Optimal Belief Revision

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Abstract

We propose a new approach to belief revision that provides a way to change knowledge bases with a minimum of effort. We call this way of revising belief states optimal belief revision. Our revision method gives special attention to the fact that most belief revision processes are directed to a specific informational objective. This approach to belief change is founded on notions such as optimal context and accessibility. For the sentential model of belief states we provide both a formal description of contexts as sub-theories determined by three parameters and a method to construct contexts. Next, we introduce an accessibility ordering for belief sets, which we then use for selecting the best (optimal) contexts with respect to the processing effort involved in the revision. Then, for finitely axiomatizable knowledge bases, we characterize a finite accessibility ranking from which the accessibility ordering for the entire base is generated and show how to determine the ranking of an arbitrary sentence in the language. Finally, we define the adjustment of the accessibility ranking of a revised base of a belief set.

Introduction

In this paper we put forward a new approach to changes in knowledge bases. Two main reasons led us to this approach to belief revision. One of these reasons originates in the intuition that in a belief revision process only part of the knowledge base has an active role in the integration of a new belief in the base. Therefore a belief revision process involving only that sub-system of beliefs can capture the essence of this process and reach the same effects as the traditional way of belief changing with a diminished processing effort. The other idea comes naturally when the belief revision process is placed in a communication environment. People are changing their minds when interacting with the world, including the others’ beliefs, and usually when they are looking for answers to their questions. Not every fact or evidence is worth incorporating in our belief systems, but only information that is relevant for us with respect to our informational needs. Consider that two intelligent agents are involved in a goal-driven dialogue. One of them, called the inquirer, is “interested” in acquiring a piece of information that would produce, by means of a belief revision process, the fulfillment of its objective or desideratum. The other agent’s role is to present facts that would help the inquirer attain its goal.

Given an incoming piece of information, one’s beliefs, and the desideratum, our purpose is to process the available information efficiently in order to fulfill the desideratum. We use terms like optimal and efficiency throughout this paper with the meaning of a comparison or ratio between the effects and effort corresponding to a belief revision process. However, these terms are not related to computational complexity. Instead, the effects represent information derived when incorporating a piece of evidence in the knowledge base; the effort required to reach such effects is mostly characterized by the notion of accessibility. A formal definition of the accessibility ordering will be provided later in this paper.

We call this way of revising belief systems optimal belief revision. Defining a way of revising a knowledge base efficiently reduces to choosing optimal contexts. The notion of context is related to and determined by three parameters: the new belief, one’s knowledge base and the desideratum, and it represents any part of the knowledge base that actually contributes to the integration of the new belief. The optimal context is a context in which the new piece of information will be the most efficiently processed such that the desideratum is met.

Both constructive and descriptive approaches to contexts are considered. In the third section we stipulate the conditions that circumscribe the set of context candidates, from which the optimal context is chosen. In the fourth section we give an explicit construction for (optimal) contexts. In the fifth section we introduce the concept of accessibility. Finally, for finitely axiomatizable knowledge bases (theories\(^2\)), we define a finite accessibility ranking from which the accessibility order-

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\(^1\)This approach was suggested by Sperber and Wilson’s relevance theory. Refer to Sperber and Wilson (1986).

\(^2\)By theory we mean any set of sentences closed under the consequence operation \(Cn\) provided by the classical propositional calculus.
ing for the entire base is generated. Also, we will see how from a finite accessibility ranking one can adjust the ranking of a repeatedly revised belief set.

Related Work

We mention three important sources related to our work. Firstly, the axiomatic and constructive approaches for contraction functions presented in Gärdenfors (1988) serve as models for characterizing (optimal) contexts from the same perspectives. Moreover, the definition and analysis of epistemic entrenchment in relation to contraction functions guided us in defining the accessibility notion and relate it to modeling optimal contexts. Secondly, relevance theory introduced in Sperber and Wilson (1986) motivated us to apply relevance theory to belief revision by placing agents in a communicative environment. Thirdly, in Williams (1998), Mary-Anne Williams uses a finite partial entrenchment ranking to generate an entrenchment ordering and describes a computational model that adjusts this ranking using an absolute measure of minimal change. Later sections of our paper focus on the same matters using the accessibility relation instead.

