On baryon-antibaryon coupling to two photons

Frank E. Close\textsuperscript{1*} and Qiang Zhao\textsuperscript{2†}

\textsuperscript{1) Department of Theoretical Physics, University of Oxford, Keble Rd., Oxford, OX1 3NP, United Kingdom and}

\textsuperscript{2) Department of Physics, University of Surrey, Guildford, GU2 7XH, United Kingdom}

We discuss recent claims that $p\bar{p} \rightarrow \gamma\gamma$ may be described by a generalized parton picture. We propose that quark-hadron duality provides a justification for the effective dominance of the “handbag” diagram assumed in recent literature, and that handbag diagrams may dominate phenomena in kinematic regions far more extensive than that might be expected from pQCD alone.

I. INTRODUCTION

Recently there has been the interesting proposal \cite{1, 2, 3, 4, 5} that exclusive proton-antiproton annihilation into two photons can, under certain kinematic conditions, be described by a generalized partonic picture. As such, this gives a new potential probe of non-forward parton distribution or double distributions \cite{6, 7}. It is argued \cite{1, 2, 3, 4, 5} that the two photons are emitted in the annihilation of a single quark and antiquark (thus via the “handbag” graph) whereby the process may be described by a generalised parton picture analogous to the “soft mechanism” in wide-angle real Compton scattering. An essential feature of the arguments is that for on-shell photons at large $s$, the limited space-like virtuality of the bound state wavefunctions constrains the active quark and antiquark to be “fast”, i.e. have $x \rightarrow 1$.

The purpose of this paper is to show that extension of recent work on quark-hadron duality \cite{8, 9, 10} may help to justify some of the arguments that enable phenomenology of the handbag diagram to be employed away from fully asymptotic regime.

References \cite{3, 4} demonstrated that a “soft-handbag” contribution (defined by re-
stricted transverse momenta and virtualities of quarks) factorises into a soft hadronic matrix element and a partonic subprocess, $\gamma\gamma \rightarrow q\bar{q}$ in the limit $x \rightarrow 1$ and large $s, -t, -u$. However, there is no general proof that such soft-handbag topology dominates for large (finite) $s$ though there are indirect hints of its importance from phenomenological fits to data in e.g. $\gamma\gamma \rightarrow \pi\pi, K\bar{K}$ [4]. The description for large intermediate $s$ implies that the spectators possess $x \leq \Lambda^2_{QCD}/s$ and that the $x$ of the active quark is not literally $\rightarrow 1$, but is in the region of $\sim 0.7$. It is plausible that soft wavefunctions can tolerate such values of $x$. However, the factorization, which is reliable as $x \rightarrow 1$, has corrections $\sim (1 - x)^n$ which may become large as $x < 1$.

In the region of $s$ that is in the hadronic continuum above the prominent resonances, we argue that quark-hadron duality provides some justification for neglecting coherent (“cats-ears” topology) that break factorization. It is also important to contrast the soft handbag for finite $s$ and $x \neq 1$ from the “hard” handbag for $s \rightarrow \infty, x \rightarrow 1$ where quarks and gluons have large $p_T$ or virtuality. We make some brief comments here in order to distinguish from the kinematics of duality and the “soft” handbag, which forms our main focus. Thus, in what follows, we propose that duality should provide a justification for the handbag diagram dominance over a wide kinematic range. In particular, for $s$ above the resonance region, the presence of destructive interference among parity-even and parity-odd states [8] in the coherent process for the $p\bar{p}$ annihilation indeed supports the assumption of the handbag diagram dominance that underpins Refs. [1, 2, 3, 4, 5] such that an effective incoherent parton interpretation can be made, but integrated over $x$. There is no necessary dominance of the $x \rightarrow 1$ domain in this process.

