Production of the $\eta_b(nS)$ states

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The rates for magnetic dipole (M1) transitions $\Upsilon(nS) \rightarrow \eta_b(n'S) + \gamma$, $n' \leq n$, are compared. The photon energies for allowed ($n' = n$) M1 transitions are very small, so hindered ($n' < n$) transitions could be more favorable for discovering the $\eta_b(1S, 2S)$. The question then arises whether $\Upsilon(2S)$ or $\Upsilon(3S)$ is a better source of $\eta_b(1S)$. Whereas one nonrelativistic model favors $\eta_b(1S)$ production from $\Upsilon(2S)$, this advantage is lost when relativistic corrections are taken into account, and is not common to all sets of wave functions even in the purely nonrelativistic limit. Thus, the prospects for discovering $\eta_b(1S)$ in $\Upsilon(3S)$ radiative decays could be comparable to those in $\Upsilon(2S)$ decays. We also discuss a suggestion for discovering the $\eta_b$ via $\Upsilon(3S) \rightarrow h_b(1P_1)\pi\pi$, followed by $h_b \rightarrow \eta_b\gamma$.

PACS Categories: 14.40.Gx, 13.20.Gd, 13.40.Hq, 12.39.Ki

The $\Upsilon b\bar{b}$ resonances have a rich spectroscopy [1]. The spin-triplet S-wave levels $\Upsilon(nS)$ with $J^{PC} = 1^{--}$ are produced by virtual photons in hadronic or $e^+e^-$ interactions, and then can undergo electric dipole (E1) transitions to the spin-triplet P-wave levels. However, to reach the spin-singlet S-wave levels $\eta_b(nS)$ from the easily-produced $1^{--}$ states, it is necessary to use either favored magnetic dipole (M1) transitions with very small photon energy, or hindered M1 transitions with change of principal quantum number. No spin-singlet $b\bar{b}$ levels have yet been seen. The mass splitting between the singlet and triplet states is a key test of the applicability of perturbative quantum chromodynamics (PQCD) to the $b\bar{b}$ system [2, 3] and is a useful check of lattice QCD results [4].

In this note we review some predictions for M1 transitions [5, 6, 7, 8, 9, 10, 11, 12] from the $\Upsilon(n^3 S_1)$ levels to the $\eta_b(n' S_0)$ states. The photon energies for allowed ($n' = n$) transitions are very small, so the hindered ($n' < n$) transitions could offer better prospects for discovering the spin-singlet states. The question then arises whether $\Upsilon(2S)$ or $\Upsilon(3S)$ is a better source of $\eta_b(1S)$. This question has taken on

1Enrico Fermi Institute preprint EFI 01-10, hep-ph/0104253. To be published in Physical Review D.

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renewed interest because of plans of the CLEO Collaboration at the Cornell Electron Storage Ring (CESR) to increase their sample of data at the Υ(3S) and possibly the Υ(2S) resonance.

The answer is very dependent on good knowledge of relativistic corrections [8, 9, 10, 12, 14, 17]. In the absence of such corrections the only source of a non-zero overlap between $nS$ and $1S$ wave functions in the hindered transitions is the variation with $r$ of $j_0(kr/2)$, where $k$ is the photon energy in the rest frame of the decaying particle and $j_0(x) \equiv (\sin x)/x$. In some (e.g., [3]) but not other (e.g, [10]) treatments, the matrix element of $j_0(kr/2)$ is much larger between $2S$ and $1S$ than between $3S$ and $1S$ wave functions. This hierarchy is largely obliterated when relativistic corrections are taken into account. The most important of these appears to be the difference between $1S_0$ and $3S_1$ wave functions due to the hyperfine interaction. The attractive spin-spin interaction in the $1S_0$ states causes the wave function to be drawn closer to the origin, leading to much less difference between the branching ratios for $\Upsilon(2S) \to \gamma \eta_b$ and $\Upsilon(3S) \to \gamma \eta_b$. More recently one group has pointed out a crucial role for exchange currents [14], which leads to very different conclusions from those obtained previously.

