Financial Modeling for Investment through Anticipated Annuities in Scenarios with Floating Interest Rate

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Abstract The aim of this work focuses on showing, through a hypothetical scenario with financial calculations, how to develop an investment with constant one-year deposit and reinvestment of dividends earned. To this end, the financial model includes a type of floating interest rate throughout the year, depending on the reference interest rate indicated by the "Banco de Mexico". For this reason in the hypothetical scenario we are considering changes in interest rates at the moment of reinvestment. Therefore, to make financial calculus, we utilized as reference interest rate, the interest rate TIIE (for its acronym in Spanish Interest rate of Internal equilibrium, which is set by the “Banco de Mexico”. We utilized the formula of an annuity anticipated with cohorts, depending on the changes in the interest rate TIIE. The results obtained show the behavior of the investment over a year in which the interest earned at all times is reinvested, generating an effect known in Mexico as “anatocismo” or compounded interest.

Keywords: anticipated annuities, floating rate interest, investment, savings, tiie, effective rate

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1. Introduction

Nowadays, due to globalization is very important to consider the factors which affect the value of money throughout the time. About these factors, we can mention the following: permanent changes or movement in the interest rates from different countries, which constitute a reference for the country, exchange rate, inflation and other factors involved in the valuation of several currencies.

Based on this information, several studies have addressed the topic of financial education, whose purpose has focused on assessing the level of financial literacy in the population. Regarding this, it is important to define financial literacy as "the way of acquiring knowledge and develop skills that we all need to make better financial decisions and thus, be able raise the level of social welfare in each of the stages of our life" (CONDUSEF, 2012).

Now if we consider the assumption that establishes "To higher financial knowledge, best financial decisions", several studies have shown that, an adequate financial education allows people to increase their savings (Bernheim and Garrett, 1996; Bernheim, Garrett and Maki, 2001; Clancy, Grinstein-Weiss and Schreiner., 2001; Moore et al 2001), to make lower interest payments by late payment (Hirad and Zorn 2001) and to the debt reduction (Elliehausen, Lundquist, and Staten, 2007).

The implementation of financial literacy programs has been mainly focused on promoting knowledge related to savings, the effect of inflation, personal finance planning and management of financial products and their implications of those products (Vitt et al, 2000). Similarly, the programs have focused on promoting awareness among population about the effects caused in long-term, over-indebtedness in their heritage (Cohen and Sebstad, 2003).

There are several studies that show deficient level of financial education in the population, for example, we can talk about the study conducted by Rooiji, Lusardi and Alessie (2011) in Holland, in a sample of two thousand households. This study showed that, the low level of financial literacy in the population, has caused uncertainty in people to invest in the stock market, thereby generating, lack of diversity regarding financial risk in their savings, and therefore, the people cannot get a higher return on their assets, as not widely used financial instruments offered by banks and financial institutions.

Similarly, among studies that have been done in Mexico, in March 2008, the Universidad Nacional Autónoma de México (National Autonomous University of Mexico, UNAM) in partnership with Banamex, carried out the first survey about Financial Culture in Mexico; the results showed that there were gaps in financial literacy, educational backwardness in financial matters (mainly rural population and lower educational sectors and
socioeconomic status), an absence in the planning of personal finance, lack of control of expenditure, income and debt; lack of savings and no knowledge regarding the calculation of interest rates (UNAM, Banamex, 2008).

Certainly, regarding the topic of financial education is very important to talk any time about their: progress, achievements, weaknesses, gaps, failures and all that surrounds this phenomenon of study, however, is during times of crisis, when it should give more importance to the aspects of financial education. We should use the evidence which has been obtained which has been reported in several studies about those lags or remaining gaps in financial education, be able to set, plans or actions to follow, in order to propose advice on what could be done in case of: the household debt or unexpected losses in the value of their financial assets, all this, due to the inadequate selection of financial instruments without a prior analysis of their needs and possibilities.

