The oral mathematical communication profile of prospective mathematics teacher in mathematics proving

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Abstract. As reported by many recent researchers, communicating mathematical ideas verbally is not an easy task, especially in mathematics proving. A qualitative research was undertaken to describe the processes of oral mathematical communication emerged when the subject did the proving task. The subject was one prospective mathematics teacher, with the high mathematical skill, selected from 48 students who took the proving tasks of geometry and algebra. Data were collected through tests, assignments, and depth interviews. The results of this study showed that the oral mathematical communication in the proving of geometry task complied with the following stages: explaining what is understood, expressing the idea in the form of drawing/symbol, explaining the idea/argument, presenting the solution steps, and reinforcing the results obtained. Furthermore, the oral mathematical communication in the proving of algebra tasks followed the following stages: explaining what is understood, explaining the idea about the form of mathematical logic, putting forward the argument, re-explaining what is understood, explaining the idea/argument, presenting the solution steps, presenting ideas in a more understandable form, putting forward the argument, and reinforcing the results obtained. In general, the subject could communicate her ideas precisely, coherently, and clearly.

1. Introduction
Oral mathematical communication is the process of conveying mathematics ideas or understanding verbally by speaking it up, the art of transmitting mathematics knowledge directly to another. Speaking is a productive skill that challenges students' capability to perform a task [1] which can take place in the form of a conversation or student talk. The exploration of mathematical ideas from various perspectives through conversation or student talk can help students enhanced their thinking and make connections [2]. Student talk is the most effective way for students to demonstrate and improve their understanding [3]. Through oral communication, students are trained to express ideas and understanding about mathematics by speaking them directly in front of others. In this paper, oral mathematical communication is defined as the process of conveying ideas and understanding of mathematics done by the prospective mathematics teacher verbally by articulating it up directly in front of other students.

Like any other parts of mathematics, the proof needs to be communicated both in writing and speaking. Communicating proof in writing means step by step verbal explanation of the truth of a mathematical statement so that it can be understood and accepted with common sense. This truth will only be acceptable if it is supported by a variety of previously proven axioms, theories, definitions, or prior knowledge. Obviously this is not an easy task. Many studies at the high school level and above have shown that developing an understanding of mathematical proof and proving remains a challenge for many students [4]. Mathematical proof is known as the most difficult topic to learn [5, 6].
problem of proof seems to be more complicated because the aim of a problem to prove is to show conclusively that a certain clearly stated assertions is true, or else to show it is false [7].

The research about oral mathematical communication has been investigated in many studies. Kabael [8] found out that junior high school mathematical teachers were less able to communicate using mathematical language. Though, mathematics is very important [9], and can be used as a tool to communicate effectively. Dewi [10] studied about mathematical communication of prospective teachers in solving mathematical problems. She also found out that prospective mathematics teacher were less able to communicate mathematical ideas and understanding. In fact, teachers are one of the key aspects of students success in learning [11]. Related to the mathematics proving, Yackel and Hanna [4] found out that developing an understanding of mathematics proving remains a challenge for many students. To communicate the proof requires a broader and more comprehensive understanding of knowledge such as conceptual understanding, procedural knowledge, proof methods, and reasoning abilities [12,13]. The various studies that have been done above emphasize the end result of communication rather than the communication process itself. In fact, communication is a process, that is the process of delivering ideas. Therefore, the process should be more emphasized.

2. Method

This research was a qualitative research. Data were collected by using tests, assignments, in-depth interviews, and questionnaire. A mathematical ability test consists of 10 questions was given to 48 students of mathematics education program of STKIP Santu Paulus Ruteng, Flores and used as a basis for determining the subject of research. Based on the results of the test, only six students have high mathematical ability. All of them are female students. One of them was chosen as a research subject. High-ability student was selected as research subject with consideration so that researchers can explore as many communication activities as possible in the proving process. In the next stage, the researcher gave the proving task to 48 students including the subject in it. The task consisted of two questions that were about geometry and algebra. After performing this task, the subject was asked to explain it verbally in front of the 11th grade students of the science program of SMAK Santu Fransiskus Xaverius Ruteng. During describing the task in front of the class, the researcher observed and recorded the performance of the research subject by using the handy-cam from beginning up to the end without interruption. The results of this recording were transcribed to serve as one of the guidelines in conducting interviews other than subject’s writing on the board. The transcribed data and the results of the interview were analysed based on the theoretical framework of oral mathematical communication. Triangulation was conducted by giving Task II which was similar to Task I with the same step to ensure that the data was valid.

