Non-equilibrium 2-Channel Kondo model for quantum dots

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We find that, under certain condition, a quantum dot with odd number of electron and coupled to two leads can be described by a non-equilibrium 2-channel Kondo model, when the two leads has a large voltage bias between them. The model is exactly soluble and can be mapped into a free fermion system, even in the presence of external magnetic field and other relevant perturbations. All (dynamical) correlation functions can be calculated. The fixed point of the 2-channel Kondo model is a non-equilibrium fixed point and is different from the usual 1-channel and 2-channel Kondo fixed point.

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In a recent experiment, Goldhaber-Gordon et al [1] and Cronenwett et al [2] observed a Kondo effect in quantum dot systems in the limit where the charge fluctuations on the dot can be ignored, and when there is a large voltage bias across the two leads. Under certain conditions as will be indicated bellow, the properties of the system is described by a 2-channel Kondo model, with non-equilibrium effect included as a perturbation. This 2-channel Kondo model can be solved exactly using a Majorana fermion approach even in the presence of some relevant perturbations, such as finite temperatures, finite magnetic fields, non-equilibrium tunneling between leads, and finite changes in the relative strength of the coupling between the two leads and the dot. Many properties of the system (including dynamical properties) can be calculated exactly. The fixed point of the 2-channel Kondo model studied here is a non-equilibrium fixed point which is different from the usual 2-channel Kondo fixed point. For example the dot-spin correlation behaves as $t^{-2}$ here instead of $t^{-1}$ for the usual 2-channel Kondo fixed point.

We start with the following model Lagrangian for the quantum dot

$$\mathcal{L} = i\psi_{i\alpha}^\dagger (\partial_t + v\partial_x + i\frac{1}{2}eV\tau^3_i)\psi_{j\alpha} - \mathbf{h} \cdot \mathbf{S} \delta(x) - 2\lambda\delta(x)\mathbf{S} \cdot (u^+\psi_{1\alpha}^+ + v^+\psi_{2\alpha}^+)\frac{\sigma_{\alpha\beta}}{2} (u\psi_{1\beta} + v\psi_{2\beta})$$ (1)

where $V$ is the voltage difference between the first and the second lead, and $\mathbf{h}$ is the coupling of external magnetic field to the dot-spin. Note that the spin-up and the spin-down Fermi surfaces in the leads remains to have the same energy even in the presence of external magnetic field, thus there is no coupling between the external magnetic field and the lead-spins in our model. $\lambda u$ and $\lambda v$ describe the strength of coupling between the dot and the two leads.

After redefine the $\psi$ field in Eq. (1): $\psi_{i\alpha} \rightarrow e^{-i\frac{1}{2}eV\tau^3_i\lambda}\psi_{j\alpha}$, the above Lagrangian can be rewritten as

$$\mathcal{L} = \mathcal{L}_{\text{Knd}} + \mathcal{L}_t + \mathcal{L}_h$$ (2)

$$\mathcal{L}_{\text{Knd}} = i\psi_{i\alpha}^\dagger (\partial_t + v\partial_x)\psi_{j\alpha} - 2\lambda\delta(x)\mathbf{S} \cdot \left( |u|^2\psi_{1\alpha}^+\frac{\sigma_{\alpha\beta}}{2} \psi_{1\beta} + |v|^2\psi_{2\alpha}^+\frac{\sigma_{\alpha\beta}}{2} \psi_{2\beta} \right)$$ (3)

$$\mathcal{L}_t = -2\lambda\delta(x)\mathbf{S} \cdot \left( e^{iVt}u^*\psi_{1\alpha}^+\frac{\sigma_{\alpha\beta}}{2} \psi_{2\beta} + \text{h.c.} \right)$$ (4)

$$\mathcal{L}_h = -\mathbf{h} \cdot \mathbf{S} \delta(x)$$ (5)

where $\lambda_t = \lambda$. We see that the finite bias is described by a time dependent term after the mapping. $\lambda_t$ is the amplitude of the spin-flip tunneling between the two leads, and $\lambda$ is the spin-exchange coupling between the dot-spin and the lead-spins.

