The Transition From Ontic Potentiality to Actualization of States in Quantum Mechanical Approach to Reality: The Proof of a Mathematical Theorem to Support it.

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Abstract: It is given a preliminary discussion on the ontic nature of quantum states to be intended as potentialities and on the central role of spin to be considered as the basic essence of quantum mechanical reality. The possible fundamental role of potentialities and of spin is evidenced in the framework of physical as well as of biological reality. After such preliminary deepening, using a quantum like scheme delineated on the basis of an algebraic structure, it is given for the first time mathematical demonstration of the transition from potentiality of states to their actualization as basic mechanism of our reality.

1. Introduction

It is known that the problem of how a mathematical superposition of manifold possibilities evolves to become a particular observable actuality, represents the basic unsolved problem of measurement in quantum mechanics. In some sense it delineates also a basic unsolved question of our science and knowledge. In fact, the present quantum problem may be considered the last more modern version of a problematicism and a debate that involved our science and philosophy from the past, starting with Empedocle, Plato and the same Aristotle who first considered that potentia and actuality are two kinds of reality and that actualities give origin to potentia which give origin to actualities. W. Heisenberg evidenced the value of quantum mechanics in this Aristotelian basic principle. On the other hand, quantum theory is not able to reach adequate evidence on the nature of such potential entities, on their qualification between their quantum actualizations and, finally, on the same mathematical and physical features regulating the transition in our reality from potential to actual entities. The aim of the present paper is to move in the framework of a quantum like formulation, giving for the first time a mathematical proof of such possible transition from potential to actual entities in our reality and simultaneously describing the mathematical and physical features that characterize such transition.

Before to proceed with the mathematical proof, we need to deepen two important questions. The first relates the nature of the quantum states and the second considers the nature of the spin. We retain that this last quantum observable represents the essence of quantum mechanics and therefore our proof will be based on the utilization at the algebraic level of basic abstract elements that, as we shall see, move in direct analogy with the notion of spin. Of course, the idea behind our algebraic quantum like representation of potentialities was inspired by the conventional quantum formalism. By the spectral postulate of quantum mechanic values belonging to spectrum and only them could be obtained as results of corresponding measurement. Spectral values are quantum potentialities encoded in a self-adjoint
operator representing a quantum observable. We emphasize that here the representation of quantum observables by self-adjoint operators in the complex Hilbert space plays the fundamental role. And the complex Hilbert space by itself is the basic structure of the conventional quantum model. We elaborate the idea about quantum potentialities in purely algebraic approach (so the complex Hilbert space is not invented from the very beginning).

One of the advantages of our approach is the possibility to present quantum like potentialities as ontic structures of the model.

2. Some Observations on the Nature of Quantum States.
Let us start with a preliminary analysis on the notion of quantum state. The problem here is well known. Let us consider a particle, as example an electron, impinging on a screen. According to quantum mechanics we cannot know where it will hit but we can always assign probabilities to potentialities of the electron to hit at different locations. These are given by the well known wave function or quantum state of the system we have in consideration. At some time, the electron impinges into some point of the screen and since it hits the screen, we have no more a matter of probability. The quantum state has collapsed into some definite point on the screen. The arising question is on the nature of the quantum state. In an epistemic interpretation we consider that the quantum state describes not the system in consideration but our status of knowledge about it. In an ontic interpretation we admit instead that the quantum states are ontic and this is to say that they describe the system as it is. The point here is to consider an ontic nature of the quantum state but the settlement of its definition is paved with conceptual difficulties. Nevertheless, some possibilities to proceed with ontic interpretation of quantum states were explored in [1], [2], [3]. In fact, it is not so easy to introduce an adequate notion of ontic potentialities in our reality. As example, it seems rather an approximation for defect to consider here a superposition state of potentialities meaning that the system can be in two or more states at the same time. In fact, we must remember here that the entity in consideration is a potential and not an actual entity. It must be considered to be real, ontologically significant, but not being actual. By the previous definition we ran the risk to consider the coexistence of potentialities as an actual like form, that is a superposition of coexisting like actualities and this is not what the quantum superposition principle admits. This is one first difficulty. One other question arises in the following manner. Let us consider two quantum non-commuting entities A and B. Quantum mechanics tells us that, if one such entity, say A, is actualized, B consequently remains an undefined potentiality. In our opinion, this is an ontic holistic process that must receive a proper general, mathematical formalization while instead in this case the traditional quantum formalism, based on the Hilbert space formulation, holds only the requirement of mutually orthogonal vectors that are representative of the mutual exclusivity of the states. The consequent mutual exclusivity of outcome states is merely an epistemic phenomenon and thus ontologically insignificant while instead we need in this case an holistic description having a full ontological explanation. This is one of the reasons because we introduce in the following section a quantum like schema, not based as traditionally on quantum linear operators and Hilbert spaces, but fixed instead on an algebraic structure and its formalization and where the auspicated ontological significance of the holistic process regarding at the same time the actualization of the entity A and the persisting potentiality of the entity B, is reached through the proof of the theorem 2. Here the notion of holism linking actuality to potentiality seems absolutely necessary. In conclusion, we like more to steer ourselves into a different definition [see also 4] considering an ontic superposition of potentialities meaning an ontic holistic entanglement where no more independently existing features of potentiality as actual like forms may be identified. In addition, linked co-existing forms of actuality and potentiality must be expected in our reality still in a whole holistic ontological framework. We would also give some evidence about such definitions. Let us admit hypothetically an existing system S that may be
represented by an actual state, that we identify by $\psi_{n,A}$, and a potential state that is given in a multiplicative manner by $\psi_{n,P} = \varphi_{n,P}X_p$ where here $X_p$ is the potential entity, a symbol whose only quantum constraint is to be $X_p^2 = 1$ so that its potentiality is to be actualized as +1 or -1. $\varphi_{n,P}$ represents instead some scalar quantity connected to $X_p$. The time evolution of such system is admitted to be given by the following map

