A Deep Learning Approach To Estimation Using Measurements Received Over a Network

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Abstract—We propose a novel deep neural network (DNN) based approximation architecture to learn estimates of measurements. We detail an algorithm that enables training of the DNN. The DNN estimator only uses measurements, if and when they are received over a communication network. The measurements are communicated over a network as packets, at a rate unknown to the estimator. Packets may suffer drops and need retransmission. They may suffer waiting delays as they traverse a network path. Works on estimation often assume knowledge of the dynamic model of the measured system, which may not be available in practice. The DNN estimator doesn’t assume knowledge of the dynamic system model or the communication network. It doesn’t require a history of measurements, often used by other works.

The DNN estimator results in significantly smaller average estimation error than the commonly used Time-varying Kalman Filter and the Unscented Kalman Filter, in simulations of linear and nonlinear dynamic systems. The DNN need not be trained separately for different communications network settings. It is robust to errors in estimation of network delays that occur due to imperfect time synchronization between the measurement source and the estimator. Last but not the least, our simulations shed light on the rate of updates that result in low estimation error.

I. INTRODUCTION

Cyber-physical systems, for example vehicle-to-everything (V2X) and public safety, are amongst the key use cases of next generation networks [1]. Such systems have one or more agents actuate in their local physical environment while requiring measurements of their larger environment. For example, a vehicle requires the positions, velocities, and accelerations of vehicles in its vicinity and not sensed by its own sensors. Measurements are communicated as packets, over a communication network, to an agent.

Networks, broadly speaking, introduce packet drops, often resulting in retransmissions, and delays that result from a packet waiting in one or more network queues along a network path, shared by other traffic. Also, network conditions vary over time, due to time-varying traffic and link conditions along any network path. The sources of measurements too choose rates of sending that may be constrained by energy budgets, network connectivity, and sensing rates.

As a result of the above stated, measurement packets arrive intermittently at an agent and are aged (were generated by a source in the past) when received. To compensate, an agent calculates estimates of measurements at its decision instants, using the history of received packets.

While the problem of estimation has been studied extensively [2]–[4], the knowledge of the dynamic model of the environment or system being measured is typically assumed. Assuming availability of such model information may not be practical. Say, for example, models for a vehicle’s (or driver’s) choice of maneuver [5]. Statistical information, for example the packet drop probability, and a model for the source generating the measurements, is also often assumed. These change over time and are unavailable in practice (see, for example [6]) to the agent calculating estimates. On the other hand, the increase in compute available at edge devices (agents) makes learning based approaches feasible in practice.

We investigate a model-free data driven approach to estimation, which will execute on an agent. Our proposed estimator is a deep neural network. It assumes no knowledge of the measurement source or the dynamic system that is measured or the communications network. Our contributions are next.

1) We propose a novel Deep Neural Network based approximation architecture, abbreviated as LAA, for the estimator, which uses the LSTM cell together with fully-connected layers. LAA takes as input the last estimate, the most recent received measurement and its age at the estimator, circumventing the need to pick a heuristic length of history of measurements received over the network.

2) We propose a learning algorithm that uses the experience replay buffer, which is commonly used in Deep Reinforcement Learning, in our problem setting. This enables us to train the proposed architecture using measurements received over the network, which may be time correlated.

3) We demonstrate that LAA doesn’t need to be trained separately for specific network instances and may be trained over a generic network with time-varying network parameters. This enables its use in the real-world where network settings may not be known upfront.

4) We show that LAA outperforms the Time Varying Kalman Filter (TVKF) and the Unscented Kalman Filter (UKF), respectively, for simulated linear and nonlinear systems, whose measurements are sent to the estimator over a network.

5) Lack of time synchronisation between an agent and a source of measurements is common in practice. This results in noisy estimates of age of received measurements because of corresponding noisy estimates of network delays. LAA’s estimates are robust to noisy estimates.

6) Simulations show that an age optimal rate of updates results in small estimation errors for all evaluated estimators.
The rest of the paper is organized as follows. We will discuss related works in Section II. This is followed by a description of the system model in Section III. Section IV details our proposed LSTM based deep neural network architecture. This is followed by the learning algorithm in Section V. Section VI describes the varied simulated environments and communication networks used for training and testing the proposed architecture. In Section VII we provide a detailed evaluation of the models we trained using the learning algorithm. We conclude in Section VIII.

