Pion Absorption Cross Section for $^2$H and $^3$He in the $\Delta$-Isobar Region: A Phenomenological Connection

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The absorption of $\pi^+$ on $^3$He in the $\Delta$-region is evaluated with exact inclusion of the final state interaction among the three emerging protons. The absorption is described by a $\pi N \rightarrow \Delta$ vertex and a $N\Delta - NN$ transition t-matrix which are calculated from a phenomenological model for $NN$ and $\pi d$ reactions. In a calculation where the initial pion scattering effects are neglected, the predicted peaks of the pion absorption cross sections for $^2$H and $^3$He lie too high in energy in relation to the data. The effect of the final state three-nucleon interaction turns out to be too small for changing the magnitude and shifting the peak position of the total absorption cross section for $^3$He. We demonstrate that the adjustment of the peak position for the deuteron cross section by small modifications of the $\Delta$-parameters, automatically leads to the correct peak position in $^3$He.

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1. INTRODUCTION

In recent years it became possible to solve the quantum mechanical three-body problem with realistic two- and three-nucleon forces [1–5]. Powerful computer facilities allowed this important step forward. Except for a few observables the theoretical predictions based on realistic $NN$ forces agree very well with the experimental data. The exact treatment of the strong rescattering among the three particles is thereby crucial.

The Faddeev equations have been applied not only to the pure $3N$ system but also to inelastic electron scattering on $^3$He [6–8]. The Faddeev formalism allowed to calculate any breakup process, exclusive [8] and inclusive [9] ones. In the same manner we apply now the Faddeev equations to investigate pion absorption phenomena. The simplest reaction is pion absorption on the deuteron requiring the study of the $\pi NN$ system for which a vast literature exists [10–21]. We shall not try to improve our understanding of this system, rather we shall present an exploratory calculation of pion absorption on $^3$He which is motivated by recent experimental studies of the reaction $\pi^+^3$He $\rightarrow 3p$ by the LADS collaboration at PSI [22–24]. In this first study we shall treat the dynamics of the incoming pion approximately. In particular we shall not allow for initial state interactions where the pion is rescattered before it is absorbed. We shall assume that pion absorption takes place in the first step by a $\Delta$-resonance mechanism and after that the nucleons interact strongly in a $3N$ state. We shall determine the effect of this final state interaction for the total absorption cross section. To the best of our knowledge this will be the first time that an exact treatment of FSI has been performed. The choice of $^3$He instead of $^2$H also allows for pion absorption on 2 nucleons not only in isospin $t = 0$ but also in $t = 1$ states. Finally choosing $\pi^+$ absorption on $^3$He generates a system of three interacting final protons, which cannot be realized in a pure $3N$ scattering process.

In this study we are interested in the $\Delta$-resonance energy range and therefore we introduce explicitly the $\Delta$ degree of freedom. We use the phenomenological $NN-N\Delta$ model of Betz and Lee [17] which treats the $\pi N\Delta$ vertex that is responsible for pion absorption in a self consistent way. In the present exploratory calculation we exclude for simplicity the contributions corresponding to propagating $\pi NN$ intermediate states. Ohta, Thies and Lee [25] applied a similar simplification of the model of Betz and Lee to heavier nuclei but did not include final state interactions and had to rely on simple model target wavefunctions, whereas here we shall use a realistic $^3$He description. A glance at the pion absorption cross section for $^2$H and $^3$He reveals immediately that it peaks around $T_{\pi} \approx 130$ MeV, whereas the elastic and inelastic cross sections peak around 170 MeV, closer to the position of the $\Delta$ resonance in free $\pi N$ scattering. We shall establish that FSI is not related to that shift in the peak position. However, it is possible to describe the energy shifts in both nuclei by a common parametrization of the underlying mechanism.
In section II we briefly outline the way we use the Faddeev equations to describe the final state interaction for pion absorption on $^3$He. Since it is similar in structure to inelastic electron scattering on $^3$He we can refer to various articles [9] for more details and show only those steps which are specific to the pion absorption process. This is presented in section III. In order to show how we treat the $\Delta$-particle we introduce the $N\Delta$ propagator in section IV. A formalism very similar in structure to ours has been presented before in [26], though no numerical application thereof is known to us. Our numerical results are shown in section V. We summarize and give an outlook in section VI.

