LMA MSW Solution from the Inverted Hierarchical Model of Neutrinos

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Abstract

We examine whether the inverted hierarchical model of neutrinos is compatible with the explanation of the large mixing angle (LMA)MSW solution of the solar neutrino problem. The left-handed Majorana neutrino mass matrix for the inverted hierarchical model, is generated through the seesaw mechanism using the diagonal form of the Dirac neutrino mass matrix and the non-diagonal texture of the right-handed Majorana mass matrix. In a model independent way we construct a specific form of the charged lepton mass matrix having a special structure in 1-2 block, which contribution to the leptonic mixing (MNS) matrix leads to the predictions $\sin^2 2\theta_{12} = 0.8517$, $\sin^2 2\theta_{23} = 0.9494$ and $|V_{e3}| = 0.159$ at the unification scale. These predictions are found to be consistent with the LMA MSW solution of the solar neutrino problem. The inverted hierarchical model is also found to be stable against the quantum radiative corrections in the MSSM. A numerical analysis of the renormalisation group equations (RGEs) in the MSSM shows a mild decrease of the mixing angles with the decrease of energy scale and the corresponding values of the neutrino mixings at the top-quark mass scale are $\sin^2 2\theta_{12} = 0.8472$, $\sin^2 2\theta_{23} = 0.9399$ and $|V_{e3}| = 0.1509$ respectively.

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1 Introduction

Neutrino physics is one of the fast developing areas of particle physics. The recent Super-Kamiokande experimental results on both solar[1] and atmospheric[2] neutrino oscillations support the approximate bimaximal mixings. Though these results favour the large mixing angle (LMA) MSW solution with active neutrinos, such interpretation is not beyond doubt at this stage[3,4,5]. We also have little idea about the pattern of the neutrino mass spectrum whether it is hierarchical or inverted hierarchical, and both possibilities are consistent with the neutrino oscillation explanations of the atmospheric and solar neutrino deficits[5,6]. The data from the long baseline experiment using a Neutrino factory will be able to confirm the actual pattern of the neutrino masses in the near future[7].

In the theoretical front the hierarchical model of neutrino masses and its generation have been widely studied and found to be consistent with the explanation of the LMA MSW solar neutrino solution[8,9]. However, the inverted hierarchical model of neutrino masses generally predicts the maximal mixing angles $\theta_{12}^\nu$ and $\theta_{23}^\nu$ close to $45^0$, and are suitable for the explanation of the vacuum oscillation (VO) solution of the solar neutrino oscillation[6,10] and the atmospheric neutrino oscillation data. The atmospheric data gives the lower bound at $\sin^22\theta_{23} \geq 0.88$ and the best-fit value at $\Delta m_{23}^2 = 3 \times 10^{-2}eV^2$. It is quite obvious that the prediction from the inverted hierarchical model fails to explain the LMA MSW solution which has upper experimental limit[4,6] $\sin^22\theta_{12} \leq 0.988$ at 95% C.L., and the best-fit values $\sin^22\theta_{12} = 0.8163$ and $\Delta m_{12}^2 = 4.2 \times 10^{-5}eV^2$. Combining LMA MSW solution and atmospheric data, the best-fit value of the mass splitting parameter is obtained[6] as $\xi = \Delta m_{12}^2/\Delta m_{23}^2 = 0.014$. It has been argued [6,10] that the diagonalisation of the charged lepton mass matrix cannot give a significant contribution to $\theta_{12}^\nu$ needed for the explanation of the LMA MSW solution. On such ground the inverted hierarchical models are assumed to be inconsistent with the LMA MSW solution. An attempt was made to explain the LMA MSW solution from the inverted hierarchical model by considering two types of charged lepton mass matrices[11] and was partially successful. We are interested here to make further investigations in this paper whether the inverted hierarchical model gives an acceptable LMA MSW solution when we include the contribution from the diagonalisation of the charged lepton mass matrix having special form in the 1-2 block, to the leptonic mixing matrix.
The paper is organised as follows. In section 2 we outline the seesaw mechanism for the generation of the neutrino mass matrix which leads to the inverted hierarchical mass pattern, and the contraction of the charged lepton mass matrix suitable for the LMA MSW solution. In section 3 we describe briefly the procedure for the analysis of the renormalisation group equations (RGEs) within the minimal supersymmetric standard model (MSSM). This is followed by a summary and discussion in section 4.

