Curvature late-time acceleration in an eternal universe

Rodrigo Maier\textsuperscript{a,1} and Ivano Damião Soares\textsuperscript{b}

\textsuperscript{a}Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, Maracanã, 20550-900, Rio de Janeiro, Brazil
\textsuperscript{b}Centro Brasileiro de Pesquisas Físicas-CBPF, Rua DR. Xavier Sigaud 150, Urca, 22290-180, Rio de Janeiro, Brazil

E-mail: rodrigo.maier@uerj.br, ivano@cbpf.br

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Abstract. We construct a FLRW universe considering an anisotropic scaling between space and time at extremely high and low energies only. In this context, Friedmann equations contain an additional term arising from spatial curvature which implements nonsingular bounces in the early Universe. The matter content of the model is a nonrelativistic pressureless perfect fluid and radiation. By breaking covariance diffeomorphism also at extreme large scales, an additional term furnishes late-time acceleration due to spatial curvature so that a cosmological constant is not needed. In order to probe the final fate of the universe we also introduce a lower order curvature term which dominates in deep IR. Given the observational parameters we obtain a concrete model in eternal recurrence in which the end of late-time acceleration takes place at a redshift $z \simeq -0.2$ and the universe recollapses at $z \simeq -0.36$.

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\textsuperscript{1}Corresponding author.
1 Introduction

Although General Relativity is the most successful theory that currently describes gravitation, it presents some intrinsic pathologies when one tries to construct a cosmological model from a proper theory of gravitation. In cosmology for instance, the $\Lambda$CDM model gives us important predictions concerning the evolution of the Universe and its current state [1]. However, let us assume that the initial conditions of our Universe were fixed when the early Universe emerged from the semi-Planckian regime and started its classical expansion. Evolving back such initial conditions using the classical Einstein field equations it results that our Universe is driven towards an initial singularity in high UV where the classical regime is no longer valid [2]. On the other hand, since 1998 observational data [3–6] are giving support to the highly unexpected assumption that our Universe is currently in a state of accelerated expansion. In order to explain this late-time acceleration phase cosmologists have been considering the existence of a new field — known as dark energy — that violates the strong energy condition in deep IR. Although the cosmological constant seems to be the simplest and most appealing candidate for dark energy, it poses a huge problem to quantum field theory on how to accommodate its observed value with vacuum energy calculations [7].

During recent decades theories of gravitation have been considered in order to solve such problems, by properly modifying General Relativity either in the high UV, or in the deep IR limit. In this context, for instance, bouncing models may circumvent the problem of the cosmological singularity, solve the flatness and horizon problems [8] and reproduce the power spectrum of primordial cosmological perturbations inferred by observations [9]. Different candidates for dark energy may also be proposed in the realm of modified theories of gravitation [10].

In an attempt to construct a quantum theory of gravitation, P. Hořava proposed a modified gravity theory by considering a Lifshitz-type anisotropic scaling between space and time at high energies [13]. In this context it has been shown [14–17] that higher order spatial curvature terms can lead to regular bounces in the early universe and also to a complex cosmological dynamics. This bounce feature is due to the presence of positive powers of the spatial curvature — in the potential of the theory — that engender nontrivial modifications of the dynamics in the high UV limit.

In this paper we intend to discuss an extension of such Hořava-Lifshitz bouncing models. This extension corresponds to also consider negative powers of the spatial curvature in the potential so that corrections in the deep IR limit are also obtained. From the theoretical point of view, we remark that such assumption characterizes a departure from Hořava-Lifshitz gravity which we intend to better address in a future analysis. Apart from such technical aspects, in this paper we perform a first phenomenological investigation of these features.
2 Field equations and the model

In the case of a 4-dimensional (1 + 3) spacetime, our basic assumption is that a preferred foliation of spacetime is a priori imposed. Therefore it is natural to work with the Arnowitt-Deser-Misner (ADM) decomposition of spacetime \[18\]

\[
ds^2 = N^2 dt^2 - (3) g_{ij} (N^i dt + dx^i)(N^j dt + dx^j) \tag{2.1}
\]

where \(N = N(t, x_i)\) is the lapse function, \(N_i = N_i(t, x_i)\) is the shift and \((3) g_{ij}(t, x_i)\) is the spatial geometry. Although the final action of the theory is not expected to be invariant under diffeomorphisms as in General Relativity, an invariant foliation preserving diffeomorphisms can be assumed. This is achieved if the action is invariant under time reparametrization together with time-dependent spatial diffeomorphisms, namely,

\[
t \to \tilde{t}(t), \quad x^i \to \tilde{x}^i(x^i, t). \tag{2.2}
\]

