Constraints on kHz QPO models and stellar EOSs from SAX J1808.4-3658, Cyg X-2 and 4U 1820-30

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ABSTRACT

We test the relativistic precession model (RPM) and the MHD Alfvén wave oscillation model (AWOM) for the kHz QPOs by the sources with measured NS masses and twin kHz QPO frequencies. For RPM, the derived NS mass of Cyg X-2 (SAX J1808.4-3658 and 4U 1820-30) is $1.96 \pm 0.10 M_\odot$ ($2.83 \pm 0.04 M_\odot$ and $1.85 \pm 0.02 M_\odot$), which is $\sim 30\%$ (100\% and 40\%) higher than the measured result $1.5 \pm 0.3 M_\odot$ ($< 1.4 M_\odot$ and $1.29^{+0.19}_{-0.07} M_\odot$). For AWOM, where the free parameter of model is the density of star, we infer the NS radii to be around 10 – 20 km for the above three sources, based on which we can infer the matter compositions inside NSs with the help of the equations of state (EOSs). In particular, for SAX J1808.4-3658, AWOM shows a lower mass density of its NS than those of the other known kHz QPO sources, with the radius range of 17 – 20 km, which excludes the strange quark matter inside its star.

Key words: stars: neutron–binaries: close–X-rays: stars–accretion: accretion disks

1 INTRODUCTION

Kilohertz quasi-periodic oscillations (kHz QPOs) are the timing phenomena that exhibit the millisecond accretion flow close to the surface of neutron star (NS) in low mass X-ray binaries (LMXBs, Lin et al. 2007). They usually occur in pairs (upper $\nu_2$ and lower $\nu_1$) and cover the frequency range from tens of Hz to nearly 1300 Hz (van der Klis 2000, 2003). Until now there are 26 sources of exhibiting the twin kHz QPOs, including 16 Atoll, 8 Z sources, as well as 2 accreting millisecond pulsars. The twin kHz QPO frequencies follow the tight correlation, showing a power-law relation (Zhang et al. 2006a). Furthermore, they also correlate with the low-frequency QPOs in a systematic way (Psaltis et al. 1994; Belloni et al. 2002).

Many authors have investigated the correlation between $\nu_1$ and $\nu_2$ (Stella & Vietri 1999) and Stella et al. 1999) propose the relativistic precession model (RPM) with the NS mass as the solely free parameter, where $\nu_2$ is the Keplerian orbital frequency and $\nu_1$ is the precession frequency of the orbit periastron. Török et al. 2008 apply this model to estimate the NS mass of Cir X-1 and give a value of $\sim 2.2 M_\odot$ (Török et al. 2011). Zhang (2004) proposes the Alfvén wave oscillation model (AWOM) with the NS density as the free parameter, where $\nu_2$ is the Keplerian orbital frequency and $\nu_1$ is the MHD Alfvén wave oscillation frequency. Stuchlik et al. 2008, 2011 interpret the high frequency QPOs as the forced resonant oscillations of the accretion disk excited by gravitational perturbations. Other models, such as the beat frequency model (Miller et al. 1998; Lamb & Miller 2001) and constant ratio model (Kluźniak & Abramowicz 2001; Abramowicz et al. 2003a,b) are not strongly supported by the subsequent observations (Méndez et al. 1998; Méndez & van der Klis 1999; Jonker et al. 2002; Wijnands et al. 2003; Belloni et al. 2003, 2007; Strohmayer & Bildsten 2006; Zhang et al. 2006a, 2008; Yin et al. 2007).

The kHz QPO phenomenon is a powerful tool to probe the physical process around the surface of NS (Zhang & Wang 2013): First, it can test Einstein’s General Relativity in a strong gravitational field because its frequency range is near the dynamical timescale of the innermost region close to NS surface (Kluźniak et al. 1994; Lai 1998; van der Klis 2000; Méndez 2002; van der Klis 2006). Second, it can constrain the NS Mass – Radius ($M – R$) relation by the kHz QPO models (Miller et al. 1998; Stella & Vietri 1999; Stella et al. 1999, Zhang 2004), which will further infer the matter state of NS with the help of
equations of state (EOSs) (Haensel et al. 2007; Zhang et al. 2007; Zhang 2009; Bhattacharyya 2010a; Petri 2011). So far, Kaaret et al. (2002) has reported a maximum frequency of 1253 Hz in source SAX J1750.8-2900 (However, Bhattacharyya (2010b) reported a possible 1860 Hz QPO in source 4U 1636-536, which may be the overtone of the kHz QPOs).

