Exploring the quantum chromodynamics landscape with high-energy nuclear collisions

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Abstract. A quantum chromodynamics (QCD) phase diagram is usually plotted as the temperature ($T$) versus the chemical potential associated with the conserved baryon number ($\mu_B$). Two fundamental properties of QCD, related to confinement and chiral symmetry, allow for two corresponding phase transitions when $T$ and $\mu_B$ are varied. Theoretically, the phase diagram is explored through non-perturbative QCD calculations on a lattice. The energy scale for the phase diagram ($\Lambda_{\text{QCD}} \sim 200\text{MeV}$) is such that it can be explored experimentally by colliding nuclei at varying beam energies in the laboratory. In this paper, we review some aspects of the QCD phase structure as explored through experimental studies using high-energy nuclear collisions. Specifically, we discuss three observations related to the formation of a strongly coupled plasma of quarks and gluons in the collisions, the experimental search for the QCD critical point on the phase diagram and the freeze-out properties of the hadronic phase.
1. Quantum chromodynamics (QCD) phase diagram

Physical systems undergo phase transitions when external parameters, such as the temperature ($T$) or a chemical potential ($\mu$), are changed. A phase diagram provides intrinsic knowledge of the structure of the matter under study. In other words, it tells us how matter organizes itself under external conditions at a given degree of freedom. The theory of strong interactions, QCD, predicts that nuclear matter at high temperature makes a transition from a state where quarks and gluons are confined and chiral symmetry is broken to a state where quarks and gluons are de-confined and chiral symmetry is restored [1–4]. QCD has several conserved quantities: baryon number ($B$), electric charge ($Q$) and strangeness ($S$). Each of these is associated with a chemical potential. As a result, the QCD phase diagram is four-dimensional. $\mu_Q$ and $\mu_S$ are relatively small when compared with $\mu_B$ (baryonic chemical potential) in high-energy nuclear collisions [2]. The $T$ and $\mu_B$ are varied in a typical QCD phase diagram, as shown in figure 1 [3]. At high temperature and density, the phase is governed by quark and gluon degrees of freedom and is commonly referred to as the quark–gluon plasma (QGP) [4]. At high densities and low temperatures, other interesting phases related to the neutron star [5] and color superconductivity [6] start to appear. These QCD transitions that occurred in the early universe have the right energy scale to be accessible by experiments. Figure 1 shows the parts of the phase diagram explored by several accelerator-based experimental programs.

Experimentally, this is done by varying the beam energy. Both $T$ and $\mu_B$ vary as a function of the center-of-mass energy ($\sqrt{s_{NN}}$) [7]. This strategy is being followed by experimental programs at the Large Hadron Collider (LHC) at CERN, the Relativistic Heavy Ion Collider (RHIC) at BNL and the Super Proton Synchrotron (SPS) at CERN, and will be followed at the Facility for Antiproton and Ion Research (FAIR) at GSI and Nuclotron-based Ion Collider fAcility (NICA) at JINR. Among these experiments, we discuss in section 2 selected results from RHIC, which provides evidence of the formation of a QGP [8].

Theoretically, finite-temperature lattice QCD calculations at $\mu_B = 0$ suggest a crossover [9] above a temperature, $T_c$, of about 170–190 MeV from a system with hadronic degrees of freedom to a QGP [10]. At large $\mu_B$, several model calculations find the quark–hadron phase transition to be of first order [11]. Going towards the smaller $\mu_B$ region, the point in the QCD phase plane ($T$ versus $\mu_B$) where the first-order phase transition ends is the QCD critical point (CP) [12]. The focus in the coming decade would be on attempts to locate the CP both...
Figure 1. Typical phase diagram of QCD. See the text for details. The figure is reproduced from [3].

experimentally and theoretically. Current theoretical calculations are highly uncertain about the location of the CP. This is primarily because the lattice QCD calculations at finite $\mu_B$ face numerical challenges. There even exists some uncertainty regarding the existence of CP [13]. The experimental plan (discussed in section 3) is to vary the $\sqrt{s_{NN}}$ of heavy-ion collisions to scan the phase plane and, at each energy, search for the signatures of the CP that might survive the evolution of the system. In the last section of the review, we discuss the thermodynamic properties of the hadronic phase. Finally, we end with a summary of our current understanding of the phase diagram and a brief outlook.