$$\psi_{n+1,A} + \varphi_{n+1,P}X_p = (\psi_{n,A} + \varphi_{n,P}X_p)(\psi_{n,A} + \varphi_{n,P}X_p) = \psi_{n,A}^2 + \varphi_{n,P}^2 + 2\psi_{n,A}\varphi_{n,P}X_p$$

In the evolution of such hypothetical system, we have an actualized entity, $\psi_{n,A}^2 + \varphi_{n,P}^2$, which at any stage of the evolution experiences both the actual ($\psi_{n,A}^2$) and the potential ($\varphi_{n,P}^2$) contributions. Instead, the potential entity is represented by the term $2\psi_{n,A}\varphi_{n,A}X_p$, where the potential symbol, $X_p$, goes on maintaining at each step, its potentiality to be +1 and -1. Finally, at some time $X_p$ may be considered to randomly assume an actualized value of +1 or of -1 giving in this case a final actualized value for the whole evolution process in consideration. This is an example of potential – actualized process that at last in principle is based on a quantum like scheme. As we see, it changes radically our traditional view on dynamics of reality. The time evolution of the system starts with the actualized value $\psi_{0,A}$. However, with basic difference respect to our traditional view on evolution processes, it has also the potentiality to be or $\psi_{0,A} + \varphi_{0,P}$ or $\psi_{0,A} - \varphi_{0,P}$. This is an intrinsic potentiality of such evolution process. In the future steps of the evolution the system maintains its potentialities that at each stage will be given directly by the basic mathematical features of the map. An occurring actualization of the process randomly at some time will attribute to $X_p$ a definite value, or +1 or -1, and this actualization will enable the evolution process to actualize a final value of the evolutive process that will account also of the previously unexpressed (non actualized) potentialities. We may also examine more articulated quantum schemes in which the potential contribution, previously expressed by $\varphi_{n,P}X_p$, will be now replaced by a more general term of the form $F(X_{1,P}, X_{2,P}, X_{3,P})$ where the $X_{i,P}$ are this time three potential symbols given as example as in the (11) of the following section and realizing in this manner a quantum like scheme in which also the contributions of the non commutativity are taken in consideration.

