Matching the observed cosmological constant with vacuum energy density in AdS

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Abstract

We calculate the vacuum energy density by taking account of different massive scalar fields in AdS spacetime. It is found that the mass spectrum of a scalar field in AdS spacetime is discrete because of a natural boundary condition. The results match well with the observed cosmological constant.

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I. INTRODUCTION

The cosmological constant has evoked much controversy in both astronomy and particle physics communities \[1,2\]. Recent observations of high-redshift supernovae seem to suggest that the global geometry of the universe may be affected by a positive cosmological constant \[3\]. And all kinds of cosmological observations, such as Cosmic Microwave Background radiation \[4,5\], redshifts of the supernovae and quasars \[6–8\], give a very tiny vacuum energy density as $10^{-48}$GeV$^4$ \[9\].

In particle physics, the vacuum is used to mean the ground state of quantum fields. A relativistic field may be thought of as a collection of harmonic oscillators of all possible frequencies, and each possible mode devotes $\frac{1}{2}h\omega$ energy to the vacuum. In this way, particle physicists \[1\] get a huge vacuum energy density as $2 \times 10^{71}$GeV$^4$, which is over 120 orders of magnitude in excess of the value allowed by cosmological observations. It is a more challenging problem to explain why the cosmological constant is so small but non-zero, than to build theoretical models where it exactly vanishes \[10\].

About twenty years ago, a number of authors discovered that Anti-de Sitter (AdS) space-time generically arose as ground state in supergravity theory, which at the time was considered to be among the most promising candidates for quantum gravity \[11–13\]. The interest on AdS spacetime was revived by a conjectured duality between string theory in the bulk of AdS and conformally invariant field theory (CFT) living on the boundary of AdS \[14\]. The AdS/CFT correspondence gives an explicit relation between Yang-Mills theory and string theory \[15,16\]. More recently, there has been a renewed interest in AdS spacetime since progresses in theories of extra dimensions present us with the enticing possibility to explain some long-standing particle physics problem by geometrical means \[17,18\].

The theoretical approaches to the cosmological constant problem can briefly be classified into three categories: (1)anthropic principle, (2)the “quintessence” \[19\], (3)fundamental partical physics and basic spacetime topology. In many scenarios, the mean value of the vacuum energy is positive, and the vacuum energy is related to the gravitational potential \[10\], or the fluctuation of the vacuum \[20\], or the wormholes which are described by string quantum cosmology.

It is no doubt that these studies are fruitful and helpful to make more progress in understanding the cosmological constant problem. However, it seems far away from achieving a natural and self-consistent explanation, which can be checked directly by the update astronomical observations.

In this Letter, we calculate the energy density of vacuum by taking account of different massive scalar fields in AdS spacetime. It is found that the mass spectrum of a scalar field in AdS spacetime is discrete because of a natural boundary condition \[21,22\]. The results match well with the observed cosmological constant.

II. EQUATIONS OF MOTION

AdS spacetime can be described as a submanifold of a pseudo-Euclidean five-dimensional embedding space with Cartesian coordinates $\xi^a$ and metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1, 1)$,

$$(\xi^0)^2 - (\xi^1)^2 - (\xi^2)^2 - (\xi^3)^2 + (\xi^5)^2 = -\frac{1}{\lambda},$$
where $\lambda$ is a constant ($\lambda < 0$). It is obvious that the symmetry group of AdS is the conformal group $SO(3, 2)$. Hence, we can define the 5-dimensional angular momentum of a free particle in AdS,

$$L^{\mu\nu} = m_0 \left( \xi^\mu \frac{d\xi^\nu}{ds} - \xi^\nu \frac{d\xi^\mu}{ds} \right),$$

(2)

the commutation relations satisfied by the ten infinitesimal generators are

$$[L_{\alpha\beta}, L_{\mu\nu}] = -i(g_{\alpha\mu}L_{\beta\nu} + g_{\beta\nu}L_{\alpha\mu} - g_{\alpha\nu}L_{\beta\mu} - g_{\beta\mu}L_{\alpha\nu}), \quad \alpha, \beta, \mu, \nu = 0, 1, 2, 3, 5.$$  

