Ultra-sensitive measurement of thermal and quantum noises

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Abstract. We have developed an experiment of ultrasensitive interferometric measurement of small displacements based on a high-finesse Fabry-Perot cavity. We have observed the internal thermal noise of mirrors and fully characterized their acoustics modes. We describe recent progress in our experimental setup in order to reach a sensitivity better than \(10^{-20}\) m/√Hz. This unique sensitivity is a step towards the first observation of the radiation pressure effects and the resulting standard quantum limit in interferometric measurements. Our experiment may become a powerful facility to test quantum noise reduction schemes such as the use of squeezed light or quantum locking of mirrors. We also present a new scheme where a cavity detuning changes the mirror dynamics in such a way that the gravitational signal is amplified by the optomechanical coupling and the interferometer sensitivity is improved.

1. Introduction

Though gravitational-wave interferometers are not sensitive to quantum effects of radiation pressure yet, this will be the case for the second-generation antennas, which will be confronted to the so-called Standard Quantum Limit (SQL)\(^1\). Number of quantum noise reduction schemes have been proposed which rely on the use of squeezed light sent into the interferometer\(^2\) or on the use of the quantum correlations induced by radiation pressure between phase and intensity fluctuations\(^3\). The possibility to implement these techniques in real interferometers gave rise to new methods such as the quantum locking of mirrors\(^4\) or the detuning of the signal recycling cavity\(^5\).

Our work is dedicated to the experimental demonstration of the quantum effects of radiation pressure and to the test of quantum noise reduction schemes. Our experimental setup is based on a high-finesse Fabry-Perot cavity used to monitor the small displacements of a mirror. The experiment already achieved a sufficient sensitivity to analyze the internal thermal noise of mirrors at room temperature and to fully characterize the optomechanical properties of the internal acoustic modes. Improvements of the setup are in progress to reach the necessary sensitivity to observe quantum effects of radiation pressure. We could then test quantum noise reduction schemes and their possible experimental implementation.

We first present in Sec. 2 the fundamental limitations enforced by light to interferometric displacement measurements, whereas our experimental results and experimental improvements

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are described in Sec. 3 and 4. A new quantum noise reduction scheme based on a detuned cavity is discussed in Sec. 5.

2. Optomechanical coupling

The optomechanical coupling is the cross-coupling between a movable mirror and the light reflected upon it. A displacement of the mirror induces a linear phase-shift of the reflected beam providing a possibility to read out the mirror motion. The response of the mirror to the radiation pressure of the incident beam disturbs the mirror motion and corresponds to the back-action of the measurement. When the mirror is part of a high-finesse optical cavity, the optical resonance enhances the optomechanical coupling, since the phase of the reflected field becomes very sensitive to the mirror displacements. At resonance, the phase-shift $\delta \varphi$ of the reflected field is given by

$$\delta \varphi = \delta \varphi_{\text{in}} + \frac{F}{\lambda} (\delta x_{\text{sig}} + \delta x_m),$$

where $\lambda$ is the optical wavelength, $F$ the cavity finesse, $\delta x_m$ the mirror displacement induced by the radiation pressure fluctuations, and $\delta \varphi_{\text{in}}$ the phase fluctuations of the incident beam. The signal $\delta x_{\text{sig}}$ can either be a physical displacement of the mirror, such as the response to an external force or the thermal noise, or an apparent variation of the cavity length due for example to a gravitational wave. The sensitivity of the measurement is limited by the phase noise of the light beam and by the radiation pressure noise. The phase noise $\delta \varphi_{\text{lin}}$ is inversely proportional to the square root $\sqrt{I}$ of the mean incident intensity. In the framework of linear response theory, the mirror response to the radiation pressure fluctuations $\delta F_{\text{rad}}$ is proportional to the mechanical susceptibility $\chi$:

$$\delta x_m[\Omega] = \chi[\Omega] \delta F_{\text{rad}}[\Omega] = 2\hbar k \chi[\Omega] \delta I[\Omega],$$

where $\delta I$ are the intracavity intensity fluctuations, proportional to $\sqrt{I}$. A compromise between phase and intensity noises then leads to the so-called Standard Quantum Limit which corresponds to a minimum measurable displacement equal to $\sqrt{\hbar \chi[\Omega]}$. This limit usually lies in the $10^{-21}$ m/$\sqrt{\text{Hz}}$ range and has never been experimentally demonstrated.

3. Thermal noise observation at the attometer level

We have developed an experiment to detect very small mirror displacements. In our setup (Fig. 1), the movable mirror is a 1-inch \times \frac{1}{4}-inch cylindrical high-quality mirror made of fused silica. For frequencies above 100 kHz, the observed mirror displacement are mainly due to the surface deformations induced by the excitation of internal acoustic modes. The mirror is included in a mm-long Fabry-Perot cavity with a finesse between 25 000 and 40 000. The light beam is provided by a frequency-stabilized Ti:Sa laser working at 810 nm. An homodyne detection monitors the phase of the field reflected by the cavity.

