Absolute Magnitudes of Seismic Red Clumps in the *Kepler* Field and SAGA: The Age Dependency of the Distance Scale

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Abstract

Red clump stars are fundamental distance indicators in astrophysics, although theoretical stellar models predict a dependence of absolute magnitudes with age. This effect is particularly strong below ~2 Gyr, but even above this limit a mild age dependence is still expected. We use seismically identified red clump stars in the *Kepler* field for which we have reliable distances, masses, and ages from the SAGA survey, to first explore this effect. By excluding red clump stars with masses larger than 1.6 M☉ (corresponding to ages younger than 2 Gyr), we derive robust calibrations linking intrinsic colors to absolute magnitudes in the following photometric systems: Strömgren *by*, Johnson *BV*, Sloan *griz*, 2MASS *JHK*, and *WISE* W1/W2/W3. With the precision achieved we also detect a slope of absolute magnitudes ~0.020 ± 0.003 mag Gyr⁻¹ in the infrared, implying that distance calibrations of clump stars can be off by up to ~0.2 mag in the infrared (over the range from 2 to 12 Gyr) if their ages are unknown. Even larger uncertainties affect optical bands, because of the stronger interdependency of absolute magnitudes on colors and age. Our distance calibrations are ultimately based on asteroseismology, and we show how the distance scale can be used to test the accuracy of seismic scaling relations. Within the uncertainties our calibrations are in agreement with those built upon local red clumps with *Hipparcos* parallaxes, although we find a tension, which, if confirmed, would imply that scaling relations overestimate the radii of red clump stars by 2 ± 2%. Data releases post *Gaia* DR1 will provide an important testbed for our results.

**Key words:** asteroseismology – stars: fundamental parameters – stars: distances – stars: late-type – surveys

1. Introduction

The clump of red giant stars is a ubiquitous feature in (nearly) equidistant stellar populations. Theoretically predicted by Thomas (1967) and Iben (1968), it was first recognized in the color–magnitude diagrams of old- and intermediate-age open clusters (Cannon 1970), and later also observed in “metal-rich” globular clusters (e.g., Hesser & Hartwick 1977), toward the Galactic bulge (e.g., Paczynski & Stanek 1998) and in nearby galaxies (e.g., Stanek & Garnavich 1998). Red clump stars (hereafter RCs) also constitute a quite remarkable and well-populated feature in the color–magnitude diagrams of nearby field stars once precise distances from *Hipparcos* are used (e.g., Girardi et al. 1998; Paczynski & Stanek 1998). It is established that RCs have nearly constant absolute magnitudes, and once identified (e.g., as an overdensity in an equidistant stellar population) they are important standard candles for deriving distances.

The identification of RCs among field stars has been difficult so far, due to the limited number of them with precise trigonometric parallaxes, combined with the fact that the *Hipparcos* “sphere” covers a rather limited volume, extending to distances of the order of 100 pc. While *Gaia* is due to shift this limit to several kiloparsecs (Lindegren et al. 2016), spaceborne asteroseismic missions such as *CoRoT* (Auvergne et al. 2009) and *Kepler* (Gilliland et al. 2010) already allow us to derive stellar distances for stars with measured solar-like oscillations, among which are RCs (e.g., Silva Aguirre et al. 2012; Miglio et al. 2013a; Casagrande et al. 2014b; Rodrigues et al. 2014). Furthermore, when period-spacing information is available, asteroseismology is also able to unambiguously distinguish between stars ascending the red giant branch (RGB) by burning hydrogen in a shell, and those (i.e., RCs) that have already ignited helium burning in their cores (e.g., Montalbán et al. 2010; Bedding et al. 2011; Stello et al. 2013).

In this work, we aim to derive color and absolute magnitude calibrations in many photometric systems for seismically identified RCs and compare these calibrations with those available in the literature for local RCs. Our goals are manifold. First, we aim to obtain a more reliable selection of RCs compared to previous studies: by taking advantage of the seismic period-spacing we can in fact precisely identify bona fide RCs. This allows us to remove contaminants from our sample (among which are stars going through the bump in the RGB), which plague other RC selection techniques. Second, we take advantage of seismic distances to derive reliable absolute magnitudes for all our RCs. Seismic distances are obtained by scaling stellar angular diameters (in our case determined from the InfraRed Flux Method; see Casagrande et al. 2014b) to seismic radii (ultimately based on scaling relations; see, e.g., Miglio et al. 2009; Stello et al. 2009). A great deal of effort is currently invested in testing the accuracy of scaling relations, and in whether they have any dependence on other parameters such as, e.g., metallicity and evolutionary phase (White et al. 2011). Radii derived from scaling relations have been shown to be accurate to about 5%, depending on evolutionary status (e.g., Huber et al. 2012; Silva Aguirre et al. 2012; White et al. 2013; Gaulme et al. 2016), although they are considerably...
less tested in the RC regime (for a summary see, e.g., Miglio et al. 2013b; Brogaard et al. 2016). Currently, uncertainty on seismic radii is one of the limiting factors in the accuracy at which seismic stellar distances can be derived. The other stems from the accuracy at which stellar effective temperatures (and thus angular diameters) can be derived from photometry (Casagrande et al. 2014a). Both sources of uncertainty, however, are distance-independent (modulo reddening), meaning that for a distance fractional error $f$, seismic distances will be superior to astrometric ones beyond $10^4 f/\omega$ pc (where $\omega$ is the parallax error in $\mu$as). Here, we use seismic distances from Casagrande et al. (2014b), which have a median uncertainty of 3.3% (assuming no systematic errors in the adopted scaling relations) and typical distances above 1 kpc. Gaia DR1 parallaxes have a systematic error of 300 $\mu$as in addition to random errors (Lindegren et al. 2016), effectively meaning that our seismic distances are always more precise than Gaia DR1. Finally, from seismology we also know the masses and ages of our RC stars, meaning that we can investigate the dependence of absolute magnitudes on these parameters, which is important for assessing the range within which RC absolute magnitude calibrations can be trusted.

