Gas accretion in a clumpy disk with application to AGNs

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Abstract

We analyze the collective gravitational interaction among gas clouds in the inner regions of galactic disks and find that it leads to accretion at a rate \( \sim M_{mc} \Omega (M_{mc}/M_t)^2 \); where \( M_{mc} \) is the molecular mass of the disk, \( M_t \) is sum of the central plus any axisymmetrically distributed mass, and \( \Omega \) is the mean angular speed of clumps. We discuss applications of this result to the mega-maser galaxy NGC 4258, for which we have observational evidence that the maser spots are concentrated in a thin molecular disk which is clumpy, and find the accretion rate to be \( \sim 1.5 \times 10^{-3} \, M_\odot \, \text{yr}^{-1} \). If the gravitational energy release of this inward falling gas were to be radiated away efficiently, then the resulting luminosity would greatly exceed the observed central luminosity of NGC 4258, indicating that most of the thermal energy of the gas is advected with the flow into the blackhole as proposed by Lasota et al. (1996).

The gravitational interactions among molecular clouds lying within the inner kpc of our galaxy give an accretion rate of \( \sim 10^{-5} \, M_\odot \, \text{yr}^{-1} \), which is consistent with the value obtained by Narayan et al. (1995) by fitting the spectrum of Sagitarrius A\*.

We also discuss possible application of this work to quasar evolution.

Subject heading: accretion disks – galaxies: individual (NGC 4258)

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1. Introduction

The gas in the inner parsec of a number of active galaxies appears to be clumpy and concentrated in a thin disk. The best evidence for this comes from the observation of discrete spots of water maser emission in active galaxies such as NGC 4258, 1068, 4945 (cf. Watson & Wallin 1994, Greenhill et al. 1995, Miyoshi et al. 1995, Greenhill et al. 1996 & 1997, Braatz et al. 1996, and references therein). The luminosity of the central source in these systems is believed to be due to the release of gravitational energy of infalling gas. The gas accretion could be a result of turbulent viscosity in the disk, however its effectiveness in a cold thin disk consisting of discrete molecular clouds in very uncertain.

Here we consider another, more likely, process driving gas accretion – the gravitational interaction among clumps, or clouds, excites epicyclic oscillation, causing clouds to collide (inelastically) and as a result some fraction of them fall inward. We treat clumps as point mass when considering their gravitational interactions, which excite epicyclic oscillations. The finite clump size is taken into consideration when determining the frequency of physical collisions among clumps. We assume that there is no dipole or low multipole order coherent density structure in the disk which induces efficient inward flow of gas. The effect of spiral density wave on accretion in NGC 4258 has been considered by Maoz and McKee in a recent paper. The process we consider has some similarity to the work of Goldreich and Tremaine (1978), however unlike the case of the planetary ring systems considered by these authors the gravitational interactions between clouds are very important and is considered in some detail here.

There is another motivation for considering the gravitational interaction among gas clumps. The mechanism for the inward transport of gas within the inner kpc of active galaxies (AGNs) to supply fuel for the central activity, is poorly understood. Tidal torques from neighboring galaxies, even in close encounters, are believed to be ineffective at this distance (but it can be important in transporting gas from several kilo-parsecs to within about a kpc). Moreover, recent observations of a sample of spiral galaxies show that the central activity is uncorrelated with the strength of the galactic bar (e.g. Ho, Filippenko and Sargent, 1997), which suggests that bars are not effective in inducing radial inflow of gas either. We estimate the efficiency of gas transport, within the inner kpc of galactic disks, due to gravitational interaction between clumps in the disk (Shlosman & Begelman 1987, also investigated this mechanism; see also Paczynski 1978, and Shlosman, Begelman & Frank 1990 and references therein).

We provide an estimate of gas accretion rate in a clumpy disk in §2 and apply it to the water maser galaxies and fueling of AGNs in §3.

2. Accretion in a clumpy disk

The accretion rate due to gravitational interaction among clumps is derived in §2.2.1. Although pressure forces are ignored in this paper, the method of §2.1 can be generalized to include pressure of the inter-clump medium. A physical, but crude, derivation of the main result is also presented in §2.2.

