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Key Points:
• We build a framework to quantitatively describe the relationship between coda-correlation and Earth structure
• We design a toy-problem experiment, a hemispherical inner core, to demonstrate a tomographic reconstruction using the proposed framework
• There is a fundamental difference between tomographic results obtained based on the formation theory and the Green’s function assumption

Supporting Information:
• Figure S1

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Seismic Event Coda-Correlation: Toward Global Coda-Correlation Tomography
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Abstract Seismic event coda-correlation is a mathematical manifestation of the seismic wavefield, and it is characterized by many prominent features that are formed due to the similarity between multiple pairs of seismic phases. This new paradigm sets a stage for extracting valuable information about Earth structure. However, earthquake coda-correlation has a fundamentally different physical mechanism from ambient-noise correlation and thus cannot be utilized in the same way as the ambient-noise correlation tomography that has been rigorously studied both in terms of theory and applications. Therefore, we are motivated to devise a new framework for the coda-correlation tomography, in which relevant features in coda-correlation are decomposed and separate constituents are individually related to Earth structure to build sensitivity kernels for tomography. Our theoretical framework is verified via a toy-problem experiment, and we compare the newly proposed method here with the one based on the assumption that an interreceiver response (Green’s function) can be obtained. We illustrate that significant differences can arise in the interpretation of results if the Green’s function is used instead of the newly proposed framework based on the understanding of the formation of coda-correlation. The proposed framework paves the way for further detailed and application-oriented method improvements and exploitation of the coda-correlation tomography in global and planetary seismology.

1. Introduction

An introduction of cross correlation in a seismic wavefield provided a momentous direction in the seismological investigations of the Earth’s interior (Campillo & Paul, 2003; Shapiro et al., 2005). In ambient-noise or seismic coda wavefield, cross-correlation stacks calculated over a long interval of time resemble interreceiver structure response (Green’s function) as if measured at one of the receivers from a virtual source at the other (Snieder, 2004, 2006; Wapenaar et al., 2010). This phenomenon is thoroughly analyzed, and the corresponding techniques are heavily exploited in ambient-noise settings (e.g., Bensen et al., 2007; Wapenaar et al., 2010; Wapenaar, Draganov, et al., 2010). There are a number of successful practices in retrieving equivalencies to surface waves propagating between two receivers for imaging subsurface structures (e.g., Bensen et al., 2008; Lin et al., 2007, 2008; Moschetti et al., 2007; Yao et al., 2006).

After similar data processing and computation routines have been adopted for the coda-correlation—the cross correlation of time series containing late-coda of large earthquakes—much attention has been gained for applications in geophysical inference on regional and global scales (e.g., Boué et al., 2014; Lin & Tsai, 2013; Pham et al., 2018; Poli et al., 2017). Coda-correlation stacks, also known as correlograms, contain stable and prominent features, particularly a class of features sensitive to the deep-Earth structure (Boué et al., 2013; Huang et al., 2015; Wang et al., 2015), which is less constrained in conventional seismic wavefield studies due to imperfect volumetric sampling than the shallow-Earth structure. If those features could be correctly interpreted, they would provide complementary constraints in resolving the Earth’s core internal structure (Tkalčić, 2015, 2017).

However, a possible coda-correlation tomography is expected to be quite different from the ambient-noise correlation applications because of their fundamentally different formation mechanisms. Unlike the ambient-noise correlation, coda-correlation does not correspond to Green’s function. Instead, the body-wave-like features in global correlograms are contributed by a few normal modes that are formed by reverberations in the earthquake coda (Poli et al., 2017). The correlogram features arise due to similarity between seismic waves of the same slowness that reverberate in multiple ways between the Earth surface and the core-mantle boundary (Pham et al., 2018). That is evident in the fact that coda-correlation stacks
do not converge to some stable functions when the source-receiver geometry is varied (Boué et al., 2014; Sager et al., 2018). Apart from that, the Green’s function method, which treats cross-correlation stacks as reconstructed Green’s function, cannot explain numerous prominent features in coda-correlation wavefield that do not exist in the seismic wavefield counterpart, such as cS-cP, cPPcP-cS, cKS-cP, cKS-cS (Phạm et al., 2018), and I2-J (Tkalčić & Phạm, 2018).