In this paper, we test the twin kHz QPO models of RPM and AWOM by the sources (Cyg X-2, SAX J1808.4-3658 and 4U 1820-30) with measured NS masses and twin kHz QPOs, and constrain the NS $M - R$ relations and infer the possible matter compositions of these NSs. Finally, we present the discussions and conclusions.

## 2 TESTING RPM AND AWOM

Both RPM and AWOM predict an approximate power law relation between $\nu_1$ and $\nu_2$ (Stella & Vietri 1999; Stella et al. 1999; Zhang 2004), then they are based on the different physical mechanisms and rely on the different parameters: RPM and AWOM rely on the NS mass and the NS density, respectively. The two models assume that the upper-frequency of the twin kHz QPOs is the Keplerian orbital frequency at radius $r$:

$$\nu_2 = \sqrt{\frac{GM}{4\pi^2 r^3}}$$

where $M$ is the NS mass. RPM predicts the following relation between $\nu_1$ and $\nu_2$ (Stella & Vietri 1999; Stella et al. 1999): \(^{2}\)

$$\Delta \nu = \nu_2 \sqrt{1 - \frac{3R_s}{r}}$$

where $R_s$ is the Schwarzschild radius (i.e. $R_s = 2GM/c^2$). Combined equation (1) and equation (2), the NS mass is the function of $\nu_1$ and $\nu_2$. AWOM predicts the relation of $\nu_1$ and $\nu_2$ as follows (Zhang 2004):

$$\nu_1 = 629(\text{Hz}) A^{-2/3} \frac{1}{\nu_{2k}^{5/3}} \sqrt{1 - \sqrt{1 - \left(\frac{\nu_{2k}}{1.85 A}\right)^{2/3}}}$$

$$A = \sqrt{\frac{m}{R_6^3}}$$

$$\rho = 4.75 \times 10^{14} A^2(\text{g cm}^{-3})$$

\(^1\) Geometrical units ($G = c = 1$) are used in RPM, where we use cgs units.
Constraints on kHz QPO models and ...

Table 1. Sources with the measured NS masses and twin kHz QPOs

| Source          | Measured Mass $(M_\odot)$ | Ref | $R_{\text{ISCO}}$ (km) | $\nu_1$ (Hz) | $\nu_2$ (Hz) | Ref | $R_K$ (km) |
|-----------------|---------------------------|-----|-------------------------|--------------|--------------|-----|------------|
| Cyg X-2         | 1.78 ± 0.23               | 1   | 15.5 ± 2.0              | 516 ± 27     | 862 ± 11     | 5   | 20.0 ± 0.9 |
|                  | 1.5 ± 0.3                 | 2   | 13 ± 3                  |              |              | 5   | 19 ± 1     |
| SAX J1808.4-3658| < 1.4                     | 3   | < 12                    | 499 ± 4      | 503.6 ± 5.3  | 6   | < 21 ± 0.1 |
| 4U 1820-30       | 1.29$^{+0.19}_{-0.07}$    | 4   | 11.4$^{+0.6}_{-0.7}$    | 764 ± 6      | 796 ± 4      | 7   | 15.6$^{+0.8}_{-0.3}$ |

Notes: The second and fourth columns show the measured NS masses and the inferred innermost stable circular orbit (ISCO) radii (i.e. $R_{\text{ISCO}} = 3R_6 = 6GM/c^2$); the fifth and sixth columns show the range of $\nu_1$ and $\nu_2$, the last column shows the Keplerian orbital radii inferred from the maxima of $\nu_2$. References: 1. Orosz & Kuulkers (1999); 2. Elebert et al (2009b); 3. Elebert et al (2009a); 4. Shaposhnikov & Titarchuk (2004); 5. Wijnands et al (1994); 6. van Straaten et al (2004); 7. Smale et al (1997).