Thus for example, were it possible to compare the $p\bar{p}$ and analogous $n\bar{n}$, the ratio of $\sum e_i^2(p)/\sum e_i^2(n)$ would be $3/2$. This corresponds to a value of $1/2$ for the fragmentation parameter $\rho$ [5].
II. HANDBAG DOMINANCE AND \( x \to 1 \)

Reference [1, 2] considers \( p\bar{p} \to \gamma\gamma \) to be Compton scattering in the crossed channel, where the exchanged system in the \( t(u) \) channel consists of at least three quarks. At large momentum transfers such a configuration should be strongly suppressed by the composite systems’ wavefunctions. Hence Refs. [1, 2] argue that the most efficient way of accommodating a large momentum transfer is via the handbag diagram (where the \( p\bar{p} \) system makes a transition to a \( q\bar{q} \) pair by exchanging a virtual “diquark” system whose spacelike virtuality is limited by the bound-state wavefunction). The \( q\bar{q} \) then annihilate into two photons by exchanging a highly virtual quark/antiquark.

There is an essential difference between the annihilation process and the forward Compton scattering that underpins deep inelastic structure functions. In the latter, the spectator (“diquark”, etc) system is effectively passive, the active quark probability being factorized out and probed by the highly virtual photon(s). Contrast this with the exclusive annihilation process, \( p\bar{p} \to \gamma\gamma \), where one may annihilate a \( q\bar{q} \) by \( q\bar{q} \to \gamma\gamma \), but must also annihilate the spectators without the emission of further radiation (be it photons, gluons or hadrons). It is this constraint in the strict limit \( s \to \infty \) that effectively requires the spectators to have null energy-momentum four-vectors, and hence the active \( q\bar{q} \) to carry the full momentum of the beams (what Refs. [1, 2] refer to as “fast” quarks and which Ref. [4] use as their driving assumption in developing phenomenology). However, there is no immediate argument to support the handbag diagram dominance. For \( s \to \infty \), where pQCD applies, one can see below that the (neglected) coherent or higher-twist contribution of Fig. 1b, is the same order of magnitude as the “handbag” diagram (Fig. 1a). We briefly review this to distinguish it from the large intermediate \( s \), where the “soft-handbag” dominance may apply, and turn to this in section III.

The nucleon wavefunction restricts the spacelike virtuality of partons to be small [1]. Production of a single real photon at large transverse momentum in parton-antiparton annihilation, then requires a rather singular kinematics, with \( p_i = (|p_T|, p_T, O(\Lambda_{QCD}/\sqrt{s})) \) for each parton (hence each \( x \to 0 \)), where \( \Lambda_{QCD} \) is the typical QCD energy scale. However, such a singular kinematics in principle can oc-
cur. A coherent amplitude, as in Fig. 1b, requires transfer of momentum so that at large \( s \) the annihilating constituents all have \( x \rightarrow 0 \). This can be achieved by means of gluon exchange but at the expense of suppression due to the large momentum flows and powers of \( \alpha_s \). (We illustrate this for a two-body system but the argument generalizes). The superscripts in Fig. 1 denote the longitudinal momentum fractions; the most favoured configuration at the hadron vertex is for the constituents to share their momenta (see e.g. Refs. [9, 10]).

The restricted kinematics \( (x \rightarrow 1) \) is unable to guarantee the dominance of the handbag diagram illustrated by Fig. 1c. The configuration in Fig. 1c, is also kinematically singular, in that the parton probabilities vanish in the strict \( x \rightarrow 1 \) limit, and the spectators must be null in order for their annihilation to contribute nothing.

One thus needs to consider how this extreme configuration arose if, as seems natural, the preferred wavefunction (denoted by the non-shaded ovals in Figs. 1a,b,d) has the partons with symmetric configuration \( x \sim 1/2 \) (for this pedagogic example of a two body system [9]). If one allows gluon exchanges as in Fig. 1a to achieve this singular kinematics, it would lead to a configuration that is generally not suppressed relative to the one in Fig. 1b. Figure 1d then shows how the intrinsic symmetric (in \( x \)) wavefunction becomes highly asymmetric. This causes Fig. 1c to be the same order in \( \alpha_s \) and momentum flow as Fig. 1b.

