Flavour symmetry breaking, baryons magnetic moments, and low energy phenomenology of hadrons

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Abstract

A priori mixings of eigenstates in physical states are quantum mechanical effects well known in several realms of physics. The possibility that such effects are also present in particle physics, in the form of mixings that break flavor and parity symmetries, is studied. Applications to non-leptonic and weak radiative decays of hyperons are discussed. The results are very encouraging but should be improved by eventually including the $W$ and $Z$ contributions, assumed small (non-enhanced) in this work.

I. INTRODUCTION

Because parity and strong flavors (strangeness, charm, etc.) symmetries are violated in nature, the physical (mass eigenstates) hadrons cannot be either parity or flavor eigenstates, i.e., the former must be admixtures of the latter. It is generally believed that the breaking of flavor global groups is caused by the mass differences of hadrons, but in such a way that parity and all flavors are conserved, i.e., the mass operator of hadrons giving rise to such breakings does not contain a piece that violates parity and flavor. The flavor and parity mixings in physical hadrons are attributed to the perturbative intervention of $W^\pm_\mu$ and $Z^0_\mu$ (parity mixing only). Precisely because such intervention is perturbative, such mixings can appear only in higher orders of perturbation theory; thus, such mixings appear, so to speak, a posteriori.

However, the possibility that the mass operator of hadrons does contain a (necessarily) very small piece that is flavor and parity violating is not excluded by any fundamental principle. If such a piece does exist, then, the parity and flavor admixtures in hadrons must come a priori, in a non-perturbative way. It is not idle to emphasize that such a piece could not be attributed to the $W^\pm_\mu$ and $Z^0_\mu$. 
Our purpose in this work is to apply the a priori mixings scheme to non-leptonic and weak radiative decays of hyperons.

II. A PRIORI MIXED HADRONS

The implementation of a priori mixings for practical applications cannot, as of today, be achieved from first principles, i.e., by starting from a model at the quark level and then performing the QCD calculations to obtain the physical hadrons and their couplings. In order to proceed we must elaborate an ansatz. We refer the reader to Ref. [1] for a complete and detailed description of this ansatz. Here, we only reproduce the expressions for the a priori mixed hadrons obtained with this ansatz (we shall restrict what follows to spin-1/2 baryons and spin-0 mesons):

\[
p_{ph} = p_s - \sigma \Sigma^+_s - \delta \Sigma^+_p + \cdots, \quad \Sigma^+_ph = \Sigma^+_s + \sigma p_s - \delta' p_p + \cdots
\]

\[
\Sigma^-_{ph} = \Sigma^-_s + \sigma \Xi^-_s + \delta \Xi^-_p + \cdots, \quad \Xi^-_{ph} = \Xi^-_s - \sigma \Sigma^-_s + \delta' \Sigma^-_p + \cdots
\]

\[
n_{ph} = n_s + \sigma\left(\frac{1}{\sqrt{2}} \Sigma^0_s + \sqrt{3} \Lambda_s\right) + \delta\left(\frac{1}{\sqrt{2}} \Sigma^0_p + \sqrt{3} \Lambda_p\right) + \cdots
\]

\[
\Lambda_{ph} = \Lambda_s + \sigma\sqrt{\frac{3}{2}}(\Xi^0_s - n_s) + \delta\sqrt{\frac{3}{2}}\Xi^0_p + \delta'\sqrt{\frac{3}{2}}n_p + \cdots
\]

\[
\Sigma^0_{ph} = \Sigma^0_s + \sigma\frac{1}{\sqrt{2}}(\Xi^0_s - n_s) + \delta\frac{1}{\sqrt{2}}\Xi^0_p + \delta'\frac{1}{\sqrt{2}}n_p + \cdots
\]

\[
\Xi^0_{ph} = \Xi^0_s - \sigma\left(\frac{1}{\sqrt{2}} \Sigma^0_s + \sqrt{3} \Lambda_s\right) + \delta\left(\frac{1}{\sqrt{2}} \Sigma^0_p + \sqrt{3} \Lambda_p\right) + \cdots
\]

\[
K^+_ph = K^+_p - \sigma \pi^+_p - \delta' \pi^+_s + \cdots, \quad K^0_{ph} = K^0_p + \sigma \frac{1}{\sqrt{2}} \pi^0_p + \delta' \frac{1}{\sqrt{2}} \pi^0_s + \cdots
\]

\[
\pi^+_ph = \pi^+_p + \sigma K^+_p - \delta K^+_s + \cdots
\]

\[
\pi^0_{ph} = \pi^0_p - \sigma \frac{1}{\sqrt{2}}(K^0_p + \bar{K}^0_p) + \delta \frac{1}{\sqrt{2}}(K^0_s - \bar{K}^0_s) + \cdots
\]

\[
\pi^-_{ph} = \pi^-_p + \sigma K^-_p + \delta K^-_s + \cdots
\]

\[
\bar{K}^0_{ph} = \bar{K}^0_p + \sigma \frac{1}{\sqrt{2}} \pi^0_p - \delta' \frac{1}{\sqrt{2}} \pi^0_s + \cdots, \quad K^-_{ph} = K^-_p - \sigma \pi^-_p + \delta' \pi^-_s + \cdots
\]