II. RELATED WORKS

There is a large body of works under the umbrella of deep reinforcement learning [7] (DRL) that learns policies for agents in a model-free setting in a data driven manner. Such works [8]–[12] are relatively limited when the environment is modelled by a time-varying dynamical system and measurements are communicated over an imperfect network. We describe a sampling of such works next.

In [10], the authors propose to use LSTM together with the twin delayed deep deterministic policy gradient algorithm to learn a control policy when measurements and not the state are available. They use a history of measurements and control inputs as an input to their neural network. They don’t assume a communications network. In [11], the authors allow for delayed measurements. They assume a known maximum delay and use this to choose the length of history of measurements and controls that are input to a Deep Q Neural network [7].

Authors in [12] and [13] use artificial neural networks to estimate localisation error in fingerprinting methods, which are a common solution for indoor positioning systems. In [14], a deep neural network is used along with the Kalman filter (assuming a linearised model for a UAV) to learn measurement noise characteristics, in order to improve state estimates.

In [15], the authors propose a Deep-Q network for learning a scheduling policy for wireless sensors over a limited number of available channels. They assume that the channel model that governs whether a packet will be successfully received or not is unknown. Also, they assume knowledge of the dynamic model of the system whose state is being estimated. Similarly, in [16] too a scheduling policy is learnt for sensors observing multiple dynamical systems. However, each sensor transmits measurements to the estimator as opposed to estimates in [15], for each observed process.

Works [8] and [9] learn the underlying system dynamic model using measurements. The relatively recent [9] learns the dynamic system model for a continuous time system using measurements of the state of the system and knowledge of the mapping between a state and the corresponding measurement. The much older work [8] uses recurrent neural networks to learn the state evolution function and the function that maps a state to a measurement for a discrete time system. These works don’t assume a communications network. Also, unlike our work, they attempt to explicitly learn the state evolution and/or measurement functions.

There are works [3]–[4] within the realm of networked control systems that consider the problem of optimal estimation and control when sensors communicate measurements to estimators over a communications network. These works assume knowledge of the dynamical model. Often assumptions are made regarding the model of packet delay and dropout over the communication network. Key to evaluation later in this paper is [17], in which the time-varying Kalman filter with an infinite time buffer is shown to be the optimal estimator for a linear system in the presence of intermittent and aged measurements and known controls. Also [18], which considers estimation using the unscented Kalman filter in presence of intermittent (but not aged) measurements.

III. SYSTEM MODEL

A measurement source generates measurements (in general, vectors) $y(t_k)$ at discrete time instants $t_k$, $k = 1, 2, \ldots$ of a certain environment (dynamical system) of interest. At any instant, it transmits a measurement with probability $0 < p < 1$, which signifies network resource constraints or energy constraints that may preclude scheduling of a generated measurement. The estimator agent calculates estimate $\hat{y}(t_k^e)$ at $t_k^e$, $k = 1, 2, \ldots$. It can only use measurements received at times $t \leq t_k^e$ to calculate $\hat{y}(t_k^e)$.

Denote the $i$th element of $y(t)$ by $y[i](t)$. The measurements are received by the estimator over a communications network and are transmitted by the source with probability $p$. As a result, the estimator will have access to aged measurements that capture an older time snapshot in the evolution of the measured dynamic system. Let $z$ denote a measurement. For example, $z$ could be $y[i]$. Let $U_i(z)$ be the timestamp (time of generation) of the most recently generated measurement $z$ available at the estimator at time $t$. Define the age of measurement $z$ at the estimator at $t$ as

$$\Delta_z(t) = t - U_i(z).$$  \hspace{1cm} (1)

To exemplify, suppose the most recent timestamp of $y[i]$ at the estimator, at time $t$, is $t'$. That is amongst all measurements of $y[i]$, which the estimator has received so far over the network, the one with a timestamp closest to $t$ is $y[i](t')$. The age of $y[i]$ is $\Delta_{y[i]}(t) = t - t'$, $t \geq t'$.

