Ameliorative Minimum Cost Flow Algorithm for Phase Unwrapping

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Abstract

Ameliorative minimum cost flow algorithm is put forward in this paper. Based on MCF, it is known that MCF is one phase unwrapping algorithm based on network flow and it is used widely because of many merits. However, the efficiency of MCF is relatively low and it demands upper computer capability during phase unwrapping. The ameliorative minimum cost flow algorithm is that interferograms are divided into small sub-areas so that it can be unwrapped respectively. After this, interferograms are fused by super wavelet based on contourlet transform. In the end, Minimum Cost Flow and its Ameliorative algorithm are validated by practical example, results indicate Ameliorative algorithm are effective phase unwrapping algorithms.

Keywords: phase unwrapping; InSAR; Minimum Cost Flow; patch algorithm; wavelet; fusion

Introduction

With the development of new theory and new technology, category of modern geodesy is manifolded, at present, Interferometric Synthetic Aperture Radar (InSAR) is one of its subjects. Interferometric SAR or InSAR, allows accurate measurements of the radiation travel path because it is coherent. Measurements of travel path variations as a function of the satellite position and time of acquisition allow generation of Digital Elevation Models (DEM) and measurement of centimeteric surface deformations of the terrain. With the launch success of COSMO-SkyMed and TerraSAR-X satellite, InSAR technique is regarded increasingly by many governments and scientists. During the course of InSAR data processing, phase unwrapping (PU) is a key flow of InSAR for DEM’s precision is influenced directly by its veracity. Almost all PU algorithms are based on the assumption that the true unwrapped phase field is ‘smooth’ and varies ‘slowly’. More precisely, neighbouring phase values are assumed to be within one-half cycle (\( \pi \) radians) of one another. Whereas, phase noise aroused by radar shadow and decorrelation sources brings the presence of some phase discontinuities (i.e. absolute phase variations between neighbouring pixels of greater than \( \pi \) radians) so that phase unwrapping is very difficult. At the present time, Almost all PU algorithms can not solved completely these problems and there

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are biggish errors for results of phase unwrapping, which give rise to some difference between DEM and terrain. How to distill useful information from worse quality data and neglect influence of phase noise for phase unwrapping is a problem to be solved imminently.

From former state, according to uniform phase unwrapping model and network optimization principle, Minimum Cost Flow (MCF) is expatiated in this paper, at the same time, its Ameliorative algorithm for Phase Unwrapping is put forward aiming at disadvantages which speed is relative small and it demands upper computer capability during phase unwrapping, that is, interferograms is divided into skit sub-area so that it can be unwrapped respectively, after this, interferograms are fused by supper wavelet based on contourlet transform. In the end, Minimum Cost Flow and its Ameliorative algorithm are validated by practical example, results indicates MCF and its Ameliorative algorithm is effective phase unwrapping algorithm. So, former problems are solved preferably by the phase unwrapping algorithm.

**Principle of Minimum Cost Flow Algorithm for Phase unwrapping**

**Phase Unwrapping Model**

In order to deal with phase discontinuities, different strategies have been followed and different algorithms have been developed. Following Ghiglia and Romero[98]and chen and Zebker[00], we will briefly describe them, using the ‘minimum Lp-norm’ framework. In fact almost all phase unwrapping algorithms seek to minimize the following cost function[1]:

\[
\min \left\{ \sum_{i,j} \omega_{i,j} \left| \Delta \phi_{i,j} - \Delta \phi_{i,j}^* \right|^p + \sum_{i,j} \omega_{i,j} \left| \Delta \phi_{i,j} - \Delta \phi_{i,j}^* \right|^p \right\}
\]

(1)

Where \( 0 \leq p \leq 2 \)

\( \Delta \) indicates discrete differentiation along range and azimuth directions respectively

\( \omega \) is user-defined weights and the summation include all appropriate rows I and j column j;

The suffix \( \omega \) to the differentiation operator \( \Delta \) indicates that the phase differences are wrapped in the interval \(-\pi \) to \( \pi \).

