Study on optical freeform surface manufacturing of progressive addition lens based on fast tool servo

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Abstract. Progressive addition lens (PAL) is currently the state-of-the-art in multifocal correction visual for freeform lens. The PAL is used to correct presbyopia by the distributing optical powers of the three zones, which are far zone, near zone and intermediate zone. As the dioptre of progressive zone varies continuously, the lenses realize simultaneously far view and near view by a pair of glasses. Because the PAL is a freeform surface with a variable radius of curvature, complex shape, processing difficulties, low efficiency, big fluctuation of interpolation error and so on. In this paper, the design surface is discretized by the constant angle method and compensates the path of the diamond tool nose radius with AKIMA interpolation method, which can identify the machine tool of the tool locus, and exploitable fast tool servo technology controls the diamond tools movement and path. MATLAB simulation results show that this method is generally more approximate to the ideal trajectory, which is not only to ensure that surface profile accuracy, but also meet the requirements of the processing efficiency and better control the surface profile.

1 Introduction

With the development of economy and society, people pay more attention to the health of vision. When people are in their mid-40s, different kinds of "presbyopia" will come up, because of the natural decline in regulating ability of the eye muscles[1]. People usually used bifocal lenses, trifocal lenses and progressive addition lens to correct vision. The progressive addition lens is designed to a non-central symmetrical aspheric lens, which is to overcome the defects mutation and jump of the optical zoom based on the bifocal lenses and trifocal lenses.

In recent years, with the development of optical processing technology, the progressive addition lens has been gradually accepted by people. Compared with the traditional bifocal lenses and trifocal lenses, the PAL lens could provide the dioptries continuous variation through the progressive zone between far zone and near zone, which is achieved by continuously change of curvature radius, so as to solve the problem of vision jump drawback[2], as shown in figure 1.
Because the progressive addition lens is a freeform surface with variable radius of curvature, complex geometry, the traditional processing methods are difficult to achieve the designed requirements. In recent years, a large number of scholars have proposed a new processing method, such as glass molding\cite{3} and plastic injection molding\cite{4,5}, fast tool servo\cite{6}, and slow slide servo\cite{7} processing etc. The injected process has the advantages of high efficiency, low cost and is suitable for mass production. However, this processing method has the unavoidable defects, such as geometric shrinkage and warping, refractive index variation, flying edge and so on. In addition, the precision of the processing method depends on the precision of the grinding tools, processing a single type is not suitable for freeform surfaces PAL individual designed requirements. The traditional processing method of freeform surface PAL, which is grinding and polishing, leading to high processing cost, long cycle, is difficult to achieve the shape accuracy. Figure 2 shows the PAL design manufacturing process. Cutting path planning for complex optical surface based on Hermite interpolation is proposed by Zhou et al\cite{8} in the method of tool radius compensation, which can decompose the high frequency motion of tool compensation to the Z coordinate. However, when solving the cutting location point of freeform surface with large curvature, the interpolation auxiliary point is far away which causes larger interpolation error. HH Feng et al\cite{9} studies freeform manufacturing of a progressive addition lens based on voice coil fast tool servo, but there is no reference to path generation method and tool nose radius compensation method. YU et al\cite{10} optimizes the traditional tool path generation method, and only the Z value is to be modified for error compensation. Therefore, the high frequency reciprocating motion of the X-slide is avoided. But the golden point separation method requires multiple iterations to the required precision, which leads to low efficiency.

In order to solve the problem of difficult machining of freeform surface, and meet PAL "personalized design" requirement, this paper studies the single point diamond ultra-precision turning machining method based on fast tool servo, and presents a method of cutter radius compensation based on AKIMA interpolation method. The MATLAB simulation analysis shows that this interpolation method can be well controlled to ensure the surface profile, the accuracy of face type surface, and improve the machining quality of freeform surface.

**2 Powers and Astigmatism of Progressive Addition Lens**

In the design of PAL lens, we should make personalized design according to the patient’s eye habits, work environments etc. Compared with other multifocal lens, freeform surface PAL doesn’t have vision jump phenomenon and a clear far-near distance resolution, which can provide clear and natural visual effects, so the wearer has no sense of vertigo during operation. Design of PAL inevitably increases the astigmatism and distortion phenomenon\cite{11}, which is related to design style and diopter.
The refractive degree is an important parameter of the progressive addition lens, which is related to the lens material and the curvature radius. Diopter can be expressed as \[^{[12]}\]:

\[
p = (1 - n)k_b + \frac{(n - 1)k_f}{1 - d(1 - \frac{1}{n})k_f}
\]  

(1)

Where \( n \) is the refractive index of the lens material, \( d \) is the thickness of the lens, \( k_f, k_b \) is the curvature of the front and back surface of the lens. If the lens is very thin, the formula can be simplified as:

\[
p = (n - 1)(k_f - k_d)
\]  

(2)

Where \( k_f - k_d \) is the curvature difference.

