Constraints on exotic dipole-dipole couplings between electrons at the micrometer scale

Shlomi Kotler,1 Roee Ozeri,2 and Derek F. Jackson Kimball3

1National Institute of Standards and Technology, 325 Broadway St., Boulder CO, 80305, USA.
2Department of Physics of Complex Systems, Weizmann Institute of Science, P. O. Box 26, Rehovot 76100, Israel.
3Department of Physics, California State University - East Bay, Hayward, California 94542-3084, USA.

(Dated: February 2, 2015)

New constraints on exotic dipole-dipole interactions between electrons at the micrometer scale are established, based on a recent measurement of the magnetic interaction between two trapped 88Sr+ ions. For light bosons (mass \( \approx 0.1\ eV \)) we obtain 90% confidence intervals on pseudo-scalar and axial-vector mediated interaction strengths of \( |g_{\pi g_{\pi}}/4\pi\hbar c| \leq 1.5 \times 10^{-3} \) and \( |g_{A g_{A}}/4\pi\hbar c| \leq 1.2 \times 10^{-17} \), respectively. These bounds significantly improve on previous work for this mass range. Assuming CPT invariance, these constraints are compared to those on anomalous electron-positron interactions, derived from positronium hyperfine spectroscopy. For axial-vector mediated interaction the electron-electron constraints are six orders of magnitude more stringent than the electron-positron constraints. Bounds on torsion gravity are also derived and compared with previous work performed at different length scales.

Extensions of the standard model which predict new particles may involve modifications to the known spin-spin interaction between fermions [11]. Examples include pseudo-scalar fields (such as the axion [2]) which naturally emerge from theories with spontaneously broken symmetries [3–5], and axial-vector fields such as para-photons [6] and extra Z bosons [1, 7], which appear in new gauge theories. Both pseudo-scalar and axial-vector fields are candidates to explain dark matter [8], dark energy [9, 10], and mysteries surrounding CP-violation [2].

Typically, to constrain the effect of a new particle of mass \( m \) it is favorable to perform an experimental investigation at a length scale \( \lambda \equiv \hbar/mc \), the reduced Compton wavelength associated with the particle. Here, \( \hbar \) is Planck’s constant divided by \( 2\pi \) and \( c \) is the speed of light.

At the macroscopic scale, there have been a number of searches for exotic dipole-dipole interactions between electrons [11−17], ranging from distance scales of a few cm to the radius of the Earth. The large scales involved contributed to the exquisite sensitivity of these endeavors, allowing for signal averaging over a large number of spins. Such an approach, however, cannot be implemented for scales significantly smaller than a millimeter. Attempting to extrapolate these bounds to the micrometer scale also fails, due to the interactions distance scaling [18].

The magnetic dipole-dipole interaction between two electron spins was measured directly for the first time only recently [19], at a previously inaccessible length scale of \( r \approx 2.4\ \mu m \). Here we exploit this recent measurement to constrain exotic spin-dependent interactions between electrons, considerably improving on previous bounds at the micrometer scale.

The new measurement reported in Ref. [19] benefits from the high controllability of trapped ions and the resulting relatively straightforward analysis of systematic errors. Briefly, two 88Sr+ ions are trapped in a harmonic potential \( \omega = 2\pi \times 2.386(2)\ MHz \) using a linear radio-frequency Paul trap. After laser cooling the ions form a Coulomb crystal with a separation of \( r = 2.407(1)\ \mu m \). Each 88Sr+ ion has a single valence electron. A magnetic field of \( \approx 0.47\ mT \) sets the quantization axis along the line connecting the two ions. The two electrons’ state is initialized to \( \left| \uparrow \downarrow \right> \) with \( P_{\text{init}} > 0.98 \) fidelity, where \( \uparrow(\downarrow) \) indicates a spin polarized along the positive(negative) magnetic field direction. The spins evolve under the spin-spin interaction for \( T = 15\ s \), ideally resulting in an entangled state \( |\Psi(T)\rangle = \cos(2\xi T)|\uparrow\downarrow\rangle + i\sin(2\xi T)|\downarrow\uparrow\rangle \).