2. The Red Clump Sample

The identification of RCs has been traditionally carried out by eye by selecting stars in the HR diagram that had a location consistent with their presence. Despite RCs having very different internal structures from stars ascending the RGB, a clean selection between the two has been impossible so far, since they occupy nearly the same position in luminosity, effective temperature, gravity, and colors within the observational uncertainties. Asteroseismology has recently allowed this limitation to be overcome, since in the $\Delta \nu$ versus $\Delta P$ diagram (here $\Delta \nu$ is the frequency shift of consecutive overtone modes of the same degree, and $\Delta P$ is the pairwise period-spacing between adjacent dipole modes), RCs are clearly separated from red giants (e.g., Stello et al. 2013). For the purposes of our work, we want a sample of seismically identified RCs that also has information on their metallicities, radii, distances, masses, and ages, as well as magnitudes in various photometric systems. This is possible thanks to the Strömgren survey for Asteroseismology and Galactic Archaeology (SAGA; Casagrande et al. 2016). Here we use seismic ages derived assuming no mass-loss: this is motivated by the fact that recent studies seem to indicate a low efficiency of mass-loss (see the discussion in Casagrande et al. 2016).

Figure 1 shows the $T_{\text{eff}}$ -- log($g$) plane for the entire SAGA sample; an overdensity of stars is present for log($g$) $\sim$ 2.5 ± 0.1 dex, but solely using this information it is impossible to single out RC stars. In SAGA, a large fraction of the objects with seismic information also have an evolutionary phase classification based on period-spacing (Stello et al. 2013). The latter tells us whether a star is evolving along the RGB with a hydrogen burning shell or if it is already in the RC phase. We thus use this information to limit our sample to objects marked as RC (green open circles in Figure 1). Also overplotted, with crosses, are seismically inferred members of the open cluster NGC 6819, some of which are RCs as well.

RCs with masses above $\geq 1.8$ $M_\odot$ ignite helium in non-degenerate conditions, which observationally results in slightly fainter luminosities and hotter effective temperatures (e.g., Girardi 1999). This feature is known as a secondary clump and is clearly visible with our data (see Figure 1, and also the discussion in Casagrande et al. 2014b). Clearly, the absolute magnitudes of secondary clump stars deviate significantly from a constant value, which in turn prevents them from being used as good distance calibrators. Because of this, first, we exclude from our sample all RCs with masses above 1.6 $M_\odot$ and ages younger than 2 Gyr. In fact, the ages of stars in the red giant phase, either RGB or RC, are largely determined by the time spent in the main sequence core-hydrogen burning phase, meaning that the mass of a red giant is also a good proxy for its age. The conversion between mass and age introduces a dependency on stellar models, which, among other things, is sensitive to overshooting during the main sequence, and mass-loss. For stars along the RGB, mass-loss mostly occurs toward the tip of the RGB and the clump phase, and thus it could potentially impact the age determination of our stars. However, recent studies suggest that mass-loss is rather inefficient (see the discussion in Casagrande et al. 2016), meaning that it only moderately affects our ages. All our ages are derived assuming no mass-loss (Reimers’ parameter $\eta$ = 0), but in Section 3.7 we discuss how our results would change in the case of an extremely efficient mass-loss ($\eta$ = 0.4, which, however, is currently disfavored by observations). Overshooting during the main sequence for $M < 1.6$ $M_\odot$ stars does not change the mass of the degenerate He core, which follows the pattern of $\Delta P$ (an important parameter for mass determination) according to Montalbán et al. (2013). With the above-mentioned selection procedure, all RCs in our sample have a very narrow range of surface gravity, $2.3 \leq$ log($g$) $\leq 2.6$ dex, while their effective temperatures vary between $\sim$4500 and 5200 K. Second, we exclude members of NGC 6819 because they are mainly red clump stars on the second sequence being a young cluster. We also exclude KIC 6206407, an RC star with a second oscillation signal in the Kepler data, indicating a likely binary (Casagrande et al. 2014b). Finally, we also exclude all stars
with a bad metallicity flag in SAGA, i.e., we keep stars with $M_{flg} = 0$ only, for a final sample of 171 stars. This is the sample of RCs that will be used in the remainder of the paper to derive our color–absolute magnitude relations in different photometric systems. Further pruning of the sample to retain the stars with the best photometric measurements in a given system will be done as described in the next section.

