Microstructure Effects on Daily Return Volatility in Financial Markets

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Abstract

We simulate a series of daily returns from intraday price movements initiated by microstructure elements. Significant evidence is found that daily returns and daily return volatility exhibit first order autocorrelation, but trading volume and daily return volatility are not correlated, while intraday volatility is. We also consider GARCH effects in daily return series and show that estimates using daily returns are biased from the influence of the level of prices. Using daily price changes instead, we find evidence of a significant GARCH component. These results suggest that microstructure elements have a considerable influence on the return generating process.

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It is a well known fact from a large number of empirical investigations that financial return series exhibit specific patterns, e.g. positive autocorrelations in volatility. Many of these patterns have been modeled using GARCH processes as introduced by Engle (1982) and Bollerslev (1986) and developed further by many other authors. Bollerslev et al. (1992) and Diebold & Lopez (1993) give an overview of these models as well as their empirical evidence.

Another empirical finding is the positive relation between return volatility and trading volume. A large number of contributions address this issue with theoretical as well as empirical investigations, see e.g. Foster & Viswanathan (1990), Foster & Viswanathan (1993a), Foster & Viswanathan (1993b), He & Wang (1993), Aoki (1999), Chen et al. (1999), Focardi et al. (1999) or Iori (1999) as few examples. A common approach in many of these models is to assign observed effects to the processing of private information.

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The model of Clark (1973) allows to relate trading volume with GARCH processes. He assumes the price change in a period of time, e.g. a trading day, to be the sum of a large number of intraday price changes. Every trade induces a minor price change as a new piece of independent information arrives at the market. Therewith return volatility in a given period is proportional to the number of trades conducted in this period. Assuming serially correlated numbers of trades, and therewith trading volume, gives rise to GARCH effects. Jones et al. (1994) show empirically that the number of trades are the most important factor influencing return volatility, rather than trading volume or trade sizes. A similar approach is taken by Andersen (1996). Lamoureux & Lastrapes (1990) show that with the inclusion of trading volume GARCH coefficients are not significant.

All these models, however, do not take into account market microstructure elements arising from nontrading periods, the bid-ask spread or inventory control of dealers, although they are frequently considered in analyzing intraday returns. It is well known from the literature that these elements induce negative serial correlation of observed returns, see Lo & MacKinlay (1990) and Roll (1984). Implications for the variance of observed returns are thus far not considered in the literature.

This paper intends to explore the effects market microstructure elements have on daily returns and return volatility. It is beyond the scope of this paper to give a full characterization of the observed effects by considering a large number of parameter constellations, we concentrate on the basic properties and leave detailed analysis for future research. We proceed as follows: the first section introduces the model which is evaluated in section 2. Section 3 investigates GARCH effects and section 4 the behavior of a measure for intraday volatility. Finally, section 5 concludes the findings and suggests directions of future research.

1 The model

This section will introduce a very general model of price formation that captures various elements from market microstructure. To reduce the complexity of our model we do not consider the information flow affecting trading behavior. Trades are only induced by investors facing a liquidity event $L_t$ in period $t$. This liquidity event is assumed to follow an AR(1) process:

\begin{equation}
L_t = \phi L_{t-1} + \varepsilon_t^L, \\
\varepsilon_t^L \sim iidN(0, \sigma^2_L).
\end{equation}

We assume further the fundamental value of the asset in the current period to be common knowledge and to follow a random walk:

\begin{equation}
P_t^* = P_{t-1}^* + \varepsilon_t^P,
\end{equation}
\[ \varepsilon_t^P \sim iidN(0, \sigma_P^2), \]

where \( \varepsilon_t^L \) and \( \varepsilon_t^P \) are independent. Within every period \( t \), \( N_t \) trading rounds are conducted, where \( N_t \) depends on the size of the liquidity event as follows:

\[ N_t = Ne^{Lt}, \]

rounded to the next integer. We assume the market to be organized as a dealer market, where dealers face inventory costs. We further suppose that only a single dealer is present in the market conducting all trades and quoting competitive prices. Inventory costs force the dealer after every trade to adjust his prices. When conducting a trade at the ask, he will increase his quote and decrease the quote when having conducted a trade at the bid, see \textit{Stoll (1978)}, \textit{Ho & Stoll (1980)} and \textit{Ho & Stoll (1981)} for inventory based models of market making.

