Staggered-immersion cooling of a quantum gas in optical lattices

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Cooling many-body systems to ultralow temperatures has revolutionized the field of quantum physics. In the nanokelvin regime, strongly correlated quantum gases in optical lattices provide a clean and controllable platform for studying complex many-body problems. However, the central challenge towards revealing exotic phases of matter and creating robust multi-particle entangled states is to further reduce the thermal entropy of such systems. Here we realize efficient cooling of ten thousand ultracold bosons in staggered optical lattices. By immersing Mott-insulator samples into removable superfluid reservoirs, thermal entropy is extracted from the system. Losing only less than half of the atoms, we lower the entropy of a Mott insulator by 65-fold, achieving a record-low entropy per particle of 0.0019 $k_B$ ($k_B$ is the Boltzmann constant). We further engineer the samples to a defect-free array of isolated single atoms and successfully transfer it into a coherent many-body state. The present staggered-immersion cooling opens up a avenue for exploring novel quantum matters and promises practical applications in quantum information science.

Ultracold atoms in optical lattices form ideal simulators for many-body phenomena owing to their excellent coherence and tunable interactions. Unfortunately, the thermal energy inherited from lattice loading of a cold gas is comparable to the elementary interactions and thus degrades the quantum effects. To circumvent the thermal fluctuations, major efforts have been focused on reduction of the entropy arising from the bulk gas or nonadiabatic loading. Nevertheless, the intrinsic heating induced by lattice laser sets a limit on the minimal entropy achieved with the above methods. Therefore, to access the underlying quantum phases, including the d-wave superconducting phase, the development of new cooling techniques for this strongly correlated system is highly demanded.

Although the thermal entropy in lattices can be transferred to other degrees of freedom of the atoms, the inherent entropy limits their further applications and an ultimate solution is to extract the thermal excitations out of the samples. For example, a removable reservoir surrounding the atomic sample is able to absorb the entropy from the system, which has enabled the realization of a Fermi-Hubbard antiferromagnet with entropy per particle $\sim 0.3 k_B$. Yet, it is difficult to achieve even lower entropy or extend to larger scales because the insulating state at the cloud center blocks the entropy transport and thereby hinders the thermalization process. In this context, theoretical schemes suggest to immerse the trapped atoms into superfluid reservoirs which could eventually carry away the thermal entropy.

However, either mixing two distinct quantum phases for efficient thermalization or realizing high-resolution addressability to remove the reservoir, pose outstanding experimental challenges.

To overcome the obstacles, we design a staggered lattice to immerse the samples alternately into superfluid reservoirs for sufficient thermalization. Meanwhile, employing a sub-lattice addressing technique, we remove all the high-entropy reservoirs in parallel and achieve the long-sought low-entropy regime. The cooling concept is sketched in Fig. 1, where the superfluid that holds a remarkably large density of states serves as a reservoir for storing the entropy of the joint system. In our two-dimensional (2D) bosonic system with a bichromatic superlattice along the $x$ direction, the competition between the kinetic energy $J$ and interaction energy $U$ leads to the formation of superfluid and Mott insulator phases. An alternating appearance of the insulator and superfluid is realized by adjusting the local chemical potentials. Based on the local transport dynamics of the atoms, we develop a thermometry method to characterize the cooling performance. The onset of superfluid phase in the reservoirs, which has large thermal conductivity and specific heat, leads to fast thermalization and significant temperature reduction, outperforming the cooling power of the normal fluid. After isolating and removing the particles in the reservoirs, the atoms of the samples (see Fig. 1b) are redistributed within the superlattice to achieve a unity-filling state with over $10^4$ sites, which can be used for the generation of multi-particle quantum entanglement. Taking advantage of the ultralow entropy, an alternative protocol for preparing quantum phases, such as the magnetically ordered states, is via adiabatic transformation. Here we optimize the adiabatic route to transfer the artificial product state into...
FIG. 1: Staggered-immersion cooling. (a) Schematic of the cooling concept. The gapped Mott insulator and gapless superfluid have substantially different density of states. For a low-temperature mixture of these phases, the entropy is mainly stored in the superfluid reservoir. (b) Experimental realization of the cooling method in an optical superlattice, where the Mott insulator samples are immersed in the superfluid reservoirs with a staggered geometry. (c) Exemplary images of the atomic densities averaged over 50 measurements. The initial cloud is created in the short-lattices without cooling. After applying the cooling sequence, we detect the odd and even subsystems separately. (d) The density profiles are azimuthal averages over the corresponding images in (c). The aspect ratio is 0.6 in calculating the samples (red) and the reservoirs (blue). The grey area containing 100 lattice sites is the effective cooling region.

a coherent superfluid phase.