§2.1 Basic equations
We consider a thin molecular disk which contains clumps of gas. We assume that the disk does not have some unstable dipole or other low order long wavelength mode which is excited to large amplitude causing efficient transport of angular momentum outward and enabling gas to fall inward. However, we assume that some local instability, gravitational or thermal, exists and is responsible for the lumpiness of the disk. The clumps move on nearly circular orbits under the influence of an axisymmetric potential $\psi(r)$ and the gravitational attraction of other clumps. The equation of motion of the clumps, neglecting pressure forces, is given by

$$\frac{d^2r}{dt^2} = -\nabla \Psi - G \nabla \int d^2r_1 \frac{\sigma(t, r_1)}{|r_1 - r|},$$

where $\sigma(t, r)$ is the surface mass density of gas, and $r$ is the center of mass of the clump. Expanding the angular dependence of the surface density $\sigma(t, r, \phi)$ in a Fourier series, and substituting the spherical harmonic expansion for $|r_1 - r|^{-1}$ in the equation above, we obtain

$$\frac{d^2r}{dt^2} = -\nabla \Psi - 2\pi G \nabla \sum_{m=-\infty}^{\infty} \int_0^\infty dr_1 r_1 K_m(r, r_1) \sigma_m(t, r_1) \exp(im\phi),$$

where

$$K_m(r, r_1) = 4\pi \sum_{\ell=|m|}^{\infty} \frac{|Y_{\ell m}(\pi/2, \phi)|^2}{2\ell + 1} \left( \frac{r_{\ell <}}{r_{\ell + 1}} \right) \approx \frac{2}{\pi} \sum_{\ell=|m|}^{\infty} \frac{1}{[\ell(\ell + 1.25) - m^2]^{1/2}} \left[ \frac{r_{\ell <}}{r_{\ell + 1}} \right],$$

$$r_{\ell <} = min(r, r_1) \quad \text{and} \quad r_{\ell >} = max(r, r_1).$$

It can be shown that the width of $K_m$ is $r/m$, and that for $m > 1$

$$\int dr_1 r_1 mK_m(r, r_1) \approx r.$$

If clumps were to move on circular orbits with angular speed $\Omega(r)$, with no change to their density or shape, then the time dependence of $\sigma_m(t, r)$ would be given by $\exp[im\Omega(r)t]$. We explicitly factor out this time dependence and using equation (2) find the torque, $T$, on a cloud of mass $m_b$ to be

$$T = 2\pi m_b G \sum_{m=-\infty}^\infty m \int_0^\infty dr_1 r_1 K_m(r, r_1) \sigma_m(t, r_1) \exp[i m \phi + i m \Omega(r_1)t].$$

Note that $(\phi + \Omega t)$ does not change as we follow a blob on a circular orbit, however, the mutual gravitational force of clumps perturbs the orbit, and therefore the torque on clumps fluctuates with time causing the amplitude of their epicyclic oscillation to undergo a random walk. We estimate the magnitude of the torque using equation (5).

The torque on a cloud, in a differentially rotating disk, is dominated by nearby clouds. To see this, consider two rings of material at radii $r_1$ and $r_2$, separated by $d_{12} = |r_1 - r_2|$. 

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The torque on a cloud in one ring due to clumps in the other ring comes from multipole moments $\sigma_m$ with $m \lesssim (r_1 + r_2)/2d_{12}$. The correlation length for $\sigma_m$ is approximately equal to the typical size of a clump, $l_b$. Thus the contribution from low order multipoles to the torque is small because within the width of the function $K_m$, $r_1/m$, we have a large number of uncorrelated rings which give positive and negative torques that tend to cancel each other out. Furthermore, the correlation time for torque in a shearing disk falls off with distance as $\Omega^{-1}(d_b/d_{12})|d \ln \Omega/d \ln r|^{-1}$ (where $d_b$ is the average separation between clouds and $\Omega(r)$ is the angular velocity). So clouds lying at radial separation much greater than $d_b$ are very inefficient in transferring angular momentum, and most of the contribution to $T$ comes from nearby clouds.

