Motion control analysis of hybrid grinding and polishing machine tool for blade finishing

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Abstract. Blade is the key component in the energy power equipment of turbine, aircraft engines and so on. Research on the process and equipment for blade finishing becomes one of important and difficult points. The motion control of hybrid grinding and polishing machine tool is different from that of traditional machine tool, and there exists motion coupling phenomenon when it moves as a whole. In order to control precisely motion of machine tool, the inverse and forward kinematics solution considering motion coupling of parallel mechanism was solved based on designed hybrid grinding and polishing machine tool for blade finishing in this paper. Firstly, the coupling factor of parallel mechanism was taken into account in the new model, and the inverse kinematics solution was modified to improve the solution accuracy. Then, the forward kinematics solution of machine tool based on inverse kinematics solution was solved. Finally, the motion simulation was made to verify the inverse and forward kinematics solution deduced is correct, which provides theoretical basis for the motion control scheme of machine tool calibration and inspection control system.

1. Introduction

Blade is a key and major component in the energy power equipment of turbine, marine propeller, aircraft engines and so on. It is characterized by complex surface structure shape, difficult processing and high precision. As a result, the finishing process of blade is one of the most important and difficult problems in the field of aerospace. In recent years, researchers have made a lot of research on precision machining equipment and process for blade, such as MTS1000-6CNC six axis belt grinding machine of IBS company, HS-196GC six axis grinding centre of Huffman company, and five axis linkage high precision vertical machining centre of C.B. Ferrari company[1-3].

Hybrid grinding and polishing machine tool is designed on the basis of the advantages of parallel mechanism such as flexible space movement, light weight of moving parts and simple configuration. Kinematics research for parallel mechanism includes forward and inverse kinematics problem solving, workspace, singularity, motion dexterity analysis and so on. The forward and inverse kinematics algorithm is used to determine the position, velocity and acceleration relationship between input and output of parallel mechanism, while it is a necessary work for machine tool configuration and motion design, and is also the basis for the study of other kinematics characteristics of parallel mechanism[4-8].

The motion control of hybrid grinding and polishing machine tool is different from that of traditional machine tool, and there exists motion coupling phenomenon when it moves as a whole. In
reference[9], the kinematics model of hybrid grinding and polishing machine tool was established by using the coordinates of workpiece trajectory points and the normal vector of the points in the profile. But the coupling motion of parallel mechanism did not be considered in this model. In this paper, the coupling factor is taken into account in the new model, and the inverse kinematics solution is modified to improve the solution accuracy. Then the forward kinematics solution of machine tool based on inverse kinematics solution is solved. Finally, the motion simulation is made to verify the inverse and forward kinematics algorithm based on kinematic coupling factor of parallel mechanism.

2. Inverse kinematics and forward kinematics solution

2.1. Structure of hybrid grinding and polishing machine tool for blade finishing

The structure of the developed grinding and polishing machine tool is shown in Figure 1. The machine tool which is composed of 3-RPS parallel mechanism, X axis, Y axis and B axis is a series parallel machine tool with five freedom degree. The linear motion range of X axis is 0 to 300 mm, the linear motion range of Y axis is 0 to 340 mm, Z and A are virtual axes which are formed by three axes motion of parallel mechanism. The linear motion range of Z axis is 590 mm to 750 mm. The A axis rotates around X axis, and the motion range is -15° to 15°. The B axis is the rotary axis of the fixture, and it rotates around the Y axis. The range of motion is 0° to 360°. The tool head system includes the driving shaft and the changeable wheel shaft. The driving shaft provides the power for the belt to rotate, and the changeable wheel shaft realizes the transformation of grinding and polishing tools of different sizes.

![Figure 1. Structure of the machine tool.](image)



2.2. Inverse kinematics solution based on kinematic coupling of parallel mechanism

The new kinematics model based on kinematic coupling of parallel mechanism is shown in Figure 2.

In Figure 2, \( O_bX_bY_bZ_b \) is the static platform coordinate system, \( OXYZ \) is the global coordinate system of machine tool, which is in the same plane as \( O_bX_bY_bZ_b \). \( O_dX_dY_dZ_d \) is the moving platform coordinate system, \( O_wX_wY_wZ_w \) is the workpiece coordinate system, \( O_fX_fY_fZ_f \) is the fixture coordinate system, which coincides with \( O_wX_wY_wZ_w \), and \( O_tX_tY_tZ_t \) is the tool head coordinate system.
Under workpiece coordinate system $O_wX_wY_wZ_w$, the contact point $P_w$ between tool head and workpiece is set as: $P_w=\begin{bmatrix} P_{wx} \\ P_{wy} \\ P_{wz} \end{bmatrix}$, and its normal vector $u_w$ is set as: $u_w=\begin{bmatrix} u_{wx} \\ u_{wy} \\ u_{wz} \end{bmatrix}$. Under tool head coordinate system $O_tX_tY_tZ_t$, the coordinate of contact point is: $P_t=[0, \ 0, \ 0]$, and its direction vector is: $u_t=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

In order to realize the processing of $P_w$ point on workpiece, it is necessary to move the machine tool axes jointly so that the contact point $P_w$ under $O_wX_wY_wZ_w$ coincides with the contact point $P_t$ under $O_tX_tY_tZ_t$. The transformation process is shown in Figure 3.