2. Establishing the partonic phase at the Relativistic Heavy Ion Collider (RHIC)

2.1. Strangeness enhancement and the formation of a gluon-rich plasma

The enhancement of strange hadron production in high-energy heavy-ion collisions [14] due to the formation of QGP is one of the four classic signatures in this field, the other three being enhanced direct photon [15] and dilepton [16] production and the suppression of $J/\Psi$ production [17] in heavy-ion collisions relative to $p+p$ collisions. In a QGP, thermal $s$ and $\bar{s}$ quarks can be produced by gluon–gluon interactions [18]. These interactions could occur very rapidly and the $s$-quark abundance would equilibrate. During hadronization, the $s$ and $\bar{s}$ quarks from the plasma coalesce to form $\phi$ mesons. Production by this process is not suppressed as per the OZI (Okubo–Zweig–Izuka) rule [19]. This, coupled with a large abundance of strange
quarks in the plasma, may lead to a dramatic increase in the production of $\phi$ mesons and other strange hadrons relative to non-QGP $p + p$ collisions.

Such predictions of relative enhancement of strange hadrons were challenged by the alternative idea that the canonical suppression of strangeness in small systems is the source of strangeness enhancement in $\Lambda$, $\Xi$ and $\Omega$ hadrons in high-energy heavy-ion collisions [20]. The strangeness conservation laws require the production of an $\bar{s}$-quark for every $s$-quark in the strong interactions. The main argument in such canonical models is that the energy and space time extensions in smaller systems may not be sufficiently large. This leads to the suppression of open-strange hadron production in small collision systems (such as $p + p$). This would then lead to a larger ratio of the yields of strange hadrons in nucleus–nucleus collisions relative to $p + p$ collisions. These statistical models also fit the data reasonably well [21]. These models predict two things: (a) strangeness enhancement in nucleus–nucleus collisions, relative to $p + p$ collisions, should increase with the valence strange quark content of the hadrons; and (b) the enhancement is predicted to decrease with increasing beam energy [22]. Discriminating between the two scenarios, QGP versus the canonical suppression, using the experimental data on $K^\pm$, $\Lambda$, $\Xi$ and $\Omega$ hadrons has been, to some extent, ambiguous. The enhancement of $\phi(s\bar{s})$ production (zero net-strangeness and hence not subjected to the canonical suppression) in heavy-ion collisions relative to $p + p$ collisions would clearly indicate the formation of a gluon-rich QGP in these collisions. This would then rule out the canonical suppression scenario.

Figure 2 shows the enhancement of strange hadron production at the RHIC [23]. The upper panel shows the ratio of strange hadron production normalized to $\langle N_{\text{part}} \rangle$ in nucleus–nucleus collisions relative to the corresponding results from $p + p$ collisions at 200 GeV. The results are plotted as a function of $\langle N_{\text{part}} \rangle$. $K^-$, $\Lambda$ and $\Xi + \Xi$ are found to exhibit an enhancement (value $>1$) that increases with the number of strange valence quarks. Furthermore, the observed enhancement in these open-strange hadrons increases with collision centrality, reaching a maximum for the most central collisions. However, the enhancement of $\phi$ meson production shows a deviation from the ordering in terms of the number of strange constituent quarks. More explicitly, this enhancement is larger than that for $K^-$ and $\Lambda$, while it is smaller than that in the case of $\Xi + \Xi$. Despite being different particle types (meson–baryon) and having different masses, the results for $K^-$ and $\Lambda$ are very similar in the entire centrality region studied. This rules out different particle types (meson versus baryon) being the reason for the deviation of $\phi$ mesons from the number of strange quarks ordering seen in figure 2 (upper panel). The observed deviation is also not a mass effect as the enhancement in $\phi$ meson production is larger than that in $\Lambda$ (which has a mass close to that of $\phi$). Further, in heavy-ion collisions, the production of $\phi$ mesons is not canonically suppressed due to its $s\bar{s}$ structure.

The observed enhancement of $\phi$ meson production then is a clear indication that the formation of a dense partonic medium is responsible for the strangeness enhancement in Au + Au collisions at 200 GeV. The idea that the observed enhancement in $\phi$ meson production is related to the medium density is further supported by the energy dependence shown in the lower panel of figure 2. The $\phi$ meson production relative to $p + p$ collisions is larger at higher beam energies, a trend opposite to that predicted in canonical models for other strange hadrons. In addition, measurements have shown that $\phi$ meson production comes not from the coalescence of $KK$ and is minimally affected by re-scattering effects in the medium [25]. Measurements also indicate that $\phi$ mesons are formed from the coalescence of seemingly thermalized strange quarks [26]. All of these observations put together indicate the formation of a dense partonic medium in heavy-ion collisions where strange quark production is enhanced. This in turn
suggests that the observed centrality dependence of the enhancement for other strange hadrons (seen in figure 2) is likely to be related to the same reason as that in the case of the φ meson: that it is due to the formation of a dense gluon-rich partonic medium in the collisions. These experimental data rule out the possibility of canonical suppression being the only source of the observed strangeness enhancement at $\sqrt{s_{NN}} = 200$ GeV.

