Reconstructing $t\bar{t}$ events with one lost jet

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Abstract

We present a technique for reconstructing the kinematics of pair-produced top quarks that decay to a charged lepton, a neutrino and four final state quarks in the subset of events where only three jets are reconstructed. We present a figure of merit that allows for a fair comparison of reconstruction algorithms without requiring their calibration. The new reconstruction of events with only three jets is fully competitive with the full reconstruction typically used for four jet events.

Keywords: top, reconstruction, partial

1. Introduction

Several problems in top quark physics require a full reconstruction of the kinematics of the top-anti-top pair. For example, to measure the forward-backward (or charge) asymmetry in $t\bar{t}$ production, it is essential to know the direction of both top and anti-top quarks.

We consider $t\bar{t}$ events where each top quark decays into a $b$ quark and a $W$ boson, and where one $W$ boson decays hadronically and one $W$ boson decays...
leptonically. We classify top quarks as “leptonic” or “hadronic”, based on the mode of the $W$-boson decay. The final state contains a lepton, a neutrino and four quarks that subsequently shower and hadronize into jets. This channel is commonly referred to as “$l$+jets”.

The four final state quarks do not always yield four reconstructed jets, which is the case, for example, when the angular separation between two of them is small. When one of the jets from top decay is lost, it is not possible to fully reconstruct the $t\bar{t}$ decay chain. In this paper we present a method to infer the direction and kinematics of the top quark and antiquark in $l$+jets events where only three jets are reconstructed, and demonstrate the application of the method to simulated $t\bar{t}$ events. We focus on $p\bar{p} \rightarrow t\bar{t}$ production at a center of mass energy of 1.96 TeV, as in the Tevatron. The method can be applied equally well to proton-proton collisions and in principle to $t\bar{t}$ events produced in lepton machines.

We discuss the selection of the events in Section 2. In Section 3 we detail the method, and how it partially reconstructs the $t\bar{t}$ pair using the simulated distributions of the invariant mass of various combinations of jets and of “$b$-tagging” observables that attempt to identify jets likely to arise from a $b$ quark [1]. We compare the performance of different reconstruction algorithms in Section 4 for which we introduce a new figure of merit (FOM).

2. Samples and selection

The results shown here are based on $t\bar{t}$ events simulated with the MC@NLO event generator [2] and processed through a detector simulation and object reconstruction that largely correspond to but are not identical to that of the
D0 experiment. In particular, some of the quality selection criteria are not applied since they are not relevant for the development of the method.

We select jets with transverse momentum $p_T > 20$ GeV and with pseudorapidity $|\eta| < 2.5$. We select leptons from electron and muon candidates with $p_T > 20$ GeV and with $|\eta_e| < 1.1$ or $|\eta_\mu| < 2.0$. We then select events with exactly one lepton and exactly three jets. We require that the transverse momentum imbalance measured by the calorimetry, $E_T$, is greater than 20 GeV. We reject events where the $E_T$ is closely aligned with the lepton and events with $E_T > 500$ GeV. These two cuts suppress multijet background and events with misreconstructed $E_T$, respectively.

We further categorize the selected events by how well the reconstructed jets match the quarks from $t\bar{t}$ decay, as that affects the quality of reconstruction. We consider a jet to be matched to a quark when their angular separation $\Delta R = \sqrt{(\Delta y)^2 + (\Delta \phi)^2}$ is less than 0.5. We classify an event as “matchable” if all $t\bar{t}$ decay products assumed to be present by the reconstruction algorithm were matched to reconstructed objects.

