The Pulsed Electromagnetic Field of Polarizable Conductive Plate and Its Integral Characteristic When Excited by a Vertical Electric Dipole

Y A Nim¹, L P Gogoleva¹ and M G Illarionova¹

¹Geological Prospecting Faculty, North-Eastern Federal University, 50 Kulakovskogo Str., Yakutsk, 677000, Russia

E-mail: gmprimpi@mail.ru

Abstract. We considered direct problem of pulsed electrical prospecting, which is based on approximation model of a polarizable electrically conductive plate (plane S), according to well-known Cole-Cole polarization model. Vertical electric lines are used as a source and receiver of a pulsed electromagnetic field, which are relatively new receiving and generating devices in pulsed electrical prospecting and fairly easy implemented in marine search and mapping of oil and gas deposits, deposits of non-ferrous metals and other mineral and raw materials sources, especially in hard-to-reach and/or deep water areas of the World Ocean, e.g. under the ice of the North Arctic ocean, in Russian exploration areas, in rift valley of Mid-Atlantic ridge. We developed analytical model of unsteady electromagnetic field of polarized plane S, excited by a vertical electric dipole. According to the Pearson hypothesis and numerous experimental data, obtained by different researchers, the polarizability of the plate is a relatively new indicator of the oil and gas deposit, which is identified by EMF induction sign change. To increase the depth of the study, we present an analytical model of integral characteristics of the pulsed electromagnetic field of polarized plate, excited by a vertical electric dipole. In this case, the magnetic flux induction, as well as EMF, changes charge.

1. Introduction

Prospects of the mineral reserves growth are mainly related to the geological study and mineral deposits development (MDD) of the World Ocean [2,3,10,15,17,18,19,20,29]. At the same time, an important source of information on the geological structure of a particularly deep-water part of the ocean floor is data obtained by geophysical studies, including electrical prospecting [2,4,6,7,9,10,14,15,17,24].

While carrying out marine electrical exploration studies, horizontal electric dipoles (lines) are usually used as sources and receivers of an electromagnetic field, but vertical electric ones can easily be used as sources and receivers in the search for oil and gas deposits, polymetallic and other deposits in the seas and oceans [8,14,17]. Moreover, to increase the depth of the investigation, it is expedient to observe the pulsed flux of magnetic induction [13,21,23,26,27].

2. Relevance

The main results of electromagnetic studies, as a rule, are the maps of longitudinal conductivity "S" of horizontally layered sections. However, according to Pierson's hypothesis, because of numerous
researches of different researchers [7,11,12,16,19,25,28] it is shown that the indicators of oil and gas deposits can serve as zone halos of hydrocarbons dispersion that generate induction-induced polarization (IIP) of rocks, marked by a change in the sign of the unsteady electromagnetic field, i.e. an almost direct criterion for the forecast of oil and gas deposits was found [5,8,9,11,16,19,25,28,29]. The same IIP also occurs with the electromagnetic excitation of copper-pyrite ores [11,24,25]. In this regard, it seems relevant to develop the technology of prospecting and exploration of MDD within the World Ocean based on a combined engineering-analytical pulse model of the magnetic induction flux of a polarizable electrically conductive formation when it is excited by a vertical electric dipole.

3. Analytical analysis technique
We represent the object of research in the form of a polarizable electrically conductive plate, approximated by the plane Sη - polarized longitudinal conductivity, an analog of the widely approved Cole-Cole model, modernized with concern to the known model of the S0 plane [12,16,22,28]:

\[ S^\eta(\omega) = S_0[1 + (i\omega\tau)^k]/[(1 - \eta)(i\omega\tau)^k] \]  

where \( S_0 \) is the longitudinal conductivity of the plate, independent to the frequency \( \omega \), \( \tau \) is the relaxation time, \( \eta \) is the polarizability of the object \((0 \leq \eta \leq 1)\), i is the imaginary unit, \( k \) is the coefficient of the relaxation time distribution, for simplicity, \( k = 1 \) (Debye model) [16,23].