2.2. Jet quenching and a highly opaque medium

One of the most exciting results to date at RHIC is the discovery of a suppression in the production of high transverse momentum ($p_T$) mesons in nucleus–nucleus collisions when compared with corresponding data from the binary collision scaled $p + p$ collisions [27]. This has been interpreted in terms of energy loss of partons in QGP. This phenomena is called jet quenching in a dense partonic matter [28]. The energy loss by energetic partons traversing the dense medium formed in high-energy heavy-ion collisions is predicted to be proportional to both the initial gluon density [29] and the lifetime of dense matter [30]. The results on high-$p_T$
suppression are usually presented in terms of the nuclear modification factor \( R_{AA} \), defined as

\[
R_{AA} = \frac{dN_{AA}/d\eta d^2p_T}{T_{AB}d\sigma_{NN}/d\eta d^2p_T},
\]

where the overlap integral \( T_{AB} = N_{\text{binary}}/\sigma_{pp}^{NN} \). \( N_{\text{binary}} \) is the number of binary collisions commonly estimated from the Glauber model calculation \([31]\), \( \sigma_{pp}^{NN} \) is the inelastic nucleon–nucleon cross-section, \( N_{AA} \) is the yield in nucleus–nucleus collisions and \( \eta \) is the pseudorapidity.

In figure 3, we show the RHIC data on the \( R_{AA}(p_T) \) for various mesons \([32]\) and direct photons \([33]\) produced in central Au+Au collisions at midrapidity. A significant suppression in high-\( p_T \) meson production is observed, and that for \( \pi^0 \)s is almost flat at \( R_{AA} \simeq 0.2 \) up to 20 GeV \( c^{-1} \). The figure also shows that the level of suppression for \( \pi^0 \)s, \( \eta \)s and \( \phi \)-mesons is very similar, which supports the conclusion that the suppression occurs in the partonic phase and not in the hadronic phase. This strong suppression of meson production is in contrast to the behavior of direct photons, also shown in the figure. The direct photons follow binary scaling (i.e. \( R_{AA} \simeq 1 \)) or no suppression. This is strong evidence that the suppression is not an initial state effect but a final state effect caused by the high-density medium with color charges created in the collision. This is further corroborated by a demonstration through a controlled experiment, using deuteron on Au ion collisions, which gave \( R_{dAu}(p_T) \sim 1 \) for \( \pi^+ \) at midrapidity and high \( p_T \) \([34]\).

The various curves in figure 3 represent different model calculations. The dashed curve shows a theoretical prediction made using the GLV parton energy loss model \([29]\). The model assumes an initial parton density of \( dN/dy = 800–1175 \), which corresponds to an energy density of approximately 5–15 GeV fm\(^{-3}\). The lower dashed curves are for higher gluon density.

**Figure 3.** Compilation of the nuclear modification factor \( (R_{AA}) \) for mesons and direct photons as measured in RHIC experiments at midrapidity for central Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. Also shown are the \( R_{dAu} \) for charged pions at \( \sqrt{s_{NN}} = 200 \) GeV. The lines are the results of various model calculations. See the text for more details.
The precision high-$p_T$ data at RHIC have been used to characterize medium density fairly accurately. The conclusion drawn was that the medium formed in central Au + Au collisions at RHIC has a high degree of opacity [35]. In addition, theoretical studies suggest that for a given initial density, the $R_{AA}(p_T)$ values are also sensitive to the lifetime ($\tau$) of dense matter formed in heavy-ion collisions [30]. The solid curves are predictions from [30] at $\sqrt{s_{NN}} = 200$ GeV with $\tau = 10$ fm c$^{-1}$ (i.e. larger than the typical system size of $\sim 6$–7 fm). The parton energy loss calculations discussed above attribute the opacity to plasma-induced radiation of gluons, much like the ordinary bremsstrahlung of photons by electrons. However, the quantitatively large suppression pattern observed at high $p_T$, for both light hadrons and those involving heavy quarks [36], showed that the mechanism of energy loss is far from being a settled issue, namely, the relative contributions of radiative and collisional forms. As an example, shown in figure 3 is a comparison of the data to theoretical results (dot-dash curves) on $R_{AA}$ from models that consider only collisional energy loss [37]. This model gives $R_{AA}$ values at high $p_T$ close to the measured values and similar to corresponding values from models having only a radiative mechanism of parton energy loss.

