Peccei-Quinn Symmetric Pure Gravity Mediation

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Abstract

Successful models of Pure Gravity Mediation (PGM) with radiative electroweak symmetry breaking can be expressed with as few as two free parameters which can be taken as the gravitino mass and $\tan\beta$. These models easily support a 125-126 GeV Higgs mass at the expense of a scalar spectrum in the multi-TeV range and a much lighter wino as the lightest supersymmetric particle. In these models, it is also quite generic that the Higgs mixing mass parameter, $\mu$, which is determined by the minimization of the Higgs potential is also in the multi-TeV range. For $\mu > 0$, the thermal relic density of winos is too small to account for the dark matter. The same is true for $\mu < 0$ unless the gravitino mass is of order 500 TeV. Here, we consider the origin of a multi-TeV $\mu$ parameter arising from the breakdown of a Peccei-Quinn (PQ) symmetry. A coupling of the PQ-symmetry breaking field, $P$, to the MSSM Higgs doublets, naturally leads to a value of $\mu \sim \langle P \rangle^2/M_P \sim \mathcal{O}(100)$ TeV and of order that is required in PGM models. In this case, axions make up the dark matter or some fraction of the dark matter with the remainder made up from thermal or non-thermal winos. We also provide solutions to the problem of isocurvature fluctuations with axion dark matter in this context.
1 Introduction

The mass of the Higgs boson $[1]$ at around 126 GeV is near the upper limit of the predictions in commonly studied models of the supersymmetric standard model such as the constrained minimal supersymmetric standard model (CMSSM) $[2,3]$. This rather large Higgs boson mass and the null results of the sparticle searches at the LHC $[6,7]$ seem to hint to rather heavy sparticles $[8,9]$, and possibly heavy top squarks with masses in the range of tens to hundreds of TeV.

Among the models with heavy sparticles, models with a mild hierarchy between sfermion and gaugino masses such as pure gravity mediation (PGM) $[10–15]$ and models with strongly stabilized moduli $[16–18]$ are very successful not only phenomenologically but also cosmologically and have spectra which are characteristic of split supersymmetry $[19]$ with anomaly mediation $[20]$. In these models, the sfermions obtain tree-level masses of the order of the gravitino mass, $m_3/2$, while the gaugino masses are dominated by one-loop masses from anomaly mediation $[20]$.

In the original version of PGM $[10]$, it was assumed that the supersymmetric Higgs mixing term, the μ-term, is generated via the tree-level couplings to an $R$-symmetry breaking sector which generates the non-vanishing vacuum expectation value of the superpotential $[21]$. In this case, the μ term and the supersymmetry breaking bilinear $B$ term are two independent parameters and are both of order the gravitino mass, $m_3/2$. In previous papers, we showed that models based on PGM with $[13]$ and without $[14]$ scalar mass universality, could explain virtually all experimental constraints with successful radiative electroweak symmetry breaking (EWSB). In this case, one can impose the supergravity boundary condition $B_0 = A_0 - m_3/2$ which is essentially $-m_3/2$ since $A_0$ is also determined by anomalies and hence small compared with $m_3/2$. The universal scalar mass, $m_0$ is fixed by $m_3/2$. The μ term is determined by the minimization of the Higgs potential. The ratio of the Higgs vacuum expectation values (vevs) can be treated as a free parameter so long as a Giudice-Masiero coupling, $c_H$, is included in the Kähler potential, $K$. The value of $c_H \lesssim 1$ is then also fixed by the minimization of the Higgs potential.

In this paper, we discuss another version of PGM, in which the μ-term originates from the breaking of a Peccei-Quinn (PQ) symmetry $[22]$ via a dimension five operator $[23]$. The size of the μ-term is determined by the PQ-breaking scale, $f_{PQ}$, and hence, it can be related to the axion dark matter density. In practice, the μ term and $f_{PQ}$ are then determined by the EWSB boundary conditions at the weak or susy scale. As in $[13]$, the $B$-term is fixed to $-m_3/2$ at the input UV scale. However, because the Higgs fields carry PQ charge in this model, a Giudice-Masiero coupling is not allowed in $K$ and therefore some departure of scalar mass universality is required $[14]$. With $c_H = 0$, tan $\beta$ must also be determined by the EWSB boundary conditions at the weak or susy scale. We will explore a three parameter version of the PGM where the three free parameters are chosen to be the gravitino mass, $m_3/2$, and the two soft Higgs masses, $m_1$ and $m_2$.