We find the medium price \( P_{t, \tau}^M \), i.e. the price in the middle between the bid and ask price, for every trading round \( \tau = 1, \ldots, N_t \) of period \( t \) by adjusting the fundamental value with a term denoted \( \eta_{t, \tau} \) representing the inventory effect.

\[ P_{t, \tau}^M = P_t^* + \eta_{t, \tau} \]

In each trading round an investor arrives at the market and decides whether to trade at the ask, at the bid or not to trade at all. This decision depends on the trading costs, \( C_{t, \tau}^a \) and \( C_{t, \tau}^b \). With \( P_{t, \tau}^a \) and \( P_{t, \tau}^b \) denoting the ask and bid prices, respectively, these trading costs are given by

\[ C_{t, \tau}^a = P_{t, \tau}^a - P_t^* \]
\[ C_{t, \tau}^b = P_t^* - P_{t, \tau}^b. \]

We suppose these costs to transform into probabilities of trading using a logit transformation:

\[ \lambda_{t, \tau}^a = \frac{1}{1 + e^{C_{t, \tau}^a}}, \]
\[ \lambda_{t, \tau}^b = \frac{1}{1 + e^{C_{t, \tau}^b}}. \]

As \( P_{t, \tau}^a \geq P_{t, \tau}^b \), it is easy to demonstrate that \( \lambda_{t, \tau}^a + \lambda_{t, \tau}^b \leq 1 \), hence with probability \( 1 - \lambda_{t, \tau}^a - \lambda_{t, \tau}^b \) no trade occurs in a trading round. Define

\[ I_{t, \tau} = \begin{cases} 
1 & \text{with probability } \lambda_{t, \tau}^a \\
0 & \text{with probability } 1 - \lambda_{t, \tau}^a - \lambda_{t, \tau}^b \\
-1 & \text{with probability } \lambda_{t, \tau}^b 
\end{cases}. \]

The trade size \( \nu_{t, \tau} \) is assumed to be log-normal distributed and independent of \( \varepsilon_t^L \) and \( \varepsilon_t^P \):

\[ \ln \nu_{t, \tau} \sim iidN(0, \sigma_{\nu}^2). \]
This trade size is supposed to effect the inventory adjustment \( \eta_{t,\tau} \) linearly with a scaling factor \( \alpha \geq 0 \). Hence we find the dynamics of inventory adjustments as

\[
\eta_{t,\tau} = \eta_{t,\tau-1} + I_{t,\tau-1} \alpha \nu_{t,\tau-1}.
\]

The gross trading volume of period \( t \) is given by

\[
V_t = \sum_{\tau=1}^{N_t} I_{t,\tau}^2 \nu_{t,\tau}.
\]

Let us further assume that the dealer cannot offset his inventory between trading periods, hence we find

\[
\eta_{t+1,1} = \eta_{t,N_t+1}.
\]

With \( s \) denoting the constant spread applied by the dealer, the transaction price is given by

\[
P_{t,\tau} = \begin{cases} 
   P_{t,\tau}^M + \frac{1}{2} I_{t,\tau} s & \text{if } I_{t,\tau} \neq 0 \\
   P_{t,\tau-1} & \text{if } I_{t,\tau} = 0
\end{cases}.
\]

We define the final transaction price of a period as the daily price:

\[
P_t = P_{t,N_t}.
\]

Therewith the relevant microstructure elements, spread, inventory control and non-trading periods have been incorporated into our model. We will now use this model of price formation to simulate intraday prices and investigate implications on volatility dynamics and its relation to trading volume from daily prices.

## 2 Numerical evaluation

We use several parameter constellations to simulate a series of daily prices and daily trading volumes using the model of section 1. For each parameter constellation 6000 trading days have been simulated eliminating the first 1000 trading days to exclude any influences from the starting values \( P_1^* = 100 \), \( L_0 = 0 \), and \( \eta_{1,1} = 0 \), so that 5000 trading days, corresponding to about 20 years of daily data, have been used for the analysis.

Throughout all simulations we assume \( N = 100 \), \( \sigma_\nu = .05 \), \( \sigma_L = .1 \), and \( \sigma_P = .05 \). In more detail the following parameter constellations have been used:

**S 1.** We observe no market microstructure elements by setting \( s = \alpha = 0 \) and the liquidity event is serially uncorrelated (\( \phi = 0 \)).

