On radiative decay of $Z'$ boson

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In the framework of the extended $SU(2)_h \times SU(2)_l$ model we study the radiative decay of an extra gauge boson $Z'$, $Z' \rightarrow \gamma Z$. The data analysis results of the CDF collaboration at the Fermilab Tevatron allow one to establish not only the lower bound of the decay probability $Z' \rightarrow \gamma Z$ normalized to Drell-Yan process $Z' \rightarrow \mu^+\mu^-$, $R(Z' \rightarrow \gamma Z/\mu^+\mu^-) > 1.4 \times 10^{-5} \cot^2 \phi$, but also to estimate the mixing angle $\xi$ between the states $Z'$ and $Z$, $|\xi| < 5 \times 10^{-2}$, taking into account the additional mixing angle $\phi$ related to the extended gauge group $SU(2)_h \times SU(2)_l$. The comparison with the decays $Z' \rightarrow \tau^+\tau^-$ and $Z' \rightarrow b\bar{b}$, $Z' \rightarrow t\bar{t}$ based on the LEP data is also presented.

1. Introduction

It is well known that various extensions of the Standard Model (SM) admit the existence of new heavy gauge bosons [1] in both the neutral ($Z'$) and the charged ($W^{\pm}$) sectors. From time to time the question of why the top quark is so heavy is addressed to the physics community. Some of the modern theoretical models can offer clarification of this problem within the new additional gauge interactions related to new gauge bosons and fermions of the third and even the fourth generations. The models operating in this direction carry out the extension of the gauge group $SU(N)$ up to $SU(N) \times SU(N)$ gauge structure [2,3]. Due to spontaneous breaking of $SU(N) \times SU(N)$ to its diagonal subgroup the relevant generators correspond to the set of massive $SU(N)$ gauge bosons interacting with fermions of any generations with different coupling strengths. The models of the $SU(N) \times SU(N)$
type predict, in fact, the existence of the $Z'$-boson interacting mainly and efficiently with fermions of the third generation. The simplest theories are based on the extensions of type $SU(2)_l \times SU(2)_h$ for weak interactions: the first two generations of fermions are correlated with the weak gauge group $SU(2)_l$, while the third generation "feels" another, "stronger" gauge structure $SU(2)_h$.

Notice that in some models containing the extended gauge group $SU(2)_l \times SU(2)_h$ the lower bound on the $Z'$-boson mass is given by the scale of 1.0-1.5 GeV [4]. In this paper, we restrict our consideration to the relatively light $SU(2)$ $Z'$-bosons of the order $O(1 \text{ GeV})$. From the phenomenological point of view the interest in these bosons is caused by the possibility of their identification at the Fermilab Tevatron with the energy $\sqrt{s} \simeq 2 \text{ TeV}$ and the $pp$-large hadron collider at the CERN with $\sqrt{s} \simeq 14 \text{ TeV}$.

The models in which the precision data on electroweak interactions admit the existence of only light new gauge bosons $Z'$ and $W^{\pm'}$ are concerned with the extended models with the technicolour [3]. The electroweak symmetry that breaks down the technicolour structure is characterized at least by two stages needed to transition from nonbreaking symmetry at high energies to the low-energy electromagnetic gauge structure. The idea is that on some scale $u$ the symmetry of two gauge groups $SU(2)$ ($SU(2)_l$ and $SU(2)_h$) is breaking to their diagonal subgroup. The break down of the rest electroweak symmetry originates at the scale $v < u$ ($v = 246 \text{ GeV}$).

The $Z - Z'$ mixing effects are defined by the angle $\xi$ within the well-known relation between the eigenstates $Z_1$ and $Z_2$ with the masses $m_{Z_1}$ and $M_{Z_2}$, respectively [1]

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos \xi & \sin \xi \\ -\sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix},$$

(1)

and

$$|\xi| = \arctan \left( \frac{m_{Z_2}^2 - m_{Z_1}^2}{M_{Z_2}^2 - m_Z^2} \right)^{1/2},$$