II. FORMALISM

Let us first describe a situation where the pion is absorbed on a nucleon converting it into a $\Delta$ particle which then together with a second nucleon undergoes an infinite number of rescatterings described by a two-body t-matrix $t_{NN,NN\Delta}$. That two-body t-matrix obeys a coupled set of Lippmann-Schwinger equations

$$(t_{NN,NN} \quad t_{NN,NN\Delta}) = \begin{pmatrix} V_{NN,NN} & V_{NN,NN\Delta} \\ V_{NN\Delta,NN} & V_{NN\Delta,NN\Delta} \end{pmatrix} + \begin{pmatrix} V_{NN,NN} & V_{NN,NN\Delta} \\ V_{NN\Delta,NN} & V_{NN\Delta,NN\Delta} \end{pmatrix} \begin{pmatrix} G^0_{NN} & 0 \\ 0 & G^0_{NN\Delta} \end{pmatrix} \begin{pmatrix} t_{NN,NN} & t_{NN,NN\Delta} \\ t_{NN\Delta,NN} & t_{NN\Delta,NN\Delta} \end{pmatrix}$$

(1)

In our model calculation we choose the transition potentials $V$ from the analysis of Betz and Lee [17]. The iteration of Eq.(1) describes the consecutive transitions between the $\Delta N$ system generated by the pion absorption and the resulting $NN$ system. The resulting amplitude has the form

$$|\Gamma\rangle = t_{NN,NN\Delta}G^0_{NN\Delta}F(\pi)|\pi,^3\text{He}\rangle$$

(2)

where $G^0_{NN\Delta}$ is the free $NN\Delta$ propagator, $F(\pi)$ is the $\pi$-absorption vertex function and $|\pi,^3\text{He}\rangle$ is the initial state. This term is depicted in Fig.1. The amplitude $\Gamma$ is the starting point for the rescattering processes among the three nucleons. Taken by itself it provides the properly symmetrised impulse approximation

$$U_{DWIA} = \frac{1}{\sqrt{3}}(1 + P)|\Gamma\rangle$$

(3)

Here DWIA means distorted waves with respect to the two-body subsystem and plane wave with respect to the third particle. We use the usual permutation operator $P$, a sum of a cyclic and an anticyclic permutation of 3 objects, which is a very convenient structural element in the Faddeev treatment of three identical particles [27]. The three nucleon rescattering amplitude

$$U_{\text{rescatt}} = \frac{1}{\sqrt{3}}(1 + P)T_{NN}|\Gamma\rangle$$

(4)

is generated by the operator $T_{NN}$, which obeys

$$T_{NN}|\Gamma\rangle = t_{NN,NN}G^0_{NNNN}P|\Gamma\rangle + t_{NN,NN}G^0_{NNNN}PT_{NN}|\Gamma\rangle$$

(5)

Here $T_{NN}$ is a three-body operator and $t_{NN,NN}$ is a two-body operator as depicted in Fig.2. Because of our simplifying assumption a re-occurrence of a $\Delta$-particle is not allowed, thus only the free 3N propagator $G^0_{NNNN}$ occurs.

Comparing Eq.(3) to the corresponding equation for inelastic electron scattering [18], we see that the driving term is modified due to the absence of the term $t_{NN,NN}G^0_{NNNN}|\Gamma\rangle$. That term would double-count the $NN$ interaction, since $|\Gamma\rangle$ contains the $NN$ interaction to infinite order in the same particle channel. In electron scattering $|\Gamma\rangle$ is driven by the electromagnetic current operator and no double counting occurs.

For the processes discussed up to now the breakup amplitude is

$$U = U_{DWIA} + U_{\text{rescatt}}$$

(6)

and this will be investigated numerically.

