Cosmological parameter fittings with the BICEP2 data

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Combining the latest Planck, Wilkinson Microwave Anisotropy Probe (WMAP), and baryon acoustic oscillation (BAO) data, we exploit the recent cosmic microwave background (CMB) B-mode power spectra data released by the BICEP2 collaboration to constrain the cosmological parameters of the ΛCDM model, especially the primordial power spectra parameters of the scalar and the tensor modes, n_s, α_s, r, n_t. We obtain constraints on the parameters for a lensed ΛCDM model using the Markov Chain Monte Carlo (MCMC) technique, the marginalized 68% bounds are 

r = 0.0307^{+0.0914}_{-0.1043},

n_s = 0.961^{+0.0061}_{-0.0061},

α_s = -0.017^{+0.0097}_{-0.0097},

n_t = 0.5198^{+0.0133}_{-0.0173}.

We find that a blue tilt for n_t is favored slightly, but it is still well consistent with flat or even red tilt. Our r value is slightly smaller than the one obtained by the BICEP group, in that we permit n_t as a free parameter without imposing the single-field slow roll inflation consistency relation. When we impose this relation, then 

r = 0.0446^{+0.0609}_{-0.2130}.

For most other parameters, the best fit values and measurement errors are not altered significantly by the introduction of the BICEP2 data.

BICEP2, B-mode, inflation

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1 Introduction

The BICEP2 experiment [1,2], dedicated to the observation of the cosmic microwave background (CMB) B-mode polarization, recently published the detection of the primordial B-mode polarization, from observations of about 380 square degrees low-foreground area of sky in the South Pole from the period of 2010–2012. The detected B-mode power is in the multipole range 30<ℓ<150, a clear excess over the base lensed-ΛCDM model at these small ℓs, this excess can not be explained by the lensing contribution, for the CMB lensing contribution to B-mode polarization peaks at ℓ~1000, while the contributed power at ℓ~100, is small. The BICEP group has also examined possible systematic error and potential foreground contaminations and excluded these as possible source of the observed B-mode power. The cross-correlation between frequency bands shows little change in the observed amplitude, implying that frequency-dependent foreground are not the dominant contributor. The presence of the B-modes induced by the primordial gravitational wave in the early universe provides a direct evidence for the inflation theory.

The tensor mode contribution to the CMB anisotropy may affect the global fitting of the cosmological parameters. The BICEP group reported their measured value of tensor-to-scalar ratio as 

r = 0.20^{+0.07}_{-0.07},

based on the lensed-ΛCDM+tensor model, and derived from importance sampling of the Planck MCMC chain using the direct likelihood
method, but did not report the constraints on other parameters. The unexpectedly large tensor-to-scalar ratio inspires a lot of interests on re-examining the inflation models [3–8] and observation datasets [9–11].

In this paper, we use the newly published BICEP2 CMB B-mode data, combined with the Planck CMB temperature data [12], the WMAP 9 year CMB polarization data [13,14], and the BAO data from the SDSS DR9 [15], SDSS DR7 [16], 6dF [17], to constrain the cosmological parameters in the lensed $\Lambda$CDM model. We derive constraints on the lensed $\Lambda$CDM model using the publicly available code COSMOMC [18], which implements a Metropolis-Hastings algorithm to perform a MCMC simulation in order to fit the cosmological parameters. This method also provides reliable error estimates on the measured variables.

Previous CMB observations from the Planck satellite, the WMAP satellite and other CMB experiments yielded a limit of much smaller tensor-to-scalar ratio $r<0.11$ (at 95% C.L.) [12], so there appears some tension between these and the BICEP result at least in the simplest lensed $\Lambda$CDM+tensors model. As pointed out by the BICEP group [1], a simple method to relax this tension is to take the running of spectral index into account, herein we will explore this possibility in our fit. There are also wide spread interests in the tensor power spectral index, as it is an important additional source of information for distinguishing inflation models [19–21], and a blue tensor power spectral index, herein we will explore this possibility in our fit. There are also wide spread interests in the tensor power spectral index, as it is an important additional source of information for distinguishing inflation models [19–21], and a blue tensor power spectrum tilt $n_s-2$ have been reported using the B-mode measurement [22]. Here we shall also investigate this problem and obtain an estimate of $n_s$ and its measurement error.

