The effective mass of the atom–radiation field system and the cavity-field Wigner distribution in the presence of a homogeneous gravitational field in the Jaynes–Cummings model

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Received 31 January 2009, in final form 24 April 2009
Published 18 June 2009
Online at stacks.iop.org/JPhysB/42/145507

Abstract
The effective mass that approximately describes the influence of a classical homogeneous gravitational field on an interacting atom–radiation field system is determined within the framework of the Jaynes–Cummings model. By taking into account both the atomic motion and the gravitational field, a full quantum treatment of the internal and external dynamics of the atom is presented. By exactly solving the Schrödinger equation in the interaction picture, the evolving state of the system is found. The influence of a classical homogeneous gravitational field on the energy eigenvalues, the effective mass of the atom–radiation field system and the Wigner distribution of the radiation field are studied, when the initial condition is such that the radiation field is prepared in a coherent state and the two-level atom is in a coherent superposition of the excited and ground states.

1. Introduction
There are several theoretical schemes for the description of the action of the cavity in quantum optics [1–4]. The effective mass approach is one of these theoretical schemes. Larson et al investigated the effect of the cavity on the atomic motion [5]. They introduced the effective mass parameter based on the Floquet theory [6–8]. In this theory, the photonic wavelength is small compared with the cavity length and the system can be treated as being approximately as periodic. The spectra of periodic Hamiltonians are known to consist of allowed energies in forms of bands, separated by forbidden gaps. These articles, which deal with the action of the cavity in quantum optics, are based on the Jaynes–Cummings model (JCM) [9]. The JCM of a two-level atom interacting with a single quantized mode of the electromagnetic field in a lossless cavity within the rotating wave approximation (RWA) is one of the few quantum-mechanical models in quantum optics which can be solved exactly. In the framework of this model many nonclassical effects of atom–radiation field interactions, such as Rabi oscillations [10, 11], collapse-revival phenomena [12–14], sub-Poissonian statistics [15] and squeezing of the radiation field [16] have been predicted.

Recent experiments with Rydberg atoms [17–19] have allowed the study of the dynamics of the two-level atom interacting with a single mode of the radiation field and the testing of the above-mentioned nonclassical effects. However, the standard JCM is not always valid. In this model, the atom is either assumed to be studied still relative to the cavity mode, or to have a large amount of kinetic energy; in both cases the atomic motion is described classically and the kinetic-energy term may be excluded from the Hamiltonian of the system. But, if the kinetic energy of the atom were of the same order of magnitude as the atom–radiation field interaction energy, the system’s dynamics would be changed significantly [20]. Thus, for every cold atom (ion) [21], the kinetic energy term for the atomic centre-of-mass motion must be treated quantum mechanically.

On the other hand, experimentally, atomic beams with very low velocities are generated in laser cooling and atomic
interferometry [22]. It is obvious that for atoms moving with a velocity of a few millimetres or centimetres per second for a period of several milliseconds or more, the influence of the Earth’s acceleration becomes important and cannot be neglected [23]. For this reason, it is of interest to study the temporal evolution of a moving atom simultaneously exposed to the gravitational field and a single-mode cavity field. Since any quantum-optical experiment in the laboratory is actually made in a non-inertial frame it is important to estimate the influence of the Earth’s acceleration on the outcome of the experiment. By referring to the equivalence principle, one can get a clear picture of what is going to happen in the interacting atom–field system exposed to a classical homogeneous gravitational field [24]. This means that the situation described below is physically equivalent to the exposure of the atom–radiation system to a gravity field: an atom is at rest or moving with constant velocity relative to an inertial system; the laboratory with the radiation field: an atom is at rest or moving with constant velocity relative to an inertial system; the laboratory with the radiation field attached to it moves with constant acceleration. The consequence is that the radiation field reaches the atom with a shifted Doppler frequency. Because of the acceleration this shift changes in time. It acts as a time-dependent detuning. A semi-classical description of a two-level atom interacting with a running laser wave in a gravitational field has been studied [25]. However, the semi-classical treatment does not permit the study of the pure quantum effects which occur in the course of the atom–field interaction. Within a quantum treatment of the internal and external dynamics of the atom, a theoretical scheme has recently been presented [26] based on an SU(2) algebraic structure to investigate the influence of a classical homogeneous gravitational field on the quantum non-demolition measurement of atomic momentum in the dispersive JCM. Also, the effects of the gravitational field on quantum-statistical properties of the lossless [24] as well as the phase-damped JCMs [27] have been investigated. It has been found that the gravitational field seriously suppresses non-classical properties of both the cavity field and the moving atom.

