The electric charge and magnetic moment of neutral fundamental particles

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Abstract

The article focuses on the issue of the two definitions of charge, mainly the gauge charge and the effective charge of fundamental particles. Most textbooks on classical electromagnetism and quantum field theory only works with the gauge charges while the concept of the induced charge remains unattended. In this article it has been shown that for intrinsically charged particles both of the charges remain the same but there can be situations where an electrically neutral particle picks up some electrical charge from its plasma surrounding. The physical origin and the scope of application of the induced charge concept has been briefly discussed in the article.

1 Introduction

The electric charge of a fundamental particle is inherently related to an underlying U(1) gauge symmetry of a theory. On the other hand the electromagnetic properties of an extended charge distribution in an arbitrary region of space-time has nothing to do with any gauge symmetry. Classically the electric and magnetic properties of the extended charge distribution is specified by the multipole expansion of the scalar and the vector potential. In quantum field theory the role of the multipole expansion of the potentials is taken by the form-factors of the electromagnetic vertex of the charge distribution.

The interesting point is that we can use the process involved to specify the various moments or form-factors of an unknown extended charge distribution to specify the electromagnetic properties of the fundamental particles themselves. It may seem that the electromagnetic form-factors or the multipole moments of a fundamental particle is an useless concept because in these cases we know their U(1) gauge charges and more over we are unsure about their spatial size. But a closer look into the quantum field theoretical foundations of particle physics tells us that although the facts stated before remain true but still they are not enough to specify the charge of the fundamental particles in an interacting theory of quantum fields. The primary reason why form-factors can play an important role in field theories is related to the virtual degrees of freedom.

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The fundamental particle which is real can always be accompanied by a lot of virtual charged particles and consequently the charge seen by a photon is effectively modified. The set of virtual charged particles near the real particle whose charge is in question gives an extended charge distribution and consequently the multipole moment analysis again becomes meaningful. As an virtually extended charge system emerges near the real on-shell particle, the multipole moments become scale dependent or in other words the electromagnetic form-factors are functions which depend upon the momentum of the probing photon. Out of all the form factors there is one which corresponds to the net charge of the system. If the fundamental particle had a U(1) gauge charge then it definitely was independent of momentum and consequently in the interacting quantum field theory there must be one form-factor which reduces to the U(1) charge of the fundamental particle in the limit of zero momentum. This form-factor is called is called the charge form-factor. Similarly it may turn up that the fundamental particle has other form-factors corresponding to magnetic moment, electric dipole moment and other esoteric moments never encountered in classical electromagnetism. If it turns out that the fundamental particle in question does not possess any U(1) gauge charge to start with then it happens that the charge form-factor vanishes in the zero momentum limit giving a consistent meaning of the charge of the fundamental particle.

The previous discussion of quantum field theory was primarily done without any mention of a background plasma of relativistic particles. In presence of such a thermal medium new physics emerges. To give one simple example which is related to the topic of this article is the emergence of screening effect. In presence of a plasma the electric charge of a charged particle gets screened by the opposite charges present. The oppositely charged particles will form an real extended charge system and consequently the effective charge of the fundamental particle gets modified and this modification is dependent on the momentum of the probing photon. From the above statements it can be inferred that the electromagnetic form-factors of a fundamental particle gets modified from its vacuum values in presence of a background plasma. One interesting feature of the thermal backgrounds is related to the emergence of an effective charge of a neutral particle. A neutral fundamental particle may have electromagnetic (as the photon) or non-electromagnetic (as the neutrino) interactions with the charged particles in the plasma and consequently it can produce a charge polarization around it. In this case as we start to probe the fundamental particle from a distance we will observe a certain amount of charges of the plasma to be in its sphere of influence and if our probing region increases indefinitely then there appears a finite number of charged particles inside the greatest sphere of influence of the fundamental particle. If the plasma is charge symmetric and the interaction of the particle in question is also charge symmetric then there will be an equal number of opposite charged particles polarized around the neutral particle and consequently the effective charge of the neutral particle vanishes. On the other hand if the plasma is not charge-symmetric or the interaction of the fundamental particle breaks charge conjugation symmetry there is a distinct possibility that there will be a net charge built up around the neutral particle. Consequently we will get a non-zero effective charge. This is a kind of inverted-screening effect, where the original charge which was to be screened is actually nonexistent. In technical language we have then an effective charge of the fundamental neutral particle in the zero momentum limit although its U(1) gauge charge is precisely zero.

