Gravitational formfactors of the $\rho$ meson in QCD sum rules

T. M. Aliev$^*$

Physics Department, Middle East Technical University, Ankara 06800, Turkey

T. Barakat$^†$

Physics Department, King Saud University, Riyadh 11451, Saudi Arabia

K. Şimşek$^‡$

Department of Physics & Astronomy, University of Rochester, Rochester, NY 14627, USA

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Abstract

By using the quark part of the energy-momentum tensor current, the gravitational formfactors of the $\rho$ meson are calculated within the light-cone sum rules method. In the considered version, the energy-momentum tensor current is not conserved and as a result, there appear nine formfactors, six (three) of which correspond to the conservation (nonconservation) of the energy-momentum tensor current. We also compare our results with the one existing in the literature.

1 Introduction

During the last years, the energy-momentum tensor (EMT) has become one of the popular research objects for better understanding the structure of hadrons [1, 2]. The gravitational formfactors (GFFs) are defined with the help of the matrix element of the symmetric EMT (see [3, 4]). The GFF relates the mass, spin, total angular momentum, etc. Understanding the EMT can help answer questions as to the origin of the nucleon mass, spin carried by quarks and gluons, and how the strong force is distributed inside hadrons. The total GFFs were introduced a long time ago for both spin-0 and spin-1/2 hadrons [5]. The GFFs for spin-1 particles have been discussed in the literature [6, 7, 8, 9, 10].

The GFFs of the $\rho$ meson was studied within the light-cone constituent quark model in [11]. It should be emphasized that the study of GFFs for hadrons started with [12] and experimental measurements through deeply virtual Compton scattering [13]. The GFFs were also calculated in chiral perturbation in [14].

In the present work, we study the GFFs of the $\rho$ meson within the light-cone sum rules. The paper is

$^*$email: taliev@metu.edu.tr
$^†$email: tbarakat@ksu.edu.sa
$^‡$email: ksimsek@ur.rochester.edu
organized as follows. In Section 2, we derive the sum rules for the GFFs under study. In Section 3, we perform the numerical analysis for the sum rules obtained in the previous section. Section 4 contains our conclusion.

2 Sum rules for the $\rho$ meson GFFs

For the calculation of the GFFs of the $\rho$ meson within the light-cone sum rules, we introduce the following correlation function:

$$\Pi_{\mu\nu\lambda} = i \int d^4x \, e^{iqx} \langle \rho(p) \mid T_{\mu\nu}(x) J_\lambda(0) \mid 0 \rangle$$  \hspace{1cm} (1)

where $J_\lambda = \bar{u}\gamma_\lambda d$ is the interpolating current of the $\rho$ meson, the $T_{\mu\nu}(x)$ is the EMT including only the contribution of quark fields,

$$T_{\mu\nu}^\rho = \frac{i}{4} \left[ \bar{\psi} (\not{D}_\mu \gamma_\nu + \not{D}_\nu \gamma_\mu) \psi - g_{\mu\nu} i \frac{2}{2} (\not{D} - m_q) \psi \right]$$  \hspace{1cm} (2)

where $\not{D}_\mu = \partial_\mu \pm igA_\mu^a \frac{\lambda^a}{2}$.

In [3], it is obtained that the second term of the EMT can be written as $g_{\mu\nu}(\not{D} - m_q) \psi \approx g_{\mu\nu}(1 + \gamma_m) m_q q\bar{q}$ where $\gamma_m$ is the anomalous dimension of the mass operator. In the present work, we are working in the chiral limit ($m_q \rightarrow 0$), hence the second term of the EMT can be neglected.

We start our consideration by computing the correlation function from the hadronic side. Its representation from the hadronic side is obtained by inserting a complete set of mesons carrying the same quantum numbers as the $\rho$ meson and, isolating the contribution of the ground state, we get

$$\Pi_{\mu\nu\lambda} = \frac{\langle \rho(p) \mid T_{\mu\nu}^\rho \mid \rho(p') \rangle \langle \rho(p') \mid J_\lambda \mid 0 \rangle}{p^2 - m_\rho^2} + \text{higher states}$$  \hspace{1cm} (3)

The matrix elements in Eq. (3) are defined as

$$\langle \rho(p') \mid J_\lambda \mid 0 \rangle = f_\rho m_\rho \epsilon_\lambda$$  \hspace{1cm} (4)

where $f_\rho$ is the $\rho$ meson decay constant, $m_\rho$ is its mass, and $\epsilon_\lambda$ is its polarization vector. The EMT of a spin-1 particle in QCD is defined as (see, for example, [10, 15])

$$\langle \rho(p) \mid T_{\mu\nu}^\rho \mid \rho(p') \rangle = 2 P_\mu P_\nu \left( - \epsilon^* \cdot \epsilon A_0(q^2) + \frac{\epsilon^* \cdot P \epsilon \cdot P}{m_\rho^2} A_1(q^2) \right) +$$

$$+ 2 [ P_\mu (\epsilon^*_\nu \epsilon \cdot P + \epsilon_\nu \epsilon^* \cdot P) + P_\nu (\epsilon^*_\mu \epsilon \cdot P + \epsilon_\mu \epsilon^* \cdot P) ] J(q^2)$$

$$+ \frac{1}{2} (g_\mu q_\nu - g_\nu q_\mu q^2) \left( \epsilon^* \cdot \epsilon D_0(q^2) + \frac{\epsilon^* \cdot P \epsilon \cdot P}{m_\rho^2} D_1(q^2) \right)$$

$$+ \left[ \frac{1}{2} (\epsilon_\mu \epsilon^*_\nu + \epsilon^*_\mu \epsilon_\nu) q^2 + (\epsilon_\mu q_\nu + \epsilon^*_\mu q_\nu) \epsilon \cdot P - (\epsilon_\nu q_\mu + \epsilon^*_\nu q_\mu) \epsilon^* \cdot P - 4 g_{\mu\nu} \epsilon^* \cdot P \epsilon \cdot P \right] E(q^2)$$

