GEOMETRIC UNCERTAINTY ANALYSIS OF A CENTRIFUGAL COMPRESSOR USING KRIGING MODEL

Shujiang Li¹ and Min Huang² and Xuejun Liu¹

¹ School of Energy and Power Engineering, Beihang University, Beijing, China
² School of Reliability and Systems Engineering, Beihang University, Beijing, China

E-mail:lshujiang0417@163.com

Abstract. Centrifugal compressor, which is an important turbomachinery, plays a significant role in the gas turbine engine. Various uncertain factors in the manufacturing process can lead to the geometric variations, which often cause noticeable variations in compressor performance. In this paper, a surrogate-based uncertainty analysis method is used to investigate the effect of geometric uncertainty on the aerodynamic performance of a centrifugal compressor. Firstly, a scaled-up version of NASA DDA's 404-III centrifugal compressor is chosen as the research object. Considering the structural features of the compressor, a parameterized model with 38 independent control parameters is established to define the geometry of the compressor, enabling rapid modification of the geometry. Then, a widely-used simulation method based on computational fluid dynamics is employed to predict the aerodynamic performance at a series of specified geometries generated using Latin Hypercube Design. Next, based on the sample data, two Kriging models are built to emulate the complex functional relationship between the control parameters and the performance of the compressor. Finally, a Kriging model-based Monte Carlo simulation method is used to perform the geometric uncertainty analysis. After specifying a reasonable probability distribution for each control parameter, the statistical distributions of the compressor performance are obtained and analyzed. The analysis results yield new insights for compressor designers.

Keywords. Centrifugal compressor, Uncertainty analysis, Kriging model.

1. Introduction

Centrifugal compressor is one of the core components of aero-engine. Its performance and reliability have a very important impact on the performance, reliability and economy of the engine. But the actual geometric parameters of blades and other components will usually have deviations from their design values because of machining error. As a result, there are performance differences between the compressors actually made.

The influence of geometric uncertainty on aero-engine performance has received widespread attention. However, most researches focus on the influence of a single parameter change on the performance of a specific position of a component, and rarely consider the comprehensive impact of
each parameter change on the performance of the compressor system. Bammert and Sandstede [1] studied experimentally the influence of the processing errors of the mounting angle, airfoil profile, blade chord length, and the surface roughness of the turbine plane cascade on airflow turning angle and cascade loss coefficient. The results show that for this turbine cascade, when the roughness error is above 5.6μm, the aerodynamic performance of the blade will be significantly affected; the contour error and the airfoil installation angle will have a greater impact on the outlet airflow angle. The effect of blade chord length machining error on the blade performance is relatively small. Moustapha et al. [2] measured the aerodynamic performance of two turbine blades with different wedge angles and leading edge radii under non-design conditions, found that the curvature continuity between the leading edge arc and the blade body profile will have a greater impact on the blade performance at non-designed attack angles. Lange and Schrapp [3] studied the effect of blade machining errors on the performance of a high-pressure compressor intermediate stage based on probabilistic method. They measured 150 blades using a three-dimensional laser scanner, analyzed the various profile parameters, and then used Monte Carlo Simulation to numerically simulate the high-pressure compressor stage. The results show that the influence of the blade leading edge thickness on the total pressure loss coefficient of the rotor and the stage isentropic efficiency of the compressor dominates, while the blade installation angle and the parameters related to the mean camber line have a major influence on the airflow turning angle.

In this paper, the effect of geometric deviation on the performance of the compressor is studied by establishing a surrogate model. The first part briefly introduces the manufacturing deviation of compressors and related research; the second part selects a scaled-up version of NASA DDA's 404-III centrifugal compressor as the research object to establish a parametric model; and then briefly introduces Kriging model and Monte Carlo Simulation; after that, the above model was used to establish a surrogate model and the uncertainty analysis of the relevant parameters; finally, the research results were summarized and analyzed.

2. Parametric model of the compressor

Establishing a parametric model of the compressor can quickly generate a discrete data model of the compressor with corresponding geometric characteristics by adjusting the values of the control parameters. In this paper, a centrifugal compressor model with public data is selected: a scale-up version of NASA DDA's 404-III type centrifugal compressor [4]. Its main parameters are shown in Table 1. Figure 1(a) (b) is a schematic diagram of a meridional flow channel and a three-dimensional structure of the compressor.

| Table 1. Basic parameters of the compressor. |
|---------------------------------------------|
| Design speed | 21789RMP |
| Mass flow | 4.39kg/s |
| Impeller | Main blades *15 & Splitter blades *15 |
| Radial diffuser | Two-dimensional wedge-shaped blade *24 |

Figure 1. Schematic diagram of the compressor.

