Joint Frequency Assignment and Flow Control for Hybrid Terrestrial-Satellite Backhauling Networks

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Abstract—In this paper, we investigate the problem of cross-layer design of the link scheduling, frequency assignment and flow control in hybrid terrestrial-satellite wireless backhauling networks. Considering network limitations and requirements, the target is to maximize the traffic that can be delivered by the network in a given period of time by deciding the active backhauling links that can transmit simultaneously over the different frequencies as well as the amount of traffic that should be forwarded in these active links. Due to the interference between the different links, the problem is NP-hard and an efficient scheme is developed to estimate the interference, determine the link capacity lower bound, and obtain the final allocation. The proposed algorithm is simulated in a realistic topology and assuming parameters of practical interest to reveal the advantage of the developed scheme.

Keywords—Cross-layer Design, Flow Control, Link Scheduling, Satellite Networks, Wireless Backhauling.

I. INTRODUCTION

Wireless backhauling provides an efficient alternative to complement the wired backhauling solution and should provide high throughput and reliable service to the end users as the wired backhauling does. Maximizing the network throughput goes beyond the isolated optimization of the physical layer, and thus a cross-layer design approach is often required to efficiently handle the link scheduling as well as the traffic that should be transmitted over these links. [1], [2]. The different backhauling nodes need to transmit/receive their own traffic in addition to route other nodes’ traffic. Accordingly, an adequate link scheduling and frequency assignment policy allows the network nodes to have simultaneous transmissions which increases the network spectrum efficiency and guarantees minimum throughput in the different links.

A survey paper that summarizes the main results regarding the frequency assignment and flow control in wireless mesh networks can be found in [3]. The problem of channel scheduling in multi-radio multi-channel network is proved to be an NP-Hard problem [4]. In [5] and [6], the problem of assigning a subset of subcarriers to a given link in wireless mesh networks is treated. The assignment is based on the channel conditions. Frequency reuse is not allowed and accordingly each frequency band is assigned to only one link at a given time. In [7], the achievable rates in multi-hop wireless mesh networks with orthogonal channels are determined where tight necessary and sufficient conditions for the achievability of the rate vector are explained. In [8], frequency reuse is enabled in orthogonal frequency division multiple access (OFDMA) based networks and heuristic algorithms for joint power control, frequency allocation and link scheduling are developed. This work is extended by the same authors in [9] where they relax part of the constrains and estimate the capacity of the links in low and high SNR regimes.

In [10], fair end-to-end transmission is achieved by developing a distributed algorithm for joint power and subcarrier allocation in OFDMA based wireless mesh networks. Based on pre-determined transmitting paths, the joint rate and power control problem is treated as a network utility maximization problem with interference consideration. In [11], interference aware resource allocation in OFDMA based wireless mesh networks is proposed. The frequencies are allocated to the different links so as to guarantee sufficient interference mitigation. A given frequency band is assumed not to be reused by nodes that are within the interference distance (the interference model protocol). Based on this work, the same authors adopted a decoupled routing and scheduling scheme [12] to maximize the network overall throughput. The interference aware resource allocation in [11] is applied and afterwards the flow is calculated in order to maximize the delivered throughput. Based on the resulted flow, the assignment of the frequency bands as well as the time slots are performed in order to evaluate the final scheduling. A general framework based on [11] and [12] is published in [13].

Hoteit et al. in [14] modeled the resource allocation problem as a bankruptcy game taking into account the interference between the different nodes and different solutions were identified based on cooperative game theory. Uddin et al. in [15] considered optimally partitioning the spectrum into a set of non-overlapping bands with non-uniform widths to allow more parallel transmission. Additionally, a low complexity heuristic algorithm is also developed to reduce the computational complexity of the optimal scheme.

Our work differs from the aforementioned papers in the fact that it considers the joint flow control and link scheduling in a multi-frequency wireless backhaul network where both terrestrial and satellite links are available in the system. The problem is solved by taking the half-duplex and uni-cast radio limitations of the network links into account. Furthermore, frequency reuse is enabled in the network under pre-determined link scheduling constraints. The optimization problem is formulated in such a way that it enables adding constraints to the
traffic path of a given node and allows the network to provide some priority between nodes if needed.