2 Generation of the inverted hierarchical neutrino mass matrix and the charged lepton mass matrix

The inverted hierarchical model of neutrinos has its origin from the low energy non-seesaw models[12], e.g., the Zee-type of model using a singly charged singlet scalar field and also the models with an approximate conserved \( L_e - L_\mu - L_\tau \) lepton number. However, it is also possible to generate the inverted hierarchical model through the seesaw mechanism at high energy scale within the framework of the grand unified theories with a chiral U(1) family symmetry[10,11]. In a model independent way, we consider the Dirac neutrino mass matrix \( m_{LR} \) and the non-diagonal form of the right-handed Majorana mass matrix \( M_R \) in the seesaw formula [13] given by

\[
m_{LL} = m_{LR} M_R^{-1} m_{LR}^T
\]

where \( m_{LL} \) is the left-handed Majorana mass matrix. The leptonic mixing matrix known as the MNS mixing matrix [14] is defined by

\[
V_{MNS} = V_{eL} V_{\nu L}^\dagger
\]

where \( V_{eL} \) and \( V_{\nu L} \) are obtained from the diagonalisation of the charged lepton and \( m_{LL} \) as

\[
m_{\text{diag}} = V_{eL} m_e V_{eR}^\dagger
\]

\[
m_{LL}^\text{diag} = V_{\nu L} m_{LL} V_{\nu L}^\dagger
\]

If the charged lepton mass matrix is diagonal, the MNS matrix (2) is simply given by

\[
V_{MNS} = V_{\nu L}^\dagger
\]
We can always express $m_{LL}$ in the basis where the charged lepton mass matrix is diagonal,

$$m'_{LL} = V_{eL} m_{LL} V_{eL}^\dagger,$$

$$m'_{LL} = V_{\nu L} m'_{LL} V_{\nu L}^\dagger,$$

$$V_{MNS} = V_{\nu L}^\dagger$$ \hspace{1cm} (5)

From the above expressions we can calculate the experimentally determined quantities as follows:

(i) the neutrino mass splitting parameter, $\xi = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{23}|}$

(ii) the atmospheric mixing angle, $S_{at} = \sin^2 2\theta_{23} = 4|V_{\nu 3}|^2 (1 - |V_{\nu 3}|^2)$

(iii) the solar mixing angle, $S_{sol} = \sin^2 2\theta_{12} = 4|V_{\nu 2}|^2 |V_{\nu 1}|^2$

(iv) the CHOOZ angle, $S_{C} = 4|V_{\nu 3}|^2 (1 - |V_{\nu 3}|^2)$ or simply $|V_{\nu 3}|$.

The $V_{fi}$ where $f = \tau, \mu, e$ and $i = 1, 2, 3$, are the elements of the MNS mixing matrix.

First, we consider the diagonal form of the charged lepton mass matrix $m_e$ given by

$$m_e = \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_\tau$$ \hspace{1cm} (6)

where the Wolfenstein parameter[15] $\lambda = 0.22$ and the ratios of the charged lepton masses are $m_\tau : m_\mu : m_e = 1 : \lambda^2 : \lambda^6$ respectively. From Eq.(6) we get $V_{eL} = 1$ and $V_{MNS} = V_{\nu L}^\dagger$ as in Eq.(4). Again we consider the diagonal form of the Dirac neutrino mass matrix $m_{LR}$ as the up-quark mass matrix[16],

$$m_{LR} = \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_t$$ \hspace{1cm} (7)

where the up-quark masses are in the ratios[17] $m_t : m_c : m_u = 1 : \lambda^4 : \lambda^8$.