Finally the coherence between \( |\uparrow\downarrow\rangle \) and \( |\downarrow\uparrow\rangle \) is quantified by rotating the spins collectively \( |\uparrow\rangle \rightarrow (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2} \) and measuring the parity observable, \( \Pi \equiv P_{\uparrow\uparrow} + P_{\downarrow\downarrow} - P_{\uparrow\downarrow} - P_{\downarrow\uparrow} \), where \( P_{ij} \) represents the probabilities to measure the spin system in respective states \( |ij\rangle \). Since the parity observable is linear in \( \sin(4\xi T) \), the coupling strength \( \xi \) can be extracted: \( \xi = 2\pi \times 1.020(95)\ MHz \), where the error is dominated by statistical uncertainty. This approach allows the experimental observable to be insensitive to spatially homogeneous magnetic field noise. The experiment also employed a spin-echo observable technique [24] to reduce the effect of magnetic field gradients to negligible levels. The measurement of \( \xi \) thus gives the interaction strength between the two bound valence electrons of two separate 88Sr+ ions. The gyro-magnetic ratio \( g \) of these bound electrons, in the presence of the field of the nucleus and the core electrons, differs from that of free electrons, \( g_{\text{free}} \), by \( \left( g_{\text{free}} - g \right) \approx 1.8 \times 10^{-5} \) (Breit formula [21]). Therefore, the fractional difference between the measured result and the result that would be obtained for two free electrons can be estimated to be \( 3.55 \times 10^{-5} \), which is well below the measurement uncertainty. This is also the case for all other considered systematic errors in the experiment, which are analyzed and detailed in the supplementary information of Ref. [19].
The measured magnetic interaction strength can easily be compared with the calculated magnetic dipole-dipole interaction between two electrons, 
\[ \xi_{\text{theory}} = \frac{\mu_0}{4\pi\lambda^2} \left( \frac{g}{r} \right)^2 \]
where \( \mu_0 \) is the magnetic permeability of vacuum and \( \mu_B \) is the Bohr magneton. The main source of error in determining \( \xi_{\text{theory}} \) is the uncertainty in \( r \) which is estimated from the measured trap frequency. The contribution of thermal fluctuations of \( r \) to the magnetic coupling average out to leading order, since they occur at the trap frequency (see supplementary information of Ref. [19]). This leads to a theoretical estimate of \( \xi_{\text{theory}} = 2\pi \times 0.931(1) \) mHz. Any deviation of the measured value from the predicted value is smaller than \( \Delta \xi/2\pi = 208 \mu \)Hz at the 90% confidence level [22].

Dobrescu and Mocioiu [1] parameterized the long-range potentials induced by the exchange of new scalar/pseudoscalar or vector/axial-vector bosons between fermions assuming rotational invariance. If the new boson is an axion-like pseudo-scalar \( (P) \) particle, there arises a dipole-dipole potential between electrons [1, 2],

\[
V_3(r) = \frac{g_P^e g_P^e}{4\pi\hbar c} \frac{\hbar^3}{m_e^2c} \left[ S_1 \cdot S_2 \left( \frac{1}{\lambda r^2} + \frac{1}{r^3} + \frac{4\pi}{3} \delta^3(r) \right) \right. \\
- (S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) \left( \frac{1}{\lambda^2 r^2} + \frac{3}{\lambda r^2} + \frac{3}{r^3} \right) e^{-r/\lambda},
\]

(1)

where \( g_P^e g_P^e/(4\pi\hbar c) \) is the dimensionless pseudo-scalar coupling constant between the electrons, \( m_e \) is the electron mass, \( S_{1,2} \) are the electron spins and \( r = \rho \hat{r} \) is the inter-particle separation [22]. If the new boson is an axial-vector \( (A) \) particle, in addition to \( V_3(r) \) there arises a Yukawa-type potential

\[
V_2(r) = \frac{g_A^e g_A^e}{4\pi\hbar c} \frac{\hbar c}{r} S_1 \cdot S_2 e^{-r/\lambda},
\]

(2)

where \( g_A^e g_A^e/(4\pi\hbar c) \) is the dimensionless axial-vector coupling constant between the electrons. (The subscripts \( i \) for the anomalous potentials \( V_i(r) \) are chosen to match the notation of Ref. [1]).

