On Finite Matrix Bi-Dimensional Formulation of $D = 4n + 2$
Classical Field Models

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Abstract

We introduce a basis for a bi-dimensional finite matrix calculus and a bi-dimensional finite matrix action principle. As an application, we analyze scalar and spinorial fields in $D = 4n + 2$ in this approach. We verify that to establish a bi-dimensional matrix action principle we have to define a Dirac-algebra-modified Leibniz rule. From the bi-dimensional equations of motion, we obtain a matrix holomorphic feature for massless matrix scalar and spinorial fields.

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1 Introduction

In the last years High Energy Physics has been centered in the research on higher dimension worlds and the models which can describe them, particularly strings, M-Theory and membranes. Many ways to reach the ordinary four dimension space-time models have been studied, where the claim is that many of these compactification procedures imply correlations between the theories mentioned above. Indeed there are claims that (Super)Membranes, M(atrix) Theory and M-theory are interconnected \([1]\) and one of the common points is that the coordinates of the extended objects present in these models are represented by matrix elements. In fact there are many indications that the matrix formulation shed light to understand what is happened in higher dimension models\([2, 3, 4, 5, 6]\). Very recently an infinite matrix behavior and the non-commutative property of the strings coordinates bring the attention of the researchers and it has been developing nowadays. We could conclude that matrix behavior is an important ingredient to these models and it is worth to analyze in its different aspects.

In this work, we are going to study simple dimensional field models in \(D = 4n + 2\) with \((2n+1, 2n+1)\) signature and we are going to exhibit its finite matrix bidimensional formulation. This structure is “naturally” observed in a \(D = 4n + 2\) massless spinorial Lagrangian when one contracts the Dirac matrices in the Weyl representation with the space-time derivatives \([7, 8]\). So, it drives us to analyze the possibility to formulate a 2D matrix space-time and introduce a finite 2D matrix calculus. Indeed, this is the main goal and motivation of this work. We wish to emphasize that we are going to develop the firsts steps towards a finite bi-dimensional matrix calculus for knowns field-theoretic models. It is not our purpose to formulate a new theory; we rather reveal some structure underneath Lagrangian models. In this new formulation type-2D, the simetries, the dynamics, and the Noether Theorem will be reassessed. We are going to apply this formulation to a massless scalar field model. As a starting point, we analyze the matrix structure of a massless spinorial and scalar fields. We obtain that some ordinary 2D features, as holomorphism, are maintained in the proposed matrix approach.

The outline of this work is in the following. In Section 2 we recollect the general Weyl representation of the Dirac matrices obtained in the work \([7]\). We contract it to the derivative in a \(D = 4n + 2\) (space-time with a signature \((s = 2n + 1, t = 2n + 1)\)) spinorial field Lagrangian and we show its natural separation into two sets of coordinates accommodated in matrix (Dirac) blocks. We also show the possible signatures of the two sets of coordinates that maintain the same “status” of the two matrix coordinates. The possible matrix Lorentz group is shown as well. In Section 3 we establish a matrix bi-dimensional action principle and the Dirac-algebra modification of the Leibniz rule (we call Dirac-Leibniz rule) necessary to the principle itself, and to compute the matrix equations of motion and energy-momentum tensors. We interpret them as a collective coordinate and/or field dynamics. In Section 4 we compute the matrix (or collective) equations of motion of the spinorial and scalar fields and we obtain a matrix holomorphism to the massless models. Finally, we conclude this work making some comments and speculating about possible future applications.

2 The Spinorial Field and The Two Matrix Coordinate Case

In this Section we are going to treat an ordinary \(D = 4n + 2\) massless spinorial model in a space-time with \((2n+1, 2n+1)\) signature, where we contract the Dirac matrices in Weyl representation...
to the derivatives. We will show a natural matrix behavior of two sets of derivatives. Indeed each of these sets which are contractions to two different \( d = 2n + 1 \) Dirac matrices we interpret as two block-derivatives. We extend this matrix (or collective) behavior to the coordinate frame in order to introduce the basis of a matrix bidimensional calculus, and the consequent extension to the fields will be explored in Section 4. As a matter of fact, it is fairly-well known that in many textbooks it has been studied the contraction of the Dirac matrices to the coordinates, particularly in four dimensions. However due to the lack of extra independent matrix elements they do not go far than the complex geometry and group analysis of space-time [9]. From our point-of-view it is because they have only one matrix coordinate and one matrix derivative. In fact, when we deal particularly with the Weyl representation of the Dirac matrices that the contraction to the derivatives reaches a matrix bi-dimensional structure as we are going to show.

