Electronic structures of transition metal dipnictides $XPn_2$ ($X=$Ta, Nb; $Pn=$P, As, Sb)

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The electronic structures and topological properties of transition metal dipnictides $XPn_2$ ($X=$Ta, Nb; $Pn=$P, As, Sb) have been systematically studied using first-principles calculations. In addition to small bulk Fermi surfaces, the band anticrossing features near the Fermi level can be identified from band structures without spin-orbit coupling, leading to nodal lines in all these compounds. Inclusion of spin-orbit coupling gaps out these nodal lines leaving only a pair of disentangled electron/hole bands crossing the Fermi level. Therefore, the low energy physics can be in general captured by the corresponding two band model with several isolated small Fermi pockets. Detailed analysis of the Fermi surfaces suggests that the arsenides and NbSb$_2$ are nearly compensated semimetals while the phosphorides and TaSb$_2$ are not. Based on the calculated band parities, the electron and hole bands are found to be weakly topological non-trivial giving rise to surface states. As an example, we presented the surface-direction-dependent band structure of the surfaces states in TaSb$_2$.

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I. INTRODUCTION

Over the last two decades, an exciting development in the condensed matter physics community has revolutionized the classification of materials. In addition to the traditional conductors/insulators/semiconductors, it has been discovered that symmetries of the crystal may lead to more delicate differences, giving rise to topologically trivial and non-trivial phases. The difference between the topological insulators (TI) and normal insulators (NI) lies at their surfaces, the respective interface with a NI, vacuum. The topological change across the interface dictates the existence of a symmetry protected metallic surface state for TI, leading to distinct magneto-electronic responses. For instance, the metallic surface state will result in a resistivity plateau in transport measurement, which is regarded as the hallmark of TI in many cases.

The study of topological materials was taken to a next level with the discovery of three-dimensional Weyl semimetals. A Weyl semimetal has separated right- and left-handed nodal points in momentum space breaking the $T \cdot P$ symmetry and can thus be realized either in centrosymmetric topological materials without the time-reversal $T$ symmetry or in noncentrosymmetric topological materials without the space inversion or parity symmetry $P$. The presence of the left and right Weyl points will lead to other intriguing phenomena including formation of disconnected Fermi arcs. In particular, due to the chiral anomaly of relativistic Weyl fermions generated by coupling to the gauge potential, the negative magnetoresistance with unusual dependence on the electric and magnetic fields may occur in these materials.

More recently, a class of transition metal dipnictides has been experimentally synthesized and characterized. All these compounds were reported to exhibit high mobilities and extremely large positive magnetoresistances (MRs) when the applied field is perpendicular to the current direction, similar to previously known Weyl semimetals. Another common feature of these compounds is the field-induced metal-insulator transition, which in some cases results in a very clear resistivity plateau at low temperatures. Interestingly, negative longitudinal MRs when the applied field is parallel to the current direction were reported in TaSb$_2$ and TaAs$_2$. All these features imply the rich physics beyond the known TIs and Weyl semimetals. On the one hand, the large positive MR is mostly attributed to semi-classical effect of electron-hole compensation, while the field-induced metal-insulator transition may be associated with the gap opening in quasi two-dimensional systems with low carrier density and high mobility. On the other hand, the field-induced resistivity plateau and negative MR should have some topological origins, although their appearances are seemingly materials-dependent. So far several limited band structure information were occasionally presented together with the experimental studies, but the reported results are far from complete and a thorough comparison for band structures of all these materials is obviously lacking. Therefore, a systematic study of electronic structures of this class of materials is urgently demanded.

In this article, we present our latest first principles results on these compounds. Our results show a number of common features of their band structures as well as several important distinctions. In particular, we shall show that the bulk electronic states of these materials can be regarded as two-band system with both electron and hole contributions. The phosphorides (TaP$_2$ and NbP$_2$) are highly uncompensated with much more holes than electrons, while TaSb$_2$ has slightly more electrons than holes. The rest arsenides and NbSb$_2$ are nearly compensated.
Moreover, both the electron and hole bands are weakly topological in all these materials, leading to protected surface states of these compounds.