$$+ \left( \epsilon_\mu \epsilon^*_\nu + \epsilon^*_\mu \epsilon_\nu - \frac{1}{2} \epsilon^* \cdot g_{\mu\nu} \right) m_\rho^2 F(q^2) + g_{\mu\nu} (\epsilon^* \cdot m_\rho^2 C_0(q^2) + \epsilon^* \cdot P \epsilon \cdot PC_1(q^2))$$  \hspace{1cm} (5)

where $P = \frac{1}{2} (p + p')$ and $q = p' - p$. Here, the superscript $q$ indicates that the considered GFF contains contributions only from the quark part of the EMT.
In Eq. (5), the first six formfactors are individually (separately for quark and gluon fields) energy-momentum conserving and the remaining three are not. Due to the sum of quark and gluon parts, the EMT conservation leads to the constraints \( \sum_{a=q,g} F^a(Q^2) = \sum_{a=q,g} C^a_0 = \sum_{a=q,g} C^a_1 = 0 \). In addition to these constraints, there are the normalization conditions \( \sum_{a=q,g} A^a_0(Q^2) = 1 \) and \( \sum_{a=q,g} J^a(Q^2) = 1 \).

Since in Eq. (5) we have nine formfactors, we need nine independent Lorentz structures for the determination of these formfactors. We choose the following structures:

\[
\Pi_{\mu\nu\lambda} = \epsilon^*_{\lambda} p_{\nu} q_{\mu} \Pi_1 + p_m u g_{\mu\nu} \epsilon^*_{q} \cdot q \Pi_2 + \epsilon^*_{\mu} p_{\nu} q_{\mu} \Pi_3 + \epsilon^*_{\lambda} q_{\mu} q_{\nu} \Pi_4 + p_{\nu} p_{\lambda} q_{\mu} \epsilon^*_{q} \cdot q \Pi_5 + q_{\mu} q_{\nu} p_{\lambda} \epsilon^*_{q} \cdot q \Pi_6 + \epsilon^*_{\mu} g_{\nu\lambda} \Pi_7 + p_{\lambda} g_{\mu\nu} \epsilon^*_{q} \cdot q \Pi_8 + q_{\mu} g_{\nu\lambda} \epsilon^*_{q} \cdot q \Pi_9 + \cdots
\]

(6)

Now, we calculate the correlation function from the QCD side. Using the explicit forms of the interpolating formfactors, we choose the following structures:

\[
\langle \bar{u}(x) \gamma_{\mu} \gamma_{5} D_{\mu} S(x) \rangle = \sum_{a=q,g} \langle \bar{u}(0) \gamma_{\lambda} D_{\mu} S(0) \rangle \cdot \langle \bar{u}(x) \gamma_{\mu} \gamma_{5} D_{\mu} S(x) \rangle
\]

(7)

From this formula, it follows that for the calculation of the correlation function from the QCD side, the expression of the quark propagator in the presence of a background field is needed. This expression was obtained in [16] as

\[
S_{q}(x) = \frac{i\gamma_{5}}{2\pi^{2}x^{4}} - \frac{ig_{s}}{16\pi^{2}x^{2}} \int_{0}^{1} du \left[ \tilde{u} \sigma_{\alpha\beta} \gamma_{5} \bar{q} \gamma_{\alpha} \gamma_{\beta} \right] + \frac{1}{4} \langle \bar{u}\gamma_{5}d \rangle 0 \]

(8)

where \( G^{\alpha\beta} \) and \( F^{\alpha\beta} \) are the gluon and photon field strength tensors, respectively. Performing relevant calculations for the correlation function from QCD, we get

\[
\Pi_{\mu\nu\lambda} = -\frac{1}{4} \left( \frac{i}{2\pi^{2}} \sum_{i} \int du \left[ \frac{1}{4} \langle \rho | \bar{u}\gamma_{i} d | 0 \rangle \right] \right.
\]

(9)

In these expressions, \( \Gamma_{i} = \{ 1, \gamma_{5}, \gamma_{\mu}, \gamma_{\mu} \gamma_{5}, \gamma_{5} \} \) is the full set of Dirac matrices. The matrix elements \( \langle \rho | \bar{u}\gamma_{i} d | 0 \rangle \) and \( \langle \rho | \bar{u}\gamma_{i} G_{\alpha\beta} d | 0 \rangle \) are expressed in terms of the \( \rho \) meson distribution amplitudes (DAs) of different twists. These DAs are the main nonperturbative parameters of the light-cone sum rules. Of the aforementioned matrix elements, only the vector and axial components survive after taking the trace, which are defined as (see [17, 18, 19, 20])

\[
\langle \rho(p) | \bar{u}(x) \gamma_{\mu} d(0) | 0 \rangle = f_{\rho} m_{\rho} \left\{ \frac{\epsilon^{*} \cdot x}{p \cdot x} \int_{0}^{1} du \ e^{\frac{i\bar{u}p_{x}x}{2m_{\rho}} A_{\parallel}(u)} + \frac{m_{\rho}^{2} x^{2}}{16} A_{\parallel}(u) \right\}
\]

\[
+ \left( \epsilon^{*} - p_{\mu} \frac{\epsilon^{*} \cdot x}{p \cdot x} \int_{0}^{1} du \ e^{\frac{i\bar{u}p_{x}x}{2m_{\rho}} g_{\perp}(u)} \right)
\]

\[
- \frac{1}{2} x_{\mu} \frac{\epsilon^{*} \cdot x}{(p \cdot x)^{2}} m_{\rho}^{2} \left( \int_{0}^{1} du \ e^{\frac{i\bar{u}p_{x}x}{2m_{\rho}} g_{\perp}(u) + \phi_{\parallel}(u) - 2g_{\parallel}(u)} \right)
\]

