From Knowledge, Knowability and the Search for Objective Randomness to a New Vision of Complexity

Paolo Allegrini\textsuperscript{1}, Martina Giuntoli\textsuperscript{2}, Paolo Grigolini\textsuperscript{2,3,4}, Bruce J. West\textsuperscript{5}

\textsuperscript{1}Istituto di Linguistica Computazionale del Consiglio Nazionale delle Ricerche, Area della Ricerca di Pisa-S. Cataldo, Via Moruzzi 1, 56124, Ghezzano-Pisa, Italy
\textsuperscript{2}Center for Nonlinear Science, University of North Texas, P.O. Box 311427, Denton, Texas, 76203-1427
\textsuperscript{3}Dipartimento di Fisica dell’Università di Pisa and INFN Piazza Torricelli 2, 56127, Pisa, Italy
\textsuperscript{4}Istituto dei Processi Chimico Fisici del CNR Area della Ricerca di Pisa, Via G. Moruzzi 1, 56124, Pisa, Italy
\textsuperscript{5}Mathematics Division, Army Research Office, Research Triangle Park, NC 27709

(Dated: October 29, 2018)

Herein we consider various concepts of entropy as measures of the complexity of phenomena and in so doing encounter a fundamental problem in physics that affects how we understand the nature of reality. In essence the difficulty has to do with our understanding of randomness, irreversibility and unpredictability using physical theory, and these in turn undermine our certainty regarding what we can and what we cannot know about complex phenomena in general. The sources of complexity examined herein appear to be channels for the amplification of naturally occurring randomness in the physical world. Our analysis suggests that when the conditions for the renormalization group apply, this spontaneous randomness, which is not a reflection of our limited knowledge, but a genuine property of nature, does not realize the conventional thermodynamic state, and a new condition, intermediate between the dynamic and the thermodynamic state, emerges. We argue that with this vision of complexity, life, which with ordinary statistical mechanics seems to be foreign to physics, becomes a natural consequence of dynamical processes.

I. INTRODUCTION

Why do things get more complicated with the passage of time?

While it may not be a mathematical theorem, it certainly seems clear that as cultures, technologies, biological species and indeed most large-scale systems, those with many interacting components, evolve over time either they become more complex or they die out. The goal of understanding the mechanisms by which evolution favors increased complexity over time is too ambitious an undertaking for us here. We do not explore these mechanisms in part because they are phenomena-specific and we are concerned with only the universal properties of complexity even though it has no clear definition. More importantly, however, complexity is very often self-generating. Herein we discuss many of the problems and paradoxes that are entangled in the concept of complexity in the restricted domain of the physical sciences. This is done because if complexity does have universal properties they should be independent of the phenomena being studied and therefore we choose the simplest context possible. For example the generation and dissipation of fluctuations involved in complex phenomena are examined through the concepts of irreversibility and randomness, but more importantly through the \textit{objectivity principle}, see for example Monod \cite{1}. The science of complexity is multidisciplinary and so it might by argued that the schema we construct, based on the paradigm of physics, is incomplete. On the other hand, we believe that the principle of objectiveness gives “hard” sciences like physics an important advantage in addressing the difficult task of understanding complex systems. As a matter of fact, we should not ignore centuries of philosophy of science and, more generally speaking, epistemology, when we refer to the concept of objectiveness. It is right in this case indeed that science and philosophy benefit by mutual exchanges. We can accept the fact that science is located in a better position in order to understand and describe the reality of things, but the problem is that we cannot still have a clear and complete definition of what we mean by nature. Furthermore, we have to stress that in some cases philosophers exert an important role of stimulus for scientists to overcome the limitations of the current views. A significant example is given by the paper of Pete Gunter, in these Proceedings \cite{2}. This paper, in our opinion, sets challenges for the scientists, and we shall discuss how to address some of these challenges. We have, in fact, three different problems related to the principle of objectiveness. The first two are connected to the scientist and the third one to nature itself.

- Since the scientists are human beings, while they investigate the things around them, it happens that they not only describe the facts the way they are, but they unavoidably corrupt and interact with the things they are studying. This will become clearer below, when we shall explain the quantum mechanics paradigm. However, this is true also in a sense that is wider than the applicability of quantum mechanics itself.

- In addition to this, the scientist him/herself is part of a natural system. For this reason it is possible that for what we call the \textit{vicious circle} the scientist cannot see all the things that he/she should see. \cite{3,4}
• Finally, we have to come back to the old and thorny problem concerning the description of reality and the methods used in order to understand it. Herein, we shall be mainly concerned with this latter problem, namely the detection of true unknowables among many unknowns, and how to deal with an unsatisfactory abundance of unknowns, through a change of perspective, which is now becoming known as the “complexity paradigm”

These three points are important to understand in order that we can accept the principle of objectiveness even if some reserves have to be made. Nevertheless the analysis and the criticism of the methods used are a pre-eminant component of whichever process toward a renewed and improved science. In spite of what we said, we, of course, value science and scientists. Nobody would deny today that scientific methods, reductionism included, make us able to describe the reality of things to a very good approximation. The main purpose of what has been said until now is just to “problematize” objectiveness, namely, to question it and to consider whether it is either a cause or a symptom of larger problems, especially within last century’s perspective, i.e. reductionism. As a matter of fact, it is not at all easy to define these two concepts, especially reductionism. The reason why this happens is because reductionism is a concept “in fieri”. We do not have a definition for reductionism that can satisfy the whole scientific and philosophical community. One of the most difficult aspects of the reductionism process to define is its connection with the nature. In fact it is not clear if reductionism is just a method used by the scientist and applied to nature in order to understand it, or if it is an attribute of nature itself. Many scientists and philosophers have written many articles and books about this topics. Steven Weinberg, for example, has been very productive from this point of view (see, e.g., his book "Dreams of a final theory" [5]). According to him, one would understand all problems starting from fundamental equations governing particle physics. Chemistry would be an application of physics, biology would be a chemistry exercise, and so on. We are not surprised that a famous biologist, Ernst Mayr, was the first who clearly defined all the different kinds of reductions [8], with the purpose of circumscribing their applicability. According to his synthesis we can classify reductionism in three categories.

1. Constitutive reductionism, namely the scientific method in which one studies the components to understand the system is no doubt the key to success of decades of science, and no scientist doubts this.

2. Theory reduction, or “the phenomenon of relatively autonomous theories becoming absorbed by, or reduced to, some other more inclusive theory” is, on the other hand, strongly criticized. A classical project is the reduction of thermodynamics to mechanics, which has never been accomplished. We shall see later in this paper that a new vision of complexity emerges from this failure in realizing this project, in the case of non-ordinary statistical mechanics.

3. Explanatory reductionism, at last, is completely rejected by Mayr. This kind of reductionism states that a complete knowledge of a system can be derived by the mere knowledge of its components. In fact, it is not immediately and totally true that the fact that we understand the single components make us able to understand the whole system that we are considering. [92]

We shall not try to resolve that controversy here, but we shall indicate where we find that the new complexity approaches based on nonlinear dynamics deviate from the traditional interpretation.

To appreciate how the complications of non-equilibrium phenomena have changed our view of the world we go back a few decades to the work on systems theory pioneered by Von Bartalanffy [8] and subsequently developed by many scientists. Systems theory adopts the perspective that determinism at best provides an inadequate description of nature and a holistic approach is more suitable for understanding phenomena in the social and life sciences. This methodology casts the scientist in the role of “problem solver” so that in order to extract information from the system the scientist must develop a "heuristic" understanding of the problem to be solved by means of metaphors. The holistic perspective assumes that the scientific knowledge is universal in that laws within a given field of study can often, if not always, be mirrored in all other fields of study. We are often convinced that holism and reductionism are two opposite concepts and that the first one belongs to philosophical approach and the other to the scientific one. This is not totally true, in fact there are many possibilities of compromising the two points of view so they can peacefully coexist.

The systems theory approach to science has proved to be effective especially at the interfaces of well-established disciplines, for example, biophysics, biochemistry, information theory and cybernetics just to name a few. However, a metatheory involving the concept of complex adaptive systems (CAS) has been invented and developed by members of the Santa Fe Institute [9]. According to Gell-Mann [10] a CAS is a system that gathers information about itself and its own behavior and from the perceived patterns which are organized into a combination of descriptions and predictions, modifies its behavior. Further, the interaction of such a CAS with the environment provides feedback with which the survival characteristics of the system are adjusted. This complicated behavior leading to an internal change in the system associated with decision making is not to be confused with the direct control envisioned in cybernetics and other early forms of systems theory.
To facilitate our discussion we adopt as a working definition of complexity that found in Reuelle's monograph, *Chance and Chaos* [11]: "Given a system and observer, the complexity of the system is measured by the difficulty encountered by the observer in extracting information from the system." One might be tempted to criticize the generality of this definition, but it is that very generality that makes it so appealing. For example, the complexity envisioned here may be applied to all manner of natural and social systems with the clear understanding that it is not objective, but depends on both the system and the observer. In addition to the subjective nature of this concept of complexity due to the explicit inclusion of the observer in its definition, there is the additional ingredient of subjectivity having to do with the "questions" the observer asks of the system. Here the notion of observer is really that of an experiment, but one could also formulate the same definition involving systems containing human beings or other conscious entities. It is difficult if not impossible, at the present time, to know if conscious and unconscious matter can be described in a unified way. Conscious matter, due to its complexity, can have emergent properties which do not imply an animistic theory, but in fact are consistent with a materialistic approach.

The theory of emergence is based on the idea that if we proceed towards organization levels more complex there are new emerging properties not present at the less complex levels (brain, consciousness, life are all examples of this). The most important problem at this point is how to consider the emergence of these new properties. They may be considered as genuinely unknown or unknown for our temporary scientific limits. The proponents of CAS would certainly argue that the existence of emergent properties, which is to say self-organization, is partly the reason for the development of their theory. However, CAS is still in its infancy and whether or not it will survive to adolescence, much less to maturity remains to be seen. One theory that has since its inception included the effect of the observer on the system observed is that of quantum mechanics (by using the case of quantum mechanics we can appreciate again how much the observer is involved in a physical fact and in its analysis). The paradigm of quantum mechanics is useful because it is accepted as the most fundamental picture of the interactions of matter in the universe and it too includes this dichotomy of the system and observer. In this general scheme the observer may also be included as part of the system and therefore the observation may or may not change the system, depending on the questions asked. For example, the observer might influence the spectral lines in the light from alpha centuri, but not the interest rates at the local bank.

So now we come down to the crucial question: Can we define a measure of complexity that will be useful across a broad spectrum of problems, from the stock market to superconductivity, from social discontent to the laughter of children? Using Ruelle’s definition one may think that it is possible to define a measure, since the concept of "difficulty" admits an ordering relation. In other words it makes sense to say; “problem A is more difficult than problem B". However, a little reflection reveals that this kind of relation is not really objective, since the difficulty of the problem depends on whom has to face it and at what time. Establishing objective criteria, for example imagining that the problems have to be faced by machines such as computers or by adopting rigid algorithms, just transfers the subjectivity to the level of the arbitrary criteria adopted. Herein we show that physical science, presumably our best effort at constructing an objective theory of reality, is not immune from these difficulties and we examine various proposals to overcome them. Because of the way we have based our discussion until now, here we decide on a point where to start, an assumption to use in order to build up our system of ideas.

We make the assumption that complexity is a property of the system and we do not address the difficulties associated with the observer, such as prejudice, limited resources and so on. Even in this restricted context of theory we hope that the measures discussed shall be of some value.

