Four-Neutrino Scenarios

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The main features of four-neutrino 3+1 and 2+2 mixing schemes are reviewed, after a discussion on the necessity of at least four massive neutrinos if the solar, atmospheric and LSND anomalies are due to neutrino oscillations.

1. Introduction

Solar and atmospheric neutrino experiments have observed for a long time anomalies that are commonly interpreted as evidences in favor of neutrino oscillations with mass squared differences
\[ 10^{-11} \text{eV}^2 \lesssim \Delta m_{\text{SUN}}^2 \lesssim 10^{-4} \text{eV}^2, \]  \[ 10^{-3} \text{eV}^2 \lesssim \Delta m_{\text{ATM}}^2 \lesssim 10^{-2} \text{eV}^2, \]
respectively (see Refs.[1,2]). More recently, the accelerator LSND experiment has reported the observation of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ appearance [3] with a mass-squared difference
\[ 10^{-1} \text{eV}^2 \lesssim \Delta m_{\text{LSND}}^2 \lesssim 10 \text{eV}^2. \]

The LSND evidence in favor of neutrino oscillations has not been confirmed by other experiments, but it has not been excluded either. Awaiting an independent check of the LSND result, that will probably come soon from the MiniBooNE experiment [3], it is interesting to consider the possibility that the results of solar, atmospheric and LSND experiments are due to neutrino oscillations. In this case, the existence of the three mass squared differences (1)–(3) with different scales implies that there are at least four massive neutrinos (three massive neutrinos are not enough because the three $\Delta m^2$’s have different scales and do not add up to zero).

Since the mass-squared differences (1)–(3) have been obtained by analyzing separately the data of each type of experiment (solar, atmospheric and LSND) in terms of two-neutrino mixing, it is legitimate to ask if three different mass squared are really necessary to fit the data. The answer is “yes”, as explained in Section 2.

Although the precise measurement of the invisible width of the $Z$ boson has determined that there are only three active flavor neutrinos, $\nu_e, \nu_\mu, \nu_\tau$, the possible existence of at least four massive neutrinos is not a problem, because in general flavor neutrinos are not mass eigenstates, i.e. there is neutrino mixing (see, e.g., Ref.[4]).

In general, the left-handed component $\nu_{\alpha L}$ of a flavor neutrino field is a linear combination of the left-handed components $\nu_{kL}$ of neutrino fields with masses $m_k$: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$, where $U$ is the unitary neutrino mixing matrix. The number of massive neutrinos is only constrained to be $\geq 3$. Following the old principle known as Occam razor, we consider the simplest case of four massive neutrinos that allows to explain all data with neutrino oscillations [3]. In this case, in the flavor basis the usual three active neutrinos $\nu_e, \nu_\mu, \nu_\tau$, are associated with a sterile neutrino, $\nu_s$, that is a singlet of the electroweak group.

Taking into account the measured hierarchy
\[ \Delta m_{\text{SUN}}^2 \ll \Delta m_{\text{ATM}}^2 \ll \Delta m_{\text{LSND}}^2, \]
there are only six types of possible four-neutrino schemes, which are shown in Fig.1. These six schemes are divided in two classes: 3+1 and 2+2. In both classes there are two groups of neutrino masses separated by the LSND gap, of the order of 1 eV, such that the largest mass-squared difference generates the oscillations observed in the LSND experiment: $\Delta m_{\text{LSND}}^2 = |\Delta m_{12}^2|$ (where $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$). In 3+1 schemes there is a
group of three neutrino masses separated from an isolated mass by the LSND gap. In 2+2 schemes there are two pairs of close masses separated by the LSND gap. The numbering of the mass eigenvalues in Fig. 1 is conveniently chosen in order to have always solar neutrino oscillations generated by $\Delta m_{21}^2 = \Delta m_{\text{SUN}}^2$. In 3+1 schemes atmospheric neutrino oscillations are generated by $|\Delta m_{31}^2| \simeq |\Delta m_{32}^2| = \Delta m_{\text{ATM}}^2$, whereas in 2+2 schemes they are generated by $|\Delta m_{13}^2| = \Delta m_{\text{ATM}}^2$.

