Study of vehicle-terrain interaction based on ANCF

G B Li $^{1,2}$, H S Wang $^2$

$^1$ Department of Mechanical Engineering, Shanghai Normal University, 100 Guilin Road, Shanghai, 200234, China
$^2$ Department of Mechanical Engineering, Shanghai Lida Polytechnic Institute, 1788 Cheting Road, Shanghai, 201609, China
liguangbu@163.com

Abstract. A brief review of the empirical and analytical models are presented. The pressure-sinkage relationship proposed by Bekker and the tractive performance (drawbar pull-slip) of vehicle is set up. A figure of drawbar pull vs. slip at different time and slip is illustrated. The modified Bekker soil models are integrated finite element (FE) Absolute Nodal Coordinate Formulation (ANCF) interpolations to determine the generalized forces. The paper discusses important fundamental issues that must be addressed when implementing terramechanics models into MBS algorithms in order to model complex tracked vehicle-soil interactions.

1. Introduction

Terramechanics is the study of the relationships between a vehicle and its environment. Some of the principal concerns in terramechanics are developing functional relationships between the design parameters of a vehicle and its performance with respect to its environment, establishing appropriate soil parameters, and promoting rational principles which can be used in the design, and evaluation of vehicles (Wong, 2010). Terramechanics models can be categorized into empirical and analytical models.

Empirical terramechanics models use established experimental measurements of appropriate parameters, properties, and behaviors of soil; these experimental results are then used to establish empirical relationships that could be used to predict at least qualitatively the response of soils under various conditions (Bekker, 1969). Parametric models, which are based on experimental work and have been widely used, offer practical means by which an engineer can qualitatively evaluate tracked vehicle performance and design.

Research in the field of terramechanics has continued over many decades. Since the tractive performance of a vehicle depends significantly on the normal and shear stress distribution at the soil-vehicle interface, many attempts have been made to predict such distributions beneath tracks and wheels (Schmid, 1995).

Bekker (1956, 1960) first proposed a theoretical study regarding the pressure-sinkage relationship of terrain, as well as the response to repeated loading and unloading. Janosi and Hanamoto (1961) proposed a model for predicting the ground pressure distribution beneath a rigid track. They assumed the normal stress distribution to be higher at the rear end of the track. The predicted ground pressure distribution was trapezoidal with higher pressure at the rear end of the track. Reece (1965) improved Bekker's model by making the parameters dimensionless. This single equation could then account for different plate shapes.
Garber and Wong (1981) developed analytical methods for predicting the pressure distribution beneath tracks taking into account all major design parameters of the vehicle and the pressure-sinkage characteristics of the soil. Wong et al. (1984) continued this work and established a model for the prediction of the distribution of ground pressure and the tractive performance of a tracked vehicle, including the effects of repetitive loading. Wong and Huang (2006) evaluated the tractive performance of wheeled and tracked vehicles assuming that the tracks on a tracked vehicle have an essentially flat and rectangular contact area with uniform normal pressure and the same contact length, as well as assuming the vehicle weight is uniformly distributed among the tracks.

2. Terramechanics based soil models

In study of vehicle-terrain interaction, terramechanics is one of the most important methods. Many attempts have been made to predict the normal and shear stress distributions beneath the tracks of moving vehicles ranging from empirical, analytical, to finite element models. Besides the tractive performance of a vehicle depends significantly upon its normal and shear stress distributions in the context of the soil-vehicle interface, and these stress distributions rely on the mechanical interactions inherent to the interface itself.

2.1 Pressure-sinkage relationship

Through above analysis, three common formulations on pressure–sinkage relationship are widely used in vehicle-terrain interaction investigation.

The first common pressure–sinkage relationship for the mineral terrain proposed by Bekker:

\[ p = (k_i \sqrt{b} + k_m) z^n = k_m z^n. \]  

(1)

\( p \) is pressure; \( b \) is the radius of a circular plate or the smaller dimension of a rectangular plate; \( n, k_i \) and \( k_m \) are pressure-sinkage parameters related to the cohesion and the angle of shearing of the material; \( k_m = (k_i / b + k_m) \), and \( z \) is sinkage.

Besides, equation (1) is now widely used in track-terrain interaction, tire-terrain interaction and wheel-terrain interaction.

Another common formulation of the pressure–sinkage relationship was proposed by Reece:

\[ p = (k_i + \theta k_m) \left( \frac{r}{b} \right)^n \]  

(2)

\( k_i \) and \( k_m \) are dimensionless soil parameters which replace Bekker’s \( k_i \) and \( k_m \) parameters.

