A Study on Grid-Size and Time-Step Calculation using the Taylor Series in Time Series Water Wave Modeling

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Received: 03 Dec 2020;
Received in revised form: 18 Jan 2021;
Accepted: 09 Feb 2021;
Available online: 26 Feb 2021

Abstract—This paper presents the time-step and grid-size calculation methods for solving time series wave differential equations using the Finite Difference Method.

The methods are formulated using the Taylor series, assuming that at a very small time-step and grid-size, the number of terms in the Taylor series starting with 2nd order with the highest order is much smaller than the order 1.

Keywords—Sinusoidal function, Taylor series

I. INTRODUCTION

The formulation of basic hydrodynamic equations, both the continuity equation and the momentum equation, is developed using the Taylor series to one derivative (Dean (1991)). It is argued that in which the time-step $\delta t$ and grid size $(\delta x, \delta z)$ is very small, calculation on the Taylor series can be done up to one order $O(\delta ^1)$.

The solution of the governing equations is numerically by using the discrete method, both the Finite Difference Method (FDM) and the Finite Element Method (FEM) by dividing the domains into grid-points. The grid-point formation requires a small grid-size under the formulation conditions, assuming the time-step $\delta t$ and grid size $(\delta x, \delta z)$ are very small.

In FDM, the equations of FDM are formulated by truncating the Taylor series, using only the first or second-order (Smith, G.D (1985)). Therefore, the application of the equations is required to use a very small time-step and grid-size where the Taylor series can be used in order 1 and order 2 only.

Thus, it takes a small time-step and grid-size following the formulation of governing equations as well as in the formulation of the FDM equation in which FDM is used in the solving governing-equations.

The accuracy of the Taylor series is determined not only by the length of the terms used but also by the grid-size used, where the smaller the grid size, the better the accuracy. At very small time-steps and grid-sizes, the number of terms of order 2 and greater is much smaller than terms of order 1. This, this condition is the basis for the development of the grid-size calculation method in this study.

The grid-size study is carried out using the sinusoidal function, where the water wave equations are the result of an analytical solution to the Laplace’s equation by using the variable separation method is a sinusoidal function in time and space.

II. THE SETTING OF TIME STEP $\delta t$ AND GRID SIZE $\delta x$

The water wave equations are obtained analytically, by solving Laplace's equation with the variable separation method in the form of multiplication between the sinusoidal function and the hyperbolic function (Dean (1991)). The horizontal axis-$x$ and time-$t$ form a sinusoidal function while the vertical axis-$z$ form a
hyperbolic function. By substituting the water wave equations into the Taylor series, the equations for the time-step \( \delta t \) and grid-size \( \delta x \) and \( \delta z \) were obtained.

The formulation is carried out by two methods. The first method is a separate method where the time-step \( \delta t \) is calculated first and then input \( \delta t \), grid-size \( \delta x \)and \( \delta z \) are calculated. The second method is to substitute the relation between \( \delta x \), \( \delta z \), and the time step \( \delta t \) in the Taylor series to obtain the equation for \( \delta t \). Then, \( \delta x \) and \( \delta z \) are calculated using the relation between \( \delta x \) and \( \delta z \) with the obtained \( \delta t \).

2.1. Separate Calculation

a. \( \delta t \) Calculation

The Taylor series form (Thomas, George B., Finney, Ross L. (1996)) for a function of time-\( t \) is,

\[
\delta f(t + \delta t) = f(t) + \delta t \frac{df}{dt} + \frac{\delta t^2}{2} \frac{d^2f}{dt^2} + \frac{\delta t^3}{6} \frac{d^3f}{dt^3} + \frac{\delta t^4}{24} \frac{d^4f}{dt^4} + \ldots \quad (1)
\]

To enable Taylor series can be used up to one derivative only, then

\[
\delta t in numerator and denominator cancel each other out,
\left| \frac{\frac{\delta^2 f}{\delta t^2} + \frac{\delta^3 f}{\delta t^3} + \frac{\delta^4 f}{\delta t^4}}{2 \frac{\delta^2 f}{\delta t^2} + \frac{\delta^3 f}{\delta t^3} + \frac{\delta^4 f}{\delta t^4}} \right| \leq \varepsilon \quad \ldots \ldots \quad (2)
\]