In conclusion, we retain that quantum potentialities, as roughly expressed by the previous model, find their principal arena in the sphere of the biological matter. In following papers, we will give some detailed examples of biological themes in which symbolic potential entities as $X_{i,P}$ may be involved. As general subjects we recall here the importance to consider the possible role of potential states, as example, in the general case of the control mechanisms in the biological sphere or also we may relate the previous discussed evolution process with the basic theme of the Neodarwinism that of course was just the object of a recent investigation in the framework of quantum potentialities [5]. In this case what radically changes respect to our traditional manner to conceive evolution is that in classical Neodarwinism we have evolution essentially intended as consequence of random variations and natural selection of what is the fittest form. Here we have selection of forms of concrete and actual matter. Instead in the case of the evolution model previously introduced and having potentialities, we have similarly a final selection of concrete and actual forms but the arena of the possible differentiation is extremely different and, in particular, forms of potentiality this time coexist with forms of actuality, and potentiality contributes to characterize actual forms at each stage of the evolution. It is sufficient to look at the previously given relation to convince that we are in fact in presence of a radically new kind of evolution mechanism where, we repeat, forms of potentiality coexist with forms of actuality. The result is in a new structure which makes possible many more possible and different pathways that result impossible in an evolution mechanism based only on
actualization. At the same time it is the basic concept of reality that changes radically in the
sense that in the traditional case it is the concrete and actual matter that constitutes the
ontological reference of a basic materialistic instance while instead in such new case matter
must be considered in its potential form to be envisaged in addition to its actual form.

3. Observations on the Nature of Spin.
W. Pauli was the physicist that had a decisive role in the elaboration of the quantum theory of
spin. Initially, he called the spin a “two-valued quantum degree of freedom”. On the basis of
this definition, we retain that initially he considered the spin as a kind of physical-logical –
informative entity linked to matter at the microphysical level. His definition of spin remained
initially rather vague and uncharacterized until R. Kroning in 1925 suggested that it would be
produced by the self-rotation of the electron. This was an idea that Pauli initially criticized
severely but Kroning’s view on spin was subsequently supported from G. Uhlenbeck and S.
Goudsmit in the same year, and finally Pauli, despite his initial objections to this idea,
formalized the theory of spin in 1927, accepting to characterize it as self-rotation of a quantum
particle. He pioneered the use of the so-called Pauli matrices as a representation of spin
operators, and he introduced a two-component spinor wave function. His spin-theory was not
relativistic. In 1928 P. Dirac described the relativistic electron with a four component spinor,
and he found that the spin is a relativistic effect that may be identified by linearization of the
Hamiltonian in special relativity [6].
At the present we are convinced that the spin represents an entity of Nature whose meaning and
role are more general, indeed universal, with respect to the rather restrictive interpretation
that was originally formulated by Kroning, Uhlenbeck and Goudsmit. We retain that the
following examples are of basic importance to accept our thesis. Quantum computing has
introduced the qubit quantum register. Here a universal unity of information is formulated in
quantum mechanical terms and information no longer results a rather abstract entity but for
the first time it is really and tangibly connected to a material object as an elementary particle. In
addition, P. O’Hara [7] obtained that the spin is introduced in a natural way into the space-time
metric, taking the square root of the metric associated with space. Also other authors, as P.
Cordero, C. Teitelboim, and R. Tabensky [8] introduced the spin into relativity but taking the
square root of the Hamiltonian. As well as in the Dirac equation the spin was obtained by
linearizing the Hamiltonian of special relativity, in [7] spin matrices were obtained by
linearizing not the Hamiltonian of relativity but rather the space-metric itself, and this result
provided the conclusion in [7] that the spin is intrinsically linked to the geometrical properties
of space-time. Such results indicate that the so-called spin must be really intended as
manifestation of a general and universal entity having in Nature an articulated, physical,
informative, and logic role. Other interesting evidences of such a conclusion may be reached
through investigation in biology and physiology. Two authors, Hu. Hu and M. Wu, [9],
introduced a theory in which the advent of consciousness is intrinsically connected to spin. They
formulated a spin-mediated consciousness theory based on pan-protopsychism. These authors
were able to discuss a well-defined neurophysiological model to support their thesis.
Considering the structure and the dynamics of the brain, they postulated that the human mind
works as follows: The nuclear spin ensembles (NSE) in both neural membranes and proteins
quantum mechanically process consciousness-related information such that conscious
experience emerges from the collapses of entangled quantum states of NSE under the influence
of the underlying spacetime dynamics. Said information is communicated to NSE through
strong spin-spin couplings by biologically available unpaired electronic spins such as those
carried by rapidly diffusing oxygen molecules and neural transmitter nitric oxides that extract
information from their diffusing pathways in the brain. In turn, the dynamics of NSE has effects
through spin chemistry on the classical neural activities such as action potentials and receptor
functions thus influencing the classical neural networks of brain [9]. The authors also gave some
supporting evidence to such a formulation introducing indications for experimental verifications. Also recently, [10], we have given direct formulation for a possible quantum mechanical model of consciousness based on the central role of the spin.