Contexts: Descriptive Approach

Consider a language \( L \) whose logic includes classical propositional logic. Beliefs and contexts are represented by closed sets of sentences (belief sets).

Let \( K \subseteq L \) be the initial belief set of the inquirer. Suppose that the inquirer’s objective/desideratum is based on a presupposition \( P \), which is a sentence in \( L \), precisely a disjunction of possible answers to a question: \( P = G_1 \lor G_2 \lor \ldots \lor G_n \).\(^3\) We consider that the presupposition is one of the beliefs in \( K \), \( P \in K \).

Each \( G_i \in L \) is called a basic goal. A disjunction of at most \( n-1 \) such basic goals is considered also a goal. Any goal achieves the desideratum, but a basic goal is preferred rather than a goal.

To attain its desideratum, the inquirer must derive one of these goals from a belief revision process. Let \( A \in L \) be the new piece of information acquired by the inquirer, which is used in the belief revision process.

In this setting, contexts are considered subsets of \( K \) that are also closed under logical consequence. Choosing a context whenever some new evidence is incorporated in \( K \) depends upon three parameters: the belief set \( K \), the new evidence \( A \) and a certain goal \( G \) or desideratum. They will appear as constants throughout our analysis. Contexts are considered optimal if for some \( K \), \( A \) and \( G \), the effort required for processing \( A \) in that context such that \( G \) is derived is as small as possible. Both the notion of accessibility and the number of sentences in a context have to be considered in assessing a certain processing effort.

The next theorem\(^4\) provides a description of (potentially optimal) contexts given the above-cited parameters. The first part (\( i \)) of Theorem 1 characterizes contexts when no desideratum or goal is assumed. The second part adds new conditions for contexts when this parameter is considered as well. It is supposed that the revision, expansion and contraction operators respect the AGM postulates.

**Theorem 1** Let \( K \) be a belief set and \( \ast \) a revision operator. Let us denote by \( \ast \) the contraction operator that is related to \( \ast \) through the Levi Identity.\(^5\)

\( i \) For any belief set \( K' \subseteq K \), if

\[
K' \setminus K'_{\neg A} = K \setminus K_{\neg A}, \text{ then (1)}
\]

\[
K_A = Cn[K'^*_A \cup (K \setminus K')]. \quad (2)
\]

\( ii \) If there is a sentence \( G \in L \), such that

\( a \) \( G \notin K \); \( b \) \( G \in K'_A \)

then there is a belief set \( K' \subseteq K \) satisfying condition (1) and:

- \( K'_{\neg A} \neq \emptyset \) if \( A \nvdash G \);
- \( G \in K'^*_A \);
- \( \neg G \in K \) then \( \neg G \in K' \).

Since in the limiting case \( K' \) can be identified with \( K \), the first part of Theorem 1 says that for any belief set \( K \) we can always identify a sub-theory \( K' \) of \( K \), whose revision with a new belief \( A \) tells us exactly what sentences are retracted from \( K \) when revising \( K \) with \( A \). \( K' \) is chosen such that exactly the same beliefs are retracted from both \( K \) and \( K' \) when revising these belief sets with the same sentence \( A \). This means that no sentences that could contribute to the derivation of \( A \) are left in the remainder \( K \setminus K' \). Moreover, attaching the revised “context” to this remainder does not lead to the negation of \( A \), unless \( \neg A \) is a tautology. The relationship between a belief set \( K \) and one of its contexts associated to \( A \) is presented in the following corollary:

**Corollary 2** For a revision operator \( \ast \) and two belief sets \( K \) and \( K' \) such that \( K' \subseteq K \) and \( K' \setminus K'_{\neg A} = K \setminus K_{\neg A} \), the following conditions hold:

1. \( K'_{\neg A} \subseteq K_{\neg A} \); (Monotony)
2. \( K'^*_A \subseteq K'^*_A \);
3. \( K'_{\neg A} \cup (K \setminus K') \not\vdash \neg A \) if \( \vdash \neg A \).

