Natural Convection from the Outside Surface of an Inclined Cylinder in Pure Liquids at Low Flux

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ABSTRACT: Many studies have investigated natural convection heat transfer from the outside surface of horizontal and vertical cylinders in both constant heat flux and temperature conditions. However, there are poor studies in natural convection from inclined cylinders. In this study, free convection heat transfer was examined experimentally from the outside surface of a cylinder for glycerol and water at various heat fluxes. The tests were performed at 10 different inclination angles of the cylinder, namely, ϕ = 0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, and 90°, measured from the horizon. Our results indicated that the average Nusselt number reduces with the growth in the inclination of the cylinder to the horizon at the same heat flux, and the average Nusselt number enhanced with the growth in heat flux at the same angle. Also, the average Nusselt number of water is greater than that of glycerol. A new experimental model for predicting the average Nusselt number is suggested, which has a satisfactory accuracy for experimental data.

1. INTRODUCTION

In a wide range of industrial processes including high solar collectors, voltage power transmission lines, and nuclear safety systems, natural convection is used from the outside surface of a cylinder for cooling and heating. A large number of studies have investigated natural convection heat transfer from the outside surface of horizontal and vertical cylinders in both constant heat flux and temperature conditions. However, there is very limited data on natural convection heat transfer for pure liquids from an inclined cylinder.5 The aim of this study was investigating the existing gap. Thus, the impact of the inclination angle was studied on natural convection heat transfer from the outside surface of a cylinder.

2. LITERATURE REVIEW

Natural convection is a type of heat transport or mechanism in which no external source is the cause of the fluid motion (such as fan, pump, etc.); however, the density diversity in the fluid occurs only because of temperature gradients.5,7 In the literature, there are numerous experimental and numerical studies about natural convection heat transfer from a vertical flat plate, vertical cylinder, inclined flat plate facing down, inclined flat plate facing up, horizontal cylinder, and horizontal flat plate, and many more correlations have been suggested. According to comprehensive research performed by Nada and Mowad,7 for free convection from a vertical flat plate, many previous researchers obtained the expressions of the average Nusselt number for different ranges of Rayleigh numbers (McAdams,8 Sparrow and Gregg,9 Warner and Arpaci,10 and Churchill and Chu11). Suryanarayana12 indicated that for a downward facing heated or an upward facing cooled inclined isothermal surface, the correlation for free convection heat transfer from a vertical flat plate can be utilized by the component of the gravitational force parallel to the plate to assess Rayleigh number. Fuji and Imura13 investigated free convection from an isothermal upward facing heated inclined plate. The results indicated that the Grashof number is affected by the boundary layer surrounding the surface. Fussey and...
Table 1. Heat Transfer Correlations for Horizontal, Vertical, and Inclined Cylinders

| reference         | correlation                                                                 | L/D  | Pr   | θ          |
|-------------------|-----------------------------------------------------------------------------|------|------|------------|
| Sedahmed and Shemilt 26 | \( \text{Nu}_L = 0.498 \left( \text{Ra}_L \cos \phi \right)^{0.28} \) 1.9 \times 10^{10} < \text{Ra}_L < 3.8 \times 10^{11} | 4.65–14 | 2300 | inclined  |
| Al-Arabi and Salman 3 | \( \frac{\text{Nu}_L}{\left( \text{Gr}_L \cos \phi \right)^{1/4}} = 0.42[1 + (1.31/L^{1/4})]^{1/2} \) | 8, 10, 16 | 0.7 | inclined  |
| Churchill and Chu 17 | \( \frac{\text{Nu}_L}{(\text{Ra}_L \cos \phi)^{1/4}} = 0.48 + 0.555 \left( \frac{D}{L \cos \phi} \right) \left( \frac{L}{L} \right)^{1/4} \) | 6, 9, 12 | 0.7 | inclined  |
| Morgan 18          | \( \text{Nu}_L = a(\text{Ra}_L)^{b} \) | 10^8 < \text{Ra}_L < 10^9 | 0.25 \times 10^8 < \text{Ra}_L < 1.8 \times 10^7 | 2.2 | horizontal |
| Le Fevre 30        | \( \text{Nu}_L = 0.67(Gr_L \text{Pr})^{0.25} \) \text{Gr}_L > 10^4 | 10^{-5} < \text{Ra}_L < 10^{12} | 3, 7, 11, 14 | vertical |
| Fouad and Ibl 31   | \( \text{Nu}_L = 0.31(Gr_L \text{Pr})^{0.28} \) \text{Gr}_L > 10^4 | 10^{-5} < \text{Ra}_L < 10^{12} | 4.65–14 | 2300 | inclined  |