II. COMPLEX SYSTEMS

There has been a substantial body of mathematical analysis regarding complexity and its measures and it is rather surprising the broad range over which mathematical reasoning and modeling has been applied. One class of problems that defines the limits of applicability of such reasoning is denoted “algorithmic complexity’. A problem is said to be algorithmically complex if to compute the solution one has to write a very long algorithm, essentially as long as the solution itself. Applications of this quite formal theory can be found in a variety of areas of applied mathematics, but even if one restricts the discussion to the physical science, it is not possible in a short space to give a fair description of the activity. Thus, we focus our attention on nonlinear dynamics, system theory and complexity in the physical sciences. Hopefully one shall be able to extrapolate from this discussion to other areas of investigation. In Section 6 we shall outline a vision of complexity whose connections with algorithmic complexity became clear in the last few years.

A system consists of a set of elements together with a defining set of relations among those elements. All the phenomena of interest to us here shall be viewed as a system. It is also possible to study a subset of elements, called a subsystem of the system. Finally, the system may interact with the observer who may be a member of the system itself or of the environment. It is also possible, and sometimes necessary, to define an environment of the environment,
and so on. As already pointed out, the complexity of a system depends on the information sought by the observer, and this depends on the purpose of the study. We imagine that a system may be studied to “understand it”, namely to describe and control it or to predict its dynamics. For example, the weather cannot be controlled, but it is very useful to make accurate short-term forecasts. Predicting the trajectory of a hurricane may save millions in dollars, not to mention the saving of lives, even if in principle we cannot know its fundamental nature. It is often crucial to study, whenever possible, the response of a complex system to external perturbations. It is the set of these responses that constitute the information that the observer tries to extract from the system and it is the difficulty encountered in understanding, controlling or predicting these responses that is intuitively used as the measure of complexity. In the last part of this paper we shall outline a vision of complexity, from which many papers of these Proceedings emerged, which seems to be convenient to address practical issues of this kind, in addition to shedding light onto the problems discussed in the first part of this paper.

It is useful to list the properties associated with the complexity of a system, because we are seeking a quantitative measure that may include an ordinal relation for complexity. We note however that in everyday usage phenomena with complicated and intricate features having both the characteristics of randomness and order are called complex. Further, there is no consensus among scientists, poets or philosophers on what constitutes a good quantitative measure of complexity. For instance, Phil Winsor in the abstract of his contribution to these Proceedings claims that his paper addresses philosophical and conceptual issues departing from the usual mode of presentation of scientific journal articles. Yet, we are convinced that his presentation reinforces the perspective of complexity that we shall outline in Section 6, and that this perspective would make it possible for us to express his views with the scientific jargon of statistical mechanics, even if anomalous statistical mechanics.

For the time being, in this preliminary exploratory phase, let us limit ourselves to remarking that any list of traits of complexity is arbitrary and idiosyncratic, but given that disclaimer the following traits are part of any characterization of complexity:

i) A complex system shall typically contain many elements. As the number of elements increases so too does the complexity.

ii) A complex system typically contains a large number of relations among its elements. These relations usually constitute the number of independent dynamical equations that determine the evolution of the system.

iii) The relations among the elements are generally nonlinear in nature, often being of a threshold or saturation character or more simply of a coupled, deterministic, nonlinear dynamical form. The system often uses these relations to evolve in a self-conscious way.

iv) The relations among the elements of the system are constrained by the environment and often take the form of being externally driven or having a time-dependent coupling. This coupling is a way for the system to probe the environment and adapt its evolution for maximal survival.

v) A complex system typically remembers its evolution for a long time and is therefore able to adapt its behavior to changes in internal and environmental conditions.

vi) A complex system is typically a composite of order and randomness, but with neither being dominant.

vii) Complex systems often exhibit scaling behavior over a wide range of time and/or length scales, indicating that no one or few scales are able to characterize the evolution of the system.

These are among the most common properties selected to characterize complex systems, see for example, and in a set of dynamical equations, these properties can often be theoretically kept under control by one or more parameters. The values of these parameters can sometimes be taken as measures for the complexity of the system. This way of proceeding is however model-dependent and does not allow the comparison between the complexities of distinctly different phenomena, or more precisely between distinctly different models of phenomena. It is also worth mentioning here that the recent results illustrated in Section 6 lead us to add to this list a further trait:

viii) Complex systems conflict with the stationary assumption and exhibit aging properties.

In the above list we included one of the most subtle concepts entering into our discussion of complexity, that is, the existence and role of randomness. Randomness is associated with our inability to predict the outcome of a process such as the flipping of a coin or the rolling of a die. It also applies to more complicated phenomena, for example, when we assume we cannot know the outcome of an athletic contest such as a basketball or football game, or more profoundly when we cannot say with certainty what the outcome of a medical operation such as the removal of a cancerous tumor will be. From one perspective the unknowability of such events has to do with the large number of elements in the system, so many in fact, that the behavior of the system ceases to be predictable. On the other hand, we now know that having only a few dynamical elements in the system does not insure predictability or knowability. It has been demonstrated that the irregular time series observed in such disciplines as economics, chemical kinetics, physics, logic, physiology, biology and on and on, are at least in part due to chaos. Technically chaos is a sensitive dependence of the solutions to a set of nonlinear, deterministic, dynamical equations on initial conditions. Practically chaos means that the solutions to such equations look erratic and may pass all the traditional tests for randomness even though they are deterministic. Therefore, if we think of random time series as complex,
then the output of a chaotic generator is complex. However, we know that something as simple as a one-dimensional, quadratic map can generate a chaotic sequence. Thus, using the traditional definition of complexity, it would appear that chaos implies the generation of complexity from simplicity. This is part of Poincaré’s legacy of paradox. Another part of that legacy is the fact that chaos is a generic property of nonlinear dynamical systems, which is to say chaos is ubiquitous; all systems change over time, and because they are nonlinear, they manifest chaotic behavior.

A nonlinear system with only a few degrees of freedom can generate random patterns and therefore has chaotic solutions. So we encounter the same restrictions on our ability to know and understand a system when there are only a few dynamical elements as when there are a great many dynamical elements, but for very different reasons. Let us refer to the former random process as noise, the unpredictable influence of the environment on the system of interest. Here the environment is assumed to have an infinite number of elements, all of which we do not know, but they are coupled to the system of interest and perturb it in a random, that is, unknown, way [19]. By way of contrast chaos is a consequence of the nonlinear, deterministic interactions in an isolated dynamical system, resulting in erratic behavior of at most limited predictability. Chaos is an implicit property of a complex system, whereas noise is a property of the environment in contact with the system of interest. Chaos can therefore be controlled and predicted over short time intervals whereas noise can neither be predicted nor controlled except perhaps through the way it interacts with the system.

The above distinction between chaos and control highlights one of the difficulties of formulating an unambiguous measure of complexity. Since noise cannot be predicted or controlled it might be viewed as being simple, thus, systems with many degrees of freedom that manifest randomness may be considered simple. This point requires an explanation. The literature on stochastic processes shows that ordinary environmental noise, assumed to be white, yields ordinary, and simple, diffusion equation, with the same second derivative with respect to space as that appearing in the ordinary Schrödinger equation of quantum mechanics. In Section 6 we shall mention conditions where the environmental fluctuations, being correlated, cannot yield simple equations. On the other hand, a system with only a few dynamical elements, when it is chaotic, might also be considered to be simple. In this way the idea of complexity is again ill posed and a new approach to its definition is required.

In the earlier papers on systems theory it is argued that the increasing complexity of an evolving system can reach a threshold where the system is so complicated that it is impossible to follow the dynamics of the individual elements, see for example, Weaver [13]. At this point new properties often emerge and the new organization undergoes a completely different type of dynamics. The details of the interactions among the individual elements are substantially less important than is the “structure”, the geometrical pattern, of the new aggregate. This is the self-aggregating behavior required in the CAS theory. Increasing further the number of elements or alternatively the number of relations often leads to a complete “disorganization” and the stochastic approach becomes a good description of the system behavior. If randomness (noise) is to be considered as something simple, as it is intuitively, one has to seek a measure of complexity that decreases in magnitude in the limit of the system having an infinite number of elements. We shall see that this attractive goal is difficult to attain and all attempts to obtain and measure noise rather than chaos either break the laws of physics or the principle of subjectivity.

III. ENTROPIES

Historically thermodynamics was the first discipline in physics to systematically investigate the order and randomness of complex systems, since it was here that the natural tendency of things to become disordered was first observed. As remarked by Schrödinger in his groundbreaking work *What is Life?* [20]: “The non-physicist finds it hard to believe that really the ordinary laws of physics, which he regards as prototype of inviolable precision, should be based on the statistical tendency of matter to go over into disorder”.

In this context the quantitative measure of “disorder” that has proven to be very valuable is entropy and the idea of thermodynamic equilibrium is the state of maximum entropy. Of course, since entropy has been used as a measure of disorder, it can also be used as a measure of complexity. If living matter is considered to be among the most complex of systems, for example the human brain, then it is useful to understand how the enigmatic state of being alive is related to entropy. Schrödinger maintained that a living organism can only hold off the state of maximum entropy, that being death, by absorbing negative entropy, or negentropy, from the environment. He points out that the essential thing in metabolism is that the organism succeeds in freeing itself from all the entropy it cannot help producing while alive.

We associate complexity with disorder, which is to say with limited knowability, and order with simplicity or absolute knowability. This rather comfortable separation into the complex and the simple, or the knowable and the unknowable, in the physical sciences will be shown to breakdown once a rigorous definition of entropy is adopted and applied outside the restricted domain of thermodynamics, in spite of Schrödinger’s best efforts. We shall see that because of the fundamental ambiguity in the definition of complexity, that even adopting the single concept of entropy...
as the measure of complexity leads to multiple definitions of entropy some of which are in conflict with one another.

We review the various definitions of entropy along with other various measures of disorder that have been used in the physical sciences. The definition of entropy as a measure of “disorder” encounters the same problems of subjectivity that we found when we defined complexity. Order is something that is difficult to define, and strictly speaking depends on the questions that are asked of the system. The process of rendering objective the concept of disorder leads to several quantitatively different definitions of entropy. Once a definition is adopted everything is somewhat “objective”. Unfortunately, as we already mentioned, the different proposals lead to different quantitative results even though they are the most objective measures of complexity we have available. The subjectivity enters through the choice of definition that is the most suitable to answer the questions of interest.

Following Cambel [21] we roughly divide entropies into three categories: the macroscopic, the statistical and the dynamical. In the first group we find the entropy stemming from thermodynamics, for example the original S-function entropy of Clausius as used by Boltzmann [22] and subsequently by Prigogine [23]. In the second category is placed the entropy stemming from the assumption of a probability distribution to characterize the system, such as the Gibbs entropy [24]. Here the activity on the microscale, or to the dynamics of the individual elements in phase space, is related to what occurs macroscopically, or at the system level. A more recent formulation of the probability entropy is that of Tsallis [25]. A special role in the statistical entropies is played by the information entropy of Shannon and Kolmogorov, that relate the physical properties of the system to the concept of information [26]. Finally, the dynamical entropies such as Kolmogorov’s are derived from the geometry of the systems dynamics in phase space. Other possible choices for categories might include phenomenological, informational or geometrical, but these would have no distinct advantage over those above and would in part overlap with the categories chosen.

A. Clausius’ Entropy

Entropy, like the length of a rod or the temperature in the room, is a physical quantity that is measurable. At the absolute zero of temperature the entropy of any piece of matter is zero. When the substance is brought into any other state by slow, reversible little steps the entropy increases by an amount that can be computed by calculating the ratio of the heat supplied to the absolute temperature at which the heat was supplied and adding up all these small contributions.