We show further that successful phenomenological models can be constructed if only the

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1 The difficulty in obtaining a 126 GeV Higgs in the CMMSSM has been relaxed recently with the inclusion of higher order corrections to the Higgs mass calculation $[4,5]$
soft mass of the up-type Higgs, $m_2$, is non-universal. Thus the family of PQ symmetric PGM models can be expressed in terms of only two parameters, $m_{3/2}$ and $m_2$. In fact, viable solutions are possible with $m_1 = m_{3/2}$ and $m_2 = 0$. Thus, if the up-type Higgs multiplet is associated with a pseudo Nambu-Goldstone boson [24] or if the Higgs soft mass is protected by a (partial) no-scale structure [25] of the Kähler potential as discussed in [14], we are reduced to a theory with one single free parameter! Finally we will show that an alternative model for breaking the PQ-symmetry based on [26] may allow full universality to be restored.

As in previous studies of PGM, we also find that the lightest supersymmetric particle (LSP) is the neutral wino. In the $R$-parity conserving case, the model allows several dark matter scenarios, including one with pure wino dark matter with possible non-thermal sources such as the late time decay of the gravitino. As we will see, even if the wino contribution to the relic density is negligible (as is the case for the thermal contribution when $\mu > 0$), it is possible that axions make up the entire dark matter, or of course it is also possible that the dark matter is an axion-wino mixture. In the scenario where a significant portion of the dark matter is the axion, the wino can be much lighter than the 3 TeV as required by thermal dark matter constraints. In fact, it could have a mass below 1 TeV and fall within the reach of the LHC at 14 TeV [12]. While it is well known that models with axion dark matter generally overproduce isocurvature fluctuations [27], it is also known that the problem may be resolved if the axion decay constant takes on values during inflation which are large compared to the nominal low energy value [28]. We show several ways, this can be implemented in the models presented.

The organization of the paper is as follows. In section 2, we summarize PGM with universal and non-Universal Higgs masses. We then generalize the model to include a PQ-symmetry in which the $\mu$-term originates from PQ-symmetry breaking. In section 3, we briefly review the properties of axion dark matter. There, we also propose several solutions to the problem of isocurvature fluctuations including a novel model of dynamical PQ-breaking. In section 4, we demonstrate that this version of PGM achieves successful EWSB. Here, we will consider the case with two non-universal Higgs soft masses, that is the three parameter version of the model. We will see however, that the down-type Higgs soft mass may remain universal, and successful models are still obtained. We will further demonstrate that even in the special case where $m_2 = 0$, viable models are possible so long as $300 \text{ TeV} \lesssim m_{3/2} \lesssim 850 \text{ TeV}$. Before concluding, we will briefly describe in section 5 an alternative model for breaking the PQ-symmetry in which full universality may be restored. The final section is devoted to discussions.

2 $\mu$-term from PQ-symmetry breaking

Pure Gravity Mediation

In pure gravity mediation models, it is assumed that all fields in the supersymmetry (SUSY) breaking sector are charged under some symmetry (e.g. an R-symmetry). Under this assumption, the gaugino masses and the $A$-terms of the chiral multiplets are suppressed at the tree-level in supergravity and they are dominated by the anomaly mediated contribu-
The soft squared masses of the scalar bosons are, on the other hand, generated at the tree-level and are expected to be of order $m^2_3/2$ with a generic Kähler potential. If we optionally assume a flat Kähler manifold for all the MSSM fields, all of the MSSM scalar fields obtain the universal soft mass squared equal to $m^2_3/2$. In our analysis, we take the model with universal scalar masses as our starting point. We will assume universality at the grand unified (GUT) scale and run all quantities down to the weak scale through 2-loop RGEs and minimize the Higgs potential. As in the CMSSM, minimization allows us to solve for $\mu$ and the bilinear supersymmetry breaking $B$-term. As we will see in our later analysis, we will be required to consider a slightly relaxed assumption where the Kähler manifold is flat for all the MSSM field except for the Higgs doublets, which leads to non-universal Higgs soft masses (NUHM) [29–31].

In the original PGM model, we assumed that the holomorphic bi-linear of the two-Higgs doublets, $H_u H_d$, has a vanishing $R$-charge and appears in the Kähler potential and the superpotential,

\begin{align}
K|_{H_u H_d} &= c_H H_u H_d + h.c., \quad (1) \\
W|_{H_u H_d} &= c'_H m^{3/2}_3 H_u H_d. \quad (2)
\end{align}

where $c_H$ and $c'_H$ are $O(1)$ coefficients [21]. From these terms, the $\mu$ and the $B$-parameters are given by,

\begin{align}
\mu &= c_H m^{3/2}_3 + c'_H m^{3/2}_3, \quad (3) \\
B\mu &= 2c_H m^{2}_3 - c'_H m^{2}_3. \quad (4)
\end{align}

In minimal supergravity, we would take $c_H = 0$, so that $\mu = c'_H m^{3/2}_3$ and $B = -m_0$, neglecting the small contribution from anomaly mediated $A$-terms. Minimization of the Higgs potential in this case allows one to solve for $\mu$ ($c'_H$) and $\tan\beta$. Inclusion of $c_H$, allows one to keep $\tan\beta$ as a free parameter solving instead for $\mu$ ($c'_H$) and $c_H$ [32]. Thus, there are two independent parameters which can be chosen as $m^{3/2}_3$ and $\tan\beta$.

