Veneziano-Yankielowicz Superpotential Terms in $\mathcal{N} = 1$ SUSY Gauge Theories

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ABSTRACT: The Veneziano-Yankielowicz glueball superpotential for an arbitrary $\mathcal{N} = 1$ SUSY pure gauge theory with classical gauge group is derived using an approach following recent work of Dijkgraaf, Vafa and others. These non-perturbative terms, which had hitherto been included by hand, are thus seen to arise naturally, and the approach is rendered self-contained. By minimising the glueball superpotential for theories with fundamental matter added, the expected vacuum structure with gaugino condensation and chiral symmetry breaking is obtained. Various possible extensions are also discussed.
1. Introduction

Recent work following a conjecture by Dijkgraaf and Vafa [1] has shown that non-perturbative information about the vacuum structure of $\mathcal{N} = 1$ SUSY gauge theories with arbitrary matter can be obtained via perturbative planar diagram computations in a matrix model.

The gauge theory/matrix model correspondence, originally established via a chain of dualities in string theory (in which the gauge theory is embedded) [1–5] is ultimately purely field-theoretic. A diagrammatic proof has been supplied in [6]; a proof based on demonstrating the equivalence between Ward identities following from a generalized Konishi [7,8] anomaly of the gauge theory (on the one side) and the loop equations of the matrix model (on the other) was given in [9].

To be precise, only a part of the contributions to the glueball superpotential have been calculated (thus far) using the matrix model correspondence; there is an additional non-perturbative contribution coming from Veneziano-Yankielowicz terms [10,11]. Previously, these have been included ‘by hand’, though in the original matrix model approach, it was noted that these terms can come from the matrix model measure [1,12]; in the Konishi anomaly approach, they correspond to an undetermined constant of integration [9].

It is shown here that the Veneziano-Yankielowicz terms, which are non-perturbative contributions coming from the gauge fields, can in fact be derived in the Dijkgraaf-Vafa context (thus rendering the approach self-contained) by the following argument. One considers the case with a classical gauge group and flavours of matter in the fundamental representation. The vacuum structure of such theories has been known for some time [13], and indeed it is re-derived below. If the matter fields have non-zero expectation values in the vacuum, then the gauge group is spontaneously broken at low energies via the Higgs mechanism.

The tree-level matter superpotential is such that, classically, there are vacuum branches in which some matter fields have zero vevs, whilst others have non-zero vevs. This allows the gauge symmetry breaking to be engineered. However, as shown in [14,15], the tree-level matter superpotential is sufficiently simple that the effective glueball superpotential can be determined exactly from the standard Konishi anomaly Ward identity (up to some constant of integration independent of the matter couplings in the tree-level superpotential). By considering two different vacua, with two different low energy gauge groups, a difference equation is obtained whose solution yields the Veneziano-Yankielowicz superpotential for the low energy pure gauge theory.

In the next section, this is performed for the gauge group $SU(N)$. Once the low energy effective superpotential for the pure gauge theory (the Veneziano-Yankielowicz part) has been obtained, the full superpotential (with the constant of integration determined) can be obtained [15] by matching it to any one of its low energy pure gauge theory limits (in which all the matter is integrated out). The vacuum structure is then determined by finding the critical points of the glueball superpotential; the expected pattern of gaugino condensation and chiral symmetry breaking is observed. In section 3, the extension to other classical gauge groups is performed. Again, the results are as expected. In section 4, the connection with the matrix model is outlined and in section 5, the results and various
possible extensions are discussed.

