PACKET-OBLIVIOUS STABLE ROUTING IN MULTI-HOP WIRELESS NETWORKS

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ABSTRACT

In this work, we study the fundamental problem of scheduling communication in multi-hop wireless networks. Packets arrive dynamically at nodes of the network together with their routes, along which they should be forwarded. Packet arrival is modeled by an adversary that injects packets, together with their routes, at nodes, in such a way that each link is to be traversed by a bounded number of injected packets; more precisely, each link occurs in no more than $\rho |T| + b$ routes injected in time interval $T$ of length $T$, where parameter $\rho < 1$ is called an injection rate and $b$ is the burstiness. The goal is to maintain system stability, i.e., to keep queues at nodes bounded at any time, for as high injection rate as possible. The challenge is, however, raised by the wireless nature of communication – if two or more neighbors of a node transmit some packets, the node could not successfully hear any transmission. Therefore, in order to achieve stability of the system for high injection rates, nodes should keep scheduling transmissions in a way that such collisions do not occur often.

We focus on packet-oblivious routing protocols; that is, algorithms that do not take into account any historical information about packets or carried out by packets. Such protocols are well motivated in practice, as real forwarding protocols and corresponding data-link layer architectures are typically packet-oblivious. We provide a local-knowledge protocol, i.e., which is working without using any topological information, except for some upper bounds on the number of links and the network’s degree, that is stable for a wide spectrum of packet injection rates. It is based on novel transmission schedules, called universally strong selectors, which, combined with some known queuing policies (LIS, SIS, NFS, FTG), makes it the best known local-knowledge packet-oblivious routing protocol regarding the injection rate for which stability is guaranteed. We also propose a global-knowledge protocol, which is stable if the packet injection rate per link is smaller than the inverse of the chromatic number of the collision graph associated with the network. Although the protocol does not take into account any historical information, it has to be seeded by some information about the network topology.

Keywords Wireless networks · routing · adversarial queuing · interference · stability · packet latency

1 Introduction

In this paper, we consider the model of radio networks [20] to study routing in multi-hop wireless networks. That allows to abstract from incidental systems details and concentrate on the essential aspects of communication that are most conducive to studying routing algorithms. One of such aspects is interferences, which provokes that when multiple packets arrive simultaneously into a node this results in an unsuccessful transmission experienced by the receiving
node. These networks pose unique challenges to design of routing algorithms because of the need to coordinate activities of the nodes whose transmissions may reach some node simultaneously.

We consider an adversarial methodology of traffic generation, which makes it possible to consider the worst-case behavior of routing. It considers the execution of a routing algorithm and the resulting traffic as determined by the adversary, who controls the injection of packets and assigns them the paths, also called routes, to follow. An adversary is constrained mostly by the injection rate, understood as a bound on the number of injected packets, thus abstracting from stochastic underpinnings of traffic generation. This, in turn, allows to consider stability, which represents bounded queues and, therefore, a smooth functioning of the communication infrastructure, without stochastic assumptions. Adversarial queuing usually allows to abstract from physical restrictions on the network, like bounded private memory at nodes used to queue packets, with the benefit of not having to deal with discarding packets and their subsequent re-injections.

Related work. The methodology of adversarial routing in wired networks was pioneered by Borodin et al. [14] and Andrews et al. [8]. Since then, a substantial effort has been invested in that area. Stability of specific scheduling policies and networks was considered for example in [12, 22, 26, 29]. A systematic account of issues related to universal stability was given in [5]. The impact of network topologies on injection rates that guarantee stability was considered in [24, 32, 54]. The model of adversarial queuing initially proposed was also extended and modified in subsequent work. In [2], it was shown how to route packets by assigning them suitable paths so that the queues at the nodes are polynomially bounded and that each packet has a polynomially bounded delay time. In [15], the authors considered networks in which links capacity to transmit packets can be slowed down or undergo variations of such capacities. Networks with nodes and links that occasionally fail were studied in [6]. Routing packets with priorities was studied in [3].

Stability in general wireless networks without explicit interferences was first studied by Andrews and Zhang [9, 10] and Cholvi and Kowalski [23]. Lim et al. [33] analyzed the stability of the max-weight protocol in wireless networks with interferences, but assuming the existence of a set of feasible edge rate vectors sufficient to keep the network stable. Chlebus et al. [21] and Anantharamu et al. [7] studied adversarial broadcasting with interferences in the case of using single-hop radio networks. Chlebus et al. [19] considered interactions among components of routing in wireless networks, which included transmission policies, scheduling policies to select the packet to transmit from a set of packets parked at a node, and hearing control mechanisms to coordinate transmissions with scheduling. In [17] the authors considered adversarial routing in wireless networks with interferences.

There is a rich study of stochastic packet queuing on a related communication model called a multiple access channel, in which a packet could leave its queue only if exactly one station was scheduled to transmit at the time. We refer the reader to the surveys by Gallager [25] and Chlebus [13] for an overview of early research. Håstad et al. [30] proved that, for any fixed injection rate smaller than 1, polynomial backoff protocols are stable and exponential backoff ones are not stable. All the previous results concerning stability were proved for expected fixed injection rates strictly smaller than 1, until recently when Bienkowski et al. [13] gave the first deterministic distributed online algorithm achieving (weak) stability also for the (highest possible) injection rate 1.

Tassiulas and Ephremides [35, 36] considered stochastic stability in general networks. A restricted case of their model, where packets were only allowed to travel to their destinations by paths of bounded lengths, was considered in [28].

There is also a study comparing the power of adaptive protocols and non adaptive ones [27] on a multiple access channel. The adaptive protocols allow stations to monitor and store some digest of the local queue history, especially its size, while the non adaptive protocols allow only to check whether the current local queue is empty or not. They used combinatorial structures called selectors [1] (earlier called superimposed codes [31]) to research the non adaptive protocols.

Our results. In this paper, we study dynamic routing in multi-hop radio networks with a specific methodology of adversarial traffic that reflects interferences. For this purpose, we define a conflict graph of the network and explore its properties from the perspective of characterizing stable packet injection rates.

We focus on packet-oblivious routing protocols; that is, algorithms that do not take into account any historical information about packets or carried out by packets. Such protocols are well motivated in practice, as real forwarding protocols and corresponding data-link layer architectures are typically packet-oblivious.

