Superconductivity of Quasi-One-Dimensional Electrons in Strong Magnetic Field

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Abstract

The superconductivity of quasi-one-dimensional electrons in the magnetic field is studied. The system is described as the one-dimensional electrons with no frustration due to the magnetic field. The interaction is assumed to be attractive between electrons in the nearest chains, which corresponds to the lines of nodes of the energy gap in the absence of the magnetic field. The effective interaction depends on the magnetic field and the transverse momentum. As the magnetic field becomes strong, the transition temperature of the spin-triplet superconductivity oscillates, while that of the spin-singlet increases monotonically.

[keywords: superconductivity, quasi-one-dimension, organic conductors]
I. INTRODUCTION

The reentrance of the superconductivity in quasi-one-dimensional electrons in strong magnetic field attracts theoretical interest \[ [1-3] \]. In the quasi-one-dimensional systems the Fermi surface is open and Landau level quantization does not exist. Lebed’ \[ 1 \] has shown that the superconducting transition temperature reaches to that in the absence of the magnetic field as the magnetic field is increased. Dupuis et al. \[ 3-5 \] have shown that there exists the cascade of the superconducting phases separated by first order transitions. The reentrance of the superconductivity is understood as follows. When the strong magnetic field is applied along the \( b \) axis, the semi-classical orbits of electrons are localized in the \( a-b \) plane, where \( a \) is the most conducting axis, i.e. the motion along the \( c \) axis is bounded. Since the orbital frustration due to the magnetic field comes from the motion of electrons in the plane perpendicular to the magnetic field, electrons can make Cooper pairs without affected by the orbital frustration and the transition temperature increases.

The reentrance of the superconductivity in two-dimensional electrons in strong magnetic field also attracts theoretical interest \[ 6,7 \]. This phenomena is similar to that in quasi-one-dimensional systems, but the origin for the reentrance of the superconductivity in two-dimension is different from that in quasi-one-dimension. In two-dimensional case Cooper pairs in strong magnetic field are formed between electrons at the lowest Landau level, whereas superconductivity is stabilized in magnetic field without Landau level quantization in quasi-one-dimensional system.

Quasi-one-dimensional electrons are realized in organic conductors, \((\text{TMTSF})_2\text{X}\), where anion \( \text{X} \) is \( \text{PF}_6 \), \( \text{ClO}_4 \) etc. The system can be described as the anisotropic tight-binding model with the hopping matrix elements, \( t_a \simeq 0.25\text{eV}, t_b/t_a \simeq 0.1 \) and \( t_c/t_a \simeq 0.01 \). Due to the smallness of \( t_c \), the magnetic field necessary to the reentrance of the superconductivity is estimated to be about 20T \[ 1,3 \], which can be realized in experiments.

The superconductivity in the quasi-one-dimensional organics, \((\text{TMTSF})_2\text{X}\), is thought to be not the conventional BCS type even in the absence of magnetic field. When the nest-
ing condition of the Fermi surface is satisfied sufficiently, which is thought to be the case at low pressure, the quasi-one-dimensional organic conductors are not superconductors but insulators due to spin density wave (SDW). Since SDW is caused by the on-site Coulomb repulsion, the on-site repulsion is expected to be large in this system. As a result the conventional s-wave spin-singlet superconductivity is not likely to be stabilized. Indeed, the NMR relaxation rate, $T_1^{-1}$ of the (TMTSF)$_2$ClO$_4$ shows little enhancement just below the transition temperature and it decreases as $T^3$ as temperature becomes low, which strongly suggests the line nodes of the energy gap in the superconducting state [8,9]. The line nodes in the superconducting states are caused by the attractive interaction between electrons which are not on the same site. When the magnetic field is applied along the $y$ direction, the $k_y$ dependence of the interaction is not affected by the magnetic field. Thus if the attractive interaction depends only on $k_y$, the magnetic field dependence of the superconducting transition temperature is the same as that in the case of the on-site attraction. However, the $k_z$ dependence of the interaction makes the different field dependence. In the quasi-one dimensional case the $k_x$ dependence of the interaction is not very important because Fermi surface consists of two planes, $k_x \approx \pm k_F$. Therefore, in this paper we assume the attractive interaction between electrons in the nearest chains in $z$ direction, which results in the line node of the energy gap in the superconducting state.

