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To cite this article: I V Kudryavtseva et al 2018 J. Phys.: Conf. Ser. 946 012118

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A new variant of a scaling hypothesis and a fundamental equation of state based on it

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Abstract. This paper deals with a fundamental equation of state (FEOS) for substances. We have suggested a new method. It allows constructing FEOS that is based on the scaling theory of critical phenomena and describes thermodynamic properties related to liquid and gas phases of a substance in a wide range of the pressures and temperatures. In the framework of the methodological approach, we have provided: (i) a transition of FEOS in a virial equation of state in the low density region; (ii) a transition of FEOS in a Widom equation of state in the critical region. The method has been tested on the example of FEOS of R218. The area of applicability of FEOS is $0 < \rho/\rho_c < 3.2$ in the density and $133 < T < 440$ K in the temperature. We have compared FEOS with some equations of state and discussed the results.

1. Introduction

The problem of constructing a fundamental equation of state (FEOS) has attracted attention of researchers considered a substance behavior in the critical region [1–12]. Especially this problem is investigated concerning some metals, which are characterized by the critical point locating at high temperatures and pressures [13–17].

We have analyzed main approaches those let us get successful results. Articles [1–3] represent a crossover equation of state written in a parametric form. There is a transition of the crossover equation to Scofield–Litster–Ho model (LM) [18] in an asymptotic vicinity of the critical point. Our analysis [19] has shown that the crossover equation [1–5] has an area of applicability: $0.4 \leq \omega \leq 1.3$ and $0.97 \leq t \leq 1.3$, here $\omega = \rho/\rho_c$ and $t = T/T_c$. Thus, an equation of state [5] represents:

- thermal data within the experimental error in the critical region;
- experimental information on the heat capacity at constant volume of argon [20] with big deviations, for example, there are deviations $\delta C_v$ up to 100% on the isochore at $\rho = 533.0$ kg/m$^3$ in the critical region.

The authors [7] proposed to solve the problem of FEOS constructing with the use of a combined equation of state in an empirical form:

$$p (\rho, T) = p_r (\rho, T) + \phi_0 (\rho, T) p_n (\rho, T).$$

This equation includes the following components:
In this work, we have intended to develop FEOS, which meets the following conditions:

1. FEOS structure: irregular component of Helmholtz free energy

2. FEOS has a satisfactory accuracy in comparison with the accuracy of a virial equation of state in a regular part of the thermodynamic surface;

3. there are no mathematical difficulties those are discussed earlier and connected with the calculations of properties \( p, F, s, i, w, C_v, C_p \) on a base of FEOS;

4. FEOS has to follow to the laws of the scaling theory of critical phenomena (ST) in the critical region.

We offer FEOS in the form:

\[
F (\rho, T) = F_{\text{reg}} (\rho, T) + F_{\text{irreg}} (\rho, T),
\]

where \( F(\rho, T) \) is the Helmholtz free energy; \( F_{\text{reg}} (\rho, T) \) is a regular function; \( F_{\text{irreg}} (\rho, T) \) is an irregular component.

Component \( F_{\text{irreg}} (\rho, T) \) is written in the form:

\[
F_{\text{irreg}} (\rho, T) = \frac{p_c}{\rho} \phi (\omega) |\Delta \rho|^{\delta+1} a (x),
\]

where \( \phi (\omega) \) is a regular function; \( p_c \) is the critical pressure; \( \Delta \rho = \omega - 1 \); \( \omega = \rho / \rho_c \); \( \delta \) is an index of the critical isotherm; \( x = \tau / |\Delta \rho|^{1/\beta} \) is a scaling variable; \( \tau = T / T_c - 1 \); \( \Delta \rho = \rho / \rho_c - 1 \).

Critical indices \( (\alpha, \alpha, \beta, \gamma) \) are interrelated by Griffith equations [23]:

\[
2 - \alpha = \beta \delta + \beta, \quad \gamma = \beta \delta - \beta.
\]

