Research Article

Measuring Performance of Ratio-Exponential-Log Type General Class of Estimators Using Two Auxiliary Variables

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1. Introduction

In survey sampling, ratio, product, exponential-ratio, log-ratio, and regression type estimators are modified or constructed by many researchers to enhance the precision of the estimators under different sampling designs by using the auxiliary variables. These estimators are commonly used by taking the advantage of correlation coefficient between the study variable and the auxiliary variable(s). Some notable work by the authors includes Olkin [1]; Mohanty [2]; Abu-Dayyeh et al. [3]; Koyuncu and Kadilar [4]; Swain [5]; Lu and Yan [6]; Lu et al. [7]; Sanaullah et al. [8]; Lu [9]; Muneer et al. [10]; Shabbir and Gupta [11]; Akingbade and Okafor [12]; Shabbir et al. [13]; Shabbir et al. [14]; Bhushan et al. [15]; Lone et al. [16]; and Kumari and Thaur [17].

Consider a finite population \( \Gamma = [\Gamma_1, \Gamma_2, \ldots, \Gamma_N] \) of N units. A sample of size \( n \) units is drawn from a population by using simple random sampling without replacement (SRSWOR). Let \( y_i \) and \( (x_i, z_i) \) be the characteristics of the study variable \( Y \) and the auxiliary variables \( (X, Z) \), respectively. Let \( \bar{y} = n^{-1} \sum_{i=1}^{n} y_i \), and \( \bar{x} = n^{-1} \sum_{i=1}^{n} x_i, \bar{z} = n^{-1} \sum_{i=1}^{n} z_i \), respectively, be the sample means corresponding to the population means \( Y = N^{-1} \sum_{i=1}^{N} y_i \), and \( \bar{x} = N^{-1} \sum_{i=1}^{N} x_i, \bar{z} = N^{-1} \sum_{i=1}^{N} z_i \). To obtain the bias and MSE expressions, we define the following error terms: \( \Xi_0 = (\bar{y} - \bar{Y}) - 1 \), and \( \Xi_1 = (\bar{x} - \bar{X}) - 1 \), \( \Xi_2 = (\bar{z} - \bar{Z}) - 1 \), such that \( E(\Xi_0) = 0 \), \( E(\Xi_1) = 0 \), \( E(\Xi_2) = 0 \), \( E(X_i^2) = \Theta X_i^2 \), \( E(Y_i^2) = \Theta Y_i^2 \), \( E(\Xi_0^2) = \Theta \Xi_0^2 \), \( E(\Xi_1^2) = \Theta \Xi_1^2 \), \( E(\Xi_2^2) = \Theta \Xi_2^2 \), \( E(\Xi_i^2) = \Theta \Xi_i^2 \). Consider a finite population \( \Gamma = [\Gamma_1, \Gamma_2, \ldots, \Gamma_N] \) of N units. A sample of size \( n \) units is drawn from a population by using simple random sampling without replacement (SRSWOR). Let \( y_i \) and \( (x_i, z_i) \) be the characteristics of the study variable \( Y \) and the auxiliary variables \( (X, Z) \), respectively. Let \( \bar{y} = n^{-1} \sum_{i=1}^{n} y_i \), and \( \bar{x} = n^{-1} \sum_{i=1}^{n} x_i, \bar{z} = n^{-1} \sum_{i=1}^{n} z_i \), respectively, be the sample means corresponding to the population means \( Y = N^{-1} \sum_{i=1}^{N} y_i \), and \( \bar{x} = N^{-1} \sum_{i=1}^{N} x_i, \bar{z} = N^{-1} \sum_{i=1}^{N} z_i \). To obtain the bias and MSE expressions, we define the following error terms: \( \Xi_0 = (\bar{y} - \bar{Y}) - 1 \), and \( \Xi_1 = (\bar{x} - \bar{X}) - 1 \), \( \Xi_2 = (\bar{z} - \bar{Z}) - 1 \), such that \( E(\Xi_0) = 0 \), \( E(\Xi_1) = 0 \), \( E(\Xi_2) = 0 \), \( E(X_i^2) = \Theta X_i^2 \), \( E(Y_i^2) = \Theta Y_i^2 \), \( E(\Xi_0^2) = \Theta \Xi_0^2 \), \( E(\Xi_1^2) = \Theta \Xi_1^2 \), \( E(\Xi_2^2) = \Theta \Xi_2^2 \), \( E(\Xi_i^2) = \Theta \Xi_i^2 \).
2. Some Existing Estimators

Some existing estimators available in the literature are essential to be discussed here.

2.1. Sample Mean Estimator. The usual sample mean estimator and its variance are given as

\[ \hat{Y}_{(0)} = \bar{Y}, \]  
\[ \text{Var}(\hat{Y}_{(0)}) = \Theta \bar{Y}^2 C_y^2. \]  

2.2. Ratio Estimators. The usual ratio estimators when using single and two auxiliary variables are given by

(i) \[ \hat{Y}^{(1)}_{(R)} = \bar{Y} \left( \frac{X}{x} \right), \]  
(ii) \[ \hat{Y}^{(2)}_{(R)} = \bar{Y} \left( \frac{Z}{x} \right), \]  
(iii) \[ \hat{Y}^{(3)}_{(R)} = \bar{Y} \left( \frac{X}{x} \right) \left( \frac{Z}{z} \right). \]  

The MSEs of ratio estimators \[ \hat{Y}^{(i)}_{(R)} (i = 1, 2, 3) \] to first order of approximation are given by

(i) \[ \text{MSE} \left( \hat{Y}^{(1)}_{(R)} \right) = \Theta \bar{Y}^2 \left[ C_y^2 + C_x^2 - 2C_{yx} \right], \]  
(ii) \[ \text{MSE} \left( \hat{Y}^{(2)}_{(R)} \right) = \Theta \bar{Y}^2 \left[ C_y^2 + C_z^2 - 2C_{yz} \right], \]  
(iii) \[ \text{MSE} \left( \hat{Y}^{(3)}_{(R)} \right) = \Theta \bar{Y}^2 \left[ C_y^2 + C_x^2 + C_z^2 - 2(C_{yx} + C_{yz}) + 2C_{xz} \right]. \]  

The ratio estimators \[ \hat{Y}^{(i)}_{(R)} (i = 1, 2, 3) \] are performing better than \[ \hat{Y}_{(0)} \] under certain conditions.

