High-sensitivity bio-sensor based on the real-splitting indirectly coupled Anti-Parity time symmetric WGMs

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Detecting the size of single nanoparticle with high precision is crucial to understanding the characteristic of the nanoparticle. In this paper, we research the single particle detection based on the Anti-parity time symmetric (APT) indirectly coupled WGMs. The results show that the Anti-parity time symmetric WGM nanoparticle sensor exhibits giant enhancement in frequency splitting compared with single WGM sensor, when the system operating at exceptional point (EP). With respect to the parity-time symmetric nanoparticle sensor, our research exhibits a real eigenfrequency splitting, which can be directly detected.

1. Introduction

In Parity-time symmetry, a non-hermitian Hamiltonian $H$, defined as $[\text{PT}, H] = 0$, can exhibit an entirely real eigenenergy spectra under the Parity-time symmetry broken phase, which has attracted much attention in recent years [1-4]. In Optical systems, optical microcavity provides an ideal platform for researching non-hermitian physics, because the complex potentials in non-Hermitian quantum systems can be easily realized, through tuning gain or loss rates of the microcavity [3,4]. Most of the novel phenomena of non-Hermitian systems are found at the Exceptional point (EP), in particular, non-Hermitian systems operating at EP can pave a new way to enhance sensitivity [5-8]. In non-Hermitian configurations, the eigenfrequency splitting is $\Delta \omega \in \varepsilon^{1/N}$, representing that the sensitivity of the system will increase with the N orders of the EP, where N is numbers of eigenvalues coalesce at EP and the $\varepsilon$ is the perturbation. Recently, the second-order and high-order EP are observed in parity–time symmetric optical microcavity systems and Hossein Hodaei has researched corresponding eigenstates coalesce to achieve higher sensitivity that is proportional to the cube root of
the perturbation [9].

Now, Anti-Parity time symmetric configurations, which obey the \{PT, H\} = 0, also has drawn interests [10-13]. Different from the Parity-time symmetry, the optical systems with APT should satisfy \(n(x)=n^*(-x)\) and the purely imaginary coupling. Anti-Parity time symmetry has been investigated in Multi-waveguides [14,15], electrical circuit resonators [16], balanced positive–negative index multilayers [12] and cold-atom lattices [13]. It is worth noting that Anti-PT-symmetric systems also exists exceptional points [15,16], although there are very limited works on EP in Anti-PT-symmetric configurations is used to enhance the sensitivity, Martino De Carlo investigated the real-splitting anti-PT-symmetric microscale optical gyroscope [17].

Traditionally, the counterpropagating modes clockwise (CW) mode and counterclockwise (CCW) mode, which have a degenerate frequency, can coexist in WGM microcavity. When a nanoparticle enters the optical mode of the cavity, the interaction between evanescent and nanoparticle will lift the degeneracy, causing frequency splitting [18-23]. The mode splitting introduced by the nanoparticle in principle is proportional to the coupled strength of perturbation \(\varepsilon\), which limits the sensitivity and detection limit of WGM nanoparticle sensor. Interestingly, optical microcavity operating at exceptional points can be used for enhanced biosensing [24]. The second-order EP system in parity-time symmetry has square-root at the complex frequency, therefore, eigenfrequency splitting induced by a perturbation \(\varepsilon\) is proportional to \((i\varepsilon)^{1/2}\) when is utilized for nanoparticle detection [25]. The eigenfrequency splitting in parity-time symmetric system induced by the perturbation is complex, which is difficult for the signal read out.

In this work, we used a structure, the indirectly coupled WGMs through two common waveguides, to realize the Anti-parity time symmetric condition. The APT nanoparticle sensor operating at the EP exhibits a real frequency splitting and a square-root dependence of the sensitivity on the perturbation, and the sensitivity can be giant enhancement compared to single WGM. With respect to the generally complex eigenfrequency splitting in Parity-time symmetric nanoparticle sensor, Anti-parity time symmetry can exhibit a real eigenfrequency splitting and this configuration can be
achieved more easily constructed than the requirement on gain in the PT-symmetric coupled microcavity.

