BRST approach to Lagrangian construction for fermionic higher spin fields in AdS space

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Abstract

We develop a general gauge-invariant Lagrangian construction for half-integer higher spin fields in the AdS space of any dimension. Starting with a formulation in terms of an auxiliary Fock space, we obtain closed nonlinear symmetry algebras of higher spin fermionic fields in the AdS space and find the corresponding BRST operator. A universal procedure for constructing gauge-invariant Lagrangians describing the dynamics of fermionic fields of any spin is developed. No off-shell constraints for the fields and gauge parameters are imposed from the very beginning. It is shown that all the constraints determining an irreducible representation of the AdS group arise as a consequence of the equations of motion and gauge transformations. As an example of the general procedure, we derive gauge-invariant Lagrangians for massive fermionic fields of spin 1/2 and 3/2 containing the complete set of auxiliary fields and gauge symmetries.

1 Introduction

Problems of higher spin field theory have attracted much attention for a long time due to the hope of finding new possibilities and approaches to the unification of the fundamental interactions. Higher spin field theory is closely related to superstring theory, which operates with an infinite tower of bosonic and fermionic higher spin fields, including massless and massive
ones. A Lagrangian formulation for an interacting higher spin field theory is one (perhaps the last) of the unsolved general problems of classical field theory (see, e.g., the reviews [1]).

The current studies in higher spin field theory concern the aspects of Lagrangian formulation in various dimensions, searches for supersymmetric generalizations, finding a correspondence with superstring and \( M \) theories and a construction of interacting Lagrangians (see [2]–[9] for massive and [10]–[18] for massless higher spin theories).

It is well-known that a space of constant curvature, in particular, the AdS space, is the simplest non-trivial background providing a consistent propagation of higher spin fields (see, e.g., [2,19]). We point out two attractive features of higher spin theory in the AdS space. First, the radius of the AdS space ensures the presence of a natural dimensional parameter for the accommodation of compatible self-interactions for massless higher spin fields [20, 21]. Second, higher spin fields in the AdS space are closely related to the conjecture [22] on the AdS/CFT correspondence between the conformal \( N = 4 \) SYM theory and the superstring theory on the \( AdS_5 \times S_5 \) Ramond–Ramond background [7].

The study of a Lagrangian formulation for massive higher spin field theories was initiated by the pioneering works of Fierz–Pauli and Singh–Hagen [23] for theories in the four-dimensional Minkowski space. These works demonstrated a specific feature of higher spin field theories: all such theories include, apart from the basic fields with a given spin, also some auxiliary fields with lesser spins, which provide a compatibility of the Lagrangian equations of motion with the constraints determining an irreducible representation of the Poincare group. Attempts to construct Lagrangian descriptions of free higher spin field theories in curved spaces and derive higher spin interactions have resulted in consistency problems which generally remain open in spite of numerous efforts.

The present work is devoted to a derivation of general gauge-invariant Lagrangian for massive and massless fields of any half-integer spin in AdS spaces of arbitrary dimensions. Our approach is based on use of the BRST–BFV construction [24] (see also the reviews [25]), which was initially developed for the quantization of gauge theories. Following a tradition accepted in string theory and in higher spin field theory, we further refer to this construction as the BRST construction, and to the corresponding BFV charge, as the BRST charge. The general application of the BRST construction to higher spin field theory consists in three steps. First, the conditions determining the representation with a given spin are regarded as first-class constraints. Second, using the algebra of these constraints, one constructs the BRST charge. Third, a higher spin field Lagrangian is constructed in terms of the BRST charge in such a way that the corresponding equations of motion reproduce the initial constraints. The final step is of fundamental importance and ensures the correctness of the Lagrangian construction. We emphasize that this approach automatically implies a gauge-invariant Lagrangian. One can expect that in the case of massive theories a Lagrangian resulting from the BRST construction should contain a number of \( \text{St"uckelberg} \) fields.

A derivation of Lagrangians for massless bosonic higher spin fields on a basis of the BRST approach has been considered in [26,27] for the flat space and in [28] for the AdS space. A generalization of the BRST approach to Lagrangians of massless fermionic higher spin fields in flat spaces of arbitrary dimensions has been given in [29]. An interaction vertex for massless bosonic higher spin fields in flat and AdS spaces has been studied within the BRST approach in [30]. The first implementation of the BRST construction for the derivation of Lagrangians for massive higher spin fields has been given in our papers for bosonic [31] and fermionic [32] models in Minkowski spaces of arbitrary dimensions and for bosonic models in AdS spaces of arbitrary dimensions [33]. Thus, the BRST approach presents a universal generating method of constructing Lagrangian formulations for higher spin fields.

In this paper, we develop a gauge-invariant approach to a Lagrangian construction for totally
symmetric fermionic higher spin fields in the AdS space of any dimension\footnote{For the sake of completeness, we note that a Lagrangian formulation for massive fields of an arbitrary half-integer spin in the AdS space subject to algebraic constraints has been recently suggested in [34].}. The conditions determining an irreducible representation of the AdS group with a given spin on tensor-spinor fields are reformulated as operator constraints in an auxiliary Fock space. These constraints form a nonlinear superalgebra with a central bosonic charge. Such an algebra has a more complex structure in comparison with its counterpart for bosonic higher spin fields in the AdS space [33], due to the presence of non-vanishing operator structure functions resolving the Jacobi identity, and therefore the construction of the corresponding BRST charge faces a problem. Some aspects of the BRST construction for bosonic nonlinear constraint algebras have been discussed in [35, 36]. We find a solution of the BRST construction for a nonlinear superalgebra in the theory under consideration and obtain a Lagrangian which reproduces the initial constraints as a consequence of the equations of motion.

The paper is organized as follows. In Section 2, we examine a closed operator superalgebra based on the constraints that determine an irreducible representation of the AdS group with a half-integer spin. In Section 3, we find a superalgebra of the additional parts for the superalgebra of a modified set of the initial operators, obtained by a linear transformation of the initial constraints, and then realize its representation in terms of new (extra) creation and annihilation operators. In Section 4, we calculate a deformed nonlinear superalgebra of constraints enlarged by the additional parts of the modified constraints. A similar construction for fermionic fields in flat space has been given in [29]. Next, we construct a BRST operator corresponding to the superalgebra of the modified enlarged constraints. The derivation of an action and of a sequence of reducible gauge transformations describing the propagation of a fermionic field of an arbitrary spin in the AdS space is realized in Section 5. In Section 6, we prove that the constructed action reproduces the correct conditions for the field that determine an irreducible representation of the AdS group with a fixed half-integer spin. In Section 7, we illustrate the procedure by constructing gauge-invariant Lagrangians for the fields of spin 1/2 and 3/2. In Conclusion, we summarize the results of the work and discuss some open problems.

In addition to the conventions of Refs. [32, 33], we use the notation $\varepsilon(A)$, $gh(A)$ for the respective values of Grassmann parity and ghost number of a quantity $A$, and denote by $\{A, B\}$ the supercommutator of quantities $A, B$, which in the case of definite values of Grassmann parity is given by $\{A, B\} = AB - (-1)^{\varepsilon(A)\varepsilon(B)}BA$.

2 Auxiliary Fock space for higher spin fields in AdS space-time

The massive half-integer spin $s = n + \frac{1}{2}$ representations of the AdS group are realized in the space of totally symmetric tensor-spinor fields $\Phi_{\mu_1...\mu_n}(x)$, the Dirac index being suppressed, satisfying to the conditions (see e.g. [4])

\begin{align}
[i\gamma^\mu \nabla_\mu - r^2(n + \frac{d}{2} - 2) - m] \Phi_{\mu_1\mu_2...\mu_n}(x) &= 0, \\
\gamma^\mu \Phi_{\mu\mu_2...\mu_n}(x) &= 0.
\end{align}

where $r = \frac{R}{d(d-1)}$, with $R$ being the scalar curvature of the space-time. We use the metric with the mostly minus signature, and the Dirac’s matrices obey the relation

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}.$$
In order to avoid an explicit manipulation with the indices, it is convenient to introduce an auxiliary Fock space \( \mathcal{H} \) generated by creation and annihilation operators with tangent space indices \((a, b = 0, 1, \ldots, d - 1)\)

\[
[a_a, a_b^+] = -\eta_{ab}, \quad \eta_{ab} = \text{diag}(+, -, \ldots, -).
\]  

(4)

An arbitrary vector in this Fock space has the form

\[
|\Phi\rangle = \sum_{n=0}^{\infty} \Phi_{a_1 \ldots a_n}(x) a^{a_1} \ldots a^{a_n} |0\rangle = \sum_{n=0}^{\infty} \Phi_{\mu_1 \ldots \mu_n}(x) a^{+\mu_1} \ldots a^{+\mu_n} |0\rangle,
\]

(5)

where \( a^{+\mu}(x) = e^\mu_a(x) a^{+a}, a^\mu(x) = e^\mu_a(x) a^a; e^\mu_a(x) \) being the vielbein. It is evident that

\[
[a_\mu, a^\nu_\nu] = -g_{\mu\nu}.
\]

(6)

We refer to the vector (5) as the basic vector. The fields \( \Phi_{\mu_1 \ldots \mu_n}(x) \) are the coefficient functions of the vector \(|\Phi\rangle\) and its symmetry properties are stipulated by the symmetry properties of the product of the creation operators. We also assume the standard relation

\[
\nabla_\mu e^a_\nu = \partial_\mu e^a_\nu - \Gamma^a_\lambda_\mu e^a_\lambda + \omega^a_\lambda_\mu e^\lambda_\nu = 0.
\]

(7)

We intend to realize relations (1), (2) as certain constraints on the vector \(|\Phi\rangle\) (5). To this end, we define an operator \( D_\mu \) acting on the vectors \(|\Phi\rangle\),

\[
D_\mu = \partial_\mu - \omega^{ab}_\mu (a^+_a a^+_b - \frac{1}{4} \gamma_{ab}), \quad \partial_\mu |0\rangle = 0, \quad \gamma_{ab} = \frac{1}{2} (\gamma_a \gamma_b - \gamma_b \gamma_a),
\]

and operators

\[
\tilde{t}_0 = i \gamma^\mu D_\mu - m - r^2 (g_0 - 2), \\
\tilde{t}_1 = \gamma^\mu a_\mu, \\
g_0 = -a^{+\mu} a^\mu + \frac{d}{2}.
\]

(8)

(9)

(10)

(11)

We can see that the constraints

\[
\tilde{t}_0 |\Phi\rangle = \tilde{t}_1 |\Phi\rangle = 0
\]

(12)

for the basic vector (5) are equivalent to equations (1), (2), with each component \( \Phi_{\mu_1 \ldots \mu_n}(x) \) in (5) subject to (1), (2), and therefore they describe a field of spin \( n + 1/2 \).

