Electron Localization in a 2D System with Random Magnetic Flux

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Fractional quantum Hall (FQH) system \(^1\) is an ideal candidate to study localization properties in strongly correlated electron systems. In a non-interacting picture, according to the scaling theory of localization \(^2\) all electrons in a two-dimensional system are localized in the absence of magnetic field. When the two-dimensional electron system is subject to a strong perpendicular magnetic field, the energy spectrum becomes a series of impurity broadened Landau levels. Extended state appear in the center of each Landau band, while states at other energies are localized. This gives rise to the integer quantum Hall effect. It has been demonstrated that the critical transitions in the center of each Landau band are universal \(^3\). In the FQH regime, electron-electron interaction plays an important role \(^4\), however, recent experiments and theories indicate that the critical properties of the plateau transition might also be in the same universality class \(^5\).

Recently, Halperin, Lee and Read \(^6\) and Kalmeyer and Zhang \(^7\) developed an effective Chern-Simons field theory to understand electronic properties of the FQH systems. In their theory the quasi-particles are weakly interacting composite fermions (as originally proposed by Jain \(^8\)) which can be constructed by attaching an even number of flux quanta to electrons under a Chern-Simons transformation. In this simple picture, the fractional quantum Hall effect can be mapped into the integer quantum Hall effect for the composite fermion system subject to an effective magnetic field \(^9\). At filling factor \(\nu_f = \frac{1}{2}\), although the effective magnetic field \(B^*\) vanishes, composite fermions are subject to the random fluctuations of the gauge field, induced by the ordinary impurities \(^10\). Thus, it is important to study the localization properties of non-interacting charged particles in a random magnetic field to understand the half-filling FQH system. The problem of charged particles moving in a random magnetic field is also relevant to the theoretical studies of high \(T_c\) models where the gauge field fluctuations play an important role \(^11\).

According to the conventional scaling theory of localization, the random flux system belongs to the unitary ensemble, which is described by a nonlinear sigma model with unitary symmetry. Since there is no net magnetic field, the topological term of the uniform magnetic field case is absent. Perturbative renormalization group calculations show that all states are localized \(^12\). However, it has been argued recently by Zhang and Arovas \(^13\) that although the constant topological term is absent, there is a term describing the long ranged interaction between the topological densities, and they conjectured that this new term could lead to a phase transition from localized to extended states. There have been a number of conflicting numerical investigations on the localization properties of the random magnetic field system. The conclusions in these studies range from all states localized \(^14\) to extended states around band center \(^15\). In Ref. \(^16\), the authors found evidence for a mobility edge, but their system is neither large enough to see good scaling nor close enough to the critical regime to obtain a conclusive critical exponent.

In this paper, we systematically investigate the localization properties of a disordered two-dimensional electron system in the presence of a random magnetic field. The localization length is calculated using a transfer matrix technique and finite-size scaling analysis. An important strength of our calculation is using system widths (upto 128) which are substantially (by a factor of four) larger than those existing in the literature. We find the following results: (1) A mobility edge \(E_c\) is observed and the localization length \(\xi\) diverges when the Fermi energy approaches the critical energy; (2) we find that the critical energy \(E_c\) shifts with increasing disorder strength; (3) \(E_c\) shifts with changing randomness in the magnetic field; (4) the critical exponent \((\nu \approx 4.5)\) remains unchanged while varying the disorder strength and the randomness of the magnetic field, indicating universality in the metal-insulator transition; (5) the mobility edge survives in the presence of weak but non-zero average random magnetic field.

We model our two-dimensional system in a very long...
strip geometry with a finite width ($M$) square lattice
with nearest neighbor hopping. The disorder potential
is modeled by the on-site white-noise potential $V_{im}$ ($i$
indicates the column index, $m$ denotes the chain index)
ranging from $-W/2$ to $W/2$. Random magnetic field is
introduced by varying the flux in each lattice plaquette
uniformly between $-\phi_r/2$ and $\phi_r/2$ (in this case the
average field is zero, we also discuss the situation of weak
but non-zero average random magnetic field in the later
part of this paper). The Hamiltonian of this system can
be written as:

$$H = \sum_i \sum_{m=1}^M V_{im}|im><im|$$

$$+ \sum_{<im;jn>} \left[ t_{im;jn}|im><jn| + t_{im;jn}^\dagger|jn><im| \right],$$

where $<im;jn>$ indicates nearest neighbors on the lattice.
The amplitude of the hopping term is chosen as the unit of the energy. A specific gauge is chosen so that
the inter-column hopping does not carry a complex phase
factor (i.e. $t_{im;i+1,m} = -1$). The only effect of random
magnetic field shows up on the phase factor of the intra-
column (inter-chain) hopping term. If the random flux
in a plaquette cornered by $(im), (i+1,m), (i+1,m+1)$
and $(i,m+1)$ is $\phi_{im}$, then

$$\frac{t_{i+1,m;i+1,m+1}}{t_{im;i,m+1}} = \exp \left[ i2\pi \frac{\phi_{im}}{\phi_o} \right],$$

where $\phi_o = hc/e$ is the magnetic flux quantum. For a
specific energy $E$, a transfer matrix $T_i$ can be easily set
up mapping the wavefunction amplitudes at column $i-1$
and $i$ to those at column $i+1$, i.e.

$$\begin{pmatrix} \psi_{i+1} \\ \psi_{i} \end{pmatrix} = T_i \begin{pmatrix} \psi_{i-1} \\ \psi_{i} \end{pmatrix} = \begin{pmatrix} H_i - E & -I \\ I & 0 \end{pmatrix} \begin{pmatrix} \psi_{i} \\ \psi_{i-1} \end{pmatrix},$$

where $H_i$ is the Hamiltonian for the $i$th column, $I$ is a
$M \times M$ unit matrix. Using a standard iteration algo-
rithm, we can calculate the Lyapunov exponents for
the transfer matrix $T_i$. The localization length $\lambda_M(E)$
for energy $E$ at finite width $M$ is then given by the inverse
of the smallest Lyapunov exponent. In our numerical calcu-
lation, we choose the sample length to be over $10^4$ so that
the self-averaging effect automatically takes care of the
ensemble statistical fluctuations. A sample of our calculated
finite width localization length for various energies is
shown in Fig.1(a).

We use the standard one-parameter finite-size scaling
analysis to obtain the thermodynamic localization length $\xi$. According to the one-parameter scaling
theory, the renormalized finite-size localization length $\lambda_M/M$ can be expressed in terms of a universal function
of $M/\xi$, i.e.

$$\frac{\lambda_M(E)}{M} = f\left( \frac{M}{\xi(E)} \right),$$

where $f(x) \propto 1/x$ in thermodynamic limit $M \to \infty$ while
approaching a constant $(\sim 1)$ at the mobility edge where
the thermodynamic localization length diverges. Numer-
ically we shift the data in Fig.1(a) onto a smooth function

![FIG. 1. (a) Renormalized finite-size localization length
($\lambda_M/M$) for different sample width $M$ in a random magnetic
field($\phi_r = 1.0$) without on-site disorder ($W = 0$); (b) Scal-
ing function and localization length in the thermodynamic limit(inset) with $\nu = 4.52 \pm 0.08$ and $E_c = -3.00$. Here dif-
f erent symbols represent different energies.]

with a least square fit. Note that we have to select data
for a large enough sample in the scaling analysis so as
to avoid severe finite-size effect. In our calculation, we
choose the data for sample width greater than 16 (includ-
ing). The thermodynamic localization length is given
by the amount of shifts on a log-log plot. A sample of the
scaling function and corresponding thermodynamic
localization length is shown in Fig.1(b). Because of the
symmetry in the problem, we only study the branch with
negative energy $E < 0$.

We first study the case with random magnetic field
characterized by fluctuation amplitude $\phi_r = 1.0$. We
find that if the Fermi energy is below $E_c = -3.0$, the finite-size localization length is well converged and always smaller than the sample width indicating that all states are localized below $-3.0$. On the contrary, for the electronic states with energy higher than $-3.0$ the inverse of the Lyapunov exponent is always larger than the sample width which is the feature of extended states.

