain. Peak velocity is increased from 2.5 to 3.5 ms$^{-1}$ due to variation in turbulent frictional parameters. Further, when flow on the slope with finite volume and constant density accelerates down under gravitation force, frictional stresses try to dominate before maximising the velocity in the transition phase. It starts merging smoothly to the horizontal plane, frictional stress dominated which ultimately increases shear stress between fluid and surface, resulting in the stopping of flow.

Figure 3. Velocity profile in different frictional combination along the runout distances

Figure 4. Effect of the turbulent friction at runout distances with time

Runout distance largely depends on the velocity of flow and bed characteristics of slope. Its assessment is necessary for the risk mapping for any type of hazard. It can be seen in Figure 4 that the
turbulent frictional parameter has a negligible effect on the runout distances, but flow stop after reaching up to maximum length from 30 to 30.9 m from case 1 to case 4 respectively. Further from Figure 4 and Table 1, it can be concluded that, if the velocity of the flow increases due to an increase in turbulent friction stresses ultimately it affects the runout distances in large catastrophic debris flows.

From experimental studies of different sediment concentrations present in debris flow across the world, the range of turbulent friction is high as 3000 to 4000 ms$^{-2}$ [13] in the natural scenario. Therefore, the large computational domain and a large range of the frictional parameter will influence the runout distances and other properties of the debris flow.

4. Conclusion
Debris flow dynamics involve the different parameters to incorporate the deriving and resisting stresses in the direction of flow. For analyzing the non-Newtonian flow such as pyroclastic flow, snow avalanches, and granular debris flow, a 2D mass and momentum conservation equation has been employed together with an appropriate rheological model to adapt the frictional characteristics of the fluid. In this regard, the Voellmy- Salm rheological model consisting of the coulomb and turbulent frictional term has been widely adopted in the literature to simulate the different types of flow hazard.

The analysis of the Voellmy-Salm rheological model using IMEX 2D software revealed that the turbulent friction parameter ($\xi$) plays a significant role in debris flow dynamics with slope and initial volume of the flow. Following are the conclusion which has been extracted from the study.

- Debris flow dynamics depend on the sediment involve in the flow, which is categorized into Snow avalanches, Pyroclastic flow, Mud slurry flow, Torrential flow, and can be defined by identifying the rheological characteristics of the flow.
- Bingham rheological model is accepted for the debris flow modeling in various case studies, but shear thickening and thinning phenomenon at high strain rate cannot incorporate in it precisely.
- Hershey-Bulky model and Voellmy- Salm rheological model improve the shortcoming of the Bingham model and can also simulate the non-Newtonian fluid behavior effectively.
- Turbulent frictional parameter varies with the sediment concentration of the fluid and influence the debris flow characteristics.
- Velocity and runout distances are important factors to characterize the debris flow and to simulate its impact on the infrastructure.
- Turbulent frictional parameter significantly influences the velocity and runout distances when use at a large scale (field condition).
- The range of the turbulent coefficient of the friction is very flexible from 3000 to 4000 ms$^{-2}$ which is responsible for uncertainties in the velocity and runout measurement.
- A small range of the turbulent coefficient friction (i.e., 100 to 300 ms$^{-2}$) has been shown a significant variation in the velocity of the flow (i.e., 2.5 to 3.5ms$^{-1}$) at constant volume and slope.
- Runout distance shows the marginal effect due to the small range of the turbulent coefficient friction and fix computational domain (i.e., 30 m to 30.9 m respectively). Therefore, care should be taken to select a suitable rheological model while analyzing the debris flow hazard on the natural terrain.

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