On amplification of light in the continuous EPR state

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Two schemes of amplification of two-mode squeezed light in the continuous variable EPR-state are considered. They are based on the integrals of motion, which allow conserving quantum correlations whereas the power of each mode may increase. One of these schemes involves a three-photon parametric process in a nonlinear transparent medium and second is a Raman type interaction of light with atomic ensemble.

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I. INTRODUCTION

Optics implementation of entangled states, particular of the continuous variable EPR type, is usually based on the light in non-classical state known as squeezed light. Because of its quantum correlations squeezed light is fragile and some special transformations can only conserve its features. It is known, that amplification of non-classical light is a hard problem. It has been shown by many authors, see, for example Caves [1], Sokolov [2], that the usual linear optics amplifier is not suitable because of its noise due from spontaneous emission. This argument can be extended into nonlinear amplifier [3] and propagation through active media considered by Paris [4] with respect to quantum information tasks. In the same time the non-classical states of light can be converted from one frequency to another perfectly in a crystal with quadratic nonlinearity [5].

The aim of this paper is to consider amplification of two-mode squeezed light of the EPR form, that can be generated in Optical Parametric oscillator (OPO). Light of such type has been used for teleportation of coherent state [6], indeed its power was week. The main idea is based on existence of integrals of motions, when light interacts with medium. The reason is that some entangled states can be eigenfunctions of the operators to be integrals, it results in conservation of quantum correlations, particular entanglement, while power of light may increase. Some examples of interactions, for which the non-classical properties of light unchange, have been discussed in ref. [7]. An thermostat, that creates and keeps atomic entanglement has been introduced by Basharov [8]. However, when light propagates trough a phase-sensitive environment, its quantum correlations degrades, as it has been found by Lee and co-workers [9].

In this paper we introduce two schemes for amplification. First involves a three-photon parametric process in transparent medium with quadratic nonlinearity, second is a Raman-type interaction of squeezed light with atomic ensemble. The peculiar property of these schemes is that they can’t generate entangled states but they conserve degree of correlations between modes or entanglement, while the power of each modes may increase. It directly results from integrals of motion, which allow achieving two tasks as conserving of properties we wish and appearance of a process we wish. We consider the time evolution here, but the presented method, based on integrals of motion, can be modified to propagation problems. It can be done with the use of the quantum transfer formalism, presented in ref. [10], where it has been discussed some examples, that show how integrals of motion work in parametric and multiphoton phenomena.

The paper is organized as follows. First some features of the continuous variables EPR states are discussed, then amplification of entangled light in nonlinear transparent medium and in resonance medium is considered.

II. SQUEEZED LIGHT AND CONTINUOUS EPR STATE

There are two ways how to introduce the continuous analog of the EPR pair. First of them includes formal definition in the eigenfunction of two operators to be a total momentum $P$ and a relative position $Q$ of a bipartite system. Second is based on a physical model of OPO, implemented in experiment. Its output is entangled state of the EPR type, that can be described by operators $P$ and $Q$ in the Heisenberg picture. In optics implementation both ways result in the same state with respect to measured values, when one finds a two-mode squeezed light.

Operators of total momentum and relative position of a bipartite system, or two particles for simplicity, can be introduced in the following way

$$Q = x_1 - \epsilon x_2,$$

$$P = p_1 + (1/\epsilon)p_2,$$

where $x_m, p_m, m = 1, 2$ are the canonical position and momentum operators of single particle, $\epsilon$ is a real c-number. Operators $P$ and $Q$ have a set of common eigenfunctions known as the continuous Bell-like states introduced by Braunstein [11]

$$Q|\Psi_{PQ}\rangle = Q|\Psi_{PQ}\rangle,$$

$$P|\Psi_{PQ}\rangle = P|\Psi_{PQ}\rangle.$$  

When eigenvalues are equal to zero, there is a continuous analog of maximally entangled EPR state

$$|\Psi_{00}\rangle = |EPR\rangle.$$  

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A simple model of OPO can be described by the effective hamiltonian of interaction $H = i\hbar (a^\dagger a^2 - h.c.)$, where $k$ is a coupling constant, $a_m$ is a photonic operator of mode $m = 1, 2$. Solution of the problem has the form $Q = Q_0 \exp(-cr), P = P_0 \exp(-cr)$, where $c = \pm 1$, $Q_0, P_0$ are input operators, $r$ is a squeezing parameter. When $r \to \infty$, one finds a continuous variables EPR state, that can be denoted as

$$Q \to 0, \quad P \to 0.$$  \hfill (4)

This is an ideal EPR pair, but it is not a physical state.

