The Cardy-Verlinde formula for 2D gravity

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Abstract

We discuss the different bounds on entropy in the context of two-dimensional cosmology. We show that in order to obtain well definite bounds one has to use the scale symmetry of the gravitational theory. We then extend the recently found relation between the Friedmann equation and the Cardy formula to the case of two dimensions. In particular, we find that in two dimensions this relation requires that the central charge $c$ of the conformal field theory is given in terms of the Newton constant $G$ of the gravitational theory by $c = 6/G$. 

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1 Introduction

In a recent paper [1], Verlinde has discussed the cosmological bounds on entropy for spacetimes of dimension $d > 2$. These are based on the holographic principle [2], which states that the entropy contained into a given region of space should be bounded by the area of the spacelike surface that encloses it. Another important result of [1] is that the Friedmann equation of cosmology for a radiation-dominated universe can be shown to be equivalent to the Cardy formula for the entropy of a conformal field theory describing the radiation. This observation has of course important implications, which have not been fully clarified yet.

The context of the original proposal of Verlinde was $d > 2$ cosmology. Although the Cardy-Verlinde formula has been generalized to describe other gravitational systems [3, 4], in particular black holes, a discussion of the $d = 2$ case is still lacking. The two-dimensional (2D) limit of the Cardy-Verlinde proposal is interesting for various reasons. From investigations of the anti-de Sitter (AdS)/Conformal Field Theory (CFT) correspondence, we know that there are 2D gravitational systems that admit 2D CFTs as duals [5, 6]. In this case one can make direct use of the original Cardy formula [7] to compute the entropy [5, 6]. A comparison of these results with a 2D generalization of the Cardy-Verlinde formula could be very useful in particular for the understanding of the puzzling features of the AdS/CFT correspondence in two dimensions [8]. Another point of interest in extending the Cardy-Verlinde formula to $d = 2$ is the clarification of the meaning of the holographic principle for 2D spacetimes. The boundaries of spacelike regions of 2D spacetimes are points, so that even the notion of holographic bound is far from trivial.

A generalization of the work of Verlinde to two spacetime dimensions presents several difficulties, essentially for dimensional reasons. First of all, in two dimensions one cannot establish a area law, since black hole horizons are isolated points. Moreover, the spatial coordinate is not a "radial" coordinate and hence one cannot impose a natural normalization on it. As we shall see later on this paper in detail, this fact is connected, at least for the 2D gravity model we consider here, to a scale symmetry of 2D gravity [9]. Related to this symmetry is also the fact that the 2D gravitational coupling constant $G$ is dimensionless, and hence one cannot even define a "Planck" length. Finally, if one works, as we do in this paper, in the context of scalar-tensor theories of gravity, the 2D cosmological equations are quite different from
their Friedmann-Robertson-Walker $d > 2$ counterparts.

Some of these problems may of course be solved if one considers gravity in two dimensions as a dimensionally reduced theory. However, if one wants to keep a purely two-dimensional point of view, one has to deal with the particular features of 2D gravity.

In this paper, we wish to extend Verlinde’s results to a radiation-dominated $1+1$ universe, in which the gravitational interaction is governed by a Jackiw-Teitelboim (JT) model \[10, 11\]. We shall see that this goal can be achieved, provided that some free parameters appearing in the solutions are fixed using the scale symmetry of the theory.

The paper is organized as follows: in sect. 2 we discuss the cosmological model derived from the JT model. In sect. 3 we discuss how the standard cosmological bounds on the entropy can be generalized to our case. In sect. 4 we use the scale symmetry of the gravitational theory to fix the free dimensionless parameters appearing in the bounds. In sect. 5 we investigate the relations between the cosmological equations and the Cardy formula. The cosmological bounds on the temperature of a radiation-dominated universe are discussed in sect. 6. Finally, in sect. 7, we present our conclusions.

