Hong–Ou–Mandel interference depends on the method of erasing the beam path information

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Abstract
We study how the information of the beam path is related to the Hong–Ou–Mandel interference with two pulsed light sources. Through a simple model in which two photons in the form of pulses pass a beam splitter and are observed at two detectors, we investigate how, during the measurement process, information about the paths of the two photons can be erased. There are two ways to clear the information of the beam path, the first being that from the beginning, during the physical measurement process, the time information is not obtained. The other is after measuring the information, to erase the temporal information in the data analyzing process. We show that Hong–Ou–Mandel interference can be obtained only when the beam path information is cleared from the physical measurement process.

Keywords Hong–Ou–Mandel interference · Quantum optics · Measurement

1 Introduction
Hong–Ou–Mandel interference, which cannot be explained from a classical point of view, has become an important cornerstone for the development of quantum optics [1]. Hong–Ou–Mandel interference is a subject that shows the pure quantum mechanical properties of light. Hong–Ou–Mandel interference was basically an experiment using two photons, but how the quantum mechanical properties of Hong–Ou–Mandel interference appear when more photons are included is constantly being studied. Four-photon [2], and three-photon [3] conditions are used to examine the Hong–Ou–Mandel interference. N identical single photon sources and measurement by N detectors in the far field system without beam splitter are also proposed to display the generalized N-photon Hong–Ou–Mandel interference [4].

There is also experimentation with Hong–Ou–Mandel interference with quantum mechanical properties, even in systems that re-composed of particles other than light. The realization of Hong–Ou–Mandel experiments using atoms instead of photons was reported [5]. The two-phonon quantum interference experiment is also performed in a system of trapped-ion phonons [6], two-particle interference in the electronic analog of the Hong–Ou–Mandel experiment in a quantum Hall conductor was investigated [7], and general bosons and fermions [8] were considered to realize Hong–Ou–Mandel interference.

Relatively recently, experiments have been conducted on the interference of two photons whose modes do not completely match. Two-photon interference was performed with uncorrelated photons with different center frequencies from a luminescent light source [9], and two-photon interference of temporally separated photons [10]. Also, a phase-randomized weak coherence state was used to perform generalized Hong–Ou–Mandel quantum interference [11]. A theoretical framework that describes Hong–Ou–Mandel interference for laser fields having arbitrary temporal waveforms and only partial overlap in time was developed [12].

We study how the information of the beam path is related to the Hong–Ou–Mandel interference with two pulsed light sources. Through a simple model in which two photons in the form of pulses pass a beam splitter and are observed at two detectors, we investigate how, during the measurement process, information about the paths of the two photons can be erased. In experiments, we usually get information from measurements and use them to analyze data. There are two ways to clear the information of the beam path: the first is that from the beginning, during the physical measurement process, the time that contains the path information is not
obtained. The other way is, after measuring the temporal information, to erase the temporal information to remove the beam path information in the data analyzing process.

In a simple model, the time width of the pulse, adjusted to the frequency width of the filter placed in front of the detector, serves to erase information about the path of the pulse causing Hong–Ou–Mandel interference. On the other hand, if we delete the beam path information by ignoring the time information after measuring a short pulse without removing the pulse path information due to the time width increase by the filter, we cannot see the Hong–Ou–Mandel interference effect. Through this, meaningful information in quantum mechanics is determined by the Hamiltonian that interacts with light when measuring. Furthermore, after the measurement by the Hamiltonian interaction has been completed, we ignore or randomly change the information, quantum phenomena such as Hong–Ou–Mandel interference are not changed.

The present paper is organized as follows. Section 2 explains the process of converting pulsed light in two different modes, which are already known, to two other lights through a beam splitter (Fig. 1). The time it takes for the two input lights to reach the beam splitter and the time it takes for the light from the beam splitter to arrive at the detector are considered. The shape of the pulse in front of the detector is calculated by taking into account the pulse width of light, which is a Gaussian pulse form on the time axis, and the time width caused by the filter placed in front of the detector. The characteristics of the light in front of the detector are calculated for the general shape of the input light.

