Suitability of material models in finite element simulation of stress relaxation for titanium sheets

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Abstract. Titanium alloys are important aerospace materials due to their low density, high strength to weight ratio and excellent corrosion resistance, but it is difficult to form them under room temperature due to poor formability and large springback, which is often formed by hot forming. And stress relaxation behaviour of material plays an important role in hot forming. In this paper, different material models are compared in simulating stress relaxation of Ti-6.5Al-2Zr-1Mo-1V sheet, and validated by experiments. A series of uniaxial tensile stress relaxation tests are performed on the Ti-6.5Al-2Zr-1Mo-1V alloy sheet over the temperature ranging from 773K to 973K, and the strain 10%. According to the result of stress relaxation tests, Time-Hardening model and Hyperbolic-Sine model are calibrated and validated. It is shown that for stress relaxation process, from 0 to 900 seconds, the maximum stress simulation error (6MPa) of Time-Hardening model is lower than Hyperbolic-Sine model (25MPa), but from 1800 to 3600 seconds, the maximum stress simulation error (15MPa) of Time-Hardening model is larger than Hyperbolic-Sine model (5MPa). In addition, it can be concluded that the stress simulation error will have little effect on shape accuracy of formed parts during the simulation.

1. Introduction
Titanium alloys are important aerospace materials due to their low density, high strength to weight ratio, good compatibility with composite materials and excellent corrosion resistance, but it is difficult to form them under room temperature due to poor formability and large springback [1]. Generally, for the parts with simple geometric shape, they are formed by the combination of cold forming and stress relaxation. For the parts with complex geometric shape, they are formed by the combination of hot forming and stress relaxation. It is one key problem that how to determine process parameters of stress relaxation, namely stress relaxation parameters, such as time, temperature and so on. Generally, finite element simulation is used to optimize process parameter, but it is still difficult to choose one suitable material model. So the different material models are compared in simulating stress relaxation of Ti-6.5Al-2Zr-1Mo-1V sheet, and validated by experiments. In this study, a series of uniaxial tensile stress relaxation tests were performed on the Ti-6.5Al-2Zr-1Mo-1V alloy sheet over the temperature ranging from 773K to 973K, and the strain 10%. And corresponding finite element models are established.

2. Material and experiment

2.1. Material
The test material is Ti-6.5Al-2Zr-1Mo-1V and its chemical composition is listed in table 1.[2] The as-received alloy was annealed after hot and cold rolling into 1mm-thick sheet. And the geometry of stress relaxation specimen is as shown in figure 1.
Table 1. The chemical composition of Ti-6.5Al-2Zr-1Mo-1V alloy (%).

|      | Al  | Zr  | Mo  | V   | Si  | Fe  | Ti  |
|------|-----|-----|-----|-----|-----|-----|-----|
|      | 5.5-7.0 | 1.5-2.5 | 0.5-2.0 | 0.8-2.5 | ≤0.15 | ≤0.25 | Bal |

Figure 1. Geometry of stress relaxation specimen.

2.2. Uniaxial tensile stress relaxation tests
The stress relaxation experiments are conducted by using a Zwick/Roell Z100 electric universal test machine equipped with a heating furnace. The furnace offers a constant temperature zone with 200mm length and the possibility to control the atmosphere temperature with ±3K precision by three independent heating zones. The designed test matrix about stress relaxation is as shown in table 2. Firstly, the specimen is heated to the desired temperature and then kept warm to make temperature uniform. Secondly, stretched the specimen to the pre strain 10%. Finally, fixed the clamp and recorded time-stress curve for 60mins.

Table 2. The designed test matrix about stress relaxation.

| Temperature | Pre-strain | Stress relaxation time |
|-------------|------------|------------------------|
| 773K        | 10%        | 60 mins                |
| 873K        | 10%        | 60 mins                |
| 973K        | 10%        | 60 mins                |

2.3. Result of uniaxial tensile stress relaxation tests
The obtained stress relaxation curves are as shown in figure 2.

Figure 2. Stress relaxation curves of Ti-6.5Al-2Zr-1Mo-1V at 773K, 873K, 973K.

As shown in figure 2, the stress relaxation curve can be divided into three parts. At first, stress declines rapidly. Subsequently, the stress relaxation goes into the transition stage. Finally, the stress relaxation goes into an apparent stable stage. It can be concluded that the increase of temperature is beneficial to accelerate the decrease of stress, which is because high temperature is beneficial to accelerate dynamic recovery and dynamic recrystallization.
3. Determination of stress relaxation model

For the simulation of stress relaxation process, stress relaxation model is essential to predict forming result. In finite element software ABAQUS, Time-Hardening model is simple and easy to solve and just has three variable to be determined. Hyperbolic-Sine model is relatively complicated and has five variable to be solved. The corresponding solving method is as follows.

