Little Groups of Preon Branes

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Abstract

Little groups for preon branes (i.e. configurations of branes with maximal (n-1)/n fraction of survived supersymmetry) for dimensions $d=2,3,...,11$ are calculated for all massless, and partially for massive orbits. For massless orbits little groups are semidirect product of $d-2$ translational group $T_{d-2}$ on a subgroup of $(\text{SO}(d-2) \times \text{R-invariance})$ group. E.g. at $d=9$ the subgroup is exceptional $G_2$ group. It is also argued, that 11d Majorana spinor invariants, which distinguish orbits, are actually invariant under $d=2+10$ Lorentz group. Possible applications of these results include construction of field theories in generalized space-times with brane charges coordinates, different problems of group’s representations decompositions, spin-statistics issues.

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1 Introduction and Conclusion

There is a strong interest to the symmetry structure of M and related theories. The space-time supersymmetry algebra of M-theory, is (\[1\] \[2\]):

\[
\{\bar{Q}, Q\} = \Gamma^\mu P_\nu + \Gamma^{\mu\nu} Z_{\mu\nu} + \Gamma^{\mu\nu\lambda\rho\sigma} Z_{\mu\nu\lambda\rho\sigma},
\]

where \(Q\) is Majorana 32-component spinor, \(Z_{\mu\nu}\) and \(Z_{\mu\nu\lambda\rho\sigma}\) are membrane and five-brane charges. This algebra is of (super)-Poincaré type, i.e. its bosonic (tensorial Poincaré) subalgebra is a semidirect product of Lorentz and Abelian subalgebras, the latter is a sum of \(P_\mu\), \(Z_{\mu\nu}\) and \(Z_{\mu\nu\lambda\rho\sigma}\). There are evidences, that this algebra can be extended. Already in the first works on extended supergravities at \(d=4\) hidden non-linearly realized symmetries have been found \[3\], which later on appeared to be connected (\[4\],\[5\]) with space-time symmetry algebra \[1\], and these observations particularly lead to suggestions on hidden space-time coordinates and symmetries of M-theory \[6\]. In some yet unknown phase of M-theory with explicit conformal symmetry the latter is assumed (\[7\], earlier references are \[3\], \[8\]) to be an \(OSp(1|64), (OSp(1|32), OSp(1|16), ...\) on lower dimensionalities) supergroup, which includes \[1\] as sub-superalgebra, just as usual Poincaré group \(SO(1,3) \ltimes T_4\) (at \(d=4\) is a subgroup of conformal group \(SO(2,4)\). Generalized space-times are suggested \[9\] for a new formulations of supergravity/superstring/M-theory, with a particular purpose of making explicit hidden symmetries of these theories. The new formulation of higher spin theories is making use of some of these generalized space-times with \(OSp(1|2n)\) symmetry \[10\] \[11\], and equivalence to theories in a usual space-time, in the free case is shown. The recently discovered \[12\] non-linear realization representation of maximal supergravities - \(d=11\) \(N=1\), \(d=10\) \(N=2a\) and \(2b\) - make use of the new space-time algebras, most general of which is the newly introduced \[13\] ”very extended” \(E_{11}\) algebra \[14\].

Correspondingly, the irreducible representations (irreps), particularly unitary irreps of these and connected algebras are attracting attention in modern literature. There are specially interesting representations among them, such as singletons of \(OSp\) (which appear in many circumstances), or preon representations of super-Poincaré, which correspond to maximally supersymmetric BPS branes \[15\],\[16\], and others.
All that is making the investigation of properties of space-time symmetry algebras of type (1) interesting and promising, both for the purpose of clarifying properties of the theory, and for seeking new formalisms for it. In the present paper we will continue investigation \cite{[17]}, \cite{[18]} of the construction of unitary irreps of algebras (1) by applying Wigner’s method of little groups (induction from the little group) \cite{[19]} \cite{[20]}. The main focus of present paper will be the preonic orbits \cite{[15]}, \cite{[16]}, i.e. orbits (configurations of branes) with maximum fraction of surviving supersymmetry, some other orbits are considered in previous papers \cite{[17]}, \cite{[21]}. We find a complete list, at dimensions 2,3,..11 of little groups for the massless representations, i.e. for those preonic configurations, for which corresponding vector $p_\mu$ is massless, and all continuous invariants are also zero. All little groups are of the form of semidirect product of compact group on group of (d-2)-dimensional translations. These results are new for d=5,6 (for symplectic-Majorana spinors),7,8,9. For massive case we find complete answers for dimensionalities 5,8,9. In this case little groups are compact, since they should be a subgroup of those for massive particles, i.e. of $\text{SO}(d-1)$. We also show that 11d spinor invariants are actually $(12=2+10)d$ invariant.

