On Newton’s Third Law

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The law of action-reaction is thoroughly used in textbooks to derive the conservation laws of linear and angular momentum, and it was considered by Ernst Mach the cornerstone of physics. We give here a background survey of several questions raised by the action-reaction law, and in particular, the role of the physical vacuum is shown to provide an appropriate framework to clarify the occurrence of possible violations of the action-reaction law. It is also obtained an expression for the general linear momentum of a body-particle in the context of statistical mechanics. It is shown that Newton’s third law is not verified in systems out of equilibrium due to an additional entropic gradient term present in the particle’s momentum.

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I. INTRODUCTION

The law of action-reaction, or Newton’s third law [1], is thoroughly used in textbooks to derive the conservation laws of linear and angular momentum. Ernst Mach considered the third law as “his most important achievement with respect to the principles” [2, 3]. However, the reasoning used primarily by Newton applies to point particles without structure and is not concerned with the motion of material bodies composed with a large number of particles, in or out of thermal equilibrium.

Ernst Mach sustained that the concept of mass and Newton’s third law were redundant; that in fact it should be enough to define operationally the mass of a given body as the unit of mass to be sure that “If two masses 1 and 2 act on each other, our very definition of mass asserts that they impart to each other contrary accelerations which are to each other respectively as 2:1” [2]. Yet philosophy has delivered us extraordinary new insights to a basic understanding of the underlying physics of force. For example, Félix Ravaisson [4] in the XIX century sustained that within the realm of the inorganic world action-equals-reaction; they are the same act perceived by two different viewpoints. But in the organic world, whenever more complex systems are at working, “Ce n’est pas assez d’un moyen terme indifférent comme le centre des forces opposées du levier; de plus en plus, il faut un centre qui, par sa propre vertu, mesure et dispense la force” [5]. So, there is in Nature the need of an “agent” that control and deliver the action from one body to another and this is, as we will see, the role of the physical vacuum, or barely just the environment of a body.

We can find in Cornille [6] a review of applications of action-reaction law in several branches of physics. In addition, Cornille introduced the concepts of spontaneous force (obeying to Newton’s third law) and stimulated force (which violates it).

In this paper we intend to show that generally in any system out of equilibrium, when entropy is velocity-dependent, Newton’s third law is violated. The need for re-examination of this problems is pressing since long-term exploitation of the cosmos face serious difficulties due to the outdated spacecraft technologies mankind possess.

Sec. II discusses the general issues in mechanics and electromagnetism related to the action-to-reaction law. Sec. III discusses the possible role of physical vacuum as a third agent that might explain action-to-reactions law violations. Secs. IV and V discusses the intrinsic violation of Newton’s third law in systems out-of-equilibrium. Sec. VI presents the conclusions that follow logically from the previous discussion.

II. BACKGROUND SURVEY

The usual derivation of the laws governing the linear and angular momentum presented in textbooks is as follows. The equation of motion of the $i$th particle is given by:

$$\mathbf{F}_i + \sum_{j \neq i} \mathbf{F}_{ij} = \frac{d\mathbf{p}_i}{dt}, \quad (1)$$

where $\mathbf{F}_i$ is an external force acting on the $i$ particle and $\mathbf{F}_{ij}$ represents the internal force exerted on the particle $i$ by the particle $j$. In the case of central forces the relation $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ is verified, a manifestation of Newton’s third law. Summing up over all the particles belonging to the
system we have from Eq. (1)
\[ \sum F_i = \sum \frac{dp_i}{dt}. \] (2)

Podolsky\cite{Podolsky} called attention to the discrepancies obtained using directly Newton’s second law, or using instead the invariance of the lagrangian under rotations. In the case of non-central forces, like a system subject to a potential function of the form \( V = r^{-1} \cos \theta \), we might expect a deviation from Newton’s third law. Indeed, angle-dependent potentials, long-range (van der Waals) forces describe rigorously the physical properties of molecular gases. We can wonder from which mechanism it comes the unbalance of forces.

