Construction of $p$-adic Hurwitz spaces

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Abstract

Moduli spaces for Galois covers of $p$-adic Mumford curves by Mumford curves are constructed using Herrlich’s Teichmüller spaces, André’s orbifold fundamental groups, and Kato’s graphs of groups encoding ramification data of charts for Mumford orbifolds.

1 Introduction

Hurwitz spaces are moduli spaces for finite branched covers of curves. Subspaces of their $p$-adic versions are moduli spaces for covers of Mumford curves, i.e. Schottky uniformisable curves. In this paper we give a description of special kinds of such spaces parametrising Galois covers of Mumford curves by Mumford curves.

These so-called Mumford-Hurwitz spaces turn out to be a finite disjoint union of equidimensional moduli spaces of $N$-uniformisable Mumford curves, where $N$ runs through some special type of finitely generated groups: Bass-Serre fundamental groups of graphs of groups. For such moduli spaces, there exists a theory of Teichmüller spaces [Her85]. This allows to “bundle together” those moduli spaces involved by examining which quotients of Y. André’s orbifold fundamental group are Bass-Serre fundamental groups for charts of a given reference orbifold. The finiteness of the number of components then boils down to giving bounds for the number of vertices of possible graphs.

Equidimensionality of all components is achieved by relating the number of branch points to the dimension of the Teichmüller spaces involved. This $p$-adic counterpart of the well-known complex result does not seem to have been mentioned in the literature, so far.

We adopt Berkovich’s language of $p$-adic strictly analytic spaces [Ber90] and make use of André’s theory of orbifold fundamental groups from [AndIII] which was developed for studying differential equations on one-dimensional orbifolds allowing covers by Mumford curves: we call these, following [Kat01], Mumford orbifolds.

In Section 2, we give a very brief overview of the theory of temperate fundamental groups, $p$-adic orbifolds and their fundamental groups.

Section 3 reviews Teichmüller spaces and moduli spaces of Mumford curves from the point of view of $p$-adic geometry. It reveals that Herrlich’s Teichmüller spaces parametrise uniformisations of Mumford orbifolds.

In Section 4, the actual construction of Mumford-Hurwitz spaces is performed, and the finiteness result is proven: a main ingredient is the quotient pasting of Kato trees, used in [Brad02a] for obtaining formulae for the number of branch points. The section ends with some examples.

The results of this paper can be found in the author’s dissertation [Brad02] of which we recommend sections 2 and 3 as a quick introduction to the theory of $p$-adic analytic spaces and to the temperate and orbifold fundamental groups from [AndIII].
2 Orbifolds and orbifold fundamental groups

2.1 The temperate fundamental group

Following [AndIII], we have

**Definition 2.1.** Let $S$ be a $p$-adic manifold. A geometric point of $S$ is an analytic map $\bar{s}: M(\Omega) \to S$, where $(\Omega, | \cdot |)$ is an algebraically closed, complete extension of $(\mathbb{C}_p, | \cdot |_p)$.

As $\Omega$ is a normed $\mathbb{C}_p$-algebra, $M(\Omega)$ is well-defined. In fact, it is a point, and with $s := \bar{s}(M(\Omega)) \in S$ we shall always mean the image of the geometric point $\bar{s}$.

In $p$-adic geometry, there are not enough topological covers of a given manifold. A remedy for this, is to extend the definition of “cover” [dJ95]: a cover of $S$ is an analytic map $f: S' \to S$, such that there is an open covering $\mathcal{U}$ of $S$ with the property

$$\forall U \in \mathcal{U}: f^{-1}(U) = \bigsqcup_i V_i$$

and all restrictions $f|_{V_i}: V_i \to U$ are finite. We see that in case all $f|_{V_i}$ are isomorphisms, we recover the well-known notion of “topological cover”.

A morphism of covers of $S$ is a commuting triangle

$$\begin{array}{ccc}
S' & \xrightarrow{\text{an}} & S'' \\
\downarrow\text{cov} & & \downarrow\text{cov} \\
S & \downarrow\text{cov} \\
\end{array}$$

The resulting category is called $\text{Cov}_S$.

This is all a bit too general, but de Jong invents “étale” covers as a special case in order to obtain a first sensible fundamental group. This turns out to be too big, so André becomes even more special:

**Definition 2.2.** A cover $g: S' \to S$ is called temperate, if there is a topological cover $T' \to T$ and a commuting diagramme

$$\begin{array}{ccc}
T' & \xrightarrow{\text{top}} & S' \\
\downarrow\text{alg} & & \downarrow\text{alg} \\
T & \downarrow g \\
\end{array}$$

where the upper horizontal arrow is a quotient over $S$, and the lower horizontal arrow is a finite étale or algebraic cover: it is a finite morphism $f$ and the $f|_{V_i}$ as above are étale maps.

The sub-category of $\text{Cov}_S$ of temperate covers is denoted by $\text{Cov}_S^{\text{temp}}$.

As fundamental groups act as permutations of fibres of covers, we define the geometric fibre of a geometric point $\bar{s}: M(\Omega) \to S$ of $S$ for the cover $f: S' \to S$ to be the set of all lifts of $\bar{s}$ to geometric points of $S'$:

$$f^{-1}(\bar{s}) := \left\{ s': M(\Omega) \to S' \mid \begin{array}{c}
\bar{s}' \xrightarrow{\bar{s}} S' \\
M(\Omega) \xrightarrow{s} S \\
\text{commutes}
\end{array} \right\}.$$
The fibre functor is the covariant functor

\[ F_{S,\bar{s}} : \text{Cov}_S \to \text{Sets}, \quad (f : S' \to S) \mapsto f^{-1}(\bar{s}). \]

**Definition 2.3.** The temperate fundamental group of \( S \) with base point \( \bar{s} : M(\Omega) \to S \) is

\[ \pi_1^{\text{temp}}(S, \bar{s}) := \text{Aut} F_{S,\bar{s}}^{\text{temp}}, \]

the automorphism group of the restricted fibre functor \( F_{S,\bar{s}}^{\text{temp}} := F_{S,\bar{s}}|_{\text{Cov}^{\text{temp}}_S} \).

