Prediction of machining deformation and reasonable design of gear blank for split straight bevel gear

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Abstract
The deformation of gear blank is serious in the machining process of the split straight bevel gear, considering the material and the design of gear blank, the relationship between the change of additional stress and bending deformation of gear blank is studied, and the calculation model of the internal additional stress and additional torque during the gear cutting is established. According to the moment-area method, the calculation formula of the bending deformation of gear blank is derived, and combined with the time-varying stiffness, the mathematical model of the gear blank deformation is obtained. The theoretical calculation, finite element analysis, and experimental results are highly consistent. Based on the above research, the internal relationships between the machining deformation and the geometric parameters such as the thickness, diameter, and gear module of the split gear blank are analyzed, and the reasonable design of the geometric parameters of the split gear blank and the reasonable dividing law of the gear blank are explored.

Keywords Split straight bevel gear · Additional stress · Finite element · Moment-area method

1 Introduction
Oversize straight bevel gear is one of the most important parts in large-scale heavy industrial equipment. It is widely used in mining, power generation, and other important fields. Extra-large bevel gear (wheel, diameter greater than 3000 mm) requires higher forming method and machining equipment, and its manufacturing cost is high, manufacturing cycle is long, and the machining is very difficult [1–4]. The structural stiffness of entire oversize straight bevel gear is small, and it is easy to deform during machining and assembly, and the machining is often limited by the size of the equipment; it is also extremely difficult to transport the machined gears. Therefore, the existing extra-large straight bevel gear mostly uses a split structure [5].

According to the engineering practice and the existing data of research group, the bending deformation of the split straight bevel gear is the most obvious after machining, which can reach several millimeters usually [5], so the structural deformation is the main factor affecting the accuracy. The deformation of split gear blank mainly lies in two aspects: one is the deformation created by the stiffness change and internal stress caused by material removal, which accounts for 92.2% of the total deformation [6–8], and the other is the structural deformation caused by surface stress, only account for 7.8% [9]. The deformation caused by the change of stiffness and internal stress is closely related to the size and cross-section design of the split gear blank.

However, in view of the process characteristics, such as dividing the gear blank first, then cutting the teeth, and large deformation, engineers are lack of effective theoretical
support on how to design and divide the wheel blank reasonably. In the existing manufacturing practice, there is a great blindness and randomness, which leads to a series of problems such as low manufacturing accuracy, long-term, difficult in assembly, and high manufacturing cost. Therefore, on the premise of considering the material and design of gear blank, building the calculation model of gear cutting deformation of split gear blank, exploring the reasonable design of geometric parameters of split gear blank, and studying the reasonable dividing law of the gear blank should be the basic problems that must be faced in the machining of split extra-large bevel gears.

According to the existing document and related reports, all the research on straight bevel gear is based on the entire gear blank, and mainly focuses on the tooth surface forming principle, machining error, tooth surface contact analysis, tooth surface correction and installation error [1–4], etc. In the research of split oversize gear, the main research object is large cylindrical gear, and mainly focuses on the machining process optimization, deformation control method, and deformation mechanism of the split cylindrical gear. However, there are few reports on the research of split straight bevel gear.

In the research of machining deformation of large-scale parts, the existing research mainly focuses on the numerical control machining deformation, deformation mechanism, and deformation prediction of aviation entire structure components, frame-shaped components, and ring form thin-walled components. Qin et al. [10] present an analysis and experimental study on the formation and distribution of machined surface residual stress in pre-stress cutting. Zhu et al. [11] using finite difference method studied the influence of the initial residual stress on the dimensional stability of manufactured components. Based on theoretical calculation and finite element simulation, Sun and Ke [12] studied the influence of the initial internal stress on the machining deformation of large entire components. Wu [13] explored the machining deformation principle of thin-walled components from the machining principle and finite element simulation. The finite element analysis model of blade deformation prediction considering the influence of residual stress is established [14], which provides an effective analysis method for deformation prediction. Lu [15] comprehensively considered the effects of initial internal stress, machining stress, temperature change, and clamping stress on the deformation of frame-shaped integral components, and established a deformation prediction model. Zhou [16] established a finite element model to predict the deformation of ring thin-walled components, analyzed the influence of machining stress on machining accuracy, and predicted the relationship between the amount of material removed and machining error. Huang [17] used theoretical calculation to analyze the stiffness change, internal stress distribution, and deformation during machining, and established the deformation prediction model of integral components. Nervi [18] established a machining deformation prediction model of components with initial internal stress, and analyzed the relationship between the distribution of initial internal stress and the deformation. Keith [19] studied the influence of machining stress on the machining deformation of components. Ratchev et al. [20] established a mathematical model to predict the machining deformation of thin-walled components on the basis of neural network, and predicted the machining error.

