Concurrence for well-formed CAFs:
Naive Semantics

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Abstract

In the area of claim-based reasoning in abstract argumentation, a claim-based semantics is said to be concurrent in a given framework if all its variants yield the same extensions. In this note, we show that the concurrence problem with respect to naive semantics is $\text{coNP}$-hard for well-formed CAFs. This solves a problem that has been left open in [2].

1 Introduction

Claim-augmented argumentation frameworks (CAFs) [4] extend abstract argumentation frameworks (AFs) [1] by a function that assigns a claim to each argument. Formally, CAFs correspond to directed labeled graphs where the nodes correspond to arguments, the labels correspond to the arguments claims, and the arcs indicate (unidirectional) conflicts between them. As with arguments in AFs, the acceptance status of claims is decided via argumentation semantics. There are often several possibilities to lift AF semantics to claim-level [3]; considering naive semantics which is based on maximization of conflict-free sets gives rise to two claim-based variants: The so-called inherited variant, which performs maximization on argument-level, and the claim-level semantics, which performs maximization over claim-sets. Deciding whether two different variants of a semantics yield the same claim-based outcome for a given CAF (the so-called concurrence problem) can be computationally challenging: As shown in [2], the concurrence problem is $\text{coNP}$-complete with respect to naive semantics.

In this note, we show that deciding concurrence with respect to naive semantics is $\text{coNP}$-hard even for well-formed CAFs, an important sub-class of CAFs that impose restrictions on the attack relation. Well-formed CAFs satisfy a quite natural behavior of conflicts: A CAF is well-formed if arguments with the same claims attack the same arguments. This closes the complexity gap in [2] for the concurrence problem.

2 Preliminaries

We introduce abstract argumentation and claim-based reasoning following [1] and [3].

Abstract Argumentation. We fix a non-finite background set $U$. An argumentation framework (AF) [1] is a directed graph $F = (A, R)$ where $A \subseteq U$ represents a set of arguments and $R \subseteq A \times A$ models attacks between them. For two arguments $a, b \in A$, if $(a, b) \in R$ we say that $a$ attacks $b$ as well as $a$ attacks (the set) $E$ given that $b \in E \subseteq A$. A set $E \subseteq A$ is conflict-free in $F$ ($E \in \text{cf}(F)$) iff for no $a, b \in E$, ...
\((a, b) \in R\). A \textit{semantics} is a function \(\sigma\) with \(F \mapsto \sigma(F) \subseteq 2^A\). In this note we focus on naive semantics.

**Definition 1.** For an AF \(F = (A, R)\), a set \(E \subseteq A\) is naive \((E \in na(F))\) iff \(E\) is \(\subseteq\)-maximal in \(cf(F)\).

**Reasoning about claims.** A \textit{claim-augmented argumentation framework (CAF)} \cite{1} is a triple \(F = (A, R, cl)\) where \(F = (A, R)\) is an AF and \(cl\) is a function which assigns a claim to each argument in \(A\). The claim-function is extended to sets in the natural way, i.e., for a set \(E \subseteq A\), we let \(cl(E) = \{cl(a) \mid a \in E\}\). A CAF is \textit{well-formed} iff \(a^+_F = b^+_F\) for all \(a, b \in A\) with \(cl(a) = cl(b)\), i.e., arguments with the same claim attack the same arguments. The literature offers several ways to extend semantics for AFs that involve claims respectively arguments to a different extent. We introduce inherited and claim-level semantics for naive semantics that perform maximization in different stages of the evaluation.

**Definition 2.** For a CAF \(F = (A, R, cl)\), \(F = (A, R)\), and a semantics \(\sigma\), we let \(\sigma_c(F) = \{cl(E) \mid E \in \sigma(F)\}\). A set \(S \subseteq cl(A)\) is

- i-naive \((S \in na_c(F))\) iff \(S \in na_c(F)\), i.e., there is \(E \subseteq A\) with \(S = cl(E)\) and \(E\) is \(\subseteq\)-maximal in \(cf(F)\);
- cl-naive \((S \in cl-naive(F))\) \(S\) is \(\subseteq\)-maximal in \(cf_c(F)\).

As shown in \cite{3}, \(cl-naive(F) \subseteq na_c(F)\).

