Nonisothermal filling of a planar channel with a power-law fluid

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Abstract. In this paper, the fountain flow of a power-law fluid during the filling of a planar vertical channel is investigated taking into account a dissipative heating. The rheological properties of the medium are defined by Ostwald de Waele power-law with an exponential temperature dependence of effective viscosity. The problem is solved numerically by means of a finite-difference method based on the SIMPLE algorithm and method of invariants for compliance with the natural boundary conditions on a free surface. The parametrical investigations of the shape of free surface as a function of the basic dimensionless criteria and nonlinearity degree of the fluid are implemented. The effect of viscous dissipation on the formation of the kinematic characteristics of the flow is shown.

1. Introduction
In processing of polymeric materials, the injection molding based on the mold filling with a liquid medium is a widespread technology. The processed composition is characterized by a complex rheological behavior and it generally flows under nonisothermal conditions. Forecasting of the quantity of products requires the prior investigation of the technological process that can be implemented using the physical and mathematical simulation methods. The mathematical simulation method is preferable as it is more efficient and informative in comparison with the physical simulation, which is used for verification of the theoretical results on the last research stage. The process of filling is characterized by both the presence of free surface, nonisothermality connected with dissipative heating, chemical conversion, heat-exchange boundary conditions, and temperature dependence of the rheological parameters of the medium. The results of a research on nonisothermal flow of the media with a complex rheology in the channels without considering the free surface are introduced in details in [1, 2]. In the articles [3–7], there is a description of computational technologies and the results of investigation of the filling process taking into account the free surface.

The aim of this work is to study the influence of the viscous dissipation on both the shape of free surface and the kinematic of a fountain flow during the filling of a planar channel with a power-law fluid.

2. Formulation of the problem
Nonisothermal filling of the planar vertical channel with the power-law fluid in a gravity field is considered. The direction of the fluid flow is opposite to those of gravity force. The flow area is shown
in figure 1. Mathematical basis of the flow includes the equations of motion, heat-transfer, and
continuity written in dimensionless variables as follows:

\[ \text{Re} \frac{d\mathbf{V}}{dt} = -\nabla p + \nabla \cdot (2 \mathbf{B} \mathbf{E}) + \mathbf{W}, \quad \text{Pe} \frac{d\theta}{dt} = \Delta \theta + \text{Br} \text{A} \theta ^2, \quad \nabla \cdot \mathbf{V} = 0. \]

Here, \( \mathbf{V} \) is the dimensionless velocity vector; \( p \) is the dimensionless pressure; \( t \) is the dimensionless
time; \( A \) is the dimensionless intensity of the strain-rate tensor \( \mathbf{E} \); \( \mathbf{W} = \{-W, 0\} \) is the dimensionless
vector; \( \theta = (T - T_0) / T_0 \) is the dimensionless temperature; \( T \) and \( T_0 \) are the dimensional temperature of the
fluid in the flow and on a solid wall, respectively. The system of equations is completed by a
rheological power-law, which includes the effective viscosity defined by the following equation [8]:

\[ B = \exp \left[ -C \theta \right] \text{A}^{m-1}. \]

Figure 1. Calculation domain

The dimensionless similarity criteria are as follows:

\[ \text{Re} = \frac{\rho L^m U_0^{2-m}}{\mu_0}, \quad \text{Pe} = \frac{\rho c U_0 L}{\lambda}, \quad \text{W} = \frac{\rho L^{m+1} g}{\mu_0 U_0^m}, \quad \text{Br} = \frac{\mu_0 U_0^{m+1} L^{1-m}}{\lambda T_0}, \quad \text{C} = k T_0. \]

Here, \( \rho \) is the fluid density; \( L \) is the half-width of the channel; \( U_0 \) is the average velocity at the inlet
section; \( m, \mu_0 \) are the power-law rheological parameters; \( k \) is the parameter of nonisothermality; \( c \) is the heat capacity; \( \lambda \) is the heat conductivity coefficient; \( g \) is the gravity acceleration.

