Fidelity of a sequence of SWAP operations on a spin chain

Robert E. Throckmorton and S. Das Sarma

Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742-4111 USA

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We consider the “transport” of the state of a spin across a Heisenberg-coupled spin chain via the use of repeated SWAP gates, starting with one of two states—one in which the leftmost spin is down and the others up, and one in which the leftmost two spins are in a singlet state (i.e., they are entangled), and the others are again all up. More specifically, we “transport” the state of the leftmost spin in the first case and the next-to-leftmost spin in the second to the other end of the chain, and then back again. We accomplish our SWAP operations here by increasing the exchange coupling between the two spins that we operate on from a base value \( J \) to a larger value \( J_{SWAP} \) for a time \( t = \pi \hbar/4J_{SWAP} \). We determine the fidelity of this sequence of operations in a number of situations—one in which only nearest-neighbor coupling exists between spins and there is no magnetic dipole-dipole coupling or noise (the most ideal case), one in which we introduce next-nearest-neighbor coupling, but none of the other effects, and one in which all of these effects are present. In the last case, the noise is assumed to be quasistatic, i.e., the exchange couplings are each drawn from a Gaussian distribution, truncated to only nonnegative values. We plot the fidelity as a function of \( J_{SWAP} \) to illustrate various effects, namely crosstalk due to coupling to other spins, as well as noise, that are detrimental to our ability to perform a SWAP operation. Our theory should be useful to the ongoing experimental efforts in building semiconductor-based spin computer architectures.

I. INTRODUCTION

The ability to transfer the state of one qubit to another is critical in quantum computation. A key part of this transfer of qubit states is the ability to perform a SWAP gate, even just between two neighboring qubits. Aside from being useful on its own for this purpose, it is also useful for performing quantum teleportation, as it would allow one to transfer entanglement from one qubit to another. A number of experimental groups have demonstrated the ability to perform such gates in electron spin-based quantum dot systems. One experiment\(^1\) has demonstrated the ability to perform a SWAP gate with a fidelity of around 90%, while another\(^2\) has implemented SWAP gates for spin eigenstates (i.e., it can only swap purely up or purely down spins) with a fidelity of 98% and for arbitrary states with a fidelity of 84%. Both of these experiments were performed on semiconductor-based electron spin qubits. In addition, there has been experimental and theoretical interest in two-qubit gates in general\(^3\) due to the fact that the ability to perform a two-qubit entangling gate is essential to universal quantum computation. The current theoretical work is on solid state spin qubits in semiconductor-based scalable platforms, where the exchange coupling between neighboring localized electrons is typically used to carry out SWAP operations. The main physics we theoretically address here is the fidelity of SWAP-induced spin transport through a sequence of qubits in the presence of non-ideal effects invariably present in real physical systems.

An essential part of the development of a quantum computer is the ability to perform any gate with a fidelity of at least 99.9% (and much higher), which is the threshold above which error-correcting codes may be implemented. We are therefore interested in characterizing the fidelity of SWAP operations in a model system. We will consider here a chain of electronic spins with Heisenberg exchange coupling. We will consider three cases—one in which there is only nearest-neighbor exchange coupling and no noise, one in which we add next-nearest-neighbor exchange (but no noise) as well, and finally one in which we also add in the magnetic dipole-dipole interaction (even though it would be very weak in actual experimental systems) and noise in the exchange couplings.

