Branes and Theta Dependence

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Abstract: We use the fivebrane of M theory to study the θ dependence of four dimensional $SU(N_c)$ super Yang-Mills and super QCD softly broken by a gaugino mass. We compute the energy of the vacuum in the supergravity approximation. The results obtained are in qualitative agreement with field theory. We also study the θ dependence of the QCD string tension via the fivebrane.

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1. Introduction

The $\theta$ dependence of four dimensional gauge theories encodes non trivial information of the dynamics. A study of this dependence in asymptotically free gauge theories requires an appropriate effective low energy description of the system, which is difficult to obtain. Recently, Witten [1] studied the $\theta$ dependence of the vacuum energy of large $N_c$ four dimensional gauge theory using the conjectured supergravity/gauge theory correspondence [2,3].

Another approach to study four dimensional gauge theories is via the fivebrane of M theory. When the fivebrane is wrapping a holomorphic curve, the effective low energy four dimensional gauge theory is supersymmetric. Using this description, many holomorphic properties of these theories have been derived [4]. In this paper we will use the fivebrane of M theory to study the theta dependence of softly broken supersymmetric four dimensional super Yang-Mills (SYM) and super QCD (SQCD).

The paper is organized as follows: In the section 2 we will consider SYM softly broken by a gaugino mass. We will first review the field theory computation of the vacuum energy and the physics encoded in the $\theta$ dependence. We will then compute the same quantity using the brane description. The two results agree qualitatively and have the same $\theta$ dependence. We will compute the QCD string tension via the fivebrane. Here we will have no field theory result to compare with. We will close the section by discussing the large $N_c$ limit. In section 3 we will consider SQCD with massive matter in the fundamental representation of the gauge group, softly broken by a gaugino mass. We will compute the vacuum energy in field theory and using the fivebrane description. Again we obtain qualitative agreement. Section 4 is devoted to a discussion and comments on the decay rate of the false vacuum.

2. Softly Broken SYM

2.1. Effective Field Theory of SYM

Consider the Veneziano-Yankielowicz (VY) effective action [5] for $SU(N_c)$ super Yang-Mills softly broken by a gaugino mass. We will follow in the discussion the conventions of

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1 For related works see refs. [5,6,7].
ref. [9]. Denote by $S$ the VY superfield and by $\varpi$ its first component. The composite superfield $S$ is defined by $S = (g^2/32\pi)W^a_{\alpha}W^{a,\alpha}$, $\alpha = 1, 2, a = 1, \ldots, N^2_c - 1$. $\varpi = (g^2/32\pi)\lambda^{a,\alpha}$ where $\lambda^{a,\alpha}$ denotes the gaugino. The effective VY action is

$$L = \frac{9}{\alpha}(SS)^{1/3} + \left[\left(S \left(N_c \log \frac{S}{|\Lambda_1|^3} - N_c - i\theta\right)\right) + h.c.\right] + [m_{\lambda}S_{\theta=0} + h.c.], \quad (2.1)$$

where the (complex) gaugino mass $m_{\lambda}$ is the renormalization group invariant supersymmetry breaking parameter. The theta angle is defined by

$$\theta = N_c\text{arg}((\Lambda_1)^3), \quad (2.2)$$

where $\Lambda_1$ is the dynamically generated scale. The scalar potential resulting from the VY action is given by

$$V = \alpha(\varpi \bar{\varpi})^{2/3} \left| N_c \log \frac{\varpi}{|\Lambda_1|^3} - i\theta \right|^2 - (m_{\lambda} \varpi + \bar{m}_{\lambda} \bar{\varpi}), \quad (2.3)$$

and the $N_c$ supersymmetric minima (when $m_{\lambda} = 0$) are located at

$$\langle \varpi \rangle_{susy,k} = |\Lambda_1|^3 \exp \left( i \frac{\theta + 2\pi k}{N_c} \right), \quad k = 0, \ldots, N_c - 1. \quad (2.4)$$

The degeneracy corresponds to the spontaneous breaking of the non anomalous $Z_{2N_c}$ discrete subgroup of $U(1)_R$ to $Z_2$.