Note that $\Delta_z(t)$ at the estimator increases at unit rate in the absence of a measurement with a more recent timestamp than already at the estimator. Further, it is reset to a smaller value in case a measurement with a more recent than available timestamp is received at $t$. $\Delta_z(t)$ is reset to the time elapsed between the generation of the measurement and it being received at $t$ [19].

We now describe the communications network. A measurement received by the estimator would have suffered delays because of queuing and (re) transmission. The delays are a function of the rate at which the network can deliver measurements and also the rate at which measurements are sent by the source. To capture the above, we simulate the network
Linear Unit, a commonly used nonlinear activation function. FC2, which is parameterized by the weight matrix $W$, is processed by two fully-connected layers to yield the estimate $\hat{y}_{t-1}$, which becomes a part of the input at the next time step.

IV. APPROXIMATION ARCHITECTURE

We propose a Long Short Term Memory (LSTM) based approximation architecture (LAA), which is a Deep Neural Network, to calculate estimates at $\hat{y}_{t-1}$. Measurements that enter the queue are serviced in a first-come-first-served manner and each packet spends a geometric($q$) time in the server, independently of other updates. That is a packet is communicated in error with probability $1 - q$ and re-transmitted till it is successfully received by the estimator. Also, a packet must wait for packets that are ahead of it in the queue to finish service. Note that $p/q$ is the utilization of the network by the source’s packets.

Algorithm 1 Learning Algorithm For LAA

1: Initialize weights $\Theta$ and replay memory $D$.
2: for episode = 1, $M$ do
3: Initialize dynamic system (environment); System state $x(1)$ is set.
4: Initialize network; set $p, q$.
5: At the estimator: Initialize $\hat{y}(0)$, $\tilde{y}(1)$, $\Delta y(1)$.
6: for $t = 1, T$ do
7: Dynamic System and Network Simulation:
8: Source generates $y(t)$, a measurement of the environment.
9: With prob. $p$ queue a packet containing $y(t)$.
10: if packet received at the estimator then
11: else
12: $\tilde{y}(t) \leftarrow \tilde{y}(t-1)$, $\Delta y(t) \leftarrow \Delta y(t-1) + 1$.
13: Dynamic system transitions to state $x(t+1)$.
14: Estimator:
15: Store experience $\{I(t), y(t)\}$ in $D$.
16: for each experience in mini-batch do
17: Forward Propagation: Calculate $\tilde{y}$ using stored $I$ and $\Theta$.
18: Calculate squared measurement residual $\epsilon_y(\Theta)^T\epsilon_y(\Theta)$.
19: Backpropagation: Update $\Theta$ by performing a gradient descent step on the squared residual averaged over the mini-batch.

Note that the LSTM cell independently updates its cell state, its hidden state, the input, forget, cell and output gates, at every time $t$. Further, the cell contains the input-hidden and the hidden-hidden weights and biases that must be learnt together with $W_{FC1}$, $b_{FC1}$, $W_{FC2}$, and $b_{FC2}$. LSTM specific details can be found in [20]. Figure 1 illustrates our architecture. In this paper, we set the hidden layer of the LSTM and FC1 to each have 64 neurons, where the size of a layer is the size of output of the layer. The size of the input layer (vector $I(t)$) and that of the output layer (FC2) is governed by the dynamic system of interest. For the linear system that we describe later, the size of the input layer is 12 and that of the output layer is 4. For the nonlinear system the sizes are 9 and 3 respectively.

Let the vector $\Theta$ comprise of all the weights and biases of our proposed architecture. Define the measurement residual as $\epsilon_y(\Theta) = y - \tilde{y}(\Theta)$. The estimator would like to minimize the mean squared measurement residual [22], with respect to $\Theta$, using measurements obtained over a certain time horizon $T$. Our optimization problem is

$$\minimize_{\Theta} \frac{1}{T} \sum_{t=1}^{T} \epsilon_y(t)(\Theta)^T \epsilon_y(t)(\Theta).$$

V. LEARNING ALGORITHM

Algorithm 1 summarizes the learning algorithm that is used to train the LAA model (solve problem (3)) using data generated by simulating the dynamic system (environment). The algorithm is executed at the estimator, which updates the weight vector $\Theta$ at time instants $t_n$, using measurements it has received up to then.

