The nonet of the light scalar tetraquarks: the mesons $a_0(980)$ and $K_0^*(800)$

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The spectroscopic parameters and partial decay widths of the light mesons $a_0(980)$ and $K_0^*(800)$ are calculated by treating them as scalar diquark-antidiquark states. The masses and couplings of the mesons are found in the framework of QCD two-point sum rule approach. The widths of the decay channels $a_0(980) \rightarrow \pi \eta$ and $a_0(980) \rightarrow K \bar{K}$, and $K_0^*(800) \rightarrow K^0 \pi^0$ and $K^*_0(800) \rightarrow K^0 \pi^0$ are evaluated using QCD sum rules on the light-cone and technical tools of the soft meson approximation. Our results for the mass of the mesons $m_{a_0} = 991^{+29}_{-27}$ MeV and $m_{K^*} = 767^{+38}_{-29}$ MeV, as well as their total width $\Gamma_{a_0} = 62.01 \pm 14.37$ MeV and $\Gamma_{K^*} = 401.1 \pm 87.1$ MeV are compared with last experimental data.

I. INTRODUCTION

The experimental investigation of the light scalar mesons and theoretical interpretation of obtained data remains one of intriguing problems in high energy physics. Experimental information on parameters of these particles suffers from large uncertainties: Their masses and widths are sometimes known with $\sim 100$ MeV accuracy [1]. The status of some of these particles is still unclear, even their existence is under question.

The theoretical interpretations of light scalars also meet with well-known troubles. Really, the nonet of scalar particles in the conventional quark-antiquark model of mesons may be realized as $1^P_0$ states. The masses of these scalars, in accordance with various model calculations are higher than 1 GeV. In fact, the isoscalar mesons $f_0(1370)$ and $f_0(1710)$, the isovector $a_0(1450)$ or isospinor $K_0^*(1430)$ states were identified as members of the $1^P_0$ multiplet. But mass of the mesons from the light scalar nonet lie below 1 GeV. Therefore, during a long time the broad scalar resonances $f_0(500)$ and $K_0^*(800)$, relatively narrow states $f_0(980)$ and $a_0(980)$ are subject of controversial theoretical hypothesis and suggestions. The main idea behind attempts to explain unusual features of these states is an assumption about four-quark (diquark-antidiquark or meson-meson) nature of these mesons [2–4]. Within this scheme quantum numbers and low masses, as well as mass hierarchy inside of the light nonet seem receive reasonable explanations. The present-day physics of the light scalars consists of different ideas, models and theories. The comprehensive information on these issues can be found in the review articles [5–8].

The diquark-antidiquark picture allows one to answer essential questions about internal organization of light scalar mesons, and calculate spectroscopic parameters and decay width of these particles [2, 6, 10]. In this model the scalar mesons emerge as the nonet of particles composed of four valence quarks. Within the nonet the $SU_f(3)$ flavor octet and singlet states may mix to create the physical mesons $f_0(500)$ and $f_0(980)$. The situation here is similar to the well-known mixing phenomenon in the $\eta, \eta'$ system of the pseudoscalar mesons. The other two scalar particles $a_0(500)$ and $K_0^*(800)$ may be identified with the isotriplet and isospinor members of the light multiplet. An interesting suggestion about the structure of the scalar mesons was proposed recently in Ref. [11]. In this picture not only light mesons but also the heavy ones are collected into two nonet of the scalar particles with diquark-antidiquark structure: The physical mesons are mixtures of the spin-0 diquarks from $[3_6, 3_f]$ representation with spin-1 diquarks from $[6_c, 3_f]$ representation of the color-flavor group.

The diquark-antidiquark model allowed one to calculate parameters of the light scalars and explore their strong and electromagnetic decay channels. To this end, different calculational schemes and methods were used. Thus, the masses of the $f_0(500), f_0(980), a_0(980)$ and $K_0^*(800)$ mesons were calculated in Ref. [12] in the framework of the relativistic diquark-antidiquark approach and nice agreements with the data were found. In the context of the four-quark Bethe-Salpeter equation the same problem was addressed in Ref. [13]. The two-pseudoscalar and two-photon decays of the mesons from the light scalar nonet were studied in Ref. [14].

Intensive investigations of the light scalar mesons as tetraquarks were performed using QCD sum rules method [15, 22]. In these papers apart from the pure diquark-antidiquarks the light scalars were treated also as mixtures of diquark-antidiquarks with different flavor structures or as superpositions of diquark-antidiquark and quark-antiquark components. The aforementioned modification were introduced by the authors mainly to achieve an agreement between theoretical predictions and experimental data.

In our work [23] we calculated the masses of the mesons $f_0(500)$ and $f_0(980)$ by considering them as states composed of scalar diquarks. We took into account the mixing of flavor octet and singlet diquark-antidiquarks that create the physical mesons and, at the same time, neglected their possible mixing with tetraquarks built of the spin-1 diquarks. Obtained in this work predictions
for the masses of the scalar mesons are in reasonable agreement with existing data. The mixing of the flavor
octet and singlet diquark-antidiquarks used in Ref. [23] to calculate spectroscopic parameters of the mesons $f_0(500)$
and $f_0(980)$ had important consequences for studying of their decay channels. Indeed, without octet-singlet
mixing the decays of different scalar mesons proceed through different mechanisms. For example, the process
$f_0(980) \rightarrow K\bar{K}$ is the superallowed Okubo-Zweig-Iizuka (OZI) decay, whereas $f_0(980) \rightarrow \pi\pi$ can proceed due
to one gluon exchange [17]. The octet-singlet mixing allows one to treat all of the light scalar mesons’ decay channels
employing the OZI mechanism, and explain differences in their partial widths by the mixing parameters. The decays
of the $f_0(500)$ and $f_0(980)$ mesons in this framework were evaluated in Ref. [24].