3.1. Determination of Time-hardening model

The Time-Hardening model is expressed as equation 1.

\[ \dot{\varepsilon} = Aq^m t^n \]  

(1)

First, the obtained stress relaxation curves are fitted by cubic delay function (equation 2).[3]

\[ \sigma = \sigma_0 + A_1 e^{-t/t_1} + A_2 e^{-t/t_2} + A_3 e^{-t/t_3} \]  

(2)

where \( \sigma \) is instantaneous stress value (MPa), \( \sigma_0 \) is ultimate stress value (MPa), t is time (s), and \( A_{1-3} \) and \( t_{1-3} \) are equation parameters. And fitting results are as shown in table 3 and figure 3, which proves that the fitting result is good.

Table 3. The fitting results of cubic delay function.

| T       | \( \sigma_0 \) | \( A_1 \) | \( A_2 \) | \( A_3 \) | \( t_1 \) | \( t_2 \) | \( t_3 \) | R-square |
|---------|---------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 773K    | 322.306       | 200.687   | 75.993    | 98.897    | 2049.55   | 26.721    | 236.517   | 0.999     |
| 873K    | 49.67         | 145.433   | 143.513   | 121.665   | 228.683   | 1563.312  | 29.274    | 0.999     |
| 973K    | 4.7           | 82.1      | 24.6      | 60.4      | 17.97     | 805.22    | 123.43    | 0.999     |

Figure 3. Fitting curves of cubic delay function at 773K, 873K, 973K.

Take the derivation of variable t by equation 2, and obtain below equation 3:

\[ \frac{d\sigma}{dt} = -\frac{A_1}{t_1} e^{-\frac{t}{t_1}} - \frac{A_2}{t_2} e^{-\frac{t}{t_2}} - \frac{A_3}{t_3} e^{-\frac{t}{t_3}} \]  

(3)

The expression of strain rate is expressed as equation 4. And equation 5 can be obtained.

\[ \dot{\varepsilon} = \frac{d\sigma}{Edt} \]  

(4)

\[ \dot{\varepsilon} = Aq^m t^n = \frac{d\sigma}{Edt} = -\frac{1}{E} \left( \frac{A_1}{t_1} e^{-\frac{t}{t_1}} - \frac{A_2}{t_2} e^{-\frac{t}{t_2}} - \frac{A_3}{t_3} e^{-\frac{t}{t_3}} \right) \]  

(5)

The variable A, n, m can be solved by fitting equation 5, and fitting result is as shown in table 4.
### Table 4. The fitting results of variable A, n, m.

| T     | A         | n     | m      |
|-------|-----------|-------|--------|
| 773K  | 4.03958e-12 | 2.615 | -0.5268 |
| 873K  | 4.83756e-11 | 2.40303 | -0.29325 |
| 973K  | 9.4804e-10  | 2.24332 | -0.08574 |

### 3.2. Determination of Hyperbolic-Sine model

The expression of Hyperbolic-Sine model is as shown in equation 6.

$$
\dot{\varepsilon} = BF(\sigma) \exp\left(-\frac{Q}{RT}\right)
$$

where

$$
F(\sigma) = \begin{cases} 
\sigma^n & \alpha \sigma < 0.8 \\
\exp(\beta \sigma) & \alpha \sigma > 1.2 \\
\left[\sinh(\alpha \sigma)\right]^{n_2} & \sigma
\end{cases}
$$

When $\alpha \sigma < 0.8$, take logarithm on both sides of equation 6 and obtain equation 7.

$$
\ln \dot{\varepsilon} = \ln B + n_1 \ln \sigma - \frac{Q}{RT}
$$

Equation 7 shows that relation between $\ln \sigma$ and $\ln \dot{\varepsilon}$ is linear, so $n_1$ under temperature 773K, 873K, 973K can be solved by linear fitting, as shown in table 5.

When $\alpha \sigma > 1.2$, take logarithm on both sides of equation 6 and obtain equation 8.

$$
\ln \dot{\varepsilon} = \ln B + \beta \sigma - \frac{Q}{RT}
$$

Equation 8 shows that relation between $\sigma$ and $\ln \dot{\varepsilon}$ is linear, so $\beta$ values under 773K, 873K, 973K can be solved by linear fitting, as shown in table 5.

The relation between $\alpha$, $\beta$ and $n_1$ can be expressed as equation 9.

$$
\alpha = \frac{\beta}{n_1}
$$

So $\alpha$ values under 773K, 873K, 973K can be solved, as shown in table 5.