![FIG. 1. The tunneling can be reduced by a change in the density of state.](image)

The above system is a 1-channel Kondo system when $eV = 0$ since $\lambda_t = \lambda$. However, when $eV$ is large, the spin-exchange term and the spin-flip tunneling term become very different. If one can make $\lambda_t \ll \lambda$ in the large $eV$ limit, then the system becomes a 2-channel Kondo problem. One way to reach this limit is to use the energy dependence of the density of states. For example, one may use n-type semiconductor as one lead and p-type as the other lead (ie put the quantum dot in the depletion layer of a diode). When $eV > E_F$, we have $\lambda_t \ll \lambda$ since the tunneling is blocked (see Fig. 1). Certainly, to see the Kondo effect, we need $E_F$ to be less then the level spacing in the dot.
Even when \( \lambda = \lambda_t \) at high energies, we can effectively have \( \lambda > \lambda_t \) at low energies. This is because at energy scales above \( eV \), the system is a 1-channel Kondo system, and \( \lambda \) and \( \lambda_t \) flow together to larger values as we lower the energy scale. When energy scales are less then \( eV \), \( \lambda_t \) can no longer flow. If the value of \( \lambda_t \) is small enough at \( eV \) (ie if \( eV \gg T_K \) where \( T_K \) is the 1-channel Kondo temperature for \( eV = 0 \)), the system behave like a 2-channel Kondo system and \( \lambda \) can continue to flow to even larger values as we lower the energy scales further. Thus at low energies, we effectively have \( \lambda > \lambda_t \). However, since \( \lambda \) and \( \lambda_t \) flow as \( \ln(E) \), it may be difficult to achieve \( \lambda \gg \lambda_t \) at low energies.

When \( \lambda_t \ll \lambda \) (at low energies), we may first set \( \lambda_t = 0 \), and study the 2-channel Kondo problem described by \( \mathcal{L}_{K\text{nd}} \). The 2-channel Kondo system \( \mathcal{L}_{K\text{nd}} \) has a 2-channel Kondo fixed point, which can be reached by tuning the relative strengths of the couplings between the dot and the two leads to \( eV \). At low energies. This is because at energy scale much lower than \( \lambda \), the system will flow as \( \ln(E) \), it may be difficult to achieve \( \lambda \gg \lambda_t \) at low energies.

In the following we are going to introduce a Majorana fermion approach (which is a combination of the Kondo coupling induces the following simple changes in the Hilbert spaces: [13]) to solve the 2-channel Kondo problem. The Majorana fermion approach used here has a full spin rotation symmetry, and is similar to the one used in Ref. [14]. The 2-channel Kondo Hamiltonian is given by

\[
H = H_f + H_K = iv \int dx \left( \psi_{i\alpha}^\dagger \frac{\sigma_{\alpha\beta}}{2} \psi_{i\beta} + \lambda \mathbf{J}_s(0) \cdot \mathbf{S} \right)
\]  

(6)

where \( H_f \) is the free fermion Hamiltonian. The above 2-channel Kondo system flows to a fixed point with coupling \( \lambda = \lambda_0 = \pi v \). In the rest of the paper we are going to only consider physics at energy scale much lower than the Kondo temperature \( T_{2K} \) and concentrate on the fixed point Hamiltonian with \( \lambda = \lambda_0 \).

