Abstract

We perform an old school, one-loop renormalization of the Abelian-Higgs model in the Unitary and $R_\xi$ gauges, focused on the scalar potential and the gauge boson mass. Our goal is to demonstrate in this simple context the validity of the Unitary gauge at the quantum level, which could open the way for an until now (mostly) avoided framework for loop computations. We indeed find that the Unitary gauge is consistent and equivalent to the $R_\xi$ gauge at the level of $\beta$-functions. Then we compare the renormalized, finite, one-loop Higgs potential in the two gauges and we again find equivalence. This equivalence needs not only a complete cancellation of the gauge fixing parameter $\xi$ from the $R_\xi$ gauge potential but also requires its $\xi$-independent part to be equal to the Unitary gauge result. We follow the quantum behaviour of the system by plotting Renormalization Group trajectories and Lines of Constant Physics, with the former the well known curves and with the latter, determined by the finite parts of the counter-terms, particularly well suited for a comparison with non-perturbative studies.
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1 Introduction

Spontaneously broken gauge theories are of great physical interest, notably because of the Brout-Englert-Higgs (or simply Higgs) mechanism [1]. Even though classically there is a simple qualitative description of the mechanism that one can find in textbooks, at the quantum level where these theories are more precisely defined, ambiguities arise. These ambiguities are related to the question of whether one can consider the quantum scalar potential itself as a physical quantity. In this work we touch on this issue approaching it from two angles, in the context of the Abelian-Higgs model [2].

One angle of approach has to do with the basis of representation for the Higgs field. Recall that in most loop calculations the (complex) Higgs field is written in a Cartesian basis: \( H = \phi_1 + i\phi_2 \) [3]. The real part of the field \( \phi_1 \) is the physical Higgs particle, while \( \phi_2 \) represents the unphysical Nambu-Goldstone (or simply Goldstone) boson. For dimensional reasons the resulting \( \beta \)-functions for the Higgs mass and quartic coupling must have the form \( \beta_{m_H}^{\text{Cart}} = c_{m_H}^{\text{Cart}} \lambda m_H^2 + \cdots \) and \( \beta_{\lambda}^{\text{Cart}} = c_{\lambda}^{\text{Cart}} \lambda^2 + \cdots \) respectively. The numerical coefficients \( c_{m_H}^{\text{Cart}} \) and \( c_{\lambda}^{\text{Cart}} \) are determined after the loop calculation has been performed. The former coefficient affects the behaviour of the Higgs mass under Renormalization Group (RG) flow and the latter that of the quartic coupling, that in turn affect triviality and in the presence of fermions, also instability bounds. Taking these bounds seriously at a quantitative level means that the scalar potential is considered to be a physical quantity. Here we perform our computations in a Polar basis, \( H = \phi e^{i\chi}/v \), where now the physical Higgs field is \( \phi = \sqrt{\phi_1^2 + \phi_2^2} \) and \( \chi \) is the Goldstone boson (\( v \) is some vacuum expectation value). In this basis one expects to find \( \beta_{m_H}^{\text{Pol}} = c_{m_H}^{\text{Pol}} \lambda m_H^2 + \cdots \) and \( \beta_{\lambda}^{\text{Pol}} = c_{\lambda}^{\text{Pol}} \lambda^2 + \cdots \) with a pending question if \( c_{m_H}^{\text{Cart}} \) agrees with \( c_{m_H}^{\text{Pol}} \) and if \( c_{\lambda}^{\text{Cart}} \) agrees with \( c_{\lambda}^{\text{Pol}} \). We find that these coefficients do not exactly agree, introducing possibly a small but computable ambiguity in the RG flows and the above mentioned bounds. The Polar basis may have though also a deeper impact on the quantum potential, having to do with its gauge (in)dependence. Some of these issues were noticed already in [4] in the context of the effective potential.

The other angle of our approach has to do with the quantization scheme. In particular, there is an infinitum of possible gauge fixing functions that one can use during quantization, each one of them introducing at least one gauge fixing parameter, say \( \xi \). A typical representative of these schemes is \( R_\xi \) gauge fixing. It is a well known fact that any sensible quantization scheme should produce a gauge independent set of physical quantities. Such quantities are for sure the masses of physical fields and the independent dimensionless couplings like \( \lambda \). Gauge independence of physical quantities in a scheme like the \( R_\xi \) gauge fixing scheme is then ensured if the corresponding \( \beta \)-functions are \( \xi \)-independent and this is indeed the case in every consistent loop calculation. The scalar effective potential com-
puted with the background field method on the other hand is notoriously known to be \(\xi\)-dependent already at one-loop, which renders its physicality (and the relevance of the precision triviality and instability bounds derived from it) a delicate matter [5, 6]. It is therefore an open issue how to define an unambiguous, physical, quantum potential in spontaneously broken gauge theories. For this reason, we choose the Unitary gauge [9] as our quantization scheme. The Unitary gauge is one where only physical fields propagate and it is commonly used in textbooks in order to demonstrate the physical spectrum of spontaneously broken gauge theories at the classical level. It is rarely used though for loop calculations, in fact we are not aware of a complete renormalization work in this gauge. A possible obstruction to completing reliably such a program may be the high momentum behaviour of the Unitary gauge boson propagator resulting in integrals that often diverge worse than quadratically with a cut-off, even in a renormalizable theory. We call these integrals, ”U-integrals”. For this reason the Unitary gauge is sometimes called a non-renormalizable gauge. A necessary condition therefore for a Unitary gauge calculation to make sense is that the physical quantities that are \(\xi\)-independent in the \(R_\xi\) gauge, to coincide with the corresponding quantities derived in the Unitary gauge. If this condition is fulfilled then through the renormalization procedure one automatically obtains a version of the scalar potential, the Unitary gauge scalar potential, that is by definition gauge independent. A question we would like to answer here is if this version of the scalar potential can be used, in principle, to derive competitive with respect to the \(R_\xi\) gauge physical predictions. Of course, an important issue is to understand the connection between the Unitary and \(R_\xi\) gauge potentials. The standard connection between these two gauges is to take the ”Unitary gauge limit” \(\xi \to \infty\) at the level of the Feynman rules, before loop integrals have been performed. Clearly this is not what we want to do here. Instead, we would like to compare the \(R_\xi\) gauge potential with the Unitary gauge potential, after loop integration. This is non-trivial, as the \(\xi \to \infty\) limit and the loop integration may not commute. For recent studies of related issues, in the context of the \(H \to \gamma\gamma\) decay, see [10, 11, 12, 13] and for some earlier works on the Unitary gauge and the Abelian-Higgs model, see [4, 14].

In this work, we consider the Abelian-Higgs model and perform the one-loop renormalization of the gauge boson mass and of the Higgs potential in both the \(R_\xi\) and Unitary gauges. In addition to the above mentioned comparison reasons, this double computation allows us to monitor and ensure the credibility of the Unitary gauge calculation. We will be able to show that as far as the \(\beta\)-functions (determined by the divergent parts of the one-loop amplitudes) is concerned, the Unitary gauge is equally consistent, in fact equivalent to the \(R_\xi\) gauge. Then we consider the scalar potential (determined by the finite parts of the amplitudes after renormalization) and ask whether it could also be physical. Let the finite part of the one-loop value of a quantity \(\star\) be defined as \((\star)_f\) with
the subscript $f$ denoting finite part. Such quantities will be the various loop corrections entering in the renormalized Higgs potential. Then schematically we have that in general

$$\left[(\ast)_f, \lim_{\xi \to \infty}\right] = \lim_{\xi \to \infty} g(\lambda, m_H, m_Z, \mu, \xi),$$  \hspace{1cm} (1.1)

where the behaviour of the function $g$ at $\xi = \infty$ is one thing we would like to understand. The background field method in the $\overline{\text{MS}}$ scheme for example gives an effective potential where $\lim_{\xi \to \infty} g(\xi) = \infty$ for a large class of gauge fixing functions [6], rendering the Unitary gauge limit after loop integration, singular. The singularity implies that the Unitary gauge is disconnected from the space of $R_\xi$ gauges and essentially that the Higgs potential is unphysical away from its extrema. Our calculation instead shows that $\lim_{\xi \to \infty} g(\xi) = 0$, perhaps the most striking result of this work, as it implies that the Higgs potential can be made gauge invariant, hence physical, even away from its extrema. For formal aspects of the quantization of the Abelian-Higgs model, see [7, 8].

In Section 2 we review the classical Abelian-Higgs model and discuss some basic conventions in our calculation. In Section 3 we perform in detail the $R_\xi$-gauge computation and in Section 4 the Unitary gauge computation. In Section 5 we renormalize the AH model. In Sections 6 and 7 we present a numerical analysis of our results. In Section 8 we state our conclusions. We also have a number of Appendices where auxiliary material can be found.

## 2 The classical theory and some basics

The bare Lagrangean of the AH model is

$$\mathcal{L}_0 = -\frac{1}{4} F_{0,\mu\nu}^2 + |D_\mu H_0|^2 + m_0^2 |H_0|^2 - \lambda_0 |H_0|^4. \hspace{1cm} (2.1)$$

Zero subscripts or superscripts denote bare quantities. As usual, the covariant derivative is $D_\mu = \partial_\mu + ig_0 A_\mu^0$ and the gauge field strength is $F_{0,\mu\nu} = \partial_\mu A_\nu^0 - \partial_\nu A_\mu^0$. The Higgs field $H_0$ is a complex scalar field. The Lagrangean is invariant under the $U(1)$ gauge transformations

$$A_\mu^0(x) \rightarrow A_\mu^0(x) + \frac{1}{g_0} \partial_\mu \theta(x)$$

$$H_0(x) \rightarrow H_0(x)e^{i\theta(x)} \hspace{1cm} (2.2)$$

with $\theta(x)$ a gauge transformation function and the discrete, global $Z_2$ symmetry

$$H_0(x) \rightarrow -H_0(x) \hspace{1cm} (2.3)$$

We assume that both $m_0$ and $\lambda_0$ are positive quantities.
As it stands, the part of the Lagrangean that corresponds to a potential
\[ V_0 = -m_0^2 |H_0|^2 + \lambda_0 |H_0|^4 \]  
(2.4)
triggers SSB. Minimization yields the vev
\[ \langle H_0 \rangle = \pm \frac{m_0}{\sqrt{2\lambda_0}} = \pm \frac{v_0}{\sqrt{2}}. \]  
(2.5)
The second of the above equations defines the bare vacuum expectation value (vev) parameter \( v_0 \). The field \( H_0 \) can be expanded around its vev as
\[ H_0(x) = \left(v_0 + \phi_0(x)\right) e^{i\chi_0(x)}/\sqrt{2}, \]  
(2.6)
where now \( \phi_0 \) is the new Higgs field fluctuation and \( \chi_0 \) is a massless Goldstone boson.

Eq. (2.6) is the so-called "Polar basis" for the Higgs field, as opposed to the "Cartesian basis" \( H_0(x) = 1/\sqrt{2}(v_0 + \phi_{1,0}(x) + i\phi_{2,0}(x)) \). The former is typically used for demonstrating the physical spectrum while the latter for quantization. Here we will stick to the Polar basis for both. Replacing the vev into Eq. (2.1), the Lagrangean takes the form
\[
\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \phi_0) (\partial^\mu \phi_0) + \frac{1}{4} (\partial_\mu \chi_0) (\partial^\mu \chi_0) + \frac{1}{2} m_Z^2 A_\mu^0 A^{\mu 0} \\
+ m_Z \partial_\mu \phi_0 A_\mu^0 + 2m_Z \sqrt{2\lambda_0} A_\mu^0 (\partial^\mu \chi_0) \phi_0 + \sqrt{2\lambda_0} (\partial_\mu \chi_0)^2 \phi_0 + \frac{2\lambda_0 m_Z^2}{m_{H_0}^2} A_\mu^0 (\partial^\mu \chi_0) \phi_0^2 \\
+ \frac{\lambda_0}{m_{H_0}^2} (\partial_\mu \chi_0)^2 \phi_0^2 + g^{\mu\nu} \frac{\lambda_0 m_Z^2}{m_{H_0}^2} A_\mu^0 A_\nu^0 \phi_0^2 + g^{\mu\nu} \frac{m_Z^2}{m_{H_0}^2} \sqrt{2\lambda_0} \phi_0 A_\mu^0 A_\nu^0 \\
- \frac{1}{2} \frac{m_{H_0}^2}{m_{H_0}} \phi_0^2 - \frac{\lambda_0}{2} \frac{m_{H_0}^2}{m_{H_0}^2} \phi_0^4 - \frac{\lambda_0}{4} \phi_0^4 + \text{const.}, \]  
(2.7)
with \( g^{\mu\nu} \) the Minkowski space metric. The bare gauge boson (\( Z \) boson) mass is defined as \( m_Z = g_0 v_0 \) and the bare Higgs mass as \( m_{H_0} = \sqrt{2} m_0 = \sqrt{2\lambda_0^2 v_0} \). As implied by the above expression, we have decided to use as independent parameters the masses and the quartic coupling \( \lambda_0 \). This means that we have eliminated the gauge coupling according to \( g_0 = m_Z \sqrt{2\lambda_0} \) and the vev according to \( v_0 = \frac{m_{H_0}}{\sqrt{2\lambda_0}} \) wherever they appear. Setting the Goldstone field \( \chi_0 \) to zero, gives us the Unitary gauge Lagrangean which will be the topic of a separate section. For now we will keep the Goldstone in the spectrum. A consequence of having a Goldstone in the spectrum is a mixing term between the \( Z \) and \( \chi_0 \) in Eq. (2.7).

We will compute all one-loop Feynman diagrams that contribute to the renormalization of the \( Z \) mass and of the scalar potential, using Dimensional Regularization (DR) [16]. We assume basic knowledge of the DR technology that we therefore do not review here, except from some necessary basic facts that can be found in the Appendices. The renormalization scale parameter of DR is denoted by \( \mu \). The small expansion parameter \( \varepsilon \) of DR is defined via
\[ \varepsilon = 4 - d. \]  
(2.8)
Since we are using DR in our renormalization scheme, a consequence is that the trace of the metric is $g_{\mu \nu} g^{\mu \nu} = d$.

Now, each diagram comes with a symmetry factor. Consider a one-loop diagram containing $n$ vertices with $k_n$ lines on each vertex. These $k_n$ lines are divided into $k_n^{in}$ and $k_n^{ext}$ for the internal and the external lines respectively. The procedure to obtain the correct symmetry factor is given for example in [15]:

- For each of the $n$ vertices, count all possible ways that the $k_n$ lines can be connected to the external legs of a given diagram. This is $k_n^{ext}$. Doing this for every vertex defines $n_O$ as the product of the $k_n^{ext}$’s. The remaining lines belong to $k_n^{in}$.
- At each of the $n$ vertices there are $k_n^{in}$ lines. Count all the possible ways the loop can be constructed using these lines. The product of the $k_n^{in}$’s defines $n_I$.
- For each of the $n$ vertices, count the number of $k_i$ lines that are equivalent. This defines $\ell_i$.
- Finally, for a given diagram count all the possible equivalent vertices of type $j$, defining $v_j$.

The symmetry factor of the diagram is then

$$S_a^b = \frac{n_O n_I}{\prod_i \ell_i! \prod_j v_j!},$$

(2.9)

where $a, b$ are indicators specifying the diagram.

A large part of the one-loop diagram expressions is dominated by the set of basic integrals called Passarino-Veltman (PV) integrals [17]. We collect in Appendix B the basics of the formulation of PV integrals, following mostly [18]. In our calculation several non-standard integrals emerge as well. The divergent ones we call collectively $U$-integrals and we compute them in Appendix C. Finite integrals and finite parts of divergent integrals are harder to classify systematically so we will be dealing with them as we proceed.

We also introduce some useful notation. Our convention for the name of a one-loop Feynman diagram $F$ is

$$F_E^{G,L}$$

(2.10)

$G = R_\xi, U$ specifies either the $R_\xi$ or the Unitary gauge where the diagram is computed. $L$ is a list containing the field(s) running in the loop in the direction of the loop momentum flow, starting from the vertex on the left side of the diagram. In case the diagram is an irreducible Box, we start the list from the upper left vertex. $E = H, Z$ specifies the external legs. In the case $E = Z$, there may be additional Lorentz indices following $E$. We
know that one-loop Feynman diagrams can be either finite or divergent. In the first case the corresponding integrals contain only finite terms and in the second case they include both divergent and finite parts. Furthermore, in each of the above cases, in the $R_\xi$ gauge, the corresponding parts could be either gauge (i.e. $\xi$) dependent or gauge independent. In the Unitary gauge there is no such distinction. Therefore, every set of one-loop diagrams contributing to the same correlator, where the sum over the index $L$ has been performed and the Lorentz indices (if present) have been appropriately contracted, can be expressed as

$$\left(4\pi\right)^{d/2} F_E^G = \mu^\varepsilon \left( \left[ F_E^G \right]_\varepsilon + \left\{ F_E^G \right\}_\varepsilon + \left[ F_E^G \right]_f + \left\{ F_E^G \right\}_f \right),$$

(2.11)

where the square brackets denote $\xi$-independent part, the curly brackets denote $\xi$-dependent part and the subscripts $\varepsilon$ and $f$ denote divergent and finite part respectively. The $1/\varepsilon$ factor is absorbed in the definitions of $\left[ F_E^G \right]_\varepsilon$ and $\left\{ F_E^G \right\}_\varepsilon$. A word of caution here is that the above separation of diagrams into $\xi$-independent and $\xi$-dependent parts is clearly not unique. Nevertheless it is a very useful notation (once we get used to it) since it allows to perform algebra with diagrams easily and also facilitates the comparison with the Unitary gauge. In fact, in most cases the $\xi$-independent part of sums of diagrams is just the corresponding Unitary gauge result. The sum of the gauge independent and gauge dependent finite parts is denoted as

$$\left( F_E^G \right)_f = \left[ F_E^G \right]_f + \left\{ F_E^G \right\}_f.$$  

(2.12)

Round, square and curly brackets appearing in any other context have their usual meaning. All entirely finite integrals are computed as described in Appendix A. We will be giving the divergent parts and the finite parts of sums of groups of diagrams explicitly in the main text, leaving some of the (increasingly cumbersome) expressions for finite (parts of) diagrams to Appendix E. There are some finite parts sitting inside the $U$-integrals too, which we compute together with their divergent parts and do not show explicitly, in order to avoid repetitions.

We perform renormalization away from the usual $\overline{\text{MS}}$ scheme. This means that our renormalization conditions force us to keep some non-trivial finite terms, that eventually enter in the renormalized Higgs potential. What finite terms are kept is of course not a unique choice. We have not checked if the gauge invariance of the potential could also be achieved in a pure $\overline{\text{MS}}$ scheme. If however $\overline{\text{MS}}$ (or any other scheme) would result in a gauge dependent potential, it would mean that in spontaneously broken gauge theories scheme dependence kicks in already at one loop. Recall that in QCD (a not spontaneously broken theory) scheme dependence appears at three loops. This would be strange because it would relate gauge dependence to scheme dependence. Our feeling is that any physical renormalization subtraction scheme applied to our calculation should produce a gauge invariant Higgs potential.
Furthermore, we perform renormalization off-shell, at zero external momenta. External momenta will be generically denoted by \( p \). In 2-point, 3-point and 4-point functions zero external momenta means \( p_i = 0 \) with \( i = 1, 2, 3 \) and \( i = 1, 2, 3, 4 \) respectively. This choice, beyond being a huge simplifying factor, is justified since we are interested in terms of the Lagrangean without derivatives. It would be equally strange if after renormalization the Higgs potential would pick up an external momentum dependence. In other words we believe that even if we had performed renormalization on-shell for example, we would have obtained the same results, alas in a more complicated way: not only individual diagrams would become much more involved, but also non-1PI diagrams may have to be added in order to arrive at gauge invariant \( \beta \)-functions. A physical argument for zero external momenta could be that if one is interested in the high energy limit, is entitled to set the masses of external particles equal to zero, that is \( p_i^2 = 0 \). But this choice, using momentum conservation in 2, 3 and 4-point functions is the same as setting the momenta themselves to zero. A subtle point of setting external momenta to zero is that in a diagram a term of the form \( p_\mu p_\nu T^{\mu\nu}(p, m_i) \), with \( p \) an external momentum, \( m_i \) mass parameters and \( T^{\mu\nu} \) a tensor (DR) integral, may appear. If we set \( p = 0 \) before integration, this term is zero. If on the other hand we do the integral first, then contract and then take the limit \( p \to 0 \), we may find a non-zero result. The latter procedure is the correct one. Expressions of the form of Eq. (2.11) in the main text will be thus given at zero external momenta. Nevertheless, for completeness and for potential future use, we collect some on-shell expressions in Appendix F.

3 \( R_\xi \) gauge

Quantization of Eq. (2.7) requires gauge fixing. The gauge fixing term

\[
\mathcal{L}_{gf} = \frac{1}{2\xi} (\partial^\mu A^0_\mu - \xi g_0 v_0 \chi_0)^2
\]

(3.1)

that removes the Goldstone-gauge boson mixing defines the \( R_\xi \) gauge and under the standard Faddeev-Popov procedure introduces the extra ghost contribution

\[
\mathcal{L}_{ghost} = (\partial_\mu \bar{\psi}) (\partial^\mu \psi) - \xi m^2_{Z_0} \bar{c}c.
\]

(3.2)

The sum \( \mathcal{L}_{R_\xi} = \mathcal{L}_0 + \mathcal{L}_{gf} + \mathcal{L}_{ghost} \)

\[
\mathcal{L}_{R_\xi} = -\frac{1}{4} F^2_{\mu\nu} + \frac{1}{2} (\partial_\mu \phi_0) (\partial^\mu \phi_0) + \frac{1}{2} (\partial_\mu \chi_0) (\partial^\mu \chi_0) - \frac{1}{2\xi} (\partial^\mu A^0_\mu)^2 + \frac{1}{2} m^2_{Z_0} A^0_\mu A^{0\mu} + 2 m_{Z_0} \sqrt{2\lambda_0} A^0_\mu (\partial^\mu \chi_0) \phi_0 + \frac{\sqrt{2\lambda_0}}{m_{H_0}} (\partial_\mu \chi_0)^2 \phi_0 + \frac{2\lambda_0 m_{Z_0}}{m^2_{H_0}} A^0_\mu (\partial^\mu \chi_0) \phi_0^2 + \frac{\lambda_0}{m^2_{H_0}} (\partial_\mu \chi_0) \phi_0^2
\]

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+ \frac{g^{\mu\nu} \lambda_0 m_{Z_0}^2}{m_{H_0}^2} A_\mu A_\nu \phi_0^2 + g^{\mu\nu} \frac{m_{Z_0}^2}{m_{H_0}} \sqrt{2\lambda_0 \phi_0 A_\mu A_\nu} - \frac{1}{2} \xi m_{Z_0}^2 \lambda_0^2 \\
- \frac{1}{2} m_{H_0}^2 \phi_0^2 - \sqrt{\frac{\lambda_0}{2}} m_{H_0} \phi_0^3 - \frac{\lambda_0}{4} \phi_0^4 + \mathcal{L}_{\text{ghost}} + \text{const.} \quad (3.3)

yields the final expression from which the $R_\xi$ gauge Feynman rules can be derived. Notice that the Goldstone has acquired an unphysical, gauge dependent mass $m_{\chi_0} = \sqrt{\xi m_{Z_0}}$. With this gauge fixing choice the ghost fields have apart from a kinetic term, a mass term equal to that of the Goldstone boson’s but they are not coupled to the Higgs field or to the gauge boson.

The Feynman rules arising from Eq. (3.3) are the following:

- **Gauge boson propagator**

  \[ \begin{array}{c}
  \gamma \\
  \end{array} \quad = \quad \frac{i \left( -g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - \xi m_{Z_0}^2} \right)}{k^2 - m_{Z_0}^2 + i\varepsilon} \]

- **Higgs propagator**

  \[ \begin{array}{c}
  \gamma \\
  \end{array} \quad = \quad \frac{i}{k^2 - m_{H_0}^2 + i\varepsilon} \]

- **Goldstone propagator**

  \[ \begin{array}{c}
  \gamma \\
  \end{array} \quad = \quad \frac{i}{k^2 - \xi m_{Z_0}^2 + i\varepsilon} \]

- **Ghost propagator**

  \[ \begin{array}{c}
  \gamma \\
  \end{array} \quad = \quad \frac{i}{k^2 - \xi m_{Z_0}^2 + i\varepsilon} \]

- **$\phi$-$\chi$-$Z$ vertex**

  \[ \begin{array}{c}
  \begin{array}{c}
  p_1 \\
  \end{array} \end{array} \quad \begin{array}{c}
  \gamma \\
  \end{array} \quad \begin{array}{c}
  \begin{array}{c}
  p_2 \\
  \end{array} \end{array} \quad = \quad -\frac{2 m_{Z_0}}{m_{H_0}} \sqrt{2\lambda_0 k_\mu} \]

- **$\phi$-$Z$-$Z$ vertex**
\[ p_1 \uparrow \quad p_3 \quad = 2ig^{\mu\nu} \frac{m_{Z_0}^2}{m_{H_0}} \sqrt{2\lambda_0} \quad p_2 \downarrow \]

- \(\phi-\phi-\phi\) vertex

\[ p_1 \quad p_3 \quad = -6i \sqrt{\frac{\lambda_0}{2} m_{H_0}} \quad p_2 \]

- \(\phi-\chi-\chi\) vertex

\[ k^a \quad p_1 \quad = 2ig^{\mu\nu} \frac{\sqrt{2\lambda_0}}{m_{H_0}} k^a_{\mu} k^b_{\nu} \]

- \(\phi-\phi-Z-Z\) vertex

\[ p_1 \quad p_3 \quad = 4i \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} g^{\mu\nu} \quad p_2 \quad p_4 \]

- \(\phi-\phi-\phi-\phi\) vertex

\[ p_1 \quad p_3 \quad = -6i \lambda_0 \quad p_2 \quad p_4 \]

- \(\phi-\phi-\chi-Z\) vertex

\[ p_1 \quad p_3 \quad = -4 \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} k_{\mu} \quad p_2 \quad k_a \]

- \(\phi-\phi-\chi-\chi\) vertex
In the above rules, $k_{a,b}$ denotes the momenta of the Goldstone bosons in a vertex. In the case that we have two Goldstones in a diagram, we choose the convention where one of them gets in and the other gets out of the vertex (since we will encounter the Goldstone only in loops).

The momentum dependence of the vertices including at least one Goldstone boson is a direct consequence of our choice to use the Polar instead of the Cartesian basis for the Higgs field, as in the latter case there are no such vertices. As a result, the corresponding loop integrals will have extra loop momentum factors in their numerator and this triggers the appearance of $U$-integrals (also) in the $R_\xi$ gauge.

In what follows, we present only the final expressions for the divergent and finite parts of the various sectors given at zero external momenta. The explicit calculation steps that we followed are given in the Appendix D.

### 3.1 Tadpoles

One-point functions are also called tadpoles. The first such diagram is

$$ p \longrightarrow \begin{array}{c} \phantom{p} \end{array} \kern 1cm k \end{array} \kern 1cm \longrightarrow \begin{array}{c} \phantom{p} \end{array} \kern 1cm = i T_{R_\xi, \phi}^R $$

and analytically evaluates to

$$ T_{R_\xi, \phi}^R = 3 \sqrt{\frac{\lambda_0}{2 m_{H_0} \mu_\xi}} A_0(m_{H_0}) . $$

The next tadpole comes with a gauge boson loop:

$$ p \longrightarrow \begin{array}{c} \phantom{p} \end{array} \kern 1cm k \end{array} \kern 1cm \longrightarrow \begin{array}{c} \phantom{p} \end{array} \kern 1cm = i T_{H}^{R_\xi, Z} $$

and it is equal to

$$ T_{H}^{R_\xi, Z} = \frac{m_{Z_0}^2}{m_{H_0}} \sqrt{2 \lambda_0 \mu_\xi} \left\{ 3 A_0(m_{Z_0}) + \xi A_0(\sqrt{\lambda m_{Z_0}}) \right\} . $$

The last tadpole has a Goldstone loop:
Using Eq. (C.2) where $U_T(m_{\chi_0})$ is calculated we obtain that the above diagram is equal to

$$T_H^{R\xi H} = -\frac{\sqrt{2\lambda_0}}{m_{H_0}} \mu^\varepsilon m_{\chi_0}^2 A_0(m_{\chi_0}).$$  (3.6)

The total tadpole value is the sum of the above three contributions:

$$T_H^{R\xi} = \mu^\varepsilon \left( 3\sqrt{\frac{\lambda_0}{2}} m_{H_0} A_0(m_{H}) + 3\sqrt{2\lambda_0 m_{Z_0}^2} A_0(m_{Z}) \right).$$  (3.7)

Exploiting our notation it can also be expressed as

$$(4\pi)^{d/2} T_H^{R\xi} = \mu^\varepsilon \left( [T_H^{R\xi}]_\varepsilon + \{T_H^{R\xi}\}_\varepsilon + [T_H^{R\xi}]_f + \{T_H^{R\xi}\}_f \right),$$

with

$$\varepsilon [T_H^{R\xi}]_\varepsilon = 6 \sqrt{\frac{\lambda_0}{2}} m_{H_0}^3 + 6 \sqrt{2\lambda_0 m_{Z_0}^4} m_{H_0},$$

$$\{T_H^{R\xi}\}_\varepsilon = 0$$  (3.8)

and

$$[T_H^{R\xi}]_f = 3 \sqrt{\frac{\lambda_0}{2}} m_{H_0}^3 + 3\sqrt{2\lambda_0 m_{Z_0}^2} m_{H_0} \ln \frac{\mu^2}{m_{H_0}^2} + 3\sqrt{2\lambda_0 m_{Z_0}^4} \ln \frac{\mu^2}{m_{Z_0}^2},$$

$$\{T_H^{R\xi}\}_f = 0.$$  (3.9)

These expressions show that the tadpole sum is $\xi$-independent both in its divergent and in its finite part. Note that tadpoles are external momentum independent objects.

A detailed descriptions of the steps that we followed is presented in Appendix [D.1](#).  

### 3.2 Corrections to the gauge boson mass

Starting the 2-point function calculations, before we compute the Higgs 2-point function, we first move out of the way the $Z$ 2-point function. It will be needed for the renormalization of the $Z$ mass.

A gauge-boson vacuum polarization amplitude can be Lorentz-covariantly split into a transverse and a longitudinal part, as we show in Appendix [D.1](#). So, following this
procedure we obtain that the quantity that enters in the renormalization of the mass of the $Z$ gauge boson is therefore

$$\mathcal{M}_Z = -\frac{1}{3} \left( g^{\mu\nu} \frac{p^\mu p^\nu}{p^2} \right) \mathcal{M}_{Z,\mu\nu}(p). \quad (3.10)$$

We now start computing the one-loop Feynman diagrams contributing to $\mathcal{M}_{Z,\mu\nu}^{R,\phi}$.

The first contributing diagram has a Higgs running in the loop:

$$p \quad \Rightarrow \quad i\mathcal{M}_{Z,\mu\nu}^{R,\phi}$$

and, in DR, it is equal to

$$\mathcal{M}_{Z,\mu\nu}^{R,\phi} = -2 g_{\mu\nu} \frac{m_{Z0}^2}{m_{H0}^2} \lambda_0 \mu^\xi A_0(m_{H0}) \quad (3.11)$$

Next we meet a couple of “sunset” diagrams. The first is

$$p \quad \Rightarrow \quad i\mathcal{M}_{Z,\mu\nu}^{R,\phi Z}$$

In DR and using Eq. (B.35), it can be expressed as

$$\mathcal{M}_{Z,\mu\nu}^{R,\phi Z} = 8 \frac{m_{Z0}^2}{m_{H0}^2} \lambda_0 \mu^\xi \left\{ -g_{\mu\nu} B_0(p, m_{Z0}, m_{H0}) + (1 - \xi) C_{\mu\nu}^1(p, m_{Z0}, m_{H0}, m_{\chi0}) \right\}. \quad (3.12)$$

Notice that the $C$-type integral above is a special PV case, computed in Appendix [B] as well.

The next sunset diagram is the last that contributes to the one-loop correction of the gauge boson propagator:

$$p \quad \Rightarrow \quad i\mathcal{M}_{Z,\mu\nu}^{R,\phi \chi}$$

and its explicit form reads

$$\mathcal{M}_{Z,\mu\nu}^{R,\phi \chi} = 8 \frac{m_{Z0}^2}{m_{H0}^2} \lambda_0 \mu^\xi B_{\mu\nu}(p, m_{\chi0}, m_{H0}) \quad (3.12)$$
Adding up all contributions we obtain

\[
\mathcal{M}_{Z,\mu\nu}^{R\xi} = \frac{m_{Z_0}^2}{m_{H_0}} \lambda_0 \mu^\varepsilon \left\{ -2g_{\mu\nu}A_0(m_{H_0}) - 8g_{\mu\nu}m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) + 8(1 - \xi) m_{Z_0}^2 C_{\mu\nu}^1(p, m_{Z_0}, m_{H_0}, m_{\chi_0}) + 8B_{\mu\nu}(p, m_{\chi_0}, m_{H_0}) \right\}. \tag{3.13}
\]

The contraction that we need is Eq. (3.10), which for general \( p \) has the form

\[
\mathcal{M}_Z^{R\xi}(p) = -\frac{1}{3} \frac{m_{Z_0}^2}{m_{H_0}} \lambda_0 \frac{\mu^\varepsilon}{16\pi^2} \left\{ -2(d + \varepsilon)A_0(m_{H_0}) - 8(d + \varepsilon)m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) + 8(1 - \xi) m_{Z_0}^2 C_{\mu\nu}^1(p, m_{Z_0}, m_{H_0}, m_{\chi_0}) + 8B_{\mu\nu}(p, m_{\chi_0}, m_{H_0}) \right\}. \tag{3.14}
\]

Specializing now to \( p = 0 \), we can express the result as

\[
(4\pi)^{d/2} \mathcal{M}_Z^{R\xi} = \mu^\varepsilon \left( [\mathcal{M}_Z^{R\xi}]_\varepsilon + \{\mathcal{M}_Z^{R\xi}\}_\varepsilon + \mathcal{M}_Z^{R\xi}_f + \{\mathcal{M}_Z^{R\xi}\}_f \right), \tag{3.15}
\]

with

\[
\varepsilon[\mathcal{M}_Z^{R\xi}]_\varepsilon = 12 \frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}, \quad \{\mathcal{M}_Z^{R\xi}\}_\varepsilon = 0. \tag{3.16}
\]

The anomalous dimension of \( Z \) can be also determined now, through the relation

\[
\delta A_R^{\xi} = -\frac{d\mathcal{M}_Z^{R\xi}(p)}{dp^2} \bigg|_{p^2=0} = \frac{\mu^\varepsilon}{(4\pi)^{d/2}} \left( [\delta A_R^{\xi}]_\varepsilon + \{\delta A_R^{\xi}\}_\varepsilon + [\delta A_R^{\xi}]_f + \{\delta A_R^{\xi}\}_f \right), \tag{3.17}
\]

with

\[
\varepsilon[\delta A_R^{\xi}]_\varepsilon = -\frac{4}{3} \lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2}, \quad \{\delta A_R^{\xi}\}_\varepsilon = 0. \tag{3.18}
\]

Evidently, all divergent parts in this sector are \( \xi \)-independent.
Regarding the finite parts, we have

\[ (\mathcal{M}_Z^{R_\xi})_f = -\frac{\lambda_0 m_{Z_0}^4 (\xi - 19)}{3m_{H_0}^2} + \frac{8\lambda_0 m_{Z_0}^6 (\xi - 1)\xi \ln \frac{m_{Z_0}^2}{m_{H_0}^2}}{3 \left( m_{H_0}^2 - m_{Z_0}^2 \right) \left( m_{H_0}^2 - m_{Z_0}^2 \xi \right)} + 2\lambda_0 m_{Z_0}^4 \ln \left( m_{H_0}^2 \right) \left( m_{H_0}^2 (\xi - 9) + 4m_{Z_0}^2 (\xi + 1) \right) \]

\[ + \frac{2\lambda_0 m_{Z_0}^4 \ln \left( m_{Z_0}^2 \right) \left( m_{Z_0}^2 \xi + m_{H_0}^2 m_{Z_0}^2 (4\xi^2 - 5\xi - 9) + 9m_{Z_0}^4 \xi \right)}{3m_{H_0}^2 \left( m_{H_0}^2 - m_{Z_0}^2 \right) \left( m_{H_0}^2 - m_{Z_0}^2 \xi \right)} - \frac{6\lambda_0 m_{Z_0}^4 \ln \mu^2}{m_{H_0}^2} + \frac{8\lambda_0 m_{Z_0}^4}{3m_{H_0}^2} - 2\lambda_0 m_{Z_0}^4 \xi \ln \xi \frac{m_{Z_0}^2}{m_{H_0}^2} \xi - \frac{4\lambda_0 m_{Z_0}^2}{3} \] (3.19)

and

\[ (\delta A^{R_\xi})_f = -\frac{2}{3} \frac{m_{Z_0}^2 \lambda_0}{\left( m_{H_0}^2 - m_{Z_0}^2 \xi \right)} \left( 1 + \ln \frac{\mu^2}{m_{H_0}^2} \right) - \frac{m_{Z_0}^2}{m_{H_0}^2} \left( 1 + \xi \ln \frac{\mu^2}{m_{Z_0}^2} \right) \right) . \] (3.20)

The finite parts here are gauge dependent and we do not separate them further to \( \xi \)-independent and \( \xi \)-dependent parts. The limit \( \xi \to \infty \) is divergent.