Defining the two matrix Dirac-algebra-preserving commuting coordinates we are in condition to do the firsts steps towards a finite matrix calculus and a matrix formulation of the fields. To start with, let us write the Weyl representation of the Dirac matrices in any even \( D \) dimension, namely

\[
\Gamma^\mu = \begin{cases} 
\sigma^y \otimes I_p \otimes \gamma^m_q = (\sigma^y \otimes I_p \otimes \gamma^m_q) \cdot (I_2 \otimes I_p \otimes \gamma^m_q) \\
\sigma^x \otimes \gamma^m_p \otimes I_q = (\sigma^x \otimes \gamma^m_p \otimes I_q) \cdot (I_2 \otimes \gamma^m_p \otimes I_q)
\end{cases}
\]

(1)

where \( \sigma_x, \sigma_y \) are Pauli matrices, \( \gamma^m_q, \gamma^m_p \) are the Dirac matrices in \( q + 1 \) and \( p + 1 \) dimensions respectively (\( p \) and \( q \) are even). For completeness we take \( \gamma^m_0 = \gamma^m_0 = I_0 = 1 \). So the dimension \( D \) is equal to \( p + q + 2 \). In particular, we are interested in working with two sets of Dirac matrices with same dimensionality which give the same “status” to the two matrix derivatives (and coordinates). It imposits that \( p = q \) and consequently \( D = 2d = 4n + 2 \).

Introducing Weyl spinors in \( D \) dimensions as

\[
\Psi = \begin{pmatrix} \psi_{a-} \\
\psi_{a+} \end{pmatrix}
\]

(2)

where \( \psi_{a-} \) and \( \psi_{a+} \) are Dirac spinors and the spinorial index \( a \) (Latin letters) run from 1 to \( 2^d \) (the squared parenthesis means that we only take the integer part), we can write the Dirac Lagrangian as

\[
\mathcal{L}_{\text{Dirac}} = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi = i \bar{\Psi} \left( \sigma^0 \partial_0 + \sigma^1 \partial_1 \right) \Psi \equiv \left( \psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+ \right)
\]

(3)

where

\[
(\partial_0, \partial_1) \equiv (\partial_0^{ab}, (\partial_1)^{cd}) = (\partial_m (\Gamma^m)^{ab}, \partial_m (\Gamma^m)^{cd})
\]

(4)

with

\[
(\Gamma^m)^{ab} = \left( I_p \otimes \gamma^m_q \right)^{ab} \quad \text{and} \quad \left( \Gamma^m \right)^{ab} = \left( \gamma^m_p \otimes I_q \right)^{ab}
\]

(5)

and \( \sigma^0 \equiv i \sigma_y, \sigma^1 \equiv \sigma \). The definitions \( (\partial_-)^{ab} = (\partial_1)^{ab} - (\partial_0)^{ab} (\equiv \partial_+ = \partial_1 + \partial_0) \) and \( (\partial_+)^{ab} = (\partial_1)^{ab} + (\partial_0)^{ab} (\equiv \partial_- = \partial_1 - \partial_0) \) represent a kind of matrix light-cone coordinates. Indeed we can observe that the two matrix derivative operators have bi-spinorial indexes. For the sake of simplicity in the last term of (2) we have dropped the spinorial indexes out. Notice that this matrix derivative formulation of the Lagrangian (3) has an explicit bi-dimensional form. We must stress that this construction is highly dependent on the “specific” Weyl representation of the \( \Gamma \) matrices. It is easy to observe that

\[
[\Gamma^m, \Gamma^{\overline{m}}] = 0.
\]

