Effect of internal magnetic flux on a relativistic spin-1 oscillator in the spinning point source-generated spacetime

Abdullah Guvendi 1*, Semra Gurtas Dogan 2†
1Department of Basic Sciences, Faculty of Science, Erzurum Technical University, 25050, Erzurum, Turkey
2Department of Medical Imaging Techniques, Hakkari University, 30000, Hakkari, Turkey

Abstract

We consider a charged relativistic spin-1 oscillator under the influence of an internal magnetic flux in a 2+1 dimensional spacetime induced by a spinning point source. In order to analyze the effects of the internal magnetic flux and spin of the point source on the relativistic dynamics of such a vector field, we seek a non-perturbative solution of the associated spin-1 equation derived as an excited state of Zitterbewegung. By performing an analytical solution of the resulting equation, we determine exact results for the system in question. Accordingly, we analyze the effects of spin of the point source and internal magnetic flux on the relativistic dynamics of the considered test field. We see that the spin of such a field can be altered by the magnetic flux and this means that the considered system may behave as a fermion or boson according to the varying values of the magnetic flux, in principle. We observe that the internal magnetic flux and the spin of the point source impact on the relativistic energy levels and probability density functions. Also, our results indicate that the spin of the point source breaks the symmetry of the energy levels corresponding to particle-antiparticle states.

Keywords: Aharonov-Bohm effect; Spin-1 oscillator; Magnetic flux; Topological defect; Spinning cosmic string.

