Sorting some fractions by using a graph area shading (GAS) technique

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Abstract. Fractions are one of the mathematics topics that have been taught starting in grade two on elementary school. For kids, understanding a fraction is not so simple, especially in sorting fractions, ascending or descending. A fraction consists of two part numerator and denominator. A common technique frequently used in sorting some fractions is making the denominators of those fractions are the same, and compare the numerator which on is greater or less than other. This technique requires a long and slightly complicated process. The researchers will study a new technique for sorting fractions, namely the Graph Area Shading (GAS) technique. By a graph, we mean a set of vertices together with edges representing a specific relation. The simplest graphs are path and cycle. We will use some other graphs such as wheel, friendship and fan graphs. This technique is considered to be pretty much easier for kids to sort the fractions.

1. Introduction

Fractions are denoted by the symbol \( \frac{a}{b} \) where \( a \) is the numerator and \( b \) is the denominator \([9]\). Given that the two fractions of the form \( x = \frac{x_1}{x_2} \) and \( y = \frac{y_1}{y_2} \) where \( x_2 \neq 0 \) and \( y_2 \neq 0 \), when we ask to the elementary school students which one is bigger? They are mostly difficult to answer. This paper will study a new technique for sorting fractions, namely the Graph Area Shading (GAS) technique for sorting fractions. A fraction plays an important role in mathematical development. In mathematics a fraction satisfies some properties, such as commutative, distributive and associative of addition and multiplication, identity operations, and others \([1, 6, 10]\). Fractions is closely related concepts with ratio and proportion \([11]\).

In Indonesia, the subject of fractions has been taught since the second grade of elementary school. A fraction does look simple, but many elementary school students still have difficulties when faced with fractions problem, especially sorting \([7]\). The fraction type consist of ordinary fractions \( \frac{a}{b} \), mixed fractions \( \frac{c}{d} \), decimal fractions \( 0.abcd \ldots \) and percent \( a\% \) \([7, 5]\). To sort fractions we should compare one fraction to another. When the two denominators are the same it is easy to handle but if they are different we need to make them the same first and it implies some difficulties for students since the students should consider the factorization process. We study a new technique for sorting fractions, namely the combination of Area Shading (AS)
technique and graph theory element. Thus, our new technique is called a Graph Area Shading (GAS) technique.

By a graph, we mean a set of vertices together with edges representing a specific relation. The simplest graphs are path and cycle. Graph theory itself was first introduced by Leonhard Euler, a Swiss mathematician in 1736, through his paper about solving the Konigsberg bridge problem [4]. Graph theory is a part of discrete mathematics, which is widely used as a tool to solve a discrete problem in real life [2]. A picture of two elements, namely vertex and edge, usually depicts a representation of graph theory element. The number of vertices and edges are respectively called order and size. The very simplest graph is path and cycle. A path is graph that number of vertices is \( n \) and the number of edges is \( n - 1 \). Whilst, a cycle is graph that number of vertices is \( n \) and the number of edges is also \( n \), as an example see Figure 1.

![Figure 1. The path of order 5, 6, 7 and the cycle of order 5, 6, 7](image)

By shading the element of graph either edge or face, we have a new technique namely Graph Area Shading (GAS) technique. Obviously we will show the algorithm of GAS technique later and compare the effectiveness of the ordinary Area Shading (AS) technique. This study is considered to be a new technique so far, and we could not find this technique in some book reference or school text book.

2. Method
In this study, we use analytic and qualitative research method. Combining with the research-based learning while we were teaching students about fraction we did a research. By means of the pattern recognition we implement the steps of research as follows: 1. Problem posing related to fraction 2. Students were asked to find some strategies to solve the problems. 3. The students obtained some initial solutions. 4. Researcher analyze the determined solution for potential general solution 5. Obtain the general pattern 6. Develop a generalization of the solution. 7. Conclude the results [8, 3].

3. Result and Discussion
3.1. Complexity of the Existing Technique
The existing technique in sorting fraction is Area Shading (AS) technique. The AS technique works by shading the area on normal bar chat. In graph theory, the object is called a ladder graph. The syntax show that the wider the shaded area, the larger the fractions value. conversely, the smaller the shaded area, the smaller the fraction value. When the two denominators of two fractions are the same, this technique is effective, but if it is not the same then it gives a complexity. It is due to the resulting partition drawn by students may not be the same. It implies that the result will be in-correct. Take the following example.

