Color-Octet Contribution and Direct CP Violation in $B \to \psi(\psi')X$

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Abstract

We study $c\bar{c}$ color-octet contribution to $B \to \psi(\psi')X$. When this contribution is included, the theoretical predictions for the branching ratios become in much better agreement with the experiment. This mechanism also enhances the partial rate asymmetries by about a factor of five. The inclusive $\psi(\psi')$ resulting from $b \to d + \text{gluon}$ can have asymmetry around a few percent whereas those from $b \to s + \text{gluon}$ has it around $4 \times 10^{-4}$. The asymmetry in the former modes should be observable, to a significance of $3\sigma$, with about $(1 \sim 10) \times 10^8 B$ mesons.
Recent experimental data from the Tevatron indicate that if only $c\bar{c}$ color-singlet contribution is included the $\psi'$ production rate at large transverse momentum predicted by QCD is about a factor of 30 below the experimental data. It has been shown by Braaten and Fleming \cite{1} and Cho and Leibovich \cite{2} that if the $c\bar{c}$ color-octet also contributes to the $\psi'$ production, the experimental data can be explained. Color-octet also has significant contribution to the $\psi$ production at the Tevatron \cite{2}. In this paper, we show that, if this mechanism is indeed the correct one, it also has important implications for $\psi(\psi')$ decays of $B$ mesons, especially for CP violating particle-antiparticle partial rate asymmetry. The point is that the penguin graph leads to an appreciable branching ratio ($\simeq 10^{-2}$) for $b \to s + \text{gluon}$. Now, in a purely perturbative approach, the formation of $\psi(\psi')$ from the gluon is severely suppressed. On the other hand, if Braaten et al.’s mechanism tends to enhance the rate for the color-octet ($c\bar{c}$ or gluon) to form the $\psi(\psi')$ then it can have important consequences for direct CP violation in inclusive $B$ decays, via $B \to \psi(\psi') + X$. This mechanism also enhances the branching ratios in these decays so that they are much closer to the experimentally measured ones.

In the SM the amplitudes for $B$ decays are generated by the following effective Hamiltonian:

$$H_{\text{eff}}^q = \frac{G_F}{\sqrt{2}}[V_{tb}V_{ts}^* (c_1 O_{1f}^q + c_2 O_{2f}^q) - \sum_{i=3}^{10} (V_{ub}V_{us}^* c_i + V_{cb}V_{cs}^* c_i + V_{tb}V_{ts}^* c_i) O_{i}^q] + H.C., \quad (1)$$

where the superscripts $u, c, t$ indicate the internal quarks, $f$ can be $u$ or $c$ quark. $q$ can be $d$ or $s$ quark depending on if the decay is a $\Delta S = 0$ or $\Delta S = -1$ process. The operators $O_i^q$ are defined as

$$O_{1f}^q = \bar{q}_a\gamma_\mu L f_B \bar{f} \gamma^\mu L b_\alpha, \quad O_{2f}^q = \bar{q}_a\gamma_\mu L f_B \bar{f} \gamma^\mu L b,$$

$$O_{3,5}^q = \bar{q}_a L b q' \gamma_{\mu} L(R) q', \quad O_{4,6}^q = \bar{q}_a \gamma_\mu L b q' \gamma_{\mu} L(R) q'_a,$$

$$O_{7,9}^q = \frac{3}{2} e q' \bar{q}_a \gamma_\mu L b q' \gamma_{\mu} R(L) q'_a, \quad O_{8,10}^q = \frac{3}{2} e q' \bar{q}_a \gamma_\mu L b q' \gamma_{\mu} R(L) q'_a,$$

where $R(L) = 1 + (-)\gamma_5$, and $q'$ is summed over $u, d, s, \text{and } c$. $O_{1,2}$ are the tree level and QCD corrected operators. $O_{3,6}$ are the strong gluon induced penguin operators, and operators $O_{7,10}$ are due to $\gamma$ and Z exchange, and “box” diagrams at loop level. The WC’s
$c_i^f$ are defined at the scale of $\mu \approx m_b$. Although the WC’s have been evaluated to the next-to-leading order in QCD \cite{3, 4}, we will use the leading order WC’s to be consistent with the matrix elements which were evaluated using NRQCD to the leading order. We give the coefficients below for $m_t = 176$ GeV, $\Lambda_4 = 0.2$ GeV, and $\mu = m_b = 5$ GeV,

