Tensor force manifestations in \textit{ab initio} study of the $^2\text{H}(d, \gamma)^4\text{He}$, $^2\text{H}(d, p)^3\text{H}$ and $^2\text{H}(d, n)^3\text{He}$ reactions

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Abstract. The $^2\text{H}(d, \gamma)^4\text{He}$ capture reaction and the $^2\text{H}(d, p)^3\text{H}$ and $^2\text{H}(d, n)^3\text{He}$ transfer reactions at very low energies are studied in an extended microscopic cluster model, in which the $^3\text{H}$, $^3\text{He}$, and $d$ clusters are given by the precise few-body wave functions with a realistic nucleon-nucleon force and the cluster relative motions are solved with the same realistic nucleon-nucleon force. Our results show that the tensor force in realistic interactions plays an essential and indispensable role to reproduce the very low energy astrophysical $S$ factor of these reactions.

1. Introduction

The microscopic cluster model (RGM) \cite{1, 2} is a successful model to study the nuclear structure and reactions between light nuclei. This model usually employs simple cluster wave functions, $S$-wave wave function for the $s$-shell clusters, and accordingly employs the effective $N-N$ interaction such as the Minnesota (MN) potential \cite{3}. In this effective potential, the effect of the tensor term and the short-range repulsion are presumed to be renormalized in the central force of the effective interaction.

Recently, we have developed a microscopic cluster model in which the cluster wave functions are given by precise few-body wave functions including higher partial waves up to the $D$-wave. The cluster wave functions and the cluster relative motion are solved with the same realistic $NN$ interaction.

In the present paper, we calculate the low energy cross sections of the $^2\text{H}(d, \gamma)^2\text{H}$, $^2\text{H}(d, p)^3\text{H}$, and $^2\text{H}(d, n)^3\text{He}$ reactions. We employ the extended microscopic cluster model and the results with the AV8' and G3RS potentials are compared with those with the MN potential in order to clarify the role of the tensor force at very low energies. Along with the radiative capture reaction, we calculate the transfer reactions, $^2\text{H}(d, p)^3\text{H}$ and $^2\text{H}(d, n)^3\text{He}$, to show the role of the tensor force in the zero energy $S$ factor of these two reactions.
Figure 1. Astrophysical S-factor of the $^2\text{H}(d, \gamma)^4\text{He}$ reaction [4]. Results calculated with the realistic (AV8', G3RS) and effective (MN) potentials are compared to experiment [12].

2. Model

Capture and transfer reactions at low energies are calculated in the microscopic cluster model [4] with the multi-channel configurations of the $^3\text{H}(1/2^+)+p$, $^3\text{He}(1/2^+)+n$, $d(1^+)+d(1^+)$, $pn(0^+)+pn(0^+)$, and $pn(0^+)+pn(0^+)$. In this model, all cluster wave functions are given by precise three- or two-body wave functions with the AV8' [5] and G3RS [6] realistic interactions. In these cluster wave functions the higher partial wave up to the D-wave are taken into account. Since it is very essential to reproduce correctly the two-body thresholds of $d+d$, $t+p$, and $h+n$ in these reaction calculations, a phenomenological three-body force [7] is added to the present Hamiltonian. The cluster wave functions are selected by the stochastic variational method [8, 9] in order to make the four-body calculation feasible. Number of the basis set is 30 for the $t$ and $h$ clusters and 8 for the $d$ cluster. For the $0^+$ state of $pn$, $pp$, and $nn$ clusters, the same $S$-wave basis set with the $d$ cluster are employed and these wave functions are given by the bound state approximation. These channels are included in order to take into account the distortion effect. The pseudo-excited states are also taken into account for all of the clusters.

With above the precise cluster wave functions, the cluster relative wave functions are solved with the same realistic $N-N$ interaction. The total wave function satisfies the Pauli principle exactly and is free from the spurious center-of-mass motion. Total angular momentum and the parity is correctly projected on the total wave function. The scattering problem is solved by microscopic $R$-matrix method (MRM) [10]. In the MRM, the cluster relative wave function is expanded with the Gaussian basis set within the channel radius and then it is connected the exact Coulomb function at the channel radius and so we get the $S$-matrix. The cross section of the capture and transfer reaction are given in Ref. [11]. Cross sections with the realistic interactions are compared with those with the MN effective potential, in which the tensor force is simulated by a renormalized central force, in order to make clear the role of the tensor force in these reactions.

