Searching for the squark flavor mixing in CP violations of $B_s \to K^+K^-$ and $K^0\bar{K}^0$ decays

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Abstract

We study CP violations in the $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$ decays in order to find the contribution of the supersymmetry, which comes from the gluino-squark mediated flavor changing current. We obtain the allowed region of the squark flavor mixing parameters by putting the experimental data, the mass difference $\Delta M_{B_s}$, the CP violating phase $\phi_s$ in $B_s \to J/\psi\phi$ decay and the $b \to s\gamma$ branching ratio. In addition to these data, we take into account the constraint from the asymmetry of $B^0 \to K^+\pi^-$ because the $B_s \to K^+K^-$ decay is related with the $B^0 \to K^+\pi^-$ decay by replacing the spectator $s$ with $d$. Under these constraints, we predict the magnitudes of the CP violation in the $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$ decays. The predicted region of the CP violation $C_{K^+K^-}$ is strongly cut from the direct CP violation of $B^0 \to K^-\pi^+$, therefore, the deviation from the SM prediction of $C_{K^+K^-}$ is not found. On the other hand, the CP violation $S_{K^+K^-}$ is possibly deviated from the SM prediction considerably, in the region of $0.1 \sim 0.5$. Since the standard model predictions of $C_{K^0\bar{K}^0}$ and $S_{K^0\bar{K}^0}$ are very small, the squark contribution can be detectable in $C_{K^0\bar{K}^0}$ and $S_{K^0\bar{K}^0}$. These magnitudes are expected in the region $C_{K^0\bar{K}^0} = -0.06 \sim 0.06$ and $S_{K^0\bar{K}^0} = -0.5 \sim 0.3$. More precise data of these CP violations provide us a crucial test for the gluino-squark mediated flavor changing current.

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1 Introduction

Recently, there have been a lot of studies to search for new physics in the low energy flavor physics such as $B_s$ decays. Actually, the LHCb collaboration has reported new data of the CP violations of the $B_s$ meson and the branching ratios of rare $B_s$ decays [1]-[12].

For many years, the CP violations in the $K$ and $B_0$ mesons have been successfully understood within the framework of the standard model (SM), so called Kobayashi-Maskawa (KM) model [13], where the source of the CP violation is the KM phase in the quark sector with three families. However, the new physics has been expected to be indirectly discovered in the flavor changing neutral current (FCNC) of the $B_0$ and $B_s$ decays at the LHCb experiment and the further coming experiment Belle II.

The LHCb collaboration presented the data of the time dependent CP asymmetry in the non-leptonic $B_s \to J/\psi \phi$ decay [4, 11, 12], which is consistent with the SM prediction. Therefore, this observed value gives us a strong constraint of the new physics contribution to the $b \to s$ transition. In addition to this result, the first measurement of time-dependent CP violation in $B_s \to K^+K^-$ decay has been reported at LHCb [14]. Some authors discussed this process and the $B_s \to K^0\overline{K}^0$ one in order to search for new physics [15]-[20], because the penguin amplitudes dominate these decays. Especially, the SM prediction of the CP violation of the $B_s \to K^0\overline{K}^0$ decay is very small, and so, the new physics contribution can be detectable in the time dependent CP asymmetry.

On the other hands, it is noticed that the $B_s \to K^+K^-$ decay is related with the $B^0 \to K^+\pi^-$ decay by replacing the spectator $s$ with $d$. Thus, the $B^0 \to K^+\pi^-$ decay associates with the processes of $B_s \to K^+K^-$ and $B_s \to K^0\overline{K}^0$ in order to search for the new physics in the $b \to s$ penguin process. It is found that the recent experimental data of the direct CP violation in $B^0 \to K^+\pi^-$ decay is well agreement with the SM prediction with the QCD factorization calculation [21, 22]. This process depends on the form factor $F(B \to K)$ and the chiral enhancement factor $(2M_K^2/m_b m_s)$ in the framework of the QCD factorization. The amplitudes of $B_s \to K^+K^-$ and $B_s \to K^0\overline{K}^0$ decays also involve the common form factor and chiral enhancement factor under neglecting the difference of masses of the $B^0$ and $B_s$ mesons.

As the new physics, we examine the sensitivity of the effect of the supersymmetry (SUSY) in the CP violation of these $B_s$ decays. Although the SUSY is one of the most attractive candidates for the new physics, the SUSY signals have not been observed yet. Since the lower bounds of the superparticle masses increase gradually, the squark and the gluino masses are supposed to be at the TeV scale [23]. While, there are new sources of the CP violation in the low energy flavor physics if the SM is extended to the SUSY model. The soft squark mass matrices contain the CP-violating phases, which contribute to the FCNC with the CP violation. Therefore, one expects the effect of the SUSY contribution in the CP-violating phenomena of the $B_s$ meson decays. We study the gluino-squark mediated flavor changing process, which is the most important process of the SUSY contribution for the $b \to s$ transition [24]- [37].

The gluino mass is expected to be larger than 1.3 TeV, and the squarks of the first and second families are also heavier than 1.4 TeV [23]. Therefore, we take the split-family scenario, in which the first and second family squarks are very heavy, $O(10-100)$ TeV, while the third family squark masses are at $O(1)$ TeV. Then, the $s \to d$ transition mediated...
by the first and second family squarks is suppressed by their heavy masses, and competing process is mediated by the second order contribution of the third family squark. In order to estimate the gluino-squark mediated FCNC for the $B_s$ meson decays, we work in the basis of the squark mass eigenstate. Then, the $6 \times 6$ mixing matrix among down-squarks and down-quarks is discussed by input of the experimental constraints.

In section 2, we present the formulation of the CP violation of the $B^0$ and $B_s$ decays in the QCD factorization. In section 3, we present the setup in our split-family scenario. In section 4, we discuss the sensitivity of the gluino-squark mediated FCNC to the CP violation of the $B^0 \to K^+ \pi^-$, $B_s \to K^+ K^-$ and, $B_s \to K^0 \bar{K}^0$ decays. Section 5 is devoted to the summary. Relevant formulations are presented in appendices A, B, and C.

2 CP violation of $B$ decays in QCD factorization

In this section, we present the formulation of the CP violation of the $B^0 \to K^+ \pi^-$, $B_s \to K^+ K^-$, and $B_s \to K^0 \bar{K}^0$ decays within the framework of the QCD factorization \[21, 22, 38, 39\].

