A quantum key distribution and identification protocol based on entanglement swapping

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A quantum key distribution and identification protocol is proposed, which is based on entanglement swapping. Through choosing particles by twos from the sequence and performing Bell measurements, two communicators can detect eavesdropping, identify each other and obtain the secure key according to the measurement results. Because the two particles measured together are selected out randomly, we need neither alternative measurements nor rotation of the Bell states. Furthermore, less Bell measurements are needed in our protocol than in the previous similar ones.

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I. INTRODUCTION

As a kind of important resource, entanglement is widely used in the research of quantum information, including quantum communication, quantum cryptography and quantum computation. Entanglement swapping, abbreviated by ES, is a nice property of entanglement. That is, by appropriate Bell measurements entanglement can be swapped between different particles. For example, consider two pairs of particles in the state of $|\Phi^+\rangle$, equivalently, $|\Phi^+\rangle_{12} = |\Phi^+\rangle_{34} = 1/\sqrt{2}(|00\rangle + |11\rangle)$, where the subscripts denote different particles. If we make a Bell measurement on 1 and 3, they will be entangled to one of the Bell states. Simultaneously, 2 and 4 will be also projected onto a corresponding Bell state. We can find the possible results through the following process:

$$
|\Phi^+\rangle_{12} \otimes |\Phi^+\rangle_{34} = \frac{1}{2}(|00\rangle + |11\rangle)|00\rangle + |11\rangle)|11\rangle = \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle)_{1234}
$$

It can be seen that there are four possible results: $|\Phi^+\rangle_{13} |\Phi^+\rangle_{24}$, $|\Phi^-\rangle_{13} |\Phi^+\rangle_{24}$, $|\Psi^+\rangle_{13} |\Psi^-\rangle_{24}$ and $|\Psi^-\rangle_{13} |\Psi^-\rangle_{24}$. Furthermore, these results appear with equal probability, that is, $1/4$. For further discussion about ES, see [22] [23] [24] [25].

Quantum cryptography is the combination of quantum mechanics and cryptography. It employs fundamental theory in quantum mechanics to obtain unconditional security. Quantum key distribution (QKD) is an important research direction in quantum cryptography. Bennett and Brassard came up with the first QKD protocol (BB84 protocol) in 1984 [17]. Afterwards, many protocols were presented [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18]. Recently, several QKD schemes based on ES were proposed [19] [20] [21] [22] [23] [24] [25] [26]. In [19] [20] [21] the author introduced a protocol without alternative measurements. It was simplified [22] and generalized [23] before long, and its security was proved in [24]. Besides, Zhao et al. presented a protocol using ES on doubly entangled photon pairs [25].

In this paper we propose a QKD protocol based on ES, which needs neither alternative measurements nor rotation of the Bell states [21] [22] [23]. Simultaneously, we also use ES to identify the legal users. The security against the attack discussed in [21] is assured by certain classical means. See Sec.II for the details of this protocol. The security is analyzed in Sec.III and a conclusion is given in Sec.IV.

II. THE QKD AND IDENTIFICATION PROTOCOL

Suppose the two legal communicators, Alice and Bob, share a bit string $ID$, which is used to identify each other. The initial $ID$ can be obtained by the means discussed in [26]. The particular process of this scheme is as follows:

1. Prepare the particles. Alice generates a sequence of EPR pairs in the state $|\Phi^+\rangle_{AB} = 1/\sqrt{2}(|00\rangle + |11\rangle)$. For each pair, Alice stores one particle and sends the other to Bob.

2. Detect eavesdropping.

(1) Having received all the particles from Alice, Bob randomly selects a set of particles out and makes Bell measurements on them by twos.

(2) Bob tells Alice the sequence numbers and measurement results of the pairs he measured.

(3) According to the sequence numbers, Alice performs Bell measurements on the corresponding pairs, and compares her results with Bob’s. For example, consider one of the pairs Bob measured, in which the sequence numbers of the two particles are $m$ and $n$. Then Alice measures her $m$-th and $n$-th particles in Bell basis, and compares the two results. As discussed in Sec.I, if these particles
were not eavesdropped, Alice and Bob should obtain the same results. Otherwise there must be an eavesdropper (Eve) in the channel, Alice will abort this communication.

