On the Width of Collective Excitations in Chiral Soliton Models

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In chiral soliton models for baryons the computation of hadronic decay widths of baryon resonances is a long standing problem. For the three flavor Skyrme model I present a solution to this problem that satisfies large-\(N_C\) consistency conditions. As an application I focus on the hadronic decay of the \(\Theta\) and \(\Theta^*\) pentaquarks.

\section{Statement of the problem}

Hadronic decays of baryon resonances are commonly described by a Yukawa interaction of the generic structure

\[ L_{\text{int}} \sim g \bar{\psi}_{B'} \phi \psi_B, \]  

where \(B'\) is the resonance that decays into baryon \(B\) and meson \(\phi\) and \(g\) is a coupling constant. It is crucial that this interaction Lagrangian is \textit{linear} in the meson field. If \(\phi\) is a pseudoscalar meson this interaction yields the decay width \(\Gamma(B' \to B\phi) \propto g^2 |\vec{p}_\phi|^3\), with \(\vec{p}_\phi\) being the momentum of the outgoing meson.

The situation is significantly different in soliton models that are based on action functionals of only meson degrees of freedom, \(\Gamma = \Gamma[\Phi]\). These action functionals contain classical (static) soliton solutions, \(\Phi_{\text{sol}}\), that are identified as baryons. The interaction of these baryons with mesons is described by the (small) meson fluctuations about the soliton: \(\Phi = \Phi_{\text{sol}} + \phi\). By pure definition we have

\[ \frac{\delta \Gamma[\Phi]}{\delta \Phi} \bigg|_{\Phi = \Phi_{\text{sol}}} = 0. \]  

Thus there is no term linear in \(\phi\) to be associated with the Yukawa interaction, Eq. (1.1). This puzzle has become famous as the Yukawa problem in soliton models. However, this does not mean that soliton models cannot describe resonance widths. On the contrary, these widths can be extracted from meson baryon scattering amplitudes, just as in experiment. In soliton models two-meson processes acquire contributions from the second order term

\[ \Gamma^{(2)} = \frac{1}{2} \phi \cdot \frac{\delta^2 \Gamma[\Phi]}{\delta^2 \Phi} \bigg|_{\Phi=\Phi_{\text{sol}}} \cdot \phi. \]  

This expansion simultaneously represents an expansion in \(N_C\), the number of color degrees of freedom: \(\Gamma = \mathcal{O}(N_C)\) and \(\Gamma^{(2)} = \mathcal{O}(N_C^0)\). Terms \(\mathcal{O}(\phi^3)\) vanish in the limit \(N_C \to \infty\). This implies that \(\Gamma^{(2)}\) contains all large-\(N_C\) information about hadronic decays of resonances. We may reverse this statement to argue about any computation of hadronic decay widths in soliton models: For \(N_C \to \infty\) it \textit{must} identically match

\begin{footnote}
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On the Width of Collective Excitations

the result obtained from \( \Gamma^{(2)} \). Unfortunately, the most prominent baryon resonance, the \( \Delta \) isobar, becomes degenerate with the nucleon as \( N_C \to \infty \). It is stable in that limit and its decay is not subject to the above described litmus-test. The situation is more interesting in soliton models for flavor \( SU(3) \). In the so-called rigid rotator approach (RRA), that generates baryon states as (flavor) rotational excitations of the soliton, exotic resonances emerge that dwell in the anti-decuplet representation of flavor \( SU(3) \).\(^1\) The most discussed (and disputed) such state is the \( \Theta^+ \) pentaquark with zero isospin and strangeness \( S = +1 \). In the limit \( N_C \to \infty \) the (would-be) anti-decuplet states maintain a non-zero mass difference with respect to the nucleon. Therefore the properties of \( \Theta^+ \) predicted from any model treatment must also be seen in the \( S \)-matrix for koan-nucleon scattering as computed from \( \Gamma^{(2)} \). This (seemingly alternative) quantization of strangeness degrees of freedom is called the bound state approach (BSA) because in the \( S = -1 \) sector the resulting equations of motion for \( \phi \) yield a bound state. Its occupation serves to describe the ordinary hyperons, \( \Lambda, \Sigma, \Sigma^* \), etc. The above discussed litmus-test requires that the BSA and RRA give identical results for the \( \Theta^+ \) properties as \( N_C \to \infty \). This did not seem to be true and it was argued that the prediction of pentaquarks would be a mere artifact of the RRA.\(^2\) Here we will show that this is a premature conclusion and that pentaquark states do indeed emerge in both approaches. Furthermore the comparison between the BSA and RRA provides an unambiguous computation of pentaquark widths: It differs substantially from previous approaches\(^3\) that adopted transition operators for \( \Theta^+ \to KN \) from the axial current.

