I. INTRODUCTION

Weak decays of heavy mesons and baryons carrying a bottom and/or a charm quark are of great interests and have been studied extensively on both experimental and theoretical sides. These decays provide useful information about the strong and electroweak interactions in the standard model (SM). Rare decays are ideal to look for new physics effects beyond SM, and recent measurements of lepton flavor universality have shown notable deviations from the standard model (see Ref. [1] for a recent brief review on the anomalies in B decays). Quite a number of physical observables like branching fractions, CP asymmetries and polarizations have been precisely measured by experiments [2–4]. On the other hand, due to our poor understanding of QCD at low energy regions, theoretical calculations of decay amplitudes are not well understood. Most of the current calculations rely on the factorization methods. Among them, many available studies are conducted at leading power in 1/m_b, while recent analyses of semileptonic and radiative processes have indicated the importance of next-to-leading power corrections [5, 6]. Apart from factorization approaches, the flavor SU(3) symmetry is a powerful tool frequently used in two-body and three-body heavy meson decays [7–20]. Although flavor SU(3) symmetry is approximate, yet it still provides very useful information about the decays. Since the SU(3) invariant amplitudes can be determined by fitting the data, the SU(3) analysis bridges experimental data and the dynamical approaches.

Among different realizations of carrying out SU(3) analysis for decay amplitudes there are two popular methods. One of them is topological diagram amplitude (TDA) method, where decay amplitudes are represented by connecting quark lines flows in different ways and then relate them by SU(3) symmetry, and another way is to construct the SU(3) irreducible representation amplitude (IRA) by decomposing effective Hamiltonian. While the TDA approach gives a better understanding of dynamics in the different amplitudes, the IRA approach shows a convenient connection with the SU(3) symmetry. These two methods looks very different in formulations, one may wonder whether they will gives a better understanding of dynamics in the different amplitudes, the IRA approach shows a convenient connection with the SU(3) symmetry. These two methods looks very different in formulations, one may wonder whether they will gives a better understanding of dynamics in the different amplitudes, the IRA approach shows a convenient connection with the SU(3) symmetry. These two methods looks very different in formulations, one may wonder whether they will gives a better understanding of dynamics in the different amplitudes, the IRA approach shows a convenient connection with the SU(3) symmetry. These two methods looks very different in formulations, one may wonder whether they will gives a better understanding of dynamics in the different amplitudes, the IRA approach shows a convenient connection with the SU(3) symmetry. These two methods looks very different in formulations, one may wonder whether they will gives a better understanding of dynamics in the different amplitudes, the IRA approach shows a convenient connection with the SU(3) symmetry. These two methods looks very different in formulations, one may wonder whether they will gives a better understanding of dynamics in the different amplitudes, the IRA approach shows a convenient connection with the SU(3) symmetry. These two methods looks very different in formulations, one may wonder whether they will
out a similar analysis for $B_c \rightarrow DP, DV$. In section V, we discuss beauty/charm baryon decays into an octet baryon and an octet pseudo-scalar meson. The expanded amplitudes and relations given these sections are useful for a global analysis when enough data is available in the future. In section VI we summarize our results. In the Appendix, we give the relations for different parametrizations in TDA and IRA methods for bottom and charmed baryon decays.

II. SU(3) PROPERTIES OF HAMILTONIAN AND HADRON STATES

A. Hadron Multiplets

Several classes of heavy hadron, containing at least one heavy quark $b$ or $c$, will be considered in this work. The involved processes include heavy SU(3) triplet mesons $B$ and $D$ decays into $PP$, $PV$, $VV$, and a $B_c$ meson decays into $DP$, $DV$. For heavy baryons, the decay processes include a heavy anti-triplet $T_{c3}$ or a $T_{c3}$ decay into a baryon in the decuplet $T_{10}$ plus a light meson, and decay into a baryon in the octet $T_8$ plus a light meson. We display the hadron SU(3) properties and their component fields in this subsection.

The $B_c$ meson contains no light quark and it is a singlet. The heavy mesons containing one heavy quark $(B_c) = (B^- (b\bar{u}), B^0 (b\bar{d}), B^0_s (b\bar{s}))$, $(D_{c}) = (D^0 (c\bar{u}), D^+ (c\bar{d}), D^+ (c\bar{s}))$, (1)

are flavor SU(3) anti-triplets.

The light pseudoscalar $P$ and vector $V$ mesons are mixture of octets and singlets so that each of them contain nine hadrons:

$$P = \begin{pmatrix} \frac{\eta_8}{\sqrt{2}} + \frac{\eta_1}{\sqrt{6}} + \frac{\eta_1}{2\sqrt{3}} & \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & -\frac{\eta_8}{\sqrt{3}} - \frac{\eta_1}{2\sqrt{6}} \\ \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \frac{\eta_8}{\sqrt{3}} - \frac{\eta_1}{2\sqrt{6}} & \frac{\eta_8}{\sqrt{2}} + \frac{\eta_1}{\sqrt{6}} \\ \frac{\eta_8}{\sqrt{3}} - \frac{\eta_1}{2\sqrt{6}} & \frac{\eta_8}{\sqrt{2}} + \frac{\eta_1}{\sqrt{6}} & -\frac{\eta_8}{\sqrt{6}} - \frac{\eta_1}{\sqrt{3}} \end{pmatrix} K^+, \quad V = \begin{pmatrix} \rho^0 + \omega & \rho^+ & K^{*+} \\ \rho^+ & \rho^0 - \frac{\rho^0 + \omega}{\sqrt{2}} & K^{*0} \\ K^{*0} & \frac{\rho^0 - \frac{\rho^0 + \omega}{\sqrt{2}}}{\sqrt{2}} & \phi \end{pmatrix},$$

(2)

where $\omega$ and $\phi$ mix in an ideal form, while the $\eta$ and $\eta'$ are mixtures of $\eta_8$ and $\eta_1$ with the mixing angle $\theta$:

$$\eta = \cos \theta \eta_8 + \sin \theta \eta_1, \quad \eta' = -\sin \theta \eta_8 + \cos \theta \eta_1.$$ (3)

Since $\eta_8$ and $\eta_1$ are not physical states, optionally one can choose the $\eta_q$ and $\eta_s$ basis for the $\eta$ mixing, which are defined so that the pseudoscalar octets $P$ has the same form of parametrization as vector octets $V$:

$$P = \begin{pmatrix} \frac{\eta_q}{\sqrt{2}} + \frac{\eta_s}{\sqrt{6}} & \frac{\eta_q}{\sqrt{6}} + \frac{\eta_s}{\sqrt{3}} & -\frac{\eta_q}{\sqrt{3}} - \frac{\eta_s}{2\sqrt{6}} \\ \frac{\eta_q}{\sqrt{6}} + \frac{\eta_s}{\sqrt{3}} & \frac{\eta_q}{\sqrt{3}} - \frac{\eta_s}{2\sqrt{6}} & \frac{\eta_q}{\sqrt{2}} + \frac{\eta_s}{\sqrt{6}} \\ \frac{\eta_q}{\sqrt{3}} - \frac{\eta_s}{2\sqrt{6}} & \frac{\eta_q}{\sqrt{2}} + \frac{\eta_s}{\sqrt{6}} & -\frac{\eta_q}{\sqrt{6}} - \frac{\eta_s}{\sqrt{3}} \end{pmatrix} \eta_q,$$ (4)

with

$$\eta_q = \frac{1}{\sqrt{3}} \eta_8 - \sqrt{\frac{1}{3}} \eta_s, \quad \eta_1 = \sqrt{\frac{2}{3}} \eta_q + \frac{1}{\sqrt{3}} \eta_s.$$ (5)

An advantage of the parametrization in Eq. (4) is that there is a one-to-one correspondence between the decay amplitudes of channels with vector final state and those of channels with pseudoscalar final state in the SU(3) limit.

A charmed or bottom baryons with two light quarks can form an anti-triplet or sextet. Most members of the sextet can decay through strong interaction or electromagnetic interactions. The only exceptions are $\Omega_b$ and $\Omega_c$ [22]. We will concentrate on anti-triplet weak decays. For the anti-triplet bottom and charmed baryons, we have the following matrix expressions:

$$\begin{pmatrix} T_{c3}^{ij} \end{pmatrix} = \begin{pmatrix} 0 & -\Lambda^+_c & \Xi^+_c \\ -\Lambda^+_c & 0 & \Xi^+_c \\ -\Xi^+_c & -\Xi^+_c & 0 \end{pmatrix}, \quad \begin{pmatrix} T_{b3}^{ij} \end{pmatrix} = \begin{pmatrix} 0 & \Lambda^0_b & \Xi^0_b \\ -\Lambda^0_b & 0 & \Xi^0_b \\ -\Xi^0_b & -\Xi^0_b & 0 \end{pmatrix}.$$ (6)

One can also contract the above matrix with the anti-symmetric tensor $\epsilon_{i,j,k}$ ($\epsilon_{123} = +1$) to have $T^{ijk}_{3,i} = \epsilon_{ijk} T^{jk}_{3}$ with

$$\begin{pmatrix} (T_{c3})_i \end{pmatrix} = \begin{pmatrix} \Xi^0_c - \Xi^+_c & \Lambda^+_c \\ \Xi^+_c & \Lambda^+_c \\ -\Xi^+_c & 0 \end{pmatrix}, \quad \begin{pmatrix} (T_{b3})_i \end{pmatrix} = \begin{pmatrix} \Xi^0_b - \Xi^0_b & \Lambda^0_b \\ \Xi^0_b & \Lambda^0_b \\ -\Xi^0_b & 0 \end{pmatrix}.$$ (7)
The lowest-lying baryon octet is given by:

\[
(T_8^i)_j = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 \\
\Sigma^- \\
-\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 \\
\Xi^- \\
-\frac{1}{\sqrt{4}} \Lambda^0 
\end{pmatrix}
\]  

One can also contract the above with \(\epsilon_{ijk}\) to have \((T_8)_ijk = \epsilon_{ijn}(T_8)_nk\).

The light baryon decuplet is given as:

\[
\begin{align*}
T_{10}^{111} &= \Delta^{++}, & T_{10}^{112} &= T_{10}^{121} = T_{10}^{211} = \frac{1}{\sqrt{3}} \Delta^+, & T_{10}^{222} = T_{10}^{102} = T_{10}^{212} = \frac{1}{\sqrt{3}} \Delta^0, \\
T_{10}^{113} &= T_{10}^{131} = T_{10}^{311} = \frac{1}{\sqrt{3}} \Sigma'^+, & T_{10}^{223} = T_{10}^{232} = T_{10}^{332} = \frac{1}{\sqrt{3}} \Sigma'^-, \\
T_{10}^{123} &= T_{10}^{132} = T_{10}^{231} = T_{10}^{231} = T_{10}^{332} = \frac{1}{\sqrt{3}} \Xi^+, & T_{10}^{333} &= \Omega^-.
\end{align*}
\]

### B. SU(3) properties of effective Hamiltonian

**Effective Hamiltonian for charmless b decays**

In the SM weak decays of charmless b decays are induced by the following electroweak effective Hamiltonian [23–25]:

\[
\mathcal{H}_{eff}^{b} = \frac{G_F}{\sqrt{2}} \left( V_{ub}V_{ub}^* [C_1 O_1 + C_2 O_2] - V_{tb}V_{tb}^* \sum_{i=0}^{10} C_i O_i \right) + \text{h.c.}
\]

Here \(G_F\) is the Fermi constant, and \(V_{ub}\) and \(V_{tb}\) are CKM matrix elements. The \(O_i\) is a four-quark operator with \(C_i\) as its Wilson coefficient. The explicit forms of \(O_i\)'s are given as follows:

\[
\begin{align*}
O_1 &= (\bar{q}^c u)_{V-A} (\bar{u}^c b)_{V-A}, & O_2 &= (\bar{q} u)_{V-A} (\bar{u} b)_{V-A}, \\
O_3 &= (\bar{q} b)_{V-A} \sum_{q'} (\bar{q'}^c q')_{V-A}, & O_4 &= (\bar{q}^c b)_{V-A} \sum_{q'} (\bar{q'}^c q')_{V-A}, \\
O_5 &= (\bar{q} b)_{V-A} \sum_{q'} (\bar{q'}^c q')_{V-A}, & O_6 &= (\bar{q}^c b)_{V-A} \sum_{q'} (\bar{q'}^c q')_{V-A}, \\
O_7 &= \frac{3}{2} (\bar{q} b)_{V-A} \sum_{q'} c_{q'} (\bar{q'}^c q')_{V-A}, & O_8 &= \frac{3}{2} (\bar{q}^c b)_{V-A} \sum_{q'} c_{q'} (\bar{q'}^c q')_{V-A}, \\
O_9 &= \frac{3}{2} (\bar{q} b)_{V-A} \sum_{q'} c_{q'} (\bar{q'}^c q')_{V-A}, & O_{10} &= \frac{3}{2} (\bar{q}^c b)_{V-A} \sum_{q'} c_{q'} (\bar{q'}^c q')_{V-A}.
\end{align*}
\]

\(q = d, s\) and \(q' = u, d, s\). Here the \(V - A\) and \(V + A\) corresponds a left-handed \(\gamma_{\mu}(1 - \gamma_5)\) and a right-handed current \(\gamma_{\mu}(1 + \gamma_5)\) respectively.

In the SU(3) group for light flavors, tree operators \(O_{1,2}\) and electroweak penguin operators \(O_{7-10}\) can be decomposed in terms of a vector \(H^{[3]}_3\), a traceless tensor antisymmetric in upper indices, \((H^{[i]}_3)^{ij}\), and a traceless tensor symmetric in upper indices, \((H^{[ij]}_3)_k\). For \(\Delta S = 0 (b \rightarrow d)\) decays, the non-zero components of the effective Hamiltonian are [8, 11, 12]:

\[
\begin{align*}
(H^{[3]}_3)^2 = 1, & \quad (H^{[ij]}_3)^2 = -(H^{[i]}_3)^2 = (H^{[ij]}_3)^2 = -(H^{[ij]}_3)^2 = 1,
2(H^{[ij]}_3)^2 = 2(H^{[ij]}_3)^2 = -3(H^{[i]}_3)^2 = -6(H^{[ij]}_3)^2 = 1, \quad 2(H^{[ij]}_3)^2 = 6.
\end{align*}
\]

For the \(\Delta S = 1(b \rightarrow s)\) decays the nonzero entries in the \(H^{[3]}_3, H^{[ij]}_3, H^{[ij]}_3\) can be obtained from Eq. (12) with the exchange \(2 \leftrightarrow 3\) corresponding to the \(d \leftrightarrow s\) exchange.

QCD penguin operators \(O_{3-6}\) behave as the \(3\) representation. For the magnetic moment operators, the color magnetic moment operator \(O_{8g} = (g_s m_b / 4\pi \lambda \epsilon_{\mu \nu \rho \sigma} T^a G^{\mu \nu}(1 + \gamma_5)b\) is an SU(3) triplet, while the electromagnetic
moment operator \( O_{7-10} = \frac{e^2 m_q}{4 \pi^2} \bar{s}g^{\mu\nu}F_{\mu\nu}(1 + \gamma_5)b \) can be effectively incorporated into the \( O_{7-10} \). Thus both of them are not included in Eq. (10) and the above decomposition is complete.

The irreducible representation amplitude (IRA) method of describing related decays is to decompose effective Hamiltonian according to the above mentioned representations and construct the amplitudes accordingly. On the other hand the the topological diagrams (TDA) method is to take the effective Hamiltonian with two light antiquarks and a light quark \( H_k^{ij} \) to represent \( \bar{q}u \bar{u}b \) with \( i = \bar{u}, k = u \) and \( j = \bar{q} \) (omitting the Lorentz indices), and then contract the indices with initial and final hadron states. In this way the decays are represented by diagrams following the quark line flows. Note that in the TDA method, the indices \( i \) and \( j \) ordering matters which are neither symmetry nor anti-symmetric. They are not traceless either.

The effective Hamiltonian have both tree and loop contributions. When strong penguin and electroweak penguin are all included the tree and loop contributions have \( 3, 6 \) and \( 1\overline{5} \) representations. The independent amplitudes have the same numbers, except that one can make one of the tree or penguin amplitude real and the rest all in principle complex. Using the unitarity property of the CKM matrix \( V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0 \), one can also rewrite c loop induced penguin contributions into amplitude proportional to \( V_{ub}V_{us}^* \) and \( V_{tb}V_{ts}^* \),

\[
\mathcal{A} = V_{ub}V_{us}^* A_u + V_{tb}V_{ts}^* A_t.
\] (13)

For simplicity, we refer to \( A_u \) as “tree” amplitude since it is dominated by tree contributions with modifications from \( u \) and \( c \) loop contributions. \( A_t \) is “penguin” amplitude with \( c \) and \( t \) loop contributions. It is necessary to stress that not all contributions in \( A_u \) are tree diagrams, and the same for \( A_t \).

Since both \( A_u \) and \( A_t \) have similar amplitudes with \( SU(3) \) representations. In our later discussions we will concentrate on the \( A_u \) amplitudes. One can easily obtain the \( A_t \) amplitudes by just changing the labels.

**Effective electroweak Hamiltonian for c hadronic decays**

For weak interaction induced \( c \) hadronic decays, the effective Hamiltonian with \( \Delta C = 1 \) and \( \Delta S = 1 \) is given as:

\[
\mathcal{H}_{eff}^c = \frac{G_F}{\sqrt{2}} \{ V_{cb}V_{cs}^* [C_1 O_{11}^{ud} + C_2 O_{22}^{dd}] + V_{cd}V_{ud}^*[C_1 O_{11}^{dd} + C_2 O_{22}^{dd}] + V_{cs}V_{us}^*[C_1 O_{11}^{us} + C_2 O_{22}^{us}] + V_{cd}V_{us}^*[C_1 O_{11}^{ds} + C_2 O_{22}^{ds}] \},
\] (14)

where we have neglected the highly suppressed penguin contributions, and

\[
O_{11}^{ud} = [\bar{s}^i \gamma_{\mu}(1 - \gamma_5)c^j][\bar{u}^i \gamma_{\mu}(1 - \gamma_5)d^j], \quad O_{22}^{dd} = [\bar{s}^i \gamma_{\mu}(1 - \gamma_5)c^j][\bar{u}^i \gamma_{\mu}(1 - \gamma_5)d^j],
\] (15)

while other operators can be obtained by replacing the \( d, s \) quark fields. Tree operators transform under the flavor \( SU(3) \) symmetry as \( 3 \otimes \overline{3} \otimes 3 = 3 \otimes \overline{3} \otimes 6 \otimes \overline{15} \).

For the Cabibbo allowed \( c \to ud \) transition, we have amplitudes proportional to \( V_{cb}V_{cs}^* \) and the Hamiltonians are:

\[
(H_6)^{31} = -(H_6)^{13} = 1, \quad (H_{15}^{15})^{31} = (H_{15}^{15})^{13} = 1.
\] (16)

For the doubly Cabibbo suppressed \( c \to du \bar{s} \) transition, we have amplitudes to be proportional to \( V_{cd}V_{us}^* \) and the Hamiltonians are:

\[
(H_6)^{21} = -(H_6)^{12} = 1, \quad (H_{15}^{15})^{21} = (H_{15}^{15})^{12} = 1.
\] (17)

For decays proportional to \( V_{cs}V_{us}^* \), we have:

\[
(H_6)^{31} = -(H_6)^{13} = 1, \quad (H_{15}^{15})^{31} = (H_{15}^{15})^{13} = 1,
\] (18)

and for decays proportional to \( V_{cd}V_{td}^* \), we have:

\[
(H_6)^{12} = -(H_6)^{21} = 1, \quad (H_{15}^{15})^{12} = (H_{15}^{15})^{21} = -1.
\] (19)

For singly Cabibbo suppressed decays, \( c \to udd \) and \( c \to u \bar{s}s \) transitions have approximately equal magnitudes but opposite signs: \( V_{cd}V_{us}^* \) \( = V_{cs}V_{us}^* - V_{cb}V_{cs}^* \approx -V_{cs}V_{us}^* \) (with \( 10^{-3} \) deviation). As a result, the contributions from the \( 3 \) representation vanish, and one has the nonzero components contributed only by \( 6 \) and \( 15 \) representations.

For the singly Cabibbo-suppressed transition, there are also loop contributions proportional to \( V_{ub}V_{us}^* \) \( = V_{tb}V_{ts}^* \). Such loop contributions are small so that we will concentrate on the dominant amplitude proportional to \( V_{cs}V_{us}^* \). However, one can include these contributions by adding a 3 representation in the Hamiltonian.

Again, we use the above \( SU(3) \) decompositions for IRA analysis and use the effective Hamiltonian \( H_k^{ij} \) with \( i = \bar{s}, j = \bar{u} \) and \( k = q \) for TDA analysis to trace the quark line flows.
III. CHARMLESS TWO-BODY $B$ DECAYS

A. $B \to PP$ decays

Let us start with the $B \to PP$ decays. The generic amplitude is decomposed according to CKM matrix elements:

$$A = V_{ub}V_{us}^* A_{u}^{IRA} + V_{tb}V_{ts}^* A_{t}^{IRA} ,$$
$$A = V_{ub}V_{us}^* A_{u}^{TDA} + V_{tb}V_{ts}^* A_{t}^{TDA} .$$

(20)

The IRA and TDA amplitudes should be equivalent, though as we have shown [21] this equivalence is not obvious.

