On the origin of the hump structure in the in-plane optical conductivity of high $T_c$ cuprates based on a SU(2) slave-boson theory

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An improved version of SU(2) slave-boson approach is applied to study the in-plane optical conductivity of the two dimensional systems of high $T_c$ cuprates. We investigate the role of fluctuations of both the phase and amplitude of order parameters on the (Drude) peak-dip-hump structure in the in-plane conductivity as a function of hole doping concentration and temperature. The mid-infrared(MIR) hump in the in-plane optical conductivity is shown to originate from the antiferromagnetic spin fluctuations of short range(the amplitude fluctuations of spin singlet pairing order parameters), which is consistent with our previous U(1) study. However the inclusion of both the phase and amplitude fluctuations is shown to substantially improve the qualitative feature of the optical conductivity by showing substantially reduced Drude peak widths for entire doping range. Both the shift of the hump position to lower frequency and the growth of the hump peak height with increasing hole concentration is shown to be consistent with observations.

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I. INTRODUCTION

High $T_c$ superconductors are believed to be the systems of strongly correlated electrons essentially in two space dimension. This strong correlation results in the peak-dip-hump structure of the optical conductivity $\sigma(\omega)$ and linear frequency dependence of the scattering rate $1/\tau(\omega)$ [1–6]. Both the appearance of the hump (which occurs at $\omega \approx 1000cm^{-1}$ in YBCO [3,4] and Bi2212 [6]) and the linear frequency dependence of the scattering rate ($1/\tau(\omega) \sim \omega$) indicate strong deviations from the Drude model prediction of conventional Fermi-liquid. Various theories have been proposed to explain these non-Fermi-liquid(NFL) like behavior in the charge dynamics(optical conductivity) of the high $T_c$ cuprates. Using the nearly antiferromagnetic Fermi-liquid theory, Stojković and Pines [7] reported a study of normal state optical conductivity for the optimally doped and overdoped regions, but not for the underdoped regions. They showed that the highly anisotropic scattering rate in different regions of the Brillouin zone leads to an average relaxation rate of the marginal Fermi-liquid form by showing $1/\tau(\omega) \sim \omega$. Their computed optical conductivity agreed well with experimental data for the normal state of an optimally doped sample. Using the spin-fermion model [8,9] and spin susceptibility parameters obtained from inelastic neutron scattering(INS) and nuclear magnetic resonance(NMR) measurements, Munzar et al. calculated the in-plane optical conductivity of optimally doped YBCO [10]. They examined the peak-dip-hump structure only at optimal doping and showed a good agreement with observation. From the computed self energy they showed that the hump is originated from the hot spot and the Drude peak from the cold spot in the Brillouin zone. Haslinger et al. reported optical conductivities $\sigma(\omega)$ of optimally doped cuprates in the normal state by allowing coupling between the fermions and the bosonic spin fluctuations [11]. They found that the width of the peak in spectral function $A_k(\omega)$ scales linearly with $\omega$ in both hot and cold spots and $\sigma(\omega)$ is inversely linear in $\omega$ up to very high frequencies. Besides these spin fluctuation theories mentioned here, by using a phenomenological form of the charge collective mode(CM) propagator and empirical parameter values Caprara et al. showed that charge-ordering instability is responsible for the hump in the optical conductivity of high $T_c$ cuprates [12] limited only to the overdoped region. The various above theories are reported in the limited range of hole doping resorting to empirical parameters by fitting INS or angle resolved photoemission spectroscopy(ARPES) measurements. Thus the applicability of these theories may be limited only to a certain range of hole doping and temperature. It is, thus, highly desirable to use a theory which does not depend on any empirical parameters for the entire range of hole doping and temperature encompassing both the pseudogap phase and the superconducting phase.

Earlier Lee and Salk [13] proposed a U(1) and SU(2) slave-boson theories and showed an arch-shaped $T_C$ line in agreement with observations. Their theory is different from other previous slave-boson theories [14–17] in that the Heisenberg term in the t-J Hamiltonian contains the contribution of coupling between the spin and charge degrees of freedom(the spinon pair and holon pair orders). Recently, using the same U(1) slave-boson theory, we [18] were able to explain the peak-dip-hump structure of the observed optical conductivity, by showing that the hump is caused by the presence of spinon pairing order formed...
from the hot spot in the Brillouin zone. In this paper, by using the SU(2) theory we report a rigorous examination on the origin of peak-dip-hump structures over the entire range of temperature \(0 < T < T_C, T_C < T < T^*\) and \(T > T^*\) and the entire (underdoped, optimally doped and overdoped) range of hole doping. In addition we discuss how the inclusion of the order parameter phase fluctuations markedly improve the predicted in-plane optical conductivity over the previous results [18].