In this paper, the effective mass of the atom–radiation field system is obtained and subsequently the influence of a classical homogeneous gravitational field on the energy eigenvalues, the effective mass of the atom–radiation field system and the Wigner distribution of the radiation field of the JCM are investigated. Working with the Jaynes–Cummings model in which the atomic motion is in a propagating light wave, a two-level atom interacting with the quantized cavity field in the presence of a homogeneous gravitational field was considered. By solving the Schrödinger equation in the interaction picture, the evolving state of the system has been presented. In section 2, a quantum treatment of the internal and external dynamics of the atom with an alternative SU(2) dynamical algebraic structure within the system is presented. Based on this SU(2) structure and by using the interaction picture, first the effective Hamiltonian describing the atom–field interaction in the presence of a classical gravity field and subsequently the effective mass of the system is obtained. Furthermore, the effective Hamiltonian is presented based on the effective mass. In section 3, the dynamic evolution of the atom–radiation field system is investigated and it is shown how the Wigner distribution of the radiation field, when initially the radiation field is prepared in a coherent state and the two-level atom is in a coherent superposition of the excited and ground states is affected by gravity. The summarized conclusions are presented in section 4.

2. The effective mass for the JCM in the presence of the gravitational field

The total Hamiltonian for the atom–field system in the presence of a classical homogeneous gravitational field and in the rotating wave approximation with the atomic motion along the position vector \( \hat{x} \) is given by

\[
\hat{H} = \frac{\hat{p}^2}{2M} - M \vec{g} \cdot \hat{x} + \hbar \omega_c \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \frac{1}{2} \hbar \omega_{eg} \hat{\sigma}_z \\
+ \hbar \lambda [\exp(-i\hat{q} \cdot \hat{x}) \hat{a}^\dagger \hat{\sigma}_z + \exp(i\hat{q} \cdot \hat{x}) \hat{a} \hat{\sigma}_z],
\]

where \( \hat{a} \) and \( \hat{a}^\dagger \) denote the annihilation and creation operators of a single-mode traveling wave with frequency \( \omega_c \), respectively, \( \hat{q} \) is the wave vector of the running wave and \( \hat{\sigma}_z \) denote the raising and lowering operators of the two-level atom with electronic levels \( |e\rangle \), \( |g\rangle \) and the Bohr transition frequency is shown as \( \omega_{eg} \). The atom–field coupling is given by the parameter \( \lambda \) and \( \hat{p}, \hat{x} \) denote the momentum and position operators of the atomic centre-of-mass motion, respectively. The Earth’s gravitational acceleration is shown as \( g \). It has been shown [26] that based on SU(2) algebraic structure as the dynamical symmetry group of the model, the Hamiltonian (1) can be transformed to the following effective Hamiltonian:

\[
\hat{H}_{\text{eff}} = \hbar \omega_c \hat{K} - \frac{3}{2} \hbar \hat{\Delta}(\hat{p}, \hat{g}, t) \hat{S}_0 + \hbar \lambda (\sqrt{\hat{K}} \hat{S}_- + \sqrt{\hat{K}} \hat{S}_+),
\]

where the operators

\[
\hat{S}_0 = \frac{1}{\sqrt{\hat{K}}} |e\rangle \langle e| - |g\rangle \langle g|,
\]

\[
\hat{S}_+ = \frac{1}{\sqrt{\hat{K}}} |g\rangle \langle e| \hat{a}^\dagger,
\]

