Modelling the trajectory of a fracture that moves under the influence of the fluid pressure in hard rock roofs of in-seam working

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Abstract. Presents the results of the solution to the problem of creation of a trajectory of movement in the roof rocks of the reservoir production artificially created cracks under the action of internal fluid pressure. The array is loaded equivalent gravitational field stresses and he is in condition of plane deformation. The emergence of cracks and admission thereto of fluid are not considered. Breeds of soil and roof of layer strong enough to deform elastically and upon reaching the limit state of collapse as the fragile materials. The problem is solved in the framework of model of geomechanical state of rock massif containing local zones in the ultimate state. The model is based on the main provisions of the solid mechanics, it is implemented by means of boundary element method and classical views of the state of the crack, its stable and unstable growth in an infinite plane of a brittle material based on the theories of Griffiths – Irwin.

1. Introduction
The issue of manifestation of different gas-geodynamic phenomena in deep depth while developing coal deposits appears to be of vital importance. Firstly, it is connected with the appearance of limit stress zones of a seam formed in selvages and in-seam workings adjacent to sidewalls and faces. The formation of such zones brings about significant incoming of methane into the workings and very often it is accompanied by sudden outbursts of fragmentary mined-rocks into workings [1–3]. Due to this, an important significant scientific and production task is in accurate predicting estimations of geo-mechanic state of an in-seam coal massif [4–6].

Depending on strength characteristics of a coal seam and adjoining country rocks their transition into limit state while being developed happens in different ways. In one case, limit stress of a roof country rock takes place in a small-sized goaf, in other cases it covers significant areas and, as a rule, causes the rock falls accompanied by dynamic phenomena [7–9].

To reduce the size of a poor caving of the roof the most effective way is to apply a method of directional hydraulic fracturing [9]. The idea of the method is in induced roof caving by means of artificial initiation of a fracture and its further development in rocks under the influence of the fluid injected into it under high pressure.

One of the first fundamental works that scientifically proved the effectiveness of hydraulic fracturing method is theoretical paper written by Khristianovitch S.A. and Zheltov Yu.P. On the bases of methods of deformable body mechanics the authors solved the issue of hydraulic fracture propagation in oil-bearing seams for the purpose of increasing the oil-well rate [10].
The basic goal for effective applying of the hydraulic fracturing method in colliery and ore undertakings is in defining optimal parameters for the initial (primordial) fracture with the purpose to obtain maximum effect for caving the roof by a growing fracture. The essential factor that influences on the trajectory of the fracture is in strata stress. Under real condition the fracture propagates near the working which changes it and this fact should be taken into the account.

By now the influence of a coal working on the trajectory of the fracture development has not been taken into account for solving similar issues [9].

2. Problem description and its solution

The results of solving the issue of fracture propagation under intrinsic pressure in a roof of in-seam working are introduced in the paper. In the process of solving the issue, the stress field in a coal massif is defined in the framework of a developed model for geo-mechanic state of a massif with in-seam working [4, 5].

The task is set as follows (figure 1). In coal rock massif under the condition of plane deformation and modeled weightless plane there is a working 1 of a rectangular cross-section with the size $b \times h$, driven at the depth $H$ along a coal seam 2 with all its thickness. The strength characteristic of a seam is less than the one of a country rocks massif but more than the one where the seam contacts with an enclosing massif loaded by gravitational pressure on the top and on the bottom $\gamma H$ ($\gamma$ – unit specific gravity of overlying strata), and at the walls $\lambda \gamma H$ ($\lambda$ – coefficient of a lateral pressure). In a seam selvage, zones of inelastic deformation 3 with the width $L$ are formed. In a working roof there is a narrow cut 4 that imitates the fracture with the length $2l$, loaded by pressure $p$ on the section with the length of $2l_0$ (index 0 refers to the initial length of the crack).

![Figure 1. Analytical model of the problem about the fracture state in a working roof.](image-url)
The cut is small compared to the output dimensions, and the coordinates of its middle in the y 0 z coordinate system, coinciding with the Central axes of the output, are indicated by \( y_t, z_t \) (index \( t \) denotes the cut). Cut inclined to the horizon at an angle \( \alpha \).