We assume that the gaugino mass softly breaks the supersymmetry, $|m_{\lambda}| < < |\Lambda_1|$. In this regime we are interested in computing quantities only to first order in $m_{\lambda}$. Consider a generic scalar potential of the form

$$V(\varphi_i) = V_{susy}(\varphi_i) - (m_{\lambda} \varpi + \bar{m}_{\lambda} \bar{\varpi}). \quad (2.5)$$

When $m_{\lambda} = 0$ the vacuum energy vanishes and

$$V_{susy}(\langle \varphi_i \rangle_{susy}) = 0, \quad \frac{\partial V_{susy}(\langle \varphi_i \rangle_{susy})}{\partial \varphi_j} = 0. \quad (2.6)$$

When supersymmetry is softly broken we look for a vacuum configuration of the form

$$\langle \varphi_i \rangle = \langle \varphi_i \rangle_{susy} + m_{\lambda} \langle \varphi_i \rangle_1 + O((m_{\lambda})^2). \quad (2.7)$$
Substituting this in eq. (2.3) and Taylor expanding for small $m_\lambda$ around $\langle \varphi_i \rangle_{\text{susy}}$ we obtain

$$V(\langle \varphi_i \rangle) = -(m_\lambda \langle \varphi \rangle_{\text{susy}} + \bar{m}_\lambda \langle \bar{\varphi} \rangle_{\text{susy}}) + O((m_\lambda)^2). \quad (2.8)$$

Thus to compute, to first order in $m_\lambda$ the vacuum energy (per unit volume) we need to know only $\langle \varphi \rangle_{\text{susy}}$. In the case at hand this is given by eq. (2.4) and we obtain \[9\]

$$E_k(\theta, N_c, m_\lambda, \Lambda_1) \simeq -2|m_\lambda| |\Lambda_1| \cos \left( \frac{\theta_p + 2\pi k}{N_c} \right), \quad (2.9)$$

where

$$\theta_p = \theta + N_c \text{arg}(m_\lambda), \quad (2.10)$$

is the physical $\theta$ angle.

The gaugino mass term breaks explicitly the $Z_{2N_c}$ symmetry and shifts the $N_c$ local minima. The true ground state is given by the value of $k$ that minimizes eq. (2.9). When $-\pi < \theta_p < \pi$ the true vacuum is at $k = 0$. At $\theta = \pi$ the $k = 0$ and $k = 1$ levels cross, and the $k = 1$ vacuum is the true ground state up to $\theta_p = 3\pi$ where it crosses the $k = 2$ vacuum, and the $k = 2$ vacuum becomes the true ground state up to $k = 3$. This pattern continues until $\theta_p = (2N_c - 1)\pi$ where we have again the $k = 0$ vacuum. Thus $\theta_p$ periodicity is $2\pi N_c$ while the physics is periodic with period $2\pi$. When $\theta_p = (2n + 1)\pi$, two vacua have the same energy and CP is spontaneously broken \[10,11\].

2.2. Brane Computation of Vacuum Energy

Consider the M5 brane with its 6d world-volume being $R^4 \times \Sigma$ embedded in $R^4 \times R^6 \times S^1$. $\Sigma$ is a real two dimensional space. We denote by $x^0, \ldots, x^3$ and $x^4, \ldots, x^9$ the coordinates on $R^4$ and $R^6$ respectively, and by $x^{10}$ the circle coordinate. The radius of the circle is taken to be $R$. Introduce the complex coordinates $v = x^4 + ix^5$, $w = x^8 + ix^9$, $t = \exp[-(x^6 + ix^{10})/R]$.

