The $\sigma \rightarrow \gamma\gamma$ Width from the Nucleon Electromagnetic Polarizabilities

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Abstract

The lightest QCD resonance, the $\sigma$, has been recently fixed in the $\pi\pi$ scattering amplitude. The nature of this state remains nowadays one of the most intriguing and difficult issues in particle physics. Its coupling to photons is crucial to discriminate its structure. We propose a new method that fixes this coupling using just available precise experimental data on the proton electromagnetic polarizabilities together with analyticity and unitarity. Taking into account the uncertainties in the analysis and in the parameter values, our result is $\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma) = (1.2 \pm 0.4)$ KeV.
The scalar-isoscalar $\sigma$ is there and is found! The history of the $\sigma$, with its ups and downs, has been most intriguing. We now know, thanks [1] to a precise analytic continuation into the complex energy plane of the $I = J = 0 \pi\pi$ partial wave scattering amplitude, that a pole does exist, signaling a state in the spectrum of hadrons. On the first sheet of the energy plane, the $S$-matrix has a zero at $E = [(441 \pm 16) - i (272 \pm 9)]$ MeV, which reflects a pole on the second sheet at the same position. Not only the $\sigma$ is confirmed as a state in the hadron spectrum but the position of its pole is located very precisely with errors of only tens of MeV. The relevance of these results has to be emphasized in view of the special rôle played by the scalars in the QCD dynamics and its vacuum with the condensates. These results have been confirmed in [2] with the position of the pole at $E = [(484 \pm 17) - i (255 \pm 10)]$ MeV. This is a zero at $E^*$ in the inverse of the $\pi\pi$ partial wave $S$-matrix $S = 1 + 2i \beta(t) T(t)$ on the first Riemann sheet. We have

$$T(t) = \frac{1}{\beta(t) \cot(\delta(t)) + i\beta(t)} \tag{1}$$

where $\delta(t)$ is the scalar-isoscalar $\pi\pi$ phase-shift, $\beta(t) = \sqrt{1 - t/4m^2}$ and $t = E^2$.

Although the pole-dominance of the $\sigma$ to the scalar-isoscalar contribution is apparent in a wide energy region around its position, the corresponding phenomenological implication is somewhat masked by the effects of its large width. For a narrow resonance, there is an observable connection between the phase dependence of the corresponding physical amplitude on the real axis and that in the complex plane, as one passes the pole. This connection is, however, lost in the case of the $\sigma$ with such a large width: one does not observe a rapid variation of the amplitude phase [3] or a Breit-Wigner type behavior around the resonance position neither. This enormous difference in the behavior of the amplitude as one moves away from the real axis is what has made the $\sigma$ existence and location so uncertain for so long.

Yet the important question about the nature of the $\sigma$ remains unanswered [4–10]. What is its rôle in the chiral dynamics of QCD? Is it a $q\bar{q}$ state? Is it a $\pi-\pi$ molecule? Is it a $(q\bar{q})-(qq)$ tetraquark? Is it a glueball state? How is it possible to distinguish these different substructures? Two photon interactions can shed some light on this question from the size of the value of the $\sigma \rightarrow \gamma\gamma$ width [11]. With this in mind, two recent calculations [12, 13] of the $\gamma\gamma \rightarrow (\pi\pi)_{I=0,2}$ amplitudes in terms of twice-subtracted dispersion relations, in order to weigh the low energy region in the dispersive integrand, have been made. Their result takes into account the now well known $\pi\pi$ final state interactions which contain the $\sigma$ pole in the scalar-isoscalar contribution. They obtain $(4.09 \pm 0.29)$ KeV [12] and $(1.8 \pm 0.4)$ KeV [13] for the width of the $\sigma$ into two-photons. Although the approach and methodology [14] are very similar in these two calculations, there is an important apparent discrepancy. Its origin is discussed in [13]. The different input used for the dispersive calculation of the production amplitudes of $\gamma\gamma \rightarrow \pi\pi$
and the use of different values for the position of the \( \sigma \) pole on the second Riemann sheet \( t_\sigma \) and its coupling to two pions \( g_{\sigma \pi \pi} \) are equally responsible. Notice that though these last two inputs are not required in the dispersive calculation, the \( \sigma \rightarrow \gamma \gamma \) width obtained in [12, 13] depends critically on them [13].

