Abstract: According to the KKLT scenario, metastable dS vacua are formed as a result of uplifting of supersymmetric AdS vacua by $D3$ branes. I describe an extended version of this scenario where metastable dS vacua appear after an uplift from a state where the potential of the volume modulus in the absence of $D3$ branes would be unbounded below. This mechanism may considerably strengthen vacuum stabilization in the early universe.
1 Introduction

According to the standard version of the KKLT scenario of vacuum stabilization in string theory, metastable dS vacua are formed due to uplifting of supersymmetric AdS vacua by $D_3$ branes [1]; see [2, 3] for a general discussion of related issues, and [4, 5] for some recent progress.

The 4d supergravity formulation of the KKLT scenario [6–8] is described by the superpotential

$$W = W_0 - Ae^{-aT} + \mu^2 X.$$  \hspace{1cm} (1.1)

Here field $T$ is the volume modulus, and $X$ is the nilpotent field representing the $D_3$ brane contribution. The nonperturbative term $-Ae^{-aT}$ in the superpotential\(^1\) may appear, for example, in the presence of a stack of $D7$ branes wrapping a homologous 4-cycle. If there are $N$ branes in the stack, one has $a = 2\pi/N$. Alternatively, this term may emerge due to instanton effects. The parameter $A$ depends on the values at which the complex structure moduli are stabilized [9–11].

If the $D3$ brane is in the bulk, which is the main case to be considered in this paper, one can describe uplifting by using the Kähler potential

$$K = -3\ln(T + \bar{T}) + X\bar{X},$$  \hspace{1cm} (1.2)

and then taking $X = 0$ after calculating the potential $V(T)$. Alternatively, if the anti-D3-brane is in a strongly warped region, its effect can be described by considering the Kähler potential $K = -3\ln(T + \bar{T} - X\bar{X})$ [12].

In section 2 we will show that supersymmetric AdS vacua do exist in the KKLT scenario [1], but only under the condition $0 < W_0/A < 1$. In section 3 we will find metastable dS vacua obtained by uplifting for $W_0/A > 1$. In this case supersymmetric AdS vacua do not exist prior to the uplifting, and the value of the volume modulus $T$ in the dS vacuum is somewhat smaller than in the standard regime $0 < W_0/A < 1$. However, we will show that the stabilized volume modulus $T$ in this model always remains greater than $a^{-1} = N/2\pi$, even for very large $W_0$. In section 4 we will discuss possible implications of our results in the cosmological context, and show that an increase of $W_0$ may significantly strengthen vacuum stabilization in the early universe.

2 Uplifting from AdS

We consider the potential $V(T)$ of the field $T$, represent the field $T$ as $T = t + i\theta$, and search for a minimum of the potential at $\theta = 0$. One can show that in this theory $V_\theta(\theta = 0) = V_{t,\theta}(\theta = 0) = 0$, and

$$V_{\theta,\theta}(\theta = 0) = \frac{a^2 Ae^{-at}W_0}{2t^2}.$$  \hspace{1cm} (2.1)

\(^1\)Traditionally, the nonperturbative term is written as $Ae^{-aT}$, but we equivalently represent it as $-Ae^{-aT}$, to simplify the description of our main results.
For definiteness, we consider $A > 0$. In this case, the state $\theta = 0$ is stable with respect to growth of perturbations of the field $\theta$ for $W_0 > 0$.

The potential at $T = t$ prior to the uplifting, i.e. for $\mu = 0$, is given by

$$V = \frac{aAe^{-2at}}{6t^2} \left( A(3 + at) - 3e^{at}W_0 \right). \quad (2.2)$$

Its derivative with respect to $t$ is

$$V_t = -\frac{aAe^{-at}}{6t^3} (2 + at) \left( A(3 + 2at) - 3e^{at}W_0 \right). \quad (2.3)$$

Comparing it with the expression for $DW$,

$$DW(t) = \frac{e^{-at}}{2t} \left( A(3 + 2at) - 3e^{at}W_0 \right), \quad (2.4)$$

one finds that any minimum of the potential prior to the uplifting automatically satisfies the condition $DW = 0$, i.e. it is supersymmetric. The potential at the minimum is negative,

$$V_{AdS} = -\frac{a^2A^2}{6t}e^{-2at}, \quad (2.5)$$

so it is a stable supersymmetric AdS minimum, which is the standard part of the KKLT construction.

Thus in the KKLT scenario all minima that we can uplift are supersymmetric AdS. But do we really need to have a stable minimum prior to the uplifting? Naively, the answer is yes, we must have it, because otherwise what exactly are we going to uplift? However, let us see whether one can relax this requirement.

