Form factors for semileptonic $B \to \pi$, $B_s \to K$ and $B_s \to D_s$ decays

Jonathan Flynn,$^{a,b,*}$ Ryan Hill,$^{a,c,*}$ Andreas Jüttner,$^d,a,b$ Amarjit Soni,$^c$ J Tobias Tsang$^f$ and Oliver Witzel$^g$

$^a$Physics & Astronomy, University of Southampton, Southampton, SO17 1BJ, UK
$^b$STAG Research Centre, University of Southampton, Southampton SO17 1BJ, UK
$^c$DISCnet Centre for Doctoral Training, University of Southampton, Southampton SO17 1BJ, UK
$^d$CERN TH Division, 1211 Genève 23, Switzerland
$^e$Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
$^f$CP3-Origins and IMADA, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark
$^g$Center for Particle Physics Siegen, Theoretische Physik 1, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, 57068 Siegen, Germany

E-mail: j.m.flynn@soton.ac.uk, r.c.hill@soton.ac.uk

We report on our determinations of $B \to \pi \ell \nu$, $B_s \to K \ell \nu$ and $B_s \to D_s \ell \nu$ semileptonic form factors. In addition we discuss the determination of $R$-ratios testing lepton-flavor universality and suggest an improved ratio. Our calculations are based on the set of 2+1 flavor domain-wall Iwasaki gauge field configurations generated by the RBC/UKQCD collaboration with three lattice spacings of $1/a = 1.78$, 2.38, and 2.79 GeV. We use the relativistic heavy quark action for $b$ quarks and charm quarks are simulated with the Möbius domain-wall fermion action.

The 38th International Symposium on Lattice Field Theory, LATTICE2021 26th–30th July, 2021
Zoom/Gather@Massachusetts Institute of Technology

Siegen preprint: SI–HEP–2021–36

*Speaker
1. Introduction

Semileptonic decays of $B(s)$ mesons play an important role in testing and constraining the Standard Model (SM) of elementary particle physics. Focusing on exclusive semileptonic decays, we report on our work for $B \to \pi \ell \nu$, $B_s \to D_s \ell \nu$ and $B_s \to K \ell \nu$ decays. Each of these processes can be described by two form factors, $f_s$ and $f_0$, which parametrize the semileptonic decay rate. For the semileptonic decay of pseudoscalar meson $B(s)$ of mass $M$ and momentum $p$ to pseudoscalar meson $P$ of mass $m$ and momentum $k$, with $q = p - k$,

$$
\frac{d\Gamma(B(s) \to P \ell \nu)}{dq^2} = \frac{G_F^4 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m^2)^2 |\tilde{k}|^2}{(q^2)^2} \left[ \left( 1 + \frac{m^2}{2q^2} \right) \tilde{k}^2 |f_s(q^2)|^2 + \frac{3m^2}{8q^2} \frac{(M^2 - m^2)^2}{M^2} |f_0(q^2)|^2 \right],
$$

where $m_\ell$ is the mass of the outgoing charged lepton $\ell$ and $\eta$ is an isospin factor. The form factors $f_s$ and $f_0$ appear in the decomposition

$$
\langle P(k) | V^\mu(0) | B(s)(p) \rangle = 2 f_s(q^2) \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) + f_0(q^2) \frac{M^2 - m^2}{q^2} q^\mu,
$$

where $V^\mu = \bar{x} \gamma^\mu b$, with $x = u$ or $c$.

Compared to our earlier results for $B \to \pi$ and $B_s \to K$ decays [1], we have added calculations of $B_s \to D_s$ form factors and have an additional, third, lattice spacing. With results for $B_s \to K$ and $B_s \to D_s$ from the same ensembles, we will be able to compute the ratio of partially integrated decay rates (minus CKM factors) in the region $q^2 \geq 7 \text{GeV}^2$ for the two decays and combine with recent LHCb results [2] to determine $|V_{ub}/V_{cb}|$. We also consider $R$ ratios of branching fractions with $\tau$ leptons in the final state to those with light final-state leptons, sensitive to violations of lepton-flavor universality (for $B \to D^{(*)} \ell \nu$ there is tension between Standard Model predictions and experimental results for $R(D^{(*)})$ [3–12]). We propose a modified ratio with smaller uncertainty when evaluated using lattice-determined form factors.

