Probing the SUSY Higgs boson couplings to scalar leptons
at high–energy $e^+e^-$ colliders

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Abstract

We discuss the production at $e^+e^-$ colliders of Higgs bosons in association with both the scalar leptons and the lightest neutralinos in the Minimal Supersymmetric extension of the Standard Model. While the rates for associated Higgs production with neutralinos and first/second generation sleptons are rather tiny, the cross section for the production of the lightest Higgs boson $h$ with scalar $\tau$ lepton pairs can reach the femtobarn level at c.m. energies at and above 500 GeV in favorable regions of the parameter space, making this process potentially detectable at a high–luminosity $e^+e^-$ collider, in particular in the $\gamma\gamma$ option. This would provide a determination of the $h\tilde{\tau}\tilde{\tau}$ coupling and opens up the possibility of measuring the parameter $\tan\beta$. 
1. Introduction

The search for Supersymmetry (SUSY) is one of the major goals of present and future high-energy colliders. Once SUSY particles are found, it would be of prime importance to study in detail their properties and interactions in order to reconstruct the SUSY Lagrangian. This will be mandatory to decide which SUSY scenario is effectively realized at the low energies probed by experiments and potentially, to derive the structure of the theory at high scales.

The SUSY Lagrangian can be reconstructed by measuring the couplings between the SUSY and the standard particles. Among these, the couplings of sparticles to Higgs bosons are of special importance since they also probe the electroweak symmetry breaking sector and might decide which Higgs scenario is at work. In the Minimal Supersymmetric Standard Model (MSSM) [1], two Higgs doublets fields [2] are needed to break the \( SU(2) \times U(1) \) symmetry, leading to a quintet of Higgs bosons, the lightest of which, the neutral CP–even scalar \( h \), should have a mass below \( \sim 130 \text{ GeV} \) [3]. The couplings of the Higgs bosons to the SUSY scalar fermions \( \tilde{f} \) and to the charginos \( \chi^\pm \) and neutralinos \( \chi^0 \) depend on the soft–SUSY breaking parameters and therefore carry informations on the fundamental SUSY theory.

In the MSSM, the Higgs boson couplings to the charginos \( \chi^\pm_{1,2} \) and neutralinos \( \chi^0_{1,...,4} \) depend on \( \tan \beta \), the ratio of the vev’s of the two Higgs fields, the higgsino parameter \( \mu \) and the bino and wino mass parameters \( M_1 \) and \( M_2 \) which are linked by the relation \( M_1 = \frac{2}{3} \tan^2 \theta_W M_2 \simeq \frac{1}{2} M_2 \) in the minimal Supergravity (mSUGRA) [4] model where the gaugino masses [as well as the scalar masses and the trilinear sfermion couplings] are unified at the GUT scale and where the electroweak symmetry is broken radiatively. For instance, the \( h \chi^0_3 \chi^0_i \) coupling which might be the first to be accessible [since the \( h \) boson is light and the neutralino \( \chi^0_1 \) is expected to be the lightest sparticle (LSP) in the MSSM] is given, in the decoupling limit [where the \( h \) boson becomes Standard Model like and all the other Higgs particles are heavy], by [5]

\[
g_{h\chi^0_3\chi^0_i} \propto (Z_{i2} - \tan \theta_W Z_{i1})(\sin \beta Z_{j3} + \cos \beta Z_{j4}) + i \leftrightarrow j
\]

with \( i = j = 1 \). Here \( Z_{ij} \) are the elements of the matrix \( Z \) which diagonalises the \( 4 \times 4 \) neutralino mass matrix. As can be seen, the \( h \) boson couples to mixtures of gaugino \((Z_{i1}, Z_{i2})\) and higgsino \((Z_{i3}, Z_{i4})\) components of the neutralinos. If the light neutralino \( \chi^0_1 \) were a pure bino [as is the case in a large part of the mSUGRA model parameter space [6], in particular when cosmological constraints are incorporated] or a pure higgsino, the coupling would vanish and thus would be hard to measure experimentally.

