Two-Proton Emission in the Hyperharmonics Approach

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Abstract. Nuclear decays into three-particle channels are considered in a few-body approach of hyperspherical harmonics with emphasis on simultaneous, or direct, emission of two protons. General conditions of direct decays are described and their main features, being experimentally established in decays of light nuclei, are reported.

The analysis method based on an expansion of decay amplitude into a series of hyperspherical harmonics is reviewed. The basis of hyperspherical harmonics functions is a generalisation of the spherical function basis in three-body systems. The method is tested on analysis of the direct decay $^6\text{Be} \rightarrow \alpha+p+p$ where the three-body components in the nuclear structure of $^6\text{Be}$ have been studied. In particular, the observed strong proton-proton correlations are treated as a manifestation of a specific three-body quantum effect: the kinematic focusing of fragments over momenta and in space.

The hyperspherical harmonics method is applied for the predictions of proton-proton correlations and life-time estimates of the nuclei $^{19}\text{Mg}$, $^{34}\text{Ca}$ and $^{48}\text{Ni}$ - candidates for two-proton radioactivity. Each direct 2p-decay should result in a set of peaks in the $E_{p-p}$ spectrum whose number and positions depend on the structure of initial nucleus, opposite to the diproton model, predicting the $^2\text{He}$ emission with one peak at $E_{p-p} \approx 0$ in all cases.

DIRECT THREE-PARTICLE DECAYS

A simultaneous emission of two protons is a genuine three-particle nuclear decay and is a complementary mode to the known sequential emission of protons via narrow intermediate states. In general, a sequential mechanism of three-particle decay is a chain of two independent binary decays via a narrow intermediate state whose width should be much smaller than the decay energy. The total decay amplitude is then a product of two binary amplitudes. When narrow intermediate states are absent the sequential decay mechanism is not plausible because it contradicts the uncertainty and causality principles. Such non-sequential decays are called direct decays.

In sequential decays information about correlations of fragments in the initial nucleus is lost because of strong final state interactions. In direct decays the fragment distributions are not distorted significantly by final state interactions and therefore reflect correlations of the respective clusters in the initial nucleus. In comparison with the two-particle case, three-particle decays are more informative due to additional degrees of freedom where more observables (e.g., energy spectra of fragments, correlations between fragments) are available. Thus, a direct three-particle decay is a promising tool to study nuclear structure.

Main features of direct decays are experimentally established in the two-proton decays of the $^6\text{Be}$ ground and first excited states and their analogs in $^6\text{He}$ and $^6\text{Li}^*(T=1)$,
(see [1] and references there). Strong nucleon-nucleon correlations are observed which are not connected with the p-p or α-p final state interactions. In particular, the measured energy spectra of α-particles from the mentioned decays display sharp peaks over broad pedestals. These peaks correspond to a strong nucleon-nucleon energy correlation which was first interpreted as emission of $^2$He, or di-proton, while the pedestals were associated with a sequential proton emission via $^5$Li [2]. However, such a model fails to explain the angular α-p correlations from the $^6$Be decay measured in the kinematical complete experiment [3].

**HYPERSPHERICAL HARMONICS METHOD**

An adequate approach describing few-body nuclear interactions is suggested by L.M. Delves, [4], who introduced the hyperspherical harmonics, or K-harmonics, basis which gives correct angular wavefunctions like the spherical harmonics in the two-particle case. Usage of K-harmonics makes it possible to write the asymptotic of the total wavefunction along three-particle channels in a way which is formally identical with that for two-particle channels. The way to solve the Schrödinger equation for three particles, which decouples the total wavefunction into radial and angular parts is illustrated below and uses the two-particle case for comparison.

