Method for Precision Test of Fine Structure Constant Variation with Optical Frequency References

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A new method for examining the possible space-time variation of the fine structure constant ($\alpha$) is proposed. The technique uses a relatively simple measurement with an optical resonator to compare atom-stabilized optical frequency references. This method does not require that the exact frequency of each reference be measured, and has the potential to yield more than a 1000-fold improvement in experimental sensitivity to changes in $\alpha$. A specific realization of an experiment using this method is discussed which can approach the precision of $\dot{\alpha}/\alpha \sim 10^{-18}/\tau$, where $\tau$ is the measurement time. Moreover, for this specific realization, a measurement of $\dot{\alpha}/\alpha \sim 10^{-15}/\text{yr}$ as a near-term goal is realistic.

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As space-time variation of fundamental constants is implicitly precluded from widely accepted physical theories, a search for such variation is a probe for physics beyond our current understanding. Although a measured variation would not necessarily disprove long-standing theories such as general relativity, quantum electrodynamics and others, it would be evidence of deeper structure in nature. Moreover, the sensitivity of these searches can best the most accurate physical measurements.

The notion that our physical description of nature is incomplete is supported with empirical evidence. Two relevant examples are: 1) the set of constants termed ‘fundamental’ are more numerous than, and hence override the absolute scales of length, mass, electric charge, etc., and 2) we have no verified theory that unifies gravity and the other basic forces.

Numerous theories currently under investigation have been devised to address these issues which either allow or necessitate space-time variation of fundamental constants such as the fine structure constant ($\alpha$). A new method for examining the possible space-time variation of the fine structure constant ($\alpha$) is discussed which can approach the precision of $\dot{\alpha}/\alpha \sim 10^{-18}/\tau$, where $\tau$ is the measurement time. Moreover, for this specific realization, a measurement of $\dot{\alpha}/\alpha \sim 10^{-15}/\text{yr}$ as a near-term goal is realistic.

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A number of workers have experimentally determined limits for the time dependence of the fine structure constant $\dot{\alpha}$ [1-3]. The sources of information used by these workers can be divided into two categories: observational data and laboratory measurements. Currently the most constraining limits for $\dot{\alpha}$ ($\equiv \text{d} \alpha/\text{d}t$) come from observational data. The first result reported in Table I was obtained from analysis of the natural fission reaction that took place in Oklo, Gabon about $2 \cdot 10^9$ years ago. The next result was obtained through analysis of astrophysical data and reports the first observation of a $4 \sigma$ deviation from $\dot{\alpha} = 0$. While this result will undoubtedly receive much scrutiny, it has highlighted the importance of other measurements to achieve the same accuracy.

The observational methods have the potential advantage of time averaging over billions of years when searching for a linear drift in $\alpha$. This advantage combined with the ability to look for both ancient and large-scale spatial variation in $\alpha$ make the observational methods valuable for the future of this research. However, these methods are not without difficulties. Most importantly, the interpretation of these data is model dependent and the data may reflect alterations by unknown mechanisms.

The final two values in Table I are the results of laboratory measurements. Laboratory searches for $\dot{\alpha}$ have several benefits over observational methods: The measurements can be made in a carefully controlled environment, are not model dependent and can be reproduced independently. Laboratory measurements have an additional benefit of being sensitive to both the present-day value of $\alpha$ and oscillations that may leave the integrated value of $\alpha$ unchanged over cosmic time scales.

The two laboratory limits reported in Table I are the results of microwave time standards comparisons. The basis for these comparisons in a search for nonzero $\dot{\alpha}$ is that configuration energies of different atoms depend differently on $\alpha$. For the microwave standards referenced in Table I the change in frequency of the “clock” transition for nonzero $\dot{\alpha}$ scales to lowest order in $\alpha$ as $(\alpha Z)^x$, where $Z$ is the atomic number and $x \sim 2$.

In general, comparison of two atom-stabilized time standards can yield a limit on the fractional change in $\alpha$ during the measurement time that is on the order of, but not better than, the fractional stability of the least precise of the standards being compared [4,5,6,7]. Table I illustrates the current status of various time standards in existence today. For these standards, the current limit for $\dot{\alpha}/\alpha$ could see an improvement of an order of magnitude in the foreseeable future. However, it is likely that further improvements on this limit will be constrained by magnetic field shifts or collisional shifts.

