Mass-loss induced instabilities in fast rotating stars

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Abstract. To explain the origin of Herbig Ae/Be stars, it has been recently proposed that strong mass-losses trigger rotational instabilities in the envelope of fast rotating stars. The kinetic energy transferred to turbulent motions would then be the energy source of the active phenomena observed in the outer atmosphere of Herbig Ae/Be stars (Vigneron et al. 1990; Lignières et al. 1996).

In this paper, we present a one-dimensional model of angular momentum transport which allows to estimate the degree of differential rotation induced by mass-loss. Gradients of angular velocity are very close to \(-2\Omega/R\) (\(\Omega\) being the surface rotation rate and \(R\) the stellar radius). For strong mass-loss, this process occurs in a short time scale as compared to other processes of angular momentum transport. Application of existing stability criteria indicates that rotational instabilities should develop for fast rotating stars. Thus, in fast rotating stars with strong winds, shear instabilities are expected to develop and to generate subphotospheric turbulent motions. Albeit very simple, this model gives strong support to the assumption made by Vigneron et al. 1990 and Lignières et al. 1996.

Key words: Instabilities – Stars: mass-loss – rotation

1. Introduction

Although far from complete, the overall picture describing the angular momentum evolution of solar type stars is well established. Despite modest mass-loss rates, \(M = 10^{-5} \text{M}_{\odot} \text{yr}^{-1}\) for the sun, magnetised stellar winds strongly brake the rotation of the star. The decrease of the angular velocity then reacts back on the magnetic field by reducing the efficiency of the dynamo process. This picture cannot be applied as such to early-type stars. First of all, the history of stellar rotation at intermediate and high masses is generally still poorly known; besides, the presence of magnetic fields in early-type stars has not been established, except for some categories of chemically peculiar stars, as well as for a couple of particular cases (Donati et al. 1997; Henrichs et al. 2000). On the other hand, we know that early type stars can experience strong angular momentum losses as stellar winds with very large mass loss rate have been observed (\(M \approx 10^{-5} - 10^{-8} \text{M}_{\odot} \text{yr}^{-1}\)). In this paper, we investigate how such angular momentum losses will affect the stellar rotation, assuming magnetic fields are not dynamically relevant.

This question has not yet received much attention although it might have important consequences for stellar structure and evolution as well as for the understanding of early-type stars activity. In the context of stellar evolution, the effect of mass-loss on rotation has to be investigated since the rotation strongly influences the stellar structure. However, current models of stellar evolution with rotation do not take this effect into account (Talon et al. 1997, Denissenkov et al. 1999). As explained below, we have been confronted to the same question while investigating the origin of the very strong activity of Herbig Ae/Be stars, a class of pre-main-sequence stars with masses ranging from 2 to 5\(\text{M}_{\odot}\).

A significant fraction of these objects is known to possess extended chromospheres, winds, and to show high levels of spectral variability. In addition, the presence of magnetic fields, first suggested by the rotational modulation of certain spectral lines (Catala et al. 1986), has been recently supported by a direct detection at the surface of the Herbig Ae star, HD 104237 (Donati et al. 1997). Detailed estimates of the non-radiative heating in the outer atmosphere of Herbig Ae/Be stars (Catala 1989, Bouret et al. 1998) compared to observational constraints on the available energy sources strongly suggest that the rotation of the star is the only energy source capable of powering such activity (see discussion in F.Lignières et al. 1996). This led Vigneron et al. (1990) to propose a scenario whereby the braking torque exerted by the stellar wind forces turbulent motions in a differentially rotating layer below the stellar surface. Then, by invoking an analogy with stellar convection zones, these turbulent motions could generate...
a magnetic field which would transfer and dissipate the turbulent kinetic energy into the outer layers of the star.

Unlike Herbig stars, there is at present no observational evidence of non-radiative energy input in the atmosphere of OB stars. Their radiatively-driven winds do not require any additional acceleration mechanism and an eventual non-radiative heating would be very difficult to detect because most lines are saturated. However, they show various forms of spectral variability. Recent observations seem to indicate that these phenomena are not due to an intrinsic variability of the wind but are instead caused by co-rotating features on the stellar surface (Massa et al. 1995). As proposed in the literature (Howarth et al. 1995; Kaper et al. 1996), these corotating features could be due to a magnetic structuration of the wind. Since OB stars rotate fast and possess strong winds, the Vigneron et al. scenario might also explain these phenomena. The recent direct detection of a magnetic field in an early Be star, $\beta$ Cep, adds some credit to this hypothesis (Henrichs et al. 2000).

