THE SIZE EFFECT REVISITED

BRANDON FLORES, TARAN GROVE, YI LIU, AND ANDREY SARANTSEV

Abstract. We compare performance of US stocks based on their size (market capitalization). We regress alpha and beta over size and other factors for individual stocks in Standard & Poor 500, for randomly generated portfolios, and for Standard & Poor 400 Mid-Cap and 600 Small-Cap. The novelty of our research is that we compare exchange-traded funds (ETFs) consisting of large-, mid- and small-cap stocks, including international ETFs. Conclusions: Size and market exposure (beta) are inversely related (strong evidence for ETFs, weak for individual stocks). No conclusive evidence about dependence of excess return (alpha) on size, or international markets.

1. Introduction

In quantitative finance, many researchers have tried to establish dependency of returns of a stock upon its market capitalization (cap), otherwise known as company size. The size effect states that stock returns are negatively correlated with size (small companies have larger returns). If small-cap stocks consistently return more given the same level of risk, this would disprove the efficient market hypothesis. The initial research [5] that observed the size effect was testing the seminal work of [18, 19] on the Capital Asset Pricing Model. This effect was incorporated in factor models in [9, 11, 13]. However, more recent research [12, 17] indicated disappearance of this effect. Reasons suggested include that this effect has been consumed by investors and is no longer relevant; that it can be explained by January anomaly; or that it is a risk effect not well reflected by usual measures, [15]. A very recent article [1] argues along these lines. A comprehensive review of the size effect literature is in (a bit dated) review [10]. In recent research [4] the size effect was used to explain why randomly simulated portfolios beat the benchmark. In [2] this observation was revisited using Stochastic Portfolio Theory. The article [3] argues that size effect still exists if you control for the quality of a company. Size effect for utility stocks is studied in [20]. Papers [6, 7] argue that the size effect is due to less information available for small stocks. International markets are studied in [8, 14, 16].

In this article, we test the size effect based on recent USA quarterly stock market data 1989–2019. Unlike some previous literature, we test dependence of both $\alpha$ and $\beta$ upon the size. We take data for constituent stocks from Standard & Poor 500 Large-Cap.

Many investors do not pick individual stocks, but invest in diversified portfolios. The size of a portfolio is defined as a weighted sum (with portfolio weights) of stock caps in this portfolio. We test whether portfolios consisting of mostly small-cap stocks have higher returns than benchmark indices, and test Standard & Poor 400 Mid-Cap and 600 Small-Cap vs 500 Large-Cap index. We also analyze large-, mid-, and small-cap exchange-traded funds issued by Vanguard and Black Rock.

One major difficulty is the well-known non-normality of quarterly stock returns. This translates into non-normality of regression residuals, thus standard statistical tests are not

Key words and phrases. Market capitalization, linear regression, efficient markets, market exposure.

JEL classification: G11, G15. AMS classification: 62P05, 91G70.
applicable. For future research, one can try generalized linear models of stochastic volatility models. With ETFs, the regression residuals are actually normal, and our research shows conclusively that $\beta > 1$ for small- and mid-cap ETFs, but $\alpha = 0$.

1.1. Raw return and equity premium. For a stock or a fund, its quarterly (raw) return can be defined in two ways:

- **End-quarter return:** Reinvest dividends paid every quarter at the end of this quarter: If $S(t)$ is the price at end of quarter $t$ (say December 31, 2018 for $t = 2018$ Q4), and $D(t)$ is the dividend paid in this quarter, then the returns is defined as
  \[
  Q(t) = \ln \frac{S(t) + D(t)}{S(t - 1)}.
  \]

- **Dynamic return:** Reinvest dividends at the day when they are paid. Invest 1 at the end of the previous quarter (say December 31, 2018) in this stock or portfolio. At the day when dividends are paid, reinvest them. At the end of this quarter, we have wealth $V$. Thus we define returns as $Q(t) = \ln V$.

Often returns are defined without logarithms; for example, instead of $Q = \ln V$ in the second version above we define $Q = V - 1$. But we believe that to define returns on the logarithmic scale is more accurate, since returns are compounded. The return of a portfolio is defined as a weighted sum (with portfolio weights) of returns of stocks in this portfolio. We compare these returns with 3-month Treasury rate $r$ at the end of previous quarter. If we invest 1 in Treasury bills at the end of previous quarter, we get $1 + r/4$ at the end of the current quarter. Thus we define returns as $Q_0 = \ln(1 + r/4)$.

