Anomalous U(1) Symmetry and Missing Doublet SU(5) Model

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Abstract

We present the supersymmetric SU(5) models which provide a simple “all order” solution to the doublet-triplet splitting problem through the missing doublet mechanism. The crucial role is played by the anomalous $U(1)_A$ gauge symmetry and no additional discrete or global symmetries are needed. Remarkably, such models can be realized even if the 75-plet Higgs is replaced by the standard 24-plet. The same $U(1)_A$ symmetry can also guarantee an exact or approximate conservation of R parity, by suppressing the B and L violating operators to the needed level. The neutrino masses and the proton decay via $d = 5$ operators are also examined. We also extend the model by incorporating $U(1)_A$ as a horizontal symmetry for explaining the fermion mass and mixing hierarchy. Interestingly, in this scheme the necessary mild violation of the troublesome SU(5) degeneracy between the down quark and the charged lepton masses can be induced by certain R-parity violating operators.
1 Introduction

Many realistic string models contain the gauge $U(1)$ factors with non-vanishing traces over the charges of the matter superfields. One can find their linear combination $U(1)_A$ which can be ‘truly’ anomalous while the other combinations are rendered traceless. Existence of such an anomalous $U(1)_A$ symmetry does not imply an anomaly in the ‘progenitor’ string theory. In the field theory limit it can be understood as a result of truncating the string spectrum to the particle spectrum, and all mixed anomalies of the matter fields are effectively canceled via the “universal” Green-Schwarz mechanism [1]. As it was shown [3], the D-term of such $U(1)_A$ symmetry gets a non-zero Fayet-Iliopoulos term $\xi$ related to the string scale $M_{str} = g M_P$ as:

$$D_A = \xi + \sum Q_i |\varphi_i|^2, \quad \xi = \frac{M_{str}^2}{192 \pi^2} \text{Tr}Q$$

where the sum includes all scalar fields $\varphi_i$ present in the theory with the nonzero $U(1)_A$ charges $Q_i$, and the relevant mass scale is the (reduced) Planck scale $M_P = (8\pi G_N)^{-1/2} \approx 2 \cdot 10^{18}$ GeV. As a result, some of the scalar fields which VEVs would vanish in the absence of the Fayet-Iliopoulos term $\xi$ now can get the non-zero vacuum expectation value (VEV) $\sim \sqrt{\xi}$. Therefore, the $U(1)_A$ symmetry breaking scale is naturally small but not too small – the ratio

$$\frac{\sqrt{\xi}}{M_P} \sim 10^{-1} - 10^{-2}$$

is just of order of the fermion mass ratios in the neighbouring families. Following this observation, in the literature anomalous gauge symmetry $U(1)$ is widely used as a horizontal symmetry for explaining the fermion mass hierarchy [4]. Recently the anomalous $U(1)_A$ symmetry was applied [5] to the solution of the doublet-triplet splitting problem by means the Goldstone boson mechanism in the supersymmetric $SU(6)$ theory [6]. This is perhaps the most economic and transparent way to get an accidental global symmetry $SU(6) \times SU(6)$ in the Higgs superpotential, among the other proposals which make use of the discrete symmetries [7].

In the present paper we show that the idea of the anomalous $U(1)_A$ gauge symmetry inspired by the string theory can be used also for achieving a simple ‘all order’ solution to the doublet-triplet splitting problem via the missing doublet mechanism (MDM) [8] in the supersymmetric $SU(5)$ theory. In particular, in sect. 2 we reproduce the original MDM by arrangement of the $U(1)_A$ charges of the Higgs 5- and 50-plets, which solution is stable against the Planck scale corrections. In the sect. 3 we suggest the improved missing doublet models which can reconcile between the proton lifetime and perturbative regime of the $SU(5)$ gauge constant above the GUT scale. Then we study the implications of this model for the neutrino masses (sect. 4), natural suppression of the dimension 3 and 4 B and L violating (R parity violating) operators (sect. 5), Planck scale induced dimension 5 B and L violating operators
(sect. 6) and then present a model incorporating also the fermion mass picture (sect. 7). Finally, in sect. 8 we briefly discuss our results.

2 The missing doublet SU(5) × U(1)\textsubscript{A} model

Consider the SU(5) model with the Higgs sector containing superfields in the following representations: \( H \sim 5, \bar{H} \sim \bar{\bar{5}}, \Phi \sim 75, \Psi \sim 50, \bar{\Psi} \sim \bar{50}, \) and a singlet \( X, \) and three families of the fermion superfields \( 10_i + \bar{\bar{5}}_i, i = 1, 2, 3. \) Let us assume that \( X \) has a negative \( U(1)\textsubscript{A} \) charge, \( Q_X = -q < 0, \) while the other charges are arranged as follows:

\( Q_\Phi = 0, \ Q_\Psi = -Q_H = -h, \ Q_\bar{\Psi} = -Q_{\bar{H}} = q + h, \ Q_{10} = -\frac{1}{2}h \) and \( Q_5 = q + \frac{3}{2}h. \) Then the most general renormalizable Higgs superpotential reads as

\[
W_{\text{Higgs}} = W_1 + W_2, \\
W_1 = M\Phi^2 + \lambda\Phi^3, \\
W_2 = \lambda_1 X\Psi\bar{\Psi} + \lambda_2 H\Phi\bar{\Psi} + \lambda_3 \bar{H}\Phi\Psi, \tag{3}
\]

where the order one constants are understood in the trilinear terms, and the mass parameter \( M \) is of order GUT scale \( M_G \simeq 10^{16} \) GeV. Note, that both the mass term \( \mu H\bar{H} \) and the coupling \( XH\bar{H} \) are forbidden by the \( U(1)\textsubscript{A} \) symmetry.