The impeller is the core component of a centrifugal compressor, and its performance will have a
major impact on the performance of the compressor. Therefore, this paper only selects the impeller of the centrifugal compressor for research, which include main blades and splitter blades, as shown in Figure 1(c).

This parametric model was built using Numeca Autoblade. Since the blade is the most important part that affects the performance of the compressor, in the model, the parameters such as the casing and the hub are set to fixed values, and the parameters related to the blade profile are used as variables. The geometrical uncertainty is reflected by adjusting the profile parameters.

2.1 Main blades.
Here we use the method of "thickness control mode", that is, first define the shape of the mean camber line, and then determine the thickness on both sides of the mean camber line from the leading edge to the trailing edge. The leading edge of the blade is arc, and the trailing edge is blunt. Since the mean camber line will not be used as a control variable in the future, in order to improve the fitting accuracy, a B-spline curve with 10 control points is used. As shown in Figure 2(a).

Asymmetric thickness distribution is adopted for the blade contour lines on both the suction side and the pressure side. The effect of blade manufacturing deviation is mainly reflected on the two contour surfaces of the blade. In the parametric model, it is mainly reflected by the change of the position of the control point on the contour line of each height. So it’s the key to choose the proper control points. Too many points will cause too many variables, which will increase the number of numerical simulation schemes and increase the difficulty of establishing surrogate model. Thus consider that the contour lines on both sides of the main blade are described by B-spline curves of 4 control points, and each control point is evenly distributed along the contour line of the blade, as shown in Figure 2(b).

2.2 Splitter blades.
The definition of the splitter blade is similar to the main blade, but it is smaller than the main blade, so we can choose fewer control points to approximate it. The Bezier curve of 10 control points is selected to define the midline of the airfoil, as shown in Figure 3(a). The contour lines on both sides of the blade are defined by B-spline curves with 3 control points, as shown in Figure 3(b).
3. Kriging model and Monte Carlo Simulation

3.1 Kriging model.
Kriging model is a method of unbiased optimal estimation of regionalized variables in a limited area based on structural analysis and variogram theory. It is also called spatial local interpolation [5].

Assume \( x^0 \) is the unobserved points that need to be evaluated, \( x_1, x_2, \ldots, x_N \) are the observation points around \( x^0 \), and \( Y(x_i) \) are the corresponding observations. Let \( \hat{Y}(x_0) \) be the estimated value of the unmeasured point, which is obtained by weighted summing the observation values of the surrounding observation points [6]:

\[
\hat{Y}(x_0) = \sum_{i=1}^{N} a_i Y(x_i)
\]

where \( a_i \) is the undetermined weighted coefficient. The key of the Kriging model is the calculation of the weight coefficient \( a_i \). There are two requirements that need to be met:

1. Unbiased estimation: Assume \( Y(x_0) \) be the true value of the valuation point. There will be variability in model space, so \( Y(x_i), \hat{Y}(x_0), Y(x_0) \) can all be regarded as random variables.

When \( \hat{Y}(x_0) \) is unbiased estimation:

\[
E[\hat{Y}(x_0) - Y(x_0)] = 0, \sum_{i=1}^{N} a_i = 1
\]

2. The variance of the difference between the estimated value \( \hat{Y}(x_0) \) and the true value \( Y(x_0) \) reaches the minimum:

\[
\sigma^2 = D(\hat{Y}(x_0) - Y(x_0)) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j C(x_i, x_j) - 2 \sum_{i=1}^{N} a_i C(x_i, x_0) + C(0)
\]

where \( C(x_i, x_j) \) represents the covariance function of the parameters. According to the conditions above, a Lagrange function can be constructed [7]:

\[
f(a_1, \ldots, a_n, \lambda) = \sigma^2 + 2\lambda(\sum_{i=1}^{n} a_i - 1)
\]

\[
\begin{align*}
\sum_{j=1}^{n} a_j &= 1 \\
\sum_{i=1}^{n} a_i C(x_i, x_j) - C(x_i, x_0) + \lambda &= 0
\end{align*}
\]

The additional parameter \( \lambda \) is a Lagrange multiplier. This equation set also called ordinary Kriging system, which contains \( n + 1 \) equations to solve for \( n \) coefficients. The solution of the system is as follows [8] :

\[
a = [C_0 + (1 - C_0^T C^{-1}) (I^T C^{-1}) I]^T C^{-1}
\]

where \( C \) is covariance matrix, \( C_0 \) is a column vector consisting of covariance between unknown points and samples. \( I \) is a column vector consisting of \( n \) ones. Bringing the results into the previous formula gives an unbiased

\[
\hat{Y}(x_0) = C_0^T C^{-1} Y + (1 - C_0^T C^{-1}) (I^T C^{-1}) (I^T C^{-1}) Y
\]

Monte Carlo Simulation

Monte Carlo Simulation (MCS) is an important numerical calculation method based on the theory of probability and statistics. It uses random numbers to solve many calculation problems. It is also called statistical simulation method [9].