The calculation of $y(t)$ and transmission of packets over the communications network, shown in Lines 6–13 of the algorithm, captures simulation of the network and generation of measurements. The processing at the estimator, starting Line 14 captures the training of LAA. Having the two execute
in sequence simplifies the learning setup and presentation. In
general, they execute independently of each other.

In our experiments, the training of LAA is executed over
$M \geq 200$ episodes with each episode $T = 40000$ time slots
long. The algorithm begins by the estimator initializing the
weight vector $\Theta$ and the replay memory (explained later)
$D$. At the beginning of every episode (Line 3), the dynamic
system is initialized and as a result its state $x(1)$ is set. The
network is initialized with a choice of $p$ and $q$. The estimator
initializes the input to LAA.

At any time step $t$ in an episode, the source generates a
measurement vector $y(t)$ of the dynamic system (see Line 7).
Next, with probability $p$, the measurement is added to the
network queue. Any update packet that completes service is
received by the estimator. The last received measurement and
its age are appropriately updated (Lines 9 – 12).

The estimator stores its experience (Line 14) at $t$ in the
replay memory $D$. The experience includes the input $I(t)$
to LAA and the ground-truth measurement $y(t)$. It must be
pointed out that the ground-truth is only needed while the
estimator is in the process of learning a good weight vector
$\Theta$, that is when LAA is being trained.

In addition, at every time $t$ (even if no new experience was
added, as may happen in practice) the estimator calculates the
average squared error over a randomly chosen mini-batch
(Lines 15 – 18) of experiences in the replay memory. Note
that the calculation of error requires forward propagation
using LAA. Specifically, for each experience we must input
the corresponding $I$ to LAA and obtain the $\hat{y}(t)$. The error is
then used to update $\Theta$ using gradient descent. This requires
calculation of gradient of the mean squared measurement
residual cost function with respect to the weight vector $\Theta$
(referred to as Backpropagation), see Line 19.

On the replay memory: The replay memory is often used to
ensure that consecutive weight updates result from experiences
that are independent of each other. Choosing experiences in
the time sequence in which they occur would result in strongly
correlated experiences when updating the weight vector, which
is detrimental to learning. The use of replay memory, when
training a deep neural network, to break correlations is common
(for example, see [7]).

On Hyperparameters and computational complexity: We
used the Adam optimiser with learning parameter $10^{-4}$ and
mini-batch size of 256. To prevent the model from over-
fitting, we use a L2 weight decay of $10^{-3}$. Our replay
memory is of size $2 \times 10^6$ experiences. The LSTM cell
has four gates, each of which computes two matrix-vector
multiplications and two additions involving vectors. For an

While we assume that the ground-truth is readily available, in practice
the said ground-truth is only available if and when the measurement $y(t)$
is received by the estimator over the communications network. Assuming
that the latter is the case, the estimator must add the experience corresponding
to the input $I(t)$ if and when $y(t)$ is received. This would result in experiences
being added to the replay memory at a slower rate than when the ground-truth
is readily available. However, the algorithm stays unchanged.

We simulated the two environments of a vehicle moving in
two dimensional space and that of an inverted pendulum (a pole)
fixed to a moving cart, often referred to as the cartpole. We
simulated different network configurations via appropriate
choices of $p$ and $q$. We detail both next.

A. Simulated Environments

The vehicle’s motion is well modeled by a linear dynamic
system that follows Newtonian kinematics. Such a model for a
vehicle is often used in autonomous vehicle path planning (see,
for example, [5 Equation (11)]) to model the motion of other
vehicles whose positions and velocities an autonomous vehicle
must estimate. The measurement vector of the vehicle at time
$t_k$ is $y_k = \begin{bmatrix} x_{k} \ y_{k} \ u_{x,k} \ u_{y,k} \end{bmatrix}$, wherein
$(p_{x,k}, p_{y,k})$ are the position coordinates, $(v_{x,k}, v_{y,k})$ are the
velocities along the $x$ and $y$ dimensions, and $(u_{x,k}, u_{y,k})$ are the
corresponding accelerations. The position coordinates are
allowed to vary over $[-1000, 1000]$ m and the velocities over
$[-10, 10]$ m/sec. At every time $t_k$, the accelerations along $x$ and $y$
are chosen uniformly and randomly from $[-3, 3]$ m/s$^2$.