The system of three particles $i, j, k$ with total energy $E$ can be characterized by 5 kinematical variables$^1$. Instead of the radius in the 2-particle centre of mass system, two Jacobi radii, $x$ and $y$, are introduced where one radius is between particles $j$ and $k$, $x=r_j-r_k$, and the another radius is between the third particle $i$ and c.m. of the selected pair, $y=(m_j r_j+m_k r_k)/(m_j+m_k)-r_i$. The respective Jacobi momenta $p_x, p_y$, Jacobi orbital momenta $\ell_x, \ell_y$, and Jacobi energies $E_x, E_y$ are defined by analogous relations (naturally, $E_x+E_y=E$). Then the 5 kinematical variables may be $\Omega_i=(\Omega_x, \Omega_y, \Omega_z)$: the directions $\Omega_x$ and $\Omega_y$ of the Jacobi momenta and the quantity $x_i=\arctan(\sqrt{E_x/E_y})$ which reflects the energy distribution between the Jacobi subsystems.

By introducing the hyperspherical coordinates, hyperradius $\rho^2=r_x^2+r_y^2$ and hyperangle $\theta=\arctan(x/y)$, one may use instead of the orbital operator $\hat{L}$, the grand orbital operator $\hat{K}$, which has eigenfunctions as the functions of hyperangle:

$$\Psi_{l,x}^{l_y} \sim \sin^{l_x}(\theta)\cos^{l_y}(\theta)P_{n}^{l_x+0.5,l_y+0.5}(\cos 2\theta),$$

where $P_n^{\alpha,\beta}$ are the Jacobi polynomials.

The respective quantum number is called hypermomentum, which minimal value is equal to the sum of the Jacobi orbital momenta, $K=\ell_x+\ell_y+n$ ($n=0,1,2,...$). This additional quantum number is a three-body analog of the orbital momentum value.

With the $\hat{K}$ eigenfunctions, one may obtain the solution of the three-particle Schrödinger equation $(T+V-E)\psi_{JM}^{l_x,l_y}=0$, with the sum of binary potentials $V=V_{ij}+V_{jk}+V_{ki}$, in a form of a hyperradial part of a total wavefunction coupled with the functions of a hyperangle:

$$\psi_{JM}^{l_x,l_y} = \sum R_{KLM}^{l_x,l_y}(\kappa\rho)Y_{KLM}^{l_x,l_y}(\Omega_i),$$

here the hyperspherical harmonics (HH) functions, or K-harmonics, are

$$Y_{KLM}^{l_x,l_y}(\Omega_i) = \Psi_{l_x}^{l_y}(\theta) \left[ Y_{l_x}(\Omega_x) \otimes Y_{l_y}(\Omega_y) \right]^{LM}_{KLM},$$

$^1$ All particles are assumed to be spinless, the general case including spins is considered in [4].
 [...]\[J^M\] denotes the \( L+S \) vector addition to form \( J \). The K-harmonics constitute an orthonormal basis like the spherical harmonics, \( Y_{lm}(\Omega) \), in the two-particle case.

After a separation of the hyperangular part in the total wave function one may obtain equations which are equivalent to the Schrödinger equation of a motion of single particle in external field.

In this approach, the centrifugal barrier of a three-body system is proportional to the factor \((K+3/2)\cdot(K+5/2)\) corresponding to the \( \ell(\ell+1) \) factor for a two-particle barrier. Usually it is much larger than the two-particle barrier and is not equal to zero even for \( K=0 \). The derived hyperradial wavefunctions display formally the same asymptotic behaviour as in the 2-body case [4].

**Expansion of a decay amplitude into a series of K-harmonics**

The hyperspherical harmonics method is applied for direct three-particle decays by B.V. Danilin *et al.*, [5], where the decay amplitude is suggested to be expanded in a series in the K-harmonics basis, in analogy with the partial wave analysis in a 2-particle case. The general formulae for this approach can be found in [6].

I will highlight the HH method for a description of energy and angular correlations of decay fragments. At low energies, the direct decays should be determined by few components in the amplitude expansion which corresponds to a minimal value of hypermomentum because of a three-particle centrifugal barrier which grows with an increase of \( K \). Thus, in the first approximation one may describe direct decays considering only few fit components with the lowest value of hypermomentum. Then an analysis of data is simple. For example, the energy spectra of fragments can be fitted by a superposition of few components with definite values of quantum numbers and with a specific energy dependence each. A weight of each component gives an information about the norm of the respective configuration in the initial nucleus \[^2\].