Used
\[
\delta t = \Delta \cos \sigma t \ldots \ldots \quad (3)
\]

\[
\sigma = \frac{2\pi}{T} \text{Angular frequency, } T \text{: period}
\]

Substitution (3) to (2) where the amplitude \( A \) on the numerator and denominator cancel each other out is done at the characteristic point and is done at the characteristic point, \( \cos \sigma t = \sin \sigma t \), in which the terms \( \cos \sigma t \) and \( \sin \sigma t \) in the numerator and denominator also cancel each other out.

\[
- \delta t \frac{\delta^2 f}{\delta t^2} - \delta t^2 \frac{\delta^3 f}{\delta t^3} - \delta t^3 \frac{\delta^4 f}{\delta t^4} \leq \varepsilon \quad \ldots \ldots \quad (2)
\]

Since the term in the absolute sign \( | \cdot | \) is negative, then the positive value of the equation becomes,

\[
\frac{\delta^2 f}{\delta t^2} - \frac{\delta^3 f}{\delta t^3} - \frac{\delta^4 f}{\delta t^4} \leq \varepsilon
\]

By taking the equal sign,

\[
\frac{\delta t}{T} \sigma - \frac{\delta^2 t}{6} \sigma^2 - \frac{\delta^3 t}{24} \sigma^3 = \varepsilon \quad \ldots \ldots \quad (4)
\]

Equation (4) can be solved by the Newton-Raphson method (Allen, Myron B.; Isaacson, Eli L. (1998)). The results of the calculation of \( \delta t \) using equation (4), for several wave periods, are presented in Table 2.1, with an accuracy level of \( \varepsilon = 0.001 \). Meanwhile, Table 2.2 presents the calculation result of \( \delta t \) for the wave period is 8 sec. with several levels of accuracy.

| \( T \) (sec.) | \( \delta t \) (sec.) | \( \delta t \) \( \frac{\varepsilon}{T} \) |
|---|---|---|
| 6 | 0.00191 | 0.00032 |
| 7 | 0.00223 | 0.00032 |
| 8 | 0.00254 | 0.00032 |
| 9 | 0.00286 | 0.00032 |
| 10 | 0.00318 | 0.00032 |
| 11 | 0.0035 | 0.00032 |
| 12 | 0.00382 | 0.00032 |
| 13 | 0.00414 | 0.00032 |
| 14 | 0.00445 | 0.00032 |
| 15 | 0.00477 | 0.00032 |

Table 2.2: The value of \( \delta t \) at several values of \( T, T = 8 \) sec.

| \( \varepsilon \) | \( \delta t \) (sec.) | \( \delta t \) \( \frac{\varepsilon}{T} \) |
|---|---|---|
| 0.1 | 0.23899 | 0.02987 |
| 0.01 | 0.0253 | 0.00316 |
| 0.001 | 0.00254 | 0.00032 |
| 0.0001 | 0.00025 | 0.00003 |

Table (2.1) shows that \( \frac{\delta t}{T} \) is constant for all periods of \( T \) for a given level of accuracy \( \varepsilon \). Meanwhile, Table (2.2) shows the change in \( \delta t \) along with the change in the level of accuracy \( \varepsilon \).

b. \( \delta x \) Calculation

The calculation of \( \delta x \) on the function \( f(x, t) \) is done using \( \delta t \) as the input obtained by the calculation method as discussed in the previous section. In this section, the Taylor Series is only used up to the 3rd differential or \( O(\delta^3) \), considering that the \( \delta t \) value obtained is quite accurate, and to facilitate writing. Taylor’s series up to the third derivative for a function \( f(x, t) \), is,

\[
f(x + \delta x, t + \delta t) = f(x, t) + s_1 + s_2 + s_3 + \ldots \ldots (5)
\]

