1. Introduction

Theoretical analysis of hadron dynamics in main part relies upon the renormalization group (RG) method. However, construction of exact solutions to the RG equation is still far from being feasible. Usually, in order to describe the strong interaction processes in asymptotical ultraviolet (UV) region, one applies the RG method together with perturbative calculations. The relevant approximate solutions to the renormalization group equation are used for quantitative analysis of high-energy experimental data. However, such solutions contain unphysical peculiarities in the infrared (IR) domain.

An effective way to overcome such difficulties consists in invoking into consideration the analyticity requirement, which follows from the general principles of local Quantum Field Theory (QFT). This idea became the basis of the so-called analytic approach to QFT, which was first formulated in late 1950’s [1]. Recently this approach has been extended to Quantum Chromodynamics (QCD) [2, 3].

2. New analytic invariant charge

The perturbative approximation for the $\beta$ function in renormalization group equation for invariant charge $\alpha(\mu^2) = g^2(\mu^2)/(4\pi)$

$$\frac{d \ln [g^2(\mu^2)]}{d \ln \mu^2} = \beta(g(\mu^2))$$

leads to unphysical peculiarities of outcoming solutions (e.g., Landau pole). In the framework of developed model [4, 5, 6] the analyticity requirement is imposed on perturbative expansion of the RG $\beta$ function for restoring its correct analytic properties. At the one-loop level the corresponding renormalization group equation can be solved explicitly:

$$\alpha^{(1)}_{an}(q^2) = \frac{4\pi}{\beta_0} \left( \frac{z - 1}{z \ln z} \right), \quad z = \frac{q^2}{\Lambda^2}.$$  

*Work partially supported by grant 00–15–96691 of the RFBR.*
At the higher loop levels only the integral representation for the analytic invariant charge (AIC) was derived (see Refs. [5, 7]). Figure 1 presents the analytic running coupling $\tilde{\alpha}_{an}(q^2) = \alpha_{an}(q^2)\beta_0/(4\pi)$ at different loop levels. The properties of the analytic invariant charge and the relevant $\beta$ function are investigated in details in Refs. [7, 8].

![Figure 1. The analytic invariant charge at different loop levels, $z = q^2/\Lambda^2$.](image1)

![Figure 2. The one-loop AIC in the spacelike and timelike regions, $z = q^2/\Lambda^2$.](image2)

All stated above relate to the spacelike values of kinematic variable ($q^2 > 0$). However, for the consistent description of some hadron processes one has to employ the running coupling in the timelike region ($s = -q^2 > 0$). Recently the continuation of the AIC to the timelike region has been performed [5]. In particular, obtained result confirms the hypothesis due to Schwinger concerning connection between the $\beta$ function and relevant spectral density (see articles [5, 9] and references therein for the details). The plots of the one-loop AIC in the spacelike and timelike regions are shown in Figure 2. In the ultraviolet limit these functions have identical behavior determined by the asymptotic freedom. But there is asymmetry between them in the intermediate energies region. The relative difference is about several percents at scale of the $Z$ boson mass, and increases when approaching the infrared domain. Apparently, this circumstance must be taken into account when one handles with experimental data.

3. Phenomenological applications

For verification of consistency of the model developed it is worth turning to its applications. Since we are working within a non-perturbative approach, the study of the non-perturbative phenomena is of a crucial importance.

It has been shown [5] that the quark-antiquark potential constructed by making use of the analytic invariant charge is confining at large distances.
The derived potential is in a quite good agreement with lattice simulation data (see Refs. [4, 6] for the details).

As known, some key non-perturbative aspects of strong interaction are described by instantons. The distribution of large-size instantons is directly related to the infrared behavior of the invariant charge. The lattice simulation of this quantity revealed severe suppression of the large-size instantons, that is compatible neither with the perturbative results, nor with the freezing of the QCD running coupling at large distances [10]. Recently the conformal inversion symmetry of this distribution was observed, which ultimately led to “rediscovery” of the analytic invariant charge [11].

The developed approach has also been applied to description of gluon condensate, inclusive τ lepton decay, and electron-positron annihilation to hadrons [6]. The consistency of estimated values of the scale parameter ($\Lambda_{QCD} \simeq 550$ MeV, one-loop level, three active flavors) testifies that the analytic invariant charge substantially incorporates, in a consistent way, both perturbative and non-perturbative aspects of Quantum Chromodynamics.

4. Conclusion

The developed model for the QCD analytic invariant charge possesses a number of profitable features. Namely, it has no unphysical peculiarities at any loop level; it contains no free parameters; it incorporates UV asymptotic freedom with IR enhancement; it has universal behavior both in UV and IR regions at any loop level; it possesses a good higher loop and scheme stability. There is considerable difference between the respective values of AIC in the low-energy domain of spacelike and timelike regions. The developed model enables one to describe various strong interaction processes both of perturbative and intrinsically non-perturbative nature.

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