Section 3 calculates the probability that one photon will be detected by each detector in two different modes for single photon input in each mode. The probability of finding a photon at \( r_c \) in the \( c \) mode detector and \( r_d \) in the \( d \) mode detector is calculated considering the time width of the input pulse and the time width of the filter placed in front of the detector. By integrating over \( r_c \) and \( r_d \), the probability of finding a single photon in each detector represents the well-known Hong–Ou–Mandel dip. However, an interesting result can be obtained by taking the time integral over one of the two detectors, that is, \( r_c \), and calculating the probability of a photon being found in the other detector as a function of time \( r_d \). If the time width of the filter in front of one detector is significantly smaller than the time width of the input light and the two input light is large enough to decompose the two input light reaching the beam splitter, the Hong–Ou–Mandel dip is not visible regardless of the time width of the detector in front of the other detector. On the other hand, if the time width by the filter in front of one detector is slightly larger than the time width of the input light, the presence or absence of the Hong–Ou–Mandel dip is determined according to the time width by the filter in front of the other detector. This is because when information on the path of a photon detected according to the time width of the filter placed in front of the detector is lost, the Hong–Ou–Mandel interference is visible; and when information on the path remains the Hong–Ou–Mandel interference is not visible. Importantly, the information on the path can be erased according to the size of the detector time window; but if there is no information on the path through the detector time window, the Hong–Ou–Mandel interference does not occur.

Section 4 summarizes the main results, and discusses their application. We discuss why the path erasure made by the time width change caused by the filter placed in front of the detector creates the Hong–Ou–Mandel interference, whereas path erasure by adjusting the size of the detector window does not make the Hong–Ou–Mandel interference.

2 Traveling pulse through a beam splitter

Let two modes \( a \) and \( b \) be coupled to \( c \) and \( d \) mode by a beam splitter, as in Fig. 1. After passing through the beam splitter, two pulses in \( a \) mode and \( b \) mode are traveling into \( c \) and \( d \) modes. It takes \( t_c \) \((t_d)\) time for a pulse in the \( a(b) \) mode to go to the beam splitter. Pulses starting from the beam splitter take \( t_c \) \((t_d)\) to go to the detector in \( c(d) \) mode. Using the time relation and the beam splitter relation [13],

\[
\begin{align*}
\hat{a}(t') &= \hat{c}(t' - t_{ca}) - \hat{r}_c(t' - t_{da}), \\
\hat{b}(t') &= \hat{r}_c(t' - t_{cb}) + \hat{d}(t' - t_{db}).
\end{align*}
\]
where $t$ and $r$ are the transmittance and the reflectance of the beam splitter, respectively. The time delay $t_d$ is defined as $t_d = t_i + t_j$ for $(i, j) = a, b, c, d$.

For simple calculation, we assume that the time behavior of the input beam is Gaussian, and we define an operator that has the same time dependence as the field, i.e.

$$\hat{a}(t') = \frac{\hat{a}}{\sqrt{\pi \delta_a^2}} e^{-t'^2/\delta_a^2},$$

$$\hat{b}(t') = \frac{\hat{b}}{\sqrt{\pi \delta_b^2}} e^{-t'^2/\delta_b^2}.$$  \hfill (2)

With the Gaussian time behavior of two modes, Eq. (refEqDefineAB) can be written

$$\hat{a} e^{-t'^2/\delta_a^2} = \hat{a} e^{-\left(t' - t_{ca}\right)^2/\delta_a^2} - r \hat{d} e^{-\left(t' - t_{da}\right)^2/\delta_a^2},$$

$$\hat{b} e^{-t'^2/\delta_b^2} = \hat{r} e^{-\left(t' - t_{cb}\right)^2/\delta_b^2} + \hat{t} \hat{d} e^{-\left(t' - t_{db}\right)^2/\delta_b^2},$$  \hfill (3)

where we assume that during traveling to the detectors, the shape of the input beam is not changed.

