GLOBULAR CLUSTERS WITH DARK MATTER HALOS. I. INITIAL RELAXATION

SERGEY MASHCHENKO AND ALISON SILLS
Department of Physics and Astronomy, McMaster University, Hamilton, ON L8S 4M1, Canada;
syami@physics.mcmaster.ca, asills@physics.mcmaster.ca
Received 2004 June 14; accepted 2004 September 26

ABSTRACT

In a series of two papers, we test the primordial scenario of globular cluster formation using results of high-resolution N-body simulations. In this first paper we study the initial relaxation of a stellar core inside a live dark matter minihalo in the early universe. Our dark matter–dominated globular clusters show features that are usually attributed to the action of the tidal field of the host galaxy. Among them are the presence of an apparent cutoff (tidal radius) or a break in the outer parts of the radial surface brightness profile and a flat line-of-sight velocity dispersion profile in the outskirts of the cluster. The apparent mass-to-light ratios of our hybrid (stars + dark matter) globular clusters are very close to those of purely stellar clusters. We suggest that additional observational evidence, such as the presence of obvious tidal tails, is required to rule out the presence of significant amounts of dark matter in present-day globular clusters.

Subject headings: dark matter — early universe — globular clusters: general — methods: n-body simulations

Online material: color figures

1. INTRODUCTION

The origin of globular clusters (GCs) is one of the important unsolved astrophysical problems. One can divide all proposed scenarios for GC formation into two large categories: (1) in situ formation (where GCs are formed inside their present-day host galaxies) and (2) pregalactic formation (where GCs are formed in smaller galaxies that later merge to become a part of the present-day host galaxy).

Pregalactic scenarios for GC formation can explain many puzzling properties of GCs in the Milky Way and other galaxies. This is especially true for so-called metal-poor GCs (with [Fe/H] \( \leq -1 \)), which form a distinctive population in many galaxies. Many of their properties are highly suggestive of their pregalactic origin: low metallicity; large age comparable to the age of the universe; isotropic orbits with an apocentric-to-pericentric distance ratio of \( R_a/R_p \approx 5.4 \pm 3.7 \) (Dinescu et al. 1999, based on data for 38 metal-poor Galactic GCs), which is consistent with the orbits of dark matter (DM) sub-halos (\( R_a/R_p \approx 5-6 \)) in cosmological \( \Lambda \)CDM simulations; and very weak or no correlation with the properties of the host galaxy.

One very interesting variation of the pregalactic picture is the primordial scenario of GC formation, which was first proposed by Peebles & Dicke (1968). In its modern interpretation, the primordial scenario assumes that GCs formed at the center of small DM minihalos in the early universe before or during the reionization of the universe, which is believed to have ended around the redshift \( z = 6 \) (Becker et al. 2001). The primordial picture has been considered by many authors, including Peebles (1984), Rosenblatt et al. (1988), Padoan et al. (1997), Cen (2001), Bromm & Clarke (2002), and Beasley et al. (2003).

It is usually assumed that during the reionization of the universe star formation can take place inside small DM halos with virial temperatures below \( \sim 10^4 \) K (or virial masses less than \( \sim 10^8 \) \( M_\odot \)), as the gas in such halos should escape the shallow gravitational potential after being heated to \( \sim 10^4 \) K by the metagalactic Lyman continuum background (Barkana & Loeb 1999). Some authors (Cen 2001; Ricotti et al. 2002) suggested, however, that the reionization of the universe can actually trigger star formation in small DM minihalos through different positive feedback mechanisms. In the scenario proposed by Cen (2001), star formation in small gas-rich minihalos is triggered by radially converging radiation shock fronts caused by the external Lyman continuum radiation field. In the cosmological simulations with radiative transfer proposed by Ricotti et al. (2002), small-halo objects constitute the bulk of mass in stars until at least redshift \( z \approx 10 \) because of the increased nonequilibrium fraction of free electrons in front of the cosmological H II regions and inside the relic cosmological H II regions, which results in more efficient \( \text{H}_2 \) formation and hence cooling of the gas.

Despite the fact that many authors considered different hydrodynamic and radiative processes that can lead to the formation of GC-like stellar clusters inside DM minihalos, there has been no detailed study on what happens to such hybrid (GC + DM halo) objects after the formation of stars from the point of galactic dynamics. Fully consistent cosmological simulations of structure formation in the universe (such as those of Ricotti et al. 2002) and even higher resolution semiconsistent simulations (like in Bromm & Clarke 2002) lack orders of magnitude in spatial and mass resolution to be able to answer the following questions: (1) How does the presence of DM modify the observable properties of GCs? (2) Are there observable features from which the presence of DM in a GC can be inferred? (3) Will DM in hybrid GCs survive tidal stripping during the hierarchical assemblage of substructure leading to the formation of large galaxies such as the Milky Way?

We try to answer the above questions in a series of two papers. In this first paper, we address the first and the second questions, with the last question being dealt with in the second paper (Mashchenko & Sills 2005, hereafter Paper II). Using the N-body tree code GADGET (Springel et al. 2001), we follow the relaxation of initially nonequilibrium stellar clusters inside live DM minihalos around the redshift \( z = 7 \). Our simulations are collisionless and can be directly compared with dynamically unevolved GCs; the impact of secular evolution (core collapse) on our results will be explored in Paper II. DM halos have either
Navarro et al. (1997, hereafter NFW) or Burkert (1995) profiles and have structural parameters taken from cosmological simulations. We use the proto-GC model from Mashchenko & Sills (2004, hereafter MS04) to set up the initial nonequilibrium configuration of stellar clusters inside DM halos. In MS04 we showed that the collapse of homogeneous isothermal stellar spheres leads to the formation of clusters with surface brightness profiles very similar to those of dynamically young Galactic GCs. In this model, all the observed correlations between structural and dynamical parameters of Galactic GCs are accurately reproduced if the initial stellar density $\rho_{i,s}$ and velocity dispersion $\sigma_{i,s}$ have the universal values $\rho_{i,s} \simeq 14 M_\odot$ pc$^{-3}$ and $\sigma_{i,s} \simeq 1.91$ km s$^{-1}$.

This paper is organized as follows. In § 2 we describe our method of simulating the evolution of a hybrid GC and list the physical and numerical parameters of our models. In § 3 we show the results of the simulations. Finally, in § 4 we discuss the results and give our conclusions.

2. MODEL

2.1. Initial Considerations

In our model, GCs with a stellar mass $m_* \geq r_{DM}$ at the center of DM halos with a virial mass $m_{DM}$ at a redshift of $z = 7$. We fix the mass ratio $\chi \equiv m_*/m_{DM}$ to 0.0088. The adopted value of $\chi$ is between the universal baryonic-to-DM density ratio $\Omega_b/\Omega_{DM} = 0.20$ (Spergel et al. 2003) and the fraction of baryons in GCs in the modern universe, $\approx 0.0025$ (McLaughlin 1999). This leaves enough room for such effects as a <10% efficiency of star formation in proto-GCs, mass losses due to stellar evolution (through supernovae and stellar winds), a decay of GC systems due to dynamical evolution of the clusters in the presence of tidal fields, and any biased mechanism of GC formation (e.g., when GCs are formed only in DM halos with a low specific angular momentum, as in Cen 2001).

2.2. DM Halos

We consider a flat ΛCDM universe ($\Omega_L + \Omega_m = 1$), with the values of the cosmological parameters $\Omega_m = 0.27$ and $H = 71$ km s$^{-1}$ Mpc$^{-1}$ (Spergel et al. 2003). In a flat universe, the critical density can be written as

$$\rho_c(z) = \frac{3H^2}{8\pi G} \left[ \Omega_m (1 + z)^3 + 1 - \Omega_m \right].$$

The virial radius of a DM halo with a virial mass $m_{DM}$ is then

$$r_{vir} = \frac{1}{1 + z} \left( \frac{2 m_{DM} G \Omega_m^2}{\Delta_c H^2 \Omega_m} \right)^{1/3},$$

where the spherical collapse overdensity $\Delta_c$ and the matter density of the universe in units of critical density at the redshift of $z$, $\Omega_m^2$, are given by (Barkana & Loeb 2001)

$$\Delta_c = 18\pi^2 + 82x - 39x^2$$

and

$$\Omega_m^2 = \left[ 1 + \frac{1 - \Omega_m}{\Omega_m (1 + z)} \right]^{-1}.$$

Here $x \equiv \Omega_m^2 - 1$.

