Spontaneous dissipation from generalized radiative corrections

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Abstract

We derive dissipative effective Hamiltonian for the unstable Lee model without any ad hoc coarse graining procedure. Generalized radiative corrections, utilizing the in-in formalism of quantum field theory, automatically yield irreversibility as well as the decay of quantum coherence. Especially we do not need to extend the ordinary Hilbert space for describing the intrinsically dissipative system if we use the generalized in-in formalism of quantum field theory.

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1 Introduction

Understanding the irreversibility in the macroscopic world is one of the most attractive issues in physics. Especially a consistent derivation of the irreversible dynamics from much fundamental microscopic law of physics would be the central issue. Our ill fortune is that the most microscopic physics are strictly reversible and a simple application of them never yield irreversibility.

A popular approach to obtain irreversibility will be to consider an open system: We decompose the total closed system into a relevant system and the remaining environmental degrees of freedom. Then by coarse graining the environmental degrees of freedom with appropriate initial conditions (projection), we obtain effective dynamics for the relevant system. The irreversibility stems from the information loss of the system into the environment. Although this pragmatic procedure is widely used in the literature, qualitative dissipative nature in general depends on how we set the separation of the total system and on the coarse graining procedure. Surely this is unfavorable nature of the theory; the irreversibility is an intrinsic nature of the system and should not be affected by the method of description. In this paper, we would like to demonstrate that for certain systems, it is possible to derive intrinsic irreversibility without specifying the separation and the coarse graining methods.

It is obvious that not all the systems show irreversibility. Then what is the essential difference between reversible and irreversible systems? According to our experience, a system should, at least, be unstable for it to show irreversibility. It is manifest that a stable state cannot change further and shows no irreversibility. On the other hand the unstable system cannot persist on the initial state.
and eventually decays into much stable state if any. However instability will not be the sufficient condition for the irreversibility. If the stable-unstable transition is simple and the system has finite recursion time, then the system cannot be irreversible. In order to obtain the infinite recursion time, we need *infinite number of degrees of freedom or chaos*. Another necessary condition for irreversibility will be a *natural averaging* procedure whatever it is implicit or explicit. One such averaging procedure will be the radiative corrections in quantum theory. Esp

a) instability b) infinite degrees of freedom and c) natural averaging, are sufficient conditions for the irreversibility, we try to use a simple model which satisfies all the above conditions. It is the unstable Lee model. This simple model of quantum field theory is exactly solved and was used in the argument of renormalizations. In this article, we use this unstable Lee model and demonstrate a possible origin of the intrinsic dissipative dynamics without any ad hoc coarse graining procedure.

The *instability* of the system is considered to be an essential ingredient for the emergence of intrinsic dissipativity. However, an unstable system can be described by hermitian Hamiltonian which includes no dissipativity nor irreversibility at least in appearance. There is a long history of the study on unstable states and its decay in quantum mechanics, in particle physics and in statistical mechanics. They faced with the complex eigenvalue for the Hamiltonian, and therefore extended the Hilbert space so that to maintain the hermiticity of the Hamiltonian. In this procedure they abandon the usual Hilbert space (a space of square integrable functions) and introduced a rigged Hilbert space (a space of distributions). According to this method, a pair of dual spaces $\Psi^\uparrow$ and $\Psi^\downarrow$ is necessary. They are dual of the space of boundary functions which are analytic in the lower (upper

We will not use this extended Hilbert space approach in the present article in order to describe the intrinsic dissipative nature of the unstable system. However, we use the generalized in-in formalism of quantum field theory, in which dissipative and irreversible properties are consistently incorporated through radiative corrections. In the ordinary quantum field theory (in-out formalism), ultra-violet divergence in the quantum system of infinite degrees of freedom necessitates the renormalization of the parameters in the Hamiltonian through the process of radiative corrections. This radiative correction is a special kind of averaging, which does not directly yield irreversibility in the stable theory. If the system is unstable, this process yields complex poles in the retarded propagator. The location of a pole is interpreted as the decay strength or the inverse of the lifetime. This dissipativity was not present in

In this paper, we first review the unstable Lee model in the usual treatment in the next section §2. Then we study the same model in the in-in formalism of quantum field theory in §3 and derive a Langevin equation for the fields. We further derive the effective Hamiltonian which describes irreversible dynamics in §4 and show that the linear entropy automatically increases. The last section §5 is for discussions and summary of our work.

