Identification of imperfections by means of dynamic response

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Abstract. The idea of using the changes of the characteristics of natural or forced vibrations (natural frequency, natural modes, damping) for the determination of the position and magnitude of damage to building structures originated in the 70s [1]. The achievement of ever higher accuracy of developing dynamic displacement sensors and evaluation instruments has made this idea increasingly applicable. Identification is a process defining the properties of a structure and the characteristics of its design model. The often required identification task of determining the rate of deterioration of structures by means of their dynamic response has been numerous lately. The paper sums up the experience of the Institute of Theoretical and Applied Mechanics (ITAM) of the Czech Academy of Sciences, with laboratory tests of large models and in-situ measurements.

1. Introduction
Attempts at expressing the scope of a structure’s damage by means of its dynamic response to excitation by a harmonic force or by some other dynamic excitation can be found in writings as early as the seventies [1]. This problem belongs to the field of modal analysis or identification and has been elaborated on theoretically for various discrete and continuous systems by numerous authors. The problem has been solved also by many experiments, especially in laboratories using beams with artificial cracks [2]. Other authors tried to determine the crack location and size in a simply supported beam by means of its dynamic response. Laboratory model tests have enabled a better definition of e.g. the theoretically introduced non-linear crack characteristics, the dependence of vibration damping on the character of crack surfaces and other factors [3]. The application of the dynamic response method and other experimental results can be found in [4], [5], [6], [7], [8] and [9].

2. Methods of Damage Size Assessment
Experimental studies of actual structures and their physical models have revealed that the changes of natural frequencies represent a sufficiently accurate indicator of changes in flexural and/or torsional rigidity.
As the natural frequency is a quantity “integrating the characteristics of the whole structure”, it cannot be used as an indicator of the location of damage or imperfection.

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Another quantity expressing the scope of damage or imperfection is the Modal Assurance Criterion (MAC) comparing the vector of the \( i \)-th natural mode in the virgin state with the vector of the same mode in the damaged state:

\[
MAC(i) = \frac{(v_{iv}^T \cdot v_{id})^2}{(v_{iv}^T \cdot v_{iv})(v_{id}^T \cdot v_{id})}
\]

Where \( v_{iv} \) – Vector of the \( i \)-th natural mode in the virgin state \( v \), \( v_{id} \) – Vector of the \( i \)-th natural mode in the damaged state \( d \), \( T \) – Transposed vector.

\( MAC = 1 \) means that the vector \( v_{iv} \) does not differ from the vector \( v_{id} \); \( MAC < 1 \) means a difference of both vectors.

3. Methods of Damage Location Determination

To determine the location of cracks in tensile regions of RC structures we use the analysis of the changes of curvature of the surfaces representing the natural vibration modes. The application of highly sensitive sensors makes it possible to determine the crack location with great probability.

By way of example we shall describe the verification of crack location in a floor slab of a department store building.

The object of the measurements was the vertical dynamic response in selected points of an RC floor slab on the 3rd floor of a department store building damaged by cracks due to volume changes (Fig. 1). The purpose of dynamic loading was to ascertain the extent at which the cracks influenced forced vibration modes.

The vibrations were excited by two rotary mechanical vibration exciters (in phase and out of phase) with directed force vector, and the accuracy of excitation frequency setting within the range of 0 – 10 Hz was 0.001Hz.

By way of example Fig. 2 shows the excited mode of stationary movement along the shown straight line produced by excitation in Points 4 and 9. At the crack location the forced mode line is not smooth. The value of amplitude deviation for the crack site (Fig. 2) is higher than 6%. Consequently, the disturbance of the smooth run of the forced vibration mode may be attributed to the crack.

![Figure 1. Ground plan of the flat-slab floor and cracks in the lower face of the floor.](image1)

![Figure 2. Excited mode of the floor.](image2)
task, i.e. determination of a design model of the structure and the stages of its deterioration, is the most frequent object of theoretical research. The Institute of Theoretical and Applied Mechanics has derived computer programmes enabling, for certain measuring procedures, the drawing of obtained vibration modes, their correction by orthogonalization and their comparison with theoretical vibration modes. Modal analysis makes it possible to determine the natural frequencies, the natural vibration modes and vibration damping from the dynamic response of the structure expressed by the transfer function $H_{k,i}$.

