Latent Space Factorisation and Manipulation via Matrix Subspace Projection

Xiao Li  
University of Aberdeen  
x.li@abdn.ac.uk  

Chenghua Lin  
University of Aberdeen  
chenghua.lin@abdn.ac.uk  

Chaozheng Wang  
University of Aberdeen  
r01cw17@abdn.ac.uk  

Frank Guerin  
University of Aberdeen  
f.guerin@abdn.ac.uk  

Abstract

This paper proposes a novel method for factorising the information in the latent space of an autoencoder (AE), to improve the interpretability of the latent space and facilitate controlled generation. When trained on a dataset with labelled attributes we can produce a latent vector which separates information encoding the attributes from other characteristic information, and also disentangles the attribute information. This then allows us to manipulate each attribute of the latent representation individually without affecting others. Our method, matrix subspace projection, is simpler than the state of the art adversarial network approaches to latent space factorisation. We demonstrate the utility of the method for attribute manipulation tasks on the CelebA image dataset and the E2E text corpus.

1 Introduction

Variational autoencoders (VAEs) have numerous applications in generating images [10, 3] or text [20, 7], or labelling data [15]. However there are two desirable abilities which are difficult to achieve: firstly the ability to control generation, for example to generate examples with desired attributes, or to change the attributes of a given example; secondly the ability to interpret the latent vector of the encoding, to understand the attributes of a given example. Both of these require the ability to fully disentangle attributes. For example, in generating images of faces, changing one attribute, like facial hair, should not also change others (such as eyes and brow) and turn a female into a male. Ideally we should be able to add facial hair or glasses without affecting other attributes of the image. Also, for interpretability, an encoder would ideally map an attribute like glasses to a single variable of the latent vector, which could then be altered to add or remove glasses.

This is an interesting and important problem, and is part of a general trend to try to move deep neural network research towards explanatory models of the world [22, 12, 14], which requires disentanglement. We need to isolate explanatory factors and build structured models, as a way to overcome the combinatorial explosion of required training examples if such factors are not isolated [22]. In images of faces for example, the training data is unlikely to present us with a feminine face with facial hair, but a good generative model should be able to ‘imagine’ novel examples that combine attributes in ways not present in training data. As noted by Higgins et al., “Models are unable to generalise to data outside of the convex hull of the training distribution… unless they learn about the data generative factors and recombine them in novel ways.” [5]

Given a dataset with labelled attributes, the difficulty here is twofold: (1) learn a latent space representation which separates these attributes from all other characteristics, and (2) fully disentangle
the attributes. If we fail in the separation part, then efforts to generate with specific attributes may conflict with other information in the latent space. If we fail in the second part then examples generated with specified attributes will also be contaminated with spurious attributes (Figure 1).

We propose a simple method which separates the labelled information from all other un-labelled information. Our variables representing labelled information are fully disentangled, with one isolated variable for each labelled variable of the training set. Therefore, when we do conditional generation, we can assign pure labels combined with other latent data which does not conflict, so that the generated pictures are no longer blurred or contaminated with spurious attributes. Although we experiment only with VAEs, any autoencoder (AE) can work with our matrix subspace projection. If the AE is a generative model (such as VAE), with our approach, it becomes a conditional generative model that generates content based on the given condition constraints.

Our key contributions are: (1) A simple generic framework for conditional generation and content replacement, directly applicable to any AE architectures (e.g., input to the encoder can be an image or a sequence such as text). (2) Strong performance on learning disentangled latent representations of attributes. (3) A principled weighting strategy for combining loss terms for training.

2 Related Work

The problem of control of generation by attributes is somewhat different to the problem of generating specified individuals (or fine-grained categories) tackled by Bao et al. [1], because individuals are mutually exclusive in the training data and do not need to be produced in combinations. We focus here on control of generation by attributes. The first approaches to this problem (conditional VAEs [9, 18, 21]) added attribute label information as an extra input to the encoder and the decoder. These approaches generate using a latent \( z \) vector and also a label \( y \), where the \( z \) vector often conflicts with the label. The \( z \) vector has not been separated into components correlated with \( y \) and components uncorrelated with \( y \), to remove any information that might conflict with \( y \). With conflicting inputs the best the VAE can do is to produce a blurry image.

