Research Article

Optimal Energy Allocation Scheme in Distributed Estimation for Wireless Sensor Networks over Rayleigh Fading Channels

Eni D. Wardihani,1,2 Wirawan,1 and Gamantyo Hendrantoro1

1Department of Electrical Engineering, Institut Teknologi Sepuluh Nopember, Surabaya 60111, Indonesia
2Department of Electrical Engineering, Politeknik Negeri Semarang, Semarang 50275, Indonesia

Correspondence should be addressed to Eni D. Wardihani; wardihani09@mhs.ee.its.ac.id

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We focus on the issue of energy and bandwidth limitations in Wireless Sensor Networks (WSN). To tackle this issue, we propose the optimal energy allocation scheme and the optimal number of quantization bits by using alternating optimization method. Firstly, we determine the optimal energy allocation scheme which minimizes the reconstruction error on fusion center by keeping the number of quantization bits per sensor fixed. Secondly, we determine the optimal number of quantization bits per sensor such that the energy allocation scheme achieves the minimum reconstruction error. To find the optimal energy allocation scheme and the optimal number of quantization bits jointly, we propose an iterative algorithm. The results show that the proposed algorithms do not only achieve better results than equal energy allocation scheme but also produce reconstruction error close to that of unquantized estimation. This paper also investigates the effects of sensor noise observation and propagation losses on the design of optimal energy allocation scheme. The optimal energy allocation scheme suggests the allocation of more energy to sensors with small variance noise of sensor observation.

1. Introduction

Wireless Sensor Networks (WSN) is one of the new technologies growing very rapidly in the last two decades. It happens because WSN is one of the solutions to challenging problems in various fields, such as environment, health, military, industrial, and residential applications. However, many technical problems have to be faced in developing WSN technology to a broader and better technology. The main challenges of WSN are the energy and bandwidth limitations, because in many instances sensor is powered by battery which has limited lifetime. In addition, communication equipments which are used to transmit data over the wireless networks have to consider the bandwidth availability and range of sensor [1].

Energy and bandwidth efficiency algorithms on WSN have been widely investigated. Various signal processing methods have been proposed [2]. Prior study of energy and bandwidth efficiency on WSN assumes that the distribution of sensor noise observation is known [3]. Unfortunately, characterizing the exact distribution of sensor noise from observation is impractical, especially for applications in a dynamic sensing environment [4].

The authors of [5] proposed a Universal Decentralized Estimation (UDES) that does not require the knowledge of sensor noise distributions. Based on this system, several algorithms to overcome energy and bandwidth limitations have been developed, such as [6, 7]. However, all of these works assume that each sensor transmits the quantized data to the fusion center without error. This is not very realistic because the links between the sensors and the FC are affected by attenuation and fading that can degrade estimation performance. In [8–11], the authors consider an imperfect channel modelled as a Binary Symmetric Channel (BSC) and an Additive White Gaussian Noise (AWGN) channel. However, the channels are not fading in either of the cases.

In contrast to the works which have been mentioned above, the main purpose of this paper is to optimize energy and bandwidth in distributed estimation system for WSN over a flat fading Rayleigh channel with path loss effects without prior knowledge of sensor noise distribution. Some
additional details regarding the contributions of the paper are listed below:

(i) we propose the optimal energy allocation scheme and the optimal number of quantization bits by using alternating optimization method. Firstly, we determine the optimal energy allocation scheme per sensor which minimizes the reconstruction error at fusion center by keeping the number of quantization bits fixed. Secondly, we determine the optimal number of quantization bits at sensor such that the optimal energy allocation scheme achieves the minimum reconstruction error at fusion center;

(ii) we present an iterative algorithm to find the optimal energy allocation scheme and the optimal number of quantization bits jointly;

(iii) this paper investigates the effect of the sensor noise observation, which has been not considered in previous efforts [6–11].

We formulate the optimal energy allocation scheme and the optimal number of quantization bits that can minimize the mean absolute reconstruction error at the fusion center as a convex optimization which results in a close form of each of them. The analytical form of the optimal number of quantization bits in a single sensor to a fusion center suggests that the optimal number of quantization bits if the energy allocation of sensor is fixed. Afterwards, we use the optimal number of quantization bits to determine the optimal energy allocation scheme in sensor by minimizing the reconstruction error.

