Impedance-matched microwave lens

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Abstract

A microwave lens with highly reduced reflectance, as compared to conventional dielectric lenses, is proposed. The lens is based on two-dimensional or three-dimensional transmission-line networks that can be designed to have an effective refractive index larger than one, while having almost perfect impedance matching with free space. The design principles are presented and an example lens is studied using commercial simulation software.
I. INTRODUCTION

Homogeneous dielectric materials having refractive index $n$ different from that in free space ($n = 1$ in free space), have been used for a long time in microwave lens applications [1, 2, 3]. The benefits of such lenses are their simple structure and straightforward design. One of the main drawbacks of such lenses is the evident impedance mismatch with the material surrounding the lens. This causes reflections from the lens and, thus, unwanted loss of power.

In this letter, we propose a novel method to overcome this drawback. The lens design which is introduced here is based on the use of transmission-line (TL) networks that have a different propagation constant than the surrounding medium, e.g., free space. Because the network impedance can be designed separately from the refractive index (unlike in a dielectric lens), the impedance matching with free space or some other medium can be obtained for different refractive indices by properly choosing the filling material and geometry of the transmission lines comprising the network. Other well-known methods to reduce reflections from a dielectric lens are e.g. restricting the refractive index to very low values, introduction of matching layers (analogous to quarter-wave transformers), and using inhomogeneous materials [1]. As compared to the method proposed here, the previous methods suffer either from very narrow operation bandwidth or complicated and expensive manufacturing.

Lens structures based on the use of transmission lines or waveguides have been widely reported in the literature, see, e.g., [3]. For example, structures consisting of stacked parallel metal plates, or arrays of parallel metal wires, behaving as artificial dielectrics with the permittivity less than one, have been used to create lenses. These lenses suffer from the same impedance matching problem as the dielectric ones. In addition, their effective refractive index strongly depends on the frequency. Other examples of previously proposed lenses (based on the use of transmission lines) use arrays of transmission lines in order to create the needed phase shift enabling the correct refraction angle at the lens surface [4, 5]. The difference with the lens studied in this letter is that the interfaces of the lenses discussed in [4, 5] behave effectively as antenna arrays and waves inside these lenses propagate only along one direction, whereas the lens discussed in this letter can be studied and modelled in a way similar to a homogeneous dielectric lens, since the period of the structure is much smaller than the wavelength and wave propagation is effectively isotropic.

Furthermore, the introduction of the transmission-line approach to the lens design
enables easy manufacturing of inhomogeneous lenses and even electrically controllable lenses, since the TL networks can be loaded by lumped elements (e.g. capacitors and/or inductors) and controllable elements such as varactors. In this letter we concentrate on a simple, i.e., an unloaded two-dimensional network, in order to demonstrate the feasibility of the proposed method.

The principle of the proposed approach to microwave lens design has been recently presented [6]. In the current letter we show in detail how a lens comprised of stacked networks of transmission lines can be designed in the same way as a traditional dielectric lens. We compare an example transmission-line lens to a reference dielectric lens and demonstrate the improvement of the impedance matching by using commercial simulation software. Also the focusing characteristics of the lenses are compared, validating the use of the lens design equations.

II. LENS DESIGN

The impedance mismatch related to traditional dielectric lenses occurs because the refractive index \( n = \sqrt{\varepsilon_r \mu_r} \) is larger than unity due to the relative permittivity \( \varepsilon_r > 1 \), while the relative permeability \( \mu_r \) is equal to unity as in free space. Therefore the wave impedance of the dielectric material differs from that in free space by the factor \( \sqrt{\mu_r \varepsilon_r} \). One solution to the impedance mismatch problem would be the use of magneto-dielectric materials having \( \mu_r > 1 \). However, in the microwave and millimeter-wave range the use of magneto-dielectrics is either impossible or prohibitively expensive.

As shown e.g. in [7, 8, 9], a two-dimensional or a three-dimensional transmission-line network can be treated as an effective medium with a certain effective wavenumber for waves travelling inside the network. As long as the period of the network is significantly smaller than the wavelength of waves in the network, the network is effectively isotropic [10]. The dispersion in an unloaded network can be derived from the rules for voltages and currents travelling in the network [7, 8, 9]. In [9] the dispersion and impedance were studied for three-dimensional unloaded and loaded networks, but in this letter we choose to use an unloaded two-dimensional network, for simplicity of consideration and design. The dispersion relation of such a network is [11]
FIG. 1: Dispersion in a transmission-line (TL) network for axial propagation ($k_y = 0, k = k_x$), in a homogeneous dielectric material with $\varepsilon_r = 4.66$ and in free space (“light line”).

$$\cos(k_x d) + \cos(k_y d) = 4 \cos^2(k_{TL} d/2) - 2,$$

(1)

where $k_{x,y}$ is the wavenumber in the network along axes $(x,y)$, $d$ is the period, and $k_{TL}$ is the wavenumber of waves in the transmission lines. The wavenumber in the network is $k = \sqrt{k_x^2 + k_y^2}$.