Because of the fermionic nature of equations (1), (2) with respect to the standard Grassmann parity, and due to the bosonic nature of the operators \( \tilde{t}_0, \tilde{t}_1: \varepsilon(\tilde{t}_0) = \varepsilon(\tilde{t}_1) = 0 \), in order to equivalently transform these operators into fermionic ones, we now introduce a set of \( d + 1 \) Grassmann-odd “gamma-matrix-like objects” \( \tilde{\gamma}^\mu, \tilde{\gamma} \) subject to

\[
\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2g^{\mu\nu}, \quad \{\tilde{\gamma}^\mu, \tilde{\gamma}\} = 0, \quad \tilde{\gamma}^2 = -1
\]

(13)

and related to the conventional gamma-matrices as follows:

\[
\gamma^\mu = \tilde{\gamma}^\mu \tilde{\gamma}.
\]

(14)

\footnote{For more details, see [33].}

\footnote{For more details, see [32].}
In terms of these gamma-matrices, we define the Grassmann-odd constraints
\begin{equation}
\bar{t}_0' = -i\tilde{\gamma}^\mu D_\mu + \tilde{\gamma} \left( m + r^{\frac{4}{3}} (g_0 - 2) \right), \quad t_1' = \tilde{\gamma}^\mu a_\mu, \tag{15}
\end{equation}
related to (9), (10) as follows:
\begin{equation}
\bar{t}_0 = -\tilde{\gamma}t_0, \quad t_1 = \tilde{\gamma}t_1. \tag{16}
\end{equation}

In order to obtain a Hermitian BRST operator, being the central object of the Lagrangian construction in the BRST approach, we need a set of first-class constraints which must be invariant with respect to Hermitian conjugation and must form a superalgebra with respect to supercommutator multiplication. To this end, we define an odd scalar product:
\begin{equation}
\langle \tilde{\Psi} | \Phi \rangle = \int d^d x \sqrt{|g|} \sum_{m,k=0}^{\infty} \langle 0 | a^{\mu} \ldots a^{\nu} \tilde{\gamma}_0 \Phi_{\mu_1 \ldots \nu_k} (x) \tilde{\gamma}_0 \Phi_{\mu_1 \ldots \nu_k} (x) a^{+\mu_1} \ldots a^{+\mu_\nu} | 0 \rangle. \tag{17}
\end{equation}
The operators $\bar{t}_0', t_1$ (15) and $t_1^+ = \tilde{\gamma}^\mu a_\mu^+$, being the Hermitian conjugate of $t_1$ with respect to the scalar product (17), generate an operator superalgebra with the central charge $\tilde{m} = (m - 2r^{\frac{4}{3}})$ and the following set of operators:
\begin{align}
\bar{t}_0' &= -i\tilde{\gamma}^\mu D_\mu + \tilde{\gamma} \left( m + r^{\frac{4}{3}} (g_0 - 2) \right), \tag{18} \\
t_1' &= \tilde{\gamma}^\mu a_\mu, \tag{19} \\
l_1' &= -ia^\mu D_\mu, \tag{20} \\
l_2' &= \frac{1}{2} a^\mu a_\mu, \tag{21} \\
g_0 &= -a^\mu a_\mu + \frac{d}{2}, \tag{22}
\end{align}
which is invariant under Hermitian conjugation. The operators (18–23) form an algebra given by Table 1 where
\begin{align}
[\bar{t}_0', t_1^+] &= 2t_1^+ + \tilde{\gamma} r^{\frac{4}{3}} t_1^+, \tag{24} \\
[\bar{t}_0', t_1] &= 2l_1 - \tilde{\gamma} r^{\frac{4}{3}} t_1, \tag{25} \\
[t_1^+, l_1] &= [l_1^+, t_1] = \bar{t}_0' - \tilde{\gamma} \left( \tilde{m} + r^{\frac{4}{3}} g_0 \right), \tag{26} \\
[\bar{t}_0', l_1] &= \tilde{\gamma} \left( 2t_1^+ l_2 + g_0 t_1 - \frac{1}{2} l_1 \right) - \tilde{\gamma} r^{\frac{4}{3}} l_1, \tag{27} \\
[l_1^+, l_0'] &= \tilde{\gamma} \left( 2t_1^+ l_2 + \frac{d}{2} g_0 t_1 - l_1 \right) - r^{\frac{4}{3}} \tilde{m} t_1, \tag{28} \\
[t_1, \bar{t}_0'] &= \tilde{\gamma} \left( 4t_1^+ l_2 + 2g_0 t_1 - t_1 \right) - r^{\frac{4}{3}} \tilde{m} t_1, \tag{29} \\
[l_1, \bar{t}_0'] &= 4r (t_1^+ l_2 + t_1^+ g_0) + 2r^{\frac{4}{3}} \tilde{m} t_1, \tag{30} \\
[l_0', l_1] &= 4r (t_1^+ l_2 + g_0 l_1) + 2r^{\frac{4}{3}} \tilde{m} l_1, \tag{31} \\
[l_0', t_1^+] &= 4r (t_1^+ l_2 + l_1^+ g_0) + 2r^{\frac{4}{3}} \tilde{m} l_1, \tag{32} \\
[l_2, \bar{t}_0'] &= 4r^{\frac{2}{3}} (r^{\frac{2}{3}} (1 + g_0) + \tilde{m}) l_2, \tag{33} \\
[l_0', l_2] &= 4r^{\frac{2}{3}} l_2 (r^{\frac{4}{3}} (1 + g_0) + \tilde{m}), \tag{34} \\
[l_1, l_1^+] &= \bar{t}_0 - \tilde{m}^2 - 2r^{\frac{4}{3}} \tilde{m} g_0 + r (r^{\frac{2}{3}} t_1^+ t_1 - \frac{1}{2} g_0 - 4l_1^+ l_2). \tag{35}
\end{align}
We call this algebra the massive half-integer higher spin symmetry algebra in the AdS space.
The method of Lagrangian construction requires an enlarging of the initial constraints \( o_i \), \( \{\vec{p}_0, t_1 \in \{o_i\}\} \), so that the enlarged Hermitian operators contain arbitrary parameters and the set of enlarged operators form an (super)algebra. A procedure of constructing of these enlarged constraints \( \tilde{O}_i = o_i + o_i' \) for the operators \( o_i \) is considerably simplified if the initial operators \( o_i \) (super)commute with its additional parts \( o_i' \); \( [o_i, o_i'] = 0 \), defined in the auxiliary operator space. In this case we can apply the method elaborated in [33]. The such realization of the enlarged first-class constraints \( \tilde{O}_i \), by means of an additive composition of the quantities \( o_i \) with additional parts \( o_i' \), requires a transformation of \( o_i \) into an equivalent set of constraints \( \tilde{o}_i \) which do not contain the \( \tilde{\gamma} \)-matrix, because the set \( o_i' \) will contain this object by construction. Thus, in order to simplify the subsequent calculations, we choose the operators \( \tilde{o}_i \) related by a nondegenerate linear transformation

\[ \tilde{o}_i = U_j \tilde{o}_j \]

with the initial constraints reproducing relations (1), (2),

\[ t_0 = -i\tilde{\gamma}^\mu D_\mu, \]
\[ l_0 = g^{\mu\nu}(D_\nu D_\mu - \Gamma^\sigma_{\mu\nu}D_\sigma) - r \left( g_0 + t_1^+ t_1 + \frac{d(d-3)}{4} \right). \]

The other constraints coincide with the initial ones.

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4Note that the case of a two-parametric set of constraints determined by the operator \( \tilde{l}_0 = -i\tilde{\gamma}^\mu D_\mu + \alpha_1\tilde{\gamma}\tilde{m} + \alpha_2\tilde{\gamma}^\perp g_0 \), analogous to the consideration for bosonic fields in the AdS space [33], from which it would be possible to construct a Lagrangian that reproduces Eq. (12) for any values \( \alpha_1, \alpha_2 \), can be realized by means of the above nondegenerate transformation of the superalgebra of the initial constraints.
The operators $\tilde{\partial}_i$ given by (19)–(22), (36), (37) with the central charge $\tilde{m}$ form a superalgebra given by Table 2, where

$$\begin{bmatrix} \downarrow, \rightarrow \end{bmatrix} \begin{array}{cccccccccc} t_0 & t_1 & t^+_1 & l_0 & l_1 & l^+_1 & l_2 & l^+_2 & g_0 & \tilde{m} \\ t_0 & -2l_0 & 2l_1 & 2l^+_1 & 0 & (38) & -41 & 0 & 0 & 0 \\ t_1 & 2l_1 & 4l_2 & -2g_0 & (39) & 0 & -t_0 & 0 & -t^+_1 & t_1 \\ t^+_1 & 2l^+_1 & -2g_0 & 4l^+_2 & -41 & t_0 & 0 & t_1 & 0 & -t^+_1 \\ l_0 & 0 & -41 & 42 & 0 & -40 & (43) & 0 & 0 & 0 \\ l_1 & -41 & 0 & -t_0 & 40 & 0 & (44) & 0 & -l^+_1 & l_1 \\ l^+_1 & 41 & t_0 & 0 & -44 & -41 & 0 & l_1 & 0 & -l^+_1 \\ l_2 & 0 & 0 & -t_1 & 0 & 0 & -l_1 & 0 & g_0 & 2l_2 \\ l^+_2 & 0 & t^+_1 & 0 & 0 & l^+_1 & 0 & -g_0 & 0 & -2l^+_2 \\ g_0 & 0 & -t_1 & t^+_1 & 0 & -l_1 & l^+_1 & -2l_2 & 2l^+_2 & 0 \\ \tilde{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Table 2: The algebra of the modified initial operators.

The operators $\tilde{\partial}_i$ given by (19)–(22), (36), (37) with the central charge $\tilde{m}$ form a superalgebra given by Table 2, where

$$\begin{align} [t_0, l_1] &= r(2t^+_1l_2 + g_0t_1 - \frac{1}{2}t_1), \\
[t_1, l_0] &= r(4t^+_1l_2 + 2g_0t_1 - t_1), \\
[l_1, l_0] &= r(4l^+_1l_2 + 2g_0l_1 - l_1), \\
[l^+_1, t_0] &= r(2l^+_1t_1 + t^+_1g_0 - \frac{1}{2}t^+_1), \\
[l_0, t^+_1] &= r(4l^+_2t_1 + 2t^+_1g_0 - t^+_1), \\
[l_0, l^+_1] &= r(4l^+_2l_1 + 2l^+_1g_0 - l^+_1), \\
[l_1, l^+_1] &= l_0 + r(g_0^2 - \frac{1}{2}g_0 - 4l^+_2l_2 + \frac{3}{2}t^+_1t_1). \end{align}$$

In terms of the operators $\tilde{\partial}_i$, equations (12) have the form

$$[t_0 + \tilde{\gamma}m + \tilde{\gamma}r\frac{1}{2} (g_0 - 2)] |\Phi\rangle = 0, \quad t_1 |\Phi\rangle = 0.$$ 

In what follows, we shall demonstrate the construction of Lagrangians by using the BRST approach developed for bosonic fields in the AdS space in [33] so as to reproduce equations (12), or, equivalently, (45). According to our approach, we have to extend the operators $\tilde{\partial}_i$ of the algebra given by Table 2 by additional parts $\partial'_i$: $\tilde{\partial}_i \rightarrow \tilde{\partial}_i = \tilde{\partial}_i + \partial'_i$, so that 1) $\tilde{\partial}_i$ must be in involution $[\tilde{\partial}_i, \tilde{\partial}_j] \sim \tilde{\partial}_k$ and 2) each Hermitian operator must contain linearly an arbitrary parameter whose values are to be determined later (for details, see [29,32,33]).

The next step of this procedure is to find the additional parts $\partial'_i$ for the operators (19)–(22), (36), (37).
3 Additional parts of operators

As stated at the end of the previous section, now we intend to find the additional parts $o'_i$ for the operators (19)–(22), (36), (37). Following the procedure described in the bosonic case in [33], we must first find the superalgebra of the additional parts. Then these additional parts are constructed from new (additional) creation and annihilation operators, as well as from the constants of the theory $r, m$, and from one of the gamma-matrix-like objects $\gamma$. Furthermore, we must introduce linearly some arbitrary constants into all of the additional parts corresponding to Hermitian operators. Since $t''_0 = -l'_0$, we cannot obtain an independent arbitrary constant in $l''_0$.

Let us briefly remind and at the same time generalize the method given in [33] to algebras including fermionic operators. The (super)commutation relations for the operators $\tilde{o}_i$ from Table 2 have the structure

$$[\tilde{o}_i, \tilde{o}_j]_s = f^{ij}_{kl} \tilde{o}_k + f^{km}_{ij} \tilde{o}_m,$$  

(46)

where the constants $f^{ij}_{kl}, f^{km}_{ij}$ obey the properties $(f^{ij}_{kl}, f^{km}_{ij}) = -(1)^{\varepsilon(Ok)\varepsilon(Om)}(f^{kl}_{ij}, f^{km}_{ij})$. Then we suppose that the operators $\tilde{o}_i$ supercommute with the additional parts $o'_i$. In this case, one can check that if we define the superalgebra of the additional parts in the form

$$[o'_i, o'_j]_s = f^{ij}_{kl} o'_k - (1)^{\varepsilon(Ok)\varepsilon(Om)} f^{km}_{ij} o'_m o'_k,$$  

(47)

then the enlarged operators $\tilde{O}_i = \tilde{o}_i + o'_i$ will form a closed superalgebra

$$[\tilde{O}_i, \tilde{O}_j]_s = f^{ij}_{kl} \tilde{O}_k - (1)^{\varepsilon(Ok)\varepsilon(Om)} f^{km}_{ij} \tilde{O}_m + f^{km}_{ij} \tilde{O}_k \tilde{O}_m,$$  

(48)

which is deformed in comparison with (46).

