INDEPENDENT (NON-ADJACENT VERTICES) TOPOLOGICAL SPACES ASSOCIATED WITH UNDIRECTED GRAPHS, WITH SOME APPLICATIONS IN BIOMATHEMATICS

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ABSTRACT: In this work, we associate a new topology to undirected graph $G = (V, E)$ which may contain one isolated vertex or more and we named it Independent (non-adjacent vertices) Topology. A new sub-basis family to generate the Independent Topology is introduced on the set of $n$ vertices $V$ and for every vertex $v$ of $V$ the number of adjacent vertices is not greater than $n - 2$ (In simple graph we can say: for every vertex $v$ of $V$, $\Delta(G) = n - 2$, where $\Delta(G)$ is the maximum degree of vertices in a graph $G$). Then we give a fundamental step toward investigation of some properties of undirected graphs by their corresponding Independent Topology which we introduce in this work. Furthermore, an application to our new model (Independent Topology) are presented, that to carry out a framework in practical life like biomathematics (applied examples in biomathematics).

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1. Introduction

In Mathematics graph theory have a long history, one branch of graph theory is a topological graph theory. The relation between graph theory and topological theory existed before and used many times by researchers to deduce a topology from a given graph. Some of them makes models defined on the set of vertices $V$ of the graph $G$ only and others made it on the set of edges $E$. They studies graphs as topologies and have been applied in almost every scientific field. Many excellent basics on the mathematics of graph theory, topological graph theory and some applications may be found in the sources [1-7].

In general graphs divided in two types; directed and undirected graph. To an undirected graph some researchers associate a topological spaces as fellow;

In 2013 [8], Jafarian et al. associate a Graphic Topology with the vertex set of a locally finite graph without isolated vertex, and they defined a sub-basis family for a graphic topology as a sets of all vertices adjacent to the vertex $v$. 
And in 2018 [9], Kilicman and Abdulkalek associate an Incidence Topology with a set of vertices for any simple graph without isolated vertex. where they defined a sub-basis family for an incident topology as a sets of all incident vertices with the edge $e$.

The previous works of topology on graphs was associated with a set of vertices without isolated vertex. Therefore, these topologies are not appropriate to be associated with graphs that have an isolated vertices.

Our motivation or target is to associate a topology on the vertex set of any undirected graph (not only simple graph or locally finite graph) and which may contain one isolated vertex or more. By introducing a new Sub-basis family defined as a sets of all vertices non-adjacent to the vertex $v$ to induce the new topology (which we named it Independent Topology), and we present a fundamental steps toward studying some main properties of undirected graphs by their corresponding topologies.

So, we have two goals for this work: First, we introduce a new model of a topology associated with graph which is most general than the previous works. Second, we apply this new model topology in some main subjects in biomathematics.

In Section 2 of the article we give some fundamental definitions and preliminaries of graph theory and topology, also In Section 3 we define our new topology (independent topology) on undirected graphs by introducing a sub-basis family for the new topology. Section 4 is devoted to some preliminaries results of independent topology.

In Section 5 some application in biomathematics of new model (independent topology) is discussed. In last Section, conclusions of this new topology on undirected graphs are presented.

2. Preliminaries

In this section we give some fundamental definitions and preliminaries of graph theory and topology. All this definitions are standard, and can be found for example in sources [2] [3] [10].

Usually the graph is a pair $G = (V, E)$, for more exactly A graph $G$ consist of a non-empty set $V$ of vertices (or nodes), and a set $E$ of edges (or arcs). If $e$ is an edge in $G$ we can write $e = v u$ ($e$ is join each vertex $v$ and $u$),where $v$ and $u$ are vertices in $V$, then $(v$ and $u$) are said adjacent vertices and incident with the edge $e$.If there is no vertex adjacent with a vertex $v$, then $v$ is said isolated vertex. the degree of the vertex $v$ denoted by $d(v)$ is the number of the edges where $v$ incident with $e$, and $\Delta(G)$ is the maximum degree of vertices in $G$. A vertex of degree 0 is isolated. An independent set in a graph $G$ is a set of pairwise non-adjacent vertices. The graph $G$ is finite if the number of the vertices in $G$ also the number of the edges in $G$ is finite, then; otherwise it is an infinite graph. If any vertex can be reached from any other vertex in $G$ by travelling along the edges, then $G$ is called connected graph and is called disconnected otherwise.

