String Inspired Neutrino Mass Textures in Light of KamLAND and WMAP

Claudio Coriano\textsuperscript{1} and Alon E. Faraggi\textsuperscript{2}\
\textsuperscript{1}Dipartimento di Fisica, Universita’ di Lecce, I.N.F.N. Sezione di Lecce, Via Arnesano, 73100 Lecce, Italy\
\textsuperscript{2}Theoretical Physics Department, University of Oxford, Oxford OX1 3NP UK

Abstract

Recent data from astrophysical and terrestrial experiments indicates large mixing angles in the neutral lepton sector and restricts the allowed regions of neutrino masses. In particular, the large mixing angles in the lepton sector are disparate from the small mixing in the quark sector. This disparity is unnatural from the point of view of grand unified theories, that are well motivated by the Standard Model multiplet structure and logarithmic running of its parameters. We examine the issue of this disparity from the perspective of string derived $SO(10)$ GUT models, in which the $SO(10)$ symmetry is broken directly at the string theory level. A characteristic feature of such models is the appearance of numerous $SO(10)$ singlet fields. We propose that the mismatch between the quark and lepton mixing parameters arises due to this extended singlet spectrum and its mixing with the right-handed neutrinos. We discuss a string inspired effective parameterization of the extended neutrino mass matrix and demonstrates that the coupling with the $SO(10)$ singlet spectrum can readily account for the neutrino flavor parameters. The mechanism implies that some $SO(10)$ singlet fields should exist at intermediate mass scales. We study the possibility of deriving the neutrino mass spectrum from string theory in a specific string derived vacuum solution, and comment on the properties that such a solution should possess.

\textsuperscript{*}E-mail address: Claudio.Coriano@le.infn.it
\textsuperscript{†}E-mail address: faraggi@thphys.ox.ac.uk
1 Introduction

The neutrino sector of the Standard Model provides another piece to the flavor enigma. Evidence for neutrino oscillations steadily accumulated over the past few years, resulting in compelling evidence for neutrino masses. This in turn points to the augmentation of the Standard Model by the right–handed neutrinos, and provides further evidence for the elegant embedding of the Standard Model matter states, generation by generation, in the 16 spinorial representation of $SO(10)$. However, in this respect the new neutrino data raises further puzzles. The observation of a zenith angle dependence of $\nu_\mu$ from cosmic ray showers at super-Kamiokande [1] provides strong evidence for oscillations in atmospheric neutrinos with maximal $\nu_\mu \rightarrow \nu_\tau$ oscillations, whereas the observations at the solar Sudbury Neutrino Observatory (SNO) [2] and at the reactor KamLAND experiment [3] favor the large mixing angle MSW solution of the solar neutrino problem [4]. The recent data from the Wilkinson Microwave Anisotropy Probe (WMAP) on cosmic microwave background anisotropies [5], combined with the 2 degree Field Galaxy Redshift Survey, CBI and ACBAR [6], restricts the amount of critical density attributed to relativistic neutrinos, and imposes that the sum of the masses is smaller than 0.75eV.

While the Standard Model data strongly supports the incorporation of the Standard Model gauge and matter spectrum in representations of larger gauge groups, the flavor sector of the Standard Model provides further challenges. In the heavy generation the consistency of the bottom–quark–tau lepton mass ratio with the experimental data arises due to the running of the strong gauge coupling. The remaining flavor data, however, must have its origin in a theory that incorporates gravity into the picture. Most developed in this context are the string theories that provide a viable perturbative framework for quantum gravity. However, a new twist of the puzzle arises due to the fact that while in the quark sector we observe an hierarchical mass pattern with suppressed mixing angles, the observations in the neutrino sector are compatible with large mixing angles that implies approximate mass degeneracy.

An elegant mechanism in the context of $SO(10)$ unification to explain the large mixing in the neutrino sector was proposed in ref. [7]. However, this mechanism utilizes the 126 of $SO(10)$, that does not arise in perturbative string theories [8]. On the other hand, string constructions offer a solution to the proton longevity problem. A doublet–triplet splitting mechanism is induced when the $SO(10)$ symmetry is broken to $SO(6) \times SO(4)$ by Wilson–lines [9]. In the stringy doublet–triplet splitting mechanism the color triplets are projected from the massless spectrum and the doublets remain light. Additional symmetries that arise in the string models may also explain the suppression of proton decay from dimension four and gravity mediated operators. String constructions also explain the existence of three generations in terms of the geometry of the compactified manifold. It is therefore important to seek other explanations for the origin of the discrepancy in the quark and lepton mass sectors. An alternative possibility to the utilization of the 126 in the seesaw mechanism is
to use the nonrenormalizable term $16 \bar{16} 16$. In this case the $B - L$ symmetry is broken along a supersymmetric flat direction by the VEVs of the neutral components of $\langle 16_H \rangle = \langle \bar{16}_H \rangle$, where $16_H$ and $\bar{16}_H$ are two Higgs multiplets, distinct from the three Standard Model generations. This term then induces the heavy Majorana mass term for the right–handed neutrino. The contemporary studies of neutrino masses in this context are based on this term. We will refer to this as the “one–step seesaw mechanism” [10]. Similarly, explorations in the context of type I string inspired models also use the “one–step seesaw mechanism” [11]. In this paper we propose that the neutrino data points to the role of $SO(10)$ singlet fields in the see–saw mass matrix.