It is well-known that universal equation of thermodynamics is applied for theoretical analysis of the strength and large displacement discontinuity in solid deformable body problem. This equation can be applied to the body with a fracture, to its adjoined states: before and after the fracture propagation on a certain value \( \Delta l \). In quasi-static process this equation looks like [11 – 13]:

\[
k_I^2 + k_{II}^2 = \frac{E \cdot P}{1 - \mu^2},
\]

where \( k_I \) – stress intensity factor due to the action of the first type of load \( p_I \) (normal wedging) acting on the banks of the crack, \( k_{II} \) – stress intensity factor from the action of the second type of shear load \( p_{II} \) acting on the banks of the crack, \( E \) – elastic modulus (Young’s elasticity modulus), \( \mu \) – Poisson’s ratio of country rock, \( P \) – density of surface energy that is necessary for the formation of the surface unit. Indices \( I, II \) – the type of load acting on the crack.

Stress intensity coefficients for the fractures are expressed by the following relationships [11 – 13]

\[
k_I = \frac{1}{\sqrt{\pi l_0}} \int_{-l_0}^{l_0} P_I \sqrt{\frac{l_0 + \xi}{l_0 - \xi}} d\xi, \quad k_{II} = \frac{1}{\sqrt{\pi l_0}} \int_{-l_0}^{l_0} P_{II} \sqrt{\frac{l_0 + \xi}{l_0 - \xi}} d\xi,
\]

where \( \xi \) - the x coordinate of the point of the crack starting from its middle.

If the fracture is situated in a uniform stress field then \( p_I, p_{II} \) do not depend on variable \( \xi \) and coefficients \( k_I \) and \( k_{II} \) after being integrated according to Eq.(2) are determined as follows:

\[
k_I = p_I \sqrt{\pi l_0}, \quad k_{II} = p_{II} \sqrt{\pi l_0},
\]

in connection with hydrofracture angled at \( \alpha \) towards the horizon in tight rocks the equations (3) are as follows:

\[
k_I = (p - p_I) \sqrt{\pi l_0}, \quad k_{II} = p_{II} \sqrt{\pi l_0}.
\]

In equations (4) \( p_I, p_{II} \) are determined by the stresses at the “distant” edges and in this case, they are connected with stresses \( \gamma H \) and \( \lambda \gamma H \) as

\[
p_I = \frac{\gamma H}{2} \left[ 1 + \lambda + (1 - \lambda) \cdot \cos 2\alpha \right], \quad p_{II} = \frac{\gamma H}{2} \left[ (1 - \lambda) \cdot \sin 2\alpha \right].
\]

Substituting equations (4) into equation (1) allows us to determine the fluid pressure, called critical pressure, and it corresponds to the moment of fracture initiation (starting of the growth). We call it the first critical pressure.

\[
p_1 = p_I + \frac{E \cdot P}{\sqrt{\pi l_0 (1 - \mu^2)}} - p_{II},
\]
where index 1 denotes the number of the first critical pressure.

To determine the fluid pressure under which the fracture grows on a certain (given) value, it is important to take into account the fact that the fracture increases its length with the velocity closely equal to sound propagation in a given medium while its filling with fluid during the growth is significantly slow. Thus, while the fluid consumes a part of the fracture with the length \(2l_0\), the length of the fracture itself is equal to \(2l_0 = 2(\Delta l + l_0)\).