In the previous paper one of the authors has shown that if the magnetic field is applied, the quasi-one-dimensional electrons in the perpendicular plane is described as the one-dimensional system and the effective interaction depends on the magnetic field and the transverse component of the wave vector [10]. When the magnetic field is applied along the $c$ axis, the perfect nesting is recovered as long as we neglect the small imperfectness of the order of $t^2_c/t_\alpha$. In that paper the on-site repulsive interaction is assumed and the field-induced spin density wave (FISDW) is studied. The quantum Hall effect in FISDW is understood as a result of the $k_y$-dependent phase of the effective interaction.

In this paper we apply the same procedure to the case of the coexistence of the on-site repulsion and the nearest-site attraction. When the magnetic field is applied in the $y$
direction, the imperfectness of the nesting of the Fermi surface is not changed and FISDW is not stabilized. In that case the superconductivity is caused by the attractive part of the interaction.

II. QUASI-ONE-DIMENSIONAL ELECTRONS IN MAGNETIC FIELD

We study the quasi-one-dimensional electrons in the magnetic field. For simplicity, we take $a$, $b$, and $c$ axes to be perpendicular each other and to be along the $x$, $y$ and $z$ directions, respectively. The generalization to the non-orthogonal lattice can be performed as in the FISDW case [11]. The Hamiltonian is written as

$$
\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_U
$$

where

$$
\mathcal{H}_0 = -t_a \sum_{\langle i,j \rangle, \sigma} e^{i\theta_{ij}} c_{i,\sigma}^\dagger c_{j,\sigma}
$$

$$
- t_b \sum_{\langle i,j \rangle, \sigma} e^{i\theta_{ij}} c_{i,\sigma}^\dagger c_{j,\sigma}
$$

$$
- t_c \sum_{\langle i,j \rangle, \sigma} e^{i\theta_{ij}} c_{i,\sigma}^\dagger c_{j,\sigma}
$$

$$
\mathcal{H}_U = \sum_{\langle i,j \rangle, \sigma, \sigma'} U_{ij} c_{i,\sigma}^\dagger c_{i,\sigma} c_{j,\sigma'}^\dagger c_{j,\sigma'}
$$

(1)

where $c_{i,\sigma}^\dagger$ and $c_{i,\sigma}$ are creation and annihilation operators of electrons, $t_a$, $t_b$ and $t_c$ are the hopping matrix elements along the $a$, $b$ and $c$ axes, respectively, $U_{ij}$ is the interaction between electrons at $i$ and $j$ sites and

$$
\theta_{ij} = \frac{2\pi}{\phi_0} \int_i^j A \mathrm{d}l.
$$

(2)

In the above $A$ is the vector potential and $\phi_0 = h c_0 / e$ is the flux quantum, where $c_0$ is the light velocity. We consider the anisotropic system with the hopping matrix elements $t_a \gg t_b \gg t_c$, which is the case in (TMTSF)$_2$X. The nesting of the Fermi surface is not perfect due to the $t_b$ term.

In eq.(1) we have neglected the Zeeman term for simplicity. The Zeeman term does not play any important role for the equal-spin-pairing case of the spin triplet. If the Zeeman
energy is taken into account, the transition temperature of the spin singlet is reduced but the superconductivity is not completely destroyed, since half of the density of states is available to make Cooper pairs for the Larkin-Ovchinikov-Frunde-Ferrell state \[3,4\]. Therefore, the effect of the Zeeman energy for the spin singlet can be taken into account by putting the density of states to be half in the strong magnetic field.