From our previous discussions we see that we can have two definitions of charge of a fundamental particle. One is related to the gauge invariance of the particle under some U(1) gauge group and the other related to its effective coupling to photons. Both of these definitions agree for a intrinsically charged particle but in nature we have electrically neutral particles also and in those cases they may pick up some charge in a plasma. Books on applications of quantum field theory generally do not shed much light on the concept of the two definitions of charges,
their physical origin and scope of applications, except some as in Refs. [1, 2]. In research level literature we find few calculations of the induced charge [3, 4] of a neutral particle in a medium. Consequently many questions remain unanswered regarding the application of electrodynamics of neutral fundamental particles. In this article we will discuss more quantitatively what has been qualitatively discussed above and try to point out the wider scope and limits of application of the effective charge concept.

The material in the article is presented in the following manner. In section 2 we distinguish between the concepts of the intrinsic or the gauge charge and the effective charge of the electrons. In section 3 we present a brief outline showing the emergence of an effective electric charge for the otherwise neutral standard model neutrinos. Section 4 contains a brief discussion on the effective induced charge of the photons and real scalars. The main points in the article are summarized in section 5.

2 The case of the electrons

2.1 The gauge charge

Generally the electric charge of any elementary particle is a concept which is intimately linked with the local gauge invariance of the theory under an Abelian gauge transformation. In QED if the electromagnetic gauge field is represented by \( A^\mu(x) \) then under a local gauge transformation

\[
A^\mu(x) \rightarrow A^\mu(x) + \partial_\mu \chi(x),
\]

\[
\psi(x) \rightarrow \psi(x) e^{-ieQ\chi(x)},
\]

where \( \chi(x) \) is a well behaved function of the space-time coordinates and \( e \) is the unit of charge, chosen as charge of the proton, and \( Q \) is a constant designating the amount of charge. The physical meaning of \( eQ \) will become transparent when we write the conserved Noether charge corresponding to the continuous symmetry of the system. Due to gauge invariance the Dirac equation for a charged fermion becomes:

\[
(i\not\partial - eQA - m)\psi(x) = 0.
\]

Using the above equation we can solve for the wave-function for various cases of the electromagnetic potential \( A_\mu \). In particular \( A_\mu \) may be a classical field and chosen in such a way that it produces a constant electric or magnetic field or both. In some particular cases we can exactly solve Eq. (3) for electric or magnetic field backgrounds. The solutions portray the behavior of a charged fermion with electric charge \( eQ \) interacting with the classical electric or a classical magnetic field. If we solve Eq. (3) in presence of such an \( A_\mu \) which produces a uniform magnetic field we will get exact non-perturbative (in the external magnetic field strength) solutions whose energy in the transverse direction of the field is Landau quantized. In this case we see that the quantum charged particle interacts with the magnetic field and does something which is analogous to the motion of a classical charged particle whose trajectory is predicted by the Lorentz force law.

In the non-relativistic limit Eq. (3) reduces to the Schrödinger-Pauli 2-component equation

\[
\left[ \frac{1}{2m} (\mathbf{p} - eQA)^2 - \frac{eQ}{2m} \mathbf{B} + eQ\phi \right] \psi_A = E \psi_A,
\]

where the Dirac spinor is written as

\[
\psi = e^{-iEt} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix},
\]
and the classical vector potential giving rise to the magnetic field $B$ is $A^\mu = (\phi, \mathbf{A})$. From the above equation it is clear that the magnetic moment of the electron is $\frac{eQ}{2m\sigma}$. The point to note is that, the charge $eQ$ is the unique coupling constant which couples the Dirac particle to the electric and the magnetic field and this coupling is prescribed by gauge invariance of the theory.