2. Special Unitary Groups

Consider the $N = 1$ supersymmetric $SU(N)$ gauge theory in four dimensions with glueball chiral superfield $S = -\frac{1}{32\pi^2}\text{tr}W^\alpha W_\alpha$. With chiral matter superfields $\Phi$ added, the theory has the Konishi anomaly [7–9]. For the chiral change of variables $\delta \Phi = \epsilon \Phi'(\Phi)$, one has

$$
\left\langle \Phi_I \frac{\partial W_{\text{tree}}}{\partial \Phi_I} + \left( \frac{1}{32\pi^2} W^K_{\alpha J} W^K_{\alpha J} \right) \frac{\partial \Phi'_K}{\partial \Phi_I} \right\rangle = 0,
$$

where the indices carry the representation of the gauge group and the tree-level matter superpotential $W_{\text{tree}} = g_k \Phi^k$ is some (gauge- and flavour-invariant) polynomial in the matter superfields. The set of such Ward identities can be solved for the vacuum expectation values of the matter superfields in a background consisting of the light degrees of freedom. The matter is assumed to be massive and so the only light degrees of freedom are the massless gauge superfields. One can then determine the effective superpotential for the massless gauge superfields by solving the partial differential equations

$$
\frac{\partial W_{\text{eff}}}{\partial g_k} = \langle \Phi^k \rangle,
$$

which follow by holomorphy and supersymmetry.

This paper considers the case where the matter sector consists of $F$ ‘quark’ flavours, viz. $F$ chiral superfields $Q^I_i$ in the fundamental representation and $F$ chiral superfields $\tilde{Q}^I_J$ in the anti-fundamental, where $i$ and $j$ are flavour indices and $I$ and $J$ are colour indices. The tree-level matter superpotential (for $F < N$) is written in terms of the $F \times F$ gauge-invariant meson matrix $M^j_i = Q^i_I \tilde{Q}^I_J$ as

$$
W_{\text{tree}} = m \text{tr}M - \lambda \text{tr}M^2.
$$

This superpotential is non-renormalizable, but this is irrelevant. It can be obtained from a renormalizable superpotential with an additional adjoint matter superfield by integrating out the extra matter. The classical equations of motion for the matter fields are

$$
m M^j_i - 2\lambda M^k_i M^j_k = 0.
$$

The meson matrix $M$ can be brought to diagonal form via a global flavour transformation; then the classical vacua have $F_-$ eigenvalues at $M^j_i = 0$ and $F_+ = F - F_-$ eigenvalues at $M^j_i = m/2\lambda$ (no sum on $i$), with the low energy gauge group broken down to $SU(N - F_+)$. The classical dynamics is modified by quantum effects. Consider the transformation $\delta Q^I_i = \epsilon Q^I_i$. From (2.1), this yields the anomalous Ward identity

$$
\left\langle m M^j_i - 2\lambda M^k_i M^j_k + \delta^j_i \frac{1}{32\pi^2} W^K_{\alpha J} W^K_{\alpha J} \right\rangle = 0.
$$

This is to be evaluated in a background consisting of the massless gauge superfields. At the classical level, it was seen that the gauge group is broken via the Higgs mechanism, with
gauge superfields corresponding to broken generators becoming massive. The background should only contain the superfields corresponding to the unbroken generators.

Let the corresponding background glueball superfield be denoted $S'$. The third term in the vacuum expectation value in (2.5) splits into two parts. The part tracing over the unbroken gauge group is by definition $S'$. The other part traces over the broken part of the gauge group. The associated superfields are massive, and their potential is (classically) quadratic and centred at the origin, such that their vevs are zero.\footnote{This is certainly true at the classical level, but it is possible that quantum corrections will modify this. However, the limit will be taken later later on in which the masses of the gauge bosons go to infinity and they decouple. In this limit, their vevs certainly are zero, and so the possibility of quantum corrections will not affect the argument below.} Furthermore, the matter expectation values factorise and so (2.5) becomes

$$m\langle M^j_i \rangle - 2\lambda \langle M^k_i M^j_k \rangle = \delta^j_i S',$$  
which represents quantum corrections to the equation of motion (2.4). Up to a global flavour rotation, (2.6) has the solution

$$\langle M^j_i \rangle = \delta^j_i \frac{m}{4\lambda} \left( 1 \pm \sqrt{1 - \frac{8\lambda S'}{m^2}} \right), \quad (2.7)$$

where the solution with the plus sign corresponds to the Higgsed vacuum in the classical limit and conversely. The eigenvalues of $M$ can be distributed as before, so that