(1)

In these expressions the subindices \(s\), and \(p\) indicate positive, and negative parity eigenstates and each physical hadron is the mass eigenstate already observed. We must point out that the previous mixings have a parallelism at the quark level so that they should be necessary to develop a formulation at that level. This particular matter will not be tried here [2].
III. APPLICATION TO NON-LEPTONIC DECAYS

If strong-flavor and parity violating pieces in the mass operator of hadrons exist they would lead to non-perturbative a priori mixings of flavor and parity eigenstates in physical (mass eigenstates) hadrons. Then, two paths for weak decays of hadrons to occur would lead to non-perturbative a priori mixings of flavor and parity eigenstates in physical hadrons. The enhancement phenomenon observed in non-leptonic decays of hyperons (NLDH) could then be attributed to this new mechanism. However, for this to be the case it will be necessary that a priori mixings produce the well established predictions of the $|\Delta I| = 1/2$ rule [34].

The a priori mixed hadrons will lead to NLDH via the parity and flavor conserving strong interaction (Yukawa) hamiltonian $H_Y$. The transition amplitudes will be given by the matrix elements $\langle B_{ph} M_{ph} | H_Y | A_{ph} \rangle$, where $A_{ph}$ and $B_{ph}$ are the initial and final hyperons and $M_{ph}$ is the emitted meson. Using the above mixings, Eqs. (I), these amplitudes will have the form $\tilde{u}_B (A - B \gamma_5) u_A$, where $u_A$ and $u_B$ are four-component Dirac spinors and the amplitudes $A$ and $B$ correspond to the parity violating and the parity conserving amplitudes of the $W_{\mu}^\pm$ mediated NLDH, although with a priori mixings these amplitudes are both actually parity and flavor conserving. As a first approximation we shall neglect isospin violations, i.e., we shall assume that $H_Y$ is an $SU_3$ scalar. However, we shall not neglect $SU_3$ breaking. One obtains for $A$ and $B$ the results:

$$A_1 = \delta' \sqrt{3} g_{p,p\pi^0}^{p,sp} + \delta(g_{s,s}^{s,s} - g_{s,p}^{s,pp})_{\Lambda,\pi^0 K^+}, \quad A_2 = -\frac{1}{\sqrt{2}}[\delta' \sqrt{3} g_{p,p\pi^0}^{p,sp} + \delta(g_{s,s}^{s,s} - g_{s,p}^{s,pp})_{\Lambda,\Sigma^+ \pi^-}],$$

$$A_3 = \delta(\sqrt{2} g^{s,sp}_{\xi^0,\pi^0} + \sqrt{3} g^{s,pp}_{\xi^0,\pi^0} + \frac{1}{\sqrt{2}} g^{s,pp}_{\Sigma^+,\Lambda^+ \pi^-}),$$

$$A_4 = -\delta' \sqrt{2} g_{p,p\pi^0}^{p,sp} + \delta(\sqrt{3} g_{\Sigma^+,\Lambda^+ \pi^-}^{s,pp} - \frac{1}{\sqrt{2}} g_{\Sigma^+,\Lambda^+ \pi^-}^{s,pp}),$$

$$A_5 = -\delta' g_{p,p\pi^0}^{p,sp} - \delta(g_{s,s}^{s,s} + g_{s,s}^{s,pp})_{\Sigma^0,\pi^0 K^-}, \quad A_6 = \delta' g_{\Sigma^+,\Lambda^+ \pi^-}^{p,sp} + \delta(g_{\Xi^-,\Lambda^- \pi^-}^{s,sp} + \sqrt{3} g_{\Xi^0,\Xi^0 \pi^0}^{s,pp}),$$

$$A_7 = \frac{1}{\sqrt{2}}\left[\delta' g_{\Sigma^+,\Lambda^+ \pi^-}^{p,sp} + \delta(g_{\Xi^-,\Lambda^- \pi^-}^{s,sp} + \sqrt{3} g_{\Xi^0,\Xi^0 \pi^0}^{s,pp})\right].$$

and

$$B_1 = \sigma(\sqrt{3} g_{p,p\pi^0}^{p,sp} + g_{\Lambda,\pi^0 K^-} - g_{\Lambda,\Sigma^+ \pi^-}), \quad B_2 = -\frac{1}{\sqrt{2}}\sigma(\sqrt{3} g_{p,p\pi^0}^{p,sp} + g_{\Lambda,\pi^0 K^-} - g_{\Lambda,\Sigma^+ \pi^-}),$$

$$B_3 = \sigma(\sqrt{2} g_{\Sigma^0,\pi^0 K^-}^{p,sp} + \sqrt{3} g_{\Xi^0,\Sigma^+ \pi^-}^{s,pp} + \frac{1}{\sqrt{2}} g_{\Sigma^+,\Lambda^+ \pi^-}^{s,pp}).$$
\[ B_4 = \sigma(\sqrt{2}g_{p,p\pi^0} + \sqrt{3}g_{\Sigma^+,\Lambda\pi^+} - \frac{1}{\sqrt{2}}g_{\Sigma^+,\Sigma^+\pi^0}), \]

