Asymmetric explosions of thermonuclear supernovae

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ABSTRACT
A Type Ia supernova explosion starts in a white dwarf as a laminar deflagration at the centre of the star and soon several hydrodynamic instabilities – in particular, the Rayleigh–Taylor (R–T) instability – begin to act. In previous work, we addressed the propagation of an initially laminar thermonuclear flame in the presence of a magnetic field assumed to be dipolar. We were able to show that, within the framework of a fractal model for the flame velocity, the front is affected by the field through the quenching of the R–T instability growth in the direction perpendicular to the field lines. As a consequence, an asymmetry develops between the magnetic polar and the equatorial axis that gives a prolate shape to the burning front. We have here computed numerically the total integrated asymmetry as the flame front propagates outward through the expanding shells of decreasing density of the magnetized white dwarf progenitor, for several chemical compositions. We have found that a total asymmetry of about 50 per cent is produced between the polar and equatorial directions for progenitors with a surface magnetic field $B \sim 5 \times 10^7$ G, and a composition $^{12}\text{C} = 0.2$ and $^{16}\text{O} = 0.8$ (in this case, the R–T instability saturates at scales $\sim 20$ times the width of the flame front). This asymmetry is in good agreement with the inferred asymmetries from spectropolarimetric observations of very young supernova remnants, which have recently revealed intrinsic linear polarization interpreted as evidence of an asymmetric explosion in several objects, such as SN1999by, SN1996X and SN1997dt. Larger magnetic field strengths will produce even larger asymmetries. We have also found that for lighter progenitors (i.e. progenitors with smaller concentrations of $^{16}\text{O}$ and larger concentrations of $^{12}\text{C}$) the total asymmetry is larger.

Key words: instabilities – MHD – supernovae: general – white dwarfs.

1 INTRODUCTION

According to the current modelling, the explosion of a Type Ia supernova (SNIa) starts as a thermonuclear deflagration of a Chandrasekhar mass white dwarf (WD) of carbon–oxygen (C+O) or oxygen–neon–magnesium (O+Ne+Mg) compositions. Due to the action of several hydrodynamic instabilities, in particular, the Rayleigh–Taylor (R–T), the initially laminar propagation does not survive as such and the combustion front rapidly develops a complex topology. A cellular stationary combustion (followed by a turbulent combustion regime) is rapidly achieved by the flame and maintained up to the end of the so-called flamelet regime when a transition to detonation may occur. As in the case of chemical laboratory flames (Gostintsev, Istratov & Shulenin 1988), thermonuclear flames probably develop a self-similar behaviour due to the action of the hydrodynamic instabilities. Under this hypothesis of a self-similar deflagration regime, the effective flame speed is well described by a fractal scaling law, as previously proposed by several authors (Woosley 1990; Timmes & Woosley 1992; Niemeyer & Hillebrandt 1995; Niemeyer & Woosley 1997; see also Ghezzi et al. 2001, hereafter Paper I), namely

$$v_{\text{frac}} = v_{\text{lamin}} \left( \frac{L_{\text{max}}}{l_{\min}} \right)^{(d-2)}.$$  \hspace{1cm} (1)

Here, $v_{\text{lamin}}$ is the laminar velocity (as obtained, for example, in Timmes & Woosley 1992), $L_{\text{max}}$ and $l_{\min}$ are the maximum and minimum R–T wavelength perturbations, respectively (see Chandrasekhar 1981) and $d$ is the fractal dimension, which may assume a range of values, $2 < d < 3$ (Woosley 1990; see also Paper I).

In a previous work (Paper I), we addressed the propagation of an initially laminar thermonuclear flame in the presence of a magnetic field assumed to be of dipolar geometry. The main result of that work was that within the framework of the fractal models for the flame

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Figure 1. Schematic representation of the propagation of the front inside a WD. Here \( \vec{B} \) indicates the direction of the dipolar magnetic field, \( \vec{V}_{\text{exp}} \) is the star’s expansion velocity, \( \vec{V}_{\text{pol}} \) and \( \vec{V}_{\text{eq}} \) are the fractional polar and equatorial velocities, and \( r_{\text{pol}} \) and \( r_{\text{eq}} \) are the polar and equatorial radii, respectively, as described in the text. We note that the combustion is isobaric to a very good approximation, and the density jump in the flame is \( \sim 10^{-1} \) times the density value.