The couplings of \( H, \bar{H} \) to the fermion fields \( 5 \) and \( 10 \) are the following:

\[
W_{\text{Yuk}} = \Gamma_{ij}^{u}10_i10_jH + \Gamma_{ij}^{d}10_i\bar{\bar{5}}_j\bar{H} \tag{4}
\]

where \( \Gamma_{ij}^{u,d}, i, j = 1, 2, 3 \) are the Yukawa coupling constants.

One has to analyze the superpotential (3) together with the D-terms

\[
g_5^2 \left( \sum \phi_r^T \phi_r \right)^2 + g_A^2 \left( \sum Q_r|\phi_r|^2 - q|X|^2 + \xi \right)^2, \tag{5}
\]

where under \( \phi_r \) we imply the scalar components of all superfields present in the theory besides \( X \) with their \( U(1)\textsubscript{A} \) charges \( Q_r, T_{a}^{(r)} \) are the SU(5) generators in the proper representations \( r, \) and \( g_5 \) and \( g_A \) respectively are the SU(5) and \( U(1)\textsubscript{A} \) gauge constants. It is easy to see that there exists a supersymmetry conserving vacuum when

\[
\langle \Phi \rangle = M_4/\lambda_1 \text{ which breaks SU(5) down to SU(3) \times SU(2) \times U(1)} \text{ symmetry of the MSSM. This VEV is normalized so that masses of the X- and Y-gauge ‘dinosaurs’ are } M_G = 24g_5^2V_0^2. \text{ As for the field } X, \text{ its non-zero VEV } \langle X \rangle = \sqrt{\xi/q} \text{ emerges from the anomalous D-term in (3) and it is essentially close to the GUT scale } M_G: \text{ modulo the factor } (\text{Tr}Q/q)^{1/2} \sim 1 - 10 \text{ we have } \langle X \rangle \sim 5 \cdot 10^{16} \text{ GeV.} \]

All other VEVs are vanishing, which is consistent with the usual F- and D-flatness conditions.

\[^1\] Usually the Higgs and fermion superfields are distinguished by introducing the matter parity: positive for Higgses and negative for fermions. Below we show that in our model we need not to introduce the ad \textit{hoc} matter parity and it can emerge as an automatic consequence of the anomalous \( U(1)\textsubscript{A} \) charges.

\[^2\] In our model the scales \( \langle \Phi \rangle \) and \( \langle X \rangle \) are actually independent and their numerical closeness seems rather accidental. Needless to say that it would be highly interesting to have a realistic mechanism which would naturally explain both values from the same origin.
For the numerical estimates in the following we take the following values for the $U(1)_A$ symmetry breaking scale: $\langle X \rangle = 10^{17}$ GeV and $\langle X \rangle = 4 \cdot 10^{17}$ GeV. Correspondingly, for the ratio $\varepsilon = \langle X \rangle / M_P$ we have $\varepsilon \simeq 1/20$ and $\varepsilon \simeq \sqrt{1/20}$. The first situation could emerge if $(\text{Tr} Q)$ has a moderate value in units of $q$. In the second one we need $(\text{Tr} Q)/q \sim 100$, which could be indeed the case in the models considered below. For the $SU(5)$ symmetry breaking scale we take a standard value $\langle \Phi \rangle = 2 \cdot 10^{16}$ GeV, though in view of the various threshold corrections it could be somewhat larger or smaller.

After substituting the VEVs $\langle \Phi \rangle$ and $\langle X \rangle$ in $W_2$, the 50-plets receive a mass $M_\Psi = \lambda_1 \langle X \rangle$ while the colour triplet components ($H_3, \bar{H}_3$) in $H, \bar{H}$ get masses via the ‘seesaw’ mixing to the triplets $\Psi_3, \bar{\Psi}_3$ from $\Psi, \bar{\Psi}$. The relevant mass matrix reads

$$M_3 = \begin{pmatrix} H_3 & \bar{\Psi}_3 \\ \Psi_3 & \lambda_3 \langle \Phi \rangle \\ \lambda_2 \langle \Phi \rangle & M_\Psi \end{pmatrix}$$

Hence, all triplet fields are massive. The triplets in 50-plets have a mass of order $\langle X \rangle > \langle \Phi \rangle$ while the ones contained in 5-plets get mass $M_T \sim \langle \Phi \rangle^2 / \langle X \rangle$.

As for the doublet components $H_2, \bar{H}_2$ in $H, \bar{H}$, they remain massless since the 50-plets do not have doublet fragments. Therefore, they can be identified with the MSSM Higgs doublets: $H_2 = H_u, \bar{H}_2 = H_d$. In this way, we have achieved a simple solution of the doublet-triplet splitting problem in the $SU(5)$ theory, in the spirit of the original MDM [8]. Note however that our solution is stable against the Planck scale corrections since the higher order operators cutoff by $M_P$

$$\frac{X^k \Phi^n}{M_P^{k+n-1}} H \bar{H}$$

are forbidden for any integer $k$ and $n$ by the $U(1)_A$ charges of the relevant fields. Indeed, the combination $H \bar{H}$ has a negative $U(1)_A$ charge $Q_H + Q_{\bar{H}} = -q$ and thus it cannot be compensated by the powers of $X$.

The MDM however has a generic problem related to the baryon number violating $d = 5$ operators [10]. Indeed, if $M_\Psi \sim \langle X \rangle \geq 10^{17}$ GeV, then the triplets $H_3, \bar{H}_3$ are too light ($M_T \leq 10^{15}$ GeV). In terms of the big matrix of the triplet masses (6) the cutoff scale for the relevant $d = 5$ operators is

$$\left( M_3^{-1} \right)_{11} = \frac{\lambda_1 \langle X \rangle}{\lambda_2 \lambda_3 \langle \Phi \rangle^2} = \frac{1}{M_T}$$

and thus they mediate too fast proton decay [14], which is excluded by the present experimental data [10].

