Abstract

In previous work it was shown that the radius of nucleus $R$ is determined by the $\alpha$-cluster structure and can be estimated on the number of $\alpha$-clusters disregarding to the number of excess neutrons. A hypothesis also was made that the radius $R_m$ of a $\beta$-stable isotope, which is actually measured at electron scattering experiments, is determined by the volume occupied by the matter of the core plus the volume occupied by the peripheral $\alpha$-clusters. In this paper it is shown that the condition $R_m = R$ restricts the number of excess neutrons filling the core to provide the $\beta$-stability. The number of peripheral clusters can vary from 1 to 5 and the value of $R$ for heavy nuclei almost do not change, whereas the number of excess neutrons should change with the number of peripheral clusters to get the value of $R_m$ close to $R$. It can explain the path of the $\beta$-stability and its width. The radii $R_m$ of the stable isotopes with $12 \leq Z \leq 83$ and the alpha-decay isotopes with $84 \leq Z \leq 116$ that are stable to $\beta$-decay have been calculated.

keywords: nuclear structure; alpha-cluster model; charge radius; matter radius; excess neutrons

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1 Introduction

It is well known that the liquid drop model gives a successful formula to calculate the charge radii of stable nuclei in dependence on the number of nucleons in a nucleus $R_{ch} \sim A^{1/3}$. In Fig. 1 the experimental radii [1,2,3,4,5] of stable isotopes are shown in dependence on $A$ and $Z$. The experimental errors which in most of the cases are within an interval from 0.006 Fm to 0.060 Fm, are not presented in the figure.

![Figure 1: Experimental radii of stable isotopes in dependence on $A$ and $Z$.](image)

One can see that the values of the radii of nuclids of different isotopes of one element in the graph of the dependance on $Z$ are gathered in short vertical columns, whereas in the other graph the values are scattered around the line $R_{ch} \sim A^{1/3}$.
The charge radii calculated by the following equation

\[ R_{ch} = r_{ch}A^{1/3}, \]  

where \( r_{ch} \) denotes the average value of the charge radius of one nucleon in a nucleus and 
\( r_{ch} = 1.080 \) Fm for the nuclei with \( 6 \leq Z \leq 11 \), 
\( r_{ch} = 1.012 \) Fm for \( 12 \leq Z \leq 23 \), 
\( r_{ch} = 0.955 \) Fm for \( 24 \leq Z \leq 83 \)

have root mean square deviation from the experimental values of the most abundant isotopes 
\( <\Delta^2>^{1/2} = 0.067 \) Fm.

The formula to calculate the radii of the nuclei in dependence on \( Z \) or in dependence on the number of \( \alpha \)-clusters \( N_\alpha \) is

\[ R_{ch} = R_\alpha N_\alpha^{1/3}, \]  

where \( N_\alpha = Z/2 \) in case of even \( Z \) and in case of odd \( Z \) the value of \( N_\alpha + 0.5 \) is used. Using 
\( R_\alpha = R_{^{4}He} = 1.710 \) Fm [1] for the nuclei with \( 3 \leq N_\alpha \leq 5 \), 
\( R_\alpha = 1.628 \) Fm for \( 6 \leq N_\alpha \leq 11 \), 
\( R_\alpha = 1.600 \) Fm for \( 12 \leq N_\alpha \leq 41 \)
gives \( <\Delta^2>^{1/2} = 0.054 \) Fm.

In the both cases three values of \( r_{ch} \) and \( R_\alpha \) have been used. The changing the slope of the functions is explained by the formation of a core, which starts growing from the nucleus with \( Z = 12 \), \( N_\alpha = 6 \). For the nuclei with \( 12 \leq Z \leq 22 \), \( 6 \leq N_\alpha \leq 11 \), the number of clusters of the core is comparable with the number of peripheral clusters having the size of nucleus \( ^{4}He \). For the other nuclei with \( Z \geq 24 \), \( N_\alpha \geq 12 \), the number of clusters of the core prevails and the mean radius of a core cluster is revealed as to be \( R_\alpha = 1.600 \) Fm.

It was shown before that the size of a nucleus is determined by the \( \alpha \)-cluster structure and the root mean square radius \( R \) of a nucleus can be estimated by a few ways on the number of \( \alpha \)-clusters in it [6,7], disregarding to the number of excess neutrons. One of the ways to estimate \( R \) is (2).