In the following, we firstly focus to derive the optimal number of quantization bits if the energy allocation of sensor is fixed. Afterwards, we use the optimal number of quantization bits to determine the optimal energy allocation scheme in sensor by minimizing the reconstruction error.

2.1. Optimal Number of Quantization Bits for Equal Energy Allocation in Each Bit. A simple scenario is adopted where all quantization bits have equal energy allocation $E_{bi} = E_b, \forall i$. The total transmission energy of all bits is fixed to $E$. When using an $L$-bit quantizer, the energy per bit ($E_b$) depends on $L$ since $E_b = E/L$. The channel between sensor and fusion center is flat fading Rayleigh channel with the error probability in high SNR ($\gamma \gg 1$) for BPSK modulation [12]:

$$P_e (y) = \frac{1}{4\gamma},$$

where $\gamma = \frac{E_b}{N_0} = \frac{E}{LN_0}$ and $N_0$ denotes noise power spectral density.

The reconstruction error in fusion center is defined as $A - \hat{A}$, written as follows:

$$A - \hat{A} = A - A_Q + A_Q - \hat{A}$$

$$= \sum_{i=1}^{\infty} b_i 2^{-i} + \sum_{i=1}^{L} b_i 2^{-i} - \sum_{i=1}^{L} \hat{b}_i 2^{-i} + \sum_{i=1}^{L} \hat{b}_i 2^{-i}$$

$$= \sum_{i=L+1}^{\infty} b_i 2^{-i} + \sum_{i=1}^{L} (b_i - \hat{b}_i) 2^{-i}. $$
By using triangle inequality, the limit absolute value of reconstruction error can be written as follows:

\[
|A - \hat{A}| \leq \sum_{i=L+1}^{\infty} 2^{-i} + \sum_{i=1}^{L} |b_i - \hat{b}_i| 2^{-i} \\
\leq 2^{-L} + \sum_{i=1}^{L} |b_i - \hat{b}_i| 2^{-i}.
\]

If we take the expectation of both sides of (6), we can obtain

\[
E[|A - \hat{A}|] \leq 2^{-L} + \sum_{i=1}^{L} E[|b_i - \hat{b}_i|] 2^{-i},
\]

where \(E[|b_i - \hat{b}_i|] = P_{\epsilon}(y)\), so that

\[
E[|A - \hat{A}|] \leq 2^{-L} + P_{\epsilon}(y) \sum_{i=1}^{L} 2^{-i} \\
\leq 2^{-L} + P_{\epsilon}(y) (1 - 2^{-L}) \\
\leq 2^{-L} + (1 - 2^{-L}) P_{\epsilon} \left( \frac{E}{LN_0} \right) := f(L).
\]

Based on (8), reconstruction error on fusion center can be minimized by determining the value of \(L\) which minimizes \(f(L)\) function. Hence, the optimal number of quantization bits is

\[
L_{\text{opt}} = \arg\min_L f(L) \\
= \arg\min_L 2^{-L} + (1 - 2^{-L}) P_{\epsilon} \left( \frac{E}{LN_0} \right).
\]

In order to determine the mean absolute reconstruction error with respect to \(\bar{p} := [p_1, \ldots, p_L]^T\), we can formulate the optimization problem as follows:

\[
\min_{\bar{p}} f_o(\bar{p}; L) := 2^{-L} + \sum_{i=1}^{L} p_i \left( \frac{p_i E}{N_0} \right) 2^{-i} \\
\text{subject to } f_i(\bar{p}) := p_i \geq 0, \quad i = 1, \ldots, L,
\]

\[
g(\bar{p}) := \sum_{i=1}^{L} p_i = 1.
\]

**Proposition 1.** (1) The objective function in (II) is a convex function.

(2) For a given \(L\), (II) admits a unique solution \(\bar{p}_L^* := [p_L^*, \ldots, p_L^*]^T\) that can minimize the objective function \(f_o(\bar{p}_L; L)\):

\[
p_i^* = \left( \frac{1}{\sqrt{4(E/N_0)2^{i\nu^*}}} \right), \quad i = 1, \ldots, L,
\]

where \(\nu^*\) is a constant chosen such that \(\sum_{i=1}^{L} p_i^* = 1\) can be fulfilled.