In quantum field theory the conserved Noether current corresponding to an infinitesimal gauge transformation as given in Eq. (2) is

$$ J^\mu_N = eQ \bar{\psi} \gamma^\mu \psi. $$

(6)

Using the Fourier expansion of the free fermionic fields we find that the conserved charge for the Abelian gauge symmetry is

$$ Q_N = \int d^3x J^0_N = eQ \int d^3x \psi^\dagger(x)\psi(x), $$

$$ = eQ \int d^3p \sum_{s=1,2} \left[ a^\dagger_s(p)a_s(p) - b^\dagger_s(p)b_s(p) \right]. $$

(7)

Here $a_s(p)$ and $b_s(p)$ are the annihilation operators for a free electron field and the free positron field respectively. The last equation implies that $eQ$ is indeed related to the charge of the fields. For one single electron $Q = -1$ and for a positron $Q = 1$. Conventionally $eQ$ is called the charge of the field $\psi$. If the field $\psi$ does not transform under a gauge transformation, is a gauge singlet, then $Q = 0$ and we do not have any interaction of the fermion with the electromagnetic field. In quantum field theory all the particles are excitations of the vacuum, which are created or destroyed by the relevant creation and destruction operators. From the above analysis we see that all these particles have a charge which is given by the eigenvalue of the charge operator $Q_N$ acting on the relevant particle states. The charge of the electron which entered the Dirac equation via minimal prescription is the eigenvalue of $Q_N$ when the charge operator acts on a free electron momentum state.

### 2.2 The effective charge

In quantum field theory, the electric charge of fermions can also be understood in an effective way. In this effective scheme it is assumed that the interaction of any off-shell photon with two on-shell fermions is of the form

$$ -J^\mu(x) A_\mu(x), $$

(9)

where $J^\mu(x)$ contains all the information about the interaction of the on-shell fermions. The matrix element of the above current is conventionally written as

$$ \langle e^-(p', s')|J^\mu(x)|e^-(p, s)\rangle = \frac{e^{-iQx}}{\sqrt{2E_pV}} \sqrt{2E_{p'}V} \bar{u}_{s'}(p') \Gamma_{\mu}(p, p') u_s(p), $$

(10)

where now $\Gamma_{\mu}(p, p')$ is an effective 4-vector vertex which can depend only upon the 4-moment’s of the electrons in the current, $V$ is some volume element and $u_s(p)$ is the Dirac spinor with.

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1. Generally Noether’s charge is defined for a global symmetry but in this case we can also define it for the local gauge invariance as the corresponding form of the Noether charge in QED is similar in form to the charge as defined in the case of global symmetries. The topic is discussed in Ref. [5].
momentum $p$ and spin $s$. Using the condition of electromagnetic current conservation we can write the most general form of $\Gamma_\mu(p,p')$ as:

$$\Gamma_\mu(p,p') = \gamma_\mu F_1(q^2) + (iF_2(q^2) + F_3(q^2)\gamma_5) \sigma_{\mu\nu} q^\nu + F_4(q^2) \left( q_\mu q^\nu - q^2 \gamma_\mu \right) \gamma_5,$$

(11)

where $q = p - p'$ and $F_1(q^2)$ is called the charge form-factor of the electron, $F_2(q^2)$ is related to the anomalous magnetic moment of the electron, $F_3(q^2)$ is related to the electric dipole moment of the electron and $F_4(q^2)$ is called the anapole moment of the electron. In the last equation

$$\sigma_{\mu\nu} = i \frac{2}{2m} [\gamma_\mu, \gamma_\nu].$$

(12)

In this article we will only concentrate on the charge form-factor and the magnetic moment form-factors leaving out the other form-factors which are excellently discussed in [6]. For the case when the initial and the final electrons have the same spin and same 4-momentum i.e., $s = s'$ and $p = p'$ we can define an effective charge of the electron from Eq. (10) and Eq. (11) as

$$Q_E = \frac{1}{2} \pi_s(p) \Gamma_0(q_0 = 0, q \to 0) u_s(p).$$

(13)

Substituting the above result in Eq. (10) and then comparing with Eq. (6) yields $Q_E = F_1(0) = Q$. The last equation serves as an effective definition of the charge of an electron and is independent of the definition of the charge as predicted from the Noether current, although in the present case both the charges are the same. The limiting procedure of taking $q_0 = 0$ and then $q \to 0$ is some times called the static limit. In the case above the static limit is irrelevant, in the sense that we could also have worked with the limiting prescription $q = 0$ and then $q_0 \to 0$, as long as Lorentz symmetry is not violated.