For each case, we will consider two initial conditions, one in which the leftmost spin is initialized in the down state and the rest in the up state (i.e., \( |\psi_{0,1}\rangle = |\uparrow \uparrow \cdots \downarrow\rangle \)), and one in which the leftmost two spins are prepared in a singlet state \( |\psi_{0,2}\rangle = \frac{1}{\sqrt{2}}(|\uparrow \downarrow . . . \downarrow \rangle - |\downarrow \uparrow \cdots \uparrow\rangle) \). The rest are all prepared in the up state (i.e., \( |\psi_{0,2}\rangle = |\uparrow \rangle \otimes |\uparrow \cdots \uparrow\rangle \)). We then consider a sequence of SWAP operations, each of which is implemented as follows. We increase the nearest-neighbor coupling between the two spins that we perform this operation on from a base value \( J \) to a larger value \( J_{SWAP} \) and maintain said value for a time \( t = \pi \hbar/4J_{SWAP} \). In the most ideal case, i.e., there is no interaction of the two spins involved with other spins and no noise in the exchange coupling, this performs a perfect SWAP operation. We will assume that the next-nearest-neighbor exchange coupling of the two spins involved to other spins, if it is included, increases in proportion to the nearest-neighbor coupling. The sequence of operations is as follows. In the case in which we start in the state, \( |\psi_{0,1}\rangle \), we “move” the down spin all the way to the right, and then back again. The sequence for the case in which we start in the state, \( |\psi_{0,2}\rangle \), is similar—we “move” the state of the right spin in the entangled pair all the way to the right, and then back again. In
this latter case, the SWAP operation has the effect of transferring entanglement with the leftmost spin to spins further right on the chain. We then numerically determine the fidelity of this sequence of operations, defined as the probability that the system, when measured, will be in the state that it would be in if the intended sequence was performed without errors, as a function of $J_{\text{SWAP}}$.

In the case that we have noise in the system, we assume that the noise is quasistatic, with all exchange couplings chosen from a Gaussian distribution with mean $J_0$ and standard deviation $\sigma_j$, which we will call the strength of the noise, truncated so that all exchange couplings are nonnegative. In all cases, we will consider chains of 4 and 6 spins. Larger number of qubits ($>6$) can be easily studied using our technique, but the current experiments are restricted only to a few spins, and therefore, we restrict our work to 6 spins at most. Larger number of spins would only suppress the calculated fidelity under the same conditions.

We find that, even in the case with only nearest-neighbor coupling, there is a loss of fidelity, due to crosstalk from the neighboring spins, but the fidelity approaches unity as $J_{\text{SWAP}}$ increases due to the shorter time that this larger exchange coupling must be maintained, which gives the crosstalk-inducing terms too little time to have a significant effect on the SWAP operation. If next-nearest-neighbor coupling is included, however, then the infidelity $(1 - F$, where $F$ is the fidelity) saturates to a nonzero value, even without noise. As noted earlier, we increase the next-nearest-neighbor exchange couplings for the two spins involved in the SWAP operation in proportion to the nearest-neighbor coupling, so that there is always significant crosstalk in this case. We then consider adding the dipole-dipole interaction and quasistatic noise. We find that, for large values of $J_{\text{SWAP}}$, the fidelity decreases monotonically as the noise strength increases, though the relationship becomes inverted for some smaller values of $J_{\text{SWAP}}$. We also indicate about how much noise the system is allowed to have to achieve the values found in the known experiments, as well as to reach the 99.9% threshold.

The rest of our paper is organized as follows. We introduce the Hamiltonian for our system in Sec. II and introduce the problem that we consider in more detail. We review how to implement a SWAP gate in the system under consideration and give our results for each case that we consider in Sec. III. We then conclude in Sec. IV.

II. HAMILTONIAN

We consider here a chain of spins coupled with both nearest- and next-nearest-neighbor Heisenberg exchange couplings and a magnetic dipole-dipole interaction. The Hamiltonian describing this system is

$$ H = - \sum_{j=1}^{L-1} J_j \vec{\sigma}_j \cdot \vec{\sigma}_{j+1} - \sum_{j=1}^{L-2} J'_j \vec{\sigma}_j \cdot \vec{\sigma}_{j+2} - \frac{\mu_0 \hbar^2}{16\pi a^3} \sum_{ij} \frac{1}{|i-j|^3} (2\sigma_{i,z}\sigma_{j,z} - \sigma_{i,x}\sigma_{j,x} - \sigma_{i,y}\sigma_{j,y}), $$

(1)

where $\vec{\sigma}_j$ is the vector of Pauli matrices acting on spin $j$, $L$ is the number of spins, $a$ is the distance between two adjacent spins and $\mu_0$ is the magnetic permeability of free space. We allow each of the exchange couplings to depend on position to allow the addition of quasistatic noise (mathematically identical to disorder).

