Gini Shortfall: A Gini mean difference-based risk measure

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Abstract. A risk is the possibility of undesirable events happening in the future. A good risk measure is needed to quantify the risks one faces. Some well-known risk measures include the Value-at-Risk (VaR) and the Expected Shortfall (ES). VaR measures the lower bound for big losses in a loss distribution tail, while ES measures the average of losses surpassing the VaR. Unfortunately, there are drawbacks in using the stated risk measures, mainly that they do not provide information regarding the variability of losses in the distribution tail. This paper will introduce and explore Gini Shortfall (GS), a more comprehensive risk measure than VaR and ES. GS provides information on the variability of data in distribution tails measured with Tail-Gini functional, a tail variability measure based on the variability measure Gini Mean Difference or Gini functional. Another superiority of GS is that under certain conditions, it can satisfy the four criteria of coherency. A coherent risk measure is useful for companies or investors to determine the right business and investing strategies. This paper will also provide explicit formulas of GS for some loss-related distributions, namely the exponential, Pareto, and logistic distributions. These formulas are then applied to calculate risks from actual stock data.

Keywords: Gini functional, loss, risk, Tail-Gini functional, variability

1. Introduction

According to the Committee of Sponsoring Organizations of the Treadway Commission Enterprise Risk Management (COSO ERM), a risk is the possibility that events will occur and affect the achievement of strategy and business objectives. Companies, especially those in the financial industry, need a good and reliable risk measure to quantify the risks they face.

Some well-known risk measures include the Value-at-Risk (VaR) and the Expected Shortfall (ES). VaR is defined as the quantile of a particular loss distribution and can be interpreted as the lower bound for a percentage of the biggest losses, while ES is defined as the expectation of a loss given that it surpasses or breaches the VaR [1]. ES is said to be a better risk measure than VaR because it provides information on the severity of losses on the distribution tail. However, usage of ES still has drawbacks since it does not explain about the variability of losses on the distribution tail.

Common variability measures used are the variance and standard deviation. These variability measures are easy to interpret for normally distributed data, but not for non-normal distributions. The variance does not represent the actual distance from an observation to the data mean because it uses a quadratic function in calculation and the standard deviation still does not fully nullify this distortion effect. Therefore, Corrado Gini introduced a new variability measure called the Gini Mean Difference or Gini functional which uses the absolute value concept instead of the quadratic function used in
variance. The Gini functional can be interpreted easier because it represents actual distances between each observation and the data mean.

Gini Shortfall (GS) is a risk measure introduced in this paper that is built from a linear combination between the Expected Shortfall and the Tail-Gini functional [2], which is a tail variability measure based on the Gini functional. GS is a coherent risk measure because it satisfies the four criteria of coherency and is also a more comprehensive risk measure compared to VaR and ES due to its ability to take into account variability in measuring risk through the use of Tail-Gini functional.

2. Materials and method

2.1. Variability measures

Variability is a term used to describe the spread of data. A large variability means the data is more spread out, while a small variability means the data is not too widely spread. A variability measure is a function \( v \) that, for random variables \( X \) and \( Y \), may fulfill these properties [2]:

a) Standardization: \( v(c) = 0 \) for all \( c \in \mathbb{R} \).

b) Location invariance: \( v(X - c) = v(X) \) for all \( c \in \mathbb{R} \).

Adding or subtracting a constant to all possible values of \( X \) will not affect its variability.

c) Positive Homogeneity: \( v(\lambda X) = \lambda v(X) \) for all \( \lambda > 0 \).

Multiplying all possible values of \( X \) by \( \lambda \) also multiplies its variability by \( \lambda \).

d) Sub-additivity: \( v(X + Y) \leq v(X) + v(Y) \).

The variability of the sum of two random variables is not greater than the sum of variabilities of each individual random variable.