The present work is an extension of our previous studies devoted to spectroscopy and decay properties of the
light scalar mesons [23, 24]. We treat them as diquark-antidiquark states composed of the scalar diquarks by ignoring
their possible mixing with spin-1 diquarks. We calculate the spectroscopic parameters of the mesons $a_0(980)$ and $K_0^*(800)$, evaluate their partial decay widths and, as a result, total widths of these particles. All investigations
are performed using QCD sum rule method: In order to calculate the mass and coupling of the mesons we employ QCD two-point sum rule approach by including into analysis quark, gluon and mixing vacuum condensates up to dimension ten [25, 26]. The sum rules for the strong couplings of the vertices $a_0(980)\eta\pi^0$, $a_0(980)K^+K^-$, $K_0^*(800)K^+\pi^-$ and $K_0^*(800)K^0\pi^0$ are derived using light-cone sum rule (LCSR) method [27] and technical tools of the soft approximation [28], which was adapted in Ref. [29] to study tetraquark-meson-meson vertices. This approach was successfully applied to evaluate strong couplings and widths of numerous decays involving tetraquarks [30, 31], including the light axial-vector meson $a_1(1420)$ [32].

This paper is organized in the following way: In the section [1] we calculate the mass and coupling of the mesons $a_0(980)$ and $K_0^*(800)$. In the section [II] we derive the sum rules to evaluate the strong couplings $g_{\eta\pi\pi}$, $g_{KKK}$, $g_{KK\pi}$ and $g_{K\pi\pi}$ of the mesons $a_0(980)$ and $K_0^*(800)$, and total widths of the mesons $a_0(980)$ and $K_0^*(800)$. Section [III] contains our concluding remarks.

II. MASS AND COUPLING OF THE MESONS

$a_0(980)$ AND $K_0^*(800)$

The mass and coupling of the mesons $a_0(980)$ and $K_0^*(800)$ can be calculated within QCD two-point sum
rule method. We consider here in details all necessary steps to find the mass and coupling of the $a_0(980)$ meson
and provide only final expressions and results for the $K_0^*(800)$ meson.

The mass and coupling of $a_0(980)$ can be extracted from the sum rule analysis of the two-point correlation
function

$$
\Pi(p) = i \int d^4x e^{ipx} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle,
$$

where $J(x)$ is the interpolating current to the $a_0(980)$ meson. In the diquark-antidiquark model it can be written in the following form

$$
J(x) = \frac{\bar{c}c}{\sqrt{2}} \left[ (u_a^T C \gamma_5 s_b) (\pi d_{\gamma5} C \sigma^T e) \\
- (d_a^T C \gamma_5 s_b) (\pi d_{\gamma5} C \sigma^T e) \right],
$$

where $C$ is the charge conjugation operator. Here we also use the short-hand notation $\bar{c}c = \epsilon_{abc}\epsilon_{dec}$ with $a, b, c, d$ and $e$ being the color indices.

In accordance with standard prescriptions of the sum rule computations the correlation function $\Pi(p)$ should be found by employing both the physical parameters of the $a_0(980)$ meson, i.e. its mass $m_{a_0}$ and coupling $f_{a_0}$ and in terms of the light-quark, gluon and mixed vacuum condensates. By matching the obtained results and benefiting from the assumption on the quark-hadron duality it is possible to extract sum rules and evaluate the physical parameters of interest.

In the case under consideration the physical side of the sum rule takes the simple form

$$
\Pi^{\text{Phys}}(p) = \frac{\langle 0 | J(a_0(p)) \langle a_0(p) | J^\dagger | 0 \rangle}{m_{a_0}^2 - p^2} + \ldots,
$$

because the $a_0(980)$ meson is the ground-state particle. The contributions coming from the excited and continuum states are shown in Eq. [1] by dots. To express $\Pi^{\text{Phys}}(p)$ in terms of the parameters $m_{a_0}$ and $f_{a_0}$ we introduce the matrix element

$$
\langle 0 | J(a_0(p)) = f_{a_0} m_{a_0},
$$

and get

$$
\Pi^{\text{Phys}}(p) = \frac{f_{a_0}^2 m_{a_0}^2}{m_{a_0}^2 - p^2} + \ldots.
$$

Effect of the excited states and continuum on the $\Pi^{\text{Phys}}(p)$ can be suppressed by means of the Borel transformation which yields

$$
\mathcal{B}\Pi^{\text{Phys}}(p) = \frac{f_{a_0}^2 m_{a_0}^2 e^{-m_{a_0}^2/M^2}}{M^2} + \ldots,
$$

where $M^2$ is the Borel parameter. In Eq. [5] by dots we again denote contributions of the excited states and continuum which will be subtracted from Borel transformation of $\Pi^{\text{OPP}}(p)$ to derive the required sum rules.
The OPE density \( \rho \) was explained numerously in existing literature, utilizing both of these approaches. These routine operations were explained numerous times in existing literature, therefore we do not elaborate on these questions here. The obtained expression for \( \Pi^{\text{OPE}}(M^2) \) has to be equated to Eq. (11), and one also has to perform the continuum subtraction. After these manipulations we find the following sum rule

\[
\begin{align*}
\Pi^{\text{OPE}}(p) = i \int d^4xe^{ipx} & \bar{\psi} \gamma_5 \slashed{D} \psi \bigg\{ \langle \bar{a} \gamma_5 S_{s(a)}(x) \rangle + (u \leftrightarrow d) \bigg\}. 
\end{align*}
\]