Leading particle measurements, such as the ones shown in figure 3, suffer from a number of limitations. (a) Leading hadrons come from a mixture of parent quarks and gluons. (b) As a fragmentation product, the energy of a leading hadron is not a perfect proxy for the energy of the parent parton as it samples a wide range of partonic energies. In future, we should look forward to, potentially, three interesting measurements. (i) The $\gamma$-jet process potentially provides access to the underlying scattered parton’s energy. Measurements of the distribution of particles from the jet opposite, in azimuth, to the tagged photon reveal how much energy was lost, and how it was redistributed, by the colored parton as it traversed the medium [38]. (ii) Another method is by the full reconstruction of jets in heavy-ion collisions [39]. Beyond producing a far better proxy for the energy of the parent parton than a leading hadron, this technique allows one to trace the evolution of energy flow in directions both longitudinal and transverse to the direction of the parent parton. Both of these methods are under active perusal at RHIC. (iii) Another important feature of jet quenching is provided by partonic identity. While it is difficult to disentangle light quarks from gluons, especially in a heavy-ion environment, charm and bottom can be easily tagged by the existence of a charmed or bottom hadron in the final state. Due to their large masses, the charm and bottom quarks are predominantly produced via hard scattering in the initial stage of the high-energy heavy-ion collision. The final state spectra can therefore serve as a sensitive tool to probe in-medium rescattering and interactions responsible for thermalization. This will also allow us to study the non-Abelian feature of QCD that results in the gluons losing more energy than do quarks in the medium [40]. Plans are in place to measure the cross-sections and transverse momentum spectra of hadrons with open and hidden heavy flavor at RHIC with new detector upgrades. This will also provide useful data to understand the different mechanisms of energy loss: collisional versus radiative.

2.3. Partonic collectivity and low viscosity

Elliptic flow, $v_2$, is an observable that is thought to reflect the conditions from the early stage of the collisions [41]. In non-central heavy-ion collisions, the initial spatial anisotropy of the overlap region of the colliding nuclei is transformed into an anisotropy in momentum space through interactions between the particles. As the system expands, it becomes more spherical; thus the driving force quenches itself. Therefore the elliptic flow is sensitive to the collision
dynamics in the early stages. It is measured by calculating $\langle \cos(2(\phi - \Psi)) \rangle$, where $\phi$ is the azimuthal angle of the produced particles and $\Psi$ is the azimuthal angle of the impact parameter, and angular brackets denote an average over many particles and events.

Figure 4 (top left panel) shows the RHIC results on the $v_2$ of light quark ($\pi$ and $p$) and strange quark ($\phi$ and $\Omega$) carrying hadrons in Au+Au collisions at 200 GeV [42]. Three very distinct experimental observations can be made. (a) At low $p_T$ ($<2$ GeV $c^{-1}$), the heavier

Figure 4. Top left plot: the elliptic flow $v_2$ for (a) light quark hadrons and (b) strange quark hadrons. The data are from the minimum bias Au + Au collisions at midrapidity for $\sqrt{s_{NN}} = 200$ GeV [42]. Top right plot: compilation of the number of constituent quarks scaled $v_2$ as a function of the scaled transverse kinetic energy [43]. Bottom plot: compilation of $\eta/s$ extracted from various measurements in heavy-ion collisions at RHIC.

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hadrons have smaller \( v_2 \). Such a mass ordering is expected in hydrodynamics calculations of \( v_2(p_T) \) for the identified particles [44]. (b) At the intermediate \( p_T \) range of 2–5 GeV \( c^{-1} \), it is observed that baryons have higher \( v_2 \) than do mesons. The \( \phi \)-meson \( v_2 \) plays a crucial role in establishing this baryon–meson difference. Such a separation of baryons and mesons in the intermediate \( p_T \) range has been observed also in measurements of the nuclear modification factor, \( R_{CP} \) [45]. These results are consistent with calculations from quark recombination models [46] implying the de-confinement of the system prior to hadronization. (c) A comparison of the \( v_2 \) results for light-quark-carrying hadrons to those for strange-quark-carrying hadrons indicates that both types of hadrons show a similar magnitude of \( v_2 \). The multi-strange hadrons (\( \phi \) and \( \Omega \)) have relatively low hadronic interaction cross-sections and freeze out early, and hence they are considered to be the most promising probes of the early stages of the collision. All of the above results indicate that a substantial amount of collectivity has been developed at the partonic stage of the heavy-ion collisions. In fact, none of the available hadronic models is able to account for the observed magnitude of \( v_2 \) at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). Only models that have additional partonic interactions explain the \( \langle v_2 \rangle \) values [47].

When elliptic flow \( v_2 \) is plotted versus transverse kinetic energy \( (m_T - m_0) \), both divided by the number of constituent quarks, the \( v_2 \) for all identified hadrons as well as light nuclei below \( (m_T - m_0) \sim 1 \text{ GeV} \) \( c^{-2} \) falls on a universal curve [48]. \( m_0 \) is the mass of the particle. This scaling behavior, as shown in figure 4, strengthens the evidence for the formation of partonic matter during the \( \text{Au} + \text{Au} \) collision process at 200 GeV. It is hard to explain this observed pattern in a scenario where only hadronic matter exists throughout the interaction, whereas the hypothesis of the coalescence of hadrons from de-confined quarks offers a ready explanation. The turn-off of the scaling at a given beam energy would indicate the hadronic side of the phase boundary.

