Transmission of a Symmetric Light Pulse through a Wide QW

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The reflection, transmission and absorption of a symmetric electromagnetic pulse, which carrying frequency is close to the frequency of an interband transition in a QW (QW), are obtained. The energy levels of a QW are assumed discrete, one exited level is taken into account. The case of a wide QW is considered when a length of the pulse wave, appropriate to the carrying frequency, is comparable to the QW’s width and it is necessary to take into account a dependence of a matrix element of an interband transition from the wave vector of light. Alongside with it the distinction in parameters of refraction of substances of a QW and barrier is taken into account. The formulas for electrical fields on the right and to the left of the QW, appropriate reflected and transmitted through a well pulses at any ratio between radiative and non-radiative broadenings of the exited energy level of the electronic system, are obtained. In figures the time dependencies of the dimensionless reflection, absorption are transmission (which are defined as modules of the appropriate energy fluxes and their sum is equal to the module of a flux of an exciting pulse energy) are represented. It is shown, that the spatial dispersion and a distinction in refraction indexes influence stronger reflection, as alongside with reflection connected with interband transitions in the QW, the additional reflection from borders of the QW takes place. In comparison with model, in which the matter is considered homogeneous and the spatial dispersion of light is not taken into account, the most radical changes take place in reflection in a case, when the radiative broadening of the exited state is large in comparison with non-radiative broadening. On the side of the large values of the QW’s width the theory is limited by a condition of existence of size-quantized energy levels.

1. INTRODUCTION

Lately in [3,6] the change of the light pulse form was investigated at its transmission through a QW. Asymmetric exciting pulses with abrupt fronts [1,2] and symmetric pulses [4,5] have been considered. It was supposed, that the exciting pulse carrying frequency $\omega_1$ is close to the electronic excitation frequency $\omega_0$ (two-level system) [4,5]. A three-level system [7] and system with many exited states [6] are investigated also. Results of these works are true for rather narrow QW’s, when the inequality

$$\kappa d \ll 1$$

is carried out, where $d$ is the QW’s width, $\kappa$ is the module of a light wave vector appropriate to the carrying frequency of the symmetric light pulse. Actually the parameter $\kappa d$ in mentioned works relied equal to zero and calculated there reflection, absorption and transmission did not depend on the QW’s width. For a numerical estimation of size $\kappa$ we use a wave length of radiation of a hetero-laser on a basis GaAs, equal $0.8\mu$. The energy, corresponding to this wave length, is as follows $h\omega_1 = 1.6 eV$. If a refraction index of a QW’s matter is $\nu = 3.5$, $\kappa = \nu \omega_1 / c = 2,8.10^5 cm^{-1}$, where $c$ is the light velocity in vacuum. For the QW width $d = 500\AA$, the parameter $\kappa d = 1.4$. Thus, for wide QWs the spatial dispersion of waves, consisting an exciting pulse, can appear essential.

For wide QWs an inequality $d \gg a_0$, where $a_0$ is the lattice constant, is very strong and at a description of a pulse transmission through a QW it is possible to use Maxwell’s equations for continuous matter. At such approach it is necessary to take into account a distinction in refraction indexes of barriers and QW. Then additional reflections should appear from borders of the QW, which decreases with reduction of the parameter $\kappa d$, but in an area $\kappa d \geq 1$ can in some cases be compared or exceed reflection caused by resonant transitions in a QW. Transmission of a light wave will change together with reflection. Thus, alongside with the account of a dependence of reflection and transmission on parameter $\kappa d$ one should take into account distinction in refraction indexes of barriers and QW. At present work the influence of these two factors on the form of reflected and transmitted pulses is taken into account.

The system consisting from a deep semiconductor QW of the I type located in an interval $0 \leq z \leq d$ and two semi-infinite barriers is considered. It is supposed, that the exciting light pulse moves along an axis $z$ from negative $z$. It is supposed also, that barriers are transparent for the pulse, and in the QW the pulse is absorbed, causing resonant interband transitions. An intrinsic semiconductor and zero temperatures are meant. Exited states are taken into account only, in which one electron from
the valence band transited in the conductivity band, thus a hole appears in the valence band. It is supposed, that \( \omega_1 \cong \omega_g \) (the energy gap in the QW is \( E_g = \hbar \omega_g \)) and a small part of valent electrons, located close to the valence band extremum (for which the method of effective mass is true) participates in absorption. For deep QWs in this case it is possible to neglect tunneling of electrons in a barrier and to consider, that electrons are absent in barriers. Besides it is possible to consider energy levels located close to the QW’s bottom in approximation of infinitely deep QWs. Considered system is inhomogeneous. As for wide QWs the inequality (1) is not performed, optical characteristics of such system are necessary to determine from the solution of Maxwell’s equations, in which current and charge densities, following from a microscopic consideration, are used \[\text{Fig.}\].

