Software Verification with PDR:
Implementation and Empirical Evaluation of the
State of the Art

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Abstract. Property-directed reachability (PDR) is a SAT/SMT-based reachability algorithm that incrementally constructs inductive invariants. After it was successfully applied to hardware model checking, several adaptations to software model checking have been proposed. We contribute a replicable and thorough comparative evaluation of the state of the art: We (1) implemented a standalone PDR algorithm and, as improvement, a PDR-based auxiliary-invariant generator for $k$-induction, and (2) performed an experimental study on the largest publicly available benchmark set of C verification tasks, in which we explore the effectiveness and efficiency of software verification with PDR. The main contribution of our work is to establish a reproducible baseline for ongoing research in the area by providing a well-engineered reference implementation and an experimental evaluation of the existing techniques.

Keywords: Software verification, Program analysis, Invariant generation, Property-directed reachability (PDR), IC3, $k$-Induction, VVT, CPAchecker

1 Introduction

Automatic software verification [26] is a broad research area with many success stories and large impact on technology that is applied in industry [2, 15, 29]. It nicely complements other general approaches to ensure functional correctness, like software testing [33] and interactive software verification [3]. One large sub-area of automatic software verification includes algorithms and approaches that are based on SMT technology. There are classic approaches like bounded model checking [11], predicate abstraction [1, 20], and $k$-induction [4, 28, 34], which are well understood and evaluated; a recent survey [5] provides a uniform overview and sheds light on the differences of the algorithms. Property-directed reachability (PDR) [13] is a relatively recent (2011) approach that is not yet included in surveys and comparative evaluations. The approach was originally applied to transition systems from hardware designs, but was also adapted to software verification in the last years [12, 13, 14, 16, 17, 27, 30, 31].

While in theory, the advantages and disadvantages of using PDR seem clear, we are interested in understanding the effect of applying PDR to a large set of verification tasks that were collected from academia and also from industrial
software, such as the Linux kernel. To achieve this goal, we implemented one PDR adaptation for software verification, and another approach that integrates a PDR-like invariant-generation module into a k-induction approach.

**PDR Adaptation for Software Verification.** PDR is a model-checking algorithm that tries to construct an inductive safety invariant by incrementally learning clauses that are inductive relative to previously learned clauses. The clause-learning strategy is guided by counterexamples to induction, i.e., each time a proof of inductiveness fails, the algorithm attempts to learn a new clause to avoid the same counterexample to induction in the future. Originally, this algorithm was designed as a SAT-based technique for Boolean finite-state systems. Every adaptation of PDR to software verification therefore needs to consider how to effectively and efficiently handle the infinite state space and how to transfer the algorithm from SAT to SMT. Furthermore, the adaptation to software has to deal with the program counter.

**PDR-like Invariant Generation.** Whenever an induction-proof attempt fails with a counterexample, the counterexample describes a state $s$ that can transition into a bad state (that violates the safety property), which means that in order to make the proof succeed, $s$ must be removed from consideration by an auxiliary invariant. From this bad-state predecessor $s$, the clause-learning strategy of PDR proceeds to generate such an auxiliary invariant by applying the following two steps: (1) $s$ is first generalized to a set of states $C$ that all transition into a bad state; (2) an invariant is constructed that is (a) inductive relative to previously found invariants \(^1\) and (b) at least strong enough to eliminate all states in $C$. If it fails to construct such an invariant and prove its inductiveness, then the steps are recursively re-applied to the counterexample obtained from the failed induction attempt.

We experimentally investigate two implementations of adaptations of PDR to software verification (CPAchecker-CTIGAR and VVT-CTIGAR), as well as several combinations that use the PDR-like invariant-generation module that we designed and implemented for this study.

**Example.** Figure 1 shows an example C program (eq2.c) that contains four unsigned integer variables $w$, $x$, $y$, and $z$. In line 10, the variable $w$ is initialized to an unknown value via the input function \(__VERIFIER_nondet_uint()\); then, its value is copied to $x$ in line 11. In line 12, variable $y$ is initialized with the value of $w + 1$, and in line 13, variable $z$ is initialized with the value of $x + 1$, such that at this point, $w$ and $x$ are equal to each other, and $y$ and $z$ are also equal to each other. Then, from line 14 to line 17, a loop with a nondeterministic exit condition (and therefore an unknown number of iterations) increments in each iteration both variables $y$ and $z$. Lastly, line 18 asserts that after the

\(^1\) An assertion $F$ is said to be inductive relative to an invariant $Inv$ if $Inv$ can be used as an auxiliary invariant for the proof of inductiveness $\forall s_j, s_{j+1} : F(s_j) \land T(s_j, s_{j+1}) \Rightarrow F(s_{j+1})$ by conjoining $Inv$ to the induction hypothesis $F(s_j)$, such that the modified induction query $\forall s_j, s_{j+1} : F(s_j) \land Inv(s_j) \land T(s_j, s_{j+1}) \Rightarrow F(s_{j+1})$ allows a proof by induction to succeed. [13]
extern void __VERIFIER_error() __attribute__((noreturn));
extern unsigned int __VERIFIER_nondet_uint(void);
void __VERIFIER_assert(int cond) {
    if (!cond) {
        ERROR: __VERIFIER_error();
    }
    return;
}
int main(void) {
    unsigned int w = __VERIFIER_nondet_uint();
    unsigned int x = w;
    unsigned int y = w + 1;
    unsigned int z = x + 1;
    while (__VERIFIER_nondet_uint()) {
        y++;
        z++;
    }
    __VERIFIER_assert(y == z);
    return 0;
}

Fig. 1: Example C program eq2.c

loop, y and z are (still) equal to each other. Since y and z are equal before the loop, and are always incremented together within the loop, the invariant $y = z$ is inductive. However, since there is no direct connection between y and z but only an indirect one via their shared dependency on w, naïve data-flow-based techniques may fail to find this invariant. In fact, we tried several configurations of the verification framework CPACHECKER, and found that many of them fail to prove this program:

- Plain $k$-induction without auxiliary-invariant generation fails, because it never checks if $y = z$ is a loop invariant and instead only checks the reachability of the assertion failure (located after loop). The reachability of the assertion failure, in turn, depends on the nondeterministic loop-exit condition. Therefore we cannot conclude from “the assertion failure was not reached in $k$ previous iterations” that “the assertion failure cannot be reached in the next iteration”. In the absence of auxiliary invariants, a valid counterexample to this induction hypothesis would always be that in the previous iterations the assertion condition was in fact violated and an assertion failure was not reached only because the loop was not exited.

- A data-flow analysis based on the abstract domain of Boxes [22] fails, because it is not able to track variable equalities.

- A data-flow analysis based on a template Eq for tracking the equality of pairs of variables fails, because while it detects the invariant $w = x$, it is unable to make the step to $y = z$ due to the inequalities between $w$ and $y$, and $x$ and $z$, respectively.
• For consistency with our evaluation, we also applied a data-flow analysis based on a template for tracking whether a variable is even or odd; obviously this is not useful for this program, and thus, this configuration also fails.

• Even combining the previous three techniques into a compound invariant generator that computes auxiliary invariants for \( k \)-induction does not yield a successful configuration for this verification task.

• The invariant generator KIPDR (the above-mentioned adaptation of PDR to \( k \)-induction, which we present in more detail in Sect. 3), however, detects the invariant \( y = z \) and is therefore able to construct a proof by induction for this verification task.

We will now briefly sketch how KIPDR detects the invariant \( y = z \) for the example verification task. At first, KIPDR attempts to prove by induction that when line 18 is reached, the assertion condition holds, which fails as discussed previously. However, this failed induction attempt yields a counterexample to induction where the values of \( y \) and \( z \) differ from each other, e.g., \( y = 0 \land z = 1 \), which is then generalized to \( y \neq z \), i.e., a set of states that includes the concrete predecessor of a bad state from the counterexample, as well as many other states that would violate the assertion, if they were reachable themselves. Then, KIPDR attempts to find an inductive invariant that eliminates all of these states, and the attempt succeeds with the invariant \( y = z \). Afterwards, KIPDR re-attempts its original induction proof to show that the assertion is never violated, which now succeeds due to the auxiliary invariant \( y = z \).

Contributions. We present the following contributions:

• We implement one adaptation of PDR to software verification (based on \([12, 21]\)) in the open-source verification framework CPAchecker, in order to establish a baseline for comparison with new ideas for improvement.

• We design and implement the algorithm KIPDR, as a new module for invariant generation that is based on ideas from PDR and use this module as an extension to a state-of-the-art approach to \( k \)-induction \([4]\).

• We conduct a large experimental study to compare several tools and approaches to software verification using PDR as a component, in order to highlight strengths and weaknesses of PDR in the domain of software verification.

• We contribute a set of small examples that need invariants that are more difficult to obtain for standard data-flow-based approaches than the invariants necessary for programs in the large benchmark set.

Related Work. While PDR (also known as IC3 for its first implementation \([13]\)) was introduced as a SAT-based algorithm for model checking finite-state Boolean transition systems \([14]\), several approaches have since then been presented to extend it to SMT and to apply it to the verification of software models: PDR has been suggested as an interpolation engine for \textsc{Impact}, but experiments have shown that it is too expensive in the general case, and is most effective if only applied as a fall-back engine for cases where a cheaper interpolation engine fails
to produce useful interpolants [16]. It also has been proposed to improve this approach by tracking control-flow locations explicitly instead of symbolically [31], thereby avoiding the problem that many iterations of the algorithm are spent only to learn the control flow, and this idea has later been extended by several improvements to the generalization step of PDR [30]. Another approach is to model the program using a Boolean abstraction, which has the advantage that it requires only few changes to the original algorithm, but the disadvantage that a refinement procedure is necessary to handle the spurious paths introduced by the abstraction: One such approach uses infeasible error paths (i.e., counterexample-guided abstraction refinement (CEGAR) [18]) to refine the abstraction [17], while another (CTIGAR) uses counterexamples to induction [12]; both of these refinement techniques use interpolation to obtain abstraction predicates; the latter of the two techniques is used in two of the configurations we compare in our evaluation (CPAchecker-CTIGAR and VVT-CTIGAR [21]). A different extension of PDR to verify infinite-state systems that does not require abstraction refinement is property-directed \( k \)-induction [27], which increases the power of the induction checks used in PDR by applying \( k \)-induction instead of 1-induction, and which uses model-based generalization in addition to interpolation to reason about potentially-infinite sets of states. Unfortunately, support for effective model-based generalization is rare in SMT solvers, making this approach impractical. In contrast, our KIPDR algorithm presented in Sect. 3 only requires support for interpolation, which is available in several SMT solvers.