**PQ-symmetric PGM**

Now, let us move on to the PQ-symmetric PGM model where the $\mu$-term is generated by the breaking of the PQ-symmetry. In this case, instead of the above potentials in Eqs. (1) and (2), the $\mu$-term is generated via,

\begin{equation}
W|_{H_u H_d} = k \frac{p^2}{M_p} H_u H_d, \quad (6)
\end{equation}

The $B\mu$ term may also obtain comparable contributions from higher dimensional operators in the Kähler potential,

\begin{equation}
\Delta K|_{H_u H_d} = \frac{Z^\dagger Z}{M_p^2} H_u H_d + h.c. \quad (5)
\end{equation}

where $Z$ denotes the supersymmetry breaking field.
where $k$ denotes a dimensionless constant and $P$ is a PQ-symmetry breaking field with PQ-charge +1. $M_P = 2.4 \times 10^{18}$ GeV refers to the reduced Planck scale. We assumed that the holomorphic bi-linear $H_uH_d$ has a PQ charge of −2 and the charges of other MSSM matter fields are appropriately assigned. As a result, the Kähler term in Eq. (1) is not allowed.

From this potential, we obtain the $\mu$ and the $B$-parameters,

$$\mu = k\frac{\langle P \rangle^2}{M_P}, \quad (7)$$

$$B\mu = -\frac{m_3}{2}\mu. \quad (8)$$

Therefore, the $\mu$-parameter is determined by the PQ-breaking scale and, for example, $\mu = O(100)$ TeV is realized for $\langle P \rangle = O(10^{12})$ GeV. It should be noted that $\mu$ cannot be much larger than the gravitino mass for successful EWSB. The $B$-parameter is, on the other hand, fixed to $-m_{3/2}$ (at the GUT scale), which should be contrasted to the above original PGM where the $B$-parameter (through $c_H$) is an independent parameter which for convenience can be exchanged with $\tan \beta$ as described above. As we will discuss later, this restricted parameter set conflicts with full scalar mass universality.

In summary, the PQ-symmetric PGM model is more restricted than the original PGM model and has effectively only one parameter

$$m_{3/2} \quad (9)$$

with full scalar mass universality. Since $B$ is fixed, the EWSB conditions amount to solving for $\tan \beta$ and the constant $k$ in Eq. (6), which has no solution. Here, we have traded the size of $\mu$-term with the $Z$-boson mass, fixed to $m_Z \simeq 91.2$ GeV. Even with the NUHM2 [30], the model has only three parameters,

$$m_{3/2}, \ m_1, \ m_2, \quad (10)$$

where $m_{1,2}$ denote the soft masses of the two Higgs doublets, $H_d, H_u$. However, as we shall see, viable solutions are possible with non-universality extended only to the up-type Higgs, ie. we will be able to keep $m_1 = m_{3/2}$. Furthermore, we will also see that the special case of $m_2 = 0$ also yields acceptable solutions, so if $H_2$ originates as a pseudo Nambu-Goldstone boson, or $H_2$ is part of a no-scale structure in $K$, we are again reduced to a one-parameter theory.

### 3 Supersymmetric axion model

**Brief review of the PQ-breaking model**

Before going on to study the parameter space for the PQ-symmetric PGM model, let us discuss the supersymmetric axion model [33] in a little more detail. As a simple example, we may take the following model of spontaneous PQ-symmetry breaking,

$$W_{PQ} = \lambda X (PQ - v_{PQ}^2), \quad (11)$$

$^3$EWSB requires either $m^2_\xi \sim \mu^2$ or $B\mu \sim \mu^2$. In either case, we get $\mu \sim m_{3/2}$.
where $\lambda$ is a dimensionless coupling, and $v_{PQ}$ a dimensionful parameter. The superfields $X$, $P$, and $Q$ have PQ-charges, 0, +1 and −1, respectively, with vacuum values,

$$\langle P \rangle = \langle Q \rangle \simeq v_{PQ}, \quad \langle X \rangle \simeq m_{3/2}/\lambda,$$

(12)

so that the PQ-symmetry is spontaneously broken and we are left with an axion, saxion and axino at an energy scale much lower than $v_{PQ}$.

