Analysis and Prediction of Reliability for Motion Error of Polish Mechanisms for Composite Box Based on Monte-carlo Method

Ya-wei ZHANG¹, Bei-zhi LI¹, Shun-zhou HUANG², Feng-bo DONG² and Ming-fang LIU²

¹College of Mechanical Engineering, Donghua University, China, 201620
²Shanghai Aerospace Equipments Manufacturer, Shanghai 200245

*Corresponding author

Key words: Reliability, Monte-carlo method, Error, Polish mechanism, Simulation.

Abstract. The polish mechanisms for composite material box is produced for polishing box within demanding error range, it is made up of gear system, shaft systems and other transmission parts, the clearance and eccentric in the assembly lead to motion error in the polish mechanism; hence, the error reliability of polish mechanisms should be investigated. The error distributions follow normal distribution, the parameter are defined according to the multi-object software ADAMS simulation experiments results. The position accuracy reliability analysis bases on the Monte-carlo method to estimate reliability of the polish equipment. Error distribution parameters are calculated with ADAMS simulation experiments instead of the real experiment, which can spare the experimental cost and time and makes it easy to predict reliability with little consumption. The error analysis focuses on the gear system and shaft system mainly, the other factor from other parts are neglected due to their little influence on the error, the reduced analysis benefits to the rapid obtaining reliability results.

Introduction

Mechanism has function of realizing motion demand and power transmission with high motion accuracy. The polish mechanisms for composite box is produced for polishing box within demanding error range, it is made up of gear system, shaft systems and other transmission parts, the clearance and eccentric in the assembly lead to motion error in the polish mechanism, at the same time, the wear due to the relative motion between parts results in the accuracy deterioration to worsen polishing quality. The reliability for motion error should be analyzed and predicted to maintain a good motion accuracy.

The reliability of equipment attracted wide attention with development of equipment technology and with complexity of equipment, Zhang et al [1] applied the fractional moment method to model probabilistic lifetime of uncertain engineering system with a small, simulated sample of remaining useful life of the system. Huang et al [2] analyzed the performance of the second-order reliability and proved the proposed second-order reliability method to estimate the probability of failure with high accuracy, computationally efficiency and convergent. A new method for hybrid reliability combines the bounding-limit-state-surface-based active learning Kriging with interval Monte Carlo simulation, which can estimate the lower and upper bounds of failure probability by interval Monte Carlo simulation based on the built Kriging metamodel [3]. Wang et al developed an approximation effective surrogate model to deal with kinematic reliability problems of steering mechanisms with high accuracy for various kinematic reliability problems, and an effective simulation tool for kinematic reliability analysis of mechanisms is given [4]. Hamiltonian Monte Carlo (HMC) method can be used to analyze reliability in the context of Subset Simulation, it can explore the probability space effectively and consistently in contrast to standard Gibbs or Metropolis-Hastings techniques [5]. ZHENG et al [6] improved the compression algorithm and sequential inference algorithm, and applied the proposed algorithms for the reliability analysis of complex multistate satellite systems, they proved to be efficient than the Bayes Net Toolbox and the Agena Risk software.

272
Monte Carlo simulation bases on repeated random sampling, possible results range and probabilities. Monte Carlo simulation generates random samples based on the mean value of the variables [7], Li et al [8] analyzed slope stability reliability with a weighted approach by Monte Carlo simulation. The stochastic Factor of Safety and the Probability of failure were modeled with efficient Coupled Markov Chain [9]. Cheng et al. [10] applied the limit equilibrium method and strength reduction method to compare the stochastic Factor of Safety and the locations of critical failure surfaces of various slopes. An advanced line sampling method is used to estimate failure probability bounds and proved to be high efficiency with moderate non-linearities [11]. The lower and upper bounds of failure probability are analyzed and estimated by subset simulation and by random set theory with high efficiency [12].

In the paper, the random sample can be realized by simulation text with the multi-object ADAMS. The whole motion error comes mainly from the gear system and shaft system in the equipment, hence, the errors in the gear system and in shaft system are furthermore should be analyzed, and the error distributions parameter can be defined according to the multi-object software ADAMS simulation experiments results. The position accuracy reliability should be estimated for the polish equipment on the Monte-carol method.