### II. THEORY

#### A. U(1) AND SU(2) SLAVE-BOSON THEORIES OF t-J HAMILTONIAN

In order to bring forth the essential physical points we briefly review the U(1) and SU(2) slave-boson theory and point out the importance of coupling between the charge and spin degrees of freedom in determining the structure of the optical conductivity [13]. The U(1) formulation is readily done by rewriting the electron operator as a composite of spinon \((f)\) and holon \((b)\) operators, \(c_{i\sigma} = f_{i\sigma} b_{i\sigma}^\dagger\) with the single occupancy constraint, \(\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} + b_{i\sigma}^\dagger b_{i\sigma} = 1\). Then we obtain the partition function,

\[
  Z = \int \mathcal{D}f \mathcal{D}b \mathcal{D}\lambda e^{-\int_0^L dt \mathcal{L}},
\]

with

\[
  \mathcal{L} = \sum_i \left( \sum_\sigma f_{i\sigma}^\dagger \partial_\tau f_{i\sigma} + b_{i\sigma}^\dagger \partial_\tau b_{i\sigma} \right) + H_{t-J} + i \sum_i \lambda_i \left( \sum_\sigma f_{i\sigma}^\dagger \partial_\tau f_{i\sigma} + b_{i\sigma}^\dagger b_{i\sigma} - 1 \right)\text{ where } \lambda_i \text{ is the Lagrange multiplier field to enforce the single occupancy constraint and } H_{t-J}, \text{ the U(1) slave-boson representation of the t-J Hamiltonian},
\]

\[
  H_{t-J} = -t \sum_{<i,j>,\sigma} \left( f_{i\sigma}^\dagger f_{j\sigma} b_{j\sigma}^\dagger b_{i\sigma} + c.c. \right) - \frac{J}{2} \sum_{<i,j>} \left( b_{i\uparrow} b_{j\downarrow} f_{i\uparrow j\downarrow}^\dagger - f_{i\uparrow j\uparrow}^\dagger f_{i\downarrow j\downarrow} \right) - \mu \sum_{i,\sigma} f_{i\sigma}^\dagger f_{i\sigma}.
\]

It is noted that the above Heisenberg interaction term (the second term) reveals coupling between the charge (holon) and spin (spinon) degrees of freedom. Such coupling is ignored in other proposed theories [14–17].

Hubbard-Stratonovich transformations for the hopping, spin pairing and holon pairing operators leads to the partition function,

\[
  Z = \int \mathcal{D}f \mathcal{D}b \mathcal{D}\Delta^f \mathcal{D}\Delta^b \mathcal{D}\lambda e^{-\int_0^L dt \mathcal{L}_{eff}},
\]

where \(\mathcal{L}_{eff} = \mathcal{L}_0 + \mathcal{L}_f + \mathcal{L}_b\) is the effective Lagrangian with

\[
  \mathcal{L}_0 = \frac{J(1-x)^2}{2} \sum_{<i,j>} \left\{ |\Delta_{ij}^f|^2 + \frac{1}{2} |\chi_{ij}|^2 + \frac{1}{4} \right\},
\]

for the order parameter Lagrangian and

\[
  \mathcal{L}_f = \sum_{i,\sigma} f_{i\sigma}^\dagger \partial_\tau f_{i\sigma} + \frac{J(1-x)^2}{4} \sum_{<i,j>,\sigma} \left\{ \chi_{ij}^\dagger f_{i\sigma}^\dagger f_{j\sigma} + c.c. \right\},
\]

for the spinon sector and

\[
  \mathcal{L}_b = \sum_i b_i^\dagger \partial_\tau b_i - t \sum_{<i,j>} \left\{ \chi_{ij}^\dagger b_i b_j + c.c. \right\},
\]

for the holon sector. Here \(\chi_{ij}, \Delta^f_{ij} \text{ and } \Delta^b_{ij}\) are the hopping, spinon pairing and holon pairing order parameters respectively. \(\mu^f(\mu^b)\) is the spinon (holon) chemical potential and \(x\), the hole concentration.

