The Search for the Sidereal and Solar Diurnal Modulations in the Total MACRO Muon Data Set

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We have analyzed 44.3M single muons collected by MACRO from 1991 through 2000 in 2,145 live days of operation. We have searched for the solar diurnal, apparent sidereal, and pseudo-sidereal modulation of the underground muon rate by computing hourly deviations of the muon rate from 6 month averages. We find evidence for statistically significant modulations with the solar diurnal and the sidereal periods. The amplitudes of these modulations are $<0.1\%$, and are at the limit of the detector statistics. The pseudo-sidereal modulation is not statistically significant.

The solar diurnal modulation is due to the daily atmospheric temperature variations at 20 km, the altitude of primary cosmic ray interactions with the atmosphere; MACRO is the deepest experiment to report this result. The sidereal modulation is in addition to the expected Compton-Getting modulation due to solar system motion relative to the Local Standard of Rest; it represents motion of the solar system with respect to the galactic cosmic rays toward the Perseus spiral arm.

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I. INTRODUCTION

MACRO is a deep underground experiment designed to search for monopoles and rare particles in the cosmic rays. Because of its great depth and sensitivity to high energy primaries, the apparatus also probes the physics of cosmic rays with energies in excess of 10 TeV. Here we describe a search for the first harmonics of the sidereal and solar diurnal modulations in the underground muon rate using the MACRO detector. Sidereal modulations result from the motion of the solar system with respect to a locally isotropic population of cosmic rays. For instance, a local source of cosmic rays could introduce a sidereal wave into the underground muon rate. Or, if the local density of halo cosmic rays were sufficiently high, a sidereal wave would be imprinted on the underground cosmic ray rate as the solar system moved through them. Solar diurnal modulations at energies greater than 10 TeV on the other hand are mostly the result of meteorological effects at or near the primary site

II. THE FIRST HARMONIC SIDEREAL AND SOLAR DIURNAL MODULATIONS

There are two primary processes that can lead to periodic modulations in the underground muon rate. One process, the Compton-Getting effect, is due to an observer’s motion with respect to a locally isotropic population of cosmic rays. The modulation is introduced as the Earth’s rotation periodically turns an observer toward and away from the direction of motion. Since the Earth rotates on its axis in a sidereal day, the Compton-Getting modulation is seen when muons are binned in sidereal time. The second process that leads to a periodic modulation in the muon rate deep underground is the result of the competition between pion decay and interaction in the upper atmosphere. As the atmosphere cools at night (or during the winter), the density increases and the pions produced in cosmic ray collisions with the atmosphere are fractionally more likely to interact than decay than during the day (or summer)
muon rate. The daily modulation introduced by solar heating and cooling is seen when muons are binned in solar-diurnal time; yearly or seasonal modulations are seen when events are binned with a period of a (tropical) year. Underground experiments measure the muon rate \( R \). So a sensitive search for sidereal and solar diurnal modulations in MACRO is a search for periodic variations in the underground muon rate as compared with the average rate,

\[
\Delta R/R_0 = \int \int (\Delta I/I_0) \, d\Omega \, dA, \tag{1}
\]

where \( \Delta R = R - R_0 \), \( R \) is the measured cosmic ray rate, \( R_0 \) is the average cosmic ray rate, \( \Delta I/I_0 = (I - I_0)/I_0 \) is the deviation from the average cosmic ray intensity, and the integration is over the acceptance of the detector.

### A. The Compton-Getting Effect

An observer moving with velocity \( \mathbf{v} \) relative to the rest frame of a cosmic-ray plasma will detect a deviation due to this motion from the average cosmic-ray intensity, an effect first described by Compton and Getting \(^4\). If the cosmic rays have a differential power-law energy spectrum, \( E^{-\gamma} \), then the first harmonic of this deviation is given by

\[
\frac{\Delta I(\psi)}{I_0} = (2 + \gamma)(v/c) \cos \beta \cos \delta \times \cos 2\pi[(\lambda - \alpha)/24], \tag{2}
\]

where \( \Delta I(\psi) = I(\psi) - I_0; \ I(\psi) \) is the cosmic ray intensity in a direction with space angle \( \psi \) between the direction of the detector’s maximum sensitivity, \( (\beta, \lambda) \), and the direction of motion, \( (\delta, \alpha) \), in celestial coordinates on the sky; \( I_0 \) is the average cosmic ray intensity; and \( (2 + \gamma)(v/c) \) is the first harmonic amplitude \(^6\). In this expression, \( \beta \) is the declination and \( \lambda \) is the right ascension (in hours) of maximum detector sensitivity, and \( \delta \) is the declination and \( \alpha \) is the right ascension (in hours) toward the direction of motion. Since a cosmic-ray detector is fixed on the rotating earth, it sweeps the sky with period equal to a sidereal day. Under these circumstances \( \lambda = \lambda(t) \), and the Compton-Getting signal is modulated with a period of a sidereal day.