When $F(\sigma) = \left[\sinh(\alpha \sigma)\right]^{n_2}$, take logarithm on both sides of equation 6 and obtain equation 10.

$$
\ln \dot{\varepsilon} = n_2 \ln \sinh(\alpha \sigma) + C
$$

where

$$
C = \ln B - \frac{Q}{RT}
$$

So $n_2$ and $C$ are slope and intercept of equation 10 respectively and the solved results are as shown in table 5.
Table 5. The solution results of variable $n_1$, $\beta$, $\alpha$, $n_2$ and $C$.

|   | $T$   | $n_1$ | $\beta$ | $\alpha$ | $n_2$ | $C$   |
|---|-------|-------|---------|----------|-------|-------|
|   | 773K  | 6.124 | 0.0176  | 2.87e-3  | 4.497 | -16.172 |
|   | 873K  | 2.67  | 0.0116  | 4.345e-3 | 2.446 | -12.65  |
|   | 973K  | 2.32  | 0.018   | 7.759e-3 | 2.254 | -10.344 |

Equation 11 shows that the relation between $C$ and $1/T$ is linear. $\ln B$ is the intersect and $-Q/R$ is the slope, so $B$ and $Q$ under temperature 773K, 873K, 973K can be solved. And the solution result of Hyperbolic-Sine model is as shown in table 6.

Table 6. The solution results of Hyperbolic-Sine model.

|   | $T$   | $B$   | $\alpha$ | $n_2$ | $Q$    | $R$   |
|---|-------|-------|----------|-------|--------|-------|
|   | 773K  | 424721| 2.87e-3  | 4.497 | 1.87e5 | 8.314 |
|   | 873K  | 424721| 4.345e-3 | 2.444 | 1.86e5 | 8.314 |
|   | 973K  | 424721| 7.759e-3 | 2.254 | 1.69e5 | 8.314 |

4. Result and discussion of simulation validation
The stress relaxation behaviour of Ti-6.5Al-2Zr-1Mo-1V under temperature 773K, 873K and 973K are simulated in ABAQUS.

4.1. Establishment of uniaxial tensile stress relaxation finite element model
The uniaxial tensile stress relaxation simulation models are established in ABAQUS, and established simulation model is as shown in figure 4. The setting of initial stress value (Predefined Field) under certain temperature is as shown in figure 5. The setting of stress relaxation model under certain temperature is as shown in figure 6. And corresponding simulation matrix is as shown in table 7.

Table 7. The solution results of Hyperbolic-Sine model.

|   | $T$   | Stress relaxation model | Initial stress value (MPa) |
|---|-------|------------------------|---------------------------|
|   | 773K  | Time-hardening          | 700                       |
|   |       | Hyperbolic-Sine         | 700                       |
|   | 873K  | Time-hardening          | 472                       |
|   |       | Hyperbolic-Sine         | 472                       |
|   | 973K  | Time-hardening          | 173                       |
|   |       | Hyperbolic-Sine         | 173                       |

Figure 4. Geometry model of simulation. Figure 5. The setting of initial stress value under certain temperature.

4.2. Result and discussion of uniaxial tensile stress relaxation finite element model
The simulation result is as shown in figure 7.
Figure 6. The setting of stress relaxation model under certain temperature.

(a) The setting of Time-Hardening model  (b) The setting of Hyperbolic-Sine model

Figure 7. The simulation result with different temperature and stress relaxation model.

(a) The prediction of Hyperbolic-Sine model  (b) The prediction of Time-Hardening model

Figure 7 shows that both of stress relaxation models can predict the stress relaxation behaviour well. From 0 to 900 seconds, the maximum stress simulation error (6MPa) of Time-Hardening model is lower than Hyperbolic-Sine model (25MPa), but from 1800 to 3600 seconds, the maximum stress simulation error (15MPa) of Time-Hardening model is larger than Hyperbolic-Sine model (5MPa). But when effect of stress simulation error value is reflected on shape accuracy of formed parts, it will be small, which is because stress simulation error value divided by elastic modulus is very small.

5. Conclusion
1. The increase of temperature is beneficial to accelerate the decrease of stress, which can make the stress decrease completely.
2. From 0 to 900 seconds, maximum stress simulation error (6MPa) of Time-Hardening model is lower than Hyperbolic-Sine model (25MPa), but from 1800 to 3600 seconds, maximum stress simulation error (15MPa) of Time-Hardening model is larger than Hyperbolic-Sine model (5MPa).
3. Stress simulation error will have little effect on shape accuracy of formed parts.

6. References
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