2 Summary of neutrino data

In this section we summarize the neutrino data. The KamLAND and SNO data are compatible with the large mixing angle solution to the solar neutrino puzzle, with

$$\Delta m_{12} \approx 7.1 \times 10^{-5} \text{eV}^2; \quad \sin^2 2\theta_{12} \geq 0.86 \quad (2.1)$$

The super-Kamiokande gives

$$\Delta m_{23} \approx 2.7 \times 10^{-3} \text{eV}^2; \quad \sin^2 2\theta_{23} \approx 1.00 \quad (2.2)$$

The $\theta_{13}$ angle is constrained by the CHOOZ experiment [12] with, $\sin \theta_{13} \leq 0.2$.

Assuming that all the light neutrinos are stable the WMAP data yields an upper bound on the sum of neutrino masses

$$\sum_i m_i < 0.71 \text{eV}. \quad (2.3)$$

We assume here a $SO(10)$ symmetry that underlies the Standard Model spectrum and interactions. This underlying symmetry may be broken directly at the string scale and need not be present in the effective low energy field theory. However, it indicates that the lepton mass matrices are related to the quark mass matrices. In particular, the mixing angles in the charged–lepton sector and the Dirac neutrino mass matrix are related to the corresponding angles in the quark sector, which are small. With the assumption we can take the charged–lepton mass matrix to be diagonal, in which case the mixing information is contained entirely in the Majorana neutrino mass matrix.

In the mass basis the neutrino mass matrix, $M_D$, is diagonal, and is related to the neutrino mass matrix in the flavor basis by a unitary transformation

$$M_\nu = U M_D U^T \quad (2.4)$$

The MNS mixing matrix $U$ relates between the flavor and mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (2.5)$$
Assuming no CP violation $U$ can be written as $U = U_{23}U_{13}U_{12}$, where $U_{ij}$ related between the $i, j$ mass eigenstates. The elements of $U_{ij}$ are constrained by the atmospheric, reactor, and solar neutrino data, and one can then obtain a simple form for the mixing matrix $U$. Given the relation (2.4), and the experimental constraints on the neutrino mass differences, the allowed patterns of the Majorana neutrino mass matrices, $M_\nu$, can then be classified [13]. These are the mass matrices that one would like to obtain from string theory.

3 General structure of the string models

In this section we discuss the general structure of string models and the new features that arise from them. A class of semi–realistic string models that preserve the $SO(10)$ embedding of the Standard Model spectrum are the heterotic–string models in the free fermionic formulations [14, 15, 16, 17, 18, 19, 20, 21]. While quasi–realistic string models that contains three generations with the correct charge assignment under the Standard Model gauge group are quite abundant, the free fermionic models preserve the $SO(10)$ embedding. This class of models serves as the prototype laboratory for phenomenological exploration of $SO(10)$ string GUTs.

In this section we summarize the general structure of the realistic free fermionic models, and of their massless spectra. The free fermionic heterotic–string formulation yields a large number of three generation models, which possess an underlying $Z_2 \times Z_2$ orbifold structure, and differ in their detailed phenomenological characteristics. We discuss here the common features of this large class of realistic string models. The discussion is qualitative and details are given in the references cited. In section (5) we analyze one specific string model in more detail.

The free fermionic models are constructed by specifying a set of boundary conditions basis vectors and the one–loop GSO projection coefficients [22]. The basis vectors, $b_k$, span a finite additive group $\Xi = \sum_k n_k b_k$. The physical massless states in the Hilbert space of a given sector $\alpha \in \Xi$, are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections.

The four dimensional gauge group in the three generation free fermionic models arises as follows. The models can in general be regarded as constructed in two stages. The first stage consists of the NAHE set of boundary conditions basis vectors, which is a set of five boundary condition basis vectors, $\{1, S, b_1, b_2, b_3\}$ [23]. The gauge group after imposing the GSO projections induced by the NAHE set basis vectors is $SO(10) \times SO(6)^3 \times E_8$ with $N = 1$ supersymmetry. The sectors $b_1, b_2$ and $b_3$ produce 48 multiplets in the 16 representation of $SO(10)$, that are are singlets of the hidden $E_8$ gauge group, and transform under the horizontal $SO(6)_j$ ($j = 1, 2, 3$) symmetries. The untwisted sector produces states in the 10 vectorial representation of $SO(10)$, that can produce electroweak Higgs doublets, and $SO(10)$ singlets that are charged under the $SO(6)^3$ symmetries. This structure is common to all the realistic free fermionic models. At this stage we anticipate that the $SO(10)$ group gives rise to
the Standard Model group factors, *i.e.* to the $SO(10)$ GUT symmetry. The 161610 $SO(10)$–invariant coupling can then gives rise to the Dirac fermion mass terms.