2.3. Exponential-Ratio Estimators. The usual exponential-ratio estimators when using single and two auxiliary variables are given by

(i) \[ \hat{Y}^{(1)}_{(E)} = \bar{Y} \exp \left( \frac{X - x}{X + x} \right), \]  
(ii) \[ \hat{Y}^{(2)}_{(E)} = \bar{Y} \exp \left( \frac{Z - z}{Z + z} \right), \]  
(iii) \[ \hat{Y}^{(3)}_{(E)} = \bar{Y} \exp \left( \frac{X - x}{X + x} \right) \exp \left( \frac{Z - z}{Z + z} \right). \]  

The MSEs of exponential-ratio estimators \[ \hat{Y}^{(i)}_{(E)} (i = 1, 2, 3) \] to first order of approximation are given by

(i) \[ \text{MSE} \left( \hat{Y}^{(1)}_{(E)} \right) \equiv \Theta \bar{Y}^2 \left[ C_y^2 + \frac{1}{4}C_x^2 - C_{yx} \right], \]  
(ii) \[ \text{MSE} \left( \hat{Y}^{(2)}_{(E)} \right) \equiv \Theta \bar{Y}^2 \left[ C_y^2 + \frac{1}{4}C_z^2 - C_{yz} \right], \]  
(iii) \[ \text{MSE} \left( \hat{Y}^{(3)}_{(E)} \right) \equiv \Theta \bar{Y}^2 \left[ C_y^2 + \frac{1}{4}(C_x^2 + C_z^2) - (C_{yx} + C_{yz}) + \frac{1}{2}C_{xz} \right]. \]  

2.4. Log-Ratio Estimators. Recently many log-type estimators have appeared in the literature in various forms when the logarithmic relationship between the study variable and the auxiliary variables exists.

The usual log-ratio estimators when using single and two auxiliary variables are given by

(i) \[ \tilde{Y}^{(1)}_{(log)} = \bar{Y} \left[ 1 + \log \left( \frac{X}{x} \right) \right], \]  
(ii) \[ \tilde{Y}^{(2)}_{(log)} = \bar{Y} \left[ 1 + \log \left( \frac{Z}{z} \right) \right], \]  
(iii) \[ \tilde{Y}^{(3)}_{(log)} = \bar{Y} \left[ 1 + \log \left( \frac{X}{x} \right) \right] \left[ 1 + \log \left( \frac{Z}{z} \right) \right]. \]  

The MSEs of log-ratio estimators \[ \tilde{Y}^{(i)}_{(log)} (i = 1, 2, 3) \] to first order of approximation are given by

(i) \[ \text{MSE} \left( \tilde{Y}^{(1)}_{(log)} \right) \equiv \Theta \bar{Y}^2 \left[ C_y^2 + C_x^2 - 2C_{yx} \right], \]  
(ii) \[ \text{MSE} \left( \tilde{Y}^{(2)}_{(log)} \right) \equiv \Theta \bar{Y}^2 \left[ C_y^2 + C_z^2 - 2C_{yz} \right], \]  
(iii) \[ \text{MSE} \left( \tilde{Y}^{(3)}_{(log)} \right) \equiv \Theta \bar{Y}^2 \left[ C_y^2 + C_x^2 + C_z^2 - 2(C_{yx} + C_{yz}) + 2C_{xz} \right]. \]  

The MSEs of log-ratio estimators \[ \tilde{Y}^{(i)}_{(log)} (i = 1, 2, 3) \] are exactly equal to the MSEs of ratio estimators \[ \hat{Y}^{(i)}_{(R)} (i = 1, 2, 3) \] but their biases are different (not shown here).

2.5. Regression Estimators. The usual regression estimators when using single and two auxiliary variables are given by

(i) \[ \bar{Y}^{(1)}_{(Reg)} = \bar{Y} + b_{yx} (X - x), \]  
(ii) \[ \bar{Y}^{(2)}_{(Reg)} = \bar{Y} + b_{yz} (Z - z), \]  
(iii) \[ \bar{Y}^{(3)}_{(Reg)} = \bar{Y} + b_{yx} (X - x) + b_{yz} (Z - z), \]  

where \[ b_{yx} = (s_{yx}/s_x^2) \] and \[ b_{yz} = (s_{yz}/s_z^2) \] are the sample regression coefficients.
The MSEs of regression estimators \( \hat{\mathbf{Y}}^{(i)} \) \((i = 1, 2, 3)\) are given by

\[
\text{(i) MSE}\left( \hat{\mathbf{Y}}^{(1)} \right) = \Theta Y^2 C_y^2 (1 - \rho_y^2),
\]
\[
\text{(ii) MSE}\left( \hat{\mathbf{Y}}^{(2)} \right) = \Theta Y^2 C_y^2 (1 - \rho_y^2),
\]
\[
\text{(iii) MSE}\left( \hat{\mathbf{Y}}^{(3)} \right) = \Theta Y^2 C_y^2 (1 - \rho_y^2 + 2 \rho_x \rho_y \rho_z p_x z).
\]

These regression estimators \( \hat{\mathbf{Y}}^{(i)} \) \((i = 1, 2, 3)\) are performing better than \( \hat{\mathbf{Y}}^{(0)} \) and \( \hat{\mathbf{Y}}^{(j)} \) \((i = 1, 2, 3; j = R, E, \log)\) under certain conditions.

