Electron-Phonon Interactions for Optical Phonon Modes in Few-Layer Graphene

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We present a first-principles study of the electron-phonon (e-ph) interactions and their contributions to the linewidths for the optical phonon modes at $\Gamma$ and $K$ in one to three-layer graphene. It is found that due to the interlayer coupling and the stacking geometry, the high-frequency optical phonon modes in few-layer graphene couple with different valence and conduction bands, giving rise to different e-ph interaction strengths for these modes. Some of the multilayer optical modes derived from the $\Gamma$-$E_{2g}$ mode of monolayer graphene exhibit slightly higher frequencies and much reduced linewidths. In addition, the linewidths of $K$-$A_1'$ related modes in multilayers depend on the stacking pattern and decrease with increasing layer numbers.

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The possibilities of developing carbon-based nanostructures for electronics applications have stimulated recent interest in graphene and its derivatives. One of the focus areas is to understand the scattering processes of electrons. In carbon nanotubes and graphite, the high currents or optical excitations have been shown to induce a significant overpopulation of the optical phonon modes of $E_{2g}$ at $\Gamma$ ($\Gamma$-$E_{2g}$) and $A_1'$ at $K$ ($K$-$A_1'$). Since these phonon modes exhibit a strong electron-phonon (e-ph) interactions, overpopulation of them leads to a dramatic reduction of the ballisitc conductance of carbon nanotubes at high bias potentials, and consequently interconnect performance deteriorates.

Understanding the phonon decays from a microscopic point of view, in particular, based on the e-ph interaction, is thus a key step to improve the transport properties of these materials and to control device performance. Furthermore, the e-ph interaction also plays a significant role in many phenomena such as the quasiparticle dynamics and anomalies in photoemission spectra, Raman scattering, and superconductivity.

Experimentally, the linewidths of the zone-center phonon modes obtained from Raman or infrared (IR) measurements contain significant contributions from the e-ph interaction. In graphene, the phonon linewidth of the $\Gamma$-$E_{2g}$ mode is estimated to be about $13$ cm$^{-1}$ based on the Raman spectra. In graphite, the graphene phonon $E_{2g}$ mode splits into two branches: the Raman-active $E_{2g}$ and IR-active $E_{1u}$ modes. The linewidth of the Raman-active mode ($11.5$ cm$^{-1}$) is almost the same as that of graphene, while IR measurements show that the linewidth of the $E_{1u}$ mode is surprisingly much smaller.

Few-layer graphene (FLG) presents an interesting system because of the possibility to tune its electronic properties. In the epitaxially grown graphene, FLG is often produced as a main product. Depending on the layer number and stacking geometry, the linear band dispersions in monolayer graphene evolve into several bands due to the interlayer coupling in FLG. Similarly, the $E_{2g}$ mode at $\Gamma$ and $A_1'$ mode at $K$ in single layer also split into several branches. It is expected that the e-ph interaction will be significantly modified as the number of layers and stacking geometry are varied.

In this work, we performed first-principles calculations of the e-ph interactions in one-, two- (AB stacking), and three-layer (ABA and ABC stackings) graphene. The $\Gamma$-$E_{2g}$ and $K$-$A_1'$ modes in graphene, which are respectively coupled to intra- and inter-valley electronic scatterings, have much stronger e-ph interactions than other modes. Here we focus on these two modes and their derivatives in FLGs in order to investigate the stacking effect. We found that while the weak interlayer interaction gives rise to only small splittings of these phonon modes, the resulting e-ph interactions for some of them are considerably suppressed because of symmetry constraints.

The e-ph matrix element $g_{\nu(k+q)j',kj}$ is defined as

$$g_{\nu(k+q)j',kj} = \sqrt{\frac{\hbar}{2M\omega_q^\nu}} |\mathbf{k} + \mathbf{q}, j'\rangle \langle \mathbf{k}, j| \frac{\delta V_{\text{scf}}}{\delta u_q^\nu} |\mathbf{k}, j\rangle,$$

where $\delta V_{\text{scf}} \equiv V_{\text{scf}}(u_q^\nu) - V_{\text{scf}}(0)$ is the variation of the self-consistent potential field due to the perturbation of a phonon with wave vector $\mathbf{q}$ and branch index $\nu$. $|\mathbf{k}, j\rangle$ is the electronic Bloch state. We further define the e-ph coupling strength between phonon mode $\nu\mathbf{q}$ and Bloch states $|\mathbf{k} + \mathbf{q}, j'\rangle$ and $|\mathbf{k}, j\rangle$ as $g^\nu_{j',j}(\mathbf{k}) = |g_{\nu(k+q)j',kj}|^2$.