The \( V_{2,3} \) potentials would contribute to the \(| \uparrow \downarrow \rangle \leftrightarrow | \downarrow \uparrow \rangle \) coherent oscillation measured in Ref. [19], according to the corresponding Hamiltonians, written in the \(| \uparrow \downarrow \rangle, | \downarrow \uparrow \rangle \) basis,

\[
H_2(r) = \frac{g_A^e g_A^e}{4\pi\hbar c} \frac{2\hbar c}{r} e^{-r/\lambda} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]

(3)

and

\[
H_3(r) = \frac{g_P^e g_P^e}{4\pi\hbar c} \frac{\hbar^3}{2m_e^2c} \left( \frac{1}{\lambda r^2} + \frac{1}{r^3} \right) e^{-r/\lambda} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
\]

(4)

The corresponding oscillation frequencies,

\[
\xi_2 = \frac{g_A^e g_A^e}{4\pi\hbar c} e^{-r/\lambda}
\]

(5)

and

\[
\xi_3 = \frac{g_P^e g_P^e}{4\pi\hbar c} \frac{\hbar^2}{2m_e^2c} \left( \frac{1}{r^3} + \frac{1}{\lambda r^2} \right) e^{-r/\lambda}
\]

(6)

are smaller than \( \Delta \xi = 2\pi \times 208 \mu \)Hz for \( r = 2.4 \mu \)m, at the 90% confidence level. This leads to the constraints plotted in Fig. 1(a) for a pseudo-scalar interaction and Fig. 1(b) for an axial-vector interaction.

Closely related experiments are the measurements of the hyperfine structure interval of the ground state in positronium (Ps) [23,25], which constrain exotic dipole-dipole interactions between electrons and positrons above.
the angstrom scale. The present agreement between the
latest experimental measurement \( \Delta E_{HF}(\text{Ps})_{\text{exp}} = 203.394.2(1.6)_{\text{stat}}(1.3)_{\text{sys}} \) MHz, and most recent theoretical calculations based on quantum electrodynamics (QED) \( \Delta E_{HF}(\text{Ps})_{\text{theory}} = 203.391.90(25) \) MHz, stands at the level of 5 MHz with a 90% confidence level \( \Delta \), dominated by the experimental uncertainty.

Similar to Ref. \( \Delta \), we can estimate the shift of positronium’s hyperfine structure interval due to \( \nu_3(r) \) using first-order perturbation theory. In this case, one must average over the spherically symmetric ground state (see, for example, Refs. \( \Delta \)). These turn out to be essentially mass independent, since in our region of interest, \( \lambda \gg a_0 \).

A similar calculation can be carried out to derive a constraint on \( \nu_2(r) \) based on the ground-state hyperfine interval in positronium \( \Delta \): the energy shift of the Ps ground-state hyperfine structure interval due to \( \nu_2(r) \) is,

\[
\Delta E_2 = \frac{\alpha^2}{4\pi\hbar c} \frac{24}{\sqrt{8\pi a_0^3}} \left( 1 - \frac{1}{1 + \lambda/a_0} \right), \tag{7}
\]

where \( \alpha \) is the fine structure constant. Demanding that \( \Delta E_2 \leq 2\pi\hbar \times 5 \) MHz leads to the constraints shown in Fig. \( \Delta \). These turn out to be essentially mass independent, since in our region of interest, \( \lambda \gg a_0 \).

The bounds derived from the \( \Delta \) Sr ion experiment and the Ps experiment are compared in Fig. \( \Delta \). For axial-vector \( \nu_2 \) coupling [Fig. \( \Delta \)], \( \Delta \) spectroscopy places a constraint which is six orders-of-magnitude stronger than the Ps counterpart. For pseudo-scalar \( \nu_2 \) coupling [Fig. \( \Delta \)], it is the Ps spectroscopy which places a stronger bound, roughly by four orders of magnitude. These differences result from the interplay between sensitivity and distance scaling. In both cases, the constraints are proportional to \( \Delta/d^3 \), where \( \Delta \) is the measurement uncertainty, \( d \) is the inter-particle distance and \( n \) is the corresponding exponent. Indeed the \( \Delta \) experiment measures energy shifts with an uncertainty \( \Delta \) and some ten orders-of-magnitude smaller than that obtained in positronium spectroscopy. This, however, is counteracted by the much larger separation \( a_0 \sim 2 \) µm between the \( \Delta \) Sr ions as compared to the \( a_0 \), angstrom-scale, separation between the electron and positron in Ps. For \( \lambda \gg 1 \) µm the axial-vector coupling scales inversely with distance \( n = 1 \) so \( d_0/a_0 \sim 10^3 \) whereas the pseudo-scalar counterpart scales inversely with the cube of the distance \( n = 3 \) so \( (d_0/a_0)^3 \sim 10^{14} \).