In this connection, to calculate the coefficients of stress intensity in Eq. (2) it is necessary to consider the above-mentioned condition and to set the integration limits correctly

\[
\begin{align*}
    k_I &= \frac{1}{\sqrt{\pi}} \int_{-l_0}^{l_0} p_I \sqrt{\frac{l + \varepsilon}{l - \varepsilon}} d\xi - \frac{1}{\sqrt{\pi}} \int_{-l}^{l} p_I \sqrt{\frac{l + \varepsilon}{l - \varepsilon}} d\xi, \\
    k_{II} &= \frac{1}{\sqrt{\pi}} \int_{-l}^{l} p_{II} \sqrt{\frac{l + \varepsilon}{l - \varepsilon}} d\xi. 
\end{align*}
\]

(7)

After integrating the relationship \(k_I, k_{II}\) appear to be the following:

\[
    k_I = \frac{2p_2 \sqrt{l}}{\sqrt{\pi}} \arctg \frac{l_0}{\sqrt{l^2 - l_0^2}} - p_I \sqrt{l/\pi}, \quad k_{II} = p_{II} \sqrt{l/\pi}.
\]

After substituting \(k_I\) and \(k_{II}\) into the equation (1) the value of the intrinsic pressure in a fracture corresponding to its propagation on a certain value \(\Delta l\) takes the following form:

\[
p_2 = \frac{\pi}{2 \arctg \frac{l_0}{\sqrt{l^2 - l_0^2}}} \left( p_I + \sqrt{\frac{EP}{\pi(1 - \mu^2)}} - p_{II}^2 \right),
\]

(8)

The value \(p_2\), determined by the relationship (8) is called the second critical value and it corresponds to the sustainable growth of the fracture. Index 2 denotes the number of the second critical pressure.

In Griffith’s theory which is referred to purely elastic materials, the characteristics \(P\) has the same value in all directions for all types of fracture deformation and its deviation from the initial direction is expected if \(k_{II} \neq 0\). Thus, the direction of the fracture propagation takes place under the angle \(\theta = \theta_c\) with respect to its initial position where the left part of the equation (1) has a maximal value out of all possible values under the given external load.

It is obvious to assume that the direction coincides with the area of maximal stress. It is known that there is no shear stress in this area and the normal stress is a principal stress and the area is called the main area.

According to this supposition the direction of the fracture development is angle \(\theta_c\) with the initial (preceding) direction of the fracture. This angle is determined in the process of solving the following trigonometric equation with respect to the angle \(\theta\) measured from the direction of the fracture at the moment of its initiation [13]

\[
k_I \left( \sin \frac{\theta}{2} + \sin \frac{3\theta}{2} \right) + k_{II} \left( \cos \frac{\theta}{2} + 3 \cos \frac{3\theta}{2} \right) = 0.
\]

(9)

To solve the equation (9) it is necessary to know values \(k_I\) and \(k_{II}\).
There are several methods for determining the coefficients of stress intensity at the ends of the fracture which is positioned in the area with “distant” edges and nonuniform stress field [13].

If the fracture is small comparing to the area dimensions and is positioned inside it then the equation (2) can be taken as one of the options for calculating the coefficient of stress intensity. Function \( p_I \) in it is equal to the stress which could be perpendicular to the fracture plane if it were shut; therefore the stress for the body without fracture is calculated analytically or numerically. Type \( II \) can be studied similarly.

In this case the stresses at the fracture edges together with its initial position can be determined in the framework of geo-mechanic model of the massif with in-seam working [5]. After that, normal and shear stresses acting at the edges of the fracture are determined according to the formula of stress component transformation under the rotation of coordinate system in the following way:

\[
p_I = \frac{\sigma_z + \sigma_y}{2} + \left(\frac{\sigma_z - \sigma_y}{2}\right)\cos2\theta + \tau_{yz}\sin2\theta, \quad p_{II} = \left(\frac{\sigma_z - \sigma_y}{2}\right)\sin2\theta + \tau_{yz}\cos2\theta. \tag{10}
\]

In equations (10) \( \sigma_z, \sigma_y, \tau_{yz} \) are the stress components determined by the solving of a boundary problem about the state of a massif with in-seam working (\( x, y, z \) – indexes represent coordinate axes).

Thus, solving the problem it is necessary to carry out a computational procedure that consists of derived series of cycles. The fracture trajectory is a polygonal line of straight-line portions where in the limits of each the stresses are constant.