The electronic states and the self-consistent field were computed using the first-principles codes Quantum ESPRESSO and the perturbation of a phonon mode was handled with the frozen-phonon approach.
FIG. 2: (Color online) (a)-(d) Optical modes at Γ, (e)-(h) band dispersions near the Fermi level, and (i)-(l) the square of the e-ph interaction strength $|g|^2$ of the optical modes at Γ for monolayer, AB bilayer, ABA and ABC trilayer graphene, respectively. Symmetry-allowed transitions from valence bands to conduction bands with nonzero strength are shown.

Fig. 1(a) shows the supercell used in the frozen-phonon calculation for the K phonons. The electronic structure was calculated with the local density approximation (LDA) within density-functional theory, and the core-valence interaction was modeled by norm-conserving pseudopotentials. Wave functions of the valence electrons were expanded in plane waves with a kinetic energy cutoff of 70 Ry. The phonon frequencies and associated eigenvectors were computed using the density-functional perturbation theory (DFPT)\textsuperscript{28}, details of which have been presented in our previous work\textsuperscript{28}. A vacuum region of 10 Å was introduced in our supercell to eliminate the artificial interaction between neighboring supercells along the z direction. The relaxed C-C bond length is 1.42 Å and the interlayer distance is 3.32 Å for all FLGs considered in this paper. Variations of the potential fields $\delta V_{\text{scf}}$ were calculated through self-consistent calculations to find the potential field for both perturbed and unperturbed systems. The following calculations were carried out on a dense 100×100 $k$-grid within a small square area enclosing point K in reciprocal space, as indicated in Fig. 1(b) in order to obtain the electronic wave functions at $k$ and $k+q$ near the Fermi level. This dense $k$-sampling was found necessary for a quantitative description of the scattering process near the Fermi level in one- and few-layer graphene. Finally, the e-ph interaction matrix elements were computed using Eq. (1).

In order to check the accuracy of our calculations, we first calculate the e-ph matrix elements over the Fermi surface and compare them with previously published results for monolayer graphene. Due to the electronic degeneracy at K, the averaged e-ph matrix elements for all possible pairs are $\langle g_{\nu}^{2} \rangle_F = \sum_{i,j} |g_{i,j}(K_{i},K_{j})|^2/4=0.0401$ eV$^2$ for the Γ-$E_{2g}$ mode, and $\langle g_{\nu}^{2} \rangle_F = \sum_{i,j} |g_{i,j}(2K_{i},K_{j})|^2/4=0.0986$ eV$^2$ for the K-$A_{1}'$ mode, respectively. These results are in excellent agreement with those in previous DFPT calculations (0.0405 and 0.0994 eV$^2$, respectively)\textsuperscript{28}.

The phonon linewidth $\gamma$ due to the e-ph coupling is defined as\textsuperscript{29}

$$\gamma_{\nu}^{q} = \frac{4\pi}{N_{k}} \sum_{k,j' \neq k} |g_{k,q+j',j}^{\nu}|^2 |f_{k,j} - f_{k+q,j'}| \times \delta\left[\varepsilon_{k,j} - \varepsilon_{(k+q),j'} + \hbar\omega_{q}^{g}\right],$$

