Using the EAS size spectra measured with the MAKET ANI array on Mt. Aragats, Armenia (3200 m a.s.l.- 700 g·cm$^{-2}$) in the range of $N_e = 10^5 - 10^7$ for different angles-of-incidence, the EAS attenuation length has been determined applying different analysis methods. The analysis is based on a data sample of $2.5 \cdot 10^6$ events collected in the period of June, 97 - April, 99. The results are compared with results deduced from data of the EAS TOP and KASCADE experiments.

1 Introduction

The intensity of Extensive Air Showers (EAS) with fixed shower sizes $N_e$ is assumed to decrease exponentially with increasing atmospheric depth of the observation level. This is considered to be due to the absorption of the particles of the EAS cascade following an exponential law

$$N_e(X) = N_e(X_0)exp\left(-\frac{X - X_0}{\Lambda}\right), \text{ with } X \geq X_0.$$  

$X_0$ is a definite initial atmospheric depth after the maximum of the longitudinal development where the number of (charged) particles is $N_e(X_0)$ and further decreasing exponentially, $N_e(X)$ is the number of particles of the EAS at the slant depth $X[g \cdot cm^{-2}]$. The quantity $\Lambda$ controls the attenuation of particles of the individual cascade (size attenuation length). It is related to the inelastic cross sections (to the mean free path length $\lambda_A$) of the interaction of the primary cosmic ray particles with air nuclei. The attenuation of the flux intensity of Extensive Air Showers is characterized by a related quantity $\lambda_N$ (intensity attenuation length, absorption), which can be directly measured by cosmic rays detector arrays. Thus measurements of the attenuation of the EAS intensity in the atmosphere are considered to be an interesting source of information about hadronic interactions, especially if extended to the ultrahigh energy region expected from the forthcoming LHC and TESLA accelerators. In addition due to the sensitivity of the cross sections to the mass of the primary, alterations of the attenuation length with the energy may be indicative for the variations of the mass composition. Measured results imply tests of the energy dependence of the extrapolated cross sections used for Monte Carlo simulations.

The investigations of the present paper are based on an EAS sample measured 1997-1999 with the MAKET ANI array on Mt. Aragats station (Armenia) and registered for different angles-of-incidence in the zenith angle interval $\Theta = 0 - 45^\circ$. The data basis of the analysis can be enlarged by published data from KASCADE (1046 g·cm$^{-2}$) and EAS TOP (810 g·cm$^{-2}$) experiments. Spectra measured by EAS TOP are given in Ref. [3]. Data and zenith angle dependence for KASCADE results are obtained by scanning the spectrum plots communicated by the KASCADE collaboration [7].

We apply different procedures to deduce the attenuation. First we consider the degradation of the
EAS flux with fixed shower size $N_e$ with increasing zenith angle i.e. increasing atmospheric thickness of the shower development (characterized by the intensity attenuation length ($\lambda_N$) \[8\]). Differently the technique of the constant intensity cut (CIC) \[9\] considers the intensity spectrum of EAS events and relates equal intensities observed at different atmospheric depths.

There is the tacit assumption that the shower size reflects the energy of the primary. The procedure can be refined by using the knee position in the $N_e$ spectrum as a bench mark for a well defined energy, so far we may associate the knee phenomenon to a feature of the primary energy spectrum of cosmic radiation.

2 Experimental spectra

The experimental basis of the present investigations are measurements of shower size spectra in the knee region and their zenith-angle dependence performed with the MAKET ANI array of the Mt. Aragats Cosmic Ray Station (3200 m a.s.l.) in Armenia. Details of the measurements and the experimental procedures taking into account the detector response are given elsewhere \[10\] \[11\]. For a detailed description of the knee region the traditional approximation with two different spectral indices below and above the knee, defining the knee position as intersection of two lines in a logarithmic presentation, appears to be insufficient. Hence a more sophisticated method has been applied with parameterization of the slope of the spectra (see Ref.\[12\]).

