Crossing the Cosmological Constant Line on the Warped DGP Brane

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Abstract

We study dynamics of the equation of state parameter for a dark energy component non-minimally coupled to induced gravity on a warped DGP brane. We show that there are appropriate domains of the model parameters space that account for crossing of the phantom divide line. This crossing, which is possible for both branches of the scenario, depends explicitly on the values of the non-minimal coupling and warp factor. The effect of warp factor appears in the value of the redshift parameter at which phantom divide line crossing occurs.

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1 Introduction

Recent observational data from CMB temperature fluctuations spectrum, Supernova type Ia redshift-distance surveys and other data sources, have shown that the universe is currently in a positively accelerated phase of expansion and its spatial geometry is nearly flat [1]. Nevertheless, there is not enough standard matter density in the universe to support this flatness and accelerated expansion. Therefore, we need either additional cosmological components or modify general relativity at cosmological scales to explain these achievements [2]. Multi-component dark energy with at least one non-canonical phantom field is a possible candidate of the first alternative. This viewpoint has been studied extensively in literature (see [5,6] and references therein). There are some datasets (such as the Gold dataset) that show a mild trend for crossing of the phantom divide line by equation of state (EoS) parameter of dark component. The equation of state parameter in these scenarios crosses the phantom divide line ($\omega = \frac{p}{\rho} = -1$) at recent redshifts and current accelerated expansion requires $\omega < -\frac{1}{3}$. In fact, recent observational data restrict $\omega(z)$ to be larger than $-1$ in the past and less than $-1$ today. The current best fit value of the equation of state parameter, using WMAP five year data combined with measurements of Type Ia supernovae and Baryon Acoustic Oscillations in the galaxy distribution, is given by $-0.11 < 1 + \omega < 0.14$ (with 95 percent CL uncertainties) [7]. It is accepted that crossing of the phantom divide line occurs at recent epoch with $z \sim 0.25$ [5,6], although this value is model dependent. Currently, models of phantom divide line crossing are so important that they can realize that which model is better than the others to describe the nature of dark energy [6]. Although this crossing cannot be explained just by one scalar field [8], generalization to multi-field case or non-minimal coupling with gravity provide enough space to achieve such a crossing [3,5,6]. Lorentz invariance violating fields are other alternative dark energy components with capability to cross the phantom divide line by EoS parameter in a fascinating manner [9]. Also it is possible to reconstruct a scalar-tensor theory of gravity in an accelerating universe where a phantom behavior can be realized [10]. The cosmological constant (with $\omega(z) = -1$) is the simplest candidate

\footnote{It is important to note that phantom fields are not consistent due to violation of the null energy condition and instabilities (see for instance [3]). However, theoretically they provide a good candidate with negative pressure to realize late-time accelerated expansion. Recently, it has been shown that phantom-like behavior can be realized without introducing any phantom matter in some specific braneworld models [4]. We note also that in our model due to its wider parameter space, it is expected essentially that the null energy condition is fulfill in at least some subspaces of the model parameter space.}
for dark energy [11,12]. However this scenario suffers from some difficulties such as lack of physical motivation, huge amount of fine-tuning to explain cosmological accelerated expansion and no dynamics for its equation of state [12]. So it seems worthwhile to probe alternative dynamical models.

Another alternative to explain current accelerated expansion of the universe is extension of the general relativity to more general theories on cosmological scales (see [13] and references therein). DGP (Dvali-Gabadadze-Porrati) braneworld scenario as an infra-red (IR) modification of general relativity, explains accelerated expansion of the universe in its self-accelerating branch via leakage of gravity to extra dimension [14]. In this model, the late-time acceleration of the universe is driven by the manifestation of the excruciatingly slow leakage of gravity off our four-dimensional world into an extra dimension [15]. In this scenario the EoS parameter of dark energy never crosses the $\omega(z) = -1$ line, and universe eventually turns out to be de Sitter phase. However, in this setup if we use a single scalar field (ordinary or phantom) on the brane, we can show that EoS parameter of dark energy can cross the phantom divide line [16]. It has been shown that DGP model with a quintom dark energy fluid (a combination of quintessence and phantom fields in a joint model) in the bulk or brane, accounts for accelerated expansion and the phantom divide line crossing [17]. In a braneworld setup with induced gravity embedded in a bulk with arbitrary matter content, the transition from a period of domination of the matter energy density by nonrelativisticbrane matter to domination by the generalized dark radiation provides a crossing of the phantom divide line [18]. In this setup there is no need to introduce additional scalar field as a dark component. Recently, phantom-like behavior in a brane-world setup with induced gravity and also curvature effects have been reported [19]. On the other hand, Gauss-Bonnent braneworld scenario with induced gravity does not need introducing any scalar field to account for this crossing. In other words, the combination of the effect of Gauss-Bonnent term in the bulk and induced gravity term on the brane behaves as dark energy on the brane [20]. Quintessential scheme can also be achieved in a geometrical way in higher order theories of gravity [21].

Crossing of the phantom divide line by a minimally coupled scalar field on the DGP braneworld has been studied by Zhang and Zhu [16]. In this setup, there are two possible cases: for ordinary scalar field EoS of dark energy crosses from $\omega > -1$ to $\omega < -1$ in normal (non self-accelerating) branch and for phantom field EoS of dark energy crosses from $\omega < -1$ to $\omega > -1$ in self-accelerating branch of DGP scenario. As an important generalization, the Randall-Sundram II model [22] combined with DGP scenario provides
a rich structure which has been called warped DGP braneworld in literature [23]. In this model, an induced curvature term appears on the brane in the RS II model.