Now if we choose an $A^\mu(x) = (0, A(x))$ and take non-relativistic fermions, it can be shown that the interaction term

$$- j_\mu A^\mu \propto \frac{e F_1(0)}{2m} \sigma \cdot B,$$

(14)

where we have taken the limit $q^2 \to 0$ and $B$ is the classical magnetic field corresponding to $A(x)$. From the above expression we see that if $F_1(0) = Q$ we get back the normal magnetic moment

$$\mu = \frac{eQ}{2m} \sigma.$$

(15)

of the electron as predicted from Eq. (11). So we see that the the effective charge and the gauge charge of the electrons behaves similarly as far as their interaction with electric or magnetic fields are concerned.

The anomalous magnetic moment term also gives a similar contribution as above except that it does not depend on $Q = F_1(0)$ but on $F_2(0)$. Consequently an uncharged particle can also have an anomalous magnetic moment. While it is obvious from the anomalous magnetic moment interaction term that it contains $\sigma_{\mu\nu}$ it can also be shown by Gordon identity that the normal magnetic moment interaction, as appearing in Eq. (14), was obtained from a term containing $\sigma_{\mu\nu}$. As

$$\bar{\psi}\sigma_{\mu\nu}\psi = \bar{\psi}_R\sigma_{\mu\nu}\psi_L + \bar{\psi}_L\sigma_{\mu\nu}\psi_R$$

(16)

where $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ and $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$ are the left-handed and right-handed fermion field projections. Consequently we can say that if due to some reason only $\psi_L$ or $\psi_R$ is present then there will be no magnetic moment of the fermions as the magnetic moment is a chirality flipping operator.
3 Electromagnetic interaction of the neutrinos

In the symmetry broken phase of the standard model the neutrinos do not have any electric charge, consequently $Q = 0$ for them. Moreover there are only left-handed neutrinos and right-handed antineutrinos in the present universe. The Dirac equation for the neutrino is:

$$i \partial \psi_L(x) = 0.$$  \hspace{1cm} (17)

As the gauge charge of this neutrino is zero the neutrino does not feel the electric or magnetic field, as an electron feels, and consequently its motion in the transverse plane of a magnetic field does not get quantized. But if we conclude from the last equation that the neutrino can never feel any electric or magnetic field then the conclusion will be wrong. The reason for the error is that the neutrino may not have a gauge coupling with the electromagnetic fields but still we have not discussed about its effective coupling with the electromagnetic field. From the interactions in the Standard Model of particle physics we know that the neutrinos can interact with the electrons and other charged particles (which have a proper gauge charge associated with them) through the charged-currents, and those charged particles can interact with the photons directly. So the neutrinos can interact with $A_\mu$ in an indirect way. This indirect interaction can also be described by Eq. (9) where now the current $J^\mu$ corresponds to the neutrino current.

3.1 The neutrino-photon effective coupling

Not repeating all the points stated before for electrons we can directly use the formula in Eq. \ref{efcharge} for the effective charge of the neutrino with a minor modification. The modification of Eq. \ref{efcharge} is related to the handedness of the Standard Model neutrinos. Now the two spinors sandwiching $\Gamma_0(q_0 = 0, q \rightarrow 0)$ in Eq. \ref{efcharge} will be replaced by the left-handed spinors, corresponding to the left-handed neutrinos. The modified equation for the neutrino effective charge becomes

$$Q_\nu = \frac{1}{2E_p} \pi_L(p) \Gamma_0(q_0 = 0, q \rightarrow 0) u_L(p).$$ \hspace{1cm} (18)

