Relativistic structure of one-meson and one-gluon exchange forces and the lower excitation spectrum of the Nucleon and the $\Delta$

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The lower excitation spectrum of the nucleon and $\Delta$ is calculated in a relativistic chiral quark model. Corrections to the baryon mass spectrum from the second order self-energy and exchange diagrams induced by pion and gluon fields are estimated in the field-theoretical framework. Convergent results for the self-energy terms are obtained when including the intermediate quark and antiquark states with a total momentum up to $j = 25/2$. Relativistic one-meson and color-magnetic one-gluon exchange forces are shown to generate spin 0, 1, 2, etc. operators, which couple the lower and the upper components of the two interacting valence quarks and yield reasonable matrix elements for the lower excitation spectrum of the Nucleon and Delta. The only contribution to the ground state nucleon and $\Delta$ comes from the spin 1 operators, which correspond to the exchanged pion or gluon in the $l=1$ orbit, thus indicating, that the both pion exchange and color-magnetic gluon exchange forces can contribute to the spin of baryons. Is is shown also that the contribution of the color-electric component of the gluon fields to the baryon spectrum is enormously large (more than 500 MeV with a value $\alpha_s = 0.65$ ) and one needs to restrict to very small values of the strong coupling constant or to exclude completely the gluon-loop corrections to the baryon spectrum. With this restriction, the calculated spectrum reproduces the main properties of the data, however needs further contribution from the two-pion exchange and instanton induced exchange (for the nucleon sector) forces in consistence with the realistic NN-interaction models.

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I. INTRODUCTION

The spectroscopy is a serious task for any model of hadron structure. It can yield a detailed information on the source of hyperfine interaction, as well as on effective degrees of freedom in the description of hadron dynamics. The so-called "three-body spin-orbit puzzle" has a long history. In the original papers of Isgur and Karl [1, 2] it was shown that the whole SU(3) baryon spectrum can be reproduced qualitatively and (with some restrictions) quantitatively in a constituent quark model based on a non-relativistic hamiltonian with a central confining potential plus effective one-gluon exchange spin-orbit forces. The hyperfine interaction was assumed to be a sum of the spin-spin, tensor and spin-orbit interaction potentials between the constituent quarks. However, the spin-orbit term yields a very large (of order 500 MeV) matrix elements in the three-body system (typical baryons), while in mesons and meson-like baryons [3] the matrix elements are small as a result of the strong cancelation between the dynamical one-gluon exchange spin-orbit forces and a pure kinematical spin-orbit forces due to the Thomas precession of the quark spin in the central confining potential. Contrary, in ordinary baryons (like $N^*$ and $\Delta^*$), there are not enough cancelation effects between the two origin spin-orbit forces. Showing these possibilities and assuming that in baryons like in mesons must be a consistent cancelation, Isgur and Karl, as a first approximation, excluded all spin-orbit forces from the hyperfine interaction potential. Since the beginning the problem was called a "three body spin-orbit puzzle" in baryons and up to now it was not resolved in a consistent way, although in the relativized version of the constituent quark models (see [4, 5, 6]) some reduction of the size of the spin-orbit interactions to acceptable levels was obtained.

There are other strong candidates for the hyperfine interaction in baryons such as Goldstone-boson exchange (GBE) forces, [7, 8] and the instanton induced exchange forces (IIE) [9]. The GBE based constituent quark model was proposed as a realization of the spontaneous breaking of the chiral symmetry of QCD, which plays a crucial role for the hadron structure. The debates between the OGE and GBE based quark models over the last decade [3, 10, 11] were focused on the "three-body spin-orbit puzzle", as well as the level ordering problem and the mixing angles in excited baryon sector. While reproducing the correct level ordering (for example $N^*(1/2^+)(1440) - N^*(3/2^-)(1520)$, $N^*(1/2^-)(1535)$), the GBE based model does not contain spin-orbit terms.

The main issue of these debates likely supports the idea that the origin of hyperfine interaction between valence quarks is a sum of the GBE and OGE forces (and possible two-pion exchange [12], and additionally IIE forces for the nucleon sector) in consistence with the theory of the low-energy NN-interaction [13]. The question is, however, how much the each exchange mechanism does contribute to the baryon spectrum? The solving of this complex problem requires many efforts to be done both theoretically and experimentally.

On the other hand, from the beginning of the proton spin crisis [14], it became clear that the non-relativistic constituent quark models are not adequate and complete picture for the hadron structure, since only about 30 percent...
of the proton spin is described by the valence quarks! The possible contribution comes from the coupling to the lower component of the Dirac spinor, one-gluon exchange and pion cloud effects. The problem can be resolved in the frame of the cloudy bag model (CBM) [15], as was claimed in the recent papers [16]. A relativistic picture of the hadron structure, while playing a decisive role for the understanding of the proton spin crisis, also should yield a consistent description of the hadron spectrum including excited baryons. The questions, concerning the source of the hyperfine splitting in baryons, the relativistic structure of the one-meson and one-gluon exchange mechanisms, the role of the lower component of the Dirac spinor of valence quarks and the contribution of the sea-quarks to baryon spectrum are still being open! The use of the bag models in baryon spectroscopy was restricted to the octet and decuplet baryons [17] while having some specific problems concerning the convergence of the self-energy and the sharp surface pion-quark coupling. The recent progress in the frame of the chiral quark-meson coupling model (CQMCM) [18] was done for the description of the N and Δ spectrum and the Nuclear Matter with inclusion both gluon and meson one-loop diagrams while restricting the intermediate baryon states to the lowest mode when considering the self-energy diagrams.

The aim of present paper is to show that a description of the baryon structure in a field-theoretical based chiral quark model [19, 20, 21, 22] can open many windows to the long-standing problems of excited baryon spectroscopy. In the last work we have calculated the lower excitation spectrum of the SU(2) baryons on the basis of one pion-loop diagrams. Further we will show that relativistic one-meson and color-magnetic one-gluon exchange forces between valence quarks generate spin 0, spin 1, spin 2 etc. operators, which can be considered as analogy of the non-relativistic spin-spin, spin-orbit, tensor etc. operators, respectively. However, all these relativistic operators couple the upper and the lower Dirac components of the two interacting valence quarks, hence have a strongly different nature. For instance, the two valence quarks in S-waves, interacting via one-gluon or one pion exchange forces are coupled only by the spin 1 operators (the analogy of the spin-orbit-forces), while in the non-relativistic models the corresponding terms have a spin-spin structure, i.e. are spin 0 operators. This finding has an important consequence for the proton spin problem, since the possible contribution to the proton spin comes not only from one-gluon exchange, but also from one-meson exchange mechanism. Additionally, in recent paper [23], meson-cloud effects were shown to be important for the description of the flavor asymmetry of the nucleon sea. We will show in a phenomenological way that a reasonable description of the lower excitation spectrum of the $N^*$ and $\Delta^*$ can be obtained in a relativistic chiral quark model, including all second-order corrections induced by pion fields (corresponding to the self energy and exchange diagrams) to the zeroth-order quark-core results with further possible improvement by including the two-pion exchange and IIE (for the nucleon sector) forces. It will be shown also that the contribution of the color-electric (Coulomb) component of the gluon fields to the baryon spectrum is enormously large at reasonable values of the strong coupling constant. To have an acceptable contribution of the gluon one-loop diagrams one needs a small value of the strong coupling constant which results the gluon-exchange contribution to the nucleon-$\Delta$ mass splitting to be negligible. This result confirm the prediction of the non-relativistic constituent quark model [10], that the one-gluon exchange mechanism does not play an important role in the description of the SU(2) baryon spectrum. We will show also that the restriction of the intermediate baryon states to the lowest mode when calculating the contributions of the self-energy diagrams is not a good approximation and in order to have convergent results, one has to include all intermediate excited quark and antiquark states with the total angular momentum up to $j=25/2$.