We consider two types of DM density profiles: NFW halos and Burkert halos. The NFW model describes reasonably well DM halos from cosmological CDM simulations. It has a cuspy inner density profile with a slope of $\gamma = -1$ and a steeper than isothermal outer density profile,

$$\rho_N(r) = \frac{\rho_{0,N}}{(r/r_s)(1 + r/r_s)^2},$$

where

$$\rho_{0,N} = \frac{m_{DM}}{4\pi r_s^3} \left[ \ln (1 + c) - \frac{c}{1 + c} \right]^{-1}.$$  \hspace{1cm} (6)

Here $r_s$ and $c \equiv r_{vir}/r_s$ are the scale radius and concentration of the halo, respectively. Burkert halos, on the other hand, have a flat core. Their outer density profile slope is identical to NFW halos ($\gamma = -3$). This model fits well the rotational curves of disk galaxies (Burkert 1995; Salucci & Burkert 2000) and has the density profile

$$\rho_B(r) = \frac{\rho_{0,B}}{(1 + r/r_s)(1 + (r/r_s)^2)}.$$ \hspace{1cm} (7)

where

$$\rho_{0,B} = \frac{m_{DM}}{2\pi r_s^2} \left[ \ln (1 + c) + \frac{1}{2} \ln (1 + c^2) - \arctan c \right]^{-1}.$$ \hspace{1cm} (8)

The concentration $c$ of cosmological halos has a weak dependence on virial mass (with less massive halos being more concentrated on average). Sternberg et al. (2002) give the following expression for the concentration of low-mass DM halos in ΛCDM cosmological simulations at $z = 0$, which was obtained from the analysis of halos with virial masses of $10^8$–$10^{11} M_\odot$: $c = 27 (m_{DM}/10^9 M_\odot)^{-0.08}$. Combining this expression with the result from Bullock et al. (2001) that the concentration scales with a redshift as $(1 + z)^{-1}$, we obtained the following formula for the concentration of low-mass halos at different redshifts:

$$c = \frac{27}{1 + z} \left( \frac{m_{DM}}{10^9 M_\odot} \right)^{-0.08}.$$ \hspace{1cm} (9)

Zhao et al. (2003a, 2003b) showed that CDM halos do not have concentrations smaller than ∼3.5; hence equation (9) becomes invalid for halos with $c \leq 4$. Sternberg et al. (2002) demonstrated that equation (9) describes very well concentrations of four dwarf disk galaxies from Burkert (1995) fitted by a Burkert profile, so it can be used for both NFW and Burkert halos.

2.3. Stellar Cores

The initial nonequilibrium configuration of proto-GCs was modeled after MS04 as a homogeneous isothermal stellar sphere with an isotropic Maxwellian distribution of stellar velocities. Stellar clusters of different mass $m_*$ initially had the same universal values of stellar density and velocity dispersion $\rho_{i,s} = 14 M_\odot$ pc$^{-3}$ and $\sigma_{i,s} = 1.91$ km s$^{-1}$ (MS04). In MS04, we use a mass parameter $\beta$ to describe different proto-GC models:

$$m_* = 10^\beta \sigma_{i,s}^3 \left( \frac{375}{4\pi \rho_{i,s} G^3} \right)^{1/2},$$ \hspace{1cm} (10)

where $G$ is the gravitational constant. The connection between the initial virial parameter $\nu$ and $\beta$ is $\nu \equiv 2K_c/W_\nu = 10^{-2/3}$. 
where $K_s$ and $W_s$ are initial kinetic and potential energies of the stellar cluster, respectively.

We use models from MS04 to plot in Figure 1 the dependence of the fraction of stars that form a gravitationally bound cluster after the initial relaxation phase of a proto-GC on the mass parameter $\beta$ (or the virial parameter $\nu$). Please note that this is the case of no DM (bare stellar cores). As one can see, there are three different regimes of the initial relaxation phase: (1) “Hot” collapse ($-0.7 \leq \beta \leq -0.35$ or $3 \leq \nu \geq 1.7$) results in a substantial loss of stars for which the initial velocity was above the escape speed for the system; only the slowest moving stars collapse to form a bound cluster. (2) “Warm” collapse ($-0.35 \leq \beta \leq 0.85$ or $1.7 \geq \nu \geq 0.27$) is very mild and nonviolent and results in virtually no escapers. (3) “Cold” collapse ($\beta \geq 0.85$ or $\nu \leq 0.27$) is violent and leads to an increasingly larger fraction of escapers for colder systems. In the last case, the nature of the unbinding of stars is very different from the hot case. During a cold collapse, the radial gravitational potential is wildly fluctuating. At the intermediate stages of the relaxation, the central potential of the cluster becomes much deeper than the final, relaxed value. As a result, a fraction of stars is accelerated to speeds that will exceed the escape speed of the relaxed cluster. Analysis shows that most of the escapers are stars that initially were predominantly in the outskirts of the cluster and moving in the outward direction.

These three regimes of the initial relaxation of proto-GCs are of a very general nature and are not restricted to the GC formation scenario of MS04. To illustrate this, let us write down an expression for the virial ratio of a homogeneous isothermal sphere, which is applicable to both the prestellar gas phase of a forming GC and the initial stellar configuration after the starburst takes place:

$$\nu = \frac{5 \langle V^2 \rangle R}{3GM}.$$  (11)

Here $M$ and $R$ are the mass and the radius of the system, and $\langle V^2 \rangle$ is the mean-square speed of either gas molecules or stars. Assuming that newly born stars have the same velocity dispersion as star-forming gas, equation (11) suggests the three following physical scenarios resulting in our hot, warm, and cold stellar configurations (in all scenarios we start with an adiabatically contracting gas cloud of a Jeans mass, which corresponds to $\nu_{\text{gas}} = 1$): (1) nonefficient star formation with a subsequent loss of the remaining gas leads to our hot case; (2) almost 100% efficient star formation results in the warm case; and (3) runaway cooling of the whole cloud on a timescale shorter than the free-fall time, leading to high-efficiency star formation, results in our cold case.

### 2.4. Physical Parameters of the Models

In this paper, we consider three initial stellar configurations, which can be considered to be typical hot, warm, and cold cases (see Fig. 1): $\beta = -0.6$ (model H, for hot), $\beta = 0.4$ (model W, for warm), and $\beta = 1.4$ (model C, for cold). We study the relaxation of stellar cores in either NFW (subscript “N”; e.g., $W_N$) or Burkert (subscript “B”) DM halos, which makes a total of six different combinations of stellar cores and DM halos. The physical parameters of these six models are summarized in Table 1.

The stellar masses $m_s$ in our models cover the whole range of GC masses (see Table 1). The initial stellar radius $r_s$ is much smaller than the scale radius of the DM halo $r_h$ in all the models. The masses of DM halos $m_{DM}$ range from $10^6$ to $10^8 M_\odot$. From Figure 6 of Barkana & Loeb (2001), these DM halos correspond to 1–1.5 $\sigma$ fluctuations collapsing at the redshift $z = 7$. As one

