Upon symmetry of baryon magnetic moments in the chiral and the quark-soliton models

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Abstract

It is shown that the baryon magnetic moment descriptions in the frameworks of the chiral model ChPT and the quark soliton χQSM one are practically identical. The main difference from the traditional unitary symmetry models proves to be in terms due to the pionic current contribution into the magnetic moments of nucleons and in the prediction for the Λ magnetic moment.

INTRODUCTION

Since appearance of a theory of unitary symmetry and then of a quark model, a problem of baryon octet magnetic moment description has been attracting many theoretical efforts. It is well known that a traditional unitary symmetry model with only two parameters [1] describes experimental data at the qualitative level. The quark models (cf., e.g., [2]) usually introduce the number of parameters ≥3 and are able to describe baryon magnetic moments quantitatively better than to 10%. In general, these quark models have strongly improved the agreement with experimental data. However, at the present time experimental data have achieved a level of 1% accuracy [3], which force theoreticians to upgrade precision of a theoretical and phenomenological description of the baryon magnetic moments. Many interesting models have been developed in order to solve this long-stayed problem from various points of view, and we are able here to cite only some of them [4-20].

Recently, baryon magnetic moments have been analyzed within two independent chiral models [21] and [24]. It has been shown there that the baryon magnetic moment symmetry is based on unitary symmetry, and the chiral model ChPT [21] and quark soliton model [24]
just describe the way in which this symmetry is broken. We would try to establish a relation between these two models.

Besides, we shall show that a phenomenological model generated by the unitary symmetry approach can be formulated in terms of the electromagnetic baryon current, which proves to be quite close to these two models.

I. CHIRAL MODEL CHPT FOR THE BARYON MAGNETIC MOMENTS

Let us briefly remind the ChPT model [21] for the baryon octet magnetic moments. It has been assumed that the leading $SU(3)_f$ breaking corrections to the magnetic moments have the same chiral transformation properties as the strange quark mass operator and the corresponding coefficients are of the order $m_s/\Lambda$, $m_s$ and $\Lambda$, being strange quark mass and chiral symmetry breaking scale, respectively. The expressions for the baryon octet magnetic moments in this model read [21]

$$
\mu(p) = \frac{1}{3}(b_1 + \alpha_4) + (b_2 + \alpha_2) + \alpha_1 + \frac{1}{3}\alpha_3 - \frac{1}{3}\beta_1 \\
\mu(n) = -\frac{2}{3}(b_1 + \alpha_4) - \frac{2}{3}\alpha_3 - \frac{1}{3}\beta_1 \\
\mu(\Sigma^+) = \frac{1}{3}(b_1 + \alpha_4) + (b_2 + \alpha_2) - \alpha_2 - \frac{1}{3}\alpha_4 - \frac{1}{3}\beta_1 \\
\mu(\Sigma^-) = \frac{1}{3}(b_1 + \alpha_4) - (b_2 + \alpha_2) + \alpha_2 - \frac{1}{3}\alpha_4 - \frac{1}{3}\beta_1 \\
\mu(\Xi^0) = -\frac{2}{3}(b_1 + \alpha_4) + \frac{2}{3}\alpha_3 - \frac{1}{3}\beta_1 \\
\mu(\Xi^-) = \frac{1}{3}(b_1 + \alpha_4) - (b_2 + \alpha_2) + \alpha_1 - \frac{1}{3}\alpha_3 - \frac{1}{3}\beta_1 \\
\mu(\Lambda^0) = -\frac{1}{3}(b_1 + \alpha_4) - \frac{5}{9}\alpha_4 - \frac{1}{3}\beta_1
$$

(1)