The SU(2) theory introduces the holon doublet

\[
  h_i = g_i \begin{pmatrix} b_i \\ 0 \end{pmatrix} = \begin{pmatrix} b_{i1} \\ b_{i2} \end{pmatrix},
\]

and the electron operator is then written,

\[
  c_{i\sigma} = \frac{1}{\sqrt{2}} h_i^\dagger \Psi_{\sigma},
\]

with the spinon doublet \(\Psi_{i\uparrow} = g_i \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow} \end{pmatrix} = \begin{pmatrix} f_{i1} \\ f_{i2} \end{pmatrix}\). Here \(g_i\) is the SU(2) rotation matrix. Inserting Eq. (8) into the t-J Hamiltonian and following same procedure with U(1) case we obtain the effective Lagrangian,

\[
  \mathcal{L}_{eff}^{SU(2)} = \mathcal{L}_0 + \mathcal{L}_f + \mathcal{L}_b,
\]

where

\[
  \mathcal{L}_0 = \frac{J(1-x)^2}{2} \sum_{<i,j>} \left\{ |\Delta_{ij}^f|^2 + \frac{1}{2} |\chi_{ij}|^2 + \frac{1}{4} \right\},
\]

\[
  + \frac{J}{2} \sum_{<i,j>} |\Delta_{ij}^b|^2 \left( \sum_{\alpha,\beta} |\Delta_{ij,\alpha\beta}^b|^2 + x^2 \right),
\]

\[
  \mathcal{L}_f = \sum_{i,\sigma} \Psi_{i\sigma}^\dagger \partial_\tau \Psi_{i\sigma},
\]

\[
  - \frac{J(1-x)^2}{4} \sum_{<i,j>,\sigma} \left\{ \Psi_{i\sigma}^\dagger \left( \chi_{ij}^\dagger - 2\Delta_{ij}^f \right) \Psi_{j\sigma} + c.c. \right\},
\]

\[
  \Psi_{i\uparrow} = g_i \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow} \end{pmatrix} = \begin{pmatrix} f_{i1} \\ f_{i2} \end{pmatrix},
\]

and

\[
  \Psi_{i\downarrow} = g_i \begin{pmatrix} f_{i\downarrow} \\ -f_{i\uparrow} \end{pmatrix} = \begin{pmatrix} f_{i2} \\ -f_{i1} \end{pmatrix}.
\]
\[ \mathcal{L}_b = \sum_i h_i^\dagger (\partial_t - \mu) h_i \]
\[ -\frac{t}{2} \sum_{<i,j>} \left( h_i^\dagger \left( \frac{\chi_{ij}^1}{\Delta_{ij}^f} - \Delta_{ij}^f \right) h_j + c.c. \right) \]
\[ -\frac{J}{2} \sum_{<i,j>} \left[ (|\Delta_{ij}^f|^2)^2 h_i^\dagger \left( \frac{\Delta_{ij}^b_{11}}{\Delta_{ij}^f_{21}} \frac{\Delta_{ij}^b_{12}}{\Delta_{ij}^f_{22}} \right) (h_j^\dagger)^2 + c.c. \right], \]
with \( \Psi_i = \left( f_i f_i^\dagger \right). \) It is quite encouraging to realize that the third term in Eqs.(6) and (12) manifests the composite nature of spinon pairing and holon pairing to allow for the formation of Cooper pairs. For the comparison of U(1) and SU(2) theory, we rewrite the holon hopping term (the second term in Eq.(12)) by using Eq.(7) as
\[ h_i^\dagger \left( \frac{\chi_{ij}^1}{\Delta_{ij}^f} - \Delta_{ij}^f \right) h_j \]
\[ = \left( b_{1i} b_{2i}^\dagger \right) \left( \frac{\chi_{ij}^1}{\Delta_{ij}^f} - \frac{\Delta_{ij}^f}{\chi_{ij}^1} \right) \left( b_{1j} b_{2j}^\dagger \right) \]
\[ = \left( b_{1i}^\dagger 0 \right) g_{ij} g_{ij} \left( \frac{\chi_{ij}^1}{\Delta_{ij}^f} - \frac{\Delta_{ij}^f}{\chi_{ij}^1} \right) \left( 0 b_{ij} \right) \]
\[ = \chi_{ij}^1 b_{ij} b_{ij}. \]
where \( \chi_{ij}^1 \) and \( \Delta_{ij}^f \) are the U(1) mean field hopping and spin pairing order parameters respectively. Thus the SU(2) order parameter matrix is related to the U(1) order parameter matrix \( U_{ij}^{U(1)} = \left( \frac{\chi_{ij}^1}{\Delta_{ij}^f} - \frac{\Delta_{ij}^f}{\chi_{ij}^1} \right) \) by gauge transformation,
\[ U_{ij}^{SU(2)} = \left( \frac{\chi_{ij}^1}{\Delta_{ij}^f} - \frac{\Delta_{ij}^f}{\chi_{ij}^1} \right) = g_i U_{ij}^{U(1)} g_j. \]
As a result, in the SU(2) theory the low energy order parameter fluctuations can be included and allows the appearance of holon doublet [16]. Such SU(2) treatment resulted in an arch shaped \( T_C \) line with the prediction of more realistic optimal doping rate than the U(1) theory [13].