Commonly used variability measures include the variance and standard deviation, which, for a random variable \( X \), are respectively defined as [3]:

\[
\text{Var}(X) = E[(X - E[X])^2]
\]

\[
= E[X^2] - (E[X])^2
\]

and

\[
\sigma_X = \sqrt{\text{Var}(X)}.
\]

However, the use of a quadratic function in the variance definition may cause some problems, as they tend to make data points which are far from the mean contribute more to the variance disproportionately, and the standard deviation does not rectify this problem either. This use of a quadratic function makes the variance and standard deviation harder to interpret since it does not represent the actual distance each data has from its mean. Moreover, variance is not a coherent variability measure, since it does not follow positive homogeneity and sub-additivity [2].

2.2. Risk measures

A risk measure is defined as a function \( \rho(X) \) that maps the loss random variable \( X \) to the set of real numbers, and is used to quantify exposure to risk. \( \rho(X) \) can be interpreted as the amount of asset or reserve a company needs to protect the company from risks regarding the loss random variable \( X \).

A typical insurance company is made up of divisions that handle different types of product, such as life, health, accident, and disability insurance. Sometimes, it might be important for the company to look at the overall risk the company faces instead of individual division risks. In this subsection, the random variables \( X \) and \( Y \) can be viewed as the loss random variables for two different divisions, while \( X + Y \) can be seen as the collective loss random variable from the two divisions [1].
Consider the set of random variables such that if $X$ and $Y$ are two members of the set, then both $cX$ and $X + Y$ are also in the set. A risk measure $\rho(X)$ is said to be coherent if it fulfills these criteria for any random variables $X$ and $Y$ [4]:

a) **Sub-additivity:** $\rho(X + Y) \leq \rho(X) + \rho(Y)$.

The risk measure for the sum of two losses is not greater than the sum of risks of each individual loss. In application, smaller insurance companies can do a merger without the worry of facing a greater risk. In stock investing, portfolio diversification (investing in more than one particular stock) will decrease the risk an investor faces.

b) **Monotonicity:** If $X \leq Y$ for all possible outcomes, then $\rho(X) \leq \rho(Y)$.

If a loss random variable always has greater losses than the other, then it also always faces a greater risk. If two investors invest in the same stock, then the investor who invests more faces a greater risk than the other.

c) **Positive homogeneity:** For any positive constant $c$, $\rho(cX) = c \rho(X)$.

If losses are multiplied by a constant $c$, than the risk a company faces will also be multiplied by $c$. For example, if an investor doubles his ownership in a particular stock, then the risk he faces is also doubled.

d) **Translation invariance:** For any positive constant $c$, $\rho(X + c) = \rho(X) - c$

Adding a risk-free investment of $c$ to a portfolio decreases its risk by the same amount. If a company owner adds capital to his company, then the company faces a smaller risk because it has more assets to cover the risks it faces.

Some well-known risk measures are the Value-at-Risk (VaR) and the Expected Shortfall (ES).

2.2.1. **Value-at-Risk (VaR).** The value-at-risk for a loss random variable $X$ at the $p$ prudence level, where $p \in (0, 1)$, is denoted as $\text{VaR}_p(X)$ or $\pi_p$, and is the $100p$-th percentile of $X$. VaR is formally defined as [2]:

$$\text{VaR}_p(X) = \inf\{x \in \mathbb{R} : F_X(x) \geq p\}.$$ 

For a continuous distribution, VaR can also be defined as the value of $\pi_p$, that satisfies [1]:

$$\Pr(X > \pi_p) = 1 - p.$$ 

The VaR satisfies monotonicity, positive homogeneity, and translation invariance, but does not satisfy sub-additivity and therefore is not a coherent risk measure [4].