To obtain IRA, one takes various representations in Eq. (12) and contracts all indices in $B_i$ and light meson $P^i_j$ with various combinations:

$$A_{u}^{IRA} = A_3^T B_i(H_3)^i_j P^k_j + C_3^T B_i(H_3)^k_j P^i_j + B_3^T B_i(H_3)^i_j P^k_j + D_3^T B_i(H_3)^j_i P^k_k$$
$$+ A_6^T B_i(H_6)^{[ij]} P^k_j P^l_j + C_6^T B_i(H_6)^{[ij]} P^k_j P^l_j + B_6^T B_i(H_6)^{[ij]} P^k_j P^l_j$$
$$+ A_5^T B_i(K_{15})^{[ij]} P^k_j P^l_j + C_5^T B_i(K_{15})^{[ij]} P^k_j P^l_j + B_5^T B_i(K_{15})^{[ij]} P^k_j P^l_j .$$

(21)

In the TDA decomposition, one has:

$$A_{u}^{TDA} = T B_i H_6^{ij} P^k_j P^l_j + C B_i H_6^{ij} P^k_j P^l_j + AB_i H_6^{ij} P^k_j P^l_j + E B_i H_6^{ij} P^k_j P^l_j$$
$$+ S B_i H_6^{ij} P^k_j P^l_j + P B_i H_6^{ij} P^k_j P^l_j + P B_i H_6^{ij} P^k_j P^l_j + S B_i H_6^{ij} P^k_j P^l_j + S B_i H_6^{ij} P^k_j P^l_j$$
$$+ E S B_i H_6^{ij} P^k_j P^l_j + A B_i H_6^{ij} P^k_j P^l_j .$$

(22)

According to this decomposition, topological diagrams for $B \to PP$ decays can be found in Fig. 1. Apart from the ordinary $T, C, A, E$, we have also included the other SU(3) irreducible amplitudes, most of which come from loop diagrams, and/or the flavor singlet diagram.

Expanding Eqs. (21,22), one obtains $B \to PP$ amplitudes in Table I. Since we have decomposed the effective Hamiltonian into irreducible representations, one may expect that there are 10 independent amplitudes for $A_u$ and similarly 10 amplitudes for $A_t$. A careful examination shows that the $A_6^T$ can be absorbed into $B_6^T$ and $C_6^T$ with a redefinition:

$$C_6^{T'} = C_6^T - A_6^T , \quad B_6^{T'} = B_6^T + A_6^T .$$

(23)

This combination can also be found explicitly from Table I. After eliminating the redundant amplitude, there are actually only 18 ($A_u$ and $A_t$ contribute 9 each) SU(3) independent amplitudes. An overall phase convention is not an observable, thus there are only 17 independent real parameters for decay amplitudes.

We list all TDA amplitudes in Table I. It is necessary to point out that the last 6 diagrams in Fig. 1 are often omitted in the SU(3) TDA analysis. However only by including them the complete equivalence of IRA and TDA can be established. One of the 10 TDA amplitudes must be redundant. Such redundancy can be understood through the following relations between the IRA and TDA amplitudes:

$$T + E = 4 A_{15}^T + 2 C_{15}^T + 4 C_{15}^T , \quad C - E = -4 A_{15}^T - 2 C_{15}^T + 4 C_{15}^T ,$$
$$A + E = 8 A_{15}^T , \quad P^u - E = -5 A_{15}^T + C_{15}^T - 4 C_{15}^T - C_{15}^T ,$$
$$P_A^u + E = A_3^T + A_{15}^T , \quad E_S^u = 2 A_{15}^T - 2 B_{15}^T + 4 B_{15}^T ,$$
$$A_S^u - E = -4 A_{15}^T + 2 B_{15}^T + 4 B_{15}^T , \quad S_S^u - E = -2 A_{15}^T + B_{15}^T - B_{15}^T .$$

(24)

We have adopted the choice in which $E$ is always in companion with another amplitude. It is also possible to replace the role of $E$ by one of the amplitudes $A, C$ or even $T$. One can also reversely obtain:

$$A_3^T = -A_3^T + B_3^T + 3 E_3^u - A_3^T , \quad C_T^T = C_T^T - 3 A - 3 E + T ,$$
$$D_T^T = S_T^u + 3 C - 3 E_T^u + 3 A_T^u - T , \quad B_T^T = 1/4 (A - E + A_T^u - E_T^u) , \quad C_T^T = C_T^T - 1/4 (A - C + E + T) ,$$
$$A_T^T = A + E , \quad B_T^T = A_T^T + B_T^T .$$

(25)
Similar analysis for the $A_t$ contributions can be obtained with the replacement for the IRA:

$$A_t^i \rightarrow A_t^P, \quad B_t^i \rightarrow B_t^P, \quad C_t^i \rightarrow C_t^P, \quad D_t^i \rightarrow D_t^P.$$  \hspace{1cm} (26)
while for TDA, we have:

\[
\begin{align*}
T &\rightarrow P_T, \quad C \rightarrow P_C, \quad A \rightarrow P_TA, \quad P^n \rightarrow P, \quad E \rightarrow P_{FE}, \\
P^n_A &\rightarrow P, \quad E^n_S \rightarrow P_{AS}, \quad A^n_S \rightarrow P_{ES}, \quad S^n_S \rightarrow P_{SS}, \quad T_S \rightarrow S.
\end{align*}
\]

(27)

1. Impact of the new TDA amplitudes

The new TDA amplitudes in Fig. 1 may play an important role in understanding CP violation (CPV) phenomena. Without the new TDA amplitudes, some decays only have terms proportional to $V_{tb}^* V_{td}$, such as $B^0 \rightarrow \bar{K}^0 K^0$ and $B^0_s \rightarrow \bar{K}^0 K^0$. For instance, in Ref. [18], the amplitudes for $B^0 \rightarrow \bar{K}^0 K^0$ read:

\[
\mathcal{A}(B^0 \rightarrow \bar{K}^0 K^0) = V_{ub} V_{td}^* \left( P - \frac{1}{2} P_{EW}^C + 2 P_A \right).
\]

(28)

This would imply the CP violating asymmetry is identically zero. However, as we have shown, these two decays receive contributions from the $P^n + 2P^n_A$ multiplied by $V_{ub} V_{td}^*$:

\[
\mathcal{A}(B^0 \rightarrow \bar{K}^0 K^0) = V_{ub} V_{td}^* (P^n + 2P^n_A) + V_{tb} V_{td}^* (P + 2P^n_A).
\]

(29)

Therefore a non-vanishing direct CP asymmetry is obtained. This would certainly affect the search for new physics in a precise CP violation measurement.

Most new TDA amplitudes in Fig. 1 arise from higher order loop corrections, and thus they are likely small in magnitude. However, sometimes they can not be completely neglected. In Ref. [12], the authors have performed a fit of $B \rightarrow PP$ decays in the IRA framework. Depending on different choices of data, four cases were considered in their analysis [12]. Here for illustration, we give their results in case 4:

\[
|C^{T}_{3}| = -0.211 \pm 0.027, \quad \delta^{T}_{3} = (-140 \pm 6)^{\circ}, \quad |B^{T}_{15}| = -0.038 \pm 0.016, \quad \delta_{B^{T}_{15}} = (78 \pm 48)^{\circ},
\]

(30)

where the magnitudes and strong phases are defined relative to the amplitude $C^{P}_{3}$. From Eq. (25), one can find that the $C^{T}_{3}$ is a mixture of $T$, $C$ and others, while the $B^{T}_{15}$ equals $(E^n_S + A^n_S)/8$. The fitted results in Eq. (30) indicate, compared to $C^{T}_{3}$, the $B^{T}_{15}$ could reach 20% in magnitude, and more notably, the strong phases are sizably different. The fact that the $B^{T}_{15}$, namely $E^n_S$ and $A^n_S$, have non-negligible contributions supports our call for a complete analysis.
2. Comparison with QCDF amplitudes

The topological amplitudes in $B \to PP$ decays can be compared to the QCDF amplitude in Ref. [26]. Such a comparison requests two remarks. Firstly, in our decomposition, we adopt the CKM matrix elements $V_{ub}V_{ub}^*$ and $V_{tb}V_{tb}^*$, while Ref. [26] used $V_{ub}V_{us}$ and $V_{tb}V_{ts}$. The unitarity of CKM matrix guarantees the equivalence of the two approaches. So we will directly compare the “tree” $A_t$ and “penguin” $A_i$ amplitudes, though some of them might be recombined in order to have the same CKM factors. Secondly, we have decomposed one part of the electroweak penguin into the QCD penguin as shown in Sec. II, and we will do so for QCDF amplitudes too.

We have the following relations for “tree” amplitudes:

$$T \to \alpha_1, \quad P^u \to \alpha_u^u + \beta_u^u, \quad C \to \alpha_2, \quad S^u \to \alpha_3^u + \beta_{S3}^u, \quad A \to \beta_2,$$

$$E \to \beta_1, \quad P_A^u \to \beta_A^u, \quad A_S^u \to \beta_{S2}, \quad E_S^u \to \beta_{S1}, \quad S_S^u \to \beta_{S4}.$$

where the notations $\alpha_i$ and $\beta_i$ are from Ref. [26]. For “penguin” ones, one can derive the relation:

$$P_T \to \alpha_{3,EW}^u, \quad P \to \alpha_3^u + \beta_3^u, \quad P_C \to \alpha_{3,EW}^u, \quad S \to \alpha_3^u + \beta_{S3}, \quad P_{TA} \to \beta_{3,EW}^c,$$

$$P_{TE} \to \beta_{3,EW}^u, \quad P_A \to \beta_3^u, \quad P_{ES} \to \beta_{S3,EW}^u, \quad P_{AS} \to \beta_{S4,EW}^u, \quad P_{SS} \to \beta_{S4}.$$

3. U-Spin relations

Some decay channels shown in Table I with $\Delta S = 0$ and $\Delta S = 1$ are related by U-spin, the $d \leftrightarrow s$ exchange symmetry. The relations will be discussed explicitly in the following. These pairs of channels include:

$B^- \to K^0K^-$ and $B^- \to \pi^-K^0$; $\overline{B}^0 \to \pi^+\pi^-$ and $\overline{B}^0 \to K^+K^-$; $\overline{B}^0 \to \pi^0K^0$ and $\overline{B}^0 \to K^0\overline{K}^0$; $\overline{B}^0 \to \pi^-K^+$ and $\overline{B}^0 \to \pi^+K^-$; $\overline{B}^0 \to \pi^0\overline{K}^0$ and $\overline{B}^0 \to \pi^0K^0$.

In the past years, there have been extensive examinations on the U-spin symmetry. One of the interesting features of these U-spin pairs is that there are CP violating relation among them. Here we consider two U-spin related decays with the same “tree” $A_u$ and “penguin” $A_i$:

$$A(B_i \to PP, \Delta S = 0) = V_{ub}V_{ub}^*A_u + V_{tb}V_{tb}^*A_t,$$

$$A(B_i \to PP, \Delta S = 1) = V_{ub}V_{ub}^*A_u + V_{tb}V_{ts}^*A_t.$$

(33)

Through the relation Im($V_{ub}V_{ub}^*V_{tb}V_{ts}$) = -Im($V_{ub}V_{ub}^*V_{tb}V_{ts}$), one can obtain the CP violating rate difference

$$\Delta(B_i \to PP, \Delta S = 0) = \Gamma(\Delta S) - \Gamma(\Delta S) \text{[9, 10, 27]}$$

$$\Delta(B_i \to PP, \Delta S = 0) = -\Delta(B_j \to PP, \Delta S = 1).$$

(34)

This leads to a relation between branching ratio and CP asymmetry $A_{CP}(\Delta S) = \Delta(B_i \to PP, \Delta S)/B(B_i \to PP)$:

$$\frac{A_{CP}(\Delta S = 0)}{A_{CP}(\Delta S = 1)} = \frac{\tau_i B(\Delta S = 1)}{\tau_i B(\Delta S = 0)}.$$  

(35)

Here $B(B_i \to PP)$ is the branching ratio of $B_i \to PP$ and $\tau_i$ is the lifetime of $B_i$.

One of the most prominent example is the case of the U-spin pair $\overline{B}^0 \to \pi^-K^+$ and $\overline{B}^0 \to \pi^+K^-$. Here we will comment on the experimental situation for this case and introduce a parameter $r_c$ to account for the deviation from

---

1 For baryonic decay modes to be discussed in the following, there are non-trivial Clebsch-Gordon coefficients, such that Eq. (33) is modified as:

$$A(\Delta S = 0) = r(V_{ub}V_{ub}^*A_u + V_{tb}V_{tb}^*A_t),$$

$$A(\Delta S = 1) = V_{ub}V_{ub}^*A_u + V_{tb}V_{ts}^*A_t.$$

The relation in Eq. (35) is changed to:

$$\frac{A_{CP}(\Delta S = 0)}{A_{CP}(\Delta S = 1)} = -r^2 \frac{\tau_i B(\Delta S = 1)}{\tau_i B(\Delta S = 0)}.$$
SU(3) symmetry.

\[
\frac{A_{CP}(B^0 \rightarrow \pi^+ K^-)}{A_{CP}(B^+_s \rightarrow \pi^- K^+)} + r_c \frac{\tau_{B^0} B(B^0 \rightarrow \pi^+ K^-)}{\tau_{B^+_s} B(B^+_s \rightarrow \pi^- K^+)} = 0.
\]  

In the SU(3) symmetry limit \( r_c = 1 \).

Using the experimental data from PDG [3, 4]:

\[
B(B^0 \rightarrow \pi^- K^+) = (5.7 \pm 0.6) \times 10^{-6}, \quad A_{CP}(B^0 \rightarrow \pi^- K^+) = (0.26 \pm 0.04),
\]

\[
B(B^0 \rightarrow \pi^+ K^-) = (19.6 \pm 0.5) \times 10^{-6}, \quad A_{CP}(B^0 \rightarrow \pi^+ K^-) = -0.082 \pm 0.006,
\]

one finds:

\[
r_c = 1.084 \pm 0.219
\]

where all errors have been added in quadrature. The resulting \( r_c \) value indicates that the U-spin symmetry is well in the case of this decay pair. The exploration in more decay pairs is helpful for further investigation on this symmetry.

Similar U-spin relations existing in other decays will be studied in the following sections. We will comment on them when specific decay channels are to be discussed.

**B. \( B \rightarrow VV \) decays**

Decay amplitudes for \( B \rightarrow VV \) channels can be obtained similarly by replacing the pseudo-scalar multiplet \( P \) by the vector multiplet \( V \) in Eq. (21) and in Eq. (22).

- Since we have chosen the same parametrization for pseudoscalar and vector mesons, the expanded amplitudes for the \( B \rightarrow VV \) channels can be obtained directly from the \( B \rightarrow PP \).

- There are three sets of amplitudes for \( B \rightarrow VV \) decays, corresponding to different polarizations. For convenience, one can choose the helicity amplitudes \( A_0, A_+, A_- \) defined as:

\[
A = S_1 \epsilon_{V_1} \cdot \epsilon_{V_2} + S_2 \frac{1}{m_B} \epsilon_{V_1}^\ast \cdot p_B \epsilon_{V_2} \cdot p_B - i S_3 \epsilon_{\mu\nu\rho\sigma} p_{V_1}^\mu p_{V_2}^\nu \epsilon_{V_1}^\ast \epsilon_{V_2}^\ast \epsilon_{\sigma\rho},
\]

with \( \epsilon_{0123} = 1 \), and

\[
A_0 = \frac{m_B^2}{2 m_{V_1} m_{V_2}} \left( S_1 + S_2 \right), \quad A_{\pm} = S_1 \mp S_3.
\]

Thus there are in total \( 3 \times 9 = 27 \) complex amplitudes for both tree and penguin, where \( "9" \) is the number of the polarization combination of final two vectors. These amplitudes correspond to \( 2 \times 54 - 1 = 107 \) real parameters in theory. Two phases cannot be measured through direct measurements of individual \( B \) and \( \bar{B} \) decays, but one of the two can be obtained through the time-dependent analysis.

- In principle, all these 107 parameters could be determined through the angular distribution studies in experiment. Each \( B \rightarrow V(\rightarrow P_1 P_2) V(\rightarrow P_3 P_4) \) channel can provide 10 observables. The angular distribution is given as:

\[
\frac{d\Gamma}{d\cos \theta_1 d\cos \theta_2 d\phi} \propto |A_0|^2 \cos^2 \theta_1 \cos^2 \theta_2 + \frac{1}{4} \sin^2 \theta_1 \sin^2 \theta_2 \left( |A_+|^2 + |A_-|^2 \right)
\]

\[
+ \frac{1}{2} \sin^2 \theta_1 \sin^2 \theta_2 \text{Re}(e^{2i\phi} A_+ A_-^*)
\]

\[
- \cos \theta_1 \sin \theta_1 \cos \theta_2 \sin \theta_2 [\text{Re}(e^{-i\phi} A_0 A_+^*) + \text{Re}(e^{i\phi} A_0 A_-^*)].
\]

Here \( \theta_1 (\theta_2) \) is defined by the flight direction of \( P_1 (P_3) \) in the rest frame of \( V_1 (V_2) \) and the flight direction of \( V_1 (V_2) \) in the \( B \) meson rest frame. \( \phi \) is the relative angle between the two decay planes.

- Unfortunately, due to the large amount of input parameters, it is a formidable task to perform a global fit, and in particular only limited data is available [3]. A realistic analysis at this stage will pick up only a limited amount of amplitudes. In this direction, the weak annihilations and hard scattering amplitudes were extracted by fitting relevant data in Ref. [28], while the authors in Ref. [29] have performed a factorization-assisted TDA analysis. This allows one to remove some suppressed amplitudes at the leading order approximation. In Ref. [30], the authors have adopted the dynamical analysis in the SCET and performed a flavor SU(3) fit of \( B \rightarrow VV \) decays. On the other hand, recent dynamical improvements exist in Refs. [31, 32] using the perturbative QCD approach and Refs. [33] in QCDF.
C. $B \to VP$ Decays

We now study the $B \to VP$ decays, whose amplitudes can be obtained by replacing one of the $P$ in Eq. (21) and in Eq. (22) by $V$ to obtain the IRA and TDA amplitudes. There are two ways to replace one of the $P$, therefore the amplitudes will be doubled compared with $B \to PP$. We have IRA and TDA for $B \to VP$ decays as follows:

$$A^\text{IRA}_{u} = A^T_{u} B_i(H_3) P^i_k V^k_j + C^T_{i} B_i(H_3) P^i_j V^j_k + C^T_{3} B_i(H_3) P^i_j V^j_k + B^T_{u} B_i(H_3) P^i_k V^j_j$$
$$+ D^T_{i} B_i(H_3) P^i_j V^j_k + D^T_{3} B_i(H_3) P^i_j V^j_k + A^T_{u} B_i(H_6) P^i_k V^j_j + A^T_{3} B_i(H_6) P^i_k V^j_j$$
$$+ C^T_{i} B_i(H_6) P^i_k V^j_j + C^T_{3} B_i(H_6) P^i_k V^j_j + P^T_{u} B_i(H_6) P^i_k V^j_j + P^T_{3} B_i(H_6) P^i_k V^j_j$$
$$+ A^T_{u} B_i(H_6) P^i_k V^j_j + A^T_{3} B_i(H_6) P^i_k V^j_j + C^T_{i} B_i(H_1) P^i_k V^j_j + C^T_{3} B_i(H_1) P^i_k V^j_j$$
$$+ B^T_{u} B_i(H_1) P^i_k V^j_j + B^T_{3} B_i(H_1) P^i_k V^j_j.$$  (41)

$$A^\text{TDA}_{u} = T_1 B_i(H^u_k) P^i_j V^j_k + T_2 B_i(H^u_k) P^i_j V^j_k + C_1 B_i(H^u_k) P^i_j V^j_k + C_2 B_i(H^u_k) P^i_j V^j_k$$
$$+ A_1 B_i(H^u_k) P^i_j V^j_k + A_2 B_i(H^u_k) P^i_j V^j_k + E_1 B_i(H^u_k) P^i_j V^j_k + E_2 B_i(H^u_k) P^i_j V^j_k$$
$$+ S_1 B_i(H^u_k) P^i_j V^j_k + S_2 B_i(H^u_k) P^i_j V^j_k + P_1 B_i(H^u_k) P^i_j V^j_k + P_2 B_i(H^u_k) P^i_j V^j_k$$
$$+ P_3 B_i(H^u_k) P^i_j V^j_k + S_3 B_i(H^u_k) P^i_j V^j_k + E_3 B_i(H^u_k) P^i_j V^j_k + E_2 B_i(H^u_k) P^i_j V^j_k$$
$$+ A_1 B_i(H^u_k) P^i_j V^j_k + A_2 B_i(H^u_k) P^i_j V^j_k.$$  (42)