Tab.1 compiles the characteristics of the size spectra measured with the MAKET ANI installation, the changes of the slopes in the knee region ($\Delta N_{e_k}$), expressed by different spectral indices below ($\gamma_1$) and above ($\gamma_2$) the knee position $N_{e_k}$ for the zenith-angle range of $\Theta = 0 - 45^\circ$. For the display and the analysis of the zenith-angle dependence, the size spectra are determined in 5 angular bins of equal $\Delta \sec \Theta$ widths. The accuracy of the zenith angle determination is estimated to be about $1.5^\circ$ \[10\]. A correction due to barometric pressure changes, which lead to small fluctuations of the atmospheric absorption, has not been made. Figure 1 displays the spectra of mean values of each atmospheric depth bin and compares with the results from EAS-TOP \[6\] and KASCADE \[7\] experiments.

| $N_e$ region | $I(N_e)$ | $\gamma_1$ | $\gamma_2$ | $\Delta(N_{e_k})$ | $N_{e_k}$ | $I(N_{e_k})$
|--------------|----------|------------|------------|-------------------|---------|----------------|
| $10^5 < N_e < 1.15 \cdot 10^6$ | $(8.95 \pm 0.18) \cdot 10^{-11} (N_e/10^5)^{\gamma_1}$ | $-2.54 \pm 0.012$ | $-2.94 \pm 0.042$ | $(1.15 \pm 0.034) \cdot 10^6$ - $(2.56 \pm 0.063) \cdot 10^6$ | $(1.75 \pm 0.05) \cdot 10^6$ | $(5.83 \pm 0.14) \cdot 10^{-14}$
| $N_e > 2.56 \cdot 10^6$ | $(3.23 \pm 0.40) \cdot 10^{-13} (N_e/10^6)^{\gamma_2}$ | $-2.54 \pm 0.012$ | $-2.94 \pm 0.042$ | $(1.15 \pm 0.034) \cdot 10^6$ - $(2.56 \pm 0.063) \cdot 10^6$ | $(1.75 \pm 0.05) \cdot 10^6$ | $(5.83 \pm 0.14) \cdot 10^{-14}$

Table 1: Flux [$m^{-2}s^{-1}sr^{-1}$] and knee region parameters of the size spectra measured with the MAKET ANI array.

Following fixed intensities of the experimental spectra (see sect.3.2) the average $N_e$ cascade development can be immediately reconstructed as shown in Figure 2. Note that the results in the range of the slant depth observed with the ANI array deviate from the exponential decrease (eq.1). That is an interesting feature which can be revealed more clearly when combining spectra accurately measured on different altitudes. In the present paper we base the formulation of the procedures estimating the attenuation on the exponential decrease (eq.1). It is our interest to explore, if this assumption applied to the ANI and KASCADE data lead to consistent results.
3 Procedures for inference of the attenuation length from size spectra

We consider the differential and integral size spectra $I(N_e, X)$ and $I(> N_e, X)$, respectively. In addition to the basic assumption of exponential attenuation of $N_e$ (eq. 1) a power-law dependence of the size spectrum

$$I(N_e, X) \propto N_e^{-\gamma}, \quad (2)$$

with the spectral index $\gamma$ is adopted.

3.1 Attenuation of the intensity of fixed $N_e$: absorption length

For different fixed values of shower size $N_e$, on different depths in the atmosphere or/and different zenith angles of incidence, from measured spectra (see vertical dotted lines on Figure 3) we obtain several values of corresponding intensities from the equivalent depths from 700 till 1280 $g \cdot cm^{-2}$. Fitting the depth dependence of the intensities by the straight line (in logarithmic scale) according to equation:

$$I(N_e, X) = I(N_e, X_0) exp \left( -\frac{X - X_0}{\lambda_N} \right) \quad (3)$$

we obtain the estimate of the absorption length $\lambda_N$. The absorption length can be estimated both by integral and differential spectra.
3.2 Constant intensity cut

The basic idea of this procedure is to compare the average size of showers which have the same rate (showers per $m^2 \cdot s \cdot sr$) in the different bins of the zenith angle of shower incidence and different slant depth, respectively [9].