3. Identify the users.

(1) Bob randomly selects a set of particles out from his left sequence, and divides them into two subsets averagely, which are denoted as $S_1$ and $S_2$, respectively.

(2) Bob makes Bell measurements on the particles in $S_1$ by twos. Note that for each pair there are two sequence numbers (because each particle has a sequence number) and one corresponding measurement result. Here two notations, $P_1$ and $R_1$, are introduced to denote all the sequence numbers and all the corresponding results, respectively. By the same means, Bob measures the particles in $S_2$. Similarly, the sequence numbers are denoted as $P_2$ and the corresponding results as $R_2$.

(3) Taking the shared information $ID$ as the key, Bob encrypts $P_1$, $R_1$ and $P_2$ with the one-time-pad cipher: $y = E_{ID}(P_1, R_1, P_2)$, and sends $y$ to Alice through the classical channel.

(4) When Alice received the ciphertext $y$, she decrypts it with the help of $ID$: $P'_1, R'_1, P'_2 = E_{ID}^{-1}(y)$. According to $P'_1$, Alice measures the corresponding particles in Bell basis and compares her results with $R'_1$. If both results coincide, Alice considers Bob is legal and the communication continues.

(5) According to $P'_2$, Alice measures the corresponding particles in Bell basis and sends the measurement results (denoted as $R'_2$) to Bob through the classical channel.

(6) Bob compares $R_2$ with $R'_2$. If they are identical, Bob considers Alice is legal. Otherwise, he stops this communication.

4. Obtain the key. Bob makes Bell measurements on his left particles by twos. It should be emphasized that each pair he measures is selected out randomly. Bob records the sequence numbers of all the pairs and sends the record to Alice. Alice then measures her corresponding particles in Bell basis. As discussed in the above paragraphs, their measurement results would be identical. Subsequently, Alice and Bob can obtain the key from these results. For example, $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$ and $|\Psi^-\rangle$ are encoded into 00, 01, 10 and 11, respectively.

5. Renew $ID$. Alice and Bob cut a little part from the key to “refuel” the shared information $ID$ and the previous one is discarded.

Thus the whole QKD and identification protocol is finished. By this process, Alice and Bob can not only get secure key but also identify each other.

III. SECURITY

The above scheme can be regarded as secure. The reasons are as follows:

1. Each user’s identity is authenticated and it is impossible for Eve to impersonate Alice (or Bob) and distribute key with Bob (or Alice). In this protocol the shared information $ID$ is only known to Alice and Bob. If Eve wants to impersonate Alice, she can not decrypt $y$ correctly and get $P_3$ in step.3. Consequently Eve can not present $R'_2$, with which Bob is satisfied. On the contrary, if Eve wants to impersonate Bob, she will be detected, too. Because Eve can not give such a “ciphertext” that Alice would obtain suited $P_1$ and $R_1$ after she used $ID$ to “decrypt” this “ciphertext”. Furthermore, Eve can not obtain $ID$ from the qubits and the classical information transmitted. Because $ID$ is used as a key of the one-time-pad cipher to encrypt some random bit strings including $P_1$, $R_1$ and $P_2$, Eve can not extract any information about $ID$, even through repeated attempts. In addition, it makes the protocol more secure that Alice and Bob would renew $ID$ when they get the key.

2. The key distributed can not be eavesdropped imperceptively. There are two general eavesdropping strategies for Eve. One is called “intercept and resend”, that is, Eve intercepts the legal particles and replaces them by her counterfeit ones. For example, Eve generates the same EPR pairs and sends one particle from each pair to Bob, thus she can judge Bob’s measurement results as Alice does in step.4. But in this case there are no correlations between Alice’s particles and the counterfeit ones. Alice and Bob will get random measurement results when they detect eavesdropping in step.2. Suppose both Alice and Bob use $s$ pairs particles to detect eavesdropping, the probability with which they obtain the same results is only $(1/4)^s$. That is, Eve will be detected with high probability when $s$ is big enough. The second strategy for Eve is to entangle an ancilla with the two-particle state that Alice and Bob are using. At some later time she can measure the ancilla to gain information about the measurement results of Bob. This kind of attack seems to be stronger than the first strategy. However, we can prove that it is invalid to our protocol as follows.

