Constraining neutrino properties with a Euclid-like galaxy cluster survey

M. Costanzi Alunno Cerbolini\textsuperscript{a,b} B. Sartoris\textsuperscript{a,b} Jun-Qing Xia\textsuperscript{c,d} A. Biviano\textsuperscript{e} S. Borgani\textsuperscript{a,b,e} M. Viel\textsuperscript{e,b}

\textsuperscript{a}Universit\`a di Trieste, Dipartimento di Fisica, via Valerio, 2, 34127 Trieste, Italy
\textsuperscript{b}INFN-National Institute for Nuclear Physics, via Valerio 2, 34127 Trieste, Italy
\textsuperscript{c}Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Science, P.O.Box 918-3, Beijing 100049, P.R.China
\textsuperscript{d}Scuola Internazionale Superiore di Studi Avanzati, Via Bonomea 265, I-34136 Trieste, Italy
\textsuperscript{e}INAF-Osservatorio Astronomico di Trieste, via Tiepolo 11, 34143 Trieste, Italy
Abstract. We perform a forecast analysis on how well a Euclid-like photometric galaxy cluster survey will constrain the total neutrino mass and effective number of neutrino species. We base our analysis on the Monte Carlo Markov Chains technique by combining information from cluster number counts and cluster power spectrum. We find that combining cluster data with Cosmic Microwave Background (CMB) measurements from Planck improves by more than an order of magnitude the constraint on neutrino masses compared to each probe used independently. For the ΛCDM+$m_\nu$ model the 2$\sigma$ upper limit on total neutrino mass shifts from $\sum m_\nu < 0.35$eV using cluster data alone to $\sum m_\nu < 0.031$eV when combined with Planck data. When a non-standard scenario with $N_{\text{eff}} \neq 3.046$ number of neutrino species is considered, we estimate an upper limit of $N_{\text{eff}} < 3.14$ (95\%CL), while the bounds on neutrino mass are relaxed to $\sum m_\nu < 0.040$eV. This accuracy would be sufficient for a 2$\sigma$ detection of neutrino mass even in the minimal normal hierarchy scenario ($\sum m_\nu \simeq 0.05$ eV). In addition to the extended ΛCDM+$m_\nu$+$N_{\text{eff}}$ model we also consider scenarios with a constant dark energy equation of state and a non-vanishing curvature. When these models are considered the error on $\sum m_\nu$ is only slightly affected, while there is a larger impact of the order of $\sim 15\%$ and $\sim 20\%$ respectively on the 2$\sigma$ error bar of $N_{\text{eff}}$ with respect to the standard case. To assess the effect of an uncertain knowledge of the relation between cluster mass and optical richness, we also treat the ΛCDM+$m_\nu$+$N_{\text{eff}}$ case with free nuisance parameters, which parameterize the uncertainties on the cluster mass determination. Adopting the over-conservative assumption of no prior knowledge on the nuisance parameter the loss of information from cluster number counts leads to a large degradation of neutrino constraints. In particular, the upper bounds for $\sum m_\nu$ are relaxed by a factor larger than two, $\sum m_\nu < 0.083$eV (95\%CL), hence compromising the possibility of detecting the total neutrino mass with good significance. We thus confirm the potential that a large optical/near-IR cluster survey, like that to be carried out by Euclid, could have in constraining neutrino properties, and we stress the importance of a robust measurement of masses, e.g. from weak lensing within the Euclid survey, in order to full exploit the cosmological information carried by such survey.

Keywords: cosmology: large-scale structure of Universe; neutrinos; galaxies: clusters.
1 Introduction

Over decades neutrino oscillation experiments have provided conclusive evidence that neutrinos have non-zero masses. Such experiments provide constraints on the neutrino mass squared difference, while they are not sensitive to the absolute scale of neutrino masses. The latest measurements, using solar, atmospheric, and reactor neutrinos, give mass difference between neutrino species of $\Delta m_{12}^2 = 7.5 \times 10^{-5} \text{eV}^2$ and $|\Delta m_{23}^2| = 2.3 \times 10^{-3} \text{eV}^2$ [e.g. 1, 2], which in turn translates to a lower bound on the sum of the three masses, $\sum m_\nu$, at 0.056(0.095) eV in the normal (inverted) hierarchy. On the other hand, cosmological data provides a tool to constrain neutrino masses due to the effects neutrinos induce on background evolution and growth of structures: relativistic neutrinos affect the Cosmic Microwave Background (CMB) anisotropies, whereas, at low redshift when they become non-relativistic, neutrinos suppress matter density fluctuations at small scales [e.g. 3, 4, for a review].

Given these multiple effects massive neutrinos leave an imprint on many cosmological observables; indeed, current neutrino constraints from cosmology rely on a combination of data from CMB experiments, Baryonic Acoustic Oscillations (BAOs) measurements, Supernovae distance moduli, galaxy clustering and galaxy cluster mass function. Recent constraints on the upper limit of the total neutrino mass lie in the range $\sum m_\nu < 0.3 – 0.8 \text{eV}$ (95% Confidence Level (CL)) [e.g. 5–11], with notable exception of Lyman–α data, which presents an even tighter bound of 0.17 eV [12].

The number of active neutrinos is known to be three to high precision through the measurement of the invisible width of the Z boson at LEP [2, 13], however, the possibility remains that additional ”sterile” species exist [e.g. 14]. In fact, in order to explain the results of short baseline neutrino oscillations experiments LSDN [15] and MiniBooNE [16], as well as the recently discovered reactor neutrino anomaly [17, 18], models with one or two light sterile neutrino have been proposed [e.g. 19–21]. Observations of the
CMB also seem to point to the same direction, favoring the presence of extra relativistic degree of freedom at the time of decoupling, in terms of the effective number of neutrino species $N_{\text{eff}}$. For instance, measurements of CMB anisotropies from the South Pole Telescope (SPT) [22] and the Atacama Cosmology Telescope (ACT) [23], combined with seven-year Wilkinson Microwave Anisotropy Probe (WMAP) data [24] and BAO and $H_0$ measurements, have measured $N_{\text{eff}} = 3.71 \pm 0.35$ and $N_{\text{eff}} = 3.52 \pm 0.39$ at 68% CL, respectively, thus suggesting values higher than those expected in the canonical scenario ($N_{\text{eff}} = 3.046$).

Among the different probes of the Large Scale Structure (LSS), many works have already proven the ability of galaxy clusters in constraining cosmological parameter, through both the evolution of their mass function [e.g. 25–32], and their large–scale clustering properties [e.g. 33–35]. Indeed, galaxy clusters supply cosmological information in different ways. The evolution of the galaxy cluster population depends on cosmological parameter both through the linear growth rate of density perturbations and the redshift dependence of the volume element. Furthermore, also clustering of galaxy clusters is sensitive to cosmological parameters through the grow rate of perturbations. These properties makes clusters an excellent probe of the growth of structure, and thus a valuable tool to constrain neutrino properties [e.g. 30, 36].

Recent constraints from galaxy cluster samples combined with CMB data, observation of BAO and supernovae Type Ia are $\sum m_\nu \lesssim 0.52$ eV, $N_{\text{eff}} = 3.6^{+1.4}_{-1.0}$ [37] and 0.72 eV, $N_{\text{eff}} \lesssim 4.6$ (95% CL) [30, 38].

While current constraints based on galaxy cluster data rely on relatively small samples of clusters identified at redshift below one [e.g. 39, 40], next generation of X-ray (e.g., eROSITA\(^1\), WFXT\(^2\)), Sunyaev-Zeldovich (e.g., CCAT\(^3\), SPT-3G) and optical (e.g., DES\(^4\), LSST\(^5\), PanSTARRS\(^6\), Euclid \(^7\)) surveys are expected to increase by orders of magnitude the number of galaxy clusters detected, further extending the probed range of redshift up to $z \sim 2$. Such large cluster surveys will provide tight constraints on cosmological parameters, independently and complementary to those recovered from other cosmological probes. In this work we explore the cosmological information contained in the cluster catalog that will be provided by the photometric redshift survey of ESA’s Euclid mission, which has been approved for launch in 2019. Specifically, we will make use of cluster number counts and cluster power spectrum to derive forecast errors on the total neutrino mass and effective number of neutrino species, for a Euclid-like galaxy cluster survey. A similar analysis based on the Fisher Matrix approach has been proposed by [41]. Even though the Fisher Matrix technique has the advantage of allowing for a quick, analytic estimate of the confidence limits, on the other hand it approximates the likelihood function as a multivariate Gaussian function of the model parameters. In general, this turns out to be a coarse approximation since the likelihood function can be highly non-Gaussian [e.g. 42]; moreover the results obtained with this technique depend on the step chosen in the calculation of numerical derivatives with respect to the parameters [e.g. 43]. For these reasons we choose to use a more robust forecast method based on the sampling of the full likelihood function using a Monte Carlo Markov Chain (MCMC) approach. Finally, unlike [41], in our work we explore also the non-standard

\(^1\)http://www.mpe.mpg.de/eROSITA
\(^2\)http://www.wfxt.eu/
\(^3\)http://www.ccatobservatory.org/index.cfm
\(^4\)http://www.darkenergysurvey.org/
\(^5\)http://www.lsst.org/lsst/
\(^6\)http://pan-starrs.ifa.hawaii.edu/public/
\(^7\)http://www.euclid-ec.org/
Figure 1: Effects of the variation of $\sum m_\nu$ and $N_{\text{eff}}$ on the linear matter power spectrum (left) and halo mass function (right) at $z = 0$; all other parameters ($\Omega_m, \Omega_\Lambda, H_0, n_s, \Delta_{200}^2, \tau$) are kept fixed to the WMAP 9-yr best-fit values for ΛCDM. See text for comments.

scenario with $N_{\text{eff}} > 3$ and $\sum m_\nu > 0$, to assess the effects of the correlation between the two parameters [see e.g. 38, 44].

The paper is organized as follows. In Section 2, we briefly explain the physical and observable effects of $N_{\text{eff}}$ and $\sum m_\nu$. Section 3 describes the formalism which we use to compute the cluster number counts ($\S$ 3.1) and power spectrum ($\S$ 3.2), while in $\S$ 3.3 we outline our forecasting procedure and specify the characteristics of the galaxy cluster survey analyzed. Our results for different cosmological model are presented in section 4, and finally in section 5 we draw our conclusions.

2 Massive neutrino effects

In this section we briefly review the physical effects of $N_{\text{eff}}$ and $\sum m_\nu$ and how they affect the halo mass function and the matter power spectrum, two quantities strictly related to the observables used to derive the forecast error (see next section).

