Elastoplastic Bending Load - Carrying Capacity of Steel Members

Pavol Juhás 1

1 Institute of Technology and Business in České Budějovice, Department of Civil Engineering, Okružní 517/10, 370 01 České Budějovice, Czech Republic

156364@mail.vstecb.cz

Abstract. Many members of steel structures are subjected mostly to bending. In these cases can be especially very useful to use by resources also plasticity of structural steels. Therefore, the new international and national standards for the design of steel structures allow the elastic and plastic calculation methods and procedures. The elastoplastic bending load-carrying capacity of steel members depends in a high degree on local stability of their webs and flanges. From local stability aspects the steel members can have compact or slender cross-sections. The compactness of steel cross-sections is only relative. It depends on the loading level or material utilization, and on the buckling resistance of their webs and flanges in the most loaded cross-sections and areas. The judgment of steel cross-sections compactness in this content is complicated stability problem. The new standards for the design of steel structures contain the specific classification of the cross-sections due to dimensions and slenderness of their compression and bending parts. The paper contains the results of the experimental-theoretical investigation of the elastoplastic local stability and load-carrying capacity of steel members. The proposed methodology enables the elastoplastic calculation of the cross-section bending resistance depending from the limit development of plastic strains.

1. Introduction

New international and national standards for the design of steel structures have integrated and equalized the elastic and plastic calculation methods and procedures that are based on elastic or plastic theory. Elastic analysis can be used in every case. However plastic analysis can be used only if the more specific material, loading, constructional, strength and stiffness assumptions and requirements are fulfilled. Sufficient local stability is one of the main assumptions and requirements of the plastic analysis and design of steel structures. The paper deals with determination of the elastoplastic bending load-carrying capacity of steel members with reflection local stability their flanges and webs following respective experimental results. The local stability aspects are taken into consideration through the limit development of plastic strains $\varepsilon$ in decided member cross-sections and areas [4, 5].
2. Local stability and classification of steel structures

The load-carrying capacity of steel members subjected mainly to bending depends in large measure on local stability of their compressed flanges and bending webs in decided cross-sections and areas. From local stability aspects steel members have compact, semi-compact or slender cross-sections, Figure 1.

The local stability, compactness and slenderness of the steel member cross-sections are relative. They depend on the loading level or material utilization, and on the buckling resistance of flanges and webs. The buckling resistance and real behaviour of the compressed flanges and bending webs depend on more parameters (material properties, geometrical dimensions and stiffness, material and geometrical imperfections, boundary conditions). Therefore, the judgment of steel cross-section compactness and slenderness is complicated stability problem that has been theoretically analyzed through respective differential equation systems [1, 2, 6]

The slenderness $\beta$ of webs and flanges (width to thickness ratio, $\beta = d / t_w$, $b / t_f$, $c / t_f$) is the characteristic parameter of buckling resistance for usual cases of steel cross-sections, Figure 1. Generally, it is economically advantageous to design the web of steel members more slender and flanges more compact. This approach has extra value for the effective design of steel members and cross-sections subjected mostly to bending.

The international standard ISO 10721-1:1997, the new European standard EN 1993-1-1:2005, and some transformed national standards for the design of steel structures contain the specific classification of the cross-sections due to dimensions and slenderness of their compression and bending parts. In these standards four classes of the cross-sections are defined:

- cross-sections of the class 1 are those in which can form a plastic hinge with the sufficient rotation capacity and with the plastic moment resistance $M_{pl}$, for the full plastic analysis;
- cross-sections of the class 2 are those in which can develop their plastic moment resistance $M_{pl}$, but they have limited rotation capacity;
- cross-sections of the class 3 are those in which the yield stress $f_y$ can be reached only in extreme fibers with the elastic moment resistance $M_{el}$;
- cross-sections of the class 4 are those in which the reduction of their moment resistance due to the effect of local buckling of the webs and flanges is necessary.

Cross-sections of the classes 1, 2 and 3 can be considered as the compact or semi compact for the defined level of material utilization. In such cases of the cross-sections full sectional dimensions can be considered in analysis. Class 4 presents slender cross-sections. In such case the effective part of cross-sections must be considered in analysis.

The standards define maximal values of the slenderness $\beta$ for individual webs and flanges of the cross-sections subjected to compression or bending, otherwise combined compression and bending:
$\beta_0$ for class 1, $\beta_0$ for class 2 and $\beta_1$ for class 3. Some differences result from comparing of the corresponding slenderness by individual standards.