with the following commutation relations:

\[
[\hat{S}_0, \hat{S}_\pm] = \pm \hat{S}_\pm, \quad [\hat{S}_-, \hat{S}_+] = -2 \hat{S}_0
\]

are the generators of the SU(2) algebra, the operator \( \hat{K} = \hat{a}^\dagger \hat{a} + |e\rangle \langle e| \) is a constant of motion which represents the total number of excitations of the atom–radiation. In addition, the operator

\[
\hat{\Delta}(\hat{p}, \hat{g}, t) = \left( \omega_c - \left( \omega_{eg} + \frac{2\hat{q} \cdot \hat{p}}{3M} + \frac{2\hat{q} \cdot \hat{g} t}{3} \right) \right)
\]

has been introduced as the Doppler shift detuning at time \( t \) [26]. The Hamiltonian (2) is in the same form as the JCM Hamiltonian, the only modification being the dependence on the conjugation momentum and the gravitational field. The eigenstates and eigenvalues of the Hamiltonian (2) are respectively given by [28]

\[
|+, n, \hat{p}\rangle = \cos \vartheta_n(\hat{p}, \hat{g}, t) |e\rangle \otimes |n\rangle \otimes |\hat{p}\rangle + \sin \vartheta_n(\hat{p}, \hat{g}, t) |g\rangle \otimes |n + 1\rangle \otimes |\hat{p}\rangle,
\]

where

\[
\vartheta_n(\hat{p}, \hat{g}, t) = \arccos \left( \cos \theta_n(\hat{p}, \hat{g}, t) \otimes |n\rangle \otimes |\hat{p}\rangle + \sin \theta_n(\hat{p}, \hat{g}, t) |g\rangle \otimes |n + 1\rangle \otimes |\hat{p}\rangle \right).
\]
\[ |\omega_c\rangle = \frac{\hbar}{\omega_c} \quad \text{for} \quad E \quad \text{and} \quad \{24-27\}. \]

In figure 1, we consider the gravitational influence of mass \( M \) exposed simultaneously to a single-mode traveling wave field and a homogeneous gravitational field will be obtained. Then, the effective Hamiltonian (2) in terms of the effective mass \( \bar{m} \) will be written. The atom with its original mass experiences both the cavity field and the gravitational field. In the effective mass approach, this means that the atom is considered to move freely with the effective mass. Larson et al. [5] showed that the effective mass is independent of time \( t \) and the time evolution of the atom–field system is governed by a time-independent Hamiltonian, when the influence of the gravitational field is not taken into account. The effective mass in the presence of the gravitational field defined as in [5], is in the form of the equation:

\[ m^* = \left| \bar{p} \right|^2 \frac{\partial E_{+n}}{\partial \bar{p}^2} \right|^{-1}. \]

Using equations (8) and (13) the effective mass of the system is obtained

\[ m^*(\bar{p}, \bar{g}, t) = \eta^{-3} \Omega_n^2(\bar{p}, \bar{g}, t), \]

where \( \eta = \left( \frac{(2\lambda t)\hbar(c\omega_c)\cos^2 \theta}{M} \right)^{1/2} \) and \( \theta \) is the angle between \( \bar{q} \) and \( \bar{p} \). Because of the gravitational field, the effective mass \( m^* \) depends on time \( t \). The time evolution of the effective mass of the system has been shown in figures 2(a)–(c) for three values of the parameter \( \bar{q} \cdot \bar{g} \). By increasing the value of the parameter \( \bar{q} \cdot \bar{g} \), the effective mass of the system decreases. Also, the effective mass will be obtained for a small gravitational field. This means that \( \bar{q} \cdot \bar{g} \) is very small, i.e., the momentum which is transferred from the laser beam to the atom is altered only slightly by the gravitational acceleration which is very small or nearly perpendicular to the laser beam. Therefore, the effective mass by considering a very small \( \bar{q} \cdot \bar{g} \) is obtained by the following form:

\[ m^*(\bar{p}) = \eta^{-3} \Omega_n^2(\bar{p}), \]
where
\[ \Omega_{\alpha}(\vec{p}) = \frac{1}{2}\sqrt{(3\Delta(\vec{p}))^2 + (4\lambda)^2(n + 1)}, \]
(16)
with
\[ \Delta(\vec{p}) = \left( \omega_c - \left( \omega_{\gamma} + \frac{2\eta \cdot \vec{g}}{3M} \right) \right). \]
(17)
The effective mass \( m^*(\vec{p}) \) is independent of time \( t \), when \( \vec{q} \cdot \vec{g} \) is very small. By using equations (2) and (14), the effective Hamiltonian \( \hat{H}_{\text{eff}} \) in terms of the effective mass is given by
\[ \hat{H}_{\text{eff}} = \hbar \omega_c K - \hbar (\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + 4\lambda^2(n + 1)^\frac{1}{2} \hat{S}_0 \]
\[ + \hbar \lambda (\sqrt{K} \hat{S}_- + \sqrt{K} \hat{S}_+). \]
(18)
In the following section, the time evolution of the atom–field system in the presence of the gravitational field in terms of the effective mass \( m^* \) will be obtained.

3. Dynamical evolution and the Wigner distribution of a field mode

In section 2, the effective mass with an effective Hamiltonian for the atom–field system in the presence of a homogeneous gravitational field was obtained. In this section, the dynamic evolution of the system will be investigated. How the gravitational field may affect the Wigner distribution of a field mode will be shown. For this purpose, the Schrödinger equation
\[ i\hbar \frac{\partial |\psi\rangle}{\partial t} = \hat{H}_{\text{eff}} |\psi\rangle, \]
(19)
for the state vector \( |\psi(t)\rangle \) with the Hamiltonian in (18) will be solved. At time \( t = 0 \) the atom is uncorrelated with the field and the state vector of the system can be written as a direct product
\[ |\psi(t = 0)\rangle = |\psi_{\text{e.m}}(0)\rangle \otimes |\psi_{\text{atom}}(0)\rangle \otimes |\psi_{\text{field}}(0)\rangle \]
\[ = \left( \int d^3 p \phi(\vec{p}) |\vec{p}\rangle \right) \otimes (c_e |e\rangle + c_g |g\rangle) \otimes \left( \sum_{n=0} w_n |n\rangle \right), \]
(20)
where we have assumed that initially the field is in a coherent superposition of Fock states, the atom is in a coherent superposition of its excited and ground states, and the wave vector for the centre-of-mass degree of freedom is \( |\psi_{\text{cm}}(0)\rangle = \int d^3 p \phi(\vec{p}) |\vec{p}\rangle \). In notation (20), we have
\[ |\psi(t = 0)\rangle = \int d^3 p \sum_{n=0} \left( \psi_{1,n}(\vec{p}, \vec{g}, t = 0) |e\rangle \otimes |n\rangle \otimes |\vec{p}\rangle \right. \]
\[ + \psi_{2,n}(\vec{p}, \vec{g}, t = 0) |g\rangle \otimes |n\rangle \otimes |\vec{p}\rangle \right), \]
(21)
where the initial conditions are found, thus
\[ \psi_{1,n}(\vec{p}, \vec{g}, t = 0) = w_n c_e \phi(\vec{p}), \]
\[ \psi_{2,n}(\vec{p}, \vec{g}, t = 0) = w_n c_g \phi(\vec{p}). \]
(22)
Whenever \( t > 0 \), the atom–field state in the presence of a homogeneous gravitational field is described by the state
