Multi-objective Optimal Sizing and Energy Management of Hybrid Energy Storage System for Electric Vehicles

Huilong Yu, Member, IEEE, Federico Cheli, Francesco Castelli-Dezza, Member, IEEE, Dongpu Cao, Member, IEEE, and Fei-Yue Wang, Fellow, IEEE

Abstract—Hybrid energy storage system (HESS) composed of lithium-ion battery and supercapacitors has been recognized as one of the most promising solutions to face against the high cost, low power density and short cycle life of the battery-only energy storage system, which is the major headache hindering the further penetration of electric vehicles. In this work, the HESS sizing and energy management problem of an electric race car is investigated as a case study to improve the driving mileage and battery cycle life performance. Compared with the existing research, the distinctive features of this work are: (1) A dynamic model and a degradation model of the battery are employed to describe the dynamic behavior and to predict the cycle life of the battery more precisely; (2) Considering the fact that the design and control problems are coupled in most cases, in order to achieve a global optimal design solution and an implementable real-time energy management system, a Bi-level multi-objective sizing and control framework based on non-dominated sorting genetic algorithm-II and fuzzy logic control (FLC) is proposed to size the HESS and to optimize the membership functions of a FLC based EMS at the same time; (3) In order to improve the optimization efficiency, a vectorized fuzzy inference system which allows large scale of fuzzy logic controllers operating in parallel is devised. At last, the Pareto optimal solutions of different HESSs are obtained and compared to show the achieved enhancements of the proposed Bi-level optimal sizing and energy management framework.

Index Terms—Hybrid energy storage system, multi-objective optimization, vectorized fuzzy interface, real-time energy management, electric vehicles.

I. INTRODUCTION

In recent years, to face the challenge of air pollution, fossil oil crisis, and greenhouse gas emissions, Electric vehicles (EVs) have gained unprecedented amount of attentions from the governments, academia and industries all over the world. After intensively developing over the last decades, the worldwide promotion and application of EVs have reached a considerable scale. However, the dynamic performance, cost, durability of an EV are still significantly influenced by the design, integration, and control of its energy storage system (ESS) [1]. It is generally known that the battery-only ESS with high cost and short cycle life has become one of the biggest obstacles hindering further penetrations of the EVs. Lithium-ion batteries with high energy density and relatively good power density dominate the most recent group of EVs in development, however, its degradation can be accelerated when there is high peak discharging/charging power demand during the acceleration/ deceleration process [2]–[4]. Alternatively, supercapacitors (SCs) can tolerate much more charging/discharging cycles and exhibit superior ability to cope with high peak power, due to their specific energy storage mechanism, but the low energy density hampers their large scale application on EVs [5], [6]. A hybrid energy storage system (HESS) composed by lithium-ion batteries and SCs which can bridge the gap between them is considered as one of the most promising solutions to solve the forgoing problems entrenched in battery-only/SC-only energy [7]–[10]. The configuration of a HESS vary with different connections of the battery, supercapacitor and DC/DC converter. The employed HESS in this work is the most studied configuration that using a bidirectional DC/DC converter to interface the supercapacitor to the DC link of the battery in parallel, where the voltage of supercapacitor can be used in a wide range [6]. Existing research has demonstrated that HESS can dramatically improve braking energy recuperation efficiency, eliminate the need for battery over-sizing, and reduce the weight and cost of the entire system [11]. However, the HESS introduces complicated sizing, energy management and integration problems [12].

The sizing problem of HESS aims to find the appropriate number of supercapacitor banks and battery cells that minimizing the cost, mass and efficiency of the HESS or maximize the battery cycle life. A multi-objective optimization problem was formulated to minimize the overall HESS size and maximize the battery cycle life, the formulated problem was solved with the sample-based global search oriented Dividing RECTangles (DIRECT) algorithm [13]. Ref. [14] proposed a multi-objective sizing approach based on non-dominated sorting genetic algorithm II (NSGA-II) for a semi-active HESS and obtained the Pareto frontier with battery capacity loss and total cost of supercapacitors as objectives. Convex optimization was introduced to solve the formulated sizing and energy management problems of different kinds of
HESSs with weighted cost as objective [1], [5].

For the energy management strategy (EMS) of HESS, both rule based approach and the optimization based approach are widely investigated [16]. A time efficient utility function-based control of a battery semi-active HESS was proposed and carried out in [17] by formulating a weighted multi-objective optimization problem, then the formulated problem is solved using the Karush-Kuhn-Tucker (KKT) conditions. Ref. [18] proposed an energy management strategy for a HESS based on fuzzy logic supervisory wavelet-transform frequency decoupling approach, which aims to maintain the state of the energy (SOE) of the supercapacitor at an optimal value, to increase the power density of the ESS and to prolong the battery lifetime. An explicit model predictive control (EMPC) system for a HESS was proposed and validated in [19] to make the HESS operating within specified constraints while allocating high and low frequency current changes to the supercapacitor and battery respectively. Ref. [20] developed a real-time predictive power management control strategy based on neural network and particle swarm optimization algorithm to minimize the total cost including battery degradation and system energy. A variable charging/discharging threshold method and an adaptive intelligence technique based on historical data was proposed in [21] to improve the power management efficiency and smooth the load of a HESS. Two real-time energy management strategies based on KKT conditions and neural network were investigated and validated with experimental work in [22] to improve the battery state of health performance of a HESS effectively.

The continuous previous efforts have improved the overall performance of HESS considerably. However, most of the aforementioned literature investigated the sizing and energy management problems separately which can not obtain the global optimal performance of the HESS since the design and control problems of it are actually coupled in practice [23]. We can only obtain the sub-optimal solution when we try to optimize one and fix the other. Some of the existing approach in literature are off-line optimization methods which are quite useful as the reference in designing real-time EMS but not appropriate for real implementation, and most of the real-time implementable EMSs are not able to achieve optimal performances. Besides, few existing work has considered the fact that the total available amount of energy could be extracted from the battery cells will vary when uses different discharging C-rates in practical cases. This dynamic situation should be considered during the optimization research especially when the HESS is operating at limit conditions. Thus, it is necessary to explore a framework that can achieve the global optimal performance by incorporating both the sizing and real-time feedback control problems of the HESS taking into account the battery dynamics.