velocity in the wrinkled and turbulent regimes, the front is affected by the field that inhibits the growth of the R–T instability in the direction perpendicular to the field lines. As a consequence, an asymmetry is established between the magnetic polar and the equatorial axis, which gives a prolate shape to the burning front (see Fig. 1). In that work, an estimate of the intrinsic asymmetry as a function of the field and the core density was presented and discussed in the context of SNIa explosions. Nevertheless, several aspects of this problem needed to be clarified as, for instance, a computation of the total integrated asymmetry, as the burning front propagates through the outer, density-decreasing shells of the magnetized expanding progenitor star, and also its meaning for the observation of SNIa remnants. We here present the results of one-dimensional numerical calculations of the integrated asymmetry through WD progenitors with different initial chemical compositions, taking into account the expansion of the star (which is assumed to be homologous).

2 INSTANTANEOUS AND CORRECTED ASYMMETRY

According to the model developed in Paper I, in the presence of a dipolar magnetic field, the ratio of the propagation velocity of the burning front in the polar and equatorial directions of the star is given by

\[
A(\rho) = \frac{v_{\text{pol}}(\rho)}{v_{\text{eq}}(\rho)} = \left(1 + \frac{B^2/8\pi}{\rho v_{\text{lam}}^2} \right)^{d-2},
\]

where \( v_{\text{pol}} \) and \( v_{\text{eq}} \) are the velocities at the polar and equator directions, respectively. This equation shows that the asymmetry of the velocity field is controlled by the quotient of the magnetic pressure \( (B^2/8\pi) \) and the ram pressure of the gas \( (\propto \rho v_{\text{lam}}^2) \). We shall label \( A(\rho) \) as the instantaneous asymmetry as a function of the density \( \rho \) for an initially spherical flame. In Paper I, we computed this quantity for different magnetic field intensities.

It is clear, however, that \( A(\rho) \) will be increasingly different from the quotient above as time elapses and the flame propagates outward through the star, because the flame in both directions will be encountering decreasing densities ahead. Therefore, a full integration of the asymmetry is needed to account for this fact.

Let us first assume that the star does not expand to derive a zeroth-order correction. Because the polar flame front is faster than the equatorial front, the former will encounter lower densities first. If we label the difference of densities in each direction as \( \Delta \rho = (\rho_{\text{eq}} - \rho_{\text{pol}}) > 0 \), the polar front faces a density \( \rho_{\text{pol}} = \rho - \Delta \rho \) when the equatorial front is at \( \rho_{\text{eq}} = \rho \). To this order, a corrected asymmetry (which takes into account the deformation of the flame) may be defined by

\[
\tilde{A}(\rho) = \frac{v_{\text{pol}}(\rho) - \Delta \rho}{v_{\text{eq}}(\rho)}.
\]

Because the fractal propagation velocity is a continuous and differentiable function of the density, we can expand \( v_{\text{pol}} \) in a Taylor series around \( \rho \) and keep the zeroth-order and first-order terms provided that \( \Delta \rho \ll \rho \), to yield

\[
\tilde{A}(\rho) = A(\rho) - \frac{1}{v_{\text{eq}}} \frac{\partial v_{\text{pol}}(\rho)}{\partial \rho} \Delta \rho.
\]

This may be rearranged to display the instantaneous asymmetry and a correction term as

\[
\tilde{A}(\rho) = A(\rho) - \frac{1}{v_{\text{eq}}} \frac{\partial v_{\text{pol}}(\rho)}{\partial \rho} \Delta \rho = A(\rho) + A_1(\rho),
\]

where

\[
A_1(\rho) = - \frac{1}{v_{\text{eq}}} \frac{\partial v_{\text{pol}}(\rho)}{\partial \rho} \Delta \rho
\]

is the correction term for the asymmetry (see below).