(6)
This commutation relation gives us the property of “splitting” the two sets of derivatives labelled by \( m \) and \( m' \) (each one with the same dimension \( d = \frac{D}{2} \)) that commute in (Dirac-matrix-)blocks. What means that we transmute the ordinary \( D \)-dimensional massless spinorial Lagrangian into a bi-dimensional matrix spinorial Lagrangian. The matrix Lagrangian has two independent matrix derivatives applied to Dirac block-spinors, \( \psi_{a+} \) and \( \psi_{a-} \), with \( 2^d \) elements. The matrix derivatives are contractions of the ordinary derivatives to different closed Dirac algebra each\(^1\). This procedure do arise a bi-dimensional but matrix form indicating a collective dynamics.

We can observe the Dirac Lagrangian (3) with the matrix derivative
\[
\frac{\partial}{\partial t} = \sigma^0 \partial_0 + \sigma^1 \partial_1
\]
which induce us to borrow a two dimension structure. The two independent matrix sets of coordinates are
\[
(X_0, X_1) \equiv \left( (X_0)^{ab}, (X_1)^{cd} \right) = \left( x_m \left( \Gamma^m \right)^{ab}, x_{m'} \left( \Gamma^{m'} \right)^{cd} \right).
\]
Notice that as the matrix derivatives \( X_0 \) and \( X_1 \) carry bi-spinorial indexes and commute between them. At this point, one can argue whether the matrix coordinates \( X_0 \) and \( X_1 \) could be interpret as really ordinary ones and therefore we could have a copy of a ordinary bi-dimensional model. This question has been partially answered in the references \([7, 8]\). Our goal in the present work is to go further to attempt to answer this question in a affirmative way paying the price of dealing with an internal (matrix) structure of the coordinates and derivatives. The main corollaries indicate a possible establishment of a finite matrix calculus, a finite matrix field theory, and a matrix functional calculus. We are going to study the preliminaries of this new feature of this kind of representation and claim that is not only a re-formulation but it reveals a new collective structure of the space-time and fields. Unfortunately the geometrical interpretation of a matrix space-time is yet obscure.

Up to now, in fact, we have only re-wrote our equations in terms of the matrix elements coming from the space-time structure dictates by the Dirac matrices defined in (1). However we have to formalize a matrix calculus in order to performing computations. To this aim we start recalling the work \([7]\), where the splitting method was shown with the correct form of the derivatives, or
\[
\partial_0 = \frac{1}{\sqrt{d}} \Gamma^m \partial_m \text{ and } \partial_1 = \frac{1}{\sqrt{d}} \Gamma^{m'} \partial_{m'}
\]
where \( d \) is the dimension of the two matrix sectors. This factor appears in order to normalize the derivatives \([3]\). This definition gives the correct free operation of the derivatives indicating the independence of the two matrix coordinates. Extending to the matrix light-cone formulation we have \( \partial_+ \) and \( \partial_- \) with the above definition of matrix derivative. Then the re-formulated Dirac Lagrangian (3) is
\[
L_{\text{Dirac}} = i \bar{\psi} \gamma^0 \gamma^1 \Psi = i d \bar{\Psi} \left( \sigma^0 \partial_0 + \sigma^1 \partial_1 \right) \Psi = i d \left( \psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+ \right).
\]
Notice that by a re-definition (re-scale) of the block-spinors, namely \( \psi \rightarrow \frac{1}{\sqrt{d}} \psi \), we obtain an ordinary 2D form.

At this point we must remark that it is important to define the signature and the group properties of the space-time involved. Indeed we want to deal with two matrix coordinates with same dimension (“status”) since we plan to borrow ordinary bi-dimensional physics aspects. We will try to convince ourselves that the matrix coordinates can behave as two block-coordinates.

