Determination of vertical derivative of gravity anomalous by upward continuation and Taylor series transform methods: application to the Southwest sub-basin of the East Vietnam Sea

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ABSTRACT

The vertical derivative of the gravity anomaly has a vital role in the methods of geological structure research such as determining fault systems and the location of the field sources. In addition, the vertical derivative is also used to calculate the downward continuation and further clarify the image of the seabed topography. However, determining the vertical derivative according to the traditional Fast Fourier Transform (FFT) method is often unstable and has low accuracy in high-order derivatives for high noise actual data. In this article, we introduce a new calculation method to determine the vertical derivative of gravity anomaly giving higher stable and accurate than traditional methods. The method is verified on synthetic model data and actual data of the Southwest sub-basin of the East Vietnam Sea.

Keywords: Gravity anomaly, vertical derivative, Taylor series, fast Fourier transform.
INTRODUCTION

Calculating the vertical derivative of gravity anomaly is an essential method in gravimetric data processing methods to determine the boundary of geological structure or fault system [1–5]. In addition, the vertical derivative is also used in calculating the downward continuation [6–12] to increase the accuracy when determining the topography of boundary surfaces such as sedimentary foundations and seabed topography [13, 14]. It is easy to see that the vertical derivative dramatically affects the accuracy of the above calculation methods. In previous studies, the authors mainly used the vertical derivative through the frequency domain, such as fast Fourier transform (FFT), Hilbert transform [15, 16], or the method of Laplace equation [17]. However, Kha and Trung [12] published a new technique using the upward continuation and Taylor series expansion methods (UCT) to calculate the vertical derivative, and showed that the calculation of the vertical derivative through the frequency domain is unstable when the data have noises, especially in case of higher-order vertical derivatives, which can affect the results of determining the structural boundary as well as the stability in the downward continuation problem. This UCT method has increased the accuracy and stability of the calculation of the higher order vertical derivatives than previous traditional methods such as FFT, Hilbert, or Laplace methods. Recently, some authors have also been used the UCT method in calculating gravity tensors [18–20], the vertical derivative in determining the structural boundaries of the Witwatersrand basin, South Africa [3]. Trung et al., (2020) [14] used the UCT method to calculate the downward continuation of the Bouguer anomaly data to the near seabed, then reverted this downward continuation gravity anomaly data to determine the sediment basement. The obtained results have higher resolution and reliability than the conventional calculation method.

To see the important application meaning of calculating the vertical derivative in the analysis of gravity data, in this article, we introduce the method of calculating the first and second vertical derivatives of the gravity anomaly according to the UCT method [12]. The results of applying the UCT method to calculate the vertical derivative of gravity anomalies have been verified on synthetic model data and actual data in the Central Basin of the East Vietnam Sea, giving results with high accuracy and stability than previous traditional methods, especially in the case of high random noise data.

THEORETICAL BASIS OF THE METHOD

Assuming \( f(x, y, z) \) is a potential field measured at an observation plane of height \( z \), in the Decartes coordinate system, the \( z \)-axis positive direction is downward. The potential field at height \( (z - \Delta h) \) is \( f(x, y, z - \Delta h) \) which can be represented by Taylor series as follows [7, 17]:

\[
f(x, y, z - \Delta h) = f(x, z) - \Delta hf'(x, y, z) + \frac{\Delta h^2}{2!} f'' + \ldots + \frac{(-\Delta h)^n}{n!} f^n (x, y, z)
\]

(1)

where: \( f'(x, y, z), f''(x, y, z), \ldots, f^n(x, y, z) \) are the vertical derivatives in order of 1, 2, \ldots, \( n \). In this study, we do not determine these vertical derivative values according to the traditional fast Fourier transform (FFT) method, but determine them through the gravity anomaly at the different upward continuation level \( f(x, y, z - \Delta h), f(x, y, z - 2\Delta h), \ldots, f(x, y, z - n\Delta h) \).

We consider equation (1) with a Taylor series expansion of \( n = 1, 2, \ldots, N \) and \( \Delta h \) is positive and small enough, then we have the following system of equations:
The equations (2) can be written in the form of matrix equation (3) as follows:

\[
\begin{pmatrix}
-\Delta h \\
-2\Delta h \\
n\Delta h
\end{pmatrix}
\begin{pmatrix}
\frac{(-\Delta h)^2}{2!} \\
\frac{(-2\Delta h)^2}{2!} \\
\frac{(-n\Delta h)^2}{2!}
\end{pmatrix}
\begin{pmatrix}
f'(x, y, z) \\
f''(x, y, z) \\
f'''(x, y, z)
\end{pmatrix}
= \begin{pmatrix}
f(x, y, z - \Delta h) - f(x, y, z) \\
f(x, y, z - 2\Delta h) - f(x, y, z) \\
\vdots \\
f(x, y, z - n\Delta h) - f(x, y, z)
\end{pmatrix}
\]

Solve the linear matrix equation (3) with the unknowns being the derivatives of order \( n = 1, 2, \ldots, N \) (\( f'(x, y, z), f''(x, y, z), \ldots, f^n(x, y, z) \)).