There are many possible applications of present results. One is the determination of the brane content of branes \cite{[18]}, i.e. decomposition of irreps w.r.t. the different subalgebras. Among these algebras can be a tensorial Poincaré with less number of tensorial generators, or a usual Poincaré, so one can consider a spectrum of usual particle representations in a given brane representation. This approach provides \cite{[18]} group-theoretic interpretation of the results of \cite{[11]}.

Another application can be the decomposition of different representation of $\text{OSp}$ (which is a ”conformal” group for tensorial Poincaré) w.r.t. the tensorial Poincaré.

Next, one has to reconsider \cite{[21]} the spin-statistics theorem, with the purpose of generalization of connection between statistics and spins. Now instead of spin we have a rich set of representations of little groups, particularly those from Tables 1 and 2, and one has to assign Fermi or Bose statistics to each of representations of little groups. Moreover, the preon case resemble strongly the usual two-dimensional situation, when there is no strong difference between fermions and bosons, both correspond to the same representation of Poincaré. Similarly, in the preon representation of super-Poincaré with tensor charges (1), e.g.) the non-zero supercharge is neutral \cite{[21]} w.r.t. the little group, supermultiplet has only two members, both in the
same representation of tensorial Poincaré and one can expect the generalized statistics, similar to those in the two-dimensional case.

Last, but not least: one of the most interesting applications (and initial motivation) of construction of unitary irreps for different orbits of tensorial Poincaré is the construction of field theories with corresponding symmetry algebra. For $d=2+q$ dimensional generalization of $\mathbb{H}$ (connected with the SO(2,10) hypothesis of [22]) the corresponding field theories are constructed for an orbit, which is a generalization of massless particle orbit and for lowest representations of corresponding little groups [17] [23]. For lowest representations of little groups of preonic orbits at dimensions 4, 6, ... the free field theories, on the level of equations of motions, are actually constructed in [10], with different motivation, coming from higher spin theories in usual space-times. The first step towards interaction in these theories is done for 2+2 dimensional theory in [24].

In the next Section Wigner’s method is briefly described and applied to the present case. Main results - the little groups for massless case, more precisely, for the zero values of all invariants (if those exist) are combined into Table 1. The additional information on little groups with non-zero invariants is combined into Table 2.

2 Little groups of preons

Modern supersymmetry algebras can be represented in a form

$$\{ Q^{j\beta} , Q_{i\alpha} \} = Z_{i\alpha}^{j\beta} \quad (2)$$

where supercharges $Q_{i\alpha}$ are subject to some constraints - Majorana, Weyl, Symplectic-Majorana etc., r.h.s. matrix $Z$ is the most generic matrix, that satisfies the same constraints. We shall consider the minimal algebras, i.e. spinors $Q_{i\alpha}$ will be the minimal possible in a given dimension, hence index $i$ of the group of R-invariance either will be absent, or will be that of $SU(2)$ doublet, in the case of symplectic spinors. Matrix $Z$ can be expanded over gamma-matrices basis and in that form r.h.s of (2) will become a usual combination of momentum $P_\mu$ and ”central charges” tensors $Z_{\mu\nu...}$. The bosonic subalgebra is a semidirect product of Lorentz and R-symmetry group, from one side, and Abelian group of momentum and central charges tensors, from the other, so for construction of unitary irreps of that algebra.
Wigner’s method \[19\][20] can be applied. The construction includes the following steps: classification of orbits of action of Lorentz $\times \mathbb{R}$ group on the numeric tensor $Z$, determination of a stabilizer (little group) of some (arbitrarily picked) point on the orbit, choice of any unitary irrep of little group and finally induction on the whole tensorial Poincaré group. The present paper is devoted to the discussion of preonic orbits, i.e. the orbits with rank one $Z$ matrix. Such $Z$’s have a form $Z = \lambda \lambda$, where $\lambda$ is a numeric spinor, and stabilizer of $Z$ is that of $\lambda$ plus transformations which change the sign of $\lambda$, (which is actually the property of full 360 degree rotation of spinor). Also, such configuration of $Z$ is providing the maximum number of surviving supersymmetries \[15\][16]. So, we come to the problem of determination of orbits of action of Lorentz group on the space of spinors $\lambda$, and calculation of corresponding little groups.

