Infrared and Ultraviolet QCD dynamics with quark mass for J=0,1 mesons.

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By using a previously developed phenomenological kernel for the study of the light quark QCD sector and dynamical chiral symmetry breaking effects we will examine the relative infrared and ultraviolet QCD dynamics for J=0,1 meson properties. For the same reasons we extend and explore a quark mass depended generalization of the kernel in the heavy quark region and we also compare with the original model. The relation between the dynamics of the quark propagator and the effective kernel with the J=0,1 QQ and qQ mesons and quarks Compton size is also discussed.

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I. INTRODUCTION

A lot of effort has been focused on studying the spectrum and the properties of light quark mesons (see [1]-[27] and references therein). In these systems non-perturbative effects are dominant, therefore they are the best candidates for understanding the mechanisms underlying confinement and dynamical chiral symmetry breaking which are fundamental elements and of crucial importance for the theory. In some of these studies the quark propagator equation has provide useful insights in the light quark sector of QCD [9], [10]. It is also interesting to explore, using that fundamental block of the theory and the quarks bound state equation, the transition to heavy quark physics.

We plan to canvass, by using an effective kernel for the gap equation, how the quark mass affects the infrared and ultraviolet dressing of the propagator and how that in turn will alter the dynamics of the bound state of quarks. A more realistic case of a quark mass dependent version of the effective kernel is also explored in the heavy quark region and we also compare with the original model. The relation between the dynamics of the quark propagator and the effective kernel with the J=0,1 QQ and qQ mesons and quarks Compton size is also discussed.

II. QUARK PROPAGATOR, MESONS BOUND STATE EQUATIONS AND RAINBOW-LADDER TRUNCATION.

The Dyson-Schwinger equation for the quark propagator (gap equation) has the form:

\[ S(p)^{-1} = Z_2(i \not{p} + m_{bm}) + Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(k) \gamma_\mu \frac{\lambda^i}{2} S(q) \Gamma_\nu^i(p,q) \]

(1)

where \( D_{\mu\nu}(k) \) is the renormalized dressed gluon propagator, \( \Gamma_\nu^i(p,q) \) is the renormalized dressed quark-gluon vertex, \( \Lambda \) is the regularization mass scale, with \( Z_1, Z_2 \) being the gluon and quark propagator renormalization constants.

Using the rainbow truncation for the gap equation and introducing \( \alpha(q^2) : \)

\[ Z_1 g^2 D_{\mu\nu}(q) \Gamma_\nu^i(p,q) \rightarrow 4\pi \alpha(q^2) D_{\mu\nu}^{\text{free}}(q) \gamma_\nu \frac{\lambda^i}{2}. \]

(2)

where \( D_{\mu\nu}^{\text{free}}(q) \) is the free gluon propagator, we can disentangle the equation from the rest of the Dyson-Schwinger equations.

The unrenormalized quark self-energy term of the gap equation in the rainbow truncation is:

\[ \Sigma'(p^2) = i \not{p} \{ A'(p^2) - 1 \} + B'(p^2) = \]

\[ \frac{4}{3} \int_\Lambda^\Lambda \frac{d^4q}{(2\pi)^4} \frac{G(k^2)}{k^2} T_{\mu\nu}(k) \gamma_\mu S(q) \gamma_\nu, \]

(3)

where we have set \( G(k^2) = 4\pi\alpha(k^2) \), \( k = p - q \) is the gluon momentum and the factor \( \frac{4}{3} \) comes from the trace over the color index. By taking the Dirac trace of the last equation we get:

\[ B'(p^2) = 4 \int_\Lambda^\Lambda \frac{d^4q}{(2\pi)^4} \frac{G(k^2)}{k^2} \sigma_s(q^2), \]

(4)

and if we multiply by \( \not{p} \) and then take the Dirac trace, we get the second equation:

\[ p^2(A'(p^2) - 1) = \frac{4}{3} \int_\Lambda^\Lambda \frac{d^4q}{(2\pi)^4} \frac{G(k^2)}{k^2} \times \]

\[ \left( p.q + \frac{2(k.p)(k.q)}{k^2} \right) \sigma_s(q^2), \]

(5)

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where we have introduced the quark propagator amplitudes $\sigma_s(q^2)$, $\sigma_v(q^2)$:
\[
\sigma_s(q^2) = \frac{1}{A(q^2)} \frac{M(q^2)}{q^2 + M^2(q^2)} \quad (a),
\]
\[
\sigma_v(q^2) = \frac{1}{A(q^2)} \frac{1}{q^2 + M^2(q^2)} \quad (b),
\]
(6)

The quark propagator in terms of $A$, $B'$ is then:
\[
S^{-1}(p) = i p A(p^2) + B(p^2) = \frac{Z_2}{p A(p^2) + \Sigma(p)} = \frac{1}{p A(p^2) + \Sigma(p)},
\]
(7)

Using the propagator renormalization condition, $S^{-1}(p)|_{p^2=\mu^2} = i \not p + m_r(\mu^2)$, we get
\[
A(\mu^2, \Lambda^2) = 1 + B'(p^2, \Lambda^2) - \Lambda'(\mu^2, \Lambda^2),
\]
(8)

\[
B(\mu^2, \Lambda^2) = m_r(\mu^2) + B'(p^2, \Lambda^2) - B'(\mu^2, \Lambda^2).
\]
(9)

where $m_r(\mu^2)$ is the renormalized current quark mass at point $\mu = 19$ GeV and it is a parameter we fit to experimental data.

The amplitude (BSA) $\Gamma^{ab}_{\lambda\lambda}(p, P)$ for a meson state of quarks of flavors a and b is given from the solution of the homogeneous Bethe-Salpeter equation (BSE):
\[
[\Gamma^{ab}(p, P)]_{\lambda\lambda} = \int d\mu d\eta d\bar{\eta} \bar{K}^{\lambda\lambda}(p, \bar{q}, P)
\times [S^a(\bar{q} + \eta P)\Gamma^{ab}(\bar{q}, P)S^b(\bar{q} - \bar{\eta} P)]_{\lambda\lambda}
\]
(10)
P is the total momentum, $\eta$ ($\bar{\eta}$) is the momentum partitioning parameter for the quark (antiquark) and $\eta + \bar{\eta} = 1$. $K^{\lambda\lambda}_{\lambda\lambda}(p, \bar{q}, P)$ is the unknown renormalized amputated irreducible quark-antiquark scattering kernel. Physical observables are independent of the partitioning parameter.