At the inlet section \( \Gamma_2 \), the velocity and temperature profiles corresponding to a fully developed
one-dimensional nonisothermal fluid flow with a specified constant flow rate in an infinite channel are
assigned. On the solid walls \( \Gamma_3 \), the no-slip conditions are imposed and the dimensionless temperature
is set to zero. On the free surface \( \Gamma_1 \), the boundary conditions are defined by both zero shear stress, the
equality of normal stress to the external pressure, which can be assumed to be zero without loss of
generality, and the absence of the heat flux. At the initial time moment, the free surface is plane and it
is located in a distance from the inlet boundary, and the temperature of fluid is equal to those of the
wall in the region of initial filling. Moreover, the free boundary moves in compliance with the
kinematic condition. The surface tension forces are not considered. The problem formulation is
dimensionless. The values of dimensionless criteria are assigned so that the stationary solution for
nonisothermal flow in the planar infinite channel could exist [8].

Consequently, the boundary conditions are written as

\[ \Gamma_1 : \frac{\partial u_s}{\partial s} + \frac{\partial u_n}{\partial n} = 0, \quad p = 2B \frac{\partial u_n}{\partial n}, \quad \frac{\partial \theta}{\partial n} = 0; \]

\[ \Gamma_2 : u = 0, \quad v = f_1(x), \quad \theta = f_2(x); \]
\[ \Gamma_3: u = 0, \quad v = 0, \quad \theta = 0; \]
\[ \Gamma_4: u = 0, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial \theta}{\partial x} = 0. \]

The conditions (1) are written in a local Cartesian coordinate system \((n, s)\), which is normally related to the free surface. The free boundary \(\Gamma_1\) moves in compliance with the kinematic condition written in the Lagrangian representation as follows:

\[ \frac{dx}{dr} = u, \quad \frac{dy}{dr} = v. \quad (2) \]

The analysis of the classical mathematical model of fluid dynamics including the motion equations, natural boundary conditions on a free surface, no-slip conditions on a moving contact line, and the values of dynamic contact angle, which is not equal to 0 and \(\pi\), indicates the singularities in determination of dynamic characteristics of the flow. These singularities lead to an infinite increase in the values of dynamic characteristics as a three-phase contact line is approached \([9, 10]\). Therefore, the problem formulation is performed assuming that there is no shear stress (1) and the normal velocity is equal to zero at the contact point C. In the small vicinity of the contact line, the tangential velocity on the wall decreases asymptotically from the value at the point C to zero. The flow regime and the interaction behavior of the phases on the contact line are supposed to provide the edge angle equal to \(\pi\) with insignificant capillary effects in comparison with the inertial, viscous, and gravitational forces in the flow. Using such a boundary conditions eliminates the particularities of traditional formulation of the problem and at the same time has a little effect on the kinematic characteristics of the flow out of a small vicinity of contact point \([11]\).

3. Method of solution

The stated problem is solved using a finite-difference method. A staggered difference grid covers the solution domain. The SIMPLE algorithm \([12]\) is applied to calculate the velocity, temperature, and pressure fields at the internal nods. Near the free boundary, the irregular nodes appear. In these nodes, the values of unknown variables are defined using linear interpolation of data from the free surface and regular nodes. The free boundary is represented as a set of particles-markers distributed uniformly along this boundary: the first marker is located on the symmetry line and the last is on the three-phase contact line. The velocity components are calculated using the method of invariants \([13]\) based on the combination of assumptions of no shear stresses and continuity equations for each point marked on the free surface at the given time moment. The particles-markers of the free surface, except for the last one, move in accordance with the discrete analogs of the kinematic condition (2). The motion of the contact point is implemented as provided by a slip condition and dynamical edge angle, which is equal to \(\pi\).

The calculation method was tested on the problem of power-law fluid flow in a planar channel at the specified flow rate with consideration for dissipative heating and exponential temperature dependence of the viscosity at \(Re=0.1, \quad Pr=100, \quad Br=1, \quad C=1, \quad W=0\). The parabolic profile of longitudinal velocity and zero temperature were set at the inlet of a channel, and the soft boundary conditions were specified at the outlet. On the solid walls, the no-slip boundary condition was realized and the temperature was set to zero. The length of the channel was selected to be sufficient for a steady state flow developed at the outlet.

