Trinification and the Strong $P$ Problem

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Models with spontaneously broken parity symmetry can solve the strong $CP$ problem in a natural way. We construct such a model in the context of SU(3)$^3$ unification. Parity has the conventional meaning in this model, and the gauge group is unified.

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1. Introduction

One of the longstanding problems of particle physics is the so-called strong CP problem, namely, why is the coefficient $\theta$ so small in the interaction

$$\mathcal{L} = \frac{\theta g^2}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} C^\mu_a C^\nu_a C^\alpha_{\beta},$$

(1.1)

where $C^\mu_a$ is the gluon field of chromodynamics, and $\epsilon_{\mu\nu\alpha\beta}$ is the totally anti-symmetric tensor. This term respects charge conjugation invariance ($C$), but violates both parity ($P$) and their combination ($CP$). When massive quarks are included, it turns out that $\theta$ itself has no physical meaning, and can be redefined by a chiral rotation of the quark fields. However, the combination

$$\bar{\theta} = \theta + \arg(\det(M_q)),$$

(1.2)

where $M_q$ is the quark mass matrix, is still physically meaningful, and violates both $P$ and $CP$. Limits on the electric dipole moment of the neutron imply \cite{1} that $\bar{\theta} < \sim 2 \times 10^{-10}$. The smallness of $\bar{\theta}$ is conventionally called the strong CP problem, but could just as well be called the strong $P$ problem.

One idea that has been pursued \cite{2} is to have $CP$ as a spontaneously broken symmetry. Parity can be used as well, and Babu and Mohapatra built a model involving parity as a softly broken symmetry. Spontaneous parity violation is just as viable an option, as pointed out by Barr, Chang, and Senjnović \cite{3}. They pointed out that spontaneous parity violation models cannot work without an extended gauge group. Their model, although it has many nice features, has the disadvantage that the symmetry they call parity has nothing to do with conventional parity; it takes standard model particles to heavy (as yet undiscovered) counterparts.

Our work differs from theirs in three ways. First, parity affects standard model particles the way you expect it to. Second, we use the extended gauge group to create a grand unified model. Third, the whole context of our model is built on a structure that has already been discussed in the literature: the trinification model of De Rújula, Georgi, and Glashow \cite{4}. In section 2 we will discuss this unification scheme and how the idea of spontaneous parity breaking applies to it. In section 3 we will work out some of the phenomenological consequences of this simple model. In section 4 we will consider some simple extensions of the idea. In section 5 we will summarize our work.
2. Trinification

Trinification, one of many unification schemes, was originally promoted \cite{4} for its relative simplicity of fermion and scalar content. The unification group is

\[ G = \text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R, \quad (2.1) \]

where $\text{SU}(3)_C$ is the standard color force, $\text{SU}(3)_L$ contains the left-handed $\text{SU}(2)_L$ force of electroweak interactions, and the remaining $\text{U}(1)$ part is distributed between $\text{SU}(3)_L$ and $\text{SU}(3)_R$. In addition, there is a cyclic $Z_3$ symmetry relating the three forces, so that they have the same coupling constants.

Both the fermions and bosons appear in a 27 dimensional representation of $G$; they transform under $G$ as the representation

\[ \Psi = \psi_L(3, \bar{3}, 1) + \psi_R(\bar{3}, 1, 3) + \psi_\ell(1, 3, \bar{3}). \quad (2.2) \]

The left-handed quarks and anti-right-handed quarks will be found in $\psi_L$ and $\psi_R$ respectively, and the leptons are found inside $\psi_\ell$. The scalars will acquire vacuum expectation values (VEV’s) which are arranged as

\[ \langle \phi_\ell \rangle = \begin{pmatrix} u & 0 & 0 \\ 0 & u & u \\ 0 & w & v \end{pmatrix}. \quad (2.3) \]

The rows and columns of this matrix are understood to indicate the transformation properties under $\text{SU}(3)_L$ and $\text{SU}(3)_R$ respectively, using the same notation as \cite{4}. The ‘constants’ $u$, $v$, and $w$ here merely represent orders of magnitude, rather than specific values. The scales $v$ and $w$ break the symmetry group down to $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$ and the standard model respectively, whereas $u$ accomplishes the electroweak breaking. If there is only one scalar field, the VEV can always be diagonalized, so that it is impossible to have the standard model at an intermediate scale. Hence it is necessary to assume at least two 27’s, and, in the interest of economy, we will assume exactly two 27’s. These fields suffice to perform all the necessary gauge symmetry breaking, both at the unification scale and the electroweak scale. To account for three generations of fermions, however, we will assume three 27’s of fermions.