### 3 The Concurrence Problem for Naive Semantics

An interesting problem that arises when considering different variants of semantics for CAFs is the so-called concurrence problem.

| \(Con^\Delta_{na_c}\), \(\Delta \in \{CAF, wf\}\) |
| --- |
| Input: A CAF \(F\) (if \(\Delta = CAF\))/a well-formed CAF \(F\) (if \(\Delta = wf\)) |
| Output: \(\text{true}\) iff \(\sigma_c(F) = cl-\sigma(F)\) |

For naive semantics, the problem can be formulated as follows: Given a CAF \(F\), is it the case that maximization on argument-level yields the same accepted sets as maximization on claim-level?

In \cite{2} it has been shown that \(Con^\Delta_{na_c}\) is \(\text{coNP}\)-complete. Since concurrence for well-formed CAFs with respect to naive semantics is a special case of CAFs we obtain upper bounds for \(Con^\Delta_{na}\), that is, \(Con^\Delta_{na}\) is in \(\text{coNP}\).

We restate the following proposition \cite{2}.

**Proposition 1.** For a CAF \(F = (A, R, cl)\), \(na_c(F) = cl-naive(F)\) if and only if \(na_c(F)\) is incomparable.

Thus it suffices to verify incomparability of \(na_c(F)\). An \(\text{NP}\) procedure for the complementary problem is by a standard guess and check procedure: Guess \(E, G \subseteq A\) and check (i) \(E, G \in \sigma((A, R))\) and (ii) \(cl(E) \subset cl(G)\). The former can be checked in time polynomial in the number of arguments in \(F\).

Next we show that verifying incomparability of \(na_c(F)\) is \(\text{NP}\)-hard even if \(F\) is well-formed. We will first define the base reduction, which we slightly extend to obtain the the reduction for the well-formed case.
Reduction 1. Let \( \varphi \) be given by a set of clauses \( C = \{c_1, \ldots, c_n\} \) over atoms in \( X \). We construct \((A, R, cl)\) with
\[
A = X \cup \bar{X} \cup C \cup \{\varphi\} \cup \{a_1, a_2\}, \quad \text{with} \quad \bar{X} = \{\bar{x} \mid x \in X\},
\]
\[
R = \{(x, cl) \mid cl \in C, x \in cl\} \cup \{x, cl\} \mid cl \in C, \neg x \in cl\}
\]
\[
\cup \{(\bar{x}, x) \mid x \in X\} \cup \{(cl_i, \varphi) \mid i \leq n\} \cup \{(\varphi, a_2)\},
\]
and \( cl(x) = x \), \( cl(\bar{x}) = \bar{x} \), \( cl(c_i) = c_i \), \( cl(\varphi) = \varphi \) and \( cl(a_i) = a \).

An example of this reduction is given in Figure 1. The intuition behind this reduction is the following: We take the smallest sub-CAF that is not concurrent. This corresponds to \((\{a_1, a_2, \varphi\}, \{(\varphi, a_2)\}, cl)\) with \( cl(a_i) = a \) and \( cl(\varphi) = \varphi \), with naive extensions \( \{a_1, \varphi\} \) and \( \{a_1, a_2\} \). Observe that the second extension requires that \( \varphi \) is not taken. We then add the additional part corresponding to a propositional formula in CNF that only allows \( \varphi \) to not be included without additional changes, when the formula is satisfiable. Thus allowing us to reduce the concurrence problem to UNSAT.

In the example in Figure 1 we see that \( \{a_1, \varphi, x_1, x_2, x_3, x_4\} \) and \( \{a_1, a_2, x_1, x_2, x_3, x_4\} \) are naive extensions because \( \{x_1, x_2, x_3, x_4\} \) is a satisfying assignment of the formula.

Generally, the proof proceeds as follows.

Proposition 2. \( Con_{na}^{cf} \) is coNP-hard.

Proof. For hardness, we present a reduction from UNSAT: Let \( \varphi \) be given by a set of clauses \( C = \{c_1, \ldots, c_n\} \) over literals in \( X \). W.l.o.g. we can assume that \( \varphi \) does not contain tautological clauses, i.e., there is no \( c_i \), \( i \leq n \) with \( x, \bar{x} \in c_i \) for any \( x \in X \). Let \((A, R, cl)\) be defined as in Reduction 1. We will show \( \varphi \) is unsatisfiable iff \( na_{cf}(\mathcal{F}) \) is incomparable.

