Modeling Galactic Rotation Curves with ultra-light scalar field dark matter

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Abstract. In this work we investigate the behaviour of SFDM at small scales, specifically its viability in reproducing galactic rotation curves. We consider a scalar field with no self-interaction. In this case, the only free parameter is the mass of the scalar field. By fitting the galactic rotational curves it is found the allowed region for the mass of the scalar field. We discuss those limits with previous bounds on the mass coming from astrophysical considerations. The galactic density profiles within the SFDM is found by solving a non linear system of ordinary differential equations for the scalar and the gravitational fields. Given the appropriate boundary conditions, the system becomes an eigenvalue problem which is solved by an inner-outer interaction using an efficient computational method. This method is faster than the predecessors, and could be implemented in parallel computing.

1. Introduction
There are two well tested theories that describe all four fundamental interactions: the Standard Model of elementary particles (SM) and the general theory of relativity. The first one explains strong, weak and electromagnetic interactions while the later explains the gravitational interaction. The combination of this two theories can explain most of the astrophysical observations at cosmological distances in a consistent model that is referred as the standard model of cosmology. This standard model of cosmology predicts that 95% of the energy density of the universe is made of two new components that are not included in the SM. They are referred as the Dark Matter (DM) and the Dark Energy (DE) and its nature is one of the most intriguing and fundamental subjects in Modern Physics. (See [1] and [2] for reviews on DM and DE respectively).

Regarding dark matter, there is a plethora of candidates, most of them coming from extensions to the SM [3]. In general, the most generic DM candidate coming form particle physics is referred as Weak Interactive Massive Particle (WIMP) and is modelled as a pressureless fluid. Another viable candidate is the axion, and recently, the interest on ultra-light axion-like dark matter candidates has been increased [4]. On the context of axion-like dark matter, one of the simplest candidates is the one which does not have any coupling with the standard model. Several flavors of this simplest candidate have been studied since more than a decade ago. They have been named as Scalar Field Dark Matter (SFDM) [5, 6, 7, 12], fuzzy dark matter [8], Bose-Einstein Condensate dark matter [26] and more recently Wave dark matter [9].

The general feature of those models is that dark matter particles form a coherent state that can be described as a massive, classical scalar field, either complex or real, minimally coupled to
gravity, endowed with a scalar field potential \( V(\Phi) \). Different potentials have been considered. For instance, \( V(\Phi) = \frac{m^2}{8\pi G\lambda^2} (\cosh(\sqrt{\frac{8\pi G}{\lambda}} \Phi) - 1) \) \([5, 6, 7]\), \( V(\Phi) = m^2\Phi^2 + \lambda\Phi^4 \) \([10, 11]\) and the simplest one that only consider the mass term \([12, 8, 9]\):

\[
V(\Phi) = m^2\Phi^2. \tag{1}
\]

With out the intension of to be exhaustive, a general description of the model with the simplest potential (1), to which we called from now on Ultra-Light Scalar Field Dark Matter Model (ULSFDM) is as follows: The thermal evolution of the bosonic particles and their condensation is studied in \([22, 13]\). Once the condensate is formed, this coherent state described by a classical scalar field with the scalar potential (1), successfully describes the dark matter evolution at cosmological scales \([14, 12]\) that is, for a homogeneous and isotropic universe. Furthermore, the ULSFDM mimics the dynamics of linear perturbations to the homogeneous distribution of dark matter \([8]\) \([23]\), and recently results, shown that the ULSFDM is able of reproduce the non linear structure formation at large scales \([25, 9, 23, 24]\).

These results indicates that ULSFDM is a robust model at cosmological scales and as viable as the more popular and well studied CDM. Even more, the ULSFDM would overcome to the CDM at smaller scales since it could not have the so called problems of CDM: the formation of singular halos and the overproduction of dwarf halos. The key piece of the ULSFDM in order to avoid the overproduction of substructure is a mass of the scalar field of the order of \( m \sim 1.1 \times 10^{-23} \) \([6]\). This value of the mass, which is the only free parameter of the model, is in agreement with results on the cosmological evolution and on the structure formation \([9]\). On the other hand, regarding ULSFDM halos, there exist self-gravitating scalar field configurations that have been proposed to play this role. Such configurations do not present singularities and, for a scalar field endowed with a potential (1) with a mass of the order of \( m \sim 10^{-23} \), have sizes comparable with galactic halos. There have been several works studying the viability of such configurations in reproducing different observables of galactic halos.