It follows that the only covariant object under spatial diffeomorphisms that contains one time derivative of the spatial metric is the extrinsic curvature \(K_{ij}\), defined as

\[
K_{ij} = \frac{1}{2N} \left( \frac{\partial g_{ij}}{\partial t} - \nabla_i N_j - \nabla_j N_i \right) \tag{2.3}
\]

where \(\nabla_i\) is the covariant derivative relative to the spatial metric \((3) g_{ij}\). Thus, to construct a general theory which is of second order in time derivatives, one needs to consider the quadratic terms \(K_{ij} K_{ij}\) and \(K^2\), where \(K\) is the trace of \(K_{ij}\). According to the previous assumptions, a preferred foliation provides enough gauge freedom that allows us to fix \(N_i = 0\). In order to simplify our analysis, we then propose the following action

\[
S = \int N \sqrt{(3) g} \left[ K_{ij} K^{ij} - \lambda K^2 - (3) R - U_{HL}(3) g_{ij}, N) - 2\kappa^2 \mathcal{L}_m \right] d^3 x \, dt \tag{2.4}
\]

where \((3) g\) is the determinant of the spatial metric \((3) g_{ij}\), \((3) R\) is the spatial Ricci scalar and \(\kappa^2\) is Einstein’s constant. \(\mathcal{L}_m\) is the lagrangian for the matter content of the model and \(\lambda\) is a constant which corresponds to a dimensionless running coupling \[13\]. Since in General Relativity the term \(K_{ij} K^{ij} - K^2\) is invariant under 4-dim diffeomorphisms we expect to recover the classical regime as \(\lambda \to 1\). It has been shown \[19\] that observational data from Type Ia Supernovae, Baryon Acoustic Oscillations, Cosmic Microwave Background and requirements of Big Bang Nucleosynthesis, furnish a constraint for the running parameter given by \(|\lambda - 1| \lesssim 0.02\). For practical purposes we are going to restrict ourselves to this domain in the remaining of the paper.

In general the potential \(U_{HL}(3) g_{ij}, N)\) can depend on the spatial metric and the lapse function due to the symmetry of the theory. It is clear that there are several invariant terms that can be included in \(U_{HL}\) — particular choices often result in different versions of the Hořava-Lifshitz gravity. However, in the following we intend to extend the Hořava-Lifshitz scenario by imposing that \(U_{HL}\) is a smooth function of \((3) R\) only, namely, \(U_{HL} = U_{HL}(3) R\). Finally \(\mathcal{L}_m\) is the Lagrangean density of the matter content of the model, which we take as noninteracting perfect fluids.

As the fundamental symmetry assumed provides enough gauge freedom to choose \(N = N(t)\) and \(N_i = 0\), we consider the case of a Friedmann-Lemaître-Robertson-Walker (FLRW) universe that in comoving coordinates \(x^i = (r, \theta, \phi)\) is expressed as

