Resolution of simple singularities yielding particle symmetries in a space-time

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Abstract

A finite subgroup of the conformal group SL(2,C) can be related to invariant polynomials on a hypersurface in C^3. The latter then carries a simple singularity, which resolves by a finite iteration of basic cycles of deprojections. The homological intersection graph of this cycles is the Dynkin graph of an ADE Lie group. The deformation of the simple singularity corresponds to ADE symmetry breaking.

A 3+1-dimensional topological model of observation is constructed, transforming consistently under SL(2,C), as an evolving 3-dimensional system of world tubes, which connect “possible points of observation”. The existence of an initial singularity for the 4-dimensional space-time is related to its global topological structure.

Associating the geometry of ADE singularities to the vertex structure of the topological model puts forward the conjecture on a likewise relation of inner symmetries of elementary particles to local space-time structure.

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1 Introduction

Relations between finite simple groups, simple singularities and simply laced Lie groups, denoted by the Cartan series A, D and E, are wellknown\textsuperscript{1−6}, but have not been used so far to tie up a particle’s inner symmetry, expressed by an ADE Lie group, explicitly to local space-time geometry near the particle. This geometry is assumed to be given as a vertex structure in a 3 + 1-dimensional topological model of observation, which is based on a network of spatial world tubes evolving in time. The present approach, inspired by string theory\textsuperscript{7}, differs from that by working a priori in 3 + 1 dimensions. Breaking of conformal symmetry is considered here as naturally given by the global topology of space-time and the structure of observation. The following section defines the topological model and discusses democracy between “possible points of observation” (PPOs) in its relation to a possible initial singularity of the space-time and its global topology.

Sec. 3 presents an overview over the conformal action of SL(2,C) and its relatives on the structures of the model.

Sec. 4 resumes known results\textsuperscript{1−5} on the finite subgroups of SL(2,C), associated simple singularities, their resolutions and homological relation of the latter to the ADE Lie groups.

Sect. 5 describes the singularities of the A and D series explicitly on characteristic algebraic hypersurfaces in $\mathbb{C}^3$, following essentially the original work of F. Klein\textsuperscript{5}.

Sec. 6 reviews deformations of simple singularities\textsuperscript{2,3,6} and presents a theorem\textsuperscript{6}, wellknown to algebraic topologists rather than physicists, which allows to decide the existence of certain deformations of the singularities by a graph theoretical method. By the latter the relation to symmetry breaking transitions between the associated ADE Lie groups becomes evident. This together with Sec. 3 has recently encouraged the author to conjecture\textsuperscript{8} a close relation of ADE symmetries and local structure of observation in space-time.

In Sec. 7 the geometric structure of the resolution of singularities of series A is described explicitly. A properly chosen real section can be identified with some vertex of the model presented in Sec. 2. This confirms the conjecture above.

Finally we discuss the perspective of results for this model.
2 Topological model of space-time observation

Starting with \( n + 1 \) vertices and demanding any of them being connected to any other one by a stringy world tube, the lowest dimension allowing the connections to be nonintersecting is \( d = 3 \), which shall be the case. Up to Sec. 6 the vertices are assumed to contain a “possible point of observation” (PPO) each. From any vertex there are \( n \) tubes going out, as the only possibility to exchange information with other vertices. The whole worldsheet shall be closed. This structure is assumed to be embedded in some spacelike section \( M^3 \) of the Lorentzian part \( M^4 \approx M^3 \times R \) of a classical space-time and to evolve in \( M^4 \) according to a synchronized time of one of the PPOs, which will be called “the observer” \( O \). Since all \( n + 1 \) PPOs are equal, anyone of them could observe a likewise system of \( n \) vertices. This fact is called “democracy”. Fig. 1 shows an example of 5 vertices connected to each other and all other ones.

