Attitude control system design using a flywheel suspended by two gimbals

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Abstract. This work presents the attitude control system design procedures for a three axis stabilized satellite in geostationary orbit, which contains a flywheel suspended by two gimbals. The use of a flywheel with two DOFs is an interesting option because with only one device it’s possible to control the torques about vehicle’s three axes; through the wheel speed control and gyrotorquing phenomenon with two DOFs. If the wheel size and speed are determined properly it’s possible to cancel cyclic torques using gas jets only periodically to cancel secular disturbance torques. The system, based on a flywheel, takes only one pitch/roll (earth) sensor to maintain precise attitude, unlike mass expulsion based control systems, which uses propellants continuously, beyond roll, pitch and yaw sensors. It is considered the satellite is in nominal orbit and, therefore, that the attitude’s acquisition phase has already elapsed. Control laws and system parameters are determined in order to cancel the solar pressure radiation disturbance torque and the torque due to misalignment of the thrusters. Stability is analyzed and step and cyclic responses are obtained.

1. Introduction

A satellite’s pictorial view is shown in figure 1 in the orbital nominal position, with the body fixed axis x, y, z - the principal axes of inertia - aligned with the orbital reference frame. The satellite parameters as well as the requirements for the torque disturbance and pointing accuracy are given in Table 1.

Table 1. Parameters and design requirements

| Parameter                         | Value                  |
|----------------------------------|------------------------|
| Satellite mass                   | 716 kg                 |
| Moments of inertia               | \( I_x = I_z = 2000 \text{Nms}^2, \ I_y = 400 \text{Nms}^2 \) |
| Attitude Accuracy requirements   | Roll (\( \phi \)) and pitch (\( \theta \)) = 0,05°, yaw (\( \psi \)) = 0,40° |
| Solar pressure torques \( t = 0 \) at 6 AM or 6 PM orbital position | \( T_x = 2 \times 10^{-5}(1 - 2 \sin \omega_o t) \) Nm |
|                                  | \( T_y = 10^{-4}(\cos \omega_o t) \) Nm |
|                                  | \( T_z = -5 \times 10^{-5}(\cos \omega_o t) \) Nm |
| Thruster misalignment torque     | \( T_F = 8,5 \times 10^{-5} \) Nm |

Where \( \omega_o \) is the geostationary orbit rate with constant value of \( 7.28 \times 10^{-5} \) rad/sec.
The nominal orientation of the momentum wheel is shown in figure 2, with the spin axis coinciding with the pitch axis. The components of the angular momentum of the wheel with respect to the principal axes of inertia are obtained with help of the figure 3.

![Figure 2. Nominal orientation of flywheel with two DOFs.](image)

**Figure 2.** Nominal orientation of flywheel with two DOFs.

**SOURCE:** Kaplan (1976), p. 243.

2. Linearized equations of motion

Separating the environmental torques in two instalments, the Euler equations of motion are given by

$$
\mathbf{T} + \mathbf{G} = \frac{d\mathbf{h}}{dt} = \left[ \frac{d\mathbf{h}}{dt} \right]_B + \mathbf{\omega} \times \mathbf{h},
$$

(1)

where \( \mathbf{T} \) are the solar radiation pressure and thrust’s misalignment torques; \( \mathbf{G} \) is the gravity gradient torque given by \( \mathbf{G} = -3\omega_0^2(I_y - I_z)\mathbf{i} - 3\omega_0^2(I_x - I_z)\mathbf{j} \); \( \mathbf{\omega} \) is the angular velocity vector of the satellite body. The vector \( \mathbf{h} \) represents the total angular momentum of the system, \( \mathbf{h} = \mathbf{h}_v + \mathbf{h}_w \), wherein \( \mathbf{h}_v \) is the angular momentum of the vehicle and \( \mathbf{h}_w \) is the angular momentum of the inertia wheel. The components of the vehicle’s angular momentum vector are given with respect to the principal axis of inertia as \( \mathbf{h}_v = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k} \). From figure 3 the components of the wheel
angular momentum with respect to $x$, $y$, $z$ are given by $h_w = (\cos \delta \sin \gamma) h_\omega i - (\cos \delta \cos \gamma) h_\omega j - (\sin \delta) h_\omega k$, where $\delta$ and $\gamma$ are the gimbals deflection angles on the roll and yaw axis, respectively.