The experiment starts with a $^{87}$Rb Bose-Einstein condensate of $\sim 8.6 \times 10^4$ atoms confined in a single well of a pancake-shaped standing wave [9]. To implement the cooling procedure, we ramp up the lattices to create a phase separation between the subsystems. The 2D cloud is adiabatically loaded into a square lattice in the $x$-$y$ plane with a period of $\lambda_s/2$, here $\lambda_s = 767$ nm denotes the wavelength of the “short” lattices. Another “long” lattice with wavelength $\lambda_l = 2\lambda_s$ is employed to construct the superlattice that separates the quantum gas into odd and even subsystems. Their lattice depths are ramped exponentially in 60 ms with a time constant of 20 ms up to $V_s = 26.1(2)E_r$ and $V_l = 10.0(1)E_r$, where $E_r = h^2/(2m\lambda_s^2)$ is the recoil energy with $h$ the Planck constant and $m$ the atomic mass. Meanwhile, the transverse trap is adjusted to set the atomic densities to $\bar{n} \simeq 1.75$ in the central area. The energy offset $\Delta$ is controlled by shifting the relative phase of the superlattice lasers [9]. As the tunneling strength decreases across the critical value, gapped Mott insulators with $\bar{n} = 2$ start to emerge from the contacting superfluid states with $\bar{n} \simeq 1.5$.

After the atoms enter the deep lattices, coherent tunneling is negligible and defects in Mott insulators are induced only by thermal fluctuations [11]. The on-site number fluctuations is probed via the photoassociation collisions, with which the occupation of a site is reduced to its odd-even parity. The odd and even subsystems are distinguished by introducing a 28 kHz splitting to their resonant frequencies [9]. Then, we perform a Landau-Zener sweep to transfer the atoms of the odd rows from the initial state $|F = 1, m_F = -1\rangle$ to $|F = 2, m_F = -2\rangle$, yielding an efficiency of 99.5(3)% (Methods). The in situ atomic densities $\bar{n}_{\text{det}}$ of the odd and even subsystems are successively recorded by absorption imaging with a high-resolution microscope. Fig. 2a shows the density profiles, where the number fluctuations of the samples decrease dramatically compared to the initial cloud before applying the staggered-immersion cooling. Nevertheless, the fidelity of the parity projection 98.7(3)% leaves a background to the measurements, which constrains the accuracy for determining the ultra-low temperature $T_f$ of the target samples ($\Delta = -U/2$) directly from the remaining densities. To overcome this imperfection, we apply thermometry on the samples via probing the mass transport between the subsystems.

The dynamics of the mass transport and the thermalization process is observed by scanning the offset of the local chemical potential. Fig. 2b shows the densities at half-integers of $\Delta/U$, where the Mott insulators emerge associated with superfluid states in the other subsystem. Changing $\Delta$ represents a small perturbation to the lattice, thus the size of the whole cloud is maintained and...
only local transport is allowed. Eight plateaus emerge around the half-integers of $\Delta/U$ in Fig. 2b, which are analogous to the concentric shell structures of the Mott insulator in a harmonic trap [11]. Intriguingly, the compressible superfluid states exhibit distinctive half-integer plateaus owing to the local particle conservation. In the absence of global mass flow, the entropy is most likely to be conserved locally and meanwhile the nonadiabaticities of the lattice loading are minimized [13]. In addition, from a time-resolved measurement of the mass transport (Methods), we find that the system can thermalize almost up to the end of the lattice loading.

Temperature of the samples is fitted based on particle and entropy conservation at different $\Delta/U$. In the entropy diagram of Fig. 3a, the cooling process is illustrated by the isentropic trajectory which links the initial $\bar{n} = 2$ Mott insulator and the final staggered subsystems. Fig. 3b shows $\bar{n}_{\text{det}}$ of the odd subsystem as a function of $\Delta/U$ at two experimental settings. The phase transition at the edge of the plateaus leads to sudden changes of the occupancies, which allows the thermometer to access ultra-low temperatures. In these samples, the lowest temperature achieved is $k_B T_f = 0.046(10)/U$. Moreover, we apply this thermometer to probe even lower temperatures by further evaporating the 2D gas. In these smaller samples, the temperatures are also determined from the density profiles with the local density approximation [13]. As shown in Fig. 3, these two thermometers are consistent, supporting the local isentropic assumption.