Given that \( v_{\text{pol}} > v_{\text{eq}} \) and both velocities increase with decreasing density,\(^1\) the difference between \( v_{\text{pol}} \) and \( v_{\text{eq}} \) is necessarily larger than the difference that they would have by imposing the same density ahead for both flames. In other words, the corrected asymmetry \( \tilde{A}(\rho) \) must be larger than the instantaneous asymptmetry \( A(\rho) \). Moreover, the value of \( A_1(\rho) \) depends on the value of \( \Delta \rho \), which in turn depends on the value of the corrected asymmetry \( \tilde{A}(\rho) \) defined in equation (5). We may then find a solution to the non-linear equation (5) by discretizing and solving it iteratively\(^2\)

\[
\tilde{A}(\rho) = A_{n-1}(\rho) - \frac{v_{\text{pol}}}{v_{\text{eq}}} \Delta \rho_{n-1}.
\]

with \( A_0 = A(\rho) \) and \( \Delta \rho_0 = 0 \). Whenever \( \Delta \rho \sim \rho \), higher-order terms become important, although \( \rho \gg \Delta \rho \) is automatically satisfied. Equation (7) converges provided that the Cauchy condition

\[
|\tilde{A}_0(\rho) - \tilde{A}_{n-1}(\rho)| \ll \epsilon
\]

is satisfied for an arbitrarily small \( \epsilon \). We shall see below how to integrate the corrected asymmetry through the star so as to estimate the accumulated effect along the whole propagation of the non-spherical flame.

2.1 Correction term of the asymmetry

The correction term for the asymmetry (equation 6) contains a derivative of the polar speed with respect to the density and, therefore, we need to know the dependence of the laminar speed with the density and composition of the fuel in order to calculate the correction term. In this subsection we calculate the correction term

\(^1\) The fractal velocity, \( v_{\text{frac}} \), increases with the density, in equation (1), because the minimum scale \( l_{\text{lam}} \) decreases faster with density than the laminar velocity, \( v_{\text{lam}} \) (see Paper I).

\(^2\) It can be checked that the Lipschitz condition is satisfied by the function \( \tilde{A}(\Delta \rho) \).
for C+O progenitors. Using the interpolation formulae given by Woosley (1986), Timmes & Woosley (1992) and Arnett (1996), we find
\[
v_{\text{lam}} = 92 \times 10^5 \left( \frac{\rho}{2 \times 10^9} \right)^{0.805} \left[ \frac{X(12\text{C})}{0.5} \right]^{0.889} \text{ cm s}^{-1}
\]
which is approximately\(^3\) valid in the density range \(0.01 \leq \rho \leq 10\). In the particular case in which \(d = 2.5\),\(^4\) the effective polar speed \(v_{\text{pol}}\) (see equation 1) does not depend on the laminar speed \(v_{\text{lam}}\), so that we could say that \(v_{\text{pol}}\) is to some extent ‘independent of the microphysics’\(^5\).

\[
v_{\text{pol}}(\rho_a) = 0.282 \rho_a^{0.8} \left( \frac{g L_{\text{max}} \delta \rho}{\rho_a^{2.6}} \right)^{0.5} \text{ cm s}^{-1}. \tag{10}
\]

Here, \(\delta \rho = \rho_f - \rho_0\) is the density jump at the flame, i.e. the difference between burned (\(\rho_f\)) and unburned (\(\rho_0\)) material densities. We note that this is, in general, different from the density difference between the polar and equatorial fronts \(\Delta \rho = \rho_{\text{eq}} - \rho_{\text{pol}}\) (see equations 3–7). Thus

\[
\frac{\partial v_{\text{pol}}}{\partial \rho_a} = 0.225 \left( \frac{g L_{\text{max}} \delta \rho}{\rho_a^{2.6}} \right)^{0.5} - 0.141 \rho_a^{2.1} \left[ (2.6 g L_{\text{max}} \delta \rho / \rho_a^{3.6}) + \left( g L_{\text{max}} / \rho_a^{2.6} \right) \right] / (g L_{\text{max}} \delta \rho)^{0.5}.
\]

Using equations (1) and (9), it is possible to obtain the polar fractal speed, \(v_{\text{pol}}\), for any C+O composition with an arbitrary fractal dimension \(d\)

\[
v_{\text{pol}} = 14.1683(0.208752)^d X(12\text{C})^{0.889} \rho_a^{0.8} \left[ \frac{g L_{\text{max}} \delta \rho}{X(12\text{C})^{0.778} \rho_a^{2.6}} \right]^{d-2}.
\]