2. Lattice calculation

We use a subset of six RBC/UKQCD 2+1-flavor domain-wall fermion (DWF) and Iwasaki gauge field ensembles with three lattice spacings $\alpha \sim 0.11, 0.08, 0.07$ fm, and pion masses spanning $267 \text{MeV} < M_\pi < 433 \text{MeV}$. The ensembles are listed in Table 1. Light and strange quarks are simulated with the Shamir DWF action with $M_5 = 1.8$. Lattice spacings are determined from combined RBC/UKQCD analyses [13–15]. Our calculations are described briefly below; for more details see [1].

Bottom quarks are simulated with the relativistic heavy quark (RHQ), action, which is the Columbia variant [18, 19] of the Fermilab heavy-quark action [20], with three nonperturbatively-tuned parameters $(m_0, c, \zeta)$ [21]. A new tuning was performed for this analysis. Charm quarks are simulated with the Möbius DWF action with $M_5 = 1.6$ [14, 15, 22, 23]. We use three masses below $m_c^{\text{phys}}$ on the C ensembles and two masses which bracket $m_c^{\text{phys}}$ on M and F.
procedure [24, 25], using

or smeared sinks.

\[ Z \] and strange quark masses and \( M_\pi \) is the unitary pion mass. Valence strange quarks are near their physical mass, with the mistuning accounted for in our systematic errors.

To find perturbatively at one-loop [26], while

Here, \( V_\mu \), and lattice, \( V_\mu \), currents are related by the ‘partially nonperturbative’ procedure [24, 25], using

\[
\langle P|V_\mu|B_\pi\rangle = Z_{V_\mu}^b \langle P|V_\mu|B_\pi\rangle,
\]

with \( Z_{V_\mu}^b = p_{V_\mu}^{bb} \sqrt{Z_{V_\mu}^{xx}} Z_{V_\mu}^{bb} \) and

\[
V_0 = V_0^0 + c_1^4 V_1^0 + c_2^1 V_1^1 + c_3^2 V_1^2 + c_4^3 V_1^3 + c_5^4 V_1^4.
\]

Here, \( p_{V_\mu}^{bb} \) and the coefficients \( c_{t,s}^n \) of the \( O(a) \) current-improvement operators are computed perturbatively at one-loop [26], while \( Z_{V_\mu}^{bb} \) is computed nonperturbatively from the forward matrix element

\[
Z_{V_\mu}^{bb} \langle B_{(s)}|V_0(0)|B_{(s)}\rangle = 2M
\]

and \( Z_{V_\mu}^{xx} \) is computed nonperturbatively using the relation \( Z_{V_\mu}^{xx} = Z_{A_\mu}^{xx} + O(am_{res}) \) for DWF fermions [17] (for our systematic error analysis for \( B_\pi \to D_\pi \) decays, we also compare to using \( Z_{V_\mu}^{xx} = Z_{V_\mu}^{n} \) [14]).

To extract the form factors we first calculate the matrix elements

\[
f_{\parallel}(E) = \frac{\langle P|V_0(0)|B_{(s)}\rangle}{\sqrt{2M}}, \quad f_{\perp}(E) = \frac{\langle P|V_i(0)|B_{(s)}\rangle}{E^i \sqrt{2M}},
\]

with a \( B_{(s)} \) meson at rest, where \( E \) is the energy of the outgoing pseudoscalar meson, from which we determine

\[
f_0(q^2) = \frac{\sqrt{2M}}{M^2 - m^2} \left[(M - E) f_{\parallel}(E) + (E^2 - m^2) f_{\perp}(E)\right],
\]

\[
f_+(q^2) = \frac{1}{\sqrt{2M}} \left[f_{\parallel}(E) + (M - E) f_{\perp}(E)\right].
\]

To find \( f_{\parallel} \) and \( f_{\perp} \), we evaluate a correlator ratio

\[
R_{3,\mu}(t, t_{sink}, \vec{k}) = \frac{C_3(\mu, t, t_{sink}, \vec{k})}{\sqrt{C_2^P(t, \vec{k}) C_2^{P_{(s)}}(t_{sink} - t, 0) \sqrt{\frac{2E}{e^{-E_1 - M(t_{sink} - t)}}}}}
\]
Form factors for semileptonic $B \rightarrow \pi, B_s \rightarrow K$ and $B_s \rightarrow D_s$ decays  

Jonathan Flynn and Ryan Hill

Figure 1: Three-point correlator used in form-factor determinations.

