CORRECTIONS OF $\mathcal{O}(\alpha_s^2)$ TO THE DECAY OF AN INTERMEDIATE-MASS HIGGS BOSON INTO TWO PHOTONS

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ABSTRACT

The QCD correction of $\mathcal{O}(\alpha_s^2)$ to the decay of the Standard Model Higgs boson into two photons is presented. We consider the contribution coming from diagrams with a heavy top quark as virtual particle. The first three terms of the expansion in the inverse top mass is calculated. Expressing the result through the on-shell top mass $M_t$, we find large coefficients for the power-suppressed terms whereas in the $\overline{\text{MS}}$ scheme the coefficients are tiny.

The Higgs boson is the only undetected particle in the Standard Model (SM). Now that top quark is well established the discovery of the Higgs boson is one of the most important aims in high-energy physics. Current experiments at LEP ruled out a Higgs boson with mass $M_H \leq 65.6$ GeV via Bjorken’s process $e^+e^- \rightarrow Z \rightarrow f\bar{f}H$. The next generation of colliders will improve this limit. Despite the fact that for an intermediate-mass Higgs boson with $M_W \leq M_H \leq 2M_W$ the decay into a $b$ quark pair will be the dominant process, a detection in this channel is unlikely with the LHC because of the large background. A very promising channel for the discovery especially for $M_H < 130$ GeV is the decay into two photons, though the branching ratio is of $\mathcal{O}(10^{-3})$. $\Gamma(H \rightarrow \gamma\gamma)$ also enters the cross section for Higgs-boson production through photon-photon fusion, which is the dominant process in future high energy $e^+e^-$ colliders. Therefore it is important to calculate higher order corrections to this process. In this note we consider the $\mathcal{O}(\alpha_s^2)$ corrections to $\Gamma(H \rightarrow \gamma\gamma)$ coming from a top quark which is heavy compared to the Higgs boson. Besides the leading term which is independent of the top-quark mass, we also calculate the first two power-suppressed corrections. Throughout this paper we work in the framework of dimensional regularisation with $D = 4 - 2\epsilon$. Large intermediate terms are treated with the help of FORM.

The coupling $H\gamma\gamma$ does not exist at Born level, so the decay of the Higgs boson into photons is a loop-induced process. At the one-loop level either fermions or $W$ bosons may be present in the loop, and the decay rate reads

$$\Gamma(H \rightarrow \gamma\gamma) = \left| \sum_f A_f(\tau_f) + A_W(\tau_W) \right|^2 \frac{M_H^3}{64\pi}, \quad (1)$$

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with \( \tau_f = M^2_f/4m^2_f \) and \( \tau_W = M^2_H/4M^2_H \). In the one-loop order \( A_W \) is dominating the correction. The contribution from light fermions is suppressed by their mass. We will focus our attention to \( A_t(\bar{\tau}_t) \) and consider QCD corrections of \( \mathcal{O}(\alpha_s^2) \) to this amplitude. It is convenient to write

\[
A_t(\bar{\tau}_t) = A_t^{(0)} + \frac{\alpha_s}{\pi} A_t^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 A_t^{(2)} + \ldots ,
\]

where \( \alpha_s \equiv \alpha_s^{(6)}(\mu) \) throughout this paper.

There is no decoupling in the limit \( M_t \to 0 \) because the \( H\bar{t}t \) coupling is proportional to the top quark mass. The exact one-loop result is well-known and reads for \( \tau_t \leq 1 \)

\[
\begin{align*}
A_t^{(0)} & = \hat{A}_t \left[ \frac{3}{2\tau_t} \left( 1 + \left( 1 - \frac{1}{\tau_t} \right) \arcsin^2 \sqrt{\tau_t} \right) \right] \\
& = \hat{A}_t \left( 1 + \frac{7}{30} \tau_t + \frac{26}{21} \tau_t^2 + \frac{512}{17325} \tau_t^3 + \frac{1216}{63063} \tau_t^4 + \frac{1216}{63063} \tau_t^5 + \mathcal{O}(\tau_t^6) \right)
\end{align*}
\]

with \( \hat{A}_t = N_C \frac{\alpha_s}{\pi} Q_t^2 \) and \( v = 2^{-1/4} G_F^{1/2} \). The second line of Eq. (3) contains the expansion for large \( M_t \).

