Quantum Statistical Mechanics of Nonrelativistic Membranes: 
Crumpling Transition at Finite Temperature

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The effect of quantum fluctuations on a nearly flat, nonrelativistic two-dimensional membrane with extrinsic curvature stiffness and tension is investigated. The renormalization group analysis is carried out in first-order perturbative theory. In contrast to thermal fluctuations, which soften the membrane at large scales and turn it into a crumpled surface, quantum fluctuations are found to stiffen the membrane, so that it exhibits a Hausdorff dimension equal to two. The large-scale behavior of the membrane is further studied at finite temperature, where a nontrivial fixed point is found, signaling a crumpling transition.

I. INTRODUCTION

Quantum oscillations, analogous to the superconducting Josephson effect, have recently been detected in samples of superfluid $^3$He [1, 2]. The apparatus involved in these experiments consists of an inner cell, filled with $^3$He, contained in an outer cell, also filled with $^3$He. The two containers are separated by a stiff membrane glued to the bottom of the inner cell, and by a softer one attached to its top. The lower membrane contains an array of small apertures allowing for exchange of atoms between the cells, equivalent to the superconducting weak link. By manipulating this membrane, the pressure between the two systems can be kept at a fixed value, and the resulting mass current is determined by the displacement of the upper membrane from its original position. Josephson current oscillations at the weak link are then observed as oscillations of this membrane.

It is thus of interest to investigate whether the membranes themselves display quantum fluctuations, and to what extent such effects may be observable.

Thermal fluctuations are known to soften fluid membranes, and to increase their surface tension [3-11]. At a temperature $T$, the membrane’s bare surface tension $r_0$ and bending rigidity $1/\alpha_0$ are renormalized at large scales as follows:

$$r = r_0 \left[ 1 + \frac{1}{4\pi} u_{th,0} \ln(\Lambda L) \right],$$

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\[ \frac{1}{\alpha} = \frac{1}{\alpha_0} \left[ 1 - \frac{3}{4\pi} u_{\text{th},0} \ln(\Lambda L) \right], \]  

(2)

where

\[ u_{\text{th},0} = k_B T \alpha_0 \]

(3)
is the (dimensionless) expansion parameter in the regime dominated by thermal fluctuations, while \( \Lambda \) is an ultraviolet wavenumber cutoff set by the inverse size of the molecules in the membrane, and \( L \) is an infrared cutoff determined by its finite size. The renormalization group equation extracted from Eq. (2) yields only the trivial fixed point \( \alpha = 0 \), implying the absence of a crumpling transition. The fixed point turns out to be unstable in the infrared. As a result, the renormalized coupling constant \( \alpha \) flows away from it, so that the bending rigidity \( 1/\alpha \) tends to zero with increasing membrane size. The length scale where the right-hand side of Eq. (2) vanishes gives the so-called de Gennes-Taupin persistence length \( \xi_{\text{th},0} = \Lambda^{-1} \exp(4\pi/3u_{\text{th},0}) \) [12], above which the bending rigidity \( 1/\alpha \) vanishes. As a consequence, such membranes are crumpled at these scales, and fill the space in which they are embedded completely. Such objects are said to have infinite Hausdorff dimension \( d_H = \infty \). The full SO(\( d \)) rotational symmetry of the \( d \)-dimensional embedding space is recovered in this way. For a smooth surface, this symmetry would be spontaneously broken to its SO(\( d-2 \)) \( \times \) SO(2) subgroup. The absence of this symmetry breaking is in accord with the Mermin-Wagner theorem [13], stating that in a two-dimensional system long-range order is destroyed at any finite temperature.

The results (1) and (2) are derived from the Canham-Helfrich model [14,15], describing stiff membranes subject to thermal undulations. The Hamiltonian reads:

\[ H_0 = \int dS \left( r_0 + \frac{1}{2\alpha_0} H^2 \right), \]

(4)

where \( dS \) is the surface element, and \( H \) corresponds to (twice) the mean curvature of the surface at each point. We omit a term proportional to the Gaussian curvature since it is only important if fluctuations change the topology of the membrane, characterized by the Euler characteristic, i.e., by the number of handles—a possibility which we shall neglect. Each point on the membrane is represented by a vector \( \mathbf{X}(\sigma_1, \sigma_2) \) in the \( d \)-dimensional bulk space, depending on two coordinates \( \sigma_1 \) and \( \sigma_2 \). Explicitly, the Hamiltonian (4) reads (for reviews, see Ref. [3])

\[ H_0 = \int d^2\sigma \sqrt{g} \left[ r_0 + \frac{1}{2\alpha_0} (\Delta \mathbf{X})^2 \right], \]