The main results on little groups of preonic orbits are combined into Tables 1 and 2. For dimensions $d = 2 \div 11$, with one time dimensions, we list the type of minimal spinor in corresponding dimension (i.e. (pseudo)Majorana=(P)M, (pseudo)Symplectic Majorana=PSM, Weyl=W, etc.), I - the number of independent invariants, which can be constructed from corresponding spinor, and the little group for the maximally non-compact case, more exactly, for the zero values of all invariants. The necessary additional information is the embedding of corresponding little group into the group of symmetry, i.e. the product of Lorentz and R-symmetry group (SO(3), if any). These embeddings are described below, as well as explicit expressions for invariants, except $d=7$ and $d=11$ cases, where we don’t know the complete minimal set of independent invariants. Nevertheless, in the last case we can claim, on the basis of some additional calculations, that these invariants are actually invariant w.r.t. the $d=12=2+10$ Lorentz group, which is well-known to be the automorphisms group of $d=11$ superalgebra. For obtaining all these results we use a combination of few methods: sometimes we calculate an algebra of stabilizer (little group) around an (arbitrary) given spinor, and calculate a dimensionality of orbit at that point by the evident formula $\dim (\text{orbit}) = (\dim G) - \dim (\text{little group})$, where $G$ is the product of Lorentz and R-symmetry groups. From this we can find the number of independent invariants, which can be constructed from a given spinor, obviously that is a codimension of an orbit in a spinor space, we denote that $I$ in the Tables. In few cases an algebraic and group-theoretic considerations are applied, particularly when considering the dimensional reductions of spinors. We use a ”mostly plus” metric, other notations and definitions of spinors and corresponding matrices
are based on [25].

Table 1. Massless preons’ little groups.

| \(d\) | Spinor type | I - invariants' number | Little Group |
|-------|-------------|------------------------|--------------|
| 2     | MW          | 0                      | 1            |
| 3     | M           | 0                      | \(T_1\)      |
| 4     | M           | 0                      | \(T_2\)      |
| 5     | PSM         | 1                      | \(SO(3) \rtimes T_3\) |
| 6     | SMW         | 0                      | \(SO(4) \rtimes T_4\) |
| 7     | SM          | 3                      | \(SO(4) \rtimes T_5\) |
| 8     | PM          | 2                      | \(SU(3) \rtimes T_6\) |
| 9     | PM          | 1                      | \(G_2 \rtimes T_7\) |
| 10    | MW          | 0                      | \(SO(7) \rtimes T_8\) |
| 11    | M           | 7                      | \(SO(7) \rtimes T_9\) |

Table 2. Massive preons’ little groups.

| \(d\) | Spinor type | I - invariants' number | Little Group |
|-------|-------------|------------------------|--------------|
| 5     | PSM         | 1                      | \(SO(4)\)  |
| 8     | PM          | 2                      | \(G_2\)      |
| 9     | PM          | 1                      | \(SO(7)\)    |