\[
ds^2 = N^2 dt^2 + (3) g_{ij} dx^i dx^j \tag{2.5}
\]
where \( t \) is the cosmological time,
\[
(3) g_{ij} = -a^2(t) \text{ diag} \left( \frac{1}{1-kr^2}, r^2, r^2 \sin^2 \theta \right),
\]
(2.6)
a\( (t) \) is the scale factor of the model and the parameter \( k \) is proportional to the curvature of the 3-dim spatial sections \( t = \text{const} \). The associated extrinsic curvature is given by
\[
K_{ij} = \frac{1}{2N} (3) \dot{g}_{ij} = -\frac{a\ddot{a}}{N} \text{ diag} \left( \frac{1}{1-kr^2}, r^2, r^2 \sin^2 \theta \right).
\]
(2.7)
From (2.4) we obtain the action of the model
\[
S = V_0 \int N a^3 \left( K_{ij} K^{ij} - \lambda K^2 - (3) R - U_{HL} - 2\kappa^2 L_m \right) dt,
\]
(2.8)
where \( V_0 \) is the spatial volume integral
\[
V_0 = \int \frac{r^2 \sin \theta}{\sqrt{1-kr^2}} drd\theta d\phi.
\]
The matter content of the model we take noninteracting dust and radiation, namely,
\[
L_m = \rho_m + \rho_r,
\]
(2.9)
where
\[
\rho_m = \rho_0 m \left( \frac{a_0}{a} \right)^3, \quad \rho_r = \rho_0 r \left( \frac{a_0}{a} \right)^4.
\]
(2.10)
The “0” subscript denotes the present epoch of our Universe. In order to simplify our analysis we will fix the natural normalization \( a_0 = 1 \). From eqs. (2.8) we then obtain
\[
S = V_0 \int L dt
\]
(2.11)
where
\[
L = \frac{3(1-3\lambda)}{N} a\ddot{a}^2 - Na^3 \left[ (3) R + U_{HL} + 2\kappa^2 (\rho_m + \rho_r) \right].
\]
(2.12)
By defining the canonical momentum
\[
p_a = \frac{\partial L}{\partial \dot{a}} = \frac{6(1-3\lambda)a\dot{a}}{N},
\]
(2.13)
the total action can be expressed as
\[
S = V_0 \int \left( \dot{a}p_a - N \mathcal{H} \right) dt,
\]
(2.14)
so that \( \delta S/\delta N = 0 \) results in the first integral of motion, the conserved Hamiltonian constraint
\[
\mathcal{H} = \frac{p_a^2}{12a(1-3\lambda)} + V(a) = 0,
\]
(2.15)
where

\[ V(a) = \frac{2\kappa^2 \rho_0 r}{a^2} + 2\kappa^2 \rho_0 m - 6ka + a^3 U_{HL}. \]  

(2.16)

As mentioned already there are several invariant terms that can be included in \( U_{HL} \). For the purposes of the present paper we will assume that \( U_{HL} \) is a smooth function of \( (3)R \) only, namely, \( U_{HL} = U((3)R) \). If we take into account that \( (3)R = -\frac{6k}{a^2} \), we see that the assumption of positive powers of \( (3)R \) in \( U_{HL} \) leads to UV corrections — this turns to be the case of Hořava-Lifshitz gravity \([14, 15]\) for bouncing cosmologies — while negative powers of \( (3)R \) lead to IR corrections. In the context of nonsingular cosmology, the core of this work is to extend the scenario of bouncing models in Hořava-Lifshitz gravity by considering terms in the potential such that corrections in the deep IR are also obtained. For that matter, we will also assume negative powers of \( (3)R \) in the potential \( U \) which may be connected to a late-time acceleration regime.

In a first analysis, we will assume here that the leading terms in the potential are

\[ U((3)R) = \alpha_0 \frac{1}{(3)R^2} + \alpha_1 \frac{1}{(3)R} + \alpha_2 (3)R^2 + \alpha_3 (3)R^3, \]  

(2.17)

where \( \alpha_i \) (\( i = 0, \ldots, 3 \)), are coupling constants. While the terms connected to \( \alpha_2 \) and \( \alpha_3 \) are due to UV corrections — they might emerge from typical Hořava-Lifshitz potentials — the terms linked to \( \alpha_0 \) and \( \alpha_1 \) are objects of extreme large scale corrections where the diffeomorphism covariance may be broken as well. Although there are no substantial constraints on the parameters \( \alpha_0, \alpha_1 \) and \( \alpha_3 \) up to date in the literature, the domain of the coupling constant \( \alpha_2 \) has been object of study in the Hořava-Lifshitz framework (see \([19, 20]\), for instance). However such constraints are dependent on different versions of Hořava-Lifshitz theories employed and we shall not be restricted on them. On the contrary, in this paper are going to restrict ourselves to (2.15)–(2.17) and introduce new constraints on the parametric space \((\alpha_0, \alpha_1, \alpha_2, \alpha_3)\) so that a concrete bouncing model with a late-time accelerated phase due to the spatial curvature may be obtained. We show that the late-time accelerated phase is connected to \( \alpha_1 \) while \( \alpha_0 \) probes the final fate of our model.

