A prescription for star formation feedback: the importance of multiple shell interactions

John Scalo and David Chappell
Astronomy Department, University of Texas, Austin, TX 78712, USA

Accepted 1999 May 7. Received 1999 May 4; in original form 1999 February 8

ABSTRACT
The relation between the star formation rate and the kinetic energy increase in a region containing a large number of stellar sources is investigated as a possible prescription for star formation feedback in larger scale galaxy evolution simulations, and in connection with observed scaling relations for molecular clouds, extragalactic giant H\textsc{ii} regions and starburst galaxies. The kinetic energy increase is not simply proportional to the source input rate, but depends on the competition between stellar power input and dissipation caused by interactions between structures formed and driven by the star formation. A simple one-zone model is used to show that, in a steady state, the energy increase should be proportional to the two-thirds power of the stellar energy injection rate, with additional factors depending on the mean density of the region and the mean column density of the fragments. The scaling relation is tested using two-dimensional pressureless hydrodynamic simulations of wind-driven star formation, in which star formation occurs according to a threshold condition on the column density through a shell, and a large number of shells are present at any one time. The morphology of the simulations resembles an irregular network or web of dynamically interacting filaments. A set of 16 simulations, in which different parameters were varied, agrees remarkably well with the simple analytical prescription for the scaling relation. Converting from the wind power of massive stars to the Lyman continuum luminosity shows that the cluster wind model for giant H\textsc{ii} regions may still be viable.

Key words: hydrodynamics – turbulence – stars: formation – ISM: bubbles – galaxies: ISM.

1 INTRODUCTION
There is a consensus that momentum and energy injection from young stars play an important role in the dynamics of interstellar gas in galaxies, ranging from the evolution of protogalaxies to the scale of the smallest interstellar clouds. For example, nearly all studies of disc galaxy formation agree that star formation (SF) feedback is a crucial ingredient in controlling the properties of the resulting model galaxy, and most explicitly recognize the extreme uncertainty in the feedback implementations now in use (e.g. Navarro & White 1993; Cole et al. 1994; Mihos & Hernquist 1994; Steinmetz & Muller 1994; Heyl et al. 1995; Steinmetz 1996; Quinn, Katz & Efstathiou 1996; Gerritsen & Ike 1997; Navarro & Steinmetz 1997; Saralund, Hensler & Theis 1997; Lia, Carraro & Chiosi 1998; Sommer-Larsen & Vedel 1998; see however Weinberg, Hernquist & Katz 1997). Basically the question is as follows. For a given stellar source energy injection rate in a localized region of a model galaxy (which is itself subject to extremely uncertain prescriptions for the star formation rate and initial mass function, which are not the subject of the present paper), how much energy should be deposited in the ambient interstellar gas? Similar considerations apply to models for the formation of elliptical galaxies (e.g. Theis, Burkert & Hensler 1992; Thomas, Greggio & Bender 1998) and the evolution of dwarf galaxies (e.g. Anderson et al. 1997; MacLow & Ferrara 1999). Feedback from star formation may also have dramatic effects on some aspects of structure formation in the universe on scales $\sim 100$ kpc and even larger; see the comparison of models presented in Ostriker & Cen (1996), where one of the models (‘CDM & GF’ in their table 1) includes SF feedback using the prescription of Cen & Ostriker (1993).

A basic assumption of nearly all this work has been that the feedback from SF is proportional to the star formation rate (SFR), with some coefficient that may depend on the ambient gas parameters. However, the regions in which star formation takes place are severely under-resolved in the simulations, and the size of each simulated region is so large that many stars or star clusters may be forming within the region. The point of the present paper is that the density of SF events is so large that interactions between shells driven by winds, H\textsc{ii} regions, supernova remnants or
superbubbles will be significant, and it is the competition between the dissipation resulting from the interactions and the driving effect of SF that determines the fraction of the star formation input on subgrid scales that is available as feedback energy at larger scales.

A closely related problem involves the supersonic linewidths observed in molecular and atomic clouds in the local Milky Way. A large number of papers have attempted to explain the linewidth–size scaling relation (see Larson 1981; Elmegreen & Falgarone 1996; Heithausen 1996 and references therein for the observed correlation; for recent theoretical discussions and references see Xie 1997; Vazquez-Semadeni, Ballesteros-Paredes & Rodríguez 1997; a number of proposals are reviewed in Scalo 1987). However the fact that the linewidth scaling correlation occurs in clouds where self-gravity is insignificant (Heithausen 1996), and recent simulation results indicating that magnetic fields are incapable of significantly retarding the decay of the ‘turbulence’ (MacLow et al. 1998; Stone, Ostriker & Cammie 1998), suggests that a stellar power source is required. (See, however, Zweibel 1998.) Such sources might involve embedded protostars (Norman & Silk 1980) or shocks external to the clouds (Kornreich & Scalo 2000). In the former case, protostellar censuses sometimes indicate that regions have more than enough energy in winds to balance gravity, but this alone does not demonstrate that YSOs can power the linewidths, because it is the balance (if attainable) between dissipation and energy production that is essential for determining the viability of the power source.