For the reconstruction of $l+\geq 4$ jets events at the Tevatron, a matchable event is the one in which the four jets of highest $p_T$ match the four final state quarks from $t\bar{t}$ decay. Only 55% of the $l+\geq 4$ jets events at the Tevatron are matchable. In the context of this paper a $l+3$ jets event is considered matchable if one jet matches the $b$ quark from of the leptonic top quark decay and the two other jets match two of the three quarks from the decay of the

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1 The rapidity $y$ is defined as $y(\theta, \beta) = \frac{1}{2} \ln [(1 + \beta \cos \theta) / (1 - \beta \cos \theta)]$, where $\theta$ is the polar angle and $\beta$ is the ratio of a particle’s momentum to its energy. The pseudorapidity is defined as $-\ln \tan \frac{\theta}{2}$. 

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hadronically decaying top quark. 20% of the $l+3$ jets events are classified as unmatchable because the $b$ jet from the leptonic top decay, which is essential to the described algorithm, is lost. In 4% of the events two jets were lost, while an extra one was gained from initial or final state radiation. Thus, 76% of the $l+3$ jets events are considered matchable.

3. Reconstructing $t\bar{t}$ in $l+3$ jets events

For almost half of the simulated $p\bar{p} \rightarrow t\bar{t}$ pairs that decay in the $l+$jets channel, only three jets are reconstructed. In our study scenario, two quarks yield a single jet due to an accidental overlap in $\approx 18\%$ of these $l+3$ jets events. One of the quarks is too forward (high $|\eta|$) to yield a selected jet in $\approx 8\%$ of the events. In the remaining $\approx 74\%$ of the events, either one of the quarks was too soft (low $p_T$) to yield a selected jet or a jet was lost due to reconstruction and identification inefficiencies.

In Fig. 1 we show a schematic of a possible $t\bar{t}$ decay process. Instead of trying to infer the kinematics of the missing or merged jet in a $l+3$ jets event, we partially reconstruct the $t\bar{t}$ system by neglecting this jet altogether. Though there is some experimental sensitivity to the presence of two quarks in a single jet, e.g., through the jet width and mass, we found it too weak to be useful. Thus we do not attempt to “unmerge” any of the jets and assign two quarks to it. Events in the $l+\geq 4$ jets channel are often reconstructed using a “kinematic fit”, which modifies the measured momenta to satisfy the known resonance masses (e.g. Ref. [3]). Given the above approximation, such refinements are of little use for $l+3$ jets events. Thus we employ a simpler approach to partially reconstruct the $t\bar{t}$ system in $l+3$ jets events.
Figure 1: Cartoon depicting an example of $t\bar{t}$ decay: the leptonic top quark
($t_l$) decays to $b$-jet $j_3$ and to a $W$ boson which decays to a lepton and a
neutrino; the hadronic top quark ($t_h$) decays to $b$-jet $j_1$ and to a $W$ boson
which decays hadronically to jets $j_2$ and $j_4$. In the depicted $l+3$ jets event,
$j_4$ is lost.

3.1. Reconstructing the leptonic $W$ boson

We start by reconstructing the leptonically decaying $W$ boson using the
lepton momentum and the $\not{E_T}$. The neutrino momentum in the plane trans-
verse to the beam direction, $\vec{q_T}$, is initially set equal to the $\not{E_T}$. The longitu-
dinal component of neutrino momentum, $q_z$, is calculated using a constraint
on the $W$-boson mass, $M_W$. The resultant quadratic equation can have two
solutions, which creates a two-fold ambiguity. Both solutions are considered.

Following Ref. [4], when the discriminant of the quadratic equation for
$q_z$ is negative, we scale $\vec{q_T}$ to satisfy the $M_W$ constraint with a discriminant
equal to zero. This results in another quadratic equation which yields two
solutions for the scale, at least one of which is positive. When both solutions
are positive, we use the one that is closer to unity.
3.2. Reconstructing the top-quark candidates

The next step is to form leptonic and hadronic top quark candidates. To do so, we assume that the lost jet is from the decay of the hadronic top quark. One of the jets is combined with the leptonic $W$ boson to form a leptonic top candidate. The two remaining jets are combined to form a “proxy” for the hadronic top quark, which serves instead of a fully reconstructed candidate. The assignment is completely defined by the choice of leptonic $b$ jet. If the previous step yielded two $q_z$ solutions, for each assignment we choose the solution where the combination of the leptonic $b$ jet, the lepton and the neutrino yields an invariant mass closer to the nominal top quark mass [5].

