Fragmentation contribution to quarkonium production in hadron collision

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ABSTRACT

We compute the contributions due to gluon and heavy-quark fragmentation to quarkonium production at large transverse momentum in \(\bar{p}p\) and \(pp\) collisions. For inclusive \(J/\psi\) production, there is a large contribution from the \(g \to \chi_c\) fragmentation. At large \(p_T\), this is comparable to the conventional charmonium model prediction via gluon fusion at the ISR and dominates over the latter at the Tevatron energy. This may help to explain, at least partly, the large \(J/\psi\) production cross-section recently observed by the CDF experiment. However, the fragmentation contribution to \(\psi'\) production is not large enough to explain the corresponding CDF data. We also present the results for \(\Upsilon\) production at ISR and Tevatron energies.
The production of quarkonia in hadron-hadron and lepton-hadron collisions has been the subject of several theoretical and experimental investigations. Experimentally, the leptonic decay channels of quarkonia provide the cleanest signals of heavy quark production. From the theoretical point of view, quarkonium production provides a very important test of perturbative QCD. Both in lepton-hadron and in hadron-hadron collisions, the quarkonia are produced mainly through gluon-initiated subprocesses – i.e. photon-gluon and gluon-gluon fusion leading to the heavy-quark pair production. Consequently these processes are also important probes for the gluon distribution in the hadron.

The colour-singlet model has been used, with reasonable success, to describe the leptoproduction \[1\] and hadroproduction \[2\] of quarkonia at large transverse momentum. In this QCD-based approach, one projects out from the full heavy-quark pair production amplitude, the part with the correct spin, parity and charge-conjugation assignments of a \[^{2S+1}L_J\] quarkonium state. The \(Q\bar{Q}\) is required to be a colour singlet, which is achieved by the radiation of a hard gluon in the final state. The matrix-element so obtained is then convoluted with the wave-function at the origin, \(|R_0|^2\) (or its derivative, in the case of \(P\)-wave quarkonia). In this approach, it is possible to compute the production cross-sections for the different resonances in a given family of quarkonia: in particular, the \(P\)-wave cross-sections are computable and their contribution to the inclusive cross-section of the \(S\)-state completely specified, once the branching ratios of the \(P\)-states to the \(S\)-state are determined experimentally. The colour-singlet model is known to provide a reasonable description of data from the EMC and NMC experiments on large-\(p_T\) leptoproduction \[3, 4\] of \(J/\psi\) and on large-\(p_T\) hadroproduction \[2, 5\] of \(J/\psi\) from ISR experiments. For a recent review of quarkonium production, and for a discussion of the \(J/\psi\) production data we refer the reader to Ref. \[6\].

The recent CDF data on large-\(p_T\) \(J/\psi\) production in 1.8 TeV \(\bar{p}p\) collisions at the Tevatron \[7\] seem to indicate, however, a serious discrepancy: the theoretical predictions of the above charmonium model are well below the data, which suggests the existence of a new contribution to \(J/\psi\) production at large \(p_T\). It has been pointed out by Braaten and Yuan \[8\] that a large contribution to quarkonium production at large \(p_T\) could come from the fragmentation of gluons and heavy quarks. Even though the fragmentation process is of higher order in the strong coupling constant, \(\alpha_s\), it can dominate over the direct quarkonium production via fusion at large \(p_T\), where terms of the order of \(p_T^2/m_Q^2\) can easily compensate for the suppression due to the extra power of \(\alpha_s\). It is important to check whether the fragmentation process can explain the discrepancy between the theoretical predictions for \(J/\psi\) production and the data from CDF.

While it is important to study the fragmentation contribution at the Tevatron energy in the light of the new CDF data, it is also interesting to study the magnitude of this contribution at lower energies. This is particularly important in view of the
fact that the low-energy $J/\psi$ production data has served as a benchmark for signals of quark-gluon plasma (QGP) formation such as $J/\psi$ suppression \textsuperscript{[4]}. Indeed, such a suppression of $J/\psi$ production has been observed \textsuperscript{[11]} by the NA38 experiment in heavy-ion collisions at SPS energies ($\sqrt{s} \approx 20$ GeV). One important feature of the QGP signal is the $p_T$ dependence of the suppression; therefore, an estimate of the magnitude of the fragmentation contribution to $J/\psi$ production at these energies is important.

In this letter, we present a complete perturbative QCD calculation of $J/\psi$ production at Tevatron energy, which includes the colour-singlet model prediction for the fusion contribution as well as the fragmentation contribution. We find that the gluon fragmentation contribution is large and can help at least partially to resolve the discrepancy between the CDF data and the earlier theoretical predictions. We also study these contributions for $pp$ collisions at $\sqrt{s} = 63$ GeV and compare them to the data from the ISR experiment \textsuperscript{[11]}. Results for $\psi'$ production at Tevatron and at ISR energies are also presented. In this case the fragmentation contribution is not large enough to explain the discrepancy between the fusion contribution and the CDF data at the Tevatron energy. This may indicate a sizable nonperturbative contribution to charmonium production, presumably due to the small charm quark mass. Therefore, we present the analogous results for $\Upsilon$ production, which may be compared with the future $\Upsilon$ production data from the Tevatron.