First, we give a new family of combinatorial structures, which we call universally strong selectors, that are used to provide a set of transmission schedules. Making use of these structures, combined with some known queuing policies such as Longest In System (LIS), Shortest In System (SIS), Nearest From Source (NFS) and Furthest To Go (FTG), we propose a local-knowledge packet-oblivious routing algorithm (i.e., which is working without using any topological
information, except for some upper bounds on the number of links and the network’s degree) that guarantees stability for certain injection rates. As far as we know, such a protocol is the best known local-knowledge packet-oblivious routing protocol regarding the injection rate for which stability is guaranteed (especially for networks with large maximum node degree).

Later, we introduce a packet-oblivious routing algorithm that guarantees stability for higher traffic, but it needs to use some global information of the system topology (so called global-knowledge).

The rest of the paper is structured as follows. Section 2 summarizes the technical preliminaries. In Section 3, we introduce and study universally strong selectors, which are the core components of the deterministic local-knowledge routing algorithm that is developed in Section 4. In Section 5, we present a global-knowledge routing algorithm that guarantees universal stability for higher traffic. In Section 6 we extend the results obtained for the Longest-In-System scheduling policy in Section 4 to other policies, mainly, SIS, NFS and FTG. This extension is based on different technical tools, mainly, on reduction to the wired model with failures studied in [4], in which SIS, NFS and FTG are stable. This reduction is only in one direction, and in particular it is an open problem whether these two models are equivalent or not. We conclude with future directions in Section 7. Some technical details regarding the reduction from Section 6 are deferred to the Appendix.

2 Model and Problem Definition

Wireless radio network. We consider a wireless radio network represented by a directed symmetric network graph $G = (V_G, E_G)$. It consists of nodes in $V_G$ representing devices, and directed edges, called links, representing the fact that a transmission from the starting node of the link could be directly delivered to the ending node. The graph is symmetric in the sense that if some $(i, j) \in E_G$ then $(j, i) \in E_G$ too. Each node has a unique ID number and it knows the number $m$ of edges in the network and the network in-degree (i.e., the largest number of links incoming to a network node).

Nodes communicate via the underlying wireless network $G$. Communication is in synchronous rounds. In each round a node could be either transmitting or listening. Node $i$ receives a message from a node $j \neq i$ in a round if $j$ is the only transmitting in-neighbor of $i$ in this round and node $i$ does not transmit in this round; we say that the message was successfully sent/transmitted from $j$ to $i$.

Conflict graphs. We define the conflict graph $H(G) = (V_{H(G)}, E_{H(G)})$ of a network $G$ as follows: (1) its vertices are links of the network (i.e., $V_{H(G)} = E_G$) and, (2) a directed edge $(u, v) \in E_{H(G)}$ if and only if a message across link $v \in E_G$ cannot be successfully transmitted while link $u \in E_G$ transmits. Note that, accordingly with the radio model, a conflict occurs if and only if the transmitter in $u$ is also a receiver in $v$ or the transmitter in $u$ is a neighbor of the receiver in $v$ (see Figure 1 for an illustrative example). If network $G$ is clear from the context, we skip the parameter $G$ in $H(G)$ (i.e., we will use $H$).

Our definition of the conflict graph is similar to the one used in [17] except that it looks only at the links of the network, instead of taking into consideration the injected packets and their paths (e.g., our conflict graph is bounded while the one used in [17] may grow with time). Note that, the links in our definition are directed in order to distinguish which transmission is blocked by which.

\[1\] It is enough if nodes know some upper bounds on these values, in which case the performance will depend on these known estimates, instead of the actual values.
Routing protocols and transmission schedules. We consider packet-oblivious routing protocols, that is, protocols which only use their hardwired memory and basic parameters of the stored packets assigned to them at injection time (such as source, destination, injection time, route) in order to decide which packet to send and when.

We distinguish between global-knowledge protocols which can use topological information given as input, and local-knowledge protocols that are given only basic system parameters such as the number of links or the network’s in-degree.

All our protocols will be based on pre-defined transmission schedules, which will be circularly repeated — the properties of these schedules will guarantee stability for certain injection rates. These schedules will be different for different types of protocols, due to the available information based on which these schedules could be created.

Adversaries. We model dynamic injection of packets by way of an adversarial model, in the spirit of similar approaches used in [14, 8, 24, 21, 23, 19, 17]. An adversary represents the users that generate packets to be routed in a given radio network. The constraints imposed on packet generation by the adversary allow considering worst-case performance of deterministic routing algorithms handling dynamic traffic.

Over time, an adversary injects packets to some nodes. The adversary decides on a path a packet has to traverse upon its injection. Our task is to develop a packet-oblivious routing protocol such that the network remains stable; that is, the number of packets simultaneously queued is bounded by a constant in all rounds. Since an unbounded adversary can exceed the capability of a network to transmit messages, we limit its power in the following way: For any time window of any length $T$, the adversary can inject packets (with their paths) in such a way that each link is traversed by at most $\rho \cdot T + b$ packets, for some $0 \leq \rho \leq 1$ and $b \in \mathbb{N}^+$. We call such an adversary a $(\rho, b)$-adversary.

3 Selectors as transmission schedulers

In this section, we introduce a family of combinatorial structures, widely called selectors [1, 20], that are the core of the deterministic routing algorithm presented in Section 4. In short, we will use specific type of selectors to provide a set of transmission schedules that assure stability when combined with suitable queuing policies.

There are many different types of selectors, with the more general one being described below:

Definition 1. Given integers $k, m$ and $n$, with $1 \leq m \leq k \leq n$, we say that a boolean matrix $M$ with $t$ rows and $n$ columns is a $(n, k, m)$-selector if any submatrix of $M$ obtained by choosing $k$ out of $n$ arbitrary columns of $M$ contains at least $m$ distinct rows of the identity matrix $I_k$. The integer $t$ is referred as the size of the $(n, k, m)$ selector.

In order to use selectors as transmission schedules, the parameter $n$ is intended to refer to the number of nodes in the network, $k$ refers to the maximum number of nodes that can compete to transmit (i.e., $k = \Delta + 1$, where $\Delta$ is the maximum degree of the network), and $m$ refers to the number of nodes that are guaranteed to successfully transmit during the $t$-round schedule. Therefore, each column of the matrix $M$ is used to define the whole transmission schedule of each node. Rows are used to decide which nodes should transmit at each time slot. In the $i$-th time slot, node $v$ will transmit iff $M_{i,v} = 1$ (and $v$ has a packet queued); the schedule is repeated after each $t$ time slots.