3 A concrete model from observational constraints

By defining the usual density parameters

\[ \Omega_{0m} = \frac{\rho_{0m}}{\rho^c_0}, \quad \Omega_{0r} = \frac{\rho_{0r}}{\rho^c_0}, \quad \Omega_{0k} = \frac{k}{a^2_0 H^2_0}, \]  

(3.1)

where \( \rho^c_0 \equiv 3H^2_0/\kappa^2 \), the Hamiltonian constraint (2.15)–(2.16), can be rewritten in the form

\[ \mathcal{H} = \frac{p^2_a}{12a(1 - 3\lambda)} + V(a) = 0 \]  

(3.2)

where

\[ V(a) = 12a^3 \times \frac{H^2_0}{2} \left( \frac{\Omega_{0m}}{a^4} + \frac{\Omega_{0r}}{a^4} - \frac{\Omega_{0k}}{a^2} + \frac{A_0 a^4}{216\Omega^2_{0k}} - \frac{A_1 a^2}{36\Omega_{0k}} + \frac{6A_2 \Omega^2_{0k}}{a^4} - \frac{36A_3 \Omega^3_{0k}}{a^6} \right). \]  

(3.3)
The coefficients $A_i$ are the respective coupling constants $\alpha_i$ rescaled:

$$A_0 = \frac{\alpha_0}{H_0^2}, \quad A_1 = \frac{\alpha_1}{H_0^2}, \quad A_2 = \alpha_2 H_0^2, \quad A_3 = \alpha_3 H_0^4. \quad (3.4)$$

It is worth noting that the term connected to $A_2$ (or $\alpha_2$) behaves like a radiation component.

From the Hamiltonian (3.2) we derive the dynamical system

$$\dot{a} = \frac{\partial H}{\partial p_a} = \frac{p_a}{6a(1 - 3\lambda)}, \quad \dot{p}_a = -\frac{\partial H}{\partial a} = \frac{p_a^2}{12a^2(1 - 3\lambda)} - \frac{dV}{da}. \quad (3.5)$$

Given the above equations we see that the structure of the phase space is organized by critical points $(a, p_a) = (a_i, 0)$, where $a_i$ are connected to the extrema of the potential $V(a)$. In fact, as we will see in the following, given the observational parameters, the phase space dynamics allow at least three critical points: two centers separated by a saddle.

In order to compare our Hamiltonian constraint to the first Friedmann equation for the $\Lambda$CDM model, it is useful to rewrite (3.2)–(3.3) as

$$\frac{1}{2} \left( \frac{p_a}{6a(3\lambda - 1)} \right)^2 + V(a) = 0 \quad (3.6)$$

where

$$V(a) = \frac{H_0^2 a^2}{3\lambda - 1} \left( \Omega_{0k} - \frac{A_0 a^4}{216\Omega_{0k}^2} + \frac{A_1 a^2}{36\Omega_{0k}} - \frac{6A_2 \Omega_{0k}^2}{a^4} + \frac{36A_3 \Omega_{0k}^3}{a^6} - \frac{\Omega_{0m}}{a^3} - \frac{\Omega_{0r}}{a^4} \right), \quad (3.7)$$

and $H \equiv \dot{a}/a$.

We now proceed to feed our model with observational parameters. In (3.7) we see explicitly that all the corrections in our model emerge from the assumption of a nonvanishing spatial curvature. As Planck data [21] still leave some room for curvature, namely,

$$\Omega_{0k} = 0.001 \pm 0.002, \quad (3.8)$$

the bounce condition $A_3 \Omega_{0k}^3 > 0$ may be satisfied for a positive or negative spatial curvature. It is worth mentioning that apart from the bounce, $A_3$ is also related to a shear component for the case of anisotropic universes. In fact, it has been shown [17] that when one considers a general Bianchi IX model with three scale factors in the framework of Hořava-Lifshitz gravity, the evolution of the anisotropy parameter shows a relative large amplification when orbits in the phase space visit a neighborhood of the bounce in high UV, where $A_3$ dominates de dynamics.