At the two-loop level also the QCD corrections are at hand. Whereas for arbitrary values of \( M_H \) and \( M_t \) only a numerical result is available in the limit of a heavy top quark also analytical results exist. We repeated this calculation and even extended it to get the first six terms in the expansion for a heavy quark. The result is given by

\[
A_t^{(1)} = \hat{A}_t \left( -1 + \frac{122}{135} \tau_t + \frac{8864}{14175} \tau_t^2 + \frac{209186}{496125} \tau_t^3 + \frac{696616}{2338875} \tau_t^4 + \frac{54072928796}{245827456875} \tau_t^5 + \mathcal{O}(\tau_t^6) \right).
\]

In order to get an impression of the quality of the expansion for large \( M_t \) we plot in Fig. 1 the one- and two-loop results for \( A_t \) against \( \tau_t \). The dotted lines are the result for infinitely large top mass. In the other curves successively higher orders of \( \tau_t \) are included. The two vertical lines indicate \( M_H = M_W \) and \( M_H = 2M_W \), respectively. Also the exact results are shown (full line). At two loops the result in the on-shell scheme (dashed line) differs from the one in the \( \overline{\text{MS}} \) scheme (dash-dotted line, \( \mu^2 = \hat{m}_t^2 \); here the variable of the abscissa is \( \bar{\tau}_t(\mu) = M^2_H/4\hat{m}_t^2(\mu) \)). This numerical difference is compensated by an appropriate change of the one-loop result. It should be noted that in this context the transition to the \( \overline{\text{MS}} \) scheme is done by replacing the on-shell top mass \( M_t \) through the running mass \( \hat{m}_t(\mu) \).

For \( M_H = \sqrt{2} M_t \approx 250 \) GeV (i.e. \( \tau_t = 0.5 \)) the curves including the \( \tau_t^5 \) term are practically indistinguishable from the exact result. But already the curves which only contain the \( \tau_t^2 \) term deliver a reasonable result and for \( M_H \approx 2M_W \) this correction is a very good approximation. For \( M_H \approx M_W \) the inclusion of the linear term only is enough to describe the exact result very well. We take these considerations as a motivation to calculate in the three-loop case the terms proportional to \( \tau_t^2 \).
Figure 1: One- and two-loop corrections to the amplitude $A_t$. Successively higher orders in $\tau_t$ are included. Dotted line: leading order result, dashed line: on-shell scheme; dash-dotted line: MS scheme, full line: exact result. The two vertical lines indicate $M_H = M_W$ and $M_H = 2M_W$, respectively.

There are different methods which allow one to calculate of $\Gamma(H \rightarrow \gamma\gamma)$ in the limit that a heavy virtual quark is present in the loop and the Higgs boson is in the intermediate mass range $\Higgs$.

A very elegant method to get the leading order (i.e. the limit $M_t \rightarrow \infty$) result is provided by the use of a low-energy theorem (LET) for the Higgs boson $\Higgs$. The basic idea is that the coupling of the Higgs boson to other particles is proportional to the mass of these particles. Both for fermions and gauge bosons the substitution $m_i \rightarrow m_i(1 + H/v)$ generates the coupling of the particle with mass $m_i$ to the Higgs boson. For a Higgs boson carrying zero four-momentum this is obtained by differentiating the respective two-point function with respect to the virtual-particle masses. To be more precise, in our case we...
can write down the effective Lagrangian
\[ \mathcal{L}_{\gamma\gamma} = -\frac{1}{4} F_{\mu\nu}^{\gamma\gamma} F_{\mu\nu}^{\gamma\gamma} \left( 1 + \tilde{\Pi}_{\gamma\gamma}^{0,t}(0) \right) \] (5)