3. Color and Magnitude Calibrations in Different Photometric Systems

In addition to our Strömgren observations, magnitudes in the following photometric systems are also available for most of the targets: $BV$ and $griz^{′}$ from APASS (Henden et al. 2009), gizr from the Kepl er Input Catalog (KIC; Brown et al. 2011), $JHK_s$ from 2MASS (Cutri et al. 2003), and $W_1W_2W_3$ from WISE (Wright et al. 2010). All seismic targets have apparent magnitudes in the range $10 \leq V \leq 14$, meaning that photometric errors are usually small in all of the above systems, with typical uncertainties varying between 0.01 and 0.03 mag. We discuss each photometric system and the quality cuts adopted on the photometry in the following subsections.

Before doing this though, reddening must be properly taken into account to derive correct intrinsic colors and absolute magnitudes. SAGA provides reddening $E(B − V)$ for all asteroseismic targets. We use these values to deredden all photometric measurements, using extinction coefficients appropriate for clump stars. These are computed as described in Casagrande & VandenBerg (2014); briefly, we apply the Cardelli et al. (1989) extinction law to a synthetic spectrum representative of clump stars ($T_{eff} = 4750$ K, $\log g = 2.0$ and [Fe/H] = 0) from which the following coefficients are derived: $R_B = 3.934$ and $R_V = 3.086$ for the Johnson system, $R_{J} = 3.669$, $R_{K} = 2.687$, $R_{H} = 2.106$, and $R_{F} = 1.517$ for the Sloan gizr system, $R_{J} = 3.846$ and $R_{K} = 3.124$ for the Strömgren system, $R_{J} = 0.894$, $R_{H} = 0.566$, and $R_{K} = 0.366$ for the 2MASS system, and $R_{W1} = 0.242$, $R_{W2} = 0.134$, and $R_{W3} = 0.349$ for the WISE system. These values are consistent with those published in the literature, but have the advantage of being computed using a reference spectrum appropriate for RCs, thus ensuring better consistency among different bands.

Once color excess and extinction coefficients are known, dereddened magnitudes in any given band $\zeta$ can be derived as

$$m_\zeta = m_\zeta - R_\zeta E(B − V),$$

from which dereddened absolute magnitudes $M_\zeta,0 = m_\zeta - 5 \log D + 5$, where $D$ is the distance (in parsecs), also determined from SAGA. Distances in SAGA are obtained by scaling angular diameters computed via the InfraRed Flux Method to asteroseismic radii. The distance of each star is then fed into empirically calibrated three-dimensional Galactic extinction models to derive its reddening, with an iterative procedure to converge in both distance and reddening. Individual uncertainties on reddening are not available, but those are expected to be of the order of $\pm 0.02$ mag on average (Casagrande et al. 2014b), given that the SAGA sample used here covers a stripe with a Galactic latitude between $8^\circ$ and $20^\circ$, where reddening is relatively low. Thus, the average colors of the stars vary by considerably less than this uncertainty. Furthermore, the zero-point of our reddening values is anchored to the open cluster NGC 6819 for which a robust value of $E(B − V) = 0.14 \pm 0.01$ is available from the literature. On average, reddening uncertainties are thus at the level of a few hundredths of a magnitude, having a negligible impact on the color–absolute magnitude relations presented later in the paper.

As a further check on the reddening values adopted from SAGA, we also derive independent estimates using the relation $E(B − V) = A_K / 0.366 = 0.918 (H − W_2 − 0.08)/0.366$, where the expression for $A_K$ is based on the Rayleigh–Jeans Color Excess method (RJCE; Majewski et al. 2011). This technique relies on the near-constancy of the infrared color $H−[4.5 \mu m])$ for evolved stars. In our case we adopt $H$ and $W_2$ magnitudes from 2MASS and WISE, respectively. The latter filter is centered on a wavelength of 4.6 $\mu$m, which is taken into account by the $−0.08$ factor in $A_K$ as reported in Majewski et al. (2011).

As an example of the different precision achieved with different sets of reddening values, Figure 2 shows the $(b − y)$ versus $M_F$ relation for RCs when adopting $E(B − V)$ from SAGA or the RJCE calibration instead. In the latter case, the scatter of the color–absolute magnitude relation is considerably larger (0.14 versus 0.10). In particular, stars belonging to the open cluster NGC 6819 cover a broad range of colors, whereas when switching to the reddening values from SAGA it narrows around $(b − y) \approx 0.63$.

