Experimental Study on Bubble Induced Velocity Fluctuations in the Boundary Layer at Transitional Reynolds Numbers

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Abstract. Our previous study showed that the frictional drag decreases with increasing void fraction at Re > 1300, while it increases at Re < 1000. Decomposition of the Reynolds shear stress also implied that bubbles induce isotropy of turbulence. In order to confirm our previous analysis and to further investigate flow fields in the vicinity of bubbles, we analyze velocity fluctuations on the quadrant space in the streamwise and transverse direction (u’ – v’ plane). Here, we focus on two specific Reynolds numbers (at Re ≈ 900 and ≈ 1410, which are close to the laminar-to-turbulent transition region) and discuss bubble effects on the sweep (u’ > 0, v’ < 0) and ejection (u’ < 0, v’ > 0) events as a function of the Reynolds number. We also illustrate the velocity fluctuations in the vicinity of an individual bubble and a swarm of bubbles on the u’ – v’ coordinates. The result shows that a bubble swarm suppresses the velocity fluctuations at Re ≈ 1410. We then rearrange the scattered PTV vectors and compute power spectra of the kinematic energy of each velocity component. It only indicates that the elongated bubble suppresses low frequency $E_{vv}$ at Re ≈ 1410.

1. INTRODUCTION

In order to explore the mechanism of drag reduction owing to microbubbles, many researchers have recently measured or simulated bubble-induced velocity fluctuations to discuss their effects on velocity distribution, vorticity, Reynolds shear stress, turbulent kinetic energy, power spectra, and other flow quantities. Xu et al. (2002) simulated the effect of different bubble sizes on the Reynolds shear stress and concluded that small bubbles tend to reduce the frictional drag. Kawamura and Kodama (2002) developed new computational methods to investigate interaction between bubbles and turbulence. Their results agreed well with the experimental studies on fully developed turbulent channel flows. Ferrante and Elghobashi (2004 and 2005) simulated microbubble-laden boundary layers over a horizontal flat plate and analyzed the bubble effects on the sweep and ejection events. Skudarnov and Lin (2007) also simulated the microbubble-laden flow. After investigating the velocity profiles and the distributions of the turbulent kinetic energy, they found that microbubbles decrease drag with increasing void fraction. Gabillet et al. (2002) experimentally studied the influence of bubbles on velocity fluctuations in the boundary layer of channel flows. They emphasized that the additional turbulent energy is caused by the relative motion of bubbles and the shear-induced turbulence. Murai et al. (2006) measured the velocity vectors using PTV in a horizontal channel and...
decomposed the Reynolds shear stress. Huang et al. (2008a) also experimentally investigated velocity fields on the streamwise and spanwise planes in boundary layers of horizontal channel flows. They discussed the bubble effect on vorticity and the power spectra. Lu et al. (2005) and Kitagawa et al. (2005) have focused on deformed bubbles and their oscillation in the boundary layer. They have investigated changes of flow fields in the vicinity of these bubbles.

All the studies mentioned above are performed under the fully developed turbulent flows at the Reynolds number of the order of $10^4$. We also believe that one of the keys to revealing the drag-reduction mechanism is to investigate flow conditions from laminar to turbulent Reynolds numbers. In our previous study (Huang et al. 2008b), we confirmed that bubbles decrease the wall shear stress with increasing void fraction at $Re > 1300$, while they increase the drag at $Re < 1000$. We then used an approach derived by Murai et al. (2006) to investigate the bubble effect in detail. One of the results implied that bubbles contribute to isotropy of turbulence.