\[ B_5 = \sigma(g_{p,p\pi^0} - g_{\Sigma^0,pK^-} - g_{\Sigma^+,\Sigma^+\pi^0}), \quad B_6 = \sigma(-g_{\Sigma^+,\Lambda\pi^+} + g_{\Sigma^-,\Lambda K^-} + \sqrt{3}g_{\Xi^0,\Xi^0\pi^0}), \]

\[ B_7 = \frac{1}{\sqrt{2}}\sigma(-g_{\Sigma^+,\Lambda\pi^+} + g_{\Sigma^-,\Lambda K^-} + \sqrt{3}g_{\Xi^0,\Xi^0\pi^0}). \]  

The subindices 1, \ldots, 7 correspond to \( \Lambda \rightarrow p\pi^-, \Lambda \rightarrow n\pi^0, \Sigma^+ \rightarrow n\pi^-, \Sigma^+ \rightarrow n\pi^+, \Sigma^+ \rightarrow p\pi^0, \Xi^+ \rightarrow \Lambda\pi^-, \) and \( \Xi^0 \rightarrow \Lambda\pi^0, \) respectively. The \( g \)-constants in these equations are Yukawa coupling constants (YCC) defined by the matrix elements of \( H_Y \) between flavor and parity eigenstates, for example, by \( \langle B_{0s}M_0|H_Y|A_{0p} \rangle = g_{A,BM}^{p,s}. \) We have omitted the upper indices in the \( g \)'s of the \( B \) amplitudes because the states involved carry the normal intrinsic parities of hadrons. In Eqs. (3) we have used the \( SU_2 \) relations \( g_{p,p\pi^0} = -g_{n,n\pi^0} = g_{p,n\pi^+}/\sqrt{2} = g_{n,p\pi^-}/\sqrt{2}, \ g_{\Sigma^+,\Lambda\pi^+} = g_{\Sigma^0,\Lambda\pi^0} = g_{\Sigma^-,\Lambda\pi^0}, \ g_{\Lambda,\Sigma^0\pi^0} = -g_{\Sigma^+,\Sigma^0\pi^0} = g_{\Sigma^-,\Sigma^0\pi^0}, \]

\[ g_{\Sigma^0,pK^-} = g_{\Sigma^-,nK^-}/\sqrt{2} = g_{\Sigma^+,\Lambda\pi^+} = g_{\Lambda,\Sigma^0\pi^0} = g_{\Sigma^-,\Sigma^0\pi^0} = g_{\Sigma^0,\Xi^0\pi^0}/\sqrt{2}, \ g_{\Lambda,\Sigma^0\pi^0} = g_{\Xi^0,\Xi^0\pi^0}/\sqrt{2}, \ g_{\Xi^0,\Xi^0\pi^0} = g_{\Xi^-,\Xi^-\pi^0}/\sqrt{2}, \ g_{\Xi^-,\Xi^-\pi^0} = g_{\Xi^0,\Xi^0\pi^0}/\sqrt{2}, \ g_{\Xi^-,\Xi^-\pi^0} = g_{\Xi^0,\Xi^0\pi^0}/\sqrt{2}, \ g_{\Xi^-,\Xi^-\pi^0} = g_{\Xi^0,\Xi^0\pi^0}/\sqrt{2}, \ g_{\Xi^-,\Xi^-\pi^0} = g_{\Xi^0,\Xi^0\pi^0}/\sqrt{2}, \ g_{\Xi^-,\Xi^-\pi^0} = g_{\Xi^0,\Xi^0\pi^0}/\sqrt{2}, \ g_{\Xi^-,\Xi^-\pi^0} = g_{\Xi^0,\Xi^0\pi^0}/\sqrt{2}, \]

Similar relations are valid within each set of upper indices, e.g., \( g_{p,p\pi^0} = -g_{n,n\pi^0}, \) etc.; the reason for this is that mirror hadrons may be expected to have the same strong-flavor assignments as ordinary hadrons. Thus, for example, \( \pi^+_p, \pi^0_p, \) and \( \pi^-_p \) form an isospin triplet, although a different one from the ordinary \( \pi^+_p, \pi^0_p, \) and \( \pi^-_p \) isospin triplet. These latter relations have been used in Eqs. (2).