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Note that for the remainder of the paper, including the figures, all colors and absolute magnitudes are always corrected for reddening, although, to make the notation more readable, the subscript 0 is not included.
In this paper linear fits are obtained using the IDL linear regression routine \textit{regress}. Uncertainties stemming from photometric and distance errors are taken into account when performing the fits. For each fit we also compute the Pearson and Spearman coefficients, to measure the strength of correlations. The results from this method are labeled as \textit{IDL-reg}. To check the robustness of our results to potential outliers, we also employ two additional methods. One method models outliers using a mixture model consisting of a straight line mixed with a broad Gaussian to capture outliers. We adapted the Python code \textit{exMix1} of Hogg et al. (2010) and label the results from this method \textit{Py-exMix1}. The other method performs a robust linear regression using an \textit{M} estimator, employing iterated re-weighted least squares, as implemented in the \textit{R} function \textit{rlm}; we label the results from using this method \textit{R-rlm}.

### 3.1. Strömgren

As shown in Figure 2, the bulk of RCs cover the color range $0.50 \leq (b - y) \leq 0.75$ mag and have absolute magnitudes between $0.4 \leq M_y \leq 1.1$ mag. Clearly, $M_y$ has a steep dependence on $(b - y)$ color, which we linearly fit (\textit{IDL-reg}), obtaining the following relation $M_y = 2.209(b - y) - 0.626$ ($\sigma = 0.10$), using 162 RCs with good-quality data. Note that stars with photometric errors in either $b$ or $y$ that are larger than 0.03 mag are excluded here. The relations of $M_y = 2.206(b - y) - 0.620$ from the \textit{R-rlm} method and of $M_y = 2.220(b - y) - 0.613$ from \textit{Py-exMix1} are very close to the \textit{IDL-reg} results, and agree within the scatter. Both the Pearson and Spearman coefficients are 0.72, indicating a strong correlation.

When the metallicity is taken into account, we find $M_y = 2.097(b - y) + 0.045[\text{Fe/H}] - 0.532$ ($\sigma = 0.102$), based on \textit{IDL-reg}. Generally, RC stars in our sample have a metallicity range of $-0.5 < [\text{Fe/H}] < 0.5$, which corresponds to a variation of $\lesssim 0.04$ mag in $M_y$. This variation is smaller than the scatter of the $M_y$ calibration. Thus, for the remainder of the paper we only provide calibrations linking absolute magnitudes to colors. We also explored whether introducing a second-order term in color improved the residual of our fit, but found this not to be the case. In fact, the major source of uncertainty in our calibration is represented by a mean uncertainty of 0.08 mag in absolute magnitude as a consequence of our typical distance uncertainties.

### 3.2. Johnson

Figure 3 shows the $(B - V)$ versus $M_V$ diagram, and the absolute magnitude distributions in the Johnson system for RC stars. In the Johnson $BV$ system, stars with APASS measurement uncertainties larger than 0.05 mag in either the $B$ or $V$ bands are removed. Most RCs are located in the range $0.6 \leq (B - V) \leq 1.3$ mag and $0.5 \leq M_V \leq 1.0$ mag. Based on 119 stars, the scatter is 0.15 mag. A more strict limit of less than 0.03 mag on the measured errors in either the $B$ or $V$ bands does not improve the correlation, and the star number of the sample is reduced to 53 stars. This scatter likely reflects the quality of the APASS magnitudes (see also the next section). Note that fits to the color and absolute magnitude relation based on the three methods, \textit{IDL-reg}, \textit{Py-exMix1}, and \textit{R-rlm}, are quite similar. The Pearson (Spearman) correlation is 0.13 (0.18), indicating a weak correlation between absolute magnitude and color, as already apparent from Figure 3.

### 3.3. Sloan

Magnitudes in the $griz$ system are available from the \textit{Kepler} Input Catalog, with typical uncertainties of 0.02 mag (Brown et al. 2011). However, KIC magnitudes are not exactly on the Sloan system, and thus have been corrected with the transformations provided by Pinsonneault et al. (2012). In addition, for a large fraction of stars $g'r'i'$ magnitudes are also available from the APASS survey; these are defined in the primed system and thus have been converted into the Sloan system using the transformations of Tucker et al. (2006). However, we also note that in the color–absolute magnitude plane the scatter is larger when using APASS magnitudes instead of KIC, thus pointing to lower precision for the former measurements. Therefore, in the following the analysis we will only use KIC $griz$ magnitudes.