We might expect that thermodynamics and statistical mechanics provide a better description of macroscopic matter. The internal energy and in particular the average total energy of a system \( \overline{E} = \sum U_i \), which includes summing up all the particles constituting the system and all storage modes, plays a fundamental role together with an equally fundamental, although less understood function, the system entropy. Interesting enough a microscopic model of friction has shown that the irreversible entropy production is drawn from the increase of Shannon information\cite{Shannon}.

This question is related to the fundamental one still not answered in physics and biophysics: how chaos in various natural systems can spontaneously transform to order? The observation of various physical and biological systems shows that a feedback is onset according to: “The medium controls the object—the object shapes the medium”\cite{Lorenz}. At the microscopic level a large class of systems generate directed motion through the interaction of a moving object with an inhomogeneous substrate periodically structured\cite{Rosenbluth}. This is the ratchet-and-pawl principle.

The apparent violation of Newton’s third law that we can find in some systems, e.g., when two equal charged bodies having equal velocities in magnitude and opposing directions, is well-known. The Lorentz’s force applied to both charges do not cancel each other since the magnetic forces do not act along a common line (see also the Onoohin’s paradox\cite{Onoohin}). The paradox is solved introducing the electromagnetic momentum \( [E \times H]/c^2 \) (values in SI units will be used throughout the text)\cite{Grassmann}.

In the domain of astrophysics, the same problem appears again. For instance, based on unexplained astrophysical observations, such as the high rotation of matter around the centers of galaxies, it was proposed a modification of Newton’s equations of dynamics\cite{Lorentz}, while more recently a new effect was reported, about the possibility of a violation of Newton’s second law with static bodies experimenting spontaneous acceleration\cite{Cohen}. In the frame of statistical mechanics, studying the effective forces between two fixed big colloidal particles immersed in a bath of small particles, it has been shown that the nonequilibrium force field is nonconservative and violates action-reaction law\cite{Calkin}.

An ongoing debate on the validity of electrolytic force law is still raging, with experimental evidence that Biot-Savart law does not obey action-reaction law (see Ref.\cite{Savart, Biot} and references therein). The essence of the problem stands on two different laws that exist in magnetostatics giving the force between two infinitely thin line-current elements \( ds_1 \) and \( ds_2 \) through which pass currents \( i_1 \) and \( i_2 \). The Ampère’s law states that this force is given by:
\[
d^2F_{2,A} = -\frac{\mu_0 i_1 i_2}{4\pi} \frac{r_{12}}{r_{12}^2} [2(ds_1 \cdot ds_2) - \frac{3}{r_{12}^2} (ds_1 \cdot r_{12})(ds_2 \cdot r_{12})].
\] (3)

This means that the force between two current elements depended not only on their distance, as in the inverse square law, but also on their angular position (in particular, implicating the existence of a longitudinal force, confirmed experimentally by Saumont\cite{Saumont} and Graneau\cite{Graneau}, and discussed by Costa de Beauregard\cite{Beauregard}). The other force generally considered is given by the Biot-Savart law, also known as the Grassmann’s equation in its integral form:
\[
d^2F_{2,BS} = -\frac{\mu_0 i_1 i_2}{4\pi} \frac{1}{r_{12}} [(ds_2 \times (ds_1 \times r_{12})].
\] (4)

Here, \( r_{12} \) is the position vector of element 2 relative to 1. While Ampère’s law obeys Newton’s third law, Biot-Savart law does not obey it (e.g., Ref.\cite{Savart, Biot, Biot}). The theory developed by Lorentz was criticized by H. Poincaré\cite{Poincare}, because it sacrificed action-to-reaction law.

