Research of Multi-objective Component Assignment Problem for Lin/con/k/n System Considering Cost

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Abstract. Component assignment problem (CAP) is widely used in engineering systems, which is investigated for enhancing the system performance by assigning switchable components to several positions. However, the traditional CAP does not consider the cost factor and has limitations for improving the system reliability. Therefore, this paper focus on a multi-objective CAP for Lin/con/k/n system to find the cost-efficient maintenance scheme. Firstly, a multi-objective mathematical optimization model is developed with the aim of maximizing the system reliability and minimizing maintenance cost, which has been proved to be a Non-deterministic polynomial hard problem and should be computed by heuristics. Secondly, a comprehensive maintenance method is adopted to improve the system reliability, which integrates improving the system reliability with changing the permutation of components. To evaluate the contribution of the component reliability on both system reliability and maintenance cost, the multi-objective Birnbaum importance (MOBI) is derived. Finally, MOBI is introduced into the non-dominated sorting genetic algorithm-II (NSGA-II) to solve the optimization model. The numerical experiments for several Lin/con/k/n Systems show that MOBI is available to solve CAP considering cost, and MOBI-based-NSGA-II algorithm is more efficient to obtain higher system reliability with lower maintenance cost.

1. Introduction

Reliability is a key response measure to examine the system’s/product’s quality. The reliability optimization has been a hot research area in the recent years regarding practical applications of industrial engineering, which can be classified by the method of enhancing the performance of the system, such as the redundancy allocation problem (RAP) [1], reliability-redundancy allocation problem (RRAP) [2], component assignment problem (CAP) [3] and complex problem [4]. Among all kinds of optimization problems, CAP has become popular for its flexible and economical property, which is frequently presented in the Lin/con/k/n systems for example pipeline system, communication system, and power distribution systems.

CAP is applied in such situations where the effectively switchable components can be assigned to several positions, the purpose is to amplify the reliability of the system by finding the optimal component assignment. Although, detailed literature has been already published to report the
solution of CAP, such as the LK-type heuristics by Lin and Kuo [5], ZK-type heuristics by Zuo and Kuo [6], Birnbaum importance method based on two-stages, and BIGLS by Yao et al [7], and BIGA by Cai et al [8]. However, the optimization of the system reliability by varying the component locations is limited to some extent. Instead of this indirect way to accelerate the system reliability, improvement in the component reliability is a direct way to enhance the system performance. However, the components reliability is constrained because of their manufacturing complexity and technology feasibility. Therefore, this work is focused on a comprehensive maintenance method by integrating the component reliability directly and changing the permutation of components, which can make up for disadvantages of each other. Because of this situation, the decision variables of the optimization model are two vectors, which are the components reliability improvement vector and the permutation of all components, respectively.

In the process of system reliability optimization, the cost factor is an important evaluation index to consider the feasibility in practical engineering. The most general reliability optimization problem aims to stimulate the reliability with the constraints of limited resources, such as cost and weight. Kuo and Wan [9] classified this kind of reliability allocation problem as problem 1. Si et al. [10] developed a nonlinear programming model such that the cost is limited. Zhao et al [11] combined the advantage of component rearrangement and component replacement to stimulate/enhance the system reliability.

In this paper, a multi-objective optimization model has been constituted with the aim of accelerating the system reliability and decelerating the maintenance cost. Previously, the reliability allocation problem and CAP was proved to be NP-hard problem, which needs to be solved by evolutionary algorithms. The optimization is usually done to perform a detailed/insight investigation of the process variables and their influence on the response measures as reported by Rehman et al. [12]. Konak et al [13] analyzed the problems during the implementation of multi-objective genetic algorithms. The range of Pareto-optimal solution should be identified more efficiently and reliably for every objective function. The size of Pareto set is found more important to attain reasonable computational effort and optimized desired results.

The multi-objective problem is solved by a great variety of algorithms for example Non-dominated sorting genetic algorithm II (NSGA-II) [14], Particle swarm optimization (PSO) [15], Coarse-grained parallel genetic algorithm (CPGA) [11]. The NSGA-II is proposed to retain an efficient solution spread, and a better convergence on the non-domestic front compared to two other elitists multi-objective EA (MOEA), as Pareto-archived evolution strategy (PAES) and Strength Pareto EA (SPEA). Deb et al. [16] suggested a unique MOEA which is based on a non-dominated sorting approach. The mechanism for diversity preservation used in NSGA-II is considered the best one among the three approaches studied here. NSGA-II is found able to come closer to the real front in comparison to other two approaches to a problem with strong interactions between parameters.