3.2.1. $\chi^2$ method

Invariant mass distributions on both the leptonic and hadronic sides have characteristic shapes as shown in Fig. 2. Both can be used to find the best jet assignment. The distributions were made using an adaptive kernel estimator [6].

A simple way to choose an assignment is to use a $\chi^2$ test statistic for the masses reconstructed for the leptonic top candidate ($m_l$) and for the proxy ($m_p$):

$$\chi^2 = \left( \frac{m_l - m_l^0}{\sigma_l} \right)^2 + \left( \frac{m_p - m_p^0}{\sigma_p} \right)^2,$$

where $m_l^0$ ($m_p^0$) and $\sigma_l$ ($\sigma_p$) are the mean and width of the Gaussian fits for leptonic (proxy) masses shown in Fig. 2. This approach picks the correct assignment in 65.9% of the cases where such an assignment exists. Below we discuss more detailed treatments that improve upon this basic technique.
3.2.2. Complete likelihood method

We improve the choice of the assignment by replacing the $\chi^2$ with a likelihood function. The likelihood formalism allows us to take into account additional information. The use of the invariant masses of the incorrect assignments, which too have distinct shapes, is detailed below. The use of “$b$-tagging” observables that attempt to identify jets likely to arise from a $b$ quark is detailed further on.

Figure 3 shows the distributions in top candidate mass on the leptonic side for three situations: when the leptonic $W$ boson is (correctly) combined with the $b$ jet from leptonic top decay ($P_{t\ell}$), when it is (wrongly) combined with the hadronic $b$ jet ($P_{th}$), and when it is (wrongly) combined with a jet from hadronic $W$-boson decay ($P_{tq}$). Using the distinct shape of a presumably “incorrect” assignment means we need to keep track of two types of assignments which may disagree. We will introduce notation for the as-
assignment used to combine the jets into the mass observables and for the assignment hypothesized to be correct.

Figure 3: Distributions of the mass of the leptonic top candidate, which comprises the lepton, neutrino and leptonic $b$-jet candidate. The distribution is shown for events where the jet assigned to the leptonic $b$ quark is the correct one ($P_{t,l}$, solid curve), the hadronic $b$ jet ($P_{t,h}$, dot-dashed curve), or a jet from hadronic $W$-boson decay ($P_{t,q}$, dashed curve).

Depending on which jet is lost and which jet is picked to form the leptonic top candidate there are four possible two-jet combinations for the proxy side. The probability distributions for the invariant mass on the proxy side are shown in Fig. 4 for hadronic and leptonic $b$ jets ($P_{p,hl}$), leptonic $b$ jet and a jet from $W$-boson decay ($P_{p,lq}$), hadronic $b$ jet and a jet from $W$-boson decay ($P_{p,hq}$), and both jets from $W$-boson decay ($P_{p,qq}$). The first two combinations are incorrect, as they include the leptonic $b$ jet. The last two combinations are correct, and under the assumption that the leptonic $b$ jet was reconstructed, they cannot both be available in the same event.

These shapes can be used to maximize the probability $P$ of selecting the
Figure 4: Distributions of the mass of the proxy for the hadronic top quark, which comprises two jets. The distribution is shown for events where the jets assigned to the proxy are the hadronic and leptonic $b$ jets ($P_{p:hl}$, solid curve), the leptonic $b$ jet and a jet from $W$-boson decay ($P_{p:lq}$, dot-dashed curve), the hadronic $b$ jet and a jet from $W$-boson decay ($P_{p:hq}$, long dashes), or both jets from the $W$-boson decay ($P_{p:qq}$, short dashes). In the last case, the $W$ resonance is clearly seen.
The correct assignment \( a \) given the data \( d \), which according to Bayes’ theorem is:

\[
P(a \mid d) = \frac{P(d \mid a) \, P(a)}{\sum_b P(d \mid b) \, P(b)} = \frac{P(d \mid a)}{\sum_b P(d \mid b)}, \tag{2}
\]

where \( b \) is any assignment and the second equality uses the fact that a priori all assignments are equally probable.