Our motivation in this paper is to consider a scalar field non-minimally coupled to induced gravity on the warped DGP braneworld as a dark energy component and investigate the roles played by non-minimal coupling and the warp effect in the dynamics of the equation of state parameter. We study dynamics of the equation of state parameter focusing on the crossing of the phantom divide line in this setup. We show that this crossing is possible for a suitable range of the model parameters and especially for some specific values of the non-minimal coupling and warp factor. More specifically, we show that for ordinary scalar (quintessence) field in the self-accelerating branch of the warped DGP model and with positive values of the non-minimal coupling, the EoS of dark energy runs from below $-1$ ($\omega_{de} < -1$) to above $-1$. In normal branch of this warped DGP model with positive and negative values of the non-minimal coupling, the EoS parameter runs from above $-1$ to below $-1$ which is supported by recent observations. For phantom field we have crossing of the phantom divide line in both branches of the warped DGP setup and in both of these cases the EoS parameter runs from above $-1$ to below $-1$. But, in self-accelerating branch of the model we have crossing behavior with positive and negative values of the non-minimal coupling in normal branch and crossing occurs just with negative values of the non-minimal coupling.

2 A Dark Energy Model on the Warped DGP Brane

2.1 Warped DGP Brane

Let us start with the action of the warped DGP model as follows

$$S = S_{bulk} + S_{brane},$$

$$S = \int_{bulk} d^5X \sqrt{-g} \left( \frac{1}{2\kappa_5^2} R + (5) L_m \right) + \int_{brane} d^4x \sqrt{-g} \left[ \frac{1}{\kappa_5^2} K^\pm + L_{brane}(g_{\alpha\beta}, \psi) \right].$$

Here $S_{bulk}$ is the action of the bulk, $S_{brane}$ is the action of the brane and $S$ is the total action. $X^A$ with $A = 0, 1, 2, 3, 5$ are coordinates in the bulk, while $x^\mu$ with $\mu = 0, 1, 2, 3$ are induced coordinates on the brane. $\kappa_5^2$ is the 5-dimensional gravitational constant. $(5) R$ and $(5) L_m$ are 5-dimensional Ricci scalar and matter Lagrangian respectively. $K^\pm$ is trace of the extrinsic curvature on either side of the brane. $L_{brane}(g_{\alpha\beta}, \psi)$ is the effective 4-dimensional Lagrangian. The action $S$ is actually a combination of the Randall-Sundrum
II and DGP model. In other words, an induced curvature term is appeared on the brane in the Randall-Sundrum II model, hence the name warped DGP Braneworld [23]. Now consider the brane Lagrangian as follows

\[ L_{\text{brane}}(g_{\alpha\beta}, \psi) = \frac{\mu^2}{2}R - \lambda + L_m, \]  

where \( \mu \) is a mass parameter, \( R \) is Ricci scalar of the brane, \( \lambda \) is tension of the brane and \( L_m \) is Lagrangian of the other matters localized on the brane. Assume that bulk contains only a cosmological constant, \( \Lambda \). With these choices, action (1) gives either a generalized DGP or a generalized RS II model: it gives DGP model if \( \lambda = 0 \) and \( \Lambda = 0 \), and gives RS II model if \( \mu = 0 \) [23]. The generalized Friedmann equation on the brane is as follows [23]

\[ H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \left[ \rho + \rho_0 \left( 1 + \varepsilon A(\rho, a) \right) \right], \]  

where \( \varepsilon = \pm 1 \) is corresponding to two possible branches of the solutions in this warped DGP model and \( A = \left[ A_0^2 + \frac{2\eta}{\rho_0} \left( \rho - \mu^2 \xi_0 \right) \right]^{1/2} \) where \( A_0 \equiv \left[ 1 - 2\eta \frac{\mu^2 \Lambda}{\rho_0} \right]^{1/2} \), \( \eta \equiv \frac{6\mu_5^6}{\rho_0 \mu^2} \) with \( 0 < \eta \leq 1 \) which is related to the warped geometry of the bulk manifold and \( \rho_0 \equiv m_\lambda^4 + 6m_5^6 \). By definition, \( m_\lambda = \lambda^{1/4} \) and \( m_5 = k_5^{-2/3} \). \( \xi_0 \) is an integration constant and corresponding term in the generalized Friedmann equation is called dark radiation term. We neglect dark radiation term in which follows. In this case, generalized Friedmann equation (4) attains the following form,

\[ H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \left[ \rho + \rho_0 + \varepsilon \rho_0 \left( A_0^2 + \frac{2\eta \rho}{\rho_0} \right)^{1/2} \right], \]  

where \( \rho \) in our case is the total energy density, including scalar field and dust matter on the brane

\[ \rho = \rho_\phi + \rho_{dm}. \]  

In which follows we construct a dark energy model on the warped DGP setup.