The relativistic quark model is based on an effective chiral Lagrangian describing quarks as relativistic fermions moving in a confining static potential. The modification of the model, the so-called Perturbative Chiral Quark model recently was applied to the ground- state baryon masses [24]. The potential is described by a Lorentz scalar and the time component of a vector potential, where the latter term is responsible for short-range fluctuations of the gluon field configurations [25]. The model potential defines unperturbed wave functions of the quarks which are subsequently used in the calculations of baryon properties. The baryons are considered as bound states of valence quarks, surrounded by a pion cloud as required by the chiral symmetry and by gluons. Interaction of quarks with a pion is introduced on the basis of the linearized $\sigma$-model [21, 26]. The residual color-magnetic quark-gluon interaction is introduced on the field-theoretical basis as prescribed by QCD. Calculations are performed perturbatively to second order in the quark-pion and quark-gluon interaction. All calculations are performed at one loop or at order of accuracy $O(1/f_s^2, \alpha_s)$. In the following we proceed as follows: we first describe the basic formalism of our approach. Then we indicate the main derivations relevant to the problem and finally present the numerical results.

II. MODEL

The effective Lagrangian of our model $\mathcal{L}(x)$ contains the quark core part $\mathcal{L}_Q(x)$ the quark-pion $\mathcal{L}_I^{(q\pi)}(x)$ and the quark-gluon $\mathcal{L}_I^{(qg)}(x)$ interaction parts, and the kinetic parts for the pion $\mathcal{L}_\pi(x)$ and gluon $\mathcal{L}_g(x)$:

$$\mathcal{L}(x) = \mathcal{L}_Q(x) + \mathcal{L}_I^{(q\pi)}(x) + \mathcal{L}_I^{(qg)}(x) + \mathcal{L}_\pi(x) + \mathcal{L}_g(x)$$
\[
\psi(x)[i \not\partial - S(r) - \gamma^0 V(r)]\psi(x) - 1/f_\pi \bar{\psi}[S(r)i\gamma^5 \tau^i \phi_i]\psi - \nonumber
-g_s \bar{\psi} A^\mu_\alpha \gamma^\mu \frac{\lambda^a}{2} \psi + \frac{1}{2} \hat{\Theta}(\mu) \lambda^a - \frac{1}{4} G^\mu_\alpha G^\mu_\alpha.
\] (1)

Here, \(\psi(x)\), \(\phi_i\), \(i = 1, 2, 3\) and \(A^\mu_\alpha\) are the quark, the pion and the gluon fields, respectively. The matrices \(\tau^i (i = 1, 2, 3)\) and \(\lambda^a (a = 1, \ldots, 8)\) are the isospin and color matrices, correspondingly. The pion decay constant \(f_\pi = 93\) MeV. The scalar part of the static confinement potential is given by

\[S(r) = cr + m\] (2)

where \(c\) and \(m\) are constants.

At short distances, transverse fluctuations of the string are dominating \(25\), with some indication that they transform like the time component of the Lorentz vector. They are given by a Coulomb type vector potential as

\[V(r) = -\alpha/r\] (3)

where \(\alpha\) is approximated by a constant. The quark fields are obtained from solving the Dirac equation with the corresponding scalar plus vector potentials

\[
[i\gamma^\mu \partial_\mu - S(r) - \gamma^0 V(r)]\psi(x) = 0
\] (4)

The respective positive and negative energy eigenstates as solutions to the Dirac equation with a spherically symmetric mean field, are given in a general form as

\[
u_\alpha(x) = \left(\begin{array}{c}
g_{N_\kappa}(r) \\
-\frac{1}{2} f_{N_\kappa}(r) \frac{\hat{\sigma} \cdot \hat{x}}{m_x}
\end{array}\right) \gamma^\nu_\kappa(\hat{x}) \chi_{\kappa m_x m_c} \exp(-iE_\alpha t)
\] (5)

\[
u_\beta(x) = \left(\begin{array}{c}
g_{N_\kappa}(r) \\
-\frac{1}{2} f_{N_\kappa}(r) \frac{\hat{\sigma} \cdot \hat{x}}{m_x}
\end{array}\right) \gamma^\nu_\kappa(\hat{x}) \chi_{\kappa m_x m_c} \exp(+iE_\beta t)
\] (6)

The quark and anti-quark eigenstates \(u\) and \(v\) are labeled by the radial, angular, azimuthal, isospin and color quantum numbers \(N, \kappa, m_x, m_t\) and \(m_c\), which are collectively denoted by \(\alpha\) and \(\beta\), respectively. The spin-angular part of the quark field operators

\[
\gamma^\nu_\kappa(\hat{x}) = [Y_\kappa(\hat{x}) \otimes \chi_{1/2}]_{m_j} j = |\kappa| - 1/2.
\] (7)

The quark fields \(\psi\) are expanded over the basis of positive and negative energy eigenstates as

\[
\psi(x) = \sum_\alpha u_\alpha(x) b_\alpha + \sum_\beta v_\beta(x) d_\beta^\dagger.
\] (8)

The expansion coefficients \(b_\alpha\) and \(d_\beta^\dagger\) are operators, which annihilate a quark and create an anti-quark in the orbits \(\alpha\) and \(\beta\), respectively.

The free pion field operator is expanded over plane wave solutions as

\[
\phi_j(x) = (2\pi)^{-3/2} \int \frac{d^3k}{(2\omega_k)^{1/2}} [a_{jk} \exp(-ikx) + a_{jk}^\dagger \exp(ikx)]
\] (9)

with the usual destruction and creation operators \(a_{jk}\) and \(a_{jk}^\dagger\), respectively. The pion energy is defined as \(\omega_k = \sqrt{k^2 + m^2_\pi}\). The expansion of the free zero mass gluon field operators is of the same form.