Unification/separation of a single pair of vertices yields separation/unification of \( n \) pairs of points on the worldsheet after pinching/before blowing up \( n \) (closed) strings respectively:

\[
\begin{align*}
\text{unify 2 PPOs} & \Rightarrow \text{n-times pinch & split} \quad S^1 \rightarrow \cdot \rightarrow : \\
\text{split 1 PPO} & \Rightarrow \text{n-times glue & blow up} \quad : \rightarrow \cdot \rightarrow S^1
\end{align*}
\]

In (2.1) the topological transformations of pinching or blowing up of a closed string \( S^1 \) are denoted resp. by \( S^1 \rightarrow \cdot \) and \( \cdot \rightarrow S^1 \), while \( \cdot \rightarrow : \) denotes the separation of a pair of points (:) by splitting a single point (\( \cdot \)) and \( : \rightarrow \cdot \) is the inverse operation, the unification of the pair, gluing it into a single point.

Fig. 1: Net of vertices of PPOs connected by stringy tubes
After O has been chosen and the connecting tubes between all observed vertices removed, the worldsheet connected to the observer can be deformed smoothly into a sphere $S^2$, centered at O, with $n$ punctures, corresponding to the observed vertices. This can be achieved by conformal rescaling of the original worldsheet metric $h_0$ to $h = e^{\Phi}h_0$ by an appropriate choice of some scalar field $\Phi$ on the tubular world sheet. The removed connections between the observed vertices would correspond to $n(n - 1)/2$ handles attached to $S^2$.

In a continuum limit $n \to \infty$ the punctures on $S^2$ are assumed to become dense on the whole $S^2$. Assuming a homogeneous distribution of punctures implies local rotation symmetry on $M^3$.

Since O is at the center and all others PPOs are on $S^2$, one might think that the original democracy is broken by observation. Whether this is so depends however on the global topology of $M^3$. If e.g. $M^3 \cong S^3$ (Fig. 2 a), it is possible to embed O together with $S^2$ in $M^3$. By shrinking $S^2$, O can be exchanged with an other PPO by an arbitrarily small deformation on $M^3$. Such a situation is given in Bianchi IX models, e.g. de Sitter space-time, which is free of an initial singularity.

If on the other hand e.g. $M^3 \cong S^2 \times \mathbb{R}$ (Fig. 2 b), embedding $S^2$ symmetrically in $M^3$ excludes O from $M^3$. Thus it cannot be exchanged with any other PPO. Such a situation appears in Kantowsky Sachs (KS) models, which have an essential singularity (i.e. one which cannot be removed by coordinate transformations) at a point or even an interval of time.

Fig. 2: Democracy a) conserved in $M^3 \cong S^3$, b) broken in $M^3 \cong S^2 \times \mathbb{R}$
Singularities of spherical space-times like those mentioned above have been classified recently in Ref. 10. One is led to the conjecture that singularities in space-time are essentially effected by a breaking of the permutation symmetry between all PPOs (vertices).

Note that the affliction of classical space-times by singularities mentioned above referred to the case of Einstein-Hilbert (EH) theory, given by a Lagrangian $L = R$. For theories with additional higher order curvature invariants in $L$ a singularity which appears for a certain model in EH theory may be evitable$^{11}$, since the strong energy condition may not be valid and then the Hawking-Penrose theorem$^{12}$ does not apply.

A resolution of scale factor singularities$^{10}$ of the classical space-time in the quantum model$^{13,14}$ corresponds$^8$ to blowing up $O$ to $S^3$. Note that quantum theory even more than special relativity (SR) does not admit a separation of $O$ from the space-time. The usual projection $S^2 \to \mathbb{C}$ used to evaluate string theory as an effective QFT in flat 2-space yields a conformal anomaly$^7$ for dimensions $D \neq 26$. Note that this projection simultaneously destroys the symmetry between $O$ and other PPOs.

Finally the alternative approach of implementing the principle of SR at the quantum level by using Quantum Frames of Reference (QFR), yields a quantized space-time$^{15}$. This should be consistent with the present multi-vertex approach by attaching a QFR to each vertex.