In general, the correction term is given by

\[
\frac{1}{v_{\text{eq}}} \frac{\partial v_{\text{pol}}}{\partial \rho_a} \Delta \rho = \frac{1}{v_{\text{eq}} g S L_{\text{max}} \delta \rho^3} \left\{ 22.6693 \times (0.208752)^d X(12\text{C})^{4.445} \left[ \frac{g L_{\text{max}} \delta \rho}{X(12\text{C})^{0.778} \rho_a^{2.6}} \right]^d \times [-1.625(d-2.30769)\rho_a^3 + (d-2.5)\rho_a^5] \right\} \Delta \rho,
\]

where the value of \(\Delta \rho\) must be calculated numerically (see the next section).

\(^3\) We should note that equation (9) is only approximate and it is used for simplicity reasons. More realistic results could probably be obtained directly interpolating the velocities given in table 3 of Timmes & Woosley (1992), and will be addressed in future calculations.

\(^4\) This is actually the maximum fractal dimension estimated in Paper I.

\(^5\) In fact, recent numerical simulations of supernovae have shown that the hydrodynamics of the explosion is independent of the microphysics of the flame (Gamezo, Khokhlov & Oran 2002). In Paper I, we found that the minimum relevant hydrodynamic scale for the flame was \(l_{\text{min}} \sim 10^8\) cm (for \(\rho \sim 10^9\) g cm\(^{-3}\)), and the flame evolution was ‘independent of the microphysics’ because \(l_{\text{min}} \gg \delta t\) almost all the way, where \(\delta t\) is the flame width. We here stop the numerical integration when \(l_{\text{min}} \sim \delta t\) or \(l_{\text{min}} \sim 20 \times \delta t\) (see below). In the particular case that \(d = 2.5\), using the fractal scaling given by equation (1) we recover the independence with microphysics, which was obtained through dimensional arguments by Khokhlov (1995), and is also consistent with the laboratory results of Taylor (see Kull 1991). In this case, the flame does not depend directly on quantities, such as the laminar velocity or the thermometric conductivity.

\section{3 INTEGRATED ASYMMETRY}

In order to compute the total asymmetry for a given set of initial conditions for the progenitor, we must integrate the following equations along both the equatorial and polar directions, respectively:

\[
\frac{dr_{\text{eq}}}{dt} = v_{\text{eq}}(\rho, t).
\]

\[
\frac{dr_{\text{pol}}}{dt} = v_{\text{pol}}(\rho, t).
\]

Here, \(v_{\text{pol}}(\rho) = \tilde{A}(\rho) v_{\text{eq}}(\rho)\), and \(v_{\text{eq}}\) is given by

\[
v_{\text{eq}} = v_{\text{lam}} \left( \frac{L_{\text{max}}}{L_{\text{eq}}} \right)^{(d-2)}.
\]

where \(L_{eq} = L_{\text{min}}\) at the magnetic equator of the star (Paper I):

\[
L_{eq} = \frac{8\pi [(B^2/8\pi) + (1/2)\rho v_{\text{lam}}^2]}{g \delta \rho}.
\]

The above equations can be integrated simultaneously in one dimension outward through the star along with equations (7) and (8) to give the integrated asymmetry \(A_{\text{int}}(\rho) = r_{eq}(\rho)/r_{pol}(\rho)\), up to a shell of density \(\rho\). The integration of equation (14) is straightforward, while equation (15) is non-linear because the value of \(\tilde{A}\) depends on the actual position of the polar front \(r_{pol}\). It is solved using the same iterative algorithm that gives the corrected asymmetry described in the previous section.

Until now, we have neglected the fact that the flame is actually propagating in an expanding star and have thus partially decoupled the equations of the burning front from the hydrodynamical motions of the full star. Because actual deflagrations are subsonic, the star will pre-expand ahead of the flame, and this will, in turn, affect the burning itself (see Fig. 1).

