Cosmic strings with self-interacting hot dark matter

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ABSTRACT
We compute the linear power spectrum of cosmic-string-seeded fluctuations in the context of neutrinos with a strong self-interaction and show that it is very similar to that obtained in the context of ‘normal’ neutrinos. We compare our results with observational data and show that for any value of the cosmological parameters $h$ and $\Omega_0$ the interacting hot dark matter power spectrum requires a scale-dependent biasing parameter.

Key words: cosmic strings – galaxies: clusters: general – dark matter – large-scale structure of Universe

1 INTRODUCTION
Observations on all scales of cosmological interest show that most of the matter in the Universe is in some form of unseen matter. On the other hand, nucleosynthesis constraints seem to imply that most of this dark matter is non-baryonic. Gravitinos, axions and neutrinos, for example, are only a few among the many candidates to constitute the dark matter.

In the standard scenario it is usually assumed that during the important epochs for structure formation, the dark matter particle candidates interact only gravitationally. However, it is possible that light neutrino-like particles could have some other kind of self-interaction during the low-energy epochs when structure formation takes place (see, for instance, Raffelt & Silk 1987; Carlson, Machacek & Hall 1992; Gradwohl & Frieman 1992; Machacek 1994; de Laix, Scherrer & Schaefer 1995).

In recent work Atrio-Barandela & Davidson (1997) have considered a light ($\sim 30$ eV) self-interacting neutrino-like particle and discussed the possibility that the dark matter in the Universe could be composed from this kind of particle. They determined the linear power spectrum of density fluctuations generated by the present time in the context of primordial Gaussian fluctuations and concluded that galaxy-sized density perturbations could survive.

Concerning the estimates for the bounds on the neutrino–neutrino cross-section very little is known. Within the framework of the standard model of particle physics, no experimental bounds have been set, and the best-known lower limit on the mean free path of the neutrinos is based on observations of the supernova SN 1987A. In fact, the neutrinos produced by SN1987A in the Large Magellanic Cloud reached the Earth after crossing the relic neutrinos of the cosmic background. Since the number of detected neutrinos agrees with the theoretical expected number from supernova models, the present mean free path of the neutrinos, $\lambda_0$, is not shorter than the distance to SN 1987A, which is of the order of $2 \times 10^{33}$ cm; for a complete analysis, see Kolb and Turner 1987 (see also Nussinov & Roncadelli 1983). In this article we will assume that the comoving value of $\lambda$ is much smaller than any scale of cosmological interest during the time when most of the structures we observe in the Universe today are generated. In this case one can treat the neutrino component with the hydrodynamic or fluid approximation.

Here we extend the work of Atrio-Barandela & Davidson (1997) to the case of non-Gaussian density perturbations induced by cosmic strings. Cosmic strings, along with other topological defects, may be produced as a result of symmetry-breaking phase transitions in the early Universe (Kibble 1976). Although some topological defects are ruled out by observations, others like cosmic strings could be the seeds which generated the large-scale structures we observe in the Universe today (see for example Silk & Vilenkin 1984; Brandenberger, Kaiser & Turok 1987b; Brandenberger et al. 1987a; Colombi 1993; Avelino 1996; Avelino et al. 1997b). They also have other interesting cosmological consequences (see for example Vilenkin 1985; Vilenkin & Shellard 1994). Among these is a non-Gaussian signature in the microwave background (on small scales), the production of double images and a primordial background of gravitational waves or other radiation.

The outline of this paper is as follows. In Section 2 we start by describing the formalism we employ in order to study the growth of density fluctuations in the context of neutrinos with a strong self-interaction. We introduce compensation in Section 3 and in Section 4 we describe the semi-analytic model we employ in order to compute the power spectrum of cosmic-string-seeded density fluctuations. In Section 5 we describe and discuss the results. We conclude in Section 6.

In this article we employ fundamental units with $\hbar = c = k_B = 1$. We consider a self-interacting neutrino with mass $m_\nu = 93\Omega_\nu h^2$ such that $\Omega_\nu = \Omega_\nu$.  

2 ‘SUBSEQUENT’ PERTURBATIONS
In a flat Friedmann–Robertson–Walker (FRW) universe with no cosmological constant and containing both cold dark matter (CDM)
and radiation fluids, the scalefactor $a$ may be written in terms of the conformal time $\eta$ as

$$a(\eta) = a_0 \exp \left[ 2(\sqrt{2} - 1) \frac{\eta}{\eta_0} + \left( 3 - 2\sqrt{2} \right) \frac{\eta^2}{\eta_0^2} \right],$$

(1)

where $a_0$ and $\eta_0$ are respectively the expansion factor and conformal time at matter–radiation equality. If the dark matter is hot but the transition from the relativistic to the non-relativistic regime occurs deep in the radiation era, equation (1) is still a good approximation.