Final results will be obtained for only discrete electron energy level in a QW. Influence of other energy levels on light reflection and absorption may be neglected, if the carrying frequency \( \omega_1 \) is close to the excitation frequency of the chosen level \( \omega_0 \), and other energy levels are located far away from the chosen energy level. Discrete energy levels in a QW in a case \( \hbar \mathbf{K}_\perp = 0 \) (where \( \mathbf{K}_\perp \) is the total wave vector of the electron-hole pair (EHP) in a QW’s plane) are excitonic energy levels in zero magnetic field, or energy levels in quantizing magnetic field directed perpendicularly to the QW’s plane. As a convenient example of an energy level we consider below the EHP energy level \( \bar{\hbar} \mathbf{g} \mathbf{e} \mathbf{v} \), without taking into account the Coulomb interaction between electron and hole, which is a weak perturbation for strong magnetic fields and not wide QWs \[\text{Fig.}\]. However, the excitonic effect will not result into basic changes of our results, but will affect only the magnitude of radiative broadening \( \gamma_r \) of the electronic excitation in a QW. The same concerns and to excitonic levels in a zero magnetic field.

### II. THE FOURIER TRANSFORM OF THE ELECTRIC FIELD INDUCED BY AN EXCITING LIGHT PULSE.

The symmetric exciting pulse, to which corresponds electrical field of circular polarization, falls on a single QW from negative \( z \)

\[
\mathbf{E}_0(z,t) = \mathbf{e}_t \, E_0 \, e^{-i \omega_1 t} \{ \Theta(p)e^{-\gamma_1 p/2} + [1 - \Theta(p)]e^{\gamma_1 p/2} \} + \text{c.c.}.
\]

(2)

Here \( E_0 \) is the real amplitude, \( p = t - \nu_1 z/c \),

\[
\mathbf{e}_t = (\mathbf{e}_x \pm i \mathbf{e}_y)/\sqrt{2}
\]

(3)
is the unit vector of circular polarization, \( \mathbf{e}_x \) and \( \mathbf{e}_y \) are unite vectors, \( \nu_1 \) is the barrier refractions index, \( c \) is the light velocity in vacuum, \( \Theta(p) \) is the Haeviside function, \( \gamma_1 \) determines the increasing and attenuation of the symmetric pulse. The Fourier transform of the function \( \mathbf{E}_0(z,t) \) looks like

\[
\mathbf{E}_0(z,\omega) = \exp(i \kappa_1 z)(\mathbf{e}_t E_0(\omega) + \mathbf{e}_t^* E_0(-\omega)),
\]

\[
E_0(\omega) = E_0 \gamma_1/[(\omega - \omega_1)^2 + (\gamma_1/2)^2],
\]

(4)

where \( \kappa_1 = \nu_1 \omega/c \).

In \[\text{Fig.}\] the task about transmission of a monochromatic electromagnetic wave through a QW is solved with taking into account its spatial dispersion. The expression for the density of a high-frequency current, induced by the transmitting electromagnetic wave, was also obtained there. For the case of the only exited energy level and circular polarization of falling waves the current density looks like

\[
\mathbf{J}(z,t) = (1/2\pi) \int_{-\infty}^{\infty} d \omega \exp(-i \omega t) \mathbf{J}(z,\omega),
\]

\[
\mathbf{J}(z,\omega) = -\mathbf{e}_t \gamma r \nu \omega/4\pi \times \left[ \frac{1}{\omega - \omega_0 + i \gamma/2} + \frac{1}{\omega + \omega_0 + i \gamma/2} \right] \times \int_0^d dz' A(z',\omega) \Phi(z') \times c.c. = \mathbf{e}_t \tilde{J}(z,t),
\]

(5)

where

\[
\hbar \omega_0 = \hbar \omega_g + \varepsilon(m_e) + \varepsilon(m_h) + \hbar \Omega(n + 1/2)
\]

(6)
is the energy of the interband transition appropriate to the chosen exited state, \( \varepsilon(m_e) (\varepsilon(m_h)) \) is the size-quantized electron (hole)energy with the quantum number \( m_e (m_h) \), \( \Omega = |e|H/\mu c \) is the cyclotron frequency, \( e \) is the electron charge, \( H \) is the magnetic field intensity, \( \mu = m_e m_h/(m_e + m_h) \), \( m_e (m_h) \) is the electron (hole) effective mass, \( n \) is the Landau quantum number, \( \gamma \) is radiative broadening of the exited state, \( \nu \) - is the QW refraction index. In approximation of an indefinitely deep QW

\[
\Phi(z) = (2/d) \sin(\pi m_e z/d) \sin(\pi m_h z/d).
\]

(7)

In Eq. (5) radiative broadening \( \gamma_r \) is introduced for the EHP in a magnetic field at \( \kappa d = 0 \)

\[
\gamma_r = (2e^2/\hbar c \nu)(p_e^2 e^2/m_0 \hbar \omega_g)\beta(\varepsilon|H|m_0 c),
\]

(8)

where \( m_0 \) is the free electron mass. The scalar \( A(z,\omega) \), connected with vector potential in the Fourier representation \( A(z,\omega) \) is as follows

\[
A(z,\omega) = \mathbf{e}_t A(z,\omega) + \mathbf{e}_t^* A(z,-\omega).
\]

(9)

The formula, similar Eq. (9), takes place and for an electric field vector \( \mathbf{E}(z,\omega) \). Eq. (5) is true for heavy holes in crystals with the zinc blend structure, if axis \( z \) is
directed along of 4-th order symmetry axis \([2,13]\). Included in \(\gamma_r\), the real constant \(p_{cv}\) is connected with an interband matrix element of a momentum for two degenerate bands:

\[
p^{I,II}_{cv} = p_{cv} (e_x \mp i e_y)/\sqrt{2}.
\]