In order to force against the drawbacks of the state-of-the-art methods, this work will explore the sizing and real-time energy management problem of HESS as a coupled problem. In particular, the HESS of an electric race car is investigated as a case study. Although win the race is the only ultimate goal on a circuit, we should try to minimize the cost for a racing team and the environmental impact caused by the waste battery as much as possible during a race or for the offline training, which can match the spirit of the electric racing better. Our goal of this work is to introduce the HESS with proper sizing parameters and optimized EMS to improve the cycle life of the battery without sacrifying the mileage of the electric race car too much. A multi-objective optimal sizing and energy management framework supported by the employed dynamic battery model, evaluation model and the proposed vectorized fuzzy inference system is proposed in this work. With this framework, one can obtain the Pareto optimal solutions of the formulated multi-objective optimal sizing and energy management problem and achieve both the optimal sizing parameters and the static parameters of the EMS at the same time for each Pareto optimal solution which are ready for further real-time implementation.

The rest of this work is organized as follows: Section 2 describes the proposed Bi-level design and energy management framework, then presents the formulation of the sizing and energy management problem. Section 3 elaborates the modellling of the battery and supercapacitor. Section 4 details the devised FLC based on vectorized fuzzy inference engine. In section 5, the simulation parameters and settings are presented in detail. Section 6 and Section 7 illustrate the obtained results and the concluding remarks respectively.

II. BI-LEVEL OPTIMAL DESIGN AND CONTROL FRAMEWORK

The proposed Bi-level optimal design and control framework is presented as Figure 1. The power demand of the driving profile $P_{dem}$, the battery state of charge $x_{SOC}$ and supercapacitor state of energy $x_{SOE}$ are the inputs of the FLC based EMS, while the outputs are requested power from the battery $P_{regbat}$ and from the supercapacitor $P_{reqsc}$. The outputs of the EMS are the inputs of the battery and supercapacitor modeled in Section III while the evaluation indexes can be calculated with the outputs of the battery and supercapacitor model.

![Fig. 1. Framework of the Bi-Level design and control](image_url)

The working scheme of the Bi-level optimal design and control is illustrated as follows. Firstly, the multi-objective algorithm will generate the sizing parameter matrix and the corresponding static tuning parameter matrix of the energy management system, in this work, the mentioned matrices are respectively the number of supercapacitor banks and the parameters of the membership functions (MFs) in different
pages of the optimization parameters. Secondly, the FLC based EMS constructed with the new membership functions will control the generated new HESS to output the demand power from the battery and supercapacitor respectively. Then, the maximum number of laps can be obtained when both the battery and supercapacitor arrives at the minimum state of charge values set in the constraints, while the capacity loss of the battery is evaluated with the average current of the battery during the whole scenario. There are quite a lot of existing literature to model the capacity loss of the lithium-ion battery. The capacity loss model is mostly validated by discharging the battery with constant current C rate, and we havent find any work that can predict the battery capacity loss dynamically with validated experimental work. Thus, we choose to estimate the capacity loss of the battery with average load as many previous work did. When the Pareto-frontier of the two evaluation indexes is obtained, the above iteration will terminate, otherwise, it will continue.

The objective of this work is to find the optimal sizing parameter \( N_{sc} \) and the parameter vector \( x_{mf} \) defining the membership functions which are respectively the key parameters of HESS design and the real-time FLC based EMS. The optimized EMS will output the requested control command series \( u(t) = [P_{reqbat}, P_{reqsc}] \) to maximize the number of traveled laps \( J_{laps} \) and battery cycle life \( J_{lifebat} \) on a given race circuit:

\[
\max J = |J_{laps}(x(t), u(t), p), J_{lifebat}(x(t), u(t), p)|
\]  

subject to:

1. the first order dynamic constraints
   \[
   \dot{x}(t) = f[x(t), u(t), t, p],
   \]
2. the boundaries of the state, control and design variables
   \[
   x_{min} \leq x(t) \leq x_{max}
   \]
3. \[
   \begin{align*}
   u_{min} & \leq u(t) \leq u_{max} \\
   p_{min} & \leq p \leq p_{max},
   \end{align*}
   \]
4. \[
   g_{min} \leq g[x(t), u(t), t, p] \leq g_{max},
   \]
5. \[
   b_{min} \leq b[x(t_0), t_0, x(t_f), t_f, p] \leq b_{max},
   \]
where \( \dot{x} \) is the first order derivative of the state variables, \( f \) is the dynamic model, \( x, u, p \) are respectively the state, control and design vector with their lower and upper bounds: \( x_{min}, u_{min}, p_{min} \) and \( x_{max}, u_{max}, p_{max} \). While \( g \) and \( b \) are the path and boundary equations respectively with their lower and upper bounds \( g_{min}, b_{min} \) and \( g_{max}, b_{max} \). The dimensions of the input and output variables in Equations (2), (3) and (5) are separately given as:

\[
\begin{align*}
  f &: \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_x} \\
  g &: \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_g} \\
  b &: \mathbb{R}^{n_x} \times \mathbb{R} \times \mathbb{R}^{n_u} \times \mathbb{R} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_b}
\end{align*}
\]

In this work, the algebraic path constraint \( g \) is eliminated by introducing a simple relaxation in Equation (16), the state variables \( x \), control variables \( u \), design parameters \( p \) and the boundary constraints \( b \) will be presented in the following paragraphs.

### III. Modelling of the HESS

In this section, the dynamic characteristics of the implemented Lithium-ion battery are analyzed first, and a dynamic battery model is employed after comparison. Then, the details of the employed battery cycle life model are presented. A simplified supercapacitor model is illustrated at the end of this section.

#### A. Dynamic Battery model

In order to obtain the optimal sizing parameters and energy management strategy for the HESS considering the characteristics of the battery in practical conditions, it is necessary to implement a proper dynamic battery model that can describe the battery dynamic behavior precisely. The most existing battery models for the simulation of battery behavior basically include the experimental, electrochemical and electrical ones [24]-[26]. However, the experimental and electrochemical model are not appropriate to simulate the battery dynamics correctly. In comparison with the experimental and electrochemical battery model, specially devised electric circuit based model are capable to represent the battery dynamics for the purpose of state of charge (SOC) estimation. In this work, a modified Shepherd model is employed to depict the dynamic characteristics of the battery during charging and discharging process [27]. The dynamic battery model are presented as Equation (6) and Equation (7) with the assumption that the internal resistance is constant and the thermal behavior of the battery is neglected.