The evolution of radiation and non-relativistic interacting hot dark matter (IHDM) density fluctuations in the synchronous gauge is given by

$$\ddot{h} + \frac{1}{a} \dot{h} - \frac{3}{2} \left( \frac{\dot{a}}{a} \right)^2 (\Omega_0 \dot{h} + 2\Omega_3 \dot{\delta}_i) - c_s^2 \nabla^2 h = 4\pi(\Theta_{00} + \Theta_{ii}),$$

(2)

$$\ddot{\delta}_i - \frac{1}{3} \alpha \dot{\delta}_i - \frac{4}{3} \dot{\delta}_i = 0,$$

(3)

where $\Theta_{0i}$ is the energy–momentum tensor of the external source, $c_s$ is the adiabatic sound speed of the interacting hot dark matter and $\Omega_0$ and $\Omega_3$ express the densities in IHDM and radiation as fractions of the critical density.

We are implicitly assuming that the strength of the neutrino–neutrino coupling is large enough for it to be a good approximation to treat the neutrino component as a perfect fluid. We also assume the radiation behaves as a fluid; this is certainly valid before recombination for the length-scales of interest, well above the radiation damping scale. Although this is not true after recombination, for $\Omega_0 h^2$ not too small the radiation no longer dominates the dynamics after the last scattering. Consequently, we expect that even then equation (3) remains a valid approximation.

The solution to the system of equations (2), (3) with initial conditions $\dot{h}_0 = \dot{h} = \dot{\delta}_i = \dot{\delta}_i = 0$ can be written in terms of Green functions as

$$\tilde{h}_0(X, \eta) = 4\pi \int_0^{\eta} d\eta' \int d^3X \, \mathcal{G}^{0i}(X; \eta, \eta') \Theta_{0i}(X', \eta'),$$

(4)

$$\mathcal{G}^{0i}(X; \eta, \eta') = \frac{1}{2\pi^2} \int_0^{\infty} \frac{\sin kX}{k} k^{2} dk.$$

(5)

Here, $\Theta_{00} = \Theta_{00} + \Theta_{0i}$ and $X = |x - x'|$. The upper index 'S' indicates that these are the 'subsequent' fluctuations, according to the notation of Veeraraghavan & Stebbins (1990), to be distinguished from the 'initial' fluctuations. In Fourier space, the Green function $\tilde{\mathcal{G}}^{0i}$ obeys the same equations as $\dot{h}_0$:

$$\ddot{\tilde{\mathcal{G}}}^{0i} + \frac{1}{a} \dot{\tilde{\mathcal{G}}}^{0i} - \frac{3}{2} \left( \frac{\dot{a}}{a} \right)^2 (\Omega_0 \tilde{\mathcal{G}}^{0i} + 2\Omega_3 \tilde{\mathcal{G}}^{0i}) + c_s^2 k^2 \tilde{\mathcal{G}}^{0i} = 0,$$

(6)

$$\ddot{\tilde{\mathcal{G}}}^{0i} - \frac{4}{3} \dot{\tilde{\mathcal{G}}}^{0i} + \frac{1}{3} k^2 \tilde{\mathcal{G}}^{0i} = 0.$$

(7)

The initial conditions at $\eta = 0$ are $\dot{\tilde{\mathcal{G}}}^{0i} = 1$, $\ddot{\tilde{\mathcal{G}}}^{0i} = 4\Omega_0 / 3$ and $\ddot{\mathcal{G}}^{0i} = 0$. Hence, given the evolution of the source stress–energy, it is possible to compute the resulting density fluctuations numerically.

We are mostly interested in the interacting IHDM inhomogeneities at late times in the matter era. In the limit $\eta \gg \eta_0$, the Green functions are dominated by the growing mode, $\propto a \eta$. Hence, the function we would like to solve for is

$$T(k; \eta) = \lim_{\eta_0 \to \infty} \frac{a_0}{a} \tilde{h}_0(k, \eta, \eta).$$

(8)

This function will be used later to construct the transfer function for the power spectrum of IHDM density fluctuations induced by cosmic strings.

