Exploiting Friction in Torque Controlled Humanoid Robots

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Abstract—A common architecture for torque controlled humanoid robots consists in two nested loops. The outer loop generates desired joint/motor torques, and the inner loop stabilises these desired values. In doing so, the inner loop usually compensates for joint friction phenomena, thus removing their inherent stabilising property that may be also beneficial for high level control objectives. This paper shows how to exploit friction for joint and task space control of humanoid robots. Experiments are carried out using the humanoid robot iCub.

I. INTRODUCTION

A humanoid robot is usually required to operate out of a protected and well-known workspace and to physically interact with a dynamic, human-centered environment. In this context, the robot is required to balance, perform manipulation tasks and – even more important – safely interact with humans. The importance of controlling the robot interaction with the environment calls for the design of torque and impedance control algorithms, capable of exploiting the forces the robot exerts at contact locations for performing dynamic tasks [1], [2]. However, despite decades of research in the subject, torque controlled humanoid robots are still a challenge for the robotics community. The variability of sensor load during locomotion, the inaccuracy of the force/torque sensing technology, and the nonlinearity of joint friction effects are only a few complexities impairing efficient robot torque control. Then, the importance of conceiving control laws ensuring a degree of robustness against some of these factors goes without saying. This paper contributes towards this direction by proposing modifications of state-of-the-art control laws that allow them to exploit the inherent stabilising nature of joint friction. The effect of these modifications is a system degree of robustness against a poor friction estimation.

Similar solutions that try to exploit the natural dynamics of the system for improving the performances and energy efficiency have been proposed in literature, e.g. for robot walking [3] or running [4]. In particular, the effect of friction at all stages of the robot mechanisms and between the robot and the environment plays an important role for the stability of the controlled system [5], [6]. Previous works already considered the possibility of exploiting the friction between the robot and the environment for controlling a robotic crawler [7], or for the locomotion of a hopping robot [4]. More generally, the passivity-based control strategies try to exploit the passivity properties of the overall system for regulation tasks [8], and can also be extended for addressing tracking problems [9].

When dealing with humanoid robots, the fixed-base assumption may be a limitation for tasks such as walking. An alternative solution is to use the floating base formalism [10], i.e. none of the robot link has an a priori constant pose w.r.t. an inertial reference frame. In this case, the control problem is further complicated by the system’s underactuation, since it forbids full state feedback linearization [11].

An effective technique for controlling floating base robots with rigid joints is the operational space control, where the control objective is often the stabilisation of the robot centroidal momentum [12]. The controllers designed for this objective are usually referred to as momentum-based controllers [13]. Momentum control can be achieved by controlling the forces the robot exerts at contact locations [14], [15], [16], and these forces are then generated by the robot joint torques. To get rid of the (eventual) actuation redundancy associated with momentum control, a lower priority task is usually added during the stabilisation of the robot momentum. This secondary task aims at imposing a desired joint robot configuration, and plays a pivotal role for the stabilization of the system zero dynamics [17].

The aim of this paper is the development of a torque control framework for a humanoid robotic platform that exploits the joint viscous and Coulomb friction for improving the system’s stability against modeling errors, and the tracking of a desired reference trajectory. As proof of concepts, we first present the control algorithm for a fixed base robot and then we extend the formulation to the control of a floating base robot.

This paper is organized as follows. Section II recalls notation, system modeling, and a classical torque control strategy for both fixed base and floating base robots. In Section III, a modification of the state-of-the-art control framework for exploiting joint friction is detailed. Experimental results on humanoid robot iCub [18] are presented in Section IV. Conclusions and perspectives conclude the paper.

II. BACKGROUND

A. Notation

- \( I \) denotes an inertial frame, with its z axis pointing against the gravity. The constant \( g \) denotes the norm of the gravitational acceleration.
- Given a matrix \( A \in \mathbb{R}^{m \times n} \), we denote with \( A^\dagger \in \mathbb{R}^{n \times m} \) its Moore Penrose pseudoinverse.
- \( e_i \in \mathbb{R}^m \) is the canonical vector, consisting of all zeros but the \( i \)-th component that is equal to one.
- We denote with \( m \) the total mass of the robot.
B. Fixed Base Robot Dynamics

