A hybrid approach for modelling dynamic behaviours of a rotor-foundation system

Z G Zhang\(^1,2\), Z Y Zhang\(^1,2\), B Jing\(^1,2\), and H X Hua\(^1,2\),
\(^1\)Institute of Vibration, Shock and Noise, Shanghai Jiao Tong University, 800 Dongchuan RD, Shanghai, 200240, China \(^2\)State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, 800 Dongchuan RD, Shanghai, 200240, China

E-mail: zzgjtx@sjtu.edu.cn

Abstract. A new hybrid approach is presented to study the dynamic behaviour of a rotor-foundation system, in which a shaft coupled with various discontinuities are connected to a flexible foundation via discrete spring subunits. By modelling the rotor with the modified transfer matrix method and describing the flexible foundation through the appropriate modal model, the proposed technique facilitates a computationally efficient modelling approach where a mixture of theoretical, numerical or experimental models can be incorporated into one overall numerical model. Particularly, the present model enables one to conveniently consider both the free and forced vibrations as well as effects of various combinations of discontinuities encountered in the rotor. Some results are compared with available results in previous publications and those from the finite element method to validate the model. Parametric studies are also performed to demonstrate the accuracy and versatility of the developed method for substructure coupling analysis.

1. Introduction

Beam-type systems have been extensively regarded as basic components to simulate many commonly used engineering structures [1-3], including turbomachinery, propulsion system, etc. However, relatively little literature is available on the dynamics of rotor-foundation systems in which the foundations are normally flexible and thus have a substantial influence on the dynamics of coupled systems, though such structures seem to be more realistic and practical.

An essentially straightforward way is to apply the finite element method (FEM) and construct the reduced order models based on the assembly process of physical elements. However, it may not always be possible to derive accurate FE expressions for foundations because of its complicated configuration and interaction with the surrounding environment [4]. A practical, and in some cases, the only feasible approach may be to characterize the foundation by measurement. The well established FRF-based substructuring method (FBSM) which provides a powerful tool for the dynamic analysis of complex combinatorial structures should be mentioned [5-7]. Its superiority is attributable to the capacity to incorporate both the theoretical and experimental FRFs of simpler subsystems into the overall system model [5, 6], which facilitates an efficient and applicatory hybrid modelling approach. As is noted above, receptance coupling and substructuring analyses [7] appears applicable to predict dynamic properties of titled assemblies from subsystem characteristics by means of FRFs. When it comes to the practical problems, the foundation is generally decided prior, and FRFs of the
foundation are typically determined from FE simulation or shaker/impact tests. With respect to the discontinuous beam, repeated modifications of properties such as positions and parameters of attachments are normally unavoidable to achieve optimized dynamics. Therefore, the FRF matrices of the multiple-discontinuity beam should be conducted for every change in design, which is rather costly and time-consuming in practice. Recently, Zhang et al.[8, 9] developed a general analytic approach based on modified transfer matrix method for dynamic analysis of single or double beams coupled with discontinuities. By organizing equilibrium equations at the interconnected point of subbeams with matrix formulations, the method is found to be more convenient to apply than standard algebraic approaches.

Despite the differences in the underlying physical concepts of the above mentioned methods, they share several similarities, namely, both the methods work in the frequency domain, and the implementations of the coupling process are both based on the enforcement of the conditions of compatibility and equilibrium at the interface DoFs. The authors notice the potential possibility to combine transfer matrix and FRF coupling as presented in [6] and [8] to establish a new hybrid synthesis method, in which a mixture of theoretical, numerical or experimental models can be incorporated as a whole to tackle the coupled problems. The proposed approach may include advantages from both the transfer matrix and FRF coupling methods, and is suitable to quickly predict the effects of modifications on the beam and those of foundation flexibility. Accordingly, this article contributes to a new hybrid approach (part analytical model, part experimental or numerical model) for determining natural frequencies and FRFs of general beam-foundation systems. It is hoped that the developed approach may offer a greater flexibility for dynamic analysis of rotor-foundation systems.