So far the rescattering parts of the diagrams were three nucleon reducible. Nonreducible diagrams shown in Fig.3 are likely to play an important role and will be investigated numerically in a forthcoming article. Here we just present the necessary formal extensions. The $\Delta$ resulting from the the absorption of the initial pion can be absorbed and
re-excited on another nucleon line, a process that can be iterated before the three nucleon final state is reached. This is incorporated in the amplitude$^1$

$$ U^{ISI} = \frac{1}{\sqrt{3}}(1 + P)T^{ISI}|\Gamma\rangle $$

where the superscript $ISI$ stands for initial state interaction and $T^{ISI}$ obeys the integral equation

$$ T^{ISI}|\Gamma\rangle = T_{NN} t_{NN,N\Delta} G_{NN,N\Delta}^0 P T_{N\Delta} F(\pi,^3\text{He}) + T_{NN} t_{NN,N\Delta} G_{NN,N\Delta}^0 P T_{N\Delta} t_{N\Delta,N\Delta} G_{N\Delta,NNN}^0 P T^{ISI}|\Gamma\rangle $$

and $T_{N\Delta}$ generates all possible $N\Delta$ pairs via

$$ T_{N\Delta} = t_{N\Delta,N\Delta} G_{N\Delta,N\Delta}^0 + t_{N\Delta,N\Delta} G_{N\Delta,NNN}^0 P T_{N\Delta}. $$

Iterating Eqs.(5,8,9) one can visualize the processes contained in $T^{ISI}$ as is shown in Figs.4,5. Fig. 3 is the simplest new diagram contained in Fig.4 representing the initial state interaction. An example of an additional final state interaction (the leading term of Eq.(8)) is shown in Fig.4.

### III. CHOICE OF COORDINATES

The amplitude $|\Gamma\rangle$ contains three steps: the pion absorption by the single particle operator $F(\pi)$, the free propagator of the (zero width) $\Delta$-particle and two nucleons, and the action of the transition operator $t_{NN,N\Delta}$ converting the $N\Delta$ system into a two-nucleon system. In our three-body context this requires the use of various sets of Jacobi momenta.

The $^3\text{He}$ wavefunction depends on the following momenta

$$ \vec{p}' = \frac{1}{2}(\vec{k}'_2 - \vec{k}'_3) $$

$$ \vec{q}' = \frac{2}{3}(\vec{k}'_1 - \frac{1}{2}(\vec{k}'_2 + \vec{k}'_3)) $$

defined in terms of the individual momenta of three nucleons. After the pion absorption on nucleon 1 we describe the system consisting of two nucleons and a $\Delta$-particle by

$$ \vec{p} = \frac{1}{2}(\vec{k}_2 - \vec{k}_3) $$

$$ \vec{q} = \frac{2M_N\vec{k}_1 - (M_N + \omega)(\vec{k}_2 + \vec{k}_3)}{3M_N + \omega}, $$

where $M_N$ is the nucleon mass and $\omega = \sqrt{\mu^2 + k^2_\pi}$ the energy of the pion in the overall CMS. We choose the single particle operator $F(\pi)$ to depend on the relative momentum $\vec{q}_0$ of nucleon 1 and the pion:

$$ \vec{q}_0 = \frac{M_N\vec{k}_x - \omega\vec{k}'_1}{M_N + \omega} = \vec{k}_x - \frac{\omega}{M_N + \omega}\vec{q}. $$

The second equality holds true in the overall CMS. The functional dependence of $F(\pi)$ related to the p-wave property of the $\Delta$-particle is given in the Appendix. We define

$$ \langle \vec{k}_1 | F | \vec{k}'_1 \vec{k}_x \rangle = F(\vec{q}_0) \delta(\vec{k}_1 - \vec{k}_1' - \vec{k}_x) $$

where $\vec{k}_1$ is the momentum of the $\Delta$-particle. Therefore we have

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1In the 3N problem this quantity would be called $U_0$ with the index denoting the three nucleon continuum channel. This is unnecessary here since $\pi^+$ absorption on $^3\text{He}$ has no other channels.