It has long been understood that certain narrow atomic transitions whose frequencies are in the optical band could be used to create frequency references, and possibly time standards, with far improved stability over the
current state-of-the-art \[20\]. The most significant obstacle in performing an experiment to compare frequency references in which one of the references is optical, or to use an optical frequency reference as a time standard, is the optical frequency metrology. Yet with significant effort, some workers have discovered clever methods to relate optical frequencies to directly measurable microwave standards \[21\], such as one of the microwave standards shown in Table \(1\). Not only are these measurements difficult, but the accuracy of the optical frequency metrology depends on the stability of the microwave standard. If the intrinsic stability of the optical reference is better than the microwave standard, this comparison adds noise into the measurement and reduces the available information about the optical frequency reference. Using this method to compare two or more such references would yield far less information than could be gained in principle.

It is therefore significant that the method proposed here makes comparisons only between optical frequency references without the potentially noisy intermediate step of comparing each to a microwave standard. The necessary comparison can be made directly between the references with a simultaneously-resonant optical cavity, or optical comparison resonator (OCR).

To illustrate, consider narrow-band sources of coherent optical radiation with frequencies \(\nu_1\) and \(\nu_2\) that are referenced to metastable transitions of suitable atoms or ions. A plot of the transmitted intensity of an optical transition in \(\nu_0\) as a function of resonator length would resemble Fig. \(\text{1}\) where the maximum separation (in Hz) between the resonance peaks of the two fields is one-half of the free-spectral-range \((v_{fsr} = c/L, \text{where } c \text{ is the speed of light and } L \text{ is the round-trip length of the resonator})\). Obviously, if the length of the resonator is stabilized by maintaining a resonance with one of the optical fields, the other field can be shifted into resonance simultaneously with this OCR with an acousto-optic modulator (AOM) or similar device where the frequency of the RF oscillator driving the AOM is less than \(\nu_{fsr}/2\). Information about changes in the relative frequencies of the atom-stabilized references can be gathered by monitoring the frequency of the RF signal.

This is important because it is sufficient to measure changes in the difference frequencies of relatively stable atomic references to measure changes in \(\alpha\). Moreover, an experiment to measure changes in \(\alpha\) with atom-stabilized frequency references does not require that the frequencies of the references be known exactly, or even that their difference frequencies be known exactly.

Calculations were made recently, with a combination of the relativistic Hartree-Fock model and many-body perturbation theory, of the relativistic energy level corrections as an expansion in powers of \(\alpha\) for In\(^+\) and Tl\(^+\) \[22\]. The results can be expressed in the form \(\dot{\omega} = \beta(\dot{\alpha}/\alpha)\). For the \(^1S_0 \leftrightarrow ^3P_0\) optical transition in In\(^+\), \(\beta = 2.6 \times 10^{14}\) Hz. While for the same transition in Tl\(^+\), \(\beta = 12 \times 10^{14}\) Hz. The large difference between In\(^+\) and Tl\(^+\) is due to the increased magnitude of the relativistic corrections for larger atomic number. The scaling rule for this effect is not simple but the magnitude of the corrections is expected to increase as \((\alpha Z)^2\). With \(Z = 49\) and 81 for In and Tl respectively, the value 3.0 for \(x\) agrees quite well with the \(\beta\) values calculated from \[23\].

Assuming the In\(^+\) and Tl\(^+\) references possess the same stability, the accuracy to which \(\dot{\alpha}/\alpha\) can be determined is 3.0 \(\sigma\) where \(\sigma\) is the fractional uncertainty of the In\(^+\) and Tl\(^+\) frequency references. As noted below, \(\sigma\) decreases as \(t^{-3/2}\). Because, to first order, a nonzero \(\dot{\alpha}\) would change linearly with time, the sensitivity of the proposed experiment to \(\dot{\alpha}\) improves as \(\approx t^{-3/2}\). At such time that the precision of the frequency references becomes limited by white noise, the sensitivity improves as \(\approx t^{-1}\).