\[ |\psi(t)\rangle = \int d^3 p \sum_{n=0} \left( \psi_{1,n}(\vec{p}, \vec{g}, t) |e\rangle \otimes |n\rangle \otimes |\vec{p}\rangle \right. \]
\[ + \psi_{2,n}(\vec{p}, \vec{g}, t) |g\rangle \otimes |n\rangle \otimes |\vec{p}\rangle \right). \]
(23)
By solving the Schrödinger equation (19) and using equations (6)–(8) we obtain
\[ \psi_{1,n}(\vec{p}, \vec{g}, t) = \phi_{1,n}(\vec{p}, \vec{g}, t) \cos \theta_n(\vec{p}, \vec{g}, t) \]
\[ + \phi_{2,n}(\vec{p}, \vec{g}, t) \sin \theta_n(\vec{p}, \vec{g}, t), \]
(24)
\[ \psi_{2,n}(\vec{p}, \vec{g}, t) = (\phi_{1,n-1}(\vec{p}, \vec{g}, t) \sin \theta_{n-1}(\vec{p}, \vec{g}, t) \]
\[ - \phi_{2,n-1}(\vec{p}, \vec{g}, t) \cos \theta_{n-1}(\vec{p}, \vec{g}, t) \],
(25)
where
\[ \sin \theta_n(\vec{p}, \vec{g}, t) \equiv \frac{2\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + (4\lambda)^2(n+1)}}{2(\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + (4\lambda)^2(n+1)}}}, \]
(26)
\[ \cos \theta_n(\vec{p}, \vec{g}, t) \equiv \frac{4\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + (4\lambda)^2(n+1)}}{2(\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + (4\lambda)^2(n+1)}}}, \]
(27)
with
\[ \phi_{1,n}(\vec{p}, \vec{g}, t) = (\phi_{1,n}(\vec{p}, \vec{g}, t = 0)A_0(\vec{p}, \vec{g}, t = 0) \]
\[ \times \exp(-i\omega_c(n + 1)t) \]
\[ \times \exp \left( i \left[ \frac{1}{2} \eta \frac{m^*(\vec{p}, \vec{g}, t)^{\frac{1}{2}}}{2\eta g} \frac{\hbar}{\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + (4\lambda)^2(n+1)}}{2(\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + (4\lambda)^2(n+1)}}}, \]
(28)
\[ \phi_{2,n}(\vec{p}, \vec{g}, t) = (\phi_{2,n}(\vec{p}, \vec{g}, t = 0)A_0(\vec{p}, \vec{g}, t = 0) \]
\[ \times \exp(-i\omega_c(n + 1)t) \]
\[ \times \exp \left( -i \left[ \frac{1}{2} \eta \frac{m^*(\vec{p}, \vec{g}, t)^{\frac{1}{2}}}{2\eta g} \frac{\hbar}{\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + (4\lambda)^2(n+1)}}{2(\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t))^\frac{1}{2} + (4\lambda)^2(n+1)}}}, \]
(29)
where
\[ A_0(\vec{p}, \vec{g}, t = 0) = \exp \left( i \left[ \frac{1}{2} \eta \frac{m^*(\vec{p}, \vec{g}, t = 0)^{\frac{1}{2}}}{2\eta g} \frac{\hbar}{\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t = 0)^{\frac{1}{2}} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t = 0)^{\frac{1}{2}} + (4\lambda)^2(n+1)}}{2(\sqrt{\eta^2 m^*(\vec{p}, \vec{g}, t = 0)^{\frac{1}{2}} - 4\lambda^2(n+1)+2\eta m^*(\vec{p}, \vec{g}, t = 0)^{\frac{1}{2}} + (4\lambda)^2(n+1)}}}, \right) \]
(30)
and
\[ \phi_{1,n}(\vec{p}, \vec{g}, t = 0) = \psi_{1,n}(\vec{p}, \vec{g}, t = 0) \cos \theta_n(\vec{p}, \vec{g}, t = 0) \]
\[ + \psi_{2,n+1}(\vec{p}, \vec{g}, t = 0) \sin \theta_n(\vec{p}, \vec{g}, t = 0), \]
(31)
\[ \phi_{2,n}(\vec{p}, \vec{g}, t = 0) = \psi_{2,n+1}(\vec{p}, \vec{g}, t = 0) \cos \theta_n(\vec{p}, \vec{g}, t = 0) \\
- \psi_{1,n}(\vec{p}, \vec{g}, t = 0) \sin \theta_n(\vec{p}, \vec{g}, t = 0), \]  
\tag{32}