The principle of the MCS is as follows: assume that the random variables \( X1 - Xn \) represent the uncertainty factors of a system, and their probability distributions are known. Randomly sample \( X1 - Xn \)
$Xn$ according to their probability distributions, then obtain the system response value corresponding
to each sampling combination plan. After all sampling calculations are completed, the probability
distribution characteristics of system response $Y1$ can be calculated (Mean, standard deviation, etc.).

4. Results

4.1 Establishment of surrogate model.

4.1.1 Input and output.
The parametric model of the compressor established earlier has a total of 193 parameters. By
analyzing the geometric characteristics represented by these parameters, 38 independent control
parameters which related to profile and tip clearance are selected from them. The range of each
parameter should be centered on the original value, and the distribution interval should be given
uniformly to both sides to ensure that it can cover and slightly exceed the parameter change interval
when the uncertainty analysis is performed later. The initial values and ranges of the input parameters
are given in Table 2.

The output selects the pressure ratio and efficiency at the design flow rate and speed to reflect the
aerodynamic performance of the compressor.

4.1.2 Sample data.
The number of sample points is related to the number of parameters and the model to be established.
The Kriging model needed in this research, according to the literature, shows that the required sample
points should be at least 10 times that of the input parameters. After comprehensively consideration,
it is planned to use the Latin hypercube method to design 550 simulation experiment schemes, of
which 500 groups are used to establish the model, 50 groups are used to test the accuracy of the model,
and are recorded as $550 \times 38$LHD. Use NUMECA to perform CFD calculation on the sample data to
obtain the training data needed to establish the kriging model.

4.1.3 Surrogate model.
From the training data, 500 sets of sample points were selected to establish the model1 with output as
the pressure ratio and the model2 with output as the efficiency by using DACE Toolbox in Matlab
[10]. Then, another 50 sets of sample points were selected from the sample data for model accuracy
testing, and the four parameters of mean, maximum, root mean square, and fitness were used as
indicators of model accuracy.

In the modeling process, various possible regression models and related function combinations
were selected and tested. The figure shows the comparison of the actual values of the pressure ratio
and efficiency with the approximate values of the final model when using 50 sets of samples for
model accuracy testing, where blue represents the actual value and orange represents the model
predicted value. It can be seen that both surrogate models have high enough accuracy and can be used
for subsequent research.

4.2 Uncertainty quantification and analysis.

4.2.1 Probability distribution of input.
Because the geometric deviation data of real centrifugal compressors could not be obtained, it was
finally determined that the contour deviation of the compressor fluctuated around $\pm 0.1$mm, and the
tip clearance deviation fluctuated around $\pm 0.03$mm by consulting relevant literatures. In this interval,
the parameter values are normally distributed usually. Therefore, a reasonable hypothesis is given for
the control parameter distribution of the compressor: assuming that the contour deviation of the compressor blade fluctuates randomly in a normal distribution within the range of $[-0.1$mm, $+0.1$mm],
according to the $3\sigma$ criterion, it can be considered the parameters $X1$-$X36$ obey the normal distribution
with its mean value and standard deviation of 0.03333mm [11].