A trapped ion time or frequency reference can offer significant improvements in stability over neutral atom references. Evidence of this appears in Table \(\text{II}\) which shows that the Hg\(^+\) reference stability is limited by magnetic field fluctuations and not by uncontrollable collisional effects. Table \(\text{II}\) presents several virtues of a trapped ion standard. For In\(^+\) in a harmonic trap with an easily obtainable frequency \(\omega_i/2\pi = 1\) MHz \[23,24\], the extent of the motional state of the ion can be described by \(\langle \dot{z}^2 \rangle^{1/2} \approx 7\) nm \[23\]. The transition of interest \(^1S_0 \leftrightarrow ^3P_0\) (Fig. \(\text{2}\)) has \(\lambda/2\pi \approx 40\) nm so that the first-order Doppler shift is clearly eliminated. Similarly, the second-order Doppler shift for In\(^+\) can be \(\Delta \omega_D/\omega_0 < 10^{-19}\). Collisional shifts can be reduced to \(\Delta \omega_C/\omega_0 < 10^{-19}\) with readily available vacuum technology.

Field shifts for In\(^+\) and Tl\(^+\) can be reduced below \(10^{-18}\) fractional frequency using standard techniques \[20,26\]. Table \(\text{IV}\) shows the field shifts of In\(^+\); values for Tl\(^+\) are similar. Magnetic fields can be reduced in a small experimental apparatus to the level of micro-Gauss with passive shielding, and the Stark shifts of the cooled indium ion in the previously described trap are \(\sim 10^{-20}\), \(\omega_i\).

Moreover, optical atomic references offer a significant advantage over microwave references in short-time stability. For an atomic reference limited by quantum statistics and using the Ramsey’s method of separated oscillatory fields \[21\], the two-sample Allan variance takes the form \(\sigma = 1/\omega_0 \sqrt{N T_R t}\) \[25\], where \(\omega_0\) is the center frequency of the reference, \(N\) is the number of (uncorrelated) atoms contributing to the signal, and \(T_R\) is the temporal extent of a single measurement, and \(t\) is the total integration time. Optical references offer an improvement over microwave references by \(\sim 10^3\) in \(\omega_0\).

There are several issues to be considered regarding the OCR for the sensitive measurement proposed in this manuscript. If a resolution of better than 1 Hz can be obtained, the OCR will not limit the accuracy to which \(\dot{\alpha}\) can be determined with this method. This criterion
suggested that the resonator linewidth should be under 10 kHz. Also, to guarantee a simultaneous resonance with the use of an AOM, the free-spectral-range of the OCR should be less than twice the AOM tuning range of a few hundred MHz. These criteria can be met for a multiply-resonant cavity with currently available optics technology.

There are shifts and broadening mechanisms other than those discussed above that can lead to systematic errors in this type of measurement. For example, Doppler shifts between the ions and the OCR could cause an anomalously positive result. These shifts arise largely from acoustical noise which exists mainly below 1 kHz. A first-order Doppler shift caused by a 30 Hz vibration with an amplitude of 20 nm will shift the OCR line center by about 1 Hz. This is small compared with the expected $\sim 10^3 - 10^4$ Hz OCR spectral width and the long-time effect of this shift is similar to a small broadening of the OCR linewidth. It does, however, obscure any information about frequency difference changes for times on the order of the inverse vibrational frequency. At a similar noise level, the second-order Doppler shift is negligible: More specifically, the amplitude of 30 Hz vibrations would have to be nearly a meter to effect a similar shift.

A change in the pressure will change the index of refraction of the air between the OCR mirrors and will alter the resonance condition. The low pressure limit of the index of refraction of standard air can be used to estimate this effect. The result is $\Delta n = 4 \cdot 10^{-7} \Delta p/\text{torr}$. Pressure changes of $10^{-8}$ torr will shift the resonance by about 1 Hz. This effect can be reduced to $< 10^{-3}$ Hz by placing the resonator in a vacuum of $10^{-11}$ torr. At this pressure, $\Delta n = -1 \cdot 10^{-19} \Delta T/K$ where $\Delta T$ is the temperature change.

Unless the intracavity power of the OCR is stabilized, the mirror coatings may heat and cool causing additional broadening of the resonance. This effect can be mitigated by using low power and stabilizing the OCR throughput.