Despite this multitude of adaptations of PDR to infinite-state systems, most implementations in practice require their input to be encoded as transition systems already. The only available software verifiers that can be applied to actual C programs and implement PDR-based techniques are CPAchecker [7], SeaHorn [23], and VVT [21].

2 Background

In this section, we briefly introduce the algorithms PDR and \( k \)-induction, which provide the core concepts on which we base our ideas. In the following description of PDR and \( k \)-induction, we use the following notation: given the propositional state variables \( s \) and \( s' \) within a state-transition system \( T \) that represents the program, predicate \( I(s) \) denotes that \( s \) is an initial state, \( T(s, s') \) that a transition from \( s \) to \( s' \) exists, and \( P(s) \) that the safety property \( P \) holds for state \( s \).

2.1 PDR

PDR maintains a list of \( k \) frames, where a frame \( F_i \) is a predicate that represents an overapproximation of all states reachable within at most \( 0 \leq i \leq k \) steps, and a queue of proof obligations, which guide invariant discovery towards invariants

\[ \text{The implementation of this approach of property-directed } k \text{-induction combines two SMT solvers, because neither of them supports all features required by the technique.} \]
relevant to prove the correctness of a safety property \( P \). For a given state \( s \), the notation \( F_i(s) \) means that the predicate \( F_i \) holds for state \( s \). The index \( i \) of a frame \( F_i \) is called its level, and the frame \( F_k \) is called the frontier, because it represents the largest overapproximation of reachable states computed by the algorithm [13]. The algorithm maintains the following invariants:

1. \( F_0(s) = I(s) \), i.e., the first frame represents precisely the initial states.
2. \( \forall i \in \{0, \ldots, k\} : F_i(s) \Rightarrow P(s) \), i.e., every frame contains only states that satisfy the safety property.
3. \( \forall i \in \{0, \ldots, k-1\} : F_i(s) \Rightarrow F_{i+1}(s) \), i.e., a frame \( F_{i+1} \) represents in addition such states that are reachable with \( i + 1 \) steps.
4. \( \forall i \in \{0, \ldots, k-1\} : F_i(s) \land T(s, s') \Rightarrow F_{i+1}(s') \), i.e., each frame is inductive relative to its predecessor.

Using these data structures and algorithm invariants, the algorithm attempts to find either a counterexample to \( P \) or a \( 1 \)-inductive invariant \( F_i \) such that \( F_i(s) \Leftrightarrow F_{i+1}(s) \) for some level \( i \in \{0, \ldots, k-1\} \). Until either of these potential outcomes is reached, PDR shifts back and forth between the following two phases:

1. If the set of states represented by the frontier \( F_k \) does not contain any predecessor states of \( \neg P \)-states (i.e., \( \forall s_j, s_{j+1} : F_k(s_j) \land T(s_j, s_{j+1}) \Rightarrow P(s_{j+1}) \), called frontier-incrementation check), a new frontier \( F_{k+1} \) is created and initialized to \( P \). Subsequently, the algorithm attempts to push forward\(^3\) each clause \( c \) of each frame \( F_i \) with \( 0 \leq i \leq k \) for which the consecution check \( F_i(s_j) \land T(s_j, s_{j+1}) \Rightarrow c(s_{j+1}) \) holds (see Fig. 2). If, on the other hand, the frontier-incrementation check fails, PDR extracts a \( \neg P \)-predecessor \( t \) in \( F_k \), which represents a counterexample to induction (CTI), from the failed query as proof obligation \( \langle t, k-1 \rangle \) (see Fig. 3, top).

2. While the queue of proof obligations is not empty, PDR processes the queue by trying to prove for each proof obligation \( \langle t, i \rangle \) that the CTI-state \( t \) is itself not reachable from \( F_i \) and therefore does not need to be considered as a relevant \( \neg P \)-predecessor. For this proof, PDR chooses some clause \( c \subseteq \neg \top \) with \( \forall s : F_i(s) \Rightarrow c(s) \). PDR then checks if \( c \) is inductive relative to \( F_i \) by performing the consecution check \( F_i(s_j) \land c(s_j) \land T(s_j, s_{j+1}) \Rightarrow c(s_{j+1}) \). If the consecution check succeeds, the frames \( F_1, \ldots, F_{i+1} \) can be strengthened by adding \( c \), thus ruling out the CTI \( t \) in these frames for the future (see Fig. 3, left). Also, unless \( i = k \), we add a new proof obligation \( \langle t, i+1 \rangle \) to the queue as an optimization to initiate forward propagation, because we expect that the CTI-state \( s \) would otherwise be rediscovered later at a higher level [12]. Otherwise, i.e., the consecution check does not succeed for clause \( c \), the algorithm extracts a predecessor \( u \) of \( t \) from the failed consecution check, which is added as a new proof obligation \( \langle u, i-1 \rangle \) if \( i > 0 \) and \( t \land I \) is unsatisfiable (see Fig. 3, right). Otherwise, \( u \) represents the initial state of a real counterexample to \( P \).

A more detailed presentation of PDR can be found in the literature [13].

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\(^3\) By “push forward”, we mean to add a clause \( c \) from frame \( F_i \) to frame \( F_{i+1} \) [13].
Fig. 2: Consecution check makes sure to only conjoin to frame $F_{i+1}$ such $c_i$ from $F_i$ that are inductive relative to $F_i$ w.r.t. transition relation $T$

Fig. 3: If phase 1 results in a proof obligation $⟨t, k−1⟩$ (top), then phase 2 resolves either by strengthening $F_k$ with $c$ (left), or by creating a new (backwards) proof obligation $⟨u, k−2⟩$ (right); if the chain of proof obligations propagates back to the initial states, then a feasible error path is found

**Example.** As an example, we will now show how PDR is applied to a verification task with

- the initial-state predicate $I(s) = (x_s = 2)$,
- the transition relation $T(s, s') = (x_{s'} = 2 \cdot x_s − 1)$, and
- the safety property $P(s) = (x_s > 0)$.

Initially, we therefore have two frames $F_0(s) = (x_s = 2)$ and $F_1(s) = (x_s > 0)$, and $k = 1$, i.e., $F_1$ is the frontier. The algorithm begins in phase 1, where we check whether the frontier contains any predecessors of $\neg P$-states. We find such a predecessor $t$ with $t = (x_t = 0.5)$, because $2 \cdot 0.5 − 1 = 0$, which violates $P$. From this CTI, we create the proof obligation $⟨(x_s = 0.5), 0⟩$, add it to our
queue of proof obligations, and switch to phase 2. In phase 2, we try to find a clause \( c \subseteq \neg t \) with \( \forall s : F_0(s) \Rightarrow c(s) \). We can find a valid clause \( c \) with \( c(s) = (x_s > 0.5) \) that represents a subset of \( \neg t \)-states and a superset of the states represented by \( F_0 \), because \( (x = 2) \Rightarrow (x > 0.5) \Rightarrow (x \neq 0.5) \), and add this clause to frames \( F_0 \) and \( F_1 \), such that \( F_0(s) = (x_s = 2 \land x_s > 0.5) \) and \( F_1(s) = (x_s > 0 \land x_s > 0.5) \), after which the algorithm switches back to phase 1. Back in phase 1, we again check if the frontier, which is still \( F_1 \), contains any predecessors of \( \neg P \)-states, which it now no longer does. Hence, we create a new frontier \( F_2 \) with \( F_2(s) = (x > 0) \). Then, we attempt to push forward each clause of \( F_0 \) and \( F_1 \), which succeeds for \( (x_s > 0.5) \) in frame \( F_0 \), which is already contained in \( F_1 \), and for \( (x_s > 0.5) \) in frame \( F_1 \), which we then add to \( F_2 \). Thus, our frames are now \( F_0(s) = (x_s = 2 \land x_s > 0.5) \), \( F_1(s) = (x_s > 0 \land x_s > 0.5) \), and \( F_2(s) = (x_s > 0 \land x_s > 0.5) \), which means that \( F_1(s) \Leftrightarrow F_2(s) \), i.e., frame \( F_1 \) represents an inductive invariant that implies \( P \) and therefore, the proof is complete.

### 2.2 k-Induction

Like PDR, \( k \)-induction attempts to prove a safety property \( P \) by applying induction. However, while PDR strengthens its induction hypothesis by using clauses extracted from specific counterexamples to induction after failed induction attempts, \( k \)-induction strengthens its induction hypothesis by increasing the length of the unrolling of the transition relation:

Starting with an initial value for the bound \( k \) (usually 1), the \( k \)-induction algorithm increases the value of \( k \) iteratively after each unsuccessful attempt at finding a specification violation (base case), proving correctness via complete loop unrolling (forward condition), or inductively proving correctness of the program (inductive-step case).

#### Base Case.

The base case of \( k \)-induction consists of running BMC with the current bound \( k \).\(^4\) This means that starting from all initial program states, all states of the program reachable within at most \( k - 1 \) unwindings of the transition relation are explored. If a \( \neg P \)-state is found, the algorithm terminates.

#### Forward Condition.

If no \( \neg P \)-state is found by the BMC in the base case, the algorithm continues by performing the forward-condition check, which attempts to prove that BMC fully explored the state space of the program by checking that no state with distance \( k' > k - 1 \) to the initial state is reachable. If this check is successful, the algorithm terminates.