The \( v_2 \) measurements of light-quark-carrying hadrons, the nuclear modification factor and \( v_2 \) for heavy-quark-carrying hadrons and the differential \( p_T \) correlations for charged hadrons have been used to extract information on a dimensionless ratio, the ratio of shear viscosity to entropy \( (\eta/s) \), for the medium formed in heavy-ion collisions at RHIC [49]. Figure 4 (bottom panel) shows the compilation of \( \eta/s \) extracted from various measurements [50]. It is observed to lie between the lowest \( \eta/s \) (\( \sim 1/4\pi \)) bound conjectured from the gauge–gravity duality (or the AdS/CFT correspondence) [51] and those for liquid helium at \( T_c \) [52]. A low value of \( \eta \) (\( \eta/s \) within a factor of 1–10 of the quantum limit) indicates that the matter formed in heavy-ion collisions at RHIC has low viscosity and hence is a strongly coupled system. However, it must be kept in mind that the \( \eta/s \) values extracted are highly model dependent and may involve assumptions that are not yet fully tested. Most of the results involve a comparison of \( v_2 \) to viscous hydrodynamic calculations that are still in the early stage of development or to transport model-inspired fits to the data [53]. The others involve model-based interpretation of \( p_T \) correlations assuming specific centrality dependence of freeze-out conditions in heavy-ion collisions. Some others involve model comparison to the heavy-flavor nuclear modification factor and \( v_2 \).

3. QCD critical point (CP) and thermalization

The CP is a landmark point in the QCD phase diagram, the observation of which will make the QCD phase diagram a reality. A close collaboration between the experiments and theory perhaps will lead to its discovery. The first step in this process is to establish an observable
for CP that can be measured experimentally and can be related to QCD calculations. In this context, it is important to recall that for a static, infinite medium, the correlation length \((\xi)\) diverges at the CP. \(\xi\) is related to various moments of the distribution of conserved quantities such as net-baryons, net-charge and net-strangeness [54]. Typically variances \((\sigma^2 \equiv \langle (\Delta N)^2 \rangle; \Delta N = N - M\); \(M\) is the mean) of these distributions are related to \(\xi\) as \(\sigma^2 \sim \xi^2\) [55]. Finite size and time effects in heavy-ion collisions put constraints on the values of \(\xi\). A theoretical calculation suggests that \(\xi \approx 2-3\) fm for heavy-ion collisions [56]. It was shown recently that higher moments of the distribution of conserved quantities, measuring the deviations from a Gaussian, have a sensitivity to CP fluctuations that is better than that of \(\sigma^2\), due to a stronger dependence on \(\xi\) [57]. The numerators in skewness \((S = \langle (\Delta N)^3 \rangle / \sigma^3\)) go as \(\xi^{4.5}\) and kurtosis \((\kappa = \langle (\Delta N)^4 \rangle / \sigma^4 - 3\)) go as \(\xi^7\). A crossing of the phase boundary can manifest itself by a change of sign of \(S\) as a function of energy density [57, 58].

The first connections between QCD calculations and experiment have recently been made [59]. Lattice calculations and QCD-based models show that the moments of net-baryon distributions are related to baryon number \((\Delta N_B)\) susceptibilities \((\chi_B = \langle (\Delta N_B)^2 \rangle / V\); \(V\) is the volume) [60, 62]. Then one can construct ratios such as

\[
S\sigma = \frac{T \chi_B^{(3)}}{\chi_B^{(2)}}, \quad \kappa\sigma^2 = \frac{T^2 \chi_B^{(4)}}{\chi_B^{(2)}} \quad \text{and} \quad \frac{\kappa\sigma}{S} = \frac{T \chi_B^{(4)}}{\chi_B^{(3)}},
\]

which do not contain the volume and therefore provide a direct and convenient comparison of experiment and theory. In the above expressions, the left-hand side of each equality can be measured in an experiment, whereas the right-hand side can be calculated by lattice QCD. Close to the CP, models predict the \(\Delta N_B\) distributions to be non-Gaussian and susceptibilities to diverge, causing the experimental observables to have large values. The experimental values should also be compared with those expected from statistics; for example, if the \(p\) and \(\bar{p}\) distributions are individually Poissonian, then \(\kappa\sigma^2\) for net-protons is unity.