A cesium reference or similar atom-stabilized oscillator will be needed to synthesize the RF drive for the frequency shifters used to achieve the simultaneous resonances in the OCR. With a fractional stability of better than $10^{-12}$ and a synthesized frequency of $10^8$ Hz, the uncertainty caused by this type of oscillator in this measurement is less than $10^{-19}$. Also, the frequency of any atom-stabilized reference will change for a nonzero $\dot{\alpha}$. This effect will not reduce the sensitivity of the proposed measurement as any such shift will be reduced by the factor of $10^5$ frequency ratio of the optical and microwave atom-stabilized oscillators.

A linear Paul trap is a convenient device to trap the indium and thallium ions simultaneously. There are several advantages to this approach: The first advantage is that the indium cooling and interrogation transition is narrow enough to allow cooling directly to the motional ground state. As the ions’ motion is coupled through their mutual repulsion, the indium ion can be used to sympathetically cool an In$^+$-Tl$^+$ two-ion crystal to the motional ground state. The similar transition in thallium is too broad to cool to this final motional state independently. Secondly, the ion separation will be about 20 $\mu$m ensuring that both ions are exposed to the same low frequency electric and magnetic fields.

A potential limitation of this technique is that neither ion will be at the zero-field point along the trap’s axis so that both ions will be exposed to a small RF electric field that can cause a quadratic Stark shift of the metastable transition. The most likely cause for this unfavorable transition is small misalignments of the trap electrodes. The magnitude of the electric field to which they could be exposed can be approximated by $E_d \approx (V_0/r_0) \cdot (d/r_0)$ where $d$ is the distance of the ions from the ideal trap center. For typical parameters, $E_d \ll 1 \text{ V/cm}$ and the fractional shift magnitude for each ion is $< 10^{-18}$ (with the same sign). Moreover, micromotion of this magnitude is easily detectable.

Due to the similarity of the $^1S_0 \leftrightarrow ^3P_0$ transition frequencies of In$^+$ and Tl$^+$, an order of magnitude of insensitivity to Doppler broadening, pressure shifts and mirror heating effects is gained over the limits discussed above. Additional benefits for comparing two IIIA ions are gained due to the similarity of the electronic configurations. For example, frequency difference measurements between In$^+$ and Tl$^+$ are less sensitive to some field shifts.

In conclusion, a test of possible space-time variation of the fine structure constant based on comparisons of optical frequency references is proposed. The required measurement of a change in the relative frequencies of the atom-stabilized references can be performed by employing a multiply-resonant optical cavity. Moreover, exact measurements of the reference frequencies are not necessary for this experiment which greatly reduces the complexity of this experiment from that of atomic clock comparisons proposed for the same purpose. The choice of In$^+$ and Tl$^+$ ions simultaneously confined in a linear trap allows the In$^+$-Tl$^+$ crystal to be cooled to the motional ground state with a single laser. This configuration does not limit the ultimate relative stability of corresponding frequency references. By using two group IIIA ions, a limit of $\dot{\alpha}/\alpha \sim 10^{-18}/\tau$ can be approached, where $\tau$ is the time over which the frequency references are compared. A measurement of $\dot{\alpha}/\alpha \sim 10^{-15}/\text{yr}$ as a near-term goal is realistic and would provide the finest laboratory resolution for $\dot{\alpha}$.

Note that a future refinement to this experiment could include other simultaneously trapped ions and comparisons of the resulting optical frequency references. The different dependencies on $\alpha$ would allow weaker requirements for measuring and eliminating systematic effects and would be especially important for quantifying any nonzero $\dot{\alpha}$, should one exist.
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Zeeman shift:
\[ \Delta \omega_{\text{Zeeman}} = 0.24 \frac{\text{mHz}}{\mu\text{G}} \]
\[ \Delta \omega_{\text{Zeeman}} / \omega_0 = 3 \cdot 10^{-20} H [\mu\text{G}] \]

Quadratic Stark shift:
\[ \Delta \omega_{\text{Stark}} \sim \frac{\text{mHz}}{(V/\text{cm})^2} \]
\[ \Delta \omega_{\text{Stark}} / \omega_0 \sim 10^{-21} T [\mu\text{K}] \]

Electric quadrupole shift:
\[ \sim 0 \text{ for } ^1S_0 \leftrightarrow ^3P_0 \]
\(10^6 \text{ less than } ^1S_0 \leftrightarrow ^3P_1\)

| TABLE IV. Field shifts of interest for an In\(^+\) reference. |