In this sense, it is very important to encourage –in the economically active population- a savings culture as a way of prevention and become in habit, as a medium to achieve goals, both personal and family (Carrillo, 2008).

On the basis of the need to expose the impact of interest rates on regular saving, we present a financial modeling in one case of investment in a year:

2. Statement of Problem

In Mexico there has been a common practice in the culture of savings relative to investment or savings in several families of the population. On this practice gets used withdraw dividends earned in each period, i.e. does not reinvested. But, what happen if the interest earned are reinvested? In a hypothetical scenario in which saver (regularly a person who saves money through a bank) has the habit of depositing constantly to increase investment, should also get used to reinvest dividends earned and thereby, at the end of some period, be able to reach accumulate a greater amount as a result of this new practice of saving.

Is for the above mentioned that we propose an investment scenario to one year, with interest floating rate. To do this, we considered the next data:

Table 1. Data to financial modeling

| Periodical quote or rent | $1,000.00 |
|-------------------------|----------|
| Investment period       | 365 days |
| Floating Interest rate  | 28 days TIIE |
| Capitalization          | every 30 days |

Firstly, we should become the interest rate TIIE to effective interest rate, to do this; we utilized the formula (1)

\[ Te = \left( 1 + \frac{i_{tie}}{365*m_c} \right)^{n_t/m_c} - 1 \times 100 \]  

Where:
- **TIIE**: 28-day (is the reference interest rate proposed by Banco de México)
- **Te**: Annual effective rate,
- **i_{tie}**: 28 days TIIE rate,
- **n_t**: period of investment,
- **m_c**: type of capitalization.

To obtain \( i_{tie \ monthly} = \frac{i_{tie}}{365*m_c} \)

Time line of anticipated annuities:

![Time line of anticipated annuities](image)

\[ FV = P_q \left( 1 + \frac{i_{er \ * 30}}{365} \right) \left[ 1 + \frac{i_{er \ * 30}}{365} \right]^{n_t/m_c} - 1 \]  

Where:
- **FV**: Future value,
- **P_q**: Periodical quote or rent,
- **i_{er}**: effective interest rate,
- **m_c**: capitalization,
- **n_t**: period of investment

After to calculate the next value (\( FV_2 \)) we use the next formula:

\[ FV_2 = FV_1 \left( 1 + \frac{i_{er \ * 30}}{365} \right) \left[ 1 + \frac{i_{er \ * 30}}{365} \right]^{n_t/m_c} - 1 \]  

Where:
- **FV_2**: Future value two,
- **FV_1**: Future value one,
- **P_q**: Periodical quote or rent,
- **i_{er}**: effective interest rate,
- **m_c**: capitalization,
- **n_t**: period of investment

And so it goes……..:

\[ FV_3 = FV_2 \left( 1 + \frac{i_{er \ * 30}}{365} \right) \left[ 1 + \frac{i_{er \ * 30}}{365} \right]^{n_t/m_c} - 1 \]  

Where:
- **FV_3**: Future value three,
- **FV_2**: Future value second,
- **P_q**: Periodical quote or rent,
- **i_{er}**: effective interest rate,
- **m_c**: capitalization,
- **n_t**: investment period

2.1. Development of the Proposed Case

To do this, we considering the next data:

Table 1. Data to financial modeling

| Property                      | $1,000.00 |
|-------------------------------|-----------|
| Periodical quote or rent      | 365 days  |
| Investment period             | 28 days TIIE |
| Floating Interest rate        | every 30 days |
| Capitalization                | every 30 days |

Firstly, we should become the interest rate TIIE to effective interest rate, to do this; we utilized the formula (1)
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\[ Te = \left(1 + \frac{0.03795}{365} \right)^{365} - 1 \times 100 \]

\[ = (1.00291123)^{3.0357143} - 1 \times 100 \]

\( Te_{January} = 3.86219942 \)

Table 2. THE (interbank equilibrium interest rate)

| Month   | THE 28 |
|---------|--------|
| January | 3.7950% |
| February| 3.7705% |
| March   | 3.7919% |
| April   | 3.8150% |
| May     | 3.7905% |
| June    | 3.7785% |
| July    | 3.3075% |
| August  | 3.3100% |
| September| 3.2860% |
| October | 3.2793% |
| November| 3.2800% |
| December| 3.2965% |

Source: own.