This presentation is based on Ref. 4) which the interested reader may want to consult for further details.

§2. The model

For simplicity we consider the Skyrme model\(^5\) as a particular example for chiral soliton models. However, we stress that our qualitative results do indeed generalize to all chiral soliton models because these results solely reflect the treatment of the model degrees of freedom.

Chiral soliton models are functionals of the chiral field, \( U \), the non-linear realization of the pseudoscalar mesons,\(^*) \( \phi_a \)

\[
U(\vec{x}, t) = \exp \left[ \frac{i}{\pi} \phi_a(\vec{x}, t) \lambda_a \right],
\]

with \( \lambda_a \) being the Gell-Mann matrices of \( SU(3) \). For a convenient presentation of the model we split the action into three pieces

\[
\Gamma = \Gamma_{SK} + \Gamma_{WZ} + \Gamma_{SB}.
\]

The first term represents the Skyrme model action

\[
\Gamma_{SK} = \int d^4x \text{tr} \left\{ \frac{f_\pi^2}{4} \left[ \partial_\mu U \partial^\mu U^\dagger \right] + \frac{1}{32 \epsilon^2} \left[ [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U] \right]^2 \right\}.
\]

\(^*)\) A remark on notation: In what follows we adopt the convention that repeated indices are summed over in the range \( a, b, c, \ldots = 1, \ldots, 8 \), \( \alpha, \beta, \gamma, \ldots = 4, \ldots, 7 \) and \( i, j, k, \ldots = 1, 2, 3 \).
Here $f_\pi = 93$ MeV is the pion decay constant and $\epsilon$ is the dimensionless Skyrme parameter. In principle this is a free model parameter. The two-flavor version of the Skyrme model suggests to put $\epsilon = 4.25$ from reproducing the $\Delta$-nucleon mass difference.\textsuperscript{4)} The QCD anomaly is incorporated via the Wess-Zumino action\textsuperscript{7)}

$$\Gamma_{WZ} = -\frac{i N_C}{240 \pi^2} \int d^5 x \ e^{\mu \rho \sigma \tau} \text{tr} \left[ \alpha_\mu \alpha_\rho \alpha_\sigma \alpha_\tau \right], \quad (2.4)$$

with $\alpha_\mu = U^\dagger \partial_\mu U$. Note that $\Gamma_{WZ}$ vanishes in the two-flavor version of the model.

The flavor symmetry breaking terms are contained in $\Gamma_{SB}$

$$\Gamma_{SB} = \frac{f_\pi^2}{4} \int d^4 x \ \text{tr} \left[ \mathcal{M} \left( U + U^\dagger - 2 \right) \right], \quad \mathcal{M} = \text{diag} \left( m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2 \right). \quad (2.5)$$

We do not include terms that distinguish between pion and kaon decay constants even though they differ by about 20% empirically. This omission is a matter of convenience and leads to an underestimation of symmetry breaking effects\textsuperscript{8)} which approximately can be accounted for by rescaling the kaon mass $m_K \rightarrow m_K f_K/f_\pi$.