Based on the above analysis, this paper intends to study from the following aspects. Firstly, the deformation and additional stress of the gear blank in gear machining are analyzed, and the calculation model of the internal additional moment of the residual material during gear cutting is established, and the calculation formula of bending deformation of the single tooth space is deduced. Secondly, according to the cumulative effect of the gear blank deformation, combined with the stiffness change, the mathematical model of the deformation is deduced, and the simulation and calculation model are verified by experiments. Then, based on the above research results, the internal relations between the machining deformation and the geometric parameters such as the thickness, diameter, and module of the gear blank are studied, and the reasonable design and dividing rules of the gear blank are preliminarily explored.

2 Analysis and calculation of the gear blank deformation

2.1 Deformation and additional moment analysis

For extra-large straight bevel gears, the root angle of the gear is very large, so the cross-section of the gear blank is approximately rectangular. Due to large diameter and small curvature of the gear blank, the entire gear blank is split into several segments, and each segment is similar to a cuboid. The length direction of the separated gear blank is set as X direction, the transverse direction is set as Y direction, and the positive direction is from the outer to the inner. Along the thickness direction of gear blank is Z-axis, and its positive direction is vertical upward.

In this study, aluminum alloy 7075-T7451 is selected as the experiment material, and the distribution of initial residual stress has been obtained according to the actual measurement. In order to apply the initial stress of the gear blank to its finite element model, according to the known distribution law of the initial residual stress, the discrete data are fitted by Fourier function, and the relationship between the initial residual stress in X and Y directions and the gear blank thickness is expressed by functions, respectively. Then, the
function expression obtained by fitting is used to program SIGINI subroutine in Visual Studio with FORTRAN language; the finite element software will call SIGINI subroutine in the calculation process to realize the automatic and continuous application of the initial residual stress. So far, the finite element model of the split gear blank can be obtained.

The essence of machining deformation of split gear blank is that the material removal in the tooth space leads to the disappearance of the internal stress contained in that part of the material, which leads to the deformation of the workpiece. Therefore, the negative additional stress can be applied on the left and right sides of the tooth space, which can be used to equivalent the effect of gear cutting on the gear blank deformation, and then the torque produced by material removal and the deformation on the gear blank can be obtained. Figure 1 is the deformation nephogram of the gear blank simulation machining. From the simulation, it can be seen that the maximum deformation is near the machining area, and the deformation of the unprocessed area is smaller. Moreover, the farther away from the cutting position, the smaller the deformation is. The deformation is a cumulative superposition result with the machining. After machined, the entire gear blank is in bending deformation state, and the deformation is symmetrical about the middle position of the gear blank length direction.

The results of finite element analysis show that the stress along the length direction of the gear blank changes most during the machining. Therefore, this paper takes the gear blank in the case of machining a single tooth space as the analysis object, and studies the additional torque and the deformation in its length direction. After first tooth space is machined, its cross-section along the X direction is nearly U-shaped, and the dimensions of the tooth space and gear blank can be determined according to the equivalent gear at the midpoint of the tooth length. The coordinate system and additional stress distribution are shown in Fig. 2. After the material in the tooth space is cut off, the residual stress is removed with the material, and the original support stress on both sides of the tooth space no longer exists. In this case, the stress $-\sigma$ is applied to the side wall of the tooth space to achieve the result of force equivalence.