There is also a conditional Generative Adversarial Network (cGAN) which trains separate encoders for the \( y \) and \( z \) vectors, but does not try to remove potentially conflicting information [17]. The cGAN authors also note that the generator can fail to generate unusual attribute combinations such as a woman with a moustache; in GAN training the discriminator discourages the generator from generating samples outside the training distribution, but a VAE decoder is not so limited.

More recent work has explicitly tackled this problem of separating the attribute information from the latent vector, using what we call an ‘adversarial’ approach [13, 3, 10]. This is inspired by Generative Adversarial Networks (GANs); a new auxiliary network is added (like the discriminator in a GAN) which attempts to guess the label of the latent vector \( z \). So long as the label can be successfully guessed then there is still label (attribute) information present in \( z \), and the encoder is penalised via an additional loss term. In this way the encoder and new auxiliary network compete, with the encoder trying to remove label information from \( z \). This has the effect of minimising the mutual information between \( z \) vector and label.

While this adversarial approach can successfully remove label information from \( z \), there is nothing to stop the decoder (generator) from associating other spurious information with the label. For example the decoder might associate the label intended to be for ‘glasses’ with an older or more masculine face. This is what we see in their results of two of the adversarial approaches (Figure 1). These examples have been reproduced from the cited papers [3] and [10], where they are used to showcase how well those systems can independently manipulate attributes. Therefore we assume they are typical examples of the level of disentanglement achieved, and not unrepresentative examples.

In [3] the label vector is a single binary variable so that the system can only be trained to control (or classify) one attribute. It is not unexpected that a generator will associate spurious information with a label if the association is present in the training data and the system has been trained only on examples labelling a single attribute, e.g., glasses. The system cannot know that it should isolate ‘wearing glasses’, and not ‘wearing glasses and older’. Lemple et al. [13] trained their system for all 40 attributes and show superior disentanglement when generating from specified attributes.

Klys et al. [10], also adversarial, take a slightly different approach in training a multi-dimensional space for an attribute, rather than a single dimension. In contrast to our focus on disentanglement,
We are interested in factorising and manipulating multiple attributes from a latent representation. Let \( x \in \mathbb{R}^n \) and \( y \in Y = \{0, 1\}^k \) representing \( k \) attributes of \( x \). Let an arbitrary AE be represented by \( z = F(x) \) and \( x' = G(z) \), where \( F(\cdot) \) is the encoder, \( G(\cdot) \) is the decoder, \( z \) is the vector of latent variables encoding \( x \), and \( x' \) is the reconstruction of \( x \) (see Figure 2). Note that when \( x' \) is a good approximation of \( x \) (i.e., \( x' \approx x \)), the attribute information of \( x \) represented in \( y \) will also be captured in the latent encoding \( z \). We would like our framework to allow manipulation of attributes in any given new data point (given only a single example), or generation of data with any specified attributes in a straightforward way. That is, for a latent vector \( z \) encoding input \( x \), associated with attribute set \( y \), we wish to replace the information of \( y \) captured by \( z \) with the information of a new set of desired attributes specified in \( y' \). Let \( K(\cdot) \) be a replacement function, then we have \( z' = K(z, y') \) and \( x'' = G(K(z, y')) \), where the information of \( y' \) can be predicted from \( x'' \) and the non-attribute information from \( x \) will be preserved. To give a concrete example, given an image of a face, \( x \), we wish to replace the information of \( y \) with the information of \( y' \).

Figure 1: Figure showing the extreme difficulty of disentangling attributes for state-of-the-art supervised ‘adversarial’ approaches [3, 10]. (a) from [3] shows significant change in the eyebrows and eyes when adding facial hair. (b,c,d) from [10]. (b) moving across the glasses and facial hair subspace, from the female on the left, brings significant changes in eyebrows and eyes, and the shape of cheeks, making the face more masculine. (c) moving in glasses subspace shows changes around the eyes and mouth, looking older. (d) also moving in glasses subspace shows a narrower smile and a more masculine lower face. Note all of the pictures are generated by VAE, none are original photographs.

Figure 2: (a) the overall structure of our model (MSP); (b) the simplified equivalent structure of MSP; (c) an visualisation example of matrix subspace projection.

their focus is on allowing control of a larger variety in generation, for example, with glasses: ‘a variety of rims and different styles of sunglasses’, or with facial hair: ‘both moustaches and beards of varying thickness’. They focused on two subsets of the CelebA dataset to train for the attributes glasses and facial hair, and their combination, but not simultaneously for all the 40 labels available.