From the above results one readily obtains the equalities:

\[ A_2 = -\frac{1}{\sqrt{2}}A_1, \quad A_5 = \frac{1}{\sqrt{2}}(A_4 - A_3), \quad A_7 = \frac{1}{\sqrt{2}}A_6, \]  

\[ B_2 = -\frac{1}{\sqrt{2}}B_1, \quad B_5 = \frac{1}{\sqrt{2}}(B_4 - B_3), \quad B_7 = \frac{1}{\sqrt{2}}B_6. \]

These are the predictions of the \( |\Delta I| = 1/2 \) rule. That is, a priori mixings in hadrons as introduced above lead to the predictions of the \( |\Delta I| = 1/2 \) rule, but notice that they do not lead to the \( |\Delta I| = 1/2 \) rule itself. This rule originally refers to the isospin covariance properties of the effective non-leptonic interaction hamiltonian to be sandwiched between strong-flavor and parity eigenstates. The \( I = 1/2 \) part of this hamiltonian is enhanced over the \( I = 3/2 \) part. In contrast, in the case of a priori mixings \( H_Y \) has been assumed to be isospin invariant, i.e., in this case the rule should be called a \( \Delta I = 0 \) rule.

It must be stressed that the results (3) and (5) are very general: (i) the predictions of the \( |\Delta I| = 1/2 \) rule are obtained simultaneously for the \( A \) and \( B \) amplitudes, (ii) they are independent of the mixing angles \( \sigma, \delta, \) and \( \delta' \), and (iii) they are also independent of particular values of the YCC. They will be violated by isospin breaking corrections. So, they should be quite accurate, as is experimentally the case.

A detailed comparison with all the experimental data available in these decays requires more space and is presented separately [3]. Nevertheless, we shall briefly mention a few very important results.
First, the experimental $B$ amplitudes (displayed in Table I) are reproduced within a few percent by accepting that the YCC are given by the ones observed in strong interactions \( \bar{B} \), an assumption which cannot be avoided in this approach. The best predictions for these amplitudes are \( B_1 = 22.11 \times 10^{-7}, B_2 = -15.63 \times 10^{-7}, B_3 = 1.39 \times 10^{-7}, B_4 = -42.03 \times 10^{-7}, B_5 = -30.67 \times 10^{-7}, B_6 = 17.45 \times 10^{-7}, \) and \( B_7 = 12.34 \times 10^{-7} \). The only unknown parameter \( \sigma \) is determined at \((3.9 \pm 1.3) \times 10^{-6}\). We quote the experimental values of the $B$ amplitudes in the natural scale of \( 10^{-7} \), see Ref. [4]. Their signs are free to choose; actually, the comparison with theoretical predictions is only meaningful for their magnitudes. The signs we display are for convenience only. This is not the case for the signs in the $A$ amplitudes.

Second, although the $A$ amplitudes involve new YCC, an important prediction is already made in Eqs. (2). Once the signs of the $B$ amplitudes are fixed, one is free to fix the signs of four $A$ amplitudes — say, $A_1 > 0, A_3 < 0, A_4 < 0, A_6 < 0$ — to match the signs of the corresponding experimental $\alpha$ asymmetries, namely, $\alpha_1 > 0, \alpha_3 < 0, \alpha_4 > 0, \alpha_6 < 0$. Then the signs of $A_2 < 0, A_5 > 0, \text{and } A_7 < 0$ are fixed by Eqs. (2) and the fact that $|A_2| \ll |A_3|$. In turn the signs of the corresponding $\alpha$’s are fixed. These three signs agree with the experimentally observed ones, namely, $\alpha_2 > 0, \alpha_5 < 0, \alpha_7 < 0$.

A detailed comparison of the $A$ amplitudes with experiment is limited by our current inability to compute well with QCD. However, one may try simple and argumentable new assumptions to make predictions for such amplitudes. Since QCD has been assumed to be common to both ordinary and mirror quarks, it is not unreasonable to expect that the magnitudes of the YCC in the $A$ amplitudes have the same magnitudes as their corresponding counterparts in the ordinary YCC of the $B$ amplitudes. The relative signs may differ, however. Introducing this assumption we obtain the predictions for the $A$ amplitudes displayed in Table II. The predictions for the $B$ amplitudes must also be redone, because determining the $A$ amplitudes alone may introduce small variations in the YCC that affect importantly the $B$ amplitudes, i.e., both the $A$ and $B$ amplitudes must be simultaneously determined, the $B$’s act then as extra constraints on the determination of the $A$’s. The new predictions for the $B$’s are also displayed in Table II. In obtaining Table II we have actually used the experimental decay rates $\Gamma$ and $\alpha$ and $\gamma$ asymmetries [4], but we only display the experimental and theoretical amplitudes.

The predictions for the $A$’s agree very well with experiment to within a few percent, while the predictions for the $B$’s remain as before. The a priori mixing angles are determined to be $|\delta| = (0.23 \pm 0.07) \times 10^{-6}, |\delta'| = (0.26 \pm 0.07) \times 10^{-6}$, and $\sigma = (4.9 \pm 1.5) \times 10^{-6}$. This last value of $\sigma$ is consistent with the previous one. The overall sign of the new YCC can be reversed and the new overall sign can be absorbed into $\delta$ and $\delta'$. This can be done partially in the group of such constants that accompanies $\delta$ or in the group that accompanies $\delta'$ or in both. Because of this, we have determined only the absolute values of $\delta$ and $\delta'$. In order to emphasize this fact we have inserted absolute value bars on $\delta$ and $\delta'$. The more detailed analysis of the comparison of the $A$’s and $B$’s with experiment is presented in Ref. [3] and it indicates that violations of the $|\Delta I| = 1/2$ rule affect the values of the a priori mixing angles and one should take more conservative ones as their estimates in NLDH, namely, $\sigma = (4.9 \pm 2.0) \times 10^{-6}, |\delta| = (0.22 \pm 0.09) \times 10^{-6}$, and $|\delta'| = (0.26 \pm 0.09) \times 10^{-6}$.

