Dynamic Response of Rigid Pavement Plate due to Localized Blast Load

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Abstract. In structural and transportation engineering applications, the dynamic response of orthotropic plates is an essential matter. Engineers did not consider the effects of dynamic loads such as those from machine vibrations or blast load. Dynamic analysis of rigid pavement plates due to local blast loads on concrete slabs in this research is modeled as concrete slabs with boundary condition that every edges of plates have a dowel-tie bar support and The rigid concrete pavement sitting on elastics Pasternak foundation is modeled by using the Kirchhoff theory of thin plates. Pasternak foundation have elastic vertical spring support and continuous shear layer. The main system responses that are observed are the transversal deflections at midspan and the internal stresses of the plate, particularly the maximum principle stress, minimum principle stress and maximum shear stress. Three loading phases are included in the analysis, namely: the positive phase, the negative phase, and the free vibration phase. Analyses are carried out utilizing a numeric approach termed the Modified Bolotin Method with two trancedental equation. The analysis is performed when the load is above the plate (0 ≤ t ≤ t0). Deflections resulting from various load positions on the set of slab models throughout all three phases are then compared side-by-side. Bending moment, shear forces, and stresses are calculated on all slab models with the Friedlander localized blast loading applied at midspan and the results are presented as stress contours that are then compared between each model.

1. Introduction

Rigid pavement are an essensial feature of the urban communication system and provide an efficient means of transportation of goods and services. To design the plate of pavement, mostly engineers do not consider the effects of other dynamic loads such as loads machine vibration or blast load. Blast loads include dynamic loads that overload structures outside ordinary conditions. Several researchers have conducted research on a rectangular orthotropic plate. [7] investigated the effects of blast loads on floor plates with perfect rigid placement to determine the response of plates with variation in plate stiffener configurations. [9] model leds to relatively simplified results, it has serious limitations. This model have deflection discontinuity between charged and the uncharged part of pavement plate. [1] looked at the dynamic analysis of rigid pavement under determination of soil parameters based on the elasticity modulus and Poisson’s ratio usually called Pasternak foundation.

There are several types of solutions for pavement plate dynamic response problems. The Bolotin method is mostly used because it can solve the problem of plates both natural frequency and free vibration. This method is used to numerically solve platen problems using trigonometric functions [11]. In the 1971 Vijakumar study entitled “A New Method for Analysis of Flexural Vibration of Rectangular Orthotropic Plates” modified the Bolotin method.
by adding terms that were ignored by Bolotin and more accurate results with varying base support conditions, especially for a higher mode vibration. The general solution of the problem is obtained from the specific properties of the Dirac delta function. This paper provides an overview of the dynamic analysis of rigid pavements response as described above.

2. Method and materials

2.1. Governing Equation

The research used an orthotropic rectangular plate resting on a Pasternak foundation that have three-parameter soil. The adjacent plate are supposed to be joined by dowels and tie bars. Research by conducting numerical analysis on Wolfram Mathematica 11.0 software by collecting data in the field without experimental testing in the laboratory.

2.2. Resolution of the problem

[Diagram: Geometry of Concrete Plate Pavement]

The damped vibration solution of the problem is set as:

\[
D_x \left( \frac{\partial^4 w}{\partial x^4} \right) + 2B \left( \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + D_y \left( \frac{\partial^4 w}{\partial y^4} \right) + yh \left( \frac{\partial w}{\partial t} \right) + \rho h \left( \frac{\partial^2 w}{\partial t^2} \right) + k_f w - G_s \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] = p_x(x, y, t)
\]

The boundary conditions (Figure 1) are modeled as follows:

The restriction of the elastic vertical translation and elastic rotation is characterized by the eight equations:

\[
-D_x \left( \frac{\partial^3 w(x, y, t)}{\partial x^3} \right) + \left( \frac{B + 2G_{xy}}{D_x} \right) \frac{\partial^3 w(x, y, t)}{\partial x \partial y^2} = k_{sx1} w(x, y, t)
\]

\[
-D_x \left( \frac{\partial^2 w}{\partial x^2} + v_y \frac{\partial^2 w}{\partial y^2} \right) = k_{rx1} \frac{\partial w}{\partial x}
\]

\[
-D_x \left( \frac{\partial^3 w(x, y, t)}{\partial x^3} \right) + \left( \frac{B + 2G_{xy}}{D_x} \right) \frac{\partial^3 w(x, y, t)}{\partial x \partial y^2} = k_{sx2} w(x, y, t)
\]