The expanded amplitudes are given in Tab. II and Tab. III. Relations between the two sets of amplitudes are derived as:

$$A^T_{u} = -\frac{1}{8}(A_1 + A_2 - 3E_1 - 3E_2) + P^{u}_{A}, \quad B^T_{u} = S^{u}_{A} + \frac{1}{8}(3E^{u}_{1} + 3E^{u}_{2} - A^{u}_{1} - A^{u}_{2})$$

$$C^T_{1} = \frac{1}{8}(3T_1 - C_1 + 3A_1 - E_1) + P^{u}_{1}, \quad C^T_{2} = \frac{1}{8}(3T_2 - C_2 + 3A_2 - E_2) + P^{u}_{2}$$

$$D^T_{1} = \frac{1}{8}(3C_1 - T_1 - E^{u}_{1} + 3A^{u}_{1}) + S^{u}_{1}, \quad D^T_{2} = \frac{1}{8}(3C_2 - T_2 - E^{u}_{2} + 3A^{u}_{2}) + S^{u}_{2}$$

$$A^T_{6} = \frac{1}{4}(A_2 - E_2), \quad A^T_{6} = \frac{1}{4}(A_1 - E_1), \quad C^T_{6} = \frac{1}{4}(T_1 - C_1), \quad C^T_{6} = \frac{1}{4}(T_2 - C_2)$$

$$A^{T}_{15} = \frac{1}{8}(A_2 + E_2), \quad A^{T}_{15} = \frac{1}{8}(A_1 + E_1), \quad C^{T}_{15} = \frac{1}{8}(T_1 + C_1), \quad C^{T}_{15} = \frac{1}{8}(T_2 + C_2)$$

$$B^{T}_{15} = \frac{1}{4}(A^{u}_{1} - E^{u}_{1}), \quad B^{T}_{15} = \frac{1}{4}(A^{u}_{2} - E^{u}_{2}), \quad B^{T}_{15} = \frac{1}{8}(E^{u}_{1} + A^{u}_{1}), \quad B^{T}_{15} = \frac{1}{8}(E^{u}_{2} + A^{u}_{2}).$$  (43)

The inverse relations are solved as:

$$A_{1} = 4A^{T}_{15} + 2A^{T}_{2}, \quad A_{2} = 2(2A^{T}_{15} + A^{T}_{1}), \quad T_{1} = 2(2C^{T}_{15} + C^{T}_{1}), \quad T_{2} = 2(2C^{T}_{15} + C^{T}_{2}),$$

$$C_{1} = 2(2C^{T}_{15} - C^{T}_{6}), \quad C_{2} = 2(2C^{T}_{15} - C^{T}_{6}), \quad E_{1} = 2(2A^{T}_{15} - A^{T}_{2}), \quad E_{2} = 2(2A^{T}_{15} - A^{T}_{2}),$$

$$A^{T}_{1} = 2(2B^{T}_{15} + B^{T}_{2}), \quad A^{T}_{2} = 2(2B^{T}_{15} + B^{T}_{2}), \quad E^{T}_{1} = 2(2B^{T}_{15} + B^{T}_{2}), \quad E^{T}_{2} = 2(2B^{T}_{15} + B^{T}_{2}),$$

$$P^{u}_{A} = -A^{T}_{15} + A^{u}_{1} - A^{T}_{2} + A^{u}_{2}, \quad S^{u}_{A} = -B^{T}_{15} + B^{T}_{2} - B^{T}_{2} + B^{T}_{2},$$

$$P^{u}_{1} = -A^{T}_{15} - A^{T}_{1} - C^{T}_{1} + C^{T}_{6}, \quad P^{u}_{2} = -A^{T}_{15} - A^{T}_{2} - C^{T}_{1} + C^{T}_{6},$$

$$S^{u}_{1} = -B^{T}_{15} - B^{T}_{2} - C^{T}_{1} + C^{T}_{6}, \quad S^{u}_{2} = -B^{T}_{15} - B^{T}_{2} - C^{T}_{1} + C^{T}_{6}.$$  (44)

Unlike the $B \to PP$ and $B \to VV$ case, we are not able to find any redundant amplitude. Thus in total, we have 18 complex amplitudes for “tree” and “penguin”, respectively. It corresponds to $2 \times 36 - 1 = 71$ real parameters in theory. A fit with all parameters is not available again, and most of the current analyses have made approximations by neglecting some suppressed amplitudes [18, 19, 34, 35].

The $B \to VP$ channels related by the $U$-spin include: $B^{-} \to \bar{K}^{0} \pi^{-}$ and $B^{-} \to K^{+}K^{-}$; $B^{-} \to \rho^{-}\bar{K}^{0}$ and $B^{-} \to K^{-}K^{0}$; $\bar{B}_{s} \to K^{+}K^{-}$ and $\bar{B}_{s} \to \rho^{+}\pi^{-}$; $\bar{B}_{s} \to K^{+}K^{-}$ and $\bar{B}_{s} \to \rho^{+}\pi^{-}$; $\bar{B}_{s} \to K^{+}K^{-}$ and $\bar{B}_{s} \to \rho^{+}\pi^{-}$ and $B^{0} \to K^{-}K^{+}$; $\bar{B}_{s} \to K^{+}\pi^{0}$ and $\bar{B}_{s} \to K^{0}\pi^{0}$; $\bar{B}_{s} \to K^{-}K^{+}$ and $\bar{B}_{s} \to \rho^{+}\pi^{-}$. However on the experimental side, there are not enough measurements to examine these relations, in particular the CPV in $B_{s}$ sector has received less consideration. We expect the situation will be improved when a large amount of data is available at LHCb, and Belle-II.
| channel | IRA | TDA |
|---------|-----|-----|
| $B^- \to \rho^0 \pi^-$ | $(A^T_6 + 3A^{T}_{15} - A^T_3) - 3A^{T}_{15} - C^T_3$ | $(-A_1 + A_2 + C_1 + P^{u2} - P^{u1} + T_2)/\sqrt{2}$ |
| $B^- \to \rho^- \pi^0$ | $(-A^T_6 - 3A^{T}_{15} + A^T_3 + 3A^{T}_{15} + C^T_3 + C^T_6 + 3C^{T}_{15})/\sqrt{2}$ | $(A_1 - A_2 + C_2 - P^{u2} + P^{v1} + T_1)/\sqrt{2}$ |
| $B^- \to \rho^- \eta_0$ | $(1/\sqrt{2})(A^T_6 + 3A^{T}_{15} + A^T_3 + 3A^{T}_{15} + 2B^{T}_{15} + 6B^{T}_{1} + C^T_3 + C^T_6 + 3C^{T}_{15} + C^T_{15} + 2D^{T}_{1})$ | $(2A^T_3 + A_1 + A_2 + C_1 + P^{u2}$ $+ P^{v1} + 2S^{u2} + T_1)/\sqrt{2}$ |
| $B^- \to K^{*0}K^-$ | $A^T_6 + 3A^{T}_{15} + C^T_3 - C^T_6 - C^T_{15}$ | $A_1 + P^{u1}$ |
| $B^- \to K^-\pi^0$ | $A^T_6 + 3A^{T}_{15} + C^T_3 - C^T_6 - C^T_{15}$ | $A_2 + P^{u2}$ |
| $B^- \to \omega \pi^-$ | $(1/\sqrt{2})(A^T_6 + 3A^{T}_{15} + A^T_3 + 3A^{T}_{15} + 2B^{T}_{15} + 6B^{T}_{1} + C^T_3 + C^T_6 + 3C^{T}_{15} + C^T_{15} + 2D^{T}_{1})$ | $(2A^T_3 + A_1 + A_2 + C_1 + P^{u2}$ $+ P^{v1} + 2S^{u1} + T_2)/\sqrt{2}$ |
| $B^- \to \phi \pi^-$ | $B^T_6 + 3B^{T}_{15} + C^T_3 - C^T_6 - C^T_{15} + 2D^T_1$ | $A^{u2}_1 + S^{u2}$ |
| $B^0 \to \rho^0 \pi^0$ | $(1/2)(A^T_6 - A^T_6 + A^T_3 - A^T_6 + A^T_{15} + C^T_3 + C^T_6 + 3C^{T}_{15})$ | $(1/2)(2P^{u}_{A} - C_1 - C_2 + E_1 + E_2 + P^{u2} + P^{v1})$ |
| $\mathcal{B}^0 \to \rho^0 \eta_0$ | $(1/2)(A^T_6 - A^T_6 + A^T_3 - A^T_6 + A^T_{15} + C^T_3 + C^T_6 + 3C^{T}_{15})$ | $(1/2)(2P^{u}_{A} - C_1 - C_2 + E_1 + E_2 + P^{u2} + P^{v1})$ |
| $\mathcal{B}^0 \to \rho^0 \eta_3$ | $(1/2)- A^T_6 + 5A^{T}_{15} - A^T_6 + 5A^{T}_{15} - 2B^{T}_{15} + 10B^{T}_{1} - C^T_3 - C^T_6 + 5C^{T}_{15} - C^T_{15} - 2D^{T}_{1}$ | $(1/2)(C_1 - C_2 + 2E^{u2} + E_1$ $+ E_2 + P^{u2} - P^{u1} - 2S^{u2})$ |
| $B^0 \to \rho^0 \pi^-$ | $A^T_6 - A^T_6 + 3A^{T}_{15} + 2A^T_3 + C^T_3 + C^T_6 + 3C^{T}_{15}$ | $P^{u}_A + E_2 + P^{v1} + T_1$ |
| $B^0 \to K^{*0}K^-$ | $A^T_6 + 2A^{T}_{15} - A^T_6 - A^T_6 + 3A^{T}_{15}$ | $P^{u}_A + E_1$ |
| $\mathcal{B}^0 \to K^{*0}K^0$ | $A^T_6 - 2A^{T}_{15} + A^T_6 - A^T_6 + 3A^{T}_{15}$ | $(P^{u}_A + P^{v2}$ $+ P^{u2} + P^{v2})$ |
| $\mathcal{B}^0 \to K^{*0}K^+$ | $A^T_6 - 2A^{T}_{15} + A^T_6 - A^T_6 + 3A^{T}_{15}$ | $P^{u}_A + E_2$ |
| $\mathcal{B}^0 \to \omega \pi^0$ | $(-B^T_6 - 5B^{T}_{15} + C^T_3 - C^T_6 - C^T_{15} + 2D^T_1)$ | $(E^{u2}_S - S^{u2})/\sqrt{2}$ |
| $\mathcal{B}^0 \to \omega \eta_3$ | $(-B^T_6 - 5B^{T}_{15} + C^T_3 - C^T_6 - C^T_{15} + 2D^T_1)$ | $(E^{u2}_S - S^{u2})/\sqrt{2}$ |
| $\mathcal{B}^0 \to \phi \pi^0$ | $(2B_6 + 2B^{T}_{15} - 2B^{T}_{15} - B^T_6 + B^T_6 + C^T_3 - C^T_{15} + D^T_1)/\sqrt{2}$ | $(E^{u2}_S + 2S^{u2} + 2S^{u2}_S)/\sqrt{2}$ |
| $\mathcal{B}^0 \to \phi \eta_3$ | $(2B_6 + 2B^{T}_{15} - 2B^{T}_{15} - B^T_6 + B^T_6 + C^T_3 - C^T_{15} + D^T_1)/\sqrt{2}$ | $(E^{u2}_S + 2S^{u2} + 2S^{u2}_S)/\sqrt{2}$ |
| $\mathcal{B}^0 \to \phi \eta_0$ | $A^T_6 + A^T_{15} - A^T_6 + A^T_{15} + A^T_6 + 3A^{T}_{15}$ | $P^{u2}_A + S^{u2}_S$ |
| $\mathcal{B}^0 \to \rho^0 K^0$ | $(A^T_6 + A^T_{15} - C^T_6 + C^T_6 + 5C^{T}_{15})/\sqrt{2}$ | $(C_1 - P^{u2})/\sqrt{2}$ |
| $\mathcal{B}^0 \to \rho^0 K^+$ | $-A^T_6 - A^T_{15} + C^T_6 + C^T_{15} + 3C^{T}_{15}$ | $P^{u1} + T_2$ |
| $\mathcal{B}^0 \to K^{*0} \pi^-$ | $-A^T_6 - A^T_{15} + C^T_6 + C^T_{15} + 3C^{T}_{15}$ | $P^{v2} + T_2$ |
| $\mathcal{B}^0 \to K^{*0} \pi^+$ | $(A^T_6 + A^T_{15} - C^T_6 + C^T_{15} + 5C^{T}_{15})/\sqrt{2}$ | $(C_2 - P^{u2})/\sqrt{2}$ |
| $\mathcal{B}^0 \to K^{*0} \eta_0$ | $-(A^T_6 + A^T_{15} + 2B^{T}_{15} + 2B^{T}_{15} - C^T_6 + C^T_6 + C^T_{15} - 2D^T_1)/\sqrt{2}$ | $(C_2 + P^{u2} + 2S^{u2})/\sqrt{2}$ |
| $\mathcal{B}^0 \to K^{*0} \eta_3$ | $-(A^T_6 + A^T_{15} + 2B^{T}_{15} + 2B^{T}_{15} - C^T_6 + C^T_6 + C^T_{15} - 2D^T_1)/\sqrt{2}$ | $(C_2 + P^{u2} + 2S^{u2})/\sqrt{2}$ |
| $\mathcal{B}^0 \to \omega K^0$ | $-A^T_6 - A^T_{15} - B^T_6 - B^T_6 + C^T_3 + C^T_{15} + C^T_6 + C^T_{15}$ | $(C_1 + P^{u1} + 2S^{u1})/\sqrt{2}$ |
| $\mathcal{B}^0 \to \phi K^0$ | $-A^T_6 - A^T_{15} - B^T_6 - B^T_6 + C^T_3 + C^T_{15} + C^T_6 + C^T_{15}$ | $(C_1 + P^{u1} + 2S^{u1})/\sqrt{2}$ |

**TABLE II:** $B \to VP$ decays induced by the $b \to d$ transition.
Using the effective Hamiltonian in Eqs. (16) and (17), one can easily obtain the \(SU(3)\) decay amplitudes in a similar fashion as that for \(B \to PP, \ VV, \ PV\). In this case there is only tree contributions which we write as
TABLE IV: Decay amplitudes for two-body $D \to PP$ decays. Decay amplitudes for two-body $D \to PP$ decays. The CKM factor should be multiplied: $V_{cs}V_{ud}^{*}$ for Cabibbo-Allowed decays; $V_{cs}V_{us}^{*}$ for singly Cabibbo-suppressed modes and $V_{cd}V_{us}^{*}$ for doubly Cabibbo-suppressed modes.

| $V_{cs}V_{ud}$ | IRA | TDA |
|----------------|-----|-----|
| $D^{0} \to \pi^{+}K^{-}$ | $-A_{6}^{0} + A_{15}^{0} + C_{6}^{0} + C_{15}^{0}$ | E + T |
| $D^{0} \to \pi^{-}K^{+}$ | $(A_{6}^{0} - A_{15}^{0} - C_{6}^{0} + C_{15}^{0})/\sqrt{2}$ | $(C - E)/\sqrt{2}$ |
| $D^{0} \to K^{0}\eta \eta \left( -A_{6}^{0} + A_{15}^{0} - 2B_{6}^{0} + 2B_{15}^{0} - C_{6}^{0} + C_{15}^{0}\right)/\sqrt{2}$ | $(C + 2E_{B}^{0} + E)/\sqrt{2}$ |
| $D^{0} \to K^{0}\eta_{s}$ | $-A_{6}^{0} - A_{15}^{0} - B_{6}^{0} + B_{15}^{0}$ | $E_{S}^{0} + E$ |
| $D^{0} \to \pi^{0}\eta$ | $2c_{15}^{0}$ | C + T |
| $D_{s}^{0} \to \pi^{+}\eta_{q}$ | $(A_{6}^{0} + A_{15}^{0} + B_{6}^{0} + B_{15}^{0})/\sqrt{2}$ | $(A_{6}^{0} + A)$ |
| $D_{s}^{0} \to \pi^{0}\eta_{s}$ | $B_{6}^{0} + B_{15}^{0} + C_{6}^{0} + C_{15}^{0}$ | $A_{S}^{0} + T$ |
| $D_{s}^{0} \to K^{+}\eta^{0}$ | $A_{6}^{0} + A_{15}^{0} - C_{6}^{0} + C_{15}^{0}$ | A + C |

| $V_{cs}V_{us}$ | IRA | TDA |
|----------------|-----|-----|
| $D^{0} \to \pi^{0}\pi^{-}$ | $A_{6}^{0} - A_{15}^{0} - C_{6}^{0} - C_{15}^{0}$ | $-E - T$ |
| $D^{0} \to \pi^{0}\pi^{0}$ | $A_{6}^{0} - A_{15}^{0} - C_{6}^{0} + C_{15}^{0}$ | C - E |
| $D^{0} \to \pi^{0}\eta_{q}$ | $-A_{6}^{0} + A_{15}^{0} - B_{6}^{0} + B_{15}^{0}$ | $E_{S}^{0} + E$ |
| $D^{0} \to \pi^{0}\eta_{s}$ | $(-B_{6}^{0} + B_{15}^{0} - C_{6}^{0} + C_{15}^{0})/\sqrt{2}$ | $(C + E_{B}^{0})/\sqrt{2}$ |
| $D^{0} \to K^{0}\eta^{0}$ | $-A_{6}^{0} + A_{15}^{0} + C_{6}^{0} + C_{15}^{0}$ | E + T |
| $D^{0} \to \eta_{q}\eta_{q}$ | $-A_{6}^{0} + A_{15}^{0} + 2B_{6}^{0} + 2B_{15}^{0} + C_{6}^{0} - C_{15}^{0}$ | $-C - 2E_{B}^{0} - E$ |
| $D^{0} \to \eta_{q}\eta_{s}$ | $(-B_{6}^{0} + B_{15}^{0} - C_{6}^{0} + C_{15}^{0})/\sqrt{2}$ | $(C + E_{B}^{0})/\sqrt{2}$ |
| $D^{0} \to \eta_{s}\eta_{s}$ | $-A_{6}^{0} + A_{15}^{0} - B_{6}^{0} + B_{15}^{0}$ | $E_{S}^{0} + E$ |
| $D^{0} \to \pi^{0}\pi^{0}$ | $\sqrt{2}c_{15}^{0}$ | $(C + T)/\sqrt{2}$ |
| $D^{0} \to \pi^{0}\eta_{q}$ | $-\sqrt{2}(A_{6}^{0} + A_{15}^{0} + B_{6}^{0} + B_{15}^{0} + C_{6}^{0} + C_{15}^{0})$ | $-(2A_{B}^{0} + 2A + C + T)/\sqrt{2}$ |
| $D^{0} \to \pi^{0}\eta_{s}$ | $-B_{6}^{0} - B_{15}^{0} - C_{6}^{0} - C_{15}^{0}$ | $C - A_{S}^{0}$ |
| $D^{0} \to \pi^{0}\eta_{s}$ | $-A_{6}^{0} - A_{15}^{0} - C_{6}^{0} + C_{15}^{0}$ | A - T |
| $D^{0} \to \eta_{q}\eta_{s}$ | $-A_{6}^{0} + A_{15}^{0} - B_{6}^{0} + B_{15}^{0}$ | $E_{S}^{0} + E$ |
| $D^{0} \to \eta_{s}\eta_{s}$ | $-B_{6}^{0} + B_{15}^{0} - C_{6}^{0} - C_{15}^{0}$ | $(A + C)/\sqrt{2}$ |
| $D^{0} \to \pi^{0}\pi^{0}$ | $\sqrt{2}c_{15}^{0}$ | $(A + C + C + T)/\sqrt{2}$ |

| $V_{cd}V_{us}$ | IRA | TDA |
|----------------|-----|-----|
| $D^{0} \to \pi^{0}\pi^{0}$ | $A_{6}^{0} - A_{15}^{0} - C_{6}^{0} + C_{15}^{0}$ | $(C - E)/\sqrt{2}$ |
| $D^{0} \to \pi^{0}K^{+}$ | $-A_{6}^{0} + A_{15}^{0} + C_{6}^{0} + C_{15}^{0}$ | E + T |
| $D^{0} \to K^{0}\eta_{q}$ | $-A_{6}^{0} + A_{15}^{0} - 2B_{6}^{0} + 2B_{15}^{0} - C_{6}^{0} - C_{15}^{0}$ | $(C + 2E_{B}^{0} + E)/\sqrt{2}$ |
| $D^{0} \to K^{0}\eta_{s}$ | $-A_{6}^{0} + A_{15}^{0} - B_{6}^{0} + B_{15}^{0}$ | $E_{S}^{0} + E$ |
| $D^{0} \to \pi^{0}K^{+}$ | $(A_{6}^{0} + A_{15}^{0} - C_{6}^{0} + C_{15}^{0})/\sqrt{2}$ | $(A - T)/\sqrt{2}$ |
| $D^{0} \to \eta_{q}\eta_{s}$ | $(A_{6}^{0} + A_{15}^{0} + 2B_{6}^{0} + 2B_{15}^{0} + C_{6}^{0} + C_{15}^{0})/\sqrt{2}$ | $(2A_{B}^{0} + A - C)/\sqrt{2}$ |
| $D^{0} \to \eta_{s}\eta_{s}$ | $A_{6}^{0} + A_{15}^{0} + B_{6}^{0} + B_{15}^{0} + 2C_{15}^{0}$ | $A_{S}^{0} + A + C + T$ |

