Inelastic neutron scattering study of magnetic excitations in Sr$_2$RuO$_4$.

M. Braden$^{a,b,c,*}$, Y. Sidis$^b$, P. Bourges$^b$, P. Pfeuty$^b$, J. Kulda$^d$, Z. Mao$^e$, Y. Maeno$^e$-f

$^a$ Forschungszentrum Karlsruhe, IFP, Postfach 3640, D-76021 Karlsruhe, Germany
$^b$Laboratoire Léon Brillouin, C.E.A./C.N.R.S., F-91191-Gif-sur-Yvette CEDEX, France
$^c$ II. Phys. Inst., Univ. zu Köln, Zülpicher Str. 77, D-50937 Köln, Germany
$^d$ Institut Laue-Langevin, Boite Postale 156, 38042 Grenoble Cedex 9, France
$^e$ Department of Physics, Kyoto University, Kyoto 606-8502, Japan
$^f$ International Innovation Center (IIC), Kyoto 606-8501, Japan

(Dated: March 22, 2022)

Magnetic excitations in Sr$_2$RuO$_4$ have been studied by inelastic neutron scattering. The magnetic fluctuations are dominated by incommensurate peaks related to the Fermi surface nesting of the quasi-one-dimensional $d_{xz}$- and $d_{yz}$-bands. The shape of the incommensurate signal agrees well with RPA calculations. At the incommensurate Q-positions the energy spectrum considerably softens upon cooling pointing to a close magnetic instability: Sr$_2$RuO$_4$ does not exhibit quantum criticality but is very close to it. $\omega/T$-scaling may be fitted to the data for temperatures above 30 K. Below the superconducting transition, the magnetic response at the nesting signal is not found to change in the energy range down to 0.4meV.

PACS numbers: 78.70.Nx, 75.40.Gb, 74.70.-b

I. INTRODUCTION

Sr$_2$RuO$_4$ is still the only superconducting layered perovskite besides the cuprates[1]: however, in contrast to the cuprate high temperature superconductors (HTSC), superconductivity in Sr$_2$RuO$_4$ develops in a well defined Fermi-liquid state [2, 3]. Nevertheless the superconducting state and the pairing mechanism in Sr$_2$RuO$_4$ are highly unconventional. The present interest in this compound goes far beyond the simple comparison with the cuprate high temperature superconductors.

The extreme sensitivity of the superconducting transition temperature on non-magnetic impurities suggests a non-phonon mechanism [4]. It is further established that superconductivity in Sr$_2$RuO$_4$ is made of spin-triplet Cooper pairs and breaks time-reversal symmetry [2, 3]. The strongest experimental argument for that can be found in the unique susceptibility measured either by the NMR-Knight-shift or polarized neutrons experiments [5, 6] and in the appearance of spontaneous fields in the superconducting state reported by $\mu$SR [7]. A spin-triplet p-wave order parameter had been proposed before these experiments [5], in the idea that superconductivity arises from a dominant interaction with ferromagnetic fluctuations in analogy to superfluid $^3$He. Rice and Sigrist stressed the comparison with the perovskites SrRuO$_4$ and CaRuO$_3$ which order ferromagnetically or are nearly ferromagnetic respectively [8]. Evidence for ferromagnetic fluctuations in Sr$_2$RuO$_4$ was inferred from NMR-experiments: Imai et al. found that $\frac{1}{T_1}$ of the O- and of the Ru-NMR exhibit a similar temperature dependence and interpreted that this could be only due to ferromagnetic fluctuations [9].

The macroscopic susceptibility in Sr$_2$RuO$_4$ is enhanced when compared with the band structure calculation but only weakly, in particular its temperature dependence remains flat [4, 10]. There exist also layered ruthenates which are very close to ferromagnetic order at low temperatures: the pure Sr$_2$Ru$_2$O$_7$ samples show meta-magnetism and samples with somehow less quality even order ferromagnetically [12]. A highly enhanced susceptibility pointing to a ferromagnetic instability is also observed in the phase diagram of Ca$_2$- Sr$_2$RuO$_4$ [13, 14] but for a rather high Ca concentration, Ca$_{1.5}$Sr$_{0.5}$RuO$_4$ [15]. In these nearly ordered layered ruthenates, the susceptibility is about two orders of magnitude higher than that in Sr$_2$RuO$_4$ and strongly temperature dependent.

Some doubt about the unique role of ferromagnetism in Sr$_2$RuO$_4$ arose from the strong moment antiferromagnetic order observed in the Ca analog [15], which inspired Mazin and Singh to perform a calculation of the generalized susceptibility based on the electronic band structure [17]. Surprisingly they found that the dominating part is neither ferro- nor antiferromagnetic but incommensurate. The Fermi-surface in Sr$_2$RuO$_4$ is well studied both by experiment [18] and by theory [19] with satisfactory agreement. Three bands are contributing to the Fermi-surface which may be roughly attributed to the three $t_{2g}$-levels, the $d_{xy}$- and $d_{xz}$-orbitals, occupied by the four 4d-electrons of Ru$^{4+}$. The $d_{xy}$-orbitals hybridize in the xy-plane and, therefore, form a band with two-dimensional character, $\gamma$-band. In contrast, the $d_{xz}$ and the $d_{yz}$ orbitals may hybridize only along the x and the y-directions, respectively, and form quasi-one-dimensional bands, $\alpha$-band and $\beta$-band, with flat sheets in the Fermi-surface, $\alpha$-sheet and $\beta$-sheet. The latter give rise to strong nesting and enhanced susceptibility for $\mathbf{q} = (0.33, q_y, 0)$ or $\mathbf{q} = (q_x, 0.33, 0)$ [17]. Along the diagonal both effects strengthen each other yielding a pronounced peak in the susceptibility at $\mathbf{q}_{nest} = (0.33, 0.33, 0)$. Using inelastic neutron scattering (INS) we have perfectly confirmed this nesting scenario [20]. The dynamic susceptibility at moderate ener-
perconductivity is transferred from the possible ones. Zhitomirsky and Rice [27] assume that surface nodes which leave horizontal line nodes as the only identity samples clearly indicates the presence of line nodes in the superconducting state [24]. Ultrasonic [24] and thermal conductivity [20] results would disagree with vertical line nodes which leave horizontal line nodes as the only possible ones. Zhitomirsky and Rice [27] assume that superconductivity is transferred from the $\gamma$-band to the $\alpha$-band and $\beta$-band by a proximity effect and get a conclusive explanation for the horizontal line nodes. In this model superconductivity should be mainly related to excitations associated with the active $\gamma$-band, which so far have not been characterized. Therefore, it appears still interesting to further deepen the study of the magnetic fluctuations in Sr$_2$RuO$_4$.