First, we begin with the effective Hamiltonian for the $\Delta B = 1$ transition as

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left[ \sum_{q'=u,c} V_{q'b} V_{q'q}^* \sum_{i=1,2} C_i O_i^{(q')} - V_{tb} V_{tq}^* \sum_{i=3-10, 6} \left( C_i O_i + \tilde{C}_i \tilde{O}_i \right) \right] ,$$

where $q = s, d$. The local operators are given as

$$O_1^{(q')} = (\bar{q}_\alpha \gamma_\mu P_L q_\beta)(\bar{q}'_\beta \gamma^\mu P_L b_\alpha), \quad O_2^{(q')} = (\bar{q}_\alpha \gamma_\mu P_L q'_\alpha)(\bar{q}'_\beta \gamma^\mu P_L b_\beta),$$

$$O_3 = (\bar{q}_\alpha \gamma_\mu P_L b_\alpha) \sum_Q (\bar{Q}_\beta \gamma^\mu P_L Q_\beta), \quad O_4 = (\bar{q}_\alpha \gamma_\mu P_L b_\beta) \sum_Q (\bar{Q}_\beta \gamma^\mu P_L Q_\alpha),$$

$$O_5 = (\bar{q}_\alpha \gamma_\mu P_L b_\alpha) \sum_Q (\bar{Q}_\beta \gamma^\mu P_R Q_\beta), \quad O_6 = (\bar{q}_\alpha \gamma_\mu P_L b_\beta) \sum_Q (\bar{Q}_\beta \gamma^\mu P_R Q_\alpha),$$

$$O_7 = \frac{3}{2} (\bar{q}_\alpha \gamma_\mu P_L b_\alpha) \sum_Q (e_Q \bar{Q}_\beta \gamma^\mu P_R Q_\beta), \quad O_8 = \frac{3}{2} (\bar{q}_\alpha \gamma_\mu P_L b_\beta) \sum_Q (e_Q \bar{Q}_\beta \gamma^\mu P_R Q_\alpha),$$

$$O_9 = \frac{3}{2} (\bar{q}_\alpha \gamma_\mu P_L b_\alpha) \sum_Q (e_Q \bar{Q}_\beta \gamma^\mu P_L Q_\beta), \quad O_{10} = \frac{3}{2} (\bar{q}_\alpha \gamma_\mu P_L b_\beta) \sum_Q (e_Q \bar{Q}_\beta \gamma^\mu P_L Q_\alpha),$$

$$O_{7\gamma} = \frac{e}{16\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} P_R b_\alpha F_{\mu\nu}, \quad O_{SG} = \frac{g_s}{16\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} P_R T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a ,$$

where $P_R = (1 + \gamma_5)/2$, $P_L = (1 - \gamma_5)/2$, and $\alpha, \beta$ are color indices, and $Q$ is taken to be $u, d, s, c$ quarks. Here, $C_i$’s and $\tilde{C}_i$’s are the Wilson coefficients at the relevant mass scale, and $\tilde{O}_i$’s are the operators by replacing $L(R)$ with $R(L)$ in $O_i$. The $\tilde{C}_i$’s are neglected in SM.

We use the value of Wilson coefficients at $\mu = m_b$ as follows:

$$C_1 = -0.185, \quad C_2 = 1.082, \quad C_3 = 0.014, \quad C_4 = -0.035,$$

$$C_5 = 0.009, \quad C_6 = -0.041, \quad C_7 = -0.002/137, \quad C_8 = 0.054/137,$$

$$C_9 = -1.292/137, \quad C_{10} = -0.262/137, \quad C_{SG} = -0.143,$$
in the SM calculations [38].

The hard scattering amplitude is given for the relevant decay modes as follows:

\[
T_p = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pu}^* V_{pb} \left[ a_1^p (\bar{q} \gamma_\mu L u) \otimes (\bar{u} \gamma^\mu L b) + a_2^p (\bar{u} \gamma_\mu L u) \otimes (\bar{q} \gamma^\mu L q') \otimes (\bar{q} \gamma^\mu L b) + a_4^p (\bar{q} \gamma_\mu L q') \otimes (\bar{q} \gamma^\mu L b) + a_5^p (\bar{q} \gamma^\mu L q') \otimes (\bar{q} \gamma^\mu L b) + a_6^p (\bar{q} \gamma^\mu L q') \otimes (\bar{q} \gamma^\mu L b) \right],
\]

where the symbol \( \otimes \) denotes \( \langle M_1 M_2 | j_2 \otimes j_1 | B \rangle \equiv \langle M_2 | j_2 | 0 \rangle \langle M_1 | j_1 | B \rangle \). The effective \( a_i^p \)'s which contain next-to leading order (NLO) coefficients and \( O(\alpha_s) \) hard scattering corrections are given as,

\[
a_1^u = 0, \quad a_i^c = a_i^u \quad (i = 3, 5, 7, 8, 9, 10, 8a, 10a), \quad a_1^u = C_2 + \frac{C_1}{N} + \frac{\alpha_s C_F}{4\pi N} C_1 F_{M_2},
\]

\[
a_2^u = C_1 + \frac{C_2}{N} + \frac{\alpha_s C_F}{4\pi N} C_2 F_{M_2}, \quad a_3^u = C_3 + \frac{C_4}{N} + \frac{\alpha_s C_F}{4\pi N} C_4 F_{M_2},
\]

\[
a_4^p = C_4 + \frac{C_3}{N} + \frac{\alpha_s C_F}{4\pi N} \left[ C_3 \left[ F_{M_2} + G_{M_2}(s_q) + G_{M_2}(s_b) \right] + C_2 G_{M_2}(s_q) \right] + (C_4 + C_6) \sum_{f=u}^b G_{M_2}(s_f) + C_{8G} G_{M_2,g},
\]

\[
a_5^u = C_5 + \frac{C_6}{N} + \frac{\alpha_s C_F}{4\pi N} C_6 (-F_{M_2} - 12), \quad a_3^u = C_3 + \frac{C_7}{N},
\]

\[
a_6^p = C_6 + \frac{C_5}{N} + \frac{\alpha_s C_F}{4\pi N} \left[ C_2 G'_{M_2}(s_p) + C_3 \left[ G'_{M_2}(s_q) + G'_{M_2}(s_b) \right] + (C_4 + C_6) \sum_{f=u}^b G'_{M_2}(s_f) + C_{8G} G'_{M_2,g} \right],
\]

\[
a_7^u = C_7 + \frac{C_8}{N} + \frac{\alpha_s C_F}{4\pi N} C_8 (-F_{M_2} - 12), \quad a_8^p = C_8 + \frac{C_7}{N},
\]

\[
a_8^u = \frac{\alpha_s C_F}{4\pi N} \left[ (C_8 + C_{10}) \sum_{f=u}^b \frac{3}{2} e_f G'_{M_2}(s_f) + C_9 \frac{3}{2} e_q G_{M_2}(s_q) + C_4 G_{M_2}(s_b) \right],
\]

\[
a_9^u = C_9 + \frac{C_{10}}{N} + \frac{\alpha_s C_F}{4\pi N} C_{10} F_{M_2}, \quad a_{10}^u = C_{10} + \frac{C_9}{N} + \frac{\alpha_s C_F}{4\pi N} C_9 F_{M_2},
\]

\[
a_{10}^p = \frac{\alpha_s C_F}{4\pi N} \left[ (C_8 + C_{10}) \sum_{f=u}^b \frac{3}{2} e_f G_{M_2}(s_f) + C_9 \frac{3}{2} e_q G_{M_2}(s_q) + C_4 G_{M_2}(s_b) \right],
\]

where \( q = d, s \quad q' = u, d, s \quad f = u, d, s, c, b \) and \( C_F = (N^2 - 1)/(2N) \) with the number of colors \( N = 3 \). In Appendix A, we present the loop integral functions \( F_{M_2}, G_{M_2,g}, G_{M_2}(s_q), G'_{M_2,g}, G'_{M_2}(s_q) \), in which the internal quark mass enters as \( s_f = m_f^2/m_b^2 \).
In this work, $C_i$ includes both SM contribution and squark-gluino one, such as $C_i = C^\text{SM}_i + C^\tilde{g}_i$, where $C^\text{SM}_i$'s are given in Ref. [40]. The Wilson coefficients of the gluino-squark contribution $C^\tilde{g}_{7,8}$ and $C^\tilde{g}_{8,G}$ are presented in Appendix B. We should also take account of the SUSY contribution in $C_i$'s($i = 3 - 10, 7, 8_G$), which are derived by replacing $L(R)$ with $R(L)$ in $C_i$. Then, $C_i$'s are replaced with $C_i - \tilde{C}_i$ in Eq.(5) for the decays $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$. The minus sign in front of $\tilde{C}_i$ is due to the parity of the final states.