The second stage of the free fermionic basis construction consists of adding to the NAHE set three (or four) additional boundary condition basis vectors, typically denoted by $\{\alpha, \beta, \gamma\}$. These additional basis vectors reduce the number of generations to three chiral generations, one from each of the sectors $b_1$, $b_2$ and $b_3$, and simultaneously break the four dimensional gauge group. The $SO(10)$ symmetry is broken to one of its subgroups $SU(5) \times U(1)$ (FSU5) [14], $SO(6) \times SO(4)$ (PS) [16], $SU(3) \times SU(2)^2 \times U(1)$ (SLM) [15], $SU(3) \times SU(2) \times U(1)^2$ (LRS) [20], or $SU(4) \times SU(2) \times U(1)$ (SU421) [21]. Similarly, the hidden $E_8$ symmetry is broken to one of its subgroups. The basis vectors $\{\alpha, \beta, \gamma\}$, combined with the NAHE–set basis vectors, give rise to additional massless sectors. In the FSU5 and PS type models two of these sectors produce the GUT Higgs representations that break the GUT symmetry, that typically appear as $16_H \oplus \overline{16}_H$ decomposed under the final $SO(10)$ unbroken subgroup. This states can therefore be used to induce the heavy Majorana mass scale for the right–handed neutrino [24], in a “one–step seesaw” mechanism. Additionally, the models contain massless states from the sectors that break the $SO(10)$ symmetry. These states cannot be embedded in $SO(10)$ representations, and carry fractional charge under either the electroweak hyper charge, or under the $SO(10)$ $U(1)_{Z'}$–subgroup [25]. In the SLM models of ref. [15, 17] the neutral component of $\overline{16}_H$ does not arise in the spectrum. Instead, the models contain Standard Model singlet states that carry $1/2$ of the $U(1)_{Z'}$ charge of the righ–handed neutrino. These states are utilized in the implementation of the seesaw mechanism in the free fermionic standard–like models [26, 27].

Subsequent to constructing the basis vectors and extracting the massless spectrum the analysis of the free fermionic models proceeds by calculating the superpotential. The cubic and higher-order terms in the superpotential are obtained by evaluating the correlators

$$A_N \sim \langle V^f_1 V^f_2 V^h_3 \cdots V_N \rangle, \quad (3.1)$$

where $V^f_i$ ($V^h_i$) are the fermionic (scalar) components of the vertex operators, using the rules given in [28]. Generically, correlators of the form (3.1) are of order $O(g^{N-2})$, and hence of progressively higher orders in the weak-coupling limit. Typically, one of the $U(1)$ factors in the free-fermion models is anomalous, and generates a Fayet–Ilioupoulos term which breaks supersymmetry at the Planck scale [29]. A supersymmetric vacuum is obtained by assigning non–trivial VEVs to a set of Standard Model singlet fields in the massless string spectrum along $F$ and $D$–flat directions. Some of these fields will appear in the nonrenormalizable terms (3.1), leading to effective operators of lower dimension. Their coefficients contain factors of order $V/M \sim 1/10$.

Pursuing this methodology the structure of the fermion mass matrices in the free fermionic models was studied [30, 31], as well as in other string models [32]. The general texture of the quark mass matrices in the superstring standard–like models
is of the following form \[31\],

\[
M_U \sim \begin{pmatrix}
\epsilon, a, b \\
\bar{a}, A, c \\
b, \bar{c}, \lambda_t
\end{pmatrix};
M_D \sim \begin{pmatrix}
\epsilon, d, e \\
\bar{d}, B, f \\
\bar{e}, \bar{f}, C
\end{pmatrix};
\] (3.2)

Due to the underlying \(SO(10)\) symmetry structure we anticipate the relations

\[
M_E \sim M_D; \quad M_N \sim M_U,
\] (3.3)

where \(M_E\) and \(M_N\) are the charged–lepton and Dirac neutrino mass matrices, respectively. In some models the top quark Yukawa coupling \(\lambda_t\) is the only one that arises at the cubic level of the superpotential and is of order one. The remaining quark and lepton Yukawa couplings arise from higher order terms in the superpotential that are suppressed relative to the leading cubic level term.

4 String inspired neutrino mass textures

In this section we discuss the string inspired two step seesaw mechanism, and identify the features that the string models should produce to accommodate this mechanism. The low energy effective field theory of our string inspired model consist of three chiral \(SO(10)\) generations decomposed under the final unbroken \(SO(10)\) subgroup. For concreteness we consider here the case of the standard–like models. In this case the unbroken \(SO(10)\) subgroup is \(SU(3) \times SU(2) \times U(1)_{T_3R} \times U(1)_{B−L}\). In addition to the three chiral generations the matter spectrum contains two electroweak Higgs doublets; the fields \(N\) and \(\bar{N}\) that break the additional \(U(1)\) symmetry to the Standard Model weak hypercharge. Additionally the model contains three \(SO(10)\) singlet fields \(\phi_m\), that obtain an electroweak or intermediate scale VEV. The relevant terms in the superpotential that contribute to the neutrino seesaw mass matrix are given by

\[
W = \cdots + \lambda_i^j N_i L_j \bar{h} + \lambda^{im}_3 N_i \tilde{N} \phi_m + \lambda^{ijk}_6 \phi_i \phi_j \phi_k.
\] (4.1)

The first term produces the neutrino Dirac mass matrix, which by the underlying \(SO(10)\) is proportional to the up–quark mass matrix, \(M_U\). The second produces the \(N\phi\) mass matrix, \(M_\chi\). The terms in this matrix are generated by VEVs that break the \(B−L\) symmetry. However, contrary to the situtation in conventional GUTs, the \(N\phi\) mass terms can vary over several orders of magnitude. The reason being that in the string models they are generically obtained from nonrenormalizable operators that arise at different orders. The third term Eq. (4.1) produces the mass terms for the \(SO(10)\) singlets. Thus, the neutrino mass matrix at the unification scale takes the general form \[33\]

\[
\begin{pmatrix}
0 & M_D & 0 \\
M_D & 0 & M_\chi \\
0 & M_\chi & M_\phi
\end{pmatrix}
\] (4.2)
and is the two–step seesaw mass matrix. The left–handed Majorana neutrino mass matrix is given by

\[ M_\nu = M_D M_\chi^{-1} M_\phi M_\chi^{-1} M_D^T. \]  