After applying of the procedure above, the superalgebra of the additional parts takes the form given by Table 3 where

$$[l'_1, l'_0] = r(2t''_1, l''_0 + g'_1 t''_1 - \frac{3}{2} t''_1),$$  

(49)

$$[l'_0, l'_1] = r(2t''_2, t'_1 + t''_1 g'_0 - \frac{1}{2} t''_1),$$  

(50)

$$[l'_0, l'_1] = r(4t''_1, l''_1 - t''_1),$$  

(51)

$$[l''_1 + l''_0] = r(4l''_1, l''_1 + 2l''_1 g'_0 - l''_1),$$  

(52)

$$[l'_0, l'_1] = r(4l''_2, l''_1 + 2g'_0 l''_1 - l''_1),$$  

(53)

$$[l''_1 + l''_0] = r(4l''_2, l''_1 + 2l''_1 + g'_0 - l''_1),$$  

(54)

$$[l'_1, l'_1] = l'_0 - r(g''_0 - \frac{1}{2} g'_0 - 4l''_2 l''_1 + \frac{3}{2} t''_1),$$  

(55)

In accordance with our method, we ascribe the following value to the additional central charge: $\tilde{m}' = -\tilde{m}$, so that the enlarged central charge $\tilde{M}$ becomes equal to zero.

Explicit expressions for the additional parts can be found by the method described in the papers [29, 36] and extended to the case of the Verma module construction for a nonlinear superalgebra given by Table 3. Omitting tedious calculations, we have

$$t''_1 = f'' + 2b''_2 f,$$  

$$g''_0 = b''_1 b_1 + 2b''_2 b_2 + f'' f + h,$$  

$$l''_1 = m_1 b''_1,$$  

$$l''_2 = b''_2.$$  

\(^5\)We have to introduce linearly some arbitrary constants into all of the additional parts corresponding to Hermitian operators. Since $t''_0 = -l'_0$, we cannot obtain an independent arbitrary constant in $l''_0$. 

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Table 3: The algebra of the additional parts for the operators.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
[t \rightarrow ] & t'_0 & t'_1 & t'_1^+ & l'_0 & l'_1 & l'_2 & l'_2^+ & g'_0 \\
\hline
\hline
t'_0 & -2l'_0 & 2l'_1 & 2l'_1^+ & 0 & -49 & (50) & 0 & 0 & 0 \\
\hline
t'_1 & 2l'_1 & 4l'_2 & -2g'_0 & -51 & 0 & -t'_0 & 0 & -t'_1^+ & t'_1 \\
\hline
t'_1^+ & 2l'_1^+ & -2g_0 & 4l'_2^+ & 52 & t'_0 & 0 & t'_1 & 0 & -t'_1^+ \\
\hline
l'_0 & 0 & (51) & -52 & 0 & (53) & (54) & 0 & 0 & 0 \\
\hline
l'_1 & (49) & 0 & -t'_0 & -53 & 0 & (55) & 0 & -l'_1^+ & l'_1 \\
\hline
l'_1^+ & -50 & t'_0 & 0 & (54) & -55 & 0 & l'_1 & 0 & -l'_1^+ \\
\hline
l'_2 & 0 & 0 & -t'_1 & 0 & 0 & -l'_1 & 0 & g'_0 & 2l'_2 \\
\hline
l'_2^+ & 0 & t'_1^+ & 0 & 0 & l'_1^+ & 0 & -g'_0 & 0 & -2l'_2^+ \\
\hline
g'_0 & 0 & -t'_1 & t'_1^+ & 0 & -l'_1 & l'_1^+ & -2l'_2 & 2l'_2^+ & 0 \\
\hline
\end{array}
\]

\[
t'_0 = 2m_1 b_1^+ f - \frac{m_1}{2} (f^+ - 2b_2^+ f) b_1^+ \sum_{k=1}^{\infty} \frac{(-2r)^k (b_2^+)^{k-1} b_1^{2k}}{(2k)!} + \tilde{\gamma} m_0 \sum_{k=0}^{\infty} \frac{(-2r)^k (b_2^+)^k b_1^{2k}}{(2k)!} \\
+ r \left( h - \frac{1}{2} \right) (f^+ - 2b_2^+ f) f b_1^+ \sum_{k=1}^{\infty} \frac{(-2r)^k (b_2^+)^{k-1} b_1^{2k+1}}{(2k+1)!},
\]

\[
t'_1 = -2g'_0 f - (f^+ - 2b_2^+ f) b_2 + \frac{1}{2} (h - \frac{1}{2}) (f^+ - 2b_2^+ f) \sum_{k=1}^{\infty} \frac{(-2r)^k (b_2^+)^{k-1} b_1^{2k}}{(2k)!} \\
+ \frac{1}{2} (f^+ - 2b_2^+ f) b_1^+ \sum_{k=1}^{\infty} \frac{(-2r)^k (b_2^+)^k b_1^{2k+1}}{(2k+1)!} \\
- \tilde{\gamma} m_0 \sum_{k=0}^{\infty} \frac{(-2r)^k (b_2^+)^k b_1^{2k+1}}{(2k+1)!},
\]

\[
l'_1 = m_2 - r \sum_{k=0}^{\infty} \frac{(-8r)^k (b_2^+)^k b_1^{2k+1}}{(2k+1)!} (1 - 4^{-k}) \\
- r b_1^+ \sum_{k=0}^{\infty} \frac{(-8r)^k (b_2^+)^k b_1^{2k+1}}{(2k+1)!} (2h - 4^{-k}) + 4r \tilde{\gamma} m_0 \sum_{k=0}^{\infty} \frac{(-2r)^k (b_2^+)^k b_1^{2k+1}}{(2k+1)!} f \\
+ r \left( h - \frac{1}{2} \right) \sum_{k=0}^{\infty} \frac{(-2r)^k (b_2^+)^{k+1} b_1^{2k+2}}{(2k+2)!} - 2r (b_1^+)^2 \sum_{k=0}^{\infty} \frac{(-8r)^k (b_2^+)^k b_1^{2k+2}}{(2k+2)!}.
\]
- 2r f^+ f \sum_{k=0}^{\infty} \left( \frac{-2r}{m_1^2} \right)^k \left\{ \frac{(h - \frac{1}{2})}{(2k)!} + \frac{b^+_1 b_1}{(2k+1)!} \right\} (b^+_2)^k b^{2k}_1 + \\
+ \frac{m_0^2 - r(h^2 - \frac{1}{4})}{2} \sum_{k=0}^{\infty} \left( \frac{-8r}{m_1^2} \right)^{k+1} \frac{(b^+_2)^{k+1} b_1^{2k+2}}{(2k+2)!}.

(60)

\[ l'_1 = -m_1 b^+_1 b_2 + \frac{m_1}{4} b^+_1 \sum_{k=1}^{\infty} \left( \frac{-8r}{m_1^2} \right)^k \left\{ \frac{2h - 4 - 4k}{(2k)!} + \frac{2b^+_1 b_1}{(2k+1)!} \right\} (b^+_2)^{k-1} b^{2k}_1 + \\
+ \frac{\tilde{\gamma} m_0}{4} (f^+ - 2b^+_2 f) \sum_{k=1}^{\infty} \left( \frac{-8r}{m_1^2} \right)^k \frac{(b^+_2)^{k-1} b^{2k}_1}{(2k)!} (1 - 4^{-k}) + \\
+ r(h - \frac{1}{2}) \sum_{k=0}^{\infty} \left( \frac{-2r}{m_1^2} \right)^k \frac{(b^+_2)^k b^{2k+1}_1}{(2k+1)!} + \frac{m_1}{2} f^+ f \sum_{k=0}^{\infty} \left( \frac{-2r}{m_1^2} \right)^k \frac{(b^+_2)^k b^{2k}_1}{(2k)!} \\
- r(h - \frac{1}{2}) f^+ f \sum_{k=0}^{\infty} \left( \frac{-2r}{m_1^2} \right)^k \frac{(b^+_2)^k b^{2k+1}_1}{(2k+1)!} - \tilde{\gamma} m_0 f \sum_{k=0}^{\infty} \left( \frac{-8r}{m_1^2} \right)^k \frac{(b^+_2)^k b^{2k}_1}{(2k)!} \\
+ \frac{m_0^2 - r(h^2 - \frac{1}{4})}{m_1} \sum_{k=0}^{\infty} \left( \frac{-8r}{m_1^2} \right)^k \frac{(b^+_2)^k b^{2k+1}_1}{(2k+1)!}.

(61)

\[ l'_2 = g'_0 b_2 - b^+_2 b^+_2 - \frac{m_0^2 - r(h^2 - \frac{1}{4})}{m_1^2} \sum_{k=0}^{\infty} \left( \frac{-8r}{m_1^2} \right)^k \frac{(b^+_2)^k b^{2k+2}_1}{(2k+2)!} + \\
- r(h - \frac{1}{2}) \sum_{k=0}^{\infty} \left( \frac{-2r}{m_1^2} \right)^k \frac{(b^+_2)^k b^{2k+2}_1}{(2k+2)!} + \frac{\tilde{\gamma} m_0}{m_1} f \sum_{k=0}^{\infty} \left( \frac{-2r}{m_1^2} \right)^k \frac{(b^+_2)^k b^{2k+1}_1}{(2k+1)!} \\
- \frac{\tilde{\gamma} m_0}{4m_1} (f^+ - 2b^+_2 f) \sum_{k=1}^{\infty} \left( \frac{-8r}{m_1^2} \right)^k \frac{(b^+_2)^{k-1} b^{2k+1}_1}{(2k+1)!} (1 - 4^{-k}) + \\
- \frac{1}{4} b^+_2 \sum_{k=1}^{\infty} \left( \frac{-8r}{m_1^2} \right)^k \left\{ \frac{2h - 4 - 4k}{(2k+1)!} + \frac{2b^+_1 b_1}{(2k+2)!} \right\} (b^+_2)^{k-1} b^{2k+1}_1 + \\
- \frac{1}{2} f^+ f \sum_{k=1}^{\infty} \left( \frac{-2r}{m_1^2} \right)^k \left\{ \frac{2h + \frac{1}{2}}{2k!} + \frac{b^+_1 b_1}{(2k+1)!} \right\} (b^+_2)^{k-1} b^{2k}_1.

(62)

In the above expressions, \( h \) is an arbitrary dimensionless constant, while \( m_0 \) and \( m_1 \) are arbitrary constants with the dimension of mass. To obtain explicit expressions for the additional parts (60), (62) in the Fock space, we have introduced two new pairs of bosonic and a new pair of fermionic creation and annihilation operators satisfying the standard commutation relations

\[ [b_1, b^+_1] = 1, \quad [b_2, b^+_2] = 1, \quad \{f, f^+\} = 1. \]

(63)

The resulting additional parts of the operators possess all the necessary properties, in particular, the additional parts which correspond to Hermitian operators contain arbitrary parameters (see the footnote 5): the operator \( t'_0 \) contains the arbitrary parameter \( m_0 \); the operator \( g'_0 \) contains the arbitrary parameter \( h \). The values of the arbitrary parameters \( h \) and \( m_0 \) will be determined later by the condition that the correct equations of motion (12), or, equivalently, (45), be reproduced.

The massive parameter \( m_1 \) remains arbitrary and can be expressed in terms of other parameters of the theory,

\[ m_1 = f(m, r) \neq 0. \]

(64)
This arbitrariness does not affect the equations for the basic vector \( \langle 5 \rangle \).