The Polish Mechanism Configuration and Its Function

The polish mechanisms for composite box is made up of machine bed, motor, indexer, self-centering fixture, float support and tail stock as shown in figure 1. the procedure of clamp and position are realized when the movable bench comes into the assemble position:

Figure 1. Polish mechanism.

(1) The composite box is lift on the floating support by the forklift;
(2) The self-centering fixture moves into the composite box with the running of the floating support, and the translating motion is limited in X direction, tail stock can move towards self-centering fixture;
(3) The floating support goes down, the self-centering fixture and tail stock support the composite box, degrees of freedom in X rotation Y rotation and Z translation direction are limited;
(4) Support bar reaches out, the degree of freedom in Y translation direction and Z rotation are limited, and the position of composite box is completed.
The Motion Error of Polish Mechanism and Its Modeling

The Error in Gear Mechanism

The error from transmission parts play an important role in position error of polish mechanism, those parts include gear, shaft and fixture.

1. The errors from gear process including the error due to geometric eccentric in gear process.
   The eccentric of drive gear is $e$, the angel between eccentric and master-slave gear center line is $\beta_1$, tooth press angel is $\alpha_n$, the error due to eccentric can be expressed:

   \[ \Delta_e = esin\beta_1 + ecos\beta_1 tan\alpha_n = \frac{esin(\beta_1+\alpha_n)}{cos\alpha_n} \]  \(\text{(1)}\)

   The formula express the linear error, this error can be translated into the angel error relative to reference circle of the drive gear, $d$ is diameter of reference circle, $mn$ is the gear normal module. $Z$ is the number of gear teeth.

   \[ \Delta_{e\phi} = \frac{\Delta_e}{2\pi} \times \frac{180+60}{\pi} = 3.44 \frac{\Delta_e}{d/2} \]  \(\text{(2)}\)

   The angel error can be formulated:

   \[ \Delta_{e\phi} = \frac{6.88}{mn} \Delta_e \]  \(\text{(3)}\)

2. The errors from gear shaft geometric eccentric in gear shaft assembly.
   The eccentric of gear shaft is $e_s$, the phase angel referring to $e$ gear center line is $\beta_2$, tooth press angel is $\alpha_n$, the error due to gear shaft geometric eccentric can be expressed:

   \[ \Delta_{es} = e sin(\beta_2+\alpha_n) \]  \(\text{(4)}\)

   The formula express the linear error, this error can be translated into the angel error relative to reference circle of the drive gear, $d$ is diameter of reference circle, $mn$ is the gear normal module. $Z$ is the number of gear teeth.

   The angel error can be formulated:

   \[ \Delta_{e\phi s} = \frac{6.88}{mn} \Delta_{es} \]  \(\text{(5)}\)

3. The errors from common normal between gears during long-term running.
   The errors from common normal is $\Delta E_w$, the gear rotation angel is $\varphi$, tooth press angel is $\alpha_n$.
   The errors from common normal is the difference between length of common normal and its nominal value, the gear rotation angel error comes from wear of gears during long-term running, and it can be expressed:

   \[ \Delta_{ew} = \Delta E_w Sin\varphi \]  \(\text{(6)}\)

   The formula express the linear error, this error can be translated into the angel error relative to reference circle of the drive gear, $mn$ is the gear normal module. The angel error can be formulated:

   \[ \Delta_{e\phi w} = \frac{6.88}{mn} \Delta_{ew} \]  \(\text{(7)}\)

4. The errors from center distance between mating gears
   The errors from center distance is $\Delta efa$, tooth press angel is $\alpha_n$. it can be expressed:

   \[ \Delta_{e\phi w} = \frac{6.88}{mn} \times 2tan\alpha_n \Delta_{efa} \]  \(\text{(8)}\)
The Error in Shaft Rotation System

(1) The error from fit clearance in shaft system and impact of round error in shaft system

The hole has a diameter of \( d_k \) and its maximum value is \( d_{k_{\text{max}}} \); the shaft has a diameter of \( d_z \) and its maximum value is \( d_{z_{\text{max}}} \), \( \Delta R_Z \) and \( \Delta R_K \) are the round error of shaft and round error of hole; \( L \) is the fit distance between hole and shaft shown as figure 2.

