LDPC code with Dynamically adjusted LLR under FSO turbulence channel

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Abstract. With the development and progress of space technology, more and more scholars begin to study FSO communication. The free-space optical communication takes the atmosphere as the medium, and the signals are susceptible to atmospheric turbulence. As an important channel code, LDPC code is applicable to almost all channels. In order to obtain better communication performance, we propose dynamically adjusted log-likelihood ratio (LLR) as the input of LDPC decoder in this paper. Compared with the traditional LLR input, the simulation results show that the LDPC code based on dynamically adjusted LLR has stronger anti-turbulence ability and better error performance under different atmospheric turbulence conditions. Coherent communication system with PM-QPSK modulation format under FSO turbulence channel is utilized in our work.

1. Introduction
With the development and progress of space technology, the information interaction between the ground and space as well as between space and space becomes larger and faster, and people's demand for information capacity and transmission speed keeps increasing. To accommodate such demand, optical communication technology emerges as the times require due to its advantages such as large capacity, high bandwidth and high security. Free space optical communication refers to the communication mode of information transmission in free space with light as carrier. Compared with traditional microwave communication [1], free space laser communication has wide bandwidth, high security, immunity to electromagnetic interference, lower power consumption, small size and large transmission capacity. Therefore, high speed free space laser communication has become a new direction of space communication development in the future, and has gotten extensive focus. However, in the free space optical communication system, it is easy to be affected by the atmospheric turbulence channel, resulting in additional turbulence channel noise in the optical signal, increasing the error rate of the system, and seriously affecting the communication rate and stability of the optical communication system. Therefore, it is necessary to study and reduce the influence of atmospheric turbulence channel on the system, and find out more effective methods.

Low density parity-check (LDPC) codes are used to improve the reliability and have received great attention. LDPC code is a block code whose check matrix contains only a small number of non-zero elements. It is the sparsity of the check matrix that ensures that the decoding complexity and the minimum code distance only increase linearly with the code length. In this paper, we choose LDPC code for Ultra-High Speed Optical Transport Networks as our channel code [2].

In this paper, the dynamically adjusted LLR is proposed as the input of the LDPC decoder, which has stronger turbulence resistance and better BER performance compared with the standard LLR input.
We first study the system model for FSO and optical coherent receiver. Then we analyze the LLR formula for FSO channel. Compared with a single AWGN channel, the traditional LLR formula based on AWGN channel may not be suitable for the joint distribution of AWGN channel and FSO channel [3]. We got the formula for LLR based on FSO channel. Meanwhile, we came up with the idea of LDPC code based on dynamically adjusted LLR. The second, we study the BER of two different LLR under different turbulence intensities. It is also compared with the case without channel coding.

The remainder of this paper is organized as follows. In Section II, we present atmospheric model, the system model and assumptions. It provides the FSO communication system architecture and Derivation of LLR based on FSO channel. Section III presents the main results of the LDPC code. Section V draws the final concluding remarks [4].

2. System model

2.1. Turbulence channel model

At present most of the studies of decoding algorithm are based on Gaussian distribution (AWGN) channel model. Gamma - Gamma probability distribution of light intensity fluctuations have a wider scope of application and can accurately describe the weak, and strong fluctuation compared with Logarithmic Normal distribution. The model parameters are relatively simple and can be connected with the physical characteristics of atmospheric turbulence, so it has been widely used in the performance analysis of free space optical communication [5].

The atmospheric turbulence channel is a discrete time channel, which is given as:

\[ y(t) = \alpha \cdot I(t) \cdot x(t) + n \]  \hspace{1cm} (1)

Where \( x(t) \) is signal to be transmitted and \( y(t) \) is signal to be received. The PM-QPSK modulation format is adopted in our system. The \( n(t) \) is the zero-mean Gaussian white noise. The \( \alpha \) is the detector responsivity of the photodetector. The \( I(t) \) is the intensity fading because of atmospheric turbulence. \( I(t) \) is a random variable.