Using (1) we can plot the dispersion curve of a network and find the value of the relative permittivity of a reference dielectric having the same dispersion in the desired frequency band. In Fig. 1 we plot the dispersion diagram for an example network with $d=8$ mm and $k_{TL} = c_0/\sqrt{2.33}$ ($c_0$ is the speed of light in vacuum, and we assume that the dielectric material filling the transmission lines of the network has the relative permittivity equal to $\varepsilon_{r,TL} = 2.33$). Fig. 1 also shows the dispersion curves for a wave in free space (the light line) and in a homogeneous dielectric material with $\varepsilon_r = 2\varepsilon_{r,TL} = 4.66$. Comparing the plots, we can conclude that at lower frequencies, i.e., below 3 GHz in this case, the refractive index of the TL network, which is defined by the wavenumber, is close to that of a dielectric material with the refractive index $n = \sqrt{\varepsilon_r} = \sqrt{4.66}$. The studied network is also effectively isotropic approximately below 3 GHz, since for the diagonal propagation ($k_x = k_y$), the dispersion curve is the same as for the dielectric material with $\varepsilon_r = 4.66$ (dashed line in Fig. 1).

The impedance of the studied network can be derived from the equations for a three-dimensional unloaded network, presented e.g. in [9]. The network impedance in the two-dimensional case reads
FIG. 2: Impedance of the studied example transmission-line network as a function of frequency.

\[ Z = \frac{jZ_{\text{TL}} \sin(k_{\text{TL}}d/2)(1 + e^{-jkd})}{\cos(k_{\text{TL}}d/2)(1 - e^{-jkd})} = Z_{\text{TL}} \frac{\tan(k_{\text{TL}}d/2)}{\tan(kd/2)}, \]  

(2)

where \( Z_{\text{TL}} \) is the impedance of isolated sections of transmission line. Using (2) we have found that in order to obtain the network impedance of \( 120\pi \approx 377 \) \( \Omega \) (the free-space impedance) approximately at 3 GHz, the impedance of the TL sections comprising the network should be designed to have impedance of \( Z_{\text{TL}} = 585 \) \( \Omega \). The resulting network impedance as a function of the frequency, plotted using (2), is shown in Fig. 2. The impedance curve demonstrates that the network impedance varies quite smoothly and, therefore, good impedance matching with free space is expected in a relatively large frequency band around 3 GHz.

III. FULL-WAVE SIMULATIONS OF A REALIZABLE LENS

As discussed above, the example TL network has the period of \( d = 8 \) mm, \( k_{\text{TL}} = c_0/\sqrt{2.33} \), and the impedance (of the TLs) of 585 \( \Omega \). To obtain this, we have decided to use TLs made of parallel metal strips, embedded in a background material with \( \varepsilon_r,_{\text{TL}} = 2.33 \). Using the simple parallel-plate approximation, as was done in [6, 11], we have found that the suitable width and separation of the TLs are 1.266 mm and 3 mm, respectively [6].

The designed lens structure has been simulated with Ansoft High Frequency Structure Simulator (HFSS) software. First, to make sure that good impedance matching is obtained, the network is simulated as a transversally infinite (infinitely periodic in \( y- \) and \( z- \)directions) slab with a normally incident electromagnetic plane wave, having electric field parallel to the
z-axis, illuminating the slab. The simulation model, with the thickness of the TL network equal to $8d$, is shown in Fig. 3.

To couple waves from free space to the network and vice versa, we have introduced short sections of gradually enlarging transmission lines, as proposed in [11]. These lines of the “transition layer” have the length of 30 mm and the ratio between the width and height equal to the TLs in the network. At the end of these lines (at the interface with free space), the width (along the $y$-axis) and separation (along the $z$-axis) of the lines are 8.0 mm and 18.96 mm, respectively.

The simulation results for the transversally infinite slab, shown in Fig. 4, demonstrate good impedance matching with free space. To illustrate the fact that changing the slab thickness does not destroy the impedance matching, we show results also for slabs with the thickness of the TL network being $4d$ and $5d$. Although the TL network impedance should be optimally matched with free space around 3 GHz, we have found the optimal frequency to be around 2.5 GHz, as seen in Fig. 4. In [6] for the same network, the optimal impedance matching was obtained at 3.5 GHz. The reason for the difference in these two cases is that the impedance of the transition layer was designed to be around 377 Ω in [6], whereas in this letter we have used the same impedance as in the TLs comprising the network ($\sim 585$ Ω). Although the transition layer is short compared to the wavelength, the specific design of the transition layer clearly affects the frequency dependence of the impedance matching and therefore it should be taken into account in the design of a lens for specific application, e.g.,

![FIG. 3: Color Online. HFSS simulation model of the transversally infinite slab. The edges of the simulation model are assigned as perfect electric conductors ($xy$-planes) and perfect magnetic conductors ($xz$-planes) to obtain periodicity. The metal strips are modelled as infinitely thin and perfectly conducting.](image)
FIG. 4: Simulated reflection ($\rho$) and transmission ($\tau$) for the slab shown in Fig. 3 (thickness of the slab being $8d$), as well as for two other slabs having different thicknesses.

by optimization using a suitable simulation software.

