Earth Gravity Field Recovered from CHAMP Science Orbit and Accelerometer Data

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ABSTRACT The earth gravity field model CDS01S of degree and order 36 has been recovered from the post processed Science Orbits and on-board accelerometer data of GFZ’s CHAMP satellite. The model resolves the geoid with an accuracy of better than 4 cm at a resolution of 700 km half-wavelength. By using the degree difference variances of geopotential coefficients to compare the model CDS01S with EIGEN3P, EIGEN1S and EGM96, the result indicates that the coefficients of CDS01S are most close to those of EIGEN3P. The result of the comparison between the accuracies of geopotential coefficients in the above models, indicates that the accuracy of coefficients in CDS01S is higher than that in EGM96. The geoid undulations of CDS01S and GGM01C up to 30 degrees are calculated and the standard deviation is 4.7 cm between them.

KEY WORDS CHAMP satellite; accelerometer and orbit data; Earth gravity field; geoid

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Introduction

Since the launch of man-made satellite early in 1957, the research for satellite gravity has been taken a wide attention in field of geodesy. Early, the ground-based satellite tracking has provided an observational data set which has been used to develop models of the global long-wavelength gravity field of the earth, and the medium- to short-wavelength gravity field has been generated from satellite altimetry and surface gravimetric data, etc. Recently, the dedicated gravity satellite mission has become into true with the development of space technology. The high-low satellite to satellite tracking mission CHAMP (challenging mini satellite payload) launched by German in July 2000, which made a key step in satellite gravity measurement. The low-low satellite-to-satellite tracking mission GRACE (gravity recovery and climate experiment mission) launched under a joint of US-German in March 2002, which shows sufficient advantage of satellite gravity measurement technology. The objectives of CHAMP and GRACE are to map the medium- to long-wavelength static and the long-wavelength time-variable gravity field of high accuracy.

Presently, many global gravity field models have been derived from CHAMP data, such as the EIGEN1S\(^1\), EIGEN2\(^2\) and EIGEN3P\(^3\) models, which have been derived from GFZ (GeoForschungsZentrum) Potsdam using satellite dynamical method. The accuracy of EIGEN2 is better than 10 cm and 0.5 mGal in terms of geoid height and gravity anomaly at \(\lambda/2 = 500\) km respectively. In theory, the dynamical method for gravity field recovery used by GFZ, which has a strictness of calculating process with a large of computing data and high complicating degree, it needs high-quality computer to sustain.

Presently, the recovery of Earth gravity field from CHAMP tracking data by use of energy integral method has arrested more attention of many scholars, the reason is that the gravity
field model based on energy method is free from the background gravity field model, and use little dynamical force models, and can provide an satellite-only Earth gravity field model directly in theory. Many scientists, such as Gerlach, Visser, and Xu Tianhe, recovered many CHAMP-derived gravity field models using the energy method. No matter using dynamical method or energy method, the CHAMP solutions have a significant improvement in long-wavelength components of gravity field when compared with the latest prior-CHAMP gravity field.

In order to fit the imminent need for military mapping, earthquake surveying and other applicable field, the satellite gravity system of China should be established. Recently, the key question is to finish the pre-research contents on the mission of satellite gravity measurement system, which includes its objective and feasibility. Therefore, we should study the high-low satellite to satellite tracking mission, like CHAMP, form software, depose data, and recover the earth gravity field, whose objective is to accelerate the step for the development of the Chinese satellite gravity system.

1 Principle of computation

The spherical harmonic expansion formulation of geopotential U is

$$U = \frac{GM}{r} \left[ 1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left( \frac{a}{r} \right)^n \left( \frac{C_n^m \cos m\lambda}{P_n^m (\sin \theta)} \right) \right] \tag{1}$$

where $GM$ is the earth gravity constant; $r$ is the geocentric distance; $\phi$ is the geocentric latitude; $\lambda$ is longitude; $a$ is the equatorial radius of the earth; $P_n^m$ is the Legendre polynomials of degree $n$ and order $m$; $C_n^m$ and $S_n^m$ are the spherical harmonic coefficients of the geopotential.

The acceleration of satellite in conventional inertial reference system is:

$$\ddot{r} = E^r \nabla U + a \tag{2}$$

where $\nabla$ represents the gradient of geopotential; $a$ represents the acceleration caused by forces except for the earth gravity; $E^r$ represents the transformation matrix from the earth-fixed frame to conventional inertial frame.