(2)

where $m_Z$ is the $Z$-boson mass. Very often the effects due to the $\xi$-angle are neglected because of its rather small value. Nevertheless, the estimation of the absolute value of $\xi$ is an important task by itself.
One of the main questions is the signature with which new gauge bosons should be displayed in the experiments. The promising modes of new heavy gauge boson decays as well as the proposals for the experimental restriction on the masses of $Z'$- and $W^{\pm'}$-bosons can be found in [5-10]. The Tevatron Run I data can allow one to estimate the lower bound on the $Z'$-boson mass at the level of $M_{Z'} > 650$ GeV as the narrow resonance with the width $\Gamma_{Z'} = 0.12 M_{Z'}$ [11]. In fact, the partial decay widths in [5-10] are defined with an accuracy up to new coupling constants which (as the rule) are unknown, e.g., the vector and axial-vector couplings of $Z'$ with the quark fields. The most promising channels to observe $Z'$-bosons are their decays into quarks and antiquarks of the third generation ($b\bar{b}, t\bar{t}$) [5-8] or even the pairs of leptons and antileptons in the current experiments at the Tevatron or forthcoming experiments at the LHC (e.g., with the help of the ATLAS detector[12]). Thus, the hadron colliders $Z'$-boson discovery potential depends on the couplings of $Z'$ with quarks and leptons.

The partial widths in the decays mentioned above are on the scale level of $O(1 \text{ GeV})$. We investigate the radiative decays of an extra gauge boson $Z'$ with production of the standard $Z$-boson ($Z' \to \gamma Z$), because the monochromatic photon could be efficiently displayed in the experiments, while the accompanying processes like $Z' \to \gamma \pm$ pseudoscalar mesons will be suppressed. The decay $Z' \to \gamma \gamma$ is forbidden because of the Bose-symmetry. For now we assume that there will be possible to observe a peak in the photon spectrum of the inclusive process $Z' \to \gamma + \text{all}$. Another important feature of the decay $Z' \to \gamma Z$ is the possibility to get the restriction on the couplings between $Z'$ and quarks. The relative width of the decay $Z' \to \gamma Z$ compared to $Z' \to \mu^+\mu^-$, $Z' \to \tau^+\tau^-$ decays is expected on the level of about $10^{-5} - 10^{-4}$, which corresponds to the partial decay width $\Gamma(Z' \to \gamma Z) \sim O(1 \text{ GeV})$. The decay $Z' \to \gamma Z$ has the peculiarity of the enhancement effect because of the factor $G_F M_{Z'}^4/m_Z^2$ due to longitudinal polarization of the $Z$-boson ($G_F$ is the Fermi weak constant). Here, for simplicity, one can neglect the effects of mixing between the physical states $Z'$ and $Z$, and one can take into account only axial couplings of $Z'$ with the quarks in the loop. We believe that all the quarks may have the identical $U'(1)$ charge. The mass of the
Z'-boson is unknown and it is generated by the own scalar singlet in the framework of the $SU(2) \times U(1)$ group.

2. General properties of $SU(2)$ Z'-bosons

Consider the physical model containing weak interactions governed by the pair of $SU(2)$ gauge groups: $SU(2)_h \times SU(2)_l$ [9]. The $SU(2)_h$ group is responsible for weak interactions with heavy leptons and quarks, and the left fermions transform as doublets while the right fermions as singlets. The group $SU(2)_l$ is related to the first two generations of leptons and quarks with the charges identical to those which come from the standard representation. The extended gauge group $SU(2)_h \times SU(2)_l$ breaks down to its diagonal subgroup $SU(2)_L$ on the scale $u$ (which is yet unknown) by some scalar field $\sigma$ with the charge $SU(2)_h \times SU(2)_l \times U(1)_Y$

$$\sigma \sim (2, 2)_0, \langle \sigma \rangle = \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix}.$$ 

The symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ takes place because of nonzero value of the vacuum expectation value of the scalar field $v = 246$ GeV.

