Electron transport on a cylindrical surface with one-dimensional leads

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A nanodevice consisting of a conductive cylinder in an axial magnetic field with one-dimensional wires attached to its lateral surface is considered. An explicit form for transmission and reflection coefficients of the system as a function of electron energy is found from the first principles. The form and the position of transmission resonances and zeros are studied. It is found that, in the case of one wire being attached to the cylinder, reflection peaks occur at energies coinciding with the discrete part of the electronic spectrum of the cylinder. These peaks are split in a magnetic field. In the case of two wires the asymmetric Fano-type resonances are detected in the transmission between the wires for integer and half-integer values of the magnetic flux. The collapse of the resonances appears for certain position of contacts. Magnetic field splits transmission peaks and leads to spin polarization of transmitted electrons.

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1. INTRODUCTION

Electron transport in curved two-dimensional nanostructures attracts considerable attention in the last decade. Transport properties of the electron gas on spherical and cylindrical nanosurfaces are intensively studied in the last few years. Those systems are of particular interest due to recent intensive experimental investigations of the coherent transport in individual carbon nanotubes and GaAs/AlGaAs heterostructures. A number of theoretical works has been focused on the electron transport in carbon nanotubes (see Ref. for review). It should be mentioned that nanotube-based multiterminal nanodevices attract recently more and more attention since they are proposed as promising units for future low-power high-speed electronics. A lot of works are devoted to the investigation of the electron transport in various interesting multiterminal nanodevices.

The conductance is usually measured for two basic geometries of contacts. Most of theoretical studies are focused on the case of end-contacted nanotubes. In this geometry a strong interaction between metal and carbon atoms is realized resulting in low contact resistance. However in the last few years much attention is devoted to side-contacted nanotubes. In this case the leads are attached to the lateral surface of the tube. The interest to these structures is stipulated by recent experiments on scanned probe microscopy of electronic transport in the nanotubes. The tip of the atomic force microscope can play the role of the side-contacted lead. Another interesting system with laterally attached leads is branched 'nanotree' reported in Ref. We mention that the side-contacted geometry is also realized in crossed carbon nanotubes. It is evident that the lateral disposition of the contacts can significantly affect the transport. In particular, the resonant transport regime is expected in this case.

Recent experiments on the transport in carbon nanotubes have reported the presence of asymmetric Fano resonances in the dependence of conductance on the Fermi energy. Being a characteristic manifestation of wave phenomena in scattering experiments resonances have received considerable attention in recent electron transport investigations. A number of papers devoted to the study of Fano resonances in the transport through quasi-one-dimensional channels with impurities. It is shown in Refs. that the same resonances occur in the conductance through a quantum nanosphere and a quantum nanotorus. Similar phenomena could be expected in the electron transport through the quantum cylinder but our analysis shows that the form of resonances differs from the Fano line shape.

The purpose of the present paper is an investigation of the electron transport through a multiterminal nanodevice consisting of a conductive cylindrical surface with one-dimensional wires attached to it. The cylinder is placed in an axial magnetic field and the wires are attached to its lateral surface. The number of wires we denote by \( N \). We consider in detail the case of one and two wires attached to the cylinder. The points of contacts on the cylinder we denote by \( \mathbf{q}_j = (z_j, \varphi_j) \), where \( z_j \) and \( \varphi_j \) are cylindrical coordinates and \( j = 1, \ldots, N \) is the number of the contact.

In our model, the electron on the cylinder is able to go away from the contact region to infinity and never returns back. We stress, that the model is valid for a realistic finite-size cylinder if its bases are immersed into absorbing electron reservoirs as shown in Fig. 1.
2. Hamiltonian and Transmission Coefficient

In the model, the wires are taken to be one-dimensional and represented by semiaxes \( \mathbb{R}_j^+ = \{ x : x \geq 0 \} \) \((j = 1 \ldots N)\). They are connected to the cylinder by gluing the point \( x = 0 \) from \( \mathbb{R}_j^+ \) to the point \( q_j \) from \( C \). We suppose \( q_i \neq q_j \) for \( i \neq j \). The scheme of the device is shown in Fig. 1.