**Remark 2.4.** The temperate fundamental group is a pro-discrete topological group. A basis of open neighbourhoods of \( 1 \) is the system of normal closed subgroups \( H \) such that \( \pi_1^{\text{temp}}(S, \bar{s})/H \) is the Galois group of the universal topological cover of some algebraic Galois cover of \( S \).

Its pro-finite completion is the algebraic fundamental group (in case \( S \) is the analytification of an algebraic variety, the latter group coincides with the usual algebraic fundamental group).

**Proof.** The first part is [AndIII, 2.1.5.]. The second follows from [AndIII, 1.4.8], as the algebraic covers of \( S \) form a full sub-category of \( \text{Cov}^{\text{temp}}_S \).

### 2.2 The orbifold fundamental group

**Definition 2.5.** A \( p \)-adic orbifold \( S = (\bar{S}, (Z_i, e_i)) \) is a \( p \)-adic manifold \( \bar{S} \) together with a locally finite family \( (Z_i) \) of irreducible divisors and natural numbers \( e_i > 0 \), and which can be covered by orbifold charts, i.e. by analytic maps \( \varphi : W \to V \subseteq \bar{S} \) with the properties

- \( W \) is a \( p \)-adic manifold.
- \( \varphi \in \text{Cov}_V \).
- Outside \( Z := \bigcup_i Z_i \cup \bar{S}_{\text{sing}} \), \( \varphi \) is a temperate Galois cover, ramified above \( Z_i \) with index \( e_i \).

A morphism \( f : S' \to S \) of orbifolds is a morphism \( \bar{S}' \to \bar{S} \) such that \( S' \) is covered with orbifold charts \( \varphi' : W' \to V' \subseteq \bar{S}' \) and there exist orbifold charts \( \varphi : W \to V \subseteq \bar{S} \) for which the diagramme

\[
\begin{array}{ccc}
W' & \xrightarrow{\varphi'} & V' \\
\downarrow & & \downarrow \quad f \\
W & \xrightarrow{\varphi} & V \\
\end{array}
\]

commutes.

The category of \( p \)-adic manifolds will be viewed as a full sub-category of the category of \( p \)-adic orbifolds.

**Definition 2.6.** A morphism \( f : S' \to S \) is called an orbifold cover if the underlying analytic map \( f : \bar{S}' \to \bar{S} \) is in \( \text{Cov}_S \), and for every orbifold chart \( \varphi' : W' \to V' \subseteq \bar{S}' \) the composition \( f \circ \varphi : W' \to f(V') \) is an orbifold chart of \( \bar{S} \).

This gives us a category \( \text{Cov}_S \). If both \( S' \) and \( S \) are manifolds, then an orbifold cover is nothing but a temperate cover.

If an orbifold has a global orbifold chart, it will be called uniformisable. Let \( S := \bar{S} \setminus Z \), where \( Z := \bigcup_i Z_i \cup \bar{S}_{\text{sing}} \).
Remark 2.7. Let $S$ be uniformisable and $(f: S' \to S) \in \text{Cov}_S$. Then the restriction $f^{-1}(S) \to S$ is a temperate cover.

Proof. [AndIII, 4.4.5]  

Important is the following notion. Let a discrete group $G$ act properly discontinuously on a connected orbifold $S = (\bar{S}, (Z_i, e_i))$. Then the orbifold quotient is the orbifold

$$S/G := (\bar{S}/G, (D \cdot G, e_D \cdot |G_D|)),$$

where $D$ runs through the $Z_i$ and the prime divisors of $\bar{S}$ with non-trivial stabiliser $G_D$, and $e_D = 1$ for $D \notin \{Z_i\}$.

Let $S$ be a uniformisable connected orbifold. Then Remark 2.7 gives us a restriction functor

$$\text{Res}: \text{Cov}_S \to \text{Cov}_{S}^{\text{temp}},$$

which is faithful [AndIII, 4.5.2].

Definition 2.8. Let $\bar{s}$ be a geometric point of $S = \bar{S} \setminus Z$ for the uniformisable orbifold $S$. Then

$$\pi_1^{\text{orb}}(S, \bar{s}) := \text{Aut}_{F_{\bar{s}}}S,$$

where $F_{\bar{s}} := F_{\bar{s}}^{\text{temp}}|_{\text{Cov}_S}$, is called the orbifold fundamental group of $S$ with base point $\bar{s}$.

The orbifold fundamental group is a pro-discrete topological group, because of [AndIII, 1.4.7], and there exists a fundamental exact sequence for finite Galois orbifold covers:

Lemma 2.9. Let $f: S'/G \to S$ be a finite orbifold quotient cover. Then for every point $\bar{s}'$ of the geometric fibre $f^{-1}(\bar{s})$, the sequence

$$1 \to \pi_1^{\text{orb}}(S', \bar{s}') \to \pi_1^{\text{orb}}(S, \bar{s}) \to G \to 1$$

is exact.

Proof. Composing an orbifold cover of $S'$ with $f$ gives an orbifold cover of $S$, so there is a natural morphism of topological groups

$$\alpha: \pi_1^{\text{orb}}(S', \bar{s}') \to \pi_1^{\text{orb}}(S, \bar{s}).$$

For the injectivity of $\alpha$, the proof of [AndIII, 1.4.12(b)] carries over: the system of all connected Galois orbifold covers factorising through $f$ is cofinal in $\text{Cov}_S$. This implies that there is an open injective morphism from $\pi_1^{\text{orb}}(S', \bar{s}') = \lim_{X' \in \text{Cov}_{S'}} \text{Aut}(X'/S')$ into

$$\lim_{X' \in \text{Cov}_{S'}} \text{Aut}(X'/S) = \lim_{X \in \text{Cov}_S} \text{Aut}(X/S) = \pi_1^{\text{orb}}(S, \bar{s}).$$

The arrow $\beta: \pi_1^{\text{orb}}(S, \bar{s}) \to G$ exists and is surjective, because $G$ is the automorphism group of an orbifold cover of $S$ and as such $G$ is a quotient of $\pi_1^{\text{orb}}(S, \bar{s})$. Here, $\ker \beta$ is the set of all $\gamma \in \pi_1^{\text{orb}}(S, \bar{s})$ fixing $f^{-1}(\bar{s})$ pointwise. But this equals $\text{im} \alpha$, as each $\gamma \in \text{im} \alpha$ lifts to a fibre automorphism of $\bar{s}'$. □
Remark 2.10. In the following, we will use this exact sequence in the special case that \(S = \bar{S}\) is a manifold: then \(\pi_{orb}^1(S, \bar{s}) = \pi_{temp}^1(S, \bar{s})\).