Experimentally measuring event-by-event the net-baryon number is difficult. However, the net-proton multiplicity \((N_p - \bar{N}_p = \Delta N_p)\) distribution is measurable. Theoretical calculations have shown that \(\Delta N_p\) fluctuations reflect the singularity of the charge and baryon number susceptibility as expected at the CP [63]. Non-CP model calculations show that the inclusion of other baryons does not add to the sensitivity of the observable [59].

Figure 5 shows the energy dependences of \(S\sigma, \kappa\sigma^2\) and \(\frac{\kappa\sigma}{S}\) for \(\Delta N_p\), in comparison to lattice QCD [60] and the Hadron Resonance Gas (HRG) model that does not include a CP [61]. The experimental values plotted are for central Au + Au collisions at \(\sqrt{s_{NN}} = 19.6, 62.4\) and \(200\) GeV. The lattice calculations, which predict a CP of \(\mu_B \sim 300\) MeV, are performed using two-flavor QCD with the number of lattice sites in imaginary time being 6 and the mass of pion being around \(230\) MeV [60]. The ratios of the nonlinear susceptibilities at finite \(\mu_B\) are obtained using Padé approximant resummations of the quark number susceptibility series. The freeze-out parameters as a function of \(\sqrt{s_{NN}}\) are taken from [64] and \(T_c = 175\) MeV.

From comparisons of the experimental data to the HRG model and the lack of non-monotonic dependence of \(\kappa\sigma^2\) on \(\sqrt{s_{NN}}\) studied, one concludes that there is no indication, from the current measurements at RHIC, of a CP in the region of the phase plane with \(\mu_B < 200\) MeV. Although it must be noted that the errors on the experimental data points at \(19.6\) GeV are quite large due to small event statistics. It is difficult to rule out the existence of CP for the entire \(\mu_B\) region below \(200\) MeV without a knowledge of the extent of the critical region of \(\mu_B\). Hence, the extent to which these results can do is guided by the theoretical work. The expected extent
of the critical region of $\mu_B$ is thought to be about 100 MeV. The results discussed here form the basis for future CP search programs at the RHIC [65]. However, the fact that the data show excellent agreement with HRG and lattice QCD, both of which assume thermalization, is another nontrivial indication of the attainment of thermalization (some other measurements are discussed in the next section) in heavy-ion collisions. Such a conclusion is drawn for the first time using fluctuation measurements.

With the idea that the rise and then the fall of observables sensitive to CP as $\mu_B$ increases should allow us to ascertain the $(T, \mu_B)$ coordinates of the CP, the beam energy scan program at the RHIC was started. The first phase of the experimental program at the RHIC is expected to be completed in 2010–2011. This phase is expected to cover a $\sqrt{s_{NN}}$ region of 39–5.5 GeV, which corresponds to a $\mu_B$ range of 112–550 MeV [66].
4. Hadronic phase

The measured hadron spectra reflect the properties of bulk matter at kinetic freeze-out after elastic collisions among the hadrons have ceased. More direct information about earlier stages can be deduced from the integrated yields of different hadron species, which change only via inelastic collisions. The point in time at which these inelastic collisions cease is referred to as the chemical freeze-out, which takes place before kinetic freeze-out. RHIC experiments have measured the $p_T$ distribution of a variety of particles over a wide range of $p_T$ values at midrapidity. A sample of the invariant yield ($\frac{d^2N}{(2\pi p_T) dy dp_T}$ (GeV $c^{-1}$)$^{-2}$) of the produced particles at the RHIC for central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV [2, 26, 32, 33, 67] is shown in figure 6. Like the $p_T$ dependence of $v_2$ discussed in section 2.3, here also we can separate the spectra into three regions based on the dominant mechanism of particle production. The low $p_T$ (<2 GeV $c^{-1}$) is explained by thermal model-based calculations [68], intermediate $p_T$ (2–6 GeV $c^{-1}$) by parton recombination based approaches [46] and high $p_T$ (>6 GeV $c^{-1}$) by including pQCD-based processes or jet production [29, 30]. The only statistical distribution that so far seems to successfully describe the $p_T$ spectra and $v_2(p_T)$ over a wide momentum range is the one based on Tsallis statistics [69].

