Two-electron coherence and time-bin entanglement detection in electron quantum optics

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Engineering and studying few-electron states in ballistic conductors is a key step towards understanding entanglement in quantum electronic systems. In this Letter, we introduce the intrinsic two-electron coherence of an electronic source in quantum Hall edge channels and relate it to two-electron wavefunctions and to current noise in an Hanbury Brown–Twiss interferometer. Inspired by the analogy with photon quantum optics, we propose to measure the intrinsic two-electron coherence of a source using low-frequency current correlation measurements at the output of a Franson interferometer. To illustrate this protocol, we discuss how it can distinguish between a time-bin entangled pure state and a statistical mixture of time shifted electron pairs.

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The initial developments of electron quantum optics [1] have concerned the generation of single to few electron excitations [2–6] in ballistic conductors. The focus was on single electron coherence studies [7, 8], either through Mach–Zehnder interferometry [9–11] or electronic distribution function measurement [12, 13]. More recently, the Hanbury Brown–Twiss (HBT) [14] and Hong–Ou–Mandel (HOM) experiments [15] have been achieved with single electron sources. By demonstrating two-particle interference effects, they are important milestones towards single electron tomography [7], mirroring optical homodyne tomography [16]. The implementation of a similar protocol in a bidimensional electron gas [17] suggests that technology has reached the point where single electron coherence can be measured using signal processing strategies [18]. These recent breakthroughs call for quantitative studies of electronic decoherence [19, 20] and decoherence protection strategies [21, 22].

Besides, designing and understanding few electron states such as the charge–n Leviton [23, 24] are crucial steps towards understanding the emergence of many-body physics in coherent electronics. The appropriate notion for studying two-particle states and their dynamics is the two-electron coherence defined in electron quantum optics [25] by analogy with photonic second order coherence [26]. Whereas the diagonal part of two-electron coherence generically encodes the statistical properties of the electron flow such as antibunching, its off-diagonal part holds the key to a better understanding of entanglement in many-body electronic systems [27]. Orbital entanglement has been widely explored both theoretically [28, 29] and experimentally [30] and in relation with Bell inequality violations [31, 32]. Although, proposals for generating time-bin entanglement in ballistic conductors exist [33], the definition of two-electron coherence [25] is quite recent. Nevertheless, accessing the intrinsic two-electron coherence of an electron quantum optics source will be instrumental to understand two-particle entanglement and, most importantly, to compare how decoherence comparatively affects single and two-electron excitations.

In this Letter, we make a step in this direction by introducing the intrinsic two-electron coherence and degree of two-electron coherence emitted by an electron source. Then we show that this intrinsic coherence can actually be measured through current correlations in a Franson interferometer like device [34] which has also been proposed to study Cooper pair coherence [35]. We also argue that this measurement method extends single electron coherence measurement through average current in Mach–Zehnder interferometry [36].

By analogy with higher order coherences in optics [26, 37], the two-electron coherence is defined as [25]:

\[ \mathcal{G}^{(2e)}(1, 2 | 1', 2') = \text{Tr} \left[ \psi(2) \psi(1) \rho \psi \dagger(1') \psi \dagger(2') \right] \]

where \( \psi \) and \( \psi \dagger \) are field operators for the electron fluid, \( \rho \) its density operator and \( 1 = (x_1, t_1), 2 = (x_2, t_2), 1' = (x_1', t_1'), 2' = (x_2', t_2') \) four space-time coordinates.

Activating a source that generates excitations on top of the Fermi sea alters all electronic coherences. A source–intrinsic contribution to the total single electron coherence \( \mathcal{G}^{(e)}(1|1') = \text{Tr} \left[ \psi(1) \rho \psi \dagger(1') \right] \) can be defined [8] with respect to the Fermi sea by

\[ \Delta \mathcal{G}^{(e)}_S(1|1') = \mathcal{G}^{(e)}(1|1') - \mathcal{G}^{(e)}_F(1|1'). \]

\[ \mathcal{G}^{(ec)}(1|1') = \mathcal{G}^{(e)}(1|1') - \mathcal{G}^{(ec)}_F(1|1'). \]
To identify the intrinsic second order coherence of the source from the full $g^{(2e)}_\rho$, we must consider all processes contributing to the co-detection of two electrons, depicted on Fig. 1. The total two-electron coherence is thus the sum of (a) the two-electron coherence of the Fermi sea; (b) two terms describing classical coincidence events in which one electron comes from the Fermi sea and the other one from the source; (c) two quantum exchange contributions involving two-particle interferences between the source and the Fermi sea; (d) an additional term that defines intrinsic two-electron coherence of the source $\Delta g^{(2e)}_S(1,2;1',2')$:

$$
\Delta g^{(2e)}_S(1,2;1',2') = g^{(2e)}_F(1,2;1',2') + g^{(2e)}_F(1,1';2,2') \Delta g^{(e)}_S(1,1') - g^{(e)}_F(2,1') \Delta g^{(e)}_S(1,2') + \Delta g^{(2e)}_S(1,2;1',2').
$$

The definition of $\Delta g^{(2e)}_S$ given by Eq. [3] is the second in a hierarchy of definitions for the n-electron excess coherence with respect to a reference situation, namely the Fermi sea. All of them make sense even in the presence of interactions since their definition does not rely on Wick theorem.

