SEARCHING FOR LIGHT ECHOES DUE TO CSM IN SN Ia SPECTRA

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ABSTRACT

We present an analytical model for light echoes (LEs) coming from circumstellar material (CSM) around Type Ia Supernovae (SNe Ia). Using this model we find two spectral signatures at 4100 Å and 6200 Å that are useful to identify LEs during the Lira law phase (between 35 and 80 days after maximum light) coming from nearby CSM at distances of 0.01-0.25 pc. We analyze a sample of 89 SNe Ia divided in two groups according to their $B - V$ decline rate during the Lira law phase, and search for LEs from CSM interaction in the group of SNe with steeper slopes by comparing their spectra with our LE model. We find that a model with LEs + pure extinction from interstellar material (ISM) fits better the observed spectra than a pure ISM extinction model that is constant in time, but we find that a decreasing extinction alone explains better the observations without the need of LEs, possibly implying dust sublimation due to the radiation from the SN.

1. INTRODUCTION

Type Ia supernovae (SNe Ia) are one of the most studied objects in astronomy. Owing to their standardizable high luminosities that make them unrivaled distance indicators up to high redshifts ($z \sim 2$, Jones et al. 2013), astronomers searching for the ultimate fate of the universe have studied them in ever greater detail, discovering several thousands of them up to now (e.g. Sako et al. 2014). Despite the success as cosmological probes, increasing observations and clues reveal that the puzzle of the nature and mechanism generating these colossal explosions is still far from reaching a conclusive solution.

As best shown in recent well-studied nearby objects (Nugent et al. 2011; Bloom et al. 2012), SNe Ia may originate from the explosion of a compact CO-rich white dwarf (WD) in a binary system. A common candidate considered for the binary companion has for long been a non-degenerate star such as a main-sequence or a red giant star, that donates mass to the WD (single degenerate, SD, scenario) either in a stable fashion so that the WD nears the Chandrasekhar mass and explodes in the Chandrasekhar mass (sub Ch-SD, e.g. Blondin et al. 2013; Sim et al. 2013; Röpke et al. 2012) or via unstable accretion leading to an initial detonation in the outer layer of the WD that triggers a subsequent detonation near the center prior to reaching the Chandrasekhar mass (sub Ch-SD, e.g. Sim et al. 2012; Kromer et al. 2010).

From an observational point of view, the SD scenario model has both evidence for and against. Among the observations that disfavor the model are: the absence of hydrogen and helium in their spectra (Lundqvist et al. 2013; Shappee et al. 2013), as well as the absence of radio and X-ray emission (Horesh et al. 2012; Chomiuk et al. 2012) which set tight constraints on mass loss from a progenitor: the non-detection of early emission from shock interaction with a companion (Bloom et al. 2013; Bianco et al. 2011; Hayden et al. 2010) and the pre-explosion non-detections (Li et al. 2011) that generally rule out red giants and He-stars; the lack of sufficient galactic X-ray emission (Gilfanov & Bogdán 2010; Di Stefano 2010) and UV radiation (Woods & Gilfanov 2013; Johansson et al. 2014) expected from mass accretion in the Ch-SD scenario; and the measured SN Ia rate as a function of redshift which challenges the modelled delay time distribution for the classical Ch-SD scenario (Maoz & Mannucci 2012; references therein). Some of these issues can be addressed through different alternative SD models (Justham et al. 2011; Di Stefano et al. 2011).

Among the observational evidence that favors the SD channel is the presence of nearby circumstellar material (CSM), presumably from mass loss in the progenitor system prior to explosion, manifested from: CSM/ejecta interaction (Hamuy et al. 2003; Silverman et al. 2013c), and most notably in the case of PTF11kx (Dilday et al. 2012); the discovery of narrow Na I D absorption lines that vary with time (Patat et al. 2007; Blondin et al. 2009; Simon et al. 2009; Sternberg et al. 2013); and the statistical preference for the interstellar lines to show blueshifts (Sternberg et al. 2011b; Maguire et al. 2013; Phillips et al. 2013). Altogether, the observational evidence suggests the possibility of multiple channels for SNe Ia.

Additionally, it has been suggested that such nearby CSM could affect the colors of SNe Ia through light scattering in the line of sight and explain in this way some of the differences in total to selective extinction ratios ($R_V$) found in SN hosts compared to the Milky Way, MW (Wang 2005; Goobar 2008; Amanullah & Goobar 2008; West 2010; Goes et al. 2013; Kaviani et al. 2013).
Heavily extincted SNe clearly show a different $R_V$ while SNe with moderate extinction show values consistent with the MW (e.g. Mandel et al. 2011, Burns et al. 2014). Cosmological studies using standard light-curve fitters obtain a luminosity-color relation that suggest reddening laws lower than the MW (e.g. Guy et al. 2005), however it is possible that SN intrinsic colors are more complicated and incorrectly modeled (e.g. Scollnic et al. 2014, Chatard et al. 2014, Conley et al. 2007).

Understanding the origin of the dispersion of SNe Ia colors not only affords the opportunity to understand their nature, but also to remove a major source of systematic uncertainty in SN Ia cosmology. As shown in Kim et al. (2013), color might indeed be the principal parameter of diversity in SN Ia light-curves, followed only by the well-known light-curve width parameter of SNe Ia. Forster et al. (2013) hereafter F13 showed that the SN Ia color evolution with time is related to the strength of the narrow sodium absorption, suggesting that at least some part of it might originate from a closer interaction with dust than with host interstellar medium (ISM). In particular, they found that redder objects at maximum light have stronger narrow absorption lines and evolve faster from red to blue during the late time evolution of the Lira law decline of 30 to 90 days past maximum light (Lira 1995; Phillips et al. 1999). Possible explanations for this are light-echoes from CSM that affect the late-time colors or, alternatively, CSM dust sublimation in the line of sight.

In this paper, we aim to investigate the results of F13 further and test the hypothesis of nearby CSM by looking for spectroscopic signatures of light-echoes in a large sample of nearby SN Ia spectra in the Lira law phase. Echoes from nearby ISM (and possible CSM) have previously been reported for SN1991T, SN1998bu, SN2006X, SN1992G and SN2014J at nebular phases (Schmidt et al. 1994, Cappellaro et al. 2001, Wang et al. 2008a, Silverman et al. 2013a, Crotts 2014). These individual studies focused on echoes generated at large distances from the SN, tens of hundreds of parsecs away, scattering hundreds of days past maximum light.