Figure 4 shows the color–absolute magnitude diagrams in different bands. A few stars are marked with red crosses, and are removed from the rest of the analysis: they are somewhat offset from the bulk of other stars, and have been identified as anomalous from their 2MASS colors (see the next section). The absolute magnitudes in each $griz$ band vary linearly with colors, and their slopes flatten as the filter moves toward longer wavelengths, i.e., from $M_g$ to $M_i$. Two stars with $(g - r) > 1.0$ seem to deviate from the linear trend of $M_g$ versus $(g - r)$. The number of points is too small to draw further conclusions; however, we caution against using the calibration at $(g - r) > 1.0$. The panels in Figure 4 display a correlation between colors and absolute magnitudes that varies depending on the filter, and decreases when moving to redder filters (see Table 1 for a list of Pearson and Spearman coefficients). We also note that the decrease of $M_i$ with $(r - i)$ is consistent with the results of Zhao et al. (2001), who found a dependence of Johnson $M_I$ with $(V - I)$. Chen et al. (2009) suggested that $i$ and $z$ are the best bands for distance calibration of red clump/red
horizontal branch stars in the Sloan system. Here we find that $M_g$ versus $(g-r)$ provides an equally good distance calibration.

### 3.4. 2MASS

We restrict ourselves to stars with $JHK_s$ errors less than 0.03 mag. Figure 5 shows the $(J-H)$ versus $M_J$, $(J-H)$ versus $M_H$, $(H-K_s)$ versus $M_H$, and $(J-K_s)$ versus $M_{K_s}$ diagrams, as well as the 2MASS color and absolute magnitude distributions for our sample stars. There is a mild slope in $M_J$ as a function of color $(J-H)$, while in the two remaining filters $M_H$ and $M_{K_s}$, there is almost no trend with color. This is quantified in Table 1 with the Pearson and Spearman correlation coefficients.

In the widely used diagram of $(J-K_s)$ versus $M_{K_s}$, RC stars cover the color range 0.5–0.7 mag, and most of them cluster at an absolute magnitude that is consistent with the value $M_{K_s} = -1.613$ obtained by Laney et al. (2012) using local RC stars (indicated in the figure with a solid line). Note that the value in Laney et al. (2012) is already converted into the 2MASS system, thus allowing a direct comparison with our results.

For some stars, color and absolute magnitude combinations in the infrared show significant deviations from the mean.
values of the whole sample, which is not compatible with the maximum allowed photometric errors of 0.03 mag set here, even after taking into account reddening and distance uncertainties on absolute magnitudes. Specifically, 10 stars marked by red crosses in Figure 5 lie beyond the dashed lines, which represent a deviation of 0.15 mag (typical maximum error) from the absolute magnitude $M_{Ks} = -1.613$ obtained by Laney et al. (2012) from local RC stars. The quoted 2MASS $JHKs$ errors for these stars cannot explain such large deviations. Most of these deviant stars are overluminous, and this would point toward the presence of some sort of infrared emission, e.g., a hot circumstellar disk. Also, although we have removed RC stars with ages below 2 Gyr from our analysis, 5 of the deviant ones have ages between 2 and 3 Gyr, and thus some residual age effect on absolute magnitudes could still be present for some of these objects. Investigating these scenarios, however, is beyond the scope of the present paper. We exclude these stars in the rest of the analysis, and we mark them with red crosses in the plots for reference.

### 3.5. WISE

Figure 6 shows the color–absolute magnitude diagrams in the WISE system. Photometric uncertainties in W1W2 are of the order of 0.02 mag, significantly smaller than those for W3W4, which are of the order of 0.02–0.10 and 0.10–0.50 mag, respectively. Because of these uncertainties, we impose a threshold on the maximum allowed photometric error, 0.03 mag on W1 and W2, and 0.05 mag on W3, while we discard W4 from the rest of the analysis.

The color and magnitude ranges in the WISE system are quite small, which indicates that these bands can be used to obtain very good distances for RC stars. We quantify in Table 1 the Pearson and Spearman correlation coefficients. Also, comparing Figures 4–6 we see that the slope of absolute magnitudes as a function of colors flattens out, and then reverses if moving to longer wavelengths.

### 3.6. The Combined Strömgren, 2MASS, and WISE Systems

It is interesting to explore two widely used combinations of color and absolute magnitudes for RC stars. Namely, $(V - K_s)$ versus $M_K$ and $(H - W2)$ versus $M_{W2}$ are widely adopted in the literature. Since $V$ magnitudes in APASS have somewhat larger errors, we adopt $y$ magnitudes in this analysis (where in fact, Strömgren $y$ was historically defined to be essentially the same as Johnson $V$). Figure 7 shows the $(y - K_s)$ versus $M_K$ and the $(H - W2)$ versus $M_{W2}$ diagrams, together with their color distributions. There is essentially flat correlation between $(y - K_s)$ and $M_K$, with a Pearson (Spearman) coefficient of −0.03 (0.08). On the contrary, $M_{W2}$ inversely correlates with $(H - W2)$, the Pearson (Spearman) coefficient being $-0.41$ ($-0.36$). Color histograms are shown in the bottom of Figure 7, and the mean values are $(y - K_s) = 2.383 \pm 0.129$ and $(H - W2) = 0.053 \pm 0.025$. In particular, the rather narrow range of $(H - W2)$ makes it a useful cryon for determining reddening in high-extinction areas using RC stars.

