A Voltage Control Model of Multi-ports DC Transformer

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Abstract. DC transformer (DCT) is basic and important power conversion equipment in DC distribution network. DC transformer adopts power electronic switches and high-frequency transformer, which can realize power conversion. Multi-ports DCT can connect many different types of power supply or load equipment, and can flexibly control the current, voltage and power flow of multiple ports, which is widely used in DC distribution network. In this paper, the output voltage control model of multi-ports DCT in steady state is established. Combined with the control model, the mechanism of controlling the phase-shift angle to realize the output voltage control is proved mathematically. The correctness of the control model is verified by simulation.

1. Introduction

In the traditional AC distribution network, the main load is passive load, and the power presents the characteristics of unidirectional flow. With the development of renewable energy technology, more and more distributed power sources and energy storage systems are included in modern AC distribution networks, showing the characteristics of bidirectional power flow. If the distributed power and energy storage facilities are directly connected to the DC power grid instead of the AC power grid, a lot of intermediate conversion links can be saved. For example, the electric energy generated by photovoltaic power generation is DC, which usually needs DC/DC and DC/AC conversion to be incorporated into the AC distribution network. If the distributed power supply can be directly connected to the DC distribution network, it usually only needs DC/DC conversion. The cancellation of DC/AC conversion link not only reduces the cost of distributed generation system, but also improves the efficiency of power conversion[1][2].

In addition to power supply and energy storage equipment, the load situation in modern distribution network is also changing. Many loads are supplied by DC network. In AC distribution network, it is generally necessary to realize the power supply of these DC loads through AC/DC conversion. In DC distribution network, these loads can be supplied directly without power conversion. For the distribution network with more distributed power supply, energy storage system and DC load, the connection mode of DC distribution network can reduce the intermediate links of new energy generation system, energy storage system and DC load access to the distribution network, to reduce the cost of access to the distribution network and improve the efficiency and power density of the whole system. In addition, compared with the AC distribution network, the DC distribution network does not need to track the voltage and frequency of the grid in terms of control and protection. The controllability and reliability of the system are greatly improved, which is more suitable for the access of distributed power and load.
In DC distribution network, DCT is a kind of basic and important power conversion equipment. DCT can realize voltage transformation and energy transmission, and the voltage transformation and energy transmission present active control mode. In recent years, the research focus on DCT mainly includes the topology and control of multi-ports DCT. Multi-ports DCT can directly interconnect multiple DC power grids. The multi-ports DCT with complete electrical isolation couples each port through a multi-winding high-frequency transformer with a common core, and the number of windings is the same as the number of ports, which is suitable for high voltage and high power applications, and has high power supply reliability. Figure 1 is a typical circuit topology of a three-ports DCT with common core.

In reference [3], three control methods of multi-ports DCT in railway locomotive electric traction are studied, which can realize the power balance of input port. In reference [4], an MMC based capacitor voltage sequencing algorithm for multi-ports DCT is studied. In reference [5], the performances of multi-ports DCT with H-bridge, neutral point clamp, flying capacitor and T-type circuit are studied and compared. In reference [6], the function relationship between the current and phase-shifting angle of the multi-ports DCT is studied, and the loss of the DC transformer is analyzed. The experiment of the DCT prototype with Si IGBT and SiC MOSFET devices shows that the highest efficiency of the 20kW prototype can reach 97.5%. In reference [7, 8], the control strategy of multi-ports DCT is studied, which can realize the effective control of power transfer between windings. In reference [9], a double closed-loop control strategy for multi-ports DCT is proposed, but the method needs to collect the AC current of transformer winding, and the calculation of phase-shift angle is greatly affected by the excitation inductance and leakage inductance parameters of transformer. In reference [10], a hardware decoupling strategy is proposed. The leakage inductance of one winding is designed to be much larger than that of the other two windings, so as to realize the power decoupling control of three windings.

In this paper, the multi-ports DCT shown in Figure 1 is taken as the research object. The output voltage model of the multi-ports DC transformer under steady-state condition is established. The correctness of the control model is verified by simulation.