But to apply the equation of the effective charge of the neutrinos we first require a form of the effective neutrino-photon vertex function $\Gamma_\mu$. From Hermiticity, electromagnetic current conservation principle and

$$q^\mu \Gamma_\mu = 0,$$ \hspace{1cm} (19)
which is the Ward identity for a particle which lacks any kind of gauge charge attached to it we can write the most general form of the effective vertex for the chiral neutrinos as [7]:

$$\Gamma_\mu(p,p') = \left(q^2 \gamma_\mu - q_\mu q\right) \left(R(q^2) + r(q^2)\gamma_5\right),$$  \hspace{1cm} (20)

where $R(q^2)$ and $r(q^2)$ are real form-factors often called the charge radius and the axial charge radius respectively. The important point now is to see whether $R(q^2)$ and $r(q^2)$ in $\Gamma_\mu$ has any poles as $q^\mu \to 0$, as only then there is a possibility of a finite $Q_\nu$. The form factors are obtained by calculations of loops which are inscribed in between two on-shell neutrinos and an off-shell photon as shown in Fig 1. In any Feynman diagram which contains an off-shell photon which is effectively coupled to two on-shell neutrinos the photon must couple to two internal lines of charged particles. If one of the lines is assigned a loop momentum $k$ then the other must carry momentum $k \pm q$. The propagator of the second line will involve the factor

$$\frac{1}{(k \pm q)^2 - m_\ell^2} = \frac{1}{q^2 \pm 2k \cdot q + (k^2 - m_\ell^2)},$$  \hspace{1cm} (21)

where $m_\ell$ is the mass of the particle in the internal line. As for any internal line $k^2 \neq m_\ell^2$, no singularity is present in the expressions of the form-factors. Consequently $\Gamma_0(q_0 = 0, q \to 0) = 0$ and from Eq. (18) we see that there is no effective neutrino charge in this case. More over as because there are no right-handed neutrinos in the Standard Model of particle physics there isn’t any magnetic moment of the neutrinos. So a Standard Model neutrino does not have a gauge charge, does not have an effective charge, does not have any normal or anomalous magnetic moment. Classically any particle having such properties will never interact with any electromagnetic field but in quantum field theory it is not so. As $\Gamma_\mu(p,p')$ is in general not zero for non-zero $q$ so the effective interaction of the neutrino with the electromagnetic fields is not zero.

### 3.2 The neutrino-photon effective coupling in a thermal background

As discussed in the introduction we know that the vertex function in presence of a background plasma has new physics in it and so heceforth in this article we will designate the neutrino-photon effective vertex in the plasma as $\Gamma'_\mu(p,p')$. The effective neutrino-photon vertex $\Gamma'_\mu(p,p')$ in this case also follows all the constraints as proposed in the last subsection as Hermiticity, current conservation and the Ward identity. The definition of effective charge of the chiral neutrino in the presence of a medium can also be written as in Eq. (18), but the static limit of the vertex function has a subtle preference here. The reason for the preference for the static limit here is related to the fact that the temperature and the chemical potential of the plasma constituents are well defined only in the rest frame of the medium and so the calculation of $\Gamma'_0$ must be done in an unique frame. This breaks the Lorentz invariance of the theory. In absence of Lorentz symmetry the limiting process $q \to 0$ looses meaning because it can be taken in two ways as first $q_0 = 0$ and then taking $q \to 0$ or taking $q = 0$ first and then $q_0 \to 0$ and these two ways produces different results in a thermal medium.

Assuming that $\Gamma'_\mu$ is analytic or at best it has a removable singularity at $q^0 = p^0 - p'^0 = 0$ we can Taylor expand $\Gamma'_\mu$ about the point $q^0 = 0$ as [4]:

$$\Gamma'_0 = G_0 + q^0 G_1 + O \left((q^0)^2\right),$$  \hspace{1cm} (22)

$$\Gamma' = H_0 + q^0 H_1 + O \left((q^0)^2\right),$$  \hspace{1cm} (23)