The vehicle’s motion is given by Equation (4), where the noise
vector $\mathbf{w}_k$ is a Gaussian noise vector of mean 0 and a diagonal
covariance matrix, with the diagonal elements set to 0.2. The
simulation time step is $\Delta t = t_{k+1} - t_k = 0.1$ s, for all $k$.

LAA estimates $\hat{y}_k = \begin{bmatrix} \hat{p}_{x,k} \ \hat{p}_{y,k} \ \hat{v}_{x,k} \ \hat{v}_{y,k} \end{bmatrix}$. Note that, the
vehicle state at any time is summarized by its position and
velocity. So while the measurement may include acceleration
information and is useful for prediction, the estimated vector
has only position and velocity.

The cartpole environment is a nonlinear dynamic system
that has a pole attached to a moving cart. Ideally, one wants
to move the cart to stabilize the pole (keep it close to the
vertical position) attached to it. We simulate the environment
as implemented by OpenAI gym api [23]. The continuous-
time dynamics are given by Equation [5], where $F$ is the force
applied to the cart, $m_c$ is mass of the cart, $m_P$ and $2l$ are

\[
\begin{bmatrix}
p_{x,k+1} \\
p_{y,k+1} \\
v_{x,k+1} \\
v_{y,k+1}
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 & \Delta t & 0 \\
0 & 1 & \Delta t & 0 \\
0 & 0 & 1 & \Delta t \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
p_{x,k} \\
p_{y,k} \\
v_{x,k} \\
v_{y,k}
\end{bmatrix} + 
\begin{bmatrix}
\Delta t^2/2 \\
\Delta t^2/2 \\
\Delta t \\
\Delta t
\end{bmatrix}
\begin{bmatrix}
u_{x,k} \\
u_{y,k}
\end{bmatrix} + \mathbf{w}_k.
\]

\[
\dot{\theta} = \frac{g \sin \theta + \cos \theta \left(-\frac{F - m_P g \theta^2 \sin \theta}{m_c + m_P}\right)}{l \left(\frac{4}{3} - \frac{m_P \cos^2 \theta}{m_c + m_P}\right)} ,
\quad \ddot{x} = F + m_P \left(\frac{\theta^2 \sin \theta - \dot{\theta} \cos \theta}{m_c + m_P}\right).
\]
respectively the mass and length of the pole, $x$ is the position of the cart, $\theta$ is the angle between the pole and the vertical, and $g$ is the acceleration due to gravity. We set $l = 1$ m, $m_c = 5$ kg and $m_p = 1$ kg. The cart’s velocity $\dot{x}$ stays in the interval $[-10, 10]$ m/sec. At every time $t_k$, we randomly choose the direction of motion of the cart to be either forward or reverse with equal probability. The simulation timestep $t_{k+1} - t_k = 0.01$ s, for all $k$.

The measurement vector of the cartpole at time $t_k$ is $y_k = [\theta_k \; \dot{\theta}_k \; \dot{x}_k \; F_k]$. LAA would like to estimate the first three quantities in the measurement vector.

### B. Communication Networks

We consider example networks with fixed $p$ and $q$ for all time and also a network in which $p$ and $q$ change over time. For when the network parameters are time-invariant, we choose $p = 0.01, 0.1, 0.297$ for $q = 0.3$, and $p = 0.01, 0.3, 0.499$ for $q = 0.5$. The choices of $p$ for each $q$ are motivated by Figure 2 that shows how the average age of updates varies at the estimator as a function of $p$ for $q = 0.3, 0.5$. As is seen in the figure, the average age is high for both $p = 0.01$ and $p = 0.297$ when $q = 0.3$. As has been shown to hold true for a wide range of network settings [19], the high age at $p = 0.01$ is explained by a low rate $p$ of sending of updates over the network. A small $p$ with respect to $q$ results in low utilisation of the queue server by the update packets. An update that arrives to the queue suffers small queue wait times before completing service. As a result, updates received by the estimator have a small age and are relatively recent. However, the average age of updates at the estimator is still high because the server receives updates infrequently.