By using these formula, we can write the decay amplitude for the $\bar{B}^0 \to K^-\pi^+$, $\bar{B}_s \to K^+K^-$ and $\bar{B}_s \to K^0\bar{K}^0$ decays, respectively, as follows:

$$\bar{A}(\bar{B}^0 \to K^-\pi^+) = \frac{G_F}{\sqrt{2}} i f_\pi (M^2_{\bar{B}^0} - M^2_K) F^{B^0\to K}_s(0)(1 - \frac{\lambda^2}{2}) |V_{cb}| \left( R_{CKM} e^{-i\gamma} [a'_4 + a_4^u + a_4^d] 
+ R_K(a_0^u + a_8^u + a_{8a}) + a_{10}^u + a_{10a}] + [a_4' + R_K(a_0^c + a_8^c) + a_{10}^c + a_{10a}] \right),$$

(6)

$$\bar{A}(\bar{B}_s \to K^+K^-) = \frac{G_F}{\sqrt{2}} i f_K (M^2_{\bar{B}_s} - M^2_K) F^{B_s\to K}_s(0)(1 - \frac{\lambda^2}{2}) |V_{cb}| \left( R_{CKM} e^{-i\gamma} [a'_4 + a_4^u + a_4^d] 
+ R_K(a_0^u + a_8^u + a_{8a}) + a_{10}^u + a_{10a}] + [a_4' + R_K(a_0^c + a_8^c) + a_{10}^c + a_{10a}] \right),$$

(7)

$$\bar{A}(\bar{B}_s \to K^0\bar{K}^0) = \frac{G_F}{\sqrt{2}} i f_K (M^2_{\bar{B}_s} - M^2_K) F^{B_s\to K}_s(0)(1 - \frac{\lambda^2}{2}) |V_{cb}| \left( R_{CKM} e^{-i\gamma} [a_4^u] 
+ R_K(a_0^u + a_8^u + a_{8a}) + a_{10}^u + a_{10a}] + [a_4' + R_K(a_0^c + a_8^c) + a_{10}^c + a_{10a}] \right),$$

(8)

where

$$R_{CKM} = \frac{\lambda}{1 - \lambda^2/2} \frac{V_{ub}}{V_{cb}},$$

and $f_{\pi(K)}$, $F^{B_s\to K}_s(0)$ are decay constants and the form factors at $q^2 = 0$, respectively. The CKM matrix elements $V_{cb}, V_{ud}$ and $V_{us}$ are chosen to be real and $\gamma$ is the phase of $V_{ub}^*$, and we take $\lambda = V_{us} = 0.22535$ and $R_K = 2M^2_K/((m_s + m_\phi)(m_b - m_q))$.

Let us discuss the time dependent CP asymmetries of $B_s$ decaying into the final state $f$, which are defined as [41]

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2},$$

(9)

where

$$\lambda_f = \frac{q}{p}, \quad \frac{q}{p} \simeq \sqrt{\frac{M^2_{f*}}{M^2_{f*}}}, \quad \bar{\rho} \equiv \frac{\bar{A}(B_s \to f)}{A(B_s \to f)}.$$

(10)

In the $B_s \to J/\psi \phi$ decay, we write $\lambda_{J/\psi \phi}$ in terms of phase factors as follow:

$$\lambda_{J/\psi \phi} \equiv e^{-i\phi_s}.$$
In the SM, the angle $\phi_s$ is given as $\phi_s = -2\beta_s$, in which $\beta_s$ is one angle of the unitarity triangle for $B_s$. The SM predicts $\phi_s$ as [42]

$$\phi_s = -0.0363 \pm 0.0017. \quad (12)$$

The recent experimental data of this phase is [4, 43]

$$\phi_s = 0.07 \pm 0.09 \pm 0.01. \quad (13)$$

This value constrains the magnitude of the new physics, which contributes to $M_{12}^s$ in Eq. (10).

For the gluino-squark contribution to $M_{12}^s$, we present the formulation in Appendix C.

The time dependent CP asymmetries of $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$ are obtained by calculating

$$\lambda_{K^+K^-} = e^{-i\phi_s} \frac{A(\bar{B}_s \to K^+K^-)}{A(B_s \to K^+K^-)}, \quad \lambda_{K^0\bar{K}^0} = e^{-i\phi_s} \frac{A(\bar{B}_s \to K^0\bar{K}^0)}{A(B_s \to K^0\bar{K}^0)}. \quad (14)$$

The new physics contribution is often sensitive in the $b \to s\gamma$ decay. The branching ratio $\text{BR}(b \to s\gamma)$ is given as [44]

$$\frac{\text{BR}(b \to s\gamma)}{\text{BR}(b \to ce\bar{\nu}e)} = \frac{|V_{ts}V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} (|C_{7\gamma}(m_b)|^2 + |\tilde{C}_{7\gamma}(m_b)|^2), \quad (15)$$

where

$$f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z, \quad z = \frac{m_{c,pole}^2}{m_{b,pole}^2}. \quad (16)$$

Here $C_{7\gamma}(m_b)$ and $\tilde{C}_{7\gamma}(m_b)$ include both contributions from the SM and the new physics. The SM prediction including the next-to-next-to-leading order correction is given as [45]

$$\text{BR}(b \to s\gamma)(\text{SM}) = (3.15 \pm 0.23) \times 10^{-4}, \quad (17)$$

on the other hand, the experimental data is obtained as [46]

$$\text{BR}(b \to s\gamma)(\text{exp}) = (3.53 \pm 0.24) \times 10^{-4}. \quad (18)$$

By inputing this experimental value, the contribution of the gluino-squark mediated flavor changing process, $C_{7\gamma}$ and $\tilde{C}_{7\gamma}$, is constrained.