2 Fitting of cosmological parameters

We explore the cosmological parameter space and obtain limits on cosmological parameters by using the MCMC technique with the CosmoMC code [18]. In our simulation we collected about 500000 chain samples, the first 1/3 of the data was used for burning in the chains and not used for the final analysis. In addition of the BICEP data [1], we use the Planck CMB temperature data [12], the WMAP 9 year CMB polarization data [13,14], and the BAO data from the SDSS DR9 [15], SDSS DR7 [16], 6dF [17] in our cosmological parameter fitting. Below, we use the following labels to denote the different data sets included in the fitting:

- Planck + WP: Planck high $\ell$, low $\ell$ [12], and WMAP9 polarization data [13, 14].
- Planck + WP + BAO: add BAO data from SDSS DR9 [15], SDSS DR7 [16], 6dF [17].
- Planck + WP + BAO + BICEP: add BICEP data [1,2].

As noted by the BICEP group, including the Planck data, the running of spectrum tilt $\alpha_s$ is needed. We shall consider a $\Lambda$CDM model, and assume that the scalar perturbations are purely adiabatic, and the scalar and tensor mode power spectra are parameterized by

$$P_s(k) = A_s \left(\frac{k}{k_0}\right)^n, \quad P_t(k) = A_t \left(\frac{k}{k_0}\right)^{n_t} + \frac{1}{2} \ln \frac{k}{k_0},$$

(1)

$$P_s(k) = A_s \left(\frac{k}{k_0}\right)^n, \quad P_t(k) = A_t \left(\frac{k}{k_0}\right)^{n_t},$$

(2)

where $k_0 = 0.05 \text{ Mpc}^{-1}$, is the pivot scale, roughly in the middle of the logarithmic range of scales probed by the WMAP and Planck experiments. The parameter $\alpha_s$ denotes the running of the scalar spectral tilt [23] with $\alpha_s = d\alpha_s/d\ln k$. The primordial tensor-to-scalar ratio is defined by $r = A_t/A_s$ at a chosen pivot scale, for example $r_{0.05}$ is defined at $k_0=0.05 \text{ Mpc}^{-1}$, and $r_{0.02}$ at $k_0=0.002 \text{ Mpc}^{-1}$. The relation between $r_{0.05}$ and $r_{0.02}$ could be inferred from eqs. (1) and (2) as thus:

$$r_{0.02} = r_{0.05} \frac{0.04^+}{0.004^+} \frac{0.04^+}{0.04^+}.$$

(3)

Throughout this paper, $r$ without the subscript (as in our plots) represents $r_{0.02}$. In the Planck Collaboration paper XVI (2013) [12], $n_s$ is assumed to be close to zero, and satisfy a single field slow roll inflation consistency relation:

$$n_s = \frac{r}{8}$$

(4)

Note that in eq. (4), $n_s$ and $r$ should be defined at the same pivot scale. The BICEP group adopted the same assumption, and applied the importance sampling method on the Planck MCMC chains [12] with the addition of the B-mode data to obtain constraints on $r$ and $n_s$ [1]. Here we study the more general case, with $n_s$ and $r$ treated as independent parameters. We fix the the number of neutrinos as $N_{\nu}=3.046$, and the sum of neutrino masses as the Planck best fit $\Sigma_m=0.06$ eV. The lensing amplitude parameter $A_L$ is fixed to 1, and we put flat priors on all fitting parameters.

The constraints on the primordial power spectrum parameters are shown in Figure 1, and constraints on other parameters in Figure 2. The marginalized 68% bounds on the parameters based on different datasets are listed in Table 1. For the scalar spectral index and running, using the Planck + WP + BAO + BICEP dataset, we obtain

$$n_s = 0.9617 \pm 0.0061, \quad \alpha_s = -0.0175_{-0.0097}^{+0.0105}$$

while for the Planck + WP + BAO data:

$$n_s = 0.9616 \pm 0.0061, \quad \alpha_s = -0.0148_{-0.0065}^{+0.0108}$$

(Table 1), the best-fit value of $\alpha_s$ is 0.027 smaller after including the BICEP2 data. The decrease in $\alpha_s$ reduces the $T\bar{T}$ angular power at small $\ell$ (Figure 3), and helps alleviate the tension between the high $r$ value obtained with BICEP
Figure 1  Joint constraints on primordial power spectrum parameters.

data and the limit derived from the large scale TT auto-correlation power from Planck experiment. The effect of decreasing $\alpha_s$ on the other power spectra is that it could lower the matter power spectrum at enormously large scale and small scales. If future galaxy surveys can probe the matter power spectrum at extremely large scales, the constraint on $\alpha_s$ can be further improved.