After this work was completed, results for $J/\psi$ production at Tevatron energy were presented in Refs. \textsuperscript{[12, 13]}. Ref. \textsuperscript{[13]} has also presented results for $\psi'$ production at Tevatron energy. Our results for $J/\psi$ and $\psi'$ production at the Tevatron energy are in agreement with the results in Refs. \textsuperscript{[12, 13]}. However, as emphasised above, we have also studied the energy dependence of the fragmentation contribution – in particular, we have studied $J/\psi$ and $\psi'$ production at ISR energies. We also present results for $\Upsilon$ production at ISR and Tevatron energies.

In computing the inclusive $J/\psi$ $p_T$ distributions, for the fusion and the fragmentation processes, the contributions of the $P$-wave $\chi_c$ states have also to be considered. The $\chi$’s are produced copiously and the $\chi_1$ and $\chi_2$ have a sizeable branching into the $J/\psi + \gamma$ mode. The large-$p_T$ production cross-section for the fusion process is given as

$$E \frac{d\sigma}{dp^2}(AB \rightarrow (J/\psi, \chi_i)X) =$$

$$\sum \int dx_1 x_1 G_{a/A}(x_1) x_2 G_{b/B}(x_2) \frac{2}{\pi} \frac{1}{x_1 - \bar{x}_T e^y} \frac{d\hat{\sigma}}{dt}(ab \rightarrow (J/\psi, \chi_i)c).$$

(1)

In the above expression, the sum runs over all the partons contributing to the subprocesses $ab \rightarrow (J/\psi, \chi_i)c$; $G_{a/A}$ and $G_{b/B}$ are the distributions of the partons $a$ and $b$ in the hadrons $A$ and $B$ with momentum fractions $x_1$ and $x_2$, respectively. Energy-
momentum conservation determines $x_2$ to be

$$x_2 = \frac{x_1 \bar{x}_T e^{-y} - 2 \tau}{2x_1 - \bar{x}_T e^y},$$

where $\tau = M^2/s$, with $M$ the mass of the resonance, $s$ the centre-of-mass energy and $y$ the rapidity at which the resonance is produced.

$$\bar{x}_T = \sqrt{x_T^2 + 4\tau} \equiv \frac{2M_T}{\sqrt{s}}, \quad x_T = \frac{2p_T}{\sqrt{s}}$$ (3)

The expressions for the subprocess cross-sections, $d\hat{\sigma}/d\hat{t}$, are explicitly given in Refs. [2] and [14]. We obtain $d\sigma/dp_T$ from Eq. 1 using

$$E \frac{d\sigma}{dp^3} = \frac{1}{\pi} \frac{1}{2p_T} \frac{d\sigma}{dy dp_T}.$$ (4)

The fragmentation contribution is computed by factorising the cross-section for the process $AB \rightarrow (J/\psi, \chi_i)X$ into a part containing the hard-scattering cross-section for producing a gluon or a charm quark and a part which specifies the fragmentation of the gluon (or the charm quark) into the required charmonium state, i.e.

$$d\sigma(AB \rightarrow (J/\psi, \chi_i)X) = \sum \int_0^1 dz \ d\sigma(AB \rightarrow cX)D_{c \rightarrow (J/\psi, \chi_i)}(z, \mu),$$ (5)

where $c$ is the fragmenting parton (either a gluon or a charm quark) and the sum in the above equation runs over all contributing partons. $D(z, \mu)$ is the fragmentation function and $z$, as usual, is the fraction of the momentum of the parent parton carried by the charmonium state. The fragmentation function is computed perturbatively at an initial scale $\mu_0$ which is of the order of $m_c$. It is then evolved to the scale typical of the fragmenting parton which is of the order of $p_T/z$, using the Altarelli-Parisi equation:

$$\mu \frac{\partial}{\partial \mu} D_{i \rightarrow (J/\psi, \chi_i)}(z) = \sum_j \int_z^1 dy P_{ij}(\frac{z}{y}, \mu) D_{j \rightarrow (J/\psi, \chi_i)}(y),$$ (6)