The additional notations are given below. The decay to the three particles \( i, j \) and \( k \) with decay energy \( Q \) may be characterized by 5 kinematical variables \( \Omega_i = (\Omega_{j-k}, \Omega_{i-jk}, x_i) \) where the quantity \( x_i = E_i MQ^{-1}(m_j + m_k)^{-1} = E_i/E_i^{max} \) is the energy of particle \( i \) in the c.m. frame of the decaying nucleus, and \( m_{i,j,k} \) are the fragment masses, \( M = m_i + m_j + m_k \). The energy \( E_i \) is related to the energies of the relative motion of particles \( j \) and \( k \) (\( E_{j-k}, \) or \( E_x \)), and of their centre of mass and particle \( i \) (\( E_{i-jk}, \) or \( E_y \)) as: \( E_i = E_y(m_j + m_k)M^{-1} \), and \( E_x + E_y = Q \). The state with spin \( J \) decays into three particles with spins \( s_i, s_j \) and \( s_k \). To describe the final state the following quantum numbers are used: The *hypermomentum* \( K = l_x + l_y + 2n \) (\( n=0,1,2,\ldots \)); the orbital angular momenta \( l_x \) and \( l_y \) conjugated to the Jacobi momenta \( p_x \) and \( p_y \) and satisfying conservation of parity \( \pi \) of the decaying state: \( \pi = (-1)^{l_x+l_y} \); the total orbital angular momentum \( \hat{L} = \hat{l}_x + \hat{l}_y \) satisfying the conservation law of the total angular momentum \( \hat{J} = \hat{L} + \hat{S} \); the total spin of all products \( \hat{S} = \hat{s}_i + \hat{s}_j + \hat{s}_k \). The total spin of any pair of particles is \( \hat{S}_{i-j} = s_i + s_j \).

The decay amplitude of a state with spin \( J \) and its projection \( M \) can be expanded in a series in an orthonormal hyperspherical harmonics basis:

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[^2]: The long-range Coulomb interaction may influence the fragment distributions and this can be taken into account in a conventional way as a final state interaction.
\[ F_M = \sum_{KLs_{l_{xy}}} B_{KLS}^{l_{xy}l_{ik}} \cdot Y_{KLM}^{l_{xy}l_{ik}}(\mathbf{p}_x, \mathbf{p}_y) \cdot C(S_{j-k}, T_{j-k}) \] (1)

The expansion coefficients \( B_{KLS}^{l_{xy}l_{ik}} \) corresponding to the hyperspherical functions \( Y_{KLM}^{l_{xy}l_{ik}} \) are determined by the decay dynamics and give, when squared, the probabilities for the observed decay modes, classified according to the sets of values of \( l_{xy}, l_{ik}, K, L, S, S_{j-k} \). The spin-isospin weight factors \( C(S_{j-k}, T_{j-k}) \) can be found in [6]. Expression (1) corresponds to the simple case where the spin projections of particles are not measured\(^3\).

In the case of two identical particles, e.g. protons, the most convenient arrangement of the Jacobi coordinates is \( (p-p, p-p \rightarrow \text{core}) \), while the set \( (p-\text{core}, p-p \rightarrow \text{core}) \) is close to coordinates in the c.m. system of a decaying nucleus which is normally used in shell-model or mean-field calculations. Conversion in the representation of (1) from one set of Jacobi coordinates \( (j-k, i-jk) \) to another set \( (k-j, i-kj) \) is accompanied by a change of the coefficients \( B_{KLS}^{l_{xy}l_{ik}} \) in accordance with the formula:

\[ B_{KLS}^{l_{xy}l_{ik}} = \sum_{l_{jk}l_{ik}} <l_{j-k}l_{i-jk}|l_{i-k}l_{j-ik}> \cdot B_{KLS}^{l_{xy}l_{ik}} \],

(2)

where \(<...|...>\) are Raynal-Revai coefficients [15].