If spin-orbital interaction is absent then spin orientation conserves and transmission coefficients \( T^\uparrow(E) \) and \( T^\downarrow(E) \) for electrons polarized in the direction of the magnetic field and in the opposite direction may be expressed in terms of the transmission coefficient \( T(E) \) for spin-free scattering

\[
T^\uparrow(E) = T(E - g \mu_B B/2), \quad T^\downarrow(E) = T(E + g \mu_B B/2),
\]

where \( g \) is electron g-factor and \( \mu_B \) is the Bohr magneton. Similar relations are valid for reflection coefficients.

Further, we will deal with the spin-free problem and use spin indices only where it is necessary.

![Diagram of the device](image)

**FIG. 1:** Scheme of the device in the case of two wires attached to the cylinder. An incident wave (IW) originating from reservoir 1 is reflected back with amplitude \( r_{11} \) and scattered to reservoir 2 with amplitude \( t_{21} \). Reservoirs 3 and 4 absorb the electron waves going away from the contact region.

A wavefunction \( \psi \) of the electron in the device consists of \( N + 1 \) parts:

\[
\psi = \begin{pmatrix} \psi_C \\ \psi_1 \\ \vdots \\ \psi_N \end{pmatrix},
\]

where \( \psi_C \) is a function on \( C \) and \( \psi_j \) \((j = 1 \ldots N)\) are functions on \( \mathbb{R}_j^+ \).

To obtain the Hamiltonian \( H \) of the whole system we use an approach based on the operator extension theory\(^{30,40}\). This method has been already used in Refs. 3\&4 for the investigation of the electron transport through the nanosphere and the nanotorus.

The Hamiltonian \( H \) of the whole system is a point perturbation of the operator

\[
H_0 = H_C \oplus H_1 \oplus \ldots \oplus H_N,
\]

where \( H_C \) is an electron Hamiltonian on the cylinder and \( H_j \) are Hamiltonians in the wires \( \mathbb{R}_j^+ \).

Using cylindrical coordinates, we can represent the Hamiltonian \( H_C \) in the form

\[
H_C = \frac{\hbar^2}{2m_c} \frac{\partial^2}{\partial \varphi^2} + \frac{\hbar^2}{2m_r} \left( i \frac{\partial}{\partial \varphi} - \Phi/\Phi_0 \right)^2,
\]

where \( p_z \) is the z-component of the momentum, \( r \) is the radius of the cylinder, \( m_c \) is the electron effective mass on the cylinder, \( \Phi = \pi r^2 B \) is the magnetic flux, and \( \Phi_0 = 2\pi \hbar c/|e| \) is the magnetic flux quantum. It is convenient to represent the Hamiltonian \( H_C \) in the form \( H_C = H_z + H_\varphi \) where \( H_z = p_z^2/2m_c \) and \( H_\varphi = \frac{\hbar^2}{2m_r} (\frac{\partial^2}{\partial \varphi^2} - \Phi/\Phi_0)^2 \). We will need below the eigenvalues

\[
E_m = \frac{\hbar^2}{2m_r} \left( m + \frac{\Phi}{\Phi_0} \right)^2
\]

and the eigenfunctions

\[
\Psi_m(\varphi) = (2\pi r)^{-1/2} \exp(i m \varphi)
\]

of the operator \( H_\varphi \).

Electron motion in each wire \( \mathbb{R}_j^+ \) is described by the Hamiltonian \( H_j = p_z^2/2m_w \), where \( p_z \) is the momentum operator and \( m_w \) is the effective mass for the electron in the wires.

To define the Hamiltonian \( H \) we use boundary conditions at points of gluing. The role of boundary values for the wavefunctions \( \psi_j \) is played, as usual, by \( \psi_j(0) \) and \( \psi_j'(0) \). The zero-range potential theory shows that to obtain a non-trivial Hamiltonian for the whole system we must consider functions \( \psi_C \) with a logarithmic singularity at the points of gluing \( q_j \)

\[
\psi_C(x) = -u_j \frac{m_w}{\pi \hbar^2} \ln \rho(x, q_j) + v_j + o(1), \quad x \to q_j.
\]

Here \( u_j \) and \( v_j \) are complex coefficients and \( \rho(x, q_j) \) is the geodesic distance on the cylinder between the points \( x \) and \( q_j \). It is known, that the most general self-adjoint boundary conditions are defined by some linear relations between \( \psi_j(0) \), \( \psi_j'(0) \), and the coefficients \( u_j \) and \( v_j \). Following Ref. 3\&4, we will write these conditions in the form

\[
\begin{aligned}
\psi_j(0) &= \sum_{k=1}^N \left[ A_{jk} u_k - \frac{\hbar^2}{2m_w} C_{jk} \psi_k'(0) \right], \\
v_j &= \sum_{k=1}^N \left[ B_{jk} u_k - \frac{\hbar^2}{2m_w} A_{jk} \psi_k'(0) \right],
\end{aligned}
\]