We have measured the thermal noise at room temperature (Fig. 2). It appears as the superposition of the response of every acoustic mode to the thermal excitation. Near the resonance frequency of a particular mode, the behavior is mainly ruled by the mechanical susceptibility of this mode, which corresponds to an harmonic oscillator:

$$\chi[\Omega] = \frac{1}{M (\Omega_M^2 - \Omega^2 - i\Gamma)},$$

where $M$ is the effective mass of the mode, $\Omega_M$ its resonance frequency and $\Gamma$ its damping rate. The thermal noise clearly exhibits thermal resonances associated to the acoustic modes of the mirror. We have calibrated the observed noise by using a frequency modulation of the laser.
Figure 1. Experimental setup. A cylindrical mirror is used as the end mirror of a high-finesse Fabry-Perot cavity. A frequency and intensity stabilized laser beam is sent into the cavity. The phase of the reflected beam is measured by an homodyne detection. An auxiliary laser beam reflected from the back of the mirror can be intensity modulated and spatially scanned over the surface to apply a controlled radiation pressure force.

The gray curve in Fig. 2 is the phase noise obtained when the cavity is out of resonance. It gives a limit of sensitivity for the measurement equal to $5 \times 10^{-20} \text{m}/\sqrt{\text{Hz}}$ beyond 400 kHz.

These measurements allow us to determine with a very good accuracy various optomechanical characteristics of each mode such as the resonance frequency, damping rate, and effective mass[11, 12]. To fully characterize the optomechanical properties of the mirror, we also used an intense auxiliary laser beam of 400 mW, reflected from the back on the mirror. The beam is intensity-modulated in order to apply a radiation pressure force at a given frequency and the beam spot can be scanned over the mirror surface by a moving lens (see Fig. 1). When the modulation is resonant with a particular mode, the amplitude of the mirror response allows us to reconstruct the spatial structure of the acoustic mode[13]. The spatial profiles of three modes are shown in Fig. 2. We have compared the resonance frequencies and the spatial profiles to the

Figure 2. Observed thermal noise spectrum of a cylindrical mirror (black curve), and corresponding phase-noise limit obtained with the cavity out of resonance (gray curve). Plots on the right are the experimental and theoretical spatial distributions of some modes.
ones computed with the program cypres developed within the Virgo collaboration by François Bondu and Jean-Yves Vinet[14]. The discrepancy between measured and computed frequencies is less than 1 %, and we obtained a very good agreement between experimental and theoretical spatial distributions, as shown in Fig. 2.

4. Towards the observation of quantum effects of radiation pressure
The observation of quantum effects of radiation pressure requires to improve the sensitivity of our experiment in order to be able to detect the displacements induced by the radiation pressure. We also have to improve the finesse and the damage threshold of the cavity in order to increase the intracavity light power and the resulting mirror displacements. We have thus designed a new cavity with two high-quality cylindrical mirrors providing a finesse better than 200 000. We have measured the mirror displacements at room temperature with this cavity. Fig. 3 presents a preliminary result showing an uncalibrated thermal noise spectrum and the corresponding phase-noise limit obtained out of resonance. As compared to the previous result of Fig. 2, we have gained a factor 10 in the sensitivity of the displacement measurement. We have also observed a thermal bistability when the incident light power is increased above 1 mW. To maintain the cavity in a stable regime we have developed a fast servoloop with an acoustooptic modulator used as a frequency shifter. A last condition to observe quantum effects is that all classical noises have to be negligible. This requires to work at low temperature. A liquid helium cryostat has been designed and will be implemented in our experimental setup.

5. Sensitivity improvement by cavity detuning
A further objective of the experiment is to test some of the quantum noise reduction schemes proposed to improve the sensitivity of gravitational-wave interferometers. We present here a scheme based on optical rigidity[15] similar to the one proposed for signal-recycled gravitational-wave detectors[10] but applied here to a single detuned cavity. This scheme could in principle be implemented without heavy extra setup.

5.1. Mirror dynamics
We consider a single-port cavity as the one used in our experimental setup (Fig. 1) and we study the response of the system to a signal $\delta x_{\text{sig}}$. We consider for simplicity that the cavity bandwidth is large enough so that the low-pass filtering by the cavity can be neglected. The cavity is usually taken at resonance to optimize the intracavity intensity and the phase shift of the beam reflected by the cavity. In this section we explore the possibility of using a detuned
cavity. The mean intracavity intensity $I$ then depends on the mean detuning $\bar{\psi}$ according to an Airy function

$$I = \frac{2\gamma}{\gamma^2 + \bar{\psi}^2}I_{\text{in}},$$

(4)

where $\gamma$ is the damping rate of the cavity related to the finesse by $F = \pi/\gamma$. In the case of a non-zero detuning, the working point of the cavity is on one side of the Airy peak and the intracavity intensity depends on the cavity length variations with a slope

$$\frac{dI}{dx} = -4k\frac{\bar{\psi}}{\gamma^2 + \bar{\psi}^2}I,$$

(5)

where $k$ is the field wavevector. Any length variation then changes the intensity inside the cavity and induces a variation of the radiation pressure exerted on the mirror. The radiation-pressure fluctuations resulting from both the mirror displacements and the signal is proportional to the slope of the Airy peak,

$$\delta F_{\text{rad}}[\Omega] = 2\hbar k \frac{dI}{dx} (\delta x_m[\Omega] + \delta x_{\text{sig}}[\Omega]),$$

(6)

and is superimposed to the quantum fluctuations of the radiation pressure given by an equation similar to eq. (2).