In the cases with \(I = 0\) the whole space of spinors (besides zero point) is an orbit of corresponding Lorentz \(\rtimes\) R-invariance group. Turning to the embeddings of little groups, we first mention that they are not simply the subgroups of direct product of Lorentz and R-symmetry groups (the later is trivial except the SM(W) cases when it is SU(2)), but actually they should be the subgroups of product of R-symmetry group with the little group of particles - \(SO(d - 2) \rtimes T_{d-2}\) for massless and \(SO(d - 1)\) for massive cases. It is obvious from the fact that in all these dimensions one can define the momenta \(p^\mu\), corresponding to \(Z = \lambda \lambda\): \(p^\mu \sim \bar{\lambda} \gamma^\mu \lambda\), and since stabilizer of \(Z\) evidently is stabilizer of \(p^\mu\), statement follows. Taking into account that Table 1 contains little groups for the massless cases only (all invariants zero, see below), we should describe embeddings of groups in the Table 1 into \(R \times SO(d - 2) \rtimes T_{d-2}\), which in turn has a well-known embedding in \(R \times SO(1, d - 1)\). Additional remark is that non-compact factors \(T_{d-2}\) in Table 1 always coincide with corresponding factor in \(R \times SO(d - 2) \rtimes T_{d-2}\), so we
need to describe embeddings of compact factors, only. Precise embedding
depends on a point on the orbit, we need to describe embeddings up to
equivalence, i.e. up to similarity transformation in the group. The Table 2
contains results for massive cases, which are also described below.

Cases d=3, 4 are evident, the absence of invariants means that the whole
space of spinors is one orbit, except the zero point. The answer for the little
group can be obtained by direct calculation, for an arbitrary given spinor.
The first non-trivial case is d=5, with pseudo-symplectic-Majorana spinors.
The massless particle little group is SO(3), the same is R-symmetry, and
SO(3) in the Table 1 is just a diagonal subgroup of their direct product. The
only independent invariant is \( m = i \lambda^i_\alpha \lambda^j_\beta C^{\alpha\beta} \epsilon_{ij} \). Evidently, for any (non-zero)
values of this invariant little groups are the same. Existence of this
invariant means that the space of non-zero spinors is not a single orbit, as
at d=4, but an infinite set of orbits, distinguished by the value of invariant.
The above little group corresponds to the zero value of that invariant. One
can show, that the mass of momenta \( p^\mu = \lambda^i_\alpha \lambda^j_\gamma C^{\alpha\beta} \gamma^\mu_\beta \epsilon_{ij} \), corresponding to
Z = \lambda \lambda, is exactly \( m^2 = -m^2 \). For an orbit with non-zero m the little
group for preons is SO(4) (without any noncompact factor) and coincide
with little group of massive particle, the only additional detail is that the
embedding of this SO(4) into SO(3) of R-symmetry, which is actually a set
of two embeddings of two SO(3) factors of SO(4) into SO(3), is non-zero,
and depends on a particular point on an orbit.

For d=6 the compact factor of massless little group is SO(4) \( \sim \) SO_L(3) \( \times \)
SO_R(3) and SO(4) group in the Table 1 is \( \sim \) SO_L(3) \( \times \) (SO_R(3) \( \times \) SO(3))_diag
where last SO(3) is the group of R-symmetry.

At d=7 for the zero values of invariants (see discussion below for d=11
case) the (compact part of the) little group is SO(4) which is the same group
described above for d=6, embedded in a natural way (e.g. as a 4x4 sub-
matrix in a left upper corner of 5x5 matrix, there is no other, non-equivalent
representation) into SO(5) group of massless particle at d=7.

At d=8 there are exactly two independent invariants: \( m_1 = i \lambda^i_\alpha \lambda^j_\beta C^{\alpha\beta} \epsilon_{ij} \)
and \( m_2 = \lambda^i_\alpha \lambda^j_\gamma C^{\alpha\beta} \gamma^\mu_\beta \epsilon_{ij} \). The momenta \( p^\mu = \lambda^i_\alpha \lambda^j_\gamma C^{\alpha\beta} \gamma^\mu_\beta \epsilon_{ij} \) square into
\( p^2 = -(m_1^2 + m_2^2) \), so for massless case we should have both masses equal to
zero. In that case the (compact part of the) little group in Table 1 is SU(3),
and is a natural subgroup of SU(4), which is isomorphic to the (compact part
of the) massless particle little group SO(6). At massive case, i.e. when one or
two of masses \( m_1, m_2 \) are non-zero, the little group is \( G_2 \). That can be shown
as follows. The little group should be a subgroup of that of massive particle - SO(7). Acting by that group in a spinor representation on the Weyl part of spinor \( \lambda \), which is an SO(7) spinor, we obtain the stabilizer \( G_2 \), since that is exactly (one of the) definition(s) of \( G_2 \). It is easy to show, that action of SO(7) on the anti-Weyl part of spinor \( \lambda \) gives the same stabilizer, since Weyl and anti-Weyl parts of \( \lambda \) are connected by (pseudo)-Majorana condition.