where \( F_{\mu\nu}^{\gamma\gamma} \) is the field strength tensor and \( \tilde{\Pi}_{\gamma\gamma}^{0,t}(0) = \Pi_{\gamma\gamma}(q^2)/q^2 |_{q=0} \). \( \Pi_{\gamma\gamma}(q^2) \) is (the transversal part of) the photon two-point function containing the top quark as virtual particle. The superscript zero indicates that we are still dealing with bare quantities. Applying the LET we get an effective Lagrangian for the \( H\gamma\gamma \) vertex
\[ \mathcal{L}_{H\gamma\gamma} = -\frac{1}{4} F_{\mu\nu}^{\gamma\gamma} F_{\mu\nu}^{\gamma\gamma} m_t^0 \frac{\partial}{\partial m_t^0} \tilde{\Pi}_{\gamma\gamma}^{0,t}(0) \frac{H}{v}. \] (6)

It is clear that after the differentiation the renormalization of the coupling constant \( \alpha_s^0 \) and of \( m_t^0 \) has to be performed.

The method of the LET has been used very extensively in the past. Besides effective \( H\gamma\gamma \) and \( Hgg \) couplings also the \( H\bar{b}b, HWW \) and \( HZZ \) couplings were considered at the two-\( [4,9] \) and even three-loop levels \( [11,12] \). In a recent work the theoretical background of the LET was considered \( [13] \). The big advantage of the LET is that much less diagrams have to be taken into account and the three-point amplitudes are simply generated by differentiating w.r.t. the top-quark mass.

The result for \( \tilde{\Pi}_{\gamma\gamma}^{0,t}(0) \) is given by \( [14] \)
\[ \tilde{\Pi}_{\gamma\gamma}^{0,t}(0) = N_C Q_t^2 \frac{\alpha_s}{4\pi} \left\{ \frac{4}{3} + \frac{4}{3} L_0 + \frac{\alpha_s}{\pi} C_F \left( \frac{13}{12} - \frac{3}{2\epsilon} - 3L_0 \right) \right. \\
+ \left. \left( \frac{\alpha_s}{\pi} \right)^2 \left[ C_F \left( -\frac{7}{16} + \frac{7}{32} \zeta(3) \right) \\
+ C_A C_F \left( -\frac{4243}{1296} - \frac{11}{18\epsilon^2} - \frac{11}{6\epsilon} L_0 - \frac{5}{108\epsilon} + \frac{223}{96} \zeta(3) - \frac{11}{12} - \frac{5}{36} L_0 - \frac{11}{4} L_0^2 \right) \\
+ C_F^2 \left( -\frac{137}{72} + \frac{17}{6\epsilon} - \frac{95}{48} \zeta(3) + \frac{17}{2} L_0 \right) \right] \\
+ C_F T n_f \left( \frac{169}{162} + \frac{2}{9\epsilon^2} + \frac{2}{3\epsilon} L_0 - \frac{5}{27\epsilon} + \frac{1}{3} \zeta(2) - \frac{5}{9} L_0 + L_0^2 \right) \right\} \] (7)

where \( N_C = 3, C_F = 4/3, C_A = 3, L_0 = \ln(\mu^2/(M_t^0)^2), T = 1/2 \) and \( n_f = 6 \) is the total number of quarks. If this expression is inserted into Eq. (6) and the top-mass and coupling-constant renormalization is performed we receive the leading-order term for the amplitude of the Higgs-boson decay into a pair of photons
\[ A_t^{lo} = A_t \left\{ 1 - \frac{\alpha_s}{\pi} C_F \frac{3}{4} \\
+ \left( \frac{\alpha_s}{\pi} \right)^2 \left[ C_F C_A \left( -\frac{7}{12} - \frac{11}{16} \right) + C_F^2 \frac{27}{32} + C_F T n_f \left( -\frac{1}{12} + \frac{1}{4} \right) - C_F T \frac{3}{16} \right] \right\} \] (8)

with \( L = \ln(\mu^2/M_t^2) \). In this paper the diagrams with the Higgs boson and the photons attached to different fermion lines are not considered. They will be treated elsewhere \( [13] \).
For this reason in order to compare the result from the LET with the explicit calculation, the contributions from the double-bubble diagram coming from the differentiation w.r.t. the inner top mass have to be subtracted. This is already done in (8).

