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The value of regulating returns for enhancing the dynamic behaviour of hybrid manufacturing-remanufacturing systems

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Abstract

Several studies have determined that product returns positively impact on the dynamics of hybrid manufacturing-remanufacturing systems, provided that they are perfectly correlated with demand. By considering imperfect correlation, we observe that intrinsic variations of returns may dramatically deteriorate the operational performance of these closed-loop supply chains. To cope with such added complexity, we propose a structure for controlling the reverse flow through the recoverable stock. The developed mechanism, in the form of a prefilter, is designed to leverage the known positive consequences of the deterministic component of the returns and to buffer the harmful impact of their stochastic component. We show that this outperforms both the benchmark push system and a baseline solution consisting of regulating all the returns. Consequently, we demonstrate that the operation of the production system is greatly smoothed and inventory is better managed. By developing a new framework for measuring the dynamics of closed-loop supply chains, we show that a significant reduction in the net stock, manufacturing, and remanufacturing variances can be achieved, which undoubtedly has implications both for stock reduction and production stabilization. Thus, the known benefits of circular economy models are strengthened, both economically and environmentally.

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1. Introduction

The world economy is currently evolving from a linear model, which extracts resources and manufactures products that are disposed of after consumption, into a circular model, which keeps resources in use for as long as possible by collecting products at the end of their life cycle (Genovese, Acquaye, Figueroa, & Koh, 2017). This has motivated the development of a new supply chain paradigm, namely, the closed-loop supply chain. In contrast to traditional open-loop supply chains, which consider a one-way movement of materials, closed-loop systems include collection and material recovery processes, such as repairing, reusing, recycling, and remanufacturing (Guide, Harrison, & Van Wassenhove, 2003). In this sense, remanufacturing has become one of the cornerstones of this emerging circular model. This may be defined as the process of restoring used products to a ‘common operating and aesthetic standard’ (Souza, 2013) and, as such, is gaining strategic importance among policy makers and businesses as a stepping stone towards both environmental and financial sustainability. The environmental benefits, which have influenced the creation of new legislation (Govindan, Soleimanli, & Kannan, 2015), are clear. For instance, Steinhilper (1998) estimated that remanufacturing typically uses 85% less energy than manufacturing. The potential financial benefits have also led to an increased attention on remanufacturing practices (Guide & Van Wassenhove, 2009). From this perspective, closed-loop systems represent a way of ‘retaining the value of products’ (Worrell, Allwood, & Gutowski, 2016).

Nonetheless, these opportunities do not come without significant challenges, in the sense that the complexity of the relevant systems, and hence the difficulty related to their management, substantially increases (Barbosa-Póvoa, da Silva, & Carvalho, 2018; De Giovanni, Reddy, & Zaccour, 2016; Hosoda, Disney, & Gavirneni, 2015; Mitra, 2012). The closed-loop model goes beyond established ideas for the traditional supply chain, as both the forward and the reverse flows must be efficiently integrated into the same business model (Guide et al., 2003). Inventory management is one of the fields where the increased complexity manifests itself more
clearly. Closed-loop systems suffer from the combined effect of demand and returns uncertainty (Goltos et al., 2018; Govindan et al., 2015; Ketzenberg, 2009; Zeballos, Gomes, Barbosa-Povoa, & Novais, 2012), which significantly challenges their performance. We refer to this as a dual-source uncertainty.

Remanufacturing closed-loop supply chains may take different forms in practice. This research focuses on hybrid systems aiming to coordinate both manufacturing and remanufacturing operations towards satisfying customer demand (Aras, Verter, & Boyaci, 2006). As the serviceable stock originates from both new and remanufactured products, such systems generally appear when both products are perfect substitutes (Souza, 2013). Single-use cameras are a traditional example of perfect substitution, as customers readily accept restored cameras with new films (Atasu, Guide, & Wassenhove, 2008). Nowadays, hybrid systems can be commonly found, for instance, in the spare parts industry (Souza, 2013). Fig. 1 presents an overview of the generic structure of a hybrid manufacturing-remanufacturing system. By highlighting the two different flows of materials (forward and reverse), the flow of orders, and the two external sources of uncertainty (consumer demand and returns) in closed-loop systems, we delimit the scope of our analysis. Note that three lead times impact on the dynamics of the system. In addition, we clarify that the recoverable inventory stores returned products from the customer.

1.1. The dynamics of closed-loop supply chains

This research looks at the behaviour of hybrid manufacturing-remanufacturing systems from the prism of the supply chain dynamics discipline. This investigates how the interaction of the various elements of the supply chain affects the time-varying response of the information and materials flows within; hence capturing relevant phenomena like the Bullwhip Effect (Lee, Padmanabhan, & Whang, 1997), which refers to the amplification of the variability of orders in the supply chain. Studies in this field generally control the variability in orders and inventories as key performance indicators, being both symptomatic of poorly performing supply chains (Hwang & Xie, 2008). First, order variability has a severe impact on capacity-related production costs, as it creates unstable production schedules that significantly decrease the efficiency of manufacturers (e.g., Metters, 1997). Second, variability in the net stock, defined as the end-of-period serviceable inventory, determines the supply chain ability to meet effectively a predetermined service level (e.g., Disney & Lombrecht, 2008).

Despite this being a mature area of operational research in open-loop supply chains (see e.g., Asgari, Nikbaksh, Hill, & Farahani, 2016), it is still emerging in closed-loop settings, as noted in the review of the Bullwhip literature by Wang and Disney (2016) and in the recent analysis of the closed-loop supply chain field by Goltos et al. (2019). Only relatively few works have provided insights into the dynamics of these systems (Braz, De Mello, de Vasconcelos Gomes, & de Souza Nascimento, 2018). Due to the complexity of the mathematical analysis involved, these contributions usually make certain assumptions about the reverse flow that only hold true in specific practical contexts. This issue is common in the broad literature of closed-loop supply chain management, as noted by Guide and Van Wassenhove (2009). Of particular interest in our field of study is that most papers model the returns as a fixed fraction (return yield) of the demand after a lead time that represents the consumption process; see e.g., Adenso-Díaz, Moreno, Gutiérrez, and Lozano (2012), Cannella, Bruccoleri, and Framinan (2016), and Tang and Naim (2004). This can be interpreted as the assumption of perfect time-lagged correlation between demand and returns (Goltos et al., 2019). Although arguably these papers offer interesting observations, they do not address the aforementioned dual-source uncertainty issue; rather, they focus on the inefficiencies generated by demand uncertainty only. Moreover, they evaluate the dynamics of closed-loop systems by using the same indicators as in open-loop systems, i.e., looking at the manufacturing orders and the serviceable inventory (see e.g., Adenso-Díaz et al., 2012; Cannella et al., 2016; Tang & Naim., 2004); hence they do not consider the behaviour of the reverse loop, which is also expected to have a major impact on the cost performance of the supply chain.

