Study of capacitor charge power supply with homopolar inductor alternator: System modelling and mode analysis

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Abstract
Capacitor charge power supply (CCPS) based on homopolar inductor alternator (HIA) has been successfully used in capacitor-based pulsed power supply (PPS), but there is still no perfect system modelling and compete system parameter design method. In this study, the system modelling and mode analysis of HIA-CCPS is developed in detail. First, a simplified CCPS model is proposed based on the voltage-behind-reactance (VBR) model of HIA and the phase base-value system (PBVS). Second, four different modes of CCPS are derived on conditions of the constant voltage load (CVL) assumption; the symbols and terms in these modes are unified. The equations derived in this study are the basis of the system parameter design and the further research of HIA-CCPS.

1 INTRODUCTION

The capacitor charge power supply (CCPS) with homopolar inductor alternator (HIA) (shown in Figure 1) has been successfully used in capacitor-based pulsed power supply (PPS) [1–4]. HIA, as shown in Figure 2, has many advantages when used as a flywheel energy storage device in CCPS due to its solid rotor structure without any windings and brushes, such as high energy-storage density and reliable performance [5, 6].

The working process of HIA-CCPS is as follows:

1. The prime mover drives the rotor of HIA to the specified speed for getting enough flywheel energy storage. Because the prime mover is size-limited and provides little energy during the fast charge process as shown in Figure 3, the capacitor charge energy is mainly provided by the flywheel energy stored in high-speed HIA rotor. Because the rotor usually has large inertia moment to obtain enough energy for multi-launch requirement, the rotor speed can be regarded as unchanged during one charge process.

2. Ensuring the no-load electromotive force (EMF) of HIA reaching the specified value after increasing the field current, the solid switches turn on and the capacitor is fast charged. When the capacitor voltage reaches the required value, the solid switches turn off. The thyristors in full bridge rectifier only control start and end of charge process without any phase control, so they are seen as uncontrolled during the charge process.

3. The capacitor discharges to the pulse load.

The cycle of (2) and (3) repeats several times until the rotor reaches the permitted lowest speed.

Although HIA-CCPS has been already built and experimented in multiple-launch tests [7–9], and can achieve excellent performance of multiple-launch in a very short time as shown in the experimental result of Figure 3 [7], there is still no available theoretical research of HIA-CCPS. The analysis model of existing research of HIA-CCPS [1] was not correct. First, the pure capacitor load was assumed as a constant current load (CCL), which should be a constant voltage load (CVL) at least. Second, there is no detail derivation of voltage-behind-reactance (VBR) model of HIA and the phase base-value system (PBVS). Second, four different modes of CCPS are derived on conditions of the constant voltage load (CVL) assumption; the symbols and terms in these modes are unified. The equations derived in this study are the basis of the system parameter design and the further research of HIA-CCPS.
can be directly adopted in capacitor charge situation and how much error between them should be analysed is not known. Second, the nomenclature of modes [18, 19] and symbols are not unified in these studies, and it is difficult to summarise the studies amongst them; and some of them are not accurate, which might confuse and mislead the readers. Third, because HIA has different transient flux path in direct- and quadrature-axes (dq) due to the field winding, its actual VBR model does not have a constant reactance in simulation [28], which makes the analytical derivation and results much complex. For the previous CVL analyses based on constant VBR model, the ac source—the transformer [11–13] or the devices—were not explained [14–19].

In this study, the foundation of HIA-CCPS study is developed, including simplified model and the mode analysis. To consider theory completeness, the derivations process and equation results are fully simplified, but only outstand some properties. This is not only a unification and correction study, but also proposes a general research method of synchronous machine-rectifier system (SMRS) and defining the variables and related terms of CCPS systematically, which makes the derivation easier to understand and guide further studies.

2 VBR MODEL OF HIA IN CCPS

VBR model with constant parameters in Figure 4 is very convenient for mode analysis in non-linear switching circuit, where the phase EMF vector is

\[
\mathbf{e}_{abc} = \begin{bmatrix} E_{pm} \cos \theta \\ E_{pm} \cos (\theta - 2\pi/3) \\ E_{pm} \cos (\theta + 2\pi/3) \end{bmatrix}
\]

where \(E_{pm}\) is the equivalent phase EMF amplitude. \(\theta\) is the electrical angle in phase-A, \(\theta = \omega_1 t + \theta_0\), \(t\) is the charge time, and \(\theta_0\) is initial charge angle.

The commutation reactance \(X_c\) is expressed as

\[
X_c = \omega_1 L_c
\]

where \(\omega_1\) is the synchronous angular frequency, and \(L_c\) is commutation inductor.

For the analyses of three-phase rectifier with CVL [11–19], the ac side is always in the form of VBR. When using the transient machine model [29], not only all winding parameters need to be considered, but also the magnetic field couplings which are very complexly expressed in switching circuit analysis. Even for the simplified dq model [4], the mode analysis must be developed by Park transformation; and the corresponding derivations are greatly complicated.

HIA can be easily expressed in VBR form for transformers, but not for the synchronous alternators due to its different transient dq flux paths. HIA belongs to electrically excited synchronous generator essentially, and it can be analysed by the two-reaction theory [30]. Although the VBR model of HIA is used in [1], the modelling process was not derived in detail, and the equations of commutation reactance \(X_c\) was not accurate.