If $M_\Psi \leq M_G$ then the model becomes strongly interacting above the scale $M_G$ since now besides the 75-plet, also the 50- and 50-plets contribute the renormalization

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3This can be the case if $\lambda_1 \ll 1$, or if 50-plets get mass from higher order operator, e.g. $\frac{X^2}{M_P^2} \Psi \bar{\Psi}$. 
group running of the $SU(5)$ gauge constant $g_5$. This contributions would drive the running $g_5$ out of the perturbative regime below the string scale.

All these put the ‘canonical’ version of the MDM [8] in a “no-go” situation unless due to GUT scale threshold uncertainties or maybe at the price of introducing some additional states at intermediate scales the $SU(5)$ unification scale $M_G$ could rise up to about $10^{17}$ GeV [9]. In the next section we present an improved version of the missing doublet model which do not suffer from the above problems.

3 Improved missing doublet model (IMDM)

We employ the proposal of ref. [13] to introduce two sets of the 5- and 50-plets: $H, H' \sim 5$, $\bar{H}, \bar{H}' \sim 50$ and $\Psi, \Psi' \sim 50$. Let us prescribe the $U(1)_A$ charges to these fields as follows:

$$Q_X = -q, \quad Q_\Phi = 0,$$
$$Q_H = h, \quad Q_{H'} = -(n+3)q-h, \quad Q_{H''} = (n+2)q + h, \quad Q_{H'''} = -q - h,$$
$$Q_\Psi = q + h, \quad Q_{\Psi'} = -(n+3)q + h, \quad Q_{\Psi''} = -(n+2)q - h,$$

where $n = 0, 1, 2, \ldots$ is an integer number. Now the Yukawa couplings (4) require the following $U(1)_A$ charges of the fermions:

$$Q_{10} = -\frac{1}{2}h, \quad Q_5 = (n+3)q + \frac{3}{2}h$$

Then the self-interaction terms $W_1$ of $\Phi$ in the Higgs superpotential are still the same as in (3) while the terms in $W_2$ are modified as:

$$W_2 = \lambda_1 X \Psi \bar{\Psi} + \lambda'_1 X \Psi' \bar{\Psi}' + \lambda_2 H \Phi \bar{\Psi} + \lambda'_2 H' \Phi \bar{\Psi}' + \lambda_3 \bar{H} \Phi \Psi' + \lambda'_3 \bar{H}' \Phi \Psi' + \lambda_3 \bar{H} \Phi \Psi' + \lambda'_3 \bar{H}' \Phi \Psi'$$

If only these couplings are left, then one can easily make sure that all triplet fields in the theory are massive, and there is no proton decay via their exchanges. However, the additional couple of the massless doublets $H'_2 + \bar{H}'_2$ emerges in the particle spectrum, which would spoil the $SU(5)$ unification of the gauge couplings.

However, now we include also the non-renormalizable operators cutoff by the Planck scale:

$$W'_2 = \alpha_1 \frac{X^{n+1}}{M_p^n} H' \bar{H}' + \alpha_2 \frac{X^{n+2}}{M_p^{n+2}} H' \Phi \bar{\Psi} + \alpha_3 \frac{X^{n+2}}{M_p^{n+2}} H' \Phi \Psi' + \alpha_4 \frac{X^{n+3}}{M_p^{n+2}} \Psi \bar{\Psi},$$

(with respect to $\Phi$ only the lowest dimensional operators are shown). These operators could emerge after integrating out the heavy states with masses $\sim M_P$ [12].

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4 We still prescribe to fermions family independent charges assuming the hierarchy of the Yukawa constants has an ad hoc origin, or it emerges as a result of the spontaneously broken (Abelian or non-Abelian) horizontal symmetry, in the spirit of refs. [12, 13].
After substituting the VEV $\langle X \rangle$, all these couplings together can be expressed in a matrix form as

$$
\begin{pmatrix}
\bar{H} & \bar{H}' & \Phi & \Phi \\
H & 0 & 0 & \Phi \\
H' & 0 & \varepsilon^{n+1}M_P & \varepsilon^{n+2}\Phi \\
\Psi & 0 & \Phi & \varepsilon M_P \\
\Psi' & \Phi & \varepsilon^{n+2}\Phi & \varepsilon^{n+3}M_P \end{pmatrix}
$$

where $\varepsilon = \langle X \rangle / M_P$, and the coupling constants are omitted. Note, that all zero elements in this expression are “all order” zeros in $X/M_P$, since the $U(1)_A$ charges of the corresponding terms are all negative. The entries $\sim \varepsilon^{n+2(3)}$ are not relevant for our further estimates and we have shown them only for demonstrating their relatively small magnitudes.

Therefore, after integrating out the heavy 50-plets at the scale $V_X$, we obtain the following mass matrices for the doublet and triplet fragments in 5-plet Higgses:

$$
\mathcal{M}_2 = H_2 \begin{pmatrix}
\bar{H}_2 & \bar{H}_2' \\
H_2 & 0 & 0
\end{pmatrix}, \quad M_D \sim \varepsilon^{n+1}M_P
$$

$$
\mathcal{M}_3 = H_3 \begin{pmatrix}
\bar{H}_3 & \bar{H}_3' \\
H_3 & 0 & M_T
\end{pmatrix}, \quad M_T, M'_T \sim \langle \Phi \rangle^2 / \langle X \rangle \sim \varepsilon_G^2 \varepsilon^{-1}M_P
$$

where $\varepsilon_G = \langle \Phi \rangle / M_P \sim 10^{-2}$. Thus, $H_2$ and $H_3$ remain massless and thus they can be identified to the MSSM Higgses $H_u,d$, while $\bar{H}_2, \bar{H}_2'$ get mass $M_D$. As for the triplet fragments, they acquire mass terms $M_T \sim \varepsilon_G \varepsilon^{-1}M_G$, i.e. below the GUT scale $M_G$. Nevertheless, the fact that $M_T < M_G$ now does not necessarily leads to unacceptably fast proton decay. In particular, if $M_D = 0$ the proton would stay stable since the Higgses $H_3$ and $\bar{H}_3$ do not match each other but get their masses rather by joining the states $\bar{H}_3'$ and $H_3'$ (in other words, $(\mathcal{M}_3^{-1})_{11}$ would vanish). However, then the extra doublets $H_2, \bar{H}_2'$ are rendered light and this would affect the gauge coupling unification.