Warneford 16 suggested two correlations for the Nusselt number with various ranges of inclination angle. Some experiments were conducted by McAdams, 8 Fujii and Imura, 13 Goldstein et al., 15 and Lloyd and Moran,16 on free convection heat transfer from upward facing and downward heated horizontal rectangular plates for horizontal flat plates, and they suggested correlations for the two cases at different ranges of Rayleigh numbers. Churchill and Chu 17 and Morgan 18 proposed comprehensive correlations for free convection from a horizontal cylinder for a broad range of Rayleigh number. Wide-ranging research on free convection from a horizontal cylinder was also conducted by Özgür Atayilmaz 19 and Teke. 20 However, Chen et al. 30 investigated the impact of geometry on the flow surrounding a cylinder in crossflow. Three various stepped-diameter circular cylinders (SDCCs) were used with different step heights.

Through using the cylinder diameter (D) and inclination angle (ϕ), Lia and Tarasuk 21 suggested the correlation \( \text{Nu}_L = m(\phi)\text{Ra}_L^{\alpha}(\phi) \). Al-Arabi and Khamis 4 suggested the correlation \( \text{Nu}_L = m(\phi)\text{Ra}_L^{\alpha}(\phi) \) using the inclination angle ϕ and cylinder length L, and the variation of L and ϕ depends on the angle of inclination. Farber and Rennat 22 conducted an experiment with a stainless steel tube with a 6 ft (1.829 m) length and 0.125 in. (3.175 mm) OD, which was heated by inserting an electric current through it for producing a constant heat flux.

The inclination angle of the tube ranged from 0° to 90°, and temperatures were achieved as high as 760 °C. Khamis 23 investigated steam-heated brass tubes at a constant temperature with different diameters and lengths. The Gr and Pr were ranged from 9.88 \times 10^7 to 2.93 \times 10^{10}, and the inclination angle of the tube ranged from 30° to 90°. Oosthuizen 24 conducted an experiment with aluminum cylinders. The length ranged from 152.4 to 304.8 mm, the diameters from 19.1 to 25.4 mm and the inclination angle from 0° to 90°. The heat transfer was obtained evaluating the amount at which the cylinders were cooled to 90 °C after being heated monotonically to 100 °C.

Many studies have recommended the natural convection heat transfer correlations for horizontal, vertical, and inclined cylinders (Table 1). Research conducted by Heo and Chung 25 indicated that the heat transfer correlations for inclined cylinders suggested by Oosthuizen, 24 Sedahmed and Shemilt, 26 Al-Arabi and Salman, 3 Buck, 27 and Stewart 28 at \( \phi = 0^\circ \) are not consistent with the correlations for a horizontal cylinder recommended by Morgan 18 and Fouad et al. 29. The suggested correlation for a vertical cylinder at \( \phi = 90^\circ \) is consistent with the Le Fevre 30 correlation for laminar conditions but is not in agreement with the Fouad and Ibl 31 correlation for turbulent conditions. Jafarpur and Yovanovich 22,33 studied an analytical method for the area’s mean
Nusselt number of free convection heat transfer, (Rayleigh number ranged from 0–10).8 Prashanna and Chhabra44,55 studied free convective heat transfer for a horizontal cylinder immersed in quiescent power-law fluids numerically. Goldstein et al.30–39 investigated the impact of geometry on the flow surrounding a cylinder in crossflow. They found that the mass/heat transfer analogy is verified experimentally for laminar, two-dimensional, and turbulent boundary layer flows over the cylinder and flat plates.