The rates for magnetic dipole transitions in quarkonium ($Q\bar{Q}$) bound states are given in the nonrelativistic approximation by [18, 19]

$$\Gamma(3S_1 \to 1S_0 + \gamma) = \frac{4}{3} \alpha \frac{e_Q^2}{m_Q^2} I^2 k^3,$$

(1)

where $\alpha = 1/137.036$ is the fine-structure constant, $e_Q$ is the quark charge in units of $|e|$ ($-1/3$ for $Q = b$), and $m_Q$ is the quark mass (which we shall take equal to 4.8 GeV/$c^2$). In all our discussions we shall assume a normal magnetic moment of the $b$ quark. The overlap integral $I$ is defined by

$$I = \langle f|j_0(kr/2)|i\rangle.$$

(2)

We summarize in Table I some predictions for mass splittings between $n^3S_1$ and $n^1S_0 \eta_b \bar{b}$ levels and the corresponding branching ratios for $\Upsilon(nS) \to \eta_b(nS)+\gamma$ entailed by Eq. (1) assuming unit overlap integral, $I = 1$. We take the total widths of the $\Upsilon(nS)$ levels to be [20] $\Gamma_{\text{tot}}[\Upsilon(1S,2S,3S)] = (52.5, 44, 26.3)$ keV. For the low-energy favored M1 transitions, the photon energies are nearly the same as the mass splittings. The wide variation in predicted hyperfine splittings leads to considerable uncertainty in predicted rates for these transitions.

For the higher-energy hindered M1 transitions, the expected photon energies $k = (M_i^2 - M_f^2)/(2M_i)$ are not so sensitive to hyperfine splittings. On the basis of present experimental values for the $\Upsilon(1S,2S,3S)$ masses [20] of (9460,10023,10355) MeV/$c^2$ and the hyperfine splittings predicted in Ref. [3], the masses for the $\eta_b$ levels from $M(1S,2S,3S) = (9403, 9997, 10336)$ MeV/$c^2$, a representative set which we shall take in further calculations. We then compare two predictions for overlap integrals and branching ratios in Table II, taking into account only the expectation value of the spherical Bessel function $j_0(kr/2)$ between initial and final
Table I: Predictions for hyperfine splittings between $n^3S_1$ and $n^1S_0$ $b\bar{b}$ levels, and corresponding predicted branching ratios $B$ for favored M1 transitions. Overlap integrals have been set equal to unity except in the second-to-last row.

| Reference  | $n = 1$ | $n = 2$ | $n = 3$ |
|------------|---------|---------|---------|
|            | $\Delta M$ (MeV) $B$ ($10^{-4}$) | $\Delta M$ (MeV) $B$ ($10^{-4}$) | $\Delta M$ (MeV) $B$ ($10^{-4}$) |
| MR83 \[5, 6\] | 57 | 1.7 | 26 | 0.19 | 19 | 0.12 |
| MB83 \[7\] | 100 | 8.9 | 40 | 0.68 | 31 | 0.53 |
| GOS84 \[8\] (a) | 67 | 2.7 | 31 | 0.32 | −3 | 0 |
| GOS84 \[8\] (b) | 78 | 4.2 | 37 | 0.54 | 27 | 0.35 |
| GI85 \[10\] (c) | 63 | 2.2 | 27 | 0.21 | 18 | 0.10 |
| PTN86 \[11\] | 35 | 0.38 | 19 | 0.07 | 15 | 0.06 |
| FY99 \[3\] | 53 | 1.3 | (d) | (d) | (d) | (d) |
| ZSG91 \[12\] | 48 | 0.99 | 28 | 0.23 | (d) | (d) |
| EQ94 \[13\] | 87 | 5.9 | 44 | 0.91 | 41 | 1.2 |
| LNR99 \[14\] (b) | 79 | 4.4 | 44 | 0.91 | 35 | 0.76 |
| LNR99 \[14\] (e) | 79 | 3.6 | 44 | 0.63 | 35 | 0.50 |
| UKQCD00 \[15\] | 42 | 0.66 | (d) | (d) | (d) | (d) |
| BSV01 \[16\] | 36–55 | 0.4–1.5 | (d) | (d) | (d) | (d) |

$^a$ Scalar confining potential (favored by fit to P-waves).

$^b$ Vector confining potential.

$^c$ The splittings are based on masses rounded to 1 MeV, not the results rounded to 10 MeV as given in Ref. \[10\].

$^d$ Not quoted. $^e$ Results for fully relativistic calculation.

Table II: Predictions for overlap integrals and branching ratios in hindered M1 transitions between $n^3S_1$ and $n^1S_0$ $b\bar{b}$ levels, neglecting relativistic corrections.

| Reference  | $|I|$ | $B$ ($10^{-4}$) | $|I|$ | $B$ ($10^{-4}$) | $|I|$ | $B$ ($10^{-4}$) |
|------------|------|----------------|------|----------------|------|----------------|
| JR83 \[5\] | 0.02 | 0.92 | < 0.002 | < 0.06 | ~ 0.005 | 0.01–0.04 |
| GI85 \[22\] | 0.017 | 0.67 | 0.0070 | 0.65 | 0.018 | 0.25 |
| ZSG91 \[12\] (a) | 0.069 | 11.0 | (b) | (b) | (b) | (b) |
| ZSG91 \[12\] (c) | 0.022 | 1.11 | (b) | (b) | (b) | (b) |
| LNR99 \[14\] (b) | 0.78 | (b) | 1.1 | (b) | 0.54 |

$^a$ Scalar-vector exchange potential of Ref. \[21\]. $^b$ Not quoted.