What is actually constrained by cosmological data is not the neutrino mass, but the neutrino energy density $\rho_\nu$, which can be related to the neutrino mass through the relation:

$$\Omega_\nu = \frac{\rho_\nu}{\rho_c} = \frac{\sum_{i}^{N_\nu} m_{\nu,i}}{93.14 \text{eV} h^2},$$  \hspace{1cm} (2.1)

where $\rho_c$ is the critical density of the Universe, and $N_\nu$ the number of massive neutrinos. In the present work, we assume three degenerate massive neutrino states, so that the total neutrino mass can be written as $\sum m_\nu = 3m_\nu$. A degenerate mass spectrum is justified on one side by the smallness of the mass difference measured, on the other by the incapability of the Euclid cluster data to probe directly the neutrino mass hierarchy, as we will explain later in $\S$ 4.2. The effective number of neutrino species parameterizes the contribution of neutrinos (or any other relativistic species) to the radiation content in the radiation dominated era through the formula:

$$\rho_\gamma = \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right] \rho_\gamma,$$  \hspace{1cm} (2.2)

$\rho_\gamma$ is the radiation density.
where \( \rho_r \) is the energy density in relativistic species and \( \rho_\gamma \) is the energy density of photons. \( N_{\text{eff}} \) can be expressed as the sum of the number of massive neutrinos \( N_\nu \) and the contribution from extra relativistic degrees of freedom, \( \Delta N_{\text{eff}} \). The standard value for the number of massive neutrinos is \( N_\nu = 3.046 \), where the 0.046 accounts for a non-instantaneous neutrino decoupling process and flavour neutrino oscillations effect [45]. Instead, any significant deviation from \( \Delta N_{\text{eff}} = 0 \) could indicate the presence of new physics beyond the standard model; adding an extra (thermalized) light fermion would contribute \( \Delta N_{\text{eff}} = 1 \), but more generally a non-integer \( \Delta N_{\text{eff}} \) value could arise from different physics, such as lepton asymmetries [46], partial thermalisation of new fermions [20], particle decay [47], non-thermal production of dark matter [48], gravity waves [49] or early dark energy [50].

Modification of these parameters has effects both on the background evolution and on structure formation. Neutrinos decouple from the primeval plasma at \( T \approx 2 - 4 \text{MeV} \), when they are still ultra-relativistic, and given their small mass, \( \sum m_\nu < 1 \text{eV} \), they become non-relativistic (only) after recombination. During this epoch, their energy density contributes as radiation rather than matter, thus changing the expansion rate (via the Friedmann equation \( H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\gamma) \)) and the time of matter-radiation equality (\( a_{\text{eq}} = \rho_0^0 / (\rho_0^0 - \rho_0^\nu) \)). The epoch of equality set the time at which sub-horizon density fluctuations start to collapse under the action of gravity and structures can evolve. When the other parameters are kept fixed, a larger value of \( \sum m_\nu \) or \( N_{\text{eff}} \) corresponds to a larger value of the radiation density, and therefore a postponed time of equality. Such modification is seen in the matter power spectrum as a shift to larger scale of its peak, which is determined by the size of the particle horizon at the time of matter-radiation equality. Moreover, since on sub-Hubble scales density fluctuations grow more efficiently during matter dominated epoch (i.e. after equality), the matter power spectrum is suppressed on small scales relatively to large scales (left panel of Fig. 1). These effects determine also a suppression of the halo mass function (right panel of Fig. 1).

After thermal decoupling neutrinos constitute a collisionless fluid, whose constituent fluid elements free-stream with a velocity, on average, equal to their thermal velocity \( v_{\text{th}} \). As the Universe expands, \( v_{\text{th}} \) decays adiabatically till neutrinos become non-relativistic. At this stage neutrinos behave as hot dark matter particles, suppressing density fluctuations on scale smaller than their free-streaming length:

\[
 k_{\text{fs}} = 0.8 \sqrt{\Omega_\Lambda + \Omega_m(1 + z)^3 \left( \frac{m}{1 \text{eV}} \right)} h \text{Mpc}^{-1} \tag{2.3}
\]

Due to their high velocity neutrinos cannot be confined on region smaller than their free-streaming length, thereby suppressing the density perturbations by a factor proportional to \( (1 - \Omega_\nu / \Omega_m) \). Furthermore, the absence of gravitational back-reaction effects from free-streaming neutrinos slow down the grow rate of CDM/baryon perturbations at late times [see e.g. 4]. Both these effects are observable in the matter power spectrum as a suppression on small scales (left panel of Fig. 1), with a constant amplitude \( \Delta P(k)/P(k) \approx -8(\Omega_\nu/\Omega_m) \) at \( k \sim 1h/\text{Mpc} \), for linear structure formation [51]. Similarly, the halo number density decreases due to the free-streaming neutrino action (right panel of Fig. 1). Again the suppression is larger for larger neutrino mass, and for a given \( \sum m_\nu \) it is more pronounced for the heaviest, late forming haloes, since the damping in power from massive neutrinos shifts the maximum cluster mass down (i.e. the scale beyond which the halo mass function is exponentially suppressed).
To actually detect the signature of massive neutrinos in the matter power spectrum and halo mass function we need a cosmic tracer of such quantities. As mentioned in the introduction we will use the cluster number counts and cluster power spectrum as observables to detect massive neutrino signature. In the next section we will describe the formalism we use to quantify such an effect on the two observable and outline the forecasting procedure adopted.

3 Methodology

We focus on the galaxy cluster sample which will be provided by a Euclid-like photometric survey. For this observable we quantify its clustering statistics in terms of the average cluster power spectrum, and its mass distribution in terms of the cluster number counts. In describing these two quantities we use the same notation as in [52]. In order to extract the cosmological parameter errors from the mock Euclid data, we perform a Bayesian likelihood analysis.

3.1 Cluster number counts

The number of clusters expected for a survey having sky coverage ∆Ω with an observed mass between \( M_{\text{ob},m} \) and \( M_{\text{ob},m+1} \) and redshift between \( z_1 \) and \( z_{1+1} \) can be expressed as:

\[
N_{l,m} = \Delta \Omega \int_{z_1}^{z_{1+1}} dz \frac{dV}{dz} \int_{M_{\text{ob},m}}^{M_{\text{ob},m+1}} \frac{dM_{\text{ob}}}{M_{\text{ob}}} \times \int_0^{\infty} dM n(M,z) p(M_{\text{ob}}||M),
\]

where \( dV/dz \) is the comoving volume element per unit redshift and solid angle, and \( n(M,z) \) is the halo mass function. In this notation \( M_{\text{ob},m=0} = M_{\text{min}}(z) \) represents the minimum value of the observed mass for a cluster to be included in the survey, and it is determined by the survey selection function (see § 3.4). The integral over the observed mass is computed within bins having width \( \Delta \log M = 0.2 \), extending from \( M_{\text{min}}(z) \) to \( 10^{15.8} h^{-1} M_\odot \). For \( n(M,z) \) we adopt the expression provided by [53], with mass function parameters obtained for overdensity \( \Delta = 200 \) with respect to the mean density of the universe (see their Table 2). Moreover, to take into account massive neutrino effects, we follow the prescription used by many authors [e.g. 54–56] neglecting the weakly clustering neutrino component when calculating the halo mass (\( M_{\text{halo}} = 4\pi r^3 \rho / 3 \), with \( \rho = \rho_m - \rho_\nu \)). Many other calibrations of the halo mass function from simulations have been presented by several authors [e.g. 57–60]. However, for the purposes of this work the choice of the best-calibrated mass function has a minor impact. Indeed the forecast errors depend primarily on the number of cluster expected for a given cosmological model, which is far more sensitive to the exponential shape of the mass function rather than to the calibration details of the mass function. The factor \( p(M_{\text{ob}}||M) \) takes into account the uncertainties that a scaling relation introduces in the knowledge of the true cluster mass. Following the prescription of [61], \( p(M_{\text{ob}}||M) \) gives the probability of assigning to a cluster of true mass \( M \) an observed mass \( M_{\text{ob}} \), as inferred from a giving scaling relation. Under the assumption of a lognormal-distributed intrinsic scatter around the nominal scaling relation with variance \( \sigma_{\ln M}^2 \), the probability can be written as:

\[
p(M_{\text{ob}}||M) = \exp\left[-\frac{x^2(M_{\text{ob}})}{2\sigma_{\ln M}^2}\right],
\]
where
\[ x(M^{ob}) = \frac{\ln M^{ob} - B_M - \ln M}{\sqrt{2\sigma_{\ln M}^2}}. \] (3.3)

Here the parameter \( B_M \) represents the fractional value of the systematic bias in the mass estimate. Moreover, according to \([62]\) we assume the following parameterization for redshift dependencies of the halo mass bias and variance (we do not consider a possible mass dependence of these parameters):
\[ B_M(z) = B_{M,0}(1 + z)^\alpha \]
\[ \sigma_{\ln M}(z) = \sigma_{\ln M,0}(1 + z)^\beta. \] (3.4)

In our formalism, we have four nuisance parameters, \( B_{M,0}, \sigma_{\ln M,0}, \alpha \) and \( \beta \), which can be allowed to vary along with the other cosmological parameters during the forecast procedure (see § 4.4).

Including Eq. 3.2 into Eq. 3.1 it follows that:
\[ N_{1,m} = \frac{\Delta \Omega}{2} \int_{z_l}^{z_{l+1}} dz \frac{dV}{dz} \int_0^\infty dM n(M, z) \times [\text{erfc}(x_{l,m}) - \text{erfc}(x_{l,m+1})], \] (3.5)

where \( \text{erfc}(x) \) is the complementary error function.

For Euclid data, the photometric redshift measurements will be calibrated using a combination of the spectroscopic survey and ground-based visual bands photometry, with an expected limiting precision of \( \sigma(z) \sim 0.05(1 + z) \) \([63]\). While this error refers to the photometric redshift of a single galaxy, the redshift of a cluster identified in the photometric survey should be reduced by a factor \( N^{1/2} \), where \( N \) is the number of galaxies assigned to the cluster. For a typical cluster at \( z \sim 1.0 \) with \( N \sim 100 \) detected galaxies the error is reduced by a factor 10, leading to \( \sigma(z) \sim 0.01 \). Thus in the following, we assume that the errors on cluster redshift measurements can be neglected.

An additional effect on the number counts is induced by line-of-sight peculiar velocities, which can scatter redshifts by \( \delta z \sim 0.003 \) for velocities of \( \sim 1000 \text{ km s}^{-1} \). However, since the redshift bins adopted in the analysis have a width of \( \Delta z = 0.2 \), i.e. far larger than the \( \delta z \) value associated to the peculiar velocities, it is a fair assumption to neglect this effect.