For example, the orbifold \(S = (\mathbb{P}^1, (0, 2), (1, 2), (\infty, 2), (\lambda, 2))\) with \(\lambda \notin \{0, 1, \infty\}\) is uniformisable by an elliptic curve \(E \xrightarrow{C_2} S\) given by the equation \(y^2 = x(x - 1)(x - \lambda)\) (we assume \(p \neq 2\)).

The exact sequence

\[
1 \rightarrow \pi_{temp}^1(E, 1) \pi_{orb}^1(S, \bar{s}) \rightarrow C_2 \rightarrow 1
\]

shows that

\[
\pi_{orb}^1(S, \bar{s}) \cong \begin{cases} \hat{Z}^2 \times C_2, & \text{if } |\lambda| - 1 = 1 \text{ (good reduction)} \\ (\mathbb{Z} \times \hat{Z}) \rtimes C_2, & \text{if } |\lambda| - 1 \neq 1 \text{ (Tate curve)} \end{cases}
\]

The temperate fundamental group of the Tate curve can be calculated from the sequence

\[
1 \rightarrow \pi_{temp}^1(G_m, 1) \rightarrow \pi_{temp}^1(E, 1) \rightarrow \pi_{top}^1(E, 1) \rightarrow 1
\]

coming from the topological universal cover \(G_m \rightarrow E\), which can be seen as a quotient orbifold cover with \(\pi_{top}^1(E, 1)\) acting freely on \(G_m\) (that is why the quotient is also a manifold) [AndIII, 2.3.2]. In Section 4.3.1, we will study the action of \(C_2\) on the temperate fundamental group of an elliptic curve.

Definition 2.11. A Mumford orbifold is a one-dimensional orbifold \(S\) uniformisable by a global Galois orbifold chart \(\bar{S}' \to S\) with a Mumford curve \(\bar{S}'\). If the orbifold \((\mathbb{P}^1, (0, e_0), (1, e_1), (\infty, e_\infty))\) is a Mumford orbifold, it will be called a Mumford-Schwarz orbifold.

In the sequel, we will be dealing mostly with Mumford orbifolds.

3 Teichmüller spaces

3.1 Mumford curves of genus \(g \geq 2\)

The following theorem, of which we will give a short proof from [Brad02, Satz 3.12] using the Berkovich geometry of one-dimensional analytic spaces, is well-known:

**Theorem 1.** If \(S\) is an irreducible non-singular projective algebraic curve defined over a large enough complete non-archimedean field \(K\), then \(\pi^1_{top}(S, \bar{s})\) is a finitely generated free group.

**Proof.** According to [Ber90, 4.3.2], \(S\) is a special quasi-polyhedron, its skeleton \(\Delta(S)\) is a subgraph of the intersection graph \(\Delta^{an}(S)\) of its stable reduction (this is where “large enough” enters), and \(\Delta(S)\) has the same Betti number as \(\Delta^{an}(S)\) (at most \(g\)). Let \(\Omega \rightarrow S\) be the topological universal cover. As \(\pi^1_{top}(S, \bar{s})\) also acts on \(\Delta(S)\) [Ber90, 4.1.8], we have a commuting diagramme

\[
\begin{array}{ccc}
\Delta(\Omega) & \xrightarrow{\cdot} & \Omega \\
/\pi^1_{top}(S, \bar{s}) \downarrow & & \downarrow /\pi^1_{top}(S, \bar{s}) \\
\Delta(S) & \xrightarrow{\cdot} & S
\end{array}
\]

Because of the retraction map \(\Omega \rightarrow \Delta(S)\), this diagramme is Cartesian. Therefore the left arrow going down is universal in the category of graph covers of \(\Delta(S)\). This implies \(\pi^1_{top}(S, \bar{s})\) is the fundamental group \(\pi_1(\Delta(S))\), a free group of rank at most \(g\). \(\square\)
Let $C$ be a Mumford curve of genus $g \geq 2$ defined over $\mathbb{C}_p$. Its stable reduction is a curve defined over $\overline{\mathbb{F}}_p$ and whose intersection graph has genus $g$. Let $\Gamma := \pi_1^{\text{top}}(C, \bar{s})$, according to Theorem [1] a free group of rank $g$, and let $\Omega \xrightarrow{\Gamma} C$ be the topological universal cover. $\Omega$ is an analytic subspace of $\mathbb{P}^1_{\mathbb{C}_p}$, and $\Gamma$ acts discontinuously on it. This gives a faithful representation $F_g \to \text{PGL}_2(\mathbb{C}_p)$ of the fundamental group as a Schottky group [GvPS].

The set $\mathcal{T}_g$ of faithful discontinuous representations $F_g \to \text{PGL}_2(\mathbb{C}_p)$ is known to be an open analytic sub-domain of the affine $\mathbb{C}_p$-variety $\mathcal{S}_g \cong \text{PGL}_2(\mathbb{C}_p)^g$ of all representations of $F_g$ into $\text{PGL}_2(\mathbb{C}_p)$ [Her84, Sect. 1]. $\text{PGL}_2(\mathbb{C}_p)$ acts on $\mathcal{T}_g$ by conjugation, and $\overline{\mathcal{T}}_g := \text{PGL}_2(\mathbb{C}_p)/\mathcal{T}_g$ is the Teichmüller space for $F_g$.

According to [Her84], $\Gamma_g := \text{Out}(F_g)$ acts on the Teichmüller space, and the quotient $\mathcal{M}_g = \overline{\mathcal{T}}_g/\Gamma_g$ is the moduli space of Mumford curves of genus $g$, which can be viewed as an analytic subspace of the moduli space $\mathcal{M}_g$ of all irreducible projective curves of genus $g$ [Lüt83].