**B. OPTICAL CONDUCTIVITY IN U(1) AND SU(2) SLAVE-BOSON THEORIES**

We obtain the optical conductivity \( \sigma(\omega) \) of an isotropic 2-D medium by evaluating the current response function \( \Pi_{xx}(\omega) \),
\[ \sigma(\omega) = \frac{\partial J_x(\omega)}{\partial E_x(\omega)} \bigg|_{E_x=0} = -\frac{1}{\omega} \frac{\partial^2 F}{\partial A_x^2} \bigg|_{A_x=0} = \frac{\Pi_{xx}(\omega)}{i\omega}. \]
Here \( J_x \) is the induced current in the \( x \) direction and \( E_x \), the external electric field. \( F \) is the free energy, \( A_x \), the electromagnetic field and \( \omega \), the frequency of applied electric field. The total response function, \( \Pi_{xx} = \Pi_{xx}^P + \Pi_{xx}^D \) is the sum of the paramagnetic response function given by the current-current correlation function \( \Pi_{xx}^P(r' - r, t' - t) \) and the diamagnetic response function associated with the average kinetic energy, \( \Pi_{xx}^D = \langle K \rangle = -t \sum_{\sigma} \langle \bar{c}_{r,x,\sigma} \bar{c}_{r+x,\sigma} + H.C. \rangle [20]. \)

Recently, using the U(1) slave-boson theory we [18] studied the current response function contributed only from spinon singlet pair excitations (the amplitude fluctuations of the spinon pairing order parameter) and U(1) gauge fields (phase of the hopping order parameter) up to the second order, by using
\[ \Pi = \Pi^b - \sum_{\alpha,\beta=\alpha,\beta} \Pi^b_{\alpha\beta} (\Pi^b + \Pi^f)_{\alpha\beta}^{-1} \Pi^b_{\alpha\beta} \]
\[ = \frac{\Pi^f \Pi^b}{\Pi^f + \Pi^b} + \frac{2 (\Pi^f + \Pi^b)^2 \Pi^f}{\Pi^f + \Pi^b} - (\Pi^f + \Pi^b \Pi^b + \Pi^f \Pi^f), \]
where \( \Pi^f(\Pi^b) \) is the spinon(holon) response function associated with the internal and external gauge field (a and A); \( \Pi^f_{\alpha\beta} \equiv -\frac{\partial^2 r_{ij}^a}{\partial A_{\alpha} \partial A_{\beta}} (\Pi^f_{\alpha\beta} \equiv -\frac{\partial^2 r_{ij}^a}{\partial \bar{A}_{\alpha} \partial A_{\beta}} \) is the spinon(holon) response function contributed from both the gauge fields and the spin pairing field and \( \Pi^f_{\alpha\beta} + \Pi^b_{\alpha\beta} \) and \( \Pi^b_{\alpha\beta} \) the response function contributed from the spinon pairing field. The first term represents the Ioffe-Larkin rule [19] which results from the back-flow condition associated with gauge field fluctuations. On the other hand the second term is attributed to the spinon singlet pair excitations. It is noted that both terms in Eq.(16) contain the effects of the coupling between the charge and spin degrees of freedom.

For completeness it is essential to include the effects of the phase fluctuations of hopping and spinon pairing order parameters in addition to the amplitude fluctuations of hopping and spin pairing order parameters. Thus we write the order parameter matrix,
\[ U_{ij}^{SU(2)} = \left( \frac{\chi_{ij}^1}{\Delta_{ij}^f} - \frac{\Delta_{ij}^f}{\chi_{ij}^1} \right) \]
\[ = \sqrt{|\chi_{ij}^1|^2 + |\Delta_{ij}^f|^2} \left( \frac{\chi_{ij}^1}{\sqrt{|\chi_{ij}^1|^2 + |\Delta_{ij}^f|^2}} - \frac{\Delta_{ij}^f}{\sqrt{|\chi_{ij}^1|^2 + |\Delta_{ij}^f|^2}} \right) \]
\[ = \eta_{ij} \left( e^{-i\alpha_{ij}} \cos \theta_{ij} - e^{i\beta_{ij}} \sin \theta_{ij} \right), \]
where \( \eta_{ij} = \sqrt{|\chi_{ij}^1|^2 + |\Delta_{ij}^f|^2} \) is the ‘amplitude’ of the order parameter matrix, \( \alpha_{ij} (\beta_{ij}) \) the phase of the hopping/spinon pairing order parameter and \( \theta_{ij} \), the relative phase angle of the hopping and spin pairing order parameters. It is noted that \( \alpha, \beta \) and \( \theta \) represent the order parameter phase fluctuations in the sense that they are low energy fluctuations. \( \eta \) represents the amplitude fluctuations of the order parameters.

