Covariant Quark Model for the Baryons

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Abstract

A family of simply solvable covariant quark models for the baryons is presented. With optimal parameter choices the models reproduce the empirical spectra of the baryons in all flavor sectors to an accuracy of a few percent. Complete spectra are obtained for all states of the strange, charm and beauty hyperons with $L \leq 2$. The magnetic moments and axial coupling constants of the ground state baryons correspond to those of conventional quark models. We construct current-density operators that are consistent with empirical nucleon form factors at low and medium energies.
1. Introduction

A theoretically consistent description of the baryons based on the concept of constituent quark should build in confinement and Poincaré invariance from the beginning. Confinement is an obvious consequence of the unbounded discrete mass spectrum. The requirements of Poincaré invariance for the mass operator are easily met by explicit construction: The Hilbert space of states $\mathcal{H}$ is the tensor product, $\mathcal{H} = \mathcal{H}_C \times \mathcal{H}_\ell$, of functions representing the center-of-mass motions with the “little Hilbert space” which is the representation space of the “little group”. The eigenfunctions of the four-momentum obtained with such a mass operator depend on a choice of boosts and kinematic variables. These eigenfunctions are related to observables only in combination with a corresponding representation of the current density operators. The construction of a Poincaré covariant quark model of the baryons thus involves both the construction of a mass operator with appropriate spectral properties and the construction of Poincaré covariant current density operators. For confined quarks the conventional assumption of a free-quark density is not compelling at low energies. Giving up that assumption greatly facilitates the construction of simple phenomenological current operators that satisfy all symmetry requirements. In this context instant-form kinematics appears to have two principal virtues: (i) It allows recovery of the familiar features of conventional quark models. In particular it facilitates implementation of the physical notion of “impulse” currents. (ii) It arises naturally when quantum mechanical models are derived from Euclidean Green functions.

Here we consider confining mass operators which provide satisfactory descriptions of the baryon spectrum in all flavor generations. The mass operators considered are sums of a confining term, which depends only on the Jacobi coordinates of the 3-quark system, and a flavor and spin dependent hyperfine correction. The former provides the basic shell structure of the spectrum and is readily diagonalized by means of hyper-spherical harmonics. The latter is built on the observation that a superior description of the baryon spectrum in all flavor generations may be achieved by assuming that the main hyperfine correction should have the flavor-spin structure $\sum_{i<j} \tilde{\lambda}_i \cdot \tilde{\lambda}_j \sigma_i \cdot \sigma_j$, which e.g. is suggested by the short range part of the Goldstone boson exchange interaction between the constituent quarks.

The aim of this work is to provide a class of simple models that can account for all states in the $S, P$ and $SD$-shells of the baryon spectra in all flavor generations. The parameter values in the models are extracted from the lowest splittings in the spectra of the successive flavor generations. The motivation for this work is the rapid progress in experimental determination of the low lying levels in the $C = +1$ charm hyperon spectra, and the parallel - if somewhat slower - progress in the case of the corresponding $B = -1$ hyperon states. We also give energy levels for the $C = 2, 3$ hyperons, which are mainly determined by the spin- and flavor independent mass operator with only small hyperfine corrections. These spectra should give fairly direct information on the effective quark confining interaction.

Quark currents consistent with the covariance conditions, lead to values for the magnetic moments and axial coupling constants of the ground state baryons similar to those obtained by conventional quark models. The 3-quark wave functions that yield satisfactory baryon spectra have small radii. These models therefore require significant quark-charge distributions to account for the observed nucleon form factors. With appropriate quark-charge and current distributions the model current operators are consistent with observed nucleon form
factors at low and medium energies. Applications to transitions are beyond the scope of the present paper.

This paper is divided into 8 sections. In section 2 we describe the model mass operator and the parameter choices. In section 3 we derive nucleon form factors as well as magnetic moments and axial coupling constants of the ground-state baryons. The current operators used are consistent with the Poincaré representation specified by the mass operator and with instant-form kinematics. The spectra of the nucleon and Δ-resonances are described in section 4, the spectra of the strange hyperons in section 5. Sections 6 and 7 contain predictions for the charm and beauty hyperons respectively. Section 8 contains a summary.

2. The Mass operator

2.1 The Hilbert Space of 3-Quark States

The states in the baryon spectrum are described by vectors in the little Hilbert space $H_\ell$, which is the representation space of the direct product of the little group, $SU(2)$, with flavor and color $SU(3)$. Concretely these states are realized by functions, $\phi$, of three quark positions $\vec{r}_i$, three spin variables $\mu_i$ and three flavor variables $f_i$, which are symmetric under permutations and invariant under translations $\vec{r}_i \to \vec{r}_i + \vec{a}$. The translational invariance is realized by expressing the wave functions in terms of Jacobi coordinates

$$\vec{r} := \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2), \quad \vec{\rho} := \sqrt{\frac{2}{3}} (\vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}).$$

Representations of the full Poincaré group obtain on the tensor product, $H := H_\ell \otimes H_c$, of the little Hilbert space $H_\ell$ with the Hilbert space $H_c$ of functions of the four-velocity $v$, which is specified by 3 independent components. Translations are generated by the four-momentum operator $P = Mv$. Any confining self-adjoint mass operator $M$, independent of $v$, satisfies all relativistic symmetry requirements if it is invariant under rotations. At the level of spectroscopy alone there is no difference between relativistic and nonrelativistic quark models because the Galilean rest energy operator satisfies all the symmetry requirements of a mass operator.

The Poincaré invariant inner product of the functions representing baryon states is defined as

$$(\Psi, \Psi) = \int d^4v 2\delta(v^2 + 1) \theta(v^0) \int d^3\kappa \int d^3k |\Psi(v, \kappa, \vec{k})|^2,$$

where $\kappa$ and $\vec{k}$ are the momenta conjugate to $\vec{\rho}$ and $\vec{r}$. Summation over spin and flavor variables is implied. Under Lorentz transformations $v \to \Lambda v$ the vectors $\kappa$ and $\vec{k}$ and the three quark spin variables $\mu_1, \mu_2, \mu_3$ undergo Wigner rotations $R_W(\Lambda, v)$:

$$R_W(\Lambda, v) := B^{-1}(\Lambda v) \Lambda B(v).$$

Note that with canonical boosts, $R_W(\Lambda v, v) = 1$, for any rotationless Lorentz transformation $\Lambda v$ in the direction of $\vec{v}$.

For the construction of quark currents it is convenient to define internal four-momenta $p$ and $q$ by

$$p := B(v)\{0, \kappa\}, \quad q := B(v)\{0, \vec{k}\},$$

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so that \( p^2 = |\kappa|^2 \) and \( q^2 = |\vec{k}|^2 \).

The construction of unitary representations of the Poincaré group sketched here naturally leads to point-form kinematics. Once the eigenfunctions of the mass operator are known, it is easy to realize unitary transformations to other forms of kinematics explicitly \(^2\). Let \( \Psi_n(v, \vec{\kappa}, \vec{k}) \) be eigenfunctions of \( M \), with eigenvalues \( M_n \). Any state \( \Psi = \sum_n \Psi_n c_n \) can be represented by functions \( \Psi(\vec{P}, \vec{p}, \vec{q}) \) normalized as

\[
(\Psi, \Psi) = \int d^3 P \int d^3 p \int d^3 q |\Psi(\vec{P}, \vec{p}, \vec{q})|^2 .
\] (2.5)

This representation is used below to calculate magnetic moments and axial coupling constants. The unitary transformation \( \Psi(v, \vec{\kappa}, \vec{k}) \rightarrow \Psi(\vec{P}, \vec{p}, \vec{q}) \) is specified by the variable transformation \( \{v, \vec{\kappa}, \vec{k}, n\} \rightarrow \{\vec{P}, \vec{p}, \vec{q}, n\} \) where \( \vec{p} = \vec{p}(v, \vec{\kappa}) \) and \( \vec{q} = \vec{q}(v, \vec{k}) \) are specified by eq. (2.5) and \( \vec{P} = M_n \vec{v} \) in each term of the sum over \( n \). Individual quark momenta are defined by

\[
\begin{align*}
\vec{p}_1 &= \frac{1}{3} \vec{P} - \frac{1}{2} \sqrt{\frac{2}{3}} \vec{p} + \sqrt{\frac{1}{2}} \vec{q}, \\
\vec{p}_2 &= \frac{1}{3} \vec{P} - \frac{1}{2} \sqrt{\frac{2}{3}} \vec{p} - \sqrt{\frac{1}{2}} \vec{q}, \\
\vec{p}_3 &= \frac{1}{3} \vec{P} + \sqrt{\frac{2}{3}} \vec{p},
\end{align*}
\] (2.6)

so that \( \vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 \).

### 2.2. The Confining Mass Operator

We shall consider the confining mass operators of the form

\[
\mathcal{M}_0 = \sqrt{3}\{\vec{k}^2 + \vec{\kappa}^2 + f(R)\} + \mathcal{K},
\] (2.7)

where \( R, \)

\[
R := \sqrt{2(\vec{r}^2 + \vec{\rho}^2)} .
\] (2.8)

is the radius of a 6-dimensional hyper-sphere, and the operator \( \mathcal{K} \)

\[
\mathcal{K} := n_S \Delta_S^2 + n_C \Delta_C^2 + n_B \Delta_B^2,
\] (2.9)

represents the flavor dependence. The integers \( n_S, n_C \) and \( n_B \) are flavor quantum numbers of strangeness, charm and beauty. The parameters \( \Delta_f \) represent the flavor dependence of the ground-state baryon masses. Mass operators of the form (2.7) satisfy all symmetry requirements and are easily diagonalized by means of hyper-spherical harmonics.\(^4\)

It is convenient to express the magnitudes \( r \) and \( \rho \) as functions of the radius \( R \) and the auxiliary variable \( z, \)

\[
z := \frac{2(r^2 - \rho^2)}{R^2} ,
\] (2.10)
so that
\[ r = \frac{R}{2}\sqrt{1 + z}, \quad \rho = \frac{R}{2}\sqrt{1 - z}. \] (2.11)

The eigenfunctions, \( \phi(r, \rho) \), of the mass operator \( \mathcal{M}_0 \) are products,
\[ \phi(r, \rho) = u_{nK}(R) \, \mathcal{Y}_K \] (2.12)
of radial functions \( u_{nK}(R) \) and the hyper-spherical harmonics \( \mathcal{Y}_K \),
\[ \mathcal{Y}_K = [Y_{\ell_1}(\hat{r}) \otimes Y_{\ell_2}(\hat{\rho})] m(1 + z)^{\ell_1/2}(1 - z)^{\ell_2/2} P_{\ell_1 + \ell_2 + \frac{1}{2}}^{\ell_1 + \frac{1}{2}}(z), \] (2.13)
where \( K = 2\nu + \ell_1 + \ell_2 \) and \( P_{\nu} \) is a Jacobi polynomial. The functions \( u_{nK}(R) \) are solutions to the radial equation
\[ \left[ 2\left\{ -\frac{1}{R^{5/2}} \frac{d^2}{dR^2} R^{5/2} + \frac{\mathcal{L}(\mathcal{L} + 1)}{R^2} \right\} + f(R) \right] u_{nK}(R) = \epsilon^2_{nK} u_{nK}(R), \] (2.14)
where \( \mathcal{L} = K + \frac{3}{2} \) is the orbital angular momentum and \( n \) is the number of nodes in the radial wave function. The normalization condition for the radial functions \( u_{nK}(R) \) are
\[ \pi \frac{128}{128} \int_0^\infty dR R^5 u_{nK}^2(R) = 1. \] (2.15)

The eigenvalues of the mass operator \( \mathcal{M}_0 \) are then
\[ \mathcal{E}_0 = \sqrt{\epsilon^2_{nK} + \mathcal{K}}. \] (2.16)

In the following we consider two models for the function \( f(R) \) in (2.7):
\[ f_1(R) = \frac{\omega^4}{2} R^2, \] (2.17)
and
\[ f_2(R) = -\frac{a}{R} + bR. \] (2.18)

The oscillator model (2.17) yields wave functions that are Gaussian in the radius \( R \), with the lowest three eigenvalues
\[ \epsilon_{00} = \sqrt{18}\omega, \quad \epsilon_{01} = \sqrt{24}\omega, \quad \epsilon_{10} = \epsilon_{02} = \sqrt{30}\omega. \] (2.19)

The “funnel” model (2.18) yields wave functions that are more sharply peaked at small \( R \) and flatter than Gaussians at large \( R \). It yields a better spectrum than the oscillator model since it splits the SD shell in the baryon spectrum, which is degenerate in the oscillator model. For the funnel model the dependence of the eigenvalues on the parameters \( a \) and \( b \) must be determined numerically. The ground state wave functions of these two models are shown in in Fig. 1 for the parameters chosen in Section 4.1.