Our objective of this study is to further investigate bubble effects on the surrounding flow fields based on the quadrant analysis. This method expresses the turbulent diffusion and the relative motion of fluid elements in boundary layers. Therefore, we can estimate the contribution of velocity fluctuations from each quadrant, i.e. in the $u'$ - $v'$ coordinates, to the Reynolds shear stress, $-\rho u'v'$, in detail. We first observe distributions of sweep ($u' > 0$, $v' < 0$) and ejection ($u' < 0$, $v' > 0$) events as a function of the transverse coordinate for both single- and two-phase flows. The results indicate that bubbles suppress the sweep and ejection events near the wall at $Re \approx 1410$. We then explore the influence of an individual bubble and a swarm of bubbles on the velocity fluctuations in comparison with those for single-phase flows. The bubble swarm suppresses velocity fluctuations at $Re \approx 1410$. After rearranging those scattered PTV vectors, there is no obvious influence from the elongated bubbles on power spectra of kinematic energy compared with those of single-phase flows. However, those bubbles enhance the kinetic energy of transverse velocity component, $E_{vv}$, at the high frequency at $Re \approx 900$, while they suppress the low frequency $E_{vv}$ at $Re \approx 1410$.

2. EXPERIMENTAL APPARATUS AND CONDITIONS

Experiments are conducted on a horizontal channel, similar to our previous study (see Huang et al. 2008b in detail). The experimental conditions are summarized in Table 1. The half height of the channel, $h = 5mm$, is adopted as the length scale. Two specific Reynolds numbers, at $Re \approx 900$ and $\approx 1410$, are chosen, which belong to the drag increase range and the decrease one, respectively. They are also close to the laminar-to-turbulent transition regime. Silicone oil (Shin-Etsu, KF96, 10cSt) is adopted as the working fluid; therefore, we can ignore the contamination disturbance and affirm the experimental reproducibility due to its stable surface property. Compressed air is injected into the channel through a needle nozzle array at $\alpha = 0.2\%$.

The test position is located at 250mm ($X/h = 50$) downstream of the injector as schematized in Fig. 1. In order to measure the flow fields surrounding bubbles in the boundary layer, a mirror is mounted

| Test position, $X/h$ | 50 | [-] |
|---------------------|----|-----|
| Bulk velocity of silicone oil, $\bar{U}$ | 2.0, 3.0 | [m/s] |
| Bulk void fraction, $\alpha$ | 0.2 | [%] |
| Temperature of silicone oil, $T$ | 21 ± 1 | [ºC] |
| Density of silicone oil, $\rho$ | 938 – 939 | [kg/m³] |
| Kinematic viscosity of silicone oil, $\nu$ | 10.6–10.9×10⁻⁶ | [m²/s] |
| Reynolds number, $Re = \bar{U}h/\nu$ | 900, 1410 | [-] |
| Friction Reynolds number, $Re_t = u_t h/\nu$ | 53 – 131 | [-] |
| Surface tension of silicone oil, $\sigma_s$ | 20.1 | [mN/m] |
on the channel window at an angle of 45º to the upper wall to reflect images of bubbles and tracer particles. Its effects on the target flow area are neglected. A high-speed video camera (Photron, FASTCAM-MAX 120 KC) is focused on the mid-span of the channel.

Using particle tracking velocimetry based on the shallow depth-of-field (Huang et al. 2008a), sufficient numbers of velocity vectors in the target depth (Δz = 0.3mm) have been detected around bubbles in the boundary layer, as shown in Fig. 2. The displacement thickness is evaluated as

\[ \delta^* (x) = 1.72 \frac{x \mu}{U \rho}, \]  

(1)

where, \( x \) is the distance from the inlet of the channel (875mm, \( x/h = 175 \)) and \( \mu \) is the dynamic viscosity. The displacement thickness is 0.77 \( y/h \) (or 40 \( y^* \)) at \( Re \approx 900 \) and 0.62 \( y/h \) (or 58 \( y^* \)) at \( Re \approx 1410 \) (\( y^* = u_\tau y/\nu \), where, \( u_\tau \) is the friction velocity, \( \nu \) is the kinematic viscosity). The spurious vectors in the PTV measurement are removed by a statistical method.