### 2.6. Some More Regression Type Estimators

Mohanty [2] suggested the following regression-type estimator:

\[
\hat{\mathbf{Y}}^{(M)} \text{ (Reg)} = \mathbf{Y} + b_{yx} (X - \bar{X}) \left( \frac{Z}{\sigma} \right).
\]

The MSE of \( \hat{\mathbf{Y}}^{(M)} \) is given by

\[
\text{MSE}\left( \hat{\mathbf{Y}}^{(M)} \text{ (Reg)} \right) = \Theta Y^2 \left[ C_y^2 (1 - \rho_y^2) + C_z^2 - 2 C_{yz} + 2C_y C_z \rho_y \rho_z p_x z \right].
\]

Swain [5] introduced the following regression-type estimator

\[
\hat{\mathbf{Y}}^{(S)} \text{ (Reg)} = [\mathbf{Y} + d_0 (X - \bar{X})] \left( \frac{Z}{\sigma} \right).
\]

The minimum MSE of \( \hat{\mathbf{Y}}^{(S)} \) at optimum value of \( d_{0\text{ (opt)}} = (\mathbf{Y} (C_y \rho_y - C_z \rho_z) / C_y) \) is given by

\[
\text{MSE}\left( \hat{\mathbf{Y}}^{(S)} \text{ (Reg)} \right)_{\text{min}} = \Theta Y^2 C_y^2 \left[ C_y^2 + C_z^2 - 2 C_{yz} \right] - \left( C_y \rho_y - C_z \rho_z \right)^2.
\]

The unbiased regression estimator when using two auxiliary variables is given by

\[
\hat{\mathbf{Y}}^{(U)} = \mathbf{Y} + d_1 (X - \bar{X}) + d_2 (Z - \bar{Z}),
\]

where \( d_1 \) and \( d_2 \) are constants.

The minimum MSE of \( \hat{\mathbf{Y}}^{(U)} \) at optimum values of \( d_{1\text{ (opt)}} = (S_y (\rho_y - \rho_z p_x z) / S_y (1 - \rho_y^2)) \) and \( d_{2\text{ (opt)}} = (S_y (\rho_y - \rho_z p_x z) / S_y (1 - \rho_z^2)) \) is given by

\[
\text{MSE}\left( \hat{\mathbf{Y}}^{(U)} \text{ (Reg)} \right)_{\text{min}} = \Theta Y^2 C_y^2 \left( 1 - R_{yx}^2 \right),
\]

where \( R_{yx}^2 = (\rho_y^2 + \rho_z^2 - 2 \rho_y \rho_z p_x z / (1 - \rho_y^2)) \) is the multiple correlation coefficient.

### 3. Proposed General Class of Estimators

We propose a ratio-exponential-log type general class of estimators in estimating the finite population mean using two auxiliary variables when some parameters of the auxiliary variables are known. We also obtain different special estimators as members of the general class of estimators which are useful in different real-life situations. The proposed estimator is the combination of three special estimators including ratio, exponential-ratio, and log-ratio by using the linear transformation as

\[
\hat{\mathbf{Y}}^{(a_1, a_2, a_3; \gamma_1; \gamma_2; \gamma_3)} = \mathbf{Y} \left[ M_1 \left( \frac{X}{\sigma_X} \right)^{a_1} \left\{ \exp \left( \frac{X - \bar{X}}{\sigma_X + \bar{X}} \right) \right\}^{a_2} \left\{ 1 + \log \left( \frac{X}{\sigma_X} \right) \right\}^{a_3} \right] + M_2 \left( \frac{Z}{\sigma_Z} \right)^{a_4} \left\{ \exp \left( \frac{Z - \bar{Z}}{\sigma_Z + \bar{Z}} \right) \right\}^{a_5} \left\{ 1 + \log \left( \frac{Z}{\sigma_Z} \right) \right\}^{a_6},
\]

where \( M_1, M_2 \) are constants, whose values are to be determined; \( a_1, a_2, a_3, a_4, a_5, a_6; \gamma_1, \gamma_2, \gamma_3 \) are scaler quantities; and \( \bar{X} = a \bar{X} + b \), \( \bar{Z} = c \bar{Z} + d \), and \( \bar{X} = \bar{X} + d \). Here \( a, b, c, d \) are the known parameters of the auxiliary variables which may be coefficients of variation \( (C_x, C_z) \), coefficients of kurtosis \( (\beta_{2x}, \beta_{2z}) \) and correlation coefficients \( (\rho_y, \rho_z) \).

Solving (25) in terms of errors to the first order of approximation, we have

\[
\hat{\mathbf{Y}}^{(a_1, a_2, a_3; \gamma_1; \gamma_2; \gamma_3)} = \mathbf{Y} \left[ M_1 \left\{ 1 + \bar{Z} - \Delta_1 g_1 \bar{X} - \Delta_2 g_2 \bar{Z}^2 \right\} + M_2 \left\{ 1 + \bar{Z} - \Omega_1 g_1 \bar{Z}_2 - \Omega_2 g_2 \bar{Z}_2 + \alpha_2 g_2 \bar{Z}^2 \right\} - 1 \right],
\]

where

\[
\Delta_1 = \frac{\Delta_2}{\Delta_1}, \quad \Omega_1 = \frac{\Delta_2}{\Omega_2}, \quad \alpha_2 = \frac{\Delta_2}{\alpha_2}.
\]
\[ g_1 = \frac{aX + b}{aX + b}, \]
\[ g_2 = \frac{cZ + d}{cZ + d}. \]
\[ \Delta_1 = \alpha_1 + \frac{1}{2}\alpha_2 + \alpha_3, \]
\[ \Delta_2 = \alpha_1\alpha_3 + \frac{1}{2}\alpha_2\alpha_3 + \frac{1}{2}\alpha_1 (\alpha_1 + 1) + \frac{1}{8}\alpha_2 (\alpha_2 + 2) + \frac{1}{2}\alpha_1\alpha_2 + \frac{1}{2}\alpha_3^2, \]
\[ \Omega_1 = y_1 + \frac{1}{2}y_2 + y_3, \]
\[ \Omega_2 = y_1y_3 + \frac{1}{2}y_2y_3 + \frac{1}{2}y_1 (y_1 + 1) + \frac{1}{8}y_2 (y_2 + 2) + \frac{1}{2}y_1y_2 + \frac{1}{2}y_3^2. \]

The bias of \( \widetilde{Y}_{(M)}^{(a_1,a_2,a_3,y_1,y_2,y_3)} \) to the first order of approximation is given by

\[ \text{Bias}\left( \widetilde{Y}_{(M)}^{(a_1,a_2,a_3,y_1,y_2,y_3)} \right) = \mathbb{E}[M_1\{1 + \Theta(\Delta_2g_1^2C_x^2 - \Delta_1g_1C_{yx})\}]
+ M_2\{1 + \Theta(\Omega_2g_2^2C_z^2 - \Omega_1g_2C_{yz})\} - 1]. \]