Here complex parameters \( A_{jk}, B_{jk}, \) and \( C_{jk} \) are the elements of \( N \times N \) matrices. The matrices \( B \) and \( C \) have to
be Hermitian because the Hamiltonian $H$ is a self-adjoint operator. To avoid a non-local tunnelling coupling between different contact points we will restrict ourselves to the case of diagonal matrices $A$, $B$, and $C$ only. According to the zero-range potential theory diagonal elements of the matrix $B$ determine the strength of point perturbations of the Hamiltonian $H_C$ at the points $q_j$ on the cylinder. These elements may be expressed in terms of scattering lengths $\lambda_j^B$ on the corresponding point perturbations: $B_{jj} = m_c \ln(\lambda_j^B)/\hbar^2$. Similarly elements $C_{jj}$ describe the strength of point perturbations at the point $x = 0$ in the wires and may be expressed in terms of scattering lengths $\lambda_j^C$ by the relation $C_{jj} = -m_w \lambda_j^C/2\hbar^2$.

For convenience, we represent parameters $A_{jj}$ in the form $A_{jj} = m_w \sqrt{\lambda_j^A} e^{i\phi_j}/\hbar^2$, where $\lambda_j^A$ has the dimension of length and $\phi_j$ is the argument of the complex number $\lambda_j^A$. We mention that the effect of the scattering lengths $\lambda_j^A$, $\lambda_j^B$, and $\lambda_j^C$ on the electron transport has been discussed in Ref. 4. In the present paper we concentrate our attention on phenomena which are independent of contact parameters.

To obtain transmission and reflection coefficients of the system one needs a solution of the Schrödinger equation for the Hamiltonian $H$. The function $\psi_1$ in this solution is a superposition of incident and reflected waves while other functions $\psi_j$ ($j = 2, \ldots, N$) represent scattered waves. The wavefunction $\psi_C$ may be expressed in terms of the Green function $G(x, x'; E)$ for the Hamiltonian $H_C$.

\[
\begin{aligned}
\psi_C(x) &= \sum_{j=1}^{N} \xi_j(E) G(x, q_j; E), \\
\psi_j(x) &= \delta_{j1} e^{-ikx} + S_j(E) e^{ikx}, \quad j = 1, \ldots, N.
\end{aligned}
\]

Here $k = \sqrt{2m_w E/\hbar^2}$ is the electron wave vector in wires, $\xi_j(E)$ are complex factors, and $S_j(E)$ is the amplitude of the outgoing wave in the wire $\mathbb{R}^+_t$.

It is well known that the Green function $G(x, x'; E)$ may be represented in the form

\[
G(x, x'; E) = \sum_{m = -\infty}^{\infty} G_z(z, z'; E - E_m) \Psi_m(\varphi) \Psi^*_m(\varphi'),
\]

where $x = (z, \varphi)$, $x' = (z', \varphi')$, and

\[
G_z(z, z'; E) = \frac{im_c}{\hbar^2 k} e^{ik|z-z'|}
\]

is the Green function of the operator $H_z$. Substituting (10) into (9), we get the following equation for $G(x, x'; E)$:

\[
G(x, x'; E) = \frac{im_c}{2\hbar^2} \sum_{m = -\infty}^{\infty} \frac{e^{ik_m|z-z'|+im(\varphi-\varphi')}}{k_mr},
\]

where $\hbar^2 k_m^2 = 2m_c(E - E_m)$, $\Re k_m > 0$ for $E > E_m$ and $\Im k_m > 0$ for $E < E_m$.