Another important point is that a study of densities (from matching CS line strengths to radiative transfer calculations), linewidths and sizes for a large sample of local regions with massive star formation yields no linewidth–size correlation at all (Plume et al. 1997). If there are ‘hidden’ correlations of certain physical variables in these data, then the appropriate combination of variables must be suggested by a theoretical model. The one discussed in the present paper predicts that the scaling relation for a stellar-powered model must jointly involve the linewidth, size, density, and mechanical luminosity of the stars.

The same problem arises in the interpretation of the power-law linewidth–luminosity (and linewidth–size) relation observed for extragalactic giant H II regions (e.g. Terlevich & Melnick 1981; see Shields 1990 and papers referred to in Tenorio-Tagle 1994 for general properties). Models based on stellar energy input (Hippelein 1986; Melnick et al. 1997) have entirely neglected the effects of dissipation on the derived scaling relations. Melnick et al. (1987) used a relation $L \propto R^2 \sigma^2$ ($R$, $L$ and $\sigma$ are Lyman continuum luminosity, H II region core size and linewidth) for the stellar-powered model, and concluded that the data agree much better with the prediction of a simple virial model, $L \propto R \sigma^2$ (see Tenorio-Tagle et al. 1993 for a detailed ‘cometary stirring’ model).

We show in the present paper that when dissipation is included, the stellar-driven model predicts a scaling relation similar to the virial model. In addition, morphological evidence (e.g. Meaburn 1984; Bruhweiler, Fitzurka & Gull 1991; Chu & Kennicutt 1994) strongly suggests that stellar wind-blown shells and filaments are important. Similar remarks apply to the correlation of supernova and wind power sources with the energy in galactic superbwinds (Leitherer, Robert & Drissen 1992) and other galactic outflows (see MacLow & Ferrara 1999, Martin 1999, and references therein), where dissipation may make some of the injected energy unavailable for outflows.

The present paper proposes a simple model for the scaling relation that should exist between the kinetic energy deposited (as measured by the gas velocity dispersion) and the energy injected (as measured by the SFR), for an equilibrium situation in which energy injection is balanced by dissipation. Two-dimensional hydrodynamic simulations of a system of wind-driven shells that interact through non-linear advection and form new stars by a column density threshold condition are presented, which, rather remarkably, verify the analytical scaling relation. The main result is that the kinetic energy gain per unit volume resulting from star formation scales as the two-thirds power of the SFR, and inversely as the four-thirds power of the mean density, although with some dependence on other parameters. The result is presented as a subgrid prescription that can be used in larger scale under-resolved simulations, but also as a possible model for the linewidth systematics of local interstellar clouds, extragalactic giant H II regions, and starburst galaxies. The application to large-scale ‘blowout’ winds from starburst galaxies is least warranted, however, because these flows are inherently anisotropic and depend on the galactic gravitational potential, effects that are not included in the present model. Besides, we are only concerned with the local energy injection in the vicinity of the star-forming region, and not with larger scale flows that this energy may drive.

2 A SIMPLE ONE-ZONE MODEL

Consider a model in which a large (‘macroscopic’) region of size $R$, mass $M$ and mean density $\rho$ contains a statistically significant number of internal stellar or cluster kinetic energy sources, which supply energy per unit volume at the rate $\dot{N}_s E$, where $\dot{N}_s$ is the total star or cluster formation rate per unit volume and $E$ is the average kinetic energy input per star or cluster. This macroscopic region could represent an unresolved cell in a numerical simulation of a galaxy, or an individual molecular cloud (idealizing the complex structure of the interstellar medium (ISM) as though it could be partitioned into discrete entities). We assume that these internal stellar energy sources drive winds that create internal substructure with a characteristic scale much smaller than $R$. This substructure is expected to be in the form of partial shells or filaments. We refer to this internal ‘microscopic’ substructure as if it can be segmented into discrete entities (‘clouds’ or ‘fragments’) with average column density $\mu_{c,cl}$, average cross-section $\sigma$ and cloud-to-cloud velocity dispersion $c$. The number of clouds per unit volume, $N_{cl}$, is assumed to be determined by the competition between production by winds and depletion caused by coalescent collisions between the clouds:

$$\frac{dN_{cl}}{dt} = \dot{N}_s - N_{cl}^3 \sigma c,$$

where the second term on the right-hand side is the rate of cloud merging per unit volume, $N_{cl}/\tau_{coll}$, where $\tau_{coll} = (N_{cl} \sigma c)^{-1}$ is the collisional time-scale. The velocity dispersion of the clouds, $c$, which is observed as the linewidth for the region of size $R$, is controlled by competition between wind energy injection and collisional dissipation:

$$\frac{dc^2/2}{dt} = \frac{\dot{N}_s E}{\rho} - \frac{1}{2} \rho c^3 N_{cl},$$

where the dissipation term is just $c^2/(2\tau_{coll})$. The division of the energy injection rate per unit volume by the average region density $\rho$ converts to an energy input per unit mass. Dissipation
resulting from drag forces could also be included, but we choose not to complicate the analysis.