2. Problem formulation

Figure 1 shows the sketch of a typical rotor-foundation system comprising of a rotor connected to a flexible foundation via $N$ discrete translational spring subunits. The upper and lower stiffness and damping coefficients of the $i$-th ($1 \leq i \leq N$) connection subunits are represented by $K_i^B$, $K_i^C$, $C_i^B$ and $C_i^C$, while $M_i^C$ denotes the lumped mass. The superscripts $B$, $C$ and $F$ refer to beam, spring connection and foundation, respectively. Meanwhile, the beam is subjected to $P$ miscellaneous discontinuities, either intermediate attachments, or geometric discontinuities. Any discontinuities considered can be idealized as a general point connecting the neutral axe of the beam with actual dimensions being assumed to be negligible. Moreover, transverse vibrations (including transverse deflections and rotation of cross sections) are concentrated on in this study for explanatory purposes.

![Figure 1. Schematic of a multi-span beam-foundation system coupled with multiple discontinuities.](image)

2.1. The transfer matrix method

The whole rotor can be subdivided into $N+1$ uniform segments with length $L_i$ ($i=1$, 2, ... $N$) by the spring connection points located at $x_i$ ($i=1$, 2, ... $N$). Each segment can be further subdivided into $p_i+1$
sub-beams by the discontinuity points. Based on the Timoshenko beam theory [9] and the method of separation of variables, the equation of motion for the \(i\)-th beam segment can be readily stated as:

\[
Y_{i}'''(x) + (\sigma_y + \tau_z) Y_i''(x) - (\alpha_i - \sigma_n \tau_n) Y_i'(x) = 0
\]

(1)

\[
\Phi_i'''(x) + (\sigma_y + \tau_z) \Phi_i''(x) - (\alpha_i - \sigma_n \tau_n) \Phi_i'(x) = 0
\]

(2)

where the prime denotes the differentiation of \(x\), and \(\sigma_y = \rho_i \omega_i^2 / E_i\), \(\tau_z = \rho_i \omega_i^2 / (\kappa_i G_i)\), \(\alpha_i = \rho_i A_i \omega_i^2 / (E_i I_i)\). \(Y_i(x)\) and \(\Phi_i(x)\) are mode shape functions with respect to the transverse displacement and the bending slope of the \(i\)-th segment. 

Each segment in the global coordinate system \((\bar{x}, \bar{y})\) can be independently described by the local coordinate system \((x_i, y_i)\). The general solution of equations (1) and (2) can then be derived in the following form as:

\[
Y_i(x) = C_{i1} \cosh \lambda_i (x_i) + C_{i2} \sinh \lambda_i (x_i) + C_{i3} \cos \bar{\lambda}_i (x_i) + C_{i4} \sin \bar{\lambda}_i (x_i)
\]

(3)

\[
\Phi_i(x) = C_{i1} q_i \sinh \lambda_i (x_i) + C_{i2} q_i \cosh \lambda_i (x_i) + C_{i3} \bar{q}_i \sin \bar{\lambda}_i (x_i) - C_{i4} \bar{q}_i \cos \bar{\lambda}_i (x_i)
\]

(4)

where \(C_{pi} (p=1, 2, 3, 4)\) is the integration constant associated with the \(i\)-th segment, and

\[
\lambda_i = \left(\sigma_y - \tau_z \frac{2}{2}\right) + \alpha_i - \sigma_n \tau_n \frac{2}{2}, \quad \bar{\lambda}_i = \left(\frac{\sigma_y - \tau_z}{2}\right) + \alpha_i + \sigma_n \tau_n \frac{2}{2}.
\]

(5)

\[
q_i = \frac{\omega_i^2 \rho_i}{\kappa_i G_i \lambda_i^2}, \quad \bar{q}_i = \frac{\omega_i^2 \rho_i}{\kappa_i G_i \bar{\lambda}_i^2} - \bar{\lambda}_i.
\]

(6)