3 Action of $\text{SL}(2,\mathbb{C})$ on the model structures

Consider now the group of invertible complex $2 \times 2$ matrices $\text{GL}(2,\mathbb{C})$. The elements of determinant 1 form a normal subgroup $\text{SL}(2,\mathbb{C})$. Division of $\text{GL}(2,\mathbb{C})$ resp. $\text{SL}(2,\mathbb{C})$ by its center $C^*$ resp. $\{ \pm E \}$ yields the group $\text{PGL}(2,\mathbb{C})$ resp. $\text{PSL}(2,\mathbb{C})$, which is the analogue in projective space, and can be identified with the automorphisms of $S^2$ (or the M"obius tranformations Aut $\mathbb{C}$) resp. the proper ortochronous Lorentz group $L^+_\uparrow$ (or the 3-dim. complex orthogonal group). One gets the following diagram with exact vertical sequences and an exact horizontal splitting one:
SO(3,C) = \text{L}^+ = \text{PSL}(2,\mathbb{C}) \\
\text{GL}(2,\mathbb{C}) \\
\text{det}^{-1} \rightarrow \mathbb{C}^* \\
\{\pm E\} \\
\mathbb{C}^* \\
\mathbb{C}^* \\
1 \\
1 (3.1)

By (3.1) transformations of \( C^2 \)-spinors, SL(2,C), project to transformations of bivectors, SO(3,C), and proper orthochronous Lorentz transformations, \( \text{L}^+ \), and they essentially constitute (modulo dilatations and phase factors) conformal transformations, PGL(2,C), on the Riemann sphere or projective plane, which correspond conformally to transformations of a tubular worldsheet\(^7\). Thus the transformation properties of the model of Sec. 2 are fixed, since all its structures are covered by the transformations above. In this paper only the worldsheet as observational structure and the 4-dimensional space-time are considered. (For a transcription to the language of spinors and their relation to the light cone see Ref. 16.)

In the following section we will deal with another very important structural property of SL(2,C), namely its finite subgroups, which are related uniquely to the simple singularities.

### 4 SL(2,C) subgroups and simple singularities

The finite subgroups F of SL(2,C), together with the properties of the related simple singularities\(^{1,2,4}\), are listed in the following table:

| F   | name             | order | \( R(X,Y,Z) \)         | basic graph | Lie G |
|-----|------------------|-------|------------------------|-------------|-------|
| \( C_n \) | cyclic           | \( n \) | \( X^n + Y^2 + Z^2 \)  | A\(_{n-1}\)  |       |
| \( D_n \) | binary dihedral | \( 4n \) | \( X^{n+1} + XY^2 + Z^2 \) | D\(_{n+2}\)  |       |
| \( T \)  | binary tetrahedral | 24    | \( X^4 + Y^3 + Z^2 \)  | E\(_6\)     |       |
| \( O \)  | binary octahedral | 48    | \( X^3Y + Y^3 + Z^2 \)  | E\(_7\)     |       |
| \( J \)  | binary icosahedral | 120  | \( X^5 + Y^3 + Z^2 \)  | E\(_8\)     |       |

Table: Properties of finite SL(2,C)-subgroups and simple singularities
The A and D series are running over integers \( n \geq 2 \). Any F is located in \( SU(2) \), the maximal compact subgroup of \( SL(2,\mathbb{C}) \) and the double cover of \( SO(3,\mathbb{R}) \). That yields (for \( F \neq C_{2j+1} \)) the fibration

\[
1 \to \{ \pm E \} \to SU(2) \to SO(3,\mathbb{C}) \to 1
\]

\[
1 \to \{ \pm E \} \to SL(2,\mathbb{C}) \to SO(3,\mathbb{C}) \to 1
\]  

(4.1)

(4.1) explains the names of the F, since PF is the symmetry group of a regular polyhedron.