1 Introduction

Analyzing the effects of curved spaces on the relativistic dynamics of the quantum mechanical systems is of great importance because such investigations may serve to build a complete theory combining the fundamental areas such as the quantum (ℏ), relativity (c) and gravity (G) theories of modern physics. In the mathematical framework, one of the useful ways to determine the effects of spacetime topology and gravity on the relativistic quantum mechanical systems is to use the relativistic wave equations such as the Dirac equation, Duffin-Kemmer-Petiau (DKP) equation, Klein-Gordon equation and fully-covariant many-body equations [1–11]. Especially for spinning particles, these equations are different possible quantum states of the same classical system because they can be derived through the canonical quantization of the action for classical spinning particle with Zitterbewegung [12]. One can see that the covariant spin-1 equation can be derived as an excited state of Zitterbewegung by applying the same quantization procedure provided that the spinor is a symmetric spinor of rank-two [12]. In its generalized form, this equation possesses spin algebra constructed through Kronecker product of the generalized Dirac matrices with the unit matrix and accordingly it is quite different from the Dirac equation. Briefly, the corresponding spin-1 field can be constructed by the direct product of two symmetric Dirac fields but the Dirac equation and the covariant spin-1 equation are equivalent to each other in the 1 + 1 dimensions (except for the mass factors). This is because we get rid of the spin effects in 1 + 1 dimensions. However, the aforementioned spin-1 equation corresponds to the spin-1 sector of the well-known DKP equation in 2 + 1 dimensions [12–15]. That is, the spin-1 equation provides serious mathematical convenience for analyzing the dynamics of the corresponding quantum mechanical systems possessing natural dynamical symmetry. It is known that one of the exactly soluble systems in the non-relativistic quantum mechanics is the harmonic oscillator and this system is described by adding a quadratic interaction potential into the free Schrodinger Hamiltonian [16]. However, generalization of a quantum oscillator is not unique in relativistic quantum theory [17]. The Dirac oscillator (DO) was described by modifying the momentum operator(s) [18] of the free Dirac Hamiltonian and it has been shown that this non-minimal interaction has important potential for many applications [19–25]. At first look, one can see that this modified Dirac equation is linear in both momentum and coordinate and its solutions yield an ordinary quantum oscillator plus a very strong spin-orbit coupling in the non-relativistic limit [18]. Hence, it is called as DO. By determining the electromagnetic potential corresponding to the DO interaction it was shown that the DO corresponds to a real physical system describing the interaction of the anomalous magnetic moment with a linearly growing electric field [20]. Due to this result, the DO system was regarded as an alternative model for the quarks in Quantum Chromodynamics [20–27]. Through some analogies, other relativistic oscillators have introduced and studied in many areas of modern physics [7–10, 25, 28–30]. For instance, such systems are used to estimate mass spectra for mesons and baryons [23–27]. Furthermore, relativistic oscillators are excellent mathematical tools for determining the effects of the background on the corresponding physi-
cal systems [31–34] because they are exactly soluble systems (in general) besides the low-energy bound state systems in Quantum Electrodynamics [1, 35] (see also [36]). Recently, by taking into account the spin algebra, the relativistic spin-1 oscillator has been introduced by adding a non-minimal interaction term into the mentioned relativistic spin-1 equation and the results of a number of applications were announced [37–41]. Moreover, if one gets rid of the spin effects, it can be seen that the announced results are exactly the same (except for the mass factors) with the previously obtained results for a 1+1-dimensional DO system [42] and for the singlet quantum state of a fermion-antifermion pair interacting through the DO coupling [23]. The other applications of the relativistic spin-1 equation can be found in the Refs. [43–46]. However, we could not find any published results based on the interaction of a charged relativistic spin-1 oscillator with the electromagnetic fields in the current literature. Hence, in this contribution, our purpose is to analyze the effect of internal magnetic flux on a relativistic spin-1 oscillator in the 2+1-dimensional background geometry spanned by a spinning point source [47]. This also allows us to discuss the influence of the background on such a vector test field. In 2+1 dimensions, the solutions of Einstein equations for static one-body and many-body sources can be obtained by applying geometrical and analytical methods [47–50]. In the one-body case, it was shown that the surface metrically lies outside the origin [33, 47–50]. Stationary spinning source and static point source generated spacetime backgrounds were first introduced in 2+1 dimensions through the mentioned solutions [47–50] and then these solutions were extended to 3+1 dimensions where there is a dynamical symmetry [47–50]. That is, dynamics of a vector field remains invariant under the Lorentz boost on the added third spatial coordinate (z-axis in the cylindrical coordinates) [31]. In 2+1 dimensions, the source locates at the spatial origin (intersection point of the string with the space-like subspace) and 2+1-dimensional cosmic string-generated geometric backgrounds are known also as point source-induced spacetime backgrounds [47–50]. These objects are responsible for some non-trivial topologies because such an induced spacetime is not flat globally even though it is flat locally. Accordingly, the cosmic strings or point sources change the dynamics of the quantum systems by altering the symmetry of the spacetime. Such a spacetime background can be characterized by two parameters. First of them is the angular deficit parameter (α ∈ (0, 1]) that relates with the tension of the source and the other is the spin parameter (σ) of the source. Furthermore, determining the effects of such a non-trivial topology on the relativistic dynamics of the physical systems are very important in the context of cosmology and particle physics. In addition, a point source-generated background can be useful to discuss the dynamics of particles in a graphene-like structures described by a surface of constant negative curvature [33]. Here, we should note that such a point source-generated spacetime describes an an-}


dimensional cosmic string-generated geometric backgrounds.
sions, the generalized form of the spin-1 equation can be written as the following [12–15]

\[
\{ \beta^\mu \mathcal{D}_\mu + i\tilde{m} \mathcal{I}_4 \} \psi(x) = 0, \quad \beta^\mu = \frac{1}{2} (\gamma^\mu \otimes \mathcal{I}_2 + \mathcal{I}_2 \otimes \gamma^\mu), \quad \tilde{m} = \frac{mc}{\hbar},
\]

\[
\mathcal{D}_\mu = \partial_\mu + i\frac{\gamma_5}{4}\hat{\Omega}_\mu, \quad \hat{\Omega}_\mu = \Omega_\mu \otimes \mathcal{I}_2 + \mathcal{I}_2 \otimes \Gamma_\mu,
\]

