Efficient Grasp Planning and Execution with Multi-Fingered Hands by Surface Fitting

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Abstract—This paper introduces a framework to plan grasps with multi-fingered hands. The framework includes a multi-dimensional iterative surface fitting (MDISF) for grasp planning and a grasp trajectory optimization (GTO) for grasp imagination. The MDISF algorithm searches for optimal contact regions and hand configurations by minimizing the collision and surface fitting error, and the GTO algorithm generates optimal finger trajectories to reach the highly ranked grasp configurations and avoid collision with the environment. The proposed grasp planning and imagination framework considers the collision avoidance and the kinematics of the hand-robot system, and is able to plan grasps and trajectories of different categories efficiently with gradient-based methods using the captured point cloud. The found grasps and trajectories are robust to sensing noises and underlying uncertainties. The effectiveness of the proposed framework is verified by both simulations and experiments. The experimental videos are available at [1].

I. INTRODUCTION

Efficient grasp planning for multi-fingered hands remains a challenging task. First, searching multiple contacts and corresponding hand configurations is a high-dimensional problem. Second, the collision should be considered in both the grasp planning and execution stages. Thirdly, the system should be robust to the uncertainties caused by the imperfect sensing, calibration and actuation.

Due to the high dimensional state space, majority of the planning researches use sampling based methods [2], [3], [4], [5], [6] by employing the precise 3D mesh model of the object. To overcome the curse of dimensionality, the hand poses in [3] were sampled around the object skeleton. To accelerate the inverse kinematics (IK) and collision detection during sampling, the object and fingertip workspace in [5] were approximated as tree structures. Without the gradient information, the sampling-based methods essentially rely on dense sampling and heavy manually-designed heuristics. Learning-based methods have been introduced to simplify the heuristics design [7], [8], [9]. To reduce the learning dimension, these methods restricted the learned policies to vertical grasps with parallel grippers. Due to the high dimensionality, it is usually intractable to learn the grasp policy for multi-fingered hand directly in configuration space. In [10], the heatmaps with high success probability were learned with a deep network, after which simulated annealing was applied for collision-free hand configurations. The planning time was over 16 secs/grasp.

It is observed that human tends to increase the contact surface to generate powerful, robust grasps. With larger contact surface, the hand provides force/torque from various positions, and the resultant wrench space can resist larger disturbances. The idea of matching the hand to the object has been explored by several researchers. In [4], the grasp quality was quantified by the distances between the key points on hand surfaces and the object mesh. The optimization was solved by the simulated annealing in absence of the analytical gradient, and the computation for the optimization was excessively heavy for real time applications. A data-driven method was proposed in [11] to find the hand pose that matches features with the object. However, it is time consuming to traverse the whole database for feature matching. In [6], the hand-object geometric fitting was achieved by sampling contacts on the object. This assignment implicitly maps the kinematics of hand to the object by manually-designed heuristics. The resultant grasps could be a narrow subset of all well-matched grasps.

To avoid excessive hand-engineering, we proposed a grasp planning method called iterative surface fitting (ISF) in [12] and increased the efficiency by combining with learning in [13]. ISF optimized for both the palm pose and finger displacements by maximizing the surface fitting score. However, the previous ISF has several limitations. First, it can only handle the grippers with one degree of freedom (DOF). Secondly, the grasps with collisions were detected and pruned after the optimization. The optimize-then-prune operation produced sub-optimal grasps.

Besides planning the desired grasp configurations, the robot should generate proper robot-finger trajectories to execute the grasps. The trajectory planning can be extremely expensive in this high dimensional space. Majority of the current grasp planning methods ignore possible collisions during the execution and simply close the fingers to execute the grasps [3], [5], [6], [14]. A method based on rapidly-exploring random tree (RRT) was introduced in [15] to plan motion and grasp simultaneously. With the manually designed heuristics, the RRT dimension was reduced to 3. These heuristics defined a potentially narrow and suboptimal subspace for RRT search. A general trajectory optimization (TrajOpt) algorithm was presented in [16] using the sequential quadratic programming (SQP) [17]. Both the RRT and TrajOpt require object mesh model during the optimization, which is generally absent in the online grasp planning scenario.

In this paper, we propose a framework to plan and execute grasps with multi-fingered hands. The framework contains
both the grasp planning and grasp imagination. The grasp planning searches for optimal grasps by deforming the hand surfaces along its feasible kinematic directions and matching towards the surface of the object, given the assumption that the large matching area produces more stable and powerful grasp. With the planned grasp configurations, the grasp imagination optimizes the robot-finger trajectories to reach the target grasps given the point cloud representation of the objects under uncertainties.