For the above reason, treating coda-correlation as means to produce a Green’s function, that is, on a par with methods developed for the ambient-noise correlation, contributes to erroneous inferences on Earth structure. A new approach is needed to utilize coda-correlation, that is, to develop the tomographic relationship between the coda-correlation and Earth structure based on the understanding of coda-correlation’s formation. That problem was not solved in previous studies (Kennett & Pham, 2018; Phạm et al., 2018; Poli et al., 2017), although they provide the conjecture and basis for this study. The solution to that problem would allow us to use a massive amount of coda-correlation observables to increase constraints on the Earth structure.

Toward that goal, our study here provides a new framework for coda-correlation tomography. Here, we decompose prominent features in coda-correlation wavefield into individual constituents that represent different cross-terms between seismic waves. Those constituents can be uniquely identified and quantitatively related to Earth structure to construct a tomography kernel. We use this framework and a “toy-problem” experiment to prove the concept and demonstrate its use in future studies.

2. Formation of Coda-Correlation for Varying Source-Receiver Geometry

We start from coda-correlation’s formation theory to develop its utilization for tomography. The coda-correlation wavefield is formed due to a similarity of seismic waves that arrive at pairs of receivers with the same slowness (Phạm et al., 2018). There can be infinite types of seismic waves in the earthquake coda, and their cross-terms interact together to form coda-correlation features. The seismic waves sample the Earth’s interior along different paths, and hence the cross-terms come with different sensitivities to Earth structure. Therefore, the relationship between the observed coda-correlation wavefield and Earth structure depends on the contributing constituents and their sensitivities to various parts of the Earth.

As shown in Figure 1, we select one of the most prominent and stable features in the coda-correlation wavefield sensitive to the deep Earth—I2* (for the nomenclature of the correlogram features, see explanations in Tkalčić and Phạm (2018))—to demonstrate the dependency on Earth structure. We alter the selection of the

Figure 1. (a) Synthetic coda-correlation stack (correlogram) using equally distributed events along the equator. (b) Coda-correlation stack at the interreceiver distance of 20°. The gray arrow denotes the coda-correlation feature I2*. 
contributing constituents through using different coda time windows and different events. For an idealized Earth model and globally uniform distribution of events (Figure 2a), synthetic coda-correlation stacks vary with different coda time windows (Figure 2b). The range of I2* time-onset variation is ~5 s. It is significant that the globally distributed events form a closed surface that surrounds the medium (Figure 2a). That type of source distribution produces the cross-correlation stacks by means of simultaneously acting sources everywhere, which can be either primary noise sources or secondary sources due to structural heterogeneity or many scatterers (Wapenaar, 2004; Wapenaar, Draganov, et al., 2010). Apart from varying the time window, the constituents and their involvement in the formation of coda-correlation depend on event-receiver geometries. We observe a similar time-onset variation for the selection of events from different regions (Figure 2c) when the time-window of 3–9 hr is used.

The above example involving a simple Earth model shows that if coda-correlation is treated as an interreceiver response—that is, the Green’s function method—the dependency on selections of coda time-windows and event locations could be falsely attributed to Earth structure anomalies and could result in erroneous tomographic images and interpretations. That time inaccuracy does not disappear for three-dimensional Earth that contains the randomness of the structure because of the lack of equipartitioning in late-coda of earthquakes (Sens-Schönfelder et al., 2015), and the inaccuracy would be worsened due to three-dimensional event distributions (Sager et al., 2018).

3. Construction of Sensitivity Kernels
3.1. Separation and Identification of Coda-Correlation Contributions

To construct the sensitivity kernels for coda-correlation tomography, we need to separate and determine individual constituents of coda-correlation features. Coda-correlation is calculated based on the conventional definition of the cross correlation between two time series $f_1(t)$ and $f_2(t)$:

$$FTCC(t) = \int_{\text{full-time}} f_1(t')f_2(t + t')dt',$$

where a full-time integral interval is used, and hence it is denoted as the full-time cross-correlation (FTCC) in this study. In calculating coda-correlation, the full-time window corresponds to the event-coda time-window (e.g., 3–9 hr after the origin time of earthquakes).

