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On $J^{PC} = 0^{--}$ exotic glueball

Abstract The mass spectrum of the gluonium with $J^{PC} = 0^{--}$ is examined in three bottom-up AdS/QCD models. The results are used to identify several production and decay modes useful for searching this state. Moreover, the properties of such glueball in a hot and dense quark medium are discussed.

Keywords QCD Phenomenology · Glueball and nonstandard multi-quark/gluon states

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1 Introduction

Glueballs, bound states of gluons arising from the non-Abelian nature of strong interactions, are an important testbed for non perturbative aspects of QCD. The main obstacle in their search is the mixing with quarkonia ($\bar{q}q$) with the same quantum numbers. A promising strategy for their identification is to focus on exotic states whose $J^{PC}$ quantum numbers are inaccessible to quark-antiquark configurations. This is the case of several glueballs with negative $C$-parity, composed of an odd number of constituent gluons ("oddballs"), for which little theoretical information is available. In particular, for $J^{PC} = 0^{--}$, the mass predictions for the lightest state span the range from $m_{0^{--}} = 2.79$ GeV in the flux-tube model [1], to $m_{0^{--}} \approx 5.166$ GeV in lattice QCD simulations [2]. Two stable $0^{--}$ oddballs, with masses $m_{0^{--}} = 3.81 \pm 0.12$ GeV and $m_{0^{--}} = 4.33 \pm 0.13$ GeV, have been predicted by QCD sum rules [3].

The mass spectrum of the $J^{PC} = 0^{--}$ oddball can be computed in a framework inspired by the AdS/CFT correspondence. This duality conjecture relates a strongly coupled gauge theory in a four dimensional (4D) Minkowski space to a semiclassical gravity theory defined in a five dimensional (5D) anti-de Sitter (AdS) geometry times a 5D sphere [4]. In Poincaré coordinates, the line element

$$ds^2 = \frac{R^2}{z^2}(dx_0^2 - dx^2 - dz^2) \quad z > 0,$$

with $z$ the fifth holographic coordinate, describes the AdS bulk metric. The original formulation of gauge/gravity duality required the 4D theory to be conformal invariant. Holographic bottom-up models, constructed to reproduce QCD properties, break such a symmetry by introducing an infrared energy scale in the bulk; the $J^{PC} = 0^{--}$ oddball mass spectrum can then be determined either in vacuum or in a quark bath at finite temperature and density. Such an approach complements the top-down methods applied to analyze, e.g., the scalar $0^{++}$ gluonium [5][6].

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2 \(J^{PC} = 0^{-+}\) mass spectrum in three holographic models of QCD

The holographic correspondence is based on a dictionary, according to which a local gauge-invariant operator in the 4D field theory is dual to a field in 5D AdS [7] [8]. In QCD, an interpolating current representing the glueball with quantum numbers \(J^{PC} = 0^{-+}\) can be written in terms of the gluon field strengths \(G_{\mu \nu}(x)\) and \(\tilde{G}_{\mu \nu}(x)\), namely

\[
J_0(x) = g_s^2 d_{abc} [\eta_{\alpha \beta}\tilde{G}_{\mu \nu}^a(x)][\partial_\alpha \partial_\beta G_{\rho \sigma}^b(x)][G_{\rho \sigma}^c(x)],
\]

with \(a, b, c\) color indices, \(d_{abc}\) the symmetric tensor defining the anticommutator of \(SU(3)_c\) generators, and \(g_s\) the strong coupling constant. The transverse \(\eta_{\alpha \beta}\) metric is defined as \(\eta_{\alpha \beta} = \eta_{\alpha \beta} - \frac{\partial_\alpha \partial_\beta}{\alpha^2}\), with \(\alpha, \beta\) (as well as \(\mu, \nu\)) 4D Lorentz indices, and \(\eta_{\alpha \beta}\) the Minkowski metric tensor [3; 9]. The operator (2) has conformal dimension \(\Delta = 8\), and its holographic dual field, \(O_0(x, z)\), has mass obtained by the relation \(M_0^2 R^2 = \Delta(\Delta - 4)\) [7] [8]. In the following, the \(AdS_5\) radius is set to \(R = 1\). The 5D action for \(O_0(x, z)\) can be written as

\[
S = \frac{1}{k} \int d^5x \sqrt{g} a(z) \left[ g^{MN} \partial_M O_0 \partial_N O_0 - M_0^2 O_0^2 \right],
\]

where \(g_{MN}\) is the bulk metric, \(g = |\det(g_{MN})|\) and \(k\) is a parameter making the action dimensionless. To account for confinement in QCD, conformal invariance must be broken in the action (3). Three different models including such effect, either by a proper choice of the function \(a(z)\) or by introducing a dynamical field in the metric, are discussed in the following. The mass spectrum can be determined solving the Euler-Lagrangian equations for the field \(O_0(x, z)\).