The Second Law of Thermodynamics, as formulated by Clausius in 1850, states that it is not possible to conduct an experiment in an isolated system whose only result is the spontaneous transfer of heat from a cold region to a hot region, since if the system is isolated work cannot be done on it. Consequently, this flow of heat defines a directionality (arrow) for time. It then took Clausius fifteen more years to prove that the thermodynamic temperature was an integrating factor for the quantity of heat transferred, so he defined the function S (entropy) in the exact differential form discussed above, meaning that the total entropy is obtained by summing the above ratio of heat to temperature over any reversible path of thermodynamic equilibrium states. Thus, this concept of entropy implies that the system is macroscopic and isolated and requires the existence of thermodynamic equilibrium. If these conditions are not fulfilled then we cannot calculate the entropy exactly but must be satisfied with the inequality $\Delta S \geq 0$ over a thermodynamic cycle, where the equality only applies for a reversible process. The inequality means that the change in entropy over a cycle, where $\Delta S$ is the difference in the entropy at the beginning and the end of the cycle, is an increase for irreversible processes or it is zero for reversible processes. The arrow of time is therefore recovered as the direction of increasing entropy for isolated systems and this in turn has been used to define what is actually irreversible.

According to the thermodynamical entropy discussed here, a completely random system would have maximum entropy and therefore maximum complexity. This concept of entropy is very useful for studying large scale physical and chemical systems at or near equilibrium. On the other hand an ordered system, such as a living organism, would have a low thermodynamical entropy and would therefore be simple under this classification scheme. Since this conclusion runs counter to our expectation that living systems are among the most complex in the universe, we cannot simply apply the definition of thermodynamical entropy to a system in order to determine its complexity. However, we do not want to abandon the notion of entropy altogether since it is tied up with the order-disorder property of a system. Thus, we shall explore some of the extensions and refinements of the entropy concept to see if these will serve our needs better. In section 6 we shall conclude our search by noticing that the ordinary form of Gibbs entropy plays a useful role, not so much to measure the complexity of a state, but rather the condition of transition from dynamics to thermodynamics.
B. Boltzmann’s Entropy

The definition of entropy as it was introduced into thermodynamics by Clausius did not rely on any statistical concepts. However in our interpretation of order and disorder fluctuations certainly played a role. It was the nineteenth century physicist Boltzmann that first attempted a synthesis of the deterministic predictability of mechanics and thermodynamics through studying the transport of large numbers of particles in gases. He developed quite complicated equations that described the fluid-like motion of gases including the collisions among the individual particles. These collisions, reasoned Boltzmann [22], would produce a randomization of the motion of the gas particles since one could not determine with absolute precision the location and size of the individual collision events. In this way he introduced a probability density function that depended on the location and velocity of each of the particles in the gas. His investigations lead him to introduce entropy in the form

$$\text{entropy} = k_B \log W .$$  \hspace{1cm} (1)

Here $k_B$ is a constant that has the appropriate dimensions for entropy and has come to be called Boltzmann’s constant and the function $W$ is a quantitative measure of the microscopic disorder in the system.

This very different looking form for the entropy, it is not Clausius’ ratio of heat to temperature, shares with the energy the property of *extensivity*, which means that if one considers two independent systems $A_1$ and $A_2$ with entropies $S_1$ and $S_2$ respectively, then the entropy of the combined system is just the arithmetical sum $S_1 + S_2$, as it would for the energy. The entropy is extensive through the logarithmic assumption which means that the measure of disorder, $W$, for the combined system, $W_{\text{com}}$, is given by the product of the individual $W$’s, i.e., $W_{\text{com}} = W_1 W_2$. The quantity $W$ indicates disorder that is in part due to heat motion in a system and in part due to the different kinds of particles that can generally intermix in a thermodynamical system. If we imagine that the phase space for an isolated system can be partitioned into a large number of cells and that each cell is statistically equivalent to each of the other cells, which is to say that the probability of a particle occupying any of the cells in phase space is equally likely, then $W$ is the volume of phase space consistent with the total energy of the system.

This definition of entropy given by (1) is fairly abstract, depending as it does on a volume element of the phase space for the microscopic elements of the dynamical systems. Boltzmann also expressed the entropy in more physical terms through the use of the continuous phase space distribution function, $\rho(x,v,t)$, where $x$ is the physical location of the all $N$ particles in configuration space $x = \{x_1, x_2, ..., x_N\}$ and $v$ is the velocity vector of all $N$ particles in velocity space $v = \{v_1, v_2, ..., v_N\}$, so this one function keeps track of where all the particles in the system are as a function of time and what they are doing. Boltzmann was then able to show that the entropy could be defined in terms of the phase space distribution function as

$$\text{entropy} = -k_B \int dx dv \rho \ln \rho \hspace{1cm} (2)$$

which is a non-decreasing function of time. He was able to show that this definition of entropy attains its maximum value when the system achieves thermodynamic equilibrium, in complete agreement with Clausius’ notion of entropy.

We shall refer to the definition of entropy as given by Boltzmann as the statistical entropy. This development reached maturity in the efforts of Gibbs [24], who was able to provide the mechanical basis of the description of thermodynamic phenomena through the formulation of statistical mechanics. Gibbs gave a probability interpretation to the phase space distribution function, and introduced the notion of ensembles into the interpretation of physical experiments.

The above statistical definition of entropy is very general and is similar to the measure of complexity we seek. In fact if the system is simple and thus we are able to measure all the coordinates and the momenta of all the particles with extreme precision, we have from Eq.\textsuperscript{2} that this entropy is a minimum. A simple system, namely one that is closed and perfectly integrable will not have any growth of entropy, due to the time-reversibility of the dynamical equations. Here “integrable” and “time-reversible” dynamics means that the particles obey Newton’s laws of motion. Even in the case where our measures are not infinitely precise, the growth rate is small, as will become clear in a subsequent section when we introduce the Kolmogorov-Sinai entropy. The probability definition of entropy also has the advantage that it recovers Clausius’ proposal in the statistical limit. Unfortunately the assumption made by Boltzmann is not easily checked, but, on the contrary the truth of Eq.\textsuperscript{2} would contradict physical law and therefore in principle can not be true. This impossibility has been proved in a variety of ways, but still *Boltzmann’s dream* puts us on the path to cross a bridge from dynamics to thermodynamics, from reversible, microscopic processes to irreversible, macroscopic processes. The latter is what we know and can directly measure, the former is a useful hypothesis that has been indirectly measured, but the connection between the two remains a mystery.
The second law of thermodynamics is so well grounded in experiment that it provides a guide to every possible definition of entropy. Thus, we know that whatever definition we choose, entropy must increase or remain constant in a closed system, or more precisely in the thermodynamical limit (where the system is described by an infinite number of variables) it must be a non-decreasing function of time for a closed system. This regression to equilibrium, where as we mentioned equilibrium is the most disordered configuration of the system, obtained by Boltzmann is the irreversibility property of the entropy identified by Clausius and defines the arrow of time. The future is therefore the direction in time that decreases the amount of order and leads towards the “heat death” of the universe. But as we well know this does not occur equally at all places and at all times; not all systems deteriorate in the short term, if they did then life would not be possible. One way to quantify the local increase in the entropy of a system was developed by Prigogine [23].

Shrödinger [20] identified negentropy as that quantity a living organism obtains from the environment in order to keep from dissipating, that is the “stuff” that enables the organism to maintain its order. Prigogine [23] was able to develop and generalize this concept through the formation of dissipative structures that are maintained through a flux of material and/or energy through the system of interest. Explicit in these ideas is that order is maintained by means of the system interacting with the environment, which means that the system is not closed as it was in the discussion of Clausius and Boltzmann. The dissipative structures of Prigogine are maintained through fluctuations providing sources of energy to the system from the environment and dissipation extracting energy from the system to the environment. This balancing of fluctuations and dissipation maintain the flux through the system which in turn supports the organization of the dissipative structure. This balance of mechanisms was expressed in terms of the changes in the statistical entropy by Prigogine:

$$\Delta S = \Delta S_I + \Delta S_E$$  \hspace{1cm} (3)

where $\Delta S$ is the change in entropy, and the subscripts refer to the internal entropy change ($I$), the entropy change in the environment ($E$) and the total entropy change ($T$) of the system. The arguments of Clausius, Boltzmann and Gibbs apply to the internal entropy of the system $\Delta S_I$ which is zero for a closed system. Thus, even though $\Delta S_I$ is non-decreasing over time, the change in entropy of the system $\Delta S_T$ can be positive, zero or negative, depending on the contribution of the environment to the entropy change, $\Delta S_E$. The system can extract what it needs from the environment to generate and/or maintain its order. Thus, the ordering of the system is more than compensated by a disordering of the environment. Consequently, as the knowability of the system increases due to $\Delta S_T < 0$ and the knowability of the environment decreases due to $\Delta S_E < 0$, indicating that the negentropy extracted from the environment to enhance the order of the system increases the disorder of the environment. Systems in which $\Delta S_T < 0$ are said to be self-organizing and must occur under conditions where the system is far from equilibrium, otherwise the internal and environmental contributions to the entropy change would just cancel one another.

It is worth remarking that the vision of complexity emerging from this paper, as discussed in section 6, does not rule out the Prigogine perspective as a source of pattern formation. However, it makes an additional possibility emerge, which is not out of equilibrium thermodynamics. As we shall see, it is a condition intermediate between dynamics and thermodynamics.

### C. Shannon and Kolmogorov-Sinai entropy

As we have seen, the thermodynamical entropy can be given a dynamical interpretation, but to do so one has to interpret the dynamics using a probability density. This procedure is questionable and has given rise to paradoxes and controversies that remain unresolved. A rigorous mathematical treatment, based either on quantum or on classical mechanics of closed (independent) systems, does not produce any regression to equilibrium, to the contrary it results in an eventual return to the initial state of the system after waiting a sufficiently long time. This is another of those paradoxes of Poincaré, a dynamical system following Newton’s laws will return to its initial state infinitely often over time. This recurrence property of Poincaré rules out the possibility of a dynamical system relaxing to an equilibrium state using only the equations of motion. The recurrence of dynamical systems has necessitated the introduction of a number of hypotheses to account for the arrow of time which is so evident on the scale of biological evolution. To explain the relaxation of a system to equilibrium and therefore to give time its direction, physicists allow for a certain uncertainty in the measured values of dynamical variables which is referred to as coarse-graining. This is traditionally done by discarding the fiction of a closed system and recognizing that every system has an environment with which it interacts. By explicitly eliminating
the environmental variables from the description of the system dynamics one obtains a description that is statistical in nature. The absolute predictability which was apparently present in the deterministic nature of Newton’s equations is abandoned for a more tractable description of the system having many fewer variables, but the price has been high and that price is predictability. Since we cannot know the environment in a finite experiment, the environment is not under our control, only the experimental system is accessible to us, the environments influence on the experimental system is unpredictable and unknowable except in an average sense when the experiment is repeated again and again. It is this repeatability of the experiment that allows us to map out all the different ways the environment influences the system through the construction of the ensemble distribution function that captures all the available information, features common to all the experiments in the ensemble.

Unfortunately there does not exist a systematic way to include all the ways the environment can interact with an arbitrary dynamical system and so the desirable situation outlined above has not as yet been attained. One way that this program has been pursued has been to make the evolution of the system uncertain by inserting a random force into the equations of motion. This procedure saves the applicability of physical theories, but it also introduces the seed of subjectivity into the evolution of the system. We have to develop mathematical tricks to treat systems with an infinite number of degrees of freedom like the environment, where the global properties of the overwhelming majority of degrees of freedom are chosen in a subjective manner, mostly for computational convenience. One such trick is to assume that the environment is already in the state of equilibrium before interacting with the system, and that it is so large that it remains in equilibrium throughout its interaction. Therefore it is assumed that one knows nothing of the environment at the start of the experiment and that one can learn nothing about the environment from the results of the experiment. This is a sad commentary on the present state of statistical physics and what it can teach us about complex systems.

We anticipate what will become clear shortly, namely that the problem with existing dynamical theories in so far as they are inconsistent with the statistical interpretation of entropy is that they are deterministic and time-reversible. These two properties imply that no probabilistic treatment of dynamics is objective, and the correctness of a statistical picture stems from subjective assumptions. Therefore there is no direct connection between dynamics and thermodynamics, because the connecting link, that being statistical mechanics, requires the introduction of probability theory. In the same vein, the Correspondence Principle, the principle according to which quantum mechanics and classical mechanics are made equivalent, is always used in a statistical sense, which without subjective assumptions cannot be correct.