\(^4\) We define the loop bound as the number of visits of the loop head, that is, with loop bound \( k = 1 \), the loop head is visited once, but there was not yet any unwinding of the loop body. This nicely matches the intuition for \( k \)-induction: 1-inductiveness means that if the invariant holds for one state (without loop unrolling), then it holds again after one loop unrolling in the successor state; \( k \)-inductiveness means that if the invariant holds for \( k \) states (\( k - 1 \) loop unrollings), then it holds again after one more loop unrolling in the successor state.
**Inductive-Step Case.** The forward-condition check, however, can only prove safety for programs with finite (and, in practice, short) loops. To prove safety beyond the bound $k$, the algorithm applies induction: The inductive-step case attempts to prove that after every sequence of $k$ unrollings of the transition relation that did not reach a $\neg P$-state, there can also be no subsequent transition into a $\neg P$-state by unwinding the transition relation once more. In the realm of model checking of software, however, the safety property $P$ is often not directly $k$-inductive for any value of $k$, thus causing the inductive-step-case check to fail. It is therefore state-of-the-art practice to add auxiliary invariants to this check to further strengthen the induction hypothesis and make it more likely to succeed. Thus, the inductive-step case proves a program safe if the following condition is unsatisfiable:

$$Inv(s_n) \land \bigwedge_{i=n}^{n+k-1} (P(s_i) \land T(s_i, s_{i+1})) \land \neg P(s_{n+k})$$

where $Inv$ is an auxiliary invariant, and $s_n, \ldots, s_{n+k}$ is any sequence of states. If this check fails, the induction attempt is inconclusive, and the program is neither proved safe nor unsafe yet with the current value of $k$ and the given auxiliary invariant. In this case, the algorithm increases the value of $k$ and starts over.

A detailed presentation of $k$-induction can be found in the literature [4, 5].

### 3 Combining $k$-Induction with PDR

Algorithm 1 shows an extension of $k$-induction with continuously-refined invariants [4] that applies PDR’s aspect of learning from counterexamples to induction and that can be applied both as a main proof engine as well as an invariant generator. This allows us to apply this extension of $k$-induction as an invariant generator to a main $k$-induction procedure, similar to the KI+$\varnothing$-KI approach [4].

**Inputs.** The algorithm takes the following inputs: The value $k_{init}$ is used to initialize the unrolling bound $k$, whereas the function inc is used to increase $k$ in line 33 after each major iteration of the algorithm, up to an upper limit of $k$ defined by the value $k_{max}$ enforced in line 3. The set of initial program states is described by the predicate $I$, the possible state transitions are described by the transition relation $T$, and the set of safe states is described by the safety property $P$. The accessor get_currently_known_invariant is used to obtain the strongest invariant currently available via a concurrently running (external) auxiliary-invariant generator. A Boolean flag $pd$ (reminding of “property-directed”) is used to control whether or not failed induction checks are used to guide the algorithm towards a sufficient strengthening of the safety property $P$ to prove correctness; if $pd$ is set to $false$, the algorithm behaves exactly like standard $k$-induction. Given a failed attempt to prove some candidate invariant $Q$ by induction, the function lift is used to obtain from a

\[Q \text{ by induction, the function lift is used to obtain from a}\]
Algorithm 1 Iterative-Deepening $k$-Induction with Property Direction

**Input:** the initial value $k_{init} \geq 1$ for the bound $k$,
an upper limit $k_{max}$ for the bound $k$,
a function $\text{inc} : \mathbb{N} \to \mathbb{N}$ with $\forall n \in \mathbb{N} : \text{inc}(n) > n$,
the initial states defined by the predicate $I$,
the transfer relation defined by the predicate $T$,
a safety property $P$,
a function $\text{get\ currently\ known\ invariant}$ to obtain auxiliary invariants,
a Boolean $pd$ that enables or disables property direction,
a function $\text{lift} : \mathbb{N} \times (S \to B) \times (S \to B) \times S \to (S \to B)$, and
a function $\text{strengthen} : \mathbb{N} \times (S \to B) \times (S \to B) \to (S \to B)$,
where $S$ is the set of program states.

**Output:** $\text{true}$ if $P$ holds, $\text{false}$ otherwise

**Variables:** the current bound $k := k_{init}$,
the invariant $\text{InternalInv} := \text{true}$ computed by this algorithm internally, and
the set $O := \{\}$ of current proof obligations.

1. while $k \leq k_{max}$ do
2. $O_{prev} := O$
3. $O := \{\}$
4. $\text{base\ case} := I(s_0) \land \bigwedge_{n=0}^{k-1} T(s_i, s_{i+1}) \land \neg P(s_n)$
5. if $\text{sat}($base\ case$)$ then
6. return $\text{false}$
7. $\text{forward\ condition} := I(s_0) \land \bigwedge_{n=0}^{k-1} T(s_i, s_{i+1})$
8. if $\neg\text{sat}($forward\ condition$)$ then
9. return $\text{true}$
10. if $pd$ then
11. for each $o \in O_{prev}$ do
12. $\text{base\ case}_o := I(s_0) \land \bigwedge_{n=0}^{k-1} T(s_i, s_{i+1}) \land \neg o(s_n)$
13. if $\text{sat}($base\ case$)_o$ then
14. return $\text{false}$
15. else
16. $\text{step\ case}_n := \bigwedge_{i=0}^{n+k-1} (o(s_i) \land T(s_i, s_{i+1})) \land \neg o(s_{n+k})$
17. $\text{ExternalInv} := \text{get\ currently\ known\ invariant}()$
18. $\text{Inv} := \text{InternalInv} \land \text{ExternalInv}$
19. if $\text{sat}(\text{Inv}(s_n)) \land \text{step\ case}_n$ then
20. $s_n := \text{satisfying\ predecessor\ state}$
21. $O := O \cup \{\text{lift}(k, \text{Inv}, o, s_0)\}$
22. else
23. $\text{InternalInv} := \text{InternalInv} \land \text{strengthen}(k, \text{Inv}, o)$
24. $\text{step\ case}_n := \bigwedge_{i=0}^{n+k-1} (P(s_i) \land T(s_i, s_{i+1})) \land \neg P(s_{n+k})$
25. $\text{ExternalInv} := \text{get\ currently\ known\ invariant}()$
26. $\text{Inv} := \text{InternalInv} \land \text{ExternalInv}$
27. if $\text{sat}(\text{Inv}(s_n)) \land \text{step\ case}_n$ then
28. if $pd$ then
29. $s := \text{satisfying\ predecessor\ state}$
30. $O := O \cup \{\text{lift}(k, \text{Inv}, P, s)\}$
31. else
32. return $\text{true}$
33. $k := \text{inc}(k)$
34. return $\text{unknown}$
concrete counterexample-to-induction (CTI) state a set of CTI states described by a state predicate $C$. An implementation of the function $lift$ needs to satisfy the condition that for a CTI $s \in S$ where $S$ is the set of program states, $k \in \mathbb{N}$, $Inv \in (S \rightarrow \mathbb{B})$, $Q \in (S \rightarrow \mathbb{B})$, and $C = lift(k, Inv, Q, s)$, the following holds:

$$C(s) \land (\forall s_n \in S : C(s_n) \Rightarrow Inv(s_n) \land \bigwedge_{i=n}^{n+k-1} (Q(s_i) \land T(s_i, s_{i+1})) \Rightarrow \neg Q(s_{n+k})),$$

which means that the CTI $s$ must be an element of the set of states described by the resulting predicate $C$ and that all states in this set must be CTIs, i.e., they need to be $k$-predecessors of $\neg Q$-states, or in other words, each state in the set of states described by the predicate $C$ must reach some $\neg Q$-state via $k$ unrollings of the transition relation $T$. We can implement $lift$ using Craig interpolation [19, 32] between $A : s = s_n$ and $B : Inv(s_n) \land \bigwedge_{i=n}^{n+k-1} (Q(s_i) \land T(s_i, s_{i+1})) \Rightarrow \neg Q(s_{n+k})$, because $s$ is a CTI, and therefore we know that $A \Rightarrow B$ holds. Hence, the resulting interpolant satisfies the criteria for $C$ to be a valid lifting of $s$ according to the requirements towards the function lift as outlined above. The function $strengthen$ is used to obtain for a $k$-inductive invariant a stronger $k$-inductive invariant, i.e., its result needs to imply the input invariant, and, just like the input invariant, it must not be violated within $k$ loop iterations and must be $k$-inductive.

**Algorithm.** Lines 4 to 6 show the base-case check (BMC) and lines 7 to 9 show the forward-condition check, both as described in Sect. 2. If $pd$ is set to $true$, lines 10 to 23 attempt to prove each proof obligation using $k$-induction: Lines 12 to 14 check the base case for a proof obligation $o$. If any violations of the proof obligation $o$ are found, this means that a predecessor state of a $\neg P$-state, and thus, transitively, a $\neg P$-state, is reachable, so we return $false$. If, otherwise, no violation was found, lines 16 to 23 check the inductive-step case to prove $o$. We strengthen the induction hypothesis of the step-case check by conjoining auxiliary invariants from an external invariant generator (via a call to `get_currently_known_invariant`) and the auxiliary invariant computed internally from proof obligations that we successfully proved previously. If the step-case check for $o$ is unsuccessful, we extract the resulting CTI state, lift it to a set of CTI states, and construct a new proof obligation so that we can later attempt to prove that these CTI states are unreachable. If, on the other hand, the step-case check for $o$ is successful, we remove $o$ from the set of unproven proof obligations $O$ in line 22. We could now directly use the proof obligation as an invariant, but instead, in line 23 we first try to $strengthen$ it into a stronger invariant that removes even more unreachable states from future consideration before conjoining it to our internally computed auxiliary invariant. In our implementation, we

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6 The formula $C$ is called Craig interpolant for two formulas $A$ and $B$ with $A \Rightarrow B$, if $A \Rightarrow C$, $C \Rightarrow B$, and all variables in $C$ occur in both $A$ and $B$.

7 Note that we do not need to check the forward condition for proof obligations, because the forward condition is unrelated to the safety property and the proof obligations, and therefore only needs to be checked once in each major iteration (i.e., once after each increment of $k$).
implement strengthen by attempting to drop components from a (disjunctive) invariant and checking if the remaining clause is still inductive. In lines 24 to 32, we check the inductive-step case for the safety property $P$. This check is mostly analogous to the inductive-step case check for the proof obligations described above, except that if the check is successful, we immediately return true.

