Cumulative post-buckling buckling behavior of smart sandwich structure

Achchhe Lal 1 and Kanif Markad 1

1Department of Mechanical Engineering, SVNIT, India
E-Mail: kmarkad13@gmail.com

Abstract: Cumulative post-buckling buckling behavior of smart sandwich structure has been presented with the combination of shape memory alloy (SMA) and shape memory polymer (SMP) in sandwich composite structure under uniform temperature distribution. The analysis is based on higher order shear deformation theory (HSDT) with von Karman nonlinearity by finite element method (FEM). Evaluation of nondimensional critical buckling temperature (NCBT) has been executed under varying plate thickness ratio. Thermal buckling performance of SMA reinforced SMP laminated composites under dynamic temperature condition with different strain rate and variation in SMA volume fraction is also presented first time to understand the cumulative post-buckling behavior.

1. Introduction

Smart materials have the characteristics to memorize a stimuli such as temperature, electric or magnetic field and correspondingly change its shape, natural frequency, stiffness, buckling, hardness and other mechanical properties. Shape memory alloys (SMA) are one of such class of smart material, which has the capability to regain its original shape/form from a deformation, when heated at an elevated temperature [1]. As compared to other smart materials, SMA has the highest recovery strain field and flexibility, due to its distinct thermo-mechanical, shape memory and pseudo-elastic properties towards the deviations in load, temperature and stresses [2]. In the past few decades, some alternatives are being explored to overcome limitations of SMA such as expense and poor fatigue. Among the various possible solutions Shape Memory Polymer (SMP) is a viable option and the further work is concentrated on it.

Buckling and post-buckling analysis of composite structures were taken into consideration by researchers, which affects stability of the structures in thermal environment [3, 4]. Embedding SMA, such as Nitinol wires, in the structure has satisfactorily enhanced the thermal buckling resistance, particularly in composites and sandwich panel subjected to thermal environment [5, 6]. Asadi et al. [7, 8] has gained analytical results for the vibration and thermal stability of hybrid composite structures strengthened with SMA fibers using FOBT technique for analytical solutions. When SMA wire reinforced composites are subjected to high temperature surplus strains are recovered, which induces internal tensions. This process create tension in structure and improves its thermal buckling and post buckling resistivity and provide hindrance to the resonance in the structure. There are few articles that deals with thermal buckling of composites or sandwich panels, which provides an opportunity to the authors to analyse the phenomenon in the current work. Analysing the phenomenon of SMP and SMA, it is noticed that the no one were
combinedly analysed the behaviour of SMA embedded SMPC and this is the reason behind the current work.

2. General Mathematical formulation

Using FEM implementation in MATLAB, the post-buckling analysis of sandwich smart composite plate is carried out by HSST adopting von-Karman kinematics in present study. Nine noded model which is having seven degrees of freedom (DOF) using \( C^0 \) continuity for the study of post-buckling behaviour of the structure.

2.1 Shape memory polymer matrix

In present study, facesheet and bottom sheet utilizes the shape memory polymer matrix having dynamic mechanical properties governed by \( T_g \) of polymer. A mathematical model is shown in Eq. (1) had been suggested to compute values of storage modulus \( E_m(T) \) of the polymers at a wide span of temperatures across \( T_g \) with a constant frequency (Mahieux and Reifsnider [9]).

\[
E_m(T) = (E_1 - E_2) \cdot \exp \left( - \left( \frac{T}{T_g} \right)^{m_1} \right) + (E_2 - E_3) \cdot \exp \left( - \left( \frac{T}{T_g} \right)^{m_2} \right) + E_3 \cdot \exp \left( - \left( \frac{T}{T_g} \right)^{m_3} \right)
\]  

Temperature dependent Poisson’s ratio (\( \mu \)) can be calculated through phase transition model for SMP material (Qi et al. [11]).

\[
\mu = \mu_p \cdot \nu_g + \mu_r(1 - \nu_g) 
\]  

\[
\nu_g = 1 - \frac{1}{1 + \exp \left[ -(T - T_m)/Z \right]} 
\]

The properties of composite material are estimated through the theory of volume averaging (Shen et al. [13]). Following equations are utilized as per the theory to calculate numerous material properties.

\[
E_{c1} = E_{f1} \cdot v_f + E_m \cdot v_m 
\]

\[
E_{c2} = (1 - C)E_{c1}^1 + C E_{c2}^1 
\]

\[
\mu_{c21} = (1 - C) \mu_{c12} + C \mu_{c21} 
\]

\[
E_{c2} = \frac{E_{c2}}{E_{c1}} 
\]

\[
G_{c12} = (1 - C) G_{c12}^1 + C G_{c12}^2 
\]

Where \( E_{c1} \) and \( E_{c2} \) are modulus of shape memory polymer composite (SMPC) along longitudinal and transverse directions respectively. \( \mu_{c12} \) and \( \mu_{c21} \) indicate Poisson’s ratio of composites along 1-2 and 2-1 plane respectively. The superscript 1 and 2 indicate values of constants in series and parallel respectively. Elastic modulus of SMP has been presented in Fig. 1 for the range of temperature from 273 K to 373 K. \( T_g \) of SMP has been investigated to be 305 K (Gu et al. [10]).
Fig. 1. Influence of variation of temperature on (a) storage modulus (a) longitudinal modulus, (b) transverse modulus (c) shear modulus of SMPC.