The mechanism of the fracture development is the following. The fracture is in unstable balance when the injected fluid reaches the pressure \( p_I \) determined by the Eq. (6). For its further propagation to \( \Delta l \) it is necessary to increase the pressure till \( p_II \) value which is calculated by the equation (8) and then the edge in a jump-like mode shifts into a nearly point. A new direction of the fracture together with the horizon forms an angle \( \alpha + \theta \). In a jump-like mode for the length of the fracture development the fluid disseminates along the increased volume under reduced pressure. Thus, in its new position both critical pressures of the fracture are also reduced. For the following stage of the fracture propagation onto a given value (distance) to the first critical pressure which corresponds to a new position of a fracture, additional fluid pressure is required. The process of fracture propagation continues till its full propagation with the number of jumps \( n \).

3. Computational experiment and analysis of the obtained results

The results of computational experiment are introduced below. The following parameters of the massif are taken as the initial data: \( \gamma=25 \text{ kN/m}^3, H=800 \text{ m}, \lambda=0.7, b=5 \text{ m}, h=3 \text{ m}, f=1, P=0.0087 \text{ MPa m}, l_0=0.1 \text{ m}, \Delta l=0.1 \text{ m}, \text{ final half-length } l_e=3.1 \text{ m}. \) Other data were being changed during the experiment.

According to the results of solving the elastoplastic problem on a seam marginal zone state it appeared that a maximum value of a bearing pressure zone is \( 1.657\gamma H \) and the size of a limit stress zone \( L \) is equal 4.17 m.

In figure 2 the trajectories of the fracture movement for some initial positions relative to the workings are built. Sections \( a, c \) with number 1 – edge of a working, 2 – limit zone edges, 3 – initial fracture, 4 – fracture trajectory are marked.

In figure 2 \( a, b \) the calculations with the following parameters of a fracture are done: \( y_c=1.5 \text{ m}, z_c=5 \text{ m}, \alpha=30^\circ \). Figure 2 \( a \) demonstrates that the trajectory of the fracture is a wavy line.

Figure 2 \( c, d \), demonstrates the obtained results \( y_c=0.5 \text{ m}, z_c=5 \text{ m}, \alpha=80^\circ \). As it can be seen from figure 2 \( c \) the fracture propagates linearly coinciding with the initial (primordial) fracture direction.

The critical pressure graphs corresponding to the growing fracture are built in figure 2 \( b, d \). Curve 1 corresponds to \( p_I \) and curve 2 is \( p_{II} \).
According to the obtained results of the research the orientation of the “primordial” fracture for its positions at the level $z_t=5$ m when the trajectory is close to the straight line are introduced.

The dependence graph of initial fracture angle of bend that corresponds to linear trajectory of its growth on initial abscissa is built in figure 3a. The graph represents gently sloping curve without extreme points.

The dependence graphs of the first critical pressure (curve 1) and the second critical pressure on the angle of bend of the initial fracture are built in figure 3b. The graph abscissa corresponds to graphs ordinates in figure 3a. As it is seen from the figure the graphs are practically equidistant and each curve is bi-curved having their lows at $\alpha_t \approx 45^\circ$ and the critical pressure aside of the working is higher than the one positioned directly above it.

The analysis of the results of the computational experiment and the graphs introduced in the figures show that the fracture trajectory has a wavy form if angle $\alpha_t$ is of arbitrary value. However, there are some values for the angle $\alpha_t$ where the fracture propagates along its initial direction only linearly.
4. Conclusions
1. The model of geo-mechanic state of a coal massif with in-seam working provides the calculation of a fracture trajectory that grows in hard rock roof under the influence of intrinsic pressure under the given medium characteristics and initial parameters of the fracture.
2. In roof rocks of an in-seam working the fracture propagates generally in curvilinear way and its trajectory depends on the parameters of the initial fracture.
3. There are some positions of the initial fracture respective to in-seam working where it develops strictly in linear way.
4. In the working surrounding there are points and directions where the linear growth of initial fracture takes place under the minimal critical pressure.

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