Considering the asymptotics (8) of $\psi_C(x)$ from (8) near the point $q_j$, we have

\[
u_j = \xi_j, \quad \nu_j = \sum_{i=1}^{N} Q_{ji}(E) \xi_i.
\]

Here $Q_{ij}(E)$ is the Krein’s $Q$-function, that is $N \times N$ matrix with elements

\[
Q_{ij} = \begin{cases}
G(q_i, q_j; E), & i \neq j; \\
\lim_{x \to q_i} \left(G(q_i, x; E) + \frac{m_c}{\pi \hbar^2} \ln \rho(q_i, x)\right), & i = j.
\end{cases}
\]

Using the elementary relation

\[
\sum_{n=1}^{\infty} \frac{\exp(-nx)}{n} = -\ln(1 - e^{-x}),
\]

we can subtract the logarithmic singularity from $G(x, x'; E)$ and get the following form for diagonal elements of $Q$-matrix

\[
Q_{jj} = \frac{m_c}{2\hbar^2} \left[\frac{i}{k_0r} + \sum_{m=1}^{\infty} \left(\frac{i}{k_mr} + \frac{i}{k_{-m}r} - \frac{2}{m} + 2 \ln m\right)\right],
\]

(13)

The similar method has been used in Ref. 41 for calculating the $Q$-function for electron Hamiltonian on a strip. It should be mentioned that Eq. (13) gives the $Q$-function for the free particle on a plain in the case of $B = 0$ and $r \to \infty$.

Let us consider the asymptotics of $\psi_C(x)$ at $z \to \pm \infty$. As it follows from (8) and (11) the wavefunction $\psi_C(x)$ is a superposition of propagating modes

\[
\tilde{\Psi}_m^\pm(\varphi, z) = \Psi_m(\varphi) \exp(\pm ik_m z).
\]

The highest and lowest numbers of the occupied modes we denote by $M^\pm = \pm \pi k r - \Phi/\Phi_0$, where $|x|$ means the integer part of $x$. Using equations (8) and (11), we obtain

\[
\psi_C(x) \sim \sum_{m=M^-}^{M^+} t_m^\pm \tilde{\Psi}_m^\pm(\varphi, z),
\]

(14)

where the sign ‘plus’ corresponds to $z \to +\infty$ and the sign ‘minus’ should be taken for $z \to -\infty$. Here $t_m^\pm$ is the partial transmission amplitude to the mode $\tilde{\Psi}_m^\pm(\varphi, z)$. As follows from (8), the amplitude is given by

\[
t_m^\pm = \frac{i}{\sqrt{2\pi \hbar k_m}} \sum_{j=1}^{N} \xi_j e^{\mp ik_m z_j - im\varphi_j},
\]

(15)

where $\xi_j = \xi_j m_w/\hbar^2$.

Denote the reflection coefficient to the wire $\mathbb{R}^+_j$ by $R_j = |S_j|^2$ and the transmission coefficient by $T_j = |S_j|^2$. The partial transmission coefficient $t_m^\pm$ to the
propagating mode \( \tilde{\Psi}_m^\pm \) is defined by \( T_m^\pm = (k_m/k)|t_m^\pm|^2 \).
We stress that the relation
\[
R_{11} + \sum_{j=2}^N T_{j1} + \sum_{m=M}^M (T_m^+ + T_m^-) = 1
\]
(16)
is valid for an arbitrary energy \( E \) that is the manifestation of the current conservation law for our system.