Here, we introduce the short-time cross-correlation (STCC) based on the definition of FTCC. The STCC is given by
\[
STCC(\tau; t) = \int_{\tau - \tau_w/2}^{\tau + \tau_w/2} H(\tau; t') f_1(t') f_2(t + t') dt',
\tag{2}
\]

where two time series are cross-correlated within a short-time window centered on time \(\tau\) with a length \(\tau_w\). For the sake of comparison with FTCC, equation 2 can be modified to

\[
STCC(\tau; t) = \int_{full - time} H(\tau; t') f_1(t') f_2(t + t') dt',
\tag{3}
\]

\[
H(\tau; t) = \begin{cases} 
1 & t \in \left[\tau - \frac{\tau_w}{2}, \tau + \frac{\tau_w}{2}\right] \\
0 & \text{others}
\end{cases}
\tag{4}
\]

The STCC allows for the decomposition of the correlation features. As shown in equations 3 and 4, the short-time window \(H(\tau; t)\) cuts out a pair of waves, one of which arrives at the time \(\tau\) in \(f_1\) and the other at the time \(\tau + t\) in \(f_2\). The \(STCC(\tau; t)\) denotes the cross-term between those two waves. Through altering the arrival time \(\tau\), cross-terms between different waves can be separated. The summation of cross-terms that have the same \(t\) but different \(\tau\) equals to a correlogram feature at the time \(t\) in FTCC. In other words, the STCC analysis can be used to decompose a correlogram feature into separate constituents. As shown in Figure 3, each of the cross-correlation features in FTCC can be decomposed into separate constituents with respect to the arrival time \(\tau\). For example, the cross-correlation feature \(X_5\) is decomposed into two constituents shown as energy islands in the STCC plot.

Figure 3. The decomposition of correlogram features (FTCC) and identification of separate constituents based on the short-time cross correlation (STCC). (a) The first time series, \(f_1\), contains three phases labeled as \(A_1\), \(B_1\), and \(C_1\). Each phase has different arrival time, \(\tau\), as indicated by black triangles. (b) The second time series, \(f_2\), contains three phases labeled as \(A_2\), \(B_2\), and \(C_2\). (c) FTCC between \(f_1\) and \(f_2\). The resulting features are labeled \(X_1, X_2, \ldots, X_8\), and they have different delay times, \(t\), denoted by black triangles. (d) STCC between \(f_1\) and \(f_2\). Amplitude islands indicate separate constituents of features in FTCC. As labeled alongside, each constituent is identified as a cross-term between two phases in \(f_1\) and \(f_2\), respectively. The identification is through matching the time \(\tau\) in STCC and the arrival time of phases in \(f_1\) and \(f_2\).
The STCC analysis allows the identification of separate constituents. Each of the constituents has a unique timestamp $\tau$. Through matching this timestamp with arrival time predictions, each constituent can be identified as a cross-term between two specific waves in $f_1$ and $f_2$. These two waves have the arrival time of $\tau$ and $\tau + t$, respectively. As shown in Figure 3, each constituent in STCC plot can be identified as the cross-terms between two phases in $f_1$ and $f_2$. For example, the two constituents of the feature $X_5$ can be identified as the cross-terms of $A_2 - A_1$ and $B_2 - B_1$, respectively, from their absolute time.

The decomposition and identification depend on the length of the short-time window $\tau_w$. Through tuning $\tau_w$, the correlogram features in different frequency bands can be routinely analyzed. Additionally, slowness analysis can further improve the accuracy of the identification if there are sufficient interreceiver distance coverages.

Figure 4 shows the isolation and identification of individual contributions to synthetic $I_{2^*}$. We set two fixed stations and random events that mimic what is realistic (Figure 4a). Few events have the epicentral distance of 0° or 180°, and most of the events have an intermediate distance (Figure 4b). Figure 4d shows the stacked $I_{2^*}$ in 3–9 hr for all events. We apply STCC to separate several constituents of the $I_{2^*}$ in 1–9 hr (Figure 4c). From the theoretical arrival time predictions, they are identified as cross-terms of body waves. For example, a short-time window around theoretical I4 and I6 arrival time is used in STCC, and then we can get the cross-term of I6-I4. We use the short-time window of 100 s, doubling the 50 s required by the Nyquist-Shannon sampling theorem. Similar separations can be done to isolate out and identify other constituents of $I_{2^*}$. The separate constituents of $I_{2^*}$ show different time picks (Figure 4c). The time deviation exists because the constituents are formed by different body waves. The body waves are sensitive to the distribution of events, and they sample the Earth structure along different paths. Those time deviations are mixed together in the stacked $I_{2^*}$ (Figure 4d). Different selections of body-wave cross-terms or alteration of the events would deviate $I_{2^*}$ in coda-correlation stacks, which explains the time inaccuracy shown in Figures 2b and 2c.