The contributions of this paper are as follows. First, the grasp planning is able to find grasps with plausible surface fitting performance efficiently. The average optimization time is 0.40 sec/grasp using the raw point cloud captured by stereo cameras. By optimizing the palm pose and finger joints iteratively, the planning algorithm is able to implement on the hands with multiple DOFs. Secondly, the collision is penalized by the gradient-based methods directly, instead of being pruned after the optimization as [12], [18]. Furthermore, the proposed method can generate both the power grasps and precision grasps by adjusting the fitting weights of fingertips. Finally, the proposed grasp imagination is able to plan collision free finger trajectories in 0.61 sec/grasp with the imperfect point cloud and underlying uncertainties.

The reminder of the paper is as follows. Section II describes the problem formulation, followed by the proposed grasping framework in Section III. The grasp planning and grasp imagination are introduced in Section IV and Section V respectively. The experimental results on a multi-fingered hand are introduced in Section VI. Section VII concludes the paper and describes the future work. The experimental videos are available at [1].

## II. PROBLEM STATEMENT

With the surface contact, the grasp planning for a multi-fingered hand problem can be formulated as:

\[
\begin{align*}
\max_{R, t, \delta q, S^f, S^o} & \quad Q(S^f, S^o) \\
\text{s.t.} & \quad S^f \subset T(\partial F; R, t, \delta q), \\
& \quad S^o = NN_{\partial O}(S^f), \\
& \quad \text{dist}(T(\partial F; R, t, \delta q), \partial O|G) \geq 0, \\
& \quad q_0 + \delta q \in [q_{\min}, q_{\max}],
\end{align*}
\]

where \( R \in SO(3), t \in \mathbb{R}^3 \) denote the rotation and translation of the hand palm, \( q_0 \in \mathbb{R}^{N_{jnt}} \) denotes the joint angle, with \( N_{jnt} \) representing the number of joints, and \( q_0 \) and \( \delta q \) represent the original and displacement of \( q \), \( S^f = [S^f_1, \ldots, S^f_{N_{cnt}}], S^o = [S^o_1, \ldots, S^o_{N_{cnt}}] \) are contact surfaces for all fingers/palms and objects, with \( S^f_i \) and \( S^o_i \) representing the \( i \)-th contact surface on the finger/palm and object, and \( N_{cnt} \) denoting the number of contact surfaces. \( Q \in \mathbb{R} \) represents the grasp quality related to \( S^f, S^o \). Constraints \((1b)\) shows that \( S^f \) is a subset of the surface transformed from the hand surface \( \partial F \) by \((R, t, \delta q)\). Constraint \((1c)\) denotes \( S^o \) is computed from the nearest neighbor \((N.N)\) of \( S^f \) on object surface \( \partial O \). Constraint \((1d)\) denotes that the transformed hand surface \( \partial F \) should not collide with the object \( \partial O \) and ground \( G \), and \((1e)\) indicates that \( q \) stays in \([q_{\min}, q_{\max}]\).

Problem (1) would be a standard grasp planning problem if all contact surfaces were degenerated into contact points. In general case, however, the problem is challenging to solve by either the sampling based methods or gradient based methods considering the inverse kinematics (IK) and collision detection with the objects of complex shapes.

We observe that human tends to match the contact surfaces during grasping in order to increase the force exerted on the object and improve the robustness to uncertainties. Therefore, the grasp quality \( Q \) is chosen as the surface fitting error between the hand contact surface \( S^f \) and object contact surface \( S^o \). More concretely,

\[
Q(S^f, S^o) = -\text{dist}(S^f, S^o).
\]

Based on this quality formulation, this paper introduces a framework to plan and execute the grasps. The technical details are explained in the following sections.

## III. THE GENERAL PLANNING FRAMEWORK

This paper introduces an optimization-based framework to plan the desired grasps and optimize robot-finger trajectories to execute the grasps. The framework is illustrated in Fig. 1. It consists of three main blocks: grasp planning by the multi-dimensional iterative surface fitting (MDISF), the grasp imagination by the grasp trajectory optimization (GTO) and the guided sampling.

Starting with an initial configuration, MDISF fits the hand surface onto the object by optimizing the palm transformation \((R, t)\) and joint displacements \(\delta q\). The registration considers the deformation from the feasible hand motion and the collision between the hand and the environment. Compared with non-rigid registration [19], [20], MDISF only deforms in the feasible directions of the joint motion. Compared with the ISF algorithm [12], MDISF is able to plan grasps for the hands with multiple DOFs, and the collision between the hand and the object/ground is penalized directly in the optimization.