Because each particle transmitted in the channel is in a maximal mixed state, there are no differences among all these particles for Eve. Furthermore, Eve does not know Bob will put which two particles together to make a Bell measurement. As a result, what she can do is to make the same operation on each particle transmitted. Let $|\varphi\rangle_{ABE}$ denote the state of the two particles and the ancilla, where the subscripts $A$, $B$ and $E$ express the particles belonging to Alice, Bob and Eve, respectively. Note that we do not limit each ancilla’s dimension, and allow Eve to build all devices that are allowed by the laws of quantum mechanics. What we wish to show is that if this entanglement introduces no errors into the QKD procedure, then $|\varphi\rangle_{ABE}$ must be a product of a two-particle state and the ancilla. This implies that Eve will gain no information about the key by observing the ancilla or, conversely, if Eve is to gain information about the key, she must invariably introduce errors.

Without loss of generality, suppose the Schmidt decomposition (27) of $|\varphi\rangle_{ABE}$ is in the form
where $|\psi_i\rangle$ and $|\phi_j\rangle$ are two sets of orthonormal states, $a_k$
are non-negative real numbers ($i, j, k = 1, 2, 3, 4$).

Because $|\psi_i\rangle$ are two-particle (four-dimensional) states, they can be written as linear combinations of $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$. Let

$|\psi_1\rangle = b_{11}|00\rangle + b_{12}|01\rangle + b_{13}|10\rangle + b_{14}|11\rangle$

$|\psi_2\rangle = b_{21}|00\rangle + b_{22}|01\rangle + b_{23}|10\rangle + b_{24}|11\rangle$

$|\psi_3\rangle = b_{31}|00\rangle + b_{32}|01\rangle + b_{33}|10\rangle + b_{34}|11\rangle$

$|\psi_4\rangle = b_{41}|00\rangle + b_{42}|01\rangle + b_{43}|10\rangle + b_{44}|11\rangle$  \(\text{(3)}\)

\[|\varphi\rangle_{ABE} = a_1|\psi_1\rangle_{AB}|\phi_1\rangle_E + a_2|\psi_2\rangle_{AB}|\phi_2\rangle_E + a_3|\psi_3\rangle_{AB}|\phi_3\rangle_E + a_4|\psi_4\rangle_{AB}|\phi_4\rangle_E \]  \(\text{(4)}\)

According to the properties of ES, we can calculate the probability with which each possible measurement-results-pair is obtained after Alice and Bob measured their particles in Bell basis. For example, observe the event that Alice gets $|\Phi^+\rangle$ and Bob gets $|\Psi^+\rangle$, which corresponds to the following item in the expansion:

\[\frac{1}{2}|\Phi^+\rangle_A|\Psi^+\rangle_B \otimes \sum_{r,s=1}^{4} (a_r b_r a_s b_s + a_r b_r a_s b_s + a_r b_r a_s b_s + a_r b_r a_s b_s)|\phi_r\phi_s\rangle_E\]  \(\text{(6)}\)

Therefore, this event occurs with the probability

\[P(\Phi^+_A\Psi^+_B) = \frac{1}{4} \sum_{r,s=1}^{4} |a_r b_r a_s b_s + a_r b_r a_s b_s + a_r b_r a_s b_s + a_r b_r a_s b_s|^2\]  \(\text{(7)}\)

However, this event should not occur. In fact, if Eve wants to escape from the detection of Alice and Bob, any results-pair other than $|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle$ and $|\Psi^-\rangle$ should not be appear. Let $P(\Phi_A^+\Psi_B^-) = 0$, we then have, from Eqs. (7) and (5),

\[v_1^T v_2 + v_2^T v_1 + v_3^T v_4 + v_4^T v_3 = 0\]  \(\text{(8)}\)

in which $v_1^T$ is the transpose of $v_1$.