In Figure 3, $^{fT_w}$、$^{dT_f}$、$^{bTd}$、$^{gT_b}$、$^{tTg}$ are homogeneous transformation matrices between coordinate systems. The transformation from $P_w$ point to $P_t$ is:

$$
^{fT_w} \cdot^{dT_f} \cdot^{bTd} \cdot^{gT_b} \cdot^{tTg} \cdot \begin{bmatrix} P_{wx} \\ P_{wy} \\ P_{wz} \\ u_{wx} \\ u_{wy} \\ u_{wz} \end{bmatrix}
$$

(1)

In (1), when calculating $^{bTd}$ from moving platform to static platform, the coupling factor of $y$ axis direction of parallel mechanism should be considered. Other homogeneous transformation matrices remain unchanged. The homogeneous transformation matrix $^{bTd}$ is:

$$
^{bTd}=\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha & \frac{1}{2}r (1-\cos \alpha) \\
0 & \sin \alpha & \cos \alpha & r_s \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(2)

The inverse kinematics analytic solution is shown in formula (3).
In (3), \( r_z \) is the displacement of moving platform along Z axis, \( \alpha \) is the rotation angle of moving platform around X axis, \( \beta \) is the rotation angle of B axis, \( r_x \) and \( r_y \) are displacement of machine tool along X axis and Y axis respectively, and \( L_1, L_2 \) and \( L_3 \) are the length of electric cylinder.

\[
\begin{align*}
\tan \beta &= \frac{u_{wz}}{u_{wx}} \\
\tan \alpha &= -\frac{u_{sy}}{u_{sx} \sin \beta - u_{sz} \cos \beta} \\
r_x &= P_x \cos \beta + P_y \sin \beta \\
r_z &= P_x \sin \beta + P_y \cos \beta - dsina + \frac{1}{2} r(1 - \cos \alpha) \\
r_y &= \sqrt{R + \frac{1}{2} r^2 - \frac{3}{2} \cos \alpha} + (rsina + r_z)^2 \\
L_1 &= \sqrt{\left(\frac{R + 1}{2} r - r_1 \cos \alpha - dh \cos \beta \right)^2 + \left(\frac{1}{2} rsina + r_z\right)^2} \\
L_2 &= L_3 = \sqrt{(r-R)^2 + \left(\frac{1}{2} rsina + r_z\right)^2} \\
\end{align*}
\]

2.3. Forward kinematics solution

Singularity inevitably occurs in any form of mechanism. Singularity means that motion is out of control, degree of freedom changes, and mechanical or kinematic performance suddenly changes when the mechanism moves to a posture. By analyzing the singularity of the hybrid grinding and polishing machine tool, it is determined that the machine tool has avoided the singularity in the range of motion. Therefore, the forward kinematics solution is the inverse function of inverse kinematics solution, and vice versa. Based on this conclusion, the forward kinematics solution of machine tool is deduced according to formula (3).

\[
r_z = \sqrt{L_2^2 - (r-R)^2} - \frac{1}{2} \cos \alpha \\
\tan \alpha = \sqrt{\left(\frac{R + 0.5r}{\left(\sqrt{(R + 0.5)^2 + (L_2^2 - (r-R)^2)}\right)}\right)^2 - \frac{3}{2} \cos \alpha} + \left(\frac{1}{2} \cos \alpha + h\right)^2 \\
\alpha = \arcsin \left[\left(\frac{L_2^2 - L_3^2}{3r} + R + \frac{1}{2} r \right) \right] - c
\]

In (5), angle \( c \) is a transition constant for mathematical solution, and can be obtained by trigonometric function transformation. From (4) and (5), the values of \( \alpha \) and \( r_z \) can be obtained, which can be substituted as known quantities into the analytical formula shown in (6), and the coordinates of contact points in workpiece coordinate system \( O_{xw}X_wY_wZ_w \) can be obtained.

\[
\begin{align*}
P_{wx} &= \frac{r_z}{\cos \beta} \left[ -r \cdot \frac{1}{2} r (1 - \cos \alpha) \right] \cos \alpha - \left[ r_z - h \right] \sin \alpha \tan \beta \\
P_{wy} &= \frac{r_z}{\cos \beta} \left[ -r \cdot \frac{1}{2} r (1 - \cos \alpha) \right] \cos \alpha - \left[ r_z - h \right] \sin \alpha \\
P_{wx} &= \frac{r_z \tan \beta}{\cos \beta} \left[ -r \cdot \frac{1}{2} r (1 - \cos \alpha) \right] \cos \alpha - \left[ r_z - h \right] \sin \alpha \tan \beta \\
&- \left[ -r \cdot \frac{1}{2} r (1 - \cos \alpha) \right] \sin \alpha - \left[ r_z - h \right] \cos \alpha \tan \beta
\end{align*}
\]