There is, of course, a wide variety of other possibilities for the confining function \( f(R) \). A version intermediate between the oscillator and the funnel may be obtained by replacing the linear powers of \( R \) in (2.18) by \( R^2 \) in both terms. That function yields energy levels between those of the oscillator and funnel models.
The baryon spectrum in the light flavor sectors is complicated by the fact that it requires hyperfine splitting sufficiently strong to mix up the shell structure of the confining well. The clearest manifestation of this is the positive parity of the $N(1440)$ and the $\Delta(1600)$ which are the lowest excited states of the nucleon and the $\Delta(1232)$ respectively. A principal advantage of the study of heavy flavor hyperons is a spectrum presumably less sensitive to the hyperfine interaction, and thus easier to interpret.

We consider the following phenomenological model for the hyperfine interaction:

$$
\mathcal{M}' = \{1 - A[(\vec{r} \times \vec{k})^2 + (\vec{\rho} \times \vec{\kappa})^2]\} H_x, \tag{2.20}
$$

where $H_x$ is the following flavor-spin operator:

$$
H_x = -\sum_{i<j} \left\{ \sum_{a=1}^{3} C_{\lambda_i^a \lambda_j^a} + \sum_{a=4}^{8} C_{S \lambda_i^a \lambda_j^a} + \sum_{a=9}^{12} C_{C \lambda_i^a \lambda_j^a} + \sum_{a=13}^{14} C_{SC \lambda_i^a \lambda_j^a} + \sum_{a=15}^{19} C_{B \lambda_i^a \lambda_j^a} + \sum_{a=20}^{21} C_{SB \lambda_i^a \lambda_j^a} \right\} \vec{\sigma}_i \cdot \vec{\sigma}_j. \tag{2.21}
$$

The matrices $\lambda_i^a$ are $SU(5)$ representations - extensions of the $SU(3)$ Gell-Mann matrices. The parameters $C, C_S, C_C, C_{SC}, C_B$ and $C_{SB}$ must be determined by the empirical spectrum. The angular momentum dependent term in eq. (2.13) is motivated by the substantial empirical splitting of the SD-shell in the light flavor sector (cf. the splitting between the $N(1440)$ and the $N(1720) - N(1680)$ multiplet).

Since the contributions $\mathcal{M}_0$ and $\mathcal{M}'$ commute, the eigenvalues of the combined mass operator $\mathcal{M}_0 + \mathcal{M}'$ are additive, that is

$$
\mathcal{E} = \mathcal{E}_0 + c, \tag{2.22}
$$

where $c$ is an eigenvalue of $\mathcal{M}'$. The hyperfine correction $c$ depends on the orbital angular momentum of the 3 quark state and on the symmetry character of the spin-flavor part of the wave function.

If the hyperfine term (2.20) is viewed as an effective representation of pseudoscalar exchange mechanisms between quarks, the matrix elements $C$ represent $\pi$, and the matrix elements $C_S, K$ and $\eta$ meson exchange. The matrix elements $C_C$ and $C_{SC}$ would represent $D$ and $D_s$ exchange mechanisms respectively and correspondingly the matrix elements $C_B$ and $C_{SB}$ would represent $B$ and $B_s$ exchange mechanisms. Because of the near degeneracy between the heavy flavor pseudoscalar and vector mesons, the matrix elements $C_S$ and $C_B$ should be viewed as representing both heavy flavor pseudoscalar and vector exchange mechanisms. The terms $a = 15$ and $a = 22$ have been left out from the sums in the hyperfine interaction (2.20), because they would represent hidden heavy flavor $\eta_C, J/\psi$ and $\eta_B, \Upsilon$ exchange mechanisms that should be far weaker than the corresponding heavy flavor pseudoscalar and vector exchange mechanisms, because of the much larger $c\bar{c}$ and $b\bar{b}$ masses.
3. The Current Operator

3.1 Covariance Conditions

The quark current density operators $I^\mu(x)$ have to satisfy the covariance conditions

$$U^\dagger(\Lambda)I^\mu(x)U(\Lambda) = \Lambda^\mu\nu I^\nu(\Lambda^{-1}x), \quad (3.1)$$

for arbitrary Lorentz transformations $\Lambda$, and

$$e^{iP^a(x)}e^{-P^a} = I^\mu(x+a). \quad (3.2)$$

for space time translations. Current conservation requires that

$$[P^\nu, I^\nu(0)] = 0. \quad (3.3)$$

Let $|M, \vec{P}, j, \sigma, \tau, \zeta>$ be eigenstates of the four momentum operator $P = \{\sqrt{\vec{P}^2 + M^2}, \vec{P}\}$ and the canonical spin, where $\sigma$ is an eigenvalue of $j_z$ and $\zeta = \pm 1$ is the intrinsic parity. Lorentz invariant form factors are matrix elements of the form

$$<\kappa', \tau', \sigma', j', \vec{P}', M'|I^\mu(0)|M, \vec{P}, j, \sigma, \tau, \kappa>, \quad (3.4)$$

Because of the covariance of the current operator and the basis states only matrix elements with $\vec{P}' = -\vec{P} \equiv \vec{Q}/2$ are required, and thus the initial and final states are related kinematically. Note that $\vec{Q}^2$:

$$\vec{Q}^2 = Q^2 + \frac{(M'^2 - M^2)^2}{Q^2 + 2(M'^2 + M^2)}, \quad (3.5)$$

is Lorentz invariant. We may assume, without loss of generality, that the $z$-axis is in the direction of $\vec{Q}$.

The current conservation condition (3.3) is then expressed in the form

$$\left[1 + \frac{4M^2}{\sqrt{\vec{Q}^2}}, \rho(0)\right] - I_L(0) = 0. \quad (3.6)$$

where the Lorentz invariant charge and longitudinal current density operators $\rho(0)$ and $I_L(0)$ are defined as

$$\rho(0) := \frac{1}{2}\{M^{-1}P_\mu, I^\mu(0)\}, \quad \sqrt{\vec{Q}^2}I_L(0) := Q \cdot I + [\sqrt{\vec{Q}^2/4 + M^2}, \rho(0)] \quad (3.7)$$

Matrix elements $<f|I^\mu(0)|a>$ of the current density are then related to the wave functions and current kernels by by

$$<f|I^\mu(0)|i> = \int d^3p_1' \int d^3p_2' \int d^3p_3' \int d^3p_1 \int d^3p_2 \int d^3p_3 \times \Psi_f^*(\vec{p}_1', \vec{p}_2', \vec{p}_3')(\vec{p}_1', \vec{p}_2', \vec{p}_3'|I^\mu(0)|\vec{p}_3, \vec{p}_2, \vec{p}_1)\Psi_a(\vec{p}_1, \vec{p}_2, \vec{p}_3). \quad (3.8)$$
Elastic electric and magnetic form factors, $G_E(Q^2)$ and $G_M(Q^2)$ are defined by

$$G_E(Q^2) := \frac{1}{2j + 1} \sum_{\sigma} \langle j, \sigma, \bar{Q}/2|\rho(0)| - \bar{Q}/2, j, \sigma \rangle ,$$

$$G_M(Q^2) = -\frac{1}{\sqrt{\eta}} \sum_{\sigma',\sigma} (-)^{j-\sigma}(j, j, \sigma', -\sigma|1, 1\rangle \langle j, \sigma', \bar{Q}/2|I_x(0) + iI_y(0)| - \bar{Q}/2, j, \sigma \rangle ,$$

where $\eta := X C Q^2 / 4 m_p^2$ and $m_p$ is the proton mass. Magnetic moments, expressed in terms of nuclear magnetons, are equal to the magnetic form factors at $Q^2 = 0$.

### 3.2 Impulse Currents

Impulse currents may be defined by kernels of the charge and transverse currents,

$$(\bar{p}_1', \bar{p}_2', \bar{p}_3' | \rho | \bar{p}_3, \bar{p}_2, \bar{p}_1) = (\bar{p}_1'|\rho_1|\bar{p}_1)\delta^{(3)}(\bar{p}_2' - \bar{p}_2)\delta^{(3)}(\bar{p}_3' - \bar{p}_3)$$

$$+(\bar{p}_2'|\rho_2|\bar{p}_2)\delta^{(3)}(\bar{p}_3' - \bar{p}_3)\delta^{(3)}(\bar{p}_1' - \bar{p}_1)$$

$$+(\bar{p}_3'|\rho_3|\bar{p}_3)\delta^{(3)}(\bar{p}_1' - \bar{p}_1)\delta^{(3)}(\bar{p}_2' - \bar{p}_2).$$

and

$$(\bar{p}_1', \bar{p}_2', \bar{p}_3' | I_{\bot} | \bar{p}_3, \bar{p}_2, \bar{p}_1) = (\bar{p}_1'|I_{\bot,1}|\bar{p}_1)\delta^{(3)}(\bar{p}_2' - \bar{p}_2)\delta^{(3)}(\bar{p}_3' - \bar{p}_3)$$

$$+(\bar{p}_2'|I_{\bot,2}|\bar{p}_2)\delta^{(3)}(\bar{p}_3' - \bar{p}_3)\delta^{(3)}(\bar{p}_1' - \bar{p}_1)$$

$$+(\bar{p}_3'|I_{\bot,3}|\bar{p}_3)\delta^{(3)}(\bar{p}_1' - \bar{p}_1)\delta^{(3)}(\bar{p}_2' - \bar{p}_2).$$

Because of the complete antisymmetry of the baryon wave functions it is sufficient to consider the current matrix elements of only one constituent, e.g. $i = 3$.

The simplest model for the electromagnetic current density for the light and strange baryons, which matches the features of conventional quark models, is specified by kernels $(\bar{p}_i'|[\hat{T}]|\bar{p}_i)$ that depend on the momentum transfer $\bar{p}_i' - \bar{p}_i = \bar{Q}$, the spin and flavor variables, but do not depend on $\bar{p}_i' + \bar{p}_i$. That is

$$(\bar{p}_i'|\rho_i|\bar{p}_i) = \left[ \frac{1}{2} \lambda_3^{(i)} f_3(\bar{Q}^2) + \frac{1}{2\sqrt{3}} \lambda_8^{(i)} f_8(\bar{Q}^2) \right] \otimes 1 .$$

$$(\bar{p}_i'|I_{\bot,1}|\bar{p}_i) = i \frac{\vec{\sigma}_i \times \vec{Q}}{2m_u} \left\{ \frac{2}{3} + \frac{1}{\sqrt{3}} \lambda_8 \right\} \left[ \frac{1}{2} \lambda_3^{(i)} g_3(\bar{Q}^2) + \frac{1}{2\sqrt{3}} \lambda_8^{(i)} g_8(\bar{Q}^2) \right] \otimes 1 .$$

The functions $f_3(\bar{Q}^2)$, $f_8(\bar{Q}^2)$ and $g_3(\bar{Q}^2)$, $g_8(\bar{Q}^2)$ represent charge and magnetization distributions of the constituent quark. The scale factors $m_u = m_d$ and $m_s$ represent quark masses.

A straightforward extension to include the charm and beauty is achieved by adding terms of the form

$$\frac{1}{6} f_C(\bar{Q}^2) [1 - \sqrt{6}\lambda_{15}^{(i)}] - \frac{1}{15} f_B(\bar{Q}^2) [1 - \sqrt{10}\lambda_{22}^{(i)}]$$

(3.14)
to the quark charge operators.