In this study, there are five assumptions in the derivation process of simplified HIA-VBR model using in CCPS. (1) The
The parameters of three-phase armature winding are symmetrical. (2) The armature resistance is much less than the commutation reactance in HIA and has little influence on the entire charge performance, which is neglected in VBR model. (3) The iron core has linear property. (4) The space harmonics produced by the machine structure are neglected. (5) Because the charge time is usually far less than the transient time constant of field winding in HIA, the field winding can be considered as superconducting during the charge process.

The voltage equation of HIA can be expressed under rotor reference direction as

\[
\begin{align*}
v_d &= R_a i_d + \omega L_a i_d + \beta l_a d \\
v_q &= R_a i_q - \omega L_a i_q + \beta l_a q \\
v_i &= R_i i_i + \beta l_i
\end{align*}
\]  

(3)

where the subscripts d and q represent the armature variables in dq frame; f refers to the field winding in direction axis. The field winding in Equation (3) is transformed to have the same equivalent turn number of armature winding. \(R_d\) and \(R_q\) are the resistors in armature and field winding, respectively. The fluxes \(\lambda_d, \lambda_d, \) and \(\lambda_i\) can be calculated as

\[
\begin{align*}
\lambda_d &= L_{ad} i_d + \lambda_{aq} \\
\lambda_d &= L_{aq} i_d + \lambda_{ad} \\
\lambda_i &= L_{ai} i_i + \lambda_{ad}
\end{align*}
\]

(4)

where \(L_{ad}\) and \(L_{ad}\) are the leakage inductors corresponding to armature and field windings.

The main fluxes \(\lambda_{aq}\) and \(\lambda_{ad}\) can be calculated by

\[
\begin{align*}
\lambda_{aq} &= L_{aq} i_q \\
\lambda_{ad} &= L_{ad} (i_d + i_i)
\end{align*}
\]

(5)

where \(L_{aq}\) and \(L_{ad}\) are the armature reaction reactors.

The equivalent transient parameters are introduced as

\[
L_{ad}' = \frac{1}{L_{ad}} + \frac{1}{L_{df}}
\]

(6)

\[
\lambda_i' = m_{df} \lambda_i
\]

(7)

where \(m_{df}\) is the main flux coefficient of field winding as

\[
m_{df} = \frac{L_{ad}'}{L_{df}} = \frac{L_{ad}}{L_q}
\]

(8)

The stator voltage equations can be converted as

\[
\begin{align*}
v_d &= R_d' i_d + \omega I_{q1} i_d + L_{q1} i_q + l_q' \\
v_q &= R_q' i_q - \omega I_{q1} i_q + L_{q1} i_d + l_q'
\end{align*}
\]

(9)

where the new variables can be expressed as

\[
\begin{align*}
R_d' &= R_d + m_{df} R_f \\
R_q' &= R_q + m_{df} R_f
\end{align*}
\]

(10)

\[
\begin{align*}
l_q' &= \omega_1 m_{df} \lambda_i = X_{ad} (i_i + m_{df} i_d) \\
l_d' &= m_{df} (v_i - R_i \lambda_i - R_f m_{df} i_d)
\end{align*}
\]

(11)

The equivalent resistors \(R_d'\) and \(R_q'\) have the same magnitude of \(R_d\) in HIA, which can be neglected by assumption in Equation (2).

Moreover, the field winding is superconductive as assumption in Equation (5), so the winding resistor \(R_{df}\) and voltage \(v_{df}\) are zero. Under the effect of flux conservation, the field winding current \(i_d\) is completely resistant to demagnetisation current \(i_d\) in armature winding as

\[
\Delta i_i = i_i - i_i^0 = -m_{df} i_d
\]

(12)

where \(i_i^0\) is the field current of no-load situation. Because \(\lambda_{df}\) remain constant during the transient process, the EMF can be derived as

\[
\begin{align*}
\epsilon_d' &= \omega_1 \lambda_{d0}' = \omega_1 L_{q1} i_0 = E_{pm0} \\
\epsilon_d' &\approx 0
\end{align*}
\]

(13)

where \(E_{pm0}\) is the no-load phase voltage amplitude value due to the Park transformation of constant phase amplitude form.

The \(\epsilon_d'\) is indeed very small during the charge process whether the field winding is assumed as superconductive. First, \(\beta \lambda_i\) only produce some ripple that has little influence on the charge effect. Second, the converted \(R_i\) is usually very small compared with the reactance \(X_{ad}\) in \(\epsilon_d'\); even if \(i_d\) is very large, \(\epsilon_d'\) is much smaller than \(\epsilon_d'\).

The voltage equations can be simplified into

\[
\begin{align*}
v_d &= \omega_1 I_{q1} i_d + L_{q1} i_q + E_{pm0} \\
v_q &= -\omega_1 I_{q1} i_q + L_{q1} i_d
\end{align*}
\]

(14)

If the above equations are directly converted into phase frame, the VBR model is still not constant due to the difference between \(L_{q1}\) and \(L_{q1}\) which is the reason why the researchers often used VBR model in transformer or asynchronous generator.