Figure 2: Extraction of $f_\parallel$ for $B_s \rightarrow K$ on the coarse ensemble C1 from the ratio $R_{3,0}$. The different colors denote different three-momenta $2\pi n/L$ injected at the current, labelled by $n^2$. The plot shows a ground-state-only fit together with a fit over an extended range of times for each momentum once excited state terms are included for the current matrix elements in the numerator of $R_{3,0}$. The horizontal bars near the left axis show the values for $f_\parallel$ from the ground-only and from the excited-state fits.

where $C_{2, P, B(s)}$ are two-point correlators and $C_{3, \mu}$ is the three-point correlator shown schematically in figure 1. For large time separations between source, sink and current insertion, we obtain

$$f_{\parallel}^{\text{bare}}(\vec{k}) = \lim_{0 \ll t \ll t_{\text{sink}}} R_{3,0}(t, t_{\text{sink}}, \vec{k}), \quad f_{\perp}^{\text{bare}}(\vec{k}) = \lim_{0 \ll t \ll t_{\text{sink}}} \frac{1}{p_\mu^t} R_{3,i}(t, t_{\text{sink}}, \vec{k}).$$

(10)

Figure 2 illustrates the determination of $f_\parallel$ for $B_s \rightarrow K$ on the coarse, C1, ensemble.

For $B_s \rightarrow K$ and $B \rightarrow \pi$ we extrapolate the renormalized lattice form factors to vanishing lattice spacing and to the physical light-quark mass, and interpolate in the kaon(pion) energy, using next-to-leading order SU(2) heavy-meson chiral perturbation theory (HM\chiPT) in the “hard-kaon(pion)”
Figure 3: Chiral-continuum extrapolation for the $B \to \pi$ form factors $f_s$ (left) and $f_0$ (right). The colored data points show the underlying data. The colored lines show the result of the fit evaluated at the parameters of the respective ensembles. The grey bands display the form factors in the chiral-continuum limit and the associated statistical uncertainty.

limit [27–29]. The function we use, with $P$ denoting kaon or pion, is

$$f^{B(s) \to P}(M_\pi, E_P, a^2) = \frac{\Lambda}{E_P + \Delta} \left[ c_0 \left( 1 + \frac{\delta f(M_\pi^{\text{sea}}) - \delta f(M_\pi^{\text{phys}})}{(4\pi f_\pi)^2} \right) \right.$$  
$$\left. + c_1 \frac{\Delta M_\pi^2}{\Lambda^2} + c_2 \frac{E_P}{\Lambda} + c_3 \frac{E_P^2}{\Lambda^2} + c_4 (a\Lambda)^2 \right],$$  

(11)

where $M_\pi^{\text{sea}}$ is the simulated pion mass on a given ensemble, $M_\pi^{\text{phys}}$ is the physical pion mass, $\Delta M_\pi^2 = (M_\pi^{\text{sea}})^2 - (M_\pi^{\text{phys}})^2$ and $\Lambda = 1 \text{ GeV}$ is the renormalization scale appearing in the one-loop chiral logarithm in $\delta f$, and is also used as a dimensionful scale to render the fit coefficients dimensionless. $\Delta = M_{B^*} - M_{B(s)}$ and the $B^*$ is a $b\bar{u}$ flavor state with $J^P = 1^+$ for $f_s$, or $J^P = 0^+$ for $f_0$. For $f_s$ this is the vector meson $B^*$ with mass $M_{B^*} = 5.32470(22) \text{ GeV}$ [30], while for $f_0$ there is a theoretical estimate for the $0^+$ state, $M_{B^*(0^+)} = 5.63 \text{ GeV}$ [31]. The term $\delta f'$ also contains an estimate for finite volume effects. Figure 3 shows the fit for $B \to \pi$ and Figure 4 shows the fit for $B_s \to K$.