Equalling the deflection, slope, bending moment, and shear force for the opposite sides of \(i\)-th connection points, analytical expressions of continuity conditions state the following matrix equations:

\[
\begin{bmatrix}
\psi_{i1}(0) C_{i1} + \psi_i(L_i) C_i \\
\chi_{i1}(0) C_{i1} + \chi_i(L_i) C_i \\
\chi_{i1}'(0) C_{i1} + \beta \chi_i'(L_i) C_i \\
V_{i1}(0) C_{i1} + \bar{R}_i^b = \eta V_i(L_i) C_i
\end{bmatrix} = 0
\]

(7)

where \(C_{i1} = [C_{1(i+1)} C_{2(i+1)} C_{3(i+1)} C_{4(i+1)}]^T\) and \(C_i = [C_{1i} C_{2i} C_{3i} C_{4i}]^T\), \(V_i(x_i) = \chi_i(x_i) - \psi_i'(x_i)\), \(R_i^b\) is the reaction force of the \(i\)-th connection point on the beam, the subscript \(i\) denotes the \(i\)-th spring subunits, the prime denotes differentiation with respect to the local coordinate \(x_i\), and

\[
\beta_i = \frac{E_i I_i}{E_{i1} I_{i1}}, \quad \eta_i = \frac{\kappa_i G_i A_i}{\kappa_{i1} G_{i1} A_{i1}}, \quad \bar{R}_i^b = \frac{R_i^b}{\kappa_{i1} G_{i1} A_{i1}}.
\]

(8)

Equation (9) can be expressed in matrix form as:

\[
T_{\beta_i} C_{i1} + \bar{R}_i^b = T_{\eta_i} C_i.
\]

(9)

where \(\bar{R}_i^b = [0 \ 0 \ \bar{R}_i^b]^T\)

Analogously to equation (9), corresponding compatibility conditions across the \(m\)-th discontinuity can also be analytically described in a matrix expression as:
\[ C_{i,m+1} = \left( T_{R,i,m} \right)^{-1} T_{L,i,m} C_{i,m}. \] (10)

One may refer to Ref.[9] for details of matrices \( T_{R,i,m} \) and \( T_{L,i,m} \). Repeated applications of equation (10) successively yield the stepwise relation of all integration constants for the \( i \)-th segment:

\[ C_{i,p_i+1} = \tilde{T}_{p_i} C_{i,p_i} = \left( \prod_{k=1}^{p_i} \tilde{T}_{p_i-k+1} \right) C_{i,1} = \tilde{T}C_{i,1}. \] (11)

where \( C_{i,1} \) and \( C_{i,p_i+1} \) associate with the leftmost and rightmost sub-beams of the \( i \)-th segment. The substitution of equation (11) into (9) leads to the transitive relationship of adjacent segments:

\[ T_{R,i} C_{i+1,1} = T_{L,i} \tilde{T} C_{i,1} - \tilde{R}^B. \] (12)

### 2.2. Description of the foundation and the connections

The transverse displacements \( \mathbf{Y}^F \) of the connection point on foundation due to interaction loads \( \mathbf{R}^F \) between spring connections and the foundation can be reasonably given by:

\[ \mathbf{Y}^F = \mathbf{H}^F \mathbf{R}^F. \] (13)

with \( \mathbf{Y}^F = [Y_1^F, Y_2^F \cdots Y_N^F]^T \), \( \mathbf{R}^F = [R_1^F, R_2^F \cdots R_N^F]^T \) and \( \mathbf{H}^F \) being the foundation FRF matrix of order \( N \times N \), where \( Y_i^F \) and \( R_i^F \) are the transverse displacement and the reaction force of the \( i \)-th (\( 1 \leq i \leq N \)) connection point on the foundation, respectively. \( \mathbf{H}^F \) can be determined either from foundation dynamics measurements directly, or calculation through the numerical model of the foundation. The reaction forces \( \mathbf{R}^F \) and \( \mathbf{R}^B \) (the reaction forces between the spring connections and the beam) can be given as the following relation:

\[
\begin{align*}
\mathbf{R}^F &= \mathbf{K}^F \mathbf{Y}^F - \mathbf{K}^M \mathbf{Y}^M, \\
\mathbf{R}^B &= \mathbf{K}^B \mathbf{Y}^B - \mathbf{K}^B \mathbf{Y}^M.
\end{align*}
\] (14)