In order to understand the significance of part \( ii \) of Theorem 1, consider the following situation: An agent makes inquiries in order to attain a certain information-goal \( G \) that would achieve its desideratum. In order to choose the right context, one should focus on two aspects:

\( \text{The proofs of all the theorems can be found in Vodislav (2000).} \)

\( \text{\ldots is, for any belief set } K, K' = (\neg A)_{\neg A}. \text{ See Gärdenfors (1988) p.69.} \)

\( \text{\ldots where the relationship between } \ast \text{ and } \ast \text{ is given through the Levi Identity.} \)

\(^3\) The notions of desideratum and presupposition originate in Hintikka’s approach to interrogatives; see Hintikka (1989), p.158.
I) the context should include those sentences that help to establish which beliefs are being retracted from the initial belief set in order to accommodate the new evidence;

II) the context should include the beliefs that together with the new one contribute to the derivation of the goal.

If one is interested in choosing contexts that satisfy the first requirement then the solution is to be found in Theorem 1 part i). Precisely, a minimal belief set $\mathcal{K}'_m$ is required. Note that for a given $\mathcal{K}$, the set of all $\mathcal{K}'$ satisfying the conditions (1) and (2) specified by the theorem is partially ordered. Each totally ordered maximal subset of this class has a minimal element, therefore the class of $\mathcal{K}'$ has at least one minimal element $\mathcal{K}'_m$.

In order to meet requirement II) the context obeying condition I) must satisfy additional restrictions, as specified in part ii) of Theorem 1. Condition ii) stipulates the following: Supposing that the goal can only be contextually derived and can be implied by neither the new evidence $\mathcal{A}$ nor the initial knowledge base $\mathcal{K}$, we want to choose such a context $\mathcal{K}'$ that does not become empty after retracting $\neg \mathcal{A}$ from it. This condition makes sense even when $\neg \mathcal{A} \notin \mathcal{K}$ since it requires that $\mathcal{K}'$ is nonempty as long as the goal is contextually derived from $\mathcal{A}$. The second part of ii) says that the goal will be obtained from the processing of the new evidence $\mathcal{A}$ in the context $\mathcal{K}'$. Also if the agent’s original base includes the negation of the goal, in order to derive the goal from the revised knowledge base, $\neg \mathcal{G}$ must be contained in the context $\mathcal{K}'$. Note that when processing a piece of information $\mathcal{A}$ in a context $\mathcal{K}'$, if that belief is already part of the agent’s old knowledge base then there is no goal $\mathcal{G}$ satisfying the conditions specified in ii). In this case, attaining goal $\mathcal{G}$ is already achieved without incorporating $\mathcal{A}$ in $\mathcal{K}$. The revision of $\mathcal{K}'$ with $\mathcal{A}$ could only result in a change in the entrenchment degree of $\mathcal{A}$, provided that an epistemic entrenchment ordering for $\mathcal{K}$ (and for $\mathcal{K}'$) is assumed.

Belief sets that satisfy conditions (1) and ii) form a partially ordered set having at least one $\subseteq$-minimal element. By choosing a minimal $\mathcal{K}'$, we ensure the minimality requirement of an optimal context. The minimality requirement is present also in the constructive approach to optimal contexts.

Theorem 1 can be reformulated for bases of belief sets. The changes consist of replacing belief set $\mathcal{K}$ with the base $\mathcal{B}_K$ and $\mathcal{K}'$ with a subset $\mathcal{B}'_K$ of $\mathcal{B}_K$, $\mathcal{A} \in \mathcal{K}$ with $\mathcal{B}_K \vdash \mathcal{A}$, etc.

Contexts: Constructive Approach

There are several ways to construct contexts: from entailment-sets of $\neg \mathcal{A}$ and $\mathcal{G}$, from maximal subsets of $\mathcal{K}$ that fail to entail $\neg \mathcal{A}$, using Grove’s system of spheres\(^7\) and from epistemic entrenchment and accessibility orderings. For lack of space, we will present only the first method here. The others can be found in Vodislav (2000).

An $A$-entailment set is a set of sentences each of which is essential for the derivation of the sentence $\mathcal{A}$.