The robot is modelled as a multi-body system composed of \( n + 1 \) rigid bodies, called links, connected by \( n \) joints with one degree of freedom each. We initially assume that the system is fixed-base, i.e. at least one of the robot links has a constant pose w.r.t. an inertial frame of reference. The joints actuation is provided by \( n \) electric brushless motors. The motors and joints dynamics can be represented by the following set of equations:

\[
\begin{align*}
M_j \ddot{s} + G_j(s) + C_j(s, \dot{s}) \dot{s} &= \tau \\
I_m \ddot{\theta} + K_c \dot{\theta} + K_v \text{sign}(\dot{\theta}) &= \tau_m - \Gamma^\top \tau \\
s &= \Gamma \theta.
\end{align*}
\]

Eq. (1a) describes the joints dynamics and is obtained by applying the Euler-Poincaré formalism [19, Ch. 13.5] to Eq. (1a) describes the joints dynamics and is obtained by applying the Euler-Poincaré formalism [19, Ch. 13.5] to

\[
\begin{align*}
M(q) \ddot{q} + C(q, \dot{q}) &= J_b \tau + \sum_{k=1}^{n_c} J_{\nu k} f_k
\end{align*}
\]

where \( M \in \mathbb{R}^{n+6 \times n+6} \) is the mass matrix, \( C \in \mathbb{R}^{n+6 \times n+6} \) is the centrifugal and Coriolis matrix, \( G \in \mathbb{R}^{n+6} \) is the gravity term, \( \Gamma \) is a selector matrix, \( \tau \in \mathbb{R}^n \) is a vector representing the joint torques, and \( f_k \in \mathbb{R}^6 \) denotes an external wrench applied by the environment to the link of the \( k \)-th contact. The Jacobian \( J_{\nu k} = J_{\nu k}(q) \) is the map between the robot’s velocity \( \nu \) and the linear and angular velocity at the \( k \)-th contact link.

As described in [22, Sec. 5], it is possible to apply a coordinate transformation in the state space \((q, \nu)\) that transforms the system dynamics (2) into a new form where the mass matrix is block diagonal, thus decoupling joint and base frame accelerations. Also, in this new set of coordinates, the first six rows of Eq. (2) are the centroidal dynamics \( \nu \). As an abuse of notation, we assume that system (2) has been transformed into this new set of coordinates, i.e.

\[
M(q) \ddot{q} + C(q, \dot{q}) = J_b \tau + \sum_{k=1}^{n_c} J_{\nu k} f_k
\]

with \( M_b \in \mathbb{R}^{6 \times 6}, M_j \in \mathbb{R}^{n \times n}, H := (H_L^\top, H_o^\top) \in \mathbb{R}^6 \) the robot centroidal momentum, and \( H_L, H_o \in \mathbb{R}^3 \) the linear and angular momentum at the center of mass, respectively.

Lastly, it is assumed that a set of holonomic constraints acts on System (2). These holonomic constraints are of the form \( c(q) = 0 \), and may represent, for instance, a frame having a constant pose w.r.t. the inertial frame. In the case where this frame corresponds to the location at which a rigid contact occurs on a link, we represent the holonomic constraint as \( J_{\nu k}(q) \nu = 0 \).

Hence, the holonomic constraints associated with all the rigid contacts can be represented as

\[
\begin{align*}
J(q) \nu &= \begin{bmatrix} J_{\nu 1}(q) & \cdots & J_{\nu n_c}(q) \end{bmatrix} \nu = [J_b \ J_j] \nu = J_b \nu_B + J_j \dot{s} = 0
\end{align*}
\]

with \( J_b \in \mathbb{R}^{6n_c \times 6}, J_j \in \mathbb{R}^{6n_c \times n} \). The base frame velocity is denoted by \( \nu_B \in \mathbb{R}^6 \), which in the new coordinates yielding a block-diagonal mass matrix is given by \( \nu_B = (\dot{p}_c, \omega_c) \), where \( \dot{p}_c \in \mathbb{R}^3 \) is the velocity of the system’s center of mass

\[\text{In the specialized literature, the term centroidal dynamics is used to indicate the rate of change of the robot’s momentum expressed at the center-of-mass, which then equals the summation of all external wrenches acting on the multi-body system [12].}

C. Floating Base Robot Dynamics

We extend now the framework presented in System (1) in case the fixed base assumption is violated and none of the robot links has a constant pose with respect to an inertial frame, i.e. the system is free floating.