Allowing both the phase and amplitude fluctuations of the order parameters, we obtain the current response function,
\[ \Pi = \Pi^b - \sum_{a,b=\alpha,\beta,\theta,\eta} \Pi^b_{\alpha\beta} (\Pi^b + \Pi^f)_{ab}^{-1} \Pi^b_{a\eta} \]
\[ = \Pi^b - \sum_{a,b=\theta,\eta} \Pi^b_{\alpha\beta} (\Pi^b + \Pi^f)_{ab}^{-1} \Pi^b_{a\eta}. \]
where $\Pi^{f}_{XY} \equiv -\frac{\omega^2}{8\pi^2 N} \left( \Pi^{b}_{YX} \equiv -\frac{\omega^2}{8\pi^2 N} \right)$ with $X,Y = \alpha, \beta, \theta$ and $\eta$. Here we used the fact that $\Pi^{b}_{A\alpha} = \Pi^{b}_{A\beta} = 0$ in the second line. Therefore there is no coupling between the external electromagnetic field $\mathbf{A}$ and the phase fluctuation modes $\alpha$ and $\beta$. For details, we refer readers to Appendix A.

III. COMPUTED RESULTS OF IN-PLANE OPTICAL CONDUCTIVITY

Fig. 1. Computed optical conductivities as a function of temperature for $x = 0.07$ (underdoped), $x = 0.12$ (optimally doped) and $x = 0.23$ (overdoped) cases with the antiferromagnetic Heisenberg coupling strength of $J=0.3t$.

We present the predicted optical conductivities by allowing the order parameter fluctuations involving the parameters $\alpha$, $\beta$, $\theta$ and $\eta$ up to second order in the framework of the SU(2) slave-boson theory of Lee and Salk [13] by choosing $J=0.3t$ for the underdoped ($x = 0.07$), optimally doped ($x = 0.12$) and overdoped ($x = 0.23$) regions (Fig. 1). Although not reported here, we find no qualitatively marked changes with the variation of $J$ in the range of $0.1t < J < 0.4t$. Here the base condensation temperature ($T_C$) and pseudogap temperature ($T^*$) are given by our previous SU(2) results [13]. It is reminded that superconductivity is characterized by the order parameters of holon pair and spinon singlet pair. The spinon singlet pairs appear at temperature below $T^*$. Below the superconducting temperature $T_C$ both the spinon singlet pair and holon pair order exist. The predicted results show the (Drude)Peak-dip-(MIR) hump structure at temperature below the pseudogap temperature $T^* (T < T^*)$ and the hump disappears above $T^*(T > T^*)$ indicating that the hump is originated from the spin singlet pair excitation, namely the antiferromagnetic spin fluctuations of the shortest possible correlation lengths. Here we would like to point out that the predicted trend of both the shift of the hump position to lower frequency and the growth of the hump peak height with increasing hole concentration is consistent with observations [21, 22]. However, quantitative results in hump position shift and hump peak height fail to agree with observations [3, 4].

To thoroughly examine the origin of the peak-dip-hump structure, below we discuss predictions made by systematic changes of various phase fluctuation modes of $\alpha$, $\beta$, and $\theta$. Here $\alpha$ and $\beta$ are the phases of hopping and spinon pairing order parameters respectively and $\theta$, the relative phase angle between the two order parameters (see Eq.(17)). Fig. 2 above is a replot of Fig. 1a only for the case of $T = 0.001t$. Without the inclusion of order parameter fluctuations only Drude peak appears. The same result is obtained with the allowance of only $\alpha$ and $\beta$ fluctuations. Thus the computed optical conductivities with and without the phase fluctuations of $\alpha$ and $\beta$ are identical. This is because $\Pi^{b}_{A\alpha} = \Pi^{b}_{A\beta} = 0$ owing to the SU(2) symmetry (see Appendix A for proof) as shown in Fig. 2 above. The $\theta$ mode fluctuations resulted in a broader Drude peak and some absorption at $200\text{cm}^{-1} < \omega < 600\text{cm}^{-1}$. The $\theta$ mode fluctuations represent an excitation mode as-

