A comment on "Discussion on the use of the strain energy release rate for fatigue delamination characterization"

M. Ciavarella (1), A. Papangelo (1), G. Cricr (2)

(1) DMMM department. Politecnico di BARI. Viale Gentile 182, 70126 Bari. Italy. Mciava@poliba.it (2) Universit di Napoli Federico II, DII dept., P.Tecchio, 80. 80125 Napoli (It)

Abstract

In a recent very interesting and illuminating proposal, Yao et al. (2014) have discussed the use of the strain energy release rate (SERR) as a parameter to characterize fatigue delamination growth in composite materials. They consider fatigue delamination data strongly affected by R-curve behaviour due to fibres bridging and argue that a better approach is to correlate the crack advance with the total work per cycle measured in the testing machine. This seems to work better than estimating the compliance as a linear fit of experimental curves from Modified Compliance Calibration ASTM standards equations for the SERR in the classical Linear Elastic Fracture Mechanics framework. We show however that if we assume indeed linear behaviour (i.e. LEFM), the approach they introduce is perfectly equivalent to the SERR one, i.e. Paris type of laws. As well known form Barenblatt and Botvina, fatigue crack growth is a weak form of scaling, and it gives Paris classical dependence only when the crack is much longer than any other characteristic sizes. Paris’ law is not a fundamental law of physics, is not an energy balance equation like Griffith, and strong size effects due to cohesive zones have been found already in concrete by Bazant. The proposal is very simple, and interesting as it would seem to suggest that a proper scaling with a cohesive model at crack tip could be predicted, although this doesn’t seem to have been attempted in the Literature. The main drawback of the present proposal is that it is not predictive, but purely observational, as it requires the actual measurement of work input during the fatigue process.

Keywords:
1. Introduction

The paper by Yao et al. (2014) discuss that the use of the strain energy release rate SERR ($G$) to characterize fatigue delamination growth, either in the form of its maximum $G_{\text{max}}$, or in the range $\Delta G$ is problematic in delamination, because of crack bridging, which leads to a crack R-curve in static tests, and therefore quite intuitively a shift of Paris curves also during fatigue loading. They find moreover that fatigue precrack or static precrack lead to different fibre bridging, so they argue that bridging is fundamentally different in static or fatigue conditions. But why they say the SERR is not an appropriate parameter?

Their proposal is to correlate the crack advance per cycle $da/dN$ with $dU/dN$, where $U$ is applied work per cycle. If we limit our discussion to the case of loading ratio $R = 0$ (althogh their paper contains experiments at $R = 0.5$), we can define as

$$U = \frac{1}{2} P_{\text{max},N} \delta_{\text{max},N}$$

where $P_{\text{max},N}$ is the maximum force at the cycle number $N$; $\delta_{\text{max},N}$ is the maximum displacement at the cycle number $N$. Therefore, $U$ is computed “with the loads and displacements measured during the fatigue tests” – unfortunately, there are no plots of the load vs displacement relationship to check whether they are really linear so as to use this simple equation, rather than a full integration of the load-displacement curve. We use force per unit thickness, so also energies are per unit thickness and we simplify the notation.

As they write for their DCB specimen, their eqt.2 shows that the SERR

$$G_{\text{max}} \propto P_{\text{max}}^2 C^{2/3}$$

where $C$ is the compliance. Also, as they say and as the ASTM D5528-01 standard suggests, the compliance is proportional to

$$C \propto a^3$$

where $a$ is crack size. $P_{\text{max}}$ varies during their test, which control the $d_{\text{max}}$. 


It seems therefore that they are essentially using an estimate of SERR which comes from a LEFM model, although it would seem that fibre bridging would make the problem non-linear, and hence in using the ASTM equation for the compliance, there is an intrinsic simplification of the SERR. Unfortunately, they don’t show also the compliance measurement, to judge how this is linear, and whether the use of ASTM Modified Compliance Calibration really is a linearized curve fit about some intermediate condition.