The separation and identification of constituents of a specific coda-correlation feature, which are cross-terms between different seismic phases, can be ambiguous in certain cases. Some seismic phases have quite similar
arrival times, and hence associated constituents cannot be isolated. For example, PcPPKIKP and PKIKPPcP possess nearly identical arrival times. Their cross-terms with PcP, PcPPKIKP−PcP and PKIKPPcP−PcP, cannot be separated from each other. Both of them contribute to the coda-correlation feature I1* although they sample the Earth’s interior along different paths. Those ambiguities should be discarded to avoid unpredictable mixtures of different waves that make the tomography kernel inaccurate. Also, the constituents that show weak energy should be discarded because they can be easily overwhelmed by noise. Only those having clear identification and prominent energy should be selected.

### 3.2. The Relationship Between the Constituents and Earth Structure

A relation between the selected constituents that are seismic wave cross-terms and Earth structure can be established. That requires a careful selection of events for each constituent. According to the stationary condition, only the events located inside the stationary volume constructively contribute to the formation of features in correlograms, and outliers distort cross-correlation functions (Kanu & Snieder, 2015; Snieder et al., 2008; Wapenaar, Draganov, et al., 2010). Arriving from a stationary point, two seismic waves measured at separate receivers have the same slowness. Theoretical analysis based on ray theory can determine the locus of the same slowness point as shown in Figure 5 for some body-wave cross-terms. Around the same slowness point, a stationary zone of a finite size can be obtained using a theoretical analysis. For example, we can select a stationary zone that is ±10° around the same slowness point for the cross-term between I6 and I4. The size of each stationary zone can be determined from theoretical arrival time predictions. Within that zone, the time variation due to different event locations is almost negligible in the sense that the variation is less than the sampling interval. Also, this finite-size zone allows many events contributing to the cross-correlation stacks to improve the signal-to-noise ratio. To use other constituents, similar analyses can be conducted for each receiver pair to determine the event stationary zone.

![Figure 5](image-url) Ray-paths for the selected I2* constituents representing the seismic wave cross-terms given two fixed receivers (triangles) on the global scale (a) and in the inner core (b). Different constituents are presented by their corresponding ray paths in color according to the color scheme. Stars indicate optimal event locations for the labeled seismic wave cross-terms. The optimal locations are determined based on the same slowness (stationary point) condition. Blue dash line denotes the ray path corresponding to the phase PKIKPPKIKP for the two selected receivers. The differential ray path of each I2* constituent is equivalent to PKIKPPKIKP if the event is in the optimal location.

After the selection of events, the sensitivities of two seismic waves to Earth structure can be determined for each cross-term. Their difference denotes the relationship between each constituent and Earth structure. For example, the travel time’s relation to the structure using ray theory is

$$t_{ij}(m) = \int_{\text{path}_i} m \, dl - \int_{\text{path}_j} m \, dl,$$

where $t_{ij}$ is the time of a cross-term between the $i$th and the $j$th seismic waves, which sample the structure, parameterized as model $m = \{v/\}$ in which $v$ denotes the seismic wave velocity, along path$_i$ and path$_j$, respectively. The two ray paths are from the same event, and they end at two receivers, respectively. For
example, Figure 5 shows ray paths and related events for some prominent constituents of I2*. Those constituents are seismic wave cross-terms of I6-I4, I7-I5, I8-I6, and I9-I7. Equation 5 is in the form of

\[ t_{ij} = g_{ij}(m), \]

where \( g_{ij} \) refers to the sensitivity between the travel time \( t_{ij} \) of a single constituent of a specific coda-correlation feature and the model \( m \). In the same manner, a sensitivity to Earth structure can be determined for each of the selected constituents. Those sensitivities compose a travel time tomography problem:

\[
\begin{pmatrix}
g_{00}(m) \\
g_{01}(m) \\
\vdots \\
g_{ij}(m) \\
\vdots
\end{pmatrix}
= 
\begin{pmatrix}
t_{00} \\
t_{01} \\
\vdots \\
t_{ij} \\
\vdots
\end{pmatrix},
\]