Due to SUSY breaking effects, the saxion and the axino obtain masses of order of the gravitino mass, and hence, they decay quickly to a pair of the gluinos and a gluino/gluon pair, respectively, and cause no cosmological problems. The axion, on the other hand, remains very light and has a very long lifetime, and can be a good dark matter candidate with a relic density [36],

$$\Omega_a h^2 = 0.18 \left( \frac{F_{PQ}}{10^{12}\text{GeV}} \right)^{1.19} \left( \frac{\Lambda}{400\text{MeV}} \right),$$

(13)

where $\Lambda$ denotes the QCD scale and the decay constant $F_{PQ}$ is defined by $v_{PQ}$ and determined by a domain wall number $N_{DW} = 6$ in this case. We assumed the initial axion amplitude $a \simeq F_{PQ}$. Thus, the axion can be the dominant dark matter component for $F_{PQ} \simeq 7 \times 10^{11}\text{GeV}$, assuming that the mis-alignment angle is of order $\pi$.

It should be noted that axion dark matter models are suspect to problems with domain walls and isocurvature fluctuations. The former problem is, however, solved relatively easily if the PQ-symmetry is broken before the primordial inflation starts and is never restored after the end of inflation. In this case, the domain walls are not formed after inflation, and hence, there is no domain wall problem.

The latter problem, the overproduction of isocurvature fluctuations [27], on the other hand, puts a severe constraint on the Hubble scale during inflation. In fact, in order to suppress the isocurvature mode in the axion dark matter enough to be consistent with the constraint set by CMB observations [37], the Hubble scale during inflation is required to be rather small (i.e. $H \lesssim 2 \times 10^7\text{GeV}$), which is much smaller than the one in more conventional models of inflation where $H \sim 10^{13}\text{GeV}$.

It is, however, possible to relax the constraint on $H$, if the axion decay constant, $F_{PQ}$, were larger than its low energy value during inflation [28]. Roughly, we would require $H/F_{PQ} \lesssim 3 \times 10^{-5}$ to resolve the problem, and hence, $F_{PQ} \gtrsim 3 \times 10^{17}\text{GeV}$ during inflation for $H \sim 10^{13}\text{GeV}$. Such a large PQ-breaking scale during inflation can be achieved, for example, when $P$ in the above model picks up a negative mass squared contribution along the $PQ$ flat direction, in a similar way that Affleck-Dine fields [38] pick up large vevs to generate a baryon asymmetry (for a concrete model in this context see [39]).

The PQ-breaking scale can also be large during inflation when the radial component of the PQ-breaking field, $\sigma \equiv P = Q$ flows into the so-called attractor solution during inflation [28]. In this case, the PQ-breaking field scales as $\lambda^{-1/2}$ where $\lambda$ is the coupling

\footnote{Here, we are assuming that the soft squared masses of $P$ and $Q$ are equal to each other. When the soft squared masses of $P$ and $Q$ are quite different from each other, the $B$-parameter in Eq.(8) obtains a sizable correction [34,35].}
in Eq. (11) [40]. This may require couplings as small as $10^{-8}$ to $10^{-12}$ depending on the particular model of inflation. Because the amplitude of the $P$ and $Q$ fields are large during inflation, it is possible that subsequent large oscillations could pass through $P = Q = 0$ which would amount to the restoration of the PQ-symmetry and could lead to domain wall formation. To determine if domain walls form, a detailed analysis of the relaxation of the $P$ and $Q$ fields is needed. It is important to note that the amount of relaxation depends on the model of inflation. For inflation models quadratic in the inflaton, the PQ fields relax too slowly and domain walls are formed, see [41].

It is also possible to relax this problem by considering a Giudice-Masiero-like term involving the PQ fields in the Kähler potential,

$$K \supset -c_{PQ} \left(PQ + P^\dagger Q^\dagger\right).$$ (14)

Since the product of $P$ and $Q$ have $R$ and $PQ$ charge zero, this additional term cannot be forbidden. During inflation, the Kähler potential now gives a correction to the scalar potential

$$\Delta V = -c_I H^2 c_{PQ} \left(PQ + P^\dagger Q^\dagger\right) + c_I H^2 \left(|P|^2 + |Q|^2\right),$$ (15)

where $c_I \approx 3$ and $H$ is Hubble’s constant. Adding this to the scalar potential for the $PQ$ fields from the superpotential, we find

$$V = |\lambda|^2 \left|PQ - \frac{c_{PQ} c_I}{|\lambda|^2} H^2 - \Lambda^2\right|^2 + c_I H^2 \left(|P|^2 + |Q|^2\right) + V_0,$$ (16)

where $V_0$ keeps track of the constant pieces in the potential. To get PQ breaking during inflation, we need $c_{PQ} > 1$. During inflation, the new minimum is around

$$P = Q \sim \left(\frac{c_{PQ} c_I}{|\lambda|^2}\right)^{1/2} H.$$ (17)

To sufficiently suppress the isocurvature perturbations, we need a somewhat small coupling,

$$\frac{|\lambda|}{\sqrt{c_{PQ} c_I}} \lesssim 3 \times 10^{-5},$$ (18)

which is effectively a constraint of $\lambda \lesssim 10^{-4}$.