$A = V_{cs/d}V_{ud/s}A_{u}^{IRA,TDA}$. For $D \to PP$ we have:

\[
A_{u}^{IRA} = A_{6}^{0}D_{i}(H_{0})_{i}^{[ij]}P_{i}^{k}P_{i}^{k} + C_{6}^{0}D_{i}(H_{0})_{i}^{[ij]}P_{j}^{k}P_{j}^{k} + B_{6}^{0}D_{i}(H_{0})_{i}^{[ij]}P_{j}^{k}P_{j}^{k} \\
+ A_{15}^{0}D_{i}(H_{0})_{i}^{[ij]}P_{i}^{k}P_{j}^{k} + C_{15}^{0}D_{i}(H_{0})_{i}^{[ij]}P_{j}^{k}P_{j}^{k} + B_{15}^{0}D_{i}(H_{0})_{i}^{[ij]}P_{j}^{k}P_{j}^{k} + E_{S}^{0}D_{i}H_{i}^{[ij]}P_{i}^{k}P_{j}^{k} + A_{S}^{0}D_{i}H_{i}^{[ij]}P_{j}^{k}P_{k}^{k}. \tag{45}
\]

\[
A_{u}^{TDA} = TD_{i}H_{i}^{[ij]}P_{i}^{k}P_{i}^{k} + CD_{i}H_{i}^{[ij]}P_{j}^{k}P_{j}^{k} + AD_{i}H_{i}^{[ij]}P_{j}^{k}P_{j}^{k} + ED_{i}H_{i}^{[ij]}P_{i}^{k}P_{k}^{k} + 2c_{15}^{0}. \tag{46}
\]
TABLE V: Decay amplitudes for two-body Cabibbo-Allowed $D \to VP$ decays.

| channel | IRA       | TDA       |
|---------|-----------|-----------|
| $D^0 \to \rho^+ K^-$ | $-A_{15}^{T1} + A_{15}^{T1} + C_{0}^{T1} + C_{15}^{T1}$ | $T_1 + E_2$ |
| $D^0 \to \rho^+ K^0$ | $(A_{15}^{T1} - A_{15}^{T1} + C_{0}^{T1} + C_{15}^{T1})/\sqrt{2}$ | $(C_2 - E_2)/\sqrt{2}$ |
| $D^0 \to K^0 \eta_s$ | $(A_{15}^{T2} - A_{15}^{T2} - C_{0}^{T2} + C_{15}^{T2})/\sqrt{2}$ | $(C_1 - E_1)/\sqrt{2}$ |
| $D^0 \to \rho^+ \pi^0$ | $-A_{0}^{T2} + A_{15}^{T2} - B_{0}^{T2} + B_{15}^{T2}$ | $E_2^2 + E_2$ |
| $D^0 \to K^+ \pi^0$ | $-A_{0}^{T2} + A_{15}^{T2} + C_{0}^{T2} + C_{15}^{T2}$ | $T_2 + E_1$ |
| $D^0 \to \omega K^0$ | $-A_{0}^{T1} + A_{15}^{T1} - 2B_{0}^{T1} + 2B_{15}^{T1} - C_{0}^{T1} + C_{15}^{T1}$ | $(C_2 + 2E_2^2 + E_1)/\sqrt{2}$ |
| $D^0 \to \phi K^0$ | $-A_{0}^{T2} + A_{15}^{T2} - B_{0}^{T1} + B_{15}^{T1}$ | $E_2^2 + E_1$ |
| $D^0 \to \rho^\pi^0$ | $C_{15}^{T1} + C_{15}^{T1} - C_{0}^{T2} + C_{15}^{T2}$ | $C_2 + T_1$ |
| $D^0 \to \rho^\pi^0$ | $-C_{0}^{T1} + C_{15}^{T1} + C_{0}^{T2} + C_{15}^{T2}$ | $C_1 + T_2$ |
| $D^+ \to \rho^+ \pi^0$ | $(A_{0}^{T1} + A_{15}^{T1} - A_{15}^{T2} - A_{15}^{T2})/\sqrt{2}$ | $(A_2 - A_1)/\sqrt{2}$ |
| $D^+ \to \rho^+ \pi^0$ | $(A_{15}^{T1} + A_{15}^{T1} + A_{15}^{T2} + A_{15}^{T2} + 2 (B_{0}^{T1} + B_{15}^{T1}))/\sqrt{2}$ | $(2A_2^2 + A_1 + A_2)/\sqrt{2}$ |
| $D^+ \to \pi^+ \eta_s$ | $B_{0}^{T2} + B_{15}^{T2} + C_{0}^{T1} + C_{15}^{T1}$ | $A_2^2 + T_1$ |
| $D^+ \to \pi^+ \eta_s$ | $(A_1 - A_2)/\sqrt{2}$ |
| $D^+ \to K^{++} K^+$ | $A_{0}^{T2} + A_{15}^{T2} - C_{0}^{T2} + C_{15}^{T2}$ | $A_2 + C_1$ |
| $D^+ \to \omega K^+$ | $(A_{15}^{T1} + A_{15}^{T1} + A_{15}^{T2} + A_{15}^{T2} + 2 (B_{0}^{T1} + B_{15}^{T1}))/\sqrt{2}$ | $(2A_2^2 + A_1 + A_2)/\sqrt{2}$ |
| $D^+ \to \phi \pi^+$ | $B_{0}^{T1} + B_{15}^{T1} + C_{0}^{T2} + C_{15}^{T2}$ | $A_2^2 + T_2$ |

The expanded amplitudes are given in Tab. IV. The amplitudes $A_{15}^T$ can be incorporated in $B_0^T$ and $C_6^T$, and then we have 5 independent amplitudes for $D \to PP$:

$$A_{15}^T = \frac{A + E}{2}, \quad B_1^T = \frac{A_2^2 + E_2^2}{2}, \quad C_{15}^T = \frac{T + C}{2}, \quad B_0^T = \frac{A_2^2 - E_2^2 + A - E}{2}, \quad C_6^T = \frac{T - C - A + E}{2},$$

(47)

with the inverse relation:

$$T + E = A_{15}^T + C_{15}^T, \quad C - E = -A_{15}^T - C_6^T + C_{15}^T, \quad A + E = 2A_{15}^T, \quad A_2^2 - E = -A_{15}^T + B_0^T + B_1^T, \quad E_2^2 + E = A_{15}^T - B_0^T + B_1^T.$$

(48)

Since one amplitude is redundant, fits with all 6 complex amplitudes should not be resolved in principle. This has been indicated by the strong correlation of parameters in the fits in Ref. [36].

Again for $D \to VP$ decays there are three sets of amplitudes similar as the $D \to PP$, and thus we have 5 independent amplitudes in total.

The IRA and TDA for $D \to VP$ decays are given as:

$$A_0^{IRA} = A_0^{T1} D_1 (H_6)_{ij}^{[i]} P_i V_j + A_0^{T2} D_1 (H_6)_{ij}^{[i]} V_i P_j + C_0^{T1} D_1 (H_6)_{ij}^{[i]} P_i V_j + C_0^{T2} D_1 (H_6)_{ij}^{[i]} V_j P_i + B_0^{T1} D_1 (H_6)_{ij}^{[i]} P_i V_j + B_0^{T2} D_1 (H_6)_{ij}^{[i]} V_j P_i + A_0^{T1} D_1 (H_7)_{ij}^{[i]} P_j V_i + A_0^{T2} D_1 (H_7)_{ij}^{[i]} V_i P_j + C_0^{T1} D_1 (H_7)_{ij}^{[i]} P_j V_i + C_0^{T2} D_1 (H_7)_{ij}^{[i]} V_i P_j + B_0^{T1} D_1 (H_{17})_{ij}^{[i]} P_j V_i + B_0^{T2} D_1 (H_{17})_{ij}^{[i]} V_i P_j + A_0^{T1} D_1 (H_{17})_{ij}^{[i]} P_j V_i + A_0^{T2} D_1 (H_{17})_{ij}^{[i]} V_i P_j,$$

(49)

$$A_0^{TDA} = T_1 D_1 H_1^{[i]} P_j V_i + T_2 D_1 H_1^{[i]} V_j P_i + C_1 D_1 H_1^{[i]} P_j V_i + C_2 D_1 H_1^{[i]} V_i P_j + A_1 D_1 H_1^{[i]} P_j V_i + A_2 D_1 H_1^{[i]} V_i P_j + E_1 D_1 H_1^{[i]} P_j V_i + E_2 D_1 H_1^{[i]} V_i P_j + E_3 D_1 H_1^{[i]} P_j V_i + E_4 D_1 H_1^{[i]} V_i P_j + A_{11} D_1 H_1^{[i]} P_j V_i + A_{12} D_1 H_1^{[i]} V_i P_j + A_{13} D_1 H_1^{[i]} P_j V_i + A_{14} D_1 H_1^{[i]} V_i P_j,$$

(50)

The expanded amplitudes are collected in Tab. V, Tab. VI and Tab. VII for the different transitions. Relations
| Channel | IRA | TDA |
|---------|-----|-----|
| $D^0 \to \rho^+ \pi^-$ | $A_6^{T_1} - A_6^{T_3} - C_6^{T_1} - C_6^{T_3}$ | $-T_1 - E_2$ |
| $D^0 \to \rho^0 \pi^0$ | $(1/2) (A_6^{T_1} - A_6^{T_3} + A_6^{T_2} - C_6^{T_1} + C_6^{T_3} + C_6^{T_2} + C_6^{T_2})$ | $(1/2) (C_1 + C_2 - E_1 - E_2)$ |
| $D^0 \to \rho^0 \eta_0$ | $(1/2) (-A_6^{T_1} + A_6^{T_1} - A_6^{T_2} + A_6^{T_2} - 2B_6^T + 2B_6^{T_2} + C_6^{T_1} + C_6^{T_3} + C_6^{T_2} - C_6^{T_3})$ | $(1/2) (C_1 - C_2 + 2E_0^2 + E_1 + E_2)$ |
| $D^0 \to \rho^0 \eta_0$ | $(-B_6^{T_2} + B_6^{T_1} - C_6^{T_2} + C_6^{T_2})/\sqrt{2}$ | $(C_2 + E_0^2)/\sqrt{2}$ |
| $D^0 \to \rho^0 \pi^0$ | $A_6^{T_2} - A_6^{T_3} - C_6^{T_2} - C_6^{T_3}$ | $-T_2 - E_1$ |
| $D^0 \to K^0 K^+$ | $-A_6^{T_1} + A_6^{T_1} + C_6^{T_1} + C_6^{T_3}$ | $T_1 + E_2$ |
| $D^0 \to K^0 K^+$ | $-A_6^{T_1} + A_6^{T_3} + A_6^{T_2} - A_6^{T_2}$ | $E_2 - E_1$ |
| $D^0 \to K^+ K^-$ | $A_6^{T_1} - A_6^{T_3} - A_6^{T_2} + A_6^{T_2}$ | $E_1 - E_2$ |
| $D^0 \to K^+ K^-$ | $-A_6^{T_1} + A_6^{T_1} + C_6^{T_1} + C_6^{T_2}$ | $T_2 + E_1$ |
| $D^0 \to \omega \pi^0$ | $(1/2) (-A_6^{T_1} + A_6^{T_1} - A_6^{T_2} + A_6^{T_2} - 2B_6^T + 2B_6^{T_2} + C_6^{T_1} + C_6^{T_3} + C_6^{T_2} - C_6^{T_3})$ | $(1/2) (-C_1 + C_2 + 2E_0^3 + E_1 + E_2)$ |
| $D^0 \to \omega \eta_0$ | $(1/2) (A_6^{T_1} - A_6^{T_1} + A_6^{T_2} - A_6^{T_2} + 2B_6^T - 2B_6^{T_2} + C_6^{T_1} + C_6^{T_3} + C_6^{T_2} - C_6^{T_3})$ | $(1/2) (-C_1 - C_2 - 2E_0^1 - 2E_0^2 + E_1 - E_2)$ |
| $D^0 \to \omega \eta_0$ | $(-2B_6^{T_2} + 2B_6^{T_2} + B_6^{T_2} - B_6^{T_2} - C_6^{T_2} + C_6^{T_2})/\sqrt{2}$ | $(C_2^2 + E_0^2)/\sqrt{2}$ |
| $D^0 \to \phi \pi^0$ | $(-B_6^{T_1} + B_6^{T_1} + C_6^{T_1} + C_6^{T_2})/\sqrt{2}$ | $(C_1 + E_0^1)/\sqrt{2}$ |
| $D^0 \to \phi \eta_0$ | $(B_6^{T_1} - B_6^{T_1} - 2B_6^{T_2} + 2B_6^{T_2} - C_6^{T_1} + C_6^{T_2})/\sqrt{2}$ | $(C_1 - E_0^3 + 2E_0^2)/\sqrt{2}$ |
| $D^0 \to \phi \eta_0$ | $(-A_6^{T_1} + A_6^{T_1} - A_6^{T_2} + A_6^{T_2} - B_6^{T_1} + B_6^{T_1} - B_6^{T_2} + B_6^{T_2})$ | $E_0^1 + E_0^2 + E_1 + E_2$ |
| $D^+ \to \rho^0 \pi^0$ | $(-A_6^{T_1} + A_6^{T_1} + A_6^{T_2} + A_6^{T_2} + C_6^{T_1} + C_6^{T_3} - C_6^{T_2} + C_6^{T_2})/\sqrt{2}$ | $(A_1 - A_2 + C_2 + T_1)/\sqrt{2}$ |
| $D^+ \to \rho^0 \eta_0$ | $-(A_6^{T_1} + A_6^{T_1} + A_6^{T_2} + A_6^{T_2} + 2B_6^T + 2B_6^{T_2} + C_6^{T_1} + C_6^{T_3} + C_6^{T_2} - C_6^{T_2})/\sqrt{2}$ | $-(2A_0^2 + A_1 + A_2 + C_2 + T_1)/\sqrt{2}$ |
| $D^+ \to \rho^+ \eta_0$ | $-B_6^{T_1} + B_6^{T_1} - C_6^{T_2} + C_6^{T_2}$ | $C_2 - A_0^2$ |
| $D^+ \to \rho^+ \pi^0$ | $(A_6^{T_1} + A_6^{T_1} - A_6^{T_2} + A_6^{T_2} - C_6^{T_1} + C_6^{T_3} + C_6^{T_2} + C_6^{T_2})/\sqrt{2}$ | $(-A_1 + A_2 + C_1 + T_2)/\sqrt{2}$ |
| $D^+ \to K^+ K^-$ | $-A_6^{T_1} + A_6^{T_1} + C_6^{T_1} + C_6^{T_3}$ | $T_1 - A_1$ |
| $D^+ \to K^+ K^-$ | $-A_6^{T_1} + A_6^{T_1} + C_6^{T_1} + C_6^{T_3}$ | $T_2 - A_2$ |
| $D^+ \to \omega \pi^+$ | $-(A_6^{T_1} + A_6^{T_1} + A_6^{T_2} + A_6^{T_2} + 2B_6^T + 2B_6^{T_2} + C_6^{T_1} + C_6^{T_3} + C_6^{T_2} - C_6^{T_2})/\sqrt{2}$ | $-(2A_0^2 + A_1 + A_2 + C_1 + T_2)/\sqrt{2}$ |
| $D^+ \to \phi \pi^+$ | $-B_6^{T_1} - B_6^{T_1} - C_6^{T_1} + C_6^{T_1}$ | $C_1 - A_0^1$ |
| $D^+ \to \phi \eta_0$ | $A_6^{T_1} + A_6^{T_1} - C_6^{T_1} - C_6^{T_2}$ | $A_1 - T_1$ |
| $D^+ \to \phi \eta_0$ | $(A_6^{T_1} + A_6^{T_1} - C_6^{T_1} + C_6^{T_1})/\sqrt{2}$ | $(A_1 + C_1)/\sqrt{2}$ |
| $D^+ \to K^+ \pi^0$ | $(A_6^{T_1} + A_6^{T_1} - C_6^{T_2} + C_6^{T_2})/\sqrt{2}$ | $(A_2 + C_2)/\sqrt{2}$ |
| $D^+ \to K^+ \eta_0$ | $(A_6^{T_1} + A_6^{T_1} + 2B_6^T + 2B_6^{T_2} + C_6^{T_1} - C_6^{T_2})/\sqrt{2}$ | $(2A_0^2 + A_1 - C_2)/\sqrt{2}$ |
| $D^+ \to K^+ \eta_0$ | $A_6^{T_1} + A_6^{T_1} + B_6^T + B_6^{T_1} + C_6^{T_1} + C_6^{T_2} + C_6^{T_2}$ | $A_0^2 + A_1 + C_2 + T_1$ |
| $D^+ \to K^+ \eta_0$ | $A_6^{T_1} + A_6^{T_1} - C_6^{T_2} - C_6^{T_2}$ | $A_2 - T_2$ |
| $D^+ \to \omega K^+$ | $(A_6^{T_1} + A_6^{T_1} + 2B_6^T + 2B_6^{T_1} + C_6^{T_1} - C_6^{T_2})/\sqrt{2}$ | $(2A_0^2 + A_1 - C_1)/\sqrt{2}$ |
| $D^+ \to \omega K^+$ | $A_6^{T_1} + A_6^{T_1} + B_6^T + B_6^{T_1} + C_6^{T_1} + C_6^{T_2} + C_6^{T_2}$ | $A_0^2 + A_1 + C_2 + T_2$ |

TABLE VI: Decay amplitudes for two-body Singly Cabibbo-Suppressed $D \to VP$ decays.
The inverse relations are solved as:

\[ A_6^{T_1} = \frac{1}{2} (A_2 - E_2), \quad A_6^{T_2} = \frac{1}{2} (A_1 - E_1), \quad B_6^{T_1} = \frac{1}{2} (A_S^{u_1} - E_S^{u_1}), \quad B_6^{T_2} = \frac{1}{2} (A_S^{u_2} - E_S^{u_2}) \]

\[ C_6^{T_1} = \frac{1}{2} (T_1 - C_1), \quad C_6^{T_2} = \frac{1}{2} (T_2 - C_2), \quad A_1^{T_1} = \frac{1}{2} (A_2 + E_2), \quad A_1^{T_2} = \frac{1}{2} (A_1 + E_1) \]

\[ B_1^{T_1} = \frac{1}{2} (E_S^{u_1} + A_S^{u_1}), \quad B_1^{T_2} = \frac{1}{2} (E_S^{u_2} + A_S^{u_2}), \quad C_1^{T_1} = \frac{1}{2} (T_1 + C_1), \quad C_1^{T_2} = \frac{1}{2} (T_2 + C_2). \]  

The inverse relations are solved as:

\[ A_1 = A_6^{T_2} + A_6^{T_2}, \quad A_2 = A_6^{T_1} + A_6^{T_1}, \quad T_1 = C_6^{T_1} + C_6^{T_1}, \quad T_2 = C_6^{T_2} + C_6^{T_2} \]

\[ C_1 = C_6^{T_2} - C_6^{T_2}, \quad C_2 = C_6^{T_1} - C_6^{T_1}, \quad E_1 = A_6^{T_2} - A_6^{T_2}, \quad E_2 = A_6^{T_1} - A_6^{T_1} \]

\[ A_S^{u_1} = B_6^{T_1} + B_6^{T_1}, \quad A_S^{u_2} = B_6^{T_1} + B_6^{T_1}, \quad E_S^{u_1} = B_6^{T_1} - B_6^{T_1}, \quad E_S^{u_2} = B_6^{T_2} - B_6^{T_2}. \]
TABLE VIII: Decay amplitudes for $B_c \to DP$ decays.

| $b \to d$ | IRA | TDA | $b \to s$ | IRA | TDA |
|-----------|-----|-----|-----------|-----|-----|
| $B_c^- \to \overline{D}^0 \pi^-$ | $A_0^d + 3A_{15}^d + B_4^d$ | $P^a + T$ | $B_c^- \to \overline{D}^0 K^- A_0^d + 3A_{15}^d + B_4^d$ | $P^a + T$ |
| $B_c^- \to D^+ \pi^0$ | $(-A_0^d + 5A_{15}^d - B_4^d)/\sqrt{2}$ | $(C - P^a)/\sqrt{2}$ | $B_c^- \to D^+ K^- A_0^d - A_{15}^d + B_4^d$ | $P^a$ |
| $B_c^- \to D^+ \eta_0$ | $(2A_0^d - A_2^d + A_{15}^d + B_4^d)/\sqrt{2}$ | $(C + P^a + 2S^a)/\sqrt{2}$ | $B_c^- \to D_s^- \pi^0 \sqrt{2} (2A_0^d - A_2^d)$ | $C/\sqrt{2}$ |
| $B_c^- \to D^- \eta_0$ | $A_2^d + A_2^{15} - A_4^{15}$ | $S^a$ | $B_c^- \to D_s^- \eta_0 \sqrt{2} (A_2^d + A_2^{15}) (C + 2S^a)/\sqrt{2}$ |
| $B_c^- \to D_s^- K^0$ | $-A_6 - A_{15} + B_4^d$ | $P^a$ | $B_c^- \to D_s^- \eta_0 A_1^d - 2A_{15}^d + B_4^d$ | $P^a + S^a$ |

Fig. 3: Feynman diagrams for tree amplitudes in the $B_c \to DP$, $DV$ decays.