In addition to the above works on disentangling labelled attributes there is also work on the more difficult problem of unsupervised learning of disentangled generative factors of data [6, 20, 11]. However the supervised approaches [13, 8, 10] generate much clearer samples of selected attributes, and superior disentanglement. An alternative approach to controlled generation is to simply train a deep convolutional network and do linear interpolation in deep feature space [19]. This shows surprisingly good results, but in changing an attribute that should only affect a local area it can affect more image regions, and can produce unrealistic results for more rare face poses.

3 Method

3.1 Problem Formulation

We are interested in factorising and manipulating multiple attributes from a latent representation learned by an arbitrary Autoencoder (AE). Suppose we are given a dataset \( D \) of elements \((x, y)\) with \( x \in \mathbb{R}^n \) and \( y \in Y = \{0, 1\}^k \) representing \( k \) attributes of \( x \). Let an arbitrary AE be represented by \( z = F(x) \) and \( x' = G(z) \), where \( F(\cdot) \) is the encoder, \( G(\cdot) \) is the decoder, \( z \) is the vector of latent variables encoding \( x \), and \( x' \) is the reconstruction of \( x \) (see Figure 2). Note that when \( x' \) is a good approximation of \( x \) (i.e., \( x' \approx x \)), the attribute information of \( x \) represented in \( y \) will also be captured in the latent encoding \( z \). We would like our framework to allow manipulation of attributes in any given new data point (given only a single example), or generation of data with any specified attributes in a straightforward way. That is, for a latent vector \( z \) encoding input \( x \), associated with attribute set \( y \), we wish to replace the information of \( y \) captured by \( z \) with the information of a new set of desired attributes specified in \( y' \). Let \( K(\cdot) \) be a replacement function, then we have \( z' = K(z, y') \) and \( x'' = G(K(z, y')) \), where the information of \( y' \) can be predicted from \( x'' \) and the non-attribute information from \( x \) will be preserved. To give a concrete example, given an image of a face, \( x \), we
wish to manipulate $x$ w.r.t. the presence or absence of a set of desired attributes encoded in $y$ (e.g., a face with or without smiles, wearing or not wearing glasses), without changing the identity of the face (i.e., preserving the non-attribute information of $x$).

3.2 Learning Disentangled Latent Representations via Matrix Subspace Projection

To tackle the problem formulated in §3.1 we propose a generic framework to factor out the information about attributes $y$ from $z$ based on the idea of performing orthogonal matrix projection onto subspaces. We now describe our Matrix Subspace Projection framework (MSP) in detail.

Given a latent vector $z$ encoding $x$ and a function $H(\cdot)$ with sufficient complexity (see Figure 2(a)), we can obtain $\hat{z} = H(z)$ such that $\hat{z}$ is a nonlinear transformation of $z$. In practice, $H$ could be implemented as an invertible neural network [8]. For a given $\hat{z}$, there exists a projection matrix $M$ such that $y$, the orthogonal projection of $\hat{z}$ onto the subspace of $M$, captures the attribute information of $\hat{z}$, and that $s$, the projection of $\hat{z}$ onto the null space of $M$ (denoted by $N$), captures the non-attribute information of $\hat{z}$ (see Figure 2(c)). Note that $N$ is the null space matrix of $M$, so we have $M \perp N$.

$$M \cdot \hat{z} = y; \quad N \cdot \hat{z} = s$$

Our main learning objective is then to estimate $M$ which is nontrivial. We turn the problem of finding an optimal solution for $M$ into an optimisation problem, in which we need to (1) enforce $y$, the orthogonal projection of $\hat{z}$ onto the subspace of $M$, to be as close to $y_T$ (i.e., the vector encoding the ground truth attribute labels) as possible; and (2) minimise $||s||^2$ so that $s$ contains as little information from $\hat{z}$ as possible. This can be formulated into the following loss function

$$L_{MSP} = L_1 + L_2 = ||M \cdot \hat{z} - y_T||^2 + ||s||^2$$

where $L_1$ and $L_2$ encode the above two constraints, respectively, and $M \cdot \hat{z}$ computes the predicted attribute labels. Given that the AE relies on the information of both $y$ and $s$ to reconstruct $x$, these two optimisation constraints essentially introduce an adversarial process: on the one hand, it discourages any information of $\hat{z}$ to be stored in $s$ due to the penalty from $L_2$; on the other hand, while there is no penalty for storing information about $\hat{z}$ in $y$, $L_1$ enforces that $y$ only captures the attribute information of $\hat{z}$. By optimising $L_{MSP}$, we cause $\hat{z}$ to be factorised, with the attribute information stored in $y$, while $s$ only retains non-attribute information.