The experimental results on the \( \gamma \gamma \rightarrow \pi \pi \) process are scarce and, in order to extract information on the \( \sigma \), unfortunately contaminated theoretically by the Born term in the charged pion channel and by the isospin \( I = 2 \) amplitude in all cases, interfering with the \( I = 0 \) amplitude in the cross section [3]. The purpose of this paper is to point out that the coupling \( g_{\sigma \gamma \gamma} \) of the \( \sigma \) meson found in the \( \pi \pi \) scattering amplitude [1, 2] is a measurable quantity, directly obtainable from the nucleon electromagnetic polarizabilities, and that it can be extracted with good precision from existing experimental values. This differs from the analysis in [15] where the properties of the \( \sigma \) meson of a Nambu–Jona-Lasinio model are used. The argument proceeds as follows. Besides the mass, charge and magnetic moment, the two structure constants, the electric \( \alpha \) and magnetic \( \beta \) polarizabilities, determine the Compton scattering amplitude [16, 17] up to second order in the energy of the photon and the differential cross section up to third order. They have been shown to determine the longest range interaction between two objects, one of them being neutral [18] and the van der Waals force [19]. The knowledge of these constants, of undoubted interest for hadron physics, is also relevant for certain implications in nuclear and astrophysical studies [20].

The available experiments of Compton scattering on protons and neutrons at low energies can be analyzed [21, 22] in terms of \( \alpha \) and \( \beta \), with the sum \( \alpha + \beta \) constrained by the sum rule obtained from the forward dispersion relation [23]. The result is \( \alpha^{\exp} = 12.0 \pm 0.6 \), \( \beta^{\exp} = 1.9 \mp 0.5 \) for protons and \( \alpha^{\exp} = 11.6 \pm 1.5 \), \( \beta^{\exp} = 3.7 \mp 2.0 \) for neutrons. All polarizabilities here and in the rest of the paper are in \( 10^{-4} \) fm\(^3 \) units.

A separate theoretical determination of \( \alpha \) and \( \beta \) needs more ingredients that the ones present in the forward sum rule. The authors of [24] investigated this problem by the use of a backward dispersion relation for the physical spin averaged amplitude. The corresponding sum rule for \( \alpha - \beta \) contains contributions from an s-channel part and a t-channel part. The first is related to the multipole content of the total photo-absorption cross section, whereas the t-channel part is related to a dispersion relation for \( t \) with the imaginary part of the amplitude, as demonstrated in [25], given by the processes \( \gamma \gamma \rightarrow \pi \pi \) and \( \pi \pi \rightarrow N \bar{N} \) via a unitarity relation. The result is the BEFT sum rule [24, 25] for \( \alpha - \beta \),

\[
\begin{align*}
\alpha - \beta &= \\
&= \frac{1}{2\pi^2} \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^2} \sqrt{1 + 2\frac{\nu}{M_p}} \left[ \sigma(\Delta \pi = \text{yes}) - \sigma(\Delta \pi = \text{no}) \right] \\
&\quad + \frac{1}{\pi^2} \int_{4m^2_\pi}^{\infty} \frac{dt}{4M^2_p - t} \left| f_0^0(t) \right| \left| F_0^0(t) \right|
\end{align*}
\]
\[
\frac{(4M_p^2 - t) (t - 4m_n^2)}{16} \left| f_2^*(t) \right| \left| F_0^*(t) \right|
\]