where the $P_{ij}$ are the splitting functions of a parton $j$ into a parton $i$. We consider the fragmentation of gluons and charm quarks alone. In principle, the contribution of the light quarks should also be considered but their contribution to the fragmentation at large $p_T$ is small and can be neglected. Further, in the evolution we consider only the $P_{gg}$ contribution in evolving the gluon fragmentation function and the $P_{cc}$ contribution in evolving the charm quark fragmentation. The effect of the non-diagonal splitting function contributions can be safely neglected. The full set of initial fragmentation functions that we need to obtain the $J/\psi$ and the $\chi$ contributions have now been computed. These are $D_{g \rightarrow J/\psi}$ [8], $D_{g \rightarrow \chi}$ [13], $D_{c \rightarrow J/\psi}$ [16] and $D_{c \rightarrow \chi}$ [7, 18].
For the fragmentation process, the cross-section is given by a formula similar to Eq. 1 but with an extra integration over $z$, or equivalently over $x_2$. We have

\[
E \frac{d\sigma}{dp^3}(AB \rightarrow (J/\psi, \chi_i)X) = \\
\sum \int dx_1 dx_2 G_{a/A}(x_1) G_{b/B}(x_2) D_{c \rightarrow (J/\psi, \chi_i)}(z) \frac{1}{\pi z} \frac{d\hat{\sigma}}{dt}(ab \rightarrow cd),
\]

with $z$ given by

\[
z = \frac{\bar{x}_T}{2} \left( \frac{e^{-y}}{x_2} + \frac{e^y}{x_1} \right).
\]

For $d\hat{\sigma}/dt(ab \rightarrow cd)$, we have used the lowest-order expressions.

Using the formalism described above, we can compute the direct (fusion) and fragmentation contributions to $J/\psi$ production. To do so, we have to choose a set of parton distributions that are compatible with all the available information on structure functions. For $J/\psi$ production at ISR energies this is not very crucial; however, at Tevatron energies the values of $x$ probed are small, and one should use a set of parton densities which are compatible with the low-$x$ structure functions measured at HERA. In our computations, we have used [19] the updated MRSD-′ parametrisations [20] for the parton densities in the nucleon, which are compatible with the most recent data on structure functions from HERA. The parton densities are evolved to a scale $Q^2 = \mu^2/4$, where $\mu$ is chosen to be $M_T$ for the case of direct $J/\psi$ production, and equal to $p_T^g = p_T/z$ for $J/\psi$ production via fragmentation. The fragmentation functions are evolved to the scale $p_T/z$. We use the above choice of parameters for all the results presented below, except when we vary these parameters to see the sensitivity of our results to this choice. Another uncertainty that enters the normalisation of the cross-section predictions is that due to the wave-function at the origin, $R_0$, and the derivative for the $P$-states, $R_1'$. In the computation of the fusion contribution these appear in the subprocess cross-sections, whereas for the fragmentation contribution, the fragmentation functions at the initial scale are proportional to these wave-function factors. In Ref. [15] the fragmentation function for $g \rightarrow \chi$ is written in terms of two parameters $H_1$ and $H_8'$, where $H_1$ is related to $R_1'$, and $H_8'$ is a parameter that describes the $g \rightarrow \chi$ fragmentation via a colour-octet mechanism. The parameter $H_8'$ is rather poorly determined. For the parameters $R_0$, $H_1$ and $H_8'$, we have used the values quoted in Refs. [5, 13] ($R_0^g = 0.8$ GeV$^3$, $H_1 = 15.0$ MeV, $H_8' = 3.0$ MeV).

The results for inclusive $J/\psi$ production for both the fusion and fragmentation contributions at ISR ($\sqrt{s} = 63$ GeV) and Tevatron ($\sqrt{s} = 1.8$ TeV) have been shown in Fig. 1. To be able to compare directly with the data from $pp$ collisions at the ISR, we have computed the invariant cross-section $BEd\sigma/dp^3$ at $y = 0$ as a function of $p_T$, where $B$ is the $J/\psi$ branching ratio into leptons ($B = 0.0594$). For $\bar{p}p$ collisions at the Tevatron, we present results for $Bd\sigma/dp_T$ integrated over a pseudo-rapidity range $|\eta| < 0.5$. At the Tevatron energy, the $b$-quark contribution to $J/\psi$ production is
substantial, but this contribution has been removed from the data by the use of a micro-
vertex detector in the CDF experiment [7]. The fusion contribution (including both
$J/\psi$ and $\chi$ contributions, shown as the dashed line in Fig. 1) is significantly below the
Tevatron data. At this energy, the gluon fragmentation contribution (again, including
$g \to J/\psi$ and $g \to \chi$ and shown as the dashed-dotted line in Fig. 1) is the dominant
contribution over almost the whole range of $p_T$ values considered. The charm-quark
fragmentation (shown as the dotted line) is insignificantly small. The sum of all three
contributions (which is shown as the full line) is consistent with the data within a factor
of $\sim 2$. The comparison with the ISR data also reveals some interesting features. The
gluon fragmentation contribution is now smaller than half the fusion contribution at
the lowest values of $p_T$ considered, but grows steadily with increasing $p_T$. Addition
of the fragmentation component is seen to improve the agreement with the ISR data
significantly.