Now, the proper choice of the elements in $M_R$, enables us to generate the inverted hierarchical neutrino mass matrix. We present here the following examples[18]:

**Example (a)**

$$m_{LL} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & \lambda^3 & 0 \\ 1 & 0 & \lambda^3 \end{pmatrix} m_0,$$ \hspace{1cm} (8)
with the choice
\[ M_R = \begin{pmatrix} -\lambda^{22} & \lambda^{15} & \lambda^{11} \\ \lambda^{15} & \lambda^{8} & -\lambda^{4} \\ \lambda^{11} & -\lambda^{4} & 1 \end{pmatrix} v_R \]

Eq.(8) yields
\[ V_{MNS} = V_{\nu L}^\dagger = \begin{pmatrix} 0.70577 & 0.70844 & -1.11 \times 10^{-16} \\ -0.50094 & 0.49906 & -0.70711 \\ -0.50094 & 0.49906 & 0.70711 \end{pmatrix} v_R \] (9)

and the neutrino mass eigenvalues \( m_i = (1.4195, 1.4089, 0.0105) m_0, \ i = 1, 2, 3 \). This gives the mass splitting parameter \( \xi = \Delta m_{12}^2/\Delta m_{23}^2 = 0.014 \), and the mixing angles \( \sin^2 2\theta_{12} = 0.9999, \sin^2 2\theta_{23} \approx 1.00, |V_{e3}| = 0.0 \).

**Example (b)**
\[ m_{LL} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -(\lambda^3 - \lambda^4)/2 & -(\lambda^3 + \lambda^4)/2 \\ 1 & -(\lambda^3 + \lambda^4)/2 & -(\lambda^3 - \lambda^4)/2 \end{pmatrix} m_0, \] (10)

with the choice
\[ M_R = \begin{pmatrix} \lambda^{23} & \lambda^{16} & \lambda^{12} \\ \lambda^{16} & \lambda^8 & -\lambda^4 \\ \lambda^{12} & -\lambda^4 & 1 \end{pmatrix} v_R \]

leading to \( m_i = (1.4195, 1.4089, 0.00239) m_0, \ \xi = 0.0151, \ \sin^2 2\theta_{12} = 0.9999, \ \sin^2 2\theta_{23} \approx 1.00, |V_{e3}| = 0.0 \).

**Example (c)**
\[ m_{LL} = \begin{pmatrix} \lambda^3 & 1 & 1 \\ 1 & \lambda^4/2 & -\lambda^4/2 \\ 1 & -\lambda^4/2 & \lambda^4/2 \end{pmatrix} m_0, \] (11)

with the choice
\[ M_R = \begin{pmatrix} 0 & \lambda^{16} & \lambda^{12} \\ \lambda^{16} & \lambda^8 & -(\lambda^4 + \lambda^{12}) \\ \lambda^{12} & -(\lambda^4 + \lambda^{12}) & 1 \end{pmatrix} v_R \]
leading to $m_i = (1.4195, 1.4089, 0.002343) m_0$, $\xi = 0.015$, $\sin^2 2 \theta_{12} = 0.9999$, $\sin^2 2 \theta_{23} \approx 1.00$, $|V_{e3}| = 0.0$.

In the above results the $V_{MNS}$ obtained from the $m_{LL}$ alone fails to explain the LMA MSW solution, and any small deviation in $m_{LL}$ will hardly affect $\sin^2 2 \theta_{12}$[6,10]. The last hope is that there could be a significant contribution to $\theta_{12}$ from the diagonalisation of the charged lepton mass matrix having special structure in 1-2 block [11]. We wish to examine how $\theta_{sol} = (\theta^e_{12} - \theta^e_{13})$ can resolve the LMA MSW solar neutrino mixing scenario [11].

We parametrise the charged leptonic mixing $V_{eL}$ by the following three rotations[19,20]

$$V_{eL} = R_{23} R_{13} R_{12}$$

$$= \begin{pmatrix}
1 & 0 & 0 \\
0 & \bar{c}_{23} & \bar{s}_{23} \\
0 & -\bar{s}_{23} & \bar{c}_{23}
\end{pmatrix}
\begin{pmatrix}
\bar{c}_{13} & 0 & \bar{s}_{13} \\
0 & 1 & 0 \\
-\bar{s}_{13} & 0 & \bar{c}_{13}
\end{pmatrix}
\begin{pmatrix}
\bar{c}_{12} & \bar{s}_{12} & 0 \\
-\bar{s}_{12} & \bar{c}_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}$$

(12)

where $\bar{s}_{ij} = \sin \theta^e_{ij}$ and $\bar{c}_{ij} = \cos \theta^e_{ij}$. Putting $\theta^e_{13} = \theta^e_{23} = 0$, Eq.(12) reduces to

$$V_{eL} = \begin{pmatrix}
\bar{c}_{12} & \bar{s}_{12} & 0 \\
-\bar{s}_{12} & \bar{c}_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}$$