**S 2.** Here also no market microstructure elements are present, but the liquidity event is serially correlated with \( \phi = .75 \).
Table 1: Parameter constellation of the simulations

|   | α  | s   | φ  |
|---|----|-----|----|
| S1| 0  | 0   | 0  |
| S2| 0  | 0   | 0.75 |
| S3| 0  | 0.25 | 0  |
| S4| 0  | 0.25 | 0.75 |
| S5| 0.1| 0   | 0  |
| S6| 0.1| 0   | 0.75 |
| S7| 0.1| 0.25| 0  |
| S8| 0.1| 0.25| 0.75 |

\[ N = 100, \sigma_\nu = 0.05 \]
\[ \sigma_L = 0.1, \sigma_P = 0.05 \]

S 3. We assume the only market microstructure element to be the spread, i.e. \( s = 0.25 \) and \( \alpha = 0 \). The liquidity event is serially uncorrelated (\( \phi = 0 \)).

S 4. We apply the same setting as in S3, but use serially correlated liquidity events (\( \phi = 0.75 \)).

S 5. We assume the only market microstructure element to be inventory control, i.e. \( s = 0 \) and \( \alpha = 0.1 \). The liquidity event is serially uncorrelated (\( \phi = 0 \)).

S 6. We apply the same setting as in S5, but use serially correlated liquidity events (\( \phi = 0.75 \)).

S 7. Here we assume both microstructure elements to be present, the spread and inventory control, i.e. \( s = 0.25 \) and \( \alpha = 0.1 \). The liquidity event is serially uncorrelated (\( \phi = 0 \)).

S 8. We apply the same setting as in S7, but use serially correlated liquidity events (\( \phi = 0.75 \)).

These parameter settings are summarized for convenience in table 1.

From the simulated time series trading volume, daily returns and squared daily returns as a measure of volatility are investigated. Daily returns are calculated by the log-ratio of prices:

\[ r_t = \ln \frac{P_t}{P_{t-1}}. \]
The results of the eight simulations are reported in table 2. Although only the results and estimates of a specific realization are reported, a large number of other realizations showed similar results, suggesting that our findings are robust. We have not reported results on the trading volume as, not surprisingly, it exhibits the same properties as the liquidity event, i.e. follows an AR(1) process.

Let us at first notice that correlated liquidity events, and therewith correlated trading volume, do not affect the results significantly. For this reason we will for the remainder of this section only consider returns arising from uncorrelated liquidity events.

**Result 1.** *Serial correlation in trading volume does not affect the properties of daily return series arising from the presence of microstructure elements.*

We further observe an increase in daily return volatility with presence of microstructure elements. This volatility increases the more elements are added. The equality of volatility between S1, S3, S5 and S7 can be rejected at the 1% significance level.

**Result 2.** *Daily return volatility increases in presence of microstructure elements.*

The result derived by Roll (1984) that the presence of a spread induces a negative first order serial correlation in returns of -0.5 for subsequent trades is confirmed from our simulations also for daily returns with intraday trading. Furthermore we do not make the assumption that trades at the bid and ask both have a probability of 0.5, when neglecting trading rounds in which no trades occur, this is only true on average. The reason for this result is that the final trade of the day can either be at the bid or the ask. A final trade one day at the bid, the next day at the ask, or vice versa, induces negative serial correlation of daily returns.

We find a similar result arising from inventory control. Suppose that to the end of a trading period a large inventory has been accumulated that could not be offset before the last trading round, hence the price is low and the return negative. The dealer begins the next trading period with a large inventory, which he now tries to reduce during the trading period, causing the price to rise the more inventory reduces and the return is positive. We therefore find negative first order autocorrelation in daily returns. Higher order autocorrelations are unlikely to be observed as the large number of trading rounds within each trading period makes it unlikely that inventory has to be reduced over several trading days. However, we can expect to find higher order autocorrelations for less frequently traded assets, i.e. assets with a small $N$.

The autocorrelation structure of daily returns suggests that they follow a MA(1) process, estimates of the coefficients for this process are given in table 3. Only estimates for serially uncorrelated liquidity events are reported, estimates from the simulations using serially correlated liquidity events show the same results. We find significant first order coefficients only in the presence of microstructure elements. The residuals
Table 2: Simulation results

These statistics are estimated from daily returns simulated by using the model developed in section 1. Values exhibiting a * are significantly different from zero at the 5% significance level.

