Meson-Nucleon Vertex Form Factors at Finite Temperature Using a Soft Pion Form Factor

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The temperature and density dependence of the meson-nucleon vertex form factors is studied in the framework of thermofield dynamics. Results are obtained for two rather different nucleon-nucleon potentials: the usual Bonn potential and the variation with a softer \( \pi NN \) form factor, due to Holinde and Thomas. In general, the results show only a modest degree of sensitivity to the choice of interaction.

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Recently \cite{b}, the temperature dependence of the meson nucleon form factors has been calculated in Thermofield Dynamics (TFD) using the Bonn \( N - N \) interaction \cite{b} that includes exchanges of \( \pi, \sigma, \rho \) and \( \omega \) mesons, with the \( \pi NN \) vertex form factor having a monopole form

\[
G_{\pi NN}(q^2) = g_{\pi NN} \frac{\Lambda^2 - m^2_{\pi}}{\Lambda^2 + q^2}, \tag{1}
\]

where \( \Lambda = 1.3 \text{ GeV} \) for the \( \pi NN \) vertex. It is anticipated that in low energy \( N - N \) scattering \( q^0 \approx 0 \) so that the 4-momentum transfer squared is usually minus the 3-momentum transfer squared. However it has been argued \cite{b} that there are several factors that point to a significantly softer form factor with \( \Lambda \approx 700 \div 800 \text{ MeV} \). It was concluded that the \( \pi NN \) form factor for a free nucleon is close to the measured axial-vector form factor, thus giving \( \Lambda \approx 500 \div 800 \text{ MeV} \). It has been suggested that a soft \( \pi NN \) form factor would make it difficult to fit the \( N - N \) scattering and, in particular the properties of the deuteron. However Holinde and Thomas \cite{b} established that a good fit can be obtained for \( \Lambda = 800 \text{ MeV} \).

The study of the temperature and density dependence of the coupling constants, form factors and the critical temperature, where the coupling constant goes to zero, is needed for this softer \( \pi NN \) form factor. This would clarify any dependence on the choice of the interaction and give us a better understanding of the equation of state that may be relevant for heavy-ion collisions. The behaviour of nuclear matter at finite temperature and density and the phase transition from the hadronic to the quark-gluon phase with subsequent hadronisation can provide valuable information about the nature of confinement in QCD.

In this brief communication we present results for the temperature and density dependence of the meson-nucleon couplings, using the OBEPTI \( N - N \) interaction \cite{b}, and compare them to the results obtained earlier with the BONN potential, OBEPT \cite{b}. The main calculational procedure has been described earlier \cite{b} and we concentrate on discussing the results in the present note.

The temperature dependence is calculated within Thermofield Dynamics, a finite temperature field theory. The finite temperature modifications of the vertex functions are calculated from the one-meson-exchange Feynmann diagrams shown in Fig. \ref{b}. The mathematical representation was explained earlier \cite{b}. To initiate the calculations, zero temperature coupling constants and vertex form factors are chosen to be either i) the Bonn potential, OBEPT, or ii) the modified Bonn potential with a soft \( \pi NN \) form factor, OBEPTI, due to Holinde and Thomas (see Table \ref{b}). It is important to note that results for \( \pi' \), appearing in OBEPTI, can be obtained from those of the \( \pi \)-meson by the following procedure

\[
f(q^2) = \frac{G_{\pi' NN}(q^2, T, p)}{G_{\pi NN}(q^2, T, p)} = \frac{g_{\pi' NN}(q^2, 0, 0)}{g_{\pi NN}(q^2, 0, 0)}
= \frac{g_{\pi' NN}}{g_{\pi NN}} \left( \frac{\Lambda_{\pi'}^2 - m^2_{\pi'}}{\Lambda^2 - m^2_{\pi}} \right) \left( \frac{\Lambda^2 + q^2}{\Lambda_{\pi'}^2 + q^2} \right).
\tag{2}
\]

The ratios of the coupling constants, \( g_{BNN}(T)/g_{BNN}(0) \), are plotted in Fig. \ref{b} Note that the \( \eta, \delta \) and \( \pi' \) mesons are present only for the potential of Holinde and Thomas. We observe that for the \( \pi, \pi', \rho, \sigma \) and \( \eta \)-mesons this ratio goes through zero at different temperature depending on the density. This temperature is called the critical temperature, \( T_c \). For \( \omega \) and \( \delta \)-mesons the behaviour of the ratio is opposite – i.e. it increases instead of going through zero. For \( \pi \)-mesons the difference between the

\[
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\]
For other applications. At small momentum transfers we can parametrize them by a monopole form

$$G_{BNN}(\vec{q}^2, T, \rho) = \frac{A_B^2(T, \rho)}{\Lambda_B^2(T, \rho) + \vec{q}^2},$$