Eq. (4.3) has the following implications. First, it is noted that it produces a double suppression with respect to the right–handed neutrinos mass scales, which allows these to be intermediate rather than at the GUT scale. This possibility is advantageous for the generation of the baryon asymmetry by leptogenesis [34]. Second, as discussed above, \( M_D \sim M_U \) and can therefore be taken to be diagonal, \( i.e. \)

\[ M_D \sim \text{Diag}(m_u, m_c, m_t) \]  

For simplicity we also take \( M_\chi \) to be diagonal, although one can consider the possibility that it is not. In this case, it is seen that the flavor structure of the left–handed neutrino mass matrix arises from the flavor structure of the matrix \( \phi \).

This result arises due to the extended singlet spectrum in the string model that is external to the \( SO(10) \) gauge group. To emphasize this point it is instructive to write the see–saw mass matrix in the form

\[ M = \begin{pmatrix} 0 & \mathbf{H} \\ \mathbf{H}^T & \mathbf{J} \end{pmatrix}. \]  

where

\[ \mathbf{H} = \begin{pmatrix} m_u & 0 & 0 & 0 & 0 & 0 \\ 0 & m_c & 0 & 0 & 0 & 0 \\ 0 & 0 & m_t & 0 & 0 & 0 \end{pmatrix} \]  

and

\[ \mathbf{J} = \begin{pmatrix} 0 & 0 & 0 & \chi_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \chi_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \chi_3 \\ \chi_1 & 0 & 0 & \phi_{11} & \phi_{12} & \phi_{13} \\ 0 & \chi_2 & 0 & \phi_{21} & \phi_{22} & \phi_{23} \\ 0 & 0 & \chi_3 & \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix}. \]  

In the limit of small left–handed neutrino masses the left–handed Majorana mass matrix is given by [35]

\[ M_\nu \approx \mathbf{H}^T \mathbf{J}^{-1} \mathbf{H}, \]  

and is identical to Eq. (4.3). The left–handed Majorana mass matrix \( M_\nu \) is then given by

\[ M_\nu = \begin{pmatrix} \frac{m_u m_c}{\chi_1 \chi_1} & \frac{m_u m_t}{\chi_1 \chi_2} & \frac{m_u m_t}{\chi_1 \chi_3} & \phi_{11} \\ \frac{m_u m_c}{\chi_2 \chi_1} & \frac{m_u m_t}{\chi_2 \chi_2} & \frac{m_u m_t}{\chi_2 \chi_3} & \phi_{21} \\ \frac{m_u m_c}{\chi_3 \chi_1} & \frac{m_u m_t}{\chi_3 \chi_2} & \frac{m_u m_t}{\chi_3 \chi_3} & \phi_{31} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix}. \]  

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The seesaw equation, eq. (4.8) on the other hand the same structure as the one–step seesaw mechanism. Namely, it has the form \( m^2/M \). This form demonstrates how the extended singlet spectrum that couples to the right–handed neutrinos results in the two–step seesaw mass matrix. We then note that choosing the appropriate values for the parameters in \( M_\chi \) and \( M_\phi \) produces the desired left–handed neutrino mass texture. For example, taking \( \chi_i = 10^8 \text{GeV} \) and

\[
M_\phi = \begin{pmatrix}
10^{13} & 10^{10} & 10^8 \\
10^{10} & 10^7 & 10^5 \\
10^8 & 10^5 & 10^3
\end{pmatrix} \text{GeV},
\]

produces a democratically left–handed neutrino mass texture, with one eigenvalue of order 1eV. Obviously, the additional freedom, in principle, admits the required neutrino mass textures. Next we turn to examine the neutrino mass textures in a specific \( SO(10) \) string–GUT model.

5 specific string model

As a concrete string model we examine the neutrino masses in the standard–like model of ref. [17]. The full massless spectrum of this model is given in Ref. [17]. A partial set which is relevant for our purposes includes the following states:

(I) There are three chiral families of quarks and leptons, each with sixteen components, including \( \bar{\nu}_R \), which arise from the twisted sectors \( b_1, b_2 \) and \( b_3 \). These transform as 16’s of \( SO(10) \) and are neutral under \( G_H \).

(II) the Neveu–Schwarz (NS) sector produces, in addition to the gravity multiplets, three pairs of electroweak doublets \( \{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\} \), three pairs of \( SO(10) \)–singlets with \( U(1)_i \) charge \( \{\Phi_{12}, \Phi_{23}, \Phi_{13}, \bar{\Phi}_{12}, \bar{\Phi}_{23}, \bar{\Phi}_{13}\} \), and three states that are singlets of the entire four dimensional gauge group, \( \{\xi_1, \xi_2, \xi_3\} \).

(III) the sector \( S + b_1 + b_2 + \alpha + \beta \) produces one additional pair of electroweak doublets \( \{h_{45}, \bar{h}_{45}\} \), one pair of color triplets \( \{D_{45}, \bar{D}_{45}\} \) and seven pairs of \( SO(10) \) singlets with \( U(1)_i \) charge \( \{\Phi_{45}, \Phi_{45}', \Phi_{12,3}', \Phi_{12,3}^\pm\} \).