It is interesting to explore the useful relations for decay widths from the amplitudes listed in Tab. V, Tab. VI and Tab. VII. For Cabibblo Allowed channels, we find $\Gamma(D_s^+ \to \rho^+\pi^0) = \Gamma(D_s^- \to \rho^0\pi^+)$. For singly Cabibblo suppressed channels, one has:

$$
\Gamma(D^0 \to \rho^+\pi^-) = \Gamma(D^0 \to K^{*-}K^-), \quad \Gamma(D_0^0 \to \rho^-\pi^+) = \Gamma(D^0 \to K^{-}\pi^-),
\Gamma(D^0 \to K^{*-}\pi^0) = \Gamma(D_s^0 \to K^{*-}\pi^+) = \Gamma(D_s^0 \to K^{*-}\pi^-),
\Gamma(D^0 \to \overline{K}^{*0}K^0) = \Gamma(D^0 \to \overline{K}^{*0}\pi^0).
$$

(53)

We refer the reader to Refs. [36-39] for some explorations of the implications on decay rates and CP asymmetries, and Refs. [40, 41] for the experimental analyses. We should point out that since the quark mass effects in charm decays might play an important role when analyzing the $D$ decays, and the SU(3) symmetry is less impressive for $D$ meson decays [36].

IV. $B_c \to DP$, $DV$ DECAYS

The effective Hamiltonian for $b$ quark decays can induce $B_c \to DP, DV$ transitions. The corresponding topological diagrams are given in Fig. 3. The IRA and TDA for $B_c \to DP$ decays are given as:

$$
A_{i,IRA}^T = A_{i,DA}^T = S^a B_c D_s H_2^T P_j^b + P^a B_c D_s H_2^T P_j^b + A_6^T B_c D_s(H_2^T P_j^b)^{1,ik} P_j^b + A_{15}^T B_c D_s(H_2^T P_j^b)^{1,ik} P_j^b,
$$

(54)

$$
A_{i,DA}^T = S^a B_c D_s H_2^T P_j^b + P^a B_c D_s H_2^T P_j^b + T B_c D_s H_2^T P_j^b + C B_c D_s H_2^T P_j^b.
$$

(55)

The expanded amplitudes can be found in Tab. VIII. Relations between the two sets of amplitudes are given as:

$$
A_{i,DA}^T = S^a - \frac{1}{8} T + \frac{3}{8} C, \quad B_{3,DA}^T = P^a + \frac{3}{8} T - \frac{1}{8} C, \quad A_{i,IRA}^T = \frac{1}{4} T - \frac{1}{4} C, \quad A_{15}^T = \frac{1}{8} T + \frac{1}{8} C.
$$

(56)

“Penguin” amplitudes are obtained similarly:

$$
A_{3,15}^T \to A_{3,15}^P, \quad B_3^T \to B_3^P, \quad S^a \to S, \quad P^a \to P, \quad T \to T, \quad C \to C.
$$

(57)

Including the “penguins”, one has 8 complex amplitudes in total.

Again decay amplitudes for $B_c \to DV$ can be obtained by replacing the pseudoscalars by their vector counterparts. The $U$-spin related channels include: $B_c^- \to \overline{D}^0 K^- \text{ and } B_c^- \to \overline{D}^0 \pi^-; B_c^- \to D^+ \overline{K}_0 \text{ and } B_c^- \to D_s^- K_0; B_c^- \to \overline{D}^0 K^{*-}$ and $B_c^- \to \overline{D}^0 \rho^-; B_c^- \to D^- \overline{K}_0$ and $B_c^- \to D_s^- K_0$. 

(59)
In Ref. [42], the LHCb collaboration has measured the product:

\[ \frac{f(B_c)}{f(B^+)} \times B(B_c^+ \to D^0 K^+) = (9.3^{+2.8}_{-3.2} \pm 0.6) \times 10^{-7}, \]

where the \( f(B_c) \) and \( f(B^+) \) are the production rates of \( B_c^+ \) and \( B^+ \), respectively. With the measured ratio [43]:

\[ \frac{f(B_c)}{f(B^+)} \sim 0.004 - 0.012, \]

one can obtain an estimated branching fraction:

\[ B(B_c^+ \to D^0 K^+) \sim 7.8 \times 10^{-5} - 2.3 \times 10^{-4}. \]

On theoretical side, model-dependent analyses give $1.3 \times 10^{-7}$ [44], and $6.6 \times 10^{-5}$ [45], while a phenomenological study implies the $B(B_c^+ \to D^0 K^+) \sim [4.4 - 9] \times 10^{-5}$ [46]. Since this transition is induced by \( b \to s \), the large branching fraction may imply a large penguin amplitude \( P \). Such a scenario can be tested by measuring the corresponding \( (B_c^+ \to D^+ K^0) \), which has the same penguin amplitude. Model-dependent calculations of other \( B_c \) decays can be found in Refs. [47, 48].

Some recent SU(3) analyses of two-body \( B_c \) decays can be found in Ref. [49, 50]. Compared to these studies, we have included all penguin amplitudes.

For the \( B_c^- \) meson, the charm quark can also decays, with the final state \( BP \) or \( BV \) [51]. Since the heavy bottom quark plays as a spectator, the decay modes are simpler. For example, for Cabibbo-allowed decay modes, there are only two channels: \( B_c^- \to \pi^- \bar{B}_s \) and \( B_c^- \to \rho^- \bar{B}_s \). Thus we expect that the SU(3) symmetry will not provide much information in these decays.

It is necessary to point out that the charmless two-body \( B_c \) decays are purely annihilation, and the typical branching fractions are below the order $10^{-6}$ [52–54]. Since there are not too many channels, it is less useful to apply the flavor SU(3) symmetry to these modes.

V. ANTITRIPLET BOTTOM BARYON DECAY INTO A BARYON AND A MESON

In this and next sections, we discuss weak decays of baryons with a heavy \( b \) and \( c \) quark. Charmed or bottom baryons with two light quarks can form an anti-triplet or a sextet. Most members of the sextet can decay via strong interactions or electromagnetic interactions. The only exceptions are \( \Omega_b \) and \( \Omega_c \). In the following we will concentrate on the anti-triplet baryons, whose weak decays are induced by the effective Hamiltonian \( H_{eff}^b \) and \( H_{eff}^c \).

A. \( T_b : (\Lambda_b, \Xi_b^0, \Xi_b^-) \) Decay into a decuplet baryon \( T_{10} \) and a light meson

The IRA amplitudes for the \( T_b \) decays into a decuplet baryon and a light meson can be parametrized as:

\[
\mathcal{A}_u^{IRA} = A_3^{T} T_{b3}^{[ij]} H_3^{[ik]} m_{(10)} P_{j}^{[l]} + A_6^{T} T_{b3}^{[ij]} (H_6)^{ijkl}_{(10)} P_{j}^{[l]} + A_7^{T} T_{b3}^{[ij]} (H_{15})^{ijkl}_{(10)} P_{j}^{[l]} P_{m}^{[l]} \\
+ B_{15}^{T} T_{b3}^{[ij]} (H_{15})^{ijkl}_{(10)} (H_{15})^{ijkl}_{(10)} P_{j}^{[l]} P_{m}^{[l]} + C_{15}^{T} T_{b3}^{[ij]} (H_{15})^{ijkl}_{(10)} (H_{15})^{ijkl}_{(10)} P_{j}^{[l]} P_{m}^{[l]} + D_{15}^{T} T_{b3}^{[ij]} (H_{15})^{ijkl}_{(10)} (H_{15})^{ijkl}_{(10)} klm P_{l}^{[l]} P_{m}^{[l]} \\
+ E_{15}^{T} T_{b3}^{[ij]} (H_{15})^{ijkl}_{(10)} (H_{15})^{ijkl}_{(10)} klm P_{l}^{[l]} P_{m}^{[l]}.
\]

The TDA amplitudes are shown in Fig. 4 with the parametrization:

\[
\mathcal{A}_u^{TDA} = a_1^{T} T_{b3}^{[ij]} H_{m}^{ijkl}_{(10)} P_{j}^{[l]} + b_1^{T} T_{b3}^{[ij]} H_{j}^{ijkl}_{(10)} (H_{15})^{ijkl}_{(10)} P_{j}^{[l]} P_{m}^{[l]} + b_2^{T} T_{b3}^{[ij]} H_{j}^{ijkl}_{(10)} (H_{15})^{ijkl}_{(10)} klm P_{l}^{[l]} P_{m}^{[l]} \\
+ b_3^{T} H_{m}^{ijkl}_{(10)} P_{j}^{[l]} + b_4^{T} H_{j}^{ijkl}_{(10)} (H_{15})^{ijkl}_{(10)} P_{j}^{[l]} P_{m}^{[l]} + b_5^{T} H_{j}^{ijkl}_{(10)} (H_{15})^{ijkl}_{(10)} klm P_{l}^{[l]} P_{m}^{[l]}.
\]

We find relations between the two sets of amplitudes as:

\[
a_1 = A_3^{T} + A_6^{T} - A_{15}^{T} - 2 B_{15}^{T} + 2 D_{15}^{T}, \quad b_1 = 4 A_{15}^{T} + 2 A_{15}^{T}, \\
b_2 = 4 A_{15}^{T} - 2 A_{15}^{T}, \quad b_3 = 8 B_{15}^{T}, \quad b_4 = 4 C_{15}^{T}, \quad b_5 = 8 D_{15}^{T}.
\]

The expanded amplitudes for individual decay modes can be found in Tab. IX.

A few remarks are given in order.

References [42, 44–51, 53, 54].
TABLE IX: Decay amplitudes for $T_s \to T_{10}P$ decays. Only those amplitudes proportional to $V_{ub}V_{us}^*$ are shown, while “penguin” amplitudes proportional to $V_{tb}V_{ts}^*$ are similar.

| $b \to d$ | IRA | TDA |
|-----------|-----|-----|
| $\Lambda_b^0 \to \Delta^+\pi^-$ | $(A_1^d - A_3^d - 5A_5^d + 6B_1^d - 6D_2^d)/\sqrt{3}$ | $(a_1 - b_1 + b_3 - b_5)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Delta^0\pi^+$ | $\sqrt{2/3}(-A_1^d + A_3^d + A_5^d - 2B_1^d + 2D_2^d)$ | $(-2a_1 + b_1 - b_2 - b_3 + b_5)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Delta^0\eta_9$ | $-4\sqrt{2/3}(A_1^d + B_1^d + 2C_5^d + D_1^d)$ | $-(b_1 + b_2 + b_3 + 2b_4 + b_5)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Delta^0\eta_8$ | $-8C_5^d/\sqrt{3}$ | $-b_4/\sqrt{3}$ |
| $\Lambda_b^0 \to \Sigma^0\eta^0$ | $-2A_5^d + 3A_7^d + 3A_8^d + 3A_9^d + 2B_1^d - 2D_2^d$ | $-a_1 - b_2$ |
| $\Lambda_b^0 \to \Sigma^0\eta^0$ | $-(-A_1^d + A_3^d + 5A_5^d + 2B_1^d + 6D_2^d)/\sqrt{6}$ | $(a_1 - b_1 - b_5)/\sqrt{6}$ |
| $\Lambda_b^0 \to \Sigma^-\pi^+$ | $(-A_1^d + A_5^d - 3A_7^d + 2B_1^d - 2D_2^d)/\sqrt{3}$ | $(-a_1 + b_2 + b_5)/\sqrt{3}$ |
| $\Xi_b^0 \to \Delta^+\pi^-$ | $(A_1^d - A_3^d - A_5^d + 6B_1^d - 6D_2^d)/\sqrt{3}$ | $(a_1 + b_1 - b_5)/\sqrt{3}$ |
| $\Xi_b^0 \to \Delta^0\pi^+$ | $(A_1^d + A_3^d - A_5^d - 2B_1^d - 8C_5^d - 6D_2^d)/\sqrt{6}$ | $(a_1 - b_1 - b_5)/\sqrt{6}$ |
| $\Xi_b^0 \to \Delta^0\eta_9$ | $(-A_1^d + A_3^d - A_5^d - 2B_1^d - 6D_2^d)/\sqrt{3}$ | $(a_1 - b_1 - b_5)/\sqrt{3}$ |
| $\Xi_b^0 \to \Xi^0\eta^0$ | $(-2A_1^d + 2A_3^d))/\sqrt{3}$ | $-b_1/\sqrt{3}$ |
| $\Xi_b^0 \to \Sigma^0\eta^0$ | $(-A_1^d + 3A_3^d + A_5^d - 6B_1^d - 2D_2^d)/2\sqrt{3}$ | $-(a_1 + b_1 + b_3 + 2b_4)/2\sqrt{3}$ |
| $\Xi_b^0 \to \Sigma^0\eta_9$ | $-(A_1^d + A_3^d + 7A_5^d + 6B_1^d + 16C_1^d + 2D_2^d)/2\sqrt{3}$ | $-(a_1 + b_1 + b_3 + 2b_4)/2\sqrt{3}$ |
| $b \to s$ | IRA | TDA |
| $\Lambda_b^0 \to \Delta^-\pi^+$ | $-(A_1^d + A_3^d - A_5^d - 2B_1^d - 6D_2^d)/\sqrt{3}$ | $-(a_1 + b_1 - b_2)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Delta^-\eta_9$ | $-(A_1^d + A_3^d - A_5^d - 2B_1^d - 6D_2^d)/\sqrt{3}$ | $-(a_1 + b_1 - b_2)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Delta^-\eta_8$ | $-(A_1^d + A_3^d - A_5^d - 2B_1^d - 6D_2^d)/\sqrt{3}$ | $-(a_1 + b_1 - b_2)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Delta^-\eta_7$ | $(A_1^d + A_3^d - A_5^d - 2B_1^d - 6D_2^d)/\sqrt{3}$ | $-(a_1 + b_1 - b_2)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\pi^+$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{6}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_9$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_8$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_7$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_6$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_5$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_4$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_3$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_2$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
| $\Lambda_b^0 \to \Xi^-\eta_1$ | $(A_1^d + A_3^d - A_5^d + 2B_1^d + 6D_2^d)/\sqrt{3}$ | $-(a_1 - b_1 - b_3)/\sqrt{3}$ |
FIG. 4: Topology diagrams for the bottom baryon decays into a decuplet baryon and a light meson.

TABLE X: U-spin relations for $T_b \to T_0V$. If the final state contains a light pseudoscalar meson, the U-spin relations can be obtained similarly except that $\eta_u$ and $\eta_d$ mix.
• As the two light quarks in the initial state are antisymmetric in the flavor space while they are symmetric in the final state. An overlap of wave functions vanishes [55], unless hard scattering interactions occur [56]. In other words, there is no “factorizable” contribution in the transition. In addition, all diagrams in Fig. 4 are suppressed by powers of 1/\( N_c \) compared to the \( T_b \rightarrow T_8 P \). This will indicate that branching fractions for these decays are likely smaller than the relevant \( B \) decays and \( T_b \rightarrow T_8 P \) decays, where \( T_8 \) represents the octet baryon.

• For the \( T_b \rightarrow T_{10} P \), one can construct the amplitudes with the spinors, and a general form is:

\[
A = p_{T_b, \mu} \bar{u}^\mu (p_{T_{10}})(A + B \gamma_5) u(p_{T_b}),
\]

where \( A \) and \( B \) are two nonperturbative coefficients containing the CKM factors, and have the same flavor structure with \( A_{u,t} \). Thus in total, one has \( 6 \times 2 \times 2 = 24 \) complex amplitudes in theory.

• Since the initial baryon and final baryons are polarized, it is convenient to express the decays with helicity amplitudes:

\[
A(S_{in} \rightarrow S_{f1} S_{f2}),
\]

where \( S_{in} \) and \( S_{f1}, S_{f2} \) are polarizations of initial and final states. The two sets of helicity amplitudes for \( T_b \rightarrow T_{10} P \) can be derived using the parametrization in Eq. (64):

\[
A \left( \frac{1}{2} \rightarrow \frac{1}{2} 0 \right) = \sqrt{\frac{2}{3} \frac{m_{T_b}}{m_{T_{10}}}} p_{cm} N_{T_{10}} N_{T_b} \left( A - B \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right),
\]

\[
A \left( -\frac{1}{2} \rightarrow -\frac{1}{2} 0 \right) = \sqrt{\frac{2}{3} \frac{m_{T_b}}{m_{T_{10}}}} p_{cm} N_{T_{10}} N_{T_b} \left( A + B \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right).
\]

Here \( E_{T_{10}} \) and \( p_{cm} \) are the energy and 3-momentum magnitude of \( T_{10} \) in the rest frame of \( T_b \). \( N_{T_{10}} \) and \( N_{T_b} \) are normalization factors of \( T_{10} \) and \( T_b \) spinors:

\[
p_{cm} = \frac{1}{2 m_{T_b}} \sqrt{(m_{T_b}^2 - (m_{T_{10}} + m_P)^2)(m_{T_b}^2 - (m_{T_{10}} - m_P)^2)}, \quad E_{T_{10}} = \frac{m_{T_{10}}^2 + m_{T_b}^2 - m_P^2}{2 m_{T_b}},
\]

\[
N_{T_{10}} = \sqrt{\frac{(m_{T_{10}} + m_{T_b})^2 - m_P^2}{2 m_{T_b}}}, \quad N_{T_b} = \sqrt{2 m_{T_b}}.
\]

• For \( T_b \rightarrow T_{10} \nu \), one can construct the amplitudes with the spinors and polarization vector 2:

\[
A = \epsilon^\nu \cdot p_{T_b} p_{T_{10}, \mu} \bar{u}^\mu (p_{T_{10}})(A' + B' \gamma_5) u(p_{T_b}) + \epsilon^\nu \nu_{T_b, \mu} \bar{u}^\mu (p_{T_{10}})(C' \gamma_\nu + D' \gamma_\nu \gamma_5) u(p_{T_b})
\]

\[+ \epsilon^\nu \cdot \bar{u}^\mu (p_{T_{10}})(E' + F' \gamma_5) u(p_{T_b}).
\]

---

2 One may expect a term which looks like \( \epsilon_{\mu \alpha \beta} \epsilon^\nu \bar{u}^\mu (p_{T_{10}})(G' \sigma^{\alpha \beta} + H' \sigma^{\alpha \beta} \gamma_5) u(p_b) \). Actually such term can be absorbed into the term \( \epsilon_{\mu} \bar{u}^\mu (p_{T_{10}})(E' + F' \gamma_5) u(p_b) \) by using the fact that the spinor-vector \( u^\mu (p_{T_{10}}) \), as an irreducible representation of \( 1/2 \otimes 1 \), must satisfy \( \gamma_{\mu} u^\mu (p_{T_{10}}) = 0 \).
There are six different polarization configurations. The helicity amplitudes are given as:

\[
A\left(\frac{1}{2} \rightarrow \frac{1}{2} 0\right) = \sqrt{\frac{2}{3}} \frac{m_T}{3 m_{T_{10}}} p_{cm} N_{T_{10}} N_{T_{b}} \left[ - \frac{m_T}{m_V} p_{cm} \left( A' - B' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right) \right] + C' \frac{p_{cm}}{m_V} \left( \frac{m_T - E_{T_{10}} - 1}{m_V m_{T_{10}}} \right) + D' \left( \frac{m_T - E_{T_{10}} - \frac{p_{cm}^2}{E_{T_{10}} + m_{T_{10}}}}{m_V} \right) + \left( - \frac{p_{cm}}{m_V m_{T_{10}}} + E_{T_{10}}(m_T - E_{T_{10}}) \frac{p_{cm}}{m_V m_{T_{10}} p_{cm}} \right) \left( E' + F' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right),
\]

\[
A\left(\frac{1}{2} \rightarrow -\frac{1}{2} 0\right) = \sqrt{\frac{2}{3}} \frac{m_T}{3 m_{T_{10}}} p_{cm} N_{T_{10}} N_{T_{b}} \left[ - \frac{m_T}{m_V} p_{cm} \left( A' + B' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right) \right] + C' \frac{p_{cm}}{m_V} \left( \frac{m_T - E_{T_{10}} - 1}{m_V m_{T_{10}}} \right) + D' \left( \frac{m_T - E_{T_{10}} - \frac{p_{cm}^2}{E_{T_{10}} + m_{T_{10}}}}{m_V} \right) + \left( - \frac{p_{cm}}{m_V m_{T_{10}}} + E_{T_{10}}(m_T - E_{T_{10}}) \frac{p_{cm}}{m_V m_{T_{10}} p_{cm}} \right) \left( E' + F' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right),
\]

\[
A\left(\frac{1}{2} \rightarrow -\frac{1}{2} 1\right) = \frac{1}{\sqrt{3}} N_{T_{10}} \left[ 2 \frac{p_{cm} m_T}{m_{T_{10}}} \left( D' - C' \frac{p}{E_{T_{10}} + m_{T_{10}}} \right) + \left( E' - F' \frac{p}{E_{T_{10}} + m_{T_{10}}} \right) \right],
\]

\[
A\left(\frac{1}{2} \rightarrow \frac{1}{2} -1\right) = \frac{1}{\sqrt{3}} N_{T_{10}} \left[ - \frac{2 p_{cm} m_T}{m_{T_{10}}} \left( D' + C' \frac{p}{E_{T_{10}} + m_{T_{10}}} \right) + \left( E' + F' \frac{p}{E_{T_{10}} + m_{T_{10}}} \right) \right],
\]

\[
A\left(\frac{1}{2} \rightarrow 3 \frac{1}{2}\right) = N_{T_{10}} \left( E' - F' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right),
\]

\[
A\left(-\frac{1}{2} \rightarrow 3 \frac{1}{2}\right) = N_{T_{10}} \left( E' + F' \frac{p_{cm}}{E_{T_{10}} + m_{T_{10}}} \right).
\]

(70)

The definitions of \(E_{T_{10}}, p_{cm}, N_{T_{10}}\) and \(N_{T_{b}}\) are the same as Eq. (68) except replacing \(m_P\) by \(m_V\). Again all these amplitudes can be determined from the angular distributions of the four-body decays \(T_b \rightarrow T_{10}(\rightarrow T_b P_1) V(\rightarrow P_2 P_3)\).