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where the coefficients of the expansion are well behaved at \( q^0 = 0 \). As Eq. \( \text{(19)} \) still holds we have,

\[
q_0 G_0 + q_0^2 G_1 = q \cdot H_0 + q_0 q \cdot H_1 .
\]

When \( q^0 \to 0 \) the above relation becomes \( q_0 (G_0 - q \cdot H_1) - q \cdot H_0 = 0 \) and as \( q_0 \) and \( q \) are independent quantities for an off-shell photon, the last equation implies \( G_0 = q \cdot H_1 \) and \( H_0 = 0 \). Using these equations in the expansion of \( \Gamma'_0 \) and \( \Gamma' \) as \( q^0 \to 0 \) we get

\[
\Gamma'_0 = q \cdot H_1 + O(q^0),
\]

\[
\Gamma' = q^0 H_1 + O((q^0)^2) .
\]

Inside a medium the term \( H_1 \) can always have a pole at \( q = 0 \) as a medium consists of real particles and they may enter the loop integrals contributing for the calculation of \( \Gamma'_0 = 0 \). For these looping real particles with 4-momentum \( k^\mu \), we may have \( k^2 = m^2 \) unlike the case in vacuum. Consequently if \( H_1 \) develops a pole at \( q = 0 \), where its form is given as

\[
H_1 \propto \frac{q}{|q|^2},
\]

then \( \Gamma'_0 \) will not be zero and will become a finite quantity as \( q^0 \to 0, q \to 0 \). In a medium this happens and consequently the neutrino develops an effective electric charge. If the effective 4-Fermi interaction is given as,

\[
L_{\text{eff}} = -\sqrt{2} G_F \left[ \bar{\psi}_{(\nu)} \gamma^\alpha L \psi_{(\nu)} \right] \left[ \bar{\psi}_{(\ell)} \gamma_\alpha (g_V + g_A \gamma_5) \psi_{(\ell)} \right],
\]

where \( L = \frac{1}{2}(1-\gamma_5) \) is the left handed projection operator, then from Fig. 1 the effective neutrino-photon vertex to one-loop level can be written as

\[
\Gamma_\mu = -\frac{\sqrt{2} G_F}{q_\ell} \gamma^\alpha L \left( g_V \Pi_{\mu \alpha} + g_A \Pi_{\mu 5} \right) .
\]

Here \( g_V \) and \( g_A \) are the vector and axial-vector couplings of the neutrinos to the leptons and \( q_\ell \) is the electric charge of the looping lepton. The term \( \Pi_{\mu \alpha} \) is exactly the expression for the vacuum polarization of the photon, and appears from the vector interaction in the effective Lagrangian. In the one-loop expression of \( \Pi_{\mu \alpha} \) if we replace one of the vector vertices by an axial-vector vertex then we get \( \Pi_{5 \mu \alpha} \). \( \Pi_{\mu \alpha} \) and \( \Pi_{5 \mu \alpha} \) arises from the vector and the axial-vector coupling of the neutrinos to charged leptons in the 4-Fermi Lagrangian. In Fig. 1 the vertex specified by the Greek letter \( \alpha \) is the 4-Fermi vertex and the Greek letter \( \mu \) stands for the vector vertex. The Feynman diagram explains the origin of the two polarization tensors. To one-loop order an exact calculation of the neutrino effective charge in a medium gives \([4]\):

\[
Q_\nu = -\frac{\sqrt{2} G_F g_V}{q_\ell} \Pi_{00} .
\]

This result shows us that in a medium there is indeed an effective electric charge of the neutrinos. In the absence of any right-handed neutrinos, the magnetic moment of the neutrino still remains zero in presence of a medium. It is to be noted that the form of the vertex function as given in Eq. \( \text{(28)} \) is a one-loop result which also holds in vacuum. The vertex function written in Eq. \( \text{(20)} \) holds only in vacuum but it is the most general form of the electromagnetic vertex function of neutrinos and holds for any order of perturbation theory.
4 A brief discussion on the electromagnetic interactions of neutral bosons