The limit web slenderness $\beta_1$ is very important for the standards and practical design. If the web slenderness $\beta = \beta_1$, than with the full elastic bending moment $M_{el}$ of the cross-section can be calculated by simple bending theory,

$$M_{el} = f_y W_{el} \text{ or } M_{el} = f_y W_{el} \gamma_{M_0},$$  \hspace{1cm} (1)

$f_y$ is the yield stress, $W_{el}$ is the elastic cross-section modulus and $\gamma_{M_0}$ is the material partial factor.

The exact estimation of the web slenderness $\beta_1$ is problematic, because it depends on the real boundary conditions and production – material and geometrical imperfections. Only the classical critical conception with idealized assumes allows clearly determination of the web slenderness $\beta_1$ [2, 6]. According to this conception the critical normal stress for the elastic limit stage of member plate element

$$\sigma_{kr} = k_\sigma \sigma_0 = 190 \text{ ksi} \ k_\sigma \beta_1^2$$  \hspace{1cm} (2)

and limit web slenderness

$$\beta_1 = 435.89 \sqrt{k_\sigma f_y} = 28.43 \sqrt{k_\sigma} \sqrt{235/f_y},$$  \hspace{1cm} (3)

$k_\sigma$ is the web buckling coefficient.

The real behavior of steel members and their webs is also affected by initial production imperfections; therefore the limit web slenderness $\beta_1$ should be smaller than by critical conceptions. In the last Slovak standard STN 73 1401:1998 was

$$\beta_1 = 130 \sqrt{235/f_y}.$$  \hspace{1cm} (4)

In the actual European standard EN 1993-1-1:2005 the unfavorable effect of initial production imperfection is takes even more, therefore the limit web slenderness $\beta_1$ is the smallest,

$$\beta_1 = 124 \sqrt{235/f_y}.$$  \hspace{1cm} (5)

The USA Specification AISC LRFD:1999 takes into consideration also the elastic constraint of the bending web into flanges, therefore by this specification the limit web slenderness

$$\beta_1 = 970 \sqrt{F_y} = 2547 \sqrt{f_y} = 166.15 \sqrt{235/f_y},$$  \hspace{1cm} (6)

$F_y$ is in ksi and $f_y$ in MPa.

Some differences result from comparing of the corresponding standard limit web slenderness’s $\beta_1$. They are relatively significant, especially between limit slenderness $\beta_1$ by EN 1993-1-1:2005 and by AISC LRFD: 1999. The actual standard EN 1993-1-1:2005 very reduces theoretical critical value of the limit web slenderness $\beta_1$ with respect to unfavourable effect of the initial imperfections.

The local stability and the limit web slenderness $\beta_{01}$ and $\beta_{02}$ depend first of all on the needed rotation capacity, or better, on the actual development of plastic strains $\varepsilon$ in the most loaded cross-sections and their areas. However, in standards the rotation capacity or plastic strains $\varepsilon$ for cross-sections of the class 1 and class 2 are not specified. Therefore, it is difficult to classify the limit web slenderness $\beta_{01}$ and $\beta_{02}$ for responsible plastic design of steel members from local stability aspects [7]. For the webs subjected to bending with pin supporting along both longitudinal borders the standards contain following limit slenderness $\beta_{01}$ and $\beta_{02}$:

$$\text{STN} \rightarrow \beta_{01} = 70 \sqrt{235/f_y}, \beta_{02} = 80 \sqrt{235/f_y}$$  \hspace{1cm} (7)
EN $\rightarrow \beta_{01} = 72 \sqrt{235 / f_y}, \beta_{02} = 83 \sqrt{235 / f_y}$. \hspace{1cm} (8)

The Specification AISC LRFD:1999 contains the following limit web slenderness for plastic design of the cross-sections:

$$\beta_0 = \frac{640}{\sqrt{F_y}} = 1680 / \sqrt{f_y} = 109.62 \sqrt{235 / f_y}.$$ \hspace{1cm} (9)

The web slenderness $\beta_{01}$ and $\beta_{02}$ according to standards STN 73 1401:1998 and EN 1993-1-1:2005 are much closed. The web slenderness $\beta_0$ by specification AISC LRFD: 1999 is very high opposite slenderness $\beta_0$ according to standards STN 73 1401:1998 and EN 1993-1-1:2005.