There is still another important reason to discuss here the real role of spin in biological dynamics.

The four bases in RNA sequences C, G, A and U (or T in DNA) may be formalized by using Pauli matrices. Note that we do not speak here of some physical feature of such molecules but of their intrinsic representation and description. Of course, the four bases of RNA (or DNA) pertain to the most universal language and description of our biological reality. Let us explain it. Bases of the same heterocyclic kind (purine or pyrimidine) have the same signs. A proper reference frame may be introduced which slides along the RNA chain from 5' to the 3' end. Let the j-th and k-th nucleotides be paired. The base pair $X_jY_k (j < k)$ is encountered twice: when the reference frame reaches position $j$, and from this position, nucleotide $X_j$ looks upright, but nucleotide $Y_k$ is upside-down; still, when reference frame reaches position $k$, from here nucleotide $Y_k$ looks upright, but nucleotide $X_j$ is upside-down. It follows that one may distinguish four base pair states of RNA since AU and UA, CG and GC, are no longer identical.

\[
\begin{align*}
A, U, C, \uparrow G, \uparrow U, \uparrow A, \downarrow G, \downarrow C,
\end{align*}
\]

Once again, may be represented by Pauli matrices that in physics are representative of the spin but here represent base-pair values and still the base-pair creation and base-pair disruption. These results were obtained by Y. Magarshak [11]. We retain that they confirm fully our thesis. The notion of spin must be intended according to a very general meaning, that one of an entity that is articulated at a physical but also informative, and logic level until as a proto quantum like potential entity, whose great importance may be identified in biological as well as in physiological studies.

Let us consider still another important result that also legitimates the reason to consider the role of spin on a more general plane. Chaotic behaviors have been identified in a consistent number of signals pertaining to physiology and biology [12]. Starting with 1999, A.Jadczyk and R. Olkiewicz [13] showed that simultaneous measurements of non commuting spin components lead to a chaotic jump on a quantum spin sphere and to generation of specific fractal images on the basis of a non linear iterated function system. Thus, once again, non commutativity, as just was outlined also by M. Zak in a previous work [14], and spin may be also responsible of chaotic behaviors.

Several authors, [9, 15] repeatedly evidenced that the spin is the essence of quantum mechanics having an ontological meaning. Hu and Wu repeatedly outlined that the driving force behind the evolution of Schrödinger equation is quantum spin and, since quantum entanglement arises from the evolution of Schrödinger equation the said spin is the genuine cause of quantum entanglement. To support this thesis we outline that recently we showed that Schrödinger equation is a manifestation of an abstract algebraic formulation in which the basic elements are given as well as in the Pauli spin formulation [16]. Quantum potentialities arise through quantum superposition principle that is admitted in Schrödinger equation.

