Schrödinger cat states prepared by Bloch oscillation in a spin-dependent optical lattice

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We propose to use Bloch oscillation of ultra-cold atoms in a spin-dependent optical lattice to prepare Schrödinger cat states. Depending on its internal state, an atom feels different periodic potentials and thus has different energy band structures for its center-of-mass motion. Consequently, under the same gravity force, the wave packets associated with different internal states perform Bloch oscillation of different amplitudes in space and in particular they can be macroscopically displaced with respect to each other. In this way, a cat state can be prepared.

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Bloch oscillation is a peculiar response of a particle in a periodic potential to a weak external force [1, 2]. Under the drag of a constant force $F$, the wave packet of the particle oscillates back and forth periodically in space without accelerating indefinitely in the direction of the force. The weirdness is that a DC bias generates an alternating current. The underlying reason is that as long as the force is weak enough, interband transitions are prohibited by the gaps between energy bands. Confined in a specific energy band by such a mechanism, the motion of the particle is captured to a good extent by the semi-classical equations [3]

$$\frac{dr}{dt} = \frac{\partial E_n(q)}{\partial q} , \quad \frac{dq}{dt} = F, \quad \text{(1)}$$

where $r$ and $q$ are the center-of-mass and wave vector of the particle, respectively, and $E_n(q)$ is the dispersion relation in the $n$-th band. In the $q$-space, the picture is that the particle traverses the Brillouin zone, which is of the topology of a circle, repeatedly at a constant rate. From these equations, one solves readily the displacement of the wave packet as

$$\Delta r(t) = \frac{1}{F} [E_n(q_0 + Ft/\hbar) - E_n(q_0)], \quad \text{(2)}$$

where $q_0$ is the initial value of $q$. We see that $\Delta r$ is a periodic function of time with the period $T = 2\pi \hbar / a F$—this states the Bloch oscillation in the semi-classical theory. Here $a$ is the period of the periodic potential and $2\pi / a$ is the size of the Brillouin zone. Note that $T$ is independent of the detailed structure of the periodic potential but depends only on its period.

Suppose initially the particle is at the bottom of the lowest energy band ($n = 0$) with $q_0 = 0$. The maximum displacement $\Delta r_m$ is reached at $t = T/2$, when the wave vector $q = \pi / a$ arrives at the edge of the first Brillouin zone. Afterwards, the velocity of the particle $dr/dt$ reverses. The value of $\Delta r_m$ is simply

$$\Delta r_m = \frac{B}{F}, \quad \text{(3)}$$

where $B \equiv E_0(\pi / a) - E_0(0)$ is the band width of the lowest band. Simple as it looks, this equation has an important implication in our work below.

So far, Bloch oscillation has been observed in a variety of systems, such as semiconductor superlattices [4, 5], cold atoms in optical lattices [6–11], and photonic lattices [12–14]. On the application side, it has found use in microwave generation [4, 5, 13], precision force measurements [8–11], and coherent transport of matter waves [16–18].

In this paper, we propose that Bloch oscillation can also be used to prepare Schrödinger cat states [19–22] in a spin-dependent optical lattice. The idea is actually very simple. In a spin-dependent optical lattice, atoms in different internal states see different potentials (e.g. of different strengths). This non-trivial fact means that they also have different energy band structures for their center-of-mass motion, which in turn means they will have different Bloch oscillation modes under the same force (see Eq. (2)). In particular, their maximum displacements will be different according to Eq. (3) since the $B$’s may differ, and this implies that the wave packets corresponding to different internal states will be displaced with respect to each other. That is, a cat state can be prepared.

First of all we need a spin-dependent optical lattice. As proposed in [24, 25] and realized in [26], such an optical lattice can be constructed by interfering two counter-propagating laser beams linearly polarized but with an angle $\theta$ between the polarization vectors. The resulting standing light field can be decomposed into a $\sigma_+$ and a $\sigma_-$ polarized one with intensities $I_+ = I_m \cos^2(kx - \theta/2)$ and $I_- = I_m \cos^2(kx + \theta/2)$, respectively. Here $k = 2\pi / \lambda$ is the wave vector of the laser beams. Such a decomposition is helpful since the dipole potential for an atom in a state $|F, m_F\rangle$ (the quantization axis of the atom is along the optical lattice) is simply the sum of the contributions of the two components. Below we will use the same system as in [26]. That is, we choose $^{87}$Rb as the atom, and $|1\rangle \equiv |F = 2, m_F = -2\rangle$ and $|0\rangle \equiv |F = 1, m_F = -1\rangle$ as the two atomic internal states. The spin dependence of the dipole potential is realized by choosing the laser frequency to resolve the fine structure of the Rubidium D line. Specifically, as in [26], by tuning the wave length of the optical lattice laser to $\lambda = 785$ nm, the dipole po-
tials for an atom in the $|1\rangle$ and $|0\rangle$ states are respectively,