**Definition 1** The set of sentences $X$ is an $A$-entailment set in a belief set $\mathcal{K}$ if and only if: a) $X \subseteq \mathcal{K}$; b) $\mathcal{A} \in \mathcal{Cn}(\mathcal{X})$; c) for any proper subset $\mathcal{Y}$ of $\mathcal{X}$, $\mathcal{A} \notin \mathcal{Cn}(\mathcal{Y})$.\(^8\)

Using the notion of “entailment set of a sentence in a belief set” we can define the context $\mathcal{K}'$ as the logical closure of the union of two subsets, one responsible for the derivation of $\neg \mathcal{A}$ and the other for the entailment of $\mathcal{G}$:

$$\mathcal{K}' = \mathcal{Cn}[\mathcal{K}'_A \cup \mathcal{K}'_G]$$

where

- $\mathcal{K}'_A = \bigcup (\mathcal{A}$-entailment set in $\mathcal{K})$
- $\mathcal{K}'_G = (\mathcal{A})X_G$ such that $X_G \subseteq \mathcal{K}$ and $X_G \cup \{\mathcal{A}\}$ is a $\mathcal{G}$-entailment set in $\mathcal{K}'_A$.

When constructing $\mathcal{K}'_G$, a construction for $\mathcal{K}'_A$ is assumed. For instance, $\mathcal{K}'_A$ can be obtained from $\mathcal{K}$ by means of the Levi Identity and partial meet contraction functions.

The union $[\mathcal{K}'_A \cup \mathcal{K}'_G]$ is not a belief set since entailment-sets for a sentence in a belief set are not closed sets. However the closure of this union (i.e. $\mathcal{K}'$) is both a belief set and a subset of $\mathcal{K}$ since it is the closure of a subset of $\mathcal{K}$.

**Theorem 3** Any subset $\mathcal{K}' \subseteq \mathcal{K}$ that is generated from entailment-sets of $\neg \mathcal{A}$ and $\mathcal{G}$ as stipulated by relation (3) satisfies Theorem 1.

Accessibility Relation

Theorem 1 describes contexts in a general manner without providing a mechanism for defining a specific (optimal) context. In this section we introduce the notion of accessibility as an additional structure for belief sets that allows us to model optimal contexts.

This relation is defined over sentences in $\mathcal{L}$, as well as over contexts/sets of sentences. The notion of accessibility is based on the idea that depending on the order in which our beliefs were acquired, we can assign them a certain degree of accessibility or we can rank them according to the rule: the last acquired belief is the most accessible one. We will use the notation $\mathcal{A} \leq _\alpha \mathcal{B}$ for “$\mathcal{B}$ is at least as accessible as $\mathcal{A}$”.

The importance of the accessibility notion in the construction of optimal contexts is supported by the ability of this concept to characterize the effort required to access pieces of information from someone’s knowledge base. Further, using the accessibility notion one can express the effort required to process some piece of information in a certain context.

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\(^7\)&nbsp;See Grove (1988).

\(^8\)&nbsp;With some changes, the definition is borrowed from Fuhrmann (1991), p.177.
A set of sentences or a context will be considered as accessible as its least accessible element. The reason is that whenever one wants to access a certain context or to process some piece of information in that context, the effort required for such an operation is determined by the effort necessary to access the least accessible sentence in that context.

**Definition 2** For any set of sentences $S$ in $L$, if $A \in S$ and $A \leq_a B$ for all $B \in S$ then $S =_a A$.\(^{9}\)

Instead of defining the accessibility of sentences in a quantitative way, we rather axiomatically define the qualitative aspect of this notion by introducing five new postulates.

**Definition 3** Given a belief set $K$ in $L$, an accessibility order related to $K$ is a relation $\leq_a$ on $L$ satisfying the following conditions:

A1) If $A \leq_a B$ and $B \leq_a C$ then $A \leq_a C$; (transitivity)

A2) For every two sentences $A, B \in L$, either $A \leq_a B$ or $B \leq_a A$;

A3) $A =_a \neg A$;

A4) $A, \neg A \notin K$ if and only if $A \leq_a B$ for all $B \in L$;

A5) Given $B_K$ such that $Cn(B_K) = K$, for any set of sentences $X_A \subseteq B_K$ such that $X_A$ is the most accessible $A$-entailment set in $B_K$ and $A \notin B_K$, the accessibility rank of $A$ is determined by the accessibility rank of $X_A$: $A =_a X_A$.