2.2.2. **Expected Shortfall (ES).** The expected shortfall, more commonly known as the Tail Conditional Expectation (TCE) or Tail Value-at-Risk (TVaR), is a risk measure that quantifies the average severity of losses beyond the VaR, also known as the distribution tail. For a loss random variable $X$, the ES is at the $100p$ % level is denoted as $\text{ES}_p(X)$, and is defined as the expectation of loss given that the loss surpasses the $100p$-th percentile of $X$ (or $\text{VaR}_p(X)$). Formally, [1]:

$$\text{ES}_p(X) = E(X|X > \pi_p)$$

$$= \frac{1}{1-p} \int_{\pi_p}^{\infty} x f(x) dx.$$ 

Besides equation 5, ES can also be defined as the average of VaRs beyond $p$ (figure 1), or [2]:

$$\text{ES}_p(X) = \frac{1}{1-p} \int_{p}^{1} \text{VaR}_u(X) du.$$ 

ES is a coherent risk measure since it satisfies the four criteria of coherency [5].
2.3. Gini functional

Corrado Gini, an Italian mathematician, noticed the drawbacks of using the variance as a variability measure, regarding the use of quadratic function that distorts the actual distances. Gini also thought that the variability of a distribution should not depend on the central tendency of the distribution, which is used in the definition of variance. Therefore, Gini proposed a new variability measure called the Gini Mean Difference or the Gini functional, which uses the absolute value concept instead of the quadratic function to measure distances. For a random variable \( X \), the Gini functional is defined as [2]:

\[
Gini(X) = E[|X^* - X^{**}|],
\]

where \( X^* \) and \( X^{**} \) are two independent and identically distributed (i.i.d.) random variables with the same pdf as \( X, f_X(x) \).

Equation 7 can also be represented as:

\[
Gini(X) = \int_0^1 \int_0^1 |F_X^{-1}(u) - F_X^{-1}(v)| dudv
\]

or in single integral form as:

\[
Gini(X) = 2 \int_0^1 F_X^{-1}(u)(2u - 1)du.
\]

Another superiority of the Gini functional compared to the variance is that it is a coherent measure of variability, since it satisfies standardization, location invariance, positive homogeneity, and sub-additivity.

2.4. Tail-Gini functional

Gini functional is a general measure of variability for a random variable. This paper is focused on the variability of the distribution tail, since it is so vital in risk management to minimize the possibility of unwanted events happening as a result of big losses. Therefore, a tail variability measure is needed. The Tail-Gini functional is a tail variability measure based on the Gini functional. The Tail-Gini functional of a random variable \( X \) at the \( 100p \) % level, \( p \in [0, 1) \), is denoted as \( TGini_p(X) \), and defined as the conditional expectation of \( X^* - X^{**} \) given that both \( X^* \) and \( X^{**} \) are greater than the \( 100p \)-th percentile [2]:

\[
TGini_p(X) = E[|X^* - X^{**}| | X^* > \pi_p, X^{**} > \pi_p].
\]
when $p = 0$, then $\text{TGini}_0(X)$ is the same measure as $\text{Gini}(X)$. The Tail-Gini functional can also be represented as:

$$\text{TGini}_p(X) = \frac{1}{(1-p)^2} \int_p^1 \int_p^1 |F_X^{-1}(u) - F_X^{-1}(v)| \, du \, dv$$

(11)

or in single integral form as:

$$\text{TGini}_p(X) = \frac{2}{(1-p)^2} \int_p^1 F_X^{-1}(u)(2u - (1+p)) \, du.$$  

(12)

The Tail-Gini functional satisfies standardization, location invariance, and positive homogeneity, but fails to satisfy sub-additivity. In other words, there exists at least a pair of random variables $X$ and $Y$ such that $\text{TGini}_p(X + Y) > \text{TGini}_p(X) + \text{TGini}_p(Y)$. As a result, the Tail-Gini functional is not a coherent variability measure.

### 2.5. Gini Shortfall

Gini Shortfall (GS) is a risk measure that combines the concepts of ES as a tail central tendency measure and TGini as a tail variability measure in a linear combination. The Gini Shortfall for a loss random variable $X$ with prudence level $p \in (0,1)$ is defined as [2]:

$$\text{GS}_p^\lambda(X) = \text{ES}_p(X) + \lambda \text{TGini}_p(X),$$

(13)

where $\lambda \geq 0$ is the loading parameter. GS is said to be a more comprehensive risk measure than VaR and ES since it provides information on the average severity as well as the variability of a loss distribution tail.