**Proof.** See Appendix A. \(\square\)

### 3. Distributed Estimation System: Multisensor to Fusion Center

The distributed estimation system assumed herein consists of a set of sensors and a fusion center with a star topology. This system is used to observe and estimate an unknown deterministic parameter \(\theta\) as shown in Figure 2. Each sensor makes an observation which is corrupted by additive noise and described by

\[
x_k = \theta + n_k, \quad k = 1, 2, \ldots, K,
\]

where \(x_k \in [-W, W]\) and \(W\) is sensor observation range that depends on sensor specification. The sensor noises \(n_k, k = 1, 2, \ldots, K\) are zero mean, spatially uncorrelated with variance \(\sigma_n^2\). The sensor observations are normalized in the range \([0, 1]\) by using linear transformation. Sensor \(k\) quantizes its observation \(x_k\) to the \(L_k\) quantization bits and the quantization results are

\[
(x_k)_Q = \sum_{i=1}^{L_k} b_{k}^{(i)} 2^{-i}. \quad (14)
\]

Bits \([b_{k}^{(i)}]_{i=1}^{L_k}\) are sent to fusion center over the wireless channel, which is again modeled as a flat fading Rayleigh channel with error probability \(P_{ext}(y)\). The signal reconstruction at the fusion center is

\[
\hat{x}_k = \sum_{i=1}^{L_k} \hat{b}_{k}^{(i)} 2^{-i}. \quad (15)
\]
The problem of interest is to design the optimal energy allocation scheme which minimizes the mean absolute reconstruction error $E[|\hat{\theta} - \theta|^2]$ when the number of quantization bits per sensor is fixed $L_k$. Then, we determine the optimal number of quantization bits per sensor such that the energy allocation scheme achieves the minimum reconstruction error $E[|\hat{\theta} - \theta|^2]$.

If the estimation error in fusion center is [13]

$$\hat{\theta} - \theta = \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{\hat{x}_k - \theta}{\sigma_k^2}$$

and we have $\theta = x_k - n_k$, then

$$\hat{\theta} - \theta = \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{\hat{x}_k - (x_k - n_k)}{\sigma_k^2}$$

$$= \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{\hat{x}_k - x_k + n_k}{\sigma_k^2}. \tag{17}$$

If reconstruction error is defined as $\hat{x}_k - x_k = \tilde{x}_k$, then

$$E[|\hat{\theta} - \theta|^2] = \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-2} E \left[ \sum_{k=1}^{K} \frac{\hat{x}_k + n_k}{\sigma_k^2} \right]^2$$

$$= \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^2 \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-2} \left( \sum_{k=1}^{K} \frac{x_k^2/\sigma_k^2}{\sigma_k^2} \right)^2$$

$$+ E \left[ \left( \sum_{k=1}^{K} \left( \frac{x_k^2}{\sigma_k^2} \right) \frac{\hat{x}_k}{\sigma_k^2} \right)^2 \left( \sum_{k=1}^{K} \frac{n_k^2}{\sigma_k^2} \right)^2 \right]$$

$$+ \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^2 \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-2} \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^2 \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-2} \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^2. \tag{18}$$

We can minimize $E[|\hat{\theta} - \theta|^2]$ by minimizing $E[\sum_{k=1}^{K} (\hat{x}_k/\sigma_k^2)^2]$. This is because for any bounded random variable $Z \in [-U, U]$ with the pdf $p(z)$, we have $E[Z]^2 = \int_{-U}^{U} |z|^2 p(z)dz = UE[Z]$. By noticing that $\sum_{k=1}^{K} (\hat{x}_k/\sigma_k^2)$ is bounded, we can instead minimize $|\sum_{k=1}^{K} (\hat{x}_k/\sigma_k^2)|^2$, where the upper bound is written as

$$\left| \sum_{k=1}^{K} \frac{\hat{x}_k}{\sigma_k^2} \right|^2$$

$$\leq \sum_{k=1}^{K} E \left| \frac{\hat{x}_k}{\sigma_k^2} \right|^2 \leq \sum_{k=1}^{K} \frac{\|\hat{x}_k\|_{\sigma_k^2} + \|x_k\|_{\sigma_k^2} + x_k}{\sigma_k^2}$$

$$\leq \sum_{k=1}^{K} \frac{\sum_{i=1}^{L_k} 2^{-i} + \sum_{i=L_k+1}^{\infty} 2^{-i}}{\sigma_k^2}$$

$$\leq \sum_{k=1}^{K} \sum_{i=1}^{\infty} 2^{-i} + \sum_{i=L_k+1}^{\infty} 2^{-i}$$

where $E[|b^{(k)}_i - b^{(k)}_j|] = P_e(y)$ is error probability of BPSK scheme in Rayleigh channels. Then, the reconstruction error can be written as

$$E[|\hat{\theta} - \theta|^2] = \sum_{k=1}^{K} \frac{2^{-L_k} + (1 - 2^{-L_k}) P_e(y)}{\sigma_k^2}. \tag{20}$$