2.2 Displacement field equation

In present analysis sandwich smart composite plate is utilized whose length ‘a’, width ‘b’ and ‘h’ is the thickness which is constant throughout. For the FEM analysis, following displacement field equation is get used.

\[ U = u_0(x, y) + g_1(z) \Omega_x(x, y) + g_2(z) \zeta_y(x, y) \]
\[ V = v_0(x, y) + g_1(z) \Omega_y(x, y) + g_2(z) \zeta_x(x, y) \]
\[ W = w_0(x, y) \]

In Eq. (9) U, V and W are the displacements along x, y and z direction respectively at any point on the plate. \( u_0, v_0, w_0 \) are the midplane displacements, \( \Omega_x, \Omega_y \) are the rotations at z=0.

2.3 Strain and displacement relation

The strain vectors can be represented as,
The displacement vector can be written as,
\[
\{ \varepsilon \} = \{ \varepsilon_x, \varepsilon_y, \varepsilon_z \}^T = \{ \varepsilon_x, \varepsilon_y, \varepsilon_z \}^T = \{ \varepsilon_x, \varepsilon_y, \varepsilon_z \}^T
\]
\[
\{ \varepsilon \} = \{ \varepsilon_x - \varepsilon_x', \varepsilon_y - \varepsilon_y', \varepsilon_z - \varepsilon_z' \}^T
\]
(10)

\[
\{ \varepsilon \} = \{ \varepsilon_x, \varepsilon_y, \varepsilon_z \}^T
\]

Displacement vector can be written as,
\[
q = \begin{bmatrix} u \\ v \\ w \\ \zeta_x \\ \zeta_y \\ \Omega_x \\ \Omega_y \end{bmatrix}
\]
The linear and nonlinear strain vector can be written as, \( \{ \varepsilon_{nl} \} = \frac{1}{2} \{ A \} \{ \phi \} \) as per Lal et al. [14].

2.4 Strain and stress relationship

The relation between stress and strain for SMA inserted composite of kth layer with reference to reference coordinate system by following relation, Kumar and Singh [12]

\[
\{ \sigma \}^k = \{ \tilde{\sigma} \}^k + \{ \sigma \}^k V^k - \{ \tilde{\sigma} \}^k \{ \alpha \}^k V^k \Delta T
\]
(11)

2.5 Finite element modeling

Displacement vector and field vector written as follows, Lal et al. [14]

\[
\{ q \}^i = \sum_{i=1}^{NE} N_i \{ q_i \}, \quad x = \sum_{i=1}^{NE} N_i \{ x_i \}, \quad y = \sum_{i=1}^{NE} N_i \{ y_i \}
\]
(12)

2.6 Strain energy in composite plate

Total strain energy in the sandwich smart composite plate is represented as,

\[
\Pi_k = \frac{1}{2} \sum_{i=1}^{NE} \{ q_i \}^T (k_i + k_{nl}) \{ q_i \} = \{ q \}^T \{ K_i + K_{nl} \} \{ q \}
\]
(13)

\( NE \) and \( i \) are the number of elements and elemental, respectively and \( q \) is the global displacement vector. Where, \( K_i \) and \( K_{nl} \) are the linear and nonlinear stiffness matrix as per Lal et al. [14]

2.7 Work done and strain energy due to thermal loading

Due to thermal load application, the work potential stored can be given as,

\[
M_T = \sum_{k=1}^{NE} \int_{z_k}^{z_{k+1}} \{ \tilde{\sigma} \} \{ \varepsilon \} \, dz; \quad M_s = \sum_{k=1}^{NE} \int_{z_k}^{z_{k+1}} \{ \tilde{\sigma} \} \{ \varepsilon \} \, dz; \quad M_p = \sum_{k=1}^{NE} \int_{z_k}^{z_{k+1}} \{ \tilde{\sigma} \} \{ \varepsilon \} \, dz
\]
(14)

\( N_{T} \) is the thermal compressive load acting over the sandwich smart composite plate.

2.8 Governing equation

The governing equation for the sandwich composite plate can be acquired by minimizing the total potential energy with respect to generalized displacements (\( \partial [\Pi, -\Pi_j] / \partial \varepsilon = 0 \)).
\[ [K]q = \lambda[K]q \]

Where, \([K] = [K_r] + [K_o] \) with \([K_{r1}] = [K_{r1}] + [K_{r12}] + [K_{r13}] \)

In the expression \( \lambda \) is the critical buckling temperature.

