THRESHOLD CORRECTIONS IN PRECISION LHC PHYSICS:
QED⊗QCD

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With an eye toward LHC processes in which theoretical precisions of 1% are desired, we introduce the theory of the simultaneous YFS resummation of QED and QCD to compute the size of the expected resummed soft radiative threshold effects in precision studies of heavy particle production at the LHC. Our results show that both QED and QCD soft threshold effects must be controlled to be on the conservative side to achieve such precision goals.

Keywords: QCD; QED; Resummation.

1. Introduction

At the LHC/ILC, the precision requirements for soft multiple gluon \( n(g) \) effects will be even more demanding than at FNAL, where the uncertainty on \( m_t \) \cite{1}, \( \delta m_t = 4.3 \text{ GeV} \), receives a soft \( n(g) \) uncertainty \( \sim 2\text{-}3 \text{ GeV} \), and soft \( n(g) \) MC exponentiation results will be an important part of the necessary theory – YFS exponentiated \( \mathcal{O}(\alpha_s^2) L \) calculations, in the presence of parton showers, on an event-by-event basis.

As many authors \cite{2} prepare the necessary results that would lead to such precision on QCD for LHC processes, the QED corrections need to be addressed as well. Estimates by Refs. \cite{3-7} show that one gets few per mille effects from QED
corrections to structure function evolution. In this paper, estimate the size of QED corrections at threshold at the LHC.

Treat simultaneously QED and QCD in the respective YFS [8, 9] exponentiation we discuss threshold effects at the LHC in the candidate luminometry [10, 11] processes \( pp \rightarrow V + n(\gamma) + m(g) + X \rightarrow \ell\ell' + n'x(\gamma) + m(g) + X \), where \( V = W^\pm, Z \), and \( \ell = \ell, \mu, \ell' = \nu, \nu, \nu, \nu \) respectively for \( V = W^+ \), and \( \ell = \nu, \nu, \nu, \nu \) respectively for \( V = W^- \).

3. Extension to QED

Here in the \( \bar{\beta} \) theory we have the replacements 2 for \( \alpha \) respectively for \( V \) and \( \bar{\beta} \) and all of the attendant IR functions, \( \{ B, B', D, S \} \), and hard photon residuals, \( \{ \bar{\beta}_n(k_1, \ldots, k_n) \} \), are specified in Refs. [9]. In Refs. [13, 14] we have extended the YFS theory to QCD: the net result is that in the analogous YFS and \( \bar{\beta} \) theory we have the replacements 2 for \( \alpha \) respectively for \( V \) and \( \bar{\beta} \) and all of the attendant IR functions, \( \{ B, B', D, S \} \), and hard photon residuals, \( \{ \bar{\beta}_n(k_1, \ldots, k_n) \} \), are specified in Refs. [9]. In Refs. [13, 14] we have extended the YFS theory to QCD: the net result is that in the analogous YFS and \( \bar{\beta} \) theory we have the replacements 2 for \( \alpha \) respectively for \( V \) and \( \bar{\beta} \) and all of the attendant IR functions, \( \{ B, B', D, S \} \), and hard photon residuals, \( \{ \bar{\beta}_n(k_1, \ldots, k_n) \} \), are free of all infrared divergences to all orders in \( \alpha_s(Q) \). The genuine non-Abelian IR physics is encoded [13] here in the \( \bar{\beta} \). The YFS resummation which we discuss here is fully consistent with that of Refs. [15, 16]. See Ref. [17] for more discussion of this point.

3. Extension to \( \text{QED} \otimes \text{QCD} \) and \( \text{QCD} \)

Simultaneous exponentiation of QED and QCD higher order effects gives [17]

\[
d\sigma_{\text{exp}} = \varepsilon_{\text{SUM} \otimes \text{QCD}} \left[ \prod_{m=1}^{\infty} \left( \int \frac{d^3 k}{k_{j_1}} \prod_{j_2=1}^{n} \frac{d^3 k'}{k'_{j_2}} \right)^2 \int d^4 y \left( \frac{2\pi}{e} \right)^4 \right] \varepsilon_{\text{SUM} \otimes \text{QCD}} \left( \prod_{m=1}^{\infty} \left( \int \frac{d^3 k}{k_{j_1}} \prod_{j_2=1}^{n} \frac{d^3 k'}{k'_{j_2}} \right) \right) \varepsilon_{\text{SUM} \otimes \text{QCD}} \left( \prod_{m=1}^{\infty} \left( \int \frac{d^3 k}{k_{j_1}} \prod_{j_2=1}^{n} \frac{d^3 k'}{k'_{j_2}} \right) \right)
\]

where the \( \bar{\beta} \) new YFS functions, \( \text{SUM} \otimes \text{QCD} \), \( \bar{\beta}_n(k_1, \ldots, k_m, k'_{1}, \ldots, k'_{n}) \), the latter has \( m \) hard gluons and \( n \) hard photons, are defined in Ref. [17]. The infrared algebra \( \text{QCD} \) [17] obtains: the average Bjorken \( x \) values for the QED and QCD emissions imply [17] that QCD dominant corrections happen an order of magnitude earlier than those for QED so that the leading \( \bar{\beta}_{0,0} \) -level gives a good estimate of the size of the effects we study.

4. \( \text{QED} \otimes \text{QCD} \) Threshold Corrections at the LHC

For the basic formula (we use the standard notation here [17])

\[
d\sigma_{\text{exp}}(pp \rightarrow V + X \rightarrow \ell\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\sigma_{\text{exp}}(x_i, x_j, s),
\]
we use the result in (1) for \( V = Z \) here with semi-analytical methods and structure functions from Ref. [24]. See also the work of Refs. [18–23]. A Monte Carlo realization will appear elsewhere [25], wherein we will ultimately use HERWIG [26], PYTHIA [27] and/or the new shower algorithm in Ref. [29] in lieu of the \( \{ F_i \} \) and thereby, in principle, improve on the shower/exact result combination in Ref. [28]. Due to its lack of the appropriate color coherence [30], we do not consider ISAJET [31] here.

We compute, with and without QED, the ratio \( r_{\text{exp}} = \frac{\sigma_{\text{exp}}}{\sigma_{\text{Born}}} \) to get the results (We stress that we do not use the narrow resonance approximation here.)

\[
r_{\text{exp}} = \begin{cases} 
1.1901, & \text{QCED} \equiv \text{QCD+QED, LHC} \\
1.1872, & \text{QCD, LHC} \\
1.1911, & \text{QCED} \equiv \text{QCD+QED, Tevatron} \\
1.1879, & \text{QCD, Tevatron}.
\end{cases}
\]

QED is at the level of .3% at both LHC and FNAL\(^a\). We agree with the results in Refs. [18–22]. The QED effect is similar in size to structure function results in Refs. [3–7].

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