\(^1\)By “closed” we mean that we include in the respective dimension its equivalent \( \gamma_5 \) Dirac matrix.
It implies to have a “metric” on these two matrix coordinates. What is dictated by the Pauli matrices $i\sigma_y$ and $\sigma_x$ in the expression $[4]$ and therefore in this case is Minkowskian. This approach promotes the matrix coordinates to a “collective time” direction and a “collective space” direction$[3]$. We can remark that it is similar to the case of two manifolds with constraints $[10]$. As an illustrative example, a 10D space-time can have the following signatures$[3]$

a) “Overall” (or $2D$ matrix) Euclidean space-times

\[
(++++++),
\]

b) “Overall” (or $2D$ matrix) Minkowskian space-times

\[
\begin{align*}
(++++++, ---) & \Rightarrow 2 \text{ Euclidean space-times}, \\
(-+++++, +---) & \Rightarrow 2 \text{ flat de Sitter space-times}, \\
(---+++ , +---) & \Rightarrow 2 \text{ flat Anti-de Sitter space-times}.
\end{align*}
\]

We bring the reader’s attention to the interesting fact that in the Minkowskian case, the two sets of coordinates have inverse (collective) metric. Which due the overall metric sign it indicates that indeed we have two sets with the same ordinary signature. Furthermore, it is well-known that the Anti-de Sitter(AdS) case (which has 3 space-like directions and 2 time-like directions), it can admit a real representation of the Dirac matrices. What implies that a two real matrix coordinates case is dictated by the inversion of the collective metric sign. Indeed it is the result of a $i$ in front of the $\sigma_y$. It could indicated that a $(5,5)$ space-time is a complex extension of a AdS $(3,2)$ space-time. We can remark the natural presence of de Sitter and Anti-de Sitter flats space-times and complex extensions are susceptible to speculations about application to strings models in AdS spaces and compactifications.

### 3 The Bi-dimensional matrix Action Principle: A Modified Leibniz Rule

In this Section, we define a matrix bi-dimensional action principle. For this purpose let us assume an Lagrangian function which matrix bi-dimensional elements,

\[ \mathcal{L} = \mathcal{L}(X_\alpha, \partial_\beta X_\alpha) \times, \]

where $\Lambda_{ab}$ is a generic matrix element. We assume that the variation of the Lagrangian commutes with the matrix derivative, so we can define a matrix functional action in two matrix dimensions as

\[ S_M = \int_M d^2X \mathcal{L}, \]

where $d^2X = dX_0dX_1 \equiv (dX_0)_{ab}(dX_1)_{cd}$. To realize the integration we have match the integrand and the integration element bi-spinorial indexes. We remark that the above action is similar to the ordinary bi-dimensional one.

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2 The interchange space-like to time-like direction is dictated by the complexification of the Euclidean Dirac matrices, as a Wick rotation procedure.

3 The direct application to the ordinary coordinates frame is present in a forthcoming paper $[11]$. 

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For completeness we verify that the derivative operations, \( \partial_0 \) and \( \partial_1 \), do not obey an ordinary Leibniz rule generally. Indeed the derivatives only obey a ordinary Leibniz rule when they do apply to a different sector element. However when we apply to the same sector element we have to bear in mind the Dirac algebra

\[
\Gamma^m \Gamma^n + \Gamma^n \Gamma^m = 2\eta^{mn} \mathbb{1}, \quad \text{and} \quad \Gamma^m \Gamma^n - \Gamma^n \Gamma^m = 4\Sigma^{mn} \mathbb{1},
\]

(12)

and equivalent to \( \Gamma^m \) sector, so we obtain a composite Dirac algebra and Leibniz rule (or shortly we can call Dirac-Leibniz rule),

\[
\partial_0 (X_0 Y_0) \Rightarrow \tilde{\partial}_0 (X_0 Y_0) = (\tilde{\partial}_0 X_0) Y_0 - X_0 (\tilde{\partial}_0 Y_0)
\]

\[
\partial_0 (X_0 Y_0) = (\partial_0 X_0) Y_0 + X_0 (\partial_0 Y_0),
\]

(13)

where we define two “new” derivatives dictated by the Dirac algebra: \( \tilde{\partial} \) does an anti-symmetrical operation and \( \partial \) does the symmetrical operation. The “1” sector has an analogous form. For the cross sectors the above two derivatives reduce to the ordinary (symmetrical) ones, or as \( \partial \). This rule will be observe when we apply this approach to ordinary vectorial elements, where we contract the element vector index to Dirac matrix index. We will treat this case in a forthcoming work\[12\]. We are going to analyze the spinorial and scalar case.