$$\langle \text{tr} M \rangle = F_+ \frac{m}{4\lambda} \left( 1 - \sqrt{1 - \frac{8\lambda S'}{m^2}} \right) + F_- \frac{m}{4\lambda} \left( 1 + \sqrt{1 - \frac{8\lambda S'}{m^2}} \right),$$  
and similarly for $\langle \text{tr} M^2 \rangle$. The partial differential equations for the effective glueball superpotential with respect to the matter sector couplings are

$$\frac{\partial W_{\text{eff}}}{\partial m} = \langle \text{tr} M \rangle,$$
$$\frac{\partial W_{\text{eff}}}{\partial \lambda} = -\langle \text{tr} M^2 \rangle,$$  

or

$$\frac{\partial W_{\text{eff}}}{\partial m} = F_- \frac{m^2}{16\lambda^2} \left( 1 - \sqrt{1 - \frac{8\lambda S'}{m^2}} \right)^2 + F_+ \frac{m^2}{16\lambda^2} \left( 1 + \sqrt{1 - \frac{8\lambda S'}{m^2}} \right)^2,$$
$$\frac{\partial W_{\text{eff}}}{\partial \lambda} = -F_- \frac{m^2}{16\lambda^2} \left( 1 - \sqrt{1 - \frac{8\lambda S'}{m^2}} \right)^2 - F_+ \frac{m^2}{16\lambda^2} \left( 1 + \sqrt{1 - \frac{8\lambda S'}{m^2}} \right)^2,$$  

which have the solution

$$W_{\text{eff}} = \frac{m^2}{8\lambda} + (F_- - F_+) \frac{m^2}{8\lambda} \sqrt{1 - \frac{8\lambda S'}{m^2}} + F S' \log m$$
$$+ \left( \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{8\lambda S'}{m^2}} \right)^{F_-} \left( \frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{8\lambda S'}{m^2}} \right)^{F_+} + c(S').\quad (2.11)$$
Here, $c$ is a constant of integration which must be independent of the matter sector couplings, but may depend on other parameters, such as $S'$. In order to fully specify $W_{\text{eff}}$ it is necessary to determine $c$. In [15], this has been done by mapping on to the known Veneziano-Yankielowicz [10, 11] form of the effective action for the low energy gauge degrees of freedom; thus the Veneziano-Yankielowicz terms are introduced by hand. In what follows, it is shown that the Veneziano-Yankielowicz effective superpotential can in fact be derived from (2.11).

To do this, first take the limit in which both the quark mass $m$ and the gauge boson mass $\sqrt{m/2\lambda}$ become large. The effective potential (2.11) becomes

$$W_{\text{eff}} = F_+ \frac{m^2}{4\lambda} - F_+ S' + F_+ S' \log \frac{S'}{m^2/2\lambda} + F_+ \frac{S'}{2} + F_+ S' \log m + c(S').$$

(2.12)

What is the meaning of this expression? In this limit, the massive degrees of freedom decouple; the effective superpotential should consist of the superpotential for the massless gauge degrees of freedom plus terms representing the contribution of the decoupled matter which has been integrated out. This decoupled matter consists of the quarks and the massive gauge bosons corresponding to the broken generators of $SU(N)$ (and their superpartners). One can calculate the contribution of the quarks to the effective superpotential as follows. The non-renormalization theorem applies to the decoupled matter sector, and the contribution is found by replacing the quark fields in the tree-level superpotential (2.3) by their vacuum expectation values. The contribution to $W_{\text{eff}}$ is thus

$$F_+ \frac{m^2}{4\lambda},$$

(2.13)

reproducing the first term in (2.12). The contribution of the massive gauge superfields to vevs (and therefore to the effective superpotential) was earlier seen to be zero. Discarding the term independent of $S'$ (the contribution of the quark superfields), what is left must represent the contribution of the massless gauge fields alone, that is, a pure gauge theory contribution. This contains the as yet unknown constant $c$, which can be removed by considering the superpotentials for two distinct vacua in which the number of Higgsed quarks, $F_+$, takes the values $F_1$ and $F_2$, but the argument $S'$ takes the same value, $T$ say, in both.\(^2\) If one then subtracts the two effective superpotential functions, the unknown constant $c$ cancels, giving

$$\Delta W_{\text{eff}} = -(F_1 - F_2)T + (F_1 - F_2)T \log \frac{T}{m^2/2\lambda}.$$