## 2 Unstable Lee model

Let us begin our argument from the unstable Lee model in the usual treatment. The system is composed from two kinds of non-relativistic fermions $N$ and $V$ and one boson field $\theta$.

Hamiltonian of the Lee model is given by

$$H = H_0 + H_{\text{int}}$$

$$H_0 = m^0_V \int \frac{d^3 p}{(2\pi)^3} N^* \dot{V} + m_N \int \frac{d^3 p}{(2\pi)^3} N^* N \dot{N} + \int \frac{d^3 k}{(2\pi)^3} \omega_k \theta^* \theta \dot{k}$$
\[
H_{\text{int}} = \lambda_0 \int \frac{d\vec{k}}{(2\pi)^3 \sqrt{2\omega_k}} \int \frac{d\vec{p}}{(2\pi)^3} f(\omega_k)(V^*_\vec{p}N_{\vec{p} - \vec{k}}\theta_{\vec{k}} + \text{h.c.}),
\]

where \(\omega_k = \sqrt{\vec{k}^2 + \mu^2}\). The states \(|N_{\vec{p}}\rangle \equiv N^*_\vec{p}|0\rangle\) and \(|a_{\vec{k}}\rangle \equiv a_{\vec{k}}^*|0\rangle\) are the eigenstates of the total Hamiltonian, however the state \(|V_{\vec{p}}\rangle \equiv V^*_\vec{p}|0\rangle\) is not.

In order to construct the eigenstate \(|\tilde{V}_{\vec{p}}\rangle\), we have to superpose the states: \(|V_{\vec{p}}\rangle\) and \(|a_{\vec{k}}N_{\vec{p} - \vec{k}}\rangle\) as

\[
|\tilde{V}_{\vec{p}}\rangle = \sqrt{Z_V} \left(|V_{\vec{p}}\rangle + \frac{d\vec{k}}{(2\pi)^3} g_{\vec{k}} |a_{\vec{k}}N_{\vec{p} - \vec{k}}\rangle\right).
\]

Then the weight \(g_{\vec{k}}\) in this form is determined from the eigenstate equation

\[
H|\tilde{V}_{\vec{p}}\rangle = m_V|\tilde{V}_{\vec{p}}\rangle,
\]

as

\[
g_{\vec{k}} = \frac{1}{m_V - m_N - \omega_{\vec{k}}} \frac{\lambda_0 f(\omega_{\vec{k}})}{\sqrt{2\omega_{\vec{k}}}}.
\]

The determination of the eigenvalue \(m_V\) is equivalent to looking for a root of the retarded propagator:

\[
G_R(E) = \left[ E - m_V^0 + i\epsilon + \int \frac{d\vec{k}}{(2\pi)^3} \frac{1}{m_N + \omega_{\vec{k}} - E - i\epsilon} \frac{\lambda_0 f^2(\omega_{\vec{k}})}{2\omega_{\vec{k}}} \right]^{-1}.
\]

A real root is found for a stable \(V\)- particle. However if unstable, the root becomes complex: \(E = m_V - i\gamma/2\). Usually \(m_V\) is interpreted as the observable mass and \(1/\gamma\) as the life-time of the physical \(V\) particle\[13\]. However, the state \(|\tilde{V}_{\vec{p}}\rangle\) turns out to have zero norm \(\langle \tilde{V}_{\vec{p}}|\tilde{V}_{\vec{p}}\rangle = 0\) simply because the Hamiltonian is a hermitian operator. Nakanishi and others \[4\] \[5\] expressed the eigenstate corresponding to this eigenvalue by introducing the notion of complex distribution. On the other hand in papers \[6\] \[7\], they extended the ordinary Hilbert space introducing the notion of the rigged Hilbert space.

### 3 Unstable Lee model in the in-in quantum field theory

We will take a conservative approach. Instead of extending the ordinary Hilbert space, we express the evolution of the unstable particle in the in-in formalism of quantum field theory \[8\] \[9\] \[10\], which is the most appropriate formalism for describing the unstable quantum system\[11\]. This is just a simple extension of the ordinary quantum theory with the doubled time-contour of integration. In this formalism, it is possible to express the statistical dissipation and fluctuations consistently with quantum field theory. Leaving the detail of this formalism for the other paper \[12\], we briefly explain this formalism here.