Here we search for light echoes (LEs) at earlier times (>30d past maximum light), coming from nearby CSM dust that is at less than a parsec from the SN. Such CSM can potentially affect the colors and the Lira law decline rate (Amanullah & Goobar 2011). Hence we search for LEs in the group of SNe analyzed in F13 that presented more extinction and a steeper than normal $B-V$ evolution (hereafter fast Lira decliners) to test the hypothesis that these may originate in regions of nearby CSM. To do this, we use SNe with a shallow Lira law slope (hereafter slow Lira decliners) as a reference set of SNe without CSM interaction.

In §2 we present our simple LE model. Then in §3 we focus on the prediction of observable spectroscopy features to look for light echoes. In §4 we present the data and the analysis. §5 summarizes the results of our search for LEs and in §6 we discuss the success of our LE model, the validity of our assumptions and other possible mechanisms that could explain fast Lira decliners. Finally, the main conclusions are summarized in §7.

2. LIGHT ECHO MODEL

The effects of the interaction of light with intervening dust from CSM causing scattering away from the line of sight, and therefore extinction and reddening, has been studied and modeled in depth in the past (e.g. Chevalier 1986, Wang 2005, Patat 2005, Patat et al. 2006, Goobar 2008). We present here a simple analytical model that is easy to implement numerically and makes clear observable predictions to directly compare with data. The CSM consists of a simple spherically isotropic shell of dust (with $R_V = 3.1$), that absorbs and scatters the light of the SN. The radius of the shell is initially fixed at 0.05 pc to produce LEs reaching the observer with time delays of ~50 days and affecting the colors during the Lira law phase, as light emitted at maximum light is observed at later epochs. At these distances we expect the temperature of the dust to be slightly lower than the sublimation temperature (2,000 K). We only consider single scattering for simplicity and because multiple scattering in the CSM becomes important when its optical depth is larger than 1 (see Patat 2005). According to our analysis all the SNe we considered have a total optical depth $\lesssim 1$ in the visible, with the exception of SN1997cw, SN1999gd, SN2003cg and SN2006X which have a total extinction $A_V > 1$ (see §3), but their $A_V$ due to the CSM extinction is lower than unity according to our models (see §5.2). Hence, ignoring multiple scattering is a reasonable approximation.

We assume that the observed flux is the sum of the light coming directly from the SN and the SN light scattered by the CSM, i.e. light echoes. The direct flux contribution from the SN at a given epoch $t$ that is extincted and scattered by intervening dust without including the contribution from LEs, can be written as:

$$f(t, \lambda) = f^0(t, \lambda) e^{-\sigma_s(\lambda)} N_{\text{tot}}$$

(1)

where $f^0$ is the intrinsic flux of the SN, $\sigma_s(\lambda)$ and $\sigma_d(\lambda)$ are the cross sections of the dust particles for scattering and absorption of photons at wavelength $\lambda$, respectively. $N_{\text{tot}}$ is the total column density of dust between the SN and the observer and is equal to $N_{\text{ISM}} + N_{\text{CSM}}$. Then, the total observed flux adding the LE contribution can be expressed as:

$$F(t, \lambda) = f(t, \lambda) + \text{LE}$$

$$= f^0(t, \lambda) e^{-\sigma(\lambda)} N_{\text{tot}} + S(t, \lambda) e^{-\sigma(\lambda)} N_{\text{CSM}}$$

(2)

where the first term is the intrinsic flux of the SN extincted by the total column density of dust (eq. 1) and the second term is the contribution from LEs, $S(t, \lambda)$, extincted by ISM dust. $\sigma(\lambda)$ is the sum of the scattering and absorption cross section. We can express $S(t, \lambda)$ as the sum of the light scattered by the CSM at different angles and epochs:

$$S(t, \lambda) = \int f^0(tr, \lambda) w(\lambda)(1 - e^{-\sigma(\lambda)} N_{\text{CSM}}) \Phi(\theta, \lambda) d\Omega$$

(3)

$$tr = t - \frac{D'(\theta) - D}{c} \approx t - \frac{R(1 - \cos(\theta))}{c}$$

(4)

where $w(\lambda)(1 - e^{-\sigma(\lambda)} N_{\text{CSM}})$ represents the fraction of light scattered by the CSM, which has a column density of $N_{\text{CSM}}$. $c$ is the speed of light, $w(\lambda)$ is the dust albedo and $tr$ is a pseudo retarded time, i.e. the time at which a scattered pulse of light is expected to be emitted to reach the
observer at the same time as a pulse emitted at time \( t \) going straight to the observer. \( D \) is the distance between the photosphere of the SN and the observer and \( D' \) is the path length travelled by a photon being scattered by the CSM to the observer at an angle \( \theta \) (see figure [1]).

\[ \Phi(\theta, \lambda) = \frac{1}{4\pi (1 + g(\lambda)^2 - 2g(\lambda)\cos(\theta))^{3/2}} \]  
(5)

where \( g \) is the degree of forward scattering. When \( g = 1 \) we have complete forward scattering and \( g = 0 \) means isotropic scattering \( (\Phi(\theta, \lambda) = 1/(4\pi)) \). We also define the parameter \( \tau_{\text{CSM}} = N_{\text{CSM}}/N_{\text{tot}} \) to simplify the notation, taking values between 0 and 1. Finally, defining a delay parameter \( \tau \equiv t - tr \), an extinction factor \( X(\lambda) = e^{-\tau(\sigma_c(\lambda)\tau + \sigma_w(\lambda))N_{\text{tot}}} \) and making a change of variable, using equation (4) we obtain

\[ S(t, \lambda) = \frac{\omega(\lambda)(1 - X^0_{\text{CSM}})}{\tau_{\text{max}}} \int_0^{\tau_{\text{max}}} f^0(t - \tau, \lambda)\Phi(\tau, \lambda)d\tau \]  
(6)

where \( \tau_{\text{max}} = 2R/c \) is the maximum delay for a light echo \( (\tau = \pi) \) and \( \Phi' (\tau, \lambda, \lambda) = 4\pi\Phi(\tau, \lambda, \lambda) \). We performed this integral numerically using the Simpson’s 1/3rd integration rule and a time step of one day to simulate spectra and to fit this model to real data.

3. LIGHT ECHO MODEL PREDICTIONS

To simulate spectra with different extinctions and LEs using eq. (6) we need to adopt a dust albedo, an extinction law, a phase function or \( g(\lambda) \), and spectral templates with no extinction nor LEs at different epochs. For the extinction law we take the parametrization proposed in Fitzpatrick (1999). We use the albedo \( w(\lambda) \) and the degree of forward scattering \( g(\lambda) \) from the MW used in Goobar (2008) which accounts for the dust properties of the CSM. We construct unreddened spectral templates at different epochs from weighted bootstrap averages of observed spectra of slow \( B - V \) Lira law decliners (see section 4.1), together with light-curve templates that we need since we normalize the spectra by their \( V \)-band flux (see section 4.1). In Figure 2 we show the different scenarios for late-time (Lira law phase) model spectra when pure extinction and simulated LEs affect the SN emission.

