The formation of a variation method for calculating the accuracy of metal cutting systems

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Abstract The paper is devoted to the topical issue of the formation of a variation method for calculating the geometric accuracy of metal cutting systems. The current state of the basic variation method for calculating the geometric accuracy of metal cutting machines is determined. A transition from considering the geometric accuracy of only the metal cutting machine to considering the accuracy of the metal cutting system is made, which includes the metal cutting tool. The proposed directions for the development of the method are recommended to be divided into groups according to the criterion of linearized and non-linear formulation when building accuracy balances. The solution of the problem posed in the paper serves as a methodological basis for work on normalization and compensation of geometrical errors of machine tools, including the creation of the foundations of standards for the accuracy of metal-cutting machines and tools. This work is useful for scientists engaged in the study of the geometric accuracy of processing on metal-cutting machines, as well as the normalization and compensation of the geometric errors of machines.

Keywords: metal cutting machine, cutting tool, geometric accuracy, variation method

1. Introduction

A significant part of the scientific results obtained in the field of precision machine tools (MT) and processing on them belongs to the scientific schools of the USSR. These scientific schools have disintegrated today, which is reflected in the introductory parts of the standards: GOST 18097-93, GOST 2110-93, etc., which are a complete authentic translation of the texts of ISO standards developed without the participation of Russian specialists.

Analysis of scientific works in the field of geometrical accuracy of machines and processing showed that the individual achievements of various scientific schools were used only partially and unsystematically in the preparation of relevant standards.

In existing standards for the norms of geometric accuracy of machines, indicators of geometric accuracy are presented, that do not influence on the accuracy of processing, and, conversely, the absence of indicators that affect the accuracy of processing.

The unified methodological basis for the development of MT accuracy standards is the variation method created by professors D.N. Reshetov and V.T. Portman. However, in the course of the research, the following fundamental disadvantages of this method were revealed:

- analysis of the relative position of the machine units is performed using coordinate systems that do not characterize the finite dimension of these units and the interaction of the units only through some of their surfaces, the connections of which coordinate systems with the overall coordinate system of the unit have
not been investigated;
- the difficulty of determining the geometrical errors of a processed part on the machine with the known geometrical errors of the machine due to the lack of direct links between them;
- considering only one coordinate system for each of the machine units, which leads to the appearance of additional errors caused by the violation of the principle of unity and combination of design, measurement and technological bases;
- the basis of this method is considering the variation of a function at a certain point, and not on a segment, which fundamentally increases the methodological error.

Thus, the fundamental goal of the «The scientific basis for the machine tools geometric accuracy standardization» project, conducted with the support of RFBR in 2016-2018, was developing a variation method for calculating the geometric accuracy of MT to eliminate the identified deficiencies.

2. **Conducted research complex**

During the work on the Project, the main obtained scientific results are presented in Figure 1. Special attention deserves the following results:
- method of robust optimization of technological machines with the joint synthesis of parameters and tolerances on them (block 1 Fig. 1);
- a unified scientific approach to building a system of standards for the MT geometric accuracy of new types and the revision of existing standards (block 3 Fig. 1);
- general mathematical models: machining accuracy on MT and accuracy of metal cutting systems (MCS) (block 16 Fig. 1);
- full development of the variation method for calculating the MT accuracy in the variation method for calculating the MCS accuracy (block 38 Fig. 1).

3. **Developing a method of robust optimization of technological machines with the joint synthesis of parameters and tolerances on them**

Considering the processes of design, manufacture, and operation of process equipment, G. Taguchi identified three main stages [1]: structural (functional) synthesis; parametric synthesis; synthesis tolerances. The development of G. Taguchi's ideas on the choice of operating conditions for technological equipment, reducing the variation in the quality indicators of its products, has generated an increased interest in optimizing tolerances when designing machines [2-7].
Figure 1. The relationship of scientific results obtained during the work on the Project in 2016-2018.
The classical formulation of the problem of multi-criteria parametric optimization is:
\[ f_i(x) \rightarrow \min_x, x \in D, i \in [1: k]; \]
\[ D = \{ x \in R^n | g_i(x) \leq 0, i \in [1: m]; g_i(x) = 0, i \in [m + 1: s]; \] (1)

where \( f_i(x) \) – set of target functions; \( D \) – set of acceptable solutions (subset of elements of \( n \)-dimensional Euclidean space \( R^n \)); \( g_i(x) \) – functional constraints; \( a_j \) and \( b_j \) – lower and upper constraints that define the range of valid parameter values \( x_j \).