These expressions can easily be rewritten in the form demonstrating that the chiral model ChPT [21] is introducing the unitary symmetry breaking terms in some definite way

$$
\mu(p) = F_N + \frac{1}{3}D_N - \frac{1}{3}\beta_1 \\
\mu(n) = -2/3D_N - \frac{1}{3}\beta_1 \\
\mu(\Sigma^+) = F_{\Sigma} + \frac{1}{3}D_{\Sigma} - \frac{1}{3}\beta_1 \\
\mu(\Sigma^-) = -F_{\Sigma} + \frac{1}{3}D_{\Sigma} - \frac{1}{3}\beta_1 \\
\mu(\Xi^0) = -\frac{2}{3}D_{\Xi} - \frac{1}{3}\beta_1 \\
\mu(\Xi^-) = -F_{\Xi} + \frac{1}{3}D_{\Xi} - \frac{1}{3}\beta_1 \\
\mu(\Lambda^0) = -\frac{1}{3}(b_1 + \alpha_4) - \frac{5}{9}\alpha_4 - \frac{1}{3}\beta_1
$$
\[ \mu(\Lambda^0) = -\frac{1}{3}D_\Lambda - \frac{1}{3}\beta_1 \]  

(2)

where

\[ F_N = b_2 + \alpha_1 + \alpha_2, \quad F_\Sigma = b_2, \quad F_\Xi = b_2 - \alpha_1 + \alpha_2, \]

\[ D_N = b_1 + \alpha_3 + \alpha_4, \quad D_\Sigma = b_1, \quad D_\Xi = b_1 - \alpha_3 + \alpha_4, \quad D_\Lambda = b_1 - \frac{8}{3}\alpha_4. \]

The main difference from other models introducing symmetry breaking mechanisms of various kinds consists in the unity operator term which explicitly breaks the octet form of the electromagnetic current in \( SU(3)_f \). Renormalization of the constants \( F \) and \( D \) can be related to the so-called middle-strong interaction contribution, which follows from the fact that for the baryons \( B(qq,q') \) it can be reduced to the form characteristic of mass breaking terms of the unitary symmetry

\[ \mu(p) = F + \frac{1}{3}D + g_1 + \beta + \pi_N \]

\[ \mu(n) = -\frac{2}{3}D + g_1 + \beta - \pi_N \]

\[ \mu(\Sigma^+) = F + \frac{1}{3}D + \beta \]

\[ \mu(\Sigma^-) = -F + \frac{1}{3}D + \beta \]

\[ \mu(\Xi^0) = -\frac{2}{3}D + g_2 + \beta \]

\[ \mu(\Xi^-) = -F + \frac{1}{3}D + g_2 + \beta \]  

(3)

\[ F = b_2, \quad D = b_1 + \alpha_1 - \alpha_2 - \alpha_3 + \alpha_4, \]

\[ g_1 = \alpha_1 - \frac{2}{3}\alpha_3 + \frac{1}{3}\alpha_4, \quad g_2 = \alpha_1 - \alpha_2 - \frac{1}{3}\alpha_3 + \frac{1}{3}\alpha_4, \]

\[ \beta = -\frac{1}{3}\beta_1 + \frac{1}{3}(\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4), \quad \pi_N = \alpha_2 + \alpha_3. \]

The results of Eq.(3) can be obtained from the following electromagnetic current (we disregard space-time indices):

\[ J^{e-m, symm1} = -F(\overline{B}_1^1 B_1^1 - \overline{B}_1^1 B_1^1) + D(\overline{B}_1^1 B_1^1 + \overline{B}_1^1 B_1^1) + 
\]

\[ g_1 \overline{B}_3^1 B_3^3 + g_2 \overline{B}_3^3 B_3^3 + (\beta - \frac{2}{3}D)Sp(\overline{B}_3^1 B_3^3) + \pi_N(\overline{B}_3^1 B_3^3 - \overline{B}_3^3 B_3^3) \]  

(4)