Direct comparison between the electron-electron and electron-positron constraints is based on the implicit assumption of CPT-invariance for the exotic interactions studied here. The potentials \( \nu_3(r) \) and \( \nu_2(r) \) are even under the parity (P) and time-reversal (T) transformations, and thus they are also even under the combined PT symmetry. Consequently, assuming CPT-invariance (where C represents charge conjugation), electrons and positrons should have the same magnitude of coupling strength to exotic pseudoscalar or axial-vector interactions: \( |\tilde{g}_p| = |\tilde{g}_p| \) and \( |\tilde{g}_e| = |\tilde{g}_e| \). It should be noted, however, that CPT invariance is not guaranteed for these exotic interactions, at least based on the current state of knowledge \( \Delta \). Thus, one can regard the \( e^- - e^- \) and the \( e^+ - e^- \) measurements discussed here as independent constraints.

The above analysis can also be used to place bounds on torsion gravity. According to general relativity, the local space-time curvature is unaffected by the presence of spin \( \Delta \). However, in extensions of general relativity based on a Riemann-Cartan spacetime, the gravitational interaction is described by a torsion tensor which can generate spin-mass and spin-spin interactions \( \Delta \). The spin-spin interaction generated by a standard propagating torsion field takes the form of \( \Delta \). with \( \lambda \rightarrow \infty \) and a coupling strength that can be parameterized in terms of a dimensionless parameter \( \beta \), such that

\[
\beta^2 = \left( \frac{\alpha^2}{4\sqrt{\pi\hbar c}} \right) \times \left( \frac{2}{Gm_e^2} \right), \tag{9}
\]

where \( G \) is the gravitational constant. The minimally coupled Dirac equation for a spin-1/2 particle \( \Delta \) predicts \( \beta = 1 \). Experimental constraints, based on the analysis presented here, are shown in table I.

| Particles | Experiment Scale | \( |\beta|^2 \leq \) | Ref. |
|-----------|-----------------|-----------------|------|
| \( n - n \) | 50 cm | \( 2 \times 10^{28} \) | \( \Delta \) |
| \( p - p \) | 50 cm | \( 2 \times 10^{31} \) | \( \Delta \) |
| \( e^- - e^- \) | 15 cm | \( 4 \times 10^{28} \) | \( \Delta \) |
| \( e^+ - e^- \) | 2 µm | \( 2 \times 10^{41} \) | this work | \( \Delta \) |
| \( n - n \) | 100 nm | \( 6 \times 10^{34} \) | \( \Delta \) |
| \( p - p \) | Å | \( 1 \times 10^{33} \) | \( \Delta \) |
| \( e^- - e^+ \) | Å | \( 3 \times 10^{38} \) | this work | \( \Delta \) |
| \( p - p, n - p \) | Å | \( 1 \times 10^{31} \) | \( \Delta \) |

**TABLE I:** Comparison of torsion gravity constraints.

Finally we note that stellar energy-loss arguments strongly constrain the pseudoscalar coupling of electrons \( \Delta \). In particular revealing that for particles with mass \( m \lesssim 10 \) keV \( \Delta \), \( |\tilde{g}_p|/\frac{\alpha^2}{4\sqrt{\pi\hbar c}} \lesssim 10^{-25} \), far exceeding the laboratory limits discussed here. In terms of torsion grav-
ity, these astrophysical constraints translate to a limit $\beta^2 \lesssim 10^{19}$. These constraints, however, do not apply to the axial-vector interactions [1].