(2.1)
in which the Greek index \( \mu \) refers to the coordinates of the curved spacetime, \( \gamma^\mu \) are the space-dependent Dirac matrices, \( \Omega_\mu \) are the spinorial affine connections for spin-1/2 field, \( \psi(x) \) is the spin-1 field, with mass of \( m \), constructed through Kronecker product of symmetric two Dirac fields [12], \( \hbar \) is the reduced Planck constant, \( c \) is the speed of light, \( x \) is the spacetime position vector, the objects \( \mathcal{I}_2 \) and \( \mathcal{I}_4 \) stand for the \( 2 \times 2 \) and \( 4 \times 4 \)-dimensional identity matrices, respectively, and the symbols \( \otimes \) are used to indicate the Kronecker (direct) products. Now, to derive the corresponding form of the fully-covariant spin-1 equation for a charged spin-1 oscillator in the 2+1 dimensional spinning cosmic string spacetime, let we start with by introducing the 2+1 dimensional spacetime generated by a spinning point source. This background can be represented through the following spacetime interval [1, 47–50]

\[
ds^2 = c^2 dt^2 + 2\varepsilon \omega dtd\phi - dr^2 - (\alpha^2 r^2 - \omega^2) d\phi^2,
\]

\[
\alpha = \left( 1 - \frac{4G\mu_s}{c^2} \right), \quad \omega = \frac{i\,\epsilon \, G}{c}
\]

(2.2)

Here, the parameter \( \alpha \), depending on the mass density \( \mu_s \) of the string, relates with the angular deficit in the background and the letter \( G \) stands for the well-known Newtonian gravitational constant. Provided 0 < \( \frac{4G\mu_s}{c^2} \) < 1, the spacetime background is not flat when viewed globally although it is flat locally. As we mentioned before, the corresponding surface becomes conical if 0 < \( \alpha \) < 1 and it becomes anti-conical when \( \alpha > 1 \) [47–50]. Also, the \( \omega \) refers to the spin of the point source and depends on the angular momentum density \( (j_s) \) of the cosmic string. It is clear that the Eq. (2.2) describes a flat Minkowski spacetime in terms of polar coordinates with the signature (+,−,−,−) if \( j_s = 0 \) and \( j_s = 0 \). This means that the mass density and spin of the cosmic string source alter the topology and accordingly it changes the symmetry of the background where the angle \( \phi \) runs in the interval \( 0 \leq \phi < 2\pi \). However, it should be said that the \( \alpha > 1 \) does not make any sense in the context of general relativity. As is mentioned, dynamics of a physical system in a cosmic string spacetime can be investigated in 2+1 dimensions without loss of generality because the dynamics of the considered system remains invariant under the Lorentz boost on the added third spatial coordinate \( (z \text{ axis in the cylindrical coordinates}) \). The spinor field must be described locally by using the rule of local Lorentz transformations. Thus, we need to find a local reference frame according to the observers. We can construct the reference frame through a non-coordinate basis \( \theta^\mu = e_a^\mu(x)dx^a \), in which the components \( e_a^\mu(x) \) are the vierbeins and satisfy the following condition \( g_{\mu\nu}(x) = e_a^\mu(x)\epsilon_b^\nu(x)\eta_{ab} \) where \( \eta_{ab} = \text{diag}(+,−,−,−) \) represents the flat Minkowski metric. Also, by using the relation: \( dx^\mu = e_a^\mu(x)\theta^a \) we can find the inverse of vierbeins. Here, we should note that the vierbeins and their inverses obey the following two conditions: \( e_a^\mu(x)e_b^\nu(x) = \delta^\mu_b \) and \( e_a^\mu(x)e_b^\nu(x) = \delta^a_b \) where \( \mu, \nu = t, r, \phi \) and \( a, b = 0, 1, 2 \). By this way, one can obtain the inverse of vierbeins as (see also [1])

\[
e^t_0(x) = \frac{1}{2}, \quad e^t_1(x) = 1, \quad e^t_2(x) = -\frac{\omega}{c}, \quad e^\phi_2(x) = \frac{1}{\alpha^t}.
\]

(2.3)