3. MATERIALS AND METHODOLOGY

3.1. Setup. Figure 1 shows the experimental equipment used in the current research. The stainless steel vessel has a cubic shape containing nearly 20 L of the test liquid. In order to maintain predetermined operating conditions, the system is persistently monitored and regulated. A PC-based data acquisition system registered the measuring parameters. Also, to change the angle of the cylinder, a hydraulic jack was used. A digital protractor was employed to read these angles. For more controllability and to reduce heat loss, the whole system is heavily insulated.

The vessel contains two heaters: (1) an auxiliary heater to enhance the bulk temperature to any set point and (2) a rod heater, which is comprised of an internally heated stainless steel rod with 15 thermocouples of stainless steel mounted at five axial locations.

Each axial location has three thermocouples that are distributed equally on the circumference of the test section near the heating surface. The injection of silicon paste into the position of each thermocouple is performed to reduce thermal contact resistance between each sheath and thermocouple.

Using the heat conduction equation for cylinders, the temperature drop is given by eq 1 due to the existence of short distance between the surface and thermocouple location.

$$\frac{1}{r} \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

where $k$ is the temperature thermal conductivity of the heater. It was approximated to a linear function of temperature.

It is estimated that the axial heat loss is less than 0.05% of the total heat transfer.30–32 However, in these sorts of experiments, uncertainty in heat flux and heat transfer coefficient measurements are very important to estimate. In this study, the method created by Jafarpur et al. was used to calculate uncertainties.33 The distance between thermocouples, thermocouple calibration, and thermal conductivity of stainless steel contributed in the unknown calculations.

There are two significant errors while doing the experiment: precision and bias errors. Precision errors are because of testing sensitive devices. Bias errors come from calibration. These errors are stated as

$$U_y = \sqrt{B_y^2 + P_y^2}$$

where $U_y$ is the uncertainty or total error, $B_y$ is the bias error, and $P_y$ is the precision error. Thermocouple calibrations, stainless steel thermal conductivity, and the distance between thermocouples were the error parameters. The thermocouples were calibrated and their correctness error was obtained statistically as ±0.2 K.

The calculation of heated surface temperature ($T_y$) was carried out by the heat flux ($q^*$) generated by the experimental heater and heater temperature ($T_{th}$) measured by the thermocouple.34 This is due to the fact that the direct measurement of temperature is linked to variations in the heated surface geometry.

$$q^* = -k \frac{dT}{dx}$$

$q^*$ is the heat flux. It was obtained using eq 4 as follows

$$q^* = \frac{V_{heater} I_{circuit}}{A_{sur}}$$

where $V_{heater}$ is the voltage, $I_{circuit}$ is the electric current of the experimental heater, and $A_{sur}$ is the surface area of the heated surface.

To calculate the pool boiling heat transfer coefficient, it was necessary to extrapolate the surface temperature of the test...
surface of the heater. The free convection heat transfer coefficient is an indicator of a fluid thermal performance, which was obtained using equation 5

\[ h = \frac{q}{T_w - T_{sat}} \]  

(5)

where \( T_{sat} \) is the saturation temperature, and \( T_w \) is the temperature at the heated surface. The indeterminacies for heat flux were obtained using equation 6.

\[ \frac{U_q}{q} = \sqrt{\sum \left( \frac{U_{\Delta T}}{\Delta T} \right)^2 + \left( \frac{U_{\Delta Z}}{\Delta Z} \right)^2 + \left( \frac{U_k}{k} \right)^2} \]  

(6)

The multimeter readings and thermocouples were repeated three times to certify data reproducibility. Table 2 depicts the indeterminacies for measurement equipment used in the present study.