The couplings of the \( h \) boson to sfermion pairs, \( g_{h\tilde{f}_i\tilde{f}_j} \), can be stronger. In the decoupling limit, and in terms of \( \tan \beta \), \( \mu \) and the trilinear coupling \( A_f \), the (normalized) diagonal and non–diagonal \( h–sfermion \) couplings read in the MSSM [\( s_{\theta_i} = \sin \theta_i \), \( c_{\theta_i} = \cos \theta_i \), etc ...]

\[
\begin{align*}
g_{h\tilde{f}_i\tilde{f}_j} &= \cos 2\beta \left[ I_f^3 \left( c_{\theta_f}^2 - e_f s_{\theta_f}^2 \right) - e_f s_{\theta_f}^2 c_{2\theta_f} \right] + \frac{m_f^2}{M_Z^2} \pm \frac{s_{2\theta_f} m_f A_f}{2 M_Z^2} [A_f - \mu (\tan \beta)^{-2 I_f^3}] \\
g_{h\tilde{f}_i\tilde{f}_j} &= \cos 2\beta \left[ e_f s_{\theta_f}^2 - I_f^3 / 2 \right] + c_{2\theta_f} m_f [A_f - \mu (\tan \beta)^{-2 I_f^3}] / (2 M_Z^2)
\end{align*}
\]

where \( I_f^3 \) is the weak isospin and \( e_f \) the electric charge of the sfermion \( \tilde{f} \) and \( \theta_f \) the mixing angle between the left and right–handed sfermions \( \tilde{f}_L \) and \( \tilde{f}_R \) [which as for the sfermion
masses $m_{f_1}$ and $m_{f_2}$, are given in terms of the three parameters above and the soft–SUSY breaking scalar masses $m_{f_{10}}$ and $m_{f_{20}}$: $s_W^2 = 1 - c_W^2 \equiv \sin^2 \theta_W$. For first and second generation sfermions, as apparent from eq. (2), these couplings are relatively tiny since $m_f$ and the mixing angle $\theta_f$ are small, and the term proportional to $\cos 2\beta$ is not enhanced.

The couplings of Higgs bosons to the third generation squarks, $\tilde{t}$ and $\tilde{b}$, can be much larger and potentially measurable in the associated Higgs+squark production process at proton or $e^+e^-$ colliders, as discussed in Refs. [6, 7]. In this paper, we will investigate the prospects of measuring the lightest CP–even Higgs boson couplings to sleptons which can be best performed in the clean environment of future high–energy $e^+e^-$ colliders\[^4\] [10]. Contrary to the squark case, the measurement of these couplings can be performed in two ways:

i) In the production of LSP pairs, $e^+e^- \rightarrow \chi^{0}_{1}\chi^{0}_{1}$, which is the first kinematically accessible SUSY process in $e^+e^-$ collisions, the $h$ boson can be emitted not only for the final $\chi^{0}_{1}$ lines but also from the selectrons which are exchanged in the $t$ and $u$ channels. The production cross section for the $e^+e^- \rightarrow \chi^{0}_{1}\chi^{0}_{1}h$ associated process thus involves the $h\tilde{e}\tilde{e}$ couplings.

ii) In the production of selectrons $e^+e^- \rightarrow \tilde{e}\tilde{e}^*$ or sneutrinos $e^+e^- \rightarrow \tilde{\nu}_e\tilde{\nu}_e^*$, the Higgs bosons can be emitted from both the final slepton lines or from, respectively, the neutralinos and charginos which are exchanged in the $t$–channels; the production cross sections are then in principle proportional to complicated combinations of the Higgs couplings to sleptons and neutralino/chargino states. In the case of smuons and staus and their corresponding sneutrinos, there is no gaugino exchange channels and the processes $e^+e^- \rightarrow \tilde{l}\tilde{l}^*$ are mediated by $s$–channel ($\gamma$)$Z$ exchange with the Higgs boson emitted from the slepton lines. Up to the small contribution of the diagrams where the $h$ boson is emitted from the $Z$–boson line [see later] the cross sections are directly proportional to the square of the $h\tilde{t}\tilde{t}$ couplings which would be then, in principle, measurable in these processes.