Implicit in the concept of entropy is the idea of uncertainty. The latter idea only makes sense in a context where there is a conscious being that is extracting information from the system, and is therefore subjective. Uncertainty means that not all the information one needs for a complete description of the behavior of a system is available. Even the term “needs” is in this sense subjective, because it depends on the questions the observer poses, which in turn depends on the ”purpose” of the observer. This is where all subjectivity enters, and we do not go further into the philosophical implications of having an observer with a purpose conducting the experiment. We only wish to be sure that a system containing conscious individuals can not be treated in a deterministic way since the objectivity stemming from determinism conflicts with the subjectivity of the individuals (free will).

However, we can safely say that entropy is a measure of uncertainty, and like uncertainty, entropy is a non-decreasing function of the amount of information available to the observer. This connection between information and thermodynamics is quite important, and at this stage of our discussion we can say that it is the uncertainty that allow us to describe dynamical systems in thermodynamical terms.

Shannon determined how to construct a formal measure of the amount of information within a system and the problems associated with the transmission of a message within a system and between systems. He expressed that information in terms of bits, or the number of binary digits in a sequence. He was able to prove that a system with $N$ possible outputs, where the output $i$ had the probability of occurring $p_i$, can be described by a function $H$ that attains its maximum value when each of the possible states of the system have the same probability of occurrence, that is the assumption of maximal randomness (maximum uncertainty) in which case $p_i = 1/N$. This result is essentially equivalent to Gibbs’ treatment of Boltzmann’s entropy, where the function $H$ is equivalent to Shrödinger’s negentropy. The analytic expressions for the entropy are exactly the same, but this informationally interpretation offers possibilities of extending the definition of entropy to situations using conditional probabilities, resulting in conditional entropies, mutual entropies, and so on. This means that it is possible to recognize two equivalent pieces of information, and to disregard the ”copy” because nothing new is learned from it. It is possible to extract the new pieces of information from a message of which the majority of the content is already known, and therefore it is useful for separating the knowable from the known.

New pieces of information decrease the level of uncertainty, and thereby increase the order of a system. As mentioned above this is precisely the mechanism discussed by Shrödinger using his concept of negentropy. This fact is highlighted by the famous paradox of Maxwell’s demon. The demon provides a mechanism by means of which a closed thermodynamic system could decrease its entropy by using information within the system itself in apparent
violation of the Second Law of Thermodynamics. The paradox was finally resolved by the physicist Leo Szilard who calculated the entropy associated with the demon’s acquisition and use of information and found that it exactly equaled the amount of entropy by which the system was reduced due to the demon’s efforts. Thus, the demon generates as much entropy as she suppresses and there is no violation of the second law.

The information entropy is closely related to the Kolmogorov-Sinai (KS) entropy. We have already quoted the relevant work of Kolmogorov. The contribution of Sinai to this entropy is given by the work of Ref. We do not address the delicate mathematical concepts behind this important kind of entropy. We limit ourselves to pointing out that the KS entropy is a trajectory property, made computable, in the case of the dynamical chaos by the Pesin theorem, which establishes this entropy to be given by the sum of the positive Lyapunov coefficients. How to relate this trajectory property to the entropy of Eq. which is, an ensemble property? This important issue has been discussed by Latora and Baranger. The very interesting work of these two authors rests on the assumption that the density time evolution can be reproduced by a bunch of trajectories, which, due to the fact that the Lyapunov coefficients are finite, tend to spread, thereby occupying an increasing number of cells of the phase space. We note that according to Petrosky and Prigogine an equation of time evolution for densities, once defined, must be considered as a law of physics on its own. Consequently, it is a problem of some interest to establish a connection between the entropy of Eq. and the KS entropy without invoking the trajectory time evolution. This delicate problem has been studied by the authors of Ref. These authors pointed out that in the case of conservative systems the entropy of Eq. increases as an effect of a coarse graining. They also noticed that in the case of intermittent randomness, even if the ergodic condition is assumed, so as to properly define the KS entropy, it is impossible to make the entropy increase of Eq. compatible with the KS entropy. This is the first example of the problems caused by anomalous statistical mechanics.

D. Rényi-Tsallis Entropy

In this section we mention briefly that in the last few years there has been a great interest for the Rényi and Tsallis entropies. The hungarian mathematician Alfréd Rényi in his treatise on probability theory, has shown that one can actually build up "information functions" that share the order relation property with Shannon’s information entropy (and therefore the metric entropy). When the set of probabilities \( \{p_i\} \) are such that \( \sum p_i \log_2 p_i \) diverges, it is possible to find a real number \( q (0 < q < 1) \) such that

\[
I_q \equiv \frac{1}{q-1} \log_2 \left[ \sum_{i=1}^{N} p_i^q \right] 
\]

converges. This is defined as the information function of order \( q \). Such information functions are useful when, going to the continuum, the probability density \( p(x, v; t) \) has long tails with diverging moments. Such distributions are quite common in the social and life sciences, and are found to be more prevalent in the physical sciences than was once believed, see for example West and Deering.

Recently Tsallis adopted a form for the entropy, which, apparently, looks similar to the Rényi form, namely

\[
S_q \equiv \frac{1 - \sum_{i=1}^{N} p_i^q}{q - 1}.
\]

Actually, this entropy violates the extensive nature of the Boltzmann entropy, discussed in Section 3.2, and, consequently departs from the Rényi entropy, which is still additive. The reasons of the success of Tsallis’ entropy is that by maximization under suitable constraints, it leads to equilibrium distributions with an inverse power law form. This is an interesting property, even though it raises the obvious criticism that this entropy is given its form, namely the form of Eq. on purpose. This means that the deviations from the standard equilibrium distribution are well known, and are a consequence of the renormalization group approach and that the entropy of Eq. is given its form for the specific purpose of yielding an inverse power law distribution.

A more satisfactory approach, in our opinion, is the derivation of the entropy form directly from dynamics. The work of Ref. proves that this is possible. An oscillator of interest playing the role of thermometer is coupled to a dynamical system, called booster for the specific purpose of keeping it distinct from the ordinary thermal baths, which already rest on the assumption that thermodynamics holds true. The authors of Ref. aimed at reaching their conclusions with no thermodynamic assumption whatsoever. They built up a Fokker-Planck equation for the oscillator of interest and used the width of the velocity distribution, expressed in terms of dynamical properties, to
measure the temperature of the booster. The interesting result of this paper is that the Boltzmann principle of Eq. (1) is recovered from dynamics in the limiting case of a booster with a large number of degrees of freedom. Note that the thermometer interacts with only one particle of the booster, called a doorway particle. For this procedure to reach the wished result, it is essential that the correlation function of the coordinate of the doorway particle undergoes a relaxation process fast enough.

It is important to point out that significant attempts at applying the same procedure in the specific case where the correlation function of the coordinate of the doorway particle undergoes an inverse power law decay have been done. The original project of deriving out of this procedure the Tsallis entropy did not yield satisfactory conclusions. A new and unexpected result emerged from these attempts. This has to do with the fact that in the ordinary case of normal statistical mechanics the transition from dynamics to thermodynamics is virtually instantaneous. We make the conjecture that the process of transition to the scaling regime in a diffusion process is an indicator of the transition from dynamics to thermodynamics. In fact, the main difference with the attainment of the ordinary form of equilibrium is that there is no feedback on the bath, the diffusion caused by the bath fluctuations being the only active process. With this perspective in mind, the main result of the work of Ref. [41] is that the process of transition from dynamics to thermodynamics lasts forever, thereby leading us to consider this condition as a new state of matter. The indication of this kind of complexity is not given by an entropy measure, but it is disclosed by the detection of multi-scaling properties. We shall be referring to this state of matter as the Living State of Matter (LSM). We shall discuss again the relevance of this perspective for complexity in Section 6. Note that the work of Allegrini et al. [43] can be regarded as a pioneering attempt in this new direction.

IV. DOES PHYSICS REALLY DESCRIBE REALITY?

A. Objectiveness and reductionism

We have seen in the previous chapter how the concept of entropy has developed into a useful tool for the study of complex phenomena. The understanding of this development is a guide in our search for a suitable measure for complexity. We learned that asking the wrong question leads one along a false trail, however, posing the right question enables one to gather information regarding the process of interest. A collection of experiments can be used to quantify a measure, and different methods of analysis provide various insights into the properties of the phenomenon using that measure. We have also seen that the concept of information offers the possibility of treating every problem with a great deal of generality, but nevertheless this approach runs the risk of confusing the physical phenomena, with any simulation of them, since they share the same amount of information. Subjectivity enters into this discussion in the sense that the computation of entropy based on the perceived complexity using that measure. We have also seen that the concept of information offers the possibility of treating every problem with a great deal of generality, but nevertheless this approach runs the risk of confusing the physical phenomena, with any simulation of them, since they share the same amount of information. Subjectivity enters into this discussion in the sense that the computation of entropy based on the perceived complexity using that measure.

The fact that we repeat the experiments and that we try new methods of analysis is important in order to see if our description of the reality is well approximated. Nevertheless we should notice that this does not mean that in this way we can suppress any subjectivity. In fact we should take into account at least two factors:

1. It is still the scientist who manipulates and interacts with the physical system.

2. The fact that we obtain many times the same results may be related to the fact that we use all the time the same methods of analysis, but these methods could be wrong for whatever unknown reason.

If we believe that these two objections are true, we should probably look for physical power of prediction elsewhere. We may reach a satisfying compromise when we test not only nature but also our methods of analysis. If, however, the laws stemming from the investigation depend on subjective limitations or other properties of the observer (any observer belonging to the human species, for instance, may be assumed to behave classically), then one may wonder if some “other observer” with different constraints would see different laws of nature. It may look strange that scientists are involved in this kind of philosophical discussion, since it may appear that a uncorrectable subjective statement or theory cannot be scientific. However, this is not true. No discipline is immune from this paradoxical logic. In fact the foundation of all of science, which by consensus is physics, has this problem at its very core: quantum mechanics. A familiar example of the paradoxes in quantum mechanics should suffice; let us consider Shrödinger’s cat. Recall that the cat is in a box and cannot be observed. There is poison gas that can be released by means of particle decay from a radioactive sample that is also within the box. Since particle decay is a quantum process, it is described by the superposition of a wave function in which the particle has decayed and one in which it has not decayed. Thus, determining whether the cat is alive or dead at any particular time is ambiguous since it would appear that the state of being of the cat is a superposition of a state in which the gas has been released and the cat is dead and a state where the gas has not been released and the cat is alive. Thus, there is a sense in which the cat is both alive and dead.
at the same time. It is not true that the quantum properties violate Aristotelian law of non-contradiction. The thing is that logic, as physics, represents different levels of approximation. An architect does not need Einstein’s relativity to build a house but just mechanics and statics. For this reason his/her world would be well represented by classical logic. We are not correct if we say that classical mechanics is not true anymore, after that quantum theory has been formulated. We are incorrect exactly in the same way if we say that classical logic has been invalidated by fuzzy logic. Every logic describes a different level of reality, or if we prefer, the same reality from different points of view.

These quantum properties violating the normal Aristotelian logic of the “excluded third” are completely objective and experimentally verified at the atomic level. However, the theory does not contain any parameter that is not observable at the level of the cat. At a certain scale size the superposition of distinct states is broken, and there is a random collapse of the total wave function onto one of the two states, thereby implying that the cat is either dead or alive but not both. This issue of measurement encapsulated in the phrase, collapse of the wave function, remains one of the mysteries of quantum mechanics. We still do not know the when and the why of wave function collapse, but it is precisely at this unknown level that the system ceases to be described in a deterministic way and a probabilistic approach becomes necessary. It should be emphasized that after the wave function collapse the uncertainty regarding the health of the cat increases, since we have passed from a deterministic picture to a statistical one, and therefore the entropy increases as well. The good news is that the measurement is irreversible, so that the arrow of time is recovered, but at this point one may argue whether or not the observer has obtained any information from the experiment. Do we have more information, or do we have more uncertainty and therefore more entropy?