(2.14)

This expression still involves the matter sector couplings $m$ and $\lambda$. These account for the required matching of the scales of the low energy $SU(N - F_{1,2})$ gauge theories (with $F_{1,2}$ Higgsed quarks and $F - F_{1,2}$ massive quarks integrated out) to the UV scale of the original

\(^2\)Of course the physical interpretation of $T$ is different in the two vacua. Here however one simply wants to determine the functional form of $W_{\text{eff}}$. $W_{\text{eff}}$ is an unconstrained function of its arguments and so the arguments may be chosen arbitrarily.
SU(N) gauge theory with F flavours. Indeed, one has [16]

\[
\Lambda_{N-F_1,0}^{3(N-F_1)} \left( \frac{m^2}{2\lambda} \right)^{F_1} = \Lambda_{N-F_2,0}^{3(N-F_2)} \left( \frac{m^2}{2\lambda} \right)^{F_2},
\]

(2.15)

where \(\Lambda_{N,F}\) denotes the scale for the gauge group \(SU(N)\) with \(F\) flavours. Using this relation, one can eliminate the matter sector couplings \(m\) and \(\lambda\) altogether from (2.14) to obtain

\[
W_{\text{eff}}(N - F_1, T, \Lambda_{N-F_1,0}) - W_{\text{eff}}(N - F_2, T, \Lambda_{N-F_2,0}) =
\]

\[
(N - F_1) \left( -T \log \frac{T}{\Lambda_{N-F_1,0}^3} + T \right) - (N - F_2) \left( -T \log \frac{T}{\Lambda_{N-F_2,0}^3} + T \right),
\]

(2.16)

where the functional dependence of \(W_{\text{eff}}\) has been indicated explicitly. This difference equation has the solution

\[
W_{\text{eff}}(N, S, \Lambda_{N,0}) = N \left( -S \log \frac{S}{\Lambda_{N,0}^3} + S \right) + f(S),
\]

(2.17)

where the full glueball superfield has been re-instated and \(f(S)\) is an arbitrary function of \(S\) alone: it cannot depend on any of the other parameters present. Furthermore, on dimensional grounds, \(f\) must be proportional to \(S\). Thus

\[
W_{\text{eff}}(N, S, \Lambda_{N,0}) = \left( -S \log \frac{S}{a\Lambda_{N,0}^3} + NS \right),
\]

(2.18)

where \(a\) is a pure number. The arbitrariness observed in \(W_{\text{eff}}\), parameterised by \(a\), corresponds precisely to the renormalisation group scheme dependence, in which one is free to shift \(\Lambda_{N,0}^{3N} \rightarrow a\Lambda_{N,0}^{3N}\). In a scheme in which \(f\) vanishes or \(a = 1\), the glueball superpotential for the pure SU(N) gauge theory is

\[
W_{\text{eff}}(N, S, \Lambda_{N,0}) = N \left( -S \log \frac{S}{\Lambda_{N,0}^3} + S \right),
\]

(2.19)

which has precisely the form suggested by Veneziano and Yankielowicz on the basis of extended \(U(1)_R\) symmetry considerations.

Now that the effective superpotential for the low energy gauge theory has been derived, one can determine \(c\) as in [15] by demanding that \(W_{\text{eff}}\) in (2.11) reproduces the correct limit as \(m^2/\lambda \rightarrow \infty\) for the vacuum with \(F_+\) Higgsed quarks and low energy gauge group \(SU(N - F_+).\)\(^3\) It is not difficult to show that the correct form is

\[
W_{\text{eff}} = S' \left( -\log \frac{S^{N}}{\Lambda_{N,F}^{3N-F} m^2} + S \right) - F' \frac{S'}{2} + F \frac{m^2}{8\lambda} + (F_+ - F_-) \frac{m^2}{8\lambda} \sqrt{1 - \frac{8\lambda S'}{m^2}}
\]

\[
+ S' \log \left[ \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{8\lambda S'}{m^2}} \right] - \frac{F_-}{2} \log \left[ \frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{8\lambda S'}{m^2}} \right].
\]