The problem of linear momentum of stationary system of charges and currents is faraway of a consensus too. Costa de Beauregard\cite{Beauregard} pointed out a violation of the action-reaction law in the interaction between a current loop I flowing on the boundary of area \( A \) with moment \( M = IA \) and an electric charge concluding that when the moment of the loop changes in the presence of an electric field a force must act on the current loop, given by \( F = |E \times M|/c^2 \). Shockley and James\cite{Shockley} attribute \( F \) to a change in “hidden momentum” \( G_I = -|E \times M|/c^2 \) carried within the current loop by the steady state power flow necessary to balance the divergence of Poynting’s vector. The total momentum is \( p = G_I + G_b \), where \( G_b = m < \hat{r}_{CM} > \) is the body momentum associated with the center of mass \( m \)\cite{vonWeißesbach, Costa}. In particular, it was shown\cite{Onoohin} that the “hidden linear momentum” has as quantum mechanical analogue the term \( \alpha \cdot E \), where \( \alpha \) are Dirac matrices appearing in the hamiltonian form \( \hat{H} \psi = i\hbar \partial \psi / \partial t \), where \( \hat{H} = -i\hbar \alpha \cdot \nabla \) is the hamiltonian operator (e.g., Ref.\cite{Dirac}). Although certainly an important issue, the concept of “hidden momentum” needs to be further clarified\cite{Barrett}.

Calkin\cite{Calkin} has shown that the net linear momentum of any closed stationary system of charges and currents is zero and can be written:
\[
P = \int \hat{d}^3r \left( \frac{\dot{u}}{c^2} \right) = M \hat{r}_{CM}.
\] (5)
where \( u \) is the energy density, \( M \) is the total mass, 
\( M = \int d^3r (u/c^2) \) and \( r_{CM} \) is the radius vector of the center of mass. He has shown, however, that the linear mechanical momentum \( \mathbf{P}_{ME} \) in a static electromagnetic field is nonzero and it is given by:

\[
\mathbf{P}_{ME} = -\int d^3r \rho \mathbf{A}. \tag{6}
\]

Here, \( \mathbf{A}^T \) denotes the transverse vector potential given by 
\( \mathbf{A}^T = (\mu_0/4\pi) \int d^3r \mathbf{J}/r \). Eq. \( 6 \) shows that \( \rho \mathbf{A} \) is a measure of momentum per unit volume.

Similar conclusion were obtained by Aharonov \textit{et al.} \cite{34} showing, in particular, that the neutron’s electric dipole moment in a external static electric field \( \mathbf{E}_0 \) experiences a force given by \( ma = -(\mathbf{v} \cdot \nabla)(\mathbf{v} \times \mathbf{E}_0) \). The experimental verification of the Aharonov-Casher effect would confirm total momentum conservation in the interactions of magnets and charges \cite{35}.

Breitenberger \cite{36} discusses thoroughly this question showing the delicate intricacies behind the subject, pointing out the conservation of canonical momentum and the “extremely small” effect of magnetic interactions through the use of the Darwin’s lagrangian derived in 1920 \cite{37}. Boyer \cite{38} applying the Darwin’s lagrangian to the system of a point charge and a magnet have shown that the center-of-energy has uniform motion. Darwin’s lagrangian is correct to the order \( 1/c^2 \) (remaining Lorentz-invariant) and the procedure to obtain it eliminates the reaction always occurs by pairs and a kind of accounting balance such as \( \mathbf{F} = -\nabla \mathbf{V} \) holds.

Although Newton’s third law of motion apparently does not hold in some situations, it is likely action and reaction always occurs by pairs and a kind of accounting balance such as \( \mathbf{F} = -\nabla \mathbf{V} \) holds.