§2.2 Estimate of velocity dispersion and accretion rate

The number of clouds in a ring of radius $r$ and thickness $l_b$ is $\sim (rl_b/d_b)^2$, and therefore $\sigma_m \sim \sigma_b(l_b^2/rd_b^2)^{1/2}$, where $\sigma_b$ is the mean surface mass density of clumps and $l_b$ is their mean size. It thus follows from equation (5) that the torque on a cloud is

$$|T| \sim Gm_b \sigma_b m_{\text{max}} \left( \frac{l_b^2}{d_b} \right) \sim Gm_b r \bar{\sigma}(r),$$

(6)

where $\bar{\sigma}(r) = \sigma_b(l_b/d_b)^2$ is the mean surface mass density of gas at $r$, and $m_{\text{max}} \sim r/d_b$. The correlation time for the torque $t_{\text{cor}} \sim \Omega^{-1}(d_b/d_t)^2$, where $d_t \sim r(m_b/M_t)^{1/3}$ is the tidal radius of the cloud, and $M_t$ is the total axisymmetrically distributed mass contained inside the radius $r$. Thus the change to the angular momentum of a clump in time $t_{\text{cor}}$ is

$$\delta L \sim \frac{Gm_b^{4/3}M_t^{2/3}}{r \Omega},$$

(7)

and the corresponding changes to the clump’s radial excursion amplitude and the radial velocity are given by

$$\delta r \sim \frac{Gm_b^{1/3}M_t^{2/3}}{r^2 \Omega^2} \sim d_b^{2/3} r^{1/3} \left( \frac{M_r}{\pi M_t} \right)^{1/3} \sim d_t,$$

(8)

and

$$\delta v_r \sim (r \Omega) \left[ \frac{M_r}{\pi M_t} \right] \left[ \frac{d_b}{r} \right]^{2/3},$$

(9)

where $M_r \equiv \pi r^2 \bar{\sigma}$ is the total cloud mass inside of radius $r$. We expect clouds to undergo collision on time scale of $t_{\text{cor}}$ when $l_b$ is not much smaller than $d_t$, and in this case the cloud velocity dispersion is equal to $\delta v_r$ given above. The expression for $\delta v_r$, in the above equation, is same as that given by Gammie et al. (1991) for the velocity dispersion of molecular clouds in the Galaxy.

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1 We are considering the case where there is no coherent large scale dipole or other low order anisotropy in the disk density distribution.
Equations (8) & (9) can also be obtained by considering the gravitational interaction between clumps and the resulting epicyclic oscillation. The change to the displacement amplitude of the epicyclic oscillation of a clump as a result of gravitational interaction with another clump, with impact parameter of \( d \cong d_t \), can be shown to be \( \sim m_b r^3/(M_t d^2) \). Here the impact parameter is the distance of closest approach for the unperturbed orbits of the two clumps. Gravitational encounters with \( d \cong d_t \), are almost adiabatic and there is little change to the epicyclic energy of clumps (two particles moving on circular orbits merely interchange their trajectories in such an encounter). Thus the dominant gravitational interactions for exciting epicyclic oscillation are those with impact parameter of \( \cong d_t \), and the change to the epicyclic amplitude in such an encounter is \( \delta r \sim m_b r^3/(M_t d^2) \sim d_t \), which is same as the expression in (8). These encounters occur on a time interval of \( \Omega^{-1}(d_b/d_t)^2 \sim t_{cor} \), and so long as the cloud size \( (l_b) \) is not much smaller than \( d_b \) the collision time is also of the same order i.e. clouds undergo physical collision on the average after having undergone one strong gravitational encounter, and so the velocity dispersion is \( \sim \Omega \delta r \), which is same as equation (9).

We have neglected the drag force of the interclump medium in our calculation which is justified because the turbulent ram force is small compared to the gravitational force so long as the density of the inter-clump medium is less than \( \bar{\sigma} \). Or in other words the drag force is small when the density contrast between clump and the interclump medium is a factor of two or greater.