The MSE of \( \widetilde{Y}_{(M)}^{(a_1,a_2,a_3,y_1,y_2,y_3)} \) to the first order of approximation is given by

\[ \text{MSE}\left( \widetilde{Y}_{(M)}^{(a_1,a_2,a_3,y_1,y_2,y_3)} \right) \approx \mathbb{E}^2\left[M_1\{1 + \Xi_0 - \Delta_1g_1\Xi_1 - \Delta_1g_1\Xi_2\Xi_1 + \Delta_2g_1^3\Xi_1^2\}
+ M_2\{1 + \Xi_0 - \Omega_1g_2\Xi_2 - \Omega_1g_2\Xi_0\Xi_2 + \Delta_2g_2^3\Xi_2^2\} - 1\}^2. \]

Solving (29), we get

\[ \text{MSE}\left( \widetilde{Y}_{(M)}^{(a_1,a_2,a_3,y_1,y_2,y_3)} \right) \approx \mathbb{E}^2\left[1 + M_1^2A_m + M_2^2B_m - 2M_1C_m - 2M_2D_m + 2M_1M_2E_m\right]. \]

where

\[ A_m = 1 + \Theta\{C_x^2(\Delta_1^2 + 2\Delta_2)g_1^2C_x^2 - 4\Delta_1g_1C_{yx}\}, \]
\[ B_m = 1 + \Theta\{C_x^2(\Omega_1^2 + 2\Omega_2)g_2^2C_z^2 - 4\Omega_1g_2C_{yz}\}, \]
\[ C_m = 1 + \Theta\{\Delta_2g_1^2C_x^2 - \Delta_1g_1C_{yx}\}, \]
\[ D_m = 1 + \Theta\{\Omega_1g_2^2C_z^2 - \Omega_1g_2C_{yz}\}, \]
\[ E_m = 1 + \Theta\{C_x^2 + \Delta_2g_1^2C_x^2 + \Omega_2g_2^2C_z^2 - 2\Delta_1g_1C_{yx} - 2\Omega_1g_1C_{yx} + \Delta_1g_1g_2C_{xz}\}. \]
Solving (30), the optimum values $M_i (i = 1, 2)$ are given as

$$
M_{1(\text{opt})} = \frac{B_m C_m - D_m E_m}{A_m B_m - E_m^2},
$$

$$
M_{2(\text{opt})} = \frac{A_m D_m - C_m E_m}{A_m B_m - E_m^2}. \tag{32}
$$

The minimum MSE of $\hat{Y}_{\text{(M)}}^{(\alpha_1, \alpha_2, \alpha_3, \gamma_1, \gamma_2, \gamma_3)}$ to the first order of approximation is given by

$$
\text{MSE}\left(\hat{Y}_{\text{(M)}}^{(\alpha_1, \alpha_2, \alpha_3, \gamma_1, \gamma_2, \gamma_3)}\right)_{\text{min}} \geq \gamma^2 \left[1 - \frac{A_m D_m^2 + B_m C_m^2 - 2C_m D_m E_m}{A_m B_m - E_m^2}\right]. \tag{33}
$$

Some special estimators as members of the proposed general class of estimators are given by

(i) Putting $\alpha_1 = \gamma_1 = 1, \alpha_2 = \alpha_3 = \gamma_2 = \gamma_3 = 0$ in (25), we get

$$
\hat{Y}_{\text{(M)}}^{(0,1,0,1,0)} = \gamma \left[M_1 \left\{ \exp\left(\frac{X^* - X^*}{X^* + X}\right) \right\} + M_2 \left\{ \exp\left(\frac{Z^* - Z^*}{Z^* + Z}\right) \right\} \right]. \tag{34}
$$

(ii) Putting $\alpha_2 = \gamma_2 = 1, \alpha_1 = \alpha_3 = \gamma_1 = \gamma_3 = 0$ in (25), we get

$$
\hat{Y}_{\text{(M)}}^{(0,0,1,0,1)} = \gamma \left[M_1 \left\{ 1 + \log\left(\frac{X^*}{X}\right) \right\} + M_2 \left\{ 1 + \log\left(\frac{Z^*}{Z}\right) \right\} \right]. \tag{35}
$$

(iii) Putting $\alpha_3 = \gamma_3 = 1, \alpha_1 = \alpha_2 = \gamma_1 = \gamma_2 = 0$ in (25), we get

$$
\hat{Y}_{\text{(M)}}^{(0,0,0,1,1)} = \gamma \left[M_1 \left\{ \exp\left(\frac{X^*}{X^* + X}\right) \right\} \right] + M_2 \left\{ \exp\left(\frac{Z^*}{Z^* + Z}\right) \right\}. \tag{36}
$$

(iv) Putting $\alpha_1 = \alpha_3 = \gamma_3 = 1, \alpha_2 = \gamma_1 = \gamma_2 = 0$ in (25), we get

$$
\hat{Y}_{\text{(M)}}^{(1,1,0,1,0)} = \gamma \left[M_1 \left\{ \frac{X^*}{X^* + X} \right\} \right] + M_2 \left\{ \exp\left(\frac{Z^*}{Z^* + Z}\right) \right\}. \tag{37}
$$

(v) Putting $\alpha_1 = \alpha_3 = \gamma_3 = 1, \alpha_2 = \gamma_1 = \gamma_2 = 0$ in (25), we get

$$
\hat{Y}_{\text{(M)}}^{(1,0,1,1,0)} = \gamma \left[M_1 \left\{ \frac{X^*}{X^*} \right\} \right] + M_2 \left\{ 1 + \log\left(\frac{X^*}{X}\right) \right\}. \tag{38}
$$
(vi) Putting \( \alpha_2 = \alpha_3 = \gamma_2 = 1, \alpha_1 = \gamma_1 = \gamma_3 = 0 \) in (25), we get

\[
\tilde{Y}^{(0,1,0,1,0)}(M) = \mathcal{Y} \left[ M_1 \left\{ \exp \left( \frac{X^* - X^*}{X^* + X^*} \right) \right\} \left\{ 1 + \log \left( \frac{X^*}{X^*} \right) \right\} + M_2 \left\{ \exp \left( \frac{Z^* - Z^*}{Z^* + Z^*} \right) \right\} \right].
\] (39)

