ORIGINAL RESEARCH PAPER

An assessment of a unmanned aerial vehicle-based broadcast scenario assuming random terrestrial user locations

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Abstract
With the advent of the new decade, unmanned aerial vehicles (UAVs) will increasingly play an active role mainly as broadband transmission relays for broadcast applications. In the present study, we consider a feasible network where a transmitter, mounted on a hill, is connected through an optical wireless link with a UAV, hovering at a certain height above the earth. The UAV provides coverage over a circular geographic area and communicates through a radio frequency (RF) link with a ground user who is randomly located inside the circular region. In this context, a mixed optical/RF link is established where the performance is thoroughly investigated in terms of the outage probability. We assume that the optical wireless link is affected by a plethora of deteriorating factors, including the presence of fog, or the atmospheric scintillation, while the RF link is subject to small-scale fading. Depending on the appearance of fog or turbulence, two different problems occur with a different mathematical formulation. For both cases, we extract closed-form expressions for the signal-to-noise ratio statistics, which are the critical component towards determining the outage probability. Then, the impact of the impairments for some typical parameter values is properly depicted in appropriate graphs.

1 | INTRODUCTION

1.1 | Background

Nowadays, a concerted amount of research by academia and industry is devoted to the possible deployment of unmanned aerial vehicles (UAVs) regarding their integration to wireless broadband networks as flying aerial base stations for broadcasting. The practical aspects and promising opportunities of such a project, its standardisation advancements, as well the regulation and security challenges are discussed in detail in ref. [1]. UAVs can be connected to a network forming a platform to provide a plethora of commercial and civil applications, including surveillance [2]. Compared to the existing infrastructures, the UAVs keep low acquisition and maintenance costs and manage to hover at a specific altitude, thus avoiding any possible obstacles. This possibility inevitably allows the establishment of line-of-sight (LOS) links to terrestrial users strengthening the signal propagation. Different types of UAVs have been proposed, including high altitude platforms (HAPs) and low altitude platforms (LAPs), as illustrated in ref. [3], with drones being the most versatile and easy to operate in practice.

As UAVs are expected to handle broadband traffic with significantly enhanced requirements for high data-rate services, laser-based technology can be employed in part as an advantageous alternative means to radio frequency (RF) and microwave transmission. Optical wireless transmission has been thoroughly investigated regarding its utilisation on terrestrial networks, where a considerable number of technical studies made its appearance in relevant academic journals and conferences [4]. On the other hand, some vicarious studies on the potential use of optical wireless links for UAV applications include the following. In ref. [5], the authors improved the performance of terrestrial relay-assisted free space optics (FSO) systems by integrating UAVs as buffer-aided moving relays. In ref. [6], a novel framework where UAVs transport the backhaul/fronthaul traffic between the access and core networks via optical wireless links, was introduced. In ref. [7], the authors discussed an architecture of an edge-computing-empowered radio access network with optical wireless fronthaul and backhaul links mounted on UAVs. In ref. [8], a
flexible configuration of delivering WiMAX traffic with a serial multi-hop network via inter-HAP optical links was proposed. Moreover, the interaction of two hovering UAVs equipped with a flat optical transmitter (Tx) array and an optical receiver (Rx) array, which is spherically curved, was investigated in ref. [9]. Positioning fluctuations have been recently examined in ref. [10], and novel statistical models for the geometric and misalignment losses have been presented. The trade-off between the UAV position and the optical beamwidth on the outage probability was considered as an optimisation problem in ref. [11]. In ref. [12] a novel probability density function (pdf) of the optical channel between two UAVs was proposed facilitating, thus, the performance analysis, whereas the performance of the ground-to-UAV, UAV-to-UAV, and UAV-to-ground networks was extensively analysed in refs. [13, 14].

An appealing research perspective is to consider mixed dual-hop relay links, where optical wireless transmission is employed in the one link and RF in the other, and investigate their performance [15, 16]. In this context, the end-to-end performance of a network, where a UAV connects mobile users via RF access links to a ground station via an optical wireless backhaul link, was recently analysed in terms of the ergodic sum rate in ref. [17]. Moreover, in ref. [18], the authors investigated the outage probability of a multi-hop mixed RF/FSO communication link using two UAVs as relays and estimated the optimal height. In ref. [19], the authors focused on the throughput maximisation in a mixed FSO/RF UAV relaying system with a buffer constraint, whereas in ref. [20], a multiuser scheduling scheme for FSO/RF links was proposed assuming satellite-UAV-terrestrial networks. All these works provide useful insight into the performance of UAV-based mixed optical/RF links which are expected to dominate in the aerial communications of the future.