In this paper we analyze the prospects of measuring the Higgs–slepton couplings at high–energy and high–luminosity $e^+e^-$ colliders. In Sections 2 and 3, we discuss the associated production of the $h$ boson with, respectively, the lightest neutralinos and selectron/sneutrino states. In Section 4 we focus on the case of stau leptons where the cross sections, in both $e^+e^-$ and $\gamma\gamma$ options of the $e^+e^-$ collider, will be shown to be potentially large. Some conclusions will be given in the final Section 5.

2. Higgs boson production in association with neutralinos

The Feynman diagrams contributing to the production of the lightest CP–even Higgs boson in association with neutralino pairs is shown in Fig. 1 [the diagrams where the $h$ boson is emitted from the electron and positron lines give negligible contributions]. A first class of contributions (1a) is formed by diagrams where the Higgs boson is emitted from the neutralino states, the latter being produced through $s$–channel $Z$ boson exchange and $t$–channel left– and right–handed selectron exchange. A second class (1b) is formed by the Higgs–strahlung production process, where the $Z$ boson is virtual and splits into two neutralinos. Finally, a third class (1c) consists of the diagrams where the Higgs boson is emitted from

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\[^4\]In this paper, we will not discuss the case of the heavier MSSM Higgs bosons since the associated production processes are less favored by phase space. In addition, we will stick to $e^+e^-$ colliders, since at hadron colliders, the cross section for (electroweak) Higgs–slepton production is relatively much smaller than the potential backgrounds, and the signal would be more complicated to extract.
the internal selectron lines. The cross section will therefore depend on the $h$ boson couplings to both the neutralinos and sleptons.

![Feynman diagrams](image_url)

Figure 1: Feynman diagrams contributing to the $e^+e^- \rightarrow h\chi_1^0\chi_1^0$ production process.

If the higgsino mass parameter $\mu$ is much larger than the bino and wino mass parameters $M_1$ and $M_2$, $|\mu| \gg M_1$, the lightest neutralino is a pure bino and its mass is given by $m_{\chi_1^0} \simeq M_1$. In this case, the neutralino couplings to the $h$ boson, eq. (1), as well as to the $Z$–boson, $g_{h\chi_i^0\chi_j^0} \propto (Z_{ij}^3 Z_{j3} - Z_{i4} Z_{j4})$, are small. The only diagram which would contribute to the production rate is then diagram (1c) where the Higgs boson is emitted from the selectron lines. The cross section will then be proportional only to the couplings $g_{h\tilde{e}\tilde{e}}$. For these couplings, one can set $m_e = 0$ and vanishing mixing angle and only the first term $\propto \cos 2\beta$ will be present. For large values of $\tan \beta$ [which are required to maximize the $h$ boson mass and to evade the experimental constraint from LEP2 searches [9], $M_h \gtrsim 113.5$ GeV in the decoupling regime] $\cos 2\beta \rightarrow -1$ and one has $g_{h\tilde{e}_i\tilde{e}_j} = -\delta_{ij} (1^2 - e_i s_W^2)/(s_W c_W)$. Note that because $s_W^2 \approx 1/4$, the $h$ couplings to left– and right–handed selectrons are almost equal [in absolute value] and equal to half of the $h\tilde{\nu}\tilde{\nu}$ coupling. [If the neutralino $\chi_1^0$ were a pure higgsino, i.e. $\mu \ll M_1$, the $h-\chi_1^0-\chi_1^0$ as well as the $\tilde{e}-\chi_1^0$ couplings would vanish; the only diagram which would contribute to the process $e^+e^- \rightarrow h\chi_1^0\chi_1^0$ would be the diagram (1b) which does not involve any Higgs coupling to superparticles.]