The time contour of integration in the in-in quantum field theory is generalized to run from \(-\infty\) to \(+\infty\) and then back to \(-\infty\) again. All the arguments of fields \(\Phi(x)\) are doubled according to this new time contour. Moreover, the Hamiltonian \(H[\Phi]\) of the system is generalized to \(\hat{H}[\Phi^\pm] = H[\Phi^+] - H[\Phi^-]\) where \(X^+(x)\) and \(X^-(x)\) mean the field quantity \(X(x)\) restricted on the forward and backward time branches, respectively. In the same manner, the Lagrangian density \(L\) is generalized.
to \( \hat{L} = L[\Phi^+] - L[\Phi^-] \). Because the standard Pauli equation for the density matrix \( i\partial\rho(t)/\partial t = [\hat{H}, \rho(t)] \) is expressed in the coordinate representation \((\Phi_+ | \rho(t) | \Phi_-) = \rho[\Phi^\pm, t]\) as

\[
i \frac{\partial \rho[\Phi^\pm, t]}{\partial t} = (H[\Phi^+] - H[\Phi^-])\rho[\Phi^\pm, t],
\]

this generalized Hamiltonian, in the coordinate representation, \( \hat{H} \) is thought to be the time translation operator for the density matrix.

The partition function is defined in the usual way except that the time-integration contour is doubled.

\[
\hat{Z}[J] \equiv \text{Tr}[T_C(\exp[i \int_C d^4x J(x) \Phi(x)])\rho] = \text{Tr}[T_+(\exp[i \int d^4x J_+(x) \Phi_+(x)])T_-(\exp[-i \int d^4x J_-(x) \Phi_-(x)])]\rho]
\]

where the suffix \( C \) in the integral means that the time integration contour is generalized so that it runs from minus infinity to plus infinity and then back to the minus infinity again. The symbol \( \rho \) is the initial density matrix. The symbol \( \Phi(x) \) represents all the quantum fields in Heisenberg picture. Generalized effective action \( \hat{\Gamma}[\Phi] \) is defined simply as the Legendre transformation of the above partition function \( \hat{Z}[J] \). Perturbation method using generalized propagators is available for calculating various quantities.

Back to the unstable Lee model, we calculate the generalized effective action. Because there is only one loop correction for the \( V \)-particle propagator and no correction for the \( N \)- and boson-particle propagators, the calculation is exactly done and the result is

\[
\hat{\Gamma} = S_N[N_+] - S_N[N_-] + S_0[\theta_+] - S_0[\theta_-] + S_{int}[N_+, V_+, \theta_+] - S_{int}[N_-, V_-, \theta_-] + \int d^4x \int d^4x' \left( \Phi_+^{\Delta}, \Phi_-^{\Delta} \right)^* \left( \Phi_+^{C}, \Phi_-^{C} \right)_{x,x'} \left( \begin{array}{cc} D - iB & i(B - A) \\ i(B + A) & -D + iB \end{array} \right) \left( \begin{array}{c} \Phi_+^{\Delta} \\ \Phi_-^{\Delta} \end{array} \right)_{x,x'}
\]

where all \( N \) and \( V \) variables are Grassmann valued fields. In the above, the first line represents the bare actions corresponding to Eq(4) but with renormalized coupling constant \( \lambda \) instead of \( \lambda_0 \). The last line is the radiatively corrected \( V \)-particle part and is further rewritten as

\[
\int d^4x \int d^4x' \left( \Phi_+^{\Delta}, \Phi_-^{\Delta} \right)^* \left( \begin{array}{cc} D^* & D + iA \\ D - iA & 0 \end{array} \right)_{x,x'} \left( \begin{array}{c} \Phi_+^{\Delta} \\ \Phi_-^{\Delta} \end{array} \right)_{x,x'}
\]

where \( \Phi_\Delta = \Phi_+ - \Phi_- \) and \( \Phi_C = (\Phi_+ + \Phi_-)/2 \). Kernels \( A, B, \) and \( D \) are induced from radiative corrections and are exactly calculated. Their Fourier transforms are given by

\[
D(E) = E - m_V^0 + \frac{d^k \lambda^2 f^2(\omega_k)}{(2\pi)^3} \frac{P}{2\omega_k} \frac{m_N + \omega_k - E}{m_N + \omega_k - E},
\]

