Study of two-spring piezoelectric harvesters

N P Georgiev\textsuperscript{1} and R P Raichev\textsuperscript{2}

\textsuperscript{1}Department of Electrical Engineering Technical University of Sofia, Branch Plovdiv, 25Tsanko Dystabanov Street, Plovdiv, Bulgaria
\textsuperscript{2}Department of Mechanical Engineering Technical University of Sofia, Branch Plovdiv, 25Tsanko Dystabanov Street, Plovdiv, Bulgaria

\textsuperscript{1}nikola.georgiev@tu-plovdiv.bg
\textsuperscript{2}rpraichev@tu-plovdiv.bg

Abstract. The paper considers two piezoelectric harvesters, consisting of two springs, attached to a piezoelectric transformer, and having a common concentrated mass. Modeling has been done by means of ANSYS, whereas the horizontal deviation of the mechanical system has been obtained, and the charge, accumulated on the electrodes of the piezo plate has been calculated. An equivalent circuit has been drawn and experimental studies have been conducted in order to validate the developed model.

1. Introduction
More and more frequently harvesters replace in practice batteries and accumulating power-supplying low-consumption electronic devices. [1]. The main advantage of these devices is that they do not require maintenance and can be used in large-scale infrastructure projects, where they convert mechanical vibrations into electrical energy [2]. Piezoelectric harvesters are among the most widely used type of them, as their construction is simple and reliable, on the one hand, and due to their small size and low price, on the other hand [3].

This paper studies and models two piezoelectric harvesters, consisting of piezoelectric transformers, rigidly fixed at their low end and having two springs with a common concentrated mass, fixed at the top. One of the harvesters has smaller springs and a smaller concentrated mass than the other. Modelling of the piezoelectric two-spring harvesters has been done by means of ANSYS R19.1 and experimental tests have been carried out to validate the models.

2. Exposition
Figure 1 presents the construction scheme of the two-spring piezoelectric harvester with the smaller springs, while Figure 2 illustrates the construction of the one with the bigger springs. The following notations are used in the schemes: 1–piezo plate; 2-brass base; 3-steel springs; 4-concentrated mass.

The piezo plates are disk-type, made of piezoelectric ceramics PZT 5A, with a diameter $D_p=20\text{mm}$ and thickness $T_p=0.3\text{ mm}$; the brass base is rectangular in shape and measures $28\times20\times0.3\text{ mm}$. The springs are two types. For the first harvester they are smaller, with wire diameter $d_s=0.4\text{ mm}$, spring diameter $D_s=4.3\text{ mm}$ and length $l_s=25\text{ mm}$, while for the second harvester they are bigger, with wire diameter $d_s=0.8\text{ mm}$, spring diameter $D_s=7.2\text{ mm}$ and length $l_s=25\text{ mm}$. The concentrated mass for the first harvester is made of steel and weighs 2, 4 and 6 grams, and for the second it weighs 3, 5, 7 and 10.5 grams, respectively.
Both studied harvesters are non-linear mechanical oscillating systems. During their simulation by means of ANSYS R19.1 the following parameters have been taken into account: the way of fixing, the influence of the force of weight of the concentrated mass, as well as the mechanical characteristics of the used springs, piezo plate and brass base. The horizontal deviation $x$ of the mechanical system, consisting of the concentrated mass, the springs, and the piezo-plate, glued to a brass base, has been obtained and presented for the harvester with the smaller springs in Figure 3, for the harvester with the bigger springs - in Fig.4. The horizontal deviation of the piezo plate, obtained during the simulation, is given in Fig. 5, while Figure 6 shows the minimum, average and maximum horizontal deviations of the piezo plate, calculated by means of ANSYS for the two-spring harvester with the smaller springs.
Figure 5. Simulation of the horizontal mechanical deviations of the piezo plate for the harvester with the smaller springs.

| Time [s] | Minimum [m] | Maximum [m] | Average [m] |
|----------|-------------|-------------|-------------|
| 1        | 2.9947e-005 | 1.0431e-003 | 4.6797e-004 |

Figure 6. Horizontal deviations of the piezo plate for the harvester with the smaller springs.