In denoting the three-quark vacuum state by \(|0\rangle\), the corresponding noninteracting many-body quark Green’s function (propagator) is given by the customary vacuum Feynman propagator for a binding potential \(27\):

\[
iG(x, x') = iG^F(x, x') = -\langle 0 | T\{\psi(x)\bar{\psi}(x')\}|0 \rangle = \sum_\alpha u_\alpha(x) \bar{u}_\alpha(x') \theta(t - t') + \sum_\beta v_\beta(x) \bar{v}_\beta(x') \theta(t' - t)
\] (10)

Since the three-quark vacuum state \(|0\rangle\) does not contain any pion or gluon, the pion and gluon Green’s functions are given by the usual free Feynman propagator for a boson field:

\[
i\Delta_{ij}(x - x') = -\langle 0 | T\{\phi_i(x)\bar{\phi}_j(x')\}|0 \rangle = i\delta_{ij} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2_\pi + i\epsilon} \exp[-ik(x - x')],
\] (11)
\[ i\Delta_{ab}^{(\mu\nu)}(x - x') = <0|T\{A_{\mu}^a(x)A_{\nu}^b(x')\}|0> = i\delta_{ab}g^{\mu\nu}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 + i\epsilon}\exp[-ik(x - x')], \]  

(in the Coulomb gauge), where we choose \( g^{\mu\nu} = \delta_{\mu\nu}g^{\mu\nu}, \) \( g^{00} = -g^{11} = -g^{22} = -g^{33} = 1. \)

Using the effective Lagrangian and the time-ordered perturbation theory one can develop a calculation scheme for the lower excitation spectrum of the nucleon and delta. In the model the quark core result \( (E_Q) \) is obtained by solving Eq. (11) for the single quark system numerically. Since we work in the independent particle model and limited with the lower excitation spectrum of the nucleon and Delta, the bare three-quark state of the \( SU(2) \)-flavor baryons corresponds to the structure \((1S_{1/2})^2(nlj)\) in the non-relativistic spectroscopic notation. The corresponding quark core energy is evaluated as the sum of single quark energies with:

\[ E_Q = 2E(1S_{1/2}) + E(nlj) \]

The result for \( E_Q \) still contains the contribution of the center of mass motion. To remove this additional term we resort to three approximate methods, which correct for the center of mass motion: the \( R = 0 \) \[28\], \( P = 0 \) \[29\] and LHO \[30\] methods.

The second order perturbative corrections to the energy spectrum of the \( SU(2) \) baryons due to the pion field \( (\Delta E^{(\pi)}) \) and the gluon fields \( (\Delta E^{(g)}) \) are calculated on the basis of the Gell-Mann and Low theorem:

\[ \Delta E = = <\Phi_0| \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int i\delta(t_1) dt_1 \ldots dt_n T[H_{\pi}(t_1) \ldots H_{\pi}(x_n)]|\Phi_0 >, \]

with \( n = 2 \), where the relevant quark-pion and quark-gluon interaction Hamiltonian densities are

\[ H_{\pi}^{(q\pi)}(x) = i \int_\pi \bar{\psi}(x)\gamma^5\tau\bar{\phi}(x)S(r)\psi(x), \]

\[ H_{\pi}^{(gg)}(x) = g_{\pi}\bar{\psi}(x)A_{\mu}^a(x)\gamma^\mu\lambda^a_2\psi(x), \]

The stationary bare three-quark state \( |\Phi_0 > \) is constructed from the vacuum state using the usual creation operators:

\[ |\Phi_0 >_{\alpha\beta\gamma} = b_\alpha^+b_\beta^+b_\gamma^+|0>, \]

where \( \alpha, \beta \) and \( \gamma \) represent the quantum numbers of the single quark states, which are coupled to the respective baryon configuration. The energy shift of Eq. (13) is evaluated up to second order in the quark-pion and quark-gluon interaction, and generates self-energy and exchange diagrams contributions.

### A. Self-energy diagrams contribution

The self-energy terms contain contribution both from intermediate quark \( (E > 0) \) and anti-quark \( (E < 0) \) states. These diagrams correspond to the case when a pion or gluon is emitted and absorbed by the same valence quark which is excited to the intermediate quark and anti-quark states. In this way one can estimate the contribution of the sea-quarks to the hadron spectrum that can not be done in non-relativistic quark models.

The pion part of the self energy term (pion cloud contribution) (see Fig.1) is evaluated as

\[ \Delta E_{s.e.}^{(\pi)} = -\frac{1}{2f_{\pi}^2}\sum_{a=1}^{3}\sum_{\alpha,\alpha'\leq \alpha} \int d^3\vec{p} \int d^3\vec{q} \left\{ \frac{1}{(2\pi)^3p_0}\left| \sum_{a} \frac{V_{\alpha\alpha'}^{a+}(\vec{p})V_{\alpha\alpha'}^{a}(\vec{p})}{E_{\alpha} - E_{\alpha'} + p_0} \right. \right. - \left. \left. \frac{1}{(2\pi)^3q_0}\left| \sum_{a} \frac{V_{\beta\alpha'}^{a+}(\vec{p})V_{\beta\alpha'}^{a}(\vec{p})}{E_{\beta} + E_{\alpha'} + q_0} \right. \right. \right\}, \]

with \( p_0^2 = \vec{p}^2 + m_{\pi}^2 \). The \( q - q - \pi \) transition form factors are defined as:

\[ V_{\alpha\alpha'}^{a}(\vec{p}) = \int d^3\vec{x} \bar{u}_a(\vec{x})\Gamma^{a}_{\alpha\alpha'}(\vec{x})u_{\alpha'}(\vec{x})e^{-i\vec{p}\vec{x}} \]

\[ V_{\beta\alpha'}^{a}(\vec{p}) = \int d^3\vec{x} \bar{u}_a(\vec{x})\Gamma^{a}_{\beta\alpha'}(\vec{x})u_{\alpha'}(\vec{x})e^{-i\vec{p}\vec{x}} \]
The vertex function of the $\pi - q - q$ and $\pi - q - \bar{q}$ transition is

$$\Gamma^a = S(r)\gamma^5a^aI_c,$$

where $I_c$ is the color unity matrix. The estimations of the $\pi - q - q$ and $g - q - q$ transition form factors are given in the Appendix A and B, respectively.

After integration over the angular part in Eq. (17), the self-energy diagrams contribution to the baryon spectrum induced by pion fields (pion cloud contribution) is evaluated as:

$$\Delta E_{\text{s.e.}}^{(\pi)} = -\frac{1}{16\pi^2f_\pi^2}\int \frac{dp}{p_0} \sum_{\alpha,\alpha' \leq \alpha} \sum_{l_n} \left\{ \left[ \int \frac{d^2r G_{\alpha\alpha'}(r)S(r)j_{ln}(pr)^2}{E_\alpha - E_{\alpha'} + p_0} \right] Q_{s.e.}(l',l',l_n,j,j') - \sum_{\beta} \left[ \int \frac{d^2r G_{\beta\alpha'}(r)S(r)j_{ln}(pr)^2}{E_\beta + E_{\alpha'} + p_0} \right] Q_{s.e.}(l,l',l_n,j,j') \right\},$$

where $j_{ln}$ is the Bessel function. The radial overlap of the single quark states with quantum numbers $\alpha = (N,l,j,m_t,m_c)$ and $\alpha'$ is defined as

$$G_{\alpha\alpha'}(r) = f_\alpha(r)g_{\alpha'}(r) + f_{\alpha'}(r)g_\alpha(r).$$

The angular momentum coefficients $Q$ are evaluated for all SU(2) baryons as

$$Q_{s.e.}(l,l',l_n,j,j') = 12\pi[l^j][l_n^j][j] \left[ C^{l_0}_{l \pm l_0} W(j_0^{l_0} l_n^{l_0}; l_0^{l_0}) \right]^2 \sum_{m_j m'_j} \left[ C^{m_j m'_j}_{j m_j l_n(m'_j - m_j)} \right]^2,$$

where $C$ and $W$ are the Clebsch-Gordan and Wigner coefficients, respectively.