Note that the additional parts do not obey the usual properties

\[
(l'_0)^+ \neq l'_0, \quad (l'_1)^+ \neq l'^+_1, \quad (l'_2)^+ \neq l'^+_2, \quad (65)
\]

\[
(t'_0)^+ \neq t'_0, \quad (t'_1)^+ \neq t'^+_1, \quad (66)
\]

if one should use the standard rules of Hermitian conjugation for the new creation and annihilation operators,

\[
(b_1)^+ = b^+_1, \quad (b_2)^+ = b^+_2, \quad (f)^+ = f^+. \quad (67)
\]

To restore the proper Hermitian conjugation properties for the additional parts, we change the scalar product in the Fock space \( \mathcal{H}' \) generated by the new creation and annihilation operators as follows:

\[
\langle \tilde{\Psi}_1|\Psi_2 \rangle_{\text{new}} = \langle \tilde{\Psi}_1|K'|\Psi_2 \rangle, \quad (68)
\]

for any vectors \( |\Psi_1\rangle, |\Psi_2\rangle \) with some, yet unknown, operator \( K' \). This operator is determined by the condition that all of the operators of the algebra must have the proper Hermitian properties with respect to the new scalar product:

\[
\langle \tilde{\Psi}_1|K'\tilde{\Psi}_0|\Psi_2 \rangle = \langle \tilde{\Psi}_2|K'\tilde{\Psi}_0|\Psi_1 \rangle^*, \quad (69)
\]

\[
\langle \tilde{\Psi}_1|K'\tilde{\Psi}_1|\Psi_2 \rangle = \langle \tilde{\Psi}_2|K'\tilde{\Psi}_1|\Psi_1 \rangle^*, \quad (70)
\]

\[
\langle \tilde{\Psi}_1|K'\tilde{\Psi}_2|\Psi_2 \rangle = \langle \tilde{\Psi}_2|K'\tilde{\Psi}_2|\Psi_1 \rangle^*, \quad (71)
\]

These relations permit one to determine the operator \( K' \) as follows:

\[
K' = Z^+Z, \quad Z = \sum_{n_1,n_2,s=0}^{\infty} \frac{1}{m_1^{n_1}(t^+_1)^{n_1}} \frac{1}{m_2^{n_2}(t^+_2)^{n_2}} \langle 0 \rangle_V \frac{1}{n_1!n_2!} (b^+_1)^{n_1} f^* (b^+_2)^{n_2}, \quad (72)
\]

where the auxiliary vector \( |0\rangle_V \) obeys the relations

\[
t'_0|0\rangle_V = t'_0|0\rangle_V = t'_2|0\rangle_V = 0, \quad V|0\rangle_V = 1 \quad (73)
\]

\[
t'_0|0\rangle_V = \tilde{\gamma} m_0|0\rangle_V, \quad t'_0|0\rangle_V = m_0^2|0\rangle_V, \quad g'_0|0\rangle_V = h|0\rangle_V. \quad (74)
\]

For low numbers \( n_i + n_f + 2n_2 \), where \( n_i \) are the numbers of “particles” associated with \( b^+_i \), and \( n_f \) is the number of “particles” associated with \( f^+ \), the operator \( K' \) reads

\[
K' = |0\rangle \langle 0| + \frac{m_0^2 - rh(h - \frac{1}{2})}{m_1^2} b^+_1|0\rangle \langle 0|b^+_1 - 2hf^+|0\rangle \langle 0|f
\]

\[
+ \frac{m_0}{m_1} \left( \tilde{\gamma} f^+|0\rangle \langle 0|b_1 + b^+_1|0\rangle \langle 0|f \tilde{\gamma} \right) + \ldots . \quad (75)
\]

This expression for the operator \( K' \) will be used later in constructing of examples in section \( \tilde{\gamma} \).

Thus, in this section, we have constructed the additional parts \( (56)-(72) \) for the constraints, which obey all the requirements. In the next section, we determine the algebra of the enlarged constraints and construct a BRST operator corresponding to this algebra.
Table 4: The algebra of the enlarged operators.

4 Deformed algebra and BRST operator

Let us turn to the algebra of the enlarged operators $\tilde{O}_i = \tilde{o}_i + o'_i$. Since the algebra is quadratic, there exist different possibilities of operator ordering in the right-hand side of the commutation relations. Analogous situation takes place in the bosonic higher spin theory as well. In [33] we have studied dependence of BRST construction on ordering prescription for the quadratic constraint algebra and proved that different ordering prescriptions lead to equivalent Lagrangians and the same solution to the equations of motion. Since the constraint algebra in the fermionic case is analogous to the bosonic one, the same consequence about the final Lagrangians and the equations of motion will be valid also for the fermionic case. Therefore we are not going to study here all the possibilities of ordering prescriptions as this was done in [33], but choose only one of them, which corresponds to the supersymmetrized ordering for the constraints and lead to the expression for the BRST operator which has nonvanishing terms of the third degree in powers of ghosts. The algebra of the enlarged operators corresponding to this BRST operator is given by Table 4, where

---

Using our results in the bosonic case [33], we see that the BRST operators corresponding to different choices of the mentioned ordering of constraints will lead to equivalent Lagrangian formulations. At the same time, we note that for the purpose of constructing Lagrangians it is sufficient to find only one BRST operator with a some fixed ordering for the constraints in its commutation relations.
Here, the coordinates corresponding to their canonically conjugate ghost momenta \( P \) of the algebra given by Table 4 can be calculated with the help of the \((CP)\)-ordering of the ghost coordinate \( C \) and momenta \( P \) operators:

\[
Q' = q_0 T_0 + q_1^+ T_1 + q_1 T_1^+ + \eta_0 L_0 + \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_2^+ L_2 + \eta_2 L_2^+ + \eta_G G_0 \\
+ i(\eta_1^+ q_1 - \eta_q q_1^+) p_0 - i(\eta_q q_1 + \eta_2 q_1^+) p_1^+ + i(\eta_G q_1 + \eta_2^+ q_1) p_1 \\
+ (q_0^2 - \eta_1^+ \eta_1) P_0 + (2q_1^+ q_1 - \eta_2^+ \eta_2) P_G + (\eta_G \eta^+ + \eta_2^+ \eta_1 - 2q_0 q_1^+) P_1 \\
+ (\eta_1^+ \eta_G + \eta_1^+ \eta_2 - 2q_1^+ q_1^+) P_1^+ + 2(\eta_G q_1^+ + \eta_1^+ q_1^+) P_2 + 2(\eta_G q_1^+ + \eta_1^+ q_1^+) P_2^+ \\
+ r(q_0 \eta_1^+ + 2q_1^+ \eta_0) \left[ \frac{1}{2} G_0 P_1 + \frac{1}{2} T_1 P_G + T_1^+ P_2 + i L_2 P_1^+ - t_1^+ P_G - i g_0 P_1 - 2 (i t_2^+ P_1 + t_1^+ P_2) \right] \\
+ r(q_0 \eta_1^+ + 2q_1^+ \eta_0) \left[ - \frac{1}{2} G_0 P_1^+ + \frac{1}{2} L_1 P_G + L_1^+ P_2 - i L_2 P_1^+ + t_1^+ P_G + i g_0 P_1^+ + 2 (i t_2^+ P_1 + t_1^+ P_2^+) \right] \\
+ 2r \eta_1^+ \eta_0 \left[ \frac{1}{2} G_0 P_1 + \frac{1}{2} L_1 P_G + L_1^+ P_2 + L_2 P_1^+ - t_1^+ P_G - g_0 P_1 - 2 (i t_2^+ P_1 + t_2^+ P_2^+) \right] \\
- 2r \eta_1^+ \eta_0 \left[ \frac{1}{2} G_0 P_1^+ + \frac{1}{2} L_1^+ P_G + L_1^+ P_2^+ + L_2 P_1^+ - t_1^+ P_G - g_0 P_1^+ - 2 (i t_2^+ P_1 + t_1^+ P_2^+) \right] \\
- r \eta_1^+ \eta_1^+ \left[ 2 L_2 P_2 + 2 L_2 P_2^+ - G_0 P_G - \frac{3}{4} T_1^+ p_1 + \frac{3}{4} T_1^+ p_1^+ \right] \\
+ r \eta_1^+ \eta_1^+ \left[ 4 (t_2^+ P_2 + t_2^+ P_2^+) - 2 g_0 P_G - \frac{3}{2} (t_1^+ p_1 + t_1^+ P_2) \right] \\
+ r^2 \eta_0 \eta_1^+ \eta_1^+ \left[ G_0 (p_1^+ p_1 + 2 P_2^+ P_2) + L_2^2 P_2^2 + L_2 (p_1^+)^2 + \frac{i}{2} (T_1^+ p_1^+ P_2 + T_1^+ P_2^+) \right] \\
+ \frac{i}{4} (T_1^+ p_1 + T_1^+ P_1^+) P_G - 2 (L_2 P_2^+ + L_2^+ P_2^+) P_G \right].
\]

Here, \( q_0, q_1, q_1^+ \) and \( \eta_0, \eta_1^+, \eta_1, \eta_2^+, \eta_G \) are, respectively, the bosonic and fermionic ghost “coordinates” corresponding to their canonically conjugate ghost “momenta” \( p_0, p_1^+, p_1, P_0, P_1, P_1^+, P_2, P_2^+, P_G \). They obey the (anti)commutation relations

\[
\{\eta_1, P_1^+\} = \{P_1, \eta_1^+\} = \{\eta_2, P_2^+\} = \{P_2, \eta_2^+\} = \{\eta_0, P_0\} = \{\eta_G, P_G\} = 1,
\]

\[
[q_0, p_0] = [q_1, p_1^+] = [q_1^+, p_1] = i
\]

and possess the standard ghost number distribution, \( gh(C) = -gh(P) = 1 \), providing the property \( gh(Q') = 1 \).
The resulting BRST operator $Q'$ is Hermitian with respect to the scalar product $\langle \ | \ \rangle$ in the tensor product of the Fock spaces $\mathcal{H}_{\text{tot}} = \mathcal{H}' \otimes \mathcal{H} \otimes \mathcal{H}_{\text{gh}}$, which is constructed as the direct product of the scalar products on $\mathcal{H}$, $\mathcal{H}'$ and $\mathcal{H}_{\text{gh}}$,

$$\langle \bar{\Psi}_1 | Q' + K | \Psi_2 \rangle = \langle \bar{\Psi}_1 | K Q' | \Psi_2 \rangle \quad (85)$$

with the operator $K$ defined in $\mathcal{H}_{\text{tot}}$ and being the tensor product of the operator $K'$ and the unit operators $\hat{1}, \hat{1}_{\text{gh}}$,

$$K = K' \otimes \hat{1} \otimes \hat{1}_{\text{gh}}. \quad (86)$$

Thus, we have constructed the Hermitian BRST operator. In the next section, this operator is applied to construct Lagrangians of fermionic higher spin fields in the AdS space.

## 5 Construction of Lagrangians

In this section, we construct Lagrangians of fermionic massive higher spin fields in the AdS space. This construction goes along the lines of [29,32]. We should first extract the dependence of the BRST operator $Q'$ (83) on the ghosts $\eta_G$, $\hat{P}_G$,

$$Q' = Q + \eta_G (\sigma + h) + B \hat{P}_G, \quad (87)$$

where

$$Q = q_0 T_0 + q_1 T_1 + q_1 T_1^+ + \eta_0 L_0 + \eta^+_1 L_1 + \eta_1 L_1^+ + \eta_2 L_2 + \eta_2 L_2^+ + i(\eta_1^+ q_1 - \eta q_1^+) p_0 - i\eta_2 q_1^+ p_1^+ + i\eta_2 q_1 p_1 + (q_0^2 - \eta_1^+ \eta_1) \hat{P}_0 + (\eta_2^+ \eta_1 - 2q_0 q_1^+) \hat{P}_1 + (\eta^+_1 \eta_2 - 2q_0 q_1) \hat{P}_1^+ - 2q_1^2 \hat{P}_2^+ - 2q_1^2 \hat{P}_2 +$$

$$+ \left[ 2 \eta_0 \eta_1^+ - \frac{i}{2} G_0 P_1 + T_1 P_2 + i L_2 p_1^+ + i \eta g_0 p_1 - 2(i\eta_2^+ p_1 + \eta_2^+ P_1) \right] + \left[ G_0 P_1 + L_1^+ P_2 + L_2 P_1^+ - g_0' P_1 - 2(l_1^+ P_2 + l_2^+ P_1^+) \right]$$

$$- 2r \eta_0 \eta_1^+ \left[ G_0 P_1 + L_1^+ P_2 + L_2 P_1^+ - g_0' P_1 - 2(l_1^+ P_2 + l_2^+ P_1^+) \right]$$

$$- r \eta_0 \eta_1^+ \left[ 2L_2 P_2 + 2L_2 P_2^+ - \frac{3i}{4} T_1^+ p_1 + \frac{3i}{4} T_1 p_1^+ \right]$$

$$+ r \eta_0 \eta_1^+ \left[ 4(l_2^+ P_2 + l_2^+ P_2^+) - \frac{3i}{2} (l_1^+ p_1 - t_1^+ P_1) \right]$$

$$+ r^2 \eta_0 \eta_1 \eta_1^+ \left[ G_0 (p_1^+ p_1 + 2P_2 p_2) + L_2 p_1^2 + L_2 (p_1^+)^2 + \left( T_1^+ P_2 + T_1 P_2^+ \right) \right], \quad (88)$$

$$\sigma = -a^+ \alpha^+ \frac{d}{2} + b^+_1 b_1 + 2b^+_2 b_2 + f^+ f$$

$$- i q_1 p_1^+ + i q_1^+ p_1 + \eta^+_1 P_1 - \eta P_1^+ + 2\eta_2^+ P_2 - 2\eta_2 P_2^+; \quad (89)$$

meanwhile the explicit expression for the operator $B$ is not essential.