Considering two different depths in atmosphere $X_1, X_2 > X_0$ the expressions of differential intensities $I(N_e, X)$ has the form

$$N_e(X_1)^{-\gamma} \exp \left[ - (\gamma - 1) \frac{X_1 - X_0}{\Lambda} \right] = N_e(X_2)^{-\gamma} \exp \left[ - (\gamma - 1) \frac{X_2 - X_0}{\Lambda} \right]$$

(4)

With simple transformations we obtain:

$$\Lambda_{\text{diff}}(I) = \frac{\gamma - 1}{\gamma} \frac{X_2 - X_1}{\ln \left( \frac{N_e(X_1)}{N_e(X_2)} \right)}$$

(5)

The attenuation lengths, obtained by integral spectra do not depend explicitly on spectral index:

$$\Lambda_{\text{int}}(I) = \frac{X_2 - X_1}{\ln \left( \frac{N_e(X_1)}{N_e(X_2)} \right)}$$

(6)

Practically the estimate of the attenuation length is obtained by fitting the $N_e$ dependence on the depth in atmosphere by the straight line according to the equation (1). The sequence of $N_e$ values is obtained according to the fixed values of the flux intensity, selected from the interpolation of the differential or integral size spectra.
For each $N_e$ value, the slope index $\gamma$ used in equation 5 is obtained by averaging over all used slant depths. Selecting equal intensities ($\approx$ primary energies) corresponding to different shower sizes $N_e$ and different depths the value of $\Lambda_{\text{diff}}(I)$ is estimated. Intensity values from $10^{-9}$ to $5 \cdot 10^{-6}$ were used for CIC method.

### 3.3 Attenuation of the size of the knee

A special variant of the constant intensity cut is to follow the decrease of the shower size at a constant primary energy in the size spectrum. Assuming that the knee phenomenon reflects a feature of the primary flux, the variation shower size at the knee with the zenith angle provides the possibility to extract the attenuation length.

Considering the assigned knee position of the data from various experiments, differences within 30% are noticed for all X-bins.

The knee positions obtained by the differential and integral spectra are a bit shifted to the smaller $N_e$ values (see Figure 4). The shift is approximately uniform over all investigated depths interval, therefore the estimates of the attenuation length by the differential and integral size spectra are very close to each other.

### 3.4 The relation between the absorption and attenuation length

We consider the quantity $I(N_e, X) dN_e$ - the number of EAS at the depth $X$ which comprise $N_e$ to $N_e + dN_e$ particles:

$$I(N_e, X) dN_e \sim N_e^{-\gamma} \exp \left[ - (\gamma - 1) \frac{X - X_0}{\Lambda} \right] dN_e \tag{7}$$

With eq.3 we obtain:

$$\Lambda_{\text{diff}}(N_e) = (\gamma(N_e) - 1) \lambda_N, \tag{8}$$

where, $\gamma(N_e)$ is the differential size spectra index (here we indicate the $N_e$ dependence of the slope index explicitly). For the integral spectra:

$$\Lambda_{\text{int}}(N_e) = \gamma(N_e) \lambda_N, \tag{9}$$

where, $\gamma(N_e)$ is integral size spectra index.

For the evaluation of the inelastic cross section and for comparison of the three methods described above we propose to use the calculated values of the attenuation length $\Lambda$ (instead of using absorption length $\lambda_N$). The attenuation of the number of particles in the individual cascade is more directly connected with the characteristics of the strong interaction and is independent from the parameters of the cosmic ray flux incident on the atmosphere. In turn the absorption length, i.e. the attenuation of the CR flux intensity, reflects also characteristics of the primary flux and is dependent on the change of the slope of the spectra.

### 3.5 Estimate of the inelastic cross section

The inelastic cross sections, of the primary nuclei with atmosphere nuclei is related by 9:

$$\sigma_{A-\text{air}}^{\text{inel}}(\text{mbarn}) = \frac{2.41 \cdot 10^4}{\lambda_A(g \cdot \text{cm}^{-2})}, \tag{10}$$

where $A$ denotes the primary nuclei. The quantity $\lambda_A$ is the interaction length of the A-nucleus in the atmosphere (note: in some publications the interaction length is denoted by $\lambda_N$, where N is
primary nuclei, in contrast in this paper \( N \) is reserved for the shower size). The interaction length \( \lambda_A \) is related with the absorption length \( \Lambda_A \) by
\[
\lambda_A = K(E) \cdot \Lambda_A
\] (11)

The \( K(E) \) coefficient reflects peculiarities of the strong interaction model used for simulation. The value of the parameter \( K \) has to be determined by simulations of the EAS development in the atmosphere. Such studies require the development of procedures for the selection of EAS initiated by primaries of a definite type (see for example in [13, 14]).

