Polarization characteristics of nuclear quasi-free (p,dπ⁺) reaction

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Abstract. Nuclear reaction ¹⁶O(p,dπ⁺)¹⁵N is considered theoretically as a prototype of a specific class of nuclear quasi-free processes where deuteron, α-particle, ³H- or ³He-cluster are produced together with pion or η-meson in proton-nucleus interactions in the intermediate energy region.

1. Introduction
To go deeper into clustering dynamics in nuclei with more attention to subnucleon aspects of nuclear structure we, starting from recent progress [1, 2] in coincidence measurements on the quasi-free process (p,dπ⁺), investigate the potential of similar quasi-free reactions with more complicated clusters such as (p,tπ⁺), (p,³He⁻), (p,³Heη) and (p,απ⁺) [3]. Here the α-particle, ³H- or ³He-cluster are expected to be produced (together with a pion or η-meson) as being composed of their lighter fragments in the very act of the in-nucleus binary collision process such as pd → tπ⁺ and so on. Taking reaction (p,dπ⁺) as a prototype of these cluster composing reactions we consider them on the same shell-model ground of our approach to quasielastic scattering reactions (p,2p), (p,pd) and (p,po) [4]–[6] when calculating spectroscopic factors for nucleons and clusters and their momentum distributions resolved over the excitation energy spectrum of the recoil nucleus. On the other hand, we take into account that corresponding two-particle free processes pp → dπ⁺, pd → tπ⁺, pd →³Heη and others become a subject of regular studies in particle and few-body nuclear physics [7]–[12].

First estimates [13] of advantages of coincidence deuteron-pion measurements on nuclear reaction (p,dπ⁺) were made immediately after pioneer experimental indication [14] to its quasi-free character. Later, the DWIA and PWIA calculations on reaction ¹²C(p,dπ⁺)¹¹B in [2] and our recent analysis of reaction ¹⁶O(p,dπ⁺)¹⁵N [15] point to promising perspectives of systematic studies of polarization characteristics of reaction (p,dπ⁺) when both produced particles - deuteron and pion - are detected in coincidence. Here we continue the line of paper [15] and concentrate on the analyzing power of differential cross section \( \frac{d^3\sigma}{d\Omega_d d\Omega_\pi dT_\pi} \) and polarization transfer parameters from the incoming proton to the produced deuteron in reaction ¹⁶O(p,dπ⁺)¹⁵N. Taking the proton energy of \( T_p = 550 \) MeV , higher than in experiments [1, 2], makes possible to simplify the procedure to calculate the proton, deuteron and pion distorted wave functions in the initial and final states of the process and to use, instead of the partial-wave expansion approach of paper [2], the eikonal (Glauber) method. We do not factorize the differential cross section \( \frac{d^3\sigma}{d\Omega_d d\Omega_\pi dT_\pi} \) into the two-body pp → dπ⁺ cross section and the
nuclear form-factor, this approximation would be too rough in the case [16]. Widely known Bugg systematization [17] for the amplitude of the elementary two-body reaction $pp \rightarrow d\pi^+$ is used throughout the paper. Paying a special attention to possible spin-orbit effects in the interaction of the incoming proton and the produced deuteron with the target and the recoil nuclei, we transform the proton and deuteron distortion factors into matrices $<\vec{p}|\hat{D}^{(\pm)}(\vec{r})|\mu>$ over magnetic quantum numbers of spin of corresponding particles. The Madison convention [18] to choose the coordinate frame and the quantization axis is used throughout considering all angular correlation and polarization aspects of the reaction.

2. Reaction $^{16}O(\vec{p}, d\pi^+)^{15}N$: differential cross section and analyzing power

We continue our analysis [15] of reaction $^{16}O(\vec{p}, d\pi^+)^{15}N$ by extending supposed geometry conditions of the reaction. As fig. 1 shows, we concentrate now near maximum of the momentum distribution of the nuclear proton due to new geometry conditions

$$E_p = 550 \text{MeV}; \theta_{\text{lab}}^d = 60^\circ; \theta_{\text{lab}}^\pi = 17^\circ; \phi_{\text{lab}}^d = 0; \phi_{\text{lab}}^\pi = 180^\circ;$$

which differ from those in [15] ($E_p = 650 \text{MeV}; \theta_{\text{lab}}^d = 52^\circ; \theta_{\text{lab}}^\pi = 12^\circ; \phi_{\text{lab}}^d = 0; \phi_{\text{lab}}^\pi = 180^\circ$).

![Figure 1](image1.png)

Figure 1. Momentum $k_R$ of the recoil nucleus $^{15}N$ in reaction $^{16}O(\vec{p}, d\pi^+)^{15}N_{g.s.}$ as a function of the pion kinetic energy $T_\pi$ in the lab-frame; dashed line - the same under the geometry conditions of [15].

