Adaptive Subcarrier, Parameter, and Power Allocation for Partitioned Edge Learning Over Broadband Channels

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Abstract

Recent years have witnessed growing interests in the deployment of AI algorithms at the network edge to exploit distributed data and computation resources, known as edge learning. Among others, a main paradigm, called partitioned edge learning (PARTEL), supports the distributed training of a large-scale AI model by dynamically partitioning the model and allocating the resultant parametric blocks to different devices for updating. Then devices upload the updates to a server where they are assembled and applied to updating the model. The two steps are iterated till the model converges. The parameter allocation provides a mechanism for controlling computation-and-communication (C2) loads. In this work, we consider the efficient joint management of parameter allocation and radio resources to reduce the learning latency of PARTEL, when deployed in a broadband system using orthogonal frequency-division multiplexing (OFDM). Specifically, the policies for joint subcarrier, parameter, and power allocation (SUPPORT) are optimized under the criterion of minimum latency. Two cases are considered. First, for the case of decomposable models (e.g., logistic regression or support vector machine), the latency-minimization problem is a mixed-integer program and non-convex. Due to its intractability, we develop a practical solution by 1) relaxing the binary subcarrier-assignment decisions and 2) transforming the relaxed problem into a convex problem of model size maximization under a latency constraint nested in a simple search for the target model size. By deriving the properties of the convex problem, a low-complexity algorithm is designed to compute the SUPPORT policy. Second, consider the case of convolutional neural network (CNN) models which can be trained using PARTEL by introducing some auxiliary variables. This, however, introduces constraints on model partitioning reducing the granularity of parameter allocation. The preceding policy is extended to CNN models by applying the proposed techniques of load rounding and proportional adjustment to rein in latency expansion caused by the load granularity constraints. Finally, experiments using real data show that joint SUPPORT can substantially reduce the latency of PARTEL for decomposable models (e.g., 31%) and CNN models (e.g., 42%).

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I. INTRODUCTION

Deploying machine learning algorithms at the network edge enables low-latency access to distributed data and computation resources, resulting in an active research area called edge learning [1]. Among a range of frameworks developed in this area, partitioned edge learning (PARTEL) considered in this work supports the efficient training of a large-scale model using distributed computation resources at mobile devices [2], [3]. To this end, the model is partitioned so that different parts are allocated to devices for updating (i.e., updating their assigned blocks of model parameters) [2]. The learning process iterates between the uploading of updated model parts from devices to a server and the broadcasting of the assembled updated model from the server to devices till the model converges. It is desirable to reduce the learning latency to reduce channel uses, exploit potentially transient computation resources at devices due to their mobility or multi-tasking, and learn to respond to “black-swan”-type events. In this work, we consider the deployment of PARTEL in a practical broadband system (e.g., LTE and 5G) and minimize its total latency by jointly managing communication resources (subcarriers and power) and computation loads (parameters requiring updating). The corresponding joint subcarrier, parameter, power allocation (SUPPORT) under the criterion of latency minimization is the focus of this work.

A. Edge Machine Learning

There exist two paradigms in distributed learning: data parallelism and model parallelism. The former refers to the simultaneous training of an AI model at multiple devices using different parts of a global dataset. On the other hand, model parallelism refers to the simultaneous training of different parts of a model at different devices. Recent research on edge learning focuses on the efficient implementation of different frameworks under these two paradigms.

1) Federated Edge Learning(FEEL): FEEL is a popular data-parallelism framework that aims at exploiting distributed mobile data while preserving privacy by avoiding sharing data [4]. Implementing the classic iterative algorithm of stochastic gradient descent (SGD), FEEL comprises multiple iterations, each of which is called a communication round. In each round, the latest global model (or equivalently its parametric set) is broadcast by a server to devices for updating; the updated models (or stochastic gradients) using local data, called local updates, are uploaded by devices to the server for aggregation to update the global model. As a typical model comprises millions to billions of parameters, the high-dimensionality of local updates transmitted by many devices over the air causes a communication bottleneck. The main research
focus in FEEL is to overcome the bottleneck by communication-efficient designs that integrate learning and wireless transmission techniques. One approach is to design simultaneous access techniques, called over-the-air computation (AirComp), that supports fast aggregation of local updates over-the-air by exploiting the waveform superposition property of a multi-access channel [5]–[7]. An alternative approach is to design vector quantization techniques for compressing high-dimensional stochastic gradients (see e.g., [8]). Besides, to reduce the communication overhead, a split learning based approach is proposed in [9], where one part of model is trained on devices and the other part is trained on server and the communication load depends on the output dimension rather than the model size. Another vein of research attempts to improve the communication efficiency of FEEL systems by radio resource management (RRM), such as multiuser scheduling [10], [11], bandwidth allocation [12], and their joint design [13], [14].

The execution of a computation intensive learning task on energy constrained devices poses another challenge for practical FEEL. Hence, another research focus in FEEL is to design energy-efficient techniques such as jointly controlling the allocation of radio resources and computation loads as well as devices’ clock frequencies [15]–[17].

2) PARTEL: The goal of PARTEL, which is a representative model-parallelism framework and also known as “parameter server” in computer science, is to leverage distributed computation resources at devices to train a large-scale model [2], [3]. To this end, the model is partitioned and its parts are allocated to different devices for updating using downloaded datasets. Implementing a SGD related algorithm called block coordinate descent, PARTEL exhibits a similar process as FEEL except for two main differences [3]. First, each device in PARTEL updates only a part of the model instead of the full model as in FEEL. Second, devices in PARTEL, which contribute computation resources but not necessarily data, usually download training datasets from the server instead of generating them locally as in FEEL.

In each round of PARTEL, a server partitions a global model under training into blocks of parameters, called parametric blocks, and allocate each of them to a single device for updating. The block sizes determine the computation-and-communication (C2) loads of individual devices. The possibility of controlling the sizes gives rise to a new research issue unique for PARTEL, namely C2 load allocation by model partitioning. Recently, it was proposed in [2] that C2 load allocation is jointly designed with bandwidth allocation for PARTEL over frequency non-selective channels. Such a joint design was found to effectively equalize the heterogeneity of channel states and devices’ computation capacities. This can substantially reduce the learning
latency under the constraint of synchronized updates [2]. However, the existing design cannot be straightforwardly extended to practical broadband systems where joint C2 control is more challenging due to frequency selectivity.

3) **PARTEL versus FEEL:** The differences in terms of goals and design of FEEL and PARTEL result in their having unique advantages and disadvantages from an implementation perspective. First, PARTEL is more communication efficient than FEEL since the former requires each device to upload its update on only a part of the model instead of the whole model in the latter. Consequently, the computation loads for devices participating in PARTEL are also lighter than those in FEEL. This is the main reason that PARTEL can support large-scale model training over many resource limited devices. On the other hand, though model partitioning in PARTEL is straightforward in the case of decomposable loss functions (e.g., logistic regression), the operation faces difficulty when the function has no such a property [e.g., a convolutional neural network (CNN)]. For CNN models, the realization of PARTEL requires the technique of introducing a set of auxiliary variables for the models so as to transform the loss function into a decomposable form [18], [19]. As a result, the number of communication rounds are doubled due to the need of training the auxiliary variables and the computation loads at devices are increased.

**B. Radio Resource Management for Broadband Systems**

Modern communication systems supporting high rates (e.g., LTE and 5G) are usually operated over broadband channels. In such systems, **orthogonal frequency-division multiplexing (OFDM)** is widely adopted to partition a spectrum into subcarriers to simplify signal processing and facilitate resource allocation. In the area of broadband RRM, extensive research has been conducted on the joint control of subcarrier allocation to multiple users, transmit power, and data rates [20]–[24].

In the pioneering work in [20], a scheme integrating subcarrier, data bit, and power allocation was proposed to minimize the total transmit power while guaranteeing the data rates requested by users in a single-cell uplink system. The key feature of the practical solution method developed in [20] for a NP-hard optimization problem is the relaxation of binary decisions on assigning individual subcarriers to users. This technique is also adopted in the design of the mentioned SUPPORT scheme in the current work. The design approach in [20] was further developed in [21] for a multi-cell system. Another vein of broadband RRM is to enhance the system throughput via adaptive transmit power control for subcarriers (see e.g., [22]). Recently, the broadband RRM techniques have been extended to more complex systems including relay systems (see e.g., [23]), full duplex systems (see e.g., [24]), and wirelessly powered systems (see e.g., [25]).
While prior work targets conventional systems providing radio-access services, the current work explores the new direction of integrating broadband RRM with computation load balancing targeting a new type of systems, namely PARTEL systems. Among others, the current design for C2 recourse management has two differences from its conventional counterparts. First, the former needs to account for not only channel states as the latter but also devices’ computation capacities. Furthermore, the load allocation in the former is more sophisticated than bit allocation in the latter as it involves model partitioning and has to address relevant issues such as model decomposability. Second, the constraint of synchronized updates in PARTEL tends to bias the RRM to favor devices with relatively weak channels and/or computation capacities. The opposite holds for the conventional systems aiming at throughput maximization (see e.g., [22]). These differences give rise to new design challenges for broadband RRM.