The most general form of the BSA for pseudoscalar mesons has four invariants while for the vector mesons has eight (see [13, 14]) and we use a four Chebychev polynomial expansion for each one of them. These amplitudes are Lorentz scalar functions of $q^2, P^2, q, P$ and the momentum partitioning parameter $\eta$. For qQ mesons that parameter help us avoid having the propagator singularities inside their mass shell BSE integration domain. Since for the mass shell momentum $P^2 = -m^2$, where $m$ is the meson mass, the quark momenta in (eq. 10) are in general complex numbers. This requires the solution of the gap equation in the appropriate parabolic region in the complex plane.

The ladder truncation for the BSE is an approximation for the equation’s kernel:
\[
[K(p, q, P)]_{\lambda\lambda} \rightarrow -4\pi \alpha(q^2) D_{\mu\nu}^\text{free}(q) \left[ \gamma^\mu \gamma^\nu \right]_{\lambda\lambda} \otimes \left[ \gamma^\lambda \gamma^\nu \right]_{\lambda\lambda},
\]
(11)

The electroweak decay constant $f_H$ of a charged pseudoscalar meson [1] expressed in terms of the meson normalized BSA and quark propagators:
\[
f_H^{PS} = \frac{Z_2 N_C}{P^2} \times \left\{ \int d^4 q \left[ P_\mu \text{Tr}_D \left[ \Gamma^{ab}(q, P) S^b(q-) \gamma_\mu S^a(q+) \right] \right] \right\}
\]
(12)

where $P^2 = -m_H^2$ and $N_C = 3$ is the number of colors, from the trace over the color indexes. Similar expression exists for vector mesons.

### III. RAINBOW-LADDER EFFECTIVE INTERACTION.

For the unknown effective running coupling we are going to use a kernel-model that has been developed within the Rainbow-Ladder truncation of Dyson-Schwinger equations. The model respects some of the most important symmetries of QCD, like Chiral symmetry and Poincare covariance, while it provides quark dressing, dynamical Chiral symmetry breaking, and most important, quark confinement. It has been used to study the physics of DCSB and related phenomena, like the spectrum of light quark mesons ([4] [5], decay constants ([6], [7], [8], [9]) and other physical observables ([10], [11], [12]), in good agreement with experimental data ([13], [14], [15]).

The so called Maris-Tandy (MT) model [4] has the form:
\[
\frac{4\pi\alpha(k^2)}{k^2} = \frac{(2\pi)^2 k^2 D}{\omega^6} \tanh^2 \frac{\omega}{2} + \frac{4(2\pi)^2 \gamma_5 F(k^2)}{\ln[\frac{\omega}{1 + \frac{m^2}{\omega^2}}]} \quad (13)
\]

For the parameters we have $\omega = 0.4$ GeV, $D = 0.93$ GeV$^2$ and $m_s = 0.5$ GeV and the u/d- and s-current quark masses at the renormalization scale $\mu = 19$ GeV, fitted to the experimental masses of pion and kaon, are $m_u/d = 0.00374$ GeV and $m_s = 0.083$ GeV. For the other two quarks, we use the masses in [16], where $m_c = 0.88$ GeV and $m_b = 3.8$ GeV. The model essentially simulates both gluon and quark-gluon vertex dressing effects. The phenomenological first term defines the behavior in the infrared region and provides the infrared enhancement necessary for the right value of the quark condensate in the chiral limit. The second term is important in the ultraviolet region and is set up so that will reproduce the 1-loop perturbative QCD running coupling behavior.

### IV. HEAVY QUARK PROPAGATOR WITHIN MESON DYNAMICS.

Recently an effort has started to extend the applications of the MT model in the heavy quarks region ([17], [18], [19]). Due to its behavior in the complex domain
the solution of the heavy quark gap equation is challenging. By changing the integration variable in the equation, from the quark internal momentum to the gluon momentum, solved that problem and the only limitation comes from the propagator’s singularities. The type and the exact location of these points is not known. Notice that the translationally invariant regularization of the integrals allows that change in the integration variable and variations of parameter $\eta$ and that our approach to numerically solve the equation is different than that in

In the case of the c quark for example with current mass $m_c = 0.88$ GeV and for the parabolic region determined by $q^2_+ = (q + \eta P)^2 - (\eta M)^2 + 2\eta \sqrt{q^2 M^2 v}$ where $q$ is the BSE integration variable, with $\eta = 0.50$ and $P^2 = -M^2 = -14$ GeV$^2$ (the peak of the region will be then at $(\eta P)^2 = -3.5$ GeV$^2$) amplitudes $\text{Re}(A)$, $\text{Re}(M)$ and $\text{Re}(\sigma_s)$ are plotted in Figures 1-3 correspondingly. For our present studies of c-quark mesons we need to solve the gap equation for $P^2$ as small as $P^2 = -M^2 \sim -9.6$ GeV$^2$, so the mass shell point (peak) is at $(\eta P)^2 \sim -2.4$ GeV$^2$ and the singularities are far from the BSE integration domain and they present no problem. This is also tested by varying parameter $\eta$.

From these plots we can see that only amplitude $\sigma_s$ (and the same is true for $\sigma_v$) has singularities, but not $A$ and $M$. We can conclude then that the singularity is the point where the denominator of $\sigma_s/v$ vanishes, i.e. $q^2_+ + M^2 (q^2_+) = 0$.

By keeping only the infrared first term of the model the solution for $\text{Re}(\sigma_s)$ for the same quark is plotted in fig. 3. In the case where we keep only the UV term in the MT model, the solution has the first singularity on the real axis (see fig. 4).

Comparing the c and b quark real part of the mass amplitude $\text{Re}(M)$, the last one appears to be almost flat near the peak of the parabolic region. $\text{Re}(A)$ also appears to vary little, just above one, in the complex plane. One may consider that such behavior of $\text{Re}(M)$ and $\text{Re}(A)$

FIG. 1: $\text{Re}(A)$ for c quark in a parabolic region in the complex plane ($P^2 = -14$ GeV$^2$, $\eta=0.50$). In this plot and in the following ones the straight lines connecting the end-points have no significance and should be ignored.

FIG. 2: $\text{Re}(M)$ for c quark in a parabolic region in the complex plane ($P^2 = -14$ GeV$^2$, $\eta=0.50$).

FIG. 3: $\text{Re}(\sigma_s)$ for c quark in a parabolic region in the complex plane ($P^2 = -14$ GeV$^2$, $\eta=0.50$) with integration over the gluon momentum (kcp). The peak of the parabolic region is at $(\eta P)^2 = -3.5$ GeV$^2$. We see indications of the existence of a pair of complex conjugate singularities near the peak of the region, approximately located at $(x_o, y_o)\sim(-2.7, \pm 3.5)$ GeV$^2$. The type of the singularities is unknown.