The set of the gauge fields looks like

$$A^\mu = \sin \Theta_W (\cos \phi W^\mu_{3h} + \sin \phi W^\mu_{3l}) + \cos \Theta_W X^\mu; \quad (3)$$

$$Z_1^\mu = \cos \Theta_W (\cos \phi W^\mu_{3h} + \sin \phi W^\mu_{3l}) - \sin \Theta_W X^\mu; \quad (4)$$

$$Z_2^\mu = - \sin \phi W^\mu_{3h} + \cos \phi W^\mu_{3l} \quad (5)$$

with the $U(1)_{em}$ group generator $Q = T_{3h} + T_{3l} + Y$ as an operator of the electric charge. Notice that the covariant derivative is getting longer with the additional term related to Z'-boson

$$D^\mu = \partial^\mu - i \frac{g}{\cos \Theta_W} Z_1^\mu (T_3 - Q \sin^2 \Theta_W) - i Z_2^\mu (-g_h \sin \phi T_{3h} + g_l \cos \phi T_{3l}) , \quad (6)$$

where $T_3 = T_{3h} + T_{3l}$, $\phi$ is the additional mixing angle, and the gauge coupling constants $g_h$ and $g_l$ have the following form:

$$g_h = \frac{g}{\cos \phi}, \quad g_l = \frac{g}{\sin \phi}.$$
The effects of the mixing $Z_1^\mu - Z_2^\mu$ are defined by the angle $\phi$, and in the leading order over $1/x = v^2/u^2$ there is the following field superposition [9]:

$$
\begin{pmatrix}
Z^0 \\
Z'
\end{pmatrix} = \begin{pmatrix}
1 - \frac{\cos^3 \phi \sin \phi}{x \cos \theta} & \frac{\cos^3 \phi \sin \phi}{x \cos \theta} \\
\frac{\cos^3 \phi \sin \phi}{x \cos \theta} & 1
\end{pmatrix}
\begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix}.
$$

(7)

3. The model

Our approach could be applied almost to all the models relevant to the $Z'$-boson (e.g., $E_6$ superstring model, etc.); however, we use the extended $SU(2)_h \times SU(2)_l$ model with the fermion loops containing up- and down- quarks including the quarks of the fourth generation. For now we assume that the couplings of $Z'$ and $Z$ with quarks $q$ are defined through the following Lagrangian density:

$$
-L = g_Z \sum_q \bar{q}(v_q - a_q \gamma_5) \gamma^\mu q Z_\mu + g_Z' \sum_q \bar{q}(v_q' - a_q' \gamma_5) \gamma^\mu q Z'_\mu,
$$

where $v_q, a_q$ and $v_q', a_q'$ are the set of vector and axial-vector coupling constants for $Z$ and $Z'$, respectively. The mixing $Z'$-$Z$ effect is given by the term $\sim \xi (M_{Z'}^2 - m_Z^2) Z'_\mu Z^\mu$ in the total Lagrangian density. Without loss of generality, the angle of the mixing $Z'$-$Z$ is not included because of its rather small value. Suppose the decay $Z' \to \gamma Z$ in the lowest order on the coupling constant is given by the loop diagrams containing the quarks mainly of the third and the fourth generations. The matrix element of this process is

$$
M = \frac{1}{16 \pi^2} e g_{Z'} g_Z F \epsilon_{\mu \nu \alpha \beta} \epsilon^\mu_q \epsilon^\nu_{Z'} \epsilon^\alpha_Z p^\beta,
$$

(8)

where $e = \sqrt{4 \pi \alpha}$ ($\alpha$ is the effective electromagnetic coupling constant to be evaluated on the scale $M_{Z'}$, $\alpha \sim 1/128$), $g_Z = g/cos \Theta_W$, $F = \sum_{q,b,t,...} e_q T_{3q}$ ($e_q$ is the electric quark charge, $T_{3q}$ is the third component of the weak isospin), $g_{Z'}$ being the free parameter, while $\epsilon^\mu_\gamma, \epsilon^\mu_{Z'}, \epsilon^\mu_Z$ are the wave functions of $\gamma$-quantum, $Z'$-, $Z$- bosons, respectively; $p^\mu$ is the 4-momentum of the $\gamma$-quantum. In the Grand Unification models $g_{Z'}$ is related to $g_Z$ in the following way (see, e.g., [13])
\[ g_{Z'} = \sqrt{\frac{5}{3}} \lambda \sin \Theta_W g_Z \simeq 0.62 g_Z, \]  

(9)

where \( \lambda \sim O(1) \). Making the replacement with \( \epsilon_Z^\mu \rightarrow q^\mu/m_Z \) in (8) one can take into account the longitudinal polarization of \( Z \)-boson in the leading order in \( 1/m_Z \) (\( q^\mu \) is the 4-momentum of \( Z \)).