In equation (13), $\lambda$ as the loading parameter represents the Tail-Gini functional contribution in the GS risk measurement. When $\lambda = 0$, then GS is the same as ES and fulfills all criteria of coherence like ES does.

The general formula of GS for a loss random variable $X$ with cdf $F_X(x)$ is defined as:

$$\text{GS}_\lambda^X(X) = \int_0^1 F_X^{-1}(u) \phi_{p,\lambda}(u) \, du$$

(14)

with

$$\phi_{p,\lambda}(u) = \frac{1}{(1-p)^2} \left( 1 - p + 4\lambda \left( u - \frac{1+p}{2} \right) \right) \mathbb{I}_{[p,1]}(u), \quad u \in [0,1]$$

(15)

where $\mathbb{I}_{[p,1]}(u)$ is the indicator function of the interval $[p,1]$:

$$\mathbb{I}_{[p,1]}(u) = \begin{cases} 1, & u \geq p \\ 0, & u < p \end{cases}$$

(16)

As previously stated, the Tail-Gini functional is not a coherent variability measure. Therefore, a large $\lambda$ means the domination of the Tail-Gini functional term in GS, and thus the monotonicity and subadditivity of GS will not hold. This implies that there may be a threshold for $\lambda$ such that coherence of GS holds [2]. As a risk measure, GS will always satisfy positive homogeneity and translation invariance for any $\lambda$. The threshold only applies for monotonicity and sub-additivity, where both criteria are only fulfilled when $\lambda \in \left[0, \frac{1}{2}\right]$.

### 2.6. GS explicit formulas for some distributions

The previous subsection has discussed about the definition of GS as a risk measure and the properties it satisfies.
2.6.1. **GS explicit formula for Exponential distribution.** The explicit formula of GS for a loss random variable that has an exponential distribution will be derived with two methods. The first method will use the linear combination in (13), while the second method will use the direct formula for GS in (14). The resulting formula from the two methods will then be compared. Let $X$ has the exponential distribution with the cdf \[ F_X(x) = 1 - e^{-x/\theta}. \] (17)

The ES with prudence level $p$ for $X$ is:

\[ \text{ES}_p(X) = \theta [1 - \ln(1 - p)]. \] (18)

The Tail-Gini functional needed for the first method will be derived. To do that, the inverse of the cdf in equation 17 is needed, which is:

\[ F_X^{-1}(x) = -\theta \ln(1 - x). \] (19)

Using (12), $\text{TGini}_p$ for $X$ can be derived as:

\[ \text{TGini}_p(X) = \frac{2}{(1 - p)^2} \lim_{a \to 1} \int_p^a \frac{-\theta \ln(1 - u) \left(2u - (1 + p)\right)}{u} du. \]

Skipping the integration steps:

\[ \text{TGini}_p(X) = \theta. \] (20)

The linear combination for GS can then be constructed:

\[ \text{GS}^\lambda_p(X) = \text{ES}_p(X) + \lambda \text{TGini}_p(X) \]

\[ = \theta [1 - \ln(1 - p)] + \lambda \theta \] (21)

\[ = \theta [\lambda - \ln(1 - p) + 1]. \]

For the second method, GS for $X$ will be derived using equation 14:

\[ \text{GS}^\lambda_p(X) = \int_0^1 F_X^{-1}(u) \phi_{p,\lambda}(u) du \]

\[ = \int_0^1 F_X^{-1}(u) \left( \frac{1}{1 - p} \right)^2 \left( 1 - p + 4\lambda \left( u - \frac{1 + p}{2} \right) \right) du \]

\[ = \lim_{a \to 1} \int_p^a \frac{-\theta \ln(1 - u) \left(2u - (1 + p)\right)}{u} \left( 1 - p + 4\lambda \left( u - \frac{1 + p}{2} \right) \right) du. \]

Skipping the integration steps:

\[ \text{GS}^\lambda_p(X) = \theta [\lambda - \ln(1 - p) + 1]. \]

It can be seen that both methods produced the same explicit GS formula for a loss random variable distributed exponentially.