(3)

where we still use the mass of the corresponding meson at zero temperature and density. In general the effective mass of a meson, $m_B$, is also temperature dependent, but that has not been considered here. From Fig. 5 we see that $T_c$ has an almost linear dependence on density, except for the case of $\sigma$-meson. However, even for this case, at moderate densities there is an almost linear dependence. Consequently, we can write the following density dependence of the critical temperature

$$T_c = T_0 \left(1 + D_T \frac{\rho}{\rho_0}\right).$$

(4)

Furthermore, using a polynomial form for the ratios

$$g_B(T, \rho) = \frac{1 + (T/T_c)\alpha_B^0 + (T/T_c)^2\beta_B^0}{1 + (\rho/\rho_0)C_B^0},$$

$$A_B(T, \rho) = \frac{1 + (T/T_c)\alpha_B^0 + (T/T_c)^2\beta_B^0}{1 + (\rho/\rho_0)C_B^0},$$

(5)

we find the values of the parameters given in Table III. Again it is clear that $G_{\pi NN}$ can be restored from $G_{\pi NN}$ as given in Eq. (2).

In conclusion, the choice of hard and soft $\pi NN$ form factors does change the behaviour of the critical temperature with density. While the changes are not, in general, dramatic, there is a qualitative change for the $\rho$ meson. When it comes to the $q^2$-dependence of the form-factors, the changes for all mesons are similar. The parametrization of the changes with temperature and density of the boson coupling constants to the nucleon, as well as the cut-off parameters in the form factors, provides a useful way to summarise the variation of these parameters. Finally it is important to realize that while the couplings of the mesons to nucleons decrease to zero at a critical temperature, it is possible that before we reach that temperature, the system may have already undergone a phase transition, leading to a deconfined quark-gluon state.

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| Meson | $I(J^P)$ | Mass (GeV) | $\Lambda_\omega$ (GeV) | $g_\omega^2/4\pi$ | $f_\omega/g_\omega$ |
|-------|---------|------------|----------------------|------------------|------------------|
| $\pi$  | 1(0^-) | 0.138      | 1.05                 | 14.9             | ...              |
| $\rho$ | 1(1^-) | 0.769      | 1.3                  | 0.99             | 6.1              |
| $\omega$ | 0(1^-) | 0.783      | 1.5                  | 20.0             | ...              |
| $\sigma$ | 0(0^+) | 0.550      | 2.0                  | 8.383            | ...              |

The behaviour of $T_c$ for various mesons using the two potentials is quite different. For the $\pi$-- and $\sigma$-- mesons, $T_c^B$ changes more rapidly for the softer pion vertex, while for the $\rho$--meson $T_c^B$ is practically constant – in contrast with the OBE potential, where it changes rapidly. The changes in $T_c^B$ show quite a significant dependence on the choice of potential.

We have also investigated the $q^2$-dependence of the meson-nucleon form factors with temperature. The results of these calculations presented in Fig. 3. In this case the pion form factor shows a large change and a stronger dependence on the choice of $T = 0$ parameter. For other mesons the form-factors show very little dependence on a hard or soft pion vertex function.

It is clear that at density $\rho_0$, the behaviour of the critical temperature for the $\rho$ meson is dramatically changed when the parameters of the OBE potential are changed. In other words, $g_{\rho NN}$ is sensitive to parameter changes. For the OBEPOTI potential $T_c^\rho(\rho)$ is almost constant, while for the OBE potential $T_c^\rho(\rho)$ decreases rapidly. For the $\pi$ meson $T_c(\rho)$ decreases even more rapidly with increasing density. For other mesons $T_c(\rho)$ has the same behaviour, increasing with increasing density. For all mesons except the pion, the critical temperature increases when one uses the OBEPOTI parametrization. Nevertheless, at moderate densities the critical temperature for the $\pi$ is also somewhat higher.

In Fig. 4 we present the form factors at several temperatures at zero density. The form factors of the $\pi NN$ and $\rho NN$ vertices are more sensitive to parameter changes, while $G_{\omega NN}$ and $G_{\sigma NN}$ are not. This is also seen from Fig. 3 where the critical temperature for the $\omega$ and $\sigma$ mesons is almost the same for both parameter sets at zero density. At high densities $G_{\tau NN}$, and $G_{\rho NN}$ fall off more rapidly at high momentum transfer, constituting quenching of the cut off parameters for corresponding meson form factors.