The cloud collisions are highly dissipative. We assume that the kinetic energy of relative motion of colliding clouds is dissipated and their orbit is circularized. Moreover, colliding clumps coalesce so long as their size is not so large that rotation prevents the merger. Clouds are likely to fragment when their size exceed the maximum length for gravitational instability, \( l_{max} \sim (r M_r)/M_t \), preventing them from continuing to grow in size when they undergo collision. In any case there is empirical evidence that gas in our galaxy, and in the disk of mega-maser galaxies, is clumpy (see §3). The case considered below is when collision between clumps are frequent so that the amplitudes of their epicyclic excursion do not grow to be too large.

§2.2.1 Accretion rate

The characteristic time for a clump to fall to the center can be estimated using equation (8) and is given by

\[
t_r \approx \Omega^{-1} \left( \frac{d_b}{r} \right)^2 \left( \frac{M_t}{m_b} \right)^{4/3},
\]

(10)

and the average mass accretion rate is

\[
\dot{M} \approx \Omega(r) M_r \left( \frac{M_r}{\pi M_t} \right)^{4/3} \left( \frac{d_b}{r} \right)^{2/3}.
\]

(11)

As pointed out earlier, this result applies so long as the density contrast between the clump and the inter-clump medium is about a factor of two or larger, and \( l_b \) is of order \( d_b \).
For \( l_b \ll d_b \), clouds undergo several strong gravitational encounters with other clouds with impact parameter \( \sim d_t \), before undergoing a physical collision, and in this time duration, \( t_{\text{coll}} \sim \Omega^{-1} d_b^2 / (d_t^{3/2} l_b^{1/2}) \), their epicyclic velocity amplitude, or the velocity dispersion, becomes

\[
\delta v_r \sim \Omega d_t \left( \frac{d_t}{l_b} \right)^{1/2} \sim (r \Omega) \frac{d_b}{(r l_b)^{1/2}} \left( \frac{M_r}{\pi M_t} \right)^{1/2},
\]

and the resulting mass accretion rate is

\[
\dot{M} \sim (\Omega M_r) \frac{d_b}{(r l_b)^{1/2}} \left( \frac{M_r}{\pi M_t} \right)^{3/2}.
\]

Since the collision time in this case is much greater than \( \Omega^{-1} \), the effective viscosity is suppressed by a factor of \( (\Omega t_{\text{coll}})^2 \) (see Goldreich and Tremaine 1978, for a discussion). The physical derivation given above automatically included this factor. Equations (12) & (13) are inapplicable in the limit \( l_b \to 0 \), when the collision time becomes very long and epicyclic amplitudes saturate at a value so that the Toore Q-parameter is about 1.

For \( l_b \sim d_b \sim l_{\text{max}} \), the accretion rate in both of the cases considered above reduces to

\[
\dot{M} \approx \Omega(r) M_r \left( \frac{M_r}{\pi M_t} \right)^2,
\]

and the cloud velocity dispersion is given by

\[
v \approx r \Omega \left( \frac{M_r}{\pi M_t} \right).
\]

Note that the relative velocity of collision between clouds is smaller than the orbital speed by a factor of the ratio of the total mass to the molecular mass, and for \( M_r \ll M_t \) collision speeds are marginally supersonic. The cooling time for gas depends on the density, composition and temperature, and it must be short or comparable to the time between collisions in order for the system to be in steady state; a specific case is discussed in §3.2.

We see from equation (10) that for \( d_b \sim l_{\text{max}} \) the effective kinematic viscosity is approximately \( l_{\text{max}}^2 \Omega \sim Q^{-2} H_z^2 \Omega \), where \( Q \sim l_{\text{max}} / H_z \) is the Toomre Q-parameter, \( H_z \sim v/\Omega \). This effective viscosity is same as in the ansatz suggested by Lin & Pringle (1987) for self-gravitating-disk. We also note that the effective \( Q \) of the disk, for the velocity dispersion given in equation (15), is of order unity, and thus our solution is internally self consistent.