Let us now consider the F-invariant polynomials on \( \mathbb{C}^2 \). They are generated by 3 fundamental F-invariants \( X, Y, Z \) subject to 1 constraint \( R(X,Y,Z)=0 \). The normal forms of \( R(X,Y,Z) \) are listed in the table above. Thus we have a quotient map

\[
q : \begin{cases} 
\mathbb{C}^2 & \to \mathbb{C}^3 \\
(z_1, z_2) & \mapsto \left( X(z), Y(z), Z(z) \right)
\end{cases}
\]

(4.2)

The image \( q(\mathbb{C}^2) = \mathbb{C}^2/F \) is a hypersurface in \( \mathbb{C}^3 \) given by the constraint \( R \). For any F a simple singularity (at 0) is just given by (the complex analytic germ of) \( \mathbb{C}^2/F \). For a complete classification list of all hypersurface singularities see Ref. 17. Here we restrict to the simple ones. Their properties have already been examined by F. Klein\(^5\). A modern overview is given by Ref. 2.

In the following we need the notion of a resolution of a singular variety \( X \) (here \( X = \mathbb{C}^2/F \)). This is given by a surjective proper map \( \pi : Y \to X \) from a regular variety \( Y \) to the singular \( X \), such that the regular subset \( X_{\text{reg}} (=X - 0 \text{ in the present case}) \) is diffeomorphic with its pre-image \( \pi^{-1} X_{\text{reg}} \), which is required to be dense in \( Y \). Normally we want \( Y \) to be a minimal resolution, demanding \( \forall \pi', Y' \exists j : Y \hookrightarrow Y' \) such that \( \pi' \circ j = \pi \).

It is now a very special property of the simple singularities that they can be resolved by a finite iteration of a deprojection procedure called blow up or \( \sigma \)-process\(^{1,4} \). For \( \mathbb{C}^3 \) the blow up at 0 is given by the canonical projection \( \beta \) of the tautological line bundle.
\[ \tau = \{(x, l) \in C^3 \times P^2 : x \in l \} \text{ onto } C^3. \] The singular fiber of \( \beta \) is given by the complex projective space \( P^2 \). Then the blow up \( \hat{X} \) of a subvariety \( X \subset C^3 \) at 0 is defined by the closure of the pre-image of \( X_{reg} = X - 0 \) under \( \beta \)

\[
\begin{array}{ccc}
C^3 & \xrightarrow{\beta} & \tau \\
\cup & \cup & \\
X & \xleftarrow{\hat{X}} & \beta^{-1}(X - 0) \\
\cup & \cup_{dense} & \\
X - 0 & \xleftarrow{\beta^{-1}(X - 0)} & 
\end{array}
\] (4.3)

Assume \( \pi = \beta^n : Y \to X \) is a resolution of a simple singularity. The exceptional set, defined as \( E = Y / \pi^{-1}X_{reg} \), is a finite union of homological cycles \( C_1, \ldots, C_r \) with each \( C_k \) isomorphic to \( P^1 \cong S^1 \cong S^2 \). So the cycles are actually 2-spheres. The first Chern class of the normal bundle of \( C_k \) in \( Y \) is \( c_1(C_k, Y) = -2 \), which is also known as the selfintersection number\(^3\) of \( C_k \) in \( Y \).

In fact the cycles \( C_k \) here either intersect in 1 point or do not intersect at all. So we can assign a basic graph to a simple singularity representing each cycle of its resolution as a dot and connecting 2 dots by an edge if the cycles intersect. It turns out that any basic graph of a simple singularity corresponds to the Dynkin graph of a simply laced Lie group. The last two columns of the table above show the basic graph and the corresponding Lie group in Cartan’s notation.