The rest of effective interest rate THE (28 days), we show in the next table:

Table 3. Effective rate THE

| Month    | 28 days THE | Effective rate THE |
|----------|-------------|--------------------|
| January  | 3.7950%     | 3.86220%           |
| February | 3.7705%     | 3.83683%           |
| March    | 3.7919%     | 3.85899%           |
| April    | 3.8150%     | 3.88292%           |
| May      | 3.7905%     | 3.85574%           |
| June     | 3.7785%     | 3.84512%           |
| July     | 3.3075%     | 3.35848%           |
| August   | 3.3100%     | 3.36105%           |
| September| 3.2860%     | 3.33631%           |
| October  | 3.2793%     | 3.32941%           |
| November | 3.2800%     | 3.33013%           |
| December | 3.2965%     | 3.34714%           |

Source: own.

Once we have effective interest rates, we can calculate the future values for the 12 months as follows:

\[ FV = P \left( 1 + \frac{i_{er}}{365} \right)^{m_c} \]

\[ \left( 1 + \frac{i_{er}}{m_c} \right)^{n} \left( 1 + \frac{i_{er}}{365} \right)^{n} \]

\[ FV_1 = P \left( 1 + \frac{i_{er}}{365} \right)^{m_c} \]

\[ \left( 1 + \frac{i_{er}}{m_c} \right)^{n} \left( 1 + \frac{i_{er}}{365} \right)^{n} \]

\[ FV_2 = FV_1 \left( 1 + \frac{i_{er}}{m_c} \right) \left( 1 + \frac{i_{er}}{365} \right)^{m_c} \]

\[ + P \left( 1 + \frac{i_{er}}{m_c} \right)^{n} \left( 1 + \frac{i_{er}}{365} \right)^{n} \left( 1 + \frac{i_{er}}{m_c} \right)^{m_c} \]

\[ FV_3 = FV_2 \left( 1 + \frac{i_{er}}{365} \right)^{m_c} \]

\[ + P \left( 1 + \frac{i_{er}}{m_c} \right)^{n} \left( 1 + \frac{i_{er}}{365} \right)^{n} \left( 1 + \frac{i_{er}}{m_c} \right)^{m_c} \]

\[ FV_4 = FV_3 \left( 1 + \frac{i_{er}}{365} \right)^{m_c} \]

\[ + P \left( 1 + \frac{i_{er}}{m_c} \right)^{n} \left( 1 + \frac{i_{er}}{365} \right)^{n} \left( 1 + \frac{i_{er}}{m_c} \right)^{m_c} \]
\[ FV_3 = $2,015.87 + $1,003.17 \]
\[ FV_3 = $3,019.04 \]  
\[ FV_4 = FV_3 \left(1 + \frac{i_{er}}{m_c} \right)^n \]
\[ + P_q \left(1 + \frac{i_{er}}{m_c} \right)^n \left[ \frac{1}{l + \frac{i_{er}}{m_c}} \right] \]
\[ FV_4 = $3,019.04 \left(1 + \frac{0.0388292 \times 30}{365} \right)^i \]
\[ + $1,000.00 \left(1 + \frac{0.0388292 \times 30}{365} \right)^i \]
\[ \times \left[ \left(1 + \frac{0.0388292 \times 30}{365} \right)^i - 1 \right] \left( \frac{0.0388292 \times 30}{365} \right) \]
\[ FV_5 = $4,031.86 \left(1 + \frac{0.0384512 \times 30}{365} \right)^i \]
\[ + $1,000.00 \left(1 + \frac{0.0384512 \times 30}{365} \right)^i \]
\[ \times \left[ \left(1 + \frac{0.0384512 \times 30}{365} \right)^i - 1 \right] \left( \frac{0.0384512 \times 30}{365} \right) \]
\[ FV_5 = $4,044.65 + $1,000.00 \times (0.0384512) \]
\[ FV_5 = $5,047.82 \]