*Equity premium* for this stock or portfolio in quarter $t$ is defined as $P(t) = Q - Q_0$. This is the return which we get above the one for non-risky assets (Treasury bills).

1.2. Excess return and market exposure. Take any benchmark, for example Standard & Poor 500 index or a corresponding mutual fund or exchange-traded fund. Denote its returns (understood as end-of-quarter returns) for a quarter $t$ by $P_0(t)$. Try to fit regression:

\[
(1) \quad P(t) = \alpha + \beta P_0(t) + \sigma \varepsilon(t), \quad \varepsilon(t) \sim \mathcal{N}(0, 1) \quad \text{i.i.d.}
\]

The intercept $\alpha$ is called *excess return*, and the slope $\beta$ is called *market exposure*. We are interested not only in coefficients $\alpha$ and $\beta$, standard error $\sigma$, and confidence intervals for $\alpha$ and $\beta$, but in $r^2$ and goodness-of-fit for this model (whether residuals are i.i.d. normal). If $\alpha = 0$ this is called Capital Asset Pricing Model, which states that the only factor determining stock returns is market exposure. This theory was challenged in the literature (some cited in the Introduction).

The main question of this article is how $\alpha$ and $\beta$ depend on size (for both stocks and portfolios). We measure size by the logarithm of capitalization, adjusted for capitalization of benchmark. Our conjecture is that $\alpha$ is negatively correlated with this measure.

Our setting is different from the classical setting by Fama & French (1993). In their article, they study only dependence of $\alpha$ on size. They also study value effect (how returns depend on fundamental factors: price to book ratio), which we do not touch in this article.

We take stock data: total quarterly returns and market caps, from *YCharts*. We take mutual and exchange-traded funds data from *YCharts*. This data is proprietary and we cannot publish it fully. However, we publish the list of all 1500 stocks and all funds which we use, on *GitHub*, see *asarantsev/SizeEffect*. We take 3-month Treasury rates from
The Size Effect Revisited

Table 1. Vanguard ETFs

| Fund   | β     | α     | σ     | $r^2$ | $T$ | $p$    |
|--------|-------|-------|-------|-------|-----|--------|
| Mega   | 0.9407| 0.0008| 0.0071| 0.9929| 45  | 0.6144 |
| Mid    | 1.1106| -0.0003| 0.0220| 0.9408| 60  | 0.7582 |
| Small  | 1.1961| -0.0025| 0.0244| 0.9374| 60  | 0.903  |

FRED (Federal Reserve Bank of Saint Louis Economic Data) and publish them on GitHub. The code for almost all computations with readme file is also available there.

In the tables below, '95% Int’ stands for confidence interval $[x_{2.5%}, x_{97.5%}]$, where $x_{α}$ is the $α$-quantile.

2. Exchange-Traded Funds

We compare large-cap, mid-cap, and small-cap ETFs for Vanguard, and Black Rock (iShares). For the latter, we compare both Standard & Poor and Morningstar classes. In many cases, these are well described by the simple linear regression. We compute quarterly end-quarter returns, without reinvesting dividends at the day they are paid.

2.1. Vanguard ETF. We compare VTI (Vanguard Total Stock Market ETF) versus MGC (Vanguard Mega Cap ETF), VO (Vanguard Mid Cap ETF), and VB (Vanguard Small Cap ETF). Let $P_0, P_G, P_M, P_S$ be equity premia for these ETFs, respectively. Let us fit (1). Results are in Table 1 with $T$ the number of quarters (data points), and the $p$-value for Shapiro-Wilk normality test.

Results show that none of these three size-based funds generate excess return $α$. But these have different market exposure: For Mega Cap ETF, $β < 1$, thus buying Mega stocks is equivalent to splitting portfolio in 94% Total ETF and 6% 3-month Treasury bills. For Mid Cap ETF, $β > 1$, and so this ETF is equivalent to shorting Treasury bills and buying Total ETF. For Small Cap ETF, $β$ is even larger than for Mid Cap ETF. Thus it is equivalent to shorting Treasury bills even more. Note that the regression fit is very good: The normality test has large $p$-value (thus we do not reject the hypothesis that residuals are normal i.i.d.). In addition, $r^2$ is very close to 1, thus almost all signal is described by this regression.