3. Results and discussion

In this study, various boundary conditions (BC) are applied which are directed as,

SSSS: \( u = v = w = \Omega x = \xi = 0 \) at \( y = 0 \) and \( b \); \( u = v = w = \Omega y = \zeta = 0 \) at \( x = 0 \) and \( a \)

CCCC: \( u = v = w = \Omega x = \xi = \Omega y = \zeta = 0 \) at \( y = 0 \) and \( b \); \( u = v = w = \Omega x = \xi = \Omega y = \zeta = 0 \) at \( x = 0 \) and \( a \)

Convergence study has been done in Figure 2 to select the number of elements (neL) to calculate the results without tempering the results. From the figure it is clear that result converging from 3x3 mesh size, so 16 elements are selected for further study.

![Convergence study of FE solution for various mesh size](image)

To validate the finite element computer program, problems that have analytical solutions are solved. The accuracy of the present HSDT model is verified by validating the results with \([5 \text{ and } 12]\) by obtaining nondimensional critical buckling temperature (NCBT) as, \( T_{cr} = \Delta T \alpha_2 \) as shown in Figure 3. Figure 4 shows NCBT variation with the SMA volume fraction and its validation with the Kumar and Singh \([12]\) performed by layerwise theory. The result indicate the same nature of variation with the available results and also shows the effect of utilization of smart composite sandwich plate under clamped boundary condition. For the study material properties are considered, which is stated as following,

\( E_s = 10.3 \text{ GPa}, E_c = E_1, E_1 = 181 \text{ GPa}, G_{12} = 7.17 \text{ GPa}, G_{13} = G_{23} = 6.21 \text{ GPa}, \nu_{12} = \nu_{13} = 0.28, \nu_{23} = 0.33, \alpha_0 = 10^{-6}, \alpha_1 = 0.02 \alpha_0, \alpha_2 = 22.5 \alpha_0 \)

Nitinol SMA Wires are utilized in the study, which has temperature dependent modulus as shown in Figure 2, adopted from the Tawfik \([16]\). \( E_o = 0.1 \text{ GPa}, G = 24.86 \text{ GPa}, \alpha_1 = 10.26 \times 10^{-6}, \nu_{12} = 0.33 \)
Figure 5 shows the effect of different prestrains induced in SMA with SMA volume fraction variation for square smart sandwich composite plate (45/SMA/45) of a/h=50 in clamped BC. Figure shows the combined effect of SMA volume fraction and induced pre-strains over performance of sandwich composite structure. With 1% pre-strain, 23% increment in NCBT is noticed when comparing between without SMA and 5% SMA volume fraction, and also 57% variation observed for 5% to 15% volume fraction variation of SMA. Similarly 27% and 38% increment is observed between 5% SMA and without SMA results. Figure 6 shows the nature of variation of NCBT with the number of layers of SMA in laminated composites with 5% volume fraction under clamped BC and amplitude ratio of 0.1. As plate thickness varies from thick to thin, the resistance against buckling also get minimized. Figure clearly shows the effect of the number of layer on the performance of sandwich composite plate against critical buckling parameter, which explains, as thickness ratio varies from 5 to 100, NCBT increases by 6% to 46%.

Figure 7 indicated the variation in NCBT for first four buckling modes for 1%, 2% and 3% strain rate with temperature variation. For this study sandwich composite square clamped plate (90/SMA/90) with thickness ratio of 100 and SMA volume fraction of 5% has been considered. Firstly, it is observed that, NCBT increased by 35%, 57% and 32% for Mode 1 to 2, Mode 2 to 3 and Mode 3 to 4 respectively. Secondly, whenever comparing $\varepsilon_r=1\%$ to $\varepsilon_r=2\%$ and $\varepsilon_r=2\%$ to $\varepsilon_r=3\%$ for 50°C NCBT varies by 6.5% and
0.5% respectively whereas total increment of 7% noted down for \( \varepsilon_r = 1\% \) to \( \varepsilon_r = 3\% \). Moreover, when comparing \( \varepsilon_r = 1\% \) to \( \varepsilon_r = 2\% \) and \( \varepsilon_r = 2\% \) to \( \varepsilon_r = 3\% \) for 60\(^0\)C, NCBT varies by 6.2% and 4.4% respectively whereas total increment of 10.4% noticed for \( \varepsilon_r = 1\% \) to \( \varepsilon_r = 3\% \). Hence it is confirmed that, with temperature and strain rate increment, resistance against buckling, that is NCBT improves.