FIG. 4: $\text{Re}(\sigma_s)$ for c quark in a parabolic region in the complex plane ($P^2 = -8$ GeV$^2$, $\eta=0.50$) keeping only the first IR term of the kernel. The peak of the parabolic region is at $(\eta P)^2 = -2.0$ GeV$^2$. The solution is different but we still have the general characteristics of the solution from the complete MT model. The first pair of singularities approximately appear to be $(x_o, y_o)\sim(-1.7,\pm 2)$ GeV$^2$. 

FIG. 3: $\text{Re}(\sigma_s)$ for c quark in a parabolic region in the complex plane ($P^2 = -14$ GeV$^2$, $\eta=0.50$) with integration over the gluon momentum (kcp). The peak of the parabolic region is at $(\eta P)^2 = -3.5$ GeV$^2$. We see indications of the existence of a pair of complex conjugate singularities near the peak of the region, approximately located at $(x_o, y_o)\sim(-2.7, \pm 3.5)$ GeV$^2$. The type of the singularities is unknown.
observe there is an increase in the quark mass dressing as we go from c to b quark using the full model. For the c quark there is an almost equal dressing from the IR and UV term of the interaction while for b quark the UV term provides more than twice the mass dressing of the IR term.

V. RESULTS FOR QUARKONIA MESONS.

The results for the masses and the decay constants of the three pseudoscalar and vector quarkonia \((QQ, Q=s,c,b)\) mesons are collected in Tables (II) and (III).

For the fictional pseudoscalar \(\bar{s}s\), the IR term gave a mass that is smaller by 18.7 \% from that of the full model, while for the decay constant we have a 17.0 \% decrease. For the \(c\bar{c}\) meson it appears the first infrared term accounts for about 80 \% of the meson mass and about 70 \% for the decay constant. Additional studies show an equal sensitivity of the decay constant for the \(s\bar{s}\) on quark’s UV dressing effects, directly and indirectly through the BSA, while for the \(c\bar{c}\) it appears to be more sensitive to UV corrections in the meson amplitude. No bound state could be reached for these two systems with the UV tail term only.

For the b quarkonium we first observe that now even the very weak, tail term, of the interaction can give us a bound state. Therefore a very weak attractive force is adequate for such heavy particles to bind them together. The mass of the system is lighter by just 0.055 GeV or 0.57 \% of the mass from the full model, while the decay constant, more sensitive to dressing details, decreases by 12.4 \%, when we keep the UV tail term only. On the other hand, since the IR term is much more attractive than the long distance term, we get a much smaller mass, a 15.4 \% relative decrease, while the decay constant decrease by more than 75 \%. This also signifies a dominance of the UV tail term over the IR long distance term, supported by the heavy quark mass behavior of the propagator.

Same observations are also true for the corresponding vector mesons data in Table III. The only difference we notice is the greater gap between the full model calculated decay constant for the b quarkonium and the one calculated by using only the UV term. The percentage difference is about twice as much that of the corresponding pseudoscalar meson and the reason is that vector mesons are more extended objects than pseudoscalar mesons, hence the IR term contributes more in the evaluation of the physical observables. The decay constant indicates a deficiency of the low momenta behavior of the UV term on the quarks relative angular momentum.

The first Chebychev moment of the dominant invariant for the equal quark pseudoscalar mesons from the different approaches appear in fig. (7). Similar behavior is observed for the corresponding vector mesons.

As the mass increases we observe that the IR amplitude has a faster decrease with momentum than the full model amplitude and the system is more delocalised, consistent
TABLE I: c and b quark propagator approximate location of the singularities in the complex plane for the full MT, IR and UV model gap solution. All data are in GeV units. For the qualitative approximate estimation of the quark mass effective dressing we assume that \((m_Q(19 \text{ GeV}) + M_Q^d)^2 \sim -x_0\).

|     | full MT | IR term | UV term |
|-----|---------|---------|---------|
|     | \((x_o, y_o)\) | \(M_o\) | \((x_o, y_o)\) | \(M_o\) |
| \(c\) | \((-2.7, \pm 3.5)\) | 0.763 | \((-1.7, \pm 2.0)\) | 0.424 |
| \(b\) | \((-24.0, \pm 12.0)\) | 1.1 | \((-17.2, \pm 7.5)\) | 0.347 |

TABLE II: \(\bar{s}s\) (fictional), \(\bar{c}c\) and \(\bar{b}b\) pseudoscalar meson masses and decay constants with their relative percentage differences from calculations where only the first infrared (IR) term or only the ultraviolet(UV) perturbative tail in the MT model is retained.

| meson          | full MT | IR only \(|\Delta M/M|\%\)| | UV only \(|\Delta M/M|\%\)| |
|----------------|---------|-------------------------------|------------------------------|
| \(\bar{s}s\)  | 0.696   | 0.565 −18.7 \(0\) | − \(0\) |
| \(\bar{c}c\)  | 1.035   | 2.41 −20.6 \(0\) | − \(0\) |
| \(\bar{b}b\)  | 0.585   | 8.106 −15.4 \(9\) | 5.30 \(−0.6\) |

| meson          | full MT | IR only \(|\Delta f/f|\%\)| | UV only \(|\Delta f/f|\%\)| |
|----------------|---------|-------------------------------|------------------------------|
| \(\bar{s} s\) | 0.182   | 0.151 −17.0 \(0\) | − \(0\) |
| \(\bar{c} c\) | 0.387   | 0.276 −28.7 \(0\) | − \(0\) |
| \(\bar{b} b\) | 0.692   | 0.172 −75.1 \(0\) | 0.606 −12.4 |

TABLE III: \(\bar{s}s\), \(\bar{c}c\) and \(\bar{b}b\) vector meson masses and decay constants with their relative percentage differences from calculations where only the first infrared (IR) term or only the ultraviolet(UV) perturbative tail in the MT model is retained.

| meson          | full MT | IR only \(|\Delta M/M|\%\)| | UV only \(|\Delta M/M|\%\)| |
|----------------|---------|-------------------------------|------------------------------|
| \(\bar{s} s\) | 1.072   | 0.949 −11.5 \(0\) | − \(0\) |
| \(\bar{c} c\) | 1.235   | 1.288 −20.0 \(0\) | − \(0\) |
| \(\bar{b} b\) | 0.658   | 8.130 −15.9 \(9\) | 5.86 \(−0.8\) |

| meson          | full MT | IR only \(|\Delta f/f|\%\)| | UV only \(|\Delta f/f|\%\)| |
|----------------|---------|-------------------------------|------------------------------|
| \(\bar{s} s\) | 0.259   | 0.274 +5.8 \(0\) | − \(0\) |
| \(\bar{c} c\) | 0.415   | 0.333 −19.8 \(0\) | − \(0\) |
| \(\bar{b} b\) | 0.682   | 0.340 −50.1 \(0\) | 0.510 −25.2 |

FIG. 7: First Chebychev moment of the dominant invariant for the \(\bar{s}s\), \(\bar{c}c\), \(\bar{b}b\) pseudoscalar mesons, from the full MT model solution and the solution with only the IR term. For the \(\bar{b}b\) quark system we also have an UV calculated amplitude since we can reach a mass shell even with only the weaker UV tail term in the interaction.