Our results based on the *IDL-reg* function are summarized in Table 1. Results from PYTHON *exMix1* and *R-rlm* are very close, and we present the color–absolute magnitude calibrations based on the *R-rlm* function in Table 2.

### 3.7. The Age Dependence of the Distance Scale

Using the age (and mass) information available from SAGA, in Figure 8 we plot the absolute magnitudes of RC stars as a function of age, in the $K_s$ and $W2$ bands, which are two of the filters displaying the least color-dependence. Stars with age errors larger than 30% are removed from this plot, although they would still follow the same trend if included. Interestingly, for stars older than 2 Gyr there is a clear dependence of $K_s$ and $W2$ absolute magnitudes on age. Fitting this trend with *IDL-reg*, we obtain:

$$M_{K_s} = (0.015 \pm 0.003) \tau - 1.715(\pm 0.016),$$

and

$$M_{W2} = (0.017 \pm 0.003) \tau - 1.682(\pm 0.016),$$

Note. The difference between mean and median values is usually only a few millimag, and never exceeds 0.01 mag. We use the scatter of the data as a conservative estimate of the errors: if we were to use the standard deviation of the mean, uncertainties would be a factor of 10 smaller. Note that all colors and absolute magnitudes are corrected for reddening.

| $(b - y)$ | 0.619 ± 0.048 | $M_y = 0.754 \pm 0.147$ | $M_y = 2.209(b - y) - 0.626$ | 0.72/0.72 |
| $B - V$ | 1.025 ± 0.140 | $M_V = 0.735 \pm 0.148$ | $M_V = 0.189(B - V) + 0.525$ | 0.13/0.18 |
| $(g - r)$ | 0.809 ± 0.078 | $M_r = 1.229 \pm 0.172$ | $M_r = 2.010(g - r) - 0.402$ | 0.88/0.89 |
| $(g - r)$ | 0.809 ± 0.078 | $M_g = 0.420 \pm 0.110$ | $M_g = 1.010(g - r) - 0.402$ | 0.67/0.72 |
| $(r - i)$ | 0.263 ± 0.029 | $M_i = 0.420 \pm 0.110$ | $M_i = 2.738(r - i) - 0.303$ | 0.65/0.71 |
| $(r - i)$ | 0.263 ± 0.029 | $M_r = 0.157 \pm 0.094$ | $M_r = 1.738(r - i) - 0.303$ | 0.45/0.55 |
| $(i - z)$ | 0.136 ± 0.021 | $M_z = 0.157 \pm 0.094$ | $M_z = 2.382(i - z) - 0.169$ | 0.53/0.52 |
| $(i - z)$ | 0.136 ± 0.021 | $M_i = 0.022 \pm 0.084$ | $M_i = 1.382(i - z) - 0.169$ | 0.34/0.34 |
| $(J - H)$ | 0.513 ± 0.034 | $M_H = -1.016 \pm 0.063$ | $M_H = 0.975(J - H) - 1.518$ | 0.50/0.47 |
| $(J - K_s)$ | 0.097 ± 0.024 | $M_K_s = -1.528 \pm 0.055$ | $M_K_s = 0.494(H - K_s) - 1.580$ | 0.15/0.18 |
| $(J - K_s)$ | 0.609 ± 0.040 | $M_K = -1.626 \pm 0.057$ | $M_K = -0.188(J - K_s) - 1.517$ | $-0.17/-0.09$ |
| $(y - K_s)$ | 2.383 ± 0.129 | $M_{K_s} = -1.626 \pm 0.057$ | $M_{K_s} = -0.003(y - K_s) - 1.625$ | $-0.03/-0.08$ |
| $(W1 - W2)$ | -0.112 ± 0.016 | $M_{W1} = -1.694 \pm 0.061$ | $M_{W1} = 0.612(W1 - W2) - 1.632$ | 0.17/0.13 |
| $(W2 - W3)$ | 0.155 ± 0.043 | $M_{W2} = -1.595 \pm 0.064$ | $M_{W2} = 0.628(W2 - W3) - 1.696$ | 0.25/0.23 |
| $(W1 - W3)$ | 0.043 ± 0.044 | $M_{W3} = -1.752 \pm 0.068$ | $M_{W3} = -0.307(W1 - W3) - 1.741$ | $-0.36/-0.20$ |
| $(H - W2)$ | 0.053 ± 0.025 | $M_{W2} = -1.581 \pm 0.060$ | $M_{W2} = -0.967(H - W2) - 1.533$ | $-0.41/-0.36$ |
where $\tau$ is the age of RC stars in Gyr. Fits using R-rlm and Py-exMix1 are very similar. $M_K = (0.016 \pm 0.002)\tau - 1.715(\pm0.008)$, $M_{W2} = (0.018 \pm 0.002)\tau - 1.683$ with the former method and $M_K = (0.015 \pm 0.003)\tau - 1.714$ and $M_{W2} = (0.017 \pm 0.004)\tau - 1.682$ with the latter. The Pearson's (Spearman) correlation coefficients are of 0.65 (0.62) for $M_K$ and 0.66 (0.63) for $M_{W2}$, indicating strong linear correlations. In Table 3 we report the slope of ages versus magnitudes for all our photometric systems. It has to be kept in mind that certain filters also display color-dependence, and thus the interdependence of ages and colors might not be straightforward to disentangle. In general, we can say that for optical colors there is a slope of $\sim 0.030 \pm 0.003$ mag Gyr$^{-1}$, whereas in the infrared the dependence is $\sim 0.020 \pm 0.003$ mag Gyr$^{-1}$ and it also displays a stronger correlation.