(IV) The sectors \( b_j + 2\gamma + (I = 1 + b_1 + b_2 + b_3) \) produce hidden–sector multiplets \( \{T_i, \bar{T}_i, V_i, \bar{V}_i\}_{i=1,2,3} \) which are \( SO(10) \) singlets but are non–neutral under \( U(1)_i \) and the hidden \( G_H \). The \( T_i(\bar{T}_i) \) are 5(\( \bar{5} \)) and \( V_i(\bar{V}_i) \) are 3(\( \bar{3} \)) of \( SU(5)_H \) and \( SU(3)_H \) gauge groups, respectively.

(V) The vectors in some combinations of \( (b_1, b_2, b_3, \alpha, \beta) \pm \gamma + (I) \) produce additional states which are either singlets of \( SU(3) \times SU(2) \times U(1)_Y \times SU(5)_H \times SU(3)_H \) or in vector–like representation of this group. The relevant states of this class \( \{H_{17} - H_{26}\} \). The states of class (V) are crucial for the seesaw mechanism in the superstring standard–like models [26].

One characteristic feature of this class of models, is that, barring the three chiral 16’s there are no additional vector–like \( 16 + \bar{16} \) pairs. As a result, elementary fields
with the quantum numbers of $N'_L \in \overline{16}$ do not exist in this class of models. Nevertheless, VEVs of products of certain condensates, which are expected to form through the hidden sector force and certain fields belonging to the set $(V)$ can provide the desired quantum numbers of sneutrino like fields – i.e $\bar{N}_R \in 16$ and $N'_L \in \overline{16}$, as, for example, in the combinations shown below:

$$\langle H_{19} \bar{T}_i \rangle \langle H_{23} \rangle \rightarrow (B - L = -1, T_{3R} = 1/2) \sim N'_L \in \overline{16}$$
$$\langle H_{20} \bar{T}_i \rangle \langle H_{26} \rangle \rightarrow (B - L = +1, T_{3R} = -1/2) \sim \bar{N}_R \in 16$$

(5.1)

Note $H_{19}$ and $T_i$ transform as 5, and $H_{20}$ and $T_i$ transform as 5, of $SU(5)_H$, respectively. Thus, $H_{19}(H_{20})$ can pair up with $\bar{T}_i(T_i)$ to make condensates at the scale $\Lambda_H$, where $SU(5)_H$ force becomes strong. In this model an effective seesaw mechanism [26] is implemented by combining the familiar Dirac masses of the neutrinos which arise through electroweak–symmetry breaking scale, with superheavy mass terms which mix $\bar{\nu}_R$ with the singlet $\phi$ fields in sets (II) and (III) [26].

The set of fields that enter the seesaw mass matrix includes the three left–handed neutrinos, $L_i$; the three right–handed neutrinos, $N_i$; and the set of Standard Model singlets. These include: the $SO(10)$ singlets with $U(1)$ and hidden charges, $\{\Phi_{45}, \Phi_{12}^{\pm}, \Phi_{13}, \Phi_{23}, \Phi_{12}, T_i, V_i\} \oplus h.c$. The set of $SU(3) \times SU(2) \times U(1)_Y$ singlets with $U(1)_{Z'}$ charge, $H_{13-14,17-20,23-26}$. The set of entirely neutral singlets $\xi_{1,2,3}$. The analysis now proceeds by analyzing the cubic level and higher order terms in the superpotential. When shifting the vacuum by the anomalous $U(1)$ cancelling VEVs, two competing processes, that depend on the set of VEVs, take place. Some non-renormalizable operators induce effective renormalizable operators. Obviously, the number of unsuppressed operators increases with the increasing number of fields with non–vanishing VEV. At the same time, the number of Standard Model singlets that receive heavy mass and decouple from the low energy spectrum also increases. In fact, a priori it is not apparent that any of the non–chiral singlets will remain light, in which case the two–step seesaw mechanism could not be phenomenologically viable. This is in a sense reminiscent of the supersymmetry $\mu$–problem. The string models, however, also exhibits cases in which some, typically undesirable states remain necessarily light. For example, in the string model of ref. [36] it is found that the right–handed neutrino are necessarily light in that model due to a local discrete symmetry. Similarly, it is has also been observed that some string models contain exotic fractionally charged states that cannot decouple from the massless spectrum. This suggests that there exist string vacua in which the mass of some of the Standard Model singlets with $U(1)_{Z'}$ charges is protected by a local discrete symmetry. These fields will play the role of the $SO(10)$ singlets in the seesaw mass matrix.