The physical meaning of intrinsic two-electron coherence can be illustrated by a few examples. We consider fixed positions for detectors so that only time variables are mentioned. Interactions are neglected.

First, we consider an ideal single electron source emitting a single electronic excitation in a wavepacket $\varphi$. Using Wick theorem, the excess single electron coherence is found to be $\Delta g^{(e)}_S(t,t') = \varphi(t) \varphi^*(t')$ whereas the intrinsic two-electron coherence vanishes as expected from the analogy with quantum optics.

Let us now focus on an ideal two-electron source that generates a state made of two single electron excitations above the Fermi sea, $\psi_1^\dagger |\varphi_a\rangle |\varphi_b\rangle |F\rangle$, where $\varphi_a, \varphi_b$ are two normalized electronic wavepackets. For simplicity, owing to the Pauli principle, we can consider orthogonal wavepackets. Wick theorem then leads to the intrinsic single and two-electron coherence emitted by this two-electron source:

$$
\Delta g^{(e)}_S(t,t') = \varphi_a(t) \varphi^*_b(t') + \varphi_b(t) \varphi^*_a(t')
$$

$$
\Delta g^{(2e)}_S(t_1,t_2|t_1',t_2') = \Phi_{ab}(t_1,t_2) \Phi^*_{ab}(t_1',t_2')
$$

where $\Phi_{ab}$ is the Slater determinant built from $\varphi_a$ and $\varphi_b$, $\Phi_{ab}(t_1,t_2) = \varphi_a(t_1) \varphi_b(t_2) - \varphi_b(t_1) \varphi_a(t_2)$.

Therefore, $\Delta g^{(2e)}_S$ gives access to the two-electron wave function emitted by the source. This is a direct extension of the single electron case, in which $\Delta g^{(e)}_S$ gives access to single particle wavefunctions $\psi\rangle$. The case of a train of single electron excitations built from mutually orthogonal wavepackets leads to an extension of Eq. (4b) in terms of two-particle Slater determinants involving all pairs of electron excitations:

$$
\Delta g^{(2e)}_S(t_1,t_2|t_1',t_2') = \sum_{\text{pairs } \{i,j\}} \Phi_{ij}(t_1,t_2) \Phi^*_{ij}(t_1',t_2').
$$

The diagonal part $\Delta g^{(2e)}_S(t_1,t_2|t_1,t_2)$ of the intrinsic two-electron coherence encodes the two-particle time correlations of the electron flow. By analogy with quantum optics, the degree of second order electronic coherence of the source can be defined as

$$
g^{(2e)}_S(t_1,t_2) = \frac{\Delta g^{(2e)}_S(t_1,t_2|t_1,t_2)}{\Delta g^{(e)}_S(t_1|t_1) \Delta g^{(e)}_S(t_2|t_2)}. \tag{7}
$$

Statistical independence of the two detection events at $t_1$ and $t_2$ corresponds to $g^{(2e)}_S(t_1,t_2) = 1$ whereas antibunching is associated with $g^{(2e)}_S(t_1,t_2) < 1$ and bunching with $g^{(2e)}_S(t_1,t_2) > 1$.

As an example, Fig. 2 presents $\Delta g^{(2e)}_S(t_1,t_2|t_1,t_2)$ for an ideal source emitting a Slater determinant built from three Landau excitations $|F\rangle \rangle$ of duration $\tau_e$ and energy $\hbar \omega_e$, which are Lorentzian in energy:

$$
\tilde{\varphi}(\omega) = \frac{1}{\sqrt{N'}} \frac{\Theta(\omega)}{\omega - \omega_e + i \frac{\gamma_e}{2}} \tag{8}
$$

where $N'$ is a normalisation constant. Excitations are separated by a time $\Delta t = 3 \tau_e$. Correlations are clearly maximal at the times associated with two electron emissions and vanish around the diagonal $t_1 \simeq t_2$ as expected from the Pauli principle.
As expected from quantum optics, \( \Delta S_{12}^{\text{out}}(t, t') \) is related to the diagonal part of \( \Delta G_{S}^{(2e)} \) by

\[
\Delta S_{12}^{\text{out}}(t, t') = (e v_F)^2 R T \Delta G_{S}^{(2e)}(t, t'|t, t') - \langle \psi_1(t) \rangle \langle \psi_2(t') \rangle .
\]

Therefore, correlation measurements in the HBT setup give a direct access to the diagonal part of \( \Delta G_{S}^{(2e)} \) and thus to \( g_{S}^{(2)} \).