Thus, the proton decay rate crucially depends on $M_D$:

$$
(\mathcal{M}_3^{-1})_{11} \sim \frac{M_D}{M_T^2} = \frac{M_D M_G}{M_T^2 M_G}
$$

Therefore, the if $M_D$ is chosen so that $M_D M_G \leq M_T^2$, then the proton lifetime in this model would not exceed the one in the minimal $SU(5)$ model.

\textsuperscript{5} It was attempted in ref. [13] to obtain this pattern by using the Peccei-Quinn symmetry. However, these authors have omitted ‘by hands’ the possible large mass terms $M_P HH'$ and $M_P H'H$ which were allowed by their $U(1)_{PQ}$ symmetry.
Table 1: Masses of extra triplets and doublets in the IMDM models

| IMDM | $X = 10^{17}$ GeV | $X = 4 \cdot 10^{17}$ GeV |
|------|-------------------|-----------------------------|
| $\Phi \sim 75$ | $M_T = 4 \cdot 10^{15}$ GeV | $M_T = 10^{15}$ GeV |
| $M_D = 2 \cdot 10^{14}$ GeV | $M_D = 6 \cdot 10^{13}$ GeV |
| $(n = 2)$ | $(n = 6)$ |
| $\Sigma \sim 24$ | $M_T = 4 \cdot 10^{11}$ GeV | $M_T = 10^{11}$ GeV |
| $M_D = 3 \cdot 10^6$ GeV | $M_D = 2 \cdot 10^5$ GeV |
| $(n = 8)$ | $(n = 18)$ |

Therefore, our theory below the GUT scale $M_G$ contains new states with masses $\ll M_G$: two couples of triplets with masses $M_T, M'_T$ and a couple of doublets with mass $M_D$. Interestingly, if $M_T$ is about a mid geometrical between $M_D$ and $M_G$, then the presence of these extra states will not affect the fact of the gauge coupling unification and the value of the unification scale itself. The couple of doublets $H'_2 + \bar{H}'_2$ contribute the gauge constants renormalization group running up from the scale $M_D$, while the two couples of triplets enter at the scale $M_T \sim \sqrt{M_D M_G}$. Therefore, at one loop approximation these extra states together contribute as a weakly split complete $SU(5)$ supermultiplet $5 + \bar{5}$ living at the scale $M_D$.

Therefore, for the given value of $\langle X \rangle$, one can always choose the enough large power $n$ for which the condition $M_D M_G \leq M_T^2$ is marginally satisfied. We obtain $\epsilon^{n+3} \leq \epsilon_G^3$, or in other words:

$$n + 3 \log \frac{M_P}{\langle X \rangle} > \log \frac{M_P}{\langle \Phi \rangle},$$

On the other hand, now 50-plets are heavy and thus their impact on the gauge coupling running from the GUT to the string scale is not anymore so dramatic. Interestingly enough, in our model the MDM can be realized by using the standard Higgs 24-plet instead of 75-plet. Indeed, one can replace $\Phi$ in eq. (13) as $\Phi \to \frac{1}{M_P} \Sigma \cdot \Sigma$, where $\Sigma \sim 24$ with a vanishing $U(1)_A$ charge has a VEV $\langle \Sigma \rangle \sim M_G$. (Hereafter such a model is refereed as IMDM24 while the one involving the 75-plet as IMDM75.) This will result only in changing the order of magnitude of the triplet masses in (14): $M_T, M'_T \sim \epsilon_G^3 \epsilon^{-1} M_P \sim \epsilon^{-1} \cdot 10^{10}$ GeV. Then the condition $M_D M_G \leq M_T^2$ translates into $\epsilon^{n+3} < \epsilon_G^7$.

Implications of both IMDM75 and IMDM24 models for the extra states populating the big desert between the electroweak and GUT scales for different values of the scale $\langle X \rangle$ are shown in the illustrative Table 1. The values of the integer $n$ for which $M_T$ appears to be about a mid geometrical between $M_D$ and $M_G$. 
4 Neutrino masses

Until now we did not fix the value of \( h \), since the needed pattern of the Higgs superpotential still allows the freedom for two uncorrelated \( U(1)_A \) charges, say \( Q_X \) and \( Q_H \). However, the charge \( h \) can be unambiguously fixed by considering the neutrino mass generating operators.

The simplest assumption would be that the neutrino masses emerge from the Planck scale operators \[17\]:

\[
\frac{\beta_{ij}}{M_P} \bar{5}_i 5_j H^2
\]

which would be allowed by the \( U(1)_A \) symmetry if \( h = -\frac{2}{5}(n + 3)q \). This operator induces the neutrino Majorana masses \( m_\nu \sim M_W^2/M_P \sim 10^{-5} \text{eV} \). This would suffice for explanation of the solar neutrino problem via the long wavelength “just-so” oscillation \( \nu_e \rightarrow \nu_{\mu,\tau} \) provided that the mixing angles are large, of order 1. The latter would be the case if all (diagonal and non-diagonal) coupling constants \( \beta_{ij} \sim 1 \), unlike the Yukawa constants of the charged fermions. However, all other existing neutrino puzzles cannot be accomodated in this case.