There are three possible jet assignments per event \((i = 1, 2, 3)\), corresponding to the choice of the candidate for the leptonic \( b \) jet. Each event is characterized by three possible masses on the leptonic side \((t_1, t_2, t_3)\) and three possible masses on the proxy side \((p_1, p_2, p_3)\). In addition to this kinematic information, \( b \)-tagging algorithms \cite{1} can also help to identify the origins of the jets. The results of the \( b \)-tagging algorithms can usually be expressed as a single continuous variable per jet, which discriminates between light and \( b \)-flavored jets. We label the \( b \)-tagging discriminant for the \( i \)-th jet as \( b_i \). Thus, data are presented by nine variables:

\[
d = (t_1, t_2, t_3; p_1, p_2, p_3; b_1, b_2, b_3) \tag{3}
\]

In matchable events the lost jet is either the hadronic \( b \) jet or a jet from hadronic \( W \)-boson decay. We label the former as \( Q = b_{qq} \) and the latter \( H = b_{bh}q \). For a matchable event, the probability for assignment \( a \) is a weighted sum of the probabilities of \( H \) and \( Q \) types:

\[
P(d \mid a) = (1 - f_Q)P(d \mid a, H) + f_QP(d \mid a, Q), \tag{4}
\]

where \( f_Q \) is the fraction of matchable events that are type \( Q \), which in our study scenario is 0.205.

Each jet assignment hypothesis specifies the type of each jet: either a \( b \) jet, or a jet from hadronic \( W \)-boson decay. The latter category includes
jets that arise from $c$ quarks, and are somewhat similar to $b$ jets \[1\]. The correlations between the $b$-tagging discriminants ($b_j$) are small, and since they are mostly independent of the true jet flavors, they are also irrelevant for our purposes. Thus, the $b$-tagging probabilities can be factorized:

$$P(d \mid a, C) = P(t_1, t_2, t_3; p_1, p_2, p_3 \mid a, C) P(b_1, b_2, b_3 \mid a, C)$$  \hspace{1cm} (5)

$$= P(t_1, t_2, t_3; p_1, p_2, p_3 \mid a, C) \prod_{j=1}^{3} P(b_j \mid a, C)$$  \hspace{1cm} (6)

where $C = H$ or $Q$ is the hypothesized class of the event. By neglecting the correlations between the remaining variables we can factorize the first two terms into six of the one-dimensional distributions shown in Figs. 3 and 4 ($P_{t_y}$ and $P_{p_y}$):

$$P(d \mid a, C) = \prod_{j=1}^{3} P_{t_j(a,C)} \prod_{j=1}^{3} P_{p_j(a,C)} \prod_{j=1}^{3} P(b_j \mid a, C)$$  \hspace{1cm} (7)

where $f(j, a, C) \in \{l, h, q\}$ gives the type of the $j$-th jet (i.e., the jet assumed to be the leptonic $b$ jet when building the $t_j$ observable) according to assignment $a$ and event class $C$, and $g(j, a, C) \in \{hq, lq, hl, qq\}$ gives the types of the non-$j$-th jets (i.e., the jets combined to form the proxy for the $p_j$ observable) according to $a$ and $C$. Though we neglected some of the correlations between the observables in Eq. 7, the structure of the likelihood preserves the dominant correlations, such as having at most one $W$-boson resonance, and the correlation between the presence of a $W$-boson resonance and the $b$-tagging variables. Using the described algorithm, the correct jet assignment is chosen for 69.1\% of the matchable events.