The first part of our loss function $L_1$ is relatively straightforward. The main obstacle here is to compute $L_2$ as $s$ is unknown. We develop a strategy to compute $||s||^2$ indirectly. Assuming that each attribute in $y$ is independent, it can be inferred that the row vectors of $M$ are linearly independent and that the block matrix $U = [M; N]$ is invertible. Taking into account Eq. 1, we can derive:

$$||s||^2 = ||s - 0||^2 = ||[y; s] - [y; 0]||^2 = ||U \cdot \hat{z} - [y; 0]||^2$$

We can further assume that $U$ is orthogonal such that the inverse of $U$ is the transpose of $U$, and the vectors of $U$ will be unit vectors and orthogonal to each other. Such a $U$ must exist because given a function $H(\cdot)$ with sufficient complexity, we can always transfer $z$ into $\hat{z}$ where $\hat{z}$ is in a similar linear space of $[y; s]$ (see §4.3 for empirical evaluation). Since $U$ is orthogonal, multiplying $U^{-1}$ by a vector does not change the norm of the vector itself. Thus, we have

$$||U \cdot \hat{z} - [y; 0]||^2 = ||U^{-1} \cdot (U \cdot \hat{z} - [y; 0])||^2 = ||\hat{z} - U^{-1} \cdot [y; 0]||^2$$

$$= ||\hat{z} - U^T \cdot [y; 0]||^2 = ||\hat{z} - [M; N]^T \cdot [y; 0]||^2$$

$$= ||\hat{z} - M^T \cdot y||^2$$

With Eq. 3 (which makes use of the properties of orthogonal matrices), we avoid computing $s$ and $N$ directly when minimising $||s||^2$, and turn the minimisation problem into optimising $M$ instead. Also note that we optimise $||s||^2$ instead of $||s||$ because the process of finding the optimal solution of $M$ can alternatively be understood as solving a least square problem for $M$, where the least square error $e^2 = ||s||^2$. Therefore, the overall loss of the MSP component can be expressed as:

$$L_{MSP} = L_1 + L_2 = ||M \cdot \hat{z} - y_T||^2 + ||\hat{z} - M^T \cdot y||^2$$

The loss function $L_{MSP}$ guarantees that after training, the resulting solution for $M$ will be part of the orthogonal matrix $U$. When $L_{MSP}$ is small, the transposition of $M$ becomes the inverse of $M$. 

4
There are several benefits to assume $U$ is an orthogonal matrix here. First, it guarantees that $M$ is orthogonal to $N$, and thus the information stored in $y$ and $s$ is decoupled, i.e., $y$ captures the label information of $z$, and $s$ the non-label information. Second, it ensures that every row vector in $M$ is orthogonal to each other. That is, for each row vector of $M$, the remaining vectors in $M$ and $N$ constitute the null space of $M$. Since each element in $y$ represents an attribute label, the orthogonality of $M$ thus ensures that every label in $y$ is independent. In other words, we will be able to completely disentangle all the individual attributes. When we change a label, it does not affect the performance of other labels (e.g., in CelebA, a female will not be changed to a male even if we add a beard to her).

Figure 2 (b) shows a simplified structure of MSP. This simplification exists because as explained earlier $H(\cdot)$ is invertible. So when the encoder and decoder have enough capacity, they can essentially absorb $H$ and $H^{-1}$. In words, rather than fitting $F$ and $G$, the encoder and decoder will fit $H \cdot F$ and $G \cdot H^{-1}$ instead. As $M$ itself is an incomplete orthogonal matrix, similar operations cannot be applied to $M$.