2.2. BlackRock Standard & Poor. We have 9 ETFs in this class, classified by two categories: Size (Small, Mid, Large) and Type (Blend, Value, Growth). The term Value refers to stocks which are underpriced relative to their fundamentals (dividends, earnings, book value, etc.) The term Growth refers to the opposite. Blend means stocks with average price relative to their fundamentals. We classify them as in Table 2 (left panel). These ETFs pay dividends once a quarter, at the same day, except when on very rare occasions an additional dividend is paid later. In such cases, we add this later dividend to next quarter’s dividend. As mentioned at the beginning of this section, we compute end-quarter returns. We have $T = 75$ quarters, from Q4 2000 to Q2 2019. For each type, we compare Large-Cap ETF with its Small-Cap and Mid-Cap counterparts. Table 2 (right panel) contains results. All 6 simple linear regressions pass the Shapiro-Wilk normality test for residuals. For all these, we cannot rule out the hypothesis that the intercept $α = 0$ (except IJJ vs IVE). In most cases $β > 1$, and $r^2 ≈ 1$ (thus most of the signal is explained using this regression).

We can also regress simultaneously each Mid-Cap ETF on its Large-Cap counterpart. There are total $T = 225$ data points, Shapiro-Wilk test still gives $p = 0.84$, $σ = 0.0321$, etc.
r² = 0.884, α = 0.0056, with confidence interval (0.001, 0.010), and β = 1.0509, with confidence interval (1.001, 1.101). For Small-Cap vs Large-Cap, we have: p = 0.378, r² = 0.831, α = 0.0073, with confidence interval (0.002, 0.013), and β = 1.0641, with confidence interval (1.001, 1.128). Note that we can reject α = 0 for significance level 5%. Thus mid- or small-cap generates additional α > 0 (of order 2-3% per year, and β > 1).

2.3. BlackRock Morningstar. Again, we have 9 ETFs in this class, with the same two categories: Type and Size, see Table 3 (left panel). The same remark about dividends for iShares applies to Morningstar. We have T = 57 quarters, from Q2 2005 to Q2 2019. See results in Table 3 (right panel). Similarly to the previous subsection, all 6 regressions have residuals which pass the Shapiro-Wilk normality test, and α = 0 cannot be ruled out. This is why we do not have a column for α.

We can also regress simultaneously each Mid-Cap ETF on its Large-Cap counterpart. There are total T = 171 data points, Shapiro-Wilk test still gives p = 0.87, σ = 0.0323, r² = 0.884, α = 0.00019, with confidence interval (–0.005, 0.005), and β = 1.117, with confidence interval (1.054, 1.180). Small-Cap ETFs vs Large-Cap ETFs gives us p = 0.378, α = –0.0027, with confidence interval (–0.009, 0.004), and β = 1.1636, with confidence interval (1.078, 1.249), σ = 0.0438, r² = 0.811. Here we cannot reject hypothesis α = 0 with significance level 5%. But we can reject β = 1. Thus going small-cap or mid-cap for iShares Morningstar ETFs gives us more exposure, but no excess return.

2.4. Recessions and expansions. Consider two time periods separately: 8 quarters Q3 2007 – Q2 2009 (Great Recession), and 40 quarters Q3 2009 – Q2 2019 (expansion).

For the Great Recession, we have 24 data points. Mid-Cap vs Large-Cap: Shapiro-Wilk p = 0.39, r² = 0.854, σ = 0.0456, α = –0.0071 with confidence interval (–0.031, 0.017), and β = 1.1149 with confidence interval (0.911, 1.319). Small-Cap vs Large-Cap: p = 0.52, σ = 0.0599, r² = 0.727, α = –0.0157 with confidence interval (–0.047, 0.016), β = 0.9870 with confidence interval (0.719, 1.255). Thus we cannot reject α = 0 or β = 1.
For the expansion, 120 data points. Small-Cap vs Large-Cap: $p = 0.77$, $r^2 = 0.81$, $\sigma = 0.0394$, $\alpha = -0.0072$ with confidence interval $(-0.015, 0.001)$, $\beta = 1.2115$ with confidence interval $(1.105, 1.318)$. Mid-Cap vs Large-Cap: $p = 0.98$, $r^2 = 0.87$, $\sigma = 0.0284$, $\alpha = -0.0002$ with confidence interval $(-0.006, 0.005)$, $\beta = 1.0926$ with confidence interval $(1.016, 1.169)$. Here, we can claim that $\beta > 1$, but we cannot reject $\alpha = 0$.