Note that Alg. 1 eagerly increases $k$, even if the set $O$ of proof obligations is not empty. This heuristic prevents the PDR part from iterating through long chains of proof obligations, it rather delegates the unrolling to the $k$-induction part.

Example. We now give an example of applying Alg. 1 to the example introduced in Fig. 1.\(^8\) For this example, we will configure the algorithm as a property-directed invariant generator. We choose $k_{init} = 1$ as the initial bound and $k_{max} = \infty$ to force the algorithm to increase the bound $k$ until a proof is found or an alarm is raised, and thus, the condition in line 1 of Alg. 1 will always evaluate to true. To increment $k$ by a value of 1 in each major iteration, we define $inc(k) = k + 1$. The set of initial states and the transfer relation are given by the input program eq2.c. As safety property $P$ we require that no call to the function \_\_VERIFIER\_error() is reachable. We will not use an external auxiliary-invariant generator, hence we define get\_currently\_known\_invariant() = true. Instead, we want to use the algorithm’s capability of property-directed invariant generation and set $pd = true$. The most important steps and variable values at some algorithm locations are summarized in Table 1.

In the first iteration, with $k = 1$, we first need to check the base case, i.e., we need to check in lines 4 to 6 of the algorithm whether the call to function \_\_VERIFIER\_error() in line 5 of the input program is reachable with zero loop iterations. Since this is not the case, we continue to lines 7 to 9 of the algorithm, where we check the forward condition, i.e., we check whether we completely unrolled the loop, that is, not more than zero unrollings of the loop are possible. Since the loop condition depends on a nondeterministic input value, this check will always fail, and so we continue to line 10 of the algorithm, where property-directed invariant generation begins, if it is switched on. Because $pd = true$, we attempt to generate invariants from our set $O$ of proof obligations. This set, however, is currently empty, and so we jump to the step-case check in lines 24 to 27 of the algorithm, where we assume for any iteration $n$ ($k$ iterations from $n$ to $n + k - 1 = n$) that the safety property holds, and from this assumption attempt to conclude that the safety property will also hold in the next iteration $n + 1$ ($n + k$). In the context of this example, this means that we assume for any iteration $n$ that we did not reach a call to the function \_\_VERIFIER\_error(), and from this assumption try to conclude that \_\_VERIFIER\_error() is also not reachable in the next iteration $n + 1$. How-

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\(^8\) For ease of presentation, we assume that $I$ encodes the beginning of the program (lines 10–13), $T$ encodes the loop body (lines 15–16), and $P$ encodes the rest of the program (which checks the property to verify).
Table 1: Relevant steps and values within Alg. 1 if applied to the example introduced in Fig. 1.

| Major iteration | 1          | 2          |
|-----------------|------------|------------|
| Line 1          | $k = 1$    | $k = 2$    |
| Line 4          | BMC finds no error in 0 iterations | BMC finds no error in up to 1 iteration |
| Line 8          | Forward-condition check fails | Forward-condition check fails |
| Line 10         | $pd = true$ | $pd = true$ |
| Line 11         | $O = \emptyset$ | $O = \{y = z\}$, $o = (y = z)$ |
| Line 13         | BMC finds no counterexample to $y = z$ in up to 1 iteration | |
| Line 17         | $ExternalInv = true$ | |
| Line 18         | $InternalInv = true$, $Inv = true$ | |
| Line 19         | Step-case check for $o$ succeeds, because $y = z$ is inductive | |
| Line 22         | $O = \emptyset$ | $O = \emptyset$ |
| Line 23         | $InternalInv = (y = z)$ | $InternalInv = (y = z)$ |
| Line 25         | $ExternalInv = true$ | $InternalInv = (y = z)$, $Inv = ((y = z))$ |
| Line 26         | $InternalInv = true$, $Inv = true$ | |
| Line 27         | Step-case check fails | Step-case check succeeds |
| Line 28         | $pd = true$ | $pd = true$ |
| Line 29         | $s = (y = 0 \land z = 1)$ | $O = \{y = z\}$ |
| Line 30         | $O = \{y = z\}$ | |
| Line 33         | $k = 2$ | |

However, we have not yet computed any auxiliary invariants, so $InternalInv = true$ and we cannot obtain any invariants from an external invariant generator either, because we defined $get\_currently\_known\_invariant() = true$, therefore also $Inv = true$ in line 26. As described in the introduction, checking the inductive-step case for the safety property $P$ will always fail without auxiliary invariants for this example, because the SMT solver can provide a model where $y \neq z$ holds before this iteration, but after this iteration, the nondeterministic loop condition evaluates to $false$ and the safety property is violated. For this example, we assume the SMT solver produces a model where before the iteration, $y = 0$ and $z = 1$, and this state becomes our satisfying predecessor state $s$ in line 29 of the algorithm. Next, we try to lift this state to a more abstract state that still satisfies the property that all of its successors violate the safety property. For this example, we assume that lifting produces the abstract state $y \neq z$. We then negate this abstract state to obtain the proof obligation $y = z$. This means that we have learned that we should prove the invariant $y = z$, such that in future induction checks, we can remove all states where $y \neq z$ from the set of predecessor states that need to be considered. Therefore, we add the proof obligation to our set $O$ of current proof obligations, such that $O := \{y = z\}$. We then increment $k$ using the function $inc(k) = k + 1$, and continue into the next major iteration of our algorithm with $k = 2$. 
Again, we check the base case in line 4, and again, the check succeeds, because we cannot reach a call to the function \_\_VERIFIER\_error() in line 5 of the input program within one loop iteration either. As explained before, the forward-condition check in line 8 of the algorithm must always fail for this input program, and so we again continue with property-directed invariant generation. This time, however, the set \( O \) of current proof obligations is not empty, and so we choose from it in line 11 of the algorithm the only proof obligation \( o \) it currently contains, \( y = z \), and try to prove it by induction. Hence, we first check the proof-obligation base case in lines 12 to 13 of the algorithm. Obviously, the check succeeds, because \( y = z \) holds before and after the first loop iteration of the program: the two variables are both initialized to 0, and are both incremented once. We still have not computed any auxiliary invariants yet, so \( \text{InternalInv} = \text{true} \), and we defined \( \text{get\_currently\_known\_invariant()} = \text{true} \), therefore also \( \text{Inv} = \text{true} \) in line 18. This means that when we check the inductive-step case in line 19 of the algorithm, we have no auxiliary invariants. Nevertheless, the check is successful, because contrary to checking the safety property \( P \), which concerns the reachability of a function call outside the loop and therefore depends on the nondeterministic loop condition, the condition that \( y = z \) before and after each loop iteration does not depend on any nondeterministic input value. Thus, we remove \( y = z \) from our set \( O \) of current proof obligations in line 22 of the algorithm. We then try to strengthen \( y = z \) to obtain a stronger \( k \)-inductive (i.e., 2-inductive, since currently \( k = 2 \)) invariant for this program. However, since \( y = z \) is already a quite strong invariant for this input program, we are unlikely to find a stronger one easily. For this example, we therefore assume that this step strengthens the invariant to \( y = z \) again. We then conjoin \( y = z \) to our current internally computed invariant \( \text{InternalInv} \), such that \( \text{InternalInv} = (y = z) \) in line 23 of the algorithm. Next, we check the inductive-step case for the safety property \( P \) in lines 24 to 27. This time, with \( \text{Inv} = (y = z) \), the check will succeed, and the algorithm returns \text{true} in line 32.

4 Evaluation

In this section, we present an extensive experimental study on the effectiveness and efficiency of adaptations of PDR to software verification.

4.1 Compared Approaches

We use the following abbreviations to distinguish between the different techniques that we evaluated:

\textbf{CTIGAR:} CTIGAR [12] is an adaptation of PDR to software verification. Our evaluation compares two implementations of CTIGAR, namely \texttt{VVT-CTIGAR} from the tool \texttt{VVT} and our own implementation \texttt{CPACHECKER-CTIGAR}. \texttt{VVT} [21] also provides a configuration that runs a parallel portfolio combination of \texttt{VVT-CTIGAR} and bounded model checking, which we call \texttt{VVT-Portfolio}.
KI: KI [4] denotes the plain \( k \)-induction algorithm without property direction and without auxiliary invariants, i.e., we configure Alg. 1 such that \( pd = false \) and \( \text{get\_currently\_known\_invariant()} \) always returns \( true \).

KIPDR: KIPDR denotes a configuration of Alg. 1 such that \( pd = true \) and \( \text{get\_currently\_known\_invariant()} \) always returns \( true \), i.e., \( k \)-induction with property direction but without additional auxiliary-invariant generation. KIPDR is, like CTTIGAR, an adaptation of PDR to software verification.

KI\( \oplus \)DF: KI\( \oplus \)DF [4] denotes a parallel combination of \( k \)-induction (without property direction) with a data-flow-based auxiliary-invariant generator that continuously supplies the \( k \)-induction procedure with invariants. Here, we configure Alg. 1 such that \( pd = false \) and \( \text{get\_currently\_known\_invariant()} \) always returns the most recent (strongest) invariant computed by the data-flow-based auxiliary-invariant generator.

KI\( \oplus \)KIPDR: Similarly to KI\( \oplus \)DF, KI\( \oplus \)KIPDR denotes a parallel combination of \( k \)-induction with an auxiliary-invariant generator — in this case, KIPDR — that continuously supplies invariants to the \( k \)-induction procedure. Here, we configure one instance of Alg. 1 such that \( pd = false \) and \( \text{get\_currently\_known\_invariant()} \) always returns the most recent (strongest) invariant computed by KIPDR (a second instance of Alg. 1 that is configured such that \( pd = true \) and \( \text{get\_currently\_known\_invariant()} \) always returns \( true \)).