The current density \(\vec{J}(z,t)\) satisfies to a condition \(\text{div} \vec{J}(z,t) = 0\) and, hence, the induced charge density \(\rho(z,t) = 0\). Then it is possible to use calibration \(\varphi(z,t) = 0\), where \(\varphi(z,t)\) is the scalar potential, and

\[
\begin{align*}
E(z,t) & = (-1/c)(\partial A/\partial t), \\
E(z,\omega) & = (i \omega/c)A(z,\omega).
\end{align*}
\]

Since \(E(z,\omega) \sim A(z,\omega)\), instead of an equation for \(A(z,\omega)\), it is more convenient to solve the similar equation for the scalar \(E(z,\omega)\), which looks like

\[
d^2 E(z,\omega)/dz^2 + \kappa^2 E(z,\omega) = - (4\pi/c)\vec{J}(z,\omega),
\]

\[
\kappa = \nu \omega/c,
\]

where in expression for \(\vec{J}(z,\omega)\) Eq. (5) it is necessary to replace, using (10), \(A(z',\omega)\) by \(E(z',\omega)\). Eq. (11) is integro-differential one. If to represent formally its solution as the sum of the general solution of the homogeneous equation and partial solution of the inhomogeneous equation, one obtains instead of Eq. (12) the Fredholm integral equation of the second kind

\[
E(z,\omega) = C_1 e^{i \kappa z} + C_2 e^{-i \kappa z} - i (\gamma_r/2)F(z) - \int_0^1 dz' E(z',\omega) \Phi(z'),
\]

which is true for \(\omega\), close to \(\omega_0\), as its conclusion in Eq. (5) for \(\vec{J}(z,\omega)\) it was not taken into account not resonant composed \(\omega + \omega_0 + i \gamma/2\). The neglect by not resonant term is equivalent to an inequality \((\omega - \omega_0)/\omega_0 \ll 1\). Thus theory becomes inexact at \(\omega - \omega_0 \approx \omega_0\), however, this area of frequencies is located far away from the resonant frequency \(\omega_0\) and does not represent any interest. In time representation discrepance of the theory appears on times \(t \leq t_0 = \omega_0^{-1}\). If \(h\omega_0 = 1.6eV\), \(t_0 = 4.10^{-16}\text{sec}\). The arbitrary constants \(C_1\) and \(C_2\) are determined from boundary conditions in planes \(z = 0\) and \(z = d\), and the function \(F(z)\) looks like

\[
F(z) = e^{i \kappa z} \int_0^z dz' e^{-i \kappa z'} \Phi(z') + e^{-i \kappa z} \int_z^d dz' e^{i \kappa z'} \Phi(z').
\]

The complex value \(\varepsilon = \varepsilon' + i \varepsilon'' = \int_0^d dz' \Phi(z')F(z')\) determines the broadening change and energy level shift, which occur due to the wave spatial dispersion. In a limiting case \(\kappa d = 0\) \(\varepsilon = \delta_{m_e m_\nu}\). In barriers, where the induced current is absent, instead of Eq. (11) it true the equation

\[
d^2 E(z,\omega)/dz^2 + \kappa_1^2 E(z,\omega) = 0,
\]

\[
(z \leq 0, z \geq d), \quad \kappa_1 = \nu_1 \omega/c,
\]

the solution of which looks like

\[
E^l(z,\omega) = \delta_{0,0}(e^{i \kappa_1 z} + C_R e^{-i \kappa_1 z}), \quad (z \leq 0),
\]

\[
E^r(z,\omega) = C_T e^{i \kappa_1 z}, \quad (z \geq 0).
\]

The first term in the expression for \(E^l(z,\omega)\) is the scalar amplitude of the Fourier-transform of the exciting pulse, \(C_R\) determines the reflected wave amplitude, \(C_T\) determines the the transmitted wave amplitude past through a QW. The factors \(C_1, C_2, C_R\) and \(C_T\), being functions of the frequency \(\omega\), are determined from the continuity conditions of \(E(z,\omega)\) and \(dE(z,\omega)/dz\) on borders \(z = 0\) and \(z = d\). It results in

\[
C_1 = (2E_0(\omega)/\Delta)e^{-i \kappa d}[1 + \zeta + (1 - \zeta)N],
\]

\[
C_2 = -(2E_0(\omega)/\Delta)(1 - \zeta)[e^{-i \kappa d} + N],
\]

\[
C_R = E_0(\omega)/(\rho/\Delta),
\]

\[
C_T = 4E_0(\omega)\zeta e^{i \kappa d}[1 + e^{-i \kappa d}N]/\Delta;
\]
\[ \Delta = (\zeta + 1)^2 e^{-i\kappa d} - (\zeta - 1)^2 e^{i\kappa d} - 2(\zeta - 1)N[(\zeta + 1)e^{-i\kappa d} + \zeta - 1], \]
\[ \rho = 2i(\zeta^2 - 1)\sin\kappa d + 2i(\zeta^2 + 1)e^{-i\kappa d} + \zeta^2 - 1]N. \]

(19)

In Eqs. (18), (19) the designations are entered

\[ \zeta = \kappa/\kappa_1 = \nu/\nu_1, \]