## 2 Two-dimensional cosmology

Let us consider the action for JT gravity minimally coupled to matter,

\[ I = \int d^2 x \sqrt{-g} \left( \eta \frac{\mathcal{R} - 2\lambda^2}{16\pi G} + L_M \right), \tag{1} \]

where $\eta$ is a scalar field (the dilaton), $G$ is the dimensionless 2D Newton constant, which could be absorbed in a redefinition of the dilaton, $\lambda^2$ is a cosmological constant and $L_M$ is the matter lagrangian. We want to discuss a radiation-dominated $1+1$ universe, in which case $L_M$ describes free (or weak interacting) massless particles. In general, $L_M$ can be given in terms of a 2D CFT. Because the matter lagrangian is that of a perfect fluid, it can be taken proportional to its density, $L_M = -\rho$ \[12\]. A constrained variation of \( I \) gives the field equations \( \mathcal{R} = 2\lambda^2, \)

\[ -(\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^2)\eta + \lambda^2 g_{\mu\nu} \eta = 8\pi T_{\mu\nu}, \tag{2} \]
where $T_{\mu\nu}$ is the standard energy-momentum tensor of a perfect fluid, $T_{\mu\nu} = p_M g_{\mu\nu} + (\rho + p_M) u_\mu u_\nu$, with $p_M$ the pressure of the fluid. The field equations (2) tell us that, independently of the matter, the spacetime has constant, positive curvature. It is therefore given by a 2D de Sitter (dS) spacetime.

We make the ansatz $ds^2 = -dt^2 + R^2(t)dx^2$, $\eta = \eta(t)$, with $0 \leq x \leq 2\pi$. Since we take $x$ periodic, we are considering a closed 1+1 universe. However, our considerations can be easily extended to a open universe. The field equations then take the form

\begin{align*}
\ddot{R} - \lambda^2 R &= 0, \\
\ddot{\eta} - \lambda^2 R\dot{\eta} &= 8\pi G R \rho, \\
\dot{\eta} - \lambda^2 \eta &= -8\pi G p_M.
\end{align*}

(3)

Combining the field equations, one obtains the energy momentum conservation in the form $\dot{\rho} = -(p_M + \rho)\dot{R}/R$. For a perfect fluid, $p_M = \gamma \rho$, and this relation can be integrated to yield $\rho R^{1+\gamma} = \text{const} = M/2\pi$. We are considering the case in which the matter is constituted of pure radiation, for which $\gamma = 1$, so that we have,

$$\rho = \frac{M}{2\pi R^2}. \quad \text{(4)}$$

The general solution of the first of Eqs. (3) is

$$R(t) = \bar{a} e^{\lambda t} + \bar{b} e^{-\lambda t}. \quad \text{(5)}$$

Depending on the relative sign of the integration constants $\bar{a}$ and $\bar{b}$, and with a suitable choice of the origin of time, the solution can assume three qualitatively different forms:

\begin{align*}
\text{I) } & R = \frac{a}{\lambda} e^{\lambda t}, \\
\text{II) } & R = \frac{a}{\lambda} \sinh \lambda t, \\
\text{III) } & R = \frac{a}{\lambda} \cosh \lambda t.
\end{align*}

(6) (7) (8)

where $a$ is a dimensionless parameter (we choose this normalization, in order to give to $R$ the physical dimension of a length).
The solutions for the scalar field are, respectively

\[ I) \quad \eta = \eta_0 e^{\lambda t} - \frac{4GM}{3a^2} e^{-2\lambda t}, \]

\[ II) \quad \eta = \eta_0 \cosh \lambda t + \frac{4GM}{a^2} \left( 1 + \cosh \lambda t \log \tanh \frac{\lambda t}{2} \right), \]

\[ III) \quad \eta = \eta_0 \sinh \lambda t - \frac{4GM}{a^2} \left( 1 + \sinh \lambda t \arctan \sinh \lambda t \right), \quad (9) \]