Here \( B_3^\gamma \) is a baryon octet, \( B_1^3 = \mu, B_3^\Xi = \Xi^0 \) etc. For the magnetic moment of the \( \Lambda \) hyperon this current gives:
\[
\mu(\Lambda)^{symm1} = -\frac{1}{3}b_1 + \frac{2}{3}\alpha_1 - \frac{2}{9}\alpha_4 - \frac{1}{3}\beta_1,
\] (5)

so that

\[
\mu(\Lambda)^{symm1} - \mu(\Lambda)^{ChPT} = \frac{2}{3}(\alpha_1 + \alpha_4).
\] (6)

For \(\pi_N = 0\) the current (4) yields a direct sum of the traditional electromagnetic current of the theory of unitary symmetry [1] and the traditional baryon current leading to the Gell-Mann-Okubo mass relation [23]. The parameter \(\pi_N\) defines the value of contribution of the pion current term and is characteristic of many versions of the chiral models. Note that with \(\alpha_1 = -\alpha_4\) the ChPT model results reduce to that given by the current of Eq.(4). As in [21] \(\alpha_1 = 0, 32, \alpha_4 = -0, 31\) (in GeV\(^{-1}\)), it goes out that the chiral model ChPT [21] results are in fact given by the phenomenological unitary current (4).

**II. QUARK SOLITON MODEL \(\chi QSM\) AND UNITARY SYMMETRY**

In [23], [24] magnetic moments of baryons were studied within the chiral quark soliton model. In this model, known also as the semibosonized Nambu-Jona-Lasinio model, the baryon can be considered as \(N_c\) valence quarks coupled to the polarized Dirac sea bound by a nontrivial chiral background hedgehog field in the Hartree-Fock approximation [24]. Magnetic moments of baryons were written in the form [24]

\[
\begin{pmatrix}
\mu(p) \\
\mu(n) \\
\mu(\Lambda) \\
\mu(\Sigma^+) \\
\mu(\Sigma^-) \\
\mu(\Xi^0) \\
\mu(\Xi^-)
\end{pmatrix}
= \begin{pmatrix}
-8 & 4 & -8 & -5 & -1 & 0 & 8 \\
6 & 2 & 14 & 5 & 1 & 2 & 4 \\
3 & 1 & -9 & 0 & 0 & 0 & 9 \\
-8 & 4 & -4 & -1 & 1 & 0 & 4 \\
2 & -6 & 14 & 5 & -1 & 2 & 4 \\
6 & 2 & -4 & -1 & -1 & 0 & 4 \\
2 & -6 & -8 & -5 & 1 & 0 & 8
\end{pmatrix}
\begin{pmatrix}
v \\
w \\
x \\
y \\
z \\
p \\
q
\end{pmatrix}
\] (7)

Here the parameters \(v\) and \(w\) are linearly related with the usual F,D coupling constants of the unitary symmetry approach. The parameters \(x, y, z, p, q \approx m_s\) are specific for the model. Upon algebraic transformations the expressions for 6 baryons \(B(qq, q')\) can be rewritten as
\[
\mu(p) = F + \frac{1}{3}D - f_1 + T - 3z \\
\mu(n) = -\frac{2}{3}D - f_1 + T + 3z \\
\mu(\Sigma^+) = F + \frac{1}{3}D + T \\
\mu(\Sigma^-) = -F + \frac{1}{3}D + T \\
\mu(\Xi^0) = -\frac{2}{3}D - f_2 + T \\
\mu(\Xi^-) = -F + \frac{1}{3}D - f_2 + T
\]  
(8)

where

\[
F = -5v + 5w - (9x + 3y + p) + z \\
D = -9v - 3w - (13x + 7y - 4q + p) + 3z \\
f_1 = 4x + 4y - 4q - z \\
f_2 = 22x + 10y - 4q + 2p - 2z \\
T = \frac{1}{3}(28x + 13y + 8q + 4p) - z .
\]  
(9)

