Multiple M-wave interaction with fluxes

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We present the equations of motion for multiple M0–brane (multiple M-wave or mM0) system in general eleven dimensional supergravity background. These are obtained in the frame of superembedding approach, but have a rigid structure: they can be restored from SO(1,1)×SO(9) symmetry characteristic for M0. BPS (Bogomol’nyi-Prasad–Sommerfield) conditions for the 1/2 supersymmetric solution of these equations have the fuzzy 2-sphere solution describing M2-brane.

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Supersymmetric extended objects, (super-)p–branes (including string for p=1, membrane for p=2 and also particle for p=0) and interacting systems of several p–branes play very important rôle in String/M-theory and in the AdS/CFT correspondence. They are used in constructing models of our Universe as a 3–brane or an intersection of p–branes in the space of higher–dimensions. Such Brane World scenarios can be developed in the frame of string/M-theory as well as independently of it. The most known examples of the later were the Randall–Sundrum models which then were incorporated in the M-theoretic context.

The most interesting p–branes are D=10 fundamental strings and Dp–branes (Dirichlet p–branes), where the fundamental string can have its ends, and D=11 M–branes with p=0,2,5. These can be described by supersymmetric solutions of 10D and 11D supergravity equations (see references therein), by the worldvolume actions and in the frame of the so-called superembedding approach (see more refs).

As far as the multiple p–brane systems are concerned, it was appreciated long ago that in the very low energy limit the dynamics of multiple Dp–brane (mDp) system is approximately described by the maximally supersymmetric U(N) super–Yang–Mills (SYM) action. In the search for a counterpart playing the rôle of SYM action, it was appreciated long ago that in the very low energy limit the dynamics of multiple Dp–brane (mDp) system is approximately described by the maximally supersymmetric U(N) super–Yang–Mills (SYM) action. In the search for a counterpart playing the rôle of SYM action, it was appreciated long ago that in the very low energy limit the dynamics of multiple Dp–brane (mDp) system is approximately described by the maximally supersymmetric U(N) super–Yang–Mills (SYM) action.

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1. To fix the basic notion and notation, we begin by a very brief description of superembedding approach to a single M0-brane in general 11D supergravity background. This requires the superfield description of 11D supergravity in terms of supervielbein one forms \( E^A := dz^M E_M A(Z) = (E^a, E^\alpha) \) (with bosonic vectorial form \( E^a, a = 0, 1, \ldots, 9, 10 \), and fermionic spinorial form \( E^\alpha, \alpha = 1, \ldots, 32 \)) which satisfy the set of superspace constraints \( \Sigma^{[1]}(32) \) of which the most important fixes the form of bosonic torsion 2-form of the curved 11D superspace \( \Sigma^{[1]}(32) \) (see [15] and refs. therein for details)

\[
T^a := DE^a = -iE^a E^\beta \Gamma^a_{\alpha\beta} .
\]

Here \( \Gamma^a_{\alpha\beta} = \Gamma^a_{\beta\alpha} \) are 11D Dirac matrices and the exterior product of differential forms is assumed \( (E^\alpha E^\beta = E^\alpha \wedge E^\beta) \). We have denoted the local coordinates of \( \Sigma^{[1]}(32) \) by \( Z^M = (x^m, \theta^q) \) \((\bar{a} = 1, \ldots, 32, m = 0, 1, \ldots, 9, 10)\).

The standard formulation of M–branes \((M p\text{branes})\) with \( p = 0, 2, 5)\) deals with embedding of a purely bosonic worldvolume \( W^{p+1} \) (worldline \( W^1 \) for M0-case of \( \Sigma \)) into the target superspace \( \Sigma^{[1]}(32) \).