(3)

It is convenient to introduce the Beltrami coordinates \(\{x^i\} (i = 0, 1, 2, 3)\) as

$$\sqrt{-\lambda}x^i = (\xi^5)^{-1}\xi^i.$$  

(4)

In terms of the Beltrami coordinates, AdS is of the form

$$\sigma(x) = 1 - \lambda \eta_{ij}x^i x^j > 0, \quad \eta_{ij} = \text{diag}(1, -1, -1, -1).$$  

(5)

The Beltrami metric can be deduced directly

$$ds^2 = (\eta_{ij}\sigma^{-1} + \lambda \eta_{ir}\eta_{js}x^r x^s \sigma^{-2})dx^i dx^j.$$  

(6)

In this coordinates, ten group generators $L_{\mu\nu}$ are simply classified into two categories, $L_{ij}$ correspond to the generators of the rotation group $SO(3)$, $P_k$ generate parallel displacements. Thus, $L_{ij}$ and $P_k$ can be respectively defined as the angular momentum and the momentum 4-vector in the Beltrami coordinates

$$P^k = \frac{1}{\sqrt{-\lambda}}L^{5k} = m_0\sigma^{-1}\frac{dx^i}{ds}, \quad L^{ij} = m_0 (x^i P^j - x^j P^i) = m_0 \sigma^{-1} \left( x^i \frac{dx^j}{ds} - x^j \frac{dx^i}{ds} \right).$$  

(7)

Einstein’s mass formula in AdS is of the form (with $\hbar = c = 1$)

$$m_0^2 = \frac{|\lambda|}{2} L^{\mu\nu} L_{\mu\nu} = E^2 - \vec{P}^2 + \lambda \vec{L}^2,$$

$$E = P^0, \quad \vec{P} = (P^1, P^2, P^3).$$  

(8)

The Beltrami metric is invariant under the coordinate transformations

$$x^i \to \bar{x}^i = \sigma^{1/2}(a)(1 - \lambda \eta_{rs}a^r x^s)^{-1}(x^j - a^j)D_j^i,$$

$$D_j^i = L_j^i + \lambda \eta_{kd}a^d \left[ \sigma(a) + \sigma^{1/2}(a) \right]^{-1}L_k^j,$$

$$(L_j^i)_{i,j=0,1,2,3} \in SO(3, 1), \quad a^i \text{ are constants}.$$  

(9)

In the coordinate $(\xi^0, x^\alpha)$, the $SO(3, 2)$ invariant metric can be written as
\[ ds^2 = \frac{1}{1 + \lambda \xi \xi^0} d\xi^0 d\xi^0 - (1 + \lambda \xi^0 \xi^0) \frac{d\mathbf{x}(I + \lambda \mathbf{x}' \mathbf{x})^{-1} d\mathbf{x}'}{1 + \lambda \mathbf{x} \mathbf{x}'}, \]  

de (10)

where the vector \( \mathbf{x} \) denotes \((x^1, x^2, x^3)\) and \( \mathbf{x}' \) the transpose of the vector \( \mathbf{x} \).

In the spherical coordinate \((x^1, x^2, x^3) \rightarrow (\rho, \theta, \phi)\), the \(SO(3, 2)\) invariant metric (10) is of the form

\[ ds^2 = \frac{1}{1 + \lambda \xi \xi^0} d\xi^0 d\xi^0 - (1 + \lambda \xi^0 \xi^0) \left[ \frac{d \rho^2}{(1 + \lambda \rho^2)^2} + \frac{\rho^2}{(1 + \lambda \rho^2)} \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right]. \]  

de (11)

The Penrose diagram of AdS shows that there is a horizon \([23]\) in the coordinate \((\xi^0, \mathbf{x})\).

We limit us in the region of \( |\xi| < \sqrt{-\lambda} \). In this region of AdS, we can introduce a timelike variable \( \tau \) as

\[ \sqrt{-\lambda} \xi^0 \equiv \sin(\sqrt{-\lambda} \tau). \]  

de (12)

Then we have a Robertson-Walker-like metric

\[ ds^2 = d\tau^2 - R^2(\tau) \left[ (1 + \lambda \rho^2)^{-2} d\rho^2 + (1 + \lambda \rho^2)^{-1} \rho^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]  

de (13)

where we have used the notation \( R(\tau) = \cos(\sqrt{-\lambda} \tau) \).