In general, an input state can be written as

$$\psi_{in} = \sum_{n,m} c_{n,m}^a |n > | a > | m > | b > \sum_{n,m} c_{n,m}^b \frac{\hat{a} \hat{b}^*}{\sqrt{n!m!}} |0 > | a > | b >$$  \hfill (4)

where we assumed that the input states at $a$ and $b$ modes are $\psi_a = \sum_n c_n^a | n >$, $\psi_b = \sum_m c_m^b | m >$, respectively. If we use the time-dependent mode relation in Eqs. (1), (3) can be written:

$$\psi_{in}(t') = \sum_{n,m} c_{n,m}^{a,b} \frac{\hat{a} \hat{b}^*}{\sqrt{n!m!}} \left\{ \frac{1}{\sqrt{\pi \delta_a^2}} \right\}^n \times \left\{ \frac{1}{\sqrt{\pi \delta_b^2}} \right\}^m$$

$$\left\{ \begin{array}{l} \hat{c} e^{-\left(t' - t_{ca}\right)^2/\delta_a^2} - \hat{r} \hat{d} e^{-\left(t' - t_{da}\right)^2/\delta_a^2} \\
\hat{r} e^{-\left(t' - t_{cb}\right)^2/\delta_b^2} + \hat{t} \hat{d} e^{-\left(t' - t_{db}\right)^2/\delta_b^2} \end{array} \right\} |0 > _c | 0 > _d .$$  \hfill (5)

The input state in Eq. (5) can be written in terms of the convolution of interference filter in Eq. (6) as follows:

$$\psi_{in}(t') = \sum_{n,m} c_{n,m}^{a,b} \frac{\hat{c} G(t', c, a) - \hat{r} G(t', d, a)}{\sqrt{2n!m!}} \times \left\{ \hat{c} G(t', c, b) + \hat{t} G(t', d, b) \right\}^m |0 > _c | 0 > _d .$$  \hfill (9)

If we put interference filters in front of the two detectors, then mode $c$ and $d$ should be changed. In the time domain, the interference filter with finite frequency width ($\Delta \omega$) can be interpreted by the Fourier transform limited time-dependent pulse with finite timewidth ($\Delta t \sim h/\Delta \omega$). We can put

$$F_i(t) = \frac{1}{\sqrt{\pi \delta_i^2}} e^{-t^2/\delta_i^2}.$$  \hfill (6)

where $i = c, d$, and $\delta_i$ is the effective time width introduced by the filter in the $(c, d)$ modes.
### 3 Time-dependent Hong–Ou–Mandel interference

For the pulse input states in $a$ and $b$ modes, we obtain the output states in $c$ and $d$ modes that travel through the beam splitter and filter. If we expand the powers in Eq. 9 using the binomial coefficient, we obtain the following:

$$\psi_{\text{out}} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c^n b_m \frac{1}{\sqrt{2^n n!}} \sum_{q=0}^{n+m} \frac{n! m!}{p!(n-p)! q!(m-q)!} \times G(t', c, a)^p G(t', d, a)^{m-p} G(t', c, b)^q G(t', d, b)^{m-q} \times (c^1)^q (d^1)^{n+m-p-q}|0 >_c |0 >_d .$$

(10)

To measure the time-dependent probability, we define the time $\tau_c$ and $\tau_d$ on two detectors in $c$ and $d$ modes, respectively. To obtain the probability that the detector measures a single photon in $c$ at $t' = \tau_c$ and $d$ mode at $t' = \tau_d$, we have to find the coefficient such that $p + q = 1$ and $n + m - p - q = 1$ in Eq. 10. Only when $n + m = 2$ can there be a nonzero coefficient, the results being

$$\langle 1(\tau_d) | < 1(\tau_c) | \psi_{\text{out}} \rangle = \sqrt{2} c^1 b^1 G(\tau_c, c, a) G(\tau_d, d, d)$$

$$+ c^1 b^1 G(t, c, b) G(\tau_d, d, d)$$

$$- c^1 b^1 G(\tau_c, c, a) G(\tau_d, d, b) - \sqrt{2} c^2 b^2 G(\tau_c, c, b) G(\tau_d, d, b).$$

(11)

where the time dependence of $G(t', i, j)$ is $G(\tau_c, i, j)$ if $i = c$ and $G(\tau_d, i, j)$ if $i = d$ since the detection time is defined by the detector on each mode.