2Following Ref. [17] the quantity $M_N + \omega$ is used interchangeably with $M_\Delta$ in the resonance energy range.
\[ \langle \bar{p}q | F | \pi^3 \text{He} \rangle = \int d\bar{p}' d q' \langle \bar{p}q | F | \bar{p}' q' \rangle \langle \bar{p}' q' | \pi^3 \text{He} \rangle = \int d q' F(q_0) \delta(q-q') - \frac{2}{3} \delta E \langle \bar{p}q | \pi^3 \text{He} \rangle . \]

The transition operator \( t_{NN,N} \) acting between particles 1 and 2 requires another set of Jacobi momenta

\[
\bar{p}'' = \frac{M_N k_1 - (M_N + \omega) k_2}{2M_N + \omega} \]

\[ q'' = \frac{(2M_N + \omega) k_3 - M_N (k_1 + k_2)}{3M_N + \omega} \]

They are related to \( \bar{p} \) and \( q \) by

\[
\bar{p} = - \frac{3M_N + \omega}{2(2M_N + \omega)} q'' - \frac{1}{2} \bar{p}'' \]

\[ q = \frac{M_N + \omega}{2M_N + \omega} q'' + \bar{p}'' . \]

Finally we use Jacobi momenta denoted by \( \bar{p}'' \) and \( q'' \) for three nucleons analogous to (15-16) describing the three-nucleon system to the left of the \( NN, N\Delta \) transition matrix, see Fig. 3. Since the transition \( t \)-matrix is diagonal in \( q'' \) and \( \bar{q}'' \) we finally get

\[
\langle \bar{p}'' q'' | t_{NN,N} G_{NNN}^0 | \pi^3 \text{He} \rangle = \int d\bar{p}'' \langle \bar{p}'' | t_{NN,N} | \bar{p}'' \rangle G_{NNN}^0 (\bar{p}'', q'') \int d\bar{p} dq \langle \bar{p}'' q'' | \bar{p}q \rangle \langle \bar{p}q | \pi^3 \text{He} \rangle . \]

**IV. DRESSING THE \( \Delta \)-PARTICLE**

So far we have introduced the momentum space representation of the \( NN - N\Delta \) transition operator and the free \( N\Delta \) propagator. In the Betz-Lee model [17] the \( N\Delta \) propagator is dressed

\[
G_{N\Delta}^0 = \frac{1}{E - (M_{\Delta}^0 - M_N) - \frac{k^2(M_N + M_{\Delta}^0)}{2M_NM_{\Delta}^0} - \Sigma_{N\Delta}(k,E)} , \]

where \( E \) is the CMS energy of the two-nucleon system, \( \vec{k} \) is the \( N\Delta \) relative momentum. The physical mass of the \( \Delta \)-particle is

\[
M_{\Delta} = M_{\Delta}^0 + \delta M \]

where \( M_{\Delta}^0 \) is the bare mass. The energy dependent self interaction \( \Sigma \) is

\[
\Sigma_{N\Delta}(k,E) = \int_0^\infty \frac{F^2(k') k'^2 dk'}{E + i\epsilon - H_{NN\pi}^0(k,k') - H_{NN\pi}^0(k')} = \delta M - i\Gamma/2 \]

with

\[
H_{NN\pi}^0(k,k') = \frac{k^2}{2M_N} + \frac{k^2}{2(M_N + \sqrt{\mu^2 + k'^2})} , \]

\[
H_{NN\pi}(k') = \frac{k'^2}{2M_N} + \sqrt{\mu^2 + k'^2} . \]

Here \( \Gamma \) is the energy dependent width of the \( \Delta \)-particle. The vertex function \( F \) contains the bare coupling constant \( F_{\Delta}^0 \) and the range parameter \( \Lambda_{\Delta} \), which are defined in the Appendix. The steps required for the partial wave representation are also described there. For the calculation below we shall allow small variations of the bare \( \Delta \) parameters \( M_{\Delta}^0 \) and \( F_{\Delta}^0 \).
V. RESULTS

In order to test the input for the pion absorption reaction on $^3$He we recalculated the total pion $\pi d$ absorption cross section on the deuteron as a function of energy in the Betz-Lee model. Table II shows the partial waves used, and the parameters for the potential are taken from [17].