In Fig. 2, $z_c$ indicates the position of neutral axis; the centroid of the section is on the neutral axis. The centroid position of the section changes constantly during the gear cutting process. When each layer of material is removed from the tooth space, different additional stress will be generated. The additional stress is superimposed with the initial residual stress in the material, resulting in the bending deformation of the gear blank. According to the curvature calculation formula in material mechanics, combined with the bending additional stress distribution during gear machining, the curvature relation, additional stress expression, and residual stress balance expression before and after gear machining can be deduced in turn; finally, the expression of the additional torque generated by the redistributed internal stress $\sigma_{11}$ on the instantaneous neutral axis is obtained:

$$M = \int_0^{z^1} \sigma_{11}(z)(z - z_c)x_2dz$$  \hspace{1cm} (1)$$

2.2 Bending deformation in machining single tooth space

In order to calculate the bending deformation of gear blank after machined a tooth space, according to the additional torque analysis, combined with the machining of single tooth space, the internal stress $\sigma_{11}$ of each layer is calculated firstly, thereafter, the dimension parameters of gear blank are substituted into Eq. (1); the additional torque of the gear blank in machining is obtained:

$$M = \int_0^{z^1} \sigma_{11}(z)(h - z_c - z)[x_3 - (x_3 - x_2)(z_c/2.25m)]dz$$ \hspace{1cm} (2)$$

where $x_1 = (L - x_3)/2$ and $m$ indicates the module of the gear.
Thereafter, substituting additional moment into the deflection calculation formula, the deformation of the gear blank after machining a single tooth space can be obtained:

$$S_1 = \int_{x_1}^{x_1+x_3} (L - x)M/(K_{11}L)dx$$  \hspace{1cm} (3)$$

where $K_{11}$ indicates the stiffness of the gear blank when machining the tooth space in the middle position of the gear blank.

In the machining simulation of single tooth space, the material in the tooth space is removed by layers, and the thickness of each layer is set to be 9 mm; the deformation nephogram is shown in Fig. 3.

According to Eq. (3), the machining deformation is calculated, and then the relationship between the gear blank deformation and the material removal thickness is drawn in the two cases of calculation method and finite element analysis. The results are illustrated in Fig. 4. It can be seen from Fig. 4 that the deformation results obtained by the two methods are basically consistent in the machining process of a single tooth space.

### 2.3 Overall bending deformation calculation of gear blank

After the first tooth space is machined, the deformation diagram of the gear blank is illustrated in Fig. 5, at this time, the angle between the bottom of the left and right sides surface and the horizontal line is $\theta_{12}$ and $\theta_{11}$, respectively. According to the previous theoretical calculation, the local maximum deformation $S_1$ can be calculated.

Here, it is set to continue cutting teeth to the right side of gear blank, and the second tooth space is machined alone; the local bending deformation of the gear blank is $S_2$, which causes the deflection angles of the right and left ends of the bottom surface to be $\theta_{21}$ and $\theta_{22}$. As illustrated in Fig. 6, coordinate system $(o_1 - x_1, z_1)$ is set up at the middle position of the split gear blank, and the $z_1$ axis is the symmetry axis of the tooth space in the middle position. The coordinate system $(o_2 - x_2, z_2)$ is developed at the position of the symmetry axis of the second tooth space; this coordinate system is obtained by translating coordinate system $(o_1 - x_1, z_1)$ firstly.
The deformation of machining the fifth tooth space is set to \( \theta_1 + \theta_2 + \theta_3 \), and then rotating the coordinate system \( \theta_1 \) degrees counterclockwise.

After the first two tooth spaces are machined, the angle between the bottom surface at the right end of the gear blank and the horizontal line is \( \theta_{11} + \theta_{21} \), the deflection angle of the left side is \( \theta_{12} + \theta_{22} \), and the deformation after superposition is \( S_x = S_x + S_y \cos \theta_{21} \). Here, the local deformation is set to \( S_x \) when machining the third tooth space, and the deflection angles of the left end and the right end are set to \( \theta_{32} \) and \( \theta_{31} \), respectively. After the right of the gear blank is machined, the deformation of the gear blank is shown in Fig. 7. This time, at the left end of the gear blank, the angle between the bottom tangent and the horizontal line is \( \theta_{32} = \theta_{12} + \theta_{22} + \theta_{32} \). At the right, the angle between the tangent line of the bottom surface and the horizontal line is \( \theta_{31} = \theta_{11} + \theta_{21} + \theta_{31} \), and the deformation after superposition is \( S_x = S_x + S_y \cos \theta_{31} \).

After the fourth tooth space, the deformation diagram of the gear blank is illustrated in Fig. 8. Here, the local deformation is set to \( S_x \), and the deflection angles of the left end and the right end are set to \( \theta_{42} \) and \( \theta_{41} \).