We use notations $K_n$, $K_{m,n}$, $P_n$ and $C_n$ for a complete graph with $n$ vertices, the complete bipartite graph when partite sets have sizes $m$ and $n$, the path on $n$ vertices and the cycle on $n$ vertices, respectively.

A topology $\mathcal{T}$ on a set $\mathcal{X}$ is a combination of subsets of $\mathcal{X}$, called open, such that the union of the members of any subset of $\mathcal{T}$ is a member of $\mathcal{T}$, the intersection of the members of any finite subset of $\mathcal{T}$ is a member of $\mathcal{T}$, and both empty set and $\mathcal{X}$ are in $\mathcal{T}$. The ordered pair $(\mathcal{X},\mathcal{T})$ is called a topological space. When the topology $\mathcal{T} = P(\mathcal{X})$ on $\mathcal{X}$ is called discrete topology while the topology $\mathcal{T} = \{\mathcal{X},\varnothing\}$ on $\mathcal{X}$ is
called indiscrete (or trivial) topology. A topology in which arbitrary intersection of open set is open called an Alexandroff space.

3. Independent topology on graphs

Now, we define our new model of topology on undirected graph $G = (V, E)$ which may contain one isolated vertex or more and we named it Independent (non-adjacent vertices) Topology. A new sub-basis family to generate the Independent Topology is introduced on the set of $V$ vertices $V$ and for every vertex $v$ of $V$ the number of adjacent vertices is not greater than $n - 2$ (in simple graph we can say: for every vertex $v$ of $V$, $\Delta(G) = n - 2$, where $\Delta(G)$ is the maximum degree of vertices in a graph $G$).

(i.e. for every vertex $v \in V$ the number of adjacent vertices is not greater than $n - 2$, where $n$ is the number of all vertices in $G$)

Suppose that $I_v$ is the set of all vertices non-adjacent (independent) to $v$. It is clear that $v \in I_u$ iff $u \in I_v$ for all $v, u \in V$ and $v \notin I_v$ for all $v \in V$.

Define $S_{IV}$ as follows: $S_{IV} = \{ I_v | v \in V \}$. Since the condition above exist and the graph $G$ can contain one isolated vertex or more, we have $V = \bigcup_{v \in V} I_v$ Hence $S_{IV}$ forms a sub-basis for a topology $T_{IV}$ on $V$, called Independent topology of $G$.

It easy to see that the independent topology of $C_n$ when $n \geq 4$ and the simple graph has $n \geq 2$ isolated vertex are discrete but the independent topology of $P_n$ is not discrete because the set contains just two vertices of degree one is open, the independent topology of $K_{n,m}$ is equal to $\{ \varphi, V, A, B \}$, where $A$ and $B$ are partite sets of $K_{n,m}$.

**Example 3.1.** Let $G = (V, E)$ be a simple graph as in Fig. 1, clearly $G$ verify the condition (for every vertex $v \in V$ the number of adjacent vertices is not greater than $n - 2$), then:

$V = \{ v_1, v_2, v_3, v_4, v_5 \}$

We have:

$S_{IV_1} = \{ v_4, v_5 \}$, $S_{IV_2} = \{ v_3, v_5 \}$, $S_{IV_3} = \{ v_2 \}$, $S_{IV_4} = \{ v_1 \}$, $S_{IV_5} = \{ v_1, v_2 \}$.

By taking finitely intersection the base obtained,

$\{ \{ v_5 \}, \{ v_4, v_5 \}, \{ v_3, v_5 \}, \{ v_2 \}, \{ v_1 \}, \{ v_1, v_2 \}, \varphi \}$

Then by taking all unions the Independent topology can be written as:

$T_{IV} = \{ \varphi, V, \{ v_1 \}, \{ v_2 \}, \{ v_3, v_5 \}, \{ v_4, v_5 \}, \{ v_2, v_3, v_5 \}, \{ v_1, v_2, v_5 \}, \{ v_1, v_3, v_5 \}, \{ v_2, v_3, v_5 \}, \{ v_1, v_3, v_5 \} \}$.