Now, we need to determine the \( \beta^\mu \) matrices as well as the spinorial affine connections \( \Omega_\mu \) for a spin-1 field in the Eq. (2.1). This requires first to construct the space-dependent Dirac matrices. To acquire this, we can use the following relation: \( \gamma^\mu = e_a^\mu \gamma^a \), in which \( \gamma^a \) are the space-independent Dirac matrices that can be written in terms of the Pauli matrices \( (\sigma^x, \sigma^y, \sigma^z) \) in 2 + 1 dimensions. According to the signature of the metric in Eq. (2.2), space-independent Dirac matrices can be chosen as \( \gamma^t = \sigma^z, \gamma^r = i\sigma^y, \gamma^\phi = \frac{1}{\alpha^t} \sigma^x \). (2.4)

To determine the spinorial connections \( \Omega_\mu \), in the Eq. (2.1), we can use the obtained spinorial affine connections for Dirac fields in the Ref. [1],

\[
\Gamma_t = 0, \quad \Gamma_r = 0, \quad \Gamma_\phi = \frac{i\alpha}{2} \sigma^x.
\]

(2.5)

Here, it is clear that there is only one non-vanishing component of the spinorial affine connections and this one is
found as $\Omega_\phi = i\alpha \text{ diag}(1, 0, 0, -1)$. As a next step, we can introduce the oscillator coupling and this can be acquired through the following non-minimal substitution [37]

$$\begin{align*}
\partial_r \rightarrow \partial_r + K_i \eta_r, & \quad K = \frac{m_\text{eff}}{\omega}, \\
\bar{\eta} = \sigma^2 \otimes \sigma^2 = \text{ diag}(1, -1, -1, 1), & \quad (2.6)
\end{align*}$$

where $\omega_\phi$ is the oscillator frequency (coupling strength) [41]. Also, the presence of an internal magnetic flux can be taken into account through the angular component of the 3-vector potential, $A_\phi = \frac{\phi_\text{int}}{2\pi c}$, where $\phi_\text{int}$ is the internal magnetic flux [51]. Here, we seek the bound state solutions for the system in question and accordingly we assume the interaction is time-independent. This allows us to factorize the spin-1 field as the following

$$\Psi(t, r, \phi) = e^{-i\omega t} e^{i\eta \phi} \begin{pmatrix} \psi_1(r) \\ \psi_2(r) \\ \psi_3(r) \\ \psi_4(r) \end{pmatrix}, \quad (2.7)$$

in which $\omega$ is the relativistic frequency and $s$ is the spin of the test field in question.

3 Altered spectrum

In this section, we derive a set of coupled equations for a charged relativistic spin-1 oscillator exposed to magnetic flux in the 2+1 dimensional spacetime generated by a spinning point source and arrive at second order wave equation in exactly soluble form. By substituting Eq. (2.4), Eq. (2.5), Eq. (2.6) and Eq. (2.7) into the Eq. (2.1) we obtain a set of coupled equations which can be rewritten as the following

$$\begin{align*}
\frac{\omega}{c} \psi_+ (r) - \tilde{m} \psi_- (r) - \delta(r) \psi_0 (r) &= 0, \\
\frac{\omega}{c} \psi_- (r) - \tilde{m} \psi_+ (r) - (\partial_r - \kappa r) \psi_0 (r) &= 0,
\end{align*}$$

where $\delta(r) = \frac{\epsilon + \Phi \omega}{\tilde{m} \rho}$, $\psi_\pm (r) = \psi_1 (r) \pm \psi_3 (r)$ and $\psi_0 (r) = \psi_2 (r) + \psi_3 (r)$ since the spinor is a symmetric spinor of rank-two. It can be seen that the first equation in the set of equations, in above, is in fully algebraic form and this makes possible to obtain easily the following expressions for the components $\psi_+$ and $\psi_-$

$$\begin{align*}
\psi_+ &= - \frac{\tilde{m} \delta (r)}{\tilde{m} \rho (\tilde{m} \rho - \tilde{m} m)} \psi_0 - \frac{(\partial_r - \kappa r)}{\omega} \psi_0 + \frac{\delta (r)}{\omega} \psi_0,
\psi_- &= - \frac{\delta (r)}{\tilde{m} \rho (\tilde{m} \rho - \tilde{m} m)} \psi_0 - \frac{(\partial_r - \kappa r)}{\omega \tilde{m} \rho} \psi_0,
\end{align*}$$