### TABLE 1

**PHYSICAL PARAMETERS OF THE MODELS**

| Model | $\beta$ | $\nu$ | $m_s$ (M$_\odot$) | $r_s$ (pc) | $\tau_s$ (Myr) | $r_{s,\text{min}}$ (pc) | $m_{DM}$ (M$_\odot$) | $c$ | $r_{\text{vir}}$ (pc) | $r_h$ (pc) | $r_{h,\text{DM}}$ (pc) | $\tau_{DM}$ (Myr) | $S$ |
|-------|--------|------|-----------------|-----------|---------------|-------------------|-------------------|---|----------------|---------|-----------------|--------------|---|
| $W_N$ | 0.4    | 0.54 | $8.8 \times 10^4$ | 11.2      | 0.49          | 2.98              | $10^7$            | 4.88 | 885            | 181     | 395             | 52           | 4.7 |
| $W_B$ | 0.4    | 0.54 | $8.8 \times 10^5$ | 11.2      | 0.49          | 2.98              | $10^7$            | 4.88 | 885            | 181     | 436             | 61           | 117 |
| $C_N$ | 1.4    | 0.12 | $8.8 \times 10^5$ | 24.2      | 0.078         | 1.57              | $10^8$            | 4.06 | 1906           | 470     | 597             | 56           | 5.8 |
| $C_B$ | 1.4    | 0.12 | $8.8 \times 10^5$ | 24.2      | 0.078         | 1.57              | $10^8$            | 4.06 | 1906           | 470     | 997             | 66           | 173 |
| $H_N$ | 0.6    | 2.5  | $8.8 \times 10^3$ | 5.21      | 17            | 4.14              | $10^6$            | 5.87 | 411            | 70      | 174             | 48           | 3.7 |
| $H_B$ | 0.6    | 2.5  | $8.8 \times 10^3$ | 5.21      | 17            | 4.14              | $10^6$            | 5.87 | 411            | 70      | 190             | 55           | 78 |

Note.—Here $\nu$ and $r_s$ are the initial virial ratio (assuming that there is no DM) and radius of the stellar core; $\tau_s$ is the crossing time at the half-mass radius for the relaxed, purely stellar models from MS04 rescaled to $\rho_c = 14 M_\odot$ pc$^{-3}$ and $\sigma_c = 1.91$ km s$^{-1}$; $r_{s,\text{min}}$ is the minimum attained half-mass radius for relaxing stellar clusters from MS04 (rescaled to the above values of $\rho_c$ and $\sigma_c$); $r_{h,\text{DM}}$ and $\tau_{DM}$ are the half-mass radius and the crossing time at the half-mass radius for DM halos; and $S \equiv m_s/m_{DM}(r_s)$ is the ratio of the stellar mass to the mass of DM within the initial stellar radius $r_s$.  

---

**Fig. 1.—Fraction of stars remaining gravitationally bound after the initial relaxation phase in the MS04 proto-GC models. Ringed circles mark models with $\beta = -0.6$ (H), $\beta = 0.4$ (W), and $\beta = 1.4$ (C).**
can see, halos in this mass range are populous enough at \( z \approx 7 \) to account for all observed GCs.

In all our models, stars initially dominate DM in the core \([S \equiv m_s/m_{DM}(r_s) > 1;\) see Table 1], which can be considered as a desirable property for star formation to take place. It is instructive to check whether the above condition \( S > 1 \) holds for other plausible values of the GC formation redshift \( z \) and stellar mass fraction \( \chi \). In Figure 2 we show the redshift dependence of the minimum mass of a stellar core in a DM halo, satisfying the condition \( S \geq 1 \), for a few fixed values of \( \chi: 0.0025, 0.0088 \), and 0.20. Both NFW (thick lines) and Burkert (thin lines) cases are shown. As one can see, only in the extreme case of an NFW halo at \( z = 20 \) with a fraction of total mass in stars \( \chi = 0.0025 \) is the minimum mass of a dominant stellar core \( m_{s, \text{min}} = 1.7 \times 10^7 M_\odot \) approaching the mass range of GCs. The conclusion that we arrive at is that, for all plausible values of \( z \) and \( \chi \), stellar cores with the universal initial density \( \rho_\odot = 14 M_\odot \, \text{pc}^{-3} \) and GC masses dominate DM at the center of DM halos.

### 2.5. Setting Up N-Body Models

DM halos in our models are assumed to have an isotropic velocity dispersion tensor. To generate \( N \)-body realizations of our DM halos, we sample the probability density function (PDF; Widrow 2000)

\[
P(E, R) \propto R^2 (\Psi - E)^{1/2} F(E).
\]

Here \( \Psi \) and \( E \) are the relative dimensionless potential and particle energy in units of \( 4\pi G m_\odot r_s^2 \) (for NFW halos) and \( \pi^2 G m_\odot \Psi r_s^2 \) (for Burkert halos), \( F(E) \) is the phase-space distribution function, and \( R \equiv r/r_s \) is the dimensionless radial coordinate of the particle. The dimensionless potential \( \Psi \) is equal to 1 at the center and 0 at infinity. For NFW halos, \( \Psi = R^{-1} \ln (1 + R) \). For Burkert halos, the corresponding expression is given in Appendix A (eq. [A1]). We use the analytic fitting formulae from Widrow (2000) to calculate \( F(E) \) for the NFW model and derive our own fitting formulae for the Burkert profile (see Appendix A). Our models are truncated at the virial radius \( r_{\text{vir}} \) (eq. [2]): \( F(E) = 0 \) for \( r > r_{\text{vir}} \). As we show in §3.2, this truncation results in an evolution in the outer DM density profile (with the profile becoming steeper for \( r \gg r_s \)) that should not affect our results.

We sample the PDF (eq. [12]) in two steps: (1) A uniform random number \( x \in [0, 1] \) is generated. The radial distance \( R \) is then obtained by solving numerically one of the two following nonlinear equations:

\[
\ln (1 + R) - R/(1 + R) = x[\ln (1 + c) - c/(1 + c)]
\]

for the NFW profile and

\[
\ln (1 + R) + [\ln (1 + R^2)]/2 - \arctan R
\]

\[
\ln (1 + c) + [\ln (1 + c^2)]/2 - \arctan c \}
\]

for the Burkert profile. (2) Given \( R \), we can calculate \( \Psi \). Instead of equation (12), we can now sample the PDF

\[
P(E) \propto \Pi \equiv \left( \frac{\Psi - E}{\Psi} \right)^{1/2} F(E) F(\Psi).
\]

As \( F(E) \) is a monotonically increasing function and \( 0 \leq E \leq \Psi \), the inequality \( 0 \leq \Pi \leq 1 \) holds. Two uniform random numbers, \( y \in [0, 1] \) and \( E \in [0, \Psi] \), are generated. If \( y \leq \Pi \), we accept the value of the dimensionless energy \( E \); otherwise, we go back to the previous step (generation of \( y \) and \( E \)).

The velocity module \( v \) can be obtained from the equation for the total energy of a particle, \( E = v^2/2 + \Psi \). Finally, two spherical coordinate angles \( \theta \) and \( \varphi \) are generated for both radius and velocity vectors using the expressions \( \cos \theta = 2t - 1 \) and \( \varphi = 2\pi u \) (here \( t \) and \( u \) are random numbers distributed uniformly between 0 and 1).

The above method to generate \( N \)-body realizations of either NFW or Burkert halos does not use a local Maxwellian approximation to assign velocities to particles. Instead, it explicitly uses phase-space distribution functions, which was shown to be a superior way of setting up \( N \)-body models (Kazantzidis et al. 2004).

Stellar cores are set up as homogeneous spheres at the center of DM halos: \( R = r_s, x^{1/3} \), where \( x \in [0, 1] \) is a uniformly distributed random number. We use equal-mass stellar particles. The velocities of the particles have a Maxwellian distribution. The components of the radius and velocity vectors are generated in the same fashion as for DM particles.

### 2.6. Numerical Parameters of the Models

In addition to the six models from Table 1, which consist of a live DM halo plus a stellar core, we also ran two more models with a static DM potential (marked with the letter S at the end). The numerical parameters for these eight models are listed in Table 2. In this table we also list parameters for the three properly rescaled stars-only models from MS04 (H0, W0, and C0), which are used as reference cases.

To run our models, we use a parallel version of the multi-stepping tree code GADGET-1.1 (Springel et al. 2001). The code allows us to handle separately stellar and DM particles, with each species having a different softening length: \( \epsilon_s \), and
The values of the softening lengths were chosen to be comparable with the initial average interparticle distance. We used the expression for the stellar particles $\epsilon = 0.77 r_h N^{-1/3}$ (Hayashi et al. 2003), where the mass-half radius $r_h$ was measured at the initial moment of time (so $r_h = r_s/2^{1/3}$). The same expression was used to calculate $\epsilon_{DM}$ for the HN, B models, whereas for the models WN, B and CN, B we used half of the value given by the equation from Hayashi et al. (2003) to achieve a better spatial resolution within the stellar core.