At \( d=9 \) situation is similar to \( d=5 \): there is one invariant \( m \equiv i\lambda_\alpha \lambda_\beta C^{\alpha\beta} \), the square of momenta \( p^\mu = \lambda_\alpha \lambda_\sigma C^{\alpha\beta} n_{\beta \mu} \) is given by \( p^2 = -m^2 \). So the whole space of spinors is a set of orbits, with different \( m \), and there are two kinds of orbits: the first one is massless, with \( m = 0 \), the little group is \( G_2 \rtimes T_7 \) and \( G_2 \) in its fundamental 7-dimensional representation is embedded in SO(7), the compact factor of massless little group. Second one is massive, with little group SO(7), naturally a subgroup of the little group of massive particle, SO(8). It is clear from the triality: the problem of finding the stabilizer of a given \( d=1+8 \) PM spinor leads to the problem of finding a stabilizer of spinor under SO(8) (the stabilizer of its momenta \( p_\mu \)), which, in turn, due to the triality property is equivalent to finding a stabilizer of eight-dimensional Euclidean vector under an SO(8) rotations, which evidently is SO(7) subgroup.

At \( d=10 \) little group is SO(7) \([18]\), embedded, in its spinor representation, into a fundamental representation of SO(8) \([20]\).

Finally, at \( d=11 \) we have little group SO(7), embedded, as at \( d=10 \), in its spinor representation into SO(8), which, in turn, naturally is a subgroup of \( d=11 \) massless SO(9). We cannot present a complete set of independent invariants (although, of course, a lot of them can be easily constructed), but we can state, that all values of those invariants are zero for an orbit of Table 1 (in our representation of gamma-matrixes, that is an orbit of a spinor with unites at first and sixth places, zero otherwise). That follows from the fact that, from one hand, these invariants are some homogeneous polynomials over \( \lambda \), and, from the other hand, we can find a Lorentz transformation, which simply rescales a given spinor on the orbit, so the non-zero value of these polynomials will be rescaled, which contradicts to their invariance. This argument actually works in other dimensions too. The interesting further remark on this \( d=11 \) case is the following. It is well-known, that \( d=11 \) algebra \([11]\) can be represented in \( d=2+10=12 \) dimensional form, since corresponding Lorentz group SO(2,10) is an automorphisms group of \([11]\). In that form momenta and second rank tensor combine into one \( d=12 \) second-rank tensor, and fifth-rank tensor \( Z_{\mu\nu\rho\sigma} \) is interpreted as 12d self-dual sixth rank tensor,
11d Majorana supercharges become 12d Majorana-Weyl supercharges, and algebra receives the form:

\[
\{ \bar{Q}, Q \} = \Gamma_{\mu \nu} P_{\mu \nu} + \Gamma^{\mu \nu \lambda \rho \sigma \delta} Z_{\mu \nu \lambda \rho \sigma \delta} \tag{5}
\]

\[
\mu \nu, ... = 0', 0, 1, ... 10
\]

One can study orbits of the same spinor under the extended group SO(2,10). The remarkable fact is that the orbit has the same dimensionality, 25, as under the action of SO(1,10) group, so coincide with 11d orbit. That follows from the fact, that although corresponding little group has more complicated structure, but its dimensionality is 41, so dimensionality of orbit is dimensionality of Lorentz group (66) minus dimensionality of little group (41), i.e. 25. So, assuming this coincidence for any spinor, we conclude that the number of independent invariants is the same, 7, they are SO(2,10) invariant, and can also serve as SO(1,10) invariants, so they are exactly an 11d invariants, so we prove that 11d Majorana spinors invariants can be chosen in 12d invariant form. In other words, 7 invariants of 11d case of Table 1 can be written in a 12d invariant form, which is non-trivial statement and provide some support for ideas of 12d invariance of M-theory.

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