To explain the paradox that both (1) and (2) can well describe the experimental radii \( R_{exp} \) the following hypothesis was proposed [8]. The \( \beta \)-stability, which the most abundant isotopes belong to, is provided by the particular number of excess neutrons in the isotope that is needed to fill in the space between the volumes occupied by the charge and the matter of the alpha clusters of the core. The size of an isotope \( R_m \) is actually determined by the volume occupied by the matter of the core \( 4/3\pi R_{m(core)}^3 \) and the volume occupied by the charge of the \( N_{\alpha pr} \) peripheral clusters

\[ R_m^3 = R_{m(core)}^3 + N_{\alpha pr} R_{^{4}He}^3. \]  

The \( R_{m(core)} \) is calculated from

\[ R_{m(core)}^3 = (N_\alpha - N_{\alpha pr})R_{m(\alpha)}^3 + \Delta N r_{m(nn)}^3, \]  

where \( R_{m(\alpha)} \) stands for the matter radius of a core \( \alpha \)-cluster, \( \Delta N \) denotes the number of excess neutrons filling the core by pairs, \( nn \)-pairs, and \( r_{m(nn)} \) stands for the radius of one neutron of the pairs. The condition

\[ R_m = R \]  

determines the particular number of excess neutrons in the \( \beta \)-stable isotopes. By fitting the values of the nuclear charge radii of the most abundant isotopes, the value of the radius of one nucleon of \( \alpha \)-cluster matter of the core \( r_{m(\alpha)} = 0.945 \) Fm, which corresponds to \( R_{m(\alpha)} = 1.500 \) Fm, and the value of \( r_{m(nn)} = 0.840 \) Fm were found [8].

In this paper the hypothesis that \( R_m = R \) provides the \( \beta \)-stability is developed. The root mean square charge radii \( R \) of nuclei are calculated by a phenomenological formula [6] with using charge radius of the core and the radius of the peripheral \( \alpha \)-cluster position \( R_p \) in the nucleus.
formula to calculate $R_p$ was obtained from an independent analysis of the differences of proton and neutron single particle binding energies in the framework of the $\alpha$-cluster model based on $pn$-pair interactions.

In the article it is shown that for the nuclei with $Z \geq 24$ the value of $R$ is almost independent of how many of the peripheral clusters $N_{\alpha_{pr}}$ are placed on the surface of the core at the same radius $R_p$ from the center of mass $N_{\alpha_{pr}} = 1 \div 5$. However, in order to have the radius of an isotope $R_m$ equal to its charge radius $R$ in the case of different numbers of peripheral clusters the different number of excess neutrons is needed. This may explain why the nuclids of different isotopes have close values of their radii and this also explains the width of the narrow path of $\beta$-stability.

2 Size of Nuclei

The radius $R$ of a nucleus is measured in electron scattering experiments as the root mean square radius $\langle r^2 \rangle^{1/2}$ of the charge distribution. The $R_{ch}$ (2) has meaning of $\langle r^3 \rangle^{1/3}$. The equality $\langle r^3 \rangle^{1/3} = \langle r^2 \rangle^{1/2}$ can be provided by the only condition that the charge density distribution $\rho(r) = \text{const}$, which is in an agreement with the results of analysis of the charge distribution of heavy nuclei, see Fig. 6.1 [5]. The value of $\rho(r)$ is approximately constant except the peripheral part with the approximately constant thickness $t \approx 2.4 \pm 0.3$ Fm for all nuclei from $^{16}$O to $^{208}$Pb.

To calculate the charge radius $R$ the following formula is used

$$N_{\alpha} < r^2 > = (N_{\alpha} - N_{\alpha_{pr}}) < r^2 >_{\text{core}} + N_{\alpha_{pr}} < r^2 >_p,$$

where $< r^2 >_{\text{core}}$ denotes mean square radius of charge distribution of the core and $< r^2 >_p$ denotes the mean square radius of the charge distribution of peripheral clusters. This equation is obtained from the definition of the root mean square radius

$$N_{\alpha} < r^2 > = 4\pi \int_0^\infty r^2 \rho(r)r^2 dr,$$

where the spherically symmetrical charge density distribution $\rho(r)$, normalized as

$$4\pi \int_0^\infty \rho(r)r^2 dr = N_{\alpha},$$

is the sum of the charge density distributions of the core $\rho_{\text{core}}(r)$ and the peripheral clusters $\rho(r)_{\alpha_{pr}}$ with the corresponding normalization

$$4\pi \int_0^\infty \rho(r)_{\text{core}}r^2 dr = N_{\alpha} - N_{\alpha_{pr}},$$

$$4\pi \int_0^\infty \rho(r)_{\alpha_{pr}}r^2 dr = N_{\alpha_{pr}}.$$

Eq. (6) is used for calculations of the charge and matter distributions of nuclei in the wave function approach with the single-particle potential model assumed in the shell model [9,10] in terms of the completed shells and the occupation numbers for the nucleons of the last not completed shell.