1.2 | Contribution

In the present study, we emphasise on some other impairments, including fog and random user placement on the ground, which are not sufficiently tackled in the previous studies. To this end, we consider a UAV-based mixed optical/RF network and examine how the outage probability is affected by the consideration of these impairments. The technical contribution of this paper is summarised in the following points:

- **Random user position:** we investigate a dual-hop relay network where the end-users are randomly placed over a circular service area. The RF link is dominated by small-scale fading. In this way, we have a more realistic approximation of the actual network operation.
- **Optical channel impairments:** we investigate two versions for the optical channel state. The first includes the impact of fog, pointing errors and angle of arrival (AoA) fluctuations whereas the second comprises atmospheric attenuation, atmospheric turbulence, pointing errors and AoA fluctuations. New closed-form expressions are extracted to describe the foggy channel state.

1.3 | System description

Figure 1 demonstrates an overall description of the network arrangement under investigation. More specifically, a dual-hop relay network is considered, which is composed of a fixed Tx mounted on a hill, a UAV hovering at a specific altitude which acts as a relay node, and a terrestrial Rx located in a suburban or urban environment. The distances between Tx-UAV and UAV-Rx are $d_1$ and $d_2$, respectively. The Tx communicates optically with the UAV, hovering at an altitude $L$, under the presence of several impairments caused by the transmission channel. The UAV forwards the information to the destination through an RF link using a decode-and-forward (DF) protocol. A wide range of fading conditions imposes further performance degradation. Furthermore, the Rx location is assumed to be random over a circular area on the ground with a radius $R$. The distance of the Rx from the centre of the circular area is $r$. The two paths of the information transmission are presented analytically in the following sections.

1.4 | Structure

The rest of the paper is organised as follows. Section 2 introduces the optical channel characteristics of the first link, which is contaminated by the deleterious effects of atmospheric attenuation, pointing error effects, fog, atmospheric turbulence, and angle of arrival (AoA) fluctuations caused by the orientation deviations of the UAV. The impact of all these impairments is mathematically formulated and considered in the analysis that follows. Based on all these deteriorations, we proceed to the comprehensive investigation of two combined channel models. The first entails fog and the second atmospheric turbulence. Finally, we derive the pdf of the
signal-to-noise ratio (SNR) assuming indirect-modulation/direct-detection (IM/DD) at the UAV. Section 3 provides a mathematical description of the channel for the second link characterised by the random positioning of a mobile user and Nakagami-m fading. Nakagami-m distribution is suitable to model small-scale fading for dense signal scatterers. The pdf of the SNR for the second link is analytically deduced. Section 4 is devoted to the outage probability determination, whereas Section 5 highlights some representative numerical results in an appropriate series of figures to get a detailed insight into the overall performance. Finally, a summary of the main findings and ways for further research are discussed in Section 6.

2 | TX-TO-UAV CHANNEL MODEL

2.1 | Impairments

We assume that the optical wireless channel is susceptible to random fluctuations and losses due to the effects of the atmospheric attenuation $b_l$, the random fog attenuation $b_f$, the atmospheric turbulence $b_a$, the pointing error loss $b_p$ and the link interruption $b_{int}$ due to the AoA fluctuations. At this point, we would like to clarify that the atmospheric turbulence is unlikely to appear when fog is present, as these factors have a strong inverse correlation [21]. As a result of this ascertainment, we investigate the effect of fog and turbulence, separately. In the sequel, we proceed to a brief description of the deteriorating factors mentioned above from a mathematical point of view.

2.1.1 | Atmospheric attenuation

The optical signal attenuates by the distance, and that loss is typically determined by the Beer-Lambert’s law [22].

$$b_l = \exp(-\sigma d_1)$$

(1)

where $\sigma$ (in km$^{-1}$) is the attenuation coefficient and $d_1$ is the distance between the Tx and the UAV.