The answer to this question may seem unsatisfactory, because in its present formulation Quantum Theory can not really be applied to the above problem, because it does not apply to individual systems, only to ensembles of systems. This means that either there are infinitely many observers each conducting the same experiment, or there is one observer conducting an infinite number of identical experiments. Either of these two perspectives enables us to resolve the paradox regarding the cat. The outcome of the experiment is uncertain so that the entropy has increased, this situation arises because for each of the experiments the system was prepared in exactly the same way, in the state of minimal entropy. The situation is different in the case of a single system, assuming that we only have one cat. If quantum theory is applied to this case it should tell us how and when the wave function collapses. We also need to know if the measurement apparatus and the observer should be included in the wave function. Even if a probabilistic approach is adopted from the very beginning, it follows that infinitely many identical systems depart from one another with alternative stories on the fate of the cat. Do we then have to assume alternative stories in infinitely many universes? If we assume that the observer and the macroscopic apparatus are classical, and therefore obey the logical principle of the “excluded third”, then these paradoxes are resolved, but a unified theory is still missing.

This brief excursion into quantum theory should be sufficient to show the unsatisfactory state of the physical theory of measurement, since it cannot explain statistical properties like entropy increase without encountering difficulties with the principle of objectivity. For this reason we shall subsequently return to an extended discussion of the influence of quantum uncertainty on macroscopic knowability.

An issue related to the information paradigm of physical understanding of nature is the principle of reductionism. This principle, in a nutshell, states that the process of understanding implies processing data for the purpose of arriving at generalizations. Such generalization are very efficient descriptions of the world, reducing what we need to remember and enhancing our ability to communicate with one another. It is much simpler to communicate a law than it is to communicate the results of thousands of experimental upon which the law is based. However, in its strong forms, reductionism states that to understand complex phenomena one needs only to understand the microscopic laws governing all the elements of the system that make up the phenomena. This reasoning implies that once one understands all the parts of a problem, one can “add them up” to understand the total. The whole is just the sum of its parts. That may sound fine in geometry but it is an incomplete description of natural phenomena. The counterpoint to reductionism is System Theory, that states that a system very often organizes itself into patterns that cannot be understood in terms of the laws governing the single elements. This self-organization constitutes the emergence of new properties, that arise, for example, in phase transitions. Living beings, too, cannot be understood using reductionism alone, but a more wholistic perspective has to be adopted. This change in perspective, from the reductionistic to the wholistic, in some ways resembles the passage from deterministic to probabilistic knowledge. In both cases the meaning of “knowledge” changes with the changing perspective. From our arguments regarding physical theory we know that a complex macroscopic system can be known in a reductionistic way in principle, but not in practice, while at the same time it can be known in a thermodynamical (wholistic) sense in practice, but not in principle.
B. Information, incompleteness and uncomputability

In addition to the arguments given above, there might also exist other reasons why, given our present state of knowledge, physical theories do not provide a satisfactory description of reality. It is not sufficient for physics to describe the world within the laboratory, it must also faithfully describe the world in which we live. It seems clear that reductionism is not enough to describe systems where the pattern of information flow often plays a role more important than that of microscopic dynamics, for example, in phase transitions. However, we still want the macroscopic rules to be consistent with the microscopic ones. If new properties emerge, even if it is impossible in practice to predict them from microscopic dynamics, they must be implicit in the microscopic equations. This weak reductionistic assumption is part of the objectivity principle. A chemical reaction is too difficult to be explained from a purely quantum mechanical perspective at the present time, but nevertheless no violation of quantum mechanics is expected to take place during a chemical reaction. The analogous situation arises at a higher level for the biological properties of a cell that cannot be understood in terms of chemistry alone. Our understanding of the global properties is achieved from a wholistic point of view, but the emerging properties have to be compatible with a weakly reductionistic perspective. Otherwise we would be tempted to imagine different laws for different space and/or time scales, or different levels of complexity. This, in turn, inhibits any possible mechanistic (objective) view of reality. We stress that in our perspective the principle of objectivity, namely the objective existence of mechanical laws, does not necessarily mean that the laws are deterministic, but a seed of randomness may be involved. Actually we shall argue that a seed of randomness must be involved in any fundamental description of reality.

We stress again that there is no randomness involved in the classical perspective, while in quantum systems randomness is triggered at the level of measurement and is ultimately the cause of all the paradoxes. Further, classical mechanics is time-reversible, and in the absence of measurement so is quantum mechanics, thus, it is therefore impossible to recover the arrow of time. The English physicist R. Penrose \[15\] stressed in a recent book, another way in which standard physical theories fail to describe reality. He (Penrose) developed an extended argument devoted to rule out the possibility of creating an artificial intelligence using standard computers. In his discussion he explains how physics is basically "computable", which is to say that the laws of physics can be faithfully implemented using computer programs, and cannot therefore explain cognitive activity. Many scientists argue that awareness and consciousness require properties that computers lack, see for example \[16\]. Penrose, however, proves that mathematical reasoning is not computable. But Penrose himself judges these (human) properties as a quality. No rules can substitute any human intuition. Namely, it is impossible for any computer to have particular mathematical knowledge available to our brain. The proof of this assertion requires one to define what a computer is meant to be, or what is called in mathematical jargon a “universal Turing machine”, and what it can or cannot do, even with unlimited time and available memory. Given these constraints it is possible to use a version of the famous incompleteness theorem of Gödel \[13\], namely, that every set of formal mathematical rules is always incomplete. In particular the knowledge itself of this incompleteness is not available to formal theories, but to us as human beings, and that is so because we are able to understand the nature of "paradox". It has been proved that the formal theories can be expressed in terms of computation and vice-versa, so that our capabilities for going beyond what is prescribed for formal theories by Gödel’s theorem is a conceptual proof of the existence of non-computable phenomena in the world.

As earlier stated, a natural application of computation theory has been to the development of a measure of complexity. This measure can be viewed as a generalization of Shannon’s information entropy. It is called “algorithmic complexity” or Kolmogorov-Chaitin complexity \[24\, 17\], after the names of the two mathematicians that independently defined it. This measure applies to binary strings and is defined as the length of the string in bits for the shortest program that is able to compute the string. Just like entropy, this function reaches a maximum if complete randomness occurs, since genuine randomness is non-computable, one has to specify the entire sequence in the program. The *Kolmogorov-Chaitin entropy*, like informational entropy, enables one to define conditional or mutual properties, to establish subadditive properties, that are the common features of complex phenomena. This measure is very useful from a conceptual point of view, but it does not have a practical use, since theorems indicate that it *cannot be computed*. This particular definition of entropy has been used as a measure of complexity in a number of different fields, including program optimization as well as image and information compression, but it is not useful for us here.

V. RANDOMNESS AND DETERMINISM IN PHYSICS

A. Reductionism and the end of physics

We have argued that what has come to be called the science of complexity is an interdisciplinary approach to the study of reality, not confined to physics, but ranging from biology to economics, and from there to psychology,
neurophysiology and the study of brain function, see for example Penrose. Schrödinger, in a recent paper in Physics Today, pointed out a crisis generated in physics by the success of renormalization group theory: "The ideas of symmetry breaking, the renormalization group and decoupling suggest a picture of the physical world that is hierarchically layered into quasiautonomous domains, with the ontology and dynamics of each layer essentially quasistable and virtually immune to whatever happens in other layers. At the height of its success, the privileged standing of high-energy physics and the reductionism that permeated the field were attacked." Reductionism was vigorously attacked early on by Anderson, and he is not the only scientist who believed reductionism had outlasted its usefulness. The renormalization group specifies a set of rules for establishing the critical coefficients of phase transition phenomena. Wilson and Kogut proved that the value of these coefficients can be assessed with theoretical models in a way that is totally independent of the detailed nature of elementary interactions. In other words, the renormalization group approach establishes the independence of the different levels of reality, and, even if in principle a man is nothing more than a collection of atoms, his behavior has to be studied, if ever possible, with scientific paradigms which do not have anything to do with atomic dynamics. The leading role of high energy physics in science was based on the implicit assumption that, once the fundamental laws of physics are established, all phenomena, at least in principle, can be explained. The advent of renormalization group theory implies that even if a final theory is possible, such as envisioned by Weinberg, it cannot be used to address the problems associated with the problems of quantifying complexity. On the other hand, this dream of a final theory might also be perceived as a nightmare by people like the present authors, who hope and believe that reality is an inexhaustible source of wonders. We share, on this issue, the same view as Leggett. We believe that the notion of strict determinism must be abandoned and that the settlement of the problem of the great unification in physics, even if it occurs, does not represent the end of physics. At the end of his book Leggett concludes: "If even a small part of the above speculation is right, then, far from the end of the road being in sight, we are still, after three hundred years, only at the beginning of a long journey along a path whose twists and turns promise to reveal vistas which at the present are beyond our wildest imagination. Personally, I see this as not pessimistic, but a highly optimistic, conclusion. In intellectual endeavour, if nowhere else, it is surely better to travel hopefully than to arrive, and I would like to think that the generation of students now embarking on a career in physics, and their children and their children's children, will grapple with questions at least as intriguing and fundamental as those which fascinate us today—questions which, in all probability, their twentieth-century predecessors did not even have the language to pose."

B. White noise as a physical source for the fulfillment of the Correspondence Principle

As Mark Twain once remarked: "The news of my death has been greatly exaggerated." The same is true of the claims made regarding the demise of the discipline of physics; that its replacement by the science of complexity may be premature especially if the paradigms necessary to understand complex phenomena have their basis in physical systems. It appears that the new paradigm upon which our understanding of complex phenomena is based is that of randomness as the key property of reality. This has presented a problem for modern physics because of the conflict between the deterministic nature of current theories and the consequent subjective character of randomness as derived from these theories as we have discussed. Weinberg argues that quantum mechanics cannot be changed, and that any possible generalization of this theory might prevent us from keeping intact the whole corpus of facts that this theory explains with such striking accuracy. On the other hand, we think that quantum mechanics can be generalized, or, and this is probably a more accurate perspective, that quantum mechanics can be recovered from a new physical principle where randomness is held to be a genuine property of nature. This can be done if the postulate of measurement in quantum mechanics, wave function collapse, is replaced by a dynamical ingredient which is genuinely stochastic such as proposed by Ghirardi, Rimini and Weber. Giovannetti et al. have shown that rather than destroying quantum mechanics altogether, a concern of Weinberg, the addition of weak stochastic forces in the microscopic domain results in physical effects difficult to detect with current technologies, which would account for its not being detected to date. However, such random forces would legitimatize the assumptions invariably made so far in deriving a unified picture of classical mechanics and thermodynamics from quantum mechanics. This strategy has been adopted by Giovannetti et al. and we are convinced that this paradigm of randomness as reality rather than its being a consequence of uncertainty or limited knowledge of initial conditions, provides the proper perspective to discuss the problems of complexity. In part because it implies that there is a fundamental limitation to what we can know with absolute certainty about the nature of reality.

Since we do not live on the microscopic level, at least consciously, it is of driving interest how to connect the macroscopic world of our everyday experience, characterized by the existence of thermodynamical processes, to the microscopic world of quantum mechanics. This problem is subtly related to that of deriving classical mechanics from quantum mechanics. This is especially true now because of the widespread conviction that chaos might enable us to realize Boltzmann’s dream of constructing a mechanical basis for microscopic processes, see for example Lebowitz.
However, Boltzmann’s dream is as elusive as the holy grail, and establishing a direct manifestation of classical chaos in quantum physics has so far not occurred despite the efforts of thousands of researchers, see the discussion by Reichl \cite{55}.