After deformation superposition, at the right end of the gear blank, the angle between the bottom tangent line of the bottom surface and the horizontal line is \( \theta_{41} = \theta_{31} + \theta_{41} \). At the left, the angle between the bottom tangent line and the horizontal line is \( \theta_{42} = \theta_{32} + \theta_{42} \), and the deformation after superposition is \( S_x = S_x + S_y \cos \theta_{42} \). Here, the local deformation of machining the fifth tooth space is set to \( S_x \), and the deflection angles of the left end and the right end are set to \( \theta_{52} \) and \( \theta_{51} \). The overall bending deformation diagram is illustrated in Fig. 9, the angles between the tangent line and the horizontal line of the bottom surface at the left and right ends of the gear blank are \( \theta_{52} = \theta_{42} + \theta_{52} \) and \( \theta_{51} = \theta_{41} + \theta_{51} \), and the total deformation is \( S_x = S_x + S_y \cos \theta_{52} \).

According to the calculation method of equivalent stiffness of the split gear blank, the stiffness at different positions can be obtained, which is set as \( K_{11}, K_{12}, K_{13}, K_{14}, K_{15} \), respectively, according to the above machining sequence. Substituting the stiffness into Eq. (3) and adjusting the integral interval, the deformation of the corresponding tooth space at different positions can be calculated. The deformation \( S_x \) has been calculated in the previous section. The deformation of machining the second tooth space separately is:

\[
S_x = \int_{x_1+x_3}^{x_1+x_3+x_4} (L - x)M/(K_{12}L)dx
\]

(4)

The deformation of machining the third tooth space separately is:

\[
S_x = \int_{x_1+x_3}^{x_1+x_3+x_4} (L - x)M/(K_{13}L)dx
\]

(5)

The deformation of machining the forth tooth space separately is:

\[
S_x = \int_{x_1-x_3}^{x_1-x_3-x_4} xM/(K_{14}L)dx
\]

(6)

The deformation of machining the fifth tooth space separately is:
In order to verify the above calculation, a gear with 20 module, 60 tooth numbers, 100 mm face width, 89° reference cone angle, 45 mm tooth depth, and 80 mm gear blank thickness is taken as an example. 1/10 of the circumference of the entire gear is taken as the split gear blank to calculate the bending deformation. The experimental material is aluminum alloy 7075-T7451. Then, the deformation and deflection angle are calculated, respectively; the results are shown in Table 1.

As indicated in Table 1, after the deformation superposition, the angle between the bottom tangent and the horizontal line at the right end is \( \theta_{12} = 0.020^\circ \), the angle at the left end is \( \theta_{22} = 0.006^\circ \), and the deformation after superposition is \( S_5 = 0.1301 \) mm. Then, the finite element simulation is carried out, and the simulation results show that the average deformation of the maximum deformation area is 0.1304 mm. Comparing the calculation results with the simulation results, it can be seen that the bending deformation obtained by the calculation is basically consistent with the simulation.

### Table 1  Deformation of gear blank when machining different tooth space

| Deformation of gear blank (mm) | Angle between bottom of right end and horizontal line | Angle between left end bottom and horizontal line |
|-------------------------------|------------------------------------------------------|---------------------------------------------------|
| \( S_1 = 0.0583 \)          | \( \theta_{11} = 0.020^\circ \)                      | \( \theta_{12} = 0.020^\circ \)                    |
| \( S_2 = 0.0249 \)          | \( \theta_{21} = 0.013^\circ \)                      | \( \theta_{22} = 0.006^\circ \)                    |
| \( S_3 = 0.0104 \)          | \( \theta_{31} = 0.011^\circ \)                      | \( \theta_{32} = 0.002^\circ \)                    |
| \( S_4 = 0.0251 \)          | \( \theta_{41} = 0.006^\circ \)                      | \( \theta_{42} = 0.013^\circ \)                    |
| \( S_5 = 0.0103 \)          | \( \theta_{51} = 0.002^\circ \)                      | \( \theta_{52} = 0.011^\circ \)                    |

\[
S_5 = \int_{x_1 - 2x_4}^{x_1 - x_3 - 2x_4} \frac{xM}{(K_1L)}dx \tag{7}
\]

2.4 Experiment of machining and deformation measurement

The material and size of the experimental gear blank are the same as simulation. The inner diameter is 1000 mm, the outer diameter is 1200 mm, and the design angle between the left and right ends surface of the gear blank is 36°.