In developing approaches to robust optimization [8], a new mathematical formulation of the problem for the synthesis of tolerances was proposed:
\[ \Delta x_i \rightarrow \max_{x \in D, l \in [1: h]} \]
\[ D = \{ x \in R | g_i(x) \geq Q_{l_{\min}}(x), \Delta Q_i(x) \leq [\Delta Q_i(x)], i \in [1: k]; \] (2)
\[ g_i(x) \leq 0, i \in [k + 1: m]; g_i(x) = 0, i \in [m + 1: s]; a_j \leq x_j \leq b_j, j \in [1: n] \}, \]

where \( \Delta x_i \) – system of objective functions representing independent parameter change intervals \( x_i \); \( Q_{l_{\min}}(x) \) – minimum admissible values of single quality indicators (SQI); \( [\Delta Q_i(x)] \) – permissible deviations of single quality indicators.

Based on (2), a multi-criteria robust tolerance optimization method has been developed for the design of engineering products, being successfully used in various engineering industries.

A combination of stages of synthesis of parameters and tolerances on them on the basis of the following statement of the problem is proposed in the paper [9]:
\[ \Delta x_i \rightarrow \max_{x \in D, l \in [1: h]} \]
\[ D = \{ x \in R | Q_i(x) \geq Q_{l_{\min}}(x), \Delta Q_i(x) \leq [\Delta Q_i(x)], i \in [1: k]; \] (3)
\[ g_i(x) \leq 0, i \in [k + 1: m]; g_i(x) = 0, i \in [m + 1: s]; a_j \leq x_j \leq b_j, j \in [1: n] \}, \]

where \( \Delta x_i \) – average values of independent parameters.

The proposed formulation of the optimization problem (3) differs from formulation (2) in that the system of objective functions includes the search for extremums of averages of independent parameters. Based on this formulation of the optimization problem, a method is proposed for robust optimization of technological machines with a joint synthesis of parameters and tolerances on them, including the following steps (without feedbacks):

- determination of the ranges of acceptable values of input parameters;
- calculating SQI values in the area defined in claim 1 - \( \{U1\} \);
- selection of sets, corresponding to the functional constraints QI and the definition of their intersection in the spaces of the input and output variables – \( \{U2\} \);
- selection of subsets in \( U2 \), corresponding to the conditions of robustness SQI – \( \{U3\} \);
- determination of tolerance values and average values of independent parameters;
- verification of tolerance values and averages of independent parameters [27-28].

Here, as in [8], independent (input) parameters are understood as parameters of dimensional relationships and relationships of material properties that are part of the functional dependences \( SQI = Q(x_1, x_2, ..., x_l) \).

The main advantage of the formulation of the problem on the basis of (3) is the ability to find optimal solutions in the absence of functional dependencies for SQI from the input parameters that are extreme in nature, but in the presence of intuitive representations of their best average values [29]. To simplify the statement of the problem (3) is also possible by reducing the criteria system to one criterion based on a weighted convolution of the system of objective functions by known methods.
4. Creating a unified scientific approach to building a system of standards for MT geometric accuracy of new types and revision of existing standards

To create a unified scientific approach to building a system of standards for MT geometric accuracy of new types and revision of existing standards, methodological foundations of standardization of parameters of geometric accuracy of machine tools were developed (block 4 Fig. 1).

Figure 2 shows the structure of the modified model of the output accuracy of the machine [10] (block 14 Fig. 1), which differs from the one proposed in [11, 12], and is based on the newly introduced concepts of the vector balance of accuracy and actually processed surfaces.

![Figure 2. The structure of the modified model of the output accuracy of the machine [10]](image)

The direct application of the definitions and interrelations introduced in [11–13] for a reasonable calculation of standards for the geometrical accuracy of machine tools, updating and revising existing standards for accuracy standards, as well as developing new standards, revealed a number of problems associated with calculating deviations of the location and shape of processed surfaces [14-16]. To solve these problems, certain provisions of the variation method for calculating the accuracy were necessary to develop (block 12, Fig. 1).