### 3.3 Applying the Matrix Subspace Projection Framework to an AE

To apply our matrix subspace projection (MSP) framework to an existing AE, one only needs to derive a final loss function by combining the loss of the AE and the loss of our MSP framework.

$$\mathcal{L} = \mathcal{L}_{AE} + \alpha \mathcal{L}_{MSP}$$

where $\alpha$ is the weight for $\mathcal{L}_{MSP}$. As illustrated in Figures 2(a) and (b), one should note that applying our framework will not change the structure of the AE, where our MSP component simply takes the latent vector $z$ of the AE as input. $\mathcal{L}_{AE}$ hopes that $s$ can store more information to reconstruct $x$, but $\mathcal{L}_{MSP}$ wants $s$ to contain less information. When $\alpha$ is small, the model becomes a standard AE. When $\alpha$ is too large, the non-label information in $z$ is reduced excessively, resulting in the generated products tending to the average of the training samples. Therefore, another challenge we face is how to set $\alpha$ appropriately.

We propose a principled strategy for effectively determining the value of $\alpha$ (this strategy is used in all experiments in this paper). Since $\mathcal{L}_{AE}$ and $\mathcal{L}_{MSP}$ essentially represent a competing relationship for $z$ resources, we specify that $\mathcal{L}_{AE}$ and $\mathcal{L}_{MSP}$ have the same influence on updating $s$. We use $\alpha$ to represent the “intensity” with which the AE updates $z$ during each back-propagation process. This intensity depends on the structure of the model and the loss function used by the model. For example, suppose an AE (for picture generation) uses CNN decoder and L2-loss. During the training process, the error of each pixel between the generated picture and the true picture is backpropagated to $z$. The sum of these gradients is the final gradient of $z$. The intensity of $z$ depends on the structure of the model and the loss function used by the model. For example, suppose an AE (for picture generation) uses CNN decoder and L2-loss. During the training process, the error of each pixel between the generated picture and the true picture is backpropagated to $z$. The sum of these gradients is the final gradient of $z$.

There are several benefits to assume $U$ is an orthogonal matrix here. First, it guarantees that $M$ is orthogonal to $N$, and thus the information stored in $y$ and $s$ is decoupled, i.e., $y$ captures the label information of $z$, and $s$ the non-label information. Second, it ensures that every row vector in $M$ is orthogonal to each other. That is, for each row vector of $M$, the remaining vectors in $M$ and $N$ constitute the null space of $M$. Since each element in $y$ represents an attribute label, the orthogonality of $M$ thus ensures that every label in $y$ is independent. In other words, we will be able to completely disentangle all the individual attributes. When we change a label, it does not affect the performance of other labels (e.g., in CelebA, a female will not be changed to a male even if we add a beard to her).

When using the cross-entropy-loss (or Nllloss etc.), which is usually for textual generative models (e.g. the seq2seq model), each generated word produces only one intensity, regardless of the word embedding size. Because,

$$\text{CrossEntropy}(x, x') = - \sum_{i} x_i \cdot \log x'_i$$

where $x$ is in one-hot encoding, and $0 \cdot \log x_i = 0$, so, there is no back propagation gradient for zero elements. Since there is only one non-zero element in $x$, for a sentence of length $k$, the strength of the entire sentence to $z$ is $k$. Thus, for cross-entropy-loss,

$$\alpha = \frac{k}{\text{count}(\text{label})}$$
3.4 Conditional Generation and Content Replacement

When the model is trained, we can complete square matrix $M$ as a standard orthogonal matrix $U$ by solving the null space (i.e. $N$) of $M$ (the null space is constituted by all the specific solutions of the equation $M \cdot n = 0$). Thus, for an input $x$, after encoding it as $z$, we can use $U = [M; N]$ to get the label vector $y$ of $x$ and the non-label information vector $s$.

$$[y; s] = U \cdot z = U \cdot F(x)$$ (8)

At this point, we can directly replace $[y; s]$ with $[y'; s]$, then calculate $z'$ from the inverse matrix of $U$ (that is, also the transpose of $U$), and then decode $z'$ into $x'$.

$$x' = G(z') = G(U^T \cdot [y'; s])$$ (9)

If the AE itself is a generative model (such as VAE), then the AE+MSP structure becomes a conditional generative model. Given a randomly sampled $s$ and a label vector $y$, the model can generate $x'$ for $y$ with Eq. [9].