In this section, we concentrate on the low-$p_T$ part of the spectra for the rest of the discussions. The transverse momentum distributions of different particles contain two components, one random and the other collective. The random component can be identified as the one that depends on the temperature of the system at kinetic freeze-out ($T_{fo}$). The collective

Figure 6. Compilation of invariant yield of produced particles at midrapidity versus $p_T$ in central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV [2, 26, 32, 33, 67]. The lines show the possibility of having three regions in the spectra where the dominant mechanism of particle productions are different. See the text for more details.
component, which arises from the matter density gradient from the center to the boundary of the fireball created in high-energy nuclear collisions, is generated by collective flow in the transverse direction and is characterized by its velocity $\beta_T$. Assuming that the system attains thermal equilibrium, the blast-wave (BW) formulation [70] can be used to extract $T_{fo}$ and $\langle \beta_T \rangle$. The transverse flow velocity of a particle at a distance $r$ from the center of the emission source, as a function of the surface velocity ($\beta_s$) of the expanding cylinder, is parameterized as $\beta_T(r) = \beta_s(r/R)^n$, where $n$ is found by fitting the data. The transverse momentum spectrum is then

$$\frac{dN}{p_T dp_T} \propto \int_0^R r \, dr \, m_T I_0 \left( \frac{p_T \sinh \rho(r)}{T_{fo}} \right) \times K_1 \left( \frac{m_T \cosh \rho(r)}{T_{fo}} \right),$$

(2)

where $I_0$ and $K_1$ are modified Bessel functions and $\rho(r) = \tanh^{-1} \beta_T(r)$.

Fits to the identified hadron $p_T$ distributions at midrapidity for Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV using equation (2) are carried out. The extracted model parameters characterizing the random (generally interpreted as a kinetic freeze-out temperature $T_{fo}$) and collective (radial flow velocity $\langle \beta_T \rangle$) flow are shown in figure 7 in terms of confidence level ($\chi^2$) contours, for various impact parameters of the collision. As the collisions become more and more central, the bulk of the system, dominated by the yields of $\pi$, $K$ and $p$, has lower kinetic freeze-out temperature and develops a stronger collective flow. On the other hand, even for the most central collisions, the spectra for multi-strange particles $\phi$ and $\Omega$ appear to reflect a higher freeze-out temperature.

Within a statistical model in thermodynamical equilibrium, the particle abundance in a system of volume $V$ can be given by

$$N_i / V = \frac{g_i}{(2\pi)^3} \gamma_S^{S_i} \int \frac{1}{\exp \left( \frac{E_i - \mu_B B_i - \mu_S S_i}{T_{ch}} \right) \pm 1} d^3 p,$$

(3)

where $N_i$ is the abundance of particle species $i$, $g_i$ is the spin degeneracy, $B_i$ and $S_i$ are the baryon number and strangeness number, respectively, $E_i$ is the particle energy, and the integral is taken over the entire momentum space [2]. The model parameters are the chemical freeze-out temperature ($T_{ch}$), the baryon ($\mu_B$) and strangeness ($\mu_S$) chemical potentials, and the ad hoc strangeness suppression factor ($\gamma_S$). The measured particle ratios are used to constrain the values of $T_{ch}$ and $\mu_B$ at chemical freeze-out.

Figure 7 compares STAR measurements of integrated hadron yield ratios for central Au + Au collisions to statistical model calculations. Excellent agreement is observed between the data and the model. These ratios, which include stable and long-lived hadrons through multi-strange baryons, are consistent with the light flavors $u$, $d$ and $s$, having reached chemical equilibrium (for central and near-central collisions only) at $T_{ch} = 163 \pm 5$ MeV. The deviations of the short-lived resonance yields (such as those for $\Lambda^*$ and $K^*$ collected near the right side of figure 7) from the statistical model fits presumably result from a hadronic rescattering after the chemical freeze-out and need to be investigated further.

The saturation of the strange sector yields, attained for the first time in near-central RHIC collisions, is particularly significant. The saturation is indicated quantitatively by the value obtained for the non-equilibrium parameter $\gamma_S$ for the strange sector in the case of central collisions. The temperature deduced from the fits is essentially equal to the critical value for a QGP-to-hadron-gas transition predicted by lattice QCD [71], but is also close to the
Hagedorn limit for an HRG, predicted without any consideration of quark and gluon degrees of freedom [72]. If thermalization is indeed achieved by the bulk matter prior to chemical freeze-out, then the deduced value of $T_{ch}$ represents a lower limit on that thermalization temperature.