The robot configuration space is now the Lie group \( Q = \mathbb{R}^3 \times SO(3) \times \mathbb{R}^n \) and it is characterized by the pose (position and orientation) of a base frame \( B \) attached to a robot’s link, and the joint positions. An element \( q \in Q \) can be defined as the following triplet: \( q = (T_B \bar{R}, b, s) \) where \( T_B \bar{R} \in \mathbb{R}^3 \) denotes the position of the base frame with respect to the inertial frame, \( \bar{R} B \in \mathbb{R}^{3 \times 3} \) is a rotation matrix representing the orientation of the base frame, and \( s \in \mathbb{R}^n \) is the joint configuration.

The velocity of the multi-body system can be characterized by the set \( V = \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^n \). An element of \( V \) is a triplet \( \nu = (\dot{T}_B \bar{R}, \dot{s}, \dot{\omega}_B) \) where \( \dot{T}_B \bar{R} \) is the angular velocity of the base frame expressed w.r.t. the inertial frame, i.e. \( \dot{T}_B B = S(\dot{T}_B \bar{R}) \dot{T}_B B \). A more detailed description of the floating base model is provided in [22].

We assume that the robot interacts with the environment by exchanging \( n_c \) distinct wrenches. The equations of motion of the multi-body system can still be described applying the Euler-Poincaré formalism:

\[
M(q) \ddot{q} + C(q, \dot{q}) \nu + G(q) = B \tau + \sum_{k=1}^{n_c} J_{\nu k} f_k
\]

\[\text{In the specialized literature, the term centroidal dynamics is used to indicate the rate of change of the robot’s momentum expressed at the center-of-mass, which then equals the summation of all external wrenches acting on the multi-body system [12].}\]
where we partitioned Eq. (2) into the floating base dynamics Eq. (6a) and the joints dynamics Eq. (6b). We define \( h := C(q, \nu \nu) + G(q) \in \mathbb{R}^{n+6} \) and its partition \( h = (h_b, h_j) \), \( h_b \in \mathbb{R}^6 \), \( h_j \in \mathbb{R}^n \). \( f := (f_1, \ldots, f_n) \in \mathbb{R}^{6n_c} \) are the set of contact forces – i.e. Lagrange multipliers – making Eq. (5) satisfied.

D. Control of a Fixed Base Robot

We assume the joint torques \( \tau \) can be used as control input of Eq. (1a). Then, being the system fully actuated, it is possible to apply a classical passivity based approach [9] for controlling the joint dynamics, e.g. by choosing \( \tau \) as follows:

\[
\tau^* = G_j + C_j \dot{s}^d + M_j \ddot{s}^d - K_P \bar{s} - K_D \ddot{s},
\]

where \( s^d(t) \in \mathbb{R}^n \) is a desired joints reference trajectory, \( \bar{s} = s - s^d \) and \( K_P, K_D \in \mathbb{R}^{n \times n} \) are two symmetric and positive definite matrices.

E. Control of a Floating Base Robot

We recall here the momentum-based control strategy for a floating base system implemented on the iCub humanoid robot [17], [23]. The control objective is the stabilization of a desired robot momentum and the stability of the associated zero dynamics.

1) Momentum control: Recall that the rate-of-change of the robot momentum equals the net external wrench acting on the robot, which in the present case reduces to the contact wrench \( f \) plus the gravity wrenches. In view of Eq. (3), the rate-of-change of the robot momentum can be expressed as:

\[
\frac{d}{dt}(M_b \dot{v_B}) = \dot{H}(f) = J_b^T f - m ge_3.
\]

Let \( H^d \in \mathbb{R}^6 \) denote the desired robot momentum, and \( \dot{H} = H - H^d \) the momentum error. Assuming that the contact wrenches \( f \) can be chosen at will, then we choose \( f \) such that [17]:

\[
\dot{H}(f) = \dot{H}^* := \dot{H}^d - K_P \bar{H} - K_I \dddot{H}, \quad \text{(9a)}
\]

\[
\dot{\bar{H}} = \begin{bmatrix} J_{G}^T(s) \\ J_{G}^T(s) \end{bmatrix} \dot{s}, \quad \text{(9b)}
\]