For $B_s \to D_s$ form factors, we combine a chiral-continuum fit with an extra-/inter-polation in the charm mass with a fit form

$$f(q^2, a, M_\pi, M_{D_s}) = \left[ c_0 + \sum_{j=1}^{n_{D_s}} c_{1j} h\left( \frac{M_{D_s}}{\Lambda} \right)^j + c_2 (a\Lambda)^2 \right] P_{a,b}(q^2/M_{D_s}^2),$$  

(12)

where

$$h\left( \frac{M_{D_s}}{\Lambda} \right) = \frac{M_{D_s}}{\Lambda} - \frac{M_{D_s}^{\text{phys}}}{\Lambda} \quad \text{and} \quad P_{a,b}(x) = \frac{1 + \sum_{i=1}^a a_i x^i}{1 + \sum_{i=1}^b b_i x^i}. \quad (13)$$

Figure 5 shows the fit. We use only one of the charm masses for F1S in this fit because of the strong correlations between results for the two charm masses on that ensemble.
Form factors for semileptonic $B \rightarrow \pi$, $B_s \rightarrow K$ and $B_s \rightarrow D_s$ decays

Jonathan Flynn and Ryan Hill

Figure 4: Chiral-continuum extrapolation for the $B_s \rightarrow K$ form factors $f_+$ (left) and $f_0$ (right). The colored data points show the underlying data. The colored lines show the result of the fit evaluated at the parameters of the respective ensembles. The grey bands display the form factors in the chiral-continuum limit and the associated statistical uncertainty.

Figure 5: Chiral-continuum extrapolation for the $B_s \rightarrow D_s$ form factors.

3. z-fits

After extrapolating our results to the continuum and physical masses, our strategy is to generate synthetic data points for the form factors, with all errors included, which can then be used in standard $z$-fits to extrapolate over the full $q^2$ range for the physical form factors. In figure 6, we illustrate the cumulative statistical plus systematic error budgets for the $f_+$ form factor for $B_s \rightarrow K$ and $B_s \rightarrow D_s$ decays. Figure 7 shows results for $z$-fits for $B \rightarrow \pi$ and $B_s \rightarrow K$ form factors. These are Bourrely-Caprini-Lellouch (BCL) fits [32], where we have included the $1^-$ $B^*$ vector meson pole for fitting $f_+$ and no pole for $f_0$ in both cases.
4. Ratios for testing lepton flavor universality

Ratios of decay rates or partially integrated decay rates with tau leptons in the final state to those with light leptons in the final state are of great interest in looking for violations of the universality of lepton couplings in the Standard Model. For the semileptonic decays considered here, such a ratio is \( R(P) \) given by

\[
R(P) = \frac{\int_{m_t^2}^{q_{\text{max}}^2} dq^2 \frac{d\Gamma(B_s \rightarrow P \ell \bar{\nu}_\ell)}{dq^2}}{\int_{m_t^2}^{q_{\text{max}}^2} dq^2 \frac{d\Gamma(B_s \rightarrow \pi \ell \bar{\nu}_\ell)}{dq^2}},
\]

\[
(14)
\]
where $\ell$ in the denominator can be $\mu$ or $e$. We have considered modifying this ratio to look for a sharper test of lepton flavor universality (see discussion in [33] on optimising observables). In particular, we try to reduce the uncertainty in the ratio coming from uncertainties in the form factors taken from our lattice simulations. To this end, we consider the modified ratio

$$ R_{\text{new}}(P) = \frac{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 \frac{d\Gamma(B_{(s)} \rightarrow P\tau\nu_{\tau})}{dq^2}}{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 \frac{\omega_{\ell}(q^2)}{\omega_{\tau}(q^2)} \frac{d\Gamma(B_{(s)} \rightarrow P\ell\nu_{\ell})}{dq^2}}, $$

following the recipe already applied to $B_{(s)} \rightarrow V$ decays (with a vector meson instead of a pseudoscalar meson in the final state) by Isidori and Sumensari [34]. The ingredients are

- Use a common integration range for numerator and denominator, with $q_{\text{min}}^2 \geq m_{\tau}^2$ [35–37].
- Reweight the integrand in the denominator to make the contributions from the vector form factor the same in numerator and denominator.