### 2.2 Quintessence Field

Now we consider a quintessence scalar field non-minimally coupled to induced gravity on the warped DGP brane as a candidate for dark energy. The action of this non-minimally coupled scalar field is given by

\[ S_\phi = \int_{\text{brane}} d^4x \sqrt{-g} \left[ -\frac{1}{2} \xi R \phi^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right], \]  

where \( \xi \) is a coupling constant, \( R \) is Ricci scalar of the bulk, \( V(\phi) \) is a potential function.
where $\xi$ is a non-minimal coupling and $R$ is Ricci scalar of the brane. We have assumed a conformal coupling of the scalar field and induced gravity. The scalar field will play the role of dark-energy component on the brane. Variation of the action with respect to $\varphi$ gives the equation of motion of the scalar field

$$\ddot{\varphi} + 3H \dot{\varphi} + \xi R \varphi + \frac{dV}{d\varphi} = 0.$$  \hspace{1cm} (8)

The energy density and pressure of the non-minimally coupled scalar field are given by

$$\rho_\varphi = \frac{1}{2} \dot{\varphi}^2 + V(\varphi) + 6 \xi H \varphi \dot{\varphi} + 3 \xi H^2 \varphi^2$$  \hspace{1cm} (9)

$$p_\varphi = \frac{1}{2} \dot{\varphi}^2 - V(\varphi) - 2\xi(\varphi \ddot{\varphi} + 2 \varphi H \dot{\varphi} + \dot{\varphi}^2) - \xi \varphi^2 (2 \dot{H} + 3 H^2)$$  \hspace{1cm} (10)

In which follows, by comparing the modified Friedmann equation in the warped DGP braneworld with the standard Friedmann equation, we deduce a definition for equation of state of dark energy component. This is reasonable since all observed features of dark energy are essentially derivable in general relativity [16,20]. The standard Friedmann equation in four dimensions is written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2}(\rho_{dm} + \rho_{de}),$$  \hspace{1cm} (11)

where $\rho_{dm}$ is the dust matter density, while $\rho_{de}$ is dark energy density. Comparing this equation with equation (5) for a spatially flat universe ($k=0$), we find

$$\rho_{de} = \rho_\varphi + \rho_0 + \varepsilon \rho_0 \left( A_0^2 + 2 \eta \frac{\rho}{\rho_0} \right)^{\frac{1}{2}}.$$  \hspace{1cm} (12)

Conservation of the scalar field effective energy density leads to

$$\frac{d\rho_\varphi}{dt} + 3H(\rho_\varphi + p_\varphi) = 0.$$  \hspace{1cm} (13)

We note that the non-minimal coupling of the scalar field to the Ricci curvature on the brane preserves conservation of the scalar field energy density$^6$.

$^6$Note that authors of Ref. [24] have treated this conservation in relatively different way. They have defined a total energy-momentum tensor consist of two parts: a pure (canonical) scalar field energy-momentum tensor and a non-minimal coupling-dependent part. The total energy density defined in this manner is then conserved. In our case, we have included all possible terms in equations (9) and (10) from beginning and evidently total energy density defined in this manner is conserved too. In fact, it is simple to show that our equations (9) and (10) are equivalent to $\rho^{tot}$ and $P^{tot}$ of Ref. [24] if we set $\alpha(\varphi) = \frac{1}{2}(1 - \xi \varphi^2)$ ( see also [25] for more detailed discussion).
Since the dust matter obeys the continuity equation and the Bianchi identity keeps valid, dark energy itself satisfies the continuity equation

$$\frac{d\rho_{de}}{dt} + 3H(\rho_{de} + p_{de}) = 0$$

(14)

where $p_{de}$ denotes the pressure of the dark energy. Note that this is just a definition and by assuming validity of this definition we can obtain effective pressure of dark energy as well as an effective equation of state. Now, the equation of state for this dark energy component can be written as follows

$$w_{de} = \frac{p_{de}}{\rho_{de}} = -1 + \frac{1}{3} \frac{d\ln \rho_{de}}{d\ln(1+z)},$$

(15)

where by using (13) and (14) we find

$$\frac{d\ln \rho_{de}}{d\ln(1+z)} = \frac{3}{\rho_{de}} \left[ \rho_{\phi} + p_{\phi} + \varepsilon \eta \left( \frac{A_0^2 + 2\eta \rho_{\phi} + \rho_{dm}}{\rho_0} \right)^{-\frac{1}{2}} \left( \rho_{\phi} + p_{\phi} + \rho_{dm} \right) \right].$$

(16)

There are three possible alternatives in this setup: if $\frac{1}{3} \frac{d\ln \rho_{de}}{d\ln(1+z)} > 0$, we have a quintessence model; if $\frac{1}{3} \frac{d\ln \rho_{de}}{d\ln(1+z)} < 0$, the model is phantom and if $\frac{1}{3} \frac{d\ln \rho_{de}}{d\ln(1+z)} = 0$, the dark component is a cosmological constant. Evidently, in this setup non-minimal coupling of the scalar field and induced gravity plays a crucial role supporting or preventing phantom divide line crossing. In this respect, the differences between the minimal and non-minimal setups will be more clear if we write the explicit dynamics of the equation of state parameter. We also discuss the effect of warp factor on the dynamics of the equation of state parameter in forthcoming arguments. We choose the following exponential potential with motivation that this type of potential can be solved exactly in the standard model

$$V = V_0 \exp(-\lambda \frac{\varphi}{\mu}),$$

(17)

where $V_0$, $\lambda$ and $\mu$ are constant.