- Branching fractions for \(T_b\) decays into a proton with three charged pion/kaons are found at the order \(10^{-5}\) in Ref. [57]. A plausible scenario is that the \(T_b \rightarrow T_{10} V\) contribute significantly to the \(T_b\) decaying into a proton and three charged light mesons. If this is true, we expect that with more data in future, a detailed analysis will determine the decay widths of \(T_b \rightarrow T_{10} V\). Then the flavor SU(3) symmetry can be examined, and meanwhile it will also shed light on the \(C P\) and \(T\) violation in baryonic transitions by using the triplet product asymmetries [58, 59].

- Through the results in Tab. IX, we can find the relations both for decays into \(T_b P\) and \(T_b V\). Here only the channels with one vector octet in final states can be listed (71), (72). For channels with one pseudoscalar in final states the relations are almost the same, obtained by replacing the vector multiplets \(V\) by the pseudo-scalar multiplets \(P\). However, \(\eta_0\) and \(\eta_8\) are unphysical states so that the decay width relations involving them should be removed.

For \(b \rightarrow d\) transitions, one has:

\[
\Gamma(\Lambda_b^0 \rightarrow \Delta^- \rho^+) = 3 \Gamma(\Lambda_b^0 \rightarrow \Sigma^- K^+), \quad \Gamma(\Xi_b^- \rightarrow \Delta^- K^0) = 6 \Gamma(\Xi_b^- \rightarrow \Sigma^- \omega),
\]

\[
\Gamma(\Lambda_b^0 \rightarrow \Delta^- \rho^+) = \frac{3}{2} \Gamma(\Xi_b^- \rightarrow \Sigma^- \rho^+), \quad \Gamma(\Xi_b^- \rightarrow \Delta^- K^0) = 3 \Gamma(\Xi_b^- \rightarrow \Sigma^- \phi),
\]

\[
\Gamma(\Lambda_b^0 \rightarrow \Delta^- \rho^+) = 3 \Gamma(\Xi_b^- \rightarrow \Sigma^- K^+), \quad \Gamma(\Xi_b^- \rightarrow \Delta^- K^0) = 3 \Gamma(\Xi_b^- \rightarrow \Sigma^- K^0),
\]

\[
\Gamma(\Lambda_b^0 \rightarrow \Sigma^- K^+) = \frac{1}{2} \Gamma(\Xi_b^- \rightarrow \Sigma^- K^+), \quad \Gamma(\Xi_b^- \rightarrow \Sigma^- \rho^+) = \frac{1}{2} \Gamma(\Xi_b^- \rightarrow \Sigma^- \rho^+),
\]

\[
\Gamma(\Xi_b^- \rightarrow \Sigma^+ K^-) = \frac{1}{2} \Gamma(\Xi_b^- \rightarrow \Xi^- K^0), \quad \Gamma(\Xi_b^- \rightarrow \Sigma^+ \rho^-) = \frac{1}{2} \Gamma(\Xi_b^- \rightarrow \Sigma^+ \rho^-),
\]

\[
\Gamma(\Xi_b^- \rightarrow \Delta^- K^0) = 6 \Gamma(\Xi_b^- \rightarrow \Sigma^- \rho^0), \quad \Gamma(\Xi_b^- \rightarrow \Sigma^- \phi) = \Gamma(\Xi_b^- \rightarrow \Sigma^- K^0).
\]

(71)
For $b \rightarrow s$ transition, we have the relations for decay widths:

$$
\Gamma(\Lambda_b^0 \rightarrow \Delta^+ \Sigma^- \pi^0) = \Gamma(\Lambda_b^0 \rightarrow \Delta^0 \Xi^-) = 2\Gamma(\Xi_b^- \rightarrow \Xi^- \omega),
\Gamma(\Lambda_b^0 \rightarrow \Sigma^- \rho^+) = \Gamma(\Lambda_b^0 \rightarrow \Xi^- \Sigma^0),
\Gamma(\Xi_b^- \rightarrow \Sigma^- \rho^+) = \Gamma(\Xi_b^- \rightarrow \Xi^- \phi),
\Gamma(\Lambda_b^0 \rightarrow \Sigma^- \rho^+) = \Gamma(\Xi_b^- \rightarrow \Xi^- \omega),
\Gamma(\Xi_b^- \rightarrow \Sigma^- \rho^+) = \frac{1}{2}\Gamma(\Xi_b^- \rightarrow \Xi^- \phi).
$$

As discussed in the previous section, charmed $b \rightarrow d$ and $b \rightarrow s$ transitions can be connected by U-spin. In Table X, we collect the $T_b \rightarrow T_{10}V$ decay pairs related by $U$-spin, while results for the final state with a light pseudoscalar meson can be obtained similarly. CP asymmetries for these pairs satisfy relation in Eq. (35).

Inspired from $B$ decay data [3, 4], we expect CP asymmetries for these decays are at the order 10%. Experimental measurements of these relations are important to test flavor SU(3) symmetry and the CKM description of CP violation in SM.

**B. $T_b(\Lambda_b, \Xi_b^0, \Xi_b^-)$ Decay into an octet baryon and a meson**

If the final state contains a baryon octet, the topological diagrams are shown in Fig. 5 where ten diagrams can be found. However unlike the decuplet baryon, the octet baryon is not fully symmetric or antisymmetric in flavor space. Thus each of the diagrams can provide more than one amplitudes. In total, one can have 26 independent TDA amplitudes:

$$
\mathcal{A}_{TDA} = \bar{a}_1 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_2 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_3 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_4 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_5 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_6 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_7 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_8 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_9 T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{10} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{11} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{12} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{13} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{14} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{15} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{16} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{17} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{18} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{19} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{20} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{21} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{22} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{23} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{24} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{25} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m + \bar{a}_{26} T_{63}^{[ij]} H_m^{kl}(T_8)_{ijkl} P_m.
$$

In the IRA approach, one can construct 14 amplitudes:

$$
\mathcal{A}_{IRA} = A_{10}^{T} (T_{63}), H_{2/3}^{[ij]} (T_8)_{ijkl} P_m + B_{10}^{T} (T_{63}), H_{3/2}^{[ij]} (T_8)_{ijkl} P_m + C_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m + D_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m + E_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m + F_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m + G_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m + H_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m + I_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m + J_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m + K_{10}^{T} (T_{63}), H_{1/2}^{[ij]} (T_8)_{ijkl} P_m.
$$

It should be noticed that in the IRA approach the antitriplet baryon and octet baryon are expressed in SU(3) representation $3$ and $3 \otimes 3$ respectively which are different from the representation $3 \otimes 3$ and $3 \otimes 3 \otimes 3$ used for TDA. Superficially this different representation contains less indexes so that it reduces the number of amplitudes from
| Channel                                   | IRA                                      | TDA                                      |
|------------------------------------------|------------------------------------------|------------------------------------------|
| $\Lambda^0 \to \Lambda^0 K^0$            | $(-2 B_3^T + D_3^T + B_6^T + 2 C_6^T + 2 E_6^T + 3 D_6^T$ \(- B_15^T + 2 C_15^T + 2 E_15^T + 3 D_15^T) \sqrt{6}$) | $(a_2 + \bar{a}_3 - 2 a_7 - 2 a_8 - 2 a_9 - 2 a_{10} - 2 a_{11} - 2 a_{12} - 2 a_{13} - 2 a_{14}) \sqrt{6}$ |
| $\Lambda^0 \to \Sigma^0 K^0$             | $(-D_4^T + B_4^T + D_6^T + B_15^T + 5 D_15^T) \sqrt{2}$) | $(a_2 + a_4 + a_6 + a_8 + b_4 + b_6 + b_7) \sqrt{2}$ |
| $\Lambda^0 \to \Sigma^- K^+$             | $D_3^T - B_3^T - D_6^T - B_{15}^T + 3 D_{15}^T$) | $a_8 + a_6 + a_7 - b_6 - b_7$ |
| $\Lambda^0 \to p\pi^-$                   | $B_3^T - C_6^T + E_6^T - C_{15}^T + 3 E_{15}^T$) | $a_2 + a_4 - a_6 - a_{10} - a_{11} + a_{12} - a_{13} + a_{14} + a_{15}$ |
| $\Lambda^0 \to n\pi^0$                   | $(-B_3^T + C_6^T - E_6^T + C_{15}^T + 5 E_{15}^T) \sqrt{2}$) | $(a_2 + a_4 + a_6 + a_{10} - a_{11} - a_{12} - a_{13} + a_{15}) \sqrt{2}$ |
| $\Lambda^0 \to n\eta$                    | $(2 A_7^T + B_3^T - 2 A_6^T - C_6^T - E_6^T - 2 A_{15}^T - C_{15}^T + E_{15}^T) \sqrt{2}$) | $(1/2 \sqrt{3})(a_2 + a_4 + a_6 - a_7 - a_8 + 2 a_{10} + 2 a_{11} - 2 a_{12} - 2 a_{13} - 2 a_{14} - 2 a_{15}) \sqrt{2}$ |
| $\Lambda^0 \to n\eta_8$                  | $A_4^T + D_4^T - A_6^T - B_6^T + E_6^T$ $+ D_8^T - A_9^T + B_9^T - E_{15}^T + D_{15}^T$ | $-(a_3 + a_5 + 2 a_7 - 2 a_9 - 2 a_{10} - 2 a_{11} - 2 a_{12} - 2 a_{13} - 2 a_{14}) \sqrt{2}$ |
| $\Xi^0 \to \Lambda^0 \pi^0$             | $(B_4^T + D_4^T + B_6^T + C_6^T + E_6^T + 3 D_6^T - 5 B_{15}^T - 5 C_{15}^T - 5 E_{15}^T + 3 D_{15}^T) \sqrt{3}$ | $(a_2 + a_4 + a_6 - a_7 - 2 a_{10} + 2 a_{11} - 2 a_{12} - 2 a_{13} + a_{15}) \sqrt{3}$ |
| $\Xi^0 \to \Lambda^0 \eta$              | $-2 A_7^T + B_7^T + 2 C_7^T + D_7^T - 6 A_6^T - B_6^T - C_6^T - E_6^T + 3 D_6^T + 6 A_{15}^T + B_{15}^T + C_{15}^T + E_{15}^T + 3 D_{15}^T \sqrt{3}$ | $(a_2 + a_4 + a_6 - a_7 - a_8 + a_{10} + a_{11} - a_{12} + a_{13} + a_{15}) \sqrt{3}$ |
| $\Xi^0 \to \Sigma^+ \pi^-$              | $(1/2)(-B_3^T - 2 C_3^T - D_3^T + B_6^T + C_6^T - E_6^T + 3 D_6^T + 3 A_{15}^T + 2 B_{15}^T + C_{15}^T + E_{15}^T - 3 D_{15}^T) \sqrt{3}$ | $(a_2 + a_4 + a_6 - a_7 - 2 a_{10} + 2 a_{11} - 2 a_{12} - 2 a_{13} + a_{14}) \sqrt{3}$ |
| $\Xi^0 \to \Xi^- \pi^+$                 | $(1/2)(-B_3^T - 2 C_3^T - D_3^T - B_6^T - C_6^T - E_6^T + 3 D_6^T + 3 A_{15}^T + 2 B_{15}^T + C_{15}^T + E_{15}^T - 3 D_{15}^T) \sqrt{3}$ | $(a_2 + a_4 + a_6 - a_7 - a_8 + a_{10} + a_{11} - a_{12} + a_{13} + a_{14}) \sqrt{3}$ |
| $\Xi^0 \to \Sigma^- \pi^-$              | $(A_3^T + A_4^T + E_4^T - 5 A_{15}^T - E_{15}^T) \sqrt{2}$ | $(-a_3 + a_5 + a_6 + a_7 - a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14}) \sqrt{2}$ |
| $\Xi^0 \to \Lambda^0 K^-$                | $-C_3^T - D_3^T + C_6^T + D_6^T + 2 B_{15}^T - 3 C_{15}^T - 3 D_{15}^T$ | $-a_3 - a_5 - a_6 - a_7 - a_8$ |
| $\Xi^0 \to nK$                           | $-C_3^T - D_3^T - C_6^T - D_6^T - 2 B_{15}^T + C_{15}^T + D_{15}^T$ | $-a_3 + a_6 + a_8 - a_9 + a_{10} + a_{11} - a_{12} + a_{13} + a_{14} + a_{15}$ |
| $\Xi^0 \to \Sigma^- K^+$                 | $-C_3^T - D_3^T - C_6^T + D_6^T + 2 B_{15}^T - C_{15}^T + D_{15}^T$ | $-a_3 - a_5 + a_6 + a_8 + a_9 + a_{10} - a_{12} + a_{13} + a_{14} + a_{15}$ |
| $\Xi^0 \to \Sigma^0 K^0$                 | $-B_3^T - C_3^T - B_6^T + E_6^T + 3 B_{15}^T + 2 C_{15}^T + E_{15}^T$ | $-a_3 - a_5 - a_6 - a_7 - a_8 - a_9 - a_{10} + a_{11} - a_{12} + a_{13} + a_{14} + a_{15}$ |
| $\Xi^0 \to \Lambda^0 \pi^-$              | $(B_4^T + D_4^T + B_6^T + C_6^T + E_6^T + 3 D_6^T + 3 A_{15}^T + 2 B_{15}^T + C_{15}^T + E_{15}^T + 3 D_{15}^T) \sqrt{3}$ | $(a_2 + a_4 + a_6 - a_7 - 2 a_{10} + 2 a_{11} - 2 a_{12} - 2 a_{13} + a_{15}) \sqrt{3}$ |
| $\Xi^0 \to \Sigma^0 \pi^-$              | $(B_4^T + D_4^T + B_6^T + C_6^T + E_6^T + 3 D_6^T + 3 A_{15}^T + 2 B_{15}^T + C_{15}^T + E_{15}^T + 3 D_{15}^T) \sqrt{3}$ | $(a_2 + a_4 + a_6 - a_7 - a_8 + a_{10} + a_{11} - a_{12} + a_{13} + a_{15}) \sqrt{3}$ |
| $\Xi^0 \to \Sigma^- \pi^0$               | $-B_3^T - D_3^T - B_6^T + C_6^T + E_6^T + 3 D_6^T + 3 A_{15}^T + 2 B_{15}^T + C_{15}^T + E_{15}^T + 3 D_{15}^T) \sqrt{3}$ | $(a_2 + a_4 + a_6 - a_7 - a_8 + a_{10} + a_{11} - a_{12} + a_{13} + a_{14} + a_{15}) \sqrt{3}$ |
| $\Xi^0 \to \Sigma^- \eta$                | $2 A_7^T + B_7^T + D_7^T + 2 A_6^T + B_6^T + C_6^T - E_6^T + 3 D_6^T + 3 A_{15}^T + 2 B_{15}^T + C_{15}^T + E_{15}^T + 3 D_{15}^T) \sqrt{3}$ | $(a_2 + a_4 + a_6 - a_7 - a_8 + a_{10} + a_{11} - a_{12} + a_{13} + a_{14} + a_{15}) \sqrt{3}$ |
TABLE XII: Decay amplitudes for two-body $T_b \to T_s P$ decays induced by the $b \to s$ transition.

| channel | IRA | TDA |
|---------|-----|-----|
| $\Lambda^0 \to \Lambda^0 \pi^0$ | $-2(E_0^\pi + B_0^\pi + C_0^\pi + 2E_1^\pi + B_1^\pi + C_1^\pi)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14})/\sqrt{2}$ |
| $\Lambda^0 \to \Lambda^0 \eta_1$ | $-2(E_1^\eta_1 + B_1^\eta_1 + C_1^\eta_1 + 2E_2^\eta_1 + B_2^\eta_1 + C_2^\eta_1)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Lambda^0 \to \Lambda^0 \eta_2$ | $-2(E_2^\eta_2 + B_2^\eta_2 + C_2^\eta_2 + 2E_3^\eta_2 + B_3^\eta_2 + C_3^\eta_2)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18} + a_{22})/\sqrt{2}$ |
| $\Lambda^0 \to \Sigma^+ \pi^0$ | $-2(E_0^\Sigma + B_0^\Sigma + C_0^\Sigma + 2E_1^\Sigma + B_1^\Sigma + C_1^\Sigma + 2E_2^\Sigma + B_2^\Sigma + C_2^\Sigma + 2E_3^\Sigma + B_3^\Sigma + C_3^\Sigma)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Lambda^0 \to \Sigma^0 \pi^0$ | $-2(E_1^\Sigma + B_1^\Sigma + C_1^\Sigma + 2E_2^\Sigma + B_2^\Sigma + C_2^\Sigma + 2E_3^\Sigma + B_3^\Sigma + C_3^\Sigma)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Lambda^0 \to \Sigma^0 \eta_1$ | $-2(E_2^\Sigma + B_2^\Sigma + C_2^\Sigma + 2E_3^\Sigma + B_3^\Sigma + C_3^\Sigma + 2E_4^\Sigma + B_4^\Sigma + C_4^\Sigma + 2E_5^\Sigma + B_5^\Sigma + C_5^\Sigma)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Lambda^0 \to \Omega^- K^0$ | $-2(E_0^\Omega + B_0^\Omega + C_0^\Omega + 2E_1^\Omega + B_1^\Omega + C_1^\Omega + 2E_2^\Omega + B_2^\Omega + C_2^\Omega + 2E_3^\Omega + B_3^\Omega + C_3^\Omega + 2E_4^\Omega + B_4^\Omega + C_4^\Omega)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Xi^- \to \Lambda^0 K^0$ | $-2(E_1^\Lambda + B_1^\Lambda + C_1^\Lambda + 2E_2^\Lambda + B_2^\Lambda + C_2^\Lambda + 2E_3^\Lambda + B_3^\Lambda + C_3^\Lambda + 2E_4^\Lambda + B_4^\Lambda + C_4^\Lambda + 2E_5^\Lambda + B_5^\Lambda + C_5^\Lambda)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Xi^- \to \Sigma^0 K^0$ | $-2(E_2^\Sigma + B_2^\Sigma + C_2^\Sigma + 2E_3^\Sigma + B_3^\Sigma + C_3^\Sigma + 2E_4^\Sigma + B_4^\Sigma + C_4^\Sigma + 2E_5^\Sigma + B_5^\Sigma + C_5^\Sigma + 2E_6^\Sigma + B_6^\Sigma + C_6^\Sigma)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Xi^- \to \Xi^- \eta_1$ | $-2(E_3^\Xi + B_3^\Xi + C_3^\Xi + 2E_4^\Xi + B_4^\Xi + C_4^\Xi + 2E_5^\Xi + B_5^\Xi + C_5^\Xi + 2E_6^\Xi + B_6^\Xi + C_6^\Xi + 2E_7^\Xi + B_7^\Xi + C_7^\Xi)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Xi^- \to \Xi^- \eta_2$ | $-2(E_4^\Xi + B_4^\Xi + C_4^\Xi + 2E_5^\Xi + B_5^\Xi + C_5^\Xi + 2E_6^\Xi + B_6^\Xi + C_6^\Xi + 2E_7^\Xi + B_7^\Xi + C_7^\Xi + 2E_8^\Xi + B_8^\Xi + C_8^\Xi)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |
| $\Xi^- \to \Xi^- \pi^0$ | $-2(E_5^\Xi + B_5^\Xi + C_5^\Xi + 2E_6^\Xi + B_6^\Xi + C_6^\Xi + 2E_7^\Xi + B_7^\Xi + C_7^\Xi + 2E_8^\Xi + B_8^\Xi + C_8^\Xi + 2E_9^\Xi + B_9^\Xi + C_9^\Xi)/\sqrt{3}$ | $(a_2 + a_4 + a_6 + a_{10} + a_{12} + a_{14} + a_{16} + a_{18})/\sqrt{2}$ |

26 to 14. This is indeed doable which is shown in Appendix A. As a result, the 14 IRA amplitudes and 26 TDA
FIG. 5: Topology diagrams for the bottom baryon decays into an octet baryon and a light meson. Since the octet baryon is not fully symmetric or antisymmetric in flavor space, there are more than one amplitudes corresponding to one topological diagram. Actually the 10 topological diagrams correspond to 26 amplitudes shown in Eq.(73).