In this paper we report on additional INS studies in Sr$_2$RuO$_4$ in the normal as well as in the superconducting phase. We present a more quantitative analysis of the incommensurate fluctuations related to the $\alpha$-sheet and $\beta$-sheet and discuss the possible contributions due to the two-dimensional $\gamma$-band.

II. EXPERIMENTAL

A. Experimental setup

Single crystals of Sr$_2$RuO$_4$ were grown by a floating zone method in an image furnace; they exhibit the superconducting transition at $T_c=0.7$, 1.35 and 1.43K and have volumes of about 450nm$^3$ each. Since most of the measurements were performed in the normal phase, where the differences in $T_c$ should not affect the magnetic excitation spectrum, we aligned the three crystals together in order to gain counting statistics in the INS experiments. Count rates in the ruthenates experiments are relatively small already due to the steeper decrease of the Ru magnetic form-factor with increasing scattering vector. For the measurements below and across the superconducting transition we mounted only the two crystals with relatively high $T_c$ together. The mounting of the two or three crystals was achieved with individual goniometers yielding a total mosaic spread below 0.5 degrees.

We used the thermal triple axis spectrometer 1T installed at the Orphée-reactor (Saclay, France) in order to further characterize the scattering in the normal state. The instrument was operated with double focusing pyrolicthic graphite (PG) monochromator and analyzer, in addition PG filters in front of the analyzer were used to suppress higher order contamination, the final neutron energy was fixed at 14.7meV. All diaphragms determining the beam paths were opened more widely than usually in order to relax $Q$-resolution, since the magnetic signals are not sharp in $Q$-space. In most experiments, the scattering plane was defined by (1,0,0) and (0,0,1) directions in order to span any directions within the (ab) plane. An additional experiment has been made with the (1,1,0) and (0,0,1) directions with the plane to follow the spin fluctuations along the c"-axis.

Studies aiming at the response in the magnetic excitation spectrum on the opening of the superconducting gap require an energy resolution better than the expected value for twice the superconducting gap. Therefore, such experiment is better placed on a cold triple-axis spectrometer even though this implies a sensitive reduction in the flux. We have made preliminary studies using the cold spectrometers 4F at the Orphée reactor and a more extensive investigation on the spectrometer IN14 at the ILL. These instruments possess PG monochromators (double at 4F and focusing at IN14) and focusing analyzers. The final neutron energy was fixed at $E_f=5$ meV on both cold source spectrometers where a Be filter has been employed to cut down higher-order wavelength neutrons. Cooling was achieved by use of a dilution- and a He$^3$-insert at 4F and IN14 respectively.

B. Theoretical background for magnetic neutron scattering

The magnetic neutron cross section per formula unit is written in terms of the Fourier transform of the spin correlation function, $S_{\alpha\beta}(Q,\omega)$ (labels $\alpha$, $\beta$ correspond to $x,y,z$) as [28]:

$$\frac{d^2\sigma}{dQd\omega} = \frac{k_i}{k_f} r_0^2 F^2(Q) \sum_{\alpha,\beta} (\delta_{\alpha,\beta} - \frac{Q_\alpha Q_\beta}{|Q|^2}) S_{\alpha\beta}(Q,\omega)$$

(1)

where $k_i$ and $k_f$ are the incident and final neutron wave vectors, $r_0^2=0.292$ barn, $F(Q)$ is the magnetic form factor. The scattering-vector $Q$ can be split into $Q = q + G$, where $q$ lies in the first Brillouin-zone and $G$ is a zone-center. All reciprocal space coordinates $(Q_x, Q_y, Q_z)$ are given in reduced lattice units of $2\pi/a$ or $2\pi/c$.

According to the fluctuation-dissipation theorem, the spin correlation function is related to the imaginary part of the dynamical magnetic susceptibility times the by one enhanced Bose-factor:

$$S_{\alpha\beta}(Q,\omega) = \frac{1}{\pi(g\mu_B)^2} \frac{\chi_{\alpha\beta}''(Q,\omega)}{1 - \exp(-\hbar\omega/k_BT)}$$

(2)

In case of weak anisotropy, which is usually observed in a paramagnetic state, $\chi_{\alpha\beta}''(Q,\omega)$ reduces to $\chi''(Q,\omega)\delta_{\alpha,\beta}$. Note that for itinerant magnets, anisotropy of the susceptibility can occur due to spin-orbit coupling. $F(Q)$ can
be described by the Ru$^+$ magnetic form factor in first approximation. Once determined, the magnetic response is converted to the dynamical susceptibility and calibrated in absolute units by comparison with acoustic phonons, using a standard procedure depicted in $[20]$. 

III. RESULTS AND DISCUSSION

A. RPA analysis of the magnetic excitations

At low temperature Sr$_2$RuO$_4$ exhibits well defined Fermi-liquid properties; therefore, it seems appropriate to assess its magnetic excitations within a RPA approach basing on the band structure. Density functional calculations in LDA were performed by several groups and yield good agreement with the Fermi-surface determined in de Haas van Alphen experiments. The bare non-interacting susceptibility, $\chi^0(q)$, can be obtained by summing the matrix elements for an electron hole excitation $[28]$

$$\chi^0(q,\omega) = -\sum_{k,i,j} M_{k+i+j} f(\varepsilon_{k+i,j}) - f(\varepsilon_{k,j}) \frac{\varepsilon_{k+i,j} - \varepsilon_{k,j} - \hbar\omega + i\epsilon}{\varepsilon_{k+i,j} - \varepsilon_{k,j}}$$  \hspace{1cm} (3)

where $\epsilon \rightarrow 0$, $f$ is the Fermi distribution function and $\varepsilon_k$ the quasiparticle dispersion relation. This was first calculated by Mazin and Singh $[17]$ under the assumption that only excitations within the same orbital character are relevant (the matrix-elements $M_{k+i+j}$ are equal to one or zero). Mazin and Singh predicted the existence of peaks in the real part of the bare susceptibility at $\omega=0$ due to the pronounced nesting of the $\alpha$-bands and $\beta$-bands. These peaks were calculated at $(0.33,0.33,0)$ and experimentally confirmed very close to this position at $q_i=(0.3,0.3,0)$, see Fig. 1. In addition to the peaks at $q_i$, this study finds ridges of high susceptibility at $(0.3, q_y,0)$ for $0.3 < q_y < 0.5$ and some shoulder for $0 < q_y < 0.3$.