Next, following the procedure of [29,32], we choose the following representation of the Hilbert space:

$$(p_0, q_1, p_1, P_0, P_G, \eta_1, P_1, \eta_2, P_2) | 0 \rangle = 0, \quad (90)$$
and suppose that the vectors and gauge parameters do not depend on $\eta_G$,

$$
|\chi\rangle = \sum_{k_i} (q_0)^{k_1} (q_1^+)^{k_2} (p_1^+)^{k_3} (\eta_0)^{k_4} (f^+)^{k_5} (\eta_1^+)^{k_6} (P_1^+)^{k_7} (\eta_2^+)^{k_8} (P_2^+)^{k_9} (b_1^+)^{k_{10}} (b_2^+)^{k_{11}} \times \alpha^{+\mu_1} \cdots \alpha^{+\mu_{k_0}} \chi_{\mu_1 \cdots \mu_{k_0}}^{k_1 \cdots k_{11}} (x) |0\rangle.
$$

(91)

The sum in (91) is taken over $k_0, k_1, k_2, k_3, k_{10}, k_{11}$, running from 0 to infinity, and over $k_4, k_5, k_6, k_7, k_8, k_9$, running from 0 to 1. Then, we derive from the equations that determine the physical vector, $Q |\chi\rangle = 0$, as well as from the reducible gauge transformations, $\delta |\chi\rangle = Q |\Lambda\rangle$, a sequence of relations:

$$
Q |\chi\rangle = 0, \quad (\sigma + h) |\chi\rangle = 0, \quad (\varepsilon, gh) (|\chi\rangle) = (1, 0),
$$

(92)

$$
\delta |\chi\rangle = Q |\Lambda\rangle, \quad (\sigma + h) |\Lambda\rangle = 0, \quad (\varepsilon, gh) (|\Lambda\rangle) = (0, -1),
$$

(93)

$$
\delta |\Lambda^{(1)}\rangle = Q |\Lambda^{(1)}\rangle, \quad (\sigma + h) |\Lambda^{(1)}\rangle = 0, \quad (\varepsilon, gh) (|\Lambda^{(1)}\rangle) = (1, -2),
$$

(94)

$$
\delta |\Lambda^{(i-1)}\rangle = Q |\Lambda^{(i)}\rangle, \quad (\sigma + h) |\Lambda^{(i)}\rangle = 0, \quad (\varepsilon, gh) (|\Lambda^{(i)}\rangle) = (i, -i - 1).
$$

(95)

The middle equation in (92) presents the equations for the possible values of $h$,

$$
h = n + \frac{d - 4}{2},
$$

(96)

with $n$ being related to spin, $s = n + 1/2$. By fixing the value of spin, we also fix the parameter $h$, according to (96). Having fixed a value of $h$, we must substitute it into each of the expressions (92)-(95); see [29] for more details.

As a next step, we have to extract the zero-mode ghosts from the operator $Q$ [88]. This operator has the structure

$$
Q = q_0 \tilde{T}_0 + \eta_0 \tilde{L}_0 + i(\eta_1^+ q_1 - \eta_1 q_1^+) p_0 + (q_0^2 - \eta_1^+ \eta_1) P_0 + \Delta Q,
$$

(97)

where

$$
\tilde{T}_0 = T_0 - 2q_1^+ P_1 - 2q_1 P_1^+ \quad 
+ n \eta_1^+ \left[ \frac{i}{2} G_0 p_1 + T_1^+ P_2 + i L_2 p_1^+ - ig_0 p_1 - 2(\varepsilon'' p_1^+ + t_1^+ P_2) \right] 
+ n \eta_1 \left[ -\frac{i}{2} G_0 p_1^+ - T_1 P_2^+ - i L_2^+ p_1 + ig_0 p_1^+ + 2(\varepsilon'' p_1^+ + t_1^+ P_2^+) \right],
$$

(98)

$$
\tilde{L}_0 = L_0 + 2q_1^+ \left[ \frac{i}{2} G_0 p_1 + T_1^+ P_2 + i L_2 p_1^+ - ig_0 p_1 - 2(\varepsilon'' p_1^+ + t_1^+ P_2) \right] 
+ 2q_1 \left[ -\frac{i}{2} G_0 p_1^+ - T_1 P_2^+ - i L_2^+ p_1 + ig_0 p_1^+ + 2(\varepsilon'' p_1^+ + t_1^+ P_2^+) \right] 
+ 2n \eta_1^+ \left[ \frac{i}{2} G_0 P_1 + L_1^+ P_2 + L_2^+ P_1^+ - g_0 P_1 - 2(t_1^+ P_2 + t_1^+ P_1^+) \right] 
- 2n \eta_1 \left[ \frac{i}{2} G_0 P_1^+ + L_1 P_2^+ + L_2 P_1^+ - g_0 P_1^+ - 2(t_1^+ P_2 + t_1^+ P_1^+) \right] 
+ r^2 n \eta_1 \left[ G_0 (p_1^+ p_1 + 2P_2 p_2) + L_2^+ p_1^2 + L_2 (p_1^+)^2 + \frac{i}{2} (T_1^+ p_1^+ P_2 + T_1 p_1 P_2^+) \right],
$$

(99)

$$
\Delta Q = q_1^+ T_1 + q_1 T_1^+ + \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_2^+ L_2 + \eta_2 L_2^+ - \eta_2 q_1^+ p_1^+ + \eta_2^+ q_1 p_1 
+ n \eta_1^+ P_1 + n \eta_1^+ P_1^+ - 2q_1^2 P_2 - 2q_1^2 P_2^+ 
- n \eta_1 \left[ 2L_2^+ P_2 + 2L_2 p_2^+ - \frac{3i}{4} T_1^+ p_1 + \frac{3i}{4} T_1 p_1^+ \right] 
+ n \eta_1 \left[ 4(t_2^+ P_2 + t_2^+ P_2^+) - \frac{3i}{2} (t_1^+ p_1 - t_1^+ P_1^+) \right].
$$

(100)
Here, $\tilde{T}_0$, $\tilde{L}_0$, $\Delta Q$ are independent of $q_0$, $p_0$, $\eta_0$, $P_0$. We also expand the state vector and gauge parameters in powers of the zero-mode ghosts:

$$|\chi\rangle = \sum_{k=0}^{\infty} q_0^k (|\chi_0^k\rangle + \eta_0 |\chi_1^k\rangle), \quad gh(|\chi_m^k\rangle) = -(m + k),$$

$$|\Lambda^{(i)}\rangle = \sum_{k=0}^{\infty} q_0^k (|\Lambda^{(i)k}_0\rangle + \eta_0 |\Lambda^{(i)k}_1\rangle), \quad gh(|\Lambda^{(i)k}_m\rangle) = -(i + k + m + 1).$$

Following the procedure of [29], we get rid of all the fields except two, $|\chi_0^0\rangle$, $|\chi_1^0\rangle$, and, hence, relations (92)–(95), with allowance for (97), (101), (102), yield two independent equations for these fields:

$$\Delta Q|\chi_0^0\rangle + \frac{1}{2} \{\tilde{T}_0, \eta_1^+ \eta_1\} |\chi_0^1\rangle = 0,$$

$$\tilde{T}_0|\chi_0^0\rangle + \Delta Q|\chi_0^1\rangle = 0,$$

where $\{A, B\} = AB + BA$ for any quantities $A, B$.

Next, due to the fact that the operators $Q$, $\tilde{T}_0$, $\eta_1^+ \eta_1$ commute with $\sigma$, we derive from (103), (104) the equations of motion for the fields with a fixed value of spin:

$$\Delta Q|\chi_0^0\rangle_n + \frac{1}{2} \{\tilde{T}_0, \eta_1^+ \eta_1\} |\chi_0^1\rangle_n = 0,$$

$$\tilde{T}_0|\chi_0^0\rangle_n + \Delta Q|\chi_0^1\rangle_n = 0,$$

where the fields $|\chi_0^0\rangle$, $|\chi_1^0\rangle$ are assumed to obey the relations

$$\sigma|\chi_0^0\rangle_n = (n + (d - 4)/2)|\chi_0^0\rangle_n, \quad \sigma|\chi_1^0\rangle_n = (n + (d - 4)/2)|\chi_1^0\rangle_n.$$

The field equations (105), (106) are Lagrangian ones and can be deduced from the following action:

$$S_n = n \langle \chi_0^0 | K_n \tilde{T}_0 |\chi_0^0\rangle_n + \frac{1}{2} n \langle \chi_1^1 | K_n \{\tilde{T}_0, \eta_1^+ \eta_1\} |\chi_0^1\rangle_n + n \langle \chi_0^0 | K_n \Delta Q |\chi_1^0\rangle_n + n \langle \chi_1^1 | K_n \Delta Q |\chi_0^0\rangle_n,$$

where the standard scalar product for the creation and annihilation operators is assumed, and the operator $K_n$ is the operator $K$ (86), where the following substitution is made: $\hbar \to - (n + (d - 4)/2)$.

The equations of motion (105), (106) and the action (108) are invariant with respect to the gauge transformations

$$\delta |\chi_0^0\rangle_n = \Delta Q|\Lambda_0^0\rangle_n + \frac{1}{2} \{\tilde{T}_0, \eta_1^+ \eta_1\} |\Lambda_0^1\rangle_n,$$

$$\delta |\chi_1^0\rangle_n = \tilde{T}_0|\Lambda_0^0\rangle_n + \Delta Q|\Lambda_0^1\rangle_n,$$

which are reducible, with the gauge parameters $|\Lambda_{0/n}^{(i)j}\rangle$, $j = 0, 1$ subject to the same conditions as those for $|\chi_{0/n}^j\rangle$ in (107),

$$\delta |\Lambda_{0/n}^{(i)0}\rangle = \Delta Q|\Lambda_{0/n}^{(i+1)0}\rangle + \frac{1}{2} \{\tilde{T}_0, \eta_1^+ \eta_1\} |\Lambda_{0/n}^{(i+1)1}\rangle, \quad |\Lambda_{0/n}^{(0)0}\rangle = |\Lambda_{0/n}^0\rangle, \quad |\Lambda_{0/n}^{(i)1}\rangle = |\Lambda_{0/n}^1\rangle, \quad |\Lambda_{0/n}^{(0)1}\rangle = |\Lambda_{0/n}^1\rangle.$$
and with a finite number of reducibility stages at \( i_{\text{max}} = n - 1 \) for spin \( s = n + 1/2 \).

We now determine the value of the arbitrary parameter \( m_0 \), using the condition that the equations (145) for the basic vector \( |\Phi\rangle \) (5) be reproduced. To this end, it is necessary that conditions (145) be implied by Eqs. (105), (106). Note that the general vector \( |\chi_0^0\rangle_n \) includes the basic vector \( |\Phi\rangle \) (5), namely,

\[
|\chi_0^0\rangle_n = |\Phi\rangle_n + |\Phi_A\rangle_n, \quad |\Phi_A\rangle_n\bigg|_{c=P=b_1^+=b_2^+=f^+=0} = 0. \quad (113)
\]

In the next section, we shall demonstrate that due to the gauge transformations and a part of the equations of motion the vector \( |\Phi_A\rangle_n \) can be completely removed, so that the resulting equations of motion have the form

\[
T_0|\Phi\rangle_n = (t_0 + \gamma m_0)|\Phi\rangle_n = 0, \quad t_1|\Phi\rangle_n = 0. \quad (114)
\]

The above relations permit one to determine the parameter \( m_0 \) in a unique way, as follows:

\[
m_0 = m - r^2 h = m + r^2 \left( n + (d - 4)/2 \right). \quad (115)
\]

It should be noted that \( m_0 \) of the present article is the AdS mass \( m_D \) of [34] corresponding to “the most-used definition of mass” (for more discussions of this point, see [34]).