Taking into account that for the nuclei with $Z \geq 24$, $N_{\alpha} \geq 12$, the radius of one core $\alpha$-cluster $R_{\alpha} = 1.600$ Fm one can write for the case when $N_{\alpha_{pr}} = 4$

$$< r^2 >_{\text{core}}^{1/2} = 1.600(N_{\alpha} - 4)^{1/3}.$$

The phenomenological formula [6] to calculate $R_p$ for the nuclei with $N_{\alpha} \geq 12$, which has meaning of $< r^2 >_p^{1/2}$, is

$$< r^2 >_p^{1/2} = 2.168(N_{\alpha} - 4)^{1/3}.$$

Then the equation (6) to calculate the charge radii $R$ becomes
\[ N_\alpha R^2 = (N_\alpha - 4)1.600^2(N_\alpha - 4)^{2/3} + 4 \times 2.168^2(N_\alpha - 4)^{2/3} \]  
and for odd nuclei \( R_1 \), taking into account that \( R_{p1} \approx R_p \) [6],

\[ (N_\alpha + 0.5)R_1^2 = (N_\alpha - 4)1.600^2(N_\alpha - 4)^{2/3} + 4.5 \times 2.168^2(N_\alpha - 4)^{2/3}. \]  

Eq. (9) and (10) were obtained [6] in the framework of the alpha cluster model of \( pn \)-pair interactions from analysis of differences between the experimental values of the binding energies of the last proton and neutron in the nuclei with \( N = Z \). It was shown that using (9) and (10) for the nuclei with \( Z \geq 24, N_\alpha \geq 12 \), gives a good fitting the experimental radii of the nuclids of the most abundant isotopes with \( \Delta^2 > 1/2 = 0.050 \text{Fm} \), see, for example Fig. 2 [6]. The deviation between \( R_{ch} \) (2) and \( R \) (9) and (10) for the nuclei with \( 24 \leq Z \leq 116 < \Delta^2 > 1/2 = 0.028 \text{Fm} \). To consider the cases with different amount of peripheral clusters, the equation (6) should be rewritten as

\[ N_\alpha R^2 = (N_\alpha - N_{\alpha pr})1.600^2(N_\alpha - N_{\alpha pr})^{2/3} + N_{\alpha pr}2.168^2(N_\alpha - N_{\alpha pr})^{2/3}. \]  

(11)

For the nuclei with odd \( Z \)

\[ (N_\alpha + 0.5)R_1^2 = N_\alpha R^2 + 0.5 \times 2.168^2(N_\alpha - N_{\alpha pr})^{2/3}. \]  

(12)

Let us analyze the function \( F(N_\alpha) = (R/(1.600^2N_\alpha^{1/3})) \). After simplifying the expression one gets

\[ F(N_\alpha) = (1 - N_{\alpha pr}/N_\alpha)^{1/3}(1 - N_{\alpha pr}/N_\alpha(1 - (2.168/1.600)^2)))^{1/2}. \]  

(13)

In Fig. 2 the graph of the function is shown with four solid lines corresponding to \( N_{\alpha pr} = 0, 3, 4, 5 \). The small crosses indicate \( F_{exp} = R_{exp}/(1.600N_\alpha^{1/3}) \). The dashed lines denote the graphs of the function \( F(N_\alpha) \) with \( N_{\alpha pr} = 4 \) and with replacement of the number 2.168 with the number 2.500 (upper line) and 1.500 (lower line), which shows that the value 2.168 obtained from an independent analysis of binding energies of light nuclei provide the function \( F(N_\alpha) \) convergent to 1 at \( N_\alpha \geq 12 \).

![Figure 2: Graphs of function \( F(N_\alpha) \) (13) with \( N_{\alpha pr} = 0, 3, 4, 5 \) (solid lines). The graphs of \( F(N_\alpha) \) with \( N_{\alpha pr} = 4 \) (dashed lines) and with the values 2.500 (upper one) and 1.500 (lower one) used instead of 2.168.](image)

This means that \( R \) for heavy nuclei does not significantly depend on the number of peripheral clusters \( N_{\alpha pr} \). This explains why the radii calculated in the \( \alpha \)-cluster model of completed shells \( R_{shl} \) [6] are close to \( R \). In the model a nucleus consists of the \( \alpha \)-clusters of completed shells (the
nuclei $^{16}$O, $^{20}$Ne, $^{40}$Ca and the nuclei placed in the right side of the Periodic Table of Elements were taken as ones with completed shells) and the number of clusters of the last not completed shell with the radius of their position in the nucleus $R_p$ (8).