$$H_{k,i} = \sum_j \frac{v_{k,j}v_{i,j}}{\left(\sum_m m_jv_{j,j}^2\right)^{\frac{1}{2}}} \left((\omega_{i,j}^2 - \omega_k^2)^2 + 4\omega_k^2\omega_j^2\right)^{\frac{1}{2}}$$

(2)

The natural vibration modes can be determined experimentally in the simplest manner by harmonic excitation. Other excitation methods use excitation by ambient unrest introduced into the structure by its foundation or excitation by pulses introduced directly into the structure. Modal analysis can also be used to determine the rigidity matrix $[k]$, provided the relation

$$\left([k] - \omega_j^2[m]_D\right)\{v_{(j)}\} = \{0\}$$

(3)

holds between the rigidity matrix $[k]$ and the mass matrix $[m]_D$, where $\{v_{(j)}\}$ is the vector of natural vibration modes and $\omega$ is a circular frequency

$$\left[v_{(j)}\right] \{k\} = \omega_j^2\left[m\right]_D \{v_{(j)}\}$$

(4)

Matrix $\{k\}$ Even if all terms of the matrix $\left[v_{(j)}\right]$, which is of the $q, p$ type, have not been ascertained experimentally. The following cases may arise:

1. $p = q$, i.e. the number of independent equations equals the number of unknown quantities,
2. $p < q$, i.e. the number of independent equations exceeds the number of unknown quantities. (We exclude the third possibility, viz. $p > q$ from our considerations).

When we multiply the matrix $\left[v_{(j)}\right]^T$ from the left eq. 3. we obtain

$$\left[v_{(j)}\right]^T \left[v_{(j)}\right] \{k\} = \left[v_{(j)}\right]^T \omega_j^2 \left[m\right]_D \{v_{(j)}\}$$

(5)

Which complies with the requirements of $p = q$.

Identification of the uniformly distributed mass of the beam $\mu$ and its bending rigidity $EJ$ can be performed by the method described in [9]. This method can be used when the experiment enables successive regular increases of applied loads in selected steps.

4. Monitoring

Monitoring of the structure’s dynamic response observes the frequency value $f_{(fun)}$ appertaining to the most energetically significant vibration mode. Wind may be excited by environmental noise, the passage of vehicles (in standard traffic conditions), by pulses or harmonically. If we base our considerations on the requirement of the permissible change of static deflection $\Delta \delta_{lim}$, we can easily derive the permissible frequency change $\Delta f_{(fun)lim}$ from the relation

$$f_{(fun)} = \frac{K}{2\pi} \left(\frac{g}{\delta}\right)^{\frac{1}{2}}$$

(6)

The magnitude of the coefficient $K$ has been dealt with by a number of authors, their survey is given in [5] It holds for beams in which $K = 1.12$ to 1.24. For other structural systems $K$ must be determined. If we introduce a new constant

$$k = K^2 \cdot \frac{g}{4\pi}$$

(7)

we can express the permissible frequency change
\[
\Delta f_{(\text{fun})}\text{lim} = \frac{k}{\delta + \Delta \delta\text{lim}} - \frac{k}{2f_{(\text{fun})}} \tag{8}
\]

Similarly, if we know the permissible change of the bending rigidity \(\Delta EJ\) of the beam, we can derive from the relation \(f_{(\text{fun})} = \sqrt{B \cdot EJ/\mu}\) the permissible frequency change

\[
\Delta f_{(\text{fun})}\text{lim} = \frac{\left( f_{(\text{fun})}^2 - B \frac{EJ - \Delta EJ}{\mu} \right)}{2f_{(\text{fun})}} \tag{9}
\]

Many structures have been selected from a great number of dynamic loading tests according to [4], [5], [6], [7] and [8], carried out in the Building Technical and Testing Institute and in the Institute of Theoretical and Applied Mechanics since 1990 until the present day.

5. Conclusion
The described methods of identification of imperfections and their location assume the knowledge of the virgin state values of the respective quantities. If the structure is monitored from a certain stage, the changes of the quantities are referred to the initial stage.

Experiments on actual structures have confirmed that the vibration-based inspection methodology for global health monitoring of structures is a promising technique enhanced by decreasing cost, good reliability and the high sensitivity of modern transducers and computer technology. The necessary step in the methodology is evaluation of the structure’s virgin state or of the state with which other changed states are compared. The advantages of using the dynamic response method are that it is non-destructive, illustrative and well applicable.

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