The superembedding approach to M–branes \([10, 12]\), following the STV (Sorokin–Tkach–Volkov) approach to superparticles and superstrings \([16]\) (see [13] for review and further refs) describes their dynamics in terms of embedding of worldvolume superspace \( \mathcal{W}^{(p+1)} \) with \( d = p + 1 \) bosonic and 16 fermionic directions into \( \Sigma^{[1]}(32) \). This embedding can be described in terms of coordinate functions \( \hat{Z}^M(\zeta) = (\hat{x}^m(\zeta), \hat{\theta}^q(\zeta)) \), which are superfields depending on the local coordinates \( \zeta^M \) of \( \mathcal{W}^{(p+1)} \),

\[
\mathcal{W}^{(p+1)} \in \Sigma^{[1]}(32) ; \quad Z^M = \hat{Z}^M(\zeta) .
\]

For \( p = 0 \) these are \( \zeta^M = (\tau, \eta^q) \), where \( \tau \) is proper time and \( \eta^q \) are 16 fermionic coordinates of the worldline superspace \( \mathcal{W}^{(1)} \), \( (\eta^q \theta^q = -\eta^q \bar{\theta}^q, \bar{q} = 1, \ldots, 16) \).

The superembedding equation states that the pull–back \( E^a := dZ^M(\zeta) E^a_M(\hat{Z}) \) of the bosonic supervielbein form \( E^a := dZ^M E_M(\hat{Z}) \) to the worldvolume superspace has no fermionic projection. For the case of M0–brane it reads

\[
\hat{E}^a_{+q} := D_{+q} \hat{Z}^M E^a_{M}(\hat{Z}) = 0 ,
\]

where \( D_{+q} \) is a fermionic covariant derivative of \( \mathcal{W}^{(1)} \), \( q = 1, \ldots, 16 \) is a spinor index of \( SO(9, 1) \) and \( + \) denotes the ‘charge’ (weight) with respect to the local \( SO(1, 1) \) group. The only bosonic covariant derivative of \( \mathcal{W}^{(1)} \) is denoted by \( D_{\pm} := D_{+q} + \text{supervielbein of } \mathcal{W}^{(1)} \) by \( e^A = d\zeta^M e^A_M(\zeta) = (e^{\#}, e^{\tau}) \). Notice that in our notation the upper plus index is equivalent to the lower minus, and vice-versa, so that one can equivalently write, for instance, \( D_{+q} = D_{q} \) and \( D_{-} = D_{-}/2 \).

The superembedding equation \([13]\) is on–shell in the sense that it contains the M0–brane equations of motion among their consequences. We refer to \([15]\) for the explicit form and the derivation of these equations. For the discussion below we will need only few details concerning the on–shell geometry of the worldline superspace \( \mathcal{W}^{(1)} \). In particular, with our conventional constraints (see \([15]\)) the bosonic torsion two form of \( \mathcal{W}^{(1)} \) reads

\[
De^{\#} = -2ie^{+q} e^{\eta} ,
\]

the Riemann curvature two form of \( \mathcal{W}^{(1)} \) vanishes, while fermionic torsion \( De^{\tau} \eta \) and curvature of the normal bundle over \( \mathcal{W}^{(1)} \) are expressed through the following components of the pull–backs of the bosonic and fermionic fluxes (field strengths or curvatures)

\[
\hat{F}_{\#ijk} := F_{\#(\hat{Z})} u^a_{\#} u^b_{\#} u^c_{\#} , \quad \hat{R}_{\#ijk} := R_{\#(\hat{Z})} u^a_{\#} u^b_{\#} u^c_{\#} , \quad \hat{T}_{\#i+q} := T_{\#(\hat{Z})} u^a_{\#} u^b_{\#} .
\]

Here \( u^a_{\#} \) and \( u^b_{\#} \) are the auxiliary moving frame superfields which obey (notice that \( u^a_{\#} \neq u^b_{\#} := u^a_{\#}/2 \))

\[
u^q_{\#} = 0 , \quad u^a_{\#} u^a_{\#} = 0 , \quad u^a_{\#} u^b_{\#} = 2 , \quad u^a_{\#} u^a_{i} = 0 , \quad u^a_{\#} u^a_{i} = 0 , \quad u^a_{\#} u^a_{j} = -\delta^i_j .
\]