(10)
\begin{align}
\langle \rho(p) | \bar{u}(x) i\gamma_\mu \gamma_5 d(0) | 0 \rangle &= -\frac{i}{4} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu\rho} x^\rho \int_0^1 du \ e^{i p x u} g_\alpha^\nu(u) \\
\langle \rho(p) | \bar{u}(x) G_{\alpha\beta} \gamma_5 d(0) | 0 \rangle &= \frac{f_{\rho \mu}}{g_\rho} \langle \rho(p) | \bar{u}(x) \gamma_\mu G_{\alpha\beta}^\nu d(0) | 0 \rangle \\
\langle \rho(p) | \bar{u}(x) G_{\alpha\beta} \gamma_i \gamma_5 d(0) | 0 \rangle &= \frac{f_{\rho \mu}}{g_\rho} \langle \rho(p) | \bar{u}(x) \gamma_\mu G_{\alpha\beta}^\nu \gamma_i d(0) | 0 \rangle
\end{align}

where \( \tilde{G}_{\alpha\beta} \) is the dual gluon field strength tensor and \( \int \mathcal{D} \alpha_i = \int d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3) \).

Using the DAs and performing Fourier and Borel transformations on the theoretical part of the correlation function, we get

\begin{align}
\Pi_1 &= \frac{1}{8} f_{\rho \mu \rho}(\mathcal{I}_1 [g^\mu_\alpha(u) + (g^\mu_\alpha(u) + 4g^\mu_\alpha(u))(-1 + u), 1] + \mathcal{I}_3[g^\mu_\alpha(u) + g^\mu_\alpha(u)(u - 4g^\mu_\alpha(u)u), 1] \\
&+ 4m^2_\rho(2\mathcal{I}_1([-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|((u), 1) + 2\mathcal{I}_3([-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|((u), 2] \\
&+ \mathcal{I}_5[(\alpha_1 + \alpha_3 - \alpha_3 u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i))(-1 + 2u), 2] - \mathcal{I}_6[(\alpha_1 + \alpha_3 u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2])}
\end{align}

\begin{align}
\Pi_2 &= \frac{1}{8} f_{\rho \mu \rho}(4\mathcal{I}_1[g^\mu_\alpha(u) - \phi_\|(u) + \phi_\|(u) - \phi_\|(u), 1] + \mathcal{I}_3[-4g^\mu_\alpha(u) + \phi_\|(u) + \phi_\|(u)u), 1] \\
&+ m^2_\rho((-1 + u) + 2\mathcal{I}_1([-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)(1 + u), 2] \\
&+ \mathcal{I}_3[\tilde{A}_i(u) + 2\mathcal{A}_i(u) + 2(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u), 2] + 4\mathcal{I}_5[(\alpha_1 + \alpha_3 - \alpha_3 u)(\mathcal{V}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2] \\
&+ \mathcal{I}_6[(\alpha_1 + \alpha_3 u)(\mathcal{V}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2])}
\end{align}

\begin{align}
\Pi_3 &= \frac{1}{4} f_{\rho \mu \rho}(2\mathcal{I}_1[g^\mu_\alpha(u) - \phi_\|(u), 1] + 2\mathcal{I}_3[-4g^\mu_\alpha(u) + \phi_\|(u), 1] \\
&+ m^2_\rho((-1 + u) + 2\mathcal{I}_3([-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)(1 + u), 2] + \mathcal{I}_3[\tilde{A}_i(u) + 2(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)u), 2])}
\end{align}

\begin{align}
\Pi_4 &= \frac{1}{8} f_{\rho \mu \rho}(-\mathcal{I}_1[g^\mu_\alpha(u) + 4g^\mu_\alpha(u), 1] + \mathcal{I}_3[g^\mu_\alpha(u) - 4g^\mu_\alpha(u)u], 1] + 4m^2_\rho(2\mathcal{I}_1([-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u), 2] \\
&- 2\mathcal{I}_3([-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u) + \phi_\|(u)u), 2] + \mathcal{I}_5[-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1 + 2u), 2] + \mathcal{I}_6[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{A}(\alpha_i)u), 2])}
\end{align}

\begin{align}
\Pi_5 &= f_{\rho \mu \rho}(2\mathcal{I}_1[\tilde{g}^\mu_\alpha(u) - \phi_\|(u)(-1 + u), 2] + 2\mathcal{I}_3[g^\mu_\alpha(u) - \phi_\|(u), 2] \\
&+ 2m^2_\rho(\mathcal{I}_1[\tilde{A}_i(u) - 2(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)(-1 + u), 2] \\
&+ \mathcal{I}_3[\tilde{A}_i(u) + 2(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)u), 3] - \mathcal{I}_5[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2] \\
&- \mathcal{I}_6[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2])}
\end{align}

\begin{align}
\Pi_6 &= \frac{1}{4} f_{\rho \mu \rho}(-\mathcal{I}_1[g^\mu_\alpha(u) + 4g^\mu_\alpha(u) - 4\phi_\|(u), 2] - \mathcal{I}_3[g^\mu_\alpha(u) - 4g^\mu_\alpha(u)u + 4\phi_\|(u), 2] \\
&+ 4m^2_\rho(\mathcal{I}_1[\tilde{A}_i(u) - 4(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)(1 + u), 3] - \mathcal{I}_3[\tilde{A}_i(u) - 4(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)u), 3])}
\end{align}

\begin{align}
\Pi_7 &= -\frac{1}{8} f_{\rho \mu \rho}(m^2_\rho(\mathcal{I}_1[A_i(u) + 2(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u), 1] + \mathcal{I}_3[A_i(u) + 2(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u), 1)] \\
&- 2\mathcal{I}_2[-g^\mu_\alpha(u) + \phi_\|(u) + 2\mathcal{I}_1[-g^\mu_\alpha(u) + \phi_\|(u))])}
\end{align}