(2.20)

\(^3\)The arguments above show that if the correct low energy limit is obtained for one value of \(F_+\), then the correct low energy limit will also be obtained for all values of \(F_+\).
Finally, one can show that the expected quantum vacuum structure is reproduced. Minimising $W_{\text{eff}}$ with respect to $S'$, one finds that

$$\log \left[ \frac{\Lambda^{3N-F} m^F}{S'^N} \left( \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{8 \lambda S'}{m^2}} \right)^{F_-} \left( \frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{8 \lambda S'}{m^2}} \right)^{F_+} \right] = 0. \quad (2.21)$$

The solution is in general non-trivial. In the limit in which quark masses and Higgs vevs become large however, it reduces to

$$\log \left[ \frac{\Lambda^{3N-F} m^F}{S'^N} \left( \frac{2 \lambda S'}{m^2} \right)^{F_+} \right] = 0, \quad (2.22)$$

implying

$$S'^{N-F_+} = \Lambda^{3(N-F_+)}_{N-F_+0}. \quad (2.23)$$

There are precisely $N - F_+$ vacua with gluino condensation and chiral symmetry breaking [13].

### 3. Orthogonal and Symplectic Groups

The extension to the other classical Lie groups is straightforward. The only differences are that i. the fundamental representation of $SO(N)(Sp(2N))$ is (pseudo-)real, and ii. the one-loop beta-function coefficients (and thus the scale matching relations (2.15)) are modified.

Because the representations are (pseudo-)real, the meson flavour matrix can be written as $M_{ij} = Q_i Q_j$, where the colour indices are implicitly contracted using the appropriate invariant tensor. The Konishi anomaly equation is thus modified to

$$2m \langle M_{ij} \rangle - 4 \lambda \langle M_{ik} M_{kj} \rangle = \delta_{ij} S', \quad (3.1)$$

and so all equations written in section 2 up to and including eq. 2.14 remain valid upon making the replacement $S' \to S'/2$.

For the $SO(N)$ gauge theory with $F < N - 4$ quarks in the fundamental (vector) representation [17], the one-loop coefficient of the beta-function is $3(N-2) - F$ [16]. The scale matching relation (2.15) is modified to

$$\Lambda^{3(N-F_1-2)}_{N-F_1,0} \left( \frac{m^2}{2 \lambda} \right)^{F_1} = \Lambda^{3(N-2)-F} m^F = \Lambda^{3(N-F_2-2)}_{N-F_2,0} \left( \frac{m^2}{2 \lambda} \right)^{F_2} \quad (3.2)$$

and the low energy glueball superpotential is

$$W_{\text{eff}}(N, S, \Lambda_{N,0}) = \left( \frac{N - 2}{2} \right) \left( -S \log \frac{S}{2 \Lambda^3_{N,0}} + S \right). \quad (3.3)$$

The only difference from the standard form is the factor of two in the logarithm. This is however renormalisation scheme dependent.
For the $Sp(2N)$ gauge theory with $F < N + 1$ flavours ($2F$ quarks) \cite{18}, the one-loop coefficient of the beta-function is $3(2N + 2) - 2F$, whence

$$W_{\text{eff}}(N, S, \Lambda_{N,0}) = (N + 1) \left( -S \log \frac{S}{2\Lambda_{N,0}^3} + S \right). \quad (3.4)$$

Again there is an additional factor of two in the logarithm, which is renormalisation scheme dependent.