In this paper, we analyze the effects of turbulence on free space laser communication on the basis of At receiver, the irradiation fluctuations \( I \) can be expressed as a multiplicative of two random variables and the random variables are subject to Gamma distribution, which can be given as:

\[ I = I_1 I_2 \]

The probability density function of irradiation fluctuations \( I \) is defined as:

\[ f_I(I) = \frac{2(\alpha\beta)}{\Gamma(\alpha)\Gamma(\beta)} I^{\alpha+\beta-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta I}) \hspace{1cm} I > 0 \]  \hspace{1cm} (3)

Where \( 1/\alpha \) and \( 1/\beta \) are the variances of the small and large scale eddies, respectively, \( \Gamma(\cdots) \) is the gamma function and \( K_{\alpha-\beta}(\cdots) \) is the modified Bessel function of the second kind.

\[ \alpha = \exp \left[ \frac{0.49\sigma^2}{(1+1.11\sigma^2)^{3/6}} \right] - 1 \]  \hspace{1cm} (4)

\[ \beta = \exp \left[ \frac{0.51\sigma^2}{(1+0.69\sigma^2)^{3/6}} \right] - 1 \]  \hspace{1cm} (5)

The Rytov variance is calculated from:

\[ \sigma^2 = 1.23C^2n^2k^{7/6}z^{11/6} \]  \hspace{1cm} (6)

where \( C^2n \) is the parameter index refraction structure, \( k \) is the optical wavenumber and \( z \) is the parameter range.
In order to obtain the turbulence effects under different channel conditions, we choose three kinds of weather conditions at different values of $C_n^2$, including: sunny: $C_n^2 = 5 \times 10^{-14} m^{2/3}$, rainy: $C_n^2 = 2 \times 10^{-13} m^{2/3}$, foggy: $C_n^2 = 5 \times 10^{-12} m^{2/3}$, and different channel spacing which are 50GHz, 100GHz, and 200GHz.

2.2. PM-QPSK Coherent optical receiver

Polarization multiplexing can be used to improve the spectrum efficiency of the system. The principle of polarization multiplexing is using the two orthogonal polarization states which can carry information to communicate with each other. With the increment of system complexity, there will be many new problems, such as polarization mode dispersion and other link damage problems.

In PM-QPSK transmitter, the optical carrier of the continuous laser is divided into polarized light with two orthogonal polarization states by the polarization beam splitter (PBS). IQ modulator is composed of two MZM modulators and a 90°PM modulator [6]. The two beams of orthonormal polarized light each go into an IQ modulator, which is parallel QPSK modulator. This creates an optical signal carrying the QPSK signal on each side. At the output side, the two beams of polarized light are combined by the polarization combiner (PBC) coupling interference beam together to form a signal, and finally the PM-QPSK signal is sent to the link for transmission. The schematic diagram of PM-QPSK transmitter is shown in figure 1.

![Figure 1: Structure of polarization diversity coherent transmitter of PM-QPSK](image)

In the coherent optical communication system, it is assumed that the electric field of the signal light $E_s(t)$ and the electric field of the local oscillator $E_l(t)$ can be expressed as:

$$E_s(t) = A_s(t) \cos(\omega_s t + \phi_s)$$  \hspace{1cm} (7)

$$E_l(t) = A_l(t) \cos(\omega_l t + \phi_l)$$  \hspace{1cm} (8)

In which $A_s(t)$ is the amplitude of signal light/dB; $A_l(t)$ is the amplitude of the local light/dB; $\omega_s$ is the angular frequency of the signal light/(rad/s); $\omega_l$ is the angular frequency of the local light/(rad/s); $\phi_s$ is the phase of the signal light/rad; $\phi_l$ is the phase of the local light/rad.

According to the relationship between the signal light frequency and the local light frequency, coherent optical communication system usually has two detection methods: coherent detection and zero
difference detector [7]. When $sL \omega - \omega_k = 0$, it is called zero-difference detection system. At this point, four-channel signal is converted into I-channel and Q-channel electrical signals through photodetector can be presented as:

\[ i_I = 2\beta A_k(t)A_i(t) \cos(\varphi_k - \varphi_i) \]  
\[ i_Q = 2\beta A_k(t)A_i(t) \sin(\varphi_k - \varphi_i) \]  

Where $\beta$ is the responsiveness of the balanced detector.