In the present study we exclude for simplicity the contribution corresponding to propagating $\pi NN$ states and the initial pion scattering effects. Thus, our result for the deuteron is different from the full unitary calculation of Ref.[17]. The dashed line in Fig.7 shows the total cross section together with the data interpolated by the solid line. The dotted line shows a calculation without the $^1D_2$ partial wave, which demonstrates the importance of that wave.

As discussed in Ref.[17], the Betz-Lee model in our simplified approximation does not quantitatively reproduce the data. The peak position is about 30 MeV too high and the cross section is too low on the rising part below the resonance. The last feature is certainly partly related to neglecting non-resonant $\pi N$ partial waves. On the other hand, the shift of the resonance has been obtained correctly in models containing explicit pion propagaton in intermediate states [2][12]. In the present paper we stick to the pure $\Delta$-model excluding explicit $NN\pi$ propagation for the reaction on the deuteron or $NN\pi N$ in the case of $^3$He. We have therefore adjusted the bare parameters $M_\Delta^0$ and $F_\Delta^0$ of the Betz-Lee model in order to reproduce the observed energy dependence of the total cross section on the deuteron. Only small changes are needed to reproduce size and position at the resonance, see Fig III

$$M_\Delta^0 = 1280 \rightarrow 1260 \text{ MeV}$$ (25)

and

$$F_\Delta^0 = 0.98 \rightarrow 1.00$$ (26)

Since the new parameters reflect pion propagation in the absorption reaction in an effective way, it is clear that the elastic $\pi N$ and the elastic $\pi d$ cross sections will not be correctly described. The same is true for the $NN$ phase shifts. As an illustration we show the effect of the new parametrization on the $^1D_2$ partial wave in Fig.9. The Betz-Lee approach which we use also neglects the diagonal potential $V_{N\Delta,N\Delta}$. We have verified that the inclusion of such a potential allows to shift the peak position downwards. For the time being, however, we restrict ourselves to treating $M_\Delta^0$ and $F_\Delta^0$ as the only effective parameters and retain $V_{N\Delta,N\Delta}=0$.

It is gratifying that the effective parameters of Fig III also improve the description of the total cross section on $^3$He as is shown in Fig IV. We therefore see that the gross feature on $^3$He falls into place once the reaction on the deuteron is properly described. In Fig V the effects of final state interactions in the $3N$ continuum state are fully included for the first time. On the scale of the figure the effect of FSI is too small to be drawn (2%). For the total cross section FSI is thus negligible. For observables and kinematics which are not dominated by the two nucleon DWIA mechanism (the quasi-deuteron process) significant modifications due to FSI are however to be expected.

VI. SUMMARY AND OUTLOOK

We formulated a model of pion absorption on $^3$He in a Faddeev scheme, which includes the final state interaction among the three outgoing nucleons and which also allows for initial state interaction where more than one $\Delta$-resonance is excited (see Fig III). The numerical evaluation in this paper is restricted to the leading quasi-deuteron absorption term with inclusion of the final state interactions, Eq. (3), between the three protons. The phenomenological Betz-Lee model for the $NN-NN$ and $NN-N\Delta$ systems is used. In the present exploratory calculation where the initial pion scattering and the contributions corresponding to propagating $\pi NN$ states are neglected, the resulting total pion absorption cross sections for $^2$H and $^3$He do not agree with the data. The most striking feature is, that the theoretical peak positions are too high in comparison to experiment. The full inclusion of the final state interaction among the 3 nucleons in our model has no visible effect for the total cross section, its contribution is only about 2%.

We introduced a very simple method to shift the peak position by treating the bare $\Delta$-mass $M_\Delta^0$ and the coupling strength $F_\Delta^0$ as parameters. We demonstrated, that a slight modification of order 2% reproduces the $^2$H and $^3$He cross sections at the same time. This points to a common mechanism for pion absorption in both nuclei.