3.3 The Form Factors of the Nucleons.

The charge density operator (3.12) yields the following expressions for the electric form factors of the proton and neutron.

\[
G_{E}^{p}(Q^{2}) = \frac{1}{2}[f_{8}(\mathcal{Q}^{2}) + f_{3}(\mathcal{Q}^{2})]F_{0}(\mathcal{Q}^{2}) ,
\]

\[
G_{E}^{n}(Q^{2}) = \frac{1}{2}[f_{8}(\mathcal{Q}^{2}) - f_{3}(\mathcal{Q}^{2})]F_{0}(\mathcal{Q}^{2}) .
\]

The function \(F_{0}(\mathcal{Q}^{2})\) is related to the ground-state wave function \(u_{0}0(R)\) by the Fourier-Bessel transform

\[
F_{0}(\mathcal{Q}^{2}) := \frac{3\pi}{16Q^{2}} \int_{0}^{\infty} dRR^{3}J_{2}(\mathcal{Q}R)u_{00}^{2}(R) ,
\]

where the variable \(\mathcal{Q}\) is defined as

\[
\mathcal{Q} := \sqrt{\frac{Q^{2}}{1 + \frac{Q^{2}}{4m_{p}^{2}}}}.
\]

The normalization of the wave function implies \(F_{0}(0) = 1\). From \(G_{E}^{p}(0) = 1\) and \(G_{E}^{n}(0) = 0\) it follows that \(f_{3}(0) = f_{8}(0) = 1\).

The proton charge radius obtains as

\[
<r_{p}^{2} >= -6 \left( \frac{dG_{E}^{p}(Q^{2})}{dQ^{2}} \right)_{Q^{2}=0} = -6 \left( \frac{dF_{0}(Q^{2})}{dQ^{2}} \right)_{Q^{2}=0} - \frac{1}{2} \left( \frac{df_{3}(Q^{2})}{dQ^{2}} + 6\frac{df_{8}(Q^{2})}{dQ^{2}} \right)_{Q^{2}=0}
\]

\[
= \frac{\pi}{768} \int_{0}^{\infty} dRR^{7}u_{00}^{2}(R) + \frac{1}{2} (<r_{8}^{2}> + <r_{3}^{2}>) .
\]

Here \(<r_{8}^{2}>\) and \(<r_{3}^{2}>)\ may be interpreted as mean square isoscalar and isovector quark charge radii respectively.

Assuming point-quark charges, \(f_{3} = f_{8} = 1\), is manifestly inadequate as the wave functions for both the oscillator or the funnel model yield much too small proton charge radii, and form factors that are too large for all values of the momentum transfer. This defect is cured by assuming appropriate quark-charge distributions.

Observed values of the e charge form factors of proton and neutron can be reproduced with the funnel model assuming

\[
f(\mathcal{Q}^{2}) := \frac{1}{2}[f_{8}(\mathcal{Q}^{2}) + f_{3}(\mathcal{Q}^{2})] = \frac{1}{1 + \frac{Q^{2}}{\Lambda^{2}}} , \tag{3.19a}
\]

and

\[
\frac{1}{2}[f_{8}(\mathcal{Q}^{2}) - f_{3}(\mathcal{Q}^{2})] = \frac{Q^{2}f(\mathcal{Q}^{2})}{4\Lambda^{2}\sqrt{1 + 7\mathcal{Q}^{2}/4\Lambda^{2}}} . \tag{3.19b}
\]

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The value $\Lambda = .7$ GeV is designed to fit the proton charge radius. The corresponding quark-charge radius is $\sqrt{\langle r_q^2 \rangle} \approx .7$ fm. The slope of $G_E^n(Q^2)$ at $Q^2 = 0$

$$\frac{dG_E^n(Q^2)}{dQ^2} = \frac{1}{4\Lambda^2} = .51 \text{GeV}^{-2} \quad (3.20)$$

is equal to the observed value [7, 8]. The $Q^2$ dependence of the proton charge form factor is illustrated in Fig. 2a in comparison with the data tabulated in ref. [9]. We also show the charge form factors calculated with point-quark charge distributions, $f_3 = f_8 = 1$, and with the wave functions of the oscillator and funnel models. The potential parameter are determined by the mass spectra in Sec. 4 below. Fig. 2b illustrates the $Q^2$ dependence of the neutron charge form factor together with and empirical fit [9], which is consistent with more recent data [10].

From the current density operator (3.13) it follows that the magnetic form factors of the nucleons depend on the the quark structure and the wave functions according to

$$G_M^p(Q^2) = \frac{m_p}{6m_u} [g_8(Q^2) + 5g_3(Q^2)] F_0(Q^2),$$

$$G_M^n(Q^2) = \frac{m_p}{6m_u} [g_8(Q^2) - 5g_3(Q^2)] F_0(Q^2). \quad (3.21)$$

Since the nucleon magnetic form factors can be approximated by the proton electric form factor multiplied with the magnetic moments [9] we choose $m_u = .336$ GeV to fit the magnetic moment of the proton with $g_8(0) + 5g_3(0) = 1$. Magnetic form factors with the correct order of magnitude obtain with $g_8(Q^2) = g_3(Q^2) = f(Q^2)$. A reasonable representation of the magnetic form factors of the nucleons is achieved by

$$g_8(Q^2) = \frac{\mu_8}{1 + Q^2/1.3\Lambda^2}, \quad g_3(Q^2) = \frac{\mu_3}{1 + Q^2/1.3\Lambda^2} \quad (3.22)$$

with $\mu_8 = .95$ and $\mu_3 = 1.01$. In Fig.3 we show the magnetic form factors of the funnel model in comparison with the empirical fit of ref. [9].

### 3.4 Magnetic Moments and Axial Coupling Constants of the Ground State Baryons

The magnetic moments that are obtained from the current operator (3.13) are the usual linear combinations of proton to quark mass ratios listed in the Table 1 along with the corresponding empirical values [11, 12] for the nucleons and the strange hyperons. The strange scale factor $m_s$ is determined by the magnetic moments of the $\Omega^-$ to be $m_s = .465$ GeV. The corresponding expressions for the magnetic moments of the heavy flavor baryons are given eq. in refs. [6, 13].

A model for the axial current density, which corresponds to that of the vector current (3.13) is

$$\vec{A}_i(Q) = g_A^q \frac{\lambda^{(i)}}{2} \vec{\sigma}_i. \quad (3.23)$$

Here $g_A^q$ represents the axial coupling constant for a single constituent quark. This model for the axial current leads to the same results as the conventional quark model for the $g_A/g_V$
ratios of the nucleons and the strange hyperons (Table 2). In the conventional representation of the $g_A/g_V$ ratios for the hyperon decays in terms of $D$ and $F$ coefficients, these take their usual quark model values

$$D = 1, \quad F = \frac{2}{3}. \tag{3.24}$$

Agreement of this model with the empirical value for the axial vector coupling for neutron decay requires the value $g_A^N = 0.76$.

4. The Spectra of the Nucleon and the $\Delta$ resonances

The spectra of the nucleon and the $\Delta$ resonances depend on the choice of the parameters $C, C_S$ and $A$ in the hyperfine interaction (2.20). The expressions for these hyperfine shifts of the states in the $S$, $P$, and $SD$ shells of these spectra are listed in Table 3, where we have employed the symmetry assignments of ref. [4]. The real values of the empirical pole positions of the known resonances given in ref. [11] have also been listed in Table 3.

With the parameter values $a = 5.66$ fm$^{-1}$ and $b = 6.09$ fm$^{-3}$ in the mass operator model (2.18) and with the parameter values $C = 28$ MeV, $C_S = 19$ MeV and $A = 0.22$ in the hyperfine term (2.21) we obtain very satisfactory energies for all the known states. The eigenvalues $\epsilon_{nK}$ determined by the first two of these values are

$$\epsilon_{00} = 1340\text{MeV}, \epsilon_{01} = 1652\text{MeV}, \epsilon_{10} = 1777\text{MeV}, \epsilon_{02} = 1867\text{MeV}. \tag{4.1}$$

The same value for $\epsilon_{00}$ obtains for the oscillator model (2.17) with $\omega = 316$ MeV. The ground state wave functions plotted in Fig. 1 have been calculated with these parameter values.

The value for $C_S$ is directly determined by the empirical mass splitting between the $\Sigma$ and $\Sigma(1385)$ hyperons (cf. Table 5 and section 5.1 below):

$$m[\Sigma(1385)] - m[\Sigma] = 10C_S, \tag{4.2}$$

The value for the parameter $C$ is then chosen so as to obtain agreement with the empirical $N\Delta$ splitting (Table 3):

$$m[\Delta(1232)] - m[N] = 12C - 2C_S. \tag{4.3}$$

Finally the value for the parameter $A$ is chosen so as to obtain agreement with the splitting between the $SD$ shell multiplets $N(1440)$ and $N(1720) - N(1680)$ multiplets.

The calculated resonance energies in Table 3 agree with the empirical pole positions of the known states to within 2% in most cases. A number of the still missing states have been identified in recent phase shift analyses. The $\frac{3}{2}^+$ member of the $L = 2$ $\Delta$ multiplet near 1750 MeV has been located in ref. [14] at 1754 MeV. In that analysis a $P_{13}$ state has also been located at 1879 MeV: this corresponds well to the $L = 0$ nucleon resonance predicted at 1881 MeV. The additional $P_{11}$ resonance predicted at 1916 MeV may be related to the additional, if somewhat lower lying $P_{11}$ resonance found in the phase shift analysis [15].
5. The Strange Hyperons

5.1 The Λ and Σ spectra

The parameter values above may be employed directly to the spectrum of the strange \( S = -1 \) hyperons. The only additional parameter required is the mass parameter \( \Delta_S \) in (2.9). With \( \Delta_S = 587 \) MeV and the confining potential (2.18) the spectrum of the \( \Lambda \)-hyperon is obtained satisfactorily as shown in Table 4. The one missing feature is the large spin-orbit splitting of the \( \Lambda(1405) - \Lambda(1520) \) multiplet, which is expected to have an anomalously large \( \bar{K}N \)-admixture [16]. The peculiar aspect of this flavor singlet multiplet is brought out by the fact that it is the only \( P \)-shell baryon multiplet for which the empirical spin-orbit splitting is not consistent with zero.

The quality of the \( \Lambda \) spectrum is similar to that of ref. [3]. The good agreement between the calculated and the empirical spectra lends credence to the quantum number and symmetry assignments made in Table 4. Note that one \( 3/2^- \) state in the \( P \) shell of the \( \Lambda \) spectrum near 1800 MeV remains to be found experimentally. This state would correspond to the \( N(1700) \) resonance in the nucleon spectrum.

The still uncertain quantum number assignments in the empirical \( \Sigma \) hyperon spectrum rule out a definite assessment of the quality of the predicted \( \Sigma \) spectrum shown in Table 5, which was obtained with the same parameter values as the \( \Lambda \) spectrum in Table 4. The energies of low lying well established positive parity \( \Sigma \) resonances \( \Sigma(1385) \) and \( \Sigma(1660) \) and the negative parity \( \Sigma(1775) \) and \( \Sigma(1915) \) resonances are however in satisfactory agreement with the empirical energies.

The calculated spectrum in Table 5 suggests that the \( \Sigma(1840) \) be the \( S = -1 \) analog of the \( \Delta(1600) \) breathing resonance. The absence of low lying \( \Sigma \) resonances in the predicted spectrum also suggests that the unassigned and unconfirmed \( \Sigma(1480) \) and \( \Sigma(1560) \) states may turn out to be spurious. The experimental information on the \( P \) shell of the \( \Sigma \) spectrum is still incomplete: both a \( 1/2^- \) and a \( 3/2^- \) state are still missing.

The interpretation of the \( \Sigma \) hyperon spectrum is complicated by the appearance of 2 negative parity resonances - the \( \Sigma(1940) \) and the \( \Sigma(2000) \) - that belong to the \( PF \) shell in the vicinity of 2 GeV. In the spectrum of the \( \Lambda \) the corresponding \( PF \) shell resonances lie at higher energies (e.g. the \( \Lambda(2100) \), \( 7/2^- \) resonance). In the nucleon spectrum the \( PF \) shell also begins at correspondingly higher excitation (e.g. the \( N(2080) \), \( 3/2^- \) resonance).