First assume \( \varphi \) is satisfiable and consider a model \( M \) of \( \varphi \). Let \( E = M \cup \{\bar{x} \mid x \notin M\} \cup \{\varphi, a_1\} \). Clearly, \( E \) is conflict-free; moreover, as \( M \) satisfies each clause \( c_i \), there is either \( x \in c_i \) with \( x \in M \) or \( x \in c_i \) with \( x \notin M \), thus \( E \) attacks each \( c_i \). Since, also \( \varphi \) and \( a_1 \) are in \( E \) we the only argument left to consider is \( a_2 \), which is however attacked by \( \varphi \) and can therefore not be included, while preserving conflict-freeness. We can conclude that \( E \) is a subset-maximal conflict-free set. Moreover, \( E' = M \cup \{\bar{x} \mid x \notin M\} \cup \{a_1, a_2\} \) is also a subset-maximal conflict-free set, since still every \( c_i \) is attacked and \( \varphi \) attacks \( a_2 \). It follows that \( na_{cf}(\mathcal{F}) \) is not incomparable since \( cl(E) = M \cup \{\bar{x} \mid x \notin M\} \cup \{\varphi, a\} \) is a strict superset of \( M \cup \{\bar{x} \mid x \notin M\} \cup \{a\} = cl(E') \) and both are contained in \( na_{cf}(\mathcal{F}) \).

Now assume that \( \varphi \) is unsatisfiable. Let \( E \) be a subset-maximal conflict-free set. If \( \varphi \in E \) it follows that none of the \( c_i \) are in \( E \). Therefore, it holds that for each \( x \) exactly one of \( x \) and \( \bar{x} \) is in \( E \). Furthermore, it always holds that \( a_1 \) is in \( E \). This means that \( cl(E) = \{\varphi, a\} \cup \{x \mid x \in X'\} \cup \{\bar{x} \mid x \notin X'\} \) for some subset \( X' \) of \( X \).
Case I: Assume there is a naive extension $E'$ such that $cl(E) \subsetneq cl(E')$. Then $E'$ must contain $cl_i$ for some $i$, $x$ for some $x \notin X'$ or $\bar{x}$ for some $x \in X'$. This is not possible, since this would imply $E \subseteq E'$, which is a contradiction to the assumption that $E$ and $E'$ are naive extensions.

Case II: Assume there is a naive extension $E'$ that contains $\varphi$ such that $cl(E') \subsetneq cl(E)$. Then there must be some $x$ with $x \in X'$ or $\bar{x}$ with $x \notin X'$ that is not contained in $E'$. This is not possible, since this would imply $E' \subseteq E$, which is a contradiction to the assumption that $E$ and $E'$ are naive extensions.

Case III: Assume there is a naive extension $E'$ that does not contain $\varphi$ such that $cl(E') \subsetneq cl(E)$. This is impossible, as $\varphi$ is unsatisfiable, which entails that there is a clause $cl_i$ that is not satisfied by $X'$. Therefore, $cl_i$ is not attacked by $E \setminus \varphi$ and also not $E'$. Thus $E'$ contains at least one $cl_i$, which means that also $cl(E')$ contains $cl_i$.

We see that any naive extension $E$ that contains $\varphi$ is incomparable to any other naive extension on claim level.

Next we show that the same holds for any naive extension $E$ that does not contain $\varphi$. It follows that $E$ contains $a_1, a_2$. Since $F' = (A \setminus \{a_1, a_2, \varphi\}, R \setminus \{(\varphi, a_2)\} \cup \{(cl_i, \varphi) | i = 1, \ldots, n\}, cl)$, the rest of the CAF, is such that all arguments have distinct claims, there is a one to one correspondence between $na_a(F')$ and $na_a(F')$, which implies that they are all incomparable with one another.

4 Conclusion

We see that even for well formed CAFs the concurrence problem is coNP-complete, as it is the case for general CAFs. This shows that while well formedness is an interesting property that only allows a fragment of the CAFs that might be deemed more reasonable, it does not lead to CAFs that are simpler, at least with respect to the concurrence problem.

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References

[1] Phan Minh Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artificial Intelligence, 77(2):321–358, 1995.

[2] Wolfgang Dvorák, Alexander Greßler, Anna Rapberger, and Stefan Woltran. The complexity landscape of claim-augmented argumentation frameworks. In AAAI 2021, Proc., pages 6296–6303. AAAI Press, 2021.

[3] Wolfgang Dvorák, Anna Rapberger, and Stefan Woltran. Argumentation semantics under a claim-centric view: Properties, expressiveness and relation to SETAFs. In KR 2020, Proc., pages 341–350. IJCAI.org, 2020.

[4] Wolfgang Dvorák and Stefan Woltran. Complexity of abstract argumentation under a claim-centric view. Artificial Intelligence, 285:103290, 2020.