A component \( a (x) \) is a scaling function in (4). It relates to Helmholtz free energy and is written as:

\[
a (x) = - \frac{u_0 k \gamma \alpha_1 x_0^{2-\alpha}}{2 \alpha \beta^2 \alpha_1 (1 - \varepsilon)} \left[ (\varphi + \varphi_1)^{2-\alpha} - \varepsilon (\varphi + \varphi_2)^{2-\alpha} \right] + \frac{u_0 x_0^2}{2 k} (\varphi + \varphi_3)^{\gamma} + C,
\]
where \( \alpha_1 = (2 - \alpha)(1 - \alpha) \); \( \gamma_1 = \gamma(\gamma - 1) \); \( \varepsilon = x_1/x_2 \); \( u_0 \) is an individual parameter; \( \varphi = x/x_0 \); \( \varphi_i = x_i/x_0 \), \( i \in \{1,2,3\} \); \( x_0 \) is an importance of \( x = \tau/|\Delta \rho|^{1/\beta} \) on the saturation line; \( b^2 = (\gamma - 2\beta) / [\gamma(1 - 2\beta)] \); \( k = [(b^2 - 1)/x_0]^\beta \); \( C \) is a constant that can be calculated from equation \( \delta + 1 \) \( a(\varphi = -1) + x_0/\beta a'(\varphi = -1) = 0 \).

In [24, 25] \( (\varphi_i) \) parameters \( (6) \) are determined only by the critical indexes. We have chosen the critical indices \( (\alpha = 0.11, \gamma = 1.24) \) on the basis of ST. The rest of the critical indices we can find from Griffiths equation \((5)\). Thus, \( (\varphi_i) \) parameters can be applied with the accuracy that correlates with the accuracy of the critical indexes. In this case, \( (\varphi_i) \) values can be expressed in the form:

\[
\varphi_1 = 2.80724769, \quad \varphi_2 = 14.4717304, \quad \varphi_3 = 5.73246825.
\]

We have determined scaling functions \( F_s \) (a scaling function \( h(x) \) related to the chemical potential, a scaling function \( f(x) \) related to the isochoric heat capacity, a scaling function \( f_z(x) \) related to the coefficient of the isothermal compressibility) by function \( (6) \) in the form

\[
h(x) = (\delta + 1) a(x) - \frac{x}{\beta} a'(x), \quad (8)
\]

\[
f(x) = -a''(x), \quad (9)
\]

\[
f_z(x) = \left( \delta h(x) - \frac{x}{\beta} h'(x) \right)^{-1}. \quad (10)
\]

Our analyses has shown these numerical values of \( F_s \) \((8)-(10)\) are in a satisfactory agreement with values related to \( F_s \), that are named as \( (h_{lm}(x), f_{lm}(x), f_{z,lm}(x)) \) and calculated with an usage of LM [18]. Thus, relative deviations of \( h_{lm}(x) \) from \( h(x) \) are placed in the range \( \pm 1\% \). This comparison shows that a scaling function \( a(x) \) \((6)\) can be recommended to construct an irregular part \( F_{nreg}(\rho, T) \).

3. FEOS structure: crossover functions and regular components

The structure of FEOS \((3)\) includes a number of components. A regular component \( F_{reg}(\rho, T) \) and a function \( \phi(\omega) \) have been established considering the following conditions. Firstly, FEOS \((3)\) can be converted to \( Z(\rho, T) \) in the field of rarefied gas. This \( Z(\rho, T) \) is a thermal equation of state that is based on equation \((3)\) and has a form:

\[
Z(\rho, T) = 1 + \omega B(T) + \omega^2 C(T) + \cdots \quad (11)
\]

where \( Z(\rho, T) = \rho/ (R\rho T) \) is the compressibility factor; \( R \) is the gas constant; \( B(T) \) and \( C(T) \) are the second and the third virial coefficients respectively.

Secondly, equation \((3)\) can be converted to a Widom equation [26] in a vicinity of the critical point:

\[
\Delta \mu = \Delta \rho|\Delta \rho|^{\delta-1} h(x), \quad (12)
\]

where \( h(x) \) is a scaling function of the chemical potential \((8)\).

Function \( F_{reg}(\rho, T) \) is represented in the form:

\[
F_{reg}(\rho, T) = RT \ln(\omega) + RT \omega \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} C_{i,j} \tau_1^i (\Delta \rho)^j. \quad (13)
\]

where \( (C_{i,j}) \) are constant coefficients; \( \tau_1 = 1 - T_c/T \).