Equilibrium self-regulation is possible for such systems, but in general a variety of sustainable non-equilibrium behaviours are possible, including limit cycles (Ikeuchi, Habe & Tunaka 1984), chaos (Scalo & Struck-Marcell 1987) and long periods of ‘incubation’ with very low star formation rates, punctuated by bursts, if there is an external energy source (e.g. a flux of small clouds, or repeated shocking; Vazquez & Scalo 1989). Here we assume an equilibrium and test the predicted scaling behaviour.

There are two key steps in obtaining a scaling relation that depends only on energy balance (equation 2).

(i) We assume that most of the mass of the system \( M \) is in the internal substructure. In that case the number of clouds in the system is \( M/m_{cl} \), where \( m_{cl} \) is the average cloud mass, and the number of clouds per unit volume is then \( N_{cl} = p/m_{cl} \), independent of the dimension of the macroscopic region (e.g. \( M \propto \rho R^2 \) in three dimensions, \( M \propto \rho R^2 \) in two dimensions).

(ii) The mass of a cloud is the average column density through the cloud times its mean cross-section: \( m_{cl} = \mu_{cl} \sigma \). Notice that this relation is independent of the shape of the cloud, within factors of order unity. Then the product \( \sigma N_{cl} = p/\mu_{cl} \), independent of the cloud size.

The equilibrium relation corresponding to equation (2) is then (to within factors of order unity)

\[
c = \left( \frac{LE_{\mu_{cl}}}{\rho} \right)^{1/3},
\]

where \( E = N_{cl}E \) is the total energy injection rate from the stellar sources per unit volume.

This result should be independent of the geometric form of the internal clouds (e.g. shells, filaments, spheres) and the geometry of the macroscopic region (e.g. two-dimensional or three-dimensional). It can be shown that if drag force, as well as collisions, is included in the energy equation, and if the effect of drag is to destroy clouds by ablation (analogous to the ‘leakage’ in the model of Norman & Silk 1980) at a rate \( N_{cl}c_{cl}/L \), where \( c_{cl} \) is the internal cloud sound speed and \( L \) is the cloud size, then the scaling relation is of the same form as equation (3), except for a factor which is of order unity as long as the drag coefficient is of order unity.

We identify \( c^2/2 \) with the kinetic energy increase per unit mass that results from a stellar source input \( E = N_{cl}E \). This energy increase depends on the average column density of the clouds and the average density of the macroscopic region, but not on the size or shape of the clouds or the dimensions of the macroscopic region. The conversion of this quantity to a net energy input rate (which is what is required in numerical simulations) requires division by an assumed characteristic time-scale for the star formation process, as discussed below.

To compare our results with observations of local clouds and extragalactic giant \( \text{H} \text{ii} \) regions, let \( L_{KE} \) be the mechanical luminosity (in erg s\(^{-1}\)). Noting that \( E/\rho \) is the energy input rate per unit mass, we have

\[
\frac{E}{\rho} = \frac{L_{KE}}{M} = \frac{L_{KE}}{\rho R^2},
\]

where \( M \) is the mass of the region of size \( R \). Writing the second factor of \( \rho \) in the denominator of equation (4) in terms of the mean column density \( \rho \) of the macroscopic region as \( \rho = \mu/R \)

gives

\[
c = \left( \frac{L_{KE}/R^2 \mu_{cl}}{\rho \mu} \right)^{1/3}.
\]

The unresolved cloud column densities \( \mu_{cl} \) are unknown, but one might assume that the ratio \( x = \mu_{cl}/\mu \) is a constant from region to region, assuming a kind of self-similarity, although this assumption is extremely uncertain. Obviously variations in this ratio will introduce scatter in any observational test of the scaling relation. Assuming that \( L_{KE} \) is proportional to the total radiative luminosity of the protostars in the macroscopic region \( L_\star \) and using \( L_\star/4\pi R^2 = F_\star \), where \( F_\star \) is the radiative flux (resulting from the internal power sources) from the macroscopic region, the predicted scaling is

\[
c \sim \frac{L_{KE}}{\rho} \left( \frac{1}{\mu R} \right)^{1/3},
\]

or, in terms of the macroscopic column density \( \mu \), size \( R \) and luminosity,

\[
c \sim \frac{L_{KE}}{\mu R} \left( \frac{1}{\mu_\star} \right)^{1/3}.
\]

However, in Section 4 below we show that, for massive stars, \( L_{KE} \) is not proportional to the radiative luminosity, so these latter two relations may be of limited use.