with \( \eta_0 \) an integration constant. All solutions are of course locally isomorphic to de Sitter spacetime, but with different parametrization, covering different regions of the 2D manifold. In particular, I and II possess a horizon at \( t = -\infty \) and \( t = 0 \), respectively. Moreover for all solutions I, II and III the scalar field \( \eta \) has a zero at a finite value \( t_0 \) of the cosmological time \( t \). Since we are dealing with a Brans-Dicke-like theory of gravity, \( \eta^{-1} \) represents a time-dependent effective Newton constant. The instant \( t = t_0 \) may therefore be interpreted as an initial singularity and we will restrict ourselves to consider only times \( t \geq t_0 \), when \( \eta \geq 0 \).

In two dimensions there is no direct analog of the \( d \)-dimensional Friedmann equation

\[ H^2 = \lambda^2 + \frac{16\pi G}{(d-1)(d-2)} \rho - \frac{1}{R^2}, \quad (10) \]

where \( H \equiv \dot{R}/R \) is the Hubble parameter (Notice that the Friedmann equation (10) is singular for \( d = 2 \)). However, an equation for \( H^2 \) can be obtained by integrating the first equation in (3):

\[ H^2 = \lambda^2 - \frac{\alpha}{R^2}. \quad (11) \]

The constant of integration \( \alpha \) for the solutions I-III is respectively, 0, \(-a^2\), \(a^2\). For a radiation-dominated universe, one can use Eq. (9) into Eq. (11), to obtain an expression formally similar to (10),

\[ H^2 = \lambda^2 + 8\pi G \rho - \frac{\bar{\alpha}}{R^2}, \quad (12) \]

where \( \bar{\alpha} = \alpha + 4GM \). Notice that we are using the arbitrariness of the integration constant \( \alpha \) to make the metric of the spacetime dependent on the matter. Consistently with the field equations (3) the effect of the matter on the metric can be only encoded in the choice of integration constants.
Our gravity model (1) has a cosmological constant different from zero. Therefore, one can assign to the vacuum an energy $E_\lambda = \lambda^2 R/4G$ and a pressure $p_\lambda = -E_\lambda$. This permits to write Eq. (12) in terms of the total energy $E = (E_\lambda + E_M)$, where $E_M = M/R$ is the energy of the matter,

$$H^2 = \frac{4GE}{R} - \frac{\bar{\alpha}}{R^2}. \quad (13)$$

### 3 Entropy bounds

In $d > 2$ a bound on the entropy of a macroscopic system, $S \leq S_B$, is believed to hold, where the Bekenstein entropy $S_B$ is defined as

$$S_B = \frac{2\pi}{d-1}ER, \quad (14)$$

with $E$ the total energy and $R$ the linear size of the system. This bound is verified for standard gases, but the numerical factor in front of $ER$ is fixed by the assumption that the bound is saturated by black holes.

A generalization of this bound to two dimensions is not straightforward. Consider for example the 2D anti-de Sitter black hole (14), which is a solution of the gravity model (1) with negative $\lambda^2$,

$$ds^2 = -(\lambda^2 x^2 - m^2)dt^2 + (\lambda^2 x^2 - m^2)^{-1}dx^2 \quad \eta = \eta_0 \lambda x \quad (15)$$

A horizon occurs at $x_0 = m/\lambda$, and one can associate to it the temperature $T = \lambda m/2\pi$ and the entropy $S = 2\pi\eta_0 m$. Moreover, by standard methods, one can assign to the black hole the ADM mass $\mathcal{M} = \eta_0 \lambda m^2/2$. Thus one gets the relation

$$S = \frac{4\pi \mathcal{M} x_0}{m^2} \quad (16)$$

If one identifies the energy $E$ of the black hole with its ADM mass and its size $R$ with the length $x_0$, one finds $S \propto ER$, but the ratio $S/ER$ grows without limit for small $m$ (in the limit $m \to 0$, however, all quantities vanish). The same situation occurs in more general two-dimensional models. The problem is connected to the fact that in two dimensions there is no radial coordinate and hence the coordinate $x$ cannot be properly normalized, or equivalently to the scale symmetry of the model, which will be discussed in the next section.
Thus, although one can envisage a Bekenstein bound of the form

\[ S \leq S_B = 2\pi\epsilon ER, \]  