An approach to PU based on network programming has received a great deal of attention, since it provides an efficient tool for a global minimization of Eq (1) under the (weighted) L1-norm (minimum absolute deviation).

**Principle of MCF**

In the Minimum Cost Flow (MCF) approach, the PU problem is equated to a general network flow problem. This reformulation of the PU optimization problem allows the use of powerful techniques developed for network optimization. Since graph theory and network programming is a mature subject of operational research, several fast optimization routines can be employed to seek the minimum cost flow[2,3,6]. Principle of Minimum Cost Flow algorithm is following:

In the grid of size of M*N, let \( \varphi \) is phase unwrapped and \( \phi \) is phase wrapped, relation of \( \varphi \) and \( \phi \) is

\[
\phi(i, j) = \varphi(i, j) + 2\pi n
\]

(2)

Where n is an integer and \( \varphi(i, j) \in [-\pi, \pi] \). phase unwrapping is a process from \( \phi(i, j) \) to \( \varphi(i, j) \).

We use the following definition of difference estimation among neighboring pixels:
\[ \Delta \phi_1(i, j) = \phi(i + 1, j) - \phi(i, j) + 2\pi n_1 \] (3)
\[ \Delta \phi_2(i, j) = \phi(i, j + 1) - \phi(i, j) + 2\pi n_2 \] (4)

Where \( n_1 \) and \( n_2 \) are integers based on primary knowledge and it can be \( \Delta \phi_1(i, j) \in [-\pi, \pi) \) (1=1,2). Owing to different integral paths, \( \Delta \phi_1(i, j) \in [-\pi, \pi) \) (1=1,2) is not always consistent with difference among neighboring points, so we define remnant as

\[
\begin{pmatrix}
k_1(i, j) \\
k_2(i, j)
\end{pmatrix} = \frac{1}{2\pi} \begin{pmatrix}
\phi(i + 1, j) \\
\phi(i, j + 1)
\end{pmatrix} - \begin{pmatrix}
\Delta \phi_1(i, j) \\
\Delta \phi_2(i, j)
\end{pmatrix}
\] (5)

Where \( k_1(i, j) \) and \( k_2(i, j) \) are very small number.

Based on these, we can compute \( k_1(i, j) \) and \( k_2(i, j) \) by the following:

\[
\min\{k_1, k_2\} \left\{ \sum_{i,j} c_1(i, j)|k_1(i, j)| + \sum_{i,j} c_2(i, j)|k_2(i, j)| \right\}
\] (6)

According to the theory of network flow, minimization of Eq.(6) can be transformed into Minimum Cost Flow to seek solutions. Inputs of MCF are every crunode’s degree (value of remnant) and expense of every flow, moreover outputs is flux of every flow, at the same time total of expense achieves minimization. After figuring out \( k_1(i, j) \) and \( k_2(i, j) \), phase grads is computed, in the end, phase is unwrapped by equation (7):

\[
\Phi(i, j) = \Phi(0, 0) + \sum_{p=0}^{i-1} (\Psi(p+1, 0) - \Psi(p, 0)) + \sum_{q=0}^{j-1} (\Psi(i, q+1) - \Psi(i, q))
\] (7)

**Unwrapping Steps of MCF**

Phase unwrapping using the Minimum Cost Flow algorithm consists of the following steps:

1. Generation of phase unwrapping validity mask for coherence maps;
2. A triangular irregular network (TIN) is built about phase and allelomorph chart is constructed according to TIN. Then remnants computed in the TIN are transformed into network;
3. In the network, every point of positive or minus remnants is connected by making use of MCF and total of minimum cost is computed;
4. Integral of phase matrix is processed according to magnitude and direction of flow so that phase is unwrapped, then integral is carried from district of high quality to district of low quality, which we can achieve phase of a global optimization.

As far as essence is concerned, MCF belongs to path track algorithm[7,9], difference between them is that MCF can distinguish effectively district of high quality to district of low quality, as a result, MCF can avoid inferior area so that precision of phase unwrapping is improved.