Conclusion: For the Great Recession, we cannot reject the hypothesis that small-cap and mid-cap ETFs are not different from large-cap ETF. But for the expansion, we can reject this hypothesis: Small and mid cap leads to larger exposure to the market.

2.5. Vanguard mutual funds. We compare quarterly returns (dynamic version) of NAESX (Vanguard Small-Cap mutual index fund) with that of VFINX (Vanguard 500 mutual index fund). Instead of interest rate as a risk-free asset, we take VMFXX (Vanguard Federal Money Market). We think this is more appropriate: One can actually invest in this. We take $T = 152$ quarters (38 years), from Q3 1981 to Q2 2019. Regression gives us $\alpha = -0.0083$, $\beta = 1.2719$, $r^2 = 0.810$, $p = 0.578$ for Shapiro-Wilk normality test applied to residuals, and $\sigma = 0.0491$. Confidence intervals for $\alpha$ and $\beta$ corresponding to 95% are $(-0.016, 0.000)$ and $(1.173, 1.371)$, respectively. For $\alpha = 0$ we have $p = 4.3\%$; thus using the standard statistical significance rules, we can reject the hypothesis that $\alpha = 0$.

2.6. Charles Schwab mutual funds. We compare quarterly returns (dynamic version) of SWSSX (Small-Cap Index) with that of SWPPX (S&P 500 Index) and use VFINX again as a risk-free asset. There are $T = 88$ quarters (22 years), and results: $\alpha = -0.0006$, with 95% confidence interval $[-0.010, 0.009]$; $\beta = 1.18$, with 95% confidence interval $[1.067, 1.293]$. Again, residuals pass Shapiro-Wilk normality test, $r^2 = 0.832$, and $\sigma = 0.0442$. Conclusion: Investing in Charles Schwab small-cap index is equivalent to shorting short-term Treasury bills (with weight 18%) and investing in Charles Schwab S&P 500 index.

2.7. International equity. We take two Invesco international equity mutual funds: OSMAX (small-cap) and QIVAX (total stock market). In place of risk-free asset, we take either Pimco international bond mutual fund: PFORX, or previously mentioned Vanguard US money market VMFXX. We have $T = 87$ quarters. We do dynamic quarterly return. In both regressions, residuals are not normal. However, we still have $r^2 = 0.649$ for PFORX and $r^2 = 0.651$ for VMFXX; thus regression explains much of the signal. We have $\alpha = 0.0181$ and $\beta = 1.1990$ for VMFXX, $\alpha = 0.0201$ and $\beta = 1.2202$ for PFORX. Actually, $\alpha$ passes the classic statistical significance test in both cases. We could conclude that international equity generates excess return if we switch to small cap; however, residuals are not normal, and this invalidates the t-tests. Reason for this failure can be complicated nature of international risk-free assets: Countries have different short-term interest rates, different from the USA Treasury rates.

3. Individual Stocks

We take Standard & Poor 500 constituent stocks as of July 7, 2019. For five of them, two share classes are shown: Alphabet Inc, Fox Corp, Under Armour, News Corp, Discovery Inc. We take only one class in each of these cases (the one which has more observations), thus bringing the total number of stocks down to 500, as fitting by the name of this index. We take 120 quarters: from Q3 1989 to Q2 2019. We measure time in quarters: $t = 0$ is end of Q2 1989, $t = 1$ is end of Q3 1989, up to $t = T := 120$ end of Q2 2019. For the first quarter, the cap data is available for 240 stocks on June 30, 1989. But as time goes on, there are more and more stocks, until for Q2 2019 all 500 stocks are available. Total, we have
Table 4. Vanguard 500 Benchmark Parameters: 30 and 10 years

| Coeff. | Estimate | p-value | 95% Int.   | Coeff. | Estimate | p-value | 95% Int.   |
|--------|----------|---------|------------|--------|----------|---------|------------|
| α₀     | 0.0282   | 0       | [0.026, 0.030] | α₀     | 0.0466   | 0       | [0.044, 0.049] |
| β₀     | 0.0162   | 0.236   | [-0.011, 0.043] | β₀     | -0.1923  | 0       | [-0.228, -0.156] |
| α₁     | -0.0149  | 0       | [-0.016, -0.014] | α₁     | -0.0122  | 0       | [-0.014, -0.010] |
| β₁     | 0.0607   | 0       | [0.045, 0.077] | β₁     | -0.0401  | 0       | [-0.068, -0.012] |

48269 stock-quarter data points. Returns are considered as dynamic returns. We also take a benchmark with quarterly returns $P₀(t)$.