Introducing the charge \( U_c(1) \), spin \( SU_s(2) \), and flavor \( SU_{ch}(2) \) densities (we will call the \( SU_{ch}(2) \) quantum numbers as flavors):

\[
J = \psi_{i\alpha}^\dagger \psi_{i\alpha}, \quad \mathbf{J}_s = \psi_{i\alpha}^\dagger \frac{\sigma_{\alpha\beta}}{2} \psi_{i\beta}, \quad J^A \equiv \psi_{i\alpha}^\dagger \frac{\mathbf{J}}{2} \psi_{i\alpha}.
\]  

(7)

we can rewrite the 2-channel Kondo Hamiltonian as

\[
H_f = \frac{\pi v}{4} J^2 + \frac{\pi v}{2} J^2_s + \frac{\pi v}{2} (J^A)^2
\]

\[
H_K = \lambda \delta(x) \mathbf{S} \cdot \mathbf{J}_s
\]  

(8)

Introducing \( \mathbf{J}_s(x) = \mathbf{J}_s(x) + \delta(x) \mathbf{S} \) which satisfies the same algebra as \( \mathbf{J}_s \), [11] we can rewrite the above Hamiltonian as

\[
H_{2CK} = \frac{\pi v}{4} J^2 + \frac{\pi v}{2} J^2_s + \frac{\pi v}{2} (J^A)^2 + \text{const.}
\]  

(9)

if \( \lambda = \lambda_0 \). In terms of \( \mathbf{J}_s \), the 2-channel Kondo Hamiltonian at the fixed point \( \lambda = \lambda_0 \) takes the same form as a free fermion Hamiltonian.

In the following, we are going to use Majorana fermions to describe our 2-channel Kondo system. Introduce eight Majorana fermions \( \sqrt{2} \text{Re} \psi_c, \sqrt{2} \text{Im} \psi_c, \eta^A_{\alpha}|A=1,2,3 \) and \( \eta^a_{\alpha}|a=1,2,3 \), [13] where \( \psi_c \) carries \( \sqrt{2} \) unit of the \( U_c(1) \) charges, \( \eta^A_{\alpha}|A=1,2,3 \) form a flavor triplet and \( \eta^a_{\alpha}|a=1,2,3 \) a spin triplet. The free Hamiltonian \( H_f \) can be rewritten as

\[
H_f = iv \int dx \left( \psi^\dagger \frac{\partial x}{2} \psi + \frac{1}{2} \eta^A_{\alpha} \frac{\partial x}{2} \eta^A_{\alpha} + \frac{1}{2} \eta^a_{\alpha} \frac{\partial x}{2} \eta^a_{\alpha} \right)
\]  

(10)

The free theory is described by the \( SU_s(2)_2 \times SU_{ch}(2)_2 \times U_c(1) \) KM algebra. In terms of the Majorana fermions, the currents \( J_c, J_s, \) and \( J^A \) take the form \( J_c = \sqrt{2} \psi^\dagger \psi_c, J_s = e^{abc} \eta^a \eta^b / 2, \) and \( J^A = e^{ABC} \eta^A \eta^B / 2. \)

Let us concentrate on the spin part described by

\[
H_s = iv \int dx \frac{1}{2} \eta^a_{\alpha} \frac{\partial x}{2} \eta^a_{\alpha}
\]  

(11)

Assume the system is finite \(-L/2 < x < L/2, \) and \( \eta^a_{\alpha} \) satisfy the anti-periodic boundary condition: \( \eta^a_{\alpha}(x) = -\eta^a_{\alpha}(x + L) \). In this case the ground state \( \langle 0 | \) is a spin singlet. The total Hilbert space contains two sectors: \( \mathcal{H}^s = \mathcal{H}_0^s \oplus \mathcal{H}_1^s \). \( \mathcal{H}_0^s \) containing even number of fermions is the space generated from \( \langle 0 | \) by applying any number of the \( \mathbf{J}_s \) operators, while \( \mathcal{H}_1^s \) containing odd number of fermions is generated by applying any number of the \( \mathbf{J}_s \) operators and one \( \eta^a_{\alpha} \) operator.