Taking into account the above-mentioned approach, selectors may be used to guarantee that during the schedule, every node will successfully receive some messages.

A $(n, k, 1)$-selector guarantees that, for each node, one of its neighbors will successfully transmit during at least 1 round per schedule cycle (that is, that node will successfully receive at least one message). However, whereas the above use of selectors is helpful in broadcasting (since there is progress every time any node receives a message from a neighbor), it happens that many neighbors may have something to send, but only one of them has something for that node. Therefore, the above presented selector guarantees that each node will receive at least one message, but not necessarily will receive the one addressed to it.

A $(n, k, k)$-selector (which is known as strong selector [1]) guarantees that every node that has exactly $k$ neighbors will receive a message from each one of them. However, it has been shown that its size $t = \Omega(\min\{n, (k^2 / \log k) \log n\})$. This means that $k$ packets will be received, but during a long amount of time.

In order to solve the above mentioned problems with known selectors, now we introduce a new type of selectors, which we call universally strong. Namely, a $(n, k, \epsilon)$-universally-strong selector of length $t$ guarantees that every node will receive $\epsilon \cdot t / k$ successful messages from every neighbor during $t$ rounds. More formally:

Definition 2. A $(n, k, \epsilon)$-universally-strong selector $S$ is a family of $t$ sets $T_1, \ldots, T_t \subseteq [n]$ such that for every set $A \subseteq [n]$ of at most $k$ elements and for every element $a \in A$ there exist at least $\epsilon \cdot t / k$ sets $T_i \in S$ such that $T_i \cap A = \{a\}$. 


3.1 Proving the existence of universally strong selectors that work in polynomial time

Clearly, universally strong selectors make sense provided they exist and their size is moderate. In the next theorem, we prove that, for any $\epsilon \leq 1/e$, there exists a $(n, k, \epsilon)$-universally-strong selector of polynomial size.

**Theorem 1.** For any $\epsilon \leq 1/e$, there exists a $(n, k, \epsilon)$-universally-strong selector of size $O(k^2 \ln n)$.

**Proof.** The proof relies on the probabilistic method.

Consider a random matrix $M$ with $t$ rows and $n$ columns, where $M_{i,j} = 1$ with probability $p$ and $M_{i,j} = 0$ otherwise. Given a row $i$ and columns $j_1, \ldots, j_k$, the probability that $M_{i,j_1} = 1$ and $M_{i,j_2} = \cdots = M_{i,j_k} = 0$ (i.e., that node $j_1$’s transmission is not interrupted by nodes $j_2, \ldots, j_k$ in round $i$) is $P = p(1-p)^{k-1}$ and is maximized with $p = 1/k$.

In further considerations, we use matrix $M$ generated with $p = 1/k$.

Given columns $C = \{j_1, \ldots, j_k\}$, let $X(C)$ be the number of “good” rows $i$ such that $M_{i,j_1} = 1$ and $M_{i,j_2} = \cdots = M_{i,j_k} = 0$.

We will use the following Chernoff bound:

$$Pr[X(C) \leq (1-\delta)E[X(C)]] \leq \exp(-E[X(C)]\delta^2/2) \quad \text{for } 0 \leq \delta \leq 1.$$ 

Using $E[X(C)] = Pt$ and $\delta = (kP - \epsilon)/(kP)$, we obtain:

$$Pr[X(C) \leq ct/k] \leq \exp(-Pt\delta^2/2).$$

Consider a “bad” event $\mathcal{E}$ such that for at least one set of columns of size at most $k$, there are few good rows. More specifically, $X(C) \leq ct/k$ for at least one set of columns $C$, where $|C| = k$. Probability $R$ of event $\mathcal{E}$ happening fulfills the following inequality:

$$R \leq k \left( \frac{n}{k} \right) \exp(-Pt\delta^2/2).$$

Therefore $R < 1$ if

$$\exp(-Pt\delta^2/2) < 1/ \left[ \binom{n}{k} \right]$$

$$-Pt\delta^2/2 < -\ln \left( \binom{n}{k} \right)$$

$$Pt \left( \frac{kP - \epsilon}{kP} \right)^2 / 2 > \ln \left( \binom{n}{k} \right)$$

Let $c = kP$. Using $\binom{n}{k} \leq \left( \frac{ne}{k} \right)^k$, provided $c \neq c$, we obtain the following:

$$t(c - \epsilon)^2/(2ck) > \ln k + \ln \left( \frac{ne}{k} \right)^k$$

$$t > \left[ 2ck \ln k + 2ck^2 \ln \left( \frac{ne}{k} \right) \right] / (c - \epsilon)^2$$

Therefore, as long as $0 \leq \delta = \frac{\epsilon - \epsilon}{c} \leq 1$ (so that we can use the Chernoff bound) and $\epsilon \neq c$, the probability of generating a random matrix $M$ such that event $\mathcal{E}$ occurs is less than 1. Thus, there exists a matrix $M$ such that, for every set of $k$ columns $j_1, \ldots, j_k$, there are at least $ct/k$ rows such that $M_{i,j_1} = 1$ and $M_{i,j_2} = \cdots = M_{i,j_k} = 0$. Trivially, such matrix $M$ guarantees the above property for any set of at most $k$ columns. Hence, $M$ represents a $(n, k, \epsilon)$-universally-strong selector, provided that $\epsilon < c = kP$. Next, we calculate which values of $\epsilon$ fulfill that inequality.

Consider a sequence $a_i = (1 + 1/i)^i$. $a_i$ is known as a lower bound on the Euler’s number $e$ (i.e., $\forall i \ a_i < e$). Note that $c = kP = (1 - 1/k)^{k-1} = 1/a_{k-1} > 1/e$ for all $k \geq 2$. This implies that any $\epsilon \leq 1/e$ fulfills the requirement of $\delta > 0$ and results in the existence of a $(n, k, \epsilon)$-universally-strong selector. □
Let $d = \log_k n$ and $q = c \cdot k \cdot d$ for some constant $c > 0$.

2. Consider all polynomials $P_i$ of degree $d$ over field $[q]$. Notice that there are $q^{d+1}$ of such polynomials.

3. Create a matrix $M$ of size $q \times q^2$. Each column will represent values $P_i(x)$ of each polynomial $P_i$ for arguments $x = 0, 1, \ldots, q - 1$. Each value $y = P_i(x)$ is represented in $q$ consecutive rows of 0s and 1s, where 1 is in $y$-th position, while on all other positions there are 0s. Notice that each column has $q^2$ rows (for each argument).

