Numerical studies on the near wall $y^+$ effect on heat and flow characteristics of the cross flow tube bank

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Abstract. The use of passive mode of heat transfer enhancement are most commonly used for the performance improvement with conservation of conventional fuels. In recent times, to visualize the insight of fluid flow analysis, the numerical simulations are mostly preferred. The number of grid points present in the in the fluid domain near the wall governs the accuracy and precision of the numerical study. Hence in the present study, three different near wall grid systems are consider to evaluate its effect on Nusselt number friction factor for the staggered tube bank arrangements. It is reported that $y^+$ as 10, insensitive to boundary layer effects at higher flow rate, while $y^+$ as 1.0, and 0.10 are in close approximation.

Keywords: Cross flow tube bank, Near wall $y^+$, Heat transfer, Friction factor

1. Introduction

Numerous types of heat exchangers are used in the industries for the effective exchange of heat and waste exhaust gas for the optimum utilization of the fuel and capital requirement. The heat exchangers are the mostly tubular type with ease in maintenance and assembly such as shell and tube type and cross-flow heat exchangers. In the cross-flow heat exchanger, the hot fluid/gas flows through the tube side, while the colder fluid flows over the tube surface [1,2]. In cross-flow tube bank, the cross tubes are arranged in either in-line or staggered layout. The cross-flow heat exchanger has the unique capability of increasing and decreasing the rate of heat transfer and pressure drop by altering the tube pitch ratio. The performance of the cross flow tube bank can be enhanced by incorporating passive enhancement techniques such as the provision of fins and vortex generators which accelerates the rate of heat transfer with additional fluid turbulence. Aiba et al. [3] performed the heat transfer investigation across in-line tubes for Reynolds number ranging from 10000- 60000 and its effect on the tube pitch ratio of 1.20 and 1.60. The use on non-circular tube section such as elliptic tubes are used so as to enhance the rate of heat transfer and to reduce the overall pumping power requirements [4,5]. The use of fins increases the net effective area available for a rate of heat transfer, along with an increase in pressure drop across tubes [6]. The thin plain continuous fin is most commonly used. The other types of fin profile such as spiral, helical, wavy fins generate the higher magnitude of pressure drop due to the formation of horse shoe vortex. Further, a higher intensity of turbulence is generated by the use of vortex generators such as delta, rectangular, and trapezoidal vortex generators which provides the secondary flow apart from the primary flow [7,8]. The use of vortex generators increases...
both the rate of heat transfer across the fluids and pressure drop associated with the cross flow fluid. The vortex generators are usually mounted over the fin surface or sometimes punched out from the fin surface itself [9].

As the computational power is increased, the use of numerical simulation is more preferred over the actual experimental analysis to save time and predict the accurate result. Buyruk numerically investigated the heat transfer characteristics on the tandem and staggered tubes in cross flow for the various pitch ratio [10]. Initially, the numerical simulation is based on the two-dimensional finite element method, whereas in recent times it is based on three dimensional with finite volume based method. Numerous investigators make use of three-dimensional finite volume based numerical analysis for the heat and flow behavior across the tube bank [12–15]. In recent times, researchers perform both RANS and LES to evaluate the performance of the tube bank [16,17]. However there is a need to study the effect of grid size near the wall on the evaluation of overall thermal performance of the tube bank. The time required for the numerical analysis is typically based on the grid size along with the computational domain. In addition to this, resolving the viscous boundary layer is the critical aspect and more number of grid points is required to capture the entire phenomenon. The finer grid size exponentially increases the computational time along with increased accuracy. Even though there are numerous works with RANS models for heat transfer and fluid flow over tube bank, no work have done till now to show the exact effect of y+ values over the calculation of thermal performance of tube bank in turbulent flow. Based on the above consideration, the present study aimed at numerical investigation of the thermal performance for the different near wall grid sizes. Hence, the thermal performance for the staggered tube bank is evaluated for three different values of y’ as 0.10, 1.0, and 10 for constant tube wall surface temperature and constant heat flux.

2. Model description and numerical solution
The near wall y+ represents the dimensionless parameter, analogous to the flow Reynolds number evaluating the influence of adjacent cell to wall are laminar or turbulent, thereby representing the part of turbulent boundary layer they resolve.

\[ y^+ = \frac{U_{cell} y}{V_{air}} \]  

(1)

The classifications of near wall zones in case of turbulent boundary layer can be further summarized as:
- y+ less than 5 : Sub-layer field is viscous
- y+ ranging from 5 to 30 : Buffer zone

The effect of viscous sub-layer and buffer zone on the Nusselt number and friction factor is estimated with three different grid sizes by considering y+ as 0.10, 1.0, and 10 respectively.