Now let us add the Kondo term \( H_K \) at the critical coupling \( \lambda_0 \). The spin part is still described by the \( SU_s(2)_2 \) KM algebra in terms of the new \( SU_s(2) \) current \( \mathbf{J}_s \). Notice that \( H_K \) conserves the even-odd fermion numbers. Thus the total Hilbert space still contains two sectors: \( \mathcal{H}^s = \mathcal{H}_0^s \oplus \mathcal{H}_1^s \), where \( \mathcal{H}_0^s \) (or \( \mathcal{H}_1^s \)) carries even-number (or odd-number) of fermions. The system can be solved using KM algebra. The ground states in each sector carry spin 1/2 due to the added dot-spin. All the states in each sector are generated by applying \( \mathbf{J}_s \)’s on the corresponding ground states and the states in \( \mathcal{H}_0^s \) and \( \mathcal{H}_1^s \) have a one-to-one correspondence. We see that the Kondo coupling induces the following simple changes in the Hilbert spaces: [13]

\[
\mathcal{H}_0^s \rightarrow \mathcal{H}_1^{s/2}, \quad \mathcal{H}_1^s \rightarrow \mathcal{H}_1^{s/2}
\]  

(12)

We can introduce new Majorana fermions to represent the new \( SU_s(2) \) current: \( \tilde{J}_a^0 = \frac{1}{2} \epsilon^{abc} \eta^a \eta^b \), \( \tilde{J}_a^0 \) carry odd fermion numbers and map between even- and odd-fermion sector. When \( x \neq 0, \tilde{J}_a^0(x) = \text{sgn}(x) \eta^a(x). \)
Note that the total Hilbert space at the fixed point has a structure $H^e = H^e_{1/2} \otimes \{ |0 \rangle, |1 \rangle \}$. We can introduce a fermion operator $d^\dagger$ that map the even-fermion state $|0\rangle$ to the odd-fermion state $|1\rangle$: $d^\dagger |0\rangle = d |1\rangle$. Note that $d$ and $d^\dagger$ commute with $J_s$, and do not appear in the fixed point Hamiltonian (which contains only $J_s$). Thus $d$ is a free fermion operator which carries no energy. Using the commutation relation (which actually defines a spin-triplet primary field)

$$[J^a_s (x), S^b] = i e^{abc} \delta (x) S^c$$

(13)

and the fact that $S$ carry even-fermion numbers, we find that the dot-spin operator $S$ is given by

$$S = \frac{\sqrt{a}}{2} \hat{\eta}_s (0)$$

(14)

where $\hat{\eta} = d + d^\dagger$. The normalization coefficient is obtained by noticing that $\{ \hat{\eta}_s^a (0), \hat{\eta}_s^b (0) \} = \delta^{ab} \delta (0)$ and $\delta (0) = 1/a$ if we choose a finite short distance cut-off $a$. With the fixed point Hamiltonian

$$H_s + H_K = iv \int dx \frac{1}{2} \hat{\eta}_s^a \partial^a \hat{\eta}_s^a$$

(15)

we can now easily calculate the $S$ correlation.

Now let us add back the non-equilibrium tunneling term $L_t$ and include the channel asymmetry term

$$S \cdot \psi_{1a}^\dagger (0) \tau_i \sigma_{\alpha \beta} \psi_{2\beta} = (0)$$

(16)