2. Analysis of power transmission characteristics

The typical topology of a multi-ports DC transformer is shown in Figure 1. The topology consists of three H-bridge circuits and a three winding high-frequency transformer. The H-bridge circuit unit connected with power supply $V_1$ outputs square wave voltage. Similarly, the H-bridge circuit unit connected with $V_2$ and $V_3$ also outputs square wave voltage. The phase shift angle between the two H-bridge circuit unit voltages connected by $V_1$ and $V_2$ is $\varphi_{12}$. The phase shift angle between the two H-bridge circuit unit voltages connected by $V_1$ and $V_3$ is $\varphi_{13}$. The phase shift angle between the two H-bridge circuit unit voltages connected by $V_2$ and $V_3$ is $\varphi_{23}$, which satisfies $\varphi_{23}=\varphi_{13}-\varphi_{12}$. The working frequency of each H-bridge circuit unit is fixed. Because the excitation inductance of the high frequency transformer is independent of the power flow, it can be ignored to simplify the analysis. Figure 2 shows the Y-type equivalent circuit of the multi-ports DCT. The amplitude of square wave
voltage is determined by the original side voltage and transformer ratio. In order to realize soft switching, the voltage transformation ratio is 1. $i_1$, $i_2$ and $i_3$ are respectively the leakage current flowing through each transformer winding and converted to the original side. It is assumed that the leakage inductance of three transformer windings after conversion is equal. Figure 3 shows the steady-state voltage and current waveform in an ideal switching cycle.

![Figure 3. The steady-state voltage and current waveform in an ideal switching cycle](image)

When the commutation process is not considered in a switching cycle, a switching cycle can be divided into six modes of $t_0$ to $t_6$. The expressions of leakage current $i_{12}(\theta), i_{23}(\theta), i_{13}(\theta)$ of three-ports DCT in six working modes are listed in Table 1, where $\theta=\omega t$. 
Table 1 Leakage current of high frequency transformer under all working models

| Working mode | $i_{12} (\theta)$ | $i_{23} (\theta)$ | $i_{13} (\theta)$ |
|--------------|-------------------|-------------------|-------------------|
| Mode 1       | $\frac{V_1 + V_2}{\omega L_{12}} \theta + i_{12} (0)$ | $-\frac{V_2 + V_3}{\omega L_{23}} (\theta + \pi - \phi_{13}) + i_{23} (\pi + \phi_{23})$ | $\frac{V_1 + V_3}{\omega L_{13}} \theta + i_{13} (0)$ |
| $0 < \theta \leq \phi_{12}$ |                  |                   |                   |
| Mode 2       | $\frac{V_1 - V_2}{\omega L_{12}} (\theta - \phi_{12}) + i_{12} (\phi_{12})$ | $\frac{V_2 + V_3}{\omega L_{23}} (\theta - \phi_{13}) + i_{23} (0)$ | $\frac{V_1 + V_3}{\omega L_{13}} \theta + i_{13} (0)$ |
| $\phi_{12} < \theta \leq \phi_{13}$ |                  |                   |                   |
| Mode 3       | $\frac{V_1 - V_2}{\omega L_{12}} (\theta - \phi_{12}) + i_{12} (\phi_{12})$ | $\frac{V_3 - V_2}{\omega L_{23}} (\theta - \phi_{23}) + i_{23} (\phi_{23})$ | $\frac{V_1 - V_3}{\omega L_{13}} (\theta - \phi_{13}) + i_{13} (\phi_{13})$ |
| $\phi_{13} < \theta \leq \pi$ |                  |                   |                   |
| Mode 4       | $-\frac{V_1 - V_2}{\omega L_{12}} (\theta - \pi) + i_{12} (\pi)$ | $\frac{V_2 - V_3}{\omega L_{23}} (\theta - \phi_{23}) + i_{23} (\phi_{23})$ | $-\frac{V_1 - V_3}{\omega L_{13}} (\theta - \pi) + i_{13} (\pi)$ |
| $\pi < \theta \leq \pi + \phi_{12}$ |                  |                   |                   |
| Mode 5       | $-\frac{V_1 + V_2}{\omega L_{12}} (\theta - \phi_{12}) + i_{12} (\phi_{12})$ | $-\frac{V_2 - V_3}{\omega L_{23}} (\theta - \phi_{23}) + i_{23} (\pi)$ | $-\frac{V_1 + V_3}{\omega L_{13}} (\theta - \pi) + i_{13} (\pi)$ |
| $\pi + \phi_{12} < \theta \leq \pi + \phi_{13}$ |                  |                   |                   |
| Mode 6       | $-\frac{V_1 + V_2}{\omega L_{12}} (\theta - \phi_{12}) + i_{12} (\phi_{12})$ | $\frac{V_3 + V_2}{\omega L_{23}} (\theta - \phi_{23}) + i_{23} (\pi + \phi_{23})$ | $-\frac{V_1 + V_3}{\omega L_{13}} (\theta - \phi_{13}) + i_{13} (\pi + \phi_{13})$ |
| $\pi + \phi_{13} < \theta \leq 2\pi$ |                  |                   |                   |