5 Explicit Description of the A and D Series

Let us now consider some instructive examples of the singularities explicitly. First the A series:
The constraint \( R = X^n + Y^2 + Z^2 = 0 \) for the cyclic singularities is equivalent to \( Q = X^n + UV = 0 \) by the substitutions

\[
\begin{align*}
U &= Y + iZ \\
V &= Y - iZ
\end{align*}
\]
respectively

\[
\begin{align*}
Y &= \frac{1}{2} (U + V) \\
Z &= \frac{1}{2i} (U - V)
\end{align*}
\]
In $\text{SL}(2,\mathbb{C})$ the cyclic group of order $n$ is represented as $C_n = \{1, E_n, \ldots, E_{n}^{n-1}\}$ with
\[ E_n = \begin{pmatrix} e^{i\frac{2\pi}{n}} & 0 \\ 0 & e^{-i\frac{2\pi}{n}} \end{pmatrix}. \]

Obviously $X = z_1 z_2$, $U = i z_1^n$, $V = i z_2^n$ are 3 independent $C_n$-invariants, satisfying $Q=0$. Thus the 3 generic $C_n$-invariant polynomials with $R=0$ are
\[
X = z_1 z_2 \\
Y = \frac{i}{2} (z_1^n + z_2^n) \\
Z = \frac{1}{2} (z_1^n - z_2^n)
\]

(5.1)

Now let us go over to the D series:
The binary dihedral group $D_n$ is represented in $\text{SL}(2,\mathbb{C})$ by combining $C_{2n}$ with
\[ B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \]

Note that $B^2 = E_{2n}^n = \{-E\} \in C_{2n}$. It is now easy to verify that
\[
X = (z_1 z_2)^2 \\
Y = \frac{i}{2} (z_1^{2n} + z_2^{2n}) \\
Z = \frac{1}{2} z_1 z_2 (z_1^{2n} - z_2^{2n})
\]

(5.2)

are 3 independent $D_n$-invariant polynomials, which satisfy just the constraint $R = X^{n+1} + XY^2 + Z^2 = 0$ given above.
The lowest order nontrivial group of this series is $D_2$. Its projection onto $L_4$ yields $PD_2 = V_4 \equiv \{E, C, P, T\}$, the Kleinian 4-group, which is the unique abelian group of order 4 with $C^2 = P^2 = T^2 = E$ and $CPT = E$.

Similar like the invariants (5.1) and (5.2) for the A resp. D series one can construct the invariants of type E. The results are already contained in the original work of F. Klein. The following section is devoted to the behavior of the ADE singularities under algebraic perturbations.
6 Deformations and symmetry breaking

For the singularity given by $R(X,Y,Z) = 0$ let us define (following Refs. 2, 3 and 6) small deformation (fiber)s in an $\varepsilon$-neighbourhood of 0 by

$$M_{S,t} = \{(X,Y,Z) \in B^3(\varepsilon) \subset C^3 \mid R(X,Y,Z) + tS(X,Y,Z) = 0\} \quad (6.1)$$

with $t$ small w.r.t. $\varepsilon$. Clearly $M_{S,0}$ is the singularity itself. $M_{1,t}$ is the Milnor fiber, which characterizes the resolution, since $H_2(M_{1,t}, Z)$ is just generated by the cycles $C_k$ introduced in sec. 4. Following theorem$^6$ holds:

*Given a simple singularity and independently a Dynkin graph $G$ of ADE type (i.e. each connected component of $G$ is of ADE type). Then the following properties are equivalent:

I. There exists a deformation $M_{S,t}$ with only simple singularities, such that the combination of these corresponds exactly to $G$.

II. $G$ is a subgraph of the basic graph of the singularity.

Similar theorems hold also for larger classes of singularities$^6$. The powerful implication of the theorem above is that it allows to decide just graph theoretically the existence of both, deformations of simple singularities and corresponding symmetry breaking transitions$^8$ between the associated ADE Lie groups (see example in Fig. 3).