(vii) Putting \( \alpha_1 = \alpha_2 = \alpha_3 = \gamma_2 = 1, \gamma_1 = \gamma_3 = 0 \) in (25), we get

\[
\tilde{Y}^{(1,1,0,1,0)}(M) = \mathcal{Y} \left[ M_1 \left\{ \frac{X^*}{X^*} \right\} \left\{ \exp \left( \frac{X^* - X^*}{X^* + X^*} \right) \right\} \left\{ 1 + \log \left( \frac{X^*}{X^*} \right) \right\} + M_2 \left\{ \exp \left( \frac{Z^* - Z^*}{Z^* + Z^*} \right) \right\} \right].
\] (40)

(viii) Putting \( \alpha_1 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_2 = \alpha_3 = 0 \) in (25), we get

\[
\tilde{Y}^{(1,0,1,1,1)}(M) = \mathcal{Y} \left[ M_1 \left\{ \frac{X^*}{X^*} \right\} + M_2 \left\{ \frac{Z^*}{Z^*} \right\} \left\{ \exp \left( \frac{Z^* - Z^*}{Z^* + Z^*} \right) \right\} \left\{ 1 + \log \left( \frac{Z^*}{Z^*} \right) \right\} \right].
\] (41)

(i) Putting \( \alpha_2 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_1 = \alpha_3 = 0 \) in (25), we get

\[
\tilde{Y}^{(0,1,0,1,1)}(M) = \mathcal{Y} \left[ M_1 \left\{ \exp \left( \frac{X^* - X^*}{X^* + X^*} \right) \right\} + M_2 \left\{ \frac{Z^*}{Z^*} \right\} \left\{ \exp \left( \frac{Z^* - Z^*}{Z^* + Z^*} \right) \right\} \left\{ 1 + \log \left( \frac{Z^*}{Z^*} \right) \right\} \right].
\] (42)

(x) Putting \( \alpha_3 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_1 = \alpha_2 = 0 \) in (25), we get

\[
\tilde{Y}^{(0,0,1,1,1)}(M) = \mathcal{Y} \left[ M_1 \left\{ 1 + \log \left( \frac{X^*}{X^*} \right) \right\} + M_2 \left\{ \frac{Z^*}{Z^*} \right\} \left\{ \exp \left( \frac{Z^* - Z^*}{Z^* + Z^*} \right) \right\} \left\{ 1 + \log \left( \frac{Z^*}{Z^*} \right) \right\} \right].
\] (43)
Note: We can generate more sub-classes of the proposed general class of estimators by using different combinations.

4. Numerical Example

We use the following four real data sets for a numerical study.

Population 1 (see [19]):

\[
\begin{align*}
Y &= \text{Area under wheat in acres in 1974}, \\
X &= \text{Area under wheat in acres in 1971}, \\
Z &= \text{Area under wheat in acres in 1973}, \\
N &= 34, \\
n &= 20, \\
\bar{Y} &= 856.412, \\
\bar{X} &= 208.882, \\
\bar{Z} &= 199.441, \\
C_Y &= 0.8561, \\
C_X &= 0.721, \\
C_Z &= 0.753, \\
\rho_{yx} &= 0.449, \\
\rho_{yz} &= 0.443, \\
\rho_{xz} &= 0.980, \\
\beta_{2x} &= 2.910, \\
\beta_{2z} &= 3.732.
\end{align*}
\] (44)

Population 2 (see [18]):

\[
\begin{align*}
Y &= \text{Output of the factory}, \\
X &= \text{Number of workers}, \\
Z &= \text{Fixed capital}, \\
N &= 80, \\
n &= 20, \\
\bar{Y} &= 5182.637, \\
\bar{X} &= 285.125, \\
\bar{Z} &= 1126.463, \\
C_Y &= 0.354, \\
C_X &= 0.948, \\
C_Z &= 0.751, \\
\rho_{yx} &= 0.915, \\
\rho_{yz} &= 0.941, \\
\rho_{xz} &= 0.988, \\
\beta_{2x} &= 0.698, \\
\beta_{2z} &= 1.050.
\end{align*}
\] (45)

Population 3 (Punjab Development Statistics (2019)):

This data is taken from Punjab development of statistics of 36 districts of Punjab, Pakistan during 2018.

\[
\begin{align*}
Y &= \text{Number of reported crimes by hurt}, \\
X &= \text{Number of reported crimes by murdered}, \\
Z &= \text{Number of reported crimes by kidnapped}, \\
N &= 36, \\
n &= 10, \\
\bar{Y} &= 421.9722, \\
\bar{X} &= 112.3889, \\
\bar{Z} &= 412.2222, \\
C_Y &= 0.5718, \\
C_X &= 0.8336, \\
C_Z &= 1.4229, \\
\rho_{yx} &= 0.7786, \\
\rho_{yz} &= 0.6900, \\
\rho_{xz} &= 0.8483, \\
\beta_{2x} &= 7.5508, \\
\beta_{2z} &= 27.5415.
\end{align*}
\] (46)

Population 4 (see [19]):

This data are based on 69 villages of Doraha development bloc of Punjab, India.

\[
\begin{align*}
Y &= \text{Number of tube wells}, \\
X &= \text{Number of tractors}, \\
Z &= \text{Net irrigated area in hectares}, \\
N &= 69, \\
n &= 10, \\
\bar{Y} &= 135.2609, \\
\bar{X} &= 21.2319, \\
\bar{Z} &= 345.7536, \\
C_Y &= 0.8422, \\
C_X &= 0.7969, \\
C_Z &= 0.8478, \\
\rho_{yx} &= 0.9118, \\
\rho_{yz} &= 0.9224, \\
\rho_{xz} &= 0.9007, \\
\beta_{2x} &= 3.7653, \\
\beta_{2z} &= 7.2159.
\end{align*}
\] (47)
The results based on Populations 1–4 are given in Tables 1–11. We use the following expression to obtain the percent relative efficiency (PRE) as

\[
\text{PRE} = \frac{\text{Var} (\hat{Y}(0))}{\text{MSE}(\cdot)} \times 100, \quad (48)
\]

where \( (\cdot) = \hat{Y}(0), \hat{Y}^{(1)}(R), \hat{Y}^{(2)}(R), \hat{Y}^{(3)}(R), \hat{Y}^{(4)}(R) \) \((i = 1, 2, 3; j = M, S, U)\).