3.1. Energy Optimization for Equal Number of Bits per Sensor

We first consider here a simple situation where each sensor transmits an equal number of quantization bits $(L_k = L)$ \forall k. The total energy of all sensors is $E_T$. Energy allocation for the kth sensor is $E_k = z_k E_T$, where $z_k$ ($k = 1, \ldots, K$) is the energy proportion of kth sensor and the energy bit of kth sensor is $b^{(k)}_i = z_k E_T$. The error probability of Rayleigh channel with BPSK modulation (4) can be expressed as

$$P_{e_k}(y) = P_{e_k} \left( \frac{E^{(k)}_b}{N_0} \right) = P_{e_k} \left( \frac{z_k E_T}{L N_0} \right). \tag{21}$$

Substituting (21) into (20), it can be obtained that

$$E[|\hat{\theta} - \theta|^2] = \sum_{k=1}^{K} \frac{2^{-L} + (1 - 2^{-L}) P_{e_k}(z_k E_T/L N_0)}{\sigma_k^2}. \tag{22}$$
The optimal energy allocation scheme is the solution for optimization problem of $\mathbf{z} := [z_1, \ldots, z_K]^T$:

\[
\min_{\mathbf{z}} f_o (\mathbf{z}; L) := \sum_{k=1}^{K} \frac{2^{-L} + (1 - 2^{-L}) P_{e_k} (z_k E_T / LN_0)}{\sigma_k^2}
\]

subject to $f_k (\mathbf{z}) := -z_k \leq 0, \quad k = 1, \ldots, K$, (23)

\[
g (\mathbf{z}) := \sum_{k=1}^{K} z_k = 1.
\]

**Proposition 2.** (1) Equation (23) is a convex function.

(2) Each sensor transmits the same fixed number of bits $L$. The channels between sensor and fusion center are flat fading Rayleigh channels. These are influenced by path loss $a_k = d_k^\alpha$, where $d_k$ is the distance between the $k$th sensor and fusion center, and $\alpha$ is path loss exponent of the wireless channel [14]. Supposing BPSK modulation the optimization problem in (23) admits a unique solution $\mathbf{z}^*_k := [z^*_1, \ldots, z^*_L]^T$ that can minimize the objective function $f_o (\mathbf{z}; L)$:

\[
z^*_k = \left( \frac{a_k L}{4 \sigma_k^2 \nu^* (E_T / N_0)} \right)^2, \quad k = 1, \ldots, K,
\]

where $\nu^*$ is a constant chosen to enforce the constraint $\sum_{k=1}^{K} z^*_k = 1$. (24)

**Proof.** See Appendix B.

Optimal number of quantization bits can be found by numerical calculations, where the value is the solution of the following problem

\[
L_{\text{opt}} = \arg \min_{L} f_{\text{opt}} (\mathbf{z}^*; L),
\]

where $f_{\text{opt}} (\mathbf{z}^*; L)$ is the optimal value of the objective function (23), if the number of quantization bits per sensor is $L$.

### 3.2. Optimal Energy Allocation Scheme for Different Number of Bits per Sensor

The $k$th sensor transmits $L_k$ quantization bits, $k = 1, \ldots, K$. The optimal energy allocation scheme that can minimize the reconstruction error at the fusion center is the solution of the following optimization problem:

\[
\min_{L_k} f_o (\mathbf{z}; L_k) := \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \left[ \frac{1}{2^{2L_k}} + \frac{1}{2^{2L_k}} P_{e_k} \left( \frac{z_k E_T}{L_k N_0} \right) \right]
\]

subject to $f_k (\mathbf{z}) := -z_k \leq 0$, (25)

\[
g (\mathbf{z}) := \sum_{k=1}^{K} z_k = 1, \quad k = 1, \ldots, K.
\]

Given a set of $L_k$, $k = 1, \ldots, K$, the optimal solution of (26) is equal to (23). The problem of interest is to obtain the minimum value of $f_o (\mathbf{z}; L_k, k = 1, \ldots, K)$ with respect to $L_k$ and $z_k$, $k = 1, \ldots, K$. Joint optimization of $L_k$ and $z_k$ can be done by the following iterative algorithm.