Aarseth et al. (1988) showed that collapsing homogeneous stellar spheres with a nonnegligible velocity dispersion $\sigma_i,s$ do not experience a fragmentation instability (and, as a consequence, preserve the orbital angular momentum of stars) if the following condition is met: $N_s \gtrsim \left[ m_s/(r_s \sigma_i,s)^2 \right]^{1/2}$, where $N_s$ is the number of stars. For our models, this adiabaticity criterion can be rewritten as

$$\log N_s \gtrsim 2 \beta + 3 \log 5.$$  \hspace{1cm} (16) 

As we discussed in MS04, if real GCs indeed started off as isothermal homogeneous stellar spheres, they should have collapsed adiabatically. We also showed that if both the condition in equation (16) and $\epsilon_s \leq 0.25 r_s, \text{min}$ are met, the results of simulations of collapsing stellar spheres do not depend on the number of particles $N_s$ and the softening length $\epsilon_s$ (here $r_s, \text{min}$ is the minimum mass-half radius of the cluster during the collapse). By comparing Tables 1 and 2, one can see that the values of $N_s$ and $\epsilon_s$ that we use in our simulations meet both the above conditions.

The evolution time $t_2$ in our runs is at least 3 times longer than the crossing time for DM, $\tau_{DM}$, and hundreds or even thousands of times longer than the crossing time for stellar particles $\tau_s$ (see Tables 1 and 2). As we see in § 3, this time is long enough for stellar and DM density profiles to converge. On the other hand, it is short enough to avoid significant dynamical evolution in the stellar clusters caused by encounters between individual particles.

The individual time steps in the simulations are equal to $\left(2\pi/\alpha a\right)^{1/2}$, where $\alpha$ is the acceleration of a particle and the parameter $\eta$ controls the accuracy of integration. We used a very conservative value of $\eta = 0.0025$ and set the maximum possible individual time step $\Delta t_{\text{max}}$ to either 20 or 0.2 Myr (see Table 2). As a result, the accuracy of integration was very high: $\Delta E/t_{\text{tot}} \lesssim 0.06\%$, where $\Delta E/t_{\text{tot}}$ is the maximum deviation of the total energy of the system from its initial value. One has to keep in mind, however, that numerical artifacts are the most pronounced in the central, densest area, where the frequency of strong gravitational interaction between particles is the highest. We estimate the severity of these effects by looking at the total energy conservation in our purely stellar models from MS04; in models H0, W0, and C0 the values of $\Delta E/t_{\text{tot}}$ are $\sim 0.05\%$, $\sim 0.85\%$, and $\sim 0.8\%$, respectively (for model W0 we used a larger value of $\eta = 0.02$; hence the relatively large errors). As one can see, although the numerical artifacts in the dense stellar cluster are more visible than in the DM + stellar core case, the magnitude of these effects is still reasonably low.

For completeness’ sake, we also give the values of other code parameters that control the accuracy of simulations: $\text{ErrTolForce} = 0.02$, $\text{ErrTolForceAcc} = 0.01$, $\text{MaxNodeMove} = 0.05$, $\text{TreeUpdateFrequency} = 0.1$, and $\text{DomainUpdateFrequency} = 0.2$ (please see the code manual for explanation of these parameters). The code was compiled with the option $\text{DBMAX}$ enabled, which allowed it to use a very conservative node-opening criterion.

### 3. RESULTS OF SIMULATIONS

#### 3.1. General Remarks

For each of our models, we generated 100–200 individual time frame snapshots. The last 20%–60% of the snapshots (corresponding to the time interval $t_1$–$t_2$; see Table 2) were used to calculate the properties of relaxed models, including radial profiles and the different stellar cluster parameters listed in Table 3. In all snapshots, we removed in an iterative manner particles whose velocity exceeds the escape velocity $V_{esc} = (-2\Phi)^{1/2}$, where $\Phi$ is the local value of the gravitational potential. This procedure affected the models from MS04 (H0, W0, and C0) and DM particles only in the models presented here. One of the reasons for removing escapers was to make the results of the simulations directly comparable with the results

---

1. Available at http://www.mpa-garching.mpg.de/gadget/.
from Paper II, where this procedure is applied to discount particles stripped by tidal forces.

The radial profiles shown in this section were obtained by averaging the corresponding profiles from individual snapshots in the time interval $t_1 - t_2$. For each profile, we only show the part that is sufficiently converged (the dispersion between different snapshots is very small). For projected quantities (such as surface brightness $\Sigma_f$ and line-of-sight velocity dispersion $\sigma_f$) we use the projection method described in Appendix B, which produces much less noisy radial profiles than a straightforward projection onto one plane or three orthogonal planes. This is crucial for the analysis of the properties of the outer, low-density parts of stellar clusters, which is the main purpose of this study.

The virial ratio $\nu$ for our models is consistent with unity within the measurement errors at the end of the simulations. All the global model parameters (listed in Table 3) converge to their final values by $t = t_2$. Similarly, radial density and velocity dispersion profiles converge to their final form over a wide range of radial distances. We explicitly checked that the evolution time $t_2$ is more than three crossing times at the largest radius for the stellar density profiles shown in Figures 3, 5, and 7. All of the above considerations let us conclude that the results presented here are collisionless steady-state solutions.

3.2. Warm Collapse

In the absence of DM, an isothermal homogeneous stellar sphere with a mass parameter $\beta = 0.4$ relaxes to its equilibrium

![Fig. 3.—Radial density profiles for warm collapse models for (a) the case of an NFW halo and (b) the case of a Burkert halo. Thick lines correspond to DM density; thin lines show stellar density. Solid lines correspond to the cases of a live DM halo + a stellar core (models W0 and W0h), long-dashed lines depict analytic DM profiles and a relaxed stellar cluster profile in the absence of DM (model W0), and dotted lines show stellar density profiles for static DM models (W0S and W0S). Vertical long-dashed, short-dashed, and dotted lines mark the values of $r_{\text{DM}}$, $r_w$, and $r_c$, respectively. [See the electronic edition of the Journal for a color version of this figure.]
state with virtually no stars lost (see Fig. 1). This makes it an interesting case to test the result from Peebles (1984), that a stellar cluster inside a static constant-density DM halo can acquire a radial density cutoff similar to that expected from the action of tidal forces of the host galaxy, for the case of live DM halos with cosmologically relevant density profiles and initially nonequilibrium stellar cores.

In Figure 3 we show the radial density profiles for a relaxed stellar core and a DM halo both for an NFW model (Fig. 3a) and a Burkert model (Fig. 3b) for the case of a warm collapse (W models). Here long-dashed lines show the density profiles for stars in the absence of DM (thin lines) and DM in the absence of stars (thick lines). Solid lines, on the other hand, show the density profiles for stars and DM having been evolved together. We also show as dotted lines the stellar radial density profiles for the case of a static DM potential.

As can be seen from Figure 3, in the absence of DM (model \( W_0 \)) the relaxed stellar cluster has a central flat core and close to a power-law outer density profile. The radial density profile has also a small “dent” outside of the core. It is not a transient feature (in the collisionless approximation), as the virial ratio for the whole cluster is equal to unity within the measurement errors (\( \nu = 1.001 \pm 0.005 \) for \( t = t_1-t_2 \), and the evolution time is >7 crossing times even at the very last radial density profile point. As we show in Paper II, collisional long-term dynamical evolution of a cluster will remove this feature, bringing the overall appearance of the density profile closer to that of the majority of observed GCs. The central stellar density in the model \( W_0 \) is \( \sim 3 \) orders of magnitude larger than the central density of DM in the undisturbed Burkert halo and becomes comparable to the density of the undisturbed NFW halo only at very small radii \( r \leq 0.1 \) pc (which is outside of the range of radial distances in Fig. 3a).