Let $A(t)$ be the set of stocks available at end of quarter $t$, and let $N(t)$ be their quantity. Let $C_i(t)$ be the cap for a stock $i \in A(t)$ at time $t$, and let $P_i(t)$ be the equity premium for a stock $i \in A(t-1)$ during quarter $t$. Define the average cap at time $t$ as

$$C(t) := \frac{1}{N(t)} \sum_{i \in A(t)} C_i(t).$$

We measure size of a stock $i \in A(t)$ as follows:

$$V_i(t) = \ln C_i(t) - \ln C(t).$$

We need to correct by subtracting the log of average cap, because the stock market has tendency to grow most of the time. Thus caps in 2019 cannot be compared with caps in 1989 without such adjustment.

We regress $α$ and $β$ in (1) over $V_i(t-1)$, the adjusted log cap of a stock $i \in A(t-1)$ at end of previous quarter $t-1$:

$$P_i(t) = (α₀ + α₁ V_i(t-1)) + (β₀ + β₁ V_i(t-1)) P₀(t) + σ₀ ε(t),$$

$i \in A(t-1), \quad t = 1, \ldots, T, \quad ε(t) \sim N(0,1).$

For a benchmark $P₀(t)$, we have two choices:

- Vanguard 500 Index Fund VFINX: The oldest existing index fund. We take its end-period returns for these 120 quarters, and compute its equity premium $B(t)$.
- average return of the stocks in $A(t-1)$:

$$P₀(t) = \frac{1}{N(t-1)} \sum_{i \in A(t-1)} R_i(t).$$

3.1. Vanguard 500 Benchmark. We get $r^2 = 0.011$ (almost nothing is described by this regression) and $σ₀ = 0.1881$. Results are in Table 4 (left panel). Residuals (judging by the Jarque-Beta test) are not i.i.d. normal. In particular, kurtosis is 65. Here $α₀ > 0$ since the equity premium for a typical stock in $A(t-1)$ is larger than the one for VFINX. The reason is that stock which are included in S&P 500 usually perform better than average. The table shows that $α₁ < 0$ and $β₁ > 0$, that is, we can reject the null hypothesis in both these cases $p = 0$ using the $t$-tests. However, we cannot apply this test anyway, since residuals are not i.i.d. normal. For the last 10 years, results are in Table 4 (right panel), and $σ = 0.144$. 
Table 5. Averaged Returns Benchmark Parameters, 30 and 10 years

| Coeff. | Estimate | p-value | 95% Int.     |
|--------|----------|---------|--------------|
| $\alpha_0$ | -0.0074 | 0       | [-0.009, -0.005] |
| $\beta_0$ | 0.8885  | 0       | [0.867, 0.910] |
| $\alpha_1$ | -0.0086 | 0       | [-0.010, -0.007] |
| $\beta_1$ | -0.1108 | 0       | [-0.123, -0.098] |

| Coeff. | Estimate | p-value | 95% Int.     |
|--------|----------|---------|--------------|
| $\alpha_0$ | -0.0036 | 0.004  | [-0.006, -0.001] |
| $\beta_0$ | 0.9121  | 0       | [0.885, 0.940] |
| $\alpha_1$ | -0.0061 | 0       | [-0.008, -0.004] |
| $\beta_1$ | -0.1171 | 0       | [-0.138, -0.096] |

3.2. Averaged returns benchmark parameters. We get $r^2 = 0.224$, but residuals (judging by the Jarque-Beta test) are not i.i.d. normal. In particular, kurtosis is 88.208. Results are in Table 5 (left panel), and $\sigma_0 = 0.1666$. This table would show convincingly that small-cap stock generate extra $\alpha$ (excess return) and extra $\beta$ (market exposure), if the residuals were normally distributed. The Jarque-Beta test shows that (3) does not fit well. Still, $\alpha_1 < 0$ and $\beta_1 < 0$, and this gives us certain evidence (although not conclusive) that small stocks have extra $\alpha$ and $\beta$. For the last 10 years, see Table 5 (right panel), and $\sigma_0 = 0.123$.

