QCD corrections to $e^+e^- \rightarrow 4$ jets

S Weinzierl†
NIKHEF, P.O. Box 41882, 1009 - DB Amsterdam, The Netherlands

Abstract. We report on the next-to-leading order QCD calculation for $e^+e^- \rightarrow 4$ jets. We explain some modern techniques which have been used to calculate the one-loop amplitudes efficiently. We further report on the general purpose numerical program “Mercutio”, which can be used to calculate any infrared safe four-jet quantity in electron-positron annihilation at next-to-leading order.

1. Motivation: LEP physics

QCD four-jet production in $e^+e^-$ annihilation can be measured at LEP and can be studied in its own right. First of all, $e^+e^- \rightarrow 4$ jets is the lowest order process which contains the non-abelian three-gluon-vertex at tree level and thus allows for an measurement of the colour factors $C_F$, $C_A$ and $T_R$ of QCD. This in turn may be used to put exclusion limits on light gluinos. Furthermore the QCD process is a background to W-pair production, when both W’s decay hadronically and to certain channels of the search of the Higgs boson like $e^+e^- \rightarrow Z^* \rightarrow ZH \rightarrow 4$ jets. The one-loop matrix elements required for an NLO study of four-jet production are also an essential input for an NNLO calculation of three-jet production. The latter one would be needed to reduce theoretical uncertainties in the extraction of the strong coupling at the Z-pole.

In general leading-order calculations in QCD give a rough description of the process under consideration, but they suffer from large uncertainties. The arbitrary choice of the renormalization scale gives rise to an ambiguity, which is reduced only in an next-to-leading order calculation. Furthermore the internal structure of a jet and the sensitivity to the merging procedure of the jet algorithm are modelled only in an NLO analysis. Both uncertainties are related to the appearance of logarithms, ultraviolet in nature in the first case, infrared in the latter, which are calculated explicitly only in an NLO calculation.

A NLO calculation proceeds in two steps: First, one needs the relevant amplitudes, in our case $e^+e^- \rightarrow 4$ partons and $e^+e^- \rightarrow 5$ partons at tree level and $e^+e^- \rightarrow 4$ partons at one loop. Among these, the one-loop amplitudes are the most complicated ones and we will comment on their calculation in the next section. The second step requires setting up a numerical Monte Carlo program which has to deal with infrared divergences. We will focus on this point in the third section. In the last section we will give some numerical results.

† Talk given at the UK Phenomenology Workshop on Collider Physics, Durham, 19-24 September 1999
QCD corrections to $e^+e^- \rightarrow 4$ jets

2. One-loop amplitudes

We used a variety of modern techniques in order to calculate the one-loop amplitudes efficiently. These include colour decomposition, where amplitudes are decomposed into simpler gauge-invariant partial amplitudes with definite colour structure, and the spinor helicity method, which consists in expressing all Lorentz four-vectors and Dirac spinors in terms of massless two-component Weyl-spinors. Their use divides the task into smaller, more manageable pieces. Also a decomposition inspired by supersymmetry proved to be useful, where the particles running around the loop are reexpressed in terms of supermultiplets. In a second step the cut technique and factorization in collinear limits are used to constrain the analytic form of the partial amplitudes.

As an example we explain in more detail the cut technique [1], which is based on unitarity. To obtain the coefficients of the basic box, triangle or bubble integrals one considers the cuts in all possible channels. Each phase-space integral is rewritten with the help of the Cutkosky rules as the imaginary part of a loop amplitude. The power of this method lies within the fact, that on each side of the cut one has a full tree amplitude and not just a single Feynman diagram. This method allows one to reconstruct the one-loop amplitude up to terms without an imaginary part. The remaining terms were obtained by examining the collinear limits.

For the reduction of tensor pentagon integrals we used a new reduction algorithm [2], based on the Schouten identity and Weyl spinors, which does not introduce artifical Gram determinants in the denominator.

The one-loop amplitudes for the first subprocess $e^+e^- \rightarrow q\bar{q}Q\bar{Q}$ were calculated in refs. [3, 4] and the amplitudes for the second subprocess $e^+e^- \rightarrow q\bar{q}gg$ in refs. [5, 6]. The calculations of the two groups agree with each other.