A second method to compute the leading-order result is provided by the use of the so-called Fock-Schwinger gauge for the external photons. In coordinate space this reads as $x^\mu A_\mu(x) = 0$. It can be shown [14] that from this condition the expansion of $A_\mu(x)$ in terms of local operators has the form

$$A_\mu(x) = \frac{1}{2} \cdot 0! x^\rho F_{\rho\mu}(0) + \frac{1}{3} \cdot 1! x^\sigma x^\rho (D_\sigma F_{\rho\mu}(0)) + \ldots .$$ (9)

In the present case we are interested in attaching two photons to the diagrams pictured in Fig. 2. If only the leading order is considered it is enough to keep the first term in Eq. (9). Then the prescription derived in [14] is as follows: (i) Route the momentum associated with the photon through the diagram to the Higgs boson; (ii) differentiate the affected propagators w.r.t. this momentum; (iii) set the momentum equal to zero and (iv) proceed in the same way with the second photon. At the three-loop level there are eight tadpole diagrams which have to be considered. A small computer program written in FORM [3] applies the above algorithm and attaches two photons. Afterwards the integration is performed. With this method the same result as in Eq. (8) is obtained.

There is also a method which leads directly to the decay rate. Thereby one has to consider $\Pi^H(q^2)$, the two point function for the Higgs boson. The imaginary part of $\Pi^H(q^2)$ coming from a two photon cut has to be extracted. In order to get the width the

Figure 2: Tadpole diagrams relevant for the method based on the Fock-Schwinger gauge. The diagram with a second light quark loop and the one containing the ghost particle are not displayed.
The most obvious method is the expansion of the triangle diagrams in their externalmomenta $q_1$ and $q_2$. One thus successively obtains higher-order terms in $M_H^2/M_t^2$ depending on how far the expansion is performed. The amplitude for the decay of the Higgs boson into two photons with polarization vectors $\epsilon_{\mu}(q_1)$ and $\epsilon_{\nu}(q_2)$ has the following Lorentz structure:

$$A_{\mu\nu} = \sum_i A_{\mu\nu}^i = \sum_i (a_{t,i} q_1 q_2 \, g_{\mu\nu} + b_{t,i} q_1^\nu q_2^\mu + c_{t,i} q_1^\mu q_2^\nu),$$

where $c_{t,i}$ has no contribution for on-shell photons. In Eq. (11) the sum runs over all diagrams relevant for the decay $H \to \gamma\gamma$. Due to gauge invariance, we have $\sum_i a_{t,i} = -\sum_i b_{t,i}$. It is easy to find projectors for $a_{t,i}$ and $b_{t,i}$:

$$a_{t,i} = \frac{A_{\mu\nu}^{t,i}}{D-2}(q_1 q_2 \, g_{\mu\nu} - q_1 \, q_2 \, q_{1\mu} q_{2\nu} - q_1 q_2 q_{2\mu} q_{1\nu}),$$

$$b_{t,i} = \frac{A_{\mu\nu}^{t,i}}{D-2}(q_1 q_2 \, g_{\mu\nu} + q_1 \, q_2 q_{1\mu} q_{2\nu} + (D-1) q_{1\mu} q_{2\nu}).$$

In fact, we calculated both, $a_{t,i}$ and $b_{t,i}$, in order to have an additional check for the correctness of our result. A $(2D$ dimensional) Taylor expansion of the triangle diagrams in the two external momenta $q_1$ and $q_2$ leads to the following structure for $a_{t,i}(q_1, q_2)$:

$$a_{t,i}(q_1, q_2) = \sum_{l,m,n=0}^{\infty} c_{i lm}^{i mn}(q_1^2)^l(q_2^2)^m(q_1 q_2)^n.$$  

(14)

After projecting out the $c_{00n}$, we end up with massive three-loop diagrams with zero external momentum. For these integrals the technique is meanwhile well established. After the decomposition of the numerator in terms of the denominator, the scalar integrals are reduced via recurrence relations to trivial ones or so-called master integrals, for which a hard three-loop calculation is necessary.