3.3. Randomly generated portfolios. For Standard & Poor data earlier, for every $t$, randomly select a vector from the simplex

$$\Delta^N(t-1) := \{ (\pi_i, i \in A(t-1)) \mid \pi_i \geq 0, \sum_{i \in A(t-1)} \pi_i = 1 \}$$

and use it for a portfolio of stocks in $A(t-1)$. Define its size by $C_{\pi}(t-1) := \sum_{i \in A(t-1)} \pi_i P_i(t)$.

We use adjusted logarithmic size, as in (2), and regress upon a benchmark, as in (3). For each quarter, we generate 10 portfolios (independent for each quarter), this brings the total number of data points to 1200. Results are in Table 6 left panel: averaged returns benchmark, right panel: VFINX (Vanguard 500) benchmark. In none of the simulations, we got normal i.i.d. residuals. For example, kurtosis for averaged returns benchmark is 6.025, and for VFINX benchmark is 65. Next, $r^2 = 0.014$ for VFINX benchmark and $r^2 = 0.988$ for averaged returns. Finally, $\sigma = 0.086$ for VFINX benchmark, and $\sigma = 0.0094$ for averaged returns benchmark.

3.4. Regression of parameters upon size. For Standard & Poor 500, select the stocks which were in the index for all 120 quarters. Compute $\alpha$ and $\beta$ for each such stock $i$; denote them by $\alpha_i$ and $\beta_i$, respectively. Then regress these estimates of $\alpha_i$ and $\beta_i$ upon the log cap $\ln C_i(0)$ at the end of Q2 1989. Since now we take only stocks in the same time span we do not need to adjust for overall market cap.

$$\alpha_i = 0.0256 - 0.0026 \ln C_i(0) + 0.0114 \varepsilon_i, \quad r^2 = 0.117;$$

$$\beta_i = 1.1998 - 0.0282 \ln C_i(0) + 0.425 \varepsilon_i, \quad r^2 = 0.011.$$
Similarly, for the last 10 years = 40 quarters, bull market:

\begin{align}
\alpha_i &= 0.0383 - 0.037 \ln C_i(0) + 0.0237 \varepsilon_i, \quad r^2 = 0.035; \\
\beta_i &= 1.6277 - 0.0636 \ln C_i(0) + 0.508 \varepsilon_i, \quad r^2 = 0.023. 
\end{align}

Residuals in (4) and (5) do not pass normality test. The \( p \)-values for both regressions for null hypothesis for intercept are below 0.01, thus if residuals were normal we could reject null hypothesis.

4. Comparison of Standard & Poor indices

We can do S&P 400 Mid-Cap or S&P 600 Small-Cap vs the benchmark: S&P 500 Large-Cap, Q3 1989 – Q2 2019. But we do not have all stocks for each index for all quarters. Instead, for every quarter, consider stocks in each index which were there on July 7, 2019. Compute returns weighted by market caps for each of three indices.

4.1. S&P 400 Mid-Cap. Regression residuals are not normal: Shapiro-Wilk test gives \( p < 3 \cdot 10^{-5} \). For S&P 400, estimates \( \alpha = -0.009828, \beta = 1.036766, \sigma = 0.04578 \). We would reject the null hypothesis \( \alpha = 0 \) with \( p = 0.03 \) but not reject the null hypothesis \( \beta = 1 \) using standard regression tests, if residuals were normal.

4.2. S&P 600 Small-Cap. Regression residuals for S&P 600 actually pass the Shapiro-Wilk normality test with \( p = 0.3467 \). For S&P 600, estimates \( \alpha = -0.019235, \beta = 1.209421, \sigma = 0.04766 \). We can reject \( \alpha = 0 \) and \( \beta = 1 \) using standard regression tests. Thus small-cap stocks have excess return \( \alpha < 0 \) but market exposure \( \beta > 1 \).

5. Conclusion

We have some weak and inconsistent evidence that small cap stocks have larger \( \beta \) (based on subsection 3.5). We cannot really say anything about dependence of \( \alpha \) on size for individual stocks. It seems strange to us that S&P 400 and S&P 600 behave so differently in subsection 3.5. This merits further research. The existing evidence is based on signs of regression coefficients (obtained by least squares method).

However, in all cases (except S&P 600 vs S&P 500) the residuals are not normal. This is true for individual stocks as well as for randomly generated portfolios. This was expected: Non-normality of stock returns is a well-established fact, and it would be highly unusual if this non-normality disappeared after inclusion of size as factor. Thus we cannot apply classical statistical significance tests. Some directions for further research would be to do generalized linear models or stochastic volatility, to account for non-normal residuals.