### 3.2 Cluster power spectrum

In order to include information from the clustering of galaxy clusters, we calculate the averaged cluster power spectrum within a given redshift interval using the expression:
\[ \bar{P}_{cl}^c(k, z) = \frac{\int_{z_l}^{z_{l+1}} dz \frac{dV}{dz} n^2(z) P_{cl}^c(k, z)}{\int_{z_l}^{z_{l+1}} dz \frac{dV}{dz} n^2(z)}, \] (3.6)

where \( n(z) = \int_0^\infty dM n(M, z) \times \text{erfc}(x_{l,m=0}) \) is the comoving number density of clusters that are included in the survey at redshift \( z \) [e.g. 64]. The cluster power spectrum \( P_{cl}^c(k, z) \) is expressed in terms of the underlying matter power spectrum \( P(k, z) \) according to \( P_{cl}^c(k, z) = b_{eff}^2(z) P(k, z) \); the term of proportionality \( b_{eff} \) is the cluster mass function averaged linear bias, defined as:
\[ b_{eff}(z) = \frac{\int_0^\infty dM n(M, z) \text{erfc}(x_{l,m=0}) b(M, z)}{\int_0^\infty dM n(M, z) \text{erfc}(x_{l,m=0})}. \] (3.7)
Figure 2: Wavenumber dependence of the relative error of the cluster power spectrum, $\sigma_{Pcl}/\bar{P}_{cl}$ defined as in Eq. 3.10, at four different redshift: $z = 0.2, 0.6, 1.0, 1.6$, from bottom to top curves, respectively.

For the linear bias of dark matter halos, $b(M,z)$, we adopt the fitting function of [65] for overdensity $\Delta = 200$ (see their Table 2). The linear matter power spectrum $P(k,z)$ is computed with the publicly available software package CAMB [66], which takes correctly into account the effect of massive neutrinos also in a mild non-linear regime [67].

As for the effect of errors in photometric redshifts, they are expected to introduce a smearing in the power spectrum at small scales (see, e.g., [68]), thus degrading the information carried by clustering analysis. Consistently with the number counts analysis we neglect in the following the effect of uncertainties in redshift measurements, thereby not accounting for the damping of the power spectrum due to photometric redshift errors. In order to avoid contribution to the matter power spectrum and scale-dependent bias introduced by non-linearities, we do not include in our analysis modes with wavenumbers larger than $k_{\text{max}} = 0.1\,\text{Mpc}^{-1}$ [69]. Although massive free-streaming neutrinos mainly affects the power spectrum at small scales (see section 2), a value of $k_{\text{max}}$ larger than $\sim 0.3\,\text{Mpc}^{-1}$ would not increase significantly the sensitivity of the survey. Indeed, given the level of Poisson noise associated to the cluster distribution (see Eq. 3.10, and Fig. 2), higher frequency modes are not adequately sampled and, therefore, adding them to the analysis does not add significant information [62].

As for the minimum value of the wavenumber we impose $k_{\text{min}} = 0.003\,\text{Mpc}^{-1}$; again, using a smaller value of $k_{\text{min}}$ does not change the final results, since extremely large scales are not sampled by the surveys. Finally, in our analysis we neglect the correction to the power spectrum due to redshift space distortion effects.

3.3 Forecasting

The forecast is based on the Bayesian inference technique, for which a likelihood function of the mock data is first constructed and then sampled in order to estimate the marginalized probability distribution of the parameters. To explore the parameter space by means of
Table 1: Fiducial parameter values.

|      | ω_b | ω_c | Θ_s | τ   | n_s | log[10^{10}A_s] | f_ν |
|------|-----|-----|-----|-----|-----|----------------|-----|
| Value| 0.02253 | 0.1122 | 1.0395 | 0.085 | 0.967 | 3.18 | 0 |

Monte Carlo Markov Chains we use the publicly available code CosmoMC\(^8\)\[70\], where we included a module for the calculation of the cluster number counts and power spectrum likelihoods.

Our most general parameter space is:

\[ Θ ≡ (ω_b, ω_c, Θ_s, τ, n_s, \log[10^{10}A_s], f_ν, N_{\text{eff}}, Ω_k, w, B_{M,0}, σ_{\ln M,0}, α, β) \] (3.8)

where the first six parameter, which define the standard ΛCDM model, are: the physical baryon \( ω_b = Ω_bh^2 \) and cold dark matter \( ω_c = Ω_ch^2 \) densities, the ratio (multiplied by 100) between the sound horizon and the angular diameter distance at decoupling \( Θ_s \), the reionization optical depth \( τ \), the scalar spectral index \( n_s \) and the amplitude of initial power spectrum \( A_s \). Besides these parameters we performed several forecasts for different extensions of the minimal cosmological model, by fitting (along with the other parameters): the neutrino density fraction \( f_ν = Ω_ν/Ω_c \), the effective number of neutrino species \( N_{\text{eff}} \), the spatial curvature \( Ω_k \) and the dark energy equation of state parameter \( w \). Finally, in order to assess the effect of the uncertain knowledge of the mass-observable relation we consider also the case in which the four nuisance parameter are treated as fitting parameters to be determined along with the cosmological ones.

Throughout this paper, our reference model is chosen to be a flat ΛCDM model with three neutrino species. The fiducial ΛCDM parameter values are listed in Table 1, consistently with the WMAP-7+BAO+\(H_0\) best-fit model by \[71\]. These fiducial parameter values are also consistent with the latest WMAP 9-yr best-fit model \[10\].

As for the nuisance parameters, clusters mass within Euclid survey will be estimated using photometric richness as a cluster mass proxy. An accurate calibration of the scaling relation between richness and mass will be provided by weak lensing mass measurements within the Euclid survey. We note that different authors calibrated the presence of a possible bias in weak lensing mass measurements by resorting to cosmological simulations of galaxy clusters \[72, 73\]. The results of these analyses converge to indicate that a small, but sizeable, underestimate in weak lensing masses, is induced by projection effects and amounts to 5–10%. For the purpose of the present analysis we prefer to assume that weak lensing provides an unbiased calibration of the mass–richness relation, thus fixing \( B_{M,0} = 0 \) as a reference value for the mass bias. For the intrinsic scatter we assume \( σ_{\ln M,0} = 0.45 \) as estimated by \[40\] by demanding consistency between available weak lensing and X-ray measurements of the maxBCG clusters, and the X-ray luminosity-mass relation inferred from the 400d X-ray cluster survey. The intrinsic scatter has the effect of increasing the number of clusters included in the survey. Indeed, the number of low-mass clusters that are up-scattered above the survey mass limit is always larger than the number of rarer high-mass clusters which are

\(^{8}\)http://cosmologist.info/cosmomc/
down-scattered below the same mass limit [e.g. 74]. Because so far there are no evidences for the evolution of the nuisance parameters we adopt $\alpha = 0$ and $\beta = 0$ as reference values, thus making the minimal assumption of constant bias and scatter with redshift. When the nuisance parameters are left free we consider two cases: one with strong prior on the evolution parameters, with $\alpha$ and $\beta$ not allowed to vary with respect to their reference value, and the other one with no prior knowledge of their value. The latter turns out to be a conservative assumption in view of the large number of clusters for which mass measurements from weak lensing will be available from Euclid [63]. As such, the corresponding uncertainties expected on cosmological parameters should be regarded as upper limits of the error introduced by the uncertainties in the relation between cluster richness and mass. The cluster power spectrum and number counts of the mock data are assumed to be equal to the theoretical cluster power spectrum and number counts of the fiducial model.

Since we are interested only in parameter error estimation, we define our likelihood functions $L$ of the observable $O$ as

$$\chi_{\text{eff}}^2 \equiv -2 \ln L = \sum_{i,j} \frac{O_{ij}^{\text{obs}} - O_{ij}^{\text{th}}}{\sigma_{ij}^2},$$

(3.9)

in such a way that $\chi_{\text{eff}}^2$ is equal to zero for the fiducial parameter values. In the previous equation $O_{ij}^{\text{obs}}$ denotes the observed cluster power spectrum, $P_{cl}^{\text{obs}}(k, z_j)$ (number counts, $N_{cl}^{\text{obs}}(M_i, z_j))$, while $O_{ij}^{\text{th}}$ is the theoretical cluster power spectrum (number counts) of Eq. 3.5 (Eq. 3.6). The statistical error associated to the observed galaxy cluster power spectrum in a bin centered on $(k_i, z_j)$ [75] reads:

$$\sigma_{P_{cl}}^2 = \frac{(2\pi)^2 (P_{cl}^{th}(k_i, z_j))}{V_{\text{sur}}(z_j) \Delta k} \left[ 1 + \frac{1}{n(z_j) P_{cl}^{th}(k_i, z_j)} \right]^2,$$

(3.10)

where $V_{\text{sur}}(z_j)$ is the comoving survey volume within the redshift bin centred on $z_j$, and $\Delta k$ is the size of the bins in wavenumber space. In this way, constraints at redshift $z$ are mostly contributed by wave-numbers $k$, which maximize the product $n(z) P_{cl}^{obs}(k, z)$ (see Fig. 2). The average cluster power spectrum is computed by integrating over redshift intervals having constant width $\Delta z = 0.2$. This choice of binning represents a compromise between the need of extracting the maximum amount of information from clustering evolution and request of negligible covariance between adjacent $z$-intervals [e.g. 76]. Indeed, the definition of Eq. 3.9 holds only if the contribution from different redshift slices carry statistically independent information. As for the statistical error of the observed number counts for a given mass and redshift bin centered in $(M_i, z_j)$, we consider only the Poissonian noise, $\sigma_{N_{cl}}^2 = N^{\text{th}}(M_i, z_j)$, neglecting the contribution from sample variance, which accounts for the clustering of clusters due to large scale structure. Given the large volume to be probed by the Euclid survey ($\sim 100 h^{-3} \text{Gpc}$) and the exponential suppression of cluster number density for mass larger than the maximum cluster mass, the shot-noise errors dominate over sample variance for the most of mass and redshift bins [77, also for a more rigorous definition of the number counts covariance matrix]. Finally, since number counts and power spectrum probe the same mass density field, the covariance between the two is expected to be different from zero. In practice, it has been shown [e.g. 78] that these two observables have in fact negligible covariance.