amplitudes are related as follows:

\[
A_3^T = \frac{1}{8} (-2\bar{a}_1 + 6\bar{a}_2 - 5\bar{a}_3 - \bar{a}_5 + \bar{a}_6 - 3\bar{a}_8 + 4\bar{a}_9 + \bar{a}_{10} - 3\bar{a}_{11} + 2\bar{a}_{13} - \bar{a}_{15} + 3\bar{a}_{16} + 3\bar{a}_{17} - \bar{a}_{18} + 4\bar{a}_{19}) \\
+ 2\bar{b}_1 + \bar{b}_3 + \bar{b}_6 + \bar{b}_7, \\
B_3^T = \frac{1}{8} (6\bar{a}_1 - 2\bar{a}_2 - 5\bar{a}_4 - 3\bar{a}_6 - \bar{a}_7 + \bar{a}_8 + 2\bar{a}_{10} + 2\bar{a}_{11} + 4\bar{a}_{12} + \bar{a}_{13} - 3\bar{a}_{14} + 3\bar{a}_{15} - \bar{a}_{16} - \bar{a}_{17} + 3\bar{a}_{18} - 4\bar{a}_{19}) \\
+ (2\bar{b}_2 + \bar{b}_4 + \bar{b}_5 - \bar{b}_7), \\
C_3^T = \frac{1}{8} (-\bar{a}_4 + 3\bar{a}_7 + \bar{a}_{10} - 3\bar{a}_{11} - 4\bar{a}_{12} - \bar{a}_{13} + 3\bar{a}_{14} + \bar{a}_{17} - 3\bar{a}_{18} + 4\bar{a}_{19} - 8\bar{b}_5) + \bar{b}_7, \\
D_3^T = \frac{1}{8} (-4 (\bar{a}_{19} + 2 (\bar{b}_6 + \bar{b}_7)) - \bar{a}_6 + 3\bar{a}_8 - \bar{a}_{10} + 3\bar{a}_{11} - 2\bar{a}_{13} - 2\bar{a}_{14} - 3\bar{a}_{17} + \bar{a}_{18}), \\
A_6^T = \frac{1}{4} (\bar{a}_3 - \bar{a}_5 - 2\bar{a}_9 - \bar{a}_{13} + \bar{a}_{14}), \\
B_6^T = \frac{1}{4} (\bar{a}_{13} - \bar{a}_{14} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}), \\
C_6^T = \frac{1}{4} (\bar{a}_4 - \bar{a}_7 - \bar{a}_{10} + \bar{a}_{11} - 2\bar{a}_{12}), \\
D_6^T = \frac{1}{4} (\bar{a}_6 - \bar{a}_8 + \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{13} + \bar{a}_{14}), \\
E_6^T = \frac{1}{4} (2\bar{a}_1 - 2\bar{a}_2 - \bar{a}_6 + \bar{a}_8 - \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} - \bar{a}_{16} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}), \\
A_{15}^T = \frac{1}{8} (\bar{a}_3 + \bar{a}_5 - \bar{a}_{13} - \bar{a}_{14}), \\
B_{15}^T = \frac{1}{8} (\bar{a}_{13} + \bar{a}_{14} - \bar{a}_{17} - \bar{a}_{18}), \\
C_{15}^T = \frac{1}{8} (\bar{a}_4 + \bar{a}_7 - \bar{a}_{10} - \bar{a}_{11}), \\
D_{15}^T = \frac{1}{8} (\bar{a}_6 + \bar{a}_8 + \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} + \bar{a}_{14}), \\
E_{15}^T = \frac{1}{8} (2\bar{a}_1 + 2\bar{a}_2 - \bar{a}_6 - \bar{a}_8 - \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18}).
\]
tudes. The redundant amplitude can be made explicit with the redefinitions:

$$A_6^{T'} = A_6^T + B_6^T, \quad B_6^{T'} = B_6^T - C_6^T, \quad C_6^{T'} = C_6^T - E_6^T, \quad D_6^{T'} = C_6^T + D_6^T.$$  \hfill (76)

In addition, this redundancy can be understood more explicitly. In this work as well as the previous work Ref. [21] we use the irreducible representation operators for IRA as \((H_6/\pi^T)_{ij}^6\). Actually there exists a simpler \(H_6\) representation introduced by Ref [74], where \(H_6\) has only two lower indexes \((H_6)_{ij}\). With the use of \((H_6)_{ij}\) we do have only 13 IRA amplitudes. However, Since the IRA operators \((H_6/\pi^T)_{ij}^6\) have the same index structure as the TDA operators. They make the derivation of IRA/TDA correspondence more directly so we will keep the use of them.

The expanded amplitudes can be found in Tab. XI for the \(b \rightarrow d\) transition and XII for the \(b \rightarrow s\) transition, respectively. Again if the final state is a vector meson, the amplitudes can be derived similarly.

A few remarks are given in order.

- At first sight, the diagrammatic approach, as depicted in Fig. 5, is more intuitive, however as we have shown in the above, it is very hard for us to determine the independent amplitudes in this approach. This will introduce subtleties to the global fit in the diagrammatic approach.

- Without including the polarization, one can see from the IRA approach, there exist 13 independent complex amplitudes with CKM factor \(V_{ub}V_{uq}^\ast\) and another 13 amplitudes accompanied by \(V_{tb}V_{tq}^\ast\).

- Two polarization configurations exist for decays into a pseudoscalar meson, while there are four possibilities for decays into a vector meson.

- The U-spin related decay pairs are given in Tab. XIII, which completely fits with the results given by Ref. [60]. Here only the case for \(T_b \rightarrow B_6P\) is listed. Since no unphysical states \(\eta_b\) and \(\eta_s\) exist in Tab. XIII. The U-spin pairs for \(T_b \rightarrow B_6V\) are similar by replacing pseudoscalar octets by vector octets.

- Some theoretical analyses of nonleptonic bottom baryon decays based on either explicit modes or the flavor symmetry can be found in Refs. [61–66], while the experimental measurements can be found in Refs. [67–69]. To date, the available measurements of two-body \(\Lambda_b\) branching fractions are [3, 4]:

$$B(\Lambda_b \rightarrow p\pi^-) = (4.2 \pm 0.8) \times 10^{-6},$$
$$B(\Lambda_b \rightarrow pK^-) = (5.1 \pm 0.9) \times 10^{-6},$$
$$B(\Lambda_b \rightarrow \Lambda\eta) = (9.2 \pm 1.0) \times 10^{-6},$$
$$B(\Lambda_b \rightarrow \Lambda\eta') < 3.1 \times 10^{-6},$$
$$B(\Lambda_b \rightarrow p\phi) = (9.2 \pm 2.5) \times 10^{-6}. \hfill (77)$$

- The CP asymmetries for \(\Lambda_b \rightarrow p\pi^-/pK^-\) [69] have been measured:

$$A_{CP}^{p\pi^-} = -0.020 \pm 0.013 \pm 0.019, \quad A_{CP}^{pK^-} = -0.035 \pm 0.017 \pm 0.020. \hfill (78)$$

Thus measuring the branching fractions and CP asymmetries for \(\Xi_b^0 \rightarrow \pi^-\Sigma^+\) and \(\Xi_b^0 \rightarrow K^-\Sigma^+\) will help us to understand the U-spin in baryonic decays.
FIG. 6: Topology diagrams for the charmed baryon decays into an octet baryon. Although there are seven topological diagrams, they correspond to 19 TDA amplitudes as given in Eq. (79).

| channel | IRA | TDA |
|---------|-----|-----|
| $\Lambda_c^+ \rightarrow \Lambda^0 \pi^+$ | $(B_6^T + C_6^T - 2E_6^T + B_{15}^T + C_{15}^T - 2E_{15}^T)/\sqrt{6}$ | $-4\bar{a}_1 + \bar{a}_4 + 2\bar{a}_6 + \bar{a}_{10} - \bar{a}_{12} + \bar{a}_{13} + 2\bar{a}_{14}$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$ | $(-B_6^T + C_6^T - B_{15}^T + C_{15}^T)/\sqrt{2}$ | $(\bar{a}_4 - \bar{a}_{10} - \bar{a}_{12} + \bar{a}_{13} + \bar{a}_{17} - \bar{a}_{19})/\sqrt{2}$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ \eta_4$ | $(2A_6^T + B_6^T + C_6^T + 2A_{15}^T + B_{15}^T + C_{15}^T)/\sqrt{2}$ | $(2\bar{a}_3 + \bar{a}_4 - 2\bar{a}_9 - \bar{a}_{10} - \bar{a}_{12} - \bar{a}_{13} - \bar{a}_{17} - \bar{a}_{19})/\sqrt{2}$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ \eta_{10}$ | $A_6^T - D_6^T + A_{15}^T + D_{15}^T$ | $\bar{a}_3 + \bar{a}_8 - \bar{a}_9 + \bar{a}_{11}$ |
| $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$ | $(B_6^T - C_6^T + B_{15}^T - C_{15}^T)/\sqrt{2}$ | $-\bar{a}_4 + \bar{a}_{10} + \bar{a}_{12} + \bar{a}_{13} - \bar{a}_{17} - \bar{a}_{19}/\sqrt{2}$ |
| $\Lambda_c^+ \rightarrow p\bar{K}^0$ | $B_6^T - E_6^T + B_{15}^T + E_{15}^T$ | $2\bar{a}_2 - \bar{a}_8 - \bar{a}_{11} + \bar{a}_{16}$ |
| $\Lambda_c^+ \rightarrow \Xi^0 K^+$ | $C_6^T + D_6^T + C_{15}^T + D_{15}^T$ | $\bar{a}_4 + \bar{a}_6 - \bar{a}_{12} + \bar{a}_{14}$ |
| $\Xi_c^+ \rightarrow \Sigma^+ K^+$ | $E_6^T + D_6^T - E_{15}^T - D_{15}^T$ | $-2\bar{a}_2 - \bar{a}_{16} - \bar{a}_{17} - \bar{a}_{19}$ |
| $\Xi_c^0 \rightarrow \Lambda^0 K^+$ | $-E_6^T - D_6^T - E_{15}^T - D_{15}^T$ | $-2\bar{a}_1 - \bar{a}_{15} - \bar{a}_{18} + \bar{a}_{19}$ |
| $\Xi_c^0 \rightarrow \Lambda^0 \pi^+$ | $(2D_6^T - C_6^T - E_6^T - 2B_{15}^T + C_{15}^T + E_{15}^T)/\sqrt{6}$ | $(2\bar{a}_2 + \bar{a}_7 - \bar{a}_8 - 2\bar{a}_{11} + \bar{a}_{12} - \bar{a}_{13} - 2\bar{a}_{14} + \bar{a}_{16} + \bar{a}_{17} + 2\bar{a}_{18} - \bar{a}_{19})/\sqrt{6}$ |
| $\Xi_c^0 \rightarrow \Sigma^0 K^+$ | $-C_6^T - D_6^T + C_{15}^T + D_{15}^T$ | $\bar{a}_7 + \bar{a}_8 + \bar{a}_{12} + \bar{a}_{13}$ |
| $\Xi_c^0 \rightarrow \Xi^0 \bar{K}^0$ | $(C_6^T - E_6^T - C_{15}^T + E_{15}^T)/\sqrt{2}$ | $(2\bar{a}_2 - \bar{a}_7 - \bar{a}_8 - \bar{a}_{12} - \bar{a}_{13} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19})/\sqrt{2}$ |
| $\Xi_c^0 \rightarrow \Xi^0 \pi^+$ | $-B_6^T + E_6^T + B_{15}^T + E_{15}^T$ | $2\bar{a}_1 - \bar{a}_6 - \bar{a}_{10} + \bar{a}_{15}$ |
| $\Xi_c^0 \rightarrow \Xi^0 \eta^0$ | $(B_6^T + D_6^T - B_{15}^T + D_{15}^T)/\sqrt{2}$ | $\bar{a}_6 + \bar{a}_{10} + \bar{a}_{18} - \bar{a}_{19}/\sqrt{2}$ |
| $\Xi_c^0 \rightarrow \Xi^0 \eta_4$ | $-2B_6^T + D_6^T + 2A_6^T + B_{15}^T + B_{15}^T + D_{15}^T)/\sqrt{2}$ | $(2\bar{a}_3 + \bar{a}_6 + 2\bar{a}_9 + \bar{a}_{10} - \bar{a}_{18} + \bar{a}_{19})/\sqrt{2}$ |
| $\Xi_c^0 \rightarrow \Xi^0 \eta_9$ | $-A_6^T - C_6^T + A_{15}^T + C_{15}^T$ | $\bar{a}_5 + \bar{a}_7 + \bar{a}_9 - \bar{a}_{11} + \bar{a}_{12} - \bar{a}_{14}$ |
| Channel | IRA | TDA |
|---------|-----|-----|
| $\Lambda_c^+ \to \Lambda^0 K^+$ | $(B_0^T - 2C_0^T - 2E_0^T - 3D_0^T + B_{15}^T - 2C_{15}^T - 3D_{15}^T)/\sqrt{6}$ | $-(a_1 + 2a_4 + a_6 - a_{10} - 2a_{12} - a_{13}) + a_{14} + 2a_{15} + a_{17} + 2a_{18} - a_{19}/\sqrt{6}$ |
| $\Lambda_c^+ \to \Sigma^+ K^+$ | $B_0^T + D_0^T + B_{15}^T - D_{15}^T$ | $-a_8 - a_{11} - a_{17} - a_{19}$ |
| $\Lambda_c^+ \to \Sigma^0 K^+$ | $(B_0^T + D_0^T + B_{15}^T + D_{15}^T)/\sqrt{2}$ | $(a_6 + a_{10} + a_{13} + a_{14} - a_{17} - a_{19})/\sqrt{2}$ |
| $\Lambda_c^+ \to p\pi^0$ | $(C_0^T - E_0^T + C_{15}^T + E_{15}^T)/\sqrt{2}$ | $(2a_2 + a_4 - a_{10} - a_{11} - a_{12} - a_{13} + a_{14} + a_{16} + a_{17} + a_{19})/\sqrt{2}$ |
| $\Lambda_c^+ \to p\eta$ | $(2A_0^T + C_0^T + E_0^T + 2A_{15}^T + C_{15}^T - E_{15}^T)/\sqrt{2}$ | $-(2a_2 - 2a_3 - a_4 - a_6 + 2a_{10} - a_{11} + a_{12} + a_{13} + a_{14} + a_{16} + a_{17} + a_{19})/\sqrt{2}$ |
| $\Lambda_c^+ \to p\eta_s$ | $A_0^T + B_0^T - E_0^T - D_0^T + A_{15}^T + B_{15}^T + E_{15}^T + D_{15}^T$ | $2a_2 + a_3 - a_6 + a_{10} + a_{12}$ |
| $\Lambda_c^+ \to n\pi^+$ | $C_0^T - E_0^T + C_{15}^T - E_{15}^T$ | $-a_8 + a_6 - a_{12} + a_{14} - a_{15} - a_{17} - a_{19}$ |
| $\Xi_c^+ \to \Lambda^0 \pi^+$ | $(B_0^T + C_0^T + E_0^T + 3D_0^T + B_{15}^T + C_{15}^T + E_{15}^T + 3D_{15}^T)/\sqrt{6}$ | $(2a_1 + a_4 + 2a_6 + a_{10} - a_{12} + a_{13} + 2a_{14} + a_{15} - a_{17} + a_{19})/\sqrt{6}$ |
| $\Xi_c^+ \to \Sigma^+ \pi^+$ | $(2A_0^T + B_0^T - E_0^T - D_0^T + 2A_{15}^T + C_0^T + E_{15}^T + D_{15}^T)/\sqrt{2}$ | $-(2a_2 + a_3 - a_6 - a_{10} - a_{12} + a_{13} + a_{16})/\sqrt{2}$ |
| $\Xi_c^+ \to \Xi^+ \eta_s$ | $2A_0^T + E_0^T + A_{15}^T + E_{15}^T$ | $-2a_2 + a_3 - a_4 - a_6 + a_{11} - a_{16} - a_{17} - a_{19}$ |
| $\Xi_c^+ \to \Xi^0 \pi^+$ | $(B_0^T - C_0^T - E_0^T - D_0^T + B_{15}^T - C_{15}^T - E_{15}^T - D_{15}^T)/\sqrt{2}$ | $-(2a_1 + a_4 - a_{10} - a_{12} + a_{13} + a_{15} + a_{17} + a_{18})/\sqrt{2}$ |
| $\Xi_c^+ \to pK^+$ | $B_0^T + D_0^T + B_{15}^T + D_{15}^T$ | $-a_8 - a_{11} + a_{17} - a_{19}$ |
| $\Xi_c^0 \to \Lambda^0 \pi^0$ | $-(B_0^T + C_0^T + E_0^T + 3D_0^T - B_{15}^T - C_{15}^T - E_{15}^T - 3D_{15}^T)/2\sqrt{3}$ | $(2a_2 - 2a_6 + a_{17} - a_8 - 3a_{10} - 2a_{11} + a_{12} - a_{13} - a_{14} + a_{16} - a_{18} + 2a_{19})/2\sqrt{3}$ |
| $\Xi_c^0 \to \Lambda^0 \eta_9$ | $(6A_0^T + B_0^T + C_0^T + E_0^T - 3D_0^T - 6A_{15}^T - B_{15}^T - C_{15}^T - E_{15}^T - 3D_{15}^T)/2\sqrt{3}$ | $(2a_2 + 3a_6 - a_7 - a_8 - 3a_{10} + 2a_{11} + a_{12} - a_{13} - a_{14} + a_{16} - 2a_{19})$ |
| $\Xi_c^0 \to \Lambda^0 \eta_s$ | $(3A_0^T + 2B_0^T + 2C_0^T - E_0^T - 3A_{15}^T - 2B_{15}^T - 2C_{15}^T + E_{15}^T + 3D_{15}^T)/\sqrt{6}$ | $(2a_2 - 3a_6 - 2a_7 - a_8 - 3a_{10} + 2a_{11} - 2a_{12} - a_{13} + a_{14} + a_{16} + a_{17} + a_{18} - a_{19})/\sqrt{6}$ |
| $\Xi_c^0 \to \Sigma^+ \eta^-$ | $C_0^T + D_0^T - C_{15}^T - D_{15}^T$ | $-a_7 - a_8 - a_{12} + a_{13}$ |
| $\Xi_c^0 \to \Sigma^0 \pi^0$ | $1/2(B_0^T + C_0^T - E_0^T + D_0^T + B_{15}^T - C_{15}^T + E_{15}^T + D_{15}^T)$ | $(1/2)(2a_2 + a_6 - a_7 - a_8 + a_{10} - a_{12} - a_{13} + a_{14} + a_{16} + 2a_{19})$ |
| $\Xi_c^0 \to \Sigma^0 \eta_9$ | $1/2(-2A_0^T - B_0^T - C_0^T + E_0^T + D_0^T + 2A_{15}^T + B_{15}^T + C_{15}^T - E_{15}^T - 3D_{15}^T)$ | $(1/2)(-2a_2 + 2a_6 + a_7 - a_8 + 2a_9 + a_{10} + a_{12} + a_{13} + a_{14} + a_{16} - a_{18} - a_{19})$ |
| $\Xi_c^0 \to \Sigma^0 \eta_s$ | $(-A_0^T - E_0^T + A_{15}^T + E_{15}^T)/\sqrt{2}$ | $(2a_2 + a_5 - a_8 + a_9 - a_{11} - a_{12} - a_{13} + a_{14} + a_{16} + a_{17} + a_{19})/\sqrt{2}$ |
| $\Xi_c^0 \to \Sigma^- \pi^+$ | $B_0^T - E_0^T - B_{15}^T - E_{15}^T$ | $-2a_1 + a_6 + a_{10} - a_{15}$ |
| $\Xi_c^0 \to pK^-$ | $-C_0^T + D_0^T + C_{15}^T + D_{15}^T$ | $a_7 + a_8 + a_{12} + a_{13}$ |
| $\Xi_c^0 \to nK^+$ | $B_0^T - C_0^T - B_{15}^T - C_{15}^T$ | $a_7 - a_8 + a_{12} - a_{13} + a_{18} - a_{19}$ |
| $\Xi_c^0 \to \Xi^- K^+$ | $-B_0^T + E_0^T + B_{15}^T + E_{15}^T$ | $2a_1 - a_6 - a_{10} + a_{15}$ |
| $\Xi_c^0 \to \Xi^0 K^0$ | $-B_0^T + C_0^T + B_{15}^T - C_{15}^T$ | $-a_7 + a_{11} - a_{12} + a_{14} - a_{18} + a_{19}$ |
TABLE XVI: Decay amplitudes for two-body Doubly Cabibbo-Suppressed charmed baryon decays.

| channel | IRA | TDA |
|---------|-----|-----|
| $\Lambda_c^+ \to pK^0$ | $-E^0_{\Lambda} - D^0_{a} + E^T_{15} + D^T_{15}$ | $2\bar{a}_2 + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{19}$ |
| $\Lambda_c^+ \to nK^+$ | $E^T_{6} + D^T_{a} + E^T_{15} + D^T_{15}$ | $2a_1 + a_{15} + a_{18} - a_{19}$ |
| $\Xi_c^+ \to \Lambda^0 K^+ - (B^0_{\Xi} - 2C^0_{a} + E^0_{a} + B^T_{15} + 2C^T_{15} + E^T_{15})/\sqrt{6}$ | $-(2a_1 - 2a_4 - a_6 + 2a_{12} + 2a_{13} - a_{14} + a_{15} - a_{17} + a_{18} - 2a_{19})/\sqrt{6}$ |
| $\Xi_c^+ \to \Sigma^+ K^0$ | $-B^0_{a} + E^0_{a} - B^T_{15} - E^T_{15}$ | $-2a_2 + a_8 + a_{11} - a_{16}$ |
| $\Xi_c^+ \to \Sigma^0 K^+$ | $-(B^0_{\Xi} + E^0_{\Xi} + B^T_{15} + E^T_{15})/\sqrt{2}$ | $(2a_1 - a_6 - a_{10} - a_{13} - a_{14} + a_{15} + a_{17} + a_{18})/\sqrt{2}$ |
| $\Xi_c^+ \to \bar{p}\pi^0$ | $-(C^0_{a} + D^0_{a} - C^T_{15} + D^T_{15})/\sqrt{2}$ | $(-a_4 + a_8 + a_{10} + a_{11} + a_{12} + a_{13})/\sqrt{2}$ |
| $\Xi_c^+ \to \bar{p}\eta_q - (2A^0_{\Xi} + C^0_{a} - D^0_{a} + 2A^T_{15} + C^T_{15} + D^T_{15})/\sqrt{2}$ | $(-2a_3 - a_4 - a_8 + 2a_9 + a_{10} - a_{11} + a_{12} + a_{13})/\sqrt{2}$ |