$$V_1(x; \theta) = V_m \cos^2 \left( kx - \frac{\theta}{2} \right),$$

$$V_0(x; \theta) = \frac{3}{2} V_m \cos^2 \left( kx + \frac{\theta}{2} \right) + \frac{1}{4} V_m \cos^2 \left( kx - \frac{\theta}{2} \right),$$

where $V_m \propto I_m$. The point is that if $\theta \neq 0$ or $\pi$, $V_1(x)$ and $V_0(x)$ are shifted relative to each other, and more importantly, have different amplitudes. The very latter effect results in different band structures as we see in Fig. 1. There the potentials $V_{1,0}$ and the corresponding energy bands for the two internal states are depicted. The parameters chosen are $V_m/E_r = 5$ and $\theta = \pi/2$, where $E_r = \hbar^2 k^2 / 2m = 2\pi\hbar \times 3.72$ kHz is the recoil energy of the atom. Note that the width of the lowest band for the $|1\rangle$ state is $2\pi\hbar \times 0.983$ kHz, while that for the $|0\rangle$ state is $2\pi\hbar \times 1.925$ kHz. The two differ almost by a factor of 2.

Now our scheme to generate a Schrödinger cat goes like this. Suppose initially the angle $\theta = 0$ (for this value of $\theta$, $V_1 = V_0$) and the atom is in the $|1\rangle$ state. As for its external state, it is assumed to be

$$\Psi_i \simeq \int_{-\pi/a}^{\pi/a} dq f(q) \phi_0(q).$$

(4)

Here $\phi_0(q)$ is the Bloch state in the lowest band with wave vector $q$. The weight function $f(q)$ is localized around $q = 0$ but otherwise unspecified. This condition is easily satisfied as long as the spatial size of the wave packet $\Psi_i$ is much larger than the lattice constant $a = \lambda/2$. Actually, the condensate wave function should satisfy this condition if the condensate is loaded adiabatically from a magnetic trap into the lattice as is usually done in cold atom experiments. It is checked that the results presented in the following are barely affected with different choices of $f(q)$, as long as the localization condition is satisfied. This fact is consistent with the semi-classical theory in which the details of the wave packets are irrelevant.

Specifically, in the simulations to be presented, $f(q)$ is of the form $f(q) \propto \exp(-q^2 / w^2)$ with $w a / \pi = 0.1 \ll 1$. This value of $w$ corresponds to a wave packet with a size on the order of 10$a$.

Then at some moment, by using a microwave pulse we can prepare the internal state of the atom into an arbitrary superposition of the $|0\rangle$ and $|1\rangle$ states. Let it be $\alpha|0\rangle + \beta|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$. Subsequently, $\theta$ is adjusted to $\pi/2$ (suddenly or smoothly, it does not matter; but in our simulation we take the sudden scenario), and the lattice is tilted by $\varphi$ with respect to the horizontal plane. The Bloch oscillation then starts. At a later time $t$, the wave function of the atom is of the form

$$\Psi(t) = \alpha|0\rangle \Psi_0(x, t) + \beta|1\rangle \Psi_1(x, t).$$

The evolution of the external wave function $\Psi_j(x, t)$ is given by

$$i\hbar \frac{\partial}{\partial t} \Psi_j = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_j(x) - F x \right) \Psi_j,$$

(5)

with the force $F = mg \sin \varphi$ and the initial condition $\Psi_j(t = 0) = \Psi_j$. Although the semi-classical theory above gives us an overall idea of the motion of the wave packets $\Psi_j(x, t)$, here we shall solve Eq. (5) numerically. Snapshots of $\Psi_j(x, t)$ are shown in Fig. 2. As expected, in the interval $0 \leq t \leq T/2$, the two wave packets move rightward and gradually depart. At the turning point $t = T/2$ (see Fig. 2II), the distance between the two reaches the maximum. Remarkably, at this point, the two wave packets are well separated—the separation between them is about $50a$, which is much larger than their sizes $(\gg 10a)$. Note that the wave packets themselves are large enough to deserve the name macroscopic. Therefore we obtain a desired Schrödinger cat state

$$\Psi(t = T/2) = \alpha|1\rangle \Psi_1(t = T/2) + \beta|0\rangle \Psi_0(t = T/2),$$