Equation (2) can be used on wheel-terrain interaction, and tire-terrain interaction. The last common formulation of the pressure-sinkage relationship was proposed by Reece:

\[ \sigma_1 = (k_i + k_m \theta) \left( \frac{r}{b} \right)^n \left[ \cos \theta - \cos \theta_m \right] \]  

(3)

\[ \sigma_2 = (k_i + k_m \theta) \left[ \cos \left( \theta - \theta_m \right) \right] \]  

(4)

\( n \) is the sinkage exponent, \( k_i \) and \( k_m \) are pressure sinkage moduli, \( b \) is the wheel width, \( r \) is the wheel radius, \( \theta \), \( \theta_m \) is an entry angle and the angular location of the maximum normal stress.

Equation (3) and (4) can be used on wheel-terrain interaction. For the flexible tire acting on the deformable terrain, Equation (3) and (4) also can be used to tire-terrain interaction if radius \( r \) is effective radius.

From above study, the pressure-sinkage relationship proposed by Bekker is the best choice for the vehicle-terrain interaction analysis.
2.2 The Tractive Performance Study of vehicle

Besides the normal pressure, terrain shear stress is another parameter to determine vehicle tractive force.

For ‘plastic’ terrain which do not exhibit a ‘hump’ of maximum shear stress, a modified version of Bekker’s equation containing only one constant was proposed by Janosi and Hanamoto and is widely used in practice:

\[ s / s_{\text{max}} = 1 - e^{-j/K} \]  \hspace{1cm} (5)

\( K \) is usually referred to as the shear deformation parameter and is a measure of the magnitude of the shear displacement required for the development of the maximum shear stress.

Based on the shear stress-shear displacement curves measured under different normal pressures, the relation between the maximum shear stress and applied normal pressure can be derived. It has been found that the Mohr-Coulomb equation describes this relation adequately in many cases:

\[ s_{\text{max}} = c + p \tan \phi. \]  \hspace{1cm} (6)

In this equation, \( s_{\text{max}} \) is the maximum shear stress, \( p \) is the normal stress, and \( c \) and \( \phi \) are the cohesion (or adhesion) and angle of shearing resistance, respectively. In deriving the values of \( c \) and \( \phi \) from measured shear data, a procedure proposed by Reece in which the effect of grouser height of the shear ring is taken into consideration should be followed.

When the normal pressure and shear stress distributions under a vehicle at a given slip have been determined, the tractive performance of the vehicle can then be predicted. The tractive performance of a vehicle is usually characterized by its motion resistance, tractive effort, and drawbar pull (the difference between tractive effort and motion resistance) as functions of slip.

If it is assumed that a track link has an essentially flat and rectangular contact area with uniform normal pressure and the same contact length \( l \), and the link and terrain is horizontal, then under steady-state operating conditions, the link tractive effort \( F \) and drawbar pull \( F_d \) at a given slip \( i \) can be expressed as following:

\[ F = b \int_0^l (c + p \tan \phi)(1 - e^{-i/K}) \, dx = bl(c + p \tan \phi) \left[ 1 - \frac{K}{il} (1 - e^{-i/K}) \right]. \]  \hspace{1cm} (7)

\[ F_d = F - R_v. \]  \hspace{1cm} (8)

\( b \) is the contact width of the track link, \( l \) is the length of track link in contact with the terrain, \( p \) is normal pressure, and \( R_v \) is the motion resistance of track link. Figure 1 illustrates a drawbar pull vs. slip at different time and slip.
3. ANCF implementation

The FE implementation of the soil mechanics plasticity equations requires the use of an approach that allows employing general constitutive models. The vehicle/soil interaction can lead to a significant change in geometry that cannot be captured using finite elements that employ only translational displacement coordinates without significant refinement. In some soil applications, such a significant change in geometry may require the use of elements that employ gradients and accurately capture curvature changes. This requirement can be met using the FE absolute nodal coordinate formulation (ANCF).

ANCF finite elements do not employ infinitesimal or finite rotations as nodal coordinates; instead, absolute slopes and displacements at the nodal points are used as the element nodal coordinates. The position vector \( r^j \) of an arbitrary point on element \( j \) can be defined in a global coordinate system \( XYZ \) as \( r^j = S^j(x^j, y^j, z^j)e^j(t) \). In this equation, \( x^j, y^j, \) and \( z^j \) are the element spatial coordinates, \( S^j \) is the shape function matrix, \( e^j \) is the vector of element nodal coordinates, and \( t \) is time. The nodal coordinate vector \( e^{jk} \) at node \( k \) can be defined as follows (Shabana, 2005)

\[
\begin{bmatrix}
\frac{\partial r^j}{\partial x^j} \\
\frac{\partial r^j}{\partial y^j} \\
\frac{\partial r^j}{\partial z^j}
\end{bmatrix}
\]

Fully parameterized ANCF finite elements allow using a general continuum mechanics approach to define the Green-Lagrange strain tensor \( \varepsilon = \frac{1}{2}(J^T J - I) \), where \( J \) is the matrix of position vector gradients. In dynamic soil problems, ANCF leads to a constant inertia matrix and to zero Coriolis and centrifugal forces. The mass matrix obtained using ANCF finite elements can always be written as \( M^j = \rho^j S^j S^j dV^j \), where \( \rho^j \) and \( V^j \) are, respectively, the initial mass density and initial volume of the finite element. ANCF finite elements allow for straightforward implementation of general constitutive models including the continuum mechanics-based soil model discussed in this paper.