(13)

This gives a special form in the 1-2 block. We then reconstruct[19] the charged lepton mass matrix using Eq.(13) from the relation,

$$m_e = V_{eL}^\dagger m_{\text{diag}} V_{eR}$$

$$= \begin{pmatrix}
\lambda^6 \bar{c}_{12}^2 + \lambda^2 \bar{s}_{12}^2 & \lambda^6 \bar{c}_{12} \bar{s}_{12} - \lambda^2 \bar{c}_{12} \bar{s}_{12} & 0 \\
\lambda^6 \bar{c}_{12} \bar{s}_{12} - \lambda^2 \bar{c}_{12} \bar{s}_{12} & \lambda^6 \bar{c}_{12}^2 + \lambda^2 \bar{s}_{12}^2 & 0 \\
0 & 0 & 1
\end{pmatrix}$$

(14)

For a specific choice of $\theta^e_{12} = 13^0$, and $\lambda = 0.22$, Eq.(14) leads to

$$m_e = \begin{pmatrix}
0.00256 & -0.01058 & 0 \\
-0.01058 & 0.04596 & 0 \\
0 & 0 & 1
\end{pmatrix}$$

(15)

which has a special form in the 1-2 block. The diagonalisation of $m_e$ in Eq.(15) gives

$$V_{eL} = \begin{pmatrix}
-0.97439 & -0.22488 & 0 \\
-0.22488 & 0.97439 & 0 \\
0 & 0 & 1
\end{pmatrix}$$

(16)
which is now completely unitary. The corresponding eigenvalues of the charged lepton mass matrix are given by

\[ m^{\text{diag}}_e = (1.182 \times 10^{-4}, 4.84 \times 10^{-2}, 1.0)m_\tau \]  

(17)

which gives almost correct physical mass ratios\[17\] \[ m_e : m_\mu : m_\tau = \lambda^6 : \lambda^2 : 1. \]

The MNS mixing matrix (2) is now calculated, using Eqs.(9) and (16), as

\[
V_{MNS} = V_{eL}V_{\nu L}^\dagger = \begin{pmatrix}
-0.5750 & -0.8025 & 0.1590 \\
-0.6468 & 0.32696 & 0.6890 \\
-0.50094 & 0.49906 & 0.70711
\end{pmatrix}
\]  

(18)

This leads to the mixing angles \( \sin^2 2\theta_{12} = 0.8517, \sin^2 2\theta_{23} = 0.9494, \) and \( |V_{e3}| = 0.159, \) and these predictions are consistent with the explanation of LMA MSW solution. The possible choice of \( \theta_{12}^e = 14^0 \) in Eq.(14) also leads to the predictions of \( \sin^2 2\theta_{12} = 0.8298, \sin^2 2\theta_{23} = 0.9415, \) and \( |V_{e3}| = 0.1710 \) while maintaining the good prediction of the ratios of the charged lepton masses. However its \( |V_{e3}| \) is above the CHOOZ and PALO VERDE experimental constraint [21] of \( |V_{e3}| \leq 0.16. \)

Taking the first result with \( \theta_{12}^e = 13^0, \) the left-handed neutrino mass \( m_{LL}' \) in the basis where the charged lepton mass matrix is diagonal(5), is now expressed for our convenience, as

\[
m_{LL}' = \begin{pmatrix}
0.437972 & -0.897698 & -0.973193 \\
-0.897698 & -0.443296 & -0.230068 \\
-0.973193 & -0.230068 & -0.005324
\end{pmatrix} m_0
\]  

(19)

where \( V_{MNS} = V_{\nu L}^\dagger \) is same as earlier given in Eq.(18), and the neutrino mass eigenvalues are

\[
m_i = (1.4196, 1.4089, 4.234 \times 10^{-7}) m_0; \quad i = 1, 2, 3
\]

which give the mass splitting parameter, \( \xi = \Delta m^2_{12}/\Delta m^2_{23} = 0.014. \)