KI\( \oplus \)DF;KIPDR: KI\( \oplus \)DF;KIPDR denotes a parallel combination of \( k \)-induction with an auxiliary-invariant generator that uses a sequential combination of a data-flow-based invariant generator and KIPDR to continuously supply \( k \)-induction with auxiliary invariants. Here, we configure one instance of Alg. 1 such that \( pd = false \) and \( \text{get\_currently\_known\_invariant()} \) always returns the most recent (strongest) invariant computed by a sequential combination of the data-flow-based invariant generator and KIPDR (a second instance of Alg. 1 that runs after the data-flow-based invariant generator finishes and is configured such that \( pd = true \) and \( \text{get\_currently\_known\_invariant()} \) always returns \( true \)).

We do not evaluate the used invariant generators as standalone verification approaches, because they are designed specifically to be used as auxiliary components and do not perform well enough in isolation. For example, data-flow based invariant-generation approaches are often too imprecise to verify tasks by themselves, whereas more precise techniques like KIPDR might spend too much time unnecessarily, resulting in too many timeouts to be competitive. Instead, we use the framework of \( k \)-induction with continuously refined invariant generation, which has been shown to be able to leverage the advantages of both quick but imprecise and slow but precise techniques [4].

4.2 Evaluation Goals

The overall goals of our experimental evaluation are to establish a baseline for experimental comparisons involving PDR in the context of software verification.
and to determine whether we can combine the strengths of PDR with those of $k$-induction. We are interested in answers to the following research hypotheses. We use the symbols ✓ and ✗ to indicate that a hypothesis will be confirmed or refuted, respectively, by our experiments on the given benchmark sets.

**Hypothesis 1:** CPAchecker is a suitable platform for implementing and evaluating PDR-based techniques. That is, we are able to implement state-of-the-art PDR techniques in this framework and obtain competitive results. ✓

**Hypothesis 2:** By providing $k$-induction with a KIPDR invariant generator, which utilizes the PDR-aspect of guiding invariant discovery by leveraging failed induction attempts, we can improve the overall effectiveness of $k$-induction. ✗

**Hypothesis 3:** On small programs, such as path programs, KIPDR is a more effective invariant generator than data-flow-based techniques and is therefore well-suited for the approach of generating path invariants [6]. ✗

**Hypothesis 4:** While KIPDR is often outperformed by simpler, data-flow-based invariant-generation techniques, there exist programs that can not be solved using the considered data-flow-based invariant-generation techniques, but can be solved using KIPDR as auxiliary-invariant generator. Furthermore, KI$\bowtie$KIPDR can be more efficient than state-of-the-art verifiers from the SV-COMP evaluation. ✓

**Hypothesis 5:** Our conclusions are relevant, because our implementation of KI$\bowtie$DF$\bowtie$KIPDR is competitive when compared to the best available implementations of PDR technology for software verification. ✓

### 4.3 Benchmark Set

The benchmark set we use in our experiments consists of verification tasks from the International Competition on Software Verification (SV-COMP) [9], in particular, we use the benchmark categories as used for SV-COMP 2018. In addition, we use a set of new verification tasks, which are (a) generated path programs from existing verification tasks, in order to obtain verification tasks that are small enough for the PDR approach to handle, and (b) manually created programs that explore the expressive power of the PDR approach. All programs that were used in the evaluations are available on the supplementary web page and in the replication package [10].

We consider only verification tasks where the property to verify is the unreachability of a program location (excluding the properties for no-overflows, memory safety, and termination, which are not in the scope of our evaluation). From the resulting set of verification tasks, we excluded the categories ReachSafety-Recursive and ConcurrencySafety, each of which is not supported by at least one of the evaluated implementations. The remaining set of categories consists of a total of 5591 verification tasks from the subcategory

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[9] https://sv-comp.sosy-lab.org/2018/benchmarks.php
[10] https://www.sosy-lab.org/research/pdr-compare/
DeviceDriversLinux64_ReachSafety of the category SoftwareSystems and from the following subcategories of the category ReachSafety: Arrays, Bitvectors, ControlFlow, ECA, Floats, Heap, Loops, ProductLines, and Sequentialized. A total of 1457 of these tasks are known to contain a specification violation, while the rest of the tasks are meant to satisfy their specification.

4.4 Verification Tools and Algorithms

We evaluate all verification tools that are (1) publicly available, (2) support C as an input-program language, and (3) implement at least one PDR-based approach. There are three such verifiers available: We use CPACHECKER [7] in revision 27742 from the trunk, SeaHorn [23] in version F16-0.1.0-rc3, and the VVT version used in the 2016 Competition on Software Verification (SV-COMP 2016) [21].

For implementing our own modules and extensions, we choose the framework CPACHECKER, because it (a) is a large open-source project that “has a well established, mature codebase maintained by a large development team” [12], and (b) seems to have efficient implementations of the core components [13].

Unfortunately, we could include neither the implementations of Cimatti and Grigio [16], nor that of Lange, Prinz, Neuhäuser, Noll, and Katoen [30, 31], in our evaluation. The former are only applicable to transition systems in SMT format and control-flow graphs in SMT-CFA format, respectively, not to C programs, and the latter is not publicly available.

4.5 Experimental Setup

For our experiments, we executed the chosen software verifiers on machines with one 3.4 GHz CPU (Intel Xeon E3-1230 v5) with 8 processing units and 33 GB of RAM each. The operating system was Ubuntu 16.04 (64 bit), using Linux 4.4 and OpenJDK 1.8. We limited each verification run to two CPU cores, a CPU run time of 15 min, and a memory usage of 15 GB. To ensure reliable and accurate measurements, we used the benchmarking framework BenchExec [8] to conduct our experiments.

4.6 Presentation

All benchmarks, tools, and the full results of our evaluation are available on a supplementary web page [10] and in the replication package [10]. All reported times are rounded to two significant digits. Because it is sometimes difficult to compare

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11 We could not use the newer version of VVT from GitHub, because it has a bug where its CTIGAR component does not report solved tasks and no fix was available.

12 Citation source: https://www.openhub.net/p/cpachecker

13 SV-COMP 2018: https://sv-comp.sosy-lab.org/2018/results/results-verified/

14 https://github.com/sosy-lab/benchexec

15 https://www.sosy-lab.org/research/pdr-compare/
results in the presence of wrong alarms or wrong proofs, we use the community-agreed schema of SV-COMP to assign quality values to each verification result, i.e., to calculate a score that quantifies the quality of the results for a verifier. For every real bug found, 1 point is assigned, for every correct safety proof, 2 points are assigned. A score of 16 points is subtracted for every wrong alarm (false positive) reported by the tool, and 32 points are subtracted for every wrong proof of safety (false negative). While this scoring scheme may not match every use case, it is not arbitrary: it follows a community consensus [9] on the difficulty of verification versus falsification and the importance of correct results, where proving correctness (computing invariants) is considered more complicated than finding bugs (computing error paths), and wrong answers are punished severely (by a factor of 16). We consider this a good fit for evaluating approaches such as $k$-induction and PDR, which focus on producing safety proofs.

4.7 Hypothesis 1: Suitability of CPACHECKER for PDR

In our first set of experiments, we establish that the CPACHECKER framework is a suitable platform for implementing PDR-based techniques by comparing the effectiveness and efficiency of our own implementation of PDR for software-model checking (CPACHECKER-CTIGAR) to the only available verifier that implements a pure PDR approach for software-model checking, which is VVT-CTIGAR [12]. The implementation in VVT has been published previously [21], whereas the implementation in CPACHECKER was newly developed for this article to serve as a baseline to compare further implementations of adaptations of PDR in CPACHECKER against. Columns two and three of Table 2 compare the results obtained by running the two implementations of CTIGAR on the whole benchmark set, and the last column of the table shows the results achieved with the standard configuration of VVT, which runs not only CTIGAR, but a portfolio analysis of CTIGAR and bounded model checking. The table lists the score achieved by each configuration and breaks it down into the amount of correct and incorrect proofs and alarms. It also lists for how many tasks a configuration exceeded the time limit of 15 min or the memory limit of 15 GB, as well as the amount of tasks a tool configuration could not solve for other reasons, such as failures in the parser, the SMT solver, or the analysis. Furthermore, the table lists the total, mean, and median CPU and wall times spent by a configuration. We observe that while CPACHECKER-CTIGAR is more effective (i.e., solves more tasks correctly) and has fewer wrong results than VVT-CTIGAR, VVT appears to be more efficient (i.e., is faster). We attribute the overall differences between the tools regarding correctly and incorrectly solved tasks to the fact that CPACHECKER as a framework is older and more mature than VVT. In fact, we see many cases where VVT fails to parse the C code of a task, likely due to limited support for some features of the C programming language. We also see

16 We chose the direct comparison with VVT over a comparison with SeaHorn here, because CPACHECKER-CTIGAR uses the same approach as VVT-CTIGAR, and because of the large number of incorrect results for SeaHorn.
that only few real bugs are detected by CPAchecker-CTIGAR (255) and VVT-CTIGAR (215), while the VVT portfolio analysis produces significantly more correct alarms (311). This confirms our expectations, because PDR is mainly aimed at computing invariants (and thus finding proofs), whereas bounded model checking is well-known as a good technique for finding bugs. When considering the union of tasks solved by CPAchecker-CTIGAR and VVT-CTIGAR, i.e., the tasks solved by either CPAchecker-CTIGAR, VVT-CTIGAR, or both (“virtual best”), we find 1474 tasks solved correctly, 1124 of which constitute proofs, and 350 of which constitute alarms. This means that on the one hand, for a large amount of tasks, CPAchecker-CTIGAR and VVT-CTIGAR both produce the same correct results, but on the other hand, there are not only many tasks CPAchecker-CTIGAR can solve correctly that VVT-CTIGAR cannot solve, but also several tasks that VVT-CTIGAR can solve but CPAchecker-CTIGAR cannot. Like the differences in the amounts of incorrect results, we can attribute this observation to the way the different frameworks model the tasks internally, where for some types of tasks, CPAchecker takes a safer approach to the trade-off between correctness and effectiveness than VVT, and where some language constructs supported by the older CPAchecker framework are not supported by the younger VVT.
To further analyze the efficiency of CPAchecker-CTIGAR and Vvt-CTIGAR, the quantile plot in Fig. 4 shows the CPU times that the two tool configurations spent on their correct results. The plot shows that for both tools, there exist approximately 610 tasks for which each of the tasks can be solved in less than approximately 5.2 s, and that in this range, Vvt is significantly faster than CPAchecker, which can partially be explained by the approximately 3 s startup time of the Java Virtual Machine that CPAchecker runs in. Beyond that point, however, CPAchecker-CTIGAR scales better than Vvt-CTIGAR.