2.1.2 | Random fog attenuation

A recent study revealed that the optical signal follows a random behaviour under fog appearance with pdf given by [23].

$$f_{b_f}(b_{fa}) = \frac{z^k}{\Gamma(k)} \left[ \ln \left( \frac{1}{b_{fa}} \right) \right]^{k-1} b_{fa}^{-\beta-1}$$

(2)

where $\Gamma(.)$ is the Gamma function [24, eqn. (8.310.1)], $0 \leq b_{fa} \leq 1, z = 4.343/(\beta d_i), k > 0$ and $\beta > 0$ are the shape and scale parameters, respectively associated with different fog densities according to the values listed in Table 1. It has to be stressed that this pdf entails the atmospheric attenuation factor since it is extracted with proper handling of the Beer-Lambert’s law.

2.1.3 | Atmospheric turbulence

The atmospheric turbulence describes both amplitude and phase random perturbations of the transmitted optical waves. In the present work, we adopt the versatile model of the Gamma-Gamma distribution which describes moderate to strong turbulence conditions [25].

$$f_{b_a}(b_a) = \frac{2 (\alpha \beta)^{\alpha \beta}}{\Gamma(\alpha) \Gamma(\beta) b_a^{\alpha+\beta-1}} K_{\alpha-\beta} \left( 2 \sqrt{\alpha \beta b_a} \right)$$

(3)

where $K_n(.)$ is the modified Bessel function of the second kind of order $n$ [[24], eqn. (8.432.2)], whereas $\alpha$ and $\beta$ are the effective number of large-scale and small-scale eddies, respectively which both depend on the Rytov variance $\sigma_R^2$ [25].

2.1.4 | Pointing errors

The LOS accuracy is frequently disturbed since wind loads induce pointing error loss due to misalignment, which considers detector aperture size, beam width and jitter variance.

Considering a Gaussian beam at the Tx and the Rx lens radius $r_a$, the pointing error loss, $b_p$, can be expressed as [26].

$$f_{b_p}(b_p) = \frac{\gamma_p^2}{A_0^2} b_p^{\gamma-1}, \quad 0 \leq b_p \leq A_0$$

(4)

where $\gamma_p = \omega d_1/2 \sigma_j, \quad \omega d_1 = \sqrt{2 \exp(-u)} u = \sqrt{2 \exp(-u)} A_0 = (\text{erf}(u))^2, \omega d_1$ is the beam waist at distance $d_1$ and $\sigma_j$ is the jitter variance at the Rx.

| Table 1 | Foggy channel parameters [23] |
|----------|-------------------------------|
| Fog thickness | Symbol | Value |
| Dense fog | $k$ | 36.05 |
| | $\beta$ | 11.91 |
| Thick fog | $k$ | 6.00 |
| | $\beta$ | 23.00 |
| Moderate fog | $k$ | 5.49 |
| | $\beta$ | 12.06 |
| Light fog | $k$ | 2.32 |
| | $\beta$ | 13.12 |
2.1.5 | AoA fluctuations

The pdf of AoA fluctuations is given by [13].

\[
f_{b_{\theta}}(b_{\theta}) = \left[ 1 - \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right) \right] \delta(b_{\theta} - 1) + \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right) \delta(b_{\theta})
\] (5)

where \( \theta_{FOV} \) is the Rx field-of-view (FOV), \( \sigma_\theta^2 \) is the variance of the Tx–Rx misalignment orientations and \( \delta(\cdot) \) is the Dirac delta function [[27], eqn.(14.03.02.0001.01)].

2.2 | Composite channel under fog

At first, we investigate the random foggy channel state in the presence of pointing errors and AoA fluctuations. In this case, the deterministic atmospheric loss factor is not taken into account since it is entailed in the foggy pdf. In this case, the channel state, \( b_F \), can be mathematically expressed as

\[
b_F = b_F' b_p b_{\theta}
\] (6)