Expanding equation (1) yields the general equation of motion

$$\begin{align*}
T &= \left[I_x \dot{\omega}_x - \delta (\sin \delta \sin \gamma) h_w + \dot{\gamma} (\cos \delta \cos \gamma) h_w + (\cos \delta \sin \gamma) h_w + \omega_y (\omega_z h_x - h_w \sin \delta) - \\
&\quad - \omega_x (\omega_y h_y - h_w \cos \delta \cos \gamma) - 3 \omega^2_z (I_y - I_z) \dot{\phi} + [I_y \dot{\omega}_y + \delta (\sin \delta \cos \gamma) h_w + \dot{\gamma} (\cos \delta \sin \gamma) h_w - \\
&\quad - (\cos \delta \cos \gamma) h_w + \omega_z (I_y h_x + h_w \cos \delta \sin \gamma) - \omega_x (I_z h_x - h_w \sin \delta) - 3 \omega^2_y (I_x - I_z) \theta] j + \\
&\quad [I_z \dot{\omega}_z - \delta (\cos \delta) h_w - (\sin \delta) h_w + \omega_x (I_y h_y - h_w \cos \delta \cos \gamma) - \omega_y (I_z h_x + h_w \cos \delta \sin \gamma)] k.
\end{align*}$$ (2)

Whereas the vehicle’s ($\phi$, $\theta$ and $\psi$) and the gimbal’s ($\delta$ and $\gamma$) angular deviations, with respect to the nominal frame shown in figure 1, are considerably smaller. Deviations from nominal wheel momentum $h_n$ are also assumed small, allowing $h_w = h_n$ and applying the Euler angles transformation (sequence 3-2-1) in the $\omega$ vector components, thus yields the linearized equations of motion

$$\begin{align*}
T_x &= I_x \ddot{\phi} + \left[4 \omega^2_\phi (I_y - I_z) + \omega_\phi h_n \phi + \left[- \omega_\phi (I_x - I_y + I_z) + h_n \right] \dot{\theta} + h_x c - \omega_o h_z c, \\
T_y &= I_y \ddot{\theta} + \left[3 \omega^2_\phi (I_x - I_z) \right] \dot{\theta} + \dot{h}_y c, \\
T_z &= I_z \ddot{\psi} + \left[2 \omega^2_\phi (I_x - I_y) + \omega_\psi h_n \psi - \left[- \omega_\psi (I_x - I_y + I_z) + h_n \right] \dot{\phi} + h_x c + \omega_o h_z c.
\end{align*}$$ (3) (4) (5)

3. Yaw axis control system design
For satellite specifications given in Table 1, it is observed that $I_x = I_z$ and thus, the pitch axis equation of motion given by equation (4) can be simplified to

$$\dot{T}_y = I_y \ddot{\theta} + \dot{h}_y c,$$ (6)

in which $\dot{h}_y c$ is the inertia wheel’s angular momentum rate of change, whose aim is to impose direct torque control about the pitch axis. A pseudorate modulator is to be used, which serves to modulate the torque control $\dot{h}_y c$ and synthesizing the angular rate of change $\dot{\theta}$, ie, the control device makes torque being dependent on the angular rate of change. A satisfactory control form is given by

$$\dot{h}_y c = K_p (\tau_p \dot{\theta} + \theta),$$ (7)

wherein the term $\dot{\theta}$ introduces the necessary damping for the system, $K_p$ is the pitch autopilot gain and $\tau_p$ is the pitch constant time. Substituting equation (7) into equation (6), applying the Laplace transform and rearranging the terms, the pitch axis control system transfer function results as being

$$\frac{\theta(s)}{T_y(s)} = \frac{1}{I_y s^2 + K_p \tau_p s + K_p}.$$ (8)

The pitch loop root locus diagram for the variable $K_p$ is shown in figure 4, and it indicates that the system is stable for all values of $K_p$. Adjusting the output to behave critically damped (no overshoot) and according to the attitude accuracy requirements given in Table 1, obtains the most suitable parameters pitch autopilot gain $K_p = 0.275$ Nm/rad and pitch constant time $\tau_p = 80$ s. Applying the parameters found in equation (8) yields the step and cyclic disturbance torques responses, shown in figures 5 and 6, respectively. The step type input is due to thruster misalignment and the cyclical type input is characteristic of the solar radiation pressure torque.