where the initial conditions \( \psi_{1,n}(\vec{p}, \vec{g}, t = 0) \) and \( \psi_{2,n}(\vec{p}, \vec{g}, t = 0) \) in (22) are defined. Also, \( \sin \theta_n(\vec{p}, \vec{g}, t = 0) \) and \( \cos \theta_n(\vec{p}, \vec{g}, t = 0) \) in (26) and (27) are defined, respectively.

Now, the influence of a classical homogeneous gravitational field on the Wigner distribution of the radiation field will be studied. Brune et al [29] showed that, at the atomic inversion half-revival time, due to the quantum interaction between the atom and the field, the Wigner distribution of the field mode has two positive blobs which are symmetrical in relation to the origin and one negative blob in the origin. Kenfack and Yezzi [30] showed that the negativity of the Wigner function is an indicator of nonclassicality. Also, according to [31] \( t = t_R/2 = 7\pi/2\lambda \), which corresponds to one-half of the revival time of the atomic inversion when the gravitational field is not taken into account. However, in these theoretical studies results are obtained only in a condition that the atomic motion is neglected and the influence of the gravitational field is not taken into account. The Wigner distribution is particularly convenient for displaying the energy and the phase information of a single mode field in a very simple and graphic form simultaneously. It also allows for a simple analysis of the field. The Wigner distribution in the presence of the gravitational field \( W(\beta, \beta^*, \vec{g}, t) \) of the complex amplitude \( \beta = X+iY \) is the real two-dimensional Fourier transformation of the symmetric characteristic function, defined as [32]

\[ W(\beta, \beta^*, \vec{g}, t) = \frac{1}{\pi^2} \int \exp \left( -\frac{\gamma^2}{2} \right) C_N(\gamma, \gamma^*, \vec{g}, t) \]
\[ \times \exp(\beta \gamma^* - \beta^* \gamma) d^2 \gamma, \]
\tag{33}

where the field normally ordered characteristic function is given by

\[ C_N(\gamma, \gamma^*, \vec{g}, t) = \langle \exp(\gamma \hat{a}) \exp(-\gamma^* \hat{a}) \rangle \]
\[ = \text{Tr}(\hat{\rho}_f(\vec{g}, t) \exp(\gamma \hat{a}) \exp(-\gamma^* \hat{a})), \]  
\tag{34}

where \( \gamma \) is a \( c \)-number variable and whenever \( t > 0 \), the reduced density operator of the cavity field is given by

\[ \hat{\rho}_f(\vec{g}, t) = \text{Tr}_a[\hat{\rho}_{a-f}(\vec{g}, t)] \]
\[ = \int d^3 p \hat{p} | \sum_{n,m} \langle \psi(t) | (\psi(t) d) \rangle | \hat{p} \]
\[ = \int d^3 p \sum_{n,m=0} \left( (\phi_{1,n}(\vec{p}, \vec{g}, t) \cos \theta_n(\vec{p}, \vec{g}, t) + \phi_{2,n}(\vec{p}, \vec{g}, t) \sin \theta_n(\vec{p}, \vec{g}, t) \right) \]
\[ \times \cos \theta_m(\vec{p}, \vec{g}, t) + \phi_{2,m}(\vec{p}, \vec{g}, t) \sin \theta_m(\vec{p}, \vec{g}, t)) \]
\[ + (\phi_{1,n-1}(\vec{p}, \vec{g}, t) \sin \theta_{n-1}(\vec{p}, \vec{g}, t) \]
\[ - \phi_{2,n-1}(\vec{p}, \vec{g}, t) \cos \theta_{n-1}(\vec{p}, \vec{g}, t)) \]
\[ \times (\phi_{2,m-1}(\vec{p}, \vec{g}, t) \sin \theta_{m-1}(\vec{p}, \vec{g}, t) - \phi_{1,m-1}(\vec{p}, \vec{g}, t) \]
\[ \cos \theta_{m-1}(\vec{p}, \vec{g}, t))|n|m \rangle, \]
\tag{35}