2.3. Dynamically adjusted LLR

2.3.1. LLR over Atmospheric turbulence channel.

Soft decision is utilized as decoder which takes log likelihood ratio (LLR) as input. For QPSK, constellation points \{ $a_0$, $a_1$, $a_2$, $a_3$ \} represents the input signals $x$={00, 01, 10, 11} separately according to formula (1). Constellation points are as:

\[ a_0 = \alpha I(t)(1 + i) \]
\[ a_1 = \alpha I(t)(-1 + i) \]
\[ a_2 = \alpha I(t)(1 - i) \]
\[ a_3 = \alpha I(t)(-1 - i) \]  

To decide which signal $y$ belongs to, there are two steps by using LLR. Firstly, we decide whether $y$ belongs to [1] or [2] by LLR(Re) that is:

\[ LLR(\text{Re}) = \ln \frac{P(y | x = a_0) + P(y | x = a_1)}{P(y | x = a_2) + P(y | x = a_3)} \]  

where $P_i$ indicates the probability and received signal is $y(t) =\alpha \cdot I(t) \cdot x(t) + n(t)$, $n$ is Gaussian White noise. Suppose the prior probabilities have the same distribution.

According to Bayes theorem, LLR (Re) can be simplified to:

\[ LLR(\text{Re}) = \ln \frac{p(y | x = a_0) + p(y | x = a_1)}{p(y | x = a_2) + p(y | x = a_3)} \]  

Where $p$ indicates the probability density function (pdf). We know that $n$ obeys the Gaussian distribution with mean 0 and variance $\sigma^2$, so we have:

\[ LLR(\text{Re}) = \ln \frac{\exp \left( \frac{\langle y, a_0 \rangle}{\sigma^2} \right) + \exp \left( \frac{\langle y, a_1 \rangle}{\sigma^2} \right)}{\exp \left( \frac{\langle y, a_2 \rangle}{\sigma^2} \right) + \exp \left( \frac{\langle y, a_3 \rangle}{\sigma^2} \right)} \]  

Where $<,>$ indicates inner product. According to approximate LLR theory in [8], LLR(Re) can be simplified to:

\[ LLR(\text{Re}) = \frac{2\alpha I(t) \text{Re} \{ y \} \sigma^2}{\sigma^2} \]  

According to LLR(Re), we can tell whether the received signal is on the positive or negative real axis. For the second step, we can gain LLR(Im) by the same procedures as get LLR(Re), and LLR(Im) can be written as:

\[ LLR(\text{Im}) = \frac{2\alpha I(t) \text{Im} \{ y \} \sigma^2}{\sigma^2} \]  

According to LLR (Im), we can tell whether the received signal is on the positive or negative imaginary axis [8]. Through these two steps, we can determine the received signal accurately.

2.3.2. LLR over Atmospheric turbulence channel

In the duration of atmospheric turbulent channel, the distribution of received signals is gaussian distribution. Formula can be simplified as:
\[ y(t) = \beta x(t) + n(t) \]  \hspace{1cm} (18)

Where \( \beta \) is a constant. Assume that \( \{y_{j1}, y_{j2}, \ldots, y_{jN}\} \) is the sequence of received signals in duration of the quasi-static channel. \( \beta \) can be given as:

\[
\beta \approx \frac{\sum_{j=1}^{N} \text{abs}(\text{real}(y_{j})) + \sum_{j=1}^{N} \text{abs}(\text{imag}(y_{j}))}{2*N} \hspace{1cm} (19)
\]

Therefore, we only need to dynamically adjust the amplitude attenuation coefficient within the holding time of the channel. In duration, \( \sigma^2 \) can be given as:

\[
\sigma^2 = \text{var(\text{abs}(\text{real}(y))) + i* \text{abs}(\text{imag}(y)))} \hspace{1cm} (20)
\]

Dynamically adjusted LLR is the input to the LDPC decoder.