Let $\Sigma$ be given by

\[2\] $v = z + \frac{\epsilon}{\bar{z}}, \quad w = \frac{\zeta}{\bar{z}}, \quad t = z^{N_c}, \quad x^7 = 2\sqrt{\epsilon} \log \left| \frac{z}{\Lambda_1} \right|$, \quad (2.11)
while the $R^4$ world-volume coordinates are $x^0, \ldots, x^3$. The flat eleven dimensional background metric is given by

$$(ds)^2 = \sum_{m,n=0}^{3} \eta_{mn} dx^m dx^n + 2G_{v\bar{v}} dv d\bar{v} + 2G_{w\bar{w}} dw d\bar{w} + 2G_{\bar{w}w} d\bar{w} dw + G_{7\bar{7}} dx^7 dx^\bar{7}, \quad (2.12)$$

where

$$G_{v\bar{v}} = \frac{(l_p)^6}{2(2\pi R)^2} \quad G_{w\bar{w}} = \frac{(l_p R)^2}{2} \quad G_{tt} = \frac{(R)^2}{2|t|^2} \quad G_{77} = \frac{(l_p)^6}{(2\pi R)^2}. \quad (2.13)$$

The effective low energy four dimensional gauge theory in $x^0, \ldots, x^3$ described by this system is $SU(N_c)$ Yang-Mills with softly broken supersymmetry \[12,5\]. The real parameter $\epsilon$ describes the soft breaking of supersymmetry: \[3\] for $\epsilon = 0$ we have a holomorphic curve and a realization of $N = 1$ supersymmetric Yang-Mills \[13,12\] while for $\epsilon \neq 0$ we get a soft breaking of supersymmetry by gaugino mass. When $\epsilon = 0$

$$\zeta_{susy} = C_\zeta \left(\frac{l_p}{R}\right)^2 (\Lambda_1)^3, \quad (2.14)$$

with $C_\zeta$ a real dimensionless constant which a priori can depend on $N_c$. When $\epsilon = 0$ \[2.11\] describes $N_c$ supersymmetric curves, which correspond to the $N_c$ supersymmetric vacua of SYM, given by the different values of the phase of $\Lambda_1$:

$$\zeta_k = |\zeta_{susy}| \exp(i\frac{\theta + 2\pi k}{N_c}), \quad (2.15)$$

where

$$\theta \equiv N_c \arg(\zeta_{susy})|_{k=0} = N_c \arg((\Lambda_1)^3)|_{k=0}, \quad k = 0, \ldots, N_c - 1. \quad (2.16)$$

The parameter $\epsilon$ is a function of the gaugino mass $m_\lambda$. It vanishes when $m_\lambda = 0$. When supersymmetry is broken, also $\zeta$ can acquire a dependence on $m_\lambda$ which we can parametrize as follows

$$\zeta(m_\lambda) = \zeta_{susy}(1 + f_\zeta(m_\lambda)), \quad (2.17)$$

3 In the supergravity approximation, the parameter $\epsilon$ is real and positive. It is possible that beyond the supergravity approximation (or for hard supersymmetry breaking) it acquires an imaginary part which can signal the disappearance of a metastable vacuum for large values of $\theta_p$, as discussed in ref. \[5\].
where $f_\zeta$ vanishes when $m_\lambda$ does. The charge assignments for the physical parameters are given by \[13\]

|         | $U(1)_{45}$ | $U(1)_{89}$ |
|---------|-------------|-------------|
| $m_\lambda$ | $-2$       | $-2$       |
| $(\Lambda_1)^3$ | $2$         | $2$        |
| $R$       | $0$         | $0$        |

(2.18)

where $U(1)_{45}$ and $U(1)_{89}$ denote the rotations in $x^4, x^5$ and $x^8, x^9$ respectively. Using dimensional analysis, the charges (2.18) and the fact that $\epsilon$ and $f_\zeta$ do not carry any charge, we obtain the following expansions to first order in $m_\lambda$

$$\epsilon(m_\lambda) = C_\epsilon m_\lambda (R)(\Lambda_1)^3 + c.c. + O((m_\lambda)^2) = 2C_\epsilon \cos(\theta_{p,k}/N_c)|m_\lambda|(R)|\Lambda_1|^3 + \cdots$$

$$f_\zeta(m_\lambda) = C_{f_\zeta} m_\lambda (R)^4(\Lambda_1)^3 + O((m_\lambda)^2),$$