II. METHOD

The calculations were performed with density functional theory (DFT) as implemented in Vienna Abinitio Simulation Package (VASP)\textsuperscript{20,21}. Plane-wave basis up to 400 eV were employed in the calculations. Throughout the calculation, the PBE parameterization of generalized gradient approximation to the exchange correlation functional was used\textsuperscript{22}. The crystal structure was fully optimized using the conventional cell with $4 \times 12 \times 5$ $\Gamma$-centered K-mesh until the force on each atom less than 1 meV/Å and internal stress less than 0.1 kbar. The subsequent electronic structure calculations were then performed with primitive cell and a $8 \times 8 \times 5$ $\Gamma$-centered K-mesh.

The topological indices $Z_2$ were calculated using the parity-check method proposed by Fu et al\textsuperscript{22}. The DFT band structures were fitted to a tight-binding (TB) model Hamiltonian using the maximally localized wannier function (MLWF) method\textsuperscript{23}, with Ta-5d and Sb-5p orbitals. The Fermi surfaces were then obtained by extrapolating the TB hamiltonian on a $100 \times 100 \times 100$ K-mesh; and the surface states were calculated using the TB hamiltonian by calculating the surface Green’s function\textsuperscript{25}.

III. RESULTS AND DISCUSSION

A. Crystal Structure and Brillouin Zone

First of all, we show the optimized geometry parameters of $XP_{n2}$ compounds with spin-orbit coupling (SOC) effect considered (TABLE I). All compounds share the same centrosymmetric base-centered monoclinic structure with space group $C_{12/m1}$, as shown in Figure 1(a). The calculated lattice constants as well as the atomic coordinates are within 5% errorbar compared to the experimental values, manifesting the validity of our calculations. Since the conventional unit cell consists of 2 primitive cells, the primitive Brillouine zone (BZ) is twice large as the conventional BZ. The BZ for the primitive cell, as well as the definition of high symmetry points, is illustrated in Fig. 1(b).

B. Bulk band structure and DOS

With the optimized structure, we calculated the bulk band structures without the SOC effect (FIG. 2). The overall band structures of these compounds resemble each other, while the systematic changes can be traced. For all the compounds, there are one electron band and one hole band which cross the Fermi level $E_F$, suggesting the two-band feature and coexistence of electron/hole carriers in these materials. In addition, two anti-crossing features can always be identified within 0.2 eV range of the Fermi level $E_F$ as indicated by the blue circles. These band inversion properties can be verified with symmetry analysis for the states near the K-points. By explicitly calculating the coordinates of the nodal k-points for the TaSb compound, we have found that these band crossing k-points form nodal lines, consistent with the general argument posed by Weng et al\textsuperscript{28,29}. For phosphorides, the two anti-crossings are energetically separated and far from $E_F$. They are closer in arsenides, and eventually become energetically almost degenerate and very close to $E_F$ in antimonides. Furthermore, the direct gap at Z from the electron to hole band is increased from a few meVs in NbP$_2$ to $\sim$ 400 meV TaSb$_2$, reflecting the increase of atom sizes and the bond lengths. Similar changes can also be seen around F, F$\prime$, and in between L and I. Thus, more anti-crossing features can be identified in either arsenides or antimonides, as indicated by the green circles in FIG. 2 (b-c) and (e-f). It is also interesting to notice that the anti-crossing features from L to I are highly asymmetric when present, different from all others.

Once the SOC effect is taken into consideration (FIG. 3), all the above nodal lines become gapped, leaving us a pair of fully gapped hole band and electron band. Furthermore, these bands are also completely separated from all other bands in the whole BZ. Therefore, $Z_2$ invariants can be calculated for each of these bands. Apart from the gap of nodal lines, some other interesting features can be identified in FIG. 3. Firstly, the size of the SOC splitting roughly follows the order of Nb<Ta and P<As<Sb, in consistent with normal expectation. Secondly, the gapped states close to Z forms two very small electron pockets for arsenides and antimonides, while the gapped...
TABLE I: Optimized geometry parameters of \( XP_n \) compounds. \( \beta \) is the angle formed by \( \mathbf{a} \) and \( \mathbf{c} \) lattice vectors. The numbers in the parenthesis are experimental values for comparison purposes, from Ref. 15, 16, 26 for \( NbP_2, NbAs_2, NbSb_2, TaP_2, TaAs_2 \) and \( TaSb_2 \), respectively; whereas the internal coordinates for antimonides and \( TaAs_2 \) are from Ref. 17, 18, respectively.