The thermal equation of state \( Z(\rho, T) \) can be written in a form:

\[
Z(\rho, T) = Z(\rho, T)_{reg} + Z(\rho, T)_{nreg}. \quad (14)
\]
The components of (14) are obtained in forms

$$Z(\rho, T)_{\text{reg}} = 1 + \omega \sum_{i=0}^{n_3} \sum_{j=2}^{j_3(i)} C_{i,j} \tau_i^1 (\Delta \rho)^{i-1} (i\omega + \Delta \rho),$$  \hspace{1cm} (15)$$

$$Z(\rho, T)_{\text{reg}} = \frac{p_c \omega}{RT} |\Delta \rho|^d \left[ \phi_0(\omega) \text{sgn}(\Delta \rho) h(x) + \phi'_0(\omega) |\Delta \rho| a(x) \right],$$ \hspace{1cm} (16)$$

where \(\phi_0(\omega) = \phi(\omega)/\omega\) is a crossover function.

Equation (14) has to follow the ST exponential laws. Hence, we have to elaborate \(Z(\rho, T)_{\text{reg}}\) in a form:

$$Z(\rho, T)_{\text{reg}} = 1 + \omega^2 y_1 + \omega y_2 + \omega (y_3 + \omega y_4) C_{1,0} + \omega (y_5 + \omega y_6) C_{2,0}$$

$$+ \omega \sum_{i=0}^{n_3} C_{i,0} (\Delta \rho)^{i-1} (i\omega + \Delta \rho) + \omega \tau_1 (2\omega - 3) C_{1,1}$$

$$+ \omega^2 \tau_1 (3\omega - 4) C_{2,1} + \omega \tau_1 \sum_{i=3}^{n_3} C_{i,1} (\Delta \rho)^{i-1} (i\omega + \Delta \rho)$$

$$+ \omega \sum_{i=0}^{n_3} \sum_{j=2}^{j_3(i)} C_{i,j} \tau_i^j (\Delta \rho)^{i-1} (i\omega + \Delta \rho),$$  \hspace{1cm} (17)$$

where \(y_1 = y'_2(\omega), y_3 = y'_4(\omega), y_5 = y'_6(\omega),\)

$$y_2 = 7.7/6 + (2.9/6) \Delta \rho - (1.1/6) \Delta \rho^2 + 0.05 \Delta \rho^3,$$  \hspace{1cm} (18)$$

$$y_3 = 5 - 4 \Delta \rho + 3 \Delta \rho^2 - 2 \Delta \rho^3 + \Delta \rho^4,$$ \hspace{1cm} (19)$$

$$y_5 = 4 - 3 \Delta \rho + 2 \Delta \rho^2 - \Delta \rho^3 + \Delta \rho^5.$$ \hspace{1cm} (20)$$

We have accepted requirements (2) and equation (11) to estimate crossover function \(\phi_0(\omega)\) included into equation (16). We have determined the following conditions for \(\phi_0(\omega)\):

$$\phi_0(\omega \to 0) \sim o(\omega^2),$$  \hspace{1cm} (21)$$

$$\phi_0(\omega \to 1) \to \text{const} + o(\Delta \rho).$$  \hspace{1cm} (22)$$

We have considered conditions (21), (22) and written \(\phi_0(\omega)\) in the form:

$$\phi_0(\omega) = D_0 \exp \left( -d(\Delta \rho)^2/\omega^q \right),$$  \hspace{1cm} (23)$$

where \(D_0\) is constant coefficient; \(d\) and \(q\) are positive constants.

Therefore, equations (14), (16) and (17) operate satisfactorily in a wide range of the pressures and temperatures including the critical region. They satisfy a number of requirements including:

- equation (3) has some properties of a virial row [27] in a low density region,
- equation (3) has a form of a Waidom equation in the critical region.

4. FEOS of R218

FEOS testing is held on R218 (the octafluoropropane) properties. Octafluoropropane has been chosen due to the fact that R218 belongs to a group of substances that are well experimentally investigated in a regular part of the thermodynamic surface and in the vicinity of the critical point [28–35].

We have considered \(F(\rho, T)\) (12) and written it the following form

$$F(\rho, T) = F^{(id)}(T, \rho) + RT \omega \tau_1 \left[ D_1 (\omega - 3) + D_2 (\omega^2 - 2\omega) \right]$$

$$+ RT \omega (y_2 + D_3 y_4 + D_4 y_6) + RT \omega \sum_{i=0}^{n_3} \sum_{j=2}^{j_3(i)} C_{i,j} \tau_i^1 (\Delta \rho)^{i-1} \omega$$

$$+ RT \omega |\Delta \rho|^d a(x),$$  \hspace{1cm} (24)$$
Coefficients and parameters of FEOS (24) are calculated on the basis of data [28–35] resulting in the following values of coefficients ($D_i$) and other parameters of equation (24): $T_c = 345.03$ K; $p_c = 2.69056$ MPa; $\rho_c = 628.0$ kg/m$^3$; $R = 44.221$ J/(K mol); $\alpha = 0.11$; $\gamma = 1.24$; $D_0 = 69056$ MPa; $\rho = 32157097995829.0$ kg/m$^3$; $a = 181.35690729944$ M; $D = 0.33358628926562$ kg/m$^3$; $\omega = 0.0893956818178$ kg/m$^3$; $\theta = 0.018631714861455$ kg/m$^3$.