The difference between the equation to calculate $R_{shl}^2$ [6] and the equation to calculate $R^2$ (9) is that the value $N_{ap}$ varies from 0 to 5. For example $N_{a}R_{shl}^2$ of nucleus $^{40}$Ca is calculated as a sum of $5R_{20}^2$ and $5R_{p}^2$. The empirical values of $R_p$ in case of the nuclei with $N_a \leq 12$ were found in the framework of the model of $pn$-pair interactions [6,7] on the values of differences between the single-particle binding energies of last proton and neutron in the nuclei with $N = Z$. The value of $R_{shl}$ for $^{52}$Cr is calculated with using the radius of nucleus $^{40}$Ca, as one with completed shells, and $N_{ap} = 2$. For the nuclei with completed shells with $N_a > 12$ the $R_{shl}$ is calculated by (2). The values of $R_{shl}$ have deviation with the experimental data of the most abundant isotopes $< \Delta^2 >^{1/2} = 0.051$ Fm [6].

So one can say that a nucleus has a dense core with $N_a - N_{ap}$ clusters and some peripheral clusters placed at the equal distances from the center of the core. The size $R$ for the nuclei with $N_a \geq 12$ does not depend significantly of the number $N_{ap} = 0 \div 5$.

The radius of a $\beta$-stable isotope $R_m$ is dependent on the volume occupied by the core. It is implied that only those nuclids stay $\beta$-stable that have proper amount of excess neutrons to make the radii $R_m$ equal to the size of the charge of the nucleus $R$.

### 3 Nuclear radii of Isotopes Stable to $\beta$-Decay

The experimental radii $R_{exp}$ of different isotopes belonging to one element differ from each other but the values are close and the differences are restricted within several hundreds of Fm. The radius of a nuclid $R_m$ (3) is calculated from the sum of cubic radii of its parts.

Let us consider first that there are four peripheral alpha clusters in the nuclei of even $Z$ (the mass number $A$ and the number of the excess neutrons $\Delta N$) and four and half peripheral alpha clusters in case of nucleus with odd $Z = Z + 1$ (the mass number $A_1$ and the number of excess neutrons $\Delta N_1$).

The excess neutron pairs are in the core and in case of odd $Z$ nuclei one excess neutron is stuck with the single $pn$-pair. The suggestion is supported by the fact that there is only one stable isotope $^{19}$F (the abundance of the isotope is 100% [11]). The link between the single pair with spin $s = 1$ and the excess neutron with $s = 1/2$ is explained by the spin correlation. The single neutron on the surface of the nucleus is not ‘seen’ by the electrons scattered on the nucleus in the experimental measurement of the radius $R_m$. Then the radius of isotopes $R_m$ and $R_{m1}$ of nuclei $A$ and $A_1$ are calculated by the following equations [8]

$$R_m^3 = 4R_{4He}^3 + R_{m(\alpha)}^3(N_\alpha - 4) + r_{m(nn)}^3\Delta N,$$  \hspace{1cm} (14)

and

$$R_{m1}^3 = 4.5R_{4He}^3 + R_{m(\alpha)}^3(N_\alpha - 4) + r_{m(nn)}^3(\Delta N_1 - 1).$$  \hspace{1cm} (15)

The equations (14) and (15) can be used to calculate charge radii $R_m$ of any $\beta$-stable isotopes.

For the $\beta$-stable isotopes with $6 \leq Z \leq 11$ $R_m = R_{ch}$. For the nuclei with $Z \geq 12$ $R_m$ is calculated by (14) and (15). For the nuclei with $6 \leq Z \leq 23$ the radii $R = R_{ch}$ (2) and for the nuclei with $Z \geq 24$ $R$ is calculated by (9) and (10).

The values of $R$ and $R_m$ for the stable nuclei with $Z \geq 12$ are presented in Table 1. The radii are given in comparison with their experimental values. For the unstable nuclei with $Z > 83$ the radii $R$ are given in Table 2 together with $R_m$ calculated for those $\alpha$-decay isotopes that are stable to the $\beta$-decay [12].

