Wing Rib Stress Analysis and Design Optimization Using Constrained Natural Element Method

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Abstract. This paper demonstrates the applicability of a novel meshless method in solving problems related to aeronautical engineering, the constraint-natural element method is used to optimize a wing rib where it presents several shape of cut-outs deals with the results findings we select the optimum design, we focus on the description and analysis of the constraint-natural element method and its application for simulating mechanical problems, the constraint natural element method is the alternative method for the finite element method where the shape functions is constructed on an extension of Voronoi diagram dual of Delaunay tessellation for non-convex domains.

1. Introduction
The wing ribs are structural elements of the airplane; with the longeron they form the skeleton for the wing skin where their functions are to maintain its shape. The ribs do not participate in the strength of the aircraft as a whole, but transmit the forces encountered by the skin to the longeron. They are subjected to loading systems. External loads applied in the plane of the rib produce a change in shear force in the wing across the rib [1]. The geometry may also include holes to reduce the weight of the wing without altering its strength. The ribs are cut to allow to fuels or equipment to pass through it [2], [3]. In this paper the wing rib of a light aircraft are analyzed with different cut-outs design.

The design of a new wing rib requires stress analysis. In engineering, stress analysis is a tool rather than a goal; the aim is to determine the stress and to predict the failure in materials subjected to forces or increase the strength of the structure without increasing the weight. Stress analysis can be performed using conventional and analytical mathematical techniques, an experimental testing or computational simulation.

Widely used in computational mechanics the finite element method is considered as the robust method to analysis mechanical phenomena [4] but this method failed to analyze some case such as large transformations problems, a localization of initiation and propagation of a crack, displacement of interfaces between solid and fluid. In the last few years there has been an increasing interest in a novel numerical method alternative to the finite element method, meshless methods have been developed with the aim to solve problems associated mesh steps linking to the finite element method. Meshless method construct the interpolating around a set of point that means no connection between nodes and no contract of shape of the element, the refining of mesh or in this case a set of point is automatically by admitting new points; however those methods suffer from a major problem where the imposition of boundary conditions is very complex and the support of the shape function must be contained a sufficient number of neighboring nodes.
A new meshless method is developed to solve this problem by using Galerkin type discretization and diagram of Voronoi. The Natural element method merge the advantage of the classic method (finite element method) and the meshless method using direct application of the boundary conditions of the Dirichlet type. It is enough to impose the values corresponding to the nodes of the edge in aim to obtain the desired solution on the edge whole [5].

On the other hand, on a non-convex edge, this property is no longer verified. In the case of points on either side of a fort non-convex domain may be natural neighbors, and therefore have an undesirable mutual influence. To solve the problem of non-convex edges, the simplest solution is to use the constrained Delaunay triangulation associated with a visibility criterion. The visibility criterion makes it possible to avoid points situated on either side of a non-convex domain each other [6], [7]. The NEM (Natural Element Method) then becomes C-NEM (Constraint-Natural Element Method).

2. Construction of function shape natural element

Introduced by Sibson, the notion of natural neighbours and natural neighbours interpolation is based on a geometric constructions known as the Voronoi diagram. The Voronoi diagram is the topological dual of the Delaunay tessellation [8]. Let’s have a cloud of separated irregular nodes \( N = \{ n_1, n_2, ..., n_N \} \) on the region \( R^2 \). The first-order Voronoi diagram divides the plane into a set of cell \( T_i \), one for each node \( n_i \), see figure 1 (a). Then the Voronoi diagram is defined by:

\[
T_i = \{ x \in R^2 : d_i < d_j \text{ and } j \neq i \}
\]

Figure 1. (a) Voronoi diagram (b) Delaunay triangulation

Where \( x \) is the coordinate of a node \( n \) and \( d_i \), \( d_j \) are the distance between \( n \) and \( n_i, n_j \) respectively; next the Delaunay tessellations are obtained by linking the nodes whose Voronoi cells have a common boundaries see figure 1 (b).

On the way to calculate the value of the Sibson interpolation, let’s added a point \( x \) to pervious region see figure 2, by extension it is possible to define a higher order Voronoi diagram (To the order \( k, k>1 \)) [10]. The second-order Voronoi cells \( T_{ij} \) is defined as:

\[
T_{ij} = \{ x \in R^2 : d_i < d_j < d_k \text{ and } k \neq i, j \}
\]

Where \( n_i \) and \( n_j \) are the first and the second closest nodes to the point \( x \).
Figure 2. First and second order Voronoi cells about x

Considered κ a Lebesgue measure (area 2D) the Sibson interpolation is given by:

\[
\Phi_i(x) = \frac{\kappa_{x_i}}{\kappa_x}
\]  
(3)

And \( \kappa_x = \sum_{i=1}^{n} \kappa_{x_i} \)  
(4)

Where \( \kappa_{x_i} \) is the cell of the second voronoi diagram of a points \( x \).

\[
\Phi_i(x) = \frac{\text{Area (cdef)}}{\text{Area (abcd)}}
\]  
(5)

Braun and Sambridge use this interpolation in a Galerkin type discretization to solve partial differential equation and give it the name of Natural element method [5].