2.6.2. **GS explicit formula for Pareto distribution.** In this subsection, the explicit GS formula for a loss random variable with Pareto distribution will be derived. Just like the previous subsection, the explicit formula will be derived using two methods: the linear combination in equation 13 and the direct GS formula in equation 14. If a loss random variable $X$ has a Pareto distribution with the parameters $\alpha$ and $\theta$, then its cdf is \[ F_X(x) = \frac{\theta}{\theta + x} \]

\[ x \geq \theta. \] (22)
Its explicit ES form is:

$$F_X(x) = 1 - \left(\frac{\theta}{x}\right)^\alpha, \quad x > \theta$$  \hspace{1cm} (22)

The inverse of the cdf in equation 22 is:

$$F_X^{-1}(x) = \frac{\theta}{(1-x)^{1/\alpha}}.$$  \hspace{1cm} (23)

After deriving the inverse of the cdf for $X$, the explicit form of Tail-Gini functional can be determined.

$$\text{TGini}_p(X) = \frac{2}{(1-p)^2} \lim_{a \to 1} \int_p^a \frac{\theta}{(1-u)^{1/\alpha}} \left(2u - (1+p)\right) du.$$  \hspace{1cm} (24)

Skipping the integration steps:

$$\text{TGini}_p(X) = \frac{2\theta}{(2\alpha + 1/\alpha - 3)(1-p)^{1/\alpha}}.$$  \hspace{1cm} (25)

with the explicit forms of ES and TGini, the linear combination for $\text{GS}_p(X)$ can be constructed.

$$\text{GS}_p(X) = \text{ES}_p(X) + \lambda \text{TGini}_p(X)$$

$$= \frac{\alpha\theta(1-p)^{-\frac{1}{\alpha}}}{\alpha - 1} + \lambda \frac{2\theta}{(2\alpha + 1/\alpha - 3)(1-p)^{1/\alpha}}$$  \hspace{1cm} (26)

GS will also be derived from equation 14 for the second method.

$$\text{GS}_p(X) = \int_0^1 F_X^{-1}(u) \phi_{p,\lambda}(u) du$$

$$= \int_p^1 F_X^{-1}(u) \frac{1}{(1-p)^2} \left(1 - p + 4\lambda \left(\frac{u + 1 + p}{2}\right)\right) du$$

$$= \lim_{a \to 1} \int_p^a \frac{\theta}{(1-u)^{1/\alpha}} \frac{1}{(1-p)^2} \left(1 - p + 4\lambda \left(\frac{u + 1 + p}{2}\right)\right) du.$$  \hspace{1cm} (27)

Skipping the integration steps:

$$\text{GS}_p(X) = \frac{\theta(2\alpha + 2\lambda - 1)}{(2\alpha + 1/\alpha - 3)(1-p)^{1/\alpha}}.$$  \hspace{1cm} (28)

Both methods also produced the same explicit GS formula for a loss random variable with Pareto distribution.

2.6.3. GS Explicit Formula for Logistic Distribution. The two previous distributions are distributions with positive support and are used to model losses that only takes positive values, such as insurance claims. In this subsection, explicit GS formula for the logistic distribution will be derived, where the
logistic distribution has an infinite support (defined on all real numbers). This property of logistic distribution is useful when trying to model losses that may take negative values, such as stock prices, since there may be an increase (negative loss) or decrease (positive loss) in prices. Let \(X\) have the logistic distribution with parameters \(\mu\) and \(s\). The cdf of \(X\) is [6]:

\[
F_X^{-1}(x) = \frac{1}{1 + e^{-(x-\mu)/s}}
\]  
(27)

while the ES of \(X\) is:

\[
ES_p(X) = \mu - s \frac{p \ln p + (1 - p) \ln(1 - p)}{1 - p}.
\]  
(28)