**Hard-wall model.** A simple way of modeling confinement in the holographic setup is by considering a slice of the \(AdS_5\) space, with a sharp cutoff at a finite distance \(z_m\) along the fifth dimension [10]. The metric \(g_{MN}\) is given by [1], and the condition \(z \leq z_m\) is implemented in the action through \(a(z) = \Theta(z_m - z)\), with \(\Theta\) the Heaviside function. The cutoff \(z_m\) sets a mass scale. The choice \(1/z_m = 346\) MeV, obtained from analyses on axial and vector mesons [10], gives the results \(m_0 = 2.80\) GeV and \(m_1 = 4.14\) GeV for the lowest-lying and the first-excited \(0^{-+}\) oddball, respectively [11].

**Soft-wall model.** In this framework, the geometry is \(AdS_5\) [1], and conformal invariance is smoothly broken by the function \(a(z) = e^{-c z^2}\) which introduces a mass scale \(c\) in the action [12]. The Regge-like mass spectrum is obtained [11]:

\[
m_n^2 = 4c^2(n + 4).
\]

Setting \(c = m_\rho/2 = 388\) MeV from the \(\rho\) meson mass computed in the model [12], one obtains \(m_0 = 1.55\) GeV and \(m_1 = 1.74\) GeV [11]. The mass spectrum (4) corresponds to the poles of the two-point correlation function of \(J_0(x)\).

\[
\Pi(p^2) = i \int d^4x e^{ipx} \langle 0 | T[J_0(x)J_0^\dagger(0)] | 0 \rangle.
\]

In the AdS/CFT dictionary, the QCD interpolating current \(J_0(x)\) is interpreted as the source of the dual field \(O_0(x, z)\). The 4D Fourier transforms of such operators, \(\tilde{J}_0(p)\) and \(\tilde{O}_0(p, z)\), are related by the bulk-to-boundary propagator \(\tilde{K}(p, z)\) of the oddball field, through \(\tilde{O}_0(p, z) = \tilde{K}(p, z)\tilde{J}_0(p)\). Holography prescribes the identification between the partition functions of the dual theories, and this allows to compute the two-point correlation function (5) as

\[
\Pi(p^2) \approx \frac{\delta^2 S_{\text{on}}}{\delta \tilde{J}_0 \delta \tilde{J}_0} |_{\tilde{J}_0 = 0},
\]

with \(S_{\text{on}}\) the 5D on-shell action. The poles of \(\Pi(p^2)\) are given in (4).
Table 1  Production and decay modes of the \( J^{PC} = 0^{--} \) oddball, for \( m_{0--} = 2.8 \) GeV.

| Radiative production | Hadronic production | Decay mode |
|----------------------|---------------------|------------|
| \( X_{c1}(3510) \rightarrow \gamma G(0^-) \) | \( X(3872) \rightarrow \omega G(0^-) \) | \( G(0^-) \rightarrow \gamma f_{1}(1285) \) |
| \( X(3872) \rightarrow \gamma G(0^-) \) | \( h_{b}(3525) \rightarrow \pi 
( I = 0) G(0^-) \) | \( G(0^-) \rightarrow \omega f_{1}(1285) \) |
| \( X_{c2}(3556) \rightarrow \gamma G(0^-) \) | \( X_{b1}(10255) \rightarrow (\omega \phi, J^{P'})G(0^-) \) | \( G(0^-) \rightarrow \rho a_{1}(1260) \) \( (I = 0) \) |
| \( X_{c2}(3927) \rightarrow \gamma G(0^-) \) | \( T_{nS} \rightarrow (f_{1}(1285), X_{c1}(3872))G(0^-) \) | 

Einstein-dilaton model. A third possibility is to consider a class of dynamical bottom-up models reproducing confinement through a distortion of the bulk geometry \[13, 14\]:

\[
\Phi(z), \text{ a dilaton field, couples to the graviton, and its profile is determined solving the Einstein equations deduced from this metric. The function } A_{s}(z) \text{ can be chosen as } A_{s}(z) = \delta^{2} \tilde{z}^{2}, \text{ with the mass scale fixed to } \delta = 0.43 \text{ GeV \[13\]. Once the profile of } \Phi(z) \text{ is obtained, the two lightest oddball states turn out to have mass } m_{0} = 2.82 \text{ GeV and } m_{1} = 4.07 \text{ GeV \[11\].}
\]

With these results, several production and decay modes can be identified, useful for the search of the \( 0^{--} \) gluonium. They are reported in Table 1 for the specific case \( m_{0--} = 2.8 \) GeV \[11\].