FIG. 2. Comparison of the optical conductivity 1) without fluctuations, with the fluctuations of 2) $\alpha, \beta, \theta$, 3) $\eta$ and 5) $\alpha, \beta, \theta, \eta$. Result of case 1) and 2) is exactly same by virtue of $\Pi^{f}_{A\alpha} = \Pi^{f}_{A\beta} = 0$ (See Appendix A).
associated with the amplitude fluctuations of both the hopping and spinon pairing order parameters which are massive excitations. The hump structure did not appear with the $\theta$ mode fluctuations. Upto now we discussed the predicted optical conductivity associated with the phase fluctuations involved with $\alpha$, $\beta$ and $\theta$. We now investigate the amplitude fluctuations of the order parameters involved with the '$n$' mode with $\eta = \sqrt{|\chi|^2 + |\Delta f|^2}$. As shown in Fig.2 only the $\eta$ mode fluctuations give a peak-dip-hump structure in the optical conductivity. However the width of the Drude peak is affected by the inclusion of the low energy excitations $\alpha$, $\beta$ and $\theta$. The phase fluctuations cause further reduction in the width of Drude peak as compared to the case with the inclusion of only $\eta$ fluctuations. Although not shown here, we observe that the phase fluctuations in the SU(2) cause a substantial reduction in the width of the Drude peak compared to the U(1) result but the position of the hump remains nearly identical. The SU(2) result is in closer agreement with observations [3,4] compared to the U(1) case. On the other hand, the height of the hump is found to be lower in the SU(2) case as compared to the U(1) case. The hump structure is largely originated from the amplitude fluctuations of the $\eta$ mode. Both the U(1) and SU(2) showed that the hump is originated from the antiferromagnetic spin fluctuations of short range order, although we cannot deduce separate contributions from the two different amplitude($|\chi|$ and $|\Delta f|$) fluctuations in this SU(2) calculation. This is because the amplitude fluctuations of both hopping order and spin(spinon) pairing order parameters occur in the background of antiferromagnetic fluctuations.

For an additional check on the origin of the hump structure in a different aspect we computed the optical conductivity using the Lanczos exact diagonalization method for a two hole doped $4 \times 4$ lattice for various values of Heisenberg antiferromagnetic coupling strength J. The peak-dip-hump structure appears only for non-zero values of J and the position of the hump increases linearly with J. This strongly indicates that the hump is originated from the spin-spin correlations. This trend is in agreement with our present calculations. In Fig.3 we show the computed results of the hump position as a function of the antiferromagnetic coupling strength J for comparison between the Lanczos exact diagonalization method(with one and two hole doped cases $4 \times 4$ square lattices) and U(1) and the SU(2) slave-boson theories.

![FIG. 3. Antiferromagnetic coupling dependence of hump position for comparison with the exact diagonalization calculations with two hole for $4 \times 4$ lattice at $T = 0K$ and our slave-boson results at $x = 0.07$(SU(2)) and 0.05(U(1)) at $T << T_c$, that is $T = 0.001t$.](image)

We note that the peak locations of the hump obtained for both cases are sensitive to the variation of the antiferromagnetic coupling strength $J$, by showing a linear increase. This implies that the antiferromagnetic spin fluctuations are responsible for the appearance of the hump structure. Thus we conclude that the hump structure in both cases is originated from the antiferromagnetic spin fluctuations, accompanying the amplitude fluctuations of the spinon pairing order parameter at the hot spots. For comparison with observation we now present the predicted hump position with the variation of hole concentration and temperature. In Fig.4 the peak position of hump is seen to remain nearly constant as a function of temperature far below $T^*$; $T^* = 0.062t$ for $x = 0.07$, $T^* = 0.05t$ for $x = 0.12$ and $T^* = 0.028t$ for $x = 0.23$. In general, the predicted hump position tends to shift to a lower frequency with increasing hole concentration and with temperature, in agreement with the optical conductivity measurements [3,21,22]. As shown in Fig.1, at temperature close to the pseudogap temperature the hump structures are seen to too rapidly disappear with increasing temperature compared to observations. This implies that the antiferromagnetic spin fluctuations of short range order other than the spin singlet pair(excitations) are important even above $T^*$. Such antiferromagnetic fluctuations of short range are consistent with NMR measurements [23] and other theoretical studies [7,10,11].