The above results, especially those of Eqs. (3) and (4) and the determination of the amplitudes, satisfy some of the most important requirements that a priori mixings must meet in order to be taken seriously as an alternative to describe the enhancement phenomenon.
In these amplitudes only contributions to first order in \( \sigma \) produce weak radiative decays via the ordinary electromagnetic interaction hamiltonian \( H_{\text{int}}^\text{em} = eJ^\mu_{\text{em}} A_\mu \), where \( J^\mu_{\text{em}} \) is the familiar e.m. current operator which is a flavor conserving Lorentz proper four-vector. That is, a priori mixings in baryons lead to weak radiative decays that in reality are ordinary parity and flavor conserving radiative decays, whose transition amplitudes are non-zero only because physical baryons are not flavor and parity eigenstates.

The radiative decay amplitudes we want are given by the usual matrix elements \( \langle \gamma, B_{\text{ph}} | H_{\text{int}}^\text{em} | A_{\text{ph}} \rangle \), where \( A_{\text{ph}} \) and \( B_{\text{ph}} \) stand for hyperons. A very simple calculation leads to the following hadronic matrix elements

\[
\langle p_{\text{ph}} | J^\mu_{\text{em}} | \Sigma^0_{\text{ph}} \rangle = \bar{u}_p \left[ \sigma \left( f_{22}^+ - f_{22}^- \right) + (\delta f_{22}^+ - \delta f_{22}^-) \gamma^5 \right] i\sigma^{\mu\nu} q_\nu u_{\Sigma^+} \\
\langle \Sigma^0_{\text{ph}} | J^\mu_{\text{em}} | \Xi^0_{\text{ph}} \rangle = \bar{u}_{\Sigma^-} \left[ \sigma \left( f_{22}^- - f_{22}^+ \right) + (\delta f_{22}^- - \delta f_{22}^+) \gamma^5 \right] i\sigma^{\mu\nu} q_\nu u_{\Xi^-} \\
\langle n_{\text{ph}} | J^\mu_{\text{em}} | \Lambda_{\text{ph}} \rangle = \bar{u}_n \left[ \sigma \left( \frac{3}{2} f_{22}^f - f_{22}^\Lambda \right) + \frac{1}{\sqrt{2}} f_{22}^{\Sigma^0} \right] \\
\frac{1}{\sqrt{2}} \left( (\delta f_{22}^f - \delta f_{22}^\Lambda) - \delta \frac{1}{\sqrt{2}} f_{22}^{\Sigma^0} \right) \gamma^5 \right] i\sigma^{\mu\nu} q_\nu u_{\Lambda} \\
\langle \Lambda_{\text{ph}} | J^\mu_{\text{em}} | \Xi^0_{\text{ph}} \rangle = \bar{u}_\Lambda \left[ \sigma \left( \frac{3}{2} f_{22}^- f_{22}^f - f_{22}^\Lambda \right) - \frac{1}{\sqrt{2}} f_{22}^{\Sigma^0} \right] \\
\frac{1}{\sqrt{2}} \left( (\delta f_{22}^- - \delta f_{22}^f) + \delta \frac{1}{\sqrt{2}} f_{22}^{\Sigma^0} \right) \gamma^5 \right] i\sigma^{\mu\nu} q_\nu u_{\Xi^0} \\
\langle \Sigma^0_{\text{ph}} | J^\mu_{\text{em}} | \Xi^0_{\text{ph}} \rangle = \bar{u}_{\Xi^0} \left[ \sigma \left( \frac{1}{\sqrt{2}} f_{22}^f f_{22}^f - f_{22}^{\Sigma^0} \right) - \frac{3}{\sqrt{2}} f_{22}^{\Sigma^0} \right] \\
\frac{1}{\sqrt{2}} \left( (\delta f_{22}^f - \delta f_{22}^{\Sigma^0}) + \delta \frac{1}{\sqrt{2}} f_{22}^{\Sigma^0} \right) \gamma^5 \right] i\sigma^{\mu\nu} q_\nu u_{\Xi^0} (6)
\]

In these amplitudes only contributions to first order in \( \sigma \), \( \delta \), and \( \delta' \) need be kept. Each matrix element is flavor and parity conserving and can be expanded in terms of charge \( f_1(0) \) form factors and anomalous magnetic \( f_2(0) \) form factors. Because the charges of the positive- and negative-parity parts of the same physical wave function are equal and such charges are controlled by the generator property of \( J_\mu \) all the \( f_1 \)'s cancel away and only the
$f_2$ contribute. The $f_2$ between $s$ and $p$ parts and between $p$ and $s$ parts can be identified with the $f_2$ between $s$ and $s$ parts, provided that a relative minus sign be present between the former two in order to respect hermicity. Notice that the amplitudes (6) are all of the form $ar{u}_B(C + D\gamma_5)i\sigma^{\mu\nu}q_\nu u_A$, where $C$ is the so-called parity conserving amplitude and $D$ is the so-called parity violating one. We stress, however, that in this model both $C$ and $D$ are parity and flavor conserving.