| r_t  | r_t^2 |
|------|-------|
| **Mean** | **Std. Dev.** |
| S1 | \(1.26 \times 10^{-5}\) | \(2.36 \times 10^{-7}\) | \(\star\) |
| | \(0.00486^*\) | \(3.35 \times 10^{-7}\) |

| Lag | Autocorrelations |
|-----|------------------|
| 1  | -.006  | .005  |
| 2  | -.009  | .008  |
| 3  | .029   | -.017 |
| 4  | -.006  | -.011 |
| 5  | -.004  | .005  |

| r_t  | r_t^2 |
|------|-------|
| **Mean** | **Std. Dev.** |
| S2 | \(5.99 \times 10^{-6}\) | \(2.23 \times 10^{-7}\) | \(\star\) |
| | \(0.00473^*\) | \(3.23 \times 10^{-7}\) |

| Lag | Autocorrelations |
|-----|------------------|
| 1  | .009  | -.018 |
| 2  | .004  | .012  |
| 3  | .005  | .000  |
| 4  | -.013 | .002  |
| 5  | .008  | -.011 |

| r_t  | r_t^2 |
|------|-------|
| **Mean** | **Std. Dev.** |
| S3 | \(-2.23 \times 10^{-6}\) | \(1.31 \times 10^{-5}\) | \(\star\) |
| | \(0.003623^*\) | \(1.35 \times 10^{-5}\) |

| Lag | Autocorrelations |
|-----|------------------|
| 1  | -.476  | -.014 |
| 2  | .012   | .007 |
| 3  | -.005  | -.003 |
| 4  | .003   | -.006 |
| 5  | .003   | -.009 |

| r_t  | r_t^2 |
|------|-------|
| **Mean** | **Std. Dev.** |
| S4 | \(-4.49 \times 10^{-6}\) | \(1.22 \times 10^{-5}\) | \(\star\) |
| | \(0.003496^*\) | \(1.26 \times 10^{-5}\) |

| Lag | Autocorrelations |
|-----|------------------|
| 1  | -.491  | .003 |
| 2  | .008   | .015 |
| 3  | -.017  | -.001 |
| 4  | .011   | .004 |
| 5  | .001   | -.004 |

| r_t  | r_t^2 |
|------|-------|
| **Mean** | **Std. Dev.** |
| S5 | \(-1.82 \times 10^{-6}\) | \(2.01 \times 10^{-5}\) | \(\star\) |
| | \(0.004481^*\) | \(2.87 \times 10^{-5}\) |

| Lag | Autocorrelations |
|-----|------------------|
| 1  | -.505  | .273 |
| 2  | .023   | .009 |
| 3  | -.005  | .020 |
| 4  | -.002  | -.009 |
| 5  | -.014  | -.008 |

| r_t  | r_t^2 |
|------|-------|
| **Mean** | **Std. Dev.** |
| S6 | \(8.12 \times 10^{-7}\) | \(2.20 \times 10^{-6}\) | \(\star\) |
| | \(0.004687^*\) | \(3.06 \times 10^{-6}\) |

| Lag | Autocorrelations |
|-----|------------------|
| 1  | -.486  | .250 |
| 2  | -.010  | .015 |
| 3  | -.012  | -.012 |
| 4  | .033   | -.009 |
| 5  | -.026  | .016 |

| r_t  | r_t^2 |
|------|-------|
| **Mean** | **Std. Dev.** |
| S7 | \(-1.25 \times 10^{-5}\) | \(2.59 \times 10^{-5}\) | \(\star\) |
| | \(0.005093^*\) | \(3.64 \times 10^{-5}\) |

| Lag | Autocorrelations |
|-----|------------------|
| 1  | -.495  | .236 |
| 2  | .010   | -.017 |
| 3  | -.031  | -.021 |
| 4  | .033   | -.008 |
| 5  | -.006  | -.011 |

| r_t  | r_t^2 |
|------|-------|
| **Mean** | **Std. Dev.** |
| S8 | \(-4.48 \times 10^{-6}\) | \(2.67 \times 10^{-5}\) | \(\star\) |
| | \(0.005172^*\) | \(3.68 \times 10^{-5}\) |

| Lag | Autocorrelations |
|-----|------------------|
| 1  | -.501  | .242 |
| 2  | .009   | .013 |
| 3  | -.012  | -.009 |
| 4  | .027   | -.008 |
| 5  | -.022  | -.007 |
Table 3: Estimates of MA(1) coefficients for daily returns

This table shows the least squares estimates of the MA(1) coefficients for daily returns, 
\[ r_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \varepsilon_t. \]  
The t-values are denoted below their estimates in parenthesis. Those estimates significant at a 5% level are marked with an \( * \).