2. Anti-PT-Symmetric single nanoparticle sensor

Fig.1 (a) The configuration of the the Anti-parity time symmetric indirectly coupled microresonators; (b) The schematic of the Anti-parity time symmetric indirectly coupled microresonators sensor, the WGM1 and the WGM2 coupled through two common waveguides.

In the case of Parity-time symmetry, eigenvalues of non-hermitian Hamiltonians can be entirely real, i.e. $[PT, H] = 0$ [1]. For Anti-parity time symmetric condition, the Hamiltonian should satisfy the condition $\{PT, H\} = 0$ [10]. Generally, we use an active cavity directly couples a passive cavity with balanced gain and loss to realize the PT-symmetric system, and both the cavities have the same resonance frequency. The APT can be implemented by two lossy cavities with an effective dissipative coupling between mode $a$ and mode $b$ showed in Fig.1(a) and the effective Hamiltonian is

$$H = \hbar (\Delta_a - i\gamma_2)a^\dagger a + \hbar (\Delta_b - i\gamma_2)b^\dagger b + \hbar (\kappa_a b + \kappa_b b^\dagger a)$$

(1)

Where $\Delta_a (\Delta_b)$ is the effective detuning, $\gamma_1 (\gamma_2)$ is the effective loss rate of the microresonator, $\kappa_1 (\kappa_2)$ is the coupling coefficient between the mode $a$ and mode $b$. The Hamiltonian is Anti-parity time symmetry, when we provide that $\Delta_a = -\Delta_b = \Delta$, $\gamma_1 = \gamma_2$, $\kappa_1 = -\kappa_2^\ast$, and the mode eigenfrequencies are

$$\omega_{1,2} = -i\gamma \pm \sqrt{\Delta^2 - \kappa^2}$$

(2)

where $\Delta = (\omega_a - \omega_b)/2$. In the Anti-Parity time symmetric phase ($\kappa > \Delta$), the eigenmodes take the degenerate resonance frequency but different decay rates. $\kappa < \Delta$ corresponds to the APT symmetric broken phase, in which eigenmodes take
nondegenerate resonance frequency but the same decay rate. The numerical verifications are depicted in Fig.2. The $\kappa_{APT} = \Delta$ is the exceptional point (EP) of the APT indirectly coupled resonators.

\[ H = H_0 + H_1 + H_2 \]
\[ H_0 = \hbar \omega_a a_\text{cw}^{\dagger} a_\text{cw} + \hbar \omega_a a_\text{ccw}^{\dagger} a_\text{ccw} + \hbar \omega_b b_\text{cw}^{\dagger} b_\text{cw} + \hbar \omega_b b_\text{ccw}^{\dagger} b_\text{ccw} \]
\[ H_1 = \sum_{m,m'=\text{cw},\text{ccw}} \hbar g a_m^{\dagger} a_{m'} \]
\[ H_2 = \hbar (\kappa_1 a_\text{cw}^{\dagger} b_\text{cw} + \kappa_2 b_\text{cw}^{\dagger} a_\text{cw}) + \hbar (\kappa_1 a_\text{ccw}^{\dagger} b_\text{ccw} + \kappa_2 b_\text{ccw}^{\dagger} a_\text{ccw}) \]

Fig.2 (a) is the real part and (b) is the imaginary part of eigenfrequencies in Anti-Parity time symmetric indirectly coupled resonators when the coupling coefficient $\kappa$ is varied.