$^c$ Scalar exchange potential of Ref. \[21\].
Table III: Predictions for overlap integrals and branching ratios in hindered M1 transitions between $n^3S_1$ and $n'^1S_0 \bar{b}b$ levels, taking into account relativistic corrections.

| Reference  | $|I|$ ($10^{-4}$) | $B$ ($10^{-4}$) | $|I|$ ($10^{-4}$) | $B$ ($10^{-4}$) | $|I|$ ($10^{-4}$) | $B$ ($10^{-4}$) |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| ZB83 [8]   | 0.080           | 15              | 0.041           | 22              | 0.095           | 7.0             |
| GOS84 [9]  | (a) 7.9         | (b)             | (b)             | (b)             | (b)             | (b)             |
| GOS84 [9]  | (c) 5.4         | (b)             | (b)             | (b)             | (b)             | (b)             |
| GI85 [10]  | (d) 0.057       | 7.4             | 0.029           | 11              | 0.054           | 2.2             |
| GI85 [22]  | (e) 0.081       | 13              | 0.043           | 25              | 0.078           | 4.7             |
| ZSG91 [12] | (f) 0.025       | 1.4             | (b)             | (b)             | (b)             | (b)             |
| ZSG91 [12] | (g) 0.001       | $\sim 0$       | (b)             | (b)             | (b)             | (b)             |
| LNR99 [14] | (h) 0.46        | (b)             | 1.4             | (b)             | 0.13            |
| LNR99 [14] | (i) 0.05        | (b)             | 0.05            | (b)             | 0.40            |

a Scalar confining potential.  b Not quoted.

b Vector confining potential.

c Based on quoted transition moments.

d Based on matrix elements between $^3S_1$ and $^1S_0$ wave functions.

e Based on quoted transition moments.

f Scalar-vector confining potential of Ref. [21].

g Scalar confining potential of Ref. [21].

h Without exchange current.  i With exchange current.

While the overlaps for the $n = 2, n' = 1$ transition are similar, the other overlaps differ significantly, suggesting that they may be sensitive to small details of the potential (as in the case of two different potentials [21] utilized by Ref. [12]).

When relativistic corrections are taken into account, the results are as shown in Table III. Several calculations [8, 10, 22] predict large branching ratios for all sets of hindered transitions. There appears to be no particular advantage in searching for the $\eta_b(1S)$ state at the $\Upsilon(2S)$; the branching ratio from the $\Upsilon(3S)$ is predicted to be slightly larger, partly compensating for the lower production cross section of the $\Upsilon(3S)$. (The cross sections for $e^+e^- \rightarrow \Upsilon(2S)$ and $e^+e^- \rightarrow \Upsilon(3S)$ are measured to be about 6.6 and 4 nb, respectively [23, 24, 25], for final states other than lepton pairs.)

The calculation of Ref. [22] is based entirely on the distortion of the spin-singlet wave functions by the strong hyperfine attraction. The spin-independent potential consists of a short distance Coulomb-type Lorentz vector and a long distance linear Lorentz scalar interaction. The hyperfine interaction is included directly in the Hamiltonian by smearing the relative coordinate over distances of the order of the inverse quark masses which has the consequence of taming the singularities present in the Breit-Fermi interaction. The resulting wave functions are compared with the corresponding spin-triplet wave functions in Fig. 1. The stronger peaking near the origin of the singlet wave functions is clearly visible.
The best of present data [23, 24, 25] may not be adequate to confront such predictions, and no published limits are reported at present. For example, at the $\Upsilon(2S)$, the transition to the $\chi_{b0}(1P)$ and a 162 MeV photon corresponds to a signal of $8637 \pm 1274$ events in CLEO data [23], for a branching ratio of $(3.4 \pm 0.5 \pm 0.6)\%$. Although a considerable extrapolation is needed to anticipate the signal of a 600 MeV photon (since the spectrum for that energy is not published), it is likely that such a photon emitted with a branching ratio of $10^{-3}$ would be lost in the combinatorial background associated with neutral pions. Similarly, in cascade decays from the $\Upsilon(3S)$ to the $\chi_{bJ}^\prime(2P)$ states followed by $\chi_{bJ}^\prime(2P) \rightarrow \Upsilon(1S)\gamma$, the 770 MeV photon corresponds to a signal of $1994 \pm 150$ events in CUSB data [25], for a branching ratio of $(2.0 \pm 0.2 \pm 0.2)\%$. This is to be compared with the sought-for signal of a $\sim 900$ MeV photon emitted with a branching ratio of about $2 \times 10^{-3}$. The corresponding spectrum for the CLEO $\Upsilon(3S)$ data, based on a smaller sample, shows similar features [26].