### Table 2. Indeterminacies of the Measurement Instruments

| Parameter                  | Instrument                  | Uncertainty |
|----------------------------|-----------------------------|-------------|
| Surface temperature        | K-type thermocouple         | 0.2 K       |
| Angle                      | digital protractor Pro 360  | 0.01°       |
| Voltage                    | Mastech MS 820SC multimeter | ±1 V        |
| Current                    | Mastech MS 820SC multimeter | ±0.1 A      |
| Bulk temperature           | Pt100 thermoresistance      | ±0.1 K      |
| Heat flux \((\text{W}\cdot\text{m}^{-2})\) |                            | ±3.32%      |

According to the measurement accuracy displayed in Table 2 and using the above method (equation 6), the maximum error for heat flux was 3.32%.

### 3.2. Experimental Procedure

The input power and inclination angle of the test section are the two independent variables in these experiments. Testing procedures for each fluid can be classified as:

A. Regulating the heater on the desired inclination angle.
B. Filling the test container with fluid.
C. Setting the bulk temperature as desired (50 °C).
D. Setting the input voltage as desired (5 V).
E. Recording data after 15 min to ensure a steady state condition so that the thermocouple temperatures are stabilized.
F. Increasing input voltage (at a constant inclination angle) at a rate of 5 V and repeating step E.
G. Repeating step F until the first vapor bubble is observed, that is, the end of the test at the set inclination angle.
H. Increasing the heater’s inclination angle to 10° and repeating steps C to G until 90°.

### 3.3. The Range of Parameters and Test Solutions

In the present study, glycerol and pure water were employed due to the following reasons:

(a) Pure water is readily available and has many resources, so its known chemical and physical properties are given; it has been one of the most widely used liquids in people’s routine life and different industries, especially those involving heat transfer equipment such as heat exchangers. Thus, it is of great value to have information about the heat transfer coefficient of pure water. (b) Glycerin has been employed in many industries including soap making, cosmetics and hygiene, making explosives, lubrication of tools and other metal installation, and antifreeze of hydraulic jacks. Thus, data on the heat transfer coefficient of glycerin can be advantageous for the abovementioned industries.

Table 3 shows the range of operating conditions in the current study, which are derived from authentic handbooks.37,38

### 4. DATA REDUCTION

The local heat transfer coefficient is calculated from the following equation:

\[ H_l = \frac{q}{(T_l - T_\infty)} \]  

(7)

The average heat transfer coefficient for a cylinder with a length \((L)\) is given as follows

\[ H_{av} = \frac{q}{(T_l - T_\infty)} \left( \frac{1}{\int_{x=0}^{x=L} Tdx} - T_\infty \right) \]  

(8)

where \( i \) is the position of wall thermocouple on the axial location of the tube, as indicated in Figure 2, and \( T_i \) is the circumferential averaged local temperature at this axial position. The average Nusselt number along the tube was calculated as

\[ Nu_{av} = \frac{h_{av} L}{(k)} \]  

(9)

The physical properties were calculated at the mean film temperature \( T_f = \frac{T_l + T_\infty}{2} \). In total, 141 tests were performed in this study, which covers the following values and ranges:

\[ Gr_L = \frac{g \beta \Delta T L^3}{\nu^2} = 4.5 \times 10^4 - 2.53 \times 10^9 \]

\[ Pr = \frac{\mu C_p}{k} = 2.7 - 0.9 \times 10^3 \]

\[ Ra_L = Pr \times Gr_L = 4.24 \times 10^5 - 7.6 \times 10^9 \]

\[ \varphi = 0^\circ \text{ (horizontal), } 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ, 80^\circ, \text{ and } 90^\circ \text{ (vertical)} \]

### 5. RESULTS AND DISCUSSION

According to the obtained results and the Rayleigh number calculated, it appears that in all angles and heat fluxes of the cylinder, the laminar flow is dominant. Figures 3 and 4 indicate
the variations of the local heat transfer coefficients of water and glycerol with the length of the cylinder for the different angles of inclination and flux of 5500 W·m⁻¹. The local heat transfer coefficients are constant in the horizontal position. In a similar way, Al-Arabi and Salman³ have reported that the local heat transfer coefficients are constant for the horizontal position. For other angles of inclination, there is a reduction in local heat transfer coefficients with increasing the axial distance measured from the bottom of the cylinder.⁵,³⁵ All the other heat fluxes behave in the same way. The reason is an increase in the boundary layer thickness, which leads to the decrease in the heat transfer coefficient. In addition, at the same axial positions, the local heat transfer coefficients decrease with the increase of inclination angle; the value is the maximum for the horizontal position of the cylinder. The results also show that, at the same axial positions, the local heat transfer coefficient of glycerol is smaller than that of water.