To study these aspects we study in some detail an explicit supersymmetric solution. The cubic level superpotential and the anomalous as well as the anomaly free, $U(1)$ combinations are given in ref. [17]. As an example, we find a solution to the $F$ and $D$ cubic level flatness constraints with the following set of fields

$$\{\bar{V}_2, V_3, H_{18}, H_{23}, H_{25}, \Phi_{45}, \bar{\Phi}_1^-, \Phi_2^+, \bar{\Phi}_3^-, \bar{\Phi}_{23}, \bar{\Phi}_{13}, \xi_1\},$$

(5.2)
having non–zero VEVs and all other fields have vanishing VEV. With this set of fields
the general solution is

$$\langle H_{23} \rangle^2 = |\langle H_{18} \rangle|^2 - \frac{1}{6} |\langle V_3 \rangle|^2 \quad (5.3)$$

$$\langle H_{25} \rangle^2 = |\langle \Phi_{13} \rangle|^2 - \frac{1}{6} |\langle V_3 \rangle|^2 \quad (5.4)$$

$$|\langle \Phi_{45} \rangle|^2 = \frac{3}{16} \frac{g^2}{\pi^2} \frac{1}{2\alpha'} + |\langle H_{18} \rangle|^2 - \frac{1}{10} |\langle V_3 \rangle|^2 \quad (5.5)$$

$$|\langle \Phi_{13} \rangle|^2 = \frac{g^2}{\pi^2} \frac{1}{2\alpha'} - \frac{1}{5} |\langle V_3 \rangle|^2 \quad (5.6)$$

$$|\langle \Phi_2^+ \rangle|^2 = \frac{g^2}{\pi^2} \frac{1}{2\alpha'} - \frac{8}{15} |\langle V_3 \rangle|^2 \quad (5.7)$$

$$|\langle \Phi_3^- \rangle|^2 = \frac{g^2}{\pi^2} \frac{1}{2\alpha'} - \frac{1}{30} |\langle V_3 \rangle|^2 \quad (5.8)$$

$$|\langle V_2 \rangle|^2 = |\langle V_3 \rangle|^2 \quad (5.9)$$

$$\langle \xi_1 \rangle = \frac{-\langle \Phi_{23} \rangle \langle H_{25} \rangle}{\langle H_{23} \rangle} \quad (5.11)$$

In this solution the VEVs of three fields, \( \{ V_3, \Phi_{23}, H_{18} \} \) remain as free parameters,
which are restricted to give a positive definite solution for the set of \( D \)-term equations.

We start with \( \{ V_3, \Phi_{23}, H_{25} \} = 0 \). In this case the set of fields with non–vanishing VEVs at the string scale contains,

\[ \{ H_{18}, H_{23}, \Phi_{45}, \Phi_1^+, \Phi_2^+, \Phi_3^-, \Phi_{13} \} \quad (5.12) \]

the cubic level terms \( h_1 \tilde{h}_3 \Phi_{23} + \tilde{h}_2 H_{15} H_{18} \) give heavy mass to \( \tilde{h}_1 \) and \( \tilde{h}_{45} \). At \( N = 3 \) we therefore have one Dirac neutrino mass term

$$\lambda_t U_1 Q_1 \tilde{h}_1 + \lambda_N N_1 L_1 \tilde{h}_1 \quad (5.13)$$

with \( \lambda_t = \lambda_N \). There are no Dirac mass terms at \( N = 4 \); only a suppressed correction
to (5.13) at \( N = 5 \), and none at \( N = 6 \). At \( N = 7 \) we get

\[ N_3 L_1 \tilde{h}_1 \Phi_{45} \tilde{\Phi}_3 T_1 \tilde{T}_3 \]
\[ N_3 L_2 \tilde{h}_1 \Phi_{45} \tilde{\Phi}_3 T_2 \tilde{T}_3 \]
\[ N_3 L_1 \tilde{h}_1 \Phi_{45} \tilde{\Phi}_3 T_2 \tilde{T}_1 \]
\[ N_1 L_3 \tilde{h}_1 \Phi_{45} \tilde{\Phi}_3 T_1 \tilde{T}_3 \]
\[ N_1 L_3 \tilde{h}_1 \Phi_{45} \tilde{\Phi}_3 T_2 \tilde{T}_3 \]

(5.14)

At \( N = 8 \) there are only corrections to \( N = 3 \). Similarly, all \( N = 9 \) order terms
are suppressed compared to lower order terms by six orders and there are new non–vanishing elements in the neutrino Dirac mass matrix. At \( N = 10 \) we have the new
non–vanishing element

\[ N_3 L_3 \tilde{h}_1 \Phi_{45} \Phi_{45} H_{18} H_{23} \tilde{\Phi}_3 T_3 \tilde{T}_3. \]

With additional VEVs turned on at the string scale, more entries in the neutrino Dirac mass matrix will be non–zero. In general we expect that the models retain some of the underlying \( SO(10) \) symmetry structure, and that the neutrino Dirac mass matrix is related to the up quark mass matrix. Next we analyze the \( N_\phi \) mixing terms. Due to the large set of Standard Model singlets that may a priori couple to the \( N_\phi \) fields, we follow the following strategy. First we make a search up to \( N = 10 \) to determine the set of fields that mix with the right handed neutrinos. We then eliminate those that receive mass from cubic level terms in the particular vacuum solution. We then determine the seesaw terms \( N_\phi \) with the set of remaining massless fields. We assume here that the hidden sector \( SU(5) \) gauge group confines at \( \Lambda \sim 10^{15} \text{GeV} \) and that the 55 combinations form condensates of the hidden \( SU(5) \) gauge group. We also take \( \langle \phi \rangle / M \sim 0.1 \) for the set of fields with non–vanishing VEVs. With these assumptions the \( N_\phi \) mixing terms are:

\[
\begin{align*}
N &= 6 \quad N_2 \Phi_2^+ \Phi_{45} H_{23} H_{19} \tilde{T}_2 & \to \ 10^{10} \text{GeV} (N_2 \phi_2^+; N_2 \Phi_{45}; N_2 H_{23}) \quad (5.15) \\
N &= 7 \quad N_3 H_{25} \Phi_{13} \Phi_{3} H_{19} \tilde{T}_3 & \to \ 10^6 \text{GeV} (N_3 H_{45}) \quad (5.16) \\
N &= 8 \quad N_2 \Phi_{13} \Phi_{3} \Phi_{2}^+ \Phi_{45} H_{23} \tilde{T}_2 H_{19} & \to \ 10^8 \text{GeV} (N_2 \Phi_{13}) \quad (5.17) \\
& \quad N_2 \Phi_2^+ \Phi_2^\prime \Phi_{45} H_{23} \tilde{T}_2 H_{19} & \to \ 10^8 \text{GeV} (N_2 \Phi_2^+) \quad (5.18) \\
& \quad N_2 \Phi_3^+ \Phi_3 \Phi_{2}^+ \Phi_{45} H_{23} \tilde{T}_2 H_{19} & \to \ 10^8 \text{GeV} (N_2 \Phi_2^+) \quad (5.19) \\
& \quad N_2 \Phi_2^- \Phi_1^+ \Phi_{45} H_{23} \tilde{T}_2 H_{19} & \to \ 10^8 \text{GeV} (N_2 \Phi_2^-) \quad (5.20) \\
N &= 10 \quad N_1 H_{25} H_{23} H_{18} \Phi_{13} \Phi_{45} \Phi_{45} \tilde{T}_1 T_2 H_{19} & \to \ 10^6 \text{GeV} (N_1 H_{25}) \quad (5.21)
\end{align*}
\]

Next we analyze the singlet mixing terms \( \phi_i \phi_j \). In the supersymmetric vacuum of eq. (5.12) there are no mixing terms at orders \( N = 4, 5, 6 \). At \( N = 7 \) we get terms of the form \( \phi_i \phi_j (T \tilde{T}) \phi^3 \) with the following terms appearing

\[
( H_{23}(\Phi_{13} + \Phi_{45} + \Phi_1^- + \Phi_2^+ + \Phi_3^+ + \Phi_3^-) + \\
\Phi_{45}(\Phi_{13} + \Phi_{45} + \Phi_1^- + \Phi_2^+ + \Phi_3^+ + \Phi_3^-) + \Phi_2^+ \Phi_2^+)(T \tilde{T}) \phi^3 + \\
(\Phi_1^- + \Phi_2^+ + \Phi_3^-) \Phi_{13} \phi^5)
\]

(5.22)

At \( N = 8 \) we get

\[
\Phi_{45} \Phi_{45} T \tilde{T} \phi^4 + \Phi_{45} H_{23} T \tilde{T} \phi^4
\]

(5.23)

At \( N = 9 \) we get terms that already appear at higher orders suppressed by additional \( \langle \text{VEV} \rangle / M \). We only list here new terms that are unsuppressed as compared to the lower order terms. These are

\[
( \Phi_1^+ \Phi_1^- + \Phi_2^+ \Phi_3^- + \Phi_2^+ \Phi_{13} + \Phi_1^- \Phi_1^- + \Phi_1^- \Phi_3^- + \Phi_1^- \Phi_2^+ + \\
\Phi_2^+ \Phi_3^- + \Phi_3^+ \Phi_2^- + \Phi_3^+ \Phi_2^+ + \Phi_{13} \Phi_{13} )T \tilde{T} \phi^5 + \\
( \Phi_{45} \Phi_{45} + H_{23} \Phi_{45} + H_{23} H_{23} ) \phi^7
\]

(5.24)
All terms that appear at $N = 10$ are suppressed compared to lower order terms and no new terms appear. The resulting neutrino mass matrix then takes the approximate form

$$
\begin{pmatrix}
L_3 & L_2 & L_1 & N_3 & N_2 & N_1 & H_{23} & H_{25} & \Phi_{13} & \Phi_{45} & \Phi^-_1 & \Phi^-_3 & \Phi^+_2 & \Phi^+_2 \\
L_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_2 & 0 & 0 & 0 & r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_1 & 0 & 0 & 0 & r & 0 & v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
N_3 & 0 & r & r & 0 & 0 & 0 & 0 & x & 0 & 0 & 0 & 0 & 0 \\
N_2 & 0 & 0 & 0 & 0 & 0 & 0 & z & 0 & u & z & u & z & u \\
N_1 & r & r & v & 0 & 0 & 0 & 0 & w & 0 & 0 & 0 & 0 & 0 \\
H_{23} & 0 & 0 & 0 & 0 & z & 0 & p & 0 & x & x & x & x \\
H_{25} & 0 & 0 & 0 & x & 0 & w & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\Phi_{13} & 0 & 0 & 0 & 0 & u & 0 & x & 0 & q & x & y & y & 0 \\
\Phi_{45} & 0 & 0 & 0 & 0 & z & 0 & p & 0 & x & p & x & x \\
\Phi^-_1 & 0 & 0 & 0 & 0 & u & 0 & x & 0 & y & x & q & q & q \\
\Phi^-_3 & 0 & 0 & 0 & 0 & u & 0 & x & 0 & y & x & q & q & q \\
\Phi^+_2 & 0 & 0 & 0 & 0 & z & 0 & x & 0 & 0 & x & q & q & 0 \\
\Phi^+_2 & 0 & 0 & 0 & 0 & 0 & 0 & x & 0 & y & x & q & q & x & q \\
\end{pmatrix}
$$