4 Evaluation

This section shows the performance of the model integrated with different types of autoencoders. We will also evaluate the orthogonality of $M$ as it is an important indicator in terms of how well our algorithm can approximate $M$.

4.1 Matrix Subspace Projection in VAE

We applied our model on the CelebA dataset [10] of approximately 200,000 images of celebrity faces with 40 labelled attributes. We used the ADAM optimiser for training where the learning rate was set to 0.0001. Each mini-batch consisted of 256 images and all the images are centralised and resized to 128 $\times$ 128 pixels. For the network architecture, our encoder consisted of 5 convolutional layers with kernel size 4 and stride 2. The decoder mirrored the encoder with 5 transposed convolutional layers. We used LeakyReLU for nonlinearity, and batch normalisation is used in between convolutional and transposed convolutional layers. The model is trained on the Tesla T4 for around 48 hours.

Figure 3 shows some results of manipulating face attributes, changing one or multiple attributes. The images in (a) show the results of moving the hairline. The first image of each group is the original image (VAE reconstruction) and the two images following it are created by setting the hairline attribute to -1 and 1. In (b) we see that the model is able to put on or take off the glasses through learning. Again, the image on the left is the original image and the image on the right is with changed attribute. In Figure 5(c) and (d) we show two groups of highly correlated attributes: (c) openness of mouth and smiling, (d) male and beard. Images with a green border are VAE reconstruction results on original attribute values, and the images beside them show other combinations of the two attributes. The translation of a specific attribute does not influence other facial features such as shape of nose and hair style. Normally, the beard is exclusive for males while in (d), we can see some bizarre but reasonable looking women with beards. This is a very rigorous test of disentanglement, due to training sets not isolating facial hair from other masculine features. The image group in bottom middle of (c) shows some weakness of the VAE method when pose and lighting conditions are atypical, resulting in blurriness.

The visual comparison between our model and the one proposed by Klys et al. [10] is shown in Figure 4. The attributes of facial hair and glasses are added to the input image simultaneously. Through comparison we can see that our model is able to keep the feminine characteristics during morphing while the male characteristics start to show up from the second or third step in their results, which indicate that the latent space of our model is better disentangled.

4.2 Matrix Subspace Projection in Seq2seq

We applied our matrix projection to a classic seq2seq model for textual content exchanging, to determine if we can replace words according the given labels and keep other words unchanged. In this task, we adopt the E2E corpus [4], which contains 50k+ reviews of restaurants (E2E dataset is developed for Natural Language Generation, but here we use it for content replacement). Each review
is a single sentence that is labelled by the attribute-value pairs, for example, “name=[The Eagle]”, “food=[French]”, and “customerRating=[3/5]”. We regard each attribute-value pair as an unique label. All the labels constitute $y$ whose entries are 1 or 0 to represent each value label. (the concrete texts of the attribute name or value are NOT used).

Both the encoder and decoder of the seq2seq model are formed by two layer LSTMs. The model is trained for 1000 epochs (on a Tesla T4 around 12 hours). After training, we reconstruct the review sentences with randomly replaced labels, for example replacing “name=[The Eagle]” by “name=[Burger King]”, “customerRating=[3/5]” by “customerRating=[1/5]”. 50% of labels are changed in each sentence. The outcomes are shown in Table 1.

We quantitatively evaluate the accuracy of the replacement by calculating the proportion of labels expressed in the corpus sentences, the vanilla seq2seq generated sentences, and our generated sentences, by a human expert evaluation. Two human experts read through all the texts to count the proportion. The outcome is shown in Table 2.

The evaluation results indicate that the performance of our model is comparable to the original seq2seq model. Our model does successfully replace content words in the sentences. Although wrong entity names were sometimes produced by our decoder, this problem is not exclusive to our model. Similar results could also be found in the original seq2seq model [4].
Table 1: Results of changing labels in E2E corpus.