The shaping function (SF) reflects the relationship between the coordinates of the points of the cutting edge of the tool in the system $S_l$ of the cutting tool and the coordinates of the same points in the coordinates $S_0$ of the workpiece:

$$r_0 = A_0 r_l,$$  \hspace{1cm} (4)

where $A_0 = \prod_{i=1}^{l} A_{l_i-l_i}$; $l$ – the number of moving parts of the shaping system of the machine; $A$ – matrices included in the product, and corresponding to one of six generalized displacements performed by the unit; $r_l$ – radius-vector of the cutting edge of the tool, as well as between generalized displacements:

$$f(q_1, \ldots, q_{n+m}) = 0; j = 1, \ldots, L,$$  \hspace{1cm} (5)

where $q_1, \ldots, q_{n+m}$ – matrix $A_{k_j}$ variables; $n$ – number of links carrying traffic shaping; $m$ – number of independent variables included in the model of the cutting tool; $L$ – number of relationships, $L=n+m-2$.

The nominal surface to be processed has the same designation $r_0$ and is obtained from (4) as a result of multiplying the matrices by the radius vector of the cutting edge of the tool, and taking into account the identified relationships in the resulting expression (5):

$$r_0 = r_0(u, v, q_0),$$  \hspace{1cm} (6)

where $u, v$ – curvilinear coordinates of the surface; $q_0$ – vector of dimensional surface parameters, $q_0 = (q_{01}, \ldots, q_{0m})^T$; $m_1$ – the number of components of the vector $q_0$; $\tau$ – transpose symbol.

It seems appropriate to give three interrelated definitions of the vector balance of accuracy $\Delta r_0$, $\Delta r_0^*$ and $\Delta r_0^{**}$.
Let the vector balance of accuracy of the machine $\Delta r_0^*$ is a variation of its shaping function, without taking into account the constraints (5) and the errors of the cutting tool, i.e.:

$$\Delta r_0 = \sum_{i=0}^{l} A_i \varepsilon_i A_i r_i, \quad (7)$$

where $\varepsilon_i$ is error matrix of the relative position of the coordinate systems:

$$\varepsilon_i = \begin{pmatrix} 0 & -\gamma_i & \beta_i & \delta_{x_i} \\ \gamma_i & 0 & -\alpha_i & \delta_{y_i} \\ -\beta_i & \alpha_i & 0 & \delta_{z_i} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

where $\delta_{x_i}, \delta_{y_i}, \delta_{z_i}$ are small displacements of the coordinate system $S_i$ along the axes $X, Y, Z$; $\alpha_i, \beta_i, \gamma_i$ are small angles of rotation of the coordinate system $S_i$ relative to the axes $X, Y, Z$.

Expression (7) takes into account all the errors of relative rotations and transfers of local coordinate systems relative to each other, starting from the coordinate system associated with the workpiece ($S_0$) to the coordinate system associated with the cutting tool ($S_l$) and is an improper vector. In other words, $\Delta r_0^*$ completely describes only the error vector of the machine $q_1 (6 \times l)$ at each point of its working space and is not associated with any particular surface to be processed.

In turn, the vector balance of the accuracy of the machine $\Delta r_0$ is a variation of its shaping function, taking into account the bond variation (5), i.e. an addition to the expression (7) is the system of equations:

$$\sum_{i=1}^{n+m} \delta q_i \delta q_i = \delta H, j = 1, 2, ..., L. \quad (9)$$

The balance of normal errors $\Delta r_n$ is used for metrological estimates, which is the projection of the vector $\Delta r_0$ onto the normal to the nominal surface $r_0$, i.e. $\Delta r_n = (\Delta r_0 n)$, where $n$ is the unit normal vector.

If two of the considered accuracy balances $\Delta r_0$ and $\Delta r_n$, although having the same designation - $\Delta r_0$, were applied in [11, 12], the concept for the vector balance of the accuracy of the machine $\Delta r_0^{**}$ is introduced for the first time as follows:

$$\Delta r_0^{**} = \Delta r_0 \delta q_i \delta q_i \delta H \delta \delta_j, \quad (10)$$

where $\delta q_i$ is the error of the position of the $i$-th unit of the machine forming system along the $j$-th generalized coordinate; $H$ is a generalized function that can be defined through the Dirac delta function, and having the property $H(x) = 1$ when $x \neq 0$, $H(x) = 0$ when $x = 0$; error values $\delta_j$ are defined in equation (9).

By introducing the generalized function $H$ into the definition for $\Delta r_0^{**}$, it is possible to take into account in the expression (10) only those errors of the units $\delta q_i$, which are included in the balance of normal errors $\Delta r_n$.