In this paper, we shall investigate the starting assumption of the Vigneron et al. scenario namely that the braking torque of an unmagnetised wind generates strong enough velocity shear to trigger an instability in the subphotospheric layers. This is a crucial step in the scenario since the onset of the instability allows to transfer kinetic energy from rotational motions to turbulent motions.

The possible connection between mass-loss and instability had been already suggested by Schatzman (1981). However, the onset of the instability is not really considered in this study since it is assumed from the start that the wind driven angular momentum losses induce a turbulent flux of angular momentum. Here, we propose a simple model of angular momentum transport by a purely radial mass flux in order to estimate the angular velocity gradients induced by mass loss. Then, the stability of these gradients is studied according to existing stability criteria. Note that, as we neglect latitudinal flows, our model is best regarded as an equatorial model.

In the absence of magnetic fields, the braking effect of a stellar wind is simply due to the fact that matter going away from the rotation axis has to slow down to preserve its angular momentum. Thus, for the star’s surface to be significantly braked, fluid parcels coming from deep layers inside the star have to reach the surface. Since radial velocities induced by mass-loss are very small deep inside the star, this is expected to take a relatively long time, not very different from the mass-loss time scale. By contrast, we shall see that the formation of unstable angular velocity gradients near the surface take place in a very short time, much smaller than the braking time scale.

The paper is organised as follows: first, we estimate the time scale characterising the braking of the stellar surface and relate it to the mass loss time scale (Sect. 2). Then, we show that radial outflows in stellar envelopes generate differential rotation and we estimate the time scale of this process (Sect. 3). The stability of these angular velocity gradients is considered (Sect. 4) and the results are summarised and discussed (Sect. 5).

2. Braking of mass-losing stars

Generally speaking, the braking of the star’s surface depends on the mass-loss mechanism and on the efficiency of angular momentum transfer inside the star. In this section, we shall make assumptions regarding these processes in order to estimate the braking time scale. However, before we consider these particular assumptions, it is interesting to show that the simple fact that the star adjusts its hydrostatic structure to its decreasing mass already implies that the star slows down as it loses mass.

Indeed, according to models of pre-main-sequence evolution (Palla & Stahler 1993), a 2$M_\odot$ pre-main-sequence star losing mass at a rate of the order of $\dot{M} = 10^{-8}M_\odot\text{yr}^{-1}$ will have lost about one percent of its mass when it arrives on the main sequence. Because mass is concentrated in the core of stars, one percent of the total mass corresponds to a significant fraction of the envelope. For pre-main-sequence models of 2 to 5$M_\odot$, this means that all the matter initially located above the radius $R_f \approx 0.63R_\odot$ is expelled during the pre-main-sequence evolution. But, during this mass-loss process, the star continuously adjusts its structure to its decreasing mass and, from the point of view of stellar structure, the loss of 1 percent of mass has a negligible effect on the stellar radius. Thus, as represented on Fig.1, the sphere containing 99% of the initial mass must have expanded significantly during its pre-main-sequence evolution. Its moment of inertia has increased and, due to angular momentum conservation, its mean angular velocity has been reduced.

Thus, we conclude that the hydrodynamic adjustment of stars ensures that mass loss is accompanied by a mean braking of the remaining matter. Now, if one wants to estimate the actual braking of the stellar surface, assumptions have to be made on the mass-loss process and on the efficiency of angular momentum transfer inside the star. In order to obtain the order of magnitude of the braking...
time scale, we assume that mass-loss is isotropic and constant in time and we consider two extreme assumptions regarding the efficiency of angular momentum transfer.