3. Result and Discussion

3.1. Data Description Based on Task I

Task I consists of two proving problems, each measuring the proving skill in the field of geometry and algebra, those are: 1) prove that if \( ABCDEF \) is a regular hexagon with the side length is \( a \) unit then the area of \( BCEF = a^2 \sqrt{3} \) units, and 2) prove that for every integer \( n \), if \( n^2 \) is odd then \( n \) is odd.

For the first problem, the subject began the proof by writing down the problem that would be proven. Based on the problem, the subject asked questions about what is known and what will be proved. According to the subject, it was known that \( ABCDEF \) is a regular hexagon with the side length is \( a \) unit, and would be proved that the area of \( BCEF \) is \( a^2 \sqrt{3} \) unit. Based on the known elements, the subject drew a regular hexagon \( ABCDEF \) (See Figure 1a). The subject also affixed the letter \( a \) on each side of the hexagon to indicate that the length of the side is \( a \) unit. Then, the subject argued that the regular hexagon consisted of six congruent equilateral triangles. Based on this understanding, the subject determined the centre point O, and connected it to each vertex with a straight line to form six congruent triangles. The subject also affixed the letter \( a \) on each side of the triangle formed to express the length of the sides. In addition, the subject also created a dashed line connecting B to F and C to E so as to form the \( BCEF \) region. To demonstrate the area of \( BCEF \), the subject drew an \( ABCDEF \)
regular hexagon which contained $BCEF$ rectangle (See Figure 1b). Then, the subject determined the length of $BE$ and used it as one element to determine the length of $CE$. The subject drew again the $BCEF$ rectangle and created diagonal lines of $BE$ and $CE$ that intersected at the center point $O$ (See Figure 1c). Based on the picture, the subject explained that the length of $BE = 2a$. Furthermore, using the Phytagoras formula, the subject affixed a little corner ($\perp$) at the angle $EFB$ to confirm that the triangle formed is a right triangle. After the length of $BE$ was obtained, using the Phytagoras formula, the subject showed that $BF = a\sqrt{3}$. Finally, the subject showed that the area of the rectangle $BCEF = a^2\sqrt{3}$.

![Figure 1](image1.png)

Figure 1. (a) The process flow of completion for the first question in task I, starting for (a), (b), (c), and the last is (d)

As in the previous question, on the second question, the subject began the proof by writing the question and then asked the students to read it together. To ensure that students understand the meaning of the theorem, the subject gave a concrete example using integer 3. The subject explained that $3^2 = 9$ is an odd and it turned out that 3 is also an odd number. However, subject said that is just an example that could not be used as a basis to make general conclusion, therefore it needed to be proven. Furthermore, with the students, the subject identified the known elements and the elements to be proven based on the question. The subject found that, the known elements are $n \in \mathbb{Z}$ and $n^2$ is odd, while the element to be proved is $n$ is odd. Then the subject explained the logic model of the implications contained in the statement. The subject said that the implication model is $p \rightarrow q$ where $p$ is “$n^2$ is odd” while $q$ is “$n$ is odd”. Moreover, the subject argued that the form of the implication $p \rightarrow q$ is equivalent to its contraposition, $\neg q \rightarrow \neg p$, which brought about the statement of the implication could be proven by using contraposition. The subject said that the contraposition of “if $n^2$ is odd then $n$ is odd” is “if $n$ is even then $n^2$ is even”. Based on this argument, the subject re-identified the elements known and the elements to be proven. According to the subject based on the contraposition statement, the known element, $n$, is even, and $n^2$, the element which was to be proved is also even. Since the known $n$ is even, the subject explained the concept of even number. According to the subject, the even number is the integer that can be divided exactly by 2, therefore if $n$ is even then $n = 2k$ where $k$ is an integer, so $n^2 = (2k)^2 = 4k^2$. Then, the subject converted $4k^2$ to $2(2k^2)$ according to the pattern of even numbers. The subject argued that since $k$ is an integer then by the closure property of the integer multiplication operation, $l = 2k^2$ is also an integer and $n^2 = 2l$. The subject argued that since $2l$ is divisible by 2 then $n^2$ is even. At the end of the proof, the subject explained that since the statement “if $n$ is even then $n^2$ is even” is true then the statement "if $n^2$ is odd then $n$ is odd" is true as well because both are equivalently related.
3.2. Data Description Based on Task II