The vertical electric dipole (VED) with the moment \( P_3 \) is placed at a height \( h = -z \) from the plane \( S^\eta \) to the origin of the cylindrical coordinate system \((r, \varphi, z)\) and directed along \( z \) axis. The medium containing the plane \( S^\eta \) is described by the Laplace equation

\[ \Delta^2 A_z = 0. \]

The boundary conditions are represented as:

\[ A_z\big|_{r=0} = A_z\big|_{r=\infty} \]

where \( I \) is current in the dipole, \( \text{dl} \) – is the element of the dipole length, \( \mu = 4\pi \times 10^{-7} \text{H/m} \).

On the boundary \( z = -h \), we have:

\[ A_{1z} = A_{2z} \quad 2. \quad \frac{\partial A_{1z}}{\partial z} - \frac{\partial A_{2z}}{\partial z} = i\omega S^\eta A_{(1,2)z} \]  

According to the field source, the vector potential \( A_z \) is introduced by the relation

\[ H_\varphi = \text{rot}A, \]

The solution of the Laplace equation on both sides of the plane \( S^\eta \) has the form:

\[ A_{1z} = P_3\int_0^\infty e^{m|z|} + C_0 e^{-mc} J_0(mr)dm, \]

\[ A_{2z} = P_3\int_0^\infty C_1 e^{mc} J_0(mr)dm, \]

where \( J_0(mr) \) is the Bessel function of zero order, \( m \) is the separation variable, \( C_0 \) and \( C_1 \) are the coefficients determined from the boundary conditions.

Solving the system (4) with allowance for the boundary conditions, we obtain:

\[ C_0 = -\frac{i\omega S^\eta}{2m + i\omega S^\eta}e^{-2mh}, \quad C_1 = -\frac{2m}{2m + i\omega S^\eta}. \]  

We enter \( C_0 \) and \( C_1 \) coefficients into Cole-Cole polarization model.
\[ C_0 = i \omega \mu S_0 \frac{(1 + i \omega \tau)}{1 + (1 - \eta) i \omega \tau} / \left[2m + i \omega \mu S_0 \frac{1 + i \omega \tau}{1 + (1 - \eta) i \omega \tau}\right] = \]
\[ = e^{-2a} \frac{p^2 + p \frac{1}{\tau}}{p^2 + \frac{2m(1 - \eta) \tau}{\mu_S \tau}} + \frac{2m}{\mu_S \tau} = e^{-2a} V W, \]

where \( p = i \omega, W = p^2 + \frac{\mu S_0 + 2m(1 - \eta) \tau}{\mu_S \tau}, \) \( \quad \frac{2m}{\mu_S \tau} = (p + a)^2 - b^2, \) \( V = p^2 + \frac{p}{\tau}. \)

By entering \( C_1 \) coefficient into equation (1), we get:
\[ C_1 = \frac{p^2 a^2 + p \beta}{(p + a)^2 - b^2}, \]

where \( a = \frac{2m(1 - \eta)}{\mu_S \tau}; \beta = \frac{2m}{\mu_S \tau}. \)

Confining momentum field determining over polarizing plane \( S'' \), substituting \( C_0 \) coefficient in the corresponding equation of the inverse Laplace-Carlson integral transformation, we get:
\[ A_v(t) = \frac{1}{4 \pi} \int_0^{\infty} e^{-\alpha} \left( chbt + \frac{\tau}{b} shbt \right) J_0(\mu r) dm = \]
\[ = \frac{1}{4 \pi} \int_0^{\infty} e^{-\alpha} \left( chbt(1 - \beta - \alpha) + shbt(\frac{\beta - \alpha}{b}) \right) J_0(\mu r) dm, \]

where \( a = \frac{2m(1 - \eta)}{2 \mu_S \tau}; \beta = \frac{m(1 - \eta)}{\mu_S \tau}; \alpha = 2h + z, t - \) observation time.