\[
B(E) = Z^{-1}_V \theta(E - m_N - \mu) \sqrt{(E - m_N)^2 - \mu^2 \lambda^2 f(E - m_N)/(4\pi)},
\]

\[
A(E) = \text{sign}(E) B(E),
\]

in the momentum representation but the three momentum is suppressed. \( D(E) \) part yields the infinite mass correction and the wave function renormalization as

\[
D(E) = (1 + C_1)E - (m_V^0 - C_0 + C_1 m_V) = Z^{-1}_V E - m_V,
\]
If we decompose the effective action as \( \Gamma = \text{Re}\Gamma + i\text{Im}\Gamma \), we can rewrite this expression by introducing auxiliary fields \( \Phi \) and \( \xi \). The above effective action can be re-expressed in a form which manifestly represents the system's unstable behavior.

The renormalized coupling constant \( \lambda \) and the wave function renormalization \( Z_V \) appeared in Eq. (10) are defined as

\[
Z_V^{-1} = 1 + C_1, \quad \lambda^2 = Z_V \lambda_0^2.
\]

The \( B(E) \) part comes from the quantum cross correlation between the forward time branch and the backward time branch. This term was absent in the usual in-out formalism. The effective action \( \Gamma \) is a statistical average of the individual effective actions \( \text{Re}\Gamma \) and \( \text{Im}\Gamma \). It is odd in the argument because we are considering specific boundary conditions.

The above kernels are in general non-local. Here we take the local approximation (setting \( E \to m_V \)) with the limit \( m_V \gg m_N, \mu \) just for simplicity. Then the kernels \( A \) and \( B \) become

\[
A(E) \approx \frac{\lambda^2 f(m_V)}{4\pi} E, \quad B(E) \approx \frac{\lambda^2 f(m_V)}{4\pi} m_V,
\]

where we have set \( Z_V = 1 \). Note that the effective action becomes complex reflecting the fact that the system is unstable. The pure imaginary term, which is proportional to \( B(t) \), is however symmetric for the exchange of the variables \( \Phi^*_F \leftrightarrow \Phi^*_B \); all other terms are anti-symmetric. Therefore if we define the hermitian conjugate including the operation of the exchange of \( \Phi^*_F \), the generalized effective action \( \hat{\Gamma} \) becomes hermitian. In fact, this hermiticity is explicitly realized later in the effective Hamiltonian.

Now we derive the generalized equations of motion for the fields. Because only the \( V\)-field shows irreversibility and dissipativity, we concentrate on this field and suppress the suffix \( V \) for the moment. The above effective action can be re-expressed in a form which manifestly represents dissipativity if we introduce auxiliary fields \( \xi(t) \) and \( \xi^*(t) \) which are also Grassmann valued fields. If we decompose the effective action as \( \Gamma = \text{Re}\Gamma + i\text{Im}\Gamma \), then the imaginary part is even in the variable \( \Phi_D(x) \):

\[
\text{Im}\hat{\Gamma}[\Phi_c, \Phi_D, \Phi_c^*, \Phi_D^*] = \int\int \Phi_D^*(x)B(x - y)\Phi_D(y).
\]

We can rewrite this expression by introducing auxiliary fields \( \xi(x) \) and \( \xi^*(x) \) which are Grassmann valued fields

\[
\exp[i\hat{\Gamma}[\Phi, \Phi^*]] = \int \left[ |\xi| |d\xi^*| P[\xi, \xi^*] \exp[i\text{Re}\Gamma + \int(i\Phi_D^*\xi - i\xi^*\Phi_D)] \right]
\]

where,

\[
P[\xi, \xi^*] = (\det B) \exp\left[ \int\int \xi^* B^{-1} \xi \right]
\]

is a normalizable positive kernel for the fields \( \xi(x) \) and \( \xi^*(x) \). Note that this weight function is purely Gaussian. It means that we may be able to interpret \( \hat{\Gamma}[\xi, \xi^*] \) as a statistical weight for the random fields \( \xi(x), \xi^*(x) \)(stochastic part). Therefore it is possible to interpret Eq. (19) that the total effective action \( \Gamma \) is a statistical average of the individual effective actions \( \text{Re}\Gamma - \int \xi^* \Phi_D + \int \Phi_D^* \xi \).

\[1\] This non-locality means that the retarded effect has finite time scale and it turns out later that the natural noise associated with the system is colored.