In order to obtain the larger neutrino masses, one can use the seesaw mechanism \[18\]. Indeed, let us introduce the \( SU(5) \) singlet neutrino states \( N_m \) \((m = 1, 2, \ldots)\), which get the large Majorana masses via couplings to \( X \) while their couplings to ”left handed” neutrinos contained in \( \bar{5} \) induce the Dirac mass terms. The relevant superotential terms can be taken as

\[
\Gamma_D^{kl} H \bar{5}_k N_m + \Gamma_M^{lr} M_P N_l N_r \left( \frac{X}{M_P} \right)^{n'}
\]

which after integrating out the heavy \( N \) states result in the following effective operators scaled by the inverse powers of \( \langle X \rangle \):

\[
\frac{\beta_{ij}}{\varepsilon^n M_P} \bar{5}_i 5_j H^2
\]

Therefore, for a given integer \( n' = 0, 1, 2, \ldots \) \((n' = 0 \text{ corresponds to case of operators } [17])\), these couplings fix the following relation between \( h \) and \( q \) charges:

\[
h = -\frac{1}{5}(2n + n' + 6)q
\]

Therefore, for \( n' = 4 \) or \( n' = 5 \) one obtains that \( m_{\nu_{\tau}} \sim 1 - 10 \text{ eV} \), which can provide the hot dark matter needed for the explanation of the large scale structure of the universe. If the neutrino masses obey the same hierarchy as the quark and lepton masses, e.g. \( \nu_e : \nu_\mu : \nu_\tau \sim u : c : t \), then the mass of \( \nu_\mu \) will emerge in the range \( \sim 3 \cdot 10^{-3} \text{eV} \) which can explain the solar neutrino problem via the MSW mechanism.

\[6\] The effective operators involving the direct powers of \( X \), \( \frac{X^l}{M_P^{n+l}} 5^2 H^2 \) would fix \( h = -\frac{1}{5}(2n + 6 - l)q \). However, in this case the neutrino masses are too small and thus of no phenomenological interest.
5 Natural R parity

Thus, the charge $h$ can be fixed in terms of $q$ by considering the neutrino mass pattern. Now we show that for certain values of $h$ one can achieve a natural suppression of the $d = 3$ and $d = 4$ B and L violating operators, without imposing an ad hoc matter parity (R-parity).

Indeed, if $h$ is given by eq. (20) then the $U(1)_A$ charge of the combinations $H^5_j$ equals to $-\frac{1}{2}n'q < 0$. Therefore, these $d = 3$ R parity breaking operators are forbidden at any power of $X/M_P$. On the other hand, the charge of the combinations $10 \cdot 5 \cdot 5$ is $\frac{1}{2}(2n - n' + 6)q$. Hence, these operators could emerge only at the level

$$10,5j,5k \left( \frac{X}{M_P} \right)^{(n+3)-n'/2}$$

(21)

Thus, for an odd $n'$ no such operators can be built and the exact R-parity conservation emerges as an automatic (accidental) consequence of the $U(1)_A$ charge content of the fields in the theory.

For an even $n'$ the R-parity breaking terms can emerge after substituting VEV of $X$, and thus they can be naturally suppressed, by the corresponding power of $\varepsilon$. By taking for example $n = 2$ and $n' = 2$, we obtain the following estimate for the constants of the R-parity breaking terms: $\lambda \sim \varepsilon^4 \sim 5 \cdot 10^{-6}$ for $\varepsilon \sim 1/20$. This is just at the border of the experimentally allowed region [21] and can be of phenomenological interest for the testing in future experiments.

In the models where the fermions carry the generation dependent $U(1)_A$ charges the R-parity breaking constants can be suppressed much stronger. For example, in the model of the section 7 with the fermion charges as in eq. (25) the following terms are allowed:

$$a_{ijk}10,5j,5k \left( \frac{X}{M_P} \right)^{n+7-i}$$

(22)

Therefore, even for the constants $a_{ijk} \sim 1$, the R-parity breaking terms involving 10 of the first family are suppressed as $\varepsilon^8$ and thus they are well below the present experimental limits [21].

6 The Planck scale $d=5$ operators

In principle, in the minimal supersymmetric $SU(5)$ theory (in fact, already in the MSSM) the nucleon decay could be induced by the Planck scale $d = 5$ operators [13] $\sim \frac{\kappa}{M_P}(10,10,10,10,5)$. The fact that these operators are cutoff by $M_P$ instead of $M_G$ as it takes place in the triplet Higgsino mediated case is no garancy that these Planck scale terms are safe unless the relevant coupling constants are further suppressed.
For example, if $\kappa \sim 1$ then the operator $(q_2 q_1)(q_1 l_k)$ would induce the proton decay with $\tau(p \rightarrow K\nu) \sim \kappa^{-2} 10^{18}$ yr, i.e. much faster than the triplet Higgsino mediated $d = 5$ operators do. Hence, in order to reconcile with the experimental bounds on the proton decay the constants $\kappa$ should be very small, $\kappa \leq 10^{-7}$.

In our model these dangerous operators are indeed suppressed by the $U(1)_A$ symmetry. For the generation blind arrangement of the fermion charges (10) for the $U(1)_A$ charge of the relevant combination we have $Q(10^3 5) = (n + 3)q$. Therefore, the lowest order term allowed is the following:

$$\frac{1}{M_P} 10_{i} 10_j 10_{l} 5_k \left( \frac{X}{M_P} \right)^{n+3} \varepsilon \left( \varepsilon^{n+3} \frac{X}{M_P} \right) 10_i 10_j 10_{l} 5_k \quad (23)$$

Then e.g. for $\varepsilon \sim 1/20$ and $n = 2$, which has been shown to be a reasonable choice for achieving the pattern for the Higgs superpotential parameters for achieving the proper suppression of the Higgsino mediated $d = 5$ operators, we obtain $\kappa \sim 10^{-7}$ which is enough to reconcile the proton lifetime with the experimental limits.