Substituting (5) into (7), we get a system of \( 2N \) linear equations for \( S_j \) and \( \xi_j \)
\[
\begin{aligned}
\sum_{i=1}^N Q_{ji} \xi_i &= B_{ji} \xi_j - \frac{-ik^2 A_{ji}}{2m_0} (S_j - \delta_{ji}) \\
S_j + \delta_{ji} &= A_{ji}^s \xi_j - \frac{4k^2 G_{ji}}{2m_0} (S_j - \delta_{ji})
\end{aligned}
\]
(17)
For convenience, we introduce the dimensionless elements of \( Q \)-matrix
\[
\tilde{Q}_{ij}(E) = (\hbar^2/m_0)(Q_{ij}(E) - B_{ij}).
\]
System (17) may be decomposed to a system of \( N \) equations for \( \xi_l \)
\[
\sum_{l=1}^N \frac{-2k \lambda_l^C \delta_{jl}}{k \lambda_l^C + 4i} \xi_l = -\frac{4k \sqrt{\lambda_l^C} e^{i\phi_l}}{k \lambda_l^C + 4i} \delta_{jl}
\]
(18)
and a similar system for \( S_l \)
\[
\sum_{l=1}^N \frac{\sqrt{\lambda_l^A(k \lambda_l^C + 4i)} e^{i\phi_l}}{\sqrt{\lambda_l^A(k \lambda_l^C - 4i)} e^{i\phi_l}} \left[ \tilde{Q}_{jl} - \frac{2k \lambda_l^A \delta_{jl}}{k \lambda_l^C + 4i} \right] S_l = \tilde{Q}_{jl} - \frac{2k \lambda_l^A \delta_{jl}}{k \lambda_l^C - 4i}.
\]
(19)
The solutions of systems (18) and (19) may be represented in the form
\[
\xi_n = \frac{\Delta_{\xi n}}{\Delta}, \quad S_n = \frac{\Delta_{Sn}}{\Delta} \frac{\sqrt{\lambda_n^A(k \lambda_n^C - 4i)} e^{i\phi_n}}{\sqrt{\lambda_n^A(k \lambda_n^C + 4i)} e^{i\phi_n}}.
\]
(20)
where
\[
\Delta = \text{det} \left[ \tilde{Q}_{jl} - \frac{2k \lambda_j^A \delta_{jl}}{k \lambda_j^C + 4i} \right],
\]
(21)
\[
\Delta_{\xi n} = \text{det} \left[ \left( \tilde{Q}_{jl} - \frac{2k \lambda_j^A \delta_{jl}}{k \lambda_j^C + 4i} \right) (1 - \delta_{nl}) + \frac{4k \sqrt{\lambda_j^A} e^{i\phi_l}}{k \lambda_j^C + 4i} \delta_{jl} \delta_{nl} \right],
\]
(22)
and
\[
\Delta_{Sn} = \text{det} \left[ \left( \tilde{Q}_{jl} - \frac{2k \lambda_j^A \delta_{jl}}{k \lambda_j^C + 4i} \right) (1 - \delta_{nl}) + \left( \tilde{Q}_{jl} - \frac{2k \lambda_j^A \delta_{jl}}{k \lambda_j^C - 4i} \right) \delta_{nl} \right].
\]
(23)

3. RESULTS AND DISCUSSION

Let us consider in detail the case of one wire attached to the cylinder. Using equations (15) and (20), we obtain
\[
T_m^\pm = \frac{8k \lambda_m^A}{\pi r k_m |2k \lambda_m^A - (k \lambda_m^C + 4i) \tilde{Q}_{11}|^2}.
\]
(24)
Note that \( T_m^+ = T_m^- \) for any energy \( E \), i.e. the scattering is isotropic in the \( z \)-direction. Reflection amplitude \( r_{11} \equiv S_1 \) may be obtained from Eq. (19)
\[
r_{11} = \frac{(k \lambda_1^C - 4i) \tilde{Q}_{11} - 2k \lambda_1^A}{(k \lambda_1^C + 4i) \tilde{Q}_{11} - 2k \lambda_1^A}.
\]
(25)

FIG. 2: Reflection coefficient as a function of the electron energy in the case of one wire attached to the cylinder at \( \lambda_1^A = \lambda_1^B = \lambda_1^C = 0.4r \), \( B = 0 \).

FIG. 3: Reflection coefficient as a function of the dimensionless parameter \( kr \). All parameters are the same as in Fig. 2

The reflection coefficient \( R_{11} = |r_{11}|^2 \) as a function of the electron energy \( E \) is represented in Fig. 2 Hereafter we use \( \varepsilon = \hbar^2/(2m_0r^2) \) for the unit of energy and suppose
The figure shows that the dependence of reflection coefficient on $E$ contains a series of sharp peaks at the points $E_m$. To study the behavior of the reflection coefficient in a vicinity of the eigenvalues $E_m$ we consider the asymptotics of $\tilde{Q}_{jl}(E)$ near these points

$$\tilde{Q}_{jl}(E) \simeq \alpha^{(m)}_{jl}/k_m + \beta^{(m)}_{jl} + O(k_m), \quad \text{as } k_m \to 0,$$

where

$$\alpha^{(m)}_{jl} = i \sum_{m'} \Psi^{*}_{m'}(\varphi_j)\Psi_{m'}(\varphi_l)$$

and $m'$ are indices for which $E_m' = E_m$. If the magnetic flux $\Phi/\Phi_0$ is integer or half-integer then the eigenvalues $E_m$ are double-degenerated and the sum in Eq. (27) contains two terms; otherwise it contains one term only.