If the input state is two pulsed number state $|1 >_a |1 >_b \rangle$, then Eq. (11) becomes

$$\langle 1, 1 | 1 >_a |1 >_b \rangle = G(\tau_c, c, b) G(\tau_d, d, d)$$

$$- G(\tau_c, c, a) G(\tau_d, d, b).$$

(12)

The joint probability that a detector in the $c$ mode measures a single photon and a detector in the $d$ mode measures a single photon becomes

$$P_j (t_a, t_b, t_c, t_d) = |\langle 1, 1 | \psi_{\text{m}} \rangle |^2$$

$$= |G(\tau_c, c, b) G(\tau_d, d, d) - G(\tau_c, c, a) G(\tau_d, d, b)|^2$$

$$= \frac{1}{2 \pi^2} \left\{ \frac{e^{-(\varpi_a - \varpi_c)^2}}{\sqrt{\delta_a^2 + \delta_c^2 + \delta_d^2}} - \frac{e^{-(\varpi_a - \varpi_d)^2}}{\sqrt{\delta_a^2 + \delta_d^2 + \delta_c^2}} \right\}^2 .$$

(13)

Figure 2 plots the joint probability $P_j$ that each detector measures a single photon at $\tau_c$ and $\tau_d$ in Fig. 2. For convenience, we use unitless time in this article; in other words, time $t$ in this article represents $t \times t_{\text{unit}}$, for example, $t_{\text{unit}} = 1$ ns. The time delays $(t_a, t_b, t_c, t_d)$ are set by $(2, 3, 2, 3)$, the time widths $(\delta_a, \delta_b)$ of two input beams and the timewidth $(\delta_c, \delta_d)$ related to the frequency filters in front of the two detectors are all the same as 1 in Fig. 2 setup. To explain the time dependence of $P_j$, we have to understand the interference effect in Eq. (11). The interference effect can be estimated if we plot the probability excluding the interference in Eq. (13). Figure 2 plots the probability $P_j^0$ excluding the interference, i.e. we plot $P_j^0 = |G(\tau_c, c, b) G(\tau_d, d, d)|^2 + |G(\tau_c, c, a) G(\tau_d, d, b)|^2$ with the same parameters. The probability is the sum of two Gaussian functions that have peaks at $(\tau_c = 4, \tau_d = 6)$ and $(\tau_c = 5, \tau_d = 5)$. The peak at $(\tau_c = 4, \tau_d = 6)$ represents the case that a pulse from $a$ mode is measured at the detector in $c$ mode and a pulse from $b$ mode is measured at the...
detector in \( d \) mode. If the two beams change, the probability will have its peak at \((\tau_c = 5, \tau_d = 5)\) accordingly. If we reduce the time widths \((\delta s)\), the two peaks can be separated and the probability between two peaks may be zero. However, the probability can be zero between the two peaks without changing the time widths. If the product of the two terms \(2G(\tau_c, c, b)G(\tau_d, d, a)G(\tau_c, c, a)G(\tau_d, d, b)\) is included, we can see the dip in the middle of two Gaussian peaks as in Fig. 2. This interference is basically related to the Hong–Ou–Mandel effect (Fig. 3).

In the usual Hong–Ou–Mandel interference measurement, the probability can be calculated after integrating the detector time \( \tau_c \) and \( \tau_d \), then

\[
P^H_j(t_a, t_b) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_j(t_a, t_b, t_c, t_d, \tau_c, \tau_d) \, d\tau_c \, d\tau_d
\]

\[
= \frac{1}{4\pi} \left\{ \sqrt{\frac{\delta^2_c + \delta^2_d}{\delta^2_a + \delta^2_d}} + \sqrt{\frac{\delta^2_c + \delta^2_d}{\delta^2_a + \delta^2_c}} + \frac{4e^{-a^2-b^2+c^2+d^2}}{\sqrt{\delta^2_a + \delta^2_c + 2\delta^2_d} \sqrt{\delta^2_a + \delta^2_d + 2\delta^2_c}} \right\}.
\]