In this paper the magnetic field $H$ is applied in the $y$ direction and the vector potential $A$ is taken as $A = (0, 0, -Hx)$. Then the non-interacting Hamiltonian is written as

$$
\mathcal{H}_0 = \int_{k_F - \frac{G}{2}}^{k_F + \frac{G}{2}} \frac{dk_x}{G} \int_{-\pi}^{\pi} \frac{dk_y}{2\pi/b} \int_{-\pi}^{\pi} \frac{dk_z}{2\pi/c} \sum_{\sigma} c_\sigma^\dagger(k) \begin{pmatrix}
\ddots & \ddots & T^* \\
\ddots & M_{-1}^* & T \\
0 & T^* & M_0 \\
T & \ddots & \ddots
\end{pmatrix} c_\sigma(k),
$$

where

$$M_n = -2t_a \{ \cos[a(k_x + nG)] - \cos(ak_F)] - 2t_b \cos(bk_y),
$$

$$T = -t_c \exp(ick_z),
$$

$$c_\sigma^\dagger = \left( \ldots, c_\sigma^\dagger(k - \mathbf{G}), c_\sigma^\dagger(k), c_\sigma^\dagger(k + \mathbf{G}), \ldots \right),
$$

and $\mathbf{G} = (G, 0, 0) = (ecH/(hc_0), 0, 0)$.

The creation operators of electrons can be written in terms of the creation operators of the eigenstates of the non-interacting Hamiltonian in the presence of the magnetic field. We consider the case that the system has the open Fermi surface and the magnetic field is not very strong in the sense that the magnetic flux per plaquette is much smaller than the flux quantum. Then we get $G \ll k_F$ and we can treat the electrons with $k_x (\approx k_F)$ and $-k_x$ independently in $\mathcal{H}_0$ as linear combinations of the eigenstates $\Psi^\dagger_l(n, \mathbf{k})$ and $\Psi^\dagger_l(n, \mathbf{k})$ as \[10\].
\[ c^\dagger(k + mG) = e^{imkz} \sum_n \varphi_r(m, n)\Psi_r^\dagger(n, k) \]
\[ c^\dagger(-k + mG) = e^{-imkz} \sum_n \varphi_l(m, n)\Psi_l^\dagger(n, -k), \tag{7} \]

where \( m \) and \( n \) are integers.

In this paper we take the interaction as

\[ U_{ij} = \begin{cases} U_0 & \text{if } r_i = r_j \\ U_1 & \text{if } r_i = r_j \pm c\hat{z} \\ 0 & \text{otherwise} \end{cases} \tag{8} \]

The Fourier-transform of the interaction is obtained as

\[ U(k, k') = U_0 + 2U_1 \cos[c(k_z - k'_z)]. \tag{9} \]

The interaction Hamiltonian is written as

\[ \mathcal{H}_U = \sum_{k,k',q,m,m',N,\sigma,\sigma'} \sum \left[ U(k_z, k'_z - q_z) c_{\sigma}(k + mG) c_{\sigma'}(-k + q - mG + NG) \right. \\
\times c_{\sigma'}(k' + m'G)c_{\sigma}(-k' + q - m'G + NG) \\
+ U(k_z, k'_z) c_{\sigma}(k + mG) c_{\sigma'}(-k + q - mG + NG) \right. \\
\times c_{\sigma'}(-k' + q - m'G + NG) c_{\sigma}(k' + m'G) \], \tag{10} \]

where \( m, m' \) and \( N \) are integers and \( k_F - G/2 \leq k_x < k_F + G/2 \). The first and the second terms are the generalization of the \( g_1 \) and \( g_2 \) terms in the \( g \)-ology of the one-dimensional system \cite{12,13} to the quasi-one-dimensional system.