Surface detectors are typically most sensitive to the declination crossing the local zenith and the right ascension crossing the meridian. For such a detector, \( \beta \) is equal to the detector’s latitude and \( \lambda \) is equal to the local sidereal time. The \( \lambda \) response of the detector therefore implies that a Compton-Getting signal will be modulated in sidereal time, \( \tau_s \), by the phase term

\[
2\pi[(\lambda - \alpha)/24] = 2\pi(\tau_s - \alpha/24), \tag{3}
\]

where the sidereal phase \( \tau_s = \tau_s/24 \).

As a deep underground detector with angular-dependent acceptance and overburden, MACRO’s directional sensitivity \( (\beta, \lambda) \) to sidereal signals differs from the ideal surface detector described above in two ways. First, the detector response may be peaked away from the zenith. For a detector whose response peaks away from the zenith, the declination of the detector’s maximum sensitivity in eq. \((2)\) is not simply its latitude and \( \beta \) needs to be replaced by an ‘effective declination’, \( \beta_{eff} \). It is evident from eq. \((2)\) that this replacement affects only the amplitude of the signal and not the sidereal hour of its maximum. For MACRO \( \beta_{eff} \) can in principle be determined by a peak in the declination distribution for the muons used in the analysis. The lower panel of Fig. 1 shows this declination distribution. The situation is clearly complicated by the broad distribution with multiple peaks at \( 15^\circ, 42^\circ, \) and \( 64^\circ \). (MACRO’s latitude = \( 42.4528^\circ \).) In the analysis presented here, we use the mean of the distribution as the effective declination, \( \beta_{eff} = 37^\circ \). It should be noted, however, that a value of \( \beta_{eff} \) between \( 15^\circ - 42^\circ \) only alters the signal amplitude by less than 30%.

Possible Compton-Getting signals with a sidereal period might result from solar motion with respect to galactic sources of cosmic rays \(^7\) or from motion with respect to halo cosmic rays residing in the galactic disk. The effect of solar system motions on the underground muon rate gives the modulated cosmic ray intensity

\[
\frac{[\Delta I(\tau_s)]/I_0}{CG} = (2 + \gamma)(v/c) \cos \beta_{eff} \cos \delta \times \cos 2\pi(\tau_s - \alpha/24). \tag{4}
\]

Thus the Compton-Getting signal expected in the underground muon rate would have the form

\[
[\Delta R(\tau_s)/R]_{\text{CG}} = K \times e^{2\pi(\tau_s - \alpha/24)}, \tag{5}
\]
One component of the Compton-Getting signal seen by MACRO in sidereal time is due to the solar motion relative to the Local Standard of Rest (LSR) \([\text{LSR}]\), the rest frame of the local cosmic rays. Adopting \(v = 16.5 \, \text{km/s} \) toward \(l = 53^\circ\), \(b = 25^\circ\) for this motion \([3]\), we compute \(K_{L}^{\text{LSR}} = 2.6 \times 10^{-4}\) for the amplitude of this wave and \(\alpha_{L}^{\text{LSR}} = 17.8^\circ\) for its right ascension.

Previous searches for this effect have given different results depending on the energy of the primaries probed. Underground muon observatories and air shower arrays \((E > 10 \, \text{TeV})\) find sidereal modulations with amplitudes in the range \(5 \times 10^{-4} - 10^{-3}\), consistent with a drift velocity of a few hundred \(\text{km/s}\) \([3]\). Shallow underground muon telescopes and neutron monitors \((E < 500 \, \text{GeV})\) observe statistically significant sidereal modulations that are explained by solar wind effects \([3]\). Underground muons detected by MACRO have energies at the surface in excess of \(E_{\mu} \geq 1.25 \, \text{TeV}\) and are due to interactions of primaries whose energy exceeds 10 TeV. Therefore, the observed muon rates are unaffected by the solar wind.