The first picture, see Figure 2, is comparing the two fractions \( \frac{4}{5} \) and \( \frac{3}{5} \). Students will obviously draw the same partition and do the shading process easily. Compare the shaded area which on the wider, it is the bigger value. However when students is given the the two fractions \( \frac{2}{3} \) and \( \frac{3}{5} \)}.
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Figure 2. The fraction $\frac{4}{5}$ and $\frac{3}{5}$ in first picture, the fraction $\frac{2}{3}$ and $\frac{3}{5}$ in second and third pictures

in second and third pictures, the individual students will not give the same results. It is due to the obtained partition is not in the same wide. Thus, the shaded area does not show the precise area. By using a new technique, we can resolve this problem. The good things with this new technique, we can extend the use of the bar chat by using any pictures. In this study we use a graph picture since it gives many picture choices and the shape is restricted to a certain shape. The students can be developed a picture by them self.

3.2. Graph Area Shading Technique

In the following section, we will describe our research results on sorting two fractions by using Graph Area Shading (GAS) Technique. We will present the results of the form of an observation followed by an illustration.

**Observation 1** Given that two fractions $\frac{3}{5}$ and $\frac{2}{4}$. By using Wheel Graph Area Shading technique, the relation is $\frac{3}{5} > \frac{2}{3}$.

**Illustration.** Consider a wheel graph. A wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle. A wheel graph with $n$ vertices will give $2(n-1)$ number of edges. By GAS technique, we firstly draw wheel graph to represent first fraction followed by second wheel representing the second fraction. Then, shade some faces as many as each numerator. Since the fraction $\frac{3}{5}$ and $\frac{2}{4}$ have respectively 3 and 2 as the numerators then we have 3 shaded faces and 2 shaded faces. Redraw the 3 shaded faces and 2 shaded faces in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded area of each fraction and compare the obtained number of shaded faces. If the total of the shaded faces for a fraction is bigger then we consider the fraction is bigger. For the illustration, we depict in Figure 3. In this case, by using wheel graph area shading technique we can conclude that $\frac{3}{5} > \frac{2}{3}$.
**Observation 2**  Given that two fractions $\frac{4}{7}$ and $\frac{5}{6}$. By using Star Graph Area Shading technique, the relation is $\frac{4}{7} < \frac{5}{6}$.

**Illustration.** Consider a star graph. A star graph is a tree on $n$ vertices with one vertex having vertex degree $n - 1$ and the others $n - 1$ having vertex degree 1. By GAS technique, we firstly draw star graph to represent first fraction followed by second star representing the second fraction. Then, shade some edges as many as each numerator. Since the fraction $\frac{4}{7}$ and $\frac{5}{6}$ have respectively 4 and 5 as the numerators then we have 4 shaded edges and 5 shaded edges. Redraw the 4 shaded edges and 5 shaded edges in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded edges of each fraction and compare the obtained number of shaded edges. Since the number of shaded edges of the first fraction is less than the second fraction, thus by using star graph area shading we can conclude that $\frac{4}{7} < \frac{5}{6}$. For the illustration, we depict in Figure 4.

![Figure 4](image-url)

**Figure 4.** The illustration of star graph representing the fraction $\frac{4}{7} < \frac{5}{6}$

**Observation 3**  Given that two fractions $\frac{3}{9}$ and $\frac{1}{8}$. By using Grid Graph Area Shading technique, the relation is $\frac{3}{9} > \frac{1}{8}$.

![Figure 5](image-url)

**Figure 5.** The illustration of grid graph representing the fraction $\frac{3}{9} > \frac{1}{8}$
Illustration. Consider a grid graph. A two-dimensional grid graph, also known as a rectangular grid graph or two-dimensional lattice graph, is a lattice graph formed by the Cartesian product of path graphs. By GAS technique, we firstly draw grid graph to represent first fraction followed by second grid graph representing the second fraction. Then, shade some faces as many as each numerator. Since the fraction $\frac{3}{9}$ and $\frac{1}{8}$ have respectively 3 and 1 as the numerators then we have 3 shaded faces and 1 shaded face. Redraw the 3 shaded faces and 1 shaded face in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded area of each fraction and compare the obtained number of shaded faces. If the total shaded faces for a fraction is bigger then we consider the fraction it is bigger. In this case, by using grid graph area shading technique we can conclude that $\frac{3}{9} > \frac{1}{8}$. For the illustration, we depict in Figure 5.

Observation 4 Given that two fractions $\frac{2}{4}$ and $\frac{1}{3}$. By using Triangular Book Graph Area Shading technique, the relation is $\frac{2}{4} > \frac{1}{3}$.