Write the differential decay rate from equation (1) in the form

$$ \frac{d\Gamma(B_{(s)} \rightarrow P\ell\nu_{\ell})}{dq^2} = \Phi(q^2) \omega_{\ell}(q^2) \left[ F_V^2 + (F_S^\ell)^2 \right], $$

where now $\ell$ can be any lepton flavor, with

$$ \Phi(q^2) = \frac{G_F^2 |V_{xb}|^2}{24\pi^3} |\vec{k}|, $$

$$ \omega_\ell(q^2) = \left( 1 - \frac{m_\ell^2}{q^2} \right) \left( 1 + \frac{m_\ell^2}{2q^2} \right), $$

$$ F_V^2 = \vec{k}^2 f_+(q^2)^2, $$

$$ (F_S^\ell)^2 = \frac{3}{4} \frac{m_\ell^2}{m_\ell^2 + 2q^2} \frac{(M^2 - m_\ell^2)^2}{M^2} |f_0(q^2)|^2. $$

If we drop the scalar contribution, $(F_S^\ell)^2$, in the denominator, with $\ell = \mu$ or $e$ again, then relying on $m_\ell^2/2q^2 \leq m_\mu^2/2m_\tau^2 = 0.002$ for the light leptons, we expect in the Standard Model,

$$ R_{\text{new,SM}}(P) = 1 + \frac{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 \Phi(q^2) \omega_{\tau}(q^2)(F_S^\tau)^2}{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 \Phi(q^2) \omega_{\tau}(q^2) F_V^2}. $$

Evaluating the new ratio from equation (15) using the $z$-fit for our lattice form factors, we find a reduced uncertainty compared to evaluating the original ratio in equation (14). It would be interesting to see how the ratios compare if evaluated using experimental differential decay rates.

**Acknowledgments**

We thank our RBC/UKQCD collaborators for helpful discussions and suggestions. Computations used resources provided by the USQCD Collaboration, funded by the Office of Science of
the US Department of Energy and by the ARCHER UK National Supercomputing Service, as well as computers at Columbia University and Brookhaven National Laboratory. We used gauge field configurations generated on the DiRAC Blue Gene Q system at the University of Edinburgh, part of the DiRAC Facility, funded by BIS National E-infrastructure grant ST/K000411/1 and STFC grants ST/H008845/1, ST/K005804/1 and ST/K005790/1. The project leading to this application has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 894103. This project has received funding from Marie Skłodowska-Curie grant 659322 (EU Horizon 2020), the European Research Council grant 279757 (EU FP7/2007–2013), STFC grants ST/P000711/1 and ST/T000775/1. RH was supported by the DISCnet Centre for Doctoral Training (STFC grant ST/P006760/1). AS was supported in part by US DOE contract DE–SC0012704. JTT acknowledges support from the Independent Research Fund Denmark, Research Project 1, grant 8021–00122. No new experimental data was generated.

References

[1] RBC/UKQCD collaboration, J. M. Flynn, T. Izubuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water et al., $B \to \pi \ell \nu$ and $B_s \to K \ell \nu$ form factors and $|V_{ub}|/|V_{cb}|$ from 2 + 1-flavor lattice QCD with domain-wall light quarks and relativistic heavy quarks, Phys. Rev. D91 (2015) 074510 [arXiv:1501.05373].

[2] LHCb collaboration, R. Aaij et al., First observation of the decay $B^0_s \to K^{-}\mu^+\nu_{\mu}$ and measurement of $|V_{ub}|/|V_{cb}|$, Phys. Rev. Lett. 126 (2021) 081804 [arXiv:2012.05143].

[3] BaBar collaboration, J. Lees et al., Evidence for an excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_{\tau}$ decays, Phys.Rev.Lett. 109 (2012) 101802 [arXiv:1205.5442].

[4] BaBar collaboration, J. P. Lees et al., Measurement of an Excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_{\tau}$ Decays and Implications for Charged Higgs Bosons, Phys. Rev. D88 (2013) 072012 [arXiv:1303.0571].

[5] Belle collaboration, M. Huschle et al., Measurement of the branching ratio of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_{\tau}$ relative to $\bar{B} \to D^{(*)} \ell^- \bar{\nu}_{\ell}$ decays with hadronic tagging at Belle, Phys. Rev. D92 (2015) 072014 [arXiv:1507.03233].