In this paper, two indirectly coupled WGMs through the common waveguides are used to realize the Anti-parity symmetry (APT) showed in Fig.1(b). When a nanoparticle enters the mode of the microresonator, the optical evanescent fields of the WGM will be scattered by the nanoparticle. The mode $a_{\text{cw}}$ and $a_{\text{ccw}}$ are coupled by backscattering in WGM2 and they indirectly coupled with mode $b_{\text{cw}}$ and $b_{\text{ccw}}$ in WGM1 through the waveguide, respectively. When we set the system at EP, a nanoparticle with perturbation strength $g$ is introduced into the WGM mode, the perturbation non-Hermitian Hamiltonian in the traveling-wave of the single nanoparticle can be written as
two WGMs. The corresponding eigenfrequencies in APT system are

\[
\omega_{\text{APT},1} = \frac{\omega_a + \omega_b}{2} - i\gamma + \sqrt{\left(\frac{\omega_a - \omega_b}{2}\right)^2 + \kappa^2},
\]

\[
\omega_{\text{APT},2} = \frac{\omega_a + \omega_b}{2} - i\gamma - \sqrt{\left(\frac{\omega_a - \omega_b}{2}\right)^2 + \kappa^2},
\]

\[
\omega_{\text{APT},3} = \frac{\omega_a + \omega_b}{2} - i\gamma + g + \sqrt{\left(\frac{\omega_a - \omega_b}{2}\right)^2 + g^2 + g\left(\frac{\omega_a - \omega_b}{2}\right) + \kappa^2},
\]

\[
\omega_{\text{APT},4} = \frac{\omega_a + \omega_b}{2} - i\gamma + g - \sqrt{\left(\frac{\omega_a - \omega_b}{2}\right)^2 + g^2 + g\left(\frac{\omega_a - \omega_b}{2}\right) + \kappa^2},
\]

(4)

Fig. 3 (a) and (c) The normalized real part and the image part of the eigenfrequencies in APT indirectly coupled resonators varies with the coupled coefficient \(\kappa\) when a single nanoparticle incoming to the WGM1, respectively. (b) and (d) The values of the normalized real and image frequency splitting between the perturbed and reference supermodes with the changing of coupling strength \(\kappa\). Here, \(g/\Delta=0.05\).

where, \(\Delta=(\omega_a-\omega_b)/2\) is the effective detuning between the two WGMs, the indirectly coupled strength is \(\kappa_{1,2} = \mu_1 \mu_2 e^{-j\theta_{1,2}}\), and the \(\mu_{1,2}\) is the real coupling between the waveguide and the WGM and the \(\theta_{1,2} = 2\pi n g L / \lambda\). The system will be APT criteria when \(\kappa_1 = -\kappa_2^* = \kappa\), where a possible solution is given by \(\theta_{1,2} = (2m+1)\frac{\pi}{2} (m \in \mathbb{N})\).

In experiment, we can choose appropriate \(L\) to implement that solution. In addition, we
need make the same effective loss rate in each resonator, i.e.,
$$\gamma = \gamma_{1,\text{ext}} - 2\mu^2 - \alpha_1 = \gamma_{2,\text{ext}} - 2\mu^2 - \alpha_2$$
where $\gamma_{1,\text{ext}}$ ($\gamma_{2,\text{ext}}$) are the external gains and $\alpha_1$ ($\alpha_2$) represent the intrinsic losses in resonator, and we need chose $\gamma_{1,\text{ext}}$ ($\gamma_{2,\text{ext}}$) to keep the $\gamma$ negative. Finally, the Heisenberg dynamics equation describing the optical modes in the APT indirectly coupled system is

$$\begin{align*}
\dot{a}_{cw} &= -i(\omega_a + g)a_{cw} - \gamma a_{cw} - i\alpha_{cw}b_{cw}, \\
\dot{a}_{ccw} &= -i(\omega_b + g)a_{ccw} - \gamma a_{ccw} - i\alpha_{ccw}b_{ccw}, \\
\dot{b}_{cw} &= (-i\omega_b - \gamma)b_{cw} - i\kappa c_{cw}, \\
\dot{b}_{ccw} &= (-i\omega_b - \gamma)b_{ccw} - i\kappa c_{ccw} - i\mu_{in},
\end{align*}$$

(5)

Where $\kappa_c = i\mu_1\mu_2$, through adjusting the coupling strength, we can make the system locate the EP. Eq. (5) shows that the nanoparticle lifts the eigenfrequency degeneracy of the supermodes in which two supermodes experience a frequency shift and linewidth change, while the other two supermodes are not affected by the nanoparticle serving as reference signals. Fig.3(b) depicts the frequency difference between the frequencies of the nanoparticle perturbed supermodes and reference supermodes, we can know that the maximum frequency splitting is obtained at the EP, due to the square-root topology. With the increasing of the coupled coefficient, the APT system will come from the APT broken phase to the APT phase, the splitting between the will not increase, in this regime the supermodes among the two WGMs will go through the equal perturbation.

