Investigation of the Taylor vortices in electrovortex flow

D A Vinogradov, Yu P Ivochkin, I O Teplyakov

Joint Institute for High Temperatures of the Russian Academy of Sciences
Russia, 125412 Moscow, Izhorskaya, 13 bld. 2

Email: igor.teplyakov@mail.ru

Abstract. The structure of the electrovortex flow appearing when the electric current passing through the liquid metal interacts with own and external magnetic fields was investigated numerically. It was shown that axial external magnetic field leads to the rotation of the liquid and generates secondary flow similar to Taylor vortex. Calculations were carried out for various ratios of electrode sizes.

1. Introduction.
An electromagnetic body vortex force appears as a result of interaction of the electric current passing through the liquid conductive media with the own magnetic field of this current and this force leads to generation of the specific flows in the volume of liquid – electrovortex flow (EVF) [1]. These flows essentially influence various technological processes in electrometallurgy, such as electroslag and electroarc remelting and welding. In particular, EVF determines the hydrodynamic structure of the flows in the working bath of arc furnaces.

This problem has been of interest from 70s of 20th century. The greatest successes were achieved in Institute of Physics of University of Latvia [1] and Joint Institute for High Temperatures RAS (Moscow, Russia) [2,3]. Currently, research is under way in Helmholtz-Zentrum Dresden-Rossendorf [3] (Germany), University of Leoben (Austria) [4], Institute of Continuous Media Mechanics (Perm, Russia) and others.

It is usually assumed that in the axisymmetric system under consideration with a central electrode without taking into account the effect of external magnetic fields, the EVF has the shape of a toroidal vortex (Fig.1a). The external axial magnetic field \( B_{\text{ext}} \) leads to an azimuthal rotation of the flow (Fig. 1b) and appearing of the secondary upward flow similar to Taylor vortex (Fig. 1c). With increasing of external magnetic field the intensity and size of secondary vortex increase too (Fig. 1d).

Electrovortex flow is very sensitive to external fields, even the Earth’s magnetic field can spin liquid metal up to velocities of about 0.1 m/s, so it is interesting to investigate flow structure on the electrical and magnetic parameters. Our goal was to obtain boundary line dividing flow with one vortex and two vortices depending on the parameters of the EVF (current \( I \) and external magnetic field \( B_{\text{ext}} \)).
Fig. 1. Structure of electrovortex flow. 1 – small electrode, 2 – liquid, 3 – container – big electrode. a) toroidal flow, b) azimuthal rotation, c) formation of the secondary flow, d) two-vortices rotating EVF.

2. Numerical method

The motion of the liquid is described by the Navier Stokes equation:

\[ \rho \left( \frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \nabla) \mathbf{U} \right) = -\nabla p + \nu \Delta \mathbf{U} + \mathbf{F}, \]

where \( \rho \) – density, \( \nu \) – kinematic viscosity and \( \mathbf{F} \) – body force driving the liquid. Electromagnetic force \( \mathbf{F} \) has the following form:

\[ \mathbf{F} = \mathbf{J} \times \mathbf{B}. \]

For moving liquid:

\[ \mathbf{J} = \sigma (\mathbf{E} + \mathbf{U} \times \mathbf{B}), \]

where \( \sigma \) – conductivity, \( \mathbf{E} \) – electric field and

\[ \mathbf{B} = \mathbf{B}_{\text{own}} + \mathbf{B}_{\text{ext}}, \]

where \( \mathbf{B}_{\text{own}} \) – own magnetic field of current passing through the liquid and \( \mathbf{B}_{\text{ext}} \) – external magnetic field. In our case \( \mathbf{B}_{\text{ext}} \) has only axial component. In a hemispherical container expression for current density is obvious (here \( R \) – radius in spherical coordinates):

\[ J_R = \frac{l}{2\pi R^2}. \]

Own magnetic field can be found from Maxwell’s equation:

\[ \nabla \times \mathbf{B}_{\text{own}} = \mu_0 \mathbf{J}, \]

and:

\[ B_{\text{own} \phi} = -\frac{\mu_0 l (1 - \cos \theta)}{2\pi R \sin \theta}. \]
At a low magnetic fields and velocities we can use the so-called electrodynamic approximation, when one can neglect the term $\mathbf{U} \times \mathbf{B}$ in Ohm’s low (1).