(7)

Equation 7 is in the form of an inverse problem, that is \( G(m) = d \), where \( G = \{g_{ij}\} \) corresponds to the tomography kernel, \( m \) is the model vector and \( d = [t_{00}, t_{01}, ..., t_{ij}, ...]^T \) is the data vector. Alternatively, the tomography problem that relates model perturbations to time perturbations is

\[
\begin{pmatrix}
g_{00}(\Delta m) \\
g_{01}(\Delta m) \\
\vdots \\
g_{ij}(\Delta m) \\
\vdots
\end{pmatrix}
= 
\begin{pmatrix}
\Delta t_{00} \\
\Delta t_{01} \\
\vdots \\
\Delta t_{ij} \\
\vdots
\end{pmatrix},
\]

(8)

where \( \Delta m \) denotes a model perturbation vector from a reference vector \( m_0 \), and \( \Delta t_{ij} \) are the time differences between observations \( t_{ij} \) and predictions \( t_{ij,0} \) that corresponds to the reference model. Linear inversion approaches can solve equations 4 and 5 and estimate the resolution and accuracy (Aster et al., 2018; Kirsch et al., 1988).

Furthermore, the inverse problem can be rewritten in the form of optimization:

\[
\text{obj}(m) = \left\| \begin{pmatrix}
g_{00}(m) - t_{00} \\
g_{01}(m) - t_{01} \\
\vdots \\
g_{ij}(m) - t_{ij} \\
\vdots
\end{pmatrix} \right\| + \alpha \| R(m) \|,
\]

(9)

where \( R(m) \) denotes the regularization or the introduction of prior information, and \( \alpha \) is the weighting factor. Through minimizing the objective function \( \text{obj}(m) \), optimal solutions accompanied by the reliability evaluations can be obtained. Based on equation 9, it is possible to build an optimal parameterization that avoids the singularity of the inverse problem. Apart from that, equation 9 allows for applying prior information in arbitrary forms, for example, the prior knowledge of model parameters that can be used to configure search ranges for different inversion parameters. That makes the optimization constrained and would regularize the inverse problem and hence increase the stability and reliability of the inversion, especially for model parameters in interested areas.

4. A Toy Problem: Tomography of a Hemispherical Inner Core

We set up a toy-problem experiment with a goal of showcasing the general framework for coda-correlation tomography. This experiment allows for a simplified Earth model and arbitrary event-receiver geometry settings. This avoids complexities such as the existence of lateral heterogeneity or theory noise but at the same time sets the stage for the coda-correlation tomography applications on a global scale.
In our toy problem, we select the correlogram feature I2* as the basis to conduct tomography experiments. I2* is one of the prominent and stable features in global cross-correlation stacks (e.g., Boué et al., 2013), and it is sensitive to the bulk of the inner core. That feature was interpreted and analyzed as PKIKPPKIKP waves in previous applications on the inner core (e.g., Huang et al., 2015; Wang et al., 2015; Wang & Song, 2018). Importantly, I2* appears in global correlograms in small interreceiver distance range, and hence, it can be easily constructed in local network settings.

Our experiment consists of two parts: (1) forward model and (2) inversion. Both parts focus on a relationship between the perturbation of the Earth model and the produced perturbation in travel times. We first generate synthetic waveforms based on an input model and formulate the time perturbations. We then build a tomography kernel and invert for the Earth structure using the synthetic data obtained through the forward model. Parallel with the newly proposed framework, we employ the Green’s function method and perform the inversions to make both qualitative and quantitative comparisons.