**Analyses of the \( ^{6}\text{Be} \), \( ^{6}\text{Li}^* \), \( ^{6}\text{He}^* \) decays into \( \alpha+N+N \).** The approach is tested in analyses of direct decays of \( A=6 \) nuclei. E.g., the energy spectrum of \( \alpha \)-particles from \( ^{6}\text{Be} \) can be fitted by the sum of two components only, with \( S_{p-p}=0 \) and \( S_{p-p}=1 \), and with \( K_{\text{min}}=2 \). The studied decays are governed by a single \( K \) value in the amplitude expansion. Thus, the hypermomentum is confirmed to be a good quantum number.

This interpretation is not unique in describing spectra of single \( \alpha \)-particles. For example, the \( \alpha \)-spectrum measured in [2] was first fitted using the model of sequential emission of protons via the intermediate nucleus \( ^5\text{Li} \). However, the two mechanisms predict quite different behaviour of \( \alpha \)-proton correlations in a kinematical complete experiment. In particular, the three-body approach predicts the different angular dependences of p-p correlations with total spins \( S_{p-p}=0 \) and \( S_{p-p}=1 \) while in the sequential model these two modes are indistinguishable. In the decisive kinematical complete experiment [1] where the \( ^{6}\text{Be} \) decay was measured by detecting both \( \alpha \)-particles and protons, the different angular distributions of the \( S_{p-p}=0 \) and \( S_{p-p}=1 \) modes were observed directly thus confirming the three-body decay mechanism.

**Nuclear structure reflected in direct decays.** The fragment spectra from studied direct decays reflect the three-body nuclear structure. For example, the \( \alpha+N+N \) correlations in \( A=6 \) nuclei calculated in [7–9] (the fragment correlations both in space and in involved angular momenta) agree quantitatively with the conclusions of data analysis. In particular, the strong momentum and space correlations between two valence protons are found in \( ^{6}\text{Be} \). These are the ‘di-proton’ and ‘cigar’ configurations. In the first case, two valent protons forms a relatively compact cluster, in the second they are mainly on opposite sides relative to the \( \alpha \)-particle. The

\(^3\) Under some conditions the decay amplitude may depend on the possible orientation of the spin of the initial state which has decayed, see [3].
observed correlations are induced by the specific three-body quantum effect: kinematic focusing of particles over momenta and in space. This phenomenon generalizes the angular dependence of a two-particle scattering with $\ell \neq 0$ to the three-body case with $K \neq 0$ and it has a universal nature.

**DIRECT TWO-PROTON EMISSION FROM NUCLEI - CANDIDATES OF TWO-PROTON RADIOACTIVITY**

This effect is expected to be essential in other nuclei with three-body structure, *e.g.* the two-proton emitters $^{19}$Mg, $^{34}$Ca and $^{48}$Ni. Their estimated 2p-decay Q-values are less than 1.5 MeV. Since the FWHM of the p-p scattering spectrum is $\sim 3$ MeV, the well-known mechanism of a sequential decay via emission of $^2$He is not plausible here. Predictions of 2p-decay modes of these nuclei done in the HH approach are presented below. The $^{19}$Mg decay is first described in details.

**Two-proton emission from $^{19}$Mg.** According to the ref. [10], the main uncertainty in predictions of $^{19}$Mg properties is given by poorly known ground-state masses of $^{19}$Mg and $^{18}$Na: 0.9(3) MeV and 1.5(7) MeV above the $^{17}$Ne+p+p threshold, respectively. Due to the large errors in mass estimates, two opposite situations are possible: i) the $^{18}$Na+p threshold is below the $^{19}$Mg ground state which decays by a sequential emission of protons via $^{18}$Na; ii) the $^{18}$Na+p threshold is well above $^{19}$Mg and a direct two-proton emission into the $^{17}$Ne+2p channel dominates. For the last case, I would like to consider a direct two-proton emission in the three-body approach which was first applied for the $^6$Be data in the ref. [5].