\[2\] Because \( A(E) \) is odd \( A(-E) = -A(E) \) and \( B(E) \) is even \( B(-E) = B(E) \), their Fourier transforms \( A(t) \) and \( B(t) \) are pure imaginary and real, respectively.
Application of the variational principle on this individual effective action yields an equation of motion for $\Phi_C(x)$ as

$$0 = \left( \frac{\delta \text{Re}\Gamma - \int \xi^* \Phi_\Delta + \int \Phi_\Delta^* \xi}{\delta \Phi_\Delta^*(x)} \right)_{\Phi_\Delta = 0}$$

$$= \left( (i - \gamma) \partial_t - m_V + \frac{\nabla^2}{2m_V} \right) \Phi_C + V' + \xi,$$

where we have used the local approximation and $V'$ is the interaction term in the total Hamiltonian $H_{\text{int}}$. We have set $J = 0$, which means there is no external source. The symbol $\gamma$ is $\lambda^2 f(m_V)/(4\pi)$. This is a renormalized Langevin type stochastic differential equation with friction and random force terms. According to this equation, the evolution of the field $\Phi_C(x)$ is partially deterministic and partially stochastic. The former part is governed by the action $\text{Re}\Gamma[\Phi]$ which include the damping effect and the latter part is induced by the random field $\xi(x)$ whose statistical properties are completely determined by $\text{Im}\Gamma[\Phi]$. Actually if we define the statistical average as

$$\langle \cdots \rangle_{\xi, \xi^*} \equiv \int [d\xi][d\xi^*] P[\xi, \xi^*] \cdots,$$

then we obtain the correlation function for the random field

$$\langle \xi^*(x)\xi(y) \rangle_{\xi} = B(y - x),$$

which becomes white noise if we take the local approximation Eq.(14). The same variational principle yields the equations of motion for the other fields $N$ and $\theta$. However there appear no new terms which show dissipativity and irreversibility even after full radiative corrections.

## 4 Effective Hamiltonian and entropy increase

We can express the dissipativity of the system in another form by constructing the effective Hamiltonian. If we apply the local approximation, then the effective action reduces to the local form $\hat{\Gamma}_V = \int d^4x \hat{L}_V$, where $\hat{L}_V$ is the generalized effective Lagrangian for the $V$-particle part:

$$\hat{\mathcal{L}}_V = i(\Phi_\Delta^* \dot{\Phi}_V^C + \Phi_V^C \dot{\Phi}_\Delta^*) + \Phi_\Delta^* \frac{\nabla^2}{2m_V} \Phi_V^C + \Phi_V^C \frac{\nabla^2}{2m_V} \Phi_\Delta^*$$

$$- \gamma(\Phi_\Delta^* \dot{\Phi}_V^C - \Phi_V^C \dot{\Phi}_\Delta^*) + i\gamma m_V \Phi_\Delta^* \Phi_\Delta^*.$$ (21)

The canonical momenta are defined by $p_\Delta^\pm \equiv \pm \partial \hat{\mathcal{L}}/\partial \dot{\Phi}_\Delta^\pm$ or

$$p_\Delta^C \equiv \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\Phi}_V^C} = (i - \gamma)\Phi_V^C, \quad p_C^\pm \equiv \frac{\partial \hat{\mathcal{L}}}{\partial \dot{\Phi}_V^\pm} = (i + \gamma)\Phi_\Delta^\pm.$$ (22)

Then the generalized effective Hamiltonian for the total system becomes

$$\hat{H} = \int d^3x [p_\Delta^\Delta \dot{\Phi}_V^\Delta + p_C^\Delta \dot{\Phi}_V^C - \hat{\mathcal{L}}]$$

$$= H[\Phi_N^+, \Phi_\theta^+] - H[\Phi_N^-, \Phi_\theta^-] + H_{\text{int}}[\Phi^+] - H_{\text{int}}[\Phi^-]$$