The action, Eq. (2.2) allows for a topologically non-trivial classical solution, the famous hedgehog soliton

$$\Phi_{sol} \sim U_0(\vec{x}) = \exp \left[ i \hat{\lambda} \cdot \hat{x} F(r) \right], \quad r = |\vec{x}| \quad (2.6)$$

embedded in the isospin subspace of flavor $SU(3)$. The chiral angle, $F(r)$ solves the classical equation of motion subject to the boundary condition $F(0) - F(\infty) = \pi$ ensuring unit winding (baryon) number. The soliton can be constructed as a function of the dimensionless variable $\epsilon f_\pi r$ and is thus not subject to $N_C$ scaling.

In the RRA baryon states are generated by canonically quantizing collective coordinates $A \in SU(3)$ that describe the (spin) flavor orientation of the soliton, $A(t)U_0(\vec{x})A^\dagger(t)$. The resultant eigenstates may be classified according to $SU(3)$ multiplets; see Ref. 9) for a review.

§3. Large $N_C$ $P$-wave channel phase shifts with strangeness

As motivated after Eq. (1.3) we introduce fluctuations $\phi \sim \eta_\alpha(\vec{x}, t)$

$$U(\vec{x}, t) = \sqrt{U_0(\vec{x})} \ \exp \left[ i \frac{\lambda_\alpha}{f_\pi} \eta_\alpha(\vec{x}, t) \right] \sqrt{U_0(\vec{x})}, \quad (3.1)$$

for the kaon fields.\textsuperscript{10)} Expanding the action in powers of these fluctuations is an expansion in $\eta_\alpha/f_\pi$ and thus a systematic series in $1/\sqrt{N_C}$. The term quadratic in $\eta_\alpha$ describes meson scattering off a potential generated by the classical soliton, Eq. (2.6). The $P$-wave mode is characterized by a single radial function

$$\left( \eta_4 + i \eta_5 \right) \left( \eta_6 + i \eta_7 \right)_p (\vec{x}, t) = \int_{-\infty}^{\infty} d\omega \ e^{i \omega t} \eta_\omega(r) \hat{x} \cdot \hat{\tau} \chi(\omega). \quad (3.2)$$

\textsuperscript{4)} To ensure that the (perturbative) $n$-point functions scale as $N_C^{1-n/26}$ we substitute $f_\pi = 93$ MeV $\sqrt{N_C/3}$ and $\epsilon = 4.25 \sqrt{3/N_C}$ in the study of the $N_C$ dependence.
In future we will omit the subscript that indicates the Fourier frequency. Furthermore $\chi(\omega)$ is a two-component iso-spinor whose components are elevated to creation- and annihilation operators upon quantization. It is straightforward to deduce the Schrödinger type equation

$$h^2 \eta(r) + \omega [2\lambda(r) - \omega M_K(r)] \eta(r) = 0 \quad \text{with} \quad h^2 = -\frac{d^2}{dr^2} - \frac{2}{r} \frac{d}{dr} + V_{\text{eff}}(r). \quad (3.3)$$

The radial functions arise from the chiral angle $F(r)$ and may be readily taken from the literature. The equation of motion (3.3) is not invariant under particle conjugation $\omega \leftrightarrow -\omega$, and thus different for kaons ($\omega > 0$) and anti-kaons ($\omega < 0$). This difference stems from the Wess-Zumino term. Equation (3.3) has a bound state solution at $\omega = -\omega_A$, that gives the mass difference between the $\Lambda$-hyperon and the nucleon in the large-$N_C$ limit. As this energy eigenvalue is negative it corresponds to a kaon, i.e. it carries strangeness $S = -1$. In the symmetric case ($m_K = m_\pi$) the bound state is simply the zero mode of $SU(3)$ flavor symmetry. The WZ-term moves the potential bound state with $S = +1$ to the positive continuum and we expect a resonance structure in that channel. The corresponding phase shift is shown in the left panel of Fig. 1. No clear resonance structure is visible; the phase shifts hardly reach $\pi/2$. The absence of such a resonance has previously lead to the premature criticism that there would not exist a bound pentaquark in the large-$N_C$ limit.