In quantum optics light can be described by the usual quadrature operators

$$X(\theta) = a^\dagger \exp(i\theta) + h.c. = 2(x \cos \theta + p \sin \theta),$$  \hfill (5)

where $a = x + ip$ is photonic operator, $[a; a^\dagger] = 1$, $[x; p] = i/2$. Any features of light can be obtained from a set of its correlation functions, particularly from variances of the quadrature operators $\langle (\Delta X)^2 \rangle = \langle X^2 \rangle - \langle X \rangle^2$, which measurement are well known. For example, in the case of coherent state $\langle (\Delta X)^2 \rangle = 1$. If

$$\langle (\Delta X(\theta))^2 \rangle < 1,$$  \hfill (6)

the light is called squeezed or in more accuracy the state is squeezed over position or amplitude, when $\theta = 0$ and it is squeezed over momentum or phase when $\theta = \pi/2$. If $\langle (\Delta X(\theta))^2 \rangle = 0$ one finds a limit squeezing. Indeed, we use terms amplitude and phase in the sense of consideration in phase space, but not with respect to operators of amplitude and phase.

The introduced quadrature operators are measured in a detection scheme including two detectors and a local field oscillator, that is mixed with the signal. Output is the difference of the detector photocurrents $i$. Assume, the quantum efficiency of both detectors is equal to 1, then there is a simple formula for spectrum of photocurrent or spectrum of the light noise. It is described by variance of the quadrature operators and has the form

$$i^2(\omega) = \int_{-\infty}^{\infty} d\tau \langle i(t)i(t + \tau) \rangle \exp(i\tau\omega)$$

$$= \int_{-\infty}^{\infty} d\tau \langle X(t)X(t + \tau) \rangle \exp(i\tau\omega).$$ \hfill (7)

Indeed, the large number of the quantum optics models results in the spectrum of noise in the low region of frequency, that reads

$$i^2(\omega \approx 0) = 1 + \langle (\Delta X(\theta))^2 \rangle_N,$$ \hfill (8)

where unit is a level of shot noise known also as standard quantum limit, subscript $N$ denotes the normal ordering of operators. It follows from (6) and (8), that squeezed light has its noise bellow the shot level and it may be suppressed even up to zero.

Operators $P$ and $Q$, given by (1), can be rewritten in terms of the quadrature operators in the following way

$$Q = (1/2)[X_1(0) - \epsilon X_2(0)],$$
$$P = (1/2)[X_1(\pi/2) - (1/\epsilon)X_2(\pi/2)],$$ \hfill (9)

where $X_m$, describes mode $m = 1, 2$. Both observables $P$ and $Q$ are measured in a detection scheme, involved a beamsplitter, that mixes two modes, and the next measurement of momentum and position of light from two outputs of the beamsplitter [6]. It follows from (2) and (4), that variances of $P$ and $Q$ are equal to zero, then equations (6) and (8) tell, that the continuous Bell-state $|\Psi_{PQ}\rangle$, together with output state of OPO, are squeezed over total momentum and relative position and their noise is suppressed below standard quantum limit.

One finds here, that these squeezed states are also entangled, but a simple observation shows, that squeezing or better non-classicality of the light state is only a necessary condition. In the same time, when light is entangled, then the measure of entanglement can be chosen as a level of the shot noise suppression.

### III. AMPLIFICATION IN PARAMETRIC PROCESS

Let an interaction of two modes be presented by hamiltonian of the form

$$V = \hbar k PQ,$$ \hfill (10)

where $k$ is a real coupling constant. Then operators of total momentum $P$ and relative position $Q$ are integrals of motion, that results in conservation of quantum correlations. Let the initial state of light be an EPR pair generated by OPO, that has the form $Z(t = 0) = Z_0 \exp(-cr)$, $Z = P, Q$. Then one finds, that $Z(t) = Z(\epsilon)$. Also any eigenstates of integrals $Z = P, Q$ are unchanged and entangled states $|\Psi_{PQ}\rangle$ are conserved, because of their evolution is reduced to multiplication to a phase factor: $\exp(-i\hbar PT) \langle \Psi_{PQ}\rangle = \exp(-i\hbar P \Omega T) \langle \Psi_{PQ}\rangle$.

In the same time each of modes can be changed wilhe the interaction. The problem, given by (10), has exact solutions for the photon number operators of modes $n_m = a^\dagger_m a_m$, $m = 1, 2$

$$n_1(t) = n_{10} + \mu^2(Q^2 + P^2) + 2\mu(QX_{10} + PX_{10}),$$
$$n_2(t) = n_{20} + \mu^2(Q^2/\epsilon^2 + P^2/\epsilon^2) + 2\mu(QX_{20}/\epsilon + PX_{20}/\epsilon),$$ \hfill (11)

where $Y_m, m = 1, 2$ are operators at $t = 0$ and $\mu = kt/2$. In accordance with (11) the maximally entangled states $|EPR\rangle$, given by (4), is unchanged with respect to number of photons or its power, because of $Q, P \approx 0$. For this case of ideal EPR pair the medium plays a role of a quantum repeater. When the EPR pair is not ideal its power may increase with a gain to be proportional to $\mu^2$.