The matrix field comes from an ordinary scalar or spinorial ones. In this case, the generic matrix field \( \Lambda \) is proportional to the identity matrix or \( \Lambda A \mathbb{1} \), where \( A \) is a generalized, but not vectorial, index. In consequence the Leibniz rule of the matrix derivative has the same rule as the ordinary one. In this case we indeed have the action similar to the ordinary bi-dimensional models, namely

\[
\delta S = \int_M d^2 X \left( \frac{\partial L}{\partial \Lambda^A} \delta \Lambda^A + \frac{\partial L}{\partial \partial_\alpha \Lambda^A} \delta (\partial_\alpha \Lambda^A) \right).
\]

(14)

Equation of Motion and The Energy-Momentum Tensor of the Matrix Spinorial and Scalar Fields

Let us analyze the above action principle applied to the spinorial and scalar fields. Taking the Lagrangian (3) and the definition (2), and assuming an equivalence to the ordinary bidimensional integral calculus, we obtain

\[
\frac{\partial L}{\partial \bar{\psi}_{\alpha}} - \partial_{\beta} \left( \frac{\partial L}{\partial (\partial_\beta \bar{\psi}_{\alpha})} \right) = 0
\]

(15)

where the index \( \bar{\alpha}, \bar{\beta} \) that run over + and −. As we start with a Weyl spinor field we indeed have similar equations as a ordinary bi-dimensional spinorial model. Then due to the expression \( \Phi \), the equations of motion are:

\[
\partial_{\alpha \beta} \psi_{b+} \equiv \partial_{-} \psi_{+} = 0, \quad \text{and} \quad \partial_{+ \beta} \psi_{b-} \equiv \partial_{+} \psi_{-} = 0.
\]

(16)

Observe that we have a matrix holomorphic characteristic of the fields imitating an ordinary bi-dimensional model. It could represent a sort of a collective integrability.

Now, taking the ordinary scalar field Lagrangian in \( D = 4n + 2 \), or \( L_{sc} = \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 \), where \( \mu = 0, \ldots, 4n + 1 \), we can rewrite the Lagrangian in the matrix version. Applying the

\[\Phi(\Lambda_{ab}) = \Phi_{ab} \text{ which in the scalar case is } \Phi \mathbb{1}.\]
same procedure used in spinorial case at the beginning of this work given by the expression (3) and (8), we have

\[ L = d^2 \eta^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi - m^2 \Phi^2, \]  

(17)

where all the operators are matrix ones, namely \( \Phi \equiv \phi \mathbb{1} \), the derivative are define in (4). The metric tensor \( \eta^{\alpha\beta} \) represents an overall metric which means a collective structure of the two sets of \( d \) coordinates. In a overall matrix Minkowskian space-time the equations will appear in the same form as an ordinary \((1 + 1)\) dimensional Klein-Gordon Lagrangian. Moreover, if we re-scale the field \( \Phi (\rightarrow \frac{1}{d} \Phi) \) and the mass term \( m (\rightarrow \frac{1}{d} m) \) we reach a matrix model with the same ordinary form. The Euler-Lagrange equation to the field \( \Phi \) is compute using the expression (15) applied to (17) where we obtain the equation of motion

\[ \Box_M \Phi = m^2 \Phi, \]  

(18)

where \( \Box_M = \eta^{\alpha\beta} \partial_\alpha \partial_\beta = \eta^{\tilde{\alpha}\tilde{\beta}} \partial_{\tilde{\alpha}} \partial_{\tilde{\beta}} = 2 \partial_+ \partial_- \) and choosing \( m = 0 \) the equation reduces to

\[ \partial_+ \partial_- \Phi = 0. \]  

(19)

What implies that the massless matrix scalar field \( \Phi \) is holomorphic in light-cone matrix coordinates. This interesting result of the collective behavior of the coordinates driven us to speculate about the possibility of collective integrability in high \((D = 4n + 2)\) dimension models.