Most of these papers claim that increasing the return yield leads to a reduced manufacturing (i.e. from virgin resources) order variability (Cannella et al., 2016; Da, Sun, & Zhao, 2008; Tang & Naim, 2004; Turrisi, Bruccoleri, & Cannella, 2013; Wang & Ding, 2009; Zhang & Yuan, 2016; Zhou & Disney, 2006; Zhou, Naim, & Disney, 2017; Zhou, Naim, Tang, & Towill, 2006). Indeed, the Bullwhip Effect may be eliminated by increasing the percentage of collected products in the closed-loop supply chain, while at the same time with a large number of echelons this phenomenon generally still persists (Cannella et al., 2016; Zhou et al., 2017). All in all, we may conclude that the closed-loop supply chain benefits from a mitigated Bullwhip Effect. Only a few contributions notably diverge from this finding. Both Huang and Liu (2008) and Jing and Gian (2009) showed order variability to be higher in closed-loop than in traditional supply chains. A noteworthy result was the one obtained by Adenso-Díaz et al. (2012), who observed a U-shaped relationship in the impact of the return rate on the Bullwhip Effect. For low values of this rate, an order smoothing effect was noticed, while for high values, the return rate tended to increase the variability of manufacturing orders. In these three articles, as also

![Fig. 1. Generic structure of a hybrid manufacturing-remanufacturing system.](image-url)
noted by Cannella et al. (2016), the return flow is not directly incorporated within the ordering policy, which may explain why the considered closed-loop systems did not benefit from the reverse materials flow.

The impact of the returns on net stock variability has been less explored but also led to different insights. While some studies supported the fact that closed-loop systems benefit from a reduced inventory variability in comparison with open-loop systems (Cannella et al., 2016; Huang & Liu, 2008; Zhou & Disney, 2006; Zhou et al., 2006; Zhou et al., 2017), others have found that the serviceable inventory of supply chains suffers from a higher volatility than that of open-loop systems (Da et al., 2008; Turrisi et al., 2013). Tang and Naim (2004) concluded that the level of information transparency is a key driver for decreasing the inventory variability in the supply chain, which helps us to understand how different assumptions result in contradictory results.

Hosoda et al. (2015) conducted the first study in the closed-loop supply chain dynamics literature that incorporates in the analysis the correlation between demand and returns. They showed that the value of advance notice of returns for improving the dynamics of the system is increasing in the correlation coefficient. Interestingly, they observed that increasing the volume of returns can have a negative impact on the dynamic performance, which was pointed out as an interesting topic for future research. To the best of our knowledge, the recent work by Hosoda and Disney (2018) is the only other work that provides insights into the impact of demand-return correlation on the dynamic behavior of hybrid manufacturing-remanufacturing systems. They showed that the performance of closed-loop systems strongly increases as the correlation grows, while their analysis was limited to the case where the returns are correlated to the demand in the previous period (which may be interpreted as the assumption of a mean return time of 1 period).

Overall, we conclude from the existing literature that in the presence of perfect correlation, returns impact positively on supply chain dynamics, especially when the availability of information is high. That is, the returns incorporate a component perfectly correlated with previous demand that enables closed-loop systems to improve their operational response by counteracting the operational variability caused by the customer requirements. However, recent works in this field suggest that the intrinsic variability of returns significantly reduces the performance of closed-loop supply chains (through an increased volatility), which may act as an important barrier to the deployment of circular economy systems in practice. At this point it should be noted that the vast majority of previous works, with the interesting exception of Zhou et al. (2006) that used a Kanban system, assume push inventory models in the remanufacturing process, i.e. the returns are processed as soon as they are collected. However, as also highlighted by Hosoda et al. (2015), managing the recoverable inventory through other policies could help real-world closed-loop supply chains to improve their dynamic behavior.

1.2. Motivation and contribution

Assuming perfect correlation constitutes a simplified feature of most closed-loop supply chain models previously developed. It means that variations in the returns series can be perfectly explained by those in the demand, after the consumption time; hence, knowing the demand translates into knowing future returns. In this case, without any intrinsic variability in the returns, the correlation coefficient is 1. While in some specific contexts the demand-return correlation is very high and this assumption may hold, in others the correlation is very low and it may even be considered that demand and returns follow independent processes. Several authors (e.g. Fleischmann & Kuik, 2003; Heisig & Fleischmann, 2001; Mitra, 2012) recognise the existence of these opposite scenarios in practice. In between these extremes (perfect correlation and independence), a wide range of real-world situations exist that may be associated with differing degrees of correlation, which would be useful towards forecasting the product returns (e.g. Agrawal, Singh, & Murtaza, 2014; Clotey, Benton, & Srivastava, 2012; Toktay, Wein, & Zenios, 2000). The lagged correlation between demand and returns may often decrease as a result of the influence of other factors, such as model obsolescence or usage patterns (Mitra, 2012). In these cases, although the returns emerge from the past demand, the demand time series lose explanatory power of the behaviour of the returns time series. For instance, this generally occurs for durable products, whose market sojourn times tend to be both long and variable (Heisig & Fleischmann, 2001).

Following from the discussion presented above, we aim to explore the operational performance of closed-loop supply chains in a wide range of practical scenarios. To this end, we consider how the demand-return correlation alter the effect of the returns on the system dynamics. In light of this, Fig. 2 represents the closed-loop scenario as a bi-dimensional space defined by the lagged correlation between demand and returns (θ) and the average return yield (β). On this plane, the traditional supply chain is defined by β = 0, while the perfectly correlated closed-loop system is defined by θ = 1. This figure illustrates the amplitude of the closed-loop field, which is divided into four quadrants. We provide examples of closed-loop archetypes that tend to operate within these quadrants: repair systems, third-party remanufacturing systems, e-commerce systems with commercial returns, and emerging industries. Hybrid manufacturing-remanufacturing systems, which we consider in this work, can operate in any of the quadrants, depending on several factors, such as those associated with the type of the product and the maturity of the industry.

In Fig. 2, the current state of knowledge is synthesised in three text-boxes, in which the degree of shading represents the level of exploration in previous research, such that the lighter the shading, the less research has been undertaken. It shows that the dynamics of traditional supply chains is a well-established discipline. Several works provide insights into the dynamic behaviour of such supply chains, since the demand variability amplification phenomenon was observed by Forrester (1961) and was labelled as the Bullwhip Effect by Lee et al. (1997). Also, Fig. 2 indicates that several research efforts have been conducted to understand the dynamics of closed-loop supply chains under the assumption of perfect correlation, with the paper by Tang and Naim (2004) generally considered as the first one (e.g. Goltos et al., 2019). Interestingly, they observe that the reverse flow of materials contributes to improve...
the dynamics of the system. Finally, this representation highlights that the 'wider' discipline of closed-loop supply chain dynamics has been little explored, although some recent works deserve a mention here, as we discussed in the previous section. In addition, we schematically pose three research questions that define the contributions of this article, which we discuss below.

Investigating the effect of the interaction between the return yield and the demand-return correlation on the performance of closed-loop supply chains represents our first contribution. To this end, we analyse separately the impact of two different components of the returns process: one that is correlated with the demand process and one that is independent. Following this, we propose a control structure for the reverse flow of materials. This is designed to avoid the intrinsic uncertainty of returns entering the closed loop and deteriorating the dynamic response of the overall system. Through this solution, which represents our second contribution, we explore the development of new business models for efficiently integrating the forward and reverse flows in remanufacturing systems. Finally, and as our third contribution, we develop—and use—a conceptual framework for evaluating the dynamics of closed-loop supply chains. Unlike previous works focusing only on the stability of the forward flow (i.e. manufacturing process and serviceable stock), we also consider the response of the reverse flow (i.e. remanufacturing process and recoverable stock). Note that these three main contributions closely relate to the main gaps identified in the previous review of the literature.