The significance of importance measure for reliability optimization lies on promoting reliability improvement by ranking rules [17]. The importance measure is first presented by Birnbaum in 1969 [18], which can well evaluate the effect of the component reliability variation on the system reliability. However, Birnbaum's importance does not consider the cost factor such that it cannot drive the reliability optimization of our multi-objective model. To that end, we extend the multi-objective importance measure (MOBI) proposed by Ma et al. [19]. The MOBI can measure the effectiveness of the component reliability on both reliability and cost, which can be introduced into NSGA-II to boost up the NSGA-II accuracy. Finally, the efficiency of the MOBI based NSGA-II is illustrated by several numerical experiments of Lin/con/k/n systems.

The work is described here in this sequence. Section 2 has provided a detailed explanation of the assumption. Section 3 has explained the optimization model of the proposed work. Section 4 has elaborated on the critical procedure of the evolutionary algorithm NSGA-II. Section 5 has discussed the obtained results. Section 6 is dedicated to describing the conclusion of this work.

2. Assumptions in maintenance optimization
A maintenance optimization problem is described here with the assumptions as reported herein:

a) There are two states to describe the component such as good state and failure state.

b) The system is also considered in working or in a failed state.

c) Both system and components are statistically mutually independent, but not inherently reliable.

d) Component’s reliability is enhanced by performing maintenance of components.

e) A system for Lin/Con/k/n: F fails when some components fail.

3. Multi-objective optimization for Lin/Con/k/n system considering cost

3.1. Optimization model

A multi-objective optimization problem is employed to perform maintenance optimization of the Lin/Con/k/n: F system, where a maintenance process including the repair cost is adopted. The main objective is the increased system (Lin/Con/k/n: F) reliability by performing maintenance of the system components. The reliability measurement expression is reported herein in Equations 1 and 2 as explained by Kuo et al. [20]

\[
R^F(n, k, p) = \begin{cases} 
R^F(n-1, k, p) - R^F(n-k-1, k, p) p_{n-k} \left( \prod_{i=n-k+1}^{n} q_i \right), & n \geq k \\
1, & n < k 
\end{cases} 
\]  

(1)

\[
R^G(n, k, p) = \begin{cases} 
R^G(n-1, k, p) + (1 - R^G(n-k-1, k, p)) q_{n-k} \left( \prod_{i=n-k+1}^{n} p_i \right), & n \geq k \\
0, & n < k 
\end{cases} 
\]  

(2)

\[
R^G(n, k, p) \quad \text{indicated the system (Lin/Con/k/n: G) reliability,} \quad p_i \quad \text{abbreviated for component} \quad i \quad \text{reliability,} \quad q_i \quad \text{illustrated the component unreliability where,} \quad q_i = 1 - p_i \quad \text{as} \quad p_0 = 1. \quad \text{The Lin/Con/k/n: G system is a mirror image of the Lin/Con/k/n: F system. Therefore, the system reliability can be enhanced by using Lin/Con/k/n: G system in the optimization model.}
\]

The second objective function depicted the system total cost of maintenance. Maintenance cost is considered a critical objective function as the reliability improvement is based on the cost which should be minimized. The expression for the calculation of the system components maintenance cost is shown in Equation 3.

\[
dc_i = 20.5 r_i + 0.5
\]  

(3)

The price of unit reliability improvement \( \frac{dc_i}{dr_i} \) possessed different values at different \( r_i \). The calculation of the cost of the component needed to repair is obtained by performing integration from \( r_{i1} \cdot r_{i2} \) of Equation 3, and its formulation is reported in Equation 4.

\[
C_i = \int_{r_{i1}}^{r_{i2}} (20.5 r_i + 0.5) dr_i \quad i = (1, 2, 3, \ldots, n)
\]  

(4)
The optimization model for Lin/con/k/n system is expressed herein by taking into account system reliability constraints, cost, and limits of component reliability.

\[
\text{Max } R^x (r_i) = R^x (n, k, r_i)
\]

\[
\text{Min } C_i = \int_{r_i}^{r_{max}} (20.5 r_i + 0.5) dr_i
\]

\[
R_i \geq R_0
\]

\[
C_i \leq C_0
\]

\[
0 \leq R_{imp} \leq R_{max} - R_i, \quad i = 1, 2, 3, \ldots, n
\]

where \(R_{imp}\) indicated the amount of improved reliability of component \(I\), \(R_{max}\) abbreviated the maximum reliability that can be achieved after the component repair, \(R_i\) presented the reliability of component after degradation. A heuristic-based evolutionary algorithm named non-dominated sorting genetic algorithm-II is employed here to analyze the optimization model by considering the maintenance cost in mind.