### 3.3 Corrections to the Higgs mass

We move on to the Higgs propagator. The first diagram we encounter is

\[ p \quad \begin{array}{ccc} k \end{array} \quad = \quad i\mathcal{M}_{H}^{R_\xi, Z} \]

and after reductions performed in Appendix [D.1] finally the above integral is written as

\[ \mathcal{M}_{H}^{R_\xi, Z} = \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^\varepsilon \left\{ 6A_0(m_{Z_0}) + 2\xi A_0(m_{\chi_0}) \right\} . \] (3.21)

The next contribution comes from the diagram

\[ p \quad \begin{array}{ccc} k \end{array} \quad = \quad i\mathcal{M}_{H}^{R_\xi, \phi} \]

and its explicit form is given by

\[ \mathcal{M}_{H}^{R_\xi, \phi} = 3\lambda_0 \mu^\varepsilon A_0(m_{H_0}) \] (3.22)

in DR.

Next comes the Goldstone loop
which, following Appendix D.1 is equal to

\[ M_{H^{\xi,\chi}}^{R} = -\frac{2\lambda_0^2}{m_{H_0}^2} \mu^\xi m_{\chi_0}^2 A_0(m_{\chi_0}) . \]  

(3.23)

It is easy to check that all of the above three diagrams are reducible Tadpoles corresponding to the three Tadpoles of Section 3.1.

A few vacuum polarization diagrams are in order. The first is

\[ M_{H^{\xi,\phi\phi}}^{R} = 9 \lambda_0 m_{H_0}^2 \mu^\xi B_0(p, m_{H_0}, m_{H_0}) . \]  

(3.24)

The Goldstone loop contribution

\[ M_{H^{\xi,\chi\chi}}^{R} = 4 \lambda_0 \mu^\xi \left\{ m_{\chi_0}^2 A_0(m_{\chi_0}) + (m_{\chi_0}^2 - p^2)g_{\mu\nu}B^\mu\nu(p, m_{\chi_0}, m_{\chi_0}) + p_\mu p_\nu B^\mu\nu(p, m_{\chi_0}, m_{\chi_0}) \right\} . \]  

(3.25)

Slightly more complicated is the gauge boson loop

\[ M_{H^{\xi,ZZ}}^{R} = i M_{H}^{R} . \]  

(3.26)
and expressing it by standard steps in terms of PV integrals, it becomes

\[
M_{\mu\nu}^{R_\xi,ZZ} = \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^\varepsilon \left\{ dB_0(p, m_{Z_0}, m_{Z_0}) - (1 - \xi) \left\{ g_{\mu\nu} C^{1\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ g_{\mu\nu} C^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} \\
+ (1 - \xi)^2 \left\{ g_{\mu\nu} C^{1\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) + p_\mu p_\nu D^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \right\} \\
+ (m_{Z_0}^2 - p^2) g_{\mu\nu} D^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) + p_\mu p_\nu D^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \right\} \right\}
\]

(3.26)

where the \(a = 1, 2, 3\) superscripts on the \(C_0\)-integrals correspond to the different combinations of the denominators according to Eq. (B.12) of Appendix B. The \(D^{\mu\nu}\) integrals are defined in Eq. (B.39).

The last contribution to the one-loop correction of the Higgs mass comes from the sunset

\[
\begin{array}{c}
\begin{array}{c}
\kappa + p \\
\downarrow \\
\mu \\
\end{array}
\end{array}
\]

where using again the standard steps, we are allowed to write this as

\[
M_{\mu\nu}^{R_\xi,\chi Z} = 8 \lambda_0 m_{Z_0}^2 m_{H_0}^2 \mu^\varepsilon \left\{ - g_{\mu\nu} B^{\mu\nu}(p, m_{\chi_0}, m_{Z_0}) \\
+ (1 - \xi) \left\{ g_{\mu\nu} B^{\mu\nu}(p, m_{\chi_0}, m_{\chi_0}) + (m_{Z_0}^2 - p^2) g_{\mu\nu} C^{1\mu\nu}(p, m_{Z_0}, m_{\chi_0}, m_{Z_0}) \right\} \\
+ p_\mu p_\nu C^{1\mu\nu}(p, m_{\chi_0}, m_{Z_0}, m_{\chi_0}) \right\} \right\},
\]

(3.27)

where \(C^{1\mu\nu}\) is defined in Eq. (B.34) in Appendix B.

Finally, summing up all contributions into \(M_{\mu\nu}^{R_\xi}\) we obtain

\[
M_{\mu\nu}^{R_\xi}(p) = \mu^\varepsilon \left\{ 3 \lambda_0 A_0(m_{H_0}) + \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 A_0(m_{Z_0}) + 2 \xi \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 A_0(m_{\chi_0}) \right\} \\
- 2 \frac{\lambda_0}{m_{H_0}^2} m_{\chi_0}^2 A_0(m_{\chi_0}) + 9 \lambda_0 B_0(p, m_{H_0}, m_{H_0}) \right\} \\
+ 4 \frac{\lambda_0}{m_{H_0}^2} \left( m_{\chi_0}^2 A_0(m_{\chi_0}) + (m_{\chi_0}^2 - p^2) g_{\mu\nu} B^{\mu\nu}(p, m_{\chi_0}, m_{\chi_0}) + p_\mu p_\nu B^{\mu\nu}(p, m_{\chi_0}, m_{\chi_0}) \right) \right\}
\]

19
\[ \begin{align*}
+ \quad 4d \frac{m_{Z_0}^4}{m_{H_0}^2} \lambda_0 B_0(p, m_{Z_0}, m_{Z_0}) - 8\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} g_{\mu\nu} B^{\mu\nu}(p, m_{\chi_0}, m_{Z_0}) \\
+ \quad \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 (1 - \xi) \left\{ -4m_{Z_0}^2 g_{\mu\nu} C^{1\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) - 4m_{Z_0}^2 g_{\mu\nu} C^{\mu\nu}(p, p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ \quad 8g_{\mu\nu} B^{\mu\nu}(p, m_{\chi_0}, m_{\chi_0}) + 8(m_{Z_0}^2 - p^2)g_{\mu\nu} C^{1\mu\nu}(p, m_{\chi_0}, m_{Z_0}, m_{\chi_0}) \\
+ \quad 8p_{\mu} p_{\nu} C^{1\mu\nu}(p, m_{\chi_0}, m_{Z_0}, m_{\chi_0}) \right\} \\
+ \quad \frac{4m_{Z_0}^2}{m_{H_0}^2} \lambda_0 (1 - \xi)^2 \left\{ g_{\mu\nu} C^{1\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ \quad (m_{Z_0}^2 - p^2)g_{\mu\nu} D^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) + p_{\mu} p_{\nu} D^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \right\} \right\}
\end{align*} \]

Note that the reduction of the \( g_{\mu\nu} D^{\mu\nu} \) and \( p_{\mu} p_{\nu} D^{\mu\nu} \) terms give only \( C_0 \) and \( D_0 \) contributions which are finite. We then have that

\[ (4\pi)^{d/2} \mathcal{M}_H^{R_\epsilon} = \mu^\epsilon \left( [\mathcal{M}_H^{R_\epsilon}]_\xi + \{\mathcal{M}_H^{R_\epsilon}\}_\xi + [\mathcal{M}_H^{R_\epsilon}]_f + \{\mathcal{M}_H^{R_\epsilon}\}_f \right) \]

with

\[ \varepsilon [\mathcal{M}_H^{R_\epsilon}]_\xi = 24\lambda_0 m_{Z_0}^2 + 36\lambda_0 \frac{m_{Z_0}^4}{m_{H_0}^2} \]

\[ \{\mathcal{M}_H^{R_\epsilon}\}_\xi = 0 \]

and

\[ [\mathcal{M}_H^{R_\epsilon}]_f = 3\lambda_0 m_{H_0}^2 + 6\lambda_0 \frac{m_{Z_0}^4}{m_{H_0}^2} + 12\lambda_0 m_{H_0}^2 \ln \frac{\mu^2}{m_{H_0}^2} + 18\lambda_0 \frac{m_{Z_0}^4}{m_{H_0}^2} \ln \frac{\mu^2}{m_{Z_0}^2} \]

\[ \{\mathcal{M}_H^{R_\epsilon}\}_f = 0 \]

at \( p = 0 \).

We are also able to compute the anomalous dimension of the Higgs, determined by

\[ \delta \phi^{R_\epsilon} = - \frac{d\mathcal{M}_H^{R_\epsilon}(p)}{dp^2} \bigg|_{p^2=0} = \frac{\mu^\epsilon}{(4\pi)^{d/2}} \left( [\delta \phi^{R_\epsilon}]_\xi + \{\delta \phi^{R_\epsilon}\}_\xi + [\delta \phi^{R_\epsilon}]_f + \{\delta \phi^{R_\epsilon}\}_f \right) \]

with

\[ \varepsilon [\delta \phi^{R_\epsilon}]_\xi = 12\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \]

\[ \{\delta \phi^{R_\epsilon}\}_\xi = 0 \]

and

\[ [\delta \phi^{R_\epsilon}]_f = 2\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} + 6\lambda_0 \frac{m_{Z_0}^4}{m_{H_0}^2} \ln \frac{\mu^2}{m_{Z_0}^2} \]
This sector turns out to be $\xi$-independent.

### 3.4 Corrections to the Higgs cubic vertex

Triangle diagrams with Higgs external legs yield corrections to the Higgs cubic vertex. Such corrections will play a crucial role in the definition of the one-loop scalar potential. The external momenta are taken to be all inflowing, thus satisfying $p_1 + p_2 + p_3 = 0$. Triangle diagrams can be split in "reducible" and "irreducible" kinds. Reducible are those that are expressible in terms of 2-point function diagrams and irreducible are those that are not.

#### 3.4.1 Reducible Triangles

The first reducible Triangle diagram is:

$$
\begin{align*}
\frac{k + P_1}{p_1} \frac{k}{k} \frac{p_3}{p_2} &= iK^{R\xi,\phi\phi}_H \\
&= \frac{18}{\sqrt{2}} m_{H_0} \mu^\xi B_0(P_1, m_{H_0}, m_{H_0}).
\end{align*}
$$

(3.34)

The factor of 3 is due to two additional diagrams, obtained from the above by cyclically permuting the external momenta. These contributions however, evaluated at either $p_1 = p_2 = p_3 = 0$ or $p_1^2 = p_2^2 = p_3^2 = m_H^2$ give an identical result.

The next diagram is one with a Goldstone in the loop:

$$
\begin{align*}
\frac{k + P_1}{p_1} \frac{k}{k} \frac{p_3}{p_2} &= iK^{R\xi,\chi\chi}_H \\
&= \frac{18}{\sqrt{2}} m_{H_0} \mu^\xi B_0(P_1, m_{H_0}, m_{H_0}).
\end{align*}
$$

(3.34)
and it is equal to Eq. (D.37) divided by $v_0$:

$$
\kappa_{H}^{R_{\xi},\chi^\chi} = 3 \cdot 8 \frac{\lambda_0^{3/2}}{m_{H_0}^3} \mu^\varepsilon \left\{ m_{\chi_0}^2 A_0(m_{\chi_0}) + \left( m_{\chi_0}^2 - P_1^2 \right) g_{\mu \nu} B_{\mu \nu} (P_1, m_{\chi_0}, m_{\chi_0}) + P_{1 \mu} P_{1 \nu} B_{\mu \nu} (P_1, m_{\chi_0}, m_{\chi_0}) \right\}.
$$

(3.35)

The factor of 3 has a similar origin as before.

The diagram with a gauge boson loop

is equal to Eq. (D.40) divided by $v_0$:

$$
\kappa_{H}^{R_{\xi},\chi^Z} = 3 \cdot 8 \frac{m_{Z_0}^4}{\sqrt{2} m_{H_0}^3} \lambda_0^{3/2} \mu^\varepsilon \left\{ dB_0(P_1, m_{Z_0}, m_{Z_0}) - (1 - \xi) \left\{ g_{\mu \nu} C_{1 \mu \nu} (p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) + g_{\mu \nu} C_{1 \mu \nu} (P_1, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} \right\}.
$$

(3.36)

The last reducible Triangle is:

is equal to Eq. (D.42) divided by $v_0$:

$$
\kappa_{H}^{R_{\xi},\chi^Z} = 3 \cdot 16 \frac{m_{Z_0}^2}{\sqrt{2} m_{H_0}^3} \lambda_0^{3/2} \mu^\varepsilon \left\{ - g_{\mu \nu} B_{\mu \nu} (P_1, m_{\chi_0}, m_{Z_0}) + (1 - \xi) \left\{ g_{\mu \nu} B_{\mu \nu} (P_1, m_{\chi_0}, m_{\chi_0}) + \left( m_{Z_0}^2 - P_1^2 \right) g_{\mu \nu} C_{1 \mu \nu} (P_1, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \right\} \right\}.
$$

(3.36)
\[ + P_{1\mu}P_{1\nu}C^{1\mu\nu}(P_1, m_{\chi_0}, m_{Z_0}, m_{\chi_0}) \]  
\[ = (\pi)^{d/2} \mathcal{K}_H^{R, \text{red.}} = \mu^\varepsilon \left( \left[ \mathcal{K}_H^{R, \text{red.}} \right]_\varepsilon + \left\{ \mathcal{K}_H^{R, \text{red.}} \right\}_f \right), \]  
(3.38)

Let us collect all reducible Triangle contributions by adding Eq. (3.34), Eq. (3.35), Eq. (3.36) and Eq. (3.37). At zero external momenta, we obtain

\[ H = \mu^\varepsilon \left( \left[ \mathcal{K}_H^{R, \text{red.}} \right]_\varepsilon + \left\{ \mathcal{K}_H^{R, \text{red.}} \right\}_f \right), \]  
(3.39)

\[ \left\{ \mathcal{K}_H^{R, \text{red.}} \right\}_f = 0. \]  
(3.40)

### 3.4.2 Irreducible Triangles

We now turn to the irreducible Triangles. All irreducible Triangle diagrams can be labelled by the momenta \( P_1 = p_1 \) and \( P_2 = p_1 + p_3 \). A simplifying consequence of renormalizing at \( p_i = 0 \) is that we can set \( P_i = 0 \), hence we can extract both divergent and finite parts, using

\[ \lim_{P_i \to 0} \mathcal{K}_H^{R, \cdots}(P_1, P_2) \equiv \mathcal{K}_H^{R, \cdots}(0, 0). \]  
(3.41)

The limit should be carefully taken, as explained in Sect. 2. Regarding denominators, from now on, we start following the notation of Eq. (A.3). Here, apart from finite integrals of the \( E \)-type, we will also see the appearance of several divergent \( U \)-integrals. All finite integrals and \( U \)-integrals here and in the following are computed (sometimes without further notice) in Appendices E and C respectively. Finite diagrams do not play a role in the renormalization program but they contribute to the scalar potential.

The first contribution to the irreducible Triangle class involves a Higgs loop and it is finite. It is the diagram

\[ k + P_1 \]
\[ k + P_2 \]
\[ k + P_3 \]
\[ = i\mathcal{K}_H^{R, \phi\phi\phi}. \]
It is equal to
\[
\mathcal{K}_H^{R_c,\phi\phi\phi} = \frac{108}{\sqrt{2}} \lambda^{3/2} m^3 H_{0} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(-i) D_1 D_2 D_3} \quad (3.42)
\]
with a symmetry factor \( S_{K_{H}}^{5} = 1 \). In DR it can be expressed as
\[
\mathcal{K}_H^{R_c,\phi\phi\phi}(P_1, P_2) = \frac{108}{\sqrt{2}} \lambda^{3/2} m^3 H_{0} \mu^e C_0(P_1, P_2, m_{H_0}, m_{H_0}, m_{H_0}). \quad (3.43)
\]
As explained,
\[
\lim_{P_i \to 0} \mathcal{K}_H^{R_c,\phi\phi\phi}(P_1, P_2) \equiv \mathcal{K}_H^{R_c,\phi\phi\phi}(0, 0). \quad (3.44)
\]
There is another finite diagram, the one with a gauge boson loop:
\[
\begin{align*}
\text{This is the first of several irreducible diagrams whose explicit form is not particularly illuminating, so we directly transfer it to Appendix E.}
\end{align*}
\]
\[
\text{Now, let us move on to diagrams that have both an infinite and a finite part. The first such diagram is}
\]

\footnote{We thank A. Chatziagapiou for pointing out a factor of 2 in this diagram that was missing in the previous version of the paper.}
Next, we have the diagram with two Goldstones and one gauge boson in the loop:

\[
\begin{align*}
\begin{array}{c}
\begin{tikzpicture}
  \draw[thick] (0,0) -- (1,0) -- (1,1) -- (0,1) -- cycle;
  \draw[thick] (-0.5,0.5) -- (0.5,0.5);
  \draw[thick] (-0.5,0.5) -- (-0.5,0);
  \draw[thick] (0.5,0.5) -- (0.5,0);
\end{tikzpicture}
\end{array}
\end{align*}
\]

It is equal to

\[
\kappa_{H, Z\chi Z}^{R, \chi Z} = 32v_0\lambda_0^2 m_0^4 m_0^4 H_0 g_{\mu
u} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{D_1} \left(-g_{\mu\alpha} + \frac{(1-\xi)k_\mu k_\alpha}{k^2 - \xi m_0^2} \right) \left(-g_{\nu\beta} + \frac{(1-\xi)(k + P_2)_{\nu}(k + P_2)_{\beta}}{(k + P_2)^2 - \xi m_0^2} \right)
\times \frac{(k + P_2)^\alpha (k + P_2)^\beta}{D_3},
\]

(3.48)

with a symmetry factor \( S_{H, Z\chi Z}^2 = 1 \). In DR it becomes

\[
\kappa_{H, Z\chi Z}^{R, \chi Z} (P_1, P_2) = -32v_0\lambda_0^2 m_0^4 m_0^4 H_0 \varepsilon \left\{ B_0(P_1, m_{Z0}, m_{Z0}) + m_0^2 C_0(P_1, m_{Z0}, m_{Z0}, m_{\chi0}) \right.
\]

\[
- (1 - \xi) \left\{ 2B_0(P_1, m_{Z0}, m_{Z0}) + 2m_0^2 C_0(P_1, m_{Z0}, m_{Z0}, m_{\chi0}) \right.
\]

\[
+ m_0^4 D_0(P_1, P_2, m_{Z0}, m_{Z0}, m_{\chi0}, m_{\chi0}) + P_1 P_2 \rho D_{\mu\nu}(P_1, P_2, m_{Z0}, m_{Z0}, m_{\chi0}, m_{\chi0}) \bigg\}
\]

\[
+ (1 - \xi)^2 \left\{ D_{\mu\nu}(P_1, P_2, m_{Z0}, m_{Z0}, m_{\chi0}, m_{\chi0}) + m_0^2 E_4(D_1, D_2, D_3, m_{\chi0}, D_5(P_1, m_{\chi0})) \right.
\]

\[
+ 2P_{\mu} P_{\nu} E_4(D_1, D_2, D_3, m_{\chi0}, D_5(P_1, m_{\chi0})) \bigg\}
\]

\[
+ P_{\mu} P_{\nu} E_4(D_1, D_2, D_3, m_{\chi0}, D_5(P_1, m_{\chi0})) \bigg\}
\}
\]

where the mass arguments of the \( D_{1,2,3} \) denominators are easily recovered from the \( Z\chi Z \) superscript structure of the diagram: \( D_1(m_{Z}) \), \( D_2(m_{\chi}) \) and \( D_3(m_{Z}) \). We are not done yet since there are three different ways to insert the Goldstone propagator in the loop. Therefore, there are two more contributing diagrams of the same kind as \( \kappa_{H, Z\chi Z}^{R, \chi Z} \). These are the diagrams

\[
\begin{align*}
\begin{array}{c}
\begin{tikzpicture}
  \draw[thick] (0,0) -- (1,0) -- (1,1) -- (0,1) -- cycle;
  \draw[thick] (-0.5,0.5) -- (0.5,0.5);
  \draw[thick] (-0.5,0.5) -- (-0.5,0);
  \draw[thick] (0.5,0.5) -- (0.5,0);
\end{tikzpicture}
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\begin{tikzpicture}
  \draw[thick] (0,0) -- (1,0) -- (1,1) -- (0,1) -- cycle;
  \draw[thick] (-0.5,0.5) -- (0.5,0.5);
  \draw[thick] (-0.5,0.5) -- (-0.5,0);
  \draw[thick] (0.5,0.5) -- (0.5,0);
\end{tikzpicture}
\end{array}
\end{align*}
\]

Performing the calculations at zero external momenta, the above diagrams have identical divergent and finite parts with \( \kappa_{H, Z\chi Z}^{R, \chi Z} \), that is

\[
\kappa_{H, Z\chi Z}^{R, \chi Z} (0, 0) = \kappa_{H, Z\chi Z}^{R, Z\chi Z} (0, 0) = \kappa_{H, Z\chi Z}^{R, \chi ZZ} (0, 0).
\]

(3.50)

Next, we have the diagram with two Goldstones and one gauge boson in the loop:
The last one-loop correction to the three-point vertex comes from the irreducible Triangle finite parts:

Again here, at zero external momenta all three diagrams have the same divergent and finite parts:

\[ K^{R_\xi,\chi Z\chi}_H = -32v_0\lambda_0^2m_Z^2m_H^2 \int \frac{d^4k}{(2\pi)^4} \frac{D_1D_2D_3}{(k + P_1)\mu(k + P_1)\nu(k + P_1)\cdot k} \]

with symmetry factor \( S_k^8 = 1 \). In DR it reads

\[ K^{R_\xi,\chi Z\chi}_H(P_1, P_2) = 32v_0\lambda_0^2m_Z^2m_H^2 \left\{ U_{K4}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) \right. \\
+ 2(P_1 + P_2)_\mu C_{K3}^{\mu\nu}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) \\
+ (P_1\mu P_1\nu + 2P_1\mu P_2\nu + P_2\mu P_2\nu) C^{\mu\nu}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) \\
+ 2P_1 \cdot P_2 g_{\mu\nu} C^{\mu\nu}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) \\
- (1 - \xi) \left\{ U_{K4}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) + m_{\chi0}^2 g_{\mu\nu} C^{\mu\nu}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) \right. \\
+ 2(P_1 + P_2)_\mu C_{K3}^{\mu\nu}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) \\
+ (P_1\mu P_1\nu + 3P_1\mu P_2\nu + P_2\mu P_2\nu) C^{\mu\nu}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) \\
+ P_1 \cdot P_2 g_{\mu\nu} C^{\mu\nu}(P_1, P_2, m_{Z0}, m_{\chi0}, m_{\chi0}) \\
\left. \right\} \right\}. \quad (3.52) \]

Now, similarly to the previous case there are three ways to insert the gauge boson in the loop, which means that there are two more diagrams of the same kind as \( K^{R_\xi,\chi Z\chi}_H \):

\[ = iK^{R_\xi,\chi Z\chi}_H(P_1, P_2) \quad \text{and} \quad \text{---} \text{---} \text{---} = iK^{R_\xi,\chi Z\chi}_H(P_1, P_2). \]

Again here, at zero external momenta all three diagrams have the same divergent and finite parts:

\[ K^{R_\xi,\chi Z\chi}_H(0, 0) = K^{R_\xi,\chi Z\chi}_H(0, 0) = K^{R_\xi,\chi Z\chi}_H(0, 0). \quad (3.53) \]

The last one-loop correction to the three-point vertex comes from the irreducible Triangle
are the finite part of this sector. The finite parts, collected according to the loop propagators, function.

We see that the irreducible Triangles do not contribute to the divergent part of the 3-point function given by the expression

\[ K_{\mathcal{H}}^{R_{\xi},\text{XXX}} = -32v_0 \frac{\lambda_0^2}{m_{H_0}^4} \int \frac{d^4k}{(2\pi)^4} \frac{-ik \cdot (k + P_1)}{D_1 D_2 D_3} (k + P_1) \cdot (k + P_2)(k + P_2) \cdot k, \]

(3.54)

with symmetry factor \( S_{\mathcal{H}}^0 = 1 \). There is only one diagram of this kind and its explicit form in DR reads

\[ K_{\mathcal{H}}^{R_{\xi},\text{XXX}}(P_1, P_2) = -32v_0 \frac{\lambda_0^2}{m_{H_0}^4} \mu^\varepsilon \left\{ U_{K_6}(P_1, P_2, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) + 2(P_1 + P_2) \mu U_{K_5}^\mu(P_1, P_2, m_{\chi_0}, m_{\chi_0}) + (P_1 P_1 + 3P_1 P_2 + P_2 P_2) U_{K_4}^{\mu \nu}(P_1, P_2, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) + P_1 \cdot P_2 U_{K_4}(P_1, P_2, m_{\chi_0}, m_{\chi_0}) + (P_1 P_1 P_2 + P_1 P_2 P_2) U_{K_4}(P_1, P_2, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) + P_1 \cdot P_2 P_1 + P_2 P_2 \right\}, \]

(3.55)

Summing up all the irreducible Triangles, we find that at zero external momenta

\[ (4\pi)^{d/2} K_{\mathcal{H}}^{R_{\xi},\text{irred.}} = \mu^\varepsilon \left( [K_{\mathcal{H}}^{R_{\xi},\text{irred.}}]_\epsilon + \{K_{\mathcal{H}}^{R_{\xi},\text{irred.}}\}_\epsilon + [K_{\mathcal{H}}^{R_{\xi},\text{irred.}}]_f + \{K_{\mathcal{H}}^{R_{\xi},\text{irred.}}\}_f \right) \]

(3.56)

with

\[
\begin{align*}
[K_{\mathcal{H}}^{R_{\xi},\text{irred.}}]_\epsilon &= 0 \\
\{K_{\mathcal{H}}^{R_{\xi},\text{irred.}}\}_\epsilon &= 0
\end{align*}
\]

(3.57)

and

\[
\begin{align*}
[K_{\mathcal{H}}^{R_{\xi},\text{irred.}}]_f &= -\frac{m_{H_0}}{\sqrt{2} \lambda_0} \left( 54\lambda_0^2 + 48 \frac{\lambda_0^2 m_{Z_0}^4}{m_{H_0}^4} \right) \\
\{K_{\mathcal{H}}^{R_{\xi},\text{irred.}}\}_f &= 0.
\end{align*}
\]

(3.58)

We see that the irreducible Triangles do not contribute to the divergent part of the 3-point function.

It is worth looking a bit closer at the cancellation of the gauge fixing parameter \( \xi \) from the finite part of this sector. The finite parts, collected according to the loop propagators, are

\[
\begin{align*}
\int & \text{Diagram} = \frac{m_{H_0}}{\sqrt{2} \lambda_0} \left( -48 \lambda_0^2 m_{Z_0}^4 m_{H_0}^4 \frac{\epsilon^2}{m_{H_0}^4} + \frac{16 \lambda_0^2 m_{Z_0}^4 \xi^2}{m_{H_0}^4} \right)
\end{align*}
\]
\[
\begin{align*}
\text{Diagram 1} & = \frac{m_{H_0}}{\sqrt{2\lambda_0}} \left( \frac{48\lambda_0^3 m_0^4 \xi^2}{m_{H_0}^2} - \frac{96\lambda_0^2 m_0^4 \xi^2}{m_{H_0}^2} \ln \frac{\mu^2}{m_{Z_0}^2} \right) \\
\text{Diagram 2} & = \frac{m_{H_0}}{\sqrt{2\lambda_0}} \left( \frac{48\lambda_0^3 m_0^4 \xi^2}{m_{H_0}^2} + \frac{288\lambda_0^2 m_0^4 \xi^2}{m_{H_0}^2} \ln \frac{\mu^2}{m_{Z_0}^2} \right) \\
\text{Diagram 3} & = \frac{m_{H_0}}{\sqrt{2\lambda_0}} \left( -\frac{80\lambda_0^3 m_0^4 \xi^2}{m_{H_0}^2} - \frac{192\lambda_0^2 m_0^4 \xi^2}{m_{H_0}^2} \ln \frac{\mu^2}{m_{Z_0}^2} \right)
\end{align*}
\]

It is easy to see the cancellation of \( \xi \) in the sum.

We now add reducible and irreducible contributions into \( K_{H}^{R_{\xi}} = K_{H}^{R_{\xi, \text{red}}} + K_{H}^{R_{\xi, \text{irred}}} \), and we have

\[
(4\pi)^{d/2} K_{H}^{R_{\xi}} = \mu^\varepsilon \left( [K_{H}^{R_{\xi}}]_\varepsilon + \{K_{H}^{R_{\xi}}\}_f + \{K_{H}^{R_{\xi}}\}_f \right)
\]

where

\[
\varepsilon [K_{H}^{R_{\xi}}]_\varepsilon = \frac{m_{H_0}}{\sqrt{2\lambda_0}} \left( 108\lambda_0^2 + 144 \frac{\lambda_0^2 m_0^4}{m_{H_0}^4} \right)
\]

\[
\{K_{H}^{R_{\xi}}\}_f = 0
\]

and

\[
[K_{H}^{R_{\xi}}]_f = \frac{m_{H_0}}{\sqrt{2\lambda_0}} \left( -54\lambda_0^2 - 24 \frac{\lambda_0^2 m_0^4}{m_{H_0}^4} + 54\lambda_0^2 \ln \frac{\mu^2}{m_{Z_0}^2} + 72 \frac{\lambda_0^2 m_0^4}{m_{H_0}^4} \ln \frac{\mu^2}{m_{Z_0}^2} \right)
\]

\[
\{K_{H}^{R_{\xi}}\}_f = 0.
\]

### 3.5 Corrections to the quartic coupling

Diagrams with four external Higgs fields contribute through their divergent parts to the running of the Higgs quartic self coupling \( \lambda \) and through their finite parts they contribute to the one-loop scalar potential. They are collectively called ”Box diagrams”, denoted as \( B_H \) and come in three classes. The first two of these classes contain reducible diagrams and the third class contains irreducible Box diagrams, called ”Square (S)-Boxes”. The reducible class is further split in two subclasses, called ”Candy (C)-Boxes” and ”Triangular (T)-Boxes”. They have the following structure:

\[
\begin{align*}
\text{Candy Box} & = B_H^C, \\
\text{Triangular Box} & = B_H^T, \\
\text{Square Box} & = B_H^S
\end{align*}
\]

Regarding the momentum flow, we take all four external momenta \( p_i = 1, 2, 3, 4 \) to be inflowing and satisfying \( p_1 + p_2 + p_3 + p_4 = 0 \). Candies and S-Boxes come in three versions, corresponding to the usual \( s, t \) and \( u \) channels, where

\[
s = (p_1 + p_2)^2, \quad t = (p_1 + p_3)^2, \quad u = (p_1 + p_4)^2
\]
$T$-Boxes come in six versions instead because they are not invariant under a reflection with respect to the axis passing through the centre of the loop in the diagram. There are two inequivalent topologies and each topology comes with $s$, $t$ and $u$ channels. Any $U$-integral that may appear is dealt with in Appendix [C] and finite integrals of the $E$, $F$, $G$ and $H$-type in Appendix [E].

### 3.5.1 Reducible Boxes

Candies have the generic momentum dependence $B_{H}^{R_{e},C}(P_{1})$, where $P_{1} = \sqrt{s}$, $\sqrt{t}$ and $\sqrt{u}$ for the three channels. Their total contribution is then a sum over $P_{1}$.