4. On the Possibility to Introduce an Algebraic Structure as Quantum Like Scheme of Our Reality.

The conclusion of the previous section is that with the term spin we should intend an entity that seems to assume a general role in our reality for the variety of the dynamics that it is able to support and for the high differentiation of the processes to which it is able to oversee. We aim to give a reason for such an entity to oversee the natural phenomena at different levels. The reason could be that the so called spin as admitted in physics is really expression of a more general and differentiated essence and modality of self-fulfillment of reality at its various levels
of manifestation. The confirmation could arise under a mathematical profile. It is known that on October 16 of 1843, a mathematician, Sir W.R. Hamilton, discovered hypercomplex numbers [17] that he initially identified as the algebra of pure time. The scientific community acknowledged lukewarmly such a new mathematical discovery. J.T. Graves stated “I have not yet any clear views on the extent to which we are at liberty to arbitrarily create new imaginaries and to endow them with supernatural properties”-such as non commutativity. In the 19th century W.M. Clifford [18] completed the work initiated by Grassman and Hamilton giving a complete formulation of such algebraic structures. A rather trivial but interesting feature is that the Hamilton algebra may be also represented by matrices and in this case we re-find Pauli matrices and the non commutativity of the basic generators of the algebra in the same manner in which they appear in quantum theory of spin. Therefore, the reason for what we have previously called the universality of spin could be explained in the fact that its mathematical counterpart regards an algebraic structure, and algebraic structures arise in the description of natural processes and they have universal character. If we identify the spin Pauli matrices of physics in the inner body of an algebraic structure, in some sense we may attempt to show that such a structure represents a rough scheme of quantum like mechanics. If so, the universality of the algebraic structure should draw directly on the possibility to retain the same quantum like expressed theory as not specialized only at the quantum microphysical level for which it was introduced in 1927. In this manner we return to consider the problem of the potentiality and actuality that constitutes the basic aim of the present paper. As outlined in the first section, quantum mechanics exhibits two basic and original features. The first is that it admits potential as well as actualized states of physical reality. The second point is that it admits that, under suitable circumstances, we have a stochastic transition from potentiality to actualization of states via the so called unknown mechanism of the wave function reduction or psi collapse. In substance, if such a strong link exists between the given algebraic structure and a rough scheme of quantum mechanics, we must re-find in the algebra the results of quantum mechanics. First of all we have to delineate in detail the basic features of such an algebraic structure, discussing in particular its basic assumptions and the manner in which this algebra may derived on the basis of its starting axiomatic points. Shown that the introduced algebraic structure, represents actually a quantum like scheme of quantum mechanics, we may attempt to take a great step forward and this is to say to give for a first time a rigorous mathematical proof of the transition from potentiality to actualization that represents the basic indemonstrable fixed focus of all the quantum mechanics. We could be entirely successful since the algebraic structure that we will use could represent a more general ontological construction respect to a more restricted realization that could be represented from traditional quantum theory. These are the objectives that are reached in the following section. Here we will utilize the great work that, starting with 1981, was developed by Y. Ilamed and N. Salingaros [19] . We will follow the same technique that these authors used in their work. We anticipate here that only two basic assumptions, quoted as (a) and (b) in the following section, seem that are required in order to formulate a rough scheme of quantum mechanics.

5. The Proof of Some Theorems.
In this section we give a rigorous proof of theorems characterizing the algebra that we employ. We will follow some basic results that were previously given by Y. Ilamed and N. Salingaros [19] in 1981, when these authors studied in detail the algebra with three anticommuting elements. Let us consider three abstract basic elements, \( e_i \), with \( i = 1,2,3 \), and the unit basis \( e_0 \equiv 1 \). Let us admit the following two assumptions:

\( a) \) it exists the scalar square for each basic element:
\[
  e_i e_i = k_i, \quad e_2 e_2 = k_2, \quad e_3 e_3 = k_3 \quad \text{with} \quad k_i \in \mathbb{R}.
\] (1)
In particular we have also that
\[ e_0 e_0 = 1. \]

b) The basic elements \( e_i \) are anticommuting elements, that is to say :
\[ e_1 e_2 = -e_2 e_1, \quad e_2 e_3 = -e_3 e_2, \quad e_3 e_1 = -e_1 e_3. \] (2)

In particular it is
\[ e_i e_0 = e_0 e_i = e_i. \]

Note that, owing to the axioms (a) and (b), the given basic elements must be considered abstract potential entities having the potentiality to simultaneously assume the numerical values \( \pm k_i^{1/2} \).

This is confirmed in particular by examining the (14) that is direct emanation of the two starting axioms.

According to [19], these are the necessary and the sufficient conditions to derive all the basic features of the algebra that we employ. To give proof, let us consider the general multiplication of the three basic elements \( e_1, e_2, e_3 \), using scalar coefficients \( \omega, \lambda, \gamma \) pertaining to some field :
\[ e_1 e_2 e_3 = \omega e_1 + \omega e_2 + \omega e_3; \quad e_2 e_3 = \lambda e_1 + \lambda e_2 + \lambda e_3; \quad e_3 e_1 = \gamma e_1 + \gamma e_2 + \gamma e_3. \] (3)

Let us introduce left and right alternation:
\[ e_1 e_2 e_3 = (e_1 e_2) e_3; \quad e_2 e_3 = e_1 (e_2 e_3); \quad e_3 = e_2 (e_3 e_1); \quad e_3 = (e_3 e_1) e_2; \quad e_1 e_2 e_3 = e_3 (e_2 e_1). \] (4)

Using the (4) in the (3) it is obtained that
\[ k_1 e_1 + k_2 e_2 + k_3 e_3; \quad k_1 e_2 + k_2 e_3 + k_3 e_1; \quad k_1 e_3 + k_2 e_1 + k_3 e_2; \quad k_1 e_1 + k_2 e_2 + k_3 e_3; \quad k_1 e_2 + k_2 e_3 + k_3 e_1; \quad k_1 e_3 + k_2 e_1 + k_3 e_2. \] (5)