VI. ANTITRIPLET CHARMED BARYON $T_c(\Lambda_c, \Xi_c^+, \Xi_c^0)$ DECAYS

For the charmed baryon decays, the $H_3$ contributions are vanishingly small, and thus we have 19 amplitudes in TDA:

$$A_{u}^{TDA} = \bar{a}_1 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_2 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_3 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_4 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m$$

$$+ \bar{a}_5 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_6 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_7 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m$$

$$+ \bar{a}_8 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_9 T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_{10} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m$$

$$+ \bar{a}_{11} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_{12} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m$$

$$+ \bar{a}_{13} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_{14} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_{15} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m$$

$$+ \bar{a}_{16} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_{17} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_{18} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m$$

$$+ \bar{a}_{19} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m + \bar{a}_{20} T_{c_3}^{[ij]} H_{m}^{kl} (\mathcal{T}_8)_{ijkl} P^m. \quad (79)$$

The corresponding Feynman diagrams are given in Fig. 6, in which 7 Feynman diagrams can be found. The analysis for independent diagrams are almost the same as that of bottom baryon decays. On the other side, 10 IRA amplitudes can be constructed as:

$$A_{u}^{IRA} = A_6^T (T_{c_3}) (H_6)^{[ik]} (\mathcal{T}_8)_{ijkl} P^l + B_6^T (T_{c_3}) (H_6)^{[ik]} (\mathcal{T}_8)_{ijkl} P^l + C_6^T (T_{c_3}) (H_6)^{[ik]} (\mathcal{T}_8)_{ijkl} P^l$$

$$+ E_6^T (T_{c_3}) (H_6)^{[ik]} (\mathcal{T}_8)_{ijkl} P^l + D_6^T (T_{c_3}) (H_6)^{[ik]} (\mathcal{T}_8)_{ijkl} P^l$$

$$+ B_6^T (T_{c_3}) (H_{15})_{ijkl} P^l + C_6^T (T_{c_3}) (H_{15})_{ijkl} P^l + D_6^T (T_{c_3}) (H_{15})_{ijkl} P^l.$$

(80)

Only 9 of them are independent, and one redundant amplitude can be made explicit with the redefinitions:

$$A_6^T = A_6^T + B_6^T, \quad B_6^T = B_6^T - C_6^T, \quad C_6^T = C_6^T - D_6^T, \quad D_6^T = C_6^T + D_6^T,$$

which is exactly the same as Eq. 76.
After a careful examination, one can also find the relations:

\[
A_6^T = \frac{1}{2} (\bar{a}_3 - \bar{a}_5 - 2\bar{a}_9 - \bar{a}_{13} + \bar{a}_{14}), \quad B_6^T = \frac{1}{2} (\bar{a}_{13} - \bar{a}_{14} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}),
\]

\[
C_6^T = \frac{1}{2} (\bar{a}_4 - \bar{a}_7 - \bar{a}_{10} + \bar{a}_{11} - 2\bar{a}_{12}), \quad D_6^T = \frac{1}{2} (\bar{a}_6 - \bar{a}_8 + \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{13} + \bar{a}_{14}),
\]

\[
E_6^T = \frac{1}{2} (2\bar{a}_1 - 2\bar{a}_2 - \bar{a}_6 + \bar{a}_8 - \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} - \bar{a}_{16} - \bar{a}_{17} + \bar{a}_{18} - 2\bar{a}_{19}),
\]

\[
A_{15}^T = \frac{1}{2} (\bar{a}_3 + \bar{a}_5 - \bar{a}_{13} - \bar{a}_{14}), \quad B_{15}^T = \frac{1}{2} (\bar{a}_{13} + \bar{a}_{14} - \bar{a}_{17} - \bar{a}_{18}),
\]

\[
C_{15}^T = \frac{1}{2} (\bar{a}_4 + \bar{a}_7 - \bar{a}_{10} - \bar{a}_{11}), \quad D_{15}^T = \frac{1}{2} (\bar{a}_6 + \bar{a}_8 + \bar{a}_{10} + \bar{a}_{11} + \bar{a}_{13} + \bar{a}_{14}),
\]

\[
E_{15}^T = \frac{1}{2} (2\bar{a}_1 + 2\bar{a}_2 - \bar{a}_6 - \bar{a}_8 - \bar{a}_{10} - \bar{a}_{11} - \bar{a}_{13} - \bar{a}_{14} + \bar{a}_{15} + \bar{a}_{16} + \bar{a}_{17} + \bar{a}_{18}).
\]  

Some further remarks are given in order.

• The flavor SU(3) symmetry in charmed baryon decays and the symmetry breaking effects have been extensively explored in Refs. [70–78], and we refer the reader to these references for detailed discussions.

• On the experimental side, BESIII collaboration has given the first measurement of decay branching fractions for the W-exchange induced decays [79]:

\[
\mathcal{B}(\Lambda_c^+ \rightarrow \Xi^0 K^+) = (5.90 \pm 0.86 \pm 0.39) \times 10^{-3},
\]

\[
\mathcal{B}(\Lambda_c^+ \rightarrow \Xi(1530)^0 K^+) = (5.02 \pm 0.99 \pm 0.31) \times 10^{-3}.
\]

It indicates that the decays into a decuplet baryon might not be power suppressed compared to those decays into an octet baryon. This introduces a theoretical difficulty to understand the charmed baryon decays.

• One can find some relations between the different channels listed in Table XIV, Table XV and Table XVI. For the charmed baryon two-body decay, there is only one relation for decay width:

\[
\Gamma(\Lambda_c^+ \rightarrow \Sigma^+\pi^0) = \Gamma(\Lambda_c^+ \rightarrow \Sigma^0\pi^+).
\]

This relation fits well with the data in Ref. [3]:

\[
\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+\pi^0) = 1.24 \pm 0.10\%, \quad \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0\pi^+) = 1.28 \pm 0.07\%.
\]

• In Ref [74], a global fit was conducted for charmed baryon decays. In that work the sextet contribution was expressed in a different representation. Relating the four coefficients in [74] with our notations, we have \(^3\):

\[
-A_6^T + D_6^T = h = (0.105 \pm 0.073) \text{ GeV}^3, \quad -B_6^T + E_6^T = a_1 = (0.244 \pm 0.006) \text{ GeV}^3,
\]

\[
-C_6^T - D_6^T = a_2 = (0.115 \pm 0.014) \text{ GeV}^3, \quad E_6^T + D_6^T = a_3 = (0.088 \pm 0.019) \text{ GeV}^3.
\]

Such a fit was conducted with the neglect of the \(H_{15^0}\) terms, which might be challenged in interpreting the \(\Lambda_c \rightarrow p\pi^0\) [71, 75, 77].

**VII. DISCUSSIONS AND CONCLUSIONS**

In this work, we have carried out a comprehensive analysis comparing two different realizations of the flavor SU(3) symmetry, the irreducible operator representation amplitude and topological diagram amplitude, to study various bottom/charm meson and baryon decays.

We find that previous analyses in the literature using these two methods do not match consistently in several ways. The TDA approach provides a more intuitive understanding of the decays, however it also suffers from a few

---

\(^3\) In Ref. [74], these parameters are denoted as \(a_1, a_2, a_3, h\). Here we add primes in order to distinguish them with the parameters used in this work.
subtleties. Using two-body $B/D$ meson decays, we have demonstrated that a few SU(3) independent amplitudes have been overlooked in TDA. Most of these amplitudes arise from higher order loop corrections, but they are irreducible in the flavor SU(3) space, and thus cannot be neglected in principle. Taking these new amplitudes into account, we find a consistent description in both approaches. We have pointed out that these new amplitudes can affect direct CP asymmetries in some channels significantly. An interesting observation is that, with these new amplitudes, the direct CP symmetries for charmless nonleptonic $b$ decays cannot be identically zero. For heavy baryon decays, we pointed out though the TDA approach is very intuitive, it suffers the difficulty in providing the independent amplitudes. On this point, the IRA approach is more helpful.

All results derived in this paper can be used to study the heavy meson and baryon decays in the future when sufficient data become available. Then one can have a better understanding of the role of flavor SU(3) symmetry in heavy meson and baryon decays.

For charm quark decays, we did not include the penguin contributions, which can also be studied in a similar manner. It is also necessary to notice that the flavor SU(3) symmetry has been applied to study weak decays of doubly heavy baryons [80–82], and multi-body $\Lambda_c$ decays [78]. The equivalence between the TDA and IRA approaches in these decay modes can be studied similarly.

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**Appendix A: Relations for bottom antitriplet baryon decay amplitudes**

In Section V, we have shown the IRA approach for bottom antitriplet decays with 14 amplitudes. Actually one can construct another form of IRA due to using a different octet baryon representation:

$$A_{T_a}^{IRA} = A_1 T^{[ij]}_{b^3} H^6_{i} \epsilon_{ijn} (T_8)_k^n P^l_l + A_2 T^{[ij]}_{b^3} H^6_{i} \epsilon_{ijn} (T_8)_k^n P^l_l + A_3 T^{[ij]}_{b^3} H^6_{i} \epsilon_{ijn} (T_8)_k^n P^l_l + A_4 T^{[ij]}_{b^3} H^6_{i} \epsilon_{ijn} (T_8)_k^n P^l_l + A_5 T^{[ij]}_{b^3} H^6_{i} \epsilon_{ijn} (T_8)_k^n P^l_l \ldots$$

These two sets of IRA are related to each other by:

$$A_1^{T_a} = 2 A_1 + A_3 + A_6 + A_7, \quad B_1^{T_a} = 2 A_2 + A_4 + A_5 - A_7, \quad C_3^{T_a} = A_7 - A_5, \quad D_7^{T_a} = -A_6 - A_7, \quad A_1^{T_b} = B_7 + B_{11} - 2 B_8, \quad B_1^{T_b} = -(B_3 + B_{11}) + 2 B_{14}, \quad C_7^{T_b} = B_7 + B_{10} - 2 B_6, \quad D_7^{T_b} = -(B_9 + B_{10} - B_{11}), \quad A_1^{T_c} = C_5 + C_8, \quad B_1^{T_c} = -(C_3 + C_8), \quad C_7^{T_c} = C_4 + C_7, \quad D_7^{T_c} = -(C_6 + C_8 + C_7), \quad E_7^{T_c} = 2 B_1 + 2 B_4 + B_2 - B_3 + B_6 + B_{10} - B_{11}, \quad E_7^{T_c} = 2 C_1 + C_2 + C_3 + C_4 + C_5 + C_7 + C_8.$$
In addition, the relation between the coefficients of 26 amplitudes in $A^{IRA}_{T_5 \rightarrow P_{T_5(u)}}$ and $A^{TDA}_{T_5 \rightarrow P_{T_5(u)}}$ is:

\[
\begin{align*}
\tilde{A}_1 &= \frac{1}{8}(-\tilde{a}_1 + 3\tilde{a}_2 - 3\tilde{a}_3 + \tilde{a}_5) + \tilde{b}_1, \\
\tilde{A}_2 &= \frac{1}{8}(3\tilde{a}_1 - \tilde{a}_2 - 3\tilde{a}_4 + \tilde{a}_7) + \tilde{b}_2, \\
\tilde{A}_3 &= \frac{1}{8}(\tilde{a}_3 - 3\tilde{a}_5 + 4\tilde{a}_9) + \tilde{b}_3, \\
\tilde{A}_4 &= \frac{1}{8}(-3\tilde{a}_6 + \tilde{a}_8 + 3\tilde{a}_{10} - \tilde{a}_{11} + 3\tilde{a}_{15} - \tilde{a}_{16}) + \tilde{b}_4, \\
\tilde{A}_5 &= \frac{1}{8}(4\tilde{a}_{12} - \tilde{a}_{17} + 3\tilde{a}_{18} + \tilde{a}_4 - 3\tilde{a}_7) + \tilde{b}_5, \\
\tilde{A}_6 &= \frac{1}{8}(3\tilde{a}_{13} - \tilde{a}_{14} + 3\tilde{a}_{17} - \tilde{a}_{18} + \tilde{a}_6 - 3\tilde{a}_8) + \tilde{b}_6, \\
\tilde{A}_7 &= \frac{1}{8}(\tilde{a}_{10} - 3\tilde{a}_{11} - \tilde{a}_{13} + 3\tilde{a}_{14} + 4\tilde{a}_{19}) + \tilde{b}_7, \\
\tilde{B}_1 &= \frac{1}{4}(\tilde{a}_1 - \tilde{a}_2), \\
\tilde{C}_1 &= \frac{1}{8}(\tilde{a}_1 + \tilde{a}_2), \\
\tilde{B}_2 &= \frac{1}{4}(\tilde{a}_{15} - \tilde{a}_{16}), \\
\tilde{C}_2 &= \frac{1}{8}(\tilde{a}_{15} + \tilde{a}_{16}), \\
\tilde{B}_3 &= \frac{1}{4}(\tilde{a}_{17} - \tilde{a}_{18}), \\
\tilde{C}_3 &= \frac{1}{8}(\tilde{a}_{17} + \tilde{a}_{18}), \\
\tilde{B}_4 &= \frac{1}{4}(\tilde{a}_{19}), \\
\tilde{B}_5 &= \frac{1}{4}(\tilde{a}_4 - \tilde{a}_7), \\
\tilde{C}_4 &= \frac{1}{8}(\tilde{a}_4 + \tilde{a}_7), \\
\tilde{B}_6 &= \frac{1}{4}(\tilde{a}_{12}), \\
\tilde{B}_7 &= \frac{1}{4}(\tilde{a}_3 - \tilde{a}_5), \\
\tilde{C}_5 &= \frac{1}{8}(\tilde{a}_3 + \tilde{a}_5), \\
\tilde{B}_8 &= \frac{1}{4}(\tilde{a}_9), \\
\tilde{B}_9 &= \frac{1}{4}(\tilde{a}_8 - \tilde{a}_6), \\
\tilde{C}_6 &= \frac{1}{8}(\tilde{a}_6 + \tilde{a}_8), \\
\tilde{B}_{10} &= \frac{1}{4}(\tilde{a}_{11} - \tilde{a}_{10}), \\
\tilde{C}_7 &= \frac{1}{8}(\tilde{a}_{10} + \tilde{a}_{11}), \\
\tilde{B}_{11} &= \frac{1}{4}(\tilde{a}_{14} - \tilde{a}_{13}), \\
\tilde{C}_8 &= \frac{1}{8}(\tilde{a}_{13} + \tilde{a}_{14}).
\end{align*}
\]

Combination of Eq. (A2) and Eq. (A4) leads to Eq. (75).

The authors of Ref. [83] have given another parametrization of IRA amplitudes, in which they have focused on the flavor non-singlet. Comparing with their results, one finds the following relations:

\[
\begin{align*}
B^T_3 &= 2b(3)_2 + d(3)_1 - e(3)_2 + c(3), \\
C^T_3 &= 2a(3) - c(3), \\
D^T_3 &= 2b(3)_1 + d(3)_2 - e(3)_1 + c(3), \\
B^T_6 &= 2(a(6)_2 - g(6) - n(6)_1 + d(6)_2 - e(6)_1 - e(6)_2), \\
C^T_6 &= 2(a(6)_1 + f(6) - n(6)_2) + d(6)_1, \\
E^T_6 &= 2(b(6)_2 - g(6)) - c(6) + d(6)_1 + d(6)_2 - e(6)_1 - e(6)_2 - g(6), \\
D^T_6 &= -2(b(6)_1 + f(6)) + c(6) - d(6)_1 - d(6)_2 + e(6)_1 + e(6)_2, \\
B^T_{15} &= 2a(T_{15})_2 + d(T_{15})_2 - e(T_{15}), \\
C^T_{15} &= 2a(T_{15})_1 + d(T_{15})_1 - e(T_{15}), \\
E^T_{15} &= 2b(T_{15})_2 + c(T_{15}) + d(T_{15})_1 - d(T_{15})_2 + e(T_{15})_1 - e(T_{15}), \\
D^T_{15} &= 2b(T_{15})_1 - c(T_{15}) - d(T_{15})_1 + d(T_{15})_2 - e(T_{15})_1 + e(T_{15}).
\end{align*}
\]

Appendix B: Relations for charmed baryon decays

One can construct another set of IRA for charmed antitriplet baryon decays with 19 amplitudes:

\[
\begin{align*}
A^{IRA}_u &= B_1 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_k)_{n}^{i} P_{m}^{j} + B_2 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + B_3 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} \\
&+ B_4 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + B_5 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + B_6 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} \\
&+ B_7 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + B_8 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + B_9 T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} \\
&+ B_{10} T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + B_{11} T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + B_{12} T^{[ij]}_{c3}(H_6)_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} \\
&+ \tilde{C}_1 T^{[ij]}_{c3}(H_{15})_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + \tilde{C}_2 T^{[ij]}_{c3}(H_{15})_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + \tilde{C}_3 T^{[ij]}_{c3}(H_{15})_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} \\
&+ \tilde{C}_4 T^{[ij]}_{c3}(H_{15})_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + \tilde{C}_5 T^{[ij]}_{c3}(H_{15})_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + \tilde{C}_6 T^{[ij]}_{c3}(H_{15})_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} \\
&+ \tilde{C}_7 T^{[ij]}_{c3}(H_{15})_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j} + \tilde{C}_8 T^{[ij]}_{c3}(H_{15})_{m}^{kl} \epsilon_{ijn}(T_8)_{n}^{i} P_{m}^{j}.
\end{align*}
\]

(B1)
The relation between the two sets of IRA is given as:

\[ A_6^T = \frac{1}{2}(B_7 + B_{11}) - \bar{B}_8, \quad B_6^T = -\frac{1}{2}(B_3 + B_{11}) + B_4, \quad C_6^T = \frac{1}{2}(B_5 + B_{10}) - \bar{B}_6, \quad D_6^T = -\frac{1}{2}(\bar{B}_9 + \bar{B}_{10} - \bar{B}_{11}) \]

\[ A_{15}^T = \frac{1}{2}(C_5 + \bar{C}_8), \quad B_{15}^T = -\frac{1}{2}(\bar{C}_3 + \bar{C}_8), \quad C_{15}^T = \frac{1}{2}(\bar{C}_4 + \bar{C}_7), \quad D_{15}^T = -\frac{1}{2}(\bar{C}_6 + \bar{C}_8 + \bar{C}_7) \]

\[ E_6^T = \bar{B}_1 + B_4 + \frac{1}{2}(B_2 - B_3 + B_9 + B_{10} - \bar{B}_{11}), \quad E_{15}^T = \bar{C}_1 + \frac{1}{2}(C_2 + C_3 + \bar{C}_6 + \bar{C}_7 + \bar{C}_8). \] (B2)

The relation between the new \( A_{T_{-}\rightarrow PT_{-}(u)} \) and \( A_{T_{+}\rightarrow PT_{+}(u)} \) can be obtained as:

\[ B_1 = \frac{1}{2}(\bar{a}_1 - \bar{a}_2), \quad C_1 = \frac{1}{2}(\bar{a}_1 + \bar{a}_2), \quad B_2 = \frac{1}{2}(\bar{a}_15 - \bar{a}_16), \quad C_2 = \frac{1}{2}(\bar{a}_15 + \bar{a}_16) \]

\[ B_3 = \frac{1}{2}(\bar{a}_17 - \bar{a}_18), \quad C_3 = \frac{1}{2}(\bar{a}_17 + \bar{a}_18), \quad B_4 = -\frac{1}{2}(\bar{a}_{19}), \quad B_5 = \frac{1}{2}(\bar{a}_4 - \bar{a}_7), \quad C_4 = \frac{1}{2}(\bar{a}_4 + \bar{a}_7) \]

\[ B_6 = \frac{1}{2}(\bar{a}_{12}, \quad B_7 = \frac{1}{2}(\bar{a}_3 - \bar{a}_5), \quad C_5 = \frac{1}{2}(\bar{a}_3 + \bar{a}_5), \quad B_8 = \frac{1}{2}(\bar{a}_9), \quad B_9 = \frac{1}{2}(\bar{a}_8 - \bar{a}_6), \quad C_6 = -\frac{1}{2}(\bar{a}_6 + \bar{a}_8) \]

\[ B_{10} = \frac{1}{2}(\bar{a}_{11} - \bar{a}_{10}), \quad C_7 = -\frac{1}{2}(\bar{a}_{10} + \bar{a}_{11}), \quad B_{11} = \frac{1}{2}(\bar{a}_{14} - \bar{a}_{13}), \quad C_8 = -\frac{1}{2}(\bar{a}_{13} + \bar{a}_{14}). \] (B3)
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