When accepted designations instead of the expression given in [11] for the actually treated surface $r$, the following expression is true:

$$r = r_0 + \Delta r_0^{**}, \quad (11)$$

which actually determines the treated surface, by the definition of $\Delta r_0^{**}$ by (10).

Further, in [11], the concept of a base surface is introduced, for which two definitions are given:

1) in the form ((5.8)-(5.11), p. 116, [11]):

$$r_b = r_0 (u, v, q), \quad (12)$$

where $q$ – base surface parameter vector.

Due to the smallness of deviations of the base surface from the nominal surface, the equation of the base surface is represented as:

$$r_b = r_0 + \Delta r_b, \quad (13)$$

where $\Delta r_b$ is the sum of the error vectors for changing the position and size of the nominal surface, i.e.: $\Delta r_b = \varepsilon_r r_0 + d r_0$, where $\varepsilon_r$ is the error matrix of the location of the coordinate system associated with the base surface.
relative to the system, in which the equation of the actually processed surface was specified in [11]. In this paper, this matrix corresponds to expression (9) when \( b \equiv i \); \( dr_0 \) is the total differential of the radius vector \( r_0 \), taken over all components of the vector \( q_0 \):

\[
dr_0 = \sum_{i=1}^{m} (\partial r_0 / \partial q_0i) \Delta q_0i.
\]  

(15)

In addition, it follows from (14) that the vector \( \Delta r_0 \) contains \( m \leq l - 2 \) size errors and \( n \leq 6 \) position errors.

2) in the form ((5.27), p. 138, [11]):

\[
r_b = r_0 + e_0 r_0 + dr_0 + \delta r_0,
\]  

(16)

where \( \delta r_0 \) is a vector characterizing the shape error of the nominal surface.

The latter of these two definitions is the most complete. Therefore, when working on the Project, it was decided to abandon the concept of a base surface introduced in [11, 13], based on the following arguments:

1. For the macrogeometrical deviations under consideration, the bases are the surface or its profile, the axis or plane of symmetry, the axis of the centers, a specific place of the geometric element, and not just the distorted nominal surface [17].

2. Tolerances of sizes and shapes are set for the nominal surface \( r_0 \), defined by expression (6), and do not require a base surface. Bases are set for location tolerances, total shape, and position tolerances, and beat tolerances [17, 18].

3. Based on the principles of unity and constancy of the bases, as a rule, the surface should not be a technological base, although there are exceptions, for example, bilateral face grinding.

4. Measurements of actual deviations (processing errors) are performed on the actually treated surface \( r \) defined by expression (11).

5. Both in modeling and in assessing the accuracy of the machine according to the accuracy of the machined surfaces, the processing of representative products is carried out on one machine, and, as a rule, in one transition, therefore the errors of basing and fixing are constant values.

6. Using the above definitions of the base surface, it is impossible to estimate the deviations from the location of two different real treated surfaces on one machine, since they have (expressions (13) or (16)) and, accordingly, different base surfaces, and also because that it does not change the technological bases [30-32].

Preserving a reasonable division of errors into the errors of size, shape, and location, we select them not at the base surface, but at the actually treated surface, defined by expression (11), as follows:

\[
r = r_0 + e_0 r_0 + \delta r_0 + dr_0,
\]  

(17)

\[
\Delta r_0^* = e_0 r_0 + \delta r_0 + dr_0.
\]  

(18)

From (17) it follows that using any of the definitions for the vector balance of accuracy \( \Delta r_0 \), \( \Delta r_0^* \) and \( \Delta r_0^{**} \), the matrix \( e_0 \) completely determines the change in the position of the workpiece when the surface \( r_0 \) is processed relative to the technological base used (\( e_0 = e_0 \)). The reason for this change is the error in the shape of the surfaces of the corresponding unit of the machine - cartridge, faceplate, table, etc., which are used to install the workpiece, and resulting in the error of its basing. The components \( dr_0 \) and \( \delta r_0 \) of expression (17), as well as in dependence (16), determine the errors of size and shape, but for a really processed surface.

Now, having dependencies (11) and (17), and using the definitions given in the standards [18, 19], it is possible to establish the relationship between the errors of size, shape, and position with the elementary errors of the machine units involved in their formation. In this case, there is no need to establish relationships between the parameters of the base surface and the machine errors [11, 13], which influence the accuracy of processing. The established dependences of the errors of size, shape, and position on the elementary errors of the machine components will be a tool for updating and revising the current standards for the accuracy of machines and the development of new standards.
5. General mathematical model of machining accuracy on MT. General mathematical model of MCS accuracy

The equation for the nominal surface to be processed is (6).