3.2. Multi-objective importance measure

The importance measure can well analyze the influence of the component reliability variation on the overall reliability. A technique proposed for the estimation of reliability importance is called Birnbaum importance (BI). BI is widely used by many scholars and experts for the solution CAP.BI for component \(i\) is expressed as:

\[
I_i^B (r(t)) = \frac{\partial R(r_1, r_2, \ldots, r_n)}{\partial r_i(t)}
\]

\[
I_i^B (r(t)) = R[r_1, r_2, \ldots, r_{i-1}, 1, r_{i+1}, \ldots, r_n] - R[r_1, r_2, \ldots, r_{i-1}, 0, r_{i+1}, \ldots, r_n]
\]

\[
I_i^B (r(t)) = \frac{\partial R(r(t))}{\partial r_i(t)} \cdot \frac{\partial r_i(t)}{\partial C_i(r(t))}
\]

\[
I_i^C (r(t)) = I_i^B (r(t)) \cdot \frac{1}{20.5 r_i + 0.5}
\]

II. The cost-reliability importance \(I_{ij}^C (r(t))\) highlights the impact on the through single components cost of maintenance reliability development.
\[ I_{pi}^r(t) = \frac{\partial c(r(t))}{\partial r_i(t)} \]  

(9)

The harmonic mean is employed to evaluate the performance improvement in resource consumption per unit [21]. The quantity of observations is divided into the reciprocal number of each number in the sequence. Therefore, the harmonic mean is the reciprocal mean of the reciprocal arithmetic.

\[
\text{Harmonic mean} = \frac{n}{\sum_{i=1}^{n} \left( \frac{1}{a_i} \right)} 
\]

(10)

III. The cost of maintenance and system reliability are highly influenced by the reliability of components which is described as MOBI. It is estimated by the formula of the harmonic mean for \( I_i^c(r(t)) \) and \( \frac{1}{I_{pi}^r(r(t))} \). The formulation of MOBI is reported herein as Equation 11.

\[
I_{mo}^r(r(t)) = \frac{2}{I_i^c(r(t)) + I_{pi}^r(r(t))} 
\]

(11)

The increase in \( I_{mo}^r(r(t)) \) is found directly proportional to the system reliability improvement and inversely proportional to the maintenance cost.

4. An evolutionary algorithm (MOBI-NSGA-II) for optimization

An optimal solution for optimization employed the genetic algorithm-II. This proposed algorithm along with the observation of advantages of non-dominated sorting is employed to solve the maintenance optimization model. The flow chart in Figure 1 provides a detailed explanation of multi-objective NSGA-II whose steps are described here in detail.

a) Determine the objective functions:

The Lin/con/k/n system reliability is considered as the optimization key function. The reliability of the required system abruptly decreased as the solution not attained the required constraints reliability. In the same way, if the cost function value is found greater as compared to C_o then it will increase.

b) Coding space, and coding method determination:

The coding space is first determined, and functions are initialized such as the enhancement of reliability, and components arrangement are included in chromosome coding. An order of from 1 to n for n components is called the permutation, and representation of reliability improvement is shown as \( 0 \leq R_{imp} \leq R_{max} - R \). The permutation of the chromosome is generated by an arbitrary sequence of n components to initialize the process for the population.

c) Non-dominated sorting:

A non-dominated sorting is conducted and grouped by fronts in the combination of parent and offspring populations because they are sorted according to an increasing degree of non-dominance. Individual of F1 is removed from the population. This process is kept on running until all individuals in the front layer are determined.
Figure 1. Flow chart to illustrate the complete methodology of multi-objective NSGA-II.

d) Crowding distance calculation:
The individuals are sorted in each objective domain for the calculation of crowding distance. The individuals in the first and the last rank are assigned the crowding distance equal to infinity. The crowding distance is calculated by the mathematical expression reported in Equations 12 and 13, respectively.