The sixteen 11D spinor superfields \( \nu^a_{\#} = \nu^a_{\#}(\zeta) \) in \([7]\) are the spinor moving frame variables which can be considered, roughly speaking, as square roots of the light–like vector \( u^a_{\#} \) in the sense of that (see \([15]\) and refs. therein)

\[
u^a_{\#} \nu^b_{\#} = \Gamma^a_{\beta\alpha} u^a_{\#} \quad \text{for } \nu^a_{\#} \text{ and the light–like moving frame vector } u^a_{\#} \text{ are covariantly constant, } D_{\nu^a_{\#}} = 0 \text{ and } D_{u^a_{\#}} = 0 . \quad \text{Then one finds}
\]

\[
D_{+q} \hat{F}_{\#ijk} = 3i\gamma_{[ij]} \nu_{\#} \nu_{+q} \hat{T}_{\#[k]} \quad ,
\]

\[
D_{+q} \hat{F}_{\#ij} = \frac{i}{2} \hat{R}_{\#ij} \nu_{\#} + \frac{i}{2} D_{\#} \hat{F}_{\#ijk} \gamma^{jk}_{\#}
\]

\[
+ \frac{1}{16} \hat{F}_{\#[ij]} \hat{F}_{\#jk} \nu_{\#} = 0 \quad \text{for } \mathcal{W}^{(1)} = \Sigma^{[1]}(32) \]
where $\gamma_{ap}^i = \gamma_{pq}^i$ are nine-dimensional Dirac matrices, 
$\gamma_{3p} + \gamma_{2p} = 2\delta^p_0 I_{10 \times 16}$ ($i = 1, \ldots, 9$). This constraint 
involves a nanoplet of $N \times N$ hermitian matrix superfields 
$\mathbf{X}^i = (\mathbf{X}^i)^\dagger$ the leading component of which provides 
the natural candidate for the field describing the relative 
motion of the M0 constituents.

Studying Bianchi identities one finds that the selfconsistency 
of the constraints [13] requires the matrix superfield 
$\mathbf{X}^i$ to obey the superembedding–like equation [15]
\begin{equation}
D_{+q} \mathbf{X}^i = 4i \gamma_{ap}^i \Psi_q .
\end{equation}

The set of physical fields of the $d=1$, $N=16$ SYM 
model defined by constraints [13] is exhausted by 
the leading component of the bosonic superfield $\mathbf{X}^i$, 
providing the non-Abelian, $N \times N$ matrix generalization 
of the Goldstone field describing a single M0-brane in 
static gauge, and by its superpartner, the leading 
component of the fermionic superfield $\Psi_q$ in [13], 
providing the non-Abelian, $N \times N$ matrix generalization of 
the fermionic Goldstone field describing a single M0-brane. 
(which can be extracted from the fermionic coordinate 
function of M0-brane by fixing the gauge with respect to 
local fermionic $\kappa$–symmetry). To be convinced in that no 
other fields appear, one can calculate the spinor covariant 
derivative of the fermionic superfield and find
\begin{equation}
D_{+p} \Psi_q = \frac{1}{2} \gamma_{pq} \mathbf{D}_{\#} \mathbf{X}^i + \frac{1}{16} \gamma_{pq} \left[ \mathbf{X}^i, \mathbf{X}^i \right] - \\
- \frac{1}{2} \mathbf{X}^i F_{\#jk} \left( \delta^{ij} \gamma^{kl} + \gamma^{ij} \gamma^{kl} \right) \psi_p .
\end{equation}

3. Equations of motion and polarization of multiple M0 by flux. 
Studying the selfconsistency condition of Eq. [15] (on the line of [13] but taking into account 
nonvanishing supergravity fluxes) we find the interacting 
dynamical equation for the fermionic matrix (super)fields
\begin{equation}
D_{\#} \Psi_q = \frac{1}{4} \gamma_{qp} \left[ \mathbf{X}^i, \psi_p \right] - \frac{1}{2} \mathbf{F}_{\#ijk} \gamma_{ap} \psi_p - \\
- \frac{1}{2} \mathbf{X}^i \hat{T}_{\#i+q} .
\end{equation}