In the expression above

\[
\bar{S}_{s(a)}(x) = CS_{s(a)}^T(x)C,
\]

where \( S_{s(a)}(x) \) are the \( s \) and \( q = u, d \) quarks’ propagators. In the present work we calculate the correlation function by taking into account nonperturbative terms up to dimension ten.

The Borel transform of the correlator \( \Pi^{\text{OPE}}(p) = \Pi^{\text{OPE}}(M^2) \) can be calculated using either the spectral density \( \rho(s) \) which is proportional to imaginary part of \( \Pi^{\text{OPE}}(p) \) or by applying the Borel transformation directly to \( \Pi^{\text{OPE}}(p) \). If necessary, \( \Pi^{\text{OPE}}(M^2) \) may be computed utilizing both of these approaches. These routine operations were explained numerous times in existing literature, therefore we do not elaborate on these questions here. The obtained expression for \( \Pi^{\text{OPE}}(M^2) \) has to be equated to Eq. (11), and one also has to perform the continuum subtraction. After these manipulations we find the following sum rule

\[
\begin{align*}
\Pi^{\text{OPE}}(p) = i \int d^4xe^{ipx} & \bar{\psi} \gamma_5 \slashed{D} \psi \bigg\{ \langle \bar{a} \gamma_5 S_{s(a)}(x) \rangle + (u \leftrightarrow d) \bigg\}. 
\end{align*}
\]

The remaining operations are standard and do not differ from ones described above in the case of the \( q_0(980) \) meson.

The numerical computations require to specify values of various parameters that enter to the quark propagators, and as a result sum rules for the mass and coupling. Among them the vacuum expectation values of the quark, gluon and mixed local operators are important ones:

\[
\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{GeV}^3, \quad \langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle, \quad m_0^2 = (0.8 \pm 0.1) \text{GeV}^2, \quad \langle \bar{q}g_s \sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle,
\]

\[
\langle \bar{a} \gamma_5 S_{s(a)}(x) \rangle \approx \langle 0 \rangle = (0.012 \pm 0.004) \text{GeV}^4. \quad (11)
\]

In the present work we neglect the masses of the \( u \) and \( d \) quarks, but set \( m_s \neq 0 \) and use in calculations \( m_s = 128 \pm 10 \text{ MeV} \). Our expressions depend also on auxiliary parameters \( M^2 \) and \( s_0 \), the choice of which has to satisfy standard restrictions. Thus, we determine the upper limit \( M^2_{\text{max}} \) of the working window \( M^2 \in [M^2_{\text{min}}, M^2_{\text{max}}] \) by requiring fulfillment of the condition imposed on the pole contribution

\[
\text{PC} = \frac{\Pi(M^2_{\text{max}}, s_0)}{\Pi(M^2_{\text{max}}, \infty)} > 0.10. \quad (12)
\]

The lower bound of the Borel parameter \( M^2_{\text{min}} \) is fixed from convergence of the operator product expansion (OPE). By quantifying this constraint we require that a contribution of the last term in OPE should be of order 5%, i.e.

\[
\frac{\Pi^{\text{Dim10}}(M^2_{\text{min}}, s_0)}{\Pi(M^2_{\text{min}}, \infty)} \approx 0.05, \quad (13)
\]

has to be obeyed. Another restriction to \( M^2_{\text{min}} \) is connected with the perturbative contribution to sum rules. In the present work we apply the following criterion: at the lower bound of \( M^2 \) the perturbative contribution has to constitute more than 70% part of the full result.

Boundaries of \( s_0 \) are fixed by analyzing the pole contribution to get its greatest accessible values. Minimal dependence of extracted quantities on \( M^2 \) while varying \( s_0 \) is another constraint that has to be imposed when choosing a region for this parameter. Performed analyses lead to the following working windows for \( M^2 \) and \( s_0 \):

\[
M^2 \in [1.1, 1.4] \text{ GeV}^2, \quad s_0 \in [1.7, 1.9] \text{ GeV}^2. \quad (14)
\]

In these regions all of constraints imposed on the correlation function are satisfied. In fact, at \( M^2_{\text{max}} \) the pole contribution PC equals to 0.115, whereas at \( M^2_{\text{min}} \) it amounts to 78% of the result. In other words, Eq. (12) determines only the lower limit for the PC: in the full interval for \( M^2 \) the pole contribution is large which should lead to reliable sum rules predictions. At the minimal allowed value of the Borel parameter contribution of Dim10 term constitutes up to 5.5% of the whole result.
And perturbative component of the correlation function II(M^2_{min}, s_0) forms its no less than 0.71 part.

In Figs. 1 and 2 we depict the sum rules results for the mass and coupling of the a_0(980) state as functions of the Borel and continuum threshold parameters. It is seen, that prediction for the coupling is rather stable against varying of both M^2 and s_0. The mass m_{a_0} demonstrates some dependence on the auxiliary parameters, which nevertheless is mild. Therefore, theoretical errors are lower than uncertainties typical for sum rules computations.