First, we assume that the transfer of angular momentum is only due to radial expansion. Then, angular momentum conservation states that the angular velocity \( \Omega_f \) of a fluid parcel located at a radius \( r = R_f \) will have decreased by \( (R_f/R_i)^2 \) when it reaches the stellar surface. This occurs when all the matter above \( r = R_i \) has been expelled so that the decrease of the surface angular velocity can be related to the mass-loss. We used the density structure of a 2\( M_{\odot} \) pre-main-sequence model (Palla & Stahler 1993) for the following calculation.

Second, we assume that an unspecified transfer mechanism enforces solid body rotation throughout the star so that angular momentum losses are distributed over the whole star and the braking of the stellar surface is less effective. In this case, global angular momentum conservation reads

\[
\frac{dJ}{dt} = \frac{2}{3} \frac{M R^2}{I} \frac{dM}{dt},
\]

where \( J = I \Omega \) is the total angular momentum. According to stellar structure models of 2 to 5\( M_{\odot} \) pre-main-sequence stars, the radius of gyration, \( I/MR^2 \) is close to 0.05 so that the angular velocity decrease is approximatively given by

\[
\frac{\Omega_f}{\Omega_i} = \left( \frac{M_f}{M_i} \right)^{40/3}
\]

To simply relate the braking to the mass-loss, we also assumed that the decrease of the moment of inertia is proportional to the mass decrease, or equivalently that mass-loss induces an uniform density decrease throughout the star. Albeit rough, this assumption is not critical for the conclusion drawn in this section.

Starting with an uniformly rotating star, Fig. 2 presents the braking of the stellar surface as a function of the percentage of mass lost assuming solid body rotation (solid line) and radial expansion (dashed line). This shows that, in the case of solid body rotation, the braking time scale is about ten times smaller than the mass loss time scale, whereas, it is hundred times smaller in the case of radial expansion (the braking time scale is defined as the time required to decrease the surface angular velocity by a factor \( e \)). We therefore conclude that the braking time scale is significantly smaller than the mass-loss time scale although both time scales does not differ by many orders of magnitude.

In the next section, we study the formation of angular velocity gradients in a stellar envelope assuming the transport of angular momentum is only due to a radial mass flux. We shall see that this process generates strong near surface gradients in a time scale smaller by many orders of magnitude than the mass-loss time scale.

3. Radial advection of angular momentum across a stellar surface

Non-uniform radial expansion tends to generate differential rotation and this is particularly true across the stellar surface where the steep density increase has to be accompanied by a steep decrease of the radial velocity. To show this let us follow the expansion of two spherical layers located at different depths in the star’s envelope, and rotating at the same rate. After a given time, the outer layer will have travelled a much larger distance than the inner one so that its angular velocity will have decreased more than that of the inner layer. A gradient of angular velocity has then appeared between both layers.

To estimate the gradient generated by this non-uniform expansion, we write down the angular momentum conservation assuming the transport of angular momentum is only due to a radial flow. Consequently, angular momentum transfers by viscous stresses, meridional circulation, gravity waves or turbulence are neglected altogether. Although latitudinal inhomogeneities in the mass-loss mechanism or Eddington-Sweet circulation are expected to induce latitudinal flows, we note that the present assumption would be still justified in the equatorial plane if the meridional circulation is symmetric with respect to the equatorial plane.

In the context of our simplified model, the angular momentum balance reads

\[
\frac{\partial}{\partial t} (\omega) + v(r,t) \frac{\partial}{\partial r} (\omega) = 0,
\]

where \( \omega \) is the specific angular momentum and \( v(r,t) \) is the radial velocity. Up to mass loss rates of the order of \( 10^{-6} M_{\odot} \) yr\(^{-1} \), radial velocities carrying out a time-independent and isotropic mass-loss are much smaller than the local sound speed. Consequently the star is al-

\[\text{Fig. 2. Braking of the stellar surface as a function of the percentage of mass lost assuming solid body rotation (solid line) or purely radial expansion (dashed line).}\]
ways very close to hydrostatic equilibrium and continuously adjusts its structure to its decreasing mass. This means in particular that the temporal variations of the density near the surface are small, because the effect of mass-loss is distributed over the whole star through hydrostatic equilibrium. Accordingly, temporal variations of the radial velocities are small and we shall neglect them as their inclusion would not modify our conclusions. Then, the radial outflows satisfies
\[ 4\pi r^2 \varrho(r) v(r) = \dot{M}, \quad (4) \]
where \( \varrho(r) \) is given by stellar structure models.

