Effect of wall-slip on natural convection in the annulus between a circular cylinder and square enclosure

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Abstract. Two-dimensional natural convection in the annulus between a hot circular cylinder and cold square enclosure is numerically investigated for Rayleigh number \(Ra=10^6\) using OpenFOAM. The Prandtl number \((Pr)\) and radius to length \((R/L)\) ratio are fixed at 0.71 and 0.2 respectively and slip condition is imposed by varying the \(Slip factor\) \((SF)\) from 0 (no-slip) to 1 (full-slip) in steps of 0.2. For each value of \(SF\), the flow patterns are visualised using streamlines and heat transfer behaviour is examined using isotherms. The full-slip boundary condition \((SF=1)\) witnesses significant phenomena like vortex splitting, shrinkage in thermally stagnant region and reduction in thermal boundary layer thickness. The heat transfer rate is found to increase with an increase in \(SF\) and results are quantified using local Nusselt number \((Nu_l)\) and average Nusselt number \((Nu_{avg})\) along the hot wall. It is noticed that for full-slip \((SF=1)\) configuration, the natural convection patterns and heat transfer characteristics undergo notable changes.

1. Introduction

Natural convection heat transfer in an annulus deserves considerable attention owing to various industrial applications in electronic cooling, nuclear engineering and safety, heat exchanging equipments etc; to cite a few. One of the earliest researches in this area was reported by Kuehn & Goldstein\[1\] wherein they studied natural convection in the horizontal annulus between two coaxial circular cylinders numerically and experimentally.

Convection analysis in rectangular enclosures with an internal heat source was primarily reported by Nasrine & Ghaddar\[2\]. The authors used a spectral element method to simulate natural convection phenomena in the annulus and established a correlation between Rayleigh number \((Ra)\) and Nusselt number \((Nu)\) that could explain the overall heat transfer characteristics. Moukalled & Acharya\[3\] investigated heat transfer inside a square enclosure containing a cylindrical heater for Rayleigh numbers \(10^4\) to \(10^7\) at three different \(R/L\) ratios 0.1, 0.2 and 0.3. They reported that the overall heat transfer increased with an increase in Rayleigh number at a constant \(R/L\) ratio however, the contribution of convection heat transfer to the overall heat transfer decreased as the \(R/L\) ratio increased. Cesini et al.\[4\] conducted numerical and experimental study of natural convection from a horizontal cylinder embedded in a rectangular cavity and studied the influence of the aspect ratio of the cavity and Rayleigh number \((Ra)\) on the thermal fields and the Nusselt number \((Nu)\). The authors also reported an enhancement in the
The effect of natural convection in the annulus for arbitrary positions of the inner cylinder was conducted by Jami et al.[5]. They reported that the heat transfer was maximum when the cylinder was kept at the centre of the enclosure with no heat generation from it. Kim et al.[6] investigated the influence of location of the inner circular cylinder relative to the outer square enclosure on the heat transfer characteristics for different Rayleigh numbers. Similar to Kim et al.[6] a study conducted by Jaddoa et al.[7] considered the effect of aspect ratio of the system along with the location of the cylinder. The results obtained in their study showed that the maximum Nusselt number was observed on top of the cylinder when it was moved diagonally and vertically. It was also noted that the surface averaged Nusselt number increased with increase in the aspect ratio of the geometry. Hussain and Hussein[8] conducted a study on natural convection which was an extension to the study done by Kim et al.[6], but in this investigation a constant heat flux per unit surface area on the inner cylinder was considered. It was reported that the flow field did not have much impact at low Rayleigh numbers but had considerable effect at high Rayleigh numbers. A nonlinear behaviour for surface averaged Nusselt number which was a function of position was also observed. The influence of Prandtl number (Pr) on natural convection inside an annulus was investigated by Liao & Lin[9] with the help of flow structure map in the Ra-Pr plane. They reported that by decreasing Pr, the flow underwent a shift from steady single to steady double thermal plume and then after a further decrease in Pr, it became unsteady.