**Discharge:**

\[
V_{batt} = E_0 - K \frac{Q_{max}}{Q_{max} - it} i - K \frac{Q_{max}}{Q_{max} - it} i - R_i + Ae^{(-B \cdot it)}
\]  

(6)

**Charge:**

\[
V_{batt} = E_0 - K \frac{Q_{max}}{Q_{max} - it} i - K \frac{Q_{max}}{it - 0.1Q_{max}} i - R_i + Ae^{(-B \cdot it)}
\]

(7)

where \( V_{batt} \) is the battery voltage (V), \( E_0 \) is the voltage constant (V), \( K \) is the polarization constant or polarization resistance, \( Q_{max} \) is the total capacity, \( i \) is the battery current, \( R_i \) is the internal resistance. The battery discharge \((i > 0)\) or charge \((i < 0)\) it is denoted as

\[
it = \int idt.
\]

(8)

The voltage amplitude \( A (V) \), time constant inverse \( B (Ah^{-1}) \) of the exponential zone, the polarization resistance \( K (\Omega) \) and the voltage constant \( E_0 (V) \) in Equations (6) and (7) are calculated as follows [28]:

\[
A = V_{Full} - V_{exp}
\]

(9)

\[
B = 3 \frac{V_{exp}}{Q_{exp}}
\]

(10)
\[ K = \frac{(V_{full} - V_{nom} + A(\exp(-BQ_{nom}) - 1))(Q - Q_{nom})}{Q_{nom}} \]  

\[ E_0 = V_{full} + K + R \cdot i - A. \]  

The typical discharge process of the lithium-ion battery is divided into three stages as demonstrated in Figure 2. In the initial stage, the battery voltage drops exponentially from the fully charged voltage \( V_{full} \), the voltage and extracted charge at the end of this stage are \( V_{exp} \) and \( Q_{exp} \) respectively. The voltage continues to decline slowly before reaching the nominal voltage \( V_{nom} \), and the corresponding discharge increases to \( Q_{nom} \) in the second stage. In the final stage, the battery voltage drops abruptly to the minimum value and the discharge reaches the maximum value \( Q_{max} \).

\[ \begin{align*} 
  &V_{bat} \\
  &V_{exp} \\
  &V_{nom} \\
  &Q_{exp} \\
  &Q_{nom} \\
\end{align*} \]

Fig. 2. Typical discharge curve of lithium-ion battery.

The state of charge of the battery \( x_{soc} \) and its derivative \( \dot{x}_{soc} \) is denoted as Equation (13) and Equation (14) respectively.

\[ x_{soc} = 100(1 - \frac{1}{3600}Q_{max}) \int_0^{t_f} i dt \]  

\[ \dot{x}_{soc} = -\frac{1}{3600}i \]  

The charging/discharging \( i \) is denoted as

\[ i = \begin{cases} 
  \frac{P_{reqbat}}{N_{bat}V_{bat}\eta_{AD}}, & P_{reqbat} \geq 0 \\
  \frac{P_{reqbat}\eta_{AD}}{N_{bat}V_{bat}}, & P_{reqbat} < 0 
\end{cases} \]  

where \( \eta_{AD} \) is the efficiency of the DC/AC converter taking into account of the motor efficiency as a constant value. In this work, the number of battery cells \( N_{bat} \) are determined by the available total mass of the HESS \( m_{HESS} \) and the number of the supercapacitor banks \( N_{sc} \), as shown in Equation (16). The total mass of the HESS is fixed in this work considering the fact that the mass of a race car is strictly limited in general.

\[ N_{bat} = \lceil (m_{HESS} - N_{sc}m_{bank})/m_{crit} \rceil \]  

Although the modified Shepherd model can represent the dynamic characteristics of the battery, it has not been employed so much in the existing literature related with the HESS design and control due to its complicity, which usually results in problems that are time-consuming and difficult to solve. There should be more efforts put in improving the optimization efficiency.

**B. Battery Cycle Life Model**

In recent years, substantial efforts have been made by both the researchers and industries to develop models that can predict the degradation of the lithium-ion batteries accurately [29]–[31]. A revised semi-empirical model based on Arrhenius equation is widely researched with large scale experiments, and this model is mostly applied in optimization and control problems related with batteries [29]. As presented from Equation (17) to Equation (20), the capacity loss of this model is expressed as a function of the discharge current rate \( C_{rate} \), temperature \( T \) and ampere-hour throughout \( A_h \).

\[ Q_{loss} = A\exp\left(\frac{-E_a}{RT}\right)(A_h)^z \]  

where \( Q_{loss} \) represents the battery capacity loss, \( A \) the pre-exponential factor, \( E_a \) the activation energy from Arrhenius law (J), \( R \) is the gas constant of 8.314, \( T \) is the absolute temperature (K), \( A_h \) is the Ah-throughput, which represents the amount of charge delivered by the battery during cycling.

The pre-exponential factor \( A \) in Equation (17) is proved to be sensitive to the discharge current rate \( C_{rate} \) with large scale experiments in [32], and it is fitted with the format as Equation (18) in [33].

\[ \ln A = a \cdot \exp(-b \cdot C_{rate}) + c \]  

The activation energy can be fitted as a linear function of discharge current rate Ref. [32].

\[ E_a = d + e \cdot C_{rate} \]  

where \( a, b, c, d, e \) are the correction parameters of the battery cycle life model.

The Ah-throughput can be expressed as

\[ A_h = \int_0^{t_f} i \frac{dt}{3600} \]  

where \( i \) is the discharge current, \( t_f \) is the end time of the current profile.

**C. Supercapacitor Model**

In this work, the capacity fading of the supercapacitor is neglected considering the fact that it has much longer cycle life than lithium-ion batteries. The supercapacitor model is simplified to a series connection of a resistance and a supercapacitor bank [6]. Also, the efficiency of the DC/DC converter between the supercapacitor and the bus is assumed to be a constant value of 0.95. The recursive supercapacitor model is deduced as

\[ \begin{align*} 
  \dot{V}_{ct} &= \begin{cases} 
  \frac{V_{ct} - \sqrt{V_{ct}^2 - 4R_{sc}P_{reqsc}/(\eta_{AD})}}{2C_{sc}R_{sc}}, & P_{reqsc} \geq 0 \\
  \frac{V_{ct} - \sqrt{V_{ct}^2 - 4R_{sc}P_{reqsc}\eta_{AD}}}{2C_{sc}R_{sc}}, & P_{reqsc} < 0 
\end{cases} 
\end{align*} \]  

(21)
where $V_{ct}$ is the total open circuit voltage of the supercapacitor pack assuming that all banks have a uniform behavior, $t_{k+1}$ is the time at step $k+1$, $R_{st}$ is the total equivalent series resistance, $P_{reqsc}$ is the demand power from the supercapacitor, $\eta_{dc}$ is the efficiency of the DC/DC converter, $C_{sc}$ is the total capacity, $SOE$ is the state of energy, $V_{ctmax}$ is the initial open circuit voltage, $V_c$ is the open circuit voltage of one supercapacitor, $N_{sc}$ is the total number of the banks, $R_s$ is the series resistance of one supercapacitor.