Note that the asteroseismic ages adopted here are obtained assuming no mass-loss. If we were to instead use asteroseismic ages derived assuming a highly efficient mass-loss, the slopes

Figure 5. Color–absolute magnitude diagram of RC stars in various combinations of 2MASS filters. The solid line in the $(J - K_s)$ vs. $M_{K_s}$ diagram is the absolute magnitude of the local RC stars from Laney et al. (2012). The dashed lines indicate $\pm 0.15$ mag with respect to the absolute magnitude of local red clump stars, as explained in the text. Stars beyond the dashed lines are marked by red crosses. Bottom panels: absolute magnitude and color distributions for our sample of stars in the 2MASS system. All colors and magnitudes are corrected for reddening.
above would increase even further. E.g., the slopes of $M_K$ and $M_{W2}$ would be $\sim -0.031 \pm 0.011$ mag Gyr$^{-1}$ and $\sim -0.035 \pm 0.011$ mag Gyr$^{-1}$. We refer to Casagrande et al. (2016) for a discussion of why a negligible mass-loss is favored by current observations.

The dependence of the absolute magnitudes of RC stars on age was theoretically predicted (see Girardi & Salaris 2001, in $I$ band) and was found in open clusters by Grocholski & Sarajedini (2002) using 2MASS photometry. Here, for the first time, we detect such a tiny trend in various optical and infrared bands in field stars thanks to the accuracy of our asteroseismic ages and distances (and hence luminosities).

4. Literature Comparison: using the Distance Scale to Test Asteroseismic Scaling Relations

In the Johnson system, $M_V$ are generally consistent with the results from Bilir et al. (2013a) based on red clump/red horizontal branch stars in globular and open clusters. They also found a weak metallicity dependence of the $M_V$, with a coefficient of 0.046 added to the magnitude versus $(B - V)$ calibration. The weak dependence of $M_V$ on [Fe/H] in our work in the Strömgren system is consistent with the results of Bilir et al. (2013a). The absolute magnitudes of RC stars are calibrated in Bilir et al. (2013b) in terms of colors for $M_K$, $M_J$, $M_K$, and $M_S$. Our color absolute magnitude diagrams are
consistent with those in Bilir et al. (2013b; their Figure 8), although our seismic selection of RC stars results in a smaller scatter. Our mean values of $M_H = -1.016 \pm 0.063$ and $M_T = -1.528 \pm 0.055$ agree within the errors with those of Laney et al. (2012), who gave $M_H = -0.984 \pm 0.014$ and $M_T = -1.490 \pm 0.015$ based on local RC stars.

The comparison with $M_K$ values from the literature indicates an overall agreement, although it deserves a discussion. Our value of $M_K = -1.626 \pm 0.057$ is consistent with that of $-1.613 \pm 0.015$ by Laney et al. (2012) for local RC stars, and that of $-1.61 \pm 0.04$ by Grocholski & Sarajedini (2002) for RC stars in clusters. The values of $-1.57 \pm 0.05$ mag by van Helshoecht & Groenewegen (2007) and of $-1.54 \pm 0.04$ mag by Groenewegen (2008) for local RC stars with Hipparcos parallaxes are fainter than our value. The comparison of our values of $M_{W1} = -1.694 \pm 0.061$ and $M_{W3} = -1.752 \pm 0.068$ with those of $M_{W1} = -1.635 \pm 0.026$ and $M_{W3} = -1.606 \pm 0.024$ in Yaz Gökçe et al. (2013) shows deviations of $-0.06$ mag in $M_{W1}$ and $-0.15$ mag in $M_{W3}$ (ours minus theirs, our absolute magnitudes being brighter). Yaz Gökçe et al. (2013) also provide $M_I$, $M_H$, and $M_K$, and again our absolute magnitudes are brighter by $\sim -0.03$ to $-0.06$ mag. From the above comparisons, we conclude that differences with the Hipparcos literature are generally within the errors, although it is noteworthy that they seem systematic, in the sense that our absolute magnitudes are brighter.