In order to prove connectedness and simple connectedness of Teichmüller space, Gerritzen dissects $\overline{\mathcal{T}}_g$ into inadmissible open parts $\mathcal{B}_g(\Gamma)$ depending only on the possible stable reduction graphs of Mumford curves. This gives the connectedness result for $\mathcal{M}_g$ which inadmissibly locally looks like $\mathcal{B}_g(\Gamma)/\text{Aut} \Gamma$ [Ger81].

From the Berkovich geometric viewpoint, the parts $\mathcal{M}_g(\Gamma) := \mathcal{B}_g(\Gamma)/\text{Aut} \Gamma$ are not disjoint—only their sets of classical points are. In fact, intersections $\mathcal{M}_g(\Gamma) \cap \mathcal{M}_g(\Gamma')$ consist of generic points of discs $D$ corresponding to families of Mumford curves parametrised by $D$ whose skeletons are either $\Gamma$ or $\Gamma'$.

### 3.2 Discontinuously uniformisable Mumford curves

Generalising the results of the preceding section, Herrlich constructs Teichmüller spaces for finitely generated groups $N$ [Her87]. In our language we ought to proceed thus: let $\mathcal{F}(N)$ be the functor

$$\text{An}(\mathbb{C}_p) \to \text{Sets}, \ S \mapsto \{\psi : N \to \text{Aut}(\mathbb{P}^1_S) \mid \psi \text{ is discontinuous}\},$$

where a discontinuous representation $\psi$ over $S$ is meant to be injective and for each geometric point $\bar{s} : K \to S$ the induced representation $\psi_{\bar{s}} : N \to \text{PGL}_2(K)$ is discontinuous.

**Proposition 3.1.** The functor $\mathcal{F}(N)$ is representable, i.e. a fine moduli space, and the following quotients

1. $\overline{\mathcal{F}(N)} := \text{PGL}_2 \setminus \mathcal{F}(N),$
2. $\mathcal{M}(N) := \overline{\mathcal{F}(N)}/\text{Out}(N)$ and
3. $\mathcal{M}(N, F) := \overline{\mathcal{F}(N)}/\text{Out}_F(N)$

are well-defined, if $N$ contains a free subgroup $F$ of rank $\geq 2$ and of finite index in $N$. In this case, the first one is representable, and the other two lead to coarse moduli spaces.

Here, $\text{Out}_F(N)$ means $\text{Aut}_F(N)/\text{Inn}(N)$, and $\text{Aut}_F(N) := \{\alpha \in \text{Aut}(N) \mid \alpha(F) = F\}$. 


Proof. The first statement is Folgerung ii) on p. 149 in \[Her87\].
1. is Folgerung on p. 151 in \[Her87\].
2. is \[Her87\], Satz 2.
3. can be proven the same way as 2. with some slight modifications.

Remark 3.2. 1. The coarse moduli space $\mathfrak{M}(N)$ parametrises so-called $N$-uniformisable covers $\varphi: \Omega' \to C$ of Mumford curves of genus $\text{rk} \ N^\text{ab}$ which are temperate outside the branch locus. If $(\zeta_i, e_i)$ are the branch points in $C$ together with the orders of ramification, then $\varphi$ is a global chart of the orbifold $\mathcal{C} = (C, (\zeta_i, e_i))$. So, the moduli space actually parametrises all $N$-uniformisable Mumford orbifolds.
2. $\mathfrak{M}(F, N)$ parametrises commuting diagrammes

\[
\begin{array}{ccc}
\Omega' & \xrightarrow{\varphi} & C' \\
\downarrow{F} & & \downarrow{\text{finite}} \\
\downarrow{\text{finite}} & & \\
\Omega & \xrightarrow{\varphi} & C \\
\end{array}
\]

and is finite over $\mathfrak{M}(N)$.

Proposition 3.3. If $N$ contains a free group of rank $\geq 2$, then $\Sigma(N)$ is a simply connected analytic space. Its finitely many components are (smooth) $p$-adic manifolds of dimension

$$3g - 3 + 2(C - c) + 3(D - d),$$

where $n$ depends only on $N$ and equals the number of branch points of the orbifold covers $\Omega(N) \to \mathcal{C}$ in $\mathfrak{M}(N)$.

Proof. The rigid analytic version of this proposition is essentially Herrlich’s habilitation thesis \[Her85\] or \[Her87\], except for the last statement.

The number $n$ does not depend on the particular graph of groups with fundamental group isomorphic to $N$:

First, $n$ is the number of cusps in the Kato graph of a realisation of $N$ as the Galois group of an orbifold cover $\varphi: \Omega \to \mathcal{C}$ of a Mumford orbifold $\mathcal{C}$ \[Brad02\, Theorem 2\]. These cusps correspond bijectively to the branch points of $\varphi$. As all vertex groups are finite, Khramtsov’s characterisation of finite graphs of groups with isomorphic fundamental groups applies \[Khr91\]: two Kato graphs with the same fundamental group differ by a finite number of admissible edge contractions or “slides” of an edge $e$ along an element $g \in N$ stabilising one of $e$’s extremities $v$. The latter means that the topological graph and all vertex and edge groups are the same, only the embedding $\alpha: N_e \to N_v$ is replaced by $c_g \circ \alpha$, where $c_g$ is conjugation by $g \in N_v$. The formula in \[Brad02\, Theorem 3\] now shows that the number of cusps are the same for all Kato graphs with the same fundamental group $N$.

4 Components of Hurwitz spaces

4.1 $G$-covers

Let $S = (\bar{S}, (\zeta_i, e_i))$ be a Mumford orbifold, and $N$ a virtually free quotient of $\pi^\text{ab}_1(S, \bar{s})$ of rank $\geq 2$ such that there exists a faithful discontinuous representation $\tau: N \to \text{PGL}_2(\mathbb{C}_p)$. Let $\Gamma^*(\tau(N))$ be the Kato graph for the global orbifold chart $\Omega^*(\tau(N)) \to S$ associated to $\tau$. 

7
**Proposition 4.1.** \( \text{Cov}_{\Gamma^*(\tau(N))}^{\text{BS}} \) is a full sub-category of \( \text{Cov}_S \).