Differentiation of the logarithm of dark energy effective density with respect to $\ln(1+z)$ yields

$$\frac{d\ln \rho_{de}}{d\ln(1+z)} = \frac{3}{\rho_{de}} \left[ \dot{\varphi}^2 - 2\xi \left( -H \dot{\varphi}^2 + \dot{H} \varphi^2 + \dot{\varphi}^2 \right) + \left[ \dot{\varphi}^2 - 2\xi \left( -H \dot{\varphi}^2 + \dot{H} \varphi^2 + \dot{\varphi}^2 \right) + \rho_{dm} \right] \right. 
\left. \varepsilon \eta \left( \frac{A_0^2 + 2\eta \rho_{\phi} + \rho_{dm}}{\rho_0} \right)^{-\frac{1}{2}} \right].$$

(18)
To study the behavior of the EoS parameter of dark energy component, we need to the explicit form of $\ddot{\varphi}$ in terms of other quantities which can be deduced from equation of motion as given by (8). On the other hand, Friedmann equation given by (5) now takes the following form

$$(\mu^2 + g)^2 H^4 + 2f(3\mu^2 + g)H^3 + \left[ f^2 - 2l(3\mu^2 + g) + 2\eta\rho_0 g \right] H^2$$

$$+ \left( -2fl + \rho_0\eta f \right) H - 2\eta\rho_0(l - \rho_0) - \rho_0^2 A_0^2 + l^2 = 0$$

(19)

where

$$g = -3\xi H^2 \varphi^2,$$

$$l = \frac{1}{2} \dot{\varphi}^2 + V(\varphi) + \rho_{dm} + \rho_0,$$

and

$$f = -6\xi H \varphi \dot{\varphi}$$

Equation (20) is a quadratic equation in terms of $H^2$ and in principle has four roots for $H$. We show these roots as $h_1$, $h_2$, $h_3$ and $h_4$. After numerical calculation, we found that two of these roots, say, $h_1$ and $h_2$ are unphysical and excluded by observational data. The other two roots, $h_3$ and $h_4$, are physical solutions corresponding to the generalized normal branch (with $\varepsilon = -1$) and the self-accelerating one (with $\varepsilon = -1$). As we will show these solutions have the capability to account for phantom divide line crossing. We introduce a new parameter defined as $s = -\ln(1 + z)$ and rewrite dark energy equation of state parameter as follows

$$w_{de} = -1 - \frac{1}{3} \frac{d \ln \rho_{de}}{d \ln s}.$$ 

(20)

Now, we analyze the behavior of $w_{de}$ versus $s$ to investigate cosmological implications of this scenario. Using equation (18), we see that in the minimal case (with $\xi = 0$) and neglecting warp effect, if we choose the sign of $\varepsilon$ to be negative, remaining terms have suitable combination of signs so that it is possible to cross the phantom divide line by the EoS parameter. In the non-minimal case, however, it is not simple to conclude that there is crossing of the phantom divide line or not just by defining the sign of $\varepsilon$, since in this case non-minimal coupling itself has a crucial role in the dynamics of the equation of state parameter. We consider $\xi$ as a fine-tuning parameter in this case. It is important to note that in the absence of the scalar field on the DGP setup, there is no crossing of the phantom divide line on the self-accelerating branch even in the warped DGP scenario. Nevertheless, normal branch accounts for crossing of the phantom divide
line in this situation. In which follows, to calculate EoS parameter \( \omega(z) \), we define some dimensionless density parameters such as (see the paper by Sahni and Shtanov in Ref. [4] and also Ref. [15])

\[
\Omega_\alpha \equiv \frac{\rho_0^{(\alpha)}}{3\mu^2 H_0^2},
\]

(21)

where we have assumed that \( \rho \) is given by the sum of the energy densities \( \rho_\alpha \) of different components labeled by \( \alpha \) with a constant EoS parameter, \( \omega_\alpha \). Also we define

\[
\Omega_k \equiv -\frac{k}{H_0^2 a_0^2}, \quad \Omega_r \equiv \frac{1}{H_0^2 r_c^2}, \quad \Omega_\lambda \equiv \frac{\lambda}{3\mu^2 H_0^2}, \quad \Omega_\Lambda \equiv -\frac{(5)\Lambda}{6H_0^2},
\]

(22)

where \( r_c \) is DGP crossover distance. In this case, Friedmann equation can be written as follows

\[
\frac{H^2(z)}{H_0^2} = \Omega_k(1+z)^2 + \Omega_{dm}[(1+z)^3 - 1] + \Omega_\phi[(1+z)^3(1+\omega) - 1] + \Omega_\lambda + 2\Omega_r
\]

\[
\pm 2\left[\Omega_r \left(\Omega_{dm}[(1+z)^3 - 1] + \Omega_\phi[(1+z)^3(1+\omega) - 1] + \Omega_\lambda + \Omega_r + \Omega_\Lambda\right)\right]^{1/2}
\]

(23)

where \( \pm \) stands for two possible embedding of the brane in the bulk. The constraint equations for cosmological parameters are given by

\[
1 - \Omega_k + \Omega_\Lambda = \sqrt{\Omega_r + \Omega_{dm} + \Omega_\phi + \Omega_\lambda + \Omega_\Lambda} \pm \sqrt{\Omega_r}
\]

(24)

We can define also \( \Omega_{ki} \) as the present value of the scalar field kinetic energy density \( \frac{1}{2}\dot{\phi}^2 \) over the critical density defined as \( \rho_c = 3H_0^2 \) (with \( 8\pi G = 1 \)). Note that \( \Omega_{ki} \) and the non-minimal coupling parameter are hidden in the definition of the \( \Omega_\phi \). If we change the values of these parameters in appropriate manner, the redshift at which crossing of the phantom divide line occurs will change since it is a model dependent quantity in this respect. In table 1, we have obtained some reliable ranges of the non-minimal coupling to have crossing of the phantom divide line in this setup. Observational data show that crossing of the phantom divide line is occurred in redshift \( 7 z \simeq 0.25 \), so we have obtained the values of \( \xi \) which are correspond to this redshift in the last column of the table 1. We have not excluded the negative values of the non-minimal coupling from our analysis. In fact, these negative values are theoretically interesting, corresponding to anti-gravitation (see [25] for further discussion). The results of the numerical calculations are shown in