SF reflects the relationship (4) between the coordinates of the cutting edge of the tool in the system SI of the cutting tool and the coordinates of the same points in the coordinates SO of the workpiece, as well as the connections (rounding, hidden and functional) between the generalized movements, such that SF (4) can be represented as follows:

\[ r_0 = r_0(u, v, q_{st}), \]

where \( q_{st} \) – machine coupling vector; \( q_{st} = (q_{s1}, \ldots, q_{sl})^T \).

Taking into account only those errors in this balance that influence on the accuracy parameters of the being treated surface, as well as the nature of this effect, a new definition of the vector balance of accuracy \( \Delta r_{0**} \) was proposed, which has the form (18).

Then the equation of the real treated surface has the form (11).

The following equation is used to determine the relationship between the geometric accuracy parameters of the machine components and the surfaces of the parts processed on these machines:

\[ \Delta r_0(u, v, \Delta q_0) = \Delta r_0^*(u, v, \Delta q_0). \]

In (20) the term \( \Delta r_0(u, v, \Delta q_0) \) is the full variation of expression (6), the term \( \Delta r_0^*(u, v, \Delta q_0) \) is the full variation of expressions (4) and (19).

Using relation (20), the correspondence between the full differentials \( dr_0(u, v, dq_0) = dr_0(u, v, dq_0) \) and between partial variations \( \delta r_0(u, v, \delta q_0) = \delta r_0(u, v, \delta q_0) \) is used, since the following expression is an identity:

\[ \varepsilon_0(r_0(u, v, q_0)) = \varepsilon_0(r_0(u, v, q_0)). \]

In addition to applying the studied SF connections: envelope, hidden and functional, one should also take into account the parameters of the cutting tool, after which SF in the expression (6) corresponds to the equation of the nominal surface to be treated:

\[ r_0 = r_0(u, v, q_1, q_2), \]

where \( q_1 \) is the machine coupling vector; \( q_2 \) – vector of dimensional links of the tool cutting edge.

The functional purpose of the MCS elements is realized with the equality of expressions (6) and (22), i.e.:

\[ r_0(u, v, q_0) = r_0(u, v, q_1, q_2). \]

Then the full variation of the SF expression:

\[ \Delta r_0 = A_{r0} \Delta r_{ri}, \]

where \( r_{ri} \) – radius-vector of tool-forming points has the form:

\[ \Delta r_0 = \Delta A_{r0} r_{ri} + A_{r0} \Delta r_{ri} + \Delta A_{r0} \Delta r_{ri}. \]

The position that the matrix \( \varepsilon_0 \) completely determines only the change in the position of the workpiece during the processing of the surface \( r_0 \) relative to the used technological base, and the components \( dr_0 \) and \( \delta r_0 \) determine the size and shape errors for the actually processed surface [20, 21], was refined in further studies on the Project. This clarification is due to the small errors included in the matrix \( \varepsilon_0 \) may, in general, include constant and variable components, i.e. this matrix in general form can be represented as \( \varepsilon_0 = \varepsilon_{0,\text{var}} + \varepsilon_{0,\text{var}} \). Then the matrix \( \varepsilon_{0,\text{var}} \) will completely determine the change in position of the workpiece, and the component \( \varepsilon_{0,\text{var}} + \delta r_0 \) determines the form error [22].

In the study of machines with parallel and hybrid arrangements, it was established that additional closed contours, implemented by parallel structure mechanisms, should be taken into account when determining the vector balance of accuracy and actual surface to be processed. For machines with sequential types of layouts of the shaping system, it is assumed that there is only one closed contour: the workpiece is machined — the cutting tool corresponds to SF, the full variation of which is used when finding the vector balance of accuracy [11]. However, even with a sequential layout type of the shaping system, there may be additional closed contours that are not taken into account when determining SF.
Experimental testing of these results allowed to obtain a system of relations between the geometrical errors of the units of the forming system of a lathe:

\[
\begin{align*}
\gamma_0 + \gamma_1 + \gamma_4 + \gamma_5 &= 0; \\
\beta_5 + (\beta_0 + \beta_1 + \beta_4)\cos \varphi + (\alpha_0 + \alpha_1 + \alpha_4)\sin \varphi &= 0; \\
\alpha_5 + (\alpha_0 + \alpha_1 + \alpha_4)\cos \varphi - (\beta_0 + \beta_1 + \beta_4)\sin \varphi &= 0; \\
\delta x_5 + (\delta x_0 + \delta x_1 + \delta x_4)\cos \varphi - (\delta y_0 + \delta y_1 + \delta y_4)\sin \varphi - l(\beta_1 \cos \varphi + \alpha_1 \sin \varphi) &= 0; \\
\delta y_5 + (\delta x_0 + \delta x_1 + \delta x_4)\sin \varphi + (\delta y_0 + \delta y_1 + \delta y_4)\cos \varphi - l(\beta_1 \sin \varphi - \alpha_1 \cos \varphi) &= 0; \\
\delta z_0 + \delta z_1 + \delta z_4 + \delta z_5 &= 0.
\end{align*}
\] (26)

Installed dependencies: (6), (11), (18), (22), (26) allowed to obtain a complete system of dimensional relationships when machining surfaces on a lathe.

Thus, on the basis of systematization of previously performed studies and the proposed method for taking into account the influence of closed contours in machine tools forming systems on the processing accuracy, an approach to building a complete system of dimensional relationships is set out (block 25, Fig. 1).

The system of dimensional relationships should have the property of completeness and non-redundancy, i.e. dependencies for errors of size, shape and location of real surfaces should be taken into account all the geometrical errors of machine tools involved in their formation [23].

The construction and analysis of the system of dimensional relationships are based on using the following principles:

- each requirement of processing accuracy has to meet at least one relationship determined on the basis of accuracy balances, and if there are more than one, these relationships must have a non-empty intersection of the ranges of acceptable values and not be linearly dependent;
- A unified approach when considering machines with sequential, parallel and hybrid layouts is ensured by using virtual coordinates, while the links of the shaping system can realize from one to six shaping movements;
- The dimensional system of the forming machine system must fully comply with the implementation of its service purpose. This correspondence being achieved at all stages of the life cycle of the machine, must be ensured during design, manufacture and testing, implemented during operation, including its modernization and repair.

The hierarchy of dimensional relationships proposed in the course of work on the Project (Fig. 3, Block 26, Fig. 1) has a 4-level structure [23].

![Figure 3. The hierarchical system of dimensional relationships in machine tools [23]](image)

At the fourth level of the hierarchy, the equations of dimensional relationships are determined without
using SF from closed and non-closed contours. The expediency of considering this level of the hierarchy of dimensional relationships and the use of expressions of the type (26) is due to the existing checks of the geometric accuracy of machines without processing sample products.

The third level of hierarchy corresponds to one equation of relations (7).

The second level of hierarchy corresponds to one expression for SF, but since the treatment of a set of different surfaces is considered here, the system of relations will be a set of expressions (18) for each surface when processing by one tool and equations of open and closed contours when they are available.

The first level of the hierarchical system of relations fully characterizes all the machine’s shaping capabilities in the processing of surface modules by various cutting tools. For this level of hierarchy, the linking system will be a set of expressions (7) for each surface when processed by an appropriate tool and equations of closed contours when they are available.

Building a complete system of dimensional relationships during processing on MT, as well as the analysis of their hierarchical system, made it possible to develop a method for multivariate synthesis of tolerances of parameters of MT geometric accuracy, the structure of which is shown in Fig. 4 [24].

![Figure 4. The structure of the method of synthesis tolerances [24]](image)

A detailed description of the stages of the proposed method is described in [24].

6. Full development of the variation method for calculating the MT accuracy in the variation method for calculating the MCS accuracy

SF, which characterizes the MCS structure, has the form (24).

After applying the function of shaping relations rounding hidden and functional, $r_0$ in (24) will correspond to that of an ideal surface to be treated.

The full variation of expression (24) has the form:

$$\Delta r_0 = \Delta A d_c r_{c1} + A_d \Delta r_{c1}, \quad (27)$$

and is the first main dependence of the variation method for calculating the MCS accuracy, because it includes in the vector balance of accuracy $\Delta r_0$ the machine error ($\Delta A d$) and the error of the cutting tool $\Delta r_{c1}$. In the variation method for calculating the accuracy of metal-cutting machines, the tool is considered either ideal or point, which is equivalent to the expression $\Delta r_{c1} = 0$. A direct consequence of this approach is the introduction of a certain factor into the resulting dependencies taking into account the fraction of the
machine’s geometric errors.