Anticipating the next section, the more suppression of the Planck scale $d = 5$ operators can take place if fermions are prescribed to have the generation dependent $U(1)_A$ charges (see e.g. below, eq. (25)). In this case we would obtain

$$\frac{1}{M_P} 10_{i} 10_j 10_{l} 5_k \left( \frac{X}{M_P} \right)^{n+14-i-j-l} \quad (24)$$

so that for $n = 2$ the terms involving first and second generations of 10 are suppressed as $\varepsilon^{12}$.

### 7 The $U(1)_A$ hierarchy of fermion masses

Below we try to join our voice to the common chorus [4] and attempt to incorporate the anomalous $U(1)_A$ symmetry for “stitching the Yukawa quilt” [20] for understanding the quark and lepton mass and mixing pattern. We consider the $SU(5)$ IMDM where the Higgs superfields possess the $U(1)_A$ charges given as in eq. (9) while the fermion charges are flavour and generation dependent. Namely, we choose the charges of $10_i = (u^c, q, e^c)_i$ and $\bar{5}_i = (d^c, l)_i$ as follows:

$$Q(10_i) = -\frac{1}{2} h + (3 - i)q, \quad Q(\bar{5}_i) = \frac{3}{2} h + (n + 5)q \quad (25)$$

where $i = 1, 2, 3$ is a family index. Thus, only the top quark can get mass via the renormalizable Yukawa coupling $10_3 10_3 H$ while the masses of other fermions will emerge from the higher order operators including powers of $X/M_P$. The effective Yukawa superpotential including the higher order operators now has the form:

$$W_{Yuk} = f_{ij} 10_i 10_j H \left( \frac{X}{M_P} \right)^{6-i-j} + g_{ik} 10_i 5_k H \left( \frac{X}{M_P} \right)^{5-i} \quad (26)$$
with constants $\sim 1$ where $f_{ij}$ is symmetric, $f_{ij} = f_{ji}$, and $g_{ik}$ can be taken in the skew-diagonal form with vanishing elements $g_{13}$, $g_{23}$ and $g_{13}$ by a proper choice of the 5-plets basis.

Therefore, after substituting the VEVs of the MSSM Higgs doublets $\langle H_u \rangle = v_u$ and $\langle H_d \rangle = v_d$ ($v_u^2 + v_d^2 = v^2$, $v = 174$ GeV, and $v_u/v_d = \tan \beta$), we would obtain the following pattern for the quark mass matrices:

$$\hat{m}^u = u_1 \begin{pmatrix} u_1^c & u_2^c & u_3^c \\ f_{11}\varepsilon^4 & f_{12}\varepsilon^3 & f_{13}\varepsilon^2 \\ f_{13}\varepsilon^2 & f_{22}\varepsilon^2 & f_{23}\varepsilon \end{pmatrix} \cdot v_u, \quad \hat{m}^d = d_1 \begin{pmatrix} d_1^c & d_2^c & d_3^c \\ g_{11}\varepsilon^4 & g_{12}\varepsilon^3 & g_{13}\varepsilon^2 \\ 0 & g_{22}\varepsilon^3 & g_{23}\varepsilon^2 \end{pmatrix} \cdot v_d$$

(27)

where we take $\varepsilon = \langle X \rangle/M_P \sim 1/20$, in accord to the estimate (2) provided that $\text{Tr} Q$ has a moderate value. Therefore, the horizontal hierarchy of quark masses exhibit the approximate scaling lows:

$$m_t : m_c : m_u \sim 1 : \varepsilon^2 : \varepsilon^4$$

$$m_b : m_s : m_d \sim 1 : \varepsilon : \varepsilon^2$$

(28)

while for the CKM mixing angles we obtain the following estimates:

$$s_{12}, s_{23} \sim \varepsilon, \quad s_{13} \sim \varepsilon^2.$$  

(29)

All these for $\varepsilon \sim 1/20$ well agrees to the observed pattern of the quark masses and mixing (actually experimental value of the Cabibbo angle $s_{12}$ is about factor of 4 above this estimate, which can have an accidental origin).

Note also that the vertical splitting between the quark masses also has a pattern which is favoured for $\tan \beta \sim 1$:

$$\frac{m_t}{m_d} \sim \tan \beta, \quad \frac{m_c}{m_s} \sim \varepsilon^{-1}\tan \beta, \quad \frac{m_t}{m_b} \sim \varepsilon^{-2}\tan \beta.$$  

(30)

Recall, that small $\tan \beta$ is also favoured by the the proton lifetime.

The problem which remains is that since $X$ is a $SU(5)$ singlet, the second operator (26) leads to exactly the same mass matrix for the charged leptons as for the down quarks: $\hat{m}^e = \hat{m}^d$ in the $SU(5)$ limit, and thus $m_{\mu,\tau} = m_{d,s,b}$.

The simplest possibility for evading these unwanted relations would be to involve the scalar $\Phi$ which breaks the $SU(5)$ symmetry into the quark and lepton mass generation. Then it woul induce different Clebsches for the down quark and lepton mass entries and thus remove these relations.