where $\tilde{m} = \frac{\tilde{m}}{m}$, $\Phi = \frac{\phi_\text{int}}{2\pi c}$. Accordingly, we can acquire a wave equation $\psi_0 + \frac{1}{\tilde{m}} \psi_0 + \tilde{Q} \psi_0 = 0$, in which

$$\tilde{Q} = \left[ \frac{\tilde{m} \delta (r)}{\tilde{m} \rho} - (\tilde{m} \rho)^2 + \frac{2\kappa r \omega + \omega}{\tilde{m} \rho} \delta (r) - \lambda (r) \right],$$

and prime ‘ denotes derivative with respect to $r$. By defining a dimensionless independent variable, $x = \kappa r^2$, and then considering an ansatz function, $\psi_0 (x) = x^{-1/2} \psi (x)$, to get rid of the term $\propto \psi' (x)$ one arrives at the following wave equation for the unknown function $\psi (x)$

$$\psi'' (x) + \left[ \frac{q_1}{x} + \frac{q_2 - q_1^2}{x^2} - \frac{1}{4} \right] \psi (x) = 0, \quad (3.2)$$

where

$$q_1 = - \frac{1}{2} + \tilde{m} (\tilde{m} \rho^2 + 2 \kappa) - 2 \tilde{m} \rho (\omega \rho + s + \Phi),$$

and

$$q_2 = \frac{s + \Phi + \omega \rho}{2 \alpha}.$$ 

Solution function of this wave equation can be expressed in terms of the confluent hypergeometric function $1_{\Phi}$ as the following

$$\psi (x) = C x^{1/2 + q_2} e^{-\frac{1}{2} x} 1_{\Phi} (\tilde{e}, \tilde{r}; x), \quad (3.3)$$

where $C$ is an arbitrary constant, $\tilde{e} = \frac{1}{2} + q_2 - q_1$ and $\tilde{r} = 1 + 2q_2$. For large values of the argument $\lambda$, the $1_{\Phi} (\tilde{e}, \tilde{r}; x)$ function behaves as

$$1_{\Phi} (\tilde{e}, \tilde{r}; x) \approx \frac{\Gamma (\tilde{r})}{\Gamma (\tilde{e})} e^{x - \frac{1}{2} x - \frac{1}{2} x^2} \left[ 1 + O (|x|^{-1}) \right],$$

where $\Gamma$ stands for the Gamma functions [56]. That is, it diverges as $x \sim x$. Here, our purpose is to determine regular solution to the wave equation and this requires that $\tilde{e} = - n$ where $n$ is the radial quantum number ($n = 0, 1, 2, \ldots$). At that rate, the solution function becomes polynomial of degree $n$ according to $x$ [56]. This termination allows us to determine the quantization condition for the considered system and accordingly we obtain an exact result for the relativistic frequency (in closed-form)

$$\omega_n / c = \frac{n \tilde{e}}{m \alpha} \tilde{\tilde{\eta}}^{-1}$$

$$\pm \tilde{m} \tilde{\eta} \left[ 1 + \frac{4\kappa \tilde{\eta}}{m \tilde{\eta}} \right] \tilde{\tilde{\eta}} + \frac{n^2 \tilde{\eta}^2}{m^4 \alpha^2 \tilde{\tilde{\eta}}^{-1}},$$

where $\tilde{\eta} = [1 + \frac{4\kappa \tilde{\eta}}{m \tilde{\eta}}]$, $\tilde{\tilde{\eta}} = [\tilde{\eta} \tilde{\eta} - \hat{S}]$, $\hat{S} = s + \Phi$ and $n^* = n + 1 + \frac{\tilde{\eta}}{m \tilde{\eta}}$. Here, it can be seen that $\tilde{\eta}$ and $\tilde{\tilde{\eta}}$ are dimensionless quantities. Also, writing an explicit expression for the relativistic energy of the considered system may be also useful to compare the findings with the previously announced results. The obtained energy spectra can be written as the following form

$$\begin{align*}
\mathcal{E}_n &= \omega_n \frac{\tilde{\eta}}{\alpha} \tilde{\tilde{\eta}}^{-1}, \\
&\pm \frac{m c^2}{\sqrt{1 + [\frac{4\kappa \tilde{\eta}}{m \tilde{\eta}} + n^*] \tilde{\tilde{\eta}} + \frac{\omega_n^2 \tilde{\eta}^2}{m^4 c^4 \alpha^2 \tilde{\tilde{\eta}}^{-1}}}, \quad (3.4)
\end{align*}$$