The transmission spectrum $s_{out}/s_{in}$ can be obtained through the relation $s_{out} = s_{in} = i\mu_2 a_{ccw} = i\mu_2 a_{cw}$.

![Fig.4 The output spectrum operating at the exceptional point of an Anti-PT symmetric](image-url)
nanoparticle sensor with the changing of the perturbation strength $g$, with $R_1=20 \, \mu m$, $R_2=19.95 \, \mu m$, $\gamma=0.5/(2\pi) \, MHz$, $\lambda=1.55 \, \mu m$ [17].

From Fig.4, the nanoparticle induced eigenfrequency splitting will move from the exceptional point to the broken phase. Intuitively, two supermodes are not affected by the nanoparticle and have two collapsing eigenfrequencies to make the peak much higher at $g=0$, because it’s like having the sum of two peaks at the same frequency. Whereas the other two eigenfrequencies are affected by the particle, more they depart from each other, the less they influence each other, thus the peaks decrease.

When the system located at the EP, i.e., $|\omega_a - \omega_b| = |2\kappa|$, from the Eq. (5), due to the nanoparticle is very small, i.e., $g \ll \Delta$, thus the eigen frequency splitting can be written as

$$w_{APT_{1,2}} = \frac{\omega_a - \omega_b}{2} - ig + g + g \mu_1 \mu_2$$

which is proportional to the square-root of the coupled coefficient and inversely proportional to the square-root of perturbation strength $g$. For a single WGM nanoparticle sensor with the same perturbation, the eigenfrequency splitting is $\Delta w_{SINGLE} = 2g$, therefore, we can define the sensitivity enhancement factor $S$

$$S = \frac{\Delta w_{APT}}{\Delta w_{SINGLE}} = \sqrt{\frac{\mu_1 \mu_2}{4g}}$$

In Fig.5, for the APT sensor, larger enhancement factor $S$ can be obtained in smaller nanoparticle. If we consider the loss induced by the nanoparticle, the corresponding eigenfrequencies will be $\Delta w_{APT} \cong g \mu_1 \mu_2 + (\gamma_s + \gamma_a) \mu_1 \mu_2$, where $\gamma_s$ represents the scattering loss of the nanoparticle and $\gamma_a$ is the absorption of the target [19]. The loss caused by nanoparticle can increase the splitting, which will lead to a better detection limit.
Fig. 5 (a) The normalized eigenfrequency splitting operating at EP varies with the perturbation strength $g$. (b) The sensitivity enhancement factor $S$ depends on the perturbation $g$. The resonance frequency $\Delta$ and the coupled coefficient $\kappa$ are fixed at 1.

Based on the Ref. [17], that coupled coefficient between the WGM with the common waveguide is $\mu_{1,2}^2 = \xi_{1,2}^2 v_g / (2\pi R_{1,2})$, $\xi$ is the fraction of the power coupled from the waveguide to the WGM, $R_{1,2}$ is the radius and $v_g$ is the group velocity. For meeting the APT condition, we design the $\xi_1=\xi_2=\xi$ and $R_1 \approx R_2$, the mode splitting and the enhancement factor $S$ are rewritten as

$$\Delta \omega_{APT} \approx \sqrt{\frac{g \xi^2 v_g}{2\pi R}}$$

$$S = \sqrt{\frac{\xi^2 v_g}{8g\pi R}}$$

(8)