We also find that the thermodynamic localization length $\xi$ diverges while approaching $E_c = -3.0$ indicating the existence of a mobility edge around $-3.0$. Our best fit analysis indeed gives a critical energy $E_c = -3.00$ with a critical exponent $\nu = 4.52$ appearing in $\xi \propto |E - E_c|^{-\nu}$ for $E < E_c$ (the critical exponent is given by the slope of the straight line in the inset log-log plot of Fig. 2(b)).

We expect that the mobility edge should shift to lower energy if the magnetic field is less random so that extended states are more favorable. This is exactly what we observe in our calculation. In Fig. 2(a) we show the typical scaling function and localization length for a less random magnetic field with $\phi_r = 0.9$. In Table I we present the critical exponents and critical energies for different randomness in the magnetic field. We find that the critical energy increases almost linearly with the random flux amplitude which is proportional to the energy fluctuation created by the random magnetic field.

We now discuss the effect due to correlation between random magnetic field and disorder potential. We consider two types of disorder potential: (i) Independent model where the distribution of disorder potential is completely independent of the random magnetic field. This model is relevant for the case with random distributed non-magnetic impurities in the sample. (ii) Correlated model in which the strength of disorder is associated with the local random magnetic field (numerically we select each on-site disorder so that it is proportional to the random flux in the neighboring plaquette). This model is relevant for the case with random distributed magnetic impurities, for example, disorder-pinned random flux lines in the sample.

In the independent model, since non-magnetic random disorder potential tends to localize all the electronic states, we expect that the mobility edge in a random magnetic field should shift to higher energy with increasing strength of disorder potential because the localized states are more favorable in this situation. In Fig. 2(b) we show the typical scaling function and localization length for a strongly disordered sample with $W = 2.0$ in a random magnetic field with $\phi_r = 1.0$. As presented in Table I, the critical energy moves to higher energy as disorder strength $W$ increases for fixed random magnetic field ($\phi_r = 1.0$) just as we expect. Presented in the last row of Table I is the data for a correlated disorder model, the behavior of the mobility edge is quite similar to that for the independent disorder model.

It is interesting to notice that even though the mobility edge is shifting for various random magnetic field and disorder strength and correlation, the critical exponent for the metal-insulator transition is more or less unchanged which indicates the universality of this critical transition. Our calculated exponent ($\nu \approx 4.5$) is quite different.

### Table I

| $\phi_r$ | $W$ | $\nu$ | $E_c$ |
|----------|------|-------|-------|
| 0.7      | 0.0  | 4.53  | -3.40 |
| 0.8      | 0.0  | 4.60  | -3.28 |
| 0.9      | 0.0  | 4.98  | -3.13 |
| 1.0      | 0.0  | 4.52  | -3.00 |
| 1.0      | 1.0  | 4.79  | -2.85 |
| 1.0      | 2.0  | 4.86  | -2.73 |
| 1.0      | 3.0  | 5.29  | -2.13 |
| 0.8      | 2.0  | 4.45  | -3.35 |
from the value obtained in Ref. [14]. However, our system is much larger (4 times larger) than the system studied in Ref. [14], our scaling is much better and our data are closer to the critical regime, thus our calculated exponent better represents the true critical exponent.

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FIG. 3. Thermodynamic localization length in the presence of a weak but non-zero average random magnetic field.

In Fig. 3 we present the localization length for a two-dimensional system subject to a random magnetic field when the average field is weak but non-zero (i.e. \( <\phi> \neq 0 \)). Clearly there exists a mobility edge at \( E_c \approx -3.0 \) for \( <\phi> = 0.01 \). It is consistent with the argument that in the vicinity of \( <B^*> = 0 \), the composite fermion system behaves as a Fermi liquid.

As mentioned earlier there is currently considerable disagreement in the literature about the nature of two-dimensional one-electron eigenstates in a random flux environment. An earlier finite size scaling analysis [12] concluded that all states are localized, but the localization lengths are exponentially large near the band center. Our largest system widths (\( =128 \)) are four times larger than those used here (\( =32 \)), which are not accessible with the currently available computers, will be needed to obtain an unambiguous \( \beta(g) \). Thus, to have a disagreement here with the conclusion of Ref. [12] and [15].

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