For m_{a_0} and f_{a_0} we find:

\[ m_{a_0} = 991^{+29}_{-27} \text{ MeV}, \quad f_{a_0} = (1.94 \pm 0.04) \cdot 10^{-3} \text{ GeV}^4. \]  

(15)

The similar analysis of the sum rules for the mass and coupling of the K^{*}_0(800) meson allows us to find the regions for the Borel and continuum threshold parameters

\[ M^2 \in [0.8, 1.0] \text{ GeV}^2, \quad s_0 \in [0.9, 1.1] \text{ GeV}^2, \]  

(16)

which lead to the following predictions:

\[ m_{K^*} = 767^{+38}_{-29} \text{ MeV}, \quad f_{K^*} = (1.71 \pm 0.07) \cdot 10^{-3} \text{ GeV}^4. \]  

(17)

Our result for the mass of the a_0(980) meson is in a nice agreement with the available experimental data m_{a_0} = 980 \pm 20 \text{ MeV} \label{eq:16}. The latest measurement of m_{K^*} performed by the BES Collaboration \cite{33} and extracted from the decay J/\psi \to K^{*}_S K^{*}_S \pi^+ \pi^- is equal to

\[ m_{K^*} = 826 \pm 49^{+49}_{-34} \text{ MeV}. \]  

(18)

From the process J/\psi \to K^{*}_S K^{*}_S \pi^+ \pi^- \text{ the same collaboration obtained (see, Ref. \cite{34})}

\[ m_{K^*} = 849 \pm 77^{+18}_{-14} \text{ MeV}. \]  

(19)

As is seen, the experimental data are not precise, and the central values for m_{K^*} are higher than our prediction. Nevertheless, within the experimental and theoretical errors they are compatible with each other.

The mass and coupling of the a_0(980) and K^{*}_0(800) mesons calculated in the present section will be used as input parameters below to find their partial decay widths.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The mass of the meson a_0(980) as a function of the Borel parameter M^2 at fixed s_0 (left panel), and as a function of the continuum threshold s_0 at fixed M^2 (right panel).}
\end{figure}

III. STRONG DECAY CHANNELS OF THE a_0(980) AND K^{*}_0(800) MESONS

In the light of the obtained results we can determine the kinematically allowed strong decay channels of the a_0(980) and K^{*}_0(800) mesons. In the present paper we restrict ourselves by studying only S-wave decays of these mesons. It turns out that the dominant S-wave strong decays of a_0(980) are processes a_0(980) \to \eta \pi^0 and a_0(980) \to K^+ K^- . For the meson K^{*}_0(800) the decays K^{*}_0(800) \to K^{*}_S \pi^0 and K^{*}_0(800) \to K^0 \pi^0 are dominant ones. All of these processes proceed through super-allowed OZI mechanism and are responsible for a main part of their full widths.

It is instructive to consider the mode a_0(980) \to \eta \pi^0 in a detailed manner. In order to calculate the strong coupling g_{\eta \pi^0} we use QCD LCSR method and start from
analysis of the correlation function

$$\Pi(p, q) = i \int d^4x e^{ip \cdot x} \langle \pi^0(q) | T \{ J^\eta(x), J^\eta(0) \} | 0 \rangle,$$

where $J(x)$ and $J^\eta(x)$ are the interpolating currents for the $a_0(980)$ and $\eta$ mesons, respectively. The interpolating current for the $a_0(980)$ is given by Eq. (2).

The situation with the choice of $J^\eta(x)$ is more subtle and deserves some explanations. The system of pseudoscalar mesons $\eta - \eta'$ has a complicated structure. In the world of the exact flavor $SU(3)$ symmetry the mesons $\eta$ and $\eta'$ can be interpreted as the octet $\eta_8$ and singlet $\eta_1$ states of the flavor group, respectively. But in the real world, where this symmetry is broken the physical particles are mixtures of the $\eta_8$ and $\eta_1$ states. Of course, the mesons $\eta$ and $\eta'$ are predominantly the $\eta_8$ and $\eta_1$ states, nevertheless the mixing phenomenon can not be ignored. This mixing can be described using the octet-singlet basis. Alternatively, the same phenomenon can be treated employing the quark-flavor basis (see, Ref. [35] for details)

$$\eta_q = \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d), \quad \eta_s = \bar{s}s. \quad (21)$$

The quark-flavor basis is more convenient to describe the mixing in the $\eta - \eta'$ system and investigate different exclusive processes involving these mesons [36]. The reason is that in this scheme with rather high accuracy the state and coupling mixing are governed by the same angle, whereas in the $\eta_1 - \eta_8$ basis one has to introduce two mixing angles for the decay constants.

In the quark-flavor basis the interpolating current of the $\eta$ meson can be obtained through mixing from the basic currents

$$J_q(x) = \frac{1}{\sqrt{2}} \left[ \bar{u}(x) i \gamma_5 u(x) + \bar{d}(x) i \gamma_5 d(x) \right],$$
$$J_s(x) = \bar{s}(x) i \gamma_5 s(x), \quad (22)$$

and reads

$$J^\eta(x) = J_q(x) \cos \varphi - J_s(x) \sin \varphi,$$

where $\varphi$ is the mixing angle.