In addition to the CP violating processes with $\Delta B = 2, 1$, the SUSY contribution is also sensitive to the electric dipole moment [47], which is the the T violation of the flavor conserving process. The experimental upper bound of the electric dipole moment of the neutron provides us the upper-bound of the chromo-EDM(cEDM) of the strange quark [48]-[51]. The cEDM of the strange quark $d_s^C$ is given in terms of the gluino-bottom-quark interactions [37]. The upper bound of the cEDM of the strange quark is given by the experimental upper bound of the neutron EDM as [51],

$$e|d_s^C| < 0.5 \times 10^{-25} \text{ ecm}. \quad (19)$$

This bound constrains the SUSY flavor mixing angles and the phases in $C_{8G}$ and $\tilde{C}_{8G}$. However, the experimental data of the direct CP violation in the $B^0 \to K^+\pi^-$ decay gives a little bit stronger constraint for $C_{8G}$ and $\tilde{C}_{8G}$ in our framework. Therefore, we omit the discussion about the cEDM in this work.
3 Setup of squark flavor mixing

Let us discuss the gluino-squark mediated flavor changing process as the dominant SUSY contribution of the $b \to s$ transition. We give the $6 \times 6$ squark mass matrix to be $M_{\tilde{q}}$ ($\tilde{q} = \tilde{u}, \tilde{d}$) in the super-CKM basis. In order to go to the diagonal basis of the squark mass matrix, we rotate $M_{\tilde{q}}$ as

$$\tilde{m}_{\tilde{q}, \text{diagonal}}^2 = \Gamma_G^{(q)} M_{\tilde{q}}^2 \Gamma_G^{(q)*},$$

(20)

where $\Gamma_G^{(q)}$ is the $6 \times 6$ unitary matrix, and we decompose it into the $3 \times 6$ matrices as $\Gamma_G^{(q)} = (\Gamma_G^{(q)L}, \Gamma_G^{(q)R})^T$ in the following expressions. Then, the gluino-squark-quark interaction is given as

$$L_{\text{int}}(\tilde{g}q\tilde{q}) = -i \sqrt{2} g_s \sum_q \overline{\tilde{q}}_i (T^a) \tilde{G}^a \left[ (\Gamma_G^{(q)L})_{ij} L + (\Gamma_G^{(q)R})_{ij} R \right] q_j + \text{h.c.},$$

(21)

where $\tilde{G}^a$ denotes the gluino field, and $L$ and $R$ are projection operators. This interaction leads to the gluino-squark mediated flavor changing process with $\Delta B = 2$ and $\Delta B = 1$ through the box and penguin diagrams.

We take the split-family scenario, in which the first and second family squarks are very heavy, $\mathcal{O}(10 - 100)$ TeV, while the third family squark masses are at $\mathcal{O}(1)$ TeV. Therefore, the first and second squark contribution is suppressed in the gluino-squark mediated flavor changing process by their heavy masses. In addition, we also assume the flavor symmetry such as $U(2)$ in order to suppress FCNC enough in the neutral K meson system. The stop and sbottom interactions dominate the gluino-squark mediated flavor changing process. Then, the sbottom interaction contributes $\Delta B = 2$ and $\Delta B = 1$ processes. We take a suitable parametrizations of $\Gamma_G^{(q)L}$ and $\Gamma_G^{(q)R}$ as follows:

$$\Gamma_G^{(d)L} = \begin{pmatrix}
1 & 0 & \delta_{13}^L c_\theta & 0 & 0 & -\delta_{13}^L s_\theta e^{i\phi} \\
0 & 1 & \delta_{23}^L c_\theta & 0 & 0 & -\delta_{23}^L s_\theta e^{i\phi} \\
-\delta_{13}^L & -\delta_{23}^L & c_\theta & 0 & 0 & -s_\theta e^{i\phi}
\end{pmatrix},$$

$$\Gamma_G^{(d)R} = \begin{pmatrix}
0 & 0 & \delta_{13}^R s_\theta e^{-i\phi} & 1 & 0 & \delta_{13}^R c_\theta \\
0 & 0 & \delta_{23}^R s_\theta e^{-i\phi} & 0 & 1 & \delta_{23}^R c_\theta \\
0 & 0 & s_\theta e^{-i\phi} & -\delta_{13}^R & -\delta_{23}^R & c_\theta
\end{pmatrix},$$

(22)

where $c_\theta = \cos \theta$ and $s_\theta = \sin \theta$, with the mixing angle $\theta$ in the $\tilde{b}_{L,R}$ sector and $\delta_{13}^L, \delta_{13}^R$ are the couplings responsible for the flavor transitions. The mixing angle $\theta$ comes from the trilinear SUSY breaking terms. If this breaking is neglected, $\theta$ vanishes. In our work, we suppose the large $\mu \tan \beta$, which leads to the non-negligible mixing angle $\theta$ in the $\tilde{b}_L - \tilde{b}_R$ sector. By using these rotation matrices, we estimate the gluino-sbottom mediated flavor changing amplitudes in the $B_s$ meson decay.

For the numerical analysis, we fix sbottom masses. The third family squarks can have substantial mixing between the left-handed squark and the right-handed one due to the large
Yukawa coupling, that is the large $\mu \tan \beta$. In our numerical calculation, we take the typical mass eigenvalues $m_{b_1}$ and $m_{b_2}$, and the gluino mass $m_{\tilde{g}}$ as follows:

$$m_{b_1} = 1 \text{ TeV}, \quad m_{b_2} = 1.5 \text{ TeV}, \quad m_{\tilde{g}} = 2 \text{ TeV},$$

(23)

where we take account of the present experimental bounds [23]. Once we fix mass eigenvalues $m_1$, $m_2$ and $\mu \tan \beta$, we can estimate the mixing angle $\theta$ between the left-handed sbottom and the right-handed one [55]. Taking $\mu \tan \beta = 20 - 50 \text{ TeV}$, we estimate $\theta$ in the range of $4^\circ - 10^\circ$, which is used in our numerical calculations. If we take $\mu \tan \beta \ll 20 \text{ TeV}$, the left-right mixing angle $\theta$ is much less than $\mathcal{O}(1^\circ)$. Then, the SUSY contribution in $C_{8G}$ and $C_{7\gamma}$ are tiny because the left-right mixing dominates $C_{8G}$ and $C_{7\gamma}$. The smaller mass difference $m_{b_2} - m_{b_1}$ gives the larger mixing angle $\theta$. However, our results does not so change since the SUSY contribution depends on the combination of $\theta$ and the mass deference as $\sin 2\theta \times (m_{b_2}^2 - m_{b_1}^2)$ in our scheme.

The relevant mixing angles are $\delta_{23}^{DL}$ and $\delta_{23}^{DR}$ for $B_s \rightarrow K^+K^-$ and $B_s \rightarrow K^0\bar{K}^0$ decays. These mixing angles are complex, and then we take

$$|\delta_{23}^{DR}| = |\delta_{23}^{DL}|,$$

(24)

for simplicity. On the other hand, the phases of $\delta_{23}^{DR}$ and $\delta_{23}^{DL}$ are free parameters, which are are constrained by experimental data.

We comment on our assumption in Eq. (24). This one may be motivated from the $SO(10)$ GUT model with SUSY apart from phases. In practice, this case of Eq. (24) give us the largest SUSY contribution in our prediction because the SUSY one is symmetric for $\delta_{23}^{DR}$ and $\delta_{23}^{DL}$ in our framework. Therefore, our predicted region of the CP violations is not changed even if this assumption is relaxed.

### 4 Numerical Results

We show predicted numerical results of the CP violation in our framework. Let us start with presenting the SM prediction of the direct CP asymmetry of the $B^0 \rightarrow K^+\pi^-$ process

$$A(B^0 \rightarrow K^+\pi^-) = \frac{|\bar{A}(B^0 \rightarrow K^-\pi^+)|^2 - |A(B^0 \rightarrow K^+\pi^-)|^2}{|\bar{A}(B^0 \rightarrow K^-\pi^+)|^2 + |A(B^0 \rightarrow K^+\pi^-)|^2}.$$

(25)

The predicted asymmetry depends on $|V_{ub}|$ and $\gamma$ in the SM. We show it versus $|V_{ub}|$ in Figure 1(a), where the recent measurements of $|V_{ub}|$ and $\gamma$ are taken as follows [56]:

$$|V_{ub}| = (3.82 \pm 0.56) \times 10^{-3}, \quad \gamma = (70.8 \pm 7.8)^\circ,$$

(26)

and other input parameters in our calculation are summarized in Table 1.