Table 2. The derived parameters by RPM and AWOM in the sources in Table 1

| Source          | $m$ (RPM) $(M_\odot)$ | $\chi^2/d.o.f.$ | $(A, \text{AWOM})$ | $\chi^2/d.o.f.$ | Radius (AWOM) (km) |
|-----------------|-----------------------|-----------------|--------------------|-----------------|--------------------|
| Cyg X-2         | 1.96 ± 0.10           | 0.62 ± 0.04     | -                  | 16.7 ± 1.1      |
| SAX J1808.4-3658| 2.83 ± 0.04           | 0.43 ± 0.01     | -                  | < 20            |
| 4U 1820-30       | 1.85 ± 0.02           | 2.76            | 0.65 ± 0.01        | 2.5 ± 1.5       | 14.5$^{+0.7}_{-0.3}$ |

Notes: The second and third (fourth and fifth) columns show the inferred NS masses (density parameter $A$ values) and the goodness of fit by RPM (AWOM), respectively. The sources Cyg X-2 and SAX J1808.4-3658 have too few data to fit, therefore we have to calculate the mean parameters for the models. The sixth column shows the inferred NS radii based on $A$ values (AWOM) and measured NS masses in Table 1.

where $\nu_2k = \nu_2/1000$ Hz, $m$ is the NS mass in $M_\odot$, $R_0 = R_{\text{ISCO}}/10^6$ cm is the NS radius $R$ in 10 km and $A$ is a parameter that presents the measurement of the NS density $\rho$.

Since the NS mass is the free parameter of RPM, we can test this model by comparing the predicted NS mass by twin kHz QPOs with the measured result. While for AWOM, the free parameter is NS density, we can infer the NS radius from the twin kHz QPOs and measured NS mass by which we can justify if this radius is a reasonable value. In summary, in order to test the two models, we need the sources to have both the measured NS masses and twin kHz QPOs. We find only three sources (Cyg X-2, SAX J1808.4-3658 and 4U 1820-30) to satisfy these conditions, as shown in Table 1, which are selected from the updated 393 pairs of twin kHz QPOs (see, e.g. van der Klis 2006; Zhang et al. 2006; Linares et al. 2005; Boutloukos et al. 2006; Zhang et al. 2006b; Homan et al. 2007; Altamirano et al. 2008; Barret et al. 2008; Strommenger et al. 2008; Boutloukos et al. 2009; Altamirano et al. 2010; Bhattacherva 2010; Homan et al. 2010; Sanna et al. 2010; Lin et al. 2011). The particular properties of these three sources are briefly summarized below:

Cyg X-2 is a persistent NS-LMXB source with the luminosity close to the Eddington limit. It locates at a distance of ~ 7 kpc (Orosz & Kuulkers, 1999) with the orbital period of 8.444 d (Casares et al. 1998) and the secondary of ~ 0.6 $M_\odot$ (Podsiadlowski & Rappaport, 2000). Both type I X-ray bursts (Smale, 1993) and twin kHz QPOs (Wijnands et al. 1998) have been observed in this source. By analyzing the high-resolution optical spectroscopy and the rotationally broadened absorption features of the secondary star, (Casares et al. 1998) determine the optical function of $f(M) = 0.69 \pm 0.03 M_\odot$ and the mass ratio of $q = M_\text{NS}/M_\text{BH} = 0.34 \pm 0.04$, based on which and adopting the inclination $i = 62.5^\circ \pm 4^\circ$, Orosz & Kuulkers (1999) find that the NS mass in this system is $M_\text{NS} = 1.78 \pm 0.23 M_\odot$. Besides this measurement, Elebert et al. (2009b) analyzed the radial velocity (RV) curve for the secondary star, derive the mass function and further imply the NS mass to be 1.5 ± 0.3 $M_\odot$.

SAX J1808.4-3658 contains a neutron star with a very low-mass companion star in a 2-h orbital period (Chakrabarty & Morgan, 1998), the estimated distance is 2.5 kpc (In't Zand et al. 1998, 2001) and the maximum luminosity is $\sim 10^{36}$ erg s$^{-1}$. This source has shown type-I X-ray bursts (In't Zand et al. 1998, 2001; Chakrabarty et al. 2003), coherent 401 Hz pulsations (Wijnands & van der Klis, 1998), and the simultaneous twin kHz QPOs (Wijnands et al. 2004). Based on the existing pulse timing measurements, Elebert et al. (2009b) constrain the mass ratio of the system and infer the mass function for the pulsar to be 0.44 $M_\odot$, from which the NS mass of this source is derived to be less than 1.4 $M_\odot$.