Within the positive and negative half cycles of a working cycle, the product of time and voltage of transformer winding shall be equal, so the leakage current shall meet the following boundary conditions:

\[
\begin{align*}
  i_{12} (0) &= -i_{12} (\pi) \\
  i_{12} (\phi_{12}) &= -i_{12} (\pi + \phi_{12}) \\
  i_{12} (\pi) &= i_{12} (\phi_{12}) = -i_{12} (\phi_{12}) \\
  i_{12} (\pi + \phi_{12}) &= i_{12} (\phi_{12}) = -i_{12} (\phi_{12}) \\
  i_{12} (\phi_{12}) &= -i_{12} (0) = \frac{V_1 + V_2}{\omega L_{12}} \phi_{12} \\
  i_{12} (\pi) &= i_{12} (\phi_{12}) = \frac{V_3 - V_2}{\omega L_{12}} (\pi - \phi_{12}) \\
  i_{12} (\pi + \phi_{12}) &= i_{12} (\phi_{12}) = \frac{V_3 + V_2}{\omega L_{12}} (\pi + \phi_{12})
\end{align*}
\]

From formula (1) and (2), the boundary conditions of $i_{12}$ can be solved:

\[
\begin{align*}
  i_{12} (0) &= \frac{V_2 - V_3}{2\omega L_{12}} (\pi - \phi_{12}) - \frac{V_2 + V_3}{2\omega L_{12}} \phi_{12} \\
  i_{12} (\phi_{12}) &= \frac{V_3 - V_2}{2\omega L_{12}} (\pi - \phi_{12}) + \frac{V_3 + V_2}{2\omega L_{12}} \phi_{12}
\end{align*}
\]

Similarly, the boundary conditions of $i_{13}$ and $i_{23}$ can be solved. In one working cycle, the power provided by port 1 is:
Where $V_1(\theta)$ is:

$$V_1(\theta) = \begin{cases} V_1, & 0 < \theta \leq \pi \\ -V_1, & \pi < \theta \leq 2\pi \end{cases}$$ (5)

The calculation method of the output power of port 2 and port 3 is similar to that of port 1. From formula (1)~(5) and Table 1:

$$P_1 = \frac{V_1 V_3}{\pi \omega L_{13}} \phi_{13} (\pi - |\phi_{13}|) + \frac{V_2 V_3}{\pi \omega L_{12}} \phi_{12} (\pi - |\phi_{12}|)$$

$$P_2 = \frac{V_1 V_3}{\pi \omega L_{12}} \phi_{12} (\pi - |\phi_{12}|) - \frac{V_2 V_3}{\pi \omega L_{23}} \phi_{23} (\pi - |\phi_{23}|)$$

$$P_3 = \frac{V_1 V_3}{\pi \omega L_{13}} \phi_{13} (\pi - |\phi_{13}|) + \frac{V_2 V_3}{\pi \omega L_{23}} \phi_{23} (\pi - |\phi_{23}|)$$ (6)

Where $P_1$, $P_2$, $P_3$ represents the active power transmitted by Ports 1, 2 and 3 respectively; the phase-shifting angles of the three square wave voltages satisfy the relation $\phi_{12}+\phi_{23}-\phi_{13}=0$. Therefore, there are only two independent phase-shifting angle variables in the power transmission equation. $\phi_{12}$ and $\phi_{23}$ are selected as power transmission control variable.