(20)

\[ N = -i(\gamma_r/2)F^2(0)/[\omega - \omega_0 + i(\gamma + \gamma_r\varepsilon)/2]. \]

(21)

The function \( E_0(\omega) \) is determined by Eq. (4). It follows from Eqs. (13), (15), that in the case \( m_\nu = m_\omega = m \) (an admitted interband transition in a limit \( \kappa d = 0 \)) \( F(z) \) and \( \varepsilon \) are equal:

\[ F(z) = iB[2 - \exp(i\kappa z) - \exp(i\kappa(d - z))] - (\kappa d/\pi m)^2\sin^2(\pi mz/d)], \]

(22)

\[ F(0) = F(d) = iB[1 - \exp(i\kappa d)], \]

\[ B = (4\pi^2 m^2/\kappa d) /[4\pi^2 m^2 - (\kappa d)^2], \]

(23)

\[ \varepsilon' = F^2(0)\exp(-i\kappa d) = 4B^2\sin^2(\kappa d/2), \]

\[ \varepsilon'' = 2B[1 - B\sin\kappa d - 3(\kappa d)^2/8\pi^2 m^2]. \]

(24)

In the Fourier-representation the electrical field vector \( E^r(z,\omega) \) to the right of a QW in agreement with Eq. (17) looks like

\[ E^r(z,\omega) = \exp(ik_1 z)[e^r C_T(\omega) + e^r C_T(-\omega)], \]

(25)

and the field vector to the left of the QW \( E^l(z,\omega) \), including the exciting pulse field Eq. (4) and the reflected wave field \( \Delta E^l(z,\omega) \) is equal

\[ E^l(z,\omega) = E_0(z,\omega) + \Delta E^l(z,\omega), \]

(26)

\[ \Delta E^l(z,\omega) = \exp(-i\kappa_1 z)[e^r C_T(\omega) + e^r C_T(-\omega)], \]

(27)

III. TRANSITION TO TIME REPRESENTATION.

In the time representation the electrical field vector of the pulse transmitted through the QW according to Eq. (17) is represented as \( p = t - z\nu_1/c \)

\[ E^r(z,t) = e^r E^r(z,t) + c.c., \]

\[ E^r(z,t) = (1/2\pi) \int_{-\infty}^{\infty} dw \exp(-i\omega p) \times C_T(\omega), \]

(28)

Similarly, the field vector of the pulse, reflected from the QW, is equal

\[ \Delta E^l(z,t) = e^l \Delta E^l(z,t) + c.c., \]

\[ \Delta E^l(z,t) = (1/2\pi) \int_{-\infty}^{\infty} dw \exp(-i\omega s) \times C_R(\omega), \]

(29)

where \( s = t + z\nu_1/c \), and the functions \( C_T(\omega) \) and \( C_R(\omega) \) (after substitution in Eq. (18) \( E_0(\omega) \) from Eq. (4) and \( N(\omega) \) from Eq. (21))  look like

\[ C_T(\omega) = \frac{4E_0 \gamma_1 \zeta \exp(-i\kappa_1 d)}{L_D} \times \frac{\omega - \omega_0 - \gamma_\varepsilon' / 2 + i\gamma/2}{(\omega - \omega_1)^2 + (\gamma/2)^2}, \]

(30)

\[ C_R(\omega) = \frac{E_0 \gamma_1 \zeta \exp(-i\kappa_1 d)}{L_D} \times \{ \omega - \omega_0 - \gamma_\varepsilon'' / 2 + i(\gamma + \gamma_\varepsilon') / 2 \} - iB_1 \gamma_\varepsilon' / 2, \]

(31)

\[ D = \omega - \omega_0 - \gamma_\varepsilon F_1 / 2 + i(\gamma + \gamma_\varepsilon F_2) / 2, \]

(32)

\[ L = (1 + \zeta)^2 \exp(-i\kappa d) - (1 - \zeta)^2 \exp(i\kappa d), \]

(33)

\[ B = -2i(1 - \zeta^2) \sin\kappa d, \]

(34)

\[ B_1 = 2[1 + \zeta^2 - (1 - \zeta^2) \exp(i\kappa d)], \]

\[ F_1 = \varepsilon'' \frac{2 \varepsilon' (1 - \zeta^2) \sin\kappa d}{1 + \zeta^2 + (1 - \zeta^2) \cos\kappa d}, \]

\[ F_2 = \frac{2 \zeta \varepsilon'}{1 + \zeta^2 + (1 - \zeta^2) \cos\kappa d}. \]

(35)

In integrals Eqs. (28), (29) poles of integrand functions are \( \omega = \omega_1 \pm i\gamma_1/2 \), and there is also the pole in the bottom half-plane \( \omega \), determined by the equation \( D = 0 \). Strictly speaking, functions \( F_1 \) and \( F_2 \) included in \( D \) (Eq. (32)) depend {

from \( \omega \) since the module of a wave vector \( \kappa = \nu_1/c \) depends from \( \omega \). However, by virtue of assumptions, made at obtaining Eq. (12) \( \omega \) should not strongly differ from frequency \( \omega_0 \) and at the solution of the equation \( D = 0 \) it is enough to be limited by the first iteration. It results in following poles in the bottom half-plane

\[ \omega = \omega_0 - \gamma_\varepsilon F_1(\omega_0) - i(\gamma + \gamma_\varepsilon F_2(\omega_0))/2. \]