The Betz-Lee model sets the transition potential $V_{N\Delta,N\Delta}$ to zero. In a forthcoming study we shall abolish that assumption and include the transition potential $V_{N\Delta,N\Delta}$, as it occurs for instance in the phenomenological AV28 potential [23]. At the same time we shall investigate the importance of initial state interactions introduced in Eq.(7). The diagonal $N\Delta$-potential is expected to be important in this context.
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APPENDIX: PARTIAL WAVE REPRESENTATIONS

Here we present the partial wave representation used. It is related to the choice of Jacobi coordinates of Fig. 3, see also Sec. III. For three particles (three nucleons or two nucleons and a Δ-particle) the partial wave basis in momentum space is

$$ | p, q, \alpha JMTM_T \rangle = | p, q, (s_2s_3)s(ls)j(\lambda s_1)i(jI)JM(\tau_2\tau_3)t(t_1)TM_T \rangle $$

(27)

where the orbital angular momenta \( l \) and \( \lambda \) are related to \( \vec{p} \) and \( \vec{q} \), and \( s_i, \tau_i \) ( \( i=1,2,3 \)) are spins and isospins, respectively.

The initial state nucleus, the \(^3\)He ground state, has \( J = \frac{2}{3} \) and \( T = \frac{1}{2} \). The final \( ppp \) state has \( T = \frac{3}{2} \). The \( \pi N \Delta \)

vertex function is written as

$$ F(q_0) = F(q_0) \sum_{m,\mu} \left( \frac{3}{2} \frac{3}{2} m \right) \sum_{m_1, m_N} \langle \frac{3}{2} m | 1m_1 \frac{1}{2} m_N \rangle Y^*_{1m_1} (q_0) \langle \frac{3}{2} \mu | \frac{1}{2} \mu_N \rangle \langle \frac{1}{2} m_N \frac{1}{2} \mu_N | $$

(28)

where \( m \) and \( m_N \) are the z-component of the spin of the \( \Delta \) and the nucleon, \( \mu \) and \( \mu_N \) are the corresponding isospin quantum numbers. The \( \pi N \Delta \) vertex function is written as

$$ F(q_0) = \frac{F_\Delta^0}{\sqrt{2(M_N + \mu)} \mu} \left( \frac{\Lambda_\Delta^2}{\Lambda_\Delta^2 + q_0^2} \right)^2 $$

(29)

where \( F_\Delta^0 = 0.98 \) and \( \Lambda_\Delta = 358 \) MeV/c. Using this operator \( F \) of Eq. (14) can be written as

$$ \langle p, q, \alpha' J' T' M'_T | F \Psi JMTM_T \rangle = \sum_{\alpha} \delta_{i,j'} \delta_{s,s'} \delta_{j,j'} \delta_{i,i'} (X_1 + X_2) \mathcal{I} $$

(30)

where

$$ X_1 = k_\pi \sqrt{\frac{3}{4\pi} (-)^{\lambda+j'+l+j-J-M} \sqrt{\lambda \lambda'I'j'j' \sum_{\lambda_1+\lambda_2=\lambda} \nu^{\lambda_1(\frac{2}{3} k_\pi)} \lambda_2 \sqrt{\frac{\lambda!}{\lambda_1!\lambda_2!}} \sqrt{\lambda_1 \lambda_2}} $$

$$ \times \sum \beta' \sum_{\ell} \tilde{L} \left\{ \begin{array}{c} \lambda_1 \\ \lambda_2 \\ \lambda' \\ \ell \end{array} \right\} \langle \begin{array}{c} \lambda_1 \\ \lambda_2 \end{array} | \begin{array}{c} \lambda' \\ \ell \end{array} \rangle \langle \begin{array}{c} \lambda_1 \lambda_2 b \end{array} | \begin{array}{c} \lambda' \ell \end{array} \rangle \rangle \langle \beta' \rangle \right\} \mathcal{I} $$