### 3. Output voltage model of port

Port 1 is connected with DC power supply, and port 2 and port 3 are connected with the load. Define that the power flow direction of port 1 is positive in the direction of inflow to the multi-port DCT; the power flow direction of port 2 and port 3 is positive in the direction of outflow from the DCT. Under the condition that the power flows from port 1 to port 2 and port 3, the phase of $V_1$ is ahead of $V_2$ and $V_3$, that is, the phase shift angle $\phi_{12}>0$ and $\phi_{13}>0$, and $\phi_{23}=\phi_{13}-\phi_{12}$.

So, from formula (6):

$$P_2 = \frac{V_1 V_3}{\pi \omega L_{12}} \phi_{12} (\pi - |\phi_{12}|) - \frac{V_2 V_3}{\pi \omega L_{23}} (\phi_{13} - \phi_{12}) [\pi - (\phi_{13} - \phi_{12})]$$ (7)

$$P_3 = \frac{V_1 V_3}{\pi \omega L_{13}} \phi_{13} (\pi - |\phi_{13}|) + \frac{V_2 V_3}{\pi \omega L_{23}} (\phi_{13} - \phi_{12}) [\pi - (\phi_{13} - \phi_{12})]$$ (8)

Ports 2 and 3 are connected with resistance load, simplified (7) and (8):

$$V_2 = \frac{R_2 V_1}{\pi \omega L_{12}} \phi_{12} (\pi - |\phi_{12}|) - \frac{R_2 V_2}{\pi \omega L_{23}} (\phi_{13} - \phi_{12}) [\pi - (\phi_{13} - \phi_{12})]$$ (9)

$$V_3 = \frac{R_2 V_1}{\pi \omega L_{13}} \phi_{13} (\pi - |\phi_{13}|) + \frac{R_2 V_2}{\pi \omega L_{23}} (\phi_{13} - \phi_{12}) [\pi - (\phi_{13} - \phi_{12})]$$ (10)

Take the output voltage $V_2$ of port 2 as an example. $V_2$ is related to the phase shift angle $\phi_{12}$ and the difference between $\phi_{13}$ and $\phi_{12}$. Generally, the phase-shift angle is the output of the PI controller through the difference between the given voltage and the feedback voltage. $\phi_{12}$ is the output of the PI controller through the difference between the given voltage $V_2^*$ of port 2 and the feedback voltage $V_2$. $\phi_{13}$ is the output of the PI controller through the difference between the given voltage $V_3^*$ of port 2 and the feedback voltage $V_3$. According to formula (9), the control block diagram of output voltage $V_2$ can be obtained, as shown in Figure 4.
The transfer function of PI controller is $k_p + \frac{k_i}{s}$; $G_1(s)$ represents the transfer function of the first term of formula (9), and $G_2(s)$ represents the transfer function of the second term of formula (9). According to Figure 4:

\[
\phi_{12} = \left(V_2^* - V_2\right) \left(k_p + \frac{k_i}{s}\right) \tag{11}
\]

\[
\phi_{13} = \left(V_3^* - V_3\right) \left(k_p + \frac{k_i}{s}\right) \tag{12}
\]

\[
V_2 = \phi_{12} G_1(s) - \left(\phi_{13} - \phi_{12}\right) G_2(s) = \phi_{12} [G_1(s) + G_2(s)] - \phi_{13} G_2(s) \tag{13}
\]

Substituting (11) and (12) into (13):

\[
V_2 = \left(V_2^* - V_2\right) \left(k_p + \frac{k_i}{s}\right) \left[G_1(s) + G_2(s)\right] - \left(V_3^* - V_3\right) \left(k_p + \frac{k_i}{s}\right) G_2(s) \tag{14}
\]

Further, formula (14) can be reduced to:

\[
V_2 = \frac{V_2^* \left(k_p + \frac{k_i}{s}\right) \left[G_1(s) + G_2(s)\right] - \left(V_3^* - V_3\right) \left(k_p + \frac{k_i}{s}\right) G_2(s)}{1 + \left(k_p + \frac{k_i}{s}\right) \left[G_1(s) + G_2(s)\right]} \tag{15}
\]