Returning to the example of Fig. 1, the following terms help identify the correct event class ($H$) and assignment ($a = 3$):
• the invariant mass formed by combining the leptonic $W$ candidate ($W_l$) and the jet $j_1$, $t_1 = m(W_l + j_1)$, should be consistent with the $P_{tch}$ distribution from Fig. [3]

• $t_2 = m(W_l + j_2)$ should be consistent with $P_{tq}$ (same figure);

• $t_3 = m(W_l + j_3)$ should be consistent with $P_{td}$ (same figure);

• the invariant mass formed by the jets $j_2$ and $j_3$, $p_1 = m(j_2 + j_3)$, should be consistent with the $P_{pdlq}$ distribution from Fig. [4]

• $p_2 = m(j_1 + j_3)$, invariant mass of leptonic $b$ jet and a light jet should be consistent with $P_{pbl}$ (same figure);

• $p_3 = m(j_1 + j_2)$, invariant mass of leptonic and hadronic $b$ jets should be consistent with $P_{plbh}$ (same figure);

• $b_1$, the $b$-tagging discriminant of $j_1$, should be consistent with the distribution for a $b$ jet;

• $b_2$ should be consistent with the distribution for a jet from hadronic $W$-boson decay;

• $b_3$ should be consistent with the distribution for a $b$ jet.

The inclusion of the rarer $Q$ events in the likelihood can distort the reconstruction of the more common case, the $H$ events. But this risk is mitigated when the likelihood contains enough information to distinguish between the two cases on an event-by-event basis. To demonstrate that, we calculate the a posteriori probability that a matchable event is of type $Q$ as:

$$P_Q = \frac{f_Q P(d \mid a, Q)}{(1 - f_Q)P(d \mid a, H) + f_Q P(d \mid a, Q)} \quad (8)$$
As Fig. 5 demonstrates the separation between the two cases is quite good. This separation is mostly due to the $b$-tagging discriminants. It is also useful to check the modeling of $P_Q$ against collider data, as all the terms in $P (d \mid a)$ also appear in $P_Q$.

![Figure 5: Distribution in the a posteriori probability for a $Q$-type event, shown for $H$- and $Q$-type events.](image)

### 3.2.3. Scaling the proxy

Given a specific jet to quark assignment we have a candidate for the leptonic top $t$ with the energy $E_t$, momentum $\vec{P}_t$ and invariant mass $m_t = \sqrt{E_t^2 - \vec{P}_t^2}$ and a proxy $p$ for the hadronic top with the energy $E_p$, momentum $\vec{P}_p$ and invariant mass $m_p = \sqrt{E_p^2 - \vec{P}_p^2}$. Since the proxy tends to underestimate the 4-vector of the hadronic top quark, the invariant mass of these two objects, $m(t+p)$, is likely to underestimate the generated invariant mass of the $t\bar{t}$ system, $m_{t\bar{t}}^{\text{gen}}$, as shown in Fig. 6. Additional scaling can be applied to the proxy 4-vector to partially correct for this underestimation. Furthermore, since the reconstructed proxy mass, $m_p$, indicates the size of the underestimation in each event, this scaling can be parametrized as a function of $m_p$. 

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For each simulated event, we define the ideal scaling of the proxy 4-vector, $\alpha$, as the scale that will bring the reconstructed $m(t + p)$ to $m_{tt}^{\text{peak}}$. Since $m_{tt}^{\text{peak}}$ is a function of $m_{tt}^{\text{gen}}$, this scale is unavailable in collider data. Instead, we reconstruct events using a scale $\hat{\alpha}$ which is an estimate of $\alpha$ based on the observable $m_p$.