In 1999 the 3+1 schemes were rather strongly disfavored by the experimental data, with respect to the 2+2 schemes [3]. In June 2000 the LSND collaboration presented the results of a new improved analysis of their data, leading to an allowed region in the $\sin^2 2\theta - \Delta m^2$ plane ($\theta$ is the two-generation mixing angle) that is larger and shifted towards lower values of $\sin^2 2\theta$, with respect to the 1999 allowed region. This implies that the 3+1 schemes are now marginally compatible with the data. Therefore, in Section 3 I discuss the 3+1 schemes, that have been recently revived [4,5]. In Section 4 I discuss the 2+2 schemes, that are still favored by the data.

2. Three $\Delta m^2$’s are necessary

Let us consider the general expression of the probability of $\nu_\alpha \to \nu_\beta$ transitions in vacuum valid for any number of massive neutrinos:

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_k U_{\alpha k}^* U_{\beta k} \exp \left( -i \frac{\Delta m_{kj}^2 L}{2E} \right) \right|^2,$$  \hspace{1cm} (5)
neutrino experiment measures a variation of the oscillation probability for \( L/E \sim 10^2 \div 10^3 \text{eV}^2 \) (see Ref.\(^\text{2}\)), there must be at least one \( \Delta m^2_{kj} \) in the range \( 10^{-3} \div 10^{-2} \text{eV}^2 \), which is out of the ranges allowed for \( \Delta m^2_{\text{SUN}} \) and \( \Delta m^2_{\text{LSND}} \). Therefore, at least a third \( \Delta m^2_{kj} \), denoted by \( \Delta m^2_{\text{ATM}} \), is needed for atmospheric neutrino oscillations. This argument is supported by a detailed calculation presented in Ref.\(^\text{1}\).

In the following sections we discuss some phenomenological aspects of the four-neutrino schemes in Fig.\(\text{1} \), in which there are three mass squared differences with the hierarchy (\(\text{3}\)) indicated by the data.

### 3. 3+1 Schemes

In 3+1 schemes the amplitude of \( \nu_\alpha \rightarrow \nu_\beta \) and \( \nu_\beta \rightarrow \nu_\alpha \) transitions in short-baseline neutrino oscillation experiments (equivalent to the usual \( \sin^2 2\theta \) in the two-generation case) is given by (see, for example, Ref.\(^\text{3}\))

\[
A_{\alpha\beta} = A_{\beta\alpha} = 4|U_{\alpha4}|^2|U_{\beta4}|^2, \tag{6}
\]

and the oscillation amplitude (again equivalent to the usual two-generation \( \sin^2 2\theta \)) in short-baseline \( \nu_\alpha \) disappearance experiments is given by

\[
B_\alpha = \sum_{\beta \neq \alpha} A_{\alpha\beta} = 4|U_{\alpha4}|^2 \left(1 - |U_{\alpha4}|^2\right) . \tag{7}
\]

Short-baseline \( \bar{\nu}_e \) and \( \nu_\mu \) disappearance experiments put rather stringent limits \( B_\alpha \leq B_\alpha^{\text{max}} \) and \( B_\mu \leq B_\mu^{\text{max}} \) for \( |\Delta m^2_{41}| \) in the LSND-allowed region. Taking into account also the results of solar and atmospheric neutrino experiments, Eq.\(\text{7}\) implies that \( |U_{e4}|^2 \) and \( |U_{\mu4}|^2 \) are small (see Ref.\(^\text{\text{3}\text{, }8}\)) and references therein):

\[
|U_{e4}|^2 \leq |U_{e4}|^2_{\text{max}} \quad \text{and} \quad |U_{\mu4}|^2 \leq |U_{\mu4}|^2_{\text{max}}, \tag{8}
\]

as shown by the dashed and dotted lines in Figs.\(\text{2} \) and \(\text{3}\). These limits imply that the amplitude \( A_{\mu\nu} \), equivalent to the usual \( \sin^2 2\theta \) in short-baseline \( \nu_\mu \rightarrow \nu_\nu \) and \( \bar{\nu}_\mu \rightarrow \bar{\nu}_\nu \) experiments, is very small:

\[
A_{\mu\nu} \leq 4|U_{\mu4}|^2_{\text{max}}|U_{e4}|^2_{\text{max}}, \tag{9}
\]

so small to be at the border of compatibility with the oscillations observed in the LSND experiment. Figure \(\text{4}\) shows the comparison of the bound \(\text{8}\) with the LSND allowed region, taking into account also the exclusion curves exclusion curves of the KARMEN \(\text{[14]} \) and BNL-E776 \(\text{[15]} \)

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**Figure 2.** 3+1 schemes. Dotted and dashed lines: \( |U_{e4}|^2_{\text{max}} \) from Bugey \(\text{[12]} \) and CHOOZ \(\text{[10]} \). Solid lines enclose the allowed regions.