We search for a way to distinguish if part of the dust found at maximum light is producing light echoes. LE spectra are integrated spectra weighted by the light-curve, and thus dominated by spectra around peak (see Figure 2 with peak template spectrum and LE spectrum). LE spectra are blue and have very strong broad emission and absorption lines, with prominent peaks at 4000, 4600, 4900 Å and minima at 4400 and 6200 Å.

When LE spectra are added to SN spectra: (1) the fact that the LE spectra are blue has a low-order effect on the observed spectra by making the colors bluer, similar to less reddening and thus difficult to differentiate; (2) the strong broad lines add an additional modulation to the observed spectra that is very distinct to the effect of reddening, since it introduces differences on scales of a couple of hundred armstrongs. By looking specifically at the wavelengths where the LE spectra has peaks or minima, it is possible to differentiate between the two scenarios. In § 4 we compare these simulations to the observed spectra of SNe Ia.

In this simulation the main signature due to LEs is found near 4100 Å. This can be seen in Figure 3 where the shape of the spectrum gets considerably modified in the LE scenario (purple and blue lines) producing a characteristic signature. On the other hand, in the pure extinction scenario if the column density is reduced (black line), it produces just a smooth change in the spectrum compared to the same spectrum with extinction (reddest line). In particular, the shape of the feature near 4100 Å will not be affected.

Figure 2.— Upper panel: simulated spectra at 50 days past maximum light extincted by different amounts of dust \( (A_V = 0.5) \) with the same \( R_V = 3.1 \) extinction law. All spectra have been normalized to the same \( V \)-band flux. Bottom panel: simulated spectra at 50 days past maximum light with the same reddening law, but also with LEs due to CSM are shown. We fixed \( A_V = 0.5 \) and varied the fractional amount of CSM \( (U_{\text{CSM}}) \) with a radius of 0.05 pc. The green line represents a typical maximum light spectrum, while in orange the LE spectrum is shown on an arbitrary scale. All models are normalized to the same \( V \)-band flux.

These features change with the distance \( R \) between the CSM and the SN. Reducing the distance is analog-
By the SN intrinsic pWs if we normalize by the pW at maximum light without extinction or LEs, while in orange the LE spectrum is shown in an arbitrary scale. All models are normalized to V-band flux.

To further investigate the LE effects on spectra, we focus on the range between 3000-5000 Å, particularly on the absorption lines and their related measurable quantities such as the equivalent width and the slope of the continuum. We define five characteristic features, four of them that are presented in Figure 3, the Ca II H&K complex between 3500-4100 Å, and two absorption features originating mainly from a blend of Mg II and Fe II at 3800-4400 Å (“line 1”) and 4250-4800 Å (“line 2”); these two together form another larger feature at 3800-4800 Å (“line 3”), equivalent to the feature pW3 in Folatelli et al. (2013). Additionally, we use the line feature around the Si II absorption around 5800-6300Å (“line 4”).

For all LE and extinction models we measure pseudo-equivalent widths (pW), i.e., with a pseudo-continuum in a similar fashion to Garavini et al. (2007b), Bronder et al. (2008), Folatelli et al. (2013). We use a semi-automatic algorithm that searches for the pseudo-continuum at specific regions defined in Table 1. The algorithm also calculates the slope of the pseudo-continuum, which we find to be another good indicator of LEs. In figure 4 we show the predicted evolution of the pWs of line 1 for a SN with pure extinction (green lines) and with LEs due to CSM (black lines). The pWs vary significantly during the Lira phase for LE models with different CSM fractions, f_{CSM}, whereas they do not for different amounts of extinction. This effect is strongest at larger CSM distances of 0.05-0.25pc. The difference between the scenario with extinction and with LEs is still clearer and less unbiased by the SN intrinsic pWs if we normalize by the pW at maximum light (hereafter pW ratio).

### Table 1

| Feature | Blueward limit range (Å) | Redward limit range (Å) |
|---------|--------------------------|-------------------------|
| Ca II   | 3500 - 3800              | 3900 - 4100             |
| line 1  | 3800 - 4100              | 4250 - 4400             |
| line 2  | 4250 - 4400              | 4400 - 4800             |
| line 3  | 3800 - 4100              | 4400 - 4800             |
| line 4  | 5800 - 6000              | 6100 - 6300             |

From the five pW ratios and respective slopes, we find that the best candidate to be a CSM indicator is line 1, the other lines show less differences in their evolution between different extinctions and CSM scenarios.
4. COMPARISON WITH OBSERVED SPECTRA

LEs are faint, and therefore they do not contribute significantly to the SN spectra around maximum. Following this reasoning, we first analyze the reddening at maximum light, to determine the total extinction or $A_V$, due to CSM and/or ISM using standard extinction laws (see §1.3). Then, analyzing the best fits to the spectra at later epochs we can discern if it is compatible with the extinction found at maximum light or if it is necessary to include LEs or, alternatively, reduce the amount of dust in the line of sight.

4.1. Data

The spectra we use were taken from the Carnegie Supernova Program CSP ([Polatelli et al. 2013], public data of the Center for Astrophysics CIAA [Blondin et al. 2012b]), the Berkeley Supernova Ia program BSNP ([Silverman et al. 2013b]) and The Online Supernova Spectrum Archive SUSPECT (see table 2). We analyzed only the subset of SNe that were already classified as fast or slow Lira decliners during the Lira Law phase in F13.

First, the spectra are corrected for Milky Way extinction using the values from [Schlafly & Finkbeiner 2011] and deredshifted to rest-frame. Then we smooth the spectra using a non-parametric fit with a velocity window of 1000 km/s and a wavelength regridding of 5 Å. We also compute the dispersion of each original spectrum with respect to its smoothed version to estimate the noise in our smoothed spectra. We normalize each smoothed spectrum to the same $V$ band flux by numerically convolving the spectrum with the filter transmission function [Bessell 1990], in order to put all spectra in the same scale and be able to compare the shape and features of the spectra instead of the absolute fluxes, which are difficult to calibrate precisely. We also adjust the shape of the spectra to match the observed colors interpolated to the given epoch ([Hsiao et al. 2007]), in order to have spectra consistent with the available photometry and with the previous work in F13.