The rest of the paper is organized as follows. Section II provides details of the system model while Section III formulates the optimization problem and discusses the solution approach. Simulation results are provided and discussed in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

The system considered in this work is a multi-hop wireless backhaul network, where there are several backhauling nodes that serve multiple users in a given geographical area. Not all backhauling nodes have direct connection to the core network. Accordingly, these distant nodes reach the core network through satellite. A schematic example of the considered multi-hop network is depicted in Fig. 1.

The network is modeled as a directed graph \( G = (\mathcal{N}, \mathcal{L}) \) where \( \mathcal{N} = \{0, 1, \ldots, N\} \) is the set of all backhauling nodes (vertices) that are assumed to be equipped with multiple antennas. Node 0 represents the core network and \( \mathcal{L} \) is the set of all available links (edges). A link exists between two nodes if they are in the transmission range of each other, i.e. \( \mathcal{L} = \{l, \Gamma(t(l), r(l)) \leq T_t(l), t(l) \in N, r(l) \in N, t(l) \neq r(l)\} \) where \( \Gamma(t(l), r(l)) \) is the distance between the transmitter on the \( l^{th} \) link, \( t(l) \), and the receiver on the \( l^{th} \) link, \( r(l) \), and finally, \( T_t(l) \) is the transmission range of the transmitting node \( t(l) \). The incidence matrix of the graph \( G \) is defined as follows

\[
I(n, l) = \begin{cases} 
1, & \text{if node } n \text{ is transmitting on } l, \\
-1, & \text{if node } n \text{ is receiving on } l, \\
0, & \text{otherwise}
\end{cases}
\]

(1)

The backhauling nodes are assumed to have access to \( K \) channels of bandwidth \( W \) Hz each. Let \( \mathcal{F} = \{f_1, f_2, \ldots, f_K\} \) denotes the set of available channels. We are interested in the problem of upstreaming the backhauling nodes traffic to the core network. Nevertheless we note that this work can be suitably adapted to operate on the downstream direction. The satellite system is assumed to share the available frequency band with the terrestrial system under the conditions that will be described in the next section.

The Multi-Commodity Flow (MCF) model \cite{16}, \cite{17} is assumed here, considering that the queuing effect is negligible at each backhauling node and the aggregate traffic per node is constant over the execution time. Let \( x_{l,d}^{f_k} \) represent the amount of flow corresponding to the commodity \( d \) that is assigned to the \( l^{th} \) link over the frequency \( f_k \). The generated traffic at each backhauling node is considered as a single commodity \( s_n \). In order to fulfill the flow conservation law, it should hold that the incoming and outgoing flows for each commodity \( d \) and at each backhauling node, should be equal, that is,}

\[
\sum_{l \in O_n^+} \sum_{f_k \in \mathcal{F}} x_{l,d}^{f_k} = \sum_{l \in O_n^-} \sum_{f_k \in \mathcal{F}} x_{l,d}^{f_k} \quad \forall n \in \mathcal{N} \setminus \{0, d\}, d \in \mathcal{N} \setminus \{0\},
\]

(2)

where \( O_n^+ \) is the set of outgoing links from node \( n \), i.e. \( O_n^+ = l : I(n, l) = 1 \) while \( O_n^- \) is the set of incoming links to node \( n \), i.e. \( O_n^- = l : I(n, l) = -1 \). The nodes 0 and \( \mathcal{N} \) are excluded in (2) as they are representing the source and destination nodes respectively. For each source backhauling node, the difference between the amount of outgoing and incoming traffic should be equal to the amount of the traffic generated in the node as

\[
\sum_{l \in O_n^+} \sum_{f_k \in \mathcal{F}} x_{l,d}^{f_k} - \sum_{l \in O_n^-} \sum_{f_k \in \mathcal{F}} x_{l,d}^{f_k} = s_n \quad \forall d \in \mathcal{N} \setminus \{0\}.
\]

(3)

The rate that should be transmitted over any link should be less or equal than the capacity of that link \( C_{l,d}^{f_k} \), which can be expressed mathematically as

\[
\sum_d x_{l,d}^{f_k} \leq C_{l,d}^{f_k} \quad \forall f_k \in \mathcal{F}, l \in, \forall d \mathcal{L}.
\]