\[
CD_i = \frac{f_m(x_{i+1}) - f_m(x_{i-1})}{f_m(x_{\text{max}}) - f_m(x_{\text{min}})}, \quad i = 1, 2, 3, \ldots, (I-1) \tag{12}
\]

\[
CD_i = M \sum_{m=1}^{M} CD_{im} \tag{13}
\]

It is understood from the above-reported equations that the solution \(i\) is preferred over \(j\) if \(R_i < R_j\) or \(R_i = R_j\) and \(CD_i > CD_j\).

e) Pareto level, and crowding distance-based ranking:
The solution in the non-dominated front 1 is ranked 1 and possesses higher fitness. The solution in the same front possessed the same rank and same fitness. The solution with a lower non-dominated rank is preferred over the other. There are some conditions of ranking the solution (a) \(j_{\text{Rank}} > i_{\text{Rank}}\); (b) \(i_{\text{Rank}} = j_{\text{Rank}}\) and \(j_{\text{distance}} < i_{\text{distance}}\) respectively. When two solutions
have the same non-dominated rank belong to the same front, which is located in a less crowded region is preferred.

f) Do crossover on the selected individuals:

Parent chromosomes are selected to produce off-springs. There existed the two types of crossover such as a single-point, and double-point crossover. Two chromosomes are elected on individual chromosome’s part regarding single-point crossover. The crossover rate is determined by the permutation process. This crossover is employed when all the genetic codes are eliminated followed by the other chromosome which endorsed the same methodology as the first one.

g) Mutation of chosen individuals:

This operator has applied the modifications to one or more "genes" randomly to produce a new off-spring. The mutation is performed on the randomly selected individual from the population using \( N_m = N \times P_m \). In these two positions, the mutation operator chose two different positions and performs genes replacement. The transformation of genes in chromosome exchanged random positions.

h) Performing maintenance components optimization:

Maintenance optimization on the components of the system is performed after performing the mutation of selected individuals. The more important component is selected to enhance reliability.

i) Getting a novel population-based on elitist strategy:

A merged population is obtained as a result of maintenance optimization, crossover, and mutation. Step 3, 4, and 5 are again performed to obtain the ranking of population. A new population \( P \) is obtained by selecting a top individual from population \( P_a \).

j) A glance at the termination condition:

The process will stop and output will be the required result as the present maximum generation number is less than the iteration number, otherwise, return to step 6 again.

5. Simulation results and discussions

The input parameters are set according to the employed F and G type systems. The detailed pump station data in the context of the system reliability is shown in Tables 1 and 2 followed by the Pareto fronts to elaborate the contrast of accuracy of the suggested algorithm (MOBI-NSGA-II) with the existing algorithm (NSGA-II) in Figs. 2 and 3. The results are comprehensively described here in the discussion section.

Table 1. Pipeline system all components process variables, along with constant factors based on F-3/10 or G-3/10 system

| Parameter          | Process variables | Constant factors |
|--------------------|-------------------|------------------|
| Maintenance cost   | Upper limit       | 200              |
| Maximum Reliability| \( R_{max} \)     | 0.9              |
| Maintenance        | Reliability Upper | 0.9              |
| Reliability        | Upper limit       | 0.9              |
| Minimum Reliability| \( R_{min} \)     | 0.6              |
| Maintenance        | Reliability Upper | 0.6              |
| Reliability        | Upper limit       | 0.6              |
| Current Reliability| \( R_0 \)         | 0.7              |
| Maintenance        | Reliability Upper | 0.7              |
| Reliability        | Upper limit       | 0.7              |
| Selected           | components        | 3                |

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Table 2. Pipeline system all components process variables, along with constant factors based on F-3/15 or G-3/15 system

| Process parameters | Pump station | Level values | Constant factors |
|--------------------|--------------|--------------|------------------|
| Maximum Reliability | $R_{\text{max}}$ | 0.92, 0.93, 0.93, 0.95, 0.94, 0.93, 0.89, 0.89, 0.92, 0.93, 0.90, 0.89, 0.88, 0.91, 0.89 | Maintenance reliability threshold value 0.95 |
| Minimum Reliability | $R_{\text{min}}$ | 0.67, 0.71, 0.73, 0.74, 0.69, 0.75, 0.69, 0.70, 0.71, 0.70, 0.72, 0.71, 0.73, 0.73, 0.70 | Total number of components 15 |
| Current Reliability | $R_0$ | 0.75, 0.74, 0.75, 0.76, 0.79, 0.77, 0.71, 0.73, 0.76, 0.75, 0.80, 0.81, 0.77, 0.74, 0.77 | Selected components 3 |