C. Summary of Contributions

In this paper, we consider a single cell broadband system where a server coordinates multiple devices, called workers, to train a model distributively based on PARTEL and using a downloaded dataset. The server communicates with devices over broadband channels based on OFDM. The computation capacity of each worker is characterized by an adjustable computation speed, and a computation power factor defined as the additional power consumption per unit increase in speed. It is assumed that the server has the knowledge of channel states of all links and the workers’ computation capacities based on feedback. In each round of PARTEL, the resources in the system are managed by the server via the following operations:

1) **Parameter Allocation**: The server controls the size of a parametric block allocated to each worker via adaptive model partitioning;

2) **Subcarrier Allocation**: The server allocates subcarriers to workers for their uploading of block updates;

3) **Power Allocation**: The server controls the computation speed and transmit power of each device over its assigned subcarriers.

The above operations are jointly controlled, termed joint SUPPORT, to minimize the latency for training a given model, or equivalently the per-round latency under the constraints of synchronized updates and per-worker power. To this end, this work focuses on optimizing the polices for joint SUPPORT. We consider both the cases of a decomposable model and a CNN model. In the latter case, the model is made decomposable using the mentioned method of
auxiliary variables so as to be trained by PARTEL \cite{18}, \cite{19}. As a result, each round is divided into two mini-rounds: one for updating the model parameters and the other for updating the auxiliary variables. The convolutional architecture of the model introduces additional constraints that reduce the granularity of C2 load allocation. Specifically, in the mini-round for updating model parameters, the load has to be a multiple of the number of parameters associated with a single neuron. In the other mini-round, each device is required to update the full set of auxiliary variables; its computation load is controlled by varying the number of samples used in the update while communication load is fixed.

It should be emphasized that the scope of the current contributions is limited to new resource-management techniques without new learning algorithms. In particular, an existing PARTEL algorithm is adopted, for which the model is assumed to converge within a finite number of rounds (see e.g., \cite{2}, \cite{3}). Under this assumption, the proposed joint SUPPORT attempts to minimize the learning latency in terms of seconds.

The main contributions are summarized as follows.

1) Joint SUPPORT for Decomposable Models: Consider decomposable models [e.g., support vector machine (SVM)]. The problem of latency minimization by joint SUPPORT is an integer program and intractable. A practical solution approach is developed using two techniques. The first is a relaxation of binary subcarrier assignment decisions. The second is the transformation of the relaxed problem into a convex problem of model size maximization under a latency constraint, which is nested in a simple search for the target model size. Considering the convex problem, the properties of three optimal resource-management operations are analyzed and then applied to design an efficient algorithm for computing the desired SUPPORT policy. Via analysis, it is found that the optimal number of parameters assigned to a worker for updating avoids high power consumption due to overloading. For this reason, the optimal number is derived to be a concave function of its speed and a monotonic decreasing function of its computation power factor. On the other hand, the optimal subcarrier assignment and power allocation over assigned subcarriers are found to favor high channel gains as expected.

2) Joint SUPPORT for CNN Models: Consider the case of CNN models. The optimization problems for joint SUPPORT in both mini-rounds are shown to have the same form as that in the preceding case except for additional load granularity constraints. This allows the extension of the joint SUPPORT policy for the case of decomposable models to the current case by rounding down the obtained loads to meet the granularity constraints. Furthermore, the remaining parameters
due to rounding are allocated over devices and subcarriers proportionally with their rounded loads, thereby reining in the latency expansion caused by the additional constraints.

II. MODELS AND METRICS

A. System Model

A single cell OFDM system is considered, as illustrated in Fig. [1](a). In the cell, there are one server equipped with a single-antenna access point (AP) and $K$ single-antenna edge devices, serving as workers. Each worker performs one task assigned by the server. The server is connected to workers via wireless links. The system bandwidth is divided into $N$ orthogonal subcarriers. The bandwidth of each subcarrier is denoted as $B$. The frequency-selective fading channel is considered, where different subcarriers will experience different channel gains. We assume that the AP has the channel state information (CSI) of all links that are useful for subcarrier allocation. Besides, the channel gains are assumed to be static in one training iteration but vary over different iterations. The uplink channel gain of worker $k$ on the subcarrier $n$ is denoted as $h_{k,n}$. We denote $\{C_{k,n}\}$ as the subcarrier allocation indicators. If the $n$-th subscriber is allocated to worker $k$, then $C_{k,n} = 1$. Otherwise, $C_{k,n} = 0$.

B. Learning Models

1) Decomposable Models: The large-scale learning tasks with decomposable objective functions (such as logistic regression) can be directly implemented using PARTEL based on the method of block coordinate descent. According to the literature (e.g., [2]), a decomposable objective function can be written as

$$\mathcal{L}(w) = \mathcal{F}(w) + \mathcal{R}(w),$$

Figure 1: PARTEL system, operations, and latencies.
where \( \mathbf{w} = \{w_1, w_2, \ldots, w_L\}^T \) is the parameter vector of the learning model, \( L \) is the size of \( \mathbf{w} \), \( \mathcal{F}(\mathbf{w}) \) is the loss function, and \( \mathcal{R}(\mathbf{w}) \) is the regularized function (e.g., \( L_1 \) regularization used to increase sparsity and \( L_2 \) regularization used to reduce overfitting). Specifically, the loss function can be written as
\[
\mathcal{F}(\mathbf{w}) = \frac{1}{M} \sum_{m=1}^{M} |y_m - \varphi(\mathbf{w}; \mathbf{x}_m)|^2,
\]
where \( M \) is the size of the dataset, \( \{\mathbf{x}_m, y_m\} \) is the \( m \)-th data sample, and \( \varphi(\cdot) \) is a smooth inference function. The regularized function is a block-separable function, given as \( \mathcal{R}(\mathbf{w}) = \sum_{i=1}^{L} \psi(w_i) \), where \( w_i \) is the \( i \)-th element of \( \mathbf{w} \) and \( \psi(\cdot) \) is the norm (e.g., \( L_1 \) or \( L_2 \) norm). During the training, the smoothness of \( \mathcal{R}(\cdot) \) decides the method to update the learning model. If \( \mathcal{R}(\cdot) \) is smooth, gradient descent algorithm is used. Otherwise, another method called proximal gradient descent, is used.

2) **CNN Models:** CNN models can not be directly implemented using PARTEL, as the nested layers therein make the gradient elements of different layers dependent. To make PARTEL feasible and efficient for CNNs, in the sequel, the method of auxiliary variables is used to decompose the CNN models into many independent subproblems [18], [19].

First, consider a CNN model with \( G \) hidden layers. The model parameter matrix is denoted as \( \mathbf{W} \) with the size of \( L \) parameters. For an arbitrary layer therein, say layer \( g \), the parameter matrix is denoted as \( \mathbf{W}_g \), the number of neurons is denoted as \( I_g \), and the \( i \)-th neuron parametric vector is denoted as \( \mathbf{w}_{g,i} \). Thereby, the objective function of the CNN model is given by
\[
\mathcal{L}(\mathbf{W}) = \sum_{m=1}^{M} |y_m - \mathcal{F}(\mathbf{x}_m; \mathbf{W})|^2,
\]
with \( \mathcal{F}(\mathbf{x}; \mathbf{W}) = f_{G+1}(\ldots f_2(f_1(\mathbf{x}; \mathbf{W}_1); \mathbf{W}_2); \ldots; \mathbf{W}_{G+1}) \),
where the model parameter matrix can be expressed as \( \mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_G, \mathbf{W}_{G+1}] \), the parameter matrix of the \( g \)-th layer can be expressed as \( \mathbf{W}_g = [\mathbf{w}_{g,1}, \mathbf{w}_{g,2}, \ldots, \mathbf{w}_{g,I_g}] \), and \( f_g(\mathbf{x}; \mathbf{W}_g) \) is the set of output (activation) functions of the \( g \)-th layer.