We emphasize that these scaling relations assume that the energy sources within the macroscopic region are numerous enough that their outflows are capable of at least partially isotropizing the driven internal motions; we do not expect the scaling relation to apply to a macroscopic region containing, say, a single energy-injecting star. The large observed luminosities of giant \( \text{H} \text{ii} \) regions suggest that they must contain many stars, but whether the number is large enough to satisfy the assumption of the present model is uncertain; see Chu & Kennicutt (1994) and Yang et al. (1996) for detailed studies of the 30 Dor region of the Large Magellanic Cloud (LMC) and the M33 \( \text{H} \text{ii} \) region NGC 604, respectively.

The equilibrium scaling relation given by equation (3) was derived for a simple ‘one-zone’ model in which the spatial degrees of freedom were suppressed. In effect the derivation can be thought of as conceptualizing the ISM as a system composed of a very large number of ‘clouds’, integrating over a kinetic equation describing the one-point probability distribution to derive ‘cloud fluid’ equations for mean variables (e.g. Scalo & Struck-Marcell 1984), ignoring gradient and advection terms, and then examining the consequences of neglecting any time dependence. This is obviously a dangerous procedure, even for the derivation of scaling relations. For this reason, in the next section we compare the equilibrium result with a large number of two-dimensional hydrodynamic simulations which were designed to study more general aspects of ISM evolution (Chappell & Scalo, 1999, hereafter CS).

### 3 SIMULATIONS OF WIND-DRIVEN STAR FORMATION

The simulations follow the evolution of a system of interacting wind-driven shells which are subject to non-linear fluid advection and, obey global mass and momentum conservation. A detailed presentation of the models and discussion of the results are given in a separate paper (CS). The calculations solve hydrodynamical
equations describing a highly compressible fluid in which advection and the corresponding ‘ram pressure’ completely dominate the thermal pressure (Mach number very large), or, equivalently, in which the effective adiabatic index $\gamma$ is zero (as might approximately apply to the ISM because of the nature of the radiative cooling curve; see Vazquez-Semadeni et al. 1996). In this case there is no energy equation to solve; the interactions of fluid elements are completely inelastic. Self-gravity and magnetic fields are neglected except that local self-gravity is artificially introduced in the form of a threshold instability criterion. Newly formed stars are assumed to inject momentum as a wind with a specified velocity. A circularly symmetric constant momentum outflow is injected locally whenever a new star forms at that site, and is assumed to last for a time $10^7\text{yr}$ (see, for example, Leitherer 1997). We also allow for a delay time $\tau_d$ between the onset of star formation and the initiation of the momentum input, treated as a constant parameter. Star formation is assumed to occur at a threshold column density corresponding to the gravitational instability criterion for an expanding shell (see Cameron & Torra 1994; Elmegreen 1994), assuming that the growth rate of the fastest growing mode is a constant. The linear perturbation analysis was generalized to include accretion and local dilatational shell stretching (see CS), but these effects turned out not to be important for these simulations, as can be understood physically (see Whitworth et al. 1994). With the growth rate of the fastest growing mode assumed constant, the criterion for star formation is simply that the column density through a filament should exceed the critical value $c_{d}\tau / G$. In the present work we simplify even further by assuming that $c_{d}$ is a constant parameter that we vary between different simulations (series ‘C’ below). Our standard model uses $c_{d} = 1\text{km s}^{-1}$, which corresponds to a critical column density of $10^{21}\text{cm}^{-2}$.

The equations describing the evolution of the system are then

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0,$$  
(8)

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla (\rho \mathbf{v} \mathbf{v}) = \sum_{x'} \frac{x - x'}{|x - x'|} N_{s}(x', t) \frac{\tau_{w}}{\tau_{w}},$$  
(9)

where $\rho$ is the gas surface density, $p_{w}$ is the total momentum input per massive star, $N_{s}(x', t)$ is the number of stars per unit area injecting momentum at position $x'$, and $\tau_{w}$ is the duration of the momentum injection ($10^7\text{yr}$ here). In practice we also experimented with models that include the source term in the continuity equation (8), accounting for the gas lost to star formation. However, because the time-scale for this gas depletion is so large compared with the phenomena of interest, and in order to calculate averages and other statistical quantities from a stationary distribution, we have omitted the mass depletion term for the calculations reported here.

The momentum source term in equation (9) requires some explanation because it only symbolically represents the finite difference procedure followed in the simulations. The position in question is $x$. The sum is over the eight nearest neighbour cells, at positions $x'$. The unit vector ensures that the momentum is directed toward the position $x$. If there is a cluster at $x'$, then $N_{c}(x')$ is the number of newly formed stars at that position, per unit area. The number of stars formed in that cluster is computed from the mass in the cluster using an adopted initial mass function (IMF). The cluster mass is computed from the mass in the simulation cell times an assumed constant star formation efficiency. The momentum input $p_{w} = m(x') v$ is calculated using a constant assumed wind velocity, $40\text{km s}^{-1}$, at a distance corresponding to the cell size (7.8 pc in the simulations reported here), and $m(x')$ is the fraction of the mass released by the cluster at $x'$ that enters the cell at $x$, assuming the morphology of the wind is circular in two dimensions. (We do not yet address the interesting question of the effect of collimated outflows rather than spherical winds.) The division by $\tau_{w}$ signifies that this mass and momentum are redistributed over the lifetime of the wind, $10^7\text{yr}$. The motion of the cluster at position $x'$ is taken into account when calculating velocities. More details of this procedure are given in CS.