with $f_{k,j}$ being the Fermi-Dirac occupation function for Bloch state $|k,j\rangle$. We used a broadening parameter of 0.01 eV for the $\delta$-function in Eq. (2), and $k_{B}T=2.5$ meV for the Fermi-Dirac distribution.
Table lists the calculated phonon linewidths due to the e-ph coupling. For the monolayer, the linewidth of the degenerate \( \Gamma-E_{2g} \) modes is 11.2 cm\(^{-1}\), in reasonable agreement with experimental observation of 15.0 cm\(^{-1}\) and previously published result of 11.5 cm\(^{-1}\). A value of 20.4 cm\(^{-1}\) is obtained for the highest optical K-\( A_1’ \) mode. In FLGs, the phonon modes split into branches with different symmetries. Consequently, their linewidths can be quite different from one another. In the AB bilayer, for example, the original highest \( \Gamma \) phonon splits into the Raman active \( E_g \) and IR-active \( E_u \) modes. While the linewidth of the \( E_g \) mode is two orders of magnitude smaller. The calculated linewidth of the \( E_g \) mode is consistent with recent experimental observation of 13.5 cm\(^{-1}\) for the \( \Gamma \) optical phonon mode in bilayer graphene. Similar results for the monolayer and bilayer graphene were also reported by Park et al. For the ABA and ABC trilayer, the results in Table show different behavior for different modes. In particular, for the ABC trilayer two out of three optical modes at \( \Gamma \) show reduced electron-phonon interaction. Most importantly, the linewidths of all K phonons in Table drop significantly as the layer number increases from 1 to 3. This indicates that the interlayer interaction can effectively suppress valley-spin decoherence via e-ph scatterings, making the FLGs more attractive as valleytronic materials.

For FLGs there are a few valence and conduction \( \pi \) bands near the Fermi energy level. The symmetry allowed interband transitions by the absorption of a phonon are indicated in the middle column of Fig. To examine the contributions of different electronic states to the e-ph coupling strengths in FLGs, we present in the right column of Fig. the absolute value squared of the e-ph coupling matrix elements for optical phonons at \( \Gamma \) as a function of electronic crystal momentum \( \mathbf{k} \) along the symmetry line \( \Gamma-K \) for all symmetry allowed transitions. Some of these matrix elements decrease monotonically with increasing \( k_z \), as in the case of monolayer; some of them show a minimum at \( K \). These features are closely related to the symmetry of the electronic states near \( K \) in the presence of interlayer interactions, and can be further quantitatively understood using a tight-binding model. Following the procedures in Refs. and , we obtain the e-ph matrix element for the in-plane optical phonon mode \( \mathbf{q} \nu \) in \( L \)-layer graphene as:

\[
\begin{align*}
g’_{(k+\mathbf{q})\lambda’,j} & = g_0 \sum_{l=1}^{L} \{ \tilde{t}(\mathbf{k}) \cdot \tilde{c}_{\alpha}(\mathbf{q}) - \tilde{t}(\mathbf{k} + \mathbf{q}) \cdot \tilde{c}_{\beta}(\mathbf{q}) \} u_{\alpha,j}(k + q) u_{\beta,j}(k) + \sum_{l=1}^{L} \{ \tilde{t}(-k - \mathbf{q}) \cdot \tilde{c}_{\alpha}^{\dagger}(\mathbf{q}) - \tilde{t}(-k) \cdot \tilde{c}_{\beta}^{\dagger}(\mathbf{q}) \} u_{\alpha,j}(k + q) u_{\beta,j}(k) \\
& = g_0 U^\dagger_{j}(k + q) \Phi’(k, q) U_{j}(k),
\end{align*}
\]

with \( U^\dagger_{j}(k) = [u_{1\alpha,j}(k), u_{1\beta,j}(k), u_{2\alpha,j}(k), u_{2\beta,j}(k), ...] \) being the tight-binding amplitudes of band \( j \) for each site in the unit cell. \( \tilde{c}_{\alpha} \) and \( \tilde{c}_{\beta} \) are the vibrational eigenvectors for the two atoms in layer \( l \). \( \tilde{t}(\mathbf{k}) = \sum_{i=1}^{3} \delta_i e^{i\mathbf{k} \cdot \mathbf{R}_i} \), where \( \delta_i \)’s \( (i=1-3) \) are the unit vectors connecting atom \( \alpha \) in layer \( l \) to its three nearest-neighbors (NNs), and \( \mathbf{R}_i \) are the lattice vectors of the unit cells in which the three NNs are located. The constant \( g_0 = J \Omega^{1/2} / (\omega_{\nu}^{e} \sqrt{M}) \) depends on the mode frequency \( \omega_{\nu}^{e} \), the e-ph interaction parameter \( J \), the area of the unit cell \( \Omega \), and the carbon atomic mass \( M \). The coupling matrix \( \Phi’(k, q) \) has the form:

\[
\Phi’(k, q) = \begin{pmatrix}
\Phi_1 \\
\Phi_2 \\
\vdots \\
\Phi_L
\end{pmatrix},
\]

with
TABLE I: Calculated phonon linewidth $\gamma$ (in cm$^{-1}$) for the high-frequency optical phonon modes at $\Gamma$ and K in monolayer, bilayer, and trilayer graphene. The mode symmetries $S$ and the frequencies $\omega$ (in cm$^{-1}$) are also listed for completeness.