The charge current should be calculated under different base inductors of \(L_{q1}\) and \(L_{q1}\) by the simplified dq model [4]. To be observed intuitively, the results are presented in average value form in Figure 5, whose base value will be defined in Section 3. The fundamental phase current amplitude \(i_{p1}\) is chosen to replace the dc charge current because their values are basically the same as in Figure 6 and the current oscillation phenomenon at the beginning of charge process in Figure 3 can be neglected.
FIGURE 5  Normalised fundamental current amplitude with base inductor $L_B = L_d'$ and different transient saliency ratio $\rho$

FIGURE 6  Amplitude of fundamental current and average charge current

It can be found in Figure 5 that when the base reactance is $X_B = X_c = X_d'$, all the initial values equal to one and the charge current only changes in shape with the transient saliency ratio $\rho = X_q/X_d'$. Usually for HIA, there is about $1 < \rho < 2$ due to its special homopolar flux structure, so the current increasing caused by $\rho$ can be used as engineering design margin, and $X_d'$ is more reasonable as the commutation reactance $X_c$ in VBR model. In [1], Ren proposed a common-mode inductor $X_{cm}' = (X_d' + X_q)/2$ as the commutation reactance $X_c$. It is obviously unreasonable, for which when the $X_{cm}'$ stays the same but the values of $X_d'$ and $X_q$ are not same, the charge currents would be quite different.

The VBR model can be obtained as

$$
\mathbf{v}_{abc} = \mathbf{k}(\theta)^{-1} \mathbf{v}_{dqp} = \mathbf{e}_{abc} - \mathbf{L}_{abc} \mathbf{\dot{i}_{abc}} = \begin{bmatrix} E_{pm0} \cos \theta \\ E_{pm0} \cos (\theta - 2\pi/3) \\ E_{pm0} \cos (\theta + 2\pi/3) \end{bmatrix} - \begin{bmatrix} I_d' \\ I_d' \\ I_d' \end{bmatrix} \mathbf{\dot{i}_{abc}}
$$

where the current is converted into generator reference. $\mathbf{k}(\theta)^{-1}$ is the pseudo-inverse of Park transform matrix. The value $\mathbf{\dot{i}_{abc}}$ is the differential operator, $d/dt$.

FIGURE 7 Circuit diagram of simplified CCPS

3 | SIMPLIFIED CCPS SYSTEM MODELLING UNDER BASE VALUE SYSTEMS (BVS)

Beside of the alternator, there are still two major parts in CCPS in Figure 1, the six-pulse rectifier and the storage capacitor load. To simplify the system model, another two assumptions as follow: (1) The stray parameters of transmission lines, rectifier, and capacitor are neglected. (2) All the solid-state switches are considered as ideal diodes. The simplified model is in Figure 7.

To make the results simple and available for different CCPS parameters, two specific BVS for CCPS are introduced in this subsection. The conventional BVS by the machine-rated values is no longer suitable for CCPS, because there is no concept of ‘rated’ in the dynamic charge process, and the rated values are not convenient for analysis.

There have been some normalisation methods proposed in CVL situation. The most commonly used is a phase base-value system (PBVS) [15, 17, 19], which can much simplify the mode analysis derivation. The base phase voltage $U_{bp}$ is peak phase EMF $E_{pm}$, and the base phase current $I_{bp}$ is the peak value of pseudo-steady-state three-phase short-circuit current $I_{scm}$ as

$$
I_{bp} = I_{scm} = E_{pm}/X_c
$$

In [16], a line base-value system (LBVS) was proposed, whose base voltage and current are $\sqrt{3}$ times to the values in PBVS. Essentially, there is no difference between them, so PBVS is adopted in derivation process in this study.

The extension base power and energy in PBVS are given by

$$
P_{bp} = U_{bp} I_{bp} = E_{pm}^2/X_c
$$

$$
E_{bp} = C_s U_{bp}^2 = C_s E_{pm}^2
$$

The ratio of them is

$$
\frac{E_{bp}}{P_{bp}} = \frac{C_s E_{pm}^2}{E_{pm}^2/X_c} = X_c/C_s = T_c
$$

where $T_c$ is defined as the time coefficient of CCPS circuit, which can measure the duration of charge process and be
chosen as the base time $t_0$. In this series, the frequency and time variables are separate, and there is no base frequency defined.

The normalised charge equation is expressed under PBVS as (* represents the normalised variables)

$$u_{dc}^* = \int i_{dc}^* \, dt$$  \hspace{1cm} (20)

Although PBVS has great advantage in derivation, it has poor representation. From the mode analysis results in this study, the values under PBVS did not change from 0% to 100%, and many values such as power and energy are even much smaller or larger than one, which is difficult to observe and analyse them intuitively. To solve this problem, a novel dc base value system (DCBVS) is proposed in this study. It is not for derivation, but for representing dc variables in values, figures and tables.

In this study, only the base voltage and energy in DCBVS are used. The base dc voltage $U_{bdc}$ and capacitor stored energy $W_{dco}$ are the maximum value that the variables can reach, which equals to the line EMF $E_{lim0}$ and $0.5C_d E_{lim0}^2$, respectively. For distinguishing the normalised dc values under PBVS and DCBVS, they are named as $u_{dc}^*$ and $u_{dc}^{**}$, respectively.

**Notation:** To simplify the representation of variables, unless it is unavoidable to use the normalised and actual values together, the asterisk superscript '*' is omitted; and for avoiding symbol confusion, the default values are all normalised in the following.

As can be seen in the averaged charge current against the charge voltage in Figure 8, the waveform of the HIA-CCPS prototype experiment and the simplified system model simulation are basically the same; and as expected, the experimental current is a bit larger than the simulation result, which can be used as an engineering design margin in preliminary parameter design.

