Dynamics of Particles Trapped by Dissipative Domain Walls

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We study the interactions of the dissipative domain walls with dielectric particles. It is shown that particles can be steadily trapped by the moving domain walls. The influence of the ratchet effect on particle trapping is considered. It is demonstrated, that the ratchet effect allows to obtain high accuracy in particle manipulation.

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1. INTRODUCTION

Nonlinear localized structures have been attracting much attention in recent time because of the two reasons. The first one is fundamental interest to their rich variety in physical systems of different natures, including hydrodynamics, plasma physics, biology and nonlinear optics, see [1–4]. The second reason of high interest in nonlinear localized structures is their potential applications in many fields, including information optical processing [5, 6], optical fiber communications [7], and optical manipulation [8, 9].

One of the most interesting localized structures are switching waves, which are also often referred as “domain walls” connecting different stationary spatially homogeneous states. In general case a domain wall moves extending the area of one of the uniform states and thus after some time the expanding state fills the cavity. The direction and the velocity of the domain wall motion strongly depends on the pumping intensity. But there is a special value of pumping intensity characterized by zero velocity of the domain wall and it is called a “Maxwell point.” Near the Maxwell point, the domain walls can create different bound states, such as bright or dark solitons [10–12].

Another important effect of domain walls is reported in [13]. It is demonstrated that under biharmonic pumping the direction and the velocity of the domain wall can be controlled by changing only the mutual phase between the harmonics, it is so-called “ratchet effect.”

In this work we suggest a new strategy of optical manipulation of small particles by dissipative domain walls. This problem is closely related to the manipulation of the particles by dissipative bright solitons considered in [8, 9]. This work is devoted to the formation, stability and the dynamics of the bound states of the particles and the domain walls. Special attention is paid to the influence of the ratchet effect on the processes of particle capturing and on the possibility to use ratchet effect for nanoparticles manipulation.

2. MATHEMATICAL MODEL

We considered a nonlinear Fabry–Pérot resonator pumped by the coherent light with a dielectric particle, confined at the surface of the resonator due to, for example, electrostatic or molecular forces. Another possible realization of such a system is a hollow-core fiber (microcapillary) with the nanoparticles placed inside the fiber. Since the present work is just a theoretical proof of concept we do not discuss the technical issues but remark that this system look feasible from the point of view of experimental realization.

It is known that the nonlinear resonators driven by external pump can exhibit bistability [14] and allows for the existence of bright solitons [10]. In the pioneering work [15], it was shown that the switching waves connecting two stable spatially uniform states can exist in the driven-dissipative resonators. Later, the structures of such a kind were found in different optical systems.

In the present work, we study how the switching waves considered in [13, 16] interact with the particles attached to the resonator. A particle on the surface of the resonator is attracted in the area of higher intensity because of the gradient force [17] and in [8, 9] it is demonstrated that dissipative solitons in considered system are able to steadily capture particles and transport them in desirable direction.

The optical field of the considered resonator is described in the slow varying amplitude approach by
the Schrödinger equation with the nonlinearity of saturable type, dissipation and pumping:

\[
\frac{\partial}{\partial t} E - i C \frac{\partial^2}{\partial x^2} E + \left( \gamma + i \delta + i \frac{\alpha}{1 + |E|^2} \right) E = (1 - e^{-(x-\eta)/\omega}) P, \]

where \( C \) is diffraction coefficient, \( E \) is a complex amplitude of optical field in the resonator, \( P \) is an amplitude of laser pumping, \( \gamma \) is decay rate, \( \alpha \) is the nonlinearity coefficient; \( \delta \) is laser detuning from the resonant frequency, \( \epsilon \) is coordinate of the nanoparticle. The parameter \( \omega \) defines the width of the particle shadow located at \( x = \epsilon / f \); \( f \) relates to the transparency of a particle: if \( f = 0 \), then the particle is transparent and if \( f = 1 \), then the particle is absolutely opaque. The motion of particle under the gradient force is described by the following equation for the particles’ coordinate:

\[
\frac{\partial}{\partial t} \epsilon = \eta \frac{\partial}{\partial x} |E(\epsilon)|^2. \tag{2}
\]

In our model we use the typical assumption that the dragging force acting on the particle is proportional to the gradient of the field intensity, the coefficient \( \eta \) accounts the interaction strength. The motion of the particle is supposed to be viscous and thus it is described by first order ordinary differential equation. Let us note that for mathematical convenience we use the dimensionless variables.

