Error of Altimetric Determination of the Wind Speed over the Ocean, Caused by Dominant Waves

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Abstract. The analysis of the error in the restoration of wind speed according to altimetry measurements is performed. For analysis, a spectral model of the field of surface waves was used. The model takes into account changes in the field of surface waves at different stages of its development. A factor that reduces the accuracy of measurements is the ambiguity of the relationship between wind speed and the variation of slopes created by long waves. It is shown that when sounding in the C band of radio waves, the error in measuring wind speed is about 2 times higher than when sounding in the Ku band. It is also shown that at a wind speed of less than 15 m/s, the influence of long waves on the accuracy of altimetry measurements of wind speed can be neglected. At a wind speed of 30 m/s, estimates of wind speed obtained from altimetry measurements may vary by 0.57 m/s when probing in the C range, and by 0.27 m/s when probing in the Ku range.

1. Introduction

Remote sensing carried out from spacecrafts is currently the only way to obtain globally information about the wind field above the sea surface [1, 2]. This approach is based on the dependence of the backscattering of electromagnetic waves on the level of roughness of the sea surface, which, in turn, depends on the wind speed [3].

If the sounding of the sea surface is carried out at small angles of incidence, the reflected radio signal is described by the equation [4]

\[ \sigma = \pi \sec^4 \theta \left| R_{eff} \right|^2 \int_{\xi_1=-\theta_1}^{\theta_1} \int_{\xi_2=-\theta_2}^{\theta_2} P_{2D} (\xi_1, \xi_2) d\xi_1 d\xi_2 \]  

(1)

where, \( \sigma \) is normalized radar cross section; \( \theta \) is the incidence angle; \( R_{eff} \) is the effective reflection coefficient [5]; \( P_{2D} \) is two-dimensional probability density function of the slopes; \( \xi_1 \) is the slope of the sea surface in the plane of sounding; \( \xi_2 \) is the slope of the sea surface in the orthogonal plane. From equation (1) it follows that the only physical mechanism that allows, based on altimetry measurements, to determine the speed of the wind is the dependence of the slopes on the wind speed. The accuracy of speed recovery depends on how unambiguously the slope characteristics are determined by the wind [6, 7].

The effect of long waves on the formation of a reflected radio signal during quasi-vertical sounding was analyzed in the work [8]. In this work, estimates of the contribution to the total dispersion of the slopes slowly adapting to waves wind changes with scales comparable to the scale of dominant waves.
were obtained. The present work is a development of these studies. The goal is to obtain quantitative estimates of the error in determining wind speed associated with changes in the characteristics of the wave field at different stages of its development.

2. Wind speed measurement errors
As a rule, to estimate the accuracy of wind speed recovery from data of satellite measurements, the root-mean-square error between wind speeds values, which are determined by remote sensing and measured from buoys, is used. Currently, the accuracy of wind speed measurement is approximately 1.5-2 m/s [7-9]. The specified accuracy was achieved during the first experiments, in which the backscattering cross section was used as the sole predictor, and has not yet been improved for one-parameter models.

Measurement accuracy is determined by technical, methodological and physical factors. The technical factor includes instrumental errors. It also includes non-synchronization of measurements from the buoy and sounding from the satellite, as well as the discrepancy between the location of the buoy and the satellite track [10]. Typically, data are used that satisfies the conditions: the buoy and the track are separated less than 50 km in space and no more than 30 minutes in time [11].

The methodical factor is associated with the comparison of the characteristics obtained by measurements at the point (buoy) and by averaging over space (radioaltimeter). The hypothesis of stationarity and ergodicity of sea waves allows, when comparing data, to replace space averaging with time averaging. However, we must take into account the following. First, this hypothesis is not true for developing and decaying waves. Second, even if it is true for a fully developed wave, there is some methodical "uncertainty" in the choice of the time of averaging the data of measurements from the buoy, adequate to spatial averaging.

The physical factor is due to the fact that the characteristics of surface waves are ambiguously dependent on wind speed. Ambiguity is associated with the existence of a large number of different physical nature of the mechanisms acting on the surface waves, as well as with different relaxation time of waves of different scales on changes in wind speed [11].