Similarly, let the probabilities of $\Phi_A^+\Psi_B^-$, $\Phi_A^-\Psi_B^+$ and $\Phi_A^-\Psi_B^-$ equal to 0, we get

\[v_1^T v_2 - v_2^T v_1 + v_3^T v_4 - v_4^T v_3 = 0\]  \(\text{(9)}\)

\[v_1^T v_2 + v_2^T v_1 - v_3^T v_4 - v_4^T v_3 = 0\]  \(\text{(10)}\)

\[v_1^T v_2 - v_2^T v_1 - v_3^T v_4 + v_4^T v_3 = 0\]  \(\text{(11)}\)
From Eqs.(8)-(11), we can obtain
\[ v_1^Tv_2 = v_2^Tv_1 = v_3^Tv_4 = v_4^Tv_3 = 0 \] (12)
That is,
\[ \begin{cases} v_1 = 0 \quad \text{or} \quad v_2 = 0 \\ v_3 = 0 \quad \text{or} \quad v_4 = 0 \end{cases} \] (13)
For the same reason, we can obtain the following results:
(1). Let the probabilities of \( \Psi^+_A \Phi^-_B \), \( \Psi^-_A \Phi^-_B \), \( \Psi^+_A \Phi^+_B \) and \( \Psi^-_A \Phi^+_B \) equal to 0, we can get
\[ \begin{cases} v_1 = 0 \quad \text{or} \quad v_3 = 0 \\ v_2 = 0 \quad \text{or} \quad v_4 = 0 \end{cases} \] (14)
(2). Let the probabilities of \( \Phi^+_A \Phi^-_B \) and \( \Phi^-_A \Phi^+_B \) equal to 0, we then have
\[ v_1^Tv_1 - v_2^Tv_2 + v_3^Tv_3 - v_4^Tv_4 = 0 \] (15)
\[ v_1^Tv_1 + v_2^Tv_2 - v_3^Tv_3 - v_4^Tv_4 = 0 \] (16)
And then
\[ \begin{cases} v_1 = \pm v_4 \\ v_2 = \pm v_3 \end{cases} \] (17)
(3). Let the probabilities of \( \Psi^+_A \Psi^-_B \) and \( \Psi^-_A \Psi^+_B \) equal to 0, we can get the same conclusion as Eq.(17).
Finally, we can obtain three results from Eqs.(13), (14) and (17):
1. \( v_1 = v_2 = v_3 = v_4 = 0 \);
2. \( v_1 = v_4 = 0 \) and \( v_2 = \pm v_3 \);
3. \( v_2 = v_3 = 0 \) and \( v_1 = \pm v_4 \).
That is, each of these results makes Eve succeed in escaping the detection of Alice and Bob. Now we can observe what the state \( |\varphi\rangle_{ABE} \) is by putting these results into Eq.(4). If the first result holds, we have \( |\varphi\rangle_{ABE} = 0 \), which is meaningless for our analysis. Consider the condition where the second result holds, \( |\varphi\rangle_{ABE} \) can be written as:
\[ |\varphi\rangle_{ABE} = (|0\rangle \pm |1\rangle)_{AB} \otimes (a_1b_{12}|\phi_1\rangle + a_2b_{22}|\phi_2\rangle + a_3b_{32}|\phi_3\rangle + a_4b_{42}|\phi_4\rangle)_E \] (18)
It can be seen that \( |\varphi\rangle_{ABE} \) is a product of a two-particle state and the ancilla. That is, there is no entanglement between Eve’s ancilla and the legal particles, and Eve can obtain no information about the key. Similarly, we can draw the same conclusion when the third result holds.

To sum up, our protocol can resist the entangle-ancilla eavesdropping strategy.

IV. CONCLUSION

We have presented a QKD and identification protocol based on ES. The security against the attack discussed in [20] is assured by a classical means, “randomly select the particles out and put together by twos”, in stead of the quantum ones such as alternative measurements [25] or rotation of the Bell states [21, 22, 23]. Furthermore, this classical means brings us another advantage. That is, it is unnecessary to randomize the initial Bell states as in [19, 21]. This in turn leads to less Bell measurements in our protocol. For instance, to distribute two bits of key, Alice and Bob make two Bell measurements in our protocol, while in [19, 21] they must make three. Therefore, we can draw a conclusion that classical means is important to the research of quantum cryptography and to some extent it is even more effective than the quantum ones. Besides, classical means is easier to be implemented. On the other hand, we have to confess that our protocol has a disadvantage, i.e., it uses a sequence of entangled states but not a single quantum system [21, 22, 23] to generate the key. Fortunately, it is not a fatal problem. Many QKD protocols work in this model, for example, the famous E91 protocol [8]. Furthermore, each pair of particles is still in one of the Bell states and can be reused in other applications after QKD.

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