We shall compare Eqs. (6) with experiment, ignoring the contributions of $W^{\pm}_f$ amplitudes. We shall do this in order to be able to appreciate to what extent a priori mixings provide on their own right a framework to describe weak radiative decays.

In principle, we have information about all the quantities that appear in these amplitudes for WRDH. The a priori angles are known from NLDH and the $f_2$ can be related to the measured total magnetic moments of spin 1/2 baryons. The latter values are displayed in the second column of Table II. However, it is important that the mixing angles be determined independently in WRDH and, accordingly, we shall use them as free parameters in the remaining of this paper. How the $f_2$'s are related to the observed total magnetic moments is a question we shall deal with in steps. As a first approximation we shall assume that the $f_2$'s are related to the $\mu$'s of Table I by the formula $\mu_A^{exp} = e_A + f_2^A$ where $A$ is a baryon and $e_A$ its charge. Thus, for example, $f_2^p$ obeys the relationship $\mu_p^{exp} = 1 + f_2^p$ (in nuclear magnetons), etc. Using this assumption we may compare with the experimental data of WRDH. The predictions obtained are displayed in the columns I of Table II.

These first results are not quite good yet but they have a qualitative value. The main point is that the a priori mixing angles come out with the same order of magnitude observed in NLDH, which is very encouraging. The predictions for the observables are some very good, some good, but some show important deviations. The latter still have qualitative value, but should be improved. The values of the $\mu$'s agree fairly well with their experimental counterparts.

As an intermediate step in this analysis it turns out to be very helpful to see what are the values of the total magnetic moments required to reproduce well the experimental observables of WRDH. This is achieved by relaxing the error bars of the measured $\mu$'s up to 10% of the corresponding central values and, then, repeating the previous step. The results are displayed in the columns II of Table II. The experimental data are very well reproduced now, but at the expense of sizable (several percent) changes in the $\mu$'s and new values for the mixing angles.

This second step clearly shows that we must accept that our first approximation—that of identifying the experimentally measured $\mu$'s with the ones that are actually related to the $f_2$'s in this approach to WRDH—must be improved. The $\mu$'s to be used for determining the $f_2$'s in the WRDH amplitudes are really transition magnetic moments. For example, the measured value of $\mu_p$ corresponds to the matrix element $\langle p_{ph}|J_{\mu_{em}}|p_{ph}\rangle \simeq \langle p_{os}|J_{\mu_{em}}|p_{os}\rangle$, where both physical wave functions carry the mass $m_p$. In contrast, the $\mu_p$ the appears in $\Sigma^+ \rightarrow p\gamma$ corresponds to a matrix element whose bra carries the mass $m_p$ and whose ket carries the mass $m_{\Sigma^+}$. So, the normalization of $\mu_p$ originating in the matrix element $\langle p_{ph}|J_\mu\Sigma_{ph}\rangle$ should be related to both masses, $m_p$ and $m_{\Sigma^+}$. It is in this sense that the magnetic moments that we must use are transition magnetic moments.

The natural normalization of magnetic moments is determined by the Gordon decomposition. Using this expansion for guidance, then, for example, $\mu_p$ should be normalized
to $m_p + m_{\Sigma^+}$ and not to $2m_p$, etc. One can see already a qualitative indication of this happening in the first column II in Table II, the changes in the $\mu$'s are systematically in this direction. $\mu_p$, $\mu_n$, $\mu_{\Xi^-}$, $\mu_{\Sigma^-}$, $\mu_{\Sigma^+}$, $\mu_{\Xi^0}$, and $\mu_{\Sigma^0}$ appear to become smaller or larger according to such changes in normalization. $\mu_A$ and $\mu_{\Sigma^0 A}$ are mixed cases because they appear in two or three decays and will be required to be reduced or to be increased in going from one case to another and, therefore, Table II cannot provide a clear cut tendency.

Our third step is to improve our approximation following the above discussion. One must change the normalization of the total magnetic moments either by applying, for example, the factor \( \frac{m_p + m_{\Sigma^+}}{m_p + m_{\Sigma^+} + m_{\Sigma^0}} \) to the experimental $\mu_p$ or the inverse factor to the theoretical $\mu_p$ related to $f^p_\alpha$. Numerically, either way leads to the same result. For definiteness, we choose the former. The corrected experimental values are displayed in column five of Table II. Then, recalculating everything lead to the predictions of columns labeled III of Table II. The values of the mixing angles appear in the bottom of the last column of this table.