The bifurcation diagram for spatially homogeneous stationary solutions under uniform pumping \( P(x) = P_0 \) is shown in Fig. 1a by black line. The spectral analysis of the linear excitations on the background of the nonlinear stationary solutions has been done and these studies show that the states belonging to the intermediate branch are unstable but the upper and the lower states are stable at the chosen parameters. A domain walls considered below are connections between the lower and the upper stable spatially uniform states. The formation of the domain walls is the result of the complex interplay of the nonlinear effect, the dispersion, the driving force (the holding beam pumping the system), and the losses. More information on the formation and the stability of these domain walls can be found in [16].

As it is mentioned before, in a general case the domain wall has non-zero propagation velocity. The velocity and the direction of the domain wall propagation can be controlled by the pumping intensity, see Fig. 1a, where the dependency of the domain wall velocity on the pump is shown by thin blue line (right axis). It should be noted, that there is a special value of pump intensity, when the domain wall is at rest. This specific pump is often referred as Maxwell point.

We performed numerical simulations with the parameters insuring the existence of the domain walls under homogeneous pumping \( P(x) = P_0 \). It was shown by direct numerical simulations that the domain walls can form from the weak initial noise if during the initial stage of the domain wall formation the pump intensity depends on coordinate, for example as it is shown in Fig. 1b. Such type of pumping (first panel in Fig. 1b) should provide the formation of the uniform state from the upper bifurcation branch in one part of the system and the state from the lowest bifurcation branch in the another part of the system. When the field intensity distribution becomes station-
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In this section we study how the presence of particles affects the dynamics of domain walls discussed in the previous section. We focus on the dynamics of the domain walls with particle under uniform time-independent pumping. Since the uniform states connected by the domain walls are not equivalent in the terms of intensities, the particle location relative to the wall is important. Let us begin with a case when the particle is placed in the area of lower field intensity. In this case several scenarios are possible.

If the decrease in the pump intensity caused by the shadow of the particle is sufficient, then the front changes the direction of propagation after collision with the particle. It can be explained by the following. The unperturbed pumping intensity is high enough and thus the system is switching from the lower spatially homogeneous stable state to the upper stable state by the motion of the domain wall connecting these states. The particle is placed far away from the domain wall and does not affect it. However, when the domain wall reaches the particle, the shadow created by the particle makes the pumping insufficient to maintain the upper state near the particle. In turn, the gradient force pulls the particle to the region of the stronger field intensity and the particle approaches the domain wall. Thus, the domain wall begins to slowly move in the opposite direction after the particle capture, as it can be seen in Fig. 2a.

It should be noted, that in case of the more opaque particle the domain wall can be almost stopped by the particle, see Fig. 2b. It happens because the pumping field near the particle is very weak and the domain wall is not able to get close enough to the particle to pull it by the gradient force.

In case if the losses introduced by the particle in pumping intensity are not so strong, then the particle is pulled into the region of the strong field behind the front. After that the front again starts to move with initial speed and the particle stays at rest, see Fig. 2c.

Now let us consider the case if the particle is placed in the area of higher intensity. In this case depending on the particle parameters also several scenarios are possible.

If pump intensity decreasing is not significant then the capture of the particle by the moving dissipative domain wall is possible. After the capture of the particle the velocity of the front increases because the particle’s shadow leads to the decrease in the pump intensity. If the force dragging the particle is strong enough to maintain the velocity of particle equal to the increased velocity of the front, then steadily trapping of a particle is possible, see Fig. 3a.

However if the drop of the pump intensity caused by the presence of the particle is large enough then the velocity of the domain wall increases beyond a threshold value and, as a result, the particle departs from the front. After that the domain wall restores its initial speed without the particle, see Fig. 3b. In case of even more opaque particle placed in the region of the higher intensity the shadow created by the particle is able to switch the field from the upper state to the low-
est in the vicinity of the particle. Thus, two more domain walls are created and the particle stays at rest in the region of the lower field intensity, see Fig. 3c.

At the end of this section let us make an important remark that in this work we present the results for the parameters providing that the field of the domain wall decays to the spatially uniform states without oscillations. This is not a general case, it is known that in the considered systems the decaying field can exhibit pronounced oscillations [10, 18, 19]. One can expect that these oscillations will affect the dynamics of the system significantly if the period of the oscillations is larger or comparable with the size of the particle. The study of this more complex case is out of the scope of the present paper and will be done elsewhere.

4. INTERACTION OF DOMAIN WALLS WITH PARTICLES UNDER THE ACTION OF THE BIHARMONIC SIGNAL

In this section we consider the influence of the biharmonic signal on the dynamics of the domain walls with particles interaction. The time-dependent spatially uniform pumping has the following form:

\[ P = P_0 + a_1 \sin(\Omega t) + a_2 \sin(2\Omega t + \theta), \]  

where \( P_0 \) is time independent component of the signal, \( \Omega \) is frequency of the first harmonic, and \( \theta \) is mutual phase difference between two harmonics.