4. Connection with the Matrix Model

In this section, the connection is made with the matrix model. The first point to note is that, in the case with $F$ fundamental flavours, the matrices $Q$ and (for $SU(N)$) $\tilde{Q}$ are $F \times \tilde{N}$, where $\tilde{N}$ is taken to be large. Formally, the partition function for the matrix model is

$$Z = \int dQd\tilde{Q} \exp -\frac{1}{g_m}W_{\text{tree}}(Q, \tilde{Q}) \quad (4.1)$$

and the required Ward Identity (2.6) can be obtained directly from

$$\int dQd\tilde{Q} \frac{d}{dQ_i} \left[ Q_j \exp -\frac{1}{g_m}W_{\text{tree}}(Q, \tilde{Q}) \right] = 0, \quad (4.2)$$

upon making the replacement $g_m \to S'$ and noting the factorisation of correlation functions in the large $\tilde{N}$ limit.

5. Discussion

In the above, the Veneziano-Yankielowicz superpotential terms for pure $\mathcal{N} = 1$ gauge theories with classical gauge group have been derived, and all results are in accord with others obtained previously.

One can now look at possible extensions of the work presented here. A first remark is that the pathological cases where the number of flavours is close to the number of colours \cite{13,17–19} were deliberately excluded. This was initially sufficient, since one sought only to derive the superpotentials for the pure gauge theory - the matter sector served only to engineer the symmetry breaking. However, one went on to determine the superpotentials and vacuum structure of the full theory (with matter), and it would be desirable to extend this analysis to the pathological cases. Presumably this can be done, and would necessitate adding baryonic terms to the tree-level matter superpotential and so on. See \cite{20–22} for work already attempted along these lines.

Secondly, it would be desirable to extend the argument to the exceptional Lie groups. At first it would seem that an analogous argument may work: starting with an exceptional gauge group with fundamental matter, one can engineer a situation in which the gauge symmetry is broken to a classical gauge group (or product thereof), for which the Veneziano-Yankielowicz superpotential terms are known. One could then match the full
superpotential onto the known Veneziano-Yankielowicz terms in the appropriate low energy limit. However, there is a problem, in that the pure gauge theory superpotentials derived herein, viz. (2.19,3.3,3.4) do not hold for the lowest-lying classical Lie groups. The reason for this is clear from [23]: the gauge group one considers in the Dijkgraaf-Vafa framework is really the supergroup $\lim_{k \to \infty} SU(N + k|k)$, for which the superpotentials derived in the present work are correct for all $N$. However, instanton effects mean that the Veneziano-Yankielowicz terms for $SU(N)$ and its supergroup extension are different for low-lying $N$ (consider for example $SU(1)$). Thus, in order to derive the Veneziano-Yankielowicz terms for the exceptional groups, one would need to fix the results for the low-lying classical gauge groups by hand first.

Thirdly, and more speculatively, it is noted that the simple case with fundamental matter discussed above may shed some light on the remarkable observation that the Veneziano-Yankielowicz terms can be obtained from the measure in the corresponding matrix model for adjoint matter. At first thought, this seems nonsensical. The Veneziano-Yankielowicz terms pertain to the pure gauge theory, so how can it be that they come from the matter sector?

Take instead the viewpoint that the Veneziano-Yankielowicz terms come from non-perturbative contributions in the pure gauge theory. Now add matter in the adjoint representation. Just as in the case with fundamental matter considered above, it is then possible to break the gauge symmetry to some smaller gauge group at low energy via the Higgs mechanism. Indeed, one can even go so far as to break the non-Abelian part of the gauge symmetry completely using the matter sector. But if one does this, the would-be Veneziano-Yankielowicz terms in the low energy glueball superpotential must somehow be removed. In order to achieve this, the matter sector must contain terms which cancel the Veneziano-Yankielowicz terms coming from the pure gauge sector. If the correspondence between the matter sector of the gauge theory and the matrix model is indeed complete, then these terms ought to come from the non-perturbative part of the matrix model, viz. the measure factor.

All of this is pure conjecture however. In order to show it, one would like to see how the argument presented here generalizes to matter in other representations (in particular the adjoint) and other superpotentials. In these more general cases, one must solve the generalized Konishi anomaly equations in closed form. Moreover, in the adjoint case, the spontaneous symmetry breaking cannot be engineered in the same way: the pattern of symmetry breaking is fixed once the form of the tree-level matter superpotential is chosen.

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