It might be argued that since the Correspondence Principle shows us how to make the transition from the quantum to the classical domains, that it should also be able to guide us to the proper interpretation of non-integrable systems in quantum mechanics. On the other hand, classical physics can be portrayed as the physics of events, and the paradoxes of quantum mechanics arise because of the lack of events in the quantum domain, as described by Blanchard and Jadczyk \cite{56}. The search for a unified picture of quantum and classical mechanics, and therefore of the macroscopic and microscopic worlds, has been undertaken by countless scientists. Many proposals for a unified theory have been put forth, and some have direct bearing on our own investigation into measures of complexity, but none have as yet emerged as clearly superior to the others.

According to quantum mechanics the dynamics of a body is described by the Shrödinger equation, and the predictions of this fundamental equation must be consistent with those provided by Newton’s equations. A microscopic body is predicted by quantum mechanics to be characterized by a wave function evolution and the average evolution must mimic the time evolution of a classical trajectory as the mass of the particle becomes macroscopic. Thus, the chaos of classical physics presents a problem for quantum mechanics in that the center of gravity of the evolution of a wave function must become erratic due to the chaotic evolution of the corresponding classical trajectory of a particle. This is actually still an unsolved problem, and the source of the difficulty may in fact be due to the physics of chaos being strongly dependent on the dynamical structures in classical phase space being fractal and self-similar, see for example Mandelbrot \cite{57} for a more complete discussion. The fractal paradigm implies that the phase space structures, when examined on any scale, look the same. There is structure within structure within structure, down to the smallest space-time intervals. This picture conflicts with quantum mechanics, where there is a natural scale limitation imposed on the quantum description by the uncertainty principle.

Consider a small volume of phase space in which an ensemble of initial trajectories (classical particles) are placed, released and allowed to evolve according to Liouville’s Theorem (Newton’s Laws). The existence of chaotic solutions imples that the initial volume will fragment, forming whirls and tendrils that interpenetrate all the available phase space without changing its volume, forming an interwoven fractal structure. The quantum picture of the same process, on the other hand, requires that the process of fragmentation cannot continue indefinitely. When the fragmentation of the wave function reaches scales on the order of Planck’s constant, it is stopped and self-similarity is broken. In the last decade scientists have wondered if this inhibiting effect of quantum mechanics might manifest itself in dynamical processes otherwise expected to be classical.

Berry \cite{58} argues that in the case where quantum mechanics is expected to recover classically chaotic trajectories, the exact correspondence between the time evolution of a single wave function and a Newtonian trajectory is lost on a time scale depending logarithmically on Planck’s constant. This time scale would make this effect, the in-equivalence between classical and quantum phenomena, experimentally accessible. In spite of this remarkable prediction, no significant effect has been found so far. According to Roncaglia et al. \cite{59} this failure is, at least in part, due to the fact that the comparison between quantum and classical predictions must be made at the statistical level, within the so-called Gibbs ensemble perspective that we discussed earlier. Roncaglia et al. \cite{59} further argue that there might be discrepancies between quantum mechanical and classical prescriptions at the statistical level if the experimental observation of the influence of chaos moves from the case of ordinary to that of anomalous diffusion. In Section 6 we shall come back to this important observation for the main purpose of proving that the birth of a new vision of Complexity accompanies the possibility of an experimental detection of spontaneous collapses.

Let us now shift perspectives from the quantum manifestations of chaos and its consequences to another paradox of quantum mechanics. As is well known, the main hurdle for a satisfactory unification of quantum and classical physics is the superposition principle, even if we assume that the Correspondence Principle could provide the proper classical limit of quantum phenomena. Let us assume that the time evolution of two distinct, narrow wave packets A and B, each reproduces a classical trajectory very well and the two trajectories are macroscopically distinct. According to the linear nature of the Shrödinger equation if A is a solution of the equation, and B is another solution of the equation, then so also is the linear superposition of A and B. It is evident that the superposition of two distinct outcomes, in this case two distinct classical trajectories, is a concept foreign to classical mechanics and indeed is essentially incompatible with our daily experience. The unfolding of the dynamics of macroscopic bodies are known to be very accurately described by Newton’s equations. We actually discussed this problem earlier in the form of the cat paradox. The resolution of the paradox was offered by the theory of Zurek \cite{60} where one takes into account the fact that there is no such thing in nature as an isolated system, there is always a certain amount of interaction with the environment. If we mimic the influence of the external environment by white noise, a totally random process with no memory, acting on the system of interest, then the correlations between the two distinct classical trajectories (states of the cat’s well being) are lost. This resolution of the paradox makes it impossible for an experimental observation, adopting the statistical view of Gibbs, to assess the simultaneous existence of the two trajectories. Thus, the environment, noise,
C. Randomness is incompatible with traditional physics

The resolution of the cat paradox by Zurek is very appealing, however, it rests on the assumption that uncorrelated random processes can be derived from within the ordinary laws of physics. This is a perspective shared by the overwhelming majority of physicists, and one of its earliest implementations was given by the celebrated golden rule of Fermi, see for example Zwanzig [61]. The golden rule is related to the possibility of turning the coherent nature of a transition process in quantum mechanics into an incoherent process strictly because of the large number of particles in the system and the correspondingly large number of quantum states participating in the transition. The uncritical acceptance of this viewpoint has turned it into a scientific dogma or prejudice rather than a description of what actually occurs.

This prejudice spread from the time of Fermi to the present in many different guises, the Pauli master equation, the van Hove master equation, see [61, 62] for a complete discussion, and transport processes based on the assumption of linear response, see Bianucci et al. [38] for a modern treatment of these ideas. Scientists have been quite satisfied because the statistical predictions resulting from these theories, which are supposed to be quantum, are completely consistent with experimental observations. However, a careful analysis of all these theories by Giovannetti et al. [54] reveals a crack in the foundation of physics in that they are all, in one way or another, based on the Markov approximation. This dependence on the Markov assumption is so fundamental to the physical theories on which our understanding of the world is based that we are forced to discuss this technical point of analysis at least at an intuitive level.

Let us begin then by following a trajectory that is given a value at the time $t=0$. Once the initial condition has been specified the trajectory, determined by Newton’s laws of motion, is fixed. The trajectory is completely disconnected from the past, the future of the system is only dependent on this initial state. This aspect of determinism is obscured if rather than studying a trajectory, we limit ourselves to considering a projection of this trajectory, much like mistaking the shadows on the wall of Plato’s cave for the actual lives of people. If we are considering projected rather than full trajectories, then two distinct projected trajectories can depart from the same initial condition, at least in so far as can be determined in the projected space. To realize that the two distinct projected trajectories are a genuine manifestation of ordinary classical mechanics, the observer has either to look at the full trajectories or at the whole history of the trajectories. In examining the full trajectories the observer will become convinced that in the full phase space, the two trajectories never intersect, or if they intersect once then they intersect infinitely often, that is, the orbit is unstable (homoclinic orbit). In studying the whole history of the trajectories the observer can remain at the projected level, only to realize that the two trajectories departing from the ”same initial condition” actually are characterized by totally different histories. Thus, we arrive at the surprising conclusion that the deterministic character of the theory adopted, including quantum mechanics if the process of measurement is included, is reflected in their non-Markov character. By their non-Markov character we mean to say that a projected representation must bear a significant dependence on the past, and that the future time evolution of the system does not depend only on the conditions of the system at the moment of the observation, but it also depends on the history of the system. This is not to be considered a special property of statistical processes that make them different from the deterministic nature of classical and quantum mechanics. On the contrary, this significant dependence on the past is the mark of the deterministic nature of the physical laws driving the time evolution of the whole Universe.

Ironically, all physicists claiming that classical physics naturally emerge from quantum, adopt the Markov approximation. Also Zurek claims that the current physical paradigms are sufficient to account for all the fundamental problems concerning the derivation of thermodynamics from classical mechanics, which in turn is derived from quantum mechanics. Thus, a Markov process is a statistical process whose time evolution is fixed only by the initial condition, so that its evolution is totally independent of the past. This property is shared by the deterministic evolution of the entire universe, but when it becomes a statistical property of a projection of the universe, it cannot be true. The Markov assumption seems to be incompatible with a rigorously quantum treatment. Thus, if it leads to plausible, and realistic results, it is a sign that unknown physical laws drive the universe. We think that these laws introduce randomness into nature, thereby making it legitimate to adopt the Markov condition.

The Markov assumption produces an exponential decay of correlations in the physical descriptions of reality. However, Fonda et al. [63] and Lee [64] proved mathematical theorems establishing that exponential decay is incompatible with both classical and quantum mechanics. Thus, the Markov assumption seems to be a subterfuge, providing an illusion of settling the fundamental problems of modern physics without any need for additional hypotheses. Ironically, the repeated use of the Markov approximation has been shown to be equivalent to a departure from traditional physics.

Thus, we see that the Markov approximation is a consequence of a previous assumption, that being the observation
that the study of the entire universe is too complex for us to address all at once. This observation implies that we examine only a projected part of the universe. This decision itself entails certain subjective elements and even if it lead to the resolution of the quantum-related problems discussed above it would still result in the Second Law of Thermodynamics being a consequence of our limited knowledge of the universe, rather than being an objective aspect of reality. The Markov approximation is therefore inconsistent with the physical laws that the advocates of this approach claim to be a complete representation of the universe.

How is it possible that such a fundamentally incorrect assumption can provide such a wealth of accurate predictions? A formal answer to this question has been given by Giovannetti et al. 153 who define the conditions for a genuine source of randomness to produce the Markov approximation with no significant departure from the predictions made using traditional physics. On the basis of the results of Giovannetti et al. we wish to make the following plausible conjecture: All the sources of complexity examined so far are actually channels for the amplification of naturally occurring randomness in the physical world.

This randomness, must not be confused with algorithmic complexity. It is a genuine property of nature independent of any experimental observation. If the algorithmic complexity is so high as to result, according to the arbitrary Markov approximation, in a very short correlation time, then the spontaneous fluctuations might have the effect of making the Markov approximation genuine. This is expected to result in predictions slightly different from those of ordinary quantum mechanics, but for practical purposes it might not have any relevant consequences. If the "subjective" source of randomness is extremely strong, an even infinitesimally small genuine source of stochasticity has the effect of making the system diffuse incoherently. This is the reason why the scientists who assume incoherent behavior without introducing objective randomness find experimental vindication of their predictions. They obtain the right answers but for the wrong reasons.

D. Information approach to complexity

We have pointed out that the concept of randomness as a consequence of a lack of information is not totally satisfactory, and that there is a need for a new concept of objective randomness, perhaps in the form of a new principle of physics. This new physical principle should be only a slight modification of traditional quantum mechanics in which it is supplemented by the inclusion of a genuine element of randomness. On the other hand, we cannot easily establish the intrinsic nature of this randomness. Is it the familiar Wiener process as Ghirardi, Rimini and Weber 52 claim? The Wiener process is ordinarily assumed to be an idealization of physical processes satisfactorily described by known physical laws. We have seen, however, that this cannot be the complete story since the white noise postulated by Zurek cannot be derived from traditional quantum mechanics. In principle we cannot rule out the possibility that the Wiener process introduced by 52 to correct ordinary quantum mechanics might have a deterministic origin similar to that generated by chaos, although produced by some still unknown deterministic mechanism.

The existence of objective randomness seems to be in conflict with the recent comments of Landauer 155, who considers the universe itself to be a computer with finite memory. This view of the world would imply that the fluctuations produced by round-off errors in ordinary computers would have a correspondence in nature resulting in fluctuations being embedded in the fabric of reality. Thus, the Markov approximation incompatible with either ordinary quantum or classical mechanics, might be produced by the round-off errors of the universe. This picture would also resolve the fundamental question swirling around the foundation of the derivation of thermodynamics from mechanics, and of classical physics from quantum as well. There are several indications that round-off errors are indistinguishable from genuine fluctuations, and that these fluctuations produce a crossover from anomalous to ordinary statistical mechanics 156 157, although at very large times, if the intensity of these fluctuations is very weak.