**Table 2.** The initial values and range of input parameters.

| NO. | parameter | Initial value (mm) | Range (mm) | NO. | parameter | Initial value (mm) | Range (mm) |
|-----|-----------|--------------------|------------|-----|-----------|--------------------|------------|
| X1  | 0% height-MB-SS-LR R | 1.27 ± 0.254 |            | X19 | 100% height-MB-SS-LR R | 0.508 ± 0.254 |           |
| X2  | 0% height-MB-SS-CP P4 | 4.75996 ± 0.254 |            | X20 | 100% height-MB-SS-CP P4 | 0.97028 ± 0.254 |           |
| X3  | 0% height-MB-SS-CP P1 | 3.49504 ± 0.254 |            | X21 | 100% height-MB-SS-CP P1 | 1.0287 ± 0.254 |           |
| X4  | 0% height-MB-SS-CP P2 | 3.89128 ± 0.254 |            | X22 | 100% height-MB-SS-CP P2 | 1.02362 ± 0.254 |           |
| X5  | 0% height-MB-SS-CP P3 | 4.08686 ± 0.254 |            | X23 | 100% height-MB-SS-CP P3 | 0.96266 ± 0.254 |           |
| X6  | 0% height-MB-PS-LR R | 1.27 ± 0.254 |            | X24 | 100% height-MB-PS-LR R | 0.508 ± 0.254 |           |
| X7  | 0% height-MB-PS-CP P4 | 3.55346 ± 0.254 |            | X25 | 100% height-MB-PS-CP P4 | 0.52578 ± 0.254 |           |
| X8  | 0% height-MB-PS-CP P1 | 3.43916 ± 0.254 |            | X26 | 100% height-MB-PS-CP P1 | 1.0287 ± 0.254 |           |
| X9  | 0% height-MB-PS-CP P2 | 3.90906 ± 0.254 |            | X27 | 100% height-MB-PS-CP P2 | 0.95758 ± 0.254 |           |
| X10 | 0% height-MB-PS-CP P3 | 3.73634 ± 0.254 |            | X28 | 100% height-MB-PS-CP P3 | 0.90678 ± 0.254 |           |
| X11 | 0% height-SB-SS-LR R | 1.143 ± 0.254 |            | X29 | 100% height-SB-SS-LR R | 0.4572 ± 0.254 |           |
| X12 | 0% height-SB-SS-CP P3 | 3.76428 ± 0.254 |            | X30 | 100% height-SB-SS-CP P3 | 0.71374 ± 0.254 |           |
| X13 | 0% height-SB-SS-CP P1 | 3.46202 ± 0.254 |            | X31 | 100% height-SB-SS-CP P1 | 1.03378 ± 0.254 |           |
| X14 | 0% height-SB-SS-CP P2 | 4.04876 ± 0.254 |            | X32 | 100% height-SB-SS-CP P2 | 0.91186 ± 0.254 |           |
| X15 | 0% height-SB-PS-LR R | 1.143 ± 0.254 |            | X33 | 100% height-SB-PS-LR R | 0.4572 ± 0.254 |           |
| X16 | 0% height-SB-PS-CP P3 | 3.29946 ± 0.254 |            | X34 | 100% height-SB-PS-CP P3 | 0.6731 ± 0.254 |           |
3.66014 ±0.254 & 100% height- & 1.02362 ±0.254 \\
3.86588 ±0.254 & 100% height- & 0.90424 ±0.254 \\
0.1524 ±0.1016 & Leading edge & 0.2032 ±0.1016 \\
0.2032 ±0.1016 & Trailing edge & tip clearance \\

(MB-main blade, SB-splitter blade, SS-suction side, PS-pressure side, LR-leading edge radius, CP-control point)

Assuming the tip clearance randomly fluctuates according to the normal distribution in the range of [-0.03mm, + 0.03mm], according to the 3σ criterion, the parameter $X_{37}$ obeys the normal distribution with a mean of 0.1524mm and a standard deviation of 0.01mm; parameter $X_{38}$ follows a normal distribution with a mean of 0.2032mm and a standard deviation of 0.01mm.

4.2.2 Uncertainty analysis of aerodynamic parameters.

The statistical analysis of pressure ratio (Y1) and efficiency (Y2) is shown in Figure 4.

![Figure 4. Distributions of pressure ratio and efficiency.](image)

It can be seen that in the case where the overall geometric change shows a normal distribution, the pressure ratio and flow rate of the compressor are centered on their original parameter values, showing a trend of normal distribution. It can also be found that the fluctuation of the pressure ratio is more severe than the efficiency.

Among the 38 input parameters, parameters $X_1, X_2, \ldots, X_{36}$ are control parameters of the blade profile, and $X_{37}$ and $X_{38}$ are control parameters of the tip clearance. Based on the above conclusions, the standard deviations of these two control parameters are changed separately, and Monte Carlo simulations are performed to study the influence of the fluctuation of the blade profile and the tip clearance on the compressor performance.

Change the range of parameters $X_1, X_2, \ldots, X_{36}$ to ± 0.15mm and ± 0.05mm, and the range of $X_{37}$ and $X_{38}$ remain the same. Then carry out Monte Carlo simulation to get the distribution of pressure ratio and efficiency, as shown in Table 3.
Table 3. Effect of blade profile on performance.

| Range  | Standard deviation | Pressure ratio mean | range       | Efficiency mean | range       | Standard deviation |
|--------|--------------------|---------------------|-------------|----------------|-------------|--------------------|
| 0.15mm | 0.05mm             | 4.8213              | [4.7950, 4.8574] | 0.0081     | 0.863       | [0.8610, 0.86520]  |
| 0.1mm  | 0.3333mm           | 4.8288              | [4.8001, 4.8565] | 0.0076     | 0.863       | [0.86140, 0.8649] |
| 0.05mm | 0.1667mm           | 4.8294              | [4.803, 4.8556]   | 0.0072     | 0.863       | [0.8618, 0.8645]  |

It can be seen that the mean of pressure ratio and efficiency changes little, and it can basically be considered that it will not be affected by the change of the profile. The range and standard deviation of the pressure ratio and efficiency increase with the increase of the range of the profile, and decrease with the decrease. The effect of the blade profile change on the efficiency is greater than the effect on the pressure ratio.

