Radiative Corrections and $Z'$

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Abstract Radiative corrections to parity violating deep inelastic electron scattering (PVDIS) are reviewed including a discussion of the renormalization group evolution (RGE) of the weak mixing angle. Recently obtained results on hypothetical $Z'$ bosons — for which parity violating observables play an important rôle — are also presented.

Keywords Radiative corrections · Extra neutral gauge bosons

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1 Effective electroweak interactions

The first two terms of the Lagrangian, \( \mathcal{L} = \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} \), of the electroweak Standard Model (SM) contain the free fermionic part and the interactions,

\[
\mathcal{L}_A + \mathcal{L}_W + \mathcal{L}_Z = -\frac{g}{2} \left( 2 \sin^2 \theta_W J_A^\mu A_\mu + J_W^\mu W^-_\mu + J_W^{\mu+} W^+_\mu + \frac{1}{\cos^2 \theta_W} J_Z^\mu Z_\mu \right),
\]

in terms of the electromagnetic current, \( J_A^\mu = \sum_{i=1}^{3} \left( \frac{2}{3} \bar{u}^i \gamma^\mu u^i - \frac{1}{3} \bar{d}^i \gamma^\mu d^i - \bar{e}^i \gamma^\mu e^i \right) \),

the weak charged current (CC), \( J_W^{\mu \pm} = \sqrt{2} \sum_{i=1}^{3} \left( \bar{u}^{i0} \gamma^\mu P_L d^{i0} + \bar{\nu}^{i0} \gamma^\mu P_L e^{i0} \right) \),

and the weak neutral current (NC), \( J_Z^\mu \equiv N_\psi \sum_{i=1}^{N_\psi} \bar{\psi}^i \gamma^\mu [g_V^i - g_A^i \gamma^5] \psi^i = -2 \sin^2 \theta_W J_A^\mu + \sum_{i=1}^{3} \left( \bar{u}^i \gamma^\mu P_L u^i - \bar{d}^i \gamma^\mu P_L d^i + \bar{\nu}^i \gamma^\mu P_L \nu^i - \bar{e}^i \gamma^\mu P_L e^i \right) \),

where \( P_L \equiv \frac{1 - \gamma^5}{2} \). At the tree-level, the NC couplings, \( g_V^i = \frac{1}{2} \tau_3^i - 2 Q^i \sin^2 \theta_W \) and \( g_A^i = \frac{1}{2} \tau_3^i \), with \( Q^i \) (\( \tau_3 \)) denoting...
Table 1 Numerical contributions to $\omega_{\text{PVDIS}}$

|                  | $2 \Delta u - \Delta d$ | $2 \Delta u - \Delta d$ | $\omega_{\text{PVDIS}}$ |
|------------------|--------------------------|--------------------------|--------------------------|
| tree + QED       | -0.7060                  | -0.0715                  | -0.7660                  |
| charge radii     | +0.0015                  | -0.0110                  | -0.0079                  |
| $\Box_{WW}$      | -0.0120                  | -0.0120                  | -0.0220                  |
| $\Box_{ZZ}$      | -0.0008                  | -0.0027                  | -0.0031                  |
| other            | -0.0009                  | -0.0011                  | -0.0018                  |
| TOTAL            | -0.7184                  | -0.0983                  | -0.8010                  |

the electric charge (third Pauli matrix), give rise to the effective 4-Fermi Hamiltonian,

$$H_{\text{eff}} = \frac{1}{2} \left( \frac{g}{2 \cos^2 \theta_W \lambda_2} \right)^2 J^\mu_Z J_{\mu Z} = \frac{G_F}{\sqrt{2}} J^\mu_Z J_{\mu Z} = \frac{G_F}{\sqrt{2}} \sum_{MNI} h_{MN}^{ij} \bar{\psi}^i \gamma^\mu \psi^j \gamma^\nu \psi^j,$$

where $\Gamma^V = \gamma^\mu$, $\Gamma^A = \gamma^\mu \gamma^5$, and $h_{MN}^{ij} = g_M^\nu g_N^\nu$. Unfortunately, there is no generally accepted notation, normalization, and sign convention for the $h_{MN}^{ij}$ in the literature. For parity violating eq interactions one defines $C_{1q} = 2h_{M}^{q}$ and $C_{2q} = 2h_{V}^{q}$. Parity violation in heavy atoms is basically driven by the $C_{1q}$, while PVDIS determines approximately the combination, $\omega_{\text{PVDIS}} = (2 \Delta u - \Delta d) + 0.84 (2 \Delta u - \Delta d)$.