Illustration. Consider a triangular book graph. The triangular book graph is a planar graph with $n + 2$ vertices and $2n + 1$ edges. It is constructed by $n$ triangles sharing a common edge. In other words the triangular book graph is the complete tripartite graph. By GAS technique, we firstly draw triangular book graph to represent first fraction followed by second triangular book graph representing the second fraction. Then, shade some faces as many as each numerator. Since the fraction $\frac{2}{4}$ and $\frac{1}{3}$ have respectively 2 and 1 as the numerators then we have 2 shaded faces and 1 shaded face. Redraw the 2 shaded faces and 1 shaded face in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded area of each fraction and compare the obtained number of shaded face. If the total shaded face for a fraction is bigger then we consider the fraction it is bigger. In this case, by using triangular book graph area shading technique we can conclude that $\frac{2}{4} > \frac{1}{3}$. For the illustration, we depict in Figure 6.

![Figure 6](Image)

Figure 6. The illustration of triangular book graph representing the fraction $\frac{2}{4} > \frac{1}{3}$

Observation 5 Given that two fractions $\frac{2}{5}$ and $\frac{5}{8}$. By using Triangular Ladder Graph Area Shading technique, the relation is $\frac{2}{5} < \frac{5}{8}$.

Illustration. Consider a triangular ladder graph. A ladder graph is graph formed by two paths where each vertex in the first path connected to associated vertex in the second path. By triangular ladder graph, we mean the ladder graph by inserting one diagonal edge on each face of the ladder. By GAS technique, we firstly draw triangular ladder graph to represent first fraction...
followed by second triangular ladder graph representing the second fraction. Then, shade some faces as many as each numerator. Since the fraction $\frac{2}{5}$ and $\frac{5}{8}$ have respectively 2 and 5 as the numerators then we have 2 shaded faces and 5 shaded faces. Redraw the 2 shaded faces and 5 shaded faces in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded area of each fraction and compare the obtained number of shaded face. If the total shaded face for a fraction is smaller then we consider the fraction it is smaller. For the illustration, we depict in Figure 7. Finally, by using triangular ladder graph area shading technique, we can conclude that $\frac{2}{5} < \frac{5}{8}$.

![Figure 7](image1.png)

**Figure 7.** The illustration of triangular ladder graph representing the fraction $\frac{2}{5} < \frac{5}{8}$

**Observation 6** Given that two fractions $\frac{6}{16}$ and $\frac{4}{14}$. By using Triangular Grid Graph Area Shading technique, the relation is $\frac{6}{16} > \frac{4}{14}$.

![Figure 8](image2.png)

**Figure 8.** The illustration of triangular grid graph representing the fraction $\frac{6}{16} > \frac{4}{14}$

**Illustration.** Consider a triangular grid graph. The triangular grid graph is the lattice graph obtained by interpreting the order—$(n + 1)$ triangular grid as a graph, with the intersection of
grid lines being the vertices and the line segments between vertices being the edges. By GAS technique, we firstly draw triangular grid graph to represent first fraction followed by second triangular grid graph representing the second fraction. Then, shade some faces as many as each numerator. Since the fraction $\frac{6}{16}$ and $\frac{4}{14}$ have respectively 6 and 4 as the numerators then we have 6 shaded faces and 4 shaded faces. Redraw the 6 shaded faces and 4 shaded faces in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded area of each fraction and compare the obtained number of shaded face. If the total shaded face for a fraction is bigger then we consider the fraction is bigger. For the illustration, we depict in Figure 8. By using triangular grid graph area shading technique we can conclude that $\frac{6}{16} > \frac{4}{14}$.

Observation 7 Given that two fractions $\frac{3}{9}$ and $\frac{\frac{3}{9}}{3}$. By using Antiprism Graph Area Shading technique, the relation is $\frac{3}{9} < \frac{\frac{3}{9}}{3}$.

Illustration. Consider a antiprism graph. An antiprism graph is a graph corresponding to the skeleton of an antiprism. Antiprism graphs are a polyhedral and planar graph. By GAS technique, we firstly draw antiprism graph to represent first fraction followed by second antiprism graph representing the second fraction. Then, shade some faces as many as each numerator. Since the fraction $\frac{3}{9}$ and $\frac{3}{7}$ have respectively 3 and 3 as the numerators then we have 3 shaded faces and 3 shaded faces. Redraw the 3 shaded faces and 3 shaded faces in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded area of each fraction and compare the obtained number of shaded face. If the total shaded face for a fraction is smaller then we consider the fraction is smaller. For the illustration, we depict in Figure 9. In this case, by using antiprism graph area shading technique we can conclude that $\frac{3}{9} < \frac{3}{7}$.