(5)

where

\[ g_{ab} = \partial_a \mathbf{X} \cdot \partial_b \mathbf{X} \]

(6)
is the metric induced by the embedding and \( g \equiv \det[g_{ab}] \). The symbol \( \partial_a (a = 1, 2) \) denotes the derivative with respect to the coordinates \( \sigma = (\sigma_1, \sigma_2) \), and \( \Delta = g^{ab} D_a D_b \) is the scalar Laplacian, where \( D_a \) is the covariant derivative associated with the metric.

By lowering the temperature, the effect of thermal fluctuations decreases, and the behavior of the membrane becomes dominated by quantum effects. To account for these, we have to include time-dependence and add a kinetic term to the classical theory. For an incompressible surface, the kinetic term reads, in euclidean spacetime,
\[ T = \frac{1}{2\nu_0} \int d^2\sigma \sqrt{g} \dot{X}^2, \quad (7) \]

where \( X \) is now time-dependent, \( 1/\nu_0 \) is the bare mass density, and the dot indicates a time derivative.

The partition function \( Z \) can be represented as a functional integral over all possible surface configurations \( X(\sigma, \tau) \):

\[ Z = \int \mathcal{D}X \exp(-S_0[X]/\hbar), \quad (8) \]

with the euclidean action

\[ S_0 = \int d\tau d^2\sigma \sqrt{g} \left[ \frac{1}{2\nu_0} \dot{X}^2 + r_0 + \frac{1}{2\alpha_0} (\Delta X)^2 \right]. \quad (9) \]

We shall in this paper study the quantum statistical mechanics of a membrane described by this action.

Specifically, in Sec. II, we give a one-loop analysis of the effect of quantum fluctuations on such a two-dimensional surface embedded in \( d = 3 \) space dimensions. We show that, contrary to thermal fluctuations, quantum fluctuations lead to a stiffening of the membrane at large scales. In the one-loop approximation, the flow equations are again found to yield only the trivial fixed point \( \nu = r = \alpha = 0 \), implying the absence of a crumpling transition. However, this fixed point is now, in contrast to the one generated by thermal fluctuations, stable in the infrared. This means that \( \alpha \) flows towards this fixed point and the bending rigidity tends to infinity with increasing membrane size, so that the surface remains two-dimensional.

We further investigate the behavior of the system at finite temperature. We find at the one-loop order a nontrivial fixed point arising here. Being unstable in the infrared, it gives rise to a crumpling transition. Below a critical temperature \( T_c \), \( \alpha \) flows to zero, implying a flat membrane, while above \( T_c \), the flow is towards infinity, implying a crumpled membrane.

**II. PERTURBATIVE EXPANSION**

We shall work in the Monge parametrization, where a point on the surface embedded in three-dimensional space is described by a displacement field \( \phi(\sigma, \tau) \) with respect to a reference plane \( \sigma = (\sigma_1, \sigma_2) \), such that

\[ X(\sigma, \tau) = (\sigma, \phi(\sigma, \tau)). \quad (10) \]

Inserting this parametrization in the action \( S_0 \), and expanding the resulting expression up to fourth order in powers of the displacement field \( \phi \), we obtain

\begin{equation}
S_0 = \int d\tau d^2\sigma \left\{ \frac{1}{2} \left[ \frac{1}{\nu_0} \dot{\phi}^2 + r_0 (\partial_0 \phi)^2 + \frac{1}{\alpha_0} (\partial^2 \phi)^2 \right] + \frac{1}{4\nu_0} \dot{\phi}^2 (\partial_0 \phi)^2 - \frac{r_0}{8} (\partial_0 \phi)^2 (\partial_\phi \phi)^2 \\
- \frac{1}{4\alpha_0} (\partial_\phi \phi)^2 (\partial^2 \phi)^2 - \frac{1}{\alpha_0} (\partial_\phi \phi)(\partial_\phi \phi)(\partial_\phi \phi)(\partial^2 \phi) \right\}. \quad (11)\end{equation}