8Actually the value of $\text{Tr} Q$ accumulated on the observable sector rather favours the larger value of $\varepsilon$, say $\varepsilon \sim \sqrt{1/20}$. For this case one can simply modify the fermion charges so that expansion in the Yukawa superpotential (26) would go in powers of $(X/M_P)^2$. This will result only in change $\varepsilon \rightarrow \varepsilon^2$ in the fermion mass matrices and thus exactly the same pattern will be kept.
Table 2: The $U(1)_A$ charges $Q$ of the superfields in units of $q = -Q_X$. The charges of the $SU(5)$ breaking fields $\Phi \sim 75$ or $\Sigma \sim 24$ in the above models are zero.

| $Q$ | $\bar{H}$ | $H$ | $\Psi$ | $\Psi'$ | $\bar{\Psi}'$ | $\bar{H}'$ | $H'$ | 5, 5, 10, 1 |
|-----|-----------|-----|--------|---------|---------------|----------|------|-----------|
| $-\frac{2(n+6)}{5}$ | $-\frac{3(n+1)}{5}$ | $-\frac{2n+7}{5}$ | $\frac{2(n+6)}{5}$ | $\frac{3(n+1)}{5}$ | $-\frac{3n-2}{5}$ | $\frac{3n-2}{5}$ | $\frac{2n+7}{5}$ | $\frac{n+21}{5}$ |

However, we find that the simplest possibility is the following. Let us assume that the "Higgs" superfield $\bar{H}'$ carries the same charge as the fermion 5-plets $\bar{5}_i$. According to eqs. (9) and (25), this requires that $-h - q = \frac{3}{2} h + (n + 5)q$ and thus $h$ is fixed as

$$h = -\frac{2}{5} (n + 6)q \quad (31)$$

The $U(1)_A$ charges of all superfields involved in game in this case is given in Table 1.

Then, if no ad hoc matter parity is introduced to distinguish the 'fermion' $\bar{5}_i$ and the 'Higgs' $\bar{H}'$ superfields are essentially the same and thus $\bar{H}'$ can be identified as a fourth fermion 5-plet: $\bar{H}' = \bar{5}_4$. Then the second 'Yukawa' term in (26) should be extended by including also $\bar{H}'$, so that the index $k$ now runs the values $k = 1, 2, 3, 4$.

On the other hand, all couplings presented for $\bar{H}'$ in the Higgs superpotential (11) and (12) now are allowed also for $\bar{5}_k$, $k = 1, 2, 3$. Then, without loss of generality, one can choose the basis in which the "fermions" $\bar{5}_i$ have no coupling to the 50-plet $\Psi$ in $W_2$ (11). In other words, $\bar{H}' = \bar{5}_4$ can be defined as a combination of all $\bar{5}_k$ 'fermions' which has the coupling $\bar{H}'\Phi\Psi$. However, in this basis the couplings

$$G_k \frac{X^{n+1}}{M_p} \bar{5}_k H', \quad k = 1, 2, 3, 4 \quad (32)$$

already cannot be rotated away and thus the doublet states in $\bar{5}_{1,2,3}$ cannot be identified anymore to the light lepton doublets $l_i$, but rather with the linear combinations of $\bar{5}_i$ and $\bar{H}'$ with the order 1 mixing angles.

The 'big' mass matrices of the down quark and lepton states now read as:

$$
\begin{pmatrix}
    d_1 \quad d_2 \quad d_3 \\
    m_{11} \varepsilon e \quad m_{12} \varepsilon e \quad m_{13} \varepsilon e \quad m_{14} \varepsilon e \\
    0 \quad m_{22} \varepsilon e \quad m_{23} \varepsilon e \quad m_{24} \varepsilon e \\
    0 \quad 0 \quad m_{33} \varepsilon e \quad m_{34} \varepsilon e \\
\end{pmatrix}
\begin{pmatrix}
    H_3' \quad H_3 \\
    M_1^D \quad M_2^D \quad M_3^D \quad M_4^D \quad M_T \\
    0 \quad 0 \quad 0 \quad M_T \quad 0 \\
\end{pmatrix}
\quad (33)
$$
where $M_k^D = G_k \varepsilon^{n+2} M_P$ and $m_{ik} = g_{ik} v_d$ (in the upper $3 \times 3$ block these terms can be still rotated to the skew diagonal form (27)).

Now we see that the mixing of the states $d_{1,2,3}$ to the heavy triplets $\bar{H}_3, \bar{H}_3'$ will not affect significantly the upper $3 \times 3$ block in the matrix (33), since the mixing angles are very small, $\sim M_D/M_T \ll 1$. Therefore, the down quark mass matrix remains practically the same as is given by eq. (27).

On the other hand, mixings of the states $e_{1,2,3}$ to $\bar{H}_2' = e_4$ are big. Hence, the doublet states in $\bar{5}_{1,2,3}$ cannot be identified anymore to the light lepton doublets $l_i$, but rather with the linear combinations of $\bar{5}_i$ and $H'$ with the order 1 mixing angles. Clearly, this mixing will deviate the lepton mass matrix from the form given by eq. (27) by the order 1 “Clebsches” in each element.

We gave up the matter parity. However, in the low energy theory (i.e. for the MSSM states) the R-parity breaking operators will be strongly suppressed. As we have already discussed in sect. 5, the $U(1)_A$ charge of the combinations $H \bar{5}_k$ is always negative, $-q$, and thus this terms cannot emerge at any order of $X/M_P$. As for the combination $10,\bar{5}_j \bar{5}_k$, its $U(1)_A$ charge is positive, $n + 7 - i$ (Note, it depends only on the generation index of 10, $i = 1, 2, 3$). Therefore, these terms can emerge in the form (22) written down in sect. 5. However it was remarked that the R-parity breaking constants are much below the experimental bounds.