4. Each row of matrix $M$ will correspond to one set $T_i$ of a universally strong selector $\{T_i\}_{i=1}^{q^2}$.

Figure 2: The Poly-Universally-Strong algorithm, given parameters $n$ and $k$.

3.2 Obtaining universally strong selectors of polynomial size

Here, we present an algorithm, which we call Poly-Universally-Strong, that computes a universally strong selector in polynomial local time. This is more efficient way of obtaining a universally-strong selector than by derandomization of the probabilistic method used in the existential result (Theorem 1); yet the obtained universally-strong selector has slightly weaker properties, i.e., slightly lower value of $\epsilon$. The algorithm, whose code is shown in Figure 2, has to be executed by each node in the network taking the same polynomials, so that all nodes will obtain exactly the same matrix that defines the transmission schedule.

The next theorem shows that, indeed, it constructs a $(n, k, \epsilon)$-universally-strong selector of polynomial size with $\epsilon = 1/(4\log_k n)$.

**Theorem 2.** Poly-Universally-Strong constructs (by using $c = 2$) a $(n, k, \epsilon)$-universally-strong selector of size $4 \cdot k^2 \cdot \log_k^2 n$ with $\epsilon = 1/(4\log_k n)$.

**Proof.** First, note that two polynomials $P_i$ and $P_j$ of degree $d$ with $i \neq j$, can have equal values for at most $d$ different arguments. This is because they have equal values for arguments $x$ for which $P_i(x) - P_j(x) = 0$. However, $P_i - P_j$ is a polynomial of degree at most $d$, so it can have at most $d$ zeroes. So, $P_i(x) = P_j(x)$ for at most $d$ different arguments $x$.

Take any polynomial $P_i$ and any $k$ polynomials $P_j$. There are at most $k \cdot d$ different arguments where one of the $k$ polynomials can be equal to $P_i$. So, for $q - k \cdot d$ different arguments, the values of the polynomial $P_i$ are unique. Therefore, if we look at rows with $1$ in column $i$ of matrix $M$ (there are $q$ of those rows, one for each argument), at least $q - k \cdot d$ of them have 0s in chosen $k$ columns. Since there are $q^2$ rows, so a fraction $(q - k \cdot d)/q^2$ of rows have the desired property (i.e., there is value 1 in column $i$ and value 0 in the chosen $k$ columns):

$$\frac{q - k \cdot d}{q^2} = \frac{(c - 1) \cdot k \cdot d}{(c \cdot k \cdot d)^2} = \frac{c - 1}{c^2 \cdot k \cdot d} \triangleq f(c).$$

Let us find the value of $c$ that maximizes the function $f$. To do it, we compute its differential

$$f'(c) = \frac{(c - 1)}{c^2 \cdot k \cdot d} = \frac{1 \cdot (c^2 \cdot k \cdot d) - (c - 1) \cdot k \cdot d \cdot 2c}{c^4 \cdot k^2 \cdot d^2} = \frac{-c^2 \cdot k \cdot d + 2c \cdot k \cdot d}{c^4 \cdot k^2 \cdot d^2} = \frac{-c + 2}{c^3 \cdot k \cdot d}.$$

So, $f'(c) = 0$ for $c = 0$ or $c = 2$. The value $c = 2$ maximizes $f$, giving $f(2) = 1/(4k \cdot d) = 1/(4k \cdot \log_k n)$. Therefore, we can construct a $(n, k, \epsilon)$-universally-strong selector with $\epsilon = f(2) \cdot k = 1/(4d) = 1/(4 \cdot \log_k n)$ of length $4k^2 \cdot \log_k^2 n$ (which means that a $f(2) = 1/(4k \cdot \log_k n)$ fraction of the selector’s sets have the desired property). \qed

4 A local-knowledge routing algorithm

In this section, we introduce a local-knowledge packet-oblivious routing algorithm that makes use of the family of universally strong selectors introduced in Section 3 as transmission schedules (i.e., the time instants when packets stored at each one node must be transmitted to a receiving node). As it has been mentioned previously, local-knowledge routing algorithms work without using any topological information, except for some upper bounds on the number of links and the network’s degree.
1. Choose \( m \) and \( \Delta \) such that \( |E(G)| \leq m \) and \( \Delta_H^L \leq \Delta \).
2. Obtain a \((m, \Delta + 1, \epsilon)\) universally strong selector (for some value of \( \epsilon \)) of some length \( t \) and use it as the transmission schedule.
3. When there are several packets awaiting in a single queue, choose the packet to be transmitted according to ALG, breaking ties in any arbitrary fashion.

Figure 3: The USS-PLUS-ALG algorithm for a network \( G \).

The code of such an algorithm, which we call USS-PLUS-ALG, is shown in Figure 3. Given a graph \( G \) with a number of links bounded by \( m \), and an in-degree of its conflict graph \( H \) (which we denote as \( \Delta_H^L \)) bounded by \( \Delta \geq 1 \), it uses a \((m, \Delta + 1, \epsilon)\)-universally strong selector as a schedule: assuming the selector is represented by matrix \( M \) with \( t \) rows, each link \( z \in E_G \) will transmit at time \( i \) iff \( M_i \mod t \cdot z = 1 \). Notice that here each link is assumed to have an independent queue, and therefore they will act as a sort of “nodes” (in terms of selectors, such as it has been stated in the previous section). This means that each individual link will have its own schedule.

Next, we show that the USS-PLUS-LIS algorithm (i.e., the USS-PLUS-ALG algorithm where ALG is the Longest-In-System scheduling policy), guarantees stability, provided a given packets’ injection admissibility condition is fulfilled.

But first, we show that LIS, combined with a transmission schedule that guarantees a number of successful transmissions in some time interval, makes the resulting routing protocol stable (these lemmas are adapted versions of analogous results about universal stability of the LIS protocol in wired network [8]).

**Definition 3.** A \((\rho, T)\)-frequent schedule for graph \( G \) is an algorithm that decides which links of graph \( G \) transmit at every round in such a way that each link is guaranteed to successfully (without radio network collisions) transmit at least \( \rho \cdot T \) times in any window of length \( T \) (provided at least \( \rho \cdot T \) packets await for transmission at the link at the start of the window).

At this point, we note that the transmission schedules provided by our universally strong selectors can be seen as \((\rho, T)\)-frequent schedules.