2.1. Physical Model

The tubes selected in the computational domain are staggered, due to higher fluid turbulence and increased rate of heat transfer. Figure 1 shows the schematic of the computational domain under consideration.
Figure 1. Computational domain with tubes in staggered arrangement

The domain comprises of five tubes with 25.44 mm outer diameter arranged in a staggered configuration with constant tube wall temperature and uniform heat flux boundary conditions with symmetric top and bottom plane. The Reynolds number for the cross fluid air is varied from 5000 to 20000. The entrance length of the domain is 3D, while the downstream length is around 10D. The entrance and exit lengths are selected in order to reduce the effect of the air inlet and pressure outlet over the flow pattern around the tube surface. Kim [18] numerically investigated the effect of pitch ratio on the convective heat transfer rate for in-line tubes under similar conditions. The diameter of the tube is 25.44 mm with the longitudinal and transverse tube pitch ratio as 2.0.

2.2. Governing Equations

It is assumed that the cross fluid air is incompressible, steady, viscous with constant physical properties. The Navier-Stoke equation and energy equation are used to resolve the fluid flow physics.

Continuity equation

\[ \frac{\partial}{\partial x_i} (\rho u_i) = 0 \]  

Momentum equation

\[ \frac{\partial}{\partial x_i} (\rho u_i u_j) = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \frac{\partial}{\partial x_j} (-\rho u_j u_i) \]  

Energy equation

\[ \frac{\partial}{\partial x_j} \left[ \rho T u_j \right] = \frac{\partial}{\partial x_j} \left[ (\tau + \tau_l) \frac{\partial T}{\partial x_j} \right] \]  

The molecular thermal diffusivity along with turbulent thermal diffusivity is represented by \( \tau \) and \( \tau_l \), respectively. Renormalization-group (RNG) k–\( \varepsilon \) turbulence model is used to estimate turbulence kinetic energy \( k \), and dissipation rate \( \varepsilon \):

\[ \frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left( \alpha_k \mu_{eff} \frac{\partial k}{\partial x_j} \right) + G_k - \rho \varepsilon \]  

\[ \frac{\partial}{\partial x_i} (\rho \varepsilon u_i) = \frac{\partial}{\partial x_j} \left( \alpha_\varepsilon \mu_{eff} \frac{\partial \varepsilon}{\partial x_j} \right) + C_{1\varepsilon} \varepsilon \frac{G_k}{k} - C_{2\varepsilon} \frac{\rho \varepsilon^2}{k} - \dot{\varepsilon} \]  

The generation of turbulent kinetic energy is represented by \( G_k \) and is given as

\[ G_k = -\rho u_i \frac{\partial u_i}{\partial x_j} \]  

The effective turbulent viscosity and turbulent viscosity is given as
\[ \mu_{\text{eff}} = \mu + \mu_i \text{ and } \mu_i = \rho C_\mu \frac{k^2}{\varepsilon} \]  

(8)

The equation constants have the following values, \( C_{1e} = 1.42 \), \( C_{2e} = 1.68 \), \( \alpha_k = 1.39 \) and \( \alpha_\varepsilon = 1.39 \).

2.3. Parameters Definition

The various parameters used in the numerical simulations are defined as follow:

The flow Reynolds number is given as

\[ \text{Re} = \frac{\rho V_{\text{max}} D}{\mu} \]  

(9)

The heat transfer coefficient, \( h = \frac{Q}{\Delta T A_p} \)  

(10)

Nusselt number, \( Nu = \frac{hD}{k} \)  

(11)

Friction factor, \( f = \frac{2.\Delta P}{\rho V_{\text{max}}^2 N_L} \)  

(12)

where \( \rho, \mu \) and \( k \) indicates the density, viscosity and thermal conductivity of the fluid, while \( \Delta P \) and \( N_L \) represents the pressure drop and number of tubes in cross direction.

3. Grid generation and validation

The uniform structured mesh is generated in ANSYS ICEM CFD. The entire computational domain comprises of air inlet, pressure outlet, entry length, exit length, and tube surfaces. The mesh size is fine near the tube surface, while the relative coarse grid is generated near the inlet and outlet section. The effect of three different \( y^+ \) as 0.10, 1.0, and 10 are considered for the grid generation and near wall height. The grid is generated by partition block method, with an aspect ratio of 1.12. The overall quality of the mesh is around 0.85 and above for all the \( y^+ \) parameters. Figure 2 indicates the structured mesh generated over the computational domain.