We have calculated the cross section $\sigma(e^+e^- \rightarrow \chi_1^0\chi_1^0h)$ and the results at $\sqrt{s} = 500$ and 800 GeV, for selected values of the LSP and selectron masses, are given in Table 1. We have assumed that the LSP neutralino is a pure bino so that $m_{\chi_1^0} = M_1$ and $Z_{11} = 1$ and we used the approximation $m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_{\tilde{e}}$; we have fixed the other parameters so that $M_h = 120$ GeV and $\cos 2\beta = -1$ as will be the case in the large $\tan \beta$ limit, $\tan \beta \sim 50$, adopted in this analysis. As can be seen, even for very small masses of these superparticles, $m_{\chi_1^0} \sim 50$ GeV and $m_{\tilde{e}_{L,R}} \sim 100$ GeV [close to the experimental bounds from negative searches at LEP2], the cross section hardly reaches the level of $10^{-2}$ fb for a c.m. energy $\sqrt{s} = 500$ GeV and even smaller at higher energies. This means that even with the integrated luminosities, $\mathcal{L} = 500$ fb$^{-1}$, expected at these machines, only a handful of events can be generated in this process. The couplings $g_{h\tilde{e}\tilde{e}}$ will therefore be very difficult to measure in this mechanism.
### Table 1: The cross sections, $\sigma(\sqrt{s})$, for the process $e^+e^- \rightarrow h\chi^0_1\chi^0_1$ [in fb] for selected values of $m_{\chi^0_1}$ and $m_{\tilde{e}}$ at c.m. energies $\sqrt{s} = 500$ GeV and 800 GeV.

| $m_{\chi^0_1}$ [GeV] | $m_{\tilde{e}}$ [GeV] | $\sigma(500)$ [fb] | $\sigma(800)$ [fb] |
|------------------------|------------------------|---------------------|---------------------|
| 50                     | 100                    | 0.010               | 0.005               |
| 50                     | 200                    | 0.001               | 0.001               |
| 100                    | 105                    | 0.005               | 0.005               |
| 100                    | 200                    | 0.0006              | 0.001               |

3. Higgs production in association with selectrons and sneutrinos

The processes $e^+e^- \rightarrow \tilde{e}\tilde{e}^*h$ and $e^+e^- \rightarrow \tilde{\nu}_{e}\tilde{\nu}_{e}^*h$ are generated by the diagrams of Fig. 2 where the neutralinos $\chi^0_{1,-4}$ and the charginos $\chi^\pm_{1,2}$ are exchanged in the $t$–channel, respectively. In the latter case, only the $Z$ boson is exchanged in the $s$–channel diagrams. If the lightest neutralinos and charginos are higgsino–like, the contributions from the $t$–channel diagrams are very small since $\chi^0_{1,2}$ and $\chi^\pm_1$ have couplings proportional to $m_e$ [only the heavier ino states would contribute but the cross sections are then suppressed since these particles are heavier]. In this case, the production cross sections are approximately the same as for associated Higgs production with $\tilde{\mu}$ and $\tilde{\tau}$ [in the case of no–mixing] and the corresponding sneutrinos, since for these particles there is no $t$–channel exchange diagram. If the $\chi^0_{1,2}$ and $\chi^\pm_1$ particles are gaugino–like all diagrams would contribute [and one can neglect the contribution of the heavier chargino and neutralino states which are higgsino–like]. In the mixed gaugino–higgsino region, where the Higgs boson couplings to neutralinos and charginos are sizeable, the last diagrams of Fig. 2a has to be taken into account; since we are interested only in the Higgs–slepton coupling, we will not discuss this region here.

![Feynman diagrams](image_url)

Figure 2: The Feynman diagrams contributing to the production of the lightest Higgs boson in association with sleptons, $e^+e^- \rightarrow \tilde{l}\tilde{l}^*h$. 

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Some values of the cross sections for the associated production processes $e^+e^- \rightarrow \tilde{e}^*h$, $e^+e^- \rightarrow \tilde{\nu}_e\tilde{\nu}_e^*h$ as well as $e^+e^- \rightarrow \tilde{\mu}_\mu\tilde{\nu}_\mu^*h$ and $e^+e^- \rightarrow \tilde{\nu}_\mu\tilde{\nu}_\mu^*h$ are displayed in Table 2 for selected values of the LSP and slepton masses at c.m. energies of 500 and 800 GeV. We have summed the cross sections over possible chiral combinations of final state sleptons and as previously, have chosen a common mass $m_{\tilde{l}}$ for all sleptons. The numbers in the cases of the $e^+e^- \rightarrow \tilde{e}^*h$ and $e^+e^- \rightarrow \tilde{\nu}_e\tilde{\nu}_e^*h$ cross sections are for bino–like LSPs; as mentioned previously; for higgsino–like LSPs, the cross sections are approximately the same as those for the processes $e^+e^- \rightarrow \tilde{\mu}_\mu^*h$ and $e^+e^- \rightarrow \tilde{\nu}_\mu\tilde{\nu}_\mu^*h$, respectively.