2 Radiative corrections

Including one-loop electroweak radiative corrections one obtains the expressions

$$2 \Delta u - \Delta d = -\frac{3}{2} \left[ \rho_{\text{NC}} - \frac{\alpha}{2 \pi} \right] \left[ 1 - \frac{20}{9} \left( \sin^2 \theta_W(0) - \frac{2\pi}{9\pi} \right) + \Box_{WW} + \Box_{ZZ} + \Box_{ZZ} \right] + \frac{5\hat{\alpha}}{9\pi} \left[ 1 - 4 \sin^2 \theta_W(M_Z) \right] \left[ \ln \frac{M_Z}{m_e} + \frac{1}{12} \right], \quad (1)$$

$$2 \Delta u - \Delta d = -\frac{3}{2} \left[ \rho_{\text{NC}} - \frac{\alpha}{6\pi} \right] \left[ 1 - 4 \left( \sin^2 \theta_W(0) - \frac{2\pi}{9\pi} \right) + \Box_{WW} + \Box_{ZZ} + \Box_{ZZ} \right] + \frac{5\hat{\alpha}}{9\pi} \left[ 1 - \frac{12}{5} \sin^2 \theta_W(M_Z) \right] \left[ \ln \frac{M_Z}{m_q} + \frac{1}{12} \right] - \frac{8\hat{\alpha}}{9\pi} \left[ \ln \frac{M_W}{m_q} + \frac{1}{12} \right], \quad (2)$$

where $\rho_{\text{NC}} \approx 1.0007$ collects various propagator and vertex corrections relative to $\mu$-decay, and the second lines are from the $e$ and $q$ charge radii. With $s^2 = \sin^2 \theta_W(M_Z)$,

$$\Box_{WW} = -\frac{9\hat{\alpha}}{8\pi s^2} \left[ 1 - \frac{\alpha_s(M_W)}{3\pi} \right], \quad \Box_{ZZ} = -\frac{3\hat{\alpha}}{4\pi} \left[ 1 - 4s^2 \right] \left[ \ln \frac{M_Z}{M_{\rho}} + \frac{3}{4} \right], \quad \Box_{ZZ} \ll \Box_{WW}$$

are the box contributions except that for $2 \Delta u - \Delta d$ the $\alpha_s$ correction to the WW-box is not yet known and $\Box_{ZZ}$ is obtained from above by replacing $4s^2$ by $28s^2/9$ and the constant $3/4$ by $5/12$. The numerical results are summarized in Table 1.

Eqs. (1) and (2) were originally obtained for atomic parity violation. For PVDIS, the one-loop expressions with the full kinematical dependence (in analogy with Ref. 5 for polarized Møller scattering) need to be computed, plus the $\alpha_s$ corrections to $\Box_{WW}$ and $\Box_{ZZ}$. In practice, one would want to define new $C_{2q}$ at these kinematics since these would supersede the ones at very low $Q^2$ with their large hadronic uncertainties.
The \( \overline{\text{MS}} \) scheme (marked by a caret) weak mixing angle enters Eqs. 1 and 2 evaluated at the renormalization scale \( \mu = 0 \). Introducing the quantity \( \hat{X} \equiv \sum_i N^C_i \gamma_i \hat{g}_i Q_i \), where \( N^C_i = 3 \) (1) for quarks (leptons) and \( \gamma_i = 4 \) (22) for chiral fermions (gauge bosons), one can show that \( d\hat{X}/X = d\hat{\alpha}/\alpha \), i.e., the RGE for \( \hat{\alpha}(\mu) \) implies that for \( \sin^2 \theta_W(\mu) \) (see Fig. 1) including experimental constraints from \( e^+e^- \) annihilation and \( \tau \) decays that enter the dispersion integral for the non-perturbative regime, provided that any one of the following conditions is satisfied: (i) no mass threshold is crossed; (ii) perturbation theory applies (\( W^\pm \), leptons, \( b \) and \( c \) quarks); (iii) equal coefficients (like for \( d \) vs. \( s \) quarks); or (iv) symmetries like \( SU(2) \) or \( SU(3) \) may be applied.