It is clear that this energy spectra becomes $\mathcal{E}_n = \pm m c^2 [1 + \frac{\omega_n \tilde{\eta}}{m \tilde{\eta}} (n + 1)]^{1/2}$ if $s = 0$, $\phi_B = 0$ and $\omega = 0$.
In this case, it can be seen that the considered system never stop oscillating even when it reaches the ground state \((n = 0)\) and the relativistic energy of such a massive vector field becomes \(E \sim \pm mc^2\) if \(\omega_v = 0\). In \(s = 0, \phi_B = 0\) and \(\varpi = 0\) case, we observe that the obtained spectrum are same (except for mass factor) as previously announced result for the singlet quantum state of a fermion-antifermion pair holding together through \(DO\) interaction \([23]\) and it is also same as the previously announced result for a one-dimensional \(DO\) \([42]\) (see also \([57]\)). However, the energy spectra in the Eq. \((3.4)\) is quite different from the obtained non-perturbative result for a \(DO\) system in the spinning cosmic string spacetime \([31]\) (see also \([58]\)) when \(s \neq 0\) and \(\varpi \neq 0\). Also, as is expected, our results show that the behaviour of a relativistic spin-1 oscillator cannot be same as the behaviour of a relativistic scalar oscillator in the cosmic string spacetime (see Eq. \((3.4)\) and \([59]\)). The non-perturbative spectrum in the Eq. \((3.4)\) allows us to discuss the effect of the internal magnetic flux and spacetime parameters besides the oscillator frequency on the relativistic spin-1 field. Now, we can discuss the results in detail. Let we start by considering the \(\varpi = 0\) and \(\Phi = 0\) case. Even in this case, we can observe that spin of the considered vector field couples with the oscillator frequency and this coupling is also altered by the string tension. In addition, the effect of the string tension on each possible spin quantum state cannot be same since there exist \(s/\alpha\) terms in the spectrum. If we consider only \(\Phi = \text{case}\), one can see that the spin of the point source couples not only with mass of the vector boson but also coupled with the oscillator frequency. Furthermore, this coupling breaks the symmetry of the energy levels (around the \(E = 0\)) corresponding to the particle-antiparticle states. On the other hand, it is clear that the magnitude of the relativistic energy increases as the oscillator frequency increases. This makes possible to use such an obtained exact result, for a relativistic spin-1 oscillator, to estimate mass spectra of massive vector bosons, in principle (see also \([23]\)). If there exists magnetic flux acting on such a spin-1 field, it is clear that the spin \((s)\) of the vector field is altered by the magnetic flux \(\Phi\) in such a way that \(s + \Phi\). This seems simple but the presence of the magnetic flux can be responsible for the behaviour of such a particle. In principle, we can realize that the considered system behaves as a fermion if \(\Phi\) equals to a half-integer. We can also conclude that the considered vector field may behave as a scalar boson or a vector boson if \(\Phi\) equals to an integer. This implies also that statistical behaviours and properties of such relativistic fields can be actively tuned by adjusting the magnetic flux, in principle.