In order to facilitate the mathematical analysis, we define the partial channel state as \( b_F' = b_F b_p \). The pdf of \( b_F' \) is obtained according to [23] as

\[
f_{b_F'}(b_F') = \frac{z^k r_p^2 b_F'^{\mu-1}}{\Gamma(k) A_0^k} \left[ \ln\left( \frac{1}{b_F'} \right) \right]^{k-1} b_F'^{\mu-1} db_F'
\] (7)

where \( \mu = z - r_F^2 \). Using [[24], eqn. (2.33.10)], the integral in the above equation yields

\[
f_{b_F'}(b_F') = \frac{z^k r_p^2 b_F'^{\mu-1}}{\Gamma(k) A_0^k} \mu^k \left[ \Gamma(k) - \Gamma\left( k, \mu \ln\left( \frac{A_0}{b_F'} \right) \right) \right]
\] (8)

where \( \Gamma(\cdot, \cdot) \) is the upper incomplete Gamma function defined in [24, eqn. (8.350.2)].

The pdf of the overall channel state, \( b_F \), is calculated by [13] as

\[
f_{b_F}(b_F) = f_{b_F'}(b_F > 0) + f_{b_F'}(b_F = 0) \delta(b_F)
\] (9)

Eventually, since,

\[
f_{b_F'}(b_F = 0) = \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right)
\] (10)

and

\[
f_{b_F}(b_F > 0) = \left[ 1 - \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right) \right] f_{b_F'}(b_F' = b_F)
\] (11)

the pdf of \( b_F \) is obtained from Equations (8) to (11) as

\[
f_{b_F}(b_F) = \left[ 1 - \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right) \right] f_{b_F'}(b_F' = b_F) + \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right)
\] (12)

Using Equation (12) and assuming IM/DD at the Rx, the pdf of the instantaneous electrical SNR, \( \gamma_F = \sqrt{\gamma^2} b_F^2 \), is evaluated after a proper variable change, as

\[
f_{\gamma_F}(\gamma_F) = \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right) \times \left[ \Gamma(k) - \Gamma\left( k, \mu \ln\left( \frac{A_0}{\sqrt{\gamma^2}} \right) \right) \right]
\] (13)

where \( \gamma_F = (\rho^2 P_t^2) / \sigma_{n_1}^2 \) is the SNR at the Tx, whereas \( P_t \) is the transmitted optical power, \( \rho \) is the electrical to optical conversion efficiency, and \( \sigma_{n_1} \) is the SD of the Gaussian noise. By integrating Equation (13) and defining \( \xi = \ln\left( \frac{A_0}{\sqrt{\gamma^2} \gamma_F} \right) \), we obtain the cumulative distribution function (cdf) as

\[
F_{\gamma_F}(\gamma_F) = \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right) + \left[ 1 - \exp\left( - \frac{\theta_{FOV}^2}{2\sigma_\theta^2} \right) \right] \left( \frac{z_k}{\mu^k (A_0 \sqrt{\gamma^2}) \gamma_F^k} - \frac{z_k^2 r_p^2}{\mu^k \Gamma(k) \int_{\ln(A_0 \sqrt{\gamma^2})}^{\infty} \exp\left( - \frac{r_p^2 \xi}{\gamma_F^2} \right) \Gamma(k, \mu \xi) d\xi} \right)
\] (14)

Finally, by properly applying the following identity [28].

\[
\Gamma(s, t) \approx (1 + t)^{-1} \exp(-t)
\] (15)

in Equation (14), we arrive at the final expression.
\[ F_{T_\gamma}(T_\gamma) \approx \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right) + \left[1 - \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right)\right] \times \]
\[ \left[ \frac{\gamma^2}{\mu^2(a_0 \sqrt{\frac{\gamma_{\theta f}}{T_\gamma}})} \frac{\exp[\gamma_p^2 + \mu]}{\mu\Gamma(k)} (\gamma_p^2 + \mu)^k \times \Gamma\left(k, \frac{\gamma_p^2 + \mu}{1 + \mu \ln\left(a_0 \sqrt{\frac{T_\gamma}{\gamma_{\theta f}}}\right)}\right) \right] \]

\[ (16) \]

### 2.3 Channel modelling under turbulence

When atmospheric turbulence takes place, the channel state, \( b_T \), can be formulated as

\[ b_T = b_1 b_2 b_3 b_4 \]