To derive this estimate, we solve for $\alpha$ in simulated events, which results in a quadratic equation:

$$\alpha^2 m_p^2 + 2\alpha \left( E_t E_p - \vec{P}_t \vec{P}_p \right) + \left( m_t^2 - m_{tt}^{\text{peak}}^2 \right) = 0.$$  

(9)

We then plot, in Fig. 7, the two-dimensional distribution of the proxy mass scaled by $\alpha(m_{tt}^{\text{gen}})$ and the unscaled $m_p$. From this distribution we parametrize the most probable value of $\alpha$ as a function of $m_p$ to find our estimated $\hat{\alpha}$. The parametrization of $\hat{\alpha}(m_p)$ was chosen from polynomial functions that were constrained so that the scaled mass, $\hat{\alpha} m_p$, is non-decreasing. Finally, we construct the invariant mass of the $t\bar{t}$ system from the sum of the 4-vector of the proxy, scaled by $\hat{\alpha}(m_p)$, and the 4-vector of the leptonic top candidate.

3.2.4. Averaging the assignments

The most significant improvement is from considering more than one jet assignment. The algorithms described so far considered only the most likely assignment, the one that minimizes the $\chi^2$ in Eq. 1 or that maximizes $P(a | d)$. It is tempting to define the ideal as $m(t + p) = m_{tt}^{\text{gen}}$, which will also calibrate the reconstructed $m_{tt}$. But it is more important to reduce the scatter in the reconstructed $m_{tt}$, and needlessly introducing the calibration lowers the effectiveness of the derived scaling.

This is enforced only at the edge of the distributions. Though the middle of the function was allowed to decrease, the best-fit function does not do so.
Figure 6: The distribution of the generated and reconstructed $t\bar{t}$ invariant masses, without any scaling of the proxy, for all selected events. The light-gray line shows a fit to the peak position of the reconstructed mass, $m_{tt}^{\text{peak}}$.

Figure 7: The distribution of the proxy mass before and after scaling by $\alpha$, for all selected events. The dashed black and white curve shows a fit to the peak position of $\alpha$, $\hat{\alpha}(m_p)$. 
in Eq. 2. But we can also use all the possible assignments weighted by their a posteriori probabilities. For example:

\[ m_{\bar{t}t} = \sum_{a} m_{\bar{t}t}^{a} P(a \mid d). \] (10)

These averaged reconstructions tend to have the advantage of a spread lower than that of the single-assignment reconstructions, and the disadvantage of a lower response. Here we define the “response” for an observable as the derivative of the average reconstructed value as a function of the true, generated value and the “spread” as the RMS of the distribution of the reconstructed value for a fixed true, generated value.

4. Performance

4.1. Definition of the figure of merit

To compare the performance of different reconstruction algorithms, we require an appropriate figure of merit. Algorithm performance is usually quantified by summarizing the distribution of the difference (or the ratio) between the reconstructed and generated observable into its RMS, or into the width of a Gaussian fit to the core of the distribution. However, this presumes that the reconstruction is unbiased and centered around the true value. For the reconstruction algorithms discussed here the difference distributions are intrinsically bimodal, since the performance differs for matchable and unmatchable events.

For matchable events, the reconstruction typically has a response that is close to one and a narrow spread, while for the unmatchable events it typically

\footnote{And also for other $tt$ reconstruction algorithms for $l+\geq4$ jets events.}
has a low response and a wide spread. Hence the average reconstruction is biased, while the peak position is almost unbiased, and the reconstruction can not be calibrated so it is both unbiased and peaks at the generated value. Some useful compromises can be made, for specific uses, but their quality varies preventing a fair comparison between different algorithms.

Instead, we contrast the reconstructed observable for two categories of events, defined by the quantiles of the generated observable. This is demonstrated in Fig. 8. Each category contains 10% of the events, and they are defined according to an offset, $s$, so that one category is generated between the $s$ and $s + 0.1$ quantiles and the other between the $0.9 - s$ and $1 - s$ quantiles (see Fig. 8a where the 2nd and 9th deciles are used). The FOM quantifies how well the reconstruction separates these two categories.