**4 MODE ANALYSIS AND BOUNDARY CONDITIONS**

Based on the simplified system model and its variation, the mode analysis can be developed to obtain the mathematical expressions of HIA-CCPS.

![Figure 8](image1.png)

**FIGURE 8** The results of prototype experiment, simplified model simulation, and CCPS-SMRS-AVM (synchronous machine-rectifier system; average value model) derivation

![Figure 9](image2.png)

**FIGURE 9** Angle diagram of 3-mode

Due to symmetrical structure of six-pulse rectifier, whether the circuit works in any mode, it strictly follows the rule that one alternator electrical period $T_1$ is six times to the dc period. Therefore, the analysis can be just carried out on $\pi/3$ electrical radical interval. In this study, the research interval is from when D3 starts conducting to D4, and the initial charge angle is assumed as $\theta_0 = 0$, which $\theta \in [\pi/3, 2\pi/3]$ is for the natural conducting situation.

In this study, the nomenclature of modes adopts 'number of switch-on' method; in the specified interval, if the switch status changes, the corresponding conducting switch number will be added. For example, in one dc cycle, if the rectifier has two different status, where the first one is commutation (three switches on), and the second one is conduction (two switches on), it will be called '32-mode'.

However, it is unrealistic to develop the mode analysis based on the capacitor charge load directly. The end value of capacitor voltage in each period need to be recorded as the initial value of the next period. The HIA in CCPS usually has very high frequency due to its high-speed rotor for energy storage, a charge process usually has thousands of periods, which costs huge calculation time, and it is impossible to analyse the mathematical relationships between the variables. Because the charge process is usually much longer than one alternator electrical period $T_1$, the capacitor can be seen as a CVL during one period.

**4.1 3-Mode**

A 3-mode corresponds to the early and middle charge stage. In 3-mode, there are always three switches on and the switch state remains same in one specified dc period. As shown in the Appendix, D2 is conducting, and D1 is commutating to D3. The mode has a constant commutation angle as $\mu = \pi/3$, so it is also called continuous commutation mode. Some researchers called it ‘33-mode’ [18], for which the switch changes on the boundary is considered, but it would mislead the readers to think that a switch changes inside the interval.

Due to the firing delay caused by the commutation inductor, D3 starts commutation delays $\alpha$ to the natural firing angle $\pi/3$, so the actual dc interval is from $\pi/3 + \alpha$ to $2\pi/3 + \alpha$. The angle diagram of 3-mode is illustrated in Figure 9.

Because the phase currents are always continuous due to the commutation inductor in each phase, whether the switches turn on or off, there is always a continuous current boundary. The continuous boundary is separated into two parts as

$$i_{abc,3}(\theta = \pi/3 + \alpha) = \begin{bmatrix} i_{dc0}^0 & -i_{dc0}^0 \end{bmatrix}^T$$ \hspace{1cm} (21)

$$i_{abc,3}(\theta = 2\pi/3 + \alpha) = \begin{bmatrix} 0 & i_{dc0}^0 \end{bmatrix}^T$$ \hspace{1cm} (22)
where $i_{dc0}$ is the intermediate variable to present the dc current at initial and final of the interval, which also shows the beginning of commutation subinterval.

Through the circuit analysis in the Appendix, the basic results of 3-mode can be expressed as

$$u_{dc} = \frac{9s}{2\pi}$$

$$i_{dc0} = \frac{\sqrt{3c}}{2}$$

where the qualities are defined as

$$s = \sin \left(2\pi/3 + \alpha\right)$$

$$c = -\cos \left(2\pi/3 + \alpha\right)$$

The instantaneous dc current is expressed as

$$i_{dc,com,3} = \sin \left(\theta - \frac{\pi}{3}\right) + 3r \left[\frac{1}{2} + \frac{(\alpha - \theta)}{\pi}\right]$$

As we can see in Figure 10, at the beginning of charge process of $u_{dc}^{net} = 0$, the commutation delay angle $\alpha$ just reaches its maximum value of 3-mode, $\pi/3$. Since the charge process starts at the critical condition between 3-mode and 43-mode as in CCL [31, 32], if the capacitor has a negative voltage, there would not be a obviously, because all the diodes are forced to conduct by the negative dc voltage simultaneously, which is a 6-mode. Therefore, the mode diagram shown in [1], including 43-mode, is not correct.

### 4.2 32-Mode

A 32-mode corresponds to the middle and advanced stage of charge process. It is also called the discontinuous commutation mode, because the commutation subinterval does not last over all one dc period, whose commutation angle $\mu < \pi/3$.

**FIGURE 11** Angle diagram of 32-mode

In CCL situation, $\alpha$ equals to zero in 32-mode; but for CVL, $\alpha$ changes around and across zero. The angle diagram of 32-mode is shown in Figure 11, where $\gamma$ is the length of conduction subinterval.

The first subinterval is commutation, where $\theta_{com} \in [\pi/3 + \alpha, \pi/3 + \alpha + \mu]$. The boundary conditions are expressed as

$$i_{abc,32,com}|_{\theta = \frac{\pi}{3} + \alpha} = \left[\hat{i}_{dc0} - \hat{i}_{dc0}\right]^T$$

$$i_{abc,32,com}|_{\theta = \frac{\pi}{3} + \alpha + \mu} = \left[0 \hat{i}_{dc1} - \hat{i}_{dc1}\right]^T$$

where $\hat{i}_{dc1}$ is the intermediate variable to express the dc current between commutation and conduction subintervals.