3. Spectral model of developing waves
The spectral model describing the changes of the wave field at different stages of the wave field development has the form [12]

$$\Psi(\omega) = a g^{2} \omega^{-5} \omega_0^{14} \exp \left\{ - \left( \frac{\omega_0}{\omega} \right)^{4} \right\} \gamma^{\Phi},$$

where \(\omega_0\) is the frequency of the spectral peak; \(g\) – gravitational acceleration; \(a = 0.006 \xi^{0.22}\); \(\gamma = \begin{cases} 1.7 & npu \quad 0.83 < \xi < 1 \\ 1.7 + 6.0 \log(\xi) & npu \quad 1 < \xi < 5 \end{cases}\); \(\Phi = \exp \left( - \frac{(\omega - \omega_0)^2}{2 \nu^2 \omega_0^2} \right)\); \(\nu = 0.08 \left(1 + 4 \xi^{-3} \right)\). Here, the parameter \(\xi\) determines the stage of development of the field of wind waves

$$\xi = W_{10} / C_0,$$

where \(W_{10}\) is wind speed at a height of 10 m; \(C_0\) is the phase speed of waves with a frequency of \(\omega_0\). Factor \(\gamma^{\Phi}\) describes the enhancement compared to the Pierson-Moskovitz spectrum [13].

The spectrum \(\Psi(\omega)\) is not explicitly dependent on wind speed. Its shape is determined by two parameters \(\xi\) and \(\omega_0\). Dominant surface waves belong to the class of gravitational waves. Gravitational waves at deep water obey the dispersion equation

$$\omega^2 = g k,$$

connecting their frequencies \(\omega\) and wave numbers \(k\). From the dispersion equation follows \(C_0 = g / \omega_0\). Given (3), we get
or

\[ \lambda_0 = 2\pi g / \omega_0^2 (\xi, W_{10}) = 2\pi W_{10}^2 / \langle \xi^2 \rangle, \]

where \( \lambda_0 \) is length of dominant wave. Equations (5) and (6) allow to estimate changes of parameters \( \omega_0 \) and \( \lambda_0 \) depending on the stage of wave field development. As shown in Figure 1, at a fixed wind speed both the frequency and the length of the dominant wave can vary widely.

![Figure 1](image-url)

Figure 1. The dependence of the frequency \( \omega_0 \) and length of the dominant developing \( \lambda_0 \) waves from the wind speed \( W_{10} \) at fixed values of the parameter \( \xi \).

4. The spectral and integral characteristics of the slope

In a linear wave field whose components obey the dispersion equation in the form (4), the slope spectrum is associated with the elevation spectrum by the equation [8]

\[ \psi(\omega) = \frac{\omega_0^4}{g^2} \Psi(\omega). \]

We will calculate the dispersion of slopes in the frequency range, from dominant waves (from frequency \( \omega_0 / 2 \)) to a some frequency \( \omega_4 \)

\[ \overline{\xi^2} = \int_{\omega_0}^{\omega_4} \frac{\omega^4}{g^2} \Psi(\omega) d\omega, \]

where \( \overline{\xi^2} \) is the variance of the total slope of the surface, equal to the sum of the variance of the slopes oriented along and across the wind direction. As will be shown below, the boundary \( \omega_4 \) depends on the wavelength of the radio altimeter, so the calculations will be carried out for its different values.

The measurement error of wind speed is determined by the ratio of correlated and uncorrelated changes in the variances of the slopes. Here we consider only the factor of ambiguity of connection with the wind characteristics of the wave field at different stages of its development.

With quasi-vertical sounding, the signal recorded on the spacecraft is formed by the slopes of the sea surface created by those waves whose length \( \lambda \) is much longer than the length of the probing radio wave \( \Lambda \) [4]. Accordingly, the equation (1) includes a two-dimensional probability density function of slopes satisfying the condition

\[ \lambda >> \Lambda, \]

\[ \omega_0 (\xi, W_{10}) = g \xi / W_{10} \]
where the condition is “much more” strictly not defined [13]. It is usually assumed that the condition is “much more” consistent with the excess of 3-5 times. Considering that the frequency and length of the surface wave (wavenumber) are related by equation (5), the variance of the slopes formed by these waves can be represented as

$$\overline{\xi^2_L} = \int_{\omega_0}^{\infty} \frac{d\omega}{\omega^2} \Psi(\omega) \, d\omega.$$  \hspace{1cm} (10)

Similarly, it is possible to determine the variance of slopes of the dominant waves $$\overline{\xi^2_L} = \int_{\omega_0}^{\infty} \frac{d\omega}{\omega^2} \Psi(\omega) \, d\omega$$.

The error of wind speed measurements is determined by the ratio between $$\overline{\xi^2_L}$$ and $$\overline{\xi^2_L}$$.