In the previous paper \cite{10} particle-hole channel is taken into account, since these terms make the instability of the Fermi surface into the spin-density-wave state if the magnetic field is applied along the \( z \) direction. Here we consider the particle-particle channel, which corresponds to the instability toward the superconductivity in the case of the attractive interaction.
With the eigenstates $\Psi_r(n, k)$ and $\Psi_l(n, k)$ the interaction Hamiltonian is written as

$$
\mathcal{H}_U = \sum_{k,k',q} \sum_{n_1,n_2,n_3,n_4} \sum_{\sigma_1,\sigma_2,\sigma_3,\sigma_4} U_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}(n_1, n_2, n_3, n_4, k_z, k'_z, q_z) \\
\times \Psi^{\dagger}_{r\sigma_1}(n_1, k) \Psi^{\dagger}_{l\sigma_2}(n_2, -k + q) \\
\times \Psi_{l\sigma_3}(n_3, -k' + q) \Psi^{\dagger}_{r\sigma_4}(n_4, k')
$$

(11)

where

$$
U_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}(n_1, n_2, n_3, n_4, k_z, k'_z, q_z) \\
= \sum_{m,m',N} \exp\left[i \left\{ 2mc(k_z - \frac{q_z}{2}) - 2m'c(k'_z - \frac{q_z}{2}) - Nc(k_z - k'_z) \right\} \right] \\
\times \varphi_r(m, n_1) \varphi_l(-m + N, n_2) \\
\times \varphi^*_l(-m' + N, n_3) \varphi^*_r(m', n_4) \\
\times \left[-U(k_z, -k'_z + q_z) \delta_{\sigma_1,\sigma_3} \delta_{\sigma_2,\sigma_4} + U(k_z, k'_z) \delta_{\sigma_1,\sigma_4} \delta_{\sigma_2,\sigma_3} \right]
$$

(12)

The coefficients $\varphi_r(m, n)$ and $\varphi_l(m, n)$ can be calculated by diagonalizing the matrix in eq.(3) numerically as we have done in the previous paper [10]. In this paper, instead of diagonalizing the matrix numerically, we use the approximation that the dispersion in $k_x$ is taken to be linear, i.e.,

$$
M_n \approx v_F(k_x + nG - k_F) - 2t_b \cos(bk_y),
$$

(13)

where $v_F = 2t_a a \sin ak_F$ is the Fermi velocity. With this approximation the eigenvalues are obtained as

$$
\xi(n, k_x, k_y) = v_F(k_x + nG - k_F) - 2t_b \cos(bk_y)
$$

(14)

and the eigenstates are given with the coefficients

$$
\varphi_r(m, n) = J_{-n+m}(z)
$$

(15)

and
\[ \varphi_{l}(m, n) = J_{n-m}(z), \quad (16) \]

where \( J_{n}(z) \) is the Bessel function and \( z = 2t_c/(v_F G) \). We can perform the \( m \) and \( m' \) summation in eq.(12) by using the identity,

\[ \sum_{m=-\infty}^{\infty} e^{imQ} J_{m-N}(z) J_{m}(z) = e^{iN(Q+\pi)} J_{N}(2z \sin \frac{Q}{2}), \quad (17) \]

and get

\[ U_{\sigma_{1},\sigma_{2},\sigma_{3},\sigma_{4}}(n_1, n_2, n_3, n_4, k, k', q_z) = \sum_{N} \exp \left[ i \left\{ (n_1 - n_2)c(k_z - \frac{q_z}{2}) + (n_3 - n_4)c(k'_z - \frac{q_z}{2}) - (n_1 + n_2 + n_3 + n_4 - 2N)\frac{\pi}{2} \right\} \right] \times J_{N-n_1-n_2}(2z \sin c(k_z - \frac{q_z}{2})) \times J_{N-n_3-n_4}(-2z \sin c(k'_z - \frac{q_z}{2})) \times [-U(k_z, -k'_z + q_z)\delta_{\sigma_1,\sigma_3}\delta_{\sigma_2,\sigma_4} + U(k_z, k'_z)\delta_{\sigma_1,\sigma_4}\delta_{\sigma_2,\sigma_3}]. \quad (18) \]

In the strong magnetic field the instability of the Fermi surface toward forming Cooper pairs occurs due to terms with \( q_x = 0, n_1 = -n_2 (= n) \) and \( n_4 = -n_3 (= n') \). Taking only these terms corresponds to the quantum limit approximation \[5\]. In this paper we take this approximation, which is justified when \( T \ll v_F G \).