The gluon part of the second order self-energy diagrams (gluon cloud) contribution is estimated in a similar fashion as

$$\Delta E_{\text{s.e.}}^{(g)} = \frac{g_s^2}{2} \sum_{\alpha,\alpha' \leq \alpha'} \int \frac{d^3\bar{p}}{(2\pi)^3} \left\{ \sum_{\alpha} \frac{V_{\alpha\alpha'}^g(\bar{p})V_{\alpha\alpha'}^{\mu}(\bar{p})}{E_\alpha - E_{\alpha'} + p} - \sum_{\beta} \frac{V_{\beta\alpha'}^g(\bar{p})V_{\beta\alpha'}^{\mu}(\bar{p})}{E_\beta + E_{\alpha'} + p} \right\},$$

where the transition form factor is evaluated with the corresponding vertex matrix

$$\Gamma^\mu = \gamma^\mu \frac{\lambda^a}{2} I_t.$$
with the isospin unity matrix $I_i$. The corresponding Feynman diagrams are given in Fig.2 where the contribution from intermediate quark and anti-quark levels have opposite signs.

After evaluation of the transition form-factors and integration over angular variables, the self-energy term induced by gluon fields can be written as a sum of color-electric (Coulomb) and color-magnetic parts:

$$
\Delta E_{\text{ex.}} = \frac{g^2}{3\pi^2} \sum_{N'j'} \sum_{(\alpha,\beta)_{L'\cdot L'\cdot 0\cdot l_n}} \left[ \int \frac{d^3p}{E_\alpha - E_\alpha' + p} \left[ \frac{R_{\alpha\alpha'}(p) + F_{\alpha\alpha'}(p)}{E_\beta + E_\alpha' + p} \right] \right] \left[ \int \frac{d^3p}{E_\alpha - E_\alpha' + p} \left[ \frac{R_{\beta\alpha'}(p) + F_{\beta\alpha'}(p)}{E_\beta + E_\alpha' + p} \right] \right]
$$

where we define function

$$
\mathcal{H}_{\alpha\alpha'1_nLL'\cdot L'\cdot 0\cdot l_n}(p) = \mathcal{H}_{\alpha\alpha'1_nLL'\cdot L'\cdot 0\cdot l_n}(p) = H_{\alpha\alpha'1_n}^2 \delta_{LL'} \delta_{JJ'} - H_{\alpha\alpha'1_n}^2 \delta_{LL'} \delta_{JJ'} + H_{\alpha\alpha'1_n}^2 \delta_{LL'} \delta_{JJ'} - H_{\alpha\alpha'1_n}^2 \delta_{LL'} \delta_{JJ'} - H_{\alpha\alpha'1_n}^2 \delta_{LL'} \delta_{JJ'}
$$

and radial integrals

$$
H_{\alpha\alpha'1_n} = H_{\alpha\alpha'1_n}(p) = \int dr \left[ r^2 f_{\alpha'}(r) g_{\alpha}^*(r) j_{\alpha'}(pr) \right],
$$

$$
R_{\alpha\alpha'1_n} = R_{\alpha\alpha'1_n}(p) = \int dr \left[ r^2 g_{\alpha'}(r) g_{\alpha}^*(r) j_{\alpha'}(pr) \right],
$$

$$
F_{\alpha\alpha'1_n} = F_{\alpha\alpha'1_n}(p) = \int dr \left[ r^2 f_{\alpha'}(r) f_{\alpha}^*(r) j_{\alpha'}(pr) \right].
$$

The angular momentum coefficients $A, B, D$ and $E$ are given in the Appendix C. We note also that the sum in Eq.(26) does not depend on the orientation of the full momentum of the valence quark $m'_j = -j', -j' + 1, \ldots, j'$.

### B. Exchange diagrams contributions

The pion exchange contribution to the baryon energy-shift (see Fig.3) is evaluated as:

$$
\Delta E_{\text{ex.}}^{(\pi)} = -\frac{1}{2f_\pi^2} \sum_{a=1}^3 \sum_{\alpha \leq \alpha'} \sum_{\alpha' \leq \alpha} \int \frac{d^3\bar{p}}{(2\pi)^3 p_0^3} \left\{ V_{\alpha\alpha'}(\bar{p}) V_{\alpha\alpha'}^*(\bar{p}) - V_{\alpha\alpha'}(\bar{p}) V_{\alpha\alpha'}^*(\bar{p}) \right\}.
$$

By using the Wick’s theorem we can write a more convenient expression for the energy shift of the SU(2) baryons from the second order pion exchange diagrams:

$$
\Delta E_{\text{ex.}}^{(\pi)} = -\frac{1}{16\pi^3 f_\pi^2} \int \frac{dp}{p_0} \sum_{l_n} \Pi_{l_n}(p)
$$

where

$$
\Pi_{l_n}(p) = \Phi_B \left[ \sum_{i \neq j} \bar{\theta}(i) \bar{\theta}(j) T_{l_n}(i) T_{l_n}(j) K_{l_n}(i) K_{l_n}^+(j) \Phi_B \right]
$$

(31)
and the operators $\bar{\tau}, T_{\ell}^{\alpha}$, and $K_{\ell}^{\alpha}$ are summed over single quark levels $i \neq j$ of the SU(2) baryon. In the quark model, the baryon wave function $|\Phi_B>$ is presented as a bound state of three valence quarks, and it can be written down commonly as

$$|\Phi_B> = |\alpha \beta \gamma > = \sum_{J_0 T_0} |\alpha; \beta; \gamma >^{T M_T(T_0)}_{J M(J_0)}$$

$$= \sum_{J_0 T_0} \hat{S} \left[ |\psi_\alpha(r_1)\psi_\beta(r_2)\psi_\gamma(r_3)Y_{J_0}^{J M}(\hat{x}_1, \hat{x}_2; \hat{x}_3) > |\chi^{T M_T(T_0)}_{12; 3} > \right] |\chi^{T M_T(T_0)}_{123} >,$$

where $J_0$ and $T_0$ are intermediate spin and isospin couplings, respectively. The states $\psi$ are the single particle states, labeled by a set of quantum numbers $\alpha$, $\beta$ and $\gamma$, excluding the color degree of freedom.
The operator $T_{l_n}$ in equation (31) is the radial integration operator:

$$<\alpha|T_{l_n}|\beta> = \int dr \left[ r^2 S(r) j_{l_n}(pr) G_{\alpha\beta}(r) \right].$$

(32)

with

$$G_{\alpha\alpha'}(r) = f_\alpha(r)g_{\alpha'}(r) + f_{\alpha'}(r)g_\alpha(r).$$

(33)

where $\alpha = (N, l, j, m_j, m_t, m_c)$ and $\alpha'$ are two sets of the single quark quantum numbers. The matrix elements of the operator $K_{l_n}$ are given by

$$<\alpha|K_{l_n}|\beta> = -\left( 4\pi |l^{\pm}(\alpha)| |l_n| |j(\alpha)| \right)^{1/2} C_{l^{\pm}(\alpha)}^{l_n,0,0} W(j(\alpha) \frac{1}{2} l_n, l(\beta); l^{\pm}(\alpha), j(\beta)) C_{j(\beta)m(\beta)}^{j(\beta)m(\beta)} N_{l_n,m_n}(m(\beta)-m(\alpha)).$$

and the Hermitian conjugation

$$<\alpha|K_{l_n}^+|\beta> = <\beta|K_{l_n}|\alpha>,$$

(34)

where $j(\alpha), l(\alpha), l^{\pm}(\alpha), m(\alpha)$ are the quantum numbers of the single quark state $<\alpha$.