The first Candy is the famous diagram

\[ p_{1} \quad k + P_{1} \quad p_{3} \quad k \quad p_{2} \quad p_{4} \]

\[ = iB_{H}^{R_{e},\phi\phi}, \quad S_{B_{H}}^{1} = \frac{1}{2} \]

which solely determines the $\beta$-function in pure scalar theories, in the case where the Higgs is expressed in the Cartesian basis, all other diagrams being finite. Here in the Polar basis we will see that this is still the case alas in a non-trivial way. The result for this diagram can be obtained from Eq. (3.34) (without the factor of 3) divided by $v_{0}$ and evaluated at $P_{1}$:

\[ B_{H}^{R_{e},\phi\phi} = 18 \lambda_{0}^{2} \mu_{\varepsilon} B_{0}(P_{1}, m_{H_{0}}, m_{H_{0}}). \]  

(3.63)

The Goldstone Candy

\[ p_{1} \quad k + P_{1} \quad p_{3} \quad k \quad p_{2} \quad p_{4} \]

\[ = iB_{H}^{R_{e},\chi\chi}, \quad S_{B_{H}}^{2} = \frac{1}{2} \]

is analogously equal to Eq. (3.35) divided by $v_{0}$ (without the factor of 3), evaluated at $P_{1}$:

\[ B_{H}^{R_{e},\chi\chi} = 8 \frac{\lambda_{0}^{2}}{m_{H_{0}}^{4}} \mu_{\varepsilon} \left\{ m_{X_{0}}^{2} A_{0}(m_{X_{0}}) + (m_{X_{0}}^{2} - P_{1}^{2}) g_{\mu\nu} B^{\mu\nu}(P_{1}, m_{X_{0}}, m_{X_{0}}) \right. \]

\[ + \left. P_{1\mu} P_{1\nu} B^{\mu\nu}(P_{1}, m_{X_{0}}, m_{X_{0}}) \right\}. \]  

(3.64)

The gauge Candy
is obtained from Eq. (3.36):

\[
\mathcal{B}^{R_c,ZZ}_H = 8 \frac{m^4_{Z_0}}{m^4_{H_0}} \lambda_0^2 \mu^\varepsilon \left\{ d\mathcal{B}_0(P_1, m_{Z_0}, m_{Z_0}) - (1 - \xi) \left\{ g_{\mu\nu} C^{1\mu\nu}(P_1, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} ight. \\
+ \left. g_{\mu\nu} C^{1\mu\nu}(P_1, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} \\
+ \left. (1 - \xi)^2 \left\{ g_{\mu\nu} C^{1\mu\nu}(P_1, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} \\
+ \left. (m^2_{Z_0} - P^2_1) g_{\mu\nu} D^{\mu\nu}(P_1, m_{Z_0}, m_{Z_0}, m_{\chi_0}) + P_{1\mu} P_{1\nu} D^{\mu\nu}(P_1, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\}.
\]

(3.65)

Finally, there is a mixed Candy obtained easily from Eq. (3.37):

\[
\mathcal{B}^{R_c,ZZ}_H = 16 \frac{m^4_{Z_0}}{m^4_{H_0}} \lambda_0^2 \mu^\varepsilon \left\{ -g_{\mu\nu} B^{\mu\nu}(P_1, m_{\chi_0}, m_{Z_0}) + (1 - \xi) \left\{ g_{\mu\nu} B^{\mu\nu}(P_1, m_{\chi_0}, m_{\chi_0}) \right\} + P_{1\mu} P_{1\nu} B^{\mu\nu}(P_1, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\}.
\]

(3.66)

The full contribution of the Candies is obtained by adding Eq. (3.64), Eq. (3.65) and Eq. (3.66) (summed over the three channels) and can be expressed as

\[
(4\pi)^{d/2} \mathcal{B}^{R_c,C}_H = \mu^\varepsilon \left[ |\mathcal{B}^{R_c,C}_H|_\varepsilon + \{ \mathcal{B}^{R_c,C}_H \}_\varepsilon + \mathcal{B}^{R_c,C}_H \right] + \{ \mathcal{B}^{R_c,C}_H \}_f
\]

(3.67)

where

\[
\varepsilon[\mathcal{B}^{R_c,C}_H]_\varepsilon = 108 \lambda_0^2 + 144 \lambda_0^2 \frac{m^4_{Z_0}}{m^4_{H_0}} + 4 \frac{s^2 \lambda_0^2}{m^4_{H_0}} + 4 \frac{t^2 \lambda_0^2}{m^4_{H_0}} + 4 \frac{u^2 \lambda_0^2}{m^4_{H_0}}
\]

30
\[ \{ \mathcal{B}_H^{R_{\xi}C} \}_{\varepsilon} = 0. \] (3.68)

and

\[ [\mathcal{B}_H^{R_{\xi}C}]_f = 54\lambda_0^2 \ln \frac{\mu^2}{m_{H_0}} + 72\lambda_0^2 m_Z^4 \ln \frac{\mu^2}{m_{Z_0}} \]

\[ \{ \mathcal{B}_H^{R_{\xi}C} \}_f = 0 \] (3.69)

for \( p_i = 0 \). We now turn to the \( T \)-Boxes that have three propagators in the loop. Each of the six channels of a given \( T \)-Box is determined by two linear combinations of the external momenta that we call \( P_1 \) and \( P_2 \). A consistent choice for \( (P_1, P_2) \) for the channels \( T_1, \ldots, T_6 \) is for example

\[ T_1 : (\sqrt{s}, p_1 + p_2 + p_3), \; T_2 : (\sqrt{t}, p_1 + p_3 + p_4), \; T_3 : (\sqrt{u}, p_1 + p_3 + p_4), \; T_4 : (\sqrt{s}, p_2 + p_3 + p_4), \; T_5 : (p_1, \sqrt{t}), \; T_6 : (p_1, \sqrt{u}). \]

The generic form of a \( T \)-Box is therefore \( \mathcal{B}_H^{R_{\xi}T}(P_1, P_2) \) and the total contribution is obtained by a sum over the six different pairs \((P_1, P_2)\). Note that the Mandelstam variables enter also via the inner products

\[ p_1 \cdot p_2 = p_3 \cdot p_4 = \frac{s}{2}, \quad p_1 \cdot p_3 = p_2 \cdot p_4 = \frac{t}{2}, \quad p_1 \cdot p_4 = p_2 \cdot p_3 = \frac{u}{2} \] (3.70)

All \( T \)-Boxes can be obtained from their corresponding irreducible Triangles. They share a common symmetry factor and analytical structure with all the difference encoded in the values that the pair \((P_1, P_2)\) can take. Hence, in this sector we give directly the formal expression in order to avoid unnecessary repetitions. To begin, we have two finite diagrams:

\[ \begin{aligned}
&\begin{tikzpicture}
\node (a) at (0,0) {$k$};
\node (b) at (-2,1) {$p_1$};
\node (c) at (2,1) {$p_3$};
\node (d) at (0,2) {$k + P_1$};
\node (e) at (-2,2) {$k + P_2$};
\node (f) at (2,2) {$p_2$};
\node (g) at (-1.5,3) {$k$};
\node (h) at (1.5,3) {$k$};
\node (i) at (-0.5,4) {$p_4$};
\draw (a) -- (b) -- (d) -- (e) -- (f) -- (i) -- (h) -- (g) -- (a);\end{tikzpicture}
\end{aligned} \]

\[ i \mathcal{B}_H^{R_{\xi},\phi\phi\phi} = \sum_{(P_1, P_2)} \frac{1}{v_0} i \kappa_{H}^{R_{\xi},\phi\phi\phi} (P_1, P_2) \]

and

\[ \begin{aligned}
&\begin{tikzpicture}
\node (a) at (0,0) {$k$};
\node (b) at (-2,1) {$p_1$};
\node (c) at (2,1) {$p_3$};
\node (d) at (0,2) {$k + P_1$};
\node (e) at (-2,2) {$k + P_2$};
\node (f) at (2,2) {$p_2$};
\node (g) at (-1.5,3) {$k$};
\node (h) at (1.5,3) {$k$};
\node (i) at (-0.5,4) {$p_4$};
\draw (a) -- (b) -- (d) -- (e) -- (f) -- (i) -- (h) -- (g) -- (a);\end{tikzpicture}
\end{aligned} \]

\[ i \mathcal{B}_H^{R_{\xi},ZZZ} = \sum_{(P_1, P_2)} \frac{1}{v_0} i \kappa_{H}^{R_{\xi},ZZZ} (P_1, P_2) \]
There are diagrams that have both divergent and finite parts. There are three with one Goldstone and two gauge bosons in the loop:

\[
\begin{align*}
\text{Diagram 1:} & \quad p_1 + P_1 \rightarrow k + P_2 \\
\text{Diagram 2:} & \quad p_1 + P_1 \rightarrow k + P_2 \\
\text{Diagram 3:} & \quad p_1 + P_1 \rightarrow k + P_2 
\end{align*}
\]

which are equal to

\[
i \mathcal{B}^\varnothing_{H} = \sum_{(P_1, P_2)} \frac{1}{v_0} i K^{R_\varnothing, \chi \chi \chi Z}(P_1, P_2), \quad i \mathcal{B}^{\chi \chi \chi Z}_{H} = \sum_{(P_1, P_2)} \frac{1}{v_0} i K^{\chi \chi \chi Z}(P_1, P_2)
\]

and

\[
i \mathcal{B}^{R_\varnothing, \chi \chi Z}_{H} = \sum_{(P_1, P_2)} \frac{1}{v_0} i K^{R_\varnothing, \chi \chi Z}(P_1, P_2) \quad \text{and} \quad i \mathcal{B}^{R_\varnothing, \chi Z}_{H} = \sum_{(P_1, P_2)} \frac{1}{v_0} i K^{R_\varnothing, \chi Z}(P_1, P_2)
\]

And there are three with one gauge boson and two Goldstones in the loop:

\[
\begin{align*}
\text{Diagram 1:} & \quad p_1 + P_1 \rightarrow k + P_2 \\
\text{Diagram 2:} & \quad p_1 + P_1 \rightarrow k + P_2 \\
\text{Diagram 3:} & \quad p_1 + P_1 \rightarrow k + P_2 
\end{align*}
\]

which are equal to

\[
i \mathcal{B}^{\chi \chi \chi Z}_{H} = \sum_{(P_1, P_2)} \frac{1}{v_0} i K^{\chi \chi \chi Z}(P_1, P_2), \quad i \mathcal{B}^{\chi \chi Z}_{H} = \sum_{(P_1, P_2)} \frac{1}{v_0} i K^{\chi \chi Z}(P_1, P_2)
\]

and

\[
i \mathcal{B}^{R_\varnothing, \chi \chi Z}_{H} = \sum_{(P_1, P_2)} \frac{1}{v_0} i K^{R_\varnothing, \chi \chi Z}(P_1, P_2) \quad \text{and} \quad i \mathcal{B}^{R_\varnothing, \chi Z}_{H} = \sum_{(P_1, P_2)} \frac{1}{v_0} i K^{R_\varnothing, \chi Z}(P_1, P_2)
\]

We finally have the Goldstone T-Box

\[
\begin{align*}
\text{Diagram 1:} & \quad p_1 + P_1 \rightarrow k + P_2 \\
\text{Diagram 2:} & \quad p_1 + P_1 \rightarrow k + P_2 \\
\text{Diagram 3:} & \quad p_1 + P_1 \rightarrow k + P_2 
\end{align*}
\]

Adding up all the T-Boxes we find that at zero external momenta

\[
(4\pi)^{d/2} \mathcal{B}^{R_\varnothing, T}_{H} = \mu^\varepsilon \left( \left[ \mathcal{B}^{T}_{H R_\varnothing} \right]_\varepsilon + \left\{ \mathcal{B}^{T}_{H R_\varnothing} \right\}_\varepsilon + \left[ \mathcal{B}^{T}_{H R_\varnothing} \right]_f + \left\{ \mathcal{B}^{T}_{H R_\varnothing} \right\}_f \right) \tag{3.71}
\]

where

\[
\varepsilon \left[ \mathcal{B}^{R_\varnothing, T}_{H} \right]_\varepsilon = -28 \lambda_0^2 \frac{s^2}{m_{H_0}^4} - 40 s t \lambda_0^2 \frac{1}{m_{H_0}^4} - 28 t^2 \lambda_0^2 \frac{1}{m_{H_0}^4} - 40 s u \lambda_0^2 \frac{1}{m_{H_0}^4} - 28 t u \lambda_0^2 \frac{1}{m_{H_0}^4} - 28 u^2 \lambda_0^2 \frac{1}{m_{H_0}^4}
\]

\[
\left\{ \mathcal{B}^{T}_{H R_\varnothing} \right\}_\varepsilon = 0. \tag{3.72}
\]

and

\[
\left[ \mathcal{B}^{R_\varnothing, T}_{H} \right]_f = -162 \lambda_0^2 - 288 \lambda_0^2 \frac{m_{Z_0}^2}{m_{H_0}^4}
\]

\[
\left\{ \mathcal{B}^{R_\varnothing, T}_{H} \right\}_f = 0. \tag{3.73}
\]
3.5.2 Irreducible Boxes

Irreducible, or S-Boxes, have a channel structure determined by three linear combinations of momenta, called $P_1$, $P_2$ and $P_3$. The resulting three independent channels are the usual $s$, $t$ and $u$ channels:

\[
\begin{align*}
\text{s-channel:} & \quad P_1 = p_1, \quad P_2 = p_1 + p_3 + p_4, \quad P_3 = p_1 + p_3 \\
\text{t-channel:} & \quad P_1 = p_3, \quad P_2 = p_1 + p_3 + p_4, \quad P_3 = p_1 + p_3 \\
\text{u-channel:} & \quad P_1 = p_1, \quad P_2 = p_1 + p_3 + p_4, \quad P_3 = p_1 + p_4
\end{align*}
\]  

(3.74)

Irreducible Boxes have therefore the generic form $B^{R_e,S}_{H}(P_1, P_2, P_3)$ and receive a contribution from the $s$, $t$ and $u$ channels. In order to take into account all channels, after considering the different diagram topologies, we sum over the $(P_1, P_2, P_3)$ according to the above rule. All S-Boxes have symmetry factor 1. Several annoying for the eye expressions for finite terms ($B^{R_e,...}_{H}$) are moved to Appendix E.

We start the computation of the S-Boxes with the finite Higgs loop diagram:

\[
B^{R_e,\phi\phi\phi}_{H} = 324\lambda_0^2 m_{H_0}^2 \int \frac{d^4k}{(2\pi)^4} \frac{(-i)}{D_1 D_2 D_3 D_4},
\]  

(3.75)

with the $D_i$ in the denominator defined in Appendix A. This is just a finite $D_0$ PV integral:

\[
B^{R_e,\phi\phi\phi}_{H} = 324\lambda_0^2 m_{H_0}^2 \mu^2 D_0(P_1, P_2, P_3, m_{H_0}, m_{H_0}, m_{H_0}, m_{H_0}).
\]  

(3.76)

The next diagram is also finite. It is
\[
\begin{align*}
    k + P_1 & \quad k + P_2 & \quad k + P_3 \\
    p_1 & \quad p_2 & \quad p_3 & \quad p_4
\end{align*}
\]

and is equal to

\[
\mathcal{B}_H^{R_\xi,ZZZZ} = 64 \frac{m_2^6 \lambda_0^2}{m_{H_0}^4} g^\mu\nu g^\alpha\beta g^\gamma\delta g^{\xi\zeta} \int \frac{d^4k}{(2\pi)^4} \left[ -i \left( -g_{\mu\zeta} + \frac{(1-\xi)k_\mu k_\zeta}{k^2 - \xi m_2^2} \right) \right] -g_{\nu\alpha} + \frac{(1-\xi)(k+P_1)_\nu(k+P_1)_\alpha}{(k+P_1)^2 - \xi m_2^2} \right) \frac{D_1}{D_2} \\
\times \left( -g_{\delta\beta} + \frac{(1-\xi)(k+P_2)_\delta(k+P_2)_\beta}{(k+P_2)^2 - \xi m_2^2} \right) \frac{D_3}{D_4}. \quad (3.77)
\]

In DR it takes the form

\[
\mathcal{B}_H^{R_\xi,ZZZZ} = (\mathcal{B}_H^{R_\xi,ZZZZ})_f. \quad (3.78)
\]

Another finite diagram is

\[
\begin{align*}
    k + P_1 & \quad k + P_2 & \quad k + P_3 \\
    p_1 & \quad p_2 & \quad p_3 & \quad p_4
\end{align*}
\]

which is equal to

\[
\mathcal{B}_H^{R_\xi,\chiZZZ} = 64 \frac{m_2^6 \lambda_0^2}{m_{H_0}^4} g^\mu\nu g^\alpha\beta g^\gamma\delta g^{\xi\zeta} \int \frac{d^4k}{(2\pi)^4} \left[ -i \left( -g_{\mu\zeta} + \frac{(1-\xi)k_\mu k_\zeta}{k^2 - \xi m_2^2} \right) \right] -g_{\nu\alpha} + \frac{(1-\xi)(k+P_1)_\nu(k+P_1)_\alpha}{(k+P_1)^2 - \xi m_2^2} \right) \frac{D_1}{D_2} \\
\times \left( -g_{\delta\beta} + \frac{(1-\xi)(k+P_2)_\delta(k+P_2)_\beta}{(k+P_2)^2 - \xi m_2^2} \right) \frac{D_3}{D_4}. \quad (3.79)
\]

and in DR to

\[
\mathcal{B}_H^{R_\xi,\chiZZZ} = (\mathcal{B}_H^{R_\xi,\chiZZZ})_f. \quad (3.80)
\]

Now, there are three additional diagrams of this type, \(\mathcal{B}_H^{R_\xi,Z\chiZZ}, \mathcal{B}_H^{R_\xi,Z\chiZ},\) and \(\mathcal{B}_H^{R_\xi,ZZ\chi},\) giving different, but finite result. The total contribution of these diagrams too is obtained
by summing over the three possible channels \((P_1, P_2, P_3)\), i.e. the \(s, t\) and \(u\) channels, according to Eq. (3.74).

We turn to the S-Boxes which have both infinite and finite parts. The first one of this type is

\[
\begin{array}{cccc}
 & & & \\
| & & & \\
\downarrow & & & \\
| & & & \\
& & & \\
\end{array}
\]

\[
P_1 \quad k + P_1 \quad \cdots \quad k + P_3 = iB_{H}^{R,ZZxx}
\]

and is given by the expression

\[
B_{H}^{R,ZZxx} = \frac{64 m_Z^4 \lambda_0^2 m_{H_0}^4}{m_{H_0}^4} g_{\mu\beta} \int \frac{d^4 k}{(2\pi)^4} \frac{-i}{D_1 D_2 D_3 D_4} \left( -g_{\mu\alpha} + \frac{(1-\xi)(k+P_3)_{\mu}(k+P_3)_{\alpha}}{(k+P_3)^2 - \xi m_Z^2} \right) k^\nu (k+P_2)^\alpha k \cdot (k+P_2).
\]

Its DR form is

\[
B_{H}^{R,ZZxx} = 64 \frac{\lambda_0^2 m_Z^4}{m_{H_0}^4} \mu^\varepsilon \left\{ D_{B4}(P_1, P_2, P_3, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \right\} + (1 - \xi) \left\{ 2D_{B4}(P_1, P_2, P_3, m_{\chi_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \right\} + (1 - \xi)^2 \left\{ D_{B4}(P_1, P_2, P_3, m_{\chi_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \right\} + (B_{H}^{R,ZZxx})_{f,1} + (1 - \xi)(B_{H}^{R,ZZxx})_{f,2} + (1 - \xi)^2 (B_{H}^{R,ZZxx})_{f,3}.
\]

A portion of the finite part of the above diagram is actually built in the \(D_{B4}\)-integrals (defined at the end of Sect. B.2) while the rest of it is given in Appendix E. There are in total six different topologies for this diagram corresponding to the different ways the two Goldstones can be distributed in the loop. All these diagrams have exactly the same divergent part as \(B_{H}^{R,ZZxx}\) but not the same finite part.

Next, we have
together with $B_{H}^{R_{\xi}Z\chi\chi\chi}$, $B_{H}^{R_{\xi}Z\chi\chi}$ and $B_{H}^{R_{\xi}Z\chi\chi\chi}$ that have also identical divergent parts but different finite parts. $B_{H}^{R_{\xi}Z\chi\chi\chi}$ is equal to

$$B_{H}^{R_{\xi}Z\chi\chi\chi} = 64 \frac{m_{20}^{2} \lambda_{0}^{2}}{m_{H_{0}}^{4}} \int \frac{d^{4}k}{(2\pi)^{4}} \frac{-i \left(-g_{\mu\nu} + \frac{(1-\xi)k_{\mu}k_{\nu}}{k^{2}-m_{20}^{2}}\right)}{D_{1}D_{2}D_{3}D_{4}} (k + P_{1})^{\mu}(k + P_{2})^{\nu} \times (k + P_{1}) \cdot (k + P_{3})(k + P_{3}) \cdot (k + P_{2})$$

and in DR evaluates to

$$B_{H}^{R_{\xi}Z\chi\chi\chi} = -64 \frac{\lambda_{0}^{2}m_{20}^{2}}{m_{H_{0}}^{4}} \mu^{\varepsilon} \begin{cases} U_{B_{6}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \\ + 2(P_{1} + P_{2} + P_{3})_{\mu}U_{B_{5}}^{\mu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \\ + (P_{1\mu}P_{1\nu} + 3P_{1\mu}P_{2\nu} + P_{2\mu}P_{2\nu}) \\ + 3P_{1\mu}P_{3\nu} + 3P_{2\mu}P_{3\nu} + 3P_{3\mu}P_{3\nu})D_{B_{4}}^{\mu\nu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \\ + (P_{1} \cdot P_{2} + P_{1} \cdot P_{3} + P_{2} \cdot P_{3})D_{B_{4}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \\ - (1 - \xi) \left(U_{B_{6}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \\ + m_{\chi_{0}}^{2}U_{B_{4}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \\ + 2(P_{1} + P_{2} + P_{3})_{\mu}U_{B_{5}}^{\mu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \\ + (P_{1\mu}P_{1\nu} + 3P_{1\mu}P_{2\nu} + P_{2\mu}P_{2\nu}) \\ + 4P_{1\mu}P_{3\nu} + 3P_{2\mu}P_{3\nu} + 3P_{3\mu}P_{3\nu})D_{B_{4}}^{\mu\nu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \\ + (P_{1} \cdot P_{2} + P_{2} \cdot P_{3})D_{B_{4}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{\chi_{0}}, m_{\chi_{0}}, m_{\chi_{0}}) \right) \end{cases} \right) + (B_{H}^{R_{\xi}Z\chi\chi\chi})_{f,1} + (1 - \xi)(B_{H}^{R_{\xi}Z\chi\chi\chi})_{f,2}, \quad (3.84)$$

where $(B_{H}^{R_{\xi}Z\chi\chi\chi})_{f,1}$ and $(B_{H}^{R_{\xi}Z\chi\chi\chi})_{f,2}$ are entirely finite, given in Appendix E. The rest of the finite part is encoded in the PV and $U$-integrals.

The last divergent $S$-Box is the one with only Goldstone bosons inside the loop:
and is equal to
\[
\mathcal{B}_H^{R_{ε,xxxx}} = 64 \frac{\lambda_0^2 m_{2\alpha}^2}{m_{H_0}^4} \int \frac{d^4k}{(2\pi)^4} \left\{ \begin{array}{c}
\frac{1}{D_1 D_2 D_3 D_4} (k + P_1) \cdot (k + P_3) \times (k + P_3) \cdot (k + P_2) \times (k + P_2) \cdot k.
\end{array} \right. 
\]
\[ (3.85) \]

Its DR form is
\[
\mathcal{B}_H^{R_{ε,xxxx}} = 64 \frac{\lambda_0^2 m_{2\alpha}^2}{m_{H_0}^4} \mu^\epsilon \left\{ \begin{array}{c}
U_{B8}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \\
+ 2(P_1 + P_2 + P_3)_{\mu} U_{B7}^{\mu}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \\
+ \left\{ P_{1\mu} P_{1\nu} + 3P_{1\mu} P_{2\nu} + P_{2\mu} P_{2\nu} \\
+ 4P_{1\mu} P_{3\nu} + 3P_{2\mu} P_{3\nu} + P_{3\mu} P_{3\nu} \right\} U_{B6}^{\mu\nu}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \\
+ (P_1 \cdot P_2 + P_2 \cdot P_3) U_{B5}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \\\n+ \left\{ P_{1\mu} P_{1\nu} P_{2\alpha} + P_{1\mu} P_{2\nu} P_{2\alpha} + 2P_{1\mu} P_{1\nu} P_{3\alpha} + 4P_{1\mu} P_{2\nu} P_{3\alpha} \right\} U_{B5}^{\mu\nu\alpha}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \\
+ \left\{ P_1 \cdot P_2 (P_1 + P_2 + 2P_3)_{\mu} \\
+ P_2 \cdot P_3 (2P_1 + P_2 + P_3)_{\mu} \right\} U_{B5}^{\mu}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \\
+ \left\{ P_{1\mu} P_{1\nu} P_{2\alpha} P_{3\beta} + P_{1\mu} P_{2\nu} P_{2\alpha} P_{3\beta} \\
+ P_{1\mu} P_{1\nu} P_{3\alpha} P_{3\beta} + P_{1\mu} P_{2\nu} P_{3\alpha} P_{3\beta} \right\} D_{\alpha\beta}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \\
+ \left\{ P_1 \cdot P_2 [P_{1\mu} P_{1\nu} + 2P_{1\mu} P_{3\nu} + P_{2\mu} P_{3\nu} + P_{3\mu} P_{3\nu}] \\
+ P_2 \cdot P_3 [P_{1\mu} P_{1\nu} + P_{1\mu} P_{2\nu} + 2P_{1\mu} P_{3\nu} + P_{2\mu} P_{3\nu}] \right\} D_{\mu\nu}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \\
+ (P_1 \cdot P_2 \times P_2 \cdot P_3) D_{B4}(P_1, P_2, P_3, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}, m_{\chi_0}) \right\}.
\]
\[ (3.86) \]

Summing up all topologies and channels, at zero external momenta, we find for the S-Boxes the relations
\[
\varepsilon[\mathcal{B}_H^{R_{ε,S}}]_e = 24 \frac{s^2 \lambda_0^2}{m_{H_0}^4} + 40 \frac{s \cdot t \lambda_0^2}{m_{H_0}^4} + 24 \frac{t^2 \lambda_0^2}{m_{H_0}^4} + 40 \frac{s \cdot u \lambda_0^2}{m_{H_0}^4} + 40 \frac{t \cdot u \lambda_0^2}{m_{H_0}^4} + 24 \frac{u^2 \lambda_0^2}{m_{H_0}^4}
\]

37
\[ \{ \mathcal{B}_H^{R_e,S} \}_\varepsilon = 0 \]  \hspace{1cm} (3.87)

and
\[ \{ \mathcal{B}_H^{R_e,S} \}_f = 162 \lambda_0^2 + 96 \frac{\lambda_0^4 m_0^4}{m_{H_0}^4} \]
\[ \{ \mathcal{B}_H^{R_e,S} \}_f = 0 \]  \hspace{1cm} (3.88)

We note the interesting facts that \( \mathcal{B}_H^{R_e,ZZ\chi\chi} \) has only a \( \{ \cdots \}_\varepsilon \) part and that
\[ \{ \mathcal{B}_H^{R_e,ZZ\chi\chi} + \mathcal{B}_H^{R_e,\chi\chi\chi} + \mathcal{B}_H^{R_e,\chi\chi\chi} \}_\varepsilon = 0, \]  \hspace{1cm} (3.89)

while \( \{ \mathcal{B}_H^{R_e,\chi\chi\chi Z} \}_\varepsilon \sim \frac{\lambda_0^2 m_0^2}{m_{H_0}^4} \) and \( \{ \mathcal{B}_H^{R_e,\chi\chi\chi} \}_\varepsilon \sim \lambda_0^2 \).

The cancellation of \( \xi \) from the finite parts is more illuminating when results from individual loop structures are shown:

\[ \begin{align*}
\frac{96 \lambda_0^2 m_0^2}{m_{H_0}^4} & + \frac{32 \lambda_0^2 m_0^2}{m_{H_0}^4} \varepsilon^2 \\
\frac{256 \lambda_0^2 m_0^2}{m_{H_0}^4} \varepsilon^2 \\
\frac{-960 \lambda_0^2 m_0^2}{m_{H_0}^4} \varepsilon^2 & + \frac{1152 \lambda_0^2 m_0^2}{m_{H_0}^4} \varepsilon^2 \ln \frac{\mu^2}{m_{Z_0}^2} \\
\frac{256 \lambda_0^2 m_0^2}{m_{H_0}^4} \varepsilon^2 & - \frac{3072 \lambda_0^2 m_0^2}{m_{H_0}^4} \varepsilon^2 \ln \frac{\mu^2}{m_{Z_0}^2} \\
\frac{416 \lambda_0^2 m_0^2}{m_{H_0}^4} \varepsilon^2 & + \frac{1920 \lambda_0^2 m_0^2}{m_{H_0}^4} \varepsilon^2 \ln \frac{\mu^2}{m_{Z_0}^2}.
\end{align*} \]

The cancellation of \( \xi \) is now evident.

The final step here is to collect all Boxes and sum them up. Adding Eq.(3.68), Eq.(3.72) and Eq.(3.87), we obtain at \( p_i = 0 \):
\[ \varepsilon \{ \mathcal{B}_H^{R_e} \}_\varepsilon = 108 \lambda_0^2 + 144 \frac{\lambda_0^4 m_0^4}{m_{H_0}^4} \]
\[ \left\{ B_{H}^{R} \right\}_{\xi} = 0. \]  

(3.90)

and

\[ [B_{H}^{R}]_{f} = -168 \frac{\lambda_{0}^{2} m_{Z_{0}}^{4}}{m_{H_{0}}^{4}} + 54 \lambda_{0}^{2} \ln \frac{\mu^{2}}{m_{H_{0}}^{2}} + 72 \frac{\lambda_{0} m_{Z_{0}}^{2}}{m_{H_{0}}^{2}} \ln \frac{\mu^{2}}{m_{Z_{0}}^{2}} \]

\[ \left\{ B_{H}^{R} \right\}_{f} = 0. \]  

(3.91)

A couple of final comments are in order. First, each block of box diagrams, Candies, T and S-Boxes is by itself \( \xi \)-independent. Moreover, looking at the results from the three sectors, one notices that they have an explicit \( s \), \( t \) and \( u \)-dependence. So, one would expect that \([B_{H}^{R}]_{\xi}\) could also be channel dependent. Nevertheless, Eq. (3.90) shows that the \( s \), \( t \) and \( u \)-dependence cancels when the full contribution of the box diagrams is taken into account.

## 4 Unitary Gauge

In the previous section, we computed one-loop processes in the Abelian Higgs model when the gauge symmetry is broken using an \( R_{\xi} \) gauge fixing term. However one of our main goals here is to investigate this model in the Unitary gauge. This is interesting since, in the Unitary gauge only physical degrees of freedom are present, in contrast to the \( R_{\xi} \) gauge. Moreover, there are statements in the literature that argue that the Unitary gauge may be problematic at the quantum level, so by comparing it to the \( R_{\xi} \) gauge, we will try to clarify the correctness of these arguments.

In the Unitary gauge, no gauge fixing is needed, therefore there is no need for ghosts. The Unitary gauge Lagrangean can be simply obtained from Eq. (3.3) by removing gauge fixing and ghost terms and setting \( \chi_{0} = 0 \) in the remaining. Doing so, we obtain the Unitary gauge Lagrangean

\[ \mathcal{L}_{AH} = -\frac{1}{4} F_{0, \mu \nu}^{2} + \frac{1}{2} (\partial_{\mu} \phi_{0}) (\partial^{\mu} \phi_{0}) \frac{1}{2} m_{Z_{0}}^{2} A_{\mu}^{0} A_{\nu}^{0} + g_{\mu \nu} \lambda_{0} m_{Z_{0}}^{2} \phi_{0}^{2} A_{\mu}^{0} A_{\nu}^{0} + \frac{1}{2} m_{Z_{0}}^{2} \phi_{0}^{2} - \frac{1}{2} \frac{\lambda_{0}}{2} m_{H_{0}}^{2} \phi_{0}^{2} - \sqrt{\frac{\lambda_{0}}{4}} \phi_{0}^{4} + \text{const.} \]  

(4.1)

from which the Feynman rules can be derived:

- Gauge boson propagator

\[ \frac{i}{k^{2} - m_{Z_{0}}^{2} + i\varepsilon} \]
• Higgs propagator

\[ \frac{i}{k^2 - m^2_H + i\varepsilon} \]

• \(\phi-Z-Z\) vertex

\[ 2ig^{\mu\nu} \frac{m_Z^2}{m_H^2} \sqrt{2\lambda_0} \]

• \(\phi-\phi-\phi\) vertex

\[ -6i \sqrt{\frac{\lambda_0}{2}} m_H \]

• \(\phi-\phi-Z-Z\) vertex

\[ 4i \frac{\lambda_0 m_Z^2}{m_H^2} g^{\mu\nu} \]

• \(\phi-\phi-\phi-\phi\) vertex

\[ -6i\lambda_0 \]

In the Unitary gauge, integrals of the \(U\)-type are ubiquitous. But such integrals we have already seen in the \(R_\xi\) gauge due to the momentum dependent vertices of the Polar basis for the Higgs field. In the Unitary gauge we do not have momentum dependent vertices, the \(U\)-integrals arise only because of the form of the propagators.

In the following sections we compute one-loop diagrams, setting the external momenta to zero at the end, as in the \(R_\xi\) gauge. Again for completeness, we present some on-shell results in Appendix F. We will directly insert symmetry factors here since they are the same as in the corresponding \(R_\xi\) calculation. In the Unitary gauge, since there is no gauge fixing parameter, we have trivially \(\{\ast\}_\varepsilon = \{\ast\}_f \equiv 0\). As before, we will consistently move nasty expressions for finite parts to Appendix E.
4.1 Tadpoles

The Higgs tadpole is

\[ p \rightarrow \begin{array}{c} k \\ \cdots \end{array} = i\mathcal{T}_H^{U,\phi} \]

\[ \mathcal{T}_H^{U,\phi} = -6\frac{1}{2} \sqrt{\frac{\lambda_0}{2}} m_{H_0} \int \frac{d^4k}{(2\pi)^4} \frac{i}{(k^2 - m_{H_0}^2)} \]  

(4.2)

and in DR reads

\[ \mathcal{T}_H^{U,\phi} = 3\sqrt{\frac{\lambda_0}{2}} m_{H_0} \mu \varepsilon A_0(m_{H_0}). \]  

(4.3)

The gauge tadpole is

\[ p \rightarrow \begin{array}{c} k \\ \cdots \end{array} = i\mathcal{T}_H^{U,Z} \]

and following Appendix D.2, this becomes

\[ \mathcal{T}_H^{U} = \mu \varepsilon \left( 3\sqrt{\frac{\lambda_0}{2}} m_{H_0} A_0(m_{H_0}) + 3\sqrt{2\lambda_0 m_{Z_0}^2} A_0(m_{Z_0}) \right). \]  

(4.4)

Adding up the two results, we have

\[ (4\pi)^d/2 \mathcal{T}_H^U = \mu \varepsilon \left( [\mathcal{T}_H^U]_\varepsilon + [\mathcal{T}_H^U]_f \right) \]  

(4.5)

with

\[ \varepsilon [\mathcal{T}_H^U]_\varepsilon = 6\sqrt{\frac{\lambda_0}{2}} m_{H_0}^3 + 6\sqrt{2\lambda_0 m_{Z_0}^2} \]  

(4.6)

and

\[ [\mathcal{T}_H^U]_f = 3\sqrt{\frac{\lambda_0}{2}} m_{H_0}^3 + 3\sqrt{2\lambda_0 m_{Z_0}^2} A_0(m_{H_0}) + 3\sqrt{\frac{\lambda_0}{2}} m_{H_0}^3 \ln \frac{\mu^2}{m_{H_0}^2} + 3\sqrt{2\lambda_0 m_{Z_0}^2} \ln \frac{\mu^2}{m_{Z_0}^2}. \]  

(4.7)

The above expressions are identical to the \( R_\xi \) expressions Eq. (3.8) and Eq. (3.9).
4.2 Corrections to the gauge boson mass

The Z boson mass receives its first correction from

$$ p \begin{array}{c} \circ \end{array} k = i \mathcal{M}_{Z,\mu\nu}^{U,\phi} $$

which is equal to

$$ \mathcal{M}_{Z,\mu\nu}^{U,\phi} = -2g_{\mu\nu} m_{Z_0}^2 \lambda_0 \mu^\varepsilon A_0(m_{H_0}). \quad (4.8) $$

Next is the Higgs sunset

$$ p \begin{array}{c} \circ \end{array} k + p = i \mathcal{M}_{Z,\mu\nu}^{U,\phi Z} $$

translating in DR to

$$ \mathcal{M}_{Z,\mu\nu}^{U,\phi Z} = -8g_{\mu\nu} m_{Z_0}^2 \lambda_0 \mu^\varepsilon B_0(p, m_{Z_0}, m_{H_0}) + 8 m_{Z_0}^2 \lambda_0 \mu^\varepsilon B_{\mu\nu}(p, m_{Z_0}, m_{H_0}). \quad (4.9) $$

The sum of these two corrections is

$$ \mathcal{M}_{Z,\mu\nu}^{U} = \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^\varepsilon \left\{ -8g_{\mu\nu} m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - 2g_{\mu\nu} A_0(m_{H_0}) \right\} + 8B_{\mu\nu}(p, m_{Z_0}, m_{H_0}) \right\}. \quad (4.10) $$

As before, the proper contraction we are after is

$$ \mathcal{M}_Z^{U} = \frac{1}{3} \left( -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} \right) \mathcal{M}_{Z,\mu\nu}^{U}(p) \quad (4.11) $$

which can be easily shown to be equal to

$$ \mathcal{M}_Z^{U}(p) = -\frac{1}{3} \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^\varepsilon \left\{ \left\{ -8(d + \varepsilon) m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - 2(d + \varepsilon) A_0(m_{H_0}) \right\} \right\} - 8m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - 2A_0(m_{H_0}) \right\} + 8 \frac{p^\mu p^\nu}{p^2} B_{\mu\nu}(p, m_{Z_0}, m_{H_0}) \right\}. \quad (4.12) $$
At $p = 0$, we have
\[(4\pi)^{d/2} M_Z^U = \mu^\varepsilon \left( [M_Z^U]_\varepsilon + [M_Z^U]_f \right) \] (4.13)

with
\[\varepsilon [M_Z^U]_\varepsilon = \frac{12 \lambda_0 m_{Z_0}^4}{m_{H_0}^2}. \] (4.14)

The wave function renormalization factor is now straightforward to compute:
\[\delta A^U = - \frac{dM_Z^U(p)}{dp^2} \bigg|_{p^2=0} = \frac{\mu^\varepsilon}{(4\pi)^{d/2}} \left( [\delta A^U]_\varepsilon + [\delta A^U]_f \right) \] (4.15)

with
\[\varepsilon [\delta A^U]_\varepsilon = - \frac{4}{3} \lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2}. \] (4.16)

Glancing back on Sect.3.2 we see the first sign of the consistency of the calculation by noticing that $M_Z^U$ and $M_Z^{R_\xi}$ have the same gauge-independent divergent parts and divergent parts of their respective anomalous dimensions.