One can see that the algebraic structures of Eq.(8) and Eq.(3) are the same. It means that magnetic moments of the octet baryons in the models \(\chi QSM\) [24] and ChPT [21] can be obtained from the unitary electromagnetic current of the form (we disregard space-time indices)

\[
J^{e-m, symm^2}_{\text{symm}} = -F(\overline{B}_1^1 B_1^{\gamma} - \overline{B}_3^1 B_3^{\gamma}) + D(\overline{B}_1^1 B_1^{\gamma} + \overline{B}_3^1 B_3^{\gamma}) -  \\
f_1 \overline{B}_2^3 B_2^{\gamma} - f_2 \overline{B}_3^3 B_3^{\gamma} + (T - \frac{2}{3}D)Sp(\overline{B}_2^\beta B_2^{\gamma}) + 3z(\overline{B}_3^2 B_2^3 - \overline{B}_3^3 B_3^2) 
\]  
(10)

With this current the magnetic moment of the \(\Lambda\) hyperon reads:

\[
\mu(\Lambda)^{\text{symm}^2} = -\frac{1}{3}D - (8x + 5y - 8q - z) 
\]  
(11)

which differs from that given by Eq.(7)

\[
\mu(\Lambda)^{\text{symm}^2} - \mu(\Lambda)^{\chi QSM} = \frac{1}{3}(16x - 8y - 7q + p) 
\]  
(12)
Eqs. (7) and (1) reduce to each other through the relations between the parameters

\[ b_1 = -(9v + 3w) - \frac{1}{2}(42x + 6y - 15q + 3p), \]
\[ b_2 = -5(v - w) - (9x + 3y + -z + p) \]
\[ \alpha_1 = -\frac{1}{4}(94x + 34y - 31q + 7p), \quad \alpha_2 = \frac{3}{2}(9x + 3y - z + p), \]
\[ \alpha_3 = -\frac{3}{2}(9x + 3y + z + p), \quad \alpha_4 = \frac{9}{4}(14x + 2y - 5q + p), \]
\[ \beta_1 = -\frac{9}{2}(8x + 2y + q + p), \]

where now

\[ 9\alpha_1 + 15\alpha_2 - 15\alpha_3 + 3\alpha_4 + 8\beta_1 = 0. \]  (13)

These formulae yield the following relation between the octet baryon magnetic moments derived in [22] and [24]

\[ -12\mu(p) - 7\mu(n) + 7\mu(\Sigma^-) + 22\mu(\Sigma^+) - 12\mu(\Lambda^0) + 3\mu(\Xi^-) + 23\mu(\Xi^0) = 0. \]  (15)

The relations (13) and (14) close our proof of the practical coincidence of the magnetic moment description in the framework of the ChPT [21] model and the \( \chi QSM \) [24] one.

### III. SUMMARY AND CONCLUSION

It has been shown that the algebraic schemes of the models [21] and [24] for the predictions of the octet baryon magnetic moments have proved to be practically identical. Moreover, the expressions for the magnetic moments \( B(qq,q') \) in these models are those given by the unitary model with the phenomenological electromagnetic current given by Eq. (1) or Eq. (10). The main difference of the models [21] and [24] from a direct sum of the traditional unitary electromagnetic and middle-strong baryon currents lies in the terms due to pion current contribution which are written explicitly in Eqs. (4) and (10). The only real difference between our phenomenological current predictions and those of [21] and [24] is in the formula for the magnetic moment of the hyperon \( \Lambda \). This difference may prove to have

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deeper meaning as Λ-hyperon being composed of all different quarks is characterized by zero values of isotopic spin and hypercharge. Quantitavely it occurs not to be very important as due to approximate equality $\alpha_1 = -\alpha_4$ the Λ magnetic moment proves to be practically the same in the phenomenological model given by Eq.(4) and ChPT [21].

In general, the analysis of the baryon magnetic moments in the framework of these models has shown once more that unitary symmetry is the basis which could be hidden in any dynamical model pretending to an adequate description of the electromagnetic properties of baryons.
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