The actual total output power of the supercapacitor is represented as

$$P_{sc} = V_{ct} \cdot \frac{V_{ct} - \sqrt{V_{ct}^2 - 4R_{s}P_{reqsc}/\eta_{dc}}}{2R_{st}}. \tag{27}$$

IV. FLC BASED ON VECTORIZED FUZZY INFERENCE ENGINE

In this work, the EMS is developed based on fuzzy logic control (FLC), which has the features of real-time, adaptive and intelligent. It allows different operators to merge non-linearities and uncertainties in the best way and incorporate heuristic control in the form of if-then rules. The developed FLC in this section are composed of the if-then fuzzy rules, fuzzification, fuzzy inference engine and defuzzification modules. To speed up the optimization and take the advantage of the powerful matrix processing capability of MATLAB, a vectorized fuzzy inference system (VFIS) presented in Figure 3 is developed for the first time according to the state-of-the-art literature. The developed VFIS is capable to handle $N_p \times N_{inp}$ dimensional inputs with $N_p$ pages of membership functions each time. This means that $N_p$ fuzzy controllers (can be hundreds of thousands depends on the performance of the utilized CPU) can work at the same time with the same page number of inputs and outputs. The following paragraph will present the detail of fuzzy rules, membership functions, vectorized fuzzification, fuzzy inference engine, and defuzzification operations of the developed vectorized FLC.

A. Fuzzy rules

Fuzzy rules are a set of if-then linguistic rules used to formulate the conditional relationships that comprise a fuzzy logic controller, for instance, a fuzzy rule can be: if SOC is Small and SOE is Big and $P_{req}$ is Positive big then $P_{sc}$ is Positive big. It is reasonable to devise the same if-then rules for the control of different sizes of HESSs since the control objectives of all the HESSs are the same in this work. The developed fuzzy rules are demonstrated as Figure 4 where the labels N, P, S, M, B means negative, positive, small, medium and big respectively. The basic idea of the fuzzy rule is to utilize the supercapacitor as a buffer to reduce the high peak power impact on battery and absorb more regenerative braking power.

B. Membership functions

The concept of membership functions was introduced by Zadeh in the first paper on fuzzy sets [34]. A membership function is a curve or a function that defines how each point of the input variables is mapped to a membership value between 0 and 1. It is quite challenging to design the optimal MFs for each HESS manually according to the engineering experiences. Besides, considering that the performance of the FLCs are sensitive to their MFs, different MFs of the FLCs with the same fuzzy rules should be devised for different sizes of HESS. Based on these considerations, the parameters of the MFs are selected as parts of the parameters to be optimized in this work.

The trapezoidal-shaped membership function is selected for the fuzzy inference engine based on the considerations that it has high flexibility, e.g., as shown in Figure 3, when $a = b$, the trapezoid MF will change its shape to triangle [12].

C. Vectorized Fuzzification

During the fuzzification stage, the input variables are identified to the fuzzy sets (membership functions) they belong to and the respective degree of membership to each relevance will be assigned. For a FIS with trapezoidal shaped MFs and a number of $N_{inp}$ inputs, the fuzzy sets of each can be described with a matrix $X_k = [a_k, b_k, c_k, d_k] \in R^{N_p \times N_{inp} \times 4}$. $N_p$ is the total page number of the inputs; $N_{k inp}$ is the number of fuzzy linguistic sets of state input $k$, $k \in \{1, 2, ..., N_{inp}\}$ and $a$, $b$, $c$, $d$ are the variables that define one trapezoid in Figure 3. The input matrix is denoted as $x_k$, its membership matrix $\mu_k \in R^{N_p \times N_{inp}}$ can be denoted as:

$$\begin{align*}
\mu_k(a_k \leq x_k < b_k) & = \frac{x_k - a_k}{b_k - a_k} \\
\mu_k(b_k \leq x_k \leq c_k) & = 1 \\
\mu_k(c_k < x_k \leq d_k) & = \frac{d_k - x_k}{d_k - c_k}
\end{align*}$$

where $a_k, b_k, c_k, d_k, I$ belong to $R^{N_p \times N_{inp}}$, and $x_k$ is denoted as:

$$x_k = [x_k, x_k, ..., x_k] \in R^{N_p \times N_{inp}}$$

The membership array $U$ for input $X$ can be constructed as:

$$U = \{\mu_1, ..., \mu_k, ..., \mu_{N_{inp}}\} \in R^{N_p \times N_{inp} \times N_{inp}}$$

D. Vectorized Fuzzy Inference Engine

Fuzzy inference is the way of mapping an input space to an output space using fuzzy logic. A FIS tries to formalize the reasoning process of human language by means of fuzzy logic (the built fuzzy if-then rules). The process of fuzzy inference involves all of the MFs, if-then rules, linguistic variables of
the inputs and outputs. Mamdani’s fuzzy inference method is
the most commonly seen fuzzy methodology. The search-able
fuzzy inference engine is able to map only one page of the
inputs to one page of the outputs. This section will give an
elaborate description of the developed powerful VFIS which
allows a large number of FLCs operating in parallel based on
Mamdani’s fuzzy inference method.

The linguistic variables are programmed with their integer
indexes from the smallest to the biggest in turn in this work.
For instance, the fuzzy sets \{NB, NM, NS, PS, PM, PB\} of
the third input in Figure 4 are correspondingly mapped to
\{1, 2, ..., N_{ti,k}\}, here \(N_{ti,k} = 6\), \(k = 3\). The fuzzy rule matrix
\(R \in \mathcal{R}_{N_r \times (N_{mi,p} + N_o)}\) is constructed with the mapped integer
indexes, \(N_r\) is the number of fuzzy rules and \(N_o\) is the number
of outputs. For instance:

\[
\text{Rule : } x_{\text{SOC}} \ x_{\text{SOE}} \ P_{\text{req}} \ P_{\text{sc}}
\]

\[
R(N_r) : \quad 1 \quad 3 \quad 6 \quad 6
\]  

(31)

where \(R(N_r)\) denotes the fuzzy rule \(N_r\), it means the rule
like: if \(SOC\) is Small and \(SOE\) is Big and \(P_{\text{req}}\) is Positive big
then $P_c$ is Positive big. The working scheme of the VFIS is illustrated as follows:

1) Repeatedly copy the membership matrix $\mu_k$ into $N_r$ blocks, and we can obtain:

$$\mu^\text{temp} = \left[ \mu_k; \mu_k; \ldots; \mu_k \right], \mu^\text{temp} \in \mathbb{R}^{N_p \times N_{t_k} \times N_r} \quad (32)$$

2) Create index matrix $L_{in} \in \mathbb{R}^{N_p \times N_{t_k} \times N_r}$ for input $k$:

$$L_{in} = \left\{ \begin{array}{c}
\begin{array}{c}
1 \ 2 \ \cdots \ \ N_{t_k} \\
1 \ 2 \ \cdots \ \ N_{t_k} \\
\vdots \ \ \ \vdots \\
1 \ 2 \ \cdots \ \ N_{t_k}
\end{array}
\end{array} \right\} \quad (33)$$

3) Repeatedly copy the $k$th column of the rule matrix $\mathbf{R} \in \mathbb{R}^{N_r \times (N_{inp} + N_o)}$ into $N_p \times N_{t_k}$ block arrangement $\mathbf{R}^\text{temp} \in \mathbb{R}^{N_p \times N_{t_k} \times N_r}$, $k \in \{1, 2, \ldots, N_{inp}\}$:

$$\mathbf{R}^\text{temp} = \left[ \begin{array}{c}
\begin{array}{c}
\mathbf{R}_k; \mathbf{R}_k; \ldots; \mathbf{R}_k \\
\mathbf{R}_k; \mathbf{R}_k; \ldots; \mathbf{R}_k \\
\vdots \ \ \ \vdots \\
\mathbf{R}_k; \mathbf{R}_k; \ldots; \mathbf{R}_k
\end{array}
\end{array} \right] \quad (34)$$

4) Get the effective membership matrix $\mu_{eff,k}$ for input $k \in \{1, 2, \ldots, N_{inp}\}$:

$$\mu_{eff,k} = \mu_k(L_{in} \equiv = \mathbf{R}^\text{temp}), \mu_{eff,k} \in \mathbb{R}^{N_p \times N_{t_k} \times N_r} \quad (35)$$

5) Combine and get the final membership matrix $U_{in} \in \mathbb{R}^{N_p \times N_r \times N_{inp}}$ for all the input $X$:

$$U_{in} = \left\{ \bigcup_{j=1}^{N_{t_k}} \mu_{eff,k}(j), \bigcup_{j=1}^{N_{t_k}} \mu_{eff,k}(j), \ldots, \bigcup_{j=1}^{N_{t_k}} \mu_{eff,k}(j) \right\} \quad (36)$$

6) Get the mapped membership matrix $U_o$ for the output fuzzy sets:

$$U_o = \bigcap_{k=1}^{N_{inp}} U_{in}(k), U_o \in \mathbb{R}^{N_p \times N_r} \quad (37)$$

7) Create index matrix $L_o \in \mathbb{R}^{N_p \times N_r \times N_{to}}$ for output the fuzzy sets:

$$L_o = \left\{ \begin{array}{c}
\begin{array}{c}
1 \ 2 \ \cdots \ \ N_{to} \\
1 \ 2 \ \cdots \ \ N_{to} \\
\vdots \ \ \ \vdots \\
1 \ 2 \ \cdots \ \ N_{to}
\end{array}
\end{array} \right\} \quad (38)$$

8) Repeatedly copy the column of output fuzzy sets in the rule matrix $\mathbf{R} \in \mathbb{R}^{N_r \times (N_{inp} + N_o)}$ into a $N_p \times N_{to}$ block arrangement $\mathbf{R}^\text{temp} \in \mathbb{R}^{N_p \times N_{t_k} \times N_r}$, $N_{to}$ is the number of fuzzy linguistic sets of output:

$$\mathbf{R}^\text{temp} = \left[ \begin{array}{c}
\begin{array}{c}
\mathbf{R}_{N_o}; \mathbf{R}_{N_o}; \ldots; \mathbf{R}_{N_o} \\
\mathbf{R}_{N_o}; \mathbf{R}_{N_o}; \ldots; \mathbf{R}_{N_o} \\
\vdots \ \ \ \vdots \\
\mathbf{R}_{N_o}; \mathbf{R}_{N_o}; \ldots; \mathbf{R}_{N_o}
\end{array}
\end{array} \right] \quad (39)$$

9) Repeatedly copy the membership matrix of the output fuzzy sets $U_o \in \mathbb{R}^{N_{t_k} \times N_r}$ into $N_{to}$ blocks $U_o^\text{temp}$:

$$U_o^\text{temp} = \left\{ U_o, U_o, \ldots, U_o \right\}, U_o^\text{temp} \in \mathbb{R}^{N_p \times N_r \times N_{to}} \quad (40)$$

10) Get the effective membership matrix $U_{eff,o}$ of all the output fuzzy sets:

$$U_{eff,o} = U_o^\text{temp}(L_o \equiv = \mathbf{R}^\text{temp}), U_{eff,o} \in \mathbb{R}^{N_p \times N_r \times N_{to}} \quad (41)$$

11) Merge the membership matrix of the output fuzzy sets in all the fuzzy rules

$$U_{o,\text{final}} = \bigcup_{i=1}^{N_r} U_{o,eff}(i), U_{o,\text{final}} \in \mathbb{R}^{N_p \times N_{to}} \quad (42)$$

By the above calculation, the membership of each trapezoid of the output fuzzy set is obtained as $U_{o,\text{final}}$, and the next step is the defuzzification.

### E. Vectorized Defuzzification

The purpose of defuzzification process is to produce a quantifiable result in crisp logic based on the given fuzzy sets and corresponding membership degrees. The defuzzification process based on center of gravity method is demonstrated as Figure 6.

![Fig. 6. Defuzzification process based on center of gravity method](image-url)

The procedure of the vectorized defuzzification is elaborated as followings:

1) Discrete the output fuzzy sets into $N_{dis}$ parts $x_o \in \mathbb{R}^{N_{dis}}$ from its minimum value $x_{o,\text{min}}$ to the maximum one $x_{o,\text{max}}$.

$$x_o = [x_{o,\text{min}} : (x_{o,\text{max}} - x_{o,\text{min}})/(N_{dis} - 1) : x_{o,\text{max}}] \quad (43)$$
where the value of $N_{dis}$ affects the accuracy of the crisp output, for instance, the increasing of $N_{dis}$ will improve the precision but will increase the computational burden.