The overall agreement is greatly improved, the experimental data are well produced while keeping the magnetic moments in very good agreement with their experimental counterparts. The only deviations that merit further discussion appear in $\Gamma_2$ and in $\mu_{\Sigma^-}$ which are intimately related. These deviations are probably due to another one of our approximations: ignoring the contributions of $W_\mu$ [9]. The agreement already obtained is probably the best one can hope for if one stays short of actually incorporating the contributions of $W_\mu$, as we have.

V. CONCLUSIONS

If a priori mixings are present, then weak decays may go via the flavor and parity conserving hamiltonians of strong and electromagnetic interactions. That is, with these mixings there would exist another mechanism to produce weak radiative, non-leptonic, and rare mode decays of hadrons, in addition to the already existing mechanisms provided by the $W_\mu$ and $Z^0_\mu$ bosons.

We are now in a position to conclude our present analysis. To extend the credibility of the a priori mixing scheme it was very important to be able to describe WRDH. As we have shown above this is achieved. However, the most important and stringent test is that the mixing angles share a universality-like property. The values for them obtained independently in WRDH are in very good agreement with the absolute values obtained for them in NLDH. It is the passing this universality-like test that lends the strongest support to the possibility that the above scheme may serve a framework for the systematic description of the enhancement phenomenon in weak decays of hadrons. Also, the contributions of $W_{\mu^\pm}$ should be included at some point at a, for consistency, small level, say, by assuming that $|\Delta I| = 1/2$ amplitudes are of the same order of magnitude as the $|\Delta I| = 3/2$ amplitudes.

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REFERENCES

[1] A. García, R. Huerta, and G. Sánchez-Colón, J. Phys. G: Nucl. Part. Phys. 24 (1998) 1207-1217.
[2] A. García, R. Huerta, and G. Sánchez-Colón, Rev. Mex. Fís. 45 (1999) 244-248.
[3] Original references for the $|\Delta I| = 1/2$ rule can be found in R. E. Marshak, Riazuddin, and C. P. Ryan, The Theory of Weak Interactions in Particle Physics (Wiley-Interscience, John Wiley and Sons, Inc., 1969).
[4] A recent review of the $|\Delta I| = 1/2$ rule in hyperon non-leptonic decays can be found in J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Rep. 131 (1986) 319.
[5] A. García, R. Huerta, and G. Sánchez-Colón, J. Phys. G: Nucl. Part. Phys. 25 (1999) 45-57.
[6] O. Dumbrajs et al., Nucl. Phys. B 216 (1983) 277. Only the squares of five YCC are quoted. We have determined their signs to match the strong-flavor $SU_3$ signs. Normalized to the pion-nucleon YCC (assumed positive), we have $g_{p,p\pi^0} = 1.0 \pm 0.0063$, $g_{\Lambda,\Sigma^+} = g_{\Lambda,\Sigma^+} = -0.897 \pm 0.074$, $g_{\Sigma^+,\Sigma^+} = 0.936 \pm 0.075$, $g_{g_\Sigma^0K^+} = g_{\Sigma^0K^-} = 0.251 \pm 0.056$, $g_{p,\Lambda K^+} = g_{\Lambda,\Lambda K^-} = 0.987 \pm 0.092$, $g_{\Sigma^0,\Sigma^0} = -0.270 \pm 0.081$, $g_{\Xi^-,\Lambda K^-} = 0.266 \pm 0.080$. We have determined the last two YCC at their $SU_3$ limit allowing 30% error bars. The values of the YCC used for the predictions of Table I are $g_{p,p\pi^0} = 1.001 = -g_{p,p\pi^0}^{-,\Lambda^+} = -0.812 = g_{\Sigma^+,\Lambda^+} = -g_{\Sigma^+,\Lambda^+}^{-,\Lambda^+}$, $g_{\Sigma^+,\Sigma^+} = 0.936 = g_{\Sigma^+,\Sigma^+}^{-,\Lambda^+}$, $g_{g_\Sigma^0K^-} = g_{\Sigma^0,\Sigma^0} = -0.270 = g_{\Sigma^0,\Sigma^0}^{-,\Lambda^+}$. These values are in reasonable agreement with the experimental counterparts. We use a chi-square method in which the experimental YCC are added as constraints.
[7] Particle Data Group, Phys. Rev. D 50 (1994) 1173-1826.
[8] A. García, R. Huerta, and G. Sánchez-Colón, J. Phys. G: Nucl. Part. Phys. 25 (1999) L1-L6.
[9] B. Bassalleck, Nucl. Phys. A 547 (1992) 299c; J. Lach and P. Żenczykowski, Int. J. Mod. Phys. A 10 (1995) 3817.
TABLE I. Predictions for the $A$ amplitudes, along with the accompanying predictions for the $B$ amplitudes, obtained by assuming that the magnitudes of the YCC of Eqs. (2) match their corresponding counterparts in Eqs. (3). The values of the YCC are listed in Ref. [6]. All amplitudes are given in units of $10^{-7}$.