**Example of Minimum Cost Flow Algorithm**

In order to validate availability of phase unwrapping with Minimum Cost Flow algorithm, an example is provided by the co-seismic fringes of the Bam Earthquake. For the example, we have chosen two IM images: 3 December 2003 and 11 June 2003., their orbit numbers are 6687 and 9192 respectively.
After baseline estimation, co-registering and interferogram computation, interferogram is unwrapped by making use of MCF, result of phase unwrapping is seen as in figure 1. The left of figure 1 is validity mask of phase unwrapping and the right is the final unwrapped interferogram. From the figure 1, the coherence of interferogram is seen as criterion evaluating phase quality so that inferior data is removed by MCF algorithm, so the interferogram of Bam is unwrapped effectively.

Be careful, although there are many merits for MCF algorithm compared with other phase unwrapping and MCF algorithm can remove influence of phase noise, MCF algorithm has also one flaw, that is, MCF algorithm requires large memory and a mass of time, for example, to unwrap a full frame of ASAR/ERS about 5GM of RAM are needed if the image is coherent everywhere. If the interferogram is so large that it exceeds memory capacity, the interferogram can not unwrapped. Therefore, in order to use MCF algorithm to unwrap interferogram quickly and effectively, Ameliorative Minimum Cost Flow algorithm is put forward in this paper, which consists of tile algorithm and image fusion.

Ameliorative Minimum Cost Flow Algorithm

Tile Algorithm

Tile Algorithm indicates that interferograms are divided into many patches, Each patch can be unwrapped respectively just as they are some unattached images. In general, the patches should be set to as large as possible within the memory constraints of the computer, and make sure that the machine is not swapping to disk while unwrapping. Intersected aim is that EMS memory resource of computer is reduced during phase unwrapping and phase unwrapping can be expedited at a certain extent.

As figure 2 indicates, there are two beelines: l1 and l2, five regions: i, j, k, m and n. l1 and l2 are went by the name of journey lines which are parallel to X axis (or Y axis). Jumping-off point of all lines can be interferogram outline or brims of equivalent quality regions, what is more, they shall be tangent lines which thread the biggest value or the smallest value of one equivalent quality region, of course, they are direction along X axis or Y axis. In addition, i, j, k, m and n are seen as tiles which be encircled by journey lines and brim of equivalent quality regions (or interferogram outline).
According to principle of tile algorithm, interferogram of section 3 is unwrapped, and we get Phase Unwrapping efficiency of tile algorithm, as in table 1. As table 1 indicates, we can make use of tile algorithm to improve speed of phase unwrapping and debase demand of computer while phase unwrapping. For all that, the primary difficulty with patch-based processing is that errors occur on the patch boundaries, as in figure 1. Errors in the patch unwrapping propagate over the entire image and can be a problem. Based on this, in order to use MCF algorithm to get the final unwrapped interferogram, fusion is processed for the final unwrapped interferogram.

Tab.2 Phase unwrapping efficiency of tile algorithm

| Parameters | Time (m) | 1  | 2   | 3  | 4  | 5  | 6  | 7  | 9  | 12 | 16 |
|------------|----------|----|-----|----|----|----|----|----|----|----|----|
| 1          | Size: 343M, 5167 x 3400 | Failure | Failure | 27.2 | 8.1 | 7.5 | 8.1 | 8.2 | 9.3 | 10.6 | 13.4 |
| 2          | Size: 225M, 3400 x 3400 | Failure | 16.59 | 5.4 | 5.3 | 6  | 6.4 | 7  | 7.6 | 8.9 | 10.8 |

Unwrapped Phase Image Fusion Based on Contourlet Transform

As inherent line and surface singularity of image can not be recognized by wavelet algorithm, and direction information which it prized is also limited, Do and Vetterli have constructed multi-scale geometry analysis tool: Contourlet transform[4,8,11]. Contourlet transform which inherits multi-resolution analysis on wavelet basis has strong capacity of image. At the same time, Contourlet transform compared with wavelet transform can express finely signal of two dimensions, owing to good multi-resolution, localization and anisotropism, it is more suitable to cope with image singal which has characters of hyperplane singularity[12]. Contourlet transform is used to Unwrapped phase image fusion, good result of phase unwrapping is obtained because geometry character of unwrapped phase image is distilled by making use of excellent speciality of contourlet transform.