Highly efficient cooling is obtained when the atomic gases in the reservoirs cross a critical point to the superfluid phase. To characterize the cooling performance, we control the initial entropy by changing the adiabaticity or duration of the lattice loading [13, 14]. For each superlattice loading sequence, a Mott insulator with central density $\bar{n} = 2$ is prepared by setting $\Delta=0$ to disable the cooling effect. Thereby the initial entropy is deduced from the parity measurement. As shown in Fig. 3d, only if the initial entropy achieves a critical value $S/N \sim 0.17 k_B$, the cooling power becomes strong and the entropy of the samples reduces significantly. This corresponds to a phase transition of the reservoir from a normal fluid to a superfluid [15], which shifts to a value below $k_B \ln 2/(2\bar{n}) = 0.20 k_B$ as the system equilibrates at non-zero tunneling strength (Methods). The experimental results are in excellent agreement with the quantum Monte-Carlo simulations. Losing less than half of the total atoms in the reservoir ($\sim 1.5/3.5$),
we lower the entropy per particle in the large samples to $S/N = 1.9^{+1.7}_{-0.4} \times 10^{-3} k_B$, corresponding to a 65-fold reduction of the initial entropy. Besides the entropy inherent in the 2D gas, the intrinsic heating induced by the lattice loading is $0.025(2) k_B$ per particle (Methods). The drastic entropy reduction shows that the cooling power has exceeded the heating rate of the optical lattices.

To create and preserve quantum coherence, a common strategy is to encode entanglement into multiple physical qubits that make up a single logical qubit [26]. In this framework, the minimum number of physical qubits, required for demonstrating a quantum computation advantage, is more than 100 [27]. Here, we implement a state engineering over $10^4$ sites to turn the low-entropy samples into a unity-filling state. This is achieved by manipulating the double-wells in parallel after cleaning the particles in the high-entropy reservoirs. Fig. 4b shows the sequences from the number state $|2,0\rangle$ to $|1,1\rangle$ in an isolated double-well unit. To avoid the level crossing to the excited band, we shift the centroid of atom pairs to match the antinodes of the long-lattice before transferring the atoms adiabatically onto its ground band. Next, we ramp up the short-lattice to split the atom pairs into a product state with one atom per site. The atoms favor to occupy different sites in the presence of the interaction blockade $U$. The achieved fidelity of 99.3(1)% is calibrated by applying multiple operations to the samples, i.e. driving the atoms between $|2,0\rangle$ and $|1,1\rangle$ back and forth as in Fig. 4b. Together with the entropy from the samples, the overall fidelity for preparing a unity filling over $10^4$ lattice sites is 99.2(1)%. Based on these quantum registers, multi-particle entanglement can be generated via superexchange interactions [9].

Numerically simulating the adiabatic passage from a product state to a coherent many-body state in a 2D system remains infeasible. Here, we experimentally investigate the adiabatic process by restoring the coherence of a superfluid state. Starting from entropy-engineered insulating states, we build the coherence by linearly ramping back the short-lattice to $5E_r$ in 30 ms, then the atoms are released to undergo a 13-ms time of flight. Without removing the high-entropy reservoirs after the cool-

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**FIG. 3:** Thermometry of the quantum gases. (a) Cooling route in the entropy diagram. The entropy per particle $S/N$ of a 1D system at $J/U = 0.02$ is given by the quantum Monte-Carlo simulations. Starting from the initial Mott insulator with $\bar{n} = 2$, the particles exhibit an adiabatic expansion to the superfluid state $\bar{n} = 1.75$. Then the isentropic system separates into an insulator sample and a superfluid reservoir as $\Delta$ varies from 0 to $U/2$. (b) Measurements of mass transport at 60 ms and 500 ms lattice ramping. The corresponding curves are isentropic fitting results. (c) Temperatures given by two independent thermometers. The temperatures $T$, corresponding to a 65-fold increase, are acquired by fitting the density distributions, whereas the $T_f$ are obtained via probing the mass transport. The dashed line denotes an equivalence of the temperatures. (d) Cooling efficiency and the critical behavior. The red curve is the theoretical prediction based on the Monte-Carlo simulations. We show the data with a logarithmic scale in the attached graph. Here the error bars represent standard deviations.
FIG. 4: Quantum state engineering in superlattice. (a) Arranging atoms to unity filling. We redistribute the atoms into the filling-1 state by controlling the double-well potential. (b) Calibration of the fidelity. Here \( \bar{n}_{\text{det}} \) represents the probability of single occupancy. We repeat the operation from \( |2, 0\rangle \) to \( |1, 1\rangle \) and its reversal in cycles, then count the atoms on \( 10^4 \) lattice sites within the ROI. Each data point is averaged over 20 measurements and the error bars denote the standard deviations. The red and blue curves are power-law fitting results. (c)-(f) Resolving the state coherence via a time of flight. The sketches above the images show the initial insulating states before lowering the lattice potential. They are (c) without removing the reservoir, (d) double occupancy on odd sites, (e) Mott insulator with unity filling and (f) redistribution of the atoms in the reservoir.