5.2 The \( \Xi \) spectrum

The spectrum of the \( S = -2 \) \( \Xi \) hyperon is given in Table 6. In the absence of empirical assignments for the excited \( \Xi \) states we have only indicated those of the well established \( \Xi \), \( \Xi(1530) \) and \( \Xi(1820) \) states. The energies of these states are in good agreement with the corresponding empirical values. The recent discovery of a state at 1690 MeV [17] agrees well with the predicted \( 1/2^+ \) state at 1695 MeV. This state is the \( S = -2 \) analog of the \( N(1440) \), \( \Lambda(1600) \) and the \( \Sigma(1660) \).

The presently known empirical \( \Xi \) spectrum contains candidates for many states of the present model in Table 6. The calculated energies are close to those predicted in ref. [3].
5.3 The $\Omega^-$ spectrum

In the present model the structure of the the $\Omega^-$ spectrum is similar to that of the $\Delta$ resonance. The spectrum in Table 7 has 3 positive parity multiplets with energies in the region 2.1 – 2.2 GeV. The empirical $\Omega^-(2250)$ very likely corresponds to one of these. The empirical $\Omega^-(2380)$ may also be a positive parity resonance in the $SD$-shell, but the $\Omega^-(2470)$ resonance is probably a negative parity resonance in the $PF$ shell.

As the mass of the $\Omega^-$ differs by 66 MeV (i.e. by $\simeq 4\%$) from its experimental value that amount should be added to the energies of the $\Omega^-$ resonances. With that shift all the excited states of the $\Omega^-$ lie above 2 GeV. This also suggests the two highest $D$-shell resonances in Table 7 may be interpreted as the $\Omega(2380)^-$ and the $\Omega(2250)^-$ respectively.

With the model (2.20)-(2.21) for the hyperfine interaction the baryon mass spectra satisfy the same relations as the model in ref. [5], e.g. the relations

$$m[\Delta(1232)] - m[N] = m[\Sigma(1385)] + \frac{3}{2}(m[\Sigma] - m[\Lambda]),$$

$$m[\Sigma(1385)] - m[\Sigma] = m[\Xi(1530)] - m[\Xi],$$

$$\frac{1}{3}(m[\Omega] - m[\Delta]) = m[\Xi(1530)] - m[\Sigma(1385)].$$

(5.1)

These relations are in good agreement with the experimental values. The Gell-Mann-Okubo and equal spacing rules are recovered in the $SU(3)_F$ symmetric limit $C = C_S$.

6. The Charm Hyperons

6.1 The $C=1$ Hyperons

6.1.1 The $\Lambda_C^+$ and $\Sigma_C$ Spectra

The hitherto observed charm hyperons all have $C = 1$, i.e. they contain one charm quark and two light quarks. While the charm hyperon spectrum in many aspects is expected to be similar to the strange hyperon spectrum, it differs from that in the much larger number of narrow states, which lie below the threshold for strong decay. In the spectrum of the $\Lambda$ there is only one state below the threshold for $\bar{K}N$ decay - the $\Lambda(1405)$. In contrast all the $7$ negative parity states in the P-shell as well as the positive parity breathing resonance of the $\Lambda_C$ are expected to lie well below the threshold for $\bar{D}N$ decay. This relative compression of the charm hyperon spectra to lower excitation energies is achieved through the flavor dependence, $\mathcal{K}$ of the mass operator $M_0$. A larger contribution $\mathcal{K}$ reduces the repulsive effect of the momentum dependent terms in $M_0$.

The spectra of the non-strange $C=1$ hyperons depend on two additional parameters: the charm quark parameter $\Delta_C$ in (2.9) and the parameter $C_C$ in the hyperfine interaction (2.20). The expressions for the energies of the states in the spectra of the $\Lambda_C^+$ and $\Sigma_C$ are listed in Tables 8 and 9 respectively.

The hyperfine interaction parameter $C_C$ - which in ref. [4] was interpreted as the matrix element of $D$ and $D^*$ meson exchange interactions - is determined by the mass difference
between the $\Sigma_C$ and the $\Sigma_C(1530)$ as (cf. Table 9):

$$m[\Sigma_C(2530)] - m[\Sigma_C] = 6C_C.$$  \hspace{1cm} (6.1)

The new value $56 \text{ MeV} \pm 3 \text{ MeV}$ \[15\] for the mass splitting between the $\Sigma_C(2530)$ and the $\Sigma_C$ then yields $C_C \simeq 10 \text{ MeV}$. With this value for $C_C$ the expression for the mass of the $\Lambda_C^+$ in Table 8 implies that the empirical value for that mass is reached with $\Delta_C = 2.2 \text{ GeV}$. We use this value here.

Given these parameter values the masses of all the other states in the spectra of the $\Lambda_C^+$ and the $\Sigma_C$ are determined. We note the agreement of the model with the average mass value of of the negative parity multiplet $\Lambda_C(2593)^+ - \Lambda_C(2625)^+$. This multiplet is the charm analog of the $\Lambda(1405) - \Lambda(1520)$ multiplet. The satisfactory simultaneous description of these two multiplets by the present model supports view that the former is a flavor singlet combination of $u$, $d$ and $c$ quarks, and the latter is a flavor singlet combination of $u$, $d$ and $s$ quarks.

At the present time no excited states of the $\Sigma_C$ beyond the $\Sigma_C(2530)$ have been found. The present prediction is that the $P-$shell states of the $\Sigma_C$ fall within the range $2.6 - 2.7 \text{ GeV}$. This is slightly higher than the corresponding predictions in \[15\]. The positive parity breathing resonances of both the $\Sigma_C$ and the $\Sigma_C(1530)$ are predicted to coincide with the lowest $P$-shell states.

6.1.2 The $\Xi_C$ Spectrum

The hyperons with $C = 1$ and $S = -1$ are denoted $\Xi_C$. In the constituent quark model these states are formed of one light, one strange and one charm quark. It is natural to divide the spectrum of the $\Xi_C$ into two separate sectors, which are respectively antisymmetric (denoted $\Xi_C^a$) and symmetric (denoted $\Xi_C^s$) under interchange of the light and the strange quark. This is in analogy with the separation of the $S = -1$ hyperon spectra into separate $\Lambda$ and $\Sigma$ sectors. The structure of the $\Xi_C^a$ spectrum should thus be similar to that of the $\Lambda$ and that of the $\Xi_C^s$ spectrum similar to that of the $\Sigma_C$ hyperon.

Both the $\Xi_C^a$ and the $\Xi_C^s$ spectra differ from those of the $\Lambda$ and the $\Sigma$ hyperons in that their entire $P$-shells are expected to lie below the threshold for the strong decay to $\bar{D}N$ (2907 MeV). Their low lying negative parity states should therefore be sharp. This expectation appears to be confirmed by the narrow width ($< 2.4 \text{ MeV}$) of the recently found $\frac{3}{2}^- \Xi_C^a$ resonance at 349 MeV excitation \[20\].

The expressions for the energies of the predicted $S$, $P$ and $SD$ shell states of the $\Xi_C$ hyperons are listed in Tables 10 and 11. These energies depend on one new parameter: $C_{SC}$, determined by the splitting between the $\Xi_C^a$ and the $\Xi_C^s$ as

$$m[\Xi_C^a] - m[\Xi_C^s] = \frac{20}{3}C_S - 2C_C - 2C_{SC}.$$  \hspace{1cm} (6.2)

With the present uncertain value of the $\Xi_C^a$ (2560 MeV) mass we have $C_{SC} = 5 \text{ MeV}$. With this value of $C_{SC}$ and those fixed previously we obtain the complete $S$, $P$ and $SD$ shell spectra of the $\Xi_C^a$ and $\Xi_C^s$. In ref. \[6\] was interpreted as the matrix element of $D_s$ and $D_s^*$ exchange interactions between the $s$ and $c$ quarks.
The $\frac{3}{2}^-$ state found in ref. [20] at an excitation energy of $349.4 \pm 0.7 \pm 1.0$ MeV above the $\Xi_0^-$ was interpreted as the strange-charm analog of the $\Lambda_C(2625)^+ \frac{3}{2}^-$ state. As in the present model the corresponding state is predicted to have the considerably lower excitation energy 269 MeV, the $\frac{3}{2}^-$ state member of the spin 3/2 negative parity multiplet is predicted to have an excitation energy of 378 MeV, we suggest that this is the more natural assignment for the $\frac{3}{2}^-$ state at 349 MeV excitation energy, than the $usc$ flavor singlet state.

6.1.3 The $\Omega_0^0$ Spectrum

The spectrum of the $\Omega_0^0$ is the only sector of the $C = 1$ hyperon spectrum, in which the $\frac{3}{2}^+$ state in the $S$-shell has yet to be found experimentally. This is expected to lie about 30 MeV above the ground state $\Omega_0^0$. In the present model the mass of the $\Omega_0^0$ is 77 MeV – i.e. by $\sim 3\%$ – smaller than the experimental value. This difference is of similar magnitude as that for the $\Omega^-$ (66 MeV), and could of course be eliminated by slightly increasing the factor multiplying the parameter $\Delta_S$ in the mass operator (2.9). While this mass difference is relatively small, it may indicate that better energies for the excited states of the $\Omega_0^0$ would obtain if an amount 77 MeV were added to the calculated energies in Table 12, where the expressions for all the states in the $S$, $P$ and $SD$ shells of the $\Omega_0^0$ are listed.

6.2 The $\Xi_{CC}$ and $\Omega_{CC}$ Spectra

The hyperons with $C = 2$ are denoted $\Xi_{CC}$. All members of this class of particles remain to be discovered. As the hyperfine interaction between the 2 charm quarks in these hyperons is expected to be of negligible magnitude compared to that between the light quark and the charm quarks, their spectrum should be easier to interpret than that of the lighter hyperons.

The present model for the hyperfine interaction (2.20), (2.19) implies that the structure of the spectrum of the $\Xi_{CC}$ hyperons should be similar to that of the $\Sigma_C$ hyperons. In the present model the hyperfine splittings of the $\Xi_{CC}$ spectrum are completely determined by the parameters $C$ and $A$ (2.20),(2.21). The masses of the $\Xi_{CC} (1/2^+)$ and the $\Xi_{CC}^* (3/2^+)$, which form the $S$ shell of the $\Xi_{CC}$ spectrum, are given as

$$m[\Xi_{CC}] = \sqrt{\epsilon_{01}^2 + 2\Delta_C^2} - 10C = 3290\text{MeV},$$

$$m[\Xi_{CC}^*] = \sqrt{\epsilon_{01}^2 + 2\Delta_C^2} - 4C = 3350\text{MeV}.$$  

(6.3)  

(6.4)

While these absolute mass values are likely to be too low by 100 – 200 MeV, the splitting (60 MeV) between the $\Xi_{CC}$ and the $\Xi_{CC}^*$ is probably a realistic prediction. The bound state version of the Skyrme model, which typically gives realistic values for the ground state hyperons [21] yields a mass of 3.5 GeV for the $\Xi_{CC}$ and value for the mass difference between the $\Xi_{CC}$ and the $\Xi_{CC}^*$ in agreement with the present one.

The baryons with $C = 2$ and $S = -1$ are denoted $\Omega_{CC}^+$. The spectrum of these particles can be determined directly from that of the $\Xi$ hyperons in Table 6. The only modification is that the constant $C_S$ in the hyperfine correction has to be replaced by the corresponding constant $C_{SC}$, which is appropriate for simultaneous charm and strangeness exchange. The
masses of the $\Omega_{CC}^+ (1/2^+)$ and the $\Omega_{CC}^{+\ast} (3/2^+)$, which form the S shell of the $\Omega_{CC}^+$ spectrum, are given as

\begin{align}
    m[\Xi_{CC}^+] &= \sqrt{\epsilon_{01}^2 + 2\Delta_C^2 + \Delta_S^2} - 10C = 3390\text{MeV}, \quad (6.5) \\
    m[\Xi_{CC}^{+\ast}] &= \sqrt{\epsilon_{01}^2 + 2\Delta_C^2 + \Delta_S^2} - 4C = 3420\text{MeV}. \quad (6.6)
\end{align}

As in the case of the $\Xi_{CC}$ we expect the absolute values to be too low by 100 – 200 MeV, but the splitting of 30 MeV between the $\Omega_{CC}^+$ and the $\Omega_{CC}^{+\ast}$ should be realistic. The Skyrme model prediction for these two states is in the region 3.7-3.8 MeV [21].