Because the full parameter space is quite large and some parameters are poorly constrained by LSS observations, we perform our forecast combining the Euclid-like cluster
catalog with Planck-like data. The mock CMB $TT$, $EE$ and $TE$ power spectra have been simulated following the procedure of [42] according to the specifications presented in the Planck Blue-Book [79, page 4, Table 1.1] based on 14 months of observations, using the three frequency channels with the lowest foreground levels at 100GHz, 143GHz and 217GHz, and a sky fraction of $f_{\text{sky}} = 0.80$. In order to avoid problems with foreground signal, beam uncertainties, etc., we cut-off the spectra at $l_{\text{max}} = 2000$.

3.4 Characteristics of the survey

Euclid is a Medium Class mission of the ESA Cosmic Vision 2015-2025 programme, planned for launch in 2019. Thanks to its three imaging and spectroscopic instruments working in the visible and near-infrared bands, Euclid will cover 15,000 square degrees of extragalactic sky with the wide survey, thereby providing high–quality images from visual imaging for more than a billion galaxies, accurate photometric redshifts from near-IR imaging photometry (in combination with ground-based data) for about $2 \times 10^8$ galaxies and about $5 \times 10^7$ spectroscopic redshift at $z > 0.7$ from near-IR slitless spectroscopy.

The most efficient method to build the Euclid galaxy cluster catalog relies on the analysis of the photometric data. To predict cosmological constraints from the expected sample of galaxy clusters, we use the analytic selection function adopted in the Euclid Red-Book [63] to forecast the contribution of the cluster survey to the cosmological constraints. The computation of the selection function is based on using the luminosity function of cluster galaxies to compute the number of galaxies expected within $R_{200c}$ down to the $H_{AB} = 24$ magnitude limit reached in the photometric survey, as a function of the cluster mass and the cluster redshift.

Specifically, we use an average of the $K_s$-band luminosity functions of nearby clusters, evaluated within $R_{500c}$ by [80], which we then evolve passively with redshift [81]. We transform the $K_s$ magnitudes into the $H_{AB}$ band by using the mean color for cluster galaxies. Integrating the luminosity function down to the apparent magnitude limit of the survey we obtain the number density of cluster galaxies within $R_{500c}$. Then, after appropriate scaling and multiplication by the corresponding sphere volume, we obtain the number of cluster galaxies within a sphere of radius $R_{200c}$. Given the direct relation between cluster mass $M_{200c}$ and radius $R_{200c}$, we obtain the number of observable galaxies for a cluster of given mass at any redshift.

In practice, this procedure is equivalent to adopting a scaling relation between a cluster mass $M_{200c}$ and richness, a relation which evolves with redshift because of passive evolution of the cluster population, and where the knowledge of the luminosity function allows the richness to be estimated down to the redshift-dependent absolute magnitude limits that correspond to the fixed apparent magnitude limit of the survey.

We then calculate the predicted number of fore-/back-ground galaxies within a cylinder of angular radius corresponding to $R_{200c}$ at the cluster redshift, and of length equal to $\pm 3$ times the photometric redshift error, with the idea that photometric redshifts will be used to reduce the fore-/back-ground. The signal-to-noise ($S/N$) for cluster detection is then obtained from the ratio between the number of cluster galaxies and the rms of the number of fore-/back-ground galaxies. The latter is contributed by both Poisson noise and cosmic variance.

\footnote{Here $R_{200c}$ is defined as the radius encompassing an average density equal to 200 times the cosmic critical density at a given redshift}
Assuming $S/N = 3$ for the fiducial limiting signal-to-noise for a reliable cluster detection turns into a selection function which provides $M_{\text{lim},200c}(z)$ defined as the limiting mass within $R_{200c}$ for a cluster to be included in the survey. As a result, one finds $M_{\text{lim},200c}(z) \simeq 1.6 \times 10^{14} M_\odot$ at $z > 0.5$, while decreasing at lower redshift, reaching $\sim 5 \times 10^{13} M_\odot$ at $z = 0.2$. Finally, to compute Eq. 3.5 we need to convert $M_{\text{lim},200c}(z)$ to $M_{\text{lim},200m}(z)$ – the limiting mass within a radius encompassing an overdensity equal to 200 times the mean density of the Universe – consistently with the chosen halo mass function (see §3.1). To this end we follow the recipe given in [77], assuming a NFW profile [82] as halo density profile and using their fitting formula (C11). Even though a $S/N = 3$ level may look optimistic the selection function adopted in this work is derived using a simplistic analytical model which does not take into account any sophisticated algorithm for cluster detection, and without making use of the full information available (e.g. from cluster density profiles, luminosity functions, red sequence and spectroscopic data). Therefore, the chosen limiting signal-to-noise is likely to represent a conservative estimate, and we can assume the cluster sample to be 100% complete and pure. Clearly, a detailed assessment of the completeness and purity of the cluster sample should require a detailed analysis of the performance of different cluster detection algorithms when applied to the Euclid survey, which is beyond the aim of this paper. Moreover, as discussed in [28], what matters in parameter estimation is not the level of the survey completeness and purity, but the uncertainty in their calibration. Thus, the assumption of a 100% pure and complete sample for $S/N \geq 3$ can be considered as assuming that purity and completeness will be accurately measured in this regime.

For the sky coverage we adopt the required area for the wide Euclid survey $\Delta \Omega = 15,000 \text{deg}^2$ [63], while the cluster number counts and power spectrum are evaluated in redshift bins of width $\Delta z = 0.2$ between $z = 0.2$ and $z = 2$. Moreover, as discussed in §3.1, the cluster number counts is computed within bins of observed mass having width $\Delta \log M = 0.2$ and extending from $M_{\text{lim},0}^{\text{ob}} = M_{\text{lim},200m}(z)$ to $10^{15.8} h^{-1} M_\odot$. From Eq. 3.5, using this survey specifics and fiducial parameter values, we expect that Euclid will find of order $1.5 \times 10^2$ cluster with a $S/N$ better than 3, between $z = 0.2$ and $z = 2.0$, with $\sim 4 \times 10^4$ having $z > 1$ (see Fig. 3).

4 Analysis and results

Having defined the reference cosmological model and the specifics of Euclid survey we now present forecast errors on neutrino parameters for various extensions of the minimal $\Lambda$CDM model. For each of the cases that we describe here below, we run four independent chains, requiring the fulfillment of the Gelman & Rubin [83] criteria with $R-1 \leq 0.03$ as convergence test.

4.1 Three massive neutrinos: $\Lambda$CDM+$m_\nu$

We start by considering the scenario with three degenerate massive neutrino species. In Table 2 we report the 68% and 95% CL bounds on $\sum m_\nu$ derived from different data sets: Planck only, cluster power spectrum only, cluster number counts and power spectrum (hereafter Euclid-Cl), and the combination of Planck and Euclid-Cl. When only cluster power spectrum data are considered we obtain a quite loose 2$\sigma$ upper limit on $\sum m_\nu$ of 1.20 eV. Otherwise, the information contained in cluster number counts alone is unable to constrain the total neutrino mass, but it greatly improves the error on $\sum m_\nu$ once added to cluster
Figure 3: The cumulative cluster redshift distribution as predicted by the reference cosmological model and the reference values for the mass nuisance parameters (see Table 1), for the Euclid cluster survey.

Table 2: Constraints on $\sum m_\nu$ for $\Lambda$CDM+$m_\nu$ model from Planck, cluster power spectrum ($P^{cl}$ only), Euclid-Cl (cluster number counts and power spectrum) data, and the combination of the two data sets Euclid-Cl+Planck. Because the parameter $\tau$ is not constrained by Euclid data, when CMB measurements are not included $\tau$ is kept fixed to its fiducial value.

| Model               | Data            | $\Lambda$CDM+$m_\nu$ |
|---------------------|-----------------|------------------------|
|                     | Planck          | $P^{cl}$-only          | Euclid-Cl        | Euclid-Cl+Planck |
| $\sum m_\nu$ [eV]   | 68% CL          | < 0.41                 | < 0.41           | < 0.17           | < 0.017 |
|                     | 95% CL          | < 0.74                 | < 1.20           | < 0.35           | < 0.031 |

power spectrum data, mainly thanks to the tight constraints provided on $\sigma_8$ (see left panel of Fig. 4). Specifically, the upper limit for $\sum m_\nu$ shrinks by a factor $\sim 4$ to 0.35 eV (95%CL). This error is comparable to the present constraints obtained combining CMB and LSS probes, and of the same order of magnitude of the error expected for Planck. Regarding parameter degeneracies for the galaxy cluster dataset, the total neutrino mass is correlated with all the cosmological parameter affecting the galaxy power spectrum shape (i.e. $\Omega_m$, $\sigma_8$, $n_s$; see red contours in Fig. 4 and Fig. 5).

The main power of constraints in cosmological parameters indeed originate from the joint analysis of galaxy cluster and CMB datasets. In this case the error on $\sum m_\nu$ is reduced to 31 meV; an improvement of more than one order of magnitude that would allow a $2\sigma$ detection of the total neutrino mass even in the minimal normal hierarchy scenario ($\sum m_\nu \simeq 0.05$ eV). The reason for such an improvement can be easily understood by looking at the right panel of Fig. 4 which shows the 68% and 95% confidence regions in the ($\sum m_\nu - \sigma_8$) plane from Planck data, Euclid-Cl data and the combination of the two. Taken independently, the CMB and galaxy cluster data exhibit significant degeneracies in this plane, but the nearly orthogonal degeneracy directions allow their combination to provide tight constraint on these parameters, and in particular on the neutrino mass. When Planck priors are added to the
When only Euclid-Cl dataset is used the parameter $\tau$, which is not constrained by this data, is kept fixed to its fiducial value 0.085.