In conclusion, we have demonstrated the staggered-immersion cooling method to dramatically reduce the entropy of a many-body strongly correlated system in optical lattices. The achieved entropy is one order of magnitude lower than the limit imposed by the intrinsic heating, opening up a new regime for studying the low-energy physics. This uniform controllable system with over ten thousand addressable qubits can be used for generating large and robust entangled states \([7, 8, 27]\), which are the key resources for quantum computation \([7, 8, 27]\). Our technique is generic and practical for cooling insulating states of other species and systems in other geometries \([23]\). For the gapless fermionic phases, our method could dramatically improve the efficiency of entropy transport \([3, 19]\), giving access to lower-temperature regimes for observing the d-wave superfluid phase. Furthermore, the defect-free platform is suitable for exploring the long-range quantum magnetic order \([24, 25]\) and resonating valence bond states \([30]\) via adiabatic transformations.

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METHODS

A. State preparation

The 2D gas is prepared by adiabatically loading a nearly pure Bose-Einstein condensate on the $5S_{1/2} |F = 1, m_F = -1 \rangle$ hyperfine state into the single layer of a pancake-shaped trap, which is a $3.0 \, \mu m$ period standing-wave generated by interfering two blue-detuned laser beams at wavelength $\lambda_s$. Then we levitate the atoms by ramping a magnetic gradient in $2.1$ seconds up to $30.5$ Gauss/cm to compensate the gravity. The depth of the pancake trap is maintained at $2.3 \, E_r$ during the levitation. The transverse confinement on the $x - y$ plane is provided by a red-detuned $1070 \, nm$ laser with a Gaussian beam waist of $116 \, \mu m$. The trap frequencies of the 2D system are typically $\omega_{x,y,z} = 2\pi \times [7.8(3), 7.8(3), 1508(10)] \, Hz$. To reduce the lattice heating caused by the movement of the center of mass, the dipole oscillation of the ensemble in this 2D hybrid trap is minimized.

Next, we perform the lattice loading by simultaneously ramping up the trap potentials in $60$ ms. The pancake trap is ramped exponentially up to $201(1) \, E_r$ with a time constant of $20$ ms. The blue-detuned lattices have anti-trapping effects, while the red-detuned Gaussian beam provides a dipole trap for adjusting the central density. In Fig.1c, the non-circular shape of the clouds results from these hybrid potentials. The superlattice potential can be expressed as,

$$V(x) = V_r \cos^2(kx) - V_i \cos^2(kx/2 + \varphi).$$

Here $k = 2\pi/\lambda_s$ is the wave number. The relative phase $\varphi$ determines the superlattice structure, and $\varphi = 0$ indicates a balanced condition for the double-well units. We keep the phase $\varphi$ constant during the ramping process, and the energy shift at the end of the dynamics is represented by the offset $\Delta$. After the lattice loading, the density distribution is frozen by increasing the short-lattices to $42 \, E_r$. We detect the \textit{in situ} densities via an absorption imaging with a microscope objective (N.A.=0.48) along the $z$ direction.

B. Manipulation of atoms in the superlattice

The superlattice phase $\varphi$ is controlled by changing the relative frequency of these lasers. The $767 \, nm$ light is provided by a titanium-sapphire laser, whose frequency is stabilized by a reference cavity. We lock the cavity onto the Rb D1 transition line to eliminate the frequency drift. The $1534 \, nm$ light comes from a fiber laser and its frequency can be tuned with a piezo. We up-convert the $1534 \, nm$ laser through a periodically-poled-lithium-niobate crystal and then beat the light with the $767 \, nm$ laser. The beat signal is detected and then converted to a feedback signal to lock the piezo of the $1534 \, nm$ laser. The $\pi$ phase shift of the superlattice is calibrated by observing the atom number of the odd or even subsystems after an adiabatic lattice loading, which gives a relative frequency shift of $532.5 \, MHz$. The beat signal is shown in Fig.5a, where the linewidth at $-3 \, dB$ below maximum is about $120 \, kHz$, corresponding to a phase broadening of $\delta \varphi = 0.7 \, mrad$.