If the web slenderness $\beta < \beta_1$, than the elastoplastic bending moment $M_{ep}$ or full /plastic bending moment $M_{pl}$ of the cross-section can be calculated,

$$M_{ep} = f_y W_{ep} \text{ or } M_{ep} = f_y W_{ep} \gamma_{MB},$$ \hspace{1cm} (10)

$$M_{pl} = f_y W_{pl} \text{ or } M_{pl} = f_y W_{pl} \gamma_{MB}. \hspace{1cm} (11)$$

$W_{ep}$ and $W_{pl}$ are the elastoplastic and full plastic modulus of the cross-section.

The elastoplastic bending moment $M_{ep}$ and full plastic bending moment $M_{pl}$ of the cross-section depend mostly on the actual material properties of applied structural steel, on the needed development of plastic strains $\varepsilon$ and buckling resistance of the bending web and compression flange.

Buckling resistance or local stability of the steel member webs subjected to bending in the elastoplastic stadium ($\beta < \beta_1$) can be sufficient accurately defined by limit strain $\varepsilon_u$ in the most loaded border of its compressed part ($\varepsilon_u > \varepsilon_y$). According the critical conception limit strain $\varepsilon_u = \varepsilon_{cr}$. For the web subjected to bending with pin supporting along both longitudinal borders critical strain $[3]$,

$$\varepsilon_{cr} = 21.582 / \beta^2$$ \hspace{1cm} (12)

More theoretical works and some results about the elastoplastic local stability and load-carrying capacity of the steel member webs already exist at present [6, 8]. But the real elastoplastic behaviour of steel members and their bending webs and compressed flanges is complicated. Therefore, the representative experimental knowledge and results about the real elastoplastic behaviour and failure mechanisms of steel members in their decisive cross-sections and areas are very important for development and précising of the elastoplastic analysis and design.

### 3. Experimental results and discussions

The experimental program included tests of 32 beams with usually welded I cross-sections of different geometrical dimensions, slenderness $\beta$ and stiffening of the web. The designed geometrical dimensions are listed in Table 1 and static schemes of the tested beams are presented in Figure 2.

| Beams | $h$ | $b$ | $t_f$ | $t_w$ |
|-------|-----|-----|-------|-------|
| N11, N21, N31, N41 | 380 | 160 | 10 | 5 |
| N12, N22, N32, N42 | 380 | 160 | 10 | 5 |
| N13, N23, N33, N34 | 380 | 160 | 10 | 5 |
| N14, N24, N34, N44 | 380 | 160 | 10 | 5 |
| C11, P11, R11, C12 | 520 | 200 | 10 | 6 |
| R12, C13, P13, R13 | 520 | 200 | 10 | 6 |
| C21, R21, C22, P22 | 520 | 200 | 10 | 6 |
| R22, C23, P23, C23 | 520 | 200 | 10 | 6 |

$\beta = (h - 2t_f) / t_w, \beta_r = (b - t_w) / 2t_f, \gamma = A_w / A$
The tested beams were made by usual production-technological process and conditions with some geometrical and material imperfections. The main aims of experimental program were investigate the elastoplastic load-carrying capacity and actual behaviour of steel members in the most loaded cross-sections and areas with accent on the development of plastic strains $\varepsilon$, local buckling of their webs $w$, global deflections $v$ and mechanisms of member failure.

The tested beams were loaded by successively increasing concentrated forces $P$, realized by hydraulic jacks. The tests continued till beginning of beam failure. All beams were horizontally supported to prevent their lateral-torsion buckling. The objectives of all realized beam tests were in order to investigate:
- successive development of the elastoplastic strains $\varepsilon$ in the most loaded points, cross-sections and stiffened web fields by tensometers and measuring exchange,
- local buckling $w$ in the most loaded stiffened web fields by static and movable deflection pickups,
- global vertical deflections $v$ in the place of end supports and in middle cross-sections of the beams by mechanical or electrical deflectometers.

Accordance with the research aims all beams failure in consequence of the local web and flange buckling in their most loaded areas and stiffened fields. Generally, the web buckling was the reason for failure, but total failure of the tested beams formatted by induced buckling of their compassion.
flanges. The typical failure mechanism of tested beams in the decided web field subjected to pure bending is presented in Figure 3.

![Figure 3. Typical web and flange failure mechanism of tested beams](image)

The measured, evaluated and analyzed values of strains $\varepsilon$, buckling $w$ and deflections $v$ have offered very important information and principal knowledge about local and global failure mechanisms of tested beams in dependency on the level and process of their loading [5].