$$c_1 = -0.249, \quad c_2 = 1.108, \quad c_3^t = 0.0116, \quad c_4^t = -0.0249, \quad c_5^t = 0.0073, \quad c_6^t = -0.0300,$$

$$c_7^t = 0.0011, \quad c_8^t = 0.0004, \quad c_9^t = -0.0092, \quad c_{10}^t = 0.0021,$$

$$c_{3,5}^{u,c} = -c_{4,6}^{u,c}/N = P_s^c/N, \quad c_{7,9}^{u,c} = P_e^{u,c}, \quad c_{8,10}^{u,c} = 0;$$

(3)

where $N$ is the number of colors. The leading contributions to $P_{s,e}^i$ are given by: $P_{s}^i = (\alpha_s/8\pi)c_2(10/9 + G(m_i, \mu, q^2))$ and $P_{e}^i = (\alpha_{em}/9\pi)(Nc_1 + c_2)(10/9 + G(m_i, \mu, q^2))$. The function $G(m, \mu, q^2)$ is given by

$$G(m, \mu, q^2) = 4 \int_0^1 x(1-x) d\ln \frac{m^2 - x(1-x)q^2}{\mu^2},$$

(4)

where $q$ is the momentum carried by the virtual gluon in the penguin diagrams. When $q^2 > 4m^2$, $G(m, \mu, q^2)$ becomes complex. In our calculation, we will use $m_u = 5$ MeV and $m_c = 1.35$ GeV. Although there is considerable uncertainty in the effective values of these masses, especially so for $m_u$, our results are rather insensitive to the numerical values of these masses.

Using factorization approximation, we have

$$M(B \to J/\psi X_s) = \frac{G_F}{\sqrt{2}} |A_1| < X_s^1 |\bar{s}\gamma_{\mu}(1 - \gamma_5)b|B > < \psi(\psi')|\bar{c}\gamma^{\mu}(1 - \gamma_5)c|X^1 >$$

$$+ 2A_8 < X_s^8 |\bar{s}\gamma_{\mu}(1 - \gamma_5)T^a b|B > < \psi(\psi')|\bar{c}\gamma^\mu(1 - \gamma_5)T^a c|X^8 >$$

$$A_1 = [V_{cb}V_{cs}^*(c_1 + \frac{c_2}{N} - c_3^c - \frac{c_4^u}{N} - c_5^u - \frac{c_6^u}{N} - c_7^c - \frac{c_8^u}{N} - c_9^u - \frac{c_{10}^u}{N})$$

$$- V_{tb}V_{ts}^*(c_3 + \frac{c_4^u}{N} + c_5^u + \frac{c_6^u}{N} + c_7^c + \frac{c_8^u}{N} + c_9^u + \frac{c_{10}^u}{N})]$$

$$A_8 = V_{cb}V_{cs}^*(c_2 - c_4^c - \frac{c_6^u}{N} - c_8^u - \frac{c_{10}^u}{N}) - V_{tb}V_{ts}^*(c_4^u + c_6^u + c_8^u + c_{10}^u).$$

(5)

where $c_i^c = c_i^c - c_{i}^u$ and $c_i^u = c_i^u - c_{i}^c$, $X_s^1 + X^1 = X_s^8 + X^8 = X_s$. The term proportional to $A_1$ is the color-singlet amplitude and the term proportional to $A_8$ is the color-octet amplitude.
If the color-octet contribution is neglected, the branching ratios for $B \rightarrow \psi(\psi')X$ are too small compared with the experimental values. In order to reproduce the experimental data, the number of colors $N$ is traditionally treated as a free parameter to parametrize the non-factorizable and the color-octet contributions. The effective number of colors $N$ is then determined from $B \rightarrow \psi(\psi')X$ to be close to 2 \cite{5}. This does not really identify where the new contributions come from. However, if the color-octet effects identified above have significant contributions, one may not need to treat $N$ as a free parameter; $N = 3$ as given by QCD may work fine. The color octet mechanism with $N = 3$ indeed improves the situation significantly. This has been pointed out by Ko, Lee and Song \cite{6}. Our calculations confirm their results. In their work the penguin contributions are not included. When penguin contributions are included, they have important implications for direct CP violation in particle-antiparticle partial rate asymmetries in these decays although their effect on the absolute rates is minimal.