Figure 4 plots the probability \( P^H_j \) that depends on the time delay \( t_b \) and the time width \( \delta_c \) and \( \delta_d \). With fixing \((t_a, t_c, t_d)\) as \((0, 1, 1)\), we change the time delay \( t_b \). As \( t_b \) approaches the value \( t_a = 0 \), we can see the Hong–Ou–Mandel dip in the probability. Since we integrate over the time \( \tau_c \) and \( \tau_d \), the probability \( P^H_j \) has no dependence on \( t_c \) and \( t_d \). In this figure, the time width of the two input beams are set as 1, and we put the timewidth related to the filters in \( c, d \) modes as the same value \( \delta \). The smaller the \( \delta \), the deeper the relative dip, and the smaller the width of the dip. It is clear that as the time width \( \delta \) increases, the width of the dip increases. On the other hand, if the time width \( \delta \) narrows, the question might arise of whether the interference effect will be erased or not. In fact, the interference effect is not erased, because the time width of the two input pulses already has sufficient interference effect at \( t_b - t_a = 2 \).
where $H$ is defined as

$$H(t_a, t_b, t_c, t_d, \delta_a, \delta_b, \delta_c, \delta_d) = \frac{(\tau_c - t_a - t_c)^2}{\delta_a^2 + \delta_c^2} + \frac{(\tau_c - t_b - t_c)^2}{\delta_b^2 + \delta_c^2} + \frac{(t_a + t_d)^2}{\delta_a^2 + \delta_d^2} + \frac{(t_b + t_d)^2}{\delta_b^2 + \delta_d^2}$$

$$- \frac{(t_b(\delta_a^2 + \delta_b^2) + t_d(\delta_b^2 + \delta_d^2) + t_d(\delta_a^2 + \delta_d^2 + 2\delta_d^2))}{(\delta_a^2 + \delta_d^2)(\delta_b^2 + \delta_d^2) + (\delta_b^2 + \delta_d^2) + 2\delta_d^2).}$$

(16)

Figure 5 plots the time-dependent joint probabilities ($P_j^D$) where the detector in $c$ mode measures a single photon at $\tau_c$ and delay time $t_d$. All time widths are set as to the same value 1, and after integration, the time-dependence $\tau_c$ has been removed. The dip in the central region ($t_b = 2$) is directly connected to the Hong–Ou–Mandel dip. As $t_b$ moves away from $t_b = 2$, as $\tau_c$ changes, we can see two peaks. For example, for $t_b = 5$, there are two peaks at $\tau_c = 4$ and $\tau_c = 7$. The peak of $\tau_c = 4$ occurs when a pulse in $a$ mode is measured in $c$ mode and a pulse in $b$ mode is measured in $d$ mode.

An interesting result can be obtained if we integrate only over $\tau_d$ for the joint probability $P_j^D(t_a, t_b, t_c, t_d, \tau_c, \tau_d)$, then

$$P_j^D(t_a, t_b, t_c, t_d, \tau_c) = \int_{-\infty}^{\infty} P_j(t_a, t_b, t_c, t_d, \tau_c, \tau_d) d\tau_d$$

$$= \frac{1}{4\pi^{3/2}} \left\{ \frac{\sqrt{2e^{-\frac{(\tau_c-t_a)^2}{\delta_a^2}}}}{(\delta_b^2 + \delta_c^2)\sqrt{\delta_a^2 + \delta_d^2}} \right.$$  

$$+ \frac{\sqrt{2e^{-\frac{(\tau_c-t_b)^2}{\delta_b^2}}}}{(\delta_a^2 + \delta_c^2)\sqrt{\delta_b^2 + \delta_d^2}} \right.$$  

$$- \frac{4}{\sqrt{(\delta_a^2 + \delta_c^2)(\delta_b^2 + \delta_c^2)(\delta_b^2 + \delta_d^2) + 2\delta_d^2)}} e^{-H(t_a, t_b, t_c, t_d, \delta_a, \delta_b, \delta_c, \delta_d)} \right\}.$$ 

(15)
The other peak at $\tau_c = 4$ is related to the case where the pulse from $a$ mode is measured in $d$ mode.