Rabani & Talebi[10] numerically simulated natural convection heat transfer in a square enclosure containing a partially heated circular cylinder and the effect of increase in heating angle, displacement heating, and Rayleigh number on the flow and temperature contours were reported. A three dimensional natural convection in the annulus between an inner hot cylinder and an outer cold cubic enclosure was studied by Souayeh et al.[11] for various tilted angles of the enclosure and Rayleigh numbers. The results obtained showed that for an inclination of 90° and Rayleigh number $10^6$, optimum average heat transfer was attained. Lee et al.[12] studied average heat transfer coefficients with increase in Ra.
the natural convection in the annulus which emphasized the effects on flow and thermal fields in the system. It was found that the alteration in the localized heated zone at the bottom of the enclosure wall affected the flow and thermal fields. The authors reported a minor variation in the thermal and flow fields due to the changes in the size of heated section. The effect on natural convection by four different polygonal enclosures with an inner cylinder was studied by Saleh et al.[13] and reported that the profile of the Nusselt number was not influenced by the polygon geometry. Mun et al.[14] investigated the natural convection in the annulus of four inner cylinders which were arranged in a diamond array within a square enclosure and reported the effect of distance between the cylinders and Rayleigh number on the time dependence of flow fields and isotherms. A three dimensional natural convection study for a sinusoidal bottom wall temperature variation in the annulus of an outer square enclosure and inner circular cylinder was conducted by Lee et al.[15]. The thermal plume was observed to be influenced by bottom wall temperature fluctuation and Rayleigh number. It was also reported that the thermal and flow fields depended on three factors such as size, count and rotational direction of the thermal plumes.

Numerous investigations have been carried out in the past which discuss the heat transfer between an eccentric cylinder and a square enclosure[16], a square enclosure with two inner cylinders at different vertical positions[17], a cylindrical heat source kept in a square enclosure with adiabatic vertical walls and varying temperature on the bottom wall[18], the effect of tilted angle of enclosure on the natural convection[19]. All the reported experimental and numerical analyses on the annular convection phenomena have used no-slip boundary condition on the hot and cold surfaces and to the best of our understanding there is hardly any report on how the natural convection phenomena is affected by the slip condition applied on the boundary walls. This paper is hence devoted to find out the effect of slip on natural convection in the annulus of a horizontal circular cylinder and a square enclosure. The results of our investigations can be insightful in industrial and academic research pertaining to wettability of surfaces especially in heat pipes where the presence of hydrophobic surfaces can limit the formation of residuals thereby improving anti-freezing characteristics.

The structure of the presentation of our investigations in the rest of the article is as follows. Numerical methodology in Section 2 describes the solver details, governing equations and the boundary conditions imposed. Results and discussion in Section 3 includes the grid independency and code validation followed by the presentation of results. The flow and thermal fields visualised using streamlines and isotherms have been illustrated followed by a thorough discussion on the heat transfer characteristics quantified using the local and surface averaged Nusselt number. Conclusions in Section 4 summarises the overall numerical findings.

2. Numerical methodology
Numerical simulations have been performed using the finite volume open source computational fluid dynamics (CFD) toolbox OpenFOAM. The generic solver buoyantBoussinesqPimpleFoam is used to solve the two-dimensional steady, laminar and incompressible governing equations in cartesian co-ordinates as given below. For the spatial discretization of diffusive and convective terms, a second order central differencing Gauss linear scheme is used.

\[
\text{Continuity : } \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}
\]

\[
X - \text{momentum : } U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \tag{2}
\]

\[
Y - \text{momentum : } U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra \Pr \Theta \tag{3}
\]
Energy: \[ E \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \] (4)

The non-dimensional variables in the governing equations are defined as follows

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad \Theta = \frac{T - T_c}{T_h - T_c}, \quad P = \frac{pL^2}{\rho \alpha^2} \] (5)

where \( u \) and \( v \) are the components of velocity in \( x \) and \( y \) directions, \( \rho \) is the density of the fluid, \( \alpha \) is the thermal diffusivity, \( T_c \) and \( T_h \) are temperatures at cold and hot walls and \( p \) is the pressure.

Rayleigh number, which is defined on the basis of the length of the enclosure is

\[ Ra = \frac{g \beta L^3 (T_h - T_c)}{\nu \alpha} \]

and Prandtl number as \( Pr = \frac{\nu}{\alpha} \) where, \( \nu \) is the kinematic viscosity of the fluid, \( g \) is the acceleration due to gravity acting downwards in the negative \( y \) direction and \( \beta \) is the coefficient of thermal expansion.