Let us now focus on the off-diagonal part of two-electron coherence emitted by the source. First of all, for a train of electronic wavepackets, Eq. (6) shows that the only off-diagonal contributions to \( \Delta G_{S}^{(2e)} \) arise from terms of the form \( \varphi_i(t_1) \varphi_j(t_2) \varphi_i(t'_1) \varphi_j(t'_2) \) where \( i \neq j \) and from their image under exchange operations \( (t_1, t_2; t'_1, t'_2) \rightarrow (t_2, t_1; t'_1, t'_2) \) or \( (t_1, t_2; t'_1, t'_2) \rightarrow (t_2, t_1; t'_2, t'_1) \). Since the antisymmetry

\[
\Delta G_{S}^{(2e)}(t_1, t_2|t'_1, t'_2) = -\Delta G_{S}^{(2e)}(t_2, t_1|t'_1, t'_2) = -\Delta G_{S}^{(2e)}(t_1, t_2|t'_2, t'_1)
\]

is purely kinematical, the physical two-electron coherence time corresponds to the decay of \( \varphi_1(t_1) \varphi_2(t_2) \varphi_1(t'_1) \varphi_2(t'_2) \) as a function of \( \tau = t_1 - t'_1 \) and \( \tau = t_2 - t'_2 \) and not from their images under the above exchange operations. Therefore, in this particular situation, the two-electron coherence time scale is directly related to the durations of the wavepackets that govern the single electron coherence time scale.

On the other hand, quantum superposition of two time shifted Slater determinants introduces a new time scale for two-electron coherence. Considering the two-electron state \( \psi^\dagger[\varphi_a][\psi^\dagger][\varphi_b][F] + \psi^\dagger[\varphi_d][\psi^\dagger][\varphi_d][F] / \sqrt{2} \) with mutually orthogonal wavepackets leads to

\[
\Delta G_{S}^{(2e)}(t_1, t_2|t'_1, t'_2) = \frac{1}{2} \sum_{\text{pairs}} \Phi_{ij}(t_1, t_2) \Phi_{kl}(t'_1, t'_2) .
\]

Terms involving identical pairs correspond to Eq. (5) and lead two-electron coherences located close to the diagonal. Assuming that \( \varphi_c \) and \( \varphi_d \) are the time-shifted by \( \Delta T_{\text{th}} \) of \( \varphi_a \) and \( \varphi_b \), terms with \( \{i, j\} \neq \{k, l\} \) contribute to off-diagonal two-electron coherences now extending over the time scale \( \Delta T_{\text{th}} \). These coherences are a signature of the time-bin entanglement \( 12^{(2e)} \) of the quantum superposition of \( \psi^\dagger[\varphi_a][\psi^\dagger][\varphi_b][F] \) and \( \psi^\dagger[\varphi_d][\psi^\dagger][\varphi_d][F] \).

In order to be captured, quantum correlations contained in the off-diagonal parts \( (t_1 \neq t'_1 \text{ and } t_2 \neq t'_2) \), have to be converted into measurable quantities. As discussed by Haack et al. [39], an ideal Mach–Zehnder interferometer (MZI) converts off-diagonal coherence in the time domain into an average current, i.e. diagonal coherence. This image naturally leads to a Franson interferometer like setup [34] depicted on Fig. 4 in which MZI are added in each outgoing arm of a beam splitter. Such
a setup has been introduced in electron quantum optics to evidence two-particle Aharonov Bohm effect and entanglement generation [44]. Here, we use it differently, namely as a measurement device, as originally proposed in photon quantum optics [34, 42, 43].

The current correlation between the two outgoing arms Lout and Rout picks up a magnetic flux dependence through the Aharonov Bohm magnetic phases $\Phi_{L,R}$. The terms depending on both magnetic fluxes, called two-electron Aharonov-Bohm terms because they involve de-localization on both MZI, are the only one that involve fully off-diagonal two-electron coherences in the time domain. They give access to the intrinsic two-electron coherence of the source since