In conclusion, our implementation was shown to be at least as good as (and even better than) the only available implementation of PDR for software model checking. Therefore, the hypothesis that CPAchecker is a suitable platform for implementing and evaluating adaptations of PDR holds.

4.8 Hypothesis 2: Augmenting k-Induction with KIPDR

We now attempt to establish that augmenting k-induction with auxiliary invariants from KIPDR improves its overall effectiveness: In the previous experiment, we presented CPAchecker-CTIGAR, which uses an adaptation of PDR as its verification engine. When we compare the results of CPAchecker-CTIGAR from
Table 2 to the results from evaluations \cite{4,5} of other techniques for previous versions of the same benchmark set, we see that neither of the two CTIGAR implementations is competitive. For example, running the $k$-induction configuration of CPACHECKER without auxiliary-invariant generation on our benchmark set, we obtain 1,239 correct proofs and 836 correct alarms, as shown in the fourth column of Table 2 (KI).

In general, however, the strength of PDR is considered to be its capability for generating safety invariants, so that it is more interesting to analyze its usefulness as an invariant generator.

For our second experiment, in which we solely compare configurations of CPACHECKER to minimize confounding effects, we therefore first took all 2,893 tasks in our benchmark set that do not contain bugs and cannot be solved by $k$-induction without an auxiliary-invariant generator, i.e., all tasks where $k$-induction might potentially benefit from auxiliary invariants. On this set of tasks, we ran three configurations of $k$-induction with auxiliary-invariant generation: KI⟲⟲⟲←−KIPDR uses the property-directed extension of $k$-induction described in Sect. 3 as an auxiliary-invariant generator to a main $k$-induction procedure, KI⟲⟲⟲←−DF uses a sequential combination of several data-flow analyses to generate auxiliary invariants, and KI⟲⟲⟲←−DF;KIPDR combines the invariant generators of the previous two. Table 3 shows that for KI⟲⟲⟲←−KIPDR the property-directed invariant-generation component KIPDR helps $k$-induction to find proofs for 449 tasks that it could not solve without auxiliary invariants. However, KI⟲⟲⟲←−DF is much more efficient and effective, and the combination of the two invariant generators provided by KI⟲⟲⟲←−DF;KIPDR does not contribute a significant improvement. The corresponding quantile plot in Fig. 5 suggests that KIPDR is slower than KI⟲⟲⟲←−DF (also compare the median CPU time), and the fact that the graph of KI⟲⟲⟲←−DF;KIPDR mostly overlaps with the graph of KI⟲⟲⟲←−DF suggests that there are only very few cases for which the behavior of the two configurations differs. Therefore, on the currently available large benchmark set, we cannot confirm Hypothesis 2, which states that the invariant generator KIPDR helps us improve the effectiveness of $k$-induction beyond the current state of the art; rather the null hypothesis holds, which states that there is no significant difference (only two more solved instances, which is not a significant improvement).

4.9 Hypothesis 3: KIPDR as a Path-Invariant Generator

While refuting Hypothesis 2, we learned that KIPDR is a less efficient and effective invariant generator than data-flow-based techniques on our set of benchmarks. One explanation may be that KIPDR is too expensive to generate invariants for whole programs within the time limit. There exists research, however, that suggests that one solution for applying expensive invariant-generation techniques is to extract a (smaller) so-called path program from the (larger) original program, apply the expensive techniques to the path program instead, and then transfer the path invariants computed for the path program back to the analysis that attempts to solve the original verification task \cite{6}.
Table 3: Results of \(k\)-induction-based configurations in CPAchecker with different approaches for generating auxiliary invariants for all 2,893 verification tasks that do not contain bugs and are not solved by \(k\)-induction without auxiliary invariants.

| Approach                        | \(KI \leftrightarrow KIPDR\) | \(KI \leftrightarrow DF\) | \(KI \leftrightarrow DF;KIPDR\) |
|----------------------------------|--------------------------------|-----------------------------|---------------------------------|
| Correct proofs                   | 449                           | 1,117                       | 1,119                           |
| Timeouts                         | 1,982                         | 1,529                       | 1,503                           |
| Out of memory                    | 143                           | 98                          | 106                             |
| Other inconclusive               | 319                           | 149                         | 165                             |

| Times for correct results        |                                |                             |                                 |
| Total CPU Time (h)               | 3.6                           | 21                          | 21                              |
| Mean CPU Time (s)                | 29                            | 68                          | 68                              |
| Median CPU Time (s)              | 12                            | 7.9                         | 8.0                             |
| Total Wall Time (h)              | 2.0                           | 11                          | 11                              |
| Mean Wall Time (s)               | 16                            | 35                          | 36                              |
| Median Wall Time (s)             | 6.4                           | 4.2                         | 4.2                             |

Fig. 5: Quantile plot of accumulated number of bug-free tasks proved correctly by different approaches for generating auxiliary invariants but not solved by \(k\)-induction without auxiliary invariants.

To determine if this approach provides a more suitable setting for KIPDR, we extract from our original benchmark set 1,167 path programs for which we are confident that they do not contain bugs. Then, we take the same approach as in the previous experiment, i.e., we exclude all 1,053 path programs that can be solved by \(k\)-induction without auxiliary invariants, and on the remaining 114 tasks, we run \(KI \leftrightarrow KIPDR\), \(KI \leftrightarrow DF\), and \(KI \leftrightarrow DF;KIPDR\).
Table 4: Results of \( k \)-induction-based configurations in CPAchecker with different approaches for generating auxiliary invariants for all 114 path programs that we assume do not contain bugs and that are not solved by \( k \)-induction without auxiliary invariants.

| Approach                  | KI+∅-KIPDR | KI+∅-DF | KI+∅-DF:KIPDR |
|---------------------------|------------|---------|---------------|
| Correct proofs            | 10         | 99      | 100           |
| Timeouts                  | 92         | 14      | 13            |
| Out of memory             | 0          | 0       | 0             |
| Other inconclusive        | 12         | 1       | 1             |

Times for correct results

- Total CPU Time (h): 0.12, 3.2, 3.4
- Mean CPU Time (s): 44, 110, 120
- Median CPU Time (s): 5.4, 26, 27
- Total Wall Time (h): 0.063, 1.7, 1.9
- Mean Wall Time (s): 23, 64, 69
- Median Wall Time (s): 2.9, 13, 14

Table 4 shows the results of this experiment. We observe again that KI+∅-KIPDR is the least effective configuration, making KIPDR an ineffective invariant generator even for this simplified set of tasks comprised of path programs. Therefore, we consider Hypothesis 3 as refuted on the given benchmark set of path programs: There is no evidence to suggest that KIPDR is an effective path-invariant generator compared to state-of-the-art techniques (only one more solved instance, which is not a significant improvement).

4.10 Hypothesis 4: KIPDR versus Data-Flow Techniques

Now we show that the higher efficiency of data-flow-based techniques is most likely due to the simple form of the invariants needed to prove the programs correct. This explains why the conceptually more advanced KIPDR is outperformed by the data-flow-based techniques.

For this experiment, we selected from the previous set of benchmarks those 449 tasks that were solved by KI+∅-KIPDR and analyzed them using specific components of the sequential combination of data-flow analyses used in KI+∅-DF. Specifically, we used one configuration that uses the abstract domain of Boxes [22], one configuration using Boxes and the template \( Eq := (x = y) \), and one configuration using Boxes, the template \( Eq \), and the template \( Mod2 := (|x| \% 2 = c) \) with \( x, y \in X \), where \( X \) is the set of program variables, and \( c \in 0, 1 \).

The results are reported in Table 5. The first configuration, Boxes, is already sufficient to verify 437 of the 449 tasks verified by KI+∅-KIPDR. Of the remaining 12 tasks, 2 can be verified using a combination of Boxes and
Table 5: Results of four $k$-induction-based configurations in CPAchecker with different approaches for generating auxiliary invariants for all 449 verification tasks that do not contain bugs and are not solved by $k$-induction without auxiliary invariants, but are solved by KI+KIPDR.

| Approach | KI+DF Boxes, Eq | KI+KIPDR Boxes, Eq, Mod2 |
|----------|-----------------|--------------------------|
| Correct proofs | 437, 438, 441    | 449                      |
| Timeouts   | 10, 10, 7       | 0                        |
| Out of memory | 1, 1, 1         | 0                        |

| Times for correct results | KI+DF Boxes, Eq | KI+KIPDR Boxes, Eq, Mod2 |
|---------------------------|-----------------|--------------------------|
| Total CPU Time (h)        | 0.81, 0.88, 0.88| 3.6                      |
| Mean CPU Time (s)         | 6.7, 7.2, 7.2   | 29                       |
| Median CPU Time (s)       | 5.9, 6.0, 6.0   | 12                       |
| Total Wall Time (h)       | 0.43, 0.46, 0.46| 2.0                      |
| Mean Wall Time (s)        | 3.5, 3.8, 3.8   | 16                       |
| Median Wall Time (s)      | 3.1, 3.2, 3.2   | 6.4                      |

Eq as an invariant generator.\(^{17}\) Of the 10 tasks that are then left, another 3 can be verified when adding the Mod2 template to the combination. The flexibility of KI+KIPDR allows for another 7 tasks to be solved: For two of these tasks, namely functions_true-unreach-call1_true-termination.i and phases_true-unreach-call1.i, the key is still tracking whether variables are even or odd, but the control flow is too complex for our simple data-flow analysis to succeed; the task ddlm2013_true-unreach-call.i requires the disjunctive invariant $a = b \lor flag = 0$; for module_get_put-drivers-net-pppox_false-termination.ko_true-unreach-call.cil.out.i.pp.i, KI+KIPDR succeeds by first proving the auxiliary invariant $ldv\_module\_refcounter > 0$ and then subsequently proving the stronger invariant $ldv\_module\_refcounter = 1$, which in turn is a sufficiently strong auxiliary invariant to solve the task; for each of the remaining three, namely s3_srvr_1_true-unreach-call_false-termination.cil.c, s3_srvr_2_true-unreach-call_false-termination.cil.c, and s3_srvr_8_true-unreach-call_false-termination.cil.c, KI+KIPDR constructs multiple large disjunctive invariants that are too complex to manually dissect, but each of which constrains the state space, such that together, they suffice as an auxiliary invariant to solve the corresponding task.