Euclid-Cl constraints, all degeneracies are either resolved or largely reduced (see blue contours in Fig. 4 and Fig. 5). Similar levels of sensibility on $\sum m_\nu$ are also expected combining Euclid galaxy or cosmic shear power spectrum measurements with Planck CMB data [e.g. 84–87] and from the combination of Planck SZ cluster survey and Planck CMB data [88].
Table 3: Constraints on $\sum m_\nu$ and $N_{\text{eff}}$ for $\Lambda$CDM+$m_\nu$+$N_{\text{eff}}$ model.

| Model                             | $\Lambda$CDM+$m_\nu$+$N_{\text{eff}}$ |
|-----------------------------------|----------------------------------------|
| Data                             | Planck     | Euclid-Cl+Planck |
| $\sum m_\nu$ [eV]                | 68% CL     | < 0.42            | < 0.022 |
|                                  | 95% CL     | < 0.78            | < 0.040 |
| $N_{\text{eff}}$                 | 95% CL     | < 3.36            | < 3.14  |

4.2 Varying $N_{\text{eff}}$: $\Lambda$CDM+$m_\nu$+$N_{\text{eff}}$

We now explore the scenario with massive neutrinos and $N_{\text{eff}}$ effective number of neutrino species. Again, we distribute the sum of neutrino masses equally among three active species ($N_\nu = 3$), and we treat additional contribution to $N_{\text{eff}}$ as massless, such that $N_{\text{eff}} = 3 + \Delta N_{\text{eff}}$, with the prior $\Delta N_{\text{eff}} \geq 0$. While the choice of keeping $N_\nu$ fixed does not affect constraints from CMB measurements (what matters is $N_{\text{eff}}$), it could change the sensitivity to $\sum m_\nu$ and $N_{\text{eff}}$ based on galaxy clusters data. Indeed, changing $N_\nu$ would change the mass of each massive neutrino and thus its free-streaming length beyond which the power spectrum is suppressed (see Eq. 2.3). We checked the case with fixed $\Delta N_{\text{eff}}$ and $N_\nu$ as free parameter and we find no qualitative changes in our results. This means that the data are not sensitive to the exact position of the break in the power spectrum induced by free-streaming neutrinos, and thus to the neutrino mass hierarchy. Table 3 shows the joint constraints on the sum of neutrino masses and on the effective number of neutrino species from Planck data alone and the combination of Planck and Euclid-Cl datasets. Looking at Planck data alone, the quality of the constraints on $\sum m_\nu$ are nearly unchanged from the single-parameter extensions discussed earlier, as it would be expected for independent parameters. Indeed, $\sum m_\nu$ and $N_{\text{eff}}$ are constrained by different features in the CMB spectra: the early integrated Sachs-Wolfe effect for $\sum m_\nu$, the damping scale and the position of the acoustic peaks for $N_{\text{eff}}$ [see e.g. 10, and references therein]. However, since both $\sum m_\nu$ and $N_{\text{eff}}$ have similar effects on the matter power spectrum (see § 2), the correlation of the two degrades the upper bound on the sum of neutrino masses inferred from Euclid-Cl+Planck data by $\sim 30\%$ to $\sum m_\nu < 0.040$[eV] at 95%CL. With this accuracy, it would still be possible a 2$\sigma$ detection of neutrino masses in the minimal normal hierarchy scenario. Constraining $N_{\text{eff}}$ is mainly achieved through CMB measurements of the redshift of the matter-radiation equality $z_{eq}$ and the baryon density $\Omega_b h^2$. However, keeping $z_{eq}$ and $\Omega_b h^2$ fixed as $N_{\text{eff}}$ increases can be achieved by increasing the cold dark matter density $\Omega_\nu h^2$, which displays a large correlation with $N_{\text{eff}}$ [89]. Euclid-Cl data alone are unable to provide constraints on the number of effective species. However, the inclusion of clusters dataset allows to significantly improve the measurements of $\Omega_m h^2$ (by constraining $\sigma_8$), thus reducing the 2$\sigma$ error on the effective number of neutrino by a factor larger than 2.5 from 0.36 to 0.14 (see right panel of Fig. 6). After the inclusion of Euclid-Cl data $N_{\text{eff}}$ still exhibits strong degeneracies with many cosmological parameters (e.g. $\Omega_m h^2$, $H_0$ and $n_s$), and a correlation of $\sim 0.5$ with $\sum m_\nu$ (see left panel of Fig. 6).

4.3 Extended models: $w$CDM+$m_\nu$+$N_{\text{eff}}$ and curved Universe

Next, we consider how the constraints on $\sum m_\nu$ and $N_{\text{eff}}$ are affected when additional degrees of freedom are introduced in the cosmological model. The effect on ($\sum m_\nu$, $N_{\text{eff}}$) of adding these degree of freedom to the $\Lambda$CDM+$m_\nu$+$N_{\text{eff}}$ model are shown in Fig. 7 and listed in Table 4. We start by considering a constant dark energy equation of state ($w \neq -1$). Neutrino
Figure 6: The 68% and 95% CL contours in the $N_{\text{eff}} - (\Omega_m, \sum m_\nu)$ planes for a $\Lambda$CDM+$m_\nu+N_{\text{eff}}$ model, from: Planck (green contours) and Planck+Euclid-Cl (blue contours) data.

Table 4: Constraints on $\sum m_\nu$ and $N_{\text{eff}}$ for the two parameter extensions $(w, \Omega_k)$ from Euclid-Cl+Planck datasets.

| Data          | Planck+Euclid-Cl |
|---------------|------------------|
| Model         | $w\Lambda$CDM+$m_\nu+N_{\text{eff}}$ | $\Lambda$CDM+$m_\nu+N_{\text{eff}}+\Omega_k$ |
| $\sum m_\nu$ [eV] | 68% CL $< 0.024$ | $< 0.024$ |
|               | 95% CL $< 0.046$ | $< 0.046$ |
| $N_{\text{eff}}$ | 95% CL $< 3.16$ | $< 3.17$ |

properties ($\sum m_\nu, N_{\text{eff}}$) and $w$ are generally degenerate because they can both affect the shape of the matter and CMB power spectra [e.g. 90]. Looking at Fig. 8 (a), we indeed see this degeneracy in the plane $N_{\text{eff}} - w$, which displays a correlation of $\sim 0.5$, whereas the parameters $w$ and $\sum m_\nu$ show almost no correlation. The Euclid clusters catalog, probing the evolution of the LSS up to $z \sim 2$, will be able to put tight constraints on the dark energy equation of state; we find for the combination of Planck and Euclid-Cl data: $-1.011 < w < -0.987$ (95% CL). Given the small uncertainty on $w$ the constraints on neutrino mass and effective number of species are only slightly degraded when $w$ is allowed to vary; the 95% CL upper limit for $N_{\text{eff}}$ is relaxed from 3.14 to 3.16 due to the degeneracy with $w$. Whereas, the 95% CL upper limit for $\sum m_\nu$ undergoes only a small degradation from 0.040 eV to 0.046 eV, caused by the weak constraints on parameters that are correlated with $\sum m_\nu$ induced by the extension of the parameter space. Secondly, we relax the prior on the curvature of the universe by considering the case $\Lambda$CDM+$m_\nu+N_{\text{eff}}+\Omega_k$. Since current data do not support departures from the flat $\Lambda$CDM model either through $\Omega_k \neq 0$ or $w \neq -1$, we introduce these parameters separately. From the combination of Planck and Euclid-Cl datasets we obtain, for the curvature parameter, the following constraint: $-0.0024 < \Omega_k < 0.0024$ (95% CL). As CMB power spectrum suffer from a well known “geometrical degeneracy” [e.g. 91, 92], Euclid-CL data considerably improves the error on $\Omega_k$ breaking such degeneracy thanks to the tight constraint on $\Omega_m$ (given by the growth information encoded in the dataset).
spatial curvature mainly affects the expansion rate via the Friedmann equation, as well as the total neutrino mass and number of effective species do. As it can be seen in Fig. 8 (b), this results in a correlation with both $\sum m_\nu$ and $N_{\text{eff}}$ of the order of $\sim 0.5$ and $\sim 0.6$, respectively. Despite these quite large degeneracies with $\Omega_k$, the small error associated to the curvature parameter leads to a slight relaxation of the constraints on neutrino properties: the upper limit for neutrino mass degrades by $\sim 10\%$, passing from 0.040 eV to 0.046 eV (95%CL), while the 2σ error on $N_{\text{eff}}$ shift from 0.14 to 0.17, a 20% degradation.

Thus, in both cases, the parameter extension entails a relaxation of the constraints on $\sum m_\nu$ and $N_{\text{eff}}$; nonetheless, given the high accuracy with which $w$ and $\Omega_k$ are expected to be measured, the survey would still allow a 2σ detection of neutrino mass in the minimal normal hierarchy scenario and reveal the presence of possible extra relativistic species.

### 4.4 Nuisance parameters

Finally, to assess the effect of an uncertain knowledge of cluster masses on $\sum m_\nu$ and $N_{\text{eff}}$ constraints, we treat the $\Lambda$CDM+$m_\nu+N_{\text{eff}}$ case with the four nuisance parameters as fitting parameters, following the so-called self-calibration method [e.g. 52, 61, 64]. The results are listed in Table 5. We start with the the over-conservative assumption of no priors on all the four nuisance parameters ($\Lambda$CDM+$m_\nu+N_{\text{eff}}$+nuis model). The uncertainties on scaling relation parameters compromise our ability to recover the halo mass function from cluster data, thus reducing the cosmological information achievable from cluster number counts. This results in a larger error for the parameters that are primarily constrained by cluster

**Figure 7**: The marginalized one-dimensional posteriors for $\sum m_\nu$ (left) and $N_{\text{eff}}$ (right) for different parameter extensions from the combination of Euclid-Cl and Planck datasets.

**Table 5**: Constraints on $\sum m_\nu$ and $N_{\text{eff}}$ for $\Lambda$CDM+$m_\nu+N_{\text{eff}}$ models with free nuisance parameters.

| Data          | Planck + Euclid-Cl | Planck + Euclid-Cl |
|---------------|--------------------|--------------------|
| Model         | $\Lambda$CDM+$m_\nu+N_{\text{eff}}$+nuis | $\Lambda$CDM+$m_\nu+N_{\text{eff}}$+nuis$_{\text{ev}}$ |
| $\sum m_\nu$ [eV] | 68% CL < 0.049 | 68% CL < 0.049 |
|               | 95% CL < 0.083 | 95% CL < 0.083 |
| $N_{\text{eff}}$ | 95% CL < 3.18  | 95% CL < 3.18  |
|               |                  |                  |
Figure 8: Joint two dimensional marginalized constraints on $w$ (a) and $\Omega_k$ (b) against $(\Sigma m_\nu, N_{\text{eff}})$ at 68% and 95% CL. The confidence regions are respectively for the extended parameter space $w\Lambda\text{CDM} + m_\nu + N_{\text{eff}}$ and $\Lambda\text{CDM} + m_\nu + N_{\text{eff}} + \Omega_k$, using data from Euclid-Cl+Planck.

number counts, in particular for $\sigma_8$, the normalization of the power spectrum. Looking at Fig. 9 (left panel), the constraints on $\sigma_8$ are relaxed by a factor of $\sim 10$ compared to the $\Lambda\text{CDM} + m_\nu + N_{\text{eff}}$ model, and the parameter recovers a large degeneration with $\Sigma m_\nu$ of the order of $\sim 0.65$. This effect, along with the degradation of other parameters errors (e.g. $\sigma(\Omega_m)$), entails a relaxation of the upper limit for $\Sigma m_\nu$ by a factor larger than two, from 0.040 eV to 0.083 eV. With these loose constraints, in the case of minimal normal hierarchy scenario, it would not be possible to have a two $\sigma$ detection of neutrino mass. Because the constraints on neutrino mass from cluster number counts rely on the evolution of the high-mass end of the mass function, $\Sigma m_\nu$ is rather degenerate with $\alpha$ and $\beta$, the two nuisance parameters which control the evolution of the systematic bias and intrinsic scatter (see Eq. 3.4). To emphasize the role played by the uncertain redshift evolution of the nuisance parameter on the determination of $\Sigma m_\nu$ we show in Fig. 9 the contours for a model with $\alpha$ and $\beta$ kept fixed ($\Lambda\text{CDM} + m_\nu + N_{\text{eff}} + \text{nuis}_{\text{ev}}$ model). In this case the degradation of the total neutrino mass constraints with respect the $\Lambda\text{CDM} + m_\nu + N_{\text{eff}}$ model is only of $\sim 40\%$, from 0.040 eV to 0.056 eV. In other words, an accurate knowledge of the redshift evolution of the nuisance parameter improves the 2$\sigma$ upper limit of $\Sigma m_\nu$ by $\sim 33\%$ compared to the previous case with no prior on the nuisance parameters.