The local stability of tested beams well characterized investigated dependencies of the load $P$ and web buckling $w$ in individual points of the most loaded areas and stiffened fields. The buckling $w$ of the web subjected prevailing to bending depends first of all on its slenderness $\beta$. Generally, the web buckling $w$ increased in accordance with increasing of the beam loading $P$ and web slenderness $\beta$. If the web slenderness $\beta$ is not too large, then the buckling $w$ is small up to some level of the loading. According to the classical stability conception the equivalent load has been accepted as the experimental critical load $P_{cr,exp}$. On the base of observed dependencies $P - w$ the experimental critical load $P_{cr,exp}$ was assigned for every relevant tested beam.

The experimental critical load $P_{cr,exp}$ can be generally less or higher than the theoretical elastic limit load $P_{el}$ or even plastic limit load $P_{pl}$. Accordingly the critical load $P_{cr,exp}$ can be elastic ($P_{cr,exp,el}$), elastoplastic ($P_{cr,exp,ep}$) or plastic ($P_{cr,exp,pl}$) experimental critical load, so

- if $P_{cr,exp} < P_{el}$ then $P_{cr,exp} = P_{cr,exp,el}$
- if $P_{el} < P_{cr,exp} < P_{pl}$ then $P_{cr,exp} = P_{cr,exp,ep}$ and
- if $P_{cr,exp} > P_{pl}$ then $P_{cr,exp} = P_{cr,exp,pl}$.

The obtained experimental results confirmed that the elastoplastic load-carrying capacity of the tested beams depends on local buckling of their webs [5].

For illustration the dependences of load $P$ and strains $\varepsilon_x$ in the individual measured points of the most loaded cross-section of the beam N12 are presented in Figure 4, $P_{el}$ and $P_{pl}$ are theoretical elastic and plastic limit load, $P_{cr,exp}$, and $P_{u,exp}$ are experimental critical and ultimate load.
Figure 4. Dependences of load $P$ and strains $\varepsilon_x$ in the individual measured points of the beam N12.

The obtained experimental knowledge about the real development of plastic strains $\varepsilon$ in the most loaded cross-sections and areas of the tested beams are very important for the determination of the elastoplastic and plastic load-carrying capacity of steel members from local stability aspects. The obtained experimental results confirmed that the load-carrying capacity of steel members and their webs depends on the development of the elastoplastic strains $\varepsilon$ in the most loaded cross-sections and their areas.

The investigated distributions of strains $\varepsilon_x$ in the cross-sections subjected to bending or bending and shear with prevailing bending well answer to assumptions of the simple bending theory. From idealized distributions of the strains $\varepsilon_x$ the experimental limit strains $\varepsilon_{cr,exp}$ and $\varepsilon_{u,exp}$ were assigned for every relevant tested web and beam.

The experimental limit strains $\varepsilon_{cr,exp}$ and $\varepsilon_{u,exp}$ has rather large random variable, but they however depend on the web slenderness $\beta$. It means that the limit development of plastic strains at steel members can be defined from local stability aspects by relation $\varepsilon_u - \beta$, where $\varepsilon_u$ is the maximal plastic strain in the compressed part of their most loaded cross-section. Therefore, according to the obtained experimental results and related theoretical results the empirical relation $\varepsilon_u - \beta$ was specified [2, 5].

The relation $\varepsilon_u - \beta$ allows the optional web classification of steel members from local stability aspects, in accordance with expected or select development of plastic strains $\varepsilon$ in the most stressed cross-sections.

The necessary development of plastic strains $\varepsilon$ for achievement of full plastic bending moment $M_{pl}$ in the most stressed cross-section of steel members is not to large. It is markedly outcome of the material hardening that in steel beams especially arises under bending loading. The necessary
development of plastic strains $\varepsilon$ depends on the static uncertainty of the member or structure and on the equivalent redistribution of the loading, that in some measure depends on the number of the plastic hinges needed for development of the total plastic failure mechanism.

It is sufficient for one plastic hinge in the most stressed cross-section with full plastic bending moment $M_{pl}$, if the maximal plastic strains $\varepsilon_{max} \approx 4 \varepsilon_y$ ($\varepsilon_y$ is the yield strain). Such plastic strains $\varepsilon$ it can be assumed and allowed in the cross-sections of class 2, according to used standard classification. For statically undetermined structures, where the loading redistribution is also utilized, the maximal plastic strains should be higher, $\varepsilon_{max} > 4 \varepsilon_y$. The real value of the maximal plastic strains $\varepsilon_{max}$ apparently depends on the measure of loading redistribution and on the effect of material hardening.