The susceptibility gets enhanced through the Stoner-like interaction which is treated in RPA by:

$$\chi(q) = \frac{\chi^0(q)}{1 - I(q)\chi^0(q)}$$ \hspace{1cm} (4)

with the $q$-dependent interaction $I(q)$. For the nesting positions Mazin and Singh get $I(q)\chi^0(q)=1.02$, which already implies a diverging susceptibility and a magnetic instability.

In Fig. 1 we show a scheme of the $(hk0)$-plane in reciprocal space. Due to the body centering in space-group I4/mmm any (hkl)-Bragg-points have to fulfill the condition $(h+k+l)$ even; $(100)$ for instance is not a zone-center but a Z-point. The boundaries of the Brillouin-zones are included in the figure. Large filled circles indicate the position of the incommensurate peaks and thick lines connecting four of them correspond to the walls of enhanced susceptibility also suggested in reference $[17]$. The dashed double arrows show the paths of the constant energy scans frequently used in this work: along $[010]$-direction – $y$-scan, transverse to the $Q$-vector, $r$-scan, and along the diagonals in $[110]$-direction, $d$-scans. Lower part: Imaginary part of the generalized susceptibility calculated by RPA along the three paths indicated in the upper part as dotted lines.

In the meanwhile several groups have performed similar calculations which all agree on the dominant incommensurate fluctuations $[22, 29, 31, 32]$. However there are serious discrepancies concerning the detailed structure of the spin susceptibility away from $q_i$. These discrepancies mainly rely on the parameters used to describe the electronic band structure, on the choice of $I(q)$ and on the inclusion of more subtle effects such as spin-orbit or Hund couplings.

In order to compare directly to the INS experiments, see equations (1) and (2), it is necessary to perform the RPA analysis by taking into account both real and imaginary part of the susceptibility. Morr et al. $[32]$ report such calculations obtained by fitting the band structure to the ARPES results $[32]$. They find in addition to the peak at $q_i$ quite strong intensity near $P_i=(0.3,0.5,0)$, (the middle of the walls, see Fig. 1), which is even com-
parable to that at \( q_l \) in the bare susceptibility. Similar results were obtained in references [3, 29]. Ng and Sigrist [29] find much less spectral weight in the ridges at \((0.3, q_y)\) for \(0.3 < q_y < 0.5\) but stronger shoulders \(0 < q_y < 0.3\). In addition, they calculate the separate contribution of the \(\gamma\)-band which does not show a particular enhancement in the ferromagnetic \(q\)-range but is little structured. Eremin et al. [22] calculate the susceptibility taking into account strong hybridization and obtain results somehow different from the other groups. They find a strong signal at \(P_l\); in addition there is some enhancement of the susceptibility related to the van-Hove singularity of the \(\gamma\)-band. This contribution occurs quite close to the zone center at \(q_{0.15}=(0.15,0,0)\).

We have performed the full RPA analysis basing on the LDA band structure reported in reference [17] in order to accompany our experimental investigations. We first calculate the bare electron hole susceptibility \(\chi_0\) from the usual expression, equation (3). For the band energies \(\epsilon(k_x, k_y, k_z)\) we use the expressions of Mazin and Singh [13] for the three mutually non hybridizing tight-binding bands in the vicinity of the Fermi level:

\[
\begin{align*}
\epsilon_{xy}(k_x, k_y, k_z) & = 400\text{meV}(-1 + 2(\cos(k_x) + \cos(k_y))) \\
& -1.2\cos(k_z)\cos(k_y)) \\
\epsilon_{xz}(k_x, k_y, k_z) & = 400\text{meV}(-0.75 + 1.25\cos(k_x) - .5\cos(k_x/2)\cos(k_y/2)\cos(k_z/2)) \\
\epsilon_{yz}(k_x, k_y, k_z) & = 400\text{meV}(-0.75 + 1.25\cos(k_y) - .5\cos(k_x/2)\cos(k_y/2)\cos(k_z/2))
\end{align*}
\]

(5)

(6)

(7)

We also used the crude approximation that matrix-elements for transitions between bands of the same character are equal to one and others zero. The \(q\)-dependent "ferromagnetic" interaction (Stoner factor) \(I(q)\) is taken to be equal to (following reference [17]): \(I(q) = 420\text{meV}^{1+0.08q^2}\). With this choice of \(I(q)\) the calculated static susceptibility \(\chi(q=0,\omega=0)\) is slightly lower than the measured one. If \(I(q)\) is chosen larger an instability appears at the incommensurate wave vector.

Our results for the imaginary part of the generalized susceptibility at an energy transfer of 6meV are given in the lower part of figure 1. Besides the dominating nesting peak near \(q_l\) there is a further contribution near \((0.15,0.15,0)\) which is related to the \(\gamma\)-sheet. In contrast we find a small susceptibility near \(P_l\) and for \((q_x,0,0)\) with small values of \(q_x\).

Since the \(q\)-position of the magnetic excitations were found not to depend on energy, it is easiest to observe the signal in INS by scanning at constant energy. The scan paths are included in Fig. 1. They are purely transverse or rocking-like, \(r\)-scan, along a \([100]\)-direction, \(x\)-scans, and in diagonal direction, \(d\)-scans.

The observed signal is rather broad and, therefore, the scans performed are extremely wide covering complete cuts through the Brillouin-zone. This further implies that the background (BG) may be non-constant at least sloping. Also, the signals are relatively weak compared to typical triple axis spectrometry problems; this implies that sample independent BG-contributions which usually are negligible play a role. Fig. 2 presents the results of \(d\)-scans at different temperatures clearly demonstrating the gain in statistics compared to the previous work [20]. The magnetic intensity shown in Fig. 2 disappears upon heating but this effect gets partially compensated by the gain through the Bose-factor.

### B. Shape of the incommensurate signal

The fact that the incommensurate signal around a zone-center, and around a \(Z\)-point, \((001)\), are equivalent already indicates that the coupling between \(\text{RuO}_2\)-planes is negligible, i.e. that in-phase and out-of-phase coupling are indistinguishable. The 2D-character has been directly documented by Servant et al. [17] who found no \(q\)-dependence at \((0.3,0.3,q_l)\) for \(q_l\) between -0.5 and 0.5. We find the same result by varying \(Q_l\) in a broader range between 2 and 5 in \((0.3,0.3,Q_l)\), see Fig. 3. This 2D character is actually surprising since the dispersion relation of the quasi-particles involved in the computation of \(\chi''(q,\omega)\) is not purely 2D [22]. One may therefore expect weak spin correlation along the \(c^*\)-direction.