In this note we investigate the allowed values of the mass \( m \) in order to fit a set of observed galactic rotation curves. We restrict ourselves to consider galaxies for which the contribution of the luminous matter to the rotation curve is negligible and to model ULSFDM halos with stable self-gravitating scalar field configurations known as ground states. We found that big galaxies demand smaller values of the mass of the scalar field than those needed for small galaxies. Some galaxies will require masses beyond the allowed regions coming from other astrophysical considerations. This multiplicity of values for the free parameter \( m \) is an issue in modeling halos with ground state configurations and then another proposals to model ULSFDM halos should be considered. In fact, one proposal has been presented in \([36]\), were non stable scalar field configurations known as excited states, have been used. However a non stable halo is not realistic. Another possibility is to use multi-state-boson-stars which are configurations where excited states coexist together with the ground state. It has been shown that there exist stable multi-state-boson-stars. \([17, 18, 19]\). The corresponding bounds on the mass of the scalar field considering excited states are left for a future work.

2. Ultra-light scalar field dark matter galactic halos

It is well known that self-gravitating configurations of a complex scalar field \( \Phi \) with a potential given by (1) can be obtained as solutions to the coupled Einstein Klein-Gordon (EKG) equations. Such configurations, known as Boson Stars (BS), have their newtonian counterparts, the so called, Newtonian Boson Stars (NBS). Because it is a good approximation to consider galactic halos as newtonian objects, it has been proposed within the ULSFDM that such NBS, instead of relativistic BS, would play the role of galactic halos.
The non-relativistic limit of the EKG equations is the Shrödinger-Poisson system:

\[\begin{align*}
    i\hbar \partial_t \psi &= -\frac{\hbar^2}{2m} \nabla^2 \psi + U \rho \psi, \\
    \nabla^2 U &= 4\pi G \rho,
\end{align*}\]

(2) (3)

where \(\rho = m^2 \psi \psi^*\), \(U\) is the gravitational potential and \(\psi\) is related to \(\Phi\) through \(\Phi = e^{-i\epsilon mc^2t/\hbar} \phi\). In fact \(\psi\) is the non relativistic part of \(\Phi\). NBS are obtained as solutions to (2-3) introducing the following ansatz

\[\psi = e^{-i\epsilon mc^2t/\hbar} \phi(r)\]

(4)

where \(\epsilon\) is the energy eigenvalue, see section (3). NBS are configurations spherically symmetric, gravitationally bounded and regular everywhere. Therefore ULSFDM halos, modeled by NBS will have such characteristics. In particular, the tangential velocity produced by such configurations, i.e., the rotation curve produced by ULSFDM halos, can be easily computed by

\[v = \sqrt{\frac{GM(r)}{r}}\]

(5)

where

\[M(r) = \int_0^r \rho dv.\]

(6)

It is important to make a comment on the gravitational stability of NBS. NBS differ qualitatively on the number of nodes of its radial function \(\phi\). The configurations for which \(\phi\) does not have nodes is called the ground state. For the first excited state \(\phi\) has one node and so on. It is well known that excited states are unstable under gravitational linear perturbations [16, 27, 28, 29, 30, 31] and then, they are not suitable to model realistic halos. In contrast, the ground state is stable and therefore in this work we will consider only this state to model ULSFDM galactic halos.

Another important point to stress is that free parameters of NBS are the central value of the scalar field \(\phi(0)\) and the mass \(m\). However, while \(m\) is a fundamental parameter of the scalar field, \(\phi(0)\) is a boundary condition that can be freely chosen. In fact relevant scales of observables for NBS depend on the pair \((\phi(0), m)\). For instance, in terms of these free parameters, the mass and the radius of the ground state are given by

\[M = \phi_c^{-1/2} 2.75 \times 10^{-10} \frac{1\text{eV}}{m} M_\odot \quad R = \phi_c^{-1/2} 2.43 \times 10^{-26} \frac{1\text{eV}}{m} \text{Kpc},\]

where \(\phi_c\) is a dimensionless quantity related with the value of \(\phi(0)\). For Newtonian systems it is expected \(\phi_c \leq 1 \times 10^{-3}\). Notice that in this regime, the boson mass \(m\) should be extremely light in order that \(M\) and \(R\) have astrophysical scales.