\begin{align}
\Pi_8 &= \frac{1}{8} f_{\rho \mu \rho}(\mathcal{I}_1[g^\mu_\alpha(u), 1] + \mathcal{I}_3[g^\mu_\alpha(u), 1] + m^2_\rho(-\mathcal{I}_1[\tilde{A}_i(u) - 4(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)(1 + u), 2] \\
&+ \mathcal{I}_3[\tilde{A}_i(u) - 4(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u)u), 2])}
\end{align}

\begin{align}
\Pi_9 &= \frac{1}{4} f_{\rho \mu \rho}(2\mathcal{I}_1[\phi_\|(u), 1] + \mathcal{I}_3[\phi_\|(u), 1] + m^2_\rho(2\mathcal{I}_1[A_i(u) + 2(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u), 2] \\
&+ \mathcal{I}_3[A_i(u) + 2(-2g^\mu_\alpha(u) + \tilde{g}_3(u) + \phi_\|(u), 2] + 2\mathcal{I}_5[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2]
\end{align}
Defining $Q^2 = -q^2$, equating the corresponding coefficients of the correlation function from the QCD and hadronic parts, and solving these equations for the nine formfactors, we get

$$A_0^2(Q^2) = \frac{1}{8} e^{m_2^2/M^2} (I_1[g_1^u(u) + (g_1^u)'(u) + 4g_1^u(u))(1 + u), 1] + I_2[g_2^u(u) + g_2^{1/2}(u))u - 4g_2^u(u)u, 1]$$

where the hat denotes integration, for example, as $\hat{f}(u) = \int_0^u dv f(v)$, and the functions $I_i[f(u), n]$, $I_j[f(u)]$, and $I_k[f(u), F(\alpha_i), n]$ are defined as

$$I_1[f(u), n] = (-1)^n \int_0^{u_{10}} du \frac{F_{1n}(u)}{(n-1)!(M^2)^{n-1}} e^{-s_1(u)/M^2}$$

$$- \left[ (-1)^{n-1} \frac{1}{(n-1)!} \sum_{\ell=1}^{n-1} \frac{1}{(M^2)^{n-\ell-1}} s_1'(u) \left( \frac{d}{du} \frac{1}{s_1'(u)} \right)^{\ell-1} F_{1n}(u) \right]_{u=u_{10}}$$

$$I_2[f(u)] = - \int_0^{u_{20}} du f(u) M^2 e^{-s_2(u)/M^2}$$

$$I_3[f(u), n] = (-1)^n \int_0^{u_{30}} du \frac{F_{3n}(u)}{(n-1)!(M^2)^{n-1}} e^{-s_3(u)/M^2}$$

$$- \left[ (-1)^{n-1} \frac{1}{(n-1)!} \sum_{\ell=1}^{n-1} \frac{1}{(M^2)^{n-\ell-1}} s_3'(u) \left( \frac{d}{du} \frac{1}{s_3'(u)} \right)^{\ell-1} F_{3n}(u) \right]_{u=u_{30}}$$

$$I_4[f(u)] = - \int_0^{u_{40}} du f(u) M^2 e^{-s_4(u)/M^2}$$

$$I_5[f(u), F(\alpha_i), n] = (-1)^n \int_0^{1} du \int_0^{u_{30}} dv f(u) \int_{(y_0-\alpha_1)/u}^{1} \int_{(y_0-\alpha_3)/u}^{1} \int_{(y_0-\alpha_1)/u}^{1} \int_{(y_0-\alpha_3)/u}^{1} \int_{(y_0-\alpha_1)/u}^{1} \int_{(y_0-\alpha_3)/u}^{1}$$

$$\frac{f(u) \tilde{F}_i(\alpha_i)}{(n-1)!(M^2)^{n-1}} e^{-s_5(u)/M^2}$$

$$I_6[f(u), F(\alpha_i), n] = (-1)^n \int_0^{1} du \int_0^{u_{30}} dv f(u) \int_{(y_0-\alpha_1)/u}^{1} \int_{(y_0-\alpha_3)/u}^{1} \int_{(y_0-\alpha_1)/u}^{1} \int_{(y_0-\alpha_3)/u}^{1} \int_{(y_0-\alpha_1)/u}^{1} \int_{(y_0-\alpha_3)/u}^{1}$$