|                   | Example 1                                                                 | Example 2                                                                 |
|-------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|
| Orig-label        | eatType[pub], customer-rating[5-out-of-5], name[Blue-Spice], near[Crowne-Plaza-Hotel] | familyFriendly[yes], area[city-centre], eatType[pub], food[Japanese], near[Express-by-Holiday-Inn], name[Green-Man] |
| Orig-text         | the blue spice pub, near crowne plaza hotel, has a customer rating of 5 out of 5. | near the express by holiday inn in the city centre is green man. it is a japanese pub that is family-friendly. |
| Rec-text          | the blue spice pub, near crowne plaza hotel, has a customer rating of 5 out of 5. | near the express by holiday inn in the city centre is green man. it is a japanese pub that is family-friendly. |
| New-label         | eatType[coffee-shop], customer-rating[5-out-of-5], name[Blue-Spice], near[Avalon] | familyFriendly[no], area[riverside], eatType[coffee-shop], food[French], near[The-Six-Bells], name[Green-Man] |
| New-text          | the blue spice coffee shop, near avalon has a customer rating of 5 out of 5. | near the six bells in the riverside area is a green man. it is a french coffee shop that is not family-friendly. |

Table 2: The proportion of sentences that express their labels for unchanged label and replaced label.

|                   | Corpus Text | Seq2seq | Seq2seq (Ours) |
|-------------------|-------------|---------|----------------|
| Unchanged Label   | 99%         | 78%     | 79%            |
| Replaced Label     | -           | -       | 76%            |

4.3 Orthogonality Evaluation

The ability to disentangle labels is ensured by the orthogonality of $M$ in our model. Instead of directly using an orthogonal matrix, we train $M$ to be orthogonal. Thus, we evaluate how close $M$ is to the orthogonal matrix. Figure 5 shows the heat map of $M^T \cdot M$ and $U^T \cdot U$, which indicates that the production is fairly close to a unit matrix. It visualises $M^T \cdot M$ in the seq2seq version of our model (Figure 5(a)) and in the VAE version (Figure 5(c)). The matrix $U^T \cdot U$ ($U$ is formed by $M$ concatenating its null space) is also visualised (in Figure 5(b) and (d)). It is clear that when the matrices are multiplied by their transposes, the products do approximate the unit matrix. Thus, $M$ is indeed trained to be a (partial) orthogonal matrix.

![Figure 5: Measuring orthogonality: Heat map of $M^T \cdot M$ and $U^T \cdot U$ for Seq2seq+MSP (a,b) and VAE+MSP (c,d)](image)

5 Conclusion

We propose a matrix projection that can be attached to various autoencoders (e.g. LSTM, VAE) to make the latent space factorised and more disentangled, based on labelled information, which ensures that manipulation in the latent space is much easier. We test the attribute manipulation ability of our model on an image dataset and text corpus, obtaining results that show clean disentanglement. In addition our model involves a simpler training process than adversarial approaches which need a long training with a very low weight on the loss coming from the discriminator that removes labelled information, to avoid the encoder being too affected by this loss term [13]. In the future, we aim to reduce blurry effects by combining our model with a generative adversarial network.
References

[1] Jianmin Bao, Dong Chen, Fang Wen, Houqiang Li, and Gang Hua. CVAE-GAN: fine-grained image generation through asymmetric training. CoRR, abs/1703.10155, 2017.

[2] Xi Chen, Yan Duan, Rein Houthooft, John Schulman, Ilya Sutskever, and Pieter Abbeel. Infogan: Interpretable representation learning by information maximizing generative adversarial nets. In D. D. Lee, M. Sugiyama, U. V. Luxburg, I. Guyon, and R. Garnett, editors, Advances in Neural Information Processing Systems 29, pages 2172–2180. Curran Associates, Inc., 2016.

[3] Antonia Creswell, Anil A. Bharath, and Biswa Sengupta. Conditional autoencoders with adversarial information factorization. CoRR, abs/1711.05175, 2017.

[4] Ondřej Dušek, Jekaterina Novikova, and Verena Rieser. Evaluating the state-of-the-art of end-to-end natural language generation: The E2E NLG Challenge. arXiv preprint arXiv:1901.11528, January 2019.

[5] Irina Higgins, Loïc Matthey, Xavier Glorot, Arka Pal, Benigno Uria, Charles Blundell, Shakir Mohamed, and Alexander Lerchner. Early visual concept learning with unsupervised deep learning. CoRR, abs/1606.05579, 2016.

[6] Irina Higgins, Loïc Matthey, Arka Pal, Christopher Burgess, Xavier Glorot, Matthew Botvinick, Shakir Mohamed, and Alexander Lerchner. beta-vae: Learning basic visual concepts with a constrained variational framework. In 5th International Conference on Learning Representations, ICLR 2017, Toulon, France, April 24-26, 2017, Conference Track Proceedings, 2017.