Both terms are spin singlet and flavor triplet. Thus the leading contribution to these operators at the 2-channel Kondo fixed point comes form the flavor triplet primary fields which have dimension 1/2. To obtain the Majorana fermion representation of the flavor triplet primary fields, let us analyze the Hilbert space of the spin and the flavor sectors (with zero-charge). Without the Kondo coupling, the Hilbert space has a form $H^s = H^s_0 \otimes H^f_0 \otimes H^s_1 \otimes H^f_1$, where $H^s_{0,1}$ are defined above for the spin sector, and $H^f_{0,1}$ are defined similarly for the flavor sector. The sector $H^s_0 \otimes H^f_0$ (or $H^f_0 \otimes H^s_0$) is generated by $J^a_s$ and $J^a_f$ from $|0\rangle$ (or $\eta_s^a \eta_f^a |0\rangle \sim \hat{\psi}_{1a} \sigma_{\alpha \beta} \hat{\psi}_{2\beta} |0\rangle$). After we include the Kondo coupling, according to Eq. (12), the Hilbert space becomes $H^s = H^s_{1/2} \otimes H^f_0 \otimes H^s_{1/2} \otimes H^f_1$. The above structure of the Hilbert space tells us that the spin triplet Majorana fermion operator $\eta_s^a$ are unphysical since they map a state in $H^s_{1/2} \otimes H^f_0$ to a state in $H^s_{1/2} \otimes H^f_0$ which is outside of our physical Hilbert space. Similarly the flavor triplet Majorana fermion operator $\eta_f^a$ are unphysical. The only physical spin triplet primary fields are $S$ and the only physical flavor triplet primary fields are $b \eta_f^a$. We note that the flavor triplet primary fields are quadratic in the fermion operators. Our model for the fixed point remains to be a free fermion theory even after we include the relevant perturbations represented by $b \eta_f^a$. This allows us to study exactly the effect of $L_t$ term and the channel asymmetry term on the 2-channel Kondo fixed point.

More generally, we find the following exactly soluble model

$$H = H_{2CK} + H_a + H'_a$$

$$+ H_t + H'_t + H_h + H'_h$$

(17)

$$H_{2CK} = -iv \int dx \left( \hat{\psi}_s^a \partial_x \psi_s^a + \hat{\psi}_f^a \partial_x \psi_f^a + \hat{\psi}_s^\dagger \partial_x \hat{\psi}_s^\dagger \right)$$

(18)

$$H_a = \frac{1}{2} \gamma (\epsilon + iV t) \hat{\psi}_s^1 (0) + h.c.$$ (19)

$$H'_a = \gamma' (\epsilon + 3AB) \eta_s^A (0) \eta_f^A (0)$$

(20)

$$H_t = \frac{1}{2} \gamma (\epsilon + iV t) \hat{\psi}_s^1 (0) + h.c.$$ (21)

$$H'_t = \frac{1}{2} \gamma' (\epsilon + iV t) \hat{\psi}_s^1 (0) + \gamma' (\epsilon + iV t) \hat{\psi}_f^1 (0) + h.c.$$ (22)

$$H_h = i \gamma_s \cdot \hat{\eta}_s (0)$$

(23)

$$H'_h = i \gamma_s \cdot \hat{\eta}_s (0)$$

(24)

$$H'_h = i \gamma_s \cdot \hat{\eta}_s (0)$$

(25)

where $\psi_f = (\eta_f^1 - i \eta_f^0) / \sqrt{2}$ and $\gamma_s = \sqrt{a} \hbar$. This Hamiltonian describes what we call the non-equilibrium 2-channel Kondo model. $H_{2CK}$ in $H$ describes the 2-channel Kondo fixed point. All other terms represent relevant or marginal perturbations around the 2-channel Kondo fixed point. $H_h$ and $H'_h$ are induced by the asymmetry in the coupling between the channel spins and the dot-spin. $H_t$ and $H'_t$ are the non-equilibrium tunneling terms caused by the finite voltage bias between the two channels. $H_e$ corresponds to the spin-flip tunneling described by $L_t$ and $H'_t$ corresponds to the direct tunneling described by $\psi_{1a} \psi_{2\alpha} + h.c.$ and $\psi_{1a} \sigma_{\alpha \beta} \psi_{2\beta} + h.c.$ $H_h$ is the coupling between a magnetic field $h$ and the dot-spin $S$.

To calculate the physical properties of the above system we also need to specify the “boundary” condition, ie how incoming fermions are distributed. The boundary condition here is that all income branches contain no Majorana fermions, ie all $k > 0$ levels are empty.

Next we simplify the Hamiltonian using the transformation $\psi_f (x, t) \rightarrow e^{iV (t+e^{-x})} \psi_f (x, t)$ which changes the $eV$ in $H$ to zero. However, the transformation also changes the boundary condition for the incoming branches: $\psi_f$ is now filled up to energy $eV$, but the $k > 0$ levels for all other Majorana fermions remain to be empty.