As can been seen from Table 2, the cross sections for $\tilde{e}, \tilde{\mu}$ and $\tilde{\nu}_\mu$ final states are also very small, below 0.03 fb at the considered c.m. energies, even for relatively small values of the LSP and slepton masses, $m_{\chi_1^0} = 50$ GeV and $m_{\tilde{l}} = 100$ GeV. Only for $\tilde{\nu}_e\tilde{\nu}_e^*h$ final states, with $m_{\chi_1^0}$ and $m_{\tilde{\nu}_e}$ values close to the bounds indicated by experimental data, that the cross sections can reach the level of $\sim 0.2$ fb. This is mainly due to the large contribution of the chargino mediated $t$–channel diagram [the charged $e^–\tilde{\nu}_e–\chi_1^\pm$ coupling is stronger than the neutral $e^-\tilde{\nu}_e–\chi_{1,2}^0$ couplings involved in selectron production] and to the fact that $g_{h\tilde{\nu}_e\tilde{\nu}_e^*} \sim 4g_{h\tilde{\nu}_e\tilde{\nu}_e^*}^2$. However, for such small masses, the sneutrino $\tilde{\nu}_e$ will dominantly decay into invisible final states, $\tilde{\nu}_e \rightarrow \nu_{e_{\chi_1^0}}$ and thus remains experimentally undetectable [recall that here, $m_{\chi_1^+} \sim 2m_{\chi_1^0}$, and the charged visible decay $\tilde{\nu}_e \rightarrow e^+\chi_1^\pm$ would be phase–space suppressed].

Thus, the prospects of measuring the Higgs-slepton couplings in these processes are rather gloomy, even for the high–luminosities, $L = 500$ fb$^{-1}$, expected at the future $e^+e^-$ machines.

| $l$ | $m_{\chi_1^0}$ [GeV] | $m_{\tilde{l}}$ [GeV] | $\sigma(500)$ [fb] | $\sigma(800)$ [fb] |
|-----|-----------------------|-------------------|------------------|------------------|
| $\tilde{e}$ | 50 | 100 | 0.021 | 0.027 |
| | 100 | 150 | 0.003 | 0.011 |
| $\tilde{\nu}_e$ | 50 | 100 | 0.127 | 0.195 |
| | 100 | 150 | 0.006 | 0.055 |
| $\tilde{\mu}$ | 50 | 100 | 0.004 | 0.004 |
| | 100 | 150 | 0.0004 | 0.002 |
| $\tilde{\nu}_\mu$ | 50 | 100 | 0.0004 | 0.001 |
| | 100 | 150 | 0.00005 | 0.0006 |

Table 2: The cross sections, $\sigma(\sqrt{s})$, for the processes $e^+e^- \rightarrow h\tilde{l}\tilde{l}^*$ [in fb] for selected values of the LSP and slepton masses at c.m. energies $\sqrt{s} = 500$ GeV and 800 GeV.

4. Higgs production in association with staus

The main reason for the smallness of the cross sections for the processes discussed in the previous sections is the smallness of the Higgs–slepton coupling itself. Indeed, compared to the $h$ boson coupling to top squarks, $g_{h\tilde{t}\tilde{t}} \sim \tan\beta/M_Z^2$ in the no–mixing case [i.e. $A_t - \mu\cot\beta \sim 0$], the Higgs coupling to selectrons is one order of magnitude smaller than its coupling to top squarks, leading to a two–order of magnitude smaller production cross section $\sigma(e^+e^- \rightarrow h\tilde{e}\tilde{e}^*)$ as compared to $\sigma(e^+e^- \rightarrow h\tilde{l}\tilde{l}^*)$. Since the latter hardly reaches the femtobarn level $\mathcal{F}$, the results for the production rates in the previous sections were to be expected.
The case of the $\tau$ sleptons is drastically different from the one of the other sleptons. Indeed, because of the relatively large value of $m_\tau$, the leading component in the $g_h \tilde{\tau}_i$ coupling, eq. (2), is the one proportional to $\sin 2\theta_\tau (A_\tau - \mu \tan \beta)$. For large values of $\mu$ and $\tan \beta$ [or and extremely large values of $A_\tau$], the mixing in the $\tau$ sector becomes very strong, $|\sin 2\theta_\tau| \simeq 1$, leading at the same time, to two important consequences:

i) The mass splitting between the two $\tilde{\tau}$ eigenstates becomes large, leading to a $\tilde{\tau}_1$ state much lighter than the other sleptons; the process $e^+ e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^* h$ will therefore be more phase space favored than the slepton processes discussed in the previous section.

ii) The $h\tilde{\tau}_1 \tilde{\tau}_1$ coupling can be strongly enhanced. For instance, for the values $\mu = 500$ GeV and $\tan \beta = 50$, leading to $|\sin 2\theta_\tau| \simeq 1$, one has $g_h \tilde{\tau}_1 \tilde{\tau}_1 \sim 2.5$ compared to $g_h \tilde{\tau}_1 \sim 0.25$ in the case of no-mixing. The cross section for $e^+ e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^* h$ can thus be much larger than those involving selectron, smuon and sneutrino final states.

The cross section for the process $e^+ e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^* h$ is similar to that of the associated production of the $h$ boson with top squarks after appropriate replacements of the couplings, charge and color factors. At high energies and when the $g_h \tilde{\tau}_1 \tilde{\tau}_1$ coupling is large, the cross section can be approximated by the sole contribution from the photon exchange diagrams with $h$ emitted from the slepton lines. This is due to the fact that both of the couplings $g_Z \tilde{\tau}_1$ and $g_h \tilde{\tau}_1 \tilde{\tau}_2$ are proportional, in the large mixing case $|\sin 2\theta_\tau| \rightarrow 1$, to $s_W^2 - 1/4$ which is close to zero for $s_W^2 \sim 0.23$, and one can safely neglect the contributions of the $Z$–exchange diagrams and those involving $\tilde{\tau}_2$ virtual states. In this case, the Dalitz plot density is given by the very simple formula:

$$
\frac{d\sigma}{dx_1 x_2} \equiv \frac{\alpha \sigma_0}{16\pi s_W^2 c_W^2} \frac{m_Z^2}{s} \frac{\beta}{g_{h \tilde{\tau}_1 \tilde{\tau}_1}} \left[ \frac{1 - 2x_1 + 4\mu s}{(1 - x_1)^2} + \frac{x_1 + x_2 - 1 + 2\mu h - 4\mu t}{(1 - x_1)(1 - x_2)} + x_1 \leftrightarrow x_2 \right]
$$

(3)

where $x_{1,2}$ are the reduced energies of the $\tilde{\tau}_1, \tilde{\tau}_1^*$ final states, $x_{1,2} = 2E_{\tilde{\tau}_1, \tilde{\tau}_1^*}/\sqrt{s}$ and $\mu_i$ the reduced mass squared, $\mu_i = M_i^2 / s$; $\sigma_0 = 4\pi \alpha^2 / 3s$ is the QED point–like cross section.

The $e^+ e^- \rightarrow h\tilde{\tau}_1 \tilde{\tau}_1^*$ cross sections in Fig. 3 are shown as a function of the $\tilde{\tau}_1$ mass for two c.m. energies, $\sqrt{s} = 500$ and 800 GeV. We have fixed the $h$ boson mass to $M_h = 120$ GeV and the SUSY parameters to: $\tan \beta = 50$, $\mu = -A_\tau = 500$ GeV and varied the soft–SUSY breaking $\tau$ masses, $m_{\tilde{\tau}_1} \simeq m_{\tilde{\tau}_2}$, to vary $m_{\tilde{\tau}_1}$. As can be seen, for relatively small $m_{\tilde{\tau}_1}$ values, the cross section can exceed $\sim 0.2$ fb, leading to more than 100 events for an expected integrated luminosity $L = 500$ fb$^{-1}$. The cross section scales quadratically with the parameters $\mu$ and $\tan \beta$ and can therefore be larger (smaller) when the values of these parameters are increased (decreased); for instance, for $\mu \sim 1$ TeV and $\tan \beta \sim 50$, $\sigma(e^+ e^- \rightarrow h\tilde{\tau}_1 \tilde{\tau}_1^*)$ reaches the femtobarn level.