**Auxiliary Variables:** The method of auxiliary variables is used by introducing one auxiliary variable per neuron per data sample: \( z_{g,i,m} = f(\mathbf{w}_{g,i}; \mathbf{z}_{g-1,m}) \), \( \forall (g,i,m) \), where \( f(\cdot) \) is the activation function, \( \mathbf{w}_{g,i} \) is the \( i \)-th neuron parametric vector in layer \( g \), \( z_{g,i,m} \) is the auxiliary variable introduced for the \( i \)-th neuron in layer \( g \) regarding data sample \( m \), \( z_{g-1,m} = [z_{g-1,1,m}, z_{g-1,2,m}, \ldots, z_{g,I_{g-1},m}]^T \) is the auxiliary variable vector for the layer \((g-1)\) regarding data sample \( m \), and \( I_{g-1} \) is the number of neurons in the \((g-1)\)-th layer. For an arbitrary data sample, say the \( m \)-th, the corresponding auxiliary matrix for the whole model is denoted as \( \mathbf{Z}_m = [\mathbf{z}_{1,m}, \ldots, \mathbf{z}_{g,m}, \ldots, \mathbf{z}_{G,m}] \), called **per-sample auxiliary matrix**. Then the overall auxiliary matrix for all samples are denoted as \( \mathbf{Z} = [\mathbf{Z}_1, \ldots, \mathbf{Z}_m, \ldots \mathbf{Z}_M] \).
Decomposed Optimization: Following [18], [19], by using the quadratic-penalty method, the problem in (2) is equivalent to minimizing

\[
L_Q(W; Z; \mu) = \sum_{m=1}^{M} |y_m - f_{G+1}(z_{G,m}; W_{G+1})|^2 + \mu \sum_{g=1}^{G} \sum_{m=1}^{M} \|z_{g,m} - f_g(z_{g-1,m}; W_g)\|^2 ,
\]  

(3)

where \(z_{0,m} = x_m\) and \(\mu \rightarrow +\infty\). In (3), the nested structure among layers is decoupled. Consequently, the gradients of any two parameters (or auxiliary variables) are independent.

Finally, the problem in (3) can be solved using the alternating optimization over \(W\) and \(Z\), i.e., sequentially solving the \(W\)-stage and \(Z\)-stage, defined below, in each training iteration.

- **W-stage**: Fixing the values of \(Z\), solve the problem of \(\min_W L_Q(W; Z; \mu)\), in which the problem of each neuron is independent and can be written as

\[
\min_{w_{g,i}} \sum_{m=1}^{M} |z_{g,i,m} - f(w_{g,i}; z_{g-1,m})|^2 , \quad \forall (g, i),
\]

(4)

where \(w_{g,i}\) and \(z_{g,i,m}\) are the parametric vector and auxiliary variable of the \(i\)-th neuron in the \(g\)-th layer, respectively, \(z_{g-1,m}\) is the auxiliary variable vector of the \((g-1)\)-th layer. Note that one device is allocated a task of updating one or more neuron parametric vectors by solving the subproblems in (4).

- **Z-stage**: Conditioned on the values of \(W\), solve the problem of \(\min_Z L_Q(W; Z; \mu)\), where the problem of optimizing each per-sample auxiliary matrix is independent of others and is given as

\[
\min_{Z_m} |y_m - f_{G+1}(W_{G+1}; z_{G,m})|^2 + \mu \sum_{g=1}^{G} \|z_{g,m} - f_g(W_g; z_{g-1,m})\|^2 , \quad \forall m,
\]

(5)

where \(Z_m\) is the per-sample auxiliary matrix corresponding to data sample \(m\). The size of the per-sample auxiliary matrix is \(\sum_{g=1}^{G} I_g\) with \(I_g\) being the number of neurons in layer \(g\). Note that one device is allocated a task of updating one or more per-sample auxiliary matrices by solving the subproblems in (5).

C. PARTEL Architecture

Consider the PARTEL system and operations in Fig. 1 that are elaborated as follows.

I) Decomposable Models: The model-parameter vector is partitioned into \(K\) disjoint parametric blocks, as \(w = \{w_1, ..., w_k, ..., w_K\}\), where \(w_k\) is allocated to worker \(k\) for update, using
a downloaded dataset\(^1\). One main benefit of PARTEL is low learning latency, as each resource-constrained worker is required to calculate and transmit the gradient or proximal gradient of only a parametric block instead of the whole parameter vector during each iteration \(^2\).

In the PARTEL framework, one training iteration of the decomposable models is called \emph{one (communication) round}. As shown in Fig. 1(b), there are three phases in each round, as follows.

- \textit{Push Phase}: The server broadcasts the whole model-parameter vector, \(w\), to all workers.
- \textit{Computation Phase}: Each worker computes the update (e.g., gradients or proximal gradients) of its allocated parametric block.
- \textit{Pull Phase}: All workers upload the updates of their corresponding parametric blocks to the server. The server updates the whole parameter vector.

The training process in Fig. 1(b) iterates when all parametric blocks are updated in the round, i.e., the tasks of all workers are synchronized in each round.

2) CNN Models: As mentioned, each round of CNN models comprises two stages: W-stage and Z-stage, described as follows.

- \textit{W-stage}: The parameter matrix \(W\) is divided into \(K\) blocks, with each being updated by one worker. To avoid inter-communication among different workers, the following load-granularity constraint is applied.

\textbf{Granularity Constraint 1} (Neuron Allocation for \(W\)-stage). In \(W\)-stage, each neuron parametric vector (e.g., \(w_{g,i}\)) defined in (4) should be allocated to one and only one worker.

- \textit{Z-stage}: The auxiliary matrix \(Z\) is divided into \(K\) blocks, with each being updated by one worker. To avoid inter-communication among workers, another load-granularity constraint is applied.

\textbf{Granularity Constraint 2} (Per-Sample Auxiliary Matrix Allocation for \(Z\)-stage). In \(Z\)-stage, each per-sample auxiliary matrix (e.g., \(Z_m\)) defined in (5) should be allocated to one and only one worker.

\textbf{Example 1}. Since the number of neurons in a CNN model and the data samples used for training are large, the sizes of each neuron problem and each per-sample auxiliary matrix problem are relatively small, compared with the whole learning tasks, making the model partitioning

\(^1\)The joint SUPPORT design of this paper can be easily extended to the case of partitioned dataset with multiple groups of workers (each with a data subset). Each group cooperatively updates a same block. The proposed joint SUPPORT can be applied in a hierarchical manner: applied for inter-group resource management and also applied for intra-group management.
meaningful. As an example, our experiments involve the CNN model “Lenet-5” trained on the MNIST dataset. A mini batch of 50 samples is used in each training iteration. In “Lenet-5”, there are 3 convolutional layers, including 142 feature maps in total. The first two convolutional layers are followed by a pooling layer and the last is followed by a fully connected layer with 84 neurons. In W-stage, the number of independent subproblems is \( I = 142 + 84 = 226 \). The size of each neuron problem is about \( \frac{1}{I} = \frac{1}{226} \) of the whole problem. In Z-stage, the size of each per-sample auxiliary matrix problem is \( \frac{1}{50} \) of the whole problem.

Each stage (W-stage or Z-stage) comprises three phases, push, computation, and pull, which are similar to those in the case of decomposable models. The main difference lies in the additional granularity constraint [1] or [2]. Each round comprises two stages and the rounds are repeated until the CNN model converges.

D. Latency and Energy Consumption Models

Consider an arbitrary communication round and an arbitrary worker, say worker \( k \). The latency and energy consumption models of each phase are described as below.

1) Push Phase: The push latency is the time for the server to broadcast the whole model-parameter vector to all workers. It is a constant identical for all workers. Besides, as the transmit power and bandwidth are very large during broadcasting, the push latency can be ignored. In this step, the energy consumption by all workers is to receive the model-parameter vector from the server and is included in the circuit energy consumption, denoted as \( \xi \).

2) Computation Phase: The computation latency of worker \( k \) depends on the size of the allocated parametric block \( L_k \) and its computation speed \( f_k \):

\[
T_{cmp}^k = \frac{L_k}{f_k}, \quad 1 \leq k \leq K,
\]

where \( f_k \) is measured by the number of parameters processed per second.

According to [26], the computation power of worker \( k \) is \( P_{cmp}^k = g_k f_k^3 \), where \( g_k \) is the computation power factor. Then, the computation energy of worker \( k \) is

\[
E_{cmp}^k = P_{cmp}^k \times T_{cmp}^k = g_k f_k^2 L_k, \quad 1 \leq k \leq K.
\]

3) Pull Phase: The pull phase consists of two parts. One is uploading gradient blocks from workers to the server. The other is the server updating the global model using the gradients sent by the workers. For the latter part, there is no energy consumption at the workers. Its latency,
denoted as $T_s$, is a constant and is same for all workers. In the sequel, we ignore the model update latency, $T_s$, as it is small and has no impact on the solution of latency minimization.