The sub-lattice addressing is implemented by introducing a spin-dependent effect onto the superlattice [31]. The short- and long-lattice are set to $84.0(6)E_r$ and $30.8(2)E_r$, respectively. As the quantization axis is along $x$ and the phase of the electro-optical modulator is tuned to $\pi/3$, we create a $28-kHz$ energy splitting between the hyperfine transition of the odd and even sites. Then a rapid adiabatical passage is employed to transfer the atoms on odd (even) sites into the hyperfine state $5S_{1/2} |F = 2, m_F = -2 \rangle$. The microwave pulse sweeps with a time-dependent Rabi frequency $\Omega_{MW}(t)$ and detuning $\delta_{MW}(t)$,

$$\Omega_{MW}(t) = \Omega_0 \text{sech} \left[ \frac{\beta}{\frac{2t}{T_p}} - 1 \right],$$

$$\delta_{MW}(t) = \frac{\sigma_{MW}}{2} \tanh \left[ \frac{\beta}{\frac{2t}{T_p}} - 1 \right].$$

Here $\Omega_0/(2\pi) = 9.85(2) \, kHz$ is the maximum Rabi frequency, $\sigma_{MW} = 20 \, kHz$ is the width of the microwave sweep. We set the pulse length to $T_p = 1 \, ms$ and the truncation factor to $\beta = 5.3$. We calibrate the fidelity of spin flips by applying a multi-pulse sequence onto the atomic ensemble. From the residual density after 10 pulses, we deduce that the fidelity of flipping the odd sites without affecting even sites is $99.5(3)\%$.

Using the site addressing technique, we calibrate the zero-point $\varphi = 0$ through the atom tunneling in double-well units [31]. We first prepare a Mott state with near unity filling in short lattices. Then the atoms in the odd sites are selectively addressed and removed from the superlattice. We quench the potentials to $16.8(1)E_r$ (short-lattice) and $10.0(1)E_r$ (long-lattice) to allow atom tunneling within double-well units for a half-period (one period is $5.0(1) \, ms$). Afterwards, we stop the tunneling by ramping up the barrier and detect the atom number of the odd sites. The tunnelling process is sensitive to the energy bias of the double-wells.

The local superlattice phases are measured via the atom tunneling. The inhomogeneity of the superlattice phases leads to the irregular stripes at the upper and
FIG. 5: Calibration of the superlattice phase. (a) Beat signal of the superlattice lasers. (b) Spatial inhomogeneity of the superlattice phase. The assembled image contains measurements at 17 different superlattice phases. Each subgraph has a size of 228 × 55 μm² at the atomic plane. The appearance of a bunch of atoms at the upper or lower boundaries is caused by the shallow lattice potential. The high atomic densities around the central area indicates the zero point of the local superlattice phase.

lower edges of the atomic cloud (Fig. 1c). We run the sequence at the superlattice depths of 13.7(1)Er (short-lattice) and 10.0(1)Er (long-lattice). The density distributions are measured after around a half-period of the atom tunneling. The high density regions in the recorded images Fig. 5(b) indicate the local degeneracy of the double-well units. Therefore, we map the superlattice phases into a quadratic-like density pattern, which is smooth in the center and sharp on the edge. In this sense, the harmonic trap is slightly modified since the energy differential is smaller than the slope of the harmonic envelope in the central region. At around 30 μm away from the cloud center, the phase inhomogeneity results in a non-monotonic density distribution. The harmonic approximation is valid when a smaller cloud is prepared to avoid the outer region.

C. Detection methods

The number fluctuations on lattice sites are acquired by applying a photoassociation (PA) laser onto the atomic sample. When the atoms in deep lattices are illuminated by the PA light, two atoms can absorb a photon to form an excited molecular state. Then the molecule quickly decays to free channels by emitting another photon [32]. The atoms escape from the lattice after this dissociation process since it gives sufficiently high kinetic energy to the particles. Therefore, only the lattice sites with odd occupations contribute one-atom signal to the final measurements, indicating a parity detection. Here, we select the PA transition to the ν=17 vibrational state in the 0₁ channel, which has a relatively large line strength to drive the intercombination transition 5S₁/₂ → 5P₃/₂. The frequency is 13.6 cm⁻¹ red-detuned to the D2 line of ⁸⁷Rb atom.