4. Preliminary result

**Proposition 4.1.** If $G = (V, E)$ is a graph, then $(V, T_{IV})$ is an Alexandroff space.

**Proof.** It is enough to prove that arbitrary intersection of members of $S_{IV}$ is open. Let $S \subseteq V$. If $v \in \cap_{u \in S} I_u$, then $v \in I_u$ for each $u \in S$. Hence $u \in I_v$ for each $u \in S$ and so $S \subseteq I_v, I_v$ and $S$ are finite sets. This means that if $S$ is infinite, then $\cap_{u \in S} I_u$ is empty, but if $S$ is finite, then $\cap_{u \in S} I_u$ is the intersection of finitely many open sets and hence $\cap_{u \in S} I_u$ is open. □

Let $G = (V, E)$ be a graph containing $v$, for each $v \in V$, the intersection of all open sets containing $v$ is the smallest open set containing $v$ we still call it $D_v$ and the family $B_{IV} = \{ D_v | v \in V \}$ is minimal basis for the topological space $(V, T_{IV})$. 

Fig. 1
Proposition 4.2. Let $G = (V, E)$ be a graph. Then we have $D_v = \cap_{u \in I_v} I_u$ and so $D_v$ is finite for every $v \in V$.

Proof. Since $D_v$ is the smallest open set containing $v$ and $S_{IV}$ is a sub-basis of $T_{IV}$ we have $D_v = \cap_{w \in S} I_w$ for some subset $S$ of $V$. This implies that $v \in I_w$ for each $w \in S$. Therefore $S \subseteq I_v$ and so $v \in \cap_{w \in S} I_w \subseteq D_v$.

Now by definition of $D_v$, the proof is complete. $\square$

Corollary 4.3. Let $G = (V, E)$ be a graph. Then for every $v, w \in V$ we have $w \in D_v$ if and only if $I_v \subseteq I_w$. Equivalently $D_v = \{ w \in V \mid I_v \subseteq I_w \}$.

Proof. By the Proposition above $w \in D_v$ if and only if $w \in I_v$ for each $u \in I_v$ if and only if $u \in I_w$ for each $u \in I_v$.

Remark 4.4. Suppose that $G = (V, E)$ is a graph, then $(V, T_{IV})$ is a discrete topological space if and only if $I_v \nsubseteq I_u$ and $I_u \nsubseteq I_v$ for every distinct pair of vertices $v, u \in V$.

Remark 4.5. We also know from Remark in [11] that an Alexandroff topological space is a compact topological space if and only if $I_v \nsubseteq I_u$ for every distinct pair of vertices $v, u \in V$. Let $T = (V, E)$ be a tree. Then $(V, T_{IV})$ is a compact space if and only if $I_v \nsubseteq I_u$ for every $v, u \in V$ such that $v \neq u$ and $\text{deg} v = \text{deg} u = 1$.

Remark 4.6. Complete graph $K_n$ does not verify the Independent Topology but if there exists an one isolated vertex or more in the same graph then it verify the Independent Topology.

Example 4.7. Let $G = (V, E)$ be a complete graph $K_3$ as in Fig.2, clearly $G$ satisfy the condition since $n = 4$ and each vertex has not greater than $n - 2$ adjacent vertices such that $V = \{v_1, v_2, v_3, v_4\}$. We have; $I_{v_1} = \{v_4\}, I_{v_2} = \{v_4\}, I_{v_3} = \{v_4\}, I_{v_4} = \{v_1, v_2, v_3\}$

Then sub-basis $S_{IV} = \{\{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}\}$

By taking finitely intersection the basis obtained $\{(v_1, v_2, v_3), \varphi\}$

Then by taking all unions the Independent Topology can be written as: $T_{IV} = \{\varphi, V, \{v_3\}, \{v_1, v_2, v_3\}\}$

Definition 4.8. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. We call $G_1$ and $G_2$ isomorphic, and write $G_1 \cong G_2$, if there exists a bijection $\xi : V_1 \rightarrow V_2$ with $v \in E_1 \Leftrightarrow \xi(x)\xi(y) \in E_2$ for all $v, u \in V_1$. Such a map $\xi$ is called an isomorphism; if $G_1 \cong G_2$, it is called an automorphism of $G_1$.