We conclude with some remarks on the production of the $\eta_b(1S)$ level through the decay $\Upsilon(3S) \rightarrow h_b(1^1P_1)\pi\pi$, which is predicted by Kuang and Yan [27] to have a branching ratio of about 0.1–1\%. [Voloshin [28] finds a much smaller value for
this quantity, less than $10^{-4}$, and suggests observing instead the isospin-violating transition $\Upsilon(3S) \to h_b(1^3P_J)\pi^0$, for which he predicts a branching ratio of $10^{-3}$.

The subsequent electric dipole decay $h_b \to \eta_b(1S) + \gamma$ is predicted [27] to have a branching ratio approaching 50%. The mass of $h_b$ is expected to be not far from the center-of-gravity of the $1^3P_J$ $\chi_b$ levels, or about 9.90 GeV/$c^2$, so the photon should have an energy of about 485 MeV in the $h_b$ rest frame. The CUSB Collaboration has searched for the $h_b(1^3P_1)\pi\pi$ signature and is able to place an upper limit at 90% c.l. [25] of $< 0.45\%$ on the combined branching ratio for an $\Upsilon(1S) - \eta_b(1S)$ splitting ranging between 50 and 110 MeV. The CLEO Collaboration [29] places an inclusive upper limit of $\mathcal{B}[\Upsilon(3S) \to \pi^+\pi^-h_b] < 0.18\%$ at 90% c.l. for $M_{h_b} = 9.900$ GeV/$c^2$ and 90% c.l. upper limits in the range of 0.1% for the cascade $\mathcal{B}[\Upsilon(3S) \to \pi^+\pi^-h_b \to \pi^+\pi^-\eta_b\gamma]$, with $M_{h_b} = 9.900 \pm 0.003$ GeV/$c^2$ and a photon energy between 434 and 466 MeV.

It appears that the richness of transitions to $\eta_b(nS)$ levels available in decays of the $\Upsilon(3S)$ make it a promising initial candidate for enhanced searches for the elusive spin-singlet levels.

[Note added: We thank R. Faustov and V. Galkin for calling attention to their work on the relativistic quark model, e.g., Refs. [30, 31]. They predict $\eta_b$ and $\eta'_b$ to lie 60 and 30 MeV below their respective $3^3S_1$ partners, and $\mathcal{B}(\Upsilon(1S) \to \eta_b\gamma) = 0.88 \times 10^{-4}$, $\mathcal{B}(\Upsilon(2S) \to \eta_b\gamma) = 1.6 \times 10^{-4}$ (private communication).]

We thank R. S. Galik for asking the question which led to this investigation, and for extensive discussions. We also thank T. Skwarnicki for helpful advice, and S. F. Tuan for reminding us of the distinction between Refs. [27] and [28]. This work was supported in part by the United States Department of Energy through Grant No. DE FG02 90ER40560 and the Natural Sciences and Engineering Research Council of Canada.

References

[1] For reviews see W. Kwong, J. L. Rosner, and C. Quigg, Ann. Rev. Nucl. Part. Sci. 37, 325 (1987); W. Buchmüller and S. Cooper, in High Energy Electron–Positron Physics, edited by A. Ali and P. Söding, World Scientific, Singapore, 1988, p. 412; D. Besson and T. Skwarnicki, Ann. Rev. Nucl. Part. Sci. 43, 333 (1993); E. Eichten and C. Quigg, Phys. Rev. D 49, 5845 (1994).

[2] Y. J. Ng, J. Pantaleone, and S.-H. H. Tye, Phys. Rev. Lett. 55, 916 (1985); J. Pantaleone, Y. J. Ng, and S.-H. H. Tye, Phys. Rev. D 33, 777 (1986); J. Pantaleone and S.-H. H. Tye, Phys. Rev. D 37, 3337 (1988).

[3] F. J. Ynduráin, Lectures at the XVII International School of Physics “QCD: Perturbative or Nonperturbative”, Lisbon, 1999, preprint FTUAM 99-32, hep-ph/9910399.