(31)

$$ X_2 = -e q \sqrt{\frac{3}{4\pi} (-)^{j'-j+l+l'-M} \sqrt{\lambda \lambda'I'j'j' \sum_{\lambda_1+\lambda_2=\lambda} \nu^{\lambda_1(\frac{2}{3} k_\pi)} \lambda_2 \sqrt{\frac{\lambda!}{\lambda_1!\lambda_2!}} \sqrt{\lambda_1 \lambda_2}} $$

$$ \times \sum \tilde{b} \tilde{b}' \sum_{\ell} \tilde{L} \left\{ \begin{array}{c} \lambda \end{array} \right\} \langle \begin{array}{c} \lambda_1 \lambda_2 b \end{array} | \begin{array}{c} \lambda' \ell \end{array} \rangle \langle \begin{array}{c} \lambda_1 \lambda_2 b \end{array} | \begin{array}{c} \lambda' \ell \end{array} \rangle \rangle \langle \beta \rangle \rangle \langle \beta' \rangle \rangle \mathcal{I} $$

(32)
\[ I = \sqrt{4T^T(-)^{\frac{1}{2}+\epsilon+T-T-M_T}} \begin{pmatrix} T & \frac{1}{2} & t \\ \frac{1}{2} & T' & 1 \end{pmatrix} \begin{pmatrix} T & 1 & T' \\ M_T & \mu & -M_T \end{pmatrix} \] (33)

and

\[ \epsilon = \frac{\omega}{M_N + \omega} \] (34)

The function \( S^2_\alpha(p, q, k_\pi) \) is defined as

\[ S^2_\alpha(p, q, k_\pi) = \int_{-1}^{1} dx P_L(x) \frac{E(|\vec{k}_\pi - \epsilon \vec{q}|) \Psi_\alpha(p, |\vec{q} - \frac{2}{3} \vec{k}_\pi|)}{|\vec{k}_\pi - \epsilon \vec{q}|} \frac{1}{|\vec{q} - \frac{2}{3} \vec{k}_\pi|^2} \] (35)

where \( x \) is the cosine between \( \vec{q} \) and \( \vec{k}_\pi \), and \( \Psi_\alpha \) is the \(^3\)He wave function in the basis (27).
TABLE I. Partial wave decomposition of $NN$ and $N\Delta$ systems.

| $NN$   | $N\Delta$ |
|--------|-----------|
| $^1S_0$ | $^5D_0$   |
| $^3P_0$ | $^3P_0$   |
| $^3P_1$ | $^3P_1$, $^5P_1$ |
| $^3P_2$, $^3F_2$ | $^3P_2$, $^5P_2$ |
| $^1D_2$ | $^5S_2$   |
| $^3F_3$ | $^5P_3$   |
| $^4G_4$ | $^5D_4$   |
FIG. 1. The amplitude $|\Gamma\rangle$ of Eq. (2) describing $\pi^+$ absorption on a nucleon in $^3$He leading to a $\Delta$-particle and followed by a deexcitation into two nucleons.

FIG. 2. The $\pi^-$-absorption on $^3$He as described in Fig. 1 followed by the complete $3N$ final state interaction $T_{NN}$.

FIG. 3. Lowest order initial state interactions, ISI.
FIG. 4. The leading term of Eq.(8) representing initial state interactions acting in the Hilbert space of two nucleons and one \Delta-particle.

FIG. 5. The general representation of the second term of Eq.(8).

FIG. 6. Jacobi momenta and related orbital angular momenta.
FIG. 7. Total cross section for $\pi + d \rightarrow pp$ as a function of the laboratory pion kinetic energy. The data (solid line) are taken from [29–41]. The dashed and dotted lines are calculated from the Betz-Lee potential with all partial wave set of Table I and without the $^1D_2$ wave, respectively.

FIG. 8. Total cross section for $\pi + d \rightarrow pp$ as a function of the incident pion energy in laboratory system. The description of the lines is the same as in Fig. 7 with the exception of the dotted line which is now the theoretical prediction based on the new parameter set.
FIG. 9. The $^1D_2$ $NN$ phase shift as a function of lab. kinetic nucleon energy. The data (solid line) are represented by the partial wave from the SAID analysis [42]. The dashed and dotted lines correspond to the Betz-Lee potential and its new parameterization, respectively.

FIG. 10. Total cross section for $\pi^{+}^3$He $\rightarrow$ ppp as a function of the laboratory pion kinetic energy. The data are from PSI [22,43] and Refs. [44–50].