If cosmic ray muons deep underground are binned in local solar diurnal time, \(t_{\odot}\), there will be a Compton-Getting signal that is a consequence of the earth’s orbital motion through the local solar system cosmic rays. Since the muons have energies at the surface \(E_{\mu} > 1.25 \, \text{TeV}\) that leave them unaffected by the solar wind, they are locally isotropic when averaged over the solar diurnal period. Ignoring the earth’s orbital eccentricity, the Compton-Getting modulation due to this orbital motion peaks near 6 am local time when the detector is sensitive in a direction most nearly parallel to the earth’s orbital velocity, and the cosmic ray intensity is modulated in solar diurnal time by the phase term

\[
2\pi(\lambda - \alpha_{\odot}) = 2\pi(\tau_{\odot} - 6/24),
\]

where the solar diurnal phase \(\tau_{\odot} = t_{\odot}/24\) \([4]\). In this case,

\[
[\Delta I(\tau_{\odot})/I_{0}]_{\text{CGG}} = (2 + \gamma)(v/c) \cos \beta_{eff} \times \cos 2\pi(\tau_{\odot} - 0.25). \tag{7}
\]

The Compton-Getting signal in solar diurnal time would have the form

\[
[\Delta R(\tau_{\odot})/R_{0}]_{CGG} = K_{CGG} \cos 2\pi(\tau_{\odot} - 0.25) = 4.55 \times 10^{-4} \cos \beta_{eff} \times \cos 2\pi(\tau_{\odot} - 0.25), \tag{8}
\]

where the first harmonic amplitude has been computed from the earth’s average orbital speed.

B. Atmospheric Effects

In addition to the Compton-Getting modulation, atmospheric effects modulate the muons in solar diurnal time, \(\Delta I(\tau_{\odot})/I_{0}\text{atm}\). This atmospheric modulation is the result of density variations at the altitude of first interaction – when cold at night, the density at the primary interaction site is fractionally higher than the daily average and pions are fractionally more likely to interact than decay, thereby giving fewer muons relative to the average underground muon rate.

During the day, the reverse occurs. The modulation of the underground muon rate in solar diurnal time due to atmospheric density variations is given by

\[
[\Delta R(\tau_{\odot})/R_{0}]_{\odot} = K_{\odot} \cos 2\pi(\tau_{\odot} - \xi_{\odot}), \tag{9}
\]

where \(\xi_{\odot}\) is the phase of maximum modulation. Atmospheric effects also modulate the underground muon intensity on a yearly time scale. This is the so-called ‘seasonal modulation’, \(\Delta I(\tau_{yr})/I_{0})_{yr}\), where \(\tau_{yr}\) is the yearly phase. If \(N_{\odot}\) is the number of solar days elapsed since the beginning of the year and \(N_{\odot}\) is the number of days in a tropical year, then \(\tau_{yr} = (N_{\odot} + t_{\odot}/24)/N_{\odot}\) and the seasonal modulation is proportional to \(\cos 2\pi(\tau_{yr} - \xi_{yr})\). The seasonal modulation has the form

\[
[\Delta R(\tau_{\odot})/R_{0}]_{yr} = K_{yr} \cos 2\pi(\tau_{yr} - \xi_{yr}). \tag{10}
\]

In Fig. 2 we show for MACRO the percentage deviations of the monthly average muon rate from the yearly average (solid circles), as a measure of \(\Delta R(\tau_{\odot})/R_{0}ryr\), for the years 1991-1997. The fit of eq.(10) to these data gives \(K_{yr} = 0.010 \pm 0.0004\) and \(\xi_{yr} = 0.54 \pm 0.06\). Superposed on the muon data are the deviations in the mean monthly effective temperature of the atmosphere from the yearly average, \(\Delta T_{eff} < T_{eff} >\) (open triangles), as defined in \([5]\). We have repeated the analysis from \([3]\) that computes the correlation of the mean monthly deviation in the muon rate with the mean monthly deviation in the effective temperature of the atmosphere,

\[
[\Delta R(\tau_{\odot})/R_{0}]_{yr} = \alpha_{T} \Delta T_{eff} < T_{eff} >. \tag{11}
\]