$$
\langle i_{L}(t_1) i_{R}(t_2) \rangle_{2AB} = \langle ev_{F} \rangle^{2} \kappa \left( \Delta \mathcal{G}^{(2c)}_{S} (t_1 - \delta t_L, t_2 - \delta t_R | t_1, t_2) e^{i(\Phi_{L} + \Phi_{R})} + \Delta \mathcal{G}^{(2c)}_{S} (t_1, t_2 | t_1 - \delta t_L, t_2 - \delta t_R) e^{-i(\Phi_{L} + \Phi_{R})} \right) + \Delta \mathcal{G}^{(2c)}_{S} (t_1 - \delta t_L, t_2 | t_1, t_2 - \delta t_R) e^{i(\Phi_{L} - \Phi_{R})} + \Delta \mathcal{G}^{(2c)}_{S} (t_1, t_2 | t_1 - \delta t_L, t_2 - \delta t_R) e^{-i(\Phi_{L} - \Phi_{R})} \right) \quad (14)
$$

where $\delta t_{L,R}$ are the time delays within the two MZI and $\kappa = RT \tau_{1} T_{1} R_{2} T_{2}$ denotes the product of the reflexion and transmission probabilities of all the beam splitters. Each contribution to the r.h.s. of Eq. (14) can be isolated by a Fourier transform of current correlation with respect to $\Phi_{L,R}$. Measuring low frequency correlations would thus give access to the real and imaginary parts of the integral over $(t_1, t_2)$ of $\Delta \mathcal{G}^{(2c)}_{S}(t_1 - \delta t_R, t_2 - \delta t_L | t_1, t_2)$.

Fig. 5 presents these quantities as functions of $\delta t_L$ and $\delta t_R$ when the source emits a pair of Landau excitations separated by $\Delta t = 3 \tau_e$. Both plots show a central peak at $\delta t_L = \delta t_R \simeq 0$ and satellite peaks at $\delta t_L \simeq -\delta t_R \simeq \pm 3 \tau_e$. The central peak arises from single electron interferences which are visible as soon as time delays $\delta t_L$ and $\delta t_R$ are shorter than the single electron coherence time. The satellite peaks are due to non-local two-electron interferences between the two time shifted electrons of the pair. Their symmetry comes from the generic antisymmetry of two-electron coherence, Eq. (12). All these peaks spread along the line $\delta t_L = -\delta t_R$ over a time scale corresponding to the total duration of the train, i.e. a classical correlation time.

Considering a superposition of time-shifted electron pairs in two time-bins, Fig. 6 immediately shows how off-diagonal two-electron coherences appear in the expected signals [12]. In this case, the two-electron coherence time corresponds to the spreading along the $\delta t_L = \delta t_R$ diagonal and corresponds to $\Delta T_{tb}$. By contrast, a statistical mixture of the two pairs produces the same output signal as a single pair (see Fig. 5) because time averaging washes out information about emission times.

To conclude, we have defined in full generality the intrinsic two-electron coherence emitted by an electronic source and related it to two-electron wavefunctions. We have shown that an ideal Franson interferometer can be used to access off-diagonal two-electron coherences from measurement of low frequency current correlations. Nonetheless the protocol presented here relies on two-electron amplitude interferences and therefore also suffers from decoherence within the detection stage, i.e. after the beam splitter. Since MZI experience important decoherence effects, the present protocol suffers from the same limitations as the one proposed by Haack et al. [36] to measure single electron coherence.

Variants of this scheme are obtained by replacing the two MZI by devices converting off-diagonal single electron coherences into measurable quantities. For example, two-electron correlations in energy could be accessed using energy filters [12], as put forward by Moskalets under a slightly different form [25]. In the same way, correlations of finite frequency currents seem promising since interaction effects on currents past the beam splitter are encoded in finite frequency admittances [45]. Finally, in the same way Hanbury Brown and Twiss [46] circumvented the challenges posed by amplitude interferometry by moving forward to intensity interferometry, two-electron coherences could be accessed through an appropriate measurement of correlations of current noise. This extends the idea of single electron tomography [7] to two-electron coherence measurement. However, although measuring correlations of current noise in mesoscopic conductors is notoriously difficult [47], we think that this method deserves further investigations.

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FIG. 4: (Color online) Franson interferometer setup used to access two-particle coherence in the time domain. The left (L) and right (R) MZI are pierced by magnetic fluxes $\Phi_{L,R}$ and have time delays $\delta t_{L,R}$. An electron source is connected to the channel 1s whereas channels 2s, $L'$ in and $R'$ in are grounded. The quantity of interest is the current correlation $\langle i_{L\text{out}}(t) i_{R\text{out}}(t') \rangle$.

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\( \int \frac{dt_1}{\tau_e} \int \frac{dt_2}{\tau_e} \Delta G^{(2e)}_S(t_1 - \Delta t_L, t_2 - \Delta t_R|t_1, t_2) \) as a function of \((\Delta t_L, \Delta t_R)\) for a source emitting a quantum superposition of two trains of two Landau excitations of duration \(\tau_e\) and energy \(\omega_e = 3/\tau_e\) separated by \(\Delta t = 3 \tau_e\). The superposed trains are separated by \(\Delta T_{tb} = 7 \tau_e\).

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