Interestingly, for \( q^2 > 0 \) timelike processes e.g. \( \gamma \gamma \rightarrow \pi\pi \), coherent diagrams such as those in Fig. 1b are anticipated in pQCD [11]. The relative strengths of various meson-pair production processes at finite \( s \), such as \( \sigma(\gamma \gamma \rightarrow \pi^0\pi^0)/\sigma(\gamma \gamma \rightarrow \pi^+\pi^-) \) do not fit well with these predictions in detail. Indeed, phenomenology seems compatible with the dominance of handbag diagrams at intermediate \( s \). This is the subject of soft-handbag dominance [1, 2, 3, 4, 5] to which we now turn.

III. SOFT HANDBAG AND DUALITY AT INTERMEDIATE ENERGIES

We shall now propose that quark-hadron duality may provide an explanation of the phenomenological success of data analyses based on the assumed dominance of the incoherent, or “handbag” diagrams at intermediate \( s \), and help underpin recent
developments based thereon \[1, 2, 4, 5\].

In quark-hadron duality the incoherent property of the handbag diagram survives in practice even though the kinematic arguments of pQCD do not necessarily apply \[8, 12\]. The underlying dynamics lead to the effective dominance of the incoherent diagrams when a suitable averaging has taken place \[8\]. We first illustrate some empirical examples and then apply the arguments to the $p\bar{p} \to \gamma\gamma$ process \[1, 2, 4, 5\].

In the deep inelastic structure functions at large $q^2$ one has $F_2^n(x)/F_2^p(x) = \sum e_i^2(n)/\sum e_i^2(p)$ in kinematic circumstances where pQCD supports the incoherent dominance, and where any $\sum_{i\neq j} e_ie_j$ contributions are higher twist and thereby suppressed. However, incoherent contributions proportional to $\sum_i e_i^2$ appear to control the cross-section ratios of Compton scattering even at low $q^2$ where the kinematic conditions are such that $\sum_{i\neq j} e_ie_j$ contributions would be anticipated. As an extreme example, consider real photons, where the non-diffractive contribution to $\sigma_{tot}(\gamma n)/\sigma_{tot}(\gamma p)$ is empirically $\sim 2/3 \equiv (2e_d^2 + e_u^2)/(2e_u^2 + e_d^2)$, even though there is no pQCD support for such a relation when $q^2 = 0$. The suppression of the $\sum_{i\neq j} e_ie_j$ contributions in this case is a result of duality in $\text{Im}(\gamma N \to \gamma N(t = 0))$ and the constraint that there are no exotic exchanges in the crossed ($t$) channel \[12\]. In a pedagogic model Reference \[8\] showed how this can arise. Furthermore, this model has been shown to realise scaling in the structure functions (“Bloom-Gilman duality”) \[10\] and to lead to a factorization of the non-forward Compton scattering \[8\].

The duality holds for real photons as $t \to 0$, and the factorisation at least for the non-forward Compton scattering of real photons. It is the latter that crosses to $p\bar{p} \to \gamma\gamma$, whereby we propose that this may also justify the dominance of the incoherent process ($\sim \sum_i e_i^2$) in $p\bar{p} \to \gamma\gamma$, but without any restriction to $x \to 1$.

In $\text{Im}(\gamma N \to \gamma N(t = 0))$ the excitation of coherent intermediate resonances includes states of positive and of negative parity. These add constructively in the $e_i^2$ contributions but are destructive in the $e_ie_j$ terms. This causes a duality between the averaged resonance excitation on the one hand, and the smooth high-energy behaviour on the other hand \[8\]. As a consequence one obtains the empirical result for the ratio of the non-diffractive pieces of $\sigma_{tot}(\gamma n)/\sigma_{tot}(\gamma p)$ even though there is no pQCD reason for the dominance of the incoherent terms.
This picture has been extended to the spin-averaged non-forward Compton amplitude \( \mathcal{A} \), where scaling and factorization properties arise. In particular, the same effective dominance of the \( \sum_i e_i^2 \) terms arose. It implies for \( \gamma(q)A \rightarrow \gamma(k)A \) with \( t \equiv (k - q)^2 \), the generalized factorization for the non-forward proton structure function or double distribution function \( \mathcal{F} \) is