VI. RELATIVE INFRARED AND ULTRAVIOLET QCD DYNAMICS FOR QUARKONIA.

From the last results become obvious that as the current quark mass increases the IR term has a marginalized important in the realization of meson observables. The question is why and how that happens. The effective kernel is the same and there is no change in the UV contribution in the quarks interaction. So essentially this is a result of the effect of the large current quark mass in the behavior of the propagator amplitudes and consequently the combined dynamical interplay with the MT effective kernel. A first naive qualitative explanation is that as we raise the quark mass the amplitudes will have smaller values, even very deep in the time-like region, and we need to get closer and closer to their singularities to notice some important increase in their values. That in turn will suppress the contribution and lessen the significance of the IR term while the UV term of the model will become more relevant in the evaluation of the observables. That is also related to the extremely fast
splitudes. For the last two cases of products involving the last product the situation is somewhere between the first and the second product. That suppression though is decreasing in the UV region in all cases we have suppression of the tail region as the current quark mass increases. For the $s$ quark system, while from the MT effect for the MT model in the IR region. Same things are true for every $p^2 > 0$ but there is a significant scale down as $p^2$ increase.

A. A first look in the quark mass dynamics of the propagator with the MT kernel.

To qualitatively see the effect of the heavy quark mass within the meson dynamics, we plot the product of the MT rainbow-ladder kernel with the two propagators amplitudes as they appear in the BSE. There are four cases: $\sigma_s(q^2_{\pi}) \cdot \sigma_s(q^2_{\eta})$, $\sigma_s(q^2_{\pi}) \cdot \sigma_c(q^2_{\eta})$ and $\sigma_s(q^2_{\pi}) \cdot \sigma_c(q^2_{\eta})$, but for convenience we consider only the real axis where we have $\text{Re}(\sigma_s/\text{Re}(q^2_{\eta})) = \sigma_s(q^2 - (\eta P)^2)$, so there are essentially only three amplitude products in the BSE integrand: $\sigma^2_s(q^2 - (\eta P)^2)$, $\sigma^2_c(q^2 - (\eta P)^2)$ and $\sigma_s(q^2 - (\eta P)^2) \cdot \sigma_c(q^2 - (\eta P)^2)$. The plots of these products with the MT kernel for the $s\bar{s}$, $c\bar{c}$, $b\bar{b}$ pseudoscalar mesons are in fig. 8.

We observe some mild increasing suppression in the IR region as the current quark mass increases for the product $\sigma_s^2$, and a much stronger increasing suppression for the second product due to the $\sigma_c^2$ amplitude. For the last product the situation is somewhere between the first two since it combines the effects of both propagator amplitudes. For the last two cases of products involving $\sigma_c$, we actually have a small enhancement in the IR region for the $s$ quark system, while from the MT-$\sigma_s^2$ plot we notice $\sigma_s^2$ has almost no effect in the IR region. For the UV region in all cases we have suppression of the tail term. That suppression though is decreasing as increasing in the current quark mass increases, somehow faster for the MT-$\sigma_s^2$ than in the MT-$\sigma_s \cdot \sigma_c$ product. Finally for the term involving $\sigma_c^2$ there is no difference for all quark systems for $q^2 > 10 \text{GeV}^2$ (the increasing IR suppression just extends a little beyond 1 GeV$^2$ as the quark mass increase). The difference in the effects between the first and the second type of terms are due to the fact that $\sigma_s$ amplitude has also the quark mass amplitude in the numerator moder-
the self-interaction term and even for the heavy b quark will vary much and not very close to the b current mass of 3.8 GeV. Dressing IR effects will also diminish and will be replaced by UV dressing effects. So it should be \( \Delta(M_{IR}) < 0, \Delta(M_{UV}) > 0 \). If one expects that overall \( \Delta(M_{\Sigma}) < 0 \) then, since \( M_{\Sigma} = M_{IR}^2 + M_{UV} \), we should have \( |\Delta(M_{IR}^2)| > |\Delta(M_{UV})| \). Notice that the estimated quark mass dressing using the singularities location of the IR and UV gap solutions for the c and b quarks (Table I) do confirm the first two expected changes in the IR and UV quark mass dressing contributions as the quark mass is raised, but don’t confirm the third case, since from these data appears the UV dressing effects increase faster than the IR decrease and the overall dressing increase. It is not clear though if this is because of a disadvantage of the MT model (IR term) or because of our approach to estimate these dressings. From the mass amplitude plots in section IV and from the quarkonia results in section V we have further evidence that the short distance dressing increases in strength, faster that the long distance dressing decreases, with the quark mass. Therefore it is possible, after certain quark mass scale, that the heavy quarks receive more dressing from the short distance than the long distance self-interaction. Another element that supports the validity of the above ideas is that the short distance analysis is essentially model independent, dictated by the perturbative QCD propagator and MT tail term. As a final note, based on the above analysis, the heavy quark bound states and confinement of their quarks, are more a result of their large mass and their relatively larger UV dressing than a result of the binding interaction between them.