Recently, it has been shown that these fluctuations freeze out into a spin density wave (SDW) ordering by a minor replacement of \(\text{Ru}\) by \(\text{Ti}\) [23]. In this ordering a very short correlation length along the \(c\)-direction nicely reflects the 2D-character of the incommensurate

![Graph showing the magnetic intensity with temperature](image-url)
in elastic signal. The SDW propagation vector finally fa-
vored in the static ordering corresponds to the out-of-
phase coupling between neighboring layers. This static
interlayer-coupling might be even due to the CDW always
coupled to a SDW, which, however, has not yet been dis-
covered so far. One may add that the stripe ordering in
SrO case in many aspects but nevertheless may be consid-
ered as a mixed SDW-CDW-ordering, occurs at the same
propagation vector [36].

The wider Q dependence of the incommensurate
signal shown in Fig. 3 can give information about the
anisotropy of the excitation, since INS measures only
the spin component perpendicular to Q. We consider the
diagonal susceptibility $\chi''_{\alpha\beta}$ with tetragonal symmetry
$\chi'_{xx} = \chi''_{yy} \neq \chi''_{zz}$. The measured intensity is
then given by:

$$\frac{d^2\sigma}{d\Omega d\omega} \propto F^2(Q)[(1 - \frac{Q_i^2}{Q^2})\chi''_{zz} + (1 + \frac{Q_i^2}{Q^2})\chi''_{\pm}]$$

which implies that the observations at high Q favor the
in-plane component of the susceptibility. For the de-
tailed analysis, one has to compare with the form fac-
tor. In the figure we show the Q dependence assuming
that spin density is localized at the Ru-site and may be
modeled by the form factor of Ru$^{1+}$ [37]. The Ru-form-
factor dependence underestimates the signal at higher
Q-values; however, the Ru$^{1+}$-form factor is certainly a
too crude approximation. Measurements of the spin-
density distribution induced by an external field have not
been very precise due to the small magnetic susceptibility
and the resulting small moment in Sr$^3$RuO$_4$ [38].

However, in Ca$_{1.5}$Sr$_0.5$RuO$_4$ which exhibits ferromagnetic
ordering below 1K and whose low temperature suscepti-
bility is about two orders of magnitude higher than that
in Sr$_2$RuO$_4$ [3, 4, 5], it has been possible to study the
field induced spin-density distribution. These experi-
ments revealed an extremely high amount of spin-density
at the oxygen position, about one third of the total mo-
moment [39] and an orbital contribution at the Ru-site. By
use of the Ca$_{1.5}$Sr$_0.5$RuO$_4$ spin-density distribution one
obtains a good description of the Q dependence given in
Fig. 3. However, the form-factor in Sr$_2$RuO$_4$ should be
even more complex. Since the main contribution
originates from the flat $d_{xy}$-orbitals [3], there must be
an anisotropy in the effective form factor which indeed
was observed in Ca$_{1.5}$Sr$_0.5$RuO$_4$ [39]. Qualitatively,
the anisotropy in the form-factor has to be compensated by
some weak anisotropy in the spin susceptibility, i.e. by
an enhanced out-of-plane component. Such susceptibility
anisotropy corresponds to the orientation of the spins in
the SDW ordering phase in Sr$_2$Ru$_{1-x}$Ti$_x$O$_4$ where
the spins are aligned parallel to the c-direction and also
to the conclusion deduced by Ng and Sigrist from the
influence of spin-orbit coupling in Sr$_2$RuO$_4$ [2].

- **Shoulder of the incommensurate peak** – In order to
to examine the shape of the incommensurate peak and to
verify the existence of the ridges of additional nesting in-
tensity or the additional peak – Shoulder of the incommensurate peak –
the SDW ordering phase in Sr$_2$Ru$_{1-x}$Ti$_x$O$_4$ [39] where
the spins are aligned parallel to the c-direction and also
to the conclusion deduced by Ng and Sigrist from the
influence of spin-orbit coupling in Sr$_2$RuO$_4$ [2].

In order to examine the shape of the incommensurate peak and to
verify the existence of the ridges of additional nesting in-
tensity or the additional peak – Shoulder of the incommensurate peak –
in Sr$_2$RuO$_4$ [3, 4, 5], reported in different
band structure analyzes [17, 19, 20, 31, 22], we have
scanned across $q_i$ along the [100] or [010]-directions,
y-scans see Fig. 1. The results are shown in Fig. 4.
One can recognize that the incommensurate peak is not
symmetric but exhibits always a shoulder to the lower
$q_x$-side in absolute units. The shoulder is seen in many
scans in reversed focusing conditions of the spectrometer
configuration excluding an experimental artefact. Our
full RPA calculation nicely agrees with such shoulder;
the thin line in Fig. 4 shows the calculated imaginary
part of the susceptibility at the energy transfer of the scan and describes the observed signal perfectly besides a minor offset in the position of the incommensurate signal. In contrast, neither the experiment nor our full RPA analysis yield significant intensity in the ridges, i.e. the range \((q_x,0.3,0)\) with \(0.3 < q_x < 0.5\), see Fig. 1 and 4. In particular an intensity at \(Q_i\) only three times weaker than that at \(q_i\) would have been easily detected experimentally. The nesting peak appears to be isolated with two shoulders along \([100]\) and \([010]\) to the lower absolute \(q_{x,y}\)-sides. These shoulders are connected to the \(\gamma\)-sheet.

\section{Possible additional magnetic excitations}

In none of the band-structure calculations there is evidence for a strong and sharp enhancement of any susceptibility exactly at the zone-center pointing to a ferromagnetic instability \cite{7, 23, 31, 32}. However, several of these calculations find some large susceptibility near the zone-center which can be associated with the van Hove-singularity of the \(\gamma\)-band closely above the Fermi-level. Eremin et al. \cite{31} report this signal at \(q=(0.1-0.2,0,0)\), other groups find a small peak along the diagonal \((q,q,0)\) \cite{24, 25, 30}. Our own analysis also yields such a signal which is found to be strongest at \((0.15,0.15,0)\) and which levels out along the \([100]\) and \([010]\) directions, see Fig. 1.