The absolute width of the decay \( Z' \rightarrow \gamma Z \) is given by

\[ \Gamma(Z' \rightarrow \gamma Z) = \frac{5 \alpha \lambda \sin^2 \Theta_W G_F^2}{96 \pi^4} F^2 m_Z^2 M_{Z'}^3 \left(1 - \frac{m_Z^2}{M_{Z'}^2}\right)^3. \]  

(10)

There is a peculiarity in (10) indicating the independence of the decay width relevant to quark masses due to the suppression factor \( \sim \left(m_q/M_{Z'}\right)^2 \) taking into account the experimental restriction on the lower bound of the \( Z' \)-boson mass.

Fig.1 Decay width \( \Gamma(Z' \rightarrow \gamma Z) \) as a function of \( M_{Z'} \) with the contribution due to only \( b \)- and \( t \)-quarks in the loop and taking into account the additional contribution extended by the quarks of the fourth generation.

In Fig.1 we present the results of calculation of \( \Gamma(Z' \rightarrow \gamma Z) \) as a function of \( M_{Z'} \) with the contribution due to only \( b \)- and \( t \)-quarks in the loop and taking into account the additional
contribution extended by the quarks of the fourth generation. It is worthy to note that the contribution, thanks to quarks of the fourth generation, leads to the 4 times enhancement of $\Gamma(Z' \to \gamma Z)$. The parameter $\lambda$ is equal to 1 in the calculations.

To use the results of the data analysis carried out at the LEP \[14,15\] and the Tevatron [9], based on the study of the influence of the low-energy effects of quark-lepton contact interactions toward the lepton pair production, we make use of the processes $Z' \to \tau^+\tau^-$ and $Z' \to \mu^+\mu^-$ as the normalized ones to the decay $Z' \to \gamma Z$ we are interested in.

The $\bar{\ell}\ell$ leptonic pair decay width of the $Z'$-boson is given by the formula

$$
\Gamma(Z' \to \bar{\ell}\ell) = \frac{g^2_{Z'} M_{Z'}}{12 \pi} \{v'^2_l (1 + 2y)^2 + a'^2_l (1 - 4y)\} \sqrt{1 - 4y},
$$

where $y = m^2_l / M^2_{Z'}$, $m_l$ means the mass of the lepton, $v'_l$ and $a'_l$ are vector and axial-vector interaction constants of $Z'$ with leptons. The parameters $v'_l$ and $a'_l$ reflect the chiral properties of the $Z'$-boson interaction to leptons as well as the relative strength of these interactions.

The ratio between the widths of the decays $Z' \to \gamma Z$ and $Z' \to \bar{\ell}\ell$ can be defined as

$$
R(Z' \to \gamma Z/\bar{\ell}\ell) \equiv \frac{BR(Z' \to \gamma Z)}{BR(Z' \to \bar{\ell}\ell)} = \frac{3 \sqrt{2} \alpha G_F F^2 M_{Z'}^2}{64 \pi^3 (v'^2_l + a'^2_l) (1 - \frac{m^2_Z}{M^2_{Z'}})^3}.
$$

Notice that the sum $v'^2_l + a'^2_l$ in (12) may be different depending of the type of leptons.

In [8], the following normalization relation $v'^2_\tau + a'^2_\tau = 0.5$ in the case of $\tau$-leptons was used. This allows us to apply this condition to the normalization process $Z' \to \tau^+\tau^-$. In Fig.2, we plot the ratio $R(Z' \to \gamma Z/\bar{\ell}\ell)$ as a function of the mass $M_{Z'}$ in the $\tau$-lepton channel ($l = \tau$) taking into account the contribution from only $b$- and $t$-quarks and with the additional loop containing the quarks of the fourth generation. The resulting calculation of $R(Z' \to \gamma Z/\mu^+\mu^-)$ (in $\mu$-meson channel) does not differ essentially in comparison with $R(Z' \to \gamma Z/\tau^+\tau^-)$ if one believes in the following coupling constant relation: $v'^2_\mu + a'^2_\mu = v'^2_\tau + a'^2_\tau$. 