We would like to consider promoting the simple $Z_3$ symmetry to the full $S_3$ permutation group symmetry. In other words, we would like to include not only cyclic
permutations of the three gauge fields, but pair switchings as well. The problem with this is that under these additional symmetries, the $27$ does not transform into itself, it transforms into a $\overline{27}$. This suggests that we should also be including complex conjugation into our symmetries. However, we know that if $\Psi$ is a fermion field that annihilates left-handed fermions, then $\Psi^\ast$ annihilates right-handed fermions. This implies that we must treat such transformations as parity transformations. Without further ado, let us write down the action of $P$, one of the three pair switching permutations, on all the fields:

\begin{align*}
\psi^A_\ell(\vec{x},t) &\to \psi^{A\dagger}_\ell(-\vec{x},t), & \phi^i_\ell(\vec{x},t) &\to \phi^{i\dagger}_\ell(-\vec{x},t), & C^\mu_a(\vec{x},t) &\to C_{a\mu}(-\vec{x},t), \\
\psi^A_L(\vec{x},t) &\to \psi^{A\dagger}_R(-\vec{x},t), & \phi^i_L(\vec{x},t) &\to \phi^{i\dagger}_R(-\vec{x},t), & L^\mu_a(\vec{x},t) &\to R_{a\mu}(-\vec{x},t), \\
\psi^A_R(\vec{x},t) &\to \psi^{A\dagger}_L(-\vec{x},t), & \phi^i_R(\vec{x},t) &\to \phi^{i\dagger}_L(-\vec{x},t), & R^\mu_a(\vec{x},t) &\to L_{a\mu}(-\vec{x},t),
\end{align*}

where $C$, $R$, and $L$ are the gauge fields, the lowering of the index $\mu$ to $\mu$ indicates reversal of the spatial components, $A = 1, 2, 3$ is a family index and $i = 1, 2$ is a gauge boson index. The daggers ($\dagger$) represent the fact that not only are each component of these matrices complex conjugated, but the SU(3)$_L$ and SU(3)$_R$ indices are exchanged as well.

$P$ is the action of one of the pair switchings on the fields; there are also two other pair switchings which can be obtained by cyclic permutations of $P$. Note that the symmetry breaking at the scale $v$ does not break $P$, though it does break the other two pair switchings. This is why $P$ corresponds to actual parity symmetry.

The symmetry $P$ does not allow the parity breaking term $[11]$ to appear in the Lagrangian; hence it doesn’t exist. However, we must take more care when considering the mass terms of the quarks; it is not immediately obvious that there is no complex determinant to the quark mass matrix, and hence a large $\bar{\theta}$. This brings our attention to the Yukawa couplings.

The Yukawa couplings responsible for the quark masses are given by

$$L_{\text{Yuk}} = -Z_3 \{ f_{iAB} \text{Tr}(\phi^i_L \psi^A_L \psi^B_R) + h.c. \} ,$$

where $Z_3$ simply implies that we must include cyclic permutations to assure the $Z_3$ symmetry is respected. Under the symmetry element $P$, we can relate the terms to their hermitian conjugates, so that $f_{iAB}^* = f_{iBA}$, or, thinking of these as matrices, $f^i_\ell = f_i$. Assuming the scalar VEV’s are real, this will result in Hermitian quark mass matrices, or $M^\dagger = M$. Since the determinant of a Hermitian matrix is real, the resulting $\bar{\theta}$ vanishes.