4U 1820-30 is a NS-LMXB source residing in the globular cluster NGC 6624. It has the extremely short orbital period of 685 s and its secondary is a low-mass helium-rich degenerate star (Stella et al. 1987). This source has an estimated distance of 6.4 ± 0.6 kpc (Vacca et al. 1998) from UV observation and 7.6 kpc from optical observations (Rich et al. 1993), and has been detected with both Type I X-ray bursts (Grindlay et al. 1978) and twin kHz QPOs (Smale et al. 1997). Based on the observed properties of X-ray bursts, Shaposhnikov & Titarchuk (2004) infer the mass-radius relationship by modeling the spectral temper-
Figure 2. The $M - R$ diagram of NS. Some representative EOS curves are shown (see also e.g. Akmal et al. 1998; Miller 2002; Bednarek et al. 2012): stars containing strange quark matter (CS1 and CS2), nucleons only (APR) (Akmal et al. 1998), nucleons and hyperons (BM$_{NH}$) (Bednarek et al. 2012), stars made of normal neutron matter (CN1 and CN2) and with pion condensate cores (CPC) (Lattimer & Prakash 2001; Cook et al. 1994). $R_s$ represents the Schwarzschild radius, and ISCO represents the innermost stable circular orbit. For each source in Table 1, we show the range of measured NS mass (see Table 1) and $M - R$ range based on $A$ value (see Table 2). (a)-(b) is for Cyg X-2 (with the NS mass $1.78 \pm 0.23 M_\odot$ and $1.5 \pm 0.3 M_\odot$, respectively), (c) is for SAX J1808.4-3658 and (d) is for 4U 1820-30.

We adopt these three sources to test the twin kHz QPO model RPM and AWOM, and the inferred parameters are shown in Table 2. As for RPM, the derived NS mass of Cyg X-2 (SAX J1808.4-3658 and 4U 1820-30) by twin kHz QPOs is $1.96 \pm 0.10 M_\odot$ ($2.83\pm0.04 M_\odot$ and $1.85\pm0.02 M_\odot$), while the corresponding measured result is $1.5\pm0.3 M_\odot$ ($<1.4 M_\odot$ and $1.29\pm0.07 M_\odot$). It can be seen that all the inferred NS masses by RPM are systematically bigger than the measured results by $\sim 50\%$, especially for SAX J1808.4-3658, whose predicted NS mass is twice of the measured result. Therefore, we can conclude that the RPM model overestimates the NS mass.

3 M-R CONSTRAIN OF NS

$M - R$ relation is helpful for understanding the nuclear compositions inside NS, and many methods are proposed to constrain it, such as by rotational broadening, redshift, Eddington limit, surface emission (Özel 2006; Zhang et al. 2007) and kHz QPOs (Miller et al. 1998). In addition, Zhang (2009) adopts AWOM to constrain $M - R$ relation by the twin kHz QPOs.

With the twin kHz QPOs and equation (4), we firstly calculate the density parameter $A$ values of the sources Cyg X-2, SAX J1808.4-3658 and 4U 1820-30, then constrain their NS radii with the measured NS masses. By comparing the measured NS mass and derived radius with the EOS curves in M-R diagram, we can infer the possible matter compositions inside these NSs, which are shown in Fig.2. For Cyg X-2, its NS is likely to contain the condensate pion core (CPC); for SAX J1808.4-3658 and 4U 1820-30, the EOS of NS may exclude...
strange quark matter (CS1 and CS2); for 4U 1820-30, its NS may contain the condensate pion core, or nucleons and hyperons (BM-NH).

Furthermore, we constrain the $M - R$ relations by the maxima of kHz QPOs, assumed to be the Keplerian orbital frequencies, for Cyg X-2, SAX J1808.4-3658 and 4U 1820-30, which are shown in Fig. 3. The constrained radii are listed in the last column of Table 1. With the certain NS mass, the EOS of NS is constrained between the line of Schwarzschild radius $R_s$ and the curve where Keplerian orbital frequency is the maxima of kHz QPOs, then this method cannot present a definite conclusion for NS EOSs. Most EOSs shown in the figure are compatible for all three sources. It can be also seen that among the EOSs shown in Fig. 3, stars made of normal neutron matter (CN2) can sustain a NS of 2 $M_\odot$.