In order to address this problem we have mapped out the intensity for \((Q_x,Q_y,0)\) with \(-0.5 < Q_x < -0.0\) and \(0.6 < Q_y < 1.5\); these scans are shown in Fig. 5 after subtraction of the scattering angle dependent background. It is obvious that the incommensurate peak is by far the strongest signal. At low temperature one may

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Mapping of the INS intensity by constant energy scans with fixed \(Q_y\) at 4.1meV energy transfer. The colour plot was obtained by adding symmetrical data after subtraction of the scattering angle dependent background.}
\end{figure}

roughly estimate any additional signal to be at least a factor 6 smaller than the nesting peak. The analysis of such weak contributions is quite delicate and demands a reliable subtraction of the background. Nevertheless the scans in the Fig. 5 indicate some scattering closer to the zone center. However, this contribution is not sharply peaked at the \(\Gamma\)-point but forms a broad square or a circle. The signal roughly agrees with the prediction that the \(\gamma\)-band yields magnetic excitations near the zone-center.

Since the sloping background is a major obstacle to analyze the additional contributions we tried to compensate these effects by scanning from \(Q=(0 1 0)\) along the four diagonals, which are illustrated in Fig. 1, and by summing the four scans. The results are given in Fig. 6. In the four single scans, Fig. 6a), one recognizes the nesting signal with its intensity being determined by the form-factor and the sloping background. The summed scans, see Fig. 6c), should have constant background and the low temperature summed scan once more documents that the nesting signal is by far the strongest one. However, upon heating additional scattering contributions seem to become enhanced in intensity in particular compared to the nesting signal which decreases; note that the Bose-factor will already strengthen any signal by a factor three in the data in Fig. 6c). Also, at higher energy the additional scattering seems to be stronger as seen in Fig. 6d) (the background is strongly sloping even in the sum due...
to a smaller scattering angle). The energy and temperature dependence of the additional contribution corresponds to that predicted by the full RPA analysis for the $\gamma$-band magnetic contribution shown in Fig. 6b). The spectral weight at $Q=(0.15,0.15,0)$ relative to that of the nesting feature increases upon increasing temperature or energy as it is expected for a signal directly related to the van Hove singularity. Due to the agreement between the INS results and the RPA calculations, we suggest to interpret the additional broad scattering as being magnetic in origin; however, a polarized neutron study would be highly desirable. This scattering further might be relevant for a quantitative explanation of the NMR-data \cite{10,20}, in particular its temperature independent part.

Sr$_2$RuO$_4$ is not close to ferromagnetic order but substitution of Sr through Ca yields such order for the concentration Ca$_{1.5}$Sr$_{0.5}$RuO$_4$ \cite{13}. This doping effect was explained in a band structure calculation \cite{40} as arising from a down shift in energy of the $\gamma$-band pushing the van Hove-singularity closer to the Fermi-level. In such samples the $\gamma$-band magnetic scattering should therefore become strongly enhanced. Indeed first INS studies on these compounds reveal broad signals similar to the additional scattering described above, but much stronger \cite{11}. This strongly supports a magnetic interpretation of the scattering in figures 5 and 6.

D. Combined temperature and energy dependence of the incommensurate signal

The energy and temperature dependence has been studied in more detail by performing $r$-scans across the incommensurate position, since in this mode the background is almost flat. However, due to phonon contributions we could not extend the measurements to energies higher than 12meV, which is slightly below the lowest phonon frequency observed at $q_i$ \cite{42}. In particular the large scans required to cover the broad magnetic signal prevent any analysis within the phonon band frequency range on a non-polarized thermal triple axis spectrometer.

The results of the scans are given in Fig. 7. At low temperature we find an energy spectrum in good agreement to that published earlier \cite{20}. In the range up to 12meV we observe at all temperatures an energy independent peak width, which, however, increases upon increase of temperature. For temperatures much higher than 160K the background considerably increases, see Fig. 2, and prevents a detailed analysis within reasonable beam-time. In Fig. 7b) we show the temperature dependence of the peak width averaged over the different energies which agrees well to the results obtained from the single scans with less statistics reported in reference \cite{20}. Even at the lowest temperature the width of the signal remains finite.

The spectral functions have been fitted by a single relaxor behavior \cite{43}:

$$\chi''(q_i,\omega) = \chi'(q_i,0) \frac{\Gamma \omega}{\omega^2 + \Gamma^2} \quad (9)$$

where $\Gamma$ is the characteristic energy and $\chi'(q_i,0)$ the amplitude which corresponds to the real part of the generalized susceptibility at $\omega=0$ according to the Kramers-Kronig relation. The $Q$-dependence of the signal may be described by a Lorentzian distribution with half width at half maximum (FWHM) corrected for the experimental resolution and temperature dependence of the square of the FWHM b). Temperature dependence of the amplitude and its inverse c) and of the characteristic energy d) in the relaxor behavior fitted to part a). Lines in b-d) are guides to the eye.

FIG. 7: Observed imaginary part of the generalized susceptibility as function of energy and temperature; lines are fits with a single relaxor a). Temperature dependence of the averaged full width at half maximum (FWHM) corrected for the experimental resolution and temperature dependence of the square of the FWHM b). Temperature dependence of the amplitude and its inverse c) and of the characteristic energy d) in the relaxor behavior fitted to part a). Lines in b-d) are guides to the eye.
Our finding that the characteristic energy in the range 6-9meV is well defined only at low temperatures can be related with the far-infrared c-axis reflectance study by Hildebrand et al. [24], since the optical spectrum shows a resonance in this energy range at low temperatures. In reference [20] we have compared the temperature dependence of the incommensurate signal with that of the spin-lattice relaxation rate $T_1$ measured by both $^{17}$O and $^{101}$Ru NMR experiments [11]. These NMR-techniques probe the low energy spin fluctuations ($\omega \to 0$ with respect to INS measurements); furthermore, they integrate the fluctuations in $q$-space. $(1/T_1 T)$ is related to the generalized susceptibility and the INS results by [15]:

$$ (1/T_1 T) \approx \frac{k_B \gamma^2 n^2}{(g \mu_B)^2} \sum_q |A(q)|^2 \frac{\chi''(q,\omega)}{\omega} \bigg|_{\omega \to 0} \quad (10) $$

with $|A(q)|$ the hyper fine fields. $(1/T_1 T)$ corresponds hence to the slope of the spectral function in Fig. 7a times the extension of the signal in $Q$-space. The new data perfectly agrees to the former result, the loss of the incommensurate signal upon heating may explain almost entirely the temperature dependent contribution to $(1/T_1 T)$ [10].