We analyze the different SN spectra at maximum light and during the Lira law phase at 5 different epochs or time windows centered at 40, 50, 60, 70 and 80 days after maximum light with a width of 10 days. To have a single representative spectrum at every time window per SN, we make weighted average spectra with the available spectra of each SN. For more details about this weighted averages check [A].

4.2. Template spectra

F13 showed that slow Lira decliners present weaker equivalents width (EW) of blended Na I D1 & D2 narrow absorption lines, while fast Lira decliners have stronger EWs and redder colors independent of environmental factors. One possible interpretation of these results is that fast Lira decliners have CSM that produces LEs or that they have nearby dust that is sublimated in time. In order to test these hypothesis and analyze the spectra of fast Lira decliners with our models, we compare them with some standard unreddened and non-evolving CSM spectral time series given in this case by the slow Lira decliners. For this, we construct different template spectra that cover the intrinsic SN Ia variability at different epochs. [Chotard et al. 2011] showed that most of the intrinsic variability of spectra in SNe Ia at 2.5 days after maximum light can be characterized with the equivalent width (EW) of Si II 4131 Å and Ca II H&K. We follow the approach and use these two lines and a stretch parametrization to describe the shape of the light curve and defined as a factor multiplying the time axis ([Perlmutter et al. 1997] [Goldhaber et al. 2001]) to construct different templates. However, we can not measure the EWs of the Ca and Si lines in all of our SNe Ia. Given than the EW of Si II 4131 is correlated with the light curve stretch parameter, we decide to use epoch and stretch as our main variables to construct templates accounting for the intrinsic variability in SN Ia spectra.

To ensure that these average templates are not heavily biased by few extreme SNe, we perform a bootstrap simulation, i.e., we constructed 100 different templates using random sets from the original. For more details see [B].

In figure 6 we compare the bootstrap templates for fast and slow Lira decline spectra at maximum light and 50 days later, and one standard deviation ($\sigma$) regions. At maximum light there is a clear difference between fast and slow Lira decliners near 4000 Å, the average difference is about 1.4$\sigma$ between 3400 - 10000 Å and 0.8$\sigma$ if we do not consider SN 2006X and SN 2003cg. Between 3400 - 5500 Å the difference is about 2.3$\sigma$ and 1.4$\sigma$ if we do not consider SN 2006X and SN 2003 cg. But this difference after 50 days decreases to 0.8$\sigma$ between 3400 - 10000 Å (0.9$\sigma$ without SN 2006X and SN 2003cg) and 0.5$\sigma$ between 3400 - 5500 Å (0.6$\sigma$ without SN2006X and SN2003cg). We also find that the dispersion among fast Lira decliners is larger than in our slow Lira decliners sample. These results confirm what was found in F13, and we stress the fact that they are valid irrespective of warping the spectra to the observed colors. Finally, we point out that the template of the slow Lira decliners is quite similar to the template by [Hsiao et al. 2007], suggesting that this is the group of more “normal” unreddened SNe Ia.

4.3. Extinction law at maximum light

To find an extinction law for each fast Lira decliner at maximum light, we use as unreddened reference a slow Lira decliner template representing the same epoch and stretch, and the extinction law described in [Fitzpatrick & Massa 1990], adopting the mean values for the parameters found in [Fitzpatrick 1999] and leaving the visual extinction $A_V$ as free parameter.

We fix the total-to-selective extinction ratio $R_V$ at the standard Milky way value of 3.1. A discussion on this is presented in [G]. For each fast Lira decliner average spectrum we fit $A_V$ and a normalization parameter $C_1$, that corrects the fact that our spectra are normalized to their V-band flux, minimizing a chi-square function (for more details see [C]).

To analyze global differences between both subsets of SNe we fit the extinction law of [Fitzpatrick 1999] to the bootstrap templates of fast Lira decliners using as reference the bootstrap templates of slow Lira decliners.
Nearby SNe Ia spectra used in this analysis (besides the data from CSP, CfA and BSNIP) from The Online Supernova Spectrum Archive (SUSPECT).

| Name      | Sources                        |
|-----------|--------------------------------|
| SN 2005hk | Phillips et al. 2007           |
| SN 1999ac | Blondin et al. 2012a           |
| SN 1998aq | Chornock et al. 2006           |
| SN 2005cf | Garavini et al. 2007a          |
| SN 2003du | Wang et al. 2009                |
| SN 2005sm | Stachiev et al. 2007           |
| SN 2006X  | Anupama et al. 2005            |
| SN 1999aa | Leonard 2007                   |
| SN 2002bo | Yamanaka et al. 2009           |
| SN 2000cx | Wang et al. 2008a               |
| SN 1994D  | Sternberg et al. 2011a         |
| SN 2003cg | Garavini et al. 2003           |
|           | Benetti et al. 2001            |
|           | Li et al. 2001                  |
|           | Elias-Rosa et al. 2006         |

Out of 31 individual spectra from different fast Lira decliners at maximum light, minimizing the $\chi^2$ of eq. (eq. 6) we obtain 24 SNe with positive $A_V$ values and 7 with nonphysical negatives values. Those SNe are excluded from our sample for the LE analysis as they present less extinction than the template at maximum light, making the posterior fit of our LE model impossible. This may be caused by some SNe in the sample of slow Lira decliners having non negligible host extinction. At maximum light the mean $A_V$ is 0.44 while excluding the highly reddened SNe, SN 2006X and SN 2003cg, it decreases to 0.27. But during the Lira law the amount of extinction starts to decrease to values near zero. In Table 3 the mean differences in $A_V$ per SN are presented with respect to the value found at maximum light excluding those SNe with negative $A_V$ at that epoch. This result confirms that fast Lira decliners are more extincted at maximum light than slow Lira decliners, and during the Lira law both groups become more similar, as shown in F13. This is also valid without the warping of the spectra to the observed colors. However, there are a few SNe that show a positive $\Delta A_V$. This could be produced by an artifact in our templates or a bad extinction law fit.

5. RESULTS

5.1. Extinctions

Using the model described in [2] plus the values for the dust albedo $w$ and the degree of forward scattering $g$, corresponding to dust with $R_V = 3.1$, we fit the LE model to observed spectra of fast Lira decliners in the Lira phase. The parameters to fit are $f_{CSM}$ and a normalization constant $C_2$ to correct for the fact that our spectra were normalized by their flux in the V band. The function to minimize for each average spectrum $i$ is

$$\chi^2_i = \sum_{\lambda} \frac{(f_i(t, \lambda) - C_2 F(t, f_{CSM}, \lambda))^2}{\delta f_i^2(t, \lambda) + (C_2 \delta F(t, f_{CSM}, \lambda))^2}$$

where

$$F(t, \lambda) = f^0(t, \lambda)10^{-0.4A_{V}} + S(t, \lambda)10^{-0.4A_{V}(1-f_{CSM})}$$

To evaluate $S(t, \lambda)$ we use eq. 6 in a slightly different version because we do not have the intrinsic fluxes in our spectra as they were previously normalized (for more details, see [3]).