The grasp imagination is to evaluate the planned grasps and generate robot-finger trajectories to execute the highly ranked grasps. The GTO algorithm is proposed to avoid collision with the environment and plan optimal finger trajectories using the incomplete point cloud and various types of uncertainties.

The guided sampling is introduced to avoid trapped in bad-performed local optima by prioritizing different initial palm placements, so that the regions with smaller fitting errors and better collision avoidance performance can be sampled more often. The introduction of guided sampling is in [12].

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**Fig. 1.** Illustration of the grasp planning and execution framework.
The MDISF algorithm and the grasp imagination will be introduced below.

IV. MULTI-DIMENSIONAL ITERATIVE SURFACE FITTING

With the surface fitting score (2) as the quality, Problem (1) can be solved with the proposed multi-dimensional iterative surface fitting (MDISF) algorithm. Similar to iterative closest point (ICP) [21], MDISF iterates between the correspondence matching and surface fitting, as shown by Fig. 2. To employ the gradient in improving the searching efficiency, the hand surface $\partial F$ is discretized into points $\{p_i, n_i^p\}_{i=1}^{N_p}$ and bounding boxes $\{B_k\}_{k=1}^{N_b}$, where $p_i \in \mathbb{R}^3$, $n_i^p \in \mathbb{S}^2$ represent the point position and normal vector pointing outward, and $N_p, N_b$ are the total number of hand surface points and boxes to cover the surface, as shown in Fig. 2(a). Only the front surface is sampled for simplification. Similarly, the object surface $\partial O$ is discretized into points $\{q_i, n_i^q\}_{i=1}^{N_q}$, where $q_i \in \mathbb{R}^3$, $n_i^q \in \mathbb{S}^2$ represent the point position and normal vector pointing outward, and $N_q$ is the total number of points on the object point cloud.

The correspondence matching finds the paired points $\{q_i, n_i^q\}_{i=1}^{N_q}$ and $\{p_i, n_i^p\}_{i=1}^{N_p}$ on object point cloud by the nearest neighbor search with duplicate/outlier removal [22]. The surface fitting minimizes the distance between the point pairs $\{p_i, q_i\}_{i=1}^{N}$, as shown in Fig. 2(b). With the point representation of the surfaces, the surface fitting error $E_{fit}$ is re-formulated as

$$E_{fit}(R, t, \delta q) = \sum_{i \in \mathcal{I}} \left(\tilde{p}_i - q_i\right)^T n_i^q + \alpha^2 \left(R n_i^p\right)^T n_i^q + 1)^2$$

where $\tilde{p}_i = R p_i + t + R \mathcal{J}_i(q) \delta q$ describes the hand surface point after the palm transformation and finger displacement, and $\mathcal{J}_i(q)$ is the Jacobian matrix at the point $p_i$ with the joint $q$. The first term describes the point distance projected to the object surface normal direction. This point-to-plane distance is broadly used in ICP [23] to allow sliding on flat surface, so that the algorithm is not sensitive to incomplete point cloud. The second term describes the alignment of the normal vectors. $\alpha$ is to balance the scale of normal alignment.

The joint displacement $\delta q$ is neglected in normal alignment due to the limited performance improvement.

With the current correspondence matching and fitting error representation in (3), the Problem (1) becomes:

$$\min_{R, t, \delta q} E_{fit}(R, t, \delta q)$$

$$\text{s.t.} \quad \text{dist} \left(\mathcal{T}(\{B_k\}_{k=1}^{N_b}; R, t, \delta q), \partial O\right) \geq 0, \quad \text{(4a)}$$

$$\text{dist} \left(\mathcal{T}(\partial F; R, t, \delta q), \mathcal{G}\right) \geq 0, \quad \text{(4b)}$$

$$\delta q + q_0 \in [q_{\min}, q_{\max}], \quad \text{(4c)}$$

where (4b) represents the collision between the object $\partial O$ and the bounding boxes of the hand $\{B_k\}_{k=1}^{N_b}$, and (4c) denotes the collision between the hand surface and the ground. Equation (4) is a non-convex programming due to the coupling term $R \mathcal{J}_i(q) \delta q$ in (4a) and the collision constraints (4b, 4c).