![Figure 5. Designed relation of the limit strains $\varepsilon_u$ to slenderness $\beta$, steel S235.](image)

Following experimental and related theoretical results the author of paper worked out the original methodology for calculation of the elastoplastic bending load-carrying capacity of the steel members $M_{ep}$ with taking account local stability aspects [4, 5]. This methodology is based on the limit development of plastic strains in the most stressed cross-section. This development is defined by limit strains $\varepsilon_u$ in the edge fibers of compressed web part subjected to bending. The limit strains result from proposed relation $\varepsilon_u - \beta$.

The author methodology was already applied in the last standard STN 73 1401:1998. The applied relation is slightly modified to respect the standard limit slenderness of the web $\beta_{01}, \beta_{02}$ and $\beta_{1}$. The standard relation is shown in Figure 5.

The presented methodology and standard relation allow to calculate the elastoplastic bending moment $M_{ep}$ of the cross-section if the web slenderness $\beta_{02} < \beta < \beta_{1}$. For symmetrical I cross-section

$$M_{ep} = M_{pl} - M_{el,w} \left( \varepsilon_y / \varepsilon_u \right)^2,$$

$M_{el,w}$ is the elastoplastic bending moment of the web.

The previous author linear relation $M_{ep} - \beta$ has had form [3]:
\[ M_{ep} = M_{pl} - (M_{pl} - M_{el})(\beta - \beta_0)/(\beta_1 - \beta_0). \]  

(14)

The similar linear relation is also applied in Specification AISC LRFD:1999. The limit slenderness \( \beta_0 \) and \( \beta_1 \) are, however, different. The particularity of this standard is that for open I cross-sections takes into consideration also residual stresses, differentially for rolled and welded cross-sections. The actual EN 1993-1-1:2005 contains already some procedure for calculation of the elastoplastic bending moment by the effective cross-section of class 2, which is presented in the Figure 6.

![Figure 6. Effective cross-section of class 2 according to EN 1993-1-1:2005.](image)

It is clear from Figure 6 that assumed effective cross-section by EN 1993-1-1:2005 is in conflict with assumptions of simple bending theory and principal experimental knowledge.

4. Conclusions

The obtained and partially presented research knowledge and results have allowed following conclusions:

- The elastoplastic load-carrying capacity of the steel members and their cross-sections at large measure depends on local stability of their webs and flanges subjected to bending and compression.
- The local stability of the steel member webs and flanges subjected to bending and compression depends on the real development of the elastoplastic strains \( \varepsilon \) in the most stressed cross-sections and areas.
- According to the experimental and theoretical results the relation for limit plastic strains \( \varepsilon_u \) and web slenderness \( \beta \) from local stability aspects has been established.
- The proposed relation \( \varepsilon_u - \beta \) allows the common web classification of steel member cross-sections from local stability aspects in accordance with expected development of plastic strains \( \varepsilon \).
- The proposed methodology enables to calculate the elastoplastic bending load-carrying capacity of steel members depending on the development of plastic strains \( \varepsilon \) in the most stressed cross-sections and areas.
- The proposed methodology does need no classification of the cross-sections for the elastoplastic and plastic calculation and design of steel members, if their needed rotation capacity or needed plastic strains \( \varepsilon \) are known or defined.
- The proposed methodology enables to calculate the elastoplastic bending load-carrying capacity of the symmetrical and unsymmetrical cross-sections depending on the development of plastic strains \( \varepsilon \).
- The proposed methodology unifies the elastic, elastoplastic and plastic calculation and design of steel members from local stability aspects.
The proposed methodology enables also calculation and judgment of the elastoplastic deflections of steel members and structures.

At the end, the proposed methodology enables also calculation and judgment of the elastoplastic deflections of steel members and structures. According to present state it appears necessary for next improvement of the elastoplastic calculation and design of steel members and structures predominantly:

- To precise the limit web slenderness $\beta_1$ for the cross-sections of steel members and structures, with taking account the real production-technological process and resulting material and geometrical imperfections.
- To precise the relation $\epsilon_u - \beta$ for web slenderness $\beta < \beta_1$ of steel members and structures, with taking account the material properties, geometrical dimensions and also interaction and stiffness of the web and flanges.
- To work out simplified procedure for identification of the real plastic strains $\epsilon_u$ in the most stressed cross-sections and areas.

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