According to the Maxwell’s theorem, the resultant of \( \mathbf{K} \) forces applied to bodies situated within a closed surface \( S \) is given by the integral over the surface \( S \) of the Maxwell stresses:

\[
\int \mathbf{T}(n)dS = \int \mathbf{f}d\Omega = \mathbf{K}. \tag{8}
\]

Here, \( \mathbf{f} \) is the ponderomotive forces density and \( d\Omega \) is the volume element. The vector \( \mathbf{T}(n) \) under the integral in the left-hand side (lhs) of the equation is the tension force acting on a surface element \( dS \), with a normal \( \mathbf{n} \) directed toward the exterior. In cartesian coordinates, each component of \( \mathbf{T}(n) \) is defined by

\[
T_\alpha(n) = t_{xx}(n,x) + t_{xy}(n,y) + t_{xz}(n,z), \tag{9}
\]

with similar expressions for \( T_y \) and \( T_z \). The 4-dimensional momentum-energy tensor is a generalization of the 3-dimensional stress tensor \( T_{\alpha\beta} \). If electric charges are inside a conducting body in vacuum, in presence of electric \( \mathbf{E} \) and magnetic \( \mathbf{H} \) fields, then Eq. \( 9 \) must be modified to the form:

\[
\int \mathbf{T}(n)dS - \mathbf{K} = \frac{1}{4\pi c} \int \frac{\partial}{\partial t} \left( \mathbf{E} \times \mathbf{H} \right) d\Omega. \tag{10}
\]

In the right-hand side of the above equation it now appears the temporal derivative of \( \mathbf{G} = \int \mathbf{g}d\Omega \), the electromagnetic momentum of the field in the entire volume contained by the surface \( S \) (with \( \mathbf{g} \) its momentum density).

In the case the surface \( S \) is filled with a homogeneous medium without true charges, Abraham proposed to write instead the following equation:

\[
\int \mathbf{T}(n)dS = \frac{\partial}{\partial t} \int \frac{\varepsilon \mu}{4\pi c} [\mathbf{E} \times \mathbf{H}] d\Omega, \tag{11}
\]

with \( \varepsilon \) and \( \mu \) the dielectric constant of the medium and its magnetic permeability.

Eq. \( 11 \) can be written on the form of a general conservation law:

\[
\frac{\partial \sigma_{\alpha\beta}}{\partial x^\beta} - \frac{\partial g_\alpha}{\partial t} = f_\alpha. \tag{12}
\]

where \( \alpha = 1, 2, 3 \), \( \sigma_{\alpha\beta} \) is the stress tensor, \( g_\alpha \) is the momentum density of the field, and \( f_\alpha \) is the total force density. After some algebra this equation can take the final form:

\[
\frac{\partial \sigma_{\alpha\beta}}{\partial x^\beta} = f_\alpha + \frac{1}{4\pi c} \frac{\partial}{\partial t} [\mathbf{D} \times \mathbf{B}]_\alpha + f_{m,\alpha}. \tag{13}
\]
Here, $f'_m$ is the total force acting in the medium (see Ref. [42]), $f^L = \rho_e E + \frac{c}{2} j \times B$ is the Lorentz force density with $\rho_e$ denoting the charge density and $j$ the current density.

Of course, field, matter and physical vacuum form together a closed system and it is usual to catch the momentum conservation law in the general form [43, 44, 45]:

$$\frac{\partial}{\partial x^\beta} \left( T^{\text{Field}}_{\alpha\beta} + T^{\text{Matter}}_{\alpha\beta} + T^{\text{Vacuum}}_{\alpha\beta} \right) = 0.$$  \hspace{1cm} (14)

Table I shows the different expressions for the energy-momentum tensors of Minkowsky, $T^{M}_{\alpha\beta}$ and Abraham, $T^{A}_{\alpha\beta}$.

The general relation between Minkowsky and Abraham momentum, free of any particular assumption, holding particularly for a moving medium, is given by

$$P^M = P^A + \int f^A dt dV.$$  \hspace{1cm} (15)

For clearness, we shall distinguish the following different parts of a system: i) the body carrying currents and the currents themselves (the structure, for short, denoted here by $K$), ii) the fields, and iii) the physical vacuum (or the medium).