(2.19)

where

$$\theta_{p,k} = \theta_p + 2\pi k = \theta + N_c \arg(m_\lambda) + 2\pi k.$$  

(2.20)

A priori the real constants $C_\epsilon$ and $C_{f_\zeta}$ can depend on $N_c$.

The classical Nambu-Goto action of the M5 brane suggests that the area of $\Sigma$ plays a role of a potential energy \[14\]. The energy (per unit volume) of the configuration is related to the area of the five-brane by $E = \text{Area}/(l_p)^6$. The area of $\Sigma$ is given by

$$\text{Area} = \int \Sigma d^2z \left\{ G_{ij}(\partial x^i \bar{\partial} x^j + \bar{\partial} x^i \partial x^j) + G_{77}\partial x^7 \bar{\partial} x^7 \right\}$$

(2.21)

where $(i, j) \in \{v, w, t\}$. However, since $\Sigma$ is non compact the area is infinite. We therefore have to define a notion of a regularized area. This will be done by subtracting the infinite area of the holomorphic (supersymmetric) curve

$$\text{Area} = \text{Area}|_{\epsilon} - \text{Area}|_{\epsilon=0}. \quad (2.22)$$

With this definition the Area of a holomorphic curve vanishes giving zero energy for the supersymmetric vacua. Computing (2.22) to first order in $m_\lambda$ we obtain for the vacuum energy

$$E_k(\theta, N_c, m_\lambda, \Lambda_1) = V_1 |m_\lambda||\Lambda_1|^3 \cos \left( \frac{\theta_p + 2\pi k}{N_c} \right) + O(|m_\lambda|^2),$$

(2.23)

where $k = 0, \ldots, N_c - 1$ and $V_1$ is given by

$$V_1 = \int_0^{+\infty} x dx \left\{ \frac{C_\epsilon}{\pi} \frac{1}{x^2} + \frac{C_{f_\zeta}}{2\pi} \frac{2}{x^4} \right\}, \quad (2.24)$$

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and $x = |z|/|\Lambda_1|$ is a dimensionless parameter.

The true vacuum is obtained by minimization of (2.23) with respect to $k$. The result (2.23) agrees with the general analysis of ref. [5] and exhibits the same qualitative behaviour and in particular the same $\theta$ dependence as the field theory result (2.9). The overall coefficient is different. In the brane computation it depends explicitly on $R$, and furthermore $V_1$ is divergent and requires a suitable regularization. However, it is by now clear that the eleven dimensional supergravity description that we are using is not capable of a reliable computation of the numerical coefficients [14,15]. Note that the result (2.23) interpolates smoothly between the SYM and ordinary Yang-Mills.

2.3. Brane computation of QCD string tension

A QCD string that carries $l \ mod \ N_c$ units of flux is realized in the brane framework as a curve connecting two points on $\Sigma$, $z_0$ and $z_0 \exp[2\pi il/N_c]$ [12]. The QCD string tension is given by the minimal length between two such points. The distance between the two points on $\Sigma$ is given by

$$\text{dist}(z_0) = \sqrt{2G_{v\bar{v}}|\Delta v|^2 + 2G_{w\bar{w}}|\Delta w|^2 + 2G_{t\bar{t}}|\Delta t|^2 + G_{7\bar{7}}(\Delta x_7)^2}.$$

(2.25)