### B. Propagator amplitudes and quark dressing dynamics with quark mass.

Although from these plots one can qualitatively and visually understand why the IR term becomes less important while the UV weak tail term becomes more relevant for the mesons physical observables as we increase the quark mass, the question is how that happens. For that reason we have to focus on the dynamics of the two amplitudes in the two regions (IR and UV) and on how that changes as we increase the current quark mass. The arguments will be again mostly qualitative. From the two coupled gap equations and using the plots we can extract some qualitative information about the relative change the IR and UV tail term will bring upon the propagator amplitudes \( M(p^2) \) and \( A(p^2) \) as we increase the mass. We notice that the integrand of the equation for \( B'(p^2) \) is just MT-\( \sigma_s \). That term is almost unchanged in the IR region. There is only a small suppression over there as we increase the quark mass but there is a decreasing UV suppression. Therefore it is possible for the UV integrated strength to dominate over the behavior of \( B'(p^2) \). On the other hand MT-\( \sigma_s \) appears in the other integral equation. There is almost no change of that function in large momenta (larger than about 9-10 GeV\(^2\)) but up to this point there is very strong suppression greatly diminishing the contribution for the values of \( A(p^2) \). As a result we expect amplitude \( A(p^2) \) to have smaller variation in the complex plane and get values closer to one as the quark mass increase. Amplitude \( M(M=B/A) \) on the other hand, since it is proportional to B, will still be receiving important contributions from

### C. A qualitative mathematical analysis.

Next we will try to qualitatively understand how a raising quark mass can have the effects we observed so far. For the light quarks we assume \( m_q << \Lambda_\chi \) where \( \Lambda_\chi \sim 1 \text{ GeV} \) is the chiral symmetry breaking scale. For the \( \sigma_s \) amplitude we have

\[
\sigma_s(q^2 - M_H^2/4) = \frac{1}{A(q^2 - M_H^2/4)} \frac{M(q^2 - M_H^2/4)}{A(q^2 - M_H^2/4)^2 [q^2 - M_H^2/4 + M^2(q^2 - M_H^2/4)]} = \frac{m_q + M_\Sigma}{A(q^2 - M_H^2/4)^2 [q^2 - M_H^2/4 + m_q^2 + M_\Sigma^2 + 2m_qM_\Sigma]} \tag{14}
\]

where \( M_H \) is the hadron mass. First we will focus on the IR region. We are very close to the peak of the parabolic region and for convenience of our analysis we take \( q^2 \sim 0 \). For the light quarks we have strong dressing effects in that region so \( m_q + M_\Sigma \sim M_\Sigma \), and the propagator
amplitude is approximately

\[ \sigma_s(q^2 \sim 0) \sim \frac{1}{A} \cdot \frac{M_S + m_q}{M_S^2 + 2m_q M_S} \]  

(15)

Although \( A \) increases near the peak, decreasing the values of \( \sigma_s \), the hadron mass \( M_H \) is mostly due to the quark IR dressing effects and therefore the difference in the denominator at the same time is getting smaller providing the very small IR enhancement we noticed for the product of \( \sigma_s \) with the MT model. Since for the other amplitude the mass function is replaced by one \((M \rightarrow 1)\) which is larger than \( m_q + M_S \), and since the denominator is the same, we expect for the same reasons as before to have stronger than the \( \sigma_s \) IR support for the MT model. For the heavy quarks we assume \( M_H \sim 2m_q \). Then

\[ \sigma_s(q^2 \sim 0) \sim \frac{1}{A} \cdot \frac{M_S + m_q}{M_S^2 + 2m_q M_S} \]  

(16)

We keep the \( M_S^2 \) in the denominator because now it is of the order of 1 GeV. \( A \) has values close to one and does not affect much the behavior of the amplitude near the peak. Because we now have the sum of two positive terms in the denominator, we expect that will lightly decrease the values of \( \sigma_s \) near the peak. The large quark mass in the numerator inhibits stronger suppressing IR effects from this amplitude but something similar does not exist in \( \sigma_v \) resulting in a stronger IR suppression. These observations help us to qualitatively understand the different IR effects we noticed in the last figures and how and why they change as the quark mass increase. We should notice at this point that the behavior of \( \sigma_s \) in the IR region is model dependent and for the heavy quarks may not present a realistic behavior, but for the other amplitude, \( \sigma_v \), we know for certain there will be a \( 1/m_q \) suppression for the heavy quarks as we increase the quark mass.

The UV region analysis is easier since we are in the perturbative region and for both light and heavy quarks we have \( m_q + M_S \sim m_q \) and \( A \sim 1 \). This is essentially model independent analysis. Notice at this point that for our analysis we are talking about dressing effects in the quark mass in the IR or UV region originating either from IR or UV interaction effects. There is though a shift in the momenta for the propagator amplitudes so in reality, when we refer to mass dressing in the IR region, we actually refer to the dressing in the quark mass near the peak (from \( \text{Re}(q^2_{\pi}) \sim -(\eta M)^2 \) to about \( \text{Re}(q^2_{\pi}) \sim -(\eta M)^2 + 2 \text{GeV}^2 \) on the real axis) of the meson mass shell parabolic region in the complex plane due to IR and/or UV interaction effects, and when we talk about mass dressing in the UV region, we actually mean the dressing in the quark mass far from the peak (approximately for \( \text{Re}(q^2_{\pi}) \sim -(\eta M)^2 + 2 \text{GeV}^2 \) on the real axis) of the meson mass shell parabolic region in the complex plane due again to IR and/or UV interaction effects. The idea of a quark receiving, at different momentum scales, both IR and UV dressing at the same time, can be accommodated by the wave-like nature of the particle. There is an insignificant quark mass dressing in the UV region (for \( q^2 > 4 - 5 \text{ GeV}^2 \) which for the propagator momenta in the BSE is for \( \text{Re}(q^2_{\pi}) \sim -(\eta M)^2 + 5 \text{ GeV}^2 \) on the real axis in the corresponding parabolic region) for all quark masses and that’s why we can assume the last approximation for the quark mass function. The reason for this is that the internal quark propagator momentum in the gap equations depend on the external momentum \( p^2 \) and as the last one increase provides through the propagator in the gap equations integrand an increasing suppression on IR and UV dressing effects for the quark mass in the UV region. For the same reason the MT model provides less quark binding as \( p^2 \) increase and we move to the UV region, the gluon momentum in the BSE is \( k = p - q \) so we have an \( e^{-p^2/\omega^2} \) term strongly suppressing IR binding forces in the UV region and through a more complicated dependence of the UV tail on \( p^2 \) we also have suppression of the UV binding effects again in the UV region. The physical interpretation is that as the quark momentum \( p^2 \) increase the particle will be more localized, receiving less IR and UV dressing and at the same time feel less of the color field of the other quark. We have then

\[ \sigma_s(q^2 > 5 \text{ GeV}^2) \sim \frac{m_q}{q^2 + m_q^2 - M_H^2/4} \]  

(17)