The order parameter is defined as \[14\]

\[ \Delta_{\sigma_{2},\sigma_{1}}(n, k_z, q_z) = -\sum_{n',k'} \sum_{\sigma_{1},\sigma_{2},\sigma_{3},\sigma_{4}} U_{\sigma_{1},\sigma_{2},\sigma_{3},\sigma_{4}}(n, -n', -n', k_z, k'_z, q_z) \times <\Psi_{l\sigma_{3}}(-n', -k' + q)\Psi_{r\sigma_{4}}(n', k')>. \quad (19) \]

The mean-field Hamiltonian is written as

\[ \mathcal{H}_{MF} = \sum_{n,k,\sigma} \left[ \xi(n, k_x, k_y)\{\Psi_{r\sigma}(n, k)\Psi_{l\sigma}(n, k) + \Psi_{l\sigma}^{\dagger}(-n, -k)\Psi_{r\sigma}^{\dagger}(-n, -k)\} \right] \]

\[ - \sum_{n,k,q,\sigma_{1},\sigma_{2}} \left[ \Delta_{\sigma_{2}\sigma_{1}}(n, k_z, q_z)\Psi_{r\sigma_{1}}^{\dagger}(n, k)\Psi_{l\sigma_{2}}^{\dagger}(-n, -k + q) + H.c. \right]. \quad (20) \]
Note that $\xi(n, k_x, k_y)$ does not depend on $k_z$, which means the one-dimensional motion in the $x$-$z$ plane in the semi-classical picture.

### III. Transition Temperature

The transition temperature is given by the linearized gap equation,

$$\Delta_{\sigma_2\sigma_1}(n, k_z, q_z) = -\sum_{n'} \int_{k_F - \frac{Q}{2}}^{k_F + \frac{Q}{2}} \frac{dk'_{x}}{G} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{dk'_{y}}{2\pi/b} \int_{-\frac{c}{2}}^{\frac{c}{2}} \frac{dk'_{z}}{2\pi/c} \sum_{\sigma_3, \sigma_4} U_{\sigma_1, \sigma_2, \sigma_3, \sigma_4}(n, -n, -n', k_z, k'_{z}, q_z) \tanh \left( \frac{\xi(n', k', q')}{2T} \right) \frac{2\pi}{\xi(n', k', q')} \Delta_{\sigma_3\sigma_4}(n', k'_{z}, q_z).$$  \hspace{1cm} (21)

This equation is simplified by defining $\Delta_{m,\sigma_1\sigma_2}$ by

$$\Delta_{\sigma_2\sigma_1}(n, k_z, q_z) = e^{2inc(k_z - \frac{Q}{2})} \sum_{m = -\infty}^{\infty} e^{inc(k_z - \frac{Q}{2})} \Delta_{m,\sigma_1\sigma_2},$$ \hspace{1cm} (22)

and using

$$\sum_{n'} \int_{k_F - \frac{Q}{2}}^{k_F + \frac{Q}{2}} \frac{dk'_{x}}{G} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{dk'_{y}}{2\pi/b} \int_{-\frac{c}{2}}^{\frac{c}{2}} \frac{dk'_{z}}{2\pi/c} \sum_{\sigma_3, \sigma_4} \frac{\xi(n', k', q')}{2T} \frac{2\pi}{\xi(n', k', q')} \Delta_{\sigma_3\sigma_4}(n', k'_{z}, q_z) = N(0) \ln \frac{2\gamma \omega_c}{\pi T},$$ \hspace{1cm} (23)

where $N(0)$ is the density of states for one spin at the Fermi energy, $\omega_c$ is the cut-off energy, and $\gamma$ is the exponential of the Euler constant. The factor $1/2$ in the right hand side of eq.(23) comes from the fact that only half of the Fermi surface ($k_x \approx +k_F$) is taken in the summation.