At $\mathcal{O}(\alpha_s^2)$, altogether $2 \times 79$ diagrams (the “2” comes from the two possible orientations of the fermion line) have to be taken into account. In Fig. 3, three exemplary graphs are pictured. Setting $C_A = 3, C_F = 4/3$ and $n_f = 6$, the result reads ($A_t = \sum_i a_{t,i}$)
Figure 3: Typical Feynman diagrams contributing to $H \to \gamma \gamma$.

$$\begin{align*}
&+ \left( \frac{\alpha_s}{\pi} \right)^2 \left( -\frac{31}{24} - \frac{7}{4} \ln \frac{\mu^2}{M_t^2} \right) \\
&+ \left( -\frac{19531913}{622080} + \frac{7}{45} \zeta(2) + \frac{14}{45} \zeta(2) \ln(2) + \frac{821063}{27648} \zeta(3) + \frac{427270}{270} \ln \frac{\mu^2}{M_t^2} \right) \tau_t \\
&+ \left( -\frac{56709666623}{2612736000} + \frac{8}{63} \zeta(2) + \frac{16}{63} \zeta(2) \ln(2) + \frac{72438107}{3317760} \zeta(3) \\
&+ \frac{2216}{2025} \ln \frac{\mu^2}{M_t^2} \right) \tau_t^2 \right) + \ldots 
\end{align*}
$$

$$\begin{align*}
= \bar{A_t} \left[ 1 + 0.2333 \tau_t + 0.09524 \tau_t^2 + 0.04952 \tau_t^3 + 0.02955 \tau_t^4 + 0.01928 \tau_t^5 \\
+ \frac{\alpha_s}{\pi} \left( -1 + 0.9037 \tau_t + 0.6253 \tau_t^2 + 0.4216 \tau_t^3 + 0.2978 \tau_t^4 + 0.2200 \tau_t^5 \right) \\
+ \left( \frac{\alpha_s}{\pi} \right)^2 \left( -1.292 - 1.75 \ln \frac{\mu^2}{M_t^2} + \left( 4.91035 + 1.58148 \ln \frac{\mu^2}{M_t^2} \right) \tau_t \\
+ \left( 5.038 + 1.094 \ln \frac{\mu^2}{M_t^2} \right) \tau_t^2 \right) + \ldots \right],
\end{align*}
$$

where the dots represent higher orders in $\tau_t$ and $\alpha_s$. The leading-order result of this method coincides with Eq. (8). To the knowledge of the author, this is the first time that the LET was tested at three-loop level. In Eq. (15), the on-shell definition of the pole mass was used. Expressing the result in terms of the $\overline{\text{MS}}$ mass $\bar{m}_t(\mu)$, the result looks like:

$$\begin{align*}
\bar{A_t} &= \bar{A_t} \left[ 1 + \frac{7}{30} \bar{\tau}_t + \frac{2}{21} \bar{\tau}_t^2 + \frac{26}{525} \bar{\tau}_t^3 + \frac{512}{17325} \bar{\tau}_t^4 + \frac{1216}{63063} \bar{\tau}_t^5 \\
+ \frac{\alpha_s}{\pi} \left( -1 + \left( \frac{38}{135} - \frac{7}{15} \bar{l} \right) \bar{\tau}_t + \left( \frac{1664}{14175} - \frac{8}{21} \bar{l} \right) \bar{\tau}_t^2 + \left( \frac{12626}{496125} - \frac{52}{175} \bar{l} \right) \bar{\tau}_t^3 \\
+ \left( \frac{40664}{2338875} - \frac{4096}{17325} \bar{l} \right) \bar{\tau}_t^4 + \left( -\frac{9128671204}{245827456875} - \frac{12160}{63063} \bar{l} \right) \bar{\tau}_t^5 \right) \\
+ \left( \frac{\alpha_s}{\pi} \right)^2 \left( -\frac{31}{24} - \frac{7}{4} \bar{l} + \left( -\frac{22326329}{622080} + \frac{4116067}{138240} \zeta(3) - \frac{769}{1080} \bar{l} + \frac{7}{120} \bar{l}^2 \right) \bar{\tau}_t \\
+ \left( -\frac{68094821183}{2612736000} + \frac{508541309}{2324320} \zeta(3) - \frac{1241}{1575} \bar{l} + \frac{3}{\bar{\tau}_t} \bar{l}^2 \right) \bar{\tau}_t^2 \right) + \ldots \right].
\end{align*}$$
\[
\begin{align*}
A_t &= 1 + 0.2333\tau_t + 0.09524\tau_t^2 + 0.04952\tau_t^3 + 0.02955\tau_t^4 + 0.01928\tau_t^5 \\
&\quad + \frac{\alpha_s}{\pi} \left( -1 + (0.2815 - 0.4667\bar{l})\tau_t + (0.1174 - 0.3810\bar{l})\tau_t^2 + (0.02545 - 0.2971\bar{l})\tau_t^3 \\
&\quad \quad + (-0.01739 - 0.2364\bar{l})\tau_t^4 + (-0.03713 - 0.1928\bar{l})\tau_t^5 \right) \\
&\quad + \left( \frac{\alpha_s}{\pi} \right)^2 \left( -1.292 - 1.75\bar{l} + (-0.09881 - 0.7120\bar{l} + 0.05833\bar{l}^2)\tau_t \\
&\quad \quad + (0.2587 - 0.7879\bar{l} + 0.4286\bar{l}^2)\tau_t^2 \right) + \ldots 
\end{align*}
\]

(16)

with \(\bar{l} = \ln \mu^2/\bar{m}_t^2(\mu)\). The leading order terms coincide in both schemes. This is the consequence of the fact that in the limit of infinitely heavy top-quark mass \(A_t\) does not depend on \(M_t\) at all. In Fig. 4, the results of \(\mathcal{O}(\alpha_s^2)\) are plotted. The curves show a similar behaviour as in the one- and two-loop cases.

Figure 4: \(\mathcal{O}(\alpha_s^2)\) corrections to \(H \rightarrow \gamma\gamma\). The same notation is adopted as in Fig. 1.

From Eqs. (15) and (16) the same observation can be made as for earlier calculations up to \(\mathcal{O}(\alpha_s^2)\), namely the coefficients of \(\alpha_s/\pi\) are much smaller in the \(\overline{\text{MS}}\) scheme than in the on-shell scheme. Consider e.g., \(\mu^2 = M_t^2\) and \(\mu^2 = \bar{m}_t^2\) in Eqs. (13) and (16). Then, the ratio of the coefficients in front of \((\alpha_s/\pi)^2\tau_t\) and \((\alpha_s/\pi)^2\tau_t^2\) is about 50 and 20, respectively. Another feature concerning the \(\overline{\text{MS}}\) scheme is already visible in the two-loop result of Fig. 1. There the expansion in \(1/M_t\) seems to converge more rapidly in the \(\overline{\text{MS}}\) than in the on-shell scheme. Actually, the curves including corrections of order \(\tau_t^3\), \(\tau_t^4\) or \(\tau_t^5\) are practically the same as the one where only the \(\mathcal{O}(\tau_t^2)\) corrections are present. The dash-dotted curves in Fig. 4 indicate a similar behaviour.

To conclude, the \(\mathcal{O}(\alpha_s^2)\) QCD corrections to the decay process \(H \rightarrow \gamma\gamma\) for an intermediate Higgs boson were calculated. We considered the contribution from a heavy top quark and evaluated the first three terms in the expansion in the inverse top mass. Inspired from the one- and two-loop case this should be at least up to \(M_H \approx 2M_W\) an
excellent approximation to the exact result. The leading term was confirmed using a low energy theorem and a method based on the Fock-Schwinger gauge.

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