1.3. Methodological approach and organisation of the paper

We conduct our research by means of employing control-theoretic techniques. As in traditional supply chains (e.g., Dejonckheere, Disney, Lambrecht, & Towill, 2004; John, Naim, & Towill, 1994; Spiegler, Naim, Towill, & Wikner, 2016), control engineering facilitates a deep understanding of the dynamic behaviour of closed-loop systems and enables the exploration of real-world solutions for increasing the efficiency of such systems. We adapt the benchmark architecture for hybrid manufacturing-remanufacturing systems developed by Tang and Naim (2004) (see also, e.g., Cannella et al., 2016; Goltsos et al., 2019; Zhou et al., 2017) to scenarios with returns uncertainty. To this end, we develop a structure for regulating the reverse flow of materials at the recoverable inventory. To confirm the insights derived from the analytical study and further explore the behaviour of the supply chain, we simulate the long-term response of the system.

The remainder of the paper is structured as follows. Section 2 describes the supply chain model, including a discussion of the modelling of the relationship between demand and returns. In addition, we present and detail two control structures for the reverse flow based on regulating the returns inventory. Section 3 analyses the static response of the supply chain in order to avoid inventory offsets that may have a detrimental impact on the service level. Section 4 investigates the dynamics of both systems, resulting from the two different control structures, by studying their response due to a unit step. We analyse the transients of the manufacturing and remanufacturing orders, as well as those of the serviceable and recoverable inventory. Section 5 presents the simulation study, in which we measure the long-term variability of orders and inventories in the forward and the reverse flows. Finally, Section 6 concludes and we reflect on the managerial implications of this research.

2. Supply chain model

In this paper, we use the following notation for the variables of the closed-loop supply chain:
- \( c_t \): manufacturing completion rate in period \( t \),
- \( d_t \): consumer demand of the product in period \( t \),
- \( d_t \): forecasted demand in period \( t \),
- \( f_{rt} \): filtered returns in period \( t \), or products retrieved from the recoverable inventory,
- \( ir_t \): incoming returns in period \( t \), or remanufacturing rate of returned products,
- \( ns_t \): actual net stock in period \( t \), or end-of-period serviceable inventory of finished products,
manufacturing order rate of original products in period $t$,
returns in period $t$, namely the products collected after consumption, with $r^D_t$ and $r^F_t$ being the deterministic and stochastic component of this variable, respectively,
returns inventory in period $t$, or end-of-period recoverable inventory of returned products,
remanufactured returns in period $t$, assuming they reach an as-good-as-new state,
target net stock in period $t$, namely, the safety stock of the serviceable inventory,
target returns inventory in period $t$, namely, the safety stock of the recoverable inventory,
target work-in-progress (WIP) in period $t$, and
WIP in period $t$, or inventory both in the manufacturing and the remanufacturing pipelines.

We employ the same notation for the variables in the discrete-time domain and in the Laplace domain, but with lowercase letters in the former, $x_t$, and uppercase in the latter, $X(s)$. In addition, we use the following notation for the parameters of the closed-loop supply chain:

- $m$ noise ratio, relating the variance of the stochastic component of the return to that of the demand,
- $T_0$ exponential smoothing parameter of the demand forecasting method, whose relation to the constant $\alpha$ of simple exponential smoothing expressed as a difference equation is estimated as $\alpha = 1/(1 + T_0)$ for a sampling interval of 1 unit of time, see John et al. (1994),
- $T_c$ average consumption time, or average time for the customer to keep the product,
- $T_f$ time constant of the prefilter, defining the fraction of returns that is released from the recoverable inventory into the remanufacturing process,
- $T_p$ time constant of the proportional controller of the net stock, or time to adjust the net stock,
- $T_m$ manufacturing lead time,
- $T_r$ re-manufacturing lead time,
- $T_w$ time constant of the proportional controller of the WIP, or time to adjust the WIP,
- $\beta$ average return yield, and
- $\theta$ time-lagged correlation coefficient between demand and returns.

2.1. Baseline model

We first build on the model by Tang and Naim (2004) representing the dynamic behaviour of closed-loop supply chains, which in turn has been developed by extending the widely used automated pipeline, inventory and order based production control system (APOBPCS) model designed by John et al. (1994) for traditional supply chains. Our baseline model is shown in Fig. 3, where the solid lines represent the forward operations and the dashed lines display the reverse logistics operations. We refer to Tang and Naim (2004), see type 2 system, for further details behind the modelling. Nonetheless, we highlight four assumptions that deserve special attention:

(i) **Order-up-to replenishment policy.** Orders are issued as the sum of three terms: the gap between the forecasted demand and the remanufactured items, a fraction of the gap between the desired and the actual net stock level, and a fraction of the gap between the desired and the actual WIP.

(ii) **High level of information transparency.** Inputs and outputs of the remanufacturing process are known and are used to estimate the overall WIP. Therefore, this system makes best use of available information from the remanufacturing process to issue the manufacturing orders.

(iii) **Perfect (positive) correlation between demand and returns.** A fixed percentage of the sold products re-enter the supply chain after the consumption time. This means that the correlation coefficient between demand and returns is +1, considering the time lag between both series.

(iv) **Push type remanufacturing system.** The returns are pushed into the remanufacturing process as soon as they are collected from the market and they end up in the serviceable inventory. In this sense, the stock is not regulated at the recoverable inventory site.

2.2. Modelling imperfect correlation between demand and returns

Following from the information above, our study emphasises order-up-to inventory models in which all the available information is employed to improve the control structure. However, we violate assumption (iii) in order to consider the impact of the time-lagged correlation between demand and returns on the dynamics of the closed-loop supply chain. As a consequence, we will also later violate assumption (iv) with the aim of enhancing the operational response of the system.

We model the returns in period $t$, $r_t$, as a fraction, the average return yield $\beta$, of the demand $T_c$ periods before, $d_{t-T_c}$, plus a white noise term, $\varepsilon_t$, following a statistical distribution with mean 0, given by

$$r_t = \beta \cdot d_{t-T_c} + \varepsilon_t = r^D_t + r^F_t.$$  \hfill (1)

Otherwise (if the mean is not 0), $\beta$ would not represent the average return yield. Thus, we assume that the demand and the white noise term of the returns are independent and identically distributed (i.i.d.) random variables; we also assume that they follow normal distributions with $d \sim N(\mu_d, \sigma_d^2)$ and $\varepsilon \sim N(0, \sigma^2_e)$. This approach allows us to link our analysis with the existing body of knowledge. Conceptually, the first addend represents the deterministic component of the returns, $r^D_t$, which symbolises the variations of returns due to variations in demand, while the second one represents the stochastic component, $r^F_t$, which considers the intrinsic variability of the returns. Note that $r^F_t > 0$ indicates that actual returns are higher than those expected according to the demand evolution and the consumption time, while $r^F_t < 0$ reflects the opposite scenario ($r^F_t = 0$ refers to actual returns being exactly as expected). The block diagram representation of this returns process can be seen in Fig. 4.