7. The Beauty Hyperons

The present model implies that the spectrum of the $B = -1$ hyperons be organized very much like that of the $C = 1$ hyperons described above in Tables 8 and 9. The expressions for the energies of the $\Lambda_B^0$ and the $\Sigma_B$ states are listed in Tables 13 and 14 respectively. These expressions are obtained replacing the hyperfine parameter $C_C$ in the energy expressions of Tables 8 and 9 throughout by the corresponding beauty quark parameter $C_B$.

The value of the parameter $C_B$ is determined by the empirical splitting between the $\Sigma_B$ and the $\Sigma_B(5870)$):

\begin{equation}
    m[\Sigma_B(5870)] - m[\Sigma_B] = 6C_B. \quad (7.1)
\end{equation}

With $m(\Sigma_B) = 5814$ MeV the value of $C_B$ is 9 MeV. The beauty quark parameter $\Delta_B$ in the mass operator $M_0$ then has to have the value 5.774 GeV in order that the $\Lambda_B^0$ take its empirical mass 5641 MeV. Given these parameter values the energies of all the excited states in the spectra of the $\Lambda_B$ and the $\Sigma_C$ are determined. These energy values are listed in Tables 13 and 14.

Heavy quark symmetry predicts that the spin of the heavy quarks decouple from the light quarks, and that therefore the baryon spectrum should fall into near degenerate multiplets with spin $S_I \pm \frac{1}{2}$, where $S_I$ is the spin of the light quarks. Heavy quark symmetry is implemented in the present model if the coefficients $C_j$ in the hyperfine interaction (2.21), which describe interactions between light and heavy quarks, vanish as the mass of the heavy quarks become infinite. The fact that the fact that the empirical splittings $m[\Sigma_C^*] - m[\Sigma_C]$ and $m[\Sigma_B^*] - m[\Sigma_B]$ are very similar indicates that the limit of heavy quark symmetry has not yet been reached in the charm and beauty sectors of the spectrum.

According to the model outlined here the spectrum of the $B = -1, S = -1 \Xi_B$ hyperons should be organized in the same manner as the corresponding spectrum of the $\Xi_C$ hyperons in Tables 10-11. The energies of the states in the $\Xi_B$ spectrum are obtained from the expressions in Tables 10-11, replacing the parameters $\Delta_C, C_C$ and $C_{SC}$ by the corresponding parameters $\Delta_B, C_B$ and $C_{SB}$. The determination of the parameter $C_{SB}$ from the mass difference

\begin{equation}
    m[\Xi_B^*] - m[\Xi_B^0] = \frac{20}{3} C_S - 2C_B - 2C_{BS}. \quad (7.2)
\end{equation}

must await the discovery of the hyperons $\Xi_B^*$ and $\Xi_B^0$.

In our model the energy of the lowest $\Xi_B$ state is

\begin{equation}
    m[\Xi_B^0] = \sqrt{\epsilon_{00}^2 + \Delta_S^2 + \Delta_B^2} - \frac{8}{3} C_S - 2C_B - 2C_{SB}. \quad (7.4)
\end{equation}
If the value of the parameter $C_{SB}$ falls in the range $0 - 5$ MeV, expected from heavy quark symmetry the mass of the $\Xi_B$ should fall in the range $5916 - 5926$ MeV. This is larger by $40 - 50$ MeV than the Skyrme model prediction [21].

8. Summary

The present formulation of Poincaré covariant quark models illustrates the possibility of satisfying all the requirements of relativistic covariance, while reproducing the good phenomenological results of conventional constituent quark models for the magnetic moments and axial couplings along with a good description of the known baryon spectrum. The emphasis in the present work is consistency with general principles and simplicity while discarding the notion of constituent quarks as a system of free particles that incidentally are confined in an infinite potential well. The key to the satisfactory description of the spectrum is a mass operator which is the sum of two commuting terms: (i) a confining mass operator with the basic point spectrum and (ii) a simple spin- and flavor dependent hyperfine term large enough to affect the level order [5, 6].

Observable form factors and transition amplitudes depend on two distinct features of a quark model that are not separately observable: (i) the wave function, which differs for different baryons and (ii) the quark-current structure that is the same for all baryons. Here we have constructed simple quark-current structures, independent of internal quark velocity, with instant form kinematics. These models satisfy all requirements of Poincaré covariance. We have shown that they are consistent with observed nucleon properties. Electromagnetic form factors and transition amplitudes of the baryons for all the states in the $S$, $P$ and $D$ shells of the baryons are well defined. A quantitative examination of these amplitudes is left to future investigations. Such investigations may indicate for configuration mixing in the wave functions.

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Table 1

Magnetic moments of the ground state baryon octet and the $\Delta^{++}$ and $\Omega^-$ (in nuclear magnetons). Column IA contains the impulse approximation expressions for the current model (3.13). Column I contains the numerical values obtained from these expressions. The empirical values ("exp") are from refs. [11, 12].

|                | IA                | exp   | I     |
|----------------|-------------------|-------|-------|
| $p$            | $\frac{m_N}{m_u}$ | +2.79 | +2.79 |
| $n$            | $-\frac{2}{3} \frac{m_N}{m_u}$ | -1.91 | -1.86 |
| $\Lambda$      | $-\frac{1}{3} \frac{m_N}{m_s}$ | -0.61 | -0.67 |
| $\Sigma^+$     | $\frac{8}{9} \frac{m_N}{m_u} + \frac{1}{9} \frac{m_N}{m_s}$ | +2.46 | +2.70 |
| $\Sigma^0$     | $\frac{2}{9} \frac{m_N}{m_u} + \frac{1}{9} \frac{m_N}{m_s}$ | ?     | +0.84 |
| $\Sigma^0 \rightarrow \Lambda$ | $-\frac{1}{\sqrt{3}} \frac{m_N}{m_u}$ | [1.61] | -1.61 |
| $\Sigma^-$     | $-\frac{4}{9} \frac{m_N}{m_u} + \frac{1}{9} \frac{m_N}{m_s}$ | -1.16 | -1.07 |
| $\Xi^0$        | $-\frac{2}{9} \frac{m_N}{m_u} - \frac{4}{9} \frac{m_N}{m_s}$ | -1.25 | -1.52 |
| $\Xi^-$        | $\frac{1}{9} \frac{m_N}{m_u} - \frac{4}{9} \frac{m_N}{m_s}$ | -0.65 | -0.59 |
| $\Delta^{++}$  | $\frac{2}{3} \frac{m_N}{m_u}$ | +4.52 | 5.58  |
| $\Delta^+ \rightarrow p$ | $\frac{2\sqrt{2}}{3} \frac{m_N}{m_u}$ | +3.1  | +2.63 |
| $\Omega^-$     | $-\frac{m_N}{m_s}$ | -2.019 | -2.019 |
Table 2

The axial coupling constants of the baryon octet. Column I gives the expressions in terms of the F and D coefficients and column II the predictions for the current operator (3.23) (with $g_A^q = 0.76$). The empirical values are from refs. \[11, 18\].

|               | I   | exp | II  |
|---------------|-----|-----|-----|
| $n \to p$     | $F + D$ | 1.26 | 1.26 |
| $\Sigma^\pm \to \Lambda$ | $\sqrt{2/3} D$ | 0.62 | 0.62 |
| $\Sigma^- \to \Sigma^0$ | $\sqrt{2} F$ | 0.67 | 0.71 |
| $\Lambda \to p$ | $-\sqrt{3}/2 (F + D/3)$ | 0.92 | 0.96 |
| $\Sigma^- \to n$ | $-(F - D)$ | 0.34 | 0.25 |
| $\Xi^- \to \Lambda$ | $\sqrt{3}/2 (F - D/3)$ | 0.31 | 0.30 |
| $\Xi^- \to \Sigma^0$ | $1/\sqrt{2} (F + D)$ | 1.36 | 0.90 |
| $\Xi^0 \to \Sigma^+$ | $F + D$ | ? | 1.26 |
| $\Xi^- \to \Xi^0$ | $F - D$ | -0.28 | -0.25 |
The nucleon and $\Delta$-states in the $S$, $P$ and $SD$ shells. The column $\epsilon$ contains the eigenvalues of the mass operator. The average over the multiplet of the real part of the empirically extracted resonance pole position is denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| nKL| f | LS Multiplet | EXP (model value) | $\epsilon$ |
|-----|-----|---------------|-------------------|------------|
| 000| [21]| $\frac{1}{2}^+, N$ | 939 | $\epsilon_{00} - 15C + C_S$ |
| 000| [3]| $\frac{3}{2}^+, \Delta$ | 1209-1211 | $\epsilon_{00} - 3C - C_S$ |
| 000| [3]| $\frac{1}{2}^+, N(1440)$ | 1346-1385 | $\epsilon_{10} - 15C + C_S$ |
| 011| [21]| $\frac{1}{2}^-, N(1535); \frac{3}{2}^-, N(1520)$ | 1496-1527 | $\epsilon_{01} - 3C + C_S$ |
| 010| [3]| $\frac{3}{2}^-, \Delta(1600)$ | 1541-1675 | $-A(9C + C_S)$ |
| 011| [3]| $\frac{1}{2}^-, \Delta(1620); \frac{3}{2}^-, \Delta(1700)$ | 1575-1700 | $\epsilon_{10} - 3C - C_S$ |
| 011| [3]| $\frac{1}{2}^-, N(1650); \frac{3}{2}^+, N(1700)$ | 1648-1710 | $\epsilon_{01} + 3C + C_S$ |
| 022| [3]| $\frac{1}{2}^+, \Delta(1750); \frac{3}{2}^+, \Delta(1754)$ | 1710-1780 | $-A(9C - C_S)$ |
| 022| [3]| $\frac{3}{2}^+, \Delta(1754); \frac{5}{2}^+, \Delta(?)$ | 1832 | $\epsilon_{02} - 3C - C_S$ |
| 022| [3]| $\frac{5}{2}^+, N(1720); \frac{7}{2}^+, N(1680)$ | 1656-1748 | $+3A(3C + C_S)$ |
| 020| [3]| $\frac{1}{2}^+, N(1710)$ | 1636-1770 | $\epsilon_{02} - 15C + C_S$ |
| 020| [3]| $\frac{3}{2}^+, N(1879)$ | ? | $+3A(15C - C_S)$ |
| 022| [3]| $\frac{3}{2}^+, N(1900); \frac{5}{2}^+, N(2000)$ | 1879-2175 | $\epsilon_{02} - 3C + C_S$ |
| 022| [3]| $\frac{5}{2}^+, N(?); \frac{7}{2}^+, N(2000)$ | 1920-2114 | $-A(9C + C_S)$ |
| 022| [3]| $\frac{1}{2}^+, \Delta(1910)$ | 1792-1950 | $\epsilon_{02} - 3C - C_S$ |
| 022| [3]| $\frac{3}{2}^+, \Delta(1920); \frac{5}{2}^+, \Delta(1905)$ | 1794-1870 | $-A(9C + 3C_S)$ |
Table 4