Figure 6 plots the probability $P^D$ as a function of $\tau_c$ and time width $\delta_d$. The time widths of the two input pulses are set to the same value 1, and we also set the time width $\delta_a$ as 0.1. For the time width $\delta_d = 0.1$, the time delay 3 between two input pulses as $\tau_c$ changes gives two peaks. This is because the time width is short enough to separate the two peaks. As $\delta_d$ increases, the overlapping part of the two pulses widens. However, the separation between the two peaks continues, even though the time width $\delta_d$ is increased by 10. If we measure photons on the detector with a time width of 10 in $d$ mode, we can not tell if the photon is from $a$ mode or $b$ mode. However, for the detector in $c$ mode whose time width is 0.1, we can check the origin of the measured photon. Therefore, in this setup, we cannot see the Hong–Ou–Mandel interference effect. The dip between the two peaks comes from the fact that these two peaks are separate independent Gaussian peaks.

Figure 7 plots the probability $P^D$ as a function of $\tau_c$ and time width $\delta_d$. The time widths of the two input pulses are set to the same value 1, and the time width $\delta_b$ is also set to 3. The time delay 3 between the two input pulses might give two peaks as $\tau_c$ changes, but the time width $\delta_d = 3$ is not short enough to make a separation of two peaks with time distance $(t_b - t_a = 3)$. Although the two probability peaks seem to be close enough to make a Hong–Ou–Mandel dip, there is no dip between the two peaks. This is because the time width $\delta_d = 0.1$ is sufficient to give the path of the photon measured at the detector in $d$ mode. Although the increase of $\delta_d$ results in a greater overlapping part of the two pulses, as $\delta_d$ increases, a dip is evident in the two peaks. To clearly see the dip in the two peaks, Fig. 8 plots the joint probability ($P^D$) that a detector in $c$ mode measures a single photon at $\tau_c$ for the time widths $\delta_a = 0.1$ and $\delta_d = 10$. The Hong–Ou–Mandel dip is associated with the interference of two photons only if the path of the two photons cannot be distinguished. The path of the photon measured at the detector in $c$ mode cannot be distinguished since the time width $\delta_c = 3$ is not short enough to separate the two paths; furthermore, the time width $\delta_d = 10$ is long enough to erase the path information. Therefore, the dip between two peaks for $\delta_d = 10$ came from the quantum interferences.

The pulse width of the two photons is $\delta_a = \delta_b = 1$, and the time distance between the two pulses is $(t_b - t_a = 3)$, so there is no chance for the two photons to meet together in the time domain. Where does the interference come from? We might say that the path information is important. Although the two pulses are well separated in the time domain, some uncertainty was introduced by the interference filter whose frequency bandwidth was related to the time width. The convolution effect erases the path information.

We now change the setup slightly. We want to add a time window for a detector in mode $d$ for the new setup. If we add all the data for that window, we might delete the arrival time of the photon; then the Hong–Ou–Mandel interference may be produced for certain conditions. In other words, for the time width $\delta = 0.1$ in Fig. 6, if we open the window much wider than the time width such as 10, we cannot obtain the arrival time of the photon. Then the path information is deleted. This might make quantum interference.

We calculate the joint probability that the time window for the detector in the $d$ mode is $(\tau_d - \tau_w, \tau_d + \tau_w)$ as follows:
The time width related to the filter in mode (\(c/d\)) is \(\delta_c = 3\) (\(\delta_d = 0.1\)). We set \((t_{ab}, t_c, t_d)\) as \((2, 5, 2, 2)\) and we also set \((\delta_c, \delta_d)\) to the same value, 1. The actual scale of \(p^W_j\) is \((\times 10^{-3})\).