Fig. 2 The ratio $R(Z' \to \gamma Z/\bar{l}l)$ as a function of the mass $M_{Z'}$ in the $\tau$-lepton channel ($l = \tau$) taking into account the contribution from only $b$- and $t$-quarks and with the additional loop containing the quarks of the fourth generation.

To estimate the lower bound on $R(Z' \to \gamma Z/\tau^+\tau^-)$, we use the results [9] of the contact 4-fermion interaction analysis in $e^+e^- \to \tau^+\tau^-$. The mass $M_{Z'}$ is given by the mass scale parameter $\Lambda$ which is typical of the scale of new physics (NP)

$$M_{Z'} = \frac{\sqrt{G_F}}{2 \sin \Theta_W},$$  \hspace{1cm} (13)

where $\Lambda > 3.8$ TeV [14] or $\Lambda > 3.9$ TeV [15]. Our numerical result is $R(Z' \to \gamma Z/\tau^+\tau^-) > 1.3 \times 10^{-5}$. Using the resulting analysis of the CDF Collaboration [9] and taking into consideration the normalization process $Z' \to \mu^+\mu^-$, we get the following estimation $R(Z' \to \gamma Z/\mu^+\mu^-) > 1.4 \times 10^{-5} \cot^2 \phi$, if $v_\mu^2 + a_\mu^2 = 0.5$ (see the $\tau$-lepton case as well).

To study the $Z'$-boson decays with the production of the third generation quarks ($Q$), $Z' \to \bar{b}b$, $Z' \to \bar{t}t$ (normalized decays), we can find lower values for $R(Z' \to \gamma Z/\bar{Q}Q)$

$$R(Z' \to \gamma Z/\bar{b}b) \equiv \frac{BR(Z' \to \gamma Z)}{BR(Z' \to \bar{b}b)} = (0.13 - 1.60) \times 10^{-6}$$  \hspace{1cm} (14)

in the mass region $0.2$ TeV $< M_{Z'} < 1.0$ TeV, and

$$R(Z' \to \gamma Z/\bar{t}t) \equiv \frac{BR(Z' \to \gamma Z)}{BR(Z' \to \bar{t}t)} = (0.61 - 1.80) \times 10^{-6}$$  \hspace{1cm} (15)
for 0.4 TeV $< M_{Z'} < 1.0$ TeV, which is natural because the QQ-channel, especially $b\bar{b}$, is more probable to be compared with leptonic decays.

Relation (2) allows us to estimate $M_{Z'}$ as a function of the mixing angle $\xi$ and the mass parameter $\Delta_M = M_{Z'} - M_Z$

$$M_{Z'} \simeq \Delta_M + \frac{1}{|\xi|} \sqrt{m_Z^2 - m_{Z_1}^2}, \ |\xi| \neq 0.$$  \hfill(16)

Following [16] we keep in mind the mixing between $Z'$ and $Z$ in the form of $\rho_{\text{mix}}$ factor in the function

$$\rho = \rho_{\text{top}} \cdot \rho_{\text{mix}},$$  \hfill(17)

occurring in the interaction constants of the SM due to the $Z' - Z$ mixing. In formula (17) one has

$$\rho_{\text{top}} = \frac{1}{1 - \delta \rho_{\text{top}}}, \ \delta \rho_{\text{top}} \simeq \frac{3 G_F}{8 \sqrt{2} \pi^2} \cdot m_{\text{top}}^2 \simeq 0.01$$  \hfill(18)

which reflects the one-loop correction due to the top-quark contribution, while

$$\rho_{\text{mix}} = 1 + \sin^2 \xi \left( \frac{M_{Z_2}^2}{m_{Z_1}^2} - 1 \right)$$  \hfill(19)

with $\rho_0 = (m_W/m_Z \cos \Theta)^2 = 1$ in the SM. Then, relation (16) is

$$M_{Z'} \simeq \Delta_M + \frac{m_Z}{|\xi|} \sqrt{1 - \frac{1}{\rho_{\text{mix}}}}.$$  \hfill(20)