As seen in Figure 1(a), the SM prediction completely agrees with the observed value $-0.0082 \pm 0.006$ [13]. The predicted asymmetry is linear dependent on $|V_{ub}|$. As far as $|V_{ub}| = (3.2 - 4.2) \times 10^{-3}$, our prediction is successful. Our prediction is not sensitive to $\gamma$ in the region of $\gamma = (70.8 \pm 7.8)^\circ$ since $\sin \gamma$ is not so changed. More precise data of the asymmetry and $|V_{ub}|$ is crucial test of our SM prediction with the QCD factorization.
\[ \alpha_s(M_Z) = 0.1184 \]
\[ m_s(2\text{GeV}) = 0.095 \text{ GeV} \]
\[ m_c(m_c) = 1.275 \text{ GeV} \]
\[ m_b(m_b) = 4.18 \text{ GeV} \]
\[ m_t(m_t) = 160.0 \text{ GeV} \text{ (MS)} \]
\[ M_{B_s} = 5.36677(24) \text{ GeV} \]
\[ \Delta M_{B_s} = (116.942 \pm 0.1564) \times 10^{-13} \text{ GeV} \]
\[ f_{B_s} = (233 \pm 10) \text{ MeV} \]
\[ f_{\pi} = (130.7 \pm 0.4) \text{ MeV} \]
\[ f_K = (156.1 \pm 1.1) \text{ MeV} \]
\[ \lambda = 0.2255(7) \]
\[ |V_{cb}| = (4.12 \pm 0.11) \times 10^{-2} \]

Table 1: Input parameters in our calculation.

We also present the CP averaged branching ratio versus the form factor \( F^{B^0 \rightarrow K}(0) \) in Figure 1(b), in which the magnitude of the form factor is taken to be \( F^{B^0 \rightarrow K}(0) = 0.26 - 0.42 \) GeV. The CP averaged branching ratio is also consistent with the observed one if \( F^{B^0 \rightarrow K}(0) = 0.37 - 0.42 \). We omit figures of the \( |V_{ub}| \) and \( \gamma \) dependences of the branching ratio because it is insensitive to \( |V_{ub}| \) and \( \gamma \).

Figure 1: Predictions of (a) the asymmetry versus \( |V_{ub}| \) and (b) the branching ratio versus the form factor \( F^{B^0 \rightarrow K}(0) \) in the \( B^0 \rightarrow K^+\pi^- \) decay. The inside between dashed red lines denotes the experimental allowed region at 90\%C.L.

The agreement between the SM prediction and the experimental data indicates that the SUSY contribution is constrained severely by the direct CP violation of \( B^0 \rightarrow K^-\pi^+ \). We have searched the allowed parameter region of \( \delta_{23}^{dL(dR)} \) by scattering the magnitude of \( \delta_{23}^{dL(dR)} \) and these phases in the region of \( 0 \sim 0.1 \) and \( -\pi \sim \pi \), respectively. These parameters are constrained by the mass difference \( \Delta M_{B_s} \), the CP violating phase \( \phi_s \) in \( B_s \rightarrow J/\psi\phi \) decay and the branching ratio of the \( b \rightarrow s\gamma \) decay. In addition to these data, the asymmetry of \( A(B^0 \rightarrow K^-\pi^+) \) constrains the magnitude of \( \delta_{23}^{dL(dR)} \). We show the predicted asymmetry

\[
\begin{array}{c}
\end{array}
\]
versus the magnitude of $\delta^{dL(dR)}_{23}$ in Figure 2, where its phase is taken in $-\pi \sim \pi$. It is found that the SUSY contribution becomes important in the region of $|\delta^{dL(dR)}_{23}| \geq 0.01$.

We also present the predicted branching ratio of the $b \to s\gamma$ decay versus the magnitude of $\delta^{dL(dR)}_{23}$ in Figure 3. The significant contribution of the SUSY effect is also seen in the region of $|\delta^{dL(dR)}_{23}| \geq 0.01$.

![Figure 2](image1.png) ![Figure 3](image2.png)

Figure 2: The predicted $A(\bar{B}^0 \to K^-\pi^+)$ versus $|\delta^{dL(R)}_{23}|$. The inside between dashed red lines denotes the experimental allowed region at 90% C.L.

Figure 3: The predicted branching ratio of $b \to s\gamma$ versus $|\delta^{dL(dR)}_{23}|$. The inside between dashed red lines denotes the experimental allowed region at 90% C.L.

Let us show the allowed region on the plane of $|\delta^{dL(R)}_{23}|$ and those phases, taking account of $\Delta M_{B_s}$, $\phi_s$ in $B_s \to J/\psi\phi$ decay, the branching ratio of $b \to s\gamma$, and the asymmetry $A(\bar{B}^0 \to K^-\pi^+)$. The input experimental data are taken at 90% C.L. We present the allowed region of $|\delta^{dL(R)}_{23}|$ versus (arg $\delta_{23}^{dL} + \text{arg } \delta_{23}^{dR}$) in Figure 4(a), and versus (arg $\delta_{23}^{dL} - \text{arg } \delta_{23}^{dR}$) in Figure 4(b) with $|\delta_{23}^{dL}| = |\delta_{23}^{dR}|$, respectively. It is found that the squark flavor mixing is allowed in the region of $|\delta_{23}^{dL}| \leq 0.02$ for all region of the phase. If two phases arg $\delta_{23}^{dL}$ and arg $\delta_{23}^{dR}$ are tuned to suppress the imaginary part, $|\delta_{23}^{dL}|$ is allowed up to 0.05.

Now we can predict the CP violations of the $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$ decays under the constraint of $\delta_{23}^{dL}$ of Figure 4. We show the predicted regions among $C_{K^+K^-}$, $S_{K^+K^-}$, $C_{K^0\bar{K}^0}$ and $S_{K^0\bar{K}^0}$ in Figures 5(a)-5(d). As seen in Figure 5(a), the predicted region of $C_{K^+K^-}$ is strongly cut by the constraint from the direct CP violation of $\bar{B}^0 \to K^-\pi^+$. Therefore, the deviation from the SM prediction of $C_{K^+K^-}$ is not found. On the other hand, $S_{K^+K^-}$ is possibly deviated from the SM prediction considerably, that is expected to be in $0.1 \sim 0.5$. The precise measurement of $S_{K^+K^-}$ is important to search for the SUSY effect.

As seen in Figure 5(b), the SM predictions of $C_{K^0\bar{K}^0}$ and $S_{K^0\bar{K}^0}$ are very small since we have

$$\frac{\bar{A}(\bar{B}_s \to K^0\bar{K}^0)}{A(B_s \to K^0\bar{K}^0)} \simeq \frac{V_{tb}V_{ts}^*}{V_{tb}V_{ts}} \, q \simeq \frac{V_{tb}V_{ts}^*}{V_{tb}V_{ts}}, \quad \lambda_{K^0\bar{K}^0} \simeq 1,$$

where the CKM matrix elements canceled out each other in $\lambda_{K^0\bar{K}^0}$. Since the SUSY contribution violates this cancellation, we expect the observation of the CP violation for both $C_{K^0\bar{K}^0}$ and

\[ \text{(27)} \]
and $S_{K^0\bar{K}^0}$ in the $B_s \to K^0\bar{K}^0$ decay. These predicted magnitudes are roughly proportional to each other in the region $C_{K^0\bar{K}^0} = -0.06 \sim 0.06$ and $S_{K^0\bar{K}^0} = -0.5 \sim 0.3$.