Computing the distance and taking the minimum with respect to $z_0$ we obtain the tension of the $l$-string as $T_l = \text{dist}(z_{0,\text{min}})/(l_p)^3$:

$$T_l = \frac{2}{l_p} \sqrt{\frac{|\xi|}{\pi} \sin(l\pi/N_c)} \left(1 + \frac{(l_p)^2}{2\pi} \frac{\pi}{|\zeta|/2\pi R^2} \right) + O(\epsilon^2).$$

(2.26)

To first order in $m_\lambda$ we get

$$T_l = \frac{2}{l_p} \sqrt{\frac{|\zeta_{\text{susy}}|}{\pi} \sin(l\pi/N_c)} \left(1 + \frac{1}{2} m_\lambda |\Lambda_1|^3 (R)^2 \left[C_{f_\xi}(R)^2 + \frac{1}{\pi} \left(\frac{l_p}{R}\right)^2 C_\epsilon \cos(\theta_{p,k}/N_c)\right]\right) + O(|m_\lambda|^2).$$

(2.27)

Unlike the vacuum energy, now we have no field theory result to compare with.

\footnote{For other work on the QCD string in the brane picture see [16,17,18,19].}
2.4. Large $N_c$ limit

Consider now the large $N_c$ limit. Since we do not know the behaviour in the large-$N_c$ limit of the higher order terms in the expansion in $m_\lambda$, we will make the assumption that these terms are not of higher order in $N_c$. The analysis of \[12\] in the $N = 1$ supersymmetric case gave $R \sim O(1/N_c)$ and $\zeta_{\text{susy}} \sim O((N_c)^2))$. Together with \(2.14\) we get $C_\zeta \sim O(1)$. Since we do not know the large $N_c$ behaviour of $\zeta$ when supersymmetry is broken we assume that $\zeta \sim \zeta_{\text{susy}} \sim O((N_c)^2)$. This implies by \(2.19\) that $C_{f_\zeta} \leq O((N_c)^2)$.

One expects for the vacuum energy that \[20,11\]

$$E_k(\theta) = C \left( \theta_p + 2\pi k \right)^2 + O(1/N_c), \quad (2.28)$$

with the coefficient $C$ being a constant independent of $N_c$. Since $m_\lambda$ and $(\Lambda_1)^3$ are proportional to $N_c$ in the large-$N_c$ limit, eq. \(2.23\) and \(2.24\) imply that $V_1 \sim O(1)$. Assuming a suitable regularization of $V_1$ it implies $C_\epsilon \sim O(1)$ while the second term in $V_1$ vanishes in the limit. Note also that $\epsilon(m_\lambda) \sim O(1)$.

The implication of the above discussion to the large $N_c$ behaviour of the QCD string tension eq. \(2.27\) is that the $m_\lambda$ term within the square brackets of this equation is $O(1)$.

3. Softly Broken SQCD

3.1. Effective Field Theory of SQCD

Consider super-QCD with gauge group $SU(N_c)$ and $N_f$ flavours. For $0 < N_f < N_c$ we consider the Taylor-Veneziano-Yankielowicz (TVY) effective action \[21\] where the mesonic matter superfields $T$ all have equal (complex) mass $m_f$ and supersymmetry is softly broken by a gaugino mass term

\[
\mathcal{L} = \frac{9}{\alpha} (\bar{S}S)^{1/3}_D + \frac{1}{\gamma} (T^i_j T^j_i) (\bar{S}S)^{-1/3}_D + \left[ S \log \left( \frac{S^{N_c-N_f} \det T}{\Lambda_1^{3(N_c-N_f)}} - (N_c - N_f) - i\theta' \right) \right]_F + h.c. + \left[ m_f T + h.c. \right] + \left[ m_\lambda S_{\theta=0} + h.c. \right],
\]

where

$$\theta' = \theta \left( 1 - \frac{N_f}{3N_c} \right). \quad (3.2)$$

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In this case

\[
(\varphi)_{s\text{usy}, k} = |\Lambda_1|^{3-N_f/N_c} |m_f|^{N_f/N_c} \exp \left( \frac{\theta' + N_f \arg(m_f) + 2\pi k}{N_c} \right),
\]