Where,

\[
s_1 = \delta t \frac{df}{dt} + \delta x \frac{df}{dx} \ldots \ldots (6)
\]

\[
s_2 = \frac{\delta^2 x}{2} \frac{df}{dx^2} + \delta t \delta x \frac{df}{dx^2} + \frac{\delta^2 x}{2} \frac{df}{dx^2} \ldots \ldots (7)
\]

\[
s_3 = \frac{\delta^3 x}{6} \frac{df}{dx^3} + \frac{\delta^2 t}{2} \delta x \frac{df}{dx^2} \ldots \ldots (7)
\]
\[
\frac{\delta x^2 + \delta t^2}{2} a^2 f - \frac{\delta x^3 a^3 f}{6} + \delta t \frac{\delta^2 f}{\delta t^2} + \delta x \frac{\delta^2 f}{\delta x^2} + \frac{\delta^3 f}{\delta x^3} 
\]
... (8)

To enable the Taylor series used with only one derivative, then
\[
\frac{s_2 + s_3}{s_1} \leq e \quad \ldots \quad (9)
\]

The function below is used, $f(x,t) = A \cos kx \cos \sigma t \ldots \ (10)$

By using the relation (6) and (7),
\[
\frac{s_2}{s_1} = \frac{\delta^2 f}{\delta t^2} + \frac{\delta^2 f}{\delta x^2} \\
\frac{s_1}{s_2} = \frac{\delta^2 f}{\delta t^2} - \frac{\delta^2 f}{\delta x^2} \ldots \ (11)
\]

In the same way, it was obtained
\[
\frac{s_3}{s_2} = \frac{\delta^2 f}{\delta t^2} - \frac{\delta^2 f}{\delta x^2} \ldots \ (12)
\]

Intuitively, it can be estimated that (11) is positive, while (12) is negative. However, considering that (11) is greater than (12), the sum of the two equations will be positive.

Substituting the two equations to (9) and assuming that the term in the absolute sign is positive, the equation is obtained.

\[
-\left( \delta t \sigma e - \frac{\delta t^2}{2} \sigma^2 + \frac{\delta t^3}{6} \sigma^3 \right) - \left( \delta t \sigma + \frac{\delta t^2}{2} \sigma^2 + e \right) k \delta x \\
+ (1 + \delta t \sigma) \frac{k^2}{2} \delta x^2 - \frac{k^3}{6} \delta x^3 = 0
\]
... (13)

\[\delta x\] can be calculated by (13) with \(\delta t\) as the input, where \(\delta t\) is obtained from the previous procedure.

Table (2.3) presents the calculation results of \(\delta x\) for several \(T\) wave periods, at a water depth of 10m. The wavelength is calculated using the dispersion equation from the linear wave theory according to Dean (1991). The level of accuracy used is \(\varepsilon = 0.001\).

| \(T\) (sec.) | \(\delta t\) (sec.) | \(\delta x\) (m) | \(L\) (m) | \(\frac{\delta x}{L}\) |
|----------------|-------------------|----------------|-------------|----------------|
| 6              | 0.00191           | 0.04643        | 48,4062     | 0.00096        |
| 7              | 0.00223           | 0.05738        | 59,8212     | 0.00096        |
| 8              | 0.00254           | 0.068          | 70,8984     | 0.00096        |
| 9              | 0.00286           | 0.07839        | 81,7267     | 0.00096        |
| 10             | 0.00318           | 0.0886         | 92,3739     | 0.00096        |
| 11             | 0.00355           | 0.09868        | 102,887     | 0.00096        |
| 12             | 0.00382           | 0.10867        | 113,299     | 0.00096        |
| 13             | 0.00414           | 0.11858        | 123,633     | 0.00096        |
| 14             | 0.00445           | 0.12843        | 133,905     | 0.00096        |
| 15             | 0.10474           | 0.13824        | 144,128     | 0.00096        |

The calculation results show that the value of \(\delta x\) obtained is quite small compared to the wavelength \(L\), which shows that the value is quite relevant to use. Since the value of \(\frac{\delta x}{t}\) might be too small, the comparison between the wave phase velocity is reviewed, \(C = \frac{L}{T}\), and \(\frac{\delta x}{t}\) (Table (2.4)).