(6)

for which the internal and external states of the atom are entangled. The point is that the latter is in two
The coherence between the two packets can be checked by applying a microwave pulse to achieve a rotation in the \{ |1\rangle, |0\rangle \} space, and then turning off the lattice and observing the momentum distribution of the atom in the |1\rangle state by the absorption imaging method \[26\]. Note that \( \Psi_{1,0}(T/2) \) are both peaked around \( q = \pm \pi/a \) in the momentum space and thus the interference pattern will primarily consist of two peaks whose amplitudes depend on the rotation as well as \( \alpha \) and \( \beta \).

Afterwards, the two wave packets move backwards and return approximately \[27\] to their original states at \( t = T \) (see Fig. 2). That is, \( \Psi_{1,0}(T) \cong \Psi_i \) up to some global phases \[28\]. The global wave function is then \( \Psi(T) \cong (\alpha |0\rangle + \beta e^{i\chi} |1\rangle)\Psi_i \), where \( \chi \) is the difference of the phases \( \Psi_{1,0} \) accumulated in a cycle. Thus the atom completes a Bloch oscillation cycle by getting its internal state rotated somehow. In the perspective of the cat state, a cycle of Bloch oscillation is a cycle of birth-growth-death. The cycle can be interrupted by putting \( F = 0 \) at an appropriate time, e.g., at \( t = T/2 \), when the cat is in its largest size. Or if the direction of the force \( F \) is reversed at \( t = T/2 \), the two wave packets will continue moving rightward instead of going back. This would help to increase the size of the cat further.

We have followed the center-of-mass motion of the wave packets in time. The results are shown in Fig. 3 (solid lines). We see that the semi-classical theory (dotted lines) is correct quantitatively. The numerically exact results deviate significantly from the semi-classical predictions only in the vicinities of \( t = 0 \) and \( t = T \). The reason is that due to the sudden change of \( \theta \), both wave packets are partially excited and some packets (not visible in the snapshots) belonging to higher bands are emitted around \( t = 0 \), which may perform Bloch oscillation also and return around \( t = T \).

We now turn to the problem of the feasibility of the
scheme in experiment. In our simulation, \( Fa/E_r = 0.005 \). It is essential to make sure that this ratio is much smaller than unity. First, the potential drop \( Fa \) between two neighboring sites should be much smaller than the gap between the zeroth and first bands (see Fig. 1), so as to suppress Zener tunneling [2]. Or equivalently, the Brillouin zone, especially its boundary, should be traversed slowly so that transition into excited bands can be neglected. Second, according to [3], the distance between the two wave packets is inversely proportional to \( F \), thus smaller \( F \) means larger separation or larger “cat”. Of course, there should be an optimal value of \( F \) since the period \( T \) is also inversely proportional to \( F \). For \( 87\text{Rb} \) and a lattice constant \( a = \lambda/2 = 392.5 \) nm, the ratio above corresponds to a tilt angle \( \varphi = 4^\circ \) and a period \( T = 53 \) ms. On the contrary, under the chosen detuning and strength of the optical lattice, the spontaneous radiation rate \( \Gamma_{\text{eff}} \) of the atom is about 0.2 s\(^{-1}\). Thus the atom is long lived enough to oscillate several cycles before incoherent processes set in.

In conclusion, we have proposed that the spin-dependent optical lattice may offer an opportunity to create Schrödinger cat states by using Bloch oscillation. Our scheme has several interesting advantages. First, the cat state experiences birth-growth-death cycles repeatedly. It would be worthy to study experimentally how this process is damped in a real optical lattice, which is believed to be well isolated from the environment. Second, if we start from a Bose-Einstein condensate and minimize the atom-atom interaction which is deleterious to the Bloch oscillation, it might be possible to create a collection of atoms condensed in a cat state. We note that some generalizations are also possible. For example, though here we focused on the one dimensional case, the scheme can be directly extended to higher dimensions [13] since two-dimensional spin-dependent optical lattices have already been demonstrated experimentally [24]. Furthermore, in contrast to the static force considered here, periodically modulations [16, 17] are worth consideration also since they may help to increase the size of the “cat”.

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