(17)

which can also be deduced from the thermodynamics of a one-dimensional gas, the coefficient \( \epsilon \) is not clearly determined. Notice that in the radiation-dominated cosmological model of the previous section \( S_B = 2\pi M \) is a conserved quantity, proportional to the matter density.

The Bekenstein bound is believed to hold when the gravitational energy of the system is small with respect to its total energy, i.e. in a weak-gravitating regime. For strong-gravitating systems, i.e. systems for which \( HR > 1 \), a different bound must be introduced. In order to establish in which regime a given system is, it is useful to define a Bekenstein-Hawking entropy \( S_{BH} \) as the Bekenstein entropy of a system with \( HR = 1 \) [1]. From the "Friedmann" equation (13) one obtains

\[ S_{BH} = \frac{V}{4GR}(1 + \bar{\alpha}) = \frac{\pi}{2G}(1 + \bar{\alpha}), \]  

(18)

where \( V = 2\pi R \) is the spatial volume. Recalling the definition of \( \bar{\alpha} \), Eq. (18) can also be written as

\[ S_{BH} = \frac{\pi}{2G}(1 + \alpha) + 2\pi M \]  

(19)

This is the sum of two constant contributions: the first depends only on the geometry, while the second, which is proportional to \( S_B \), depends only on the matter content and is not present in \( d > 2 \). In absence of matter, the second contribution vanishes. The appearance of a factor \( \alpha \) proportional to \( a^2 \) is again a consequence of the scale invariance of the theory, which does not fix the scale of the spatial coordinate in the solutions.

Notice that the bound \( S \leq S_{BH} \) is a truly holographic bound for a 2D spacetime. The boundary of a spatial section of a 2D universe are two points: thus the holographic principle states that the entropy can only depend on the Newton constant \( G \) and on a dimensionless parameter.

In a cosmological contest and when \( HR > 1 \) the Bekenstein bound must be replaced by a holographic bound. However, it has been argued that the Bekenstein-Hawking bound \( S \leq S_{BH} \) is not the right choice. A suitable bound is given by the Hubble entropy \( S_H \), defined as the entropy of a universe filled with black holes of the size of a particle horizon [13]. Later, a weaker
definition of $S_H$ was proposed, in which the maximal size of the black holes is the Hubble radius $H^{-1}$ [14]. In our 2D context, $S_H$ can be calculated as follows [17]. From Eq. (18), a black hole of radius $H^{-1}$ has entropy $(1 + \bar{\alpha})H V_H/4G$. Since the universe can contain $N_H = V/V_H$ black holes, one obtains

$$S_H = \frac{V H}{4G} (1 + \bar{\alpha}) = \frac{\pi R H}{2G} (1 + \bar{\alpha})$$

(20)

Notice that, while the solution I and II have a particle horizon of the size of the Hubble radius, in case III the size of the particle horizon grows exponentially with time.

To conclude, although in the 2D case one may be able to obtain different entropy bounds, they do not seem to be universal. The entropy bounds $S \leq S_{BH}$ and $S \leq S_H$ depend on the arbitrary, dimensionless parameters $\bar{\alpha}$ and $\epsilon$. They appear to be defined up to arbitrary scales. In the next section we will show that this fact is a consequence of a scale symmetry of our 2D gravity model. This scale symmetry is a peculiarity of two-dimensional gravity and is related to the impossibility of defining an area law for the entropy.