3 Oddball in thermalized and dense medium

The AdS/QCD duality allows to examine the properties of a \( J^{PC} = 0^{--} \) oddball in a quark thermal bath, at finite temperature \( T \) and chemical potential \( \mu \). Stability of the gluon configuration against thermal and density fluctuations can be investigated in comparison with other quark and gluon bound states. In-medium effects can be incorporated in the holographic description using the action \[1\], with an appropriate bulk geometry. I explicitly discuss the soft-wall case. The 5D line element

\[
d s_{(ED)}^{2} = \frac{e^{2A_{s}(z) - \frac{4}{z^{2}}}}{z^{2}} \left[ d x_{0}^{2} - d x_{0} d x_{0} - d z^{2} \right] . \tag{7}
\]

\( \Phi(z) \), a dilaton field, couples to the graviton, and its profile is determined solving the Einstein equations deduced from this metric. The function \( A_{s}(z) \) can be chosen as \( A_{s}(z) = \delta^{2} \tilde{z}^{2} \), with the mass scale fixed to \( \delta = 0.43 \) GeV \[13\]. Once the profile of \( \Phi(z) \) is obtained, the two lightest oddball states turn out to have mass \( m_{0} = 2.82 \) GeV and \( m_{1} = 4.07 \) GeV \[11\].

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\[
d s^{2} = \frac{1}{z^{2}} \left( f(z) d x_{0}^{2} - d x_{0} - \frac{d^{2}}{f(z)} \right) \tag{8}
\]

is characterized by a metric with \( f(z) \) which can be related to temperature and density of the boundary theory. The 5D Reissner-Nordström AdS geometry (AdS/RN), with

\[
f(z) = 1 - \left( \frac{1}{z_{h}^{2}} + q^{2} z_{h}^{2} \right) z^{4} + q^{2} z^{6} , \tag{9}
\]

can be used as the holographic dual of a thermalized and dense medium in the deconfined phase \[15, 16\].

This charged black hole bulk metric has an outer horizon \( z = z_{h} \) and charge \( q \) which can be related to the temperature \( T \) and chemical potential \( \mu \) of the dual boundary theory. Defining \( Q = q z_{h}^{4} \) and imposing the condition \( 0 \leq Q \leq \sqrt{2} \), the black-hole temperature is

\[
T = \frac{1}{4 \pi} \left| \frac{d f}{d z} \right|_{z = z_{h}} = \frac{1}{\pi z_{h}} \left( 1 - \frac{Q^{2}}{2} \right) . \tag{10}
\]

In the QCD generating functional, \( \mu \) multiplies the quark number operator \( O_{q}(x) = q^{T}(x) q(x) \). Thus, it can be interpreted, according to gauge/gravity correspondence, as the source of a bulk field dual to \( O_{q}(x) \), the time component of a \( U(1) \) gauge field \( A_{M}(x, z) \). By rotational invariance, the spatial components \( A_{i} \) (with
\[ \mu = \kappa \frac{Q}{2z_h}, \]

with \( \kappa \) a dimensionless parameter that will be fixed to 1, a choice determining the quark chemical potential \( \mu \) up to a numerical factor. Important information on the QCD bound states in a quark thermal bath can be inferred from the spectral function \( \rho (\omega^2) \), the imaginary part of the retarded Green function. The spectral function of the \( 0^- \) oddball is represented in vacuum by an infinite number of delta functions centered at the eigenvalues of the mass spectrum \( (4) \). Fig. 1 shows the changes of the in-medium \( \rho (\omega^2) \) as temperature and chemical potential are switched on. At any finite and fixed value of \( \mu \), the peaks of the spectral function broaden and move towards lower values of \( \omega^2 \) when \( T \) increases. An analogous behaviour is observed by keeping the temperature fixed, and increasing the chemical potential. The \( (T, \mu) \)-dependence of the lightest oddball’s squared mass and width is shown in Fig. 2. The values of the temperature and chemical potential at which the lowest-energy peak becomes indistinguishable in the profile of the spectral function, are smaller than those obtained for light vector \( [17; 18] \), scalar \( \bar{q}q \) mesons \( [19] \), the lightest scalar glueball \( [20] \), and hybrid mesons \( [21] \). Hence, \( 0^- \) oddballs are found to suffer of larger in-medium instabilities with respect to other hadrons.

For points in the \( T - \mu \) plane below the QCD deconfinement transition, the state described by AdS/RN metric is metastable \( [16] \). The thermal-charged AdS metric (tcAdS), with line element \( (3) \) and deformation

\[ f(z) = 1 + q^2 z^6. \]

is proposed as a dual of the confined phase of QCD at small temperature and finite chemical potential \( [16; 22] \). This geometry is related to the AdS/RN one by the Hawking-Page transition, which describes deconfinement in the holographic framework. The chemical potential, related to the finite density of the hadronic medium,
grows linearly with $q$, while the temperature is implemented through a periodicity in the Euclidean time coordinate $\tau = i\xi_0$ [22]. The results, depicted in Fig. 3, reveal that the masses of the two lightest oddballs increase with $\mu$, a different behaviour with respect to what is found in the deconfined phase [11].

4 Conclusions

In AdS/QCD, the mass of the lowest-lying $0^{--}$ oddball is found to be lighter than as computed by other approaches. The effect of deconfined quark matter can be modeled by the AdS/RN geometry: oddballs are found to suffer from larger thermal and density instabilities with respect to other hadrons. On the other hand, the holographic description of a confined medium, obtained using the tcAdS metric, gives a mass which increases with the chemical potential.

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