![FIG. 4. Temperature dependence of hump position as a function of hole concentration $x$ with antiferromagnetic coupling $J=0.3t$.](image)

**IV. SUMMARY**

In the present study, we studied qualitative aspects of physics involved with the optical conductivity for the two-dimensional systems of strongly correlated electrons for a wide range of both hole doping(under-, optimal and over-doping) and temperature($T < T_c$, $T_c < T < T^*$, and $T^* < T$) without resorting to any empirical parameters. By observing the linear increase of the hump position with the antiferro-
magnetic Heisenberg coupling strength J, we showed that the hump structure in the optical conductivity is originated from the antiferromagnetic spin fluctuations which accompany the spin(spinon) singlet pair excitations (i.e., the antiferromagnetic fluctuations of the shortest possible correlation length) and that the AF fluctuations of larger correlation lengths are also important for causing the hump structure. This observation is consistent with NMR measurements [23] and other theoretical studies [7,10,11]. The shift of the hump position to lower frequency and the increase of the hump peak height with increasing hole concentration is seen to be consistent with observations. We have demonstrated that the \( \theta \) mode fluctuations which represents the amplitude fluctuations of both the hopping and spinon pairing order parameters sharpens the Drude peak of the optical conductivity, yielding markedly better agreement with observations compared to the U(1) theory and that the hump structure is not affected by the \( \alpha, \beta \) and \( \theta \) mode fluctuations, that is, the phase fluctuations involved with the hopping and spin singlet pairing order parameters. As pointed out earlier, our computed results fail to agree with observations in the magnitude of hump height.

To achieve quantitative accuracy in the future study it is desirable to include the next nearest hopping term (\( t' \) term) in the \( t-J \) Hamiltonian and the effects of the AF fluctuations of larger correlation lengths.

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APPENDIX A: PROOF OF \( \Pi_{\alpha}^{B} = \Pi_{\alpha}^{B} = 0 \) IN THE SU(2) THEORY

From Eq. (12), the holon action in the presence of the external electromagnetic field is written,

\[
S_{h}(h, \alpha, \beta, \theta) = \int_{0}^{\beta} d\tau \sum_{i} h_{i}^{\dagger}(\partial_{\tau} - \mu)h_{i} + \frac{t}{2} \sum_{<i,j>} \left\{ e^{iA_{ij} \eta_{ij}}h_{i}^{\dagger} \left( e^{-i\alpha_{ij}} \cos \theta_{ij} - e^{i\beta_{ij}} \sin \theta_{ij} \right) h_{j} + \text{c.c.} \right\} - \frac{J}{2} \Delta'^{2} \sum_{<i,j>} \left\{ \eta^{2}_{ij} \sin^{2} \theta_{ij} h_{i}^{\dagger} \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} \right) \left( h_{j}^{\dagger} \right)^{T} + \text{c.c.} \right\}, \tag{A1}
\]

where we used \( \chi_{ij} = \eta_{ij} e^{i\alpha_{ij}} \cos \theta_{ij} \) and \( \Delta'^{i} j = \eta_{ij} e^{i\beta_{ij}} \sin \theta_{ij} \). Under the local SU(2) transformation holon field and the order parameters are transformed as

\[
h_{i}^{\prime} = g_{i}h_{i}, \tag{A2}
\]

\[
U_{ij}^{\prime} = g_{i}U_{ij}g_{j}^{\dagger}, \tag{A3}
\]

where \( g_{i} = e^{iS_{i}^{\theta} \Theta_{i}} \) is the SU(2) transformation matrix and \( U_{ij} = \left( \begin{array}{cc} \chi_{ij}^{\dagger} & -\Delta'^{i} j_{ij} \\ -\Delta'^{i} j_{ij} & -\chi_{ij} \end{array} \right) \), the order parameter matrix. Here \( \vec{r} \) are Pauli spin matrices and \( \vec{\Theta}_{i} \), the rotation angle. Considering the SU(2) rotation of \( \pm \pi \) around the 2-axis the SU(2) transformation matrix is given by

\[
g_{i} = (-1)^{j_{x}^{i} + j_{y}^{i}} \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \] and \( U_{ij} \) is transformed as