A general calculation for any geometry and either linear or non-linear behaviour would give

\[- \frac{dU_{\text{max}}}{dN} = - \frac{dU_{\text{max}}}{da} \frac{da}{dN}\]  \hspace{1cm} (4)

so if we attempt a power law fit

\[\frac{da}{dN} = C_1 \left( - \frac{dU_{\text{max}}}{dN} \right)^{m_1} = C_1 \left( - \frac{dU_{\text{max}}}{da} \frac{da}{dN} \right)^{m_1}\]  \hspace{1cm} (5)

we are really writing

\[\frac{da}{dN} = C_1^{-1 - m_1} \left( - \frac{dU_{\text{max}}}{da} \right)^{m_1 / (1 - m_1)} \hspace{1cm} (6)\]

But from the definition of SERR \(G_{\text{max}} = - \frac{dU_{\text{max}}}{da}\). Hence, if

\[\frac{da}{dN} = C_2 G_{\text{max}}^{m_2} = C_2 \left( - \frac{dU_{\text{max}}}{da} \right)^{m_2}\]  \hspace{1cm} (7)

it results the Delft proposal is perfectly equivalent to the classical SERR one, and the correspondence between the two power law is precisely that

\[C_2 = C_1^{-1 - m_1/1 - m_1}; \quad m_2 = \frac{m_1}{1 - m_1} \hspace{1cm} (8)\]

Figure 1 shows the fatigue resistance curves plotted as \(\frac{da}{dN} = C_2 G_{\text{max}}^{m_2}\) with different pre-crack lengths for Specimen F.2 (45//45 interface) of the Yao et al(2014) paper, showing the data converge for large precrack size with a \(m_2 = 11.2\), but for the small precrack the exponent is also rather different. Fig.2 shows the same data but in terms of \(\frac{da}{dN} = C_1 \left( - \frac{dU_{\text{max}}}{dN} \right)^{m_1}\) show a \(m_1 = 0.8\). Hence, with our prediction we should have \(m_2 = \frac{0.8}{0.2} = 4\) — the general trend we predict is correct, although we have made the calculation very simple with assuming \(R = 0\) which is not really their case of \(R = 0.5\).
Fig. 1 - The data from Yao et al. show \( \frac{da}{dN} = C_2 G_{\text{max}}^{m_2} \) (is \( G_{\text{min}} = 0? \)) with \( m_2 \) varying from \( m_1 = 18 \) to \( m_1 = 11 \) when pre-crack is long enough.

Fig. 2 - The data collapse for various laminates using \( \frac{da}{dN} = C_1 \left( \frac{dU_{\text{max}}}{dN} \right)^{m_1} \) with \( m_1 = 0.8 \).

2. Discussion and non-linear models

When dealing with crack fibre bridging, we cannot estimate \( G \) using LEFM. As Barenblatt and Botvina clarified very clearly (see Ciavarella et al., 2008), clear dimensional analysis arguments elucidate that Paris’ power-law is a weak form of scaling, and Paris’ parameters \( C \) and \( m \) should not be taken as true “material constants”. Indeed, they are expected to depend on
all the dimensionless parameters of the problem, and can be considered close to be “constants” only for example when crack length is much larger than any other intrinsic characteristic of the problem, including material length scales – for a cohesive zone model, the length of it should be much smaller than the crack size. Size-scale dependencies of $m$ and $C$ like those reported by Yao et al (2014) were known much earlier in concrete in his studies of concrete (see see Ciavarella et al., 2008).

We attempt here a very preliminary and qualitative use of the Barenblatt-Dugdale cohesive model for a crack from $x = 0$ to $x = a$, for which we have a ”process zone” of size $b$ and this is such to eliminate the stress singularity at the fictitious crack tip, $x = a + b$. Therefore, there are two SIFs, one due to external loading, and one due to cohesive stresses $\sigma_c$ (like fibre bridging stresses, or yield stress in Dugdale material, or true cohesive stress in the Barenblatt original model). $G$ can then be computed as

$$G = \sigma_c \delta$$

where $\delta$ is the COD at the mouth of the real crack $x = a$, i.e. at the end of the cohesive zone. It is clear that for the SIF corresponding to the external loading, the linear equations apply. But the cohesive stress region introduces a non-linearity, the bigger the cohesive region is. And the true $G$ is less than that computed from the external loading condition only. When a crack develops a large cohesive zone compared to its size, i.e. when the crack is small, we have the bigger deviation from linearity, whereas when the crack is sufficiently long, the cohesive zone becomes small enough for SSY to occur and the cohesive model converges essentially to the LEFM linear predictions.