The temperature dependence of the magnetic excitation spectrum at the incommensurate position may be analyzed within the results of the self consistent renormalization theory described in reference [13]. In an antiferromagnetic metal the transition is governed by a single parameter related to the Stoner-enhancement at the ordering wave-vector, $\delta = 1 - I(q_1)\chi_0(q_i)$. The characteristic entities of the magnetic excitations are then given by:

$$ \kappa^2 \propto \delta \quad (11) $$

$$ \Gamma \propto \delta \quad (12) $$

$$ \frac{1}{\chi'(q_1,0)} \propto \delta . \quad (13) $$

When the system approaches the phase transition, the unique parameter $\delta$ diminishes, which behavior should be observable in all three parameters. Equations (11-13) imply a sharpening of the magnetic response in $q$-space as well as in energy and a divergence of the susceptibility at the ordering vector. Fig. 7 qualitatively confirms this picture. All the relevant parameters, see the right scales in Fig 7, decrease towards low temperature. Therefore, one may conclude that Sr$_2$RuO$_4$ is approaching the SDW transition related to the nesting effects upon cooling. However, all these parameters do not vanish completely but remain finite even at the lowest temperatures in agreement with the well known fact that Sr$_2$RuO$_4$ does not exhibit magnetic ordering. In particular the magnetic scattering remains rather broad in $q$-space implying a short correlation length of just 3-4 lattice spacings. The temperature dependence of the magnetic excitations corroborate our recent finding that only a small amount of Ti is sufficient to induce SDW magnetic ordering [25].

Since Sr$_2$RuO$_4$ is close to a quantum critical point it is tempting to analyze whether the excitation spectrum is governed by some $\omega^n$ scaling, as it has been claimed for the high temperature superconductor La$_{2-\delta}$Sr$_\delta$CuO$_4$ [10, 17, 18] and for CeCu$_{5.9}$Au$_{0.1}$ [19]. One would expect that the susceptibility is given by:

$$ \chi''(q_1,\omega, T) \propto T^{-\alpha} g(\omega/T) . \quad (14) $$

In Fig. 8 we plot the $\omega^n \chi''(q_1, \omega)$-data of Fig. 7 and that obtained previously as a function of temperature [20] against $\omega/T$ for $\alpha=0.75$ and 1.0. Only the data at higher temperatures agree with the scaling concept, demonstrating that Sr$_2$RuO$_4$ is not a quantum critical point. The schematic inset may illustrate the phase diagram, where the magnetic transition is determined by some parameter $r$ (external pressure or composition). At the critical transition one would observe quantum criticality in the entire temperature range, whereas for $r$-values where the transition is suppressed quantum criticality is observed only at higher temperatures. One then may expect a cross-over temperature $T^*$ where the system transforms from an unconventional metal at high temperatures towards a Fermi-liquid at low temperatures [50]. Only in the temperature range above $T^*$ the magnetic excitations should exhibit the related $\omega^n$ scaling. Our data clearly shows that such scaling can be fitted to the data only for the three higher temperatures studied. The description with the scaling concept seems to be slightly better for the exponent $\alpha=0.75$. The temperature dependent data suggests a crossover near 30K. This cross-over agrees very well to that seen in electronic transport properties, where well defined Fermi-liquid behavior is only observed below about 25K [1, 3].

**E. Magnetic scattering in the superconducting phase**

As emphasized by Joynt and Rice [51], the wavevector- and energy-dependent spin susceptibility in superconductors reflects directly the vector structure of the superconducting (SC) gap function, allowing a complete identification of the SC order parameter symmetry. Inelastic neutron scattering experiments have the potential, in principle, to determine the superconducting order parameter. In high-$T_c$ superconductors, the spin-singlet $d$-wave symmetry SC gap induces a striking modification of the spin susceptibility in the superconducting state. As a consequence, the so-called ”magnetic resonance peak” has been observed in the superconducting state of various copper oxide superconductors by INS [22, 23]. Therefore, a similar experiment in Sr$_2$RuO$_4$ would certainly be instructive about the SC gap symmetry.
The enhanced spin susceptibility has been calculated in [32, 54, 55], considering a spin-triplet p-wave superconducting state with $d(k)=z(k_x \pm ik_y)$. Note that in such a case, the superconducting gap is isotropic due to the particular shape of the Fermi-surface in Sr$_2$RuO$_4$. For the wavevector $q_i$, Kee et al. [54] and Morr et al. [32] predicted that below $T_c$ spectral weight is shifted from below twice the superconducting gap, $\Delta$, into a resonance like feature close to $2\Delta$. Morr et al. find the resonance in the $zz$-channel yielding an enhancement of the magnetic excitation intensity by a factor of 9 in the superconducting state as compared to the normal state [32]. The difference between the in-plane and out-of-plane susceptibility in the superconducting state arises from the coherence-factor.

Similar theoretical framework is currently used to describe the spin excitation spectrum in spin-singlet HTSC cuprate superconductors [56]. Theoretical works show that the opening of a d-wave order gap together with the exchange interaction lead to the appearance of a similar resonant feature below $2\Delta$ at the antiferromagnetic wavevector. These theories successfully account for the observation by INS of the magnetic resonance peak in the superconducting state.

Using unpolarized INS, one measures the superposition of the out-of-plane and in-plane components of the susceptibility (see Eq. 3) and both components are equally weighted, when performing the measurements $Q_1=0$. Thus, the predicted resonance feature should be observable, at the value of $\sim 2\Delta$, if one obtains an experimental arrangement which allows to study the inelastic magnetic signal in this energy range. Due to the almost linear decrease of $\chi''(q_i, \omega, T)$ towards low energies, see Fig. 7, and due to the higher required resolution which implies less neutron flux, these experiments are extremely time demanding. We have analyzed the magnetic excitations in the superconducting phase on the cold triple axis spectrometer IN14 at the ILL using a two crystal assembly, the results are shown in Fig. 8.

The right part of Fig. 8 shows a scan across the incommensurate peak at $\omega=4$ meV, i.e. the range already studied with thermal neutrons. This signal can be determined with little beam time. Performing the same scan at 1.6 and 0.8 meV requires considerably more time but still exhibits a well defined signal which seems not to experience any change in the superconducting state at $T=0.35$ K. The results of constant-$Q$-scans at $Q=(0.7,0.3,0)$ are shown in the left part of Fig. 9. As there is no visible difference between the results obtained at 0.35 and 0.80 K (both in the SC state), we have added the two scans below $T_c$ in the lower left part of Fig. 9. Importantly and despite efforts to get rather high statistics, there is no change visible in the energy spectra above and below the superconducting transition in the energy range of the superconducting gap. The spin susceptibility is not modified appreciably across the superconducting transition; our data even do not show any opening of a gap. One can describe the energy dependence presented in Fig. 9 with the single relaxor using the same fitted parameters as the low temperature data in Fig. 5.a. However, the detailed shape of the spin susceptibility (Fig. 9) does not exactly match such a simple linear behavior but rather seems to indicate some anomaly near $2\Delta$ which requires further experimental work.
There is actually little known about the value of the superconducting gap in Sr$_2$RuO$_4$. Laube et al. \cite{Laube} have reported an Andreev reflection study where the opening of the gap is clearly visible in an astonishingly large energy range. The quantitative analysis of the spectra is quite involved; assuming a p-wave order parameter Laube et al. obtain $2\Delta=2.2$ meV which may be compared to the value expected within BCS-theory $2\Delta=3.55$ $k_B T_c = 4.97$ K = 0.43 meV. Our data shows that there is no change in the excitation spectrum for energies well below the reported value of $2\Delta$, but it has not been possible to investigate the lowest energies due to the strong elastic incoherent signal. With further increased resolution ($k_f = 1.3\AA^{-1}$) we have scanned the energy range 0.3–0.7 meV at 0.80 K again without evidence for a resonant feature. Furthermore, the comparison of two scans at constant $Q=(1,0,0)$ did not yield any difference below and above $T_c$, meaning no ferromagnetic spin susceptibility enhancement.