However, for index funds (both mutual and exchange-traded), the evidence is very strong and consistent: \( \beta \) increases for mid- and small-cap, but there is no additional \( \alpha \). Thus going small-cap results in excess exposure to the market, but no excess returns. Another way to state this is: Buying small stocks is equivalent to shorting short-term Treasury bills and investing in S&P 500 or a similar large-cap index.

Since the overall market goes up most of the time, this implies higher average long-term returns. We stress that this does not disprove the efficient market hypothesis: Small-cap stocks have higher long-term returns, but also higher risk (measured by volatility).

Finally, international markets do not provide us with conclusive evidence one way or the other. This seems an important topic for further research.
REFERENCES

[1] Ron Alquist, Ronen Israel, Tobias Moskowitz (2018). Fact, Fiction, and the Size Effect. The Journal of Portfolio Management, 45 (1), 34–61.

[2] Robert D. Arnott, Jason Hsu, Vitali Kalesnik, Phil Tindall (2013). The Surprising Alpha from Malkiel’s Monkey and Upside-Down Strategies. The Journal of Portfolio Management 39 (4), 91–105.

[3] Clifford Asness, Andrea Frazzini, Ronen Israel, Tobias J. Moskowitz, Lasse H. Pedersen (2018). Size Matters, if You Control Your Junk. The Journal of Financial Economics 129 (3), 479–509.

[4] Adrian Banner, Robert Fernholz, Vassilios Papathanakos, Johannes Ruf, David Schofield (2018). Diversification, Volatility, and Surprising Alpha. arXiv:1809.03769

[5] Rolf W. Banz (1981). The relationship between return and market value of common stocks. The Journal of Financial Economics 9 (1), 3–18.

[6] Christopher B. Barry, Stephen J. Brown (1984). Differential Information and the Small Firm Effect. Journal of Financial Economics 13 (2), 283–294.

[7] Gregory A. Brauer (1986). Using Jump-Diffusion Return Models to Measure Differential Information by Firm Size. Journal of Financial and Quantitative Analysis 21 (4), 447–458.

[8] Nusret Cakici, Yi Tang, An Yan (2016). Do the Size, Value, and Momentum Factors Drive Stock Returns in Emerging Markets?

[9] Mark M. Carhart (1997). On Persistence in Mutual Fund Performance. The Journal of Finance 52 (1), 57–82.

[10] Michael A. Crain (2011). A Literature Review of the Size Effect. SSRN:1710076.

[11] Eugene F. Fama, Kenneth R. French (1993). Common Risk Factors in the Returns on Stocks and Bonds. The Journal of Financial Economics 33 (1), 3–56.

[12] Eugene F. Fama, Kenneth R. French (2011). Size, Value, and Momentum in International Stock Returns. The Journal of Financial Economics 105 (3), 457–472.

[13] Eugene F. Fama, Kenneth R. French (2015). A Five-Factor Stock Market Model. The Journal of Financial Economics 116 (1), 1–22.

[14] Rizaldi Fauzi, Imam Wahyudi (2016). The Effect of Firm and Stock Characteristics on Stock Returns: Stock Market Crash Analysis. The Journal of Finance and Data Science 2 (2), 112–124.

[15] Irwin Friend, Larry H. P. Lang (1988). The Size Effect on Stock Returns: Is it Simply a Risk Effect not Adequately Reflected by the Usual Measures? The Journal of Banking and Finance 12 (1), 13–30.

[16] Matthias X. Hanauer, Martin Linhart (2015). Size, Value, and Momentum in Emerging Market Stock Returns: Integrated or Segmented Pricing? Asia-Pacific Journal of Financial Studies 44, 175–214.

[17] Haim Levy, Moshe Levy (2011). The Small Firm Effect: A Financial Mirage? The Journal of Portfolio Management 37 (2), 129–138.

[18] JohnLintner (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. The Review of Economics and Statistics 47 (1), 13–37.

[19] Willian F. Sharpe (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. The Journal of Finance 19 (3), 425–442.

[20] Thomas M. Zepp (2003). Utility Stocks and the Size Effect — Revisited. The Quarterly Review of Economics and Finance 43 (3), 578–582.

University of Nevada in Reno, Department of Mathematics and Statistics
E-mail address: asarantsev@unr.edu