The second state is conduction, where $\theta_{cond} \in [\pi/3 + \alpha + \mu, 2\pi/3 + \alpha]$, and the boundary conditions are

$$i_{abc,32,cond}|_{\theta = \frac{\pi}{3} + \alpha + \mu} = \left[0 \hat{i}_{dc1} - \hat{i}_{dc1}\right]^T$$

$$i_{abc,32,cond}|_{\theta = \frac{2\pi}{3} + \alpha} = \left[0 \hat{i}_{dc0} - \hat{i}_{dc0}\right]^T$$

The boundary conditions are still lacking because $\alpha$ is not fixed. In previous mode analysis of CVL, there are only current boundary conditions mentioned. Although it is exactly that, for the uncontrolled diode, the switch conduction state can be calculated directly by current [17]; but the current only decide the switching off position, and the switching on position should be determined by voltage.

In the conduction subinterval, when the voltage of non-conduction phase reaches the commutation voltage as shown in the Appendix, the corresponding diode turns on and all the phase voltages in this moment are continuous. Therefore, the voltage boundary condition can be expressed as

$$v_{abc,32,cond}|_{\theta = \frac{\pi}{3} + \alpha} = v_{abc,32,com}|_{\theta = \frac{\pi}{3} + \alpha} = \frac{1}{3} \left[u_{dc} u_{dc} - 2u_{dc}\right]^T$$

There is no continuous voltage condition when the switch turns on in 3-mode, because the switch state only changed with the off action caused by the current, not the voltage. This is also the reason why CCL situation has no voltage boundary condition.

The basic results of 32-mode can be shown as

$$u_{dc} = 3c$$

$$\mu = 3r$$
The instantaneous dc currents are shown in Equations (37) and (38). The point between 3-mode and 32-mode can be calculated by plugging \( \mu = \pi/3 \) into (38) as \( u_{\text{dc}}^{\text{nor}} \approx 74.63\% \) and \( \alpha \approx -0.078 \).

\[
i_{\text{dc}0} = -\pi c + \sqrt{3} \cos \alpha \tag{35}
\]

\[
i_{\text{dc}1} = \pi c - \sqrt{3} \cos (\mu + \alpha) \tag{36}
\]

From the trend of dc current in Figure 12, there would be a zero current happens in the middle of conduction subinterval at the end of 32-mode, so the next mode should be '3202-mode'. The critical point between 32-mode and 3202-mode can be solved by finding zero-crossing of the extremely minimum dc current, whose angle and current are expressed as

\[
i_{\text{dc},3202,\text{cond}1} \bigg|_{\theta = \pi/3 + \alpha + \beta_1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \tag{40}
\]

Because the second conduction subinterval contains a switch-on situation, the conditions are in Equations (31), (41) and (42)

\[
i_{\text{dc},3202,\text{cond}2} \bigg|_{\theta = \pi/3 + \alpha + \beta_2} = \begin{bmatrix} 0 & i_{\text{dc}1} - i_{\text{dc}1} \end{bmatrix}^T \tag{41}
\]

\[
v_{\text{abc},3202,\text{cond}2} \bigg|_{\theta = \pi/3 + \alpha + \beta_2} = \begin{bmatrix} \cos(\theta) \cos(\theta - 2\pi/3) \cos(\theta + 2\pi/3) \end{bmatrix}^T \tag{42}
\]

The basic results of 3202-mode can be derived as

\[
u_{\text{dc}} = 3\epsilon \tag{43}
\]

\[
\sin \left( \frac{\pi}{3} - \alpha + \gamma_2 \right) = -\sqrt{3}\epsilon \tag{44}
\]

\[
i_{\text{dc}0} = \sin(\gamma_2 - \alpha) - \frac{3}{2}\gamma_2\epsilon \tag{45}
\]

\[
i_{\text{dc}1} = -\mu \epsilon + s - \sin \left( \frac{2\pi}{3} + \alpha + \mu \right) \tag{46}
\]

\[
\sqrt{3} \left[ \cos \left( \frac{\pi}{3} + \alpha + \beta_1 \right) - \cos \left( \frac{\pi}{3} + \alpha + \mu \right) \right]
- 2 \left[ \sin \left( \frac{2\pi}{3} + \alpha + \mu \right) + s \right] + 3(\gamma_1 + n)\epsilon = 0 \tag{47}
\]
The instantaneous dc currents of different modes are shown in Figure 14 and expressed in Equations (49) to (51).

The instantaneous dc currents of different $\nu_{dc}^{nor}$ are shown in Figure 14 and expressed in Equations (49) to (51).

The angle diagram of 20-mode is shown in Figure 16. The boundary conditions are expressed as

$$i_{abc,20,cond} \mid \theta = \frac{\pi}{3} + \beta_2 = 0 \begin{bmatrix} \hat{i}_{dcl} - \hat{i}_{dc0} \end{bmatrix}^T$$

4.4 20-Mode

The 20-mode corresponds to the final stage of the charging process. Although it is the most easily analyzable mode because only the simple conduction subinterval should be analyzed here, it is the worst unified mode, because the names of mode and angles are not the same in different studies [17–19].