For simplicity, let us drop the sub-leading terms $H''s$. Among the eight Majorana fermions in $H$ (now with $eV = 0$) only one linear combination couples to $b$:

$$H_b = \int dx (-iv) \eta_b \partial_x \eta_b + i \gamma_b \eta_b (0)$$

(26)

where
\[ \eta_b = \gamma^{-1}[\alpha \eta_f + \text{Re}(\gamma_1) \eta_f + \text{Im}(\gamma_1) \eta_f^* + \gamma_s \tilde{\eta}_s] \]  
(27) 
\[ \gamma = \sqrt{\frac{\alpha^2}{\alpha^2} + |\eta|^2 + \gamma_s^2} \]  
(28) 

The equation of motion are [17] 
\[ \partial_t \eta_b(x, t) = -v \partial_x \eta_b(x, t) - \gamma \hat{b}(t) \delta(x) \]  
(29) 
\[ \partial_t \hat{b}(t) = -2i \eta_b(0, t) - \gamma \eta_b(0^+, t) + \eta_b(0^-, t) \]  
(30) 

If we expand \( \eta_b(x) = \sum_k \eta_k^-(k) e^{ikx} \) for \( x < 0 \) and \( \eta_b(x) = \sum_k \eta_k^+(k) e^{ikx} \) for \( x > 0 \), then the scattering is simply an energy dependent phase shift: \( \eta_k^\pm(k) = e^{i\phi_k} \eta_k^\pm(k) \) where 
\[ e^{i\phi_k} = \frac{\tilde{\eta}_s^* k - \gamma^2}{\tilde{\eta}_s^* k + \gamma^2} \]  
(31) 

This allows us to determine how the Majorana fermions, \( (\eta_1, \ldots, \eta_s) = (\sqrt{2} \text{Re}\psi_e, \sqrt{2} \text{Im}\psi_e, \eta_f^*, \tilde{\eta}_s) \), scatter across the \( d \) level at \( x = 0 \): 
\[ \eta_{I+}(k) = M_{IJ} \eta_{I-}(k), \quad M = 1 + (e^{i\phi_k} - 1) \frac{\gamma_I^J \gamma_I^J}{\gamma} \]  
(32) 

where \( (\gamma_1, \ldots, \gamma_s) = (0, 0, \text{Re}(\gamma_1), \text{Im}(\gamma_1), \eta_f^*, \tilde{\eta}_s) \) are the Fourier components of \( \eta_b(x) \) for \( x > 0 \) and \( x < 0 \). The large phase shift \( \phi_k = \pi \) at \( k = 0 \) indicates a resonance at the Fermi level. This setup allows us to easily calculate the total tunneling current [17] 
\[ I_t = e v \int \frac{dk}{2\pi} \left( \psi_{I+}^*(k) \psi_{I+}(k) - \psi_{I-}^*(k) \psi_{I-}(k) \right) \]  
(33) 

where \( n(e) = 1/(e^{\epsilon/T} + 1) \). Unfortunately \( I_t \) having a very smooth dependence on \( V \) cannot reveal the resonance at the Fermi surface when \( eV \) is large. One need to use the noise in \( I_t \) near \( \omega = eV \) to probe the resonance. [18] 

The effect of \( H' \) is to generate additional energy independent rotations among \( \eta_b \)'s. It changes \( M \) to \( M = 1 + (e^{i\phi_k} - 1) \gamma_I^J \gamma_I^J \), where \( \gamma^\pm \) are two unit vectors. 