Elongated bubbles are considered to strongly relate to the shear stress (Rust and Manga, 2002). At \( Re \approx 900 \), as shown in Fig. 2 (a), the axial ratio of the bubble is about 3.4 (\( l/b \), where \( l \) and \( b \) are the semi-major and semi-minor axes of the sheared bubble, respectively) with 31° of the inclination angle,

![Fig. 1 Cross section of the flow visualization system for PTV. Y is the transverse direction, and z is the spanwise direction.](image1)

![Fig. 2 The PTV velocity fields around an elongated bubble. (a) at Re \( \approx 900 \) (\( \bar{U} = 2.0 \) m/s ); (b) at Re \( \approx 1410 \) (\( \bar{U} = 3.0 \) m/s )](image2)
\(\theta\), while the axial ratio is 4.1 and \(\theta = 22^\circ\) at \(Re \approx 1410\), as shown in Fig. 2 (b). Their capillary numbers are about 0.24 at \(Re \approx 900\) and 0.80 at \(Re \approx 1410\), both of which are less than unity. At these \(Ca\) numbers, bubbles maintain steady elongated shapes. The \(Ca\) number is defined as

\[
Ca = \frac{\mu U_b}{\sigma_s},
\]

where, \(U_b\) is the velocity of bubbles and \(\sigma_s\) is the surface tension. It represents the ratio of the shear stress, which deforms bubbles, to the surface tension.

3. RESULTS AND DISCUSSION

3.1 Our Previous Results
In our previous studies (see Huang et al. 2008b in detail), we measured the wall shear stress, \(\tau_w\), with a shear force transducer (SSK Co., LTD. S10W-05) at three test positions \(X/h = 50, 200, 800\) downstream of the bubble injector. The bulk void fraction, \(\bar{\alpha}\), ranged from 0.2% to about 5%. After converting the wall shear stress to the skin friction coefficient, \(C_f\), we compared the modified skin friction ratios of two-phase flows, \(C_f\), to single-phase flows, \(C_{f0}\), to minimize the uncertainty induced from injected bubbles and temperature variation. Fig. 3 displays the skin friction ratio at the test positions verse the bulk void fraction at different Reynolds numbers. The data points can be categorized into two types based on the Reynolds number. One is that the frictional ratios are greater than unity at \(Re \approx 900\), which indicates the drag increase for two-phase flows; the other is that the ratios are less than unity at \(Re \approx 1410\) indicating drag reduction. The ratios are monotonically decreased with increasing void fraction to about 20% when \(\bar{\alpha} \approx 5.0\%\) at \(X/h = 50\). According to the response frequency of the transducer \(f_n = 30\) Hz, noise at the higher frequency is filtered to ensure the measurement accuracy.

We also decomposed the Reynolds shear stress of bubbly flows into three components \((\tau_1 = -\rho f\overline{u'v'} , \tau_2 = -\rho f\overline{u'u'} , \tau_3 = -\rho f\overline{u'v'}\) where \(f\) is the volume fraction of the continuous phase) based on the two-fluid model (Murai et al. 2006). Fig. 4 displays the normalized stress components together with the Reynolds shear stress, \(-\rho u'v'\), for single- and two-phase flows at the two Reynolds numbers. The injected bubbles decrease the stress components, \(\tau_1\), by reducing the mixture density. \(\tau_2\) generates negative momentum exchange near the wall at \(Re \approx 1410\) and dissipates the fluid energy. \(\tau_3\) maintains small positive stress across the upper half of the channel. This phenomenon implied that the

![Fig. 3 Ratios of the skin friction coefficient vs. bulk void fractions at different Reynolds numbers.](image-url)
positive $u' v'$ of the continuous phase in the vicinity of the bubbles is increased, which contributes to the isotropy of the turbulence. However, in order to confirm our analysis, it is still important to further investigate instantaneous flow fields in the vicinity of bubbles.