There are two mis-nomenclatures in this mode. First, since the commutation subinterval is disappeared, the firing delay angle $\alpha$ has already no physical meaning, and should not be used anymore. Second, due to the definition of one specified dc cycle, which is from D3 start conducting to D4, the dc period obviously starts from the conduction subinterval and ends after the non-conduction subinterval in this mode, which means the naming of 020-mode and 02-mode has no reason. Besides, the above 3202-mode had also been designated as 2023-mode [18].

The angle diagram of 20-mode is shown in Figure 16. The boundary conditions are expressed as

$$i_{abc,20,cond} \mid \theta = \frac{\pi}{3} + \beta_2 = 0 \begin{bmatrix} \hat{i}_{dcl} - \hat{i}_{dc0} \end{bmatrix}^T$$
The basic results of 20-mode can be derived as

$$u_{dc} = \sqrt{3} \sin \left( \frac{\pi}{3} + \beta_2 \right)$$

(55)

The instantaneous dc current is shown in Figure 17 and (57).

$$i_{dc,cond,20} = \frac{\sqrt{3}}{2} \left( \frac{\pi}{3} + \beta_2 - \theta \right) \sin \left( \frac{\pi}{3} + \beta_2 \right)$$

$$\quad + \frac{\sqrt{3}}{2} \left[ \cos \left( \frac{\pi}{3} + \beta_2 \right) - \cos(\theta) \right]$$

(57)

4.5 Mode analysis summary

There are totally four modes during the whole charge process. The 3-mode and 32-mode belong to CCM, which account for about 95% of the charge voltage; 3202-mode and 20-mode are DCMs. The mode analysis results are summarised in Table 1. In addition, there is still a 6-mode for the negative dc voltage situation.

As can be seen in Figure 18, $\alpha$ changes small in 32-mode, which is the reason why Ren considers $\alpha = 0$ in CCPS as the CCL situation [1]; if the work on DCM were developed, the boundary conditions would be contradictory to each other.

Since the beginning of 20-mode is rewritten as $\frac{\pi}{3} + \beta_2$, which is different from the other three modes as $\frac{\pi}{3} + \alpha$, a start angle $\xi$ is introduced, which equals to $\beta_2$ in 20-mode and $\alpha$ in other three modes, to unify them.

4.6 Mode analysis experiment

In order to prove the mode analysis theory, the experiment of HIA-CCPS is carried out. The motor speed is 2000 r/min and the excitation current is 3 A.

Figure 20 presents the experimental platform of HIA-CCPS. The original design sizes and parameters of prototype motor are shown in Table 2. Figure 21–22 show the waveforms in the whole charging process. The waveforms of line-to-line voltage, DC side charging voltage, excitation voltage and excitation current are shown on Figure 21. Figure 22 shows the waveforms of three-phase current and DC side charging current. Figures 22 and 23 show details of the phase current and line-to-line voltage at 0.1 and 0.2 s. It is obvious that the experimental results are consistent with the theoretical results.
| Mode | $u_{dc}$ | $w_{dc}$ | $\alpha$ | $\beta$ |
|------|--------|----------|--------|--------|
| 3    | 0% ∼ 74.6% | 0% ∼ 55.7% | 60° ∼ −4.47° | 60° |
| 32   | 74.6% ∼ 94.96% | 55.7% ∼ 90.17% | −4.47° ∼ 3.25° | 60° ∼ 3.19° |
| 3202 | 94.96% ∼ 95.77% | 90.17% ∼ 91.72% | 3.25° ∼ 3.57° | 3.19° ∼ 0° |
| 20   | 95.77% ∼ 100% | 91.72% ∼ 100% | − | 0° |

$w_{dc}$ is capacitor stored energy that equals to $u_{dc}^2$, under DC phase base-value system (BVS).

In conclusion, as the phase current and line-to-line voltage of the HIA coincide well with the theoretical waveforms, the accuracy of the mode analysis results is confirmed.
CONCLUSION

In this study, the main foundation work of CCPS is developed, including simplified system modelling and mode analysis. The modelling assumption include the field winding flux conservation during fast capacitor charge process, and the special properties of transient salient ratio $1 < \rho < 2$ in HIA. As a conclusion, the parameters of machine VBR model is $E_{pm} = E_{pm0} = X_{ad} \theta_0$ and $X_c = X_c'$. The simplified model can well present the experimental results in Figure 8 and be adopted in preliminary design.

For the mode analysis based on CVL assumption, there are totally 4 + 1 possible modes, where 3-mode and 32-mode belong to CCM and they account for most of charge process about $\eta_{dc} < 95\%$, and the last 6-mode is according to the negative dc voltage situation.

At the same time, the above analysis results are verified by experiments. The experimental results are in perfect agreement with the theoretical calculation, which proves the accuracy of mode analysis.

However, the instantaneous charge current expressions are still not convenient to be calculated in the charge in Equation (20) directly, because the instantaneous current is not only a function of capacitor voltage, but also relative to electrical angle $\theta$. The current parameters are decided by the initial value of capacitor voltage in each period, which costs much calculation during the charge process. The parameters are changed only once in a period, so the current change lags behind the capacitor voltage, which makes accumulative error overall the charge process.

To solve this problem, the average value model (AVM) method can be used, where the angle factor in the charge current can be eliminated, and the averaged current can track capacitor voltage during the charge process immediately. Essentially, the capacitor charging itself indeed consists function of average value calculation. When the dc voltage remains constant, the integrand results are the same, whether the instantaneous or the average values, in a period.