We will consider two problems. First, let us initialize the system so that the leftmost spin is down, but all other spins are up (i.e., the initial state is $|\psi_{0,1}\rangle = |\downarrow\uparrow\cdots\uparrow\rangle$). We then apply a series of SWAP gates to "move" the down spin all the way to the right, and then all the way back to the left. We will then calculate the fidelity of this sequence of moves, defined as the probability that the state that we measure the system in is that which we would obtain ideally (in this case, this would just be the initial state):

$$ F = |\langle \psi_0 | U^\dagger R | \psi_0 \rangle |^2, $$(2)

where $|\psi_0\rangle$ is the initial state, $R$ is the ideal sequence of operations (in our case, just the identity operation), and $U$ is the actual sequence of operations performed under the influence of error-causing effects. When we consider the effects of noise, we will average this fidelity over different realizations of noise and report the noise-averaged fidelity $\bar{F}$. The second problem that we will consider is similar—we initialize the system so that the leftmost two spins are in a singlet state, $|S\rangle = \frac{1}{\sqrt{3}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$, while the rest are up (i.e., the full initial state is now $|\psi_{0,2}\rangle = \frac{1}{\sqrt{3}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \otimes |\uparrow\uparrow\cdots\uparrow\rangle$), and then consider a chain of SWAP operations that "move" the state of the second spin all the way to the right, and then back again to the second spin. The effect of each of these SWAP operations in this case is to transfer the second spin’s entanglement with the first to each of the other spins to its right.

III. RESULTS

We now present the results that we obtain. Before doing so, however, we first review how we perform SWAP gates in our model in the most ideal case. We consider a system consisting of just two exchange-coupled spins (we ignore the dipole-dipole interaction),

$$ H = -J \vec{\sigma}_1 \cdot \vec{\sigma}_2. $$

(3)

If we write this in matrix form in the basis, $(|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle)$, we get

$$ H = \begin{bmatrix} J & 0 & 0 & 0 \\ 0 & -J & 2J & 0 \\ 0 & 2J & -J & 0 \\ 0 & 0 & 0 & J \end{bmatrix}. $$

(4)
Note that this is a block diagonal matrix; this allows us to use the identity for $2 \times 2$ Hermitian matrices,
\[
e^{i\vec{v} \cdot \vec{\sigma}} = \cos v \mathbb{1} + i \hat{v} \cdot \vec{\sigma} \sin v,
\] (5)
where $v = | \vec{v} |$ and $\hat{v} = \vec{v}/v$, to obtain the corresponding time evolution operator,
\[
U(t) = e^{-iHt/\hbar} = \begin{bmatrix}
e^{-i\tau} & 0 & 0 & 0 \\
e^{i\tau} \cos (2\tau) & -ie^{i\tau} \sin (2\tau) & 0 & 0 \\
0 & -ie^{i\tau} \sin (2\tau) & e^{i\tau} \cos (2\tau) & 0 \\
0 & 0 & 0 & e^{-i\tau}
\end{bmatrix},
\] (6)
where $\tau = \frac{t}{\hbar}$.

If we now let $t = \pi \hbar/4J$, this becomes
\[
U \left( \frac{\pi \hbar}{4J} \right) = e^{-i\pi/4} \begin{bmatrix}1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1\end{bmatrix} = e^{-i\pi/4} \text{SWAP}. 
\] (7)

We see that this Hamiltonian implements a SWAP operation up to an (unimportant) overall phase factor.

Note, however, that this analysis assumes just two coupled qubits—coupling to additional qubits will negatively affect the SWAP operation, as we will demonstrate shortly.