Change the range of parameters $X37$ and $X38$ to $\pm 0.1$mm and $\pm 0.01$mm, and the range of $X1$ , $X2$ , ... $X36$ remain the same. The results are shown in Table 4.

It can be seen that the mean of pressure ratio and efficiency is basically unchanged, and it can basically be considered that it will not be affected by the amplitude of the tip clearance. The range and standard deviation of the pressure ratio and efficiency increase with the increase of the tip clearance, and decrease with the decrease. The effect of tip clearance change on pressure ratio and efficiency is greater than the effect of blade profile change.

Table 4. Effect of tip clearance on performance.

| Range  | Standard deviation | Pressure ratio mean | range       | Efficiency mean | range       | Standard deviation |
|--------|--------------------|---------------------|-------------|----------------|-------------|--------------------|
| 0.1mm  | 0.0333mm           | 4.8288              | [4.7403, 4.9168] | 0.024       | 0.8631      | [0.8588, 0.8677]  |
| 0.03mm | 0.01mm             | 4.8288              | [4.8001, 4.8556] | 0.0076     | 0.863       | [0.8614, 0.8649] |
| 0.01mm | 0.0033mm           | 4.8288              | [4.8148, 4.8415]   | 0.0034     | 0.863       | [0.8620, 0.8641]  |

5. Conclusion

In this paper, a scaled-up version of NASA DDA's 404-III centrifugal compressor is selected as the research object, and the impact of manufacturing deviation on the performance of the compressor is studied by means of uncertainty quantification. A parametric model and a kriging model of the compressor was established, then the Monte Carlo simulation was used to carry out quantitative research on uncertainty. The results show that when the geometric deviation of the compressor is normally distributed, its performance is also normally distributed. The effect of blade profile on pressure ratio is greater than its effect on efficiency, but their mean changes little. The effect of tip clearance on pressure ratio and efficiency is greater than the effect of blade profile, but the effect on their mean is little either. The change trend of pressure ratio and efficiency is the same as the change trend of the blade profile and tip clearance.
6. Reference

[1] Bammert K and Sandstede H., 1976 "Influences of Manufacturing Tolerances and Surface Roughness of Blades on the Performance of Turbines". *J. Journal of Engineering for Gas Turbines and Power*, 9(1) 29–36

[2] Benner M W. 2003, "The influence of leading-edge geometry on profile and secondary losses in turbine cascades". *Pediatric Pulmonology*, Vol. 65, No. 02, pp.08–57

[3] Lange, A., Voigt, M., Vogeler, K., Schrapp, H., & Johann, E. 2010. “Probabilistic CFD simulation of a high-pressure compressor stage taking manufacturing variability into account”. *In ASME Turbo Expo 2010: Power for Land, Sea, and Air*, pp 617–28

[4] Mckain T F , Holbrook G J. 1997 "Coordinates for a High Performance 4:1 Pressure Ratio Centrifugal Compressor"

[5] Sacks, J., Welch, W. J., Mitchell, T. J. & Wynn, H. P. 1989. Design and Analysis of Computer Experiments. *J. Statistical Science*, 4(4) 409–23

[6] Le, N. D., Zidek, J. V. (2006), "Statistical analysis of environmental space-time processes". *J. Springer Science & Business Media*, 101–34

[7] Wackernagel, H. 2013 Multivariate geostatistics: an introduction with applications. *J. Springer Science & Business Media*

[8] Cressie, N. 1990. The origins of kriging. *Mathematical geology*, Vol.22, No.3, pp.239–52

[9] Binder, K., Heermann, D., Roelofs, L., Mallinckrodt, A. J. & McKay, S. 1993. Monte Carlo simulation in statistical physics. *J. Computers in Physics*, 7(2) 156–7

[10] Lophaven, S. N., Nielsen, H. B. & Sondergaard, J. 2002. *DACE-A Matlab Kriging toolbox*, version 2.0

[11] Grafarend, E. W. 2006 *Linear and nonlinear models: fixed effects, random effects, and mixed models*. de Gruyter