|       | Monolayer |          | AB       |          | ABA      |          | ABC      |          |
|-------|-----------|----------|----------|----------|----------|----------|----------|----------|
|       | $S\omega\gamma$ | $S\omega\gamma$ | $S\omega\gamma$ | $S\omega\gamma$ | $S\omega\gamma$ | $S\omega\gamma$ | $S\omega\gamma$ | $S\omega\gamma$ |
| $\Gamma$ | $E_{2g}$  | 1586.0   | 11.2     | $E_g$    | 1587.0   | 8.6      | $E_u$    | 1592.0  | 0.1      | $E''_g$  | 1586.0   | 9.7      | $E_{g,a}$ | 1586.0   | 7.2      |
|        | $E_u$    | 1592.0   | 0.1      |          |          |          |          |          |          | $E''_u$  | 1588.0   | 11.0     | $E_u$    | 1589.0  | 0.0      |
|        | $E''_b$  | 1593.0   | 2.8      |          |          |          |          |          |          |          |          |          |          |          | 0.3      |
| $K$   | $A'_1$   | 1306.0   | 20.4     | $E$      | 1318.0   | 9.0      |          |          |          | $E'_1,E''_1$ | 1316.0   | 8.4      | $E$      | 1318.0  | 2.8      |
|         |          |          |          |          |        |          |          |          |          | $E'_2$   | 1324.0   | 3.6      | $A_1$    | 1325.0  | 2.2      |

\[ \tilde{\nu}(\mathbf{k}) \cdot \tilde{\nu}_{l\alpha}^{(\mathbf{q})} - \tilde{\nu}(\mathbf{k} + \mathbf{q}) \cdot \tilde{\nu}_{l\beta}^{(\mathbf{q})} \]

\[ 0 \]

FIG. 4: (Color online) Electronic momentum-resolved contributions of all scattering processes to the phonon linewidth for the doubly degenerate (a) $E_{2g}$ mode in monolayer graphene and (b) $E_g$ mode in bilayer graphene. Note that only $k_x, k_y \leq 0.009$ for monolayer and $-0.021 \leq k_x, k_y \leq 0.021$ for bilayer are shown. $k_x$ and $k_y$ are in $2\pi/a_0$.

It is clear that the e-ph interaction strength depends on the orbital characteristics of the initial and final electronic states as well as the displacement pattern of the phonon mode. Note that $\Phi_l$ is an off-diagonal matrix, coupling the electronic component of one sublattice to that of the other sublattice in each layer through phonon displacements in the same layer. The coupling effects in different layers will be summed up in either a constructive or a destructive way, depending on the relative phase of the phonon displacements in different layers and the relative phase of the tight-binding amplitudes in the initial and final electronic states involved.

As an example, Figure 3 schematically shows the symmetry-allowed transitions through the $E_g$ and $E_u$ modes at $\Gamma$ between valence and conduction bands in bilayer graphene. The four relevant energy bands shown in Fig. 2(f) are indexed as 2, 1, 1’, and 2’ from lower valence bands to higher conduction bands. For the $E_g$ mode, the in-plane vibrations of $\alpha$ and $\beta$ atoms in neighboring planes are in phase. Taking into account the symmetry of electronic wave function in each layer, allowed transitions turn out to be $1 \rightarrow 1'$ and $2 \rightarrow 2'$. As $k$ approaches $K$, the four bands become parabolic and the wave-function amplitudes tend to be localized on one sublattice of each layer. Since the transition matrix elements for $1 \rightarrow 1'$ and $2 \rightarrow 2'$ contain product of the electronic wave functions on both sublattices, small wave function amplitudes give rise to vanishingly small matrix elements near $K$, as shown in Fig. 2(j). On the other hand, the $E_u$ mode involves in-plane vibrations of $\alpha$ and $\beta$ atoms in neighboring planes that are out of phase. The symmetry allowed transitions are $2 \rightarrow 1'$ and $1 \rightarrow 2'$. Even though the wave function amplitude at one sublattice may become small when $k$ approaches $K$, the products with wave function amplitudes at the other sublattice are still finite. Therefore, the matrix elements exhibit similar linear behavior as in monolayer graphene.