For uploading, worker $k$ transmits over a set of assigned subcarriers. We denote $T_{k,n}^{\text{com}}$ as the uploading latency of worker $k$ on subcarrier $n$. If subcarrier $n$ is not allocated to $k$, i.e., $C_{k,n} = 0$, $T_{k,n}^{\text{com}} = 0$. Otherwise,

$$T_{k,n}^{\text{com}} = \frac{L_{k,n} \tau}{R_{k,n}}, \quad \forall C_{k,n} = 1,$$  

(8)

where $L_{k,n}$ is the number of parameters uploaded by worker $k$ on subcarrier $n$, $\tau$ is the number of bits per gradient element, and $R_{k,n}$ is the channel capacity of worker $k$ on subcarrier $n$. The channel capacity is given by $\{R_{k,n} = B \log_2 \left(1 + \frac{P_{k,n}^{\text{com}} h_{k,n}}{\sigma^2}\right), \forall (k,n)\}$, where $B$ is the subcarrier bandwidth, $\sigma^2$ is the power of additive white Gaussian noise, $P_{k,n}^{\text{com}}$ is the transmit power, and $h_{k,n}$ is the channel gain of worker $k$ on subcarrier $n$, respectively. It follows that

$$P_{k,n}^{\text{com}} = \frac{(2^{R_{k,n} / B} - 1) \sigma^2}{h_{k,n}}, \quad \forall (k,n).$$  

(9)

Then, the overall uploading latency of worker $k$ is decided by the slowest subcarrier:

$$T_k^{\text{com}} = \max_n T_{k,n}^{\text{com}}, \quad 1 \leq k \leq K.$$  

(10)

The uploading energy consumption of worker $k$ is modeled as follows. Let $E_{k,n}^{\text{com}}$ denote the transmit energy consumption of worker $k$ on subcarrier $n$. If subcarrier $n$ is not allocated, i.e., $C_{k,n} = 0$, $E_{k,n}^{\text{com}} = 0$. Otherwise,

$$E_{k,n}^{\text{com}} = C_{k,n} P_{k,n}^{\text{com}} T_{k,n}^{\text{com}}, \quad \forall (k,n).$$  

(11)

By substituting the transmit power density $P_{k,n}^{\text{com}}$ in (9) and the uploading latency $T_{k,n}^{\text{com}}$ in (8), $E_{k,n}^{\text{com}}$ can be further derived as

$$E_{k,n}^{\text{com}} = C_{k,n} \frac{(2^{R_{k,n} / B} - 1) \sigma^2 L_{k,n} \tau}{h_{k,n} R_{k,n}}, \quad \forall (k,n).$$  

(12)

The total uploading energy consumption of worker $k$ is the sum of uploading energy consumption over all subcarriers: $\{E_k^{\text{com}} = \sum_{n=1}^{N} E_{k,n}^{\text{com}}, 1 \leq k \leq K\}$. By substituting $E_{k,n}^{\text{com}}$ in (12),

$$E_k^{\text{com}} = \sum_{n=1}^{N} C_{k,n} \frac{(2^{R_{k,n} / B} - 1) \sigma^2 L_{k,n} \tau}{h_{k,n} R_{k,n}}, \quad 1 \leq k \leq K.$$  

(13)

Next, the total latency and energy consumption of worker $k$ are defined as follows. The latency of worker $k$ is the sum latencies of the two phases:

$$T_k = T_k^{\text{cmp}} + T_k^{\text{com}}, \quad 1 \leq k \leq K,$$  

(14)
where $T_{k}^{cmp}$ is the computation latency defined in (6), $T_{k}^{com}$ is the uploading latency defined in (10). The energy consumption of worker $k$ is given by:

$$E_k = E_{k}^{cmp} + E_{k}^{com} + \xi, \quad 1 \leq k \leq K,$$

where $\xi$ is the constant circuit energy consumption when there is no computation and transmission, $E_{k}^{cmp}$ defined in (7) and $E_{k}^{com}$ defined in (13) are the computation and uploading energy consumption of worker $k$, respectively.

### III. Problem Formulation

We aim at minimizing the overall learning latency of the PARTEL system, which depends on two factors: the per-round latency and the number of rounds for model convergence. The overall learning latency is defined as the total latency of all rounds till model convergence. In [2] for narrowband channels, it is proved that the overall learning latency minimization is equivalent to separately minimizing the per-round latency. The result can also apply to the current case of broadband channels, as stated below.

**Lemma 1** (Equivalent Per-Round Latency Minimization [2]). The overall learning latency minimization is equivalent to separately minimizing the latencies for all rounds, as the distributed learning algorithms implemented using PARTEL are equivalent to the corresponding centralized ones in terms of convergence rate as measured by the required number of communication rounds. Specifically, for distributed learning, the values of updates (e.g., gradients and proximal gradients) calculated in each round and the number of rounds required for model convergence are independent of SUPPORT.

Using this result, we formulate the equivalent per-round latency-minimization problem. For an arbitrary round, we aim to minimize its latency, denoted as $T$, under the constraints on subcarrier assignment, latency requirement, parameter allocation, and power control, described as follows.

1) **Subcarrier Assignment Constraints:** Each subcarrier can be allocated to one worker:

$$\begin{cases} C_{k,n} \in \{0, 1\}, & \forall (k,n), \\ \sum_{k=1}^{K} C_{k,n} = 1, & 1 \leq n \leq N, \end{cases}$$

where $C_{k,n} = 1$ represents that the subcarrier $n$ is allocated to worker $k$. 
2) **Per-Round Latency Constraints:** As all parametric blocks should be updated in one round, all workers’ latencies, say \( \{T_k\} \), should not exceed the overall one-round latency \( T \):

\[
T_k \leq T, \quad 1 \leq k \leq K. \tag{17}
\]

As mentioned, \( T \) is the latency for an arbitrary round and can be different over different rounds. By substituting \( T_k \) in (14), the constraints in (17) can be derived as

\[
T_{\text{cmp}} + T_{\text{com}}^k \leq T, \quad 1 \leq k \leq K, \tag{18}
\]

which, by substituting the uploading latency \( T_{\text{com}}^k \) in (10), are equivalent to

(C2: Per-Round Latency Constraint) \( T_{\text{cmp}}^k + T_{\text{com}}^{k,n} \leq T, \quad \forall C_{k,n} = 1, \tag{19} \)

where \( T_{\text{cmp}}^k \) defined in (6) is the computation latency of worker \( k \) and \( T_{\text{com}}^{k,n} \) defined in (8) is the uploading latency of worker \( k \) on subcarrier \( n \).

3) **Parameter Constraints:** The parameter constraints are two tiers. On the one hand, the total updatable number of parameters by all workers should be no smaller than the size of the model:

(C3: Inter-Worker Parameter Constraint) \( \sum_{k=1}^{K} L_k \geq L, \tag{20} \)

where \( L_k \) is the size of the parametric block allocated to worker \( k \) and \( L \) is the size of the model-parameter vector (or matrix). On the other hand, for each worker, the total uploaded number of parameters on all subcarriers should be no smaller than its allocated parametric-block size:

(C4: Intra-Worker Parameter Constraint) \( \sum_{n=1}^{N} C_{k,n} L_{k,n} \geq L_k, \quad 1 \leq k \leq K, \tag{21} \)

where \( L_{k,n} \) is the number of parameters uploaded by worker \( k \) on subcarrier \( n \). In the sequel, \( \{L_k\} \) and \( \{L_{k,n}\} \) are relaxed to be continuous for simplicity. In practice, the solved \( \{L_k^*\} \) and \( \{L_{k,n}^*\} \) will be rounded for implementation and the loss caused by the rounding operation can be ignored, since the values of \( \{L_k\} \) and \( \{L_{k,n}\} \) are typically large.

For the case of CNN models, granularity constraints 1 and 2 can be written mathematically as follows.

(C\textsubscript{cnn}: Additional Parameter Constraint for CNN Models) \( \frac{L_k}{L_{\text{sub}}} \in \mathbb{N}^+, \quad 1 \leq k \leq K, \tag{22} \)

where \( \mathbb{N}^+ \) is the set of positive integers and \( L_{\text{sub}} \) is the size of the subproblems, i.e., neurons or per-sample auxiliary matrices. For \( W \text{-stage} \), the size of all neurons, say \( L_{\text{sub}} \), are assumed the same for simplicity, which has little impact on the solution, since the size of each neuron is
much smaller than that of the whole problem, as mentioned in Example 1. For Z-stage, the size of each per-sample auxiliary matrix is the total number of neurons, say \( L_{\text{sub}} = \sum_{g=1}^{G} I_g \) with \( I_g \) being the number of neurons in layer \( g \).

4) **Power Constraints**: The power consumption of each worker is constrained as

\[
\frac{E_k}{T_k} \leq P_k, \quad 1 \leq k \leq K,
\]

(23)

where \( E_k \) defined in (15), \( T_k \) defined in (14), and \( P_k \) are the energy consumption, latency, and maximal permitted power of worker \( k \), respectively.

5) **Latency-Minimization Problem**: Under these constraints, the per-round latency-minimization problem by joint SUPPORT can be formulated as

\[
\min \{C_{k,n}\}, \{L_k\}, \{L_{k,n}\}, \{R_{k,n}\}, T \quad \text{s.t.} \quad (C1) \sim (C5) \quad \text{for a CNN Model.}
\]

(24)

IV. **Joint SUPPORT for Decomposable Models**

In this section, joint SUPPORT is designed by developing a tractable approach for solving Problem (P1).