The theory presented by Morr et al. \cite{Morr} should be considered as being quite reliable in the case of Sr$_2$RuO$_4$, since the RPA approach to the magnetic excitations is so successful in the normal state, and since Sr$_2$RuO$_4$ exhibits well defined Fermi-liquid properties at temperatures below 25 K. Therefore, the data in Fig. 4 gives strong evidence against a simple p-wave order parameter in Sr$_2$RuO$_4$ with a maximum value of the gap of the order of the reported value \cite{Laube}. However, in the meanwhile there are several indications that the order parameter is more complex. The recent specific heat data on the highest quality single crystals \cite{Laube} points to the existence of line nodes in the gap function which were then shown to be aligned parallel to the a,b-plane (horizontal line nodes). Such line nodes were explained by Zhitomirsky and Rice through a proximity effect between the active $\gamma$-band and the more passive one-dimensional bands \cite{Zhitomirsky}. A modulation of the gap function along the c-direction will wipe out the resonance predicted for the non-modulated p-wave gap, since for the $Q$-position analyzed, $(0.7, 0.3, 0)$, the electron hole excitation involves parts of the Fermi surface which are fully, partially or not gapped at all. In this sense the absence of any temperature dependence in the magnetic excitation spectrum is consistent with the presently most accepted shape of the gap function. Further theoretical as well as experimental studies are required to clarify the possibility of a resonant feature at other positions in ($Q$, $\omega$)-space.

IV. CONCLUSIONS

Using assemblies of several crystals of Sr$_2$RuO$_4$ we have analyzed the magnetic excitations by INS. The incommensurate signal arising from the nesting between the one-dimensional bands shows an asymmetry between the $\gamma$-band and the more passive one-dimensional bands \cite{Zhitomirsky}. A modulation of the gap function along the c-direction will wipe out the resonance predicted for the non-modulated p-wave gap, since for the $Q$-position analyzed, $(0.7, 0.3, 0)$, the electron hole excitation involves parts of the Fermi surface which are fully, partially or not gapped at all. In this sense the absence of any temperature dependence in the magnetic excitation spectrum is consistent with the presently most accepted shape of the gap function. Further theoretical as well as experimental studies are required to clarify the possibility of a resonant feature at other positions in ($Q$, $\omega$)-space.

The energy dependence of the incommensurate signal varies with temperature and exhibits a general softening of the spectrum upon cooling. This behavior indicates that Sr$_2$RuO$_4$ is approaching the corresponding SDW instability at low temperature even though this compound is not at a quantum critical composition. This interpretation is confirmed by the fact that the generalized susceptibility exhibits some $\omega/T$-scaling only above $\sim$30 K, i.e. in the temperature range where also the transport properties indicate non-Fermi-liquid behavior.

The analysis of magnetic excitations besides the nesting ones shows only minor contributions. There is some evidence for additional magnetic scattering closer to the zone-center but still not peaking at the zone-center. This interpretation gets support from the fact that similar scattering is observed in nearly ferromagnetic Ca$_1.5$Sr$_{0.5}$RuO$_4$ and from various RPA calculations which find excitations mainly related to the $\gamma$-band in this $Q$-range.

The magnetic excitations in Sr$_2$RuO$_4$ may be compared to the distinct types of magnetic order which have been induced by substitution. The dominant excitations reflect the SDW reported to occur in Sr$_2$Ru$_{1-x}$Ti$_x$O$_4$ at small Ti concentrations. The less strong excitations situation more closely to the zone-center and most likely related to the $\gamma$-band become enhanced through Ca-substitution which drives the system towards ferromagnetism, but only for rather high Ca-concentration.

Upon cooling through the superconducting transition we do not observe any change in the magnetic excitation spectra, which combined with recent calculations \cite{Morr} indicates that the order-parameter in Sr$_2$RuO$_4$ does not possess simple p-wave symmetry. These experimental findings are still in agreement with a p-wave order parameter modulated by horizontal line nodes.

Acknowledgements. We would like to thank O. Friedt, H.Y. Kee, D. Morr and R. Werner for stimulating discussions and P. Boutrouille (LLB) and S. Pujol (ILL) for technical assistance. Work at Cologne University was supported by the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 608. Work at at Kyoto was supported by a grant from CREST, Japan Science and Technology Corporation.

* electronic mail : braden@ph2.uni-koeln.de

\begin{thebibliography}{9}
\bibitem{Maeno} Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J.G. Bednorz, and F. Lichtenberg, Nature (London) \textbf{372}, 532 (1994).
\end{thebibliography}
[2] Y. Maeno, K. Yoshida, H. Hashimoto, S. Nishizaki, S.-I. Ikeda, M. Nohara, T. Fujita, A. P. Mackenzie, N. E. Hussey, J. G. Bednorz and F. Lichtenberg, J. Phys. Soc. of Japan 66, 1405 (1997).

[3] A. P. Mackenzie, R. K. W. Haselwinner, A. W. Tyler, G. G. Lonzarich, Y. Mori, S. Nishizaki and Y. Maeno, Phys. Rev. Lett. 80, 161 (1998).

[4] Y. Maeno, T.M. Rice and M. Sigrist, Physics Today January 2001.

[5] K. Ishida, H. Mukuda, Y. Kitaoka, K. Asayama, Z. Q. Mao, Y. Mori, and Y. Maeno, Nature (London) 396, 658 (1998).

[6] J. A. Duffy, S. M. Hayden, Y. Maeno, Z. Mao, J. Kulda, and G. J. McIntyre, Phys. Rev. Lett. 85, 5412 (2000).