A. Collision Handling

To address the collision term, we employ the point representation of the object $\{q_i, n_i^q\}_{i=1}^{N_q}$ and check the inclusion of the points in $\{B_k\}_{k=1}^{N_b}$. As for the hand-ground collision, we represent the ground by a point on ground $q_g$ and normal vector $n_g$, and check the ground collision by the sign of $(p_i - q_g)^T n_g$. Penalty method [24] is introduced to avoid collision and ensure that the hand can move smoothly in the space occupied by the object. More concretely, the collision error is formulated as:

$$E_{col}(R, t, \delta q) = \sum_{l \in \mathcal{L}_a} \|\tilde{p}_l - q_l\|^2 + \sum_{l \in \mathcal{L}_g} \left((\tilde{p}_l - q_g)^T n_g\right)^2$$

where $\{q_l\}_{l \in \mathcal{L}_a}$ denotes the object points that are in collision with the bounding boxes. $\{p_l\}_{l \in \mathcal{L}_a}$ denotes the corresponding points on the box front or back surfaces, and $\tilde{p}_l = R p_l + t + R \mathcal{J}_l(q) \delta q$. To ensure that (5) reduces all types of collision, we choose the front or the back surfaces that $q_l$ paired with, as shown in Fig. 3.

With the penalty method, the surface fitting (4) becomes

$$\min_{R, t, \delta q} E(R, t, \delta q)$$

$$\text{s.t.} \quad \delta q + q_0 \in [q_{\min}, q_{\max}], \quad \text{(6b)}$$

where $E(R, t, \delta q) = E_{fit}(R, t, \delta q) + w E_{col}(R, t, \delta q)$ represents the overall error during the surface fitting of the
current correspondence. \( w \) denotes the penalty weight of the collision.

**B. Iterative Palm Finger Optimization (IPFO)**

Problem (6) is a discretization of (1) under the current correspondence and collision penalty, and is solved by the iterative palm finger optimization (IPFO) revised from [12]. The IPFO algorithm iteratively optimizes the palm transformation \((R, t)\) and the finger displacements \(\delta q\).

1) **Palm Optimization**: The palm optimization searches for optimal \((R, t)\) by fixing the finger joint configuration:

\[
\min_{R, t} E(R, t, 0) = \min_x \| Ax - b \|^2
\]

where \( x = [p^T, t^T]^T \in \mathbb{R}^6 \) is a local parameterization of the palm transformation, and \( F \) is the axis-angle vector to approximate \( R \) in small rotation angle assumption, i.e. \( R \approx I + F \), where \( \bullet \) is a skew-symmetric representation of cross product. The matrix \( A = [a_{p,1}^T, \ldots, a_{p,m}^T, \ldots, a_{p,m}^T] \in \mathbb{R}^{2(\mathcal{I} \cup \mathcal{L}) \times 6} \) with \( a_{p,i} = (p_i \times n_i)^T \) as the point-to-plane fitting error, and \( a_{n,i} = \alpha(n_i \times n_i)^T, 0_3^T \) as the normal alignment error. \( a_{c,\text{obj},l} \) includes hand-object collision \( b_{\text{obj},l} = w_p[p - q_i] \) and hand-ground collision \( b_{\text{gnd},l} = w_p[p - q_i] \) as the point-to-plane fitting error, and \( a_{n,i} = \alpha(n_i \times n_i)^T, 0_3^T \) as the normal alignment error.

\[
\text{Equation (7) is a least squares problem and is solved analytically by:}
\]

\[
x^* = (A^T A)^{-1} A^T b
\]

2) **Finger Optimization**: The finger optimization fixes the palm transformation \((R, t)\) and searches for optimal finger displacements \(\delta q\):

\[
\begin{align}
\min_{\delta q} E(R*, t*, \delta q) &= \min \| C\delta q - d \|^2 \\
\text{s.t.} & \quad \delta q + q \in [q_{\min}, q_{\max}],
\end{align}
\]

where \( C = [c_{p,1}^T, \ldots, c_{p,m}^T, \ldots, c_{p,m}^T] \in \mathbb{R}^{2(\mathcal{I} \cup \mathcal{L}) \times N_{\text{jnt}}} \), with \( c_{p,i} = (n_i^T)^T J_i(q) \) as the point-to-plane fitting error, \( c_{c,\text{obj},l} \) includes hand-object collision \( c_{\text{obj},l} = w^R c_{\text{obj},l}(q) \) and hand-ground collision \( c_{\text{gnd},l} = (n_i^T)^T J_i(q) \). Similarly, \( d = \frac{d_{p,1}^T, \ldots, d_{p,m}^T, \ldots, d_{p,m}^T} \in \mathbb{R}^{2(\mathcal{I} \cup \mathcal{L}) \times 6} \), with \( d_{p,i} = R^* p_i + t* - q_i \) and \( d_{gnd,l} = w_p[R^* p_i + t* - q_i] \).