Consider a vector-valued function: \( u(x) \subset R^2 \rightarrow R^2 \) the interpolation scheme is in the form:

\[
u(x) = \sum_{i=1}^{n} \Phi_i(x)u_i
\]  
(6)

Where \( u_i (i = 1, 2, \ldots, n) \) are the vectors of nodal displacements at the \( n \) natural neighbours, and \( \Phi_i(x) \) are the shape functions associated with each node.

3. Extension of natural element method for non-convex domain

In the case of a fort non-convex domain several points in the either sides may be neighbours and therefore have an undesirable mutual influence see figure 3 (a). Yvonnet and al [6] via a constrained Voronoi diagram a dual of constrained Delaunay triangulation results from the application of a visibility criterion define the interpolation functions for a non-convex domain and keeping the main properties of the NEM interpolation. A point \( x \) is visible from a node \( n_i \) inside the domain if a segment drawn between \( n_i \) and \( x \) does not cross the boundary of the domain \( \Gamma \) see figure 3 (b).

Figure 3. (a) Voronoi diagram and Delaunay triangulation for a non-convex domain.
(b) constrained-Voronoi diagram and Delaunay triangulation for a non-convex domain.

If the visibility criterion is introduced in the NEM, the natural neighbours become constrained natural neighbors. Thus, the new functional approximation can be written as:

\[
u(x) = \sum_{i=1}^{v} \Phi_i^C(x)u_i
\]  
(7)

Where \( v \) is the number of natural neighbours visible from point \( x \) and \( \Phi_i^C \) is the constrained natural neighbor shape function [6].

4. Case study

For this task, we use the constrained natural element method to optimize a wing rib; this wing rib is attached to the front spar, rear spars and wing skins see figure 4 (a). In this particular case the weight of the rib is imposed, we want to increase the strength of rib without increasing its weight.
The rib is made from Aluminium 7075-T73 a very high strength material used for highly stressed structural parts, the material properties are shown in the table 1.

| Properties           | Values | Properties             | Values    |
|----------------------|--------|------------------------|-----------|
| Density              | 2810 kg m⁻³ | Shear Modulus         | 26.9 GPa  |
| Modulus of Elasticity| 72 GPa | Tensile Strength, Yield| 435 MPa   |
| Poisson’s Ratio      | 0.33   | Shear Strength         | 300 MPa   |

Figure 4. Wing ribs synoptic scheme. (a) Wing box. (b) Wing rib design variables

To determine the optimal configuration, The geometry will be simplified by ignoring the curvature of the rib edges and we assume that the front spar loads the rib in shear and the wing rib is built in the rear spar, the shear forces q=10 KN see figure 4 (b). We will analyze three different designs where the shapes of the holes are modeled in several form as shown in the figure 5.

Figure 5. Modeling of rib structure. (a) With circular holes. (b) With isosceles trapezoid holes. (c) With triangular holes.

The first design include three circular holes with a varied diameter where R1=127mm, R2=101.6mm and R3= 76.2mm see figure 4 (a). The second design consist two isosceles trapezoid holes where the first isosceles trapezoid had for bases B1=215mm, B1’=183.6mm and the legs Lg1=286.6mm, The second isosceles trapezoid had for bases B2=165mm, B2’=140.9mm and the legs Lg2=192.4mm see figure 4 (b).The last design represent four triangular holes with varying lengths and bases where two are isosceles triangle and have for bases Bt1=274mm, Bt2=203.2 mm and lengths Lt1=236.7mm, Lt2=249.4mm, the last two triangle had the same shape where the base Bt3=413mm and the lengths Lt3=88.5mm see figure 4 (c).

5. Governing equation

Let’s consider a linear elastic body, homogeneous and isotropic, which undergoes small deformations and small displacements. It occupies a domain $V$ in which we denote the surface boundary by $S$ and subjected to the body forces $b_i$ in $V$, to the surface traction $t_i$ on $S_i$ and displacement $u_i$ imposed on $S_u$. Where $S_u \cup S_b = S$ (see figure 6).
The equation of equilibrium described by the linear elastic body is given in indicial form as follow:

\[ \sigma_{ij,j} + b_i = 0 \]  

(8)

The constitutive relation is given according to the stress – strain relation (Hooke’s law) in matrix form as:

\[ \{\sigma\} = [C]\{\varepsilon\} \]  

(9)

And \([C]\) is the material property matrix.