From Mitra et al. (2012), the correlation coefficient $\theta$ between demand and returns, considering the time lag of the average consumption time $T_c$, is

$$\theta = \frac{1}{\sqrt{1 + \frac{m}{\sigma_r^2}}}.$$  \hfill (2)

where $m$ is the noise ratio, defined as the quotient of the variances of both noise sources,

$$m = \frac{\sigma^2_r}{\sigma^2_e}.$$  \hfill (3)

Of course, $\sigma^2_e = 0$ results in $\theta = 1$, which is the situation of the baseline model. However, for a given $\sigma^2_r$ and $\beta$, $\theta$ is a decreasing function of $\sigma^2_e$. That is, increasing the variability of the stochastic component of the returns (white noise term) decreases the demand-return correlation at lag $T_c$. 

2.3. Two control structures for the reverse flow

From a control-theoretic perspective, the stochastic component of the returns represents a second, independent, disturbance in the closed-loop system. This adds an additional exogenous variable that needs to be accommodated. To mitigate the potential negative impact of this new source of variability, and hence reduce the volatility in the supply chain, we develop a control structure for the reverse flow based on building stock in the returns inventory and releasing a fraction at a time. In control theory, this element, which aims to smooth the signal entering the feedback loop, is referred to as a prefilter. This is very common in control structures, as it adds a degree of freedom to the design problem. Besides smoothing the signal, it may have other purposes, such as adjusting the overall gain of the system or mitigating signals within a predefined range of frequencies.

Thus, not all returns are directly pushed into the remanufacturing process, as assumed in previous studies, but rather they go through a recoverable inventory which regulates the collected products to be processed at each time. This is commonly the case in many industries where there is a build-up of returned products (e.g., Difrancesco & Huchzermeier, 2016; French, 2008; Guide, Jayaraman, Srivastava, & Benton, 2000). Through this strategy, we aim to improve the dynamics of the hybrid system.

We evaluate two different control structures for the reverse flow, representing two alternatives for placing the prefilter. First, the control-theoretic approach would directly place the prefilter on the external disturbance, that is, the white noise. This would be aimed at leveraging the positive impact of the deterministic component of the returns on the closed-loop supply chain, at the same time that the consequences of the stochastic one are mitigated. Second, we consider a conventional—maybe the most intuitive—approach, which would control all the returns received. Under this scenario, the prefilter would accommodate both components of the returns. Both control structures are displayed in Fig. 5, where \( R(s) \) refers to the position of the returns inventory, \( IR(s) \) represents the remanufacturing rate of returns, and \( FR(s) \) accounts for the returns recovered from the recoverable stock. Note that Fig. 5 also displays the enumerated transfer function of the prefilter.

Eqs. (4a)–(6a) present the discrete form of the first control structure, in which a portion of the returns (defined by its deterministic term) is directly pushed into the remanufacturing process, while the rest (its stochastic term) builds up stock, forming the recoverable inventory. We call this the stochastic-returns-inventory (SRI) representation.

\[
\begin{align*}
    r_i &= r_{i-1} + r_i^f - fr_t \\
    i_r &= r_i^f + fr_t \\
    f_t &= \frac{1}{I_f} \left( r_i - tr_i \right) = \frac{1}{I_f + 1} \left( r_{i-1} + r_i^f - tr_i \right)
\end{align*}
\]

On the other hand, Eqs. (4b)–(6b) represent the second control structure, in which both components of the returns pass through the recoverable inventory. We refer to this as the all-returns-inventory (ARI) representation. We note that the term \( T_f \beta \mu_d \) is required to compensate the effect of the deterministic component of the returns in the recoverable inventory.

\[
\begin{align*}
    r_i &= r_{i-1} + r_i^f + r_i^f - fr_t \\
    i_r &= fr_t
\end{align*}
\]
\[ f_T = \frac{1}{T_f} \left( r_i + T_f \beta \mu_d - \tau r_i \right) \]

\[ = \frac{1}{T_f + 1} \left( r_i + r_i^0 + T_f \beta \mu_d - \tau r_i \right) \]  \hspace{1cm} (6b)

Appendix A provides more detail on the rationale behind both control policies for the returns inventory by providing a practical example of how they may be implemented in practice.

2.4. Block diagrams

The overall block diagram representing both closed-loop supply chain models, which only differ in the control structure of the returns flow, is shown in Fig. 6. We name them the SRI- and ARI-APIOBPCS models. We note that we develop the study in the Laplace domain to ensure consistency, and for benchmarking, with the foundational works in this area (Tang & Naim, 2004; Zhou & Disney, 2006; Zhou et al., 2006). This approach allows us to focus on the time-varying characteristics of the hybrid system and explore the impact of shock and random stimuli (Zhou et al., 2017).

A significant contribution of this research paper is that we focus not only on the dynamics of the forward flow but also consider those of the reverse flow. That is, besides exploring the response of \( O(s) \) and \( N(s) \), we analyse that of \( IR(s) \) and \( RL(s) \). Thus, to investigate the impact of both inputs (i.e. \( D(s) \) and \( R^2(s) \)) on the four outputs, we have obtained eight transfer functions for each model, which are detailed in Appendix B. To develop such transfer functions, as usual in this kind of studies, we adopt the following simplification for the modelling of lead times in the closed-loop system: \( e^{-T_f s} \equiv 1/(1 + Ts) \), where \( T \) is the time lag. This has been shown to be a sufficient representation of real-world lead-time distributions in supply chains (Wikner, 2003; Wolstenholme, 1990; Zhou et al., 2017).

3. Stability and static analysis

To delimit the behavioural boundaries for both closed-loop supply chain models, we first analyse their stability. This depends upon the position of their poles in the complex plane. The transfer functions shown in Appendix B illustrate that the poles of both systems are the same. These can be obtained by equating the following factors to 0; (1) \( T_0 s + 1 \); (2) \( T_1 T_0 s^2 + (T_1/T_0)(T_0 + T_0) s + 1 \); (3) \( T_0 s + 1 \); (4) \( T_0 s + 1 \); and (5) \( T_0 s + 1 \). To verify the stability condition, all the poles must be negative real numbers or complex numbers with negative real parts. Poles of factors (1), (3), (4), and (5) can be directly obtained; while for factor (2), we apply the Routh–Hurwitz criterion (Dorf & Bishop, 1995) for second-order systems. This states that if and only if the coefficients of the polynomial are greater than 0, all the poles are placed on the left side of the imaginary axis.

Assuming that parameters are not null, the following conditions apply as necessary and sufficient to ensure the stability of both systems: \( T_0, T_1, T_2, T_2 > 0, T_0 T_1 > 0 \), and \( (T_1/T_0)(T_0 + T_0) > 0 \). This defines three pathways to achieve a stable model: (a) \( T_0, T_1, T_2, T_2, T_2 > 0, T_0 T_1 > 0 \), \( T_0 T_1 > 0, T_0 T_1 > 0, T_0 T_1 > 0 \), \( T_0 T_1 > 0 \), \( T_0 T_1 > 0 \). Given that the pathway (b) holds mathematically but is meaningless in practice, as it would imply a negative manufacturing lead time, we do not consider this option. Note that the average return yield \( \beta \) and the estimated pipeline lead time \( T_0 \) are the only structural parameters in the supply chain that do not determine its stability.