\( K_P, K_I \in \mathbb{R}^{6 \times 6} \) two symmetric, positive definite matrices and

\[
\tilde{J}_G(s) := -M_b J_b^T J_j = \begin{bmatrix} J_{G}^T(s) \\ -J_{G}^T(s) \end{bmatrix} \in \mathbb{R}^{6 \times n}. \quad \text{(10)}
\]

If \( n_c > 1 \), there are infinite contact wrenches \( f \) that satisfy Eq. (9a). We parametrize the set of solutions \( f \) to (9a) as:

\[
f = f_1 + N_b f_0
\]

with \( f_1 = J_b^T \left( \dot{H}^* + m ge_3 \right), \) \( N_b \in \mathbb{R}^{6n_c \times 6n_c} \) the projector into the null space of \( J_b^T \), and \( f_0 \in \mathbb{R}^{6n_c} \) the wrench redundancy that does not influence \( \dot{H}(f) = \dot{H}^* \). To determine the control torques that instantaneously realize the contact wrenches given by (11), we use the dynamic equations (2) along with the constraints (5), which yields:

\[
\tau = \Lambda^1(JM^{-1}(h - J^T f) - \dot{J} \nu) + N_A \tau_0
\]

where \( \Lambda = J_j M_j^{-1} \in \mathbb{R}^{6n_c \times n}, \) \( N_A \in \mathbb{R}^{n \times n} \) the projector onto the nullspace of \( \Lambda \), and \( \tau_0 \in \mathbb{R}^n \) a free variable.

2) Stability of the Zero Dynamics: The stability of the zero dynamics is attempted by means of a so called postural task, which exploits the free variable \( \tau_0 \) in (12). A choice of the postural task that ensures the stability of the zero dynamics on one foot balancing is [17]:

\[
\tau_0 = h_j - J_j^T f + u_0
\]

where \( u_0 := -K_p^J N_A M_j(s - s^d) - K_q^J N_A M_j \dot{s}, \) and \( K_P^J \in \mathbb{R}^{n \times n} \) and \( K_q^J \in \mathbb{R}^{n \times n} \) two symmetric, positive definite matrices. An interesting property of the closed loop system (2)–(11)–(12)–(13) is that in view of the choice (13) of the postural control, the closed loop joint space dynamics \( \ddot{s} \) does not depend upon the wrench redundancy \( f_0 \).

In the language of Optimization Theory, we can rewrite the control strategy as a single optimisation problem as follows:

\[
f^* = \arg\min_f |\tau^*(f)|^2
\]

s.t.

\[
Af < b, \quad \text{(14b)}
\]

\[
\dot{H}(f) = \dot{H}^*, \quad \text{(14c)}
\]

\[
\tau^*(f) = \arg\min_{\tau} |\tau - \tau_0(f)|^2
\]

s.t.

\[
\dot{J}(q, \nu \nu) \nu + J(q) \nu = 0, \quad \text{(14e)}
\]

\[
\dot{\nu} = M^{-1}(B \tau + J^T f - h), \quad \text{(14f)}
\]

\[
\tau_0 = h_j - J_j^T f + u_0. \quad \text{(14g)}
\]

The constraints (14b) ensure the satisfaction of friction cones, normal contact surface forces, and center-of-pressure constraints. The control torques are then given by \( \tau = \tau^*(f^*) \).

F. Motor Control

The joint torques \( \tau^* \) that stabilize Eq. (1a) and (6a)–(6b) must be related to the actual control input of the system, namely the motor torques \( \tau_m \), that are proportional to the motors PWM. An inner control loop is responsible for stabilizing any desired torque reference signal upon the desired values. The control input for the inner loop is of the form:

\[
\tau^*_m = K_c \dot{\theta} + K_s \text{sign}(\dot{\theta}) + \Gamma^T \tau^* - K_I \int (\tau - \tau^*), \quad \text{(15)}
\]
where the joint friction is compensated inside the feedforward term, and $K_f$ is a symmetric and positive definite matrix.

III. EXPLOITING FRICTION

As proof of concepts, we present a control framework for fixed base robots that exploits the viscous and Coulomb friction in Eq. (1b) to improve the tracking of a desired joint trajectory. This framework is then extended to the control of the floating base system Eq. (6a)–(6b), for improving the tracking of the robot momentum dynamics.

