Surface Water Wave Topography Construction using Free Surface Synthetic Schlieren Method for Demonstration of Ripple Tank Wave Phenomena

S Yapo¹, E Seesomboon², N Chattrapiban³,⁵ and N Pussadee³,⁴,⁵

¹ Master’s Degree Program in Teaching Physics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand
² Department of Mechanical Engineering, Faculty of Engineering, Chiang Mai University, Chiang Mai 50200, Thailand
³ Department of Physics and Materials Science, Chiang Mai University, Chiang Mai 50200, Thailand
⁴ Research Center in Physics and Astronomy, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand
⁵ Thailand Center of Excellence in Physics, Commission on Higher Education, 328 Si Ayutthaya Road, Bangkok 10400, Thailand

E-mail: nirut.p@cmu.ac.th

Abstract. The ripple tank is a popular water wave phenomena demonstration tool for secondary school students. The projected wave images are observed in bright - dark patterns on a screen. This tool, however, falls short in presenting the amplitude of the waves which is another important parameter in explaining the wave phenomena. The free surface synthetic schlieren (FS-SS) method presents an immense technical tool for solving this problem. FS-SS is an optical method based on light refraction in determining the surface gradient field from the motion of a random dot pattern when the water surface is perturbed. The surface height of the wave is constructed using the inverse gradient operation on the displacement gradient field of the random dot pattern. In this work, Wave propagation, reflection, diffraction, and interference pattern surface construction were performed to visualize wave phenomena in 3D.

1. Introduction

Ripple tanks are commonly used in demonstrating wave phenomena in which wave propagation, reflection, diffraction, and interference can be visualized as projected bright - dark patterns on a screen. These 2D demonstrations do not, however, reveal the true 3D nature of wave phenomena. If the amplitude of the wave is not known, phenomena such as superposition cannot be discussed in great depth. The free surface synthetic schlieren (FS-SS) method presents an immense technical tool for solving this problem. FS-SS is an optical method based on light refraction in determining the surface gradient field from the motion of a random dot pattern when the water surface is perturbed. The surface height of the wave is constructed using the inverse gradient operation on the displacement gradient field of the random dot pattern. In this study, applying the FS-SS technique in bridging
the gap between 2D ripple tank demonstrations and actual 3D wave phenomena to improve student understanding of wave phenomena.

2. Free surface synthetic schlieren method

Surface topography construction using the FS-SS technique relies on refraction at the interface of two media with different indices of refraction. There are two main steps in constructing the surface. The displacement gradient field of a random dot pattern \( \delta \vec{r}(x, y) \) is first calculated, then the comparison between images of the pattern from the perturbed surface and flat surface is made using a digital image correlation (DIC) algorithm. The height topography of the surface is then obtained from inverse gradient computation of the displacement field. The surface topography \( h(x, y) \) can be obtained from equation [1]:

\[
h(x, y) = h_p - \nabla^{-1} \left( \frac{\delta \vec{r}(x, y)}{\left( \frac{1}{1-n'/n} \right) h_p - \frac{1}{H}} \right)
\]  

where \( n = 1 \) and \( n' = 1.33 \) are the refractive indices of air and water respectively, \( h_p \) is the depth of the water and \( H \) is the distance of the camera from the bottom of the tank. Equation (1) originates from small angle approximation, if the surface curvature of the wave and therefore its amplitude is too high, ray crossing will occur leading to errors in the \( \delta \vec{r} \) calculation.

3. Experimental setup

The experiment was performed in an ITL-1071 model ripple tank (Intellect, Thailand) with a 30.0 cm \( \times \) 30.0 cm viewing area. The tank was filled with water up to 1.95 cm in depth. A random pattern made of 0.4 mm diameter black dots with 50% of the bottom of the tank was placed underneath the tank. The wave propagation generated by 10 Hz oscillation spherical and bar dippers were recorded at 50 fps using a digital camera (SONY RX10M3) with 3,840 pixels \( \times \) 2,160 pixels resolution placed 90.0 cm above the dot pattern. Absorbent fabric was used to line the tank’s perimeter to reduce wave reflection off of the walls. The ripple generator system had to be separated from the tank as minute vibrations could result in small capillary wave generation on the water’s surface. The MATLAB PIVlab toolbox [2] was used to perform displacement field gradient calculations from reference and recorded images. The computed gradient data was imported into the MATLAB PIVmat toolbox [1] to compute surface construction using the inverse gradient operation.

4. Result and discussion

The summary of water-wave surface construction using FS-SS method is shown in Figure 1.

![Figure 1](image-url)

**Figure 1.** The surface topography construction using a DIC algorithm of (a) reference and (b) refracted images to obtain (c) displacement field gradients and subsequently using the inverse gradient operation to obtain (d) surface height.
Surface wave generated by a circular dipper is illustrated in Figure 2. It is noteworthy that the wave displays significant attenuation with a constant of $1.1 \times 10^{-2} \text{ mm}^{-1}$.

For reflection of plane waves off of flat surfaces, shown in Figure 3, not only do the two FS-SS constructed image shows the reflected nature of the wave, it also signifies the superposition of incident and reflected waves. When compared with the amplitude of the plane wave arriving at the position of the flat obstacle, the amplitude of the constructed superposed waves are twice of the incident wave.

It can be seen from Figure 4 (a) that for the wavelength to opening ratio of 37, the wave spreads out of the opening as circular wave pattern. In Figure 4 (b) for wavelength to opening the ratio of 0.5, the plane wave emerges from the opening except for around the edge, of the opening that the circular wave was observed.

With two in-phase waves generated by two simultaneous oscillating spherical dippers, the interference pattern can be constructed with FS-SS technique. Figure 5 shows a FS-SS snapshot image of waves at 0.36 s after first wavefront, were generated. Amplitudes of individual circular wave at of points A and B are 0.18 mm and 0.18 mm respectively. The constructive superposed wave at point C has 0.40 mm amplitude, approximately twice of those at points A and B.
Figure 4. 3D and 2D constructed surfaces of diffracted wave pattern of a 18.5 mm wavelength plane wave passing through (a) 37 and (b) 0.5 wavelength to the opening ratio.

Figure 5. 3D (a) and 2D (b) of interference pattern $t = 0.36$ s after dippers started to oscillate.

5. Conclusion
The surface water-wave topography was successfully constructed for wave propagation, reflection, diffraction, and interference phenomena. The amplitude value of the wave was used in explaining the wave superposition principle. The results of this work will be further developed into a wave visualization tool kit for future secondary school class demonstration.

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