Table 1. ($C_{i,j}$) coefficients of FEOS (26).

| $i$ | 0   | 1   | 2   | 3   |
|-----|-----|-----|-----|-----|
| 0   | 0   | 0   | 2.5617691416775 | -6.9861966820767 |
| 1   | 0   | 0   | -1.6139150254152 | 7.7886566171272 |
| 2   | 0   | 0   | -7.6763359406852 | 3.6943056002531 |
| 3   | 0   | -1.9083187225931 | 6.8045361816143 | -4.791135667487 |
| 4   | 0   | -0.34558255453005 | 5.6854455471724 | 0.72483101219932 |
| 5   | 0   | 1.8135690729944 | -7.411215987246 | -0.0893956818178 |
| 6   | -0.32157097995829 | 0.33358628926562 | 0 | 0 |
| 7   | 0.18282438203348 | -2.7597382038695 | 1.7191647772076 | 0 |
| 8   | 0.49620712531941 | 1.053647067233 | 0 | 0 |
| 9   | -0.55964848046682 | 1.9258863635011 | -0.0753385684843 | 0 |
| 10  | -0.14839676191572 | -1.9469699891719 | -0.2374157112432 | 0 |
| 11  | 0.55663092874474 | 0.44054971854678 | 0.1095619269401 | 0 |
| 12  | -0.37553492267565 | 0.20429804433639 | 0 | 0 |
| 13  | 0.11178156989302 | -0.12445028592387 | -0.00471669671239 | 0.00010388597496 |
| 14  | -0.01284627125068 | 0.018631714861455 | 0 | 0 |

where $F^{(id)} (T, \rho)$ is an ideal gas component of $F(\rho, T)$.

This component is calculated on the base of a $C_{p}^{(id)}$ formula [29]:

$$ F^{(id)} (T, \rho) = (0.117 - R) (T \ln T - T) + 0.14547T^2 - 0.000371T^3 + RT \ln \omega. \quad (25) $$

The thermal equation $Z(\rho, T)$ and the heat capacity are written on the basis of equations (24) and (25) as follows:

$$ Z(\rho, T) = 1 + y_1 \omega^2 + y_2 \omega + D_3 (y_3 \omega^2 + y_4 \omega) + D_4 (y_5 \omega^2 + y_6 \omega) $$

$$ + \omega \sum_{i=0}^{14} \sum_{j=0}^{7} C_{i,j} \tau_i^j \Delta \rho^{i-1} (i \omega + \Delta \rho) + D_1 \omega \tau_1 (2 \omega - 3) + D_2 \omega^2 \tau_1 (3 \omega - 4) $$

$$ + \omega |\Delta \rho|^\delta t^{-1} \left[ \phi_0 (\omega) \text{sgn} (\Delta \rho) \left( (\delta + 1) a (x) - \frac{x}{\beta} a' (x) \right) \right] $$

$$ + \phi_0' (\omega) |\Delta \rho| a (x), \quad (26) $$

$$ C_v = -R \omega t^{-2} \sum_{i=0}^{14} \sum_{j=0}^{7} C_{i,j} (j - 1) \tau_i^j \Delta \rho^j $$

$$ - R t \phi_0 (\omega) |\Delta \rho|^{-\alpha/\beta} a'' (x) \frac{1}{(117 + 2.908 T - 0.002226 T^2 - R)}. \quad (27) $$

Coefficients and parameters of FEOS (24) are calculated on the basis of data [28–35] resulting in the following values of coefficients ($D_i$) and other parameters of equation (24): $T_c = 345.03$ K; $p_c = 2.69056$ MPa; $\rho_c = 628.0$ kg/m$^3$; $R = 44.221$ J/(K mol); $\alpha = 0.11$; $\gamma = 1.24$; $D_0 = \ldots$
Table 2. \((C_{i,j})\) coefficients of FEOS \((26)\).