Likewise, the forecast error on $N_{\text{eff}}$ is influenced by the loss of constraining power of the cluster number counts data, even if to a lesser extent than the bounds on $\Sigma m_\nu$, since the constraints on $N_{\text{eff}}$ are primarily contributed by CMB measurements. The 2$\sigma$ upper limit
shifts from $N_{\text{eff}} < 3.14$ to $N_{\text{eff}} < 3.16$ and $N_{\text{eff}} < 3.18$, in the model with strong evolution prior and free nuisance parameters, respectively. In these case the degradation is mainly due to the larger error associated to $\Omega_m h^2$, which is highly degenerate with $N_{\text{eff}}$ as explained in section 4.2 and shown in Fig. 9 (right panel). We remind that the results for the model with no prior have to be regarded as an upper limit on the error introduced by the uncertain knowledge of the scaling relation; nevertheless, these results highlight the importance of having robust calibration of the scaling relation, and in particular of their evolution with redshift, to fully exploit the cosmological information contained in the Euclid cluster catalog.

5 Conclusions

In this paper, we presented forecasts on the capability of a future photometric galaxy cluster survey such as Euclid, in combination with Planck-like data, to provide constraints on neutrino properties. Specifically, we rely on two observables: the cluster number counts and their power spectrum. Our analysis is based on the Markov Chain Monte Carlo methods rather than the Fisher Matrix technique, which results in more reliable error bars. We start by considering a reference ΛCDM model in agreement with the recent results of WMAP 9-yr.

In order to study possible degeneracies with $\sum m_\nu$, besides the ΛCDM model with massive neutrino, we also consider models with $N_{\text{eff}}$ effective number of relativistic species, a constant dark energy equation of state $w$ and curvature. Following the self-calibration approach, along with the other cosmological parameter, we decide to explore also the effect of leaving free the nuisance parameters that describe the relation between cluster optical richness and mass, its scatter and redshift evolution.

Our results can be summarized as follows:
From the combination of Euclid number counts and clustering data we obtain a 2σ upper limits for the total neutrino mass of $\sum m_\nu < 0.35 \text{ eV}$, comparable with present constraints from the combination of CMB and LSS probes [e.g. 7, 10, 38]. When Planck data are added to the Euclid-Cl ones the error on $\sum m_\nu$ is reduced by a factor larger than 10 to $\sum m_\nu < 0.031 \text{ eV}$. With this accuracy the total neutrino mass could be detected at 2σ level even in the minimal normal hierarchy scenario. The large improvement is due to the different degeneracies present between Euclid and Planck that are broken once the two experiments are combined.

Because the effective number of neutrino species is degenerate with the sum of neutrino masses, varying $N_{\text{eff}}$ entails a relaxation of 2σ error bars on $\sum m_\nu$ by $\sim 30\%$ in the Planck+Euclid-Cl case. Still, the 2σ error is lower than the minimum neutrino mass admitted by neutrino oscillation experiments. The Euclid-Cl dataset is unable to constraints $N_{\text{eff}}$ by itself, but improves the 2σ upper limits on $N_{\text{eff}}$ from 3.36 using Planck-only, to 3.14 in the Planck+Euclid-Cl case. The improvement is mainly due to the tighter constraints on $\Omega_m h^2$ provided by the Euclid-Cl datasets.

In models with varying $w$ or $\Omega_k$ the 2σ error on $\sum m_\nu$ is relaxed only by $\sim 10\%$. In both cases the high accuracy with which $w$ or $\Omega_k$ are constrained by the Planck+Euclid-Cl data prevents the error on $\sum m_\nu$ from being largely degraded. As for $N_{\text{eff}}$, the parameter shows a correlation of the order of $\sim 0.5$ with both $w$ and $\Omega_k$, which shifts the 2σ upper limit for $N_{\text{eff}}$ to 3.16 and 3.17, respectively.

When nuisance parameters are considered in a conservative way (no prior) our ability to recover the halo mass function from cluster data is compromised. The degradation of cosmological information results in a $\sim 2$ times larger 2σ error for neutrino masses ($\sum m_\nu < 0.083 \text{ eV}$) and a degradation of $\sim 30\%$ of the 2σ error on the effective number of neutrinos ($N_{\text{eff}} < 3.18$). In this case the accuracy would not be sufficient for detecting the total neutrino mass with good significance in the minimal normal hierarchy scenario. Whereas, assuming a perfect knowledge of the redshift evolution of the nuisance parameters we partially recover the informations contained in cluster number counts data. In this case the 2σ upper limit for $\sum m_\nu$ is degraded only by 40% to $\sum m_\nu < 0.056 \text{ eV}$, while the 2σ error on the effective number of neutrinos degrades by $\sim 15\%$ to $N_{\text{eff}} < 3.16$.

It is worth reminding that in our analysis we did not include the effect of redshift space distortions in the distribution of galaxy clusters induced by peculiar velocities. This effect should be in principle included when forecasting the cosmological constraining power of future cluster surveys; indeed, as demonstrated by [52], the inclusion of redshift space distortions carries significant cosmological information through the growth rate of density perturbations.

As a concluding remark, we emphasize once again the importance to provide an accurate calibration of the scaling relation between the observable quantity on which cluster selection is based, optical richness in this case, and cluster mass. Thanks to the exquisite imaging quality expected for the Euclid survey, weak lensing masses for individual objects will be available for a significant fraction of the clusters identified from the photometric selection. At the same time, staking analysis will provide an accurate calibration of the relation between weak lensing masses and richness. For this reason, the Euclid cluster survey will represent a powerful complement to galaxy clustering and cosmic shear analyses to constrain cosmology through the growth of perturbations.
Note added: After the submission of this work cosmological results from the first 15.5 months of Planck operations has been published [93]. The fiducial values adopted in this work are found to be consistent within 2σ with the mean values obtained by Planck Collaboration. For this data release the authors did not use polarization spectra, so we can not make a direct comparison of our forecast with the actual constraints from Planck. However, for a ΛCDM+$m_\nu$ model, using Planck temperature power spectrum in combination with a WMAP-9yr polarization low-multipole likelihood the authors obtained $\sum m_\nu < 0.933$ (95%CL), compatible with our expected error.

Acknowledgments

We thank Lauro Moscardini and Jochen Weller for useful comments, and an anonymous referee for constructive criticisms that helped improving the presentation of the results. We acknowledge financial support from the agreement ASI/INAF I/023/12/0. This work has been supported by the PRIN-INAF09 project “Towards an Italian Network for Computational Cosmology”, by the PRIN-MIUR09 “Tracing the growth of structures in the Universe”, and by the PD51 INFN grant. MV is supported by the ERC Starting Grant CosmoIGM. JX is supported by the National Youth Thousand Talents Program and the grants No. Y25155E0U1 and No. Y3291740S3.

References

[1] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and et al., “Global analysis of neutrino masses, mixings, and phases: Entering the era of leptonic CP violation searches,” Physical Review D 86 no. 1, (July, 2012) 013012, arXiv:1205.5254 [hep-ph].

[2] D. V. Forero, M. Tórtola, and J. W. F. Valle, “Global status of neutrino oscillation parameters after Neutrino-2012,” Physical Review D 86 no. 7, (Oct., 2012) 073012, arXiv:1205.4018 [hep-ph].

[3] A. D. Dolgov, “Neutrinos in cosmology,” Physics Reports 370 (Nov., 2002) 333–535, arXiv:hep-ph/0202122.

[4] J. Lesgourgues and S. Pastor, “Massive neutrinos and cosmology,” Physics Reports 429 (July, 2006) 307–379, arXiv:astro-ph/0603494.

[5] S. A. Thomas, F. B. Abdalla, and O. Lahav, “Upper Bound of 0.28 eV on Neutrino Masses from the Largest Photometric Redshift Survey,” Physical Review Letters 105 no. 3, (July, 2010) 031301, arXiv:0911.5291 [astro-ph.CO].

[6] M. Viel, M. G. Haehnelt, and V. Springel, “The effect of neutrinos on the matter distribution as probed by the intergalactic medium,” Journal of Cosmology and Astroparticle Physics 6 (June, 2010) 15, arXiv:1003.2422 [astro-ph.CO].

[7] G.-B. Zhao, S. Saito, W. J. Percival, A. J. Ross, F. Montesano, and et al., “The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: weighing the neutrino mass using the galaxy power spectrum of the CMASS sample,” ArXiv e-prints (Nov., 2012), arXiv:1211.3741 [astro-ph.CO].

[8] J.-Q. Xia, B. R. Granett, M. Viel, S. Bird, L. Guzzo, and et al., “Constraints on massive neutrinos from the CFHTLS angular power spectrum,” Journal of Cosmology and Astroparticle Physics 6 (June, 2012) 10, arXiv:1203.5105 [astro-ph.CO].
S. Riemer-Sorensen, D. Parkinson, T. Davis, and C. Blake, “Simultaneous constraints on the number and mass of relativistic species,” ArXiv e-prints (Oct., 2012), arXiv:1210.2131 [astro-ph.CO].

G. Hinshaw, D. Larson, E. Komatsu, D. N. Spergel, C. L. Bennett, and et al., “Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results,” ArXiv e-prints (Dec., 2012), arXiv:1212.5226 [astro-ph.CO].

S. Joudaki, “Constraints on Neutrino Mass and Light Degrees of Freedom in Extended Cosmological Parameter Spaces,” ArXiv e-prints (Jan., 2012), arXiv:1202.0005 [astro-ph.CO].