**Lemma 1.** If there exists a \((\rho', T)\)-frequent schedule \( S \), then using LIS as the queueing policy guarantees stability of the resulting routing protocol against any \((\rho, b)\)-adversary for \( \rho < \rho' \).

Before we prove this lemma, we will introduce some additional notations and auxiliary lemmas.

Let \( L \) be the length of the longest route in the system. Let us denote by class \( i \) the set of packets injected during \( i \)-th window. A class \( i \) is said to be active during a window \( w \) if and only if at some time during window \( w \) there is some packet in the system of class \( i' \leq i \).

Consider some packet \( p \) injected during window \( W_0 \), whose path crosses links \( e_1, e_2, \ldots, e_L \), in this order. We use \( W_i \) to denote the window, during which \( p \) crossed link \( e_i \). Let \( c_w \) denote the number of active classes during window \( w \). We define \( c = \max_{w \in \{W_0, W_L\}} c_w \). Then, we can bound the number of windows to deliver \( p \).

**Lemma 2.**

\[
W_L - W_0 \leq \frac{1 - \left(1 - \frac{\rho}{\rho'}\right)^L}{\rho \cdot T} \cdot (b - 1) + c \cdot \left[1 - \left(1 - \frac{\rho}{\rho'}\right)^L\right].
\]

**Proof.** The packet \( p \) reaches link \( e_i \) for the first time in window \( W_{i-1} \). Since \( p \) is in the system, during window \( W_{i-1} \) all classes \( \{W_0, W_{i-1}\} \) are active. Therefore, according to the definition of \( c \), there are at most \( c - (W_{i-1} - W_0) \) active classes with packets older than packet \( p \). Packets in those classes are the only packets that take priority over packet \( p \) on link \( e_i \). The oldest such packet was injected during window \( w_{\text{first}} = W_0 - c - (W_{i-1} - W_0) = W_{i-1} - c \). Since its injection, at most \( (W_0 - w_{\text{first}}) \cdot \rho \cdot T + b = [c - (W_{i-1} - W_0)] \cdot \rho \cdot T + b \) packets older than \( p \) could be injected into the system. Therefore, there are at most \([c - (W_{i-1} - W_0)] \cdot \rho \cdot T + b - 1\) packets that will take priority over packet \( p \) on link \( e_i \). Since each link transmits at least \( \rho' T \) times per window, the number of windows until \( p \) transgresses link \( e_i \) is at most

\[
W_i - W_{i-1} \leq \frac{\rho \cdot T \cdot (c + W_0 - W_{i-1}) + b - 1}{\rho' \cdot T}.
\]

Hence,

\[
W_i \leq \left(1 - \frac{\rho}{\rho'}\right) W_{i-1} + \frac{\rho}{\rho'} (c + W_0) + \frac{b - 1}{\rho' \cdot T}.
\]
Therefore, solving the recurrence, we get
\[
W_L \leq (1 - \frac{\rho}{\rho'})^L \cdot W_0 + \sum_{i=0}^{L-1} (1 - \frac{\rho}{\rho'})^i \left[ \frac{\rho}{\rho'} (c + W_0) + \frac{1}{\rho' T} \right] = (1 - \frac{\rho}{\rho'})^L \cdot W_0 + \left[ 1 - (1 - \frac{\rho}{\rho'})^L \right] (c + W_0) + \frac{1 - (1 - \frac{\rho}{\rho'})^L}{\rho' T} \cdot \frac{b - 1}{\rho' T},
\]
which proves the lemma.

Now we have a bound on how long packet \( \rho \) can be in the system, depending on value \( c \). We will show that \( c \) is bounded by a constant, depending only on network and adversary parameters, i.e., \( L, \rho \) and \( b \), and value \( \rho' \) from Lemma 1.

**Lemma 3.** There are never more than
\[
(b - 1) \cdot \frac{1 - (1 - \frac{\rho}{\rho'})^L}{(1 - \frac{\rho}{\rho'})^L \cdot \rho \cdot T}
\]
active classes in the system.

**Proof.** Let \( c' = (b - 1) \cdot \frac{1 - (1 - \frac{\rho}{\rho'})^L}{(1 - \frac{\rho}{\rho'})^L \cdot \rho \cdot T} \). Assume, by contradiction, that a window \( w \) is the first window during which there are at least \( c' + 1 \) active classes. Hence, at the end of window \( w - 1 \), there is a packet \( q \) that was in the system for \( c' \) windows, and no more than \( c' \) classes were active until the end of window \( w - 1 \).

According to Lemma 2, packet \( q \) is delivered in at most
\[
c' \left[ 1 - (1 - \frac{\rho}{\rho'})^L \right] + \frac{1 - (1 - \frac{\rho}{\rho'})^L}{\rho \cdot T} \cdot (b - 1) = c' - 1
\]
windows, which gives a contradiction.

Now that we have proven that any packet \( \rho \) spends bounded time in the system, we can prove Lemma 1.

**Proof of Lemma 1.** In Lemma 3, it has been shown that \( c \) is bounded. By Lemma 3, this implies that \( W_L - W_0 \) is also bounded. This result guarantees that each packet spends a bounded time in the system. That means that such system is stable against any \((\rho, b)\)-adversary, provided that \( \rho' > \rho \), which completes the proof of the lemma.

We now proceed with the main result in this section.

**Theorem 3.** Given a network \( G \), the USS-PLUS-LIS algorithm is stable against any \((\rho, b)\)-adversary, for \( \rho < \frac{\epsilon}{\Delta+1} \).

**Proof.** Let us take any arbitrary link \( z \in \mathcal{E}_G \) and consider the set of all other links that conflict with link \( z \), of which there are at most \( \Delta \). This means that there exist at least \( \epsilon \cdot \frac{t}{\Delta+1} \) rows \( i \) in \( M \) such that \( M_{i \mod t, z} = 1 \) and \( M_{i \mod t, c_z} = \cdots = M_{i \mod t, c_j} = 0 \). Therefore, at time \( i \), link \( z \) will transmit a message, and no link that conflicts with the link \( z \) will transmit. This guarantees that each link will successfully transmit, at least, \( \epsilon \cdot \frac{t}{\Delta+1} \) messages during any schedule of length \( t \) (i.e., we obtained a \((\epsilon/(\Delta+1), t)\)-frequent schedule \( S \)). Then, we can apply the result Lemma 1 to deduce that such an algorithm is stable against any \((\rho, b)\)-adversary, where \( \rho < \frac{\epsilon}{\Delta+1} \).