3 \ Renormalisation effects in MSSM

It is desirable to inspect how the values of \( \sin^2 2\theta_{12}, \sin^2 2\theta_{23}, \) \( |V_{e3}| \) and \( \xi \) evaluated at the unification scale where the seesaw mechanism is operative, respond to the renormalisation group analysis on running from higher scale
\( (M_u = 2 \times 10^{16} GeV) \) down to the top quark mass scale \((\mu = m_t) \) [22]. We consider the renormalisation group equations (RGEs) for the three gauge couplings \((g_1, g_2, g_3) \) and the third family Yukawa couplings \((h_t, h_b, h_\tau) \) in the minimal supersymmetric standard model (MSSM) in the standard fashion [23]. At high scale \( \mu = 2 \times 10^{16} GeV \), we assume the unification of gauge couplings as well as third generation Yukawa couplings for large \( \tan \beta \) [23]. We choose the input \( \alpha_2 = (5/3)\alpha_1 = \alpha_3 = 1/24 \), and \( h_t = h_b = h_\tau = 0.7 \) corresponding to large \( \tan \beta = v_u/v_d \).

We express \( m_{LL} \) in terms of \( K \), the coefficient of the dimension five neutrino mass operator[24,25,26,27] in a scale-dependent manner,

\[
m_{LL}(t) = v_u^2(t)K(t) \tag{20}
\]

where \( t = \ln(\mu) \) and \( v_u(t) \) is the scale-dependent[27] vacuum expectation value (VEV) \( v_u = v_0 \sin \beta, v_0 = 174 GeV \). In the basis where the charged lepton mass matrix is diagonal, we can write Eq.(20) as [25,27]

\[
m'_{LL}(t) = v_u^2(t)K'(t) \tag{21}
\]

where \( K'(t) \) is the coefficient of the dimension five neutrino mass operators in the basis where the charged lepton mass matrix is diagonal. The evolution equations are given by[27]

\[
\frac{d}{dt} ln v_u(t) = \frac{1}{16\pi^2} \left[ \frac{3}{20} g_1^2 + \frac{3}{4} g_2^2 - 3h_t^2 \right], \tag{22}
\]

\[
\frac{d}{dt} ln K'(t) = -\frac{1}{16\pi^2} \left[ \frac{6}{5} g_1^2 + 6g_2^2 - 6h_t^2 - h_\tau^2 \delta_{i3} - h_t^2 \delta_{j3} \right], \tag{23}
\]

The evolution equation of \( m'_{LL}(t) \) in Eq.(21) simplifies[27] to

\[
\frac{d}{dt} ln m'_{LL}(t) = \frac{1}{16\pi^2} \left[ -\frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 + h_\tau^2 \delta_{i3} + h_t^2 \delta_{j3} \right]. \tag{24}
\]

Upon integration from high scale \((t_u = \ln M_u) \) to lower scale \((t_0 = \ln m_t) \) where \( t_0 \leq t \leq t_u \) and \( t = \ln \mu, \) we get[27]

\[
m'_{LL}(t_0) = e^{\frac{2}{\pi} I_{g_1}(t_0)} e^{\frac{2}{\pi} I_{g_2}(t_0)} \times \begin{pmatrix}
  m'_{LL11}(t_u) & m'_{LL12}(t_u) & m'_{LL13}(t_u) e^{-I_\tau(t_u)} \\
  m'_{LL21}(t_u) & m'_{LL22}(t_u) & m'_{LL23}(t_u) e^{-I_\tau(t_u)} \\
  m'_{LL31}(t_u) e^{-I_\tau(t_u)} & m'_{LL32}(t_u) e^{-I_\tau(t_u)} & m'_{LL23}(t_u) e^{-2I_\tau(t_u)}
\end{pmatrix} \tag{25}
\]
where
\[ I_{g_i}(t_0) = \frac{1}{16\pi^2} \int_{t_0}^{t_u} g_i^2(t) dt, \quad i = 1, 2, 3; \]
\[ I_f(t_0) = \frac{1}{16\pi^2} \int_{t_0}^{t_u} h_j^2(t) dt, \quad f = t, b, \tau. \]  

(26)