### 3 Uplift from a bottomless well

Equation $DW = 0$ describing the position of the AdS minimum can be represented as follows:

$$\frac{W_0}{A} = e^{-x} \left(1 + \frac{2}{3}x\right), \quad (3.1)$$

where $x = at$. The r.h.s. of this equation increases with the decrease of $x$, and approaches 1 at $x = 0$. This means that the potential does not have any minimum for $W_0/A > 1$. For $W_0/A > 1$ the potential prior to the uplifting is unbounded below, and in the small $t$ limit it falls to $-\infty$ as

$$V = -\frac{W_0 - A}{2t^2} + \ldots \quad (3.2)$$

However, this does not mean that this singular potential cannot be stabilized and uplifted. The full expression for the potential, taking into account uplifting, is

$$V = \frac{\mu^4}{8t^3} + \frac{aAe^{-2at}}{6t^2} \left( A(3 + at) - 3e^{at}W_0 \right). \quad (3.3)$$
The first term is a positive $D^3$ contribution to $V$ in the theory (1.1), (1.2),
\[ \Delta V = \frac{\mu^4}{8t^3}. \] (3.4)
This contribution immediately makes the potential bounded below. At small $t$ the total potential
\[ V = \frac{\mu^4}{8t^3} - \frac{W_0 - A}{2t^2} + ... \] (3.5)
is dominated by its first, positive term, which stabilizes the potential, and uplifts its minimum.\(^2\)

To study this effect in a more detailed way, one should find the values of $W_0$ and $\mu$ required for the existence of a metastable dS minimum with a negligibly small positive value (cosmological constant) $V_{dS} = \Lambda \approx 0$ at a given point $x = at$ for given $a$, $A$. These conditions yield:
\[ \frac{W_0}{A} = \frac{e^{-x}(2x^2 + 4x - 3)}{3(x - 1)}, \] (3.6)
\[ \frac{\mu^4}{A^2} = \frac{4x^2e^{-2x}(2 + x)}{3(x - 1)}. \] (3.7)

These results imply that for each set of parameters $a$ and $A$ one can find a set of parameters $W_0$ and $\mu$ such that the potential $V$ has a metastable dS minimum with a very small positive $V$, at any desirable value of $x = at > 1$, i.e. at $T = t > 1/a$.

Equations (3.6) and (3.7) are equally valid for $W_0/A > 1$ and for $W_0/A < 1$. From this perspective, there is nothing special about the dS states uplifted from the supersymmetric AdS minima with $W_0/A < 1$, as compared to the dS states which appear after adding the positive $D^3$ contribution (3.4) to the potential (2.2), which is unbounded below at small $t$ for $W_0/A > 1$.

For $W_0 < A$ one has $at > 1.569$, and the potential prior to the uplifting has a supersymmetric AdS minimum, which is then uplifted to dS. On the other hand, all dS vacua with a minimum at $1 < at < 1.569$ are obtained by adding the uplifting $D^3$ contribution (3.4) to the potential (2.2), which is unbounded below for $W_0 > A$ in the absence of the positive $D^3$ contribution (3.4). Note that the position of the minimum always remains at $t > 1/a$, and it gradually approaches $1/a$ only in the limit $W_0 \to \infty$.

In the limit $at - 1 \ll 1$ one has
\[ at - 1 = \frac{e^{-1}A}{W_0} = \frac{4e^{-2}A^2}{\mu^4}. \] (3.8)
Therefore uplifting to a dS state with a tiny positive cosmological constant and $W_0 \gg A$ requires uplift with
\[ \mu^4 = \frac{4}{e}AW_0. \] (3.9)

\(^2\)This resembles the explanation of stability of the hydrogen atom in quantum mechanics. The energy of interaction of a proton and an electron is $-\frac{e^2}{r}$, which is a bottomless well. But electron does not fall to $r = 0$ because the energy required for compression of its wave function to $\Delta x \sim r$ is proportional to $\frac{1}{r^2}$.
The moduli mass squared of the field $t$ after the uplift with $W_0 \gg A$ is

$$m_t^2 = \frac{a^3}{3} AW_0,$$

and the gravitino mass is

$$m_{3/2} = \left(\frac{a}{2}\right)^{3/2} W_0.$$  

Fig. 1 shows the potential $V(T)$ with a dS minimum with a tiny cosmological constant for a particular case $A = 1$, $W_0 = 100$, $a = 2\pi/100$. The minimum is very close to the limiting value of $T = t = 1/a \approx 15.9$.