The advection terms are differenced according to a variant of a Van Leer (1977) first-order scheme. Modifications were made to minimize anomalous anisotropic effects associated with the numerical viscosity in the scheme, which would otherwise introduce artificial density and velocity fluctuations in expanding shells. Details are given in CS. The boundary conditions were doubly periodic. The initial conditions consisted of a uniform density field and a Gaussian velocity field with prescribed power spectrum. We examined the effects of varying the initial power spectrum and the resolution ($128^2$, $256^2$ and $512^2$). The scales were normalized such that the lattice spacing was $7.8\text{pc}$, so these resolutions correspond to total region sizes of 1, 2 and 4 kpc, but we expect the essential results to apply to smaller or larger scales if the size and velocity scaling are adjusted. A series of $256^2$ simulations with initial energy spectrum given by $E(k) \propto k^4 \exp(k^2/k_0^2)$ and $k_0 = 4$ is presented here. The simulations were integrated for about 2 Gyr, long enough for initial transients to disappear and to study the temporal evolution of the system. We point out that these very long integrations were made possible by neglect of physical processes beside advection, and by the adoption of a first-order difference scheme. A detailed presentation is given in CS. Here we are only concerned with checking the scaling relation given by equation (3).

All the simulations evolve into a network of irregularly shaped filaments (in two dimensions) which cover a large range of sizes and which are the products of the distortion of the originally symmetric star-forming shells by interactions with other shells and by advection (the distinction is not clear-cut, because most of the mass ends up in the filaments). Sometimes filament interactions lead to nearly spherical ‘clumps’ which may or may not be dense enough to form a star. Often an expanding shell produced by one ‘star’ or ‘cluster’ compresses gas along the filament in which it was horn, stimulating further star formation and sometimes resulting in groups or chains of clusters. The overall filamentary structure is not dependent on the existence of the wind energy input, but is an inevitable result of the high compressibility brought about by the absence of pressure; in fact, simulations without stellar forcing develop similar structure, although of course with no input the structure is eventually concentrated on large scales, and the velocities monotonically decrease with time. The ubiquity of similar networks of filamentary structure in simulations that involve the inclusion of different physical processes (e.g. cooling, pressure, self-gravity, magnetic fields, different types of star formation ‘laws’; see Bania & Lyon 1980; Chiang & Prendergast 1985; Chiang & Bregman 1988, Vazquez-Semadeni, Passot & Pouquet 1995 and Passot, Vazquez-Semadeni & Pouquet 1995) suggests, considering the present results, that such filamentary structures are primarily the result of highly compressible advection.

An example of the gas density field at six different times is shown in Fig. 1. The stellar distributions were discussed in Scalo
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& Chappell (1999), where it was shown that the simulated model can account for the observed power-law correlation functions of young stars in a number of star-forming regions. The simulations do qualitatively resemble the morphology observed in well-resolved giant H\textsc{ii} regions like 30 Dor (Chu & Kennicutt 1994, who describe the structure as a ‘complex network of expanding systems’), in the entire LMC as mapped in H\textsc{i} (Kim et al. 1998) and in some local molecular clouds that exhibit a ‘network of filaments’ (see Mizuno et al. 1995 for Taurus), perhaps being ‘churned’ by protostellar outflows (e.g. Bally et al. 1999 for the Circinus cloud).

Out of 43 simulations we have run, we examine here a set of 16 simulations, all with resolution 256$^2$, in which the following parameters were varied.

(i) In the series labelled ‘C’ the star formation threshold was varied by changing the assumed internal gas velocity dispersion in the shells, which is equivalent to varying the critical column density for gravitational instability; the variation covered a factor of 6. These runs effectively alter the overall star formation rate.

(ii) In the series labelled ‘D’ the time delay between the onset of gravitational instability and stellar heating was varied between zero and 10$\text{yr}$. Increasing the time delay increases the star formation activity and makes the spatial and temporal behaviour more coherent. The primary reason for this result is that the wind from a young cluster in a filament usually disrupts the filament. A larger time delay increases the likelihood that the column density of a filament, which is sweeping up ambient gas, will reach the

Figure 1. Snapshots of the density field at six times for a typical two-dimensional simulation.
star formation threshold before it is disrupted. The net result is that more star clusters will form, on average, per filament.

(iii) In the series labelled ‘E’ the assumed energy input per star formed was varied over a factor of 16.

All of these series are different physical ways of altering the overall level of star formation activity.