\textsuperscript{7}Note that this is a model dependent value but this value is suitable for our purposes in forthcoming arguments.
Table 1: Acceptable range of $\xi$ to have crossing of the phantom divide line with quintessence field (constraint by the age of the universe).

| $\varepsilon$ | $\xi$ | Acceptable range of $\xi$ | The value of $\xi$ for $z=0.25$ |
|---------------|-------|---------------------------|-------------------------------|
| +1 negative   |       | $-0.605 < \xi \leq 0$     | -0.438                        |
| +1 positive   |       | no crossing                | -                              |
| -1 negative   |       | $-0.83 < \xi \leq 0$       | -0.522                        |
| -1 positive   |       | $0 \leq \xi < 0.148$       | 0.124                         |

Figures 1, 2, 3 and 4 for two branches of this DGP-inspired model and with different values of the non-minimal coupling $\xi$. In this figures, the best ranges of the values for $\xi$ to have a reliable model in comparison with observational data are obtained. Note that in all of our numerical calculations we have assumed $\Omega_{ki} = 0.01$, $\Omega_{rc} = 0.01$, $\Omega_m = 0.3$, $\Omega_k = 0$, $\Omega_\Lambda = \Omega_{de} = 0.7$, $\Omega_A = 1$, $A_0 = 1$, $H_0 = 1$, $\mu = 1$ and $\eta = 0.99$.

Figure 1(a) shows that for self-accelerating branch of the model (with $\varepsilon = +1$) and with negative values of the non-minimal coupling, there is a crossing of the phantom divide line by the equation of state parameter (note that self-accelerating branch of this DGP-inspired model accounts for late-time accelerated expansion). As we know, the EoS of a single scalar field in standard relativity never crosses the phantom divide line (see for instance the paper by Vikman in Ref. [8]). Also, in DGP scenario with a canonical scalar field on the brane, there is no crossing of the phantom divide line in the self-accelerating branch of the model (see [16] and [27]), but in our model we have such a crossing due to the existence of the non-minimal coupling and warp effect. In our case, if we choose for instance $\xi = -0.438$, we have a phantom divide line crossing at the point with $s = -0.22$ corresponding to $z = 0.25$ in agreement with observations. However, this crossing occurs for negative values of the non-minimal coupling. On the other hand, here the EoS runs from below $-1$ to above $-1$ (from phantom to quintessence phase) and therefore avoids big-rip singularity.

The effect of the warp factor on the dynamics of the dark energy component can be explained by the variation of $\eta$ parameter as shown in figure 1(b). As this figure shows, for sufficiently small values of $\eta$, equation of state parameter, $\omega$, crosses the phantom divide line in relatively small values of redshift. In figure 1(c) we plotted $\omega_{de}$ for $DGP^{(+)}$ branch.
Figure 1: a) In the self-accelerating branch of the model and with negative values of the nonminimal coupling, the EoS parameter crosses the phantom divide line. For instance, with $\xi = -0.438$ this crossing occurs at $s = -0.22$ or $z = 0.25$. b) The role played by $\eta$ (which is related to warp effect) on the crossing of the phantom divide line. For sufficiently small values of $\eta$, equation of state parameter, $\omega$, crosses the phantom divide line in relatively small values of redshift. For example, with $\xi = -0.438$, the EoS of dark energy crosses $\omega = -1$ line with $\eta = 0.99$ at $s \approx -0.22$ or $z = 0.25$, while for $\eta = 0.50$ this crossing occurs at $s \approx -0.212$ or $z = 0.236$ and for $\eta = 0.10$ this crossing occurs at $s \approx -0.198$ or $z = 0.218$. c) Equation of state parameter, $\omega$, versus $s$ and $\eta$ with $\xi = -0.438$ in a three dimensional plot and within self-accelerating branch. In self-accelerating branch with negative non-minimal coupling, crossing of the phantom divide line occurs from phantom to quintessence phase.
of the model with $\xi = -0.438$ with respect to the parameters $s$ and $\eta$. In this figure, $\eta$ is restricted to the interval $0 < \eta \leq 1$.

In figure 2 we see that for self-accelerating branch of the model and with positive values of the non-minimal coupling, there is no crossing of the phantom divide line.

Figure 3(a) shows that for normal branch (with $\varepsilon = -1$) of the model and with negative values of $\xi$, there is crossing of the phantom divide line also. For instance, with $\xi = -0.522$, we have a crossing at the point with $s = -0.22$ corresponding to $z = 0.25$ in agreement with observational data. The other note is that EoS transits from $\omega_{de} > -1$ to $\omega_{de} < -1$ which is supported by recent observation. Figure 3(b) shows that by decreasing the values of $\eta$, equation of state parameter, $\omega$, crosses the phantom divide line in relatively small values of redshift. In figure 3(c) we plotted $\omega_{de}$ for $DGP^{(-)}$ branch with $\xi = -0.522$ and with respect to the parameters $s$ and $\eta$.