Experimental design: firstly, the split gear blank is processed, then the initial measurement of the gear blank is carried out on the coordinate measuring machine; secondly, the first tooth space in the middle is machined, thereafter, the second measurement is carried out; thirdly,
the other right tooth spaces are machined at one time, and the third measurement is carried out; after that, the tooth space in the left half were machined at one time, and fourth measurements are made.

Measurement items: bottom surface flatness, perpendicularity of left and right end surfaces, inner and outer circle diameter, angle between left and right end surfaces. Figure 10 is the first measurement before machining tooth space after the split gear blank is ready. Figure 11 is the machining of the first tooth space, and Fig. 12 is the second measurement.

Figure 13 shows the remaining tooth spaces machining on the right side, and Fig. 14 is the third measurement. Figure 15 shows the remaining tooth spaces machining on the left side, and Fig. 16 is the fourth measurement.

The measurement results are illustrated in Table 2. It can be seen from the data that when the gear blank was first measured, the gear cutting process has not been done, so the measurement data 2 is small. However, with the continuous machining, the shape and position tolerance and inner and outer diameter of the gear blank have changed significantly. The angle between the left and right end surfaces of the gear blank decreases continuously in the process of machining, and the angle after machining is 0.02° less than that before machining, which indicates that the opening deformation of the gear blank occurs during the machining process; this deformation makes the wheel blank tend to be less curvature. The inner and
outer diameter increases about 0.070 mm after machining, which further indicates that the above deformation occurred. The perpendicularity tolerance of the left and right sides relative to the bottom surface has been increasing slightly, the bottom surface flatness also keeps increasing during the processing, and the flatness tolerance is much larger than the perpendicularity tolerance.

### 3 Reasonable design of split gear blank

The previous research and engineering practice show that the deformation of the split straight bevel gear is related to the thickness, length, width, and the size of the tooth space of the split gear blank. When the combination of the above factors is different, the deformation is different, and there are many ways of the combination. Therefore, it is difficult to get the deformation law through the machining practice, and then guide the gear blank design. This should be the main reason for the lack of relevant theoretical results for many years.

The theoretical research and experiments in the early stage of this research show that the deformation calculation method and finite element analysis model for the split straight bevel gear are correct and credible, which provides a feasible way for reasonable design of the split gear blank. In the previous study, it was found that the thickness and size had the greatest influence on the deformation of the gear blank, so in the next part of the paper, we will try to start from the two aspects to preliminarily explore the reasonable design of the split wheel blank.

#### 3.1 Thickness design of split gear blank

In this section, we will explore the internal relationship between the thickness change and Z direction deformation of split gear blank. The teeth number in the entire circle of the analysis model is 90, and the teeth number in the split gear blank is 6. The thickness (excluding the height of the tooth space) of the gear blank is gradually increased from 1.2 times of the tooth depth to 3.0 times. The gear modules are 20, 25, 30, 36, and 40, respectively. Different modules, different gear blank thicknesses are selected, and then finite element analysis is carried out. For space limitation, Fig. 17 shows only the displacement nephogram of module 20, and the thickness is 1.3 times, 1.6 times, 2.0 times, and 3.0 times of the tooth depth, respectively.

Table 3 shows that the bending deformation in other cases, $m$ is the gear module, $f$ is the maximum deformation, and $t$ is the ratio between the thickness and tooth depth.

After fitting the results in above table, the deformation curves with different modules and thickness can be obtained, and the change trend is shown in Fig. 18.

It can be seen from Fig. 18 that when the number of teeth of the split gear blank is constant, the deformation increases first and then decreases during the gear blank thickness increasing gradually, and the change trend with different modules is the same. When the ratio of the gear

| Bottom surface flatness (mm) | Perpendicularity of left end surface (mm) | Perpendicularity of right end face (mm) | Outer diameter (mm) | Inner diameter (mm) | Angle between two ends |
|-----------------------------|------------------------------------------|----------------------------------------|---------------------|---------------------|-----------------------|
| The 1st measurement         | 0.038                                    | 0.016                                  | 1200.001            | 1000.001            | 36.009°               |
| The 2nd measurement         | 0.077                                    | 0.019                                  | 1200.028            | 1000.002            | 36.005°               |
| The 3rd measurement         | 0.121                                    | 0.025                                  | 1200.049            | 1000.049            | 35.996°               |
| The 4th measurement         | 0.161                                    | 0.028                                  | 1200.072            | 1000.071            | 35.989°               |