#### A. spin singlet

We can treat the spin singlet case and the spin triplet case separately. In this section we consider the spin singlet case. For the spin singlet case the order parameter is defined by

$$\Delta^{(s)}_m = \frac{1}{2}(\Delta_{m,\uparrow\downarrow} - \Delta_{m,\downarrow\uparrow})$$ \hspace{1cm} (24)

and the transition temperature is determined by the equation,

$$\Delta^{(s)}_m = N(0) \ln \frac{1.13 \omega_c}{T} \sum_{m' = -\infty}^{\infty} M^{(s)}_{m,m'} \Delta^{(s)}_{m'},$$ \hspace{1cm} (25)
where

\[
M_{m,m'}^{(s)} = -\int_{-\pi}^{\pi} \frac{dk_z}{2\pi/c} \int_{-\pi}^{\pi} \frac{dk'_z}{2\pi/c} e^{-i(c(k_z-m'k'_z))} \\
\times J_0(2z\{\sin ck_z - \sin ck'_z\}) \\
\times [U_0 + 2U_1 \cos ck_z \cos ck'_z]
\]

(26)

If the maximum eigenvalue of the matrix \(M_{m,m'}^{(s)}\), which we define \(g^{(s)}\), is positive, the transition temperature is given by

\[T_c^{(s)} = 1.13\omega_c \exp\left(-\frac{1}{N(0)g^{(s)}}\right).\]  

(27)

Using the identity for the Bessel function

\[J_N(z) = \int_{0}^{2\pi} d\theta \exp\left[i(N\theta - z \sin \theta)\right],\]

(28)

we get

\[M_{m,m'}^{(s)} = -U_0L_{m,m'} - U_1C_{m,m'},\]

(29)

where

\[L_{m,m'} = \int_{0}^{2\pi} d\theta J_m(2z \sin \theta)J_{m'}(2z \sin \theta),\]

(30)

and

\[C_{m,m'} = \frac{1}{2}(L_{m+1,m'+1} + L_{m-1,m'-1} + L_{m+1,m'-1} + L_{m-1,m'+1}).\]

(31)

First we consider the case \(U_0 < 0\) and \(U_1 = 0\) (i.e. on-site attraction), which has been studied by Lebed\(^1\),\(^2\) and Dupuis et al.\(^3\)\(-5\). Noting that

\[L_{m,m'} = \sum_{N=-\infty}^{\infty} T_{m,N}^* T_{N,m'}\]

(32)

where

\[T_{N,m'} = \int_{-\pi}^{\pi} d\theta' e^{iN\theta} \sin(2z \sin \theta')\]

\[= \int_{-\frac{\pi}{c}}^{\frac{\pi}{c}} \frac{dk_z}{2\pi/c} e^{i(m'c)k_z} J_N(2z \sin ck_z)\]

(33)
we can show that all eigenvalues of the matrix $L_{m,m'}$ are positive or zero. Since $L_{m,m'} = 0$ if one of $m$ and $m'$ is odd and the other is even, the matrix $(M_{m,m'})$ is divided into two parts, even and odd. When the magnetic field is changed, the maximum eigenvalue is obtained by the even part or the odd part, resulting in the cascade transition into the superconducting state, as shown by Dupuis et al. \[3–5\]. In Fig.1 we plot the effective coupling constant $g^{(s)}/|U_0|$ as a function of $h = H/H_0$, where $H_0 = 2t_ehc_0/(v_{Febc})$. When we take the parameters as $t_a \approx 3000\text{K}$, $t_c \approx 20\text{K}$, $a \approx 7\text{Å}$, $c \approx 14\text{Å}$ and $a k_F = \pi/4$, we get $H_0 \approx 10\text{T}$.