In the next section, we shall demonstrate that the action actually reproduces the correct equations of motion (12). Thus, we have constructed Lagrangians for fermionic fields of any fixed spin using the BRST approach.\(^8\)

### 6 Reduction to the initial irreducible relations

Let us show that the equations of motion (1), (2) [or equivalently (145)] can be obtained from the Lagrangian (108) after gauge-fixing and removing the auxiliary fields by using a part of the equations of motion. Let us start with gauge-fixing.

#### 6.1 Gauge-fixing

Let us consider a field of spin \( s = n + 1/2 \). Then we have a reducible gauge theory with \( n - 1 \) reducibility stages. Due to restriction (107) and the ghost number restriction [see the right-hand formulae in (102)], the lowest-stage gauge parameters have the form

\[
|\Lambda_0^{(n-1)0}\rangle_n = (p_1^+)^{n-1}\left\{P_1^+|\lambda\rangle_0 + p_1^+|\lambda_1\rangle_0\right\}, \quad (116)
\]

\[
|\Lambda_0^{(n-1)1}\rangle_n \equiv 0, \quad (117)
\]

with the subscripts of the state vectors being associated with the eigenvalues of the corresponding state vectors (107). It can be verified directly that one can eliminate the dependence on the ghost \( \mathcal{P}_2^+ \) from the gauge function \( |\Lambda^{(n-2)0}\rangle_0 \) of the \((n-2)\)-th stage (the gauge function \( |\Lambda^{(n-2)1}\rangle_0 \) has no \( \mathcal{P}_2^+ \) dependence). It is then possible to verify that one can remove the dependence of \( |\Lambda^{(n-3)0\rangle_0}, |\Lambda^{(n-3)1}\rangle_0 \) on \( \mathcal{P}_2^+ \) with the help of the remaining gauge parameters (which do not depend on \( \mathcal{P}_2^+ \)) \( |\Lambda^{(n-2)0}\rangle_0, |\Lambda^{(n-2)1}\rangle_0 \).

\(^8\)The construction of a Lagrangian describing the propagation of all fermionic fields in the AdS space simultaneously is analogous to the case of the flat space [29] and we do not consider it here. We only note that the necessary condition for resolving this problem is to replace in \( Q', Q, K \) the parameter \(-h\) by the operator \( \sigma \) in an appropriate way and discard the condition (107) for the fields and gauge parameters.
We now suppose that we have removed the dependence on \( P^+_2 \) from the gauge functions of the \((i+1)\)-th stage \( |\Lambda^{(i+1)0}_0\rangle, \ j = 0, 1 \), i.e., we have \( \eta_2 |\Lambda^{(i+1)0}_0\rangle = 0 \). Let us show that these restricted gauge functions can be used to remove the dependence on \( P^+_2 \) from the gauge functions \( |\Lambda^{(i)j}_0\rangle \). We introduce the following notation for the gauge parameters, related to their decomposition in ghosts \( P^+_2, P^+_2 \):

\[
|\Lambda^{(i)j}_0\rangle = |\Lambda^{(i)j}_{00}\rangle + P^+_1 |\Lambda^{(i)j}_{01}\rangle + P^+_2 |\Lambda^{(i)j}_{02}\rangle + P^+_1 P^+_2 |\Lambda^{(i)j}_{03}\rangle. \tag{118}
\]

Here and elsewhere, we omit the subscripts of the vectors which are associated with the eigenvalues of the operator \( \sigma \). Then, using (111) and (112), we find a gauge transformation for \( |\Lambda^{(i)j}_{02}\rangle \) and \( |\Lambda^{(i)j}_{03}\rangle \), being coefficients at \( P^+_2 \),

\[
\delta|\Lambda^{(i)0}_{02}\rangle = -2q^2 |\Lambda^{(i+1)0}_{00}\rangle + 2r \eta_1^+(L_2 - 2l_2') |\Lambda^{(i+1)0}_{01}\rangle, \tag{119}
\]
\[
\delta|\Lambda^{(i)0}_{03}\rangle = 2q^2 |\Lambda^{(i+1)0}_{01}\rangle, \tag{120}
\]
\[
\delta|\Lambda^{(i)1}_{02}\rangle = -2q^2 |\Lambda^{(i+1)1}_{00}\rangle + 2r \eta_1^+(L_2 - 2l_2') |\Lambda^{(i+1)1}_{01}\rangle + r(T_1 - 2t_1') |\Lambda^{(i+1)0}_{01}\rangle \tag{121}
\]
\[
\delta|\Lambda^{(i)1}_{03}\rangle = 2q^2 |\Lambda^{(i+1)1}_{01}\rangle. \tag{122}
\]

Using (119)–(122), we can see that the dependence on \( P^+_2 \) in \( |\Lambda^{(i)j}_0\rangle \) can be removed with the help of the gauge transformations. To this end, we should first make gauge transformations with \( |\Lambda^{(i+1)0}_{01}\rangle \) and \( |\Lambda^{(i+1)1}_{01}\rangle \), removing \( |\Lambda^{(i)0}_{02}\rangle \) and \( |\Lambda^{(i)0}_{03}\rangle \), respectively. Then we should make gauge transformation with the parameter \( |\Lambda^{(i+1)0}_{00}\rangle \), removing \( |\Lambda^{(i)0}_{02}\rangle \). Finally, we should make gauge transformation with the parameter \( |\Lambda^{(i+1)1}_{00}\rangle \), removing \( |\Lambda^{(i)1}_{02}\rangle \). Thus, we have shown that the dependence on \( P^+_2 \) can be eliminated from \( |\Lambda^{(i)j}_0\rangle \).

This procedure works perfectly well until the terms linear in \( p^+_1 \) appear in the gauge functions \( |\Lambda^{(i+1)}\rangle \). When these terms are present, some of the gauge parameters remain unused after eliminating the \( P^+_2 \) dependence. Due to the presence of \( r \)-dependent terms in (119), (121), it is obvious that if one should make a gauge transformation with such a parameter the terms depending on the ghost \( P^+_2 \) may appear again. Therefore, one should make gauge transformations with parameters being linear in \( p^+_1 \) or independent of it, before removing the \( P^+_2 \) dependence. The first gauge function where such a term appear is \( |\Lambda^{(1)}\rangle \). Let us consider gauge transformation with this gauge function more carefully.

Suppose that the dependence on the ghost \( P^+_2 \) in \( |\Lambda^{(1)}\rangle \) has been removed by a gauge transformation. Let us decompose the gauge functions \( |\Lambda^{(1)}\rangle \) and \( |\Lambda^{(0)}\rangle \) as follows:

\[
|\Lambda^{(1)0}_0\rangle = -ip^+_1 P^+_1 |\omega\rangle + (p^+_1)^2 (\ldots), \tag{123}
\]
\[
|\Lambda^{(1)1}_0\rangle = (p^+_1)^2 (\ldots), \tag{124}
\]
\[
|\Lambda^{(0)0}_0\rangle = P^+_1 |\varepsilon\rangle - ip^+_1 |\varepsilon_1\rangle - i\eta_1^+ p^+_1 P^+_1 |\varepsilon_2\rangle - iq^2 p^+_1 P^+_1 |\varepsilon_3\rangle - in^2 p^+_1 P^+_1 |\varepsilon_4\rangle + (p^+_1)^2 (\ldots) + P^+_2 (\ldots), \tag{125}
\]
\[
|\Lambda^{(0)1}_0\rangle = -ip^+_1 P^+_1 |\varepsilon_5\rangle + (p^+_1)^2 (\ldots) + P^+_2 (\ldots). \tag{126}
\]

As has been shown, we have to make a gauge transformation with a parameter linear in \( p^+_1 \) (parameters which do not depend on \( p^+_1 \) are absent from \( |\Lambda^{(1)}\rangle \)). Therefore, we use \( |\omega\rangle \) to make such a gauge transformation. Since

\[
\delta|\varepsilon\rangle = -T^+_1 |\omega\rangle, \tag{127}
\]

we use \( |\omega\rangle \) to eliminate the dependence on \( b^+_2 \) and \( f^+ \) from \( |\varepsilon\rangle \),

\[
b_2 |\varepsilon\rangle = f |\varepsilon\rangle = 0, \tag{128}
\]

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and then we remove the $\mathcal{P}_2^+$ dependence from $|\Lambda^{(0)}\rangle$ as described above.

Let us turn to the gauge-fixing of the fields. We decompose the field in ghosts $\mathcal{P}_1^+$, $\mathcal{P}_2^+$ by analogy with the gauge functions,

$$|\chi^0_{0j}\rangle = |\chi^{j}_{00}\rangle + \mathcal{P}_1^+|\chi^{j}_{01}\rangle + \mathcal{P}_2^+|\chi^{j}_{02}\rangle + \mathcal{P}_1^+\mathcal{P}_2^+|\chi^{j}_{03}\rangle, \quad j = 0, 1, \quad (129)$$

with each $|\chi^0_{0m}\rangle$ being an expansion in $p_1^+$,

$$|\chi^0_{0m}\rangle = \sum_{k=0}^{\infty} \langle p_1^+ \rangle^k |\chi_{0m}\rangle. \quad (130)$$

By analogy with the gauge-fixing of $|\Lambda^{(0)}\rangle$, we have to use $|\varepsilon_i\rangle$ and then remove the $\mathcal{P}_2^+$ dependence from the fields $|\chi^0_{0j}\rangle$.

Substituting (125), (126) into (109), (110), we find a gauge transformation for the fields independent of the ghost $p_1^+$,

$$\delta|\chi^0_{00}\rangle = L_1^+|\varepsilon\rangle + T_1^+|\varepsilon_1\rangle, \quad (131)$$

$$\delta|\chi^0_{01}\rangle = \eta^0_1 \left(-T_1^+|\varepsilon_2\rangle - L_1^+|\varepsilon\rangle - |\varepsilon_5\rangle\right) + \eta^0_2 \left(-T_1^+|\varepsilon_4\rangle - L_2^+|\varepsilon\rangle - |\varepsilon_3\rangle\right) + q^0_1 \left(-T_1^+|\varepsilon_3\rangle - T_1^+|\varepsilon\rangle\right), \quad (132)$$

$$\delta|\chi^0_{10}\rangle = -T_0^+|\varepsilon_1\rangle - 2|\varepsilon_1\rangle - T_1^+|\varepsilon_5\rangle. \quad (133)$$

Thus, we can first remove the $b_2^+$ and $f^+$ dependence of $|\chi^{00}_{00}\rangle$, using $|\varepsilon_1\rangle$, and then remove the $b_2^+$ dependence of $|\chi^{00}_{00}\rangle$, using the restricted (128) gauge parameter $|\varepsilon\rangle$. We then remove the $b_2^+$ and $f^+$ dependence of $|\chi^{00}_{00}\rangle$ and $|\chi^{00}_{01}\rangle$, using the gauge parameters $|\varepsilon_5\rangle$, $|\varepsilon_2\rangle$, $|\varepsilon_3\rangle$, $|\varepsilon_4\rangle$. After this, we remove the $\mathcal{P}_2^+$ dependence of the fields. Now, all the gauge parameters have been used, and we have the following conditions for the fields:

$$b_1|\chi^{00}_{00}\rangle = b_2|\chi^{00}_{00}\rangle = f|\chi^{00}_{00}\rangle = 0, \quad (134)$$

$$b_2|\chi^{00}_{01}\rangle = f|\chi^{00}_{01}\rangle = 0, \quad j = 0, 1, \quad (135)$$

$$\eta_2|\chi^{00}_{00}\rangle = \eta_2|\chi^{00}_{01}\rangle = 0. \quad (136)$$

Since $gh(|\chi^{00}_{00}\rangle) = 0$, it does not depend on ghost “coordinates” and due to (134) we conclude, after gauge-fixing, that

$$|\chi^{00}_{00}\rangle = |\Phi\rangle, \quad (137)$$

where $|\Phi\rangle$ is the physical field (5).

Let us turn to removing the auxiliary fields with the help of a part of the equations of motion.