Fig. 17 Displacement nephogram of the gear blank in Z direction
blank thickness to the tooth depth is constant, the larger the module is, the greater the deformation is. When the deformation reaches peak value, the deformation decreases rapidly with the increase of the thickness, and then the reduction speed tends to be slow. When the thickness is about 1.325 times of the tooth depth, the deformation reaches the peak value; when the ratio is 1.45 times, the speed of deformation decrease begins to slow down; and when the ratio is 1.8 times, the speed of the gear deformation decrease begins to stabilize.

To sum up, in order to control the deformation of split gear blank, the thickness of about 1.325 times tooth depth should be avoided in the design of gear blank. Considering the deformation control and manufacturing cost, the thickness of 1.8 times tooth depth is more appropriate.

### 3.2 Size design of split gear blank

#### 3.2.1 Size design with fixed module

The module of the analysis model is 40 mm, the thickness is 1.8 times of the tooth depth, and the number of teeth in the entire circle is changed from 70 to 110, which will lead to the change of gear diameter. For gears with different diameters, different gear blank divided ratios are selected, respectively, and then machining simulation is carried out. Due to the limited space, Fig. 19 only shows the displacement nephogram when the teeth number is 80 and the teeth number ratio of the split gear blank to the entire gear ring is 3/20, 1/10, and 1/20, respectively.

The simulation machining deformation values of different gear blank diameter and different split block size are illustrated in Tables 4, 5, 6, 7, and 8, respectively.

By analyzing the data in the table above, it can be seen that when the module is constant, the smaller the teeth number ratio is, the smaller the deformation is. When changing the diameter of the gear and the teeth number ratio of the split gear blank, as long as the teeth numbers are the same, the deformation are basically the same. The relationship between the deformation of the split gear blank and teeth number contained in it is drawn into a line graph, as shown in Fig. 20. It can be seen from the graph that when the teeth number is less than 8, the deformation increases slowly with the increase of the teeth number and when the teeth number continues to increase, the deformation increases faster and faster.

Obviously, the shorter the split gear blank is, the smaller the deformation is; however, the split block is too small, the assembly is more difficult. Considering comprehensively, the teeth number contained in each split block should be between 5 and 8, which is more suitable.

#### 3.3 Size design with fixed diameter

The diameter of the model gear is 3600 mm, and gear module changes from 36 to 50. For gears with different module, different teeth number ratio is selected to simulate machining. Due to the limited space, Fig. 21 only shows the displacement nephogram when the teeth number is 36 and the teeth number ratio is 3/20, 1/10, and 1/25, respectively.

In other cases, the simulation results of the maximum negative displacement of the split gear blank in Z direction are illustrated in Tables 9, 10, 11, and 12.
Fig. 19  Displacement nephogram in Z direction of the split gear blank

Table 4  Deformation of gear blank with 70 teeth

| Teeth number ratio | 1/7 (10) | 1/10 (7) | 3/35 (6) | 1/14 (5) | 2/35 (4) | 3/70 (3) |
|--------------------|----------|----------|----------|----------|----------|----------|
| Z direction deformation (mm) | 0.777    | 0.363    | 0.267    | 0.183    | 0.119    | 0.068    |

Table 5  Deformation of gear blank with 80 teeth

| Teeth number ratio | 3/20 (12) | 1/10 (8) | 3/40 (6) | 1/16 (5) | 1/20 (4) | 3/80 (3) |
|--------------------|-----------|----------|----------|----------|----------|----------|
| Z direction deformation (mm) | 1.130    | 0.477    | 0.266    | 0.183    | 0.119    | 0.068    |

Table 6  Deformation of gear blank with 90 teeth

| Teeth number ratio | 1/6 (15) | 1/10 (9) | 1/15 (6) | 1/18 (5) | 2/45 (4) | 1/30 (3) |
|--------------------|----------|----------|----------|----------|----------|----------|
| Z direction deformation (mm) | 1.812    | 0.604    | 0.267    | 0.183    | 0.119    | 0.069    |