| \(i\) | \(j\) | 4 | 5 | 6 | 7 |
|-------|-------|---|---|---|---|
| 0     | 6     | -5.9964777446567 | -1.0256258034188 | 0.14872269607323 | -0.007132339751594 |
| 1     | 6     | -1.0987570101407  | 0              | 0              | 0              |
| 2     | 6     | 6.4843652187108   | 0              | 0              | 0              |
| 3     | 6     | -1.814225240777   | 0              | 0              | 0              |

Figure 1. Relative deviations \(\delta \rho = 100 \left( \rho_{\text{exp}} - \rho_{\text{cal}} \right) / \rho_{\text{exp}}\); \(\rho_{\text{cal}}\) corresponds to values calculated with a help of \((26)\) \((T = T_{\text{exp}}, p = p_{\text{exp}})\) in a single phase region; \(\rho_{\text{exp}}, T_{\text{exp}}, p_{\text{exp}}\) are from data [33] over isothermal lines: 1—133.15 K; 2—153.15 K; 3—173.15 K; 4—193.15 K; 5—213.15 K; 6—233.15 K; 7—253.15 K; 8—273.15 K; 9—293.15 K; 10—313.15 K.

5.3413236741558; \(D_1 = 0.8062439924; D_2 = 0.982021514026; D_3 = -8.01043426513 \times 10^{-3}; D_4 = 0.0879297351782; d = 2.0; q = 0.183; u_0 = 1.\)

The values of coefficients \(C_{i,j}\) are given in tables 1 and 2.

We have determined properties \(Y_1 = (p, \rho, C_v, C_p, \cdots)\) calculated on the base of equations \((26)\) and \((27)\). First, we have determined local deviations \((\%))\), \(\delta Y_1 = 100 \left( Y_{\text{exp}} - Y_1 \right) / Y_{\text{exp}},\) here \(Y_{\text{exp}}\) are experimental data [32]. These deviations \(\delta Y_1\) are shown:

- in figure 1 for the densities related to one phase region;
- in figure 2 for the pressures related to one phase region.
Figure 2. Relative deviations $\delta p = 100 \left( \frac{p_{\text{exp}} - p_{\text{cal}}}{p_{\text{exp}}} \right)$; $p_{\text{cal}}$ corresponds to values calculated with a help of (26) ($T = T_{\text{exp}}$, $\rho = \rho_{\text{exp}}$) in a single phase region; $\rho_{\text{exp}}$, $T_{\text{exp}}$, $p_{\text{exp}}$ are from data [33] over isothermal lines: 1—133.15 K; 2—153.15 K; 3—173.15 K; 4—193.15 K; 5—213.15 K; 6—233.15 K; 7—253.15 K; 8—273.15 K; 9—293.15 K; 10—313.15 K.

Figure 3. Relative deviations $\delta \rho = 100 \left( \frac{\rho_{\text{exp}} - \rho_{\text{cal}}}{\rho_{\text{exp}}} \right)$; $\rho_{\text{cal}}$ corresponds to values calculated with a help of (26) ($T = T_{\text{exp}}$, $p = p_{\text{exp}}$) in a single phase region; $\rho_{\text{exp}}$, $T_{\text{exp}}$, $p_{\text{exp}}$ are from data [33] over isothermal lines of 133.15, 153.15, 173.15, 193.15, 213.15, 233.15, 253.15, 273.15, 293.15 and 313.15 K; 1—$\delta \rho$ calculated with a help of (26); 2—$\delta \rho$ calculated with a help of [36].
Figure 4. Relative deviations $\delta p = 100 \left( \frac{p_{\text{exp}} - p_{\text{cal}}}{p_{\text{exp}}} \right)$; $p_{\text{cal}}$ corresponds to values calculated with a help of (26) and [36] ($T = T_{\text{exp}}, \rho = \rho_{\text{exp}}$) in a single phase region; $p_{\text{exp}}, T_{\text{exp}}, p_{\text{exp}}$ are from data [33] over isothermal lines of 133.15, 153.15, 173.15, 193.15, 213.15, 233.15, 253.15, 273.15, 293.15 and 313.15 K; 1 — $\delta p$ calculated with a help of (26); 2 — $\delta p$ calculated with a help of equation [36].