| \(T\) (sec.) | \(\frac{\delta x}{t}\) (m/sec.) | \(C\) (m/sec) | \(\frac{\delta x}{t} / C\) |
|----------------|-------------------|-------------|----------------|
| 6              | 24.3257           | 8.0677      | 3.01519        |
| 7              | 25.7675           | 8.54589     | 3.01519        |
| 8              | 26.7215           | 8.86229     | 3.01519        |
| 9              | 27.3802           | 9.08074     | 3.01519        |
| 10             | 27.8525           | 9.23739     | 3.01519        |
| 11             | 28.2022           | 9.35337     | 3.01519        |
| 12             | 28.4682           | 9.44158     | 3.01519        |
| 13             | 28.6751           | 9.51022     | 3.01519        |
| 14             | 28.8392           | 9.56465     | 3.01519        |
| 15             | 28.9716           | 9.60854     | 3.01519        |

Table (2.4) shows that \(\frac{\delta x}{t}\) is much greater than \(C\) there the ratio \(\frac{\delta x}{t} / C = 3.01519 \times \frac{\delta x}{t} = 3.01519 C \delta t\). This is following with the Courant criteria, \(\delta x = 3.0 C \delta t\) (Courant (2928)).

2.2. \(\delta t\) calculation by substituting \(\delta x\).
In this section $\delta t$ and $\delta x$ are processed using the same procedure as the previous one where $\delta x$ is substituted with $\delta t$, in which

$$\delta x = yC\delta t = \gamma \frac{\delta t}{\delta x}$$  \ldots (14)  

$C = \frac{L}{T}$ is wave celerity  

$L = \frac{2\pi}{k}$ Wavelength  

$y$: Coefficient (when used the results in Table (2.4), $y = 3.01519$)

Furthermore, using the relation (6) and (7), where $\delta x$ is substituted by (14) to obtain:

$$\frac{\delta x}{\delta y} = \left(\frac{1/2 - \gamma + \frac{\gamma^2}{2}}{1 + \gamma}\right) \sigma^2 \delta t^2$$  \ldots (15)  

By using the relations (6) and (8) and by substituting $\delta x$ with (14),

$$\frac{\delta x}{\delta y} = -\left(\frac{1/2 - \gamma + \frac{\gamma^2}{2}}{1 + \gamma}\right) \sigma^2 \delta t^2$$  \ldots (16)  

Substitute (15) and (16) into (9) and taking a value equal to,

$$-\left(\frac{1/6 + \frac{\gamma}{2} + \frac{\gamma^2}{6}}{1 + \gamma}\right) \sigma^2 \delta t^2$$  

$$+ \left(\frac{1/2 - \gamma + \frac{\gamma^2}{2}}{1 + \gamma}\right) \sigma \delta t - \varepsilon = 0$$  \ldots (17)

By inputting $\sigma$, $\varepsilon$, $\delta t$ can be calculated with (14). Meanwhile, $\gamma$ can be obtained using the previous calculation results, $\gamma = 3.01519$.

The calculation of $\delta x$ in the table (2.5) is done in the water depth $h = 10$ m, while the wave number $k$ is calculated using the dispersion equation of the linear wave theory (Dean (1991)). It shows that $\delta t$ and $\delta x$ in Table (2.5) are more or less the same as the calculation results in Table (2.3). However, the calculation of $\delta t$ and (17) was preferable considering that there is a clear interaction between $\delta t$ and $\delta x$.

Table 2.5: The results of the calculation of $\delta t$ and $\delta x$ with (17), for $\gamma = 3.01519$.

| $T$ (sec) | $\delta t$ (sec) | $\delta x$ (m) | $\delta x/L$ |
|----------|-----------------|----------------|-------------|
| 6        | 0.00191         | 0.04619        | 0.00095     |
| 7        | 0.00223         | 0.05709        | 0.00095     |
| 8        | 0.00255         | 0.06766        | 0.00095     |
| 9        | 0.00286         | 0.07799        | 0.00095     |

2.3. $\delta z$ calculation in the function $f(x, z, t)$

This section formulates the calculation method $\delta z$ for a function $f(x, z, t)$. The calculation method is as discussed in the previous section, where $\delta t$ and $\delta x$ are the inputs obtained from the previous procedure on a sinusoidal function $f(x, t)$.