with \( \mathbf{K}^F = \text{diag}\{K_1^F + i\omega \xi_1^F\} \cdots (K_N^F + i\omega \xi_N^F)\}, \mathbf{K}^B = \text{diag}\{(K_1^B + i\omega \xi_1^B) \cdots (K_N^B + i\omega \xi_N^B)\}, \mathbf{Y}^B = [Y_1, Y_2 \cdots Y_N]^T, \text{ and } Y_i = Y_{i,p_i+1}(L_{i,p_i+1}) \text{ is the transverse displacement of the } \( i \)-th (\( 1 \leq i \leq N \)) connection point on the beam.

The governing equation of lumped mass \( M_i^C \) in the \( i \)-th spring connection can be given as:

\[ (\mathbf{K}^F + \mathbf{K}^B - \omega^2 \mathbf{M}) \mathbf{Y}^M = \mathbf{K}^F \mathbf{Y}^F + \mathbf{K}^B \mathbf{Y}^B. \] (15)

where \( \mathbf{M} = \text{diag}\{M_1^C, M_2^C \cdots M_N^C\}, \mathbf{Y}^C = [Y_1^C, Y_2^C \cdots Y_N^C]^T, \text{ and } Y_i^C \text{ is the transverse deflection of the } (1 \leq i \leq N) \text{ lumped mass}. \text{ Defining } \mathbf{K}^c = \mathbf{K}^F + \mathbf{K}^B - \omega^2 \mathbf{M} \text{ and rearranging, leads to the expression of } \mathbf{R}^B \text{ in } \mathbf{Y}^B:

\[ \mathbf{R}^B = \left( \mathbf{K}^B - \mathbf{K}^B \left[ \mathbf{K}^c - \mathbf{K}^F (\mathbf{H}^F \mathbf{K}^F - \mathbf{I})^{-1} \mathbf{H}^F \mathbf{K}^F \right]^{-1} \mathbf{K}^B \right) \mathbf{Y}^B = \mathbf{H} \mathbf{Y}^B. \] (16)

### 2.3. The coupled structure

In order to adequately characterizes the coupling effects, equation (16) can be rewritten as:

\[ \mathbf{R}^B = \mathbf{H} \sum_{m=1}^{N} H_{m} Y_{m} = \sum_{m=1}^{N} H_{m} Y_{m}, \quad 1 \leq m \leq N, \] (17)
where $H_m$ is the $m$-th column of $H$.

Substituting equation (3) into (17), leads to the expressions of $R_B$ in terms of $C_{m,1}$:

$$R_B = \sum_{m=1}^{N} \left[ H_m \psi_{m,p_m+1} (L_{m,p_m+1}) \overline{T}_{m} \right] C_{m,1} = \sum_{m=1}^{N} \left( W_m C_{m,1} \right),$$

with $W_m = H_m \psi_{m,p_m+1} (L_{m,p_m+1}) \overline{T}_{m}$ and $p_m$ being the number of discontinuities included in the $m$-th beam segment. The transitive relationship of adjacent segments across the $i$-th connection point can then be rewritten as follows:

$$C_{i+1} = T_{L_i}^{-1} T_{L_i} T_{C_{i,1}} - \sum_{m=1}^{N} \left[ T_{R_i}^{m} \overline{W}_{m,1} C_{m,1} \right], \quad 1 \leq i \leq N,$$

where $\overline{W}_{m,j} = [0 \ 0 \ 0 \ \cdots]^{T}$ and $W_{m,i}$ is the $i$-th row of $W_m$.