A. Fixed Base Robot

Recall the kinematic relation Eq. (1c) between the joints position $s$ and motors position $\theta$:

$$s = \Gamma \theta,$$

and rewrite the motors dynamics Eq. (1b) by substituting $\dot{s}$, $\dot{\theta}$ with $\ddot{s}$ and $\ddot{\theta}$:

$$\begin{align*}
I_m \Gamma^{-1} \ddot{s} &= \tau_m - K_f(\Gamma^{-1} \ddot{s}) - \Gamma^{-\top} \tau_s \\
(16)
\end{align*}$$

where $K_f(\Gamma^{-1} \ddot{s})$ is a diagonal matrix that collects the Coulomb and viscous friction coefficients. The elements along the diagonal of $K_f$ are all positive, and of the form:

$$k_{f, i} = k_{v, i} + \frac{k_{c, i}}{\epsilon},$$

with $e_i$ is the canonical vector, consisting of all zeros but the $i$-th component that is equal to one, and the $\epsilon \in \mathbb{R}^+$, $\epsilon << 1$. Then, by multiplying (16) times $\Gamma^{-\top}$ and by summing up motors and joints equations (1a)-(16), one has:

$$\begin{align*}
\bar{M}_f \ddot{s} + C_f \ddot{s} + G_j &= u - \bar{K}_f \ddot{s}, \\
(17)
\end{align*}$$

where $\bar{M}_f = M_f + \Gamma^{-\top} I_m \Gamma^{-1}$, $\bar{K}_f = \Gamma^{-\top} K_f \Gamma^{-1}$ are two symmetric and positive definite matrices, while $u = \Gamma^{-\top} \tau_m$. The term $\Gamma^{-\top} I_m \Gamma^{-1}$ is usually referred as motor reflected inertia [9].

Now, the system (17) can be controlled using $u$ (i.e., the motor torques $\tau_m$) as control input instead of the joint torques. In particular, a choice of $u$ that guarantees the asymptotic stability of the joint dynamics Eq. (1a) along a desired trajectory $s^d$ is:

$$u^* = G_j + C_j \ddot{s} + \bar{M}_f \ddot{s} - K_f \ddot{s} - K_D \dot{s} + \bar{K}_f \ddot{s} = (21a)$$

The proof is in the Appendix VI. Then, the closed-loop joints dynamics becomes:

$$\begin{align*}
\bar{M}_f \ddot{s} + C_f \ddot{s} &= \bar{M}_f \ddot{s} - K_f \ddot{s} - (K_D + \bar{K}_f) \dot{s}, \\
(19)
\end{align*}$$

and the associated motor torques can be computed as:

$$\tau^*_m = \Gamma^{-\top} u^*. $$

(20)

If an estimation of the motor torques is available, Eq. (20) may be corrected with the additional feedback term $-K_f \ddot{\theta} \Gamma^{-\top} (u - u^*)$. Note that the control law (20) does not compensate for friction effects, and the effect of the additional term $\bar{K}_f \ddot{s}^d$ in the closed-loop dynamics is similar to that of a feedback term on the joint velocity error. The control law Eq. (7)–(15) instead compensates for the Coulomb and viscous friction in (15). Then, artificial damping is added to the joint dynamics by means of the term $K_D \dot{s}$ in Eq. (7).

When implementing the two algorithms on a real robot, the control input (18) may provide a couple of advantages:

- the closed-loop system is more robust w.r.t. friction modelling errors. In case the friction coefficients are overestimated, stability of the closed-loop system (19) can still be retained with a proper tuning of feedback gains. Nothing can be done instead to avoid instability in case the control law (7)–(15) is applied;
- if the joint velocity is obtained e.g. by numerical differentiation of the encoders signal, the resulting estimation may be affected by noise. This limits the tuning of the velocity feedback gains, because too high values of $K_D$ may lead to numerical instability due to the noisy measurements. On the other hand, filtering the signal introduces a phase delay that affects the trajectory tracking performances. The additional term $\bar{K}_f \ddot{s}$ in the closed loop dynamics Eq. (19) contributes to increase the system damping without modifying $K_D$, therefore the tracking performances can be improved without the necessity of increasing the velocity feedback gains.