[6] Belle collaboration, S. Hirose et al., Measurement of the $\tau$ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^* \tau^- \bar{\nu}_{\tau}$, Phys. Rev. Lett. 118 (2017) 211801 [arXiv:1612.00529].

[7] Belle collaboration, S. Hirose et al., Measurement of the $\tau$ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^* \tau^- \bar{\nu}_{\tau}$ with one-prong hadronic $\tau$ decays at belle, Phys. Rev. D97 (2018) 012004 [arXiv:1709.00129].

[8] Belle collaboration, A. Abdesselam et al., Measurement of $R(D)$ and $R(D^*)$ with a semileptonic tagging method, arXiv:1904.08794.

[9] Belle collaboration, G. Caria et al., Measurement of $R(D)$ and $R(D^*)$ with a semileptonic tagging method, Phys. Rev. Lett. 124 (2020) 161803 [arXiv:1910.05864].
Form factors for semileptonic $B \to \pi$, $B \to K$ and $B \to D_\tau$ decays

Jonathan Flynn and Ryan Hill

[10] LHCb collaboration, R. Aaij et al., Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \to D^{*-}\tau^+\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \to D^{*-}\mu^+\bar{\nu}_\mu)$, Phys. Rev. Lett. 115 (2015) 111803 [arXiv:1506.08614], [Erratum: Phys. Rev. Lett. 115, no.15,159901(2015)].

[11] LHCb collaboration, R. Aaij et al., Measurement of the ratio of branching fractions $B^0 \to D^*\tau^+\nu_\tau$ and $B^0 \to D^*\mu^+\nu_\mu$ using three-prong $\tau$-lepton decays, Phys. Rev. Lett. 120 (2018) 072013 [arXiv:1711.02505].

[12] LHCb collaboration, R. Aaij et al., Test of lepton flavor universality by the measurement of the $B^0 \to D^\tau\tau\nu_\tau$ branching fraction using three-prong $\tau$ decays, Phys. Rev. D97 (2018) 072013 [arXiv:1711.02505].

[13] RBC/UKQCD collaboration, T. Blum et al., Domain wall QCD with physical quark masses, Phys. Rev. D93 (2016) 074505 [arXiv:1411.7017].

[14] RBC/UKQCD collaboration, P. A. Boyle, L. Del Debbio, A. Jüttner, A. Khamseh, F. Sanfilippo and J. T. Tsang, The decay constants $f_D$ and $f_{D_s}$ in the continuum limit of $N_f = 2 + 1$ domain wall lattice QCD, JHEP 12 (2017) 008 [arXiv:1701.02644].

[15] RBC/UKQCD collaboration, P. A. Boyle, L. Del Debbio, N. Garron, A. Jüttner, A. Soni, J. T. Tsang et al., SU(3)-breaking ratios for $D_{(s)}$ and $B_{(s)}$ mesons, arXiv:1812.08791.

[16] RBC/UKQCD collaboration, C. Allton et al., Physical Results from 2+1 Flavor Domain Wall QCD and SU(2) Chiral Perturbation Theory, Phys. Rev. D78 (2008) 114509 [arXiv:0804.0473].

[17] RBC/UKQCD collaboration, Y. Aoki et al., Continuum Limit Physics from 2+1 Flavor Domain Wall QCD, Phys.Rev. D83 (2011) 074508 [arXiv:1011.0892].

[18] N. H. Christ, M. Li and H.-W. Lin, Relativistic heavy quark effective action, Phys.Rev. D76 (2007) 074505 [arXiv:hep-lat/0608006].

[19] H.-W. Lin and N. Christ, Non-perturbatively determined relativistic heavy quark action, Phys.Rev. D76 (2007) 074506 [arXiv:hep-lat/0608005].

[20] A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Massive fermions in lattice gauge theory, Phys. Rev. D55 (1997) 3933 [arXiv:hep-lat/9604004].

[21] RBC/UKQCD collaboration, Y. Aoki, N. H. Christ, J. M. Flynn, T. Izubuchi, C. Lehner, M. Li et al., Nonperturbative tuning of an improved relativistic heavy-quark action with application to bottom spectroscopy, Phys. Rev. D86 (2012) 116003 [arXiv:1206.2554].