Table 1. Radii of nuclei. Abundance is given in %, radii in Fm
|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   |   |   |   |   |   |
| 12 | 24 | 78.6 | 2.985(30) | 2.958 | 2.991 | 48 | 112 | 24.1 | 4.635 | 4.595 |
| 13 | 27 | 100.0 | 3.06(9) | 3.038 | 3.060 | 49 | 115 | 95.8 | 4.611(10) | 4.664 | 4.634 |
| 14 | 28 | 92.2 | 3.14(4) | 3.114 | 3.112 | 50 | 120 | 33.0 | 4.630(7) | 4.700 | 4.684 |
| 15 | 31 | 100.0 | 3.24 | 3.186 | 3.195 | 50 | 118 | 24.0 | 4.700 | 4.666 |
| 16 | 32 | 95.0 | 3.240(11) | 3.256 | 3.224 | 51 | 121 | 57.3 | 4.63(9) | 4.728 | 4.703 |
| 17 | 35 | 75.5 | 3.335(18) | 3.322 | 3.302 | 52 | 130 | 34.5 | 4.721(6) | 4.762 | 4.787 |
| 18 | 40 | 99.6 | 3.393(15) | 3.386 | 3.398 | 52 | 124 | 4.61 | 4.762 | 4.734 |
| 19 | 41 | 0.34 | 3.327(15) | 3.386 | 3.328 | 53 | 127 | 100.0 | 4.737(7) | 4.790 | 4.771 |
| 20 | 40 | 97.0 | 3.482(25) | 3.507 | 3.427 | 54 | 130 | 4.08 | 4.823 | 4.801 |
| 21 | 45 | 100.0 | 3.550(5) | 3.565 | 3.529 | 55 | 133 | 100.0 | 4.806(11) | 4.851 | 4.837 |
| 22 | 48 | 74.0 | 3.59(4) | 3.620 | 3.583 | 56 | 136 | 7.81 | 4.882 | 4.866 |
| 23 | 51 | 99.8 | 3.645(5) | 3.618 | 3.669 | 57 | 140 | 88.5 | 4.883(9) | 4.940 | 4.913 |
| 24 | 52 | 3.680(11) | 3.649 | 3.729 | 58 | 138 | 0.25 | 4.940 | 4.897 |
| 25 | 55 | 91.7 | 3.77(7) | 3.763 | 3.809 | 59 | 141 | 27.1 | 4.993(35) | 4.996 | 4.943 |
| 26 | 56 | 67.8 | 3.760(10) | 3.836 | 3.829 | 60 | 142 | 26.6 | 5.095(30) | 5.052 | 5.036 |
| 27 | 59 | 100.0 | 4.115 | 4.112 | 3.985 | 61 | 150 | 7.47 | 5.052 | 5.021 |
| 28 | 64 | 89.1 | 4.163(79) | 4.146 | 4.130 | 62 | 153 | 47.8 | 5.150(22) | 5.078 | 5.053 |
| 30 | 74 | 36.7 | 4.230 | 4.205 | 65 | 159 | 27.1 | 4.993(35) | 4.996 | 4.943 |
| 31 | 76 | 60.2 | 4.027 | 4.050 | 66 | 164 | 28.2 | 5.222(30) | 5.158 | 5.138 |
| 32 | 80 | 49.8 | 4.142(3) | 4.115 | 4.141 | 67 | 165 | 100.0 | 5.210(70) | 5.184 | 5.170 |
| 33 | 84 | 56.9 | 4.198 | 4.205 | 68 | 166 | 33.4 | 5.243(30) | 5.210 | 5.181 |
| 34 | 86 | 11.56 | 4.193 | 4.189 | 69 | 169 | 100.0 | 5.226(4) | 5.235 | 5.212 |
| 35 | 90 | 100.0 | 4.28(2) | 4.278 | 4.252 | 70 | 174 | 31.8 | 5.312(60) | 5.260 | 5.251 |
| 36 | 92 | 4.624(8) | 4.635 | 4.613 | 71 | 175 | 79.4 | 5.378(30) | 5.285 | 5.267 |
| 37 | 97 | 27.9 | 4.180 | 4.230 | 4.230 | 72 | 180 | 35.2 | 5.339(22) | 5.310 | 5.306 |
| 38 | 102 | 31.6 | 4.480(22) | 4.500 | 4.411 | 73 | 181 | 100.0 | 5.500(200) | 5.334 | 5.321 |
| 39 | 103 | 100.0 | 4.27(2) | 4.309 | 4.292 | 74 | 186 | 28.4 | 5.42(7) | 5.358 | 5.345 |
| 40 | 106 | 51.5 | 4.28(2) | 4.355 | 4.308 | 75 | 184 | 30.6 | 5.358 | 5.349 |
| 41 | 108 | 100.0 | 4.317(8) | 4.385 | 4.352 | 76 | 187 | 62.9 | 5.383 | 5.374 |
| 42 | 109 | 23.8 | 4.391(26) | 4.429 | 4.409 | 77 | 192 | 41.0 | 5.412(22) | 5.406 | 5.412 |
| 43 | 110 | 16.7 | 4.429 | 4.388 | 78 | 190 | 26.4 | 5.406 | 5.399 |
| 44 | 112 | 31.6 | 4.480(22) | 4.500 | 4.411 | 79 | 194 | 32.9 | 5.366(22) | 5.453 | 5.436 |
| 45 | 114 | 100.0 | 4.510(44) | 4.529 | 4.488 | 80 | 202 | 29.8 | 5.499(17) | 5.499 | 5.500 |
| 46 | 116 | 100.0 | 4.541(33) | 4.569 | 4.521 | 81 | 205 | 70.5 | 5.484(6) | 5.522 | 5.528 |
| 47 | 118 | 100.0 | 4.542(10) | 4.598 | 4.543 | 82 | 208 | 52.3 | 5.521(29) | 5.544 | 5.550 |
| 48 | 120 | 28.9 | 4.624(8) | 4.635 | 4.613 | 83 | 209 | 100.0 | 5.568 | 5.564 |
Table 2. Radii of $\alpha$-decay isotopes stable to $\beta$-decay. $\Delta N$ stands for number of excess neutrons.