The inverse of the cdf of \(X\) in equation 27 is:

\[
F_X^{-1}(x) = \mu + s \ln \left( \frac{x}{1-x} \right),
\]  
(29)

which can be used to determine the Tail-Gini functional of \(X\):

\[
T\text{Gini}_p(X) = \frac{2}{(1-p)^2} \lim_{a \to 1} \int_p^a \left( \mu + s \ln \left( \frac{u}{1-u} \right) \right) (2u - (1 + p)) du.
\]

Skipping the integration steps:

\[
T\text{Gini}_p(X) = \frac{2s}{1-p} \left( \frac{p \ln p}{1-p} + 1 \right).
\]  
(30)

Using the explicit form for ES in equation 28 and TGini in equation 30, the linear combination for GS can be constructed:

\[
GS^\lambda_p(X) = ES_p(X) + \lambda T\text{Gini}_p(X)
\]

\[
= \mu - s \frac{p \ln p + (1 - p) \ln(1 - p)}{1 - p} + \lambda \frac{2s}{1-p} \left( \frac{p \ln p}{1-p} + 1 \right)
\]

\[
= \mu + \frac{s}{1-p} \left[ 2\lambda \left( \frac{p \ln p}{1-p} + 1 \right) - p \ln p - (1 - p) \ln(1 - p) \right].
\]  
(31)

For the second method, the direct formula for GS in equation 14 will be used:

\[
GS^\lambda_p(X) = \int_0^1 F_X^{-1}(u) \phi_p \lambda(u) du
\]

\[
= \int_0^1 F_X^{-1}(u) \frac{1}{(1-p)^2} \left( 1 - p + 4\lambda \left( u - \frac{1 + p}{2} \right) \right) du
\]

\[
= \lim_{a \to 1} \int_p^a \left( \mu + s \ln \left( \frac{u}{1-u} \right) \right) \frac{1}{(1-p)^2} \left( 1 - p + 4\lambda \left( u - \frac{1 + p}{2} \right) \right) du.
\]

Skipping the integration steps:

\[
GS^\lambda_p(X) = \mu + \frac{s}{1-p} \left[ 2\lambda \left( \frac{p \ln p}{1-p} + 1 \right) - p \ln p - (1 - p) \ln(1 - p) \right].
\]

Again, the same results are obtained from the two methods of deriving the explicit GS formula for a loss random variable with logistic distribution.
3. Results and discussion

3.1. Data Description
The data used for this simulation is the monthly closing price of PT Bank Central Asia Tbk stock (code BBCA in Indonesia Stock Exchange) obtained from the website Investing.com. The observed data period is April 2004 to November 2019, and consists of 187 observations. The monthly loss is then obtained by subtracting the previous month closing price from the observed month closing price (figure 2).

Based on table 1, it can be seen that the mean and median is not far apart, which indicates that the distribution of this loss data may have a skewness close to 0 [1]. Also, note that the maximum and minimum values of the data are quite far from the mean and median, which might indicate that the distribution may be a heavy-tailed one. These properties are fulfilled by the logistic distribution, so a test needs to be done in order to check if the data actually follows logistic distribution.

First, assume the data follows logistic distribution. The maximum likelihood method can determine the best-fitted parameter for this data if it follows logistic distribution. These parameters are then used to do a Kolmogorov goodness-of-fit test to check if the above assumption is true and the data follows logistic distribution. Tests are run on R version 3.5.2, and it is concluded that the data follows logistic distribution with parameters $\mu = -137.2076$ and $s = 308.3856$.

3.2. Risk measurement
After determining the distribution that fits the data, the explicit formulas for logistic distribution obtained in the previous section can be used to calculate various variability and risk measures. Measurement will use three different prudence levels and three different loading parameters. Table 2 shows a summary of the results.