U. Seljak, A. Slosar, and P. McDonald, “Cosmological parameters from combining the Lyman-α forest with CMB, galaxy clustering and SN constraints,” Journal of Cosmology and Astroparticle Physics 10 (Oct., 2006) 14, arXiv:astro-ph/0604335.

The ALEPH Collaboration, the DELPHI Collaboration, the L3 Collaboration, the OPAL Collaboration, and the LEP Electroweak Working Group, “Precision Electroweak Measurements on the Z Resonance,” ArXiv High Energy Physics - Experiment e-prints (Sept., 2005), arXiv:hep-ex/0509008.

G. Steigman, “Equivalent Neutrinos, Light WIMPs, and the Chimera of Dark Radiation,” ArXiv e-prints (Feb., 2013), arXiv:1303.0049 [astro-ph.CO].

A. Aguilar, L. B. Auerbach, R. L. Burman, and et al., “Evidence for Neutrino Oscillations from the Observation of Electron Anti-neutrinos in a Muon Anti-Neutrino Beam,” Physical Review D 64 no. 11, (Dec., 2001) 112007, arXiv:hep-ex/0104049.

M. Maltoni and T. Schwetz, “Sterile neutrino oscillations after first MiniBooNE results,” Physical Review D 76 no. 9, (Nov., 2007) 093005, arXiv:0705.0107 [hep-ph].

A. Melchiorri, O. Mena, S. Palomares-Ruiz, S. Pascoli, A. Slosar, and et al., “Sterile neutrinos in light of recent cosmological and oscillation data: a multi-flavor scheme approach,” Journal of Cosmology and Astroparticle Physics 1 (Jan., 2009) 36, arXiv:0810.5133 [hep-ph].

J. Kopp, M. Maltoni, and T. Schwetz, “Are There Sterile Neutrinos at the eV Scale?,” Physical Review Letters 107 no. 9, (Aug., 2011) 091801, arXiv:1103.4570 [hep-ph].

Z. Hou, C. L. Reichardt, K. T. Story, B. Follin, R. Keisler, and et al., “Constraints on Cosmology from the Cosmic Microwave Background Power Spectrum of the 2500-square degree SPT-SZ Survey,” ArXiv e-prints (Dec., 2012), arXiv:1212.6267 [astro-ph.CO].

J. L. Sievers, R. A. Hlozek, M. R. Nolta, V. Acquaviva, G. E. Addison, and et al., “The Atacama Cosmology Telescope: Cosmological parameters from three seasons of data,” ArXiv e-prints (Jan., 2013), arXiv:1301.0824 [astro-ph.CO].

D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. R. Nolta, and et al., “Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and...
WMAP-derived Parameters,” *Astrophysical Journal Supplement* **192** (Feb., 2011) 16, arXiv:1001.4635 [astro-ph.CO].

[25] S. Borgani, P. Rosati, P. Tozzi, S. A. Stanford, P. R. Eisenhardt, and *et al.*, “Measuring Ω_m with the ROSAT Deep Cluster Survey,” *Astrophysical Journal* **561** (Nov., 2001) 13–21, arXiv:astro-ph/0106428.

[26] T. H. Reiprich and H. Böhringer, “The Mass Function of an X-Ray Flux-limited Sample of Galaxy Clusters,” *Astrophysical Journal* **567** (Mar., 2002) 716–740, arXiv:astro-ph/0111285.

[27] A. Vikhlinin, A. V. Kravtsov, R. A. Burenin, H. Ebeling, W. R. Forman, and *et al.*, “Chandra Cluster Cosmology Project III: Cosmological Parameter Constraints,” *Astrophysical Journal* **692** (Feb., 2009) 1060–1074, arXiv:0812.2720.

[28] E. Rozo, R. H. Wechsler, E. S. Rykoff, J. T. Annis, M. R. Becker, and *et al.*, “Cosmological Constraints from the Sloan Digital Sky Survey maxBCG Cluster Catalog,” *Astrophysical Journal* **708** (Jan., 2010) 645–660, arXiv:0909.4329 [astro-ph.CO].

[29] S. W. Allen, A. E. Evrard, and A. B. Mantz, “Cosmological Parameters from Observations of Galaxy Clusters,” *Annual Review of Astronomy & Astrophysics* **49** (Sept., 2011) 409–470, arXiv:1103.4829 [astro-ph.CO].

[30] D. Rapetti, C. Blake, S. W. Allen, A. Mantz, D. Parkinson, and *et al.*, “A combined measurement of cosmic growth and expansion from clusters of galaxies, the CMB and galaxy clustering,” *ArXiv e-prints* (May, 2012), arXiv:1205.5556 [astro-ph.CO].

[31] A. Mantz, S. W. Allen, D. Rapetti, and H. Ebeling, “The observed growth of massive galaxy clusters - I. Statistical methods and cosmological constraints,” *Monthly Notices of the Royal Astronomical Society* **406** (Aug., 2010) 1759–1772, arXiv:0909.3098 [astro-ph.CO].

[32] A. Vikhlinin, A. V. Kravtsov, R. A. Burenin, H. Ebeling, W. R. Forman, and *et al.*, “Chandra Cluster Cosmology Project III: Cosmological Parameter Constraints,” *Astrophysical Journal* **692** (Feb., 2009) 1060–1074, arXiv:0812.2720.

[33] S. Borgani and L. Guzzo, “X-ray clusters of galaxies as tracers of structure in the Universe,” *Nature* **409** (Jan., 2001) 39–45, arXiv:astro-ph/0012439.

[34] P. Schuecker, H. Böhringer, C. A. Collins, and L. Guzzo, “The REFLEX galaxy cluster survey. VII. Omega_m and sigma_8 from cluster abundance and large-scale clustering,” *Astronomy and Astrophysics* **398** (Feb., 2003) 867–877, arXiv:astro-ph/0208251.

[35] A. Mana, T. Giannantonio, J. Weller, B. Hoyle, G. Huetsi, and *et al.*, “Combining clustering and abundances of galaxy clusters to test cosmology and primordial non-Gaussianity,” *ArXiv e-prints* (Mar., 2013), arXiv:1303.0287 [astro-ph.CO].

[36] A. V. Kravtsov and S. Borgani, “Formation of Galaxy Clusters,” *Annual Review of Astronomy & Astrophysics* **50** (Sept., 2012) 353–409, arXiv:1205.5556 [astro-ph.CO].

[37] A. Mantz, S. W. Allen, and D. Rapetti, “The observed growth of massive galaxy clusters - IV. Robust constraints on neutrino properties,” *Monthly Notices of the Royal Astronomical Society* **406** (Aug., 2010) 1805–1814, arXiv:0911.1788 [astro-ph.CO].

[38] R. A. Burenin and A. Vikhlinin, “Cosmological parameters constraints from galaxy cluster mass function measurements in combination with other cosmological data,” *Astronomy Letters* **38** (June, 2012) 347–363, arXiv:1202.2889 [astro-ph.CO].

[39] R. A. Burenin, A. Vikhlinin, A. Hornstrup, H. Ebeling, H. Quintana, and A. Mescheryakov, “The 400 Square Degree ROSAT PSPC Galaxy Cluster Survey: Catalog and Statistical
Calibration,” *Astrophysical Journal Supplement* **172** (Oct., 2007) 561–582, arXiv:astro-ph/0610739.

[40] E. Rozo, E. S. Rykov, A. Evrard, M. Becker, T. McKay, and *et al.*, “Constraining the Scatter in the Mass-richness Relation of maxBCG Clusters with Weak Lensing and X-ray Data,” *Astrophysical Journal* **699** (July, 2009) 768–781, arXiv:0809.2794.

[41] C. Carbone, C. Fedeli, L. Moscardini, and A. Cimatti, “Measuring the neutrino mass from future wide galaxy cluster catalogues,” *Journal of Cosmology and Astroparticle Physics* **3** (Mar., 2012) 23, arXiv:1112.4810 [astro-ph.CO].

[42] L. Perotto, J. Lesgourgues, S. Hannestad, H. Tu, and Y. Y Y Wong, “Probing cosmological parameters with the CMB: forecasts from Monte Carlo simulations,” *Journal of Cosmology and Astroparticle Physics* **10** (Oct., 2006) 13, arXiv:astro-ph/0606227.

[43] L. Wolz, M. Kilbinger, J. Weller, and T. Giannantonio, “On the validity of cosmological Fisher matrix forecasts,” *Journal of Cosmology and Astroparticle Physics* **9** (Sept., 2012) 9, arXiv:1205.3984 [astro-ph.CO].

[44] E. Giusarma, R. de Putter, and O. Mena, “Testing standard and non-standard neutrino physics with cosmological data,” *ArXiv e-prints* (Nov., 2012), arXiv:1211.2154 [astro-ph.CO].

[45] G. Mangano, G. Miele, S. Pastor, T. Pinto, O. Pisanti, and *et al.*, “Relic neutrino decoupling including flavour oscillations,” *Nuclear Physics B* **729** (Nov., 2005) 221–234, arXiv:hep-ph/0506164.

[46] S. Hannestad, I. Tamborra, and T. Tram, “Thermalisation of light sterile neutrinos in the early universe,” *Journal of Cosmology and Astroparticle Physics* **7** (July, 2012) 25, arXiv:1204.5861 [astro-ph.CO].

[47] S. Palomares-Ruiz, S. Pascoli, and T. Schwetz, “Explaining LSND by a decaying sterile neutrino,” *Journal of High Energy Physics* **9** (Sept., 2005) 48, arXiv:hep-ph/0505216.

[48] D. Hooper, F. S. Queiroz, and N. Y. Gnedin, “Nonthermal dark matter mimicking an additional neutrino species in the early universe,” *Physical Review D* **85** no. 6, (Mar., 2012) 063513, arXiv:1111.6599 [astro-ph.CO].

[49] T. L. Smith, E. Pierpaoli, and M. Kamionkowski, “New Cosmic Microwave Background Constraint to Primordial Gravitational Waves,” *Physical Review Letters* **97** no. 2, (July, 2006) 021301, arXiv:astro-ph/0603144.

[50] E. Calabrese, D. Huterer, E. V. Linder, A. Melchiorri, and L. Pagano, “Limits on dark radiation, early dark energy, and relativistic degrees of freedom,” *Physical Review D* **83** no. 12, (June, 2011) 123504, arXiv:1103.4132 [astro-ph.CO].

[51] W. Hu, D. J. Eisenstein, and M. Tegmark, “Weighing Neutrinos with Galaxy Surveys,” *Physical Review Letters* **80** (June, 1998) 5255–5258, arXiv:astro-ph/9712057.