2) Repeatedly copy $x_o \in \mathcal{R}^{N_{dis}}$ and output fuzzy set $S_o = [a_o, b_o, c_o, d_o] \in \mathcal{R}^{N_{io} \times N_p \times N_{dis} \times 4}$, we can obtain $x_{o, temp} \in \mathcal{R}^{N_{io} \times N_p \times N_{dis}}$ and $S_{o, temp} \in \mathcal{R}^{N_{io} \times N_p \times N_{dis} \times 4}$ respectively:

$$x_{o, temp} = \left[ \begin{array}{cccc} x_o & x_o & \ldots & x_o \\ x_o & x_o & \ldots & x_o \\ \vdots & \vdots & \ddots & \vdots \\ x_o & x_o & \ldots & x_o \end{array} \right] \times N_p$$ (44)

$$S_{o, temp} = [S_o, S_o, \ldots, S_o]$$ (45)

3) Calculate the membership matrix of $x_{o, temp}$ based on the output fuzzy set $S_{o, temp} = [a_o, temp, b_o, temp, c_o, temp, d_o, temp] \in \mathcal{R}^{N_{io} \times N_p \times N_{dis} \times 4}$:

$$\mu_o(a_o, temp \leq x_{o, temp} < b_o, temp) = \frac{x_{o, temp} - a_o, temp}{b_o, temp - a_o, temp}$$

$$\mu_o(b_o, temp \leq x_{o, temp} \leq c_o, temp) = 1$$

$$\mu_o(c_o, temp < x_{o, temp} \leq d_o, temp) = \frac{d_o, temp - x_{o, temp}}{d_o, temp - c_o, temp}$$ (46)

4) Repeatedly copy the membership matrix $U_{o, final}$ in to $N_{dis}$ blocks $U_{o, temp} \in \mathcal{R}^{N_{io} \times N_p \times N_{dis}}$:

$$U_{o, temp} = \left[ U_{o, final}, U_{o, final}, \ldots, U_{o, final} \right]$$ (47)

5) Find the effective membership matrix:

$$U_{eff, o} = U_{o, temp} \bigcap \mu_o, U_{eff, o} \in \mathcal{R}^{N_{io} \times N_p \times N_{dis}}$$ (48)

6) Merge the membership matrix obtained in last step:

$$U_{o, x} = \bigcup_{i=1}^{N_i} U_{eff, o}(i), U_{o, x} \in \mathcal{R}^{N_p \times N_{dis}}$$ (49)

7) Calculate the crisp output matrix for all the input matrices:

$$y = \sum_{i=1}^{N_{dis}} x_o(i) \odot U_{o, x} (x_o(i)) \sum_{i=1}^{N_{dis}} U_{o, x} (x_o(i)) , y \in \mathcal{R}^{N_p}$$ (50)

In order to design the fuzzy rules and membership functions conveniently, the devised vectorized FLC modules illustrated above are developed in MATLAB with standard and user friendly interfaces.

V. SIMULATION PARAMETERS AND SETTINGS

The state variables includes the battery state of charge $x_{SOC}$ and state of energy of the supercapacitor $x_{SOE}$, $x = [x_{SOC}, x_{SOE}]$. The control variable output by the FLC in this work is the requested power from the supercapacitor $u = P_{reqsc}$, the demand power from the battery can be calculated by $P_{reqbat} = P_{dem} - P_{reqsc}$. The design parameter vector is $p = \{N_{sc}, x_{mf}\}$. As demonstrated in Figure 7, there are 28 parameters of the devised membership functions plus one design parameter of the HESS in one page of parameters to be optimized. The design vector $p$ is constrained by defining $p_{min}$ and $p_{max}$.

The operating profile of an electric race car is of great difference with the one of conventional electric vehicle running on a city road. Thus, standard driving cycles are not suitable for the research on electric race car. The real driving cycle of a race car in Nurburgring circuit is chosen as the test scenario. The demand driving/braking power is calculated by Equation (51). The corresponding velocity profile, acceleration profile and demand power are demonstrated as Figure 8.

$$P_{dem} = \left( \frac{1}{2} \rho C_d A v^2 + f m_r v + m_g a \right) v$$ (51)

The detail simulation parameters of the race car, 53 Ah high energy lithium-ion battery, 2.85V/3400F high performance supercapacitor and the converters are illustrated in Table I.

In the FLC based EMS, the SOE and current of the supercapacitor are constrained between 0.1 and 0.99, -2000 A and 2000A respectively. While the SOC of the lithium-ion battery is constrained between 0.2 and 0.9, the current is regulated by adjusting the requested power from the battery. When the lithium-ion battery is exhausted, the simulation of one iteration will be terminated and the objective functions will be correspondingly evaluated. The temperature is for sure very important in any kind of vehicle equipped with batteries since it can affect the performance of the batteries directly. However, it is very difficult to model the heat generation, dispassion and the thermal control system of the energy storage system on an
that the temperature is controlled at a constant value (23 °C) electric vehicle precisely. Actually, it is reasonable to assume population even if their fitness values are relatively lower. individuals that can assist to improve the diversity of the nondominated fronts, the controlled elitist GA also favors optimization problem. Instead of only choosing the top-ranking NSGA-II [36] is implemented to solve the multi-objective op-

time constant inverse of the battery cell (s) \( h \)

\[
\begin{array}{lll}
\text{Parameters} & \text{Symbol} & \text{Value} \\
\text{Vehicle mass (kg)} & m_\text{v} & 570 \\
\text{Aerodynamics coefficient (h^2 N/km^2)} & \rho C_d A & 0.075 \\
\text{Rolling resistance coefficient} & f & 0.016 \\
\text{Mass of the battery cell (kg)} & m_\text{cell} & 1.15 \\
\text{Voltage constant of the battery cell (V)} & E_0 & 3.43 \\
\text{Total capacity of the battery cell(Ah)} & Q_{\text{max}} & 55 \\
\text{Polarization resistance of the battery cell (Ω)} & K & 8.85 \times 10^{-5} \\
\text{Internal resistance of the battery cell (Ω)} & R & 1.33 \times 10^{-3} \\
\text{Voltage amplitude of the battery cell (V)} & A & 0.761 \\
\text{Time constant inverse of the battery cell (s)} & B & 0.040 \\
\text{Fitting parameter of pre-exponential factor} & a & 1.345 \\
\text{Fitting parameter of pre-exponential factor} & b & 0.2563 \\
\text{Fitting parameter of pre-exponential factor} & c & 9.179 \\
\text{Fitting parameter of activation energy} & d & 46688 \\
\text{Fitting parameter of activation energy} & e & -470.3 \\
\text{Mass of the supercapacitor bank (kg)} & m_{\text{bank}} & 0.52 \\
\text{Supercapacitor bank capacity(F)} & C_{\text{bank}} & 3400 \\
\text{Supercapacitor equivalent series resistance (Ω)} & R_s & 2.2 \times 10^{-4} \\
\text{DC/DC converter efficiency} & \eta_{\text{dc}} & 0.95 \\
\text{DC/AC converter efficiency} & \eta_{\text{AC}} & 0.96 \\
\end{array}
\]