![Figure 2. BBCA monthly loss graph](image)

| Table 1. Descriptive statistics for BBCA monthly loss |
|---------------------------------|------------------|
| Descriptive statistics         | Value            |
| Minimum                        | -2,400.00        |
| Maximum                        | 1,350.00         |
| Mean                           | -162.70          |
| Median                         | -100.00          |
| Variance                       | 345,764.35       |
| Standard Deviation             | 588.02           |


Table 2. Risk measurement summary

| Measure | Prudence level |
|---------|----------------|
|         | 90 % | 95 % | 99 % |
| VaR     | 540.38 | 770.81 | 1,279.86 |
| ES      | 865.30 | 1,087.18 | 1,589.80 |
| TGini   | 319.21 | 313.66 | 309.42 |
| GS – λ = 0.1 | 897.22 | 1,118.54 | 1,620.74 |
| GS – λ = 0.25 | 945.10 | 1,165.59 | 1,667.15 |
| GS – λ = 0.5 | 1,024.91 | 1,244.01 | 1,744.51 |

Figure 3. VaR, ES, and TGini on various prudence levels

The results obtained in table 2 will be visualized in graphs as follows (figure 3). As seen on figure 3, it is clear that a higher prudence level used will result in higher VaR and ES values. This is obvious since the VaR represents the lower bound for the biggest 100(1 – p) % of losses while the ES represents the average of those losses surpassing the VaR. The lower bound for 1 % of the biggest losses is of course greater that the lower bound for 10 % of the biggest losses, and the same thing applies for the average of those big losses.

A different trend can be seen on the Tail-Gini functional, where a higher prudence level results in a smaller value of TGini. This is because the Tail-Gini functional represents the variability in the biggest 100(1 – p) % of losses. The variability for 1 % of the biggest losses is obviously greater than the variability of 10 % of the biggest losses.

As an example, the numbers for the 90 % prudence level will be used. The value of VaR₀.₉₀(ₓ) is Rp 540.38, which means for every 10 observed months, there may be 1 month in which the price of BBCA stocks decline by more than Rp 540.38. For the other 9 months, it is predicted that price will not go down further than this number. ES₀.₉₀(ₓ) is the average of losses in months where losses exceed the VaR of Rp 540.38. In this case, the average loss for those months is Rp 865.30.
Meanwhile, the value of $\text{TGin}_q(X)$ represents the average deviation of each loss in the distribution tail from the tail mean or the ES. On average, losses in the 90% distribution tail deviate as much as Rp 319.21 from the tail mean which is Rp 865.30.

For GS, it can be seen from figure 4 that risk measures are higher when using a larger loading parameter. The use of a small loading parameter means that the company or investor believes that the price fluctuation is not too significant, while the use of a large loading parameter means the opposite. On figure 4, the distances between the orange and green lines represent the amount of added risk when variability is taken into account when measuring risk. As an example, for prudence level $p = 99\%$ and loading parameter $\lambda = 0.25$, there will be an additional Rp 945.10 − Rp 865.30 = Rp 79.8 risk when variability is taken into account in GS compared to when it is not, in ES. It is important to remember that the use of a loading parameter greater than 0.5 will result in lack of coherency.

4. Conclusion

The Gini Shortfall is a more comprehensive risk measure compared to the VaR and ES because it takes into account the variability of losses in the distribution tail. This is because GS is constructed through a linear combination of the Expected Shortfall as a tail central tendency measure and the Tail-Gini functional as a tail variability measure. Moreover, based on the simulation done in this paper, the resulting values for GS are always greater than ES, which may help in providing more security to a company in calculating reserves. GS is also a coherent risk measure under certain conditions, which might prove to be a useful property for companies and investors in making business and investing decisions. Explicit GS formulas have been derived in this paper and can be used to calculate risks that has either the exponential, Pareto, or logistic distribution.

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