Next we consider the case $U_0 > 0$ and $U_1 < 0$. Since

$$L_{m,m'} = (-1)^m L_{-m,m'} = (-1)^{m'} L_{m,-m'}$$

and

$$C_{m,m'} = -(-1)^m C_{-m,m'} = (-1)^{m'} C_{m,-m'},$$

we get $LC = CL = 0$, i.e. the eigenstates of $M$ are the simultaneous eigenstates of $L$ and $C$ and at least one of the eigenvalues of $L$ and $C$ for each eigenstate is zero. All eigenvalues of $C_{m,m'}$ are positive or zero, which can be shown by noting

$$C_{m,m'} = \sum_{N=-\infty}^{\infty} \frac{1}{2} (T^*_N T_{m+1,N} + T^*_N T_{m-1,N}) (T^*_N T_{m+1,N} + T^*_N T_{m-1,N}).$$

Thus $g^{(s)}$ is obtained by calculating the maximum eigenvalue of $C_{m,m'}$ in the case of $U_0 > 0$ and $U_1 < 0$. In this case the transition temperature does not depend on $U_0$. In Fig.2 $g^{(s)}/|U_1|$ is plotted as a function of $h$. In contrast to Fig.1 the effective coupling constant does not oscillate as the magnetic field is increased. Only in the small region of $h \approx 0.2$ the largest eigenvalues for the even part and the odd part of the matrix $C_{m,m'}$ intersect.

**B. spin triplet**

In this section we consider the spin triplet pairing case. The linearized gap equation for the spin triplet pairing is given as
\[ \Delta_m^{(t)} = N(0) \ln \frac{1.13 \omega_c}{T} \sum_{m'=-\infty}^{\infty} M_{m,m'}^{(t)} \Delta_m^{(t)}, \]  

(37)

where \( \Delta_m^{(t)} \) is either \( \Delta_{m,\uparrow\uparrow}, (\Delta_{m,\downarrow\uparrow} + \Delta_{m,\uparrow\downarrow})/2 \) or \( \Delta_{m,\downarrow\downarrow} \) and

\[ M_{m,m'}^{(t)} = -\frac{U_1}{2} (L_{m+1,m'+1} + L_{m-1,m'-1} - L_{m+1,m'-1} - L_{m-1,m+1}) \]  

(38)

If the maximum eigenvalue of the matrix \( M_{m,m'}^{(t)} \) is positive, we define it as \( g^{(t)} \) and we get the transition temperature as

\[ T_c^{(t)} = 1.13 \omega_c \exp(-\frac{1}{N(0)g^{(t)}}) \]  

(39)

In Fig. 3 we plot \( g^{(t)}/|U_1| \) vs. \( h \). The effective coupling constant is given by the eigenvalue of the even part or odd part of the matrix \( M_{m,m'}^{(t)} \) depending on \( h \), as in the case of the spin singlet for the on-site attractive interaction.

IV. CONCLUSION

In this paper we have studied the transition to superconductivity in the quasi-one-dimensional systems in the magnetic field applied along the \( y \) direction. The magnetic field per plaquette is much smaller than the flux quantum but strong enough so that we can apply the quantum limit approximation, which is realized in quasi-one-dimensional organic superconductors in the magnetic field of the order of 10T. With this condition the system is treated as one-dimension with the effective interaction depending on the magnetic field. We studied the case that the interaction between electrons is repulsive for the on-site electrons and attractive for electrons at the nearest sites in the \( z \) direction. We obtained that the effective coupling constant for the spin triplet oscillates as the magnetic field is increased, while that for the spin singlet increases monotonically.

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FIGURES

FIG. 1. The effective coupling constant for the spin singlet in the case of on-site interaction ($U_0 < 0$) as a function of the magnetic field $h = H/H_0$. The effective interaction is given by the eigenvalues for the even part of the matrix (solid lines) and the odd part (broken lines).

FIG. 2. The effective coupling constant for the spin singlet in the case of attraction interaction between nearest sites ($U_1 < 0$) as a function of the magnetic field. The solid lines show the largest and the second largest eigenvalues for the even part of the matrix and the broken lines show the eigenvalues for the odd part.

FIG. 3. The effective coupling constant for the spin triplet in the case of attraction interaction between nearest sites ($U_1 < 0$) as a function of the magnetic field. The solid lines show the largest and the second largest eigenvalues for the even part of the matrix and the broken lines show the eigenvalues for the odd part.