Framework of two filters is adopted by contourlet transform, firstly, Unwrapped phase image is decomposed by Laplace (LP) decomposability algorithm to capture point singularity, then sub-strip of low frequency continue to use LP to decompose alternately it, like this, one unwrapped phase image can be decomposed as sub-strips of low and high frequency. Secondly, sub-strips of high frequency is analyzed with direction.
In this paper, based on contourlet transform of region characters, contourlet transform is used for unwrapped phase image fusion after original interferogram is unwrapped by MCF algorithm, unwrapped phase image fusion of contourlet transform consists of the following steps:

(1) Contourlet decompose; unwrapped phase image is decomposed by using contourlet transform, as a result, we can get corresponding contourlet coefficients. If two unwrapped phase images are IA and IB respectively, each unwrapped phase image is decomposed repeatedly for m by using LP, and direction number of each scale is l(j), decompose process of unwrapped phase image is the following:

\[
\begin{align*}
I_A(i, j) & \rightarrow (c_1(A), c_2(A), \ldots, c_{m-1}(A), c_m(A), d_m(A)) \\
I_B(i, j) & \rightarrow (c_1(B), c_2(B), \ldots, c_{m-1}(B), c_m(B), d_m(B)) \\
\end{align*}
\]

\[
c_m(n) = \{ b_{m,1}(n), b_{m,2}(n), \ldots, b_{m,l(j)}(n) \}, n = A, B
\]

Where \( d_m \) is sub-strip of low frequency; \( c_m \) is direction sub-strips of scale m; \( b_{m,k} \) is sub-strip of high frequency for scale m and the kth direction.

(2) unwrapped phase image fusion; As to sub-strip of low frequency and sub-strip of high frequency after decompose, contourlet coefficients after image fusion is gained with image fusion after they are distinguished and fused according to fusion rules.

(3) Contourlet compose; Compose is inverse process of decompose, that is, contourlet coefficients after fusion is processed with inverse contourlet transform so that syncretic image is achieved. Of course, the phase image contains information of many images. The process can be expressed as following:

\[
( c_1^F, c_2^F, \ldots, c_{m-1}^F, c_m^F, d_m^F ) \rightarrow I_{AB}(i, j)
\]

According to above steps of image fusion, different unwrapped phase images(In this paper, unwrapped phase images are two images: 6 and 12 patched) which interferogram is unwrapped by using tile algorithm are fused by means of contourlet transform, so that syncretic phase image is gained, as figure 4. As figure 3 indicates, image after fusion is slippery and phases are matched at the overlapping tile boundaries.

Therefore, Ameliorative Minimum Cost Flow algorithm not only do not influence result of phase unwrapped with the MCF algorithm, but also debase demands for computer capability and improve speed of phase unwrapping.

Fig.3. Phase image after fusion
Conclusions

Phase unwrapping is a key step of InSAR, on the other hand, as far as many other phase unwrapping algorithm, unwrapped results is influenced easily by phase noise. So, Ameliorative Minimum Cost Flow algorithm is put forward in this paper based on MCF, in the end, the algorithm is validated by example. There are the following conclusions:

(1) MCF algorithm is one effective phase unwrapping algorithm which can neglect regions of a great deal phase noise and unwrapped result is very reasonable;
(2) Ameliorative Minimum Cost Flow algorithm consists of tile Algorithm and unwrapped phase image fusion based on contourlet transform;
(3) Ameliorative Minimum Cost Flow algorithm is also one effective algorithm;
(4) Ameliorative Minimum Cost Flow algorithm can debase demands for computer capability and improve speed of phase unwrapping.

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