$$\frac{f(u) \tilde{F}_i(\alpha_i)}{(n-1)!(M^2)^{n-1}} e^{-s_6(u)/M^2}$$

with

$$s_1(u) = s_2(u) = m_2^2 u - \frac{u}{u} q^2$$

$$s_3(u) = s_4(u) = m_2^2 \bar{u} - \frac{\bar{u}}{\bar{u}} q^2$$

$$s_5(u) = s_6(u) = m_2^2 \bar{y} - \frac{\bar{y}}{\bar{y}} q^2$$

and

$$F_{1n}(u) = \frac{f(u)}{u^n}$$

$$F_{3n}(u) = \frac{f(u)}{u^n}$$

$$\tilde{F}_i(\alpha_i) = \frac{\bar{F}(\alpha_i)}{y^n}, \quad i = 5, 6$$

and

$$y_5 = \alpha_1 + \bar{u} \alpha_3$$

$$y_6 = \alpha_1 + u \alpha_3$$

$u_{10} = u_{20}$ is a solution of the equation $m_2^2 u - u q^2 / \bar{u} = s_0$ and $u_{30} = u_{40}$ is a solution of the equation $m_2^2 \bar{u} - \bar{u} q^2 / u = s_0$, and finally, $y_0$ is a solution of the equation $m_2^2 \bar{y} - \bar{y} q^2 / y = s_0$. Note that in Eqs. (23) and (25), the contributions of the surface terms in the leading-twist amplitudes are taken into account.
\[ A_1^q(Q^2) = \frac{1}{Q^2} e^{m_2^2/M^2} m_2^2 (I_q [g_1^q(u) + 4 \hat{g}_1^2(u) - 4(\hat{\phi}_q(u) + \phi_q(u))(1-u) + g_1^q(u)(1-u) + 4g_2^q(u)(1-u), 1]
+ I_3 [g_1^q(u) - 4 \hat{g}_1^2(u) + 4 \hat{\phi}_q(u) + (g_1^q(u) - 4g_2^q(u) + 4\phi_q(u))u, 1]
+ m_2^2 [-I_q [\hat{A}(u) + 2A(u)(1-u) + 4(\hat{\phi}_q(u) - g_3(u) - \hat{\phi}_q(u) + 2\hat{g}_1^2(u)(1-u) - 2\hat{\phi}_q(u)(1-u) + 4\phi_q(u)]
+ 2\hat{\phi}_q(u)(1-1-u) + 2(\hat{g}_1^2(u) + g_3(u) + \hat{\phi}_q(u) - 1-1-u)^2, 3]
+ I_3 [u^2(\hat{A}(u) + 2(-2\hat{\phi}_q(u) + g_3(u) + \hat{\phi}_q(u))u, 2)]
+ 4I_3 [(a_1 + a_3 - a_3)u(-V(a_1) + A(a_1)(1-1-u), 2)] - 4I_6 [(a_1 + a_3)u(-V(a_1) + A(a_1)(1-1-u), 2)] )
\] (38)

\[ C_0^q(Q^2) = \frac{1}{32m_2^2} e^{m_2^2/M^2} (2[I_q [g_1^q(u) + 2 \hat{g}_1^2(u) - 2g_2^q(u)(1-u) + \hat{g}_2^2(u)(1-u), 1] + I_2 [g_1^q(u) + 2 \hat{g}_1^2(u) - 2\phi_q(u)]
+ I_3 [A(u) + 1(u) + 2u] + 4\phi_q(u) + (4g_2^q(u) + 2(1-u)) + 2\phi_q(u) + 2\hat{\phi}_q(u)]
+ 4I_3 [A(u)(1-u) + 2m_2^2(u) - A(u)(1-u) + 32g_3(u)]
+ 4I_3 [4g_2^q(u) + 2 \hat{g}_1^2(u) - 2g_2^q(u) + 2(1-u)] + 16g_2^q(u) + 2(1-u)^2, 2]
+ 16m_2^2 (I_q [1-1-u] + \hat{\phi}_q(u)(2m_2^2 + Q^2)(1-1-u) - 2(2\hat{\phi}_q(u) + g_3(u) + \hat{\phi}_q(u))(2m_2^2 + Q^2)(1-1-u) + Q^2(1-1-u) + 2m_2^2)
+ Q^2(1-1-u) + 2m_2^2(1-u)]
\] (39)

\[ C_1^q(Q^2) = -\frac{1}{2Q^2} e^{m_2^2/M^2} (I_q [g_1^q(u)(2m_2^2 - Q^2) + g_2^q(u)(4m_2^2(-1-u) - Q^2(1-u)) + 4m_2^2(4\hat{g}_1^2(u) + 2\hat{\phi}_q(u)]
\] (40)
\[
- 4A_\parallel(u)m_\rho^2(-1 + u) + 4(-2g_\parallel^\prime(u) + g_3(u) + \hat{\phi}_\parallel(u))(m_\rho^2(-1 + u) - Q^2(1 + u) + 4\tilde{g}_\parallel^\prime(u)(4m_\rho^2(-1 + u)^2 + Q^2(3 + (-2 + u)u), 2) + 4m_\rho^2(\mathcal{I}_3[-2(-2g_\parallel^\prime(u) + g_3(u) + \hat{\phi}_\parallel(u)) (4m_\rho^2 + Q^2) - 2g_\parallel^\prime(u) - 4g_\parallel^\prime(u)(4m_\rho^2 + Q^2) + 4\tilde{g}_\parallel^\prime(u)) (4m_\rho^2 + Q^2)^2 + 4\tilde{g}_\parallel^\prime(u)(4m_\rho^2 + Q^2)^2 + 4A_\parallel(u)m_\rho^2 u + 4g_\parallel^\prime(u)(4m_\rho^2 + Q^2)^2 + 4(-2g_\parallel^\prime(u) + g_3(u) + \hat{\phi}_\parallel(u))(Q^2(-2 + u) - m_\rho^2 u - 4\tilde{g}_\parallel^\prime(u)(2Q^2 + (4m_\rho^2 + Q^2)^2) + 4m_\rho^2(Q^2)^2, 2) + 4\mathcal{I}_5[-V(\alpha)(Q^2(\alpha + \alpha_3 - \alpha_3 u)(2m_\rho^2 - Q^2 u) - A(\alpha)(Q^2 - 2Q^2 u + \alpha_1(m_\rho^2(2 - 4u) + Q^2 u + (2Q^2 - 2Q^2 u + m_\rho^2(2 - 2 + 4u) + \alpha_3 u(Q^2 - Q^2 u + m_\rho^2(2 - 2 + 4u))))), 2) - 4\mathcal{I}_6[-V(\alpha)(Q^2(2 + 2Q^2 (1 + u) + Q^2 u + (2Q^2 - 2Q^2 u + m_\rho^2(2 - 2 + 4u) + \alpha_3 u(Q^2 - Q^2 u + m_\rho^2(2 - 2 + 4u)))), 2)])
\]