\[
F_2(x, \xi, t) = \sum_i e_i^2 \left( \frac{x - \xi}{x^2} \right) F_2(x) F_{el}(t) ,
\]

where \( F_{el}(t) \) is the elastic form factor satisfying \( F_{el}(t = 0) = 1 \). In the particular limit of \( \xi \rightarrow 0 \) and \( -t/Q^2 << 1 \), this factorization also satisfies Ji and Radyushkin’s sum rule \([6, 7]\).

The analogous analysis can be applied to the crossed channel, \( \gamma \gamma \rightarrow M \bar{M} \), where \( M \) refers to a two-body “meson”. The essential results can then be generalised to \( \gamma \gamma \rightarrow p \bar{p} \) of Ref. \([1, 2, 4, 5]\). The most general consequence is again that terms proportional to \( \sum_{i \neq j} e_i e_j \) are suppressed by destructive interference (in the \( t \)-channel). The corresponding effect is that the dominant process in the \( s \)-channel is \( \gamma \gamma \rightarrow M(q\bar{q}) \rightarrow p\bar{p} \), where the intermediate meson states \( M \) have been summed over, and all “exotic” \( qq\bar{q}q \) intermediate states been suppressed. This underpins the phenomenology of Ref. \([3]\). In particular, such arguments also immediately imply \( \sigma(\gamma \gamma \rightarrow \pi^+\pi^-) \simeq \sigma(\gamma \gamma \rightarrow \pi^0\pi^0) \) in the hadronic continuum.

We can compare our distribution function with that used in Ref. \([1]\). There, a factorization ansatz was made for the double distribution function:

\[
F_\alpha(x, \alpha; s) = f_\alpha(x) h_\alpha(x, \alpha) S_\alpha(x, \alpha, s) ,
\]

separating the soft and hard contributions with \( S_\alpha(x, \alpha, s = 0) = 1 \) and \( \int h(x, \alpha) d\alpha = 1 \). Furthermore, it was assumed that \( h_\alpha(x, \alpha) \equiv \delta(\alpha) \).

If we now impose an analogous set of ansatz on our form [Eq. (1)]:

\[
F(x) h(x, \xi) S(x, \xi, t) \text{ with } S(x, \xi, t = 0) = 1, \int h(x, \xi) d\xi = 1 \text{ and } h(x, \xi) \equiv \delta(\xi) ,
\]

and then cross \( t \) to \( s \) channel, we obtain analogues of their Eqs. (6) and (7) \([1]\). Namely, the factorization satisfies

\[
F_2(x, \xi \rightarrow 0, s) = \sum_i e_i^2 F_1(x) F_{el}(s) = \sum_i e_i^2 x q(x) F_{el}(s) ,
\]

and

\[
\frac{1}{2} \int_{\xi = -1 + |x|}^{1 - |x|} d\xi \frac{x}{\xi} \frac{F_2(x, \xi, s = 0)}{x} = \sum_i e_i^2 [\theta(x) q(x) - \theta(-x) \bar{q}(-x)] ,
\]
where $\xi \to 0$ forces us throw away the term proportional to $\xi^2$; $q(x)$ is the unpolarized quark (antiquark) distribution function, and $\theta(x)$ is the step function. The factor $1/2$ comes from the re-definition of $\xi \equiv x_{bj} - x_{bj}^{fin}$ in contrast with $\xi \equiv (x_{bj}^{fin} - x_{bj})/2$ in Ref. [1], and the relation $-\theta(x)F_2(x) + \theta(-x)F_2(-x) = 0$ according to the momentum sum rule has been used. Taking this factorization scheme, the integration ranges for $x$ and $\xi$ become $-1 < x < 1$ and $-1 + |x| < \xi < 1 - |x|$, which are consistent with the convention used in the literature [1, 2].