For verification of the computational technique, the results of calculations were compared with the numerical solution of equivalent one-dimensional problem of nonisothermal power-law fluid flow in an infinite channel at the specified flow rate. As a result of testing the approximation convergence, the grid step for the following calculations was set to \(1/40\), which yielded the relative errors 0.08% and 0.03% for the calculations of velocity and temperature on the symmetry line of the channel, respectively \([14]\).
4. Results and discussion

The results of nonisothermal filling of the channel at low Reynolds number and high Peclet number that are typical for the flow of polymeric fluids during the processing by a casting method are considered. The free boundary of the studied power-law fluid flow is characterized by the parameter \( \chi = \Delta y/L \), which indicates the position of the point B on the symmetry line relative to the point C on three-phase contact line (figure 1). The effect of nonlinearity degree \( m \) on the parameter \( \chi \) at various \( Br, C, \) and \( W \) is shown in figure 2. In the case of dilatant fluid \( (m>1) \) at \( W=0 \) (figure 2a), the parameter \( \chi \) decreases insignificantly with an increase in \( m \). In the case of pseudoplastic fluid \( (m<1) \), the convexity of the free surface increases as the \( m \) decreases. In figure 2a, b the solid lines correspond to the isothermal filling of the planar channel with the power-law fluid [15]. As the \( W \) increases the behavior of functional connection \( \chi(m) \) changes.

![Figure 2](image)

**Figure 2.** Parameter \( \chi \) as a function of nonlinearity degree \( m \) at \( Re=0.1, Pe=100 \): (a) \( C=1 \); (b) \( Br=2 \)

In considered flow, the effect of parameter \( Br \) on parameter \( \chi \) is shown in figure 3. At the upper half of the figure, the curves 1, 2 represent the functional connection \( \chi(\text{Br}) \) at \( W=0 \), and at the lower half, at \( W=5 \) (curves 3, 4). In the case of \( W=0 \), the value of convexity property \( \chi \) of the free surface for pseudoplastic fluid is higher than for dilatant fluid, but when \( W=5 \), the \( \chi \) is higher for dilatant fluid in comparison with that for pseudoplastic fluid.

![Figure 3](image)

**Figure 3.** Parameter \( \chi \) as a function of \( \text{Br} \) at \( Re=0.1, Pe=100 \): curves 1, 4 \(- m=0.7 \); curves 2, 3 \(- m=1.3 \)
The fields of transverse and longitudinal velocity, and effective viscosity for both isothermal and nonisothermal filling of the planar channel with the fluid are performed in figure 4. The behavior of velocity distribution confirms the flow separation into a two-dimensional fountain flow zone in the vicinity of a free boundary and a one-dimensional flow located far away from the free boundary [6]. Heating of the fluid due to the viscous dissipation yields the viscosity decrease over the flow region (figure 4e, f). As viscosity decreases, the intensity of the fluid spreading towards the solid walls in the vicinity of free boundary increases. It in turn leads to reduction of the distance between three-phase contact point and top of the flow on the symmetry line of the channel. Dissipative heating provides the increase in dimensionless maximum of transverse and longitudinal velocity from 0.28 to 0.48 and from 1.4 to 1.6, respectively (figure 4a, b, c, d). The length of two-dimensional flow zone becomes longer than that in the case of isothermal filling of the channel. The effect of a viscous dissipation and rheology on the temperature distribution was demonstrated in [14].

![Figure 4. Distributions of transverse velocity, longitudinal velocity, and effective viscosity during the channel filling with the power-law fluid at Re=0.1, Pe=100, W=5, m=0.7: a, c, e – isothermal filling [15]; b, d, f – nonisothermal filling at Br=2, C=1](image)

5. Conclusions
As a result of investigation implemented, the effect of the Brinkman number Br, the W criterion, the parameters of the rheological model C and m on the kinematics of the power-law fluid flow in the planar infinite channel has been shown. Increase in the value of W from 0 to 5 leads to the changes in behavior of the functional dependence between the characteristic \( \chi \) of a free boundary and nonlinearity
degree $m$ and the level of intensity of mechanical energy dissipation in the flow have been performed. The comparison of the results with that in the case of isothermal filling of the planar channel with non-Newtonian fluid demonstrates the increase in the length of a two-dimensional flow zone as a consequence of the decrease in the viscosity due to the fluid heating.

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