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2Note that large values of $\mu \sim O(1$ TeV) can be obtained naturally in mSUGRA from the requirement of radiative electroweak symmetry breaking, while very large values of $\tan \beta \sim O(50)$ are favored if one requires Yukawa coupling unification at the GUT scale; see Ref. 3.

3Here we will only deal with the continuum cross section. Another possibility to generate $\tilde{\tau}_1 \tilde{\tau}_1^* h$ final states would be to produce mixed $\tilde{\tau}_1 \tilde{\tau}_2^*$ pairs in $e^+ e^-$ collisions, with the heavier $\tilde{\tau}_2$ decaying into a $\tilde{\tau}_1 h$ final state. This two–step process, however, needs large enough phase space so that the heavier $\tilde{\tau}_2$ eigenstate can be produced; in addition the branching ratio for the decay $\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 h$ is small since in the large mixing case, the coupling $g_h \tilde{\tau}_1 \tilde{\tau}_2$, eq. (2), is close to zero.

4For the numerical analysis, we have used the complete formula for the cross section, including the small contributions from the diagrams where the $h$ boson is emitted from the $Z$–line and with the exchange of $\tilde{\tau}_2$. 

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In the case where the main decay mode of the stau would be $\tilde{\tau}_1 \rightarrow \tau \chi^0_1$, the final state would consist of a $b\bar{b}$ pair [since the main decay mode of the $h$ boson is $h \rightarrow b\bar{b}$] peaking at $M_h$ [which would be precisely measured in the main Higgs production processes], two tau leptons and a fair amount of missing energy [when the $m_{\tilde{\tau}_1} - m_{\chi^0_1}$ difference is substantial]:

$$e^+e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_1 h \rightarrow \tau^+\tau^- + b\bar{b} + \not{E}_T$$

(4)

This signal would be not too difficult to detect in the clean environment of $e^+e^-$ colliders. A detailed analysis taking into account background and detection efficiencies, which is beyond the scope of this paper, is nevertheless required to assess in which part of the MSSM parameter space this final state can be isolated experimentally.

Figure 3: The cross sections for associated $\tilde{\tau}_1 \tilde{\tau}_1^* h$ production as a function of $m_{\tilde{\tau}_1}$ in $e^+e^-$ collisions at $\sqrt{s} = 500$ and 800 GeV; $\mu = -A_{\tau} = 500$ GeV and $\tan \beta = 50$.

Figure 4: The cross sections for associated $\tilde{\tau}_1 \tilde{\tau}_1^* h$ production as a function of $m_{\tilde{\tau}_1}$ in $\gamma\gamma$ collisions at $\sqrt{s_{\gamma\gamma}} = 400$ and 650 GeV; $\mu = -A_{\tau} = 500$ GeV and $\tan \beta = 50$. 
For completeness, we have also studied the associated production of the $h$ boson with $\tilde{\tau}$ sleptons in photon–photon collisions, $\gamma\gamma \rightarrow h\tilde{\tau}_1\tilde{\tau}_1^*$, since future $e^+e^-$ linear colliders can be turned into high–energy $\gamma\gamma$ colliders [with the photons coming from Compton back–scattering of laser beams] which may have $\sim 80\%$ of the c.m. energy and $\sim 50\%$ of the luminosity available at the original $e^+e^-$ machine. The process $\gamma\gamma \rightarrow h\tilde{\tau}_1\tilde{\tau}_1^*$ is generated by two diagrams: one with $\tilde{\tau}_1$ exchanged in the $t$–channel and one involving the quartic $\gamma\gamma\tilde{\tau}_1\tilde{\tau}_1^*$ coupling, the $h$ boson being emitted from the external and internal slepton lines.