Remark 4.9. It is easy to check, If $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic graphs, then topological spaces $(V_1, T_{IV_1})$ and $(V_2, T_{IV_2})$ are homeomorphic. The converse is not true, in general. For example $C_n$ when $n \geq 4$ and the simple graph has $n \geq 2$ isolated vertex, are not isomorphic graphs, but their corresponding independent topologies are both discrete and hence homeomorphic.

Proposition 4.10. Let $G = (V, E)$ be a graph. Then $(V, T_{IV})$ is a compact Independent topological space if and only if $V$ is finite.
Proof. By Proposition 4.2, $D_v$ is finite for every $v \in V$, hence if $V$ is infinite, then $B_{iv}$ is an open covering of $(V, T_{iv})$ which has no finite sub cover. □

**Definition 4.11.** In the graph $G$ if $F \subseteq V(G)$, then we write $G - F$ for the sub graph obtained by deleting the set of vertices $F$. A cut-vertex of $G$ is a vertex whose deletion increases the number of components of $G$, i.e. a vertex $v \in V(G)$ such that $G - \{v\}$ has more components than $G$. A vertex cut of a connected graph $G$ is a set $H \subseteq V(G)$ such that $G - H$ has more than one component. A vertex cut $H$ of $G$ is said to be minimal if every proper subset of $H$ is not a vertex cut.

It is obvious that, if $v$ be a cut vertex in a graph $G = (V, E)$ (not necessarily connected). Then $\{v\} \notin T_{iv}$.

**Example 4.12.** Let $G = (V, E)$ be a graph as in Fig. 3 such that $V = \{v_1, v_2, v_3, v_4, v_5\}$. We have;

$I_{v_1} = \{v_4, v_5\}, I_{v_2} = \{v_4, v_5\}, I_{v_3} = \{v_5\}, I_{v_4} = \{v_1, v_2\}, I_{v_5} = \{v_1, v_2, v_3\}$.

Then $S_{iv} = \{(v_5), (v_1, v_2), (v_4, v_5), (v_1, v_2, v_3)\}$

By taking finitely intersection the basis obtained

$\{\varphi, \{v_5\}, \{v_4, v_5\}, \{v_1, v_2\} \}$

Then by taking all unions the Independent Topology can be written as:

$T_{iv} = \{\varphi, V, \{v_5\}, \{v_4, v_5\}, \{v_1, v_2\}, (v_1, v_2, v_3), (v_1, v_2, v_4, v_5), (v_1, v_2, v_3, v_5)\}$

It is clear in this example $\{v_3\}$ is a cut vertex but $\{v_5\} \notin T_{iv}$.

Now, the connected graph is a tree if and only if every vertex of degree greater than one is a cut-vertex. Therefore, if $T = (V, E)$ is a tree and $v \in V$ with $deg v \geq 2$, then $\{v\} \notin T_{iv}$.

**Example 4.13.** Let $T = (V, E)$ be a graph as in Fig. 4 such that $V = \{v_1, v_2, v_3, v_4, v_5\}$. We have;

$I_{v_1} = \{v_3, v_4, v_5\}, I_{v_2} = \{v_4, v_5\}, I_{v_3} = \{v_1\}, I_{v_4} = \{v_1, v_2, v_5\}, I_{v_5} = \{v_1, v_2, v_3\}$.

Then $S_{iv} = \{(v_3, v_4, v_5), (v_4, v_5), (v_1), (v_1, v_2, v_3), (v_1, v_2, v_4)\}$

By taking finitely intersection the basis obtained

$\{\varphi, \{v_3, v_4, v_5\}, \{v_4, v_5\}, \{v_1\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_1, v_3, v_4, v_5\}\}$

Then by taking all unions the Independent Topology can be written as:

$T_{iv} = \{\varphi, V, \{v_3, v_4, v_5\}, \{v_4, v_5\}, \{v_1\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_1, v_3, v_4, v_5\}\}$

Clearly, $\{v_2\}$ and $\{v_3\}$ are cut vertex in the graph but both of them do not belong to $T_{iv}$.