The $S$, $P$ and $SD$ shell states in the $\Lambda$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplets of of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| nKL[f]_FS[f]_F[f]_S | LS Multiplet | EXP (model value) | $\epsilon\sqrt{\frac{\Delta^2_{nK}}{\epsilon_{nK}^2}} - c$ |
|-----------------------|--------------|-------------------|---------------|
| 000[3]_FS[21]_F[21]_S | $\frac{1}{2}^+, \Lambda$ | 1116 (1116) | $\hat{\epsilon}_{00} - 9C - 5C_S$ |
| 011[21]_FS[111]_F[21]_S | $\frac{1}{2}^-, \Lambda(1405)$ | 1405-1520 (1535) | $\hat{\epsilon}_{01} - 3C - 5C_S$ |
| | $\frac{3}{2}^-, \Lambda(1520)$ | 1560-1700 (1524) | $-A(3C + 5C_S)$ |
| 100[3]_FS[21]_F[21]_S | $\frac{1}{2}^+, \Lambda(1600)$ | 1600-1695 (1640) | $\hat{\epsilon}_{10} - 9C - 5C_S$ |
| 011[21]_FS[21]_F[21]_S | $\frac{1}{2}^-, \Lambda(1670)$ | 1670-1720 (1773) | $\hat{\epsilon}_{01} + 3C - 5C_S$ |
| | $\frac{3}{2}^-, \Lambda(1800); \frac{3}{2}^-, \Lambda(?)$ | 1720-1830 (1739) | $-2A(3C + 5C_S)$ |
| 020[21]_FS[111]_F[21]_S | $\frac{1}{2}^+, \Lambda(1810)$ | 1750-1850 (1839) | $\hat{\epsilon}_{02} - 3C - 5C_S$ |
| 022[3]_FS[21]_F[21]_S | $\frac{3}{2}^+, \Lambda(1890)$ | 1815-1910 (1839) | $\hat{\epsilon}_{02} - 9C - 5C_S$ |
| | $\frac{1}{2}^+, \Lambda(1820)$ | 1890-1960 (1890) | $+3A(9C + 5C_S)$ |
| 020[21]_FS[21]_F[3]_S | $\frac{1}{2}^+, \Lambda(?)$ | ? (1844) | $\hat{\epsilon}_{02} - 3C + 5C_S$ |
| 020[21]_FS[21]_F[21]_S | $\frac{3}{2}^+, \Lambda(?)$ | ? (1977) | $\hat{\epsilon}_{02} + 3C - 5C_S$ |
| 02[21]_FS[21]_F[3]_S | $\frac{1}{2}^+, \Lambda(?)$; $\frac{3}{2}^+, \Lambda(?)$ | ? (1977) | $\hat{\epsilon}_{02} + 3C - 5C_S$ |
| | $\frac{3}{2}^+, \Lambda(?)$; $\frac{5}{2}^+, \Lambda(2110?)$ | ? (2001) | $\hat{\epsilon}_{02} - 3C + 5C_S$ |
| 02[21]_FS[21]_F[21]_S | $\frac{3}{2}^+, \Lambda(?)$; $\frac{5}{2}^+, \Lambda(2110?)$ | 2090-2140 (1916) | $\hat{\epsilon}_{02} - 3C + 5C_S$ |
| | $\frac{3}{2}^+, \Lambda(?)$; $\frac{5}{2}^+, \Lambda(2110?)$ | 2090-2140 (1959) | $+\frac{3}{2}(21C + 35C_S)$ |
Table 5

The $S$, $P$ and $SD$ shell states in the $\Sigma$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplet of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones. The unconfirmed unassigned low lying $\Sigma(1450)$ and $\Sigma(1560)$ resonances still included in ref. [11] have been omitted from this table.

| \(nK\ell[f]_{FS}[f]_{F}[f]_{S}\) | LS Multiplet | EXP (model value) | $\epsilon$ |
|----------------------------------|--------------|------------------|------------|
| 000[3]_{FS}[21]_{F}[21]_{S}      | \(\frac{1}{2}^+, \Sigma\) | 1189-1192 (1188) | $\hat{\epsilon}_{00} - C - 13C_S$ |
| 000[3]_{FS}[3]_{F}[3]_{S}        | \(\frac{3}{2}^+, \Sigma(1385)\) | 1379 (1378) | $\hat{\epsilon}_{00} - C - 3C_S$ |
| 100[3]_{FS}[21]_{F}[21]_{S}      | \(\frac{1}{2}^+, \Sigma(1660)\) | 1660-1690 (1596) | $\hat{\epsilon}_{10} - C - 13C_S$ |
| 011[21]_{FS}[21]_{F}[21]_{S}     | \(\frac{1}{2}^-, \Sigma(1620)\); \(\frac{3}{2}^-\), $\Sigma(1580)$ | 1580-1640 (1676) | $\hat{\epsilon}_{01} + C - 3C_S$ |
| 100[3]_{FS}[3]_{F}[3]_{S}        | \(\frac{3}{2}^+, \Sigma(1840)\) | 1720-1925 (1786) | $\hat{\epsilon}_{10} - C - 3C_S$ |
| 011[21]_{FS}[3]_{F}[21]_{S}      | \(\frac{1}{2}^-, \Sigma(1750)\); \(\frac{3}{2}^-\), $\Sigma(?)$ | 1730-1780 (1749) | $\hat{\epsilon}_{01} - C + 3C_S$ |
| 022[3]_{FS}[3]_{F}[3]_{S}        | \(\frac{1}{2}^+, \Sigma(1775)\); \(\frac{3}{2}^+\), $\Sigma(?)$ | 1740-1790 (1928) | $\hat{\epsilon}_{02} - C - 3C_S$ |
| 022[3]_{FS}[21]_{F}[21]_{S}      | \(\frac{1}{2}^+, \Sigma(1840)\); \(\frac{5}{2}^+\), $\Sigma(1915)$ | 1720-1937 (1864) | $\hat{\epsilon}_{02} - C - 13C_S$ |
| 020[21]_{FS}[21]_{F}[21]_{S}     | \(\frac{1}{2}^+, \Sigma(1880)\) | 1826-1885 (1880) | $\hat{\epsilon}_{02} + C - 3C_S$ |
| 020[21]_{FS}[3]_{F}[3]_{S}       | \(\frac{3}{2}^+, \Sigma(?)\) | ? (1953) | $\hat{\epsilon}_{02} + C + 3C_S$ |
| 022[21]_{FS}[21]_{F}[21]_{S}     | \(\frac{3}{2}^+, \Sigma(?)\); \(\frac{5}{2}^+\), $\Sigma(?)$ | ? (1971) | $\hat{\epsilon}_{02} + C - 3C_S$ |
| 022[21]_{FS}[21]_{F}[3]_{S}      | \(\frac{1}{2}^+, \Sigma(?)\); \(\frac{3}{2}^+\), $\Sigma(?)$ | 2025-2040 (1984) | $\hat{\epsilon}_{02} - C + 3C_S$ |
| 020[21]_{FS}[3]_{F}[21]_{S}      | \(\frac{1}{2}^+, \Sigma(?)\) | ? (1986) | $\hat{\epsilon}_{02} + C + 3C_S$ |
| 022[21]_{FS}[3]_{F}[21]_{S}      | \(\frac{3}{2}^+, \Sigma(2080)\); \(\frac{5}{2}^+\), $\Sigma(2070)$ | 2051-2091 (2014) | $\hat{\epsilon}_{02} + C - 3C_S$ |
Table 6

The $S$, $P$ and $SD$ shell states in the $\Xi$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplets of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| $nK\ell[f]_{FS}[f]_{F}[f]_{S}$ | $LS$ Multiplet | EXP (model value) | $\sqrt{\epsilon_{nK}^2 + 2\Delta_S^2} - c$ |
|-------------------------------|----------------|------------------|--------------------------------------|
| $000[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+$, $\Xi$ | 1314-1321         | $\epsilon_{00} - 14C_S$             |
|                                |                 | (1310)           |                                      |
| $000[3]_{FS}[3]_{F}[3]_{S}$   | $\frac{3}{2}^+$, $\Xi$(1530) | 1532-1534       | $\epsilon_{00} - 4C_S$             |
|                                |                 | (1500)           |                                      |
| $100[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+$, $\Xi$(1690?) | 1690            | $\epsilon_{10} - 14C_S$           |
|                                |                 | (1695)           |                                      |
| $011[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^-$, $\Xi$(?); $\frac{3}{2}^-$, $\Xi$(1820) | ?               | $\epsilon_{01} - 2C_S$           |
|                                |                 | (1769)           | $-10AC_S$                             |
|                                |                 | (1885)           | $\epsilon_{10} - 4C_S$             |
| $011[21]_{FS}[3]_{F}[21]_{S}$ | $\frac{3}{2}^-$, $\Xi$(?) | ?               | $\epsilon_{01} + 4C_S$           |
|                                |                 | (1875)           | $-12AC_S$                             |
| $011[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{1}{2}^-$, $\Xi$(?); $\frac{3}{2}^-$, $\Xi$(?) | ?               | $\epsilon_{01} + 2C_S$           |
|                                |                 | (1853)           | $-8AC_S$                              |
| $022[3]_{FS}[3]_{F}[3]_{S}$   | $\frac{1}{2}^+$, $\Xi$(?); $\frac{3}{2}^+$, $\Xi$(?) | ?               | $\epsilon_{02} - 4C_S$           |
|                                |                 | (1953)           | $+12AC_S$                             |
| $022[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+$, $\Xi$(?); $\frac{5}{2}^+$, $\Xi$(?) | ?               | $\epsilon_{02} - 14C_S$           |
|                                |                 | (2017)           | $+42AC_S$                             |
| $020[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+$, $\Xi$(?) | ?               | $\epsilon_{02} - 2C_S$           |
|                                |                 | (1963)           | $-10AC_S$                             |
| $020[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+$, $\Xi$(?) | ?               | $\epsilon_{02} + 2C_S$           |
|                                |                 | (2048)           | $-8AC_S$                              |
| $022[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+$, $\Xi$(?); $\frac{5}{2}^+$, $\Xi$(?) | ?               | $\epsilon_{02} - 2C_S$           |
|                                |                 | (2051)           | $+11AC_S$                             |
| $022[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+$, $\Xi$(?); $\frac{5}{2}^+$, $\Xi$(?) | ?               | $\epsilon_{02} + 2C_S$           |
|                                |                 | (2073)           | $-2AC_S$                              |
| $020[21]_{FS}[3]_{F}[21]_{S}$ | $\frac{1}{2}^+$, $\Xi$(?) | ?               | $\epsilon_{02} + 4C_S$           |
|                                |                 | (2069)           | $-12AC_S$                             |
| $022[21]_{FS}[3]_{F}[21]_{S}$ | $\frac{3}{2}^+$, $\Xi$(?); $\frac{5}{2}^+$, $\Xi$(?) | ?               | $\epsilon_{02} + 4C_S$           |
|                                |                 | (2094)           | $-6AC_S$                              |
Table 7

The $S$, $P$ and $SD$ shell states in the $\Omega^-$ hyperon spectrum. The column $\hat{\epsilon}$ contains the eigenvalues of the mass operator. The averages over the multiplets of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| $nKL[f]_{FS}[f]_{F}[f]_{S}$ | $LS$ Multiplet | EXP (model value) | $\epsilon$ \(\sqrt{\epsilon_{nK}^2 + 3\Delta_S^2} - c\) |
|-----------------------------|----------------|-------------------|----------------------------------|
| 000[3]$_{FS}[3]_{F}[3]_{S}$ | \(\frac{3}{2}^+; \Omega^-\) | 1672 (1606) | \(\hat{\epsilon}_{00} - 4C_S\) |
| 100[3]$_{FS}[3]_{F}[3]_{S}$ | \(\frac{3}{2}^+; \Omega^-(?\) | ? (1971) | \(\hat{\epsilon}_{10} - 4C_S\) |
| 011[21]$_{FS}[3]_{F}[21]_{S}$ | \(\frac{1}{2}^-; \Omega^-(?\); \(\frac{3}{2}^-; \Omega^-(?\) | ? (1953) | \(\hat{\epsilon}_{01} + 4C_S\) |
| 022[3]$_{FS}[3]_{F}[3]_{S}$ | \(\frac{1}{2}^+; \Omega^-(?\); \(\frac{3}{2}^+; \Omega^-(?\) | ? (2100) | \(\hat{\epsilon}_{02} - 4C_S\) |
| 020[21]$_{FS}[3]_{F}[21]_{S}$ | \(\frac{1}{2}^+; \Omega^-(?\) | ? (2262) | \(\hat{\epsilon}_{02} + 4C_S\) |
| 022[21]$_{FS}[3]_{F}[21]_{S}$ | \(\frac{5}{2}^+; \Omega^-(?\); \(\frac{3}{2}^+; \Omega^-(?\) | ? (2177) | \(\hat{\epsilon}_{02} + 6AC_S\) |
The $S$, $P$ and $SD$ shell states in the $\Lambda_C^+$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplets of of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| $nKL[f]_{FS}[f]_{F}[f]_{S}$ | $LS$ Multiplet | EXP (model value) | $\epsilon$ |
|---|---|---|---|
| 000$[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Lambda_C^+$ | 2285 | $\sqrt{\epsilon_{00}^2 + \Delta_C^2} - c$ |
| 011$[21]_{FS}[111]_{F}[21]_{S}$ | $\frac{1}{2}^-, \Lambda_C(2593)^+$ | 2593-2625 | $\epsilon_{01} - 3C + \frac{1}{3}C_S - 4C_C$ |
| 010$[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Lambda_C(2625)^+$ | (2610) | $-A(3C - \frac{1}{3}C_S + 4C_C)$ |
| 011$[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^-, \Lambda_C(?)$ | (2537) | $\epsilon_{10} - 9C + C_S - 6C_C$ |
| 011$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{1}{2}^-, \Lambda_C(?)$ | (2656) | $-A(3C - \frac{1}{3}C_S + 10C_C)$ |
| 011$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{1}{2}^-, \Lambda_C(?)$ | (2656) | $-A(3C - \frac{1}{3}C_S - 2C_C)$ |
| 002$[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Lambda_C(?)$ | (2787) | $\epsilon_{02} - 9C + C_S - 6C_S$ |
| 020$[21]_{FS}[111]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Lambda_C(?)$ | (2743) | $+A(27C - 3C_S + 18C_C)$ |
| 020$[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Lambda_C(?)$ | (2790) | $-A(3C - \frac{1}{3}C_S + 10C_C)$ |
| 020$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+, \Lambda_C(?)$ | (2906) | $\epsilon_{02} + 3C - \frac{1}{3}C_S - 2C_C$ |
| 022$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{1}{2}^+, \Lambda_C(?)$ | (2906) | $-A(6C - \frac{2}{3}C_S + 2C_C)$ |
| 022$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+, \Lambda_C(?)$ | (2926) | $\epsilon_{02} + 3C - \frac{1}{3}C_S - 2C_C$ |
| 022$[21]_{FS}[111]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Lambda_C(?)$ | (2958) | $-A(6C - \frac{2}{3}C_S - 7C_C)$ |
| 022$[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Lambda_C(?)$ | (2885) | $\epsilon_{02} - 3C + \frac{1}{3}C_S + 2C_C$ |
| 022$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{5}{2}^+, \Lambda_C(?)$ | (2885) | $+A(\frac{21}{2}C - \frac{2}{3}C_S - C_C)$ |
Table 9