By using the selectors provided by the POLY-UNIVERSALLY-STRONG algorithm in USS-PLUS-LIS, we have the following result:

**Corollary 1.** Given a network \( G \), the USS-PLUS-LIS algorithm using a universally strong selector computed by the POLY-UNIVERSALLY-STRONG algorithm is a stable algorithm against any \((\rho, b)\)-adversary, for \( \rho < \frac{1}{4(\Delta+1) \log_{\Delta+1} m} \).

If instead of the selectors provided by the POLY-UNIVERSALLY-STRONG algorithm, we use a selector from Theorem 1 we have that:
Corollary 2. Given a network $G$, there exists a universally strong selector that, used in the USS-PLUS-LIS algorithm, provides a stable algorithm against any $(\rho, b)$-adversary, for $\rho < \frac{1}{\sqrt{\Delta}}$.

As it can be readily seen, the USS-PLUS-LIS algorithm for a network $G$ requires some knowledge of the value of the in-degree of its conflict graph $H$ (i.e., of $\Delta_{in}^H$). In order to obtain $H$ it is necessary to gather the whole topology of $G$. However, as the next lemma shows, $\Delta_{in}^H$ can be bounded by the in-degree of the network $G$ (denoted $\Delta_G$).

Lemma 4. $\Delta_{in}^H \leq \Delta_G^2 + \Delta_G - 1$, provided $\Delta_G > 0$.

Proof. If $\Delta_{in}^H = 0$, then the lemma is trivially true. Otherwise, consider a vertex $e$ in $H$ of maximum in-degree $\text{deg}(e) = \Delta_{in}^H$. Since $\Delta_{in}^H \neq 0$, there is at least one edge $(e', e) \in H$ such that, in $G$, $e$ cannot successfully transmit at the same time instant when $e'$ transmits. Let us denote $e = (u, v)$ and $e' = (u', v')$, and let us consider the different scenarios where $e$ and $e'$ may conflict.

Now, we make a case analysis regarding the possible conflicts in $G$ (note that its in-degree is equal to its out-degree, since $G$ is symmetric):

1. $u' = u$ and $v' \neq v$ (a node $u = u'$ cannot transmit messages to 2 different receivers): there are at most $\Delta_G - 1$ such links $e'$, given fixed link $e$.
2. $u' = v$ (if $u'$ transmits, it cannot listen at the same time): there are at most $\Delta_G$ such links $e'$, given fixed link $e$.
3. $u' \neq u$ is a neighbor of $v$ (i.e., $v$ can hear both from $u$ and $u'$): there are at most $\Delta_G = 1$ neighbors of $v$ different than node $u$, and each of them has, at most, $\Delta_G$ different links. This gives $\Delta_G^2$ such links $e'$, given fixed link $e$.

Therefore, in overall there are at most $(\Delta_G - 1) + \Delta_G + (\Delta_G^2 - \Delta_G) = \Delta_G^2 + \Delta_G - 1$ such links.\[\square\]

The previous lemma shows that USS-PLUS-LIS can be seen as a local-knowledge algorithm, in the sense that it only requires some knowledge about two basic system parameters: the number of links and the network’s in-degree. In Section 5, we will look at a solution that also requires some global-knowledge of $G$.

Comparison with other local-knowledge routing protocols In the past, two approaches have been considered for local-knowledge packet-oblivious routing in wireless networks: either by using selectors as transmission schedules, or by using transmission schedules defined in terms of only one guaranteed successful transmission per a given number of time steps, e.g., \[19\].

The latter approach is more general than the previous approaches based on selectors, and thus subsumes them. Let us compare with the previous approach of using transmission schedules defined in terms of only one guaranteed successful transmission per a given number $h$ of time steps. In \[11\] it has been shown that the values of $h$ are lower bounded by $\Omega((k^2 / \log k) \log n)$, where $k$ denotes in our case the largest node degree in the network. Since, regardless of the used packet scheduling policy, in that setting the injection rates that guarantees stability are upper bounded by $\rho_3 = 1/h$, then we have that the ratio $\rho_2 / \rho_3$ is lower bounded by $\Omega(\log n / \log k)$ (i.e., $\rho_2 \geq \rho_3$).

It follows that our results are better, in terms of stability guarantees for wider range of injection rates, than the provided by all previously proposed local-knowledge packet-oblivious protocols. Moreover, the bigger the maximum node degree of the network the wider stability range is provided by our approach – the improvement is linear in terms of the maximum degree.

5 A global-knowledge routing algorithm

In this section, we introduce a global-knowledge packet-oblivious routing algorithm, which we call COLORING-PLUS-ALG, that is based on using graph coloring as transmission schedules. Similar to the USS-PLUS-ALG algorithm, the COLORING-PLUS-ALG algorithm does not take into account any historical information. However, it has to be seeded by some information about the network topology.

Before we introduce the above-mentioned routing algorithm, we state the following fact regarding the relationship between vertex coloring in a conflict graph, and its use as a transmission schedule.

Fact 1 (\[17\]). Vertex coloring of the conflict graph $H(G)$ using $x$ colors is equivalent to a schedule of length $x$ that successfully transmits a packet via each directed link of network $G$.\[9\]
On the other hand, taking the $\rho \Delta$ the two concerned nodes), we have that the networks (i.e., starting from an undirected one, each link was replaced by two oppositely directed links between 4 network $G$ of the COLORING PLUS algorithm requires global-knowledge of the structure of the graph). As it has been shown in Theorem 4, the CCOLORING PLUS protocol is stable against adversaries whose injection rate is upper bounded by $\rho < 1/\chi(H)$, where $\chi(H)$ is the chromatic number of the conflict graph $H$ of the network $G$.

Proof. Let us split the vertices $V_H$ of the graph $H$ into sets $V_H^i$ for $i = 0, 1, \ldots, k - 1$, where every vertex in $V_H^i$ is assigned the $i$-th color in the vertex coloring of graph $H$. Each link in the graph $G$ is represented by one vertex in $V_H$, and therefore each link is assigned a unique color. According to the definition of the conflict graph $H$, if there is no edge $\langle u, v \rangle \in E_H$, then links $u \in E_G$ and $v \in E_G$ can deliver their packets simultaneously, without a collision. Therefore, if at a given round $t$ only links of $(t \mod i)$-th color transmit, then no collision occurs. Since each link has a color $i \in \{0, 1, \ldots, k - 1\}$ assigned to it, then each link will successfully transmit a packet once each $k$ consecutive rounds (as far as there is one packet waiting in its queue).