We denote the distributions of the reconstructed observable for these categories $f_L$ and $f_H$. An example is shown in Fig. 8b. Were these distributions Gaussian and identical, it would be natural to quantify the separation in terms of $N_\sigma$, the number of standard deviations between their peaks. To generalize this concept to arbitrary distributions and to focus on the possible misclassification of events between the two categories, we define $T(x)$ as the overlap between these distributions at observable value $x$ and the minimal overlap $M$:

$$M = \min_x T(x), \quad T(x) = \max \left( \int_x^{\infty} f_L(x') \, dx', \int_{-\infty}^x f_H(x') \, dx' \right).$$

(11)

These too are shown in Fig. 8b. Smaller $M$ values indicate less misclassification and hence better performance of the reconstruction algorithm.

We can translate $M$ to the more familiar “number of $\sigma$s” by considering $M$ for two Gaussian distributions of width one, whose means are separated
Figure 8: An example of the minimal overlap as a figure of merit for a reconstruction. The events of $f_L$ are shaded in (a) and shown with the solid curve in (b); the events of $f_H$ are hatched in (a) and shown with the dashed curve in (b). Overflows are shown in the edge bins. The offset is $s = 0.1$, the point of minimal overlap is $m_{t\bar{t}} = 427$ GeV and $M = 0.093$, as shown in the lower panel of (b), and $N_\sigma = 2.65$. 
by $N_\sigma$:

$$M (N_\sigma) = \int_{\frac{1}{2}N_\sigma}^{\infty} G(x) \, dx = \frac{1}{2} \left( 1 - \text{erf} \left( \frac{N_\sigma}{2\sqrt{2}} \right) \right),$$  \hspace{1cm} (12)

where $G$ is the normal distribution (see Fig. 9a). By inverting this relationship (see Fig. 9b), we can present the minimal overlap in terms of $N_\sigma$.

![Figure 9: The minimal overlap and the number of $\sigma$s.](image)

This FOM has another, incidental advantage. Unlike RMS values, it can be interpreted without referring to the width and shape of the expected generated distribution.

4.2. Comparison of the algorithms

Figures 10 and 11 compare the reconstruction of different classes of events with the new algorithm. For ease of display, a rough linear calibration of $m_{\ell\ell}$ is used when displaying the resolutions of the partial reconstruction algorithm. Both classes of matchable events (case $H$ and case $Q$) are reconstructed well, and the reconstruction of unmatchable events is not much worse. As 76% of the events are matchable, the reconstruction for all events is almost as good as for matchable events. The reconstruction of the hadronic-top rapidity is especially weak for events of type $Q$, indicating that a missing
“hadronic” $b$ jet is more problematic than a missing jet from $W$-boson decay. The reconstruction of the leptonic-top rapidity is especially weak for unmatchable events, since for most of these events the “leptonic” $b$ jet is lost.

We now compare the performance of the partial reconstruction algorithm described in this paper with that of Ref. [3], a kinematic fit algorithm which modifies the measured momenta within their experimental uncertainties while satisfying the top-quark and $W$-boson invariant mass constraints. This algorithm was used to fully reconstruct the $l+\geq4$ jets events in many top measurements (e.g. in Refs. [7] and [8]). As with the new algorithm, we can either use the most likely assignment from the kinematic fit or use a weighted average of all assignments. The relative weight of each assignment is $\exp \left( -\chi^2 / 2 \right)$, as in Ref. [7]. The distributions of the differences and ratio between reconstructed and generated observables for these two algorithms, shown in Fig. [12] illustrate that the partial reconstruction provide a performance similar in quality to that of the full reconstruction.