$$- \Phi_\Delta^* \frac{\nabla^2}{2m_V} \Phi_V^C - \Phi_V^C \frac{\nabla^2}{2m_V} \Phi_\Delta^* - i\gamma m_V \Phi_\Delta^* \Phi_\Delta^*.$$ (23)
where \( H[\Phi^+, \Phi^-] \) and \( H_{\text{int}}[\Phi^\pm] \) are, respectively, the free \( N \theta \)-particle part and the interaction part of the original Hamiltonian Eq.(1). There is no radiative corrections for these parts except \( Z_V \) and \( \lambda \). Note that the effective Hamiltonian, even after the local approximation, is hermitian in the sense \( H[\Phi^-, \Phi^-]^* = H[\Phi^+, \Phi^-] \). Therefore if we write down the generalized Pauli equation as \( i \partial \rho[\Phi^\pm]/\partial t = H[\Phi^\pm] \rho[\Phi^\pm] \), the total probability is conserved (\( \text{Tr} \rho = \text{const.} \)). We now demonstrate this in an operator form of the Pauli equation.

We rewrite the above Pauli equation for the density matrix in an operator form. Remember that we have been using the representation: \( \Phi(\vec{x})|\Phi\rangle = \Phi(\vec{x})|\Phi\rangle \), \( \langle \Phi_+|\rho(t)|\Phi_- \rangle = \rho[\Phi^\pm, t] \), and so on. Therefore for example, the operator form of \( \Phi \Delta \rho[\Phi^\pm, t] \) is \( \Phi[\Phi^+, \rho(t)] \). In the similar way, the operator form of the Pauli equation becomes

\[
i \frac{\partial \rho(t)}{\partial t} = [H_R, \rho(t)] - i \gamma m_V [\Phi_V^*, [\Phi_V, \rho(t)]] ,
\]

where \( H_R \) is the total Hamiltonian Eq.(1) with renormalizations. It is easy to see the conservation of probability from this equation:

\[
\frac{\partial}{\partial t} \text{Tr} \rho(t) = 0.
\]

Further we define the linear entropy \( S(t) \equiv -\text{Tr} \rho(t)^2 \), which has values in \([-1, 0]\). This measures how the system possesses coherence or the amount of classicality; \( S(t) = -1 \) for a pure quantum state, and the coherence is much destroyed for larger values of \( S(t) \)[14]. Time evolution of \( S(t) \) is governed by the last term on the RHS in Eq.(24),

\[
\frac{\partial S(t)}{\partial t} = 2 \gamma m_V \text{Tr}[\rho(t), [\Phi_V^*, \rho(t)]] = 2 \gamma m_V \text{Tr}[[\Phi_V, \rho(t)]^2] > 0.
\]

Therefore the linear entropy perpetually increases. This manifestly shows irreversibility of the system. We emphasize that we did not use any ad hoc averaging method such as partial trace on the environment; we do not have any environment at all. Therefore the irreversibility represented by Eq.(26) is intrinsic to the system. Moreover this result does not rely upon ad hoc approximations such as the truncation of the sequence of correlation functions[3]; the Lee model is exactly solved. This decoherence term stems from the quantum interference between the fields on the forward time branch and those on the backward time branch, and was not exist in the usual quantum field theory of in-out formalism.

### 5 Discussions and summary

In this article, we have arrived at the dissipative expressions Eq.(18) Eq.(23) Eq.(24) for the dynamics of unstable Lee model. We would like to emphasize the following points for the origin of intrinsic irreversibility of the model.

1. The starting point of our study has been the bare Hamiltonian Eq.(1), in which no dissipativity is manifest. However this bare Hamiltonian itself does not correctly describe the real system; we need to take into account the radiative corrections and remove the divergences in the theory. These radiative corrections and renormalizations are essential for defining a feasible

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3We have used non-relativistic approximations, neglected the recoil of the heavy \( V \)-particle, and took local approximations. These are all our approximations.
At the same time, these radiative corrections automatically induce the dissipative kernels in Eq. (10) if we use the in-in formalism of quantum field theory. Note that there is no ad hoc coarse graining process at all in this procedure of radiative corrections. Actually no information included in the bare Hamiltonian is lost in the process of radiative corrections.

2. The system can be consistently expressed in the density matrix formalism with the ordinary Hilbert space. We do not have to extend the original Hilbert space anymore. We found that the $V$-particle state spontaneously decoheres. This is reasonable because the decay of a $V$-particle means not only a diffusion of energy but also a diffusion of information.

It will be interesting to compare our approach with the others concerning irreversibility and dissipativity in quantum theory.