3. Some Applications

In this section we apply the results of §2 to molecular clouds within the inner kpc of our galaxy (§3.1), molecular disks in mega-maser galaxies (§3.2), and the evolution
of AGNs (§3.3). In all these cases, we estimate an accretion rate and compare it with previously proposed specific model of the system.

§3.1 Molecular clouds in the Galaxy

As a final example we consider molecular clouds within the inner kilo-parsec of our own galaxy. The amount of gas within the inner kiloparsec of our galactic center is \( \sim 10^8 \, M_\odot \). From the molecular and HI line emissions we know that the gas distribution is very lumpy and much of it is contained in molecular clouds of radius about 10 pc. The total stellar mass within the inner kiloparsec is about \( 2 \times 10^{10} \, M_\odot \). Thus using equation (14) we estimate the accretion rate to be \( \sim 10^{-5} \, M_\odot \, \text{yr}^{-1} \) which is consistent with the value of \( 10^{-5} \alpha/\text{year} \) obtained by Narayan et al. (1995) from a fit to the observed spectrum of Sagitarrius A*, from radio to hard x-ray wavelengths, in their advection dominated flow model.

§3.2 Mega-maser galaxies

Water maser emission has been detected from about a dozen galaxies, of which NGC 4258 is perhaps the best studied case (Braatz, Wilson & Henkel, 1996). The maser spots in NGC 4258 fall almost along a straight line in the sky from which it is inferred that these sources lie in a thin disk the width of which is observationally estimated to be less than about\(^2\) 0.003 pc, and the radii of the inner and the outer edges of the disk are 0.13pc and 0.24pc respectively. From the observed acceleration of masing spots the central mass is inferred to be \( 3.6 \times 10^7 \, M_\odot \) (Mayoshi et al. 1995, Greenhill et al. 1996). The dynamical range of observation for NGC 4258 is about \( 10^3 \) and the maser amplification factor is estimated to be \( \sim 10^4 \) (cf. Greenhill et al. 1995, Herrnstein et al. 1997), from which we infer that the density contrast between the maser spots and the mean disk is about a factor of three.\(^3\) The linear drift in the observed velocities of the systemic spots in NGC 4258, over a period of almost 10 years, is direct evidence that the masing spots are discrete clumps which maintain their identity as they move on circular orbits (Haschick et al. 1994, Greenhill et al. 1995, Miyoshi et al. 1995). The density of the clouds must be at least \( 1.5 \times 10^{10} \, \text{cm}^{-3} \) in order that the masing spots are not disrupted by the tidal field of the central mass. This is less than the maximum density of \( 10^{11} \, \text{cm}^{-3} \) at which the upper energy state of H\(_2\)O is collisionally de-excited and the maser amplification is quenched (cf. Reid & Moran 1988, Greenhill et al. 1995). The total molecular mass of the disk, using

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\(^2\) The disk thickness is expected to be \( \sim 0.002 \, \text{pc} \) if the molecular gas at a temperature of \( 10^3 \, \text{K} \) is in vertical hydrostatic equilibrium.

\(^3\) There is considerable uncertainty in estimating the density contrast from maser emission because the amplification factor for the extragalactic masers, if unsaturated, is highly uncertain (the density contrast would be \( \sim 2 \) if the gain factor for NGC 4258 were to be \( 10^8 \)) and because the observed maser flux depends on the path length through the disk over which the line of sight velocity is nearly constant. A density contrast of 2 to 3 is obtained by assuming that the effective path length does not vary rapidly across a maser spot, which is reasonable, but for which we do not have independent observational support. However, we note that if the fluid velocity were to vary on the length scale of a spot size then it will necessarily lead to significant density inhomogeneity.
the above parameters, is thus inferred to be about $2 \times 10^5 \, M_\odot$. The size of the maser spots, $\sim 10^{-3} \, \text{pc}$, is approximately the maximum allowed size for gravitational instability ($l_{\text{max}}$), which is consistent with the suggestion made in the last section that clumps grow in size when they undergo collision until fragmentation limits their size to $\sim l_{\text{max}}$.