This leaves as the only problem area the treatment of the \( u \) vs. the \( (d, s) \) quark thresholds, or—considering that \( m_s \neq m_d \approx m_u \) — the separation of the \( s \) quark from the \( (u, d) \) doublet. Our strategy [6] is to define threshold masses (absorbing QCD matching effects), \( \bar{m}_q = \xi_q M_1 S/2 \), in terms of \( 1S \) resonance masses. The \( \xi_q \) are between 0 (chiral limit) and 1 (infinitely heavy quarks). One expects \( \xi_b > \xi_c > \xi_s > \xi_d > \xi_u \) and we explicitly verified \( \xi_b > \xi_c \) in perturbative QCD. Now, \( \xi_s = \xi_c \) defines the heavy quark limit for the \( s \) quark, implying \( \bar{m}_s < 387 \text{ MeV} \). On the other hand, \( \xi_s = \xi_d \approx \xi_u \) together with the dispersion result for the three-flavor RGE for \( \hat{\alpha} \) below \( \mu = \bar{m}_c \), \( \Delta \hat{\alpha}^{(3)}(\bar{m}_c) \), yields an upper limit on the \( s \) quark contribution and \( \bar{m}_s > 240 \text{ MeV} \). Besides parametric uncertainties from the input values of \( \bar{m}_b \), \( \bar{m}_c \), and \( \hat{\alpha} \), this procedure introduces an experimental error through \( \Delta \hat{\alpha}^{(3)}(\bar{m}_c) \) (\( \pm 3 \times 10^{-5} \)), \( SU(3)_F \) breaking masses, \( \bar{m}_u = \bar{m}_d \neq \bar{m}_s (\pm 5 \times 10^{-5}) \), and \( SU(2)_I \) breaking masses, \( \bar{m}_u \neq \bar{m}_d (\pm 8 \times 10^{-6}) \). Starting at three-loop order there is also the (OZI rule violating) singlet (QCD annihilation) contribution to the RGE for \( \hat{\alpha} \) (but by virtue of \( Q_u + Q_d + Q_s = \tau_3^{uu} + \tau_3^{dd} = 0 \) not present in \( \hat{X} \)) introducing another \( \pm 3 \times 10^{-5} \) error.
3 $Z'$ physics: the search for a fifth force

Extra $Z'$ bosons are predicted in virtually all scenarios for TeV scale physics beyond the SM, including grand unified theories, left-right models, superstrings, technicolor, large extra dimensions and little Higgs theories and in all these cases one expects $M_{Z'} = \mathcal{O}(\text{TeV})$ and 100 (1,000) fb$^{-1}$ of LHC data will explore $M_{Z'}$ values up to 5 (6) TeV [4]. Angular distributions of leptons may help to discriminate spin-1 ($Z'$) against spin-0 (sneutrino) and spin-2 (Kaluza-Klein graviton) resonances [5]. The LHC will also have some diagnostic tools to narrow down the underlying $Z'$ model by studying, e.g., leptonic forward-backward asymmetries and heavy quark final states [9,10].