In Table 1, we observed the following:

(i) The ratio and log-ratio estimators \( \hat{Y}^{(i)}(R), \hat{Y}^{(i)}(\text{log}) \) in population 2, \( (\hat{Y}^{(2)}(R), \hat{Y}^{(2)}(\text{log})) \) in populations 2 and 3, and \( (\hat{Y}^{(3)}(R), \hat{Y}^{(3)}(\text{log})) \) in all four populations are performing poorly as compared to \( \hat{Y}(0) \).

(ii) The exponential-ratio estimator \( \hat{Y}^{(3)}(E) \) in Populations 2 and 3 is not performing good.

(iii) Mohanty [2] regression estimator \( \hat{Y}^{(M)}(\text{Reg}) \) in populations 1–3 and Swain [5] estimator \( \hat{Y}^{(3)}(\text{Reg}) \) in Population 3 are not efficient as compared to \( \hat{Y}(0) \).

(iv) Among all the estimators above discussed, the performance of \( \hat{Y}^{(U)}(\text{Reg}) \) is the best.

5. Comparison of Estimators

Now we compare the proposed class of estimators with other existing estimators.

**Condition 1.** By (2) and (33), \( \text{MSE}(\hat{Y}^{(i)}(M))_{\min} < \text{Var} (\hat{Y}(0)) \) if

\[
\Theta C_y^2 + \frac{O_1}{O_2} - 1 > 0, \quad (49)
\]

where \( O_1 = A_m D_m^2 + B_m C_m^2 - 2C_m D_m E_m \) and \( O_2 = A_m B_m - E_m^2 \).

**Condition 2.** By ((4)–(6)) or ((12)–(14)) and (33), \( \text{MSE}(\hat{Y}^{(i)}(M))_{\min} < \text{MSE}(\hat{Y}^{(i)}(R), \hat{Y}^{(i)}(\text{log})) \) \((i = 1, 2, 3)\) if

\[
\begin{align*}
\Theta \left( C_y^2 + C_x^2 - 2C_{yx} \right) + \frac{O_1}{O_2} - 1 > 0, \\
\Theta \left( C_y^2 + C_z^2 - 2C_{yz} \right) + \frac{O_1}{O_2} - 1 > 0, \\
\Theta \left( C_y^2 + C_x^2 + C_z^2 - 2(C_{yx} + C_{yz}) + 2C_{xz} \right) + \frac{O_1}{O_2} - 1 > 0.
\end{align*}
\]

\[(50)\]

**Condition 3.** By ((8)–(10)) and (33), \( \text{MSE}(\hat{Y}^{(i)}(M))_{\min} < \text{MSE}(\hat{Y}^{(i)}(E)) \) \((i = 1, 2, 3)\) if

\[
\Theta \left( C_y^2 + \frac{1}{4} C_x^2 - C_{yx} \right) + \frac{O_1}{O_2} - 1 > 0,
\]

\[
\Theta \left( C_y^2 + \frac{1}{4} C_z^2 - C_{yz} \right) + \frac{O_1}{O_2} - 1 > 0,
\]

\[
\Theta \left( C_y^2 + \frac{1}{4} C_x^2 + C_z^2 - (C_{yx} + C_{yz}) + \frac{1}{2} C_{xz} \right) + \frac{O_1}{O_2} - 1 > 0.
\]

\[(51)\]

**Condition 4.** By ((16)–(18)) and (33), \( \text{MSE}(\hat{Y}^{(i)}(M))_{\min} < \text{MSE}(\hat{Y}^{(i)}(\text{Reg})) \) \((i = 1, 2, 3)\) if

\[
\Theta \left( C_y^2 + \frac{1}{4} C_x^2 - C_{yx} \right) + \frac{O_1}{O_2} - 1 > 0,
\]

\[
\Theta \left( C_y^2 + \frac{1}{4} C_z^2 - C_{yz} \right) + \frac{O_1}{O_2} - 1 > 0,
\]

\[
\Theta \left( C_y^2 + \frac{1}{4} C_x^2 + C_z^2 - (C_{yx} + C_{yz}) + \frac{1}{2} C_{xz} \right) + \frac{O_1}{O_2} - 1 > 0.
\]

\[(51)\]
Table 3: PRE of proposed estimator when \((\alpha_3 = \gamma_2 = 1, \alpha_1 = \alpha_5 = \gamma_1 = \gamma_3 = 0)\).

| a | b | c | d   | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|---|---|---|-----|-------|-------|-------|-------|
| 1 | 0 | 1 | 0   | 127.090 | 956.748 | 264.876 | 311.868 |
| 1 | \(C_x\) | 1 | \(C_z\) | 127.073 | 956.817 | 264.643 | 310.282 |
| \(\beta_{2x}\) | \(C_x\) | \(\beta_{2z}\) | \(C_z\) | 127.084 | 956.854 | 264.849 | 311.339 |
| \(\beta_{2x}\) | \(\beta_{2z}\) | \(C_z\) | 126.980 | 956.775 | 261.317 | 319.839 |
| 1 | \(\rho_{yx}\) | 1 | \(\rho_{yz}\) | 127.079 | 956.808 | 264.675 | 310.291 |
| 1 | \(\beta_{2x}\) | 1 | \(\beta_{2z}\) | 127.011 | 956.783 | 261.624 | 314.469 |
| \(\rho_{yx}\) | \(C_x\) | \(\rho_{yz}\) | \(C_z\) | 127.075 | 956.803 | 264.642 | 310.422 |
| \(\rho_{yx}\) | \(\rho_{yz}\) | \(\beta_{2z}\) | \(\rho_{yz}\) | 126.900 | 956.788 | 260.260 | 316.213 |