4 Application to the data

The mean values of the attenuation lengths obtained by various methods from data of the ANI and KASCADE installations, as well as for the joint ANI & KASCADE data by the differential (\( \Lambda_{diff} \)) and integral spectra (\( \Lambda_{int} \)) are compiled in the Tables 2,3,4.

| Min. depth | MAKET ANI | ANI+KASCADE | KASCADE |
|------------|-----------|-------------|---------|
| \( X_0, g \cdot cm^{-2} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) |
| 700        | 248 ± 27  | 247 ± 42    | 203 ± 10 | 203 ± 13 | –           | –           |
| 758        | 236 ± 32  | 237 ± 51    | 195 ± 8  | 196 ± 12 | –           | –           |
| 816        | 211 ± 43  | 218 ± 70    | 186 ± 9  | 188 ± 13 | –           | –           |
| 1020       | –         | –           | –        | –        | 181 ± 14    | 182 ± 23    |

Table 2: Attenuation lengths for the data from the MAKET ANI and KASCADE installations estimated by the CIC method from differential and integral size spectra

| Min. depth | MAKET ANI | ANI+KASCADE | KASCADE |
|------------|-----------|-------------|---------|
| \( X_0, g \cdot cm^{-2} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) |
| 700        | 239 ± 14  | 240 ± 15    | 191 ± 11 | 193 ± 13 | –           | –           |
| 758        | 232 ± 13  | 228 ± 19    | 186 ± 10 | 184 ± 17 | –           | –           |
| 816        | 213 ± 14  | 219 ± 27    | 179 ± 11 | 181 ± 24 | –           | –           |
| 1020       | –         | –           | –        | –        | 181 ± 7     | 183 ± 11    |

Table 3: Attenuation lengths for the data from the MAKET ANI and KASCADE installations estimated by the recalculation from the absorption length for differential and integral size spectra

| Min. depth | MAKET ANI | ANI+KASCADE | KASCADE |
|------------|-----------|-------------|---------|
| \( X_0, g \cdot cm^{-2} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) | \( \Lambda_{int} \) | \( \Lambda_{diff} \) |
| 700        | 302 ± 71  | 295 ± 83    | 241 ± 17 | 237 ± 15 | –           | –           |
| 758        | 272 ± 51  | 263 ± 42    | 242 ± 20 | 221 ± 17 | –           | –           |
| 816        | –         | –           | 225 ± 21 | 225 ± 19 | –           | –           |
| 1020       | –         | –           | –        | –        | 232 ± 26    | 222 ± 28    |

Table 4: Attenuation lengths for the data from the MAKET ANI and KASCADE installations, estimated by the “attenuation of knee position” method from differential and integral size spectra

The alternative estimates of the attenuation length reflect the inherent uncertainties of the methods and the statistical errors, as well as the fluctuations of cascade development in the atmosphere, the
energy dependence of the inelastic cross section and possible changes in mass composition. As obvious in Figure 2, the values corresponding to the minimal equivalent depths of used MAKET ANI data, deviate significantly from the exponential decrease. The observations reflects the flattening of the cascade curve just after the shower maximum in the altitude $500 - 600 \text{ g} \cdot \text{cm}^{-2}$. Due to these features the attenuation lengths calculated by MAKET ANI data appear to be significantly larger than those derived for the KASCADE data (Tables 2, 3).

Therefore, for the combined analysis of the KASCADE and ANI data we omitted the first and the second zenith angle bins of MAKET ANI and calculate the attenuation lengths by the remaining 9 (minimal equivalent depth $758 \text{ g} \cdot \text{cm}^{-2}$) and 8 (minimal equivalent depth $816 \text{ g} \cdot \text{cm}^{-2}$) angular bins. The dependences of estimated values of attenuation length on the shower size and flux intensity for different amount of the angular bins used, are displayed in Figures 5 (note, that higher intensities on the X axes correspond to the lower primary energies) and 6. The attenuation length estimates obtained from the differential and integral spectra agree fairly well. The results of
both CIC and recalculation from absorption length agree within the error bars. The results obtained by the “attenuation of knee position” are larger for MAKET ANI and KASCADE. As pointed out by S. Ostapchenko [15] it is the consequence of the large EAS fluctuations with the tendency to shift the knee position to the lower energies (and correspondingly to higher fluxes) in a way to ”slow down” the cascade curve attenuation.