**Algorithm 3** (iterative algorithm). Consider the following.

(1) The step $l$, with $L_k, k = 1, \ldots, K$ finds $\mathbf{z}^l = [z_1^l, \ldots, z_K^l]$ as the optimal solution of (26).

(2) Update $L_k$ for $L_k^{l+1}$ based on iteration

\[
L_k^{l+1} = \arg \min_{L_k} \left[ \frac{1}{2^{L_k}} + \left( 1 - 2^{-L_k} \right) P_{e_k} \left( \frac{z_k E_T}{LN_0} \right) \right].
\]

(3) Back to step ($l+1$).

If $P_{e_k} (y)$ is convex and $(L_k)^K$ are fixed, then the problem in (26) is definitively convex. Therefore, the energy allocation $z_k$ can be found by numerical calculation. Step (1) of Algorithm 3 can be easily resolved. It can be proved that the objective function is always decreasing from one iteration to the next:

\[
\begin{align*}
&f_o (\mathbf{z}^{l+1}, L_k^{l+1}) \leq f_o (\mathbf{z}^l, L_k^l), \quad k = 1, \ldots, K \quad (28)
\end{align*}
\]

Simulation result shows that Algorithm 3 converges after 3-4 iterations. In Section 4, the approach to Algorithm 3 will be used to obtain the optimal values of $L_k$ and $z_k$, $k = 1, \ldots, K$.

### 4. Numerical Result

This section provides numerical examples to corroborate the analytical result derived in previous sections. Parameters that will be used in simulations are summarized in Table 1.

#### 4.1. Single Sensor to Fusion Center

In the first system, a single sensor transmits its quantized observation to the fusion center over flat fading Rayleigh channels. In the equal energy allocation scheme, along with the increasing number of quantization bits, the reconstruction error will increase. This does not happen in the optimal scheme as shown in Figure 3. The explanation for this behaviour is that as $L$ increases, the equal scheme increases the probability of error for all transmitted bits. The optimal scheme can be found by solving the convex optimization problem in (11) using the interior point method [15]. The result shows that the optimal scheme provides greater energy on a smaller bit index (Most Significant Bit) as shown in Figure 4. This is in agreement with that of [5].

#### 4.2. Distributed Estimation System: Multisensor to Fusion Center

In the second system, suppose that $K = 10$ sensors are deployed with variance noise of sensor observation denoted by $(\sigma_1, \sigma_2, \ldots, \sigma_{10})$. The distances of the sensors to the fusion center $(d_k, k = 1, \ldots, K)$ are varied. The path loss exponent of wireless channel is assumed $\alpha = 3.5$. The quantized observations are transmitted over AWGN and Rayleigh channel. The total energy budget was set to $E_T / N_0 = 200$. 

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Table 1: Parameters in simulation.

| Parameters | Specifications | Corresponding figure |
|------------|----------------|-----------------------|
| \( K \)    | 10             | Figures 3–8           |
| \( \alpha \) | 3.5            | Figures 3–8           |
| \( E_r/N_0 \) | 200           | Figures 3–8           |
| \( L = L_{opt} \) | 7, \( \forall k, k = 1, \ldots, K \) | Figure 4 |
| \( \sigma_k^2 \) | \( 0.01 \times k \) | Figures 5 and 6 |
| \( d_k \)   | \( d_k \in [1,10] \) |                       |

| Case | 1 | 2 | 3 |
|------|---|---|---|
| \( L_{opt} \) | 6, \( \forall k \) | 6, \( \forall k \) | 6, \( \forall k \) |
| \( d_k \) | 1, \( \forall k \) | \( k/4 \) | \( d_k \in [1,10] \) |
| \( \sigma_k^2 \) | \( 0.01 \times k \) | 0.01, \( \forall k \) | 0.01 \( \times k \) |

Figure 3: Reconstruction error with equal energy allocation scheme among bits as in (8) and optimal energy allocation scheme among bits as in (II) for single sensor to FC over Rayleigh channel.