| $Z$ | $A$ | $\Delta N$ | $R_{ch}(2)$ | $R(9,10)$ | $R_m(14,15)$ | $Z$ | $A$ | $\Delta N$ | $R_{ch}(2)$ | $R(9,10)$ | $R_m(14,15)$ |
|-----|-----|-----------|-----------|-----------|-----------|-----|-----|-----------|-----------|-----------|-----------|
| 84  | 212 | 44        | 5.562     | 5.589     | 5.587     | 100 | 256 | 56       | 5.895     | 5.921     | 5.929     |
| 85  | 215 | 45        | 5.584     | 5.612     | 5.613     | 101 | 259 | 57       | 5.914     | 5.942     | 5.953     |
| 86  | 216 | 44        | 5.606     | 5.633     | 5.622     | 102 | 260 | 56       | 5.934     | 5.960     | 5.961     |
| 87  | 219 | 45        | 5.627     | 5.656     | 5.649     | 103 | 263 | 57       | 5.953     | 5.981     | 5.985     |
| 88  | 222 | 46        | 5.649     | 5.676     | 5.670     | 104 | 266 | 58       | 5.972     | 5.999     | 6.004     |
| 89  | 225 | 47        | 5.670     | 5.698     | 5.696     | 105 | 269 | 59       | 5.991     | 6.019     | 6.027     |
| 90  | 228 | 48        | 5.691     | 5.718     | 5.717     | 106 | 272 | 60       | 6.010     | 6.037     | 6.046     |
| 91  | 231 | 49        | 5.712     | 5.741     | 5.742     | 107 | 275 | 61       | 6.029     | 6.057     | 6.068     |
| 92  | 234 | 50        | 5.733     | 5.760     | 5.763     | 108 | 278 | 62       | 6.048     | 6.074     | 6.087     |
| 93  | 237 | 51        | 5.754     | 5.782     | 5.788     | 109 | 281 | 63       | 6.067     | 6.094     | 6.109     |
| 94  | 238 | 51        | 5.774     | 5.801     | 5.797     | 110 | 284 | 64       | 6.085     | 6.111     | 6.128     |
| 95  | 241 | 51        | 5.795     | 5.823     | 5.821     | 111 | 287 | 65       | 6.103     | 6.131     | 6.150     |
| 96  | 240 | 52        | 5.774     | 5.801     | 5.809     | 112 | 290 | 66       | 6.122     | 6.148     | 6.168     |
| 97  | 243 | 53        | 5.795     | 5.823     | 5.833     | 113 | 293 | 67       | 6.140     | 6.167     | 6.190     |
| 98  | 244 | 52        | 5.815     | 5.842     | 5.842     | 114 | 296 | 68       | 6.158     | 6.184     | 6.208     |
| 99  | 247 | 53        | 5.835     | 5.863     | 5.866     | 115 | 299 | 69       | 6.176     | 6.203     | 6.229     |
| 100 | 250 | 54        | 5.855     | 5.882     | 5.886     | 116 | 302 | 70       | 6.194     | 6.219     | 6.247     |
| 101 | 253 | 55        | 5.875     | 5.903     | 5.910     | 117 | 305 | 71       | 6.212     | 6.239     | 6.268     |

Eq. (14) and (15) mean that the nuclei with $Z$ and $Z_1$ have one core with the same amount of excess neutrons in it $\Delta N_{core} = \Delta N = \Delta N_1 - 1$, which leads to the relation between the mass numbers of neighboring nuclei $A_1 - A = 3$. Indeed, there is always an isotope with even $Z = Z_1 - 1$ with the mass number $A$ less than $A_1$ on 3 in the $\beta$-stability path.