From the sizing point of view, using different number of supercapacitors means different compromises between high power density and high energy density. As it is demonstrated in Figure 9 utilizing more supercapacitors can assist to reduce the average current of the Lithium-ion battery which is beneficial for longer cycle life of the battery, but cut down the energy density of the HESS which results in shorter driving mileage. When less supercapacitors are used, the results will be opposite. It is also observed from Figure 9 that HESS with the same design solutions (makers filled with the same color) may achieve different values of both objective functions, which means that for the same HESS with uniform fuzzy rules, the parameters of the membership functions will determine whether we can achieve the Pareto optimal solutions. Thanks to the proposed Bi-level optimal sizing and control framework, the corresponding sizing parameter \( N_{sc} \) of each HESS and the membership function parameters \( x_{mf} \) of the related EMS are coupled and obtained at the same time for all the solutions including those on the Pareto frontier.

Moreover, this work has investigated the optimal sizing and control results of HESSs with different total mass. From Figure 10 we can drawn the following basic conclusions: 1) HESSs with smaller total mass will cover fewer number of available laps, but the available cycle life of the battery are longer due to their shorter operating mileage; 2) We can achieve a pretty decent compromised solution that can enhance both objective functions with only about 40 supercapacitor banks and the optimized membership functions.

In order to analyze the reason of the exhibited advantages of the proposed Bi-level optimal sizing and control framework, one solution from the Pareto frontier in Figure 9 \((N_{sc} = 32)\) is compared with the solution with same sizing parameter but the initial devised membership functions. Figure 11 demonstrates the initial and optimized membership functions with the dotted lines and solid lines respectively.

The achieved available number of laps of the initial and optimized solutions are very similar which are respectively 17.88 and 17.98. This is mostly due to the fact that the two cases are implemented with the same HESS and the available
mileage is mainly determined by the sizing parameters rather than the control parameters. However, the available cycle life of the battery are different which are respectively 6082 and 6463. This means that HESS with the optimized membership functions improved the cycle life by 6.3%. Figure 12 presents the interested variables between 0-200s, as it is illustrated in Figure 12 (a) and Figure 12 (b), the EMS with initial devised membership functions tends to request more high peak power from the battery and less from the supercapacitors which will accelerate the degradation of the battery. This phenomenon can be explained with the curve of SOE in Figure 12 (c). We can see that EMS with the initial devised membership functions tends to exhaust the supercapacitors very fast at a few seconds after starting the operation and the average SOE is under 20% during the simulation which is not capable to provide long-time high peak power to protect the battery. While EMS with the optimized membership functions tends to maintain the SOE of the supercapacitors above 50%, which helps to play the role of shaving the peak and filling the valley very well during the whole driving profile. For instance, the curves in the dotted box in Figure 12 (c) demonstrate that the requested power from the battery is less after optimizing the MFs since the SOE is maintained at a relatively high level due to the optimized EMS.

VII. CONCLUSIONS

More supercapacitors do not always guarantee a better overall performance especially when the total mass of the HESS is limited. However, we are able to obtain a pretty good balanced performance with less supercapacitors and the optimized EMS by the proposed optimization framework. The proposed Bi-level optimal sizing and control framework in this work makes it possible to obtain the global optimal solutions since it enables the optimization algorithm to search both the design and control parameters simultaneously. The user could choose the favored sizing solution from the obtained Pareto frontier packaged with the optimal membership functions based on a preferred compromise between the two objectives. The obtained global optimal sizing parameters and optimal parameters of the real-time controller on the Pareto frontier can be put into real-time implementations. In addition to the Bi-level optimal sizing and control framework, the devised
vectorized fuzzy logic controller with standard interfaces can be used in other kinds of real time feedback control problems, in particular, it can assist to dramatically improve the computational efficiency when need to optimize the parameters of fuzzy logic controller.

REFERENCES AND FOOTNOTES

REFERENCES

[1] X. Hu, L. Johannesson, N. Murgovski, and B. Egardt, “Longevity-conscious dimensioning and power management of the hybrid energy storage system in a fuel cell hybrid electric bus,” Applied Energy, vol. 137, pp. 913–924, 2015.

[2] G. Suri and S. Onori, “A control-oriented cycle-life model for hybrid electric vehicle lithium-ion batteries,” Energy, vol. 96, pp. 644–653, 2016.

[3] F. Sun, X. Hu, Y. Zou, and S. Li, “Adaptive unscented Kalman filtering for state of charge estimation of a lithium-ion battery for electric vehicles,” Energy, vol. 36, no. 5, pp. 3531–3540, 2011.

[4] Y. Zou, X. Hu, H. Ma, and S. E. Li, “Combined state of charge and state of health estimation over lithium-ion battery cell lifespan for electric vehicles,” Journal of Power Sources, vol. 273, no. Supplement C, pp. 793–803, 2015.

[5] A. Burke, “Ultracapacitors: why, how, and where is the technology,” Journal of power sources, vol. 91, no. 1, pp. 37–50, 2000.

[6] L. Zhang, X. Hu, Z. Wang, F. Sun, and D. G. Dorrell, “A review of supercapacitor modeling, estimation, and applications: A control/management perspective,” Renewable and Sustainable Energy Reviews, vol. 81, pp. 1878–2018, 2018.

[7] S. M. Lukic, S. G. Wirasinha, F. Rodriguez, J. Cao, and A. Emadi, “Power management of an ultracapacitor/battery hybrid energy storage system in anhev,” in 2006 IEEE Vehicle Power and Propulsion Conference, Sept 2006, pp. 1–6.

[8] J. Cao and A. Emadi, “A new battery/ultracapacitor hybrid energy storage system for electric, hybrid, and plug-in hybrid electric vehicles,” IEEE Transactions on power electronics, vol. 27, no. 1, pp. 122–132, 2012.

[9] T. Ma, H. Yang, and L. Lu, “Development of hybrid batteriesupercapacitor energy storage for remote area renewable energy systems,” Applied Energy, vol. 153, pp. 56–62, 2015.