Figure 4(a) shows that EoS parameter of dark energy crosses the phantom divide line in the normal branch ($\varepsilon = -1$) of the model with positive $\xi$. For $\xi = 0.124$ we have a crossing at the point with $s = -0.22$ corresponding to $z = 0.25$. Here, the EoS of dark energy transits from $\omega_{de} > -1$ to $\omega_{de} < -1$ in agreement with the recent observation which show crossing from quintessence to phantom phase. Figure 4(b) shows that by reduction of the values of $\eta$, $\omega$ crosses the phantom divide line in relatively lower values of redshift. In figure 4(c) again we plotted $\omega_{de}$ for $DGP^{(-)}$ branch with $\xi = 0.124$ with respect to the parameters $s$ and $\eta$. 
Figure 2: With a quintessence field on the warped DGP brane, there is no crossing of the phantom divide line in self-accelerating branch of the model with positive values of the non-minimal coupling. This means that inclusion of the warp factor cannot produce crossing of the phantom divide in this case.
Figure 3: a) In the normal branch of the model, there is a crossing of the phantom divide line with negative values of the nonminimal coupling. This crossing occurs from quintessence to phantom phase. For instance, the EoS parameter crosses the $\omega = -1$ line for $\xi = -0.522$ at $s = -0.22$ or $z = 0.25$. b) The role played by $\eta$ factor. For sufficiently small values of $\eta$, equation of state parameter, $\omega$, crosses the phantom divide line in relatively small values of redshift. For example, with $\xi = -0.522$, the EoS of dark energy crosses $\omega = -1$ line with $\eta = 0.99$ at $s \approx -0.22$ or $z = 0.25$, while for $\eta = 0.7$ this crossing occurs at $s \approx -0.203$ or $z = 0.225$ and for $\eta = 0.5$ occurs at $s \approx -0.189$ or $z = 0.208$. c) Equation of state parameter $\omega$, versus $s$ and $\eta$ with $\xi = -0.522$ in a three-dimensional plot and for normal branch of the model. Note that in this case, crossing runs from quintessence to phantom phase.
Figure 4: The same as figure 3 but now with positive values of the non-minimal coupling. a) In this case crossing of the phantom divide line runs from quintessence to phantom phase. For $\xi = 0.124$ this crossing occurs at $s = -0.22$ or $z = 0.25$. b) The role played by $\eta$ on the crossing of the phantom divide line. For sufficiently small values of $\eta$, equation of state parameter crosses the phantom divide line in relatively small values of redshift. For instance, if $\xi = 0.124$, the EoS of dark energy crosses $\omega = -1$ line for $\eta = 0.99$ at $s \approx -0.22$, or $z \approx 0.25$, while for $\eta = 0.7$ this crossing occurs at $s \approx -0.16$ or $z \approx 0.173$ and for $\eta = 0.5$ occurs at $s \approx -0.10$ or $z \approx 0.105$. c) Equation of state parameter, $\omega$, versus $s$ and $\eta$ with $\xi = 0.124$ in a 3-dimensional plot.
2.3 Phantom field

Now we investigate dynamics of a phantom field non-minimally coupled to induced gravity on the warped DGP brane. Most of the techniques and discussions for this case are similar to the previous subsection. The action of the model is

\[ S_\sigma = \int_{\text{brane}} d^4x \sqrt{-g} \left[ -\frac{1}{2} R \sigma^2 + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - V(\sigma) \right], \]  

(25)

where \( \sigma \) is a phantom field. Variation of the action with respect to \( \sigma \) gives the equation of motion of the phantom field

\[ \ddot{\sigma} + 3H \dot{\sigma} - \xi R \sigma - \frac{dV}{d\sigma} = 0. \]  

(26)

The energy density and pressure of this phantom field are given as

\[ \rho_\sigma = -\frac{1}{2} \dot{\sigma}^2 + V(\sigma) + 6\xi H \dot{\sigma} \sigma + 3\xi H^2 \sigma^2, \]  

(27)

and

\[ p_\sigma = -\frac{1}{2} \dot{\sigma}^2 - V(\sigma) - 2\xi (\sigma \ddot{\sigma} + 2\sigma H \dot{\sigma} + \dot{\sigma}^2) - \xi \sigma^2 (2\dot{H} + 3H^2). \]  

(28)

To compare with the results corresponding to the quintessence field, we assume the same type of potential

\[ V(\sigma) = V_0 \exp(-\frac{\sigma}{\mu}) \]  

(29)

where \( V_0, \lambda \) and \( \mu \) are constant. Differentiation of the effective energy density of phantom field with respect to \( \ln(1 + z) \) is given by

\[ \frac{d\rho_{de}}{d \ln (1 + z)} = \frac{3}{\rho_{de}} \left[ -\dot{\sigma}^2 - 2\xi \left( -H \sigma \dot{\sigma} + \dot{H} \sigma^2 + \sigma \ddot{\sigma} + \dot{\sigma}^2 \right) + \left[ \dot{\sigma}^2 - 2\xi \left( -H \sigma \dot{\sigma} + \dot{H} \sigma^2 + \sigma \ddot{\sigma} + \dot{\sigma}^2 \right) + \rho_{dm} \right] \right. \]

\[ \left. \left[ \varepsilon \eta \left( A_0^2 + 2\eta \frac{\dot{\sigma}^2 + V(\sigma) + 6\xi H \sigma \dot{\sigma} + 3\xi H^2 \sigma^2 + \rho_{dm}}{\rho_0} \right)^{-\frac{1}{2}} \right] \right], \]  

(30)

where \( \dot{\sigma} \) can be deduced from equation of motion of \( \sigma \), (26). On the other hand, Friedmann equation now takes the following form

\[ (\mu^2 + g)^2 H^4 + 2f(3\mu^2 + g)H^3 + \left[ f^2 - 2l(3\mu^2 + g) + 2\eta \rho_0 g \right] H^2 \]

\[ + \left( -2fl + \rho_0 \eta f \right) H - 2\eta \rho_0 (l - \rho_0) - \rho_0^2 A_0^2 + l^2 = 0 \]  

(31)

where by definition

\[ g = -3\xi H^2 \sigma^2, \]
Table 2: Acceptable range of $\xi$ to have crossing of the phantom divide line with just one phantom field (constraint by the age of the universe).