In case of stable nuclei $R$ and $R_m$ have been calculated for the most abundant isotopes and for those nuclei that have mass numbers $A = A_1 - 3$. For the unstable nuclei the charge and matter radii have been calculated only for stable to $\beta$-decay nuclei with $84 \leq Z \leq 116$ [12] with $A = A_1 - 3$.

In the Table the data for the nucleus with $Z = 117$ is given also in a supposition that it is $\beta$-stable.

In the Table the calculated values are given in comparison with the experimental values. The deviation between $R_m$ and $R_{exp}$ is $\Delta^2 > 1/2 = 0.048$ Fm. One can see from the tables that the values of $R$ and $R_m$ for the nuclei with $11 \leq Z \leq 117 < \Delta^2 > 1/2 = 0.031$ Fm.

Unlike the even nuclei the odd nuclei have only one, rarely two $\beta$-stable isotopes [12]. The alpha cluster model based on $pn$-pair interactions [6,7] provides some reasonable explanation for it. The analysis of the nuclear binding energies made in the framework of the model shows that the single proton-neutron pair on the nucleus periphery has six meson bonds with the six pairs of three nearby $\alpha$-clusters with the energy about 15 MeV. This may constitute one big cluster which consists of three and half or four and half $\alpha$-clusters. The four clusters on the periphery may be preferable due to $\alpha$-cluster’s features to have three meson bonds with the three nearby clusters, which corresponds to nucleus $^{16}$O, and the single $pn$-pair ties up the three nearby clusters with the six meson bonds.

In Fig. 3 a distribution of the number of excess neutrons of $\beta$-stable isotopes on $Z$ is shown.

The chain of squares indicates the number of excess neutrons in the $\beta$-stable isotopes with their mass numbers $A = A_1 - 3$. The smaller squares scattered around the chain indicate the numbers of the excess neutrons of even $\beta$-stable nuclei with the biggest and the smallest $A$. The solid line indicates the number of neutrons calculated by the following equation

$$\Delta N = (R^3 - R_{3He}^3 N_{\alpha pr} - R_{m(\alpha)}^3 (N_\alpha - N_{\alpha pr}))/r_{m(nn)}^3,$$ (16)
\[ \Delta N_1 = \Delta N + 1. \] (17)

The short dashed line indicates the number of excess neutrons calculated by (16) and (17) with \( R = 1.600N_\alpha^{1/3} \text{Fm} \). The line \( \Delta N \) corresponding to \( R = R_{\text{shl}} \), see Fig. 2 [8], is not presented here. In Fig. 3 it would be between the solid and the dashed lines and for \( Z > 85 \) it goes above the chain of the squares on several neutrons.

If in (16) one takes \( R = R_m \) calculated by (14) and (15) for the isotopes \( A \) and \( A_1 \) being in the relation \( A_1 = A + 3 \), it is possible to estimate the width of the \( \beta \)-stability path. In Fig. 3 two long dashed lines indicate \( \Delta N \) calculated by (16) with \( N_{\alpha_{pr}} = 6 \) (lower one) and with the \( N_{\alpha_{pr}} = 2 \) (higher line).

This fact that the stable isotopes of one nucleus have close values of radii is illustrated on stable isotopes of \( ^{20}\text{Ca} \). The size of the nucleus has been estimated by a few ways. These are \( R = R_{\text{ch}} = 3.507 \text{ Fm} \), Table 1, the value of \( 1.600 \times 10^{1/3} = 3.447 \text{ Fm} \) and \( R_{\text{shl}} = 3.480 \text{ Fm} \) [6,7].

The value of \( R_{\text{shl}} \) was calculated in the \( \alpha \)-cluster shell model, where nucleus \( ^{40}\text{Ca} \) is considered as a nucleus \( ^{20}\text{Ne} \) plus five \( \alpha \)-clusters above it.

The radii \( R_m \) of nuclei \( ^{40}\text{Ca}, ^{42}\text{Ca}, ^{44}\text{Ca}, ^{46}\text{Ca}, ^{48}\text{Ca} \) calculated (14) with the number of peripheral clusters 5, 5, 5, 3 and 2 the values of \( R_m \) correspondingly are \( R_{40\text{Ca}} = 3.473 \text{ Fm}, R_{42\text{Ca}} = 3.505 \text{ Fm}, R_{44\text{Ca}} = 3.537 \text{ Fm}, R_{46\text{Ca}} = 3.481 \text{ Fm}, R_{48\text{Ca}} = 3.468 \text{ Fm} \). The radius \( R_{43\text{Ca}} = R_{42\text{Ca}} \), because the last single excess neutron is not in the core consisted of zero spin objects, which are \( \alpha \)-clusters and \( nn \)-pairs. It is supposed to be on the surface of the nucleus and is not seen by the light charge particle scattered on the nucleus.