In the case of a live DM halo coevolving with the stellar core, the DM density profile is significantly modified within the central area dominated by stars (see Fig. 3). In both NFW and Burkert halos, DM is adiabatically compressed by the potential of the collapsing stellar cluster to form a steeper slope in the DM radial density profile. The slope of the innermost part of the DM density profile is \( \gamma \approx -1.5 \) for the NFW halo and \( \gamma \approx -1.0 \) for the Burkert halo. The lack of resolution does not allow us to check whether the slope stays the same closer to the center of the DM halos.

In the outer parts of the DM halos (beyond the scale radius \( r_s \), which is marked by vertical dotted lines in Fig. 3), the DM density profile becomes steeper than the original slope of \( \gamma = -3 \). This can be explained by the fact that our DM halo models are truncated at a finite radius \( r_{vir} \) and hence are not in equilibrium. It should bear no effect on our results, as in all our models stellar clusters do not extend beyond \( r_s \).

As one can see in Figure 3, the impact of the presence of a live DM halo on the stellar density profile is remarkably small (especially for the Burkert halo). In the presence of DM, the central stellar density becomes somewhat larger (by \( \sim 60\% \) and \( \sim 10\% \) for the NFW and Burkert halos, respectively; see Table 3).

The most interesting effect is observed in the outer parts of the stellar density profiles, where the slope of the profiles becomes significantly larger than that of a purely stellar cluster, which is similar to the result of Peebles (1984). The density profile starts deviating from the profile for the model \( W_0 \) somewhere between the radius \( r_p \) and the radius \( r_m \) (where the enclosed masses for stars and DM become equal; see Table 3).

There is a simple explanation for the steepening of the stellar density profile seen in Figure 3 around the radius \( r_m \). In the absence of DM, warm collapse of a homogeneous stellar sphere (our model \( W_0 \)) is violent enough to eject a number of stars into almost-radial orbits, forming a halo of the relaxed cluster. In the case when the DM halo is present, the dynamics does not change much near the center of the cluster, where stars are the dominant mass component. Around the radius \( r_m \) the enclosed DM mass becomes comparable to that of stars. As a result, gravitational deceleration, experienced by stars ejected beyond \( r_m \), more than doubles, resulting in a significantly smaller number of stars populating the outer stellar halo at \( r > r_m \). This should lead to a significantly steeper radial density profile beyond \( r_m \), as observed in our \( W_{N,B} \) models.

As in the Peebles (1984) model, in our model the steepening of the stellar density profile is caused by the fact that at large radii the gravitational potential is dominated by DM. The principal difference between the two models is that in Peebles (1984) the stellar cluster is assumed to be isothermal and in equilibrium; our clusters are not in equilibrium initially, and their final equilibrium configuration is not isothermal (stars are dynamically colder in the outskirts of the cluster).

To allow a direct comparison of the model results with observed GCs, in Figure 4 we plot both the surface brightness (\( \Sigma_{\nu} \)) profile (Fig. 4a) and the line-of-sight velocity dispersion (\( \sigma_{\nu} \)) profile (Fig. 4b) for the warm collapse models. The surface brightness \( \Sigma_{\nu} \) in units of mag arcsec\(^{-2} \) is calculated using the formula

\[
\Sigma_{\nu} = V_0 - 2.5 \log (\zeta/\Gamma_{GC}) + 2 - 2 \log (3600 \times 180/\pi) \, .
\]

(17)

Here \( V_0 = 4.87 \) mag is the absolute \( V \)-band magnitude of the Sun, \( \zeta \) is the projected surface mass density in units of \( M_\odot \) pc\(^{-2} \), and \( \Gamma_{GC} \) is the assumed \( V \)-band mass-to-light ratio for baryons in GCs. For \( \Gamma_{GC} \) we adopt the observationally derived averaged value of 1.45 from McLaughlin (1999), which is also between the two stellar synthesis model values (1.36 for the Salpeter and 1.56 for the composite [with a zero slope for stellar masses \( < 0.3 M_\odot \)] initial mass functions) of Mateo et al. (1998).

As can be seen in Figure 4, the presence of a DM halo introduces an outer cutoff in the surface brightness radial profile of the stellar cluster, which makes the profile look very similar to a King model profile. Interestingly, a stellar cluster evolving in a live NFW DM halo acquires even steeper outer density and brightness profiles than for the case of a static DM potential (see Figs. 3a and 4a).

Analysis of the line-of-sight velocity dispersion profiles for warm collapse models (Fig. 4b) shows that the presence of a DM halo slightly inflates the value of \( \sigma_{\nu} \) in the core of the cluster (by \( \approx 20\% \); see Table 3). The outer dispersion profile either stays unchanged (as in the case of the Burkert halo) or becomes slightly steeper (for the NFW model). The static DM models present profiles of an intermediate type.

A slight increase of \( \sigma_{\nu} \) in the central part of the cluster can be misinterpreted observationally as a GC with no DM that has a slightly larger value of the baryonic mass-to-light ratio \( \Gamma \). To quantify this effect, we apply a core-fitting (or King’s) method of finding the central value of \( \Gamma \) in spherical stellar systems (Richstone & Tremaine 1986):

\[
\Gamma = \frac{9 \sigma_{\nu}^2}{2 \pi G I_0 R_{sub}} \, .
\]

(18)

Here \( I_0 \) is the central surface brightness in physical units. We assume here that \( I_0 = \zeta_0/\Gamma_{GC} \), where \( \zeta_0 \) is the projected surface brightness.
mass density at the center of the model cluster. The core-fitting method assumes that the mass-to-light ratio is independent of radius, which is obviously not true for our stars + DM models. In our models the stellar particles have the same mass, so there is no radial mass segregation between high- and low-mass stars caused by the long-term dynamical evolution of the cluster. As a result, equation (18) should not be used to compare our models with real GCs. Instead, we use it to check whether there is a change in the apparent mass-to-light ratio $\Upsilon$ between our purely stellar models and the models containing DM. We list the values of $\Upsilon$ for different models in Table 3. As one can see, in the case of the NFW halo (model $W_N$) the presence of the DM halo leads to a $\sim$25% increase in the value of the apparent mass-to-light ratio. Such a small increase is well within the observed dispersion of $\Upsilon$ values for Galactic GCs (Pryor & Meylan 1993). In the case of the Burkert halo, the apparent central mass-to-light ratio is only $\sim$6% larger than for the purely stellar model.

3.3. Cold Collapse

A principal difference between warm and cold collapse models from MS04 is that in the cold collapse case a significant fraction of stars becomes unbound after the initial relaxation phase (see Fig. 1). In our model $C_0$ with the mass parameter $\beta = 1.4$, the fraction of the stars lost is 30%. The radial density profile for the $C_0$ model is similar to that of the $W_0$ model (see Fig. 5). One can intuitively expect the escapers from the model $C_0$ to be trapped inside our models containing DM, forming a distinctive feature in the outer density and velocity dispersion profiles.

As can be seen in Figures 5 and 6, our cold collapse models do show such features. Both density and surface brightness profiles become more shallow in the outer parts of the stellar clusters, where DM dominates stars. This is valid for both NFW and Burkert halos. For the NFW and Burkert cases, the slope of the outer stellar density profile is $\gamma = -2.6$ and $-2.2$, respectively. For the purely stellar case (model $C_0$) the slope is $\gamma = -3.8$, so the relative change in the slope is $\Delta \gamma = 1.2$ for the NFW and 1.6 for the Burkert case. This behavior is mimicked by the corresponding surface brightness profiles (Fig. 6a). It is interesting to note that for the $C_N$ model the radial $\Sigma_r$ profile exhibits a significant steepening of the slope in the outermost parts of the cluster, creating the appearance of a tidal cutoff (similarly to the warm collapse cases).

Even more pronounced are features in the line-of-sight velocity dispersion profiles (Fig. 6b). In both the NFW and Burkert cases, there is a plateau in the radial $\sigma_r$ profiles around the radius $r_{\text{fr}}$, where DM becomes the dominant mass component. The apparent break in the surface brightness profile and accompanying flattening of the radial $\sigma_r$ profile seen in our models containing DM can be misinterpreted observationally as the presence of extratidal stars heated by the tidal field of the host galaxy.