\[
\text{Equation (9) is a least-squares problem with box constraints, and is solved by initializing } \delta q_0 = \frac{C^T C}{\gamma} \frac{C^T d}{\gamma} \text{ and iterating between}
\]

\[
\begin{align}
\delta q_m &= \delta q_{m-1} - \gamma C^T (C\delta q_{m-1} - d) \\
\delta q_{m+1} &= \max(\min(\delta q_m, q_{\max} - q), q_{\min} - q)
\end{align}
\]

until converge, where \( \gamma \) is the step size for gradient descent and is set as 0.1 \( N_{\text{jnt}} \), \( \gamma = \sqrt{\text{trace}(C^T C)} \). Equation (10) is able to converge around 10 ~ 50 iterations.

IPFO is summarized in Alg. (1). The Alg. (1) feeds as inputs \( \partial \bar{F} \) represented by \( \{p_i, n_i\}_{i=1}^{N_{\text{jnt}}} \), \( \partial \bar{D} \) represented by \( \{q_i, n_i\}_{i=1}^{N_{\text{jnt}}} \), the surface fitting indices \( \mathcal{I} \) and collision avoidance indices \( \mathcal{L} \). The corresponding points for fitting and collision are then sampled in Line (4-5). The palm optimization and finger optimization are shown in Line (6-10).

**Theorem 1**: The IPFO algorithm in Alg. (1) converges to a local optimum of Problem (6).

**Proof**: The convergence of IPFO is proved based on the global convergence theorem [24]. First, \( R, t, \delta q \in D = SE(3) \times \mathbb{R}^{N_{\text{jnt}}} \) is a compact set. Second, the function \( E(R, t, \delta q) \) in (6) is a continuous function. With the construction of IPFO in Line (6-7) of Alg. (1), we claim that the function \( E(R, t, \delta q) \) is a local optimum of Problem (6). Lastly, the IPFO algorithm computed by the palm optimization \( PO \) (Line 6) and finger optimization \( FO \) (Line 7) is a closed mapping, since \( PO \) is continuous and point-to-point, and \( FO \) is closed in \( PO(R, t, \delta q) \). Therefore, IPFO described by Alg. (1) converges to a local optimum under the current correspondence. **[End of Proof]**

**C. Fitting Weights Reshaping**

The current MDISF algorithm assumes that all points have equivalent importance. With this assumption, MDISF may produce unsatisfying power grasps which either prevent the hand from closing fingers if it matches to the region close to hinge, or easily collide with the ground if the object is flat. To generate natural power grasps, we shape the weights of points on different regions of the hand surface with Gaussian, as shown in Fig. 4(a). With this shaping, the central regions of palms and links are emphasized since these regions have better robustness to uncertainties and allow large-scale joint motion. To produce precision grasps for flat objects, we emphasize the fitting of points on fingertips, as shown in...
The correspondence (Line 6-7) and searching optimal hand configuration hierarchically using the multi-resolution pyramid, as shown in Alg. (2). MDISF iterates between matching the correspondence (Line 6-7) and searching for optimal transformation and finger displacements \( R, t, \delta q \) with IPFO (Line 8).

V. GRASPING IMAGINATION

In this section, the found grasps are first ranked based on the proposed quality metric, after which the grasp trajectories are planned to reach the highly ranked grasps.

1) Grasp Quality Evaluation: The grasp quality is evaluated based on the grasp wrench space (GWS) [25]. In this paper, GWS \( \mathcal{P} \) is constructed by 1) finding contact points by the nearest neighbor of the final hand surface on the object, 2) removing the contacts with large normal alignment error, 3) extracting the center points and the average normals by K-means, and 4) building \( \mathcal{P} \) based on the extracted grasp points and normals using the soft finger model [26]. With the GWS \( \mathcal{P} \), three quality features are calculated. The first feature \( Q_{\text{is}} \) is a bool type variable indicates the ability to resist arbitrary small disturbance by checking the inclusion of origin in \( \mathcal{P} \). The second feature \( Q_{\text{col}} \) indicates the magnitude of disturbance resistance by computing the volume of \( \mathcal{P} \). The third feature \( Q_{\text{cond}} \) indicates the isotropy of the disturbance resistance by the condition number of the \( WW^T \), where \( W \in \mathbb{R}^{6 \times N_f} \) is the vertex matrix of convex hull \( \mathcal{P} \).