The equation of motion for a massive scalar field in AdS spacetime is of the form

\[ \partial_{\tau}^2 U - \frac{2}{\rho} \partial_{\rho} U + \frac{k^2}{(1 + \lambda \rho^2)^2} \frac{1}{\rho^2} \partial_{\rho}^2 U - \frac{l(l+1)}{\rho^2 (1 + \lambda \rho^2)} U = 0, \]  

\[ R^2 \frac{d^2 T}{d\tau^2} + 3R \frac{dR}{d\tau} \frac{dT}{d\tau} + (m_0^2 R^2 + k^2) T = 0, \]  

\[ R^2 \frac{\partial^2 Y_{lm}}{\partial \theta^2} + \cot \theta \frac{\partial Y_{lm}}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2 Y_{lm}}{\partial \phi^2} + l(l+1) Y_{lm} = 0. \]  

III. DISCRETE MASS SPECTRUM

We can solve the equation of motion, which was obtained in the last section, for a massive scalar field \([21, 24]\) by writing

\[ \Phi(\tau; \rho, \theta, \phi) = T(\tau) U(\rho) Y_{lm}(\theta, \phi). \]  

de (15)

The reduced equations of motion in terms of \( T(\tau), U(\rho) \) and \( Y_{lm}(\theta, \phi) \) are of the form

\[ \frac{\partial^2 U}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial U}{\partial \rho} + \frac{k^2}{(1 + \lambda \rho^2)^2} \frac{1}{\rho^2} \frac{\partial^2 U}{\partial \rho^2} - \frac{l(l+1)}{\rho^2 (1 + \lambda \rho^2)} U = 0, \]  

\[ R^2 \frac{d^2 T}{d\tau^2} + 3R \frac{dR}{d\tau} \frac{dT}{d\tau} + (m_0^2 R^2 + k^2) T = 0, \]  

\[ R^2 \frac{\partial^2 Y_{lm}}{\partial \theta^2} + \cot \theta \frac{\partial Y_{lm}}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2 Y_{lm}}{\partial \phi^2} + l(l+1) Y_{lm} = 0. \]  

It is obvious that the solutions of the angular part of equation of motion are the spherical harmonic functions \( Y_{lm}(\theta, \phi) \).
The radial function determines the evolution of the model universe. For convenience, setting \( \lambda = -a^{-2} \) and \( \varrho = a^{-1} \rho \), we can rewrite the radial equation as

\[
\frac{d^2 U(\varrho)}{d\varrho^2} + \frac{2}{\varrho} \frac{dU(\varrho)}{d\varrho} + \left[ \frac{k^2 a^2}{(1 - \varrho^2)^2} - \frac{l(l+1)}{\varrho^2(1 - \varrho^2)} \right] U(\varrho) = 0 .
\] (17)

It is obvious that, at \( \varrho = 0, \pm 1 \), the radial function is singular. One can set \( U(\varrho) \) as following

\[
U(\varrho) = \varrho^l(1 - \varrho^2)^{1/2} F(\varrho) ,
\] (18)

where \( \mu \) is a solution of the index equation \( \mu(\mu - 2) + k^2 a^2 = 0 \). \( F(\varrho) \) is a solution of the hypergeometric equation

\[
(1 - \varrho^2) \frac{d^2 F}{d\varrho^2} + \left[ \frac{2(l+1)}{\varrho} - 2(l + \mu + 1) \right] \frac{dF}{d\varrho} + \left[ \frac{1}{4} - \left( \mu + l + \frac{1}{2} \right)^2 \right] F = 0 .
\] (19)

Therefore, we get the radial function of the form

\[
U(\rho) = C \left( \frac{\rho}{a} \right)^l \left( 1 - \frac{\rho^2}{a^2} \right)^{\frac{1}{2} + \frac{l}{2} + \frac{1}{2} \sqrt{1 - k^2 a^2}} \times _2F_1 \left( \frac{1}{2}(l + \sqrt{1 - k^2 a^2} + 2), \frac{1}{2}(l + \sqrt{1 - k^2 a^2} + 1), l + \frac{3}{2}; \frac{\rho^2}{a^2} \right) ,
\] (20)

where \( C \) is the normalization constant.