4. Roll/yaw control system design
In order to simplify the equation of motion for roll and yaw axis, it is assumed that $h_n \gg \max [I_x \omega_o, I_y \omega_o, I_z \omega_o]$, so the equations (3) and (5) are rewritten as

$$\begin{align*}
T_x &= I_x \ddot{\phi} + \omega_o h_n \phi + h_n \dot{\psi} + \dot{h}_x c - \omega_o h_z c, \\
T_z &= I_z \ddot{\psi} + \omega_o h_n \psi - \omega_o (I_x - I_y + I_z) + h_n \dot{\phi} + h_x c + \omega_o h_z c.
\end{align*}$$ (9)
Again using a pseudorate modulator and based on the same principles used to prepare the pitch axis control system, the used roll/yaw control laws are

\[ M_{xc} = \dot{h}_{xc} - \omega_0 h_{zc} + \omega_0 h_n \dot{\phi} + h_{zc} + \omega_0 h_{xc}, \]

\[ M_{zc} = \dot{h}_{zc} + \omega_0 h_{zc} - h_n \dot{\phi} = -k K (\tau \dot{\phi} + \phi), \]

where \( M_{xc} \) and \( M_{zc} \) represent the roll and yaw control moments, respectively, \( K \) is the roll autopilot gain, \( \tau \) is the roll time constant and \( k \) is the yaw-to-roll gain ratio.

\[ T_z = I_z \ddot{\psi} + \omega_0 h_n \dot{\psi} - h_n \dot{\phi} + h_{zc} + \omega_0 h_{xc}. \]  

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{root_locus.png}
\caption{Pitch loop root locus diagram for variable \( K_p \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{step_response.png}
\caption{Pitch axis output for a step input.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{cyclic_response.png}
\caption{Pitch axis output for a cyclical input.}
\end{figure}

Replacing control law in equations (9) and (10), applying the Laplace transform and rearranging the terms, the angular responses are obtained

\[ \Phi(s) = \frac{T_z(s) (I_z s^2 + \omega_0 h_n) - T_z(s) h_n s}{I_z s^4 + K I_z s^3 + (K I_z + I_z \omega_0 h_n + K K h_n s) s^2 + (\omega_0 h_n K \tau + K K h_n) s + \omega_0 h_n K^2}. \]
The roll/yaw system root locus diagram for the variable $K$ is shown in figure 7. Adjusting the output to behave critically damped (no overshoot) and according to the attitude accuracy requirements given in Table 1, obtains the most suitable parameters for roll autopilot gain $K = 1.56$ Nm/rad, roll constant time $\tau = 80$ s and yaw-to-roll gain ratio $k = 0.054$. Due to the coupling between the motions, a disturbance on one axis (roll or yaw) directly affects the motion of both axes. The figures 8 and 9 show the output of the roll axis due to a step disturbance in the roll and yaw axes, respectively, caused by the thrust misalignment; while the figures 10 and 11 show the output of the yaw axis due to a step disturbance in the roll and yaw axes, respectively, caused by the same effect. The figures 12 and 13 show the roll and yaw axes responses due to a disturbance caused by a cyclic solar pressure torque type.

\[
\Psi(s) = \frac{T_2(s)(I_x s^2 + K t s + K) + k K (t s + 1) T_2(s)}{I_x I_z s^4 + K I_z t s^3 + (K I_z + I_x \omega_0 h_\omega + k k h_\omega) s^2 + (\omega_0 h_\omega K t + k K h_\omega) s + \omega_0 h_\omega K}.
\]

Figure 7. Roll/yaw system root locus diagram for variable $K$.

Figure 8. Roll axis output for a step input in the roll axis.

Figure 9. Roll axis output for a step input in the yaw axis.
Figure 10. Yaw axis output for a step input in the roll axis.

Figure 11. Yaw axis output for a step input in the yaw axis.

Figure 12. Roll axis output for a cyclical input.

Figure 13. Yaw axis output for a cyclical input.

5. Conclusions
The linearized equation of motion for the system under consideration was obtained, which in turn separate the pitch motion of roll and yaw motions, thus simplifying the application of control system theory. Unlike most designs, which use three inertia wheels, one for each axis, this design has achieved successful results in accordance with any attitude of accuracy requirements only with one inertia wheel, minimizing costs, improving the dynamics and reducing satellite weight.

References
[1] Kaplan M H 1976 Modern Spacecraft Dynamics & Control (New York: John Wiley & Sons)