The first two postulates define $\leq$ as a total transitive binary relation over sentences in $L$. Conditions (A3) and (A4) are better understood if we consider that for a belief set $K$, the accessibility rank of a sentence $A \in L$ is determined by the lowest accessibility level to which we need to go in $K$ in order to determine whether or not $A$ is the case (given $K$).\(^{10}\) If $A \in K$, for instance, then the rank of $A$ is already known. If $\neg A \in K$ then $A$ has the same accessibility level as $\neg A$ since finding out that $\neg A$ is the case leads to finding out that $A$ is not the case. This explains postulate (A3). Also, if neither $A$ nor $\neg A$ are in $K$ then it is impossible to find out given $K$ whether $A$ (and similarly, $\neg A$) is the case or not. Therefore, for this last case, $A$ would be among those sentences whose level of accessibility is the lowest. This gives us a reason for formulating the fourth postulate.

Note that condition (A5) does not hold in case $A$ is a sentence in $B_K$. Since sentences in $B_K$ are independently acquired, they are ranked according only to the order in which they have been incorporated in the base, regardless of their logical relationship. In contrast, the accessibility ranking of the information that is derived from the base will be ranked only according to the ranks of the sentences from which it is obtained by derivation.

Formally, we can describe the relation between the accessibility ordering and optimal contexts as follows.

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\(^{9}\) $A =_a B$ if $A \leq_a B$ and $B \leq_a A$.

\(^{10}\) By definition, we say that sentence $A$ is the case given $K$ if and only if $A \in K$; $A$ is not the case given $K$ if and only if $A \notin K$. $\neg A \in K$, and $A$ is undetermined given $K$ if and only if $A, \neg A \notin K$.

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**Definition 4** Consider some belief set $K$, a sentence $A \in L$, and two sub-theories of $K$: $K_1', K_2'$, both satisfying conditions (1) and (ii) as specified in Theorem 1. Then $K_1'$ is preferred over $K_2'$ as a potentially optimal context if and only if $K_1'$ is more accessible than $K_2'$.

The postulates suggest the following conclusion and motivate the next two definitions.

The number of accessibility levels in a base of a belief set is the same as the number of accessibility levels in the belief set generated from that base. Moreover, the accessibility ordering of the base determines the accessibility ordering of the belief set generated from the base. This property results from the last postulate, (A5).

**Definition 5** For a belief set $K$, an accessibility ordering $\leq_a$ and a sentence $A \in K$, we define $cut_{\leq_a}(A, K) = \{ B | B \in K, A \leq_a B \}$.

**Definition 6** For a belief set $K$, a base $B_K$ of $K$, an accessibility ordering $\leq_a$ and a sentence $A \in K$, we say that $cut_{\leq_a}(A, K)$ is a bad cut if there is a sentence $C \in B_K$ such that $C \in Cn[cut_{\leq_a}(A, K)]$ but $C \notin cut_{\leq_a}(A, K)$.

Note then the following result:

**Theorem 4** For an accessibility ordering $\leq_a$, a belief set $K$ and sentence $A \in K$, if $cut_{\leq_a}(A, K)$ is not a bad cut then $cut_{\leq_a}(A, K)$ is a belief set.

This theorem provides a solution for the construction of optimal contexts. Precisely, this result allows us to conclude that the subset $K'$ which contains $K$’s most accessible sentences and satisfies conditions (1) and (ii) as stipulated in Theorem 1 is or includes an optimal context. However, if $K'$ represents a bad cut, an optimal context is a subset of the logical closure of $K'$.

**Generating an Accessibility Ordering from a Finite Accessibility Ranking**

In this section we define a finite accessibility ranking from which we will then generate a finitely representable accessibility ordering. The reason for this action is to overcome the representation problem whenever an accessibility ordering ranks an infinite number of sentences. A finite accessibility ranking ranks the sentences of a finite knowledge base. We will see that for axiomatizable theories/belief sets we can generate an ordering that ranks all the sentences of that theory/belief set from that finite ranking.

**Definition 7** A finite accessibility ranking is an integer function $af : S \rightarrow [0, n]$ such that range(af) = $[1, n]$; where $S$ is a finite set of sentences in $L$ and natural numbers 0 to n designate ranks.