The equilibrium scaling relation equation (3) involves the column density $\mu_{cl}$ that corresponds to the filaments in the present simulations. After testing several approaches, the following straightforward method was adopted for identification of filaments. The average density in each cell was compared with the densities in the nearest eight neighbours. If the central lattice site was a maximum along at least two of the four possible directions, the site was considered to lie on a filament ridge. Requiring that a cell be a local maximum in four or more directions would locate cloud peaks, while virtually all sites are local maxima in at least one direction (tangent to the local density contour). The local directionality of a filament was then computed by estimating the subgrid position of the filament ridge based on the density at the neighbouring lattice sites, and then least-squares fitting a line to the positions of neighbouring lattice sites that were also found to lie on the filament ridge. This method was found to give excellent results, both by tests on density rings with a range of widths and radii, and by visual comparison of the filament results with the density field of the simulations.

Fig. 2 plots the time-averaged velocity dispersion for each of these simulations as a function of the quantity $N_\star E \mu_\star \rho^2$. It is seen that the simulations conform remarkably well with the scaling index of $1/3$ predicted for the equilibrium one-zone model. Although this result is encouraging, it must be remembered that the simulation quantities plotted are long-time averages, and we have not studied the magnitude of the predicted fluctuations about the mean scaling relation. The agreement also should not be interpreted as support for uncritical use of equilibrium one-zone models. However, in the present case it does appear that the long-time averages are controlled by a balance between stellar energy injection and dissipative shell/filament interactions. Another consideration is the fact that the cloud or filament size does not enter the scaling relation, a cancellation that allowed us to ignore the equation governing the number of clouds.

### 4 DISCUSSION

We have shown that both a simple analytical model for the balance of SF energy injection and dissipation and numerical simulations of wind-driven, interacting shells result in a scaling relation between the resulting velocity dispersion $c$ and the energy injection rate per unit volume $\dot{N}_\star E$, the average column density of `clouds' $\mu_{cl}$ and the mean density $\rho$, given by equation (3). We interpret $\epsilon = c^2/2$ as the kinetic energy that results from a given SFR $\dot{N}_\star$, assuming that the average energy injection per star or star cluster, $E$, is a constant. Then

$$\epsilon \sim (N_\star \mu_\star)^{2/3} / \rho^{1/3}. \quad (10)$$

Obviously, the rate of energy feedback (which is what is needed for galaxy simulation prescriptions) depends on an assumed timescale, the duration of the SF event. If we assume that this timescale is proportional to the collapse timescale $\tau \sim \rho^{-1/2}$, then the energy dependence of the energy injection rate is $\rho^{-5/6}$. However, local molecular clouds are believed to have lifetimes in excess of their free-fall times. Furthermore, the simulations result in continual, ongoing star formation because of the large number of uncorrelated star formation sites. For these reasons, it might be more appropriate to take the characteristic timescale as a constant. On the other hand, Elmegreen (1998) argues that a number of physical processes should result in a characteristic time-scale that scales as $\rho^{-1/2}$. The important point is that the SF energy feedback, or feedback rate, should not scale linearly with the SFR, but as the SFR to the $2/3$ power.

The additional forms of the predicted scaling relations (equations 5–7) should be more useful in interpreting the linewidths of local clouds (e.g. Plume et al. 1997) when the radiation flux or luminosity is the primary observed variable. For example, in terms of the mechanical luminosity, equation (7) gives $\epsilon \propto (L/\mu R)^{2/3}$, where $\mu$ is the macroscopic column density, if the parameter $x$ is assumed to be constant.

Observations of giant H\textsc{ii} regions give a scaling relation between velocity dispersion (linewidth) and Lyman continuum luminosity $L$ of the form $\epsilon \sim L^{1/4}$ to $L^{1/2}$ (see Hippelein 1986; Melnick et al. 1987; Melnick, Terlevich & Moles 1988). However, the observed scaling is bivariate, with an additional dependence on

![Figure 2](https://academic.oup.com/mnras/article-abstract/310/1/1/971925/971925)

**Figure 2.** Scaling of the filament velocity dispersion with the quantity $N_\star E \mu_\star \rho^2$ as predicted by equation (3). Open circles: runs C, which vary the assumed filament internal velocity dispersion, and hence star formation threshold. Crosses: runs D, which vary the time delay between filament gravitational instability and the onset of wind energy input. Boxes: runs E in which the assumed kinetic energy input per massive star was varied. The slope of the solid line shows the scaling predicted by the analytical model derived in the text.
the size of the H\textsc{ii} region, \( R \). The scaling relations proposed here (equations 3–7) are in terms of the source kinetic energy injection rate, not the radiative luminosity. We identify the source kinetic energy rate with the wind power of a massive star, \( L_{\text{w}}(m) \propto M V_{\infty}^2 \), where \( M \) is the mass-loss rate and \( V_{\infty} \) is the terminal wind speed, both at stellar mass \( m \), and then integrate over an IMF. In order to compare the predicted scaling relations with the observed scaling relation, we therefore need to derive a relation between the IMF-averaged wind power \( L_{\text{w}} \) and the IMF-averaged Lyman continuum luminosity \( L_{\text{c}} \).