### 3.2 Quadrant Analysis

In this study, we first analyze velocity fluctuation components in the streamwise and transverse directions, referred to as the quadrant analysis, to discuss the bubble-induced velocity fluctuations in detail. The streamwise and transverse components of the velocity fluctuations are defined as

\[ u' = u - \overline{u} \quad \text{and} \quad v' = v - \overline{v}, \]

(3)

where, the over-bars denote the time-averaged variables and the primes indicate the fluctuation components. The quadrant analysis divides $u' - v'$ into four quadrants according to the sign of $u'$ and $v'$. The first quadrant, $u' > 0$ and $v' > 0$, contains outward motion of high-speed flow; the second one, $u' < 0$ and $v' > 0$, contains the motion associated with ejection of low-speed flow away from the wall; the third, $u' < 0$ and $v' < 0$ contains inward motion of low-speed flow; and the fourth quadrant, $u' > 0$ and $v' < 0$, contains an inrush of high-speed flow, generally referred as a sweep event (Wallace et al.).

![Fig. 5 The schematic of the division of $u' - v'$ coordinates and the hole event.](image)
1972; Kim et al. 1987), as schematized in Fig. 5 together with the “hole event”, which will be discussed in the next section. The sweep and ejection events contribute to the positive Reynolds shear stress, $-\rho u'v'$, while the other two events corresponds to the negative part.

Fig. 6 shows the distribution of the normalized $u'v'$ from each quadrant as a function of $y^+$-location. In each flow condition, the intensity of the ejection event, $[u'v']_2$, (the second quadrant, $u' < 0$ and $v' > 0$) dominates at $y^+ \approx 20$, while that of the sweep event, $[u'v']_4$, (the forth quadrant, $u' > 0$ and $v' < 0$) is greater than other events away from the wall, at $y^+ \approx 20$. Here, the subscripts 2 and 4 indicate the quadrant numbers. At Re $\approx 900$, as shown in Fig. 6 (a), injected bubbles greatly enhance the intensity of the sweep and ejection events in the vicinity of the wall, $y^+ < 25$. The location where the intensities of the two events become equivalent ($[u'v']_2 = [u'v']_4$) also shifts from $y^+ \approx 25$ for single-phase flows, to the wall, at $y^+ \approx 20$ for two-phase flows. These events induced by bubbles have the potential to increase the shear stress on the wall (as derived in Gabillet et al. 2002). This is also consistent with our previously measured wall shear stress as shown in Fig. 3. In contrast, as shown in Fig. 6 (b), bubbles slightly suppress the intensity of the ejection and sweep events, especially the ejections, while they enhance the outward motion in the first quadrant ($u' > 0$ and $v' > 0$) and the inward motion in the third quadrant ($u' < 0$ and $v' < 0$). Therefore, the Reynolds shear stress can be attenuated due to isotropy of turbulence.

![Figure 6](image)

Fig. 6 $u'v'$ from each quadrant normalized by the frictional velocity of single-phase flows, $u_{\tau_0}$. (a) at Re $\approx 900$; (b) at Re $\approx 1410$. Lines express the experimental data for single-phase flows (S-p); symbols (+, ◊, ×, Δ) represent those for two-phase flows (T-p).

### 3.3 Fractional Contribution of the Four Quandrants

Besides displaying the distribution of different quadrant events, it is also important to quantitatively evaluate their relative importance in generating the Reynolds shear stress across transverse planes in the boundary layer. Here, we utilize the “hole event” (Lu and Willmarth, 1973; Hurther and Lemmin, 2000; Mazumde and Ojha, 2007), as illustrated by the cross-hatched region in Fig. 5, to separate the strength of turbulent diffusion. With this scheme, large contributors to Reynolds shear stress, $u'v'$, can be extracted by leaving the smaller fluctuating $u'v'$ signal in the “hole region”. We compute the contribution of each quadrant, $I$, to the Reynolds shear stress as a function of the threshold level of the hyperbolic hole region, $H$, as follows:

$$
\langle u'v' \rangle_{H,I} = \lim_{I \to \infty} \frac{1}{T} \int_0^T u'(t)v'(t) \delta_{i,H}(u',v') dt,
$$

(4)
where, the angle brackets denote a conditional average. The indicator function \( \delta_{i,H} \) obeys
\[
\delta_{i,H}(u',v') = \begin{cases} 
1, & \text{if } u'u' \geq H|u'|, \\
0, & \text{otherwise},
\end{cases}
\] (5)
namely, only when \((u', v')\) is in the \(i\)th quadrant and \(|u'| \geq H|u'|\), \(\delta_{i,H}\) becomes unity.