The equation of motion for \( \tilde{b} \) also implies that \( \tilde{b} = -i \int \frac{dk}{2\pi} (1 + e^{i\phi_k}) \eta^\pm_k(k) \) which satisfies \( \tilde{b}^2 = 1 \). This expression allows us to calculate the response function 
\[ iD_f(t) = \Theta(t) \langle [F^3(t), F^3(0)] \rangle \]  
(34) 

where \( F^3 = i\tilde{b} \tilde{b}^3 \) (which comes from Eq. [11]). We find that (assuming \( \gamma_\alpha = \gamma_s = 0 \) and \( eV \to \infty \) for simplicity) 
\[ \text{Im} D_f(\omega) = \frac{1}{2\pi} \int d\nu \frac{\gamma^2 v^{-1}}{\omega^2 + \gamma^2 v^{-2}} (n(\nu - \omega) - n(\omega - \nu)) \]  
(35) 

\[ D_f(\omega) = -\frac{1}{\pi} \int d\nu \frac{\text{Im} D_f(\nu)}{\omega - \nu + i0^+} \]  

We see that \( C = \left( \frac{d\gamma_s}{dV} \right)^2 \text{Re} D_f(\omega) \) is the capacitance, and \( \sigma = \omega \left( \frac{d\gamma_s}{dV} \right)^2 \text{Im} D_f(\omega) \) is the conductance in parallel with the capacitance. We find that, for \( \omega > \gamma^2 v^{-1} \), \( C \sim \text{Im}(\nu T_{2K}/\omega) \) and \( \sigma \propto \omega \), while for \( \omega < \gamma^2 v^{-1} \), \( C \sim \text{Im}(\nu T_{2K}/\gamma^2) \) and \( \sigma \propto \omega^2 \). Measuring \( C \) and \( \sigma \) by applying an AC component in the gate voltage \( V_g \) may allow us to probe the resonance associated with the fixed point. The behavior for \( \omega > \gamma^2 v^{-1} \) is the behavior for usual 2-channel Kondo fixed point.

Similarly we can also calculate the dot-spin correlation exactly. In the presence of the \( L_1 \) term, the correlation for dot-spin has the usual 2-channel Kondo behavior for \( t < v/\gamma^2 \), and crosses over to \( 1/t^2 \) behavior for \( t > v/\gamma^2 \).

We would like to repeat that all of the above results are valid only when \( \gamma^2 v^2 \) and \( T \) are much less than the Kondo temperature \( T_{2K} \).

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[1] D. Goldhaber-Gordon, et al, Nature, 391, 156 (1998); LANL archives cond-mat/9807233.
[2] S. M. Cronenwett, et al, Science, 281, 540, (1998).
[3] T. K. Ng and P. A. Lee, Phys. Rev. Lett., 61, 1768 (1988).
[4] L. I. Glazman and M. E. Raikh, JETP Lett., 47, 452 (1988).
[5] A. Kawabata, J. Phys. Soc. Jpn., 60, 3222 (1991).
[6] Y. Meir, et al, Phys. Rev. Lett., 66, 3048 (1991); 70, 2601 (1993).
[7] H. Hershfield, et al, Phys. Rev. Lett., 67, 3720 (1991).
[8] A. L. Yeyati, et al, Phys. Rev. Lett., 71, 2991 (1993).
[9] N. S. Wingreen and Y. Meir, Phys. Rev. B, 49, 11040 (1994).
[10] Y. Wan, et al, Phys. Rev. B, 51, 14782 (1995).
[11] I. Affleck Nucl. Phys. B336, 517 (1990).
[12] I. Affleck and A.W.W. Ludwig Nucl. Phys. B360, 641 (1990).
[13] J.M. Maldacena and A.W.W. Ludwig, Nucl. Phys. B506, 565 (1997).
[14] V.J. Emery and S. Kivelson, Phys. Rev. B 46, 10812 (1992).
[15] Jinwu Ye, cond-mat/9612029 Nucl. Phys. B512, 543 (1998).
[16] P. Coleman, L. Ioffe and A.M. Tsvelik, Phys. Rev. B52, 6611 (1995).
[17] C. de C. Chamon, D.E. Freed, and X.-G. Wen, Phys. Rev. B 53, 4033 (1996).
[18] A. Lopatnikova and X.G. Wen, to appear.