**Proof.** To a cover of graphs of groups

\[
\Delta^*(\tau(N)) \longrightarrow (\Gamma, H_\bullet)
\]

\[
\downarrow /N \quad \downarrow \text{cov}
\]

\[
\Gamma^*(\tau(N))
\]

corresponds a unique subgroup \( H \) of \( \tau(N) \) giving rise to a unique orbifold cover

\[
\Omega \longrightarrow S'
\]

\[
\downarrow /\tau(N) \quad \downarrow \text{orb}
\]

\[
S
\]

where \( S' = (\Omega^*/H, (\zeta_{ij}, e_{ij}) \), with \( \zeta_{ij} \) above \( \zeta_i \) and \( e_{ij} \) the ramification index of \( \zeta_{ij} \) \( (i = 0, \ldots, n) \). This gives a functor \( \text{Cov}_{\Gamma^*(\tau(N))}^{\text{BS}} \rightarrow \text{Cov}_S \) embedding the first category into the second as a full sub-category.

\( \square \)

Étale covering theory yields

**Corollary 4.2.** There exists a canonical homomorphism of topological groups \( \psi: \pi_1^{\text{temp}}(\bar{S}', \bar{s}') \rightarrow \pi_1^{\text{BS}}(\Gamma^*(\tau(N))) \) with dense image. As \( \pi_1^{\text{BS}}(\Gamma^*(\tau(N))) \) is discrete, \( \psi \) is surjective.

We observe that all (finite) Galois global orbifold charts \( \varphi: \bar{S}' \rightarrow S \) with a Mumford curve \( S' \) lead to covers of graphs: let \( \Omega' \rightarrow S' \) be the universal topological cover of \( S' \), and let \( N_\varphi := \text{Aut}(\Omega', S) \). Then

\[
\Omega' \longrightarrow S
\]

is a Galois orbifold cover, and the following diagramme commutes with exact rows \([\text{AndIII}, 4.5.8]\):

\[
\begin{array}{ccccccc}
1 & \longrightarrow & \pi_1^{\text{temp}}(S', s') & \longrightarrow & \pi_1^{\text{orb}}(S, \bar{s}) & \longrightarrow & G & \longrightarrow & 1 \\
\downarrow & & \downarrow & & \downarrow & & \downarrow & & 1 \\
1 & \longrightarrow & \pi_1^{\text{top}}(S', s') & \longrightarrow & N_\varphi & \longrightarrow & G & \longrightarrow & 1
\end{array}
\]

The vertical arrows are surjective (left and middle: dense image, target discrete; right: map is the identity).

**Lemma 4.3.** \( N_\varphi \) is a fundamental group of a graph of groups.

**Proof.** As \( F := \pi_1^{\text{top}}(S', s') \) is free and of finite index in \( N_\varphi \), \( \Omega' \subseteq \mathbb{P}^1_{\mathbb{C}_p} \) is the set of ordinary points of \( N_\varphi \). If \( \Delta' := \Delta(\Omega') \) is the skeleton of \( \Omega' \), then there is a retraction map \( \Omega' \rightarrow \Delta' \) \([\text{Ber90}, 4.1.6]\), and \( F \) acts on \( \Delta' \) \([\text{Ber90}, 4.1.8]\). \( N_\varphi \) does the same, and viewing \( \Delta' \) as a graph, this leads to the following commuting diagramme

\[
\begin{array}{ccc}
\Omega' & \longrightarrow & S' \\
\downarrow & & \downarrow \\
\Delta' & \longrightarrow & \bar{S}
\end{array}
\]

\[
\begin{array}{ccc}
\Delta' & \longrightarrow & \bar{S} \\
\downarrow /N_\varphi & & \downarrow \\
\Gamma(N_\varphi) & \longrightarrow & S
\end{array}
\]
The vertical arrows are all retraction maps, \( \Gamma' \) is a graph with trivial vertex and edge stabilisers, and \( \Gamma(N_\varphi) \) is the graph of groups \( (\Delta'/N_\varphi, N_\varphi \ast) \). Because \( \Delta' \) is a tree, \( N_\varphi \cong \pi^{BS}(\Gamma(N_\varphi)) \).

As we can replace \( \Gamma(N_\varphi) \) by the Kato graph \( \Gamma^*(N_\varphi) \) of (a representation of) \( N_\varphi \), we obtain

**Lemma 4.4.** Let \( S \) be a Mumford orbifold and \( \varphi: \bar{S}' \xrightarrow{\mathcal{I}_G} S \) a global chart with a Mumford curve \( \bar{S}' \) together with its universal topological cover \( \psi: \Omega' \to \bar{S}' \). Then the following diagramme commutes and has exact rows and columns:

\[
\begin{array}{c}
1 \\
1 \\
1 \\
1 \\
1
\end{array}
\xrightarrow{\text{\( \pi^\text{temp}_1(\Omega, \bar{\omega}) \)}}
\begin{array}{c}
\pi^\text{temp}_1(\Omega, \bar{\omega}) \\
\pi^\text{temp}_1(\bar{S}', \bar{s}') \\
\pi^\text{top}_1(\bar{S}', \bar{s}') \\
1 \\
1
\end{array}
\xrightarrow{\text{\( \pi^\text{orb}_1(\bar{S}, \bar{s}) \)}}
\begin{array}{c}
\pi^\text{orbi}_1(\bar{S}, \bar{s}) \\
G \\
1 \\
1 \\
1
\end{array}
\xrightarrow{G}
\begin{array}{c}
G \\
1 \\
1 \\
1 \\
1
\end{array}
\]

Here, \( N \) is the deck group of \( \varphi \circ \psi: \Omega \to S \).

**Proof.** The two bottom rows are identical to the diagramme with exact rows above, only \( N_\varphi \) has been replaced by the Bass-Serre fundamental group \( \pi^{BS}_1(\Gamma^*(N_\varphi)) \). The middle and left columns are exact by the fundamental exact sequence in Lemma 2.9. The commutativity of the upper two boxes is clear, as the upper left horizontal arrow is the identity map.