Figure 5. Relative deviations $\delta \rho = 100 \left( \frac{\rho_{\text{exp}} - \rho_{\text{cal}}}{\rho_{\text{exp}}} \right)$; $\rho_{\text{cal}}$ corresponds to values calculated with a help of (26) and [36] ($T = T_{\text{exp}}, \rho = \rho_{\text{exp}}$) in a single phase region; $\rho_{\text{exp}}, T_{\text{exp}}, p_{\text{exp}}$ are from data [30] over isochors of 618.71, 564.6, 479.9, 375.3 and 187.8 kg/m$^3$; 1 — $\delta \rho$ calculated with a help of (26); 2 — $\delta \rho$ calculated with a help of equation [36].

Second, properties $Y_2 = (p, \rho, C_v, C_p, \cdots)$ are calculated on the base of FEOS [36]. We have determined local deviations ($\%$), $\delta Y_2 = 100 \left( \frac{Y_{\text{exp}} - Y_2}{Y_{\text{exp}}} \right)$, here $Y_{\text{exp}}$ are data [32].
Figure 6. Relative deviations $\delta \rho = 100 \left( \frac{\rho_{\text{exp}} - \rho_{\text{cal}}}{\rho_{\text{exp}}} \right)$; $\rho_{\text{cal}}$ corresponds to values calculated with a help of (26) and [36] ($T = T_{\text{exp}}$, $p = p_{\text{exp}}$) in a single phase region; $\rho_{\text{exp}}$, $T_{\text{exp}}$, $p_{\text{exp}}$ are from data [34] over quasi-isochors of 1794, 1746, 1686, 1612, 1545, 1496, 1428, 1346 and 1205 kg/m$^3$:
1 — $\delta \rho$ calculated with a help of (26); 2 — $\delta \rho$ calculated with a help of equation [36].

Figure 7. Behavior of the isochoric heat capacity in a single-phase region over the isochor (995.7 kg/m$^3$): 1 — experimental points [32]; 2 — $C_v$ calculated with a help of (27); 3 — $C_v$ calculated with a help of the equation [36]; 4, 5 — crossing points related to the phase equilibrium curve.

These deviations $\delta Y_2$ are shown:
• in figure 3 for the densities related to one phase region;
• in figure 4 for the pressures related to one phase region.

Remarkable deviations can be seen for the data related to [33] in the field of a dense fluid (figures 3 and 4). A similar pattern is observed in the description of $p$-$\rho$-$T$ data [30] in the wide neighborhood of critical point (figure 5). Third, remarkable deviations (figure 6) can be seen in a comparison with experimental $p$-$\rho$-$T$ data [34]. This effect can be explained by the fact that data [34] are not consistent with the experimental data [33]. The data [34] are obtained in a much narrower range of parameters of state than the data [33]. Fourth, experimental data
on $C_v$ [31, 32] is described with equation (27) within the experimental error (figures 7 and 8) in a wide neighborhood of the critical point. Remarkable deviations are realized for properties $Y_2 = C_v$ based on FEOS [36] in this region of parameters of state.

5. Conclusions
We have developed a new version of the scaling hypothesis [6]. Applying this approach we have used:

- the equation of state in the form of LM;
- a Benedek hypothesis.

It has been estimated, firstly, that FEOS (3) works satisfactorily in a wide range of the pressures and temperatures including the critical region.

Secondly, FEOS is converted in a virial series [27] in a regular part of the thermodynamic surface. Thirdly, FEOS is converted in a Widom equation in the critical region.

This methodical part allows to develop FEOS (17) related to R218 and agrees satisfactorily with the experimental data in wide intervals of the pressures and temperatures including the critical region. Comparing $Z(\rho, T)$ (26) is fulfilled jointly with FEOS [36] and experimental data. It has revealed the following results. The maximum deviation $\delta Y_1$ of the density from equation (26) takes the following values:

- $\delta \rho_{\text{max}} = 0.1\%$ if experimental data [33] are analyzed in the liquid dense region;
- $\delta \rho_{\text{max}} = 4.5\%$ if experimental data [30, 33] are analyzed in the vicinity of the critical point.

It is estimated that the maximum deviation, $\delta Y_2$, of the density from FEOS [36] equals $\delta \rho_{\text{max}} = 0.3\%$ in the fluid dense region and $\delta \rho_{\text{max}} = 35\%$ in the vicinity of the critical point.

Acknowledgments
This work was partially financially supported by the Government of the Russian Federation, grant 074-U01.
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