[4] Some recent lattice QCD results for hyperfine splittings in the upsilon system are given by G.S. Bali, K. Schilling, and A. Wachter, Phys. Rev. D 56, 2566 (1997); T. Manke et al., Phys. Rev. D 62, 114508 (2000).
[5] J. L. Rosner, in *Experimental Meson Spectroscopy – 1983*, AIP Conference Proceedings No. 113, edited by S. J. Lindenbaum (AIP, New York, 1984), p. 46.

[6] P. Moxhay and J. L. Rosner, Phys. Rev. D **28**, 1132 (1983).

[7] R. McClary and N. Byers, Phys. Rev. D **28**, 1692 (1983).

[8] V. Zambetakis and N. Byers, Phys. Rev. D **28**, 2908 (1983).

[9] H. Grotch, D. A. Owen, and K. J. Sebastian, Phys. Rev. D **30**, 1924 (1984).

[10] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[11] Pantaleone, Tye, and Ng [2].

[12] X. Zhang, K. J. Sebastian, and H. Grotch, Phys. Rev. D **44**, 1606 (1991).

[13] Eichten and Quigg [1].

[14] T. A. Lähde, C. J. Nyfält, and D. O. Riska, Nucl. Phys. A**645**, 587 (1999).

[15] UKQCD Collaboration, L. Marcantonio *et al.*, [hep-lat/0011053](http://arxiv.org/abs/hep-lat/0011053) (unpublished).

[16] N. Brambilla, Y. Sumino, and A. Vairo, Tohoku University preprint TU-607, [hep-ph/0101303](http://arxiv.org/abs/hep-ph/0101303) (unpublished). We thank N. Brambilla for communicating predicted values of the $\Upsilon(1S)–\eta_b(1S)$ splitting.

[17] G. Feinberg and J. Sucher, Phys. Rev. Lett. **25**, 1740 (1975); J. Sucher, Rep. Prog. Phys. **41**, 1781 (1978);

[18] V. A. Novikov *et al.*, Phys. Rep. **41C**, 1 (1978).

[19] J. D. Jackson, “Lecture on the New Particles,” in *Proceedings of the Summer Institute on Particle Physics, August 2–13, 1976*, edited by M. C. Zipf, Stanford Linear Accelerator Center Report SLAC-198, November 1977, p. 147.

[20] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[21] S. N. Gupta, W. W. Repko, and C. J. Suchyta III, Phys. Rev. D **39**, 974 (1989).

[22] Wave functions based on calculations in Ref. [11].

[23] CLEO Collaboration, K. W. Edwards *et al.*, Phys. Rev. D **59**, 032003 (1999): (488 ± 18) K produced $\Upsilon(2S)$ based on 73.6 pb$^{-1}$. Earlier smaller samples have been obtained by: CUSB Collaboration, C. Klopfenstein *et al.*, Phys. Rev. Lett. **51**, 160 (1983) (153 K); CLEO Collaboration, P. Haas *et al.*, Phys. Rev. Lett. **52**, 799 (1984) (125 K); Crystal Ball Collaboration, R. Nernst *et al.*, Phys. Rev. Lett. **54**, 2195 (1985) (193 K); ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **160**, 331 (1985).

[24] CLEO Collaboration, R. Morrison *et al.*, Phys. Rev. Lett. **67**, 1696 (1991): 410 K produced $\Upsilon(3S)$ based on 116 pb$^{-1}$. 

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[25] CUSB Collaboration, M. Narain et al., Phys. Rev. Lett. 66, 3113 (1991): 990 K produced Υ(3S) based on 217.3 pb⁻¹.

[26] CLEO Collaboration, presented by T. Skwarnicki, Proceedings of the 25th International Conference on High Energy Physics, Singapore, Aug. 2–8, 1990, edited by K. K. Phua and Y. Yamaguchi (Southeast Asia Physics Association, 1991), p. 550, based on 112 ± 11 pb⁻¹.

[27] Y.-P. Kuang and T.-M. Yan, Phys. Rev. D 24, 2874 (1981).

[28] M. B. Voloshin, Yad. Fiz. 43, 1571 (1986) [Sov. J. Nucl. Phys. 43, 1011 (1986)].

[29] CLEO Collaboration, F. Butler et al., Phys. Rev. D 49, 40 (1994).

[30] V. O. Galkin and R. N. Faustov, Yad. Fiz. 44, 1575 (1986) [Sov. J. Nucl. Phys. 44, 1023 (1986)]; in Proceedings of the International Seminar Quarks ’88 (Tbilisi, USSR, May 1988) (World Scientific, Singapore, 1989), p. 624.

[31] D. Ebert, R. Faustov, and V. Galkin, Phys. Rev. D 62, 034014 (2000); hep-ph/0110190 (unpublished).