The contribution of the second-order gluon-exchange terms to the baryon spectrum (see Fig.4) is given by

$$\Delta E_{ex}^{(g)} = -\frac{g^2}{2} \sum_{a} \sum_{\alpha \leq \alpha'} \sum_{\alpha \leq \alpha'} \int \frac{d^3p}{(2\pi)^3 p^2} \left\{ V_{\alpha\alpha'}^{a\mu} (\hat{p}) V_{\alpha'\alpha'}^{a\nu} (\hat{p}) - V_{\alpha\alpha'}^{a\mu} (\hat{p}) V_{\alpha'\alpha'}^{a\nu} (\hat{p}) \right\} g^{\mu\nu}.\,$$

(35)

By using the Wick’s theorem we can write more convenient expression for this equation

$$\Delta E_{ex}^{(g)} = -\frac{g^2}{2} \int_0^\infty dp \sum_{l_n,m_n} Q_{l_n,m_n}(p)$$

(36)

with the corresponding color-electric (Coulomb) and color-magnetic parts:

$$Q_{l_n,m_n}(p) = <\Phi_B | \sum_{i \neq j} \frac{\tilde{\xi}(i) \tilde{\xi}(j)}{2} T_{l_n}^{(g)}(i) T_{l_n}^{(g)}(j) \tilde{F}_{l_n,m_n}(i) \tilde{F}_{l_n,m_n}^+(j) | \Phi_B >$$

$$- <\Phi_B | \sum_{i \neq j} \frac{\tilde{\xi}(i) \tilde{\xi}(j)}{2} T_{l_n}^{(g)}(i) T_{l_n}^{(g)}(j) \tilde{F}_{l_n,m_n}(i) \tilde{F}_{l_n,m_n}^+(j) \tilde{\alpha}(i) \tilde{\alpha}(j) | \Phi_B >.\,$$

(37)

The operator $T_{l_n}^{(g)}$ is the radial integration operator with the factor $j_{l_n}(pr)$. The operators $\tilde{F}_{l_n,m_n}(i)$ and $\tilde{F}_{l_n,m_n}^+(j)$ are the angular integration operator with the factors $Y_{l_n,m_n}(\hat{x}_i)$ and $Y_{l_n,m_n}^*(\hat{x}_j)$ respectively. All these operators are summed over single quark levels $i \neq j$ of the SU(2) baryon.

We note that the relativistic one-pion and one-gluon (color-magnetic) exchange forces have strongly different structure from corresponding non-relativistic exchange forces: the exchanged pion or gluon in the $l_n$-orbit couple the upper (lower) component of $i$-valence quark with the lower (upper) component of another $j$-valence quark (see above equations) and generate spin 0, 1, 2, etc. operators. As we see below, the only contribution to the g.s. $N$ and $\Delta$ comes from the pion exchange and gluon exchange (color-magnetic part) via the spin 1 operator (an analog of the spin-orbit operator), while in the non-relativistic case the corresponding terms are spin 0 (spin-spin) operators. Another important point is that the both exchange operators are symmetric under the permutation of the upper and the lower components of the interacting valence quarks in $i$- and $j$-orbits. This fact indicates the chiral invariance of the Lagrangian in the matrix elements level.

III. NUMERICAL RESULTS

In order to account for the finite size effect of the pion, we introduce a one-pion vertex regularization function in the momentum space, parameterized in the dipole form as

$$F_{p}(p^2) = \frac{\Lambda^2 - m_{\pi}^2}{\Lambda^2 + p^2}.\,$$
From the flux-tube study [25] we fix $\alpha_s = 0.26 \approx \pi/12$. The strong coupling constant $g_s^2 = 4\pi\alpha_s$ with the value $\alpha_s = 0.65$. The parameters of the confining potential ($c$ and $m$) are chosen to reproduce the correct axial charge $g_A$ of the proton (and the empirical pion-nucleon coupling constant $G_{\pi NN}^2/4\pi = 14$ via the Goldberger-Treiman relation) and a normal value for the quark core RMS radius of the proton (see [20]).

In Table 1 we indicate the model parameters together with the corresponding single-quark energies for the models $A$ and $B$. By examining the two model parameters we can check the sensitivity of the baryon spectrum on the description of the static properties of the proton.

Table 2 contains the quark core results [20] for the static properties of the proton with the correction on the center of mass motion (CM). One can note from the table that a larger value of the strength parameter $c$ (model $B$) of the confining potential yields a smaller value for the proton RMS radius. Further we will see that the calculated spectrum is quite sensitive on the choice of the the strength parameter, hence on the proton RMS radius.

In Table 3 we give the perturbative corrections to the valence quarks energy shifts in the $1S$, $2S$, $1P_{1/2}$, $1P_{3/2}$ orbits from the self-energy diagrams induced by pion and gluon fields. Contributions from intermediate quark and anti-quark states are included with the full momentum up to $j = 25/2$ which enables convergent results opposite to the bag models (see [21]). The columns with $I = 0$ corresponds to the special cases where an intermediate quark is in the same orbit as the initial and final quark. These cases are useful to check the limitation of the self energy in the literature with the intermediate ground state of the baryon. As one can note from the table, the self-energy results with the restriction $I = 0$ are negative for the both pion and gluon (color-magnetic part) fields, while are positive without the restriction (due to the intermediate anti-quark states). Thus the restriction $I = 0$ decreases the baryon energy contrary to the complete sum of intermediate quark and anti-quark states.

The contributions of the exchange diagrams to the energy-shift of the lower SU(2) baryon states are given in Tables 4 and 5. As was pointed out above, the relativistic structures of the pion and gluon exchange mechanisms are different from the corresponding non-relativistic operators. In first case the both pion and gluon exchange operators couple the upper (lower) and lower (upper) Dirac components of the two interacting valence quarks. One can see from Table 4 that in the ground and radially excited states of the $N$ and $\Delta$ where all three valence quarks are in the $S$-orbits, the exchanged pion or gluon can be only in the $l_n = 1$ orbit, i.e. generate spin 1 operators (analogy of the spin-orbit operator). These operators, naturally, can contribute to the proton spin like the lower component (p-wave) of the Dirac wave function. The non-relativistic pion- and gluon-exchange operators between valence quarks in the ground state have a spin-spin structure, i.e. are spin 0 operators and as a result do not contribute to the proton spin. For the baryon states with the last excited quark in the $1P_{1/2}$-orbit, the contribution comes from the both $l_n = 0$ (spin-spin) and $l_n = 1$ (spin-orbit) terms. For baryons with a structure $(1S)^21P_{3/2}$ the corresponding exchange forces generate the $l_n = 1$ (spin-orbit) and $l_n = 2$ (tensor) operators. From the Table 5 one can note that the color-electric forces yield large spin-independent and small spin-dependent matrix elements. In the case when the intermediate quark states are restricted to the ground state in the $N(939)$ and $\Delta(1232)$, the color-electric component of the gluon-cloud
contribution is exactly canceled by the corresponding exchange term in consistence with the MIT bag-model result. The analysis of the results presented for the excited baryon states with the structure $(1S)^21P_{1/2}$ and $(1S)^21P_{3/2}$ indicates that the spin-orbit matrix elements generated by the both pion- and gluon-exchange (color-magnetic) forces are of acceptable order contrary to the non-relativistic one-gluon exchange spin-orbit matrix elements \[1,2\].