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It remains only to confirm that the scalar VEV’s are real. To do so, we must study the portion of the scalar potential responsible for the symmetry breaking. Since only the $\phi_\ell$ portions acquire VEV’s, we will focus on these. The portion of the scalar potential that is relevant is given by

$$-\mathcal{L}_\ell = m^2_{ij} \text{Tr}(\phi_\ell^{i\dagger} \phi_\ell^j) + \{\gamma_{ijk}\epsilon_{\alpha\beta\gamma}\epsilon^{\delta\rho\sigma} \phi_\ell^{i\alpha} \phi_\ell^{j\beta} \phi_\ell^{k\gamma} + \text{h.c.}\} + \lambda_{ijkl} \text{Tr}(\phi_\ell^{i\dagger} \phi_\ell^j) \text{Tr}(\phi_\ell^{k\dagger} \phi_\ell^l) + \eta_{ijkl} \text{Tr}(\phi_\ell^{i\dagger} \phi_\ell^j \phi_\ell^{k\dagger} \phi_\ell^l) . \tag{2.6}$$

We now apply the parity symmetry together with the demand of Hermiticity to show that, in fact, all of these constants are forced to be real. Hermiticity implies

$$m^2_{ij} = m^2_{ji}^*, \quad \lambda_{ijkl} = \lambda_{jilk}^*, \quad \text{and} \quad \eta_{ijkl} = \eta_{jkil}^* , \tag{2.7}$$

whereas parity implies

$$m^2_{ij} = m^2_{ji} , \quad \gamma_{ijk} = \gamma_{ikj}^* , \quad \lambda_{ijkl} = \lambda_{jilk} , \quad \text{and} \quad \eta_{ijkl} = \eta_{lijk} . \tag{2.8}$$

It is obvious that these conditions together imply all the coefficients are real with the possible exception of $\eta_{ijkl}$. Using the equations (2.7) and (2.8), it is easy to show that all of the following are equal:

$$\eta_{ijkl}^* = \eta_{kijl} = \eta_{ilkj} = \eta_{jilk} = \eta_{lkji} . \tag{2.9}$$

The first two expressions force $\eta_{ijkl}$ to be real if $i = k$ or $j = l$. Hence we need consider only when $i \neq k$ and $j \neq l$. Because the indices take on only the values 1 or 2, these conditions imply either $i = j$ and $k = l$ or $i = l$ and $j = k$. Then the last two expressions assure that $\eta_{ijkl}$ is real, and thus the whole potential (2.6) is real. This makes it likely that the VEV’s will be real, resulting in Hermitian quark mass matrices and vanishing $\bar{\theta}$, at least at tree level. We will discuss $\bar{\theta}$ at one loop level in the next section.

Thus we see that spontaneous parity violation in the context of trinification provides a natural solution to the strong $CP$ problem. It should be noted, incidentally, that even though the potential involving only $\phi_\ell$ is real, complex numbers occur both in the quark Yukawa couplings and in the scalar potential in terms like $\text{Tr}(\phi_\ell^{i\dagger} \phi_\ell^j \phi_\ell^{k\dagger} \phi_\ell^l)$. Thus the theory does not have $CP$ symmetry, only parity.
3. Phenomenology

We now wish to do some more specific calculations. To start with, let’s consider the three scales $u$, $v$, and $w$; we will assume the hierarchy $u \ll w \ll v$. At the unification scale $v$, the gauge couplings are all equal, so that $\alpha_C = \alpha_L = \alpha_R$. The electroweak interactions are as strong as strong interactions and the weak angle is given by $\sin^2 \theta_W = \frac{3}{8}$. As we scale down first to $w$ and then to $u$, the symmetry is broken down first to $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$, then to the standard model, and finally to $\text{SU}(3)_C \times \text{U}(1)_Q$. Meanwhile, the coupling constants will evolve according to the equation

$$\frac{d}{dt} \left( \frac{4\pi}{\alpha_i} \right) = \frac{\pi^2}{3} t_2(V) - \frac{8}{3} t_2(F) - \frac{2}{3} t_2(S),$$

(3.1)

where $t_2(V)$, $t_2(F)$, and $t_2(S)$ are the Casimir operators for the gauge bosons, fermions, and scalars respectively, and $t$ is the logarithm of the renormalization scale.