Delamination with fibre bridging shows the typical elastic-plastic fracture mechanics Irwin’s crack extension resistance curve (R-curve). This is why for larger precracks, Yao et al. find much lower Paris curves coefficient $C$. However, R-curve depends on the geometry and $G$ may be difficult to calculate. Also, Yao et al (2014) discuss, the cohesive model would need to predict a different cohesive zone size in static and fatigue condition for the same crack size.

For illustrative purposes, let us consider the Barenblatt-Maugis model the length of this cohesive zone for a crack in a infinite sheet (Cornetti et al., 2016) is, under SSY (Small Scale Yielding) ($b << a$)

$$b = \frac{\pi}{8} \left( \frac{K_I}{\sigma_c} \right)^2$$

(9)
while $G = \sigma_c \delta$. We retain however at least a second order term in the COD (Crack Opening Displacement)

$$\delta = \frac{8\sigma_c a}{\pi E} \ln \left( \frac{a + b}{a} \right) \approx \frac{8\sigma_c a}{\pi E} \left( \frac{b}{a} - \frac{b^2}{2a^2} \right)$$  \hspace{1cm} (10)

Then,

$$G = \frac{K^2}{E} \left( 1 - \frac{\pi K^2}{16 \sigma_c^2 a} \right) = G_{el} \left( 1 - \frac{\pi E G_{el}}{16 \sigma_c^2 a} \right)$$  \hspace{1cm} (11)

The first term is the classical linear term, while the second term gives a reduction which depends on cohesive stress, and size of the crack — disappearing for large cracks (for which there is no ”bridging” in our composite delamination case). Therefore, a Paris law in this case could be written as

$$\frac{da}{dN} = C_3 G_{m3}^{m3} = C_3 \left( 1 - \frac{\pi E G_{el,max}}{16 \sigma_c^2 a} \right)^{m3} \frac{G_{m3}^{m3}}{G_{el,max}} = C_{3,eff} G_{el,max}^{m3}$$  \hspace{1cm} (12)

so $C_{3,eff} = C_3 \left( 1 - \frac{\pi E G_{el,max}}{16 \sigma_c^2 a} \right)^{m3}$ would be reduced for shorter cracks, as it appears indeed in the data of Yao et al.(2014) when they plot Paris plots in terms of $G_{el,max}$. 

Cohesive models for fatigue delamination usually are of different type, they try to model the damage occurring during fatigue see (Pascoe et al., 2013), and we are not aware of a simple model like the one we are describing. But it may well predict more closely the results Yao et al.(2014) seem to observe.

3. Conclusion

The proposal by Yao et al.(2014) to compute the crack driving force in fatigue from the actual work done in the loading testing machine is not in contrast with a Strain Energy Release Rate (SERR) approach, but only different from the SERR computed from a linear model. In the classical problem where we have no deviations from a Small Scale Yielding (SSY) assumption, we have derived the relationship between the Yao et al.(2014) Paris-type proposal and the classical Paris type equation on $G_{max}$. The observation from the group of Benedictus is anyway extremely simple and useful. It clarifies important aspects of fatigue crack propagation, with particular reference to materials with R-curve behaviour like delamination in composites. There
are some not very clear points in the paper by Yao et al.(2014) who at some point seem to confuse an energy balance approach like in Griffith fracture, with a fatigue process which is only a weak form of scaling with the Irwin Stress Intensity Factor (at least in the original Paris form), or in terms of more general crack driving forces, in later generalizations. The main drawback of the Delft proposal is perhaps that it is not predictive, as it requires the actual measurement during the fatigue process of the work input in the specimen by the testing machine. It rather suggest, instead, that cohesive models which appear not to have been attempted in a simple form as we are discussing here, would be more successful than LEFM models in predicting fatigue delamination crack growth.

4. References

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