Using the numerical values of \( I_{g_i}(t) \) and \( I_f(t) \) at different energy scales \( t_0 \leq t \leq t_u \) the left-handed Majorana mass matrix \( m'_{LL}(t) \) in Eq.(25) is estimated at different energy scales from the value of \( m'_{LL}(t_u) \) given in Eq.(19). At each scale the leptonic mixing matrix \( V_{MNS}(t) = V'_{\nu L}(t) \) is calculated and this in turn, gives mixing angles \( \sin^2 2\theta_{12}, \sin^2 2\theta_{23} \) and \( |V_{e3}| \). For example, at the top-quark mass scale \( t_0 = \ln m_t = 5.349 \), we have calculated \( I_r(t_0) = 0.100317 \) and the leptonic mixing matrix
\[
V_{MNS} = \begin{pmatrix}
-0.56962 & -0.80795 & 0.15085 \\
-0.67061 & 0.35075 & -0.65364 \\
-0.4752 & 0.47349 & 0.74161
\end{pmatrix}
\]  

(27)

which leads to the low-energy predictions: \( \sin^2 2\theta_{12} = 0.8472 \) and \( \sin^2 2\theta_{23} = 0.9399 \). There is a mild reduction from the values estimated at the high energy scale \( t_u \). This feature shows the compatibility of the inverted hierarchical model with the explanation of the LMA MSW solution. The parameter \( |V_{e3}| = 0.15085 \) meets the CHOOZ constraint \( |V_{e3}| \leq 0.16 \) [21].

The neutrino mass eigenvalues at low-energy scale are obtained as \( m_i = (1.3533, 1.3436, 3.8376 \times 10^{-7}) m_0 \). However the mass splitting parameter \( \xi = \Delta m^2_1/\Delta m^2_3 = 0.01449 \) remains almost constant. The running of the mixing angles \( S_{sol} = \sin^2 2\theta_{12} \) and \( S_{at} = \sin^2 2\theta_{23} \) is shown in Fig.1 by the solid-line and dotted-line respectively. Both parameters decrease with decrease in energy scale \( t \). This is a desirable feature for maintaining the stability condition of the inverted hierarchical model. Note that there is no exponential term which depends on the top-quark Yukawa coupling integration in Eq.(25). This differs from the expressions calculated in earlier papers[11,25]. The absence of such term increases the stability criteria of the mass matrix at low \( tan\beta \) region and this is made possible only when we consider the scale-dependent vacuum expectation value in Eq.(22). As discussed before, if the CHOOZ constraint is relaxed to \( |V_{e3}| \leq .2 \), then we would be able to get even lower value of \( \sin^2 2\theta_{12} \) suitable for the explanation of the best-fit value of the LMA MSW solution.
Fig. 1. Variations of $S_{sol} = \sin^2 2\theta_{12}$ and $S_{at} = \sin^2 2\theta_{23}$ with energy scale $t = \ln(\mu)$, which are represented by solid-line and dotted-line, respectively.

We now briefly discuss the analytic solution for the evolution of the neutrino mixings in MSSM. The equation for the evolution of the neutrino mixing angle $\sin^2 2\theta_{ij}$ can be approximated as two flavour mixing in terms of the neutrino mass eigenvalues\cite{26} $m_1, m_2, m_3$ by a generalised evolution equation,

$$16\pi^2 \frac{d}{dt} \sin^2 2\theta_{ij} = -2 \sin^2 2\theta_{ij} (1 - \sin^2 2\theta_{ij}) (h_j^2 - h_i^2) \frac{m_j + m_i}{m_j - m_i}$$  \hspace{1cm} (28)

where $i < j$ and $i, j = 1, 2, 3$; or $e, \mu, \tau$. For solar mixing angle $\theta_{12}$, we have

$$16\pi^2 \frac{d}{dt} \sin^2 2\theta_{12} = -2 \sin^2 2\theta_{12} (1 - \sin^2 2\theta_{12}) (h_\mu^2 - h_e^2) \frac{m_2 + m_1}{m_2 - m_1}$$  \hspace{1cm} (29)
and for atmospheric mixing angle $\theta_{23}$ we have

$$16\pi^2 \frac{d}{dt} \sin^2 2\theta_{23} = -2 \sin^2 2\theta_{23}(1 - \sin^2 2\theta_{23})(h_\tau^2 - h_\mu^2) \frac{m_3 + m_2}{m_3 - m_2}$$

(30)

Since $h_\tau^2 > h_\mu^2 > h_e^2$ and $m_1 > m_2 > m_3$, both $S_{\text{sol}}$ and $S_{\text{at}}$ in Eqs.(29) and (30) increase with increasing energy scale. These features are plainly visible in the Fig.1.