In order to obtain non-zero partial rate asymmetry, it is necessary to have non-zero CP violating phases and CP conserving strong phases due to the final state rescattering for different amplitudes. In the above case, the CP violating phases are provided by the phases in the CKM matrix elements $V_{ub}V_{cs}^*$ and $V_{tb}V_{ts}^*$. We will use the Wolfenstein parametrization such that the element $V_{ub}$ is given by $|V_{ub}|e^{-i\gamma}$. For the strong phases, we will use the phases generated at the quark level by appealing to quark-hadron duality. This should be at least a good indication for the size of the phases \cite{6}. Naively the strong phases are generated by exchanging $u$ and $c$ quarks in the loop. However, CPT theorem dictates that the phases generated by the $c$ quark in the loop not to contribute to the rate asymmetry for the production of $\psi(\psi')$ \cite{8,9,10}. Both the strong penguin WC’s $c_{3,4,5,6}^u$ and the electroweak penguin WC’s $c_{7,8,9,10}^u$ contribute to the strong phases. It is, however, interesting to note that if the color-octet amplitude is neglected, the strong penguin do not generate non-zero strong phases because the combination $c_3^u + c_4^u/N + c_5^u + c_6^u/N$ in $A_1$ is identically zero as can be seen from Eq.3. The leading non-zero strong phases are then generated by electroweak penguin and are therefore small. If these phases are the only ones, the partial rate asymmetry is
predicted to be very small as shown in Figure 1 \[10\]. Here we have used \( N = 2 \) since no color-octet contributions have so far been included as discussed before. If the color-octet contribution turns out to be significant, the situation can become dramatically different.

Including the color-octet contribution, we have

\[
|M(B \rightarrow \psi(\psi')X_s)|^2 = G_F^2 \text{Tr}(P_s + m_s)\gamma_\mu(P_b + m_b)\gamma_\nu(1 - \gamma_5) \\
\times \left(-g_\mu\nu + \frac{P_\mu^{\psi(\psi')}P_\nu^{\psi(\psi')}}{m_\psi^{\psi(\psi')}}\right) \frac{2m_c}{3} \left< O_1^{\psi(\psi')(3S_1)} \right> \\
\times \left[ |A_1|^2 + \frac{2}{N} |A_8|^2 \left< O_8^{\psi(\psi')(3S_1)} \right> \right],
\]

where the operators \( O_1, O_8^{\psi(\psi')(3S_1)} \) are defined in Ref. \[11,12\].

From eq.6, the branching ratios and the CP violating partial rate asymmetries can be easily calculated. To quantitatively assess the importance of the color-octet contribution we recall that it has been shown by Ref. \[1,2\] that even with a small matrix element for color-octet to produce a \( \psi(\psi') \), the experimental data from the Tevatron can be understood. Fitting the experimental data from the Tevatron, Cho and Leibovich obtain \[2\]

\[
< O_8^{\psi(\psi')(3S_1)} > = 1.2 \times 10^{-2}\text{GeV}^3, \\
< O_8^{\psi'(3S_1)} > = 7.3 \times 10^{-3}\text{GeV}^3.
\]

The color-singlet matrix elements determined from leptonic decays of \( \psi \) and \( \psi' \) are \[3\]

\[
< O_1^{\psi(3S_1)} >= 1.32\text{GeV}^3, \\
< O_1^{\psi'(3S_1)} >= 0.53\text{GeV}^3.
\]