(36)

At use of the approached value of the pole Eq. (36) we shall receive, that

\[ \kappa = \kappa_0 = \nu_0 \omega/c, \]

\[ \kappa_1 = \kappa_{10} = \nu_1 \omega_0/c. \]
On the other hand, the poles \( \omega = \omega_I \pm i \gamma_I / 2 \) lead to \( \kappa = \kappa_I = \nu \omega_I / c, \quad \kappa_1 = \kappa_{1I} = \nu_1 \omega_I / c. \) As the theory is true at performance of an inequality \((\omega - \omega_O) / \omega_O \ll 1, \) further we consider, that \( \kappa_{I} = \kappa_0 = \kappa, \quad \kappa_{1I} = \kappa_{10} = \kappa_1. \)

After integration on \( \omega \) scalar functions \( E^r(z,t) \) and \( \Delta E^l(z,t) \) accept a kind

\[
E^r(z,t) = (4\zeta E_0 / \mathcal{L}) \exp(-i(\omega_I p + \kappa_1 d)) \times \{(1 - \Theta(p)) \exp(\gamma_I p/2) W_T(\gamma_I) + \Theta(p) \epsilon_T\};
\]

\[
\Delta E^l(z,t) = (E_0 / \mathcal{L}) \exp(-i\omega_I s) \times \{(1 - \Theta(s)) \exp(\gamma_I s/2) W_R(\gamma_I) + \Theta(s) \epsilon_R\};
\]

(38)

(39)

where the functions \( \epsilon_T \) and \( \epsilon_R \) are represented by the uniform formula

\[
\epsilon_T(R) = e^{-\nu(p)s/2} W_T(R)(-\gamma_I)
- e^{i(\Delta \omega - \gamma_r, F_1 / 2) p(s) W_T'(R)} \times e^{-(\gamma + \gamma_F s)p(s)/2}.
\]

(40)

In Eqs. (38) - (40) designations are entered

\[
\Delta \omega = \omega_I - \omega_0
\]

(41)

\[
W_T(\gamma_I) = [\Delta \omega - \gamma_r \epsilon''/2 + i(\gamma + \gamma_I / 2)] / \Omega(\gamma_I)
\]

(42)

\[
W_R(\gamma_I) = \{B[\Delta \omega - \gamma_r \epsilon''/2 + i(\gamma + \gamma_I / 2) - i\gamma r \epsilon / B_1 / 2] / \Omega(\gamma_I)\}
\]

(43)

\[
W_T'(R) = -i(\gamma r / 2) [F_2 - i(\epsilon'' - F_1)]
\times \left( \frac{1}{\Omega(-\gamma_I)} - \frac{1}{\Omega(\gamma_I)} \right)
\]

(44)

\[
W_R' = -i(\gamma r / 2) \times \left\{B[\epsilon' - F_2 + i(\epsilon'' - F_1)] + \epsilon' B_1 \right\}
\times \left( \frac{1}{\Omega(-\gamma_I)} - \frac{1}{\Omega(\gamma_I)} \right)
\]

(45)

\[
\Omega(\gamma_I) = \Delta \omega - \gamma_r F_1 / 2
+i(\gamma + \gamma_I + \gamma_r F_2) / 2.
\]

(46)

Let us notice, that account of dependence \( \kappa \) from \( \omega \) results into replacement in the expression for \( E^r(z,t) \) (38) variable \( p \) on \( p' = p + t_1 \), where \( t_1 = \nu_1 d / c \) corresponds to time, during which light passes a QW. Thus, account of dependencies \( \kappa \) from \( \omega \) will have an effect only for \( p \leq t_1 \). If \( d = 500 \text{A}, \nu_1 = 3, \quad t = 5.10^{-16} \text{c.} \equiv t_0. \) Since \( t_1 \leq t_0 \), account of dependence \( \kappa \) from \( \omega \) at the calculation of Eqs. (28), (29) is excess of accuracy, as results in amendments of the same order, which were not taken into account in Eq. (12). The obtained expressions for \( E^r(z,t) \) and \( E^l(z,t) \) are very large and their analytical research is complicated. Therefore two limiting cases represent some interest, when these expressions become essentially simpler. If the matter is homogeneous, i.e. \( \nu_1 = \nu \), then

\[
\kappa_1 = \kappa, \quad \mathcal{L} = 4 \exp(-i \kappa d), \quad B = 0,
\]

\[
B_1 = 4, \quad F_1 = \epsilon'', \quad F_2 = \epsilon'
\]

and Eqs. (38) and (39) pass in

\[
E^r(z,t) = E_0(z,t) + \Delta E^r(z,t) = E_0(z,t)
- E_0(i \gamma r \epsilon' / 2) \exp(-i \omega_I p) \times \{(1 - \Theta(p)) \exp(\gamma_I p/2) / \Omega(\gamma_I) + \Theta(p) \epsilon\},
\]

(47)

\[
\Delta E^l(z,t) = -E_0(i \gamma r \epsilon' / 2) \exp(-i \omega_I s - \kappa d)) \times \{(1 - \Theta(s)) \exp(\gamma_I s/2) / \Omega(\gamma_I) + \Theta(s) \epsilon\},
\]