In Table 6 we give the mass values for the g.s. $N(939)$ with and without CM correction for the both Model A and B. The restriction to the intermediate ground state when estimating the self-energy ($I=0$) yields too small values for the nucleon energy (and also for other baryons). Contrary, the inclusion of all excited intermediate states results large mass values. Model B yields even larger value for the mass of the nucleon. This means that larger values of the strength parameter $c$ of the confining potential is not likely. Another important point is that the inclusion of the gluon-exchange and self-energy diagrams do not improve the situation. For the model A, the contribution of the pion fields to the $N(939)$ is about 200 MeV, while the gluon fields yield more than 500 MeV shift with the strong coupling constant $\alpha_s = 0.65$.

One can note that the three methods for the correction of center of mass motion agree within 50 MeV which seems too large. However, these three methods always give corrections with systematic differences. For example, the LHO method yields correction larger than the $P = 0$ method, but smaller than the $R = 0$ method. Thus, we can fix one of these methods and go to the excited sector.

And finally in last Table 7 we compare the theoretical energy values including the CM correction in the LHO method with experimental data from PDG [21]. We assume that the lower negative parity excited N and $\Delta$ states correspond to the excited last valence quark in the $1P_{3/2}$-orbit, while the radially excited Nucleon $N^*(1440)$ (Roper) and Delta $\Delta^*(1600)$ resonances are assigned with the radially (in 2S-state) excited valence quark.

We note, that one can recalculate the whole gluon-loop corrections to the SU(2) baryon spectrum by changing the value of the strong coupling constant (we use $\alpha_s = 0.65$) in order to have smaller contribution to the mass-shift values. However, this way yields smaller values for the splittings between SU(2) baryon states. The situation is not helpful for the $N - \Delta$ splitting, since we have only 63 MeV with the above value of the strong coupling constant. Thus, we come to the conclusion that the gluon field corrections to the mass-shift values of the SU(2) baryon states must be small and confirm the results of the non-relativistic CQM results [10].

When ignoring the gluon loop corrections, we have an overall good description of the SU(2) baryon spectrum. The Nucleon g.s. and the Roper resonance $N^*(1440)(1/2^+)$ are overestimated by more than 200 MeV, while the orbital excitations of the Nucleon, $N^*(1520)(3/2^-)$ and $N^*(1535)(1/2^-)$ are slightly underestimated by about 40-60 MeV.

The situation in the Delta sector is quite different. Unlike the Nucleon sector, the $\Delta(1232)(3/2^+)$ resonance and its first radial excitation $\Delta(1600)(3/2^+)$ are overestimated slightly by 50-90 MeV in the Model A. But, the orbitally excited Delta resonances $\Delta(1620)(1/2^-)$ and $\Delta(1700)(3/2^-)$ are also underestimated by small values (like the Nucleon sector).

In order to check whether the gluon field contributions can be replaced by the multiple pion exchange mechanism, we repeat the calculations of the ground state Nucleon and $\Delta(1232)(3/2^+)$ with smaller and larger values of the pion decay constant. For the $f'_\pi = 1.5 f_\pi$ we obtained $m(N) = 1055$ MeV and $m(\Delta) = 1118$ MeV which have to be compared with the numbers 1106 MeV and 1310 MeV from the Table 7, respectively. For the value $f'_{\pi} = 0.5 f_\pi$ we have $m(N) = 1766$ MeV and $m(\Delta) = 2342$ MeV, which are very large comparing the corresponding estimations calculated with the experimental pion decay constant $f_\pi$.

The different level of the estimations for the Nucleon and Delta sectors indicates on the necessity of the strong Instanton induced exchange mechanism, which does not change the Delta spectrum, while effecting the Nucleons essentially. Additionally, the two-pion loop corrections are needed for the both sector, which would yield small corrections.

### IV. SUMMARY AND CONCLUSIONS

We have developed a relativistic chiral quark model with the inclusion of the gluon and pion fields contribution to the mass spectrum of the SU(2) baryons. These fields were shown to generate the second-order self-energy and exchange corrections. The overall good description of the lower excitation spectrum of the $N$ and $\Delta$ states are obtained when restricting to the pion fields corrections. The color-electric components of the gluon fields give enormously large contributions to the baryon spectrum (about 500 MeV with $\alpha_s = 0.65$ for the nucleon g.s.). The exchanged pion or gluon (color-magnetic component) generates the spin 0 (spin-spin), spin 1 (spin-orbit) and spin 2 (tensor) operators with reasonable values of the matrix elements for the SU(2) baryons energy shifts, thus indicating no any “spin-orbit problem” in the excited Nucleon and Delta sectors. The only contribution to the g.s. Nucleon and Delta comes from the spin 1 (spin-orbit) operator which can contribute also to the baryon spin.

When using a small value of the strong coupling constant or completely ignoring the gluon loop corrections, the model yields a quite good description of the Delta resonances mass-spectrum within 50-60 MeV. However, the calcu-
lated Nucleon sector differs from the experimental values essentially.

In conclusion, the developed model still needs an additional mechanism for the description of the excited Nucleon and ∆ spectrum. Possible candidates are the two-pion exchange and the Instanton induced exchange (for the nucleon sector) forces.