\[
-D_x \left( \frac{\partial^2 w}{\partial x^2} + v_y \frac{\partial^2 w}{\partial y^2} \right) = k_{rx2} \frac{\partial w}{\partial x}
\]

\[
-D_x \left( \frac{\partial^3 w(x, y, t)}{\partial y^3} \right) + \left( \frac{B + 2G_{xy}}{D_y} \right) \frac{\partial^3 w(x, y, t)}{\partial y \partial x^2} = k_{sy1} w(x, y, t)
\]

\[
-D_x \left( \frac{\partial^2 w}{\partial y^2} + v_x \frac{\partial^2 w}{\partial x^2} \right) = k_{ry1} \frac{\partial w}{\partial y}
\]
\[-D_x \left( \frac{\partial^3 w(x, y, t)}{\partial y^3} + \frac{B + 2G_{xy}}{D_y} \frac{\partial^3 w(x, y, t)}{\partial y \partial x^2} \right) = k_{sy} \frac{\partial^2 w}{\partial y^2} \]

\[-D_x \left( \frac{\partial^2 w}{\partial y^2} + v_x \frac{\partial^2 w}{\partial x^2} \right) = k_{rx} \frac{\partial w}{\partial y} \]

3. Results and discussion

The analysis in this study was to obtain values of body forces and deflection. The value of body forces includes moment and shear. The rigid pavement plates studied were modeled as elastic, homogeneous, and orthotropic plates. The dimensions of the concrete plate used for analysis correspond to the dimensions of the standard concrete plate used by PT Jasa Marga for the construction of highway rigid pavements. At the intersection of the edge of one plate with the other in the direction of the x-axis and the y-axis the transfer tool is used (load transfer device) in the form of dowel-tie bars. The plate module has dimensions of 3500 x 5000 mm with supports on all four sides in the form of dowels and tie bars. The calculated dynamic response of the structure is deflection and main stress in the entire plate area.

3.1. Material Properties

| Description                              | Notation | Value (Unit) |
|------------------------------------------|----------|--------------|
| Plate length (x axis parallel)           | a        | 5 m          |
| Plate width (parallel to the y axis)     | b        | 3.5 m        |
| Plate thickness                          | h        | 0.23–0.25 m  |
| Concrete density                         | \( \rho \) | 2400 N/m³   |
| Modulus of elasticity of concrete x-axis | \( E_x \) | 27.8 x 10⁹ N/m² |
| Modulus of elasticity of concrete y-axis | \( E_y \) | 30.0 x 10⁹ N/m² |
| Rotational stiffness in the x direction  | \( k_{rx} \) | 1 x 10⁶ N/m/grad |
| Rotational stiffness in the y direction  | \( k_{ry} \) | 1 x 10⁶ N/m/grad |
| Vertical translational stiffness x-axis  | \( k_{sx} \) | 2.0 x 10⁸ N/m |
| Vertical translational stiffness y-axis  | \( k_{sy} \) | 2.0 x 10⁸ N/m |
| Damping Coefficient                      | \( \gamma \) | 0.05         |

3.2. Variations in Research Case Studies

| Kasus | a [m] | b [m] | h [m] | \( x_0 \) [m] | \( y_0 \) [m] | k [MN/m] | Gs [MN/m²] |
|-------|-------|-------|-------|--------------|--------------|----------|------------|
| 1 A   | 5     | 3.5   | 0.25  | 2.5 (4/8*a)  | 1.75 (b/2)   | 27.25    |            |
| B     |       |       |       |              |              | 54.5     |            |
| C     |       |       |       |              |              | 81.75    |            |
| D     |       |       |       |              |              | 109      |            |
| 2 A   | 5     | 3.5   | 0.25  | 1.875 (3/8*a)| 1.75 (b/2)   | 27.25    | 9.52       |
| B     |       |       |       |              |              | 54.5     |            |
| C     |       |       |       |              |              | 81.75    |            |
| D     |       |       |       |              |              | 109      |            |
| 3 A   | 5     | 3.5   | 0.25  | 1.25 (2/8*a)| 1.75 (b/2)   | 27.25    |            |
| B     |       |       |       |              |              | 54.5     |            |
| C     |       |       |       |              |              | 81.75    |            |
| D     |       |       |       |              |              | 109      |            |
### Table