The phenomenological side of the sum rule is obtained by expressing $\Pi(p, q)$ in terms of the strong coupling $g_{\eta \pi}$ and physical parameters of the $a_0(980)$ and $\eta$ mesons

$$\Pi(p, q) = \frac{\langle 0 | J^\eta(0) | \eta(p) \rangle}{p^2 - m_\eta^2} \langle \eta(p) \pi^0(q) | a_0(p') \rangle \times \langle a_0(p') | J^\eta(0) | 0 \rangle,$$

where $m_\eta$ is the mass of $\eta$ the dots being stood for contributions of excited states. The matrix element $\langle a_0(p') | J^\eta(0) \rangle$ has been introduced in the previous section, and the vertex $\langle \eta(p) \pi^0(q) | a_0(p') \rangle$ can be written down in the following form

$$(\eta(p) \pi^0(q) | a_0(p') \rangle = g_{\eta \pi} p \cdot p', \quad (24)$$

where $g_{\eta \pi}$ is the coupling corresponding to the strong vertex $a_0(980) \eta \pi^0$. The last element in Eq. (23) $\langle 0 | J^\eta(0) | \eta(p) \rangle$ is defined by the expression

$$\langle 0 | J^\eta(0) | \eta(p) \rangle = \frac{1}{2m_s} \left( h^\eta \cos \varphi - h^\eta_s \sin \varphi \right) \quad (25)$$

and differs from the similar matrix elements of conventional pseudoscalar mesons: here relevant comments are in order. It is known that the axial-anomaly modifies the matrix elements of the $\eta$ and $\eta'$ mesons. Indeed, for $h^\eta_s$ we have

$$h^\eta_s = m_\eta^2 f^\eta(q) - \langle 0 | \alpha_\eta G^A_{\mu \nu} G^A_{\mu \nu} | \eta(p) \rangle,$$

where $\langle 0 | \alpha_\eta G^A_{\mu \nu} G^A_{\mu \nu} | \eta(p) \rangle$ is the matrix element appeared due to the $U(1)$ axial-anomaly. The quantities...
can be expressed in terms of the parameters \(h_s, h_q\) and mixing angle \(\varphi\)

\[
h_q^s = -h_s \sin \varphi, \quad h_q^q = h_q \cos \varphi
\]

which modifies Eq. (25)

\[
\langle 0 | J^0 | \eta(p) \rangle = -\frac{H^q}{2m_s},
\]

where we introduce the short-hand notation \(H^q = (h_q \cos^2 \varphi + h_s \sin^2 \varphi)\). In calculations we employ the numerical values of \(h_q\) and \(h_s\) (in GeV³)

\[
h_q = 0.0016 \pm 0.004, \quad h_s = 0.087 \pm 0.006
\]

extracted from analysis of experimental data. The same phenomenological analyses predict \(\varphi = 39.3^\circ \pm 1^\circ\). Then the physical side of the sum rule can be recast into the form

\[
\Pi^{\text{phys}}(p) = -\frac{H^q f_{q0} m_{q0}}{2m_s} \frac{m^2}{(p^2 - m^2)^2} + \ldots,
\]

where \(m^2 = (m_{q0}^2 + m_q^2)/2\).

In the last equality we take into account that \(p = p'\) and \(q = 0\), which is required when considering a vertex composed of a tetraquark and two conventional mesons [29]. In the case of vertices containing only ordinary mesons calculation of the corresponding strong coupling can be performed in the context of the LCSR method’s full version: the limit \(q = 0\) is known there as the soft approximation. For tetraquark-meson-meson vertices the full LCSR method reduces to its soft approximation, which is only way to compute the strong couplings. Therefore, we use here technical tools elaborated in the soft approximation by bearing in mind that in our case this is only available approach to evaluate \(g_{\alpha \gamma \eta}\). In the limit \(q = 0\) the correlation function \(\Pi^{\text{phys}}(p)\) depends on a variable \(p^2\), as a result we have to fulfill the one-variable Borel transformation which yields

\[
\mathcal{B}\Pi^{\text{phys}}(p) = -\frac{H^q f_{q0} m_{q0} m^2}{2m_s} \frac{e^{-m^2/M^2}}{M^2} + \ldots
\]

We proceed by computing the QCD side of the sum rule. It is easily seen that \(J_q(x)\) does not contribute to the correlation function \(\Pi(p, q)\). Indeed, by substituting the current \(J_q(x)\) into Eq. (20) and performing contractions of the \(\bar{\tau}u\) and \(d \bar{d}\) fields from \(J_q(x)\) with relevant parts of \(J(x)\) we get apart from light \(u, d\)-quark propagators matrix elements of the local operators \(\Gamma_j\) (here, \(\Gamma^j = 1, \gamma_5, \gamma_\lambda, i\gamma_5\gamma_\lambda, \sigma_{\lambda \rho}/\sqrt{2}\) is the full set of Dirac matrices) sandwiched between the \(\pi^0\) and vacuum

\[
\langle \pi^0 | \bar{\pi}(0) \Gamma_j s(0) | 0 \rangle,
\]

which are identically equal to zero. In other words, only \(-\sin \varphi J_q(x)\) component of the \(\eta\) meson’s current contributes to the correlation function \(\Pi(p, q)\).