Regarding the finite parts we have
\[[M_Z^U]_f = - \frac{4}{3} \frac{m_{H_0}^2 m_{Z_0}^2 \lambda_0}{m_{H_0}^2 - m_{Z_0}^2} + 10 \frac{m_{Z_0}^4 \lambda_0}{3} \frac{m_{H_0}^2 - m_{Z_0}^2}{m_{H_0}^2 - m_{Z_0}^2} - 26 \frac{m_{Z_0}^6 \lambda_0}{3} \frac{m_{H_0}^2 m_{H_0}^2}{(m_{H_0}^2 - m_{Z_0}^2)} + 16 \frac{m_{Z_0}^4 \lambda_0}{m_{H_0}^2} \ln \frac{2 \mu^2}{m_{H_0}^2} + 2 \frac{m_{Z_0}^4 \lambda_0}{3} \frac{m_{H_0}^2 - m_{Z_0}^2}{m_{H_0}^2 - m_{Z_0}^2} \ln \frac{2 \mu^2}{m_{Z_0}^2} - 6 \frac{m_{Z_0}^6 \lambda_0}{m_{H_0}^2 (m_{H_0}^2 - m_{Z_0}^2)} \ln \frac{\mu^2}{m_{Z_0}^2} \] (4.17)

and
\[[\delta A^U]_f = - \frac{2}{3} \frac{m_{Z_0}^2 \lambda_0}{m_{H_0}^2 - m_{Z_0}^2} \left( 1 + \ln \frac{\mu^2}{m_{H_0}^2} \right) - \frac{m_{Z_0}^2}{m_{H_0}^4} \left( 1 + \ln \frac{\mu^2}{m_{Z_0}^2} \right). \] (4.18)

It is interesting to notice that the last two expressions for the finite parts can be obtained from the corresponding $R_\xi$ gauge expressions in Eq. (3.19) and Eq. (3.20) for $\xi = 1$ and not for $\xi \to \infty$.

### 4.3 Corrections to the Higgs mass

The first correction to the Higgs mass comes from
\[\begin{array}{c}
p \in\nabla \\
\end{array} = iM_H^{U,Z} \]
and in DR it becomes
\[
M_{H}^{U,Z} = \mu^{\varepsilon} \left\{ 2d \frac{\lambda_{0} m_{Z_{0}}^{2}}{m_{H_{0}}^{2}} A_{0}(m_{Z_{0}}) - 2 \frac{\lambda_{0}}{m_{H_{0}}^{2}} U_{\tau}(1, m_{Z_{0}}) \right\}
\]
\[
= \mu^{\varepsilon} 6 \frac{\lambda_{0} m_{Z_{0}}^{2}}{m_{H_{0}}^{2}} A_{0}(m_{Z_{0}}). \tag{4.19}
\]

Next is
\[
p \begin{array}{c}
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \hline
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \end{array}
\end{array}
\]
\[= iM_{H}^{U,\phi} \]

In DR it reads,
\[
M_{H}^{U,\phi} = 3\lambda_{0} \mu^{\varepsilon} A_{0}(m_{H_{0}}). \tag{4.20}
\]

The Higgs vacuum polarization diagram
\[
p \begin{array}{c}
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \hline
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \end{array}
\end{array}
\]
\[= iM_{H}^{U,\phi\phi} \]

which in DR is equal to
\[
M_{H}^{U,\phi\phi} = 9\lambda_{0} m_{H_{0}}^{2} \mu^{\varepsilon} B_{0}(p, m_{H_{0}}, m_{H_{0}}). \tag{4.21}
\]

The corresponding gauge loop is
\[
p \begin{array}{c}
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \hline
\phantom{=} \\
\phantom{=} \\
\phantom{=} \\
\phantom{=} \end{array}
\end{array}
\]
\[= iM_{H}^{U,ZZ} \]

and in DR
\[
M_{H}^{U,ZZ} = \mu^{\varepsilon} \left\{ 4d \frac{\lambda_{0} m_{Z_{0}}^{2}}{m_{H_{0}}^{2}} B_{0}(p, m_{Z_{0}}, m_{Z_{0}})
\right.
\]
\[
- 4 \frac{\lambda_{0} m_{Z_{0}}^{2}}{m_{H_{0}}^{2}} g_{\mu\nu} B_{k+p}^{\mu\nu}(p, m_{Z_{0}}, m_{Z_{0}}) + 4 \frac{\lambda_{0}}{m_{H_{0}}^{2}} m_{Z_{0}}^{2} A_{0}(m_{Z_{0}})
\]
\[
+ 4 \frac{\lambda_{0}}{m_{H_{0}}^{2}} \mu p_{p} p_{\nu} B_{k+p}^{\mu\nu}(p, m_{Z_{0}}, m_{Z_{0}})
\left.
\right\} \tag{4.22}
\]
where we have defined
\[ g_{\mu\nu}B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) = \int \frac{d^4k}{(2\pi)^4} \frac{-i(k + p)^2}{(k^2 - m_{Z_0}^2)((k + p)^2 - m_{Z_0}^2)}. \]  

Adding up Eq. (4.19), Eq. (4.20), Eq. (4.21) and Eq. (4.22) we obtain
\[ M_{\mu H}^U(p) = \mu^\varepsilon \left\{ 6 \frac{\lambda_0^2}{m_{H_0}^2} A_0(m_{Z_0}) + \frac{2\lambda_0 m_{Z_0}^4}{m_{H_0}^2} + 3\lambda_0 A_0(m_{H_0}) + 9\lambda_0 m_{H_0}^2 B_0(p, m_{H_0}, m_{H_0}) + \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} \right\}, \]
\[ -4g_{\mu\nu}B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) + 4A_0(m_{Z_0}) - 4p_{\mu}^2 g_{\mu\nu}B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) + 4m_{Z_0}^2 + \frac{4p_{\mu}p_{\nu}}{m_{Z_0}^2} B^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) \right\}. \]  

Using the reduction formulae in Appendix B and summing up all contributions we can extract
\[ (4\pi)^{d/2}M_{H}^U = \mu^\varepsilon \left[ [M_{H}]_\varepsilon + [M_{H}^U]_f \right] \]
with
\[ \varepsilon[M_{H}]_\varepsilon = 24\lambda_0 m_{H_0}^2 + 36 \frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2} \]
and
\[ [M_{H}^U]_f = 3\lambda_0 m_{H_0}^2 + 6\frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} + 12\lambda_0 m_{H_0}^2 \ln \frac{H_0}{m_{H_0}} + 18 \frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2} \ln \frac{H_0}{m_{Z_0}}. \]
The anomalous dimension is then
\[ \delta\phi^U = \frac{dM_{H}^U(p)}{dp^2} \bigg|_{p^2=0} = \frac{\mu^\varepsilon}{(4\pi)^{d/2}} \left( [\delta\phi^U]_\varepsilon + [\delta\phi^U]_f \right) \]
with
\[ \varepsilon[\delta\phi^U]_\varepsilon = 12\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2}. \]
and
\[ [\delta\phi^U]_f = 2\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} + 6\lambda_0 \frac{m_{Z_0}^4}{m_{H_0}^2} \ln \frac{H_0}{m_{Z_0}}. \]
Comparing to the result of Sect. 3.3 we can see that Eq. (4.26) and Eq. (4.27) of $M_{H}^U$ are identical to Eq. (3.30) and Eq. (3.31) of $M_{H}^{\xi}$ and (the $\xi$-independent) anomalous dimension of the $R_\xi$ gauge is equal to $\delta\phi^U$. 

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4.4 Corrections to the Higgs cubic vertex

As before we split the diagrams in two classes. The first class consists of the reducible Triangle diagrams, while the second one consists of the irreducible Triangles. The contribution of the various topologies and channels is contained in their $P_1$ and $(P_1, P_2)$ dependence respectively, exactly as in the $R_\xi$-gauge calculation.

4.4.1 Reducible Triangles

The first reducible Triangle is

\[ p_1 \rightarrow k + P_1 \rightarrow k \rightarrow p_3 = iK_{H}^{U,\phi \phi} \]

and it is the same loop as in Eq. (4.21) with $p \rightarrow P_1$, with the same symmetry factor, divided by $v_0$:

\[ K_{H}^{U,\phi \phi} = 18 \frac{\lambda_0^{3/2}}{\sqrt{2}} m_{H_0} \xi B_0(P_1, m_{H_0}, m_{H_0}) , \quad (4.31) \]

where the three different channels are obtained by summing over $P_1$, with $P_1 = p_1, p_2, p_3$ (resulting to an overall factor of 3, as in the $R_\xi$-gauge).

Next is the gauge loop

\[ p_1 \rightarrow k + P_1 \rightarrow k \rightarrow p_3 = iK_{H}^{U,ZZ} \]

which is Eq. (4.22), divided by $v_0$:

\[ K_{H}^{U,ZZ} = \frac{1}{v_0 m_{H_0}^2} \mu_\xi \left\{ dm_{Z_0} B_0(P_1, m_{Z_0}, m_{Z_0}) \right. \]

\[ - m_{Z_0}^2 g_{\mu \nu} B_{k+p_1}^\mu \xi (P_1, m_{Z_0}, m_{Z_0}) + m_{Z_0}^2 A_0(m_{Z_0}) + 4m_{Z_0}^2 \]

\[ - P_1^2 g_{\mu \nu} B_{k+p_1}^\mu \xi (P_1, m_{Z_0}, m_{Z_0}) + P_{1 \mu} P_{1 \nu} B_{k+p_1}^\mu \xi (P_1, m_{Z_0}, m_{Z_0}) \right\} . \quad (4.32) \]

There are again two more channels with $P_1 = p_2$ and $P_1 = p_3$, yielding identical contributions.
Adding up all the reducible Triangles, we have

\[(4\pi)^{d/2} K^{U,\text{red.}}_H = \mu^\varepsilon \left( [K^{U,\text{red.}}_H]_e + [K^{U,\text{red.}}_H]_f \right) \]  

(4.33)

with

\[\varepsilon [K^{U,\text{red.}}_H]_e = \frac{m_{H_0}}{\sqrt{2}\lambda_0} \left( 108\lambda_0^2 + 144\frac{\lambda_0^2 m_{Z_0}^2}{m_{H_0}^2} \right) \]  

(4.34)

and

\[[K^{U,\text{red.}}_H]_f = \frac{m_{H_0}}{\sqrt{2}\lambda} \left( 54\lambda_0^2 \ln \frac{\mu^2}{m_{H_0}^2} + 72\frac{\lambda_0^2 m_{Z_0}^2}{m_{H_0}^2} \ln \frac{\mu^2}{m_{Z_0}^2} \right) \]  

(4.35)

at \( p_i = 0 \).

### 4.4.2 Irreducible Triangles

The first irreducible Triangle is

\[ k + P_1 \quad - \quad p_3 \]
\[ \quad p_1 \quad \mapsto \quad k + P_2 \quad = i K^{U,\phi\phi}_H \]

\[ K^{U,\phi\phi}_H = \frac{108}{\sqrt{2}} \lambda_0^{3/2} m_{H_0}^3 \int \frac{d^4k}{(2\pi)^4} (-i) \frac{1}{D_1 D_2 D_3}. \]  

(4.36)

and in DR

\[ K^{U,\phi\phi}_H = \frac{108}{\sqrt{2}} \lambda_0^{3/2} m_{H_0}^3 \mu^\varepsilon C_0 (P_1, P_2, m_{H_0}, m_{H_0}, m_{H_0}). \]  

(4.37)

In the Unitary gauge there is only one more divergent irreducible Triangle, which is

\[ k + P_1 \quad - \quad p_3 \]
\[ \quad p_1 \quad \mapsto \quad k + P_2 \quad = i K^{U,ZZZ}_H \]

\[ K^{U,ZZZ}_H = -16\sqrt{2} \frac{m_{Z_0}^6}{m_{H_0}^3} \lambda_0^{3/2} g^{\mu\nu} g^{\alpha\beta} g^{\gamma\delta} \int \frac{d^4k}{(2\pi)^4} (-i) \left( -g_{\mu\gamma} + \frac{k_\mu k_\gamma}{m_{Z_0}^2} \right) \left( -g_{\nu\alpha} + \frac{(k + P_1)_\nu (k + P_1)_\alpha}{m_{Z_0}^2} \right) \]  

(4.38)
\[
\times \left( -g_{\delta \beta} + \frac{(k + P_2)_{\delta}(k + P_2)_{\beta}}{m_{Z0}^2} \right) 
\]

and in DR

\[
\kappa^{U,zzz}_{H} = -32v_0 m_{Z0}^6 \lambda_0^2 \mu^\varepsilon \left\{ -dC_0(P_1, P_2, m_{Z0}, m_{Z0}) + 3B_0(P_1, m_{Z0}, m_{Z0}) \right\}
\]

\[
+ \frac{1}{m_{Z0}^2} \left\{ (3m_{Z0}^2 + P_1 \cdot P_1 + P_2 \cdot P_2)C_0(P_1, P_2, m_{Z0}, m_{Z0}) + 3B_0(P_1, m_{Z0}, m_{Z0}) \right\}
\]

\[
+ \frac{1}{m_{Z0}^2} \left\{ 3U'K_4(P_1, P_2, m_{Z0}, m_{Z0}) + 4(P_1 + P_2)\mu C_{K3}^\mu(P_1, P_2, m_{Z0}, m_{Z0}) \right\}
\]

\[
+ 2(P_1 \mu P_1 \nu + P_1 \mu P_2 \nu + P_2 \mu P_2 \nu)C^{\mu \nu}(P_1, P_2, m_{Z0}, m_{Z0}) + 2P_1 \cdot P_2 B_0(p_1, m_{Z0}, m_{Z0})
\]

\[
+ 2P_1 \cdot P_2 (P_1 + P_2, \mu C^{\mu}(P_1, P_2, m_{Z0}, m_{Z0}) + P_1 \cdot P_2 C_0(P_1, P_2, m_{Z0}, m_{Z0}) \right\}
\]

\[
+ \frac{1}{m_{Z0}^2} \left\{ U'K_4(P_1, P_2, m_{Z0}, m_{Z0}) + 2(P_1 + P_2)\mu U'K_5(P_1, P_2, m_{Z0}, m_{Z0}) \right\}
\]

\[
+ (P_1 \mu P_1 \nu + 3P_1 \mu P_2 \nu + P_2 \mu P_2 \nu)U'K_4(P_1, P_2, m_{Z0}, m_{Z0})
\]

\[
+ P_1 \cdot P_2 U'K_4(P_1, P_2, m_{Z0}, m_{Z0}) + (P_1 \mu P_1 \nu_0 + P_1 \mu P_2 \nu_0)C^{\mu \nu_0}(P_1, P_2, m_{Z0}, m_{Z0})
\]

\[
+ P_1 \cdot P_2 (P_1 + P_2)C_{K3}^\mu(P_1, P_2, m_{Z0}, m_{Z0})
\]

\[
+ P_1 \cdot P_2 P_1 \mu P_2 \nu C^{\mu \nu}(P_1, P_2, m_{Z0}, m_{Z0}) \right\} \right\},
\]

where in this case \( P_1 = p_1 \) and \( P_2 = p_1 + p_3 \). The contracted PV integral \( C_{K3}^\mu \) is defined in Appendix B.2.

In total, we find for the irreducible Triangles

\[
(4\pi)^{d/2} [\kappa_{H}^{U,\text{irred.}}]_\varepsilon = \mu^\varepsilon \left( [\kappa_{H}^{U,\text{irred.}}]_\varepsilon + [\kappa_{H}^{U,\text{irred.}}]_f \right)
\]

with

\[
[\kappa_{H}^{U,\text{irred.}}]_\varepsilon = 0
\]

and

\[
[\kappa_{H}^{U,\text{irred.}}]_f = -\frac{m_{H0}}{\sqrt{2\lambda}} \left( \frac{54\lambda_0^2}{\lambda_0^2 + \frac{4 \lambda_0^2 m_{Z0}}{m_{H0}^4}} \right).
\]

Finally, adding reducible and irreducible contributions, we have

\[
(4\pi)^{d/2} \kappa_{H}^{U} = \mu^\varepsilon \left( [\kappa_{H}^{U}]_\varepsilon + [\kappa_{H}^{U}]_f \right)
\]

where

\[
\varepsilon [\kappa_{H}^{U}]_\varepsilon = \frac{m_{H0}}{\sqrt{2\lambda_0}} \left( 108\lambda_0^2 + \frac{144 \lambda_0^2 m_{Z0}}{m_{H0}^4} \right)
\]

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\[ [\mathcal{K}_{HT}]_f = \frac{m_{H_0}}{\sqrt{2\lambda}} \left( -54\lambda_0^2 - \frac{24\lambda_0^2 m_{Z_0}^4}{m_{H_0}^2} + 54\lambda_0^2 \ln \frac{\mu^2}{m_{H_0}^2} + 72\lambda_0^2 m_{Z_0}^4 \ln \frac{\mu^2}{m_{Z_0}^2} \right). \quad (4.45) \]

These are the same results as the ones found in Eq. (3.60) and Eq. (3.61) in the $R_\xi$-gauge.

### 4.5 Corrections to the quartic coupling

The separation of the Box diagrams into $C$, $T$ and $S$-Boxes in the Unitary gauge holds exactly like in the $R_\xi$ gauge. The same goes for the labelling of the various channels by the momenta $P_1$ (for Candies), $(P_1, P_2)$ for $T$-Boxes and $(P_1, P_2, P_3)$ for $S$-Boxes.

#### 4.5.1 Reducible Boxes

The Higgs Candy

\[ \begin{array}{c}
\begin{array}{c}
\vec{p}_1
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{k} + P_1
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{p}_3
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{k}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{p}_2
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{p}_4
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
= i B_{H}^{U,\phi \phi}
\end{array}
\end{array}\]

has been computed already in the $R_\xi$ gauge with the result

\[ B_{H}^{U,\phi \phi} = 18\lambda_0^2 \mu B_0(P_1, m_{H_0}, m_{H_0}). \quad (4.46) \]

The gauge Candy is slightly different due to the different gauge boson propagator. The diagram is

\[ \begin{array}{c}
\begin{array}{c}
\vec{p}_1
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{k} + P_1
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{p}_3
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{k}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{p}_2
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\vec{p}_4
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
= i B_{H}^{U,ZZ}
\end{array}
\end{array}\]

It evaluates in DR to

\[ (4\pi)^{d/2} B_{H}^{U,ZZ} = 8 \frac{m_{Z_0}^2}{m_{H_0}^4} \lambda_0^2 \mu^2 \left\{ dm_{Z_0}^2 B_0(P_1, m_{Z_0}, m_{Z_0}) - g_{\mu \nu} B_{k+P_1}^{\mu \nu}(P_1, m_{Z_0}, m_{Z_0}) + A_0(m_{Z_0}) + 2m_{Z_0}^2 + \frac{P_1^2}{m_{Z_0}^2} g_{\mu \nu} B^{\mu \nu}(P_1, m_{Z_0}, m_{Z_0}) + \frac{P_1^2}{m_{Z_0}^2} B^{\mu \nu}(P_1, m_{Z_0}, m_{Z_0}) \right\}. \quad (4.47) \]
These are the only two contributions in this sector and adding them up, we have

\[(4\pi)^{d/2} B_{H}^{U,C} = \mu^\varepsilon \left( [B_{H}^{U,C}]_\varepsilon + [B_{H}^{U,C}]_f \right) \tag{4.48} \]

where

\[
\varepsilon [B_{H}^{U,C}]_\varepsilon = 108\lambda_0^2 + 14\frac{\lambda_0^2 m_{Z_0}^2}{m_{H_0}} + 4\frac{s^2 \lambda_0^2}{m_{H_0}^4} + 4\frac{t^2 \lambda_0^2}{m_{H_0}^4} + 4\frac{u^2 \lambda_0^2}{m_{H_0}^4} \tag{4.49} \]

and

\[
[B_{H}^{U,C}]_f = 54\lambda_0^2 \ln \frac{\mu^2}{m_{H_0}^2} + 72\frac{\lambda_0^2 m_{Z_0}^4}{m_{H_0}^4} \ln \frac{\mu^2}{m_{Z_0}^2}. \tag{4.50} \]

We turn to the \( T \)-Boxes. In the Unitary gauge we have only two contributing diagrams, one finite and one divergent. The finite diagram is

\[
B_{H}^{U,\phi\phi} = 108\lambda_0^2 m_{H_0}^2 \mu^\varepsilon C_0(P_1, m_{H_0}, m_{H_0}, m_{H_0}). \tag{4.51} \]

The \( t \) and \( u \) channels can be obtained as explained in the \( R_\zeta \)-gauge calculation. The divergent diagram in this sector is

\[
B_{H}^{U,ZZZ} = -32\frac{m_{Z_0}^6}{m_{H_0}^4} \lambda_0^2 \mu^\varepsilon \left\{-dC_0(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \right. \\
+ \frac{1}{m_{Z_0}^2} \left\{(3m_{Z_0}^2 + P_1 \cdot P_1 + P_2 \cdot P_2)C_0(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) + 3B_0(P_1, m_{Z_0}, m_{Z_0}) \right\} \\
+ \frac{1}{m_{Z_0}^2} \left\{3U_{K4}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) + 4(P_1 + P_2)\mu C_{K3}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \right. \\
+ \left\{2(P_1 \mu P_1 + P_1 \mu P_2 + P_2 \mu P_2)C_{\mu
u}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) + 2P_1 \cdot P_2 B_0(p_1, m_{Z_0}, m_{Z_0}) \right\} \right\} \\
+ \left. \left\{2P_1 P_2 B_0(p_1, m_{Z_0}, m_{Z_0}) \right\} \right\} \tag{4.39} \]

and can be easily be obtained from Eq.\[4.39\] by dividing by \( v_0 \):
\[ \begin{align*}
+ \ & 2 P_1 \cdot P_2 (P_1 + P_2) \mu C^\mu (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) + P_1 \cdot P_2 C_0 (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \\
+ \ & \frac{1}{m_{Z_0}^6} \left\{ U_{K_0} (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) + 2 (P_1 + P_2) \mu U_{K_5} (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \\
+ \ & (P_1 \mu P_1 + 3 P_1 \mu P_2 + P_2 \mu P_2) U_{K_4} (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \\
+ \ & P_1 \cdot P_2 U_{K_4} (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \\
+ \ & (P_1 \mu P_1 P_2 + P_2 \mu P_2 P_1) C^{\mu \nu} (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \\
+ \ & P_1 \cdot P_2 (P_1 \mu + P_2 \mu) C_\mu (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \\
+ \ & P_1 \cdot P_2 P_1 \mu P_2 \nu C_\mu (P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \right\} .
\end{align*} \]

(4.52)

After summing over all channels by summing over all \((P_1, P_2)\), we obtain

\[
(4\pi)^{d/2} B_{H}^{U,T} = \mu^\varepsilon \left( [B_{H}^{U,T}]_\varepsilon + [B_{H}^{U,T}]_f \right) \tag{4.53}
\]

where

\[
\varepsilon [B_{H}^{U,T}]_\varepsilon = -28 s^2 \lambda_0^2 m_{H_0}^4 - 40 s \cdot t \lambda_0^2 m_{H_0}^4 - 28 t^2 \lambda_0^2 m_{H_0}^4 - 40 s \cdot u \lambda_0^2 m_{H_0}^4 - 40 t \cdot u \lambda_0^2 m_{H_0}^4 - 28 u^2 \lambda_0^2 m_{H_0}^4 \tag{4.54}
\]

and

\[
[B_{H}^{U,T}]_f = -162 \lambda_0^2 - 288 \lambda_0^2 m_{Z_0}^4 \tag{4.55}
\]

at \(p_i = 0\).

### 4.5.2 Irreducible Boxes

The irreducible, \(S\)-Boxes in the Unitary gauge are only two, one finite and one divergent. The finite diagram is the Higgs \(S\)-Box

\[
B_{H}^{U,\phi\phi\phi\phi} = 324 \lambda_0^2 m_{H_0}^2 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{D_1 D_2 D_3 D_4}. \tag{4.56}
\]
In DR

\[ \mathcal{B}_{H}^{\phi \phi \phi \phi} = 324 \lambda_{0}^{2} m_{H_{0}}^{2} \mu^{\epsilon} D_{0}(P_{1}, P_{2}, P_{3}, m_{H_{0}}, m_{H_{0}}, m_{H_{0}}, m_{H_{0}}). \]  

(4.57)

The divergent S-Box is the one with a gauge loop

\[ \Rightarrow i \mathcal{B}_{H}^{\phi \phi \phi \phi} \]

which is equal to

\[ \mathcal{B}_{H}^{\phi \phi \phi \phi} = 64 \frac{m_{Z_{0}}^{8}}{m_{H_{0}}^{4}} \lambda_{0}^{2} g_{\mu \nu} g_{\alpha \beta} g_{\gamma \delta} g_{\epsilon \zeta} \int \frac{d^{4}k}{(2\pi)^{4}} \frac{-i}{D_{1}D_{2}D_{3}D_{4}} \left( -g_{\alpha \beta} + \frac{(k + P_{\mu})(k + P_{\mu})}{m_{Z_{0}}^{2}} \left( -g_{\delta \epsilon} + \frac{(k + P_{3})_{\delta}(k + P_{3})_{\epsilon}}{m_{Z_{0}}^{4}} \right) \right), \]

(4.58)

and in DR

\[ \mathcal{B}_{H}^{\phi \phi \phi \phi} = 64 \frac{\lambda_{0}^{2} m_{Z_{0}}^{8}}{m_{H_{0}}^{4}} \mu^{\epsilon} \left\{ dD_{0}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}}, m_{Z_{0}}, m_{Z_{0}}) \right\} \]

\[- \left( \frac{1}{m_{Z_{0}}^{2}} \left\{ 4g_{\mu \nu} D_{\mu}^{\nu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}}, m_{Z_{0}, m_{Z_{0}}}) \right\} \right) \]

\[ + 2(P_{1} + P_{2} + P_{3})_{\mu} D_{\mu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}}, m_{Z_{0}, m_{Z_{0}}}) \]

\[ + (P_{1}^{2} + P_{2}^{2} + P_{3}^{2}) D_{0}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}, m_{Z_{0}}}) \}

\[ + \frac{1}{m_{Z_{0}}^{4}} \left\{ 6D_{B_{4}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}, m_{Z_{0}}}) \right\} \]

\[- \left( \frac{1}{m_{Z_{0}}^{2}} \left\{ 4U_{B_{6}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}, m_{Z_{0}}}) \right\} \right) \]

\[ + 6(P_{1} + P_{2} + P_{3})_{\mu} U_{B_{5}}^{\mu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}, m_{Z_{0}}}) \]

\[ + 3(2P_{1 \mu} P_{2 \nu} + P_{2 \mu} P_{3 \nu} + 2P_{1 \mu} P_{3 \nu} + 2P_{2 \mu} P_{3 \nu}) \]

\[ + P_{3 \mu} P_{3 \nu} + P_{1 \mu} P_{1 \nu}) D_{B_{4}}^{\mu \nu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}, m_{Z_{0}}}) \]

\[ + 2(P_{1} \cdot P_{2} + P_{1} \cdot P_{3} + P_{2} \cdot P_{3}) D_{B_{4}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}, m_{Z_{0}}, m_{Z_{0}}}) \}

\[- \left( \frac{1}{m_{Z_{0}}^{4}} \left\{ U_{B_{8}}(P_{1}, P_{2}, P_{3}, m_{Z_{0}}, m_{Z_{0}, m_{Z_{0}}}) \right\} \right) \]

\[ + 2(P_{1} + P_{2} + P_{3})_{\mu} U_{B_{7}}^{\mu}(P_{1}, P_{2}, P_{3}, m_{Z_{0}, m_{Z_{0}}, m_{Z_{0}}}) \]
\[
+ \left\{ P_{1\mu}P_{1\nu} + 3P_{1\mu}P_{2\nu} + P_{2\mu}P_{2\nu} \right. \\
+ 4P_{1\mu}P_{3\nu} + 3P_{2\mu}P_{3\nu} + P_{3\mu}P_{3\nu} \right\} U_{B_6}^{\mu\nu}(P_1, P_2, P_3, m_{z_0}, m_{\chi_0}, m_{Z_0}, m_{\gamma_0}) \\
+ (P_1 \cdot P_2 + P_2 \cdot P_3)U_{B_6}(P_1, P_2, P_3, m_{z_0}, m_{Z_0}, m_{\gamma_0}, m_{\gamma_0}) \\
+ \left\{ P_{1\mu}P_{1\nu}P_{2\alpha} + P_{1\mu}P_{2\nu}P_{2\alpha} + 2P_{1\mu}P_{1\nu}P_{3\alpha} + 4P_{1\mu}P_{2\nu}P_{3\alpha} \right. \\
+ P_{2\mu}P_{2\nu}P_{3\alpha} + 2P_{1\mu}P_{3\nu}P_{3\alpha} + P_{2\mu}P_{3\nu}P_{3\alpha} \right\} U_{B_5}^{\mu\nu\alpha}(P_1, P_2, P_3, m_{z_0}, m_{Z_0}, m_{\gamma_0}, m_{\gamma_0}) \\
+ \left\{ P_{1\mu}P_{2\nu}[P_1 + P_2 + 2P_3]_{\mu} + P_{2\nu}P_{3}[2P_1 + P_2 + P_3]_{\mu} \right\} U_{B_5}^{\mu\nu}(P_1, P_2, P_3, m_{z_0}, m_{Z_0}, m_{\gamma_0}, m_{\gamma_0}) \\
+ \left\{ P_{1\mu}P_{1\nu}P_{2\alpha}P_{3\beta} + P_{1\mu}P_{2\nu}P_{2\alpha}P_{3\beta} \right. \\
+ P_{1\mu}P_{1\nu}P_{3\alpha}P_{3\beta} + P_{1\mu}P_{2\nu}P_{3\alpha}P_{3\beta} \right\} D^{\mu\nu\alpha\beta}(P_1, P_2, P_3, m_{z_0}, m_{Z_0}, m_{\gamma_0}, m_{\gamma_0}) \\
+ \left\{ P_{1\mu}P_{2\nu}[P_{1\mu}P_{1\nu} + 2P_{1\mu}P_{3\nu} + P_{2\mu}P_{3\nu} + P_{3\mu}P_{3\nu}] \right. \\
+ P_{2\nu}P_{3}[P_{1\mu}P_{1\nu} + P_{1\mu}P_{2\nu} + 2P_{1\mu}P_{3\nu} + P_{2\mu}P_{3\nu}] \right\} D_{B_4}(P_1, P_2, P_3, m_{Z_0}, m_{Z_0}, m_{Z_0}) \\
+ \left\{ (P_1 \cdot P_2 \times P_2 \cdot P_3)D_{B_4}(P_1, P_2, P_3, m_{z_0}, m_{z_0}, m_{Z_0}, m_{Z_0}) \right\} \right) \\
+ (B_H^{U,ZZZZ})_{f,1} + (B_H^{U,ZZZZ})_{f,2} + (B_H^{U,ZZZZ})_{f,3}. \tag{4.59}
\]

The \((B_H^{U,ZZZZ})_{f,1}, (B_H^{U,ZZZZ})_{f,2}\) and \((B_H^{U,ZZZZ})_{f,3}\) are finite integrals, moved to Appendix \[E\]. The \(U\)-integrals are dealt with in Appendix \[C\] and the \((P_1, P_2, P_3)\) as in the \(R_\epsilon\) gauge. The divergence structure of the sum of the \(S\)-Boxes at zero external momenta is revealed through the relation

\[
(4\pi)^{d/2}B_H^{U,S} = \mu^\varepsilon \left( [B_H^{U,S}]_\varepsilon + [B_H^{U,S}]_f \right) \tag{4.60}
\]

where

\[
\varepsilon[B_H^{U,S}]_\varepsilon = 24 s^2 \lambda_0^2 + 40 s \cdot t \lambda_0^2 + 24 s^2 \lambda_0^2 + 40 s \cdot u \lambda_0^2 + 40 s \cdot m_{z_0}^4 \lambda_0^2 + 24 s \cdot u \lambda_0^2 + 24 s \cdot m_{z_0}^4 \lambda_0^2 \\
+ 24 t^2 \lambda_0^2 + 40 t \cdot u \lambda_0^2 + 40 t \cdot m_{z_0}^4 \lambda_0^2 + 24 t^2 \lambda_0^2 + 24 t \cdot u \lambda_0^2 + 24 t \cdot m_{z_0}^4 \lambda_0^2 \\
+ 24 u^2 \lambda_0^2 + 40 u \cdot v \lambda_0^2 + 40 u \cdot m_{z_0}^4 \lambda_0^2 + 24 u^2 \lambda_0^2 + 24 u \cdot v \lambda_0^2 + 24 u \cdot m_{z_0}^4 \lambda_0^2
\]

and

\[
[B_H^{U,S}]_f = 162 \lambda_0^2 + 96 \lambda_0^2 m_{z_0}^4 \tag{4.62}
\]

The total sum of the Boxes then satisfies

\[
\varepsilon[B_H^{U,S}]_\varepsilon = 108 \lambda_0^2 + 144 \lambda_0^2 m_{z_0}^4 \tag{4.63}
\]

and

\[
[B_H^{U,S}]_f = -168 \lambda_0^2 m_{z_0}^4 + 54 \lambda_0^2 \ln \frac{\mu^2}{m_{z_0}^4} + 72 \lambda_0^2 m_{z_0}^4 \ln \frac{\mu^2}{m_{z_0}^4} \tag{4.64}
\]

53
In the 4-point function as well, we observe a sector by sector agreement between the $R_\xi$ and Unitary gauges.

5 Renormalization

In this section we will renormalize the Abelian-Higgs model in both the $R_\xi$ and Unitary gauges. In fact, since we will be concerned here mainly with the $Z$-mass and scalar potential we will be able to perform the renormalization program simultaneously for both.

One of the results of the calculation of the previous two sections is that the one-loop corrections to the $Z$-mass and the scalar potential have identical divergent parts in the $R_\xi$ and Unitary gauges. This means that the two gauges have the same $\beta$-functions for the masses and quartic coupling. Now the Lagrangean in the Unitary gauge can be obtained from the $R_\xi$-gauge Lagrangean by dropping the gauge fixing and ghost terms and simply setting the Goldstone field to zero. Since we are interested here only in the common subsector that consists of the $Z$-mass term and the scalar potential, we can carry out the renormalization program on the Unitary gauge Lagrangean and only when we arrive at the stage where we analyse the finite, renormalized scalar potential where finite corrections become relevant, we may have to distinguish between the two gauges if necessary. Thus for now and until further notice, we drop the ”U” superscript that denotes Unitary gauge.