The theoretical satellite orbit $r_t$ is calculated from the selected force models by numerical integral of the satellite acceleration $\ddot{r}$. Since there exist the observed and calculated errors, the error in the calculating process for $r_t$, the force model errors, as well as the uncertainties in gravity field harmonic coefficients and constants, the observational orbit $r_o$ is not equal to the calculated orbit $r_t$, namely, $\Delta r = r_o - r_t \neq 0$.

The equation can be obtained by linear disposal:

$$\Delta r = \sum_{i=1}^{n} \frac{dr}{dE_i} \Delta E_i + \sum_{i=1}^{n} \sum_{m=0}^{n} \left( \frac{dr}{dC_n^m} \Delta C_n^m + \frac{dr}{dS_n^m} \Delta S_n^m \right) + \frac{dr}{d\Delta \varepsilon} \Delta \varepsilon + V \tag{3}$$

where $\mu = GM$; $\Delta E_i, \Delta \mu, \Delta C_n^m, \Delta S_n^m$ and $\Delta \varepsilon$ represent the correction of Earth rotation, gravitational constant, geopotential coefficients and other parameters respectively; $V$ is the residual.

Since the corrections of the solved parameters in Eq. (3) are independent of one and another, so we can form the normal equation by the observational residual between the computed orbit $r_t$ and the observed orbit $r_o$ derived by the precise orbit determination, and then solve the geopotential coefficients with least square method.

The partial derivatives in Eq. (3) are calculated when the numerical integral of satellite acceleration $\ddot{r}$ is made. In the solving program, the $r$ is separated into $x$, $y$ and $z$ along three orientations to establish normal equation.

The accuracy of the earth gravity field is usually assessed by analysis of the estimation of geopotential coefficients errors or the corresponding geoid height errors. If the $n$ and $m$ represent the degree and order of geopotential coefficients respectively, the degree error variance in geoid height is defined as:

$$\sigma_{m,n} = a_n \sqrt{\sum_{m=0}^{n} \sigma_{n,m}^2 + \sum_{n=1}^{\infty} \sigma_{m,n}^2} \tag{4}$$

where $\sigma_{n,m}$ are the standard deviation of normalized geopotential coefficients.

Furthermore, the differences between two gravity fields are also compared from the analysis of the different geopotential coefficients, the
degree difference variance is defined as
\[ \Delta_n = \sum_{n=0}^{\infty} \Delta C_{nm} + \sum_{m=1}^{\infty} \Delta S_{nm} \]  
(5)
where \( \Delta C_{nm} \), \( \Delta S_{nm} \) represent the differences in geopotential coefficients two models.

2 Accelerometer calibration

According to the description of CHAMP data format, the correction of the accelerometer data can be calculated by use of the following formula:
\[ a_i = f_i(f_i - b_i) \quad (i = 1, 2, 3) \]  
(6)
where \( i = 1, 2, 3 \) represent the linear accelerometer data vector components of \( x \), \( y \) and \( z \) respectively; \( a_i \) represents the corrected accelerometer data in \( i \)-direction; \( f_i \), \( b_i \), and \( k_i \) represent uncorrected accelerometer measurement data (including the correction of Lorentz force and model), bias and scale factor parameters in \( i \)-direction respectively.

The accelerometer data derived from Eq. (6) is listed in accelerometer instrument fixed system, but it usually is used under the conventional inertial system, so it should be transformed from instrument fixed system to conventional inertial system, the transform program is shown in Reference [7].

Since the CHAMP accelerometer data is transformed from electric signal, there exist the instrument excursion problem, so the scale factor and bias are not constants. Furthermore, there are some problems in one electrode for measurement of radial accelerometer data, although an algorithm has been found to overcome the problem recently, it still affects the veracity of non-conservative force measurement directly, so it needs to be re-calibrated before recovery of the earth gravity field. During the calibrating process, we first design the partial derivatives of scale factor and bias of accelerometer data according to Eq. (6), and then take the GFZ-derived precise CHAMP orbit as the observational value, and establish the observational equation, and calibrate the parameters with satellite dynamical method. Fig. 1 gives the difference between GFZ-derived precise orbit and the calculated orbit about the accelerometer parameters re-calibrated before and after.