Task II was given as an effort to check the consistency of the process of oral mathematical communication that emerged during the subject proved the statements of mathematics. In doing so, the two questions in Task II were similar to the two questions in Task I. The two given questions were: (1) prove that the regular hexagonal area with a side length is $a$ is $\frac{3}{2}a^2\sqrt{3}$ units, and (2) prove that for every integer $n$, if $n^2$ is even then $n$ is even.

![Figure 2](image_url)

**Figure 2.** (a) The process flow of completion for the first question in task II, starting for (a), (b), (c), and the last is (d).

To deal with the first question, the subject began the proof by writing the question then asked the students to read it together. Based on the question, the subject asked questions about what is known and what will be proved. Through this way, the subject conveyed what she understood. According to the subject, it is known that there is a regular hexagon with the side length is $a$ unit, and will be proved that its area is $\frac{3}{2}a^2\sqrt{3}$. Since the known is the hexagon, the subject drew a regular hexagon named $ABCDEF$ by writing the constituent letters at each vertex (See Figure 2a). The subject also affixed a letter $a$ on each side of the hexagon to indicate that the length of the sides is $a$ unit. The subject explained that the sides of a regular hexagon have the same length. Then the subject determined the centre point O, and connected it to each hexagon angle with a straight line to form six triangles. The subject argued that the regular hexagon consisted of six congruent equilateral triangles so that the side lengths of the triangles is equal to the side lengths of the regular hexagon. Therefore, the subject affixed the letter $a$ on each side of the formed triangle to express the side length. Furthermore, the subject determined the angle of the triangle right at the centre point O. The subject drew again the regular hexagon $ABCDEF$, determined its midpoint O and connected it to each vertex on the hexagon. In addition, the subject also created a circle centred at the point O (See Figure 2b). The subject explained that the degree measure of circle is $360^\circ$, so that the degree measure of every angle of the triangle coincided with the centre point O is $\frac{360^\circ}{6} = 60^\circ$. Furthermore, the subject determined the area of triangle $AOB$. For that reason, the subject drew the triangle $AOB$ (See Figure 2c) and affixed the letter $a$ on its sides. Then the subject argued that because the regular hexagon is made up of six equilateral triangles, the area of the regular hexagon is equal to six times the area of one of its constituent triangles. Using this concept, the subject then showed that the regular hexagonal area is $= \frac{3}{2}a^2\sqrt{3}$. At the end of the proof, the subject briefly confirmed the results obtained.