Graphical trends for F-3/10 and F-3/15 system

Figure 2. Parametric trends to demonstrate the comparison of NSGA-II, and MOBI-NSGA-II algorithm in context of the maintenance cost, and system reliability for two different F-type systems. (a) Representation of Pareto fronts to do the comparison of two algorithms in the F-3/10 system, (b) Illustration of pointed portions of (a) to highlight the areas at which both algorithms possess the
same level of accuracy in terms of cost and reliability, (c) Visual presentation of Pareto fronts to compare two algorithms such as NSGA-II and MOBI-NSGA-II in F-3/15 system, (d) Demonstration of the critical points of both algorithms in F-3/15 system from (c) at which same reliability-cost curve is obtained.

The part(a) of Figure 2 for the F-3/10 system explained that the system reliability of MOBI-NSGA-II is more at little maintenance cost compared to NSGA-II. This resulted that the algorithm depicted with red color in Figure 2 is more reliable than a blue-colored one. Similar results were found in Figure 2(c) for the F-3/15 system although there existed a minute uniformity variation in cost, and reliability. The comparison performed at the system level inferred that the F-3/10 is more efficient in the context of the two algorithms variation based on reliability, and maintenance cost. Thus, it is inferred that the minimum is the maintenance cost, the more reliable is the system. Therefore, it has resulted that MOBI-NSGA-II is proved more efficient compared to the simple NSGA-II.

The existing algorithm NSGA-II can be employed to gain a graphical trend for cost and reliability. However, the comparison of the accuracy of the existed algorithm (NSGA-II) with the proposed one (MOBI-NSGA-II) is demonstrated in Figure 3.

**Graphical trends for G-3/10 and G-3/15 system**

![Graphical trends for G-3/10 and G-3/15 system](image)

**Figure 3.** Pareto fronts to express the accuracy of the proposed algorithm MOBI-NSGA-II compared to the existing algorithm in terms of the maintenance cost, and system reliability for two different G-type systems. (a) Depiction of Pareto fronts for the sake of comparison of two algorithms in G-3/15 system, (b) Description of pointed portions of (a) to signify the areas at which both algorithms possess indistinguishable accuracy regarding cost and reliability, (c) Graphical
illustration to do the comparison of two previously stated algorithms in G-3/10 system, (d) Indication of the close points of both algorithms in G-3/10 system from (c) at which the proposed and the existing algorithm are at the same level of accuracy concerning maintenance cost and system reliability.

The analysis of the Pareto fronts indicated in Figure 3(a) for the G-3/15 system signified a comparative less accuracy of the proposed algorithm (shown in red circles) compared to the existing one (pointed in blue steric signs) that the system reliability of MOBI-NSGA-II is more at the minimum cost compared to NSGA-II. The part(c) of Figure 3 for the G-3/10 system has inferred that the proposed algorithm MOBI-NSGA-II is more precise and accurate in terms of both factors such as reliability and cost. This resulted that the algorithm viewed with red color in Figure 3(c) is a bit more reliable in contrast to the blue color. The system-level comparison of both G-type systems has resulted that the precision and accuracy of the 1<sup>st</sup> G-type system(G-3/10) is improved as compared to the 2<sup>nd</sup> one(G-3/15). As a result, it is understood that an efficient algorithm is one that provides a little cost along with a significant increase in reliability. Therefore, it is concluded that the proposed algorithm MOBI-NSGA-II is better than the simple NSGA-II.

6. Conclusions
This work was established to resolve the multi-response component assignment problem for Lin/con/k/n system to boost up the reliability, and maintenance cost reduction of the overall system. The comparison of two genetic algorithms like NSGA-II and MOBI-NSGA-II was performed in Pareto fronts based on maintenance cost, and system reliability in different F and G type systems. The comparison of two F-type systems like F-3/10 and F-3/15, and two G-type systems such as G-3/10 and G-3/15 was performed to analyze that which system serves better to improve the overall system efficiency. The conclusions made as a result of this work include:

a) The proposed algorithm MOBI-NSGA-II is proved more efficient and reliable at a little maintenance cost collate to the conventional algorithm NSGA-II.

b) Reliability and cost are inter-related with each other. Therefore, an adequate combination of both these response measures is highly desired to stimulate the system’s efficiency.

c) A comparison of various systems is inferred that F-3/10 is more efficient in contrast to F-3/15. Similarly, G-3/10 is more accurate and reliable in contrast to G-3/15.

d) Future work may be performed on the comparison of other different variety of algorithms with the proposed algorithm MOBI-NSGA-II to analyze the system reliability and maintenance cost in a wide variety of systems, and this optimization model should be practically implemented in reconfigurable systems.