Without the color-octet contributions, and with \( N = 3 \), the branching ratios predicted, are several times smaller than the experimental values: \( Br(B \rightarrow \psi X) = (0.8 \pm 0.08)\% \), and \( Br(B \rightarrow \psi' X) = (0.34 \pm 0.05)\% \). The inclusion of the color-octet contributions improves the situation with the branching ratios now predicted to be: \( Br(B \rightarrow \psi X) = 0.54\% \), and \( Br(B \rightarrow \psi' X) = 0.25\% \), with \( N = 3 \). These numbers are in good agreement with Ko et al \[3\] and they are also much closer to the experimental values.
Unfortunately the results are very sensitive to $c_1$ and $c_2$. The dominant color-singlet contributions are from operators $O_{1,2}$ which are proportional to $c_1 + c_2/N$. There is an accidental cancellation here. Had one used the next-to-leading coefficient for $c_{1,2}$, the cancellation is even more severe. If one adjusts the scale $\mu$ and $\Lambda_4$ for the leading coefficient, one can get larger values for the branching ratios. There are other uncertainties in the evaluation of the branching ratios, namely, there are more operators which may contribute to the branching ratio. For example, there may be contributions from $O_{1,8}(1^S_0)$. The value for $<O_1(1^S_0)>$ is expected to be much smaller than $<O_1(3^S_0)>$. Its contribution to the branching ratios is expected to be small. The contributions from $O_8(1^S_0)$ may be potentially large because $<O_8(1^S_0)>$ may be not too much smaller than $<O_8(3^S_0)>$. If this is indeed the case, the experimental branching ratios can be easily reproduced.

When the color-octet contributions are included the strong penguin also generate strong phases through the coefficient $A_8$ in Eq. 4. These phases are much larger than the ones generated by the electroweak penguins, and therefore much larger partial rate asymmetries result. The results are shown in Figure 2. In the figure, we used $q^2 = m_{\psi(\psi')}^2$ because the $\psi(\psi')$ carries most of the momentum from the virtual gluon. We also set $\alpha_s$ at $q^2 = m_{\psi(\psi')}^2$ and the corresponding value $\alpha_s(m_{\psi(\psi')}^2) = 0.27$ for $\Lambda_4 = 0.2$ GeV. If larger $\alpha_s$ is used, the asymmetries become bigger. The asymmetry for $B \rightarrow \psi'X_s$ is slightly larger than that for $B \rightarrow \psi X_s$. This is due to the fact that the ratio of the color-octet matrix element to color-singlet is larger for the $\psi'$. We also considered the contribution from dipole penguin operators, $O_{11} = (g_s/16\pi^2)m_b\bar{s}\sigma_{\mu\nu}RT^a b G_{a\mu\nu}$ and $O_{12} = (e/16\pi^2)m_b\bar{s}\sigma_{\mu\nu} R b F_{a\mu\nu}$. Here $G_{a\mu\nu}$ and $F_{a\mu\nu}$ are the gluon and photon field strengths, respectively. It has been shown that the operator $O_{11}$ can enhance certain $B$ decay branching ratios by as much as $30\%$ \[14\]. However, its contribution to $B \rightarrow \psi'(\psi')X_s$ branching ratio is less than $10^{-4}$ and to partial rate asymmetry is less than $10^{-5}$. $O_{12}$ contributions are even smaller. We remark that even if the operator $O_8(1^S_0)$ contributes significantly to the branching ratios, it will not introduce new strong phases in the amplitude because its contributions are proportional to $(c_4^{u(t)c} - c_6^{u(t)c})$ which generate vanishing absorptive amplitudes. And therefore, it will not
affect the asymmetries evaluated here.

It is clear from comparison of Fig.1 and Fig.2 that inclusion of the color-octet enhances the asymmetries. The asymmetries are bigger by about a factor of five. At present the CP violating phase $\gamma$ is not well determined; $\sin \gamma$ can vary from 0.1 to 1. If we use the best fit value from the experimental data, $\gamma$ is about $70^\circ [15]$. With this value, the rate asymmetries for $B \to \psi(\psi')$ are about $4 \times 10^{-4}$ ($6 \times 10^{-4}$). In order to observe the asymmetries at the $3\sigma$ level in $B \to \psi X_s$ and $B \to \psi' X_s$, one would need about $4 \times 10^9 B$ decays. This number does not include any factor(s) for experimental efficiencies. So the number of B’s needed is likely to be even higher depending on the specific final states of the $\psi(\psi')$ that are accessible.