3. Numerical solutions for the Schrödinger-Poisson system

Taking the ansatz (4) and imposing boundary conditions of regularity and asymptotic flatness, the system (2-3) becomes a Sturm-Liouville two-point boundary value problem: given a central value for the radial function \(\phi(0)\), there are discrete values \(\{\epsilon_i, \phi_i(r), U_i(r)\}\) for which solutions \(\{\epsilon_i, \phi_i(r), U_i(r)\}\) satisfy the boundary conditions. Usually the shooting method is used to solve this eigenvalue problem [28, 29, 30, 18]. However we solved it using an algorithm developed in Bernstein et al [32].
Figure 1. Best fit velocity profile for different low surface brightness galaxies.

System (2-3) can be written as:

\[ \frac{\partial^2 \bar{f}}{\partial \bar{r}^2} = 2\bar{f}(\bar{u} - \bar{\epsilon}) \]  \hspace{1cm} (8)  

\[ \nabla^2 \bar{u} = \frac{1}{\bar{r}^2} \bar{f}^2 \]  \hspace{1cm} (9)

where we have re-scaled the system and we have defined \( \bar{f} = \sqrt{\frac{4\pi}{m^3}} r \phi (\frac{m_p}{m})^3 \), \( \bar{\epsilon} = (\frac{m}{m_p})^4 m \epsilon \), \( \bar{u} = (\frac{m}{m_p})^4 U \).

The numerical scheme for computing solutions to (8-9) consist in: 

i) to supply an initial guess for \( \bar{f} \) and to solve Eqn. (9) for \( \bar{u} \) and to solve the eigenvalue problem for \( \bar{\epsilon} \) and \( \bar{f} \).

ii) to substitute \( \bar{u} \) in (8) and to solve the eigenvalue problem for \( \bar{\epsilon} \) and \( \bar{f} \).

iii) to substitute the new \( \bar{f} \) in Eqn. (9) and solve it for \( \bar{u} \).

iv) to iterate the previous two steps until the value of \( \bar{\epsilon} \) converge sufficiently.

Equation (8) can be solved using the finite differences, while equation (9) can be solved using quadratures.

4. Comparison with observations

Once we have found a solution for the Schrödinger-Poisson system, we can compute the theoretical velocity \( v_{\text{theo}}(\phi(0), m) = v(r_i) = \sqrt{GM(r_i)/r_i} \), with \( M(r) \) as defined as eq. (6). Here we perform a single \( \chi^2 \) analysis. There are two free parameters: the amplitude of the scalar field at the origin and the mass of the scalar field. Thus

\[ \chi^2(\phi(0), m) = \sum_i \frac{(v_{\text{theo}}(\phi(0), m) - v_{\text{exp}})^2}{\delta v_{\text{exp}}} \]  \hspace{1cm} (10)
where $i$ runs for all data points, $\delta v_{exp}$ is the error in the determination of the rotational velocity and $v_{exp}^i$ is the central value of the rotational velocity of the stars around the galaxies.

As an example we have used the data points of the galaxies ESO 3020120, ESO 4880049, ESO3050090, U11616, U11557 and U4115 taken from [33]. The best fit together with the data points are shown in Fig. 1. As can bee seen, there are values of $m$ and $\phi(0)$ such as we have a good match between theory and observations. The allowed regions for $\phi(0)$ and $m$ at 68% and 90% C.L. are shown in Fig. 2. There $\phi(0) \propto \lambda$.

5. Discussion

There are different constraints for the mass of the scalar field within the ULSFDM. For instance in [6], the model predicts a cut in the power spectrum that will alleviate the problems that CDM posses if $m \sim 1.1 \times 10^{-23}$ eV. Another interesting constraint was derived in [34] where in order to explain that a relic cluster of star had survived without been disrupted by the gravitational field of the halo as the observations indicate in the interior of the dwarf galaxy Ursa Minor, then $3 \times 10^{-23} eV < m < 1 \times 10^{-22} eV$. Finally, masses between $4 \times 10^{-24} eV < m < 1.6 \times 10^{-23}$ have been derived by fitting a Universal Rotational Curve (URC) [35]. We have included those limits together with the allowed regions for the mass of the scalar field in Fig. 2.

As it can bee seen, big galaxies with high velocities requires small masses of the scalar field, as small as those needed in [6, 35]. But in general there is a conflict with small galaxies. In particular Dwarf galaxies [34] requires bigger masses. Just the galaxy U4115, the one with the smallest radial distance and smaller velocities has compatible masses with all constraints. Is it possible to have the same mass for all galaxies? That would be a benchmark for the model, but
what we see from this exercise is that it is not possible. One way to overcome this may be to perform the fit for big galaxies by using not only ground state BS but multi state boson stars [17, 18, 19]. Those configurations have bigger masses and extend longer distances, such as the mass could be lowered. Further studies are needed.

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