Integrating of $N$ transfer matrices provided by equation (19) and rearranging, results in the assembled matrix of the form:

$$\begin{align*}
C_L &= U_{\text{sum}} C_R,
\end{align*}$$

where $C_L = [0 \ 0 \ \cdots \ C_{N+1,1}^{T}]^{T}$, $C_R = [C_{1,1}^{T} \ C_{1,2,1}^{T} \ \cdots \ C_{1,1,N}^{T} \ C_{1,1,1}^{T}]^{T}$, and

$$U_{\text{sum}} = \begin{bmatrix}
-T_{R_1}^{1} \overline{W}_{N,1} & -T_{R_1}^{1} \overline{W}_{N,1} & \cdots & -T_{R_1}^{1} \overline{W}_{N,1} & -T_{R_1}^{1} \overline{W}_{N,1} & -T_{R_1}^{1} \overline{W}_{N,1} \\
-T_{R_2}^{1} \overline{W}_{N,2} & -T_{R_2}^{1} \overline{W}_{N,2} & \cdots & -T_{R_2}^{1} \overline{W}_{N,2} & -T_{R_2}^{1} \overline{W}_{N,2} & -T_{R_2}^{1} \overline{W}_{N,2} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
-(T_{R_N}^{1})^{-1} \overline{W}_{N,N} & -(T_{R_N}^{1})^{-1} \overline{W}_{N,N} & \cdots & -(T_{R_N}^{1})^{-1} \overline{W}_{N,N} & -(T_{R_N}^{1})^{-1} \overline{W}_{N,N} & -(T_{R_N}^{1})^{-1} \overline{W}_{N,N}
\end{bmatrix}.$$

Rewriting $U_{\text{sum}}$ in the form of block matrices, in which $U_1$, $U_2$, $U_3$ and $U_4$ are, respectively, submatrices of $U_{\text{sum}}$ of order $4(N-1)\times4(N-1)$, $4(N-1)\times4$, $4\times4$ and $4\times4$, functional relations among integration constants $C_{N+1,1}, C_{1,1,1}$ and $C_R$ can be given as:

$$\begin{align*}
C_{N+1,1} &= (U_4 - U_3 U_1^{-1} U_2) C_{1,1} \\
C_{1,1,1} &= (U_3 - U_4 U_1^{-1} U_1) C_R,
\end{align*}$$

where $C_R = [C_{N,1}^{T} \ C_{N+1,1}^{T} \ \cdots \ C_{2,1}^{T}]^{T}$.

2.4. Derivation of frequency response functions (FRFs)

Considering a harmonic force $e^{i\omega t}$ applied at the left-hand or right-hand end of the beam, the boundary conditions can be rewritten as:

$$\begin{align*}
\begin{bmatrix}
B_L \\
B_R \overline{T}_{N+1} (U_4 - U_3 U_1^{-1} U_2)
\end{bmatrix} C_{1,1} = B_{4\times4} C_{1,1} = F_E,
\end{align*}$$

with $F_E = [0 \ 0 \ 0 \ 0 \ \cdots]^{T}$ or $F_E = [0 \ 0 \ 0 \ 0 \ \cdots]^{T}$ depending on the position of the applied load, $B_L$ and $B_R$ the coefficient matrices associated with the corresponding ends.

The corresponding FRFs for the leftmost sub-beam can then be obtained as follows:
\[ f_{1,1}(x_{1,1}) = \psi_{1,1}(x_{1,1})B_{4\times4}^{-1}F_E, \]  

(23)

Considering a point force \( e^{i\Omega t} \) applied at an arbitrary position of the beam, an artificial spring of zero stiffness and damping is inserted between the beam and foundation at the position of the applied force. If the station numbering for a concentrated force is \( u \), the transfer matrix expression of the connected segments across the artificial connection point can be given as follows:

\[ C_{u+1,1} = T_{Ru}^{-1}T_{La}^{-1}C_{u,1} - \sum_{m=1}^{N} \left[ T_{Ru}^{-1}W_{m,u}C_{m,1} \right] + T_{Ru}^{-1}F_E, \]  

(24)

where \( F_E = [0 \ 0 \ 0 \ \ldots \ 0 \ \ldots \ 0]^{\top} \) is the force vector. By reconstructing equation (20), one obtains the relation between integration constants \( C_{N+1,1} \) and \( C_{1,1} \) as:

\[ C_{N+1,1} = (U_4 - U_3 U_1^{-1} U_2)C_{1,1} - U_3 U_1^{-1} \bar{F}_E, \]  

(25)

where \( \bar{F}_E = [0 \ 0 \ 0 \ \ldots \ U_4 \ \ldots \ 0]^{\top} \), and \( T_{Ru}^{-1}F_E \) is the \( u \)-th column submatrix of \( \bar{F}_E \).