To simplify the mathematical analysis, we define the partial channel state as \( b_T' = b_1 b_2 b_3 \). The pdf of \( b_T' \) can be obtained according to \([13, 26]\) as

\[ f_{b_T'}(b_T') = \frac{2\gamma_p^2(a\beta)^{-\frac{3}{2}}b_T'^{-\frac{3}{2}}}{\Gamma(\alpha)\Gamma(\beta)\left(A_0 b_1\right)^{\gamma_{\theta f}}} \times \int_{a_0}^\infty b_4^{-\alpha/2} - \gamma_p^{-1} K_{\alpha-\beta}\left(2\sqrt{a\beta \gamma_p} \right) db_4 \]

\[ (18) \]

The integral in Equation (18) can be solved if the modified Bessel function \( K_{\alpha-\beta} \) is replaced by the Meijer G function [27, eqn. (03.04.26.0006.01)]. Then, by properly applying [27, eqn. (07.34.21.0085.01)] and [24, eqn. (9.31.5)], the pdf of \( b_T' \) occurs as

\[ f_{b_T'}(b_T') = \frac{\gamma_p^2}{\Gamma(\alpha)\Gamma(\beta)} b_T'^{-1} G_{1,3}^{3,0}\left(\frac{a\beta}{A_0 b_1} b_T' \left| \frac{1 + \gamma_p^2}{\gamma_p^2, \alpha, \beta} \right. \right) \]

\[ (19) \]

In a similar way as in the previous subsection, the pdf of \( b_T \) is obtained as \([13]\).

\[ f_{b_T}(b_T) = f_{b_T}(b_T > 0) + f_{b_T}(b_T = 0) \delta(b_T) \]

\[ (20) \]

Hence as,

\[ f_{b_T}(b_T = 0) = \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right) \]

\[ (21) \]

\[ f_{b_T}(b_T > 0) = \left[1 - \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right)\right] \]

\[ (22) \]

Equation (20) is finally deduced as

\[ f_{b_T}(b_T) = \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right) + \left[1 - \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right)\right] \times \]

\[ \frac{\gamma_p^2}{\Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left(\frac{a\beta}{A_0 b_1} \left| \frac{1 + \gamma_p^2}{\gamma_p^2, \alpha, \beta} \right. \right) \]

\[ (23) \]

The pdf of the instantaneous electrical SNR, \( \gamma_T = \frac{\gamma_{\theta f}^2 b_T^2}{\gamma_{\theta f}^2} \), is evaluated after a proper variable change in Equation (23), as

\[ f_{\gamma_T}(\gamma_T) = \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right) + \left[1 - \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right)\right] \times \]

\[ \frac{\gamma_T}{\Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left(\frac{a\beta}{A_0 b_1} \left| \frac{1 + \gamma_T^2}{\gamma_T^2, \alpha, \beta} \right. \right) \]

\[ (24) \]

where \( \gamma_{\theta f} = (\rho^2 P_{1a})/\sigma_{\theta f}^2 \). Then, by integrating Equation (24) and using [27, eqn. (07.34.21.0084.01)], we obtain a closed form expression for the cdf as

\[ F_{\gamma_T}(\gamma_T) = \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right) + \left[1 - \exp\left(-\frac{\theta_{FOV}^2}{2\sigma_\theta^2}\right)\right] \times \]

\[ \frac{\gamma_T}{\Gamma(\alpha)\Gamma(\beta)} G_{2,4}^{3,0}\left(\frac{a\beta}{A_0 b_1} \left| \frac{1 + \gamma_T^2}{\gamma_T^2, \alpha, \beta, 0} \right. \right) \]

\[ (25) \]

### 3 UAV-TO-RX CHANNEL MODEL DESCRIPTION

#### 3.1 Impairments

The UAV-to-Rx channel has some unique features compared to the terrestrial ones, such as a high likelihood of LOS propagation, which decreases transmit power requirements. The UAV-to-Rx link faces many other challenges due to arbitrary mobility patterns and diverse types of communication applications [29]. Several channel models have been proposed, as summarised in the up-to-date surveys of refs. [29, 30], and potential researchers can refer to them. In what follows, we consider that a randomly
positioned ground user experiences small-scale fading effects modelled by the Nakagami-\(m\) distribution.