Well below the shower development maximum starting from $816 g \cdot cm^{-2}$ KASCADE and MAKET ANI data could be fitted with one decay parameter (see Figure 7). There is a concentration of the knee positions on the curve showing the dependence of the attenuation of the flux intensity ($\approx$ primary energy). In turn, the curve displaying the dependence of the attenuation length on the shower size demonstrates a rather large dispersion of the ”knee positions”. These observations in size and energy scales may be interpreted as an indication of the astrophysical nature of the knee phenomenon.

5 Conclusion

Experimental studies of EAS characteristic like the depth of the shower maximum $X_{max}$, the elongation rate $dX_{max}/dlog_{10}E$ and the attenuation length $\Lambda$ are of particular importance, since they map rather directly basic features of the hadronic interaction. Strictly, however, the interpretation of these quantities in terms of hadronic cross sections cannot bypass the necessity of detailed calculations of the shower development. Nevertheless these type of EAS quantities, if compared with Monte Carlo simulation results, provide stringent tests of the interaction model ingredients of the simulations.

The recent results of various experimental installations are sufficiently accurate to enable relevant studies of this kind, and combining the data from arrays situated on different altitudes (like MAKET ANI and KASCADE) allows a large span in the atmospheric slant depth for reconstructing the development of the charge particle size. In turn such studies, if using a sufficiently large data sample, could be continued in a more detailed manner by separating the muon component and taking into account the deviations from the exponential shape of the cascade decline. The penetrating muon component contributes with smaller attenuation to the development of the considered charged particle component, but hardly with an exponential degrading (according to eq.1). Actually by use of methods in progress to isolate different primary groups (“pure nuclear beams”) of the size spectra [14] [16], these kind of interaction studies would get of extreme interest.

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References

[1] S. Hayakawa, *Cosmic Ray Physics*, Interscience Monographs and Texts in Physics and Astronomy, V. 22, Wiley-Interscience, 1969

[2] V.V. Avakyan et al., Jadernaya Fiz. 56 (1993) 182

[3] G.G. Hovsepyan for the ANI collaboration., Proc. of the Workshop ANI 98, eds. A.A. Chilingarigan, H.Rebel, M. Roth, M.Z. Zazyan, FZKA 6215, Forschungszentrum Karlsruhe 1998, 45

[4] H.O. Klages et al. - KASCADE collaboration, Nucl. Phys. B (Proc. Suppl.) 52B (1997) 92

[5] M. Aglietta et al., Nucl. Instrum. and Meth. A336 (1993) 310

[6] M. Aglietta et al., Astropart. Phys. 10 (1999) 1

[7] R. Glasstetter et al. - KASCADE collaboration, Proc. 16th ECRS (Alcala, 1999), 564

[8] G.B. Khristiansen, G. Kulikov, J. Fomin, *Cosmic Rays of Superhigh Energies*, Verlag Thiemig, München, 1979

[9] M. Nagano et al., Journ. Phys. G: Nucl. Phys. 10 (1984) L235; Gaisser T.K, *Cosmic Rays and Particle Physics*, Cambridge Univ. Press, 1992

[10] G.V. Gharagyozyan for the ANI collaboration, Proc. of the Workshop ANI 98, eds. A.A. Chilingarigan, H.Rebel, M. Roth, M.Z. Zazyan, FZKA 6215, Forschungszentrum Karlsruhe 1998, 51

[11] S.V. Blokhin, V.A. Romakhin, G.G. Hovsepyan, these proceedings

[12] S.H. Sokhoyan S.H. et al. - ANI collaboration, Proc. of the Workshop ANI 98, eds. A.A. Chilingarigan, H.Rebel, M. Roth, M.Z. Zazyan, FZKA 6215, Forschungszentrum Karlsruhe 1998, 55

[13] A.A. Chilingarian, H.Z.Zazyan, Yad. Fiz. 54 (1991) 128

[14] A. Vardanyan et al., these proceedings

[15] S. Ostapchenko, private communication, 1999

[16] G.V. Gharagyozyan et al., these proceedings