5.1 Counter-terms

We introduce the counter-terms

\[\begin{align*}
m_0^2 &= m^2 + \delta m \\
g_0 &= g + \delta g \\
\lambda_0 &= \lambda + \delta \lambda
\end{align*}\]  

(5.1)

In the Abelian-Higgs model there is a non-zero anomalous dimension for the scalar (as opposed to $\phi^4$ theory and the linear sigma model) and likewise a non-zero anomalous dimension for the gauge boson. Therefore, we also introduce

\[\begin{align*}
H_0 &= \sqrt{Z_\phi} \phi = \sqrt{1 + \delta \phi} \phi \\
A_0 &= \sqrt{Z_A} A = \sqrt{1 + \delta A} A
\end{align*}\]  

(5.2)

Substituting the above in Eq. (2.1) and then expressing the Higgs in the Polar basis, we obtain

\[\mathcal{L} = \mathcal{L}^{\text{tree}} + \mathcal{L}^{\text{count.}},\]  

(5.3)
with $\mathcal{L}^{\text{tree}}$ the renormalized tree-level Lagrangean

$$\mathcal{L}^{\text{R.tree}} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\partial_{\mu} \phi) (\partial_{\nu} \phi) + \frac{1}{2} m_Z^2 A_{\mu} A^\mu + g_{\mu\nu} \frac{\lambda m_Z^2}{m_H^2} A_{\mu} A_{\nu} \phi^2$$

$$+ g_{\mu\nu} \frac{m_Z^2}{m_H} \sqrt{2\lambda} \phi A_{\mu} A_{\nu} - \frac{1}{2} m_H^2 \phi^2 - \sqrt{\lambda} m_H \phi^3 - \frac{\lambda}{4} \phi^4 \quad (5.4)$$

and $\mathcal{L}^{\text{count.}}$, the counterterm Lagrangian

$$\mathcal{L}^{\text{count.}} = \frac{1}{2} \left\{ -(p^2 g_{\mu\nu} - p_{\mu} p_{\nu} - g_{\mu\nu} m_Z^2) \delta A + g_{\mu\nu} m_Z^2 \phi^2 + g_{\mu\nu} \frac{2m_Hm_Z}{\sqrt{2\lambda}} \delta g + \mathcal{M}_{Z,\mu\nu} \right\} A_{\mu} A_{\nu}$$

$$+ g_{\mu\nu} \left\{ \frac{\sqrt{2\lambda} m_Z}{m_H} \delta g + \frac{\lambda m_Z^2}{m_H^2} \delta A + \frac{\lambda m_Z^2}{m_H^2} \delta \phi + \mathcal{M}_{Z3} \right\} A_{\mu} A_{\nu}$$

$$+ g_{\mu\nu} \left\{ 2m_Z \delta g + \frac{\sqrt{2\lambda} m_Z^2}{m_H} \delta A + \frac{\sqrt{2\lambda} m_Z^2}{m_H} \delta \phi + \mathcal{M}_{Z3} \right\} \phi A_{\mu} A_{\nu}$$

$$+ \frac{m_H}{\sqrt{2\lambda}} \left\{ -\frac{1}{2} m_H \frac{\delta \phi}{\delta \lambda} \frac{m^2}{2\lambda} + \frac{m_H^2}{2\lambda} \delta \phi + m_H + T_H \right\} \phi$$

$$+ \frac{1}{2} \left\{ (p^2 - \frac{5}{2} m_H^2) \delta \phi + m_H - \frac{3m_H^2}{2\lambda} \delta \lambda + \mathcal{M}_H \right\} \phi^2$$

$$- \frac{m_H}{\sqrt{2\lambda}} \left\{ \delta \lambda + 2\lambda \delta \phi - \frac{\sqrt{2\lambda} K_H}{m_H} \right\} \phi^3 - \frac{1}{4} \left\{ \delta \lambda + 2\lambda \delta \phi - \frac{B_H}{6} \right\} \phi^4 \quad (5.5)$$

with the computed one-loop corrections also added with appropriate factors. Corrections to interaction terms between the Higgs and the $Z$ ($\mathcal{M}_{Z3}$ and $\mathcal{M}_{Z4}$) we have not computed but we are not concerned with those here.

The Feynman rules for the counterterms deriving from the above expression are

- Higgs 1-point function

$$\begin{array}{c}
\bullet \\
\end{array} = i \frac{m_H}{\sqrt{2\lambda}} \left[ -\frac{1}{2} m_H^2 \delta \phi + \delta m - \frac{m_H^2}{2\lambda} \delta \lambda \right]$$

- Gauge boson 2-point function

$$\begin{array}{c}
\begin{array}{c}
\circ \circ \circ \circ \circ \circ \circ \\
\end{array}
= i \left[ -(p^2 g_{\mu\nu} - p_{\mu} p_{\nu} - g_{\mu\nu} m_Z^2) \delta A + g_{\mu\nu} m_Z^2 \delta \phi + g_{\mu\nu} \frac{\sqrt{2m_Hm_Z}}{\sqrt{\lambda}} \delta g \right]
\end{array}$$

- Higgs 2-point function
\[ = i \left[ (p^2 - \frac{5}{2} m_H^2) \delta \phi + \delta m - \frac{3 m_H^2 \delta \lambda}{2\lambda} \right] \]

- For completeness, the \( \phi-Z-Z \) counterterm vertex

\[ = ig^{\mu\nu} \left[ 4 m_Z \delta g + 2 \sqrt{2} \lambda m_Z^2 \delta A + 2 \sqrt{2} \lambda m_Z^2 \delta \phi \right] \]

- \( \phi-\phi-\phi \) vertex counterterm

\[ = -6i \frac{m_H}{\sqrt{2}\lambda} [\delta \lambda + 2\lambda \delta \phi] \]

- For completeness, the \( \phi-\phi-Z-Z \) vertex counterterm

\[ = ig^{\mu\nu} \left[ 4 \sqrt{2} \lambda m_Z \delta g + \frac{4 \lambda m_Z^2}{m_H^2} \delta A + \frac{4 \lambda m_Z^2}{m_H^2} \delta \phi \right] \]

- \( \phi-\phi-\phi-\phi \) vertex counterterm

\[ = -6i \left[ \delta \lambda + 2\lambda \delta \phi \right] \]

The renormalization conditions are in order. All conditions are imposed at zero external momenta. Regarding the gauge boson sector, our renormalization condition is that the mass of the gauge boson be \( m_Z = g v_0 \). Diagrammatically this condition is

\[ + \quad = 0 \]

and as an equation

\[ M_Z - \frac{1}{3} \left( g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2} \right) \left( -p^2 g_{\mu\nu} + p_{\mu} p_{\nu} + g_{\mu\nu} m_Z^2 \right) \delta A + g_{\mu\nu} m_Z^2 \delta \phi + g_{\mu\nu} \frac{2 m_H m_Z}{\sqrt{2} \lambda} \delta g = 0 \]
This is an independent condition from the rest that can be directly solved for $\delta g$:

$$2v_0\delta g = \frac{M_Z}{m_Z} - \left( m_Z - \frac{p^2}{m_Z} \right) \delta A - m_Z \delta \phi$$  \hspace{1cm} (5.7)

At zero external momentum, this becomes

$$v_0\delta g \equiv \delta m_Z = \frac{\mu^2}{(4\pi)^2} \left[ \frac{2}{3} \frac{\lambda m_Z^2}{m_H^2} + \frac{1}{2} \left( \frac{M_Z}{m_Z} - m_Z \delta A - m_Z \delta \phi \right) \right],$$  \hspace{1cm} (5.8)

where we have absorbed in $\delta g$ all the finite parts. Just one comment on the Higgs-$Z$ interaction terms that we have not computed here: they become finite provided that

$$\frac{M_Z}{m_Z^2} + \mathcal{M}_{Z3,4} = \text{finite}. \hspace{1cm} (5.9)$$

Now we turn to the scalar sector. In order to avoid the tadpoles contaminating the one-loop vev for the Higgs field, we impose a vanishing tadpole condition. Diagrammatically it is

$$\begin{array}{c}
\quad \downarrow \\
\quad + \quad \bullet \\
= 0
\end{array}$$

and as an equation

$$\mathcal{T}_H + \frac{m_H}{\sqrt{2\lambda}} \left( -\frac{1}{2} m_H^2 \delta \phi + \delta m - \frac{m_H^2}{2\lambda} \delta \lambda \right) = 0. \hspace{1cm} (5.10)$$

The second condition is the requirement that the only term that remains in the quadratic part of the potential, is $m_H$. This means that in the quadratic term of the potential we absorb, together with the divergent part, the entire finite term as well:

$$\begin{array}{c}
\quad \downarrow \\
\quad + \quad \bullet \\
= 0
\end{array}$$

or, in equation at $p = 0$,

$$\mathcal{M}_H - \frac{5}{2} m_H^2 \delta \phi + \delta m - \frac{3m_H^2}{2\lambda} \delta \lambda = 0. \hspace{1cm} (5.11)$$

These two conditions fix completely our freedom. The solution to the system of Eq. (5.10) and Eq. (5.11) is

$$\delta m = \frac{\mu^2}{(4\pi)^2} \left[ \frac{1}{\varepsilon} \left( 6\lambda m_Z^2 - 12\lambda m_Z^2 \right) + \frac{1}{2} \left( \mathcal{M}_H - m_H^2 \delta \phi - \frac{3\sqrt{2\lambda}}{m_H} \mathcal{T}_H \right) \right].$$  \hspace{1cm} (5.12)
\[
\delta \lambda = \frac{\mu^2}{(4\pi)^2} \left[ \frac{1}{\varepsilon} \left( 18\lambda^2 - 24\lambda^2 \frac{m_Z^2}{m_H^2} + 24\lambda^2 \frac{m_Z^4}{m_H^4} \right) + \frac{\lambda}{m_H^2} \left( M_H - 2m_H^2 \delta \phi - \frac{\sqrt{2} \lambda}{m_H} \right) \right]_f
\]

(5.13)

For later reference note that
\[
\delta m_H = 2\delta m. \quad (5.14)
\]

Divergences must cancel automatically from the rest. Absence of divergences in the cubic coupling means

\[
\begin{array}{c}
\text{gauge} \\
\text{vertex} \\
= \text{finite}
\end{array}
\]

or
\[
\frac{\mathcal{K}_H}{6} - \frac{m_H}{\sqrt{2}\lambda} (2\lambda \delta \phi + \delta \lambda) = \text{finite}. \quad (5.15)
\]

Substituting Eq. (5.12) and Eq. (5.13) in the above, we find that the divergent part cancels as expected and we are left with the finite terms
\[
C_{\phi^3} = \frac{1}{16\pi^2} \left( \frac{\mathcal{K}_{Hf}}{6} - \frac{1}{2} \frac{\sqrt{2} \lambda}{m_H} \mathcal{M}_{Hf} + \frac{\lambda}{m_H^2} T_{Hf} \right). \quad (5.16)
\]

Absence of divergences from the quartic coupling on the other hand requires

\[
\begin{array}{c}
\text{gauge} \\
\text{vertex} \\
= \text{finite}
\end{array}
\]

or
\[
\frac{\mathcal{B}_H}{6} - (2\lambda \delta \phi + \delta \lambda) = \text{finite}. \quad (5.17)
\]

Substituting again Eq. (5.12) and Eq. (5.13) in the above, we again observe the cancellation of the divergent part and we collect the finite piece
\[
C_{\phi^4} = \frac{1}{16\pi^2} \left( \frac{\mathcal{B}_{Hf}}{6} - \frac{\lambda}{m_H^2} \mathcal{M}_{Hf} + \frac{\lambda\sqrt{2}\lambda}{m_H^3} T_{Hf} \right). \quad (5.18)
\]

The subscript \( f \) in the various one-loop quantities denotes finite part, in either the \( R_\xi \) or the Unitary gauge. For example, \( T_{Hf} = [T_H^{R_\xi}]_f + \{T_H^{R_\xi}\}_f \) in the \( R_\xi \) gauge \( T_{Hf} = [T_H^U]_f \) in the Unitary gauge.

The finite one-loop Higgs potential we are left with after renormalization is then
\[
V_1(\phi) = \frac{1}{2} m_H^2 \phi^2 + \left[ \sqrt{\frac{\lambda}{2}} m_H + C_{\phi^3} \right] \phi^3 + \frac{1}{4} [\lambda + C_{\phi^4}] \phi^4. \quad (5.19)
\]
The Higgs field anomalous dimension has apparently cancelled from the renormalized potential. We are now ready to minimize this potential. There are three extrema. Our preferred "physical" solution for the global minimum is

\[ \langle \phi \rangle \equiv v = 0 , \] (5.20)

for which the potential satisfies \( V_1''(v) = m_H^2 \). The quantities \( C_{\phi^3} \) and \( C_{\phi^4} \) are examples of the \( \star \) quantities in Eq. (1.1) of the Introduction. Note finally that \( V_1(v) = 0 \).

### 5.2 Physical quantities and the \( \beta \)-functions

Let us denote by \( \alpha_0 \) a generic bare coupling. Its counter-term \( \delta \alpha(\mu) \) is introduced via the relation

\[ \alpha_0 = \alpha(\mu) + \delta \alpha(\mu) \] (5.21)

where \( \alpha(\mu) \) is the renormalized running coupling. At one-loop the counter-term has the form

\[ \delta \alpha(\mu) = \frac{\mu^\varepsilon}{(4\pi)^2} \left( \frac{C_\alpha}{\varepsilon} + \sum_k f_{\Delta A_0}^k \ln \frac{\mu^2}{m_k^2} + \sum_{k,i} f_{\Delta B_0}^k \int_0^1 dx \ln \left( \frac{\mu^2}{\Delta_k^i(m_k,m_i)} \right) \right) . \] (5.22)

The indices \( i \) and \( k \) are counting the fields running in the loop. The entire divergent part has been collected in the term proportional to \( C_\alpha \). In the notation of the previous sections, we can identify \( (\mu^\varepsilon/16\pi^2)C_\alpha = \varepsilon(\delta \alpha)_\varepsilon = \varepsilon[\delta \alpha]_\varepsilon + \varepsilon\{\delta \alpha\}_\varepsilon \) and \( (\delta \alpha)_f = [\delta \alpha]_f + \{\delta \alpha\}_f \) containing the finite logarithms and the non-logarithmic finite term. As already noted, in the Unitary gauge we have \( \{\delta \alpha\}_\varepsilon = \{\delta \alpha\}_f = 0 \) by definition.

It is useful to review the calculations of \( \beta \)-functions in the presence of multiple couplings. We introduce the boundary condition

\[ \alpha(\mu = m_{\text{phys.}}) \equiv \alpha . \] (5.23)

We also use the following standard definitions

\[ \delta_\alpha \equiv \frac{\delta \alpha(\mu)}{\alpha(\mu)} \]

\[ \beta_\alpha \equiv \frac{d}{d\mu} \alpha(\mu) \]

\[ \tilde{\beta}_\alpha \equiv \frac{\beta_\alpha}{\alpha} \] (5.24)
For a general coupling we recall the successive relations

\[
0 = \mu \frac{d}{d\mu} \alpha_0 = \mu \frac{d}{d\mu} \left\{ \mu^\varepsilon \alpha(\mu)(1 + \delta_\alpha) \right\} = \mu \frac{d}{d\mu} \left\{ \mu^\varepsilon \alpha(\mu) + \mu^\varepsilon \delta \alpha(\mu) \right\} \Leftrightarrow \\
0 = \mu \left\{ \varepsilon \alpha(1 + \delta_\alpha) + (1 + \delta_\alpha) \mu \frac{\partial \alpha}{\partial \mu} + \alpha \mu \frac{\partial \delta_\alpha}{\partial \mu} \right\} \Leftrightarrow \\
\beta_\alpha(1 + \delta_\alpha) = -\varepsilon \alpha(1 + \delta_\alpha) - \alpha \mu \frac{\partial \delta_\alpha}{\partial \mu} \Leftrightarrow \\
\beta_\alpha = -\varepsilon \alpha - \alpha \mu \frac{\partial \delta_\alpha}{\partial \mu} (1 + \delta_\alpha)^{-1} \Leftrightarrow \\
\beta_\alpha = -\varepsilon \alpha - \alpha \mu \frac{\partial \delta_\alpha}{\partial \mu} \Leftrightarrow , \quad (5.25)
\]

where, since \( \delta_\alpha \sim O(h) \), we have performed an expansion in \( h \) in order to get rid of terms of \( O(h^2) \) like \( \delta_\alpha \frac{\partial \delta_\alpha}{\partial \mu} \). In the case of the AH model where we have three couplings, we will have a system of equations:

\[
\beta_\lambda = -\varepsilon \lambda - \lambda \left\{ \beta_\lambda \frac{\partial \delta_\lambda}{\partial \lambda} + \beta_{m_H} \frac{\partial \delta_\lambda}{\partial m_H} + \beta_{m_Z} \frac{\partial \delta_\lambda}{\partial m_Z} \right\} \\
\beta_{m_H} = -\varepsilon m_H^2 - m_H^2 \left\{ \beta_\lambda \frac{\partial \delta_{m_H}}{\partial \lambda} + \beta_{m_H} \frac{\partial \delta_{m_H}}{\partial m_H} + \beta_{m_Z} \frac{\partial \delta_{m_H}}{\partial m_Z} \right\} \\
\beta_{m_Z} = -\varepsilon m_Z^2 - m_Z^2 \left\{ \beta_\lambda \frac{\partial \delta_{m_Z}}{\partial \lambda} + \beta_{m_H} \frac{\partial \delta_{m_Z}}{\partial m_H} + \beta_{m_Z} \frac{\partial \delta_{m_Z}}{\partial m_Z} \right\} . \quad (5.26)
\]

This system can be rewritten as

\[
\beta_\lambda(1 + \frac{\partial \delta_\lambda}{\partial \lambda}) + \lambda \left\{ \beta_{m_H} \frac{\partial \delta_\lambda}{\partial m_H} + \beta_{m_Z} \frac{\partial \delta_\lambda}{\partial m_Z} \right\} = -\varepsilon \lambda \\
\beta_{m_H} (1 + \frac{\partial \delta_{m_H}}{\partial m_H}) + m_H^2 \left\{ \beta_\lambda \frac{\partial \delta_{m_H}}{\partial \lambda} + \beta_{m_H} \frac{\partial \delta_{m_H}}{\partial m_H} + \beta_{m_Z} \frac{\partial \delta_{m_H}}{\partial m_Z} \right\} = -\varepsilon m_H^2 \\
\beta_{m_Z} (1 + \frac{\partial \delta_{m_Z}}{\partial m_Z}) + m_Z^2 \left\{ \beta_\lambda \frac{\partial \delta_{m_Z}}{\partial \lambda} + \beta_{m_H} \frac{\partial \delta_{m_Z}}{\partial m_H} + \beta_{m_Z} \frac{\partial \delta_{m_Z}}{\partial m_Z} \right\} = -\varepsilon m_Z^2 , \quad (5.27)
\]

or in matrix form as

\[
\begin{pmatrix}
1 + \frac{\partial \delta_\lambda}{\partial \lambda} & \lambda \frac{\partial \delta_\lambda}{\partial m_H} & \lambda \frac{\partial \delta_\lambda}{\partial m_Z} \\
\frac{\partial \delta_{m_H}}{\partial \lambda} & 1 + \frac{\partial \delta_{m_H}}{\partial m_H} & \frac{\partial \delta_{m_H}}{\partial m_Z} \\
\frac{\partial \delta_{m_Z}}{\partial \lambda} & \frac{\partial \delta_{m_Z}}{\partial m_H} & 1 + \frac{\partial \delta_{m_Z}}{\partial m_Z}
\end{pmatrix}
\begin{pmatrix}
\beta_\lambda \\
\beta_{m_H} \\
\beta_{m_Z}
\end{pmatrix}
=
\begin{pmatrix}
-\varepsilon m_H^2 \\
-\varepsilon m_Z^2
\end{pmatrix} \quad (5.28)
\]

Inverting the matrix we obtain

\[
16\pi^2 \beta_\lambda = \lambda^2 \frac{\partial \alpha}{\partial \lambda} + \lambda m_H^2 \frac{\partial \alpha}{\partial m_H} + \lambda m_Z^2 \frac{\partial \alpha}{\partial m_Z} 
\]
Thus, all that we need to do in order to obtain the various $\beta$-functions, is to identify from the explicit form of the counterterms the quantities $C_\alpha$ defined in Eq. (5.22) and build its $\beta$-function, according to Eq. (5.29).

Moreover, solving the differential equation for the running coupling yields the Renormalization Group flow of the coupling $\alpha$:

$$\alpha(\mu) = \frac{\alpha}{1 + \beta_\alpha \ln \left( \frac{m_{\text{phys}}}{\mu} \right)}$$  \hspace{1cm} (5.30)

The Landau pole associated with the coupling $\alpha$ is

$$\mu_L^\alpha = m_{\text{phys}} e^{\frac{\alpha}{\beta_\alpha}}.$$  \hspace{1cm} (5.31)

In the AH model we found from our one-loop calculation that $C_{m_Z} = \frac{2}{3} \frac{\lambda m_{H}^4}{m_{H}^2}$, $C_{m_H} = 6\lambda m_{H}^2 - 12\lambda m_{Z}^2$ and $C_\lambda = 18\lambda^2 - 24\lambda^2 \frac{m_{Z}^2}{m_{H}^2} + 24\lambda^2 \frac{m_{Z}^4}{m_{H}^4}$ that immediately determine

$$\beta_{m_Z} = \frac{1}{16\pi^2} \left( \frac{2 \lambda m_{Z}^2}{3 m_{H}^2} \right)$$

$$\beta_{m_H} = \frac{1}{16\pi^2} \left( 6\lambda m_{H}^2 - 12\lambda m_{Z}^2 \right)$$

$$\beta_\lambda = \frac{1}{16\pi^2} \left( 18\lambda^2 - 24\lambda^2 \frac{m_{Z}^2}{m_{H}^2} + 24\lambda^2 \frac{m_{Z}^4}{m_{H}^4} \right).$$  \hspace{1cm} (5.32)

The above expressions hold for both $R_\xi$ and Unitary gauges. It is interesting to notice that these $\beta$-functions are not identically the same as those that one would compute in a Cartesian basis for the Higgs. For a comparison see the next section.

### 6 The one-loop Higgs potential

To summarize our result regarding the one-loop Higgs potential, in Eq. (5.19) we found that it is determined by the quantities $C_{\phi^3}$ and $C_{\phi^4}$ in Eq. (5.16) and in Eq. (5.18) respectively, yielding the finite expression

$$V_1(\phi) = \frac{1}{2} m_H^2 \phi^2 + \left[ \sqrt{\frac{\lambda}{2}} m_H - \frac{m_H}{16\pi^2 \sqrt{2\lambda}} \left( 9\lambda^2 + \frac{8\lambda^2 m_{Z}^2}{m_{H}^4} \right) \right] \phi^3$$

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\[ + \frac{1}{4} \left[ \lambda - \frac{1}{16\pi^2} \left( \frac{32\lambda^2 m_H^2}{m_H^2} \right) \right] \phi^4. \] (6.1)

The form of the potential is such that the \( \xi \)-independent part of its \( R_\xi \) gauge expression is the same as the Unitary gauge expression. Moreover, using the standard prescription to compute the \( U_T \) integral, see Eq. (C.2), the \( \xi \)-dependent part is made to vanish. In addition, the potential has no explicit \( \mu \)-dependence. All the \( \mu \)-dependence is implicit, through the dependence on \( \mu \) of the renormalized quantities \( \lambda, m_H \) and \( m_Z \) via their RG evolution. In the following numerical analysis one can either interpret the potential in Eq. (6.1) as a \( \xi \)-independent \( R_\xi \) gauge potential or simply as a Unitary gauge potential, as these are the same independently of prescriptions.

Before we proceed with the study of the gauge invariant Higgs potential Eq. (6.1), we recall the standard result used to extract physics from the Higgs potential. The usual method to compute the Higgs potential is in the context of the background field method, where a functional integration yields the so called Higgs effective potential. We will not review the details of this well known calculation; we assume that the reader has some familiarity with it. The derivation of an effective scalar potential via the background field method at one-loop is much simpler than computing Feynman diagrams. The simplicity is related among other reasons to the Cartesian basis representation of the Higgs field because in the Cartesian basis the Gaussian path integral involved, is almost trivial. The result of the calculation for the scalar potential in the Abelian-Higgs model (see for example [6]) with the same normalization of the mass and quartic coupling as in Eq. (2.1), in \( \overline{\text{MS}} \) scheme with Fermi gauge fixing, is \( (H_0 \to \phi/\sqrt{2}) \)

\[ V_{\text{1,eff.}}^{\overline{\text{MS}}} = -\frac{1}{2} m_A^2 \phi^2 + \frac{\lambda}{4} \phi^4 \]

\[ - \frac{1}{64\pi^2} \left\{ 3m_A^4 \left( \ln \frac{\mu^2}{m_A^2} + \frac{5}{6} \right) + m_B^4 \left( \ln \frac{\mu^2}{m_B^2} + \frac{3}{2} \right) + \sum_{\pm} m_{C,\pm}^4 \left( \ln \frac{\mu^2}{m_{C,\pm}^2} + \frac{3}{2} \right) \right\} \] (6.2)

with

\[ m_A^2 = g^2 \phi^2 \]
\[ m_B^2 = 3\lambda \phi^2 - m^2 \]
\[ m_{C,\pm}^2 = \frac{1}{2} \left[ (\lambda \phi^2 - m^2) \pm \sqrt{(\lambda \phi^2 - m^2)^2 - 4\xi g^2 \phi^2 (\lambda \phi^2 - m^2)} \right]. \] (6.3)

The derivation of the \( \beta \)-function is also simple, provided that a separate diagram calculation has yielded the also well known result

\[ 16\pi^2 \gamma = g^2 (-3 + \xi) \] (6.4)
Figure 1: The RG evolution of the $Z$ (left) and Higgs (right) mass.

for the anomalous dimension $\gamma$. Then, one finds

$$16\pi^2 \beta_{m^2} = 8\lambda m^2 - 6g^2 m^2$$
$$16\pi^2 \beta_\lambda = 20\lambda^2 - 12\lambda g^2 + 6g^4$$
$$16\pi^2 \beta_g = \frac{1}{3} g^4$$

(6.5)

for the $\beta$-functions. These Cartesian basis results are not identically the same as the Polar basis results in Eq. (5.32). Despite however the fact that some coefficients are not the same, their physical content is similar.

In Figs. 1 and 2 we plot the RG evolution of the Higgs and the $Z$ mass as well as that of the coupling $\lambda$, as a function of the Renormalization scale $\mu$, as determined in Eq. (5.32). We do not produce separate figures for Eq. (6.5) because the numerical differences are quite small. The physical values at $\mu = 125$ GeV we use are 125 GeV for the Higgs mass, 91 GeV for the $Z$-mass and 0.12 for $\lambda$. We stop the evolution at a certain scale

$$\mu_I \simeq 3.03 \cdot 10^{16} \text{GeV}$$

(6.6)

whose physical meaning will be discussed below. We observe the usual perturbative, logarithmic evolution of the couplings.

The potential in Eq. (6.1) is gauge invariant and the one in Eq. (6.2) is manifestly gauge dependent. To quantify the effect of $\xi$, the different basis for the Higgs and the different subtraction schemes, we compare numerically the two results. The comparison that follows should be clearly taken with a grain of salt, as the two objects are quite different. In Fig. 3 we plot the difference between the one-loop Higgs potential Eq. (6.1) and the effective potential Eq. (6.2) at $\mu = 125$ GeV, for $\xi = 0.001, 1, 10, 100, 100000$. The running values of $m_H$, $m_Z$ and $\lambda$ in the potential $V_1$ are determined from Eq. (5.32), while those of $m$, $g$ and $\lambda$ in $V_{1,\text{eff}}^{\overline{\text{MS}}}$ are determined using Eq. (6.5). A regime around $\phi = 0$ in the plot is missing because there the effective potential is imaginary. There seem to be ways to fix this [19] but this is not our concern here. $V_1$ remains instead always real. What we
Figure 2: The RG evolution of $\lambda$.

Figure 3: The difference $V_1(\phi) - V_{1,\text{eff}}^{\text{MS}}(\phi)$ between the 1-loop and effective Higgs potentials at $\mu = 125$ GeV for various values of $\xi = 10^{-3}, 1, 10, 100, 10^5$. Curves taking larger values on the vertical axis correspond to larger $\xi$.

observe is that as $\xi$ increases, the regime where the two expressions agree shrinks. There seems to be a value of $\xi$ for which the best agreement is achieved, which from the figure is around $\xi < O(10)$. The limit $\xi \to \infty$ of the effective potential is clearly singular: in Eq. (1.1) we have $\lim_{\xi \to \infty} g(\xi) = \infty$.

From now on we concentrate on $V_1(\phi)$ in Eq. (6.1) and we analyze it numerically. First,
in Fig. 4 we plot it at the physical scale $\mu = 125$ GeV. We observe that it has two minima, of which the one at $\phi = 0$ is the global minimum, as claimed. The tilt in the mexican hat shape implies that the global $Z_2$ symmetry $H_0 \rightarrow -H_0$ of the classical potential has been spontaneously broken by quantum effects. The breaking is small and the vacuum in the interior of the Higgs phase is stable. Going to higher scales, we observe a big desert. To illustrate this, in Fig. 5 we plot the potential for $\mu = 10^{12}$ GeV and $\mu = 10^{19}$ GeV respectively. The Abelian-Higgs model does not care about 'low' intermediate scales including the Planck scale. It roams through them perturbatively to much higher scales. The first qualitative change observed is around $\mu \simeq 10^{40}$ GeV where the local minimum at negative values of $\phi$ becomes the global one, see the left of Fig. 6. For the next five or so orders of magnitude in $\mu$, the new global minimum becomes deeper while the local

Figure 4: The gauge invariant 1-loop Higgs potential $V_1(\phi)$ at $\mu = 125$ GeV.

Figure 5: The 1-loop Higgs potential $V_1(\phi)$ at $\mu = 10^{12}$ GeV (left) and $\mu = 10^{19}$ GeV (right).
one, at $\phi = 0$, becomes shallower. Nevertheless, if we zoom in near $\phi = 0$ we will see that the local minimum is still there. Just below $\mu_I$ the picture of the potential remains the same, the local minimum becomes even deeper and there is no restoration of the global $Z_2$ symmetry, see right of Fig. [6]. At this scale, $\lambda \simeq 1.8$, i.e. still perturbative. In Fig. [7] we plot the potential just above $\mu_I$, at $3.0 \cdot 10^{45}$ GeV; we observe that $\mu_I$ is the approximate scale where the potential develops an instability. A related question that arises is what is the scale where perturbation theory breaks down, that is which scale produces a quartic coupling of $\lambda \simeq 4\pi$ (according to the usual perturbative argument based on the fact that each loop introduces an additional factor of $\frac{\lambda^2}{16\pi^2}$). This scale turns out to be

$$\mu_{NP} \simeq 1.5 \cdot 10^{49} \text{ GeV},$$

(6.7)
a scale slightly larger than the instability scale and remarkably close to the Landau pole.
of \( \lambda, \mu_{\lambda}^{L} \simeq 4.3 \cdot 10^{19} \) GeV. The three orders of magnitude in \( \mu \) between \( \mu_I \) and \( \mu_{NP} \) is just an order of magnitude or less on a logarithmic scale. In a gauge invariant cut-off regularization such as the lattice, this is typical in the vicinity of a phase transition and can be achieved by a moderate change of the bare couplings. We could say that an interval of scale evolution, long in perturbative time can be short in (cut-off) non-perturbative time. More on this in the next section.

In Fig. 8 we plot once more the potential in the vicinity of the Landau pole. On the left, we show the global form of the potential, where we can see the flattening of the \( \phi = 0 \) regime. On the right, we zoom in the \( \phi = 0 \) regime and observe that the local minimum is still there, even though the closer one gets to the Landau pole the more one must zoom in to see it, as the flattening gets closer to forming a saddle point. Note that the gauge coupling \( g \) changes very little during these scale changes thus remains perturbative. This means that beyond the Landau pole where \( \lambda \) turns negative lays the Coulomb phase.

The physics that we extract from the numerical analysis is that the Abelian-Higgs model remains perturbative almost all the way to the Landau pole \( \mu_{\lambda}^{L} \) of \( \lambda \). At a scale approximately \( \mu_I \simeq 7 \cdot 10^{-4} \mu_{\lambda}^{L} \) an instability develops, the onset of the system trying to pass from the Higgs phase into the Coulomb phase. Above \( \mu_I \) there are two vacua, one is the old global minimum that has turned into a local minimum and the unstable vacuum, corresponding to the Coulomb phase. Up to the scale of \( 0.35 \mu_{\lambda}^{L} \) the system is trying perturbatively to stabilize in the Coulomb phase and this is finally achieved at the Landau pole where in the last few tiny scale seconds, approximately between \( [0.35-1.00] \cdot \mu_{\lambda}^{L} \), non-perturbative evolution takes over. The breadth of the unstable regime reflects perhaps the fact that the phase transition between the Higgs and the Coulomb phase is a strong first order phase transition [20]. It is likely that in a Monte Carlo simulation, as the phase transition is crossed, one would observe the system tunelling back and forth between the
two vacua.

7 Lines of Constant Physics

The following section has a more speculative character in comparison to the previous ones in an attempt to make a connection to non-perturbative properties of the Abelian-Higgs model. In a non-perturbative approach one of the first thing one does is to map the phase diagram of a model via Monte Carlo simulations. For the Abelian-Higgs model the phase diagram is defined by three axes in the space of the three bare parameters \( g_0, \lambda_0 \) and \( m_0 \). For instance in a lattice regularization, the three axes can be chosen to be

\[
\beta \equiv \frac{1}{g_0^2}, \quad \kappa \equiv \frac{1 - 2\lambda_0}{1 + \frac{1}{2}a^2m_0^2}, \quad \lambda_0, \quad (7.1)
\]

with \( a \) the lattice spacing. The \( \beta \) above should not be confused with a beta-function, it is the conventional notation for a lattice gauge coupling. The mapping of the phase diagram has been done for the Abelian-Higgs model to some extent in the past \[20\] and more recently, for \( \lambda_0 = 0.15 \), in \[21\]. It seems to depend weakly on \( \lambda_0 \). Here we reproduce it (semi) qualitatively:

![Phase Diagram](image)

We also note that the above phase diagram has been constructed using \( \kappa \equiv \frac{1 - 2\lambda_0}{8 + a^2m_0^2} \), a slightly different normalization for \( \kappa \) in Eq. (7.1). This just corresponds to choosing a different normalization for the bare vev parameter \( v_0 \). Here we will use the normalization of Eq. (7.1) and \( v_0 = 246 \text{ GeV} \).

A Line of Constant Physics (LCP) of the AH model is defined by the curve on its phase diagram all of whose points satisfy the constraint that \( m_H, m_Z \) and \( \lambda \) are some
chosen constants. We can choose for example
\[ m_H = 125 \text{ GeV}, \quad m_Z = 91 \text{ GeV}, \quad \lambda = 0.12, \quad (7.2) \]
and then the points of the LCP can be defined perturbatively to be determined by the equations
\[
\begin{align*}
    m^2_{H_0}(\mu) &= 125^2 + (\delta m_H)_f(m_H = 125, m_Z = 91, \lambda = 0.12; \mu) \\
    m^2_{Z_0}(\mu) &= 91^2 + (\delta m_Z)_f(m_H = 125, m_Z = 91, \lambda = 0.12; \mu) \\
    \lambda_0(\mu) &= 0.12 + (\delta \lambda)_f(m_H = 125, m_Z = 91, \lambda = 0.12; \mu) \quad (7.3)
\end{align*}
\]
where
\[
\begin{align*}
    (\delta m_H)_f(m_H = 125, m_Z = 91, \lambda = 0.12; \mu) &= -19557.3 + 5625 \ln \frac{\mu^2}{15625} - 5962.32 \ln \frac{\mu^2}{8281} \\
    (\delta \lambda)_f(m_H = 125, m_Z = 91, \lambda = 0.12; \mu) &= -0.0305 + 0.01296 \ln \frac{\mu^2}{15625} - 0.043 \ln \frac{\mu^2}{8281} \\
    (\delta m_Z)_f(m_H = 125, m_Z = 91, \lambda = 0.12; \mu) &= 2537.18 + 6723.04 \ln \frac{\mu^2}{15625} - 52.0632 \ln \frac{\mu^2}{8281} \quad (7.4)
\end{align*}
\]
are the finite parts of the Unitary gauge counter-terms, according to Eq. (5.12), Eq. (5.13) and Eq. (5.8). We recall that $m_{H_0} = 2m_0$, $m_{Z_0} = g_0v_0$ and correspondingly $\delta m_H = 2\delta m$, $\delta m_Z = v_0\delta g$.