The average age is high at $p = 0.297$ because the rate of sending updates is close to $q$. The high utilisation $p/q$ of the server has updates experience high delays due to large queue waiting times. The estimator receives updates at a high rate $p$ but with a high average age. The high average ages when $p = 0.01, 0.499$ for $q = 0.5$ can be explained similarly. The network settings of $p = 0.1, q = 0.3$ and $p = 0.3, q = 0.5$ result in close to minimum average age.

### VII. Evaluation

We used the learning algorithm, described in Algorithm 1 to train a LAA model for each of the following network and dynamic system settings. For both the linear system and the cartpole, we trained a model for each $(p, q)$ in the list $(0.01, 0.3), (0.1, 0.3), (0.297, 0.3), (0.01, 0.5), (0.3, 0.5), (0.499, 0.5)$. We also trained a model, where in each episode of training we randomly chose $q$ in $(10^{-2}, 1)$ and $p$ in $(10^{-3}, q)$.

![Fig. 2: Average age as a function of utilization for $q = 0.3, 0.5$.](image)

Note that it is essential for queue stability to choose $p < q$. We will refer to the former settings as fixed network and the latter as time-varying network. The two are compared in Section VII-B.

Further, for each fixed-network setting, we perform an ablation study to answer whether the age $\Delta y(t)$ at the input of LAA (see Equation (2)) makes a significant difference to the estimation error. To do so, we train a LAA model, for each fixed network setting, without age as an input. We will refer to this as the No Age setting, which is evaluated in Section VII-B.

Last but not the least, for each fixed-network setting, we train a LAA model, for when the current values of $u_{x,k}, u_{y,k}$ and $F_k$ are assumed to be known is referred to as the known control setting.

In Section VII-C we test the trained models that take age as an input for when the true age is available and also when a noisy estimate of the same is available. We also test the fixed network models that don’t take age as input. The performance metric of interest is the estimation error, which is the square-root of the average squared measurement residual defined in Section IV.

We show a comparison of the estimation error achieved by the LAA models, for the linear dynamic system, with the commonly used Time-Varying Kalman Filter (TVKF), and for the cartpole, with the Unscented Kalman Filter (UKF). The average error is computed over 200 episodes, wherein each episode is 40000 time steps long.

#### A. Baselining LAA using the known control setting

We begin by evaluating LAA for the known control setting. For this setting, the baseline algorithms of Time-Varying Kalman Filter (TVKF) and the Unscented Kalman Filter (UKF) for the non-linear dynamic system are expected to

4We sample uniformly from the range of $\log_{10}$ probabilities to ensure a good representation of probabilities $<1$. 
Controls are communicated over the network for the estimation error obtained by LAA (fixed network over the network). For the linear system, Figure 4a compares recent measurement packet containing controls is received TVKF and the UKF use the last known control till a more accurate estimate of age. Including age at the input of LAA is always better estimation error wise.

Figures 3a, 3b, 4a, 4b, 5a, 5b, 6a, 6b, 7a, 7b show that the testing performance of the model trained using a time-varying network is in fact almost as good as that trained for the fixed network setting. There is no benefit in training different LAA models for different fixed network settings. For Figure 6a, the testing was done for q = 0.3 and a linear system. In Figure 6a, the chosen system does well. Both TVKF and the UKF assume knowledge of the dynamic system model.

The TVKF with infinite time buffer is in fact known to be the optimal estimator [17], for linear dynamic systems, when the controls actions are always known to the estimator. This gives us an opportunity to benchmark the data-driven LAA against a known optimal estimator. We compare (see Figure 3a) the estimation error obtained using the LAA models trained for the known control setting with that obtained using the TVKF for the linear system. The algorithms are compared for when q = 0.3 and p takes values of 0.01 (low rate and high age, see Figure 2), 0.1 (close to the age optimal rate), and 0.297 (high rate, delay, and age). As is seen from the figure, LAA performs almost as well as the optimal TVKF.

For a nonlinear dynamic system, we were able to find a UKF implementation [18] that works with intermittent measurements. However, an update when received is assumed current (fresh, age 0). We modify the UKF implementation in the following manner. When a measurement is not received, we propagate the prior state estimate and error covariance according to the system dynamic model and set the posterior of the state and error covariance to their respective priors. When a new measurement arrives, we use it to recalculate the posterior estimates corresponding to the timestamp of the update, and propagate the estimates up to the current time using the model. Figure 3b compares LAA and UKF. LAA compares well with UKF. LAA performs almost as well as the optimal TVKF.