The natural question is whether the hamiltonian (10) can describe any real process. To answer the question let rewrite
the hamiltonian in terms of photonic operators
\[ V = i \frac{k}{4} a_1^{12} - a_2^{12} - a_1^{12} + a_2^{12} \]
\[ + \frac{1}{\epsilon} (a_1^{12} - a_1 a_2)(1 - \epsilon^2) \]
\[ + \frac{1}{\epsilon} (a_2^{12} - a_1^{12} a_2)(1 + \epsilon^2)]. \tag{12} \]
Assume \( \epsilon = \pm 1 \), then one finds three-photon parametric phenomena in transparent medium with quadratic nonlinearity. In fact there are three processes of the frequency conversions, presented in \[ \text{ref. [12], [13].} \]

To consider a resonance interaction of light with an ensemble of \( N \) two-level identical atoms let introduce an effective hamiltonian
\[ H = \hbar \omega_0 B - S_{01} B^\dagger, \tag{13} \]
where atomic operators read \( S_{xy} = \sum_a s_{xy}(a) \), \( s_{xy}(a) = |x\rangle_a \langle y|, x, y = 0, 1 \) and \( \langle 0 |a| 1 \rangle \) are ground and excited levels of an atom \( a \). In the hamiltonian e.m. field is presented by its operators \( B, B^\dagger \).
Assume, three modes \( M = 1, 2, 3 \) at frequencies \( \omega_M \), described by operators \( a_M \), interact with atoms by Raman type so that \( \omega_1 = \omega_0 \) and \( \omega_3 = \omega_0 \), where \( \omega_0 \) is a frequency of the atomic transition \( 0 \to 1 \). The operators \( B \) takes the form \( B = g a_1 - f a_3 a_2^\dagger \), where \( g, f \) are coupling constants. Let the mode at frequency \( \omega_2 \) be a classical wave, then \( B = g(a_1 + \nu a_2^\dagger) \), where \( \nu = f a_3 / g \). Assuming \( \nu = \epsilon = \pm 1 \), the field operators read
\[ B = g(a_1 - \epsilon a_2^\dagger) = Q + i P, \]
\[ B^\dagger = g(a_1^\dagger - \epsilon a_2) = Q - i P. \tag{14} \]

Equation \[ (14) \] tells, that operators \( P \) and \( Q \), where \( \epsilon^2 = 1 \), are integrals of motion. Thanks to these integrals quantum correlations of the entangled states of the EPR type will be unchanged under evolution, given by hamiltonian \[ (13), (14). \]

To discuss amplification of modes \( \omega_1, \omega_2 \) introduce the master equation for density matrix of field \( \rho \). In the first approximation over hamiltonian of interaction it has the form
\[ \dot{\rho} = \frac{-N_0}{\gamma_\perp}(B^\dagger B \rho + B \rho B^\dagger + h.c.) \]
\[ - \frac{N_1}{\gamma_\perp}(B B^\dagger \rho + B^\dagger \rho B + h.c.), \tag{15} \]
where \( N_0 \) and \( N_1 \) are occupations of the ground and excited atomic levels, \( N_0 + N_1 = N \) and the transversal decay rate \( \gamma_\perp \) is introduced. It follows from \[ (15), \] that equations for the photon numbers of single modes have the form
\[ \langle \dot{n}_1 \rangle = \frac{g^2}{\gamma_\perp}(N_1 - N_0)(\langle n_1 \rangle - \langle n_2 \rangle + \langle B B^\dagger \rangle) \]
\[ + \frac{g^2}{\gamma_\perp}(N_0 + N_1), \]
\[ \langle \dot{n}_1 \rangle - \langle \dot{n}_2 \rangle = \frac{2g^2}{\gamma_\perp}(N_1 - N_0)(\langle B B^\dagger \rangle). \tag{16} \]

There are two kind of amplification here. If \( N_1 = N_0 \), then the power of each of modes increases with the gain to be proportional to total number of atom \( N \), but the relative power of modes is unchanged. Let \( \langle B B^\dagger \rangle = 0 \), it means, that, for example, the initial state is EPR states for which \( Q \to 0, P \to 0 \). Then \( \langle n_0 \rangle = n_0 + (g^2 t / \gamma_\perp)\langle (N_0 - N_0)(n_10 - n_20) + N_1 + N_0 \rangle \). It results in amplification of both modes while its quantum correlations are conserved, if these modes have the same number of photons initially.

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