4.4. Light Echo fit

The function to minimize for each average spectrum $i$ is

$$\chi^2 = \sum_{\lambda} \frac{(f_i(t, \lambda) - C_2 F(t, f_{CSM}, \lambda))^2}{\delta f_i^2(t, \lambda) + (C_2 \delta F(t, f_{CSM}, \lambda))^2}$$

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TABLE 3
Mean and uncertainty of $\Delta AV = AV(t) - AV(max)$ for fast Lira decliner SNe at different epochs.

|          | 40 days | 50 days | 60 days | 70 days | 80 days |
|----------|---------|---------|---------|---------|---------|
| $-0.26 \pm 0.07$ | $-0.19 \pm 0.09$ | $-0.54 \pm 0.11$ | $-0.21 \pm 0.17$ | $-0.29 \pm 0.18$ |

TABLE 4
Comparison of ISM, LE and DS models with overall spectra: fraction of fast Lira decliner SNe with positive $AV$ at maximum light that have a BIC value which favors the LE or DS models vs the other models at different time windows.

|          | 40 days | 50 days | 60 days | 70 days | 80 days |
|----------|---------|---------|---------|---------|---------|
| Number of SNe | 16 | 11 | 8 | 10 | 7 |
| LE vs ISM | 0.50 | 0.36 | 0.63 | 0.5 | 0.57 |
| LE vs ISM & DS | 0.19 | 0.18 | 0.13 | 0.2 | 0.14 |
| DS vs ISM & Echo | 0.44 | 0.27 | 0.63 | 0.3 | 0.43 |

TABLE 5
Comparison of ISM, LE and DS models with specific spectral regions: fraction of fast Lira decliner SNe with positive $AV$ at maximum light that have a BIC value which favors the LE or DS models vs the other models at 50 days after maximum light for different wavelength regions.

|          | 3900-10000 Å | 3000-6500 Å | 6500-10000 Å | 3600-4800 Å | 5900-6400 Å |
|----------|--------------|-------------|--------------|-------------|-------------|
| LE vs ISM | 0.36 | 0.45 | 0.45 | 0.27 | 0.18 |
| LE vs ISM & DS | 0.18 | 0.36 | 0.45 | 0.27 | 0.18 |
| DS vs LE & ISM | 0.27 | 0.18 | 0.0 | 0.0 | 0.0 |

Fig. 7.—— $AV$ fit results at maximum light vs $B - V$ slopes of the Lira law from F13.

5.2. Light echoes

Once an $AV$ per fast Lira decliner SN is found, we fit our light echo model to the different fast Lira decliner spectra to search for spectral evidence of LEs. We look for the minimum of eq. (7) by varying $f_{CSM}$ and $C_2$. We fix $RV$ to 3.1 and the radius of the CSM shell to 0.05 pc (more on this in §).

We also fit the data with two other models to compare the goodness of fit of the light echo model applied to the observations. The three models that we compare are:

1. The light echo model in which we fit for $f_{CSM}$ and $C_2$. It also includes ISM extinction to get a total extinction consistent with the amount of extinction found at maximum light.

2. A pure ISM model that just uses a late time spectral template with the same extinction found at maximum light. In this scenario we just fit a normalization constant $C_1$.

3. A dust sublimation (DS) model consisting of a new pure extinction fit during the Lira law, with a lower $AV$ value than the one found at maximum light.

Then we calculate the Bayesian Information Criterion (BIC) parameter in order to compare the goodness of the three models applied to the same data. The BIC parameter is obtained as

$$BIC = \chi^2 + k \cdot \ln(n)$$

where $k$ is the number of free parameters to fit in each model and $n$ is the number of data points or wavelengths with measured flux values. The best model is the one with the lowest BIC. It is a combination of $\chi^2$ plus a function that penalizes having too many free parameters overfitting the data. With the BIC parameter we judge whether the LE model is able to explain the observed spectra better than a simple extinction or DS model. The results of the fits to the entire spectra and comparison between the three models are shown in Table 4.

According to our results, the LE model works better than using just the extinction law found at maximum light in 50% of the cases. But when we compare it to the dust sublimation model, the fraction of favorable cases drops to values lower than 20%, giving larger weight to a scenario where $AV$ simply decreases with time.

We also fit the three models to the spectra in particular wavelength ranges: blue region (3000 - 5000 Å); red region (6500 - 10000 Å); signature I (3600 - 4800 Å).
and signature II (5900 - 6400 Å). The results are shown in Table 5. The analysis in wavelength ranges is not favourable for the DS model, and the simple extinction found at maximum light works better than the LE model in most cases. However, this can be explained by the fact that when we look at particular regions of a spectrum a degeneracy appears between the normalisation and the extinction, as there is not enough wavelength range to anchor the reddening, obtaining a better BIC with the constant extinction model than with the DS model which has more parameters.

For each late-time SN spectrum, we derive the value for the CSM fraction $f_{\text{CSM}}$. Multiplying this value with the $A_V$ found at maximum light we can infer the fraction of the extinction due to CSM under the hypothesis of LE. For the four SNe with $A_V > 1$ at maximum light, we obtain that their $A_V$’s due to CSM are lower than 0.5, validating our single scattering assumption.

In Figure 8 we show $f_{\text{CSM}} \times A_V$ for different SNe at different epochs. Only epochs in which the LE model was the best according to its BIC value are plotted. If the CSM were not disturbed by the supernova we would expect $f_{\text{CSM}} \times A_V$ to stay almost constant for each SN. Unfortunately, only one object, SN 2003W, has more than one late-time spectrum consistent with LE to allow us to perform this test. The three resulting $f_{\text{CSM}} \times A_V$ values range between 0.03-0.23, which we consider a satisfactory agreement considering the simplifications of the LE model.

![Figure 8](image)

**Fig. 8.** $A_V \times f_{\text{CSM}}$ vs time since maximum light. Only cases in which the LE model works better than the pure extinction and DS models are shown. The error bars represent three standard deviation errors on the parameters that minimize the $\chi^2$ function under our LE model assumption.

The number of SNe for which the LE model has a favorable BIC in comparison with the ISM and DS models is small (7 SNe), and just SN2003W appears in more than one epoch. Even in such cases, a visual inspection of the spectra does not reveal the signatures expected from LEs. This poses serious questions on the LE scenario.