To develop the steady-state study, we focus on the concept of static gain. For stable systems, it represents the ratio of the (variation in the) output and the (variation in the) input under steady-state conditions, i.e. it reports the final value of the output when there is a unit step in the input. Thus, it allows us to check that the system behaves in the long term as desired. For a generic function \( G(s) \), the static gain is obtained by \( G(0) = \lim_{s \to 0} G(s) \). The final value of the unit step response could also be obtained by applying the Final Value Theorem of the Laplace transform (Aseltine, 1954). For the 16 functions, the static gains are represented in Table 1.

| Table 1 |
|---|
| Final value of the state variables when facing a unit step in the two inputs. |
| \( O(s) \) | \( N(s) \) | \( IR(s) \) | \( RL(s) \) |
| SRI-APIOBPCS | \( D(s) \) | \( 1 - \beta \) | \( (T_1/T_0)(T_2 - (1 - \beta)T_0 - \beta T_1) \) | \( \beta \) | 0 |
| \( R^2(s) \) | \( -1 \) | \( (T_1/T_0)|T_0 - T_1| \) | 1 | \( T_f \) |
| ARI-APIOBPCS | \( D(s) \) | \( 1 - \beta \) | \( (T_1/T_0)(T_2 - (1 - \beta)T_0 - \beta T_1) \) | \( \beta \) | \( \beta T_f \) |
| \( R^2(s) \) | \( -1 \) | \( (T_1/T_0)|T_0 - T_1| \) | 1 | \( T_f \) |
(a) Supply chain with the prefilter for stochastic returns, or SRI-APIOBPCS model.

(b) Supply chain with the prefilter for all returns, or ARI-APIOBPCS model.

Fig. 6. Block diagram of the closed-loop supply chain with imperfect demand-return correlation.
The impact of both inputs on manufacturing and remanufacturing processes, i.e. on \(O(s)\) and \(IR(s)\), is straightforward in the two supply chain models. As can be expected, a sudden unit increase in demand will provoke a long-term increase of \((1 - \beta)\) units in the manufacturing orders and \(\beta\) units in the remanufacturing rate, which will eventually return to the recoverable inventory. Both closed-loop supply chains also behave appropriately when facing a unit step in the stochastic component of the returns, which would eventually translate into a unit decrease in the manufacturing orders as well as into a unit increase in the remanufacturing rate.

As underlined by Tang and Naim (2004), there is a potential offset set of the steady-state value of the net stock for both supply chain models—this occurs when the static gain of the serviceable inventory is not 0—, caused by long-term variations in demand, unless it is verified that

\[
T_p = (1 - \beta)T_m + \beta T_r. \tag{7}
\]

That is, this net stock offset is avoided if and only if the estimated pipeline lead time is obtained as a weighted average of the manufacturing and remanufacturing lead times. As per Eq. (7), the average return yield \(\beta\) defines the balance between both lead times. Table 1 shows that this applies for both closed-loop supply chain models. As discussed by Disney and Towill (2005), such an offset tends to decrease significantly the inventory performance of the supply chain.

In contrast, the offset in the net stock caused by a long-term increase in the stochastic component of the returns cannot be avoided by enumerating an appropriate decision parameter. This offset is positively related to the difference between the manufacturing and remanufacturing lead times and the ratio of the time constant of the inventory controller to that of the WIP controller. Only if \(T_r = T_m\) (a relatively unlikely case since these constitute physical parameters) will an offset be prevented. However, due to the white noise condition of this stochastic component—that is, a long-term increase or decrease in the volume of returns would impact on the mean of the deterministic component (through the average return yield) rather than on the mean of the stochastic component—, this offset will not cause a significant long-term inventory drift in the closed-loop supply chain.

Table 1 reveals that the only difference between the steady-state behaviour of both systems emerges in the response of the recoverable stock to a demand change. A sudden demand increase tends to generate an inventory offset when all returns go through the recoverable inventory, which is avoided by prefiltering exclusively the stochastic component of the returns. This explains why an additional term is required in the serviceable inventory model, as per Fig. 5(b); however, this may cause volatility in the supply chain when the relevant parameters \((\beta, T_r, \mu_d)\) suffer from changes over time. Therefore, the steady-state impact of demand on the recoverable stock highlights the first advantage of the SRI representation with respect to the ARI one. Finally, an inventory offset is also caused by stochastic returns on the returns inventory of both closed-loop models, but, again, its impact is expected to be less damaging as this component of the returns represents a white noise with mean 0.

4. Dynamic analysis

In this section, we study the response of the supply chain when it faces a unit step in both inputs. Due to its analytical value, the step response provides relevant insights into the dynamics of control systems. Hence, this classic rich picture has been widely shown to facilitate a firm understanding of the long-term behaviour of supply chains (Towill, Zhou, & Disney, 2007). First, we analyse the impact of stochastic returns. This will allow us to find out how the prefilter improves the dynamics of the hybrid system. It should be noted that, as can be seen in the block diagrams represented in Fig. 6, this input has the same consequences on both closed-loop supply chain models. Second, we investigate the effect of the demand with the aim of comparing both control structures for the returns inventory.

For this analysis, the decision parameters \(T_p\), \(T_r\), and \(T_m\) have been set based on recommendations put forward by John et al. (1994), i.e. \(T_p = 16\), \(T_r = 8\), and \(T_m = 8\) (periods). This configuration has been widely understood as a ‘trade-off setting’ for the inventory control system of supply chains, minimising the variance in orders and inventory by slowing the system response but not at the expense of recovering from demand changes. Similar to Tang and Naim (2004) and Zhou et al. (2017), we have selected \(T_m = 8\), \(T_r = 4\), and \(T_p = 32\) (periods) for the lead times. This represents real-world scenarios, where remanufacturing generally takes less time than manufacturing, and consumption tends to be the longest process. Moreover, we assume that, on average, 40% of the products return to the supply chain, that is, \(\beta = 0.4\). We will later consider the impact of the return yield by exploring other values for this variable. Finally, the WIP policy has been designed to avoid the long-term net stock offset caused by changes in demand; hence, \(T_p = 6.4\) (periods); see Eq. (7).

4.1. Exploring the impact of stochastic returns on the closed-loop supply chain

As previously highlighted, the relationship between the uncorrelated component of the returns and the four state variables defining the dynamics of the supply chain (i.e. \(O(s)\), \(NS(s)\), \(IR(s)\), and \(R \in L(s)\)) is the same in the SRI- and ARI-APIOBPCS models. In light of this, we investigate the transients of the behaviour of the variables when subjected to a unit step in the stochastic component of the returns at \(t = 0\) to understand the impact of the prefilter on the dynamics of the system. Fig. 7 depicts the step responses. These need to be interpreted in relative terms to the initial state of the system (as indicated by the symbol \(\Delta\)); thus representing increases and/or decreases with respect to this initial state. For ease of analysis, the initial values for all relevant variables have been set as 0.