(4)

Each backhauling node is assumed to be half-duplex; thus implying that the node cannot receive and transmit simultaneously over the same channel. The backhauling nodes can communicate with multiple nodes simultaneously. However, it’s assumed that the backhauling node cannot transmit to different nodes over the same frequency, i.e. there is no broadcasting or multicasting. Additionally, the backhauling node cannot receive from multiple nodes over the same frequency. These constraints in addition to the previous one on capacity given in (4) can be expressed mathematically as

\[
\sum_{l \in O_n^+} \sum_{l \in O_n^+} x_{l,d}^{f_k} + \sum_{l \in O_n^-} \sum_{l \in O_n^-} x_{l,d}^{f_k} \leq \gamma \forall f_k \in \mathcal{F}, \forall n \in \mathcal{N} \setminus \{0\}.
\]

(5)

where \( \gamma = 2/3 \) defines a sufficient condition for feasible schedule as discussed in \cite{7}, \cite{9}, \cite{18} while \( \gamma = 1 \) defines a necessary condition. \( x_{l,d}^{f_k}/C_{l,d}^{f_k} \) represents the fraction of time the channel \( f_k \) is active on the \( l^{th} \) link.

The capacity of the channel is given by \( C_{l,d}^{f_k} = W \log_2 \left(1 + SINR_{l,d}^{f_k}\right) \), where \( W \) is the bandwidth of the
where $\Delta$ is a positive constant that is multiplied by the sum of flows of the commodities in the set $D$ that are transmitted over the links in the set $S$. Assuming that $S$ is representing the satellite system links and $D$ is representing the nodes with delay sensitive traffic, traffic generated by the nodes in $D$ can be prevented from being routed through the satellite links in order to avoid possible excessive propagation delays. Note that $\Delta$ can be used to activate and deactivate the second part of the objective function as well as calibrating the ratio of the flow that can be transmitted over the links in set $S$, $C1$ and $C2$ enforce the fulfillment of the flow conservation law while $C3$ represents the capacity and the link scheduling constraints. Constraint $C4$ ensures that only one frequency is used on the link at a given time. This constraint can be removed in case multi-frequency transmission is allowed. Lastly, constraint $C5$ ensures positive values of the amount of flow.

The objective function as well as the constraints $(C1, C2, C5)$ are linear. Constraints $(C3, C4)$ are nonlinear constraints thus making (8) a nonlinear optimization problem. In order to be able to solve the problem, one can try to find an estimate value of the link capacities in order to linearize the constraints $(C3, C4)$. The capacity of the link depends on the transmit power of the link and the interference received from other nodes sharing the same frequency. If the transmit power of all the backhauling nodes is fixed to a pre-defined value, the interference introduced to the link can be evaluated if the links that share the same frequency are identified. To go forward with the interference calculation, we start by evaluating the interference introduced by a given link to the rest of the links if they are assumed to share the same frequency. Accordingly, the matrix $I_{int}$ is constructed where the element $I_{int}(l^*,l)$ represents the received interference on the $l^{th}$ link due to the transmission occurring on the $l^*$th link. That is, the elements of matrix $I_{int}$ represent the amount of interference introduced to the row-link when the column link is transmitting on the same frequency. The matrix $I_{int}$ can be written as

$$
I_{int} = \begin{bmatrix}
0 & \cdots & I_{int}(1,L) \\
\vdots & \ddots & \vdots \\
I_{int}(L,1) & \cdots & 0
\end{bmatrix}.
$$

Afterwards, the links that have high interference to each other are considered as conflict links. Accordingly, an additional constraint is added to the original formulation to count for this conflict in order to enforce the system not to assign the same frequency to these links. To do so, the conflict matrix $L_{conf}$ should be constructed where the element $L_{conf}(l^*,l)$ is equal to one when the links $l^*$ and $l$ are in conflict and should not share the same frequency. If the incoming and outgoing links are added to the $L_{conf}(l^*,l)$ matrix as conflict links as well, the constraint $C3$ can be replaced by the following constraint which counts for both the link scheduling constraints as well as the links conflict constraints