The energy-momentum tensor is compute as an example of the matrix calculus. To the matrix spinorial field we take the Lagrangian (3), we obtain the improved matrix energy-momentum tensor:

\[ T^{\text{Dirac}}_{\alpha\beta} = \frac{i}{2} \left( \overline{\Psi} \sigma_\alpha \partial_\beta \Psi + \overline{\Psi} \sigma_\beta \partial_\alpha \Psi - \eta_{\alpha\beta} \overline{\Psi} \sigma_\gamma \partial_\gamma \Psi \right) \]  

(20)

bearing in mind that the bi-dimensional flat metric tensor \( \eta_{\alpha\beta} \) is an overall one which can be treated similarly as the ordinary one. To the matrix scalar field, we get

\[ T^{\text{scalar}}_{\alpha\beta} = 2 \left[ \eta^{\sigma\rho} \eta_{\alpha\beta} \partial_\sigma \Phi \partial_\rho \Phi - \partial_\alpha \Phi \partial_\beta \Phi \right]. \]  

(21)

Notice that the above expression have the same form as the ordinary 2D ones.

4 Conclusions

In this work we have introduced the firsts steps to the interpretation of a finite matrix bi-dimensional structure of simple field theories. It is essentially based on the contraction of the Dirac \( \Gamma \) matrices in the Weyl representation in \( D = 4n + 2 \) dimensions to the derivative in the spinorial Lagrangian (3). It was shown that in this representation the spinorial Lagrangian is re-formulated as a matrix bi-dimensional theory.

We have dealt with the particular \( D = 4n + 2 = 2d \) dimensions of space-times with signature \((\tilde{s}, \tilde{t}) = (2n + 1, 2n + 1)\). Which in an “overall” matrix Minkowskian metric case it can be separated in two \( d \) dimensional Dirac-preserving-algebra space-times with inverse metric sign and same dimension. This inversion of metric sign could represent a kind of “mirror” characteristic of the separation. The Lorentz Group with a Minkowskian overall metric \( SO(1, 1) \) with a signature \((\tilde{s}, \tilde{t})\) has two nested lower ordinary Lorentz (sub-)Groups with signatures \((s, t)\) where \( d = s + t \). We expect that this mirror characteristic have to do with analicity of the matrix
spaces in spite how obscure is the geometric interpretation of a matrix space-time. Indeed we had observed that in the particular AdS space-time where we can have real representation of the Dirac matrices this approach could represent that the $D = 4n + 2$ space-time is a complex extension of the $d = 2n + 1$ with signature $(n + 1, n)$.

We have defined a matrix bi-dimensional action principle and a Dirac algebra extension to the Leibniz rule to the matrix derivative, we called Dirac-Leibniz rule. This action principle is related to a matrix or collective dynamics of the two Dirac-preserving algebra sets of elements (coordinates, functions, fields, etc.) accommodated in matrices. As an application we obtain the collective equations of motion of a matrix spinorial and scalar fields.

We had obtained an collective holomorphism for the massless spinorial and scalar fields what emphases the bi-dimensional collective behavior. This interesting collective and bi-dimensional feature will allow us to go on into speculations about the formulation of extended objects. Indeed we expect that this matrix bi-dimensional feature can possibly represent a “mirror” characteristic of the space-time which could imply to a possible alternative procedure to the compactification scheme. Consequently it could reflect that we can “live” in one face of the mirror and the mass term is our contact to the “reflected face space-time. It has the same spirit as the Randall-Sundrum\cite{13} and the TeV Gravitation\cite{14} approaches, and the claim on folding of the branes too\cite{7} but without compactification.

Remarkably this alternative mirror vision can be only present when the two sets of coordinates are in an overall Minkowski space-time. So if we start with this collective features to a $D = 10$ model, we have two ways to reach five dimensional models and preserve a closed Dirac algebra: or compactifying five dimensions directly as one matrix, or interpret the other set of elements as a hidden one. Anyway in these two visions we have only one way to reach four dimensions.

Moreover, we can speculate that a string model can represent a collective behavior of some p-brane\cite{6}. Finally we have reasons to believe that this formalism have connections with supersymmetry.

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