4.1. Forward Model

We set two Earth models and simulate seismic wavefield on a global scale. We focus on P wave velocity perturbations between those two models. The first model is a spherically symmetric Earth model, ak135 (Kennett et al., 1995). It is the reference model (m0) in equation 8 that provides the time predictions (tij,0). The second model is identical to the first model, but it has a hemispherical inner core. According to that model, the entire eastern hemisphere of the inner core has a P wave speed perturbation of +1% relative to the reference model (Figure 6). We note that the setting in our experiment is fundamentally different from a hemispherical dichotomy that has been empirically found for the outermost part of the inner core although the amplitude of fluctuation is similar (e.g., Shearer, 1994; Song & Helmberger, 1998; Tanaka & Hamaguchi, 1997; Waszek & Deuss, 2015). In our simulation, we use the SPECFEM method (Komatitsch et al., 2010; Komatitsch & Tromp, 2002) and employ global events with the depth of 50 km to suppress the excitation of surface waves in the late-coda. We use equidistant events along the equator to build a data set for synthetics. The event separation is 0.5°. To make the computation feasible for so many events, the simulation is two-dimensional. Furthermore, we set receiver pairs with small and intermediate interreceiver distances (Figure 6). This setting imitates the receiver-pair settings in regional seismic networks. The focal mechanism is of an explosive type, and the source time function is Gaussian, following Pham et al. (2018). We do not consider variations of focal mechanism and source time function for simplicity.

After generating synthetic seismic wavefields, we compute STCCs and interreceiver cross-correlation stacks. The STCCs are used to verify the newly proposed method, and the cross-correlation stacks are in conjunction with the Green’s function method. We extract coda wavefield in 1–9 hr to calculate the STCCs and in 3–9 hr...
to calculate interreceiver cross-correlation stacks. The earthquake coda in 3–9 hr makes prominent features in coda-correlation stacks (Lin & Tsai, 2013; Phảm et al., 2018; Poli et al., 2017). We used 1–9 hr in STCC because coda-correlation constituents appear after 1 hr, as shown in Figure 4c. The short-time window is 100 s in STCC computation based on the analysis shown in Figure 4. We apply temporal normalization to suppress surface waves and spectral whitening to balance energy across the entire frequency band following Phảm et al. (2018).

In synthetic STCCs, the constituents of I2* can be separated and identified (Figure 4c). Figure 5 shows the ray paths for significant constituents for a receiver pair. They are cross-terms of I6–I4, I7–I5, I8–I6, and I9–I7, and they have clear identifications (Figure 4c). Out of those constituents, we randomly select a single constituent at each receiver pair. For example, we select the cross-term I6–I4 and discard other cross-terms at some receiver pairs. This selection resembles the absence of constituents due to limited event distributions. After the selection, we determine the optimal event location for each constituent based on the same slowness (stationary principle) condition, and we choose a finite-size zone that is ±10° around the optimal location to select the events. For all events within that finite-size zone, the constituents of the same type are stacked to improve the signal-to-noise ratio.

We measure the time perturbations of the selected constituents. The time perturbation is defined as a result of the P wave velocity deviations in the inner core, and hence it is denoted as ΔtIC. We only focus on the inner core structure for ΔtIC, although the ray paths of the I2* constituents sample the whole Earth. If the whole Earth were included, the relationship between the inner core and the measurement would be overwhelmed by the structure outside the inner core, and hence the inverse problem would be unsolvable. The ill-condition is obvious concerning that the inner core volume is less than 1% of the whole Earth. ΔtIC is the time difference between any two constituents that correspond to the hemispherical model (m) and the reference model (m₀), respectively. ΔtIC can be measured through matching waveforms of the two constituents. Figure 7a shows the measured time perturbations that are compared with the theoretical values. For each constituent, the theoretical time perturbation is calculated using the ray theory based on equation 5.

Figure 7. Toy-problem time perturbations at the fixed receiver pairs (Figure 6a) due to the introduction of a hemispherical structure in the inner core. A theoretical (gray cross) and measured (black cross) time perturbations for the selected I2* constituents using the new theoretical framework proposed in this study. (b) The histogram of measurement errors for the new method. The solid line denotes the Gaussian estimation of error distribution. (c) Theoretical PKIKPKIKP time perturbations (gray crosses) and measured time perturbations (black crosses) for the stacked I2*. In the Green’s function method, the stacked I2* using all events and all seismic coda in 3–9 hr are treated as reconstructed PKIKPKIKP. (d) The histogram of measurement errors for the Green’s function method.
We measure the time perturbations for I2*. Similarly, this perturbation is the time difference between two I2* features that correspond to the hemispherical model \( m \) and the one-dimensional reference model \( m_0 \), respectively. In the Green's function method, I2* is treated as PKIKPPKIKP phase, and hence we denote it as \( \Delta t_{\text{IC}}^{gf} \). Figure 7c shows the measurements \( \Delta t_{\text{IC}}^{gf} \). The measurements are compared with the theoretical time perturbations for the interreceiver PKIKPPKIKP phase.