Taking into account the Planck data [21], we fix:

$$\Omega_{0m} = 0.315, \quad \Omega_{0r} = 10^{-5}, \quad H_0 = 67.4 \text{ km/s/Mpc}. \quad (3.9)$$

Furthermore, current observations [21, 22] give support to the following constraints: (i) $V(a = 1) = -H_0^2/2$; (ii) the deceleration parameter is given by $q_0 = -(a\ddot{a}/\dot{a}^2)|_0 \simeq -0.54$; (iii) the predicted value of the scale factor $a_e$ — at the end of the matter era — corresponds to a redshift $z \simeq 0.4$ (or, $a_e \simeq 0.7$). The latter constraint implies that a bouncing epoch followed by a decelerated phase with a graceful exit to a late-time accelerated regime can only be obtained as long as the potential $V(a)$ has at least two local extrema — one local minimum $a_1$ and one local maximum $a_e$ — so that $a_1 < a_e$ and $V(a_e) < 0$. 


Taking into account (3.9), it can be easily seen that conditions (i), (ii) and (iii) allows one to write $A_0$, $A_1$ and $A_2$ as simple functions of $(A_3, \Omega_{0k}, \lambda)$. In this sense observations constrain our model to the 3-dimensional parametric space $(A_3, \Omega_{0k}, \lambda)$. As mentioned before, we are interested in bouncing configurations so that we are restricted to the domain $A_3 \Omega_{0k}^2 > 0$. Bearing this condition in mind, we now proceed to examine the sign of $A_0$ in order to probe the final fate of our model. In fact, by a simple inspection of the potential (3.7), we see that the leading term in the deep IR is connected to $A_0$. For $A_0 > 0$ our model expands forever. However, when $A_0 < 0$ an inevitable collapse takes place instead. As the classical regime is expected to be recovered as $\lambda \rightarrow 1$, we start by fixing $\lambda = 1$. In figure 1 we plot the domain of the parametric space $(A_0, A_3, \Omega_{0k}, \lambda = 1)$ so that conditions (i), (ii) and (iii) are automatically satisfied. In the left (right) panel we consider the case $\Omega_{0k} < 0$ ($\Omega_{0k} > 0$) for which the bounce condition reads $A_3 < 0$ ($A_3 > 0$). These plots were generated considering the whole range (3.8) allowed by Planck data [21]. In both plots we see that $A_0 < 0$ so that a decelerated phase is predicted after the late-time accelerated regime and the universe inexorably recollapses. It can be numerically shown that this feature is maintained for any value of $\lambda$ in the domain $|\lambda - 1| \lesssim 0.02$ predicted by [19]. In order to better understand this behaviour, it can be easily seen that condition (iii) furnishes

$$A_0 \approx -238.095 \Omega_{0k}^2 (-0.6274 + 1.0667\lambda) - 238.095 \Omega_{0k}^3 (1.143 - 186.813 A_3 \Omega_{0k}^2). \quad (3.10)$$

For $\lambda = 1$, we have just shown that $A_0 < 0$ within the range allowed by Planck data [21] as long as the bounce condition is satisfied. If we now remember that $|\Omega_{0k}| \lesssim 10^{-3}$, we see that the first (and leading) term in (3.10) is still negative for $|\lambda - 1| \lesssim 0.02$. Therefore, the overall behaviour expected for our model is a universe in eternal recurrence regardless the numerical value of $\lambda$ in the domain $|\lambda - 1| \lesssim 0.02$. In order to simplify our analysis, we are going to restrict ourselves to $\lambda = 1$ in the remaining of the paper. As an illustration, we fix

**Figure 1.** The domain of $A_0$ — as a function of $A_3$ and $\Omega_{0k}$ for $\lambda = 1$ — given the observational constraints (i), (ii) and (iii), see text. In both plots we see that $A_0 < 0$ so that a decelerated phase is predicted after the late-time accelerated regime and the universe inexorably recollapses. As we are considering only bouncing configurations, the overall behaviour of our model is a universe in eternal recurrence within the range of $\Omega_{0k}$ allowed by Planck data.
The behaviour of the potential $V(a)$ (left panel) and the phase space (right panel). For the purpose of illustration, here we have fixed $A_3 = 2 \times 10^5$, $\Omega_{0k} = 0.001$ and $H_0^2 = 1$. For $p_a > 0$ we characterize three distinct regions: (i) the point $a = a_1$ sets up the transition from an early universe to a decelerated radiation/matter era; (ii) $a = a_e$ defines the transition to a late-time accelerated regime; (iii) $a = a_2$ determines the end of late-time acceleration and the universe starts its own recollapse towards an eternal recurrence configuration.