Now fix a chart \( \varphi \) as above, and let

\[
\mathfrak{T}(\varphi) := \begin{cases} 
\pi^\text{orbi}_1(\bar{S}, \bar{s}) & \longrightarrow \text{PGL}_2(\mathbb{C}_p) \\
\downarrow & \downarrow \\
N_\varphi & \text{faithful discont.}
\end{cases}
\]

be the set of all representations of \( \pi^\text{orbi}_1(\bar{S}, \bar{s}) \) factorising over faithfully discontinuous representations of \( N_\varphi \). The isomorphism theorem shows that the induced map

\[
\mathfrak{T}(\varphi) \to \mathfrak{T}(N_\varphi)
\]

is bijective, so \( \mathfrak{T}(\varphi) \) inherits from \( \mathfrak{T}(N_\varphi) \) the structure of a (generally unconnected) manifold. Let us fix the finite Galois group \( G \), and let \( \mathfrak{K}_G(S) \) be the set of all isomorphism classes of global charts \( \varphi: \bar{S}' \xrightarrow{\mathcal{I}_G} S \) with \( \bar{S}' \) a Mumford curve and \( \text{Aut}(\bar{S}', S) \cong G \). According to the Riemann-Hurwitz formula, the genus \( g' \) of \( \bar{S}' \) is the same for all \( \varphi \in \mathfrak{K}_G(S) \). If \( g \) denotes the genus of \( \bar{S} \), we can define:

\[
\mathfrak{H}_G(S) := \bigcup_{\varphi \in \mathfrak{K}_G(S)} \mathcal{M}(N_\varphi, F_{g'g}),
\]
where $F_{g'}$ is the free group in $g'$ generators sitting in the exact sequence

$$1 \to F_{g'} \to N_\varphi \to G \to 1$$

The Hurwitz space for covers of Mumford orbifolds of genus $g$ is obtained in the following way: let

$$\text{MOrb}_g(\mathcal{L})$$

be the sub-category of the category of orbifolds consisting of the one-dimensional Mumford orbifolds of genus $g$ with ramification indexing sequence $\mathcal{L} := (e_1, \ldots, e_n)$.

**Definition 4.5.** The space

$$\mathfrak{H}_g(G; e_1, \ldots, e_n) := \mathfrak{H}_g(G, \mathcal{L}) := \bigcup_{S \in \text{MOrb}_g(\mathcal{L})} \mathfrak{H}_G(S)$$

is called the Mumford-Hurwitz space for $g$-covers with signature $\mathcal{L}$ of Mumford curves of genus $g$.

In the following subsection, we will show that $\mathfrak{H}_G(S)$ is essentially finite, and that this gives only finitely many components for the Mumford-Hurwitz space.

### 4.2 The chart number

Let $\Gamma^* := \Gamma^*(N)$ be a Kato graph. A stable model of $\Gamma^*(N)$ is the graph obtained by stabilising the finite part of the Kato graph, i.e. by contracting edges $e$ in $\Gamma^*_{\text{fin}} := \Gamma^* \setminus \partial \Gamma^*$, whenever $N_e$ is isomorphic to the stabiliser $N_v$ of an extremity $v$ of $e$ and the valency of $v$ in $\Gamma^*_{\text{fin}}$ is less than 3.

We will call, for convenience, the number of stable models for Kato graphs of charts $\varphi \in \mathfrak{H}_G(S)$ the chart number for the Mumford orbifold

$$S = (\bar{S}, (\zeta_1, e_1), \ldots, (\zeta_n, e_n))$$

with Galois group $G$ and $n$ marked points.

Quite obvious, but essential, is

**Lemma 4.6.** If $\varphi : \bar{S}' \xrightarrow{|G|} S$ is a finite global chart for $S$, and if $\bar{S}'$ is a Mumford curve, then all vertex (and edge) stabilisers of the corresponding Kato graph are subgroups of $G$.

**Proof.** With the notations from the previous subsection, the diagramme

$$\begin{align*}
T \xrightarrow{\varphi} \Gamma \\
\downarrow_{\varphi} \quad \quad \quad \quad \quad \quad \downarrow_{|G|} \\
\Gamma^* \xrightarrow{|N_\varphi|} \Gamma^*_{\text{fin}} \(N_\varphi\)
\end{align*}$$

shows that vertex stabilisers occur only in the vertical map, because $F$ is a free group: The maps with source $T$ are both universal covers of the graph without groups $\Gamma$ on the one hand, and of the Kato graph on the other hand. \qed
**Theorem 2.** The total chart number of all $S \in \text{MOrb}_g(e)$ is finite, if stable models of Kato graphs are counted without multiplicities.

**Proof.** We shall show that the number of vertices for graphs of fixed Betti number with $n$ cusps is bounded from above. As the size of vertex stabilisers is also bounded by Lemma 4.6, this proves the theorem.

Let first $S$ be a rational Mumford orbifold, $\varphi : \tilde{S}' \to S$ a chart with deck group $G$, and $\tilde{S}'$ a Mumford curve. Abbreviating $N := N_{\varphi}$, we assume further that the Kato tree $\Gamma^* := \Gamma^*(N)$ is stable and that $n \geq 3$.

In case $\Gamma^*$ is irreducible, i.e. no edge stabiliser is trivial, [Brad02a, Proposition 15] implies that the valency of any vertex in $\Gamma^*$ is at most 3.

Let $v$ be a vertex in $\Gamma^*_\text{fin}$, and $\text{val}(v)$ its valency in that tree. There are the following possibilities:

1. $\text{val}(v) = 1$.
2. $\text{val}(v) = 2$ and the $N_e$ for both edges $e$ emanating from $v$ are cyclic.
3. $\text{val}(v) = 2$ and $N_e$ for an edge $e$ emanating from $v$ is non-cyclic.
4. $\text{val}(v) = 3$.

For proving boundedness in each case, we rely on how $\Gamma^*$ is obtained by glueing Kato trees for finite groups. This is explained in the proof of [Brad02a, Theorem 2]. These trees have two cusps if the group is cyclic, and three cusps otherwise.