As follows from Eq. (26), the denominator in Eq. (24) has a root singularity at $E = E_m$ while the numerator remains finite. Therefore all transmission coefficients $T_{m}$ vanish and the reflection coefficient $R_{11}$ reaches a unity. The reflection coefficient has a kink in a vicinity of each point $E_m$ stipulated by the root singularity of the Green function on the cylinder. Using equations (25) and (26), we obtain the following asymptotics for $R_{11}(k)$ near $\kappa_m = \sqrt{2m }/E_m$.

$$R_{11}(k) = \begin{cases} 1 - a_1(\kappa_m - k) + o(k_m^2), & \text{as } k \to \kappa_m - 0 \\ 1 - a_2\sqrt{k^2 - \kappa_m^2} + o(k_m), & \text{as } k \to \kappa_m + 0, \end{cases}$$

where $a_1$ and $a_2$ are positive numbers. The form of the reflection coefficient in a vicinity of the point $E_3$ is shown in Fig. 3

The magnetic field splits double degenerated energy levels of the operator $H_e$ and peaks on the plot $R_{11}(kr)$ transform into doublets (Fig. 4). If the magnetic flux $\Phi/\Phi_0$ is half-integer then the levels $E_m$ are double-degenerated and the peaks are singlet as for the case of integer flux, although their positions are shifted.

The reflection coefficient as a function of the magnetic field is shown in Fig. 5. Peaks on the plot correspond to the coincidence of the electron energy with the values $E_m$. Note that the function $R_{11}(\Phi)$ is periodic with a period $\Phi_0$ that causes the Aharonov–Bohm oscillations in the transport. If the value of $kr$ is integer or half-integer then there is only one peak of $R_{11}(\Phi)$ on the period, otherwise there are two peaks on each cycle.

Let us turn to the case of two wires attached to the cylinder. Using Eq. (23), we obtain

$$t_{21} = \frac{16i k \sqrt{\lambda_1^A \lambda_2^A} e^{i(\varphi_1 - \varphi_2)} \tilde{Q}_{21}}{(k \lambda_1^C + 4i)(k \lambda_2^C + 4i) \Delta}, \quad \text{Eq. (28)}$$

where

$$\Delta = \det \tilde{Q} - \frac{2k\lambda_1^A}{k \lambda_1^C + 4i} \tilde{Q}_{22} - \frac{2k\lambda_2^A}{k \lambda_2^C + 4i} \tilde{Q}_{11} + \frac{4k^2 \lambda_1^A \lambda_2^A}{(k \lambda_1^C + 4i)(k \lambda_2^C + 4i)}, \quad \text{Eq. (29)}$$

The transmission coefficients $T_{21} = |t_{21}|^2$ as a function of the electron energy is shown in Fig. 6. The figure corresponds to the case when the contacts are placed on different generatrices ($\varphi_1 \neq \varphi_2$) and shifted along the axis of the cylinder ($z_1 \neq z_2$) in the zero magnetic field. One can see a series of zeros at $E = E_m$. Transmission coefficient has a peak in the neighborhood of each zero. The behavior of the transmission amplitude in a vicinity of the eigenvalues $E_m$ depends strongly on contact position and applied magnetic field. If the points $q_i$ are placed on the cylinder in a random manner then the transmission coefficient vanishes at the double-degenerated values $E_m$. The denominator in Eq. (28) has a pole at $E = E_m$ while the numerator has a root singularity only. Hence, the transmission coefficient vanishes in these points.

Let us consider in detail the form of the transmission coefficient in the vicinity of $E_m$. Using the asymptotic expression (26) for $\tilde{Q}_{jl}(E)$, we obtain the following

FIG. 4: Reflection coefficient at $\Phi/\Phi_0 = 0.1$. Other parameters are the same as in Fig. 2.

FIG. 5: Reflection coefficient as a function of the magnetic field. Solid line: $k = 4/r$; dash line: $k = 4.2/r$. 