![Figure 1. KKLT potential (3.3) with a metastable dS vacuum at $t \approx 16$, $\theta = 0$. It was obtained by adding the $D3$ contribution (3.4) with $\mu = 3.4786$ to the potential (2.2) in the KKLT model (1.1), (1.2) with $a = 2\pi/100$, $A = 1$, $W_0 = 100$.](image)

### 4 Discussion

Historically, there were several opposite arguments concerning the value of $W_0$ in the KKLT scenario. One of the arguments was that in order to have low-scale supersymmetry breaking one would need to have an extremely small value of $|W_0|$. However, a subsequent investigation has shown that the universe in the KKLT scenario tends to decompactify for the Hubble constant greater than the gravitino mass, which resulted in the stability constraint $H \lesssim m_{3/2}$ [13, 14]. A similar constraint in the LVS models is even stronger, $H \lesssim m_{3/2}^{3/2}$ [15]. The simplest way to avoid vacuum destabilization in the early universe is to consider a generalization of the KKLT construction, disentangling the strength of the vacuum stabilization and the magnitude of supersymmetry breaking [13, 16–18]. In such models, the smallness of $W_0$ is not required for the smallness of supersymmetry breaking. This approach turned out very helpful for dS vacua stabilization in a broad class of type IIB and type IIA string theory inspired models, and in M-theory [19–21].

An independent argument in favor of small $W_0$ is that the large values of the volume modulus $T$ are required for suppression of $\alpha'$ corrections and validity of the effective
supergravity approach. In the KKLT scenario, the volume modulus $T$ tends to increase, though only logarithmically, at small $W_0/A$. Finding realistic models with $W_0 \ll 1$ turned out difficult, but they do exist; see e.g. an example of such model with $W_0 \sim 10^{-8}$ recently found in [22]. However, once again, small values of $W_0/A$ in combination with large values of the volume modulus make dS vacua more vulnerable with respect to decompactification in the early universe, so one may try to find some regime where the volume modulus is not too large even if $W_0/A \geq 1$.

To illustrate this issue, we show the KKLT potential $V(t)$ for $\theta = 0$, $A = 1$ and $a = 2\pi/100$, for two very different values of $W_0$: $W_0 = 10^{-2}$ (left panel of Fig. 2) and $W_0 = 10^2$ (right panel of Fig. 2).

In the first case, for $W_0 = 10^{-2}$, uplifting occurs from a supersymmetric AdS vacuum (the green line). The dS minimum of the uplifted potential shown by the red line is at $t \sim 103$. The barrier stabilizing the vacuum state with a small cosmological constant has the height $\sim 10^{-11}$ in the Planck density units. If one considers a more significant uplifting, which is similar to what happens in the early universe at large energy density [13, 14], the dS minimum disappears and the universe decompactifies. This happens for energy density greater than $2 \times 10^{-11}$, which is an order of magnitude below the energy density during inflation in many popular inflationary models, such as the Starobinsky model, the Higgs inflation, and the simplest versions of $\alpha$-attractors.

The right panel (see also Fig. 1) shows the model with $W_0 = 100$, which is unbounded below prior to the uplifting. In this case the uplifted potential shown by the red line has a minimum at $t \approx 16$, which is close to the limiting value of $t = 1/a = 100/2\pi \approx 15.9$. The stabilizing barrier is 9 orders of magnitude higher than in the case $W_0 = 10^{-2}$, and the dS vacuum is stable in the early universe at energy density up to $2 \times 10^{-4}$.

To understand the general pattern revealed by these two figures, let us study the standard regime with $W_0 \ll A$ and AdS minimum with the depth given by (2.5). Its uplifting is achieved by adding to the potential a function rapidly decreasing at large $t$. Therefore the height of the barrier stabilizing the dS minimum after the uplifting is always

![Figure 2](image-url). A comparison of KKLT uplifting for $A = 1$, $a = 2\pi/100$, for $W_0 = 10^{-2}$ (left panel) and $W_0 = 10^2$ (right panel). The green lines show the potential before the uplifting, the red lines show the potential with a dS minimum with a tiny positive cosmological constant. The blue lines illustrate the disappearance of the minimum if the uplifting is too large, which may result in decompactification of 6 extra dimensions in the early universe.
smaller than the depth of the AdS minimum \[13\],

\[ V_{\text{barrier}} < \frac{a^2 A^2}{6t} e^{-2at} . \] (4.1)

If \( W_0 \) is small and the minimum of the potential is at \( t \gg 1/a \), then this expression in combination with (2.3) implies that

\[ V_{\text{barrier}} < \frac{3 W_0^2}{8t^3} . \] (4.2)

The maximal height of uplifting of a dS vacuum shown by the inflection point of the blue lines in the left panel Fig. 2 is just a little bit higher than \( V_{\text{barrier}} \). Thus the traditional approach results in the suppression of the height of the protective barrier by two factors: by \( W_0^2 \), which was supposed to be very small, and by \( t^{-3} \), where \( t \) was supposed to be very large.