For the stars in a giant H\textsc{ii} region, we assume a power-law differential mass spectrum of the form \( n(m) = A m^{-\gamma} \), where \( \gamma \) is a parameter. If \( \gamma > 1 \) (which is a very reasonable condition), then, if the Lyman continuum luminosity varies with mass as \( m^a \), the IMF-averaged \( L \) is dominated by the term \( A m_{\text{m}}^{a-\gamma+1} \), where \( m_{\text{m}} \) is the upper mass limit. Similarly, if the wind luminosity varies with mass as \( m^b \), the IMF-averaged wind luminosity is proportional to \( A m_{\text{m}}^{b-\gamma+1} \). Eliminating \( m_{\text{m}} \) gives \( L_{\text{w}} \sim L^5 \), where

\[
\delta = \frac{\beta - \gamma + 1}{\alpha - \gamma + 1}.
\]  

(11)

The values of \( \alpha \) and \( \beta \) were estimated using the wind luminosities and Lyman continuum luminosities as a function of mass (between 23 and 87 \( M_{\odot} \)) tabulated by Leitherer (1997, table 6). We evaluated the exponent \( \delta \) in equation (11) for luminosity classes (LC) V, III and I, although there is not much variation in \( \delta \) between these classes. In all cases we find that \( \delta \) is significantly larger than unity. For LC V, \( \alpha = 3.2 \), \( \beta = 2.6 \) and \( \delta \) varies between 1.3 and 1.6 for IMF indices between 1 and 2.5.

For example, taking \( \delta = 1.5 \), equation (5) gives

\[
L \sim \left( \frac{\rho}{\mu \mu_{\text{cl}}} \right)^{2/3} R^{1/3} c^3.
\]  

(12)

Earlier treatments of wind-powered velocity dispersions, which neglected dissipation, gave \( L \sim R^3 c^3 \). Melnick et al. (1987) showed that the observed bivariate distribution does not agree with this prediction, but gives better agreement with the virial prediction \( L \sim R c^3 \). However, the results including dissipation (equation 12) give a scaling that is close to the virial scaling, although additional parameters (mean density \( \rho \) and column density parameter \( \mu_{\text{cl}}/\mu \)) enter the scaling relation. Expressed in terms of the mean column density \( \mu \) (equation 6), we find, for \( \delta = 1.5 \), \( L \sim (R \mu/\mu_{\text{cl}})^{2/3} c^3 \), again close to the virial result. Considering that Melnick et al. (1987) found a better result, but still not exact, fit with the virial result [they found \( L \sim (R c)^{0.36} \)], it appears that the wind model, including dissipation, is still viable, although the dependence on mass density or column density remains to be studied. According to Kennicutt (1984), densities are roughly constant among giant extragalactic H\textsc{ii} regions. The virialization/bow shock, or 'cometary stirring', model of Tenorio-Tagle et al. (1993) remains attractive; we are only pointing out that the difference between the scaling relations predicted by the cometary stirring model and the stellar wind model is not nearly as large as previously claimed.

For local molecular clouds with massive star formation, we attempted to compare the predicted scaling (equation 6 and 7) with the results of Plume et al. (1997), but found that the empirical values of the luminosity, density and size were too uncertain to afford a meaningful comparison. Luminosity estimates were only available for 12 of the 150 regions studied, and a plot of linewidth versus \( L/\rho R^2 \) (see equation 5) for these regions yielded only a very rough correlation dominated by scatter. In addition, we have no indication of how to transform observed luminosity to kinetic energy input in this case (they were assumed to be proportional in equations 6 and 7). The IMF-averaged wind–radiative luminosity conversion derived earlier predicts a linewidth that scales as \( L^{1/2}/(\rho R^2)^{1/3} \). When the Plume et al. linewidths (averages from three transitions) are plotted versus this quantity, using their tabulated luminosities, sizes and densities (from C\textsuperscript{34}S except in the case GL 490, for which only the CS density was available), a fairly good correlation is found for most sources, but with strong deviations for W51(OH) (linewidth too large compared with prediction) and CRL 2591 (linewidth too small compared with prediction). However, many of the source luminosities are so small that the energy input cannot include stars as massive as assumed in the IMF averages, or indicate that the number of stellar energy sources is of order unity, violating the assumption of our model. More observational studies aimed at testing the prediction of the present paper are needed. The important point is that we showed that, in a stellar-driven model, the scaling relation involves several variables, all of which must be estimated. These predicted results should apply just as well to clouds driven by outflows from low-mass young stellar objects (YSOs). We speculate that much of the scatter found in attempts to reconstruct the ‘Larson relations’ (Larson 1981) is simply the result of variations in these additional variables. On the other hand, an obvious weakness of the stellar-powered model is that Larson-type scaling is inferred for diffuse clouds without internal stellar sources (Heithausen 1996). This suggests that the power source for these regions may be stellar-driven shocks external to the clouds, along the lines proposed by Kormreich & Scalo (2000). Conversion of magnetic energy into kinetic energy by ambipolar diffusion (Zweibel 1998) is unlikely for these clouds, because the ionized fraction should be relatively large.