As usual in supersymmetric theories, the higher 
components in decomposition of the superfield version 
of the fermionic equations over the Grassmann coordinates 
of $W^{(1)}$[16] give the bosonic equations of motion. In the case of 
our multiple M0 system these are the Gauss constraint
\begin{equation}
\left[ \mathbf{X}^i, D_{\#} \mathbf{X}^i \right] = 4i \left\{ \Psi_q, \Psi_q \right\}
\end{equation}
and proper equation of motion
\begin{equation}
D_{\#} D_{\#} \mathbf{X}^i = \frac{1}{16} \left[ \mathbf{X}^i, \left[ \mathbf{X}^i, \mathbf{X}^i \right] \right] + i \gamma_{pq} \left\{ \Psi_q, \psi_p \right\} + \\
+ \frac{1}{8} \mathbf{F}_{\#ijk} \left[ \mathbf{X}^j, \mathbf{X}^k \right] + \frac{1}{4} \mathbf{X}^j \hat{R}_{\#j} - 2i \Psi_q \hat{T}_{\#i+q} .
\end{equation}
The third term in the r.h.s. of the bosonic equation [15], 
$\mathbf{F}_{\#ijk} \left[ \mathbf{X}^i, \mathbf{X}^k \right]$, is essentially non-Abelian and typical 
for ‘dielectric coupling’ characteristic for the Emparan-
Myers ‘dielectric brane effect’ [19, 20]. The fourth term is 
the mass term for $N \times N$ matrix SO(9) vector 
superfield $\mathbf{X}^i$ with the mass matrix given by the projection 
of Riemann tensor $\hat{R}_{\#i} (\equiv \hat{R}_{\#i} \#i)$ defined in Eq. [9].

4. Actually, using only the SO(1,1)$\times$SO(9) symmetry 
of our mM0 system one can not only find all the terms in the 
r.h.s.’s of Eqs. (16)–(18), but also conclude that only 
other two contributions might be possible but are absent. 
The reason beyond this rigid structure of the multiple M0 equations 
is that all the basic superfields and projections of the background 
fluxes interacting with mM0 constituents carry positive SO(1,1) weights (‘charges’).

Indeed, the SO(1,1) weights are +2 for the bosonic 
superfield $\mathbf{X}^i := \mathbf{X}^i_{+1,+3}$ for the fermionic $\Psi_q := \Psi_{++q}$, 
and +2, +3 and +4, respectively, for the 11D supergravity 
fluxes $\hat{F}_{\#jkl} := \hat{F}_{++jkl}$ [5], $T_{++j} := T_{++j}$ [7] 
and $R_{i++j} := R_{++i++j}$ [6]. Then, as the covariant 
derivative $D_{\#} := D_{++}$ has the weight +2, the fermionic 
and bosonic equation of motion are $N \times N$ matrices with 
the SO(1,1) weights +5 and +6, respectively. Now, 
taking into account also the SO(9) index structure of the 
basic superfields and fluxes, one sees that, if we do not 
allow ourselves using the inverse and fractional powers 
of $R_{i+}$ of matrix superfield $\mathbf{X}^i \mathbf{X}^i$, very few terms 
can be written in addition to ones already present in [16]. 
Moreover, all but two of these actually vanish.