For \( n = 8 \), solving equation (3) we get the vertical derivatives from the 1st to 8th order. Here is the definite formula for the vertical derivatives of the 1st, 2nd and 3rd order:

\[
f'(x, y, z) = \frac{2283}{840\Delta h} f(x, y, z) - 6720 f(x, y, z - \Delta h) + 11760 f(x, y, z - 2\Delta h) - 15680 f(x, y, z - 3\Delta h) + \ldots
\]

\[
+ \frac{14700}{840\Delta h} f(x, y, z - 4\Delta h) - 9408 f(x, y, z - 5\Delta h) + 3920 f(x, y, z - 6\Delta h) - 960 f(x, y, z - 7\Delta h) + 105 f(x, y, z - 8\Delta h)
\]

\[
f''(x, y, z) = \frac{29531}{5040\Delta h^3} f(x, y, z) - 138528 f(x, y, z - \Delta h) + 312984 f(x, y, z - 2\Delta h) - 448672 f(x, y, z - 3\Delta h) + \ldots
\]

\[
+ \frac{435330}{5040\Delta h^3} f(x, y, z - 4\Delta h) - 284256 f(x, y, z - 5\Delta h) + 120008 f(x, y, z - 6\Delta h) - 29664 f(x, y, z - 7\Delta h) + \frac{3267}{5040\Delta h^3} f(x, y, z - 8\Delta h)
\]

\[
f'''(x, y, z) = \frac{2403}{240\Delta h^3} f(x, y, z) - 13960 f(x, y, z - \Delta h) + 36706 f(x, y, z - 2\Delta h) - 57384 f(x, y, z - 3\Delta h) + \ldots
\]

\[
+ \frac{58280}{240\Delta h^3} f(x, y, z - 4\Delta h) - 39128 f(x, y, z - 5\Delta h) + 16830 f(x, y, z - 6\Delta h) - 4216 f(x, y, z - 7\Delta h) + \frac{469}{240\Delta h^3} f(x, y, z - 8\Delta h)
\]

The 1st, 2nd, and 3rd vertical derivative in the formulas (7), (8), and (9) are defined from the gravity anomalies upward continued at the elevation \( \Delta h, 2\Delta h, \ldots, 8\Delta h \). Calculation results using these formulas give us a more stable and accurate vertical derivative value than other
conventional calculation methods, especially in the case of noisy data [12].

**APPLY ON SYNTHETIC MODEL**

The theoretical model investigated in this study is a rectangular prism with physical parameters, as shown in Table 1. Gravity anomalies on the observed surface of the rectangular prism are shown in Figure 1a. The first, second, and third order vertical derivatives of the gravity anomaly of the rectangular prism are calculated by equations (4), (5), and (6) for both correct gravity anomaly data (without random noise) and plus 5% random noise. The results of calculating the vertical derivative by formulas (4), (5) and (6) are compared with the result of calculating the vertical derivative by the traditional FFT method.

**Table 1. Physical parameters of the prism model**

| $X_1$ (km) | $X_2$ (km) | $D_x$ (km) | $Z_1$ (km) | $Z_2$ (km) | Mật độ dư (g/cm$^3$) |
|------------|------------|------------|------------|------------|-------------------|
| 60         | 65         | 0.5        | 3          | 6          | 0.2               |

**Figure 1.** a) Gravity anomaly of rectangular prisms; b) Rectangular prism model with physical parameters in Table 1

The calculation results of the first, second, and third order vertical derivatives in Figure 2 show that in the case of data without noise (exact data), the difference between the vertical derivative is calculated by the FFT method and the UCT method is a minimal error and almost the same (Table 2 and Figure 2). The vertical derivative values have minor errors compared with the theoretical values in all three cases of first, second, and third derivatives (Table 2). Thus, with correct data, the vertical derivatives are calculated by the FFT method, and the UCT method does not see the difference.

**Table 2. Root mean square error (RMSE) of the first, second and third order vertical derivatives are calculated by FFT and UCT methods in the case of the correct gravity anomaly**

| Root mean square error | The 1$^{st}$ vertical derivative (mGal) | The 2$^{nd}$ vertical derivative (mGal) | The 3$^{rd}$ vertical derivative (mGal) |
|------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| FFT method             | 0.00267                                | 0.00086                                | 0.00154                                |
| UCT method             | 0.00267                                | 0.00086                                | 0.00144                                |

When the gravity anomaly is added 5% random noise, the vertical derivative calculated by the UCT and the FFT methods are very different (Figure 3). The vertical derivative calculated by the FFT method gives very poor results. The vertical derivative value is noisy and unstable: The first order vertical derivative (red line in Figure 3a) appears relatively large sawtooth pulses at the high anomalous amplitude region. The 2$^{nd}$ and 3$^{rd}$ order derivatives (red lines in Figures 3b and 3c) are instability, the
vertical derivative value is strongly perturbed, and the amplitude is completely different from its true value (amplitude is larger 10 times for the second-order derivative and 100 times for the third-order derivative). The vertical derivative calculated by the UCT method gives very good results in the first-order vertical derivative (Figure 3a), and even with the second-order vertical derivative, the value is consistent with the theoretical value (3b). For the third order vertical derivative (Figure 3c), although there is a discrepancy between the theoretical and calculated results, the shape of the vertical derivative graph is quite consistent with the theoretical derivative.