After some manipulations we get

\[
\Pi^{\text{OPE}}(p) = \sin \varphi \int d^4xe^{ip \cdot x} \frac{e^x}{\sqrt{2}} \left\{ \gamma_5 \bar{s}^b(x) \gamma_5 \right\}
\]

\[
\times \bar{S}^a_s(-x) \gamma_5 \alpha \beta \left( \langle \pi^0 | \bar{u}^\alpha \gamma_5 u^\beta (0) \rangle - \langle \pi^0 | \bar{d}^\alpha \gamma_5 d^\beta (0) \rangle \right),
\]

where \(\alpha\) and \(\beta\) are spinor indices.

Calculations of the correlation function in accordance with recipes described in a rather detailed form in Ref. [29] reveal that the matrix elements of the pion which contributes to \(\Pi^{\text{OPE}}(p)\) are \((0 | m \gamma_5 u | \pi^0\rangle)\) and \((0 | d \gamma_5 d | \pi^0\rangle)\) given, for example, in the form

\[
\sqrt{2}(0 | \pi i \gamma_5 u | \pi^0\rangle) = f_\pi \mu_\pi, \quad \mu_\pi = -\frac{2(q \eta)}{f_\pi^2}.
\]

In Eq. (33) \(f_\pi\) and \(\langle \pi \eta \rangle\) are the pion decay constant and the quark vacuum condensate, respectively. Then the Borel transform of \(\Pi^{\text{OPE}}(p) = \Pi^{\text{OPE}}(M^2)\) which is necessary to derive the sum rule reads

\[
\Pi^{\text{OPE}}(M^2) = \frac{f_\pi \mu_\pi}{16 \pi^2} \sin \varphi \int_0^\infty ds \frac{e^{-s/M^2}}{M^4} ds\frac{e^{-s/M^2}}{M^2}
\]

\[
- \sin \varphi \left( \frac{f_\pi \mu_\pi}{16 \pi^2} \frac{\alpha_s G_s^2}{\pi} - \frac{f_\pi \mu_\pi m_s}{6} \right) \]

Equating the Borel transforms \(\Pi^{\text{phys}}(p)\) and \(\Pi^{\text{OPE}}(M^2)\) we get the unsubtracted sum rule. But the sum rule applicable to evaluate \(g_{\alpha \gamma \eta}\) can be obtained only after subtracting the contributions of excited states and continuum. In the soft approximation an additional problem in this procedure is connected with contributions to \(\Pi^{\text{phys}}(p)\) of excited states, some of which even after Borel transformation remain unsuppressed [29], and should be removed by applying the operator \(\mathcal{P}(M^2, m^2)\) (see, Ref. [37])

\[
\mathcal{P}(M^2, m^2) = \left( 1 - \frac{d}{dM^2} \right) M^2 e^{m^2/M^2}.
\]

As a result we derive our final sum rule for the strong coupling

\[
g_{\alpha \gamma \eta} = -\frac{2m_s}{H^q f_{q0} m_{q0} M^2} \mathcal{P}(M^2, m^2) \Pi^{\text{OPE}}(M^2, s_0),
\]

where \(\Pi^{\text{OPE}}(M^2, s_0)\) is given by Eq. (34) where the upper limit of the integral \(\infty\) is replaced by \(s_0\).

The decay process \(s_0(980) \rightarrow K^+ K^-\) is investigated by the same manner. The differences here are connected with the correlation function

\[
\Pi_K(p, q) = i \int d^4xe^{ip \cdot x} \langle K^+ (q) | T \{ J_{K^-} (x) J_S^+ (0) \} | 0 \rangle,
\]

with the interpolating current \(J_{K^-}(x)\)

\[
J_{K^-}(x) = \bar{\pi}(x)i\gamma_5 s^I(x),
\]
and also the matrix element of the $K$ mesons

$$\langle 0|\pi K^0|K^-(p)\rangle = \frac{f_K m_K^2}{m_s}.$$  \hspace{1cm} (39)$$

In Eq. (39) $m_K$ and $f_K$ are the $K^\pm$ mesons' mass and decay constant, respectively. After relevant replacements the phenomenological side of sum rule is obtained from Eq. (23), whereas for $\Pi^{\text{OPE}}_K(p, q)$ we get

$$\Pi^{\text{OPE}}_K(p, q) = \int \! d^4 x e^{ipx} \frac{e^2}{\sqrt{2}} \left[ \tilde{\gamma}_5 S^{ib}_s(x) \tilde{\gamma}_5 S^{*ab}_u(-x) \gamma_5 \right]_{\alpha\beta} \times (K^+(q)|\pi^*_{0\alpha}(0)|s(0)|0). \hspace{1cm} (40)$$

The following operations are standard manipulations, therefore we write down only the final sum rule for the strong coupling $g_{aKK}$

$$g_{aKK} = \frac{m_s}{m_{a0} f_{a0} m_K f_K m^2} \mathcal{P}(M^2, \bar{m}^2) \Pi^{\text{OPE}}_K(M^2, s_0),$$  \hspace{1cm} (41)$$

where $\bar{m}^2 = (m^2_{a0} + m^2_K)/2$ and

$$\Pi^{\text{OPE}}_K(M^2, s_0) = -\frac{f_K m^2_K}{16\sqrt{2} \pi m_s} \int_{m^2}^{s_0} ds s^{-1/2} \left( \frac{\alpha_s G^2}{\pi} - \frac{f_K m^2_K}{12} \frac{\alpha_s G^2}{\pi} (2(\bar{m}u) - (\bar{s}s)) \right). \hspace{1cm} (42)$$