With the above assumptions, the specific angular momentum evolves like a passive scalar advected in a one dimensional stationary flow \( v(r) \). The mathematical problem can be readily solved and we will do so in the following for a radial flow corresponding to a 2\( M_\odot \) star with a mass loss rate of \( \dot{M} = 10^{-8} M_\odot \text{yr}^{-1} \). But first we derive useful properties by studying the evolution of the angular momentum gradient in the general case.

Except when \( \omega \) or \( v \) are uniform in space, advection always modifies the distribution of the conserved quantity. While decelerated flows tend to sharpen \( \omega \)-gradients, accelerated flows smooth them out. To illustrate the process of gradient smoothing in an accelerated flow, the angular momentum evolution of two neighbouring fluid elements \( A \) and \( B \) has been represented in Fig.3. Following the motions, the angular momentum gradient between these two points decreases because their separation \( \Delta r \) increases while the angular momentum difference \( \Delta \omega \) is conserved.

This simple sketch can also be used to quantify the gradient decrease. We first note that \( A \) and \( B \) travel the distance separating \( r_1 + \Delta r_1 \) from \( r_2 \) in the same time. Thus, the time interval required by \( A \) to go from \( r_1 \) to \( r_1 + \Delta r_1 \) is the same as the one used by \( B \) to go from \( r_2 \) to \( r_2 + \Delta r_2 \). This is expressed by, \( \Delta r_2 / v(r_2) = \Delta r_1 / v(r_1) \), and then,
\[ v(r_1) \frac{\Delta \omega}{\Delta r_1} = v(r_2) \frac{\Delta \omega}{\Delta r_2}, \quad (5) \]
Taking the limit of vanishing separation between \( A \) and \( B \), we conclude that the product \( v(r) \partial \omega / \partial r \) is conserved following the motions, a property which can be readily verified by calculating the Lagrangian derivative of \( v \partial \omega / \partial r \).

This property implies that angular momentum gradients can be completely smoothed out if they are advected in a strongly enough accelerated flow. This is particularly relevant near stellar surfaces where steep density gradients induce steep radial velocity gradients.

To specify this effect we express the conservation of \( v \partial \omega / \partial r \) for the radial outflow given by Eq. (4). Then, the evolution of the logarithmic gradients of the angular velocity following the flow between radii \( r_1 \) and \( r_2 \) reads
\[ \partial \ln \Omega / \partial \ln r (r_2) = -2 + \frac{\varrho(r_2)}{\varrho(r_1)} \left( \frac{r_2}{r_1} \right)^3 \left( \partial \ln \Omega / \partial \ln r (r_1) + 2 \right), \quad (6) \]
where the quantity \( \partial \ln \Omega / \partial \ln r + 2 \) measures departures from uniform specific angular momentum.

In this expression, \( \varrho(r) r^3 \) decreases almost like \( \varrho(r) \) in the vicinity of the photosphere since the density decreases considerably over distances small compared to the stellar radius. Then, the above expression shows that any departure from constant specific angular momentum will have decreased by a factor \( e^a \) after crossing \( a \) density scale heights. Moreover, initial departures can not be too large otherwise the corresponding differential rotation would be subjected to powerful instabilities. Then, for any realistic values of the initial angular velocity gradients, fluid elements reaching the surface will have an angular velocity gradient close to \( -2 \Omega / R \).

We confirmed the validity of this simple picture by solving the advection problem for the density profile corresponding to the stellar structure model of a pre-main-sequence 2\( M_\odot \) star (Palla & Stahler 1993). Again, the mass-loss rate has been fixed to \( \dot{M} = 10^{-8} M_\odot \text{yr}^{-1} \). Starting from a solid body rotation, the evolution of the rotation profile in the vicinity of the stellar surface is presented in Fig.4. The different curves correspond to increasing advection times, \( 10^{-1} \), 1, 10, 10\(^2\), 10\(^3\) and 10\(^4\) years, respectively.
We observe that the evolution is first rapid and then much slower. This is because the evolution starts with the rapid expansion of the outer layers which tends to set-up a profile of uniform specific angular momentum $\Omega \propto 1/r^2$. Once this is done, the evolution takes place on much larger time scales as it involves the much slowly expanding inner layers. The gradual set-up of the $\Omega \propto 1/r^2$ profile is confirmed by the evolution of the angular velocity logarithmic gradients. As shown on Fig. 5, specific angular momentum gradients are rapidly smoothed out in the outer layers while the inner ones become affected after a long time.