**Proposition 4.14.** Let $G = (V, E)$ be a connected graph and $M$ is a minimal vertex cut in $G$. Then $M \in T_{iv}$.
Proof. Suppose that $G - M$ has $k \geq 2$ components, say $G_i = (V_i, E_i)$ for $i = 1, 2, \ldots, k$. Every vertex $v \in M$ must be adjacent to vertices of at least two different components, say $G_1$ and $G_2$, because $M$ is a minimal vertex cut.

Suppose that $\{u_1, \ldots, u_{k_1}\} = I_0 \cap V_1$ and $\{w_1, \ldots, w_{k_2}\} = I_0 \cap V_2$, then we have $v \in \cap_{i=1}^{k_1} I_{ul} \subseteq M \cup V_1$ and $v \in \cap_{i=1}^{k_2} I_{wl} \subseteq M \cup V_2$ and so $v \in (\cap_{i=1}^{k_1} I_{ul}) \cap (\cap_{i=1}^{k_2} I_{wl}) \subseteq M \cup (V_1 \cap V_2) = M$ that is $v$ is an interior point of $M$. □

5. Application of Independent Topology in biomathematics.

We apply the above definition on a bio-mathematical applications. We conclude that the undirected graph must be connected for modifying the bio-mathematical state.

5.1. In a possible genetic for the inheritance of blood group.

There are four main blood groups (types of blood) A, B, AB and O, your blood group is determined by the genes you inherit from your parents. Everyone has an (ABO) blood type just like eye or hair color. Each biological parent donates one of two (ABO) genes to their child, the A and B genes are dominant and the O gene is recessive [12].

![Diagram of blood groups and genetic inheritance](image)

**Fig. 5:** diagram of a possible genetic for the inheritance of blood group and it is graph.

By a graph above, $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$, where $v_1 = AO$, $v_2 = BO$, $v_3 = A$, $v_4 = O$, $v_5 = B$, $v_6 = AB$, $v_7 = OO$. We have;

$I_{v_1} = \{v_2, v_5, v_6, v_7\}$, $I_{v_2} = \{v_1, v_3, v_6, v_7\}$, $I_{v_3} = \{v_2, v_4, v_5, v_7\}$, $I_{v_4} = \{v_3, v_5, v_6\}$

$I_{v_5} = \{v_1, v_3, v_4, v_7\}$, $I_{v_6} = \{v_1, v_2, v_4, v_7\}$, $I_{v_7} = \{v_1, v_2, v_3, v_5, v_6\}$

Then $S_{IV} = \{\{v_2, v_5, v_6, v_7\}, \{v_1, v_3, v_6, v_7\}, \{v_2, v_4, v_5, v_7\}, \{v_3, v_5, v_6\}, \{v_1, v_3, v_4, v_7\}, \{v_1, v_2, v_4, v_7\}, \{v_1, v_2, v_3, v_5, v_6\}\}$

By taking finitely intersection the basis obtained
5.2. In general shape of Bipolar neuron.

Neurons are the cells that make up the brain and the nervous system. They are the fundamental units that send and receive signals which allow us to move our muscles, feel the external world, think, form memories and much more.

Just from looking down a microscope, however, it becomes very clear that not all neurons are the same. So just how many types of neurons are there? And how do scientists decide on the categories? For neurons in the brain, at least, this isn’t an easy question to answer. For the spinal cord though, we can say that there are three types of neurons: sensory, motor, and interneurons.

Most neurons can be anatomically characterized as: Unipolar, Bipolar, Multipolar. Bipolar, these neurons have two processes arising from a central cell body, typically one axon and one dendrite. These cells are found in the retina [12].