Our expression, which was generalized to the Compton scattering on the proton, then leads directly to the vector form factor for the proton defined in Ref. [1] for the $p\bar{p} \to \gamma\gamma$:

$$R_V(s) \sim \sum_i e_i^2 F_{el}(s).$$

Although the generalization of the factorization to the physical nucleon scattering has not been justified, we find that the above analytical expression from the factorization is rather interesting in light of the numerical results of Ref. [1, 2]. Given a dipole feature to the elastic form factor $F_{el}(s) = 1/(1+ s/\lambda^2)^2$, we plot the $R_V(s)$ in Fig. 2, which exhibits a similar feature as found in Ref. [1, 2].

Meanwhile, one consequence is that the cross section ratios between the $p\bar{p}$ and $n\bar{n}$ annihilation are governed by the constituent charges, namely, $\sigma(p\bar{p})/\sigma(n\bar{n}) = (2e_u^2 + e_d^2)/(2e_d^2 + e_u^2)$. There is no restriction to specific extremes of $x$, nor is there a freedom of fragmentation parameter $\rho$.

IV. SUMMARY

In the spirit of Refs. [8, 12] as illustrated for DVCS and wide angle Compton scattering (WACS) in Ref. [9], one can argue for the effective dominance of the incoherent diagrams, and the ability to interpret wide-angle $p\bar{p} \to \gamma\gamma$ by non-forward distributions, as in Refs. [1, 2, 4, 5]. There is, however, no dynamical reason to require the dominance of $x \to 1$ configurations.

Meanwhile, note the link between the effective destruction of $\sum_{i\neq j} e_i e_j$ terms and the absence of exotic exchanges in the crossed channel [8, 12]. This necessarily imposes constraints on the relative magnitudes of $\gamma\gamma \to AA$. In particular, under the kinematic circumstances of Refs. [4, 11] $\sigma(\gamma\gamma \to \pi^+\pi^-)/\sigma(\gamma\gamma \to \pi^0\pi^0) = 1$. These results can be generalized to physical baryons and give the relations for listed
in Ref. [4], but where the above arguments would imply that \( \rho \) in Eq. (43) of [4] simply counts the relative number of \( d \) and \( u \) valence quarks in a proton. Assuming SU(3) flavour symmetry, one thus would have \( \rho = 1/2 \) in Eq. (43) of Ref. [5].

In summary, duality enables us to generalise the results of Refs. [1, 2, 4, 5]. In regions where pQCD is able to support “local” dominance of leading twist “handbag” diagrams, quark-hadron duality shows that such diagrams dominate certain ratios of cross sections if averaged over \( s \) such that direct channel resonances of opposite symmetry types destructively cancel. As \( s \to \infty \) where the density of states is high, this is expected to happen locally, and the results of pQCD arise. At the other extreme, namely the region where individual resonances dominate locally, cancelling only when the resonance region is suitably averaged [8, 10], such cancellations will not arise and the coherent effects dominate. At intermediate \( s \), as here, where the density of resonances rapidly saturates, the cancellations are rather local and the handbag dominance becomes effective. Thus in summary, we support the hypothesis that the incoherent probability description is valid. In particular, there is no necessary restriction to the \( x \to 1 \) limit.

It is also worth noting that our model is manifestly symmetric by construction and implicitly has \( \rho = 1/2 \) [5] and hence the ratio of \( p\bar{p} \) to \( n\bar{n} \) is 3/2. A realistic picture would require extension to include spin and single gluon exchange, or symmetry breaking effects. However, this goes beyond our immediate aims which were to give some underpinning of the arguments for application of handbag phenomenology to \( p\bar{p} \to \gamma\gamma \).

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FIG. 1: Schematic diagrams for (a) incoherent photon emission and (b) coherent photon emission; Fig. (c) is the “handbag” which is the same order as (a); Fig. (d) illustrates the kinematical access of the soft “handbag” via gluon exchange.
FIG. 2: Squared form factor for $p\bar{p} \rightarrow \gamma\gamma$ predicted based on duality.