| \( \beta \) (\( {\AA} \)) | \( x_1 \) | \( x_2 \) | \( z_1 \) | \( z_2 \) | \( x_{Pn,I} \) | \( z_{Pn,I} \) | \( x_{Pn,II} \) | \( z_{Pn,II} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( NbP_2 \) | 8.902 (8.872) | 3.290 (3.266) | 7.584 (7.510) | 0.1000 (0.107) | 0.4029 (0.399) | 0.1000 (0.112) | 0.1334 (0.143) | 0.5258 (0.531) |
| \( NbAs_2 \) | 9.454 (9.354) | 3.418 (3.381) | 7.884 (7.795) | 0.1065 (0.107) | 0.4050 (0.404) | 0.1166 (0.107) | 0.1414 (0.140) | 0.5286 (0.526) |
| \( NbSb_2 \) | 10.359 (10.233) | 3.676 (3.630) | 8.423 (8.328) | 0.1516 (0.154) | 0.4050 (0.404) | 0.1012 (0.112) | 0.1489 (0.142) | 0.5358 (0.526) |
| \( TaP_2 \) | 8.892 (8.861) | 3.290 (3.268) | 7.543 (7.488) | 0.1598 (0.154) | 0.4050 (0.404) | 0.1012 (0.112) | 0.1489 (0.142) | 0.5358 (0.526) |
| \( TaAs_2 \) | 9.441 (9.329) | 3.422 (3.385) | 7.843 (7.753) | 0.1571 (0.154) | 0.4050 (0.404) | 0.1012 (0.112) | 0.1489 (0.142) | 0.5358 (0.526) |
| \( TaSb_2 \) | 10.356 (10.223) | 3.697 (3.645) | 8.383 (8.292) | 0.1489 (0.142) | 0.4050 (0.404) | 0.1012 (0.112) | 0.1489 (0.142) | 0.5358 (0.526) |

states close to \( Y \) forms two small hole pockets for \( NbP_2 \) and \( NbAs_2 \). The formation of these small closed pockets can also be evidenced from the Fermi surface (FS) plot of these compounds (FIG. 2). The DOS at the Fermi level \( n(E_F) \) are estimated to be 0.099 (\( NbP_2 \)), 0.546 (\( NbAs_2 \)), 1.084 (\( NbSb_2 \)), 1.115 (\( TaP_2 \)), 0.583 (\( TaAs_2 \)), and 0.803 (\( TaSb_2 \)) states per unit cell per eV, respectively. The systematic increase of \( n(E_F) \) from phosphides to antimonides is consistent with the increase of FS area; and proportional to the carrier density in these compounds and the metallicity thereafter.

Closer examination of the FS plots reveals further details of these compounds. From phosphides to antimonides, the increase of \( n(E_F) \) is due to increase of both hole FS (blue/green sheets) and electron FS (cyan/red sheets) (FIG. 2). We can also identify the electron/hole DOS contribution by evaluating the second derivative of band energy \( m_{e,h} = \frac{d^2\epsilon_k}{dk_a dk_b} \) for all bands crossing the Fermi level. It has been proposed that the large MR observed was due to nearly compensated electron/hole density.16,17 As the current direction is usually aligned along \( \mathbf{b} \) axis, we classified the contribution according to \( m_{eb} \). The electron/hole DOS ratio for these compounds are 1:2.4 (\( NbP_2 \)), 1:1.13 (\( NbAs_2 \)), 1:0.53 (\( NbSb_2 \)), 1:1.9 (\( TaP_2 \)), 1:1.16 (\( TaAs_2 \)), and 1:2.51 (\( TaSb_2 \)), respectively. Thus, the phosphides are highly uncompensated with much more holes than electrons, and \( TaSb_2 \) has slightly more electrons than holes, while all other compounds...
(a) NbP$_2$
(b) NbAs$_2$
(c) NbSb$_2$
(d) TaP$_2$
(e) TaAs$_2$
(f) TaSb$_2$

FIG. 3: Bulk band structure of $XPn_2$ calculated with SOC. The Fermi energy is aligned at 0.