For the tensor-to-scalar ratio $r$, the BICEP group reported a value of $r = 0.20^{0.07}_{0.05}$ based on their fit to the CMB power spectrum, with the consistency relation eq. (4). Using the Planck + WP + BAO + BICEP dataset, we obtain $r = 0.0307^{0.0914}_{0.1043}$, where we have taken $n_t$ and $r$ as independent free parameters. If we fix $n_t$ to the single-field inflation consistency relation value as the Planck and BICEP2 group did, we will obtain a higher value of $r$ and can also place a tighter constraint on $r$. We compare the two dimensional likelihood contour of $r$ vs $n_t$ with and without single-field inflation consistency relation in Figure 4. Blue dashed curves in Figure 4 show the result if we impose the single-field slow roll inflation consistency relation $n_t = -r/8$ in the fitting, with $r = 0.2130^{0.0466}_{0.0466}$ (1$\sigma$ error bar). Note that with the consistency relation, the $r$ value is significantly higher than the one without.

For the tensor spectrum index $n_t$, using the Planck + WP + BAO + BICEP dataset and without consistency relation, we obtain $n_t = 0.4515^{0.4579}_{0.5198}$. This result shows that a blue tilt is slightly favored, but a flat or even red tilt is still consistent with the current data.

Thus, without imposing the consistency relation, the best fitting value of the $r_{0.002}$ and $n_t$ we obtain are $r \sim 0.1$ and $n_t \sim -0.52$, while the BICEP group reported $r \sim 0.2$ and $n_t \sim -0.024$ (fixed by single-field slow-roll inflation consistency
Figure 2  Constraints on other cosmological parameters.

Figure 3  Comparison of CMB and matter power spectra for different running index $\alpha$.

Figure 4  Comparison of the two dimensional likelihood contour obtained by using and without using single-field inflation consistency relation. The dataset used is Planck + WP + BAO + BICEP. Relation prior) [1]. We plot the CMB power spectra and matter power spectra according to these two fits in Figure 5. The red curves and blue curves overlap each other for most
Figure 5  Comparison of power spectra results predicted by two $n_t$ and $r$ parameter sets. One set is our best fitting value(red curves), another is from BICEP2 paper [1] (blue dotted curve).

$\ell$-values, the primary difference is at the very large scales ($\ell<15$), where the statistics are poor, and it is difficult to distinguish the two cases with the current the current observational data.

Constraints on the other cosmological parameters are shown in Figure 2 and Table 1. By combining the BAO data, we obtain tighter constraints on all parameters, and help break the parameter degeneracy. Nevertheless, there is not significant difference for these parameters after the addition of the BICEP data.

3  Conclusion

In this paper, we use the newly published BICEP2 CMB B-mode data, Planck CMB temperature data [12], the WMAP 9 year CMB polarization data [13,14] to constrain the base lensed $\Lambda$CDM model. In addition to the CMB data, we also use the BAO data from SDSS DR9 [15], SDSS DR7 [16], 6dF [17], which help to break parameter degeneracy.

For most parameters, we find that the best fit values and measurement errors are not altered much by the introduction of the BICEP2 data. The most affected parameters are $r$, $\alpha_s$ and $n_t$. Combining Planck + WP + BICEP+ BAO dataset, we obtain marginalized 68% bounds on some interested parameters such as:

$$r = 0.1043^{+0.0307}_{-0.0914}.$$  \hspace{1cm} (5)

$$n_s = 0.9617^{+0.0061}_{-0.0061}.$$  \hspace{1cm} (6)

$$\alpha_s = -0.0175^{+0.0105}_{-0.0097}.$$  \hspace{1cm} (7)

We find that a blue tensor tilt ($n_t>0$) is slightly favored, but a flat or red tilt is consistent with the data. The best fitting value of $r$ we obtain is slightly smaller than BICEP2 group obtained, and the constraint on $r$ is also looser than BICEP2 group obtained. This result is reasonable, as we have not imposed the single-field-slow-roll inflation con-

| Parameter | Planck + WP + BAO + BICEP | Planck + WP + BAO | Planck + WP |
|-----------|---------------------------|-------------------|-------------|
| $n_s$     | 0.9618 ± 0.0080           | 0.9616 ± 0.0085   | 0.9614 ± 0.0082 |
| $r_{0.002}$ | 0.0634 ± 0.0070           | 0.0015 ± 0.0050   | 0.0017 ± 0.0050 |
| $\alpha_s$ | -0.0080 ± 0.0175          | -0.0129 ± 0.0148  | -0.0090 ± 0.0150 |
| $n_t$     | 0.7293 ± 0.0189           | 0.6230 ± 0.0184   | 0.6920 ± 0.0184 |
| $\Omega_m$ | 0.3015 ± 0.0362           | 0.3080 ± 0.0362   | 0.3095 ± 0.0362 |
| $\Omega_b$ | 0.6965 ± 0.0974           | 0.6920 ± 0.0974   | 0.6920 ± 0.0974 |
| $\sigma_8$ | 0.8173 ± 0.0898           | 0.8218 ± 0.0898   | 0.8303 ± 0.0898 |
| $H_0$     | 68.2990 ± 0.4320          | 68.2599 ± 0.4320  | 68.2224 ± 0.4320 |
| $100\theta_{MC}$ | 1.0417 ± 0.0006 | 1.0415 ± 0.0006 | 1.0415 ± 0.0006 |
consistency relation on $n_s$, and treated it as an independent parameter. If we impose this relation, we will obtain $r = 0.2130^{+0.0609}_{-0.0446}$ ($1 \sigma$ error) instead.