Fig. 3: Example of possible symmetry breaking

Considering also the actions of SL(2,C) on space-time and tubular worldsheet according to Sec. 3, the author has conjectured$^8$ an intimate relation between space-time and observational structure on the on hand and internal (Yang Mills like) symmetries on the other.
In the following section we describe the geometric structure of some simple singularities explicitly and relate them to the structure of the model of Sec. 2, thus confirming the conjecture above.

7 Geometry of the singularities in the model

Let us consider the geometry of the most simple singularity $A_1$ given by the constraint

$$-X^2 + (iY)^2 + (iZ)^2 = 0. \quad (7.1)$$

This surface is a double cone with double point singularity at 0, the 2 cones corresponding to $X = \pm R$. It is resolved by a single blow up (4.3) yielding a surface isomorphic to $T^*P^1$ and the singular fiber is given by the zerosection $P^1$ (see Fig. 4). $T^*P^1$ is the unique line bundle of Chern class -2 and thus it is the standard building block for the resolutions of higher ADE singularities involving more fundamental cycles.

Fig. 4: Resolution of $A_1$
More generally an $A_n$ singularity is given by a surface $X^{n+1} = R^2$, with $R = i(Y^2 + Z^2)^{1/2}$ like above. This surface is built by $n + 1$ cones given by $X = e^{2\pi ik/n+1} R^{2/n+1}$ with $k = 1, \ldots, n + 1$, their tops joining in the singularity 0. The resolution can be found by $n$ blow ups (4.3), with $n - i$ cones left after the $i$th blow up. The resolution is given by a vertex of $n + 1$ joining tubes $T^*P^1$ rather than cones. The singular fiber of the resolution is given by a joining chain of $n$ spheres $P^1 \vee \cdots \vee P^1$.

This is now demonstrated in detail for $n = 2$ (see Fig. 5). The surface is given by 3 cones joining in a point. The first blow up yields an $X$ like a $T^*P^1$ tube with a cone attached to the image of the zero section $P^1$ in a point $\hat{0}$. This is however not yet a resolution, since $\beta^{-1}X_{reg}$ is singular in $\hat{0}$. Applying (4.3) once more $\hat{0}$ is blown up to a further $P^1$ intersecting the first one just in a point. This yields the resolution, given by 3 regularly joining $T^*P^1$ tubes. The singular fiber is the join $P^1 \vee P^1$ of 2 spheres.

Fig. 5: Resolution of $A_2$
In order to relate these complex structures to the real structures of our topological model, let us take a proper real section, such that $P^1 \cong S^2$ becomes restricted to $S^1$ and the complex cotangent bundle $T^*P^1$ restricts to the real cotangent bundle $T^*S^1 \cong S^1 \times \mathbb{R}$. Thus we can identify the tubular strings between PPOs as real sections of resolutions of $A_1$ singularities, which themselves might have been created each by 2 points of the worldsheet identifying to a double point. So $A_1$ singularities and resolution appear naturally in processes like (2.1).

Until now we have considered any single PPO as vertex with $n$ tubes attached. Note however that this point of view can be transformed into a different one by a smooth deformation of the worldsheet replacing any $n$-tube vertex replaced by $n-k+1$ vertices with maximally $k$ external tubes ($3 \leq k \leq n$). Taking such a vertex as PPO, the associated 2-sphere would have only $k$ punctures. The most refined decomposition is the tree level with $k = 3$. At this level any vertex corresponds to a real section of an $A_2$ resolution. Generally at level $k$ any maximal vertex corresponds to an $A_{k-1}$ resolution.

However at any refined level democracy among vertices is broken, since no longer each of them is connected to any other one. Democracy of PPOs can be retained if we consider only a whole cluster of $n-k+1$ “confined” vertices, rather than a simple vertex, as PPO.

Note that the resolution by passing in (6.1) from $M_{1,0}$ to $M_{1,t}$ with $t \neq 0$ is similar like in QFT shifting particles off the singular light cone by giving them a (small) nonzero mass. In fact the geometry of the $A_1$-resolution corresponds to a complex mass shell.