\[D_0^2(Q^2) = \frac{1}{8} e^{m^2/2M^2} [\mathcal{I}_4[g^\alpha_\parallel(u)(8m_\rho^2 - Q^2) + g_\parallel^\alpha(u)(8m_\rho^2(-1 + u) - Q^2(1 + u)) + 4((8m_\rho^2 - Q^2)(g_\parallel^\alpha(u) - \hat{\phi}_\parallel(u) + \phi_\parallel(u) - \phi_\parallel(u) u) + \hat{g}_\parallel^\alpha(u)(8m_\rho^2(-1 + u) - Q^2(1 + u)), 1] - \mathcal{I}_5[g_\parallel^\alpha(u)(8m_\rho^2 - Q^2) + 4g_\parallel^\alpha(u)(Q^2(-2 + u) - 8m_\rho^2 u - 4(8m_\rho^2 - Q^2)(\hat{g}_\parallel^\alpha(u) - \hat{\phi}_\parallel(u) - \phi_\parallel(u) u) + \hat{g}_\parallel^\alpha(u)(2Q^2 + 8m_\rho^2 u - Q^2 u), 1] + m_\rho^2(\mathcal{I}_3[8\hat{\phi}_\parallel(u)m_\rho^2 + 4g_\parallel^\alpha(u)Q^2 - \hat{\phi}_\parallel(u)Q^2 + 16\hat{g}_\parallel^\alpha(u)Q^2 - 32\hat{g}_\parallel^\alpha(u)Q^2 + 16g_\parallel^\alpha(u)Q^2 + 16\hat{\phi}_\parallel(u)Q^2] - 16\hat{\phi}_\parallel(u)Q^2 + 16A_\parallel(u)m_\rho^2(-1 + u) + 64\hat{g}_\parallel^\alpha(u)m_\rho^2(-1 + u) - 32g_\parallel^\alpha(u)m_\rho^2(-1 + u) - 32\hat{\phi}_\parallel(u)m_\rho^2(-1 + u) + 2A_\parallel(u)Q^2(-1 + u) + 8\hat{\phi}_\parallel(u)Q^2(-1 + u) + 4\hat{g}_\parallel^\alpha(u)m_\rho^2(-1 + u) - 24\hat{\phi}_\parallel(u)Q^2(-1 + u) + 24\hat{\phi}_\parallel(u)Q^2(-1 + u)^2, 1) + \mathcal{I}_5[-\hat{\phi}_\parallel(u)m_\rho^2 u + 32g_\parallel^\alpha(u)m_\rho^2 u + 8\hat{g}_\parallel^\alpha(u)Q^2 u - 4g_\parallel^\alpha(u)Q^2 u + 2A_\parallel(u)(-8m_\rho^2 + Q^2 u) + 64\hat{g}_\parallel^\alpha(u)m_\rho^2 u + 24g_\parallel^\alpha(u)Q^2 u^2 + 24\hat{\phi}_\parallel(u)(4Q^2 + 8m_\rho^2 u - Q^2 u), 1] + 8m_\rho^2(\mathcal{I}_1[-2(2\hat{g}_\parallel^\prime(u) + g_3(u) + \hat{\phi}_\parallel(u)(-4Q^2 + (8m_\rho^2 + 3Q^2 u)^2) + \hat{A}_\parallel(u)(2Q^2 + (8m_\rho^2 + 3Q^2 u)^2), 3]) - 4\mathcal{I}_5[A(\alpha)(-2Q^2 - (\alpha_1 + \alpha_3)(8m_\rho^2 + 3Q^2) + 8(2\alpha_1 + 3\alpha_3)m_\rho^2 u + (4 - 2\alpha_1 + 4(1 + u)Q^2 u + 2\alpha_3(-8m_\rho^2 + Q^2)(u^2) - V(\alpha)(2Q^2 + (8m_\rho^2 + Q^2(3 - 8u))(\alpha_1 + \alpha_3 - \alpha_3 u)), 2) + 4\mathcal{I}_5[A(\alpha)(\alpha_1 Q^2(5 - 2u) + 8\alpha_3 m_\rho^2 u(1 + 2u) + 8\alpha_3 m_\rho^2 u(1 + 2u) + Q^2(2 + 4(1 + \alpha_3(5 - 2u)))) - V(\alpha)(2Q^2 + (1 + \alpha_3)(8m_\rho^2 + 3Q^2(5 + 8u))), 2)])
\]

\[E_0^2(Q^2) = \frac{1}{8Q^2} e^{m^2/2M^2} [4(\mathcal{I}_1[g^\alpha_\parallel(u)m_\rho^2 - 4\hat{\phi}_\parallel(u)m_\rho^2 + \hat{g}_\parallel^\alpha(u)(4m_\rho^2 + Q^2) - \hat{\phi}_\parallel(u)Q^2 - g_\parallel^\prime(u)m_\rho^2(1 - u) + 4m^2(\phi_\parallel(u)(1 - u) - \phi_\parallel(u)Q^2(1 - u)) + \mathcal{I}_5[g_\parallel^\alpha(u)m_\rho^2 + g_\parallel^\alpha(u)m_\rho^2 u - 4g_\parallel^\alpha(u)m_\rho^2 u - 4m_\rho^2 - Q^2)(\hat{g}_\parallel(u) - \hat{\phi}_\parallel(u) - \phi_\parallel(u) u), 1] + m_\rho^2(-\mathcal{I}_5[\hat{A}_\parallel(u)(4m_\rho^2 - Q^2)^2(2 u) + 2(-2\tilde{g}_\parallel^\prime(u) + g_3(u) + \hat{\phi}_\parallel(u)(4m_\rho^2 + Q^2)^2 + 8(\hat{g}_\parallel^\prime(u) - \hat{\phi}_\parallel(u)2m_\rho^2 + Q^2)(1 - u)), 2)] + \mathcal{I}_5[\hat{A}_\parallel(u)(4m_\rho^2 - Q^2) + A_\parallel(u)(8m_\rho^2 u - 2Q^2 u) + 4u(-2\hat{g}_\parallel^\prime(u) + g_3(u) + \hat{\phi}_\parallel(u))(4m_\rho^2 + Q^2)]
\]