A schematic of the computational domain is given in Figure 1(a) which consists of a square enclosure of length \( L \) with an inner cylinder (\( R = 0.2L \)) located at its centre. The inner cylinder is kept at a constant high temperature \( \Theta = 1 \) whereas the outer square enclosure is maintained at a relatively low temperature \( \Theta = 0 \).

Slip conditions on the walls are varied using the partialSlip boundary condition (BC) available in OpenFOAM. partialSlip represents a mixed boundary condition which is an implementation of the Navier slip law as expressed below. For a 2-D flow, the first order approximation for the velocity slip boundary is given by

\[ U + \lambda \frac{dU}{dY} = 0 \] (6)

here \( \lambda \) is the slip length as defined in Legendre et al.[20], Silvia and Bodgan[21].

Let \( U_0 \) be the tangential velocity on the wall face boundary and \( U_1 \) the tangential cell center velocity at the boundary cell. Equation (6) can be re-written in terms of \( U_0, U_1 \) and \( D \) as follows

\[ U_0 + \frac{\lambda (U_0 - U_1)}{D/2} = 0 \] (7)

where \( D \) is the size of the boundary cell. The above equation after re-arranging becomes

\[ U_0 = \frac{2\lambda}{D + 2\lambda} U_1 \] (8)

Here, we introduce a new term Slip factor (SF) defined as

\[ SF = \frac{2\lambda}{D + 2\lambda} \] (9)

Hence equation (8) becomes

\[ U_0 = (SF) U_1 \] (10)

For \( \lambda \rightarrow 0 \), we have \( SF = 0 \), which indicates no-slip condition. For \( \lambda \rightarrow \infty \), we have \( SF = 1 \), which indicates full-slip condition.
The rate of heat transfer is computed at each wall and is expressed in terms of local Nusselt number \( (Nu_l) \) and average Nusselt number \( (Nu_{avg}) \).

\[
Nu_l = \frac{\partial \Theta}{\partial n}, \quad Nu_{avg} = \frac{1}{W} \int_0^W Nu_l \, dS
\]  

(11)

here \( n \) is the unit normal vector to the wall and \( W \) is the surface area of the wall.

Conditions imposed at the walls other than the slip boundary conditions are as follows:
Left, right, top and bottom walls : Dirichlet boundary condition, \( \Theta = 0 \)
Inner cylinder wall : Dirichlet boundary condition, \( \Theta = 1 \)

The radiation heat transfer is considered negligible and the fluid properties are assumed to be constant, except for the variation in density which is obtained by the Boussinesq approximation.

3. Results and discussion
Computations have been carried out for \( Ra = 10^6 \) and \( Pr = 0.71 \) by changing the slip conditions, \( 0 \leq SF \leq 1 \). Section 3.1 discusses the grid sensitivity studies and details of validation of the present numerical code is explained in Section 3.2. Section 3.3 and Section 3.4 are dedicated to the illustration of flow and thermal fields and the heat transfer characteristics are examined in Section 3.5 by plotting the local Nusselt number \( (Nu_l) \) distribution and average Nusselt number \( (Nu_{avg}) \) along the walls.

3.1. Grid independency
Grid independency tests are conducted to minimise the influence of number of cells on the computational results and to optimise the computational cost. In order to accurately capture
the boundary layer phenomenon, finer mesh is adopted near the walls and coarser mesh for other regions.

Table 1: Grid independency data for Ra=10^5, Pr=0.71, SF=0

| Number of cells | Nu_{avg}  |
|-----------------|-----------|
| 5100            | 7.977     |
| 8000            | 7.937(-0.501%) |
| 11500           | 7.930(-0.088%) |
| 15600           | 7.928(0.025%)  |
| 20300           | 7.899(-0.365%) |
| 25600           | 7.905(0.076%)  |

Table 1 shows the variation in Nu_{avg} while increasing the number of cells. It is evident that, the percentage difference is minimum (0.025%) when the cell count is 15600 beyond which an increase in grid count is accompanied by accumulation of numerical errors hence an optimum grid structure comprising of 15600 cells is employed throughout our computations.