1. T. Petrosky et al. [6] and I. E. Antoniou et al. [7] proposed the extension of the ordinary Hilbert space in order to express the semi-group property of the evolution of unstable state with hermitian Hamiltonian. In our case, instead of extending the representation space, we extended the field variables making the time integration contour double and introduced the density matrix in the in-in formalism of quantum field theory. The generalized Hamiltonian Eq. (23) is guaranteed to be hermitian even if the system is dissipative and irreversible. A new feature, which was not discussed in the work [6] [7], is the destruction of quantum coherence associated with the instability as is expressed in the last term of Eq. (21) and Eq. (23). This term directly increases the entropy of the system as we have seen in Eq. (26).

In the work [6] [7], they tried to connect the deterministic time-reversible theory and the statistical time-irreversible theory by a star-unitary operator. In our case, this kind of transformation is the process of the generalized radiative corrections in the in-in formalism of quantum field theory. According to the work [6] [7], a pair of dual spaces was necessary in order to separately represent the future-decaying and past-decaying states. In our case, this pair corresponds to the extension of the variables introducing the doubled time contour; the time evolution on the forward-time branch represents the future-decaying state, and vice versa for past-decaying state.

2. Laplace et al. [16] and Umezawa [17] introduced a notion of dynamical map which relates bare fields and the radiatively corrected asymptotic fields. This map specifies, among many equivalent representation of the canonical commutation relation, one representation suitable for the description of the actual system. From this point of view, the dynamical map, in the in-in formalism of quantum field theory, gives an ensemble of equivalent representations as we see in Eq. (16); the total effective action $\hat{\Gamma}[\Phi, \Phi^*]$, which corresponds to the effective Hamiltonian Eq. (23), is the statistical average of dynamics each of which has deterministic evolution. In this sense, our dynamical map yields one-to-many correspondence instead of one-to-one.

3. Arimitsu et al. [18] derived a general effective Hamiltonian in the thermo-field dynamics. Our effective Hamiltonian Eq. (23) is similar to that derived by Arimitsu et al. [18]. In their formalism, it was necessary to double the dynamical degrees of freedom, $\Phi$ fields and $\tilde{\Phi}$ fields, in order to describe dissipative quantum field theory within the ordinary Hilbert space. The situation is almost the same in our case; we had to introduce $\Phi_+$ as well as $\Phi_-$ fields in order to describe dissipative quantum field theory within the ordinary Hilbert space.

\[ \text{In general, friction term reduces the entropy and the term which induce the quantum decoherence (diffusion term) increases the entropy.} \]
4. An usual method to derive quantum-dissipative dynamics is to use the influence functional method. The influence functional is the induced action by partial trace of an environmental degrees of freedom and is technically almost the same as our Eq. In our case, we simply considered radiative corrections for all the fields (full trace in Eq.) not introducing the environment. Moreover the dissipative properties we obtained have nothing to do with the truncation of the BBGKY hierarchy of greens functions because the Lee model is exactly solved; one-loop graph is the whole radiative correction. These facts also suggest that the dissipative properties are intrinsic to the system.

We summarize our work. We studied the unstable Lee model constructing the radiatively corrected effective action Eq. and Hamiltonian Eq. in the in-in formalism of quantum field theory. From the effective action Eq., we derived a Langevin equation for the V-field Eq. which explicitly shows damping and fluctuation of the state. From the effective Hamiltonian Eq., we derived perpetually increasing entropy Eq. The irreversibility and dissipativity were not manifest in the original bare Hamiltonian Eq.. However we have to make radiative corrections (dynamical map) which is an indispensable process to define the asymptotic fields and a feasible theory. Through this process we obtained the effective action and Hamiltonian in which the irreversibility and dissipativity are manifest. Technically the radiative correction process is regarded as an averaging process. This averaging process yields the dissipativity. However this averaging process is the unique procedure and does not include any arbitrariness in principle. Therefore the dissipativity and irreversibility we derived are intrinsic to the unstable Lee model. The increase of entropy is due to the last term in Eq. which destroys the quantum coherence (=decoherence). This term appears due to the interference of fields and , which is specific to the in-in formalism of quantum field theory. In this way irreversibility is consistently described within the ordinary Hilbert space. If we force to ascribe a wave function for the unstable state, then the norm of the wave function would vanish. We introduced a density matrix and allowed mixed state for the unstable particle. Then the probability is conserved despite the irreversibility.

We would like to report extension of our formalism of intrinsic irreversibility and dissipativity in our future publications.

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