On the theoretical ground exposed above, the impulse transmitted to the structure should be given by the following equation:

$$P^K = \int f^A dt dV = P^M - P^A.$$  \hspace{1cm} (16)

Here, $f^A$ denotes the Abraham’s force density [46, 47]:

$$f^A = \frac{\varepsilon_r \mu_r - 1}{4\pi c} \partial \left[ E \times H \right] \frac{\partial}{\partial t}.$$  \hspace{1cm} (17)

This is in agreement with experimental data [48] and was proposed by others [49, 50]. As this force is acting over the medium, it is expected nonlinearities related to the behavior of the dielectric to different applied frequencies, temperature, pressure, and large amplitudes of the electric field when a pure dielectric response of the matter is no longer proportional to the electric field (see Ref. [51] on this topic).

The momentum conservation law can be rewritten under the general form (e.g., Ref. [42]):

$$\frac{\partial \sigma_{\alpha\beta}}{\partial x^\beta} = f^L_{\alpha} + \frac{1}{4\pi c} \frac{\partial}{\partial t} \left[ D \times B \right]_{\alpha} + f^m_{\alpha},$$  \hspace{1cm} (18)

with $f^m$ denoting the force acting on the medium. The second term in the r.h.s. of above equation could possible be called vacuum-interactance term [52] - in fact, Minkowski term. Already according to an interpretation of Einstein and Laub [53], the integration of above equation over all space, the derivative over stress tensor gives a null integral and the Lorentz forces summed over all the universe must be balanced by the quantity

$$\int_\infty \varepsilon_0 \mu_0 \frac{\partial [E \times H]}{\partial t} dV$$  in order Newton’s third law be preserved. It is important to remark that the field momentum $[D \times B]$ is equivalent to $\rho A$, the first term is related to the stress-tensor representation, while the second one is related to the “fluid-flow” representation [54].

As is well known, Maxwell’s classical theory introduces the idea of a real vacuum medium. After being considered useless by Einstein’s special theory of relativity, the “ether” (actually replaced by the term vacuum or physical vacuum) was rehabilitated by Einstein in 1920 [55]. In fact, general theory of relativity describes space with physical properties by means of ten functions $g_{\mu\nu}$ (see also [56]). According to Einstein,

The “ether” of general relativity is a medium that by itself is devoid of all mechanical and kinematic properties but at the same time determines mechanical (and electromagnetic) processes.

Dirac felt the need to introduce the idea of “ether” in quantum mechanics [57]. In fact, according to quantum field theory, particles can condense in vacuum giving rise
TABLE I: Expressions for the energy-momentum tensors of Min kowksy $T_{\alpha,\beta}^M$ and Abraham $T_{\alpha,\beta}^A$, using $i, k = 1, 2, 3, 4$; $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 =ict$. The Poynting’s vector is $S = [E \times H]$ and the energy for a system at rest is $w = \frac{1}{8\pi}(\epsilon E^2 + \mu H^2)$.

| Minkowsky | Abraham |
|-----------|---------|
| $T_{\alpha,\beta}^M = \begin{pmatrix} \sigma_{\alpha,\beta} & -icg^\alpha \\ -icg^\beta & \frac{w}{2} \end{pmatrix}$ | $T_{\alpha,\beta}^A = \begin{pmatrix} \sigma_{\alpha,\beta} & -icg^\alpha \\ -icg^\beta & \frac{w}{2} \end{pmatrix}$ |
| $g^M = \frac{1}{2}(E \times H)$ | $g^A = \frac{1}{2}(E \times H)$ |

It was assumed that all particles have the same drift velocity and they turn all at the same angular velocity $\omega$. The center of mass of the body moves with the same macroscopic velocity and the body turns at the same angular velocity $\omega$. The last term of Eq. (21) represents the gradient of the entropy in a nonequilibrium situation and $S$ is the transformed function defined by:

$$p_i = m_i v_i + q_i A + m_i [\omega \times r_i] - m_i T_i \frac{\partial S}{\partial p_i}.$$
where \( \mathbf{a} \) and \( \mathbf{b} \) are Lagrange multipliers.