### A. Noisless case

Let us first consider the noisless case. In this case, all $J_j = J$ and $J_{j'} = J'$. We will consider in this subsection cases in which $J' = 0$ (i.e., nearest-neighbor coupling only) and $J' = 0.01J$. We also assume that the “base” value of $J$ is 150 kHz, or $10^{-5} \text{Hz}$, the value used in the simulations in Ref. 11 therefore, the largest value of $J_{\text{SWAP}}$ that we consider here is exactly the value used in this reference. This results in the dipole-dipole interaction being negligible (on the order of tens of Hz for nearest neighbors for a typical experimental system), so we will neglect it for now. When we perform a SWAP operation, we assume that the next-nearest-neighbor exchange couplings involving the two spins undergoing the SWAP increase in proportion to the nearest-neighbor coupling. As an example, if we perform a SWAP operation on spins 2 and 3, then the couplings between spins 1 and 3 and between 2 and 4 will also increase proportionally. This assumed proportionality between the nearest-neighbor ($J$) and the next-nearest neighbor ($J'$) spin-spin exchange coupling is reasonable for fixed localized spins with the proportionality constant typically being small, depending on the inter-qubit spacings in the system. If the exchange coupling falls off exponentially with interqubit spacing, which is approximately the situation for quantum dot or donor based spin qubit architectures, then $J' = 0.01J$, as assumed in our calculations, is most likely a very optimistic estimate for the next-nearest-neighbor spin coupling strength hindering the SWAP operation. If we assume exponential localization of the electronic wave function within a quantum dot, so that $J'/J \sim e^{-d/a}$, where $d$ is the distance between quantum dots and $a$ is the width of the dots, then we would find that $J' \approx 0.13J$ in the experiment of Ref. 11 resulting in a much larger next-nearest-neighbor coupling. A visual estimate of the relevant dimensions yields $d \approx 200 \text{ nm}$ and $a \approx 100 \text{ nm}$. On the other hand, if we assume that the wave function is Gaussian, so that $J'/J \sim e^{-3(d/a)^2}$, then we find that the next-nearest-neighbor exchange coupling is much smaller, $J' \approx 6.14 \times 10^{-6}J$. Therefore, we see that the magnitude of $J'$ relative to $J$ varies depending on the details of how the wave function falls off with distance. Our assumption of $J' = 0.01J$ here falls between these two values, so we expect that it is a fair estimate of the actual value.

We present our results for the case in which we start in the state, $|\psi_{01}\rangle$, in Figs. 1 and 2, for values of $J_{\text{SWAP}}/J$ from 1 to 1,000 and for chains of 4 and 6 spins. We see these that, even in the absence of noise, there is error in the overall SWAP operations, especially for smaller values of $J_{\text{SWAP}}$. This is due to crosstalk, which results from the fact that one can never completely turn off the exchange coupling between any two given spins, and thus interactions of the two spins involved in the SWAP operation with spins not involved in it will have an effect on the overall operation. We also note that, in the cases in which there is next-nearest-neighbor coupling, the infidelity appears to saturate to a finite, non-zero, value, while it appears to decline to arbitrarily small values without next-nearest-neighbor coupling, approaching zero as $J_{\text{SWAP}}$ grows. This is not surprising, as, in the nearest-neighbor-only case, a large $J_{\text{SWAP}}$ results in the other interaction terms becoming negligible compared to that between the two spins being swapped, and the shorter time over which the pulse is applied means that there is less time for crosstalk to have a significant effect on the fidelity. On the other hand, when next-nearest-neighbor coupling is present, we will never have all of the other interaction terms become negligible, and thus there will always be some noticeable crosstalk effects, hence the saturation of the infidelity to a finite, nonzero, value. We see similar effects in the case in which the leftmost two spins start in the singlet state, as we show in Figs. 3 and 4.

### B. Noisy case

We now include quasistatic noise in our system. We model this noise as follows. The “base” values of the nearest-neighbor exchange couplings $J_j$ are chosen from a Gaussian distribution $\sim e^{-(J_j-J_0)^2/2\sigma_j^2}$, truncated so that all $J_j/J_0 \in [0, \infty)$. We assume here that $\sigma_j \propto J$, an assumption used in other work on correction of noise-induced errors. In practice, we implement the linearity of $\sigma_j$ in $J$ by assuming that the actual coupling scales...
FIG. 1: Plot of infidelity $1 - F$ of “transporting” a down spin from left to right and back again as a function of $J_{SWAP}/J$ for 4 spins in the absence of noise and dipole-dipole coupling. The system is initialized in the state, $|\psi_0,1\rangle = |↓↑↑↑\rangle$. The plot on the left assumes a nearest-neighbor coupling only, i.e., $J' = 0$, while that on the right assumes a next-nearest-neighbor coupling $J' = 0.01J$.