For the strong couplings $g_{K\pi\pi}$ and $g_{K^0\pi\pi}$ we obtain:

$$g_{K\pi\pi} = \frac{m_s}{m_K f_{KK} m^2} \mathcal{P}(M^2, m^2) \Pi^{\text{OPE}}_1(M^2, s_0),$$  \hspace{1cm} (43)$$

and

$$g_{K^0\pi\pi} = \frac{m_s}{m_{K^0} f_{K^0} m_{K^0} f_{K^0} m^2} \mathcal{P}(M^2, m^2) \Pi^{\text{OPE}}_2(M^2, s_0),$$  \hspace{1cm} (44)$$

where $m^2 = (m^2_{K^\pm} + m^2_K)/2$ and $m^2 = (m^2_{K^\pm} + m^2_K)/2$, respectively. The correlation functions in Eqs. (43) and (44) are given by the expressions

$$\Pi^{\text{OPE}}_1(M^2, s_0) = -\frac{f_\pi m_\pi}{16\pi} \int_{m^2}^{s_0} ds s^{-1/2} \left( \frac{\alpha_s G^2}{\pi} - \frac{f_\pi m_\pi}{12} (2(\bar{m}u) - (\bar{s}s)) \right), \hspace{1cm} (45)$$

and

$$\Pi^{\text{OPE}}_2(M^2, s_0) = \Pi^{\text{OPE}}_1(M^2, s_0)/\sqrt{2}.$$  \hspace{1cm} (46)$$

Sum rules obtained for the strong couplings $g_{a\pi\pi}$, $g_{aKK}$, $g_{K\pi\pi}$ and $g_{K^0\pi\pi}$ will be used to determine the partial decay widths of the mesons $a_0(980)$ and $K^*_0(800)$.

IV. NUMERICAL ANALYSIS

In numerical computations of the strong couplings for the quark and gluon condensates we utilize their values presented in Eq. (11). Apart from these parameters we also employ the masses and decay constants of the $\pi$ and $K$ mesons: for the pion $m_{\pi^\pm} = 139.57061\pm0.00024$ MeV, $m_{\pi^0} = 134.9770\pm0.0005$ MeV and $f_\pi = 131$ MeV and for the $K$ meson $m_{K^\pm} = 493.677\pm0.016$ MeV, $m_{K^0} = 497.611\pm0.013$ MeV and $f_K = 155.72$ MeV.

We have employed the different working regions for the Borel parameter $M^2$ and continuum threshold $s_0$ when considering decays of the $a_0(980)$ and $K^*_0(800)$ mesons: these windows have been chosen in accordance with standard constraints of the sum rule computations explained in the section [11].

For the strong couplings $g_{a\pi\pi}$ and $g_{aKK}$ the Borel and continuum threshold parameters are varied within the limits

$$M^2 \in [1.1 - 1.4]$ GeV^2, \ s_0 \in [1.9 - 2.1]$ GeV^2. \hspace{1cm} (47)$$

The corresponding sum rules lead to the following predictions (in units of GeV$^{-1}$)

$$g_{a\pi\pi} = 5.36 \pm 1.41,\ \ g_{aKK} = 9.10 \pm 2.76. \hspace{1cm} (48)$$

It is known that a stability of the obtained results on $M^2$ and $s_0$ is one of the important constraints imposed on sum rule computations. As an example, in Fig. 3 we plot the coupling $g_{a\pi\pi}$ as a function of $M^2$ and $s_0$. It is evident that $g_{a\pi\pi}$ depends on $M^2$ and $s_0$, which generates essential part of uncertainties in the evaluated quantities. It is also seen that these ambiguities amount to $\sim 30\%$ of the central values which is acceptable for the sum rules computations.

For the partial decay width of the processes $a_0(980) \rightarrow \eta\pi^0$ and $a_0(980) \rightarrow K^+K^-$ we get

$$\Gamma \left[ a_0(980) \rightarrow \eta\pi^0 \right] = 50.57 \pm 13.87$ MeV, \hspace{1cm} (49)$$

$$\Gamma \left[ a_0(980) \rightarrow K^+K^- \right] = 11.44 \pm 3.76$ MeV. \hspace{1cm} (50)$$

The total width of the meson $a_0(980)$ is formed mainly due to the decay channels $a_0(980) \rightarrow \eta\pi^0$ and $a_0(980) \rightarrow K^+K^-$; we assume that $P$-wave decays do not modify it considerably. Therefore it seems reasonable to compare $\Gamma_{\text{th.}} = 62.01 \pm 14.37$ MeV which is the sum of two partial decay widths with the available information on $\Gamma_{\text{exp.}} = 50 - 100$ MeV noting a full overlap of these results. As we have noted above, experimental data for the total width of the light scalar mesons suffer from large uncertainties. Therefore, we can state that our theoretical prediction does not contradict to the present-day experimental data.