3.1.1 | Random position

We assume that the UAV hovers at altitude \(L\) where there exists a user who is randomly located over a circular area with radius \(R\). In such case the radical distance, \(r\), follows a uniform distribution with pdf as [31].

\[
f_r(r) = \frac{2r}{R^2}, \quad 0 \leq r \leq R \tag{26}
\]

After a simple random variable change, the pdf of the distance between the user and the UAV, \(d_2\), is readily derived as

\[
f_{d_2}(d_2) = \frac{2d_2}{R^2}, \quad L \leq d_2 \leq \Lambda \tag{27}
\]

where \(\Lambda = \sqrt{L^2 + R^2}\).

3.1.2 | Nakagami-\(m\) fading

Under the assumption that the received signal is the superposition of both diffuse and specular scattering, the received signal power \(p\) follows a Nakagami-\(m\) distribution as [[32], eqn. (3.39)]

\[
f_p(p) = \left(\frac{m}{\bar{p}}\right)^m \frac{p^{m-1}}{\Gamma(m)} \exp\left(-\frac{mp}{\bar{p}}\right) \tag{28}
\]

where \(\bar{p} = P_t/d_2^2\) is the mean signal power, \(P_t\) is the transmitted RF power, \(m \geq 1/2\) is the Nakagami-\(m\) fading parameter, and \(\nu\) is the path loss exponent. The conditional pdf of the received signal power with respect to distance \(d_2\) is given by

\[
f_{p|d_2}(p|d_2) = \left(\frac{md_2^2}{P_t}\right)^m \frac{p^{m-1}}{\Gamma(m)} \exp\left(-\frac{mpd_2^2}{P_t}\right) \tag{29}
\]

In order to obtain the unconditional distribution of the received signal power, Equation (29) is averaged as

\[
f_p(p) = \int_0^\Lambda f_{p|d_2}(p|d_2)f_{d_2}(d_2)dd_2 = \frac{2p^{m-1}}{R^2\Gamma(m)} \int_0^\Lambda \left(\frac{md_2^2}{P_t}\right)^m \exp\left(-\frac{pmd_2^2}{P_t}\right) d_2dd_2 \tag{30}
\]

Using the variable change, \(Md_2^2 = \omega\), where \(M = \frac{P_t}{\bar{p}}\), Equation (30) becomes

\[
f_p(p) = \frac{2p^{m-1}}{vR^2M^2\Gamma(m)} \int_{ML}^{MA} \omega^{m+\frac{1}{2}-1} \exp(-p\omega)d\omega \tag{31}
\]

The pdf of the instantaneous SNR \(\gamma_2 = p/P_{n_2}\) using Equation (31) is obtained as

\[
f_{\gamma_2}({\gamma_2}) = \frac{2p^{m-1}}{vR^2M^2\Gamma(m)} {\gamma_2}^{m-1} \int_{ML}^{MA} \omega^{m+\frac{1}{2}-1} \exp(-p\omega){\gamma_2}d\omega \tag{32}
\]

where \(P_{n_2}\) is the RF noise power. By integrating the above equation and using the variable change, \(P_{n_2}\omega = \psi\), we obtain the cdf as

\[
F_{\gamma_2}(\gamma_2) = \frac{2\gamma_2^{2\nu}}{vR^2m^{2\nu}\Gamma(m)} \times \int_{\gamma_2}^{\gamma_2^{\nu}} \int_{\psi}^{\infty} \psi^{m+\frac{1}{2}-1} \exp(-\psi\omega)d\psi d\gamma_2 \tag{33}
\]

where \(\gamma_2 = P_t/P_{n_2}\) is the average SNR at the RF Tx. After some simple algebraic operations, interchanging the order of integration, and using [24, eqn. (2.3.3.10)] in Equation (33), we finally arrive at

\[
F_{\gamma_2}(\gamma_2) = 1 - \frac{1}{R^2\Gamma(m)} \times \left[\Lambda^2\Gamma\left(m, m\Lambda\gamma_2/\bar{\gamma}_2\right) - L^2\Gamma\left(m, ml\gamma_2/\bar{\gamma}_2\right)\right] - \frac{1}{R^2m^{2\nu}\Gamma(m)} \left(\bar{\gamma}_2\right)^{2\nu} \times \left[\Gamma\left(m + \frac{2}{\nu}, ml\gamma_2/\bar{\gamma}_2\right) - \Gamma\left(m + \frac{2}{\nu}, m\Lambda\gamma_2/\bar{\gamma}_2\right)\right] \tag{34}
\]