We can therefore conclude that starting with any realistic angular velocity profile in a stellar envelope, radial expansion will generate angular velocity gradients very close to $-2\Omega/R$ near the surface.

Whether such gradients are actually present in the subphotospheric layers of mass-losing stars depends on how the characteristic time for the formation of these gradients compares with the time scales of other angular momentum transport processes. The angular velocity gradients being produced by radial acceleration, the associated time scale is

$$t_G = \frac{1}{\frac{d\Omega}{dr}} = \frac{H_\rho}{v(r)} = \frac{4\pi r^2 \rho(r) H_\rho}{M} t_M,$$  

where $H_\rho$ is the density scale height and $t_M = M/\dot{M}$ is the mass-loss time scale. Fig. 5 shows that no more than one month is necessary to form gradients of the order of $\Omega/R$ for a mass loss rate equal to $\dot{M} = 10^{-8} M_\odot$ yr$^{-1}$. This time scale is smaller by many orders of magnitude than the mass-loss time scale or the braking time scale. This is because the formation of gradients requires radial displacements corresponding to a density scale height whereas a significant braking requires radial displacements of the order of the stellar radius.

4. Stability of the angular velocity gradients

In this section, we investigate whether the differential rotation induced by the radial mass-loss is sufficient to trigger a hydrodynamical instability.

The uniform angular momentum profile being marginally stable with respect to Rayleigh-Taylor instability, we consider its stability with respect to shear instabilities. When, as it is the case here, the vorticity associated with the velocity profile does not possess extreme, the study of shear instabilities is much complicated by the fact that finite amplitude perturbations involving non-linear effects have to be taken into account. Existing stability criteria for such velocity profiles rely on laboratory experiments which shows critical Reynolds numbers above which destabilization occurs. Although the critical Reynolds number depends on the particular flow configuration, its value is generally of order of 1000.

J.P. Zahn (1974) proposed the following instability criterion

$$\frac{d\Omega}{dr} > \left(PrRe_{\text{crit}}\right)^{1/2}N,$$  

where $Re_{\text{crit}}$ is the critical Reynolds number, $Pr = \nu/\kappa$ is the Prandtl number comparing the thermal diffusivity $\kappa$ to the viscosity $\nu$ and $N$ is the Brunt-Väisälä frequency which measures the strength of stable stratification in radiative interiors. The combined effect of the stable stratification and the thermal diffusion has been derived on phenomenological grounds. However, the way the criterion depends on this effect has been recently supported in the context of the linear stability theory (Lignières et al. 1999).

Applying the criterion to an uniform angular momentum profile, we find that instability occurs if the rotation period at the surface is smaller than

$$P \leq 18.4 \frac{g_\odot}{g} \text{days},$$

where the actual temperature gradient has been taken equal to $\nabla = 0.25$, a typical value for radiative envelopes according to Cox (1968), and the radiative viscosity has been assumed to dominate the molecular one which is true in the envelope of intermediate mass stars. Note also that this expression holds for a mono-atomic completely ionised gas in a chemically homogeneous star. When radiation pressure is taken into account, the upper limit of the rotation period has to be multiplied by $1/\sqrt{1-3\beta}$, where $\beta$ is the ratio between the gas pressure and the total pressure. This factor remains very close to one for pre-main-sequence models from 2 to $5M_\odot$.