3 Fabrication of the Progressive Addition lens

3.1 Freeform Surface Machining Technology

The machining of optical parts with freeform surfaces is performed on a diamond turning machine with fast tool servo (FTS). Fast tool servo machining is one of the single point diamonds turning method. In general, the three axis single point diamond lathe is "T" type distribution, including two straight axis--X axis and Z axis, and the main spindle (also called C axis) is installed on the X axis. When the work-piece is processed, the work-piece is clamped on the C axis by a vacuum chuck with the main spindle to rotate, and the FTS servo system is installed in the Z axis to achieve reciprocating motion. The relative motion of X axis and Z axis are driven by the servo motor and piezoelectric actuator, thus the processing of any free surface can be realized. Figure 3 shows the developed FTS diamond turning machine principle, the piezoelectric actuator can drive diamond tool following high-acceleration trajectories at frequencies up to several multiples of the spindle frequency, therefore, the higher spindle speed can be selected in the machining process.

![Figure 3 Schematic of FTS processing principle](image)

3.2 Cutter Contact Path Planning

The turning is the process of the interaction between the tool and the work-piece. The tool removes excess material on the work-piece along a specific tool path, which can be obtained requirements the shape, precision, and surface quality\[^{[13]}\]. It is crucial to generate efficient and accurate tool path in the production process\[^{[14]}\], the tool moves alone the Archimedes spiral which is consisted with Polar axis \( \rho \) and azimuth \( \theta \) in the free surface turning process, and then by calculating the value of point coordinate Z series, the ideal free surface can be obtained, as shown in figure 4, the freeform surface equation \( z = f(x, y) \) is transformed to polar coordinated equation \( z = f(\rho \cos \theta, \rho \sin \theta) \) by the column coordinate transformation formula \( x = \rho \cdot \cos(\theta), y = \rho \cdot \sin(\theta) \).
At present, tool step size is mainly determined by constant-angle or constant arc-length. The constant arc-length method leads to the number of cutting contacts point. The distribution of the cutter contact point is sparse and the machining error is larger in the center region. With the constant-angle method, generating tool contacts point is Sparse in outer ring, but the center of the work-piece is concentrate. This method is applied to the kind of the lens with low precision requirement in the outer ring and high center precision. At the same time, the calculation is simple, and the angle is the same between the adjacent points, so C axis uniform moves. Then we adopt the constant-angle method to discrete cutting contacts in this paper:

\[
\left\{
\begin{align*}
\rho &= \frac{D}{2} - \frac{\alpha}{2\pi} \theta \\
\theta &= \omega \cdot t \\
z &= f(\rho \cos \theta, \rho \sin \theta)
\end{align*}
\right.
\tag{3}
\]

\(\theta\) is the included angle, which is the between arbitrary cutter contact and the origin point, the increment of \(\theta\) is \(\Delta \theta = \frac{360}{N}\). \(N\) is the number of cutter contacts point. The spiral line is shown in figure 4.

### 3.3 Tool Nose Radius Compensate Based on AKIMA Interpolation

In the actual process, the machine program is only to identify the cutting location point, so the cutting contact point which calculates the tool nose radius compensation should be modified for cutting location point. According to the equation of the isometric line(4), the isometric cutting location point trajectory is obtained along the normal direction, then with the AKIMA interpolation curve \(z = A(\rho)\) instead of the isometric line the cutter of the contact trajectory line \(z = f(\rho)\).

\[
F(\rho) = \left\{
\begin{align*}
\rho - r \cdot \sqrt{1 + [f'(\rho)]^2} \\
f(\rho) + r \cdot \sqrt{1 + [f'(\rho)]^2}
\end{align*}
\right.
\tag{4}
\]

As shown in figure 5, the tool path with the tip p as the reference point will cause excessive cutting during the actual process. In order to avoid over cut, the paper presents the tool nose radius compensation. Because the main load of the machine tool is on the X axis, its dynamic response ability is limited. In order to prevent the X axis reciprocating motion, the phenomenon of over cutting cannot be working. Therefore, the AKIMA interpolation is used to compensate tool nose radius on the dynamic response of the Z axis. AKIMA interpolation establishes three degree polynomial fitting curve by any adjacent two points\(^{[15]}\). Assuming that the intersection equation is \(z = f(\rho)\) among any plane, the included angle with X axis is \(\theta\) and parallels to the Z axis and the machining plane. Known
cutting point \( P_i (\rho_i, z_i) \) \((i=0, 1...n-1)\), in any sub-interval \([\rho_i, \rho_{i+1}]\)\((i=0,1...n-1)\) to meet the following conditions:

\[
\begin{align*}
  z_i & = f(\rho_i) \\
  z_{i+1} & = f(\rho_{i+1}) \\
  z'_i & = g_i \\
  z'_{i+1} & = g_{i+1}
\end{align*}
\]

In order to construct the AKIMA interpolation curve, two points should be taken in the vicinity of \( P_3 \) and \( P_4 \) interpolation points. The isometric point of the cutting point \( P_i \) is obtained by the isometric formula, and the tangent vector the point of the cutting point is the same as that of the \( P_i \). \( g_i \) is the slope of the cutting point \( P_i \), which can be obtained by the equation:

\[
g_i = \left\{ \begin{array}{ll}
\frac{1}{2} (U_{i-1} + U_i), \text{ when } U_{i-2} = U_{i-1} \neq U_i = U_{i+1} \\
\frac{|U_{i+1} - U_i| U_{i-1} + |U_{i-1} - U_{i-2}| U_i}{|U_{i+1} - U_i| + |U_{i-1} - U_{i-2}|}
\end{array} \right. \quad (6)
\]