![Figure 2. Uniform structured mesh over the computational domain](image)

The three different grids system with near wall \( y^+ \) as 0.10, 1.0, and 10 are used in the present numerical study. In general \( y^+ \) values can be calculated only after completion of the numerical simulations. But at the same time the first cell thickness of the grids near the wall, effect the simulations significantly. A normal practice that is adopted in the CFD domain is to pre-estimate the \( y^+ \) and thereby first cell thickness using the correlations of skin friction coefficients for Flat plate boundary layers. So in the current study near wall shear stresses are estimated before simulation using Schlichting skin friction Correlation. Using the above calculated values of wall shear stress the friction velocity \( u^* \) is calculated for each Reynolds number. Substituting the above \( u^* \) value in equation (1) and with prefixed values of \( y^+ \) the first cell thickness of grids are calculated for corresponding Reynolds Number. Table 1 show the value of first cell thickness estimated using the above mentioned method for each case and the actual value of \( y^+ \) after simulations. It can be noted that the actual values of \( y^+ \) are close to the pre-estimated values of \( y^+ \) in the case of Enhanced wall treatments.
Table 1. Estimated value of First Cell thickness before numerical simulation and actual value of y+ after numerical simulation

| Re  | Pre-estimated y+ = 0.1 (Enhanced wall treatment) | Pre-estimated y+ = 1 (Enhanced wall treatment) | Pre-estimated y+ = 10 (Enhanced wall treatment) |
|-----|-----------------------------------------------|----------------------------------------------|-----------------------------------------------|
|     | First cell thickness (mm) | Actual y+ after simulation | First cell thickness (mm) | Actual y+ after simulation | First cell thickness (mm) | Actual y+ after simulation |
| 5000 | 6.4x10^{-3} | 0.1064 | 6.4x10^{-2} | 1.0719 | 6.4x10^{-1} | 9.9817 |
| 8000 | 4.2x10^{-3} | 0.0985 | 4.2x10^{-2} | 1.0054 | 4.2x10^{-1} | 9.6910 |
| 11000 | 3.2 x10^{-3} | 0.0946 | 3.2x10^{-2} | 0.9722 | 3.2x10^{-1} | 9.6701 |
| 14000 | 2.6 x10^{-3} | 0.0920 | 2.6x10^{-2} | 0.9437 | 2.6x10^{-1} | 9.6023 |
| 17000 | 2.2 x10^{-3} | 0.0917 | 2.2x10^{-2} | 0.9213 | 2.2x10^{-1} | 9.5210 |
| 20000 | 1.9 x10^{-3} | 0.0890 | 1.9x10^{-2} | 0.9005 | 1.9x10^{-1} | 9.3360 |

Figure 3 illustrates the enlarged view of grid generated for y+ as 0.1, 1.0 and 10 at Reynolds Number 20000. Similarly grids are generated for each case of y+ and corresponding Reynolds Number as per the first cell thickness mentioned in table 1.

![Figure 3. Enlarge view of structured mesh with near wall y+ as 0.10, 1.0, and 10 for Re 20000](image)

The number of grid points critically governs the precision of the numerical model. Hence a sufficient number of grid points are required in the computational domain to consider all the physical phenomena involving in case of tube bank. Figure 4 indicates the grid independency test for the numerical analysis of staggered tube bank along with validation of the numerical scheme with the analytical co-relation proposed by Zukauskas [11].
The grid independency test is performed at flow Reynolds number of 20000, as higher turbulence is generated at peak intensity of flow. The deviation in the Nusselt number and friction factor above 51281 number of elements is relatively non-significant. Hence grid system with 51281 is selected for the numerical analysis. In a similar way, various numerical models are selected and compared with the analytical Zukauskas co-relation. The RNG k-ε turbulence scheme is closely mapped with the analytical finding even with increased Reynolds number of flow.

4. Results

The effect of near wall $y^+$ on the thermal performance is evaluated on the basis of constant heat flux and constant tube wall surface temperature. The Nusselt number and friction factors were evaluated for the different range of flow Reynolds number. This study is limited to enhanced wall treatments only.