(48)

where function \( \Omega(\gamma_I) \), determined in Eq. (40), turns in

\[
\Omega(\gamma_I) = \Delta \omega - \gamma_r \epsilon''/2 + i(\gamma + \gamma_I + \gamma_r \epsilon')/2
\]

(49)

and the function (40) accepts a kind

\[
\epsilon = \exp(-\gamma_I t/2) / \Omega(-\gamma_I)
- \exp(i(\Delta \omega - \gamma_r \epsilon''/2) \epsilon \exp(-\gamma r, \epsilon' / 2) / \Omega(-\gamma_I - \Omega(\gamma_I))\]

(50)

For \( E^r \) parameter \( t = p, \) for \( \Delta E^l \) \( t = s \). Function \( \Delta E^r(z,t) \) determines distortion of an exciting pulse, transmitted the QW.

It is seen from Eqs. (47) and (48) that the account of a spatial dispersion in a case of homogeneous matter results in a frequency shift of \( \omega_0 \) on magnitude \( \gamma_r \epsilon'' / 2 \) and to replacement \( \gamma_r \) by \( \tilde{\gamma}_r = \gamma_r \epsilon'. \) The value \( \tilde{\gamma}_r \) coincides with radiative broadenings of the EHP in a strong magnetic field at \( M_\perp = 0 \) in case of any value \( \kappa d \) calculated in \( \mathbb{H} \). If the spatial dispersion is not taken into account, i.e. \( \kappa d = 0, \) then, according to Eq. (24), \( \epsilon' \to 1, \quad \epsilon'' \to 0 \) and Eqs. (47) and (48) pass into expressions obtained in \( \mathbb{H} \) for homogeneous matter in absence of the spatial dispersion. Eqs. (47) and (48) coincide with similar expressions obtained in \( \mathbb{H} \) (Eq. (15), if there under frequency transition \( \omega_0 \) to understand \( \omega_0 + \gamma_r \epsilon''/2, \) and under \( \gamma_r \) the value \( \tilde{\gamma}_r \)).

A limiting case of a weak spatial dispersion, when \( \kappa d \to 0, \) but the matter is inhomogeneous, i.e. \( \nu_1 \neq \nu \) represents also some interest. It can take place for comparatively narrow QWs. Believing in Eqs. (38) and (39) \( \kappa d = 0, \) we shall obtain, that \( \mathcal{L} = 4 \zeta, \quad B = 0, \quad B_1 = 4 \zeta^2, \quad F_1 = 0, \quad F_2 = \tilde{\zeta} \) and

\[
\Delta E^r(z,t) = (-i E_0 \gamma_r \zeta / 2) \exp(-i \omega_I p) \times \{(1 - \Theta(p)) \exp(\gamma_I p/2) / \Omega(\gamma_I) + \epsilon' (p) \Theta(p)\}
\]

(51)
\begin{align*}
\epsilon'(p) &= \exp(-\gamma_{\text{f}} p/2)/\Omega(-\gamma_{\text{f}}) \\
&- \exp[i\Delta E p - (\gamma + \gamma_{\text{r}} \zeta) p/2] \\
&\times (\Omega(-\gamma_{\text{f}})^{-1} - \Omega(\gamma_{\text{f}})^{-1}).
\end{align*}

In this case
\begin{equation}
\Omega(\gamma_{\text{f}}) = \Delta \omega + i(\gamma + \gamma_{\text{r}} + \gamma_{\text{f}} \zeta/2),
\end{equation}
and \(\Delta E'(z,t)\) differs from Eq. (51) by the replacement \(p\) by \(s\). One can see, that the matter heterogeneity without taking into account the spatial dispersion results only into replacement \(\gamma_{\text{r}}\) by \(\gamma_{\text{f}} \zeta\), i.e. to the replacement in Eq. (8) for \(\gamma_{\text{f}}\) \(\nu\) on \(\nu_{\text{r}}\). Eqs. (51) and (52) coincide with obtained in [1], if \(\gamma_{\text{f}}\) there is replaced by \(\zeta \gamma_{\text{f}}\). Since in real systems \(\zeta \approx 1\), the changes, which are brought in only by heterogeneity of matter, are insignificant. The limit transition \(\gamma_{\text{f}} \to 0\) means a transition to a monochromatic exciting wave. In this limiting case Eqs. (38) and (39) pass in expressions obtained in [1].

IV. REFLECTION AND TRANSMISSION OF AN EXCITING PULSE.

The energy flux \(S(p)\), appropriate to the electrical field of the exciting pulse, is equal
\begin{equation}
S(p) = (e_z/4\pi)(c/\nu_1)(E_0(z,t))^2 = e_z S_0 P(p),
\end{equation}
where \(S_0 = cE_0^2/(2\pi \nu_1)\), \(e_z\) is the unit vector in a direction \(z\). The dimensionless function \(P(p)\) determines the spatial and time dependence of the energy flux of the exciting pulse
\begin{equation}
P(p) = (E_0(z,t))^2/S_0 = \Theta(p)e^{-\gamma p} + [1 - \Theta(p)]e^{\gamma p}.
\end{equation}
The transmitted flux to the right of the QW, by analogy with Eq. (54), looks like
\begin{equation}
S^r(z,t) = (e_z/4\pi)(c/\nu_1)(E^r(z,t))^2 = e_z S_0 T(p).
\end{equation}
For the reflected flux (to the left of a QW) we obtain
\begin{equation}
S^l = -(e_z/4\pi)(c/\nu_1)(\Delta E^l(z,t))^2
= -e_z S_0 R(s).
\end{equation}
The dimensionless functions \(T(p)\) and \(R(s)\) determine the shares of the transmitted and reflected energy of the exciting pulse.