In terms of the variable \( \zeta (\equiv \sin \frac{\tau}{a}) \), the timelike evolution function \( T \) satisfies the equation

\[
(1 - \zeta \zeta) \frac{d^2 T}{d\zeta^2} - 4\zeta \frac{dT}{d\zeta} + \left( a^2 m_0^2 + \frac{a^2 k^2}{1 - \zeta \zeta} \right) T = 0 .
\] (21)

By introducing \( T(\zeta) = (1 - \zeta \zeta)^{-\frac{1}{2}} P(\zeta) \), we transform the timelike evolution equation as the standard associated Legendre equation

\[
(1 - \zeta \zeta) \frac{d^2 P}{d\zeta^2} - 2\zeta \frac{dP}{d\zeta} + \left( a^2 m_0^2 + 2 - \frac{1 - a^2 k^2}{1 - \zeta \zeta} \right) P = 0 .
\] (22)

Therefore, the solutions of the timelike evolution equation can be presented as

\[
T_1(\tau) \propto \frac{1}{\cos \frac{\tau}{a}} P^{\frac{1 - k^2 a^2}{2} + \frac{1}{2} + \frac{1}{2} \sqrt{1 + m_0^2 a^2 + 2}} \left( \sin \frac{\tau}{a} \right) ,
\]
\[
T_2(\tau) \propto \frac{1}{\cos \frac{\tau}{a}} Q^{\frac{1 - k^2 a^2}{2} + \frac{1}{2} + \frac{1}{2} \sqrt{1 + m_0^2 a^2 + 2}} \left( \sin \frac{\tau}{a} \right) ,
\] (23)

where \( P^N_1(\zeta) \) and \( Q^N_1(\zeta) \) are associated Legendre functions. Because \( Q^N_1(\zeta) \) become infinite on the boundary \( |\xi^0| = \frac{1}{\sqrt{-\lambda}} \), we would ignore \( Q^N_1(\zeta) \). The natural boundary condition of \( P^N_1(\zeta) \) on \( |\xi^0| = \frac{1}{\sqrt{-\lambda}} \) requires \( I, N \) to be integers. This gives the discrete mass spectrum of scalar fields in AdS spacetime.
\[a^2 m_0^2 + 2 = I(I + 1) , \]
\[-k^2 a^2 + 1 = N^2, \quad |N| \leq I . \]  

The wave functions of scalar fields in AdS spacetime can be written as following
\[
\Phi_{N Ilm}(\tau; \rho, \theta, \phi) \propto U^N_l(\rho)\left(\cos \frac{\tau}{a}\right)^{-1} P^N_l\left(\sin \frac{\tau}{a}\right) Y_{lm}(\theta, \phi) .
\]  

In order to get a scalar field theory on AdS spacetime, we now construct a Hilbert space by defining the inner product on the whole AdS manifold
\[
(\Phi^*_N I l^\prime m^\prime, \Phi_{N Ilm}) = \int_V \Phi^*_N I l^\prime m^\prime \Phi_{N Ilm} R^3(\tau)(1 + \lambda \rho^2)^{-2} \rho^2 \sin \theta d\tau d\rho d\theta d\phi .
\]

Because of the orthogonality of the associated Legendre functions \(P^N_l(\zeta)\) and the spherical harmonic functions \(Y_{lm}(\theta, \phi)\), these \(\Phi_{N Ilm}(\tau; \rho, \theta, \phi)\) form a complete orthonormal basis of the Hilbert space
\[
(\Phi^*_N I l^\prime m^\prime, \Phi_{N Ilm}) = \delta_{NN^\prime} \delta_{II^\prime} \delta_{ll^\prime} \delta_{mm^\prime} .
\]