When studying the steady-state control strategy, if only $V_2$ is taken as the controlled object, $V_1$ can be regarded as working in the steady-state state, that is, $V_3 = V_3^*$. The formula (15) can be rewritten as:

\[
V_2 = \frac{V_2^* \left(k_p + \frac{k_i}{s}\right) \left[G_1(s) + G_2(s)\right]}{1 + \left(k_p + \frac{k_i}{s}\right) \left[G_1(s) + G_2(s)\right]} \tag{16}
\]

Let $s = j\omega$. Because the working voltage of the converter is DC, $\omega = 0$. And then:

\[
\left(k_p + \frac{k_i}{s}\right) = \left(k_p + \frac{k_i}{j\omega}\right) \rightarrow +\infty \tag{17}
\]

Formula (17) is substituted into formula (13). Since the gains of $G_1(s)$ and $G_2(s)$ are both finite, the calculation result is shown below:

\[
V_2 = V_2^* \tag{18}
\]

According to formula (18), although the voltage and power of each port of the multi-port DCT are coupled, the precise control of each port voltage can be realized by PI controller. Similarly, it can be proved that by controlling the phase shift angle $\phi_{13}$ with PI controller, the output voltage $V_3$ can be controlled to make it completely consistent with the given voltage. Therefore, the control system diagram of multi-port DCT in steady state can be obtained, as shown in Figure 5.
Figure 5. Control system diagram of multi-port DCT in steady state

$V_2^*$ compared with the given voltage $V_2^*$, through PI controller, the output is $\varphi_{12}$, which is taken as phase shift angle. $V_3^*$ compared with the given voltage $V_3^*$, through PI controller, the output is $\varphi_{13}$, which is taken as phase shift angle. Finally, according to the generated phase-shift angle, the delay between different ports is generated to drive the corresponding power semiconductor devices on and off.

4. Simulation and experiment results

To verify the correctness of the theoretical analysis, a simulation model of multi-ports DCT is built in PSIM software. The corresponding parameters are shown in Table 2. Among them, the DC power supply is the ideal power supply and the load is the resistance. The multi-ports DCT has three ports, among which port 1 is fixed as DC power supply, and port 2 and port 3 are connected with resistance load. The simulation is carried out according to the following conditions:

Operating condition 1: port 2 is connected with resistance load which is 12 $\Omega$ and port 3 is connected with resistance load which is 18 $\Omega$. At this time, the output power of port 2 is greater than that of port 3.

Operating condition 2: The first 0.25s is the same as operating condition 1. After 0.25s, port 3 is connected to resistance load which is 18 $\Omega$ in parallel. At this time, the two parallel 18 $\Omega$ load resistances of port 3 are equivalent to a 9 $\Omega$ load resistor. Port 2 is connected to resistance load which is 12 $\Omega$ and remains unchanged. The output power of port 3 is greater than that of port 2.

Table 2 Parameters of multi-ports DCT simulation model

| Name of parameter                              | Values of parameters |
|------------------------------------------------|----------------------|
| DC power supply Voltage                        | $V_1 = 40$ V         |
| output load voltage                            | $V_2 = V_3 = 40$ V   |
| ratio of HFT                                    | 1:1:1                |
| Leakage inductance of HFT                      | $L_1 = L_2 = L_3 = 40$ $\mu$H |
| Output capacitance of DC side                  | $C_1 = C_2 = C_3 = 6.56$ mF |
| Initial voltage of output capacitance of DC side| 40 V                 |
| switching frequency                            | 5 kHz                |
| step time                                       | 0.2 $\mu$s           |

Under operation condition 1, the output DC voltage of port 2 is $V_2$, and the output DC voltage of port 3 is $V_3$. Their waveforms are shown in Figure 6. Under operation condition 2, the output DC voltage of port 2 is $V_2$, and the output DC voltage of port 3 is $V_3$. Their waveforms are shown in Figure 7.
According to the analysis of Figure 6 and Figure 7, the output voltage of this port can be stabilized and kept constant only by controlling the phase-shifting angle. The simulation results show that the port voltage control model studied in this paper is correct.

5. Conclusion
In this paper, the voltage control model of multi-port DCT is studied. The power transmission characteristics of multi-ports DCT is analyzed. On this basis, the output voltage control model of multi-ports DCT in steady state is established. Combined with the control model, it is proved that the output voltage can be controlled by controlling the phase shift angle. The correctness of the control model is verified by simulation.

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