| Kasus | a (m) | b (m) | h (m) | x₀ (m) | y₀ (m) | k (MN/m) | Gs (MN/m²) |
|-------|-------|-------|-------|--------|--------|----------|------------|
| 4 A   | 5     | 3.5   | 0.25  | 0.625 (1/8*a) | 1.75 (b/2) | 27.25    | 54.5       |
|       | B     |       |       |        |        | 81.75    | 109        |
|       | C     |       |       |        |        |          |            |
|       | D     |       |       |        |        |          |            |
| 5 A   | 5     | 3.5   | 0.23  | 2.5 (4/8*a) | 1.75 (b/2) | 27.25    | 54.5       |
|       | B     |       |       |        |        | 81.75    | 109        |
|       | C     |       |       |        |        |          |            |
|       | D     |       |       |        |        |          |            |

### 3.3. Time History Plate Deflection

Dynamic loads in the form localized blast loads are carried out in each case variation according to table 2, so that the value of the load and structural response will change with time. Deflection of the plate in the middle span compared to time so as to produce a history of the deflection of the plate within a certain duration. The time duration displayed is a time from 0 seconds to 0.25 seconds so that all load phases can be displayed in full. The structural response for each phase turns out to have a unique value. For the positive phase occurs at 0 < t ≤ 0.0018 seconds, the negative phase occurs after the positive phase which occurs at 0.0018 < t ≤ 0.0054 seconds, then the free vibration phase occurs after the negative phase which occurs at 0.0054 < t ≤ 0.25 seconds.

![Figure 2. Time History Each Model Case Studies](image-url)
From Time History deflection shows that each model of structural variation is underdamped because in the free vibration phase the amplitude of the deflection decreases gradually and will reach zero deflection after several cycles. This is in accordance with the theory of structural dynamics which states structures with a damping ratio of less than 1 are underdamped.

Figure 2 shows that the maximum deflection occurs in the free vibration phase. Likewise for the influence of plate thickness, the less the thickness of the plate (h), the deflection that occurs will be even greater. While the influence of the stiffness of the supporting layer (k), if the k value is greater then the deflection is smaller.

3.4. Body Forces

The bending moment of the x and y directions is the second derivative of the deflection function w (x, y, t). In addition, deflection of deflection function w (x, y, t) if it is lowered three times, the shear forces on the x and y axes will be obtained. This analysis parameter puts the largest moment in the middle of the span where x (0) = and y (0) =. The bending moment value of the x direction plate and the y direction for the case study with load in the center of the span or center of the plate (x (0) = and y (0) =) at the time interval during the free vibration phase (0.0054 <t ≤ 0.25 s) for both 230 mm plate thickness and 250 mm plate thickness, where at that phase the maximum moment occurs. The moment value can be seen in the following table.

| Table 3. Value Bending Moment and Shear Forces h_{plate} = 250 mm |
|-------------------|--------|--------|--------|--------|
| Moment            | Case   | 1A     | 1B     | 1C     | 1D     |
| Moment X          | k = 27.25 MN | 20816.8 | 19409.4 | 18440.2 | 17782.1 |
| Mx [N.m] Max      | k = 54.5 MN | -15704.9 | -16853.1 | -17848.9 | -18705.9 |
| Moment Y          | k = 81.75 MN | 58145.1 | 54566.6 | 51661.2 | 49237.5 |
| My [N.m] Max      | k = 109 MN | -34641.8 | -35871.9 | -36857.3 | -37620.6 |
| Shear X           | k = 27.25 MN | 91895.44 | 89398.92 | 87018.55 | 84732.87 |
| Qx [N] Max        | k = 54.5 MN | -110276.05 | -106805.90 | -103488.74 | -100304.28 |

| Table 4. Value Bending Moment and Shear Forces h_{plate} = 230 mm |
|-------------------|--------|--------|--------|--------|
| Moment            | Kasus  | 5A     | 5B     | 5C     | 5D     |
| Moment X          | k = 27.25 MN | 36100.8 | 34590.5 | 33170.3 | 31822.9 |
| Mx [N.m] Max      | k = 54.5 MN | -29557.0 | -33777.5 | -36464.7 | -37860.9 |
| Moment Y          | k = 81.75 MN | 58291.9 | 54580.2 | 51120.9 | 47881.9 |
| My [N.m] Max      | k = 109 MN | -69889.0 | -77039.9 | -81690.7 | -84229.1 |
| Shear X           | k = 27.25 MN | 102203.38 | 99208.66 | 96356.92 | 93651.78 |
| Qx [N] Min        | k = 54.5 MN | -143666.35 | -138276.25 | -133237.20 | -128483.81 |