4 | OUTAGE PROBABILITY

The end-to-end SNR, \(\gamma\), is defined for a dual-hop DF link as \(\gamma = \min(\gamma_1, \gamma_2)\). Hence, the outage probability of \(\gamma\) is derived, for a given threshold SNR, \(\gamma_{th}\), as [15, eqn. (8.13)]

\[
P_{out} = Pr(\min(\gamma_1, \gamma_2) \leq \gamma_{th}) = F_{\gamma}(\gamma_{th}) \tag{35}
\]

where \(\gamma_1 = \{\gamma_F, \gamma_T\}\). Note that according to [[33], eqn. (6.81)], the cdf of \(\gamma\) is obtained using

\[
F_{\gamma}(\gamma) = F_{\gamma_1}(\gamma) + F_{\gamma_2}(\gamma) - F_{\gamma_1}(\gamma)F_{\gamma_2}(\gamma) \tag{36}
\]

where \(F_{\gamma_1}(\gamma)\) is determined by (16) or (25), and \(F_{\gamma_2}(\gamma)\) by (34).
2 Parameter values [13, 26]

| Parameter                                      | Value        |
|-----------------------------------------------|--------------|
| Optical wavelength, $\lambda$                | 1550 nm      |
| Distance for the turbulent link, $d_1$        | 1 km         |
| Distance for the foggy link, $d_1$            | 200 m        |
| Atmospheric attenuation at distance $d_1 = 1$ km, $b_1$ | 0.9          |
| Rx diameter, $2r_a$                          | 20 cm        |
| Transmit divergence at $1/e$, $\theta_T$      | 2.5 mrad     |
| Beam radius for the turbulent link, $w_{d_1}$ | $\pm 2.5$ m  |
| Beam radius for the foggy link, $w_{d_1}$     | $\pm 0.5$ m  |
| Pointing errors, $\theta_s$                  | 1 mrad       |
| Jitter standard deviation for the turbulent link, $\sigma_1$ | $\approx 30$ cm |
| Jitter standard deviation for the foggy link, $\sigma_s$ | $\approx 6$ cm |
| SNR at the RF Tx, $\overline{T}$             | 110 dB       |
| SNR at the optical Tx, $\overline{T} = \overline{f}$ | 100 dB       |
| UAV altitude, $L$                            | 20 m         |
| Coverage radius, $R$                         | 50 m         |
| Path loss exponent, $\kappa$                 | 4            |
| Rylov variance, $\sigma_R$                   | 1            |
| Rx field-of-view (FOV), $\theta_{FOV}$        | 7 mrad       |
| Standard deviation of AoA fluctuations, $\sigma_\theta$ | 1 mrad       |

**5 | NUMERICAL RESULTS**

**5.1 | SNR statistics**

In the following, a set of numerical results is presented to facilitate the performance analysis in terms of the outage probability. Most of the model parameter values are indicated in Table 2 [13, 26]. The set $m = \{1, 4/3\}$ of fading conditions is assumed, which corresponds to exclusive NLOS power (Rayleigh distribution) and an equal amount of power for NLOS/LOS signal components (Rice distribution with Rician fading parameter $K = 1$, respectively [33]. Note that the parameters $m$ and $K$ are related according to $m = (K + 1)^2/(2K + 1)$. The distance $d_1$ between the Tx and the UAV is assumed to be 200 m for the random foggy channel and 1 km when turbulence with Rylov variance $\sigma_R = 1$ is present. The different distance values are justified since the fog deteriorates the link performance much more than turbulence, as will be highlighted in the sequel. Due to the above fact and in order to proceed with a fair comparison, the values of $w_{d_1}$, and $\sigma_s$ have been properly adapted.

The outage probability performance for the foggy channel is illustrated in Figures 2–4 against the SD of the Tx-Rx misalignment orientation $\sigma_\theta$, the radius of the covered area $R$, and the fog condition. It has to be stressed that the results in Figures 2 and 3 were obtained under the presence of light fog conditions.