Indeed, one could add $\mathbf{X}^i \gamma^{ij} T_{++j}$ to the Dirac equation [16], 
however this term can be expressed through the 
already present $\mathbf{X}^i T_{++i}$ using (12a). This equation is 
also responsible for vanishing, $\Psi_q \gamma^{ap} T_{++p} = 0$, 
of the only possible contribution to Eq. (17), and for 
that the possible fermionic contribution $\Psi_q \gamma^{ap} T_{++p}$ to Eq. (18) can be expressed in terms of $\Psi_q T_{++i}$ already present there. As far as the pure bosonic contributions 
to (15) are concerned, the already present terms could be 
completed by the $\gamma^{ij} \hat{F}_{\#ijkl}$ and $\gamma^{ij} \hat{F}_{\#ijkl}$ 
(due to (12b)), $\hat{F}_{\#ijkl} \hat{F}_{\#ijkl} \propto R_{\#ijkl}$. Thus the only results of 
the explicit calculations in the frame of superembedding 
approach are the absence of these two contributions to 
the mass matrix of the N×N matrix superfield $\mathbf{X}^i$ and 
the exact values of the nonvanishing coefficients in (16)–(18).

Such a rigid structure of the mM0 equations (16)–
(18), which suggests their universality, comes from 
SO(1,1)$\times$SO(9) symmetry of our mM0 system. 
This originates in that M0-brane is actually the massless 
11D superparticle the momentum of which is light–like and 
has a small group which is essentially SO(1,1)$\times$SO(9) 
(see [27] and refs. therein for a more precise statement). 
Such a rigidity cannot be seen from observing the mD0 
equations (14) as they are: as D0-brane is a massive 10D 
superparticle, the (spatial) symmetry of the relative motion 
of mD0 system of (14) is restricted to SO(9), the 
small group of the timelike 10D momentum. Thus to see 
the rigid structure of mD0 equations (14), one needs 
to appreciate their 11D origin, i.e. their appearance as 
a result of dimensional reduction of our mM0 equations.

5. The BPS conditions for the supersymmetric bosonic 
solutions of Eqs. (18) and (17) can be obtained from Eqs. 
(11) and (15). For 1/2 supersymmetric configurations 
these (1/2 BPS equations) read
\begin{equation}
D_{+p} \hat{T}_{\#i+q} \mid 0 = 0 \quad (a) , \quad D_{+p} \Psi_q \mid 0 = 0 \quad (b) .
\end{equation}
Eq. (19b) restricts the 3-form flux pull–back to be constant.
\( D_\# \hat{F}_{\# ijk} = 0 \), and, up to \( SL(9) \) transformations, to have the form \( \hat{F}_{\# ijk} = 3/4 \delta_i^j \delta_j^k \epsilon^{IJK} \) \((I = 1, 2, 3)\). Then Eq. (19b) has the fuzzy 2–sphere solution
\[
X^I = \delta_I^J f T^J, \quad [T^I, T^J] = \epsilon^{IJK} T^K.
\] (20)

This configuration was known to solve the pure bosonic equations of [19], but in our case it appears as describing the M2–brane as a supersymmetric configuration of mM0 fluxes, with also the relation of the special form of the flux \( \hat{F}_{\# ijk} \) with preservation of 16 supersymmetries becomes manifest. Curiously, the famous Nahm equation
\[
D_\# X^I + i \epsilon^{IJK} [X^J, X^K] = 0
\] (28), which also has fuzzy–two–sphere–related solution, appears as an SO(3) invariant 1/4 BPS condition for the case of vanishing 4-form flux pull–back, \( \hat{F}_{\# ijk} = 0 \).

6. Giving a covariant and supersymmetric description of the Matrix model interaction with nontrivial 11D supergravity fluxes, our approach might provide a new framework for studying M–theory. The first of the promising directions is to search for other supersymmetric solutions of the mM0 equations, representing more complicated M-brane and D-brane configurations.

For the development of our approach it is important to clarify whether our superembedding description can be generalized for multiple Mp-branes and Dp-branes with higher \( p \). An important problem is also to find an action functional for the embedding functions and matrix superfields \( X^I, \Psi_q \) which reproduced our mM0 equations. To this end the application of the ‘Ectoplasm–like’ technique of restoring the action from superembedding approach (see [22] and refs. therein) looks promising. Another challenge is to understand whether one can develop a counterpart of the (string-inspired and hence seemingly ten dimensional) boundary fermion approach [22] for the eleven dimensional multiple M0–system.

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