Taylor series for a function $f(x, z, t)$ is,

$$f(x + \delta x, z + \delta z, t + \delta t) = f(x, z, t) + s_1 + s_2 + s_3$$  \ldots (19)

$$s_1 = \delta t \frac{\partial f}{\partial t} + \delta x \frac{\partial f}{\partial x} + \delta z \frac{\partial f}{\partial z}$$  \ldots (20)

$$s_2 = \delta t^2 \frac{\partial^2 f}{\partial t^2} + \delta t \delta x \frac{\partial^2 f}{\partial t \partial x} + \delta x^2 \frac{\partial^2 f}{\partial x^2}$$

$$+ \delta t \delta z \frac{\partial^2 f}{\partial t \partial z} + \delta z \delta x \frac{\partial^2 f}{\partial x \partial z} + \delta z^2 \frac{\partial^2 f}{\partial z^2}$$  \ldots (21)

$$s_3 = \delta t^3 \frac{\partial^3 f}{\partial t^3} + \delta t^2 \delta x \frac{\partial^3 f}{\partial t^2 \partial x}$$

$$+ \delta t \delta z \delta x \frac{\partial^3 f}{\partial t \partial z \partial x} + \delta z^2 \frac{\partial^3 f}{\partial z^2 \partial x}$$

$$+ \delta t \delta z \delta x \frac{\partial^3 f}{\partial t \partial z \partial x} + \delta z^3 \frac{\partial^3 f}{\partial z^3}$$  \ldots (22)

$\delta z$ will be calculated using equation (9).

As a function $f(x, z, t)$, the following form of function is used

$$f(x, z, t) = \cosh(k(x + \delta x)) \cos(kx) \cos(t + \delta t)$$  \ldots (23)

At the characteristic point, the sinusoidal terms in the numerator and denominator might cancel each other out. The equation is done $at = \eta$, where $\eta$ is water surface elevation and is done in deep water where $\tan\eta(h + \eta) = 1 \cos\eta = \cosh(h + \eta)$. Thus, the hyperbolic function of the numerator and the denominators might eliminate each other. By using the wave-number...
conservation law (Hutahaean (2020)), the relation \( \tan(k(h + \eta)) = 1 \) remains valid in shallow waters.

After removing the sinusoidal and hyperbolic elements, the equations of \( s_1 \), \( s_2 \) and \( s_3 \) are,

\[
s_1 = -\delta \sigma - \delta \eta \delta x - \delta \eta \delta \xi \ldots (24)
\]
\[
s_2 = -\frac{\delta t^2}{2} \sigma^2 + \delta \sigma \delta x \delta k - \delta \eta \delta x \delta k
\]
\[
-\delta \sigma \delta z \delta k - \delta \eta \delta x \delta z \delta k^2 + \frac{\delta \sigma^2 k^2}{2} + \frac{\delta \eta \delta x \delta z \delta k^2}{2} + \frac{\delta \sigma^3 k^3}{6} + \frac{\delta \eta \delta x \delta z \delta k^3}{6}
\]
\ldots (25)

\[
s_3 = \frac{\delta t^3}{6} \sigma^3 + \frac{\delta t^2}{2} \delta x \sigma^2 k - \frac{\delta t^2}{2} \delta \sigma \delta x k
\]
\[
+\delta \delta x \sigma k^2 - \delta \delta \eta \delta x \delta k^2
\]
\[
-\delta \delta z \delta x \delta z \delta k^2 + \frac{\delta \delta x^2 k^2}{2} + \frac{\delta \delta \eta \delta z \delta x \delta z k^2}{2} + \frac{\delta \delta x^3 k^3}{6} + \frac{\delta \delta \eta \delta z \delta x \delta z k^3}{6}
\]
\ldots (26)

Substituting \( s_1, s_2, \) and \( s_3 \) to (9) assuming that \( \frac{s_2 + s_3}{s_1} \) is positive and using the equal sign, the equation for \( \delta z \) is in the form of a 3-degree polynomial.

The results of the calculation of \( \delta z \) where \( \delta t \) and \( \delta x \) are calculated using the procedure in sub-chapter (2.2) are presented in Table (2.6), where the calculation at a water depth of 10m using the accuracy level \( \delta z = 0.00012 \)is constat for all wave periods. Thus, it can be assumed that:

\[
\delta z = \frac{\gamma^2}{2} \delta t = \gamma^2 \delta t \ldots (27)
\]

Where \( \gamma = 3 \) can be used.