In Fig. 2, we show the magnitude of the variation of the total contribution, due to
changing the scale from $\mu/2$ to $2\mu$. These correspond to the upper and the lower lines
respectively. It should be mentioned here that the former choice is favoured by several
QCD tests. We have also studied the effect of varying the choice of parton densities
by using GRV densities [21] instead of MRSD’, since both are compatible with the
most recent structure function measurements. The effect of this change on the total
$J/\psi$ cross-section is not significant.

Our results for $\psi'$ are shown in Fig. 3, again for both the energies. For $\psi'$, the $P-
state contributions are, of course, absent. At the lower ISR energy, the fragmentation
contributions are much too small so that the total contribution is equal to the direct
contribution. For the Tevatron energy, we find that the charm quark fragmentation
contribution dominates at large $p_T$. The $\psi'$ data is also contaminated at this energy
by the $b$-decay contributions. We use the $b$-inclusive data from CDF and subtract the
theoretical $b$-quark contribution to get the data points shown in Fig. 3. But the sum
of all the contributions is clearly not anywhere close to explaining the data. This is
clearly an indication that some new mechanism is needed to explain the $\psi'$ data. It
may be relevant to note here that the $\psi'$ has only the $S$-wave contribution, whereas
the $\psi$ is dominantly produced from the decays of the $P$-wave $\chi$ states. A significant
contribution from any new mechanism to $S$-state production will therefore lead to a
relatively larger enhancement for $\psi'$ compared to $\psi$. In view of the modest value of the
charm quark mass one may speculate such a contribution to arise from nonperturbative
effects.

Fig. 4 shows our predictions for the case of $\Upsilon$ production. As in the case of $J/\psi$
production, we have included the contributions due to the decays of the $P$-states.
At the ISR energy, the $\Upsilon$ comes largely from direct production via fusion while the
fragmentation contributions are negligibly small. For the Tevatron energy, the gluon
fragmentation contribution starts dominating only beyond a $p_T$ of 15 GeV; the rela-
tively larger value of this cross-over point reflects the relatively larger mass of bottom
quark compared to charm. This also implies that any nonperturbative contribution will be relatively small for the ϒ. Thus it will be important to test this perturbative QCD prediction with the ϒ production data from the Tevatron.

To conclude, we have studied both the fusion and fragmentation contributions to J/ψ, ψ′ and ϒ production at Tevatron and ISR energies. We find that at the Tevatron energy, the J/ψ’s are produced dominantly via gluon fragmentation. The sum of the J/ψ production cross-sections from the fusion and fragmentation processes is compatible with the data from CDF to within a factor of ∼2. For ψ′, however, even the addition of the fragmentation contribution is not sufficient to restore agreement with the CDF data. This may indicate a significant nonperturbative contribution to charmonium production, presumably due to the modest value of the charm quark mass. Therefore it will be important to test the predicted ϒ production cross-section with the future Tevatron data.

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Figure captions

Fig. 1 Upper figure: The cross-section $BE\sigma/dp^3$ (at $y = 0$) for the process $pp \to J/\psi X$ as a function of $p_T$ at $\sqrt{s} = 63$ GeV. The data are taken from Ref. [11].

Lower figure: The cross-section $Bd\sigma/dp_T$ (integrated over the pseudorapidity range $-0.5 < \eta < 0.5$) for the process $\bar{p}p \to J/\psi X$ as a function of $p_T$ at $\sqrt{s} = 1.8$ TeV. The data are taken from Ref. [7]. The different curves (in both the upper and the lower figures) correspond to the direct production via fusion (dashed line), the gluon fragmentation contribution (dashed-dotted line), the charm quark fragmentation term (dotted line) and the sum of all contributions (solid line).

Fig. 2 The scale dependence of the $J/\psi$ cross-section as a function of $p_T$ for $pp$ collisions at $\sqrt{s} = 63$ GeV (upper figure), and for $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV (lower figure). The solid curve in both figures is for the scale $\mu/2$ and the dashed curve is for the scale $2\mu$.

Fig. 3 The $\psi'$ cross-section as a function of $p_T$ for $pp$ collisions at $\sqrt{s} = 63$ GeV (upper figure), and for $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV (lower figure), with the different contributions shown as in Fig. 1. The $\sqrt{s} = 1.8$ TeV data are obtained as explained in the text.

Fig. 4 The $\Upsilon$ cross-section as a function of $p_T$ for $pp$ collisions at $\sqrt{s} = 63$ GeV (upper figure), and for $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV (lower figure), with the different contributions shown as in Fig. 1. The $\sqrt{s} = 63$ GeV data are taken from Ref. [11].