| Decay          | $B_{\text{exp}}$ | $B_{\text{th}}$ | $A_{\text{exp}}$ | $A_{\text{th}}$ |
|---------------|------------------|-----------------|------------------|-----------------|
| $\Lambda \to p\pi^-$ | $-22.09 \pm 0.44$ | $-22.38$ | $-3.231 \pm 0.020$ | $-3.262$ |
| $\Lambda \to n\pi^0$ | $15.89 \pm 0.1$ | $15.83$ | $2.374 \pm 0.027$ | $2.307$ |
| $\Sigma^- \to n\pi^-$ | $1.43 \pm 0.17$ | $1.34$ | $-4.269 \pm 0.014$ | $-4.264$ |
| $\Sigma^+ \to n\pi^+$ | $-42.17 \pm 0.18$ | $-42.09$ | $-0.140 \pm 0.027$ | $-0.152$ |
| $\Sigma^+ \to p\pi^0$ | $-26.86 \pm 1.10$ | $-30.72$ | $3.247 \pm 0.089$ | $2.907$ |
| $\Xi^- \to \Lambda\pi^-$ | $-17.47 \pm 0.50$ | $-17.27$ | $4.497 \pm 0.020$ | $4.521$ |
| $\Xi^0 \to \Lambda\pi^0$ | $-12.29 \pm 0.70$ | $-12.21$ | $3.431 \pm 0.055$ | $3.197$ |

TABLE II. Experimental values of the observables in WRDH and of the magnetic moments (m. m.) of hyperons along with the predictions of the three cases considered. The indeces 1,2,....,5 on the observables correspond, respectively, to the decays $\Sigma^+ \to p\gamma$, $\Xi^- \to \Sigma^-\gamma$, $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma^0\gamma$. The numbers in parenthesis in the m. m. indicate the decay in which they appear. The values of the a priori mixing angles in column eight come from NLDH. The mixing angles are in $10^{-6}$, the decay rates are in $10^6$ sec$^{-1}$, and the m. m. are in nuclear magnetons. The only m. m. that has not been measured is $\mu_{20}$. We have taken for it its $SU(3)$ estimate with a 10% error bar.

| Magnetic moments | Transition m. m. | Observables and angles |
|------------------|------------------|------------------------|
|                 | Exp. | I | II | III | Exp. | I | II | III |
| $\mu_p(1)$       | 2.793 | 2.793 | 2.745 | 2.463 | 2.463 | $\Gamma_1$ | 15.65 $\pm$ 0.88 | 11.55 | 15.60 | 14.62 |
| $\mu_n(3)$      | $-1.913$ | $-1.913$ | $-1.654$ | $-1.750$ | $-1.750$ | $\Gamma_2$ | 0.77 $\pm$ 0.14 | 1.37 | 0.81 | 1.30 |
| $\mu_{\Xi^-}(2)$ | $-0.651 \pm 0.003$ | $-0.652$ | $-0.747$ | $-0.683 \pm 0.003$ | $-0.685$ | $\Gamma_3$ | 6.65 $\pm$ 0.57 | 7.13 | 6.67 | 6.16 |
| $\mu_{\Sigma^-}(2)$ | $-1.160 \pm 0.025$ | $-1.018$ | $-0.868$ | $-1.103 \pm 0.024$ | $-0.958$ | $\Gamma_4$ | 3.66 $\pm$ 0.56 | 5.26 | 3.68 | 4.38 |
| $\mu_{\Lambda}(3)$ | $-0.613 \pm 0.004$ | $-0.611$ | $-0.553$ | $-0.665 \pm 0.004$ | $-0.665$ | $\Gamma_5$ | 12.1 $\pm$ 1.4 | 5.23 | 11.8 | 10.13 |
| $\mu_{\Sigma^+}(4)$ | $-0.563 \pm 0.004$ | $-0.562$ | $a_1$ | $-0.76 \pm 0.08$ | $-0.78$ | $a_2$ | $-0.09$ | 0.56 | 0.20 |
| $\mu_{\Xi^0(1)}$ | 2.458 $\pm$ 0.010 | 2.430 | 2.553 | 2.748 $\pm$ 0.011 | 2.763 | $a_3$ | $-0.09$ | $-0.83$ | 0.76 |
| $\mu_{\Xi^0(4)}$ | $-1.250 \pm 0.014$ | $-1.271$ | $-1.502$ | $-1.353 \pm 0.015$ | $-1.357$ | $a_4$ | $-0.40 \pm 0.40$ | $-0.86$ | 0.18 | 0.27 |
| $\mu_{\Xi^0(5)}$ | $-1.311 \pm 0.015$ | $-1.305$ | $a_5$ | $0.20 \pm 0.32$ | $-0.46$ | $0.22 \pm 0.09$ | 0.03 $\pm$ 0.08 | $-0.40 \pm 0.19$ | $-0.11 \pm 0.08$ |
| $\mu_{\Xi^0(6)}$ | $0.649 \pm 0.065$ | 0.499 | 0.624 | 0.617 $\pm$ 0.062 | 0.500 | $\delta$ | $0.26 \pm 0.09$ | 0.10 $\pm$ 0.07 | $-0.28 \pm 0.18$ | $-0.25 \pm 0.08$ |