In conclusion, the choice of hard and soft $\pi NN$ form factors does change the behaviour of the critical temperature with density. While the changes are not, in general, dramatic, there is a qualitative change for the $\rho$ meson. When it comes to the $q^2$-dependence of the form-factors, the changes for all mesons are similar. The parametrization of the changes with temperature and density of the boson coupling constants to the nucleon, as well as the cut-off parameters in the form factors, provides a useful way to summarise the variation of these parameters. Finally it is important to realize that while the couplings of the mesons to nucleons decrease to zero at a critical temperature, it is possible that before we reach that temperature, the system may have already undergone a phase transition, leading to a deconfined quark-gluon state.
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[1] A.M. Rakhimov, U.T. Yakhshiev, F.C. Khanna. Phys. Rev. C61, 024907 (2000).
[2] R. Machleidt, Adv. Nucl. Phys. 19, 189 (1989).
[3] A.W. Thomas, K. Holinde. Phys. Rev. Lett. 63, 2025 (1989); P. A. Guichon, G. A. Miller and A. W. Thomas, Phys. Lett. B 124, 109 (1983).
[4] K. Holinde, A.W. Thomas, Phys. Rev. C42, R1195 (1990).

| Meson | $\alpha^g$ | $\beta^g$ | $C^g$ | $\alpha^\Lambda$ | $\beta^\Lambda$ | $C^\Lambda$ | $D$ | $T_c$ |
|-------|------------|------------|-------|------------------|-----------------|-----------|-----|------|
| **OBEP parametrization** |
| $\pi$ | 1.329 | -2.390 | 0.033 | -0.230 | -0.400 | -0.008 | -0.008 | 406 |
| $\rho$ | 0.066 | -1.058 | -0.002 | -0.070 | -1.976 | -0.007 | -0.008 | 285 |
| $\omega$ | 0.138 | 0.808 | -0.008 | 0.008 | -0.524 | 0.006 | 0.031 | 170 |
| $\sigma$ | 0.559 | -1.547 | 0.010 | -0.018 | -0.024 | -0.003 | 0.005 | 93 |
| **OBETI parametrization** |
| $\pi$ | 1.018 | -2.029 | 0.031 | -0.142 | -0.369 | -0.007 | -0.016 | 429 |
| $\rho$ | 0.108 | -1.079 | -0.001 | -0.081 | -2.012 | -0.006 | 0.001 | 281 |
| $\omega$ | 0.062 | 0.904 | -0.013 | 0.005 | -0.547 | 0.008 | 0.058 | 171 |
| $\sigma$ | 0.554 | -1.528 | 0.016 | -0.008 | -0.043 | -0.002 | 0.007 | 93 |
| $\eta$ | 1.449 | -2.442 | -0.020 | -0.963 | 14.468 | -0.003 | 0.012 | 478 |
| $\delta$ | 0.211 | 0.724 | -0.014 | 0.233 | -0.364 | 0.019 | 0.045 | 152 |

TABLE II: Parameters of vertex form factors in Eqs. (4)-(5). The last column is the critical temperature, $T_c$, at zero density, $\rho = 0$. 
FIG. 1: Feynman diagrams for the pion-nucleon vertex. The solid line indicates the nucleon, while the dashed, dotted, dot-dashed, wavy, dot-dot-dashed, dot-dot-dot-dashed and dashed-dashed-dot lines indicate the $\pi^-$, $\rho^-$, $\omega^-$, $\sigma^-$, $\pi'$, $\eta^-$ and $\delta^-$ mesons, respectively.

FIG. 2: The ratio of meson-nucleon coupling constants at finite temperature, $T$, to those at $T = 0$, as a function of temperature at $\rho = 0$ (stars and fivestars), $\rho = \rho_0$ (squares and triangles), $\rho = 5\rho_0$ (dotted circles and simple dots). Stars, squares and dotted circles correspond to calculations using the OBEP potential. Fivestars, triangles and dots correspond to calculations using the OBEPTI potential. Results for the $\pi'$ coincide with the OBEPTI part of figure a (see Eq. (2)).
FIG. 3: Density dependence of the critical temperature for several (a: π, b: ρ, c: ω, d: σ, e: η, δ) mesons. In figures a - d the solid lines correspond to the calculation based on the OBEP potential \cite{2}, while the dashed lines correspond to calculations using the OBEPTI potential \cite{4}. In figure e the solid line corresponds to the η and the dashed line to the δ. Calculations in figure e are made using the OBEPTI potential. The results for the π' coincide with the OBEPTI part of figure a.

FIG. 4: Meson-nucleon form factors at several temperatures as a function of $q^2$ using OBEP \cite{2} (stars, squares and dotted circles) and OBEPTI potentials \cite{4} (fivestars, triangles and dots) at $\rho = 0$. Results for the π' can be extracted from the OBEPTI part of figure a, taking into account the scaling function, $f(q^2)$ (see Eq. \ref{eq:2}).