RANS computations for identification of 1-D cavitation model parameters: application to full load cavitation vortex rope

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Abstract. Due to the massive penetration of alternative renewable energies, hydropower is a key energy conversion technology for stabilizing the electrical power network by using hydraulic machines at off design operating conditions. At full load, the axisymmetric cavitation vortex rope developing in Francis turbines acts as an internal source of energy, leading to an instability commonly referred to as self-excited surge. 1-D models are developed to predict this phenomenon and to define the range of safe operating points for a hydropower plant. These models require a calibration of several parameters. The present work aims at identifying these parameters by using CFD results as objective functions for an optimization process. A 2-D Venturi and 3-D Francis turbine are considered.

1. Introduction
The extensive development of new renewable energy sources provokes electrical power flow fluctuations. In order to manage these fluctuations, hydraulic power plants, taming also green energy, are used due to their ability to respond quickly to a variation of the load. However, in order to inject the suitable amount of power in the grid, the hydraulic turbines have to operate far from their best efficient point. In the case of Francis turbines, the turbine off design operating conditions yield to the development of flow instabilities. One of them, that develops at full load, is the cavitation vortex rope leading to self-excited pressure and discharge oscillations [1]. One-dimensional (1-D) unsteady cavitation models are used to investigate such instabilities [2]. However, the 1-D cavitation model requires a calibration of the physical parameters. The calibration can be achieved by experimental measurements [3] or Computational Fluid Dynamics (CFD) simulations ([4],[5]). However, the calibration of the second viscosity by CFD is still challenging [6].
In the present paper, an approach is described to compute the aforementioned parameters by comparing the system dynamics response of both the CFD and 1-D unsteady models. The calibration is achieved by using the CFD results as objective functions for the 1-D model.
2. 1-D cavitation model

The modelling of the draft tube cavitation flow is based on both continuity and momentum equations (see Equations 1 and 2) including the convective terms and the divergent geometry [2]:

\[
\frac{dQ}{dt} = \frac{-dV_c}{dt} \approx C_c \frac{dh}{dt} + \chi \frac{dQ}{dt} = \frac{-dV_c}{dt} \approx C_c \frac{dh}{dt} + \chi \frac{dQ}{dt}
\]  

(1)

\[
\frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{Q}{gA^2} \frac{\partial Q}{\partial x} - \frac{Q^2}{gA^3} K_x + \frac{\partial h}{\partial x} + \tau_0 \pi D \frac{\rho g A}{\rho g A} + \frac{\mu''}{\rho g A} \frac{\partial^2 Q}{\partial x^2} + S_h = 0
\]

(2)

This set of equations involve four cavitation vortex rope parameters to be assessed:

• the local wave speed \( a \) in a control volume of length \( dx \), which yields the value of the cavitation compliance \( C_c = \frac{gA dx}{a^2} = -\frac{\partial V_c}{\partial t} \bigg|_{Q=\text{cste}} \);  

• the mass flow gain factor \( \chi = -\frac{\partial V_c}{\partial t} \bigg|_{h=\text{cste}} \) corresponding to a variation of the cavitation volume as function of the inlet discharge by keeping the pressure constant;  

• the second viscosity \( \mu'' \) introducing dissipation due to the phase change;  

• the momentum excitation source \( S_h \). This parameter is not considered since the phenomenon of interest at full load conditions is of self-excited nature.

3. Methodology

The proposed methodology to find the 1-D cavitation model parameters is presented in Figure 1. Three parameters need to be found: the mass flow gain factor \( \chi \), the wave speed \( a \) and the second viscosity \( \mu'' \). The mass flow gain factor is determined with a quasi-static approach: several CFD simulations are performed by keeping constant the pressure downstream the cavitation volume for different inlet discharges. The variation of the cavitation volume as a function of the inlet discharge yields the mass flow gain factor. For identification of the wave speed and the second viscosity, unsteady simulations are performed with an imposed outlet pressure fluctuation. The resonance frequency and the pressure fluctuations at a given location are then used as the objective functions for the optimization process. The optimization process allows the determination of the set of 1-D cavitation parameters that minimizes the errors for the two objectives between the 1-D and the CFD models.
4. Cases studies
Two test cases are considered: a 2-D Venturi geometry and a simplified 3-D Francis turbine for which only the runner and the axisymmetric simplified draft tube cone are considered, see Figure 2.

5. Numerical set up
5.1. CFD modelling
The CFD simulations are carried out with the Ansys CFX 15.0 software. The flow is modelled using the Unsteady Reynolds-Averaged Navier-Stokes (URANS) equations write for a homogeneous two-phase mixture. The cavitation model is the one proposed by Zwart [8] with the default coefficient values. For the 2-D Venturi, a structured mesh is set up with 15'000 nodes. For the 3D Francis turbine, a structured mesh, with 4 million nodes, is set up in the runner and the cone domains, whereas an unstructured coarse mesh is used for the draft tube extension. This extension avoids that the vortex rope reaches the outlet section. In both cases, the velocity field is imposed at the inlet and a sinusoidal static pressure is specified at the outlet to excite the system.