Figure 7. Simulation of the horizontal mechanical deviations of the piezo plate for the harvester with the bigger springs.

| Time [s] | Minimum [m] | Maximum [m] | Average [m] |
|----------|-------------|-------------|-------------|
| 1        | 2.4557e-004 | 2.0107e-003 | 1.1098e-003 |

Figure 8. Horizontal deviations of the piezo plate for the harvester with the bigger springs.

Figure 7 shows the horizontal deviation of the piezo plate for the harvester with the big springs, obtained during the simulation, while Figure 8 illustrates the minimum, average and maximum deviations of the piezo plate for the same harvester, calculated by means of ANSYS.
The force, acting on the middle area of the piezo plate is proportional to the average horizontal deviation \( x_{avr}(t) \), calculated by means of ANSYS, where \( r_p \) is the radius of the piezo plate, \( Y \) is the Young modulus and \( J \) is the moment of inertia [4]

\[
F(t) = \frac{3YJ}{r_p^3} x_{avr}(t)
\]  

(1)

The average horizontal deviation \( x_{avr}(t) \) is a variable, changing in a sinusoidal way, with an amplitude \( x_m \) and circular frequency \( \omega \)

\[
x_{avr}(t) = x_m \sin \omega t
\]  

(2)

With the help of the force \( F(t) \), acting on the middle area of the piezo plate, the idle run voltage on the electrodes is defined [5], where \( g_{31} \) is the transverse voltage constant

\[
u(t) = \frac{g_{31}}{r_p} F(t)
\]  

(3)

By means of (1), (2) and (3) the voltage at idle run of the piezo plate is found

\[
u(t) = \frac{3g_{31}YJ}{r_p^4} x_m \sin \omega t
\]  

(4)

The amplitude of the voltage at idle run \( U_m \) can be calculated with the help of the amplitude of the average horizontal deviation \( x_m \)

\[
U_m = \frac{3g_{31}YJ}{r_p^4} x_m
\]  

(5)

The maximum charge, accumulated on the electrodes of the piezo plate with capacity \( C_p \), is equal to

\[
Q_m = C_p U_m = \frac{3g_{31}YJ C_p}{r_p^4} x_m
\]  

(6)

The equivalent electric circuit of the piezo plate is presented in Figure 9, where \( J_p \) denotes the source of electricity, proportional to the accumulated charge, \( C_p \) is the capacity of the piezo plate, and \( R_L \) is the active load, connected to the harvester.

\[
J_p = j\omega Q_m
\]  

(7)

Figure 10 presents the equivalent electric circuit of the piezoelectric harvesters with a voltage doubling amplifier at active load. By \( R_L \) here the active load resistance is denoted, and \( U \) is the rectified voltage.
The active power at direct-current mode is calculated with the help of the charge on the electrodes of the piezo plate \( Q \), the voltage on the germanium diode \( U_D \) and the parameters of the equivalent circuit.

\[
P_L = \frac{1,41}{R_L} \left( \frac{\omega Q R_L \sqrt{1 + (\omega C_p R_L)^2} - U_D}{} \right)^2 \frac{1}{R_L} \tag{8}
\]

3. Experimental studies

Figure 11 presents the characteristics of the relationship between the rectified voltage at idle run and the frequency for the harvester with the smaller springs at concentrated masses of 2, 4, and 6 grams. It can be seen from the characteristics that with the increase in the concentrated mass the resonant frequencies of the harvesters decrease, and the maximum effective voltages at idle run also decrease. The harvester with a concentrated mass of 2 grams has the highest voltage and two resonant frequencies (of 18 and 6 Hz) are obtained for it.

Figure 12 presents the characteristics of the relationship between the rectified voltage at idle run and the frequency of the harvester with the bigger springs at a concentrated mass of 3, 5, 7 and 10.5 grams, respectively. It can easily be seen from the graph that when the concentrated mass increases, the basic resonant frequencies, for which there are maximum voltages at idle run, decrease. For these frequencies together with the increase in the mass, the voltages at idle run go down. Second resonant frequencies appear for harvesters with big springs, which are lower than the basic ones and
have lower voltages at idle run. A harvester with a concentrated mass of 3.5 grams at a frequency of 16 Hz has the highest voltage.