In conclusion, the results of a new direct measurement of the magnetic interaction between two electrons at the micrometer scale were used to place bounds on exotic forces. We have improved constraints on axial-vector mediated spin-spin interactions between electrons by six orders of magnitude at the micrometer scale (equivalently for masses below $\sim 1$ eV). These measurements are consistent with constraints from Positronium spectroscopy for the case of pseudo-scalar mediated interactions and complement previous torsion gravity bounds. Note that as can be seen from Eq. [1] and Eq. [2], extrapolating these bounds to the micrometer scale deteriorates their sensitivity roughly by a factor of $e^{\pi}/x$, where $x$ is the ratio between the measurement scale and the extrapolated scale, i.e. $x \geq 1 \text{nm}/1 \mu m = 1000$. That renders bounds obtained at the millimeter scale useless at the micrometer scale.

We thank Dmitry Budker again for careful reading of our manuscript. This work was supported in part, by the U.S. National Science Foundation under grant PHY-1307507 and by I-Core: the Israeli excellence center “circle of light” and the European Research Council (consolidator Grant Ionology-616919).

[1] B. A. Dobrescu and I. Mocioiu, J. High Energy Phys. 11, 5 (2006).
[2] J. E. Moody and F. Wilczek, Phys. Rev. D 30, 130 (1984).
[3] S. Weinberg, Phys. Rev. Lett. 40, 223 (1978).
[4] F. Wilczek, Phys. Rev. Lett. 40, 279 (1978).
[5] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977).
[6] B. A. Dobrescu, Phys. Rev. Lett. 94, 151802 (2005).
[7] T. Appelquist, B.A. Dobrescu and A.R. Hopper, Phys. Rev. D 68, 035012 (2003).
[8] Gianfranco Bertone, Dan Hooper, and Joseph Silk, Physics Reports 405, 279 (2005).
[9] A. Friedland, H. Murayama, and M. Perelstein, Phys. Rev. D 67, 043519 (2003).
[10] V. Flambaum, S. Lambert and M. Pospelov, Phys. Rev. D 80, 105021 (2009).
[11] P. V. Vorobyov and Ya. I. Gitarts, Phys. Lett. B 208, 146 (1988).
[12] R. C. Ritter, C. E. Goldblum, W.-T. Ni, G. T. Gillies, and C. C. Speake, Phys. Rev. D 42, 977 (1990).
[13] V. F. Bobrakov, Yu. V. Borisov, M. S. Lasakov, A. P. Serebrov, R. R. Tal'daev, and A. S. Trofimova, Pis'ma Zh. Eksp. Teor. Fiz. 53, 283 (1991) [JETP Lett. 53, 294 (1991)].
[14] D. J. Wineland, J. J. Bollinger, D. J. Heinzen, W. M. Itano, and M. G. Raizen, Phys. Rev. Lett. 67, 1735 (1991).
[15] T. C. P. Chui and W. T. Ni, Phys. Rev. Lett. 71, 3247 (1993).
[16] L. Hunter, J. Gordon, S. Peck, D. Ang, and J.-F. Lin, Science 339, 928 (2012).
[17] B. R. Heckel, W. A. Terrano, and E. G. Adelberger, Phys. Rev. Lett. 111, 151802, (2013).
[18] As can be seen from Eq. [1] and Eq. [2],
[42] Y. N. Obukhov, A. J. Silenko, and O. V. Treyaev, Phys. Rev. D 90, 124068 (2014).
[43] R. T. Hammond, Phys. Rev. D 52, 6918 (1995).
[44] E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl, S. Schlamming, Prog. in Part. and Nucl. Phys. 62, 102 (2009).
[45] D. E. Neville, Phys. Rev. D 21, 2075 (1980).
[46] D. E. Neville, Phys. Rev. D 25(2), 573 (1982).
[47] S. M. Carroll and G. B. Field, Phys. Rev. D 50(6), 3867 (1994).
[48] G. Vasilakis, J. M. Brown, T. W. Kornack, and M. V. Romalis, Phys. Rev. Lett. 103, 261801 (2009).
[49] D. F. Jackson Kimball, arXiv:1407.2671 (2014).
[50] Changbo Fu and W. M. Snow, arXiv:1103.0659 (2011).
[51] N. F. Ramsey, Physica (Amsterdam) 96A, 285 (1979).
[52] G. Raffelt, Phys. Rev. D 86, 015001 (2012).
[53] G. Raffelt and A. Weiss, Phys. Rev. D 51, 1495 (1995).