A. **Equivalent Latency Requirement**

First, the following necessary condition for the equivalent latency requirement can be derived to simplify Problem (P1). Note that in [2], similar equivalent latency property can be derived in the PARTEL design for frequency non-selective channels. However, for OFDM systems considered in this paper, the binary subcarrier allocation among workers and the corresponding inter-subcarrier power and parameter allocation for each device make the problem much more complicated.

**Lemma 2** (Equivalent Latency for All Workers). To achieve the optimal solution of (P1), the following latency condition should be satisfied:

\[
T_{k,\text{cmp}} + T_{k,\text{com}} = T, \quad \forall C_{k,n} = 1,
\]

(25)

where \( T_{k,\text{cmp}} \) defined in (6) is the computation latency of worker \( k \), \( T_{k,\text{com}} \) defined in (8) is the uploading latency of worker \( k \) on subcarrier \( n \).

**Proof**: See Appendix A.

The result in Lemma 2 yields the following insights. First, it requires all workers the same latency with the overall latency \( T \). Second, for each worker, the uploading latency on all allocated subcarriers should be equal.
By substituting the computation latency $T_{k}^{\text{cmp}}$ defined in (6) and the uploading latency $T_{k,n}^{\text{com}}$ defined in (8) into the necessary condition in Lemma 2, we can derive the number of parameters uploaded by worker $k$ on subcarrier $n$, say $L_{k,n}$, as

$$L_{k,n} = \frac{C_{k,n}R_{k,n}}{\tau} \left( T - \frac{L_{k}}{f_{k}} \right), \quad \forall (k,n),$$  \hspace{1cm} (26)

where $T$ is the per-round latency, $C_{k,n} \in \{0, 1\}$ is the subcarrier-allocation indicator, $L_{k}$ is the parametric-block size allocated to worker $k$, $R_{k,n}$ is the channel capacity of $k$ on subcarrier $n$. By substituting $L_{k,n}$ defined in (26) and the necessary condition in Lemma 2, Problem (P1) can be simplified as:

$$\begin{align*}
\min_{\{C_{k,n}\}, \{L_{k}\}, \{R_{k,n}\}, T} & \quad T, \\
\text{s.t.} & \quad (C1), (C3), \\
& \sum_{n=1}^{N} \frac{C_{k,n}R_{k,n}}{\tau} \left( T - \frac{L_{k}}{f_{k}} \right) \geq L_{k}, \quad 1 \leq k \leq K, \\
& \quad E_{k} \leq P_{k}T, \quad 1 \leq k \leq K,
\end{align*}$$  \hspace{1cm} (27)

where $E_{k}$ defined in (15) is the energy consumption of worker $k$. By substituting $L_{k,n}$ in (26), $E_{k}$ can be expressed as

$$E_{k} = g_{k}f_{k}^{2}L_{k} + \sum_{n=1}^{N} C_{k,n} \left( \frac{2R_{k,n}/B - 1}{h_{k,n}} \right) \frac{\sigma^{2}}{\tau} \left( T - \frac{L_{k}}{f_{k}} \right) + \xi.$$  \hspace{1cm} (28)

**B. Equivalent Convex Problem**

Problem (P2) is a mixed integer non-convex problem and is NP-hard. In the sequel, two steps are used to tackle it. First, following the standard approach to tackle integer programming (see e.g., [20]), linear programming relaxation is used to relax the subcarrier-allocation indicators in Problem (P2) to be continuous, i.e., $\{C_{k,n} \in [0, 1], \forall (k, n)\}$. Then, following the method in [2], the relaxed problem can be equivalently converted to the problem of updatable model size maximization. However, it remains non-convex and difficult to tackle due to the intra-worker parameter constraint and the power constraint. In the sequel, the problem of updatable model size maximization is derived and solved.
Given the one-round latency $T$ for an arbitrary round, let $\hat{L}^*(T)$ denote the maximum size of a model that can be updated within the round. Then $\hat{L}^*(T)$ solves the following problem of model size maximization:

\[
\hat{L}^*(T) = \max_{\{C_{k,n}\}, \{L_k\}, \{R_{k,n}\}} \sum_{k=1}^{K} L_k,
\]

subject to:

\[
0 \leq C_{k,n} \leq 1, \quad \forall (k, n),
\]

\[
\sum_{k=1}^{K} C_{k,n} = 1, \quad 1 \leq n \leq N,
\]

\[
\sum_{n=1}^{N} C_{k,n} R_{k,n} \left( T - \frac{L_k}{f_k} \right) \geq L_k, \quad 1 \leq k \leq K,
\]

\[
E_k \leq P_k T, \quad 1 \leq k \leq K,
\]

where $C_{k,n}$ is the subcarrier-allocation indicator, $L_k$ is the parametric-block size allocated to worker $k$, $R_{k,n}$ is the channel capacity of worker $k$ on subcarrier $n$, $T$ is the one-round latency, $E_k$ defined in (28) is the energy consumption of worker $k$. Note that solving Problem (P2) via utilizing the problem of model size maximization in Problem (P3) follows the method in [2]. However, new challenges arise from the subcarrier allocation among workers and the inter-subcarrier power and parameter allocation for each worker, leading to the non-convexity and a much larger size of Problem (P3).

**Lemma 3** (Relation of Maximal Model Size and Latency). $\hat{L}^*(T)$ defined in Problem (P3) is a monotonously increasing function of $T$.

**Proof:** See Appendix B.

It follows from the result in Lemma 3 that the solution of Problem (P2) is the minimal latency, say $T^*$, which makes the updatable model size $\hat{L}^*(T^*)$ no less than the target size $L$. This suggests a method to solve Problem (P2) by searching $T^*$ using the criterion $\hat{L}^*(T) \geq L$, which will be elaborated in the later subsection.

To get the maximum updatable model size $\hat{L}^*(T)$ requires solving Problem (P3). To this end, the following variables are used to transform Problem (P3) into a convex problem.

\[
\begin{align*}
\varphi_k &= \left( T - \frac{L_k}{f_k} \right)^{-1}, \\
\tilde{R}_{k,n} &= C_{k,n} R_{k,n},
\end{align*}
\]
By substituting the variables in (30) and $E_k$ defined in (28), Problem (P3) can be written as

$\hat{L}^*(T) = \max_{\{C_{k,n}\}, \{\phi_k\}} \sum_{k=1}^{K} f_k \left( T - \frac{1}{\phi_k} \right),$

s.t. $0 \leq C_{k,n} \leq 1, \quad \forall (k, n),

(P4)

$\sum_{k=1}^{K} C_{k,n} = 1, \quad 1 \leq n \leq N,$

$\sum_{n=1}^{N} \frac{\tilde{R}_{k,n}}{\tau} \geq f_k (T\phi_k - 1), \quad 1 \leq k \leq K,$

$\sum_{n=1}^{N} C_{k,n} \alpha^2 \left( \frac{2\tilde{R}_{k,n}}{h_{k,n}} - 1 \right) + g_k f_k^3 (\phi_k T - 1) \leq (P_k T - \xi)\phi_k, \quad 1 \leq k \leq K.$

Lemma 4. Problem (P4) is a convex problem.

Proof: See Appendix C

C. Properties of Optimal Policies

Based on the results in the previous subsection, the optimal policies of Problem (P2) with relaxed subcarrier-allocation indicators are proposed, as described in the following.

As (P4) is convex, the primal-dual method can be used to get the optimal solution:

$max_{\{\mu_n\}, \{\lambda_k\}, \{C_{k,n}\}, \{\tilde{R}_{k,n}\}} \min_{\{\nu_k\}} \mathcal{L}_{P4}, \tag{31}$

where $\mathcal{L}_{P4}$ is the Lagrange function of Problem (P4), given as

$\mathcal{L}_{P4} = -\sum_{k=1}^{K} f_k \left( T - \frac{1}{\phi_k} \right) + \sum_{n=1}^{N} \mu_n \left( 1 - \sum_{k=1}^{K} C_{k,n} \right) + \sum_{k=1}^{K} \lambda_k \left[ f_k (T\phi_k - 1) - \sum_{n=1}^{N} \frac{\tilde{R}_{k,n}}{\tau} \right]$

$+ \sum_{k=1}^{K} \nu_k \left[ \sum_{n=1}^{N} C_{k,n} \left( \frac{2\tilde{R}_{k,n}}{h_{k,n}} - 1 \right) \times \frac{\sigma^2}{h_{k,n}} + g_k f_k^3 (T\phi_k - 1) - (P_k T - \xi)\phi_k \right], \tag{32}$

and $\{\mu_n\}, \{\lambda_k \geq 0\}, \text{ and } \{\nu_k \geq 0\}$ are Lagrangian multipliers.