FIG. 2: Plot of infidelity $1 - F$ of “transporting” a down spin from left to right and back again as a function of $J_{SWAP}/J$ for 6 spins in the absence of noise and dipole-dipole coupling. The system is initialized in the state, $|\psi_0,1\rangle = |↓↑↑↑↑↑\rangle$. The plot on the left assumes a nearest-neighbor coupling only, i.e., $J' = 0$, while that on the right assumes a next-nearest-neighbor coupling $J' = 0.01J$.

proportionately to the intended (i.e., without noise) coupling. For each value of $\sigma_J/J_0$, we use 10,000 realizations of noise and determine the noise-averaged fidelity. We plot the average infidelity for chains of 4 and 6 spins, with the system initialized in the state $|\psi_0,1\rangle$, Figs. 5 and 6. We also show dashed lines corresponding to the fidelities reported in the experiments of Refs. 1 and 2 as well as to 99.9% fidelity. We find that, for $\sigma_J \leq 0.2J$, the effect of noise on the fidelity is very hard to discern visually; we illustrate this in Fig. 7. We see that, at least for large values of $J_{SWAP}$, the noise-averaged infidelity increases with $\sigma_J$, as expected. However, the relationship becomes more complicated for smaller values of $J_{SWAP}$—for some smaller values of $J_{SWAP}$, the fidelity may actually slightly increase for increasing noise strength. We have also performed a similar calculation for the case in
FIG. 3: Plot of infidelity $1 - F$ of “transporting” one of two entangled spins from left to right and back again as a function of $J_{\text{SWAP}} / J$ for 4 spins in the absence of noise and dipole-dipole coupling. The system is initialized in the state, $|\psi_{0,2}\rangle = |S\rangle \otimes |\uparrow\uparrow\rangle$, where $|S\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$. The plot on the left assumes a nearest-neighbor coupling only, i.e., $J' = 0$, while that on the right assumes a next-nearest-neighbor coupling $J' = 0.01J$.

FIG. 4: Plot of infidelity $1 - F$ of “transporting” one of two entangled spins from left to right and back again as a function of $J_{\text{SWAP}} / J$ for 6 spins in the absence of noise and dipole-dipole coupling. The system is initialized in the state, $|\psi_{0,2}\rangle = |S\rangle \otimes |\uparrow\uparrow\uparrow\uparrow\rangle$, where $|S\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$. The plot on the left assumes a nearest-neighbor coupling only, i.e., $J' = 0$, while that on the right assumes a next-nearest-neighbor coupling $J' = 0.01J$.

which we start with the state $|\psi_{0,2}\rangle$, and we present the results in Figs. 8 and 9. We find similar results as in the previous case. Overall, our results imply that current experiments are unlikely to achieve the required 99.9% fidelity needed in order to implement error correction assuming the value of the next-nearest-neighbor exchange coupling that we assumed. We also find that the current experimental fidelities reported, both for four spin systems, correspond to higher noise strengths than considered here, if we assume that $J_{\text{SWAP}}$ is tuned to the highest value considered, $J_{\text{SWAP}} = 1,000J$. 
IV. CONCLUSIONS

We investigated the fidelity of a sequence of SWAP operations performed on a chain of spins. We considered three cases—one in which only nearest-neighbor exchange coupling existed among the spins, one in which we included next-nearest-neighbor coupling as well, and finally one in which we also added the magnetic dipole-dipole coupling (though it is very small) and quasistatic noise in the exchange couplings. In order to implement the SWAP gate between two neighboring spins, we increased the coupling between those spins to a larger value $J_{\text{SWAP}}$ and maintained this value for a time $t = \pi \hbar / 4J$; in the cases in which we include next-nearest-neighbor coupling, we assume that these additional couplings involving the two spins undergoing the SWAP operation increase in proportion to the nearest-neighbor coupling. We also considered two initial states for the system—a state in which the leftmost spin is down, while the rest are up (i.e., $|\psi_{0,1}\rangle = |\downarrow \uparrow \uparrow \cdots \uparrow \rangle$), and one in which the leftmost two spins are in a singlet state while the rest are up (i.e., $|\psi_{0,2}\rangle = |S\rangle \otimes |\uparrow \cdots \uparrow \rangle$, where $|S\rangle = \frac{1}{\sqrt{2}} (|\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle)$. For each, we determined the fidelity of a sequence of SWAP gates that moved the state of the leftmost spin (starting in the state $|\psi_{0,1}\rangle$) or the second spin from the left (starting in the state $|\psi_{0,2}\rangle$) all the way to the right and then back again as a function of $J_{\text{SWAP}}$ for chains of 4 and 6 spins.