Using following scales: for linear size – $R_2$, for current density – $J_0 = \frac{I}{R_2^2}$, for magnetic field - $B_0 = \frac{\mu_0 I}{R_2}$, for electromagnetic force: $F_0 = \frac{\mu_0 I^2}{R_2}$, for velocity – $U_0 = \left(\frac{F_0 R_2}{\rho}\right)^{\frac{1}{2}}$, for time – $t_0 = \frac{R_2}{u_0}$, for pressure $- p_0 = \rho U_0^2$, Reynolds number – $Re = \frac{I}{v} \left(\frac{\mu_0}{\rho}\right)^{\frac{1}{2}}$, parameter of electrovortex flow – $S = \frac{Re^2}{\rho v^2}$, parameter of external magnetic field influence $N = \frac{BR_2}{\rho v^2}$ which can be interpreted as electromagnetic Taylor number and parameter $\alpha = \frac{B_{\text{ext}}}{B_{\text{own}}} = \frac{N}{S}$.

Finally we obtain dimensionless equation to describe electrovortex flow in axial external magnetic field:

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \nabla) \mathbf{U} = -\nabla p + S^{-\frac{1}{2}} \Delta \mathbf{U} + \frac{\cos \theta - 1}{4\pi^2 R^3 \sin \theta} \mathbf{e}_\theta - \frac{N \sin \theta}{S} \frac{2\pi R^2}{\sin \theta} \mathbf{e}_\varphi.$$

Calculations were carried out with the finite volume method with SIMPLE-scheme on unstructured grid. 2D axisymmetric laminar model was used. We were interested in a stationary solution but the flow was rather unstable so we had to carry out unsteady calculations.

We calculated several regimes for various ratios of small and big electrodes ($R_1/R_2$): 1:10, 1:37.6, 1:100. Ratio 1:37.6 was chosen in accordance to electrode sizes ($R_1=2.5$ mm, $R_2=94$ mm) in experimental setup with eutectic alloy In-Ga-Sn in JIHT RAS [2,3,6]. To get boundary line the following technique was used. For some $S$ we set parameter $\alpha$ and carried out unsteady calculations to see streamlines (Fig. 2.).

Fig. 2. Streamlines of electrovortex flow, $I=400A$, $R_1/R_2=1:37.6$ a) one-vortex structure, $B_{\text{ext}}=1e-5$ T; b) two-vortices structure, $B_{\text{ext}}=1e-4$ T.

In case of a) parameter $\alpha$ was being increased until approach case b) and vice versa. Then step of changing $\alpha$ was being decreased and process repeated until 5% accuracy of determination $\alpha$ for the
point on the boundary line one vortex-two vortices. Similar calculations were performed for $S$ in the range of $10^3 \div 10^9$ for all radiuses ratios of small and big electrodes.

3. Results
Obtained boundary line is presented in Fig. 3. in dimensionless and dimension form. With growing $S$, a less $\alpha$ is required to form upward vortex. In dimension form it means that with increasing electric current passing through the liquid we need bigger external magnetic field and with growing size of small electrode - a smaller magnetic field.

![Fig. 3. a) Boundary line in coordinates $\alpha$-$S$; b) in coordinates $B_{\text{ext}}$-$I$ (for eutectic alloy In-Ga-Sn).](image)

4. Conclusions
As a result of carried out numerical calculations a boundary line dividing one-vortex and two-vortices zone in parameters $\alpha$-$S$ and $B$-$I$ was obtained. This result can be used for a priori estimate of the flow structure.

5. Acknowledgments
This work was supported by Russian Scientific Fund, grant №17-19-01745.

References
[1] Bojarevich V, Frejbergs J, Shilova E I, Shcherbinin E V 1989. *Electrically induced vortical flows* (Kluwer Acad. Publ., Dordrecht, Boston, London)
[2] Zhilin V G, Ivochkin Y P., Igumnov V S, Oksman A A 1995 Experimental investigation of the electrovortex flows in hemispherical volume. *High Temperature* 33(1) p 1 – 4.
[3] Zhilin V G, Ivochkin Y P, and Teplyakov I O 2011 The problem of swirling of axisymmetric electrovortex flows. *High temperatures* 49(6) p 927-929
[4] Shatrov V, Gerbeth G 2012 Stability of the electrically induced flow between two hemispherical electrodes *Magnetohydrodynamics* 48(3) p 469-484.
[5] Kharicha A, Teplyakov I, Ivochkin Y, Wu M, Ludwig A, Guseva A 2015 Experimental and numerical analysis of free surface deformation in an electrically driven flow. *Experimental Thermal and Fluid Science* 62 p 192–201.
[6] Prokhorenko V Y, Ratushyak E A, Stadnyk B I, Lakh V I, Koval A M 1970 Physical properties of thermometric alloy In-Ga-Sn. *High temperature* 8(2) p 346-350.