4 Summary and discussions

In this contribution, we have investigated the effects of an internal magnetic flux and spin of a point source on a charged relativistic spin-1 oscillator in a 2+1-dimensional background generated by a spinning point source. To do this, we have solved the associated spin-1 equation for a charged spin-1 field with a non-minimal coupling under the influence of an internal magnetic flux in the mentioned spacetime background and have acquired an exact energy spectrum in closed-form. The obtained spectrum is given by the Eq. \((3.4)\). This energy expression shows that the topology of the background geometry effects differently the pos-
sible spin quantum states ($s = 0, \pm 1$) of the system since there exist $s/\alpha$ terms in the obtained spectra. We have also observe that the spin of the considered vector test field is altered by the internal magnetic flux ($\propto \phi_B$) as $s + \frac{e\phi}{2\pi c}$ according to the elementary electrical charge ($e$) of the considered relativistic particle. Furthermore, our results show that the altered spin ($\tilde{S}$) of the considered vector field couples with the oscillator frequency ($\omega_0$) and spin parameter ($\varepsilon$) of the point source. Here, it is also clear that the info about the string tension is carried by the spin of the spin-1 vector boson and hence the effects of the background on the energy levels of such a test field do not appear if one looks $s = 0$ states provided that $\varepsilon = 0$. Even though the internal magnetic flux affects the spin of the considered vector field, this coupling does not mix the positive and negative energy states. We can also see that the oscillator frequency is changed by the altered spin since the resulting spectrum includes $\tilde{S}_{\alpha\beta}$ terms. This means that the resulting oscillator may oscillate with a different frequency if there exists an internal magnetic flux acting on the relativistic spin-1 oscillator even if the spacetime is globally and locally flat ($\alpha = 1$, $\varepsilon = 0$). In the Figure 1, the alterations on the relativistic energy levels can be seen for varying values of the $\phi_B$ and $\varepsilon$ and this figure shows that the magnitude of the relativistic energy levels is changed by $\phi_B$ besides the $\varepsilon$. Here, one can also observe that the $\varepsilon$ break the $\tilde{S}$ symmetry of the energy levels around the zero energy. In the parts of the Figure 2, we have shown the space dependence of the probability density function(s) for different $\phi_B$ and $\varepsilon$ values and these parts show that the $\phi_B$ and $\varepsilon$ parameters can change the magnitude of the probability density function(s). This also implies that the presence of an internal magnetic flux and spin of the point source can impact on particle production (see also $[60]$).

Acknowledgements

The authors thank the kind reviewers for valuable comments and constructive suggestions.

Data availability

This manuscript has no associated data or the data will not be deposited.

Conflicts of interest statement

There is no conflict of interest declared by the authors.