**IV. COSMOLOGICAL CONSTANT**

During the last three decades, several approaches of quantizing fields in AdS had been acquired. Since AdS is a homogeneous space of the conformal group \(SO(3, 2)\), Fronsdal adopted a group-theoretic approach to get canonical quantization of fields \([24–27]\). The covariant self-consistent quantization scheme was devised by considering the information variance through AdS timelike spatial infinity \([22]\). For our aim in this paper, we just treat the AdS timelike spatial infinity as a natural boundary constraint to obtain the discrete Hamiltonian eigenvalues. In this case, a Hermitian free scalar field operator is defined by
\[
\Phi(x) = \sum_{N, I, l, m} \left(\Phi_{N Ilm}(x)\hat{a}_{N Ilm} + \Phi^*_N I l^\prime m^\prime(x)\hat{a}^+_N I l^\prime m^\prime\right) ,
\]
where the sign \(x\) denotes the Beltrami coordinates \((x^0, x^1, x^2, x^3)\), and the operators \(\hat{a}_{N Ilm}, \hat{a}^+_N I l^\prime m^\prime\) satisfy
\[
[\hat{a}_{N Ilm}, \hat{a}^*_N I l^\prime m^\prime] = [\hat{a}^+_N I l^\prime m^\prime, \hat{a}^+_N I l^\prime m^\prime] = 0 ,
\]
\[
[\hat{a}_{N Ilm}, \hat{a}^+_N I l^\prime m^\prime] = \delta_{NN^\prime} \delta_{II^\prime} \delta_{ll^\prime} \delta_{mm^\prime} ,
\]
\[
\hat{a}_{N Ilm} |0\rangle = 0 , \quad |N, I, l, m\rangle = \hat{a}^+_N I l^\prime m^\prime |0\rangle ,
\]
here \(|0\rangle\) denotes the vacuum state. Thus, the \(\hat{a}_{N Ilm}\) and \(\hat{a}^+_N I l^\prime m^\prime\) are the annihilation and creation operators on the Fock space, which is constructed as an infinite tensor product of the simple-harmonic-oscillator Hilbert spaces. In quantum field theory, the vacuum is used to mean the ground state of quantum fields. We can reasonably require that the ground states are spherically symmetric, \(i.e., l = m = 0\) \([21]\). Hence, the vacuum energy is simply the sum of ground energy of the states \(|N100\rangle\). The parameter \(k\) in the Eq.(21) is a wave vector because the timelike evolution equation should reduce to the evolution equation in
Minkowskian spacetime when $|\lambda| \rightarrow 0$. Now we have an upper limitation on the quantum number $N$ ($N^2 = 0, 1$). According to (8), the ground state energy of a simple harmonic oscillator is

$$E^2 = m_0^2 + k^2 + \lambda(l + 1) = m_0^2 + k^2 .$$  

(30)

In the case of continuous wave vector spectrum in Minkowskian spacetime, summing the ground state energy $|\frac{1}{2}\hbar \omega\rangle$ up to a wave number cutoff $\Lambda \gg m_0$ yields a vacuum energy density,

$$\langle \rho \rangle = \frac{1}{2} \int_0^\Lambda \frac{4\pi k^2 dk}{(2\pi)^3} \sqrt{m_0^2 + k^2} \simeq \frac{\Lambda^4}{16\pi^2} .$$  

(31)

If we believe general relativity up to the Planck scale \[1\], then we might take $\Lambda \simeq (8\pi G)^{-\frac{1}{2}}$, which would give

$$\langle \rho \rangle \simeq 2^{-10} \pi^{-4} G^{-2} = 2 \times 10^{71} \text{GeV}^4 .$$  

(32)

It shows that the energy density of vacuum got by the quantum field theory in Minkowskian spacetime is over 120 orders of magnitude in excess of the value of astronomical observations \[1\].