5. Errors of altimetry measurement of wind speed due to dominant waves

The error of measurements of wind speed by the radio altimeter analyzed here depends on the limits within which the dispersion $$\overline{\xi^2_L}$$ can change at a fixed wind speed at different stages of the wave field development. As the wave field develops, the frequency $$\omega_0$$ shifts to lower values. This extends the frequency range $$\left(\omega_0/2, \omega_0\right)$$ in which the slope dispersion $$\overline{\xi^2_L}$$ is determined. At the same time in the spectral model (2) there is a decrease in the multiplier $$\alpha$$, as well as reduced multiplier $$\gamma^\Phi$$. The resulting changes of the slopes dispersion at the change in the stage $$\zeta$$ development are shown in Figure 2.

![Figure 2](image_url)

**Figure 2.** Dependences of slopes dispersions on the stage of development of the wave field at a fixed wind speed: a – slope dispersion is determined for waves longer than 5 m; b – for waves longer than 10 m. Curves 1-3 correspond to the wind speed of 10, 15 and 20 m/s.

The calculations were carried out at a fixed wind speed, i.e. different stages of the development of the wave field corresponded to different frequencies (lengths) of dominant waves. Since the earlier stages of development correspond to the larger values of the parameter $$\zeta$$, the orientation of the axis $$OX$$ is changed.

To estimate the parameter $$\overline{\xi^2_L}$$, we use the results of [14, 15], where it was shown that the parameter $$\overline{\xi^2_L}$$ can be described as part of the total dispersion of slopes $$\overline{\xi^2_{all}}$$ created by waves of all scales. The total dispersion of the slopes is determined by optical measurements data [16, 17]. The wavelengths of the optical range are much less than the lengths of the shortest waves present on the
sea surface. Therefore, statistical moments of the sea surface determined by aerial photographs or optical scanners describe slopes at all scales [18, 19].

The variation of the slopes of the sea surface $\xi_\lambda^2$, which determine the level of the backward reflected radio signal, are presented in Figure 3a. It can be seen that with increasing frequency of the radio wave (decreasing its length), the parameter increases. The curves shown in figure 3a correspond to the three most common radio bands at the altimetric measurements of in which the sensing is carried out. According to the IEEE Standart 521-2002, these bands are denoted as C, X and Ku. The average frequencies of these bands are 6, 10 and 15 GHz, respectively.

![Figure 3](image)

**Figure 3.** Dependences of slope dispersion on wind speed: a – dependence $\xi_\lambda^2 = \xi_\lambda^2(W_{10})$, curves 1, 2 and 3 correspond to radio waves C, X and Ku ranges; b – dependence $\xi_L^2 = \xi_L^2(W_{10})$, curves 1, 2 and 3 correspond to the dominant wavelengths of 40 m, 80 m and 120 m.

The spread of values $\xi_\lambda^2$ showed in figure 3b determines the error of altimetric measurement of wind speed. Values $\xi_\lambda^2$ vary between $\pm 0.0035$ the average $\xi_\lambda^2$ at a wind speed of 10 m/s and $\pm 0.0093$ the average $\xi_L^2$ at a wind speed of 30 m/s. These changes lead to the scatter of the measured values of wind speed during sounding in the C, X and Ku ranges $\pm 4.3\%$, $\pm 2.8\%$ and $\pm 2.0\%$ accordingly, if $W_{10} = 10$ m/s. If $W_{10} = 30$ m/s the spread is $\pm 1.9\%$, $\pm 1.3\%$ and $\pm 0.9\%$.

6. Conclusion

In this paper the analysis of the physical limitations of accuracy of reconstruction of the wind speed above the sea surface according altimetry measurements. A factor that reduces the accuracy of measurements is the ambiguity of the relationship between wind speed and the variation of sea surface slopes created by long waves. The spectral model of the surface wave field is used for the analysis. The indicated model takes into account changes in the field of surface waves at different stages of its development. It is shown that the largest error is observed during sounding in the C band of radio waves. It is about 1.5 times higher than when probing in the X band and 2 times higher than when probing in the Ku band. It is also shown that at a wind speed of less than 15 m/s, the influence of long waves on the accuracy of altimetry measurements of wind speed can be neglected. At a wind speed of 30 m/s, the wind speed estimates obtained from the altimetric measurements can deviate by 0.57 m/s.
when probing in the C range, by 0.39 m/s when probing in the X range and by 0.27 m/s when probing in the Ku range.

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