The situation with $B \to \psi(\psi') X_d$ ($X_d$ denotes states without strangeness number) is better. The analysis is similar to $B \to \psi(\psi') X_s$ case. One only needs to change the relevant CKM matrix elements $V_{cb}V^*_{cs}$ and $V_{tb}V^*_{ts}$ to $V_{cb}V^*_{cd}$ and $V_{tb}V^*_{td}$, respectively in Eq. 5. The results are shown in Figure 3. The asymmetries can be as large as 1.5%. They are about 1% (1.5%) for $B \to \psi(\psi') X_s$ with $\gamma = 70^\circ$. It is interesting to observe that these asymmetries are similar to those obtained in Ref. [16] by considering absorptive contribution from rescattering of $c\bar{c}$ color-octet states. For the $X_d$ final state the branching ratios are smaller by a factor of $|V_{cd}/V_{cs}|^2$ compared with $B \to \psi(\psi') X_s$. Therefore to observe the asymmetries in $B \to \psi(\psi') X_d$ at the $3\sigma$ level, one would need about $1 \times 10^8 B$ decays. Assuming an effective efficiency of 0.2 (i.e. including the branching ratio into some specific final state(s)) would make the actual number be more like $5 \times 10^8$.

The number of B’s needed is clearly rather large so that even B factories may have a difficult time. On the other hand the $\psi$ and $\psi'$ tend to give distinctive signal which may even be accessible in a hadronic environment with a B-detector.

In the above, we have considered the parton level processes, $b \to s(d)\psi(\psi')$. If one considers the parton level processes, $b \to s(d)Y$ with $Y$ being $\psi$ and other $c\bar{c}$ states which can materialize into $\psi$, and do not isolate each individual $Y$ and let it decay into $\psi$, then the hadron level process, $B \to \psi X$ will have more sources to provide strong phases. In addition to the parton level strong phases discussed above, there is also the possibility of generating
strong phases from resonant effects [17]. The partial rate asymmetries may be even larger than what we obtained here. We will discuss the results from these mechanisms elsewhere.

One might think that the same mechanism will also enhance the rate asymmetry in $B \rightarrow \eta_c X_s(X_d)$. It turns out that this is not true here if only $O_{1,8}(^3S_0)$ operators are included. In this case, for the same reason as for $B \rightarrow \psi(\psi')X$, the color-singlet only generates strong phases through electroweak penguins. However, the inclusion of color-octet contributions will not improve the situation because the analogous color-octet parameter $A_8(\eta_c)$ is different than the $A_8$ parameter in Eq.5. The strong penguin contribution in $A_8(\eta_c)$ is proportional to $(c_4^{u(t)c} - c_6^{u(t)c})$ which generates vanishing strong phase too. However, if the operator $O_8(^1S_0)$ contributes significantly, strong phases will be generated by the strong penguins because the contributions are then proportional to $(c_4^{u(t)c} + c_6^{u(t)c})$. There may be sizable rate asymmetries which also need further study.

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FIG. 1. The partial rate asymmetry for $B \to \psi(\psi')X_s$ without color-octet contribution with $|V_{cb}| = 0.04$, $|V_{ub}/V_{cb}| = 0.08$ and $|V_{us}| = 0.22$. The vertical axis is the asymmetry and the horizontal axis is the value in degree for the phase angle $\gamma$ in the Wolfenstein parametrization.

FIG. 2. The partial rate asymmetry for $B \to \psi(\psi')X_s$. The solid and dashed lines are for $B \to \psi X_s$, and $B \to \psi' X_s$, respectively.
FIG. 3. The partial rate asymmetry for $B \to \psi(\psi')X_s$. The solid and dashed lines are for $B \to \psi X_d$ and $B \to \psi' X_d$, respectively.