A few more comments on our choice of the texture of the charged lepton mass matrix are also presented. Here we examine the forms of the texture of the charged lepton mass matrix $m_e$ and its diagonalisation matrix $V_{eL}$ in Eqs.(15)and (16). For simplicity of our analysis we express them in the following approximate forms

$$m_e \sim \begin{pmatrix} \lambda^4 & -\lambda^3 & 0 \\ -\lambda^3 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(31)

and

$$V_{eL} \sim \begin{pmatrix} -1 & -\lambda & 0 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(32)

It is interesting to note that the position of the zeros in the mass matrix in Eq.(31) have the same structure with those of lepton mass matrix obtained by Ibanez and Ross[29] in the gauge theory of the standard model with an horizontal $U(1)$ gauge factor. Such form of the texture of the charged lepton mass matrix is also proposed by Georgi and Jarlskog[30] in $SU(5)$ model, and this can be realised in a model based on $SU_YSO(10) \times U(2)$ using a 126-dimensional Higgs[31]. The CKM matrix of the quark mixings defined by $V_{CKM} = V_{uL}V_{dL}^\dagger$, is usually parametrised by[15]

$$V_{CKM} \sim \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

(33)

where $\lambda = 0.22$ and $|A| = 0.90$. For our choice of the diagonal up-quark mass matrix in Eq.(7), we have $V_{uL} = 1$ leading to $V_{CKM} = V_{dL}^\dagger$. Since $m_d = m_e^T$, 

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we have $V_{eL} = V_{dL}^\dagger = V_{CKM}$. Neglecting higher power of $\lambda$ in Eq.(33), we have

$$V_{eL} \sim \begin{pmatrix} 1 & \lambda & 0 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(34)

which is almost same as $V_{eL}$ given in Eq.(32) except the difference in sign before some entries. The positions of the zeros are the same. Such linkage gives partial justification to our motivation for the choice of the charged lepton mass matrix (15).

4 Summary and Discussion

The left-handed Majorana neutrino mass matrix $m_{LL}$ which explains the inverted hierarchical pattern of neutrino masses, has been generated from the seesaw mechanism using non-diagonal texture of the right-handed Majorana neutrino mass matrix $M_R$ and diagonal form of the Dirac neutrino mass matrix. We have explained the leptonic mixing matrix generated from the diagonalisation of $m_{LL}$ of the inverted hierarchical model and the mixing angles so far obtained $\sin^2 \theta_{12} \approx 0.999$, is too large for the explanation of the LMA MSW solution. Such high value of $\sin^2 \theta_{12}$ can be toned down by considering the contribution from the charged lepton mass matrix having special structure in the 1-2 block. With such consideration, the predictions on the mixing angles at the high scale are $\sin^2 2\theta_{12} = 0.8517$, $\sin^2 2\theta_{23} = 0.9494$ and $|V_{e3}| = 0.159$ which are consistent with the LMA MSW solution.

The above results which are calculated at the high energy scale (say, $\mu = M_u = 2 \times 10^{16} GeV$) where the seesaw mechanism operates, decrease with the decrease in energy scale, under the quantum radiative corrections within the framework of the MSSM. This is a good feature at least for $\sin^2 2\theta_{12}$ in this inverted hierarchical model as it gives the stability under quantum radiative corrections and shows complete consistency of the model with the explanation of the LMA MSW solution. The present finding fails to support the claim that an arbitrary mixing at the high scale can get “magnified” to a large mixing, and even possibly maximal mixing at the low scale [28]. Experimental data from a Neutrino factory may confirm the pattern of the neutrino masses in near future, and hence the sign of $\Delta m^2_{23}$. Such confirmation of the detailed pattern of neutrino masses and their mixing angles is very
important as it may give a clue to the understanding of quark masses and their mixing angles within the framework of an all-encompassing theory[5].

Though we have constructed both \( m_{LL} \) and \( m_e \) in a model independent way and have shown how the inverted hierarchical model of neutrinos can explain the present experimental data particularly LMA MSW solution, the present work is expected to be an important clue for building models from the grand unified theories with the chiral U(1) symmetry.

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