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References
[1] Ramirez-Marquez J E and Coit D W 2004 A heuristic for solving the redundancy allocation problem for multi-state series-parallel systems Reliability Engineering & System Safety 83 341–9
[2] Wu X and Wu X 2017 An importance based algorithm for reliability- redundancy allocation of phased mission systems IEEE International Conference on Software Quality Reliability and Security 152–9
[3] Lin YK and Yeh CT 2011 Multistate components assignment problem with optimal network reliability subject to assignment budget Appl. Math. Comput. 217 10074–86
[4] Pant S, Anand D, Kishor A and Singh S B 2015 A particle swarm algorithm for optimization of complex system reliability International Journal of Performability Engineering 11(1)
33–42

[5] Lin FH and Kuo W 2002 Reliability Importance and Invariant Optimal Allocation Journal of Heuristics 8 155–71

[6] Zuo MJ and Kuo W 1990 Design and performance analysis of consecutive k-out-of-n structure Naval Research Logistics 37 203–30

[7] Yao Q, Zhu X and Kuo W 2011 Heuristics for component assignment problems based on the Birnbaum importance IIE Transactions 43 633–46

[8] Cai Z, Si S, Sun S and Li C 2016 Optimization of linear consecutive-k-out-of-n system with a Birnbaum importance-based genetic algorithm Reliab. Eng. Syst. Saf. 152 248–58

[9] W. Kuo and R. Wan 2007 Recent advances in optimal reliability allocation IEEE Trans. Syst., Man, Cybern. A, Syst. Human 37 143–156

[10] Si S, Liu M, Jiang Z, Jin T and Cai Z 2019 System Reliability Allocation and Optimization Based on Generalized Birnbaum Importance Measure IEEE Transactions on Reliability 68 831–43

[11] Zhao J, Si S and Cai Z 2019 A multi-objective reliability optimization for reconfigurable systems considering components degradation Reliab. Eng. Syst. Saf 183 104–15

[12] M. Rehman, S.A. Khan and R. Naveed 2020 Parametric optimization in electric wire discharge machining of DC53 steel using gamma phase coated wire Journal of Mechanical Science and Technology 34 1–7 https://doi.org/10.1007/s12206-020-05

[13] Konak A, Coit D.W and Smith A.E 2006 Multi-objective optimization using genetic algorithms: A tutorial Reliab. Eng. Syst. Saf 91 992–1007

[14] Abouei Ardakan M and Rezvan M T 2018 Multi-objective optimization of reliability-redundancy allocation problem with cold-standby strategy using NSGA-II Reliability Engineering & System Safety 172 225–38

[15] Coello CAC, Pulido GT and Lechuga MS 2004 Handling multiple objectives with particle swarm optimization IEEE Trans Evolution Comput 8 256–79

[16] Deb K, Pratap A, Agarwal S and Meyarivan T 2002 A Fast and Elitist Multi-objective Genetic Algorithm IEEE Trans. on evolutionary computation 6 182–97

[17] Si S, Zhao J, Cai Z and et al 2020 Recent advances in system reliability optimization driven by importance measures Frontiers of Engineering Management 7 335–58

[18] Birnbaum ZW 1969 On the importance of different components in a multicomponent system In: Krishnaiah PR, editor. Multivariate analysis II. New York: Academic Press 581–92

[19] Ma C, Wang W, Cai Z and Zhao J 2020 Maintenance optimization of reconfigurable systems based on multi-objective Birnbaum importance Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability Doi: 10.1177/1748006X20901983

[20] Kuo W, Zhang W and Zuo M 1990 A Consecutive-k-out-of-n: F System: The Mirror Image of a Consecutive-k-out-of-n: F System IEEE Transaction on Reliability 39 244–53

[21] Shen E 1931 A note on the definition of the harmonic mean The Journal of Educational Psychology 22 311–2