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References

[1] Isgur N and Karl G 1977 Phys. Lett. 72B 109
[2] Isgur N and Karl G 1978 Phys. Rev. D18 4187
[3] Isgur N 1999 Phys. Rev. D60 054013
[4] Capstick S and Isgur N 1986 Phys. Rev. D34 2809
[5] Stancu F and Stassart P 1990 Phys. Rev. D41 916
[6] Stancu F and Stassart P 1991 Phys. Lett. B269 243
[7] Glozman L Y and Riska D O 1996 Phys. Rept. 268 263
[8] Glozman L Y, Plessas W, Varga K and Wagenbrunn R F 1998 Phys. Rev. D58 094030
[9] Löring U, Kretzschmar K, Metsch B C and Petry H R 2001 Euro.Phys. Lett. A10 309
[10] Löring U, Metsch B C and Petry H R 2001 Euro.Phys. Lett. A10 447
[11] Glozman L Y, Papp Z, Plessas W, Varga K and Wagenbrunn R F 1999 Phys. Rev. D60 054013
[12] Glozman L Y and Riska D O 2001 Nucl. Phys. A543 231
[13] Holinde K 1992 Nucl. Phys. A543 143
[14] Ashman J et al 1998 Phys. Lett. B266 346
[15] Thomas A W, Theberge S and Miller G A 1980 Phys. Rev. D22 2838; 1981 Phys. Rev. D24 216
[16] Thomas A W 2008 Prog. Part. Nucl. Phys. 61 219
[17] Saito K 1984 Prog. of Theor. Phys. V71 775
[18] Nagai S, Miyatsu T and Saito K 2008 Phys. Lett. B666 239
[19] Oset E, Tegen R and Weise W 1984 Nucl. Phys. A 426 456
[20] Gutsche T 1987 Ph. D. Thesis, Florida State University, 249 p. (unpublished)
[21] Gutsche T and Robson D 1989 Phys. Lett. B229 333
[22] Tursunov E M 2005 J. Phys. G: Nucl. Part. Phys. 31 617
[23] Bijker R and Santopinto E 2008 arXiv:0809.2299 (nucl-th)
[24] Inoue T, Lyubovitskij V E, Gutsche T and Faessler A 2006 Int. J. Mod. Phys. E 15 121
[25] Lüscher M 1981 Nucl. Phys. B180 317
[26] Gell-Mann M and Levy M 1960 Nuovo Cim. 16 1729
[27] Fetter A I and Walecka J D 1971 Quantum theory of many particle systems (McGraw-Hill, New York)
[28] Lu D H, Thomas A W and Williams A G 1998 Phys. Rev. C57 2628
[29] Tegen R, Brockmann R and Weise W 1982 Z. Phys. A307 339
[30] Wilets L 1989 "Non-Topological Solitons (World Scientific, Singapore)
[31] Yao W M et al (Particle Data Group) 2006 J. Phys. G33 1
Putting explicit expression of the vertex matrix $\Gamma^\alpha(\vec{x})$ from Eq.(19) into Eq.(17) we receive next equation:

$$V^\alpha_{\alpha'}(\vec{p}) = -i \int d\vec{r}^2 \left[ g_\alpha(r)f_{\alpha'}(r) + g_{\alpha'}(r)f_\alpha(r) \right] S(r)$$

$$\int d\vec{r} \left[ Y_{jl'}^m(\vec{r}) (\vec{\sigma} \vec{r}) Y_{jl}^m(\vec{r}) e^{-i\vec{p} \vec{r}} \right] < m_\lambda | r^\alpha | m'_\lambda > < m_\epsilon | I_c | m'_\epsilon >$$

(38)

Now using

$$Y_{jl'}^m(\vec{r}) (\vec{\sigma} \vec{r}) = -Y_{jl'}^m(\vec{r})$$

which couples the lower orbital momentum to the spin, and expanding the exponential function over spherical Bessel functions and integrating over angular part of the variable $\vec{r}$, we get next equation for the integral

$$\int d\vec{r} \left[ Y_{jl'}^m(\vec{r}) (\vec{\sigma} \vec{r}) Y_{jl}^m(\vec{r}) e^{-i\vec{p} \vec{r}} \right] = \sum_{l_n} (-i)^l_n j_{l_n}(pr) Y_{l_n}^{m_j'l_j}(\vec{p}) F(l^\pm, l'_n, j, j', m_j, m_j'),$$

where coefficients $F$ are defined as

$$F(l^\pm, l'_n, j, j', m_j, m_j') = - \left( 4\pi |l^\pm||l_n|j \right)^{1/2} C_{l^\pm l_n 0} W(j l_n, l'; l^\pm, j') C_{j l_n 0}^{m_j'm_j}.$$

For the transition form-factor now it is easy to write the next expression:

$$V^\alpha_{\alpha'}(\vec{p}) = \sum_{l_n} (-i)^{l_n+1} \int d\vec{r}^2 \left[ g_\alpha(r)f_{\alpha'}(r) + g_{\alpha'}(r)f_\alpha(r) \right] S(r) j_{l_n}(pr)$$

$$Y_{l_n}^{m_j'-m_j}(\vec{p}) F(l^\pm, l'_n, j, j', m_j, m_j') < m_\lambda | r^\alpha | m'_\lambda > < m_\epsilon | I_c | m'_\epsilon > .$$

(39)

The Hermitian conjunction of the transition form factor

$$V^{\alpha'}_{\alpha}(\vec{p}) = \sum_{l_n} (i)^{l_n+1} \int d\vec{r}^2 \left[ g_\alpha(r)f_{\alpha'}(r) + g_{\alpha'}(r)f_\alpha(r) \right] S(r) j_{l_n}(pr)$$

$$Y_{l_n}^{m_j+m_j}(\vec{p}) F(l^\pm, l'_n, j, j', m_j, m_j') < m_\lambda | r^\alpha | m'_\lambda > < m_\epsilon | I_c | m'_\epsilon > .$$

(40)

Appendix B: g-q-q transition form factor

The gluon-quark-quark transition form-factor is defined as

$$V^{\alpha\mu}_{\alpha'}(\vec{p}) = \int d^3 x \bar{u}_\alpha(\vec{x}) \Gamma^\alpha_{\mu'} u_{\alpha'}(\vec{x}) \exp(-i\vec{p} \vec{x}),$$

where the vertex matrix

$$\Gamma^\alpha_{\mu} \equiv \gamma^\mu \frac{\lambda^\alpha}{2} I_t.$$  

Using the properties of the Dirac matrices

$$\gamma^0 \gamma^\mu = \delta_{\mu 0} I + \hat{\alpha}_k \delta_{\mu k}$$

(41)

one can write:

$$V^{\alpha\mu}_{\alpha'}(\vec{p}) = \delta_{\mu 0} \int d^3 x \bar{u}_\alpha(\vec{x}) \frac{\lambda^\alpha}{2} I_t u_{\alpha'}(\vec{x}) \exp(-i\vec{p} \vec{x}) + \delta_{\mu k} \int d^3 x \bar{u}_\alpha(\vec{x}) \frac{\lambda^\alpha}{2} I_t \hat{\alpha}_k u_{\alpha'}(\vec{x}) \exp(-i\vec{p} \vec{x})$$

(42)

The last expression is convenient for the estimation of the exchange diagrams.
For the self-energy diagrams we use an alternative expression of the transition form-factors. Putting the quark wave functions with further integration over the radial part of the spatial coordinate one can write for the transition form-factor next equation:

$$V_{\alpha\alpha'}^{\mu}(\hat{p}) = \sum_{l_a,m_a} \sum_{LL',m_L,m_L',m_{L'}} \left[ \frac{[L][l_a](4\pi)}{[L']^2} \right] \delta(-i)^a Y_{l_a,m_a}(\hat{p}) \begin{pmatrix} M_{m_s,m_s'}^{L} \end{pmatrix}^{L'}_{L} C_{L0l_a0}^{L'} \begin{pmatrix} \frac{C_{m_L}^{L'} m_L' + C_{m_sL}^{L'} m_s'}{C_{Lm_LL'!m_a}} \end{pmatrix} \int r^2 R_{\mu L L'}^{\alpha \alpha'}(r) j_{l_a}(pr) dr <m_s|I_1|m_s'| + m_s|\frac{\lambda}{2}|m_s'>, \quad (43)$$

where the spin transition matrices

$$M_{m_s,m_s'}^{0} = \delta_{m_s,m_s'},$$

and

$$M_{m_s,m_s'}^{k} = \sum_{k'=\pm 1,0} h_{kk'} \left[ \delta_{k'} \delta_{m_s1/2} \delta_{m_s'(-1/2)} + \delta_{k'}(-1) \delta_{m_s,-(-1/2)} \delta_{m_s'1/2} + 2m_s \delta_{k0} \delta_{m_s,m_s'} \right]$$

with the only nonzero expansion coefficients $h_{1,+1} = h_{1,-1} = h_{3,0} = 1$, and $h_{2,+1} = -h_{2,-1} = -i$.