Similarly to warm collapse models, the apparent mass-to-light ratio $\Upsilon$ for cold collapse models in the presence of a live DM halo is very close to the purely stellar case $C_0$ (see Table 3). Interestingly, the half-mass radii for warm and cold collapse models with DM are almost identical. This is consistent with the properties of the observed GCs, which have comparable half-mass radii for a wide range of cluster masses.

DM density profiles for the cold collapse cases (Fig. 5) exhibit a similar behavior to the warm collapse models in the star-dominated central region. For both NFW and Burkert halos, the innermost slopes of the density profiles become steeper ($\gamma \approx -1.8$ and $-0.6$, respectively) in the presence of a stellar core. The NFW DM density profile shows a break around the radius $r_{\text{fr}}$.

As one can see in Table 3 and Figure 5, the final stellar cluster half-mass radius is smaller than the DM softening length $r_{\text{fr}} \sim 5$ pc. We reran models $C_N$ and $C_B$ with a much smaller value of $r_{\text{fr}} = 1$ pc to test the possibility that our results were influenced by the fact that DM is not resolved on the stellar cluster scale. For both $C_N$ and $C_B$, the relaxed stellar profiles are found to be virtually identical to the cases with larger $r_{\text{fr}}$. In particular, a “kink” seen in Figures 5a and 5b around the radius...
DM is also present at the same location in the simulations with much smaller value of $\epsilon_{DM}$ and is definitely not a numerical artifact. In the case of the NFW halo, the central stellar velocity dispersion becomes slightly larger, which results in somewhat larger value of $\gamma = 1.88$. This could be in part because of artificial mass segregation, which should be more pronounced in the case of $\epsilon_{DM}$ being significantly lower than the optimal value (in our cold collapse models, DM particles are ~20 times more massive than stellar particles). As a result, the actual $\gamma$ value could be even closer to the baryonic value. In the case of Burkert halo with $\epsilon_{DM} = 1$ pc, the core mass-to-light ratio $\gamma = 1.45$, which is identical to the case of no DM. We conclude that our choice of $\epsilon_{DM}$ did not affect the main results presented in this section.

3.4. Hot Collapse

It is generally assumed that in an equilibrium star-forming cloud no bound stellar cluster will be formed after stellar winds...
and supernova explosions expel the remaining gas if the star formation efficiency is less than 50% (which corresponds to the initial virial parameter for the stellar cluster \( \nu > 2 \)). In MS04 we showed that this is not true for initially homogeneous stellar clusters that have a Maxwellian distribution of stellar velocities. In such clusters with the initial virial parameter as large as 2.9 (corresponding to the mass parameter \( \beta \) as low as \(-0.7\)), a bound cluster containing less than 100% of the total mass is formed by the slowest moving stars. (A similar conclusion was reached by Boily & Kroupa [2003] for initially polytropic stellar spheres.)

In our model H0 (\( \beta = -0.6, \nu = 2.5 \)), \( \sim 57\% \) of all stars stay gravitationally bound, but less than 50% (which corresponds to the initial virial parameter for the stellar cluster \( \nu > 2 \)). In MS04 we showed that this is not true for initially homogeneous stellar clusters that have a Maxwellian distribution of stellar velocities. In such clusters with the initial virial parameter as large as 2.9 (corresponding to the mass parameter \( \beta \) as low as \(-0.7\)), a bound cluster containing less than 100% of the total mass is formed by the slowest moving stars. (A similar conclusion was reached by Boily & Kroupa [2003] for initially polytropic stellar spheres.)

In our model H0 (\( \beta = -0.6, \nu = 2.5 \)), \( \sim 57\% \) of all stars stay gravitationally bound, but less than 50% (which corresponds to the initial virial parameter for the stellar cluster \( \nu > 2 \)). In MS04 we showed that this is not true for initially homogeneous stellar clusters that have a Maxwellian distribution of stellar velocities. In such clusters with the initial virial parameter as large as 2.9 (corresponding to the mass parameter \( \beta \) as low as \(-0.7\)), a bound cluster containing less than 100% of the total mass is formed by the slowest moving stars. (A similar conclusion was reached by Boily & Kroupa [2003] for initially polytropic stellar spheres.)

In the case of the NFW halo, the central stellar density and central velocity dispersion become larger in the presence of DM by factors of 35 and 3, respectively. In the Burkert halo case, the increase is not as dramatic but still significant: the central density and the velocity dispersion become 7 and 2 times larger than those in the absence of DM, respectively. Interestingly, as in the cold collapse cases, the presence of DM brings the half-mass radius of the stellar core in the HN, B models closer to the half-mass radius of the warm collapse models (see Table 3). The apparent central mass-to-light ratio \( \Upsilon \) is noticeably inflated by the presence of DM (especially for the case of NFW halo) by 76% and 36% for NFW and Burkert models, respectively.

As can be seen in Figure 7a, in the model HN stars have a comparable density to DM in the core and become less dense than DM outside of the core radius \( r_0 \). The situation is different (and more in line with warm and cold collapse models) for the HB model (Fig. 7b), where stars are the dominant (although not by a large margin) mass component in the core. There are no obvious features in the density and surface brightness profiles caused by the presence of DM: the profiles look very similar to those of the W0 and \( \Theta \) models, which do not have DM.

Similarly to the warm and cold collapse cases, the presence of a stellar cluster makes DM denser in the central area. In the case of the NFW halo, the innermost slope of the DM density profile is comparable to the slope \( \gamma = -1 \) for the undisturbed halo. For the Burkert halo, the slope becomes steeper, reaching the value \( \gamma \sim -0.4 \) at the innermost resolved point.

The most interesting behavior is exhibited by the line-of-sight velocity dispersion profiles (Fig. 8b). In the presence of DM, the profiles become remarkably flat, changing by a mere 0.2–0.3 dex over the whole range of radial distances (out to \( \sim 15 \) apparent half-mass radii). The combination of the low mass, low central surface brightness, large apparent mass-to-light ratio, and almost flat velocity dispersion profiles seen in...
models H_{N, B} can be mistakenly interpreted as a GC observed at the final stage of its disruption by the tidal forces of the host galaxy.

4. DISCUSSION AND CONCLUSIONS

A significant hindrance to a wider acceptance of the primordial scenarios for GC origin is an apparent absence of DM in Galactic GCs. Many observational facts have been suggested as evidence for GCs having no DM, including the presence of such features in the outer parts of the GC density profiles as apparent tidal cutoffs or breaks, relatively low values of the apparent central mass-to-light ratio $C_7$ that are consistent with purely baryonic clusters, the flat radial distribution of the line-of-sight velocity dispersion in the outskirts of GCs believed to be a sign of tidal heating, and the nonspherical shape of the clusters in their outer parts.

Here we present the results of simulations of stellar clusters relaxing inside live DM minihalos in the early universe ($z = 7$). We study three distinctly different cases that can correspond to very different gasdynamic processes forming a GC: a mild warm collapse, a violent cold collapse resulting in a much denser cluster with a significant fraction of stars escaping the GC in the absence of DM, and a hot collapse resulting in a lower density cluster with many would-be escapers. We show that GCs forming in DM minihalos exhibit the same properties as one would expect from the action of the tidal field of the host galaxy on a purely stellar cluster: King-like radial density cutoffs (for the case of a warm collapse) and breaks in the outer parts of the density profile accompanied by a plateau in the velocity dispersion profile (for a cold collapse). In addition, the apparent mass-to-light ratio for our clusters with DM is generally close to the case of a purely stellar cluster. (The special case of a hot collapse inside a DM halo, which produces inflated values of $T$, can be mistaken for a cluster being at its last stage of disruption by the tidal forces.)

We argue that the increasingly eccentric isodensity contours observed in the outskirts of some GCs could be created by a stellar cluster relaxing inside a triaxial DM minihalo and not by external tidal fields, as it is usually interpreted. Indeed, cosmological DM halos are known to have noticeably nonspherical shapes; a stellar cluster relaxing inside such a halo would have close to a spherical distribution in its denser part, where the stars dominate DM, and would exhibit isodensity contours of increasingly larger eccentricity in its outskirts, where DM becomes the dominant mass component.