3 Numerical analysis

In this section, we numerically analyze the light-cone sum rules for the GFFs of the ρ meson by using Package X [23]. In the sum rules, we took the mass and decay constant of the ρ meson to be $m_\rho = 0.77$ GeV and $f_\rho = 0.20$ GeV, respectively. Another set of essential input parameters are the ρ meson DAs of different twists. The relevant DAs are given as follows [17, 18, 19, 20]:

\begin{align}
\phi_\parallel(u) &= 6 u \bar{u} \left[ 1 + 3 a_\parallel \xi + \frac{3}{2} \left( 5 \xi^2 - 1 \right) \right] \\
g_3(u) &= 1 + \left( -1 - \frac{2}{7} a_\parallel^2 + \frac{40}{3} \xi_3 - \frac{20}{3} \xi_4 \right) C_2^{1/2}(\xi) + \left[ - \frac{27}{28} a_\parallel + \frac{5}{4} \xi_3 - \frac{15}{16} \xi_3^4 (\omega_3^3 + 3 \omega_3^\prime) \right] C_4^{1/2}(\xi) \\
g_3^a(u) &= 6 u \bar{u} \left\{ 1 + a_\parallel \xi \right\} + \frac{1}{4} a_2 \xi + \frac{5}{4} \xi_3 \left( 1 - \frac{3}{16} \omega_3^3 \right) + \frac{35}{4} \xi_3 \left( 5 \xi^2 - 1 \right) \\
\end{align}

(46) (47) (48)
Figure 1: The GFFs of the $\rho$ meson: $A_0^q(Q^2)$. The results of Sun and Dong [11] in the light-cone constituent quark model, Abidin et al. [21] from the AdS/QCD approach, and Freese et al. [22] in the NJL model are also shown.

Figure 2: The GFFs of the $\rho$ meson: $A_1^q(Q^2)$. The results of Sun and Dong [11] in the light-cone constituent quark model and Freese et al. [22] in the NJL model are also shown.
Figure 3: The GFFs of the \( \rho \) meson: \( D_q^0(Q^2) \). The results of Sun and Dong \cite{11} in the light-cone constituent quark model and Freese et al. \cite{22} in the NJL model are also shown.

Figure 4: The GFFs of the \( \rho \) meson: \( D_q^1(Q^2) \). The results of Sun and Dong \cite{11} in the light-cone constituent quark model and Freese et al. \cite{22} in the NJL model are also shown.
Figure 5: The GFFs of the $\rho$ meson: $E^q(Q^2)$. The results of Sun and Dong [11] in the light-cone constituent quark model and Freese et al. [22] in the NJL model are also shown.

Figure 6: The GFFs of the $\rho$ meson: $J^q(Q^2)$. The results of Sun and Dong [11] in the light-cone constituent quark model, Abidin et al. [21] from the AdS/QCD approach, and Freese et al. [22] in the NJL model are also shown.
\[ g_λ^v(u) = \frac{3}{4}(1 + \xi^2) + a_1^∥ \frac{3}{2} \xi + \left( \frac{3}{7} a_2^∥ + 5 \zeta_3^A \right) (3 \xi^2 - 1) + \left( \frac{9}{112} a_2^∥ + \frac{105}{16} \zeta_3^V - \frac{15}{64} \zeta_3^A \right) (3 - 30 \xi^2 + 35 \zeta_4^A) \]  \( \xi = 1 - u, \) and \( \zeta = 2u - 1. \) The values of the parameters inside the DAAs at the renormalization scale of \( \mu = 1 \) GeV are \( a_1^∥ = 0, \ a_2^∥ = 0.18, \ \zeta_3^A = 0.032, \ \zeta_4 = 0.15, \ \omega_3^V = -2.1, \end{equation} \( \omega_3^V = 3.8, \) and \( \zeta_3^V = 0.013. \)

From the sum rules for the GFFs, we see that besides the input parameters, they contain two auxiliary parameters, namely the Borel mass parameter, \( M^2, \) and the continuum threshold, \( s_0. \) Apparently, the measurable GFFs should be independent of them. We find the working region of \( M^2 \) to be

\[ 1.0 \text{ GeV} < M^2 < 2.0 \text{ GeV} \]  \( s_0 = 1.4 \text{ GeV}^2 \)  \( \xi = 0, \) and \( \zeta = 2u - 1. \) The values of the parameters inside the DAAs at the renormalization scale of \( \mu = 1 \) GeV are \( a_1^∥ = 0, \ a_2^∥ = 0.18, \ \zeta_3^A = 0.032, \ \zeta_4 = 0.15, \ \omega_3^V = -2.1, \) \( \omega_3^V = 3.8, \) and \( \zeta_3^V = 0.013. \)

We present our results for the six GFFs that lead to the conservation of the EMT in Figs. 1–6. The GFFs \( A_0(q^2) \) and \( J(q^2) \) are related to the mass and charge conservation, hence are subject to the constraint at zero-momentum transfer \( A_0(0) = 1 \) and \( J(0) = 1 \) \( [24, 25, 21, 26]. \) It is crucial to note that in this work, we took into account only the contribution from the quarks in the EMT.