B. LAA’s performance for the fixed network setting, with and without age inputs

As in typically the case, the control actions, unlike for baselining, are obtained over the network. We assume that the TVKF and the UKF use the last known control till a more recent measurement packet containing controls is received over the network. For the linear system, Figure 4a compares the estimation error obtained by LAA (fixed network setting), LAA with No Age, and the TVKF. Figure 4b does the same for the cartpole with the UKF chosen instead of the TVKF. It is clear that LAA with the age inputs does the best and in fact outperforms the TVKF and the UKF. LAA without the age inputs performs fairly well when p is set close to the age optimal rate but suffers otherwise. Keeping the age inputs Δu and Δv clearly benefits estimation. Also, see Figures 3a and 4a, all estimators give a much smaller estimation error when the chosen p is the age optimal rate.

C. LAA’s performance when age estimates are noisy

In practice, getting an accurate estimate of age requires the source of the measurement and the estimator to be time-synchronized. This is often imperfect and typically not available. A typical workaround is to estimate one-way delays using round-trip-time (RTT) (between source and estimator) measurements. The one-way delay estimates can then be used to estimate age, which recall is the time elapsed between generation of a measurement and its reception at the estimator.

For the purpose of evaluating the impact of noisy age estimates on LAA’s performance, we scale the true age by a continuous uniform random variable that takes values in (0, 2), where a scaling of 2 approximates a RTT based estimate of age. To the scaled value we add a Gaussian with mean 0 and standard deviation equal to 10% of the true age. Figure 5 shows the impact of noisy age estimates on the estimation error. Note that LAA was trained as earlier with true age values at its input. Noisy age estimates were only provided when testing. The estimation performance of LAA is not very impacted, unlike the TVKF and the UKF that see a significant increase in estimation error in comparison to when the true age is available.

D. Does LAA train well over a time-varying network setting? Or do we need to train it separately for every fixed network setting of interest?

To have to train LAA separately for fixed network settings would make it impractical. Often, the network parameters aren’t known and change over time. Figures 6a, 6b, 7a, 7b compare the testing performance of LAA trained over a time-varying network with the testing performance of LAA trained for the same fixed network parameters that are used for testing. Figures 6a, 6b, 7a, 7b show that the testing performance of the model trained using a time-varying network is in fact almost as good as that trained for the fixed network setting. There is no benefit in training different LAA models for different fixed network settings. For Figure 6a, the testing was done for q = 0.3 and a linear system. In Figure 6b, the chosen system
We proposed a deep neural network model to learn estimates of state measurements in a model-free setting using intermittent and aged state measurements received over the network. We demonstrated its efficacy by comparing with the baselines of the TVKF and the UKF for different network settings and example linear and nonlinear systems. Our model performed well even in the presence of noisy age estimates and doesn’t need to be trained separately for different network configurations, making it more suitable for real world settings. We showed that the age of updates as an input results in smaller estimation error. Also, all estimators see smaller errors when the utilization of the network minimizes the average age of updates at the estimator.

VIII. CONCLUSION

![Fig. 6](image1)

![Fig. 7](image2)

Fig. 6: Figures compare the test performance of the LAA model trained over a time-varying network setting with the performance of a model trained for the specific network setting used for the test. Controls are communicated over the network. (a) Linear system, \( q = 0.3 \) and the \( p \) shown in the figure are the different test network settings (b) Nonlinear, \( q = 0.5 \) and the \( p \) shown in the figure are the different test network settings. The corresponding results for \( q = 0.5 \) and \( q = 0.3 \) are similar. The LAA model trained over a time-varying network does almost as well as the models trained for specific network settings.

Fig. 7: The evaluation setup is akin to that for Figure 6. We set \( q = 0.007 \). (a) Linear system (b) Nonlinear System. Again, the LAA model trained over a time-varying network performs very well.

is the nonlinear cartpole and \( q = 0.5 \). Figure 7 shows the estimation error for very small values of \( p \) and \( q \). The LAA model trained over a time-varying network is seen to work well over a wide range of \( p \) and \( q \).

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