Next, the necessary conditions for achieving the optimal solution of the inner loop are used to derive the optimal policies. The inner loop problem is given by

$\min_{\{C_{k,n}\}, \{\tilde{R}_{k,n}\}, \{\phi_k\}} \mathcal{L}_{P4}, \quad \text{given } \{\mu_n\}, \{\lambda_k\}, \{\nu_k\}. \tag{33}$
The first necessary condition is
\[
\frac{\partial L_{P4}}{\partial \tilde{R}_{k,n}} = -\frac{\lambda_k}{\tau} + \nu_k 2\tilde{R}_{k,n} \ln 2 \times \frac{\sigma^2}{B\tilde{h}_{k,n}} = 0, \quad \forall C_{k,n} \neq 0, \tag{34}
\]
which gives the following optimal scheme for calculating the channel capacity:
\[
R^*_k,n = \begin{cases} \tilde{R}^*_k,n \in C^*_k,n \Rightarrow B \log_2 \left( \frac{\lambda_k B}{\nu_k \tau \ln 2} \right) + B \log_2 \left( \frac{h_{k,n}}{\sigma^2} \right), & \forall C_{k,n} \neq 0, \\ 0, & \text{otherwise} \end{cases} \tag{35}
\]
By substituting \( R^*_k,n \) in (35) into the transmission power in (9), the optimal power-allocation scheme can be derived, as in the following lemma.

**Lemma 5** (Optimal Power Allocation). The optimal power-allocation scheme is
\[
P^{}_{\text{com}}^{\text{opt}} = \begin{cases} \frac{\lambda_k B}{\nu_k \tau \ln 2} - \frac{\sigma^2}{h_{k,n}}, & \forall C_{k,n} \neq 0, \\ 0, & \text{otherwise} \end{cases} \tag{36}
\]
The water-filling like result in (36) shows that for each worker, more power should be allocated on the subcarrier with high channel gain, say \( h_{k,n} \).

The second necessary condition to achieve the optimum of the inner loop problem in (33) is
\[
\frac{\partial L_{P4}}{\partial \varphi_k} = -\frac{f_k}{\varphi^2_k} + \lambda_k f_k T + \nu_k g_k f_k^2 T - \nu_k (P_k T - \xi) = 0. \tag{37}
\]
By substituting the variable transformations in (30) into (37), we can achieve the optimal inter-worker parameter allocation scheme, as follows.

**Lemma 6** (Optimal Parameter Allocation among Workers). The optimal inter-worker parameter-allocation scheme is
\[
L^*_k = \left[ T - \sqrt{\lambda_k T + \nu_k g_k f_k^2 T - \nu_k (P_k T - \xi) / f_k} \right] f_k, \quad 1 \leq k \leq K. \tag{38}
\]
In (38), the optimal parametric-block size allocated to worker \( k \), say \( L^*_k \), is a concave function of the computation speed \( f_k \) and a monotone decreasing function of the computation power factor \( g_k \). On one hand, large \( f_k \) can reduce the computation latency. On the other hand, the computation energy increases as a square function of \( f_k \). The optimal load in (38) balances the two aspects.

Substituting the parameter-allocation scheme in (38) and the channel capacity in (35) into the intra-worker parameter-allocation scheme \( \{ L_{k,n} \} \) in (26), gives the following lemma.
Lemma 7 (Optimal Parameter Allocation Among Subcarriers). The optimal intra-worker parameter allocation scheme is given by

$$I^*_{k,n} = \begin{cases} \sqrt{\lambda_k T + \nu_k g_k f_k^2 T - \nu_k (P_k T - \xi)/f_k} \times B \log_2 \left( \frac{\lambda_k Bh_{k,n}}{\nu_k \tau \sigma^2 \ln 2} \right), & \text{if } C_{k,n} \neq 0, \\ 0, & \text{otherwise.} \end{cases}$$  \hspace{1cm} (39)

From (39), more parameters should be assigned to the channel with high gain.

The third necessary condition to achieve the optimum of the inner loop problem in (33) is

$$\frac{\partial L_{P4}}{\partial C_{k,n}} = -\mu_n + I_{k,n} = 0, \quad \forall (k,n),$$  \hspace{1cm} (40)

where $$I_{k,n}$$ is the indicator function given by

$$I_{k,n} = \nu_k \sigma^2 \left( \frac{2R^*_{k,n}/B - 1}{h_{k,n}} \times R^*_{k,n} \frac{2R^*_{k,n}/B \ln 2}{B} \right), \quad \forall (k,n).$$  \hspace{1cm} (41)

Note that $$I_{k,n}$$ is determined when $$R^*_{k,n}$$ is known. Let $$\mu_n = \min_k I_{k,n}$$. If $$I_{k,n} > \mu_n$$, $$C_{k,n} = 0$$, as the condition in (40) can not be satisfied. If $$I_{k,n} = \mu_n$$ for a unique worker, say $$k$$, then $$C_{k,n} = 1$$. If $$I_{k,n} = \mu_n$$ for multiple workers, then $$C_{k,n} \in (0,1)$$ for these workers. And in the last case, it is easy to show that the values of the non-zero $$\{C_{k,n}\}$$ won’t influence the value of the Lagrange function $$L_{P4}$$ defined in (32), as long as the subcarrier assignment constraint, say $$\left\{ \sum_{n=1}^{N} C_{k,n} = 1, \quad 1 \leq n \leq N \right\}$$, are satisfied.

The optimal subcarrier allocation is summarized in the following lemma.

Lemma 8 (Optimal Subcarrier Allocation). The optimal subcarrier allocation is given as:

$$C^*_{k,n} = \begin{cases} 0, & \text{if } I_{k,n} > \mu_n, \\ \in (0,1), & \text{if } I_{k,n} = \mu_n \text{ for multiple workers,} \\ 1, & \text{if } I_{k,n} = \mu_n \text{ for a unique worker} W_k, \end{cases}$$  \hspace{1cm} (42)

where $$I_{k,n}$$ is the indicator function defined in (41), $$\mu_n = \min_k I_{k,n}$$, and $$R^*_{k,n}$$ is the optimal channel capacity in (35).

In (42), a high channel gain leads to a small value of $$I_{k,n}$$ and thus a high possibility to make $$C_{k,n} \neq 0$$. That means the subcarrier with higher channel gain has larger possibility to be allocated to the worker. Note that in the optimal scheme in Lemma 8, some subcarrier-allocation indicators may be fractions. The standard approach is to round those to be binary (see, e.g., [20]), which will be elaborated in the later subsection.
Algorithm 1 Updatable Model Size Maximization

1: **Input:** channel gains \( \{h_{k,n}\} \), computation speeds \( \{f_k\} \), computation power factors, \( \{g_k\} \), and the given one-round latency \( T \).

2: **Initialize** \( \{\lambda_k^{(0)}\}, \{\nu_k^{(0)}\} \), and \( i = 0 \).

3: **Loop**

4: Update the multipliers as
\[
\begin{align*}
\lambda_k^{(i+1)} &= \max \left\{ \lambda_k^{(i)} + \eta_{\lambda_k} \frac{\partial L_{P4}}{\partial \lambda_k}, \ 0 \right\}, \ 1 \leq k \leq K, \\
\nu_k^{(i+1)} &= \max \left\{ \nu_k^{(i)} + \eta_{\nu_k} \frac{\partial L_{P4}}{\partial \nu_k}, \ 0 \right\}, \ 1 \leq k \leq K,
\end{align*}
\]

5: Solve \( \{L_k^*\}, \{R_{k,n}^*\}, \) and \( \{C_{k,n}^*\} \) using (38), (35), and (42), respectively.

6: Get \( \{\varphi_k^*\} \) and \( \{\tilde{R}_{k,n}^*\} \) with (30).

7: **Until Convergence**

8: \( \hat{L}^*(T) = \sum_{k=1}^{K} L_k^* \).

9: **Output:** \( \hat{L}^*(T), \{L_k^*\}, \{R_{k,n}^*\}, \) and \( \{C_{k,n}^*\} \).

D. Optimal Policy Computation

In this subsection, the joint SUPPORT algorithm to solve the original Problem (P1) is proposed. First, we solve the convex Problem (P4) by the primal-dual method using the closed-form results in Lemmas 6-8. Some notation is described as follows. \( \{\eta_{\lambda_k}\} \) and \( \{\eta_{\nu_k}\} \) denote the step sizes of gradient descent. \( L_{P4} \) and \( \mu, \ {\lambda_k \geq 0}, \ \{\nu_k \geq 0\} \) are the Lagrange function and Lagrangian multipliers defined in (32). With the notation, the application of the primal-dual method yields Algorithm 1 for solving Problem (P4).