The $S$, $P$ and $SD$ shell states in the $\Sigma_C$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplet of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| $nKL[f]_{FS}[f]_{F}S[f]_{S}$ | $LS$ Multiplet | EXP (model value) | $\epsilon$ |
|-------------------------------|----------------|-------------------|-----------|
| $000[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Sigma_C$ | 2452-2455         | $\hat{\epsilon}_{00} - C - \frac{1}{3}C_S - 10C_C$ |
| $000[3]_{FS}[3]_{F}[3]_{S}$   | $\frac{3}{2}^+, \Sigma_C(2530)$ | 2518              | $\hat{\epsilon}_{00} - C - \frac{1}{3}C_S - 4C_C$ |
| $100[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{10} - C - \frac{1}{3}C_S - 10C_C$ |
| $011[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^-, \Sigma_C(?)$, $\frac{3}{2}^-, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{01} + C + \frac{1}{3}C_S - 2C_C$ |
| $100[3]_{FS}[3]_{F}[3]_{S}$   | $\frac{3}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{10} - C - \frac{1}{3}C_S - 4C_C$ |
| $011[21]_{FS}[3]_{F}[21]_{S}$ | $\frac{1}{2}^-, \Sigma_C(?)$, $\frac{3}{2}^-, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{01} + C + \frac{1}{3}C_S + 4C_C$ |
| $011[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{1}{2}^-, \Sigma_C(?)$, $\frac{3}{2}^-, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{01} - C - \frac{1}{3}C_S + 2C_C$ |
| $002[3]_{FS}[3]_{F}[3]_{S}$   | $\frac{3}{2}^+, \Sigma_C(?)$, $\frac{5}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{02} - C - \frac{1}{3}C_S - 10C_C$ |
| $002[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Sigma_C(?)$, $\frac{5}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{02} - C - \frac{1}{3}C_S - 4C_C$ |
| $020[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{02} - C - \frac{1}{3}C_S + 2C_C$ |
| $020[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{02} - C - \frac{1}{3}C_S + 2C_C$ |
| $022[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Sigma_C(?)$, $\frac{5}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{02} + C + \frac{1}{3}C_S - 2C_C$ |
| $022[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+, \Sigma_C(?)$, $\frac{5}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{02} - C - \frac{1}{3}C_S + 2C_C$ |
| $020[21]_{FS}[3]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{02} + C + \frac{1}{3}C_S + 4C_C$ |
| $022[21]_{FS}[3]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Sigma_C(?)$, $\frac{5}{2}^+, \Sigma_C(?)$ | ?                 | $\hat{\epsilon}_{02} + C + \frac{1}{3}C_S + 4C_C$ |
The $S$, $P$ and $SD$ shell states in the $\Xi^0_C$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplets of of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

Table 10

| $nKL[f]_{FS}[f]_F[f]_S$ | LS Multiplet | EXP (model value) | $\epsilon$ |
|--------------------------|--------------|------------------|------------|
| 000[3]_{FS}[21]_{F}[21]_S | $\frac{1}{2}^+; \Xi^0_C$ | 2465-2470 | $\hat{\epsilon}_{00} - 8C_S - 3C_C - 3C_{SC}$ |
| 011[21]_{FS}[111]_{F}[21]_S | $\frac{1}{2}^+; \Xi^0_C(?)$ | ? | $\hat{\epsilon}_{01} - \frac{8}{3}C_S - 2C_C - 2C_{SC}$ |
| 100[3]_{FS}[21]_{F}[21]_S | $\frac{3}{2}^-; \Xi^0_C(?)$ | (2716) | $-A(\frac{8}{3}C_S + 2C_C + 2C_{SC})$ |
| 011[21]_{FS}[21]_{F}[21]_S | $\frac{1}{2}^-; \Xi^0_C(?)$ | ? | $\hat{\epsilon}_{10} - 8C_S - 3C_C - 3C_{SC}$ |
| 011[21]_{FS}[21]_{F}[3]_S | $\frac{5}{2}^-; \Xi^0_C(?)$ | (2716) | $-A(\frac{8}{3}C_S + 5C_C + 5C_{SC})$ |
| 020[11]_{FS}[111]_{F}[21]_S | $\frac{5}{2}^+; \Xi^0_C(?)$ | (2814-2819) | $\hat{\epsilon}_{02} - \frac{8}{3}C_S - 2C_C - 2C_{SC}$ |
| 022[3]_{FS}[21]_{F}[21]_S | $\frac{3}{2}^+; \Xi^0_C(?)$ | ? | $\hat{\epsilon}_{02} - 8C_S - 3C_C - 3C_{SC}$ |
| 020[21]_{FS}[21]_{F}[21]_S | $\frac{5}{2}^+; \Xi^0_C(?)$ | (2879) | $+3A(8C_S + 3C_C + 3C_{SC})$ |
| 020[21]_{FS}[21]_{F}[3]_S | $\frac{3}{2}^-; \Xi^0_C(?)$ | (2882) | $\hat{\epsilon}_{02} - \frac{8}{3}C_S - 2C_C - 2C_{SC}$ |
| 022[21]_{FS}[21]_{F}[3]_S | $\frac{3}{2}^+; \Xi^0_C(?)$ | (2976) | $\hat{\epsilon}_{02} - \frac{8}{3}C_S - 2C_C - 2C_{SC}$ |
| 022[21]_{FS}[21]_{F}[3]_S | $\frac{5}{2}^+; \Xi^0_C(?)$ | (2970) | $\hat{\epsilon}_{02} - \frac{8}{3}C_S - 7C_C - \frac{7}{2}C_{SC}$ |
| 022[21]_{FS}[111]_{F}[21]_S | $\frac{3}{2}^+; \Xi^0_C(?)$ | (2927) | $\hat{\epsilon}_{02} - \frac{8}{3}C_S - 2C_C - 2C_{SC}$ |
| 022[21]_{FS}[21]_{F}[21]_S | $\frac{5}{2}^+; \Xi^0_C(?)$ | (2987) | $\hat{\epsilon}_{02} - \frac{8}{3}C_S - 7C_C - \frac{7}{2}C_{SC}$ |
| 022[21]_{FS}[21]_{F}[21]_S | $\frac{5}{2}^+; \Xi^0_C(?)$ | (2987) | $\hat{\epsilon}_{02} - \frac{8}{3}C_S - 7C_C - \frac{7}{2}C_{SC}$ |
Table 11

The $S$, $P$ and $SD$ shell states in the $\Xi^s$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplet of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| $nKL[f]_{FS}[f]_{F}[f]_S$ | LS Multiplet | EXP (model value) | $\epsilon$ |
|--------------------------|-------------|------------------|------------|
| 000$[3]_{FS}[21]_{F}[21]_S$ | $\frac{1}{2}^+; \Xi^s_C$ | 2560 (2543) | $\epsilon_00 - \frac{4}{3}C_S - 5C_C - 5C_{SC}$ |
| 000$[3]_{FS}[3]_{F}[3]_S$ | $\frac{3}{2}^+; \Xi^s_C$ | 2642 (2589) | $\epsilon_00 - \frac{4}{3}C_S - 2C_C - 2C_{SC}$ |
| 100$[3]_{FS}[21]_{F}[21]_S$ | $\frac{1}{2}^+; \Xi^s_C (?)$ | ? (2798) | $\epsilon_{10} - \frac{4}{3}C_S - 5C_C - 5C_{SC}$ |
| 011$[21]_{FS}[21]_{F}[21]_S$ | $\frac{1}{2}^+; \Xi^s_C (?)$; $\frac{3}{2}^-; \Xi^s_C (?)$ | ? (2798) | $\epsilon_{01} + \frac{4}{3}C_S - C_C - C_{SC}$ |
| 100$[3]_{FS}[3]_{F}[3]_S$ | $\frac{3}{2}^+; \Xi^s_C (?)$ | ? (2835) | $\epsilon_{10} - \frac{4}{3}C_S - 2C_S - 2C_{SC}$ |
| 011$[21]_{FS}[3]_{F}[21]_S$ | $\frac{1}{2}^-; \Xi^s_C (?)$; $\frac{3}{2}^-; \Xi^s_C (?)$ | ? (2833) | $\epsilon_{01} + \frac{4}{3}C_S + 2C_C + 2C_{SC}$ |
| 011$[21]_{FS}[21]_{F}[3]_S$ | $\frac{1}{2}^-; \Xi^s_C (?)$; $\frac{3}{2}^-; \Xi^s_C (?)$ | ? (2795) | $\epsilon_{01} - \frac{4}{3}C_S + C_C + C_{SC}$ |
| 022$[3]_{FS}[3]_{F}[3]_S$ | $\frac{1}{2}^+; \Xi^s_C (?)$; $\frac{3}{2}^+; \Xi^s_C (?)$ | ? (2927) | $\epsilon_{02} + \frac{4}{3}C_S - 2C_C - 2C_{SC}$ |
| 022$[3]_{FS}[21]_{F}[21]_S$ | $\frac{1}{2}^+; \Xi^s_C (?)$; $\frac{3}{2}^+; \Xi^s_C (?)$ | ? (2912) | $\epsilon_{02} - \frac{4}{3}C_S - 5C_C - 5C_{SC}$ |
| 020$[21]_{FS}[21]_{F}[21]_S$ | $\frac{1}{2}^+; \Xi^s_C (?)$ | ? (2900) | $\epsilon_{02} + \frac{4}{3}C_S + 3C_C + 3C_{SC}$ |
| 020$[21]_{FS}[21]_{F}[3]_S$ | $\frac{3}{2}^+; \Xi^s_C (?)$ | ? (2926) | $\epsilon_{02} - \frac{4}{3}C_S + C_C + C_{SC}$ |
| 022$[21]_{FS}[21]_{F}[21]_S$ | $\frac{3}{2}^+; \Xi^s_C (?)$; $\frac{5}{2}^+; \Xi^s_C (?)$ | ? (2963) | $\epsilon_{02} + \frac{4}{3}C_S - C_S - C_{SC}$ |
| 022$[21]_{FS}[21]_{F}[3]_S$ | $\frac{1}{2}^+; \Xi^s_C (?)$; $\frac{3}{2}^+; \Xi^s_C (?)$ | ? (2963) | $\epsilon_{02} - \frac{4}{3}C_S - 2C_C - 2C_{SC}$ |
| 022$[21]_{FS}[21]_{F}[3]_S$ | $\frac{3}{2}^+; \Xi^s_C (?)$; $\frac{5}{2}^+; \Xi^s_C (?)$ | ? (2902) | $\epsilon_{02} + \frac{4}{3}C_S - 2C_C - 2C_{SC}$ |
| 022$[21]_{FS}[21]_{F}[3]_S$ | $\frac{1}{2}^+; \Xi^s_C (?)$ | ? (2952) | $\epsilon_{02} + \frac{4}{3}C_S + 2C_C + 2C_{SC}$ |
| 022$[21]_{FS}[3]_{F}[21]_S$ | $\frac{3}{2}^+; \Xi^s_C (?)$; $\frac{5}{2}^+; \Xi^s_C (?)$ | ? (2989) | $\epsilon_{02} + \frac{4}{3}C_S + 2C_C + 2C_{SC}$ |
Table 12