As we noted that, the correlation function from QCD side can be calculated at sufficiently large negative values of \( Q^2. \) The formfactors can be reliably determined at \( Q^2 \geq 1 \text{ GeV}^2 \) domain. The LCSR method is not applicable for smaller values of \( Q^2, Q^2 < 1 \text{ GeV}^2. \) In order to extend the results for the formfactors to \( Q^2 = 0 \) point, we look for a parametrization of them in such a way that at large \( Q^2 \) domain, the parametrization coincides with the sum rules predictions. Our numerical analysis shows that the best parametrization for the formfactors is as follows:

\[ A_0^v(Q^2) = 0.583 \left( 1 + \frac{Q^2}{3.451} \right)^{-0.603} \]  \( \xi = 1 - u, \) and \( \zeta = 2u - 1. \) The values of the parameters inside the DAAs at the renormalization scale of \( \mu = 1 \) GeV are \( a_1^∥ = 0, \ a_2^∥ = 0.18, \ \zeta_3^A = 0.032, \ \zeta_4 = 0.15, \ \omega_3^V = -2.1, \) \( \omega_3^V = 3.8, \) and \( \zeta_3^V = 0.013. \)

\[ A_1^v(Q^2) = 2.247 \left( 1 + \frac{Q^2}{1.042} \right)^{-1.329} \]  \( \xi = 0, \) and \( \zeta = 2u - 1. \) The values of the parameters inside the DAAs at the renormalization scale of \( \mu = 1 \) GeV are \( a_1^∥ = 0, \ a_2^∥ = 0.18, \ \zeta_3^A = 0.032, \ \zeta_4 = 0.15, \ \omega_3^V = -2.1, \) \( \omega_3^V = 3.8, \) and \( \zeta_3^V = 0.013. \)

\[ D_0^v(Q^2) = -3.086 \left( 1 + \frac{Q^2}{0.160} \right)^{-0.554} \]  \( \xi = 0, \) and \( \zeta = 2u - 1. \) The values of the parameters inside the DAAs at the renormalization scale of \( \mu = 1 \) GeV are \( a_1^∥ = 0, \ a_2^∥ = 0.18, \ \zeta_3^A = 0.032, \ \zeta_4 = 0.15, \ \omega_3^V = -2.1, \) \( \omega_3^V = 3.8, \) and \( \zeta_3^V = 0.013. \)

\[ D_1^v(Q^2) = 21.04 \left( 1 + \frac{Q^2}{0.904} \right)^{-1.936} \]  \( \xi = 0, \) and \( \zeta = 2u - 1. \) The values of the parameters inside the DAAs at the renormalization scale of \( \mu = 1 \) GeV are \( a_1^∥ = 0, \ a_2^∥ = 0.18, \ \zeta_3^A = 0.032, \ \zeta_4 = 0.15, \ \omega_3^V = -2.1, \) \( \omega_3^V = 3.8, \) and \( \zeta_3^V = 0.013. \)

\[ E^v(Q^2) = 0.769 \left( 1 + \frac{Q^2}{2.603} \right)^{-1.473} \]  \( \xi = 0, \) and \( \zeta = 2u - 1. \) The values of the parameters inside the DAAs at the renormalization scale of \( \mu = 1 \) GeV are \( a_1^∥ = 0, \ a_2^∥ = 0.18, \ \zeta_3^A = 0.032, \ \zeta_4 = 0.15, \ \omega_3^V = -2.1, \) \( \omega_3^V = 3.8, \) and \( \zeta_3^V = 0.013. \)

\[ J^v(Q^2) = 0.413 \left( 1 + \frac{Q^2}{2.541} \right)^{-1.351} \]  \( \xi = 0, \) and \( \zeta = 2u - 1. \) The values of the parameters inside the DAAs at the renormalization scale of \( \mu = 1 \) GeV are \( a_1^∥ = 0, \ a_2^∥ = 0.18, \ \zeta_3^A = 0.032, \ \zeta_4 = 0.15, \ \omega_3^V = -2.1, \) \( \omega_3^V = 3.8, \) and \( \zeta_3^V = 0.013. \)

From Figs. 1–6, we deduce the following results: The contributions from the quark sector of the EMT to the GFF for \( A_0^v \) seem to make up nearly 60% of the total contributions. For the GFF \( J, \) 40% of the total contribution consists of the quark part. The results for both formfactors agree with the expectation that
the quark sector in the EMT will add up to almost half of the total contributions to the GFFs. For the rest of the GFFs displayed in Figs. 1–6, we see that the quark contributions seem to be larger than the total obtained in the literature, which would indicate that the gluon contributions should have largely negative contributions.

At the end of this section, we present our predictions for the average mass radius, \( \langle r^2 \rangle_{\text{mass}} \), and the quadrupole moment of the \( \rho \) meson. The average mass radius \( \langle r^2 \rangle_{\text{mass}} \) is obtained in the light-cone frame in Ref. [22] as

\[
\langle r^2 \rangle_{\text{mass}} = 4 \left. \frac{dA_0(q^2)}{dq^2} \right|_{q^2=0} + \frac{1}{3m^2} [2A_0(0) + A_1(0) - 2J(0) + 2E(0)]
\]

(61)

In our work, we obtain

\[
\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \text{ fm}
\]

(62)

while it was found to be 0.41 fm in [11]. Finally, the gravitational quadrupole moment is given in terms of the GFFs as [18]

\[
Q_{\text{mass}} = -\frac{1}{m} \left[ -A_0(0) + \frac{1}{2}A_1(0) + 2J(0) - E(0) \right]
\]

(63)

In our work, we obtain

\[
Q_{\text{mass}} = -0.0512 \ m_\rho \cdot \text{fm}^2
\]

(64)

while it was obtained to be \(-0.0322 \ m_\rho \cdot \text{fm}^2\) in [11].

4 Conclusion

In this work, we studied the GFFs of the \( \rho \) meson within the light-cone QCD sum rules approach by taking into account only the quark part in the EMT. The GFFs \( A_0 \) and \( J \) are related to the mass and charge conservation and they are subject to the constraint \( A_0(0) = 1 \) and \( J(0) = 1 \) at zero transfer momentum. We have shown that the quark contributions make up nearly 60% and 40% of the aforementioned GFFs, respectively. The mass radius was obtained to be 0.32 fm and the gravitational quadrupole moment of the \( \rho \) meson was found to be \(-0.0512 \ m_\rho \cdot \text{fm}^2\). Finally, we compared our results to the ones in the literature, and considering the fact that the gluon part has been neglected in the EMT, the results do not differ significantly.

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