In this scenario, after inflation the $P$ and $Q$ fields will relax back to the true minimum set by $\Lambda$. The masses of the $P$ and $Q$ fields during inflation are of order $\sqrt{(c_{PQ} - 1)c_I H}$. Therefore, during their relaxation back to the true minimum, $P$ and $Q$ will track the minimum (so long as $c_{PQ}$ is not very close to 1) and no domain walls will be formed, that is, the PQ-symmetry remains broken.
Dynamical PQ-symmetry breaking

We may also consider an alternative mechanism for realizing a large PQ-breaking scale during inflation. If PQ-breaking scale is generated dynamically it is possible to get this dynamical scale large during inflation. To illustrate this mechanism, let us consider a supersymmetric SU(2) gauge theory with four fundamental chiral fields, \( P_{1,2} \) and \( Q_{1,2} \), and four singlet fields \( S_{ij} \) \((i, j = 1 \cdots 2)\). The superfields \( P, Q \) and \( S \) have PQ-charges +1, −1 and 0, respectively, and they are coupled through the superpotential,

\[
W = \sum_{i,j=1,2} \lambda S_{ij} P_i Q_j ,
\]

where \( \lambda \) is again a dimensionless coupling. Below the dynamical scale of \( SU(2), \Lambda_{PQ} \), the model can be described by the six composite mesons,

\[
M_{PP} \simeq P_1 P_2 / \Lambda_{PQ}, \quad M_{QQ} \simeq Q_1 Q_2 / \Lambda_{PQ}, \quad M_{PQ_{ij}} \simeq P_i Q_j / \Lambda_{PQ},
\]

whose effective superpotential terms are roughly given by,

\[
W_{\text{eff}} \simeq \sum_{ij} \lambda \Lambda_{PQ} S_{ij} M_{PQ_{ij}} + X(\text{Pf}(M) - \Lambda_{PQ}^2).
\]

Here, \( X \) denotes the Lagrange multiplier to impose the quantum deformed moduli constraint on the mesons \([42]\),

\[
\text{Pf}(M) = M_{PP} M_{QQ} + \frac{1}{2} \sum_{ijkl} \epsilon_{ij} \epsilon_{kl} M_{PQ_{ik}} M_{PQ_{jl}} = \Lambda_{PQ}^2.
\]

By noting that the \( M_{PQ} \) obtain masses of \( \mathcal{O}(\lambda \Lambda_{PQ}) \) from the first term in Eq. (21), we find that the PQ-symmetry is spontaneously broken by,

\[
\langle M_{PP} \rangle = \langle M_{QQ} \rangle \simeq \Lambda_{PQ}, \quad \langle X \rangle \simeq m_{3/2}, \quad \langle M_{PQ} \rangle = 0,
\]

Therefore, this model is a dynamical realization of the previous model defined in Eq. (11).

Now let us assume that the gauge coupling constant of \( SU(2) \) is given by a vev of a singlet field \( \phi \) through the gauge kinetic function, \( f = (1/g_0^2 + \phi^2) \), assuming \( g_0 = 4\pi \) and \( \langle \phi \rangle = O(1) \) at the Planck scale. Then the effective gauge coupling at the Planck scale is perturbative, \( 1/g^2 = (1/g_0^2 + \langle \phi \rangle^2) \), with \( \langle \phi \rangle \) chosen so that the condensation scale is \( \Lambda_{PQ} \simeq 10^{12} \text{GeV} \). However, if \( \phi \) gets a large positive mass during inflation, the vev of \( \phi \) will be suppressed, \( \langle \phi \rangle \ll 1 \). Since the vev independent part of the gauge coupling is already strong at the Planck scale, we get \( \Lambda_{PQ} = M_P \) during inflation. In this way, we can easily realize a large PQ-breaking scale during inflation, and hence, the isocurvature fluctuations in axions can be suppressed. \( \phi \) can be given a mass by adding the following interactions,

\[
W = \lambda' Y \left( \phi^2 - M_P^2 \right).
\]

At the minimum of the potential, \( \phi \) has a Planck scale vev which generates a mass for \( \phi \) of order \( \lambda' M_P \) avoiding a moduli problem \([ \ref{footnote5} ] \).

\[\footnote{This model is close to the model of dynamical PQ-breaking proposed in \([\ref{footnote44}]\), where the PQ-symmetry and supersymmetry are broken simultaneously.}\]

\[\footnote{If \( \phi \) gets a positive Hubble induced mass \( \mathcal{O}(\lambda' M_P)^2 \), \( \phi \) will be stabilized at its origin \( \phi = 0 \).}\]
4 Results

We are now in a position to explore the necessary parameter space for the PQ-symmetric PGM model. We begin with the three-parameter version of the model defined by the gravitino mass, and the two soft Higgs masses. All other scalars are assumed to be universal at the GUT scale (the renormalization scale where the two electroweak couplings are equal). All masses and couplings are run down to the weak scale, where the Higgs potential is minimized, thus determining $\mu$ (or $k$ in this context) and $\tan \beta$. Gaugino masses and $A$-terms assume their anomaly mediated values, and $B_0 = -m_{3/2}$. As we have seen previously [14], PGM solutions with $c_H = 0$ are possible so long as we allow the Higgs soft masses to depart from universality.