### 2.1 The IHDM sound speed

The adiabatic sound speed of the self-interacting neutrinos, $c_s^2$, is given by

$$c_s^2 = \frac{\dot{\rho}}{\rho},$$

(9)

where $\rho$ stands for pressure, $\rho$ is the mass density of the neutrinos and the subscript, $S$, on the right-hand side means that the entropy per particle, $S$, is constant.

The neutrinos have a Fermi–Dirac distribution function. Therefore, the neutrino number density $n$, mass density $\rho$ and pressure $p$ are given respectively by

$$n = A \int_m^\infty \frac{dE}{E^2} \left( \frac{E^2 - m^2}{E^2 - m^2 + 1} \right)^{1/2} \exp \left[ \frac{(-E - m)^2}{2E} \right] + 1 dE,$$

(10)

$$\rho = A \int_m^\infty \frac{dE}{E^2} \left( \frac{E^2 - m^2}{E^2 - m^2 + 1} \right)^{1/2} \exp \left[ \frac{(-E - m)^2}{2E} \right] + 1 dE,$$

(11)

$$p = \frac{A}{2} \int_m^\infty \frac{dE}{E^2} \left( \frac{E^2 - m^2}{E^2 - m^2 + 1} \right)^{3/2} \exp \left[ \frac{(-E - m)^2}{2E} \right] + 1 dE,$$

(12)

where $E = (p^2 + m^2)^{1/2}$ is the energy, $p$ is the momentum, $m$ is the mass, $T$ is the temperature, $\mu_\nu = \mu_p(T)$ is the chemical potential, $A = g(2\pi)^3$, and $g$ is the spin degeneracy factor of the neutrinos. Using the constraint

$$S = \frac{1}{T} \left( \frac{p + \rho}{n} - \mu_p \right) = \frac{7\pi^4}{135} (\frac{T}{3})^3,$$

(13)

where $S$ is the entropy per neutrino, we numerically calculated $n, \rho, p$ and $\mu_p$ as functions of $T$. We then obtained the adiabatic sound speed $c_s^2 = (\partial p/\partial \rho)_S$, and fitted the obtained result by

$$c_s(a) = \frac{\sqrt{3}}{3} \left[ 1 + \left( \frac{a}{a_{eq}} \right)^2 \right]^{-1/2},$$

(14)

where $\alpha = 0.19$ is the best-fitting parameter. In Fig. 1 we represent the numerical plots of $T$ and $\mu_p$ (in units of $m$), and $c_s$, as functions of the scalefactor $a$, normalized to unity at the epoch of equality between matter and radiation.

### 3 Compensation

The linear perturbations induced by cosmic strings are the sum of initial and subsequent perturbations:

$$\dot{h}_0(k; \eta) = \dot{h}_0^0(k; \eta) + \dot{h}_0^S(k; \eta) = 4\pi(1 + \epsilon_0) \int_0^\infty d\eta' T(k; \eta') \Theta_{00}(k; \eta').$$

(15)

The transfer function for the subsequent perturbations, those generated actively, was obtained in the previous section. To include compensation for the initial perturbations, $\delta_i$, we make the subtraction (Albrecht & Stebbins 1992a)

$$T_i(k; \eta) = \left[ 1 + (k/k_s)^2 \right]^{-1} T(k; \eta),$$

(16)

where $k_s$ is a long-wavelength cut-off at the compensation scale. This results from the fact that local physical processes cannot produce perturbations on scales much larger than the horizon size (Traschen 1985; Veeraraghavan & Stebbins 1990). Consequently, the power spectrum of density perturbations is bounded by a $k^2$ spectrum on large scales ($k < k_s$), assuming that background
fluctuations are uncorrelated for points which have never been in causal contact. In the I-model of strings, which most resembles the Bennett–Bouchet (Bennett & Bouchet 1990; Bennett 1990; Bouchet 1990) and Allen–Shellard simulations (Allen & Shellard 1990; Shellard & Allen 1990), \( k_c = 2\pi/\eta \).

4 POWER SPECTRUM

The analytic expression of Albrecht & Stebbins (1992a, 1992b) for the power spectrum of density perturbations induced by cosmic strings in a \( \Omega = 1 \) FRW universe with no cosmological constant is

\[
P(k) = 16\pi^2 (1 + z_0^2)^2 (G\mu)^2 \times \int_{\eta_0}^{\infty} \frac{d\eta}{\eta} \mathcal{F}[k\tilde{\xi}(\eta)]T_s(k, \eta)^{1/2},
\]

where the \( \rho + 3p \) part of the string stress–energy tensor is modelled by \( \mathcal{F} \), given by

\[
\mathcal{F}[k\tilde{\xi}(\eta)] = \frac{2}{\pi^2} \beta^2 \frac{1}{\xi^2} \left[ 1 + 2(k\chi(\eta))^2 \right]^{-1}.
\]