We show the correlations between $C_{K^0\bar{K}^0}$ and $C_{K^+K^-}$ in Figures 5(c), and between $S_{K^0\bar{K}^0}$ and $S_{K^+K^-}$ in Figures 5(d), respectively. While the predicted value of $C_{K^+K^-}$ is restricted around 0.1, $C_{K^0\bar{K}^0}$ is expected in the region of $-0.06 \sim 0.06$. On the other hand, $S_{K^0\bar{K}^0}$ is roughly proportional to $S_{K^+K^-}$, which gives us a crucial test for the SUSY contribution.

5 Summary

In order to search for the gluino-squark mediated flavor changing effect, we have studied the CP violations in the $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$ processes, in which the $b \to s$ transition penguin amplitudes dominate the decays. We have searched for the allowed region of the flavor mixing $\delta_{23}^{dL}$, by putting the experimental data the mass difference $\Delta M_{B_s}$, the CP violating phase $\phi_s$ in $B_s \to J/\psi \phi$ decay and the $b \to s\gamma$ branching ratio. In addition to these data, we have taken into account the constraint from the asymmetry of $B^0 \to K^+\pi^-$ because the $B_s \to K^+K^-$ decay is related with the $B^0 \to K^+\pi^-$ decay by replacing the spectator $s$ with $d$. We have obtained the constraint of $|\delta_{23}^{dL}| \leq 0.05$.

Under the constraint, we have predicted the CP violations in the $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$ decays. The predicted region of the CP violation $C_{K^+K^-}$ is strongly cut by the constraint from the direct CP violation of $\bar{B}^0 \to K^-\pi^+$, which is well agreement with the SM prediction with the QCD factorization calculation. Therefore, the deviation from the SM prediction of $C_{K^+K^-}$ is not expected. On the other hand, $S_{K^+K^-}$ is possibly deviated from the SM prediction considerably, in the region of $0.1 \sim 0.5$. Since the SM predictions of $C_{K^0\bar{K}^0}$ and $S_{K^0\bar{K}^0}$ are tiny, the SUSY contribution is expected to be detectable in $C_{K^0\bar{K}^0}$ and $S_{K^0\bar{K}^0}$. These expected magnitudes are in the region $C_{K^0\bar{K}^0} = -0.06 \sim 0.06$ and $S_{K^0\bar{K}^0} = -0.5 \sim 0.3$. We expect more precise data of the CP violations in these decays, which provide us a crucial test for the SUSY contribution.
Figure 5: The predicted CP violations of (a) $C_{K^+K^-} - S_{K^+K^-}$, (b) $C_{K^0\bar{K}^0} - S_{K^0\bar{K}^0}$, (c) $C_{\bar{K}^0\bar{K}^0} - C_{K^+K^-}$, and (d) $S_{\bar{K}^0\bar{K}^0} - S_{K^+K^-}$. The inside between dashed red lines denotes the experimental allowed region at 90% C.L., and yellow regions denote the SM predictions.

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Appendix

A Loop integral in penguins

The loop integrals in Eq. (5) are given as follows [38, 39]:
$$F_{M_2} = -12 \ln \frac{\mu}{m_b} - 18 + f^I_{M_2} + f^I_{M_2},$$

$$f^I_{M_2} = \int_0^1 dx \, g(x) \phi_{M_2}(x), \quad g(x) = \frac{3(1 - 2x)}{1 - x} \ln x - 3i\pi,$$

$$f^I_{M_2} = \frac{4\pi^2}{N} \frac{f_M f_B}{f^0_{+m} M_B^2} \int_0^1 dz \frac{\phi_B(z)}{z} \int_0^1 dx \frac{\phi_{M_1}(x)}{x} \int_0^1 dy \frac{\phi_{M_2}(y)}{y},$$

$$G_{M_2,g} = -\int_0^1 dx \frac{2}{x} \phi_{M_2}(x),$$

$$G_{M_2(s_g)} = \frac{2}{3} - \frac{4}{3} \ln \frac{\mu}{m_b} + 4 \int_0^1 dx \phi_{M_2}(x) \int_0^1 du \, u \bar{u} \ln[s_g - u \bar{x} - i\epsilon],$$

$$G'_{M_2,g} = -\int_0^1 dx \frac{3}{2} \phi_{M_2}(x) = -\frac{3}{2},$$

$$G'_{M_2(s_g)} = \frac{1}{3} - \ln \frac{\mu}{m_b} + 3 \int_0^1 dx \phi^0_{M_2}(x) \int_0^1 du \, u \bar{u} \ln[s_g - u \bar{x} - i\epsilon],$$

where $\bar{x} = 1 - x$ and $\bar{u} = 1 - u$. The internal quark mass in the penguin diagrams enters as $s_f = m_f^2/m_b^2$. The functions $\phi(x)$ and $\phi^0(x)$ are meson’s leading-twist distribution amplitude and twist-3 distribution amplitude, respectively. For $\pi$ and $K$ mesons, we use well known form [58, 59]:

$$\phi_{\pi,K}(x) = 6x(1 - x), \quad \phi^0_{\pi,K}(x) = 1. \quad (29)$$

For the $B$ meson, we use [60, 61, 62]

$$\phi_B(x) = N_B x^2(1 - x)^2 \exp \left[ -\frac{M_B^2 x^2}{2 \omega_B^2} \right], \quad (30)$$

where $\omega_B = 0.4\text{GeV}$, and $0.5\text{GeV}$ for the $B^0$ and $B_s$ mesons, respectively, and $N_B$ is the normalization constant to make $\int_0^1 dx \phi_B(x) = 1$.

**B Squark contribution in $\Delta B = 1$ process**

The Wilson coefficients for the gluino contribution in Eq.(11) are written as [63]

$$C_{7\gamma}(m_\tilde{g}) = \frac{8 \sqrt{2} \alpha_s \pi}{3 2 G_F V_{tb} V_{tq}^*} \times \left[ \left( \frac{\Gamma_{GL}^{(d)}}{m^2_{d_3}} \right)^* \left( \frac{\Gamma_{GR}^{(d)}}{m^2_{d_3}} \right)^* \left( \frac{1}{3} F_2(x^3_{\tilde{g}}) \right) + \frac{m_\tilde{g}}{m_b} \left( \frac{\Gamma_{GR}^{(d)}}{m^2_{d_3}} \left( \frac{1}{3} F_4(x^3_{\tilde{g}}) \right) \right) \right] + \frac{\Gamma_{GL}^{(d)}}{m^2_{d_3}} \left( \frac{\Gamma_{GL}^{(d)}}{m^2_{d_3}} \left( \frac{1}{3} F_2(x^3_{\tilde{g}}) \right) + \frac{m_\tilde{g}}{m_b} \left( \frac{\Gamma_{GR}^{(d)}}{m^2_{d_3}} \left( \frac{1}{3} F_4(x^3_{\tilde{g}}) \right) \right) \right), \quad (31)$$