[52] B. Sartoris, S. Borgani, P. Rosati, and J. Weller, “Probing dark energy with the next generation X-ray surveys of galaxy clusters,” *Monthly Notices of the Royal Astronomical Society* **423** (July, 2012) 2503–2517, arXiv:1112.0327 [astro-ph.CO].

[53] J. Tinker, A. V. Kravtsov, A. Klypin, K. Abazajian, M. Warren, and *et al.*, “Toward a Halo Mass Function for Precision Cosmology: The Limits of Universality,” *Astrophysical Journal* **688** (Dec., 2008) 709–728, arXiv:0803.2706.

[54] J. Brandbyge, S. Hannestad, T. Haugbolle, and Y. Y. Y. Wong, “Neutrinos in non-linear structure formation – the effect on halo properties,” *Journal of Cosmology and Astroparticle Physics* **9** (Sept., 2010) 14–+,
[55] F. Marulli, C. Carbone, M. Viel, L. Moscardini, and A. Cimatti, “Effects of massive neutrinos on the large-scale structure of the Universe,” 
*Monthly Notices of the Royal Astronomical Society* (Sept., 2011) 1424, 
*arXiv:1103.0278 [astro-ph.CO]*.

[56] F. Villaescusa-Navarro, S. Bird, C. Peña-Garay, and M. Viel, “Non-linear evolution of the cosmic neutrino background,” *ArXiv e-prints* (Dec., 2012), *arXiv:1212.4855 [astro-ph.CO]*.

[57] R. K. Sheth and G. Tormen, “Large-scale bias and the peak background split,” 
*Monthly Notices of the Royal Astronomical Society* 308 (Sept., 1999) 119–126, 
*arXiv:astro-ph/9901122*.

[58] A. Jenkins, C. S. Frenk, S. D. M. White, J. M. Colberg, S. Cole, A. E. Evrard, H. M. P. Couchman, and N. Yoshida, “The mass function of dark matter haloes,” 
*Monthly Notices of the Royal Astronomical Society* 321 (Feb., 2001) 372–384, 
*arXiv:astro-ph/0005260*.

[59] M. S. Warren, K. Abazajian, D. E. Holz, and L. Teodoro, “Precision Determination of the Mass Function of Dark Matter Halos,” 
*Astrophysical Journal* 646 (Aug., 2006) 881–885, 
*arXiv:astro-ph/0506395*.

[60] M. Crocce, P. Fosalba, F. J. Castander, and E. Gaztañaga, “Simulating the Universe with MICE: the abundance of massive clusters,” 
*Monthly Notices of the Royal Astronomical Society* 403 (Apr., 2010) 1353–1367, 
*arXiv:0907.0019 [astro-ph.CO]*.

[61] M. Lima and W. Hu, “Self-calibration of cluster dark energy studies: Observable-mass distribution,” 
*Physical Review D* 72 no. 4, (Aug., 2005) 043006–+, *arXiv:astro-ph/0503363*.

[62] B. Sartoris, S. Borgani, C. Fedeli, S. Matarrese, L. Moscardini, and et al., “The potential of X-ray cluster surveys to constrain primordial non-Gaussianity,” 
*Monthly Notices of the Royal Astronomical Society* 407 (Oct., 2010) 2339–2354, 
*arXiv:1003.0841 [astro-ph.CO]*.

[63] R. Laureijs, J. Amiaux, S. Arduini, J. Augu`eres, J. Brinchmann, R. Cole, M. Cropper, C. Dabin, L. Duvet, A. Ealet, and et al. et al., “Euclid Definition Study Report,” *ArXiv e-prints* (Oct., 2011), *arXiv:1110.3193 [astro-ph.CO]*.

[64] S. Majumdar and J. J. Mohr, “Self-Calibration in Cluster Studies of Dark Energy: Combining the Cluster Redshift Distribution, the Power Spectrum, and Mass Measurements,” 
*Astrophysical Journal* 613 (Sept., 2004) 41–50, *arXiv:astro-ph/0305341*.

[65] J. L. Tinker, B. E. Robertson, A. V. Kravtsov, A. Klypin, M. S. Warren, and et al., “The Large-scale Bias of Dark Matter Halos: Numerical Calibration and Model Tests,” 
*Astrophysical Journal* 724 (Dec., 2010) 878–886, *arXiv:1001.3162 [astro-ph.CO]*.

[66] A. Lewis, A. Challinor, and A. Lasenby, “Efficient Computation of Cosmic Microwave Background Anisotropies in Closed Friedmann-Robertson-Walker Models,” 
*Astrophysical Journal* 538 (Aug., 2000) 473–476, *arXiv:astro-ph/9911177*.

[67] S. Bird, M. Viel, and M. G. Haehnelt, “Massive neutrinos and the non-linear matter power spectrum,” 
*Monthly Notices of the Royal Astronomical Society* 420 (Mar., 2012) 2551–2561, 
*arXiv:1109.4416 [astro-ph.CO]*.

[68] G. Hütsi, “Power spectrum of the maxBCG sample: detection of acoustic oscillations using galaxy clusters,” 
*Monthly Notices of the Royal Astronomical Society* 401 (Feb., 2010) 2477–2489, 
*arXiv:0910.0492 [astro-ph.CO]*.

[69] W. J. Percival and M. White, “Testing cosmological structure formation using redshift-space
distortions,” *Monthly Notices of the Royal Astronomical Society** 393 (Feb., 2009) 297–308, arXiv:0808.0003.

[70] A. Lewis and S. Bridle, “Cosmological parameters from CMB and other data: A Monte Carlo approach,” *Physical Review D* 66 no. 10, (Nov., 2002) 103511, arXiv:astro-ph/0205436.

[71] E. Komatsu et al., “Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation,” *Astrophysical Journal Supplement* 192 (Feb., 2011) 18–+, arXiv:1001.4538 [astro-ph.CO].

[72] M. R. Becker and A. V. Kravtsov, “On the Accuracy of Weak-lensing Cluster Mass Reconstructions,” *Astrophysical Journal* 740 (Oct., 2011) 25–+, arXiv:1201.1681 [astro-ph.CO].

[73] E. Rasia, M. Meneghetti, R. Martino, S. Borgani, A. Bonafede, K. Dolag, S. Ettori, D. Fabjan, C. Giocoli, P. Mazzotta, J. Merten, M. Radovich, and L. Tornatore, “Lensing and x-ray mass estimates of clusters (simulations),” *New Journal of Physics* 14 no. 5, (May, 2012) 055018, arXiv:1201.1569 [astro-ph.CO].

[74] C. Cunha, “Cross-calibration of cluster mass observables,” *Physical Review D* 79 no. 6, (Mar., 2009) 063009, arXiv:0812.0583.

[75] H. A. Feldman, N. Kaiser, and J. A. Peacock, “Power-spectrum analysis of three-dimensional redshift surveys,” *Astrophysical Journal* 426 (May, 1994) 23–37, arXiv:astro-ph/9304022.

[76] W. Hu and A. V. Kravtsov, “Sample Variance Considerations for Cluster Surveys,” *Astrophysical Journal* 584 (Feb., 2003) 702–715, arXiv:astro-ph/0203169.

[77] Y.-T. Lin, J. J. Mohr, and S. A. Stanford, “Near-Infrared Properties of Galaxy Clusters: Luminosity as a Binding Mass Predictor and the State of Cluster Baryons,” *Astrophysical Journal* 591 (July, 2003) 749–763.

[78] J. F. Navarro, C. S. Frenk, and S. D. M. White, “A Universal Density Profile from Hierarchical Clustering,” *Astrophysical Journal* 490 (Dec., 1997) 493, arXiv:astro-ph/9611107.

[79] A. Gelman and D. B. Rubin, “Inference from Iterative Simulation Using Multiple Sequences,” *Statistical Science* 7 no. 4, (Nov., 1992) 457–472.

[80] C. Carbone, L. Verde, Y. Wang, and A. Cimatti, “Neutrino constraints from future nearly all-sky spectroscopic galaxy surveys,” *Journal of Cosmology and Astroparticle Physics* 3 (Mar., 2011) 30, arXiv:1012.2868 [astro-ph.CO].

[81] B. Audren, J. Lesgourges, S. Bird, M. G. Haehnelt, and M. Viel, “Neutrino masses and cosmological parameters from a Euclid-like survey: Markov Chain Monte Carlo forecasts including theoretical errors,” *ArXiv e-prints* (Oct., 2012), arXiv:1210.2194 [astro-ph.CO].

[82] J. Hamann, S. Hannestad, and Y. Y. Y. Wong, “Measuring neutrino masses with a future
galaxy survey,” *Journal of Cosmology and Astroparticle Physics* 11 (Nov., 2012) 52, arXiv:1209.1043 [astro-ph.CO].

[87] J.-Q. Xia, H. Li, G.-B. Zhao, and X. Zhang, “Probing for the Cosmological Parameters with Planck Measurement,” *International Journal of Modern Physics D* 17 (2008) 2025–2048, arXiv:0708.1111.

[88] D. S. Y. Mak and E. Pierpaoli, “Constraints on Neutrino Mass from Sunyaev–Zeldovich Cluster Surveys,” arXiv:1303.2081 [astro-ph.CO].

[89] Z. Hou, R. Keisler, L. Knox, M. Millea, and C. Reichardt, “How Additional Massless Neutrinos Affect the Cosmic Microwave Background Damping Tail,” ArXiv e-prints (Apr., 2011) , arXiv:1104.2333 [astro-ph.CO].

[90] J.-Q. Xia, G.-B. Zhao, and X. Zhang, “Cosmological neutrino mass limit and the dynamics of dark energy,” Physical Review D 75 no. 10, (May, 2007) 103505, arXiv:astro-ph/0609463.

[91] J. R. Bond, G. Efstathiou, and M. Tegmark, “Forecasting cosmic parameter errors from microwave background anisotropy experiments,” *Monthly Notices of the Royal Astronomical Society* 291 (Nov., 1997) L33–L41, arXiv:astro-ph/9702100.

[92] M. Zaldarriaga, D. N. Spergel, and U. Seljak, “Microwave Background Constraints on Cosmological Parameters,” *Astrophysical Journal* 488 (Oct., 1997) 1, arXiv:astro-ph/9702157.

[93] Planck Collaboration, P. A. R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown, F. Atrio-Barandela, J. Aumont, C. Baccigalupi, A. J. Banday, and et al. et al., “Planck 2013 results. XVI. Cosmological parameters,” ArXiv e-prints (Mar., 2013) , arXiv:1303.5076 [astro-ph.CO].

– 26 –