REFERENCES

1. Ren, Z., et al.: Performance of homopolar inductor alternator with diode-bridge rectifier and capacitive load. IEEE Trans. Ind. Electron. 60(11), 4891–4902 (2013)
2. Tang, P., et al.: Research on the excitation control of brushless doubly-fed alternator in a novel pulse capacitor charge power supply. IEEE Trans. Plasma Sci. 45(7), 1288–1294 (2017)
3. Yu, K., et al.: Design consideration of eddy-current loss for rotor of HIA with rectifier and capacitive loads. IEEE Trans. Plasma Sci. 46(8), 2949–2953 (2018)
4. Yu, K., et al.: A novel critical analysis method of homopolar inductor alternator for preliminary design in capacitor charge power supply. IEEE Trans. Plasma Sci. 47(5), 2354–2361 (2019)
5. Lou, Z., et al.: Analysis of homopolar inductor alternator for high reliability high power density applications. In: 2009 IEEE 6th International Power Electronics and Motion Control Conference, Wuhan pp. 841–844. (2009)
6. Ren, Z., et al.: Investigation of a novel pulse CCPS utilizing inertial energy storage of homopolar inductor alternator. IEEE Trans. Plasma Sci. 39(1), 310–315 (2011)
7. Ye, C., et al.: Optimal design and experimental research of a capacitor-charging pulsed alternator. IEEE Trans. Energy Convers. 30(3), 948–956 (2015)
8. Xin, Q., et al.: Repetition pulse charging characteristics for homopolar inductor alternator with rectified capacitive load. IEEJ Trans. Electr. Electron. Eng. 10(1), 44–49 (2015)
9. Liu, L., et al.: Analysis and test efficiency of a high-power pulsed power supply based on HIA. IEEE Trans. Plasma Sci. 47(5), 2293–2301 (2019)
10. Hancock, M.: Rectifier action with constant load voltage: Infinite-capacitance condition. Proc. IEEE 120(12), 1529–1530 (1973)
11. Mayordomo, J.G., et al.: A new frequency domain arc furnace model for iterative harmonic analysis. IEEE Trans. Power Delivery 12(4), 1771–1778 (1997)
12. Mayordomo, J.G., et al.: A unified theory of uncontrolled rectifiers, discharge lamps and arc furnaces. I. An analytical approach for normalized harmonic emission calculations. In: 8th International Conference on Harmonics and Quality of Power. Proceedings (Cat. No. 98EX227), Athens, Greece vol. 2, pp. 740–748. (1998)
13. Caliskan, V., et al.: Analysis of three-phase rectifiers with constant-voltage loads. In: 30th Annual IEEE Power Electronics Specialists Conference. Record. (Cat. No.99CH36321), Charleston vol. 2, pp. 715–720. (1999)
14. Bleijs, J.A.M.: Continuous conduction mode operation of three-phase diode bridge rectifier with constant load voltage. Electr. Power Appl. 152(2), 359–368 (2005)
15. Pejović, P., Kolar, J.W.: Exact analysis of three-phase rectifiers with constant voltage loads. IEEE Trans. Circuits Syst. II Express Briefs 55(8), 743–747 (2008)
16. Chen, L., Xie, Y.: Analysis of three-phase bridge rectifier with constant voltage loads. In: International Conference on Electrical and Control Engineering, Wuhan pp. 3347–3350 (2010)
17. Pejović, P., Kolar, J.W.: An analysis of three-phase rectifiers with constant voltage loads. In: Proceedings of Papers 5th European Conference on Circuits and Systems for Communications (ECCSC’10), Belgrade pp. 119–126. (2010)
18. Gerlano, A.D., et al.: Comprehensive steady-state analytical model of a three-phase diode rectifier connected to a constant DC voltage source. IET Power Electron. 6(9), 1927–1938 (2013)
19. Zhao, Z., et al.: Boundary and optimum of constant-voltage-load three-phase bridge rectifier. In: 17th International Conference on Electrical Machines and Systems (ICEMS), Hangzhou, pp. 3192–3198. (2014)
20. Ray, W.F., et al.: The three-phase bridge rectifier with a capacitive load. In: Third International Conference on Power Electronics and Variable-Speed Drives, London, pp. 153–156. (1988)
21. Grozdzich, M., Redmann, R.: Line current harmonics of VSI-fed adjustable-speed drives. IEEE Trans. Ind. Appl. 36(2), 683–690 (2000)
22. Carpinelli, G., et al.: Analytical modeling for harmonic analysis of line current of VSI-fed drives. IEEE Trans. Power Delivery 19(3), 1212–1224 (2004)
23. Dhombane, G.A., et al.: High-power-factor operation with improved line current harmonics of three-phase AC-to-DC Converter. In: Annual IEEE India Conference, Kanpur pp. 437–440. (2008)
24. Lian, K.L., et al.: Harmonic analysis of a three-phase diode bridge rectifier based on sampled-data model. IEEE Trans. Power Delivery 23(2), 1088–1096 (2008)
25. Iacchetti, M.F., et al.: Operation and design issues of a doubly fed induction generator stator connected to a dc net by a diode rectifier. IET Electr. Power Appl. 8(8), 310–319 (2014)
26. Mayordomo, J.G., et al.: A detailed procedure for harmonic analysis of three-phase diode rectifiers under discontinuous conduction mode and nonideal conditions. IEEE Trans. Power Delivery 33(2), 741–751 (2018)
27. Chiniforosh, S., et al.: Steady-state and dynamic performance of front-end diode rectifier loads as predicted by dynamic average-value models. IEEE Trans. Power Delivery 28(3), 1533–1541 (2013)
28. Pekarek, S.D., et al.: An efficient and accurate model for the simulation and analysis of HIA/converter systems. IEEE Trans. Energy Convers. 13(1), 42–48 (1998)
29. Krause, P., et al.: Analysis of Electric Machinery and Drive Systems, 3rd edition. Wiley-IEEE Press: Hoboken (2013)
30. Lou, Z., et al.: Two-reaction theory of homopolar inductor alternator. IEEE Trans. Energy Convers. 25(3), 424–438 (2010)
31. Tzeng, Y., et al.: Modes of operation in parallel-connected 12-pulse uncontrolled bridge rectifiers without an interphase transformer. IEEE Trans. Energy Convers. 13(1), 677–679 (2010)
32. Witzke, R.L., et al.: Influence of A-C reactance on voltage regulation of how to cite this article: Yu K, Yao J, Guo S, Xie X. Study of capacitor charge power supply with homopolar inductor alternator: System modelling and mode analysis. IET Power Electron. 2021;14:14–26. https://doi.org/10.1049/pe12.12002