A regression analysis shows that \(\alpha_{T} = 0.91 \pm 0.07\), or equivalently 0.42 \pm 0.03 \%/K, consistent with previous findings. In this case, the atmosphere’s effective temperature is given by

\[
\frac{\Delta R(\tau_{\odot})/R_{0}}{\Delta T_{eff}} = \alpha_{T} \Delta T_{eff} < T_{eff} > = 0.91 \pm 0.07. \tag{12}
\]
C. The Pseudo-Sidereal Modulation

As first pointed out by Compton and Gettings [5], atmospheric effects can mimic a sidereal modulation if the yearly (seasonal) wave beats with a solar diurnal wave. Under these circumstances, the beats due to atmospheric effects have the form

\[
\langle 2A_\rho \rangle_{\text{beats}} = K_\odot K_\text{yr} \times \\
\cos 2\pi ((N_{\text{sol}} + 1)\tau_\text{yr} - \xi_\odot - \xi_\text{yr}) \\
+ K_\odot K_\text{yr} \times \\
\cos 2\pi ((N_{\text{sol}} - 1)\tau_\text{yr} - \xi_\odot + \xi_\text{yr})
\]

\[= K_\odot K_\text{yr} \times \\
\cos 2\pi (\tau_\text{yr} - \xi_\odot - \xi_\text{yr}) \\
+ K_\odot K_\text{yr} \times \\
\cos 2\pi (\tau_{\text{pseudo}} - \xi_\odot + \xi_\text{yr}). \quad (12)
\]

In this expression, the first modulation, with a period of \((N_{\text{sol}} + 1)\) solar days, represents a pseudo-sidereal modulation resulting from atmospheric effects. This modulation was named the ‘antisidereal’ modulation by Farley and Storey [10], and is often called the ‘antisidereal’ time, \(t_{\text{pseudo}} = N_{\text{sol}} \times t_\odot/(N_{\text{sol}} - 1)\), and is purely the result of the beating of the yearly and daily solar modulations.

To correct for the pseudo-sidereal effect, the method described by Farley and Storey [10] is often used. In this method, the amplitude of the pseudo-sidereal wave, which is equal to the amplitude of the pseudo-sidereal wave, is determined by binning the data in pseudo-sidereal time. The phase of the pseudo-sidereal wave is determined by reflecting the pseudo-sidereal wave vector about the solar wave vector.

III. DATA ANALYSIS

This analysis presented here requires only the arrival time of each muon. In principle, the analysis is simple: the arrival time of each muon is binned separately in histograms of sidereal, solar diurnal, and pseudo-sidereal time; an accurate account of the live-time in each histogram bin is accumulated; at the end, the number of events in each bin is divided by the live-time in that bin to give a muon rate. The results are based upon separate analyses of the muon rate in each histogram bin compared with the average rate for the entire period.

For the sidereal analysis, it must be emphasized that the results are not based on tracking information, that is, the determination of the right ascension and declination of each muon. Here is the aim of the sidereal analysis. If the muon rate were completely uniform over the celestial sphere, the detector would measure a uniform rate underground as a function of sidereal hour. If there were an excess of muons from some particular celestial direction, MACRO would measure an increase in the muon rate as the detector’s maximum response swept over that direction. In the sidereal analysis, we are searching for this excess.

A. Run/Event selection

In this analysis we included data runs starting from the beginning of MACRO data taking with 6 supermodules in November 1991 through May 2000. (A complete description of the detector during its running is given in [3].) A typical data run had a duration of 6-12 hours, and over 9,500 data runs were included in this analysis. The analysis proceeded by first dividing these individual data runs into 17 run sets of approximately 6 months duration during which the detector acceptance remained constant. We then filtered the data to include only muons with single tracks in both views. In this analysis, we accept events that fire at least 3 planes in the lower detector. For events crossing only 3 planes, the trigger efficiency is approximately 72%; for events crossing more than 3 planes, the trigger efficiency is > 99%. Once filtered, we compiled for each run set a histogram of the single muon rate per hour for all runs in that run set and then fitted a Gaussian to the resulting distribution.

We implemented run cuts as follows. A run was excluded from further analysis if:

- not all 6 supermodules were active, or
- the wire efficiencies were < 90% and/or the strip efficiencies were < 80% during the run, or
- the single muon rate for the run was > 5σ from the mean single muon rate for that run set.

For the efficiency cuts, wire and strip efficiencies were determined for each run by computing the average number of wires and strips recording hits for all single muons crossing 10 planes.