### 5.3. Line comparison

Another way to test our LE model is to use the line diagnostics presented in section 3. For this we calculate for all our spectra the pW and pseudo-continuum slope for the four diagnostic lines previously defined, in the same way we measure them in the simulated spectra. We obtain these features at all available epochs and also normalize the pW curves by their values at maximum light in order to study their evolution. Finally we compare the results for the slow and fast Lira decliners with the simulated spectra. Apart from some slight differences between the two samples, we find that in general the populations are consistent within the errors. Both groups of SN Ia show similar trend and dispersion in their evolution. As an example, Figure 9 shows the distribution of pW at 55 days after maximum light for the two SN samples. In the context of the LE model, we would expect both a difference in the dispersion of pWs and their mean value, neither of which is observed.

To measure possible statistical differences we perform a K-S test. We find that for line 1 the two populations, slow and fast Lira decliners, have a 99% probability of being drawn from the same distribution. We check all of the other lines for which we obtain low K-S values, yet we do not find any hint of LE signatures among fast Lira decliners according to our predictions (KS values between 0.2 and 0.99). The pW distributions are consistent for both populations during the Lira phase. However, some pseudo-continuum slopes show very low KS values (lower than 0.01). This difference in both SN samples can be explained by the greater color dispersion of fast Lira decliners which also tend to be redder at maximum light, as shown in F13. As a matter of fact, the trend goes in the opposite direction of what is predicted by our LE models.

![Figure 9](image)

**Fig. 9.** Histograms of the ratio of pseudo-equivalent width at maximum light and at 55 days past maximum light for the diagnostic echo line 1 for the sample of slow (blue) and fast (red) Lira decliners. Vertical dashed lines show the median of the population: 5.4±1.70 for slow and 5.7±1.57 for fast Lira decliners, respectively. The KS test for these distributions gives a probability of 99% of being drawn from the same distributions.

Although we do not find a correlation between the evolution of lines and fast Lira decliners, our CSM model predictions, especially for line 1, can be important to diagnose the presence of LEs in other samples of SNe Ia.

### 6. Discussion

#### 6.1. Extinction laws and $R_V$
To calculate the extinction at maximum, we also use other reddening laws such as Cardelli et al. (1989) with the inclusion of O’Donnell (1994), obtaining similar results to Fitzpatrick (1999) when keeping $R_V$ fixed. We also explore the reddening law proposed in Goobar (2008), but we discard it for two reasons: the reddening law becomes degenerate with our normalization constant $C_1$ and also because it accounts for the observed reddening in the context of CSM without considering the evolution of the radiated spectrum. In contrast, we aim to find an intrinsic reddening law and the time evolution of the spectrum caused by LEs.

To be consistent in our predictions and in our fits we used extinction laws with a fixed $R_V$ of 3.1 as we use standard MW values for the albedo and phase function of interstellar dust. Therefore our analysis is restricted to a statistical point of view more than the study of particular cases. The $R_V$ value can vary depending on the properties of the dust (e.g. grain size distribution and composition) and seems to be different from the MW in the line of sight towards some SNe Ia (Mandel et al. 2011 Burns et al. 2014). It is not very clear what range of values are consistent with circumstellar dust surrounding a SN. It is very important to differentiate in extinction laws analysis between the intrinsic $R_V$, which comes from the dust properties, and the observed $R_V$ when a pure extinction law is assumed, omitting more complex interactions. A future improvement in our model is to compute and use the specific opacities, albedo and phase function given any dust grain size distribution and composition.

6.2. Light echo models

We have explored the possibility of detecting LEs due to CSM in SN Ia spectra. Our results show that LEs are not a global phenomenon on fast Lira decliner SNe during the Lira law phase. Even though we find that for ~50% of the spectra the LE model works better than the extinction law derived at maximum light, the number of favorable cases drop to values near 15% when we compare them with the DS models.

We also fit the LE model using a CSM radius $R$ of 0.01 and 0.25 pc, instead of 0.05pc. With the smallest $R$ we obtain a lower fraction of favorable cases for the LE model compared to the original results with $R = 0.05$ pc, even lower than 50% when the LE model is compared with just the pure extinction model (ISM). On the other hand, when we fix $R = 0.25$ we recover almost the same results than the original CSM scenario. Therefore, if CSM is present, larger radii of 0.05-0.25pc are favoured.

In principle, it is possible to fit at the same time $R$, $f_{\text{CSM}}$ and $C_2$, but this is computationally expensive and could overfit the data.

Our CSM model consists of an isotropic spherical shell in the limit of negligible thickness. We did not consider multiple scattering, which at optically thin scenarios is negligible. Nevertheless, we know that multiple scattering could become important at optical wavelengths when the optical depth is larger than 1, i.e. $A_V \geq 1$. Therefore we are unable to analyze SNe with an expected $A_V$ due to CSM larger than one, but according to our results none of the SNe in our sample presented an $A_V$ due to CSM larger than 1 (including SN2003eg and SN2006x). A model that includes multiple scattering is necessary to predict the effect on the light curves and spectra when an optically thick CSM is present (Amanullah & Goobar 2011 Patat 2005).

The CSM geometry may probably be different from a spherical shell, e.g. non-isotropic disk or ring geometries formed from a planetary nebula have been proposed recently to model time variable Na I D absorption (Soker 2014). The predicted LE signatures might vary depending on the CSM geometry and orientation to the observer.

For an optically thin shell, a rough estimate of the total dust mass in our models can be obtained for a given CSM radius, an $A_V$ and a typical ISM dust opacity:

$$M_\text{d} = 6.4 \times 10^{-5} \left( \frac{R}{0.01 \text{ pc}} \right)^2 \left( \frac{A_V}{0.1} \right) \left( \frac{\kappa}{8.55 \times 10^4 \text{ cm}^2 \text{g}^{-1}} \right)^{-1} \text{M}_\odot$$

(10)

This is a very high mass in form of dust for a CSM. However, we are assuming a spherical shell and a specific opacity corresponding to interstellar extinction in the MW. If we consider a different geometry for the CSM or a larger specific opacity, i.e. smaller grains, the inferred mass will vary by orders of magnitude.

6.3. Light echo effects on the light curves

Using Monte Carlo simulations, Amanullah & Goobar (2011) found that different radius or $E(B-V)$ for the CSM could affect the $B-V$ evolution during the Lira law phase in different ways. In order to test the hypothesis made in F13 that LEs could increase the $B-V$ decline rate during the Lira law phase, we investigate whether our CSM model affects the behaviour of the light curve during the Lira law phase. We find that LEs make the opposite, they tend to smooth the color evolution. This goes in the opposite direction of our goal of finding light echoes in fast Lira decliners spectra and favors the DS model.