\[ FV_6 = FV_5 \left(1 + \frac{i_{er}}{m_c} \right)^n \]
\[ + P_q \left(1 + \frac{i_{er}}{m_c} \right)^n \left[ \frac{1}{l + \frac{i_{er}}{m_c}} \right] \]
\[ FV_6 = $5,047.82 \left(1 + \frac{0.0384512 \times 30}{365} \right)^i \]
\[ + $1,000.00 \left(1 + \frac{0.0384512 \times 30}{365} \right)^i \]
\[ \times \left[ \left(1 + \frac{0.0384512 \times 30}{365} \right)^i - 1 \right] \left( \frac{0.0384512 \times 30}{365} \right) \]
\[ FV_6 = $5,063.77 + $1,000.00 \times (0.0384512) \]
\[ FV_6 = $6,066.93 \]

\[ FV_7 = FV_6 \left(1 + \frac{i_{er}}{m_c} \right)^n \]
\[ + P_q \left(1 + \frac{i_{er}}{m_c} \right)^n \left[ \frac{1}{l + \frac{i_{er}}{m_c}} \right] \]
\[ FV_7 = $6,066.93 \left(1 + \frac{0.0383584 \times 30}{365} \right)^i \]
\[ + $1,000.00 \left(1 + \frac{0.0383584 \times 30}{365} \right)^i \]
\[ \times \left[ \left(1 + \frac{0.0383584 \times 30}{365} \right)^i - 1 \right] \left( \frac{0.0383584 \times 30}{365} \right) \]
\[ FV_7 = $6,666.93 \times (0.0383584) \]
\[ FV_7 = $6,066.93 \left(1 + 0.0027604 \right)^7 \]
\[ +$1,000.00 \left(1 + 0.0027604 \right)^7 \left( \frac{(1 + 0.0027604)^7 - 1}{0.0027604} \right) \]
\[ FV_7 = $6,066.93 \left(1 + 0.0027604 \right)^7 \]
\[ +$1,000.00 \left(1 + 0.0027604 \right)^7 \left( \frac{0.0027604}{0.0027604} \right) \]  \hspace{1cm} (12.1)
\[ FV_7 = $6,083.68 + $1,000.00 \left(1 + 0.0027604 \right)^7 \]
\[ FV_7 = $6,083.68 + $1,002.76 \]
\[ FV_7 = $7,086.44 \]

\[ FV_8 = FV_7 \left(1 + \frac{i_{er}}{m_c} \right)^\frac{n}{m_c} \]
\[ + P_q \left(1 + \frac{i_{er}}{m_c} \right)^\frac{n}{m_c} \left( \frac{1 - \left(1 + \frac{i_{er}}{m_c} \right)^{\frac{n}{m_c}}}{\frac{i_{er}}{m_c}} \right) \]  \hspace{1cm} (13)
\[ FV_8 = $7,086.44 \left(1 + \frac{0.0336105}{365} \right)^{30} \]
\[ +$1,000.00 \left(1 + \frac{0.0336105}{365} \right)^{30} \]
\[ \times \left[ \frac{\left(1 + \frac{0.0336105}{365} \right)^{30} - 1}{\frac{0.0336105}{365} \times 30} \right] \]
\[ FV_8 = $7,106.01 + $1,000.00 \left(1 + \frac{0.0336105}{365} \right)^{30} \]
\[ FV_8 = $7,106.01 + $1,002.76 \]
\[ FV_8 = $8,108.78 \]