**Figure 3.** 3+1 schemes. Dotted and dashed lines: \( |U_{\mu4}|^2_{\text{max}} \) from CDHS \(\text{[13]} \) and Super-Kamiokande \(\text{[16]} \). Solid lines: allowed regions.
experiments. One can see that there are four regions that are marginally allowed, denoted by R1, R2, R3, R4.

Let us denote by $A_{\mu e}^{\text{min}}$ the lower limit for $A_{\mu e}$ in the four allowed regions in Fig. 4. Then, from $A_{\mu e} = 4|U_{\mu 4}|^2|U_{e 4}|^2$ and the upper bounds (8), one can derive lower limits for $|U_{e 4}|^2$ and $|U_{\mu 4}|^2$:

$$|U_{e 4}|^2 \geq \frac{A_{\mu e}^{\text{min}}}{4|U_{\mu 4}|^2_{\text{max}}}, \quad |U_{\mu 4}|^2 \geq \frac{A_{\mu e}^{\text{min}}}{4|U_{e 4}|^2_{\text{max}}}. \quad (10)$$

The upper and lower limits (8) and (10) for $|U_{e 4}|^2$ and $|U_{\mu 4}|^2$ determine the allowed regions enclosed by solid lines in Figs. 2 and 3.

Summarizing the general properties of 3+1 schemes obtained so far, from Fig. 2 we know that $|U_{e 4}|^2$ is very small, of the order of $10^{-2}$, and from Fig. 3 we know that in the regions R2, R3, R4 $|U_{\mu 4}|^2$ is also very small, of the order of $10^{-2}$, whereas in the region R1 $|U_{\mu 4}|^2$ is relatively large, $0.33 \lesssim |U_{\mu 4}|^2 \lesssim 0.55$. On the other hand, the mixing of $\nu_\tau$ and $\nu_4$ with $\nu_4$ are unknown.

The authors of Ref. 5 considered the interesting possibility that

$$1 - |U_{\nu 4}|^2 \ll 1, \quad (11)$$

i.e. that the isolated neutrino $\nu_4$ practically coincides with $\nu_s$. Notice, however, that $|U_{\nu 4}|^2$ cannot be exactly equal to one, because LSND oscillations require that $|U_{e 4}|^2$ and $|U_{\mu 4}|^2$ do not vanish, as shown in Figs. 2 and 3, and unitarity implies that $1 - |U_{\nu 4}|^2 \geq |U_{e 4}|^2 + |U_{\mu 4}|^2$. The possibility (11) is attractive because it represents a perturbation of the standard three-neutrino mixing in which a mass eigenstate is added, that mixes mainly with the new sterile neutrino $\nu_s$ and very weakly with the standard active neutrinos $\nu_e, \nu_\mu, \nu_\tau$. In this case, the usual phenomenology of three-neutrino mixing in solar and atmospheric neutrino oscillation experiments is practically unchanged: the atmospheric neutrino anomaly would be explained by dominant $\nu_\mu \rightarrow \nu_\tau$ transitions, with possible sub-dominant $\nu_\mu \rightarrow \nu_e$ transitions constrained by the CHOOZ bound, and the solar neutrino problem would be

Figure 4. 3+1 schemes. Very thick solid line: allowed regions. Thick solid line: disappearance bound. Dotted line: LSND 2000 allowed regions at 90% CL. Solid line: LSND 2000 allowed regions at 99% CL. Broken dash-dotted line: Bugey exclusion curve at 90% CL. Vertical dash-dotted line: CHOOZ exclusion curve at 90% CL. Long-dashed line: KARMEN 2000 exclusion curve at 90% CL. Short-dashed line: BNL-E776 exclusion curve at 90% CL.

Figure 5. 3+1 schemes with $|U_{\nu 4}|^2 \ll 1$. Solid lines enclose the allowed regions. Long dashed line: CHORUS exclusion curve at 90% CL. Short dashed line: NOMAD exclusion curve at 90% CL. Dotted line: CDHS exclusion curve at 90% CL.
explained by an approximately equal mixture of $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ transitions (see, for example, Ref. [4]). An appealing characteristic of this scenario is the practical absence of transitions of solar and atmospheric neutrinos into sterile neutrinos, that seems to be favored by the latest data (see [18, 2, 19]).

Figure 6. 2+2 schemes. See caption of Fig.4.