## 4 Scale symmetry and entropy bounds

It is well known that 2D AdS space has $SL(2,R)$ as isometry group (see for instance Ref. [5]). The spacetime metric is therefore invariant under the subgroup of $SL(2,R)$ describing dilatations, which for the ground state $m = 0$ in Eq. (15) is realized as

$$x \rightarrow \nu x, \quad t \rightarrow t/\nu.$$  

(21)

Under this scale transformation the dilaton $\eta$ is not invariant, $\eta \rightarrow \nu \eta$, but the scale factor $\nu$ can be absorbed in a different definition of the integration constant $\eta_0$ appearing in Eq. (15). It is evident that this scale transformation is a classical symmetry of the theory because under $\eta \rightarrow \nu \eta$ the action for pure gravity changes just for an overall constant factor.

This scale symmetry is also a classical invariance of our (de Sitter) action (1) in absence of matter. In the matter-coupled case the scale transformation just changes by a constant factor the Newton constant $G$.

It is not difficult to realize that the presence of the integration constants $\eta_0, m$ (in the AdS solution) and of $\eta_0, a$ (in the dS solution) is a consequence
of the scale symmetry. The transformation (21) maps one solution of the fields equations characterized by \( \eta_0, m \) into an other solution with different values of the integration constants, \( \eta'_0, m' \). It is therefore evident that we can use the scale symmetry (21) to write the entropy bounds in a form that is independent of the dimensionless parameters \( \bar{\alpha}, \epsilon \).

Instead of directly working on the cosmological solution, it is more instructive to fix these parameters by considering the two-dimensional black hole. Since the 2D cosmological solutions are the analytical continuation \( \lambda \rightarrow i\lambda \) of the black hole solutions (15), one can fix \( \bar{\alpha}, \epsilon \) using the latter as the maximum entropy configuration.

The problem reduces then to fix the dependence on \( m \) of the thermodynamical parameters \( E = M, T, S \) of the AdS\( _2 \) black hole (15). Introducing the length scale \( L = 1/\lambda \) and the central charge \( c = 12\eta_0 \) of the thermal CFT arising in the AdS\( _2 \)/CFT correspondence one has [5, 9]

\[
T = \frac{1}{2\pi} \frac{m}{L}, \quad S = 2\pi \frac{c}{12} m, \quad E = \frac{c}{24} m^2.
\]  

For a generic black hole solution the behavior under the scale transformations has been given in Ref [9]. The AdS\( _2 \) black hole metric (15) is invariant under the scale transformations,

\[
x \rightarrow \nu x, \quad t \rightarrow \frac{t}{\nu}, \quad M \rightarrow \nu^2 M.
\]

The dilaton transforms as

\[
\eta \rightarrow \nu \eta,
\]

whereas, \( T, S, E \) scale as

\[
T \rightarrow \nu T, \quad S \rightarrow \nu S, \quad E \rightarrow \nu^2 E.
\]

The physical meaning of this scale invariance of the theory can be easily understood. It is a general feature of all metric theories of gravity that lengths (or masses) can be only measured with reference to an (asymptotic) reference frame. For asymptotically Minkowskian solutions this frame is given by Minkowski space with the usual normalization \( (ds^2 = -dt^2 + dx^2) \). Owing to the dilatation isometry of AdS\( _2 \), for solutions which are asymptotically AdS there is no such “preferred” reference frame. We are free to change the length of our rule using the scale transformations (23), without changing the physics. We have a sort of gauge symmetry, stating that black hole solutions
connected by the scale transformations (23) are physically equivalent. However, the energy $E$ and entropy $S$ change under the scale transformation, they are not gauge-invariant quantities. For this reason, although we cannot find an absolute upper bound for $S$, every $m$-dependent bound of the form $S(m) \leq S_H(m)$ has a gauge-invariant meaning. Thus, fixing the gauge we can remove from the entropy bounds the dependence on the dimensionless parameters $\alpha, \epsilon, m$. This can be easily done by using Eqs. (25) with $\nu = 1/m$ into Eqs. (22) to remove the $m$-dependence of $T, S, E$,