\(H\) can be selected as \(H = 0.5, 1.0, 1.5, 2.0, \ldots\). The stress fraction of the \(i\)-th quadrant is normalized by the averaged intensity of \(u'v'\), as
\[
S_{i,H} = \langle u'v' \rangle_{i,H} / |u'|. 
\] (6)
Their summation also satisfies
\[
\sum_{i=1}^{4} |S_{i,0}| = 1, 
\] (7)
at \(H = 0\). Figs. 7 and 8 respectively display the fractional contribution of each quadrant at the two Reynolds numbers at different \(y\)-locations.

As shown in Fig. 7 (a), the total fractions of the inward motion and ejection event \((|S_{1,0}| \text{ and } |S_{2,0}| \text{ at } H = 0)\) are suppressed in two-phase flows, while that of the sweep event, \(|S_{4,0}|\), is greatly enhanced at \(Re \approx 900, y^+ = 10\). Furthermore, for strong fluctuating \(u'v'\) signals, such as at \(H > 1\), the relative fraction of the sweep event becomes greater than that of the ejection \((|S_{4,0}| > |S_{2,0}|, H > 1\)). This phenomenon is counter to single-phase flows, in which the ejection dominates the fractional contribution to the Reynolds shear stress \((|S_{2,0}| > |S_{4,0}|, H \geq 0\)). Therefore, the velocity in this layer is enhanced by the injection of bubbles. However, the result is a little different at \(y^+ = 20\), as shown in Fig. 7 (b). Bubbles enhance the total fractions of the ejection and sweep events and their fractional contributions are almost identical with each other \((|S_{2,0}| \approx |S_{4,0}|, H \geq 0\).

As shown in Fig. 8, bubbles slightly suppress the fraction of the sweep event \((|S_{4,0}|, H \geq 0)\) at \(Re \approx 900, y^+ = 20\).
Fig. 8 The fractional contribution of each quadrant on the Reynolds shear stress against the hole size, $H$, at $Re \approx 1410$. (a) at $y^+ = 10$; (b) at $y^+ = 30$.

$\approx 1410$. However, it is difficult to distinguish their influence on the other three quadrants due to the low bulk void fraction ($\alpha = 0.2\%$), especially at $y^+ = 30$. At this $y$-location, the total fractions of the sweep and ejection events are almost the same, about 40\% ($|S_{2,0}| \approx |S_{4,0}| \approx 40\%$, $H = 0$). But, the intensity of $u'v'$ for strong fluctuations, such as that at $H \geq 1$, is essentially contributed from ejection events ($|S_{2,0}| > |S_{4,0}|$, $H \geq 1$).

### 3.4 Velocity Fluctuation in the Vicinity of Bubbles

From the consecutive frames, we observe that there exist series of individual bubbles and bubble swarms in the boundary layer whose effects on the surrounding flows appear to be different. Therefore, we analyze the contribution of these types of bubbles to the quadrant events in details. For individual bubbles, the distance to the nearest bubble is generally much greater than its diameter and the bubble-bubble interaction is negligible. A swarm of bubbles consists of several bubbles and the typical distance between them is comparable with their sizes. Therefore, we should consider the influence of the bubble-bubble interaction on the surrounding flow fields.

Fig. 9 displays the velocity fluctuations of the continuous phase (silicone oil) in a thin shear layer including the peak Reynolds shear stress at each Reynolds number ($y/h = 0.28$ or $y^+ = 15$ at $Re \approx 900$ and $y/h = 0.2$ or $y^+ = 20$ at $Re \approx 1410$) for two-phase flows as well as for single-phase flows. The focused layer is nearly inside the free-stream rather than the bubble location ($0.05 < y/h < 0.2$) at $Re \approx 900$. In contrast, it is closer to the bubble centroid ($y/h \approx 0.18$) at $Re \approx 1410$. Individual bubbles are elongated, as displayed in Fig. 2.