5. Summary and outlook

In summary, the current understanding of the QCD phase diagram is depicted in figure 8. From the QCD calculations on a lattice, it is now established theoretically that the quark–hadron transition at $\mu_B = 0$ is a cross-over. The critical temperature of a quark–hadron phase transition lies within the range of 170–190 MeV. Most calculations on lattice also indicate the existence of QCD CP at $\mu_B > 160$ MeV. The exact location of CP is not yet known unambiguously. Two such predictions computed on a lattice are shown in figure 8 for a $T_c$ of 176 MeV [12]. New distinct signatures have been predicted by QCD model calculations to locate the CP in the phase diagram. The specific suggestion for a CP search is to look for non-monotonic variation in the products of the higher moments of net-proton and net-charge distributions, which are related to susceptibilities, as a function of $\sqrt{s_{NN}}$ (or $T$, $\mu_B$). At the top RHIC energies, fluctuations in net-proton numbers are found to be consistent with expectations from HRG and lattice...
Figure 8. Temperature versus baryon chemical potential ($\mu_B$) for heavy-ion collisions at various $\sqrt{s_{NN}}$ values [65]. The $\mu_B$ values shown were estimated at chemical freeze-out. The kinetic and chemical freeze-out parameters, extracted using models assuming thermal and chemical equilibrium from the midrapidity particle ratio and $p_T$ spectra measurements in heavy-ion collisions, are shown. The range of critical temperatures ($T_c$) of the cross-over quark–hadron phase transition at $\mu_B = 0$ and the QCD CP from two different calculations by lattice QCD are also indicated [12]. Model-based estimates of the range of initial temperature ($T_{\text{initial}}$) achieved in heavy-ion collisions based in part on direct photon data at the top RHIC [73] and SPS [74] energies are also shown. The range of $\mu_B$ to be scanned in the RHIC beam energy scan program corresponding to $\sqrt{s_{NN}} = 5.5$–39 GeV as well as experiments at SPS and CBM are indicated as a shaded ellipse. The solid point around $T \sim 0$ and $\mu_B = 938$ MeV represents nuclear matter in the ground state.

QCD calculations [59]. These measurements formed the basis of the CP search in the heavy-ion collision program, placed constraints on the location of CP in the QCD phase plane and strengthened the evidence for thermalization in central Au + Au collisions at the RHIC.

High-energy heavy-ion collision experiments have seen distinct signatures that suggest that the relevant degrees of freedom in the initial stages of the collisions at top RHIC energies are quarks and gluons [8]. Three such signatures related to strangeness enhancement, jet quenching and partonic collectivity are discussed in this paper. The initial temperatures ($T_{\text{initial}}$) achieved at top RHIC and SPS energies are obtained from models [75] that explain the direct photon measurements from the PHENIX experiment at the RHIC [73] and from the WA98 experiment at SPS [74]. From these models, which assume that thermalization is achieved in the collisions within a time range of 0.1–1.2 fm $c^{-1}$, the $T_{\text{initial}}$ extracted is greater than 300 MeV at the RHIC and greater than 200 MeV at SPS. Further, the understanding of suppression in high-$p_T$ hadron...
production in heavy-ion collisions relative to $p + p$ collisions at the RHIC requires a medium energy density $\gg 1$ GeV fm$^{-3}$ which is the critical energy density from the lattice for a phase transition. This also shows that the medium has a high degree of opacity to propagation of color charges. In addition, the measurement of elliptic flow and the observation of the number of constituent quark scalings demonstrate that substantial collectivity has been developed in the partonic phase. The magnitude of the flow across several hadronic species and a small value of the viscosity to entropy ratio extracted from the data support the idea of the formation of a strongly coupled system in heavy-ion collisions. This then also supports the notion of creating a liquid with low viscosity in high-energy nuclear collisions [76].

The experiments have also measured the temperature at which the inelastic collisions cease (chemical freeze-out) and elastic collisions cease (kinetic freeze-out). These temperatures (as shown in figure 8) are extracted from the measured particle ratios and transverse momentum distributions using model calculations that assume that the system is in chemical and thermal equilibrium.

New experimental programs at the RHIC, SPS, FAIR and NICA facilities have been designed to explore a large part of the QCD phase diagram, covering a $\mu_B$ range of 20–600 MeV, whereas the experimental program at the LHC (probing the cross-over region of $\mu_B \sim 0$ MeV of the phase diagram) have started to provide a unique opportunity to understand the properties of matter governed by quark–gluon degrees of freedom at unprecedented high initial temperatures (higher plasma lifetime) achieved in the Pb + Pb collisions at 2.76–5.5 TeV [77]. Both at the LHC and at the RHIC, one specific observable that has the potential to provide a better understanding of the system formed in heavy-ion collisions is dileptons. Theoretically, dileptons are from the virtual photons and are different from real photons in having a mass. The dilepton mass opens up a new dimension and can be used to study the time evolution of the system in heavy-ion collisions. For example, recently it was reported that virtual photon (dilepton) interferometry provides access to the development of collective flow with time [78]. Studying the $p_T$ dependence of the elliptic flow and nuclear modification factor for dileptons of masses corresponding to various hadrons and beyond will help us to understand partonic collectivity and medium opacity. A comparison of the spectral functions of resonances decaying to dileptons and hadrons will let us know more about the medium effects, while the slope of the dilepton $p_T$ distributions will provide information about the development of radial flow and will provide direct evidence for thermal radiation of partonic origin in high-energy nuclear collisions [79].

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