$Z'$ models based on the gauge group $E_6$ without kinetic mixing correspond to extending the SM by a $U(1)' = \cos \beta U(1)_X + \sin \beta U(1)_Y$ ($-90^\circ < \beta \leq 90^\circ$). Particular values for $\beta$ give $Z'$ models of special interest, namely (i) $\beta = 0^\circ \Rightarrow Z'$ and is defined by the breaking of $SO(10) \rightarrow SU(5) \times U(1)_X$; (ii) $\beta = 90^\circ \Rightarrow Z_\psi$ defined by the breaking of $E_6 \rightarrow SO(10) \times U(1)_Y$; (iii) $\beta \approx -52.2^\circ \Rightarrow Z_\eta$ and appears in a class of heterotic string models compactified on Calabi-Yau manifolds; (iv) $\beta \approx 37.8^\circ \Rightarrow Z_{1/2} \perp Z_\eta$ and is hadrophobic in that it doesn’t couple to up-type quarks; (v) $\beta \approx 23.3^\circ \Rightarrow Z_S$ and gives rise to the so-called secluded $U(1)'$ breaking model addressing both the little hierarchy problem ($M_Z \ll M_{Z'}$) [11] and electroweak baryogenesis [12]; and (vi) $\beta \approx 75.5^\circ \Rightarrow Z_N$ with no couplings to right-handed neutrinos and therefore allowing the (ordinary) see-saw mechanism. Adding kinetic mixing is equivalent to considering the more general combination, $Z' = \cos \alpha \cos \beta Z_X + \sin \alpha \cos \beta Z_Y + \sin \beta Z_\psi$. Then the values (vii) $(\alpha, \beta) \approx (50.8^\circ, 0^\circ) \Rightarrow Z_R$ defined by the breaking of $SU(2)_R \rightarrow U(1)_R$; (viii) in left-right symmetric models appears the $Z_{LR} \approx 1.53 Z_R - 0.33 Z_{B-L}$, where $(\alpha, \beta) \approx (-39.2^\circ, 0^\circ) \Rightarrow Z_{B-L} \perp Z_R$; while (ix) $(\alpha, \beta) \approx (28.6^\circ, -48.6^\circ) \Rightarrow Z_E$ with no couplings to charged leptons and left-handed neutrinos. Finally, (x) the sequential $Z_{SM}$ couples like and could be an excited state of the ordinary $Z$ boson.

$Z'$ bosons can have various effects on precision observables. The $Z-Z'$ mixing angle, $\theta_{ZZ'}$, is strongly constrained by the $M_W$-$M_Z$ interdependence (even for the $Z_E$) and by the $Z$-pole (because $\theta_{ZZ'}$ affects the very precisely measured $Z$ couplings to fermions). Conversely, if $\theta_{ZZ'} = 0$ the $Z$ pole observables are rather blind to $Z'$ physics because the $Z$ and $Z'$ amplitudes are almost completely out of phase and one needs to go off-peak, i.e., to LEP 2 and low energies. There are also loop effects which are small but not necessarily negligible. E.g., the $M_W$-$G_F$ relation, parametrized by $\Delta \hat{r}_W$, is shifted,

$$\delta(\Delta \hat{r}_W) = \frac{5}{2} \frac{\alpha}{\pi \cos^2 \theta_W} \lambda^\ell_\psi \epsilon_\mu \epsilon^\mu_\ell \frac{M^2_W}{M^2_{Z'}} \ln \frac{M^2_{Z'}}{M^2_W} \ln \frac{M^2_{Z'}}{M^2_W} \ln \frac{M^2_{Z'}}{M^2_W} \ln \frac{M^2_{Z'}}{M^2_W} \ln \frac{M^2_{Z'}}{M^2_W} \ln$$

where the $\epsilon^\mu_\ell$ denote $U(1)'$ charges and $\lambda$ is a model dependent parameter of $\mathcal{O}(1)$. $Z'$ bosons would also yield an apparent violation of first row CKM unitarity, $\delta(U^2_{ud} + V^2_{us} + V^2_{ub})$, given by the r.h.s. of Eq. (3) upon replacing $\epsilon^\mu_\ell$ by $-2(\epsilon^\mu_\ell - \epsilon^\mu_\mu)$. Finally, the muon anomalous magnetic moment [13] would receive a (usually tiny) correction, $\delta a_\mu = 5/36 \alpha/\pi \cos^2 \theta_W \lambda (V^2_{\mu} - 5 A^2_{\mu}) m^2_{\mu}/M^2_{Z'}$, with some interest for the $Z_\psi$ which is insensitive to most other precision data (since it does not possess any vector couplings $V_\mu$) while the axial coupling $A_\mu$ comes enhanced in $\delta a_\mu$.