Under the action of a biharmonic pumping signal, it is possible to control the velocity of domain wall not only by changing the amplitude of the pump but also by changing the mutual phase of the harmonics, see [13]. This effect is especially important in the vicinity of the Maxwell point when the domain wall is at rest. If time-independent part of pump is close to the Maxwell point, then by changing the mutual phase \( \theta \) it is possible to change not only velocity of the domain wall, but also its direction of propagation. In case if \( \theta = 0 \) the domain wall propagates in the direction of extension of the area of higher field intensity, and in case if \( \theta = \pi/2 \) the domain wall moves in the opposite direction, see Fig. 4.

Let us consider a case when the domain wall moves in the direction of increasing area of the upper spatially homogeneous state under the influence of the ratchet effect. When the domain wall reaches the particle and the particle is opaque enough then the wall changes its direction of propagation and the particle starts to move slowly with the domain wall, see Fig. 4a. The effect is similar to the one shown in Figs. 2a and 2b.

In case if the particle is transparent enough the domain wall pulls the particle in the region of the higher intensity and restores the initial average velocity, see Fig. 4b, what is similar to Fig. 2c.

Now let us consider an opposite case, when the domain wall moves in the direction of extension of the area of lower intensity because of the ratchet effect. From Fig. 4c, it can be seen that after the domain wall reaches the particle the domain wall captures it and they move together, because the particle is transparent enough. Due to the viscous motion, the particle trajectory has very small oscillating component and it can be said that the particle is moving with the velocity equal to the average velocity of the domain wall, see Fig. 4c. The capture is similar to the one from Fig. 3a. If the transparency of the particle gets lower than the capturing regime is analogous to the case illustrated in Fig. 3b, in other words if the particle is quite opaque, then the particle breaks away from the wall, see Fig. 4d.

Let us remark that we have studied the stability of the bound state of a particle and a domain wall moving with a constant velocity. It was found that the eigenval-
ues describing the relaxation of the particle to its equilibrium point is pure real and this means that the system does not have internal modes that can be resonantly excited because of the periodical variation of the velocity or the profile of the domain wall. In other words, no resonant excitation of the particle oscillations is possible in the most realistic case when the motion of the particle is viscous. It should be mentioned that if the inertia of the particle becomes of importance then the dynamics can become more complicated exhibiting different resonances. However, these effects are out of the scope of the present work.

Let us emphasize the possible importance of the ratchet effect for the manipulation of the nanoparticles by the nonlinear domain wall. The velocity caused by the ratchet effect can be made very low and the particle shifts by a known distance over each of the period of the pump intensity oscillation. This way the shift of the particle can be controlled with high accuracy sim-

Fig. 4. (Color online) Interaction of the particle with the domain wall driven in motion by the ratchet effect. Parameters: \( P = P_1 + a_1 \sin(\Omega t) + a_2 \sin(2\Omega t + \theta) \), where \( a_1 = a_2 = 0.1 \) and \( \Omega = 0.05 \), other parameters are the same as in Fig. 2. (a) Initially, the domain wall propagates in the direction of increase in the higher state area because \( \theta = 0 \), but after collision with a particle \( (f = 0.002) \) it changes the direction of propagation. (b) Particle is pulled in the area of higher intensity, parameters are the same as in (a), but \( f = 0.0005 \). (c) Domain wall propagates in the direction of increase in the lower state area because \( \theta = \frac{\pi}{2} \) and after collision with a particle \( (f = 0.005) \) increases the speed of propagation. (d) Particle breaks away from the wall and stays in the area of lower intensity, parameters are the same as in (a), but \( f = 0.02 \).
ply by counting the number of the pump intensity oscillations.

5. CONCLUSIONS

Now let us briefly summarize the main results of the paper. The dynamics of a particle under the action of an optical nonlinear wave in a pumped dissipative resonator with saturable nonlinearity is considered. The considered nonlinear waves are the domain walls connecting two different dynamically stable spatially uniform states of the system. The dependence of the domain walls velocity on the pump is found numerically.

The interaction of the moving domain wall with the particle under the spatially uniform stationary pumping field is considered. It is shown that different scenarios of the mutual dynamics of the particle and the dissipative front are possible depending on the particle's transparency and its position in respect to the domain wall. It is demonstrated that the stable capture of the particle by the domain wall can be realized.

The influence of the ratchet effect on the domain wall with the particle interaction is considered. The possibility to use ratchet effect to set the particle into the controllable motion is shown. It is worth mention that the ratchet effect can help to achieve high manipulation accuracy.

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