By using the parameters in Table 1, Figure 5 shows the comparison between equal energy allocation scheme and optimal energy allocation scheme in (23) for distributed estimation system over AWGN and Rayleigh channels. This comparison is done for a variable number of bits \( L \) by defining a specific set of values for \( \{d_k\}_{k=1}^{10} \) and \( \{\sigma_k^2\}_{k=1}^{10} \). Both types of channels show the benefits of optimal energy allocation scheme. In Rayleigh channel, the optimal energy allocation scheme is effective to decrease the reconstruction error significantly. It is observed that the increasing number of quantization bits for each sensor decreases the reconstruction error to the floor. The optimal number of quantization bits in (25) ranges from 5 to 10. After the optimal number of quantization bits, the increasing number of quantization bits will not influence the reconstruction error at the fusion center.

Figure 4: Optimal energy allocation scheme for single sensor to FC with \( L = L_{opt} = 7 \).

Figure 6 compares equal energy allocation scheme, optimal energy allocation scheme in (23), and joint optimization by using iterative algorithm method in (26). It shows that Algorithm 3 has the best performance. Simulation is done assuming the total number of quantization bits is equal in every sensor.

As explained in Section 3.2, we can find the optimal value of \( z_k \) and \( L_k, k = 1, \ldots, K \), jointly by using Algorithm 3. The resulting optimal energy allocation scheme and the optimal number of quantization bits per sensor are depicted in Figures 7 and 8. Figure 7 shows the optimal energy allocation scheme among sensors if we choose the optimal value of \( L \) in (25), \( L_k = L_{opt} \). The appropriate optimal energy allocation scheme is the numerical solutions of the convex problem in (23). We also investigated the effect of variance noise of sensor toward the optimal energy allocation scheme. In case the distance between sensors and fusion center is equal, the optimal energy allocation scheme will allocate low energy to sensors which have high variance noise of sensor observation.
Figure 5: Reconstruction error of objective function in (23) with equal and optimal energy allocation schemes in distributed estimation system over AWGN and Rayleigh fading channel, for $K = 10$, $\sigma_k^2 = 0.01 \times k$, $k = 1, 2, \ldots, K$, and $d_k = 1$, $\forall k$.

Figure 8 shows the optimal number of quantization bits per sensor based on iteration in (27). It shows that the sensors with high variance noise of sensor observation will allocate less number of quantization bits.

5. Conclusion

Energy and bandwidth are serious limitations for the wide development of WSN. This has motivated us to propose the optimal energy allocation scheme and the optimal number of quantization bits for WSN over Rayleigh fading channels with path loss effects. We introduce a concept of alternating optimization and iterative algorithm to facilitate the solution. We formulate the optimal energy allocation scheme and the optimal number of quantization bits as a convex optimization and present these closed form. We also have found the optimal number of quantization bits that can minimize reconstruction error when the energy allocation for each bit is equal. The optimal energy allocation scheme has allocated more energy for most significant bit. We observe that the optimal energy allocation scheme decreases the reconstruction error to the floor when the number of quantization bits increases. It is in contrast with the equal energy allocation scheme, because in this scheme, the increasing number of quantization bits increases the probability of error per bit transmitted.

In distributed estimation system, we have shown that the optimal energy allocation scheme per sensor depends on the number of quantization bits per sensor, path loss, and variance noise of sensor observation. The optimal number of quantization bits per sensor can be found with the help of our convex optimization formula. In Rayleigh channel, the link between sensors and fusion center are modelled as flat fading Rayleigh channel, for $K = 10$, $\sigma_k^2 = 0.01 \times k$, $k = 1, 2, \ldots, K$, and $d_k = [1, 10]$.
Figure 8: Optimal number of quantization bits per sensor: \( L_k, k = 1, \ldots, K \). Case 1: \( d_k = 1, \forall k, \sigma^2_k = 0.01 \times k, k = 1, \ldots, K \), case 2: \( d_k = k/4, \sigma^2_k = 0.01, \forall k \), and case 3: \( d_k \in [1, 10] \), \( \sigma^2_k = 0.01 \times k \), \( k = 1, \ldots, K \).