B. Floating base robot

We rewrite the motors dynamics in system (6) as in Eq. (16). Again, by multiplying (16) times $\Gamma^{-\top}$ and by summing up motors and joints equations (6b)-(16), one has:

$$\begin{align*}
M_b \ddot{\theta}_B + h_b &= J_b^\top f \\
(21a)
\end{align*}$$

As for Eq. (2), system (21) can be compactly rewritten as follows:

$$\begin{align*}
\bar{M} \dot{\nu} + h &= J^\top f + Bu - B \bar{K}_f \ddot{s}, \\
(22)
\end{align*}$$

where we recall that $h = (h_b, h_j)$, $B = (0_{n \times 6}, 1_n)^\top$, while the mass matrix $\bar{M}$ is given by:

$$\bar{M} = \begin{bmatrix}
M_b & 0_{n \times 6} \\
0_{n \times 6} & \bar{M}_j
\end{bmatrix}. $$

Recall the rate of change of the momentum can be written as $\dot{H}(f) = J_b^\top f - mge_3$. A relation between the contact wrenches $f$ and the friction component $\bar{K}_f \ddot{s}$ can be obtained by substituting the dynamic equations (22) in the contact constraint equations (5), which yields:

$$J \bar{M}^{-1}(J^\top f - h + Bu - B \bar{K}_f \ddot{s}) + J \dot{\nu} = 0. $$

(23)

Writing explicitly the contact wrenches from Eq. (23) gives:

$$f = f_m + D \bar{K}_f \ddot{s}, $$

(24)

where we defined $f_m, D$ as:

$$f_m = (J \bar{M}^{-1} J^\top)^{-1} (J \bar{M}^{-1} (h - Bu) - J \dot{\nu}) $$

$$D = (J \bar{M}^{-1} J^\top)^{-1} J \bar{M}^{-1} B. $$

Substitute now the contact wrenches $f$ in the momentum dynamics with the right-hand side of Eq. (24):

$$\dot{H}(f) = J_b^\top f_m + J_b^\top D \bar{K}_f \ddot{s} - mge_3. $$

(25)
In order to come up with a formulation similar to that of Eq. (17), we split the joint velocity \( \dot{s} \) into two components:

\[
\dot{s} = -D^\top J_b H + (1_n + D^\top J_b \tilde{J}_G) \dot{s},
\]

where \( \tilde{J}_G \) is the reduced centroidal momentum matrix \(^{[17]}\) and it is defined by Eq. (10). We then exploit the component \(-D^\top J_b H\) for trajectory tracking as in Eq. (17), and we compensate instead for its complement. The desired rate of change of the robot momentum can be now written as:

\[
\dot{H}^* = \dot{H}^d - K_fh - K_iH + TH^d.
\]

(26)

where \( T = J_b^\top D\tilde{K}_f D^\top J_b \in \mathbb{R}^{6 \times 6} \) is a symmetric and positive definite matrix. The previous control law (11)–(12) can now be applied as before, by substituting \( \tau \) with \( u \) and \( f \) with \( f_m \). To achieve a closed loop joints dynamics similar to that of system (2)–(11)–(12)–(13), we choose the postural task \( \tau_0 \) as follows:

\[
\tau_0 = h_j - J_j^\top f + \tilde{K}_f \dot{s}^d + u_0,
\]

(27)

where we again exploited the effect of friction for improving also the postural task tracking performances.

IV. EXPERIMENTAL RESULTS

We applied the control algorithms presented in Sec. II–III on the iCub humanoid robot \(^{[24]}\). For the purpose of this paper, iCub is endowed with 23 degrees of freedom. The inner control loop Eq. (15) and Eq. (20), running at 1kHz, is responsible for stabilizing any desired torque reference signal. On the real robot, the presence of delays allow us to approximate the feedback term on the integral error as \( \approx -K_i(\tau^* - \tau) \). During the experiments, we only considered the effect of the viscous friction in the harmonic drive gearboxes, that on iCub gives the major contribution to friction effects while the robot is moving. Approximately, the order of magnitude of the elements of \( \tilde{K}_f \) on iCub is \( \approx 10 \).