![Graph showing voltage at idle run vs. frequency for different masses](image1)

**Figure 12.**

Figure 13 shows the characteristics of both the measured and the calculated in the process of modeling direct-current powers for the harvester with the smaller springs at the three different weights of the concentrated mass: 2 grams and resonant frequency $f=18$ Hz; 4 grams and resonant frequency $f=8$ Hz; and 6 grams and resonant frequency $f=7$ Hz. It can be seen from the characteristics that together with the increase in the concentrated mass both the measured and the obtained by modeling powers decrease. When the active load increases, the output powers also go up.

![Graph showing active power vs. load resistance for different masses](image2)

**Figure 13.**
The characteristics of both the measured and the obtained by modeling direct-current powers of the harvester with the bigger springs for a concentrated mass of 3.5 grams and resonant frequency $f=16$ Hz, for a mass of 7 grams and $f=12$ Hz, as well as for a mass of 10.5 grams and $f=10$ Hz, are shown in Figure 14. It can be seen that with the increase in the concentrated mass both the measured and the obtained by modeling powers decrease. When the active load increases, the output powers increase.

![Graph showing the relationship between active power (mW) and load resistance (kOhm) for different masses and resonant frequencies.]

Table 1 presents the measured maximum direct-current voltages and powers and the resonant frequencies for the harvesters with small springs at their three types of concentrated mass. A harvester with a concentrated mass of 2 grams has the highest measured voltage and power, while one with a mass of 6 grams has the lowest measured voltage and power.

Table 2 gives the measured maximum direct-current voltages and powers, as well as the resonant frequencies for the harvesters with big springs at their three types of concentrated mass. The highest measured voltage and power characterize a harvester with a concentrated mass of 3.5 grams, while the lowest voltage and power are measured for a harvester with a concentrated mass of 10.5 grams.

### Table 1. Measured voltages, power and resonant frequencies for harvesters with small springs.

| Mass (gr) | Maximum voltage (V) | Maximum power (μW) | Resonant frequency (Hz) |
|-----------|---------------------|--------------------|------------------------|
| 2         | 0.62                | 1.04               | 18                     |
| 4         | 0.43                | 0.46               | 8                      |
| 6         | 0.352               | 0.31               | 7                      |
### Table 2. Measured voltages, power and resonant frequencies for harvesters with big springs

| Mass (gr) | Maximum voltage (V) | Maximum power (µW) | Resonant frequency (Hz) |
|-----------|----------------------|--------------------|------------------------|
| 3.5       | 1.91                 | 9.57               | 16                     |
| 7         | 1.32                 | 4.49               | 12                     |
| 10.5      | 0.91                 | 2.21               | 10                     |

### 4. Conclusions

Theoretical derivations, programming simulations and experimental studies of two harvesters, consisting of a transformer, two springs and a concentrated mass, have been done. One of the harvesters is with smaller springs and a smaller concentrated mass, the other is with bigger ones.

Modeling of the two-spring piezoelectric harvesters has been done by means of ANSYS R19.1 and the maximum horizontal deviations $x_m$ of the mechanical systems have been found. An equivalent electric circuit of the piezo plate has been drawn and an expression for the maximum power at active load has been obtained. From the frequency characteristics it can be seen that with the increase in the concentrated mass the resonant frequencies of both types of harvesters decrease and so do the rectified voltages at idle run. Both the measured and the obtained by modeling maximum powers decrease together with the increase of the concentrated mass for the two types of harvesters. When the active load increases, the voltages and the powers increase for the two types of harvesters again.

The maximum voltages for the harvester with big springs are nine times higher than for the harvester with smaller springs. The powers of the harvester with big springs are nine times higher than the measured values for a harvester with small springs. Therefore a harvester with big springs is more appropriate for use in practice.

### Acknowledgements

The authors would like to thank the Research and Development Sector at the Technical University of Sofia for the financial support.

### References

[1] Kubba C and Jiang K 2014 *Sensors.* 14
[2] Lu F, Lee H and Lim S 2004 *Smart Mater. Struct.* 13
[3] Dhakara L, Liuva H, Tayb F and Leea Ch 2013 *Sensors and Actuators.* A199
[4] https://www.fs.vsb.cz/export/sites/fs/330/.content/files/technical_vibration_2.pdf
[5] Zizys D 2018 *Doctoral Thesis Engineering Sciences, Mechanical Engineering. Kaunas university of technology*