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Frames over finite fields: equiangular lines in orthogonal geometry. (English) Zbl 07472504
Linear Algebra Appl. 639, 50-80 (2022)

Summary: We investigate equiangular lines in finite orthogonal geometries, focusing specifically on equian-
gular tight frames (ETFs). In parallel with the known correspondence between real ETFs and strongly
regular graphs (SRGs) that satisfy certain parameter constraints, we prove that ETFs in finite orthogonal
geometries are closely aligned with a modular generalization of SRGs. The constraints in our finite field
setting are weaker, and all but 18 known SRG parameters on \( v \leq 1300 \) vertices satisfy at least one of
them. Applying our results to triangular graphs, we deduce that Gerzon’s bound is attained in finite
orthogonal geometries of infinitely many dimensions. We also demonstrate connections with real ETFs,
and derive necessary conditions for ETFs in finite orthogonal geometries. As an application, we show that
Gerzon’s bound cannot be attained in a finite orthogonal geometry of dimension 5.

MSC:
51E26 Other finite linear geometries
51A50 Polar geometry, symplectic spaces, orthogonal spaces
05E30 Association schemes, strongly regular graphs
42C15 General harmonic expansions, frames
51F25 Orthogonal and unitary groups in metric geometry

Keywords:
equiangular lines; equiangular tight frames; finite fields; strongly regular graphs

Software:
GRAPE

Full Text: DOI

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