Another interesting possibility has been considered in Ref. [8]:

$$|U_{e4}|^2 \ll 1.$$  \hspace{1cm} (12)

This could be obtained, for example, in the hierarchical scheme I (see Fig. 1) with an appropriate symmetry keeping the sterile neutrino very light, i.e. mostly mixed with the lightest mass eigenstates. Notice that nothing forbids $|U_{e4}|^2$ to be even zero exactly. The possibility (12) is interesting because if it is realized there are relatively large $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\tau$ transitions in short-baseline neutrino oscillation experiments, that could be observed in the near future. This is due to the fact that the unitarity of the mixing matrix implies that $|U_{\tau 4}|^2$ is large $(1 - |U_{\tau 4}|^2 \ll 1$ in the regions R2, R3, R4 and $0.45 \lesssim |U_{\tau 4}|^2 \lesssim 0.67$ in the region R1). Therefore, the amplitudes $A_{\mu\tau} = 4|U_{\mu 4}|^2|U_{\tau 4}|^2$ and $A_{e\tau} = 4|U_{e 4}|^2|U_{\tau 4}|^2$ of short-baseline $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\tau$ oscillations are suppressed only by the smallness of $|U_{\mu 4}|^2$ and $|U_{e 4}|^2$ and lie just below the upper limits imposed by the negative results of short-baseline $\nu_\mu$ and $\bar{\nu}_e$ disappearance experiments. Figure 8 shows the allowed regions in the $A_{\mu\tau} - |\Delta m_{31}^2|$ plane. One can see that the region R4 is excluded by the neg-
ative results of the CHORUS [10] and NOMAD [4] experiments. The other three regions are possible and predict relatively large oscillation amplitudes that could be observed in the near future, especially the two regions R2 and R3 in which $A_{\mu e} \sim 4 \times 10^{-2} - 10^{-1}$. An unattractive feature of this scenario is its predictions of large $\nu_\mu \rightarrow \nu_\tau$ transitions of atmospheric neutrinos, that appear to be disfavored by the latest data (see [24]).

4. 2+2 Schemes

The two 2+2 schemes in Fig. 1 are favored by the data because they do not suffer the constraint imposed by the thick solid line in Fig. 4 that is valid only in 3+1 schemes. Therefore, all the part of the LSND region in the $A_{\mu e} - \Delta m_{43}^2$ plane that is not excluded by other experiments is allowed, as shown in Fig. 4. For this reason, the phenomenology of 2+2 schemes has been studied in many articles [3].

Figures 7 and 8 show the limits on the mixing of $\nu_\tau$ and $\nu_\mu$ obtained from the results of short-baseline, solar and atmospheric experiments [19]. From Fig. 4 one can see that the mixing of $\nu_\tau$ with $\nu_3$ and $\nu_4$, whose mass-squared difference $\Delta m_{43}^2$ generates atmospheric neutrino transitions, is very small, leading to a suppression of oscillations of $\nu_\tau$’s in atmospheric and long-baseline experiments [20].

The mixing of $\nu_\tau$ and $\nu_\alpha$ is almost unknown, with weak limits obtained in recent fits of solar [22] and atmospheric data [22,24]. For example, it is possible that both solar $\nu_\tau$’s and atmospheric $\nu_\alpha$’s oscillate into approximately equal mixtures of $\nu_\tau$ and $\nu_\alpha$’s.

In the future it may be possible to exclude the scheme A if it will be established with confidence that the effective number of neutrinos in Big-Bang Nucleosynthesis is less than four. In this case $|U_{s3}|^2 + |U_{s4}|^2 < 1$ [22] and solar and atmospheric neutrino oscillations occur, respectively, through the decoupled channels $\nu_\tau \rightarrow \nu_\theta$ and $\nu_\mu \rightarrow \nu_\tau$. It has been shown that in this scenario the small mass splitting in scheme A between $\nu_3$ and $\nu_4$ is incompatible with radiative corrections [22] and the effective Majorana mass in neutrinoless double-beta decay in scheme A is at the border of compatibility with the experimental limit [27].

5. Conclusions

Four-neutrino mixing is a realistic possibility (if the solar, atmospheric and LSND anomalies are due to neutrino oscillations). It is rather complicated, but very interesting, both for theory and experiments, because: it has a rich phenomenology; the existence of a sterile neutrino is far beyond the Standard Model, hinting for exciting new physics; there are several observable oscillation channels in short-baseline and long-baseline experiments; CP violation may be observable in long-baseline experiments [3].

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