Since $\chi(H)$-coloring is an optimal coloring of graph $H$, we have the following result.

Corollary 3. An optimal coloring of collision graph $H$ provides a $(1/\chi(H), \chi(H))$-frequent schedule.

Once we have made it clear that coloring of a collision graph can be used to obtain a transmission schedule, the code of the COLORING-PLUS-LIS algorithm is shown in Figure 4.

As we did in the previous case, here we show that COLORING-PLUS-LIS (i.e., the COLORING-PLUS-LIS where ALG is the Longest-In-System scheduling policy), guarantees stability, provided a given packets’ injection admissibility condition is fulfilled. Although such an admissibility condition is less restrictive than the corresponding for the USSPLUS-LIS protocol (i.e., it is possible to guarantee stability for a wider range of injection rates), the COLORINGPLUS-LIS algorithm requires global-knowledge of the structure of the graph (note that the USSPLUS-LIS protocol only requires local-knowledge of the structure of the graph).

Theorem 4. The COLORING-PLUS-LIS algorithm is stable provided $\rho < 1/\chi(H)$, where $\chi(H)$ is the chromatic number of the conflict graph $H$ of the network $G$.

Proof. We start the proof with referring to Corollary 3 which shows that coloring of a collision graph can be used to obtain a $(1/\chi(H), \chi(H))$-frequent schedule $C$.

Let us take any $\rho = 1/\chi(H) - \epsilon$, for some $\epsilon > 0$. We can use Lemma 1 with $S = C$ (so, $\rho' = 1/\chi(H)$) to show that COLORING-PLUS-LIS is stable against any $(\rho, b)$-adversary in the radio network model.

Observe that the COLORING-PLUS-LIS algorithm requires global-knowledge of the structure of the graph: first, to construct $H$, and then to obtain its optimal coloring.

Global-knowledge vs local-knowledge routing protocols As it has been shown in Theorem 4 the COLORINGPLUS-LIS protocol is stable provided $\rho < 1/\chi(H)$, where $\chi(H)$ is the chromatic number of the conflict graph $H$ of network $G$. By the Brooks’ theorem [16], we have that $\chi(H) = \Delta_H + 1$, and taking into account how we have defined the networks (i.e., starting from an undirected one, each link was replaced by two oppositely directed links between the two concerned nodes), we have that $\Delta_H = \Delta_{in}^H$. Since $\Delta_{in}^H = k - 1$ then the injection rates for which stability is guaranteed is upper bounded by $\rho_1 = 1/k$.

On the other hand, taking the $(n, k, \epsilon)$ universally strong selector in Theorem 2 with $k = \Delta_{in}^H + 1$ and $\epsilon = 1/(4k \log_k n)$, we have that $4k \log_k n$ packets are guaranteed to succeed per $4k^2 \log_k^2 n$ time slots. This means that the USSPLUS-LIS protocol is stable against adversaries whose injection rate is upper bounded by $\rho_2 = 1/(4k \log_k n)$.

Therefore, we have that $\rho_2/\rho_1 = k/(4k \log_k n) = 1/(4 \log_k n)$. If, instead of the selector in Theorem 2 we consider the selector in Theorem 1 with $k = \Delta_{in}^H + 1$ and $\epsilon = 1/e$, we have that $\rho_2 = 1/(e \cdot k)$ and, therefore, $\rho_2/\rho_1 = 1/e$. 

Figure 4: The COLORING-PLUS-LIS algorithm for graph $G$. 

Note that every set of vertices of same color can be extended to a maximal independent set. The resulting family of independent sets is still a feasible schedule that guarantees no conflicts and is no worse than just coloring. In fact, it may allow some links to transmit more than once during the schedule, without increasing the length of the schedule.

Following, we show that coloring of a collision graph can be used to obtain a transmission schedule, where each link is guaranteed to regularly transmit.

Lemma 5. A $k$-coloring of collision graph $H$ provides a $(1/k, k)$-frequent schedule.

Proof. Let us split the vertices $V_H$ of the graph $H$ into sets $V_H^i$ for $i = 0, 1, \ldots, k - 1$, where every vertex in $V_H^i$ is assigned the $i$-th color in the vertex coloring of graph $H$. Each link in the graph $G$ is represented by one vertex in $V_H$, and therefore each link is assigned a unique color. According to the definition of the conflict graph $H$, if there is no edge $\langle u, v \rangle \in E_H$, then links $u \in E_G$ and $v \in E_G$ can deliver their packets simultaneously, without a collision. Therefore, if at a given round $t$ only links of $(t \mod i)$-th color transmit, then no collision occurs. Since each link has a color $i \in \{0, 1, \ldots, k - 1\}$ assigned to it, then each link will successfully transmit a packet once each $k$ consecutive rounds (as far as there is one packet waiting in its queue). 

Since $\chi(H)$-coloring is an optimal coloring of graph $H$, we have the following result.

Corollary 3. An optimal coloring of collision graph $H$ provides a $(1/\chi(H), \chi(H))$-frequent schedule.
The previous result implies that, by using the COLORING-PLUS-LIS protocol, it is possible to guarantee stability for a wider range of injection rates than by using the USS-PLUS-LIS protocol. In other words, there is a price that must be paid in order to use a local-knowledge protocol instead of a global-knowledge one: namely, the injection rate for which stability is guaranteed will be $\epsilon$ times smaller.

6 Extension of the results to other scheduling policies

In this section, we show that the results obtained in Section 4 for local-knowledge routing combined with LIS (Longest In System) can be extended to other scheduling policies; namely, NFS (Nearest-From-Source), SIS (Shortest-In-System) and FTG (Farthest-To-Go). Indeed, for such a scheduling policies, Theorems 5 and 6 respectively parallelize the analogous results in Theorems 3 and 4 obtained for LIS.

Theorem 5. Given a network $G$, the USS-PLUS-ALG algorithm, where $\text{ALG} \in \{NFS,SIS,FTG\}$, is stable against any $(\rho, b)$-adversary, for $\rho < \frac{\epsilon}{\Delta + 1}$.

Proof. The proof is similar to that in Theorem 3. The only difference is that, instead of Lemma 1, we can apply the results in Lemma 6 for NFS, SIS and FTG (see Appendix A) to deduce that such an algorithm is stable against any $(\rho, b)$-adversary, where $\rho < \frac{\epsilon}{\Delta + 1}$.