In this way, we can obtain the absolute value of $|\xi|$

$$|\xi| = \frac{m_Z}{C_{\text{exp}} - \Delta_M} \sqrt{1 - \frac{1}{\rho_{\text{mix}}}},$$  \hfill(21)

where $C_{\text{exp}}$ is the minimal value of the massive scale parameter of NP, which has been estimated at the LEP [14,15] and the Tevatron [11] under the contact four-fermion interaction analysis in $e^+e^- \rightarrow Z' \rightarrow \tau^+\tau^-$ and $\bar{p}p \rightarrow Z' \rightarrow e^+e^-, \mu^+\mu^-$, respectively: $C_{LEP}^{\text{exp}} = 0.355$ TeV [14], $C_{LEP}^{\text{exp}} = 0.365$ TeV [15], $C_{CDF}^{\text{exp}} = 0.345 \cdot \cot \phi$ TeV (electrons channel) [11] and $C_{CDF}^{\text{exp}} = 0.380 \cdot \cot \phi$ TeV (muons channel) [11].
To conclude this section, we derive an estimation on the mixing angle $\xi$. Actually, one can use the fact that $\rho_{\text{mix}}$ could be extracted from (17), where the $\rho_{\text{top}}$-factor is already known, and the full factor $\rho$ enters in its turn in the redefined constant $g_Z$ of the interaction of the $Z$-boson with the fermions, $g_Z = (g/\cos \Theta_W) \cdot \rho^{1/2}$ or even in the updated value of $\sin^2 \Theta_W$.

$$
\sin^2 \Theta_W = \frac{1}{2} - \left[ \frac{1}{4} - \frac{\pi \alpha(m_Z)}{\sqrt{2} G_F \rho m_Z^2} \right]^{1/2}.
$$

The numerical analysis shows that the upper limit on $|\xi|$, obtained within the CDF data [11] is comparable with those based on the LEP data [14,15] only under the following condition $\sin \phi \simeq \cos \phi$. In this case our estimations lead to the following restriction on $|\xi|$:

$$
0 < |\xi| < 5 \times 10^{-2} \quad (22)
$$

at $0 < \delta < 0.05$, where $\delta = \rho_{\text{mix}} - 1$. The increase in the mixing angle $\phi$ up to the physically grounded one $\sin \phi = 0.85$ [9] leads to the insignificant rise of the upper limit of $|\xi|$, $0 < |\xi| < 6 \cdot 10^{-2}$, in the above-mentioned interval for $\delta$.

The ratio between two scales $u/v$ of break down of the initial symmetry in the $SU(2)_h \times SU(2)_l$ - model has the following lower bound:

$$
\frac{u}{v} > \Lambda \frac{\cos^2 \phi \sqrt{\alpha}}{2 \sin \Theta_W m_W}. \quad (23)
$$

To get the estimation on the upper limit of the scale $u$ one can take into consideration the fact that in the framework of the model $SU(2)_h \times SU(2)_l$ the minimal value of the full width $\Gamma_{Z'}$ of the $Z'$-boson decay is achieved at $\sin \phi \simeq 0.85$ [9]. Moreover, $\sin \phi$ is restricted by the value $\sim 0.85$, where the sharp rise of $\Gamma_{Z'}$ begins because of the dominant contribution of the gauge coupling constant $g_h = g/(1 - \sin^2 \phi)^{1/2} (g = e/\sin \Theta_W)$. Thus, the values $\sin \phi$ can be restricted in the window $0.6 \leq \sin \phi \leq 0.85$, where the contribution of the constant $g_l = g/\sin \phi (1/g^2 = 1/g_h^2 + 1/g_l^2)$ becomes insignificant compared to $g_h$. The numerical estimations indicate $1.16 \leq (u/v) \leq 2.98$, where the lower bound corresponds to $\sin \phi = 0.85$ while the upper limit is achieved at $\sin \phi = 0.6$. 