3. Results

Figure 1 shows our astrophysical $S$ factor for the $^2\text{H}(d, \gamma)^4\text{He}$ capture reaction, and a comparison with the experimental data. We consider only the dominant $E2$ transition, that is, the transition from the $2^+$ continuum state to the $0^+$ ground state of $^4\text{He}$. Our results with the AV8' and
Figure 2. Astrophysical $S$-factors of the $^2\text{H}(d,\gamma)^4\text{He}$ reaction with the AV8$'$ potential, and contributions of the three incoming $dd$ channels, $^5S_2$, $^1D_2$, and $^5D_2$ [4].

G3RS potentials reproduce the experimental data very well, especially the flat behaviour below 0.3 MeV whereas the MN potential fails to reproduce the data below 0.3 MeV. In order to make clear the role of the tensor force, we decomposed the $S$ factor with the AV8$'$ potential into the three different entrance $d+d$ channels, $^5S_2$, $^1D_2$, and $^5D_2$, as shown in figure 2. This shows that the $S$ factor below the 0.3 MeV is dominated by the $^5S_2$ entrance channel which gives the flat behaviour at low energies, while the $^1D_2$ channel plays a dominant role above 0.3 MeV. Through the $E2$ transition, the $^5S_2$ entrance channel transits into the $D$-wave component of the $0^+$ ground state where the tensor force of the $N$-$N$ interaction is indispensable to produce this $D$-wave component. However, since the MN potential gives a pure $S$-wave component ($L=0$, $S=0$) in the $0^+$ ground state, the $E2$ transition from the $^5S_2$ entrance channel is forbidden. Therefore the $S$ factor below the 0.3 MeV with the MN potential comes from the the $^1D_2$ entrance channel. As a result, the MN potential fails to reproduce the data below 0.3 MeV.

Figure 3 shows our astrophysical $S$ factors of the $^2\text{H}(d,p)^3\text{H}$ and $^2\text{H}(d,n)^3\text{He}$ transfer reactions, in comparison with experimental data. We have taken into account up to the $2^\pm$ states in the present calculation. The results with the AV8$'$ and G3RS potentials reproduce the experimental data very well whereas the MN potential underestimates the $S$ factor. In figure 4(a), the $S$ factor of the $^2\text{H}(d,p)^3\text{H}$ with the AV8$'$ potential is decomposed into the different spin/parity states $J^\pi$. Below 0.1 MeV, the most important contribution to the $S$ factor comes from the $2^+$ state. In figure 4(b), this $2^+$ partial $S$ factor is further decomposed into the three dominant transitions, $d+d\ ^5S_2\rightarrow t+p\ ^3D_2$, $d+d\ ^5S_2\rightarrow t+p\ ^3D_2$, and $d+d\ ^1D_2\rightarrow t+p\ ^1D_2$. This figure shows that the $^5S_2\rightarrow ^3D_2$ transition gives dominant contribution below 0.1 MeV where the tensor force of the $N$-$N$ interaction is indispensable for this transition. Therefore, this transition is forbidden in the MN potential and the $2^+$ contribution comes mainly from the $^1D_2\rightarrow ^1D_2$ transition. This is the reason why the MN potential fails to reproduce the data in this transfer reaction.

4. Conclusion
The radiative capture reaction, $^2\text{H}(d,\gamma)^4\text{He}$ and the transfer reactions, $^2\text{H}(d,p)^3\text{H}$ and $^2\text{H}(d,n)^3\text{He}$, which are of astrophysical interest, are calculated in the extended microscopic cluster model with a realistic nucleon-nucleon interaction in order to make a clear the role of the
Figure 3. Astrophysical $S$-factors of the $^2\text{H}(d,p)^3\text{H}$ and $^2\text{H}(d,n)^3\text{He}$ reactions [4]. Results calculated with the realistic (AV8', G3RS) and effective (MN) potentials are compared to experiment [12, 13].

Figure 4. (a) Contributions of the different $J^\pi$ states to the astrophysical $S$-factor of the $^2\text{H}(d,p)^3\text{H}$ reaction [4]. (b) Decomposition of the $2^+$ contribution according to the incoming $dd$ and outgoing $tp$ channels. The AV8' potential is used.

tensor force in these reactions. The AV8' and G3RS potentials can reproduce the astrophysical $S$ factor of these reactions at very low energies, while the MN potential fails to reproduce the experimental data. In the capture reaction, the $E2$ transition from the $2^+$ $d+d$ $S$ wave continuum state to the $D$ wave component of the $0^+$ ground state is dominant. And the transition from the $d+d$ $S$ wave continuum state to the $t+p(^4\text{He}+n)$ $D$ wave channel in the $2^+$ state is essential in the transfer reactions. The tensor force is indispensable for these transitions but the transitions are forbidden in the calculation with the effective potential since the tensor force is not explicitly included in the MN potential. Our results show that the tensor force plays an important role in the astrophysical reactions involving four nucleons.
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