There are 44.3M muons in the total data set of 341,000 runs. The total live-time for the entire period from the beginning of MACRO data taking was 88,230 days. The total number of hours of live-time was 13.7 M hr.
cluded runs is 2,145 days. The average single muon rate in the total data set is 860.53 muons/hour/6 supermodules.

B. Histograms of deviations from the mean solar diurnal, apparent sidereal, and pseudo-sidereal muon rates

We searched for the solar diurnal, apparent sidereal, and pseudo-sidereal modulations as follows. First, event histograms for each run were constructed for the three periods by binning the arrival time of each muon according to its: (1) local solar diurnal time at the Gran Sasso; (2) local sidereal time; and (3) local pseudo-sidereal time. The periods of these modulations are: solar diurnal day = 86,400 seconds; sidereal day = 86,164.09892 seconds [11]; the pseudo-sidereal day was assumed to be longer than a solar day by the same fraction that a sidereal day is shorter than a solar day, or 86,636.54693 seconds. This pseudo-sidereal wave has zero phase at the autumnal equinox, when the sidereal time, the solar time, and the pseudo-sidereal time are coincident [11]. In this analysis, the pseudo-sidereal time was computed relative to the 1988 autumnal equinox, September 22, 1988, 19h 29m UT. Second, the live time in each run was similarly binned into three histograms. The live time for a run was computed as the difference between the arrival times of the first and last muons. The live time was added to the histograms from the time of the first muon to the time of the last muon. The rate histograms for each run were then computed by dividing the contents of its corresponding live time histogram by the rate binned in solar diurnal time, sidereal time, and pseudo-sidereal time. The contents of each bin in these summary histograms were then normalized by the number of entries in that bin. The resulting histograms of the normalized fluctuations, \( \Delta R/R_0 \), for the solar diurnal, sidereal, and pseudo-sidereal periods are shown in Figures 3, 5, and 6, respectively.

IV. PHASE ANALYSIS

We searched for the first harmonics of the solar diurnal, apparent sidereal, and pseudo-sidereal modulations by fitting the histograms of the fluctuations to the form:

\[
\Delta R/R_0 = K \cos 2\pi[(t - t_{\text{max}})/24],
\]

where \( \Delta R/R_0 \) is the deviation from the average muon rate; \( K \) is the amplitude of the modulation; \( t \) is the solar diurnal, sidereal time, or pseudo-sidereal time; and \( t_{\text{max}} \) is the time of maximum. The results of a \( \chi^2 \)-fit to the solar diurnal and sidereal histograms are given in Table 1. The fitted curves are superposed on the histograms in Figures 3 and 5. In Table 1, a correction of -0.3 to \( t_{\text{max}} \) has been applied to the sidereal wave fit to account for MACRO’s acceptance and overburden, as described in §II.A. For the pseudo-sidereal histogram, the expected pseudo-sidereal modulation from eq. (11) is given in Table 1, with errors computed by adding the errors in the yearly and solar diurnal modulations in quadrature. This expected wave has been fitted to the pseudo-sidereal histogram and the results shown in Figure 6.

In addition, Table 1 gives the results of fitting the null hypothesis to the three data sets – the hypothesis that no modulation is present in the deviations from the average muon rate.

A. Solar diurnal modulation

Table 1 shows that the solar diurnal modulation is a statistically significant effect. The origin of this modulation is most likely the daily atmospheric temperature variations at approximately 20 km, the altitude of the primary cosmic ray interactions with the atmosphere.

We have used the atmospheric temperature data from the European Centre for Medium-Range Weather Forecasts (ECMWF) at (0h, 6h, and 12h) to calculate the temperature dependence of the primary cosmic ray flux. The temperature dependence of the cosmic ray flux is given by the formula:

\[
\frac{dN}{dE} = N_0 \left( \frac{t}{t_0} \right)^{-\alpha} e^{-\frac{E}{\Delta E}}
\]

where \( N_0 \) is the cosmic ray flux at time \( t_0 \) and energy \( E_0 \), and \( \alpha \) and \( \Delta E \) are constants. We have used the temperature data from ECMWF to calculate the expected solar diurnal modulation from eq. (12) and compared it to the observed modulation. The resulting solar diurnal wave fit is shown in Figure 3, and Table 1 gives the fitted values of the solar diurnal modulation for the three periods.