From the (5), using the assumption (b), we obtain that
\[ \omega \frac{e_1 e_2}{k_2} + \omega \frac{e_2 e_3}{k_3} = \gamma \frac{e_1 e_2}{k_3} - \gamma \frac{e_2 e_3}{k_3} + \gamma e_3; \quad \omega \frac{e_1 e_2}{k_1} + \omega \frac{e_3}{k_3} = -\frac{\lambda_1}{k_3} e_1 + \lambda_2 e_2 + \lambda e_3; \quad \gamma \frac{e_1 e_2}{k_1} + \gamma \frac{e_3}{k_3} = -\frac{\lambda_1}{k_2} e_1 + \lambda_2 e_2 + \lambda_3 e_3. \] (6)

For the principle of identity, we have that it must be
\[ \omega_1 = \omega_2 = \omega_3 = \lambda_1 = \lambda_2 = \lambda_3 = \gamma_1 = \gamma_2 = 0 \] (7)
and
\[ -\lambda_1 k_1 + \gamma_1 k_2 = 0 \quad \gamma_2 k_2 - \omega_1 k_3 = 0 \quad \lambda_3 k_1 - \omega_2 k_1 = 0 \] (8)

The (8) is an homogeneous system admitting non trivial solutions since its determinant \( \Lambda = 0 \), and the following set of solutions is given:
\[ k_1 = -\gamma_2 \omega_3, \quad k_2 = -\lambda_1 \omega_3, \quad k_3 = -\lambda_1 \gamma_2 \] (9).
Admitting \( k_1 = k_2 = k_3 = +1 \), it is obtained that
\[
\omega_j = \lambda_j = \gamma_j = i
\]  
(10)

Using the (3), the theorem is proven, showing that the basic features of the considered algebra
are given in the following manner
\[
e_i e_j = -e_j e_i = i e_j \quad ; \quad e_i e_j = -e_j e_i = i e_i \quad ; \quad e_i e_j = -e_j e_i = i e_j
\]
(11)

The content of theorem 1. is thus established: given three abstract basic elements as defined in
(a) and (b), an algebraic structure is established with four generators \((e_0, e_1, e_2, e_3)\).

Note that the (11) represents one of the most basic relations in quantum mechanics. It has been
here derived only on the basis of two algebraic assumptions, given respectively in (a) and (b).

We may now add some comments to the previous formulation.

The activity of scientific knowledge in two hundred years of development unequivocally shows
that some algebraic structures arise naturally in the description of natural entities and
phenomena. Thus, it is quite natural to attempt to identify the phenomenological counterpart of
the algebraic structure given in (11). From (1) we have that
\[
e_1^2 = 1 \quad , \quad e_2^2 = 1 \quad , \quad e_3^2 = 1
\]
(12)

The (12) evidences that, being the \( e_i \) abstract potential entities, we may choose to attribute
them the numerical values of \( \pm 1 \). Admitting to be \( p_i (+1) \) the probability to attribute the value
\(+1\) to \( e_1 \) and \( p_i (-1) \) that one for \(-1\), considering the corresponding notation for the two
remaining basic elements, we may introduce the following mean values:
\[
< e_1 >= (+1)p_1 (+1) + (-1)p_1 (-1), \quad < e_2 >= (+1)p_2 (+1) + (-1)p_2 (-1),
\]
\[
< e_3 >= (+1)p_3 (+1) + (-1)p_3 (-1).
\]

It has been shown elsewhere [20] that
\[
< e_1 >^2 + < e_2 >^2 + < e_3 >^2 \leq 1
\]
(14)

Let us observe that the (14) may be considered to represent a general principle of ontic
potentialities and, in particular, it indicates that we never can attribute simultaneously definite
numerical values to two basic elements \( e_i \). In conclusion, as seen by the axioms (a) and (b),
by the (11), by the (13) and the (14), we have delineated a rough scheme of quantum like theory
through an algebraic structure. In this algebraic scheme some principles of the basic theoretical
framework result to be represented.

These principles are that the given algebraic structure reflects an intrinsic indetermination and
an ontic potentiality for its abstract elements. This means that, in absence of a direct numerical
attribution, such basic elements are symbols that act in the algebra as such symbols, having an
intrinsic indetermination and an ontic potentiality. This is to say that, in absence of attribution of
a given numerical value, the basic elements \( e_i \) operate in the given algebraic structure
preserving the potentiality to assume a direct, possible, numerical value at any stage of the
algebraic operations. In addition, let us consider, as example, to be \( < e_1 >= 1 \) (attribution of \(+1\)
to \( e_1 \)), the (14) unequivocally shows that both \( e_2 \) and \( e_3 \) remain in the superposition of potential
states of \(+1\) and \(-1\).