The simple data-flow-based techniques, however, are much more efficient (see CPU times). This explains why over the whole set of benchmarks, which apparently contains many tasks for which simple invariants are sufficient, the data-flow based techniques are more successful.

\(^{17}\) However, due to the higher precision after adding the Eq template, there is also a task that is no longer solved, thus, the total number of proofs increases only by one.
Table 6: Results of four $k$-induction-based configurations in CPAchecker with different approaches for generating auxiliary invariants for seven manually crafted verification tasks that do not contain bugs and are not solved by $k$-induction without auxiliary invariants; an entry “T” means that the CPU-time limit was exceeded, an entry “M” means that the memory limit was exceeded, and all other entries represent the CPU time a configuration spent to correctly solve the task.

| Task     | Fig. | KI←DF Boxes | KI←DF Boxes, Eq | KI←DF-KIPDR |
|----------|------|--------------|-----------------|-------------|
| const.c  | 6    | 3.3 s        | 3.3 s           | 3.2 s       | 3.8 s       |
| eq1.c    | 7    | T            | 3.2 s           | 3.3 s       | 4.9 s       |
| eq2.c    | 1    | M            | M               | M           | 3.9 s       |
| even.c   | 8    | T            | T               | 3.5 s       | 3.9 s       |
| odd.c    | 9    | T            | T               | 3.4 s       | 4.1 s       |
| mod4.c   | 10   | T            | T               | T           | 3.6 s       |
| bin-suffix-5.c | 11 | M            | M               | M           | 3.6 s       |

To further explore the differences between the approaches regarding their expressive power, we manually created seven additional verification tasks that outline the strengths and weaknesses of the configurations discussed above. We list all discussed example programs as figures in this section, except for eq2.c, which is already listed in Fig. 1. We do not repeat lines 1 to 9, because they are the same as in Fig. 1 for all example programs. Table 6 shows the results we obtained for these tasks. All configurations were able to prove safety for the task const.c (Fig. 6), which was crafted such that it is sufficient to detect the invariant $s = 0$. The task eq1.c shown in Fig. 7 requires the invariant generators to compute the invariant $u = x \land y = z$, for which the abstract domain of Boxes is not strong enough by itself, but which is trivial for the configurations using the Eq template and for KIPDR. The task eq2.c shown in Fig. 1, which has already been discussed as an example in the introduction,

```c
int main(void) {
    unsigned int s = 0;
    while (__VERIFIER_nondet_uint()) {
        if (s != 0) {
            ++s;
        } else if (__VERIFIER_nondet_uint()) {
            __VERIFIER_assert(s == 0);
        }
    }
    return 0;
}
```

Fig. 6: Program `const.c`
int main(void) {
  unsigned int w = __VERIFIER_nondet_uint();
  unsigned int x = w;
  unsigned int y = __VERIFIER_nondet_uint();
  unsigned int z = y;
  while (__VERIFIER_nondet_uint()) {
    if (__VERIFIER_nondet_uint()) {
      ++w; ++x;
    } else {
      --y; --z;
    }
  }
  __VERIFIER_assert(w == x && y == z);
  return 0;
}

Fig. 7: Program eq1.c

int main(void) {
  unsigned int x = 0;
  while (__VERIFIER_nondet_int()) {
    x += 2;
  }
  __VERIFIER_assert(!(x % 2));
  return 0;
}

Fig. 8: Program even.c

requires a similarly simple equality invariant, $y = z$, but the only way to prove it is to first prove that at least initially, $x = w \land y = (w + 1) \land z = (x + 1)$ holds, for which the simple abstract domains are too weak, but which is once again easy for KIPDR. The tasks even.c (Fig. 8) and odd.c (Fig. 9) were specifically designed to be solved only using invariant generators that could provide the invariants $|x| \% 2 = 0$ and $|x| \% 2 = 1$, respectively, so it is not surprising that of the configurations using data-flow-based invariant generators only the one using the Mod2-template was able to solve the task. The task mod4.c shown in Fig. 10, however, while being a simple adaptation of even.c that requires the invariant $|x| \% 4 = 0$, is only solved by KIPDR, as is the task bin-suffix-5.c shown in Fig. 11, which requires an invariant $(x \& 5) = 5$ where ‘&’ is the binary AND operator. While these last two tasks are conceptually not more difficult to solve than the tasks that are solved using templates, each of them would require a new template to be specified (and implemented), whereas the flexibility of KIPDR allows the invariant generator to discover the invariant without any help. This shows that while a fitting data-flow-based invariant generator may be efficient if it is available, KIPDR is generally more powerful, because it does not require specific templates.

The conclusion of our study on hand-crafted verification tasks is that there are programs for which KIPDR is the superior technique, and that one of the following assumptions might be true: (a) the large benchmark set from the sv-benchmarks repository is not diverse enough to represent situations that occur
int main(void) {
  unsigned int x = 1;
  while (__VERIFIER_nondet_int()) {
    x += 2;
  }
  __VERIFIER_assert(x % 2);
  return 0;
}

Fig. 9: Program \texttt{odd.c}

int main(void) {
  unsigned int x = 0;
  while (__VERIFIER_nondet_int()) {
    x += 4;
  }
  __VERIFIER_assert(!(x % 4));
  return 0;
}

Fig. 10: Program \texttt{mod4.c}

in practice, or, (b) the vast majority of programs needs only ‘simple’ invariants
(that can be constructed using data-flow techniques).

On the chosen benchmark set, our experiments support the hypothesis that
KIPDR can be very strong and efficient on tasks that other approaches can not
solve (tasks with ‘interesting’ invariants). It is important to note that this is
an ‘exists’ statement and can not be generalized, as shown by the results that
KIPDR is often outperformed by simpler, data-flow-based invariant-generation
techniques. ✓

Further Discussion. The seven example programs\footnote{https://github.com/sosy-lab/sv-benchmarks/tree/svcomp19/c/loop-invariants/} were added to the bench-
mark collection that was also used for SV-COMP 2019, and thus, results are
available for all verifiers that participated in the competition\footnote{See the last seven rows in this table: https://sv-comp.sosy-lab.org/2019/results/results-verified/ReachSafety-Loops.table.html}. Table 7 summa-
rizes the results of the best six verifiers in comparison with the KI⟲⟲⟲←−
KIPDR approach that we created for the study in this paper. Those verifiers are, in
alphabetical order, Skink, Ultimate Automizer, Ultimate Kojak, Ultimate
Taipan, VeriAbs, and VIAP. Please note that the results of those six verifiers
were obtained in a slightly different environment than the one for the evaluations
we performed in this paper, because while the same hardware configuration was
used for both sets of results, the operating-system version used in SV-COMP 2019
was Ubuntu 18.04 based on Linux 4.15.

As should be expected, we observe that there are several approaches other
than KI⟲⟲⟲→−KIPDR that can also solve the tasks we crafted. For example,
because each loop in our seven manually crafted examples can be replaced by a
single linear-arithmetic computation, VeriAbs is able to apply loop-acceleration
to solve the tasks. We also looked into the reasons why Ultimate Automizer,
int main(void) {
    unsigned int x = 5;
    while (__VERIFIER_nondet_int()) {
        x += 8;
    }
    __VERIFIER_assert((x & 5) == 5);
    return 0;
}

Fig. 11: Program bin-suffix-5.c

Table 7: Results of SV-COMP 2019 for the six verifiers that performed best on our seven manually crafted verification tasks, compared to the results of KI–KIPDR approach previously shown in Table 6; an entry “T” means that the CPU-time limit was exceeded, an entry “M” means that the memory limit was exceeded, an entry “O” means that the verifier gave up deliberately for other reasons, and all other entries represent the CPU time a verifier configuration spent to correctly solve the task; note that SV-COMP 2019 used Ubuntu 18.04 based on Linux 4.15, whereas our evaluation of KI–KIPDR used Ubuntu 16.04 based on Linux 4.4; otherwise, the evaluation environment was the same.

| Task       | Skink | UAutomizer | UKojak | UTaipan | VeriAbs | VIAP | KI–KIPDR |
|------------|-------|------------|--------|---------|---------|------|----------|
| const.c    | 4.2 s | 8.7 s      | 9.1 s  | 8.2 s   | 13 s    | 110 s| 3.8 s    |
| eq1.c      | 290 s | 7.8 s      | 7.6 s  | 8.3 s   | 14 s    | 57 s | 4.9 s    |
| eq2.c      | 4.1 s | 8.1 s      | 8.6 s  | 7.6 s   | 14 s    | 4.7 s| 3.9 s    |
| even.c     | 3.7 s | 7.4 s      | 8.2 s  | 8.6 s   | 140 s   | 4.5 s| 3.9 s    |
| odd.c      | O     | 9.6 s      | T      | 11 s    | 140 s   | 4.6 s| 4.1 s    |
| mod4.c     | 4.0 s | 8.4 s      | 8.4 s  | 7.7 s   | 140 s   | 4.5 s| 3.6 s    |
| bin-suffix-5.c | O | 14 s  | T      | 13 s    | 13 s    | 4.7 s| 3.6 s    |

which uses automata-based trace abstraction [25]. Interestingly, Ultimate Automizer, which uses the SMT solver Z3, can solve all seven tasks, a reimplementation of the same algorithm in CPAchecker can also solve them if configured to use Z3, but the same implementation cannot solve them using MathSAT5, because no suitable interpolants are generated, i.e., in the case of Ultimate Automizer, the results appear to be related more to the choice of SMT solver rather than the algorithm itself. While we already know from our previous experiments that in general, data-flow analyses can be very efficient invariant generators, this comparison between KI–KIPDR and the best verifiers from SV-COMP 2019 reconfirms our observation from Table 6 that there are tasks where other approaches are necessary and available, and that one such approach is KI–KIPDR, which performs at least as good as the best available verifiers for all seven examples, in some cases even significantly better.