where \( \sin \theta_n(\vec{p}, \vec{g}, t) \), \( \cos \theta_n(\vec{p}, \vec{g}, t) \), \( \phi_{1,n}(\vec{p}, \vec{g}, t) \) and \( \phi_{2,n}(\vec{p}, \vec{g}, t) \) in (26)–(29) were defined, respectively. It is assumed at \( t = 0 \), the two-level atom is in a coherent superposition of the excited state and the ground state with \( c_1(0) = \frac{1}{\sqrt{2}} \), \( c_2(0) = \frac{1}{\sqrt{2}} \). The influence of a classical homogeneous gravitational field on the Wigner distribution of the radiation field is considered when at \( t = 0 \), the cavity field is prepared in a Glauber coherent state, \( w_0(0) = \exp(-\frac{\alpha^* \alpha}{2}) \exp(\frac{\alpha \alpha^*}{2}) \), where \( \alpha \) is the eigenvalue of the coherent state with \( \alpha = |\alpha| \). In figure 3, the Wigner distribution of the radiation field as a function of the scaled time \( \lambda \tau \) is plotted. We consider \( \phi(\vec{p}) = \frac{1}{\sqrt{\lambda \tau}} \exp(\frac{\alpha \alpha^*}{2}) \) with \( \alpha_0 = 1 \), \( \alpha = 5 \) and \( t = t_R/2 = 7\pi/2\lambda \). In figure 3(a), the Wigner distribution of the radiation field with small gravitational influence will be considered. In figure 3(a), the Wigner distribution has both negative and positive values for \( \vec{q} \cdot \vec{g} = 0 \). In figures 3(b) and (c), the evolution of the Wigner distribution of the radiation field for \( \vec{q} \cdot \vec{g} = 0.5 \times 10^7 s^2 \) and \( \vec{q} \cdot \vec{g} = 1.5 \times 10^7 s^2 \) will be considered, respectively. In figures 3(b) and (c), with the increasing value of the parameter \( \vec{q} \cdot \vec{g} \), the nonclassical behaviour of the Wigner distribution of the cavity field is suppressed.

4. Summary and conclusions

The effective mass that approximately describes the effect of a classical homogeneous gravitational field on an interacting atom–radiation field system was determined within the framework of the Jaynes–Cummings model. By taking into account both the atomic motion and the gravitational field, a full quantum treatment of the internal and external dynamics of the atom were presented. By solving the Schrödinger equation precisely in the interaction picture, the evolving state of the system was found. The influence of a classical homogeneous gravitational field on the energy eigenvalues, the effective mass of the atom–radiation field system and the Wigner distribution of the radiation field when, initially the radiation field is prepared in a coherent state and the two-level atom is in a coherent superposition of the excited and ground

Figure 3. The Wigner distribution of the cavity field versus X = Re(β) and Y = Im(β) with the same corresponding data used in figure 1, \( \alpha = 5 \) and \( t = \frac{t_R}{2} = \frac{7\pi}{2\lambda} \). (a) For \( \vec{q} \cdot \vec{g} = 0 \), (b) for \( \vec{q} \cdot \vec{g} = 1 \times 10^7 s^2 \) and (c) for \( \vec{q} \cdot \vec{g} = 1.5 \times 10^7 s^2 \).
states were studied. The results are summarized as follows:
(1) the energy levels $E_+$ and $E_-$ are not intersected when the
detuning increases with time $t$, (2) by increasing value of the
parameter $\vec{q} \cdot \vec{g}$, the effective mass of the system decreases and
(3) with the increase of the gravitational field influence, the
nonclassical behaviour of the Wigner distribution of the cavity
field is suppressed.

Acknowledgments

The author wishes to thank The members of the Office of
Research of the Islamic Azad University-Shahreza Branch for
their support.

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