It is important to underline that \(T_r = 0\) represents the hybrid system without the control structure for the returns inventory; see Eqs. (4a)–(6a) and (4b)–(6b) resulting in \(i_r = r_l\) (for a fixed \(i_r\)). In light of this, Fig. 7 illustrates how this baseline closed-loop system deals poorly with the stochastic component of returns. Fig. 7(a) and (b) reveals that the system amplifies the variability of the orders issued to deal with stochastic returns, and it also generates variability in the serviceable stock. It represents another form through which the Bullwhip Effect phenomenon manifests itself in the supply chain, which will add to the traditional one caused by uncertainty in demand.

Both negative consequences (on manufacturing orders and serviceable inventory) can be mitigated by building up a returns inventory, through which the release of products to the remanufacturing process is regulated. As can be seen in the graphs under concern, increasing the value of \(T_r\) smooths the behaviour of the supply chain. The overshoot in both responses (slightly greater than 10% for \(T_r = 0\) in both cases) can be reduced or even avoided by an appropriately regulated prefilter. Therefore, we can expect that this mechanism will positively impact the long-term operational response of the supply chain in terms of orders and net stock variability, which will be verified in Section 5.

By inspection of Fig. 7(c) and (d), we observe the dynamics of the reverse flow of materials. It can be seen how the proposed control structure smooths the remanufacturing process at the expense of creating variability at the recoverable inventory. Hence, the only drawback of the prefilter can be explained in terms of this
Indeed, this variable does not exist in the baseline system (i.e. \( r_t = 0 \)), in which returns are directly remanufactured. However, a priori, building up a returns inventory is much less costly for the hybrid manufacturing-remanufacturing system than an end-product inventory. Note that the latter directly responds to customer requirements; hence, its variability will directly impact on the service level of the closed-loop supply chain.

Overall, increasing \( T_f \) has a positive effect from three perspectives, but it negatively impacts on the recoverable stock. This underscores the need for finding the right balance between its desired effects and its counter effect, as is the case with other decision variables in the supply chain.

### 4.2. Exploring the impact of demand on the closed-loop supply chain

As seen, the control of the recoverable inventory mitigates the harmful effect of returns uncertainty on closed-loop supply chain dynamics. But how does it affect the relationship between demand and the four state variables? As per Fig. 6, the prefilter does not impact on this relationship in the SRI-APIOBPCS model, but on the contrary it does in the ARI-APIOBPCS model. Thereby, \( T_f = 0 \) does not only represent the baseline supply chain but also (only in this analysis) the SRI-APIOBPCS model. This allows us to compare both control structures for the reverse flow of materials. To this end, Fig. 8 represents the unit-step response of the ARI-APIOBPCS model when facing a change in demand at \( t = 0 \). Again, we assume the initial values of the four relevant variables are 0.

Fig. 8(a) and (b) suggests that adding the prefilter for all the returns deteriorates supply chain dynamics by increasing both manufacturing order and net stock variability. The maximum of the overshoot (in absolute value) of both responses increases as \( T_f \) grows. Nonetheless, the sensitivity of the net stock response of the ARI-APIOBPCS model to changes in \( T_f \) seems to be relatively low.

In light of this, the SRI-APIOBPCS model outperforms the ARI-APIOBPCS model from a perspective relying on the dynamics of the forward flow of materials. In addition, Fig. 8(d) shows that
prefiltering all the returns increases the variability and mean of the serviceable inventory position. On the contrary, Fig. 8(c) illustrates the only positive effect of the ARI-APIOBPCS model. As can be expected, prefiltering all the returns (and not only their stochastic component) right before the remanufacturing process decreases the variability of the remanufacturing rate.

In summary, only the remanufacturing process, through a reduced variability, benefits from prefiltering all the returns; but this upgrade occurs at the expense of deteriorating the dynamics of the manufacturing orders, the serviceable inventory, and the recoverable inventory. Thus, although regulating all the returns seems to be the most intuitive option for controlling the reverse flow, this alternative can be enhanced by regulating only the independent component of the returns. We interpret this interesting result in the following manner. As demonstrated by previous works (e.g. Cannella et al., 2016; Tang & Naim, 2004; Zhou et al., 2017), the deterministic component of the returns has a positive impact on the dynamics of closed-loop supply chains. Thus, smoothing their entry into the system may buffer this enhancement; hence losing the value of the correlated returns. If, on the contrary, only the stochastic component of the returns is regulated through the returns inventory: (1) the dynamic improvement prompted by the deterministic component in the closed-loop system is preserved; at the same time that (2) the negative impact of the new source of variability, i.e. the stochastic component, on the dynamics of the closed-loop system is widely mitigated.

Appendix C offers an in-depth understanding of the underlying causes of the dynamic impact of the prefiler on both supply chain models by looking at the position of poles and zeros in the complex plane.

5. Operational study

The step response illustrates how the closed-loop supply chain responds when it faces a sudden change in its inputs. It is called the ‘shock lens’ through which the dynamics of supply chains can be analysed (Towill et al., 2007). To explore its performance in a realistic scenario and verify the previous insights, we resort to
simulating the long-term supply chain behaviour when both inputs are i.i.d. normally distributed variables. This approach is the so-called 'variance lens' (Towill et al., 2007). To this end, we simulate consumer demand through a normal distribution with mean of 100 units and standard deviation of 20 units, i.e. $d \sim N(100, 20^2)$. The use of normal distribution to simulate customer behaviour is very common in the literature, e.g. Tang and Naim (2004) and Hallkos, Kevork, and Tziourtzioumis (2018). Since the stochastic component of the returns is considered to be a white noise process, it follows a normal distribution, $\epsilon \sim N(0, m \cdot 20^2)$, where $m$ is the previously defined noise ratio, as per Eq. (3). Unless otherwise stated, we use the same values for the supply chain parameters as in the previous section.

In this section, we delve into the amplification of variability in the hybrid manufacturing-remanufacturing system and the use of the prefilter to mitigate it. We first propose a conceptual framework for evaluating the dynamic performance of closed-loop supply chains, considering both the forward and reverse flows of materials. Next, we calculate the proposed metrics as functions of the time constant of the prefilter $T_f$. Finally, we investigate the effect of the correlation between demand and returns by examining different values of the noise ratio $m$ and the average return yield $\beta$.

5.1. An integrative framework for assessing the dynamics of closed-loop systems

As previously discussed, the vast majority of the relevant papers in the field of closed-loop supply chain dynamics evaluate the operational performance of these systems by using the same metrics as in open-loop supply chains. That is, they consider the (manufacturing) order variance ratio,

$$\text{MOV} = \frac{\sigma^2_d}{\sigma^2_r},$$

which is also known as the Bullwhip ratio, and the (serviceable) inventory variance ratio,

$$\text{SIV} = \frac{\sigma^2_i}{\sigma^2_r}.$$

The former greatly contributes to production costs and the latter significantly impacts on inventory costs (Cannella, Barbosa-Póvoa, Friman, & Relvas, 2013; Disney & Lambrecht, 2008). In this sense, they represent a key trade-off faced by supply chain managers (Disney, Farasyn, Lambrecht, Towill, & Van de Velde, 2006).