FIG. 6: Transmission coefficient $T_{21}$ as a function of the electron energy.
representation for \( t_{21}(k) \)

\[
t_{21}(k) \simeq c_m \frac{k_m}{f_m + k_m}, \quad (30)
\]
as \( E \to E_m \). Here \( c_m \) is a normalization factor and

\[
f_m = \frac{\det \alpha^{(m)}}{\gamma_m} \quad (31)
\]
is a complex number with

\[
\gamma_m = \alpha_{11}^{(m)} \left( \beta_{22} - \frac{2 \kappa_m \lambda_{2}^A}{(\kappa_m \lambda_{2}^A)^2 + 16} \right) + \\
\alpha_{22}^{(m)} \left( \beta_{11} - \frac{2 \kappa_m \lambda_{1}^B}{(\kappa_m \lambda_{1}^B)^2 + 16} \right) - \\
\alpha_{12}^{(m)} \beta_{21} - \alpha_{21}^{(m)} \beta_{12}.
\]

One can see, that the behavior of transmission coefficient in the vicinity of \( E_m \) is determined by the value \( f_m \). Two curves for different positions of the contacts corresponding to different \( f_m \) are represented in Fig. 7. If \( f_m \to 0 \) then the transmission coefficient has a peak in a vicinity of the zero \( k = \kappa_p \). The distance between the peak and the zero decreases with decreasing of \(|f_m|\) while the peak value remains finite. The form of the graph in this region (dash line in Fig. 7) resembles the form of the Fano resonance, but it is significant that the Fano curve is smooth in contrast to function (30). If \( f_m = 0 \) then the peak and the zero of transmission coincide and cancel each other (solid line in Fig. 7). We note that \( f_m \) equals zero only if \( \det \alpha^{(m)} = 0 \) as it follows from Eq. (31). Therefore the form of the transmission coefficient in the vicinity of \( E_m \) is determined by the degree of degeneracy of \( E_m \) and by the symmetry of contact location. In particular \( \det \alpha^{(m)} = 0 \) for all positions of contacts if the eigenvalue \( E_m \) is non-degenerated as it follows from Eq. (27). Therefore the zeros do not appear in a magnetic field with non-integer value of \( 2\Phi/\Phi_0 \). The magnetic field splits double degenerated energy levels \( E_m \) and removes transmission zeros. The dependence \( T_{21}(E) \) for non-integer value of magnetic flux is represented in Fig. 8. One can see that the peaks on the plot \( T_{21}(E) \) transform into doublets.

The value \( \det \alpha \) for integer \( 2\Phi/\Phi_0 \) is given by

\[
\det \alpha^{(m)} = -(\pi r)^{-2} \sin^2 \left((m + \Phi/\Phi_0)(\varphi_2 - \varphi_1)\right). \quad (32)
\]
If \( \sin((m + \Phi/\Phi_0)(\varphi_2 - \varphi_1)) = 0 \) then the value \( f_m \) vanishes and the zero at the point \( E_m \) disappears. This phenomenon is similar to the collapse of the Fano resonance in the transmission through a quantum sphere. The disappearance of the zeros is associated with the symmetry of the contact location. We note that all zeros disappear if the points \( q_1 \) and \( q_2 \) are placed on the same generatrix \( (\varphi_1 = \varphi_2) \) or on the opposite generatrices of the cylinder \((|\varphi_1 - \varphi_2| = \pi)\). It is significant that the

FIG. 6: Transmission coefficient \( T_{21} \) as a function of the electron energy at \( \varphi_1 - \varphi_2 = 0.08\pi \), \( z_1 - z_2 = 0.2r \), and \( \lambda_{2}^A = \lambda_{2}^B = \lambda_{2}^C = 0.4r \).

FIG. 7: Transmission coefficient \( T_{21} \) at \( B = 0 \), \( z_1 - z_2 = 0.1r \), \( \lambda_{2}^A = \lambda_{2}^B = \lambda_{2}^C = 0.4r \). Solid line: \( \varphi_1 - \varphi_2 = 0 \); dash line: \( \varphi_1 - \varphi_2 = 0.08\pi \).

FIG. 8: Transmission coefficient \( T_{21} \) at \( \lambda_{2}^A = \lambda_{2}^B = \lambda_{2}^C = 0.4r \), \( z_1 - z_2 = 0.05r \), \( \varphi_1 - \varphi_2 = 0.05\pi \). Solid line: \( B = 0 \); dash line: \( \Phi/\Phi_0 = 0.1 \).
positions of all zeros are independent of the scattering lengths $\lambda_A^i$, $\lambda_B^j$, and $\lambda_C^l$.