where \( M_p \) is the proton mass, the partial wave helicity amplitudes \( f_0^+(t) \) and \( f_2^+(t) \) for \( NN \rightarrow \pi\pi \) are Frazer and Fulco's [26] and the partial wave helicity amplitudes \( F_0^0(t) \) and \( F_0^2(t) \) for \( \gamma\gamma \rightarrow \pi\pi \) are defined as in [27]. The absorptive part in the s-channel contribution is obtained from that of the forward physical amplitude by changing the sign of the non parity flip multipoles \( (\Delta \pi = \text{no}) \). A reliable evaluation of this s-channel integrand [22] gives \((\alpha - \beta)^s = -5.0 \pm 1.0\) for protons and neutrons. The importance of the t-channel contribution was already emphasized in [25] and the connection of \((\alpha - \beta)^t\) to the isoscalar s-wave \( \gamma\gamma \rightarrow \pi\pi \) amplitude pointed out. The “experimental” \((\alpha - \beta)^t\) is thus \(15.1 \pm 1.3\) for protons and \(12.9 \pm 2.7\) for neutrons, compatible with the isoscalar selection imposed by the t-channel sum rule. It is furthermore remarkable that the products of helicity amplitudes which appear in Eq. (2) are the products of their moduli, which might take negative values if the phase of the corresponding amplitude differs from the \( \pi\pi \) phase shift in an odd number of \( \pi \)'s. From the factor outside the bracket in the integrand of Eq. (2) one sees the strong dominance of the low and intermediate regions. The d-wave contribution is much smaller than the s wave one, so that we take it fixed by the Born term in the crossed channel [30], leading to \((\alpha - \beta)_2^t = -1.7\). As a consequence, the “experimental” quantity to be compared with the integral term containing \( F_0^0(t) \) in Eq. (2) is \((\alpha - \beta)_0^t = (16.8 \pm 1.3)\). The input Gourdin-Martin’s \(|F_0^0(t)|\) amplitude in that integral is what we want to fix from this “experimental” value. The Frazer-Fulco’s \(|f_2^+(t)|\) amplitude is, on the contrary, well known [29].

On the physical sheet, we write

\[
F_0^0(t) = A(t) T(t)
\]

[3, 12, 28] where \( T(t) \) is the scalar-isoscalar \( \pi\pi \) scattering amplitude of Eq. (1). In this way we ensure that unitarity and analyticity are explicit and that the \( \sigma \) pole position and the coupling to two pions are exact inputs to construct \( F_0^0(t) \). Here we shall use a simple analytic expression for \( T(t) \), compatible with Roy’s equations, which takes a three parameter fit from [2] including both low energy kaon data and high energy data. This fit is valid up to values of \( t \) of the order of 1 GeV\(^2\), which is enough in the integrand of the polarizability sum rule in Eq. (2).

The factor \( A(t) \) is real for physical values of \( t \). At the \( \sigma \) pole position on the first Riemann sheet [3, 12, 13]

\[
A(t_\sigma) = e^2 \sqrt{6} \frac{g_{\sigma\gamma\gamma}}{g_{\sigma\pi\pi}},
\]

where \( e \) is the electron electromagnetic charge, \( g_{\sigma\pi\pi}^2 \) is the residue of the \( \pi\pi \) scattering amplitude at the \( \sigma \) pole on the second Riemann sheet and \( g_{\sigma\gamma\gamma} g_{\sigma\pi\pi} \) is
proportional to the residue of the $\gamma\gamma \rightarrow \pi\pi$ scalar-isoscalar scattering amplitude on the second Riemann sheet. The proportionality factors are such that $g_{\sigma\pi\pi}$ and $g_{\sigma\gamma\gamma}$ agree with the ones used in [3,12]. The pole width is [3,12,28]

$$\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma) = \frac{\alpha^2|\beta(t_{\sigma}) g_{\sigma\gamma\gamma}|}{4M_{\sigma}}$$

that agrees, modulo normalizations, with the one given in [13]. This is not the observable radiative width that would be associated with a possible Breit-Wigner resonance in the physical $\gamma\gamma \rightarrow (\pi\pi)_{I=0}$ amplitude. However, in order to discuss the structure of the $\sigma$, one has to move around the pole and $\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma)$ is the appropriate one.

Due to Low’s low energy theorem [16], the amplitude $F_0^0(t)$ is given by the Born term at low energies. With this behavior in mind, we consider the Born contribution to the crossed channel describing the Compton scattering on the pion $\gamma\pi \rightarrow \gamma\pi$. Compatible with unitarity and analyticity, this Born contribution leads [14,30], through (3), to a dressed with $\pi\pi$ final state interactions Born amplitude $F_0^0(t)|_B$ in the annihilation channel $\gamma\gamma \rightarrow \pi\pi$ which includes thus the $\sigma$. Its expression is given here as

$$F_0^0(t)|_B = e^2 \left[ \cot(\delta(t)) \frac{4m^2_\pi}{t} \log \left( \frac{1 + \beta(t)}{1 - \beta(t)} \right) - \frac{2}{\pi} \left( 1 + \frac{m^2_\pi}{t} \left[ \log^2 \left( \frac{1 + \beta(t)}{1 - \beta(t)} \right) - \pi^2 \right] \right) \right] T(t)$$