The displacement field describes the undulations of the surface. Its spectrum, \( \omega^2 = c_s^2 q^2 \), is gapless with \( c_s^2 = r_0 \nu_0 \) the velocity of the transversal waves.
In the one-loop approximation, the exponent in (8) may be expanded up to second order around a background configuration \( \Phi(\sigma, \tau) \) extremizing \( S_0 \). The resulting integral is Gaussian and yields the effective action

\[
S_{\text{eff}}[\Phi] = S_0[\Phi] + S_1[\Phi] = S_0[\Phi] + \frac{\hbar}{2} \text{Tr} \ln \left[ \frac{\delta^2 S_0}{\delta \phi(\sigma, \tau) \delta \phi(\sigma', \tau')} \right]_{\Phi},
\]

where the expression in square brackets is a functional matrix given by the second functional derivative of \( S_0 \), and \( \text{Tr} \) denotes the functional trace, i.e., the integral \( \int d\tau d^2\sigma \), as well as the integral \( \int d\omega dq/(2\pi)^3 \) over the (angular) frequency \( \omega \) and the wavevector \( q \).

Using a derivative-expansion method due to Fraser [16], we expand the one-loop correction \( S_1 \) in Eq. (12) in powers of the derivatives of the field \( \phi(\sigma, \tau) \), as in Ref. [17]. To obtain the renormalization of the parameters \( r_0, \alpha_0 \), and \( \nu_0 \), it suffices to keep only the first three terms of the expansion:

\[
S_1 = \frac{\hbar}{2} \int d\tau d^2\sigma \left[ I_1 \phi^2 + I_2 (\partial_\alpha \phi)^2 + I_3 (\partial^2 \phi)^2 + \ldots \right],
\]

with

\[
I_1 = \frac{1}{2\nu_0} \int \frac{d\omega \ dq}{(2\pi)^2} \frac{\omega^2}{\nu_0 + r_0 q^2 + q^4/\alpha_0},
\]

\[
I_2 = \int \frac{d\omega \ dq}{2\pi (2\pi)^2} \left( \frac{\omega^2}{\nu_0 + r_0 q^2 + q^4/\alpha_0} \right)^2,
\]

\[
I_3 = -\frac{3}{2\alpha_0} \int \frac{d\omega \ dq}{2\pi (2\pi)^2} \frac{\omega^2}{\nu_0 + r_0 q^2 + q^4/\alpha_0}.
\]

After the integrals over the loop energy \( \omega \) have been carried out, the resulting momentum integrals in Eqs. (14)–(16) diverge in the ultraviolet. To regularize them, we introduce a wavenumber cutoff \( \Lambda \). Contributions proportional to positive powers of \( \Lambda \) are irrelevant, and will be ignored. In dimensional regularization, where the number \( D \) of space dimensions of the membrane is analytically continued to be less than two, \( D = 2 - \epsilon \), these powerlike divergences never appear in the first place. Only logarithmic divergences arise as poles in \( 1/\epsilon \). Substituting the results of the integration into Eq. (13), we obtain the effective action

\[
S_{\text{eff}} = \frac{1}{2} \int d\tau d^2\sigma \left[ \frac{1}{\nu} \phi^2 + r (\partial_\alpha \phi)^2 + \frac{1}{\alpha} (\partial^2 \phi)^2 + \ldots \right],
\]

with the renormalized inverse mass density \( \nu \), surface tension \( r \), and inverse rigidity \( \alpha \)

\[
\frac{1}{\nu} = \frac{1}{\nu_0} \left[ 1 - \frac{1}{16\pi} u_{\text{qm},0} \ln (\Lambda \xi_{\text{qm},0}) \right],
\]

\[
r = r_0
\]

\[
\frac{1}{\alpha} = \frac{1}{\alpha_0} \left[ 1 + \frac{3}{16\pi} u_{\text{qm},0} \ln (\Lambda \xi_{\text{qm},0}) \right],
\]

where the dimensionless parameter \( u_{\text{qm},0} = h r_0 \alpha_0^{3/2} \nu_0^{1/2} \) is the (bare) expansion parameter in the quantum regime. Note that the surface tension is not renormalized by quantum fluctuations at
this order. The parameter $\xi_{\text{qm},0} = 2/\sqrt{r_0\alpha_0}$ in the argument of the logarithm in Eqs. (18) and (20) defines a characteristic length scale of the problem. It sets the scale at which the tension and stiffness terms in the action (17) become equally important. At larger scales, the second term in the expansion (17) becomes more important and the undulations are dominated by tension, while at smaller scales, the third term dominates and the undulations are controlled by stiffness.