In the near future, Planck and other experiments will provide more data on CMB polarization, and help improve the constraint on these parameters.

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1 BICEP2 Collaboration, Ade P, et al. BICEP2 I: Detection of B-mode polarization at degree angular scales. arXiv:1403.3985 [astro-ph.CO]
2 BICEP2 Collaboration, Ade P A R, et al. BICEP2 II: Experiment and three-year data set. arXiv:1403.4302 [astro-ph.CO]
3 Hertzberg M P. Inflation, symmetry, and B-modes. arXiv:1403.5253 [hep-th]
4 Choudhury S, Mazumdar A. Reconstructing inflationary potential from BICEP2 and running of tensor modes. arXiv:1403.5549 [hep-th]
5 Ma Y Z, Wang Y. Reconstructing the local potential of inflation with BICEP2 data. arXiv:1403.4585 [astro-ph.CO]
6 Gong Y. The challenge for single field inflation with BICEP2 result. arXiv:1403.5716 [gr-qc]
7 Xia J Q, Cai Y F, Li H, et al. Evidence for bouncing evolution before inflation after BICEP2. arXiv:1403.7623 [astro-ph.CO]
8 Cai Y F, Gong J O, Pt S. Conformal description of inflation and primordial B-modes. arXiv:1404.2560 [hep-th]
9 Zhao W, Cheng C, Huang Q G. Hint of relic gravitational waves in the Planck and WMAP data. arXiv:1403.3919 [astro-ph.CO]
10 Zhao W, Grishchuk L. Relic gravitational waves: latest revisions and preparations for new data. Phys Rev D, 2010, 82: 123008
11 Zhang J F, Li Y H, Zhang X. Sterile neutrinos help reconcile the observational results of primordial gravitational waves from Planck and BICEP2. arXiv:1403.7028 [astro-ph.CO]
12 Planck Collaboration, Ade P, et al. Planck 2013 results. XVI. Cosmological parameters. arXiv:1303.5076 [astro-ph.CO]
13 Hinshaw G, Larson D, Komatsu E, et al. Nine-year Wilkinson microwave anisotropy probe (WMAP) observations: cosmological parameter results. Astrophys J Suppl Ser, 2013, 208(2): 19
14 Bennett C L, Larson D, Weiland J L, et al. Nine-year Wilkinson microwave anisotropy probe (WMAP) observations: final maps and results. Astrophys J Suppl Ser, 2013, 208(2): 20
15 Anderson L, Aubourg E, Bailey S, et al. The clustering of galaxies in the SDSS-III baryon oscillation spectroscopic survey: Baryon acoustic oscillations in the Data Release 9 spectroscopic galaxy sample. Mon Not R Astron Soc, 2013, 427(4): 3435–3467
16 Padmanabhan N, Xu X, Eisenstein D J, et al. A 2 percent distance to $z = 0.35$ by reconstructing baryon acoustic oscillations—I. Methods and application to the Sloan Digital Sky Survey. Mon Not R Astron Soc, 2012, 427(3): 2132–2145
17 Beutler F, Blake C, Colless M, et al. The 6dF Galaxy Survey: baryon acoustic oscillations and the local Hubble constant. Mon Not R Astron Soc, 2011, 416(4): 3017–3032
18 Lewis A, Bridle S. Cosmological parameters from cmb and other data: a monte-carlo approach. Phys Rev, 2002: 103511
19 Abazajian K N, Aslanyan G, Easther R, et al. The Knotted Sky II: Does BICEP2 require a nontrivial primordial power spectrum? arXiv:1403.5922 [astro-ph.CO]
20 Gong J O. Non-trivial running of the primordial tensor spectrum. arXiv:1403.5163 [astro-ph.CO]
21 Brandenberger R H, Nayeri A, Patil S P. Closed string thermodynamics and a blue tensor spectrum. arXiv:1403.4927 [astro-ph.CO]
22 Gerbino M, Marchini A, Pagano L, et al. Blue gravity waves from BICEP2? arXiv:1403.5732 [astro-ph.CO]
23 Kosowsky A. Turner M S. Cmb anisotropy and the running of the scalar spectral index. Phys Rev D, 1995, 52: 1739. arXiv:astro-ph/9504071