\[ FV_9 = FV_8 \left(1 + \frac{i_{er}}{m_c} \right)^\frac{n}{m_c} \]
\[ + P_q \left(1 + \frac{i_{er}}{m_c} \right)^\frac{n}{m_c} \left( \frac{1 - \left(1 + \frac{i_{er}}{m_c} \right)^{\frac{n}{m_c}}}{\frac{i_{er}}{m_c}} \right) \]  \hspace{1cm} (14)
\[ FV_9 = $8,108.78 \left(1 + \frac{0.0333631}{365} \times 30 \right) \]
\[ +$1,000.00 \left(1 + \frac{0.0333631}{365} \times 30 \right) \]
\[ \times \left[ \frac{\left(1 + \frac{0.0333631}{365} \times 30 \right)^{30} - 1}{\frac{0.0333631}{365} \times 30} \right] \]
\[ FV_9 = $8,131.01 + $1,000.00 \left(1 + \frac{0.0333631}{365} \times 30 \right)^{30} \]
\[ FV_9 = $8,131.01 + $1,002.74 \]
\[ FV_9 = $9,133.75 \]

\[ FV_{10} = FV_9 \left(1 + \frac{i_{er}}{m_c} \right)^\frac{n}{m_c} \]
\[ + P_q \left(1 + \frac{i_{er}}{m_c} \right)^\frac{n}{m_c} \left( \frac{1 - \left(1 + \frac{i_{er}}{m_c} \right)^{\frac{n}{m_c}}}{\frac{i_{er}}{m_c}} \right) \]  \hspace{1cm} (15)
\[ FV_{10} = $9,133.75 \left(1 + \frac{0.0332941}{365} \times 30 \right) \]
\[ +$1,000.00 \left(1 + \frac{0.0332941}{365} \times 30 \right) \]
\[ \times \left[ \frac{\left(1 + \frac{0.0332941}{365} \times 30 \right)^{30} - 1}{\frac{0.0332941}{365} \times 30} \right] \]
\[ FV_{10} = $9,158.75 + $1,000.00 \left(1 + \frac{0.0332941}{365} \times 30 \right)^{30} \]
\[ FV_{10} = $9,158.75 + $1,002.74 \]
\[ FV_{10} = $10,161.49 \]
\[
FV_{11} = FV_{10} \left( 1 + \frac{i_{er}}{m_c} \right)^{n mc} + P_q \left( 1 + \frac{i_{er}}{m_c} \right)^{n mc} \left[ \frac{1}{m_c} - l \right]
\]
\[
FV_{11} = $10,161.49 \left( 1 + \frac{0.0333013}{365} \right)^{365} \left( 1 + \frac{0.0027371}{365} \right)^{-1} + $1,000.00 \left( 1 + \frac{0.0333013}{365} \right)^{365} \left( 1 + \frac{0.0027371}{365} \right)^{-1}
\]
\[
FV_{11} = $10,161.49 \left( 1 + \frac{0.0333013}{365} \right)^{365} + $1,000.00 \left( 1 + \frac{0.0333013}{365} \right)^{365}
\]
\[
FV_{12} = $11,192.04 \left( 1 + \frac{0.0333013}{365} \right)^{365} + $1,000.00 \left( 1 + \frac{0.0333013}{365} \right)^{365}
\]

3. Conclusions

The total amount that will be received at the end of the investment, will be $12,225.58; less $12,000.00 corresponding to all the deposits made throughout of the year, so, we would have $225.58 which correspond to interest accrued during one year of investment, therefore, we may say that average interest rate is a 1.879833% ($225.58 / $12,000.00)

As we can see, the mathematical model proposed to evaluate an investment project allow to make decision among different proposals of investment, which offer the Sistema Financiero Mexicano (México)

Following the theoretical implications, we can see in a study conducted by Bernheim, Garret and Miki (2001), who demonstrated that having an adequate financial literacy, allows to the population to increase in a higher proportion their savings, considering if the people know evaluate the effect of interest rates on savings or investment project.

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