To a very good approximation, we can assume a homologous expansion of the WD and this allows a simple description of the expansion as a function of time, in which the radius of the \(n\)th zone at a given time \(t\) evolves according to

\[
r_n(t) = a(t) r_n^{0*},
\]

where \(r_n^{0*}\) gives the distance to the centre in the initial condition (i.e. in hydrostatic equilibrium), and \(a(t)\) is the homology factor satisfying \(a(0) = 1\). The density will then evolve according to

\[
\rho_n(t) = \rho_n^{0*}/a^2(t),
\]

where \(\rho_n^{0*}\) is the density at the \(n\)th zone in hydrostatic equilibrium, so that in the comoving frame of each zone the density will decrease with time.\(^6\)

It is easy to see that the stellar expansion should produce a slower flame front than in the stationary case. This is due to the fact that, when the deflagration reaches the radius \(r_n\), the density there will be lower than it is without expansion. Therefore, the propagation speed will be modified accordingly. The distance between zones is

\(^6\) We note that some results of a perfect homologous expansion should be expected mainly due to the fact that, as the flame is accelerated downstream under the action of hydrodynamical instabilities, shock and pressure waves may travel ahead of the flame and disturb locally the stellar structure. None the less, because the deflagration evolves under approximate isobaric conditions, the actual expansion is expected to be nearly close to the homology assumed here.
The computed progenitor models were resolved in the comoving frame of the flow. To compute the variation of the magnetic field strength, we have assumed magnetic flux conservation. The computed progenitor models were resolved in $10^3$ zones, giving a physical resolution $\sim 1.5$ km for a star with a radius $R = 10^8$ km. The integration was stopped when the polar combustion front reached a shell with density $\rho \sim 10^7$ g cm$^{-3}$ for which the transition to detonation is believed to occur (Khokhlov 1995). This is a first step towards a more complete calculation, but nevertheless contains all the essential ingredients necessary to address the total asymmetry $A_{\text{tot}}$, namely

$$A_{\text{tot}} = r'_{\text{pol}}/r'_{\text{eq}}.$$  \hspace{1cm} (20)

Here, generally speaking, $r'_{\text{eq}} > r_{\text{eq}}$ and $r'_{\text{pol}} > r_{\text{pol}}$.

### 4 RESULTS

The results of the calculations are shown in Figs 4–8. The instantaneous asymmetry near the stellar surface (for a density $\rho = 5 \times 10^7$ g cm$^{-3}$) is shown in Fig. 4 for several progenitors with C–O compositions as a function of the surface magnetic field.

Integrated asymmetries are depicted in Fig. 5 for three different compositions (see captions) and a maximum surface magnetic field $B = 10^9$ G. It is apparent from Figs 4 and 5 that heavier progenitors develop larger instantaneous asymmetries, but the lighter ones achieve the largest integrated asymmetries. Hence, it is important to understand the reasons behind such behaviour for future applications.

The key aspect for this behaviour may be traced back to saturation effects of the flame as follows. The value of $I_{\text{min}}$ (see equation 17) indicates that it must decrease as the density decreases, but it clearly cannot be arbitrarily small. In fact, there is a minimum value associated with the width of the flame $\delta_t$, which we will call the saturation scale, $I_{\text{sat}}$.

The asymmetry between the polar and equatorial directions begins to decrease when the instabilities in the polar front reach the saturation limit $I_{\text{sat}}$. The polar front will then stop accelerating, while the equatorial front will still be accelerating. This will give place to a symmetrization phase that will last until the flame is quenched, or a transition to detonation occurs. We cannot predict exactly this symmetrization effect here, but if a transition to detonation occurs at densities $\sim 10^7$ g cm$^{-3}$, then the errors in our present calculation must be small. On the other hand, if no transition to detonation takes place, then the uncertainties in our estimates will depend upon