Hence, a calculation method might be formulated where \( \delta x \) and \( \delta z \) are substituted by \( \delta t \) using the relations in (14) and (27) to (24), (25), and (26).

\[
s_1 = (-1 + \gamma^2) \sigma \delta t \ldots (28)
\]
\[
s_2 = \left( -\frac{1}{2} + \gamma - \gamma^2 - \frac{1}{2} y^3 - \frac{1}{2} y^4 \right) \frac{\sigma^2 \delta t^2}{} \ldots (29)
\]
\[
s_3 = \left( \frac{1}{6} + \frac{1}{2} \gamma + \frac{1}{6} y^3 - \frac{1}{2} y^4 + \frac{1}{6} y^6 \right) \frac{\sigma^3 \delta t^3}{} \ldots (30)
\]

Substituting (28), (29), and (30) to (9) and assuming that the term in the absolute sign is positive, the quadratic equation of \( \delta t \) is obtained as follows,

\[
a \delta t^2 + b \delta t + c = 0 \ldots (31)
\]
\[
a = \left( \frac{1}{6} + \frac{1}{2} \gamma + \frac{1}{6} y^3 - \frac{1}{2} y^4 + \frac{1}{6} y^6 \right) \frac{\sigma^2}{} \ldots (32)
\]
\[
b = \left( -\frac{1}{2} + \gamma - \gamma^2 - \frac{1}{2} y^3 - \frac{1}{2} y^4 \right) \frac{\sigma}{} \ldots (33)
\]
\[
c = -(-1 + \gamma + \gamma^2) \frac{\sigma}{\ldots (34)}
\]

The results of the calculation of \( \delta t \) with (31) for several wave periods, at a water depth of 10m with an accuracy level of \( \delta t = 0.0012 \), while \( \delta z = 0.00012 \)are presented in Table (2.7).

| \( T \) (sec) | \( \delta t \) (sec) | \( \delta x \) (m) | \( \delta z \) (m) | \( \frac{\delta z}{\delta x} \) |
|-----|-----|-----|-----|-----|
| 6   | 0.00191 | 0.04619 | 0.13859 | 3.00012 |
| 7   | 0.00223 | 0.05709 | 0.17127 | 3.00012 |
| 8   | 0.00255 | 0.06766 | 0.20298 | 3.00012 |
| 9   | 0.00286 | 0.07799 | 0.23398 | 3.00012 |
| 10  | 0.00318 | 0.08815 | 0.26447 | 3.00012 |
| 11  | 0.00352 | 0.09819 | 0.29457 | 3.00012 |
| 12  | 0.00382 | 0.10812 | 0.32438 | 3.00012 |
| 13  | 0.00414 | 0.11798 | 0.35396 | 3.00012 |
| 14  | 0.00445 | 0.12779 | 0.38337 | 3.00012 |
| 15  | 0.00477 | 0.13754 | 0.41264 | 3.00012 |

There is a relatively small difference between the results in Table (2.7) and the results in Table (2.6). It can be said that with (31) simultaneous calculations on\( \delta t, \delta x \) and \( \delta z \),

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the results of calculation with (31), in Table (2.7) will be more reliable than the results of calculations in (2.6).

It is important to note that \( \frac{\delta x}{\delta t} \) and \( \frac{\delta z}{\delta t} \) correlate with the velocity of the C wave phase, in equations (14) and (27), in interpreting these two parameters.

III. AIRY’S LONG WAVE EQUATION COMPLETION

In this section, Airy’s Long Wave Equation or also known as the Shallow Water Equation is resolved. The equation is actually for small amplitude and longwave. To be used for short and high amplitude waves, a modification of the equation might be used. The formulation is not presented in this study considering that this study does not aim to develop a water wave model, but only to test the reliability of the time-step and grid-size produced in the previous sections.