By a Graph of Bipolar neuron general shape, $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and;

\[ I_{v_1} = \{v_2, v_4, v_5, v_6, v_7, v_8\}, \quad I_{v_2} = \{v_1, v_4, v_5, v_6, v_7, v_8\}, \quad I_{v_3} = \{v_5, v_6, v_7, v_8\}, \]
\[ I_{v_4} = \{v_1, v_2, v_6, v_7, v_8\}, \quad I_{v_5} = \{v_1, v_2, v_3, v_7, v_8\}, \quad I_{v_6} = \{v_1, v_2, v_3, v_4\}, \]
\[ I_{v_7} = \{v_1, v_2, v_3, v_4, v_5, v_8\}, \quad I_{v_8} = \{v_1, v_2, v_3, v_4, v_5, v_7\}. \]

Then;
\[ S_p = \{ v_2, v_4, v_5, v_6, v_7, v_8 \}, \{ v_1, v_4, v_5, v_6, v_7, v_8 \}, \{ v_1, v_2, v_6, v_7, v_8 \}, \{ v_1, v_2, v_3, v_7, v_8 \}, \{ v_1, v_2, v_3, v_4, v_5, v_8 \}, \{ v_1, v_2, v_3, v_4, v_5, v_7 \}, \}

By taking finitely intersection the basis obtained:
\[
\{ \phi, \{ v_2, v_4, v_5, v_6, v_7, v_8 \}, \{ v_1, v_4, v_5, v_6, v_7, v_8 \}, \{ v_1, v_2, v_6, v_7, v_8 \}, \{ v_1, v_2, v_3, v_6, v_7, v_8 \}, \{ v_1, v_2, v_3, v_5, v_7 \}, \{ v_1, v_2, v_3, v_4, v_5, v_7 \}, \{ v_1, v_2, v_3, v_4, v_5, v_6 \}, \{ v_1, v_2, v_3, v_4, v_5, v_8 \}, \{ v_1, v_2, v_3, v_4, v_6, v_8 \}, \{ v_1, v_2, v_3, v_4, v_7, v_8 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_5 \}, \{ v_1, v_2, v_3, v_4, v_6, v_5 \}, \{ v_1, v_2, v_3, v_4, v_6, v_7 \}, \{ v_1, v_2, v_3, v_4, v_7, v_5 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \{ v_1, v_2, v_3, v_4, v_7, v_6 \}, \}
\]

Then by taking all unions the independent topology can be written as:
\[
S_p = \{ \phi, V, \{ v_1 \}, \{ v_2 \}, \{ v_4 \}, \{ v_5 \}, \{ v_7 \}, \{ v_1, v_2 \}, \{ v_1, v_3 \}, \{ v_1, v_4 \}, \{ v_1, v_5 \}, \{ v_1, v_6 \}, \{ v_1, v_7 \}, \{ v_2, v_3 \}, \{ v_2, v_4 \}, \{ v_2, v_5 \}, \{ v_2, v_6 \}, \{ v_2, v_7 \}, \}
\]

5.3. In connections of the renal artery of human kidney.

The kidneys are a pair of bean-shaped organs on either side of your spine, below your ribs and behind your belly. Each kidney receive blood from the paired renal arteries; blood exits into the paired renal veins. Each kidney is attached to a ureter, a tube that carries excreted urine to the bladder, and has around a million tiny filters called nephrons.

The kidneys' job is to filter your blood. They remove wastes, control the body's fluid balance, and keep the right levels of electrolytes. All of the blood in your body passes through them several times a day [12].

![Renal Artery Graph](image1.png) ![Anatomy of Human Kidney](image2.png)

**Fig. 7:** Anatomy of Human Kidney and Renal Artery Graph.
Now, let $G = (V,E)$ be a graph represents the associations (connections) points of the renal artery of human kidney (which is a non-simple graph because it has two multiple edges $(v_4, v_5)$ and $(v_{12}, v_{13})$) as in Fig.7 such that: $V = \{v_1, v_2, \ldots, v_{16}\}$

We have a sub-basis family of the Independent topology as fellow:

$\mathcal{I}_1 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$,

$\mathcal{I}_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$,

$\mathcal{I}_3 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$,

$\mathcal{I}_4 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$,

$\mathcal{I}_5 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$,

$\mathcal{I}_6 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$.

Then, by taking finitely intersection we find the basis, and after that find the all unions, the Independent Topology will obtained.

Conclusions:
A synthesis between graph theory and topology has been made. A topology with the set of vertices for any undirected graph has been associated, called independent topology. The study of some properties of this new model of topology has been presented. It has been shown that this topology is an Alexandroff topology. Useful applications of independent topology in biomathematics have been introduced. Therefore, this article can be considered as a point of applying another topological concept of graphs in scientific fields, which could lead to another significant applications in the future.

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