We had chosen real sections, like e.g. in (7.1), such that they correspond to the worldsheet structure in a synchronized spacelike section $M^3$. Note however that the complex structure of the resolution contains various possibilities for real sections and choices of signatures. Recently it was pointed out, how a change of space-time signature may be possible and interpreted, where instantons play a crucial role. A full space-time analysis of the model of Sec. 2 provides an instanton interpretation of geometric structures related to ADE singularities.
8 Discussion

Using a specific model of observation with special emphasis to microscopic democracy of PPOs (“possible points of observation”) in space-time, conjectures about an intimate relation between inner symmetries of the Yang Mills type and local structure of observation in space-time near a particle are confirmed. Explicitly the resolution of singularities of the A series appear naturally as vertices of the tubular network which is transporting the information between PPOs. The A\textsubscript{1} singularity and its resolution (corresponding to the SU(2) symmetry of electroweak interaction) appears in any unification/separation process of (PPO) vertices as pinch/blow up of specific strings on the tubular worldsheet.

Abelian U(1) factors are not considered in the ADE theory but might be represented by (counter)clockwise twistings of the string-tubes around their axis according to the polarisation of a lightwave propagating along some of them.

The resolutions of the A\textsubscript{k-1} singularities for \(k \geq 3\) (corresponding to a SU(\(k\)) Yang Mills symmetry) have real sections isomorphic to a \(k\)-tube vertex. Confinement might be related to the necessity to consider clusters rather than single vertices as PPOs in order to restore democracy at the most refined level \(k = 3\).

We have coined the term PPO for an undetermined location somewhere in the interior at a vertex of an extended observer carrying the frame of reference. In contrast to a quantization of space-time into disconnected parts\textsuperscript{15}, our model assumes (at least in the initial phase considered here) all PPOs to be located in a common connected region, namely the interior of the closed tubular worldsheet. Regions connected only by stringtubes with diameter very much below Plancklength \(L_{Pl}\) should be considered as quasidisconnected, which yields the correspondence to the QFR (Quantum Frame of Reference) approach\textsuperscript{15}.

String pinchings according to (2.1) can be performed equivalently around interior and exterior of the tubular worldsheet w.r.t. the embedding space, both yielding the same unification of vertex tubes. Note the resemblance of the present approach to knot theory and neural networks.
Assuming a mean extension of the vertices of only few orders of magnitude above $L_{Pl}$ and a mean stringtube diameter of order $L_{Pl}$ or below, the PPOs become sharp only for $L_{Pl} \to 0$ in the macroscopic continuum limit $n \to \infty$. In this limit there should appear a connection to the gauge bundle framework.

By the theorem\textsuperscript{6} of Sec. 6, the Dynkin graphs describe the possible ADE symmetry breaking transitions and deformations of singularities. Notably in our model all this happens a priori in a space-time of $D = 4$ rather than decending from $D = 26$.

For an evolutionary process there remains therefore the crucial question, whether a classical space-time (initial) singularity appears. This question is closely related to the question of PPO democracy via the global topology of $M^3$. Furthermore a resolution of a scale-factor singularity corresponds to a deprojection process, similar to those involved in the ADE resolutions. All that indicates a unified description of both the singularities \textit{in} and \textit{of} space-time. This and a more detailed evaluation of the physical predictions of the model will be topic of a forthcoming paper.

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Figure captions:

Fig. 1: Net of vertices of PPOs connected by stringy tubes

Fig. 2: Democracy a) conserved in $M^3 \cong S^3$, b) broken in $M^3 \cong S^2 \times R$

Fig. 3: Example of possible symmetry breaking

Fig. 4: Resolution of $A_1$

Fig. 5: Resolution of $A_2$

Table caption:

Table: Properties of finite SL(2,C)-subgroups and simple singularities
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http://arxiv.org/ps/gr-qc/9404032v1
This figure "fig1-2.png" is available in "png" format from:

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