The optimal energy allocation scheme is effective to decrease the reconstruction error significantly. It is observed that the increasing number of quantization bits for each sensor decreases the reconstruction error to the floor. After the optimal number of quantization bits, the increasing number of quantization bits does not influence the reconstruction error at the fusion center. The optimal number of quantization bits produces reconstruction error close to that of unquantized estimation. The optimal energy allocation scheme suggests the allocation of more energy to sensors with small variance noise observation. Joint optimization using our iterative algorithm has the best estimation performance compared to equal and optimal energy allocation schemes, individually.

Appendices

A. Proof of Proposition 1

(1) Problem (II) is convex if and only if \( P_e(\gamma) \) is convex. In special cases, for example, BPSK over flat fading Rayleigh channel, error probability in high SNR is as in (4). \( P_e(\gamma) \) is convex if and only if \( \nabla^2 P_e(\gamma) \geq 0 \), which reduces to the simple condition \( P_e''(\gamma) \geq 0 \). The second derivative of \( P_e(\gamma) \) to \( \gamma \) is

\[
P_e''(\gamma) = \frac{1}{2\gamma^3}.
\]

(2) \( \overline{p}_L := [p^*_1, \ldots, p^*_L]^T \) is the optimal solution of \( f_o(\overline{p}_L; L) \) when transmitting \( L \) bits and \( \overline{p}^*_L := [p^*_1, p^*_2, \ldots, p^*_L]^T \) is the optimal solution of \( f_o(\overline{p}_L; \tilde{L}) \) when transmitting \( \tilde{L} \) bits.

Because \( L > \tilde{L} \), we can construct the following \( L \) dimensional vector

\[
\overline{p}_L := [p^*_1, \ldots, p^*_L]^T = \left[ \overline{p}^T_L, 0, \ldots, 0 \right]^T \ .
\]

By construction, \( \overline{p}_L \) is a feasible point for the optimization problem in (II) when transmitting \( L \) bits because

\[
p_i \geq 0, \quad i = 1, \ldots, L, \quad \sum_{i=1}^L p_i = \sum_{i=1}^{\tilde{L}} p^*_i = 1.
\]

As \( \overline{p}_L \) is the optimal solution when transmitting \( L \) bits, we have

\[
f_o(\overline{p}_L; L) \geq f_o(\overline{p}_L; \tilde{L}).
\]

We can write down \( f_o(\overline{p}_L; L) \) explicitly as [8]

\[
f_o(\overline{p}_L; \tilde{L}) = 2^{-\tilde{L}} + \sum_{i=1}^{\tilde{L}} p_i E N_0 \]

\[
= 2^{-\tilde{L}} + \sum_{i=1}^{\tilde{L}} p_i E N_0 2^{-i} + \sum_{i=\tilde{L}+1}^{L} p_i E N_0 2^{-i}
\]

\[
= 2^{-\tilde{L}} - 2^{-\tilde{L}+1} + f_o(\overline{p}_L, \tilde{L}) + \frac{1}{2} \sum_{i=\tilde{L}+1}^{L} 2^{-i}
\]

\[
= 2^{-\tilde{L}-1} - 2^{-\tilde{L}} + f_o(\overline{p}_L, \tilde{L}) < f_o(\overline{p}_L, L).
\]

If \( P_e(0) = 1/2 \) from (A.4) and (A.5), it follows readily that \( f_o(\overline{p}_L; L) < f_o(\overline{p}_L; \tilde{L}) \).

The objective function (II) is convex and \( \overline{p}_L^* := [p^*_1, \ldots, p^*_L]^T \) is the optimal solution that can minimize \( f_o(\overline{p}_L; L) \). Then, we can use Karush Kuhn Tucker (KKT) condition, which shows that there are \( \{\lambda^*_i\}_{i=1}^L \) and \( \nu^* \) until [15]

\[
p_i^* \leq 0, \quad \lambda^*_i \geq 0, \quad p_i^* \lambda^*_i = 0, \quad i = 1, \ldots, L, \quad (A.6)
\]

\[
\sum_{i=1}^L p^*_i = 1, \quad (A.7)
\]

\[
\forall f_o, \left( p^*_i; L \right) + \sum_{i=1}^L \lambda^*_i f_i (p^*_i) + \nu^* \nabla g_i (p^*_i) = 0, \quad (A.8)
\]

where \( \nabla \) is the gradient of (A.8). It follows from (A.6)–(A.8) that the \( \{p^*_i\}_{i=1}^L \) must meet

\[
2^{-i} E \frac{dP_e(\gamma)}{dy} \bigg|_{\gamma=(E/N_0)p_i^*} - \lambda^*_i + \nu^* = 0, \quad (A.9)
\]

\[i = 1, \ldots, L.\]
We take the special case of BPSK with error probability as in (4). Derivative of \( P_e(y) \) to \( y \) can be calculated as follows:

\[
\frac{dP_e(y)}{dy} = -\frac{1}{4y^2}. \tag{A.10}
\]