Let us define, by analogy with [1], the absorbed energy flux \(S^a\) as a difference of the flux \(S + S^l\) at \(z = 0\) entered in the QW from the left, and flux \(S^r\) at \(z = d\) leaving the QW, on the right at the same moment of time \(t\):
\begin{equation}
S^a(t) = S(t) + S^l(t) - S^r(t).
\end{equation}
Using the definitions Eqs. (54) - (58) let us present \(S^a(t)\) as
\begin{equation}
S^a(t) = e_z S_0 \left[ P(t) - R(t) - T(t) \right].
\end{equation}
Defining a share of the absorbed energy \(A(t)\) by equality \(S^a(t) = e_z S_0 A(t)\) we shall obtain, that
\begin{equation}
A(t) = P(t) - R(t) - T(t).
\end{equation}
The Eq. (60) can be generalized if to remove planes, in which are observed flows on distance \(z = -z_0\) (to the left of well) and on \(z_0\) to the right of a QW \(z_0 > 0\). Then instead of Eq. (60) we shall obtain
\begin{equation}
A(x) = P(x) - R(x) - T(x),
\end{equation}
where \(x = p = s = t - \nu_1 |z_0|/c\). Expressions for the values \(T, R\) and \(A\), which are determined by scalars \(E^r(z,t)\) and \(\Delta E^l(z,t)\) on general Eqs. (38) and (39) are not shown here in view of their extreme size. The values \(P(t), T(t)\) and \(R(t)\) are always positive, \(A(t)\) can be of any sign. The negative absorption at some moment of time \(t\) means, that the electronic system of a QW gives back energy saved in previous moments of time.

V. TIME DEPENDENCE OF REFLECTION, TRANSMISSION AND ABSORPTION IN THE RESONANCE \(\omega_L = \omega_0\).

Let’s consider at first the limiting case \(\gamma \gg \gamma_{\text{r}}\). Then fields \(E^r(z,t)\) and \(\Delta E^l(z,t)\) from Eqs. (38) and (39) can be represented as a decomposition to a row
\begin{equation}
E^r(z,t) = E^r_0(z,t) + (\gamma_{\text{r}}/\gamma)E^r_1(z,t) + ..., \tag{62}
\end{equation}
\begin{equation}
\Delta E^l(z,t) = \Delta E^l_0(z,t) + (\gamma_{\text{r}}/\gamma)\Delta E^l_1(z,t) + ..., \tag{63}
\end{equation}
where
\begin{align*}
E^r_0(z,t) &= e_t (4\zeta E_0/L) \exp[-i(\omega_1 p + \kappa_1 d)] \\
&\times \{1 - \Theta(p)\} \exp(\gamma_{\text{f}} p/2) + \Theta(p) \exp(-\gamma_{\text{f}} p/2) \\
&+ \text{c.c.}, \tag{64}
\end{align*}
\begin{align*}
\Delta E^l_0(z,t) &= -e_t (BE_0/L) \exp(-i \omega_1 s) \\
&\times \{1 - \Theta(s)\} \exp(\gamma_{\text{f}} s/2) + \Theta(s) \exp(-\gamma_{\text{f}} s/2) \\
&+ \text{c.c.} \tag{65}
\end{align*}
correspond to fields of transmitted and reflected pulses at \(\gamma_{\text{p}} = 0\), i.e. when the absorption in a QW is absent.

In limiting cases \(\kappa d \neq 0, \zeta = 1 = \kappa d = 0, \zeta \neq 1\) \(\Delta E^l_0(z,t) = 0\), since, according to Eq. (34), \(S = 0\). In the first case it is connected with that the matter becomes homogeneous, in the second case - that a substance in QW becomes very little and the transmitting
wave does not react to it. In these limiting cases, as it follows from Eq. (57), \( R(t) \sim (\gamma_r/\gamma)^2 \), i. e. is a small value. At transition to a general case \( \kappa d \neq 0 \), \( \zeta \neq 1 \), \( R(t) \) looks like

\[
R(t) = S_0^{-1}[(\Delta E_0^1(s))^2 + 2(\gamma_r/\gamma)(\Delta E_0^0(s) \Delta E_1^1(s))], \tag{66}
\]

what will result in essential increase of reflection at the expense of first term in Eq. (66). As to transmission \( T(t) \), in limiting cases \( \kappa d = 0 \) or \( \zeta = 1 \), \( T(t) = P(t) \). At transition to a general case transmission changes poorly, as a multiplier \( 16\zeta/|L|^2 \) here does not too strongly differ from unit.