In Fig. 1, we show examples of the $m_1, m_2$ plane for fixed values of $m_{3/2} = 60, 150, 300$, and 400 TeV. The red dot-dashed curves show contours of the light Higgs mass, $mh$ from 122 - 130 GeV in 1 GeV intervals. The region with 124 GeV $< mh < 128$ GeV is shaded green. In all cases, we have assumed the supergravity boundary condition of $B_0 = -m_{3/2}$ and $c_H = 0$, and calculate $\mu$ and $\tan \beta$. The solid black contours show $\mu/m_{3/2}$ and as one can see, this ratio is close to one over much of the displayed planes. The exception occurs when $m_2^2$ is large and positive causing $\mu$ to become small. In the figure, when $m_{3/2} = 60$ TeV, the region shaded blue at the top right of the figure has $\mu^2 < 0$ indicating the lack of an EWSB solution. The region at low and negative $m_1^2$ is also excluded as there the Higgs pseudo-scalar mass, $m_A^2 < 0$. Note the sign of the soft Higgs masses in the figure refers to the sign of the mass squared. The gray dotted curves show the calculated values of $\tan \beta$ which are typically around 4-5 when $m_{3/2} = 60$ TeV, and are closer to 2 at larger $m_{3/2}$.

As one can see all of the viable solutions displayed in the figures require some degree of non-universality. In each case displayed, forcing Higgs mass universality would either require a value of $m_1$ too small corresponding to a light Higgs with mass $< 124$ GeV, or a value of $m_2$ too large to allow EWSB solutions. As one can see, $m_1$ can be made universal, when the gravitino mass is $\gtrsim 300$ TeV. For $m_{3/2} = 300$ TeV as shown in the lower left panel, $m_1 = m_{3/2}$ is very close to the $m_h = 124$ GeV contour when $m_2$ is relatively small. When $m_{3/2} = 400$ TeV as shown in the lower right panel, There is no problem is obtaining a suitable Higgs mass. But for $m_2 \gtrsim 300$ TeV, $\mu^2$ quickly runs negative and we lose the ability to satisfy the EWSB boundary conditions. At smaller, $m_2$ the results are in fact quite insensitive to the particular value of $m_2$. Therefore in what follows, we will set $m_2 = 0$.

The vanishing of the up-type Higgs soft mass at the universality scale can be explained if either this Higgs field was part of a no-scale structure [25] of the Kähler potential as in

$$K = yy^* - 3 \log \left(1 - \frac{1}{3}(H_2 H_2^* + K^{(Z)})\right) + \log |W|^2$$

(24)

where $Z$ is the field(s) which breaks supersymmetry, and $y$ represents all other fields including $H_1$. The resulting soft masses for the Higgs doublets in this case is $m_Z^2 = 0$. Alternatively, it is possible that the up-type doublet appears as a Nambu-Goldstone boson described by the coset space, $U(3)/SU(2) \times U(1)$ [24].

Having fixed $m_2 = 0$, it is possible to display the parameter space on a single two-dimensional $m_1, m_{3/2}$ plane, as in Fig. 2. As one can see, for low(er) values of $m_{3/2}$, the value
Figure 1: The \((m_1, m_2)\) plane for fixed \(m_{3/2} = 60, 150, 300, \) and \(400 \text{ TeV}\). Shown are the contours for the light Higgs mass, \(m_h\) (red, dot dashed) from 122 - 130 GeV in 1 GeV intervals. The region with a Higgs mass between 124 and 128 GeV is shaded green. The gray dotted curves show the calculated values of \(\tan \beta\). The range spans 2 - 15 when \(m_{3/2} = 60 \text{ TeV}\), and 2-3 when \(m_{3/2} = 400 \text{ TeV}\). Also shown are contours of \(\mu/m_{3/2}\) (solid, black) which are typically close to 1. The blue shaded regions (when shown) correspond to regions where no EWSB is possible.

of \(m_1\) needed to obtain \(m_h\) between 124 and 128 GeV (shown as the shaded green region) requires non-universality in the Higgs soft masses and \(m_1 > m_{3/2}\). When \(m_{3/2} \gtrsim 300 \text{ TeV}\),
solutions with $m_1 = m_{3/2}$ become possible. In all cases, we find $2 \lesssim \tan \beta \lesssim 4$.