![Figure 9](image)

Observation 8 Given that two fractions $\frac{5}{10}$ and $\frac{7}{12}$. By using Sunflower Graph Area Shading technique, the relation is $\frac{5}{10} < \frac{7}{12}$.

Illustration. Consider a sunflower graph. The sunflower graph obtained from wheel graph having a cycle and by adding $n$ new vertices. By GAS technique, we firstly draw sunflower
graph to represent first fraction followed by second sunflower graph representing the second fraction. Then, shade some faces as many as each numerator. Since the fraction \( \frac{5}{10} \) and \( \frac{7}{12} \) have respectively 5 and 7 as the numerators then we have 5 shaded faces and 7 shaded faces. Redraw the 5 shaded faces and 7 shaded faces in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded area of each fraction and compare the obtained number of shaded face. If the total shaded face for a fraction is smaller then we consider the fraction is smaller. In this case, by using sunflower graph area shading technique we can conclude that \( \frac{5}{10} < \frac{7}{12} \).

![Figure 10](image.png)

**Figure 10.** The illustration of sunflower graph representing the fraction \( \frac{5}{10} < \frac{7}{12} \)

**Observation 9** Given that two fractions \( \frac{2}{5} \) and \( \frac{3}{6} \). By using Jahangir Graph Area Shading technique, the relation is \( \frac{2}{5} < \frac{3}{6} \).

**Illustration.** Consider a jahangir graph. A jahangir graph is a graph on \( nm + 1 \) vertices and \( m(n + 1) \) edges for all \( n \geq 2 \) and \( m \geq 3 \). The jahangir consists of a cycle with one additional vertex which is adjacent to \( m \) vertices of \( C_{n,m} \) at distance to each other. By GAS technique, we firstly draw jahangir graph to represent first fraction followed by second jahangir graph representing the second fraction. Then, shade some faces as many as each numerator. Since the fraction \( \frac{2}{5} \) and \( \frac{3}{6} \) have respectively 2 and 3 as the numerators then we have 2 shaded faces and 3 shaded faces. Redraw the 2 shaded faces and 3 shaded faces in the second row as many as the denominator of each fraction by considering in crossing position of the two fractions. Finally, do manual counting of the whole shaded area of each fraction and compare the obtained number of shaded face. If the total shaded face for a fraction is smaller then we consider the fraction is smaller. For the illustration, we depict in Figure 11. Finally, by using jahangir graph area shading technique we can conclude that \( \frac{2}{5} < \frac{3}{6} \).

3.3. Discussions

We have described our novel technique for sorting fractions. Which fractions is bigger? This question is difficult for some elementary school students to solve it. There are a lot of techniques to solve this problem, one of them is Area Shading (AS) technique by using a normal bar chat. However, this technique show a complexity and less accuracy. In this study, we have found new
Figure 11. The illustration of jahangir graph representing the fraction $\frac{2}{5} < \frac{3}{6}$

Algorithm for Sorting Fractions

START: Given two fractions.
Choose a special graph.
Consider the nominator and denominator of each fraction.
Draw two graphs which size or faces associating with the fractions denominator.
Consider the number of edges or faces of the graphs.
Set that the number of edges or faces as many as the denominator.
Do area shading on some edges or faces.
Make sure the shaded area as many as each nominator.
Redraw the shaded areas as many as the denominator in the second row.
Set that the number of the shaded area in second row as many as the denominator.
Consider that in crossing position of the two fractions
Count the total shaded area of each fraction.
END. Consider the bigger sum to decide the relation.

By Graph Area Shading (GAS) technique, the elementary school students will be able to compare fractions easily. They can also consider which type of graph that they want to use it. In this study, some various graphs are presented to educate school students to create their own graph. In this technique, the graph chosen is not necessarily in the same size. The nice thing we compare the number of the duplication of shaded area for each fractions. Thus, the mathematics operation involved in this cognitive process is not complex, they only need the mathematical counting process. However, by GAS, it will improve students creative thinking skills in solving a sorting fraction problem, since students can consider the unique graph which is different from other students. Obviously it will give much attraction on the student learning activities, then it will impact to the rise of higher order thinking skills.
4. Conclusions

Based on the results and discussion, we can conclude that the GAS technique has some advantages, namely efficient technique, accuracy, and very attractive for students especially in the early elementary school students. Furthermore, this technique can improve their creative thinking skills when they are allowed to consider any graph and shade their faces according to the numerator of each fraction. Since it is a novel of research in sorting two fractions, we have found any result yet if we need to compare more than two fractions. Therefore, we propose the Graph Area Shading (GAS) technique to sort at least three fractions for the open problem.

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