**Remark 1 (Low Complexity of Updatable Model Size Maximization).** The computation complexity of Algorithm 1 is \( O(K^2 N) \) with \( K \) being the number of workers and \( N \) being the number of subcarriers, as the closed-form results in Lemmas 6-8 makes the updating of corresponding variables more efficient. As a comparison, directly solving the non-convex Problem (P3) has a computational complexity of at least \( O(K^3 N^3) \) and is suboptimal.

Then, as mentioned in the preceding subsection, Problem (P2) with relaxed subcarrier-allocation indicators can be solved by nesting a one-dimensional search over the latency \( T \) and solving the convex Problem (P4). Based on the monotonicity of \( \hat{L}^*(T) \) in Lemma 3, the search can be efficiently implemented by bisection method. While the solution of Problem (P4) is presented in
Algorithm 2 Joint SUPPORT
1: **Input:** channel gains \( \{h_{k,n}\} \), computation speeds \( \{f_k\} \), and computation power factors, \( \{g_k\} \).
2: **Select** \( T = T_u \) that makes \( \hat{L}^*(T_u) \) defined in Problem (P4) larger than \( L \).
3: **Select** \( T_l = T_l \) that makes \( \hat{L}^*(T_l) < L \).
4: **While** \( T_u \neq T_l \)
5: Let \( T_m = (T_u + T_l)/2 \).
6: Input \( \{h_{k,n}\}, \{f_k\}, \{g_k\} \) and \( T = T_m \) into Algorithm 1 to solve (P4).
7: Obtain \( \hat{L}^*(T_m), \{L^*_k\}, \{R^*_k,n\}, \) and \( \{C^*_k,n\} \).
8: **If** \( \hat{L}^*(T_m) \geq L \)
9: \( T_u = T_m \).
10: **Else**
11: \( T_l = T_m \).
12: **End if**
13: **End while**
14: \( T^* = T_m \).
15: **Output:** \( T^*, \{L^*_k\}, \{R^*_k,n\}, \) and \( \{C^*_k,n\} \).

Algorithm 1 Then the optimal policy to solve Problem (P2) with relaxed subcarrier-allocation indicators is presented in Algorithm 2 by nesting the bisection search and Algorithm 1.

Finally, based on Algorithm 2 the joint scheme of SUPPORT without relaxation is proposed to solve the original Problem (P1). Note that not all subcarrier-allocation indicators solved by Algorithm 2 are integers, i.e., \( C^*_k,n \in (0, 1) \) for some \( (k, n) \). For these subcarriers, a practical subcarrier-allocation scheme following [20] is determined as

\[
C^*_{k_1,n} = 1, \quad k_1 = \arg \max_k L^*_k,n, \quad 1 \leq n \leq N,
\]

where the subcarrier is allocated to the worker with the largest value. Then, given the subcarrier-allocation scheme \( \{C^*_k,n\} \), the latency-minimization problem is a special case of Problem (P1), whose solution can also be solved by Algorithm 2.

V. JOINT SUPPORT FOR CNN MODELS

In this section, CNN models are considered. Since Problem (P1) is not tractable in this case with the additional constraint (\( C_{cnn} \)), we propose an approximate solution method that leverages the result for decomposable model case, described as follows.
1) For both $W$-stage and $Z$-stage, solve the joint scheme of SUPPORT using the method in Section [V] without considering granularity constraints [1] and [2].

2) Given the subcarrier-allocation scheme, round the parameter allocation for each worker to satisfy granularity constraint [1] for $W$-stage and granularity constraint [2] for $Z$-stage.

The challenges lie in Step 2) and are two-fold. On one hand, how should the rounding indicator be designed to minimize the rounding loss. On the other hand, as each worker’s number of parameters changes, the corresponding channel-capacity (or power) allocation and intra-worker parameter allocation among the assigned subcarriers should be redesigned. To tackle these challenges, in the sequel, we first propose a joint scheme of SUPPORT for CNN models. Then, the rounding scheme is designed accordingly and the resultant latency increase is analyzed.

1) Joint SUPPORT for CNN Models: Denote the solved one-round latency as $T^*$, the subcarrier-allocation policy as $\{C_{k,n}^*\}$, the spectrum efficiencies as $\{R_{k,n}^*\}$, the number of parameters of worker $k$ as $L_k^*$, the number of parameters uploaded by worker $k$ on subcarrier $n$ as $L_{k,n}^*$. Consider an arbitrary worker, say worker $k$. If its number of parameters is rounded down to satisfy $(C_{cnn})$, the reduced number of parameters is denoted as $\Delta L_k^d \geq 0$. If its number of parameters is rounded up, the additional number of parameters to be uploaded is denoted as $\Delta L_k^u \geq 0$. Note that if worker $k$’s number of parameters is rounded down, no influence is caused to the one-round latency. Hence, only the case of being rounded up is considered in the sequel.

Our aim is to design rounding scheme to minimize the resulted additional one-round latency.

Next, the joint scheme of SUPPORT is designed as

\[
\begin{align*}
C_{k,n} &= C_{k,n}^*, \quad R_{k,n} = R_{k,n}^*, \\
\Delta L_{k,n} &= L_{k,n}^* \times \frac{\Delta L_k^u}{L_k^*},
\end{align*}
\]

where $\Delta L_{k,n}$ is the number of additional parameters allocated to subcarrier $n$ for uploading, which is proportional to its currently uploaded number of parameters $L_{k,n}^*$. In (44), the allocation of subcarriers $\{C_{k,n}\}$ and the channel capacities $\{R_{k,n}\}$ of the assigned subcarriers remain the same. Two concerns motivate us to design the joint SUPPORT scheme as (44). First, the assigned subcarrier that can currently upload more updates of parameters can upload more additional parameters in the same additional latency. Second, the proportional additional parameter allocation together with the unchanged allocation of subcarriers and channel capacities can yield a simple upper bound of the additional latency for each worker, as shown in the following lemma.
Lemma 9 (Additional Latency). Consider an arbitrary worker, say worker $k$, the design in (44) results in an upper bound of the minimum additional latency:

$$
\Delta T_k \leq T^* \times \frac{\Delta L^u_k}{L^*_k},
$$

(45)

where $T^*$ is the solved latency in Step 1), $\Delta T_k$, $\Delta L^u_k$, and $L^*_k$ are the additional latency, the number of additional parameters after the rounding operation, and the solved number of parameters in Step 1) of worker $k$, respectively.

The proof of Lemma 9 is straightforward and hence omitted. Two observations can be made from Lemma 9. On one hand, as mentioned in Example 1, the size of the subproblems are far smaller than the problems of $W$-stage and $Z$-stage, i.e., $\Delta L^u_k \ll L^*_k$. Therefore, the additional latency $\Delta T_k$ is small for all workers. On the other hand, the round-up indicator, denoted as $I_k$, should be the ratio $I_k = \frac{\Delta L^u_k}{L^*_k}$.

2) Parameter Rounding Scheme: Note that Lemma 9 only gives the additional latency for one worker. To minimize the additional one-round latency, the rounding scheme is designed to make the workers with least $I_k$ to round up and the others to round down, described as follows.

1) Sort the round-up indicators $\{I_k\}$ from the least to the biggest and the new permutation is indexed by $k'$, i.e., $\{I_{k'}\}$ is sorted from the least to the largest.

2) Find the least $K'_1$ following the new permutation $\{I_{k'}\}$, which satisfies

$$
\sum_{k'=1}^{K'_1} \Delta L^u_{k'} \geq \sum_{k'=K_1+1}^{K} \Delta L^d_{k'},
$$

(46)

where $L^u_{k'}$ is the additional number of parameters of worker $k'$ when being rounded up and $\Delta L^d_{k'}$ is the reduced number of parameters when being rounded down. (46) means that by rounding up $K'_1$ workers with least round-up indicators, the parameters of all workers can satisfy granularity constraints 1 and 2.

3) The additional one-round latency is $\Delta T \leq T^* \times I_{K'_1}$, where $T^*$ is the solved one round latency without considering granularity constraints 1 and 2.

VI. EXPERIMENTAL RESULTS

A. Experiment Setup

The experimental settings are specified as follows unless specified otherwise. In the OFDM based PARTEL system, there are $K$ workers and $N$ subcarriers. The bandwidth of each subcarrier is $B = 312.5$ kHz. The subcarrier channel gains $\{H_{k,n}\}$ are assumed to be i.i.d. Rayleigh fading
with the average path loss of $10^{-3}$. The noise power density is set as $10^{-9}$ W/Hz. The workers’ computation speeds $\{f_k\}$ are uniformly selected from the set $\{0.1, 0.2, ..., 1.0\} \times 10^6$ parameters processed per second in one local computation iteration. The corresponding computation power factors $\{g_k\}$ are uniformly selected from the set $\{0.1, 0.2, ..., 1.0\} \times 10^{-16}$. The maximum power consumed by workers $\{P_k\}$ is set as 8 W. Both decomposable models and CNN models are trained using the PARTEL framework. The learning settings are as follows.