To obtain the flow equations, we apply Wilson’s procedure [18]. Integrating out a momentum shell $\Lambda/s < q < \Lambda$, rescaling the coupling constants $\hat{\nu} = s\nu - 3\nu, \hat{r} \rightarrow s^{-3}r, \hat{\alpha} \rightarrow s^{-1}\alpha$, we arrive at

$$
\beta_\nu(\nu, r, \alpha) = s\frac{\partial \hat{\nu}}{\partial s}\bigg|_{s=1} = (\epsilon - 3)\nu + \frac{1}{16\pi}u_{\text{qm}}\nu,
$$

(21)

$$
\beta_r(\nu, r, \alpha) = s\frac{\partial \hat{r}}{\partial s}\bigg|_{s=1} = -(\epsilon - 3)r,
$$

(22)

$$
\beta_\alpha(\nu, r, \alpha) = s\frac{\partial \hat{\alpha}}{\partial s}\bigg|_{s=1} = (\epsilon - 1)\alpha - \frac{3}{16\pi}u_{\text{qm}}\alpha,
$$

(23)

where $\epsilon = 2 - D$ is assumed to be small and will be set to zero at the end of the calculation. The coefficients of the first terms at the right-hand sides denote the scaling dimension of the scaling fields. For small $\epsilon$, the scaling fields $\nu$ and $\alpha$ are irrelevant, while $r$ is relevant. Criticality is obtained by setting the relevant fields to zero, i.e., $r = 0$ in this case. Starting somewhere on the critical surface $r = 0$, the system flows towards the trivial fixed point $\nu = \alpha = 0$. This guarantees the stiffness of the membrane at large scales. There is no crumpling transition at the absolute zero of temperature; the membrane is always flat, where the normal vectors to its surface are strongly correlated. More specifically, the correlation function between the normal vectors to the surface behaves at large scales as

$$
\langle \partial_a X(\sigma, \tau) \cdot \partial_b X(\sigma', \tau) \rangle \sim \frac{\delta_{ab}}{|\sigma - \sigma'|^3}.
$$

(24)

This algebraic fall-off implies the absence of a persistence length which would define the length scale above which the normals become uncorrelated and the surface becomes crumpled.

To investigate this further, let us calculate the Hausdorff dimension $d_H$ of the membrane. It can be defined by the relation between its mean surface area $\langle A \rangle$, where

$$
A = \int d^2\sigma \sqrt{g},
$$

(25)

and the frame, or projected area $A_0 = \int d^2\sigma$. This relation is

$$
\langle A \rangle \sim A_0^{d_H/2},
$$

(26)

so that the Hausdorff dimension is given by

$$
d_H = 2\frac{\partial \ln \langle A \rangle}{\partial \ln A_0}.
$$

(27)

Since the frame area $A_0$ scales with the cutoff $\Lambda$ as
\[ A_0 = \int d^2 \sigma \sim \Lambda^{-2}, \]  

Eq. (27) can also be written in the form

\[ d_H = -\frac{\partial \ln \langle A \rangle}{\partial \ln \Lambda}. \]  

At the one-loop level, the mean surface area is

\[ \langle A \rangle = \left\langle \int d^2 \sigma \left[ 1 + \frac{1}{2} (\partial_a \phi)^2 + \ldots \right] \right\rangle = A_0 \left( 1 + \frac{\hbar}{2} \int \frac{d\omega}{2\pi} \frac{d^2 q}{(2\pi)^2} \frac{q^2}{\omega^2/\nu_0 + r_0 q^2 + q^4/\alpha_0} \right), \]  

so that we obtain from relation (29) the Hausdorff dimension

\[ d_H = 2 + \frac{1}{16\pi} u_{qm}. \]  

For large membranes, \( u_{qm} \to 0 \), implying a Hausdorff dimension \( d_H = 2 \). Expressed in group theoretic terms, the \( \text{SO}(d) \) rotational symmetry of \( d \)-dimensional space is spontaneously broken to its \( \text{SO}(d-2) \times \text{SO}(2) \) subgroup. Since we are now at zero temperature, where the membrane has an extra (time) dimension, this spontaneous symmetry breaking does not violate the Mermin-Wagner theorem. The algebraic decay found in Eq. (24) is an immediate consequence of this symmetry breaking and identifies the two resulting Goldstone modes.