**Figure 2.** The vertical derivative of gravity anomaly of the rectangular prism calculated by the FFT method and the UCT method almost coincides with the theoretical value; a) the first order vertical derivative; b) the 2nd order vertical derivative; c) the 3rd order vertical derivative

**Figure 3.** Vertical derivative of gravity anomaly of a prism (Figure 1) when the gravity anomaly is added 5% random noise; a) the 1st order vertical derivative; b) the 2nd order vertical derivative; c) the 3rd order vertical derivative
Thus, it can be seen that, in the case of measured data with random noise, the calculation of the vertical derivative by the method proposed by Kha and Trung (2020) [12] gives very high accurate and stable results while the results calculated by the FFT method have low accuracy and unstable results, especially in the case of second and third order derivatives.

VERTICAL DERIVATIVE OF GRAVITY ANOMALY IN THE SOUTHWEST OF THE CENTRAL BASIN, EAST VIETNAM SEA

The study area is located in the southwest of the Central Basin of the East Vietnam Sea (Figure 4), including the area of the Southwest sub-basin in the center, a part of the Hoang Sa islands in the North and a part of the Truong Sa islands in the South (Figure 4a). Figure 4b is the Bouguer gravity anomaly calculated from the free-air satellite gravity anomaly data with 1’ × 1’ resolution (https://topex.ucsd.edu/cgi-bin/get_data.cgi), V29.1 [22] and bathymetry data from GEBCO source with 15” × 15” resolution. (https://www.gebco.net/data_and_products/gridded_bathymetry_data/). The results of calculating the first-order vertical derivative by the UTC method (Figure 5a) have stable values, except for some areas Northeast of the Southwest sub-basin. The first order vertical derivative map has sharp positive and negative anomalies, clearly reflecting the geological structure system in the study area, such as spreading ridge axis, continental - oceanic crust boundary, NE-SW fault system, and sub-meridian, sub-latitude faults (Figure 4a and Figure 5a). The first order vertical derivative map calculated by the FFT method gives much worse results: in the Southwest sub-basin and the boundary of the oceanic-continental crust. The first order vertical derivative appears in many speckled spots showing unstable perturbation of the calculation results (Figure 5c). The calculations of the second vertical derivative show that for the UTC method, the derivative value also appears a little unstable in some sub-regions of the southwest basin, such as along the spreading ridge axis in the northeast (Fig. 5b). However, this map has reflected quite well the structural elements in the study area, such as the spreading ridge axis, NE-SW, and sub-meridian fault systems. On the contrary, looking at the results of calculating the second derivative by the FFT method (Figure 5d), we see that the obtained results are fuzzy. The second derivative values fluctuate very strongly, forming unstable value regions, and the gravity field image does not reflect the structural elements in the study area.

Figure 4. (a) Diagram of fault system [21] and location of the study area; (b) Bouguer gravity anomaly map of the study area
Some methods for determining the horizontal boundary of the field sources using vertical derivatives [1–4, 22–24] calculated by the FFT method will inevitably lead to instability in the calculation results. For example, below is the result of calculating the logistic function of the total horizontal gradient (LTHG) [3] using the vertical derivative of the entire horizontal gradient:
\[ LTHG = \left[ 1 + \exp \left( -\frac{\partial THG}{\partial z} \right) \right]^{1/2} \]

We see the LTHG formula that if we use the FFT method to calculate the vertical derivative, the result will be far different from the UTC method. Figure 6a is a map of LTHG calculated by the UTC method with very stable results, the sequence of peak points is obvious.

Meanwhile, the LTHG map calculated by the FFT method (Figure 6b) gives bad results in the Southwest sub-basin. It is clear that the accuracy of the vertical derivative has a significant influence on the quality of the LTHG map.

![Figure 6](image)

**Figure 6.** Comparison of results of calculating LTHG by UTC method (a) and by FFT method (b)

**CONCLUSION**

Equations (4), (5), and (6) allow for the calculation of the first, second, and third derivatives of potential field anomalies with higher stability and accuracy than the conventional methods, especially in case of the noisy data. Calculating the first and second order vertical derivatives of the gravity anomaly by the UTC method achieves high accuracy, making an important contribution to the processing and interpretation of gravity data using vertical derivative: the obtained results have high accuracy and stability easier to determine the horizontal geological structure boundaries. The UTC method can be used to calculate the downward continuation over some deep water areas to increase the detail of local, shallow geological structures, thereby contributing to clarifying the image of geological structure in the study area.

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