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\[ C_{8G}(m_{\tilde{g}}) = \frac{8}{3} \sqrt{2} \alpha_s \pi \frac{\Gamma^{(d)}_{GL} k_3}{m_{d}^2} \left\{ \left( \Gamma^{(d)}_{GL} \right)_{33} \left( -\frac{9}{8} F_1(x_{\tilde{g}}^2) - \frac{1}{8} F_2(x_{\tilde{g}}^2) \right) + \frac{m_{\tilde{g}}}{m_b} \left( \Gamma^{(d)}_{GR} \right)_{33} \left( -\frac{9}{8} F_3(x_{\tilde{g}}^3) - \frac{1}{8} F_4(x_{\tilde{g}}^3) \right) \right\} \]

where \( k = 2, 1 \) correspond to \( b \rightarrow q \) \((q = s, d)\) transitions, respectively. The loop functions \( F_i(x_{\tilde{g}}^I) \) are given as

\[ F_1(x_{\tilde{g}}^I) = \frac{x_{\tilde{g}}^I \log x_{\tilde{g}}^I}{2(x_{\tilde{g}}^I - 1)^4} + \frac{(x_{\tilde{g}}^I)^2 - 5x_{\tilde{g}}^I - 2}{12(x_{\tilde{g}}^I - 1)^3}, \]
\[ F_2(x_{\tilde{g}}^I) = -\frac{(x_{\tilde{g}}^I)^2 \log x_{\tilde{g}}^I}{2(x_{\tilde{g}}^I - 1)^4} + \frac{2(x_{\tilde{g}}^I)^2 + 5x_{\tilde{g}}^I - 1}{12(x_{\tilde{g}}^I - 1)^3}, \]
\[ F_3(x_{\tilde{g}}^I) = \log x_{\tilde{g}}^I \frac{(x_{\tilde{g}}^I - 1)^3}{(x_{\tilde{g}}^I - 1)^2} + \frac{x_{\tilde{g}}^I - 3}{2(x_{\tilde{g}}^I - 1)^2}, \]
\[ F_4(x_{\tilde{g}}^I) = -\frac{x_{\tilde{g}}^I \log x_{\tilde{g}}^I}{(x_{\tilde{g}}^I - 1)^3} + \frac{x_{\tilde{g}}^I + 1}{2(x_{\tilde{g}}^I - 1)^2} = \frac{1}{2} g_{2[1]}(x_{\tilde{g}}^I, x_{\tilde{g}}^I), \]

with \( x_{\tilde{g}}^I = m_b^2/m_{d}^2 \) \((I = 3, 6)\). The NLO of these Wilson coefficients are omitted. We also omit other Wilson coefficients which are the NLO contributions to our numerical calculations. The Wilson coefficients \( C_{8G}(m_{\tilde{g}}) \)'s are obtained by replacing \( L(R) \) with \( R(L) \) in \( C_{8G}(m_{\tilde{g}}) \)'s.

The Wilson coefficients of \( C_{7G}(m_b) \) and \( C_{8G}(m_b) \) at the \( m_b \) scale are given at the leading order of QCD as follows \[40\]:

\[ C_{7G}(m_b) = \zeta C_{7G}(m_{\tilde{g}}) + \frac{8}{3} (\eta - \zeta) C_{8G}(m_{\tilde{g}}), \]
\[ C_{8G}(m_b) = \eta C_{8G}(m_{\tilde{g}}), \]

where

\[ \zeta = \left( \frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_t)} \right)^{\frac{16}{27}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{16}{27}}, \quad \eta = \left( \frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_t)} \right)^{\frac{14}{27}} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{\frac{14}{27}}. \]

### C Squark contribution in \( \Delta B = 2 \) process

The \( \Delta B = 2 \) effective Lagrangian from the gluino-sbottom-quark interaction is given as

\[ \mathcal{L}_{\text{eff}, \Delta B=2} = -\frac{1}{2} \left[ C_{V LL} O_{V LL} + C_{V RR} O_{V RR} \right] \]
\[ -\frac{1}{2} \sum_{i=1}^{2} \left[ C^{(i)}_{S LL} O^{(i)}_{S LL} + C^{(i)}_{S RR} O^{(i)}_{S RR} + C^{(i)}_{S LR} O^{(i)}_{S LR} \right], \]

13
then, the $P^0$-$P^0$ mixing, $M_{12}$, is written as
\begin{equation}
M_{12} = -\frac{1}{2m_P}\langle P^0|\mathcal{L}_{\text{eff}}^{F=2}|P^0 \rangle. \tag{37}
\end{equation}

The hadronic matrix elements are given in terms of the non-perturbative parameters $B_i$ as:
\begin{align}
\langle P^0|\mathcal{O}_{VLL}|P^0 \rangle &= \frac{2}{3}m_p^2f_p^2B_1, \quad \langle P^0|\mathcal{O}_{VRR}|P^0 \rangle = \langle P^0|\mathcal{O}_{VLL}|P^0 \rangle, \\
\langle P^0|\mathcal{O}_{SLL}^{(1)}|P^0 \rangle &= -\frac{5}{12}m_p^2f_p^2B_2, \quad \langle P^0|\mathcal{O}_{SRR}^{(1)}|P^0 \rangle = \langle P^0|\mathcal{O}_{SLL}^{(1)}|P^0 \rangle, \\
\langle P^0|\mathcal{O}_{SLL}^{(2)}|P^0 \rangle &= \frac{1}{12}m_p^2f_p^2B_3, \quad \langle P^0|\mathcal{O}_{SRR}^{(2)}|P^0 \rangle = \langle P^0|\mathcal{O}_{SLL}^{(2)}|P^0 \rangle, \\
\langle P^0|\mathcal{O}_{SLR}^{(1)}|P^0 \rangle &= \frac{1}{2}m_p^2f_p^2B_4, \quad \langle P^0|\mathcal{O}_{SLR}^{(2)}|P^0 \rangle = \frac{1}{6}m_p^2f_p^2B_5, \tag{38}
\end{align}

where
\begin{equation}
R_P = \left(\frac{m_P}{m_Q + m_q}\right)^2, \tag{39}
\end{equation}

with $(P, Q, q) = (B_d, b, d), (B_s, b, s)$.