(5.25)

where

$$
r \sim 10^{-6}\text{GeV} \quad v \sim 10^2\text{GeV} \quad w \sim 10^6\text{GeV} \quad q \sim 10^7\text{GeV} \quad u \sim 10^8\text{GeV} \\
x \sim 10^9\text{GeV} \quad z \sim 10^{10}\text{GeV} \quad p \sim 10^{11}\text{GeV} \quad y \sim 10^{13}\text{GeV}
$$

We next proceed to examine the mass spectrum of (5.25). We emphasize, however, that our aim is to study qualitatively the possible role of the $SO(10)$ singlet fields in generating the neutrino flavor parameters, rather than to find a vacuum solution that produces a realistic neutrino spectrum. Indeed, the solution given by eq. (5.2) cannot produce realistic mixing also in the quark sector [31]. Our aim is to demonstrate the complication of trying to extract useful information from the string models in regard to the neutrino spectrum. In this respect we note that the vacuum solution given by (5.12) is a highly simplified solution, which is adequate for our purpose here. In this regard we note from Eq. (5.25) that the main difficulty in implementing the “two-step” seesaw mechanism in the string theory is need to keep at least three of the $SO(10)$ singlet fields light down to relatively low energies, whereas from (5.25) we note that the general expectation is for for the additional singlets to get mass terms which are at least comparable to their mass terms with the right–handed neutrinos. We note, however, from the analysis of the noneremalizable terms that the $SO(10)$ singlet mass terms are spread over several energy scales. Thus, the...
mass eigenstates of (5.25) produce several degenerate states with large mixing. The neutrino spectrum itself is, however, not realistic in this vacuum with the lightest eigenstate being a $L_3$ that does not mix with the other states, and is of order 10eV. We also note that in this vacuum the charm mass terms vanishes. A slightly more realistic spectrum can be obtained by turning $\bar{\phi}_{23} \neq 0$, in which case at $N = 5$ we have the Dirac mass term $N_2 L_2 \bar{h}_{45} \Phi_{45} \bar{\Phi}_{23}$. The mass matrix (5.25) then has the mass eigenvalues \{1.7 \times 10^{13}, 1.7 \times 10^{13}, 2 \times 10^{11}, 1 \times 10^{10}, 9 \times 10^9, 1 \times 10^9, 5 \times 10^6, 101, 101, 17.5, 17.5, 0.02, 2.4^{-8}\} GeV. Thus, the lightest eigenvalue, which is predominantly $L_3$, is of of order 10eV. The next lightest states are two nearly degenerate states of order 17.5 GeV that contain $\sim 70\%$ mixture of $\Phi_1^- and \Phi_3^-$ with order 10% mixing with $L_2$. The remaining spectrum is readily analyzed and contains mixtures of the right-handed neutrinos and the $SO(10)$ singlets. The detailed analysis is not particularly revealing, so we do not elaborate on it here. The main lesson from our analysis is the demonstration that although a simple and elegant reasoning for the neutrino spectrum can be motivated from string theory in the form of (4.9), obtaining it from string models is an entirely different story. The main difficulty from the perspective of the string model construction is to understand how the singlet masses can be protected from being too massive. Furthermore, the analysis is complicated due to the proliferation of $SO(10)$ singlets in the massless string spectrum and the lack of an apparent guiding symmetry. While the vacuum solution, eq. (5.12) that we used for the analysis is somewhat simple, it does illustrate the primary difficulty in incorporating the two-step seesaw in the string models. More complicated solutions will allow more detailed structure for the neutrino Dirac mass matrices, and will allow more $SO(10)$ singlets to effectively mix with the right-handed neutrinos through nonrenormalizable terms. At the same time, however, they will also generate more mass terms the $SO(10)$ singlets and hence the primary difficulty remains. Finally, we also comment that, in general we may also try to implement the “one-step” seesaw in the string models. We also remark that another possibility is to implement the “one-step” seesaw in the string models. In the class of models under consideration this necessitates the utilization of the exotic $H$ fields that carry $-1/2Q(Z')$ with respect to the charge of the right-handed neutrino. The relevant terms are then of the form $NNHHHH\phi^n$. In the specific model under investigation such terms were not found up to $N = 14$ and hence cannot induce the seesaw mechanism. Another important observation is that due the fact the right-handed neutrino mass terms are obtained from nonrenormalizable terms, we cannot generically assume that the seesaw scale $m_\chi$ is of the order of the GUT or string scale, and the right-handed neutrino majorana mass matrix can in general involve several disparate scales.

6 Conclusions

The neutrino mass and mixing spectrum that emerged over the past few years adds to the flavor puzzle. The new twist, however, is the large mixing versus the small mix-
ing that we anticipated from the quark sector and Grand Unification. In this paper we proposed that this disparity between the quark and lepton sector can be readily understood in the context of string theories due to the extensive $SO(10)$ singlet spectrum that exists in these models. This leads to the “two-step” seesaw mechanism that indeed can easily account for the flavor parameters in the neutrino sector. Implementing the seesaw mechanism in string constructions poses a far greater challenge. Short of deriving the seesaw mechanism from string models there are nevertheless many interesting physics issues that the “two-step” seesaw mechanism presents. Primarily, with respect to the possibility that the new sterile states may be sufficiently light, *i.e.* of the order $10^{-100}\text{TeV}$, and produce observable phenomenological or cosmological effects. We will return to these issues in future publications.

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