At $Re \approx 900$, velocity fluctuation in single-phase flows is weak, and data points are dense around $u' = v' = 0$ (in the range of hole event, $H < 1$). However, the injected bubbles promote turbulence. Individual bubbles induce the sweep event, while bubble swarms generate the ejection. At $Re \approx 1410$, the magnitude of velocity fluctuations induced by the individual bubble is comparable with that of single-phase flows. Individual bubbles generate the ejection, opposed to the sweep event in the single-phase case. In contrast, bubble swarms suppress velocity fluctuations by decreasing both ejection and sweep events.

### 3.5 Rearranged Velocity Field and Power Spectrum of Kinetic Energy

According to the discussion on the relation between elongated bubbles and the shear stress (Rust and Manga 2002), we subsequently rearrange the scattered PTV vectors with the Spatio-temporal
Fig. 9 Distribution of velocity fluctuations of continuous phase (silicone oil), $u', v'$, in the quadrant. indicates the velocity fluctuations around the individual bubbles shown in Fig. 2.

Biquadratic-ellipsoidal Equation Rearrangement (ST-BER) method developed by Ido et al. (2002) to obtain instantaneous flow fields around bubbles, which retain the third-order spatial continuity.

Fig. 10 shows the contours of the velocity magnitude at the two Reynolds numbers overlaying the relative velocity vectors of continuous phase to the elongated bubble illustrated in Fig. 2. The silicone oil flows from left to right. White parts express bubble images. Although these bubbles are elongated at each Reynolds number as described in Sec. 2, we find no remarkable effect on flow fields. We only observe that the velocity magnitude is slightly enhanced behind the bubble at Re $\approx 900$ and velocity vectors indicate that flow elements move along the major axis of the bubble toward free-stream, which generates clockwise vorticity in front of bubbles. At Re $\approx 1410$, the velocity magnitude behind the bubble attenuates. The relative velocity of the continuous phase to the bubble also directs along the major axis but toward the wall, especially in the range of $y^+ < 10$. These phenomena are consistent with the previous study by Kitagawa et al. (2005).

Subsequently, averaged power spectra of the kinetic energy for each velocity component ($E_{uu} = \frac{u'^2}{2}$ and $E_{vv} = \frac{v'^2}{2}$) are plotted in Fig. 11 as a function of $y^+$. The velocity in the bubble occupied area is interpolated by the Laplace equation. The interpolation is applied only inside the bubble area; therefore, the original measurement information of the continuous phase is unchanged. The abscissa is the frequency normalized by the frame rate of the high-speed camera. In both figures, the magnitudes of the power spectra increase with distance away from the wall. Their magnitudes keep constant in the low frequency, while decline greatly at the high frequency region. The slope at the high frequency part roughly matches -5/3 of Kolmogorov’s law. We also notice that at Re $\approx 900$, as shown in Fig. 11 (a), the kinetic energy of $E_{vv}$ on $y^+ = 5$, close to the centroid of the bubble, is enhanced at $K' > 0.25$. Its magnitude is comparable with that of the $E_{uu}$ there. In contrast, as shown in Fig. 11 (b), $E_{vv}$ on $y^+ = 20$ at Re $\approx 1410$, where is also close to the centroid of the bubble, slightly decreases at $K' < 0.2$. The decrease indicates that the elongated bubble suppresses the low frequency kinetic energy of the transverse velocity component and relatively large eddies, whose wavelength has the inverse proportion to frequency.

4. CONCLUSIONS

In this study, we experimentally investigated the bubble-induced velocity fluctuation at two specific Reynolds numbers (Re $\approx 900$ and $\approx 1410$), which belonged to the drag increase range and the decrease one, respectively. They were also close to the laminar-to-turbulent transition regime. From our
experiment, the following remarks were concluded:

1) Based on the quadrant analysis for the continuous phase, we found that the intensity of the ejection (the second quadrant, $u' < 0$ and $v' > 0$) and sweep (the forth quadrant, $u' > 0$ and $v' < 0$) events is greatly enhanced by bubbles at $Re \approx 900$; in contrast, bubbles suppress those events at $Re \approx 1410$. 