A home-built diode laser generates the PA light. This laser is locked onto a Fabry-Pérot cavity, which is stabilized by the ⁸⁷Rb repumper laser. The PA laser beam has a Gaussian beam waist of 82 μm at the atomic plane. We filter the laser with a hot vapor cell of Rb to elimi-
nate the influence of the sidebands at the resonant frequencies, achieving a 30 dB-attenuation. After atoms are pinned in lattices, we shine the PA light onto the cloud with an intensity of 0.67 W/cm$^2$ and then monitor the remaining atoms. Figure 4 shows the decay of a Mott state with initial double filling, giving a decay rate of 5.6(2) kHz. The one-body scattering rate of the light is 47 Hz. We can estimate the remaining atoms by taking account of the competition between one- and two-body losses. The measurements indicate a fidelity of 98.7(3)% for this parity detection, which agrees well with the theoretical prediction. Apart from the finite efficiency of the PA collision, the detection also suffers from the unavoidable shot noise of the absorption imaging. The duration of the imaging pulse is 10 μs, giving rise to a standard deviation of 0.6 on the counting number per site. We count the region of interest that contains ~1700 pixels to suppress the density error to 0.015.

The entropy of Mott insulator states can be deduced from the parity measurements. The von Neumann entropy of a quantum system with the density matrix $\rho$ is $S \equiv -\text{Tr}\{\rho \log \rho\}$. In a Mott insulator with $\bar{n} = 2$, the lowest-lying excitations are states with filling $n = 3$ (particle) and $n = 1$ (hole). We can estimate its entropy using the “particle-hole approximation” by neglecting the higher-energy excitations [34, 35]. In the sample without applying the cooling sequence, the temperature of the cloud is mainly controlled by extending the superlattice ramping time. The entropy rises as the duration gets longer (Fig. 7). This is mainly due to the intrinsic heating of the lattice laser arising from the incoherent light scattering [14]. We fit the data with a linear curve and find the heating rate to be 0.42(3) $k_B/s$ per particle.

D. Quantum Monte Carlo simulations

The ultracold Bose gas in optical lattices is well described by the Bose-Hubbard model [36]. Here we use a “worm” quantum Monte-Carlo (QMC) algorithm to simulate the equilibrium system [37, 38]. The simulations are performed on a homogenous 2D square lattices with a size of $L \times L$, or a 1D chain with a length of $L$. The system is considered as a grand canonical ensemble. After thermalization, we continue to perform the QMC sweeps for at least 10$^7$ times to get the results under each setting. When the temperature $T$ and chemical potential $\mu$ are fixed, the atom number fluctuates in the outcome. For a fixed particle number $\bar{n}$, the chemical potential $\mu$ serves as a variable.

The thermodynamical quantities can be fully determined in this simulation. From the specific heat $C$, we deduce the entropy density $S(T)$ as following,

$$S(T) = S(T \mu) - \int_T^{T_H} \frac{C}{T} dT' .$$  \hspace{1cm} (3)

To avoid the calculation difficulty at ultra-low temperatures, the integration is carried out from a high temperature $T_H = U$ instead of zero temperature. The entropy $S(T \mu = U)$ is calculated based on the grand canonical distribution by neglecting the hopping term. Generally, the simulations are performed in a 2D parameter plane. We choose the chemical potential $\mu/U$ from 0.8 to 1.6 with a 0.01 interval, and the temperature $k_B T/U$ from 0.01 to 0.4 with 40 steps. The entropy per particle at other values on the plane are deduced by performing a 2D interpolation. The lower boundary of the entropy $S/N = 1.9^{+0.4}_{-0.7} \times 10^{-3}$ is limited by the accuracy of the simulation at ultra-low temperatures.

The mean density and number statistics are also obtained. Comparing to the experimental results at different $\Delta/\mu$, we fit the measurements based on the particle and entropy conservations. To acquire the simulation results of Fig. 2, we perform the calculation of entropy, mean densities and probabilities of odd occupancies in a larger parameter plane. The chemical potential $\mu/U$ is from -1 to 3.56. The temperature $k_B T/U$ starts at 0.01 and end up with 0.3. At temperature $k_B T_f/U = 0.05$, we find the isentropic cooling trajectory on the entropy and density diagrams. Vice versa, we can fit the temperature of a cold sample when the mean density and probabilities of the site occupancies are known.