Substitution of (A.10) into (A.9) results in the form of optimal energy allocation scheme as follows:

\[
P_i^* = \left( \frac{1}{\sqrt{4(E/N_0)^2 v^*}} \right), \quad i = 1, \ldots, L. \tag{A.11}
\]

**B. Proof of Proposition 2**

(1) Problem (23) is convex if and only if \( P_i(y) \) is convex. The proof is like that of Proposition 1.

(2) If the objective function (23) is convex and \( \frac{\partial}{\partial \gamma} P_i(y) \) exists, then (A.1) becomes

\[
\frac{1}{\sigma_k^2 LN_0} \frac{dP_e(y)}{dy} \bigg|_{y=(E_i/LN_0)\lambda_k} = -\lambda_k^* + v^* = 0, \quad k = 1, \ldots, K. \tag{A.12}
\]

where \( \nabla \) is the gradient of (B.3). From (B.1)–(B.3), the following must be satisfied

\[
\frac{1}{\sigma_k^2 LN_0} \frac{dP_e(y)}{dy} \bigg|_{y=(E_i/LN_0)\lambda_k} = -\lambda_k^* + v^* = 0, \quad k = 1, \ldots, K. \tag{B.4}
\]

If the channel between \( k \)th sensor with FC is influenced by path loss \( a_k = d_k^\alpha \), where \( d_k \) is the distance between the \( k \)th sensor and FC and \( \alpha \) is path loss exponent of the wireless channel [14], then (A.1) becomes

\[
P_e(y) = \frac{a_k}{4y}. \tag{B.5}
\]

Its derivative to \( y \) can be calculated as follows:

\[
\frac{dP_e(y)}{dy} = -\frac{a_k}{4y^2}. \tag{B.6}
\]

By substituting (B.6) into (B.4), we can obtain the optimal energy allocation scheme as follows:

\[
z_k^* = \left( \frac{a_k L}{4\sigma_k^2 v^* (E_i/LN_0)} \right), \quad k = 1, \ldots, K. \tag{B.7}
\]

**Notations**

- \( A \): Sensor observation in single sensor to fusion center system
- \( \theta \): The deterministic parameter to be estimated
- \( n \): Sensor noise observation
- \( A_Q \): The quantization result of \( A \)
- \( A_0 \): The estimation of \( A \)
- \( L \): The number of quantization bits
- \( i \): Index of bit
- \( b \): The \( i \)th quantization bits
- \( E_i \): The energy per bit
- \( E_{bi} \): The energy allocation for \( i \)th bit
- \( E \): The total energy of all bits
- \( \gamma \): Signal-to-noise ratio
- \( N_0 \): Noise power spectral density
- \( P_e \): The error probability
- \( P_i \): The energy proportion of \( i \)th bit
- \( P_i^* \): The optimal energy proportion of \( i \)th bit
- \( K \): Total number of sensor
- \( k \): Index sensor
- \( W \): Sensor observation range
- \( x_k \): The observation of \( k \)th sensor
- \( (x_k)_Q \): The quantization result of \( x_k \)
- \( b_k \): Error probability of \( k \)th sensor
- \( E_{bi} \): The energy allocation for \( i \)th bit
- \( E_{bi} \): The energy proportion of \( i \)th bit
- \( E_{bi}^k \): The total energy of all bits
- \( x_k \): The number of quantization bit of \( k \)th sensor
- \( E_{rk} \): The number of quantization bit of \( k \)th sensor
- \( z_k^* \): The energy proportion of \( k \)th sensor
- \( z_k^* \): The optimal energy proportion of \( k \)th sensor
- \( E_{rk} \): The energy per bit of \( k \)th sensor
- \( d_k \): The distance between \( k \)th sensor and FC
- \( \alpha \): Path loss exponent
- \( \alpha_k \): Path loss.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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