The next definition introduces a way of extending the domain of the finite ranking $af$ to all the sentences in the language. The degree of accessibility of a sentence in the language represents the effort required to access someone’s beliefs used to find out whether or not that sentence is accepted by that person.
Definition 8 Let \( \alpha \) be a finite accessibility ranking. The degree of accessibility of a sentence \( p \in L \) is given as follows:

a) \( \text{deg}_\alpha(p) = n \) if \( \neg p \);

b) \( \text{deg}_\alpha(p) = \max_i(\min_q(\alpha(q) | q \in X_i^p)) \) if \( p \in Cn[\text{dom}(\alpha)] \) and \( \neg p \); where for each \( i \), \( X_i^p \) is a \( p \)-entailment set in \( Cn[\text{dom}(\alpha)] \)

c) If \( p \notin Cn[\text{dom}(\alpha)] \), then \( \text{deg}_\alpha(p) \) is determined inductively from the degree of accessibility of sentences in \( p \in Cn[\text{dom}(\alpha)] \) using the following rules:

1) \( \text{deg}_\alpha(p) = \text{deg}_\alpha(\neg p) \);

2) \( \text{deg}_\alpha(p) = 0 \) iff \( p \notin Cn[\text{dom}(\alpha)] \).

The following theorem shows how a finite accessibility ranking can generate an accessibility ordering using degrees of accessibility.

Theorem 5 Let \( \alpha \) be a finite accessibility ranking and \( p, q \) sentences in \( L \). Define \( \leq_\alpha \) by \( p \leq_\alpha q \) iff \( \text{deg}_\alpha(p) \leq \text{deg}_\alpha(q) \). Then \( \leq_\alpha \) is an accessibility ordering related to \( Cn[\text{dom}(\alpha)] \).

Note that for \( \text{dom}(\alpha) = B_K \), where \( B_K \) is a finite set of sentences such that \( Cn(B_K) = K \), \( \leq_\alpha \) is the accessibility ordering related to \( K \). Since \( B_K \) is finite, \( K \) is a finitely axiomatizable theory. In this case the finite accessibility ranking \( \alpha \) generates an accessibility ranking \( \leq_\alpha \) over \( K \).

Iterated Optimal Revision

In the introduction it was mentioned that the inquirer’s desideratum can be fulfilled by several goals. Therefore, iterative revisions allow the agent to get closer with each step to the fulfillment of its desideratum. For each iteration a certain goal is attained. Each such goal can either override the previously derived goal(s) or represent a sub-goal of a final goal that is achieved after a number of such iterations.

The adjustment that can be brought to the accessibility ranking of the beliefs in a base of a belief set can be described as follows.

Definition 9 Let \( B_K \) be a finite set of sentences in \( L \), \( p \in B_K \) and \( \alpha \) a finite accessibility ranking assigned to \( B_K \). We define the adjustment of \( \alpha \), when \( B_K \) is revised with \( A \in L \), to be a function \( \alpha^* \) such that:

\[
\begin{align*}
\alpha^*(p) = \\
&= \begin{cases} \\
&n + 1 \quad \text{if } p = A \text{ or } p = \neg A \\
&\alpha(p) \quad \text{if } p \notin B_K \setminus A \\
&0 \quad \text{otherwise}
\end{cases}
\end{align*}
\]

This means that the newly acquired evidence is placed at the most accessible level. All the beliefs in the original base are assigned their initial level of accessibility. The sentences that are retracted from \( B_K \) when \( A \) is incorporated are assigned the accessibility ranking/degree as stipulated in part c) of Definition 8.

Conclusions

We have defined a way to change knowledge bases such that the same effects as those achieved in a conventional belief revision process are obtained, only with a smaller processing effort. This approach to belief revision is founded on notions such as optimal context and accessibility. For the sentential model of belief states we provide both a formal description of contexts as sub-theories determined by three parameters and a method to construct contexts. Next, we introduce an accessibility ordering for belief sets, which we then use for selecting the best (optimal) contexts with respect to the processing effort involved in the revision. Then, for finitely axiomatizable knowledge bases, we characterize a finite accessibility ranking from which the accessibility ordering for the entire base is generated and show how to determine the ranking of an arbitrary sentence in the language. Finally, we define the adjustment of the accessibility ranking of a revised base of a belief set.

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