The energy input scaling derived here may also have relevance to starburst galaxies. Melnick et al. (1988) showed that the scaling in small starburst ‘H\textsc{ii} galaxies’ is similar to that observed in giant H\textsc{ii} regions. Also interesting is the semi-empirical correlation derived by Leitherer et al. (1992) between the kinetic energy of superwind’ outflows from starburst galaxies and the derived energy input from winds and supernovae for a number of starburst galaxies. The derived correlation is steeper than linear, compared with the less than linear prediction presented here. However the sizes and densities of these starburst regions need to be included for a proper comparison with the predicted relation. Besides these effects, a dependence of burst duration on metallicity could be involved, as suggested by Leitherer (1997). Clearly more work is needed to compare the present model with observations of superwinds.

We want to emphasize that the application of the present model to superwinds and outflows from starbursting dwarf galaxies may be inappropriate, because ‘blowout’ may be most effective for an essentially single central wind source, and involves a fight against the vertical gravitational field of the galaxy (e.g. the recent numerical simulations of MacLow & Ferrara 1999). Furthermore, as mentioned earlier, the present model only provides a prescription for the feedback in the local vicinity of the star formation event, and has nothing to say about larger scale flows driven by the feedback.

For simulations of galaxy formation and evolution, the present result gives an easy-to-implement subgrid star formation feedback prescription. The prescription depends on known macroscopically available variables, the star formation rate and average density,
except for the presence of the term involving the cloud column densities.

Finally, we emphasize that the present results depend fundamentally on the fact that the shells and shell fragments in the model systems lose energy primarily by interactions with ambient material, in our case other shells. For this situation to occur, the SFR must be large enough that the mean separation of shell-producing events is smaller than the size of the shells when radiative losses become important. For example, if the shells were caused by supernova remnants expanding into a smooth medium, rather than the cluster wind supershells assumed here, radiative losses would begin to dominate when the shell radius is only a few pc for the fiducial density (using the analytical results given in Franco et al. 1994), leading to the formation of a dense shell behind the shock. Most of the shell energy might be dissipated before an interaction with another shell occurs, if the average separation of SNR sources is large enough. In this case, the one-zone self-regulation model would be similar to previous studies of non-interacting shells (e.g. Franco, Santillan & Marcos 1995), and a case could be made that the energy injection rate is proportional to the first power of the SFR. However, as at least Type II supernovae, which have a significantly higher rate than Type I events, should explode within their parent cluster, wind-driven bubbles should be the dominant energy-injecting process, and models for these superbubbles remain adiabatic to sizes much larger than the mean separation of clusters in our simulations, so shell interactions should dominate the dissipation. In the case of smaller scale winds from protostars, the estimates given by Norman & Silk (1980) indicate again that the shell radius at which radiative losses become important is larger than the mean separation of protostars, based on several estimates of the number density of protostellar sources in molecular clouds. For these reasons we feel that shell interactions should dominate in a variety of star-forming environments over a broad range of scales, although more quantitative work is required to establish the validity of this conclusion. A more detailed discussion of the implications of multiple shell interactions is given elsewhere.

As far as we know, the only previous studies of one-zone models with properties that depend on the process of shell interactions (in a galactic star formation context) are those of Norman & Silk (1980) and Franco & Cox (1983) for low-mass stars, and Franco & Shore (1984) for massive stars. The Norman & Silk model is conceptually similar to both the analytic and simulation models presented here, because both envision star formation as occurring after multiple coalescence of shell fragments. However, we are unable to deduce from their paper how the overall velocity dispersion of the region depends on the SFR. The Franco & Cox (1983) and Franco & Shore (1984) models are conceptually very different from the present approach. They estimated the number of shells that would be required to maintain support against collapse and to fill a given volume of star formation activity as the criterion for self-regulation. They then used this number to estimate the SFR of either low-mass or massive stars. They were mostly concerned with the dependence of the resulting SFR on the gas density, and it is not clear how their results could be used to derive the scaling between kinetic energy injection and the SFR. A basic difference between their model and the present model is that, in the former, shell interactions are assumed to inhibit SF by preventing shells from expanding to column densities above which stars could form, while in the present model star formation is allowed to be induced by the shell interactions; SF does not occur because the shells have sufficient time to develop large column densities before interaction. The agreement we find between the present one-zone models and the simulations, which allows for both types of star formation mechanisms, suggests that the latter, positive feedback effect, dominates.

ACKNOWLEDGMENTS

This work was supported by NASA Grant NAG5-3107 and a grant from Cray Research. We thank the referee, Anthony Whitworth, for comments and suggestions which improved the presentation. We also thank Pepe Franco for pointing out the possibility that radiative losses during shell expansion might dominate dissipation by shell interactions, and for helpful correspondence.

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