§4. Constraint fluctuations

To study the coupling between the fluctuations and the collective excitations we generalize Eq. (3.1) to

$$U(\vec{x}, t) = A(t)\sqrt{U_0(\vec{x})} \exp \left[ \frac{i}{\pi} \lambda_\alpha \tilde{\eta}_\alpha(\vec{x}, t) \right] \sqrt{U_0(\vec{x})} A^\dagger(t). \quad (4.1)$$

These fluctuations dwell in the intrinsic system as they rotate along with the soliton. The kaon $P$-wave is subject to the modified integro-differential equation

$$h^2 \tilde{\eta}(r) + \omega [2\lambda(r) - \omega M_K(r)] \tilde{\eta}(r) = -z(r) \left[ \int_0^\infty r'^2 dr' z(r') 2\lambda(r') \tilde{\eta}(r') \right]$$

Fig. 1. Large $N_C$ $P$-wave phase shifts with strangeness $S = +1$ as function of the kaon momentum. Left panel: unconstrained; right panel: constrained to be orthogonal to the collective rotation.
for the flavor symmetric case.\textsuperscript{4) }The radial function \( \tilde{\eta}(r) \) is defined according to Eq. (3.2) and \( z(r) = \sqrt{4\pi f} \left( \frac{f}{\sqrt{\Theta K}} \right) \frac{F(r)}{2} \) is the collective mode wave-function normalized with respect to the moment of inertia for flavor rotations into strangeness direction, \( \Theta_K = \frac{f^2}{\pi} \int d^3 r M_K(r) \sin^2 \frac{F(r)}{2} = \mathcal{O}(N_C) \). The non-local terms without \( X_{A,\Theta} \) reflect the constraint \( \int d^3 r z(r) M_K(r) \tilde{\eta}(r) = 0 \) which avoids double counting of rotational modes in strangeness direction. The interesting coupling is contained in the interaction Hamiltonian

\[
H_{\text{int}} = \frac{2}{\sqrt{4\pi \Theta K}} \delta_{\alpha\beta} D_{\gamma\alpha} R_{\beta} \int d^3 r z(r) \left[ 2\lambda(r) - \omega_0 M_K(r) \right] \hat{x}_i \xi_\gamma(\vec{x}, t),
\]

where \( \xi_a = D_{ab} \tilde{\eta}_b \) are the fluctuations in the laboratory frame, that we actually detect in \( KN \) scattering. The collective coordinates are parameterized via the adjoint representation \( D_{ab}(A) = \frac{1}{2} \text{tr} \left[ \lambda_a A \lambda_b A^\dagger \right] \) and the \( SU(3) \) generators \( R_a \). Integrating out the collective degrees of freedom by means of standard perturbation theory induces the separable potential

\[
\frac{|\langle \Theta | H_{\text{int}} | (KN)_{I=0} \rangle |^2}{\omega_\Theta - \omega} + \frac{|\langle A | H_{\text{int}} | (KN)_{I=0} \rangle |^2}{\omega_A + \omega}.
\]

These matrix elements concern the \( T \)-matrix elements in the laboratory frame. Since the laboratory and intrinsic \( T \)-matrix elements are identical for the \( \Theta^+ \) channel,\textsuperscript{11) }we may add the exchange potential, Eq. (4.4) in the intrinsic frame. We define matrix elements of collective coordinate operators

\[
\langle \Theta^+ | d_{3a\beta} D_{\alpha+} R_{\beta} | n \rangle =: X_{\Theta} \sqrt{\frac{N_C}{32}}, \quad \langle A | d_{3a\beta} D_{\alpha-} R_{\beta} | p \rangle =: X_A \sqrt{\frac{N_C}{32}},
\]