The principle of virtual displacements (PVD) presents an indirect way to express the equilibrium of a body. This PVD states that the total internal virtual work is equal to the total external virtual work.

\[ \int_v \{\sigma\}^T \{\varepsilon\} dv = \int_v \{P_v\}^T \{\ddu\} dv + \int_{S_f} \{P_s\}^T \{\ddu\} ds \]  

(10)

According to this, substituting the approximation functions by the C-NEM scheme (7) in the preceding equation (10), we obtain a system of linear equation, the governing equations is given in this form:

\[ [K]\{\delta\} = \{F\} \]  

(11)

\(\{\delta\}\) is the vector containing the nodal displacements, with the stiffness matrix \([K]\) is given by:

\[ [K] = \int_v [B]^T [C][B] dv \]  

(12)

And

\[ \{F\} = \int_v [N]^T \{b\} dv + \int_{S_f} [N]^T \{\ddv\} ds \]  

(13)

\([N]\) is the matrix containing the shape functions

\[ [N] = \begin{bmatrix} \phi_1 & 0 & \phi_2 & 0 & \cdots & \phi_N & 0 \\ 0 & \phi_1 & 0 & \phi_2 & \cdots & 0 & \phi_N \end{bmatrix} \]  

(14)

And \([B]\) is the matrix containing the derivatives of shape functions C-NEM.

\[ [B] = \begin{bmatrix} \phi_{1,x} & 0 & \phi_{2,x} & 0 & \cdots & \phi_{N,x} & 0 \\ 0 & \phi_{1,x} & 0 & \phi_{2,x} & \cdots & 0 & \phi_{N,x} \\ \phi_{1,y} & \phi_{1,x} & \phi_{2,y} & \phi_{1,x} & \cdots & \phi_{N,x} & \phi_{N,x} \end{bmatrix} \]  

(15)

6. Numerical simulation and discussion

The objective is to determine the stress field in the structure. Concluding the maximum stress and maximum vertical displacement of the rib where the maximum stress must be below the yield stress of the rib material and the maximum vertical movement of the rib should not exceed 2.5mm (design criterias). This is a selective study of the best performing configuration.
Figure 7. Von-mises stress distribution in the wing rib (a) with three circular holes (b) with two trapezoid holes (c) with four triangular holes.

Figure 7 represents the simulation results of the Von-mises stress distribution for the three proposed designs. From this figure, we can notice that the stressed area is located in the upper rear spar. By applying a shear load, for the given design varieties, the Von-mises stress around the cut-outs does not represent a significant intensity. Changing the shape of the holes, the stressed zone still located on the rear spar, the form of the cut-out does not influence the distribution of stress whether it is circular, triangular or trapezoid.

Figure 8. Displacement field of the wing rib structure (a) with three circular holes (b) with two trapezoid holes (c) with four triangular holes.

Figure 8 shows the displacement field of the wing rib structure under shear forces. We notice no considerable difference between the three designs except for the trapezoid form. It is explained by the reason that the frontal opening is large and endures such displacement. The overall simulation results are shown in the table 2.

Figure 9. XX stress distribution along the rear spar width

Table 2. Stress and displacement values.

| Configuration  | Von-Mises stress (Mpa) | Max stress (Mpa) | Max displacement (mm) |
|----------------|------------------------|-----------------|-----------------------|
| Configuration A | 1.6113                 | 1.4654          | 2.1996                |
| Configuration B | 1.3812                 | 1.3474          | 2.3888                |
As concluded from the previous section, the form of the holes does not affect the destabilization process around the circumference of the cut-out itself though; the initiation of cracks is located in the upper rear spar area. The form of the hole take part to sustain the load and diminish the initiation of cracks as it is the case of trapezoid form, the stability performance are given at maximum comparing to a rib without opening see figure 9.

7. Conclusion
In this paper we used the constrained natural element method to optimize the cut-out in the wing ribs of a light-aircraft by adopting three different configurations. This novel work shows that changing the form of the cut-out will increase the strength of the wing rib. For this particular case, the configuration with trapezoid holes leads to optimal stability performance to sustain shear load additionally, it gives a big opening, which allow passing and maintaining the wing interior equipments. The constrained natural element method was applied successfully to solve problem in analyses of stress and design of new structure in aeronautic engineering and can be the alternative of the finite element method in commercial software where, the proposed approach reveals various applications in mechanical engineering. As a perspective to this work, extend the study to define the influence of buckling on wings ribs structure and the initiation of the cracks in the structure.

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