$$
\sum_{l \in L} \sum_{d \in d} \frac{x_{l,d}^{f_k}}{C_l} L_{conf}(l^*,l) \leq \gamma, \forall l^* \in L, \forall f_k \in F,
$$

where

$$
L_{conf} = \begin{bmatrix}
1 & \cdots & L_{conf}(1,L) \\
\vdots & \ddots & \vdots \\
L_{conf}(L,1) & \cdots & 1
\end{bmatrix}.
$$
TABLE I. TERRESTRIAL SYSTEM PARAMETERS

| Parameter                | Value                        |
|--------------------------|------------------------------|
| Antenna Pattern          | ITU-R F.1245-2               |
| Max. antenna gain        | 38 dB                        |
| Channel Model            | Free Space Path Loss         |
| Noise Density            | -139 dBW/MHz                 |
| Transmit Power Density   | -38.13 dBW/MHz               |
| Total bandwidth          | 17.7-19.7 GHz                |
| Rate per backhauling node| 420 Mbps                     |

By assuming the worst case scenario, defined as the case in which all the channels with no conflict with a given link are using the same frequency, the total interference introduced to a given link can be evaluated. Accordingly, the capacity of each link can be calculated and the nonlinear constraints can be converted into linear ones which makes the problem convex and easier to solve.

IV. SIMULATION RESULTS

To evaluate the performance of the proposed scheme, we consider an extension of a real topology being used in Finland, which is depicted in Fig.2. The changes on the original topology correspond to the inclusion of hybrid nodes and the addition of extra links. In particular and as depicted in Fig.2, the changes are: 9 new bi-directional links have been included (in green), 1 bi-directional link between node 1 and node 5 has been removed (in red) while node 4 and node 12 have satellite transmission capabilities. Note that these nodes, 4 and 12, are far apart from each other and represent alternative offloading points from the terrestrial backhaul.

The satellite node is forwarding the incoming traffic and has no generated traffic and the interference from the satellite system to the terrestrial one is assumed to be negligible as the outgoing and incoming links to the hybrid backhauling node do not share the same frequency used by the satellite segment. Additionally, node 8 and the satellite are assumed to be connected to the core network. In total, the final extended network topology has $N = 15$ terrestrial nodes that are interconnected via $L = 44$ unidirectional communication links. The hybrid nodes are indicated with a violet dot in Fig.2.

The terrestrial topology shown in Fig.2 is the same as the one in Fig. 1. The real topology uses 8 carriers of $B_c = 56$ MHz and allocates the different terrestrial links as given by Finnish communications regulatory authority (FICORA). Carriers are divided into two blocks of 4 carriers each: one block from 17700 to 17924 MHz and another block from 18708 and 18934 MHz. The allocation is extended to be used for generating the benchmark related to the extended network. Accordingly, the related interference and rate information is also evaluated based on the information regarding the altitude and height of the different nodes that corresponding to the new links. For the proposed scheme, the worst case SINR is considered by assuming that all the links that have no conflict with a given link are sharing the same frequency and inducing interference to it. The links to the core are assumed to have sufficient capacity to deliver the data arrived at node 8 and at the satellite. This is assumed to avoid the case that the performance of the network under study is limited by the capacity of the connection to the core rather than by the characteristics of the network links. The traffic per node is assumed to be 420 Mbps for the backhauling nodes. This value is chosen so that compact figures with $\alpha \in [0, 2]$ are obtained. The proposed technique can have a solution for any positive traffic value. A summary of the terrestrial system parameters can be found in Table I.

### A. Performance Evaluation Considering 56 MHz Channels

In this part of the simulation, we fix the channel bandwidth to 56 MHz as applied in the benchmark scheme. Starting with active satellite links, i.e. $\Delta = 0$, and by solving the net flow maximization problem for the benchmark scheme, the delivered data rate is equal to 4888.10 Mbps. Considering that the benchmark scheme uses 8 channels, each with 56 MHz, the total used bandwidth is 448 MHz. Accordingly, the benchmark spectrum efficiency (SE) = 4888.10/448 = 10.91 bps/Hz. For the proposed scheme, the delivered rate and the SE as function of the number of the used channel is summarized in Table II.