Using equation (15), we find that the mean random velocity of clumps in NGC 4258 molecular disk should be $\sim 2 \, \text{km s}^{-1}$, which is consistent with the scatter in the observed velocity after the best fitting Keplerian velocity has been subtracted from the data (Mayoshi et al. 1995). Cloud collisions at this speed are barely supersonic (the gas temperature is expected to be between 500 and $10^3 \, \text{K}$) and molecules are not dissociated, nor do we expect much density enhancement in such a collision. The time between collisions can be shown to be about $(d_b/l_{\text{max}})^{2/3}$ times an orbital period provided that $l_b \ll l_{\text{max}}$. Thus we expect a cloud to undergo a collision once every few orbits; these collisions last for about $\Omega^{-1}(l_b M_t/r M_\ast) \sim$ a few orbits (here $l_b$ is the cloud size). Colliding clouds lose energy via rotational and vibration transitions of several molecules of which H$_2$O and CO are particularly important for the NGC 4258 disk. Using the cooling functions computed by Neufeld and Kaufman (1993) we find that the H$_2$O rotation transitions dominate the cooling at temperature of $\sim 10^3 \, \text{K}$ and H$_2$ number density of $\sim 10^{10} \, \text{cm}^{-3}$, inspite of the fact that these lines are optically thick, and the resulting cooling time for colliding clouds is a few tens of years which is smaller than the time between collisions. The average energy loss rate resulting from cloud collision in NGC 4258 disk is $\sim 10^{38} \, \text{erg s}^{-1}$.

We estimate the mass accretion rate for NGC 4258 using equation (14) and find it to be $1.5 \times 10^{-3} \, M_\odot$ per year. If the entire gravitational potential energy release associated with this mass accretion rate is radiated away then the resulting bolometric luminosity should be about $2 \times 10^{43} \, \text{erg s}^{-1}$, which is much greater than the observed x-ray luminosity in 2-10 kev band of $2 \times 10^{40} \, \text{erg s}^{-1}$ and the upper limit to the optical luminosity of $10^{42} \, \text{erg s}^{-1}$. However, the observed luminosity is consistent with the accretion rate estimated here if most of the gravitational energy release of the infalling gas is advected into the black hole as suggested by Lasota et al. (1996). For a general discussion of the advection dominated accretion flow model, see Narayan and Yi (1994 & 1995).

The H$_2$O maser emission from NGC 1068 is confined to a disk between radii 0.65 pc and 1.1 pc, and the mass of the central blackhole is estimated to be about $1.5 \times 10^7 \, M_\odot$ (Greenhill & Gwinn, 1997); the uncertainty in these parameters, however, is larger than for the NGC 4258 system. If the density of the molecular gas in the disk is similar to that of NGC 4258, then the disk mass should be about $10^6 \, M_\odot$ and the peculiar velocity of clumps, determined from equation (15), comes out to be $\sim 10 \, \text{km s}^{-1}$ which is close to the scatter observed in the velocity of the maser knots. Equation (14) yields an accretion rate of $0.05 \, M_\odot \, \text{yr}^{-1}$, which is consistent with the observed total luminosity of $\sim 4 \times 10^{44} \, \text{erg s}^{-1}$ (Pier et al. 1994) provided that the gravitational energy release of the gas is efficiently radiated away.