Differential cross section for reaction $^{16}O(p, d\pi^+)^{15}N$ induced by a non-polarized proton beam and the analyzing power $A_0(T_\pi)$ in reaction $^{16}O(\vec{p}, d\pi^+)^{15}N$ are shown in fig. 2 and fig. 3 when calculated without (solid line) and with (dashed line) factorization approximation.

![Figure 2](image2.png)

Figure 2. Differential cross section for reaction $^{16}O(p, d\pi^+)^{15}N$ in channels $^{16}O\rightarrow^{15}N(1p_{1/2})$ and $^{16}O\rightarrow^{15}N(1p_{3/2})$ as a function of the pion kinetic energy $T_\pi$; dashed lines are corresponding to the factorization approximation.

In former case, results for two reaction channels $^{16}O\rightarrow^{15}N(1p_{1/2})$ and $^{16}O\rightarrow^{15}N(1p_{3/2})$ differ inconsiderably from each other: their $T_\pi$ dependence is practically the same while their ratio to
each other which is close 1:2 is not more than the statistical weight ratio for corresponding $(1p_{1/2}^{-1})$ and $(1p_{3/2}^{-1})$ recoil nucleus states. The figures show also that the factorization approximation leads to practically the same results as obtained without this approximation. Another situation takes place as concerns the analyzing power $A_0(T_\pi)$ (fig. 3). Its $T_\pi$-dependence is different in channels $^{16}O\rightarrow^{15}N(1p_{1/2}^{-1})$ and $^{16}O\rightarrow^{15}N(1p_{3/2}^{-1})$. Contrary to the differential cross section, the analyzing power calculated when ignoring the factorization approximation differs considerably from that obtained within this approximation.

3. Polarization of the produced deuteron

General character and degree of polarization of the produced deuteron are determined by spin dependent characteristics of the free two-body process $pp \rightarrow d\pi^+$ and polarization of the incoming proton beam, by kinematic and geometry conditions of the pion and deuteron detection and also by the initial state and final state distortions of the wave functions of the incoming and outgoing particles. Components of vector and tensor polarization of the deuteron are calculated using density matrix (statistical tensors $\rho^{(d)}_{k\ell qd}(S_d,S_d) = \rho^{(d)}_{k\ell qd}(1,1)$) of the deuteron spin after they are averaged over orientation of the angular momentum $\vec{J}_R$ of the recoil nucleus, e.g.: 

$$P_y^{(d)} = \sqrt{2/3} \frac{\rho^{(d)}_{10}(1,1) >}{\rho^{(d)}_{00}(1,1) >} \quad (1)$$

(quantization axis - along axis y). Each of the components of statistical tensors $\rho^{(d)}_{k\ell qd}(1,1)$ is a sum of two terms where the first of them does not depend on polarization of the incoming proton (the induced polarization) while the other (the transferred polarization) is proportional to the polarization degree $P^{(in)}$ of the proton beam:

$$\rho^{(d)}_{kq}(1,1) = \left(\rho^{(d)}_{kq}\right)^{\text{unpol}} + \Delta\rho^{(d)}_{kq} \cdot P^{(in)},$$

where $\left(\rho^{(d)}_{kq}\right)^{\text{unpol}}$ and $\Delta\rho^{(d)}_{kq}$ are bilinear forms of the Bugg amplitudes [17].

We calculate vector polarization of the deuteron in reaction $(\vec{p}, d\pi^+)$ as

$$\vec{P}^{(d)} = \frac{\vec{P}^{(unpol)} + \sum_{ij} \vec{e}_i K_{ij} P^{(in)}_{j}}{1 + \vec{P}^{(in)} \cdot \vec{R}}, \quad (2)$$

where $\vec{e}_i$ with $i = x,y,z$ are the unit vectors along the axes of the coordinate frame. Other parameters of this formula are calculated as Trace of bilinear combinations of the T-matrix of
the reaction over orientation of the angular momentum of the recoil nucleus). Among them are: $\vec{P}^{(unpol)} \sim Tr \left( \hat{T}^+ \hat{S}^{(d)} \hat{T} \right)$ - vector of deuteron polarization in the case of non-polarized proton beam; and $K_{ij} = \frac{1}{2} Tr \left( \hat{S}_i^{(d)} \hat{T} \sigma_j^{(p)} \hat{T}^+ \right)$ - matrix of coefficients of polarization transfer from proton to deuteron (see [15] for more details).

Starting from equation (2), consider first the case when the incoming proton is either non-polarized or directed along the normal to the reaction plane: $\vec{P}^{(in)} = (0, P_{y}^{(in)}; 0)$. Under this condition the initial state of the system is symmetric relative to the reaction plane and remains the same in course of the reaction. Hence, the deuteron polarization vector is also perpendicular to the reaction plane: $\vec{P}^{(d)} = (0, P_{y}^{(d)}; 0)$. Fig. 4 demonstrates variation of the polarization degree $P_{y}^{(d)}$ with pion energy $T_\pi$ in three cases of the non-polarized protons and the protons totally polarized in two opposite directions along the normal to the reaction plane: $P_{y}^{(in)} = \pm 1$.