Fig. 10 Rearranged instantaneous contours of velocity magnitude for the streamwise component overlaid with relative velocity fluctuations to bubbles.

Fig. 11 Power spectra of the kinetic energy for each velocity component ($u$ and $v$).
For the hole event, whose threshold value is $H \geq 1$, the absolute stress fraction for the sweep events, $|S_{14}|$, in two-phase flows becomes greater than that for the ejections, $|S_{21}|$, on $y^+ = 10$ at $Re \approx 900$. This phenomenon is counter to that for single-phase flows, in which the stress fraction for the ejections is greater than the sweep events.

By comparing the velocity fluctuations in the vicinity of individual bubbles and bubble swarms at $Re \approx 900$ and $\approx 1410$, we observe that the swarm of bubbles suppresses the ejection and sweep events at $Re \approx 1410$.

But, after rearranging the instantaneous flow fields around the elongated bubbles, we found that those bubbles enhance the magnitude of the power spectrum for $E_{vv}$ at the high frequency at $Re \approx 900$, while they suppresses the low frequency $E_{vv}$ at $Re \approx 1410$.

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NOMENCLATURE

- $b$: semi-minor axis of the elongated bubble [m]
- $Ca$: capillary number [-]
- $C_f$: skin-friction coefficient [-]
- $E_{uu}$, $E_{vv}$: kinematic energy for each velocity component per unit mass [m$^2$/s$^2$]
- $f$: volume fraction of the continuous phase [-]
- $f'$: fluctuations of volume fraction for the continuous phase [-]
- $f_n$: response frequency of the transducer [Hz]
- $H$: threshold level of the hyperbolic hole region [-]
- $h$: half height of the channel, used as the length scale [m]
- $K_*$: normalized frequency [-]
- $l$: semi-major axis of the elongated bubble [m]
- $Re$: Reynolds number [-]
- $Re_c$: friction Reynolds number [-]
- $S$: Normalized fraction contribution of each quadrant [-]
- $T$: temperature of silicone oil [ºC]
- $t$: time period [s]
- $\overline{U}$: bulk velocity of silicone oil [m/s]
- $U_b$: bubble velocity [m/s]
- $u$, $v$: streamwise and transverse velocity components [m/s]
- $\overline{u}$, $\overline{v}$: time-averaged streamwise and transverse velocity components [m/s]
- $u'$, $v'$: streamwise and transverse components of velocity fluctuations [m/s]
- $u_\tau$: friction velocity [m/s]
- $X$: test position from the bubble injector [mm]
- $x$: distance from the inlet of the channel [mm]
- $y$: transverse distance from the upper wall of the channel [m]
- $y^+$: wall unit [-]
- $z$: spanwise distance [mm]
- $\Delta z$: target depth [mm]

Greek Letters

- $\alpha$: bulk void fraction [%]
- $\delta$: displacement thickness [mm]
\( \delta_{ii} \) indicator function of the hole event

\( \mu \) dynamic viscosity of silicone oil [Pa·s]

\( \nu \) kinematic viscosity of silicone oil [m²/s]

\( \theta \) inclination angle of bubbles to the streamwise direction [°]

\( \rho \) density of silicone oil [kg/m³]

\( \sigma_s \) surface tension of silicone oil [mN/m]

\( \tau_w \) wall shear stress [Pa]

\( \tau_1, \tau_2, \tau_3 \) decomposed Reynolds shear stress components for two-phase flows [Pa]

Subscripts

\( i \) \text{-th quadrant}, \( i = 1, 2, 3 \text{ and } 4 \)

\( n \) \text{-th test position downstream of the bubble injector}, \( n= 1, 2, 3 \)

\( 0 \) parameters for single-phase flows

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