Results from a global analysis [14] are shown in Table 2. Some $Z'$ models give a fairly low minimum $\chi^2$, especially the $Z_\psi$ and $Z_R$. Technically, there is a 90% C.L. upper bound on the $Z_R$ mass of about 29 TeV. Of course, at present there is little significance to this observation since there are two additional fit parameters ($M^2_{Z'}$ and
Table 2 95% C.L. lower mass limits (in GeV) on extra $Z'$ bosons and lower and upper limits for $\theta_{ZZ'}$ from electroweak precision data, assuming 114.4 GeV $< M_H < 1$ TeV. Also shown are for comparison (where applicable) the limits obtained by CDF (they assume that no supersymmetric or exotic decay channels are open; otherwise the limits would be moderately weaker) and LEP 2 (constraining virtual $Z'$ bosons by their effects on cross sections and angular distributions of di-leptons, hadrons, $b\bar{b}$ and $c\bar{c}$ final states). CDF sees a significant excess at a di-electron invariant mass of 240 GeV, but this is not confirmed in the $\mu^+\mu^-$ channel. The result for the leptophobic $Z_E$ (in parentheses) in the electroweak column assumes a specifically chosen Higgs sector. The CDF number refers to the $Z_{SM}$ limit from the di-jet channel and should give a rough estimate of the sensitivity to our specific $Z_L$. The various mass limits are highly complementary (e.g., unlike Tevatron limits, electroweak and LEP 2 limits scale with the coupling strength). The last column indicates the $\chi^2$ minimum for each model.

| $Z'$   | electroweak | CDF   | LEP 2  | $\theta_{\min}^{ZZ'}$ | $\theta_{\max}^{ZZ'}$ | $\chi^2_{\min}$ |
|--------|-------------|-------|--------|-----------------------|-----------------------|-----------------|
| $Z_N$  | 1,141       | 892   | 673    | $-0.0016$             | 0.0006                | 47.3            |
| $Z_N'$ | 147         | 878   | 481    | $-0.0018$             | 0.0009                | 46.5            |
| $Z_L$  | 427         | 982   | 434    | $-0.0047$             | 0.0021                | 47.7            |
| $Z_I$  | 1,204       | 789   |        | $-0.0005$             | 0.0012                | 47.4            |
| $Z_Z$  | 1,257       | 821   |        | $-0.0013$             | 0.0005                | 47.3            |
| $Z_R$  | 623         | 861   |        | $-0.0015$             | 0.0007                | 47.4            |
| $Z_{LR}$ | 442         |       |        | $-0.0015$             | 0.0009                | 46.1            |
| $Z_{R'}$ | 998         | 630   | 804    | $-0.0013$             | 0.0006                | 47.3            |
| $Z_{R'}$ | (803)       | (740) |        | $-0.0094$             | 0.0081                | 47.7            |
| $Z_{SM}$ | 1,403       | 1,030 | 1,787  | $-0.0026$             | 0.0006                | 47.2            |

$\theta_{ZZ'}$ and various adjustable charges (like the angles $\alpha$ and $\beta$). Still this surprises given that the SM fit is quite good with $\chi^2_{\text{min}} = 48.0/45$ (with $M_H$ unconstrained). It is interesting that the improvement, $\Delta \chi^2_{\text{min}} = -2.9$, is mainly from PAVI observables, namely from polarized Møller [15] ($-1.7$) and $e^-$-hadron scattering [16] ($-0.9$). The best fit with $M_{Z'} = 667$ GeV implies shifts in the so-called weak charges, $|\delta Q_W(e,p)| = -0.0073$, corresponding to 6.6$\sigma$ and 2.5$\sigma$, respectively, for the proposed MOLLER [17] and Qweak [18] experiments at JLab. Similarly, expect $|\delta |\omega_{PVDIS}| = -0.0200$ ($4.2\sigma$).

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