\[
p^W_j(t_{ab}, t_c, t_d, t_w) = \int_{t_{ab}-t_w}^{t_{ab}+t_w} P_j(t_{ab}, 0, 0, \tau_c, \tau_d')d\tau_d' = \frac{1}{8\sqrt{2\pi 1/2}(1 + \delta_c^2)} e^{-\frac{2(t_{ab} - \tau_c)^2}{(1 + \delta_c^2)(1 + \delta_d^2)}}
\]

\[
+ e^{\frac{2(t_{ab} - \tau_c)^2}{(1 + \delta_c^2)(1 + \delta_d^2)}} G(t_w, \tau_d, 1 + \delta_d^2)
\]

\[
- e^{\frac{2(t_{ab} - \tau_c)^2}{(1 + \delta_c^2)(1 + \delta_d^2)}} G(2t_w, t_{ab} - 2\tau_d, 4(1 + \delta_d^2)) \tag{17}
\]

where

\[
G(t_1, t_2, \Delta) = \frac{(t_2 + t_1)}{\sqrt{(t_2 + t_1)^2}} \operatorname{Erf}\left(\sqrt{\frac{2(t_2 + t_1)^2}{\Delta}}\right) - \frac{(t_2 - t_1)}{\sqrt{(t_2 - t_1)^2}} \operatorname{Erf}\left(\sqrt{\frac{2(t_2 - t_1)^2}{\Delta}}\right) \tag{18}
\]

and the function \(\operatorname{Erf}(x)\) is defined by the value \(\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt\).

We also set \(t_i = 0\) for all the time delays, except \(t_{ab} = t_w\), as for simplicity, we also assume that the initial input pulse has time width \(\delta_a = \delta_b = 1\).

Figure 9 plots the joint probability for the time window \((\frac{t_w}{2} - t_w, \frac{t_w}{2} + t_w)\). The time widths \(\delta_c\) and \(\delta_d\) are (3 and 10), respectively. Figure 8 shows the same condition for the graph with \(\delta_d = 10\). The same result can be obtained if the time window \(t_w\) increases by 10. If the time window \(t_w\) is small, the joint probability is small since we integrate all data within the time window \((\frac{t_w}{2} - t_w, \frac{t_w}{2} + t_w)\). The dip in the middle of two peaks is comes from the quantum interference for the same reason explained in Fig. 7. Actually, the path information is already lost since regardless of the time window, the time width \(\delta_d\) is 10.

Figure 10 plots the joint probability for the time window \((\frac{t_w}{2} - t_w, \frac{t_w}{2} + t_w)\). The time width \(\delta_c\) and \(\delta_d\) are (3 and 0.1), respectively. Even though the time width \(\delta_d\) is 0.1, when we integrate all the data from the window \((\frac{t_w}{2} - t_w, \frac{t_w}{2} + t_w)\), we cannot obtain the path information of the two input pulses. If \(t_w\) is 10, then the information from the detector in \(d\) mode is simply the fact that a photon was measured. There is no information as to whether the measured photon came from \(a\) mode or \(b\) mode. However, the calculation shows that even if the path information of the measured photons is not known, there is no quantum interference. The result is very similar to the data in Fig. 8 for the graph with \(\delta_d = 0.1\).

At first glance, we think opening the time window up to \(t_w = 10\) can lead to quantum interference, because we do not know the path information of the detector’s photons in \(d\) mode. From a photon’s standpoint, it is not important whether the exact time that the photon was measured is recorded or not. The important thing is that the photon was measured at the detector in \(d\) mode after passing the filter with time width \(\delta_d = 0.1\).

### 4 Conclusion and discussion

We investigated how information of the paths of two photons can be erased during the measurement process, with a simple model in which two pulse-shaped photons pass a beam splitter, and are measured at two detectors.

First, we calculate the time-dependent joint probability \(P_j\) that each detector measures a single photon at a different time. The joint probability is not a simple sum of Gaussian probability, as there is the interference effect caused by the uncertainty of the beam path of the two pulses. If we integrate the time-dependent joint probability over the detection time in each detector, we can obtain the normal Hong–Ou–Mandel interference effect. In this calculation, we confirm the two-photon interference [14, 15] that Hong–Ou–Mandel interference is caused not by the fact that two photons arrive at the beam splitter at the
same time, but by the fact that the paths of the measured photons are unknown.