![Fig. 10. — Simulated light curves using our LE model. In black is the original light curve without extinction. The reddest curve represents a light curve with pure extinction ($A_V=0.5$ and $R_V=3.1$) and the rest are scenarios with different fractions of CSM.](image)

In Figure 10 we present these simulated light curves. As expected, in the pure extinction scenario the $B$ and
V light curves (red line) are just uniformly shifted downward with respect to the original light curve without extinction (black line). On the other hand, the presence of LEs due to CSM modify the shape of these curves, increasing the brightness in B and V at later epochs as blue light from maximum light is reaching the observer. However, the slope of the B – V evolution actually becomes shallower during the Lira law, see Table 6. This can be explained qualitatively as our LE model adds light emitted at previous epochs to the intrinsic emitted light, including late time emission with small time delays. These contributions make the observed light curve evolve slowly.

The wiggles in the B and B – V light curves are not real. They are caused by the way we compute the B magnitudes from the template spectra, which depend on the available SN spectra at each time window. On the other hand, the V magnitudes match the observed photometry by construction. Despite this, the general shape of the light curves is clear.

| Table 6 |

| $f_{\text{CSM}}$ | Slope ± error [mag/day] |
|-----------------|-------------------------|
| 0.0             | −0.010 ± 0.002          |
| 0.2             | −0.008 ± 0.001          |
| 0.4             | −0.008 ± 0.001          |
| 0.6             | −0.007 ± 0.001          |
| 0.8             | −0.004 ± 0.001          |
| 1.0             | −0.005 ± 0.001          |

6.4. Dust sublimation and its effects on the light curve

A decreasing extinction or opacity could occur if the CSM dust that was extincted at maximum light got sublimated by an increment of temperature due to the SN radiation, reducing the total opacity in the line of sight. This sublimation could also change the observed $R_V$ as it might change the grain size distribution and composition, explaining the evolution found in F13. If this happens during the Lira law phase, it will be reflected in a steeper $B - V$ slope. To test this hypothesis we simulated light curves with a decreasing $A_V$, and variable $R_V$. Figure 11 presents these three different scenarios. We found that a decreasing $A_V$ or increasing $R_V$ can make the $B - V$ evolution become steeper.

A smaller $R_V$ makes the extinction law more sensitive to the blue than the red, which is expected if the grain size distribution favors small sizes. An increasing $R_V$ could be produced if the smaller grains disappear as sublimation occurs. Another possibility is that the intrinsic CSM $R_V$ was lower than the ISM $R_V$, therefore as the circumstellar dust is being sublimated, the total $R_V$ increases reaching values similar to the interstellar $R_V$. This hypothesis is also consistent with the mean $\Delta A_V$ found at different epochs for fast Lira decliners.

Within this context, it is important to note that recently Goobar et al. (2014) showed that SN2014J, an extended red SN Ia with low $R_V$ (Foley et al. 2014) and with strong narrow absorption features (Welty et al. 2014), has possible signatures of cooling from shocked material from nearby CSM of dimensions larger than 1

![Fig. 11.— Simulated light curves using extinction laws that vary over time. In black is the original light curve without extinction. The blue curve is a model with $A_V$ decreasing from 0.5 to 0.0 and $R_V = 3.1$ held constant. The red curve represents a light curve with $A_V = 0.5$ and $R_V$ evolving from 2.0 to 3.1. The green curve represents a light curve with $A_V = 0.5$ and $R_V$ evolving from 3.1 to 2.0.

$R_0$. But the scales of this CSM are smaller than the ones considered in this paper.

Another possibility is that the radiation pressure (RP) of the SN is blowing away the CSM dust particles. If the CSM radius is increased, the observed extinction will decrease as the column density decreases, also producing blueshifted absorption lines. A rough estimation of the time scales in which the RP could produce an observable change in the CSM extinction can be calculated as

$$\tau_{\text{RP}} = \frac{R_{\text{CSM}}}{\Delta v}$$

(11)

$$\tau_{\text{RP}} = \frac{m_d R_{\text{CSM}} c}{f_{\text{SN}} \sigma_a \Delta t}$$

(12)

where $R$ is CSM radius, $\Delta v$ is a characteristic velocity of the dust grains after they absorb linear momentum from the SN incident radiation flux $f_{\text{SN}}$ that can be calculated using the well known relation between the radiation pressure and flux of an electromagnetic wave. We consider spherical dust particles of radius $a$, internal density of 1 gr/cm$^3$, mass $m_d$, and with a cross section $\sigma_a$ calculated using the Mie theory. Assuming a typical SN Ia luminosity and a time range $\Delta t = 25$ days, centered at maximum light, in which the RP injects momentum to the dust particles, we can obtain an estimate of the time scale in which this effect could be observed:

$$\tau_{\text{RP}} = 1.4 \times 10^2 \left( \frac{a}{1 \mu m} \right) \left( \frac{R}{0.01 \text{pc}} \right) \text{days}$$

(13)

Thus, if the radiation pressure is the cause of the decreasing extinction at $\tau_{\text{RP}} \sim 80$ days past explosion (Lira law), the CSM dust particles should be smaller than 1 $\mu$m or be at distances of $\sim 10^3$ astronomical units. At these distances we expect that the sublimation time scales of these smaller grains are much shorter than the calculated
above.

Therefore, even if dust sublimation time scales are too long to account for the changes in extinction and $R_V$, during the Lira law phase, we expect that the CSM expansion due to RP should increase the observed $R_V$ and decrease the extinction, as smaller grains are blown away.

An alternative scenario to the CSM sublimation and RP that could explain the decrease in extinction is the transversal expansion of the ejecta in a non-homogeneous ISM. This possibility is explored in F13 where they conclude that this scenario could explain the change in the average column density as the photospheric radius increases, but it does not explain the change in the observed $R_V$ over time.

7. CONCLUSIONS

A CSM model producing light echoes (LEs) has been developed. It is simple enough to be computed quickly and be used in our fitting routines, but with enough complexity to account for the albedo of the dust and the scattering phase function. Our model predicts two spectral signatures produced by LEs at 4100 and 6200 Å that can help us discriminate between a pure extinction or extinction+LEs scenario. These features appear within small wavelength ranges as opposed to overall color changes that can also be produced by reddening. The evolution of these signatures is another tracer of the presence of CSM producing LEs.