![Diagram](image)

Figure 2: The $(m_1, m_{3/2})$ plane for fixed $m_2 = 0$. Shown are the contours for the light Higgs mass, $m_h$ (red, dot dashed) from 122 - 132 GeV in 4 GeV intervals. The region with a Higgs mass between 124 and 128 GeV is shaded green. The gray dotted curves show the calculated values of $\tan \beta$. The black solid line shows the down-type universal case, where $m_1 = m_{3/2}$. The blue shaded regions (when shown) correspond to regions where no EWSB is possible.

We can go further and insist on universality of the down-type Higgs soft mass. Results for this restrictive case are shown in Fig. 3 where we plot $\mu/m_{3/2}$, $\tan \beta$ (left) and $m_h$ (right) as a function of $m_1 = m_{3/2}$ for $m_2 = 0$. As one can see, in this case, for a wide range of values of $m_{3/2}$, $\mu/m_{3/2} \simeq 1$ and $\tan \beta \simeq 2.2$. However, in order to obtain $m_h > 124$ GeV, we need $300$ TeV $\lesssim m_{3/2} \lesssim 850$ TeV.

5 Universality With PQ-symmetry Breaking

By generalizing the model for PQ-symmetry breaking, it may be possible to restore full scalar mass universality. The particular model we consider was first presented in [26] and relates the PQ scale with see-saw scale for generating neutrino masses. The superpotential for this model is

$$W_{PQ} = \frac{f}{M_P} P^3 Q + \frac{g}{M_P} H_u H_d PQ + \frac{1}{2} h_{ij} P N_i N_j ,$$ (25)
where $P,Q$ are the fields responsible for breaking the PQ-symmetry with charges $(-1,3)$, $N_i$ are the right handed neutrinos with PQ charge $1/2$, and $H_u,d$ have PQ charge $-1$. In this model the right handed neutrino masses are generated by PQ-symmetry breaking, which is of order $10^{12}$ GeV as discussed above. In pure gravity mediation models, these fields will also get supersymmetry breaking parameters. The relevant supersymmetry breaking soft masses and $A$-terms are

$$-\mathcal{L}_{\text{soft}} \supset m_Q^2 |Q|^2 + m_P^2 |P|^2 + m_N^2 |N|^2 + f \frac{m_A}{M_P} P^3 Q + \text{h.c.}$$

with

$$m_Q^2(\mu_{\text{GUT}}) = m_P^2(\mu_{\text{GUT}}) = m_N^2(\mu_{\text{GUT}}) = m_{3/2}^2 \quad m_A(\mu_{\text{GUT}}) = m_{3/2}$$

The RG equations for these soft masses are dominated by the couplings $h_{ij}$. In fact, if the number of neutrinos is large enough and the $h_{ij}$ are large enough, $m_P^2$ will be driven negative and the PQ-symmetry is broken. The vevs of $\langle P \rangle$ and $\langle Q \rangle$ will effectively be free parameters determined by $h_{ij}$ and $f$. Breaking the PQ-symmetry in this manner will generate independent vevs for $\langle Q \rangle$ and $\langle P \rangle$. Because both $\langle P \rangle$ and $\langle Q \rangle$ are none zero, the $F$-terms for $P,Q$ will also be non-zero and independent. With this set up, $\mu$ and $B\mu$ are linearly independent as in Eq. (3) and (4) and in contrast to the case considered in Eq. (7) and (8). This additional freedom in $\mu$ and $B\mu$ makes universal soft mass for the MSSM fields possible. For this scenario, the parameter space, defined by $m_{3/2}$ and $\tan \beta$, will be identical to that considered in [13].

There is another way to get universality for the scalar masses with the PQ breaking symmetry. If we again include Giudice-Masiero mixing term for the $P$ and $Q$ in the Kähler potential, the relationship for $B$ of the Higgs bosons changes to

$$B\mu = -m_{3/2} (1 - 2c_{PQ}) \mu .$$

Figure 3: Solutions for $\mu/m_{3/2}$ and $\tan \beta$ (left) and for $m_h$ (right) in the one parameter version of the PQ-symmetric PGM model. Here $m_1 = m_{3/2}$ and $m_2 = 0$. 

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Due to the freedom in $B$ from the $c_{PQ}$ term, we can independently define $\mu$ and $B$. This allows for solutions to the EWSB conditions to be found by solving for $k$ and $c_{PQ}$ leaving $\tan \beta$ as a free parameter.