4. Research results
We define the vertical component and its integral model of the pulsed electromagnetic field of a vertical electric dipole. According to the laws of electrodynamics, we have:
\[ E_z(t) = -\frac{\partial^2 A_v}{\partial z^2}; \]
\[ \Phi_z(t) = \int_0^t E_z(t) 2\pi R; \]

where \( R \) is the radius of the contour, permeated by the magnetic flux.

Let's find:
\[ E_z(t) = \frac{Idl}{8\pi R} \left( -\frac{1}{2\pi} e^{-\frac{t}{\tau}} \left( \frac{\mu S_0}{4(1-\eta)\tau} \frac{a}{(a^2 + r^2)^\frac{3}{2}} - \frac{\mu S_0}{4(1-\eta)\tau} \frac{(a + 2kt)}{(a + 2kt)^2 + r^2} \right) \right. \]

\[ - \frac{2(a + 2kt)^2 + r^2}{2} \left( \frac{(a + 2kt) - \frac{r^2}{2}}{(a + 2kt)^2 + r^2} \right) + e^{-\frac{t}{\tau}} \left( -\frac{\mu S_0}{4(1-\eta)\tau} \frac{8k(a + 2kt)^2 + 2kr^2}{[(a + 2kt)^2 + r^2]^\frac{3}{2}} \right) \]

\[ \frac{6k(a + 2kt)^3 + kr^2(a + 2kt)}{[(a + 2kt)^2 + r^2]^\frac{5}{2}} \mu S_0 \left( \frac{5Rr^2(a + 2kt) + 2kr^2 - 8k(a + 2kt)^2}{[(a + 2kt)^2 + r^2]^\frac{3}{2}} \right) \]

\[ \phi_z(t) = \frac{Idl}{2} \text{Re} \left( e^{-\frac{t}{\tau}} \left( \frac{\mu S_0}{4(1-\eta)\tau} \frac{a}{(a^2 + r^2)^\frac{3}{2}} - \frac{\mu S_0}{4(1-\eta)\tau} \frac{(a + 2kt)}{(a + 2kt)^2 + r^2} \right) \right. \]

\[ - \frac{2(a + 2kt)^2 + r^2}{2} \left( \frac{(a + 2kt) - \frac{r^2}{2}}{(a + 2kt)^2 + r^2} \right) \]

\[ \left. \frac{[(a + 2kt)^2 + r^2]^\frac{5}{2}}{[(a + 2kt)^2 + r^2]^\frac{3}{2}} \right) \]

where \( k = \frac{1-\eta}{\mu S_0}, \alpha = 2h + z, \mu = 4\pi \cdot 10^{-7} H/m. \)

Figures 1 and 2 show the curves \( E_z(t) \) and \( \phi_z(t) \), respectively.

**Figure 1.** Unsteady electromagnetic field of electrical component of vertical electric \( E_z(t) \).
It can be seen from the figure that both functions change the sign of the pulsed electromagnetic field, noting the object polarization. According to the principle of permutational duality, the components of the pulsed electromagnetic field of an electrically conductive layer excited by a vertical electric dipole are equivalent to the corresponding known field components when the same magnetic layer is excited by a vertical magnetic dipole. In this case, the field $E_z(t)$ is equivalent to $B_z(t)$ and $\Phi_z(t)$ to $\Phi_\phi(t)$[23].

5. Conclusion
Combined engineering and analytical model of technological support of VED is shown with reference to observations of unsteady field in elementary functions: induced EMF and flux of magnetic induction of an electrically conductive polarizing layer. The model can be used for prospecting and mapping studies of the World Ocean bottom.