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4. Results

In conclusion let us formulate the main results. In the paper, the radiative decays of new extra gauge bosons the $Z'$ accompanying the production of the $Z$-boson are investigated in the framework of the extended $SU(2)_h \times SU(2)_l$ gauge model. We investigated the induced amplitude of the radiative decay $Z' \rightarrow Z\gamma$ with the finite value of the relative decay width which can be measurable and thus, the relations between $Z'$ and fermions should be clarified. The results presented here are based on the consideration of the sensitivity of $M_{Z'}$ to $\Lambda$ in the framework of the contact quark-lepton interaction. The resulting truth depends mainly on the mixing angle $\phi$, and the interaction constants $g_{Z'}$, $v'_l$ and $a'_l$. The results obtained give a promising chance for an indirect observation of the $Z'$-bosons in their decays in lepton-antilepton pairs or even in their resonances in the photonic spectra because of the radiative decays. The extended model $SU(2)_h \times SU(2)_l$ admits the strengthening interaction between the third generation fermions with the extended gauge sector. In this case, the study of the final states $\mu^+\mu^-$, $\bar{b}b$, $\bar{t}t$ is most important. However, the decay modes $Z' \rightarrow \bar{t}t$ and $Z' \rightarrow \bar{b}b$ are rather difficult for observation because of their big invariant mass, e.g., for $\bar{t}t$-pair, as well as the large QCD background in the case of $\bar{b}b$-pair. The most probable channel is the production of two $\tau$-leptons, $\bar{p}p(pp) \rightarrow Z' \rightarrow \tau^+\tau^- + X$. The precise measurement of the angle distributions for the pairs of leptons and antileptons ($\mu^+\mu^-$ or $\tau^+\tau^-$) will allow one to clarify the very important and instructive information on the structure of the interaction constants between $Z'$ and fermions, and hence, to get an indirect confirmation of existence of a new heavy gauge boson, or even a set of gauge bosons, as well as to restrict the domain of their investigation, e.g., over their masses.

The resulting CDF data analysis for the ”gauge” process $Z' \rightarrow \mu^+\mu^-$ can allow one to define the lower bound on the decay probability $R(Z' \rightarrow \gamma Z/\mu^+\mu^-) > 1.4 \times 10^{-5} \cot^2 \phi$ in the case $v'^2_\mu + a'^2_\mu = v'^2_\tau + a'^2_\tau \simeq 0.5$ we have proposed, and to give the estimation on the mixing angle $\phi$ of the extended gauge group $SU(2)_h \times SU(2)_l$. In addition, the quarks of the fourth generation can lead to 4 times increase in the decay width $\Gamma(Z' \rightarrow \gamma Z)$. The CDF data in application to four-fermion contact interaction at $\cot \phi \simeq 1$ in order to estimate the mixing
angle $\xi$ for the system $Z' - Z$ are comparable with the data given by LEP, thus leading to the following restriction $|\xi| < 5 \times 10^{-2}$. Here the special role of the group $SU(2)_h$ relevant to top-quarks (and hence $\tau$-leptons) shows up, which is significant to current experiments at the Tevatron. We emphasize that the group $SU(2)_h$ is just a conductor of weak interactions where top-quarks and $\tau$-leptons are involved. As noted in Chapter 3, the LEP data relevant to decays $Z' \to \tau^+\tau^-$ and $Z' \to \bar{b}b$, $Z' \to \bar{t}t$ are taken into account when we carried out the comparison between the Tevatron and the LEP data analyses.

It turns out that a probable investigation of radiative decays of $Z'$-bosons at the acting hadron collider - the Fermilab Tevatron, would allow one to clarify, e.g., the question of the efficiency of the gauge group $SU(2)_h \times SU(2)_l$, and to give the estimation on the vector and axial-vector coupling constants of $Z'$-boson interaction with fermions.

Undoubtedly, for a further deeper study of $Z'$-bosons the most urgent step would be the investigation of decays like $Z' \to \tau^+\tau^- + X$ with the restriction on the parameter $\Lambda$ itself, and hence, on the mass $M_{Z'}$. Each of $\tau$-leptons (in the final state) decays into the hadrons ($BR(\tau \to \text{hadrons}) \approx 0.65$) or even into charge leptons and neutrinos $\tau^+\tau^- \to e^+e^-\nu$, $\tau^+\tau^- \to \mu^+\mu^-\nu$, $\tau^+\tau^- \to e^+\mu^-\nu(e^-\mu^+\nu)$ ($BR(\tau \to \text{leptons}) \approx 0.35$). Therefore, new data on the processes like $\bar{p}p \to Z' \to \tau^+\tau^- + X$ at the Tevatron with the energy $\sqrt{s} \approx 2$ TeV would be very important.

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