20 A thorough evaluation of the impact of the choice of the SMT solver, and the used theory, on verification results was done by Wendler [35].
The results for Hypotheses 2 and 3 suggested to reconfirm the insight by Cimatti and Griggio that PDR is most effective if only applied as a fall-back engine for cases where a cheaper interpolation engine fails to produce useful interpolants [16]. The results of Table 7 draw a more optimistic picture.

4.11 Hypothesis 5: Relevancy

To conclude our evaluation, we establish the relevancy of our previous conclusions by showing that the best of our configurations that use PDR is competitive when compared to the best available tool implementations of adaptations of PDR to software verification, and by comparing KI→DF;KIPDR against the best verifiers in the subcategory ReachSafety-Loops from SV-COMP 2019, a category that is well known to contain many tasks that require effort to be spent on generating loop invariants.

Comparison against PDR-Based Verification Tools. The last three columns of Table 2 give an overview over the best configurations of three different software verifiers that use adaptations of PDR. For CPAchecker, we selected KI→DF;KIPDR. For SeaHorn, we used the same configuration as submitted by the developers to the 2016 Competition on Software Verification (SV-COMP 2016) [24]. For Vvt, we used the portfolio configuration. We observe that SeaHorn achieves the highest number of correct proofs, but also has a significant amount of incorrect proofs. CPAchecker is the slowest of the three tools and finds fewer proofs than SeaHorn, but CPAchecker has no wrong proofs,
and also closely leads in the amount of found bugs. The score-based quantile plot of these results displayed in Fig. 12 visualizes the effects of incorrect results on the computed score. While the graph for SeaHorn is longer, i.e., shows that it solved the most tasks, it is offset to the left by a total penalty of $-3344$ points, such that in the end, KI+$\emptyset$-DF;KIPDR accumulates the highest score because it has a smaller penalty of only $-32$ points. The plot also shows again, as in Fig. 4, that the Java-based CPACHECKER has a much higher startup time (about 3 s) than the other two tools, which return results almost immediately for some tasks.

These results confirm our hypothesis that our previous conclusions are relevant, because they are supported by an implementation that is competitive when compared to the best available PDR-based tool implementations.

**Comparison against the Best Participants of SV-COMP 2019 in the Category ReachSafety-Loops.** Some of the previous experiments appeared to suggest that KIPDR is a slow invariant generator that causes KI+$\emptyset$-KIPDR to be an inferior choice in many cases. While Table 7 only shows a few anecdotal counterexamples to this interpretation, where KI+$\emptyset$-KIPDR is marginally faster than the best verifiers from SV-COMP 2019 on six out of seven selected example tasks, we will now consider how KI+$\emptyset$-KIPDR compares to the best verifiers from SV-COMP 2019 in the subcategory ReachSafety-Loops, which is known to contain many tasks that require effort to be spent on generating loop invariants.

For a fair comparison, we re-executed our new implementations in the same execution environment, i.e., not only on the same hardware configuration but also, unlike our prior experiments, on Ubuntu 18.04 based on Linux 4.15. In this newer execution environment, the Java version used was OpenJDK 11.0.

Table 8 shows the results for all 208 verification tasks of the SV-COMP 2019 subcategory ReachSafety-Loops, 59 of which contain bugs, while the other 149 are considered to be safe, for the best three verifiers in that category, namely ULTIMATE AUTOMIZER, ULTIMATE TAIPIAN, and VERIABS, as well as for KI+$\emptyset$-KIPDR and KI+$\emptyset$-DF;KIPDR. While the SV-COMP competitors clearly solve more tasks correctly and obtain a significantly higher score than KI+$\emptyset$-KIPDR and KI+$\emptyset$-DF;KIPDR, we notice that for the tasks they can solve correctly, the two PDR-based $k$-induction configurations have significantly lower median CPU times than the SV-COMP competitors, and that KI+$\emptyset$-KIPDR uses only about half as much CPU time in the arithmetic mean than the other configurations. For the same set of benchmarks, i.e., the subcategory ReachSafety-Loops of SV-COMP 2019, Fig. 13 directly compares the CPU times spent on tasks by both VERIABS (x-axis), which was the best verifier in that subcategory, and KI+$\emptyset$-KIPDR (y-axis), to visualize the differences in efficiency between the two configurations. We observe that there are more tasks solved by VERIABS for which KI+$\emptyset$-KIPDR exceeds its time limit than there are tasks solved by KI+$\emptyset$-KIPDR for which VERIABS exceeds its time limit, but we also see that for the majority of tasks that were solved by both verifiers, KI+$\emptyset$-KIPDR is faster than VERIABS, in a significant amount of cases even by more than an order of magnitude. This shows that the invariant generator KIPDR is not nec-
Table 8: Results for all 208 verification tasks of the SV-COMP 2019 subcategory ReachSafety-Loops, 59 of which contain bugs, while the other 149 are considered to be safe, for the best three verifiers in that category (Ultimate Automizer, Ultimate Taipan, and VeriAbs), as well as for KI⟲⟲⟲←−KIPDR and KI⟲⟲⟲←−DF;KIPDR.

| Verifier       | SV-COMP 2019 | KI⟲⟲⟲←−KIPDR | KI⟲⟲⟲←−DF;KIPDR |
|---------------|--------------|---------------|-----------------|
| Score         | 221 234 307  | 192 215       |                 |
| Correct results| 144 150 182 | 114 126       |                 |
| Correct proofs | 109 116 125 | 78 89         |                 |
| Correct alarms | 35 34 57    | 36 37         |                 |
| Wrong proofs   | 1 1 0       | 0 0           |                 |
| Wrong alarms   | 0 0 0       | 0 0           |                 |
| Timeouts       | 56 52 18    | 85 76         |                 |
| Out of memory  | 0 0 2       | 2 1           |                 |
| Other inconclusive | 7 5 6      | 7 5           |                 |

Times for correct results

|                  | SV-COMP 2019 | KI⟲⟲⟲←−KIPDR | KI⟲⟲⟲←−DF;KIPDR |
|------------------|--------------|---------------|-----------------|
| Total CPU Time (h)| 1.1 1.1 1.9 | 0.44 1.2      |                 |
| Mean CPU Time (s) | 28 27 97     | 14 34         |                 |
| Median CPU Time (s)| 9.7 10 27   | 5.5 6.2       |                 |
| Total Wall Time (h)| 0.77 0.72 3.8 | 0.23 0.60    |                 |
| Mean Wall Time (s)| 19 17 75    | 7.3 17        |                 |
| Median Wall Time (s)| 3.4 3.7 11  | 2.9 3.2       |                 |

essarily slower than other approaches, and that on the contrary, it can even be significantly faster than other approaches, depending on the benchmark set.

In conclusion, compared to the PDR-based verifiers and compared to the state-of-the-art verifiers that participated in SV-COMP, our PDR-based invariant generator obtains promising results. ✓

4.12 Threats to Validity

External validity: We identify the following threats to the external validity of our experiments: (1) We used the largest and most diverse publicly available benchmark for software verification, but the risk remains that our conclusions are limited to the kinds of programs represented by the benchmark. As there were only two verifiers with support for PDR participating in the competition, this benchmark might contain few or no verification tasks that are easy for PDR and difficult for the other techniques. We tried to limit this bias towards large C programs by using path programs [6] (a way to reduce the complexity for the invariant generator), and we also added a few “hand-made” programs that explore the boundaries of expressive power of the approaches. (2) Our benchmark set contains only C programs, because this is the only language supported by all of the evaluated tools. Therefore, there is no guarantee that our results can be transferred to programs written in other programming languages, for example software
written in functional programming languages. Moreover, it might be possible that PDR-based model checkers for hardware perform better on programs that are converted from C to transition systems that those model checkers use. 

Internal validity: In our benchmarking environment, we execute up to four processes in parallel on each machine to make our large experimental study feasible. A drawback of the resulting shared usage of hardware resources (such as caches or buses) [8] is that this setup may cause some minor noise in the measurements. However, we use the benchmarking framework BenchExec to properly account for the resources (CPU time and memory) of each process and its subprocesses and to ensure that for each physical core of the execution machines, both of its processing units (virtual cores) are allocated to the same process to avoid the measurement errors occurring from shared CPU resources or from dynamic relocation of processes by the OS scheduler, and to avoid performance influences from non-uniform memory access (NUMA) by ensuring that all executions only use memory belonging to their exclusively assigned CPU cores.

5 Conclusion

Property-directed reachability (a.k.a. IC3) is a verification approach that is popular and successful in some fields of formal verification (e.g., hardware designs, Horn clauses). Unfortunately, there is a large gap between this success story and the applicability in practical software verification. We are closing this gap by (a) providing a well-engineered implementation of one published adaptation of PDR to software verification, (b) designing and implementing an invariant
generator based on the ideas of PDR, and (c) providing an evaluation of all applicable tools and approaches on the largest available benchmark set of C verification tasks. This provides a good foundation as baseline for ongoing research in this area.

The results of our comparative evaluation extend the knowledge about PDR for software verification in the following ways: (1) Our implementation outperforms the existing implementation of PDR (Vvt) and is more precise than the other software verifier that uses PDR (SeaHorn). Thus, our implementation can serve as a reference implementation for further research on PDR for software verification. (2) On most of the programs in the widely used sv-benchmarks collection of verification tasks, other techniques are more effective (solve more problems) and more efficient (solve the problems faster). (3) PDR can be an effective and efficient technique for computing invariants that are difficult to obtain: there are programs for which our PDR-based approach is more efficient than the best invariant generator from SV-COMP in the subcategory ReachSafety-Loops.

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