To describe the decay of the $^{19}$Mg ground state which has an unknown spin-parity, one should assume some structure of $^{19}$Mg. Since $^{19}$Mg has 12 protons, two "valent" protons are likely in the $d_{5/2}$ shell and should have the total spin $S_{p-p}=0$. Thus, the spin-parity of $^{19}$Mg are probably $1/2^-$, the same as the $^{17}$Ne ground state. If one assumes that the $^{19}$Mg.g.s. has mainly the $^{17}$Ne+p+p cluster structure, it decays directly into the same fragments. As the mentioned transition ends in the $^{17}$Ne($1/2^-$)+2p channel, the lowest value of the hypermomentum $K$, allowed by momentum and parity conservation, is zero. One may estimate the three-particle decay width of the suggested $^{19}$Mg state using the three-body model [11] where the "generalized R-matrix" approach is suggested. The ordinary R-matrix formula for decaying states is there replaced by a similar expression $\Gamma_K(E)=2P_{K+3/2}(E)\gamma_K^2$, where the penetrability for a three-particle decay is practically the same function as in the R-matrix approach: $P_{K+3/2}(E)=\frac{\kappa \rho}{F_{K+3/2} + G_{K+3/2}}$. The physical meaning of the partial reduced width $\gamma_K^2$ is the same as in the two-body case, characterizing the spectroscopic factor for the three-particle exit channel with a hyperradius $\rho$. Functions $F_{K+3/2}$ and $G_{K+3/2}$ are calculated as regular and irregular Coulomb functions. The Wigner limit of a reduced width is assumed. The value of the calculation parameter, the radius of channel, is chosen of 16 f using an extrapolation of the systematic behaviour of a channel radius extracted from the fitted known widths of nuclei with dominated direct three-particle decay channels: $^6$Be ($\sim 8$ f), $^{10}$He ($\sim 11$ f), $^{16}$Ne ($\sim 16$ f) *etc.*, according to [12].

In Tables 1–3 the results of $^{19}$Mg width calculations are shown for varied values of the ground state position respective to the $^{17}$Ne+p+p threshold, the channel radius $\rho$ and the hypermomentum $K$, [13]. As one may see, the estimated width of the $^{19}$Mg
TABLE 1. Calculated widths of the $^{19}$Mg ground state for the decay energy $Q=0.89$ MeV. The main component in the $^{19}$Mg wave function is assumed with the hypermomentum $K=0$.

| Radius of channel, f | Penetrability | Width (MeV) |
|---------------------|--------------|-------------|
| 14.                 | 5.40·10$^{-7}$ | 3.13·10$^{-6}$ |
| 16.                 | 2.10·10$^{-6}$ | 0.139·10$^{-4}$ |
| 18.                 | 7.27·10$^{-6}$ | 0.542·10$^{-4}$ |
| 20.                 | 0.226·10$^{-4}$ | 0.000187 |

TABLE 2. The same as in Table 1, except $Q$-values.

| Channel radius, f | Penetrability Width (MeV) | Penetrability Width (MeV) |
|-------------------|---------------------------|----------------------------|
| Q=0.54 MeV        | Q=0.54 MeV                | Q=1.24 MeV Q=1.24 MeV     |
| 14.               | 3.84·10$^{-11}$ 1.73·10$^{-10}$ | 0.595 0.000499 |
| 16.               | 1.65·10$^{-10}$ 8.57·10$^{-10}$ | 0.673 0.00198 |
| 18.               | 6.40·10$^{-10}$ 3.72·10$^{-9}$ | 0.731 0.00679 |
| 20.               | 2.24·10$^{-9}$ 1.45·10$^{-8}$ | 0.773 0.0203 |

TABLE 3. The same as in Table 1, except dominating hypermomentum value, $K$.

| Channel radius, f | Penetrability Width (MeV) | Penetrability Width (MeV) |
|-------------------|---------------------------|----------------------------|
| $K=2$             | $K=2$                     | $K=4$ $K=4$               |
| Q=0.54 MeV        | Q=0.54 MeV                | Q=1.24 MeV Q=1.24 MeV     |
| 14.               | 2.35·10$^{-8}$ 1.36·10$^{-7}$ | 1.95·10$^{-10}$ 1.13·10$^{-9}$ |
| 16.               | 1.18·10$^{-7}$ 7.87·10$^{-7}$ | 1.42·10$^{-9}$ 9.45·10$^{-9}$ |
| 18.               | 5.13·10$^{-7}$ 3.82·10$^{-6}$ | 8.45·10$^{-9}$ 6.30·10$^{-8}$ |
| 20.               | 1.94·10$^{-6}$ 0.161·10$^{-4}$ | 4.23·10$^{-8}$ 3.50·10$^{-7}$ |