We see that if a still unknown principle of statistics, requiring that nature is fundamentally random and irreversible, then the unsatisfactory aspects of the current definitions of complexity are resolved. This is true in spite of the ambiguity in the meaning of randomness in this new context. This is where the physics paradigm suitably extended may play a crucial role in the development of measures of complex phenomena.

VI. CONCLUSIONS: OBJECTIVE RANDOMNESS INDUCING A NEW VISION OF COMPLEXITY

The discussion of the earlier sections is enough for us to reach a conclusion fitting the Penrose’s view about "why a new physics is needed to understand the mind" 168. However, in the last few years there have been many new results, on which many of the papers of these Proceedings are based, which are also suggesting a new vision of complexity, which, hopefully, affords convincing answers to many of the question discussed in this paper. In addition to those mentioned in the earlier sections, other groups are also looking for a picture of reality where randomness is already present at the fundamental level.
Let us quote some relevant cases. An interesting proposal has been made \cite{69} for a realistic setting for Feynman paths. This is an attempt at a realistic interpretation of the amplitudes, rather than probabilities, in the Feynman interpretation of quantum mechanics with the path-integral formalism. This new formulation rests on the dynamics of a pair of entwined trajectories. The particles move on entwined-pair trajectories in space time therefore generating the impression of unitary time evolution, with dynamic rules, though, that are as random as the random walker prescriptions of classical mechanics. This is in a sense the reverse of the assumption implicitly made by the advocates of decoherence, whose philosophy would lead us to conclude that wave-function collapses, and with them the second principle of thermodynamics itself, are an illusion of observers forced by their human limitation to look at a limited portion of the Universe. The authors of Ref. \cite{69} conjecture that from their theory a realistic interpretation of the wave-function collapse might emerge. This is quite possible, due to the fact that the new physics that they propose is essentially random and non-unitary.

Another approach to quantum mechanics moving from thermodynamics, with the second law regarded as being a fundamental law of nature, rather than an illusion of the human observer, has been proposed by El Naschie\cite{70}. El Naschie proves that the Cantorian space can serve as a geometrical model for a spacetime support of the thermodynamical approach. Additional work at uncovering some unsuspected connections between the abstract algebra of wild topology and high energy physics has been more recently found by the same author\cite{71} Using the same perspective, the three Nicolis\cite{72} explained the two-slit delayed experiment without using the Wheeler interpretation of the "observer particpancy", setting doubts on the independent existence of the Universe.

However, at the end of this paper devoted to looking for a satisfactory definition of complexity, it is convenient to discuss the consequence that a new physics might have on this specific issue. The main problem with the work of Ref. \cite{55} is that the conclusions might be more satisfactory from a philosophical point of view, since the resulting diffusion equation, with the characteristics of normal diffusion, is not the mere result of a contraction procedure, equivalent to interpreting the second principle as a human illusion, but the second principle is true, independently of the existence of an observer. However, from a practical point of view the advocates of de-coherence theory might conclude that the same result is obtained with simpler calculation, and, consequently, applying the Ockham principle\cite{68}, is true. For this reason, it is important to mention the work of ref. \cite{55} that yields a remarkable result, this being a different experimental result, according to the perspective adopted. Another way to express the same conclusion is as follows. As pointed out in earlier sections, the de-coherence wisdom rests on the division of the Universe into two parts, the system of interest and its environment. If the environment is the source of uncorrelated fluctuations, the resulting Markov equation yields results that from a statistical point of view are equivalent to those where real collapses, and events, unpredictable events, take place. If we move from this safe condition to a condition where the bath is responsible for correlated fluctuations the statistical equivalence of the two pictures is not longer guaranteed. At the time of this writing, the research work on these delicate issues is not yet completed. However, there are strong indications that the breakdown of the equivalence between density and trajectories noticed by the authors of Ref. \cite{55} is provoked by the occurrence of aging \cite{72, 73}. This brings us back to the conception of complexity as LSM.

The vision of LSM emerging from the dynamic model of Refs. \cite{40, 41, 76}, according to the authors of Ref. \cite{77}, has a biological relevance and represents a vision that, without conflicting with that of the Prigogine school\cite{4}, affords additional arguments to support the view that life is not foreign to nature, as misrepresented by the conventional equilibrium statistical mechanics. Even in the absence of a flow of energy from outside, we can notice a natural tendency to the emergence of properties, such as aging, that are conventionally attributed to living systems. We have to notice that this vision of complexity emerges from the dynamic approach of Ref. \cite{55} extended to the case where long-range correlation and memory are present. In this case the transition from dynamics to thermodynamics is infinitely slow thereby suggesting that this condition as a new state of matter, the earlier mentioned concept of LSM.

This dynamic approach yielding the vision of LSM, on the other hand, is at the basis of new techniques of analysis of time series \cite{78}, which are currently used with success to assess the complexity of the systems, from which these time series are generated. As we have earlier mentioned the Kolmogorov complexity is not computable, and these techniques, directly or indirectly related to the concept of a Kolmogorov complexity, yield a computable measure of complexity. It is interesting to remark that, although these techniques are accurate, distinguishing with their help biotic from a-biotic systems remains a challenging issue \cite{78, 80}. It is interesting to notice that the field of complexity is reversing the current perspective. While, as pointed out by Gunter\cite{2}, explaining why rocks and life emerged at the same time, in the geological scale, is a challenging issues for ordinary physics (this meaning for us, essentially, ordinary statistical mechanics), from within the field of complexity it is rather becoming challenging the distinction between biotic and a-biotic systems, given the widespread tendency in nature to establish long-range correlations.

Due to the importance of the vision of complexity suggested by the paper of Ref. \cite{41} before ending this paper is convenient to devote some more comments to it. It is based on the concept of Lévy walk, a process characterized by...
ininitely extended time memory, and so time non-locality, which is slowly converted into a Lévy flight, namely, as pointed out by the authors of Ref. 53 into space non-locality. However, it takes an infinite time for this transition to occur. Throughout this transition process, lasting for an infinite time, the dynamic process is multi scaling, rather than mono scaling. Thus, we are led to conclude that the condition of scaling, a quite mono-scaling condition, departing from the ordinary scaling of Brownian motion, is not an indication of complexity. It is rather the inscription on a grave signaling that the system was complex when it was alive. In fact, an exact mono-scaling condition indicates that the Markov condition has been recovered, this meaning that many uncorrelated and objective jumps occurred. Notice that this condition of thermal death, occurring without the influence of environmental noise, which probably would make Gaussian the resulting death, is an idealization of reality. However, this idealization serves the desirable purpose of illustrating the dynamic perspective of LSM. In this ideal condition, the system would age forever without ever dying. Furthermore, no simple generalization of diffusion equation is known, for a fair representation of this process, thereby really implying the breakdown of the simplicity condition. What about a quantum derivation of LSM? We note that the work of 76 refers to a real experiment, on the so called blinking quantum dots (see the important paper of Jung, Barkai and Silbey 82 for details on this fundamental aspect). We are inclined to believe that the jumps from the light to the darkness state and vice-versa are triggered by the spontaneous GRW collapses. To have a non-Poisson statistics for these collapses we probably need to generalize the work of Tessieri et al. 83 to the non-Ohmic condition. The authors of Ref. 83 studied the case when the de-coherence of the system of interest is produced by the interaction with a bath of bosons, undergoing the GRW collapses. The calculation must be extended to the case of a non-Ohmic bath: an interesting research program.

In conclusion, we have to acknowledge that there are significant attempts at reconciling general relativity to quantum mechanics 54, 55, 56, 57, 58, 59 using fractal geometry, namely, one of the theoretical ingredient of complexity. Furthermore, as earlier remarked, the assumption of randomness as an essential ingredient of the new physics 60, 70, 71, 72 makes it natural to perceive the second principle as real rather than as an illusion, as it is subtlety implied by de-coherence theory. We think that all these authors are doing remarkable work to properly address the challenge of Gunter 2 who correctly perceives quantum mechanics, general relativity and quantum mechanics, as three different theories, with no connections. We stress that within this context the dynamic approach to complexity, moving from the earlier work of Ref. 33, is producing some specific benefits, although at more limited level of establishing a relation between dynamics and thermodynamics, with two major results. The first is the discovery of a promising direction to project experiments aiming at turning a philosophical controversy about randomness and wave-function collapses into a real scientific issue 73. The second is the proposal 70 of a new view of complexity as a state of transition from dynamics to thermodynamics, denoted as LSM, with the important effect of abolishing the perspective of ordinary statistical mechanics that would make life foreign to physics.

Acknowledgments PG acknowledges support from ARO, through Grant DAAD19-02-0037.