APPENDIX A: CIRCUIT ANALYSIS

A.1 | General equations

Whether the process is in commutation, conduction, or non-current, there are general equations from D3 starts conducting to D4 as

\[ i_a + i_b + i_c = 0 \]

\[ i_{dc} = -i_c \]

A.2 | Commutation subinterval

The addition circuit equation is

\[ i_a = i_b = i_c + i_{dc} \]

The voltage equations can be expanded as

\[ \cos(\theta) - \frac{d_i}{d\theta} = \cos(\theta - \frac{2\pi}{3}) \]

\[ -\frac{d_i}{d\theta} = \cos(\theta + \frac{2\pi}{3}) - \frac{d_i}{d\theta} + i_{dc} \]

The phase currents can be solved as

\[
\begin{align*}
\dot{i}_a &= \sin(\theta) - \frac{1}{3} i_{dc} \theta + C_{a,com} \\
\dot{i}_b &= -\sin(\theta + \frac{\pi}{3}) - \frac{1}{3} i_{dc} \theta + C_{b,com} \\
\dot{i}_c &= -\sin(\theta - \frac{\pi}{3}) + \frac{2}{3} i_{dc} \theta + C_{c,com}
\end{align*}
\]

where \( C_{a,com}, C_{b,com}, \) and \( C_{c,com} \) are the integral constant terms of commutation process and their sum is zero.
The phase voltages can be solved as

\[
\begin{align*}
    v_a &= \cos(\theta) - \frac{d}{d\theta} = \frac{1}{3} u_{dc} \\
    v_b &= \cos\left(\theta - \frac{2\pi}{3}\right) - \frac{d_{ib}}{d\theta} = \frac{1}{3} u_{dc} \\
    v_c &= \cos\left(\theta + \frac{2\pi}{3}\right) - \frac{d_{ic}}{d\theta} = -\frac{2}{3} u_{dc}
\end{align*}
\]

A.3 | Conduction subinterval

The addition circuit equation is

\[
i_a = 0
\]

\[
v_b = v_c + u_{dc}
\]

The voltage equations can be expanded as

\[
\cos\left(\theta - \frac{2\pi}{3}\right) - \frac{d_{ib}}{d\theta} = \cos\left(\theta + \frac{2\pi}{3}\right) - \frac{d_{ic}}{d\theta} + u_{dc}
\]

The phase currents can be solved as

\[
\begin{align*}
    i_b &= -\frac{\sqrt{3}}{2} \sin\left(\theta + \frac{\pi}{2}\right) - \frac{1}{2} u_{dc} \theta + C_{b,cond} \\
    i_c &= \frac{\sqrt{3}}{2} \sin\left(\theta + \frac{\pi}{2}\right) + \frac{1}{2} u_{dc} \theta + C_{c,cond}
\end{align*}
\]

where \( C_{b,cond} \) and \( C_{c,cond} \) are the integral constant terms, and their sum is zero.

The phase voltages can be solved as

\[
\begin{align*}
    v_a &= \cos(\theta) - \frac{d}{d\theta} = \cos(\theta) \\
    v_b &= \cos\left(\theta - \frac{2\pi}{3}\right) - \frac{d_{ib}}{d\theta} = -\frac{1}{2} \cos(\theta) + \frac{1}{2} u_{dc} \\
    v_c &= \cos\left(\theta + \frac{2\pi}{3}\right) - \frac{d_{ic}}{d\theta} = -\frac{1}{2} \cos(\theta) - \frac{1}{2} u_{dc}
\end{align*}
\]

A.4 | Non-conduction subinterval

The addition circuit equation is

\[
i_a = i_b = i_c = 0
\]

The phase voltages can be solved as

\[
\begin{align*}
    v_a &= \cos(\theta) - \frac{d}{d\theta} = \cos(\theta) \\
    v_b &= \cos\left(\theta - \frac{2\pi}{3}\right) - \frac{d_{ib}}{d\theta} = \cos\left(\theta - \frac{2\pi}{3}\right) \\
    v_c &= \cos\left(\theta + \frac{2\pi}{3}\right) - \frac{d_{ic}}{d\theta} = \cos\left(\theta + \frac{2\pi}{3}\right)
\end{align*}
\]