![Figure 2. Impact of fit clearance and from round error in shaft system](image)

The radical error from fit clearance in shaft system and from round error in shaft system can be formulated:

\[
\Delta_{\text{ef}} = \frac{2(\Delta R_Z + \Delta R_K) + (d_k - d_z)}{2}
\]

(9)

The angle error translated from the error above is then expressed:

\[
\Delta_{e\phi f} = \frac{2(\Delta R_Z + \Delta R_K) + (d_k - d_z)}{L} \times \frac{180}{\pi}
\]

(10)

(2) Coaxially error from the hole and shaft

Coaxially error is the maximum distance between actual position and ideal position for shaft or for hole. The coaxially errors for shaft and for the hole are expressed with \( \Delta f_Z \) and \( \Delta f_K \) shown as figure 3, the angle error can be formulated as:

\[
\Delta_{e\phi a} = \frac{(\Delta f_Z \sin \varphi + \Delta f_K)}{L_x} \times \frac{180}{\pi}
\]

(10)

The Error Distribution Analysis Basing on Simulation of Polish Mechanisms with ADAMS

The gear systems and shaft systems in the polish system are first modeled with CAD software.
Pro/Engineering for preprocess, then the model are put into multi-body simulation software ADAMS/View for forth analysis.

In this process, the parameters of gear system should be defined: such as modules, tooth number, tooth depth coefficients, top clearance factor, pressure angle, tooth width. The contact area between gears is regarded as elliptical area, its stiffness coefficient is calculated with HERTZ theory. The interpolation error have to be controlled to avoid the long running time in ADAMS, the Dstiff integration solver is adopted in Integrator option in ADAMS for high interpolated accuracy. The rotation angles of master gear and slave gear are measured in ADAMS, the slave gear has an angle error due to eccentric error when master gear maintains a constant rotational speed. When gear ratio is i, the changes of angle displacements for master gear and slave gear are $\Delta \varphi_1$ and $\Delta \varphi_2$, the angle error in ADAMS can be expressed with

$$\Delta \varphi = \varphi_2 - \frac{\varphi_1}{i}$$  \hspace{1cm} (11)

Figure 4. Rotation angles of master gear and slave gear.

The Simulation is conducted by putting cmd file into ADAMS, and the results can be obtained by postprocess in ADAMS. The simulation experiments are repeatedly to obtain much results by change the parameters in cmd files.

The position coordinates for shaft and hole can be parameterized by embedding program with C language into ADAMS, and the shaft centerline position change can be simulation with random distribution in the error circle. The error distribution for gear systems in the polish mechanisms can be calculated as following:

1. The errors distribution from gear process $\Delta e_{\varphi p}$ including the error due to geometric eccentric in gear process
   $\Delta e_{\varphi p}$ follows a normal distribution, the error tolerance denotes $\delta p$, then the mean value $\mu_{\varphi p}=0$, standard deviation
   $$\sigma_{\varphi p} = \frac{6.88}{m_{n}z} \times \frac{\sigma_{p}}{6}$$  \hspace{1cm} (12)

2. The errors distribution $\Delta e_{\varphi s}$ from gear shaft geometric eccentric in gear shaft assembly.
   $\Delta e_{\varphi s}$ follows a normal distribution, the error tolerance denotes $\delta s$, then the mean value $\mu_{\varphi s}=0$, standard deviation
   $$\sigma_{\varphi s} = \frac{6.88}{m_{n}z} \times \frac{\sigma_{s}}{6}$$  \hspace{1cm} (13)