The final grasp quality metric \( Q_{\text{gsp}} \) is represented as:

\[
Q_{\text{gsp}} = Q_{\text{col}} + \frac{3}{Q_{\text{cond}}} + 11Q_{\text{in}} \tag{11}
\]

The parameters of \( (11) \) is obtained by regression using the standard Ferrari-Canny metric [27] on 200 grasps from 10 objects. Equation \( (11) \) is able to rank the found collision-free grasps more efficiently than the Ferrari-Canny metric with comparable accuracy. Compared with Ferrari-Canny metric, the computation time of \( (11) \) reduced by 98.77\% from 2.43 secs/grasp to 0.034 sec/grasp. The top-1 score is 70\% and top-3 score is 90\%, out of 20 classes to be ranked. Highly ranked grasps are fed for trajectory generation.

2) Grasp Trajectory Optimization: This paper presents a two-step procedure to plan the robot-finger trajectories. First, the hand keeps half-closed and approaches the pre-grasp pose with [28]. The object is represented by its bounding box in this step. The pre-grasp position is defined by lifting the hand 0.3 m from the final grasp, and the rotation is defined as the closest canonical orientation from the final grasp pose. Second, we optimize for the finger trajectories while predefining the palm trajectory by interpolation. The grasp trajectory optimization (GTO) becomes:

\[
\min_{q_1, \ldots, q_S} \sum_{s=1}^{S-1} \| q_{s+1} - q_s \|^2 \tag{12a}
\]

s.t.
\[
dist (\mathcal{T}(\partial \mathcal{F}_s; q_s - q_0^s), \partial \mathcal{G}) \geq 0, \tag{12b}
\]
\[
|q_{s+1} - q_s| \leq \Delta_q, \quad s = 1, \ldots, S - 1 \tag{12c}
\]
\[
q_s \in [q_{\min}, q_{\max}], \quad s = 1, \ldots, S \tag{12d}
\]
\[
q_1 = q_{\text{pregrasp}}, \quad q_S = q_{\text{final}}. \tag{12e}
\]

where \( s \) is the sample index, \( S \) is the number of samples on the trajectory, and \( \mathcal{T}(\partial \mathcal{F}_s; q_s - q_0^s) \) denotes the transformed hand surfaces after the joint displacement at the \( s \)-th sample. Optimization \( (12a) \) is to minimize the total length of the trajectory \( (12a) \) from the pre-grasp to final grasp \( (12e) \) and avoid collision with both the object and ground \( (12b) \).

Similar to MDISF, the collision constraints are penalized in the cost. We adopt the formulation in TrajOpt [16]:

\[
\text{col term} = |\text{safe} - sd(\partial \mathcal{G}, \mathcal{T}(\partial \mathcal{F}_s; q_s - q_0^s))| \tag{13}
\]

where \( \text{safe} \) denotes the safety distance, \( sd(A, B) \) denotes the signed distance between A and B, and \( |x| = \max(x, 0) \).

We propose an approach to compute the signed distance \( sd(\partial \mathcal{G}, \mathcal{T}(\partial \mathcal{F}_s; q_s - q_0^s)) \) in absence of the 3D mesh and convex decomposition of the object, as shown in Fig. 5.

We first inflate the bounding boxes \( \{B_k^i\}_{k=1}^{N_k} \) at sample \( s \)
by \( d_{\text{check}} \) and check the inclusion of object points. For each interior point \( q_{k,s} \), we calculate the signed distance by projecting \( q_{k,s} \) to surfaces of \( B_k^s \) and filtering out the points with \( (q_{k,s} - p_i)^T n_i^s < 0 \) for those \( q_{k,s} \in B_k^s \). The closest point to \( q_{k,s} \) is denoted as \( p_i^s \), as shown in Fig. 3(a). The point-box signed distance \( sd(B_k^s, q_{k,s}) = (p_i^s - q_{k,s})^T n_i^s \) and \( n_i^s \) is a normal vector with direction \( q_{k,s} - p_i^s \) if \( q_{k,s} \in B_k^s \) or reverse otherwise. Therefore, \( sd(B_k^s, p_i^s) = \min_j sd(B_k^s, q_{j,k}) \) with the critical index \( l_k^s = \text{argmin}_i sd(B_k^s, q_{j,k}) \), as shown in Fig. 3(b). The hand-object collision indexes \( L_{s,o} = \{l_k^s \}_{k=1}^{N} \). Similarly, the hand-ground collision indexes \( L_{s,g} \) includes all the points that have potential collision with ground.