E. Dynamics during the cooling process

In the cooling samples, we perform a time-resolved measurement on the number statistics across the superfluid-Mott insulator phase transition. For the averaged filling $\bar{n} = 2$, the critical point of the phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading. Fig. 8(a) shows the parity measurements as the $J_t/U$ and entropy $S$ measured on the number statistics across the superfluid-Mott insulator phase transition in a 2D square system is $(J/U)_c = 0.035$ [39, 40]. The anisotropic tunneling of this staggered system allows us to define a total tunneling strength by neglecting the inter-well coupling $J_t = J_x + 2J_y$. The strength is $J_t/U = 0.013$ at the end of the lattice loading.
FIG. 8: Phase transitions and critical values. (a) The red points are measurements at different lattice depth during the superfluid to Mott insulator transition. The solid curves are QMC simulations at four temperatures. (b) The final atomic density is monitored after a sudden change of the energy offset during the entropy transport process. The red and blue data correspond to the parity projection of the odd and even rows, respectively. The measurements suggest that the dynamics of mass transport occurs until $J_x/U \sim 0.02$. (c) Cooling power at different thermalization conditions. The solid lines are QMC simulations. The cooling is more efficient when the system can thermalize up to small tunneling strength.

FIG. 9: Thermometry with two methods. (a)-(d) show the cooling samples and thermometers based on two distinct methods. We apply the averaging along $x$ direction to get the density profiles among a region of interest, which are marked by green dashed rectangles. The systems have not achieved a global thermal equilibrium, especially for the low-temperature samples. From the density profiles, we obtain the temperatures of (a)-(d) are $k_B T_f/U = 0.037(4), 0.045(3), 0.060(3)$ and $0.083(8)$, respectively. From the transport behavior, we get the corresponding temperatures as $k_B T_f/U = 0.038^{+0.007}_{-0.007}, 0.044^{+0.011}_{-0.011}, 0.073^{+0.008}_{-0.013}$ and $0.087^{+0.007}_{-0.010}$, respectively. These two thermometers are consistent with each other.

tems. When the change is applied at low lattice depth and the tunneling is larger than $J_x/U = 0.1$, the sys-
tem fully equilibrates and the even rows enter the Mott insulator. The mass transport becomes slower as the phase change is applied at deeper lattice. The local mass varies until the tunneling strength becomes $J_{t}/U = 0.02$, where the lattice ramping just has 3 ms left. In the cooling sequence, we maintain the superlattice phase during the lattice ramping to redistribute the entropy adiabatically. The $\Delta /U$ in Fig. 2 is calculated under the condition of the final lattice depths $V_x = 26.1(2)E_r$, and $V_t = 10.0(1)E_r$. The consistence between the experimental results and theoretical predictions (Fig. 2b) confirm that the system can thermalize up to the ending time of the lattice loading. Here we evaluate the entropy conservation in the staggered system at $J_{r}/U = 0.04$.

Since the cooling power strongly depends on the thermalization rate [41], we present a comparison between the experimental results and theoretical simulations. For the half-integer filling $\bar{n} = 1.5$, the minimum entropy of a classical state is $S_c = k_B \ln 2$. While this is the upper limit for a superfluid phase due to its underlying quantum correlations. The value of entropy per particle $S_c/(2\bar{n})$ defines a critical point for superfluid to normal fluid transition [15]. Only when the initial entropy becomes lower than this critical value, the onset of the superfluid state in the reservoir leads to strong cooling power. If the system can only achieve thermal equilibrium at non-zero tunneling strengths, the critical value of the initial entropy would becomes smaller [23]. Fig. 3(c) shows the cooling effect at different tunneling strengths. The experimental data are deduced from the parity measurements, which is independent on the newly-developed thermometer. Compared to the QMC simulations, the measurements support that our system has a fast thermalization and can be characterized as the 1D system at $J/U = 0.02$.

F. Thermometry

Here we present three methods for measuring the temperature. When the many-body wave functions are projected to number states on deep lattices, we can estimate the temperature from the parity measurements. At high temperatures, such as the initial entropy of Fig. 3, these values are deduced from the probabilities of the odd occupancies. However, limited by the fidelity of the parity detection, we probe the low-temperature system with two other approaches. One is based on the mass transport and the other relies on the shell structure of the Mott insulator state $|\bar{n}\rangle$. Fig. 3 shows the consistency of these two methods, and the experimental details are presented here.