- **Decomposable Model**: A $L_1$-regularized logistic regression task is considered, which trains a news-filtering model using the News20 dataset collected in [27]. The model size is $1.24 \times 10^6$. The training and test datasets have 15936 and 3993 samples respectively. $K = 50$ workers with $N = 80$ subcarriers are used to complete the task.

- **CNN Model**: The CNN model “LeNet-5” is trained on the MNIST dataset. In the “LeNet-5” model, there are 60,000 parameters in total. The method of auxiliary variables in [19] is used to train the “LeNet-5” model at the PARTEL framework. In each training iteration, a mini batch of 50 data samples is used. There are 469,400 auxiliary variables in total. $K = 30$ workers with $N = 50$ subcarriers are used to complete the task.

For comparison, three communication schemes are considered, described as follows.

- **Joint SUPPORT**: The joint schemes of SUPPORT proposed in Section IV or V.

- **Baseline**: The number of parameters computed by each worker is first allocated proportional to their computation capacity. Then, the subcarriers are allocated, which is a special case of the joint SUPPORT scheme.

- **Greedy Scheme for FEEL**: The training samples are equally distributed among workers. Thereby, the computation latency and energy of each worker is determined. The subcarrier allocation follows a greedy way. The subcarriers are randomly indexed and sequentially allocated from the 1st to the $N$-th. The $i$-th subcarrier is allocated to the worker whose latency is currently the longest. Note that the latency minimization of one worker given the subcarrier allocation is simple and omitted.

**B. Decomposable Models**

The learning performance of training the logistic regression model is compared in Fig. 2. As observed, the model trained in PARTEL with the proposed joint SUPPORT converges much faster than the one trained in FEEL with the greedy communication scheme, in which each worker uploads the updates of all parameters. Besides, the joint SUPPORT outperforms the
Figure 2: Learning performance versus (communication-plus-computation) latency.

Figure 3: Latency performance versus (a) a varying number of workers and (b) subcarriers.

baseline in terms of model convergence with a latency reduction of 31.06% on average. That’s because the allocations of parameters and subcarriers are sequentially designed in the baseline.

Fig. 3 shows the impacts of number of workers and subcarriers on the per-round latency. As observed, the per-round latencies of both schemes decrease as the number of workers or subcarriers increases. The reasons are as follows. More workers can provide more computation capacity and hence reduce the computation latency. Moreover, more subcarriers allocated to workers can reduce the uploading latency.
C. CNN Models

The learning performance of training the LeNet-5 is compared in Fig. 4. Although two stages (rounds) complete one training iteration in PARTEL using the joint scheme, it outperforms the FEEL using the greedy scheme in terms of model convergence, as the latter has to upload the updates of all parameters in each round. Besides, the joint SUPPORT can reduce latency by 42.11% compared to the baseline for the similar reason in the decomposable model case.

The impacts of the number of workers and subcarriers on the latency performance of training LeNet-5 is compared in Fig. 5. As shown in the figure, the latencies of the two schemes for both W-stage and Z-stage decrease with the number of workers and subcarriers for the same
reasons in the case of decomposable models.

The experimental results above show that our proposed joint scheme of SUPPORT has the best performance regarding learning latency and verifies our analysis.

APPENDIX

A. Proof of Lemma 2

KKT conditions are used to show Lemma 2. The Lagrange function of Problem (P1) is in (47), where \( \{ \mu_n \}, \lambda \geq 0, \{ \nu_k \geq 0 \}, \{ \alpha_k \geq 0 \}, \) and \( \{ \beta_{k,n} \geq 0 \} \) are multipliers.

\[
\mathcal{L} = T + \sum_{n=1}^{N} \mu_n \left( 1 - \sum_{k=1}^{K} C_{k,n} \right) + \lambda \left( L - \sum_{k=1}^{K} L_k \right) + \sum_{k=1}^{K} \nu_k \left( L_k - \sum_{n=1}^{N} C_{k,n} L_{k,n} \right) \\
+ \sum_{k=1}^{K} \alpha_k \left( \frac{E_k}{T_k} - P_k \right) + \sum_{k=1}^{K} \sum_{n=1}^{N} \beta_{k,n} C_{k,n} \left( T_{k}^{\text{cmp}} + T_{k,n}^{\text{com}} - T \right),
\]

(47)

Then, consider an arbitrary subcarrier-allocation scheme \( \{ C_{k,n} \} \), KKT conditions are necessary to solve the problem. Some related KKT conditions are given below:

\[
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial T} &= 1 - C_{k,n} \beta_{k,n} = 0, \quad 1 \leq k \leq K, \\
\beta_{k,n} C_{k,n} \left( T_{k}^{\text{cmp}} + T_{k,n}^{\text{com}} - T \right) &= 0, \quad \forall (k,n),
\end{aligned}
\]

(48)

From the first condition in (48), we can show that \( \{ \beta_{k,n} \neq 0, \forall C_{k,n} = 1 \} \), which, together with the second condition in (48), can show that \( T_{k}^{\text{cmp}} + T_{k,n}^{\text{com}} = T, \forall C_{k,n} = 1 \). Note that the above condition is necessary for arbitrary subcarrier-allocation schemes. Hence, it is a necessary condition to solve (P1).

B. Proof of Lemma 3

First, we show that the equality of the third and forth constraints in Problem (P3) should be achieved. The Lagrange function of (P3) is

\[
\mathcal{L} = \sum_{k=1}^{K} L_k + \sum_{n=1}^{N} \mu_n \left( \sum_{k=1}^{K} C_{k,n} - 1 \right) + \sum_{k=1}^{K} \lambda_k \left[ L_k - \sum_{n=1}^{N} \frac{C_{k,n} R_{k,n}}{\tau} \left( T - \frac{L_k}{f_k} \right) \right] \\
+ \sum_{k=1}^{K} \nu_k \left( E_k - P_k T \right),
\]

(49)

where \( \{ \mu_n \}, \{ \lambda_k \geq 0 \}, \) and \( \{ \nu_k \geq 0 \} \) are multipliers. Using KKT conditions and the similar approaches in Appendix A, we can show that \( \{ \lambda_k \neq 0, 1 \leq k \leq K \} \) and \( \{ \nu_k \neq 0 1 \leq k \leq K \} \) and the equalities of the third and forth constraints in Problem (P3) should be achieved.
Then, consider $T_1 < T_2$. When $T = T_1$, denote the optimal solution of (P3) as $\{C_{k,n,1}^*\}$, $\{L_{k,1}^*\}$, $\{R_{k,n,1}^*\}$, and the maximum updatable model size as $L^*(T_1)$.

Next, let $T = T_2$, $\{C_{k,n,2} = C_{k,n,1}^*\}$, and $\{R_{k,n,2} = R_{k,n,1}^*\}$. By substituting $L_{k,1} = L_{k,1}^*$ into the third and forth conditions in Problem (P3), the equalities are not achieved. This shows that the updatable number of parameters by each worker, denoted as $\{L_{k,2}\}$, can be larger, i.e., $L_{k,2} > L_{k,1}^*$. It follows that $\sum_{k=1}^K L_{k,2} > \sum_{k=1}^K L_{k,1}^* = L^*(T_1)$. Furthermore, the optimal solution for $T = T_2$ satisfies $L^*(T_2) \geq \sum_{k=1}^K L_{k,2}$. Hence, we have $L^*(T_2) > L^*(T_1)$.

C. Proof of Lemma 4

First, the third constraint in Problem (P3), by dividing $(T - L_{k}/f_k)$ on both sides and substituting the variable transformations in \((30)\), can be derived as the third constraint in Problem (P4): $\left\{\sum_{n=1}^N \tilde{R}_{k,n}/\tau \geq f_k (T\varphi_k - 1), \ 1 \leq k \leq K\right\}$. Obviously, the feasible region of the above constraint is a convex set. Then, by substituting $E_k$ in \((28)\), dividing $\left(T - L_{k}/f_k\right)$ on both sides, and substituting the variable transformations in \((30)\), the forth constraint in Problem (P3) can be equally derived as the forth constraint in Problem (P4):

$$
\sum_{n=1}^N C_{k,n} g^2 \left(\frac{\tilde{R}_{k,n}}{h_{k,n}} - 1\right) + g_k f_k^3 (\varphi_k T - 1) \leq (P_k T - \xi) \varphi_k, \ 1 \leq k \leq K. \tag{50}
$$

In \((50)\), the first term is a convex function as $f(x, y) = xe^{y/x}$ is convex. Thereby, the feasible region of the constraint in \((50)\) is a convex set. Besides, the objective function and other constraints are convex. Thus, Problem (P4) is convex.

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