For a comparison, the time perturbations in the new framework are more accurately predicted than those under the Green's function assumption. The former is much closer to the theoretical values than the latter (Figures 7a and 7c), and the former presents a sharper and thinner error distribution with a shape being closer to the Gaussian centered at zero than the latter (Figures 7b and 7d). A wider error distribution of the latter (Figure 7c) is due to the stacking of many body-wave cross-terms that have varied times due to event locations and ray path differences (Figures 4c and 4d). That makes I2* fundamentally different than PKIKPPKIKP. The new method relies on the selection of separate constituents of I2* and events under the same slowness condition. For the selected constituents, for instance, I7-I5, the common paths from a given event to the receivers are canceled, and the remainder is equal to PKIKPPKIKP between those two receivers. Therefore, the isolated constituents show better time prediction of PKIKPPKIKP than I2*.

4.2. Inversion

We carry out tomographic inversions using two methods: a new method within the framework proposed in this study and a method that assumes the Green’s functions concept is accurate. For both methods, we use the same parameterization for the inner core; that is, it is divided into eight discrete segments (Figure 8a).

The tomographic kernels are fundamentally different between the two methods. As for the new method, we build structural sensitivity kernels for the selected constituents, as explained in section 4.1. The construction of sensitivity kernels involves a high-frequency approximation based on equation 5. Those sensitivities...
contribute to the tomography kernel that represents the sampling of the inner core along many differential ray paths. Figures 6a and 6b show the geometrical coverage of the differential ray paths for the selected constituents. It is clear that differential ray paths exhibit uniform distributions and sample various azimuths in the inner core (Figure 6b). In contrast, for the Green’s function method, we build a tomography kernel by treating I2* as an interreceiver PKIKPPKIKP phase using the ray theory.

After building tomographic kernels, we pose a linear inverse problem based on equation 8. The inverse problems for both cases establish a connection between P wave velocity perturbations in the inner core (Δm) and the time perturbations measured through the forward model (ΔtIC or ΔtICf). The velocity perturbations mark the difference between the model m and the reference model (m0).

We employ the damped least-squares method to solve the inverse problem. The damping is necessary here because measurement errors cannot be avoided, although the toy-problem is in an idealized case without noise (Figure 7). The errors can be related to the ray approximations in equation 5, the rounding and floating errors in simulations and computations, and the measurement errors. The damping moderates the singularity of the tomography kernel and hence suppresses the back projection of errors from the data space to the parameters in the model space, which would overwhelm the solution recovery. As shown in Figure S1 in the supporting information, we select the optimal damping factor based on the L-curve method (Aster et al., 2018). The damping weakens the singularity of the toy-problem inversion in the sense that it reduces the distance between the maximal and the minimal singular values (Figure S1). Therefore, the inversion results primarily depend on the accuracy of the tomography kernels and time perturbation measurements.

Figures 8b and 8c show the recovered inner core structure by the new method and by the Green’s function method, respectively. The former approach presents a much better recovery of the synthetic model than the latter. The P wave velocity contrast between the two hemispheres is sharper in the former than in the latter. The error plots (Figures 8d and 8e) highlight the difference between the two approaches in the recovery ability. The new method demonstrates an overall relative error less than 10%, while the Green’s function method yields relative errors more significant than 60% in some parts. This difference, combined with the time perturbation comparison achieved through modeling (Figure 7), demonstrates the accuracy of the framework proposed here for the coda-correlation tomography.