3.2. Validation
Validation of the solver for the present computations is performed by comparing the Nusselt number data available in the literature. Table 2 compares the Nu_{avg} values with no-slip condition at the walls obtained using the present code with the existing data. The percentage deviation of the computed Nu_{avg} values with those reported in the literature are indicated in brackets. The results obtained are found to be in good agreement and the maximum deviation is less than 3%.

Table 2: Validation: Nu_{avg} comparison with previous studies for Pr=0.71, SF=0

| Ra   | Present study | Park et al.[16] | Kim et al.[6] | Kim et al.[22] | Moukalled & Acharya[3] |
|------|---------------|-----------------|---------------|----------------|------------------------|
| 10^4 | 5.147         | 5.128(-0.374%)  | 5.108(-0.767%)| 5.113(-0.669%) | 5.286(-2.625%)         |
| 10^5 | 7.93          | 7.836(-1.203%)  | 7.767(-2.102%)| 7.75(-2.326%)  | —                      |
| 10^6 | 14.377        | 14.462(-0.589%) | 14.11(-1.89%) | 14.2(-1.245%)  | 14.228(-1.242%)        |

3.3. Streamlines
Figure 2 illustrates the flow fields for various slip configurations visualised using streamlines. The fluid in the vicinity of the hot wall gets heated up, becomes less dense and begins to ascend. As soon it reaches the top wall, it interacts with the cold surface, migrates outwards, becomes more dense and moves down. While descending, some of the cold fluid again warms up due to the interaction with the hot surface and its neighbourhood thus giving rise to a pair of recirculating convection currents manifested as vortices. The physical mechanism of the formation of natural convection currents is explained in Moukalled & Acharya[3] and Kim et al.[6].

The no-slip condition represented by SF=0 witnesses two recirculating primary vortices symmetric about the vertical axis. The cores of these vortices are found to occupy the upper half of the domain indicating strong convection currents. In addition to the occurrence of counterclockwise primary vortices on either side of the vertical axis, small recirculating secondary vortices are also visible near to the bottom wall.
With increase in slip factor, the counter-rotating secondary vortices are seen to enlarge gradually and consistently upto SF=0.8. In all these cases, the pair of primary vortices continue to exist with their core at the upper half. Introduction of full-slip (SF=1) on the surface of hot and cold walls brings notable changes in the convection currents. The core of the counter-rotating primary vortices on either side of the vertical axis splits into two thereby giving rise to two additional inner vortices however, the circulating outer envelope remains in position. Also, the tiny recirculating secondary vortex pair near to the bottom wall vanishes. This drastic variation in convection has significant contribution to the thermal fields as discussed below.

3.4. Isotherms

Previous literature have observed the dominance of convection currents in the annular region at Ra=10^5 and above. Due to this, the thermal boundary layer on the top half of the hot cylinder gets separated and develops into a plume in the upward direction before the flow becomes steady. The effect of the plume formation is felt in the adjacent layers finally giving way to the distorted isotherms. The thermal gradients at the upper half of the domain is very high compared to the other half due to the formation and progression of the thermal plume. This is exactly the same reason why the cores of the primary vortices belong to the upper half as evidenced in the streamlines. Figure 3 illustrates the temperature contours (isotherms) for various slip configurations. The isotherms at no-slip condition (SF=0) is in excellent match with previously recorded literature (Kim et al.[6], Sheikholeslami et al.[23]). The increased thermal stagnation at Ra=10^6, which is predominant in the lower half of the domain, acts as a hindrance to the penetration of flow to the bottom of the annulus. This is another reason for the occurrence of weak thermal gradients and convection currents at the bottom.

Figure 2: Streamlines for various slip configurations at Ra=10^6
With increase in slip factor, close observations reveal the plume shifting slightly vertically upward towards the top wall upto SF=0.8. Visual investigations in this range, however, reveal little variation in the thermally stagnant region near to the bottom wall. Introduction of full-slip conditions on the hot and cold walls brings changes everywhere. The hot plume shrinks and extends further upward whose impact reaches the lower half. The stagnant region suddenly shrinks and occupies the space between the bottom wall and lower half of the inner cylinder. The boundary layer on the lower hot surface becomes thinner influencing the heat transfer characteristics which is discussed in the next section.