6 Discussions

The relatively large Higgs mass determined at the LHC coupled with the lack of discovery of any superpartners indicates that the scale of supersymmetry must be higher than originally thought if it is realized at low energy at all. While it remains possible that the discovery of supersymmetry is around the corner at scales close to 1 TeV and well within reach on a 14 TeV LHC collider \cite{5,9}, it is also possible that the supersymmetry scale is significantly higher and sits in the range of 100 - 1000 TeV as expected in PGM models \cite{10,15}, models with strongly stabilized moduli \cite{16,18}, or in so called models of mini-split supersymmetry \cite{43}.

In either case, we are forced to address the question regarding the scale of supersymmetry breaking. In models where the SUSY scale is upwards of 100 TeV, gaugino masses are generally generated at the 1-loop level through anomalies. In that case, the lightest supersymmetric particle is usually the wino. For $\mu > 0$, the wino relic density is too small to account for the dark matter of the universe, but the wino might be a viable candidate when $\mu < 0$ and the SUSY scale is of order 500 TeV (see e.g. \cite{13}). However, even in that case, there are strong constraints against wino dark matter from higher energy gamma-ray observations \cite{45}. In this context the axion becomes an attractive dark matter candidate.

In addition to the dark matter problems, most low energy supersymmetric models suffer from the $\mu$-problem. Even in models where the scale of supersymmetry breaking is generated spontaneously, the $\mu$ term, being supersymmetric, is typically put in by hand as a bilinear in the superpotential. Therefore, the dynamical generation of the $\mu$ term, through the coupling of the MSSM Higgs doublets to Standard Model singlets with non-zero PQ charge \cite{23} offers an attractive solution to potentially both the $\mu$ term and dark matter problems.

From Eq. \cite{13}, it is clear that a PQ scale of close to $10^{12}$ GeV is needed, if axions are to be the dominant form of dark matter in the universe. Thus if axions make up any significant component of the dark matter (say, at least 10% of the dark matter), the $\mu$ term is expected to be at least several TeV. If axions are the dominant form of dark matter, then the $\mu$ term is expected to be of order 100 TeV or more.

In this paper, we have shown how a model of PGM can be constructed in the context of supergravity with a PQ coupling to the MSSM Higgs doublets. Because of this coupling, a Giudice-Masiero like term in the Kähler potential is not allowed and we must deviate slightly from pure scalar mass universality. Allowing for the possibility of non-universal Higgs masses, we have shown that viable models exist with (non-Higgs) scalar mass universality which respect minimal supergravity boundary conditions for the $B$-term. Gaugino masses and $A$-terms are assumed to arise from anomaly mediated contributions. All soft terms and couplings are run down from the universality scale (assumed to be the GUT scale) and minimization of the Higgs potential is used to determine the $\mu$ term and $\tan \beta$. If axions are present, there will be some restrictions on this parameter space from the fact that $c_{PQ} > 1$. 

\footnote{There will be some restrictions on this parameter space from the fact that $c_{PQ} > 1$.}
the dominant form of dark matter, the coupling of the Higgs doublets to the PQ fields thus generates a $\mu$ term of order of 100 TeV which sets the scale for supersymmetry breaking. While it is possible to find solutions with $\mu \ll m_{3/2}$ (for example when $m_2$ is large and $\mu^2$ is driven to 0), it is not possible to find solutions with $\mu \gg m_{3/2}$. Thus fixing $\mu \gtrsim O(100)$ TeV in order to obtain a significant axion relic density forces us into the domain where supersymmetry is broken at a similarly high scale.

Fortuitously, it also possible to find solutions of the type just described with a Higgs mass in the range determined at the LHC. For relatively low $m_{3/2} \lesssim 300$ TeV, we require $m_1 \gtrsim m_{3/2}$. While for larger $m_{3/2}$ up to 850 TeV, solutions with $m_1 = m_{3/2}$ are possible. Our results are not particularly sensitive to $m_2$, though $m_2 < m_{3/2}$ is quite generic. If the up-type Higgs is a pseudo Nambu-Goldstone boson or is part of a no-scale structure so that $m_2 = 0$, we are left with a particularly simple model with one free parameter, $m_{3/2}$. For $m_{3/2} > 300$ TeV, we have $m_h > 124$ GeV, and $\mu/m_{3/2} \simeq 1$ and $\tan \beta \simeq 2.2$.

We have also shown several mechanisms which suppress isocurvature fluctuations despite having a dominant component of axion dark matter. The simplest possibility, which may not apply here, is that described in \cite{28} and requires only a small coupling $\lambda$ in Eq. (11). The constraint on $\lambda$ can be significantly relaxed if a Giudice-Masiero-like term is added to the Kähler potential as in Eq. (14). Finally, we have proposed a novel dynamical mechanism for the generation of the axion decay constant which allows $F_{PQ} \simeq M_P$ during inflation, and smaller values ($O(10^{12})$ GeV) at low energy.

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