We found that, even in the case in which there is just nearest-neighbor coupling, the fidelity of this sequence of operations is reduced due to crosstalk, though it tends to unity as we increase $J_{\text{SWAP}}$. If we introduce next-nearest-neighbor coupling as well, then we find that the fidelity saturates at a value less than unity, no matter how large we make $J_{\text{SWAP}}$. This is due to the fact that we assume that the next-nearest-neighbor couplings involving the spins undergoing the SWAP operation increase in proportion to the nearest-neighbor coupling, so that there is always significant crosstalk. We then added in the magnetic dipole-dipole interaction and quasistatic noise. We found that, for large values of $J_{\text{SWAP}}$, the fidelity decreased monotonically for increasing noise strength, as expected, but that it could slightly increase with increasing noise strength for some smaller values of $J_{\text{SWAP}}$. In each case, we show what level of noise we could expect in different experiments given the reported fidelities in each.

Our results imply that, while current experiments are unlikely to achieve the necessary 99.9% fidelity required for error correction techniques to be employed, it is still possible to achieve such a fidelity if noise in the exchange couplings were to be reduced. Despite this, methods for combating the effects of noise and crosstalk are still of great interest. One means by which higher fidelities may be achieved is through error-correcting pulse sequences that cancel out the effects of error-inducing terms to a given order, similar to those described in Refs. [10] and [11]. Another method is to reduce the noise in the exchange couplings, which could be achieved by reducing noise in the voltage on the gates used to define the quantum
FIG. 7: Plot of noise-averaged infidelity $1 - \bar{F}$ of “transporting” a down spin from left to right and back again as a function of $J_{\text{SWAP}}/J_0$ for 4 spins for low noise strengths, $0 \leq \sigma_J \leq 0.2J$. The solid black curve corresponds to no noise, while the solid red curve corresponds to $\sigma_J = 0.2J$. The system is initialized in the state, $|\psi_0,1\rangle = |\downarrow\uparrow\uparrow\uparrow\rangle$. We include a next-nearest-neighbor coupling $J' = 0.01J$, dipole-dipole coupling, and with noise in $J$ with strength (i.e., standard deviation) $\sigma_J$. The red dashed line corresponds to 99.9% single SWAP fidelity, the blue dashed line to the experiment of Ref. [1] and the black dashed line to the experiment of Ref. [2].

dots in experimental systems. It should be noted, however, that our conclusions assume that the next-nearest-neighbor couplings are 1% of the nearest-neighbor couplings; if this percentage is lower, then higher fidelities will be possible. We should also note that we deal here solely with errors due to noise in the exchange couplings, presumably itself due to noise in the voltage sources used to define the quantum dots that the electrons reside in; another potential source of error, the treatment of which is beyond the scope of this work, are sources intrinsic to the semiconductor system, such as spin-orbit coupling 12.

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FIG. 8: Plot of noise-averaged infidelity $1 - \bar{F}$ of “transporting” one of two entangled spins from left to right and back again as a function of $J_{\text{SWAP}}/J_0$ for 4 spins. The system is initialized in the state, $|\psi_{0,2}\rangle = |S\rangle \otimes |\uparrow\uparrow\rangle$, where $|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$. We include a next-nearest-neighbor coupling $J' = 0.01J$, dipole-dipole coupling, and with noise in $J$ with strength (i.e., standard deviation) $\sigma_J$. The red dashed line corresponds to 99.9% single SWAP fidelity, the blue dashed line to the experiment of Ref. 1, and the black dashed line to the experiment of Ref. 2.

FIG. 9: Plot of noise-averaged infidelity $1 - \bar{F}$ of “transporting” one of two entangled spins from left to right and back again as a function of $J_{\text{SWAP}}/J_0$ for 6 spins. The system is initialized in the state, $|\psi_{0,2}\rangle = |S\rangle \otimes |\uparrow\uparrow\uparrow\uparrow\rangle$, where $|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$. We include a next-nearest-neighbor coupling $J' = 0.01J$, dipole-dipole coupling, and with noise in $J$ with strength (i.e., standard deviation) $\sigma_J$. The red dashed line corresponds to 99.9% single SWAP fidelity, the blue dashed line to the experiment of Ref. 1, and the black dashed line to the experiment of Ref. 2.