3. Numerical implementation: Mercutio

The second major part of a general purpose NLO program for four jets is coding the one-loop amplitudes and the five parton tree-level amplitudes in a numerical Monte Carlo program. At leading order the task is relatively simple: One parton corresponds to one jet. At NLO however, a jet can be modeled by two partons. At NLO the cross section receives contributions from the virtual corrections and the real emission part. Only the sum of them is infrared finite, whereas when taken separately, each part gives a divergent contribution. Several methods to handle this problem exist, such as the phase-space slicing method [7], the subtraction method [8] and the dipole formalism [9]. We have chosen the dipole formalism. Within the dipole formalism one subtracts and adds again a suitably chosen term:

$$\sigma^{NLO} = \int n+1 (d\sigma^R - d\sigma^A) + \int n \left( d\sigma^V + \int l d\sigma^A \right)$$  \hspace{1cm} (1)

The approximation term $d\sigma^A$ has to fullfill the following two requirements: First, $d\sigma^A$ must be a proper approximation to $d\sigma^R$, with the same pointlike singular behaviour in $D$ dimensions as $d\sigma^R$. Secondly, $d\sigma^A$ must be analytically integrable in $D$ dimensions over the one-parton subspace leading to the soft and collinear divergences.

Let me now turn to the details of the Monte Carlo integration. The heart of any Monte Carlo integration is the random number generator. Among other things,
it should have a long period and should not introduce artificial correlations. As the default random number generator we use
\[ s_i = (s_{i-24} + s_{i-55}) \mod 2^{32}. \] (2)
It was proposed by Mitchell and Moore and has a period of \(2^5(2^{55} - 1)\), where \(0 \leq f < 2^5\). Massless fourmomenta are generated with the help of the RAMBO-algorithm [10]. This algorithm generates events with a uniform weight. Adaptive importance sampling is implemented using the VEGAS-algorithm [11]. A naive implementation of the dipole formalism will give large statistical errors when performing a Monte Carlo integration over the real corrections with dipole factors subtracted. In order to improve the efficiency of the Monte Carlo integration we remap the phase space to make the integrand more flat. A simplified model for the term \(d\sigma^R - d\sigma^A\) would be
\[ F = \int_0^1 dx \left( \frac{f(x)}{x} - \frac{g(x)}{x} \right) \] (3)
where \(f(0) = g(0)\) is assumed. \(f(x)/x\) corresponds to the original real emission part with a soft or collinear singularity at \(x = 0\), \(g(x)/x\) corresponds to the subtraction term of the dipole formalism. Eq. (3) can be rewritten as
\[ F = \int_0^{y_{\text{min}}} dx \frac{f(x)}{x} - g(x) + \int_{\ln y_{\text{min}}}^0 dy \left( f(e^y) - g(e^y) \right), \] (4)
where \(y_{\text{min}}\) is an artificial parameter separating a numerically dangerous region from a stable region. Using the Taylor expansion for \(f(x) - g(x)\), one sees that the first term gives a contribution of order \(O(y_{\text{min}})\). In the second term the \(1/x\) behaviour has been absorbed into the integral measure by a change of variables \(y = \ln x\), and the integrand tends to be more flat. It should be noted that there is no approximation involved.

4. Numerical results

Numerical programs for \(e^+e^- \rightarrow 4\) jets have been provided by four groups: MENLO PARC [12], DEBRECEN [13], EERAD2 [14] and MERCUTIO [15]. Various cross-checks have been performed among these programs and they agree within statistical errors. Here we report on the numerical program “Mercutio”, which was written in C++. The four-jet fraction is defined as
\[ R_4 = \frac{\sigma_{4\text{-jet}}}{\sigma_{\text{tot}}}. \] (5)
The values obtained for the four-jet fraction for the DURHAM algorithm with \(y_{\text{cut}} = 0.01\) for various energies are given in table [1]. The decrease with energy is mainly due to the running of the strong coupling.