![Fig. 1 Residuals of scale factor and bias parameters re-calibrated before and after](image)

Fig. 1 shows that there exist considerable differences between the accelerometer data re-calibrated and not. After it has been re-calibrated, the RMS between the GFZ-derived orbit and the calculated orbit reduces from about 80 cm to 7 cm, so it is more important to re-calibrate the scale factor and bias of accelerometer data. If the parameters of accelerometer data are not re-calibrated, the non-conservative forces can not be removed effectively, and the recovery of the earth gravity field will have a low accuracy.

3 Earth gravity field recovery

Application of Eq. (3), an Earth gravity field model CDS01S to the degree and order 36 has been generated from 238 days of CHAMP tracking data with the satellite dynamical method, the estimable parameters include the geopotential coefficients and force model parameters, as well as the orbit state parameters and other solved parameters in the least square normal equation.

In order to understand the accuracy of CDS01S, the gravity field models of EIGEN3P, EIGEN1S and EGM96 are chosen for the comparison which is to calculate out the geopotential coefficients differences between the CDS01S and all of the mentioned gravity field models respectively. Fig. 2 gives the degree difference variances of the geopotential coefficients to the degree and order 36 between CDS01S and
EIGEN3P, EGM96 or EIGEN1S.

Fig. 2 Degree difference variance of geopotential coefficients between two models

Fig. 2 shows that the $36 \times 36$ degree geopotential coefficients of CDS01S are closest to those of EIGEN3P, and CDS01S has larger difference when compared with EGM96. For the degree less than 20 in long-wavelength gravity field, the geopotential coefficients of EIGEN3P is also close to those of EIGEN1S.

Since the geoid height is also usually to describe Earth gravity field model in geodesy, so the degree error variances in term of geoid heights between CDS01S and EIGEN3P, EGM96 or EIGEN1S solutions can be shown in Fig. 3.

Fig. 3 Degree error variance in geoid height

Fig. 3 shows that the accuracy of CDS01S is better than that of EGM96. When the degree is less than 25, the accuracy of CDS01S is close to that of EIGEN1S, but it is better than EIGEN1S over 25 degrees. It may be related with the selected CHAMP tracking data set, the CDS01S is derived from 238-day data, but only 88-day data is used in the EIGEN1S. The degree error variances in term of geoid heights are 6 cm, 2 cm, 9 cm, 10 cm for CDS01S, EIGEN3P, EGM96 and EIGEN1S model at the degree 36 respectively.

In order to improve the knowledge of the accuracy of CDS01S, the geoid undulations have been calculated by use of the geopotential coefficients of CDS01S and GGM01C to the degree and order 30, the differences between them are showed in Fig. 4.

Fig. 4 Geoid difference between CDS01S and GGM01

From Fig. 4, there is not much difference of the geoid heights between CDS01S and GGM01C solutions, and they are in good agreement in some regions, the largest difference is about 45 cm, and the standard deviation is about 4.7 cm. The result shows that the geopotential coefficients to $30 \times 30$ degree between the CDS01S and GGM01C are close to one and another relatively.

4 Conclusions

The scale factor and bias of on-board accelerometer data should be re-calibrated before the recovery of gravity field by use of CHAMP satellite tracking data. Comparing the gravity field models between CDS01S and EIGEN3P, EGM96 or EIGEN1S to the degree and order 36, we find the geopotential coefficients of CDS01S are closest to those of EIGEN3P, and the accuracy of CDS01S is better than that of EGM96, the result shows the long-wavelength part of gravity field model can be significant improved by use of CHAMP data. Fig. 4 shows that there is little geoid undulation difference between CDS01S and GGM01C solutions, which farther shows the reliability of geopotential coefficients in CDS01S model.

It is more important to determine Earth gravity filed by use of GFZ's CHAMP satellite data for development of the gravity satellite system of
China, because the gravity field result is used to assess the reliability of our processing satellite data, establishing model and forming software. Analysis of the energy or dynamic method for gravity field recovery using satellite tracking data, the result indicate, for energy method, the key question in Earth gravity field recovery is how to re-calibrate the parameters of accelerometer data, but for the dynamical method, it is easy to calibrate the parameters of accelerometer data by use of satellite precise orbit data. Furthermore, the dynamical method can be developed into dynamical method completely, namely, it can simultaneously solve the parameters of satellite orbit, geopotential coefficients, accelerometer constant bias, GPS ambiguity and receiver clock biases at the same, and the result will have many virtues of high accuracy, stability and litter system errors, etc, but there exist many solved parameters, a great deal of calculated data, high complication and time consuming cost in the solved process, so it need high quality computer to sustain.

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Publications

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