On the other hand, in the limit \( W_0 \gg A \), instead of the calculating the height of the barrier one can obtain a direct analytical estimate of the maximal height of uplifting of a dS vacuum, shown by the inflection point in the right panel of Fig. 2. The inflection point appears at \( t \approx \sqrt{2}/a \), and its height is

\[ V_{\text{max}} \approx \frac{\sqrt{2} - 1}{12} e^{-\sqrt{2} a^3 A W_0} \sim 10^{-2} a^3 A W_0 . \] (4.3)

In the example shown in Fig. 2 we see that one can increase the range of stability of compactification by 9 orders of magnitude while still having a reasonably large value of the volume modulus. Equation (4.3) shows that one can further enhance stability of compactification by increasing \( A \) and \( W_0 \) while preserving the same value of the volume modulus \( T \), which does not depend on \( W_0 \) in the regime when \( W_0 \gg A \).

These results have been obtained in the version of the KKLT scenario with the Kähler potential \( K = -3 \ln(T + \bar{T}) + X \bar{X} \) describing uplift due to the \( D3 \) brane in the bulk. If one considers uplift due to the \( D3 \) brane in the strongly warped region, using the Kähler potential \( K = -3 \ln(T + \bar{T} - X \bar{X}) \), the results change. Stable dS vacua may still exist for \( W_0 > A \) remains possible, but only for a rather limited range of values of \( W_0 / A \). This suggests that it is easier to achieve strong vacuum stabilization in the models where the \( D3 \) brane is in the bulk.

Similar results can be obtained in a more general class of theories. In particular, one may consider the KL version of the KKLT scenario, which allows to have small supersymmetry breaking compatible with strong moduli stabilization \[13, 16–18\]. The basic idea was to find a supersymmetric Minkowski vacuum without flat directions. Any small deformation of such vacuum due to a change of model parameters transforms it into a supersymmetric AdS vacuum, which can be subsequently uplifted. If the deformations of the original state are sufficiently small, one obtains a strongly stabilized dS vacuum with a controllably small supersymmetry breaking.
In a recent series of papers [19–21] this approach was generalized and used for finding stable dS vacua in a broad class of type IIB and type IIA string theory models, and in M-theory. In addition to many dS vacua obtained by small deformations of the original supersymmetric Minkowski vacua, Ref. [19–21] also found stable dS vacua produced by a very large increase of \( W_0 \) accompanied by a large uplift. General theorems describing small deformations of a supersymmetric Minkowski vacuum state [16, 19, 20] did not make any predictions about existence and stability of vacua of this type, and yet such dS vacua were present. Moreover, they were stabilized by potential barriers which could be many orders of magnitude higher than the barriers stabilizing the original Minkowski vacua. In particular, one can show that the strongly stabilized dS vacua in the KL scenario with the racetrack superpotentials shown in Fig. 15 of [20] have been obtained in the model which in the absence of uplifting would have a potential \( V(T) \) unbounded from below.

One should note that our results are obtained not in the full string theory context, but in the effective 4d supergravity approach. String theory considerations may affect, in particular, the choice of the parameters \( W_0, A \) and \( \mu \) used in our numerical examples. In this respect it may be important that the value of the volume modulus \( T \) at the minimum of the potential does not change if one simultaneously rescales \( W_0 \to cW_0, A \to cA \) and \( \mu^2 \to c\mu^2 \). Moreover, the value of the volume modulus \( T \) in the KKLT model (1.1), (1.2) practically does not depend on \( W_0 \) for \( W_0 \gg A \).

The main qualitative result of this investigation is that the existence of a supersymmetric AdS (or Minkowski) vacuum is not a necessary precondition for the existence of stable dS vacua. In particular, when finding dS vacua using equations (3.6), (3.7) we did not make any assumptions about the behavior of the potential in the absence of the uplifting contribution of the \( D3 \) brane. Instead of that, we simply analyzed the behavior of the system for all possible relations between the model parameters, taking into account uplifting. If, instead, we would divide the procedure into two parts, first finding a supersymmetric AdS vacuum, and then uplifting it, then we would find dS vacua only for \( W_0 < A \) and miss all strongly stabilized dS states with \( W_0 > A \), which may play an important role in describing string theory vacua in the very early universe.