| $\varepsilon$ | $\xi$ | Acceptable range of $\xi$ | The value of $\xi$ for $z=0.25$ |
|---------------|-------|---------------------------|-------------------------------|
| +1 negative   | $-0.485 < \xi \leq 0$ | -0.366                     |
| +1 positive   | $0.055 \leq \xi \leq 0.170$ | 0.088                       |
| -1 negative   | no crossing                | —                           |
| -1 positive   | $0 \leq \xi < 0.166$     | 0.166                       |

$l = -\frac{1}{2}\dot{\sigma}^2 + V(\sigma) + \rho_{dm} + \rho_0,$

and

$f = -6\xi H\dot{\sigma}\dot{\sigma}.$

Similar to the last subsection, there is a fourth order equation for $H$ and in principle this equation has four roots. Two of these roots are unphysical but two remaining solutions are physical and corresponding to two branches of solutions in this DGP-inspired model. In table 2, we have obtained some acceptable ranges of non-minimal coupling to have crossing of the phantom divide line in this setup and constraint by the age of the universe (that is, we have assumed that the age of the universe is 13 Gyr).

Figure 5(a) shows that for self-accelerating branch of the model (with $\varepsilon = +1$) and with negative values of the non-minimal coupling, there is a crossing of the phantom divide line by the equation of state parameter. For $\xi = -0.366$, we have a crossing at the point with $s = -0.22$ corresponding to $z = 0.25$ in agreement with observations. Here the EoS runs from above $-1$ to below $-1$ (from quintessence to phantom phase). Figure 5(b) shows that for sufficiently small values of $\eta$, equation of state parameter, $\omega$, crosses the phantom divide line in relatively large values of redshift. In figure 5(c) we plotted $\omega_{de}$ for $DGP^\text{(+)}$ branch with $\xi = -0.366$ and with respect to the parameters $s$ and $\eta$.

Figure 6(a) shows that for self-accelerating branch of this model and with positive values of $\xi$, there is a crossing of the phantom divide line too. For example, with $\xi = 0.088$, we have a crossing at the point with $s = -0.22$ corresponding to $z = 0.25$ in agreement with observational data. As another important point, the EoS parameter transits from
Figure 5: Dynamics of the equation of state parameter with a phantom field on the self-accelerating branch of the model. a) With negative values of the nonminimal coupling, the EoS of dark energy crosses $\omega = -1$ line running from quintessence to phantom phase. For instance, with $\xi = -0.366$, this crossing occurs at $s = -0.22$ or $z = 0.25$. b) The role played by $\eta$ in equation of state of phantom field on the self-accelerating branch of the model. For sufficiently small values of $\eta$, equation of state parameter crosses the phantom divide line in relatively large values of redshift (in contrast to the case with quintessence field). For example, with $\xi = -0.366$, the EoS of dark energy crosses $\omega = -1$ line with $\eta = 0.99$ at $s \approx -0.22$ or $z = 0.25$, while for $\eta = 0.50$ this crossing occurs at $s \approx -0.250$ or $z \approx 0.280$ and for $\eta = 0.10$ this occurs at $s \approx -0.297$ or $z = 0.345$. c) Equation of state parameter, $\omega$, versus $s$ and $\eta$ with $\xi = -0.366$ in self-accelerating branch of the model in a 3-dimensional plot. In the self-accelerating branch with phantom field, $\omega = -1$ line crossing runs from quintessence to phantom phase.
\( \omega_{de} > -1 \) to \( \omega_{de} < -1 \). Figure 6(b) shows that by decreasing the values of \( \eta \), equation of state parameter, \( \omega \), crosses the phantom divide line in relatively large values of redshift. In figure 6(c) we plotted \( \omega_{de} \) for \( DGP^{(+)} \) branch with \( \xi = 0.088 \) and with respect to the parameters \( s \) and \( \eta \). In figure 7 we see that for normal or non self-accelerating branch of the model and with negative values of the non-minimal coupling, there is no crossing of the phantom divide line.

Figure 8(a) shows the phantom divide line crossing of EoS parameter for normal branch \( (\varepsilon = -1) \) of the model with positive values of \( \xi \). Here, the EoS of dark energy transits from \( \omega_{de} > -1 \) to \( \omega_{de} < -1 \). Figure 8(b) shows that by reduction of the values of \( \eta \), \( \omega \) crosses the phantom divide line in relatively smaller values of redshift. In figure 8(c) again, we plotted \( \omega_{de} \) for normal branch of the model with \( \xi = 0.166 \) versus the parameters \( s \) and \( \eta \).
Figure 6: The self-accelerating branch of the model with positive values of the non-minimal coupling in the presence of just one phantom field on the brane. The situation is similar to previous figure and phantom divide line crossing runs from quintessence to phantom phase. a) With positive values of the nonminimal coupling, the EoS of dark energy crosses \( \omega = -1 \) line for \( \xi = 0.088 \) at \( s = -0.22 \) or \( z = 0.25 \). b) The role of \( \eta \) on the crossing of the phantom divide line. As previous figure and contrary to quintessence case, for sufficiently small values of \( \eta \) equation of state parameter crosses the phantom divide line in relatively large values of redshift. For example, with \( \xi = 0.088 \), the EoS of dark energy crosses \( \omega = -1 \) line with \( \eta = 0.99 \) at \( s \approx -0.22 \) or \( z = 0.25 \), while for \( \eta = 0.5 \) this crossing occurs at \( s \approx -0.29 \) or \( z = 0.33 \) and for \( \eta = 0.1 \) this occurs at \( s \approx -0.44 \) or \( z = 0.55 \). c) A 3-dimensional plot of the equation of state parameter, \( \omega \), versus \( s \) and \( \eta \) with \( \xi = 0.088 \).
Figure 7: With a phantom field on the warped DGP brane, there is no crossing of the phantom divide line in normal (non self-accelerating) branch of the model with negative values of the non-minimal coupling. Comparison with figure 2 for a quintessence field on the warped DGP brane, shows the differences between two situation.
Figure 8: In the normal branch of the model with a phantom field on the brane, crossing of the phantom divide line runs from quintessence to phantom phase. This is a general behavior for equation of state of phantom field on the brane independent of the signs of non-minimal coupling and \( \varepsilon \). This is supported by observations too. However, as we have shown in figures 1, 3 and 4, the situation for quintessence field on the warped DGP brane is different and crossing of the phantom divide line depends on the signs of the non-minimal coupling and \( \varepsilon \). That is, while with phantom field on the warped DGP brane, crossing of the phantom divide line runs always from quintessence to phantom phase, in the case of quintessence field on the brane this running of crossing depends on the sign of the non-minimal coupling and \( \varepsilon \).
3 Summary and Conclusion