Below we present some other interesting R-parity breaking terms allowed in our model. The ones involving only the light (MSSM) states are:

$$\frac{1}{M_P} 10, 10_j 10_k \bar{H} \left( \frac{X}{M_P} \right)^{12-i-j-k}$$

with an impact for the lightest neutralino stability. All other possible operators involve the superheavy states. E.g. the coupling

$$10, \bar{H} H' \left( \frac{X}{M_P} \right)^{5-i}$$

which can even faster destabilize the lightest neutralino, via the diagram induced by the exchange of the colour triplet Higgsino with a mass $M_T \sim \langle \Phi \rangle^2/\langle X \rangle \sim 10^{15}$ GeV: $n \to udd$ or $n \to d u c\bar{c}$. Therefore, if these processes are active in this model, the cold dark matter of the universe cannot consist of the lightest neutralino and another candidate should be found.
8 Discussion

In this paper we have revised the missing doublet $SU(5)$ model. A key role in our picture is played by the anomalous gauge symmetry $U(1)_A$. We have shown some examples of supersymmetric $SU(5) \times U(1)_A$ models which could provide an appealing simultaneous solution to the various “hot” puzzles in the SUSY GUT philosophy as are the gauge hierarchy and doublet-triplet splitting problem, problem of fermion mass hierarchy, origin of matter parity (or R parity) conservation and so long lifetime of proton. In particular, the $U(1)_A$ charge content of superfields in the theory can be arranged so that R parity breaking operators will be forbidden at any order in $M_P^{-1}$. In other words, the exact conservation of R parity can be an accidental consequence of the gauge symmetry. In more interesting cases, however, the R parity can be only approximate symmetry, with implications for the future search of the R-parity breaking phenomena and certainly with a great impact on the stability of the lightest neutralino.

We have also extended a picture for the fermion masses by involving $U(1)_A$ as a horizontal symmetry. The fermion mass hierarchy as well as the magnitudes of the CKM mixing angles can be naturally understood in terms of small parameter ($\varepsilon \sim 1/20$) or not so small ($\varepsilon \sim \sqrt{1/20}$), with a proper choice of the fermion $U(1)_A$ charges. We have also shown a simple mechanism for explaining the origin of the about factor of 3 splitting between the down quark and lepton masses in the same family. Interestingly, this mechanism is organically related to the features needed for achieving the realistic IMDM pattern for the doublet-triplet splitting.

The actual value of the $U(1)_A$ symmetry breaking scale cannot be deduced unambiguously. The Green-Schwarz mechanism implies the following relations between the mixed $U(1)_A$ anomaly coefficients and the Kac-Moody levels:

$$\frac{C_5}{k_5} = \frac{C_A}{k_A} = \frac{C_g}{k_g}$$

In our models all anomaly coefficients are typically $O(100)$. Among these the mixed $U(1)_A - SU(5)$ anomaly $C_5$ is contributed solely by the $SU(5)$ superfields in the game unless some other states are also introduced. The value $\text{Tr}Q/q$, which is essentially the mixed $U(1)_A$-gravity anomaly $C_g$ remains arbitrary, since besides the observable ($SU(5)$) matter it can be contributed by some ‘hidden’ matter singlet with respect to $SU(5)$: $\text{Tr}Q = \text{Tr}Q_{\text{obs}}+\text{Tr}Q_{\text{hid}}$. These additional states will contribute also the $U(1)_A^3$ anomaly coefficient $C_A \sim \text{Tr}(Q^3)$. In the IMDM models the ‘observable’ portion of $C_g$ is rather large, $\text{Tr}Q_{\text{obs}} \sim 100$, and it would favour the ‘not so small’ value of $\varepsilon$ (say $\sim \sqrt{1/20}$). If the hidden matter gives a big negative contribution, then $\varepsilon \sim 1/20$ can be also possible. There can be also a case that the field $X$ shares the VEV (2) with some other singlets so that its its own portion in $\sqrt{\lambda_i}$ is small. In any case, the Diophantian equations to be satisfied for having the Green-Schwarz mechanism valid for the universal cancellations of all mixed anomalies through the shift of the
axion Ims do not restrict much the charge content in our scheme unlike the cases of the MSSM based models [4], since now the MSSM is embedded in the SU(5). In particular, the GUT scale value of the Weinberg angle \( \sin^2 \Theta_W = 3/8 \) now directly follows from \( SU(5) \) without specific matching of the mixed anomalies for each factor in \( SU(3) \times SU(2) \times U(1) \) separately.

We have not discussed neither the origin of the supersymmetry breaking nor the \( \mu \)-problem. It was implicitly assumed that the supersymmetry breaking occurs via the incognito mechanism in some hidden sector and then transmitted through the gravity to the observable sector. Perhaps there can be found a clever mechanism which relates the origin of the supersymmetry breaking to the same anomalous \( U(1)_A \) symmetry, in the spirit recently discussed in refs. [22]. We have tried some not so clever possibilities but did not find a one worth of publishing.

The interesting features of the IMDM models make us to think that maybe neither the missing doublet mechanism nor the concept of the anomalous \( U(1)_A \) stringy gauge symmetry are not that bad ideas if the two are working together.

Of course, there is a big question whether the models presented here or alike can indeed emerge from the string theory. No explicit string construction exists at the Kac-Moody level \( k_5 \geq 4 \) which is required for having 50-plets in the \( SU(5) \) stringy GUT [23]. However, if the the IMDM like models can be indeed found in a string theory context, they would have the interesting feature that the string constant \( g_{str}^2 = k_5 g_5^2 = k_A g_A^2 \) could be enough large, say \( \sim 5 \). As we already remarked, in the IMDM models (perhaps more interesting is IMDM24), the \( SU(5) \) gauge constant running up from the GUT scale can be prevented from being ‘exploded’ to the strong coupling regime below \( M_P \). However, its value at energies \( \sim M_P \) should be significantly larger than in the minimal \( SU(5) \), due to contributions of the 50-plets and 75- or 24-plet. Therefore, it is rather probable that \( \alpha_{str} \) can have a magnitude of about 0.5-1, which can leave open the possibility to prevent the runaway behaviour and find the true vacuum values for the dilaton and the moduli fields. This feature might be of certain interest since by the duality arguments neither the weak regime \( \alpha_{str} \ll 1 \) nor the strong one \( \alpha_{str} \gg 1 \) seem to be good for stabilizing the vacuum state [24].

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