5. Discussion and Conclusion

In this study, we propose a new framework toward accurate coda-correlation tomography based on the understanding of coda-correlation’s formation. In this new framework, we build travel time’s sensitivity to Earth structure for tomography. Our toy-problem experiment demonstrates that the new framework is feasible and accurate and allows building the relationship between coda-correlation features and Earth structure. The sensitivity kernel derived in this study can be extended with Fréchet kernels based on finite-frequency methods (Dahlen et al., 2000a, 2000b; Marquering et al., 1999). Although high-frequency approximation is effective in resolving the hemispherical dichotomy of the bulk of the inner core, as shown in the toy-problem experiment, finite-frequency approaches would further improve tomographic resolution because the features in global correlograms are prominent in intermediate periods (Kennett & Pham, 2018; Pham et al., 2018; Poli et al., 2017). We, however, look forward to incorporating finite frequency kernel into coda-correlation tomography in future studies. We speculate that fine-scale structures such as the layering of the inner core (e.g., Garcia & Souriau, 2000; Shearer, 1994; Su et al., 1996) could also be resolved.

The new framework distinguishes coda-correlation from noise-correlation through a decomposition of features in correlograms into separate constituents and builds relationships between these constituents and Earth structure. In contrast, noise-correlation methods treat cross-correlation stacks as reconstructed interreceiver responses. This comparison reflects the difference in the formation of noise-correlation and coda-correlation. The former converges to interreceiver response in a diffusive noise field (Lobkis & Weaver, 2001; Snieder et al., 2006; Snieder & Sens-Schönfelder, 2015; Wapenaar, 2004; Wapenaar, Draganov, et al., 2010), whereas the latter is far removed from a reconstructed interreceiver response because of the lack of equipartitioning in earthquake late-coda (Sens-Schönfelder et al., 2015). Specifically, coda-correlation is dominated by cross-terms between seismic waves that reverberate
between the free surface and the core-mantle boundary. Therefore, it is essential to treat coda-correlation differently from reconstructed interreceiver response. The same considerations are necessary when an arbitrary waveform that is cross-correlated is not completely diffuse (e.g., Fichtner et al., 2017).

The new framework allows for the use of the Earth’s correlation waveform to infer Earth structure. This complements the constraints made by the utilization of the seismic waveform methods. For example, most inferences on the inner core structure are confined in the outermost part due to sparse seismic observations (Tkalcic, 2017). We are confident that the future applications of coda-correlation tomography will constrain the entire inner core structure, as demonstrated in the toy-problem experiment.

The data coverage seems to be limited due to the uneven global distribution of earthquakes. Sometimes, there are no events inside the stationary zone for a specific coda-correlation constituent. However, it is always possible to consider other constituents. For example, I3–I1, a constituent of I2*, requires events that are close to the antipodal points of the receivers, but I6–I4 and I7–I5 allow for the use of the events in an intermediate range of epicentral distances. In the conventional tomography, the reverberations in the late-coda time-window (e.g., I6 and I7) are never used simply because they have not yet been observed. In the new framework, however, it is possible to utilize those exotic seismic phases and hence increase constraints on Earth structure via analyzing their cross-terms.

Apart from the above, there are novel opportunities to constrain Earth structure, that is, via multiple features in correlograms that do not exist as the seismic waveform counterparts, for example, cS–cP (Pham et al., 2018) and I2–J (Tkalcic & Pham, 2018). Other features can also be analyzed for their constituents, as we have done for I2* (Figure 4), and based on that analysis and the varying source-receiver geometry consideration, individual constituents can be related to Earth structure. Furthermore, the new framework allows for a dense volumetric coverage of Earth. There are numerous constituents of a single correlogram feature, and each of these constituents can be employed in a tomographic inversion. For example, I2* can be decomposed into many constituents (Figure 4) corresponding to different ray paths. Including all of them would decrease the singularity of the inverse problem, and hence it would improve the stability and reliability of the tomographic inversion.

The new framework requires accurate separations and identifications of constituents for a given correlogram feature. Inaccurate or ambiguous identifications would lead to incorrect inversion results. It is necessary to tune parameters such as the short-time window in STCC and the frequency band. Theoretical experiments, such as the toy-problem designed for this study, allow for the selection and validation of those parameters. Additionally, slowness analyses can improve the accuracy of the separation and identification of cross-terms between seismic waves. Slowness analysis is feasible in cases of dense seismic arrays. Therefore, combining dense arrays with coda-correlation has a strong potential to advance the field of global seismology in the coming decades.

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