With \( L_{s,o}, L_{s,g} \), the collision for the \( s \)-th sample is penalized as:

\[
E_{\text{col},s} = \sum_{l_k^s \in L_{s,o}} \left( |d_{sa,fe} - (p_i^s - q_{k,s})^T n_i^s| \right)^2 + \sum_{l_k^s \in L_{s,g}} \left( |d_{sa,fe} - (p_i^s - q_{k,s})^T n_i^s| \right)^2 \tag{14}
\]

where \( p_i^s = p_i^s + J_i^s(q_0^s) \delta q_s \). Therefore, GTO \((12)\) can be reformulated as:

\[
\min_{\delta q_1, \ldots, \delta q_S} \sum_{s=1}^{S-1} \|q_{s+1}^0 + \delta q_{s+1} - q_s^0 - \delta q_s\|^2 + cE_{\text{col},s} \tag{15a}
\]

\[\text{s.t. } \begin{align*}
&|q_{s+1}^0 + \delta q_{s+1} - q_s^0 - \delta q_s| \leq \Delta_q, \quad (15b) \\
&q_0^s + \delta q_s \in [q_{\text{min}}, q_{\text{max}}], \quad s = 1, \ldots, S \quad (15c) \\
&\delta q_1 = 0, \delta q_S = 0, \quad (15d) \\
&|\delta q_s| \leq \Delta_q, \quad s = 1, \ldots, S \quad (15e)
\end{align*}
\]

Optimization \((15)\) solves for optimal joint displacements \( \{\delta q_s^S\}_{s=1}^{S} \) using the current joint samples and collided points. The \( \{\delta q_s^S\}_{s=1}^{S} \) then updates joint samples \( q_0^s \leftarrow q_0^s + \delta q_s^S \), hand surfaces \( \partial \mathcal{F}_s \leftarrow T(\partial \mathcal{F}_s, \delta q_s^S) \), collision penalty \( c \leftarrow \mu c \) and indexes \( L_{s,o}, L_{s,g} \). Optimization \((15)\) iterates until no collision or reaching the maximum iterations.

VI. SIMULATION AND EXPERIMENT

This section presents the simulation and experiment. The experimental videos are available at [1].

A. Parameter Lists

For MDISF, \( \alpha = 0.03 \). \( N_p = 450, N_b = 7, L = 4, I_0 = 200, \epsilon_0 = 0.02 \). IPFO used \( \Delta = 1e-5, T_{\text{max}} = 20 \). The power grasp used Gaussian \( \Sigma = \text{diag}(l/2, w/0.1) \) with mean at the link center, where \( l, w \) are the link length and width, and the base weights for palm, proximal and distal link were 0.1, 0.1, 1. The precision grasp used Gaussian \( \Sigma = \text{diag}(l/5, w/0.1) \) with mean at the fingertip and base weights 0.01, 0.01, 0.1.

As for GTO, \( d_{\text{check}} = 0.03 \) m and \( d_{sa,fe} = 0.01 \) m, \( S = 30, \Delta_{\text{by}} = [0.2, 0.2, 0.2, 0.4], \Delta_q = [0.4, 0.4, 0.4, 0.4, 0.4] \). The maximum iterations for \((15)\) was 20. The starting collision penalty \( \epsilon_0 = 1 \) and \( \mu = 2 \).

B. Simulation and Experiment Results

The simulation was conducted on a desktop with 32GB RAM and 4.0GHz CPU. The grasps were computed by Matlab and visualized by V-REP. The BarrettHand BH8-282 was used to test the effectiveness of the algorithm. The visualization of the MDISF iterations are shown in Fig. 6. MDISF considers both the collision avoidance and the surface fitting in each IPFO iteration. MDISF started from a random pose around the object (Fig. 6(b-h)) and optimized for palm pose and joint displacements to reduce the fitting error and penalize the collision (Fig. 6(a) and optimized for palm pose and joint displacements to reduce the maximum iterations.

![Fig. 6. Visualization of MDISF iterations on a dragon object. (a) Initial hand configuration. The initial pose has collision and large fitting error. (b-h) Error reduction in different MDISF iterations.](image)