The Wilson coefficients for the gluino contribution in Eq. (39) are written as
\begin{align}
C_{VLL}(m_{\tilde{g}}) &= \alpha_s^2 \frac{m_{\tilde{g}}}{g^2} \sum_{I,J=1}^{6} (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GLL}^{(d)})_{ji} \left[ \frac{11}{18} g_{2[1]}(x_I^g, x_J^g) + \frac{2}{9} g_{1[1]}(x_I^g, x_J^g) \right], \\
C_{VRR}(m_{\tilde{g}}) &= C_{VLL}(m_{\tilde{g}})(L \leftrightarrow R), \\
C_{SRR}^{(1)}(m_{\tilde{g}}) &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{I,J=1}^{6} (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GLL}^{(d)})_{ji} \frac{17}{9} g_{1[1]}(x_I^g, x_J^g), \\
C_{SLL}^{(1)}(m_{\tilde{g}}) &= C_{SRR}^{(1)}(m_{\tilde{g}})(L \leftrightarrow R), \\
C_{SRR}^{(2)}(m_{\tilde{g}}) &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{I,J=1}^{6} (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GLL}^{(d)})_{ji} \left( -\frac{1}{3} \right) g_{1[1]}(x_I^g, x_J^g), \\
C_{SLL}^{(2)}(m_{\tilde{g}}) &= C_{SRR}^{(2)}(m_{\tilde{g}})(L \leftrightarrow R), \\
C_{SLR}^{(1)}(m_{\tilde{g}}) &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{I,J=1}^{6} \left\{ (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GLL}^{(d)})_{ji} \left( -\frac{11}{9} \right) g_{2[1]}(x_I^g, x_J^g) + (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GLL}^{(d)})_{ji} \left[ \frac{14}{3} g_{1[1]}(x_I^g, x_J^g) - \frac{2}{3} g_{2[1]}(x_I^g, x_J^g) \right] \right\}, \\
C_{SLR}^{(2)}(m_{\tilde{g}}) &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \sum_{I,J=1}^{6} \left\{ (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GLL}^{(d)})_{ji} \left( -\frac{5}{3} \right) g_{2[1]}(x_I^g, x_J^g) + (\lambda_{GLL}^{(d)})_{ij} (\lambda_{GLL}^{(d)})_{ji} \left[ \frac{2}{9} g_{1[1]}(x_I^g, x_J^g) + \frac{10}{9} g_{2[1]}(x_I^g, x_J^g) \right] \right\}. \tag{40}
\end{align}
where
\[
(\lambda_{GLL})_{ij}^d = (\Gamma_{GL})_{i}^d K_{GL} (\Gamma_{GL})_{j}^d, \quad (\lambda_{GR})_{ij}^d = (\Gamma_{GR})_{i}^d K_{GR} (\Gamma_{GR})_{j}^d, \\
(\lambda_{GLL})_{ij}^f = (\Gamma_{GL})_{i}^f K_{GL} (\Gamma_{GL})_{j}^f, \quad (\lambda_{GR})_{ij}^f = (\Gamma_{GR})_{i}^f K_{GR} (\Gamma_{GR})_{j}^f.
\]
(41)

Here we take \((i, j) = (1, 3), (2, 3)\) which correspond to the \(B^0\) and \(B_s\) mesons, respectively. The loop functions are given as follows:

- If \(x_I^g \neq x_J^g\) \((x_{I,J}^g = m_{d_{I,J}}^2/m_{b}^2)\),
  \[
g_{1I}(x_I^g, x_J^g) = \frac{1}{x_I^g - x_J^g} \left( \frac{x_I^g \log x_I^g}{x_I^g - 1} - 1 - \frac{x_J^g \log x_J^g}{x_J^g - 1} + \frac{1}{x_I^g - 1} + \frac{1}{x_J^g - 1} \right),
\]
  \[
g_{2I}(x_I^g, x_J^g) = \frac{1}{x_I^g - x_J^g} \left( \frac{x_I^g \log x_I^g}{x_I^g - 1} - 1 - \frac{x_J^g \log x_J^g}{x_J^g - 1} + \frac{1}{x_I^g - 1} + \frac{1}{x_J^g - 1} \right). \tag{42}
\]

- If \(x_I^g = x_J^g\),
  \[
g_{1I}(x_I^g, x_I^g) = -\frac{(x_I^g + 1) \log x_I^g}{(x_I^g - 1)^3} + \frac{2}{(x_I^g - 1)^2},
\]
  \[
g_{2I}(x_I^g, x_I^g) = -\frac{2x_I^g \log x_I^g}{(x_I^g - 1)^3} + \frac{x_I^g + 1}{(x_I^g - 1)^2}. \tag{43}
\]

In this paper, we take \((I, J) = (3, 3), (3, 6), (6, 3), (6, 6)\), because we assume the split-family. The effective Wilson coefficients are given at the leading order of QCD as follows:

\[
C_{VLL}(m_b) = \eta_{VLL}^B C_{VLL}(m_{b}), \quad C_{VRR}(m_b) = \eta_{VRR}^B C_{VLL}(m_{b}), \\
(C^{(1)}_{SLL}(m_b), C^{(2)}_{SLL}(m_b)) = (C^{(1)}_{SLL}(m_{b}), C^{(2)}_{SLL}(m_{b})) X_{LL}^{-1} \eta_{LL}^B X_{LL}, \\
(C^{(1)}_{SRR}(m_b), C^{(2)}_{SRR}(m_b)) = (C^{(1)}_{SRR}(m_{b}), C^{(2)}_{SRR}(m_{b})) X_{RR}^{-1} \eta_{RR}^B X_{RR}, \\
(C^{(1)}_{SLR}(m_b), C^{(2)}_{SLR}(m_b)) = (C^{(1)}_{SLR}(m_{b}), C^{(2)}_{SLR}(m_{b})) X_{LR}^{-1} \eta_{LR}^B X_{LR}, \tag{44}
\]

where

\[
\eta_{VLL}^B = \eta_{VRR}^B = \left( \frac{\alpha_s(m_{b})}{\alpha_s(m_t)} \right)^{\frac{6}{21}} \left( \frac{\alpha_s(m_{t})}{\alpha_s(m_{b})} \right)^{\frac{6}{21}},
\]
\[
\eta_{LL}^B = \eta_{RR}^B = S_{LL} \left( \frac{d_{LL}^{d_{LL}}}{d_{LL}^{d_{LL}}} \frac{d_{LL}^{d_{LL}}}{d_{LL}^{d_{LL}}} \right) S_{LL}^{-1}, \quad \eta_{LR}^B = S_{LR} \left( \frac{d_{LR}^{d_{LR}}}{d_{LR}^{d_{LR}}} \frac{d_{LR}^{d_{LR}}}{d_{LR}^{d_{LR}}} \right) S_{LR}^{-1}, \\
\eta_{b_{g}} = \left( \frac{\alpha_s(m_{b})}{\alpha_s(m_{t})} \right)^{\frac{1}{24}} \left( \frac{\alpha_s(m_{t})}{\alpha_s(m_{b})} \right)^{\frac{3}{48}}, \tag{45}
\]

15
\[ d_{LL}^1 = \frac{2}{3}(1 - \sqrt{241}), \quad d_{LL}^2 = \frac{2}{3}(1 + \sqrt{241}), \quad d_{LR}^1 = -16, \quad d_{LR}^2 = 2, \]
\[ S_{LL} = \begin{pmatrix} \frac{16+\sqrt{241}}{60} & \frac{16-\sqrt{241}}{60} \\ 1 & 1 \end{pmatrix}, \quad S_{LR} = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}, \]
\[ X_{LL} = X_{RR} = \begin{pmatrix} 1 & 0 \\ 4 & 8 \end{pmatrix}, \quad X_{LR} = \begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}. \]

(46)

For the parameters $B_i^{(a)}(i = 2 - 5)$ of the $B_s$ meson, we use values in [64] as follows:
\[ B_2^{(B_s)}(m_b) = 0.80(1)(4), \quad B_3^{(B_s)}(m_b) = 0.93(3)(8), \]
\[ B_4^{(B_s)}(m_b) = 1.16(2)(^{+5}_{-7}), \quad B_5^{(B_s)}(m_b) = 1.75(3)(^{+21}_{-6}). \]

(47)

On the other hand, we use the most updated value for $\hat{B}_1^{(a)}$ as [56, 65]
\[ \hat{B}_1^{(B_s)} = 1.33 \pm 0.06. \]

(48)

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