In quantum field theory photons are the Abelian local gauge fields and do not carry any gauge charge. In other words photons do not interact with photons directly. But this statement does not rule out the effective coupling of photons with photons. As for the case of electrons and neutrinos here also we can write the effective electromagnetic current of the photons which interacts with another off-shell photon via intermediate virtual charged particles. The matrix element of the electromagnetic current of the photon, in vacuum, can be written analogously as

\[ \langle A^\alpha(p', \epsilon')| J_\mu(x) | A^\alpha(p, \epsilon) \rangle = \frac{e^{-iq \cdot x}}{\sqrt{2E_p V}} \epsilon^\dagger \times (p') \epsilon\alpha(p) \Gamma_{\alpha\alpha\mu}(p, p') \]

where \( \epsilon^\dagger\alpha(p') \) and \( \epsilon\alpha(p) \) are the polarization vectors of the on-shell photons with energies \( E_{p'} \) and \( E_p \). In the above equation \( V \) is the volume element in which the events occur and in the end of the calculation we can take it to be infinite. The momentum change of the on-shell photons is given by \( q = p' - p \). In the above equation \( \Gamma_{\alpha\alpha\mu}(p, p') \) is the effective vertex function of three photons. The expression of the electromagnetic vertex of neutral spin-1 particles shades some light on the electromagnetic interaction of photons in vacuum, and this topic has been discussed in general in Ref. [9]. Without doing any explicit calculation it can be said that in QED this effective vertex vanishes as any charged fermion loop with odd number of photon vertices vanishes due to charge conjugation symmetry, a statement formally known as Furry’s theorem in QED. Consequently in vacuum odd number of photons do not interact with themselves in any order of perturbations, unless we include fields which breaks charge conjugation symmetry as neutrinos. But the situation changes if we have a background thermal medium which breaks charge symmetry in a trivial way. One of the simplest example is the plasma atmosphere of stars or planets whose temperature is less than 0.5MeV. In these kind of atmospheres there will be more electrons than positrons. In this case the photon electromagnetic effective vertex does not vanish and odd number of photons start to interact with each other in an effective way. The effective charge of the photon is related with \( \Gamma_{\alpha\alpha\mu}(q_0 = 0, q \to 0) \) and in principle it exists. A calculation of the effective charge of the photon has been done using techniques of plasma physics and statistical mechanics in Ref. [10].

As for the neutral photons the real scalar fields do not have any direct electromagnetic interactions. But these fields can interact with other charged scalar fields or can interact with charged fermions via Yukawa interactions. The charged scalar or fermions will interact with photons giving an indirect linkage of the real scalars with the electromagnetic field. The effective vertex of the real scalars with the photons is given as

\[ \langle \phi(p')| J_\mu(x) | \phi(p) \rangle = \frac{e^{-iq \cdot x}}{\sqrt{2E_p V}} \Gamma_\mu(p', p), \]

where all the terms in the above equation has the conventional meaning. Using the conservation condition of the electromagnetic current, the on-shell property of the scalar fields the effective electromagnetic vertex of the scalar field can be written as [11]:

\[ \Gamma_\mu(p, q) = a[p \cdot q q_\mu - q^2 p_\mu], \]

where \( \Gamma_\mu(p, q) \) has been expressed in terms of \( p \) and \( q = p' - p \) instead of \( p' \) and \( p \) alone and \( a \) is a constant. As in the case of the neutrinos in vacuum in this case also we do not expect
any singularities of $a$ as $q \to 0$. So the above equation implies that $\Gamma_\mu(q_0 = 0, q \to 0) = 0$ and consequently there will be no effective charge of the real scalar fields in vacuum. In a background medium we can also write the electromagnetic vertex of the scalars as done in Ref. [11]. In presence of a background medium the real scalars can in principle acquire an induced charge analogous to the case of the neutrinos.