Expression (27) is the main dependence of the variation method for calculating the MCS accuracy, because here $\Delta r_{ri} \neq 0$. Obviously, the study of the influence of only the cutting tool on the machining accuracy will correspond to the condition $\Delta A_{0r} = 0$. Expression (27) is the vector balance of accuracy (vector field), which fully characterizes the influence of the geometrical errors of the machine and tool on the machining accuracy of any surface that can be machined on this MCS.

For those MCS, in which it is possible to change the location and orientation of the workpiece without the use of a device for the workpiece, characterized by the matrix $A_{ro}$, expression (24) has the following form:

$$ r_0 = A_{ro} A_{strr_i}, $$

and the full expression (28) is:

$$ \Delta r_0 = \Delta A_{ro} A_{strr_i} + A_{ro} \Delta A_{strr_i} + A_{ro} A_{str} \Delta r_i, $$

and is the second main dependence (vector field) of the variation method for calculating the MCS accuracy, which has the same properties as expression (27).

The third main dependence, which takes into account the influence of the geometrical errors of the machine and tool on the machining accuracy of a particular surface to be machined, given the radius by the vector $r_0$ has the following form:

$$ \Delta_r n,k = \sqrt{(n \Delta r_0)^2 + (k_1 \Delta r_0)^2 + (k_2 \Delta r_0)^2}, $$

where $n, k_1$ and $k_2$ are improper vectors of normal and tangents to the equation of the nominal surface being machined, respectively.

The main dependencies of the variation method for calculating the MCS accuracy should also include the projections of expressions (27) and (28) on the normal $n$, without taking into account the second-order components of smallness, characterized by the projections of expressions (27) and (28) on the tangents $k_1$ and $k_2$.

In the classical formulation, the variation method for calculating the MT accuracy is based on the consistent linearization of the SF variation. With the development of the variation method for calculating the MT accuracy in the variation method for calculating the MCS accuracy, sources of nonlinearities were found with variations of SF, which increases the accuracy of the calculations [25].

The first source of nonlinearities is associated with the determination of the error matrix of the relative position of the coordinate systems $\varepsilon_i$. If this matrix is determined by the decomposition of trigonometric functions as $\sin \delta \approx \delta$ and $\cos \delta \approx 1$ in the linear formulation, then taking quadratic terms into account, the decomposition of the form $\sin \delta \approx \delta$ and $\cos \delta \approx 1 - \delta^2/2$ is used.

The second source is related to the determination of the vector balance of the MT accuracy. When taking quadratic terms into account, the vector balance of MT accuracy should be determined according to the following dependence [26]:

$$ \Delta r_0 = \left( \sum_{i=0} A_{0,lt} \varepsilon_i A_{t,l} + \sum_{j=1} \sum_{i=0} A_{0,lt} \varepsilon_i A_{t,i+1} \varepsilon_{i+1} A_{t,i+1,l} e^4 \right) e^4. $$

The third source is related to the consideration of nonlinearities in determining the total variation of SF MCS, which has the form:

$$ \Delta r_0 = \Delta A_{0,rt} + A_{0,lt} \Delta r_t + \Delta A_{0,lt} \Delta r_t. $$

Thus, expressions (24), (27) - (32) are the main dependencies in the development of the variation method for calculating the MT accuracy in the variation method for calculating the MCS accuracy.

### 7. Conclusions

During the work on the Project, a set of tasks was solved to achieve the goal of the research, which made it possible to obtain the most important scientific results presented in this paper. Also, it is important to note that directions for further research have been identified in the areas of machining accuracy on MT in the
presence of functional dependences of geometric errors on the parameters of coordinate movements of the machine units during shaping; MCS accuracy when machining complex surfaces with shaped cutting tools; nonlinear theory of geometric precision MT.

The obtained scientific results make it possible to carry out scientifically-based rationing of the accuracy characteristics of MT serial, parallel and hybrid arrangements, i.e. to develop recommendations for setting tolerance values for machine accuracy parameters that influence on machining accuracy. The obtained variation method for calculating the MCS accuracy is intended for revising the existing and preparing drafts of new standards for the geometrical accuracy norms MT; development of technical specifications for the design of new types of machines and types with a given processing accuracy; diagnosing MCS; prediction of MCS; securing MT upgrades.

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