We calculated the probability $P_j^D$ obtained by integrating $P_j$ over the detecting time $t_d$ in the $d$ mode as a function of $\tau_r$ and time width $\delta_d$. The time widths of the two input pulses are set to the same value 1 and the time distance $(t_b - t_a = 3)$. When the time width $\delta_c = 0.1$ as in Fig. 6, the time delay 3 between two input pulses gives two peaks as $\tau_r$ changes for the time width $\delta_d = 0.1$. This is because the time width is short enough to separate the two peaks. As $\delta_d$ increase, the overlapping part of two pulses widens. However, the separation between the two peaks continues, even though the time width $\delta_d$ increases by 10. If we measure photons on the detector with a time width of 10 in $d$ mode, we cannot tell if it is from $a$ mode or $b$ mode. However, the detector in $c$ mode whose time width is 0.1 can check the origin of the measured photon. Therefore, in this setup, we cannot see the Hong–Ou–Mandel interference effect. The dip between the two peaks came from the fact that these two peaks are separate independent Gaussian peaks.

On the other hand, when the time width $\delta_c = 3$, as in Fig. 7, the time delay 3 between two input pulses might give two peaks as $\tau_r$ changes, but the time width $\delta_c = 3$ is not short enough to make a separation of two peaks with time distance $(t_b - t_a = 3)$. Although the two probability peaks seem to be close enough to make a Hong–Ou–Mandel dip, there is no dip between the two peaks for the time width $\delta_d = 0.1$. This is because the time width $\delta_d = 0.1$ is sufficient to give us the path of the photon measured at the detector in $d$ mode. Although the increase of $\delta_d$ increases the overlapping part of the two pulses in the middle of the two peaks, we can see a clean dip between the two peaks as $\delta_d$ increases. In Fig. 8, we can see the dip between the two peaks as $\tau_r$ changes. The Hong–Ou–Mandel dip is associated with the interference of two photons only if the path of the two photons cannot be distinguished. The path of photon measured at the detector in $c$ mode cannot be distinguished since the time width $(\delta_c = 3)$ is not short enough to separate the two paths; furthermore, the time width $(\delta_d = 10)$ is long enough to erase the path information. Therefore, the dip between the two peaks for $\delta_d = 10$ comes from the quantum interferences.

Although both the pulse width of the two photon is $\delta_a = \delta_b = 1$ and the time distance between the two pulses is $(t_b - t_a) = 3$, this gives no chance for the two photons meet together in the time domain, as some uncertainty was introduced by the interference filter whose frequency bandwidth was related to the time width. The lost path information gives Hong–Ou–Mandel interference.

We changed the setup and added a time window for a detector in mode $d$. For the time width $\delta_c = 3$, $\delta_d = 0.1$ in Fig. 10, we increased the width of the time window up to $t_w = 10$; we then cannot obtain the arrival time of the measured photon in $d$ mode, in other words, we have no beam path information of the measured photon in $d$ mode. However, we cannot see the Hong–Ou–Mandel interference. For the same time window $t_w = 10$, the Hong–Ou–Mandel interference can be seen for the time width $\delta_c = 3$, $\delta_d = 10$ as in Fig. 9. The change in the size of the window only affects the change in the size of the signal and does not change the shape of the signal.

If the time width of the pulse modified by the filter placed in front of the detector is large enough to erase the beam path information, Hong–Ou–Mandel interference can be made although the pulse width of the initial pulse is short enough to separate the two pulses at the detector. On the other hand, if we delete the beam path information by ignoring the time information after measuring short pulses, we cannot see the Hong–Ou–Mandel interference effect.

Through this, meaningful information in quantum mechanics is determined by the Hamiltonian that interacts with light when measuring. Furthermore, even if we randomly ignore or change the information after the measurement by the Hamiltonian interaction has been completed, quantum phenomena, such as the Hong–Ou–Mandel interference, are not changed. In our study, we considered a situation where none of the characteristics of the detector were changed, depending on the wavelength of light.

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