We compare our models with observed SN spectra. We find that LEs from CSM at 0.01-0.25pc are not a global phenomenon in fast Lira decliner SNe when they are compared to slow Lira decliners. ISM or CSM dust being sublimated (DS) at later epochs explains better the observed spectra when the models are fitted using the overall spectrum. Additionally, we find no evidence for LEs based on the narrow spectral diagnostics predicted by our model.

We explore the effects on the light curve of circumstellar dust being sublimated or blown away by radiation pressure, finding that both scenarios could produce a faster $B - V$ decline during the Lira law and a change in $R_V$, although a more rigorous physical modeling is needed to explore these possibilities.

We laid out several ways to improve our models: adding a $R_V$ as a free parameter in our fits; using different CSM geometries to test our predictions; and a Monte Carlo radiative transfer simulation to see if the LE signatures remain in the multiple scattering scenario. The analysis can also be improved using a larger sample of SNe, particularly those highly extincted and with possible CSM characteristics.

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APPENDIX

AVERAGE SPECTRA PER SN PER TIME RANGE

For the jth spectrum of the ith SN we defined the following weights:

$$w_j^i = e^{-(t_j-t_0)^2/2\sigma_0^2}$$  (A1)

where $t_0$ is the epoch that we want to represent and $\sigma_0 = 5.0$ days is half of the time window. The average spectrum for a particular SN at the given epoch $t_0$ was then calculated according to

$$f_i^i(\lambda) = \frac{\sum_j w_j^i f_j^i(\lambda)/\delta f_j^i(\lambda)^2}{\sum_j w_j^i /\delta f_j^i(\lambda)^2}$$  (A2)

where $f_j^i(\lambda)$ and $\delta f_j^i(\lambda)$ are the normalized flux of the jth spectrum and its error. The sum goes over all the available spectra of the ith SN with measured fluxes in the given time range.

To ensure that these average templates are not heavily biased by few extreme SNe, we performed a bootstrap simulation. Then computing the mean template and the dispersion around it, we obtained a “bootstrap template” $(A1)$.

We chose $\sigma_0 = 1.5$ days and $\sigma_s = 0.11$ in order to reproduce a specific epoch and stretch. We do not have strong arguments to choose particular values, thus we use the standard deviation of stretches in our sample for $\sigma_s$ and to $\sigma_0$ value smaller than our time windows, but large enough to reproduce smooth light curves. Defining $\alpha_i = w_i^t \times w_i^s$ the template spectrum with a certain epoch $t_0$ and stretch $s_0$ is

$$F(\lambda) = \sum_i \alpha_i f_i(\lambda)/\delta f_i(\lambda)^2$$  (B3)

where the sum goes over all the available already averaged spectra from slow Lira decliners with measured fluxes at wavelength $\lambda$. Finally we normalized the template spectrum by its flux in the $V$ band.

To construct template spectra we define two weight factors for time and stretch of the ith SN we defined the following weights:

$$w_j^i = e^{-(t_j-t_0)^2/2\sigma_0^2}$$  (B1)

$$w_s^i = e^{-(s_j-s_0)^2/2\sigma_s^2}$$  (B2)

We chose $\sigma_0 = 1.5$ days and $\sigma_s = 0.11$ in order to reproduce a specific epoch and stretch. We do not have strong arguments to choose particular values, thus we use the standard deviation of stretches in our sample for $\sigma_s$ and a $\sigma_0$ value smaller than our time windows, but large enough to reproduce smooth light curves. Defining $\alpha_i = w_i^t \times w_i^s$ the template spectrum with a certain epoch $t_0$ and stretch $s_0$ is

$$F(\lambda) = \sum_i \alpha_i f_i(\lambda)/\delta f_i(\lambda)^2$$  (B3)

where the sum goes over all the available already averaged spectra from slow Lira decliners with measured fluxes at wavelength $\lambda$. Finally we normalized the template spectrum by its flux in the $V$ band.

EXTINCTION LAW FIT

To fit an extinction law we minimize a chi-square function for each average spectrum $i$

$$\chi_i^2 = \sum_\lambda \frac{(f_i^i(\lambda) - f_0(\lambda)10^{-0.4A(\lambda)/C_A})^2}{\delta f_i(\lambda)^2 + (\frac{F_i(\lambda)}{f_0(\lambda)}\delta f_0(\lambda))^2}$$  (C1)

where $f_i^i(\lambda)$ is the normalized flux of the ith fast Lira decliner SN at maximum light and $f_0(\lambda)$ is an unreddened template representing the same intrinsic flux.

To ensure that these average templates are not heavily biased by few extreme SNe, we performed a bootstrap simulation. Then computing the mean template and the dispersion around it, we obtained a “bootstrap template” and its error. In figure [12] we compare a template constructed using weighted averages and a template using the bootstrap simulation. In both cases we used all the slow Lira decliners sample. There are slight differences between both spectra, but these are not significant and the estimated errors are very similar.

LIGHT ECHO FIT

To fit our light echo model we introduce a new factor $N$ to force the spectra to evolve consistently with the light curve in the $V$ band. Hence we calculated $S(t, \lambda)$ as

$$S(t) = \frac{w(1-10^{-0.44(\lambda)/C_A})}{t_{\text{max}}} \int_0^{t_{\text{max}}} N(t, \tau) f_0(t-\tau) \Phi'(\tau) d\tau$$  (D1)

$$N(t, \tau) = 10^{-0.4(V(t-\tau)-V(t))}$$  (D2)

where we omitted the wavelength dependency of $S$, $f_0$, $\omega$, $\Phi'$ and $A$ (the extinction law found at maximum light). $f_0(t)$ is a template spectrum representing the same intrinsic flux at time $t$. We multiply each spectrum by $N(t, \tau)$,
Fig. 12.— Template spectrum of a slow Lira decliner SN using weighted averages vs using the bootstrap technique at 50 days after maximum light and using a stretch of 0.98.

Fig. 13.— Extinction law fit for SN1999gd at maximum light using Fitzpatrick (1999) extinction law. The best fit with $R_V = 3.1$ yields $A_V = 1.280 \pm 0.003$. In red we present the observed spectrum of SN1999gd at maximum light, in black a constructed template representing the same epoch and stretch. The blue line is the dereddened spectrum and in green, the extinction factor $e^{-\tau \lambda}$ or $10^{-0.4 A_\lambda}$ shape.

which is the $V$ light curve normalized in $t$, to correct for the fact that all our templates are normalized by their flux in the $V$ band. This factor is computed using weighted averages of the $V$ magnitudes of slow Lira decliners considering epoch and stretch as we did for the template spectra. The $V$ magnitude data was taken from SiFTO fits to the data (Conley et al. 2008).