4 DISCUSSIONS AND CONCLUSIONS

By exploiting the three sources, Cyg X-2, SAX J1808.4-3658 and 4U 1820-30, with the measured NS masses and twin kHz QPOs, we test the models of twin kHz QPOs (RPM and AWOM), and constrain their NS EOSs. The summaries and discussions are listed below:

1. We test the twin kHz QPO models RPM and AWOM by the sources with the measured NS masses and twin kHz QPOs. From the results in Table 2, it can be seen that the inferred NS masses by RPM are all systematically bigger than the measured results by ~ 50%. In particular, the inferred mass (2.83 ± 0.04) of SAX J1808.4-3658 is twice of the measured result (<1.4 $M_\odot$). It can be seen from Fig. 4 that most data points of the twin kHz QPOs center at the predicted curve by RPM with NS mass of 2 $M_\odot$. RPM overestimates the NS mass of Cyg X-2 (1.96 ± 0.10 $M_\odot$) by ~ 30% than the measured result (1.5 ± 0.3 $M_\odot$). We take this scale factor as the reference and infer the mean value of $M_\odot$ to be 1.54 $M_\odot$, which is close to the statistical result by [Zhang et al. 2011] (1.57 ± 0.35 $M_\odot$ for the measured NS masses of MSPs). The predicted NS mass by RPM for XTE 1807-294 is near 3.2 $M_\odot$ (see also Fig. 4). The twin kHz QPO properties of XTE 1807-294 and SAX J1808.4-3658 are similar, so we take the scale factor ~ 100% to infer the NS mass of XTE 1807-294 to be 1.6 $M_\odot$. In the construction of RPM, only the ideal environments are considered, such as vacuum condition and test particle, which is too simplified to satisfy the reality of the accreting NS, and we think that the modification of this model is necessary by considering the interaction between the NS and accretion disk, the effect of the magnetic field. The inferred NS radii of the three sources (Cyg X-2, SAX J1808.4-3658 and 4U 1820-30) by AWOM are around 10 – 20 km (see Table 2), which satisfy the theoretical expectations of NS EOSs.

2. We constrain the NS $M - R$ relations of the sources in Table 1 by twin kHz QPOs and AWOM, and infer the possible matter compositions of these NSs with the help of EOSs, which are shown in Fig. 2. The measured NS mass of SAX J1808.4-3658 is < 1.4 $M_\odot$ with the inferred density parameter $A = 0.43 ± 0.01$ by AWOM, which is less than the values of the other sources with $A ∼ 0.6$. The EOS of this NS may exclude the strange quark matter (see Fig. 2) comparing with the relative reference by [Li et al. 1999]. By adopting the NS mass range of 1 – 1.4 $M_\odot$ (1 $M_\odot$ is the measured lower limit of the NS mass, see [Miller 2002, Zhang et al. 2011]) and $A$ value, we infer its NS radius range to be 17 – 20 km, and induce that the EOS of its NS is close to the state with condensate pion core (CPC). Similarly, the EOS of Cyg X-2 is also near CPC.

For 4U 1820-30, Shaposhnikov & Titarchuk (2004) infer its NS mass of 1.29$^{+0.12}_{-0.07} M_\odot$ and radius of 11.2$^{+0.5}_{-0.4}$ km according to the observed properties of X-ray bursts. The authors adopt the photospheric radius expansion model to fit the NS mass, radius and some other parameters simultaneously, in which some theoretical assumptions are set, such as spectral formation, dynamic evolution of NS-disk geometry and the helium abundance $Y_H$ (to be 1.0). Based on AWOM, we infer the NS density parameter $A (0.65±0.01)$ of 4U 1820-30, where two theoretical assumptions (Keplerian orbital frequency for $v_2$ and MHD Alfvén wave oscillation frequency for $v_1$) are employed. Furthermore, the NS radius of this source is inferred to be 14.5$^{+0.5}_{-0.5}$ km by adopting the NS mass of 1.29$^{+0.12}_{-0.07} M_\odot$ (Shaposhnikov & Titarchuk 2004), which is bigger than their result (11.2$^{+0.5}_{-0.4}$ km). Considering the model dependence of the two results, we cannot make a certain conclusion which radius is more reasonable.

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