\[
\begin{align*}
U_{ij}^{\prime} &= g_{i}U_{ij}g_{j}^{\dagger} \\
&= (-1)^{j_{x}^{i} + j_{y}^{i}} \left( \begin{array}{cc} -\Delta'^{i} j_{ij} & 0 \\ 0 & \Delta'^{i} j_{ij} \end{array} \right) \left( \begin{array}{cc} e^{-i\alpha_{ij}} \cos \theta_{ij} & e^{i\beta_{ij}} \sin \theta_{ij} \\ -e^{i\beta_{ij}} \sin \theta_{ij} & e^{-i\alpha_{ij}} \cos \theta_{ij} \end{array} \right) \\
&= \left( \begin{array}{cc} e^{-i\alpha_{ij}} \cos \theta_{ij} - e^{i\beta_{ij}} \sin \theta_{ij} \\ -e^{i\beta_{ij}} \sin \theta_{ij} - e^{-i\alpha_{ij}} \cos \theta_{ij} \end{array} \right) \left( \begin{array}{cc} e^{-i\alpha_{ij}} \cos \theta_{ij} & e^{i\beta_{ij}} \sin \theta_{ij} \\ -e^{i\beta_{ij}} \sin \theta_{ij} & e^{-i\alpha_{ij}} \cos \theta_{ij} \end{array} \right) \left( \begin{array}{c} e^{-i\alpha_{ij}} \cos \theta_{ij} \\ -e^{i\beta_{ij}} \sin \theta_{ij} \end{array} \right) \left( \begin{array}{c} e^{i\beta_{ij}} \sin \theta_{ij} \\ -e^{-i\alpha_{ij}} \cos \theta_{ij} \end{array} \right), \tag{A4}
\end{align*}
\]

where \( j = i + x \) or \( j = i + y \) and \((-1)^{j_{x}^{i} + j_{y}^{i}} \times (-1)^{j_{x}^{j} + j_{y}^{j}} = -1 \). Thus we obtain \( \alpha_{ij}^{\prime} = -\alpha_{ij}, \beta_{ij}^{\prime} = -\beta_{ij}, \theta_{ij}^{\prime} = \theta_{ij} \) and \( \eta_{ij}^{\prime} = \eta_{ij} \). Then the holon action is given by

\[
S_{h}(h, A, \alpha, \beta, \theta, \eta) = \int_{0}^{\beta} d\tau \sum_{i} h_{i}^{\dagger}(\partial_{\tau} - \mu)h_{i} + \frac{t}{2} \sum_{<i,j>} \left\{ e^{iA_{ij} \eta_{ij}}h_{i}^{\dagger} \left( e^{-i\alpha_{ij}} \cos \theta_{ij} - e^{i\beta_{ij}} \sin \theta_{ij} \right) h_{j} + \text{c.c.} \right\} - \frac{J}{2} \Delta'^{2} \sum_{<i,j>} \left\{ \eta^{2}_{ij} \sin^{2} \theta_{ij} h_{i}^{\dagger} \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} \right) \left( h_{j}^{\dagger} \right)^{T} + \text{c.c.} \right\} - \frac{J}{2} \Delta'^{2} \sum_{<i,j>} \left\{ \eta^{2}_{ij} \sin^{2} \theta_{ij} h_{i}^{\dagger} \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} \right) \left( h_{j}^{\dagger} \right)^{T} + \text{c.c.} \right\}
\]

\[
= S_{h}^{b}(h', A, -\alpha, -\beta, \theta, \eta). \tag{A5}
\]

Thus we find that the holon free energy is the even function of \( \alpha \) and \( \beta \):

\[
F_{h}(A, \alpha, \beta, \theta, \eta) = -\frac{1}{\beta} \ln \int \mathcal{D}h e^{-S_{h}(h, A, \alpha, \beta, \theta, \eta)} = -\frac{1}{\beta} \ln \int \mathcal{D}h' e^{-S_{h}(h', A, -\alpha, -\beta, \theta, \eta)} = F_{h}(A, -\alpha, -\beta, \theta, \eta), \tag{A6}
\]

where we used the fact that the holon free energy is invariant under the local SU(2) transformation. It is then obvious that the holon current response function \( \frac{\partial^{2} F_{h}}{\partial A \partial \alpha} \) satisfies

\[
\Pi_{\alpha}^{B} = -\frac{\partial^{2} F_{h}(A, \alpha, \beta, \theta, \eta)}{\partial A \partial \alpha} \bigg|_{\alpha=\beta=0} = -\frac{\partial^{2} F_{h}(A, -\alpha, -\beta, \theta, \eta)}{\partial A \partial \alpha} \bigg|_{\alpha=\beta=0} = -\Pi_{\alpha}^{B}. \tag{A7}
\]
Thus we obtain $\Pi^\beta_{A\alpha} = 0$. Similarly $\Pi^\alpha_{A\beta} = 0$. The physical meaning of $\Pi^\beta_{A\alpha} = \Pi^\alpha_{A\beta} = 0$ is that there is no coupling between the external electromagnetic field $A$ and the order parameter phase fluctuations $\alpha$ and $\beta$. Thus $\alpha$ and $\beta$ mode fluctuations are not affected by the external electromagnetic field and the external electromagnetic field can not screen the fluctuations of these modes.

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