Table[1] uses the FOM introduced in Section 4.1 to quantitatively compare the performance of the two algorithms. The performance of simpler versions of the new algorithm, corresponding to Sections 3.2.1, 3.2.2, 3.2.3, and 3.2.4, are also compared. A constant offset, $s$, was chosen for each observable ($m_{t\bar{t}}, y_l, y_h$ and $\Delta y$). The offsets were chosen so the resulting $N_\sigma$ values are $\approx 2$, a level of separation where further improvements are still useful (see Fig. [8b]). Though the tail behavior of the reconstructions varies, the variations are limited to a fraction of events much smaller than the 10% we consider in each category. Thus the choice of offsets has little effect on the comparison of
Figure 10: Resolution in top-quark rapidity on the leptonic side (a,b) and the proxy side (c,d), and in $\Delta y$ (e,f). The y axis in the left-hand plots is on a logarithmic scale, while the right-hand plots show the peak region on a linear scale. Events where one of the jets from $W$-boson decay is lost (case $H$) are shown by the long-dashed curves, events where the hadronic $b$ jet is lost (case $Q$) are shown by the dashed-dotted curves, unmatchable events are shown by the dashed curves, and the solid curves show all events. As we expect symmetric resolution functions, we construct all curves to be symmetric.
Figure 11: Resolution in $m_{t\bar{t}}$. The y axis in the left-hand plots is on a logarithmic scale, while the right-hand plots show the peak region on a linear scale. Events where one of the jets from $W$-boson decay is lost (case $H$) are shown by the long-dashed curves, events where the hadronic $b$ jet is lost (case $Q$) are shown by the dashed-dotted curves, unmatchable events are shown by the dashed curves, and the solid curves show all events.

reconstruction techniques. We find that the partial reconstruction of $m_{t\bar{t}}$ and $\Delta y$ in $l+3$ jets sample is fully competitive with that of the full reconstruction in the $l+\geq4$ jets events.

The $l+3$ jets channel has the obvious disadvantage of missing a jet. On the other hand, it has the advantage of fewer jets from initial state radiation, and for the algorithm outlined here, of fewer unmatchable events. These advantages compensate quite well for the missing jet. It may be that the reconstruction of $l+\geq4$ jets can also be improved by considering additional types of events, in particular, events where one jet is lost and a jet from initial state radiation was selected.
Figure 12: Resolution in (a) $y_l$, (b) $y_h$, (c) $\Delta y$, and in (d) $m_{tt}$ for $l+3$ jets events (solid curve) and for $l+\geq 4$ jets events reconstructed with a kinematic fitter [3] (dot-dashed curve). In both cases, the weighted average of all assignments is used.
Table 1: Performance of the various reconstruction algorithms for all selected events

|                | $m_{t\bar{t}}$ | $\Delta y$ | $y_l$ | $y_h$ |
|----------------|----------------|------------|-------|-------|
| offset         | 0.1            | 0          | 0.2   | 0.2   |
| **Separation power in $N_\sigma$** |                |            |       |       |
| $l+3\text{ jets}$ |                |            |       |       |
| $\chi^2$ based | 2.52           | 2.10       | 2.00  | 2.70  |
| complete likelihood | 2.53           | 2.21       | 2.05  | 2.69  |
| scaled proxy   | 2.60           | "          | "     | "     |
| averaged       | 2.65           | 2.61       | 2.26  | 2.92  |
| $l+\geq4\text{ jets}$ |                |            |       |       |
| best assignment | 2.45           | 1.68       | 1.57  | 2.26  |
| averaged       | 2.56           | 2.51       | 1.86  | 2.70  |
5. Summary

We present an algorithm that partially reconstructs $t\bar{t}$ events in the $l+$jets channel in the case when one of the jets is lost, resulting in a $l+3$ jets topology. Probabilities for correct and incorrect jet assignment are formed based on $b$-tagging discriminants and on all possible mass combinations on the leptonic and hadronic sides. The algorithm can be applied to measure the forward-backward asymmetry in $t\bar{t}$ production, the invariant mass spectrum of the $t\bar{t}$ system and for most other analyses that require a full reconstruction. The performance of the partial reconstruction algorithm is competitive with that commonly achieved for fully reconstructed $l+\geq4$ jets events. The inclusion of $l+3$ jets events roughly doubles the amount of usable signal for measurements of top pair production properties.

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