to end up with Eq. (4.2). The first factor in the coefficient \( \omega_0 = 2 \left( \frac{2}{\sqrt{\Theta K}} \right) \sqrt{\frac{N_C}{32}} = \frac{N_C}{4\Theta K} \) arises in the equation of motion because the potential, Eq. (4.4) is quadratic in the fluctuations. The remaining (squared) factors stem from the definitions of \( X_{\Theta,A} \) and the constant of proportionality in \( H_{\text{int}} \). The \( X_{\Theta,A} \) must be computed with the methods provided in Ref. 12) but generalized to arbitrary (odd) \( N_C \).\textsuperscript{4) }For \( N_C \to \infty \) we have \( X_{\Theta} \to 1 \) and \( X_A \to 0 \). From the orthogonality conditions of the equation of motion (3.3) we straightforwardly verify that

\[
\tilde{\eta}(r) = \eta(r) - az(r) \quad \text{with} \quad a = \int_0^\infty dr r^2 z(r) M_K(r) \eta(r),
\]

solves Eq. (4.2) for large \( N_C \). This is essential because, as \( z(r) \) is localized in space, \( \eta \) and \( \tilde{\eta} \) have identical phase shifts! Hence the litmus-test discussed in the introduction is indeed satisfied. The physics behind \( \tilde{\eta} \) is best understood when introducing

\textsuperscript{4) }The more complicated case \( m_K \neq m_\pi \) is at length discussed in Ref. 4).
reduced wave-functions $\bar{\eta}(r)$ that solve Eq. (4.2) modified with $X_\Theta \equiv X_A \equiv 0$. These wave-functions are still orthogonal to the collective modes and actually lead to the background phase shift shown in Fig. 1. Having obtained the reduced wave-functions we may switch on the exchange contributions, Eq. (4.4). The additional separable potential may be treated by standard $R$-matrix techniques and augments the phase shift by

$$\tan(\delta_R(k)) = \frac{\Gamma(\omega_k)/2}{\omega_\Theta - \omega_k + \Delta(\omega_k)}.$$  (4.7)

Here $\omega_\Theta = \frac{N_C+3}{4\Theta K}$ is the RRA result for the excitation energy of $\Theta$. This phase shift exhibits the canonical resonance structure with the width and the pole shift

$$\Gamma(\omega_k) = 2k\omega_0X_\Theta^2 \left| \int_0^\infty r^2 dr \, z(r)2\lambda(r)\bar{\eta}_\omega(r) \right|^2,$$  (4.8)

$$\Delta(\omega_k) = \frac{1}{2\pi\omega_k} \mathcal{P} \int_0^\infty q dq \left[ \frac{\Gamma(\omega_q)}{\omega_k - \omega_q} + \frac{\Gamma(-\omega_q)}{\omega_k + \omega_q} \right],$$  (4.9)

respectively. We have numerically verified that in the large-$N_C$ limit with $X_\Theta^2 = 1$, the phase shift from Eq. (4.7) is identical to what is labeled resonance phase shift in Fig. 1, that we calculated as the difference between the total ($\eta$) and background ($\bar{\eta}$) phase shifts. For finite $N_C$ we have $X_\Theta \neq 1$ and $X_A \neq 0$ so the $R$-matrix formalism becomes two-dimensional ($A$ and $\Theta^+$ exchange). Contrary to earlier criticisms the large $N_C$ pentaquark phase shift indeed resonates!

§5. Results

In Fig. 2 we show the resonance phase shift computed from Eq. (4.7) for various values of $N_C$. First we observe that the resonance position quickly moves towards larger energies as $N_C$ decreases. This is mainly due to the strong $N_C$ dependence of $\omega_\Theta$: For $N_C = 3$ it is twice as large as in the limit $N_C \to \infty$. The pole shift $\Delta$ is actually quite small (some ten MeV) so that $\omega_\Theta$ is indeed a reliable estimate of the resonance energy. Second, the resonance becomes shaper as $N_C$ decreases. To major parts this is caused by the reduction of $X_\Theta$.