The corresponding evaluation of the sum rule in Eq. (2) from this $F_0^0(t)|_B$ leads to a value $(\alpha - \beta)_0|_B = 5.9 \pm 1.1$, smaller than the “experiment”. The main reason for this small value is the presence of a zero in the integrand of the sum rule (2) at a moderate $t$-value $t_0 \simeq 0.29 \text{GeV}^2$, as shown in Fig. 1. Such an amplitude $F_0^0(t)|_B$, when analytically continued to complex $t$, has a $\sigma$ pole
in the second Riemann sheet at $t = t_{\sigma} = \{(474 \pm 6) - i (254 \pm 4)\} \text{ MeV}^2$ with $g_{\sigma\pi\pi} = \{(452 \pm 4) + i (224 \pm 2)\} \text{ MeV}$ and on the first Riemann sheet $A(t_{\sigma})|_B = -(0.107^{+0.002}_{-0.006}) + i (0.085 \pm 0.006)$ which leads to $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi} = -(0.48^{+0.01}_{-0.03}) + i (0.38 \pm 0.03)$ and the corresponding width $\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma)|_B = (2.5 \pm 0.2) \text{ KeV}$. These results are, however, not adequate to reproduce the experimental electromagnetic polarizabilities of the nucleon, as we have seen, and the input needed in the Compton scattering on the pion has to go beyond the Born contribution, with a modification of the $A(t)$ factor in Eq. (3).

It is known [31] that, at moderate values of $t$, the main addition to the dressed Born amplitude is the inclusion of a term proportional to $t/(t-t_A)$ in $A(t)$ where $1/(t-t_A)$ cancels the Adler zero present in $T(t)$. Adding this term also gives very good fits to $\gamma\gamma \rightarrow \pi\pi$ data at the intermediate energies [31] relevant for (2). Its dynamical origin can be traced from the next exchanges to the Compton scattering amplitude, being the leading ones $\gamma\pi \rightarrow a_1, \rho, b_1 \rightarrow \gamma\pi$. Although the low energy limit of these contributions in Compton scattering would correspond to the pion polarizability, our new term is an effective one for moderate higher values of $t$ in the annihilation channel and not connected to the pion polarizability. Therefore, we write

$$A(t) = A(t)|_B + C \frac{t}{t-t_A}$$

with $C$ a real constant to be determined phenomenologically. We fix it by requiring that the “experimental” value of $(\alpha - \beta)^0_t$ is reproduced. This procedure leads to $C = e^2 (1.07^{+0.10}_{-0.28})$ and the integrand of the sum rule is given in Fig. 1 as a continuous line. It is noticeable that the zero at $t_0$ in the dressed Born amplitude has clearly disappeared. Notice that $C$ has to be positive in order to match the “experimental” value of $(\alpha - \beta)^0_t$.

In spite of the fundamental dynamics imposed by the presence of the $\sigma$ resonance in this $t$-channel polarizability sum rule, its presence is not apparent when going to the physical real $t$ axis: there is no trace of a resonant Breit-Wigner type behavior in Fig. 1.