For the Parity-time symmetry sensor, the frequency difference $\Delta \omega_{PT} = \sqrt{-i g}$ is complex splitting [25]. The APT nanoparticle sensor we proposed can exhibit a real splitting, comparing to the complex splitting in the Parity-time symmetric sensor, which means that the eigenfrequency splitting can be directly detected. Considering the practical parameters, we set the parameters $R_1=20\mu m$, $R_2=19.95\mu m$, $\xi=0.1,0.5,1$, $\gamma=0.5 / (2\pi) MHz$ [17]. Fig.6 shows the enhancement of the splitting of an APT nanoparticle sensor at the EP with respect to a single WGM sensor. The sensitivity enhancement of five orders of magnitude at the exceptional point.
Fig. 6 Comparison the eigenfrequency splitting at the EP between the APT nanoparticle sensor and the splitting. The parameters are set as $R_1=20\mu m$, $R_2=19.95\mu m$, $\xi=0.1, 0.5, 1$, $\gamma_1=\gamma_2=0.5/(2\pi)\text{MHz}$.

3. The detection limit of the Anti-PT-Symmetric nanoparticle sensor

Now, we research the detection limit of the Anti-PT-Symmetric nanoparticle sensor. Here, we introduce the frequency splitting quality $Q_{sp}[24]$, 

$$Q_{sp} = \frac{\text{Re}(\omega_{APT,+}) - \text{Re}(\omega_{APT,-})}{\text{Im}(\omega_{APT,+}) - \text{Im}(\omega_{APT,-})}$$ (9)

If $Q_{sp} > 1$ the eigenfrequency splitting can be resolved easily in experiment, when $Q_{sp} = 1$, the detection limit $g_{\text{min}}$ can be obtained. We chose $\omega_{APT,3}$ as $\omega_{APT,+}$ and the $\omega_{APT,1}$ as the $\omega_{APT,+}$, because $\omega_{APT,1}$ corresponding supermodes not experiences a frequency shift and linewidth change can serve as reference signals. Thus, the detection limit of the perturbation $g$ is
\[ Q_{\text{sp, APT}} = \frac{2g \mu_1 \mu_2}{2\gamma} \]
\[ g_{\text{min}} = \frac{4\gamma^2}{\mu_1 \mu_2} \]

The theoretical detection limit of the perturbation \( g \) is proportional to the effective loss rate of the cavity and is inversely proportional to the indirectly coupled coefficient between the two cavities. Fig. 7 demonstrates that the higher loss rate of the cavity, the lower precision. In practice, the backscattering strength is

\[ g = -\text{Re}[\alpha f^2(\tilde{r})]\omega / 2V_m, \]

where \( f(r) \) is the cavity mode function, \( V_m \) is the mode volume and

\[ \alpha = 3V_p \epsilon_1 (\epsilon_p - \epsilon_l) / (\epsilon_p + 2\epsilon_l) \]

is the polarizability of a particle, where \( V_p \) is the volume of the nanoparticle, \( \epsilon_p \) and \( \epsilon_l \) denote dielectric permittivities of the nanoparticle and the medium, respectively. Therefore, we can derive the particle size through \( g \).

Fig. 7 In different loss rate \( \gamma \), the detection limit of the perturbation \( g \) varies with the indirectly coupled coefficient at EP. Here, \( \gamma_1 = \gamma_2 = 0.7/(2\pi) \text{MHz}, -0.5/(2\pi) \text{MHz}, -0.3/(2\pi) \text{MHz} \).

4. Conclusions

We investigated an integrated Anti-parity symmetric indirectly coupled WGM nanoparticle sensor. As a result, the configuration operating at the exceptional point shows a significant enhancement with respect to the classical nanoparticle sensor,
achieving an eigenfrequency splitting the 5 orders of magnitude higher than a single WGM sensor. With regards to the Parity-time symmetric nanoparticle sensor, the Anti-PT-symmetric solution is more suitable for nanoparticle detecting, because it exhibits a real frequency splitting. Compared with balancing the gain and loss in parity-time symmetric nanoparticle senor, Anti-PT-symmetric sensor just requires to adjust the detuning between the two WGMs, keeping the configuration at the exceptional point more reliable. We believe that the APT sensor at EP could pave a way to realize the ultra-high sensitivity integrated optical nanoparticle sensor.

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