To demonstrate the functionality of the proposed lens, a model illustrated in Fig. 5 was simulated by illuminating the lens with a normally incident plane wave. Here we use such boundary conditions that we effectively simulate a structure which is infinitely periodic in the vertical direction, i.e., along the $z$-axis. Also, a perfect magnetic conductor (PMC) boundary is assigned to the $xz$-plane in the center of the lens to reduce the simulation time.

The lens curvature is designed using the well-known lens design equations for dielectric lenses [3] to obtain a lens that focuses a plane wave to a line on the other side of the lens. One side of the lens (the side from where the plane wave impinges on the lens) is flat and the other side has a certain curvature. Here we have assumed that the lens material has a refractive index of $n = \sqrt{4.66}$. The width of the lens along the $y$-axis is $64d = 512$ mm and with this width and curvature, the focal point is at a distance of $80d$ from the lens [3]. For the practical implementation of the TL network, the optimal curvature is approximated by a stepwise structure as shown in Fig. 5.

The lens with the given dimensions (shown in Fig. 5) having the previously described network structure and transition layers was modelled with HFSS software. The simulation model is shown in Fig. 6 together with the HFSS model of a reference dielectric lens having the same curvature and the relative permittivity of $\varepsilon_r = 4.66$. Three edges of the simulation model are terminated with a perfectly matched layer (PML) and in the symmetry plane (center of the lens) there is a PMC boundary. First, the lenses shown in Fig. 6 were
FIG. 5: Analytically calculated lens curvature (solid black line) and the transmission-line network (each square represents one unit cell of the network) having approximately the same curvature. Only half of the lens is shown along the $y$-axis since the simulation model is cut in half by a PMC boundary.

illuminated by a plane wave propagating in the $+x$-direction and having electric field along the $z$-axis. The phase of the simulated electric field in both systems is plotted in Fig. 6 for the frequency 2.4 GHz.

Clearly the proposed transmission-line lens refracts the incident plane wave similarly as the homogeneous dielectric lens. The TL lens reflectance, as compared to the reference

FIG. 6: Color Online. Phase of the simulated electric field (at the frequency 2.4 GHz) for (a) the transmission-line lens and (b) reference dielectric lens. A plane wave propagating in the $+x$-direction illuminates the lenses. For clarity, the phase is not plotted inside the lenses.
FIG. 7: Simulated reflectance of the lenses shown in Fig. 6. The plane wave illuminating the lenses travels in the $+x$-direction.

The reflectance of the lens, is lowered significantly (by 4 dB or more) in a relative bandwidth of approximately 16.7 percent, see Fig. 7. In order to make a fair comparison between the two lenses shown in Fig. 6, the losses inside the both lenses are equal, i.e., the both dielectrics are lossless and the transmission-line sections at the edges of the proposed TL lens (which are not connected to the transition layer) are left open.

To further demonstrate the focusing effect and the operation of the designed TL lens, a line source was introduced at the expected focal point. The expected focal point is located at the distance $80d$ away from the lens as shown in Fig. 5 (the reader should note that the used simulation software allows sources to be placed outside the finite simulation model). Snapshots of the simulated electric field distributions are shown in Fig. 8, demonstrating that the cylindrical wavefronts radiated by the source (with electric field along the $z$-axis) are refracted in the correct way, creating a plane wave (approximately) on the other side of the lens that travels to the $-x$-direction. To illustrate the good impedance matching of the TL lens, only the scattered fields are plotted on the source sides of the lenses.

IV. DISCUSSION AND CONCLUSIONS

We have proposed a novel method for realizing a microwave lens with highly reduced reflections as compared to traditional lenses made of homogeneous dielectric materials. The lens is based on the use of transmission-line networks that are matched to free space. Since the refractive index of the network can be designed separately from the network impedance, the network can be impedance matched with free space while having high values of the
FIG. 8: Color Online. Simulated electric field at the frequency 2.4 GHz for (a) the transmission-line lens and (b) the reference dielectric lens. A line source positioned at the focal point (see Fig. 5) illuminates the lenses. On the source sides only the scattered field is plotted in order to compare the magnitude of the reflections from the lens interfaces. For clarity, the fields are not plotted inside the lenses.

effective refractive index of the lens. We have presented simple design equations for the dispersion and impedance of the transmission-line networks, and have demonstrated the feasibility of the proposed approach by simulations of a lens that focuses incoming plane waves similar to a dielectric lens of the same size and shape. In a certain frequency band the transmission-line lens exhibits strongly mitigated reflection of power, as compared to the reference dielectric lens.

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