TABLE II: Parities of bands at time-reversal invariant momenta (TRIM) $\Gamma$: (0,0,0), $N$: ($\pi$, 0, 0), $N'$: (0, $\pi$, 0), Y: ($\pi$, $\pi$, 0), $Z$: (0, 0, $\pi$), M: ($\pi$, 0, $\pi$), $M'$: (0, $\pi$, $\pi$) and L: ($\pi$, $\pi$, $\pi$). $\Pi_n$ is the multiplication of the parities for bands 1 to $n$. The highest occupied band at each TRIM is indicated with $^\circ$.

|        | $XP_2$ |        |        | $XAs_2$ |        | $XSb_2$ |
|--------|--------|--------|--------|---------|--------|---------|
| $\Pi_20$ | $\Pi_{21}$ | $\Pi_{22}$ | $\Pi_{23}$ | $\Pi_{20}$ | $\Pi_{21}$ | $\Pi_{22}$ | $\Pi_{23}$ |
| $\Gamma$ |  +  |  $^-$ |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  +  |  +  |  +  |  +  |  +  |  +  |  +  |
| $N$     |  +  |  $^-$ |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  $^\circ$ |  +  |  +  |  +  |
| $N'$    |  +  |  $^-$ |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  $^\circ$ |  +  |  +  |  +  |
| Y       |  +  |  $^-$ |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  $^\circ$ |  +  |  +  |  +  |
| Z       |  +  |  $^-$ |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  $^\circ$ |  +  |  +  |  +  |
| M       |  +  |  $^-$ |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  $^\circ$ |  +  |  +  |  +  |
| $M'$    |  +  |  $^-$ |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  $^\circ$ |  +  |  +  |  +  |
| L       |  +  |  $^-$ |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  +  |  $^\circ$ |  +  |  $^\circ$ |  +  |  +  |  +  |

are nearly compensated. Although such classification is very crude indeed, it agrees with previous experimental results.$^{15-17,19}$

C. $Z_2$ invariants

Since the structure has an inversion center, the topological indices can be easily calculated by examining the parities of occupied bands at the time-reversal invariant momenta (TRIM).$^{21}$ Noticing that both the 21st and 22nd bands cross the Fermi level $E_F$, we calculated four sets of $Z_2$ numbers $\Pi_{20}$, $\Pi_{21}$, $\Pi_{22}$ and $\Pi_{23}$, where $\Pi_n$ is the multiplication of the parities for bands 1 to $n$. The $\Pi_{20}$, $\Pi_{21}$ and $\Pi_{22}$ are exactly the same for all these com-

FIG. 4: Fermi surfaces of $XPn_2$ calculated with SOC (using MLWF fitted TB Hamiltonians). In all compounds, there are 3 isolated pockets, including 2 large ones and 1 small one.
FIG. 5: Cartoon illustration of possible surface states. The rows from top to bottom are phosphorides, arsenides, and antimonides, respectively. The bottom left corner of each panel is $\Gamma$, and the surface indices are illustrated at the center of the panel. The blue/red circles represent $+/−$ polarizations, respectively; and open circles indicate overwhelming bulk states due to occupations. Topological surface states can be present between solid circles with different colors.

FIG. 6: Surface states of TaSb$_2$. The surface direction is labeled in conventional cell, thus [110] surface is indeed [100] surface of the primitive cell.

In conclusion, we have performed a systematic study of the electronic structures and topological properties of transition metal dipnictides $XPn_2$ ($X=$Ta, Nb; $Pn=P$, As, Sb) using first-principles calculations. Nodal line features can be identified in these compounds, and the inclusion of spin-orbit coupling gaps out all the anticrossing features. Small electron (hole) pockets due to the gapped nodal states can be identified in arsenides and antimonides (NbP$_2$ and NbAs$_2$), respectively. The DOS at $E_F$ systematically increases in the order of phosphorides < arsenides < antimonides. Furthermore, the NbAs$_2$, NbSb$_2$ and TaAs$_2$ are nearly compensated semimetals; the phosphorides have much more holes than electrons; while the TaSb$_2$ has slightly more electrons than holes. By calculating the band parities, we found both the elec-
tron and hole bands are weakly topological and thus shall give rise to surfaces states, although the presence of these states are not as robust as the ones emerging from strongly topological insulators and depends on details including the electron polarizations at TRIMs and the electron occupations.

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All calculations were performed at the High Performance Computing Center of Hangzhou Normal University College of Science.

Note Added: Before the submission of this manuscript, we became aware of the work by Shen et al. They have also observed negative MR in NbAs$_2$, and their band structure results are in good agreement with ours. We notice that the three Fermi pockets in their calculation is due to the folded BZ for the conventional cell instead of the primitive one. The $F_p$ frequency from their experiment may be due to the two small pockets close to the Z point in our calculation which become indistinguishable because of the identical shape and small sizes.
The geometry parameters obtained without SOC differs by less than 1% compared to those obtained with SOC.