Let us explain this last point in detail. The algebraic structure given in (1), (2), and (11) admits
idempotents. Let us consider two of such idempotents:
\[
\psi_1 = \frac{1 + e_3}{2} \quad \text{and} \quad \psi_2 = \frac{1 - e_3}{2}
\]
(15)

It is easy to verify that \( \psi_1^2 = \psi_1 \) and \( \psi_2^2 = \psi_2 \). Let us examine now the following algebraic
relations:
\[
e_3 \psi_1 = \psi_1 e_3 = \psi_1
\]
(16)
\[ e_3 \psi_2 = \psi_2 e_3 = -\psi_2 \]  

(17)

Similar relations hold in the case of \( e_1 \) or \( e_2 \). The relevant result is that the (16) establishes that the given algebraic structure, with reference to the idempotent \( \psi_1 \), attributes to \( e_3 \) the numerical value of +1 while the (17) establishes that, with reference to \( \psi_2 \), the numerical value of -1 is attributed to \( e_3 \).

The conclusion is very important: The conceptual counterpart of the (16) and (17) is that we are in presence of a self-referential process. On the basis of such self-referential process, as given in (16) and in (17), this algebraic structure is able to attribute a precise numerical value to its basic elements. Each of the three basic elements is able to make a transition from the condition of pure potentiality to a condition of actuality, that is to say in mathematical terms from the pure symbolic representation of the given abstract elements to that one of a real number. Let us remember that, on the basis of the (14), this self-referential process may regard each time one and only one of the three basic elements. In brief, for the first time we are analyzing an algebraic structure that represents a rough quantum like scheme and that, at the same time, as repeatedly admitted also in usual quantum mechanics, evidences, on the basis of a self-referential process, that it is possible a transition from potentiality to actualization as we discussed it in the first section of this paper.

Note also the importance of the (16) and the (17) from the viewpoint of the logic. Through the self-referential process given in (16) and (17), our algebra recovers two first principles of logic that are the Principle of non-Contradiction and the Principle of the excluded Middle. Obviously, in order to reach a rigorous formulation of such matter, the central question that mathematically arises is that we must give proof that it does exist and it may be carefully defined an algebraic structure that initially is given as by the (1), (2), and (11) and then it is characterized by the numerical attribution to one of its basic elements, as example to \( e_3 \), of one numerical value, say of +1 or of -1. The same conclusion holds if we consider a numerical attribution to \( e_1 \) or to \( e_2 \).

Let us consider the following argument. If

\[ e_3 \rightarrow +1 \]  

(18)

we should have that

\[ \psi_1 \rightarrow +1, \ \psi_2 \rightarrow 0, \]  

(19)

and, in the (11),

\[ e_i e_2 = i, e_2 e_1 = -i, e_i j = -e_i, i e_2 = e_1, e_1 i = e_2, i e_1 = -e_2 \]  

(20).

In other terms, if we attribute the numerical value of +1 to \( e_3 \) a new algebraic structure arises with new generators whose rules are given in (20) instead of in (11). Therefore, the arising central problem is to proof the real existence of such new algebraic structure. Note that, in the case of the starting algebraic structure we showed that it exists in the following manner

\[ e_1^2 = 1, e_2^2 = 1, e_3^2 = 1, i = e_1 e_2 e_3, e_i e_2 = -e_i e_1 = i e_3, e_2 e_3 = -e_2 e_3 = i e_1, e_1 e_i = e_i e_1 = -e_i e_3 = i e_2. \]

In the present case, with \( e_3 \rightarrow +1 \), we have to show that it exists in the following manner

\[ e_1^2 = 1, e_2^2 = 1, i^2 = -1, e_1 e_2 = i, e_2 e_1 = -i, e_i j = -e_i, i e_2 = e_1, e_1 i = e_2, i e_1 = -e_2 \]  

(22).