Table 4: PRE of proposed estimator when \((\alpha_3 = \gamma_3 = 1, \alpha_1 = \alpha_2 = \gamma_1 = \gamma_2 = 0)\).

| a | b | c | d   | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|---|---|---|-----|-------|-------|-------|-------|
| 1 | 0 | 1 | 0   | 106.402 | 186.926 | 127.151 | 919.696 |
| 1 | \(C_x\) | 1 | \(C_z\) | 106.664 | 184.941 | 129.254 | 921.254 |
| \(\beta_{2x}\) | \(C_x\) | \(\beta_{2z}\) | \(C_z\) | 106.488 | 183.610 | 127.443 | 920.619 |
| \(\beta_{2x}\) | \(\beta_{2z}\) | \(C_z\) | 107.724 | 186.467 | 149.441 | 873.745 |
| 1 | \(\rho_{yx}\) | 1 | \(\rho_{yz}\) | 106.555 | 185.296 | 129.179 | 921.052 |
| 1 | \(\beta_{2x}\) | 1 | \(\beta_{2z}\) | 107.357 | 186.124 | 144.723 | 889.198 |
| \(\rho_{yx}\) | \(C_x\) | \(\rho_{yz}\) | \(C_z\) | 106.615 | 183.553 | 129.618 | 920.360 |
| \(\beta_{2x}\) | \(\rho_{yz}\) | \(C_z\) | 106.941 | 184.727 | 129.828 | 921.127 |
| \(\rho_{yx}\) | \(\rho_{yz}\) | \(\beta_{2z}\) | \(\rho_{yz}\) | 108.305 | 186.007 | 149.286 | 883.271 |

Table 5: PRE of proposed estimator when \((\alpha_1 = \alpha_2 = \gamma_2 = 1, \alpha_5 = \gamma_1 = \gamma_3 = 0)\).

| a | b | c | d   | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|---|---|---|-----|-------|-------|-------|-------|
| 1 | 0 | 1 | 0   | 126.769 | 914.223 | 141.181 | 807.850 |
| 1 | \(C_x\) | 1 | \(C_z\) | 126.777 | 914.152 | 141.896 | 794.946 |
| \(\beta_{2x}\) | \(C_x\) | \(\beta_{2z}\) | \(C_z\) | 126.771 | 914.197 | 141.216 | 804.456 |
| \(\beta_{2x}\) | \(\beta_{2z}\) | \(C_z\) | 126.818 | 913.997 | 150.724 | 732.960 |
| 1 | \(\rho_{yx}\) | 1 | \(\rho_{yz}\) | 126.774 | 914.109 | 141.569 | 793.158 |
| 1 | \(\beta_{2x}\) | 1 | \(\beta_{2z}\) | 126.807 | 914.066 | 153.920 | 747.374 |
| \(\rho_{yx}\) | \(C_x\) | \(\rho_{yz}\) | \(C_z\) | 126.775 | 914.048 | 141.497 | 789.553 |
| \(\beta_{2x}\) | \(\rho_{yz}\) | \(C_z\) | 126.786 | 914.150 | 142.202 | 793.730 |
| \(\rho_{yx}\) | \(\beta_{2x}\) | \(\rho_{yz}\) | \(\beta_{2z}\) | 126.770 | 914.155 | 141.204 | 803.975 |
| \(\rho_{yx}\) | \(\beta_{2x}\) | \(\rho_{yz}\) | \(\beta_{2z}\) | 126.849 | 914.058 | 159.313 | 741.821 |

Table 6: PRE of proposed estimator when \((\alpha_1 = \alpha_3 = \gamma_2 = 1, \alpha_2 = \gamma_1 = \gamma_3 = 0)\).

| a | b | c | d   | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|---|---|---|-----|-------|-------|-------|-------|
| 1 | 0 | 1 | 0   | 126.737 | 912.457 | 139.553 | 845.147 |
| 1 | \(C_x\) | 1 | \(C_z\) | 126.743 | 912.383 | 140.132 | 836.605 |
| \(\beta_{2x}\) | \(C_x\) | \(\beta_{2z}\) | \(C_z\) | 126.739 | 912.423 | 139.573 | 842.896 |
| \(\rho_{yx}\) | \(C_x\) | \(\rho_{yz}\) | \(C_z\) | 126.779 | 912.237 | 147.425 | 797.474 |
| 1 | \(\rho_{yx}\) | 1 | \(\rho_{yz}\) | 126.741 | 912.342 | 139.830 | 835.452 |
| 1 | \(\beta_{2x}\) | 1 | \(\beta_{2z}\) | 126.769 | 912.303 | 150.655 | 806.104 |
| \(\beta_{2x}\) | \(\rho_{yz}\) | \(C_z\) | \(\rho_{yz}\) | 126.742 | 914.284 | 139.744 | 833.140 |
| \(\rho_{yx}\) | \(C_x\) | \(\rho_{yz}\) | \(C_z\) | 126.752 | 912.380 | 140.393 | 835.815 |
| \(\beta_{2x}\) | \(\rho_{yz}\) | \(C_z\) | \(\rho_{yz}\) | 126.738 | 914.383 | 139.562 | 842.580 |
| \(\rho_{yx}\) | \(\rho_{yz}\) | \(\beta_{2z}\) | \(\rho_{yz}\) | 126.805 | 912.295 | 155.475 | 802.724 |
### Table 7: PRE of proposed estimator when \((\alpha_2 = \alpha_3 = \gamma_2 = 1, \alpha_1 = \gamma_1 = \gamma_3 = 0)\).

| a  | b  | c  | d  | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|----|----|----|----|--------|--------|--------|--------|
| 1  | 0  | 1  | 0  | 126.710| 920.174| 140.021| 784.443|
| 1  | C_x| 1  | C_y| 126.715| 920.076| 140.691| 775.612|
| \(\beta_{2x}\)| \(\beta_{2y}\)| \(\beta_{2z}\)| \(\beta_{2z}\)| 126.711| 920.126| 140.050| 782.185|
| \(\rho_{yx}\)| \(\rho_{xz}\)| \(\rho_{xz}\)| \(\rho_{xz}\)| 126.743| 919.891| 149.088| 727.261|
| \(\rho_{yx}\)| \(\rho_{xz}\)| \(\rho_{xz}\)| \(\rho_{xz}\)| 126.713| 920.025| 140.371| 774.362|
| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| \(\rho_{yz}\)| 126.735| 919.976| 152.300| 739.153|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 126.714| 919.950| 140.293| 771.816|
| \(\rho_{yx}\)| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 126.763| 919.965| 157.553| 734.610|