![Figure 7. Variation in NCBT with strain rate for different Modes. (M stands for Mode)](Image)

![Figure 8. Variation in nondimensional critical buckling temperature under the influence of temperature and strain rate over SMPC and SMA](Image)

Figure 7 highlights the abrupt decline which is due to the volatile material behavior of SMP matrix and SMA fiber reinforcements in the similar temperature span. It can be observed that instantaneous drop is regulated by modulus of SMP matrix, which regulates the profile of buckling load. Tawfik [16] indicates that, in a region 0\(^0\)C to 30\(^0\)C, temperature dependent elastic modulus of SMA fiber doesn’t show much variation as well as recovery stresses remains zero during same interval, but after this range dynamic change in modulus is takes place. Also at the same time, dynamic variation in elastic modulus of SMPC is observed as shown in Figure 1. Figure 8 indicates conclusive comparison of positive effect of combined shape memory properties of SMA and SMP on buckling resistance of material at elevated temperature. This is primarily due to the recovery stresses of SMA generated at elevated temperatures which enhances the stability of material as compared to normal SMPC or laminated composite structure.

### 4. Conclusions

Under induced uniform thermal loading, non-linear thermal post-buckling analysis of laminated composite plate, smart sandwich composite plate and for first time the combined effect of SMA and SMP is analyzed using higher order shear deformation theory and von Karman kinematics. Study also shows the combined effect of SMA volume fraction and induced pre-strains over performance of sandwich composite structure. With increasing SMA volume fraction, increment is noted in NCBT. Study also confirmed that, with temperature and strain rate increment, resistance against buckling, that is NCBT improves.

### References:

[1] Lagoudas D.C. 2008 *Shape memory alloys: modeling and engineering applications*. Springer Science & Business Media
[2] Asadi H, Akbarzadeh A H, and Wang Q 2015 Nonlinear thermo-inertial instability of functionally graded shape memory alloy sandwich plates Composite Structures 120 pp 496–508

[3] Mansouri M H, Shariyat M 2014 Thermal buckling predictions of three types of high-order theories for the heterogeneous orthotropic plates using the new version of DQM Composite Structures 113 pp 40–55

[4] Li Z M 2014 Thermal postbuckling behavior of 3D braided beams with initial geometric imperfection under different type temperature distributions Composite Structures 108 pp 924–936

[5] Lee J J, Choi S 1999 Thermal buckling and postbuckling analysis of a laminated composite beam with embedded SMA actuators Composite Structures 47 pp 695–703. https://doi.org/10.1016/S0263-8223(00)00038-6

[6] Lal A, Markad K 2020 Static buckling analysis of shape memory alloy reinforced composite laminated plate IOP Conf. Series: Materials Science and Engineering 814 p 012009

[7] Asadi H, Bodaghi M, Shakeri M, Aghdam M M 2013 On the free vibration of thermally pre/post-buckled shear deformable SMA hybrid composite beams Aerospace Science and Technology 31 pp 73–86

[8] Asadi H, Kiani Y, Shakeri M and Eslami M R 2014 Exact solution for nonlinear thermal stability of hybrid laminated composite Timoshenko beams reinforced with SMA fibers Composite Structures 108 pp 811–822. https://doi.org/10.1016/j.compstruct.2013.09.010

[9] Mahieux C A and Reifsnider K L 2001 Property modeling across transition temperatures in polymers: a robust stiffness–temperature model Polymer 42 pp 3281–3291.

[10] Gu J, Leng J, Sun H, Zeng H and Cai Z 2019 Thermomechanical constitutive modeling of fiber reinforced shape memory polymer composites based on thermodynamics with internal state variables Mechanics of Materials 130 pp 9–19. https://doi.org/10.1016/j.mechmat.2019.01.004

[11] Qi H J, Nguyen T D, Castro F, Yakacki C M and Shandas R 2008 Finite deformation thermomechanical behavior of thermally induced shape memory polymers Journal of the Mechanics and Physics of Solids 56 pp 1730–1751. https://doi.org/10.1016/j.jmps.2007.12.002

[12] Kumar C N and Singh B N 2009 Thermal Buckling and Post-Buckling of Laminated Composite Plates with SMA Fibers using Layerwise Theory International Journal for Computational Methods in Engineering Science and Mechanics 10(6) pp 423–429.

[13] Shen G L, Hu G and Liu B 2006 Mechanics of composite materials. Science and Technology, Beijing, China.

[14] Lal A, Singh B N and Kale S 2011 Stochastic post buckling analysis of laminated composite cylindrical shell panel subjected to hygro-thermo-mechanical loading Composite Structures 93 pp 1187–1200.

[15] Lee J 1997 Thermally induced buckling of laminated Composites by a layerwise theory Computers & Structures 65(6) pp 917–922.

[16] Tawfik M 1999 Suppression of post-buckling deflection and panel-flutter using shape memory alloy. MSc thesis, Aerospace department, old dominion university, Norfolk V A.