The exact results depend on the precise ways in which various masses appear at each symmetry breaking. The most naive assumption is that all particles that acquire mass from a given symmetry breaking are immediately removed from (3.1). Just above the electroweak scale $u$, there is no reason to assume that any particles are light except for the three standard fermion families and one Higgs doublet. A right-handed neutrino and three more doublets must be present above the scale $w$, and of course above the scale $v$ all particles are massless. Treating the onset of each scale as sudden, and using the weak scale values $\alpha_C(M_Z) = 0.1134$, $\alpha_{em}(M_Z) = \frac{1}{128}$, and $\sin^2 \theta_W(M_Z) = 0.2325$, we find the approximate unification scales

$$w = 4 \times 10^{10} \text{ GeV} \quad \text{and} \quad v = 2 \times 10^{16} \text{ GeV}.$$  

(3.2)

Note that below the unification scale $v$, baryon number is conserved, provided all of the colored scalar particles are very heavy. Baryon number violation proceeds through Yukawa couplings of the colored scalars, which acquire masses at the scale $v$. Because of the smallness of Yukawa couplings and the largeness of $v$, proton decay limits are well within experimental bounds.

It should be recognized that the scales in (3.2) are not precise. In particular, the heavy leptons acquire masses from the same Yukawa couplings which are responsible for the light lepton masses. The lightness of the standard leptons may well be reflected in the
lightness of these heavy leptons, which will introduce threshold effects at the high scales. This results in a substantial change in these scales to

\[ w = 1.2 \times 10^{12} \text{ GeV} \quad \text{and} \quad v = 3 \times 10^{15} \text{ GeV}. \]  

(3.3)

The cancelling of \( \bar{\theta} \) at tree level in models with spontaneous symmetry breaking does not, in general, persist at higher loop levels. In particular, loops of colored scalar particles will result in complex effective quartic interactions involving two scalar fields transforming as \((1, 2, 2, 0)\) and two scalars transforming as \((1, 2, 1, +1)\) under the gauge group \(\text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}\) below the scale \(v\). When the VEV \(w\) turns on, this will result in a complex mixing among the fields transforming as \((1, 2, 2, 0)\), some part of which is destined to become the standard Higgs doublet. This results in a phase mismatch between the portion of the field coupling to the up-type quarks and the portion coupling to the down-type quarks, so that there will be a resulting phase in the quark determinant and hence non-vanishing \(\bar{\theta}\). We consider this a major problem with this theory in its simplest form. It is, however, impossible to estimate the magnitude of this contribution to \(\bar{\theta}\) because we know nothing about the quartic couplings in the original theory which lead to this effect.

Most other contributions to \(\bar{\theta}\) are either small or vanish. Because \(P\) is still a good symmetry between the scales \(v\) and \(w\), \(\bar{\theta}\) vanishes to all orders above the scale \(w\). This causes most diagrams involving colored scalars to be suppressed by two or more powers of \(w/v\). Contributions arising from the Yukawa couplings tend to be very small and arise only at high loop level; indeed, we have not yet found any nonvanishing contributions. Basically, this occurs because it is possible to use a vector unitary transformation to redefine the quark fields such that only one phase appears among all the Yukawa couplings. The situation is exactly analogous to the standard model, where the Cabibbo-Kobayashi-Maskawa (CKM) matrix has only one phase.