$$\rho = 5 \times 10^7$$ g cm$^{-3}$

We have computed the total asymmetry by taking into account the stellar expansion effect upon the density, for different initial chemical compositions of the WD progenitor. We have integrated equations (14) and (15) together with equations (18) and (19), using a subroutine to interpolate step by step the physical properties of the deflagration front, i.e. the laminar velocity and the density jump, and the value of the magnetic field. We have employed an analytical function for $a(t)$, namely $a(t) = 0.25 t + 1$ obtained from a numerical fit to the homologous expansion solutions of Goldreich & Weber (1980). The magnetic field strength has been assumed to decrease approximately linearly with the radial coordinate (see Wendell, Van Horn & Sargent 1987; Paper I). The initial density and gravity of the progenitor are depicted in Figs 2 and 3, respectively, as functions of the stellar radius. To compute the variation of the magnetic field in the moving frame of the flame, we have assumed magnetic flux conservation. The computed progenitor models were resolved in $10^3$
The density at which the flame quenches and will be as large or as small as the density is. Due to the same reason, we predict smaller symmetrization effects on lighter progenitors, i.e. progenitors with $X^{(12C)} = 0.5$ and $X^{(16O)} = 0.5$ will reach the saturation limit later at lower densities and will, therefore, develop larger integrated asymmetries (see Fig. 5).

It is known that if $l_{sat} = l_{min} \sim \delta_l$ thermodynamic effects must be taken into account. In fact, in most of the laboratory flames $l_{sat} \geq 20 \delta_l$ is observed (Bychkov, Kovalev & Liberman 1999). Because no definitive answer to this question exists, we have calculated the integrated asymmetry for different values of the saturation scale between these two extremes (i.e. from $l_{sat} = \delta_l$ to $20 \delta_l$) in Fig. 6, for the progenitor (a) of Fig. 5. In Fig. 7, we have fixed the saturation scale at $l_{sat} = 20 \delta_l$ in order to compare the integrated and the instantaneous asymmetry variations with the surface magnetic field for the same progenitor. Analysing these figures, we find that the integrated asymmetry is enormous if $l_{sat} \sim \delta_l$, and can be larger than 1000 per cent if the magnetic field is $\gtrsim 10^9$ G. When $l_{sat} \sim 20 \delta_l$ is imposed, the asymmetries are smaller but can still be very large. For instance, the progenitor shows a maximum asymmetry of $\sim 50$ per cent, for $l_{sat} \sim 20 \delta_l$ (Fig. 8), which is in the ballpark of what polarimetric observations suggest ($\sim 20$ per cent or less, see the next section; Leonard, Filippenko & Matheson 2000; Wang, Wheeler & Höflich 1997; Howell et al. 2001).

5 DISCUSSION AND CONCLUSIONS

It has been generally assumed that supernovae explosions are spherical. However, recent polarimetric studies of several supernovae have revealed intrinsic linear polarization that seems to be evidence of asymmetric explosion (see Branch et al. 2000; Leonard et al. 2000; Howell 2001; Howell et al. 2001). In the present work, we have investigated the effects of the magnetic field of the WD progenitor on the propagation of the burning front of thermonuclear supernovae. We have found that an asymmetry develops that could, in principle, help to explain observed asymmetries in these systems.

The magnetic field strengths inferred for isolated WDs are between $3 \times 10^4$ and $10^9$ G, while for WDs in AM Her binaries, they are in the range $10^7$ to $2 \times 10^8$ G (Wickamasinghe & Ferrario 2000) and could be further amplified by compression during the accretion phase (Cumming 2002). Magnetic fields have not been taken into account in previous studies of thermonuclear supernovae because their pressure is found to be much smaller than the gas pressure. None the less, our studies have revealed that the magnetic fields act upon the hydrodynamical instabilities that develop in the flame front (where the magnetic pressure is larger than the ram pressure; see equation 2) and quench their growth in the direction.
Figure 8. Integrated asymmetries for progenitor \( A^{12}\text{C} = 0.2, A^{16}\text{O} = 0.8 \), with a surface field \( B = 5 \times 10^7 \) Gauss, \( A^{12}\text{C} = 0.2, A^{16}\text{O} = 0.8 \), and saturation scales \( I_{sat} = \delta_t \) (dashed line) and \( 20\delta_t \) (full line). This represents the set of parameters that better fits observational data from SNIa spectropolarimetric studies and experimental knowledge from laboratory flames. We have stopped the calculations when the flame vanishes. At this time, expansion can start symmetrizing the flame, although more efficient symmetrization must be expected during the coasting phase of the remnant due to enhanced turbulent diffusion.