For light quarks the numerator is very small and as \( q^2 \) increase the amplitude will decrease very fast. For \( \sigma_v \) though the numerator is one which is larger than the quark mass and that decrease will be weaker. For the heavy quarks on the other hand since \( M_H \sim 2m_q \) the amplitude is further simplified to \( \sigma_s(q^2 > 5 \text{ GeV}^2) \sim m_q/q^2 \). As the current mass further increase will provide more support for the values of \( \sigma_s \) in large momenta. If we replace that mass with one to get the other amplitude, since now the quark masses are larger than 1 GeV, there will be less UV enhancement than with the \( \sigma_s \). The simple form of the denominator also makes possible for the product of the two functions with the MT model to follow closer the changes in the behavior of the model, in fig. (9) notice the obvious bending at about 2-3 GeV\(^2\) of the c and b quark cases of products. Finally notice that we have very strong increase in the suppression effects in the IR region as the quark mass is raised and that affects the propagator amplitude products in a small area near the tip of the parabolic region, while there is a comparatively mild decrease in the suppression effects in the UV region as the quark mass is raised in a comparatively much larger area of the mass shell parabolic region in the complex plane. Therefore, based on our analysis at the end of previous subsection B, the integrated strength of the increase in the UV dressing and binding effects should be slightly more that the integrated strength of the decrease in the IR dressing and binding effects so that at the end will have an overall increase in the quark mass UV dressing and binding energy in both IR and UV regions.
D. Physical interpretation and other aspects of the relative effects of the UV and IR dynamics.

Summing up, we found that as the current quark mass increase the propagator amplitude $\sigma_v$ and less $\sigma_s$ will suppress mass dressing and binding IR contributions while initially for light quarks $\sigma_v$ will enhance IR contributions. At the same time $\sigma_v$ will provide the support that will enhance UV dressing and binding effects as the current mass is raised. From the location of the singularities of the c quark propagator, from the full model calculations and when we keep the IR or UV term (see Table I), and from plots (9, 10) qualitatively we may conclude that this is the current quark mass region where this transition, in dominance of dressing and binding from UV over IR region contributions, takes place. In terms of physics, the long distance interaction between the quarks (IR term) is inhibited by their large mass and now they interact mostly through the exchange of short-range (large momentum) gluons. Notice at this point the unique feature of the theory where the interaction degrees of freedom (gluons) depend on the mass of their source. As the mass is raised becomes increasingly more difficult for the quarks to move further apart with the large current quark mass gradually replacing in that way the effect of the strong IR term of the interaction. In other words, the increase in the current mass now replaces the strong quark mass dressing UV effects of the model (enhancing at the same time UV dressing), making that term almost unnecessary, and with only a small attraction we can have a bound state.

From the c quark amplitude plot in fig. (4) and a similar one for the b quark amplitude, becomes obvious that the MT kernel, because of the behavior of the infrared term, does not support a single pole-mass constituent-like behavior and actually, from the b quark plot, it appears we are moving further away from that type of behavior as the current quark mass increase. We can assume then that a constituent-like behavior can be supported by the effective kernel, as we increase the quark mass, only if there is an important decrease in the strength of the IR term of the kernel.

For the solution of the gap equation one starts by assuming a free quark boundary condition at some large space-like momentum scale, well inside the perturbative region, and this solution is modified because of the interaction of the quark with the vacuum, mostly in the nonperturbative region for the light quarks. If we ignore the strong nonperturbative effects, expressed through the first term of the MT kernel, and keep only the perturbative UV tail term, then from perturbative QCD we know that the pole of the propagator, will retain some of its initial features determined by the current quark mass. The fact that we still have a first singularity on the real axis (fig. 6 and similar plot for the b quark) is a verification of that. On the other hand as we go to heavier quarks the IR dressing effects will become comparatively smaller, the location of the singularities will be mostly determined by the current quark mass (and UV dressing) and they will be very close, along the real axis, to the mass-pole singularity of the free propagator. Therefore the dynamics of the interaction, as expressed through the MT model (specifically the first part), will have a lesser role in the location of the singularities as the current quark mass is raised, but will be always responsible for their type (confinement excludes poles) which still has a very subtle and important role for the calculation of certain meson observables. Therefore we expect that a small variation of the parameters of the model, will have insignificant impact in the location of the heavy quark propagator singularities. As a consequence the values of the physical observables will also show indifference to these variations and will be mostly determined by the quark masses.

Reversing the reasoning, if the IR term of the model had an explicit quark mass dependence, we should expect a decreasing dependence of the strength of the model on the mass as we go to heavier quarks.

VII. AN EFFECTIVE KERNEL WITH QUARK MASS DEPENDENCE.

Higher order diagrams in the expansion of the BSE kernel contain internal quark propagators which will introduce a quark mass dependence to it. The parameters of an effective kernel that is applicable over a wide range of quark mass can be expected to have an explicit quark mass dependence. The way the parameters should vary with the quark mass is not easy to determine. In ref. 35 it was noted that diagrams higher order than ladder-rainbow provide an attractive effect on mesons which decreases with increasing quark mass. Qualitative estimates of this effect for quarkonia were used to produce an $m_q$-dependent effective kernel (of rainbow-ladder format) whose role was to produce meson bound state quantities that left room for subsequent and explicit corrections from higher order kernel processes. The IR strength and range of such a core kernel was fitted to reproduce the expected behavior of eq.(9) in ref. 35 in the low quark mass region. With the MT-model form used as a template for the core kernel, the IR strength was fitted to light meson properties leaving room for higher order terms effects. On the other hand the IR strength of the MT effective kernel was fitted to light quark meson properties absorbing higher order effects. Since the MT-model parameters are fixed these effects will be still present in the heavy quarks region. The extrapolation of the core kernel to the heavy quark region provides an interesting point of comparison with the MT-model.

So far we had numerous indications that the MT effective kernel provides too much dressing for the heavy quarks. At the same time we believe it gives a BSE kernel that is too attractive and that cancels to some degree the overdressing effects in the quark mass. To make things more complicated, we found that the heavy quark propagator itself weakens to a certain degree the model’s IR
effects. Therefore it is not so easy to determine the net effect of the MT interaction kernel in the heavy quark region from the calculated meson observables. Since the heavy quark region was not considered in ref. 35, we extend its application in the c and b quark region and we examine any benefits for studying heavy quark mesons.