[10] O. Ahmed and J. Bleijs, “An overview of dcde converter topologies for fuel cell-ultracapacitor hybrid distribution system,” Renewable and Sustainable Energy Reviews, vol. 42, no. Supplement C, pp. 609–626, 2015.

[11] S. J. Moura, F. B. Argomedo, R. Klein, A. Mirtabatabaie, and M. Krstic, “Battery state estimation for a single particle model with electrolyte dynamics,” IEEE Transactions on Control Systems Technology, vol. 25, no. 2, pp. 453–468, 2017.

[12] H. Chaoui and C. C. Ibe-Ekeocha, “State of charge and state of health estimation for lithium batteries using recurrent neural networks,” IEEE Transactions on Vehicular Technology, vol. 66, no. 4, pp. 8773–8783, 2017.

[13] O. Tremblay, “Experimental validation of a battery dynamic model for ev applications experimental validation of a battery dynamic model for ev applications,” in 24th International Battery, Hybrid and Fuel Cell Electric Vehicle Symposium and Exhibition 2009, EVS 24, vol. 2, 2009, pp. 930–939.

[14] A. Cordoba-Arenas, S. Onori, Y. Guezennecl, and G. Rizzoni, “Capacity and power fade cycle-life model for plug-in hybrid electric vehicle lithium-ion battery cells containing blended spinel and layered-oxide positive electrodes,” Journal of Power Sources, vol. 278, pp. 473–483, 2015.

[15] R. Deshpande, M. Verbrugge, Y.-T. Cheng, J. Wang, and P. Liu, “Battery Cycle Life Prediction with Coupled Chemical Degradation and Fatigue Mechanics,” Journal of the Electrochemical Society, vol. 159, no. 10, pp. A1730–A1738, aug 2012.

[16] R. Wang, Y. Chen, D. Feng, X. Huang, and J. Wang, “Development and performance characterization of an electric ground vehicle with independently actuated in-wheel motors,” Journal of Power Sources, vol. 196, no. 8, pp. 3962–3971, 2011.

[17] J. Shen, A. Hasanazadeh, and A. Khaligh, “Optimal power split and sizing of hybrid energy storage system for electric vehicles,” in 2014 IEEE Transportation Electrification Conference and Expo (ITEC), 2014, pp. 1–6.

[18] L. Zadeh, “Fuzzy sets,” Information and Control, vol. 8, no. 3, pp. 338 – 353, 1965.

[19] B. Hredzak, V. G. Agelidis, and G. Demetriades, “Application of explicit model predictive control to a hybrid battery-ultracapacitor power source,” Journal of Power Sources, vol. 277, pp. 84–94, 2015.

[20] Q. Zhang, W. Deng, and G. Li, “Stochastic Control of Predictive Power Management for Battery/Supercapacitor Hybrid Energy Storage Systems of Electric Vehicles,” IEEE Transactions on Industrial Informatics, pp. 1–1, 2017.

[21] K. Jia, Y. Chen, T. Bi, Y. Lin, D. Thomas, and M. Sumner, “Historical-Data-Based Energy Management in a Microgrid With a Hybrid Energy Storage System,” IEEE Transactions on Industrial Informatics, vol. 13, no. 5, pp. 2597–2605, 2017.

[22] J. Shen and A. Khaligh, “Design and real-time controller implementation for a battery-ultracapacitor hybrid energy storage system,” IEEE Transactions on Industrial Informatics, vol. 12, no. 5, pp. 1910–1918, 2016.

[23] H. Yu, F. Castelli-Dezza, and F. Cheli, “Optimal powertrain design and control of a 2-Idw electric race car,” in 2017 International Conference of Electrical and Electronic Technologies for Automotive, June 2017, pp. 1–7.

[24] Y. Cao, R. C. Kroeze, and P. T. Krein, “Multi-timescale parametric electrical battery model for use in dynamic electric vehicle simulations,” IEEE Transactions on Transportation Electrification, vol. 2, no. 4, pp. 432–442, 2016.

[25] J. Shen and A. Khaligh, “Design and real-time controller implementation for a battery-ultracapacitor hybrid energy storage system,” IEEE Transactions on Industrial Informatics, vol. 12, no. 5, pp. 453–468, 2017.

[26] H. Chaoui and C. C. Ibe-Ekeocha, “State of charge and state of health estimation for lithium batteries using recurrent neural networks,” IEEE Transactions on Vehicular Technology, vol. 66, no. 4, pp. 8773–8783, 2017.

[27] O. Tremblay, “Experimental validation of a battery dynamic model for ev applications experimental validation of a battery dynamic model for ev applications,” in 24th International Battery, Hybrid and Fuel Cell Electric Vehicle Symposium and Exhibition 2009, EVS 24, vol. 2, 2009, pp. 930–939.

[28] A. Cordoba-Arenas, S. Onori, Y. Guezennecl, and G. Rizzoni, “Capacity and power fade cycle-life model for plug-in hybrid electric vehicle lithium-ion battery cells containing blended spinel and layered-oxide positive electrodes,” Journal of Power Sources, vol. 278, pp. 473–483, 2015.

[29] R. Deshpande, M. Verbrugge, Y.-T. Cheng, J. Wang, and P. Liu, “Battery Cycle Life Prediction with Coupled Chemical Degradation and Fatigue Mechanics,” Journal of the Electrochemical Society, vol. 159, no. 10, pp. A1730–A1738, aug 2012.

[30] R. Wang, Y. Chen, D. Feng, X. Huang, and J. Wang, “Development and performance characterization of an electric ground vehicle with independently actuated in-wheel motors,” Journal of Power Sources, vol. 196, no. 8, pp. 3962–3971, 2011.

[31] J. Shen, A. Hasanazadeh, and A. Khaligh, “Optimal power split and sizing of hybrid energy storage system for electric vehicles,” in 2014 IEEE Transportation Electrification Conference and Expo (ITEC), 2014, pp. 1–6.

[32] L. Zadeh, “Fuzzy sets,” Information and Control, vol. 8, no. 3, pp. 338 – 353, 1965.

[33] S. Ebbesen, P. Elbert, and L. Guzzella, “Battery state-of-health perceptive energy management for hybrid electric vehicles,” IEEE Transactions on Vehicular Technology, vol. 61, no. 7, pp. 2893–2900, Sept 2012.

[34] K. Deb, “Multi-objective optimization using evolutionary algorithms: an introduction,” Multi-objective evolutionary optimisation for product design and manufacturing, pp. 1–24, 2011.