This approach is based on the forward flow and provides us with relevant information on the performance of closed-loop supply chains but does not look at the problem (system) in its entirety. The same rationale can be easily applied to the reverse flow, in which case variability can also be assumed to trigger production- and inventory-related costs in the closed-loop supply chain. Thus, we define the remanufacturing order variance ratio,

$$\text{ROV} = \frac{\sigma^2_d}{\sigma^2_r},$$

and to assess the variability in the remanufacturing rate in comparison with the variability in demand, and the recoverable inventory variance ratio,

$$\text{RIIV} = \frac{\sigma^2_i}{\sigma^2_r},$$

to measure the variability in the returns inventory over the variability in demand.

These four metrics constitute collectively a more complete scorecard to explore the dynamics of closed-loop supply chains. To better interpret the metrics, we may split them into two parts by considering the two noise sources in the supply chain, i.e. demand and stochastic returns. To do so, the principle of superposition for linear systems allows us to express any state variable, $x_t$, as the sum of the impact of the demand, $x^d_t$, and the stochastic returns, $x^r_t$, on this variable. That is, $x_t = x^d_t + x^r_t$ (with the other inputs shown in Fig. 6 being 0). As both external variables are assumed to be independent (and their effects on $x_t$ can also be assumed to be so), $\sigma^2_x = \sigma^2_{x^d} + \sigma^2_{x^r}$. Dividing by the variance of the demand and using the noise ratio, $m$, we obtain $\frac{\sigma^2_d}{\sigma^2} = \frac{\sigma^2_{x^d}}{\sigma^2} + \frac{\sigma^2_{x^r}}{\sigma^2} = \frac{\sigma^2_{x^d}}{\sigma^2} + \frac{\sigma^2_{x^r}}{\sigma^2} + m \frac{\sigma^2_{x^r}}{\sigma^2}.$

Thereby, in the system under consideration, the overall amplification of variability in any state variable over the demand can be expressed as the intrinsic amplification caused by the demand uncertainty (when $\sigma^2_{x^r} = 0$) plus $m$ times the intrinsic amplification caused by the returns uncertainty, which arises from the stochastic component (when $d_t = 0$). For example, for the manufacturing orders, $\text{MOV} = \text{MOV}^d + m \cdot \text{MOV}^r$, using superscripts to illustrate the impact of each input on this variable. As previously discussed, the second addend remains largely unexplored in the literature.

5.2. On how the prefilter mitigates the variability amplification

Under the previously defined conditions and initially employing $m = 1$ (that is, the same variability in both noise inputs), we simulate the behaviour of the SRI- and ARI-APIOBPCS models over 200,000 time periods for $T_f = \{0, 1, 2, 3, \ldots, 16\}$. This has proven to be a large enough time horizon to ensure the stability of the response and the consistency of the results (e.g. Ponte, Wang, de la Fuente, & Disney, 2017). For each simulation run, we measure the four defined operational metrics, i.e. MOV, SIV, ROV, and RIV. Fig. 9 represents these results. To facilitate their interpretation, this figure also displays the amplification of variability generated by the demand and the stochastic component of the returns (e.g. in the first graph $\text{MOV}^d$ and $\text{MOV}^r$, respectively). Note that, as previously discussed, the second term (i.e. the impact of stochastic returns) is the same for both supply chain models.

We first refer to the manufacturing variance. Fig. 9(a) shows that the variability in orders caused by the stochastic returns, represented by a dotted line, is greatly alleviated by the proposed control structure. The dashed lines show that the variability in orders caused by the demand is independent of $T_f$ in the SRI-APIOBPCS model, while it is slightly increasing in $T_f$ in the ARI-APIOBPCS model. Accordingly (note that for $m = 1$, MOV results in the sum of both individual ratios), the solid line reveals that the manufacturing variability can be significantly reduced by controlling the returns inventory. The higher the parameter $T_f$, the lower the ratio $\text{MOV}^R$. In addition, the benefits of the prefilter are accentuated, from this perspective, for the SRI-APIOBPCS model. All these observations are strongly in line with the insights derived from the analysis of the step response.

Similar conclusions can be drawn from the analysis of the serviceable inventory in Fig. 9(b). First, SIV/R can be significantly reduced by the use of the prefilter. This improvement also springs from controlling the recoverable inventory, which mitigates the variability that enters into the closed loop due to the intrinsic uncertainty on returns. Second, the SRI-APIOBPCS model again outperforms the ARI-APIOBPCS model, given that regulating all the returns worsens the dynamic impact of demand on the variability of the serviceable inventory. These findings also confirm those insights derived from exploring the step response of the net stock.

We now consider the variability in the remanufacturing rate. Consistent with the step response study, ROV/R is the only operational metric benefiting from the ARI-APIOBPCS model in comparison to the SRI-APIOBPCS model. Nonetheless, Fig. 9(c) shows that
the difference is relatively small in the context studied here, as the stochastic returns are the main source of variability in the reverse flow (note that the difference between both models arises through the deterministic component). In any case, this reverse flow becomes dramatically more stable as $T_f$ increases; hence, the control policy for the returns stock substantially enables stability in the operation of the closed-loop supply chain.

Finally, Fig. 9(d) plots the variability in the recoverable inventory. If all the returns are directly remanufactured (i.e. the baseline push system, with $T_f = 0$), $RIV = 0$. However, when the pre-filter is considered, variability appears in the recoverable inventory and it increases proportionally with $T_f$. It can be noted that, again, the SRI-APIOBPCS model performs better than the ARI-APIOBPCS model. These insights also fit perfectly with those obtained from the step response analysis.

In short, we observe how the demand and the stochastic component of the returns add to the generation of variability in the closed-loop supply chain. From this perspective, Fig. 9 clearly reveals that regulating the reverse flow of materials enables a major enhancement in the dynamics of hybrid manufacturing-remanufacturing systems. However, this occurs at the expense of creating variability in the recoverable stock, which stresses the need for appropriately setting the prefilter so that the operational performance of the system is optimised. Overall, we may also conclude that these benefits are stronger if only stochastic returns are regulated at the recoverable inventory. This improvement of the SRI- over the ARI-APIOBPCS model can be observed from the perspective of manufacturing orders, serviceable inventory, and recoverable inventory; however, remanufacturing order variability is slightly higher.

5.3. On how the demand-return correlation influences performance

To explore the impact of demand-return correlation on the system dynamics, we carry out the same analysis for, first, a range of noise ratios $m$ and, second, several average return yields $\beta$. As per
Eq. (2), these are the parameters that determine the value of the lagged correlation $\theta$.

First, we modify the value of the standard deviation of the stochastic component of the returns. Besides considering the previous results obtained for $\sigma_x = \sigma_d = 20$ ($m = 1, \theta = 0.371$), we simulate $\sigma_x = 0.5\sigma_d = 10$ ($m = 0.25, \theta = 0.625$), which illustrates a scenario in which returns uncertainty is relatively low and demand-return correlation is high, and $\sigma_x = 2\sigma_d = 40$ ($m = 4, \theta = 0.196$), which represents the opposite scenario. These results are displayed in Fig. 10. For the sake of simplicity, and once we have understood how this variability is generated from its two components, we only plot the values of the four metrics for each considered noise ratio.