We next investigate how the high-temperature regime dominated by thermal fluctuations, where the membrane is found to be always crumpled, goes over into the low-temperature regime dominated by quantum fluctuations, where the membrane remains flat. To investigate this temperature dependence, we adopt the imaginary-time approach to thermal field theory [19, 20]. It can be derived from the corresponding (euclidean) quantum theory at zero temperature by restricting the euclidean time to the finite interval \( 0 \leq \tau \leq \hbar/k_B T \), and substituting

\[ \int \frac{d\omega}{2\pi} g(\omega) \to \frac{k_B T}{\hbar} \sum_n g(\omega_n), \]  

where \( g \) is an arbitrary function, and \( \omega_n \) denote the Matsubara frequencies,

\[ \omega_n = 2\pi n k_B T/\hbar, \quad n = 0, \pm 1, \pm 2 \cdots. \]  

Using this substitution in the zero-temperature integrals (14)–(16), as well as the formula [21]

\[ \frac{k_B T}{\hbar} \sum_n \frac{1}{\omega_n^2 + a^2} = \frac{1}{2a} \coth \left( \frac{h a}{2k_B T} \right) \]  

to carry out the sum over the Matsubara frequencies, we arrive at

\[ \frac{1}{\nu} = \frac{1}{\nu_0} \left[ 1 + \frac{1}{8\pi} u_{qm,0} F_1(T, \Lambda') \right], \]  

\[ r = r_0 \left[ 1 - \frac{1}{2\pi} u_{qm,0} F_2(T, \Lambda') \right], \]  

\[ \frac{1}{\alpha} = \frac{1}{\alpha_0} \left[ 1 - \frac{3}{8\pi} u_{qm,0} F_3(T, \Lambda') \right], \]  

\[ F_1(T, \Lambda') = \int \frac{d^2 \sigma}{(2\pi)^2} \left[ 1 + \frac{1}{2} (\partial_a \phi)^2 \right], \]  

\[ F_2(T, \Lambda') = \int \frac{d^2 \sigma}{(2\pi)^2} (\partial_a \phi)^2, \]  

\[ F_3(T, \Lambda') = \int \frac{d^2 \sigma}{(2\pi)^2} (\partial_a \phi)^4. \]
with

\[ F_1(T, \Lambda') = \int_{\Lambda'}^{\Lambda} dq' q'^2 \frac{\coth(\frac{1}{2} \gamma_0 q' \sqrt{1 + q'^2})}{\sqrt{1 + q'^2}}, \]

\[ F_2(T, \Lambda') = \int_{\Lambda'}^{\Lambda} dq' (\frac{3}{4} q'^2 + \frac{q'^4}{4}) \frac{\coth(\frac{1}{2} \gamma_0 q' \sqrt{1 + q'^2})}{\sqrt{1 + q'^2}}. \]

Here, we rescaled the integration variables: \( q' = q \xi_0 m_0 / 2 \), \( \Lambda' = \Lambda \xi_0 m_0 / 2 \), and \( \gamma_0 \) stands for \( \gamma_0 = \frac{\hbar r_0}{u_\text{th}} / k_B T \). This dimensionless parameter can be expressed as the ratio of the expansion parameters in the quantum and classical regime,

\[ \gamma_0 = \frac{u_{\text{qm},0}}{u_{\text{th},0}}. \]

The integrals in Eqs. (39) and (38) diverge in the ultraviolet as \( \Lambda' \to \infty \). As before, we disregard powerlike divergences, and consider only the logarithmically diverging terms. We thus arrive at:

\[ \frac{1}{\nu} = \frac{1}{\nu_0} \left[ 1 + \frac{1}{4\pi} \left( u_{\text{th},0} - \frac{1}{4} u_{\text{qm},0} \right) \ln(\Lambda') \right], \]

\[ r = r_0 \left[ 1 + \frac{1}{4\pi} u_{\text{th},0} \ln(\Lambda') \right], \]

\[ \frac{1}{\alpha} = \frac{1}{\alpha_0} \left[ 1 - \frac{3}{4\pi} \left( u_{\text{th},0} - \frac{1}{4} u_{\text{qm},0} \right) \ln(\Lambda') \right]. \]

One may check that as the temperature \( T \) and, consequently, \( u_{\text{th},0} \propto T \) tend to infinity, Eqs. (42) and (43) reproduce, up to finite terms, the high-temperature results (1) and (2). On the other hand, in the limit \( u_{\text{th},0} \propto T \to 0 \), we recover the zero-temperature results (18)–(20).