In collecting the contributions from different $k$ states to calculate the phonon linewidth in Eq. (2), energy conservation is controlled by the delta function. For the $E_g$ mode, this condition can be satisfied in the transition $1 \rightarrow 1'$ but not in the transition $2 \rightarrow 2'$. Nevertheless, the phonon linewidth listed in Table I is about 3/4 of the value for monolayer graphene. Figure 4 presents the $k$-resolved linewidth contributions for the doubly degenerate $E_{2g}$ mode in monolayer graphene and the $E_g$ mode in bilayer graphene. Clearly, the contributions are almost isotropic in $k$ space for the $E_{2g}$ mode in monolayer graphene. The shell shape is a result of energy conservation for the scattering events. Due to the trigonal symmetry of the constant-energy surface in bilayer graphene, the $k$-dependent contributions show a threefold symmetry for the $E_g$ mode. In contrast, the symmetry allowed transitions of $2 \rightarrow 1'$ and $1 \rightarrow 2'$ for the $E_u$ mode cannot satisfy the required energy conservation because the energy separation of the initial and final states is too large. This leads to a vanishingly small probability for the $E_u$ mode to decay through the e-ph interaction. These results are a reminiscence of the properties of $\Gamma-E_{2g}$ and $\Gamma-E_{1u}$ phonons in bulk graphite regarding their e-ph interactions.
For the ABA trilayer, the four energy bands crossing or touching the Fermi energy are again indexed as 2, 1, 1′ and 2′ in Fig. 2. When a Γ-E_0 phonon is absorbed, both symmetry and energy allowed transitions are 2→2′ and 1→1′, which manifest themselves as the inner and outer rings respectively in Fig. 5(a). When a Γ-E_0 phonon is absorbed, the symmetry and energy allowed transitions are 2→1′ and 1→2′. Incidentally, the energy differences between bands 2 and 1′ and between bands 1 and 2′ are equal for any k. Therefore, the rings associated with these two different transitions overlap in Fig. 5(b). The electronic states involved span an energy range of 0.2 eV about the Fermi level. A Γ-E_0 phonon obeys the same selection rule as a Γ-E_0 phonon since both bear the same group representation. The e-ph coupling strength of the former is, however, much weaker than that of the latter.

For the ABC trilayer, the relevant energy bands are indexed as 1 and 1′ in Fig.2. The three Γ phonons derived from the E_2g mode in monolayer graphene are respectively denoted by Γ-E_{g,a}, Γ-E_{a}, and Γ-E_{g,b} in the order of increasing frequencies. Interestingly, only the Γ-E_{g,a} phonons show an appreciable e-ph coupling corresponding to the transition of 1 → 1′.

In summary, we have studied the phonon linewidths of the high-frequency optical phonon modes in few-layer graphene. We found that there is a strong suppression of the e-ph interaction for these modes resulting from stacking patterns. The split optical phonon modes in few-layer graphene are shown to only couple with the electronic bands of specific orbital symmetry and exhibit various e-ph interaction strengths. These features are well illustrated using a tight-binding model.