|  | \( \alpha_0 \) | \( \alpha_1 \) | \( R^2 \) |
|---|---|---|---|
| S1 | \( 1.26 \times 10^{-5} \) (1.8416) | -0.0062 (-.3985) | .000 |
| S3 | \( -3.01 \times 10^{-6} \) (-.3813) | -0.8042* (-95.614) | .3837 |
| S5 | \( -1.40 \times 10^{-6} \) (-.1992) | -0.8540* (-116.006) | .4208 |
| S7 | \( -1.21 \times 10^{-5} \) (-1.725) | -0.871* (-125.213) | .4304 |

of this regression are not serially correlated as the Durbin-Watson statistic as well as the Breusch-Godfrey test (both not reported here) suggest. Including higher order moving average coefficients or autoregressive elements gives us no significant new coefficients, but a poorer goodness of fit, so that we can confirm daily returns to follow a MA(1) process.

**Result 3.** *Microstructure elements induce negative first order serial correlation of daily returns, which follow a MA(1) process.*

A final property which can be observed from table 2 is that inventory induces positive first order serial correlation of squared returns, i.e. volatility. The reason for this finding is the same as for the negative first order autocorrelation of daily returns. A high return is in most cases associated with a large change in inventory holdings, the next trading day the dealer offers incentives for offsetting orders to arrive at the market, hence we will likely also find a high return, although of a different sign, causing the volatility to be high again. Through the large number of intraday trades this inventory offsetting is likely to be completed after one trading day, for which reason we find no significant evidence of higher order autocorrelations.

**Result 4.** *Inventory control causes positive first order serial correlation of daily return volatility.*

Throughout all parameter combinations we find no significant evidence that daily return volatility and trading volume are correlated. We therefore have not reported these crosscorrelations here.
Table 4: Estimates of GARCH(1,1) coefficients for daily returns

This table shows the maximum likelihood estimates of the GARCH(1,1) model for daily returns:

\[ r_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \varepsilon_t \]

\[ \varepsilon_t \sim N(0, h_t) \]

\[ h_t = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 + \gamma_2 h_{t-1} \]

The associated z-values are shown below their estimates in parenthesis and estimates significant at the 5% level are marked with a *.

|   | \( \alpha_0 \)    | \( \alpha_1 \)    | \( \gamma_0 \)    | \( \gamma_1 \)    | \( \gamma_2 \)    | \( R^2 \)   |
|---|------------------|------------------|------------------|------------------|------------------|----------|
| S1 | 1.26 \times 10^{-3} | .005             | 7.39 \times 10^{-8} | .150*            | .600*            | -.0001   |
|    | (1.001)          | (.169)           | (1.116)          | (3.074)          | (3.878)          |          |
| S3 | 7.05 \times 10^{-6} | -.611*           | 4.86 \times 10^{-6} | -.183*           | .613*            | .3481    |
|    | (.611)           | (-42.171)        | (4.813)          | (-6.801)         | (5.459)          |          |
| S5 | 2.58 \times 10^{-6} | .005             | 5.59 \times 10^{-6} | .150*            | .600*            | -.0051   |
|    | (.043)           | (.287)           | (4.432)          | (7.557)          | (10.561)         |          |
| S7 | -1.25 \times 10^{-3} | -.877*           | 2.85 \times 10^{-6} | -.001            | .818*            | .4307    |
|    | (-1.905)         | (-137.602)       | (.798)           | (-1.093)         | (3.408)          |          |

Result 5. Microstructure elements do not cause correlations between daily return volatility and trading volume.

We can summarize our results in the following proposition:

Proposition 1. The presence of microstructure elements gives rise to negative first order serial correlation in daily returns and positive first order correlation in daily return volatility, but not of any correlation between daily return volatility and trading volume. These properties are not affected by serial correlated trading volumes.

3 GARCH effects

In nearly all financial return series evidence of GARCH effects have been reported. We therefore have estimated the GARCH(1,1) model for the return series generated from our simulations. The results are reported in table [4].