In order to explore the behavior of the membrane at large length scales, we compute the flow equations corresponding to the three parameters of the theory, as we did above. They are given by:

\[ \beta_\nu(\nu, r, \alpha) = (\epsilon - 3) \nu - \frac{1}{4\pi} \left( u_{\text{th}} - \frac{1}{4} u_{\text{qm}} \right) \nu, \]

\[ \beta_r(\nu, r, \alpha) = -(\epsilon - 3) r + \frac{1}{4\pi} u_{\text{th}} r, \]

\[ \beta_\alpha(\nu, r, \alpha) = (\epsilon - 1) \alpha + \frac{3}{4\pi} \left( u_{\text{th}} - \frac{1}{4} u_{\text{qm}} \right) \alpha. \]

This system of equations admits two possible fixed points (see Fig. 1), viz. the trivial one at \( \nu = r = \alpha = 0 \) which we already found above at \( T = 0 \), and a new one at \( \nu = r = 0, \alpha = \alpha^* \), with

\[ \alpha^* = \frac{4\pi}{3k_B T} (1 - \epsilon). \]
FIG. 1. Flow diagrams in the \((\alpha, \nu)\)-plane. As \(T\) becomes finite (middle panel), a nontrivial fixed point \((\alpha^* \neq 0)\) starts to move to the right away from the origin, and disappears at infinity when \(T\) tends to zero (right panel).

Note that this fixed point exists even for a two-dimensional membrane \((\epsilon = 0)\). The scaling field \(r\) is found to be relevant, while \(\nu\) is found to be irrelevant with respect to both fixed points. Criticality is obtained by setting \(r = 0\). The scaling field \(\alpha\) behaves differently: it is irrelevant with respect to the trivial fixed point, but relevant with respect to the new one. The presence of the unstable fixed point implies the existence of a crumpling transition for a two-dimensional membrane at a critical temperature

\[
T_c = \frac{4\pi}{3k_B} \frac{1}{\alpha^*}. \tag{48}
\]

For \(T < T_c\), \(\alpha\) flows away from \(\alpha^*\) and towards the trivial fixed point \(\alpha = 0\). The correlation between the normals to the surface is long-ranged, and the membrane remains flat. For \(T > T_c\), on the other hand, \(\alpha\) flows away from \(\alpha^*\) in the other direction, that is \(\alpha \to \infty\). In this case, thermal fluctuations dominate. The correlation between the normals to the surface is short-ranged, and the membrane is found to be crumpled. As the temperature tends to infinity, the time dimension shrinks to a point, making the integration \(\int d\tau\) disappear from the action \((9)\). This implies that the parameter \(\epsilon\) in the flow equations \((44)\)–\((46)\) must be set equal to \(\epsilon = \epsilon' + 1\) \([23,24]\), with \(\epsilon' = 0\) for \(D = 2\). We see that the fixed point \((47)\) reduces in the limit \(T \to \infty\) to the trivial one for a two-dimensional membrane. A similar nontrivial fixed point to the one in \((47)\) was found in Refs. \([5,22]\) for a \((2 + |\epsilon'|)\)-dimensional membrane \((\epsilon' < 0)\) described by the classical Canham-Helfrich model embedded in \(3 + |\epsilon'|\) dimensions. In our case, due to the presence of the extra time dimension, the nontrivial fixed point \((47)\) exists also in two dimensions.

III. CONCLUSIONS

We have analyzed the large-scale behavior of a membrane subject to thermal and quantum fluctuations. At the absolute zero of temperature, the membrane is found to be flat, and the vectors
normal to the surface are found to be strongly correlated. This behavior, being qualitatively different from that at high temperatures, led us to investigate how these two regimes are related, by studying the system at finite temperature. We found that it exhibits a crumpling transition: above a critical temperature $T_c$, thermal fluctuations turn the membrane into a crumpled surface, while for temperatures below $T_c$, quantum fluctuations render the membrane flat.

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