In formula (3), when solving normal vector, there are three unknown quantities in two equations, which cannot solve the exact value, but the significance of finding normal vector is to find the
principal normal direction vector of the contact point, without the modulus of the normal vector being invariable, so we can add constraints to the normal solution: let one of the three coordinate components of normal vector be a constant value, and set it here as: $u_{wz}=1$, so as to simplify the solution of normal vector. The analytic formula of unit normal vector is obtained as shown in (7).

$$
\begin{align*}
    u_{wx} &= \frac{\tan \beta}{\sqrt{1 + (\tan \beta)^2 + [\tan \alpha (\tan \beta \sin \beta + \cos \beta)]^2}}, \\
    u_{wy} &= \frac{\tan \alpha (\tan \beta \sin \beta + \cos \beta)}{\sqrt{1 + (\tan \beta)^2 + [\tan \alpha (\tan \beta \sin \beta + \cos \beta)]^2}}, \\
    u_{wz} &= \frac{1}{\sqrt{1 + (\tan \beta)^2 + [\tan \alpha (\tan \beta \sin \beta + \cos \beta)]^2}}.
\end{align*}
$$

(7)

It is known that the initial length of three electric cylinders of 3-RPS parallel mechanism is 620 mm, the maximum elongation is: $\Delta L=160$ mm, and the rotating angle range of the moving platform is: $\alpha \in [-15^\circ, 15^\circ]$. From Figure 2, it is known that $X$ axis coordinates of the centre point $O_d$ of the moving platform are equal to 0, and the space trajectory of the point $O_d$ is the same as its projection in the plane $YOZ$, as shown in Figure 4. From formula (4) and (5), the rotating angle $\alpha$ of moving platform of the parallel mechanism around $X$ axis can be calculated. The $\alpha$ is represented in the same coordinate system with its corresponding point $O_d$ coordinate trajectory, as shown in Figure 5.

![Figure 4. The motion range of moving platform center point $O_d$.](image)

![Figure 5. The point $O_d$ position and rotation angle of moving platform.](image)

From Figures 4 and Figures 5, we can know the space position range of $O_d$ point. The $Z$ axis limit position is 587mm to 754mm, and the $Y$ axis motion is the accompanying motion of parallel mechanism, its limit position is 0mm to 4.3mm.

3. Kinematics simulation

It can be seen that the parallel mechanism of the hybrid grinding and polishing machine tool will appear accompanying motion in the $Y$ axis direction by singularity analysis, and the model is more complex. So motion simulation is made to verify the inverse and forward kinematics algorithm based on kinematic coupling factor of parallel mechanisms deduced above by simulation software ADAMS.

In simulation analysis, the initial position and posture of the parallel mechanism are as follows: the moving platform is horizontal, that is: $\alpha=0^\circ$, the height of the moving platform is 682.8mm, the initial length of the three electric cylinders is 711.5mm.

Example 1:
Three electric cylinders are set up for uniform elongation at 5mm/s for 10 seconds.

Example 2:
The electric cylinder $L_1$ is assumed to shorten uniformly with -5mm/s, and $L_2$ and $L_3$ are assumed to elongate uniformly with 5mm/s for 10s.

In Example 1 and Example 2, the point $O_d$ trajectory can be obtained by simulation software as shown in Figure 6 and Figure 7 respectively, and the $Z$ direction coordinate of point $O_d$ can be obtained as shown in Figure 8 and Figure 9 respectively.

In Figure 6 and Figure 7, the white vertical line on the moving platform is the point $O_d$ trajectory. In Example 1, the $Z$ axis motion range is 682.8 mm to 734.8 mm by using formula (4) and (5) within the expansion range of the electric cylinder, which is in agreement with simulation results in Figure 8. In Example 2, the $Z$ axis motion range is 682.8 mm to 697.5 mm by using formula (4) and (5) within the expansion range of the electric cylinder, which is in agreement with simulation results in Figure 9.

4. Conclusions
Considering motion coupling of parallel mechanism, inverse and forward kinematics solution is solved based on designed hybrid grinding and polishing machine tool for blade finishing in this paper. Firstly, the coupling factor of parallel mechanism is taken into account in the new model, and the inverse kinematics solution is modified to improve the solution accuracy. Then, the forward kinematics solution of machine tool based on inverse kinematics solution is solved. Finally, the motion simulation is made to verify the inverse and forward kinematics algorithm based on kinematic coupling factor of parallel mechanisms by simulation software ADAMS. The simulation result shows that the inverse
and forward kinematics solution established is correct, which prepares for the motion control scheme of machine tool calibration and inspection control system.

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