All in all, these plots show the strength of the negative impact of returns uncertainty on the dynamics of closed-loop supply chains. That is, although the operation of supply chains may significantly benefit from reverse logistics, a high intrinsic variability in the returns may outweigh this improvement leading the supply chain to a poor dynamic performance. This occurs especially when a push policy is employed in the recoverable inventory (i.e. for $T_f = 0$). Under these circumstances, while variance ratios for perfect correlation are lower than in open-loop supply chains, if $m$ was large enough, the enhancement against the traditional system would be lost. Hence, the need for controlling the recoverable inventory—and the benefits of regulating the time constant $T_f$ at high levels—grows as $m$ increases, i.e. as the (time-lagged) correlation between demand and returns decreases.

Secondly, we return to $m = 1$ and consider several yields. This can be also interpreted as the impact of the collection strategy efficiency in the supply chain (Hong, Govindan, Xu, & Du, 2017). To this end, besides employing the previous value of $\beta = 0.4$ ($\theta = 0.371$), we analyse $\beta = 0.1$ ($\theta = 0.100$) and $\beta = 0.7$ ($\theta = 0.573$). Interestingly, this allows us to explore the evolution from open- to closed-loop supply chain archetypes through increased return rates. We display these results in Fig. 11.

We observe that increasing the return yield has a positive impact on the manufacturing order variability. A positive effect of
the return yield can also be obtained in the variability of the serviceable stock. Nonetheless, the strength of this effect is relatively low, as noted by Tang and Naim (2004). However, and interestingly, Fig. 11(b) reveals that when $T_f$ is high in the ARI-APIOBPCS model, the impact of increasing the return yield is negative in terms of inventory variability (note the order of the curves is inverted for high values of $T_f$), which is another argument in favour of regulating only the stochastic component of the returns. Finally, we underline that the dynamics of the reverse flow of materials are less influenced by the return yield, especially in the SRI-APIOBPCS model.

In general terms, in this research we have considered a dual-source uncertainty problem in closed-loop supply chains, in which the noise ratio $m$ and the return yield $\beta$ determine the lagged correlation $\theta$ between demand and returns. As our results have revealed, the dynamics of the supply chain significantly improve as $m$ decreases and $\beta$ increases, that is, as the correlation coefficient $\theta$ increases. In this sense, the closed-loop supply chain may perform better or worse than the traditional system depending on these parameters. For low $m$ and high $\beta$, the operation of the closed-loop system can significantly benefit from the reverse flow of materials, while for high $m$ and low $\beta$, the dynamics of the supply chain may be seriously damaged by returns uncertainty. We underscore that when returns uncertainty is high, regulating the reverse flow through the recoverable stock becomes essential to prevent returns variability from significantly deteriorating the performance of closed-loop systems.

6. Conclusions and implications

Remanufacturing is an area gaining increasing attention due to the environmental and business opportunities it entails. This motivates the need for understanding the dynamic behaviour of closed-loop supply chains in the same way as that of traditional supply chains. However, as we noted in our literature review, very little prior research has been undertaken to shed light on how the interaction between demand and returns uncertainty impact on the performance of these systems. Hence, relevant critical questions
are still unexplored, and some of them are addressed in this paper.

The rather limited research in this area indicates that increasing the return yield may alleviate the variability in the supply chain. This has led researchers to conclude that the reverse flow of materials results in improved dynamic behaviour in closed-loop supply chains, which for example becomes evident through a reduced Bullwhip Effect. While we have also observed this impact of the return yield in a hybrid manufacturing-remanufacturing system, our research establishes that the dynamics of these closed-loop systems are strongly influenced by the correlation between demand and returns. From this dual-source uncertainty perspective, we reveal that the operational performance of such systems may enormously suffer from intrinsic variations in the collected products.

To avoid this second source of uncertainty entering into the closed loop, we investigate the potential of controlling the reverse flow of materials through an inventory policy for the recoverable items. In this sense, we develop a control structure for building-up and regulating the returns inventory that can significantly improve the economic performance of the system. The proposed mechanism, a prefilter, prevents the returns variability from deteriorating the operation of the system. Accordingly, the manufacturing and remanufacturing processes are smoothed, while at the same time inventory availability is increased. Hence, this solution effectively copes with the added complexity that closed-loop systems entail. In this sense, our study establishes the importance of managing the inventories of returns instead of pushing them into the supply chain, as assumed by previous literature.

Through an integrative framework for analysing the dynamics of closed-loop supply chains, we reveal that our proposed solution yields potential increases in absolute values of, and variability in, the inventory of returned goods. We have observed this build-up of returns inventory in several sectors, such as remanufactured automotive engines (Briggs, 2017). Although it is often seen by practitioners as an operational inefficiency that should be eliminated, this research reveals that the recoverable product inventory may absorb the intrinsic variation of returns, which smooths the dynamics and improves the operational performance of the closed-loop supply chain.

It is interesting to underline that, like previous studies in this field, we explore a linear supply chain model. While we have designed the baseline scenario to avoid entering into conflicting regions of the parameter design, this may be interpreted as a limitation of our study. In this sense, understanding the impact of different sources of nonlinearities, such as capacity constraints or non-negativities in the relevant variables, would emerge as an interesting avenue for future research, especially as recent developments in this field would facilitate the analysis (e.g. Spiegler et al., 2016).

By means of establishing a conceptual difference between the deterministic and stochastic component of returns, we demonstrate that the closed-loop supply chain performs better when the control structure is only applied to the latter. This generates what may be interpreted as a parallel return inventory control policy in the reverse flow of materials, defining a new conceptual approach for regulating returns inventories. This has been shown to outperform the alternative control policy based on regulating all the returns at the recoverable inventory site as well as the baseline push system that processes the returns as soon as they arrive. Under this parallel control architecture, the value of the deterministic component for improving the system performance can be leveraged at the same time that the overall system becomes robust to uncertainty in returns.

This perspective emphasizes the role of returns forecasting in the process of developing efficient closed-loop supply chains. Assumptions made in previous studies in the remanufacturing field do not include the need to estimate returned products; however, returns forecasting entails an enormous opportunity to enhance the performance of circular economy inventory and production systems, and hence constitutes an increasing concern in real-world applications (Shaw, 2017; Tsiliyannis, 2018). Note that the better these estimations in practice are, the lower the relevance of the uncertainty associated to the returns, which as has been seen results in a reduced variability in the supply chain.

Delving into these solutions for managing closed-loop systems from a holistic perspective is a main topic in our future research agenda. We work closely with several UK remanufacturers, the research and innovation agenda of which demonstrates both the opportunities derived from, and the complexity of, integrating all the processes, decisions, and structures in resilient closed-loop supply chains. Developing such systemic dynamic properties to deal with a growing variety of trade-off requirements, dependent on type of product, manufacturing and remanufacturing processes, cost structures, and market conditions, is a major challenge towards the widespread deployment of environmentally and financially sustainable remanufacturing systems.

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Supplementary materials

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