The experimental values of the radii of the isotopes are: for \( ^{40}\text{Ca} \) (96.97%) \( R_{40\text{Ca}} = 3.450(10) \div 3.482(25) \text{ Fm} \) [2], 3.478 Fm [4], for \( ^{42}\text{Ca} \) (0.64%) \( R_{42\text{Ca}} = 3.508 \text{ Fm} \) [4], for \( ^{44}\text{Ca} \) (2.06%) \( R_{44\text{Ca}} = 3.518 \text{ Fm} \) [4], for \( ^{46}\text{Ca} \) (0.0033%) \( R_{46\text{Ca}} = 3.498 \text{ Fm} \) [4], for \( ^{48}\text{Ca} \) (0.185%) \( R_{48\text{Ca}} = 3.479 \text{ Fm} \) [4], 4.70 Fm and 3.51 Fm [2], for \( ^{43}\text{Ca} \) (0.145%) \( R_{43\text{Ca}} = 3.495 \text{ Fm} \) [4]. Both experimental and calculated values show that the radii of the stable isotopes are close within an accuracy of the
experimental data deviation of 0.03 Fm.

The data for the most explored nucleus $^{40}$Ca reveal the real accuracy of the data obtained from the electron scattering measurement of the root mean square radius of a nucleus. The deviation between the data taking into account the measurement errors can be up to 0.06 Fm. It is because the results of the analysis of the experimental data are still model dependent. It should define the minimal deviation expected to be obtained from theoretical calculations. From this point of view the most appreciated measurement [4] has been carried out with one model of analysis of data for several isotopes, revealing close but different values of the radii.

4 Conclusion

The size of a nucleus $R$ can be estimated by a few ways in the framework of $\alpha$-cluster model based on $pn$-pair interactions. These are $R_{shl}$ [6], $R_{ch}$ (2), $R$ (9) and (10), and $R_m$ (14) and (15). All these estimations have deviation with the experimental radii of the most abundant isotopes $<\Delta^2>^{1/2}=0.051, 0.054, 0.050$ and 0.048 Fm correspondingly. The average deviation between the values is within 0.035 Fm.

In calculations of root mean square radii $R_{shl}$ and $R$ a representation of a core and a few peripheral clusters placed on the surface of the core at the same distances $R_p$ from the center of mass is used. The number of peripheral clusters $N_{\alpha_{pr}}$ is varied from 0 to 5 in calculations of $R_{shl}$ and $N_{\alpha_{pr}}=4$ in case of $R$.

The values of $R_{shl}$, $R_{ch}$ and $R$ have been calculated on the number of $\alpha$-clusters disregarding to the number of excess neutrons. The values of $R_m$ is calculated from a supposition that the core occupies some volume determined by the alpha cluster matter and the matter of neutron-neutron excess pairs.

The $pn$-pair interaction model [6] provides an explanation of the fact that the nuclei with odd $Z_1=Z+1$ have only one, rarely two $\beta$-stable isotopes $A_1$, whereas the nuclei with even $Z$, have considerably bigger verity of $A$. The single proton-neutron pair has six meson bonds with the three peripheral $\alpha$-clusters with a large energy $\sim 15$ MeV, which constitutes one big peripheral cluster of three and half or four and half $\alpha$-clusters with one excess neutron stuck with the single $pn$-pair due to spin correlations between the pair with $s=1$ and the neutron with $s=1/2$. Therefore the nuclei $A_1$ and $A=A_1-3$ have one core and this chain determines the $\beta$-stability path.

It is shown that values of $<R$ do not depend significantly on the number of peripheral clusters. For the case of different numbers of peripheral clusters the different numbers of excess neutrons is needed to have the radius $R_m = R$. This can determine the width of the $\beta$-stability path for the even nuclei.

The equations (14) and (15) can be used to calculate charge radii $R_m$ of any $\beta$-stable isotopes. The radii $R_m$ of the most abundant isotopes are given in comparison with their experimental values. The radii for the isotopes stable to $\beta$-decay with $A$ and $A_1$ related with the equation $A=A_1-3$ for all nuclei with $12 \leq Z \leq 116$ are presented in Tables. The deviation between $R_m$ and $R$ $<\Delta^2>^{1/2}=0.031$ Fm.

For calculation of the radii two parameters have been used. This is the matter radius of a core $\alpha$-cluster 1.500 Fm and the radius of one neutron of the core $nn$-pairs 0.840 Fm. The values may differ for various nuclei. However, as it is shown here, they do not change considerably.

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