In Figure 2, we observe that for low SNR thresholds, the impact of $\sigma_\theta$ by 1 mrad significantly degrades the outage probability. For example, by considering the SNR threshold equal to 10 dB, the transition from 1 to 2 mrad increases more than 20 times the outage probability, whereas a further increment to 3 mrad induces an additional increment of outage probability by 30 times. On the other hand, for SNR thresholds above 35 dB, this impairment becomes less critical since the other ones are dominating.

Furthermore, the impact of the coverage radius $R$ on outage probability is illustrated in Figure 3. It can be observed that as the radius $R$ gets doubled, the corresponding SNR threshold decreases by almost 8 to 10 dB for a specific value of outage probability. The outage probability lowers significantly as the coverage area becomes small.

Finally, Figure 4 presents the impact of the fog conditions on the outage probability. It is highlighted that for SNR threshold values below 10 dB and $m = 4/3$ the system operation under light fog achieves at least 30 times lower outage probability compared to the operation under moderate fog and 3000 times lower compared to the operation under thick fog. On the contrary, considering $m = 1$, the outage probability under light fog reduces 10 times compared to the presence of moderate fog and 1000 times in case of thick fog.

On the other hand, Figures 5–7 depict the outage probability performance for the turbulent channel against $\sigma_\theta$, $R$ and the ratio $\sigma_s/r_a$. In the first two figures, the same qualitative conclusions can be inferred as in the foggy channel model, with a notable deviation in the SNR threshold values. More precisely, the impact of $\sigma_\theta$ becomes less critical for SNR thresholds above 25 dB in this case, whereas the impact of radius $R$ is rather similar for the SNR threshold below 20 dB.

In addition, the impact of the ratio of $\sigma_s/r_a$ is demonstrated in Figure 7. The outage probability is quite similar for the two fading conditions. Furthermore, the impact of this parameter gets more severe as $\sigma_s/r_a$ increases.

Moreover, a comparative overview in Figures 2–7 indicates a quite similar performance between light fog and turbulence conditions for the selected set of parameter values presented in Table 2. As the distance $d_1$ under fog was selected to be five times smaller than in the presence of turbulence, that remark is of particular importance and implies that the atmospheric conditions need to be exhaustively studied during the initial network deployment.

Finally, it is worth mentioning the impact of $m$ value, which characterises the RF channel fading conditions on the overall performance. When a LOS component is present ($m = 4/3$), the outage probability decreases compared with the case where only the NLOS component exists ($m = 1$). That is obvious since the presence of the LOS component enhances the quality of the RF link.
FIGURE 2  The impact of $\sigma_\theta$ on the outage performance for the random foggy channel ($d_1 = 200$ m)

FIGURE 3  The impact of $R$ on the outage performance for the random foggy channel ($d_1 = 200$ m)

FIGURE 4  The impact of fog condition on the outage performance for the random foggy channel ($d_1 = 200$ m)
FIGURE 5  The impact of $\sigma_0$ on the outage performance for the turbulent channel ($d_1 = 1$ km)

FIGURE 6  The impact of $R$ on the outage performance for the turbulent channel ($d_1 = 1$ km)

FIGURE 7  The impact of $\sigma_s/r_\alpha$ on the outage performance for the turbulent channel ($d_1 = 1$ km)
6 CONCLUSIONS AND FURTHER RESEARCH

The performance of a mixed optical/RF link with a UAV acting as a relay was thoroughly investigated in terms of the outage probability. In the scenario under consideration, the UAV transfers information from a fixed Tx on a hill to a user located on the earth surface. The Tx-UAV optical wireless link is susceptible to atmospheric path loss, fog or atmospheric turbulence appearance, pointing error effects and AoA fluctuations. The RF UAV-user link experiences small-scale fading, whereas the terrestrial user is randomly located in a circular region with uniform distribution. The various impairments under consideration lead to an intriguing mathematical problem, which eventually leads to tractable mathematical expressions with the aid of special functions. Appropriate numerical results depicted in suitable figures accompany the study to shed light on the impact of the impairments on the overall performance.

The present work may trigger the launch of relevant studies involving different broadcast scenarios for investigation. Possible extensions include the adoption of any random UAV movement, mobility effects for the ground users, diversity techniques, different relay protocols and possible integration of satellite links. As the integration of UAVs in the next generation communication systems will be further increased shortly, a thorough performance evaluation of these scenarios at a theoretical level becomes much more imperative.

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