3. RESULTS AND DISCUSSION

Figure 2 shows the simulation system of the 112Gbit/s coherent DP-QPSK FSO Communication, and the simulation is carried out using Optisystem 15. The performance of the system is analyzed by using optical power meter, BER analyzers, electrical constellation analyzers, optical spectrum analyzers. The simulation parameters for the FSO system are listed in Table 1 and Table 2. The aperture diameter of transmitter and receiver are 5cm and 30cm, respectively. The beam divergence is set as 1mrad, and the index refraction structure using three conditions, weak turbulence \(1.4745e^{-16} m^{-2/3}\), moderate turbulence \(9.5840e^{-16} m^{-2/3}\), and strong turbulence \(9.5840e^{-16} m^{-2/3}\).

| Parameter                  | Value                   | Units     |
|----------------------------|-------------------------|-----------|
| Range                      | 10                      | km        |
| Attenuation                | 0.2                     | dB/km     |
| Transmitter aperture diameter | 5                      | cm        |
| Beam divergence            | 1                       | mrad      |
| Receiver aperture diameter | 30                      | cm        |
| Index refraction structure | 1.4745e-16 (weak)       | \(m^{-2/3}\) |
|                           | 9.5840e-16 (moderate)   | \(m^{-2/3}\) |
|                           | 2.5803e-15 (strong)     | \(m^{-2/3}\) |
| Frequency                  | 1550                    | nm        |
Table 2 Transmitter/Receiver

| Parameter       | Value | Units |
|-----------------|-------|-------|
| Format          | DP-QPSK |       |
| Laser linewidth | 0.01  | MHz   |
| LO linewidth    | 0.01  | MHz   |
| LO power        | 0     | dBm   |
| Responsivity    | 0.64  | A/W   |
| Dark current    | 10    | nA    |

At the same time, based on the results, we find that the intensity of turbulence has a greater impact on the system, which is shown in the Figure 3. Because of atmospheric turbulence, the constellations look like stripes instead of gauss. As the intensity of turbulence increasing [9], constellation points close to the origin, which means that some of the data points are in the wrong quadrant.

![Fig.3 the constellation of receiver under different index refraction structure](image)

(a) weak turbulence (b) moderate turbulence (c) strong turbulence

The BER performance of the 112 Gbit/s FSO system at different turbulence conditions is shown in the Fig.4 (a)-(c). First of all, LDPC code can enhance receiver sensitivity and increase link margin in atmospheric turbulence channel. When the BER is $10^{-4}$, LDPC code can boost 6dB receiver sensitivity under weak turbulence, and 13dB under moderate turbulence and 15.9dB under strong turbulence. Secondly, LDPC code based on Dynamically adjusted LLR performs better than LDPC code based on LLR under traditional AWGN channel. LDPC code based on Dynamically adjusted LLR can enhance 13% the receiver sensitivity than LLR under traditional AWGN channel under weak turbulence and 7.4% under moderate turbulence [10].

The BER performance of the 112 Gbit/s FSO system at different turbulence conditions is shown in the Fig.5 (a)-(c). When the BER is $10^{-4}$, we find that the intensity of turbulence has a greater impact on the system. The receiver sensitivity for no channel coding is approximately 10dB budget between weak turbulence and moderate turbulence. The receiver sensitivity for LDPC code based on Dynamically adjusted LLR is approximately 3dB budget between weak turbulence and moderate turbulence. The receiver sensitivity for LDPC code based on LLR under traditional AWGN channel is approximately 3dB budget between weak turbulence and moderate turbulence. The results show that LDPC code can reduce the effect of atmospheric turbulence on the system [11]. Meanwhile, LDPC code based on Dynamically adjusted LLR has stronger turbulence resistance.
Fig. 4 The BER performance of the system using LDPC code with different LLR information and no channel coding.

(a) LDPC code based on Dynamically adjusted LLR under traditional AWGN channel
(b) LDPC code based on LLR
(c) No channel coding

Fig. 5 The BER performances of the system using LDPC code with different turbulence

4. Conclusion
A 112Gbit/s coherent free space optical communication system based on PM-QPSK modulation technologies and LDPC code based on Dynamically adjusted LLR under different atmospheric are
proposed in our paper. The proposed 112Gbit/s FSO communication system is investigated in terms of BER, received power, the receiver sensitivity of different intensity of turbulence. The simulation results show that LDPC code based on Dynamically adjusted LLR can enhance 13% the receiver sensitivity than LLR under traditional AWGN channel under weak turbulence and 7.4% under moderate turbulence. Meanwhile, the performance of the system shows that LDPC code based on Dynamically adjusted LLR has stronger turbulence resistance than LDPC code based on LLR under traditional AWGN channel.

Acknowledgments
The paper was supported by the Guangxi Natural Science Foundation of China (project number: 2018GXNSFAA294056)

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