3. The errors distribution $\Delta e_{\varphi w}$ from common normal between gears during long-term running.
   $\Delta e_{\varphi w}$ follows a normal distribution, the error tolerance denotes $\delta w$, then the mean value $\mu_{\varphi w}=0$, standard deviation
   $$\sigma_{\varphi w} = \frac{6.88}{m_{n}z} \times \frac{\sigma_{w}}{6}$$  \hspace{1cm} (14)
(4) The errors distribution $\Delta_{\psi w}$ from common normal between gears during long-term running. $\Delta_{\psi w}$ follows a normal distribution, the error tolerance denotes $\delta w$, then the mean value $\mu_{\psi w}=0$, standard deviation

$$\sigma_{\psi w} = \frac{6.88}{m_n^2} \times \frac{\sigma_w}{6}$$ (15)

The error distribution for shaft systems in the polish mechanisms can be calculated as following:

(1) The errors distribution $\Delta_{\psi f}$ from fit clearance in shaft system and from round error in shaft system $\Delta_{\psi f}$ follows a normal distribution, the error tolerance denotes $\delta f$, then the mean value $\mu_{\psi f}=0$, standard deviation

$$\Delta_{\psi f} = \frac{\rho R}{L} * \frac{R}{6}$$ (16)

(2) The coaxially error distribution $\Delta_{\psi a}$ from the hole and shaft $\Delta_{\psi a}$ follows a normal distribution, the error tolerance denotes $\delta a$, then the mean value $\mu_{\psi a}=0$, standard deviation

$$\Delta_{\psi a} = \frac{\rho L}{F} * \frac{F}{6}$$ (4-7)

![Figure 5. Self-centering fixture.](image)

Self-centering fixture adopt centering control, and this fixture mode is inclined to eccentric between fixture centerline and the box workpiece centerline, the analysis for shaft coaxially error is same as for the Self-centering fixture.

According to the simulation experiments results, the parameters of error distribution in polish mechanisms can be defined. The concrete error destitution results is shown as following table:

| The name of error | parts | The errors distribution |
|-------------------|-------|-------------------------|
| The errors from gear process $\Delta_{\psi p}$ | Gear in the indexer | N(0, 0.61731²) |
| The errors from gear shaft geometric eccentric in gear shaft assembly | Gear in the indexer | N(0, 0.07926718²) |
| The errors from common normal between gears during long-term running | Gear in the indexer | N(0, 0.3984726²) |
| The errors from center distance between mating gears | Gear in the indexer | N(0, 0.55662²) |
| The error from fit clearance in shaft system and from round error in shaft system | Shaft in indexer | N(0, 0.0438291²), |
| | Shaft in tail stock | N(0, 0.0537829²) |
### Reliability Prediction of Polish Mechanisms Motion Basing on Monte-Carlo

The Monte-Carlo method is adopted for analyzing motion errors of equipment, the reliability of polish mechanisms bases on following principle:

The position error of polish mechanisms can be defined as following function

\[ G(x) = g(x_1, x_2, \cdots, x_n) \]  

(17)

State function \( g(x) = 0 \) divides basic variable space into failure space and reliable space, the probability of survival \( P_R \) can be expressed as

\[ P_R = \int \cdots \int f_x(x_1, x_2, \cdots, x_n) \, dx_1 \, dx_2 \cdots \, dx_n \]  

(18)

In which \( f_x(x_1, x_2, \cdots, x_n) \) is joint probability density function of basic vector of random variable \( X = [x_1, x_2, \cdots, x_n] \). Every variable is independent, and

\[ P_R = \int \cdots \int f_x(x_1, x_2, \cdots, x_n) \, dx_1 \, dx_2 \cdots \, dx_n \]  

(19)

In which \( f_x(x_i) (i = 1, 2, \cdots, n) \) is probability density function of random variable \( x_i \).

The concrete computation procedure can be realized:

According to probability distribution of error parameters for position system, \( N \) groups of random vector samples \( \Delta_j = [\Delta_{\phi Fj}, \Delta_{\phi sj}, \cdots, \Delta_{tj}] \) \((j = 1, 2, \cdots, N)\) are produced.

(2) Random vector sample \( \Delta_j \) is put into state function \( g_1(x) = 0.04 - \Delta \) and \( g_2(x) = 0.04 + \Delta \), the function \( I_R(\Delta_j) \) is accumulated according to status indicator.