With the numerical program for \(e^+e^- \rightarrow 4\) jets one may also study the internal structure of three-jets events. One example is the jet broadening variable defined as
\[ B_{\text{jet}} = \frac{1}{n_{\text{jets}}} \sum_{a} \frac{|p_{a}^\perp|}{\sum_{a} |p_{a}|}, \] (6)
Here \(p_{a}^\perp\) is the momentum of particle \(a\) transverse to the jet axis of jet \(J\), and the sum over \(a\) extends over all particles in the jet \(J\). The jet broadening variable is calculated
QCD corrections to $e^+e^- \rightarrow 4$ jets

| $\sqrt{Q^2}$ | $R_{4j}^{LO}$ | $R_{4j}^{NLO}$ |
|-------------|--------------|---------------|
| $m_Z$ GeV   | $(2.98 \pm 0.01) \cdot 10^{-2}$ | $(4.72 \pm 0.01) \cdot 10^{-2}$ |
| 135 GeV     | $(2.65 \pm 0.01) \cdot 10^{-2}$ | $(4.12 \pm 0.01) \cdot 10^{-2}$ |
| 161 GeV     | $(2.53 \pm 0.01) \cdot 10^{-2}$ | $(3.89 \pm 0.01) \cdot 10^{-2}$ |
| 172 GeV     | $(2.48 \pm 0.01) \cdot 10^{-2}$ | $(3.81 \pm 0.01) \cdot 10^{-2}$ |
| 183 GeV     | $(2.44 \pm 0.01) \cdot 10^{-2}$ | $(3.73 \pm 0.01) \cdot 10^{-2}$ |
| 189 GeV     | $(2.42 \pm 0.01) \cdot 10^{-2}$ | $(3.69 \pm 0.01) \cdot 10^{-2}$ |

Table 1. The four-jet fraction at LO and NLO for the DURHAM algorithm with $y_{cut} = 0.01$ and various energies.

Figure 1. The $B_{jet}$ distribution at NLO (diamonds) and LO (crosses).

for three-jet events defined by the DURHAM algorithm and $y_{cut} = 0.1$. This choice is motivated by a recent analysis of the Aleph collaboration [16]. Figure 1 shows the distribution of the jet broadening variable.

References

[1] Zvi Bern, Lance Dixon, David C. Dunbar, and David A. Kosower. Nucl. Phys., B435:59–101, 1995.
[2] S. Weinzierl. Phys. Lett., B450:234, 1999.
[3] Zvi Bern, Lance Dixon, David A. Kosower, and Stefan Weinzierl. Nucl. Phys., B489:3–23, 1997.
[4] E. W. N. Glover and D. J. Miller. Phys. Lett., B396:257–263, 1997.
[5] Zvi Bern, Lance Dixon, and David A. Kosower. Nucl. Phys., B513:3, 1998.
[6] J. M. Campbell, E. W. N. Glover, and D. J. Miller. Phys. Lett., B409:503–508, 1997.
[7] W. T. Giele and E. W. N. Glover. Phys. Rev., D46:1980–2010, 1992.
[8] S. Frixione, Z. Kunszt, and A. Signer. Nucl. Phys., B467:399–442, 1996.
[9] S. Catani and M. H. Seymour. Nucl. Phys., B485:291–419, 1997.
[10] R. Kleiss, W. J. Stirling, and S. D. Ellis. Comput. Phys. Commun., 40:359, 1986.
[11] G. Peter Lepage. J. Comput. Phys., 27:192, 1978.
[12] Lance Dixon and Adrian Signer. Phys. Rev., D56:4031–4038, 1997.
[13] Zoltan Nagy and Zoltan Trocsanyi. Phys. Rev., D59:014020, 1999.
[14] J. M. Campbell, M. A. Cullen, and E. W. N. Glover. Eur. Phys. J., C9:245, 1999.
[15] Stefan Weinzierl and David A. Kosower. Phys. Rev., D60:054028, 1999.
[16] R. Barate et al. Measurements of the structure of quark and gluon jets in hadronic Z decays. CERN-EP-98-016.