By satisfying the boundary conditions, \( C_{1,1} \) can be readily determined as:

\[ C_{1,1} = B_{4\times4}^{-1} \begin{bmatrix} 0_{2\times4} \\ B_{4\times4}^{-1}T_{R_{N+1}}U_3 U_1^{-1} \bar{F}_E \end{bmatrix} = B_{4\times4}^{-1} \bar{F}_E, \]  

(26)

The corresponding FRFs for the leftmost sub-beam are then given as:

\[ f_{1,1}(x_{1,1}) = \psi_{1,1}(x_{1,1})B_{4\times4}^{-1} \bar{F}_E, \]  

(27)

Once \( C_{1,1} \) is determined, the FRFs for other sub-beams can be conveniently obtained by employing transitive relation of equation (19). The same procedure can be easily extended to derive the FRFs for a harmonic couple \( e^{i\Omega t} \) for both cases, and the repetitious details are not given here for simplicity.

3. Numerical validation

A numerical example is designed to demonstrate the reliability and applicability of the preceding formulations. As shown in figure 2, the model under study is a coupled structure consisting of a simple supported rectangular plate taking as the foundation and a rotor carrying a concentrated mass \( M^B \).

Figure 2. Schematic of the coupled beam–plate system, where the plate is considered as the foundation.
The plate and rotor are connected through three translational spring subunits and share the following material properties: Young’s modulus $E = 2.1 \times 10^{11}$ Pa; mass density $\rho = 7800$ kg/m$^3$; Poisson ratio $\nu = 0.3$. The cross section of the rotor is a solid circle with the radius $r = 0.01$ m and the thickness of the plate is $h = 0.01$ m. Parameters of spring connections and the mass are adopted as: $M^b = 50$ Kg, $M^c = 20$ Kg, $K^B_i = K^F_i = 10^8$ N/m and $C^B_i = C^F_i = 100$ N·s/m ($i = 1, 2, 3$).

### 3.1. Validation of the present formulation

The following calculations study the transverse responses under a harmonic force $e^{i\Omega t}$ at the right end of the rotor. Input data are FRFs of the plate producing by finite element analysis, as shown in figure 3. The present results are compared with the ‘exact’ FRFs which are generated from the modal analyses of the FE model of the whole system. As to the FE simulations, the 4-node structural shell elements are used to model the plate, and the 2-node Timoshenko beam elements constrained to planar motion are utilized to analyse the rotor. A 20×60 mesh is used for the plate, and the rotor is discretized with 120 finite elements after analysing convergence on natural frequencies.

![Figure 3. Transverse FRFs between the 1st connection point (P1) and the i-th point (Pi) on the plate.](image)

![Figure 4. Transverse FRFs between the excitation point and the 2-nd connection point on the rotor.](image)

Consider, for example, the FRFs of coupled points on the rotor and the foundation with regard to the 2-nd spring subunit. Figures 4 and 5 displays the similarity among the ‘exact’ FRFs from FE analysis, the analytical FRFs and the FRF-based substructuring method. It is clear that the present analysis yields a very good result. However, the problem-solving efficiency of FEM demonstrably depends upon the order $N_s \times N_c$ of the system matrices and $N_c$ is large in general, whereas that of the present analysis merely lies in the number $N$ of connection springs and the sizes of matrices are no larger than $4N \times 4N$ ($4N$ is far less than $N_c$). That will lead to a noticeable computational advantage.
Moreover, the established model allows one to conveniently access effects of parametric variability of
the connections and discontinuities on vibration behaviours. When it comes to the FBSM, the FRF
matrices of the multiple-discontinuity beam should be reconducted for every change in beam, which
may be tedious in practices.