At the end of this process, after all runs were analyzed, there resulted a set of three histograms with fluctuations from the mean muon rate binned in solar diurnal time, sidereal time, and pseudo-sidereal time. The contents of each bin in these summary histograms were then normalized by the number of entries in that bin. The resulting histograms of the normalized fluctuations, \( \Delta R/R_0 \), for the solar diurnal, sidereal, and pseudo-sidereal periods are shown in Figures 3, 5, and 6, respectively.
B. Sidereal modulation

Table 1 shows that the sidereal modulation is also a statistically significant result. The observed amplitude of $K = 8.1 \times 10^{-4}$ is significantly larger than expected for the Compton-Getting effect due to the solar system motion with respect to the LSR, $K = 2.6 \times 10^{-4}$; the maximum observed phase of $t_{max} = 23.2^h$ is also significantly different from the expected $t_{max} = 17.8^h$. This suggests that we have found another modulation in addition to expected Compton-Getting modulation due to solar system motion.

C. Pseudo-sidereal modulation

As shown in Table 1, the fit of eq. (12) to the pseudo-sidereal histogram has the same statistical significance as the null hypothesis. This result is not unexpected. Since the significance of the solar diurnal and sidereal modulations are detections at or near the limit of the detector statistics, the pseudo-sidereal modulation is too small for a statistically significant signal.

V. RESULTS

Since the pseudo-sidereal modulation is of low statistical significance, the correction to the apparent sidereal modulation for the pseudo-sidereal modulation is not required. After correcting the sidereal modulation for the solar motion relative to the LSR, the final value for the sidereal modulation of the underground muon rate is $K_* = (8.2 \pm 2.7) \times 10^{-4}$ and a phase of maximum of $t_{max} = 0.4 \pm 1.1$ hr.

The comparison of the result found in this investigation with the results of other experiments in the primary cosmic ray energy range $E_p \geq 1$ TeV are shown in Fig. 7. In the upper panel the amplitudes of the modulations are shown. To compare the results of different experiments, the reported amplitudes have been corrected for the effective declination of the experiment, c.f. eq.(3) and to the LSR. Included in the amplitude is a systematic error of 7% to account for the uncertainty in $\beta_{eff}$. In the lower panel, the sidereal hour of the maximum amplitude of the modulation, or the right ascension of the maximum signal, is compared with results from other experiments. Again, the phase has been corrected to the LSR. In this figure, the y-axis labels for $18^h$ to $24^h$ have been replaced by $-6^h$ to $0^h$. These figures clearly show that the results found in this investigation are consistent with other experimental determinations in the energy range $10^{12} - 3 \times 10^{14}$ eV, and that these results provide a bridge between underground experiments and extensive air shower arrays.

The interpretation of the sidereal wave is not straightforward due to the complicated nature of cosmic ray propagation through the galactic magnetic field at particle energies in the range $10^{12} - 10^{15}$ eV. However, some understanding can come from mapping the observed sidereal wave into galactic coordinates. We have realized this mapping by first recognizing that MACRO is most sensitive to the declination equal to $\delta \sim \beta_{eff} \sim 37^\circ$, and then tracing out the path of this declination in sidereal time. Once this path has been determined, we transform it into galactic coordinates. This path is shown in Fig. 8. On this path we superpose the sidereal wave as follows: positive wave amplitudes are shown as crosses and negative wave amplitudes are shown as open circles. The position on this path of the maximum wave amplitude is shown as a filled star. To show the effect of the largest possible variation in $\beta_{eff}$, the position of the maximum wave amplitude for $\beta_{eff} = 15^\circ, 64^\circ$, the outlying peaks in the declination distribution in Fig. 1, are shown as filled triangles. In addition we show the direction toward which the sun is moving as a result of differential galactic rotation as a filled circle and the direction of the Perseus spiral arm as a filled square. This figure shows that the wave intensity is positive over a wide range in galactic longitude, $l = 80^\circ - 150^\circ$, when MACRO is pointing towards the galactic plane, and is negative when MACRO is pointing to high galactic latitudes. This pattern is typical of many detectors in the northern hemisphere.
However, it is difficult to ascribe the wave to a specific direction and/or “source”, as for instance, the direction of galactic rotation or the direction of the local magnetic field. This is primarily because the wave is at the very limit of MACRO statistics. Among the possibilities for the origin of the wave are: particle capture in magnetic traps, a close-by supernova, or the motion of the solar system through a spherical distribution of halo cosmic rays.