### 6.2 Removing auxiliary fields by equations of motion

Let us decompose equation (105) in $\mathcal{P}_1^+$, $\mathcal{P}_2^+$ and consider the coefficients of $\mathcal{P}_2^+$-dependent parts:

$$-2\mathcal{P}_2^+\mathcal{P}_1^+ : \quad q^0_1|\chi^{00}_{00}\rangle = 0, \quad (138)$$

$$-2\mathcal{P}_2^+ : \quad q^0_2|\chi^{00}_{00}\rangle - r\eta^+_1 (L_2 - 2l_2')|\chi^{00}_{01}\rangle = 0. \quad (139)$$

Using (138), we conclude that $|\chi^{00}_{01}\rangle$ contains no terms of an order larger than the first order in $p_1^+$,

$$|\chi^{00}_{01}\rangle = |\chi^{00}_{01}\rangle + p_1^+|\chi^{01}_{01}\rangle. \quad (140)$$
Substituting (140) to (139), we find
\[ |\chi_{00}\rangle = \sum_{k=0}^{3} (p_{1}^{+})^{k} |\chi_{00}^{0k}\rangle, \]
where
\[ |\chi_{00}^{02}\rangle = -\frac{r}{2} \eta_{1}^{+}(L_{2} - 2l_{2}')|\chi_{00}^{00}\rangle, \quad |\chi_{00}^{03}\rangle = -\frac{r}{6} \eta_{1}^{+}(L_{2} - 2l_{2}'|\chi_{00}^{01}\rangle. \]

In a similar way, we decompose equation (106) and obtain
\[ -2p_{2}^{+}p_{1}^{+} : \quad q_{1}^{2}|\chi_{01}\rangle = 0, \]
\[ -2p_{2}^{+} : \quad q_{1}^{2}|\chi_{00}\rangle - r\eta_{1}^{+}(L_{2} - 2l_{2}')|\chi_{01}\rangle - \frac{r}{2}(T_{1} - 2t_{1}')|\chi_{01}\rangle = 0. \]

These equations have the following solution:
\[ |\chi_{01}\rangle = |\chi_{01}^{0}\rangle + p_{1}^{+}|\chi_{01}^{1}\rangle, \quad |\chi_{00}\rangle = \sum_{k=1}^{3} (p_{1}^{+})^{k} |\chi_{00}^{1k}\rangle, \]
where
\[ |\chi_{00}^{12}\rangle = -\frac{r}{2} \eta_{1}^{+}(L_{2} - 2l_{2}')|\chi_{01}^{10}\rangle - \frac{r}{4}(T_{1} - 2t_{1}')|\chi_{01}^{00}\rangle, \]
\[ |\chi_{00}^{13}\rangle = -\frac{r}{6} \eta_{1}^{+}(L_{2} - 2l_{2}')|\chi_{11}\rangle - \frac{r}{12}(T_{1} - 2t_{1}')|\chi_{01}\rangle. \]

Let us now consider a part of equations (105), (106) containing the physical field \( |\chi_{00}^{0}\rangle = |\Phi\rangle \)
(137),
\[ (t_{0} + \hat{g}_{m_{0}})|\Phi\rangle + iT_{1}^{+}|\chi_{00}^{11}\rangle + T_{1}^{+}|\chi_{01}^{10}\rangle - r(L_{2}^{+} - 2L_{2}')|\chi_{00}^{01}\rangle = 0, \]
\[ q_{1}^{+} : \quad t_{1}|\Phi\rangle + iT_{1}^{+}|\chi_{00}^{11}\rangle + T_{1}^{+}|\chi_{01}^{10}\rangle - r(L_{2}^{+} - 2L_{2}')|\chi_{00}^{01}\rangle = 0, \]
\[ \eta_{1}^{+} : \quad t_{1}|\Phi\rangle - iT_{1}^{+}|\chi_{01}^{01}\rangle + L_{1}^{+}|\chi_{01}^{10}\rangle - r(L_{2}^{+} - 2L_{2}')|\chi_{01}^{01}\rangle = 0, \]
\[ \eta_{2}^{+} : \quad t_{2}|\Phi\rangle - iT_{1}^{+}|\chi_{00}^{01}\rangle + T_{1}^{+}|\chi_{01}^{01}\rangle - r(L_{2}^{+} - 2L_{2}')|\chi_{00}^{01}\rangle = 0, \]
where we have decomposed the fields \( |\chi_{01}^{0}\rangle \) and \( |\chi_{00}^{1}\rangle \) as follows:
\[ |\chi_{01}^{0}\rangle = q_{1}^{+}|\chi_{01}^{01}\rangle + \eta_{1}^{+}|\chi_{01}^{01}\rangle + \eta_{2}^{+}|\chi_{01}^{02}\rangle, \quad |\chi_{00}^{1}\rangle = q_{1}^{+}|\chi_{00}^{01}\rangle + \eta_{1}^{+}|\chi_{01}^{01}\rangle + \eta_{2}^{+}|\chi_{00}^{02}\rangle. \]

Acting on (149) by the operators \( f \) and \( b_{2} \) and taking into account the gauge-fixing condition (134), (135), we obtain, respectively,
\[ fT_{1}^{+}|\chi_{00}^{01}\rangle = 0, \quad b_{2}T_{1}^{+}|\chi_{00}^{01}\rangle = 0. \]

These equations yield
\[ |\chi_{00}^{01}\rangle = 0. \]

\[ \text{The term } |\chi_{00}^{0}\rangle \text{ corresponding to } (p_{1}^{+})^{0} \text{ is absent in the sum of } (145) \text{ due to the ghost number restriction (101).} \]
Acting on \((151)\) by \(f\) and \(b_2\), we obtain
\[
f T_1^+ |\chi_{002}^{010}\rangle = b_2 T_1^+ |\chi_{002}^{010}\rangle = 0 \quad \implies \quad |\chi_{002}^{010}\rangle = 0. \tag{155}
\]

We now act on equation \((148)\) by the operator \(f\),
\[
f T_1^+ |\chi_{00}^{11}\rangle = 0, \tag{156}
\]
and then presenting the vector \(|\chi_{00}^{11}\rangle\) as a power series in \(f^+\)
\[
|\chi_{00}^{11}\rangle = |a_{00}^{11}\rangle + f^+|b_{00}^{11}\rangle, \tag{157}
\]
we have
\[
|a_{00}^{11}\rangle = t_1^+|b_{00}^{11}\rangle. \tag{158}
\]
Substituting this result into equation \((145)\) and acting twice by the operator \(b_2\) on the resulting equations, we arrive at
\[
b_2 |b_{00}^{11}\rangle = 0. \tag{159}
\]
Finally, equation \((148)\) acquires the form
\[
(t_0 + \gamma m_0) |\Phi\rangle + 2iL_2^+ |b_{00}^{11}\rangle + L_1^+ |\chi_{01}^{10}\rangle - r(L_2^+ - 2t_2^+)|\chi_{010}^{001}\rangle = 0. \tag{160}
\]
Let us present the state vectors \(|b_{00}^{11}\rangle\), \(|\chi_{01}^{10}\rangle\), \(|\chi_{010}^{001}\rangle\) as a power series in \(b_1^+\),
\[
|b_{00}^{11}\rangle = \sum_{k=0}^{n-2} (b_1^+)^k |b_{00}^{11}\rangle^k, \quad |\chi_{01}^{10}\rangle = \sum_{k=0}^{n-1} (b_1^+)^k |\chi_{01}^{10}\rangle^k, \quad |\chi_{010}^{001}\rangle = \sum_{k=0}^{n-2} (b_1^+)^k |\chi_{010}^{001}\rangle^k, \tag{161}
\]
and then, presenting equations \((160)\) and \((149)\) as a power series in \(b_1^+\) and \(b_2^+\), we have
\[
(b_1^+)^n : \quad m_1 |\chi_{01}^{10}\rangle^{n-1} = 0, \quad \tag{162}
(b_1^+)^{n-1} : \quad m_1 |\chi_{01}^{10}\rangle^{n-2} = -t_1^+ |\chi_{01}^{10}\rangle^{n-1}, \tag{163}
1 \leq k \leq n-2 \quad (b_1^+)^k : \quad m_1 |\chi_{01}^{10}\rangle^{k-1} = -t_1^+ |\chi_{01}^{10}\rangle^k - 2it_2^+ |b_{00}^{11}\rangle^k + rt_2^+ |\chi_{010}^{001}\rangle^k, \tag{164}
(b_1^+)^0 : \quad (t_0 + \gamma m_0) |\Phi\rangle = -2it_2^+ |b_{00}^{11}\rangle^0 - t_1^+ |\chi_{01}^{10}\rangle^0 + rt_2^+ |\chi_{010}^{001}\rangle^0, \tag{165}
\]
\[
0 \leq k \leq n-2 \quad b_2^+ (b_1^+)^k : \quad |b_{00}^{11}\rangle^k = \frac{ir}{2} |\chi_{010}^{001}\rangle^k. \tag{166}
\]
\[
(b_1^+)^{n-1} : \quad m_1 |\chi_{010}^{001}\rangle^{n-2} = |\chi_{010}^{10}\rangle^{n-1}, \quad \tag{167}
1 \leq k \leq n-2 \quad (b_1^+)^k : \quad m_1 |\chi_{010}^{001}\rangle^{k-1} = |\chi_{010}^{10}\rangle^k - t_1^+ |\chi_{010}^{001}\rangle^k, \tag{168}
(b_1^+)^0 : \quad t_1 |\Phi\rangle = |\chi_{010}^{10}\rangle^0 - t_1^+ |\chi_{010}^{001}\rangle^0. \tag{169}
\]
Using \((162)\) and \((163)\), we have \(|\chi_{01}^{10}\rangle^{n-1} = |\chi_{01}^{10}\rangle^{n-2} = 0\). Substituting this result into \((167)\) and then into \((168)\) for \(k = n - 2\), we obtain \(|\chi_{010}^{001}\rangle^{n-2} = |\chi_{010}^{001}\rangle^{n-3} = 0\). Turning to equation \((166)\) for \(k = n - 2\) and \(k = n - 3\), we conclude that \(|b_{00}^{11}\rangle^{n-2} = |b_{00}^{11}\rangle^{n-3} = 0\). We now repeat the procedure starting from equation \((163)\) for \(k = n - 2\) and \(k = n - 3\). Finally, we obtain
\[
(t_0 + \gamma m_0) |\Phi\rangle = 0, \quad t_1 |\Phi\rangle = 0, \tag{170}
|\chi_{01}^{10}\rangle = |\chi_{010}^{001}\rangle = 0, \quad |b_{00}^{11}\rangle = 0 \quad \implies \quad |\chi_{00}^{11}\rangle = 0. \tag{171}
\]
Using \((170)\), we can see that the physical state \((5)\) satisfies \((1)\), \((2)\), or equivalently \((45)\), provided that condition \((115)\) is taken into account.