![Figure 5](image)

**Figure 5.** Transverse FRFs between the excitation point and the 2-nd connection point on the foundation.

3.2. Effects of the measured noise
The developed method is expected to allow the direct use of the measured FRFs, thus it is essential to
evaluate the sensitivity of assembly responses to noise in the foundation FRFs, which is carried out by
repeating previous analyses in the presence of normally-distributed, 3% and 8% random noise.
Considering the FRFs of coupled point on the beam again, calculation results with and without noise
effects are compared and illustrated in figures 6 and 7. For the case of 3% noise level, little noise of
the predicted FRF is observed. If 8% random noise is added, however, some noise can be found near
the resonance and anti-resonance peaks across the frequency range 50 Hz to 200 Hz, which is most
likely due to the matrix inversion in equation (21). Practically, it is meaningful to reduce the effects of
the measured noise level. An effective practice is to smooth out the noisy FRFs of the foundation with
the Savitzky-Golay smoothing filters [11] before the coupling analysis. Smoothing the FRFs added 8%
random noise by applying a cubic polynomial filter to data frames of length 27, it is found that the
result in figure 7 can be greatly improved, as shown in figure 8.

![Figure 6](image)

**Figure 6.** Transverse FRFs between the excitation point and the 2nd connection point on the beam for the cases of 3% noise added in foundation FRFs.
Figure 7. Transverse FRFs between the excitation point and the 2nd connection point on the beam for the cases of 8% noise added in foundation FRFs.

Figure 8. Transverse FRF between the excitation point and the 2nd connection point on the beam when foundation FRFs with 8% noise are filtered before the coupling analysis.

4. Conclusions
A hybrid modelling approach is applied to the analysis of a rotor-foundation system. The formulation is on the basis of the modified transfer matrix method and the receptance coupling. The proposed technique facilitates an efficient modelling method where a mixture of theoretical, numerical or experimental models can be incorporated into an overall model, which contributes to a great flexibility in dynamic analyses of coupled rotor-foundation structures. Discontinuities encountered in applications and coupling effects between the beam and the foundation can be systematically characterized, which provides a significantly simplified computer programming. Moreover, the sensitivity of responses to noise in the foundation FRFs is evaluated. The data smoothing procedure is employed to handle noisy FRFs of foundation and the results can be greatly improved.

Acknowledgments
This work was supported by the National Natural Science Foundation of China (NSFC) [grant number 51505281].

References
[1] Posiadala B 1997 Free vibrations of uniform Timoshenko beams with attachments J. Sound. Vib 204 359
[2] Cha P D 2005 A general approach to formulating the frequency equation for a beam carrying miscellaneous attachments J. Sound. Vib 286 921
[3] Yavari A and Sarkani S 2001 On applications of generalized functions to the analysis of Euler–Bernoulli beam–columns with jump discontinuities Int. J. Mech. Sci 43 1543
[4] Feng N S and Hahn E J 1995 Including foundation effects on the vibration behaviour of rotating machinery Mech. Syst. Signal. Pr 9 243
[5] Ren Y and Beards C F 1995 On substructure synthesis with FRF data J. Sound. Vib 185 845
[6] Liu W and Ewins D J 2002 Substructure synthesis via elastic media J. Sound. Vib 257 361
[7] De Klerk D, Rixen D J and Voormeeren S N 2008 General framework for dynamic substructuring: history, review, and classification of techniques AIAA. J 46 1169
[8] Zhang Z G, Chen F, Zhang Z Y, and Hua H X 2014 Vibration analysis of non-uniform Timoshenko beams coupled with flexible attachments and multiple discontinuities Int. J. Mech. Sci 80 131
[9] Zhang Z G, Huang X C, Zhang Z Y, and Hua H X 2014 On the transverse vibration of Timoshenko double-beam systems coupled with various discontinuities Int. J. Mech. Sci 89 222
[10] Timoshenko S P 1974 Vibration problems in engineering (New York: John Wiley & Sons)
[11] Orfanidis S J 1995 Introduction to signal processing (New Jersey: Prentice-Hall)