### Table 8: PRE of proposed estimator when \((\alpha_1 = \alpha_2 = \alpha_3 = \gamma_2 = 1, \gamma_1 = \gamma_3 = 0)\).

| a  | b  | c  | d  | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|----|----|----|----|--------|--------|--------|--------|
| 1  | 0  | 1  | 0  | 126.768| 907.889| 139.833| 902.503|
| 1  | C_x| 1  | C_y| 126.776| 907.822| 140.396| 890.351|
| \(\beta_{2x}\)| \(\beta_{2y}\)| \(\beta_{2z}\)| \(\beta_{2z}\)| 126.770| 907.856| 139.852| 899.195|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 126.818| 907.695| 147.416| 843.179|
| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| \(\rho_{yz}\)| 126.772| 907.787| 140.103| 888.744|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 126.806| 907.753| 150.567| 852.694|
| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| \(\rho_{yz}\)| 126.774| 907.736| 140.019| 885.566|
| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| \(\rho_{yz}\)| 126.785| 907.820| 140.649| 889.257|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 126.769| 907.821| 139.842| 898.731|
| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| \(\rho_{yz}\)| 126.850| 907.746| 155.212| 848.913|

### Table 9: PRE of proposed estimator when \((\alpha_1 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_2 = \alpha_3 = 0)\).

| a  | b  | c  | d  | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|----|----|----|----|--------|--------|--------|--------|
| 1  | 0  | 1  | 0  | 123.139| 145.452| 159.789| 669.527|
| 1  | C_x| 1  | C_y| 123.185| 146.707| 161.602| 702.708|
| \(\beta_{2x}\)| \(\beta_{2y}\)| \(\beta_{2z}\)| \(\beta_{2z}\)| 123.155| 147.296| 160.040| 678.643|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 123.384| 146.324| 178.354| 820.621|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 123.168| 146.636| 161.528| 707.218|
| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| \(\rho_{yz}\)| 123.317| 146.313| 174.823| 798.447|
| \(\beta_{2x}\)| \(\beta_{yz}\)| \(\rho_{yz}\)| \(\beta_{yz}\)| 123.179| 146.668| 161.898| 716.133|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 123.240| 146.826| 162.099| 705.743|
| \(\beta_{2x}\)| \(\beta_{yz}\)| \(\rho_{yz}\)| \(\beta_{yz}\)| 123.149| 147.206| 160.025| 679.938|
| \(\rho_{yx}\)| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| 123.523| 146.397| 178.726| 807.318|

### Table 10: PRE of proposed estimator when \((\alpha_2 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_1 = \alpha_3 = 0)\).

| a  | b  | c  | d  | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|----|----|----|----|--------|--------|--------|--------|
| 1  | 0  | 1  | 0  | 127.304| 483.947| 272.896| 931.187|
| 1  | C_x| 1  | C_y| 127.306| 485.394| 272.843| 952.519|
| \(\beta_{2x}\)| \(\beta_{2y}\)| \(\beta_{2z}\)| \(\beta_{2z}\)| 127.304| 486.023| 272.627| 955.357|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 127.314| 485.060| 274.208| 928.019|
| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| \(\rho_{yz}\)| 127.305| 485.341| 272.869| 952.032|
| \(\beta_{2x}\)| \(\beta_{yz}\)| \(\rho_{yz}\)| \(\beta_{yz}\)| 127.311| 485.006| 273.258| 933.369|
| \(\rho_{yx}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| \(\rho_{yz}\)| 127.306| 485.413| 272.947| 952.016|
| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| \(\rho_{yz}\)| 127.304| 485.948| 272.626| 955.247|
| \(\rho_{yx}\)| \(\beta_{2x}\)| \(\rho_{yz}\)| \(\beta_{2z}\)| 127.318| 485.105| 272.896| 931.187|
Table 11: PRE of proposed estimator when \((\alpha_3 = y_1 = y_2 = y_3 = 1, \alpha_1 = \alpha_2 = 0)\).

| \(a\) | \(b\) | \(c\) | \(d\) | Pop. 1 | Pop. 2 | Pop. 3 | Pop. 4 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 1     | 0     | 122.150 | 178.926 | 180.747 | 649.616 |
| 1     | \(C_x\) | 1     | \(C_x\) | 122.196 | 180.406 | 181.991 | 682.237 |
| \(\beta_{2x}\) | \(C_x\) | \(\beta_{2x}\) | \(C_x\) | 122.167 | 181.159 | 180.961 | 658.386 |
| \(C_x\) | \(\beta_{2x}\) | \(C_x\) | \(\beta_{2x}\) | 122.392 | 179.818 | 192.910 | 797.503 |
| 1     | \(\rho_{xy}\) | 1     | \(\rho_{yz}\) | 122.179 | 180.286 | 182.139 | 686.675 |
| \(\beta_{2x}\) | \(\rho_{xy}\) | \(\beta_{2x}\) | \(\rho_{yz}\) | 122.324 | 179.859 | 188.365 | 776.097 |
| \(C_x\) | \(\rho_{xy}\) | \(C_x\) | \(\rho_{yz}\) | 122.191 | 180.278 | 182.524 | 695.431 |
| \(\rho_{xy}\) | \(\beta_{2x}\) | \(\rho_{yz}\) | \(\beta_{2x}\) | 122.529 | 179.955 | 190.802 | 784.693 |

The proposed sub-classes of general estimators are performing well as compared to their competitor estimators. We have generated 10 sub-classes from the proposed general estimators with different combinations which all are efficient in different situations as compared to SRS. So, the proposed general class of estimators is preferable in further study.

### Data Availability

The data used to support the findings of this study are included within the article.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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