$$T = \frac{1}{2\pi L}, \quad S = \frac{c}{12} 2\pi E = \frac{c}{24} 1 L,$$

and choosing the value of $m$ to fix, by means of Eq. (24), $\eta_0$ (hence the central charge $c$) in terms of the Newton constant $G$,

$$\eta_0 = \frac{c}{12} = \frac{1}{2G}. \quad (27)$$

With this choice, one has for the two-dimensional black hole

$$S_B = 2\pi ER, \quad (28)$$

fixing $\epsilon = 1$ in the Bekenstein bound (17). Moreover, if one identifies $S_{BH}$ with the entropy of the AdS$_2$ black hole given by Eq. (26), one finds $\bar{\alpha} = 1$. It follows immediately

$$S_{BH} = \frac{\pi}{G}, \quad (29)$$

and hence

$$S_H = \frac{\pi RH}{G}. \quad (30)$$

With these definitions, one has the relation

$$S^2_H = S_{BH}(2S_B - S_{BH}). \quad (31)$$

Once the value of $\bar{\alpha}$ has been fixed, $\bar{\alpha} = 1$ the cosmological equation (13) takes the Friedmann form

$$H^2 = \frac{4GE}{R} - \frac{1}{R^2}. \quad (32)$$

\(^1\)Also the geodesic motion is unaffected by a rescaling of $m$, which simply shifts the definition of the energy of a test particle.
5 Cardy-Verlinde formula for 2D cosmology

Recalling the definition of the Bekenstein-Hawking entropy, one can define a Bekenstein-Hawking energy $E_{BH}$, as the energy corresponding to the condition $S_B = S_{BH}$, at which the gravitational system becomes strong-coupled:

$$E_{BH} = \frac{1}{2RG}$$  \hspace{1cm} (33)

Using Eqs. (30), (33) and (32) one gets a first form of the 2D Cardy-Verlinde formula

$$S_H = 2\pi R \sqrt{E_{BH}(2E - E_{BH})}$$  \hspace{1cm} (34)

Following Verlinde [1] it is now useful to split the total energy $E$ in extensive $E_E$ and non-extensive (Casimir) part $E_C$/2\footnote{The Casimir energy is of course $E_C/2$. We adopt this rather odd notation in order to conform to ref. [1] and to simplify some of the following formulas.}

$$E = E_E + \frac{E_C}{2},$$  \hspace{1cm} (35)

where $E_C$ is defined as

$$E_C \equiv E + pV - TS$$  \hspace{1cm} (36)

with $p = p_M + p_\lambda$.

The scaling behavior of $E_C$ follows from general arguments (for instance the form of the Casimir energy of a CFT on the cylinder), $E_C(\Lambda S, \Lambda V) = \Lambda^{-1}E_C(S, V)$. Moreover, conformal invariance implies that $ER$ is independent of the volume $V = 2\pi R$. Combined with the scaling behavior of $E_E$, which by definition is $E_E(\Lambda S, \Lambda V) = \Lambda E_E(S, V)$, this gives

$$E_E = \frac{b}{4\pi R} S^2, \quad E_C = \frac{d}{2\pi R}$$  \hspace{1cm} (37)

where $b, d$ are arbitrary constants. Using this equation and (33) one gets

$$S = \frac{2\pi R}{\sqrt{bd}} \sqrt{E_C(2E - E_C)}$$  \hspace{1cm} (38)

Eq. (38) becomes the Cardy formula if we take $bd = 1$ and consider a 2D CFT on the cylinder. In fact, in this case the total and Casimir energy are given by

$$E = \frac{l_0}{R}, \quad \frac{E_C}{2} = \frac{c}{24R},$$  \hspace{1cm} (39)
where \( c \) and \( l_0 \) are the central charge and the eigenvalue of the Virasoro operator \( L_0 \) of the CFT and \( R \) is the radius of the cylinder. Inserting the previous equations into Eq. (38) one finds the Cardy formula

\[
S = 2\pi \sqrt{\frac{c}{6} \left( l_0 - \frac{c}{24} \right)}. \tag{40}
\]