[22] RBC/UKQCD collaboration, P. Boyle, A. Jüttner, M. K. Marinkovic, F. Sanfilippo, M. Spraggs and J. T. Tsang, An exploratory study of heavy domain wall fermions on the lattice, JHEP 04 (2016) 037 [arXiv:1602.04118].

[23] P. A. Boyle, L. Del Debbio, A. Jüttner, A. Khamseh, J. T. Tsang and O. Witzel, Heavy domain wall fermions: The RBC and UKQCD charm physics program, EPJ Web Conf. 175 (2018) 13013 [arXiv:1712.00862].
Form factors for semileptonic $B \rightarrow \pi$, $B_s \rightarrow K$ and $B_s \rightarrow D_s$ decays

Jonathan Flynn and Ryan Hill

[24] S. Hashimoto, A. X. El-Khadra, A. S. Kronfeld, P. B. Mackenzie, S. M. Ryan and J. N. Simone, *Lattice QCD calculation of $\bar{B} \rightarrow D\pi$ decay form-factors at zero recoil*, Phys. Rev. D61 (1999) 014502 [arXiv:hep-ph/9906376].

[25] A. X. El-Khadra et al., *The Semileptonic decays $B \rightarrow \pi l \nu$ and $D \rightarrow \pi l \nu$ from lattice QCD*, Phys.Rev. D64 (2001) 014502 [arXiv:hep-ph/0101023].

[26] C. Lehner. Private communication, 2014.

[27] RBC Collaboration, UKQCD Collaboration collaboration, J. Flynn and C. Sachrajda, *SU(2) chiral perturbation theory for $K(l3)$ decay amplitudes*, Nucl.Phys. B812 (2009) 64 [arXiv:0809.1229].

[28] J. Bijnens and I. Jemos, *Hard pion chiral perturbation theory for $B \rightarrow \pi l \nu$ decays and a determination of $|V_{ub}|$*, Phys.Rev. D79 (2009) 013008 [arXiv:0807.2722].

[29] D. Bećirević, S. Prelovsek and J. Zupan, *$B \rightarrow \pi$ and $B \rightarrow K$ transitions in standard and quenched chiral perturbation theory*, Phys.Rev. D67 (2003) 054010 [arXiv:hep-lat/0210048].

[30] Particle Data Group collaboration, M. Tanabashi et al., *Review of particle physics (with 2019 update)*, Phys. Rev. D98 (2018) 030001.

[31] W. A. Bardeen, E. J. Eichten and C. T. Hill, *Chiral multiplets of heavy-light mesons*, Phys.Rev. D68 (2003) 054024 [arXiv:hep-ph/0305049].

[32] C. Bourrely, I. Caprini and L. Lellouch, *Model-independent description of $B \rightarrow \pi l \nu$ decays and a determination of $|V_{ub}|$*, Phys.Rev. D79 (2009) 013008 [arXiv:0807.2722].

[33] D. Atwood and A. Soni, *Analysis for magnetic moment and electric dipole moment form-factors of the top quark via $e^+ e^- \rightarrow t$ anti-$t$*, Phys. Rev. D 45 (1992) 2405.

[34] G. Isidori and O. Sumensari, *Optimized lepton universality tests in $B \rightarrow Vl\bar{\nu}$ decays*, Eur. Phys. J. C 80 (2020) 1078 [arXiv:2007.08481].

[35] M. Freytsis, Z. Ligeti and J. T. Ruderman, *Flavor models for $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}$*, Phys. Rev. D 92 (2015) 054018 [arXiv:1506.08896].

[36] F. U. Bernlochner and Z. Ligeti, *Semileptonic $B_s^{(*)}$ decays to excited charmed mesons with $e, \mu, \tau$ and searching for new physics with $R(D^{(*)})$*, Phys. Rev. D 95 (2017) 014022 [arXiv:1609.09330].

[37] RBC/UKQCD collaboration, J. M. Flynn, R. C. Hill, A. Jüttner, A. Soni, J. T. Tsang and O. Witzel, *Nonperturbative calculations of form factors for exclusive semileptonic $B_s^{(*)}$ decays*, PoS ICHEP2020 (2021) 436 [arXiv:2012.04323].