(3.3)

with \( k = 0, \cdots, N_c - 1 \), and the vacuum energy (per unit volume) is given by

\[
E_k(\theta, N_c, N_f, m_\lambda, m_f, \Lambda_1) \simeq -2|m_\lambda| |\Lambda_1|^{3-N_f/N_c} |m_f|^{N_f/N_c} \cos \left( \frac{\theta_p + 2\pi k}{N_c} \right),
\]

(3.4)

where

\[
\theta_p = \theta' + N_f \arg(m_f) + N_c \arg(m_\lambda).
\]

(3.5)

When \( N_F \geq N_c \) extra baryonic superfields must be added to the TVY effective action. Notice that in the regime we are considering the energy of the vacuum configuration depends only on the supersymmetric expectation value of \( \varphi \) (3.3) (see the discussion in section 2.1). We therefore expect that the vacuum energy is still given by (3.4). As we will see this is indeed the result of the brane computation.

3.2. Brane Computation of Vacuum Energy

We will now add massive matter in the fundamental representation of the gauge group. For simplicity we assume that all \( N_f \) matter multiplets have the same mass \( m_f \). \( \Sigma \) is given by

\[
v = z + \frac{\epsilon}{z}, \quad w = \frac{\zeta}{z},
\]

\[
t = \frac{z^{N_c}}{(z - z_-)^{N_f}}, \quad x^7 = 2\sqrt{\epsilon} \log \left| \frac{z}{\Lambda_1} \right|.
\]

(3.6)

In the supersymmetric case (\( \epsilon = 0 \)) the parameters of the curve are related to the physical parameters by

\[
\zeta_{\text{susy}} = C_\zeta \frac{(l_p)^2}{R} (\Lambda_1)^{3-N_f/N_c} (m_f)^{N_f/N_c}
\]

\[
z_{\text{susy}} = -m_f.
\]

(3.7)

When \( \epsilon \neq 0 \) supersymmetry is softly broken. As discussed in ref. [22], this soft breaking corresponds in field theory to introducing only a gaugino mass \( m_\lambda \). The parameters in \( \Sigma \) can be written as

\[
\zeta(m_\lambda) = \zeta_{\text{susy}} (1 + f_\zeta(m_\lambda))
\]

\[
z_-(m_\lambda) = -m_f (1 + f_{z_-}(m_\lambda)),
\]

(3.8)
where \( f_\zeta(m_\lambda), f_{z-}(m_\lambda) \) and \( \epsilon(m_\lambda) \) vanish when \( m_\lambda \) does.

Using the charge assignments

| \( m_\lambda, m_f \) | \( U(1)_{45} \) | \( U(1)_{89} \) |
|-----------------|-------|-------|
| \( m_\lambda \) | -2    | -2    |
| \( m_f \)      | 2     | 0     |
| \( \Lambda_1^{3N_c-N_f} \) | \( 2N_c - 2N_f \) | \( 2N_c \) |
| \( R \)        | 0     | 0     |

(3.9)

and the fact that \( \epsilon, f_{z-} \) and \( f_\zeta \) do not carry any charge, we obtain the following expansions to first order in \( m_\lambda \)

\[
\begin{align*}
\epsilon(m_\lambda) &= 2C_\epsilon(R)^2|m_\lambda||\Lambda_1|^{3-N_f/N_c}|m_f|^{N_f/N_c}\cos\left(\frac{\theta_p + 2\pi k}{N_c}\right) + O(|m_\lambda|^2) \\
f_\zeta(m_\lambda) &= C_{f_\zeta}m_\lambda(R)^4(\Lambda_1)^{3-N_f/N_c}|m_f|^{N_f/N_c} + O((m_\lambda)^2) \\
f_{z-}(m_\lambda) &= C_{f_{z-}}m_\lambda(R)^4(\Lambda_1)^{3-N_f/N_c}|m_f|^{N_f/N_c} + O((m_\lambda)^2),
\end{align*}
\]