(3) The reliability RMCS can be calculated as

\[ R = \frac{1}{N} \sum_{j=1}^{N} I_R(X_j) = \frac{N_R}{N} \]  

(20)
The polished composite box can be produced 4 times every hour, considering work time 2400 hours per year and the equipment of polish mechanism lifetime with 40 years, the polished composite need to be processed round $5 \times 10^5$ times in the whole equipment lifetime, the reliability of polish mechanisms can be calculated with value 98.58% with Software MATLAB basing on Monte-Carlo method.

**Conclusion**

In this paper, the motion error is analyzed based on the polish mechanism configuration and its function. The whole motion error comes mainly from the gear system and shaft system in the equipment, hence, the errors in the gear system and in shaft system are furthermore analyzed and the formula for calculating error are given; the error distributions follow normal distribution, the parameter are defined according to the multi-object software ADAMS simulation experiments results. The position accuracy reliability analysis bases on the Monte-Carlo method to estimate reliability of the polish equipment. The following points should be emphasized:

Error distribution parameters are calculated with ADAMS simulation experiments instead of the real experiment, which can spare the experimental cost and time and makes it easy to predict reliability with little consumption.

The error analysis focuses on the gear system and shaft system mainly, the other factor from other parts are neglected due to their little influence on the error, the reduced analysis benefits to the rapid obtaining reliability results.

**Acknowledgment**

This paper is supported by “Mega-project of High-grade NC Machine Tools and Basic Manufacturing Equipment (2017ZX04005001)” and by “Shanghai Science and Technology Innovation Action Plan” (Project number 17DZ2281000).
Reference

[1] Xufang Zhang, Wei He, Yimin Zhang, Mahesh D. Pandey, An effective approach for probabilistic lifetime modelling based on the principle of maximum entropy with fractional moments, Applied Mathematical Modelling 55 (2018) 68–80.

[2] Xianzhen Huang, Yuxiong Li, Yimin Zhang, Xufang Zhang, A new direct second-order reliability analysis method, Applied Mathematical Modelling 55 (2018) 68–80.

[3] Jinhao Zhang, Mi Xiao, Liang Gao, Sheng Chu, A bounding-limit-state-surface-based active learning Kriging method for hybrid reliability analysis under random and probability-box variables, Mechanical Systems and Signal Processing 134 (2019) 106310.

[4] Lei Wang, Xufang Zhang, Yangjunjian Zhou, An effective approach for kinematic reliability analysis of steering mechanisms, Reliability Engineering and System Safety 180 (2018) 62–76.

[5] Ziqi Wang, Marco Broccardo, Junho Song, Hamiltonian Monte Carlo methods for Subset Simulation in reliability Analysis, Structural Safety 76 (2019) 51–67.

[6] Xiaohu Zheng, Wen Yao, Yingchun Xu, Xianqi Chen, Improved compression inference algorithm for reliability analysis of complex multistate satellite system based on multilevel Bayesian network, Reliability Engineering and System Safety 189 (2019) 123–142.

[7] Rashki M, Miri M, Azhdary Moghadam M. A new efficient simulation method to approximate the probability of failure and most probable point. Struct Saf 2012; 39:22–9.

[8] Li DQ, Zhang FP, Cao ZJ, Zhou W, Phoon KK, Zhou CB. Efficient reliability updating of slope stability by reweighting failure samples generated by Monte Carlo simulation. Comput Geotech 2015; 69:588–600.

[9] Li DQ, Qi XH, Cao ZJ, Tang XS, Phoon KK, Zhou CB. Evaluating slope stability uncertainty using coupled Markov chain. Comput Geotech 2016; 73:72–82.

[10] Cheng YM, Lansivaara T, Wei WB. Two-dimensional slope stability analysis by limit equilibrium and strength reduction methods. Comput Geotech 2007; 34(3):137–50.

[11] M. de Angelis, E. Patelli, M. Beer, Advanced line sampling for efficient robust reliability analysis, Struct. Saf. 52 (2015) 170–182.

[12] D.A. Alvarez, F. Uribe, J.E. Hurtado, Estimation of the lower and upper bounds on the probability of failure using subset simulation and random set theory, Mech. Syst. Signal Proc. 100 (2018) 782–801.