A. Fixed base joints tracking

The first experimental setup is carried out with the robot pelvis fixed on a pole. The robot is free to move its torso, legs and arms. A desired sinusoidal reference trajectory of amplitude \( 15[\text{deg}] \) and frequency \( 0.5Hz \) is applied to each controlled joint. A video showing the experiment is attached to the paper.

The control algorithms are the one presented in II, D and III. A for fixed base robots. Figure 1 shows the norm of the joint position errors while executing the task. The control law that exploits friction (EF) shows a considerable improvement in the tracking performances, that confirms the effectiveness of the proposed approach.

B. Floating base momentum tracking

The second experimental setup is carried out with the robot balancing on its feet. The robot moves its CoM from the left to the right foot, following a sinusoidal trajectory of amplitude \( 4[\text{cm}] \) and frequency \( 0.5Hz \). A video of this second experiment is also presented. Figure 2 represents the tracking error of the CoM, and the reference signal versus the real movement of the robot. As for the fixed base experiment, the control law that exploits friction shows an improvement in tracking performances. Figures 3-4 show instead the linear and angular momentum error during left and right movements. While with the EF controller the linear momentum tracking error improved, the angular momentum is worse than with the previous control algorithm. A tentative explanation of this (unexpected) result is that this phenomenon is due to the coupling of linear and angular momentum dynamics introduced by the non-diagonal term \( TH^d \) in Eq. (26). An order of magnitude of difference in the
feedback gains for linear and angular momentum dynamics may explain the poor disturbance rejection of the closed-loop angular momentum dynamics. Future works may investigate if a proper gain tuning procedure is enough for reducing this coupling effect.

C. The effect of motors reflected inertia

The control algorithms presented in Section II, E and III, B require the inversion of the system’s mass matrix $M$. However, the mass matrix of a humanoid robot may be ill-conditioned because of the presence of links with very different mass and inertia. On iCub, the condition number of the mass matrix is cond($M$) $\approx$ 20000. An interesting advantage of the control framework presented in III, B is that the mass matrix is summed up with the so called motors reflected inertia. This term acts as a regularization parameter of the system mass matrix, improving the numerical stability of the control algorithm without introducing modeling errors. On iCub, the order of magnitude of the motors reflected inertia is about $\approx$ 0.1. With the addition of motors reflected inertia, the condition number of the mass matrix decreased to cond($\tilde{M}$) $\approx$ 800. A video showing the role of motors reflected inertia for performing very fast dynamic movements is attached to the paper.

V. CONCLUSIONS

This paper proposed a modification of state-of-the-art torque control of humanoid robots that allows to exploit the inherent stabilising nature of joint friction. In particular, the joint viscous and Coulomb friction is used for improving the tracking of a desired reference trajectory. As proof of concepts, we first presented the control algorithm for a fixed base robot and then we extended the formulation to the control of a floating base robot. Experimental results that show the effectiveness and the limitations of the proposed approach have been presented. Future works may focus on further developing the theoretical framework and on the design of new experiments for validating the proposed approach in different scenarios.

VI. APPENDIX

Consider the following (valid) Lyapunov function candidate:

$$V = \frac{1}{2} \hat{s}^\top \tilde{M} \hat{s} + \frac{1}{2} \hat{s}^\top K P \hat{s}. \quad (28)$$

The time derivative of $V$ is given by:

$$\dot{V} = \dot{\hat{s}}^\top (\tilde{M} \ddot{s} + \hat{\tilde{M}} \ddot{s} + K P \dot{s}). \quad (29)$$

By substituting the joint dynamics Eq. (17) and the control input Eq. (18) into $\dot{V}$, one is left with:

$$\dot{V} = \dot{\hat{s}}^\top (\frac{1}{2} \tilde{M} \dot{s} - C_j - K_D - K_f) \dot{s}. \quad (30)$$

Over all possible representations of the Coriolis matrix, we choose $C_j$ such that $\tilde{M} \dot{s} - 2C_j \dot{s} = \tilde{M} \dot{s} - 2C_j \dot{s}$ is a skew symmetric matrix. Therefore, Eq. (30) remains:

$$\dot{V} = -\dot{\hat{s}}^\top (K_D + K_f) \dot{s} \leq 0. \quad (31)$$

It is then straightforward to verify the convergence of $s$ to $s^d$ by resorting to the LaSalle’s invariance principle.

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