ground state is of 10 eV judging purely by assumption that the $K_{min}=\ell_{p-p}+\ell_{Ne-pp}=0$ configuration dominates the $^{19}$Mg wave function. However, due to Pauli principle, such a component should be suppressed because the valent protons have to be in the $d_{5/2}$ shell (or $\ell_{p-Ne}=\ell_{p-pNe}=2$). The configuration with next hypermomentum, $K=2$, (the estimated width $\sim1$ eV) does not match to the assumed $d_{5/2}$ shell structure as well. Finally, the configuration with $K=4$ is allowed by the Pauli principle (because the assumed ($d_{5/2})^2$ wave in $^{19}$Mg overlaps by 90% with the $K=4$, $\ell_{p-p}=\ell_{Ne-pp}=0$ component, using eq.(2)) and the respective width of the ground state is of 0.01 eV. The last value corresponds to a very long life-time, of $10^{-14}$ s, and thus the decay could be classified as a radioactivity phenomenon.

On the basis of the assumed structure of $^{19}$Mg one may predict properties of its direct three-particle decay (see details in Appendix). If the $K=0$ and $K=2$ components in $^{19}$Mg are suppressed by the Pauli principle completely, the decay amplitude and spectra of the fragments should only be defined by the $K_{min}=4$ configuration with $\ell_{p-p}=\ell_{Ne-pp}=0$. For example, the corresponding $E_{p-p}$ spectrum is shown in fig.1, on left, by a solid curve. Dashed and dotted curves are the results of a calculation of the suppressed decay modes with $K=2$ and 0, respectively.

The long-range Coulomb repulsion must be taken into account when a more realistic approximation is required. In this case the final state interaction model can be used. The full decay amplitude $F$ is then factorized as $|F|^2=|F_3|^2\cdot|F_{FSI}|^2$, where $|F_3|$ is the three-particle decay amplitude, and $|F_{FSI}|$ - the final state interaction.

4) The $^{17}$Ne spectrum can easy be obtained from the $E_{p-p}$ spectrum using the formulae $E_{Ne}=-\frac{2}{19}E_{Ne-pp}$ and $E_{Ne-pp}+E_{p-p}=Q$. 


On left: possible $E_{p-p}$ spectra from the direct decay $^{19}\text{Mg} (1/2^{-}) \rightarrow ^{17}\text{Ne} + p + p$ where $\ell_{p-p} = \ell_{Ne-pp} = 0$. The p-p energy is given as a fraction of the maximum $E_{p-p}$ value. The solid, dashed and dotted curves correspond to the decay modes with $K=4, 2$ and $0$, respectively. On right: the same components as shown to the left, and in addition the Coulomb repulsion of the fragments is taken into account. The dot-dashed curve is the result of diproton model.

The calculated $E_{p-p}$ spectra from $^{19}\text{Mg}$ are shown in fig.1, on right: the dotted, dashed, solid curves corresponds to the $K=0, 2, 4$ components, respectively. As one can see, the final state interaction influences significantly the $E_{p-p}$ spectrum of the $K=0$ mode only. The considered configurations in $^{19}\text{Mg}$ have very different probabilities to be observed in its decay. One may quantitatively compare the calculated penetrabilities of the $K=0, 2, 4$ modes for the $^{19}\text{Mg}$ direct three-particle decay, which values are $2 \cdot 10^{-6}$, $10^{-7}$, $10^{-9}$, respectively (see Tables 1–3, the row with the radius of channel of 16 f).