[1] J. Monod, Chance and Necessity; an essay on the natural philosophy of modern biology, New York, Vintage Books (1972).
[2] P. Gunter, "Analysis and its Discontents: Nonlinearity and the Way Things Aren’t", Chaos, Solitons and Fractals, These Proceedings.
[3] K. Gödel, Uber formal unentscheibare Satze der Principa Mathematica und verwander Systeme I, Monothese fur Mathematik und Physik 38, 173-98 (1931); K. Gödel, On undecidable propositions of formal mathematics systems, Institute for Advanced Study, Princeton (1934).
[4] I. Prigogine and I. Stenger, Order out of chaos: man’s new dialogue with nature, Bantam Books, New York (1984).
[5] S. Weinberg, Dreams of a Final Theory, Pantheon Books, New York (1992).
[6] E. Mayr, "The limit of reductionism", Nature, 311, p. 475 (1988).
[7] John D. Barrow, Theories of Everything: The Quest for Ultimate Explanation, Clarendon Press, Oxford (1991).
[8] L. von Bertalanffy, General Systems Theory, G. Braziller, New York (1968).
[9] G.A. Cowen, D. Pines and D. Metzler, Editors, Complexity, Metaphors, Models and Reality, Addison-Wesley, Reading, Mass. (1994).
[10] M. Gell-Mann, "Complex Adaptive Systems", in R.
[11] D. Ruelle, Chance and Chaos, Princeton University Press, Princeton, New Jersey (1991).
[12] P. Winsor, "Complexity in the Experimental Audio/Visual Arts", Chaos, Solitons and Fractal, These Proceedings
[13] W. Weaver, "Science and Complexity", American Scientist 36, 536-44 (1948).
[14] R.L. Flood and E.R. Carson, Dealing with Complexity, 2nd Edition, Plenum Press, New York (1993); 1st Edition (1988).
[15] M. Peterson, Ergodic Theory, Cambridge University Press, Cambridge (1983).
[16] V.I. Arnold and A. Avez, Ergodic Problems of Classical Mechanics, W.A. Benjamin, New York (1968).
[17] M.C. Mackey, Time’s Arrow, Springer-Verlag, New York (1992)
[18] A. Lasota and M.C. Mackey, Chaos, Fractals and Noise, Springer-Verlag, New York (1994).
[19] J. Zernike, Entropy, the devil on the pillow, Kuwer-Deventer, Amsterdam (1972).
[20] E. Schrödinger, *What is Life?*, Cambridge University Press, New York (1995) first published in 1944.
[21] A.B. Cambel, *Applied Chaos Theory*, Academic Press, Boston (1993).
[22] L. Boltzmann, *Wissenschaftliche Abhandlungen*, editor Fr. Hasenohrl, three volumes, Leipzig (1909).
[23] I. Prigogine, *Thermodynamics of Irreversible Processes*, 2nd revised edition, Wiley Interscience, New York (1961); 1st edition (1955).
[24] J.W. Gibbs, *Elementary Principles in Statistical Mechanics*, Ox Bow Press, Woodbridge, Conn. (1981) first published in 1901.
[25] C. Tsallis, S.V.F. Levy, A.M.C. Souza and R. Maynard, "Statistical-Mechanical Foundation of the Ubiquity of Lévy Distributions in Nature,” Phys. Rev. Lett. 75, 3589-92 (1995).
[26] A.N. Kolmogorov, *Doklady Akademii Nauk* 119, 861 (1958).
[27] C.E. Shannon, "A mathematical theory of communication", The Bell System Journal XXVII, 379-423 and 623-56 (1948).
[28] L. Sztizl, "On the decrease of entropy in a thermodynamical system by the intervention of intelligent beings", Behavioral Science 9, 301-310 (1964); first published in Zeitschrift für Physik 53, 840-56 (1929).
[29] Ya.G. Sinai, Dokl. Acad. Sci.USSR 124, 768 (1959).
[30] Ya. B. Pesin, Adv. Geophys. 32, 55 (1977).
[31] V. Latora and M. Baranger, "Kolmogorov-Sinai Entropy versus Physical Entropy", Phys. Rev. Lett. 82, 520 (1999).
[32] T. Petroski and I. Prigogine, Chaos, Solitons and Fractals 7, 441 (1996).
[33] T. Petroski and I. Prigogine, Chaos, Solitons and Fractals 11, 373 (2000).
[34] M. Bologna, P. Grigolini, M. Karagiogis, and A. Rosa, Phys. "Trajectory versus probability density entropy”, Phys. Rev. E 64, 016223 (2001).
[35] A. Renyi, *Probability Theory*, North Holland, Amsterdam (1970).
[36] B.J. West and W. Deering, *The Lure of Modern Science: Fractal Thinking*, Studies of Nonlinear Phenomena in Life Science, Vol. 3, World Scientific, New Jersey (1995).
[37] B.J. West, "Comments on the Renormalization Group, Scaling and Measures of Complexity", Chaos, Solitons and Fractals, these Proceedings.
[38] M. Bianucci, R. Mannella, B.J. West and P. Grigolini, "From dynamics to thermodynamics: linear response and statistical mechanics", Phys. Rev. E 51, 3002 (1995).
[39] M. Annunziato, P. P. Grigolini, and B.J. West, "Canonical and noncanonical equilibrium distribution, Phys. Rev. E 64, 011107 (1-13) (2001).
[40] M. Ignaccolo, P. Grigolini, A. Rosa, Sporadic randomness: The transition from the stationary to the nonstationary condition, Phys. Rev. E64 026210 (1-11) (2001).
[41] P. Allegrini, J. Bellazzini, G. Bramanti, M. Ignaccolo, P. Grigolini, and J. Yang, Scaling breakdown: A signature of aging, Phys. Rev. E, 66, 015101 (1-4) R (2002).
[42] M. Buiatti, P. Grigolini, A. Montagnini, "A Dynamic Approach to the Thermodynamics of Superdiffusion", Phys. Rev. Lett. 82, 3383-3387 (1999).
[43] P. Allegrini, P. Grigolini and B.J. West, "Dynamical approach to Lévy processes", Phys. Rev. E 54, 4760-67 (1996).
[44] B. De Witt and R.N. Graham, *The Many Worlds Interpretation of Quantum Mechanics*, Princeton University Press, Princeton, New Jersey (1973).
[45] R. Penrose, *Shadows of the Mind, a Search for the Missing Science of Consciousness*, Oxford University Press, Oxford (1994).
[46] P. Lowenhard, "The Mind-Body Problem: Some Neurobiological Reflections", in *Reductionism and System Theory in the Life Sciences*, Editors, P. Høyning-Huene and F.M. Wuketits, Kluver Academic Pub., Dordrecht (1989).
[47] G. Chaitin, *Algorithmic Information Theory*, Addison-Wesley, New York (1987).
[48] S.S. Schweber, Physics Today 46 (11), 34 November (1993).
[49] P.W. Anderson, Science 177, 393 (1972).
[50] K.G. Wilson and J. Kogut, Phys. Rep. 12, 75 (1974).
[51] A.J. Leggett, *The Problems of Physics*, Oxford University Press, Oxford (1987).
[52] G.C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D 34, 470 (1986).
[53] V. Giovannetti, P. Grigolini, G. Tesi and D. Vitali, "Wave function collapse versus objective randomness”, Phys. Lett. A 224, 31 (1996).
[54] J. Lebowitz, in *Physical origins of time asymmetry*, edited by J.J. Halliwell, J. Perez-Mercader, and W.J. Zurek, Cambridge University Press, Oxford (1987).
[55] L. E. Reichl, *The Transition to Chaos*, Springer-Verlag, New York (1992).
[56] Ph. Blanchard and A. Jadczyk, Phys. Lett. A 203, 260 (1995).
[57] B.B. Mandelbrodt, *The Fractal Geometry of Nature*, W.H. Freeman and Co., San Francisco (1977).
[58] M.V. Berry, NY Ann Phys 131, 163 (1981).
[59] R. Roncaglia, L. Bonci, B.J. West and P. Grigolini, "Anomalous Diffusion and the Correspondence Principle", Phys. Rev. E 51, 5524 (1995).
[60] W.H. Zurek, Phys. Today 44 (10), 36 (1991).
[61] R. Zwanzig, in *Lectures in Theoretical Physics*, Vol.3, edited by W.E. Brittin et al., Interscience, New York (1961).
[62] R. Zwanzig, in *Quantum Statistical Mechanics*, ed. P.H.E. Meijer, Gordon and Breach, pp.139 (1966).
[63] L. Fonda, G.C. Ghirardi and A. Rimini, Rep. Prog. Phys. 41, 587 (1977).
[64] M.H. Lee, Phys. Rev. Lett. 51, 1227 (1983).
[65] R. Landsauer, Phys. Lett. A 217, 188 (1996).
[66] R. Bettin, R. Mannella, B.J. West, P. Grigolini, "Influence of the Environment on Anomalous Diffusion", Phys. Rev. E 51, 212 (1995).

[67] E. Floriani, R. Mannella, P. Grigolini, "Noise-induced transition from anomalous to ordinary diffusion: the crossover time as a function of noise intensity" Phys Rev E 52, 5910 (1995).

[68] R. Penrose, "Why new physics is needed to understand the mind", in What is Life? The Next Fifty Years, editors M.P. Murphy and L.A.J. O’Neill, Cambridge University Press, Cambridge (1995).

[69] G.N. Ord and J.A. Guaitieri, "A realistic setting for Feynman paths". Chaos, Solitons and Fractals, 14, 929 (2002).

[70] M.L. El Naschie, "Time symmetry breaking, duality and Cantorian spacetime". Chaos, Solitons and Fractals 7, 499 (1996).

[71] M.L. El Naschie, "Wild topology, hyperbolic geometry and fusion algebra of high energy particle physics", Chaos, Solitons and Fractals 13, 1935 (2002).

[72] J.S. Nicolis, G. Nicolis and C. Nicolis, "Nonlinear dynamics and the two-slit delayed experiment". Chaos, Solitons and Fractals 12, 407 (2001).

[73] M. Bologna, P. Grigolini Luigi Palatella, Marco Pala, Decoherence, wave function collapses and non-ordinary statistical mechanics, Chaos, Solitons and Fractals, 17, 601-608 (2003).

[74] M. Bologna, P. Grigolini and B. J. West, "Strange kinetics:conflict between density and trajectory description", 284, 115 (2002).

[75] P. Allegrini, P. Grigolini, L. Palatella, A. Rosa, "The conflict between trajectory and density description: the statistical source of disagreement", submitted to Phys. Rev. E, [cond-mat/0212614].

[76] P. Allegrini, G. Aquino, P. Grigolini, L. Palatella, A. Rosa, "Breakdown of the Onsager Principle as a sign of aging", submitted to Phys. Rev. E, [cond-mat/0304506].

[77] M. Buiatti, M. Buiatti, "Towards a characterisation of the living state of matter", Chaos, Solitons and Fractals, These Proceedings.

[78] P. Allegrini, V. Benci, P. Grigolini, P. Hamilton, M. Ignaccolo, G. Menconi, L. Palatella, G. Raffaelli, N. Scafetta, M. Virgilio, J. Yang, Compression and diffusion: a joint approach to detect complexity, Chaos, Solitons and Fractals, 15, 517 (2003).

[79] M. C. Storrie-Lombardi, F. A. Corsetti, M. Ignaccolo, P. Grigolini, M. Ignaccolo, P. Allegrini, S. Galatolo, G. Tinetti, Complexity Analysis to Explore the Structure of Ancient Stromatolites, Chaos, Solitons and Fractals, These Proceedings.

[80] M. Ignaccolo, A. Schwettmann, R. Failla, M. Storrie-Lombardi, P. Grigolini, "Stromatolites: why do we care?", Chaos, Solitons and Fractals, These Proceedings.

[81] V. Seshadri, B.J. West, Proc. Natl. Acad. Sci. US 79, 4051 (1982).

[82] Y.J. Jung, E. Barkai, R. J. Silbey, "Lineshape theory and photon counting statistics for blinking quantum dots: a Lévy walk process, Chem. Phys. 284, 181 (2002).

[83] L. Tessieri, D. Vitali, P. Grigolini, "Quantum Jumps as an Objective Process of Nature", Phys. Rev. A 51, 4404 (1995).

[84] L. Nottale, Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity, World Scientific, Singapore (1993).

[85] M.S. El Naschie, "Superstrings, knots and non-commutative geometry in space" Int. J. Theor. Phys., 37 2935 (1998).

[86] M. S. El Naschie, "Non-linear dynamics and infinite dimensional topology in high energy particle physics", Chaos, Solitons and Fractals 17, 591 (2003).

[87] M. S. El Naschie, "Quantum loops, wild topology and fat Cantor sets in transfinite high-energy physics", Chaos, Solitons and Fractals, 13, 1167 (2002).

[88] B.G. Sidharth, "The nature of quantum space-time and the Cantorian E(∞) proposal", Chaos, Solitons and Fractals 14 1325 (2002).

[89] M. Agop, P.D. Ioannou, C. Gh. Buzea, "Cantorian E(∞) space-time, gravitation and superconductivity", Chaos, Solitons and Fractals 13, 1137 (2002).

[90] In a certain way this reminds us of Gödel and his analysis of logical systems. If we consider his two theorems on undecidability in closed formal systems in a wider way and not strictly from a mathematical and philosophical point of view, then it is easy to see that together with the impossibility of proving a logical system "totally true from the inside", we have the uneasiness of analysing a system that looks at itself from the inside expecting to find the truth.

[91] This problem is also source of paradoxes in quantum mechanics, but, even in the classical realm, it touches the core problem of determinism in a strict sense, as well explained by the "egg’s paradox", reported in [7]. This paradox, due to Diderot, refers to the impossibility of classical Newtonian mechanics to describe the development of living beings. In this paper we shall discuss some recent developments that lead us to interpret complexity as a condition intermediate between dynamics and thermodynamics, and make, consequently, easier to re-conciliate life with statistical mechanics.

[92] Even the majority of physicists reject this kind of reductionism. John Barrow [7], for example, in his "Theories of everything. The quest for ultimate explanation" makes the debate on reductionism popular and within the reach of everybody. He actually takes the example of a calculator and tries to understand if this calculator can be considered just a certain amount of atoms or more than that. This kind of process is very clear to all physicists who deal with nonlinear science, the science where input and output are not proportionally and constantly connected. This idea of nonlinearity naturally leads to the theory of emergence.

[93] We are referring to William of Ockham, a well known philosopher of the 14th century. The medieval rule of parsimony, or principle of economy, frequently used by Ockham came to be known as Ockham’s razor. The rule, which said that plurality should not be assumed without necessity (or, in modern English, keep it as simple as you can), was used to eliminate many pseudo-explanatory entities.