The expression for the core effective kernel of ref. 35 is:

$$\frac{4\pi\alpha(k^2)}{k^2} = C(\omega, \hat{m}) \left( \frac{2\pi k^2}{\omega^2} e^{-\frac{k^2}{\omega}} + \frac{2(2\pi)^2\gamma_m\Delta^2 F(k^2)}{\ln[\tau + \left( 1 + \frac{k^2}{\Lambda_{QCD}^2}\right)^2]} \right).$$ (18)

The midpoint value of $\omega$ in the minimal sensitivity region, from studies going as far as about the $s$ quark mass, was found to have the following dependence on the range of quark masses is mapped into a very small domain $\hat{m}$. The midpoint value of $\omega(0) = 0$ is:

$$\omega(\hat{x}) = 0.38 + \frac{0.17}{1 + \hat{x}}, \quad \hat{x} = \frac{\hat{m}}{\hat{m}_0}, \quad \hat{m}_0 = 0.12 \text{ GeV},$$ (19)

and in this case the product of the two core-model parameters, $D$ and $\omega$, will depend only on $\hat{x}$:

$$C(\hat{x}) = \omega D = C_0 + \frac{0.86}{1 + C_2 \hat{x} + C_3 \hat{x}^2}$$ (20)

with $C_0 = 0.11$, $C_2 = 0.885$, $C_3 = 0.474$.

If we assume these relations are true for all current masses then we notice that from the chiral limit all the way to infinite quark mass, parameter $\omega(\hat{x})$ varies from $\omega(0) = 0.55 \text{ GeV}$ to $\omega(\infty) = 0.38 \text{ GeV}$ and $C(\hat{x})$ from $C(0) = 0.97 \text{ GeV}$ to $C(\infty) = 0.11 \text{ GeV}$ or for $D(\hat{x})$ from $D(0) = 1.763$ to $D(\infty) = 0.289$. So the infinite range of quark masses is mapped into a very small domain of the parameters $\omega(\hat{x})$ and $C(\hat{x})$. From the plot in fig. 12 we observe there is a very fast variation of $C(\omega)$ for quark masses up to $\hat{m} \sim 2 - 3 \text{ GeV}$ and then above that the parameters rapidly approach their limit values and they don’t vary much. The limiting kernel parameters $C$, $\omega$ are a possible definition of a rainbow -ladder kernel for very massive quarks. As a qualitative comparison with the MT-model at high quark mass we present some meson results from use of the new effective kernel which we call a core model.

Since the $c$ quark mass of 0.88 GeV at scale $\mu = 19 \text{ GeV}$ corresponds to a $\hat{m}$ of about 1 GeV, we obtain $D = 0.308$ and $\omega = 0.39 \text{ GeV}$ for this core kernel. For the $b$ meson studies we use the core model limit values (D=0.289, $\omega = 0.38 \text{ GeV}$).

The results for the $c$- and $b$-quarkonia masses and their decay constants from the core model for comparison with the MT model results are collected in table (IV). Summing up, the relative percentage changes in the parameters of the model and the resulting relative percentage changes in the quarkonia masses and decay constants, for the $c$ quarkonia are:

$$\frac{\Delta M_c}{M_c} = \left( -66.88, -2.5 \right)\%$$

$$\frac{\Delta f_{c}}{f_{c}} = \left( -3.3, -13.7 \right)\%$$

$$\frac{\Delta \eta_{c}}{\eta_{c}} = \left( -5.7, -23.1 \right)\%$$

while for the $b$ systems:

$$\frac{\Delta M_b}{M_b} = \left( -68.92, -5.0 \right)\%$$

$$\frac{\Delta f_{b}}{f_{b}} = \left( -5.0, -8.5 \right)\%$$

$$\frac{\Delta \eta_{b}}{\eta_{b}} = \left( -0.75, -13.6 \right)\%.$$

It is obvious that the decay constants, especially those of the vector mesons, since they are more extended particles, are much more sensitive to parameters changes. Vector meson masses also appear to be relatively more sensitive than the pseudoscalar ones, for $b$ quarkonia though the relative mass change indicate insensitivity to the IR dynamics of the model. Finally by comparing the core model results for the $b$ quarkonium with those obtained with the UV term only in the model, we see that masses, especially that of the vector meson, are closer to the experimental ones but the vector meson decay constant at the same time becomes even smaller.

VIII. RECOVERY OF A qQ MESON MASS SHELL.

A qQ meson mass shell can not be obtained with a full model dressing for both quarks, and that reveals the limitations of the approach for light-heavy meson studies. Given that the core model of an effective BSE kernel is $m_q$-dependent, and that its use for qQ mesons containing quarks of different masses is not defined, we decided to explore the point of view that the DSE kernel for the dressing of the heavy quark propagator could be too strong in the infrared. Hence we performed qQ meson calculations in which the IR term of the MT-model kernel is removed. The BSE kernel for the interaction of the
light and heavy quark thoug should be approximately unchanged since there is no significant change in the size of the qQ meson compared to the size of the light quark mesons. Solving the DSE for the b quark propagator with only the UV tail term of the MT kernel, where the full MT kernel was used for the calculation of the light quark propagator and the solution of the BSE. All data are in GeV units.

| qB | ω(D, ω(GeV)) | M_{h}^b(GeV) | I_{h}^b(GeV) | M_{c}^b(GeV) | I_{c}^b(GeV) | M_{f}^b(GeV) | I_{f}^b(GeV) | Υ_{J/ψ}^b(GeV) | f_{η}^b(GeV) | Υ_{ψ}^b(GeV) | f_{η}^b(GeV) | Υ_{ψ}^b(GeV) | f_{η}^b(GeV) | Υ_{ψ}^b(GeV) | f_{η}^b(GeV) | Υ_{ψ}^b(GeV) | f_{η}^b(GeV) | Υ_{ψ}^b(GeV) | f_{η}^b(GeV) |
|----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| nb | (0.93, 0.40) | 4.720 | 4.465 | 12.16 | 0.167 | 0.133 | 24.4 |
| sb | (0.90, 0.39) | 4.730 | 4.478 | 11.59 | - | 0.164 |
| cb | (0.86, 0.286) | 5.831 | 7.238 | - | 0.453 |

The size of the QQ mesons becomes smaller as quark mass increases and the dressed quark quasi-particles themselves become smaller in size. The infrared component of the kernel relates to large distance gluons; such components should be less physically relevant for internal dynamics of heavy, small Compton size, particles. This is supported by our finding that the heavier QQ quarkonia receive diminishing contributions from the infrared component of our model kernel; the UV component alone provides a very good description of the bb states. We stress that this refers not only to the binding interaction but also to the quark self-energy dressing.