Table 7  Deformation of gear blank with 100 teeth

| Teeth number ratio | 3/20 (15) | 3/25 (12) | 1/10 (10) | 2/25 (8) | 3/50 (6) | 1/25 (4) |
|--------------------|-----------|-----------|-----------|----------|----------|----------|
| Z direction deformation (mm) | 1.784    | 1.100    | 0.765    | 0.471    | 0.266    | 0.119    |

Table 8  Deformation of gear blank with 110 teeth

| Teeth number ratio | 3/22 (15) | 1/10 (11) | 1/11 (10) | 4/55 (8) | 3/55 (6) | 2/55 (4) |
|--------------------|-----------|-----------|-----------|----------|----------|----------|
| Z direction deformation (mm) | 1.756    | 0.899    | 0.757    | 0.470    | 0.265    | 0.120    |
Fig. 20 Relationship between the deformation and teeth number

Fig. 21 Displacement nephogram in Z direction of the split gear blank

**Table 9** Deformation with module 36 and teeth number 100

| Tooth number ratio | 3/20 (15) | 3/25 (12) | 1/10 (10) | 2/25 (8) | 3/50 (6) | 1/25 (4) |
|--------------------|-----------|-----------|-----------|----------|----------|----------|
| Z direction deformation (mm) | 1.598 | 0.989 | 0.647 | 0.425 | 0.239 | 0.108 |

**Table 10** Deformation with module 40 and teeth number 90

| Tooth number ratio | 1/6 (15) | 1/10 (9) | 1/15 (6) | 1/18 (5) | 2/45 (4) | 1/30 (3) |
|--------------------|----------|----------|----------|----------|----------|----------|
| Z direction deformation (mm) | 1.812 | 0.604 | 0.267 | 0.183 | 0.119 | 0.069 |

**Table 11** Deformation with module 45 and teeth number 80

| Tooth number ratio | 3/20 (12) | 1/10 (8) | 3/40 (6) | 1/16 (5) | 1/20 (4) | 3/80 (3) |
|--------------------|-----------|----------|----------|----------|----------|----------|
| Z direction deformation (mm) | 1.283 | 0.538 | 0.299 | 0.206 | 0.137 | 0.076 |

**Table 12** Deformation with module 50 and teeth number 72

| Tooth number ratio | 1/6 (12) | 1/9 (8) | 1/12 (6) | 5/72 (5) | 1/18 (4) | 1/24 (3) |
|--------------------|----------|---------|----------|----------|----------|----------|
| Z direction deformation (mm) | 1.451 | 0.603 | 0.334 | 0.229 | 0.149 | 0.084 |
When the gear diameter is constant, the module changed, the number of teeth in single split gear blank is different, and the deformation is shown in Fig. 22.

It can be seen from the figure that when the diameter is the same but the module is different, the more teeth the gear blank contains, the greater the deformation is. When the number of teeth exceeds 8, the deformation begins to increase rapidly. In addition, it can be found from the figure that when the diameter and the number of teeth contained in a split gear blank are fixed, the larger the module is, the greater the deformation is. In order to avoid excessive deformation, the teeth number in a split gear blank should be between 5 and 8.

4 Conclusion

The major research results and conclusions are as follows:

1. For the split straight bevel gear, through the calculation of bending deformation, finite element simulation, and experiment, it shows that the superposition calculation method of machining deformation proposed in this paper is feasible, and the deformation calculation model is correct. The finite element simulation technology can effectively predict the machining deformation caused by residual stress of the complex workpiece similar to split gear blank.

2. The bending deformation of the gear blank in gear machining is the largest when the design thickness is about 1.32 times of the tooth height. Considering both the deformation and the size of the split gear blank, it is more appropriate when the thickness of the gear blank is 1.8 times of the tooth depth. Moreover, it is appropriate to take 5 to 8 teeth in each gear blank, and this value has no obvious effect on the deformation when the diameter and the module change.

Author contribution Bin Wang and Chenxiao Yan mainly carried out the theoretical research and finite element simulation and wrote this manuscript. Peiyao Feng and Shuaipu Wang designed most of the experiments and performed most experiments. Shuo Chen and Xuemei Cao analyzed the results.

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Fig. 22 Curve diagram of gear blank deformation with different modulus and tooth number
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