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FIGURE CAPTIONS

FIG. 1. Upper panel: Hour angle distribution for the muons used in this analysis. The mean of the distribution falls at $-7.7^\circ$. Lower panel: The declination distribution for the muons used in this analysis. The broad distribution has peaks at $\delta = 15^\circ$, $42^\circ$, and $64^\circ$. The mean of the distribution falls at $\delta = 37^\circ$.

FIG. 2. Percentage deviations of the monthly average muon rate from the yearly mean, $[\Delta R/\langle R \rangle_{yr}]_{\odot}$ (solid circles), computed from data collected by MACRO during the years 1991-1997, compared with percentage deviations of the monthly average effective temperature from the yearly mean, $\Delta T_{eff}/\langle T_{eff} \rangle$ (open triangles), during the same period from balloon data provided by Ispettorato Tele comunicazioni ed Assistenza Volo Dell’Aeronautica Italiana. The computation of $T_{eff}$ is described in [2]. A small offset between the monthly means has been introduced for clarity.

FIG. 3. Deviations of the muon rate from the mean muon rate binned according to the local solar diurnal time at the Gran Sasso. Superposed is the best-fit curve of the form eq.(13) representing the modulation.

FIG. 4. Superposed as open triangles onto the solar diurnal modulation of the mean muon rate, $\Delta R/R$, shown in Fig. 3 are the deviations from the daily mean of the effective temperature, $\Delta T_{eff}/T_{eff}$, at 0, 6, 12, and 18 hours. The temperature data are taken from the European Centre for Medium-Range Temperature Forcasts (ECMWF) on a grid centered on the Gran Sasso.

FIG. 5. Deviations of the muon rate from the mean muon rate binned according to the local sidereal time at the Gran Sasso. Superposed is the best-fit curve of the form eq.(13) representing the modulation.

FIG. 6. Deviations of the muon rate from the mean muon rate binned according to the local pseudo-sidereal time at the Gran Sasso. Superposed is the expected pseudo-sidereal modulation from eq.(12).

FIG. 7. Summary of sidereal wave searches at energies $\gtrsim 10^{12}$ eV. Upper panel: measurements of the amplitude; Lower panel: measurements of the phase maximum of the first harmonic in the sidereal variations. Filled symbols represent underground experiments and open symbols represent EAS experiments. Symbol key: large filled circle = MACRO (this paper); inverted filled triangle = Poatina [13]; filled square = Matsushiro [14]; filled triangle = Utah [5]; asterisk = Hong Kong [15]; small filled circle = Baksan [16]; filled star = Tibet [17]; open cross = Baksan EAS [18]; open triangle = Kamioka [7]; open square = Norikura [19]; open diamond = Musala Peak [20]; open circle = EAS-TOP [21]; open star = Linsley [22].

FIG. 8. The sidereal wave detected by MACRO mapped into Galactic coordinates. Positive wave amplitudes are shown as crosses; negative wave amplitudes are shown as open circles. The wave’s maximum amplitude, corrected for the motion of the sun with respect to the Local Standard of Rest [6] is shown as a filled star, assuming $\delta = 37^\circ$. The upper filled triangle shows the maximum wave amplitude assuming $\delta = 64^\circ$; the lower filled triangle shows the maximum wave amplitude for $\delta = 15^\circ$. The direction toward which the sun is moving as a result of differential Galactic rotation is shown as a filled circle and the direction of the Perseus spiral arm is shown as a filled square.
TABLE I: Results of Histogram Fits

| Period            | $K$             | $t_{max}$ (hr) | $\chi^2$/DoF | $\chi^2$/DoF (null hypothesis) |
|-------------------|-----------------|----------------|--------------|---------------------------------|
| Solar Diurnal     | $(0.88 \pm 0.26) \times 10^{-3}$ | $17.8 \pm 1.2$ | 3.4/5        | 14.6/7                          |
| Apparent Sidereal | $(0.82 \pm 0.27) \times 10^{-3}$ | $23.2 \pm 1.3$ | 4.6/5        | 13.8/7                          |
| Pseudo-sidereal   | $(0.0097 \pm 0.0029) \times 10^{-3}$ | $4.8 \pm 2.5$  | 6.9/7        | 6.9/7                           |

† corrected for MACRO response in sidereal hour