Let us now turn to equations \((150)\), \((151)\). Taking into account \((154)\), \((155)\), \((171)\), we can see that these two equations read

\[
\begin{align*}
l_1|\Phi\rangle - iT_1^+|\chi_{010}\rangle + L_1^+|\chi_{011}\rangle - 2r(L_2^+ - 2P_2^+)|\chi_{012}\rangle = 0, \\
l_2|\Phi\rangle + L_2^+|\chi_{012}\rangle - |\chi_{011}\rangle = 0.
\end{align*}
\]

They are analogous to \((148)\) and \((149)\), where \(|\chi_{011}\rangle = 0\) in \((151)\) is taken into account. Therefore, we can repeat the procedure that has been carried out with equations \((148)\), \((149)\), and thus we arrive at the conclusion that

\[
\begin{align*}
l_1|\Phi\rangle = l_2|\Phi\rangle = 0, \\
|\chi_{010}\rangle = |\chi_{012}\rangle = |\chi_{011}\rangle = 0.
\end{align*}
\]

Equations \((174)\) are consequences of \((170)\); they do not impose any additional restrictions on the physical state \((5)\). Collecting \((154)\), \((155)\), \((171)\), \((175)\) and \((152)\), we have

\[
|\chi_{10}\rangle = 0, |\chi_{11}\rangle = 0, |\chi_{00}\rangle = 0.
\]

Observing \((140)\), \((141)\), \((142)\) and \((145)\), \((146)\), \((147)\), we can see that it remains to show that \(|\chi_{01}\rangle = |\chi_{11}\rangle = 0\). To prove this fact, we decompose \((105)\) and \((106)\) in the ghosts \(p_1^+\), \(P_1^+\) and consider the equations which are the coefficients at \((p_1^+)^0P_1^+\). With allowance for \((176)\), these equations read

\[
\begin{align*}
iT_1^+|\chi_{01}\rangle - \eta_2 p_1|\chi_{01}\rangle = 0, \\
iT_1^+|\chi_{11}\rangle - \eta_2 p_1|\chi_{11}\rangle = 0,
\end{align*}
\]

respectively. Decomposing \(|\chi_{01}\rangle\), \(|\chi_{11}\rangle\) in ghost “coordinates”,

\[
\begin{align*}
|\chi_{01}\rangle = q_1^+|\chi_{010}\rangle + \eta_1^+|\chi_{011}\rangle + \eta_1^+|\chi_{012}\rangle, \\
|\chi_{11}\rangle = q_1^+|\chi_{110}\rangle + \eta_1^+|\chi_{111}\rangle + \eta_1^+|\chi_{112}\rangle,
\end{align*}
\]

and substituting the result into \((177)\), we obtain equations for the coefficients of \((178)\) in the form

\[
T_1^+|\chi_{01m}\rangle = 0 \quad \Rightarrow \quad |\chi_{01m}\rangle = 0.
\]

Therefore, all the coefficient in decompositions \((178)\) of \(|\chi_{01}\rangle\), \(|\chi_{11}\rangle\) are equal to zero, and we conclude that

\[
|\chi_{01}\rangle = |\chi_{11}\rangle = 0.
\]

Thus, we have shown that after the gauge fixing \((134)\)–\((136)\) all the auxiliary fields become equal to zero, \((176)\), \((180)\), and the physical state \(|\Phi\rangle = |\chi_{00}\rangle\) \((137)\) obeys equations \((1)\), \((2)\).

Let us now consider some examples of the Lagrangian construction procedure.

### 7 Examples

Here, we shall illustrate the general procedure of gauge-invariant Lagrangian construction by using the examples of fermionic fields of spin 1/2 and 3/2.
7.1 Spin-1/2 field

For a fermionic field of spin \( s = \frac{1}{2} \), we have \( h = 2 - \frac{d}{2} \). Then the only nonvanishing vector \( |\chi_0^0\rangle_0 \) subject to condition (107) and having the proper ghost number (92) has the form

\[
|\chi_0^0\rangle_0 = \psi(x)|0\rangle, \quad a|\chi^0_0\rangle = a(0)|\psi^+(x)\rangle_0. \tag{181}
\]

Then, due to (75), (86) for \( K_0 = |0\rangle\langle 0| + \ldots \), (115), the action implied by (108) has the form

\[
S_0 = a|\chi^0_0\rangle_0 T_0|\chi^0_0\rangle_0 = -\int d^dx \sqrt{|g|} \bar{\psi} \left\{ i\gamma^\mu \nabla_\mu - m - r^{\frac{d}{2}} \left( \frac{d}{2} - 2 \right) \right\} \psi. \tag{182}
\]

Here, we have applied the definition (14) for the conventional gamma-matrices and have introduced the Dirac-conjugate spinor \( \bar{\psi} = \psi^+ \gamma^0 \). Thus, we can see that the action (182) reproduces equation (1) for \( n = 0 \), which corresponds to spin-1/2 field.

7.2 Spin-3/2 field

In the case of a spin-3/2 field, we have \( n = 1, h = 1 - d/2, m_0 = m + r^{\frac{d}{2}}(d/2 - 2) \). Since \( n_{\text{max}} = 0 \), the corresponding Lagrangian formulation is an irreducible gauge theory. Due to \( gh(|\Lambda^0_0\rangle_1) = -2 \), the nonvanishing fields \( |\chi_0^0\rangle_1, |\chi_1^0\rangle_1 \) and the gauge parameter \( |\Lambda_0^0\rangle_1 \), for \( |\Lambda_0^0\rangle_1 \equiv 0 \), possess the following Grassmann grading and ghost number distributions:

\[
(\varepsilon, gh) (|\chi_0^0\rangle_1) = (1, 0), \quad (\varepsilon, gh) (|\chi_1^0\rangle_1) = (1, -1), \quad (\varepsilon, gh) (|\Lambda^0_0\rangle_1) = (0, -1). \tag{183}
\]

These conditions determine the dependence of the fields and gauge parameter on the oscillator variables in a unique form:

\[
|\chi_0^0\rangle_1 = \left[ -i a^+ \psi(x) + f^+ \bar{\psi}(x) + b_1^+ \phi(x) \right] |0\rangle, \quad |\chi_1^0\rangle_1 = \left[ P_1^+ \bar{\gamma} \chi(x) + i p_1^+ \chi_1(x) \right] |0\rangle, \tag{184}
\]

\[
|\Lambda^0_0\rangle_1 = \left[ P_1^+ \xi_1(x) - i p_1^+ \gamma \xi_2(x) \right] |0\rangle. \tag{186}
\]

Substituting (184), (185) into (108), we find the action (up to an overall factor) for a spin-3/2 field interacting with the AdS background:

\[
S_1 = -\int d^dx \sqrt{|g|} \left\{ \bar{\psi} \left\{ i\gamma^\mu \nabla_\mu - m_0 \right\} \psi - \nabla_\mu \chi + i\gamma^\mu \chi_1 \right\} - \left\{ \left[ (d-2)\bar{\psi} - \frac{m_0}{m_1} \phi \right] \left\{ i\gamma^\mu \nabla_\mu + m_0 \right\} \psi - \frac{r(d-1)}{2m_1} \phi - \chi_1 \right\} - \left\{ \frac{M^2}{m_1^2} \bar{\psi} + \frac{m_0}{m_1} \phi \right\} \left\{ i\gamma^\sigma \nabla_\sigma - m_0 \phi - 2m_1 \psi - m_1 \chi_1 \right\} + \bar{\chi} \left\{ i\gamma^\mu \nabla_\mu + m_0 \chi + \chi_1 + \nabla^\mu \psi_\mu + m_0 \psi + m_1 \phi \right\} + \bar{\chi}_1 \left\{ i\gamma^\mu \psi_\mu + (d-2)\psi - \chi - \frac{m_0}{m_1} \phi \right\}, \tag{187}
\]

where \( M^2 = m_0^2 - \frac{1}{4} r(d-1)(d-2) \). To obtain the action (187), we have used the expressions for the operators \( \hat{K}_1 \) (75), (86). Substituting (184)–(186) into (109), (110), we find the gauge transformations

\[
\delta \psi_\mu = \nabla_\mu \xi_1 + i\gamma_\mu \xi_2, \quad \delta \psi = \xi_2, \quad \delta \phi = m_1 \xi_1 \tag{188}
\]

\[
\delta \chi = \left[ i\gamma^\mu \nabla_\mu - m_0 \right] \xi_1 - 2\xi_2, \quad \delta \chi_1 = \left[ i\gamma^\mu \nabla_\mu + m_0 \right] \xi_2 - \frac{r}{2} (d-1) \xi_1. \tag{189}
\]
Let us present the action in terms of one physical field $\psi_\mu$. To this end, we get rid of the fields $\varphi, \psi$, by using their gauge transformations and the gauge parameters $\xi_1, \xi_2$, respectively. Having expressed the field $\chi$, using the equation of motion $\chi = i\gamma^\mu\psi_\mu$, we can see that the terms with the Lagrangian multiplier $\chi_1$ turn to zero. As a result, we obtain

$$\mathcal{L}_{RS} = \bar{\psi}_\mu(i\gamma^\sigma \nabla_\sigma - m_0)\psi_\mu - i\bar{\psi}_\mu(\gamma_\nu \nabla_\mu + \gamma_\mu \nabla_\nu)\psi^\nu + \bar{\psi}_\mu\gamma_\nu(i\gamma^\sigma \nabla_\sigma + m_0)\gamma^\mu \psi_\nu.$$

This is a generalization of the Rarita–Schwinger Lagrangian to a $d$-dimensional AdS space.

8 Conclusion

We have constructed a gauge-invariant Lagrangian formulation of half-integer totally symmetric higher spin fields in the AdS space of any dimension in the “metric-like” formulation. The results of this study are most general and apply to both massive and massless fermionic higher spin fields in the AdS, Minkowski, and dS$^{10}$ spaces.

Starting from embedding the fermionic higher spin fields into vectors of an auxiliary Fock space, we treat the fields as components of these Fock-space vectors, and, as a result, we reformulate the theory in terms of such vectors. We realize the conditions that define an irreducible representation of the AdS group with a given mass and spin in terms of differential operators acting in this Fock space. The mentioned conditions are interpreted as constraints imposed on the Fock space vectors and generate a closed higher spin nonlinear symmetry superalgebra being the basic object of this study.

It is shown that the derivation of a correct Lagrangian formulation requires a transition to another constraint basis for the original symmetry algebra, which is algebraically equivalent to the initial basis and does not contain the constraints that define an irreducible representation of the AdS group. This set of constraints provides an additive extension of the initial algebra to another operator algebra. Then, as shown in [33], it is necessary to deform the initial algebra. As a result, one obtains a nonlinear superalgebra of enlarged constraints, whose construction by means of the additional operators realizes a special conversion of the initial system of first- and second-class operator constraints into a system of first-class ones with a preservation of the initial algebraic structure for the deformed constraints (see [37] for the elaboration of conversion methods). Due to the nonlinearity of the underlying algebra of the enlarged constraints, the corresponding BRST operator is defined ambiguously, and we construct it in an exact form with a non-vanishing term of third order in powers of ghosts for the case of a supersymmetric ordering of the constraints in commutator relations. It is shown that the resulting BRST operator yields a consistent Lagrangian dynamics for fermionic fields of any spin. The corresponding Lagrangian formulation is constructed in a concise form in terms of a Fock space and proves to be a reducible gauge theory with a finite number of reducibility stages, the number growing with the spin value. It is interesting to observe that the methods developed for the quantization of gauge theories turn out to be extremely efficient for deriving classical gauge-invariant Lagrangians for higher spin field theories, thus reflecting one more side of the BV–BFV duality concept [38], which permits one to construct, by means of a Hamiltonian BFV–BRST charge, the objects used in Lagrangian formalism.

We have proved that the Lagrangian equations of motion $[105], [106]$, after a partial gauge-fixing, reproduce the equations corresponding to the relations determining the irreducible representation of the AdS group. This proof completes the derivation of a correct gauge-invariant

\[10\] In the case of HS fields on dS space the corresponding Lagrangians lose the property of reality due to $r^{1/2}$. 

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Lagrangian formulation for higher spin fermionic fields in the AdS space. As examples demonstrating the general scheme of Lagrangian construction, we have obtained gauge-invariant Lagrangian descriptions of fields with spin-1/2 and spin-3/2 in an explicit form. In principle, the constructed description permits one to obtain explicit Lagrangians for any other half-integer-spin fields.

The basic results of the present work are given by relations (108), where the Lagrangian action for a field with an arbitrary half-integer spin is constructed, and by (109)−(112), where the gauge transformations for the fields are presented as well as the sequence of reducible gauge transformations for the gauge parameters.

Concluding, we shall discuss two points. First, the gauge-invariant formulation for massive higher spin field theories in the AdS space fields can have very interesting applications in the calculation of a quantum effective action for higher spin massive fields in the AdS space. The Lagrangians of all such models have a gauge-invariant kinetic term and a mass term violating the gauge invariance. Such a structure of a Lagrangian leads to some problems of quantum calculations in a curved space-time. To avoid these problems, it is natural to construct a gauge-invariant formulation by using appropriate Stückelberg fields. Then, one can impose gauge-fixing conditions removing the gauge degeneration of the kinetic term in the Lagrangian of the basic fields and apply the standard techniques for the calculation of an effective action (see the examples of such calculations in [39]). We emphasize that the BRST approach leads automatically to a gauge-invariant Lagrangian with the entire set of appropriate Stückelberg fields. As a result, one has a basis for constructing the effective action of massive higher spin fields in the AdS space. Second, the BRST approach can provide a systematic method of constructing interaction vertices for higher spin fields in the AdS space [30]. Therefore, one can hope that the BRST construction developed in this paper will be useful in deriving interaction vertices for massive higher spin fermionic fields in the AdS space.

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