Interestingly, we also find significant coefficients when no microstructure elements are present and hence daily returns are determined by changes in the fundamental value, which are iid distributed. When rewriting the definition of the return in conventional
form,
\[ \bar{r}_t = \ln \frac{\tilde{P}_t^*}{P_{t-1}} \approx \frac{\tilde{P}_t^* - \tilde{P}_{t-1}^*}{P_{t-1}} = \frac{\tilde{\varepsilon}_t}{\tilde{P}_{t-1}^*}, \]
we see that the return is also influenced by the level of prices, \( \tilde{P}_{t-1}^* \), which follows a MA(1) process according to (2). The conditional variance of (15) is given by
\[ \text{Var} \left[ \bar{r}_t | P_{t-1}^* \right] = \left( \frac{1}{P_{t-1}^*} \right)^2 \sigma_P^2. \]

Therewith the conditional variance follows also a MA(1) process as does \( \left( \frac{1}{P_{t-1}^*} \right)^2 \) and exhibits a positive first order autocorrelation. This effect in combination with the well known difficulties in estimating GARCH models causes the significance of the coefficients in S1 and S2. As the same problem also arises in presence of microstructure elements, the other estimates will also be biased as we can see from the estimates of the mean equation. Seemingly this problem is less pronounced in presence of a spread.

Thus far the literature has not considered the properties of conditional variances arising from return series due to changing levels of prices. It is beyond the scope of this paper to analyze this aspect further, instead we focus on daily price changes, \( \delta_t = P_t - P_{t-1} \), rather than returns, which we know to be iid distributed for the fundamental value. The estimates for the GARCH(1,1) model using daily price changes are reported in table 5.

We observe that the results of the mean equation are very close to those neglecting GARCH effects as reported in table 3. We have not reported these estimates for the daily price changes, but they are very close to those of daily returns. The estimates for the ARCH component \( \gamma_1 \) shows no significance for any simulation. However, with presence of microstructure elements the GARCH component \( \gamma_2 \) is significant. Therewith our findings suggest daily price changes to follow a GARCH(0,1) process with a MA(1) process for the mean.

Although the origin of this behavior in presence of inventory control is the positive first order autocorrelation of daily return volatility (Result 4), this explanation cannot be used with the spread being the only microstructure element.

**Proposition 2.** With presence of microstructure elements daily price changes follow a GARCH(0,1) process.

The estimates derived for our simulation are not too different from those observed in empirical investigations. Most empirical investigations report a relatively small ARCH component and a dominant GARCH component, which both sum to about .9. When considering the above mentioned biases from analyzing returns and the well
Table 5: Estimates of GARCH(1,1) coefficients for daily price changes

This table shows the maximum likelihood estimates of the GARCH(1,1) model for daily price changes:

\[ \delta_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \varepsilon_t \]

\[ \varepsilon_t \sim N(0, h_t) \]

\[ h_t = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 + \gamma_2 h_{t-1} \]

The associated z-values are shown below their estimates in parenthesis and estimates significant at the 5% level are marked with a *.

|     | \( \alpha_0 \) | \( \alpha_1 \) | \( \gamma_0 \) | \( \gamma_1 \) | \( \gamma_2 \) | \( R^2 \) |
|-----|---------------|---------------|---------------|---------------|---------------|-----------|
| S1  | .001          | -.005         | .002          | .006          | .237          | .000      |
|     | (1.860)       | (-.380)       | (.346)        | (.453)        | (.108)        |           |
| S3  | -.000         | -.794*        | .017          | -.045         | .831*         | .383      |
|     | (.154)        | (-83.845)     | (1.110)       | (-1.783)      | (4.620)       |           |
| S5  | -.000         | -.854*        | .013          | .011          | .879*         | .421      |
|     | (-1.911)      | (-115.000)    | (.920)        | (1.201)       | (7.037)       |           |
| S7  | -.001         | -.870*        | .017          | -.009         | .891*         | .430      |
|     | (-1.905)      | (-137.602)    | (.798)        | (-1.093)      | (3.408)       |           |

known statistical problems associated with the estimation of GARCH models, we see that at least a considerable part of the found GARCH effects may be attributed to microstructure elements.