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1 K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva, A. A. Firsov, Science 306, 666 (2004).
2 C. Berger, Z. M. Song, T. B. Li, X. B. Li, A. Y. Ogtsazghi, R. Feng, Z. T. Dai, A. N. Marchenkov, E. H. Conrad, P. N. First, and W. A. de Heer, J. Phys. Chem. B 108, 19912 (2004).
3 Y. Zhang, J. P. Small, W. V. Pontius, and P. Kim, Appl. Phys. Lett. 86, 073104 (2005).
4 Z. Yao, C. L. Kane, and C. Dekker, Phys. Rev. Lett. 84, 2941 (2000).
5 M. Lazzeri, S. Piscanec, F. Mauri, A. C. Ferrari, and J. Robertson, Phys. Rev. Lett. 95, 236302 (2005).
6 T. Kamprath, L. Perfetti, F. Schapper, C. Frischkorn, and M. Wol, Phys. Rev. Lett. 95, 187403 (2005).
7 N. Bonini, M. Lazzeri, N. Marzari, and F. Mauri, Phys. Rev. Lett. 99, 176802 (2007).
8 G. Grimvall, The Electron-Phonon Interaction in Metals (North-Holland, Amsterdam, 1981).
9 A. Bostwick, T. Ohta, J. L. McChesney, T. Seyller, K. Horn, and E. Rotenberg, Solid State Comm. 143, 63 (2007).
10 J. González and E. Perfetto, Phys. Rev. Lett. 101, 176802 (2008)
11 M. Calandra and F. Mauri, Phys. Rev. B 76, 205411 (2007).
12 S. Y. Zhou, D. A. Siegel, A. V. Fedorov, A. Lanzara, Phys. Rev. Lett. 101, 086402 (2008).
13 E. H. Hwang and S. Das Sarma, Phys. Rev. B 77, 081412(R) (2008).
14 A. C. Ferrari, Solid State Comm. 143, 47 (2007).
15 F. Giustino, J. R. Yates, I. Souza, M. L. Cohen, and S. G. Louie, Phys. Rev. Lett. 98, 047005 (2007).
16 J. Menendez and M. Cardona, Phys. Rev. B 29, 2051 (1984).
17 J. Yan, Y. Zhang, P. Kim, and A. Pinczuk, Phys. Rev. Lett. 98, 166802 (2007).
18 M. Lazzeri, S. Piscanec, F. Mauri, A. C. Ferrari, and J. Robertson, Phys. Rev. B 73, 155426 (2006).
19 R. J. Nemanich, G. Lucovsky, S. A. Solin, Solid Stat. Comm. 23, 117 (1977).
20 T. Ohta, B. Bostwick, T. Seyller, K. Horn, and E. Rotenberg, Science 313, 951 (2006).
21 S. Latil, and L. Henrard, Phys. Rev. Lett. 97, 036803 (2006).
22 T. Ohta, A. Bostwick, J. L. McChesney, T. Seyller, K. Horn, and E. Rotenberg, Phys. Rev. Lett. 98, 206802 (2007).
23 J. A. Yan, W. Y. Ruan, and M. Y. Chou, Phys. Rev. B 77, 125401 (2008)
24 We note there are some further discussions of Kohn anomaly in one and bilayer graphene recently. See, e.g., W.-K. Tse, Ben-Yu-Kuang Hu, and S. Das Sarma, Phys. Rev. Lett. 101, 066401 (2008). E. H. Hwang, and S. Das-Sarma, Phys. Rev. Lett. 101, 156802 (2008).
25 S. Baroni, S. de Gironcoli, and A. Dal Corso, Rev. Mod.
Phys. 73, 515 (2001).
26 S. Baroni, A. Dal Corso, S. de Gironcoli, and P. Giannozzi, http://www.pwscf.org.
27 N. Troullier and J. L. Martins, Phys. Rev. B 43, 1993 (1991).
28 S. Piscanec, M. Lazzeri, F. Mauri, A. C. Ferrari, and J. Robertson, Phys. Rev. Lett. 93, 185503 (2004).
29 P. B. Allen, Phys. Rev. B. 6, 2577 (1972); P. B. Allen and R. Silberglitt, ibid. 9, 4733 (1974).
30 J. Yan, E. A. Henriksen, P. Kim, and A. Pinczuk, Phys. Rev. Lett. 101, 136804 (2008).
31 C.-H. Park, F. Giustino, M. L. Cohen, and S. G. Louie, Phys. Rev. Lett. 99, 086804 (2007); Nano Lett. (to be published) (2008).
32 A. Rycerz, J. Tworzydło, C. W. J. Beenakker, Nature Physics 3, 172 (2007).
33 A. R. Akhmerov and C. W. J. Beenakker, Phys. Rev. Lett. 98, 157003 (2007).
34 L. M. Woods, and G. D. Mahan, Phys. Rev. B 61, 10651 (2000).
35 G. D. Mahan, Phys. Rev. B 68, 125409 (2003).