6. References
[1] Balakin A I, Myasnikov I F and Sajganov 1979 Rol otricatelnkykh anomaliy MPP pri poiskakh sulfidnogo orudieniya: Razvedka i okhrana nedr pp 35-37
[2] Barsukov P O and Fainberg E B 2013 A mobile time domain sounding for shallow water First Break 31 pp 53-63
[3] Budangov A G 2007 Shelf Rossii: neftegazovyy potentials strany Geofizika 3 pp 51-54
[4] Chave A and Cox C 1982 Controlled electromagnetic sources for measuring electric conductivity beneath the oceans Forward problem and model study J Geophys Res 87 P 5327-5338
[5] Constable S and Srnka L An introduction to marine controlled-Source electromagnetic method for hydrocarbon exploration Geophysics 72 2 WA3 WA12
[6] Dobrynin S I and Bobrovnikov L Z 2015 Razrabotka teoreticheskikh osnov dlya sozdaniya metoda poiskov i razvedki mestorozhdenij poleznyh iskopaemyh na osnove beskontaktnogo obnaruzheniya signalov estestvennyh ehlektromagnitnyh polej s borta letatel nogo apparata Proc. XII Int. Sci. Conf. Novye idei v naukah o Zemle Vol. 1 (Moscow: MGRI-MGGRU) pp 261-262
[7] Egorov I V and Pospeev A V 2015 Sravnitel nyj analiz istochnikov nestacionarnogo ehlektromagnitnogo polya Geofizika 1 pp 26-30
[8] Goldman M, Mogilatov V, Haroon, Levi E and Tezkan B 2015 Signal detectability of marine electromagnetic methods in the exploration of resistive targets *Geophysical Prospecting* **63** pp 192-210

[9] Goryunov A S, Kisilev E S and Oviarenko A V 2010 Prognoz neftegazonosnosti ehlektrorazvedchnymi metodami *Geofizika* **5** pp 24-28

[10] Hatylla A and Dmitriev D 1981 On the use of transient EM-method over a sulphide are body under sallow sea water *Proc. Geol geshem and Geophys Investig Eastern Part Balt Shield Pap 10 Gen Meet* (Helsinki) pp 189-195

[11] Zhuravleva R B, Ulitin R V, Krestanin B A and Usanin V L 1972 Vliyanie vyzvannoj polyarizacii na krivye stanovleniya MPP na primere mednokolchedannogo mestorozhdeniya *Razvedka geofizika* **78** pp 57-61

[12] Kamenetsky F M, Stettler E H and Trigubovich G M 2010 *Transient Geo-electromagnetics* 304

[13] Kamenetsky F M, Vaškulič A M, Drabich P P and Timofeev V M 1985 Issledovanie integralnych perekhodnych harakteristik v impulsnom indukcionnom ehlektrozavedke *Preprint №88*

[14] Kaufman A A, Alekseev D A and Orištalo M O 2016 *Principy ehlektromagnitnych metodov nazemnoj geofiziki* p 558

[15] Molovichko M S 2008 Sravnenie statisticheskih svojstv pomekh pri morskih izmereniyah ustannavlivayushchihsya ehlektricheskih polej *Geofizika* **5** pp 59-64

[16] Ozhogina E G, Yakushina O A and Astahova Yu M 2015 Glubokovodnye polimetallicheskie sulfidnye rudy potencialnyj istochnik cvetnyh metallov *Proc. XII Int. Conf. Novye idei v naukah o Zemle Vol I* (Moscow: MGRI-MGGRU) pp 39-41

[17] Sheymann S M, Isaev G A and Poletaeva N G 1981 Vliyanie polyariznosti gornyh porod v metode perekhodnych protsessov v rudnoi geofizike *Metody razovedchnoi geofiziki teorii i praktika interpretatsii v rudnoi geofizike* pp 40-53

[18] Tikshaev V V and Sidorov V A 1973 Integralnyi sposob postroeniia krivykh stanovleniia polia *Prikladnaia geofizika* **71** pp 122-128
[27] Timofeev V M, Mamaev V A and Kamenetskii F M 1985 Izmerenie potoka magnitnoi induktsii neustanovivshegosia polya Razvedochnaia geofizika 101 pp 58-63
[28] Veltsov J N, Epov M J and Antonov E Yu 2002 Reconstruction of Cole-Cole parameters from IP induction foundling data J. of the Balkan Geophys. Soc. 5 pp 15 -20
[29] Weidelt P 1982 Response characteristics of coincident loop transient electromagnetic systems Geophysics 48 pp 1325-1330
[30] White J R 1987 Geomagnetism 355 p