It appears that this can be the result of physical properties of glycerol such as viscosity and thermal conductivity; the viscosity of glycerol is greater and its thermal conductivity is smaller than that of water. Consequently, this decreases the Ra number of glycerol as compared to water. Figures 5 and 6 show the variations of \( h_{av} \) with cylinder length for the same runs. Figures 5 and 6 indicate that at the same value of cylinder length, \( h_{av} \) is the maximum for the horizontal cylinder and decreases with the growth of inclination angle. Also, as cylinder length increases, the average heat transfer coefficient decreases. \( h_{av} \) is constant for the horizontal position.

Figure 3. Variation of \( h \) for water with cylinder length.

Figure 4. Variation of \( h \) for glycerol with cylinder length.

Figure 5. Variation of \( h \) for water with cylinder length.

Figure 6. Variation of \( h \) for glycerol with cylinder length.

Figure 7. Comparison of the average heat transfer coefficient with the local heat transfer coefficient. Experimental data demonstrate that for the horizontal position, \( h_{av} \) is equal to the local heat transfer coefficient. However, for other angles of inclination, the average heat transfer coefficient is greater than the local heat transfer coefficient. Experimental data for water shows the same characteristics.

Figure 8 indicates the impact of the inclination angle on the local and average Nusselt number (heat flux: 5000 W·m⁻², \( x = L = 0.1 \text{ m} \)). As indicated in the figure, with the increase in the
inclination angle of the cylinder, the average and local Nusselt number decrease. It is true for other heat fluxes and cylinder lengths (x). Also, for glycerol, the variations of the Nusselt number with the inclination angle are smaller than those of water.

5.1. Comparison with Literature. Comparing the equations obtained for the inclined, horizontal, and vertical cylinders with the generally accepted ones is significantly helpful. This is performed to assess the accuracy of the apparatus. Comparing this study against those in the literature is difficult, because there is no study available in the literature for the present geometry, orientations, and surface material.

5.1.1. Horizontal. Figure 9 shows the evaluation of the experimental average Nusselt number (horizontal cylinder) with the prior studies for water. Our results are consistent with all the previous investigations. Churchill and Chu correlation demonstrate the best prediction for experimental data (Churchill and Chu: 18.7%, Morgan: 21.5%, Fand et al.: 25%).17,18,29

5.1.2. Vertical. Figure 10 indicates the evaluation of the experimental average Nusselt number for the presented vertical cylinder in this study against previous ones for water and glycerol. Our results are consistent with all the previous investigations shown in Figure 10. However, in fluxes less than 8000 W·m⁻², it was observed that there is a significant diversion between experimental data and previous correlations; however, in higher fluxes, the diversion is less than 15%.

5.1.3. Inclined. Figure 11 shows the evaluation of the experimental average Nusselt number for the inclined cylinder with the prior studies for water. Our results are consistent with all the previous investigations. Stewart and Buck’s correlation indicated the most diversion, and Stewart’s correlation showed the least diversion.

The absolute average error is given using eq 10:

$$\text{AAE\%} = \left| \frac{\text{Nu}_{\text{predicted}} - \text{Nu}_{\text{experimental}}}{\text{Nu}_{\text{experimental}}} \right| \times 100$$  \hspace{1cm} (10)
It is worthy of noting that Oosthuizen’s correlation can be neither used in a horizontal position nor a vertical position. Also, correlations of Stewart and Buck, Stewart, and Sedahmed and Shemilt cannot be employed in the vertical position. Considering this point and Figure 12, the correlation of Churchill and Chu for the horizontal position, Fouad and Ibl’s correlation for the vertical position and for the other inclination angles, Stewart’s correlation, and Oosthuizen’s correlation are consistent with experimental data. The diversity between the proposed relations (for example, Stewart and Buck27) and empirical data is not indicative of their imprecision. This may be due to variant experimental conditions including heater geometry, roughness, working fluid, heater material, and range of Prandtl number.