It is also important to remember that few Galactic GCs show clear signs of a tidal cutoff in the outer density profiles (Trager et al. 1995). The tidal features of an opposite nature, breaks in the outer parts of the radial surface brightness profiles in some GCs, are often observed at or below the inferred level of contamination by foreground/background objects and could be, in many cases, an artifact of the background subtraction procedure, which relies heavily on the assumption that the background objects are smoothly distributed across the field of view. A good example is that of the Draco dwarf spheroidal galaxy. Irwin & Hatzidimitriou (1995) used simple nonfiltered stellar counts from photographic plates followed by a background subtraction procedure to show that this galaxy appears to have a relatively small value of its radial density tidal cutoff $r_t = 28 \pm 2.4$ and a substantial population of extratidal stars. Odenkirchen et al. (2001) used a more advanced approach of multicolor filtering of Draco stars from the Sloan Digital Sky Survey images and achieved a much higher signal-to-noise ratio than in Irwin & Hatzidimitriou (1995). New, higher quality results were supposed to make the "extratidal" features of Draco much more visible. Instead, Odenkirchen et al. (2001) demonstrated that Draco’s radial surface brightness profile is very regular down to a very low level (0.003 of the central surface brightness) and suggested a larger value for the King tidal radius of $r_t \simeq 50''$. 

---

Fig. 8.—Radial profiles for hot collapse models of (a) surface brightness $\Sigma_V$ and (b) line-of-sight velocity dispersion $\sigma_v$. Thick lines correspond to NFW cases; thin lines correspond to Burkert cases. Solid lines show profiles for models H$_0$ and H$_B$; long-dashed lines correspond to model H$_N$. Vertical short-dashed lines mark the value of $r_m$ (only the Burkert case is shown, as in the NFW case the enclosed DM mass is larger than that of stars at any radius); vertical dotted lines correspond to $r_s$. [See the electronic edition of the Journal for a color version of this figure.]
We argue that the qualitative results presented in this paper are very general and do not depend much on the fact that we used the MS04 model to set up the initial nonequilibrium stellar core configurations or on the particular values of the model parameters (such as \( \rho_{st}, \sigma_{st}, \) and \( \chi \)). As we discussed in § 3, the appearance of tidal or extratidal features in our warm and cold collapse models is caused by two reasons: (1) at radii \( r > r_m \), the potential is dominated by DM, whereas in the stellar core the potential is dominated by stars from the beginning until the end of simulations; and (2) the collapse is violent enough to eject a fraction of stars beyond the initial stellar cluster radius.

As we showed in § 2.4, for the initial stellar density \( \rho_{st} = 14 M_\odot \text{pc}^{-3} \) (from MS04) the whole physically plausible range of stars-to-DM mass ratios \( \chi \) and GC formation redshifts \( z \) satisfy the above first condition. For warm and cold collapse, this condition can be reexpressed as \( S \equiv m_s/m_{DM}(r_s) > 1 \). One can easily estimate \( S \) for other values of \( \rho_{st} \).

The second condition is more difficult to quantify. In MS04 we showed that collapsing homogeneous isothermal spheres produce extended halos for any values of the initial virial ratio \( \nu \) (except for \( \nu > 2.9 \) systems, which are too hot to form a bound cluster in the absence of DM). Roy & Perez (2004) simulated cold collapse for a wider spectrum of initial cluster configurations, including power-law density profile, clumpy, and rotating clusters. In all their simulations an extended halo is formed after the initial violent relaxation. It appears that in many (probably most) stellar cluster configurations that are not in detailed equilibrium initially, our second condition can be met.

Of course, not every stellar cluster configuration will result in an extended halo after the initial violent relaxation phase. Spitler & Thuan (1972) demonstrated that a warm (\( \nu = 0.5 \)) collapse of a homogeneous isothermal sphere produces clusters with large cores (their models D and G). Adiabatic collapse of homogeneous isothermal spheres produces clusters in which the surface density profiles are very close to those of dynamically young Galactic GCs (MS04). Roy & Perez (2004) concluded that any cold stellar system that does not contain significant inhomogeneities relaxes to a large-core configuration. The results from Roy & Perez (2004) can also be used to estimate the importance of adiabaticity for core formation. Indeed, their initially homogeneous models H and G span a large range of \( \nu \) and include both adiabatic cases (\( \nu > 0.16 \)) for their number of particles \( N = 3 \times 10^8 \), from eq. [16]) and nonadiabatic ones (\( \nu < 0.16 \)). It appears that all their models (adiabatic and nonadiabatic) form a relatively large core. The issue is still open, but it appears that the adiabaticity requirement (our eq. [16]) is not a very important one for our problem (although this requirement is met automatically for real GCs for the values of \( \rho_{st} \) and \( \sigma_{st} \) derived in MS04).

An important point to make is that the simulations presented in this paper describe the collisionless phase of GC formation and evolution and cannot be directly applied to GCs that have experienced significant secular evolution due to encounters between individual stars. In MS04 we showed that such collisionless simulations of purely stellar clusters describe very well the surface brightness profiles of dynamically young Galactic GCs (such as NGC 2419, NGC 5139, IC 4499, Arp 2, and Palomar 3; see Fig. 1 in MS04). In Paper II we will address (among other things) the issue of long-term dynamical evolution of hybrid GCs. We will demonstrate that, at least for the warm-collapse case, secular evolution does not change our qualitative results presented in this paper.

In the light of the results presented in this paper and the above arguments, we argue that additional observational evidence is required to determine with any degree of confidence whether GCs have any DM presently attached to them or whether they are purely stellar systems truncated by the tidal field of the host galaxy. Decisive evidence would be the presence of obvious tidal tails. A beautiful example is Palomar 5 (Odenkirchen et al. 2003), where tidal tails were observed to extend over \( 10^4 \) in the sky.

Even if a GC is proven not to have a significant amount of DM, it does not preclude it having been formed originally inside a DM minihalo. In the semiconsistent simulations by Bromm & Clarke (2002) of dwarf galaxy formation, proto-GCs were observed to form inside DM minihalos, with the DM being lost during the violent relaxation accompanying the formation of the dwarf galaxy. Unfortunately, their simulations did not have enough resolution to clarify the fate of DM in proto-GCs. In Paper II we will address the issue of the fate of DM in hybrid proto-GCs experiencing severe tidal stripping in the potential of the host dwarf galaxy.

We would like to thank Volker Springel for his help with introducing an external static potential in GADGET. S. M. is partially supported by SHARCNet. The simulations reported in this paper were carried out on the McKenzie cluster at the Canadian Institute for Theoretical Astrophysics.

**APPENDIX A**

**DISTRIBUTION FUNCTION FOR BURKERT HALOS**

The dimensionless potential \( \Psi \) of Burkert halos with the density profile given by equations (7) and (8) is

\[
\Psi = 1 - \frac{2}{\pi} \left\{ (1 + R^{-1})(\arctan R - \ln(1 + R)) + \frac{1}{2}(1 - R^{-1})\ln(1 + R^2) \right\}.
\]  

(A1)

Here \( R \equiv r/r_s \) is the radial distance in scale radius units, and \( \Psi \) is in \( \pi^2 G \rho_{0,B} r_s^2 \) units. The potential \( \Psi \) is equal to 1 at the center of the halo and zero at infinity.

The phase-space distribution function for a Burkert halo with an isotropic dispersion tensor can be derived through an Abel transform (Binney & Tremaine 1987, p. 237; we skip the second part of the integrand, which is equal to zero for Burkert halos):

\[
F(E) = \frac{1}{\sqrt{8\pi^2}} \int_0^E d^2 \rho \frac{d\Psi}{d\Psi^2} (E - \Psi)^{1/2}.
\]  

(A2)

Here \( E \) is the relative energy in the same units as \( \Psi \), and \( \rho \) is the density in \( \rho_{0,B} \) units, which results in \( (\pi r_s)^2 \rho_{0,B} G^{3/2} \) units for \( F \).
The function $d^2\rho/d\Psi^2$ for a Burkert halo has the same asymptotic behavior for \( R \to 0 \) and \( R \to \infty \) as the model I from Widrow (2000), which has the density profile
\[
\frac{d^2\rho}{d\Psi^2}(R \to 0) \propto R^{-3}, \quad \frac{d^2\rho}{d\Psi^2}(R \to \infty) \propto \frac{\Psi}{(-\ln \Psi)^3}.
\]
This allowed us to use the analytic fitting formula from Widrow (2000) for the distribution function of Burkert halo,
\[
F(E) = F_0 E^{1/2} (1 - E)^{-1} \left( \frac{-\ln E}{1 - E} \right)^q e^p.
\]
Here \( P = \sum_i p_i E^i \) is a polynomial introduced to improve the fit.