Remarkably, it can be noticed that the proposed scheme can achieve the delivered rate by the benchmark scheme by using only four frequency bands. In particular, the SE gain that can be achieved corresponds to 2.47X compared to the benchmark scheme. Additionally, considering the same bandwidth for both system, i.e. 8 channels, the SE is improved by a factor of nearly 1.6. Fig. 3 depicts the ratio of the delivered traffic per node $\alpha$ against the number of used 56 MHz channels. The delivered rate increases as the number of channel increases due to the additional transmission bands and reduced interference. Note that no increment is achieved by increasing the number of channels beyond 9 as the system is able to manage the link scheduling problem in the network. Recall that every link can use only one frequency and the value of the capacities of a given link over the different frequencies is equal assuming.

### TABLE II. DELIVERED RATE AND SE VS. NO OF 56 MHz CHANNELS WITH ACTIVE SATELLITE LINKS

| No. of 56 MHz Channels | Delivered data rate (Mbps) | SE (bps/Hz) |
|------------------------|-----------------------------|-------------|
| 1                      | 1508.57                     | 26.94       |
| 2                      | 3017.14                     | 26.94       |
| 3                      | 4524.54                     | 26.93       |
| 4                      | 5309.61                     | 23.70       |
| 5                      | 6074.37                     | 21.69       |
| 6                      | 6776.30                     | 20.16       |
| 7                      | 7477.50                     | 19.07       |
| 8                      | 8179.39                     | 18.25       |
that a similar worst case SINR scheme is applied in all the frequencies.

The previous simulations are repeated after assuming that the satellite links are disabled, i.e. considering a high value of $\Delta$. By solving the net flow maximization problem for the benchmark scheme, the delivered data rate is equal to 2772 Mbps and hence the benchmark SE = $2772/448 = 6.1875$ bps/Hz. For the proposed scheme, the delivered rate and the SE as a function of the number of the used channel is summarized in Table III.

The proposed scheme can achieve the delivered rate provided by the benchmark network by using only four frequency bands, and the SE gain goes up by factor of 2.34 when considering 2 channels only. Additionally, considering the same bandwidth for both system, i.e. 8 channels, the SE is improved by factor of nearly 2. Fig. 4 depicts $\alpha$ against the number of used 56 MHz channels. It can be observed that no rate increment can be achieved by using more than 8 channels. Additionally, one can note that -as expected- disabling the satellite links will reduce the total amount of delivered traffic as the number of links connecting the network to the core is reduced.

### B. Performance Evaluation Considering Variable Channel Width

In this part, we assume that the satellite links are active. Additionally, we assume that the channel bandwidth is varying with the number of channels considered in the system. However, all the links are using equal-width channels. The total bandwidth is considered to be $8 \times 56$ MHz $= 448$ MHz and the per channel bandwidth is 448 $/K$. The delivered data rates as well as the SE are summarized in Table IV. The SE versus the number of channels is depicted in Fig 5.

It is remarkable that the SE decreases with the increase in the number of channels as the frequency is reused less. As the number of channels becomes more than 9, the system is using only 9 channels as anticipated from the results in Fig. 3 and hence, the SE of the system keeps constant. The data rate of the lower number of channels is higher than that of higher number of channels due to the use of more transmission power which increases the capacity of the links if they are scheduled adequately. This can be deduced from Fig. 6 where the energy efficiency (EE) of the terrestrial links of the system is depicted against the number of the channels. It is remarkable that EE increases with the increment of the number of the channels. It can be noted also that the benchmark scheme has EE =12.3365 Mbps/mW while the terrestrial links of proposed scheme with the same channels and bandwidth has EE = 20.6416 Mbps/mW.

### V. Conclusions

In this paper, the problem of cross layer optimization to obtain the frequency assignment, link scheduling and the links flow in hybrid terrestrial-satellite backhauling network is considered. The problem is formulated as an optimization problem with nonlinear constraints that relates the capacity and per link flow. As the original problem is hard to solve, an estimation of the link capacities is proposed in order to linearize the constraints. The estimation is based on the knowledge of the transmit power over the links as well as the interference introduced to that link. The interference is evaluated based on developed conflict matrix that controls the sharing of the frequencies between the different links. The proposed algorithm achieves an SE gain between 1.6 and 2 with respect to the classical benchmark solution.
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