We now have all the ingredients to plot a first "perturbative LCP". In Fig. 9 we plot the LCP, defined by Eq. (7.3) and Eq. (7.2) with $v_0 = 246$ GeV, for the range $\mu \in [125, 5000]$ GeV. The part (lower left) of the line where the points are denser corresponds to larger values of $\mu$. There, the parameter $\beta = 1/g^2_0$ (that decreases quite rapidly as $\mu$ increases) approaches 1, which means that most likely we are about to enter into either the Confined or the Coulomb phase. There is potentially a non-trivial message in this so it is worth redoing it this time using Eq. (7.1), i.e. to try to actually put the perturbative LCP on the non-perturbative phase diagram. The only obstacle is to relate the lattice spacing to some scale parameter in the perturbative calculation. This can be quite an involved operation, depending on our desired level of precision. For a discussion of this issue, see [22]. Here we will simplify the discussion and make the naive identification $a = 1/\Lambda$ with $\Lambda$ a momentum cut-off. We need to do some extra work though, as we have to estimate $\delta m_H$ in a cut-off regularization. Fortunately this is not too hard, it yields to leading order
\[
m^2_{H_0}(\Lambda) = 125^2 + \frac{6\lambda}{16\pi^2} \Lambda^2 + \cdots \quad (7.5)
\]
that should replace the first of Eq. (7.3) in Eq. (7.1). The dots stand for small corrections. For $m_Z$ and $\lambda$ on the other hand that have a logarithmic cut-off dependence we keep the
same relations as before. This is not a completely well defined identification but it is
good enough for the information we want to extract. We call the resulting line on the
$\beta - \kappa$ plane, the ”cut-off LCP”. On the left of Fig. 10 we plot the cut-off LCP defined
by Eq. (7.2) for $v_0 = 246$ GeV and projecting the $\lambda_0$ dependence on the $\beta - \kappa$ plane.
The values of the cut-off range in $\Lambda \in [125, 700]$ GeV and they increase as we move down
and left along the curve. By comparing to the Monte Carlo phase diagram we can now
clearly see that around the upper limit of the $\Lambda$-range the system hits the Higgs-Coulomb
phase transition. Some representative values of the parameters can be seen on Table 1.

On the right of Fig. 10 we compare the cut-off LCP to the one we would have obtained

| $\Lambda$ | $\beta$ | $\kappa$ | $\lambda_0$ | $q$ |
|----------|--------|----------|-------------|----|
| 125      | 5.6    | 0.45     | 0.07        | 1.7|
| 250      | 3.0    | 0.32     | 0.19        | 3.8|
| 375      | 2.4    | 0.25     | 0.26        | 7.3|
| 500      | 2.0    | 0.2      | 0.31        | 12.1|
| 625      | 1.9    | 0.15     | 0.35        | 18.3|
| 750      | 1.7    | 0.12     | 0.38        | 25.9|

Table 1: Scale and corresponding bare parameters of the LCP defined by $m_Z = 91$ GeV,
$m_H = 125$ GeV and $\lambda = 0.12$ with the lattice spacing identified as $a = 1/\Lambda$.  

Figure 9: The perturbative LCP defined by Eq. (7.2) for $v_0 = 246$ GeV.
Figure 10: The cut-off LCP that identifies $\Lambda = 1/a$ for $v_0 = 246$ GeV projected on the $\beta - \kappa$ plane (left) and a comparison with the corresponding LCP with the identification $\mu = 1/a$ (right). The lower curve is the cut-off LCP.

Table 2: Scale and corresponding bare parameters of the LCP defined by $m_Z = 91$ GeV, $m_H = 125$ GeV and $\lambda = 0.12$ with the lattice spacing identified as $a = 1/\mu$.

| $\mu$ | $\beta$ | $\kappa$ | $\lambda_0$ | $q$  |
|-------|---------|----------|-------------|------|
| 125   | 5.6     | 1.11     | 0.07        | 0    |
| 250   | 3.0     | 0.65     | 0.19        | 0.0014 |
| 375   | 2.4     | 0.48     | 0.26        | 0.0023 |
| 500   | 2.0     | 0.38     | 0.31        | 0.0029 |
| 625   | 1.9     | 0.30     | 0.35        | 0.0033 |
| 750   | 1.7     | 0.23     | 0.38        | 0.0037 |

by the identification $\mu = 1/a$. Some representative values of the parameters for this identification can be seen on Table 2. The numbers change but the conclusion remains: If we want to construct an LCP that keeps the values of the physical quantities close to their "experimentally measured values" (we are not in the Standard Model but the point we would like to make should be clear), we can not take the cut-off too high, since around 1 TeV or so we are forced to cross into the Coulomb phase. Notice that the analysis of the RG trajectories and of the potential in the previous section did not warn us about this. This has potentially consequences for the fine tuning of the Higgs mass. In Tables 1 and 2 we introduced the unsophisticated fine tuning parameter

$$q = \frac{|(\delta m_H)_f|}{m_H}$$

from which we can see the increase of fine tuning as the scale increases, with the $\Lambda = 1/a$ identification naturally worse. Even so though, since the cut-off stops at a maximum...
value of about 1 TeV, it can not take very large values.

8 Conclusions

We carried out the renormalization of the Higgs potential and of the Z-mass in the Abelian-Higgs model in the $R_\xi$ and Unitary gauges. The fact that this is a sector of the action not containing derivatives, suggested us to renormalize at zero external momenta. The renormalization conditions we used imply subtractions that are necessarily different than $\overline{\text{MS}}$. In this context, we showed that these two gauges are completely equivalent both at the level of the $\beta$-functions and at the level of the finite remnants after renormalization, that determine the one-loop Higgs potential. Moreover, we showed that after the renormalization procedure we obtain automatically the gauge independence of the potential. The cancellation of the gauge fixing parameter $\xi$ from the scalar potential obtained via Feynman diagram calculations in an $R_\xi$ gauge fixing scheme is intriguing as the corresponding effective potential obtained via the background field method is manifestly $\xi$-dependent. Its equivalence to the Unitary gauge on the other hand could open a new direction in Standard Model calculations, as the number of diagrams in the Unitary gauge is significantly smaller. Apart however from simplifying already known results, computations such as done here could also render currently ambiguous results based on the dynamics of the scalar potential away from its extrema, more robust. Since this work is just a demonstration of gauge independence of the Higgs potential by construction it would be certainly worth trying to construct some sort of a more formal ‘proof’.

We have analyzed numerically the Higgs potential and saw that the system generically behaves perturbatively until quite close to the Landau pole of $\lambda$. It becomes metastable already in the perturbative regime and only during its last steps towards the phase transition into the Coulomb phase turns non-perturbative. Finally, we constructed a couple of Lines of Constant Physics in the Unitary gauge and placed them on the Abelian-Higgs phase diagram in order to enhance the perturbative analysis by some non-perturbative input. This suggested us that in a Higgs-scalar system the cut-off can not be generically driven to the highest possible scales that the RG flow or the stability of the potential allows, if we want to keep the physics constant. This is because it is possible that way before those scales are reached, a phase transition may be encountered.

Regarding generalizations of this work, simple bosonic extensions should work in a similar way, without any surprises. In the presence of fermions some extra care may be needed to handle the $U$-integrals but this is expected to be a straightforward extra technical step. In this case though it is also expected that a new instability will appear at some intermediate scale and it would be interesting to work out in our framework how
this modifies the bosonic system. Finally, if the observed consistency between the two gauges breaks down at higher loops then there is something about the quantum internal structure of spontaneously broken gauge theories that needs to be understood better.

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Appendices

A One-loop integrals

The general form of a Feynman diagram $F_{E}^{G,L}$ (see Eq. (2.11) in the text for the explanation of the notation) is

$$(4\pi)^{d/2} F_{E}^{G,L} = \mu^\varepsilon \left( [F_{E}^{G,L}]_\varepsilon + \{F_{E}^{G,L}\}_\varepsilon + [F_{E}^{G,L}]_f + \{F_{E}^{G,L}\}_f \right). \tag{A.1}$$

Here we are interested in the finite part of this expression, the general form of which is

$$\frac{1}{(4\pi)^{d/2}} \left( F_{E}^{G,L} \right)_f = \frac{1}{(4\pi)^{d/2}} \left( [F_{E}^{G,L}]_f + \{F_{E}^{G,L}\}_f \right) = \int \frac{d^d k}{i(4\pi)^{d/2}} \sum_n \left( \frac{N^{(n)}_{F_{E}^{G,L}}}{D_1 D_2 \cdots D_n} \right)_f \tag{A.2}$$

where the range of the sum over $n$ depends on the diagram. The $n$'th term of the numerator has a denominator with $n$ factors, yields a finite result after integration over the loop momentum $k$ and it is denoted as $\left( \frac{N^{(n)}_{F_{E}^{G,L}}}{D_1 D_2 \cdots D_n} \right)_f$. It is also diagram dependent.

In the Unitary gauge, even though this notation is redundant, we will still use it for extra clarity. The denominators are defined as

$$D_i(P_{i-1}, m_i) = (k + P_{i-1})^2 - m_i^2, \quad i = 1, \cdots, n \tag{A.3}$$

and $P_0 \equiv 0$. Notice that the proper argument of the denominators is $D_i(P_{i-1}, m_i)$. Nevertheless, we do not always show both arguments systematically in the main text, apart from cases where otherwise their absence could cause ambiguities. Such a case is some finite diagrams, which generally have more than four denominators. In other cases where there can be no confusion, we show none, or only one of the arguments.

Feynman parametrization is implemented using

$$\frac{1}{D_1 D_2 \cdots D_n} = (n - 1)! \int_0^1 D^\alpha x \frac{1}{(D_1 x_1 + D_2 x_2 + \cdots + D_n x_n)^n}. \tag{A.4}$$
where we defined
\[ \int_0^1 dx_1 \cdots \int_0^1 dx_n \delta(1 - \sum_{i=1}^n x_i) \equiv \int_0^1 \mathcal{D}^n x. \] (A.5)

Passing now to Dimensional Regularization, we introduce the useful shorthand notation
\[ \int \frac{d^d k}{i (4\pi)^{d/2}} (\cdots) \equiv \langle \cdots \rangle. \] (A.6)

We can then express Eq. (A.2) in DR as
\[ \frac{1}{(4\pi)^{d/2}} (F^{G,L}_E)_{f} = \sum_n \Gamma(n) \int_0^1 \mathcal{D}^n x \left\{ \left( \frac{N^{(n)}_{F^{G,L}_E}}{(k^2 - \Delta_{F_{0,n}})^{n}} \right)_f (k \to k - \sum_{i=1}^{n-1} P_{i} x_{i+1}) \right\}, \] (A.7)

with
\[ \Delta_{F_{0,n}}(x_1, \cdots, x_n) = - \sum_{i=1}^{n-1} P_{i}^2 x_{i+1} + \left( \sum_{i=1}^{n-1} P_{i} x_{i+1} \right)^2 + \sum_{i=1}^n x_{i} m_{i}^2. \] (A.8)

In a few simple cases we use the conventional notation
\[ \Delta_{F_{0,2}} \equiv \Delta_{B_0} \]
\[ \Delta_{F_{0,3}} \equiv \Delta_{C_0} \]
\[ \Delta_{F_{0,4}} \equiv \Delta_{D_0} \] (A.9)

\section{Passarino - Veltman}

In this Appendix we collect some standard integrals of the Passarino-Veltman type, encountered in the text.

\subsection{Scalars}

The simplest PV integral is the scalar tadpole integral
\[ A_0(m) = \int \frac{d^d k}{i (2\pi)^d} \frac{1}{k^2 - m^2}, \] (B.1)

naively quadratically divergent with a cut-off. In Dimensional Regularization, it can be expressed as (attaching the factor \( \mu^\varepsilon \))
\[ \mu^\varepsilon A_0(m) = - \frac{\mu^\varepsilon}{(4\pi)^{d/2}} \Gamma \left( 1 - \frac{d}{2} \right) \frac{1}{m^{1-d/2}} \] (B.2)

and by expanding in \( \varepsilon \) finally as
\[ \mu^\varepsilon A_0(m) = \frac{1}{(4\pi)^2} m^2 \left( \frac{2}{\varepsilon} + \ln \frac{\mu^2}{m^2} + 1 \right). \] (B.3)
The scalar, naively logarithmically divergent PV integral is

\[ B_0(P_1, m_1, m_2) = \int \frac{d^4k}{i(2\pi)^4} \frac{1}{D_1 D_2} \ . \]  

(B.4)

It can be computed explicitly using the formulation of Appendix A. It corresponds to \( n = 2 \) and a numerator equal to 1. In \( d \)-dimensions, this integral is

\[ B_0(P_1, m_1, m_2) = \frac{1}{(4\pi)^{d/2}} \int_0^1 dx \Gamma \left( \frac{2}{2} - \frac{d}{2} \right) \left( \frac{1}{\Delta B_0} \right)^{2-d/2} \ . \]  

(B.5)

Expanding in \( \epsilon \) we get

\[ B_0(P_1, m_1, m_2) = \frac{1}{(4\pi)^2} \left( \frac{2}{\epsilon} + \int_0^1 dx \ln \frac{4\pi \epsilon e^{-\gamma_E}}{\Delta B_0} \right) \]  

(B.6)

and finally

\[ \mu^\epsilon B_0(P_1, m_1, m_2) = \frac{1}{(4\pi)^2} \left( \frac{2}{\epsilon} + \int_0^1 dx \ln \frac{\mu^2}{\Delta B_0} \right) \ . \]  

(B.7)

We note that this integral is symmetric under \( m_1 \leftrightarrow m_2 \). An important special case is when \( P_1 = 0 \) in which case

\[ B_0^1(m_1, m_2) \equiv B_0(P_1 = 0, m_1, m_2) = A_0(m_1) - A_0(m_2) \]  

(B.8)

The finite scalar integral that appears in Triangles and Boxes is of the form

\[ C_0(P_1, P_2, m_1, m_2, m_3) = \int \frac{d^4k}{i(2\pi)^4} \frac{1}{D_1 D_2 D_3} \ . \]  

(B.9)

In DR, this integral becomes

\[ C_0(P_1, P_2, m_1, m_2, m_3) = 2 \int_0^1 D^3x \left( \frac{1}{(k^2 - \Delta C_0)^3} \right) \ . \]  

(B.10)

Expanding in \( \epsilon \) it reduces to

\[ \mu^\epsilon C_0(P_1, P_2, m_1, m_2, m_3) = -\frac{\mu^\epsilon}{(4\pi)^{d/2}} \int_0^1 D^3x \frac{\Gamma \left( 3 - \frac{d}{2} \right)}{\Delta C_0^{3-d/2}} \]  

\[ = -\frac{\mu^\epsilon}{16\pi^2} \int_0^1 D^3x \frac{1}{\Delta C_0} \ . \]  

(B.11)

We also define some special cases of \( C_0 \) integrals that appear in the main text in various places:

\[ C_0^1(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \equiv C_0(P_1 = p, P_2 = 0, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \]
\[ C_0^2(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \equiv C_0(P_1 = p, P_2 = p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \]
\[ C_0^3(p, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \equiv C_0(P_1 = 0, P_2 = p, m_{Z_0}, m_{\chi_0}, m_{\chi_0}). \]  

(B.12)

The last scalar is the finite Box integral
\[ D_0(P_1, P_2, P_3, m_1, m_2, m_3, m_4) = \int \frac{d^4k}{i(2\pi)^4} \frac{1}{D_1 D_2 D_3 D_4}. \]  

(B.13)

The integral in \(d\)-dimensions becomes
\[ \mu^4 D_0(P_1, P_2, P_3, m_1, m_2, m_3, m_4) = 6\mu^\varepsilon \int_0^1 D^4 x \left\langle \frac{1}{(k^2 - \Delta D_0)^4} \right\rangle \]
\[ = \frac{\mu^\varepsilon}{(4\pi)^{d/2}} \int_0^1 D^4 x \Gamma \frac{4 - d}{\Delta^4 D_0} \]
\[ = \frac{\mu^\varepsilon}{16\pi^2} \int_0^1 D^4 x \frac{1}{\Delta^2 D_0}. \]  

(B.14)

### B.2 Tensors

Standard tensor PV integrals can be algebraically reduced to scalar integrals. Actually in one-loop diagrams only contractions of tensors with the metric and external momenta occur. The tadpole integral has no standard PV extension. It has an extension of the \(U\)-type, naively quartically divergent with a cut-off and it will be computed in Appendix C.

Let us introduce some shorthand notation. First, define for any \(P\):
\[ B_0(1, 2) \equiv B_0(P, m_1, m_2) \]  

(B.15)

and for any \(P_1\) and \(P_2\)
\[ B_0(1, 3) \equiv B_0(P_2, m_1, m_3) \]
\[ B_0(2, 3) \equiv B_0(P_2 - P_1, m_2, m_3). \]  

(B.16)

The simplest \(B\)-tensor is the linearly divergent
\[ B^\mu(P, m_1, m_2) = \left\langle \frac{k^\mu}{(k^2 - m_1^2)((k + P)^2 - m_2^2)} \right\rangle \]  

(B.17)

and it can be contracted only by a momentum
\[ P_\mu B^\mu(P, m_1, m_2) = \frac{1}{2} (f_1(P) B_0(1, 2) + A_0(m_1) - A_0(m_2)), \]  

(B.18)

where
\[ f_1(P) = m_2^2 - m_1^2 - P^2. \]  

(B.19)
The other PV tensor extension of the $B$-type is of the form

$$B^{\mu\nu} = \frac{k^\mu k^\nu}{(k^2 - m_1^2)((k + P)^2 - m_2^2)} \tag{B.20}$$

and it is quadratically divergent. Its contraction with the metric is

$$g_{\mu\nu}B^{\mu\nu}(P, m_1, m_2) = m_1^2 B_0(P, m_1, m_2) + A_0(m_2) \tag{B.21}$$

while its contraction with $P_\mu P_\nu$ is

$$P_\mu P_\nu B^{\mu\nu}(P, m_1, m_2) = \frac{m_2^2 - m_1^2 - P^2}{4} A_0(m_1) + \frac{m_1^2 - m_2^2 + 3P^2}{4} A_0(m_2) + \frac{(m_1^4 + m_2^4 - 2m_1^2m_2^2)}{4} + \frac{P^2(2m_1^2 - 2m_2^2 + P^2)}{4} B_0(P, m_1, m_2). \tag{B.22}$$

The simplest $C$-tensor is

$$C^{\mu}(P_1, P_2, m_1, m_2, m_3) = \frac{k^\mu}{D_1D_2D_3}. \tag{B.23}$$

It contracts either as

$$R_1^{[c]} \equiv P_\mu C^{\mu} = \frac{1}{2} \left( f_1(P_1) C_0(P_1, P_2, m_1, m_2, m_3) + B_0(1, 3) - B_0(2, 3) \right) \tag{B.24}$$

or as

$$R_2^{[c]} \equiv (P_2 - P_1)_\mu C^{\mu} = \frac{1}{2} \left( f_2(P_1, P_2) C_0(P_1, P_2, m_1, m_2, m_3) + B_0(1, 2) - B_0(1, 3) \right) \tag{B.25}$$

where

$$f_2(P_1, P_2) = m_3^2 - m_2^2 - P_2^2 + P_1^2. \tag{B.26}$$

The second $C$-tensor, logarithmically divergent, integral is

$$C^{\mu\nu}(P_1, P_2, m_1, m_2, m_3) = \frac{k^\mu k^\nu}{D_1D_2D_3}. \tag{B.27}$$

Its contraction with the metric is

$$g_{\mu\nu}C^{\mu\nu} = m_1^2 C_0(P_1, P_2, m_1, m_2, m_3) + B_0(P_2 - P_1, m_2, m_3). \tag{B.28}$$

There are three different momentum contractions that can appear. Following [18], we define the matrix

$$G_2^{-1} = \frac{1}{\det G_2} \begin{pmatrix} (P_2 - P_1) \cdot (P_2 - P_1) & -P_1 \cdot (P_2 - P_1) \\ -P_1 \cdot (P_2 - P_1) & P_1 \cdot P_1 \end{pmatrix}$$
and using this matrix the quantities

\[
\begin{pmatrix}
C_1 \\
C_2
\end{pmatrix} = G_2^{-1} \begin{pmatrix}
R_1^{[c]} \\
R_2^{[c]}
\end{pmatrix}.
\] (B.29)

We also define

\[
B_1(1, 2) \equiv B_1(P_1, m_1, m_2) = \frac{1}{2P_1^2} (f_1(P_1)B_0(1, 2) + A_0(m_1) - A_0(m_2))
\]

\[
B_1(1, 3) \equiv B_1(P_2, m_1, m_3) = \frac{1}{2P_2^2} (f_1(P_2)B_0(1, 3) + A_0(m_1) - A_0(m_3))
\]

\[
B_1(2, 3) \equiv B_1(P_2 - P_1, m_2, m_3) = \frac{1}{2(P_2 - P_1)^2} (f_1(P_2 - P_1)B_0(2, 3) + A_0(m_2) - A_0(m_3))
\]

\[
C_{00}(1, 2, 3) = \frac{1}{2(d - 2)} \left( 2m_1^2C_0 - f_2(P_2, P_1)C_2 - f_1(P_1)C_1 + B_0(2, 3) \right)
\] (B.30)

out of which we construct the four quantities

\[
R_{1}^{[c]} = \frac{1}{2} (f_1(P_1)C_1 + B_1(1, 3) + B_0(2, 3) - 2C_{00}(1, 2, 3))
\]

\[
R_{2}^{[c]} = \frac{1}{2} (f_2(P_2, P_1)C_1 + B_1(1, 2) - B_1(1, 3))
\]

\[
R_{1}^{[c]} = \frac{1}{2} (f_1(P_1)C_2 + B_1(1, 3) - B_1(2, 3))
\]

\[
R_{2}^{[c]} = \frac{1}{2} (f_2(P_2, P_1)C_2 - B_1(1, 3) - 2C_{00}(1, 2, 3)).
\] (B.31)

This data determines the quantities

\[
C_{11} = \frac{1}{\det G_2} \left\{ (P_2 - P_1)^2 R_{1}^{[c]} - P_1 \cdot (P_2 - P_1) R_{2}^{[c]} \right\}
\]

\[
C_{12} = C_{21} = \frac{1}{\det G_2} \left\{ P_2^2 R_{2}^{[c]} - P_1 \cdot (P_2 - P_1) R_{1}^{[c]} \right\}
\]

\[
C_{22} = \frac{1}{\det G_2} \left\{ P_2^2 R_{2}^{[c]} - P_1 \cdot (P_2 - P_1) R_{1}^{[c]} \right\}
\] (B.32)

and in terms of these we can express $C_{\mu \nu}$ itself:

\[
C_{\mu \nu} = \frac{g_{\mu \nu}}{2(d - 2)} \left( 2m_1^2C_0(P_1, P_2, m_1, m_2, m_3) - f_2(P_2, P_1)C_2 - f_1(P_1)C_1 + B_0(2, 3) \right)
\]

\[
+ P_1^\mu P_1^\nu C_{11} + (P_1^\mu (P_2 - P_1)^\nu + (P_2 - P_1)^\mu P_1^\nu) C_{12} + (P_2 - P_1)^\mu (P_2 - P_1)^\nu C_{22}
\] (B.33)

and from the above expression it is straightforward to compute all its momentum contractions.
We sometimes encounter $C$-integrals with $\det[G_2] = 0$. Such integrals are computed with direct Feynman parametrization and DR (i.e. without algebraic reduction). To give an example, we may stumble on

\[
C^{1}_{\mu\nu}(p, m_1, m_2, m_3) \equiv C_{\mu\nu}(P_1 = p, P_2 = 0, m_1, m_2, m_3) = \frac{k_\mu k_\nu}{(k^2 - m_1^2)((k + p)^2 - m_3^2)(k^2 - m_3^2)} = 2 \int_0^1 D^3 x \left( \frac{k_\mu k_\nu + x_2 k_\mu p_\nu + x_2 k_\nu p_\mu + x_2^2 p_\mu p_\nu}{(k^2 - \Delta C_0)^3} \right),
\]  
(B.34)

which in DR, after expanding in $\varepsilon$, becomes

\[
16\pi^2 \mu^\varepsilon C^{1}_{\mu\nu}(d = 4, \varepsilon \to 0) = \frac{g_{\mu\nu}}{4} \left( \frac{2}{\varepsilon} + 2 \int_0^1 D^3 x \ln \frac{\mu^2}{\Delta C_0} \right) + 2 \int_0^1 D^3 x \frac{x_2^2 p_\mu p_\nu}{\Delta C_0} .
\]  
(B.35)

Let us now consider the linearly divergent rank 3 integral

\[
C^{\mu\nu\alpha}(P_1, P_2, m_1, m_2, m_3) = \left< \frac{k^\mu k^\nu k^\alpha}{D_1 D_2 D_3} \right> .
\]  
(B.36)

This integral could be reduced in principle algebraically, following [18]. Here, as it is only linearly divergent, we will compute it in a brute force way with Feynman parametrization. It is a case with $n = 3$ and $N^{(3)}_{C^{\mu\nu\alpha}} = k^\mu k^\nu k^\alpha$ in the language of Appendix A. It explicitly evaluates to

\[
16\pi^2 \mu^\varepsilon C^{\mu\nu\alpha}(P_1, P_2, m_1, m_2, m_3) = -2 \frac{g^{\mu\nu}}{d} \int_0^1 D^3 x [P_1 x_2 + P_2 x_3]^\alpha B_0^K + 2 \frac{g^{\mu\alpha}}{d} \int_0^1 D^3 x [P_1 x_2 + P_2 x_3]^\nu B_0^K + 2 \frac{g^{\nu\alpha}}{d} \int_0^1 D^3 x [P_1 x_2 + P_2 x_3]^{\mu} B_0^K + \int_0^1 D^3 x [P_1 x_2 + P_2 x_3]^{\mu}[P_1 x_2 + P_2 x_3]^\nu[P_1 x_2 + P_2 x_3]^\alpha \frac{\Delta C_0}{D_0^K} .
\]  
(B.37)

where

\[
B_0^K = \frac{2}{\varepsilon} + \ln \frac{\mu^2}{\Delta C_0} .
\]  
(B.38)

It is now easy to compute any contraction of the above expression with the metric and/or momenta. A useful contraction is $C^{\mu}_{\alpha 3} \equiv g_{\nu\alpha} C^{\mu\nu\alpha}$.

All box tensor integrals are computed directly as in Appendix A since they are at most logarithmically divergent. $D^\mu$, $D^{\mu\nu}$, $D^{\mu\nu\alpha}$ and $D^{\mu\nu\alpha\beta}$ are computed as

\[
D^{\mu, \nu, \rho, \omega, \mu\nu\rho\omega} = 6 \int_0^1 D^4 x \left( N^{(4)}_{D^{\mu, \nu, \rho, \omega, \mu\nu\rho\omega}} (k - \sum_{i=1}^4 P_i x_{i+1}) \right) D_1 D_2 D_3 D_4,
\]  
(B.39)

with $N^{(4)}_{D^\mu} = k^\mu$, $N^{(4)}_{D^{\mu\nu}} = k^\mu k^\nu$, $N^{(4)}_{D^{\mu\nu\alpha}} = k^\mu k^\nu k^\alpha$ and $N^{(4)}_{D^{\mu\nu\alpha\beta}} = k^\mu k^\nu k^\alpha k^\beta$. Two useful contractions are $D^{\mu\nu\alpha}_{B_4} \equiv g_{\alpha\beta} D^{\mu\nu\alpha\beta}$ and $D_{B_4} \equiv g_{\mu\nu} D^{\mu\nu}_{B_4}$.  

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C  $U$-integrals

$U$-integrals are linearly, quadratically, cubically or quartically divergent diagrams that
do not appear when the Higgs is expressed in a Cartesian basis and an $R_\xi$ gauge fixing
is performed. If either a Polar basis for the Higgs is used and/or the computation is
performed in the Unitary gauge, these integrals do appear. Clearly, the standard PV
reduction formulae must be extended. At each level of $n$-point functions we meet at least
an integral of the $U$-type.

C.1  $U$-integrals in Tadpoles

Indeed, during the Tadpole calculation in section 3.1 we find the quartically divergent
contraction

$$U^{\mu\nu}_T(m) = \left\langle \frac{k^\mu k^\nu}{k^2 - m^2} \right\rangle = \frac{g_{\mu\nu}}{d} \left\langle \frac{k^2}{k^2 - m^2} \right\rangle \Leftrightarrow$$

$$U_T(m) = g_{\mu\nu}U^{\mu\nu}_T(m) = \frac{d}{d} \left\langle \frac{k^2}{k^2 - m^2} \right\rangle = \left\langle \frac{m^2}{k^2 - m^2} \right\rangle + \left\langle \frac{k^2 - m^2}{k^2 - m^2} \right\rangle = m^2 A_0(m) + \mathcal{V}. \quad (C.1)$$

where $\mathcal{V}$ is the DR volume of space-time. The usual prescription is $\mathcal{V} = 0$, in which case,
the tensor-Tadpole $U$-integral reduces to

$$U_T(m) = m^2 A_0(m). \quad (C.2)$$

C.2  $U$-integrals in mass corrections

The basic $U$-integral with two denominators is the quartically divergent with the cut-off
integral

$$U_{M4}(P_1, m_1, m_2) = \left\langle \frac{k^4}{D_1 D_2(P_1)} \right\rangle \quad (C.3)$$

with $D_1 = k^2 - m_1^2$ and $D_2(P_1) = (k + P_1)^2 - m_2^2$ as defined in Eq. (A.3). We show only
the momentum argument $P_1$ as we would like to follow it. Adding and subtracting a term,
the above integral can be rewritten as

$$U_{M4}(P_1, m_1, m_2) = \left\langle \frac{(k^2 - m_1^2)k^2}{D_1 D_2(P_1)} \right\rangle + \left\langle \frac{k^2 m_1^2}{D_1 D_2(P_1)} \right\rangle$$
\[ \left\langle \frac{k^2}{D_2(P_1)} \right\rangle = \left\langle \frac{k^2}{D_2(0)} \right\rangle - \left\langle \frac{2P_1 \cdot k}{D_2(0)} \right\rangle + \left\langle \frac{P_1^2}{D_2(0)} \right\rangle \]

\[ = (m_2^2 + P_1^2)A_0(m_2) + \frac{1}{16\pi^2}m_2^4, \quad \text{(C.5)} \]

Now the second term is a standard PV integral, while the first term is still a \( U \)-integral (it is still quartically divergent) but we can compute it straightforwardly as

\[ \left\langle \frac{k^2}{D_2(0)} \right\rangle = \left\langle \frac{k^2}{D_2(0)} \right\rangle - \left\langle \frac{2P_1 \cdot k}{D_2(0)} \right\rangle + \left\langle \frac{P_1^2}{D_2(0)} \right\rangle \]

\[ = (m_2^2 + P_1^2)A_0(m_2) + \frac{1}{16\pi^2}m_2^4, \quad \text{(C.5)} \]

where we have performed the shift \( k \to k - P_1 \) and we have dropped the \( 2k \cdot P_1 \) term because it is odd under \( k \to -k \). Also, we have used Eq. (C.2) for the rational part. From (C.4) and (C.5) and using Eq. (B.21), we get

\[ U_{M4}(P_1, m_1, m_2) = (m_1^2 + m_2^2 + P_1^2)A_0(m_2) + m_1^4B_0(P_1, m_1, m_2). \quad \text{(C.6)} \]

The reduction of \( U_{M4} \) was carried through using at some point a loop momentum shift. We know that in DR and in the absence of fermions, momentum shifts are ok, as long as the integral is up to naively quadratically divergent with the cut-off. \( U \)-integrals are however often cubically or quartically divergent. In order to check the validity of momentum shifts, we will now re-compute \( U_{M4} \) without momentum shift and compare the results.

Starting with Eq. (C.3), by adding and subtracting terms, we construct in the numerator \( D_2(P_1) \) obtaining

\[ U_{M4}(P_1, m_1, m_2) = \left\langle \frac{(k + P_1)^2 - m_2^2}{D_1D_2(P_1)} k^2 \right\rangle - 2 \left\langle \frac{k^2P_1 \cdot k}{D_1D_2(P_1)} \right\rangle + \left\langle \frac{m_2^2 - P_1^2}{D_1D_2(P_1)} \right\rangle \]

\[ = \left\langle \frac{k^2}{D_1} \right\rangle - 2 \left\langle \frac{k^2P_1 \cdot k}{D_1D_2(P_1)} \right\rangle + \left(m_2^2 - P_1^2 \right) g_{\mu\nu}B^{\mu\nu}(P_1, m_1, m_2). \quad \text{(C.7)} \]

The second term in the last expression is

\[ -2 \left\langle \frac{k^2P_1 \cdot k}{D_1D_2(P_1)} \right\rangle = -2 \left\langle \frac{(k + P_1)^2 - m_2^2}{D_1D_2(P_1)} P_1 \cdot k \right\rangle \]

\[ + 4 \left\langle \frac{P_1 \cdot kP_1 \cdot k}{D_1D_2(P_1)} \right\rangle - 2(m_2^2 - P_1^2) \left\langle \frac{P_1 \cdot k}{D_1D_2(P_1)} \right\rangle \]

\[ = -2 \left\langle \frac{P_1 \cdot k}{D_1} \right\rangle + 4P_{1\mu}P_{1\nu}B^{\mu\nu}(P_1, m_1, m_2) \]

\[ - 2(m_2^2 - P_1^2)P_{1\mu}B^\mu(P_1, m_1, m_2). \quad \text{(C.8)} \]

The first term of the above relation is zero since \( P_1 \cdot k \) term is odd under \( k \to -k \). The third term of Eq. (C.7) and the second and third terms of Eq. (C.8) are standard PV
integrals. The first term of Eq. [C.7] however is still a $U$-integral (and still quartically divergent) but we can reduce it easily as

$$\left\langle \frac{k^2}{D_1} \right\rangle = \left\langle \frac{k^2 - m_1^2}{D_1(0)} \right\rangle = m_1^2 A_0(m_1). \quad (C.9)$$

Combining Eq. [C.7] with Eq. [C.8] and Eq. [C.9] and using Eq. [B.18], Eq. [B.21] and Eq. [B.22] we obtain

$$U_{M4}(P_1, m_1, m_2) = (m_1^2 + m_2^2 + P_1^2) A_0(m_2) + m_1^4 B_0(P_1, m_1, m_2), \quad (C.10)$$

which is the same as Eq. [C.6]. Clearly, we have traded loop momentum shifts in highly divergent integrals for adding and subtracting infinities, a slightly less disturbing operation. In any case, the final results will justify or not these manipulations.

### C.3 $U$-integrals in Triangles

In the Triangle sector we first meet

$$U^\mu_{K4}(P_1, P_2, m_1, m_2, m_3) = \left\langle \frac{k^2 k^\mu}{D_1 D_2 D_3} \right\rangle. \quad (C.11)$$

It can be reduced easily as

$$U^\mu_{K4}(P_1, P_2, m_1, m_2, m_3) = \left\langle \frac{(k + P_2)^2 - m_3^2}{D_1 D_2 D_3} k^\mu k^\nu \right\rangle - 2P_2 \alpha C^\mu\nu\alpha(P_1, P_2, m_1, m_2, m_3)$$

$$+ (m_3^2 - P_2^2) C^\mu\nu(P_1, P_2, m_1, m_2, m_3)$$

$$= B^\mu\nu(P_1, m_1, m_2) - 2P_2 \alpha C^\mu\nu\alpha(P_1, P_2, m_1, m_2, m_3)$$

$$+ (m_3^2 - P_2^2) C^\mu\nu(P_1, P_2, m_1, m_2, m_3) \quad (C.12)$$

and with PV reduction further on.