We arrive at the proof of theorem 2: given the algebraic structure A, fixed as in the (1), (2), and (11), it exists an algebraic structure B, that we call a subalgebra of A, with basic elements (generators) given in (22). To proof, consider that we now attribute to \( e_3 \) the numerical value of +1 and so it is dismissed from the basic scheme of the three anticommuting basic elements. It is now replaced by \( i \). Rewriting the (3) and performing calculations we arrive to the solutions of the (8) that are given in the following manner:

\[ k_1 = -\gamma_2 \omega_3, k_2 = -\lambda_3 \omega_3, k_3 = -\lambda_3 \gamma_2 \]  

(22)
where this time it must be \( k_1 = k_2 = +1 \) and \( k_3 = -1 \). The solutions are given for
\[
\omega_j = +1, \lambda_j = -1, \gamma_j = -1 \tag{23}
\]
and consequently the (22) are proven as expected. Therefore it is shown that in the case
\( e_3 \rightarrow +1 \), the subalgebra \( B \) exists having the basic features given in (22).

The theorem 2 may be shown also in the case in which we attribute to \( e_1 \) the numerical value of
\( -1 \). We have
\[
e_3 \rightarrow -1 \tag{24}
\]
and
\[
\psi_1 \rightarrow 0, \psi_2 \rightarrow -1 \tag{25}
\]
and the subalgebra \( B \) is given in the following terms:
\[
e_1^2 = 1, e_2^2 = 1, i^2 = -1, e_1e_2 = -i, e_2e_1 = i, e_2i = e_1, ie_2 = -e_1, e_1i = -e_2, ie_1 = e_2 \tag{26}
\]
The solutions of the (22) are given in this case by \( \omega_3 = -1, \lambda_1 = +1, \gamma_2 = +1 \). The theorem is shown also in this case. In a similar way it is obtained the proof when considering the case of attribution of a numerical value to \( e_1 \) or to \( e_2 \).

In this manner we have reached the central aim of the paper. Also if using an algebraic structure, this is the first time in which we are able to show the manner in which it is realized the passage from potentiality to actualization and it has been demonstrated by using a rigorous formulation based on two mathematical theorems. Since, as previously said, the counterpart exists in natural processes of the algebraic structures arising during their description, we expect that the two theorems demonstrate the passage from potentiality to actualization in our reality.

6. An Application in \( \psi \)-collapse of Quantum Mechanics.

It is well known that one of the basic unsolved problems of quantum mechanics resides in the so called process of reduction of wave function or \( \psi \)-collapse.

Consider a two state quantum system \( S \) with connected quantum observable \( \sigma_3 \). It is known that we have
\[
\psi = c_1 \varphi_1 + c_2 \varphi_2 \quad \text{with} \quad \varphi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \varphi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{27}
\]
and
\[
|c_1|^2 + |c_2|^2 = 1 \tag{28}
\]
It is still known that we may represent the state of such system by a density matrix \( \rho \) given in the following terms
\[
\rho = a + be_1 + ce_2 + de_3 \tag{29}
\]
with
\[
a = \frac{|c_1|^2}{2}, \quad b = \frac{|c_2|^2}{2}, \quad c = \frac{i(c_1c_2^* - c_1^*c_2)}{2}, \quad d = \frac{|c_1|^2 - |c_2|^2}{2} \tag{30}
\]
where in matrix notation, \( e_1, e_2 \), and \( e_3 \) are the well known Pauli matrices
\[
e_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{31}
\]
It is also easily verified and well known that we may find a \( 2 \times 2 \) matrix representation of the algebra \( A \) (as well as of the subalgebra \( B \)), given in the previous section, by using the same matrix configuration, given in (31). In conclusion we may write the (30) in explicit form in one of the two equivalent forms:
\[
\rho = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(c_1 c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - e_2) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3
\]

or
\[
\rho = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(c_1 c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - e_2) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3
\]

Let us admit now that we make a measurement of \( \sigma_j \) with result +1. Admitting in this case that the subalgebra \( B \) obtained in the previous section is valid, we have that the (18), the (19), and the (22) are valid in the (32), and thus we have that the quantum interference terms disappear and the matrix density is reduced to
\[
\rho_M = |c_1|^2 \times I
\]

In the case of the measurement of \( \sigma_j \) is performed with result -1, the same subalgebra holds where now the (24), the (25), and the (26) are valid. Applied to the (33), still the interference terms disappear, and they give this time that
\[
\rho_M = |c_2|^2 \times I
\]

As expected, the subalgebra \( B \), introduced in the previous section, describes the collapse of the wave function in quantum mechanics.

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