The author is grateful to Renata Kallosh for many enlightening discussions, and to N. Cribiori, S. Kachru, L. McAllister, C. Roupec and Y. Yamada for useful comments. I am supported by SITP and by the US National Science Foundation Grant PHY-1720397, and by the Simons Foundation Origins of the Universe program (Modern Inflationary Cosmology collaboration), and by the Simons Fellowship in Theoretical Physics.

References

[1] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, De Sitter vacua in string theory, Phys. Rev. D68 (2003) 046005 [hep-th/0301240].

[2] M. R. Douglas and S. Kachru, Flux compactification, Rev. Mod. Phys. 79 (2007) 733 [hep-th/0610102].
3. Y. Akrami, R. Kallosh, A. Linde and V. Vardanyan, *The Landscape, the Swampland and the Era of Precision Cosmology*, Fortsch. Phys. 67 (2019) 1800075 [1808.09440].

4. Y. Hamada, A. Hebecker, G. Shiu and P. Soler, *Understanding KKLT from a 10d perspective*, JHEP 06 (2019) 019 [1902.01410].

5. S. Kachru, M. Kim, L. McAllister and M. Zimet, *de Sitter Vacua from Ten Dimensions*, 1908.04788.

6. S. Ferrara, R. Kallosh and A. Linde, *Cosmology with Nilpotent Superfields*, JHEP 10 (2014) 143 [1408.4096].

7. R. Kallosh and T. Wrase, *Emergence of Spontaneously Broken Supersymmetry on an Anti-D3-Brane in KKLT dS Vacua*, JHEP 12 (2014) 117 [1411.1121].

8. E. A. Bergshoeff, K. Dasgupta, R. Kallosh, A. Van Proeyen and T. Wrase, *D3 and dS*, JHEP 05 (2015) 058 [1502.07627].

9. C. P. Burgess, A. de la Macorra, I. Maksymyk and F. Quevedo, *Supersymmetric models with product groups and field dependent gauge couplings*, JHEP 09 (1998) 007 [hep-th/9808087].

10. D. Baumann, A. Dymarsky, I. R. Klebanov, J. M. Maldacena, L. P. McAllister and A. Murugan, *On D3-brane Potentials in Compactifications with Fluxes and Wrapped D-branes*, JHEP 11 (2006) 031 [hep-th/0607050].

11. D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov and L. McAllister, *D3-brane Potentials from Fluxes in AdS/CFT*, JHEP 06 (2010) 072 [1001.5028].

12. R. Kallosh, F. Quevedo and A. M. Uranga, *String Theory Realizations of the Nilpotent Goldstino*, JHEP 12 (2015) 039 [1507.07556].

13. R. Kallosh and A. D. Linde, *Landscape, the scale of SUSY breaking, and inflation*, JHEP 12 (2004) 004 [hep-th/0411011].

14. W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *Maximal temperature in flux compactifications*, JCAP 0501 (2005) 004 [hep-th/041109].

15. J. P. Conlon, R. Kallosh, A. D. Linde and F. Quevedo, *Volume Modulus Inflation and the Gravitino Mass Problem*, JCAP 0809 (2008) 011 [0806.0809].

16. J. J. Blanco-Pillado, R. Kallosh and A. D. Linde, *Supersymmetry and stability of flux vacua*, JHEP 05 (2006) 053 [hep-th/0511042].

17. R. Kallosh, A. Linde, K. A. Olive and T. Rube, *Chaotic inflation and supersymmetry breaking*, Phys. Rev. D84 (2011) 083519 [1106.6025].

18. A. Linde, Y. Mambrini and K. A. Olive, *Supersymmetry Breaking due to Moduli Stabilization in String Theory*, Phys. Rev. D85 (2012) 066005 [1111.1465].

19. R. Kallosh and A. Linde, *Mass Production of Type IIA dS Vacua*, 1910.08217.

20. N. Cribiori, R. Kallosh, A. Linde and C. Roupec, *Mass Production of IIA and IIB dS Vacua*, 1912.00027.

21. N. Cribiori, R. Kallosh, A. Linde and C. Roupec, *de Sitter Minima from M theory and String theory*, 1912.02791.

22. M. Demirtas, M. Kim, L. McAllister and J. Moritz, *Vacua with Small Flux Superpotential*, 1912.10047.