An alternative approach to explain current positively accelerated phase of the universe expansion is to use a multi-component dark energy with at least one non-canonical phantom field. The analysis of the properties of dark energy from recent observations mildly favor models where $\omega = \frac{\mathcal{E}}{\rho}$ crosses the phantom divide line, $\omega = -1$, in the near past. In this paper, we have considered a scalar field non-minimally coupled to induced gravity on the warped DGP braneworld as a dark energy component and we have investigated the roles played by the non-minimal coupling and the warp effect on the dynamics of the equation of state parameter. In this respect, we have studied the dynamics of equation of state parameter focusing on the crossing of the phantom divide line in this setup. As it is well-known, in the absence of scalar field there is no crossing of the phantom divide line in self-accelerating branch of the DGP setup and this is the case even in the warped DGP scenario. However, in the presence of a scalar field (minimally or non-minimally coupled to induced gravity), it is possible to realize this crossing. We have shown that this crossing is possible for a suitable range of the model parameters and especially for some specific values of the non-minimal coupling and parameter $\eta$ related to the warp effect in this DGP-inspired scenario. In the first stage, we have considered a canonical (quintessence) scalar field non-minimally coupled to the induced gravity on the warped DGP brane. In this case, we have shown that crossing of the cosmological constant line by the EoS parameter of the quintessence field occurs in both self-accelerating and normal branches of the model. For self-accelerating branch of the model (with $\varepsilon = +1$), crossing of the cosmological constant line occurs with negative values of the non-minimal coupling parameter and this crossing runs from phantom to quintessence phase. There is no crossing behavior in the self-accelerating branch of the model with positive values of the non-minimal coupling. On the other hand, for normal branch of the model (with $\varepsilon = -1$), the equation of state parameter of dark energy crosses the phantom divide line with negative values of the non-minimal coupling as well as its positive values, but the crossing behavior is completely different from the former one (the self-accelerating branch). Indeed, in this case the EoS parameter of dark energy crosses the phantom divide line in a different direction; from quintessence to phantom phase and this is supported by recent observations. By investigating the role played by the parameter $\eta$ (which is related to the warp effect) in both branches of the model, we found that decreasing of the effect of $\eta$ factor leads to the result that the EoS parameter of dark energy crosses the
cosmological constant line in relativity smaller values of redshift. We should stress here that negative values of the non-minimal coupling are interesting at least theoretically since they show anti-gravitation. However, recent observational constraints on the values of the non-minimal coupling favor positivity of this factor (see for instance [28] and [29]).

In the next stage, we have considered a phantom field non-minimally coupled to the induced gravity on the warped DGP brane. We have shown that in the self-accelerating branch of the model, crossing of the phantom divide line by the EoS parameter of dark energy occurs with both signs of the non-minimal coupling and this crossing runs always from quintessence to phantom phase. Finally, we have shown that with a phantom field on the warped DGP brane, there is no crossing of the phantom divide line in the normal (non-self-accelerating) branch of the model with negative values of the non-minimal coupling. In the normal branch with phantom field, by considering positive values of the non-minimal coupling parameter, the EoS parameter of dark energy crosses the phantom divide line from quintessence to phantom phase supported by observations. With a phantom field on the self-accelerating branch of the model and for both signs of the non-minimal coupling, reduction of the values of $\eta$ leads to phantom divide line crossing in relatively larger values of redshift, but in normal branch reduction of $\eta$ leads to crossing in smaller values of redshift.

In summary, with positive values of the non-minimal coupling which is physically more relevant, we have shown that: in the self-accelerating branch of this warped-DGP setup with just one quintessence field, crossing the phantom divide line cannot be realized. However, it is possible to realize phantom divide line crossing in the normal branch of the model and this crossing runs from phantom to quintessence phase. With just one phantom field on the brane, it is possible to realize phantom divide line crossing with positive values of the non-minimal coupling in both branches of this DGP-inspired model and this crossing runs from quintessence to phantom phase. Although with a phantom field on the warped DGP brane, crossing of the phantom divide line runs always from quintessence to phantom phase, in the case of quintessence field on the brane this running depends on the sign of the non-minimal coupling and $\varepsilon$. Finally, we should stress that self-accelerating branch of the DGP scenario suffers from ghost instabilities (see [30,31]). Incorporation of new degrees of freedom such as the non-minimal coupling of the scalar field and induced gravity and also warp geometry of the bulk provides a wider parameters space in our setup and this wider parameter space may provide a suitable basis to treat ghost instabilities.
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