Whenever the system is in thermodynamic equilibrium the canonical momentum is obtained for each composing particle:

\[
p_i = p_{rel} + m_i [\omega \times r_i] + q_i \mathbf{A}_i. \tag{23}
\]

Otherwise, when the system is subjected to forced constraints in such a way that entropic gradients in momentum space do exist, then a new expression for the particle momentum must be taken into account, that is, Eq. 21.

Summing up over all the constituents particles of a given thermodynamical system pertaining to the same aggregate (e.g., body or Brownian particle), we obtain:

\[
P = M \mathbf{v}_c + \sum_i m_i [\omega \times r_i] + Q \mathbf{A} - \sum_i m_i T_i \frac{\partial S}{\partial p_i}. \tag{24}
\]

To simplify we can assume that all the particles inside the system have the same random kinetic energy, \( T_i = \zeta \):

\[
P = M \mathbf{v}_c + \sum_i m_i [\omega \times r_i] + Q \mathbf{A} - \zeta \sum_i \frac{\partial S_{ne}}{\partial r_i}. \tag{25}
\]

where by \( S_{ne} \) we denote the entropy when the system is in a state out of equilibrium. The first term on the right-hand side is the bodily momentum associated with motion of the center of mass \( M \); the second term represents the rotational momentum; the third is the momentum of the joint electromagnetic field of the moving charges \( \mathbf{A} \); finally, the last term is a new momentum term, that can be physically understood as a kind of “entropic momentum” since it is ultimately associated to the information exchanged with the medium on the the physical system viewpoint (e.g., momentum that eventually is radiated by the charged particle). Lorentz’s equations don’t change when time is reversed, but when retarded potentials are applied the time delay of electromagnetic signals on different parts of the system do not allow perfect compensation of internal forces, introducing irreversibility into the system \( \mathbf{A} \). This is always true whenever there is time-dependent electric \( \mathbf{E} \) and magnetic fields \( \mathbf{B} \). Cornish \( \mathbf{A} \) obtained a solution of the equation of motion of a simple dumbbell system held at fixed distance and have shown that the effect of radiation reaction on an accelerating system induces a self-accelerated transverse motion. Obara and Baba \( \mathbf{A} \) have discussed the electromagnetic propulsion of a electric dipole system and they have shown that the propulsion effect results from the delay action of the static and inductive near-field created by one electric dipole on the other. These are examples of irreversible (out of equilibrium) phenomena that do not comply with action-reaction law.

At this stage, we can argue that the momentum is always a conserved quantity provided that we add the right term, making Newton’s third law verified. This apparent “missing symmetry” might result because matter alone does not form a closed system, and we need to include the physical vacuum in order symmetry be restored. So, when we have two systems 1 and 2 interacting via some kind of force field \( \mathbf{F} \), the reaction from the vacuum must be included as a sort of bookkeeping device:

\[
\mathbf{F}^{\text{matter}}_{12} = -\mathbf{F}^{\text{matter}}_{21} + \mathbf{F}^{\text{vacuum}}. \tag{26}
\]

We may assume the existence of a physical vacuum probably well described by a spin-0 field \( \phi(x) \) whose vacuum expectation value is not zero:

\[
\text{vacuum} \sim \phi(x), \tag{27}
\]

and at its lowest-energy state to have zero 4-momentum, \( k_\mu = 0 \) (e.g., Ref. \( \text{[49]} \)).

This new state out of equilibrium can be constrained by applying an external force on the system (e.g., set all system into rotation about its central axis at the same angular velocity \( \omega \)).