[7] J. I. I. Mazin and D.I. Singh, Phys. Rev. Lett. 84, 2666 (2000).

[8] T. M. Rice and M. Sigrist, J. Phys. Cond. Matt. 7, L643 (1995).

[9] A. Callaghan, C. W. Moeller, and R. Ward, Inorg. Chem. 5, 1572 (1966); J. M. Longo, P. M. Raccach, and J. B. Goodenough, J. Appl. Phys. 39, 1372 (1968).

[10] T. Imai, A.W. Hunt, K.W. Thurber and F.C. Chou, Phys. Rev. Lett. 81, 1572 (1998).

[11] T. Kuwabara and M. Ogata, Phys. Rev. Lett. 69, 572 (1997).

[12] A. Gukasov and M. Braden, unpublished results.

[13] Y. Maeno, cond-mat/0206156.

[14] M. Braden, O. Friedt, Y. Sidis, P. Bourges, M. Minakata and Y. Maeno, Phys. Rev. Lett. 88, 197002 (2002).

[15] T. Takimoto, Phys. Rev. B 62, 3505 (2000).

[16] G. M. Luke, Y. Fudamoto, K. M. Kojima, M. I. Larkin, J. A. Duffy, S. M. Hayden, Y. Maeno, Z. Mao, Y. Mori, H. Nakamura, and M. Sigrist, Nature (London) 394, 558 (1998).

[17] T. Nomura and K. Yamada, J. Phys. Soc. Jpn. 69, 865 (1995).

[18] M. Braden, W. Reichardt, S. Nishizaki, Y. Mori, and Y. Maeno, Phys. Rev. B 57, 1236 (1998).

[19] M. Braden, O. Friedt, and M. Braden, unpublished results.

[20] I. I. Mazin and D.I. Singh, Phys. Rev. Lett. 79, 733 (1997).

[21] H. Yoshizawa, T. Kakeshita, R. Kajimoto, T. Tanabe, T. Katsufuji and Y. Tokura, Phys. Rev. B 83, 854 (2000).

[22] H. Yoshizawa, T. Kakeshita, R. Kajimoto, T. Tanabe, T. Katsufuji and Y. Tokura, J. Phys. Soc. of Japan 66, 572 (1998).

[23] R. W. Erwin, M.A. Kastner and G. Shirane, Phys. Rev. Lett. 67, 227002 (2001).

[24] Y. Maeno, K. Yoshida, H. Hashimoto, S. Nishizaki, S.-I. Ikeda, M. Nohara, T. Fujita, A. P. Mackenzie, N. E. Hussey, J. G. Bednorz and F. Lichtenberg, J. Phys. Soc. of Japan 66, 1405 (1997).

[25] J. A. Duffy, S. M. Hayden, Y. Maeno, Z. Mao, J. Kulda, and G. J. McIntyre, Phys. Rev. Lett. 85, 5412 (2000).

[26] T. Takimoto, Phys. Rev. B 62, 14641 (1999).

[27] Y. Maeno, cond-mat/0206156.

[28] M. Braden, O. Friedt, and M. Braden, unpublished results.

[29] M. Braden, W. Reichardt, S. Nishizaki, Y. Mori, and Y. Maeno, Phys. Rev. B 57, 1236 (1998).

[30] T. M. Rice and M. Sigrist, J. Phys. Cond. Matt. 7, L643 (1995).

[31] A. Callaghan, C. W. Moeller, and R. Ward, Inorg. Chem. 5, 1572 (1966); J. M. Longo, P. M. Raccach, and J. B. Goodenough, J. Appl. Phys. 39, 1372 (1968).

[32] G. M. Luke, Y. Fudamoto, K. M. Kojima, M. I. Larkin, J. A. Duffy, S. M. Hayden, Y. Maeno, Z. Mao, Y. Mori, H. Nakamura, and M. Sigrist, Nature (London) 394, 558 (1998).

[33] T. Imai, A.W. Hunt, K.W. Thurber and F.C. Chou, Phys. Rev. Lett. 81, 1572 (1998).

[34] T. Kuwabara and M. Ogata, Phys. Rev. Lett. 69, 572 (1997).

[35] A. Gukasov, M. Braden, unpublished results.

[36] A. Gukasov, M. Braden, R. Papoular, S. Nakatsuji and Y. Maeno, cond-mat/0206156.

[37] Z. Fang and K. Terakura, Phys. Rev. B 64, 020209 (2001).

[38] T. Moriya, Spin fluctuations in itinerant electron magnetism, Solid-State Sciences 56, Springer Verlag, Berlin Heidelberg (1985).

[39] M.G. Hildebrand, M. Reedyk, T. Katsufuji and Y. Tokura, Phys. Rev. Lett. 87, 227002 (2001).

[40] C. Berthier et al., J. Phys. I France 6, 2205 (1996); R.E. Walstedt, B.S. Shastry and S.W. Cheong, Phys. Rev. Lett. 72, 3610 (1994).

[41] G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, and J. Kulda, Science, 278, 1432 (1997).

[42] B. Keimer, R.J. Birgeneau, A. Cassanho, Y. Endoh, R.W. Erwin, M.A. Kastner and G. Shirane, Phys. Rev. Lett. 67, 1930 (1991).

[43] S. Sachdev and J. Ye, Phys. Rev. Lett. 69, 2411 (1992).

[44] A. Schroeder, G. Aeppli, E. Bucher, R. Ramazashvili, and P. Coleman, Phys. Rev. Lett. 80, 5623 (1998).

[45] S. Sachdev, Science 288, 475 (2000).

[46] R. Joynt and T.M. Rice, Phys. Rev. B 38, 2345 (1988).

[47] J. Rossat-Mignod, L.P. Regnault, C. Vettier, P. Bourges, P. Burlet, J. Bossy, J.Y. Henry and G. Lapertot, Physica C, 185-189, 185 (1991).

[48] H.F. He, P. Bourges, Y. Sidis, C. Ulrich L.P. Regnault, S. Pailhès, N.S. Berzigiarova, N.N. Kolesnikov, and B. Keimer, Science, 295, 1045 (2002) and references therein.

[49] H.Y. Kee, J. Phys. Condens. Matter 12, 2279 (2000).

[50] D. Fay and L. Tewordt Phys. Rev. B 62, 4036 (2000).

[51] See e.g., F. Onufrieva and P. Pfeuty, Phys. Rev. B 65, 054515 (2002).

[52] F. Laube, G. Goll, H. v. Löhneysen, M. Fogelström, and F. Lichtenberg, Phys. Rev. Lett. 84, 1595 (2000).