In the second question, the subject began the proof by writing the question then asked the students to read it together. Furthermore, together with the students, the subject identified the known elements and the elements to be proven based on the question. From the results of the identification it was found that, the known element is “$n$ is an integer and $n^2$ is even”, while the element to be proved is “$n$ is
even”. Then the subject explained the logic model of the implications contained in the problem. The subject explained that the implication model is \( p \rightarrow q \). Moreover, the subject argued that the form of implication \( p \rightarrow q \) is equivalent to its contraposition, \( \neg q \rightarrow \neg p \), which caused that the statement of implication can be proved by using contraposition. Based on this argument, the subject re-identified and re-explained the known elements and the elements to be proved based on its contraposition. According to the subject, the known element is “\( n \) is odd”, whereas the element to be proved is “\( n^2 \) is odd”. According to the subject, the odd number is an integer that is not divisible by 2, so that if \( n \) is odd then \( n = 2k + 1 \) where \( k \) is an integer, so \( n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4(k^2 + k) + 1 = 4l + 1 \) with \( l = k^2 + k \). The subject argued that since \( k \) is an integer then based on the closure property, \( l = k^2 + k \) is an integer as well. To bring \( 4l + 1 \) to an odd number form, the subject manipulated it to \( 2(2l) + 1 = 2m + 1 \) with \( m = 2l \). The subject argued that since \( l \) is an integer then \( m = 2l \) is also an integer. Furthermore, the subject argued that since \( 2m + 1 \) met the notation of the odd number pattern then \( n^2 = 2m + 1 \) is also a member of an odd number. At the end of the proof, the subject briefly explained: “it is proven that if \( n \) is odd, then \( n^2 \) is odd, so it is true that “if \( n^2 \) is even then \( n \) is even”.

3.3. Discussion

The results of this study showed that the subject always started her oral explanation by rewriting the problem on the board. This is a good thing in communication. By rewriting the questions, the teacher ensures that the student knows clearly what he or she will learn. In addition, in this way, students have the opportunity to mobilize the various prerequisite knowledge needed to understand what the teacher will explain. Subsequently, the subject explained what she understands from a matter that includes known elements and elements to be proved. This step is very important in the process of proof. Polya asserts that in the early stages of proof we must identify the main points in the matter, such as the hypothesis (the known element), and the conclusion (the element to be shown) [7]. The hypothesis will be the assumption underlying the subsequent reasoning process to show that conclusions are acceptable. In this study, the subject could explain the main elements correctly. During the process of explanation, the subject also always put forward ideas to justify the reasoning process that she did. This is consistent with what the NCTM emphasizes that the subject should be able to explain the logical and systematic reasons for each reasoning process chosen [2]. In this study, using such logical reasoning, the subject could present the steps of completion systematically. In the process of completion, the subject often manipulate ideas and present them in other more easily understood forms. This is also one of the communication skills that needs to be developed. By re-envisioning ideas in a more suitable form, solving the problem would be much easier. At the end of her explanation, the subject always reaffirmed the results already obtained to confirm that the conclusion is proved. This also fits the rules of writing proof [7].

In addition to the results described above, it also appears that there is a difference in the communication strategy the subject chooses in explaining both types of questions given. For geometry problems, the subject tend to communicate her ideas by using images as symbolic representations of the concepts to be explained. This is in line with other findings which showed that in the geometric proof the subject tended to present the idea in the form of images [12,14]. The images are needed to form a mental model of the problem to be solved. The images make it easier for the subject to explain important elements and to find mathematical relationships that enable the subject to sequentially and systematically summarize the argument. Therefore, the completion steps made by the subject always refers to the image that has been made. As for communicating algebraic problems, subject rely on their mastery of various knowledge of mathematical logic. This is in line with previous findings which indicate that mastery of content knowledge is very important in solving mathematical problems [15]. In both given algebraic questions, the subject uses her knowledge of implication and contraposition. The subject knows that the contraposition of a statement is equivalent to its implications. Based on that knowledge, the subject presents the next steps of settlement by referring to the contraposition of the given statement.
4. Conclusion
Based on the results of the research described above, it can be concluded that the subject's mathematical communication in proving begins by writing the problem, explaining the main elements in the matter, constructing the argument and presenting the settlement steps, explaining ideas in a more understandable form, and ending by reaffirming the results obtained. Subject can communicate her authentication ideas correctly, systematically and logically. In addition, the subject also uses various communication strategies, such as by using images and other mathematical representations. The strategy chosen depends very much on the type of problem given.

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