\( U_i \) is the slope between two adjacent cutting points, which can be obtained:

\[
U_i = \frac{z_{i+1} - z_i}{\rho_{i+1} - \rho_i}, i=1,2...n-1
\]

In order to calculate the slope at the end point of the curve, two auxiliary points are needed at the end point, as shown in figure 6:

\[
\begin{align*}
  U_0 &= 2U_1 - U_2 \\
  U_{n+1} &= 2U_n - U_{n-2} \\
  U_{n-1} &= 2U_{n-4} - U_3 \\
  U_n &= 2U_{n-1} - U_{n-3}
\end{align*}
\]

Then, a three degree polynomial can be uniquely defined on this interval:

\[
Z(\rho) = g_i (\rho - \rho_i) + \frac{(3\rho_i - 2\rho_{i+1} - \rho_{i+2})}{\rho_{i+2} - \rho_i} \cdot (\rho - \rho_i)^2 + \frac{2(3\rho_i - 2\rho_{i+1} - \rho_{i+2})}{\rho_{i+2} - \rho_i} \cdot (\rho - \rho_i)^2 + z_i
\]

In order to avoid over cutting, the cutting location point needs to move from the \( P_i \) point to \( P_i' \) along the \( Z \) axis, until tool profile is tangent to surface contour. According to the formula (4) ~ (9), when the abscissa is \( \rho_i \), \( Z \) coordinates can be obtained by the tool center value is \( A(\rho_i) \). The cutter path curve generated is more close to the design surface by this equation. The tool nose compensation is \( A(\rho_i) - r \) along the direction of the \( Z \) axis.

![Figure 5: Tool radius compensation](image)
4 Result Analyses

The aspheric surface is a special case of freeform surface, which is rotational symmetry. The aspheric surface has better radius of curvature, which can maintain a good phase difference correction and obtain better performances. The freeform surface equation is defined as:

\[
 z = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}} + \sum_{j=1}^{n} A_j x^j y^j \tag{10}
\]

Where \(c\) is the curvature, \(r\) is the radial coordinate, \(k\) is the taper coefficient, each parameter value in Table 1, when \(k > 0\) is a flat ellipsoid, \(k = 0\) is spherical, \(1 < k < 0\) to ellipsoid, \(k = 1\) is parabolic, \(k < 1\) is hyperboloid, figure 7 shows a progressive lens theoretical designed model.

In the calculation of tool path, assuming that the spindle speed is 600rpm, the feed rate is \(f = 0.2 \text{mm/rev}\), the discrete angle between adjacent blade contacts is \(\Delta \theta = 5^\circ\). Selecting the tool nose radius \(r = 0.5\text{mm}\), the rake angle is \(0^\circ\) and clearance angle is \(10^\circ\) of the diamond cutting tool.

| parameter       | value               |
|-----------------|---------------------|
| \(c/\text{mm}^{-1}\) | 0.01009472          |
| \(K/\text{mm}^{-1}\) | -6.112959           |
| \(A03\)         | -2.5526374X10-5    |
| \(A21\)         | -3.4195423X10-5    |
| \(A23\)         | -2.3625646X10-8    |
| \(A31\)         | -1.5621045X10-6    |
| \(A05\)         | -4.0519426X10-9    |

Table 1: The progressive lens parameter values

Figure 8 shows the 3D tool path when tool is machining tool progressive addition lens, the required cutter location points is \(N = \frac{360 \times D}{2 \times f \times \Delta \theta} = 7200\), according to the progressive lens equation to be the tool.
path in the X-C plane as shown in Figure 9, in a processing cycle the cutting tool appears twice peaks and troughs amplitude that gradually decays to zero from the outer to the inner ring.

![3D Tool Path](image1)

**Figure 8: 3D Tool Path**

![X-C Flat Tool Path](image2)

**Figure 9: X-C Flat Tool Path**

Figure 10 shows the 3D surface topography of progressive addition lens turning machining, so that the clear image is displayed, we need to delete some cutter location points. From the Figure10, we can clearly see the tool path line, processing texture and other characteristics of morphology, and the optimized tool path significantly improves the processing accuracy.

![3D Topography Simulation](image3)

**Figure 10: 3D Topography Simulation**

### 5 Conclusions
Progressive addition lens (PAL) is currently the state-of-the-art in multifocal correction visual for freeform lens. This paper introduces the design principle and processing method of progressive addition lens. With the focuses on the optimized method based on fast tool servo technology for progressive addition lens processing path, the processing fast tool servo is a high-precision, high-efficiency single point diamond turning methods. This paper adopts discrete cutter contact angle method and makes the arc radius compensation for the diamond cutter. The simulation results of the
surface morphology show that the optimized tool path can evidently improve the machining accuracy and machining efficiency, and reduce the deviation of surface profile.

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