5.2. Hydro-acoustic modelling
The Venturi and Francis turbine test cases are modelled with the SIMSEN software which includes the cavitation model described previously [2]. The cavitation zone, where cavitation parameters must be identified, is spatially discretized with a lumped model in the case of the Venturi geometry, whereas a distributed model is used for the Francis turbine draft tube. In the cavitation free region, the wave speed is set to 1’000 m/s. Finally, the boundary conditions of the 3-D model are reproduced in the 1-D model: a constant discharge source at the inlet and a sinusoidal static pressure at the outlet.

6. Cavitation model parameters
6.1. Mass flow gain factor
To derive the mass flow gain factor, several 3-D computations are performed by keeping the pressure, as close as possible to the cavitation volume, constant for different inlet discharges. Using a linear regression, the value of the mass flow gain factor $\chi_1$ is determined: $\chi_1 = -0.001$ s for the Venturi test case and $\chi_1 = -0.034$ s for the Francis turbine test case.

6.2. Wave speed and second viscosity
The CFD simulation of the Venturi test case features a resonance frequency value of $f_n = 2H z$ where a maximum of pressure and cavitation volume fluctuations are experienced [6]. Time
Figure 3. Pressure fluctuations in out of resonance conditions $f_s = 6\text{Hz}$ (left) and in resonance conditions $f_s = 2\text{Hz}$ (right). CFD results, Venturi test case.

Table 1. 1-D cavitation model parameters for the Venturi and the Francis turbine test cases.

|                | Venturi                  | Francis Turbine          |
|----------------|--------------------------|--------------------------|
| $\alpha$       | 4.3 $m \cdot s^{-1}$     | 85.8 $m \cdot s^{-1}$    |
| $\mu''$        | 800 $Pa \cdot s$         | 17,723 $Pa \cdot s$      |
| $\chi$         | -0.001 $s$               | -0.034 $s$               |

history of dimensionless pressure fluctuations in Plane2 and in Plane4 are plotted and compared to the excitation outlet pressure set to a frequency value of $f_s = 6\text{Hz}$ and to the resonance frequency $f_s = 2\text{Hz}$ in Figure 3. For out of resonance conditions, the amplitude of pressure fluctuations decreases from the outlet to the cavitation sheet. For resonance conditions the amplitude near the cavitation sheet is higher than the outlet excitation source. Moreover, pressure fluctuations do not feature sinusoidal shape anymore. This different behaviour between resonance and non-resonance conditions are also observed for the Francis turbine test case. In Figure 4, pressure fluctuations at the interface of the draft tube are compared to the excitation outlet pressure set respectively to $f_s = 30\text{Hz}$ and $f_s = 20\text{Hz}$. In resonance conditions, the non-linearity of the system response is observed.

The resonance frequency $f_n$ and the amplitude of pressure fluctuations $h^*$ at the location of interest are set as the objective functions of the global optimization process to derive the wave speed and the second viscosity parameters. The set of 1-D cavitation model parameters that match the eigenfrequency and the amplitude of pressure fluctuations are given in Table 1 for the two test cases. For each test case, two identifications have been performed with and without mass flow gain factor to assess its influence on the wave speed and the second viscosity values. In the case of the Venturi, the low value of the mass flow gain factor barely influences the parameters. In the case of the Francis turbine, the parameters are slightly modified. To compensate the stabilizing effect of the mass flow gain factor and to keep the same eigenfrequency, the wave speed parameter is increased with the second viscosity.

In Figure 5, the forced response of the 1-D model is plotted along the system abscissa. The CFD pressure fluctuation amplitudes at planes Plane2 and Plane4 for the Venturi and at the interface for the Francis turbine are compared for the two investigated frequencies of each case study. At the resonance conditions (surrounded symbol), for which the identification process has been carried out, the 1-D model matches the CFD results. Out of resonance, a difference is observed between the 1-D model and the CFD results. This difference observed for out of resonance conditions could be explained by either frequency dependent cavitation parameters or non-linearities of the system at resonance conditions.
Figure 4. Pressure fluctuations in out of resonance conditions $f_s = 30\, Hz$ (left) and in resonance conditions $f_s = 20\, Hz$ (right). CFD results, Francis turbine test case.

Figure 5. Forced response of the system for different excitation frequency obtained with 1-D model. Venturi test case (left) and Francis turbine test case (right).

7. Conclusion
A calibration process of the parameters involved in a 1-D model designed for stability analysis of power plants has been described. This process is based on an optimization procedure that uses CFD results as objective functions for the 1-D model. Firstly, a 2-D test case has been considered to prove that the second viscosity can be identified from URANS computations. Then, the methodology has been applied to a Francis turbine. The obtained values are close to the mentioned parameters available in the literature. The identified parameters are not intrinsic to the cavitation model dynamics of the RANS model. An investigation of the influence of the grid and the cavitation model would be also valuable in the future.

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