5.1.4. New Empirical Model. The functional equation for the average heat transfer coefficient in natural convection can be represented as below:

\[ h = f(k, \beta, |T_w - T_\infty|, L_c, \delta, \alpha, g, \phi) \]  

(11)

In this research, new dimensionless groups were made from all of the available influencing parameters except the inclination angle, and effects of the inclination angle on the characteristic length and experimental constants were considered.

The influencing parameters include the average heat transfer coefficient, thermal conductivity |T_w - T_\infty|, characteristic length, kinematics viscosity, thermal diffusivity, volume expansion coefficient, and acceleration of gravity. In total, there are eight parameters with five dimensions including power, length, temperature, time, and mass. The following dimensionless have been acquired using Buckingham’s \( \pi \) theorem:

\[ \pi_1 = \frac{Nu_L}{k}, \pi_2 = \frac{\beta \Delta T}{L_c}, \pi_3 = \frac{g \Delta T L_c^3}{\delta \alpha} \]

(12, 13, 14)

It appears that \( \pi_2 \) and \( \pi_3 \) usually emerge as a product. This product is called the Rayleigh number as

\[ \pi_4 = \pi_2 \times \pi_3 = \frac{g \beta \Delta T L_c^3}{\delta \alpha} \]

(15)

so we expect data are correlated with functional equations of the form

![Figure 12. Comparison of the experimental average Nusselt number for all inclination angles with the previous studies.](image)
Thus of the inclination angle as shown in eq 17:

\[ \frac{Nu}{L_c} = A(Ra_{c})^{\beta} \]  

where the Nusselt number and Rayleigh number are based on the characteristic length \( L_c \). Researchers have suggested different values of characteristic length for different geometry and surface positions, but this parameter has not been discussed in any study for inclined cylinders. In the present study, the characteristic length \( L_c \) is considered as a function of the inclination angle as shown in eq 17:

\[ L_c = f(\phi) = \frac{L}{1 + \sin \phi} \]  

The variations of the average Nusselt number in terms of the average Ra for the horizontal position are shown in Figure 13.

![Figure 13. Variation of Nu with Ra.](image)

As shown in Figure 13 and eq 16, \( A = 0.174 \) and \( B = 0.326 \). Thus

\[ Nu_{L_c} = 0.174(Ra_{L_c})^{0.326} \]  

Almost the same \( A \) is obtained by the other angles of inclination, but \( B \) is different. \( B \) can be a function of \( \sin \phi \). Figure 13 indicates the variations of \( B \) with \( \sin \phi \) for heat flux (for example, heat flux is $550 \text{ W m}^{-2}$).

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**NOMENCLATURE**
- \( \text{Nu} \): Nusselt number
- \( \text{Ra} \): Rayleigh number
- \( \text{L} \): Cylinder diameter (m)
- \( \text{L}_{\text{c}} \): Cylinder length (m)
- \( \text{P} \): Pressure (Pa)
- \( \text{K} \): Thermal conductivity (W m$^{-1}$ K$^{-1}$)
- \( \text{C}_{\text{p}} \): Specific heat at constant pressure (J kg$^{-1}$ K$^{-1}$)
- \( \text{L}_{\text{c}} \): Characteristic length (m)
- \( \text{h}_{\text{tab}} \): Heat transfer coefficient (W m$^{-2}$ K$^{-1}$)
- \( \text{q}_{\text{tab}} \): Heat flux (W m$^{-2}$)
- \( \text{T}_{\text{tab}} \): Temperature (K)
- \( \rho_{\text{tab}} \): Density (kg m$^{-3}$)
- \( \beta \): Angle of inclination (°) \( \phi = 0 \), horizontal
- \( \beta_{\text{tab}} \): Volume expansion coefficient (m$^{3}$ K$^{-1}$)
- \( \Delta \): Difference
- \( \mu_{\text{tab}} \): Dynamic viscosity (Pa s)

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