It was shown that the entropy must increase with a small displacement from a previous referred state \( \text{[44, 50]} \). Considering that the entropy is proportional to the logarithm of the statistical weight \( \Omega \sim \exp(S/k_B) \) and considering that \( S = S_{eq} + S_{ne} \) we can expect an increase of the nonequilibrium entropy \( S_{ne} \) with a small increase of the \( i \)th particle’s velocity \( \mathbf{v}_i = \mathbf{r}_i \), since with an increase of particle’s speed (although in random motion) the entropy must increases altogether. Therefore, we must always have:

\[
T_i \frac{\partial S_{ne}}{\partial r_i} \geq 0, \forall i = 1, \ldots N. \tag{28}
\]

In conditions of mechanical equilibrium the equality must hold, otherwise condition \( \text{[28]} \) can be considered a \textit{universal criterion of evolution}. Considering that the entropy is an invariant \( \text{[81]} \) there is no extra similar term when the momentum is transferred to another inertial frame of reference.

Quite withstanding, there is an important theorem derived by Baierlin \( \text{[82]} \) showing that the Gibbs entropy for a system of free particles with kinetic energy \( K \), density \( \rho \) and absolute temperature \( T \), \( S(K, \rho, T) \), is greater than the entropy associated to the same system.
subject to arbitrary velocity-independent interactions $V$, $S(K + V, \rho, T)$, such as $S(K + V, \rho, T) \leq S(K, \rho, T)$.

At the electromagnetic level, Maxwell conceived a dynamical model of a vacuum with hidden matter in motion. As it is well-known, Einstein’s theory of relativity eradicated the notion of “ether” but later revived its interest in order to give some physical mean to ity eradicated the notion of “ether” but later revived its interest in order to give some physical mean to Minkowski obtained as a mathematical consequence of the Maxwell’s mechanical medium that the Lorentz’s force should be exactly balanced by the divergence of the Maxwell’s tensor in vacuum $T_{\text{vac}}$ minus the rate of change of the Poynting’s vector:

$$\rho E + \mu_0 [J \times H] = \nabla \cdot T_{\text{vac}} - \frac{\partial}{\partial t} \epsilon_0 \mu_0 [E \times H].$$

(29)

Einstein and Laub remarked [53] that when Eq. 29 is integrated all over the entire Universe the term $\nabla \cdot T_{\text{vac}}$ vanish which means that the sum of all Lorentz forces in the Universe must be equal to the quantity $\int_{\infty}^{\mu_0} \epsilon_0 \mu_0 \partial/\partial t [E \times H] \, dv$ in order to comply with Newton’s third law (see Ref. [82]). But, this long range force depends on the constant of gravitation $G$. Einstein accepted the Faraday’ point of view of the reality of fields, and this gravitational field according to him would propagate all over the entire space without loss, locally obeying to the action-reaction law. But nothing can reassure us that the propagating wave through the vacuum will be lost at infinite distances [84]. Poincaré [53] also argues about the possible dissipation of the action on matter due to the absorption of the propagating wave in the context of Lorentz’s theory.

By Noether’s theorem, energy conservation is related to translational invariance in time $(t \rightarrow t + \alpha)$ and momentum conservation is related to translational invariance in space $(r_1 \rightarrow r_1 + b_i)$. This important theorem thus implies that the law of conservation of momentum (not equivalent to the action-equals-reaction principle) is always valid, while the law of action and reaction does not always holds, as shown in the previous examples.

Some kind of relationship must therefore exists between entropy and Newton’s third law, as it was through the combined equation with the first and second law of thermodynamics that our main result were obtained. This idea was verified recently through a standard Smoluchowski approach and on Brownian dynamic computer simulation of two fixed big colloidal particles in a bath of small Brownian particles drifting with uniform velocity along a given direction. It was shown that, in striking contrast to the equilibrium case, the nonequilibrium effective force violates Newton’s third law, implying the presence of nonconservative action showing a strong anisotropy [60]. This result reminds our Eq. 26.