Theorem 6. The COLORING-PLUS-ALG algorithm, where $\text{ALG} \in \{NFS,SIS,FTG\}$, is stable provided $\rho < \frac{1}{\chi(H)}$, where $\chi(H)$ is the chromatic number of the conflict graph $H$ of the network $G$.

Proof. We will reduce the packet scheduling in radio network problem to the problem of packet scheduling in the wired failure model [4], in which these policies are known to be stable.

We start the proof with referring to Corollary 3 which shows that coloring of a collision graph can be used to obtain a $(1/\chi(H), \chi(H))$-frequent schedule $C$.

Let us take any $\rho = 1/\chi(H) - \epsilon$, for some $\epsilon > 0$. Now, we can use Lemma 6 with $S = C$ (so, $\rho' = 1/\chi(H)$) and $\text{ALG} \in \{NFS,SIS,FTG\}$ (with $\rho'' = 1 - \epsilon$) to show that we can build an algorithm that is stable against any $(\rho, b)$-adversary in the radio network model (see Appendix A). Note that COLORING-PLUS-ALG is a special case of the algorithm built in the proof of Lemma 6 with $S = C$. Therefore COLORING-PLUS-ALG with $\text{ALG} \in \{NFS,SIS,FTG\}$ is stable against any $(\rho, b)$-adversary in the radio network model.

7 Future work

A natural direction would be to study impossibility results to show that our results are tight (what, we conjecture, is the case, at least asymptotically). Other classes of protocols are also interesting for a study, for instance, when packets are injected without pre-defined routes. Universally strong selectors are interesting on its own right – finding more tight polynomial construction and more applications for them is a promising open direction. Finally, exploring the reductions between various settings of adversarial routing could lead to new discoveries, as demonstrated in the last part of this work.

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[28] Paweł Garnarek, Tomasz Jurdziński, and Dariusz R. Kowalski. mmwave wireless backhaul scheduling of stochastic packet arrivals. In *forthcoming Proceedings of the 33rd IEEE International Parallel and Distributed Processing Symposium (IPDPS)*. IEEE, 2019.
First, let us explain the (wired) failure model \([4]\). Given is a network graph \(G\). A \((\rho, b)\)-adversary in the failure model injects paths (packets) into \(G\) and generates failures in such a way that in any interval \(I\) the following inequality holds:

\[
\text{Arr}_e(I) + \text{Fail}_e(I) \leq |I| + b,
\]

where \(\text{Arr}_e(I)\) is the number of packets injected during interval \(I\) that pass through edge \(e\) and \(\text{Fail}_e(I)\) is the number of failures on edge \(e\) generated during interval \(I\). Each link \(e\) that has some packets waiting in its queue can transmit a packet in every round, i.e., there are no collisions between edges.

There are known stable algorithms for packet scheduling in the failure model, such as NFS (Nearest-From-Source), SIS (Shortest-In-System), or FTG (Farthest-To-Go) against \((\rho, b)\)-adversary with any \(\rho < 1\) \([3]\).

**Lemma 6.** Suppose we have a stable algorithm \(\text{ALG}\) against any \((\rho'', b)\)-adversary \(\text{ADV}_{\text{fail}}\) in the failure model on graph \(G\). Suppose we have a \((\rho', T)\)-frequent schedule \(S\).

Then we can build a stable algorithm \(S^{\text{PLUS-ALG}}\) against any \((\rho, b)\)-adversary \(\text{ADV}_{\text{RN}}\) in the radio network model on graph \(G\), for any \(\rho\) such that \(\rho < \rho'\) and \(\rho'' \geq 1 + \rho - \rho'\).

**Proof.** The stable algorithm in each round has two steps:

1. Determine which links transmit, according to a \((\rho', T)\)-frequent schedule \(S\) for some parameters \(\rho'\) and \(T\),
2. Determine, for each link \(e\), which packet awaiting in a queue of link \(e\) to transmit, according to \(\text{ALG}\).

We can think of rounds when \(S\) does not successfully transmit a packet via link \(e\) due to a collision as failures on link \(e\) in the failure model. Schedule \(S\) guarantees that each link \(e\) has at most \((1 - \rho')T\) transmission blocked in any interval \(I\) of length \(T\). This means that each link \(e\) has at most \(\text{Fail}_e(I) \leq (1 - \rho')T\) failures during \(I\). Furthermore, \(\text{ADV}_{\text{RN}}\) can inject at most \(\text{Arr}_e(I) \leq \rho T + b\) packets passing through each edge \(e\) during \(I\).

\[
\text{Arr}_e(I) + \text{Fail}_e(I) \leq \rho T + b + (1 - \rho')T = T(1 + \rho - \rho') + b
\]

Therefore, the graph \(G\) with packet arrivals from \(\text{ADV}_{\text{RN}}\) and failures being collisions generated by \(S\) is an instance of the failure model with a \((1 + \rho - \rho', b)\)-adversary. That means that using \(\text{ALG}\) to compute which packet to choose for each link at each round guarantees stability, provided \(\rho'' \geq 1 + \rho - \rho'\).

\(\square\)

Note that using a schedule obtained from coloring in place of \(S\) in the algorithm constructed above provides \text{COLORING-PLUS-ALG} algorithm.
Lemma 7. Having a $(\rho', T)$-frequent schedule $S$ and using NFS, SIS or FTG to determine which of the awaiting packets is transmitted, gives an algorithm that is stable against $(\rho, b)$-adversary in radio network $G$, for any $\rho < \rho'$.

Proof. As mentioned earlier, NFS (Nearest-From-Source), SIS (Shortest-In-System) and FTG (Farthest-To-Go) policies are stable against $(\rho'', b)$-adversary with any $\rho'' < 1$ in the failure model.

Assume we have a $(\rho', T)$-frequent schedule $S$ for some $\rho'$. Let us consider any $(\rho, b)$-adversary in radio network $G$ such that $\rho = \rho' - \epsilon$ for some $\epsilon > 0$. Then $1 + \rho - \rho' = 1 - \epsilon$. NFS, SIS and FTG are stable against $(1 - \epsilon, b)$-adversary in the failure model. According to Lemma 6, taking $A_L G \in \{\text{NFS, SIS, FTG}\}$ and $\rho'' = 1 - \epsilon$, $S$-PLUS-ALG is stable against any $(\rho, b)$-adversary in radio model, provided $\rho < \rho'$.