For a finite element or deformable body, the principle of virtual work can be written using the reference configuration as
\[ \int_V \rho \mathbf{r}^T \delta \mathbf{r} \, dV + \int_V \mathbf{\sigma}_{p2} : \delta \mathbf{\varepsilon} \, dV - \int_V \mathbf{f}_b \delta \mathbf{r} \, dV = 0 \]  

(10)

In this equation, \( V \) is the initial or reference volume, \( \rho \) is the initial mass density, \( \mathbf{r} \) is the global position vector of an arbitrary point, \( \mathbf{\sigma}_{p2} \) is the second Piola Kirchhoff stress tensor, \( \mathbf{\varepsilon} \) is the Green-Lagrange strain tensor, and \( \mathbf{f}_b \) is the vector of body forces. The second term in the preceding equation can be recognized as the virtual work of the elastic forces, it can be rewritten to define the generalized elastic forces, that is

\[ \delta W_e = \int_V \mathbf{\sigma}_{p2} : \delta \mathbf{\varepsilon} \, dV = \mathbf{Q}_e \delta \mathbf{e} \]  

(11)

\( \delta \mathbf{e} \) is the virtual change in the nodal coordinates associated with a particular ANCF finite element or a body, and \( \mathbf{Q}_e \) is the vector of the generalized elastic forces. The vector of elastic forces often takes a fairly complicated form, especially in the case of plasticity formulations, and is obtained using numerical integration methods. The principle of virtual work leads to the following equations of motion:

\[ \mathbf{M} \ddot{\mathbf{e}} + \mathbf{Q}_e = \mathbf{0} \]  

(12)

\( \mathbf{M} \) is the symmetric mass matrix, and \( \mathbf{Q}_e \) is the vector of body applied nodal forces.

Knowing the strains, the soil properties, yield function, and the flow rule; the state of soil deformation (elastic or plastic) can be determined. Knowing the state of deformation, the constitutive model appropriate for this state can be used to determine the elastic force vector \( \mathbf{Q}_e \).

4. Integration with MBS Algorithms

The FE implementation of the soil mechanics plasticity equations requires the use of an approach that allows employing general constitutive models. The vehicle/soil interaction can lead to a significant change in geometry that cannot be captured using finite elements that employ only translational displacement coordinates without significant refinement. In some soil applications, such a significant change in geometry may require the use of elements that employ gradients and accurately capture curvature changes. This requirement can be met using the FE absolute nodal coordinate formulation (ANCF). The objective of this paper is to present a review of soil mechanics formulations that can be integrated with computational MBS algorithms used for the virtual prototyping of vehicle systems. These algorithms allow for modeling rigid, flexible, and very flexible bodies. The small deformation of flexible bodies in vehicle systems are often examined using the floating frame of reference (FFR) formulation. Therefore, efficient modeling of complex vehicle system dynamics requires the implementation of different formulations that can be used for rigid body, small deformation, and large and plastic deformation analyses. A Newton-Euler or Lagrangian formulation can be used to model rigid bodies, the FFR formulation that employs two sets of coordinates (reference and elastic) can be used to model small deformations, and ANCF finite elements can be used to model large and plastic deformations including soil deformations.

MBS algorithms are designed to exploit the sparse matrix structure of the resulting dynamic equations. Because ANCF finite elements lead to a constant inertia matrix, Cholesky coordinates can be used to obtain an identity generalized mass matrix, leading to an optimum sparse matrix structure. Computational MBS algorithms are also designed to solve a system of differential and algebraic equations. The differential equations define the system equations of motion, while the algebraic equations define the joint constraints and specified motion trajectories. Using the constraint equations and the equations of motion, the augmented form of the equations of motion can be got as (Shabana, 2005).
5. Summary

In this paper, several simple models including Bekker’s model are discussed. Bekker’s model as well as other parametric and analytical terramechanics models have been used in the study of track/soil interaction and can be implemented in MBS algorithms using simple discrete force elements. The absolute nodal coordinate formulation (ANCF) which allows for the study of vehicle/soil interaction is also discussed.

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