The model of Zel’dovich et al. for the mode–mode interactions of burned cells explains the saturation of the exponential growth of the hydrodynamic instabilities and gives the lower cut-off assumed for the instability which depends mainly on the saturation scale as explained above (see equation 17 and also equation 4 of Paper I).
Finally, we should mention that other models have been suggested in the literature to explain the asymmetries in SNIa explosions, although all of them have their problems (see, for example, Howell et al. 2001). The model investigated here also presents limitations that could, at least in part, be solved with the help of future multi-dimensional numerical calculations and larger samples of observed asymmetric astrophysical objects. Laboratory experiments of the deflagration of mixtures of gases in the presence of magnetic fields could also provide alternative tests of the asymmetry effect in the flame front and also of the fractal model examined. The configuration of these experiments must be settled carefully in order to ensure the growth of the R–T instability over a wide spectrum of perturbations, and the geometry of the apparatus must be chosen in such a way that its minimum size is larger than the minimum wavelength of the perturbations (Ghezzi 2002).8

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REFERENCES

Arnett W. D., 1996, Supernovae and Nucelosynthesis. Princeton Univ. Press, Princeton, NJ
Blinnikov S. I., Sasorov P. V., 1996, Phys. Rev. E, 53, 5
Branch D., 2000, (astro-ph/0012300)
Bychkov V. V., Kovalev K. A., Liberman M. A., 1999, Phys. Rev. E, 60, 2897

8 We are of course supposing that the fractal model can explain the turbulent regime of a supernova flame. Previous laboratory experiments (Gostintsev et al. 1988) and numerical simulations of fractal flames in the wrinkled regime (Filyand, Sivashinsky & Frankel 1994; Blinnikov & Sasorov 1996) seem to support this hypothesis.

Chandrasekhar S., 1981, Hydrodynamic and Hydromagnetic Stability. Dover, New York
Cumming A., 2002, MNRAS, 333, 589
Filyand L., Sivashinsky G. I., Frankel M. L., 1994, Physica D, 72, 110
Gamezo V. N., Khokhlov A. M., Oran E. S., 2002, Am. Astron. Soc. Meeting, 200, 14.01
Ghezzi C. R., 2002, PhD thesis, Univ. São Paulo, Brazil
Ghezzi C. R., de Gouveia Dal Pino E. M., Horvath J. E., 2001, ApJ, 548, L193 (Paper I)
Goldreich P., Weber S. V., 1980, ApJ, 238, 991
Gostintsev Yu. A., Istratov A. G., Shulenin Yu. V., 1988, Combustion Explosions Shock Waves, 24, 70
Howell D. A., 2001, ApJ, 554, L193
Howell D. A., Höflich P., Wang L., Wheeler J. C., 2001, ApJ, 556, 302
Jun B., Norman M. L., Stone J. M., 1995, ApJ, 453, 332
Khokhlov A., 1995, ApJ, 449, 695
Kull H. J., 1991, Phys. Rep., 206, 197
Leonard D. C., Filippenko A. V., Matheson T., 2000, in Holt S. S., Zhang W. W., eds, AIP Conf. Proc. 522, Cosmic Explosions. Am. Inst. Phys., Melville
Niemeyer J. C., Hillebrandt W., 1995, ApJ, 452, 769
Niemeyer J. C., Woosley S. E., 1997, ApJ, 475, 740
Timmes F. X., Woosley S. E., 1992, ApJ, 396, 649
Wang L., Wheeler J. C., Höflich P., 1997, ApJ, 476, L27
Wendell C. E., Van Horn H. M., Sargent D., 1987, ApJ, 313, 284
Wickamasinghe D. T., 2000, PASP, 112, 873
Woosley S. E., 1986, in Hauck B., Maeder A., Meynet G., eds, Nucleosynthesis and Chemical Evolution, Vol. 1, Swiss Soc. Astrophys. Astron., Geneva Obs., Geneva
Woosley S. E., 1990, in Petschek A. G., ed., Supernovae. Springer-Verlag, New York, p. 182
Zel’dovich Ya. B., 1966, J. Appl. Mech. & Tech. Phys., 7, 68
Zel’dovich Ya. B., Istratov A. G., Kidin N. I., Librovich V. B., 1980, Combustion Sci. Technol., 24, 1

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