(3.10)

where

\[
\theta_p = \left(1 - \frac{N_f}{3N_c}\right)\theta + N_c\text{arg}(m_\lambda) + N_f\text{arg}(m_f).
\]

(3.11)

The computation of the energy (per unit volume) is done as in softly broken SYM. We get to order \( O(m_\lambda) \)

\[
E_k(\theta, N_c, N_f, m_\lambda, m_f, \Lambda_1) = V_2|m_\lambda||\Lambda_1|^{3-N_f/N_c}|m_f|^{N_f/N_c}\cos\left(\frac{\theta_p + 2\pi k}{N_c}\right),
\]

(3.12)

with \( k = 0, \ldots, N_c - 1 \) and

\[
V_2 = \int_0^{+\infty} xdx \left\{ \frac{1}{\pi} \frac{C_\epsilon}{x^2} + C_{f_\zeta}(C_\zeta)^2(R)^4|\Lambda_1|^{4-2N_f/N_c}|m_f|^{2N_f/N_c} \frac{2\pi}{x^4} \right. \\
- \left. C_{f_{z-}}(N_f)^2 \left( \frac{R}{l_p} \right)^6 \frac{|\Lambda_1|}{|m_f|} \frac{2\pi}{(1 - x^2)^2} \right\}.
\]

(3.13)

Comparing (3.12) and the field theory results (3.4) we see that they have the same qualitative behaviour and \( \theta \) dependence.

Taking the limit \( m_f \to \infty \) while keeping \( (\Lambda_1)^3 = (\Lambda_1)^{3-N_f/N_c}(m_f)^{N_f/N_c} \) fixed (with \( N_f < 3N_c \)) and rescaling appropriately the integration variables, one decouples all the matter fields and obtains the results of the previous section, as expected.
4. Discussion and Conclusions

We computed, in the supergravity approximation, the vacuum energy of $N = 1$ SYM and SQCD with massive matter, softly broken by a gaugino mass using their realization via the fivebrane of M theory. The results we obtained matched qualitatively the field theory expectations. In particular the $\theta$ dependence and the physics associated with it were in agreement. This supports the hope that the field and brane theories are in the same universality class. Our analysis is valid for finite $N_c$ with soft supersymmetry breaking parameter $m_\lambda$. Note in comparison, that in [1] the analysis was carried out for large $N_c$ and hard supersymmetry breaking.

We also computed the QCD string tension via the fivebrane realization. It would be interesting to perform a field theory computation to see if there is a qualitative agreement with the brane result of the $\theta$ angle dependence.

Another interesting issue is the decay rate of the false vacuum. In the semiclassical approximation it is given by

$$\Gamma \sim \frac{1}{|\Lambda_1|^4} \exp \left[ -\frac{27}{2} \pi^2 \frac{(T_D)^4}{|\Delta E|^3} \right],$$

where $T_D$ is the tension of the domain wall separating the two vacua and $\Delta E$ is the difference of their energies. Since $\Delta E$ is of order $m_\lambda$ while $(T_D)^4 = (T_{D,\text{susy}})^4 + 4m_\lambda T_{D,\text{broken}}(T_{D,\text{susy}})^3 + O((m_\lambda)^2)$, the dominant term in the decay rate for small $m_\lambda$ is $T_{D,\text{susy}}^4$. Therefor the decay rate predicted by field theory and the brane theory agree qualitatively and have same $\theta$ dependence. It would also be interesting to see whether the agreement continues to hold when including the $O(m_\lambda)$ correction to the domain wall tension. In this case, both the field theory and the brane theory results are not known. In order to get the decay rate (4.1) purely from brane considerations we need to use the realization of the domain wall connecting two adjacent vacua as a three manifold interpolating between the two dimensional real manifolds $\Sigma, \Sigma'$ [24].

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