1. In this case, $v$ has a cusp emanating from it in $\Gamma^*$.
2. Here, $N_v$ is not cyclic, and $v$ has a cusp in $\Gamma^*$.
3. Let $v = o(e)$, where $e$ is an edge with $N_e$ not cyclic. Then $v(t(e)) = 3$ in $\Gamma^*$, because $N_{t(e)}$ is not cyclic. So, here is a cusp going out of $v$.
4. For each vertex $v$ with $\text{val}(v) = 3$, there is at least one extremal vertex in $\Gamma^*_\text{fin}$. For such vertices, we are in case 1.

From this, we see that the number of vertices in $\Gamma^*$ is bounded.

In case $\Gamma^*$ contains edges with trivial stabiliser, each maximal subtree without trivial edge groups has at least one cusp going out. Such a subtree is called an irreducible component of $\Gamma^*$. The number of irreducible components is therefore bounded.

Let now $S$ be of arbitrary genus. The considerations above for a fundamental domain, viewed as a tree of groups, of $\Gamma^*$ in its universal covering tree prove the theorem in this general case.

**Corollary 4.7.** The Mumford-Hurwitz space $\mathfrak{H}_g(G,e)$ is a coarse moduli space parametrising $G$-covers with signature $e = (e_1, \ldots, e_n)$ of Mumford covers of genus $g$, and has only finitely many components of equal dimension $3g - 3 + n$.

**Proof.** The moduli space property is given by Proposition 3.1. Equidimensionality has been proven in Proposition 3.3. The finiteness property of the Mumford-Hurwitz space follows from the connectedness of each $\mathcal{M}(N_{\varphi})$ (of which $\mathcal{M}(N_{\varphi}, F_{g'})$ is a finite cover) and from the finiteness of the chart number (Theorem 2).
4.3 Examples

4.3.1 Tate orbifolds

Here we study Galois covers of the Tate orbifold $S_\lambda = (\mathbb{P}_1, (0, 2), (1, 2), (\infty, 2), (\lambda, 2))$. This means that $G = C_2$, the cyclic group of order two. Let $p \neq 2$. The rational orbifold $S_\lambda$ can be defined for all $\lambda \neq 0, 1, \infty$, but it is a global orbifold chart of Mumford type if and only if $|\lambda||\lambda - 1| \neq 1$. In fact, the equation

$$y^2 = x(x - 1)(x - \lambda)$$

defines a chart $\varphi: T \to S_\lambda$ with Galois group $C_2$, and $T$ is known to be a Tate elliptic curve if and only if $|\lambda||\lambda| \neq 1$. We have

$$S_0(C_2; 2, 2, 2, 2) = \{\lambda \in \mathbb{C}_p^* \setminus \{1\} \mid |\lambda'||\lambda' - 1| \neq 1\} = \mathcal{M}(C_2 * C_2, \mathbb{Z}).$$

The moduli space $\mathcal{M}(C_2 * C_2, \mathbb{Z})$ is given by the chart

$$\mathbb{C}_p^* / C_2 * C_2 \rightarrow S_\lambda$$

coming from the universal topological cover of the Tate curve; it is a Galois cover whose group $\pi_1^{BS}(\Gamma) \cong C_2 * C_2$ comes from the cover of analytic skeletons viewed as graph with groups

$$\begin{array}{c}
\begin{array}{c}
\mathbb{Z} \\
\downarrow \\
\mathbb{Z}
\end{array}
\end{array}$$

In order to understand the associated surjection $\pi_1^{\text{orb}}(S_\lambda, \bar{s}) \to \pi_1^{\text{BS}}(\Gamma)$, we observe

**Proposition 4.8.** Let $S_\lambda$ be as above. Then:

1. If $S_\lambda$ is a Tate orbifold, then the isomorphisms

$$\pi_1^{\text{orb}}(S_\lambda, \bar{s}) \cong (\mathbb{Z} \times \hat{\mathbb{Z}}) \rtimes C_2 \cong \hat{\mathbb{Z}} \rtimes (C_2 * C_2)$$

hold.

2. If $S_\lambda$ is not a Tate orbifold, then

$$\pi_1^{\text{orb}}(S_\lambda, \bar{s}) \cong \hat{\mathbb{Z}}^2 \rtimes C_2.$$
Proof. The first isomorphy in 1. as well as the isomorphy of 2. is shown in [AndIII, Remarks 4.5.6.(c)], while the second isomorphy in 1. follows from Lemma 4.4: the diagramme is in our case

\[ \begin{array}{ccc}
1 & 1 & 1 \\
\downarrow & \downarrow & \downarrow \\
1 & \mathbb{Z} & 1 \\
\downarrow & \downarrow & \downarrow \\
1 & \mathbb{Z} \times \mathbb{Z} & \pi_1^{\text{orb}}(S_\lambda, \tilde{s}) \\
\downarrow & \downarrow & \downarrow \\
1 & \mathbb{Z} & Z_2 * Z_2 \\
\downarrow & \downarrow & \downarrow \\
1 & 1 & 1
\end{array} \]

For the action of $C_2$, let $\varphi_n: \Omega_n \to T$ be a tempered cover of the elliptic curve $T$. In the case that $T$ has good reduction, $\varphi_n$ is an isogeny. If $T$ is a Tate curve, we may assume that $\varphi_n$ is the composition of the universal cover of a Tate curve with an isogeny. In either case, $\varphi_n$ is a homomorphism of Abelian groups. Let $\sigma_n$ be the involution of $\Omega_n$ lifting the involution $\sigma: t \mapsto t^{-1}$ of $T$ (we shall write $T$ as a multiplicative Abelian group for obvious reasons). Any other lift of $\sigma$ is of the form

$\sigma_n \cdot \varepsilon_n: \Omega_n \to \Omega_n, \omega \mapsto \omega^{-1} \cdot \varepsilon_n(\omega)$

with $\varepsilon_n(\omega) \in \ker \varphi_n$. As $\varepsilon_n$ is continuous and $\ker \varphi_n$ discrete, we find

$\varepsilon_n = \text{const.} = \sigma_n(1)$.