The cross sections for the subprocess are shown in Fig. 4 for two c.m. energies, $\sqrt{s}_{\gamma\gamma} = 400$ and 650 GeV, with the same inputs as in Fig. 3. These are almost an order of magnitude larger than the corresponding ones at the $e^+e^-$ mode of the collider for relatively small $m_{\tilde{\tau}_1}$, and reaches the femtobarn level for $m_{\tilde{\tau}_1} \sim 100$ GeV for both c.m. energies. For comparable luminosities of the $\gamma\gamma$ and $e^+e^-$ colliders, and despite the smaller $\gamma\gamma$ c.m. energy, a sizeable number of events might be collected for small $m_{\tilde{\tau}_1}$ and large $\tan\beta, \mu$ values. For instance, at $\sqrt{s}_{\gamma\gamma} = 650$ GeV, stau masses up to $m_{\tilde{\tau}_1} \sim 250$ GeV can be probed for the inputs of Fig. 4, if one requires $\sim 50$ events for detectability at a luminosity $L_{e^+e^-} \sim L_{\gamma\gamma} \sim 500$ fb$^{-1}$.

5. Conclusions

We have studied the associated production of the lightest $h$ boson along with a neutralino or a slepton pair at future $e^+e^-$ colliders in the context of the MSSM. The cross sections for these processes are proportional to the Higgs–slepton couplings and, hence, would allow for their measurements if they are large enough.

It turns out, mainly because of the fact that the couplings of Higgs bosons to first and second generation charged sleptons as well as to sneutrinos are rather tiny, the cross sections for these associated production processes are too small to generate a sufficient number of events, even with the large luminosity expected at these colliders. An exception might be the process $e^+e^- \rightarrow \tilde{\nu}_e\tilde{\nu}_e^* h$ which, because of the large contribution from the $t$–channel chargino exchange, might have cross sections at the level of $\sim 0.2$ fb, for $\chi_1^\pm$ and $\tilde{\nu}_e$ mass values not much beyond the present experimental bounds.

In contrast, the couplings of the $h$ boson to $\tilde{\tau}$ pairs can be sizeable for large values of the parameters $\tan\beta$ and $\mu$, leading to reasonably large $e^+e^- \rightarrow h\tilde{\tau}_1\tilde{\tau}_1^*$ production cross sections for not too heavy $\tilde{\tau}_1$, in particular at the $\gamma\gamma$ option of the $e^+e^-$ collider. The determination of the $h\tilde{\tau}\tilde{\tau}$ coupling would provide a very important information on the SUSY Lagrangian. Indeed, the fact that this coupling is proportional to $\tan\beta$, can be exploited to measure this fundamental parameter which is, otherwise [in contrast to the case of the parameter $\mu$, for instance], very difficult to be determined in other processes when it is rather large$^5$.

$^5$The cross section has, in principle, to be convoluted with the photon spectra; but here for illustration, we will simply tune the energy and the luminosity of the $\gamma\gamma$ collider at the maximum. Note that, in the expression of the differential cross section for $\gamma\gamma \rightarrow h\tilde{\tau}_1\tilde{\tau}_1^*$ given in Ref. $^5$, a color factor is missing so that the cross section is three times larger than shown in the corresponding figure.

$^6$Indeed, the parameter $\tan\beta$ can be directly measured only in the associated production of the pseudoscalar $A$ boson with $b\bar{b}$ pairs $^5$ for small $A$ masses, $M_A \lesssim 100$ GeV. The value of $\tan\beta$ is difficult to determine from chargino/neutralino production $^5$ for $\tan\beta \gtrsim 10$ since, in this case, the observables depend only on $\cos 2\beta$ which becomes flat for $\beta \rightarrow \pi/2$. In sfermion, in particular $\tilde{\tau}$, production $^5$, besides the fact that other parameters [such as the soft–SUSY breaking scalar masses] enter the analysis, one needs enough phase–space to produce both $\tilde{\tau}_1$ and $\tilde{\tau}_2$ which have a large mass splitting in the large $\tan\beta$ and $\mu$ scenario.
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