The term corresponding to $\delta x_m$ in eq. (6) is somewhat equivalent to the action of an external feedback loop. Its effect is to change the mechanical response of the mirror to an external force. The resulting mirror response is now described by an effective mechanical susceptibility $\chi_{\text{eff}}$ given by

$$\chi_{\text{eff}}^{-1}[\Omega] = \chi^{-1}[\Omega] + 8\hbar k^2 I \frac{\bar{\psi}}{\gamma^2 + \bar{\psi}^2}.$$  

(7)

The effect of this force is then a modification of the spring constant of the mechanical resonator, known as optical rigidity [15, 16]. Depending on the signs of $\bar{\psi}$ and $\chi$, it can be used to amplify the response of the mirror to an external force.

5.2. Sensitivity improvement

The second term in eq. (6) is proportional to $\delta x_{\text{sig}}$ so that the mirror becomes sensitive to the signal. The response of the mirror can then produce an amplification or an attenuation of the signal, according to the sign of $\chi_{\text{eff}}$ and to the slope of the Airy peak. One can take advantage of this amplification to improve the sensitivity to a signal.

For frequencies above the mechanical resonance frequency, the mechanical susceptibility $\chi$ decreases as $1/\Omega^2$. For a resonant cavity, the SQL is reached for a single frequency $\Omega_{\text{SQL}}$ depending on the intracavity intensity. Curve $a$ of Fig. 4 shows the noise spectrum for a resonant cavity, normalized to the SQL at $\Omega_{\text{SQL}}$. For frequencies below $\Omega_{\text{SQL}}$, the response to radiation pressure fluctuations increases and produces an excess noise; for frequencies above, the sensitivity is limited by the phase fluctuations and tends toward the shot-noise limit.

Curves $b$, $c$ and $d$ show the noise spectra computed for increasing positive detunings and increasing intracavity intensities. Although these curves exhibit a larger noise at low frequency than in the resonant case, one gets a noise reduction below the standard quantum limit in the intermediate frequency domain, which becomes larger and larger as the detuning and the intracavity intensity increase.

6. Conclusion

We have presented an experiment of ultrasensitive measurement of a mirror motion using a high-finesse cavity and a very stable laser source. We have observed the thermal noise of a
cylindrical mirror with a sensitivity of $5 \times 10^{-20} \text{m}/\sqrt{\text{Hz}}$ and made a full characterization of the optomechanical properties of its internal acoustic modes. Recent experimental progresses have allowed us to improve the sensitivity, by using a cavity with a finesse larger than 200 000. The SQL then should be reached by low-temperature operation. We have also presented a new quantum noise reduction scheme based on a detuned cavity. The amplification of the signal and the modification of the mirror dynamics by the internal radiation pressure may improve the sensitivity of the measurement beyond the SQL.

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Reference
[1] C.M. Caves, Phys. Rev. D 23, 1693 (1981)
[2] M.T. Jaekel, S. Reynaud, Europhys. Lett. 13, 301 (1990)
[3] V.B. Braginsky, F. Ya. Khalili, Quantum measurement (Cambridge University Press, Cambridge, 1992)
[4] P. Fritschel, Proc. SPIE 4856-39, 282 (2002)
[5] M. Xiao, L. Wu, H.J. Kimble, Phys. Rev. Lett. 59, 278 (1987)
[6] P. Grangier, R. E. Slusher, B. Yurke, A. LaPorta, Phys. Rev. Lett. 59, 2153 (1987)
[7] K. McKenzie, D.A. Shaddock, D.E. McClelland, B.C. Buchler, P.K. Lam, Phys. Rev. Lett. 88, 231102 (2002)
[8] H.J. Kimble, Y. Levin, A.B. Matsko, K.S. Thorne, S.P. Vyatchanin, Phys. Rev. D 65, 022002 (2002)
[9] J.M. Courty, A. Heidmann, M. Pinard, Phys. Rev. Lett. 90, 083601 (2003)
[10] A. Buonanno, Y. Chen, Phys. Rev. D 64, 042006 (2001)
[11] M. Pinard, P.-F. Cohadon, T. Briant, A. Heidmann, Phys. Rev. A 63, 013808 (2000)
[12] T. Briant, P.-F. Cohadon, M. Pinard, A. Heidmann, Eur. Phys. J. D 22, 131 (2003)
[13] T. Briant, P.-F. Cohadon, A. Heidmann, M. Pinard, Phys. Rev. A 68, 033823 (2003)
[14] F. Bondu, J.Y. Vinet, Phys. Lett. A 198, 74 (1995)
[15] V.B. Braginsky, F.Ya. Khalili, P.S. Volikov, Phys. Lett. A 287, 31 (2001)
[16] A. Buonanno, Y. Chen, Phys. Rev. D 65, 042001 (2002)