§3.3 Activity in galactic centers

The nuclear activity in the centers of galaxies (AGNs) is believed to be caused by gas accretion onto a massive central black hole. There is some evidence that star formation and nuclear activity in the inner regions of galaxies can be triggered or enhanced by
the presence of a close companion. Close tidal interactions can remove sufficient angular momentum and enable gas to fall from the outer parts of the galaxy to within about 1 kpc of the center. However, tidal interactions are ineffective at smaller distances. If the gas within this region is cold and clumpy, as for instance is the case in our own galaxy and some quasar hosts, then, as discussed in §2, gravitational interaction among clumps cause some fraction of the gas to fall inward. Since the accretion rate due to this process scales as \( \Omega M_c(M_c/M_t)^2 \) (where \( \Omega \) is the angular velocity at \( \sim \) kpc, and \( M_c \) & \( M_t \) are the molecular and the total mass contained within a kpc), the accretion decreases rapidly as the gas is depleted provided that the standard turbulent viscosity is unimportant at \( \sim \) kpc scale. As the fractional gas mass decreases with time (part of the gas falls into the blackhole and some fraction is converted into stars), the accretion rate falls significantly below the Eddington value and in the advection dominated regime where most of the gravitational energy release is carried with the gas inside the horizon (Narayan and Yi, 1995) causing the luminosity of AGNs to evolve more rapidly than the cubic dependence of accretion rate on disk mass. Assuming that the star formation efficiency in the central kpc of active galaxies is same as the giant molecular clouds in our galaxy, we find the time scale for quasar luminosity evolution to be about \( 5 \times 10^8 \) yrs.

The rapid evolution of the AGN luminosity function with redshift could perhaps be a consequence of the accretion dominated by the gravitational interaction between clumps. Since the accretion rate, due to this process, falls off as inverse square of the blackhole mass we expect quasars with more massive black holes to have a shorter activity life, provided that gas is converted into stars at a fixed rate and therefore more of the gas mass is used up in stars instead of fueling the central activity.

4. Conclusion

We have calculated gas accretion, in a cold clumpy molecular disk, due to gravitational interaction among the clumps which causes fluctuating torque on clouds. Molecular clumps, unlike stellar systems, undergo frequent highly inelastic collisions which damps their epicyclic oscillation and causes a net inward flow of gas. We find that the accretion rate due to gravitational interaction among clumps is \( \sim \Omega M_{mc}(M_{mc}/\pi M_t)^2 \) when clumps have size \( \sim l_{max} \); here \( l_{max} \) is the maximum length for gravitational instability in a shearing disk, \( M_{mc} \) is the gas mass of the disk and \( M_t \) is the sum of the central and disk mass.

We have applied this result to water mega-maser galaxies, and find that the accretion rate for NGC 4258 should be about \( 1.5 \times 10^{-3} \) M\( \odot \) yr\(^{-1} \) which supports the advection dominated accretion flow model for this system suggested by Lasota et al. (1996) i.e. most of the gravitational energy release of the accreting gas is advected with the flow instead of being radiated away. Maoz and McKee (1997) have arrived at a similar accretion rate in their model of NGC 4258 by assuming spiral shock in the circumnuclear disk.

The total mass in molecular clouds within the central kpc of our galaxy is \( 10^8 \) M\( \odot \). We find that the gravitational interaction among these molecular clouds gives rise to an

\[^{4}\text{Recent CO observation of the cloverleaf quasar at } z=2.56 \text{ suggests that there is about } 4 \times 10^9 \text{ M}\odot \text{ of molecular gas within 800 pc of the center (Barvainis 1997).}\]
accretion rate of $\sim 10^{-5} \, M_\odot \, \text{yr}^{-1}$, which is consistent with the value obtained by Narayan et al. (1995) from a fit to the spectrum of Sagitarrius A* in their advection dominated model.

If the accretion rate in AGNs were to be set by the gravitational viscosity discussed here (at a distance of about 1 kpc from the center), then their luminosity would undergo vary rapid evolution, as observed, since the accretion rate is proportional to $M_{mc}^3$.

We have assumed in this paper that the disk is not subject to large scale dipole or other low order harmonic instability that can otherwise efficiently redistribute angular momentum and lead to accretion. Moreover, we have assumed that cloud size does not grow monotonically when they undergo collision; clouds fragment when their size exceeds $l_{max}$. Thus a combination of merger and fragmentation could maintain a steady state distribution of molecular clump size. We have also neglected pressure forces and other hydrodynamical effects of cloud interaction. A fully self consistent calculation, without these assumptions, is desirable and perhaps will have to be attempted using a numerical hydrodynamical simulation of molecular disks.

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