4 Intraday Volatility

We can approximate intraday volatility by comparing the highest and lowest transaction price within a trading day, \( P_{t}^{\text{max}} \) and \( P_{t}^{\text{min}} \). We therefore define

\[ \sigma_t^{\text{intra}} = \ln \frac{P_{t}^{\text{max}}}{P_{t}^{\text{min}}}. \]

The statistics for \( \sigma_t^{\text{intra}} \) are presented in table 6. We see that without microstructure elements all transactions are conducted at the fundamental value, which is constant throughout the day, hence the highest and lowest price coincide. In presence of the spread as the only microstructure element we find intraday volatility to be highly persistent over time, a unit root cannot be rejected at any reasonable level of significance using the Augmented Dickey-Fuller test. This behavior can be explained by the constant absolute difference of these prices. Every transaction takes place either at the bid or at the ask, hence the difference of the highest and lowest price is the
Table 6: Simulation results for intraday volatility

This table shows the descriptive statistics and autocorrelations of the intraday volatility measure $\sigma_{\text{intra}}$ as well as the cross correlations with trading volume. Values being different from zero at the 5% significance level are marked with a *.

| Mean | Std. Dev. | Corr($\gamma_t, \gamma_{t-j}$) |
|------|-----------|--------------------------------|
|      |           | j=1  | j=2  | j=3  | j=4  | j=5  |
| S1   | 0         |      |      |      |      |      |
| S2   | 0         |      |      |      |      |      |
| S3   | .0051*    | $4.42 \times 10^{-5}$* | .998* | .996* | .994* | .992* | .990* |
| S4   | .0049*    | $6.54 \times 10^{-5}$* | .999* | .997* | .996* | .995* | .994* |
| S5   | .0108*    | .0024* | .055* | -.018 | -.010 | -.024 | -.012 |
| S6   | .0114*    | .0025* | .109* | .035  | .026  | .002  | .023  |
| S7   | .0135*    | .0023* | .062* | .006  | .007  | .005  | .028  |
| S8   | .0135*    | .0024* | .083* | .046  | .039  | .020  | -.001 |

| Mean | Std. Dev. | Corr($\gamma_t, V_{t-j}$) |
|------|-----------|--------------------------|
|      |           | j=0 | j=1 | j=2 | j=3 | j=4 | j=5 |
| S1   | -         | -   | -   | -   | -   | -   | -   |
| S2   | -         | -   | -   | -   | -   | -   | -   |
| S3   | .0043     | -.0051 | -.0047 | -.0049 | -.0054 | -.0056 |
| S4   | .0089     | .0080  | .0077  | .0075  | .0072  | .0066 |
| S5   | .1338*    | .0040  | -.0181 | -.0286 | -.0164 | -.0079 |
| S6   | .2306*    | .1692* | .1287* | .1151* | .1015* | .0974* |
| S7   | .1406*    | -.0049 | .0123  | .0069  | .0296  | .0000 |
| S8   | .2460*    | .1683* | .1450* | .1034* | .0681* | .0404 |

spread, changes in $\sigma_{\text{intra}}$ are only the result of changes in the level of prices, i.e. the fundamental value, which exhibits a unit root by construction.

Inventory control, however, causes small but significant positive first order serial correlation of intraday volatility. The argument for this finding is the same as for the positive serial correlation of daily return volatility, although it is of smaller magnitude. The correlation has to be smaller, because intraday volatility is also affected by large inventory changes reversed within the same trading day, so that volatility changes have to be less correlated. We also find a positive correlation between current trading volume and intraday volatility. This can easily be explained in analogy to the model of Clark (1973). A higher liquidity event causes a larger number of trading rounds and therewith, on average, a larger number of trades and a higher trading volume. The larger number of trades causes the inventory adjustment, $\eta_{t,\tau}$, to vary more within a trading day, hence the highest and lowest transaction prices are likely to differ more, i.e. intraday volatility is higher. The observed correlations of higher order are only the result of correlated trading volume, but not of any persistence in the correlation...
of intraday volatility. This result gives rise to our final proposition:

**Proposition 3.** *Intraday volatility and trading volume are positively correlated.*

## 5 Conclusions

We simulated a series of daily returns incorporating market microstructure elements and investigated the properties of these returns. Most important we found negative first order autocorrelation of daily returns, positive first order autocorrelation of daily return volatility, GARCH effects and a positive correlation between trading volume and intraday volatility, but no correlation with daily return volatility.

Many of these results can also be observed empirically, therefore the model awaits empirical tests to specify the influence of microstructure elements on daily returns. Future research may focus on the causes of the observed GARCH effects and the bias in GARCH estimates of return series arising from the influence of the price level. The generality of the model developed here allows to apply a large variety of parameter constellations and explore the effects arising from microstructure elements in much more detail than has been possible here. We then may get a better understanding of the return generating process.