In Fig. 1 the time dependencies of the dimensionless transmission \( T \), absorption \( A \) and reflection \( R \) are represented for different values of parameters \( \kappa d \) and \( \zeta \). In Fig. 1 it is visible, that curves \( T(t) \) practically coincide for \( \kappa d = 0 \), \( \zeta = 1 \) and \( \kappa d = 1.5 \), \( \zeta = 1.1 \). The same takes place for absorption \( A(t) \). On the other hand, as it is visible in Fig. 1b, the reflection, being a small value, essentially depends on parameter \( \zeta \) for \( \kappa d = 1.5 \): at change \( \zeta \) from 1 up to 1.3 \( R(t) \) grows in 8 times.

In a limiting case \( \gamma_r \gg \gamma \) the induced fields are comparable on value with a field of an exciting pulse and consequently the form of the transmitted through a QW a pulse varies very strongly. It is visible in Fig. 2, where transmission is small, and the reflection \( R \) dominates. In the concept of the special points on time curves \( T \), \( A \) and \( R \) was entered. In particular, one of these points (the point of total reflection of the first type) was determined by a condition \( R(t_0) = P(t_0), T(t_0) = A(t_0) = 0 \) (see Fig. 2). If to take into account, that \( \zeta \neq 1 \), \( \kappa d \neq 0 \) (fig. 2b), then in a point of total reflection other condition takes place, namely \( T(t_0) + A(t_0) = 0 \), \( R(t_0) = P(t_0) \). It means, that \( A(t_0) < 0 \), i. e. the generation of radiation takes place, which was saved by system at earlier moments of time. Arisen at transition to a general case the special point according to the classification of is the special point of total reflection of the second type. Let’s note also, that in Fig. 2b transmission in some times is more, than in Fig. 2, i. e. and in this case the heterogeneity of matter and spatial dispersion strongly influence only on small values, which in the given limiting case is \( T(t) \).

VI. A DEVIATION OF CARRYING FREQUENCY FROM RESONANT.

In it was shown, that the carrying frequency deviation \( \Delta \omega \) results into time oscillations of \( A(t) \) and \( R(t) \). However, oscillations were distinct only for small values. On the other hand, taking into account the matter heterogeneity and spatial dispersion results in occurrence additional reflections from borders of a QW, which can exceed the oscillating component \( R(t) \). In Fig. 3 the examples of influence of the matter heterogeneity and spatial dispersion of light wave on function \( R(t) \) are given. The most significant changes take place in the case \( \gamma_r/\gamma \ll 1 \), i. e. for the short exciting pulse. It is visible in Fig. 3 that the value \( R(0) \) is increased in comparison with a case \( \zeta = 1 \), \( \kappa d = 0 \) more than in 300 times, oscillations here are indiscernible in view of their small amplitude. For an intermediate case \( \gamma_r = \gamma_l \) (Fig. 3b) the changes are insignificant and oscillations on a curve, corresponding to \( \zeta = 1.1 \), \( \kappa d = 1.5 \) are well visible. In Fig. 3 reflection \( R(0) \) is increased in 22 times, oscillations are still distinct. As to absorption, oscillating curves \( A(t) \) poorly change at transition to an inhomogeneous matter. It is explained by the fact that absorption is caused by quantum transitions in QW, which poorly depend on a refraction index.

In summary in Fig. 4 the curves \( R(t) \) for a case \( \gamma_r \gg \gamma_l \) are given (a long exciting pulse), when \( \Delta \omega \neq 0 \), but oscillations of reflection practically are imperceptible. It follows from Fig. 4 that the account only of the spatial dispersion, reduces reflection in comparison with a case \( \kappa d = 0 \). It is explained by reduction of radiative broadening \( \gamma_r \varepsilon' \), since \( \varepsilon' \) is a decreasing function of parameter \( \kappa d \). The transition to inhomogeneous matter results in increasing of reflection by that greater, than more the parameter \( \zeta \).

On the basis of obtained results it is possible to make a general conclusion, that the account of the matter heterogeneity and spatial dispersion of plane waves, consisting the exciting pulse, influences strongly only reflection. Changes are most significant in that case, when reflection, connected with interband transitions in a QW, is small and it masks by stronger reflection from borders of a QW. It takes place in the limiting case \( \gamma \gg \gamma_r \) (it was demonstrated above for the exact resonance \( \Delta \omega = 0 \)) and at a deviation of a carrying frequency from resonant in the other limiting case \( \gamma \ll \gamma_r \). It is necessary to note also strong dependence of reflection from the parameter \( \nu/\nu_1 \), which is increased for the account reflections from borders of a QW. The change of transmission also takes place only if it is not small.

In real semiconductor heterostructures the impure electrons of a barrier transit in a QW, deforming near to borders its rectangular form. Therefore advanced above theory is true for pure substances and wide QWs, when the size of the deformed frontier areas is small in comparison to a QW width. Besides the theory is true for deep QWs, the position of first energy levels in which and wave functions, appropriate to them, differ weakly from the position of energy levels and wave functions in indefinitely deep QWs. Since in the theory one exited level is taken into account only, the neighbour energy levels in a QW should be located further, than broadening of a considered energy level, and broadening of an exciting light pulse should be less than distances between neigh-
bour levels. These requirements impose restriction from above on a QW width. For example, for \( d = 500 \ang \) and \( m = 0.06 m_0 \) the difference of two lowest size-quantized energy levels is \( \approx 10^{-3} \text{eV} \).

VII. ACKNOWLEDGEMENTS

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