In these equations, \( \eta_0 \) is the conformal time today, \( \eta_i \) is the conformal time when the string network was formed, \( \chi \) is the typical curvature scale of the wakes,

\[
\xi = (\rho_s/\mu)^{1/2}, \quad \beta = (\sigma^2)^{1/2},
\]

\[
\Sigma = \frac{\mu_\chi}{\mu} \gamma_b \beta_b + \frac{1}{2\gamma_b \beta_b} \left( \frac{\mu_\chi^2 - \mu^2}{\mu_\chi \mu} \right),
\]

where \( \rho_s \) is the energy density in long strings, \( \mu \) is the string mass per unit length, \( \sigma \) is the microscopic velocity of the string, \( \beta_b \) is the macroscopic bulk velocity of the string, \( \gamma_b = (1 - \beta_b^2)^{1/2} \), and \( \mu_\chi \) is the renormalized mass per unit length. We note that \( \Sigma \beta^2 \sim 1 \) and \( \chi/a \sim 2\xi/a \sim \eta/3 \) both in the matter and radiation eras (Bennett & Bouchet 1990; Allen & Shellard 1990) and consequently the structure function \( \mathcal{F} \) is always well-approximated by

\[
\mathcal{F}[k\eta] = \frac{8}{\pi^2} \left[ 1 + \frac{2}{9} (k\eta)^2 \right]^{-1}.
\]

We substitute this in equation (17) in order to calculate the IHDM power spectrum.

5 RESULTS

In Fig. 2 we compare the power spectrum of the perturbations generated by cosmic strings for both CDM (dot–dashed line), HDM (dotted line) and IHDM (solid line) for \( h = 1 \) and \( \Omega_0 = 1 \). The curves are normalized to the COBE DMR observations (Allen et al. 1997). We can see that the HDM and IHDM power spectra are very similar, the IHDM power spectrum having a slightly larger amplitude on small scales. The absence of oscillations reflects the incoherent nature of string perturbations, which is implicit in the way we add the perturbations generated by the cosmic strings at different times to calculate the power spectrum. Hence, it seems that the IHDM cosmic string scenario suffers from similar problems to the standard HDM scenario for cosmic strings, namely a shortage of power mainly on large scales. We note that the CDM and HDM results we obtained here are consistent with other recent calculations using either simplified cosmic string models (Mähönen 1996; Abel et al. 1997; Avelino, Caldwell & Martins 1997a; Albrecht, Battye & Robinson 1997) or high-resolution string network simulations (Avelino et al. 1997b).

5.1 Open models and \( \Lambda \) models

The generalization of these results for open models of structure formation can be made using the same techniques employed by Avelino et al. (1997a). In Fig. 3 we compare the power spectrum of the perturbations generated by cosmic strings in the context of IHDM with the Peacock & Dodds (1994) linear power spectrum reconstruction inferred from various galaxy surveys.

The IHDM power spectrum is again normalized to the COBE DMR observations (Allen et al. 1997). In an open universe, perturbations are damped on too large scales, which is in conflict with the observational data. In a flat universe with a cosmological constant the power spectrum requires a slightly lower biasing on large scales than in an open universe with the same matter density (Avelino et al. 1997a,b). However, because the shape of the power spectrum of density fluctuations induced by cosmic strings remains the same this model is also in conflict with observations.

6 CONCLUSIONS

In this paper we calculated the power spectrum of density fluctuations induced by cosmic strings in the context of neutrinos with strong self-interactions. We concluded that because gravitational instabilities can only be established on scales larger than the Jeans
In opposition to the case studied by Atrio-Barandela & Davidson (1997), we note the absence of oscillations, which is due to the incoherent nature of the cosmic string source [this was also found by Ferreira (1995) in the context of a purely baryonic universe]. The gravitational force which the strings, long-lived seed perturbations, exert on the interacting hot dark matter drives subhorizon pressure waves in a random way which prevents oscillations in the IHDM power spectrum from emerging. In fact, the results obtained for HDM and IHDM are very similar, the IHDM spectrum having a slightly larger amplitude on small scales. In an open universe or a flat universe with a cosmological constant, perturbations are damped on even larger scales than in a $\Omega = 1, \Lambda = 0$ universe. Hence, we conclude that it seems difficult to reconcile these results with observations unless there is some physical mechanism which is capable of generating a scale-dependent biasing.

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