When $A(t)$ is analytically continued to the complex plane, at $t_{\sigma}$ on the first Riemann sheet $A(t_{\sigma}) = -(0.007^{+0.028}_{-0.007}) + i (0.088^{+0.008}_{-0.006})$, which due to the value of $C$ has a very much reduced real part and an increased imaginary part when compared with $A(t_{\sigma})|_B$. This leads to $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi} = -(0.03^{+0.12}_{-0.03}) + i (0.39^{+0.04}_{-0.03})$ and $\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma)(0) = (1.0^{+0.3}_{-0.2}) \text{ KeV}$. This is the main result of this paper. The error quoted for $\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma)$ is that induced by the uncertainty in the “experimental” value of $(\alpha - \beta)^0_t$ and by its extraction from the sum rule (2) only. In order to obtain the rest of the uncertainty, we proceed to modify either the form of $A(t)$ in Eq. (3) or the $\sigma$ properties in the pion scattering amplitude $T(t)$. In the first case, we remind the reader that at low energies a constant term in $A(t)$ is fixed by the Born contribution and a linear term in $t$ fixed by the pion electromagnetic polarizabilities $(\pi - B)_{\pi \pm}$ or equivalently by $L_9 + L_{10} = (1.4 \pm 0.3) \cdot 10^{-3}$ in chiral perturbation theory [32, 33]. As a consequence, a natural new parametrization of
where $D$ parametrizes the new effective quadratic term in the amplitude for the intermediate region of $t$. When we fix $D$ to reproduce the “experimental” value of $(\alpha - \beta)_0^I$, we obtain an effective value $D = e^2 (2.0^{+0.3}_{-0.6})$ MeV$^{-2}$. With this value, the integrand of $(\alpha - \beta)_0^I$ in (2) goes close to the continuous line in Fig. 1, somewhat below first and somewhat above after. When (8) is analytically continued to $t_\sigma$, we get $A^{II}(t_\sigma) = -(0.044^{+0.015}_{-0.007}) + i (0.042^{+0.022}_{-0.015})$, leading to the coupling $g^{II}_{\sigma\gamma\gamma}/g_{\sigma\pi\pi} = -(0.21^{+0.05}_{-0.03}) + i (0.20^{+0.10}_{-0.07})$ and the width $\Gamma^{II}_{\text{pole}}(\sigma \rightarrow \gamma\gamma) = (0.6^{+0.6}_{-0.3})$ KeV. Since (7) gives already a good fit for $\sigma\pi\pi$ values slightly modified in order to reproduce the “experimental” value of $(\alpha - \beta)_0^I$ for $\sigma\pi\pi$ at the energies that are relevant for (2), it has no sense to go higher in the degree of the effective term in $A(t)$ and we just use this result to estimate the final uncertainty. If, instead, we still use the parametrization (7), with the three parameter fit formula including low energy kaon data and high energy data for $\cot(\delta(t))$ in [2] but with parameter values slightly modified in order to reproduce the $\sigma$ pole position $t_\sigma = ((438 \pm 6) - i (267 \pm 4))$ MeV$^2$ found in [1], we get $g_{\sigma\pi\pi} = [(472 \pm 4) + i (191 \pm 2)]$ MeV. With this $T(t)$ and the dressed Born amplitude in (6), one gets $(\alpha - \beta)_0^I|_B = 5.6 \pm 1.0$, $A(t_\sigma)|_B = -(0.133 \pm 0.005) + i (0.095 \pm 0.007)$ and $\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma)|_B = (3.9 \pm 0.4)$ KeV. The effective value of $C$ in (7) moves to $C = e^2 (1.03^{+0.14}_{-0.24})$ when fixed to reproduce the “experimental” value of $(\alpha - \beta)_0^I$. With this new $C$, the analytic continuation to the new $t_\sigma$ gives $A(t_\sigma) = -(0.019^{+0.003}_{-0.016}) + i (0.096 \pm 0.006)$, so that $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi} = -(0.09^{+0.01}_{-0.01}) + i (0.43 \pm 0.03)$ and $\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma) = (1.4 \pm 0.2)$ KeV. In this case, the integrand of $(\alpha - \beta)_0^I$ in (2) is again very similar to the continuous curve in Fig. 1.

As final result for the electromagnetic pole width of the $\sigma$ found in the $\pi\pi$ scattering amplitude, we quote

$$\Gamma_{\text{pole}}(\sigma \rightarrow \gamma\gamma) = (1.2 \pm 0.4) \text{ KeV}$$

which is the weighted average between the result of using $A(t)$ in (7), with the three-parameter fit formula for $\cot(\delta(t))$ from [2] including both low energy kaon data and high energy data, and the result of using the same $A(t)$ but varying the parameters of the fit for $\cot(\delta(t))$ found in [2] in order to mimic the pole position found in [1]. The result of using the $A^{II}(t)$ in (8) is taken into account just in the final uncertainty in Eq. (9).

To conclude, we have shown that the scalar-isoscalar $\gamma\gamma \rightarrow \pi\pi$ amplitude $F_0^\sigma(t)$ may be fixed using analyticity, unitarity and experimental information on the proton electromagnetic polarizabilities. This is possible and direct because this component is projected out in the sum rule (2) for the proton polarizabilities. When $F_0^\sigma(t)$ is analytically continued to the complex plane, the $\sigma$ pole position and its $g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}$ and $g_{\sigma\pi\pi}$ residues become fixed.

\[ A(t) \text{ would be} \]
\[ A^{II}(t) = A(t)|_B + 64\pi e^2 (L_0 + L_{10}) \frac{t}{t - t_A} + D \frac{t^2}{t - t_A} \]
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