5 Discussion and summary

In this article we have shown that the concept of electric charge is not solely related with Noether’s current or an Abelian gauge symmetry. Although whenever some one speaks about charge an Abelian gauge symmetry naturally comes to our mind. Precisely for this reason the point about the induced charge requires some understanding because of the various notions we unconsciously attach with the gauge charge of the particles compels us to think about the induced charge in a wrong way. When ever we find that a particle is charged we want to understand how does two charged particles interact with each other? How does the charged particle respond to a classical electric or a magnetic field? To properly answer these questions we have to understand the origin of the induced charge. As we see that none of the neutral elementary particles acquire an effective charge in vacuum consequently they only aquire it from a medium which is comprised of charged particles (and antiparticles). Thus the induced charge is a byproduct of a collective phenomenon. In presence of an external static electric field (where $E = -\nabla \phi(x, t)$) these induced charge acts like a normal gauge charge and will get repelled or attracted according to the situation. This happens because the definition of the effective charge presented in this article is based on the interaction of the net acquired charge and the electrostatic potential of an external photon. But the induced charge does not respond to any classical magnetic field via the Lorentz force law. This is because in the quantum level the Lorentz force law is an outcome of the gauge invariance of the theory (where the gauge field comes in the scene through minimal prescription) and in the case of particles with zero gauge charge there will be no effect. Similarly the induced charge of the particles does not impart any dipolar interactions. For dipolar interactions the relevant quantity required is the electric dipole moment form-factor or the magnetic dipole moment form-factor of the neutral particle. To interact with the dipole moments with an external magnetic or electric field the neutral particle must acquire a net magnetic or electric dipole moment from the thermal bath and as long as it does not have that the test particle will not interact with any field through its dipoles. In the presence of a strong classical magnetic field the property of the charged particles of the medium themselves changes and consequently now the charge build up around the neutral particle will happen in a different way as compared to the charge build up in the absence of the magnetic field. In presence of an external magnetic field the charged particles of the medium will get Landau quantized and henceforth their way of accumulation around, or dispersion from, the neutral particle will change and consequently the effective charge built up will also change [12].

In conclusion we can say that the intrinsic charge of a fundamental particle has multiple roles to play. It appears in the various other electromagnetic couplings of the particle as in the magnetic moment, electric dipole moment etc, etc. But the effective induced charge of any fundamental neutral particle, which only becomes important in presence of a thermal background of intrinsically charged particles, has a much lower status. To find out the proper electromagnetic coupling of a neutral particle we have to calculate the relevant electromagnetic form factors appearing in the effective electromagnetic vertex and only then can we be certain of the various electromagnetic couplings of the neutral elementary particle.
References

[1] G. G. Raffelt, “Stars As Laboratories For Fundamental Physics: The Astrophysics Of Chicago, USA: Univ. Pr. (1996) 664 p

[2] R. N. Mohapatra and P. B. Pal, “Massive neutrinos in physics and astrophysics. Second edition,” World Sci. Lect. Notes Phys. 60, 1 (1998) [World Sci. Lect. Notes Phys. 72, 1 (2004)].

[3] T. Altherr and P. Salati, Nucl. Phys. B 421, 662 (1994) [arXiv:hep-ph/9312204].

[4] J. F. Nieves and P. B. Pal, Phys. Rev. D 49, 1398 (1994) [arXiv:hep-ph/9305308].

[5] H. A. Al-Kuwari and M. O. Taha, Am. J. Phys. 59, 363 (1991).

[6] A. Lahiri and P. B. Pal, “A first book of quantum field theory,” Harrow, UK: Alpha Sci. Int. (2005) 380 p

[7] J. F. Nieves, Phys. Rev. D 26, 3152 (1982).

[8] J. C. D’Olivo, J. F. Nieves and P. B. Pal, Phys. Rev. D 40, 3679 (1989).

[9] J. F. Nieves and P. B. Pal, Phys. Rev. D 55, 3118 (1997) [arXiv:hep-ph/9611431].

[10] J. T. Mendonca, A. M. Martins and A. Guerreiro, Phys. Rev. E 62, 2989 (2000).

[11] J. F. Nieves and P. B. Pal, Phys. Rev. D 77, 113001 (2008) [arXiv:0712.4345 [hep-ph]].

[12] K. Bhattacharya, A. K. Ganguly and S. Konar, Phys. Rev. D 65, 013007 (2002) [Erratum-ibid. D 66, 119902 (2002)] [arXiv:hep-ph/0107259].