The next case is the cubically divergent

$$U^\mu_{K5}(P_1, P_2, m_1, m_2, m_3) = \left\langle \frac{k^4 k^\mu}{D_1 D_2 D_3} \right\rangle. \quad (C.13)$$

Following similar steps as before

$$U^\mu_{K5}(P_1, P_2, m_1, m_2, m_3) = \left\langle \frac{(k + P_2)^2 - m_3^2}{D_1 D_2 D_3} k^\mu k^\nu \right\rangle - 2P_2 \alpha U^\mu_{K4}(P_1, P_2, m_1, m_2, m_3)$$

$$+ (m_3^2 - P_2^2) C^\mu_{K3}(P_1, P_2, m_1, m_2, m_3)$$

$$= \left\langle \frac{k^2 k^\mu}{D_1 D_2} \right\rangle - 2P_2 \alpha U^\mu_{K4}(P_1, P_2, m_1, m_2, m_3)$$

$$+ (m_3^2 - P_2^2) C^\mu_{K3}(P_1, P_2, m_1, m_2, m_3).$$
Only the first term is new (and also a $U$-integral) but it is easy to compute it:

$$\left\langle \frac{k^2 k^\mu}{D_1 D_2} \right\rangle = \left\langle \frac{(k^2 - m_1^2) k^\mu}{D_1 D_2} \right\rangle + m_1^2 \left\langle \frac{k^\mu}{D_1 D_2} \right\rangle$$

$$= \left\langle \frac{k^\mu}{D_2(P_1)} \right\rangle + m_1^2 B^\mu(P_1, m_1, m_2)$$

$$= \left\langle \frac{k^\mu}{D_2(0)} \right\rangle - P_1^\mu A_0(m_2) + m_1^2 B^\mu(P_1, m_1, m_2)$$

$$= -P_1^\mu A_0(m_2) + m_1^2 B^\mu(P_1, m_1, m_2) \quad (C.15)$$

where in the third line above we have shifted the loop momentum and then neglected the $k$-odd term, as before. It total,

$$U_5^\mu(P_1, P_2, m_1, m_2, m_3) = -P_1^\mu A_0(m_2) + m_1^2 B^\mu(P_1, m_1, m_2) - 2P_2^\nu U_{5\nu}^\mu(P_1, P_2, m_1, m_2, m_3)$$

$$+ (m_3^2 - P_2^2) C_{53}^\mu(P_1, P_2, m_1, m_2, m_3). \quad (C.16)$$

The last Triangle $U$-integral is quartically divergent, It is successively reduced as

$$U_6^\mu(P_1, P_2, m_1, m_2, m_3) = \left\langle \frac{k^6}{D_1 D_2 D_3} \right\rangle$$

$$= \left\langle \frac{((k + P_2)^2 - m_3^2) k^4}{D_1 D_2 D_3} \right\rangle - 2P_2^\nu U_{5\nu}^\mu(P_1, P_2, m_1, m_2, m_3)$$

$$+ (m_3^2 - P_2^2) U_{54}^\mu(P_1, P_2, m_1, m_2, m_3)$$

$$= \left\langle \frac{k^4}{D_1 D_3} \right\rangle - 2P_2^\nu U_{5\nu}^\mu(P_1, P_2, m_1, m_2, m_3)$$

$$+ (m_3^2 - P_2^2) U_{54}^\mu(P_1, P_2, m_1, m_2, m_3)$$

$$= U_{44}(P_1, P_2, m_1, m_2) - 2P_2^\nu U_{5\nu}^\mu(P_1, P_2, m_1, m_2, m_3)$$

$$+ (m_3^2 - P_2^2) U_{54}^\mu(P_1, P_2, m_1, m_2, m_3). \quad (C.17)$$

where we have defined $U_{44} = g_{\nu\nu} U_{5\nu}^\mu$.

### C.4 $U$-integrals in Boxes

The lowest Box $U$-integrals have five momenta in their numerator. One is

$$U_{55}^\mu(P_1, P_2, P_3, m_1, m_2, m_3, m_4) = \left\langle \frac{k^4 k^\mu}{D_1 D_2 D_3 D_4} \right\rangle$$

$$= C_{53}^\mu(P_1, P_2, m_1, m_2, m_3) - 2P_3^\nu D_{54}^{\nu\mu}(P_1, P_2, P_3, m_1, m_2, m_3, m_4)$$

$$+ (m_3^2 - P_3^2) g_{\nu\nu} D_{54}^{\nu\mu}(P_1, P_2, P_3, m_1, m_2, m_3, m_4)$$

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and the other is
\[ U_{B5}^{\mu\nu\alpha}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) = \left\langle \frac{k^2 k^\mu k^\nu k^\alpha}{D_1 D_2 D_3 D_4} \right\rangle \]
\[ = C^{\mu\nu\alpha}(P_1, P_2, m_1, m_2, m_3) - 2P_{3\beta}D^{\mu\nu\alpha\beta}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) \]
\[ + (m_4^2 - P_3^2)D^{\mu\nu\alpha}(P_1, P_2, P_3, m_1, m_2, m_3, m_4). \]  
(C.19)

The pattern should start becoming obvious by now. For example, we have
\[ U_{B6}^{\mu\nu}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) = \left\langle \frac{k^4 k^\mu k^\nu}{D_1 D_2 D_3 D_4} \right\rangle \]
\[ = U_{K4}\left(P_1, P_2, m_1, m_2, m_3\right) \]
\[ - 2P_{3\alpha}U_{B5}^{\mu\nu\alpha}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) \]
\[ + (m_4^2 - P_3^2)D^{\mu\nu\alpha}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) \]  
(C.20)

and its contracted version \( U_{B6} \equiv g_{\mu\nu}U_{B6}^{\mu\nu} \). The cubically divergent Box \( U \)-integral is
\[ U_{B7}^{\mu}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) = \left\langle \frac{k^6 k^\mu}{D_1 D_2 D_3 D_4} \right\rangle \]
\[ = U_{K5}\left(P_1, P_2, m_1, m_2, m_3\right) \]
\[ - 2P_{3\alpha}U_{B6}^{\mu\nu}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) \]
\[ + (m_4^2 - P_3^2)U_{B5}^{\mu\nu}(P_1, P_2, P_3, m_1, m_2, m_3, m_4). \]  
(C.21)

Finally we have the quartically divergent
\[ U_{B8}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) = \left\langle \frac{k^8}{D_1 D_2 D_3 D_4} \right\rangle \]
\[ = U_{K6}\left(P_1, P_2, m_1, m_2, m_3\right) \]
\[ - 2P_{3\alpha}U_{B7}^{\mu\nu}(P_1, P_2, P_3, m_1, m_2, m_3, m_4) \]
\[ + (m_4^2 - P_3^2)U_{B6}(P_1, P_2, P_3, m_1, m_2, m_3, m_4). \]  
(C.22)

### D Explicit calculation of the diagrams

In this Appendix we present the explicit calculation of the one-loop Feynman diagrams, concentrating on the Tadpole and the two-point function categories, in both \( R_\xi \) and Unitary gauges. The Triangle and Box contributions can be straightforwardly computed following similar steps, therefore there is no need to calculate them explicitly here.
D.1 $R_{\xi}$-diagrams

In order to be synchronised with our main text, we start form the first set of diagrams which corresponds to the one-point functions. The first such diagram is

$$ p \quad \quad \quad \quad \quad k \quad \quad \quad \quad \quad = \quad i T^R_{H,\phi} $$

and analytically evaluates to

$$ i T^R_{H,\phi} = -6S^1_T \sqrt{\frac{\lambda_0}{2}} m_{H_0} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - m_{H_0}^2}. \quad (D.1) $$

The symmetry factor is $S^1_T = \frac{1}{2}$ since $(n_O, n_l, \ell_1, v_1) = (3, 1, 3, 1)$. In DR this integral takes the form

$$ T^R_{H,\phi} = 3 \sqrt{\frac{\lambda_0}{2}} m_{H_0} \mu^\epsilon A_0(m_{H_0}). \quad (D.2) $$

The ”tadpole integral” $A_0$ is defined in Appendix B. It has mass dimension 2 and it is (external) momentum independent.

The next tadpole comes with a gauge boson loop:

$$ p \quad \quad \quad \quad \quad k \quad \quad \quad \quad \quad = \quad i T^R_{H,Z} $$

It is equal to

$$ T^R_{H,Z} = \frac{m_{Z_0}^2}{m_{H_0}} \sqrt{2\lambda_0} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - m_{Z_0}^2} \left[ (1 - \xi) \frac{m_{Z_0}^2}{m_{H_0}} \sqrt{2\lambda_0} g_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{ik^\mu k^\nu}{(k^2 - m_{Z_0}^2)} - \xi \frac{m_{Z_0}^2}{m_{H_0}} \right]. \quad (D.3) $$

with symmetry factor $S^2_T = \frac{1}{2}$ since $(n_O, n_l, \ell_1, v_1) = (1, 1, 2, 1)$. Using the relation $k^\mu k^\nu = \frac{g_{\mu\nu}}{d^4k} k^2$ under the integral it simplifies to

$$ T^R_{H,Z} = \frac{m_{Z_0}^2}{m_{H_0}} \sqrt{2\lambda_0} \mu^\ell \left\{ 4A_0(m_{Z_0}) - (1 - \xi) A_0(\sqrt{\xi} m_{Z_0}) - (1 - \xi) m_{Z_0}^2 B^1_0(m_{Z_0}, \sqrt{\xi} m_{Z_0}) \right\}, \quad (D.4) $$

where the $B_0$ scalar integral appears at $p^2 = 0$ and we call it $B^1_0$. Using Eq.(B.8) the gauge tadpole becomes

$$ T^R_{H,Z} = \frac{m_{Z_0}^2}{m_{H_0}} \sqrt{2\lambda_0} \mu^\ell \left\{ 3A_0(m_{Z_0}) + \xi A_0(\sqrt{\xi} m_{Z_0}) \right\}. \quad (D.5) $$

The last tadpole has a Goldstone loop:
\[ p \longrightarrow k \] = \( i \mathcal{T}_{H}^{R_{\xi}} \)

It is equal to

\[ \mathcal{T}_{H}^{R_{\xi}} = -\sqrt{2} \lambda_{0} \int \frac{d^{4}k}{m_{H_{0}}} \frac{-ik^{2}}{(k^{2} - \left(m_{\chi_{0}}^{2}\right))} \]  

with \( S_{T}^{3} = \frac{1}{2} \) as \((n_{O}, n_{l}, \ell_{1}, v_{1}) = (1, 1, 2, 1)\). Notice that the integral in Eq. \( \text{(D.6)} \) is a \( U \)-integral (called \( U_{T} \)), the first in the class of highly divergent integrals that we will often encounter in the Unitary gauge. As already mentioned, the origin of such integrals emerging also in the \( R_{\xi} \) gauge can be traced to our Polar basis choice to represent the Higgs field. Using Eq. \( \text{(C.2)} \) to calculate \( U_{T}(m_{\chi_{0}}) \) we obtain that in DR,

\[ \mathcal{T}_{H}^{R_{\xi}} = -\sqrt{2} \lambda_{0} \mu^{\epsilon} m_{H_{0}^{2}} A_{0}(m_{\chi_{0}}). \]  

Finally, the total tadpole value is the sum of the above three contributions:

\[ \mathcal{T}_{H}^{R_{\xi}} = \mu^{\epsilon} \left( 3 \sqrt{\frac{\lambda_{0}}{2}} m_{H_{0}} A_{0}(m_{H}) + 3 \sqrt{2} \lambda_{0} m_{Z_{0}}^{2} A_{0}(m_{Z}) \right). \]  

Next we have the \( Z \) 2-point function. A gauge-boson vacuum polarization amplitude can be Lorentz-covariantly split into a transverse and a longitudinal part

\[ \mathcal{M}_{Z,\mu\nu} = \left( -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^{2}} \right) \Pi^{T}(p^{2}) + \frac{p_{\mu}p_{\nu}}{p^{2}} \Pi^{L}(p^{2}). \]  

Contracting with \( p^{\mu}p^{\nu} \) both sides fixes

\[ \Pi^{L}(p^{2}) = \frac{p^{\mu}p^{\nu}}{p^{2}} \mathcal{M}_{Z,\mu\nu}. \]  

Contracting with \( g^{\mu\nu} \) gives on the other hand

\[ g^{\mu\nu} \mathcal{M}_{Z,\mu\nu} = -(d - 1) \Pi^{T} + \Pi^{L} \]  

that can be easily solved for the transverse part in \( d = 4 \)

\[ \Pi^{T} = \frac{1}{3} \left( -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^{2}} \right) \mathcal{M}_{Z}^{\mu\nu}. \]  

Now, the Schwinger-Dyson equation that the dressed \( Z \)-propagator

\[ G_{\mu\nu} = -g_{\mu\nu} G(p^{2}) + \frac{p_{\mu}p_{\nu}}{m_{Z_{0}}^{2}} L(p^{2}) \]  

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obeys is written as

\[ G_{\mu\nu} = G_{\mu\nu} + D_{\mu\rho} M_Z^{\rho\sigma} G_{\sigma\nu} \]  \hspace{1cm} (D.14)

with \( D_{\mu\rho} \) the tree level gauge boson propagator

\[ D_{\mu\rho} = \frac{\left( -g_{\mu\rho} + \frac{p_{\mu}p_{\rho}}{m_{Z_0}^2} \right)}{p^2 - m_{Z_0}^2}. \]  \hspace{1cm} (D.15)

So, performing the contractions the Schwinger-Dyson equation becomes

\[ -g_{\mu\nu} G + \frac{p_{\mu}p_{\nu}}{m_{Z_0}^2} L = \left( \frac{-g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{m_{Z_0}^2}}{p^2 - m_{Z_0}^2} \right) \Pi^T G + \frac{\left( -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^2} \right)}{p^2 - m_{Z_0}^2} \Pi^T G. \]  \hspace{1cm} (D.16)

Contracting again with \( p^\mu p^\nu \) we have that

\[ -G + \frac{p^2}{m_{Z_0}^2} L = \frac{1}{m_{Z_0}^2} \left[ 1 - \Pi^L (G - L) \right] \]  \hspace{1cm} (D.17)

while contracting with the metric gives

\[ -dG + \frac{p^2}{m_{Z_0}^2} L = \frac{-d + \frac{p^2}{m_{Z_0}^2}}{p^2 - m_{Z_0}^2} + \frac{-d + 1}{p^2 - m_{Z_0}^2} \Pi^T G - \frac{1}{m_{Z_0}^2} \Pi^L (G - L). \]  \hspace{1cm} (D.18)

The solution of the above equation is

\[ G(p^2) = \frac{1}{p^2 - m_{Z_0}^2 - \Pi^T (p^2)} \]

\[ L(p^2) = G(p^2) \left[ 1 - \frac{\Pi^T}{p^2 - \Pi^L} \right]. \]  \hspace{1cm} (D.19)

The quantity that enters in the renormalization of the mass of the \( Z \) gauge boson is therefore

\[ M_Z = -\frac{1}{3} \left( g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2} \right) M_Z^{\mu\nu}(p). \]  \hspace{1cm} (D.20)

We now start computing the one-loop Feynman diagrams contributing to \( M_Z^{R_{\epsilon,\phi}} \).

The first contributing diagram has a Higgs running in the loop:

\[ p \quad \begin{array}{c}
\hline
k \\
\hline
\end{array} \quad \Rightarrow \quad i M_{Z,\mu\nu}^{R_{\epsilon,\phi}} \]
and it is equal to
\[
\mathcal{M}_{Z,\mu\nu}^{R_\xi,\phi} = -2g_{\mu\nu} \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - m_{Z_0}^2},
\]
where we used that \( S_\xi^1 = \frac{1}{2} \) because \((n_O, n_I, \ell_1, \ell_2, v_1) = (2, 1, 2, 2, 1)\). In DR this integral is
\[
\mathcal{M}_{Z,\mu\nu}^{R_\xi,\phi} = -2g_{\mu\nu} \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu \epsilon A_0(m_{H_0}).
\]
Next we meet a couple of "sunset" diagrams. The first is
\[
\begin{array}{c}
\text{\hphantom{\quad \quad \quad} p} \\
\text{\hphantom{\quad \quad \quad} \mapsto} \\
\text{\hphantom{\quad \quad \quad} k + p} \\
\text{\hphantom{\quad \quad \quad} \mapsto} \\
\text{\hphantom{\quad \quad \quad} k}
\end{array}
= \mathcal{M}_{Z,\mu\nu}^{R_\xi,\phi Z}
\]
that evaluates to
\[
\mathcal{M}_{Z,\mu\nu}^{R_\xi,\phi Z} = -8g_{\mu\alpha}g_{\nu\beta} \frac{m_{Z_0}^4}{m_{H_0}^2} \lambda_0 \int \frac{d^4k}{(2\pi)^4} \left( (k^2 - m_{Z_0}^2) \left( (k + p)^2 - m_{H_0}^2 \right) \right) \frac{-ig^{\alpha\beta}}{(k^2 - m_{Z_0}^2) \left( (k + p)^2 - m_{H_0}^2 \right) \left( k^2 - m_{\chi_0}^2 \right)}
\]
\[
\text{(D.23)}
\]
with \((n_O, n_I, \ell_1, \ell_2, v_1) = (2 \times 2 \times 2, 1, 2, 2, 2)\) and \( S_\xi^2 = 1 \). In DR and using Eq. (B.35), it can be expressed as
\[
\mathcal{M}_{Z,\mu\nu}^{R_\xi,\phi Z} = 8 \frac{m_{Z_0}^4}{m_{H_0}^2} \lambda_0 \mu \epsilon \left\{ g_{\mu\nu} B_0(p, m_{Z_0}, m_{H_0}) + (1 - \xi) C_{\mu\nu}^1(p, m_{Z_0}, m_{H_0}, m_{\chi_0}) \right\}.
\]
Notice that the \( C \)-type integral above is a special PV case, computed in Appendix B as well.

The next sunset diagram is the last that contributes to the one-loop correction of the gauge boson propagator:
\[
\begin{array}{c}
\text{\hphantom{\quad \quad \quad} p} \\
\text{\hphantom{\quad \quad \quad} \mapsto} \\
\text{\hphantom{\quad \quad \quad} k + p} \\
\text{\hphantom{\quad \quad \quad} \mapsto} \\
\text{\hphantom{\quad \quad \quad} k}
\end{array}
= \mathcal{M}_{Z,\mu\nu}^{R_\xi,\chi_0}
\]
It is equal to
\[
\mathcal{M}_{Z,\mu\nu}^{R_\xi,\chi_0} = 8 \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \int \frac{d^4k}{(2\pi)^4} \left( (k^2 - m_{\chi_0}^2) \left( (k + p)^2 - m_{H_0}^2 \right) \right) \frac{-i k_\mu k_\nu}{(k^2 - m_{Z_0}^2) \left( (k + p)^2 - m_{H_0}^2 \right)}
\]
\[
\text{(D.24)}
\]
with $S_3^i = 1$ from $(n_O, n_t, \ell_1, \ell_2, v_1) = (1 \times 1 \times 2, 1, 1, 1, 2)$. In DR it is

$$
\mathcal{M}_{Z, \mu \nu}^{R_{\xi, \phi}} = \frac{m^2_{Z_0}}{m^2_{H_0}} \lambda_0 \mu \varepsilon B_{\mu \nu}(p, m_{\chi_0}, m_{H_0}).
$$

(D.25)

Adding up all contributions we obtain

$$
\mathcal{M}_{Z, \mu \nu}^{R_{\xi}} = \frac{m^2_{Z_0}}{m^2_{H_0}} \lambda_0 \mu \varepsilon \left\{ -2g_{\mu \nu}A_0(m_{H_0}) - 8g_{\mu \nu}m^2_{Z_0}B_0(p, m_{Z_0}, m_{H_0}) 
\right.
\right.
\left.\right.
\left.\right. + 8(1 - \xi)m^2_{Z_0}C^1_{\mu \nu}(m_{Z_0}, m_{H_0}, m_{\chi_0}) + 8B_{\mu \nu}(p, m_{\chi_0}, m_{H_0}) \right\}.
$$

(D.26)

Next, we consider the one-loop contributions to the Higgs propagator. The first diagram here is

Next contribution comes from the diagram
with explicit form

\[ M^{R_{\xi,\phi}}_H = 3\lambda_0 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - m_{H_0}^2} \]  
(D.30)

with \((n_O, n_l, \ell_1, v_1) = (4 \times 3, 1, 4, 1)\) and \(S^2_{M_H} = \frac{1}{2}\). This is just

\[ M^{R_{\xi,\phi}}_H = 3\lambda_0 \mu^\epsilon A_0(m_{H_0}) \]  
(D.31)
in DR.

Next comes the Goldstone loop

\[ p \quad \begin{array}{c} \hline \end{array} \quad k \quad \begin{array}{c} \hline \end{array} \quad = iM^{R_{\xi,\chi}}_H \]

that is equal to

\[ M^{R_{\xi,\chi}}_H = -2\frac{\lambda_0}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - m_{\chi_0}^2} \]  
(D.32)

with \((n_O, n_l, \ell_1, \ell_2, v_1) = (2, 1, 2, 2, 1)\) and \(S^3_{M_H} = \frac{1}{2}\). This is

\[ M^{R_{\xi,\chi}}_H = -\frac{2\lambda_0}{m_{H_0}^2} \mu^\epsilon m_{\chi_0}^2 A_0(m_{\chi_0}). \]  
(D.33)

A few vacuum polarization diagrams are in order. The first is

\[ p \quad \begin{array}{c} \hline \end{array} \quad \begin{array}{c} \hline \end{array} \quad \begin{array}{c} \hline \end{array} \quad k + p \quad \begin{array}{c} \hline \end{array} \quad = iM^{R_{\xi,\phi\phi}}_H \]

and it is equal to

\[ M^{R_{\xi,\phi\phi}}_H = 9\lambda_0 m_{H_0}^2 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2 - m_{H_0}^2)(k^2 - m_{H_0}^2)} \]  
(D.34)

and finally in DR to

\[ M^{R_{\xi,\phi\phi}}_H = 9\lambda_0 m_{H_0}^2 m^\epsilon B_0(p, m_{H_0}, m_{H_0}), \]  
(D.35)

where \((n_O, n_l, \ell_1, \ell_2, v_1) = (3 \times 3 \times 2, 2, 3, 3, 2)\) and \(S^4_{M_H} = \frac{1}{2}\).

The Goldstone loop contribution
Slightly more complicated is the gauge boson loop with symmetry factor $S$ can be expanded as

\[ iM^{R_{e,xx}}_H = \int_0^{2\pi} d\phi \delta^{\mu
u}(k_0^2 - m_{\chi_0}^2) \left( -g_{\mu\alpha} + \frac{1 - \xi k_0}{k^2 - \xi m_{\chi_0}^2} \right) \left( -g_{\nu\beta} + \frac{1 - \xi (k + p)_{\nu}}{(k + p)^2 - \xi m_{\chi_0}^2} \right) \]

(D.36)

with symmetry factor $S^5_{M_H} = \frac{1}{2}$ from $(n_O, n_l, \ell_1, \ell_2, v_1) = (2 \times 1, 2, 2, 2, 2)$. In DR it becomes

\[ M^{R_{e,xx}}_H = 4 \frac{\lambda_0}{m_{H_0}^2} \mu^2 \left\{ \frac{m_{\chi_0}^2 A_0(m_{\chi_0}) + (m_{\chi_0}^2 - p^2)g_{\mu\nu}B^{\mu\nu}(p, m_{\chi_0}, m_{\chi_0}) + p_{\mu}p_{\nu}B^{\mu\nu}(p, m_{\chi_0}, m_{\chi_0})}{(k^2 - m_{\chi_0}^2)(k + p)^2 - m_{\chi_0}^2} \right\} \]

(D.37)

Slightly more complicated is the gauge boson loop

\[ iM^{R_{e,ZZ}}_H \]

evaluating to

\[ M^{R_{e,ZZ}}_H = 4g^{\mu\nu}g^{\rho\beta} m_{Z_0}^2 \lambda_0 \int_0^{2\pi} d\phi \delta^{\mu\nu}(k_0^2 - m_{Z_0}^2) \left( -g_{\mu\alpha} + \frac{1 - \xi k_0}{k^2 - \xi m_{Z_0}^2} \right) \left( -g_{\nu\beta} + \frac{1 - \xi (k + p)_{\nu}}{(k + p)^2 - \xi m_{Z_0}^2} \right) \]

(D.38)

with $(n_O, n_l, \ell_1, \ell_2, v_1) = (2 \times 1, 2, 2, 2, 2)$ and $S^6_{M_H} = \frac{1}{2}$. The numerator of this diagram can be expanded as

\[ N = g^{\mu\nu}g^{\rho\beta} \left( -g_{\mu\alpha} + \frac{1 - \xi k_0}{k^2 - \xi m_{Z_0}^2} \right) \left( -g_{\nu\beta} + \frac{1 - \xi (k + p)_{\nu}}{(k + p)^2 - \xi m_{Z_0}^2} \right) \]

\[ = d - (1 - \xi) \left( \frac{k^2}{k^2 - \xi m_{Z_0}^2} + \frac{2k \cdot p + p^2}{(k + p)^2 - \xi m_{Z_0}^2} \right) \]

\[ + (1 - \xi)^2 \left( \frac{k^4 + 2k^2 k \cdot p + (k \cdot p)^2}{(k^2 - \xi m_{Z_0}^2)((k + p)^2 - \xi m_{Z_0}^2)} \right) \]

(D.39)

and then it is a standard step to express it in terms of PV integrals as

\[ M^{R_{e,ZZ}}_H = 4 \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^2 \left\{ dB_0(p, m_{Z_0}, m_{Z_0}) - (1 - \xi) \left\{ g_{\mu\nu}C^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} \right\} \]
\[
\begin{align*}
+ \ g_{\mu\nu}C^{\mu\nu}(p, p, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \\
+ \ (1 - \xi)^2 \left\{ g_{\mu\nu}C^{1\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ \ (m_{Z_0}^2 - p^2)g_{\mu\nu}D^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) + p_\mu p_\nu D^{\mu\nu}(p, m_{Z_0}, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \right\} \right\} \\
\end{align*}
\]

(D.40)

where the \( a = 1, 2, 3 \) superscripts on the \( C_0 \)-integrals correspond to the different combinations of the denominators according to Eq. (B.12) of Appendix B. The \( D^{\mu\nu} \) integrals are defined in Eq. (B.39).

Finally, the last contribution to the one-loop correction of the Higgs mass comes from the sunset

\[
\begin{align*}
\begin{array}{c}
\begin{array}{c}
\hline
k + p \\
\hline
\end{array}
\end{array}
\end{align*}
\]

\[
\begin{array}{c}
p \\
\hline
k
\end{array}
\]

\[
= i M_{H}^{R_{\xi,\chi Z}}
\]

with explicit form

\[
M_{H}^{R_{\xi,\chi Z}} = -8\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{i k\mu k\nu}{k^2 - m_{\chi_0}^2} \left( -g_{\mu\nu} + \frac{(1 - \xi)(k+p)\mu(k+p)\nu}{(k+p)^2 - m_{Z_0}^2} \right)
\]

\[
= -8\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} g_{\mu\nu}B^{\mu\nu}(p, m_{\chi_0}, m_{Z_0})
\]

\[
+ 8(1 - \xi)\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{-i (k^4 + 2k^2 k \cdot p + (k \cdot p)^2)}{(k^2 - m_{\chi_0}^2)((k+p)^2 - m_{Z_0}^2)((k+p)^2 - m_{\chi_0}^2)}
\]

(D.41)

where \((n_O, n_l, \ell_1, \ell_2, v_1) = (2 \times 1, 1, 1, 1, 2)\) and \(S_{M_H}^7 = 1\). Standard steps allow us to write this as

\[
M_{H}^{R_{\xi,\chi Z}} = 8\lambda_0 \frac{m_{Z_0}^2}{m_{H_0}^2} \left\{ -g_{\mu\nu}B^{\mu\nu}(p, m_{\chi_0}, m_{Z_0}) \\
+ (1 - \xi) \left\{ g_{\mu\nu}B^{\mu\nu}(p, m_{\chi_0}, m_{\chi_0}) + (m_{Z_0}^2 - p^2)g_{\mu\nu}C^{1\mu\nu}(p, m_{Z_0}, m_{\chi_0}, m_{\chi_0}) \\
+ p_\mu p_\nu C^{1\mu\nu}(p, m_{\chi_0}, m_{Z_0}, m_{\chi_0}) \right\} \right\}
\]

(D.42)

where \(C^{1\mu\nu}_{\mu\nu}\) is defined in Eq. (B.34) in Appendix B.
D.2  Unitary gauge diagrams

Similarly with the previous subsection, we consider first the Higgs tadpoles starting from

\[ p \rightarrow \begin{array}{c} \infty \\ \hline \end{array} \]

\[ = i \mathcal{T}_H^{U,\phi} \]

\[ \mathcal{T}_H^{U,\phi} = -6 \frac{1}{2} \sqrt{\frac{\lambda_0}{2}} m_{H_0} \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m_{H_0}^2} \]  \hfill (D.43)

and in DR is

\[ \mathcal{T}_H^{U,\phi} = 3 \sqrt{\frac{\lambda_0}{2}} m_{H_0} \mu^\varepsilon \int \frac{d^d k}{(4\pi)^d/2} \frac{1}{i\pi^{d/2} (k^2 - m_{H_0}^2)} \]

or simply

\[ \mathcal{T}_H^{U,\phi} = 3 \sqrt{\frac{\lambda_0}{2}} m_{H_0} \mu^\varepsilon A_0(m_{H_0}). \]  \hfill (D.44)

Next is the gauge tadpole is

\[ p \rightarrow \begin{array}{c} \infty \\ \hline \end{array} \]

\[ = i \mathcal{T}_H^{U,Z} \]

\[ \mathcal{T}_H^{U,Z} = d \frac{\sqrt{2\lambda_0 m_{Z_0}^2}}{m_{H_0}} \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m_{Z_0}^2} + \frac{\sqrt{2\lambda_0}}{m_{H_0}} \int \frac{d^4 k}{(2\pi)^4} \frac{i k^2}{k^2 - m_{Z_0}^2} \]  \hfill (D.45)

where we have expanded the numerator, used that \( g_{\mu\nu} g^{\mu\nu} = d \) and that under the integral

\( k^\mu k^\nu = g_{\mu\nu} k^2. \)  \hfill In DR

\[ \mathcal{T}_H^{U,Z} = \mu^\varepsilon \left( d \frac{\sqrt{2\lambda_0 m_{Z_0}^2}}{m_{H_0}} A_0(m_{Z_0}) - \frac{\sqrt{2\lambda_0}}{m_{H_0}} U_T(m_{Z_0}) \right). \]

Using Eq. (C.2) to calculate \( U_T(m_{Z_0}) \), this becomes

\[ \mathcal{T}_H^{U} = \mu^\varepsilon \left( 3 \sqrt{\frac{\lambda_0}{2}} m_{H_0} A_0(m_{H_0}) + 3 \frac{\sqrt{2\lambda_0 m_{Z_0}^2}}{m_{H_0}} A_0(m_{Z_0}) \right). \]  \hfill (D.47)

Next, we present the explicit calculation of the one-loop Z boson mass corrections starting from,

\[ p \rightarrow \begin{array}{c} \infty \\ \hline \end{array} \]

\[ = i \mathcal{M}_Z^{U,\phi} \]
\[ \mathcal{M}^{U,\phi}_{Z,\mu\nu} = -2g_{\mu\nu} \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{-i}{k^2 - m_{Z_0}^2} \right\} \] (D.48)

which is equal to

\[ \mathcal{M}^{U,\phi}_{Z,\mu\nu} = -2g_{\mu\nu} \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^\varepsilon A_0(m_{H_0}) . \] (D.49)

Next is the Higgs sunset

\[ k + p \]
\[ \mu \nu \]
\[ \frac{d^4k}{(2\pi)^4} \left\{ \frac{-i}{k^2 - m_{Z_0}^2} \right\} \]

\[ \mathcal{M}^{U,\phi}_{Z,\mu\nu} = -8g_{\mu\alpha}g_{\nu\beta} \frac{m_{Z_0}^4}{m_{H_0}^2} \lambda_0 \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{-i}{k^2 - m_{Z_0}^2} \right\} \] (D.50)

translating in DR to

\[ \mathcal{M}^{U,\phi}_{Z,\mu\nu} = -8g_{\mu\nu} \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^\varepsilon B_0(p, m_{Z_0}, m_{H_0}) + 8 \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^\varepsilon B_{\mu\nu}(p, m_{Z_0}, m_{H_0}) . \] (D.51)

The sum of these two corrections is

\[ \mathcal{M}^{U}_{Z,\mu\nu} = \frac{m_{Z_0}^2}{m_{H_0}^2} \lambda_0 \mu^\varepsilon \left\{ -8g_{\mu\nu} m_{Z_0}^2 B_0(p, m_{Z_0}, m_{H_0}) - 2g_{\mu\nu} A_0(m_{H_0}) \right\} \]
\[ + 8 B_{\mu\nu}(p, m_{Z_0}, m_{H_0}) \] (D.52)

Now, we deal with the one-loop corrections of the Higgs mass in Unitary gauge. The first comes from

\[ p \]
\[ \mu \nu \]
\[ \frac{d^4k}{(2\pi)^4} \left\{ \frac{-i}{k^2 - m_{Z_0}^2} \right\} \]

\[ \mathcal{M}^{U}_{Z,\mu\nu} = \frac{1}{2} 4d \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{-i}{k^2 - m_{Z_0}^2} \right\} \]
\[ + \frac{1}{2} \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{-i}{k^2 - m_{Z_0}^2} \right\} \] (D.53)

In DR it becomes

\[ \mathcal{M}^{U}_{Z,\mu\nu} = \mu^\varepsilon \left\{ 2d \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} A_0(m_{Z_0}) - 2 \frac{\lambda_0}{m_{H_0}^2} U_T(1, m_{Z_0}) \right\} \]
\[ = \mu^\varepsilon \frac{6 \lambda_0 m_{Z_0}^2}{m_{H_0}^2} A_0(m_{Z_0}) . \] (D.54)

Next is
In DR, \[ \mathcal{M}^{U,\phi}_H = \frac{3}{2} \lambda_0 \mu A_0(m_{H_0}). \] (D.56)

The Higgs vacuum polarization diagram

\[ p \quad \begin{array}{c} \hline k \end{array} \quad k + p = i \mathcal{M}^{U,\phi}_H \]

\[ \mathcal{M}^{U,\phi}_H = \frac{1}{2} 18 \lambda m_{H_0}^2 \int \frac{d^4k}{(2\pi)^4} \frac{-i}{\left(k^2 - m_{H_0}^2\right)\left((k+p)^2 - m_{H_0}^2\right)} \] (D.57)

in DR is equal to

\[ \mathcal{M}^{U,\phi}_H = 9 \lambda_0 m_{H_0}^2 \mu B_0(p, m_{H_0}, m_{H_0}). \] (D.58)

The corresponding gauge loop is

\[ p \quad \begin{array}{c} \hline k \end{array} \quad k + p = i \mathcal{M}^{U,ZZ}_H \]

\[ \mathcal{M}^{U,ZZ}_H = \frac{1}{2} 8 g^{\mu\nu} g^{\alpha\beta} \lambda_0 m_{Z_0}^4 \int \frac{d^4k}{(2\pi)^4} \frac{-i \left(-g_{\mu\alpha} + \frac{k\mu k\alpha}{m_{Z_0}^2}\right) \left(-g_{\nu\beta} + \frac{(k+p)\nu (k+p)\beta}{m_{Z_0}^2}\right)}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)} \] (D.59)

Expanding the numerator it becomes

\[ \mathcal{M}^{U,ZZ}_H = \frac{d}{m_{h_0}^2} \lambda_0 m_{Z_0}^4 \int \frac{d^4k}{(2\pi)^4} \frac{-4i}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)} + 4 \frac{\lambda_0 m_{Z_0}^2}{m_{h_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{ik^2}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)} + 4 \frac{\lambda_0 m_{Z_0}^2}{m_{h_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{i(k+p)^2}{\left(k^2 - m_{Z_0}^2\right)\left((k+p)^2 - m_{Z_0}^2\right)} \]
Adding up Eq. (D.54), Eq. (D.56), Eq. (D.58) and Eq. (D.61) we obtain
for the arguments of integrals, we define expressions are quite long, we use for simplicity some shorthand notation. In particular, parts of the divergent ones, in both $R_\xi$ and Unitary gauges. Since the corresponding expressions are quite long, we use for simplicity some shorthand notation. In particular, for the arguments of integrals, we define

\[-4 \frac{\lambda_0}{m_{Z_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{ik^2(k+p)^2}{(k^2 - m_{Z_0}^2)((k+p)^2 - m_{Z_0}^2)} \]
\[+ 4 \frac{\lambda_0}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{i p^2 k^2}{(k^2 - m_{Z_0}^2)((k+p)^2 - m_{Z_0}^2)} \]
\[-4 \frac{\lambda_0}{m_{H_0}^2} \int \frac{d^4k}{(2\pi)^4} \frac{i (k \cdot p)^2}{(k^2 - m_{Z_0}^2)((k+p)^2 - m_{Z_0}^2)} \] (D.60)

and in DR

\[ M_{H}^{u,zz} = \mu^\epsilon \left\{ 4d \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} B_0(p, m_{Z_0}, m_{Z_0}) \right. \]
\[- 4 \frac{\lambda_0 m_{Z_0}^2}{m_{H_0}^2} g_{\mu\nu} B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) + 4 \frac{\lambda_0}{m_{H_0}^2} m_{Z_0}^2 A_0(m_{Z_0}) + \left. 4 \frac{\lambda_0}{m_{H_0}^2} m_{Z_0}^4 \right. \]
\[- 4 \frac{\lambda_0}{m_{H_0}^2} p^2 g_{\mu\nu} B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) + 4 \frac{\lambda_0}{m_{H_0}^2} p_\mu p_\nu B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) \} \]. (D.61)

where we have defined

\[ g_{\mu\nu} B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) = \int \frac{d^4k}{(2\pi)^4} \frac{-i(k+p)^2}{(k^2 - m_{Z_0}^2)((k+p)^2 - m_{Z_0}^2)}. \] (D.62)