$A_3 = 2 \times 10^5$ and $\Omega_{0k} = 0.001$. In figure 2 we show the behaviour of the potential $V$ (left panel) and the phase space (right panel). In order to qualitatively grasp their overall shape, we have used the normalization $H_0^2 \equiv 1$.

Restoring the observational value of the Hubble constant according to [21], we now proceed to put a better limit on the bounce parameter $A_3$ — as expected, the smaller is $|A_3|$, smaller is the bounce scale. For that matter we are going to restrict ourselves to the case $\Omega_{0k} = 0.001$. Considering the evolution of quantum cosmological perturbations, it has been shown [23, 24] that in order to obtain amplitudes and wavelength spectra compatible with CMB data, one must satisfy the condition $l_c \gtrsim 10^3 \times l_p$, where $l_c \propto 1/\sqrt{R}$ is the curvature scale at the bounce and $l_p$ is the Planck length. For that matter we infer the lower limit $A_3 \gtrsim 10^{-52}$. On the other hand, if one intends not to spoil the predicted high-redshift events like the cosmic neutrino background [25] — at a redshift $z \simeq 10^{10}$ — we fix the upper limit $A_3 \lesssim 10^{-14}$. Therefore, for every value of $A_3$ in the domain

$$10^{-52} \lesssim A_3 \lesssim 10^{-14},$$

we obtain a phase space orbit with the same shape of that one depicted in the right panel of figure 2. It is worth noticing that its shape is maintained for 38 orders of magnitude. Given the physical range of interest (3.11) of $A_3$, from (3.9) and conditions (i), (ii) and (iii), we obtain:

$$A_0 \simeq -0.0000944641, \quad A_1 \simeq -0.037594, \quad A_2 \simeq 13174.2.$$  \hspace{1cm} (3.12)

As $A_0 < 0$ and $A_1 < 0$, we obtain a concrete cosmological model with a late-time accelerated regime in eternal recurrence, as expected. In order to better illustrate this behaviour, let us consider the Friedmann equation as a function of the redshift $z$:

$$H^2 + \mathcal{U}(z) = 0$$  \hspace{1cm} (3.13)
Figure 3. $H$ as a function of the redshift for $A_3 = 10^{-20}$. In the left panel we show the behaviour of $H$ in a neighbourhood our present era until the universe recollapses at $z \simeq -0.36$. In the right panel we show the behaviour of $H$ in a neighbourhood of the bounce. Here we have fixed $H_0 = 7.28 \times 10^{-29}\text{cm}^{-1}$ in units $c = G = 1$, so that $\alpha_0 \simeq -7.6 \times 10^{-503}\text{GeV}^{-6}$, $\alpha_1 \simeq -3.2 \times 10^{-334}\text{GeV}^{-4}$, $\alpha_2 \simeq 1.4 \times 10^{170}\text{GeV}^{2}$, $\alpha_3 \simeq 1.1 \times 10^{312}\text{GeV}^{4}$.

where

$$U(z) = H_0^2 \left[ \Omega_{0k}(1+z)^2 - \frac{A_0}{216(1+z)^2\Omega_{0k}} + \frac{A_1}{36(1+z)^2\Omega_{0k}} - 6A_2(1+z)^4\Omega_{0k}^2 ight. \left. + 36A_3(1+z)^6\Omega_{0k}^3 - \Omega_{0m}(1+z)^3 - \Omega_r(1+z)^4 \right]. \quad (3.14)$$

In figure 3 we show the behaviour of $H$ as a function of the redshift for $A_3 = 10^{-20}$. In this context, the domain of breaking diffeomorphism invariance — where the model becomes invariant over foliations which preserve diffeomorphism — is given by $z \lesssim -0.2$ and $10^{10} \lesssim z$. It is worth noting that the end of late-time acceleration should take place at a redshift $z \simeq -0.2$ and universe recollapses at $z \simeq -0.36$. Converting both epochs into times, we obtain that the end of late-time acceleration takes place about two billion years from now, before the universe recollapses in 10 billion years from its present state.