[7] Zhiting Hu, Zichao Yang, Xiaodan Liang, Ruslan Salakhutdinov, and Eric P. Xing. Toward controlled generation of text. In Doina Precup and Yee Whye Teh, editors, Proceedings of the 34th International Conference on Machine Learning, volume 70 of Proceedings of Machine Learning Research, pages 1587–1596, International Convention Centre, Sydney, Australia, 06–11 Aug 2017. PMLR.

[8] Jörn-Henrik Jacobsen, Arnold Smeulders, and Edouard Oyallon. i-revnet: Deep invertible networks. arXiv preprint arXiv:1802.07088, 2018.

[9] Durk P Kingma, Shakir Mohamed, Danilo Jimenez Rezende, and Max Welling. Semi-supervised learning with deep generative models. In Z. Ghahramani, M. Welling, C. Cortes, N. D. Lawrence, and K. Q. Weinberger, editors, Advances in Neural Information Processing Systems 27, pages 3581–3589. Curran Associates, Inc., 2014.

[10] Jack Klys, Jake Snell, and Richard Zemel. Learning latent subspaces in variational autoencoders. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, editors, Advances in Neural Information Processing Systems 31, pages 6444–6454. Curran Associates, Inc., 2018.

[11] Abhishek Kumar, Prasanna Sattigeri, and Avinash Balakrishnan. Variational inference of disentangled latent concepts from unlabeled observations. In 6th International Conference on Learning Representations, ICLR 2018, Vancouver, BC, Canada, April 30 - May 3, 2018, Conference Track Proceedings, 2018.

[12] Brenden M. Lake, Tomer D. Ullman, Joshua B. Tenenbaum, and Samuel J. Gershman. Building machines that learn and think like people. CoRR, abs/1604.00289, 2016.

[13] Guillaume Lample, Neil Zeghidour, Nicolas Usunier, Antoine Bordes, Ludovic Denoyer, et al. Fader networks: Manipulating images by sliding attributes. In Advances in Neural Information Processing Systems, pages 5967–5976, 2017.

[14] Yann LeCun. The power and limits of deep learning. ResearchTechnology Management, 61(6):22–27, 2013.

[15] Yang Li, Quan Pan, Suhang Wang, Haiyun Peng, Tao Yang, and Erik Cambria. Disentangled variational auto-encoder for semi-supervised learning. Information Sciences, 482:73–85, 2019.
[16] Ziwei Liu, Ping Luo, Xiaogang Wang, and Xiaoou Tang. Deep learning face attributes in the wild. In Proceedings of the IEEE international conference on computer vision, pages 3730–3738, 2015.

[17] Guim Perarnau, Joost van de Weijer, Bogdan Raducanu, and Jose M. Álvarez. Invertible conditional gans for image editing. CoRR, abs/1611.06355, 2016.

[18] Kihyuk Sohn, Honglak Lee, and Xinchen Yan. Learning structured output representation using deep conditional generative models. In C. Cortes, N. D. Lawrence, D. D. Lee, M. Sugiyama, and R. Garnett, editors, Advances in Neural Information Processing Systems 28, pages 3483–3491. Curran Associates, Inc., 2015.

[19] Paul Upchurch, Jacob R. Gardner, Geoff Pleiss, Robert Pless, Noah Snavely, Kavita Bala, and Kilian Q. Weinberger. Deep feature interpolation for image content changes. In 2017 IEEE Conference on Computer Vision and Pattern Recognition, CVPR 2017, Honolulu, HI, USA, July 21-26, 2017, pages 6090–6099, 2017.

[20] Yijun Xiao, Tiancheng Zhao, and William Yang Wang. Dirichlet variational autoencoder for text modeling. CoRR, abs/1811.00135, 2018.

[21] Xinchen Yan, Jimei Yang, Kihyuk Sohn, and Honglak Lee. Attribute2image: Conditional image generation from visual attributes. In Computer Vision - ECCV 2016 - 14th European Conference, Amsterdam, The Netherlands, October 11-14, 2016, Proceedings, Part IV, pages 776–791, 2016.

[22] Alan L. Yuille and Chenxi Liu. Deep nets: What have they ever done for vision? CoRR, abs/1805.04025, 2018.