In the case of $f_m = 0$ the transmission coefficient $T_{21}$ may be represented near $E_m$ in the form

$$T_{21}(k) \simeq |c_m|^2|1 + g_m k_m|^2,$$

where $c_m$ is a normalization factor and $g_m$ is a complex number depending on the position of contacts and scattering lengths. The smoothness of the curve $T_{21}(E)$ at the point $E = E_m$ is determined by the number $g_m$. Indeed, the left-hand derivative is infinite in this point if $\text{Im} \ g_m \neq 0$ and the right-hand derivative is infinite for $\text{Re} \ g_m \neq 0$. If the one-sided derivatives are different then the transmission coefficient has a kink at the point $E_m$. The solid line in Fig. 7 corresponds to the case of infinite derivatives.

According to Landauer–Büttiker formalism the ballistic conductance $G$ of the device at the zero temperature is determined by transmission probabilities $T_{21}^e$ and $T_{21}^h$

$$G = G_0 (T_{21}^e + T_{21}^h),$$

where $G_0 = e^2/(2\pi\hbar)$ is the conductance quantum. The conductance as a function of the electron energy is shown in Fig. 9. The dependence of the transmission coefficient on electron spin orientation in the magnetic field results in additional splitting of conductance peaks (Fig. 9) and partial spin polarization of transmitted electrons. It should be mentioned that the spin splitting $g \mu_B B$ is independent of magnetic quantum number $m$ while the splitting of the eigenvalues $E_m$ is proportional to $m$ hence the peaks are not equidistant. It is essential that the spin polarization can be changed either by magnetic field or by electron energy. The complete polarization is possible for integer and half-integer values of magnetic flux $\Phi/\Phi_0$.

The conductance oscillates as a function of energy with the period

$$\Delta E \simeq \frac{\pi}{L} \frac{dE}{dk_m} = \frac{\pi \hbar^2 k_m}{4 m_c L},$$

if the longitudinal distance $L = |z_1 - z_2|$ between the points $q_1$ and $q_2$ is much larger than the radius $r$ (see Fig. 10). The oscillations are stipulated by the interference of electron waves on the cylinder. It should be noted that similar oscillations have been observed in recent experiments with carbon nanotubes. The geometry of the experiment differs from ours, in particular, contacts are not point-like. But some results are valid for our system as well, in particular, the estimation for the period of oscillations cared out in Ref. 5 is in agreement with Eq. (34).

4. CONCLUSION

Electron transport in a nanodevice consisting of a conductive cylinder with one-dimensional wires connected to its lateral surface is considered. The one-particle Hamiltonian of the system is obtained using linear boundary conditions at the points of contact. An explicit form for transmission and reflection coefficient as a function of electron energy is found solving the Schrödinger equation. The general case of arbitrary number of wires and arbitrary disposition of contacts is considered. Two cases corresponding to a single wire and two wires attached to the cylinder surface are studied in detail. It is found that reflection peaks occur at energies coinciding with the discrete part $E_m$ of the electron spectrum on the cylinder. The form of reflection peaks is discussed.

A similar analysis of the two-wire case shows that the transmission coefficient equals zero at energies $E_m$. We have found that asymmetric Fano-type resonances appear in a vicinity of the zeros. The zeros exist only if
the number of magnetic flux quanta through the cylinder is integer or half-integer. They exist for all positions of contacts $q_1$ and $q_2$ except some specific points. It is shown that the zero at the point $E_m$ disappears if the value det $\alpha^{(m)}$ defined by Eq. (32) vanishes. The behavior of the transmission coefficient in this case resembles the collapse of the Fano resonances discussed in earlier studies.

The conductance of the device is investigated using Landauer–Buttiker formalism. The resonances in transmission coefficient lead to appearance of conductance oscillations. The magnetic field split conductance peaks and cause spin polarization of transmitted electrons. The complete spin polarization is possible for integer and half-integer values of the magnetic flux.

The results of the paper may be useful for the study of electron transport in single-wall carbon nanotubes and rolled GaAs/AlGaAs heterostructures. The experimental observation of the discussed effects should become possible involving leads thin enough, like the tip of the scanning tunnel microscope. The geometry of the device in the case of one wire resembles the geometry of experiments on scanned probe microscopy of carbon nanotubes. Experimental setup using two tips on the same nanotube seems in principle feasible, although perhaps difficult to realize. In the case of multi-mode leads the interference of electron waves from different modes will most probably result in additional transmission peaks and minima. We stress that most of the obtained results reflect the intrinsic properties of electron motion on the cylinder. Therefore they are expected to remain valid qualitatively even in the case of realistic non-one-dimensional wires.

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