4.1. With constant tube wall temperature

The Nusselt number and friction factor is evaluated for the staggered tube bank for the constant tube wall temperature for the flow Reynolds number ranging from 5000 to 20000. The effect of near wall $y^+$ on heat transfer rate and friction factor is depicted in figure 5.

Figure 4. Grid independency and validation of the numerical scheme

Figure 5. Effect of $y^+$ on Nusselt number and friction factor with constant wall temperature
The Nusselt number increases with flow Reynolds number for all the $y^+$ cases. For the lower range of Reynolds number up to 8000, Nusselt number for $y^+$ as 0.10 and 1.0 is almost of the same magnitude, while the magnitude of Nusselt number is higher by 10% for $y^+$ as 10. With the increase in the flow Reynolds number, the deviation in the Nusselt number further increases for $y^+$ as 10. The magnitude of Nusselt number for $y^+$ as 0.10 and 1.0 are in close approximation for the entire range of Reynolds number with the maximum variation of 1.50% at the peak intensity of Reynolds number. The maximum variation of 16% is observed in Nusselt number for $y^+$ as 10 at Reynolds number of 20000. Hence it can be inferred that, the grid points with $y^+$ 1.0 are sufficient to consider the flow across number of tubes, as tubes walls are subjected to higher temperature. With increasing number of near wall grid points the variation in Nusselt number is negligible and can increase the computational time requirement.

Similarly, in case of friction factor, the variation in friction factor is relatively low for the $y^+$ as 0.10, and 1.0 respectively. The magnitude of friction factor is highest for $y^+$ of 10 and lowest for 0.1, with $y^+$ of 1.0 represents the intermediate magnitude. The variation in friction factor for $y^+$ of 10 decreases with an increase in flow Reynolds number beyond 11000. The maximum variation in the friction factor for the $y^+$ as 10 is observed at the lower range of Reynolds number rather than at the higher range of Reynolds number as in case of Nusselt number. The near wall $y^+$ is more influenced by thermal boundary layer rather than the hydrodynamic boundary layer.

4.2. With constant heat flux

In case of constant tube wall temperature, the numerical analysis was also performed under constant heat flux with the effectively same temperature as in the earlier case. The effect of near wall $y^+$ on Nusselt number and friction factor is illustrated in figure 6.

![Figure 6](image-url)  
**Figure 6.** The effect of $y^+$ on Nusselt number and friction factor with constant heat flux.

As in case of constant tube wall temperature, the trend in the Nusselt number is similar for the constant heat flux conditions. The Nusselt number for all the $y+$ values increases with Reynolds number. The magnitude of Nusselt number for $y+$ as 10 relatively higher than $y+$ of 0.10, and 1.0 for all the values of flow Reynolds number. Here, again the magnitude of Nusselt number for $y+$ as 0.1 and 1.0 are in close proximity for the entire range of Reynolds number. A maximum variation of 12-15% in Nusselt number is observed for $y+$ as 10 at higher range of Reynolds number. The trend in friction factor is exactly same as in case of constant tube wall temperature representing more...
dominating effect of Nusselt number rather than friction factor. The overall magnitude of Nusselt number with constant heat flux condition is relatively lower than constant tube wall condition.

5. Conclusion
The present numerical analysis is performed to study the effect of near wall $y^+$ grid size on the thermal and hydraulic performance of the cross flow tube bank. The effect of three different $y^+$ grids on the Nusselt number and friction factor for constant tube wall and constant heat flux is as follows:

- The effect of near wall $y^+$ remarkably affects the Nusselt number and friction factor. The $y^+$ as 0.10 and 1.0 are in good proximity with the analytical finding by Zukauskas, while $y^+$ as 10 indicates the variation of 5-16% with increased Reynolds number.
- In case of friction factor, the magnitude of friction factor for $y^+$ of 10 is 5-8% higher than $y^+$ of 0.10, and 1.0 respectively with increased Reynolds number.
- At lower range of Reynolds number, the variation on both Nusselt number and friction factor is not significant. This is due to the more dominating viscous forces hence even with fewer grid points are capable to consider the effect of boundary layer formation over the tubes.
- As, the Reynolds number increases, the thickness of boundary layer shrinks, hence more grid points are required to capture the boundary layer effects. This can only be achieved with near wall $y^+$ as 1.0, and 0.1.
- Furthermore, the variation in Nusselt number and friction factor is not prominent with $y^+$ as 1.0, and 0.10. So grid size with $y^+$ as 1.0 can be selected which often require less computational time for the RANS simulations.

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