The radial functions are defined as

$$R_{\mu L L'}^{\alpha \alpha'}(r) = \delta_{\mu;0} \delta_{L;L'} \left( g_{\alpha} g_{\alpha'} + f_{\alpha} f_{\alpha'} \right) + i \delta_{\mu;k} \left( \delta_{L;L'} \left( g_{\alpha} f_{\alpha'} + f_{\alpha} g_{\alpha'} \right) - \delta_{L;L'} \left( g_{\alpha'} f_{\alpha} + f_{\alpha'} g_{\alpha} \right) \right)$$

**Appendix C: Recoupling of the Clebsch-Gordan coefficients**

$$A_{LL'L';L'a}^{m_{l}m'_{l}} \equiv \sum_{m_{L'},m_{l},m_{a}} C_{m_L}^{m_{L'}} C_{m_sL}^{m_{L'}} C_{m_{L'}m_L}^{L'} C_{m_sL}^{L'} C_{m_{L'}m_sL}^{m_{L'}} C_{m_{L'}m_sL}^{m_{L'}} C_{m_{L'}m_sL}^{L'} C_{m_{L'}m_sL}^{L'} \delta_{m_{s},m_{s}'} \delta_{m_{s},m_{s}'} \delta_{L;L} \delta_{L';L'}$$

The coefficients $B, D, E$ are sums of the Clebsch-Gordan coefficients with the corresponding delta factors (instead of the delta factors of the coefficient $A$): $\delta_{m_s,m_s'}, \delta_{m_s,m_s'}, \delta_{m_s,m_s'}, \delta_{m_s,m_s'}, \delta_{m_s,m_s'}, \delta_{m_s,-m_s'}, \delta_{m_s,m_s'}, \delta_{m_s,m_s'}, \delta_{m_s,-m_s'}$, respectively.
TABLE I: Parameter sets for the models A and B and corresponding single quark energies in MeV

| Model | c, Gev | m, Gev | A, Gev | αs | αs | E(1S) | E(2S) | E(1P3/2) | E(1P1/2) |
|-------|--------|--------|--------|-----|-----|-------|-------|----------|----------|
| A     | 0.16   | 0.06   | 1.0    | 0.26| 0.65| 571.7 | 986.7 | 822.8    | 860.7    |
| B     | 0.20   | 0.07   | 1.2    | 0.26| 0.65| 641.4 | 1105.7| 922.0    | 964.4    |

TABLE II: Quark core contributions (with CM-correction) to the static properties of the proton

| Model | g_A | μ_p, N.M. | RMS radius, fm | G_{NN}^2/(4π) |
|-------|-----|-----------|----------------|---------------|
| A     | 1.26| 1.58      | 0.32           | 13.919        |
| B     | 1.26| 1.41      | 0.47           | 13.984        |

TABLE IV: Second order perturbative corrections from one-pion and one-gluon exchange operators with different l_n for the energy shift of lowest N and Δ states in MeV for the Model A

| (J, T) | (3/2, 3/2) | (3/2, 1/2) | (1/2, 3/2) | (1/2, 1/2) | (1/2, 1/2) | (1/2, 3/2) |
|--------|------------|------------|------------|------------|------------|------------|
| l_n = 1 | -179.5     | -31.6      |            |            |            |            |
| l_n = 1 | -120.35    | 13.4       |            |            |            |            |
| l_n = 0 | 15.11      | -24.95     | -37.78     | -12.47     | 7.56       | 62.37      |
| l_n = 1 | -11.42     | -24.62     | -31.32     | 24.62      | -31.32     | -17.60     |
| l_n = 1 | -70.64     | -23.46     | -110.93    | -17.34     | 23.21      | 30.94      |
| l_n = 2 | -62.14     | -7.77      | -39.41     | -5.45      | -12.43     | -2.12      |

TABLE V: Second order perturbative corrections induced by the color-electric component of the one-gluon exchange operator for the energy shift of lowest N and Δ states in MeV for the Model A

| (J, T) | (3/2, 3/2) | (3/2, 1/2) | (1/2, 3/2) | (1/2, 1/2) | (1/2, 3/2) |
|--------|------------|------------|------------|------------|------------|
| l_n = 0 | -443.7     | -443.7     |            |            |            |
| l_n = 0 | -411.4     | -411.4     |            |            |            |
| l_n = 1 | -410.6     | -410.6     | 3.5        | -18.2      | -7.0       |
| l_n = 1 | -389.4     | -389.4     | -389.4     | -389.4     | -389.4     |
| l_n = 1 | -7.6       | -29.8      | 23.4       | 15.3       | -7.6       |

TABLE VI: The mass value of the g.s. nucleon in MeV with and without center of mass (CM) correction

| Model | No CM | R=0.28 | Y=0.29 | LHO=0.30 |
|-------|-------|--------|--------|----------|
| A     | 1715  | 940    | 985    | 966      |
| B     | 1924  | 1057   | 1110   | 1088     |

TABLE III: Second order perturbative corrections from self energy diagrams induced by pion and gluon fields for the energy shift of the valence quarks in MeV

| Model | 1S | 1S | 2S | 2S | 1P1/2 | 1P1/2 | 1P3/2 | 1P3/2 |
|-------|---|---|---|---|------|------|------|------|
| π     | 126.5 | 53.86 | 550 | 13.7 | 248   | -1.3  | 262   | -37.4 |
| g(cm) | 47  | -31.7 | 70  | -6.5 | 72    | -15.5 | 48    | -27.6 |
| A     | 289 | 147.9 | 315 | 99.1 | 308   | 119.4 | 307   | 119.6 |
| B     | 306.5 | 165.3 | 330 | 110.2 | 320   | 133.4 | 320   | 133.9 |
TABLE VII: Estimations for the energy values of the lowest $N$ and $\Delta$ states in MeV for the Model A with the CM correction in the LHO method

| State                  | $E_Q + \Delta E(\pi)$ | $E_Q + \Delta E(\pi) + \Delta E(g)$ | Exp. [31]|
|------------------------|------------------------|--------------------------------------|----------|
| $N(939)(1/2^+)$        | 1166                   | 1698                                 | 938 ± 939|
| $N(1440)(1/2^+)$       | 1684                   | 2314                                 | 1430 ± 1470|
| $N(1520)(3/2^-)$       | 1473                   | 2058                                 | 1515 ± 1530|
| $N(1535)(1/2^-)$       | 1490                   | 2089                                 | 1520 ± 1555|
| $\Delta(1232)(3/2^-)$  | 1310                   | 1905                                 | 1230 ± 1234|
| $\Delta(1600)(3/2^+)$  | 1781                   | 2461                                 | 1550 ± 1700|
| $\Delta(1620)(1/2^-)$  | 1615                   | 2259                                 | 1615 ± 1675|
| $\Delta(1700)(3/2^-)$  | 1605                   | 2242                                 | 1670 ± 1770|