Our results suggest that microstructure elements should not be ignored in analyzing daily returns. Extensions to weekly or monthly returns are straightforward and do not change the results as the employed model does not restrict the length of a time period. Considering these aspects may help to find a more appropriate model for the behavior of asset prices.
References

ANDERSEN, T. (1996): Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility. In: Journal of Finance, 51, 169–204.

Aoki, M. (1999): Analysis of an Open Model of Share Markets with Several Types of Participants. In: Proceedings of the 4th Workshop on Economics with Heterogeneous Interacting Agents. University of Genoa, Italy.

BOLLERSLEV, T. (1986): Generalized Autoregressive Conditional Heteroskedasticity. In: Journal of Econometrics, 31, 307–327.

BOLLERSLEV, T., CHOU, R. & K. KRONER (1992): ARCH Modelling in Finance: A review of the Theory and Empirical Evidence. In: Journal of Econometrics, 52, 5–59.

BREUSCH, T. (1978): Testing for Autocorrelation in Dynamic Linear Models. In: Australian Economic Papers, 17, 334–355.

CAMPBELL, J., LO, A. & MACKINLAY, C. (1997): The Econometrics of Financial Markets. Princeton: Princeton University Press.

CHEN, S., LUX, T. & MARCHESI, M. (1999): Testing for Nonlinearity in an Artificial Financial Market. In: Proceedings of the 4th Workshop on Economics with Heterogeneous Interacting Agents. University of Genoa, Italy.

CLARK, P. (1973): A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices. In: Econometrica, 41, 131–156.

DIEBOLD, F. & LOPEZ, J. (1995): Modeling Volatility Dynamics. Federal Reserve Bank of New York Research Paper 9522.

ENGLE, R. (1982): Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation. In: Econometrica, 50, 987–1008.

FOCARDI, S., CINCOTTI, S. & MARCHESI, M. (1999): Self-organized Criticality and Large Crashes in Financial Markets. In: Proceedings of the 4th Workshop on Economics with Heterogeneous Interacting Agents. University of Genoa, Italy.

FOSTER, D. & VISWANATHAN, S. (1990): A Theory of the Interday Variations in Volume, Variance, and Trading Costs in Securities Markets. In: Review of Financial Studies, 3, 593–624.

FOSTER, D. & VISWANATHAN, S. (1993a): The Effect of Public Information and Competition on Trading Volume and Price Volatility. In: Review of Financial Studies, 6, 23–56.

FOSTER, D. & VISWANATHAN, S. (1993b): Variations in Trading Volume, Return Volatility, and Trading Costs: Evidence on Recent Price Formation Models. In: Journal of Finance, 48, 187–211.

GOODFREY, L. G. (1978): Testing against General Autoregressive and Moving Average Error Models when the Regressors Include Lagged Dependent Variables. In: Econometrica, 46, 1293–1302.

GOODFREY, L. G. (1988): Specification Tests in Econometrics: The Lagrange Multiplier Principle and other Approaches, vol. 16, Econometric Society Monographs Series. Cambridge, UK: Cambridge University Press.

HAMILTON, J. (1994): Time Series Analysis. Princeton: Princeton University Press.

HE, H. & WANG, J. (1995): Differential Information and Dynamic Behavior of Stock Trading Volume. In: Review of Financial Studies, 8, 919–972.

HO, T. & STOLL, H. (1980): On Dealership Markets under Competition. In: Journal of Finance, 35, 259–267.
Ho, T. & Stoll, H. (1981): Optimal Dealer Pricing under Transactions and Return Uncertainty. In: Journal of Financial Economics, 9, 47–73.

Iori, G. (1999): A Microsimulation of Traders Activity in the Stock Market: The Role of Heterogeneous Expectations, Agents Interaction and Information Flow. In: Proceedings of the 4th Workshop on Economics with Heterogeneous Interacting Agents. University of Genoa, Italy.

Jones, C., Kaul, G. & Lipson, M. (1994): Transactions, Volume, and Volatility. In: Review of Financial Studies, 7, 631–651.

Lamoureux, C. & Lastrapes, W. (1990): Heteroskedasticity in Stock Return Data: Volume versus GARCH Effects. In: Journal of Finance, 45, 221–229.

Lo, A. & MacKinlay, C. (1990): An Econometric Analysis of Nonsynchronous-Trading. In: Journal of Econometrics, 45, 181–212.

Roll, R. (1984): A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. In: Journal of Finance, 39, 1127–1140.

Stoll, H. (1978): The Supply of Dealer Services in Securities Markets. In: Journal of Finance, 22, 1133–1151.