Comparing Eq. (34) with Eq. (38) (\( bd = 1 \)) we find that they agree if we take \( S = S_H \) and \( E_{BH} = E_C \). We are therefore led to a cosmological bound for the Casimir energy

\[
E_C \leq E_{BH}, \tag{41}
\]

analogous to the one proposed for higher-dimensional cosmology [1]. Using the Friedmann Equation (32), one easily finds that for \( HR \geq 1 \), \( E \geq E_{BH} \). Hence for strongly gravitating systems we have

\[
E_C \leq E_{BH} \leq E. \tag{42}
\]

This bound shares some nice features with its higher-dimensional counterpart: (a) It is always valid and does not break down for \( HR \leq 1 \). (b) Its physical meaning is that the cosmological energy by itself is not sufficient to produce a black hole of the size of the entire universe. In fact, the AdS\(_2\) black hole saturates the bound. (c) For \( HR > 1 \) it is equivalent to the Hubble bound \( S < S_H \). (d) When the bound is saturated \( E_C = E_{BH} \) and the cosmological equation (32) becomes the Cardy formula (40). The translation table between 2D cosmology and 2D CFT is given by

\[
\begin{align*}
\frac{c}{12} & \leftrightarrow \frac{1}{2G}, \\
l_0 & \leftrightarrow ER, \\
S & \leftrightarrow \frac{\pi RH}{G}.
\end{align*}
\]

\[
(43)
\]

### 6 Limiting temperature

The cosmological equation (3) can be used, in conjunction with Eq. (32), to give a lower bound for temperature in a radiation-dominated universe. From (3) and (32) follows

\[
\dot{H} = \lambda^2 - \frac{4GE}{R} + \frac{1}{R^2} = \frac{1 - 4GM}{R^2}. \tag{44}
\]
Defining the Hubble temperature

\[ T_H = -\frac{1}{2\pi} \frac{\dot{H}}{H}, \quad (45) \]

and using the definition of \( S_H \) and \( E_{BH} \), Eq. (44) becomes

\[ E_{BH} = -T_H S_H + E + pV. \quad (46) \]

Comparing (46) with (36) and using the bounds \( S \leq S_H, E_C \leq E_{BH} \) one obtains a lower bound for \( T \),

\[ T \geq T_H. \quad (47) \]

7 Conclusions

We have shown that the analysis of Verlinde on cosmological entropy bounds and their relations with CFT can be extended to a two-dimensional model, in spite of the difficulties related to the definition of a holographic principle in two dimensions. The identification of the Friedmann equation with the Cardy formula requires the use of the scaling invariance of the theory in order to fix some dimensionless parameters.

The most striking feature of the mapping between 2D cosmology and 2D CFT is the identification of the Newton constant in terms of the central charge of the CFT. The correspondence between cosmological equations and the Cardy formula requires \( c = 6/G \). This relation has an obvious holographic nature. In higher dimensions the holographic principle requires \( c \propto V/GR \). Extended to the 2D case where \( V = 2\pi R \) this relation reproduces our result. Further support to the holographic origin of this relation, comes from the fact that it can also be deduced using the AdS/CFT correspondence in two dimensions \[3, 7, 18\]. In this context, it has been found that the central charge of the CFT living on the boundary of AdS\(_2\) is given by \( c = 12\eta_0 \), which is our present result, because \( \eta_0 \), is proportional to the inverse of the 2D Newton constant. Owing to the dilatation symmetry of our model, under which \( \eta_0 \) scales as in Eq. (24), the coefficient of proportionality between \( \eta_0 \) and \( G^{-1} \) depends on the dilatation-gauge we choose.
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