4 Conclusions

In this paper we construct a closed FLRW universe by imposing an anisotropic scaling between space and time at extremely high and low energies. From the Hamiltonian constraint we obtain a first integral corresponding to a modified Friedmann equation which contain additional terms arising from curvature. Such terms implement nonsingular bounces in the early Universe together with a late-time acceleration regime. The matter content of the model is a nonrelativistic pressureless perfect fluid and radiation. Considering the breaking of covariance diffeomorphism also at extreme large scales, we introduce a higher order term to probe the final fate of the universe. Given observational parameters we constrain the model so that an eternal recurrence regime is inexorable. According to observational parameters the model predicts that the end of late-time acceleration should take place at a redshift $z \simeq -0.2$ (or in 2 Gyrs) and the universe recollapses at $z \simeq -0.36$ (in 10 Gyrs). It is worth to remark that this feature is maintained for a domain of 38 orders of magnitude of the parametric space.
Our treatment in the paper is based strongly on the Hamiltonian formulation, with a conserved Hamiltonian constraint. By the use of canonical variables we were able to make a global examination of the phase space so that appropriate critical points provide the bounce and a late-time acceleration regime. Both epochs are due to the breaking of diffeomorphism covariance which occurs at redshift $z \lesssim -0.2$ and $10^{10} \lesssim z$, respectively. In such domains the model becomes invariant over foliations which preserve diffeomorphism instead.

The breaking of diffeomorphism invariance has been a topic of interest during the last decade. In fact, several authors have argued that such a feature is relevant for theories in which General Relativity is an emergent phenomenon from a more fundamental theory. By comparing our model to the Hořava-Lifshitz scenario — the most investigated framework in which diffeomorphism invariance can be broken — we would be in a position to better understand the origin of the terms which provide the bounce together with late-time acceleration in this paper.

In the framework of Hořava-Lifshitz, although the projectable version seems to be plagued with an extra scalar degree of freedom which is either classically unstable or a ghost in the IR [26], there is still some room for a healthy version of a proper theory of gravitation in which diffeomorphism covariance can be broken. In fact, in the non-projectable version of Hořava-Lifshitz one may also include invariant contractions of $\partial \ln N / \partial x^i$ in the potential $U$. Connected to the lowest order invariant $\partial_i \ln N \partial^i \ln N$, there is a coupling parameter $\sigma$ which defines a “safe” domain of the theory [26, 27]. Although in this case there is also an extra scalar degree of freedom when one linearizes the theory in a Minkowski background, for $0 < \sigma < 2$ this mode might not be a ghost nor classically unstable (as long as detailed balance is not imposed). Notwithstanding the non-projectable version also to exhibit a strong coupling [27]–[29], it has been argued that its scale is too high to be phenomenologically accessible from gravitational experiments [26].

Although the terms connected to the coupling constants $\alpha_2$ and $\alpha_3$ are genuine Hořava-Lifshitz potential corrections — despite its versions — terms such as those linked to $\alpha_0$ and $\alpha_1$ are not. In fact, the latter might turn the Hořava-Lifshitz framework nonrenormalizable by power-counting. As a future perspective we aim to better understand, from the theoretical point of view, what are the physical consequences/issues of such terms, characterizing this departure from Hořava-Lifshitz gravity.

Finally we should mention that we have not examined the question of perturbations in the eternal FLRW cosmological model in the framework of the above Horava-Lifshitz corrections where the gravitational part of the Lagrangean, containing higher order curvature terms, implement infinitely many bounces in the dynamics of the background universe. These perturbations would correspond to perturbations in the matter content of the model and/or in the gravitational part of the Lagrangean which contains higher order curvature terms. The exam of the stability of the perturbations is a fundamental and involved question, in particular concerning the infinite recurrence of the background model, demanding a careful numerical treatment which is beyond the object of the present paper and will be considered in a future work. Our experience with such a problem comes from the numerical exam of perturbations made for one bounce models in braneworld cosmologies with a deSitter bulk or with a perturbed bulk, where the amplitudes of the perturbations remain sufficiently small and bounded relative to the background values (cf. [30]). For eternal universes some of these features may be maintained only for early times (typically of the order of the first bounce) and eternal solutions may be highly unstable configurations considering the background model of this paper. Concerning the present problem in a distinct framework (an eternal FLRW
universe in the realm of a Horava-Lifshitz theory) preliminary numerical tests indicate that scalar perturbations may remain sufficiently small and bounded at least for a certain finite number of bounces and for a parameter subdomain of the domain \((A_0, A_3, \Omega_{0k})\) of figure 1, within the range allowed by Planck data.

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