The strong decays of the meson $K^*_0(800)$ can be analyzed in the same way. In the case of the $K^*_0(800)$ meson’s decays we use

$$M^2 \in [0.8 - 1.0]$ GeV^2, \ s_0 \in [1.2 - 1.5]$ GeV^2, \hspace{1cm} (51)$$

and find for the strong couplings (in GeV$^{-1}$)

$$g_{K\pi\pi} = 19.46 \pm 5.64,\ \ g_{K^0\pi\pi} = 13.47 \pm 3.91. \hspace{1cm} (52)$$

The partial decay widths are equal to

$$\Gamma \left[ K^*_0(800) \rightarrow K^+\pi^- \right] = 270.39 \pm 78.42$ MeV,

$$\Gamma \left[ K^*_0(800) \rightarrow K^0\pi^0 \right] = 130.69 \pm 37.91$ MeV. \hspace{1cm} (53)$$
Then using these two decay modes for the total width of \( K^*_0(800) \) we get \( \Gamma_{\text{th}} = 401.1 \pm 87.1 \) MeV. Experimental data borrowed from Refs. [33, 34] predicts \( \Gamma_{\text{exp.}} = 449 \pm 156 +144_{-81} \) MeV and \( \Gamma_{\text{exp.}} = 512 \pm 80 +92_{-44} \) MeV, respectively, which have rather imprecise nature. Comparing our results with these data we are able to state that they do not contradict to each other.

V. CONCLUDING NOTES

Investigation of the scalar mesons \( a_0(980) \) and \( K^*_0(800) \) by modeling them as diquark-antidiquarks carried out in the present work has allowed us to explore the suggestion about exotic nature of these resonances. Using the well-known QCD sum rule method we have calculated their masses and total widths. Our result for the mass of the \( a_0(980) \) agrees with experimental data. Its total width evaluated using two \( S \)-wave dominant strong decay channels is also in accord with the data, because our result lies entirely in the experimental region \( \Gamma_{\text{exp.}} = 50 - 100 \) MeV. The situation with experimental information on the parameters of the \( K^*_0(800) \) meson is worse than in the case of \( a_0(980) \). Thus, available data on both the mass and total width of this scalar meson in rather imprecise and suffers from large uncertainties. The predictions obtained in the present work do not contradict to last experimental measurements, nevertheless reliable conclusions can be made only on basis of a more precise experimental information. In other words, problems in interpretation of the light scalar mesons are connected with lack of precise experimental information rather than with deficiency of theoretical investigations.

VI. ACKNOWLEDGMENTS

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[1] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, 100001 (2016) and (2017).
[2] R. L. Jaffe, Phys. Rev. D 15, 267 (1977).
[3] J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982).
[4] J. D. Weinstein and N. Isgur, Phys. Rev. D 41, 2236 (1990).
[5] C. Amsler and N. A. Tornqvist, Phys. Rept. 389, 61 (2004).
[6] D. V. Bugg, Phys. Rept. 397, 257 (2004).
[7] R. L. Jaffe, Phys. Rept. 409, 1 (2005).
[8] E. Klempt and A. Zaitsev, Phys. Rept. 454, 1 (2007).
[9] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. Lett. 93, 212002 (2004).
[10] G. ’t Hooft, G. Isidori, L. Maiani, A. D. Polosa and V. Riquer, Phys. Lett. B 662, 424 (2008).
[11] H. Kim, K. S. Kim, M. K. Cheoun and M. Oka, arXiv:1711.08213 [hep-ph].
[12] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 60, 273 (2009).
[13] G. Eichmann, C. S. Fischer and W. Heupel,
[14] F. Giacosa, Phys. Rev. D 74, 014028 (2006).
[15] J. I. Latorre and P. Pascual, J. Phys. G 11, L231 (1985).
[16] S. Narison, Phys. Lett. B 175, 88 (1986).
[17] T. V. Brito, F. S. Navarra, M. Nielsen and M. E. Bracco, Phys. Lett. B 608, 69 (2005).
[18] Z. G. Wang and W. M. Yang, Eur. Phys. J. C 42, 89 (2005).
[19] H. X. Chen, A. Hosaka and S. L. Zhu, Phys. Rev. D 76, 094025 (2007).
[20] J. Sugiyama, T. Nakamura, N. Ishii, T. Nishikawa and M. Oka, Phys. Rev. D 76, 114010 (2007).
[21] T. Kojo and D. Jido, Phys. Rev. D 78, 114005 (2008).
[22] Z. G. Wang, Eur. Phys. J. C 76, 427 (2016).
[23] S. S. Agaev, K. Azizi and H. Sundu, [arXiv:1711.11553 [hep-ph]], to be published in Phys. Lett. B.
[24] S. S. Agaev, K. Azizi and H. Sundu, [arXiv:1804.01726 [hep-ph]].
[25] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979).
[26] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 448 (1979).
[27] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B 312, 509 (1989).
[28] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D 51, 6177 (1995).
[29] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 93, 074002 (2016).
[30] S. S. Agaev, K. Azizi and H. Sundu, Eur. Phys. J. C 77, 836 (2017).
[31] S. S. Agaev, K. Azizi and H. Sundu, Eur. Phys. J. C 78, 141 (2018).
[32] H. Sundu, S. S. Agaev and K. Azizi, Phys. Rev. D 97, 054001 (2018).
[33] M. Ablikim et al. [BES Collaboration], Phys. Lett. B 698, 183 (2011).
[34] M. Ablikim et al., Phys. Lett. B 693, 88 (2010).
[35] T. Feldmann, Int. J. Mod. Phys. A 15, 159 (2000).
[36] S. S. Agaev, V. M. Braun, N. Offen, F. A. Porkert and A. Schäfer, Phys. Rev. D 90, 074019 (2014).
[37] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B 232, 109 (1984).