FIG. 2: Schematic of the self-accelerated device.

V. IS IT VERIFIED THE ACTION-EQUALS-REACTION LAW IN A THERMODYNAMICAL SYSTEM OUT-OF-EQUILIBRIUM?

The maximizing entropy procedure proposed in Ref. [72, 73] suggest the following “gedankenexperiment”. This problem bears some resemblance to the Leo Szilard’s thermodynamic engine with one-molecule fluid (e.g., Ref. [51]), although we are not concerned here with nqeutropy issues.

Let us suppose a system consisting of a spherical body made of $N$ number of particles closed in a box and moving along one direction (see Fig. 2). The left side is at temperature $T_2$, the right side is at temperature $T_3$, while the body particle itself is at temperature $T_1$ (and in equilibrium with their photonic environment). Furthermore, we assume that both surfaces and the body particle are all thermal reservoirs, and hence their respective temperatures do not change. Let us suppose that the onset of nonequilibrium dynamics can be forced by some means in the previously described device. When the particle collides with the top its momentum varies according to:

$$\delta p_i = -mv_1 + mv_1 + (T_3 - T_1) \partial_v S.$$  

(30)

Here, $\partial_v S$ denotes the (nonequilibrium) entropy gradient in velocity space. After the collision the particle goes back to hit the right surface at temperature $T_3$. The momentum variation after the second collision is given
by:
\[ \delta p_1 = mv_1 - mv_1' + (T_2 - T_1) \partial_v S. \]  
(31)
We assume that the body attain thermal equilibrium with the environment (which must remain at constant temperature \( T_0 \)) fast enough before the next hit against the wall of the thermal reservoir. The total balance after a complete loop, back and forth, is given by
\[ \delta p_1 = - \delta p_1 - \partial_v S(T_2 + T_3 - 2T_1) = - \delta p_1 - \Delta \zeta \nabla_v S. \]  
(32)
To make it more clear, we might write Eq. 32 under the form
\[ \delta p_1 = - \delta p_1 - \delta p_1^a, \]  
(33)
where we denote by \( \delta p_1^a \equiv \Delta \zeta \nabla_v S \), the change in momentum by the physical vacuum (others, would call “inertial space”). Therefore, it is clear from the above analysis that in systems out of equilibrium Newton’s third law is not verified. The conservation of canonical momentum, however, is well verified, as it must be according to Noether’s theorem. Otherwise, when the temperatures are equal for all thermal bath in contact, such as \( T_1 = T_2 = T_3 \), Newton’s third law is verified:
\[ \delta p_1 = - \delta p_1. \]  
(34)
In the frame of nonlinear dynamics and statistical approach Denisov has shown [88] that a rigid shell and a nucleus with internal dynamic asymmetric can perform self unidirectional propulsion. It seems now certain that depletion forces between two fixed big colloidal particles in a bath of small particle exhibits nonconservative and strongly anisotropic forces that violate action-reaction law [59] (see also Ref. [89]). Also, internal Casimir forces between a circle and a plate in nonequilibrium situation violates Newton’s law [90].

VI. CONCLUSION
The purpose of this study is to examine how the action-reaction law is faced in the literature in the domain of mechanics, electrodynamics and statistical mechanics, and to offer a methodological approach in order to tackle the fundamental aspect of the problem suggesting that a third part should be included in the analysis of forces, what we called here for the sake of conciseness, the physical vacuum. Furthermore, a general procedure lead us to an expression for the general linear (canonical) momentum of a body-particle in the framework of statistical mechanics. Theoretical arguments and numerical computations suggest that Newton’s third law is not verified in out-of-equilibrium systems due to an additional entropic gradient term present in the particle’s canonical momentum. Although Noether’s theorem guaranty the conservation of canonical momentum, the actions-equality-reaction principle can be restored in nonequilibrium conditions only if a new force term representing the action of the medium on the particles is taken into account.

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