Governing equations consists of two equations, namely

a. Water surface equation
\[
\frac{\partial \eta}{\partial t} = - \frac{1}{0.5\pi} \frac{\partial}{\partial x} \left( 0.025U \frac{\partial \eta}{\partial x} \right) \quad (35)
\]

b. Momentum equation
\[
\frac{\partial U}{\partial t} = - \frac{1}{9} \left( \frac{\partial}{\partial x} \left( U \frac{\partial U}{\partial x} + g \frac{\partial \eta}{\partial x} \right) \right) \quad (36)
\]

\( \eta \): water surface elevation

\( U \): depth average particle velocity

\( h \): still water depth

\( g \): gravitation acceleration

Both equations are solved by modifying MacCormack’s predictor-corrector method (Anderson, J.D. Jr.(1994)). The original predictor-connector method is a combination of the forward difference with the backward difference in solving the time differential (Anderson, J.D. Jr.(1994)). In this study, a combination of central and backward differences was used.

The simulation results for waves with a period of 9 sec., at constant still water depth \( h = 10 \) m, wave amplitude 0.8 m on execution for 10 times the wave period or 80 sec., are presented in Fig. (2). Time-step\( t \) and grid-size \( \delta x \) are generated using the accuracy level of \( \varepsilon = 0.001 \).

Fig. (2) shows that the wave curve is stable at execution for 80 sec. Thus, the wave crest elevation \( \eta_{\text{max}} \) was stable. This shows the stability of the model, the numerical method used, as well as the good time-step and grid-size. In case the time-step and grid-size are not correct, for example, the grid-size is too large, and then a reduction in waveamplitude or wave height might occur.

Given the wavelength of the model is shorter than the wavelength of the linear wave theory, the calculation of the grid-size \( \delta x \) used the following dispersion equation to obtain the appropriate grid-size:
\[
\sigma^2 = gk \tanh(kh) \quad (37)
\]

However, this study does not propose a new dispersion equation, but only the calculation of the grid-size.

Furthermore, the model is worked on the sloping bottom where the initial still water depth is 10m, while the final still water depth is 1m. The bottom slope is \( \frac{9}{300} \) with a channel length of 300m. The wave period is 8 sec, with a wave amplitude of 0.8 m. The model results are presented in Fig (3).
Fig (3) visualizes the occurrence of shoaling. Meanwhile, in the very shallow waters, there is a large increase in wave crest which possibly breaking the model. The concern, in this case, is the stability of the model in unstable wave conditions, at the time of breaking, which also shows that the time-step and grid-size used are quite good.

IV. CONCLUSION

By using the right time-step and grid-size, the Taylor series can be cut using only one derivative (order 1). Thus, the equations in FDM are exact. Moreover, the use of the right time-size and grid-size, the solution of the governing equations of the hydrodynamic equations is under the conditions of the formulation, where generally the governing-equations are formulated by cutting the Taylor series in order 1 only.

It is important to note that the results of this study are that the grid-size on the Taylor series in a sinusoidal function is correlated with the phase velocity instead of the particle velocity or the current velocity. This should be considered in formulating the governing equations of the wave model, especially in the formulation of the momentum equation.

REFERENCES

[1] Dean, R.G., Dalrymple, R.A. (1991). Water wave mechanics for engineers and scientists. Advance Series on Ocean Engineering.2, Singapore: World Scientific. ISBN 978-981-02-0420-4. OCLC 22907242.

[2] Thomas, George B., Jr., Finney, Ross L. (1996), Calculus and Analytic Geometry (9th ed.), Addison Wesley, ISBN 0-201-53174-7.

[3] Courant, R., Friedrichs, K., Lewy, H. (1928), Uber die partiellen Differentialgleichungen der mathemtischen Physik. Mathematische Annalen (in German). 100 (1): 32-74, Bibcode:1928. MathAn 100.32.C, doi:10.1007/BF01448839. JFM 54.0486.01 MR 1512478.

[4] Hutahaean, S/ (2020). Study on The Breaker Height of Water Wave Equation Formulated Using Weighted Total Acceleration Equation. Jurnal Teknik Sipil, Vol. 27 No.1, April 2020, ISSN 0853-2982, eISSN 2549-2659

[5] Anderson, J.D. Jr (1994). Computational Fluid Dynamics: The Basic With Applications, McGraw Hill.