We now turn to more quantitative results for which we also include flavor sym-
metry breaking effects. Then the resonance position changes to

\[ \omega_\Theta = \frac{1}{2} \left[ \sqrt{\omega_0^2 + \frac{3\Gamma}{2\Theta} + \omega_0} \right] + \mathcal{O}\left(\frac{1}{N_C}\right), \tag{5.1} \]

where \( \Gamma = \mathcal{O}(N_C) \) is a functional of the soliton that is proportional to the meson mass difference, \( m_K^2 - m_\pi^2 \). The \( \mathcal{O}(1/N_C) \) piece is sizable for \( N_C = 3 \) and we compute it in the scenario of Ref. 12. We then find \( \omega_\Theta \approx 700 \text{ MeV} \) and due to model dependencies we expect the pentaquark to be about 600...900 MeV heavier than the nucleon.

For the width calculations there are two principle differences. First, the interaction Hamiltonian acquires an additional term

\[ H_{\text{int}}^{sb} = (m_K^2 - m_\pi^2) d_{i\alpha\beta} D_{\gamma\alpha} D_{8\beta} \int d^3r \: z(r) \gamma(r) \tilde{\xi}(\vec{x}, t) \hat{x}_i. \tag{5.2} \]

The radial function \( \gamma(r) \) is again given in terms of the chiral angle. Second, the \( X_A \) does not vanish as \( N_C \to \infty \) and the \( R \)-matrix formalism is always two dimensional. Nevertheless, the large-\( N_C \) solution is always of the form (4.6) and the BSA phase shift is recovered. The width function turns to

\[ \Gamma(\omega_k) = 2k\omega_0 \left| \int_0^\infty r^2 dr \: z(r) \left[ 2X_\Theta \lambda(r) + \frac{Y_\Theta}{\omega_0} (m_K^2 - m_\pi^2) \right] \eta_{\omega_k}(r) \right|^2, \tag{5.3} \]

where \( X_\Theta \) and \( Y_\Theta = \sqrt{8N_C/3} \langle \Theta^+ | d_{3\alpha\beta} D_{+\alpha} D_{8\beta} | n \rangle \) are to be computed in the RRA approach with full inclusion of flavor symmetry breaking effects.

This width function is shown (for \( N_C = 3 \)) in Fig. 3 for \( \Theta \) and its isovector partner \( \Theta^* \). The latter merely requires the appropriate modification of the matrix elements in Eq. (4.5). Most importantly, the \( k^3 \) behavior of the width function, as suggested by the model, Eq. (1.1) is reproduced only right above threshold, afterwards it levels off. Second, and somewhat surprising, the width of the non-ground state pentaquark is smaller than that of the lowest lying pentaquark. Our particular model yields \( \Gamma_\Theta \approx 40 \text{ MeV} \) and \( \Gamma_{\Theta^*} \approx 20 \text{ MeV} \). We note that there are certainly model ambiguities in the actual data.

Fig. 3. Model prediction for the width, \( \Gamma(\omega) \) of \( \Theta^+ \) (left) and \( \Theta^{*+} \) (right) for \( N_C = 3 \) as function of the momentum \( k = \sqrt{\omega^2 - m_K^2} \) for three values of the kaon mass. Note that the two figures have different scales.
§6. Conclusions

We have discussed the chiral soliton model approach to $KN$ scattering in the $S = +1$ channel which contains the potential\(^{13}\) $\Theta^+$ pentaquark, a state predicted as a flavor rotational excitation of the soliton. Though the exactly known large $N_C$ phase shift suggests otherwise, the $\Theta$ emerges as a genuine resonance. Our central result is the width function for $\Theta \to KN$. In the flavor symmetric case it contains only a single collective coordinate operator and is thus very different from estimates that extract an effective Yukawa coupling from the axial current matrix element.\(^3\) Since our approach matches the exact large $N_C$ result, we must conclude that those axial current scenarios are erroneous\(^{14}\) and that the cancellation among contributions to this matrix element is an invalid argument for a small pentaquark width.

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