Our absolute magnitudes are based on seismic distances $D$, which are derived from scaling angular diameters obtained via the InfraRed Flux Method to the stellar radii $R$ obtained from scaling relations (Silva Aguirre et al. 2012; Casagrande et al. 2014b). We have carried out an extensive comparison of our angular diameters with interferometric measurements in Casagrande et al. (2014a). If we assume our angular diameter scale to be correct, then any change in stellar radii due to the scaling relation directly translates into a change of distances.
Thus, we can compare the absolute magnitudes of clump stars we derive (i.e., based on distances relying on scaling relations) with those available in the literature and obtained with independent methods. From distance moduli, it follows that for a given photometric system a difference \( \Delta M = M - M_l \) in absolute magnitudes between our calibrations \( (M) \) and those available in the literature \( M_l \) corresponds to a fractional change in distance,

\[ \frac{D}{D_l} = 10^{-0.2\Delta M}, \]  

which can then be used to constrain how much a seismic radii \( R \) should vary to agree with the distance scale in the literature. In this way, we can thus put an upper limit to the precision of scaling relations.

For this purpose, we compare the difference of the absolute magnitudes with those in the literature for \( M_{Ks} \), the filter showing the least dependence on colors, and only mildly affects from reddening. The offsets are \(-0.013\) mag with respect to the absolute magnitude in Laney et al. (2012), \(-0.056\) mag with respect to van Helshoecht & Groenewegen (2007), \(-0.086\) mag with respect to Groenewegen (2008), and \(-0.016\) mag with respect to Grocholski & Sarajedini (2002).

This corresponds to our seismic distances being \( 10^{-0.2\Delta M} \), i.e. 1.006, 1.026, 1.040, and 1.007 larger. If we assume that radii from scaling relations are entirely responsible for the difference, then radii from scaling relations are overestimated by an amount that goes from \( 0.6 \pm 2.7\% \) to \( 4.0 \pm 3.2\% \), depending on the literature calibration we use for comparison. The average offset is \( 2 \pm 2\% \). This comparison indicates a mild tension between our seismic distance scale and that deduced from Hipparcos parallaxes. If confirmed, it would put a limit on the accuracy at which scaling relations are applicable.

**Table 3**

| Band | Slope \( \pm \) Correlation |
|------|----------------------------|
| \( y \) | 0.032 ± 0.003 0.43/0.43 |
| \( V \) | 0.028 ± 0.003 0.40/0.41 |
| \( g \) | 0.036 ± 0.003 0.40/0.42 |
| \( r \) | 0.029 ± 0.003 0.52/0.50 |
| \( i \) | 0.026 ± 0.003 0.55/0.53 |
| \( z \) | 0.025 ± 0.003 0.61/0.60 |
| \( J \) | 0.020 ± 0.003 0.63/0.62 |
| \( H \) | 0.018 ± 0.003 0.71/0.69 |
| \( K_s \) | 0.015 ± 0.003 0.65/0.62 |
| \( W1 \) | 0.016 ± 0.003 0.62/0.59 |
| \( W2 \) | 0.017 ± 0.003 0.66/0.63 |
| \( W3 \) | 0.017 ± 0.004 0.51/0.48 |

**Note.** The last column indicates Pearson and Spearman correlation coefficients.
to red clump stars. However, a few caveats must be remembered. Our comparison assumes no color-dependence (i.e., we only compare mean absolute magnitudes). Also, one may wonder whether the age distribution underlying our sample is different from that of other calibrations. Since all calibrations are based on nearby stars, or open clusters, it is reasonable to assume that the underlying age distributions are comparable. However, calibrations using field stars in the literature have no age information, and likely include many stars younger than 2 Gyr, which instead we have excluded. If we were to include stars of all ages in our calibration, the mean absolute magnitude for our sample would be $M_K = -1.628 \pm 0.130$, i.e., the difference would stay the same, but the scatter would increase significantly.

5. Summary

Based on the reddening and distance estimates for a sample of $\sim 170$ seismically identified red clump stars in Casagrande et al. (2014b), we have investigated color and absolute magnitude distributions of RC stars in Strömgren $BV$, Sloan $griz$, 2MASS $JHK_s$, and WISE $W1/W2/W3$ photometric systems. For the first time, we find a clear trend between absolute magnitudes and ages in field RC stars. The absolute magnitudes of RC stars deviate significantly from a constant value at ages below 2 Gyr, which indicates that RC stars can be reliably used as distance indicators only for populations older than this age. Even so, a statistically significant correlation between absolute magnitudes and ages remains, which, in the worst case, can introduce a bias up to $\sim 0.3$ mag in the optical and $\sim 0.2$ mag in the infrared if ages of clump stars are not known. Our absolute magnitudes for RC stars in the 2MASS and WISE systems are generally consistent within the errors with those obtained from local RC stars with accurate Hipparcos parallaxes. However, possible tension at the level of a few percent is identified. Assuming that seismic scaling relations are responsible for this difference, this would imply that seismic radii for red clump stars are overestimated by $2 \pm 2\%$ when using scaling relations. Our methodology, along with improvements to the calibration of the distance scale with future Gaia data releases, will be able to shed light on this issue.

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