Now, $\sigma := (\sigma_n) \in \pi_1^{\text{orb}}(S_\lambda, \tilde{s}) \setminus \pi_1^{\text{temp}}(T, \bar{1})$, $\varepsilon := (\varepsilon_n) \in \pi_1^{\text{temp}}(T, \bar{1})$, and the involution $\sigma$ acts on the Abelian group $\pi_1^{\text{temp}}(T, \bar{1})$ by conjugation

$\pi_1^{\text{temp}}(T, \bar{1}) \to \pi_1^{\text{temp}}(T, \bar{1}), \gamma \mapsto (\sigma \varepsilon) \cdot \gamma \cdot (\sigma \varepsilon)^{-1} = \sigma \gamma^{-1} = \gamma^{-1}$.

The last equality can be checked for each $\varphi_n$: Here the fibre $\varphi_n^{-1}(1)$ equals the Abelian deck group $G_n$. Then each $\gamma_n$ from $\gamma = (\gamma_n) \in \pi_1^{\text{temp}}(T, \bar{1})$ is the translation $x \mapsto x \cdot \gamma_n$, and for all $x \in G_n$ we have

$\gamma_n^{\sigma_n}(x) = \sigma_n \gamma_n \sigma_n(x) = \sigma_n \gamma_n(x^{-1}) = \sigma_n(x^{-1} \cdot \gamma_n) = \gamma_n^{-1} \cdot x = \gamma_n^{-1}(x)$,

in other words, $\gamma_n^{\sigma_n} = \gamma_n^{-1}$.

As $C_2 * C_2$ is the free product of two copies of $\langle \sigma \rangle$, the exact diagramme from the beginning of the proof gives the last assertion. \[\square\]

Remark 4.9. If $T$ has good reduction, then the temperate fundamental group is the algebraic fundamental group, $\pi_1^{\text{temp}}(T, \bar{1}) \cong \pi_1^{\text{alg}}(T, \bar{1})$, as temperate covers of $T$ are all finite. In this case, it is known that $C_2$ inverts each of the two topological generators of $\mathbb{Z}^2$, for example by using the fact, that this is the case in the complex situation for the (complex) topological fundamental group and that the pro-finite completion of the latter is the algebraic fundamental group. Thus, Proposition 4.8 can be viewed as a generalisation of the good reduction case.
4.3.2 Triangle groups

A special type of rational Mumford orbifolds are *Mumford-Schwarz orbifolds*

\[ S(e_0, e_1, e_\infty) = (\mathbb{P}^1_{\mathbb{C}_p}, (0, e_0), (1, e_1), (\infty, e_\infty)). \]

Such an orbifold \( S(e_0, e_1, e_\infty) \) is a quotient of a Mumford curve \( X \), and if the genus of \( X \) is \( \geq 2 \), it is called *hyperbolic*. The Bass-Serre fundamental group of a corresponding Kato graph is called a *\( p \)-adic triangle group of Mumford type* in the hyperbolic case.

Kato has proven that \( p \)-adic triangle groups of Mumford type exist only for \( p \leq 5 \) \cite{Kat00}, more exactly:

**Theorem 3 (Kato, 1999).** *There are infinitely many hyperbolic Mumford-Schwarz orbifolds for \( p \leq 5 \), and none for \( p > 5 \). For given ramification indices \( e_0, e_1 \) and \( e_\infty \) there are only finitely many \( p \)-adic triangle groups \( \Delta(e_0, e_1, e_\infty) \) of Mumford type.*

This also implies the finiteness of the chart number for Mumford-Schwarz orbifolds, a special case of Theorem 2. In any case, we partially recover from Corollary 4.7:

**Corollary 4.10.** *The Mumford-Hurwitz space \( \mathcal{H}_0(G; e_0, e_1, e_\infty) \) is a finite set.*

1. If \( p > 5 \), then it is non-empty, only if \( G \) equals one of the finite \( \Delta(e_0, e_1, e_\infty) \).

2. It is non-empty for finite quotients of infinitely many infinite triangle groups, if \( p \leq 5 \).

4.3.3 Quadrangle groups

The formula in \cite[Theorem 2]{Brad02a} and its proof imply that the only way to obtain Kato tree with four cusps is by pasting two three-cusped Kato graphs along an elementary Kato graph with two cusps:

\[ \Delta \quad e \quad + \quad e \quad \Delta' \quad \sim \quad \Delta \quad C_e \quad \Delta' \]

By Theorem 3, \( \Delta \) and \( \Delta' \) are both finite, if \( p > 5 \). In other words, The fundamental group is an amalgam of two finite groups. This has also been observed in \cite{vdPV01} by different means. In the case that \( p \leq 5 \), one has only to check in Kato’s list of amalgams \cite[Theorem (2)]{Kat00}, where exactly the cusps emanate and what their decomposition groups are (an easy task). Then “glue together” two cusps with equal stabiliser \( C_e \) (as then one has an allowed segment of Herrlich’s list \cite{Her82}).

4.3.4 Cyclic covers

From Lemma 4.6 we get immediately

**Proposition 4.11.** *Let \( S \) be any Mumford orbifold and \( \varphi \in \mathcal{R}_{C_e}(S) \). Then all vertex groups in the Kato graph for \( \varphi \) are cyclic, and all edge groups are either trivial or equal to the stabilizer of one of its extremities.*
The Kato graph reveals the relative position of the branch points \([\text{Brad02a}, \text{Theorem 4}]\), so we have

**Corollary 4.12.** If a Mumford orbifold \(S = (\bar{S}, (\zeta_1, e_1), \ldots, (\zeta_m, e_m))\) admits a chart \(\varphi \in K_{C_n}(\bar{S})\), then \(m\) is even, and the branch points are separated by a pure affinoid covering of \(S\) into pairs of branch points with equal decomposition group.

The meaning of Proposition \([4.11]\) in case \(S\) is of genus zero is that, if one stabilises the irreducible components of a Kato tree of \(S\), there remains only one vertex, i.e. one has an elementary Kato tree for the corresponding decomposition group.

In order to find the relative distances of the pairs of branch points in \(S\), one has to calculate the lengths of the edges joining the vertices from which cusps emanate. The subtleties lie in the case that \(p\) divides the order of a vertex group. This dealt with in Sections 6.3 and 6.4 of \([\text{Brad02}]\).

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