Adding up Eq. (D.54), Eq. (D.56), Eq. (D.58) and Eq. (D.61) we obtain

\[ M_{H}^{u}(p) = \mu^\epsilon \left\{ 6 \frac{\lambda m_{Z_0}^2}{m_{H_0}^2} A_0(m_{Z_0}) + 2 \frac{\lambda m_{Z_0}^4}{m_{H_0}^2} + 3 \lambda_0 A_0(m_{H_0}) \right. \]
\[ + 9 \lambda_0 m_{H_0}^2 B_0(p, m_{H_0}, m_{H_0}) + \left. \lambda_0 m_{Z_0}^2 \right\} \]
\[ - 4g_{\mu\nu} B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) + 4A_0(m_{Z_0}) \left. - 4 \frac{p^2}{m_{Z_0}^2} g_{\mu\nu} B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) \right. \]
\[ + 4m_{Z_0}^2 + 4 \frac{p_\mu p_\nu}{m_{Z_0}^2} B_{k+p}^{\mu\nu}(p, m_{Z_0}, m_{Z_0}) \} \}. \] (D.63)

### E Finite parts

In this Appendix we present the explicit form of the finite diagrams along with the finite parts of the divergent ones, in both $R_\xi$ and Unitary gauges. Since the corresponding expressions are quite long, we use for simplicity some shorthand notation. In particular, for the arguments of integrals, we define

\[ D_4 \equiv (D_1, D_2, D_3, D_4). \] (E.1)
In addition, in agreement with our notation in Appendix A, we define the following integrals

\[ E_{\mu, \nu, \rho, \sigma, \tau} = 4! \int_0^1 D^5 x \left\langle \frac{N^{(5)}_{E_{\mu, \nu, \rho, \sigma, \tau}}(k - \sum_{i=1}^5 P_i x_{i+1})}{D_1 D_2 D_3 D_4 D_5} \right\rangle \]

\[ F_{\mu, \nu, \rho, \sigma, \tau} = 5! \int_0^1 D^6 x \left\langle \frac{N^{(6)}_{F_{\mu, \nu, \rho, \sigma, \tau}}(k - \sum_{i=1}^6 P_i x_{i+1})}{D_1 D_2 D_3 D_4 D_5 D_6} \right\rangle \]

\[ G_{\mu, \nu, \rho, \sigma, \tau} = 6! \int_0^1 D^7 x \left\langle \frac{N^{(7)}_{G_{\mu, \nu, \rho, \sigma, \tau}}(k - \sum_{i=1}^7 P_i x_{i+1})}{D_1 D_2 D_3 D_4 D_5 D_6 D_7} \right\rangle \]

\[ H_{\mu, \nu, \rho, \sigma, \tau} = 7! \int_0^1 D^8 x \left\langle \frac{N^{(8)}_{H_{\mu, \nu, \rho, \sigma, \tau}}(k - \sum_{i=1}^8 P_i x_{i+1})}{D_1 D_2 D_3 D_4 D_5 D_6 D_7 D_8} \right\rangle \]

(E.2)

**E.1 \( R_\xi \) gauge**

In the \( R_\xi \) gauge we have the following finite parts:

Finite parts of the Triangle diagrams

\[ (K_{R_\xi, 3Z}^{H_{3Z}})_f = 16 \sqrt{2} \frac{m_{Z_0}^3}{m_{H_0}^3} \mu^\varepsilon \left\{ (4 - \varepsilon) C_0(P_1, m_{Z_0}, m_{Z_0}, m_{Z_0}) \right. \]

\[ - (1 - \xi) \left\{ g_{\mu \nu} D^\mu(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \right. \]

\[ + g_{\mu \nu} D^\mu(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) + 2 P_1^\mu D^\mu(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \]

\[ + 2 P_1^\mu D^\mu(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) + P_1^2 D_0(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \]

\[ + P_2^2 D_0(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}) \}

\[ + (1 - \xi)^2 \left\{ E_4(D_4, D_5(0, m_{Z_0})) = E_4(D_4, D_5(P_1, m_{Z_0})) \right. \]

\[ + E_4(D_1, D_2, D_3, D_4(0, m_{Z_0}), D_5(P_1, m_{Z_0})) \]

\[ + 2 P_1 P_2 E_3^\mu(D_4, D_5(0, m_{Z_0})) + 2 P_1 P_2 E_3^\mu(D_1, D_2, D_3, D_4(0, m_{Z_0}), D_5(P_1, m_{Z_0})) \]

\[ + 2 P_1 P_2 E_3^\mu(D_4, D_5(P_1, m_{Z_0})) + 2 P_1 P_2 E_3^\mu(D_1, D_2, D_3, D_4(0, m_{Z_0}), D_5(P_1, m_{Z_0})) \]

\[ + 2 P_1 P_2 E_3^\mu(D_1, D_2, D_3, D_4(0, m_{Z_0}), D_5(P_1, m_{Z_0})) \]

\[ + P_2 P_2 E_3^\mu(D_1, D_2, D_3, D_4(0, m_{Z_0}), D_5(P_1, m_{Z_0})) \]

\[ + 2 P_1 P_2 E_3^\mu(D_1, D_2, D_3, D_4(0, m_{Z_0}), D_5(P_1, m_{Z_0})) \]

\[ + 2 P_1 P_2 E_3^\mu(D_1, D_2, D_3, D_4(0, m_{Z_0}), D_5(P_1, m_{Z_0})) \]

\[ + (P_1 P_2)^2 E_0(D_1, D_2, D_3, D_4(0, m_{Z_0}), D_5(P_1, m_{Z_0})) \]
\[
F_0(D_4, D_5(0, m_{\chi_0}, D_6(P_1, m_{\chi_0})) \\
+ 2(P_1 + P_2) \mu F_5^\mu(D_4, D_5(0, m_{\chi_0}, D_6(P_1, m_{\chi_0})) \\
+ (P_1 \mu P_{1\nu} + 3P_{1\mu} P_{2\nu} + P_{2\mu} P_{2\nu}) F_4^{\mu
u}(D_4, D_5(0, m_{\chi_0}, D_6(P_1, m_{\chi_0})) \\
+ (P_1 \mu P_{1\nu} P_{2\alpha} + P_{2\mu} P_{2\nu} P_{1\alpha}) F_3^{\mu\nu\alpha}(D_4, D_5(0, m_{\chi_0}, D_6(P_1, m_{\chi_0})) \\
+ P_1 \cdot P_2(P_1 + P_2) \mu F_3^\mu(D_4, D_5(0, m_{\chi_0}, D_6(P_1, m_{\chi_0})) \\
+ P_1 \cdot P_2 P_1 P_2 P_2 F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}, D_6(P_1, m_{\chi_0})) \\
+ P_1 \cdot P_2 F_4(D_4, D_5(0, m_{\chi_0}, D_6(P_1, m_{\chi_0}))) \}
\]

\[
(B_{H}^{R, zzzz})_f = 64 \frac{m_8^{s_0} \lambda_0^2}{m_{H_0}^2} \mu \left\{ dD_0(D_1, D_2, D_3, D_4) \\
- (1 - \xi)^2 \left( F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) + F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
+ F_4(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) + F_4(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_1 \mu F_3^\mu(D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ 2P_1 \mu F_3^\mu(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_2 \mu F_3^\mu(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ 2P_2 \mu F_3^\mu(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_1 \mu F_3^\mu(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
+ 2P_2 \mu F_3^\mu(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_3 \mu F_3^\mu(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ 2P_3 \mu F_3^\mu(D_4, D_5(P_3, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_3 \mu F_3^\mu(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ P_1 \mu P_1 \nu \left\{ F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) + F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
+ F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \right\} + 2P_1 \mu P_2 \nu F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) \\
+ P_2 \mu P_2 \nu \left\{ F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) + F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
+ F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \right\} \\
+ P_3 \mu P_3 \nu \left\{ F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) + F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \right\} \right\}
\]

Finite parts of the Box diagrams

\[
(B_{H}^{R, zzzz})_f = 64 \frac{m_8^{s_0} \lambda_0^2}{m_{H_0}^2} \mu \left\{ dD_0(D_1, D_2, D_3, D_4) \\
- (1 - \xi)^2 \left( F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) + F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
+ F_4(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) + F_4(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_1 \mu F_3^\mu(D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ 2P_1 \mu F_3^\mu(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_2 \mu F_3^\mu(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ 2P_2 \mu F_3^\mu(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_1 \mu F_3^\mu(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
+ 2P_2 \mu F_3^\mu(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_3 \mu F_3^\mu(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ 2P_3 \mu F_3^\mu(D_4, D_5(P_3, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_3 \mu F_3^\mu(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
+ P_1 \mu P_1 \nu \left\{ F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) + F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
+ F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \right\} + 2P_1 \mu P_2 \nu F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) \\
+ P_2 \mu P_2 \nu \left\{ F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) + F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
+ F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \right\} \\
+ P_3 \mu P_3 \nu \left\{ F_2^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) + F_2^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \right\} \right\}
\]

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\[\begin{align*}
&+ F_2^{\mu \rho}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_{1,\rho}P_{2,\nu}F_2^{\mu \nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) \\
&+ 2P_{1,\mu}P_{3,\nu}F_2^{\mu \nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(0, m_{\chi_0})) \\
&+ 2P_{2,\mu}P_{3,\nu}F_2^{\mu \nu}(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
&+ 2\left\{P_1 \cdot P_2 F_2(D_4, D_5(P_1, m_{\chi_0}), D_6(0, m_{\chi_0}))ight\} \\
&+ P_1 \cdot P_3 F_2(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
&+ P_2 \cdot P_3 F_2(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
&+ 2P_1 \cdot P_2 \left\{P_{1,\mu} + P_{2,\mu}\right\} F_4^{\mu}(D_4, D_5(P_1, m_{\chi_0}), D_6(0, m_{\chi_0})) \\
&+ 2P_1 \cdot P_3 \left\{P_{1,\mu} + P_{3,\mu}\right\} F_4^{\mu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
&+ 2P_2 \cdot P_3 \left\{P_{2,\mu} + P_{3,\mu}\right\} F_4^{\mu}(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
&+ (P_1 \cdot P_2)F_0(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0})) \\
&+ (P_1 \cdot P_3)F_0(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
&+ (P_2 \cdot P_3)F_0(D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \\
&- (1 - \xi)^3 \left(G_6(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) + G_6(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) + G_6(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0}))ight) \\
&+ \sum_{l=1}^3 2P_{l,\mu}G_5^{\mu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \\
&+ \sum_{l=1}^3 2P_{l,\mu}G_5^{\mu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l=1}^3 2P_{l,\mu}G_5^{\mu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1}^3 2P_{l,\mu}P_{m,\nu}G_4^{\mu \nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \\
&+ \sum_{l,m=1}^3 2P_{l,\mu}P_{m,\nu}G_4^{\mu \nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1}^3 2P_{l,\mu}P_{m,\nu}G_4^{\mu \nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m,n=1}^3 P_{l,\mu}P_{m,\nu}P_{n,\alpha}G_3^{\mu \nu \alpha}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \\
&+ \sum_{l,m,n=1}^3 P_{l,\mu}P_{m,\nu}P_{n,\alpha}G_3^{\mu \nu \alpha}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
\end{align*}\]
\[\begin{align*}
&+ \sum_{l,m,n=1}^{3} P_{l,m} P_{m,n} P_{n,l} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1, m \neq l}^{3} P_{l,m} G_4(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1, m \neq l}^{3} P_{l,m} G_5(D_4, D_5(0, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l=1}^{3} P_{l,m} G_6(D_4, D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \\
&+ \sum_{l=1}^{3} P_{l,m} G_7(D_4, D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l=1}^{3} P_{l,m} G_8(D_4, D_6(P_3, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1}^{3} P_{l,m} P_{m,n} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1, m \neq l}^{3} P_{l,m} P_{m,n} P_{n,l} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1, m \neq l}^{3} P_{l,m} P_{m,n} P_{n,l} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1, m \neq l}^{3} P_{l,m} P_{m,n} P_{n,l} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1, m \neq l}^{3} P_{l,m} P_{m,n} P_{n,l} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(P_3, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1}^{3} P_{l,m} P_{m,n} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \sum_{l,m=1}^{3} P_{l,m} P_{m,n} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(P_3, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
&+ \left( P_{l,m} P_{m,n} G_{\mu\nu}^{\alpha\beta}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \right) \\
&+ (1 - \xi)^{3} \left( H_8(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0}), D_8(P_3, m_{\chi_0})) \right) \\
&+ 2(1 + P_2 + P_3)^{\mu} H_{7}^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0}), D_8(P_3, m_{\chi_0}))
\end{align*}\]
\[
B_{H}^{R_{i}^{c}\chi \frac{zzz}{\mathcal{Z}}}_f = \frac{64m_{Z_0}^{6}\chi_0^{2}}{m_{H_0}^{4}}\mu^{\epsilon} \left\{ -C_0(D_1, D_2, D_3) - m_{Z_0}^{2} D_0(D_1, D_2, D_3, D_4) \right\} \\
+ (1 - \xi) \left( E_4(D_4, D_5(0, m_{\chi_0})) + E_4(D_4, D_5(0, m_{\chi_0})) \right) \\
+ E_4(D_4, D_5(P_1, m_{\chi_0})) + E_4(D_4, D_5(P_1, m_{\chi_0})) \\
+ E_4(D_4, D_5(0, m_{\chi_0})) + 2P_{1,\mu}E_3^{\mu}(D_4, D_5(0, m_{\chi_0})) \\
+ 2P_{2,\mu}E_3^{\mu}(D_4, D_5(0, m_{\chi_0})) + 2P_{3,\mu}E_3^{\mu}(D_4, D_5(P_1, m_{\chi_0})) \\
+ P_{1,\mu}P_{1,\nu}\left\{ E_2^{\nu}(D_4, D_5(0, m_{\chi_0})) + 2P_{1,\mu}P_{2,\nu}E_2^{\nu}(D_4, D_5(0, m_{\chi_0})) \right\} \\
+ P_{2,\mu}P_{2,\nu}E_2^{\nu}(D_4, D_5(0, m_{\chi_0})) + E_2^{\nu}(D_4, D_5(P_1, m_{\chi_0})) \\
+ P_{3,\mu}P_{3,\nu}E_2^{\nu}(D_4, D_5(0, m_{\chi_0})) + 2P_{1,\mu}P_{2,\nu}E_2^{\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0})) \right\} \\
+ 2P_{1,\mu}P_{3,\nu}E_2^{\nu}(D_4, D_5(P_1, m_{\chi_0})) \\
+ 2P_1 \cdot P_2 E_2(D_4, D_5(P_1, m_{\chi_0}))
\]

(E.4)
\[ + \ 2P_1 \cdot P_2 \left\{ P_{1,\mu} + P_{2,\mu} \right\} E_4^\mu (D_4, D_5(P_1, m_{\chi_0})) \]
\[ + \ (P_1 \cdot P_2)^2 E_0 (D_4, D_5(P_1, m_{\chi_0})) \]
\[ - \ (1 - \xi)^2 \left( F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \right) \]
\[ + \ F_4(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + F_4(D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ F_4(D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_{1,\mu} F_3^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ 2P_{2,\mu} F_3^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_{3,\mu} F_3^\mu (D_4, D_5(P_1, m_{\chi_0})) \]
\[ + \ 2P_{2,\mu} F_3^\mu (D_4, D_5(P_2, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_{3,\mu} F_3^\mu (D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ P_{1,\mu} F_1^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_{1,\mu} F_5^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ 2P_{2,\mu} F_5^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_{3,\mu} F_5^\mu (D_4, D_5(P_1, m_{\chi_0})) \]
\[ + \ 2P_{1,\mu} F_2^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) + 2P_{2,\mu} F_2^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ 2P_{1,\mu} F_3^\mu (D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ 2P_{1,\mu} F_2^\mu (D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ 2P_{1,\mu} F_3^\mu (D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ (P_1 \cdot P_2)^2 E_0 (D_4, D_5(P_1, m_{\chi_0}), D_6(P_3, m_{\chi_0})) \]
\[ + \ (1 - \xi)^3 \left( G_6(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \right) \]
\[ + \ G_6(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \]
\[ + \ G_6(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \]
\[ + \sum_{l=1}^{3} 2P_{1,\mu} G_5^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \]
\[ + \sum_{l=1}^{3} 2P_{1,\mu} G_5^\mu (D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \]
\[ + \sum_{l=1}^{3} 2P_{1,\mu} G_5^\mu (D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \]
\[ + \sum_{l,m=1}^{3} 2P_{1,\mu} G_4^{au} (D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \]
\[ + \sum_{l,m=1}^{3} 2P_{1,\mu} G_4^{au} (D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \]
\[ + \sum_{l,m=1}^{3} 2P_{1,\mu} G_4^{au} (D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \]
\[\begin{align*}
\sum_{l,m,n=1}^{3} & P_{l\mu} P_{m\nu} P_{n\alpha} G_{3}^{\mu\nu\alpha}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \\
+ & \sum_{l,m,n=1}^{3} P_{l\mu} P_{m\nu} P_{n\alpha} G_{3}^{\mu\nu\alpha}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m,n=1}^{3} P_{l\mu} P_{m\nu} P_{n\alpha} G_{3}^{\mu\nu\alpha}(D_4, D_5(0, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m=1, m\neq l}^{3} P_{l} \cdot P_{m} G_{4}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m=1, m\neq l}^{3} P_{l} \cdot P_{m} G_{4}(D_4, D_5(0, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l=1}^{3} P_{l\mu} G_{3}^{\mu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_2, m_{\chi_0})) \\
+ & \sum_{l=1}^{3} P_{l\mu} G_{3}^{\mu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l=1}^{3} P_{l\mu} G_{3}^{\mu}(D_4, D_5(0, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m=1}^{3} P_{l} \cdot P_{m} P_{m\mu} G_{3}^{\mu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m=1}^{3} P_{l} \cdot P_{m} P_{m\mu} G_{3}^{\mu}(D_4, D_5(0, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m=1, m\neq l}^{3} P_{l} \cdot P_{m} P_{m\mu} G_{2}^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m=1, m\neq l}^{3} P_{l} \cdot P_{m} P_{m\mu} G_{2}^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m=1, m\neq l}^{3} P_{l} \cdot P_{m} P_{m\mu} G_{2}^{\mu\nu}(D_4, D_5(0, m_{\chi_0}), D_6(P_3, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \left( P_{1} \cdot P_{2} P_{2} P_{3} + P_{2} \cdot P_{3} P_{1} \cdot P_{3} \right) G_{2}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m,n=1, n\neq m\neq l}^{3} P_{l} \cdot P_{m} P_{m\mu} P_{n}(P_m + P_n) G_{1}^{\mu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m,n=1, n\neq m\neq l}^{3} P_{l} \cdot P_{m} P_{m\mu} P_{n}(P_m + P_n) G_{1}^{\mu}(D_4, D_5(0, m_{\chi_0}), D_6(P_1, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \sum_{l,m=1, m\neq l}^{3} P_{l} \cdot P_{m} P_{m\mu} G_{2}^{\mu\nu}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
+ & \left( P_{1} \cdot P_{2} P_{2} P_{3} P_{1} \cdot P_{3} G_{0}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \right) \right) \\
+ & P_{1} \cdot P_{2} P_{2} P_{3} P_{1} \cdot P_{3} G_{0}(D_4, D_5(P_1, m_{\chi_0}), D_6(P_2, m_{\chi_0}), D_7(P_3, m_{\chi_0})) \\
\end{align*}\]
\[(B_{H}^{R_{\xi}zz\chi\chi})_{f,1} = 64 \frac{m_{\Omega}^{4} \lambda_{0}^{2}}{m_{H_0}^{4}} \mu^{\varepsilon} \left\{ 2P_{2,\mu}g_{\nu\alpha}D^{\nu\alpha}(D1, D2, D3, D4) + P_{2,\mu}P_{2,\nu}D^{\mu\nu}(D1, D2, D3, D4) \right\} \]

\[(E.6)\]

\[(B_{H}^{R_{\xi}zz\chi\chi})_{f,2} = 64 \frac{m_{\Omega}^{4} \lambda_{0}^{2}}{m_{H_0}^{4}} \mu^{\varepsilon} \left\{ -2P_{3,\mu}E_{5}^{\mu}(D4, D5(P_{1}, m_{\chi_0})) + (m_{\chi_0}^{2} - P_{2}^{2})E_{6}(D4, D5(P_{1}, m_{\chi_0})) + m_{\chi_0}^{2}E_{6}(D4, D5(0, m_{\chi_0})) + E_{4}(D4, D5(P_{1}, m_{\chi_0})) + E_{4}(D4, D5(0, m_{\chi_0})) + 2P_{2,\mu}E_{3}^{\nu}(D4, D5(P_{1}, m_{\chi_0})) + 2P_{1,\mu}E_{3}^{\nu}(D4, D5(0, m_{\chi_0})) + P_{1,\mu}P_{1,\nu}E_{2}^{\mu\nu}(D4, D5(0, m_{\chi_0})) + 2P_{1,\mu}P_{2,\nu}E_{2}^{\mu\nu}(D4, D5(0, m_{\chi_0})) + P_{2,\mu}P_{2,\nu}\left\{ E_{2}^{\mu\nu}(D4, D5(0, m_{\chi_0})) + E_{2}^{\mu\nu}(D4, D5(P_{1}, m_{\chi_0})) \right\} \right\} + P_{3,\mu}P_{3,\nu}E_{2}^{\mu\nu}(D4, D5(0, m_{\chi_0})) + 2P_{1,\mu}P_{2,\nu}E_{2}^{\mu\nu}(D4, D5(P_{1}, m_{\chi_0})) + 2P_{1,\mu}P_{3,\nu}E_{2}^{\mu\nu}(D4, D5(P_{1}, m_{\chi_0})) + 2P_{1,\mu}P_{2,\nu}E_{2}^{\mu\nu}(D4, D5(P_{1}, m_{\chi_0})) + 2P_{1} \cdot P_{2}\left\{ P_{1,\mu} + P_{2,\mu} \right\}E_{1}^{\mu}(D4, D5(P_{1}, m_{\chi_0})) \}

\[(E.7)\]

\[(B_{H}^{R_{\xi}zz\chi\chi})_{f,3} = 64 \frac{m_{\Omega}^{4} \lambda_{0}^{2}}{m_{H_0}^{4}} \mu^{\varepsilon} \left\{ F_{4}(D4, D5(0, m_{\chi_0}), D_{6}(0, m_{\chi_0})) + F_{4}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + F_{4}(D4, D_{5}(P_{1}, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + F_{4}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{2,\mu}F_{3}^{\mu}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{2,\mu}F_{3}^{\mu}(D4, D_{5}(P_{1}, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{2,\mu}F_{3}^{\mu}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{2,\mu}F_{3}^{\mu}(D4, D_{5}(P_{1}, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{2,\mu}F_{3}^{\mu}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + P_{1,\mu}P_{1,\nu}F_{2}^{\mu\nu}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{1,\mu}P_{2,\nu}F_{2}^{\mu\nu}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + P_{2,\mu}P_{2,\nu}F_{2}^{\mu\nu}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + P_{2,\mu}P_{2,\nu}F_{2}^{\mu\nu}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{1,\mu}P_{2,\nu}F_{2}^{\mu\nu}(D4, D_{5}(P_{1}, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{1,\mu}P_{2,\nu}F_{2}^{\mu\nu}(D4, D_{5}(0, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{1} \cdot P_{2}F_{2}(D4, D_{5}(P_{1}, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) + 2P_{1} \cdot P_{2}\left\{ P_{1,\mu} + P_{2,\mu} \right\}F_{1}^{\mu}(D4, D_{5}(P_{1}, m_{\chi_0}), D_{6}(P_{2}, m_{\chi_0})) \}

\[(E.8)\]
\[ (B_{H}^{R:zzzz})_{f,1} = 64 \frac{m_{Z_0}^2 \lambda_0^2}{m_{H_0}^4} \mu \epsilon \left\{ g_{\mu \nu} D^{\mu \nu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) + g_{\mu \nu} D^{\mu \nu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) + 2P_1^\mu D^{\mu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) + 2P_2^\mu D^{\mu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) + P_1^2 D_0(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} \] (E.8)

\[ (B_{H}^{R:zzzz})_{f,2} = 64 \frac{m_{Z_0}^2 \lambda_0^2}{m_{H_0}^4} \mu \epsilon \left\{ E_4(D_4, D_5(0, m_{\chi_0})) + E_4(D_4, D_5(P_1, m_{\chi_0})) + E_4(D_1, D_2, D_3, D_4(0, m_{\chi_0}), D_5(P_1, m_{\chi_0})) + 2P_1^\mu E_3^\mu(D_4, D_5(P_1, m_{\chi_0})) + 2P_2^\mu E_3^\mu(D_1, D_2, D_3, D_4(0, m_{\chi_0}), D_5(P_1, m_{\chi_0})) + 2P_1^\mu P_2^\nu E_2^\mu(D_4, D_5(P_1, m_{\chi_0})) + P_2^\mu P_2^\nu E_2^\mu(D_1, D_2, D_3, D_4(0, m_{\chi_0}), D_5(P_1, m_{\chi_0})) + 2P_1^\mu P_2^\nu(D_1, D_2, D_3, D_4(0, m_{\chi_0}), D_5(P_1, m_{\chi_0})) + 2P_1^\mu P_2^\nu(D_1, D_2, D_3, D_4(0, m_{\chi_0}), D_5(P_1, m_{\chi_0})) + (P_1 \cdot P_2)^2 E_0(D_1, D_2, D_3, D_4(0, m_{\chi_0}), D_5(P_1, m_{\chi_0})) \right\} \] (E.9)

E.2 Unitary gauge

In the Unitary gauge the finite parts are:

Finite parts of the Box diagrams

\[ (B_{H}^{U:zzzz})_{f,1} = 64 \frac{m_{Z_0}^4 \lambda_0^2}{m_{H_0}^4} \mu \epsilon \left\{ 6g_{\nu \alpha}(P_1 + P_2 + P_3)\mu D^{\mu \nu \alpha}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} + \left( 3P_{1}^\nu P_{1}^\nu + 2P_{1}^\nu P_{2}^\nu + 3P_{2}^\nu P_{2}^\nu \right) + \left( 2P_{1}^\mu P_{3}^\nu + 2P_{2}^\mu P_{3}^\nu + 3P_{3}^\mu P_{3}^\nu \right) + \left( 2(P_1 \cdot P_2 + P_1 \cdot P_2 + P_1 \cdot P_2)g_{\mu \nu} \right) D^{\mu \nu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) + \left( 2P_1 \cdot P_2(P_1^\mu + P_2^\mu) + 2P_1 \cdot P_2(P_1^\mu + P_2^\mu) \right) \]

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\[ (B^ {u,zzzz}_H \right)_{f,2} = 64 \frac{m^2_{Z_0}}{m_{H_0}^4} \mu^\varepsilon \left\{ \left( 2 \sum_{l,m,n=1}^{3} P_{l,\mu} P_{m,\nu} P_{n,\alpha} \right) + \sum_{l,m,n=1,n \neq m \neq l}^{3} 2(P_1 \cdot P_m) \right. \\
+ P_1 \cdot P_n + P_n \cdot P_m) (P_1 + P_m + P_n) \mu \right\} D^{\mu\nu\alpha}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ P_1 \cdot P_2 \sum_{l,m=1}^{3} P_{l,\mu} P_{m,\nu} D^{\mu\nu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ P_2 \cdot P_3 \sum_{l,m=1}^{3} P_{l,\mu} P_{m,\nu} D^{\mu\nu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ \sum_{l,m,n=1,n \neq m \neq l}^{3} P_1 \cdot P_m P_1 \cdot P_n g_{\mu\nu} D^{\mu\nu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ (P_1 \cdot P_2 P_2 \cdot P_3(P_1 + P_3) \mu \\
+ P_1 \cdot P_3 P_2 \cdot P_3(P_1 + P_3) \mu) D^{\mu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ P_1 \cdot P_2 P_1 \cdot P_3 P_2 \cdot P_3 D_0(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} \] (E.12)

\[ (B^ {u,zzzz}_H \right)_{f,3} = 64 \frac{\lambda^2_0}{m_{H_0}^4} \mu^\varepsilon \left\{ \left( P_1 \cdot P_2(P_{1,\mu} P_{2,\nu} P_{3,\alpha} + P_{1,\mu} P_{3,\nu} P_{3,\alpha}) + P_2 \cdot P_3(P_{1,\mu} P_{1,\nu} P_{3,\alpha} \\
+ P_{1,\mu} P_{2,\nu} P_{3,\alpha}) + P_1 \cdot P_2 g_{\mu\nu}(P_1 + P_3) \mu \right) D^{\mu\nu\alpha}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \\
+ P_1 \cdot P_2 P_2 \cdot P_3 P_{1,\mu} P_{3,\nu} D^{\mu\nu}(P_1, P_2, m_{Z_0}, m_{Z_0}, m_{Z_0}, m_{\chi_0}) \right\} \] (E.13)

### F On-shell results

For completeness, in this Appendix we demonstrate the results for the divergent parts of the two- three- and four-point one-loop functions calculated on-shell, in both \( R_\xi \) and Unitary gauges. We start with the case of \( R_\xi \) gauge where we will demonstrate the results for the two-, three- and four-point functions calculated at on-shell, i.e. at \( p^2 = m^2_Z \) for the \( Z \)-mass and at \( p^2_i = m^2_{H_i} \) for everything else. Tadpoles are external momentum independent objects.
• Vacuum polarization of the $Z$-boson

$\begin{align*}
\text{Graph} & = [M^R_Z]_\epsilon + \{M^R_Z\}_\epsilon + [M^R_Z]_f + \{M^R_Z\}_f \\
\end{align*}$

with

$\begin{align*}
\epsilon[M^R_Z]_\epsilon & = \frac{40 \lambda_0 m^4_{Z_0}}{3 m^2_{H_0}}, \\
\{M^R_Z\}_\epsilon & = 0
\end{align*}$ \quad (F.1)

and

$\begin{align*}
\epsilon[\delta A^R]_\epsilon & = \frac{4}{3} \lambda_0 \frac{m^2_{Z_0}}{m^2_{H_0}}, \\
\{\delta A^R\}_\epsilon & = 0
\end{align*}$ \quad (F.2)

• One-loop corrections to the Higgs propagator

$\begin{align*}
\text{Graph} & = [M^R_H]_\epsilon + \{M^R_H\}_\epsilon + [M^R_H]_f + \{M^R_H\}_f \\
\end{align*}$

with

$\begin{align*}
\epsilon[M^R_H]_\epsilon & = 26 \lambda_0 m^2_{H_0} - 12 \lambda_0 m^2_{Z_0} + 36 \frac{\lambda_0 m^4_{Z_0}}{m^2_{H_0}}, \\
\epsilon[\{M^R_H\}_\epsilon] & = 12 \frac{\lambda_0 m^4_{Z_0}}{m^2_{H_0}} \epsilon^2
\end{align*}$ \quad (F.3)

and

$\begin{align*}
\epsilon[\delta \phi^R]_\epsilon & = -4 \lambda_0 + 12 \lambda_0 \frac{m^2_{Z_0}}{m^2_{H_0}}, \\
\{\delta \phi^R\}_\epsilon & = 0
\end{align*}$ \quad (F.4)

• One-loop corrections to the Higgs three-point vertex

$\begin{align*}
\text{Graph} & = [K^R_H]_\epsilon + \{K^R_H\}_\epsilon + [K^R_H]_f + \{K^R_H\}_f \\
\end{align*}$

where

$\begin{align*}
\epsilon[K^R_H]_\epsilon & = \epsilon[K^R_H, \text{red.}, + K^R_H, \text{irred.}]_\epsilon = \frac{m_{H_0}}{2 \lambda_0} \left( 84 \lambda_0^2 + 144 \frac{\lambda_0^2 m^4_{Z_0}}{m^4_{H_0}} \right) \\
\{K^R_H\}_\epsilon & = \{K^R_H, \text{red.}, + K^R_H, \text{irred.}\}_\epsilon = 0.
\end{align*}$ \quad (F.5)

• One-loop corrections to Higgs quartic coupling

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with
\[ \varepsilon [B_{R\xi}^e]_\varepsilon = 252\lambda_0^2 + 144\frac{\lambda_0^2 m_{Z_0}^4}{m_{H_0}^4}, \quad \{B_{R\xi}^e\}_\varepsilon = 0. \] (F.6)

Next we list on-shell results for the Unitary gauge.

• Vacuum polarization of the $Z$-boson

\[ = [M_Z^U]_\varepsilon + \{M_Z^U\}_\varepsilon + \{M_Z^U\}_f + \{M_Z^U\}_f \]

with
\[ \varepsilon [M_Z^U]_\varepsilon = \frac{40\lambda_0 m_{Z_0}^4}{3 m_{H_0}^2}, \quad \{M_Z^U\}_\varepsilon = 0 \] (F.7)

and
\[ \varepsilon [\delta A^U]_\varepsilon = \frac{4}{3} m_{Z_0}^2 \frac{m_{Z_0}^4}{m_{H_0}^2}, \quad \{\delta A^U\}_\varepsilon = 0 \] (F.8)

• One-loop corrections to the Higgs propagator

\[ = [M_H^U]_\varepsilon + \{M_H^U\}_\varepsilon + \{M_H^U\}_f + \{M_H^U\}_f \]

with
\[ \varepsilon [M_H^U]_\varepsilon = 26\lambda_0 m_{H_0}^2 - 12\lambda_0 m_{Z_0}^2 + 36\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2}, \quad \{M_H^U\}_\varepsilon = 12\frac{\lambda_0 m_{Z_0}^4}{m_{H_0}^2} \xi^2 \] (F.9)

and
\[ \varepsilon [\delta \phi^U]_\varepsilon = -4\lambda_0 + 12\lambda_0 m_{Z_0}^2 \frac{m_{Z_0}^4}{m_{H_0}^2}, \quad \{\delta \phi^U\}_\varepsilon = 0 \] (F.10)

• One-loop corrections to the Higgs three-point vertex
\[ K^U_H \varepsilon + \left\{ K^U_H \right\}_\varepsilon + [K^U_H]_f + \left\{ K^U_H \right\}_f \]

where

\[
\varepsilon[K^U_H]_\varepsilon = \varepsilon[K^U_H, \text{red.}] + \varepsilon[K^U_H, \text{irred.}]_\varepsilon = \frac{m_{H_0}}{\sqrt{2}\lambda_0} \left( 84\lambda_0^2 + 144\frac{\lambda_0^2 m_{Z_0}^4}{m_{H_0}^4} \right)
\]

\[
\left\{ K^U_H \right\}_\varepsilon = \left\{ K^U_H, \text{red.} + K^U_{H,U} \right\}_\varepsilon = 0.
\] (F.11)

- One-loop corrections to Higgs quartic coupling

\[ B^U_H \varepsilon + \left\{ B^U_H \right\}_\varepsilon + [B^U_H]_f + \left\{ B^U_H \right\}_f \]

with

\[ \varepsilon[B^U_H]_\varepsilon = 252\lambda_0^2 + 144\frac{\lambda_0^2 m_{Z_0}^4}{m_{H_0}^4}, \quad \left\{ B^U_H \right\}_\varepsilon = 0. \] (F.12)

We observe that the results of the one-loop diagrams in $R_\xi$ and in the Unitary gauge are the same.

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