In detail, as the quark mass increases, due to its smaller Compton size, UV dressing will increase and IR dressing will be strongly suppressed. Model independent evidence indicate that the short distance dressing will be, after certain mass scale, more than the long distance dressing, and at a higher mass scale, it may be over and above the long distance dressing of the lightest u/d quarks. The heavy quarks will be limited in a smaller area in space and the binding force will be provided by the exchange of short distance gluons, signifying a smaller overall binding energy. The heavy quark mass and the now relatively stronger UV dressing, triggered by that large mass, make feasible, with a very weak force, to bind the particles together.

On the other hand, the qQ mesons have a size that does
not diminish significantly with increasing heavy quark mass. So the infrared sector of the binding interaction should remain relevant. However the self-energy dressing of the heavy quark should not receive strong contributions from large distance gluons. This becomes evident by our finding that a suppression of the infrared contributions from large distance gluons. This becomes evident should remain relevant. However the self-energy dressing mass. So the infrared sector of the binding interaction does not diminish significantly with increasing heavy quark.

In a similar way as the quark momentum increases the quark dressing kernel allows a physical $q_b = u/d, s, c$ meson state even though the binding effective interaction and the dressing of the light quarks remains unchanged.

Aspects of this study may be related to Brodsky and Shrock’s suggestion of a maximum wavelength of quarks and gluons in hadrons. The present work may actually suggest a possible scenario for the natural realization of such scales in QCD. Further studies are necessary in that direction.

References

[1] L. X. Gutierrez-Guerrero, A. Bashir, I. C. Cloet and C. D. Roberts, arXiv:1002.1968 [nucl-th].
[2] L. Chang, I. C. Cloet, B. El-Bennich, T. Klahn and C. D. Roberts, Chin. Phys. C 33, 1189 (2009) arXiv:0906.4304 [nucl-th].
[3] M. S. Bhagwat, A. Hoell, A. Krassnigg, C. D. Roberts and S. V. Wright, Few Body Syst. 40, 209 (2007) arXiv:nucl-th/0701009.
[4] P. Maris and P.C. Tandy, Phys. Rev. C 60, 055214 (1999) arXiv:nucl-th/9905056.
[5] P. Maris and C.D. Roberts, Int. J. Mod. Phys. E 12, 297 (2003) arXiv:nucl-th/0301049.
[6] P. Maris and C.D. Roberts, Phys. Rev. C 56, 3369 (1997) arXiv:nucl-th/9708029.
[7] P. Maris, C.D. Roberts and P.C. Tandy, Phys. Lett. B 420, 267 (1998) arXiv:nucl-th/9707003.
[8] P. Maris, AIP Conf. Proc. 892, 65 (2007) arXiv:nucl-th/0611057.
[9] M. S. Bhagwat, A. Krassnigg, P. Maris and C. D. Roberts, Eur. Phys. J. A 31, 630 (2007) arXiv:nucl-th/0612027.
[10] L. Chang, Y. X. Liu, M. S. Bhagwat, C. D. Roberts and S. V. Wright, Phys. Rev. C 75, 015201 (2007) arXiv:nucl-th/0605058.
[11] S. J. Brodsky and R. Shrock, Phys. Lett. B 666, 95 (2008) arXiv:0806.1355 [hep-th].
[12] L. Holl, A. Krassnigg and C.D. Roberts, Phys. Rev. C 70, 042203 (2004) arXiv:nucl-th/0406030.
[13] D. Jarecke, P. Maris and P. C. Tandy, Phys. Rev. C 67, 035202 (2003) arXiv:nucl-th/0208019.
[14] A. Krassnigg and P. Maris, J. Phys. Conf. Ser. 9, 153 (2005) arXiv:nucl-th/0412058.
[15] C.D. Roberts, Nucl. Phys. A 605, 475 (1996) arXiv:hep-ph/9406233.
[16] P. Maris and P. C. Tandy, Phys. Rev. C 62, 055204 (2000) arXiv:nucl-th/0005015.
[17] M.S. Bhagwat, M.A. Pichowsky, C.D. Roberts and P.C. Tandy, Phys. Rev. C 68, 015203 (2003) arXiv:nucl-th/0304003.
[18] P. Maris and P.C. Tandy, Phys. Rev. C 61, 045202 (2000) arXiv:nucl-th/9910033.
[19] P. Maris and P. C. Tandy, Nucl. Phys. Proc. Suppl. 161, 163 (2006) arXiv:nucl-th/0511017.
[20] M. Bhagwat, M.A. Pichowsky, P. C. Tandy Phys. Rev. D 67, 054019 (2003) arXiv:hep-ph/0212276.
[21] S. Eidelman et al. [Particle Data Group Collaboration], Phys. Lett. B 592, 1 (2004).
[22] M.A. Pichowsky Published in *Pittsburgh 2002, Physics of excited nucleons* 83-92 arXiv:nucl-th/0302026.
[23] A. Holl et al., Phys. Rev. C 71, 065204 (2005) arXiv:nucl-th/0503043.
[24] P. Maris and C.D. Roberts, Phys. Rev. C 58, 3659 (1998) arXiv:nucl-th/9804062.
[25] P. Maris, Nucl. Phys. Proc. Suppl. 90, 127 (2000) arXiv:nucl-th/0008048.
[26] P. Maris and P.C. Tandy, Phys. Rev. C 65, 045211 (2002) arXiv:nucl-th/0201017.
[27] P. Maris, AIP Conf. Proc. 892, 65 (2007) arXiv:nucl-th/0611057.
[28] A. Krassnigg, Phys. Rev. D 80, 114010 (2009) arXiv:0909.4016 [hep-ph].
[29] N. A. Souchlas: Quark dynamics and constituent masses in heavy quark systems, Phd Thesis 2009.
[30] Lei Chang, Yu-Xin Liu, Mandar S. Bhagwat, Craig D. Roberts, Stewart V. Wright. Phys. Rev. C 75, 015201 (2007).
[31] R. Alkofer, P. Watson and H. Weigel, Phys. Rev. D 65, 094026 (2002) arXiv:hep-ph/0202053.
[32] R. Alkofer, L. von Smekal and P. Watson, arXiv:hep-ph/0105142.
[33] P. Maris, A. Raya, C. D. Roberts and S. M. Schmidt, Eur. Phys. J. A 18, 231 (2003) arXiv:nucl-th/0208071.
[34] S.R. Amendolia et al. [NA7 Collaboration], Nucl. Phys. B 277, 168 (1986).
[35] G. Eichmann, R. Alkofer, I. C. Cloet, A. Krassnigg and C. D. Roberts, arXiv:0802.1948 [nucl-th].
[36] S. J. Brodsky and R. Shrock, arXiv:0803.2531 [hep-th].
[37] S. J. Brodsky and R. Shrock, arXiv:0803.2531 [hep-th].