Based on Table 3 and Table 4, it can be concluded that the bending moment of the x direction and y direction shows the same behavior as deflection, the bending moment gets smaller when the greater the stiffness value of the supporting soil layer k, and the bending moment gets bigger when the plate thickness gets smaller. In the case of plate thickness h = 250 mm, the calculation of the bending moment in the x direction, the stiffness of the supporting soil layer k = 27.25 MN is 20816.8 Nm. The stiffness of the supporting soil layer k = 54.5 MN reduces the bending moment by 6.76%. Supporting soil stiffness k = 81.75 MN reduces the bending moment by
11.42%. The stiffness of the supporting soil layer $k = 109$ MN reduces the bending moment by 14.58%.

In the case of plate thickness $h = 230$ mm, the calculation of the bending moment in the $x$ direction, the stiffness of the supporting soil layer $k = 27.25$ MN is 36100.8 Nm. The stiffness of the supporting soil layer $k = 54.5$ MN reduces the bending moment by 4.18%. The stiffness of the supporting soil layer $k = 81.75$ MN reduces the bending moment by 8.12%. The stiffness of the supporting soil layer $k = 109$ MN reduces the bending moment by 11.85%.

In comparison with the same stiffness value of the supporting soil layer $k$, with a value of $k = 27.25$ MN, if the thickness of the plate varied from plate thickness of 230 mm to 250 mm, the bending moment of $x$ direction was reduced by 42.34%. From these results it can be concluded that adding plate thickness is far more effective than adding stiffness of the supporting soil layer.

4. Conclusion

Based on the results of the analysis of the response of several types of plates to the local blast load that has been done in this study, the following conclusions can be drawn:

a. The largest structural dynamic response occurs in the free vibration phase, not in the positive or negative phases. This is evident from the measurement of deflection response and stress in each precise case both in the position of the load in the middle span, in a quarter span, and in the eighth, also in each variation of the stiffness of the supporting layers and each plate thickness that has been studied. Even though the load is no longer working on the plate, the initial condition that occurs due to the end of the previous phase’s loading duration can actually cause a deflection value greater than the deflection during the duration of blast load.

b. The addition of plate thickness can reduce the moment value and plate shear force. In the 0.23 m thick plate model the stiffness of the supporting soil layer $k = 27.25$ MN and the work load in the middle produce the greatest moment of $x$ direction of 36100.8 Nm and the largest shear force of the $x$ direction of 143.666.35 N. Whereas the model with plate thickness of 0.25 m with the same variation produces the largest moment of $x$ direction by 20,816.8 Nm and the largest shear force of $x$ direction is 110,276.05 N. There is a moment reduction when the plate thickness is increased by 20 mm by 42.34% and there is a reduction in force shear of 23.24%.

c. Adding the value of the stiffness of the supporting layers can reduce the value of the moment and the shear force of the plate. At a plate thickness of 250 mm and load position in the middle of the span, the maximum moment value of $x$ direction is 20,816.8 Nm and the maximum shear force of $x$ direction is 110,276.05 N occurs in the case of plates with stiffness of the supporting soil layer $k = 27.25$ MN, whereas a maximum moment value of 1484.57 Nm and a maximum shear force of 84732.87 N occur in the case of a 250 mm thick plate and load position in the center of the span with stiffness of the supporting soil layer $k = 109$ MN.

d. Load position has a significant effect on the dynamic response of the plates. In the 250 mm thick plate model with a stiffness of the supporting soil layer $k = 27.25$ MN and the work load in the middle of the span produces the largest deflection of 0.616 mm while the load with a position in the eighth span produces the smallest deflection, which is 0.380 mm. This means that the closer the position of the load to the pedestal, the smaller the amount of deflection that occurs.
Plate thickness has a significant effect on the dynamic response of the plates. In the model with a plate thickness of 0.23 m the plate with stiffness of the supporting soil layer $k = 27.25$ MN and the work load in the middle produces the largest deflection of 1,788 mm. While the model with a plate thickness of 0.25 m with the same variation produces the largest deflection of 0.616 mm. A deflection reduction occurs when the plate thickness is increased by 20 mm by 65.55%.

In general, if reviewed based on the most significant effect in reducing deflection, plate thickness is the first sequence followed by the influence of the position of the explosion when it occurs further to the edge of the plate, then finally the addition of the stiffness of the supporting soil layer.

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