We solve equation (A2) numerically for the interval of radial distances \( R = 10^{-6} \) to \( 10^6 \). We obtained the following values for the fitting coefficients in equation (A4):
\[
\begin{align*}
F_0 &= 5.93859 \times 10^{-4}, & q &= -2.58496, & p_1 &= -0.875187, & p_2 &= 24.4945, & p_3 &= -147.8711, \\
p_4 &= 460.351, & p_5 &= -747.347, & p_6 &= 605.212, & p_7 &= -193.621.
\end{align*}
\]
We checked the accuracy of the fitting formula (eq. [A4]) with the above values of the fitting coefficients by comparing the analytic density profile of the Burkert halo (eq. [7]) with the density profile derived from the distribution function (Binney & Tremaine 1987, p. 236),
\[
\rho(R) = 2\sqrt{8}\pi^2 \int_0^{\Psi} F(E)(\Psi - E)^{1/2} dE.
\]
The deviation of \( \rho(R) \) in equation (A6), with \( F(E) \) given by equations (A4) and (A5), from the analytic density profile was found to be less than 0.7% for the interval of the radial distances \( R = 10^{-6} \) to \( 10^6 \).

APPENDIX B

PROJECTION METHOD

For \( N \)-body models of spherically symmetric stellar systems, such as the GC models presented in the paper, one can produce radial profiles of projected observable quantities (such as surface mass density or velocity dispersion) by averaging these quantities over all possible rotations of the cluster relative to the observer. Radial profiles created in this way preserve a maximum of information, which is very important for studies of the outskirts of the clusters and for clusters simulated with a relatively small number of particles.

Let us consider the \( i \)th stellar particle \( P_i \) in a spherically symmetric cluster, which is located at the distance \( r_i \) from the center of the cluster (Fig. 9). We want to find its averaged contribution to one projection bin, corresponding to the range of projected distances

![Diagram of projection method](image-url)
Projected radial distance of the particle equal to the radial distance of the center of the bin $R$ averaged value of the quantity. The probability that the particle will make no contribution to the projection bin. In our method, every particle with velocity dispersion $\sigma_z$ is transformed into the vector $(V'_x, V'_y, V'_z)$ with the components

$$V'_x = V'_{xz} \sin \psi', \quad V'_y = V_{xy} \sin \varphi', \quad V'_z = V'_{xz} \cos \psi'.$$

The parameters $V'_{xz}, V_{xy}, \psi',$ and $\varphi'$ can be obtained after a series of the following calculations:

$$r \equiv (x^2 + y^2 + z^2)^{1/2}, \quad r_{xy} \equiv (x^2 + y^2)^{1/2}, \quad V_{xy} \equiv (V_x^2 + V_y^2)^{1/2}, \quad \sin \varphi = V_y/V_{xy},$$

$$\cos \varphi = V_x/V_{xy}, \quad \sin \alpha = y/r_{xy}, \quad \cos \alpha = x/r_{xy}, \quad \varphi' \equiv \varphi - \alpha,$$

$$V'_{xz} \equiv (V_{xy}^2 \cos^2 \varphi' + V_z^2)^{1/2}, \quad \sin \psi = V_{xy} \cos \varphi'/V'_{xz}, \quad \cos \psi = V_z/V'_{xz},$$

$$\sin \theta = r_{xy}/r, \quad \cos \theta = z/r, \quad \psi' \equiv \psi - \theta + \arcsin \kappa.$$  

Here $\kappa$ is equal to $R/r$ if $R/r < 1$ and is equal to 1 otherwise; $r, r_{xy}, V_{xy}, \varphi, \alpha, \theta,$ and $\psi$ are intermediate variables.

If the vector $(V'_{xz}, V'_y, V'_z)$ is the velocity vector, then substituting $V''_{xz}$ in place of $Q_i$ in equation (B3) gives us the value of the square of the line-of-sight velocity dispersion $\sigma_z$, averaged over all possible rotations of the cluster, for the bin $R_1 - R_2$. Similarly, replacing $Q_i$ with $V''_x$ and $V''_y$ produces the averaged values of the square of the radial and tangential components of the proper motion dispersion, respectively.

REFERENCES

Aarseth, S. J., Lin, D. N. C., & Papaloizou, J. C. B. 1988, ApJ, 324, 288
Barkana, R., & Loeb, A. 1999, ApJ, 523, 54
Beasley, M. A., Kawata, D., Pearce, F. R., Forbes, D. A., & Gibson, B. K. 2003, ApJ, 596, L187
Becker, R. H., et al. 2001, AJ, 122, 2850
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press)
Boily, C. M., & Kroupa, P. 2003, MNRAS, 338, 665
Bromm, V., & Clarke, C. J. 2002, ApJ, 566, L1
Bullock, J. S., Kolatt, T. S., Sigad, Y., Somerville, R. S., Kravtsov, A. V., Klypin, A. A., Primack, J. R., & Dekel, A. 2001, MNRAS, 321, 559
No. 1, 2005  GLOBULAR CLUSTERS WITH DARK MATTER HALOS. I.  257

Burkert, A. 1995, ApJ, 447, L25
Cen, R. 2001, ApJ, 560, 592
Dinescu, D. I., Girard, T. M., & van Altena, W. F. 1999, AJ, 117, 1792
Hayashi, E., Navarro, J. F., Taylor, J. E., Stadel, J., & Quinn, T. 2003, ApJ, 584, 541
Irwin, M., & Hatzidimitriou, D. 1995, MNRAS, 277, 1354
Kazantzidis, S., Magorrian, J., & Moore, B. 2004, ApJ, 601, 37
Mashchenko, S., & Sills, A. 2004, ApJ, 605, L121 (MS04)
———. 2005, ApJ, 619, 258 (Paper II)
Mateo, M., Olszewski, E. W., Vogt, S. S., & Keane, M. J. 1998, AJ, 116, 2315
McLaughlin, D. E. 1999, AJ, 117, 2398
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Odenkirchen, M., et al. 2001, AJ, 122, 2538
———. 2003, AJ, 126, 2385
Padoan, P., Jimenez, R., & Jones, B. 1997, MNRAS, 285, 711
Peebles, P. J. E. 1984, ApJ, 277, 470
Peebles, P. J. E., & Dicke, R. H. 1968, ApJ, 154, 891
Pryor, C., & Meylan, G. 1993, in ASP Conf. Ser. 50, Structure and Dynamics of Globular Clusters (San Francisco: ASP), 357
Richstone, D. O., & Tremaine, S. 1986, AJ, 92, 72
Ricotti, M., Gnedin, N. Y., & Shull, J. M. 2002, ApJ, 575, 49
Rosenblatt, E. I., Faber, S. M., & Blumenthal, G. R. 1988, ApJ, 330, 191
Roy, F., & Perez, J. 2004, MNRAS, 348, 62
Salucci, P., & Burkert, A. 2000, ApJ, 537, L9
Spergel, D. N., et al. 2003, ApJS, 148, 175
Spitzer, L. J., & Thuan, T. X. 1972, ApJ, 175, 31
Springel, V., Yoshida, N., & White, S. D. M. 2001, NewA, 6, 79
Sternberg, A., McKee, C. F., & Wolfire, M. G. 2002, ApJS, 143, 419
Trager, S. C., King, I. R., & Djorgovski, S. 1995, AJ, 109, 218
Widrow, L. M. 2000, ApJS, 131, 39
Zhao, D. H., Jing, Y. P., Mo, H. J., & Börner, G. 2003a, ApJ, 597, L9
Zhao, D. H., Mo, H. J., Jing, Y. P., & Börner, G. 2003b, MNRAS, 339, 12