Thus, there could be three exotic modes of the 2p-decay of $^{19}\text{Mg}$. First, if the $K=0$ component is not forbidden strictly by the Pauli principle and its admixture to the dominating $K=4$ configuration is more than 0.1%, the $K=0$ mode should mainly be observed, with weak p-p correlations as shown in fig.1 by dotted curves. Second, if the admixture of the suppressed $K=2$ component in $^{19}\text{Mg}$ is more than 1%, the strong p-p correlations (like in the di-proton model) should be detected as well as strong p-p anticorrelations, see dashed curves in fig.1. Third, if the Pauli principle suppresses the mentioned components strongly, very exotic p-p correlations with three peaks in $E_{p-p}$ spectrum should appear, see solid curves in fig.1. Combinations of these three basic decay modes are possible as well.

One should also consider the decay branch $^{19}\text{Mg} (1/2^{-}) \rightarrow ^{17}\text{Ne} (1/2^{-}) + p + p$, with $L=1$. In this decay with $K_{\text{min}}=2$, in order to conserve the momentum and parity, two protons have to be with $S_{p-p}=1$ and the relative orbital momenta - with $\ell_{p-p} = \ell_{Ne-pp} = 1$. This component corresponds with 100% probability to the $\ell_{p-Ne} = \ell_{p-pNe} = 1$ configuration, which agrees with two valent protons being in the factor, see Appendix.
p-shell of $^{19}$Mg and with $S_{p-p}=1$. This contradicts to the assumed $(d_{5/2})^2$ structure of $^{19}$Mg and therefore such a component is unlikely compete with the decay modes considered above.

Decay modes and life-time estimates of $^{34}$Ca and $^{48}$Ni. The decay properties of $^{34}$Ca are expected to be similar to those of $^{19}$Mg. The $^{34}$Ca ground state is calculated to be bound respective to the single proton emission ($Q_p=0.9$ MeV) and unbound respective to the $^{32}$Ar+$p$+$p$ decay with $Q_2p=0.75$ MeV [16]. According to a conventional shell-model, $^{34}$Ca has a complete proton $d$-shell. Therefore its spin-parity is $0^+$ and one may apply all considerations valid for the $^{19}$Mg case. The only significant differences are the higher Coulomb barrier and the smaller $Q_2p$-value which result in much larger estimated life-times: $\sim 10^{-10}$ s, $10^{-9}$ s, $5 \cdot 10^{-7}$ s for the dominating K=0,2,4 components, respectively [13]. The corresponding 2p-decay modes are shown as $E_{p-p}$ spectra in fig.2, on left.

The recently discovered (see J. Giovinazzo et al., Proc. of PROCON99) double-magic nucleus $^{48}$Ni($0^+$) with a complete proton $f_{7/2}$-shell is estimated to be bound respective to the 1p-emission ($Q_p=0.46$ MeV) and it may decay into the $^{46}$Fe+$p$+$p$ channel with $Q_2p=1.36$ MeV [16]. Since two valent protons are expected to be in the $f$-shell, the lowest hypermomentum allowed by the Pauli principle is K=6. The corresponding single term in the HH expansion of the decay amplitude is $Y_{00}^{00}$ (see notation in Appendix, eq.(4)) and the respective impressive 2p-decay correlations are shown by solid curve in fig.2, on right. If a small admixture of the $(p_{1/2})^2$-wave, of 1%, is present in $^{48}$Ni, then the K=2 component should dominate in its decay (like in the $^{19}$Mg case) with the $E_{p-p}$ spectrum shown by the dashed curve in fig.2, on right. The result of diproton model is shown by dash-dotted curve for a comparison. The high Coulomb barrier for a 2p-decay causes a very large life-times: $\sim 10^{-6}$ s and
\( \sim 10^{-4} \) s for the K=2 and K=6 components, respectively [13].

**SUMMARY**

Two-proton decay is a three-body problem in a case of non-sequential, or direct two-proton emission. As direct decays may be studied by expanding a decay amplitude in a series of hyperspherical harmonics functions, the fragment spectra can be fitted by a few components determined by relative orbital momenta of fragments and a single, minimal value of hypermomentum.

Strong correlations of fragments, observed in direct decays of the A=6 nuclei, reflect exotic three-body configurations in the mother nuclei. These modes are induced by a three-body quantum effect of general nature: kinematic focusing of fragments over momenta and in space due to K\( \neq 0 \).