![Figure 4. Polarization degree $P_{y}^{(d)}$ of the deuteron in reaction $^{16}O(p, d\pi^+)^{15}N$ with protons polarized along the normal to the reaction plane: $P_{y}^{(in)} = -1$ (solid lines); $P_{y}^{(in)} = 0$ (dash-dot lines); $P_{y}^{(in)} = 1$ (dashed lines).](image)

Now consider the case when the incoming proton is polarized along its momentum: $\vec{P}^{(in)} = (0, 0, P_{z}^{(in)})$. Here polarization transfer takes place in two directions: along the incoming beam (axis $z$) and, also in the reaction plane, normally to this direction (axis $x$). Scalar product $\vec{P}^{(d)} \cdot \vec{R}$ in the denominator of (2) vanishes, so the whole equation (2) is reduced to

$$\vec{P}^{(d)} = \vec{e}_y P^{(unpol)} + (\vec{e}_x K_{xz} + \vec{e}_z K_{zz}) P_{z}^{(in)}.$$

The first component of this polarization vector, normal to the reaction plane, represents polarization induced under particle-nucleus interactions in the initial and final states of the reaction; within the PWIA approach it vanishes. Polarization transfer coefficients $K_{xz}$, $K_{zz}$ show the magnitude and direction of the in-plane components of $\vec{P}^{(d)}$. Fig. 5 shows their dependence on the pion energy $T_\pi$ in channel $^{16}O(\vec{p}, d\pi^+)^{15}N$ of the reaction $^{16}O(\vec{p}, d\pi^+)^{15}N$.

4. Sensitivity of the deuteron polarization to parametrization of the two-body $pp \rightarrow d\pi^+$ amplitude

A long time ago an understanding was formed about dominant contribution of $s$- and $p$-wave components $a_0$, $a_1$, $a_2$ of the two-body $pp \rightarrow d\pi^+$ amplitude to its multipole-spin decomposition (see [19] and table 1). At present, this concept continues to serve as a key in current investigations
Table 1. Nomenclature [19] for partial amplitudes $< L_{\pi d}|t_f(E_{pp'}^{c.m.})|L_{pp'}', S_{pp'}'>$

| amplitude | $S_{pp'}$ | $L_{pp'}$ | $L_{\pi d}$ | $J$ |
|-----------|-----------|-----------|-------------|-----|
| $a_0$     | 0         | 0         | 1           | 0   |
| $a_1$     | 1         | 1         | 0           | 1   |
| $a_2$     | 0         | 2         | 1           | 2   |
| $a_3$     | 1         | 1         | 2           | 1   |
| $a_4$     | 1         | 1         | 2           | 2   |
| $a_5$     | 1         | 3         | 2           | 2   |
| $a_6$     | 1         | 3         | 2           | 3   |

of the problem. Detailed measurements of the angular distribution of reaction $pp \rightarrow d\pi^+$ performed recently in Julich with protons of energy 320 MeV [7] confirmed the weakness of $d$-components of the amplitude at this energy. Similar conclusion was made in [9] where spin transfer in reaction $\vec{p}\vec{p} \rightarrow d\pi^+$ was investigated in the proton energy range of $350 - 400$ MeV. So, experiments with higher proton energy are needed to know more about $d$-waves in the process under consideration. Calculations [15] showed that the role of next $a_3$, $a_4$, $a_5$ and $a_6$ components of table 1 could be more pronounced in both the two-body $pp \rightarrow d\pi^+$ process and nuclear reaction $(p, d\pi^+)$ if coming to $500$ MeV of TRIUMF measurements [2] and $650$ MeV of paper [15]. Our present results for reaction $^{16}O(\vec{p}, d\pi^+)^{15}N$ shown in fig. 5 under new geometry conditions confirm this expectation.
5. Conclusion
Calculation results of the present work combined with those of our preceding paper [15] show that, taking into account the shell-model advantages of $^{16}$O target against $^{12}$C in earlier studies [1, 2], coincidence measurements on reaction $^{16}$O$(p,d\pi^+)^{15}$N with polarized protons can be an important contribution to understand deeper general features of nuclear reaction $(p,d\pi^+)$ as a specific sort of nuclear quasi-free processes. General approach and our computer codes elaborated in course of the work can be used for analogous calculations within wider program of $(p,d\pi^+)$ experiments including differential cross section and analyzing power measurements for the ground and excited states of the recoil nuclei together with polarization characteristics of the produced deuteron (and, in perspective, of the excited recoil nucleus) in a wide range of conditions concerning target nuclei, energy of the beam and geometry conditions of the measurements. We hope that coordinated experimental and theoretical $(p,d\pi^+)$ studies can serve as a bridge from fundamental experiments on quasi-elastic cluster knockout reactions, such as $(p,pd)$ or $(p,p\alpha)$, to systematic coincidence measurements on pion and $\eta$-meson production in quasi-free cluster composing reactions.

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