As shown in the previous section, the natural boundary conditions at the points $\xi^0 = \pm \sqrt{-\lambda}$ assure that the wavevector $k$ be discrete. Therefore, the integration in the formula (31) should be alternated by summation in this discrete energy Fock space. In fact, any scalar field of arbitrary mass $m_0$ gives contributions to the energy density of vacuum. Because the mass spectrum in the quantum field theory on Minkowskian spacetime is continuous, it is difficult to get a sum over different mass modes. Now a discrete mass spectrum has been obtained for scalar fields in AdS spacetime, thus we can sum the contributions of all scalar fields with different mass. We, therefore, get the energy density of AdS vacuum

$$\langle \rho \rangle = \frac{2\pi}{(2\pi)^2} \sum_{m_0} \sum_k \frac{k^2}{(2\pi)^3} \delta k \sqrt{m_0^2 + k^2} ,$$  

(33)

where $\delta k = |k(N = 1) - k(N = 0)| = \frac{1}{a}$ is the wavevector difference of two eigen states. Equation (24) is used to get the energy density of vacuum as following

$$\langle \rho \rangle = \frac{1}{(2\pi)^2} \sum_{I=1}^{I_{\text{max}}} \sum_{N=0}^{1} \frac{1 - N^2}{a^4} \sqrt{I(I + 1) - N^2 - 1} ,$$  

(34)

where $I_{\text{max}}$ is the cutoff of mass spectrum. We would estimate $I_{\text{max}}$ as the Planck scale, $E_{\text{Planck}} \approx 10^{19} \text{GeV}$, which is widely accepted as a point where conventional field theory breaks down due to quantum gravitational effects. The maximal energy $E_{\text{max}}$ will be the Planck energy corresponding to the cutoff $I_{\text{max}}$,

$$E_{\text{max}} = E_{\text{Planck}} = \frac{1}{a} \sqrt{I_{\text{max}}(I_{\text{max}} + 1) - N^2 - 1} , \quad N = 0 .$$  

(35)

In terms of the Planck energy, we obtain a relation of the energy density of vacuum with the radius $a$ of AdS spacetime
The radius of AdS spacetime can be determined by the observed Hubble constant

\[ a = \frac{1}{H_0}. \tag{37} \]

Thus, we get the energy density of vacuum by the Equation (36)

\[ \langle \rho \rangle = \frac{1}{8\pi^2} \frac{(E_{\text{Planck}})^2}{a^2}. \tag{36} \]

which is in good agreement with the present astronomical observational value \[3,5,9\]. Furthermore, we acquire the cosmological constant

\[ \Omega_\Lambda = \frac{8\pi G \langle \rho \rangle}{3H_0^2} \approx \frac{1}{3\pi}. \tag{39} \]

What we obtain here is in agreement with the hints of the redshift observations of supernovae and quasars. References \[1,28,29\] gave an upper limitation of the curvature \( \lambda \) from the estimation of the universe age and the Hubble constant: \( |\lambda| \simeq 10^{-56} \text{ cm}^{-2} \). Our result is also in agreement with this upper limitation for the radius of AdS spacetime got by different ways.

V. CONCLUDING REMARKS

Since de Sitter found the de Sitter solution of Einstein’s equation in 1917, de Sitter and anti-de Sitter spacetime has been studied extensively by physicists and astronomers. Recent developments in AdS physics include the AdS/CFT correspondence and theory of extra dimensions. In this paper, we try to give a new understanding to the long standing cosmological constant problem. We assumed that the topological structure of the whole universe is AdS spacetime. This is consistent with the Randall-Sundrum model, where a slice of five-dimensional AdS was used \[17\]. We got a Robertson-Walker-like metric which keeps the spacelike submanifold of AdS invariant under the spacelike subgroup transformation \( SO(3) \). Equations of motion for massive scalar fields were solved exactly by variables separating method. Solutions indicate that the mass spectrum of scalar fields is discrete and possible normal modes of scalar fields are limited. These facts tell us clearly that we can sum the zero-point energies of all kinds of scalar fields with different mass. At last, an intrinsic relation between the energy density of vacuum and the curvature of AdS spacetime was obtained, and the cosmological constant was calculated by using \( c/H_0 \) for the radius of AdS spacetime, which matches well with the observational cosmological constant. It should be pointed out that our results is not dependent explicitly on dimensions of spacetime \[30\]. Only for definiteness, we presented the formalism for AdS\(_4\).

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