These two thermometries are carried out on a small cloud with $\sim 4 \times 10^4$ atoms. The transverse confinement is provided by a Gaussian potential. With the local density approximation, the chemical potentials of the lattice sites are $\mu(y) = \mu(0) - V(y)$, where $\mu(0)$ is the chemical potential at the trap center. Fig. 3 shows the thermometry of these two methods performed on the cooling samples. The temperature of the system is controlled via the duration and adiabaticity of the lattice ramping. We find that the system has lower temperatures when the lattice loading has smaller modifications on the cloud size. This is because the adiabaticity is improved as the particles undergo less transport [13]. The whole cloud does not reach a global thermal equilibrium. The thermometry based on the transport behavior is robust to the irregular trap potential and can be used to probe systems without a global thermal equilibrium. Furthermore, via counting the number statistics of the Mott insulator state, we can apply this thermometry onto a superfluid phase by coupling it with an insulating state.

G. Quantum state engineering in superlattice

In the insulating region, the defects on lattice sites could be reduced by engineering the Hubbard interactions within few-body systems [13, 37]. Here, we redistribute the atom pairs into single occupancy within double-well units. After the cooling sequence, the particles in the superfluid reservoir are removed by applying site-selective addressing sequences, leaving behind a filling-2 Mott state at odd rows. Then we perform an operation to arrange the $|2,0\rangle$ to $|1,1\rangle$, as shown in Fig. 10. The atoms are first transferred from the odd sites into long-lattice potential, secondly the atoms are split into single occupancy. In the first part, tuning the short-lattice potential to match the wave functions of the long-lattice can avoid the atom excitations to higher bands. In the second part, the inhomogeneity of the superlattice phase leads to a competition between the interaction blockade and the energy bias at the regions away from the cloud center, which explains the appearance of fringes in the attached graphs of Fig. 1(b). In addition, the intrinsic heating of the lattice laser plays a role as the operation time becomes longer. Finally, we optimize the operation by suppressing these excitations.

We calibrate the fidelity of the operation with parity detection method. The parity projection remove the atom pair of $|2,0\rangle$, while it has no impact on the $|1,1\rangle$ state. Since $|2,0\rangle$ has negligible decay during the operations, the error of the operation mainly comes from the crosstalk between different double-well units. This error accumulates and its fidelity $\eta$ can be evaluated from the final states after applying multi-cycles $N_l$. The decay of the state $|1,1\rangle$ follows a power law function $\eta^{N_l}$ and the $|2,0\rangle$ changes in the form of $1 - \eta^{N_l}$. We fit the measurements with these functions and obtain a fidelity of $\eta = 99.3(1)\%$. The final fidelity of the unity filling among $10^4$ sites is 99.2% by taking account of the 99.9% occupation of the cooling samples.

To prepare a coherent many-body state from the engineered product state, we decrease the lattice depths in 30 ms to enter the superfluid regime. The harmonic confinement is ramped up simultaneously with the lattices. While the pancake trap decreases to reduce the on-site
FIG. 10: Experimental sequence for atom redistribution in the double-well units. After the superlattice potential are ramped up, we adjust the EOM to $\pi/2$ to shift the atom pairs to the potential minimum of the long-lattice. Then the short-lattice is reduced to map the atom pairs into the ground state of the long-lattice. Afterwards, the quantization axis is changed from $x$ to $z$ direction. Making use of the on-site interaction, we split the atoms into double wells by ramping the lattices in 3 ms.

FIG. 11: Restoring the superfluid from engineered product states. The insulating states are prepared at different energy offsets $\Delta/U$. After entering the superfluid regime by lowering down the lattice depths to $7Er$, the state coherence is revealed by the atomic interference after a time of flight.

interaction. Afterwards, we detect the coherence of the state by observing the interference pattern after a 13 ms time of flight. The coherence of the final state is revealed by the condensate fraction. Fig. 11 shows the restoring patterns after we redistribute the odd rows of the insulating state prepared at different initial energy offsets $\Delta$. The condensate fraction is 0.47 at $\Delta/U = -0.5$ but weaker at the other side $\Delta/U = 0.5$ (condensate fraction is 0.15). At the positive side of $\Delta$, the inherent entropy in the initial product state leads to excitations on the final many-body state. For the negative $\Delta$, the low-density edges of the cloud contribute to high entropy that reduces the condensate fraction. Unlike the staircase-like behavior of the mass transport in Fig. 2b, the condensate fraction has smooth change as we vary the $\Delta$. This indicates that the defects on the initial state not only produce entropy for the final state, but also influence the adiabaticity of the transformation process.

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