The $S$, $P$ and $SD$ shell states in the $\Omega^+_C$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplets of the of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| $nK\ell[f]_FS[f]_F[f]_S$ | LS Multiplet | EXP (model value) | $\epsilon$ |
|---------------------------|--------------|------------------|-----------|
| 000[3]_FS[21]_F[21]_S   | $\frac{1}{2}^+, \Omega^+_C$ | 2710             | $\hat{\epsilon}_{00} - \frac{4}{3}C_S - 10C_{SC}$ |
| 000[3]_FS[3]_F[3]_S     | $\frac{3}{2}^+, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{00} - \frac{4}{3}C_S - 4C_{SC}$ |
| 100[3]_FS[21]_F[21]_S   | $\frac{1}{2}^+, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{10} - \frac{4}{3}C_S - 10C_{SC}$ |
| 011[21]_FS[21]_F[21]_S  | $\frac{1}{2}^-, \Omega^+_C(?)$, $\frac{3}{2}^-, \Omega^+_C$ | ?                | $\hat{\epsilon}_{01} + \frac{4}{3}C_S - 2C_{SC}$ |
| 100[3]_FS[3]_F[3]_S     | $\frac{3}{2}^+, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{10} - \frac{4}{3}C_S - 4C_{SC}$ |
| 011[21]_FS[3]_F[21]_S   | $\frac{1}{2}^-, \Omega^+_C$, $\frac{3}{2}^-, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{01} + \frac{4}{3}C_S + 4C_{SC}$ |
| 011[21]_FS[21]_F[3]_S   | $\frac{1}{2}^-, \Omega^+_C(?)$, $\frac{3}{2}^-, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{01} - \frac{4}{3}C_S + 2C_{SC}$ |
| 022[3]_FS[3]_F[3]_S     | $\frac{1}{2}^+, \Omega^+_C(?)$, $\frac{3}{2}^+, \Omega^+_C$ | ?                | $\hat{\epsilon}_{02} - \frac{4}{3}C_S - 4C_{SC}$ |
| 022[3]_FS[21]_F[21]_S   | $\frac{3}{2}^+, \Omega^+_C(?)$, $\frac{5}{2}^+, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{02} + \frac{4}{3}C_S - 2C_{SC}$ |
| 020[21]_FS[21]_F[21]_S  | $\frac{3}{2}^+, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{02} - \frac{4}{3}C_S + 2C_{SC}$ |
| 020[21]_FS[21]_F[3]_S   | $\frac{3}{2}^+, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{02} - \frac{4}{3}C_S + 2C_{SC}$ |
| 022[21]_FS[21]_F[21]_S  | $\frac{3}{2}^+, \Omega^+_C(?)$, $\frac{5}{2}^+, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{02} + \frac{4}{3}C_S - 2C_{SC}$ |
| 022[21]_FS[21]_F[3]_S   | $\frac{3}{2}^+, \Omega^+_C(?)$, $\frac{5}{2}^+, \Omega^+_C(?)$ | ?                | $\hat{\epsilon}_{02} - \frac{4}{3}C_S + 2C_{SC}$ |

Note: $\Omega_C^+$ and $\Omega_C^-$ states are listed in Table 12 for the $S$, $P$ and $SD$ shell states in the $\Omega^+_C$ hyperon spectrum. The table contains the model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20). The empirical values are denoted EXP (model value). The eigenvalues $\epsilon$ are calculated using the equation $\sqrt{\epsilon_{\text{model}}} + 2\Delta_s^3 + \Delta_C^3 - c$. The values are compared with the empirical values listed in Table 12.
The $S$, $P$ and $SD$ shell states in the $Λ_b^0$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplets of of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

Table 13

The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplets of empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| $nKL[f]_{FS}[f]_{F}[f]_{S}$ | $LS$ Multiplet | EXP (model value) | $\epsilon$ |
|-----------------------------|-----------------|-------------------|------------|
|                             |                 |                   | $\sqrt{\epsilon^2_{\pi K} + \Delta^2_B} - \epsilon$ |
| 000$[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+$, $Λ_b^0$ | 5641 (5641) | $\hat{\epsilon}_{00} - 9C + C_S - 6C_B$ |
| 011$[21]_{FS}[111]_{F}[21]_{S}$ | $\frac{3}{2}^-$, $Λ_b^0(?)$ | ? (5867) | $\hat{\epsilon}_{01} - 3C + \frac{1}{3}C_S - 4C_B$ |
| 100$[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+$, $Λ_b^0(?)$ | ? (5754) | $\hat{\epsilon}_{10} - 9C + C_S - 6C_B$ |
| 011$[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^-$, $Λ_b^0(?)$ | ? (5909) | $\hat{\epsilon}_{01} + 3C - \frac{1}{3}C_S + 10C_B$ |
| 011$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^-$, $Λ_b^0(?)$; $\frac{3}{2}^-$, $Λ_b^0(?)$ | ? (6027) | $\hat{\epsilon}_{02} - 3C + \frac{1}{3}C_S - 2C_B$ |
| 020$[3]_{FS}[111]_{F}[21]_{S}$ | $\frac{3}{2}^+$, $Λ_b^0(?)$ | ? (5930) | $\hat{\epsilon}_{02} - 9C + C_S - 6C_B$ |
| 022$[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+$, $Λ_b^0(?)$ | ? (5971) | $\hat{\epsilon}_{02} - 3C + \frac{1}{3}C_S + 2C_B$ |
| 020$[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+$, $Λ_b^0(?)$ | ? (5972) | $\hat{\epsilon}_{02} - 3C + \frac{1}{3}C_S + 10C_B$ |
| 020$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+$, $Λ_b^0(?)$ | ? (6090) | $\hat{\epsilon}_{02} + 3C - \frac{1}{3}C_S - 2C_B$ |
| 022$[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+$, $Λ_b^0(?)$; $\frac{3}{2}^+$, $Λ_b^0(?)$ | ? (6108) | $\hat{\epsilon}_{02} + 3C - \frac{1}{3}C_S - 7C_B$ |
| 022$[21]_{FS}[111]_{F}[21]_{S}$ | $\frac{3}{2}^+$, $Λ_b^0(?)$; $\frac{3}{2}^+$, $Λ_b^0(?)$ | ? (6042) | $\hat{\epsilon}_{02} - 3C + \frac{1}{3}C_S - 4C_B$ |
| 022$[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+$, $Λ_b^0(?)$; $\frac{5}{2}^+$, $Λ_b^0(?)$ | ? (6066) | $\hat{\epsilon}_{02} - 3C + \frac{1}{3}C_S + 14C_B$ |
The $S$, $P$ and $SD$ shell states in the $\Sigma_b$ hyperon spectrum. The column $\epsilon$ contains the eigenvalues of the mass operator. The averages over the multiplet of the empirically extracted mass values are denoted EXP. The model values obtained with the mass operators (2.7) with the confining well (2.18) and the hyperfine interaction (2.20) are listed below the empirical ones.

| $nKL[f]_{FS}[f]_{F}[f]_{S}$ | $LS$ Multiplet | EXP (model value) | $\epsilon$ |
|-----------------------------|-----------------|------------------|------------|
| $000[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Sigma_b$ | 5814 (5801) | $\epsilon_{00} - C - \frac{1}{3}C_S - 10C_B$ |
| $000[3]_{FS}[3]_{F}[3]_{S}$ | $\frac{3}{2}^+, \Sigma_b(?)$ | 5870 (5857) | $\epsilon_{00} - C - \frac{1}{3}C_S - 4C_B$ |
| $100[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Sigma_b(?)$ | (5916) | $\hat{\epsilon}_{10} - C - \frac{1}{3}C_S - 10C_B$ |
| $011[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^-, \Sigma_b(?)$, $\Sigma_b(?)$, $\Sigma_b(?)$ | (5987) | $\hat{\epsilon}_{01} + C + \frac{1}{3}C_S - 2C_B$ |
| $100[3]_{FS}[3]_{F}[3]_{S}$ | $\frac{3}{2}^+, \Sigma_b(?)$ | (5971) | $\hat{\epsilon}_{10} - C - \frac{1}{3}C_S - 4C_B$ |
| $011[21]_{FS}[3]_{F}[3]_{S}$ | $\frac{1}{2}^-, \Sigma_b(?)$, $\Sigma_b(?)$, $\Sigma_b(?)$ | (6030) | $\hat{\epsilon}_{01} - C - \frac{1}{3}C_S + 2C_B$ |
| $011[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{1}{2}^-, \Sigma_b(?)$, $\Sigma_b(?)$, $\Sigma_b(?)$ | (5977) | $-6AC_B$ |
| $022[3]_{FS}[3]_{F}[3]_{S}$ | $\frac{1}{2}^-, \Sigma_b(?)$, $\Sigma_b(?)$, $\Sigma_b(?)$ | (6044) | $\hat{\epsilon}_{02} - C - \frac{1}{3}C_S - 4C_B$ |
| $022[3]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^-, \Sigma_b(?)$, $\Sigma_b(?)$, $\Sigma_b(?)$ | (6026) | $+A(3C + C_S + 12C_B)$ |
| $020[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Sigma_b(?)$ | (6050) | $\hat{\epsilon}_{02} + C + \frac{1}{3}C_S - 2C_B$ |
| $020[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{3}{2}^+, \Sigma_b(?)$ | (6040) | $-A(3C + C_S + 6C_B)$ |
| $022[21]_{FS}[21]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Sigma_b(?)$, $\Sigma_b(?)$, $\Sigma_b(?)$ | (6091) | $\hat{\epsilon}_{02} + C + \frac{1}{3}C_S - 2C_B$ |
| $022[21]_{FS}[21]_{F}[3]_{S}$ | $\frac{1}{2}^+, \Sigma_b(?)$, $\Sigma_b(?)$, $\Sigma_b(?)$ | (6069) | $\hat{\epsilon}_{02} + C - \frac{1}{3}C_S + 3C_B$ |
| $020[21]_{FS}[3]_{F}[21]_{S}$ | $\frac{1}{2}^+, \Sigma_b(?)$ | (6092) | $+A(3C + C_S - 3C_B)$ |
| $022[21]_{FS}[3]_{F}[21]_{S}$ | $\frac{3}{2}^+, \Sigma_b(?)$, $\Sigma_b(?)$, $\Sigma_b(?)$ | (6115) | $\hat{\epsilon}_{02} + C + \frac{1}{3}C_S + 4C_B$ |

Table 14
**Figure Captions**

Fig. 1. The ground state wave functions $u_{00}(R)$ for the confining models (2.17) (oscillator) and (2.18) (funnel) as functions of the radius $R$. The wave functions have been calculated with the parameter values described in section 4.1.

Fig. 2a The proton charge form factor of the funnel model (solid line) compared with data [9]. The dash-dot and dash lines show the form factors of the funnel and oscillator models with point-quark charge distributions.

Fig. 2b The neutron charge form factor of the funnel model (solid line) in comparison with an empirical fit [9] (dashed line).

Fig. 3 Proton and neutron magnetic form factors of the funnel model compared to empirical fits (dashed lines).
Fig. 2a
Fig. 3