Finite neutrino masses are another interesting consequence of this theory. At the scale \(v\), the fields \(\psi_{\ell 33}\) will acquire masses at one loop level of the order \(f^2v/16\pi^2\), where \(f\) is a generic quark-type Yukawa coupling. At the scale \(w\) these heavy neutrinos mix with \(\psi_{\ell 32}\) with a fraction \(w/v\), giving them see-saw small masses of order \(f^2w^2/16\pi^2v\). At the symmetry breaking scale \(u\), the lightest \(\psi_{\ell 32}\) will acquire a Dirac mass of order \(hu\) connecting it with the standard neutrino \(\psi_{\ell 23}\), where \(h\) is a standard Yukawa coupling responsible for lepton masses. This will, by the see-saw mechanism, result in Majorana
masses for the physical neutrinos of order $16\pi^2 u^2 v h^2 / w^2 f^2$. Plugging in values from (3.3), and guessing $h/f \approx m_\tau / m_t \approx 10^{-2}$, neutrino masses of the order 1 keV are likely. Such neutrino masses are possible for the muon or tau neutrino, and could have very interesting cosmological consequences. Once again, it is impossible to determine these masses with sufficient accuracy to make phenomenological predictions.

4. Extensions

Several possible extensions of our work here seem worthy of note. The simplest and most obvious is to increase the number of scalar fields to match the fermion fields. Unfortunately, the constraints (2.7) and (2.8) are then no longer sufficient to assure the $\eta_{ijkl}$ is necessarily real.

Additional structure is required to avoid problems. For example, if we impose an additional $S_3$ symmetry among the families, where we simultaneously permute the scalar and fermion family numbers, our solution is restored.

Seemingly more attractive is the idea of supersymmetrizing the theory, since Higgs and fermions have the same 27-dimensional representation. Also, our desired VEV’s, $v$ and $w$, seem to coincide with the F-flat direction of a general superpotential. Upon close scrutiny, however, these nice features evaporate quickly. Although the F-terms are flat, the D-terms are positive definite and therefore favor $v = w = 0$. To avoid this, it is necessary to introduce additional $\overline{27}$’s to cancel these positive definite terms. Supersymmetric $SU(3)^3$ models containing $n + 3$ 27’s and $n \overline{27}$’s have been built for other reasons [5], and do have a certain number of pleasant features [5] [6], but all the phenomenological details have not been worked out. Besides problems common in other supersymmetric GUT’s, the values of $v$ and $w$ remain controversial. Renormalization group calculation based on the latest LEP data [7] suggests that $v = w = 10^{16.0 \pm 0.3}$ GeV. It is difficult, however, to obtain such large VEV’s via the usual soft-supersymmetry-breaking-versus-nonrenormalisable-term mechanism without fine-tuning [8]. Even if one is willing to fine-tune, $v = w = 10^{16}$ GeV is not compatible with the fact that neutrino masses are small [9]. Furthermore, the phases of $v$ and $w$ are not determined, and in general, can assume any values. Because these phases contribute directly to $\bar{\theta}$, they will rule out all attempts to solve the strong $CP$ ($P$) problem with spontaneous $CP$ ($P$) breaking.

All in all, the problems seem to proliferate just as fast as the solutions.
5. Conclusion

We have demonstrated the viability of SU(3)$^3$ to explain the strong CP problem in terms of spontaneous parity breaking. Doubtless there are other unification schemes which work as well.

Several problems remain, however. The existence of wide disparities between unification and electroweak scales (the hierarchy problem) is, of course, still unexplained. Spontaneous parity breaking shares with spontaneous CP breaking the problem of domain walls coming from the breaking of discrete symmetries [10]. Inflation might well solve such problems [11]. However, this problem existed in the trinification model with only a $Z_3$ symmetry, so the extension to an $S_3$ symmetry does not necessarily make the problem any worse.

There is no explanation of why there should be three generations of fermions and two of bosons. Adding a third boson requires additional structure to avoid the reintroduction of $\bar{\theta}$. Supersymmetry has certain desirable features when applied to this model, but the problems seem to proliferate faster than their solutions.

It is difficult to make definite phenomenological predictions because of the numerous parameters appearing in the theory. However, it seems likely that neutrino masses might lie in experimentally accessible regions.

We feel that our suggested solution to the strong CP problem deserves more attention. In particular, other unification schemes may also allow parity symmetry with the possibility of spontaneous breaking and consequent vanishing $\bar{\theta}$. We hope this idea will be fully explored.
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