3.5. Variation in Nusselt number
Figure 4(a) shows the distribution of local Nusselt number ($Nu_l$) along the right half of the hot wall. Due to the symmetric nature of flow and thermal fields, the heat transfer characteristics too are same on either side of the vertical axis hence, the $Nu_l$ variation along the left surface is same as that of the right side of which the later is considered for analysis.
Moving from $\theta=0^\circ$ to $\theta=180^\circ$ in the clockwise direction, the development of plume increases the boundary layer thickness in its vicinity thereby reducing the rate of heat transfer. The influence of plume is less felt on the lower half and the comparatively thinner boundary layer enhances the heat transfer characteristics. The local Nusselt number ($N_u_l$) monotonically increases between $\theta=0^\circ$ and $\theta=150^\circ$ irrespective of the slip conditions. Between $\theta=150^\circ$ and $\theta=180^\circ$, the local Nusselt number is nearly constant (except for $SF=1$) as the heat transfer takes place mainly due to conduction since the flow is restricted from penetrating to the bottom of the enclosure. $N_u_l$ profile for $SF=0$ matches with previous studies such as Kim et al.$[6]$ and Aithal$[24]$. The trend shown in the $N_u_l$-$\theta$ plot is same for slip values in the range $SF=0$ to $SF=0.8$, however, the heat transfer characteristics for the full-slip boundary condition receives special attention. For $SF=1$, the slope of $N_u_l$-$\theta$ curve between $\theta=0^\circ$ to $\theta=30^\circ$ is large due to greater heat transfer rate. Due to the splitting up of inner core of the vortex, the convection currents are evenly distributed which enhances the overall heat transfer characteristics. This is reflected in the higher $N_u_l$ values throughout the hot surface. Between $\theta=120^\circ$ and $\theta=180^\circ$, the $N_u_l$-$\theta$ curve is more steeper owing to the shrinkage in thermal stagnation and decrease in thermal boundary layer thickness. This can also be visualised in the corresponding temperature profile (Figure 3(f)).

Figure 4(b) shows the average Nusselt number ($N_u_{avg}$) for the entire range of slip conditions. The $N_u_{avg}$ value at no-slip condition matches well with the previously reported values by Kim et al.$[6]$. It is noticed that the heat transfer rate increases with increase in slip factor. Between $SF=0$ and $SF=0.8$, the introduction of slip conditions, even though marginal, enhances the heat transfer rate. However, there is steep rise in $N_u_{avg}$ with the introduction of full-slip on the hot and cold walls. The shrinkage of thermal plume and thermal stagnation together with the distribution of convection currents to the lower side contribute to the enhancement in overall heat transfer characteristics.

4. Conclusions
Numerical analysis is conducted to investigate the effect of wall-slip on natural convection in the annulus between a square enclosure and circular cylinder. It is observed that increase in slip
factor from SF=0 to SF=0.8 brings marginal variation in flow and thermal fields and overall heat transfer rate however, introduction of full-slip (SF=1) induces stronger convection currents. The other inferences are given below

- Between SF=0 and 0.8, the flow field witnesses two counter-rotating primary vortices symmetric about the vertical axis with the vortex cores occupying the upper half. The boundary layer near to the bottom wall separates giving rise to a pair of tiny secondary vortices.
- At full-slip (SF=1), the cores of the primary vortices on either side breaks into 2 vortices thereby evenly distributing the convection currents.
- For all cases investigated, the thermal boundary layer on the top half of the hot circular cylinder develops into a plume projecting upwards.
- Thermal stagnation is predominant for slip values between SF=0 and SF=0.8. For full-slip (SF=1) condition, the stagnation shrinks and occupies the space between the bottom wall and lower half of the circular cylinder.
- The increase in slip factor from SF=0 to 0.8 is accompanied by a monotonic increase in local Nusselt number ($Nu_{lt}$) up to $\theta=150^\circ$ beyond which the variation is nearly constant. For full-slip condition, the distribution of convection currents and shrinkage in thermal stagnation bring about significant enhancement in the overall heat transfer rate.
- The $Nu_{avg}$ marginally increases between SF=0 and 0.8 and the introduction of full-slip brings a steep increase in the heat transfer rate.

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