Funding

There is no funding regarding this article

References

[1] Guvendi, A., Sucu, Y., Physics Letters B 811 135960 (2021)
[2] Gurtas Dogan S., Annals of Physics, 169344 (2023)
[3] Zare, S., de Montigny, M., Chen, H., Hassanabadi, H., and J. Phys. A, 54, 365201 (2021)
[4] Zare, S., Hassanabadi, H., Junker, G., General Relativity and Gravitation 54 1–8 (2022)
[5] Zare, S., Hassanabadi, H., Rampho, G.J., Ikot, A.N., The European Physical Journal Plus 137 1–7 (2022)
[6] Zare, S., Hassanabadi, H., de Montigny, M., The European Physical Journal Plus 135 1–13 (2020)
[7] Hassanabadi, H., Chung, W.S., Zare, S., Sobhani, H., The European Physical Journal Plus 137 1–7 (2022)
[8] Hassanabadi, H., Zare, S., de Montigny, M., General Relativity and Gravitation 54 1–8 (2022)
[9] Zare, S., Hassanabadi, H., De Montigny, M., International Journal of Modern Physics A 37 2250099–154 (2022)
[10] Zare, S., Hassanabadi, H., Junker, G., Modern Physics Letters A 37 2250099–154 (2022)
[11] Zare, S., Hassanabadi, H., Guvendi, A., Chung, W.S., International Journal of Modern Physics A 37 2250099–154 (2022)
[12] Barut, A.O., Physics Letters B 237 436–439 (1990)
[13] Ünal, N., Foundations of Physics 27 731–746 (1997)
[14] Dernek, M., Gurtas Dogan S., Sucu, Y., Ünal, N., Turkish Journal of Physics 135 1–7 (2021)
[15] Guvendi, A., Zare, S., Hassanabadi, H., European Physical Journal A 57 1–8 (2021)
[16] Greiner, W., Quantum mechanics: an introduction, fourth edition, 512 (Springer, Berlin, 2001)
[17] Carvalho, J., Furtado, C., Moraes, F., Physical Review A 84 032109 (2011)
[18] Moshinsky, M., Szczepaniak, A., Journal of Physics A: Mathematical and General 26 731–746 (1993)
[19] Bueno, M.J., Lemos de Melo, J., Furtado, C., de M Carvalho, A.M., European Physical Journal A 6 365201 (2021)
[20] Bentez, J., y Romero, R.P.M., Núñez-Yépez, H.N., Salas-Brito, A.L., Physical Review A 71 041801 (2007)
[21] Franco-Villafane, J.A., Sadurni, E., Barkhofen, S., Kuhl, U., Mortessagne, F., Physical Review A 79 041801 (2007)
[22] Rozmej, P., Arvieu, R., Journal of Physics A: Mathematical and General 36 571647 (2003)
[23] Guvendi, A., European Physical Journal C 81 1–7 (2021)
[24] Zare, S., Hassanabadi, H., Guvendi, A., European Physical Journal Plus 136 1–18 (2022)
[25] Guvendi, A., Boumali, A., European Physical Journal Plus 136 1–18 (2022)
[26] Moreno, M., Zentella, A., Journal of Physics A: Mathematical and General 23 197–210 (1990)
[27] Bakke, K., General Relativity and Gravitation 45 1847–1859 (2013)
[28] Bermudez, A., Martin-Delgado, M.A., Solano, E., Physical Review A 77 012106 (2008)
[29] Hosseinpour, M., Hassanabadi, H., de Montigny, M., European Physical Journal C 81 1–7 (2021)
[30] Ahmed, F., Annals of Physics 415 168113 (2020)
[31] Guvendi, A., International Journal of Modern Physics A 36 2150144 (2021)
[32] Guvendi, A., Hassanabadi, H., International Journal of Modern Physics A 36 2150144 (2021)
[33] Guvendi, A., Sahin, R., Sucu, Y., European Physical Journal B 94 1–7 (2021)
[34] Guvendi, A., Zare, S., Hassanabadi, H., Few-Body Systems 62 1–8 (2021)
[41] Guvendi, A., Sakarya University Journal of Science 25 834–840 (2021)
[42] Pacheco, M.H., Landim, R.R., Almeida, C.A.S., Physics Letters A 311 93–96 (2003)
[43] Gecim, G., Sucu, Y., European Physical Journal Plus 132 1–8 (2017)
[44] Tekincay, C., Dernek, M., Sucu, Y., European Physical Journal Plus 136 1–13 (2021)
[45] Sucu, Y., Tekincay, C., Astrophysics and Space Science 364 1–7 (2019)
[46] Gurtas Dogan, S., Few-Body Systems 63 1–10 (2022)
[47] Clement, G., Annals of Physics 201 241–257 (1990)
[48] Deser, S., Jackiw, R., Hooft, G., Annals of Physics 152 220–235 (1984)
[49] Deser, S., Jackiw, R., Physical review letters 68 267 (1992)
[50] Gott, J.R., Alpert, M., General Relativity and Gravitation 16, 243–247 (1984)
[51] Ahmed, F., Scientific Reports 11 1–9 (2021)
[52] Chen, H., Long, Z.W., Ran, Q.K., Yang, Y., Long, C.Y., Europhysics Letters 132 50006 (2021)
[53] Yang, Y., Long, Z.W., Chen, H., Zhao, Z.L., Long, C.Y., Modern Physics Letters A 36 2150059 (2021)
[54] Vitória, R.L.L., Bakke, K., European Physical Journal Plus 133 490 (2018)
[55] Zare, S., Hassanabadi, H., de Montigny, M., International Journal of Modern Physics A 35 2050195 (2020)
[56] Guvendi, A., Gurtas Dogan, S., Classical and Quantum Gravity 40 025003 (2022)
[57] Mandal, B. P., Verma, S., Physics Letters A 374 1021–1023 (2010)
[58] Cunha, M.M., Dias, H.S., Silva, E.O., Physical Review D 102 105020 (2020)
[59] Santos, L.C.N., Barros, C.C., The European Physical Journal C 78 1–8 (2018)
[60] Belbaki, B., Bounames, A., arXiv preprint arXiv:2210.08827 (2022)