Because of this three-body phenomenon, strong p-p correlations are expected in direct two-proton emission of other nuclei, eg two-proton radioactivity candidates \(^{19}\text{Mg},^{44}\text{Ca},^{48}\text{Ni}\). The life-time and decay properties of these nuclei considered in a three-body approach depend strongly on the mass and the structure of ground states. Exotic decay modes, eg two-proton radioactivity with strongly oscillating p-p correlations, may appear making these drip-line nuclei attractive objects for experimental studies.

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**Appendix. Direct \( 2p \)-decays of \( 0^+ \) nuclear states.**

Explicit formulae for a simultaneous emission of two protons from the ground state of \(^{19}\text{Mg}\) are presented here, namely, for the transitions with \( \Delta J^\pi \) of \( 0^+ \) and \( 1^+ \).

The probability of the \(^{19}\text{Mg}\) decay into the \(^{17}\text{Ne}+2p\) channel can be derived in analogy with the \(^6\text{Be} \) 2p-decay, [1]. The \(^{17}\text{Ne}\) spectrum differs from the \(^6\text{Be}\) case by normalization coefficients only and can be fitted by the expression

\[
\frac{\partial^2 P}{\partial E_{Ne} \partial \Omega_{Ne}} = \frac{19}{16\pi Q} \sqrt{x_{Ne}(1-x_{Ne})} \cdot |F|^2 \cdot |F_{FSI}|^2, \tag{3}
\]

where \( x=E_{Ne}/E_{Ne}^{\text{max}} \). For the direct two proton decay with \( \Delta J^\pi=0^+ \), the amplitude approximation \( F \) has few components, e.g. the expansion with the lowest possible hypermomentum values of 0, 2, 4 and 6 is:

\[
F(p-p,Ne-pp) \sim B_{000}^{00}Y_{00}^{00} + B_{200}^{00}Y_{20}^{00} + B_{400}^{00}Y_{40}^{00} + B_{600}^{00}Y_{60}^{00} = B_{000}^{00} + B_{200}^{00}(2x-1) + B_{400}^{00}(16x^2 - 16x + 3) + B_{600}^{00}(2x-1)(8x(x-1)+1). \tag{4}
\]

The decay amplitude (4) expressed via other Jacobi coordinates \( (p-Ne,p-pNe) \) is:
where new norms are calculated using the Raynal-Revai transformation (2) of the expansion coefficients $B_{K00}^{00}$ from (4).

As the coordinates $(p-Ne, p-pNe)$ are almost the same as the proton coordinates in the $^{19}\text{Mg}$ c.m. system, the equation (5) can be used for estimates of the single-proton configurations in $^{19}\text{Mg}$ corresponding to the respective p-p modes in the eq.(4). In particular, the first component in the eq.(4), with $K=0$, $\ell_{p-p}=\ell_{Ne-pNe}=0$, with the 100% probability corresponds to the $\ell_{p-Ne}=\ell_{p-pNe}=0$ component in eq.(5) which matches the valent protons being in the $s$-shell of $^{19}\text{Mg}$. The second term in (4), when $K=2$, with the $(0.9985)^2=0.997$ probability coincides with the $\ell_{p-Ne}=\ell_{p-pNe}=1$ component in eq.(5) which represents valent protons in the $p$-shell. And the last term in (4), with $K=4$, has the $(0.94)^2=0.884$ probability to overlap with the $\ell_{p-Ne}=\ell_{p-pNe}=2$ component which matches the valent protons in the $d$-shell.

In eq. (3), the final state interaction factor $F_{FSI}$ was used (as in the Migdal-Watson model applied in the $^{6}\text{Be}$ decay case) to take into account the Coulomb repulsion of charged particles and the p-p attraction in a $S=\ell_{pp}=0$ wave: $|F_{FSI}|^2 = |P_{Ne-p1} P_{Ne-p2} |^2 |\Phi_{p-p} |^2$ where $P_{i-j}(E_{i-j}) = (F_0^2 + G_0^2)^{-1}$ is the Coulomb penetration factor for particles $i,j$ and $|\Phi_{p-p} |^2$ is a p-p interaction factor taken in the effective range expansion [14].

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