![Fig. 7. Profile of the error reduction during MDISF. (Left) Overall error, (Middle) Average surface fitting error after outliers/duplicate removal, (Right) Collision error without multiplying the penalty weight.](image)
TABLE I
NUMERICAL RESULTS OF THE GRASPING FRAMEWORK

| Object | collision-free# | Time (secs) | Qualities | $Q_{gsp}$ |
|--------|-----------------|-------------|-----------|-----------|
|        | total samples   | ISF | GTO | Sum | FC |
| Bunny  | 8/10 4/5        | 2.8 | 3.2 | 6.0 | 7/8 | 6.86 |
| Screwdriver | 9/10 1/5  | 2.2 | 3.0 | 5.2 | 9/9 | 9.56 |
| Gun    | 2/10 1/2       | 2.5 | 1.8 | 4.3 | 2/2 | -9.41 |
| Kettle | 8/10 5/5        | 2.1 | 2.2 | 4.3 | 8/8 | 3.39 |
| Goblet | 10/10 5/5       | 2.4 | 1.7 | 4.1 | 10/10 | 13.66 |
| Dorae mon | 9/10 5/5      | 2.1 | 1.6 | 3.7 | 9/9 | 9.27 |
| Hand   | 1/10 1/1       | 2.3 | 0.6 | 2.9 | 1/1 | -12.32 |
| Banana | 4/10 3/4        | 2.2 | 2.5 | 4.7 | 4/4 | -1.71 |
| Mug    | 8/10 5/5        | 3.0 | 2.2 | 5.2 | 8/8 | 8.18 |
| Oscar  | 3/10 3/3        | 2.9 | 1.4 | 4.3 | 3/3 | -6.80 |
| Average|                 | 2.45| 2.02| 4.47| 2.068|

Fig. 8. Visualization of the MDISF on ten objects.

The $Q_{gsp}$ reflects the graspability (difficulty of being grasped) of the objects with the Barrett hand. Goblet and screwdriver were the top-2 objects with the highest graspability due to the proper size and simple structure. Hand model and gun were the top-2 objects with the lowest graspability since they are flat and close to ground, and Barrett hand can easily collide with ground/object and get trapped in the infeasible local optima.

Figure 9 compares the precision grasps and power grasps generated by MDISF. The precision mode and power mode generated 8 and 5 collision-free grasps out of 10 samples on Bunny object, respectively. The hand tended to collide with the object in power grasp mode since the fitting of palm/proximal links were emphasized and hand stayed closer to object, as shown in Fig. 9(b).

Figure 10 shows the result of GTO on kettle object. The trajectory started from the top and reached the desired grasp with one finger in narrow space. The fingers collided with the object when grasping with the predefined finger motion, as shown in Fig. 10(Top). GTO planned finger trajectories to avoid collision and reached the target grasp in narrow space, as shown in Fig. 10(Bottom).

Figure 11 shows the experimental results using the FANUC LRMate 200iD/7L manipulator and BarrettHand BH8-282 on ten objects. The scene was captured by two IDS Ensenso N35 stereo cameras. The observed point cloud and the optimized grasp are shown in the left, and the executed grasp after GTO is shown in the middle. Extra amount of finger motion was executed in order to provide necessary force to clamp the object, as shown in the right. The observed point cloud was incomplete and noisy, and the system also contained uncertainties in calibration ($\sim 3$ mm for robot-camera frame alignment), positioning ($\sim 1^\circ$ TCP-palm alignment, $\sim 2.8^\circ$ finger joint tracking error) and communication (0.1 sec robot-hand command synchronization error). The system exhibited certain robustness and was able to plan and execute grasps under the unsatisfying point cloud and various types of uncertainties, as shown in Fig. 11(1-9).

We also include three failed grasps to reflect three failure modes, as shown in Fig. 11(10-12). The first failure mode was raised from slippage. Without force optimization, the exerted force might fall out of the friction cone, as shown in Fig. 11(10). The second failure mode was raised from the asymmetric distribution of contact forces during clamping, as shown in Fig. 11(11). The third failure mode was raised from the internal disturbance. The proximal link accidentally collided with object during the clamp stage and introduced a large disturbance, as shown in Fig. 11(12).

VII. CONCLUSION

This paper has proposed an efficient framework for grasp generation and execution. The framework includes a multi-dimensional iterative surface fitting (MDISF) and a grasp trajectory optimization (GTO). The MDISF algorithm searches for optimal grasps by minimizing the hand-object fitting error and penalizing the collision, and the GTO algorithm plans finger trajectories for grasp execution with the point cloud representation of the object. The MDISF-GTO exhibits certain robustness to the incomplete/noisy point cloud and various underlying uncertainties. In average, it took 0.40 sec for MDISF to find a collision-free grasp, and took 0.61 sec for GTO to optimize the trajectory to reach the grasp.
The current implementation clamps objects by simply closing fingers. This may cause slippage, uneven force distribution and introduce internal disturbances to the system. Future work will optimize the grasping force [29] to have better slippage resistance and disturbance robustness.

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