The above instability condition being easily met by early type stars, we conclude that mass-loss tends to impose an unstable differential rotation below the surface of early-type stars.
5. Discussion and conclusion

We have studied the effect of mass-loss on the rotation of stellar envelopes in the simplified context of an unmagnetised spherically symmetric outflow. In the same way as an inhomogeneous distribution of a passive scalar tends to be smoothed out in an expanding flows, the radial gradients of specific angular momentum are smoothed out by radial expansion in stellar envelopes. This process becomes more and more effective as one approaches the surface because expansion becomes stronger and stronger. As shown in Figs 4 and 5, a profile of uniform angular momentum, $\Omega \propto 1/r^2$ is rapidly set-up in the outermost layers of the star and then pervades towards the interior on much larger time scales. These time scales being proportional to the mass-loss time scale, they vary very much from stars to stars. For a typical Herbig star ($M = 10^{-8} M_\odot$ yr$^{-1}$), an angular velocity gradient close to $-2\Omega/R$ appears in one month. By contrast, for a solar-type mass-loss rate, $M = 10^{-14} M_\odot$ yr$^{-1}$, it would be $10^6$ times longer to reach the same level of differential rotation. According to existing stability criteria, the uniform angular momentum profile is subjected to shear instabilities provided the rotation period is shorter than $18.4$ g$_\odot$/g days.

The present one-dimensional model is admittedly not realistic at least because latitudinal variations occur in rotating stars and give rise to a meridional circulation. Nevertheless, as already mentioned, neglecting latitudinal flows may be justified in the equatorial plane. In the following we discuss to which extent the neglected processes can prevent the formation of turbulent differentially rotating layers below the surface of rapidly rotating mass-losing stars. We first consider the angular momentum transport and then the stability problem.

Latitudinal variations could be inherent to the mass-loss mechanism as proposed for radiatively driven wind emitted from fast rotating stars (Owocki & Gayley 1997). Such variations are likely to generate a meridional circulation (Maeder 1999) but, at the latitudes where outflows occur, we still expect radial expansion to play a major role in shaping the angular velocity profile.

The Eddington-Sweet circulation driven by departure from sphericity operates on time scale larger than the Kelvin-Helmholtz time scale of the star. This time scale is of the order of the order of $10^7$ years for a $2M_\odot$ Herbig stars. Consequently the Eddington-Sweet circulation is unlikely to prevent the formation of the differentially rotating subphotospheric layer for strong mass-loss rate.

Turbulent motions like those generated in thermal convection zones might prevent the formation of the differentially rotating layer. In radiative envelopes however, it is not clear whether turbulent motions (not generated by the mass-loss process) would be vigorous enough.

Strong magnetic fields could in principle prevent the formation of these gradients. Note that this objection is not relevant in the context of the Vigneron et al. scenario where the shear layer is first produced by an unmagnetised wind.

Finally, we also neglected the effect of pre-main-sequence contraction. This is justified because the negative radial velocities associated with the contraction are much smaller than the positive radial velocities induced by mass-loss in the vicinity of the stellar surface.

In what concerns the stability of the uniform angular momentum profile, one has to remind that the stability criterion results from an extrapolation of laboratory experiments results. Despite recent numerical simulations which contradict the validity of this extrapolation for a differential rotation stable with respect to the Rayleigh-Taylor instability (Balbus et al. 1996), analysis of the experimental results reveal that for large values of the Reynolds number the properties of the turbulent flow do not depend on its stability or instability with respect to the Rayleigh-Taylor criterion (Richard & Zahn 1999). The solution of this debate would have to wait numerical simulations with higher resolution or specifically designed laboratory experiments.

The fact that the shear layer is embedded in an expanding flow may also affect the stability because expansion is known to suppress the turbulence. For example, this phenomenon is observed in numerical simulations of turbulent convection near the solar surface (Stein & Nordlund 1998). However, the growth time scale of the instability being of the order of the rotation period ($\approx 1$ day for Herbig stars), we do not expect the expansion to be rapid enough to suppress shear turbulence.

We conclude that the present model of angular momentum advection by a radial flow supports the assumption made by Vigneron et al. (1990) and F. Lignières et al. (1996) that unmagnetised winds tend to force an unstable differential rotation in the subphotospheric layers of Herbig Ae/Be stars. Note that this work is part of an ongoing effort to assess the viability of the Vigneron et al. scenario. F. Lignières et al (1996) and F.Lignières et al. (1998) have already studied the step further, once turbulent motions are generated at the top of the radiative envelope, their work pointing towards the formation and inwards expansion of a differentially rotating turbulent layer. In addition, three-dimensional numerical simulations are being performed to investigate whether magnetic fields can be generated in such a layer.

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