PROBING DARK ENERGY WITH THE CMB: PROJECTED CONSTRAINTS FROM MAP AND PLANCK

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Submitted to ApJL

ABSTRACT

We investigate the accuracy attainable by forthcoming space-based observations of the cosmic microwave background (CMB) temperature and polarization anisotropy in constraining the dark energy density parameter $\Omega_Q$ and equation of state $w_Q = p_Q/\rho_Q$. Despite degeneracies among parameters, it is possible for high precision observations such as those from MAP and Planck to provide interesting information on the nature of the dark energy. Furthermore, we show that imposing a flat universe constraint makes it possible to obtain tight limits in the space of dark energy parameters even from the CMB alone.

Subject headings: Cosmic microwave background — Cosmology: observations — Cosmology: theory — equation of state

1. INTRODUCTION

Recent cosmological observations have shed new light on the cosmic budget problem. There is now strong evidence from CMB anisotropy measurements (see e.g., Benoit et al. 2002 and references therein) that the universe has a total energy density close to critical (and therefore a flat large scale geometry) and that matter (either luminous or dark) can only account for about 30% of the total density (see e.g., Turner 2002). On the other hand, direct evidence for cosmic acceleration from high redshift type Ia supernovae observations (Riess et al. 1998; Perlmutter et al. 1999) can be interpreted as indication for the existence of a smooth dark energy component with equation of state $w_Q \equiv p_Q/\rho_Q < -1/3$, that would be responsible for the remaining 70% of the critical density. The nature of this dark energy component, however, remains a mystery. In particular, little is known about its equation of state. The simplest possible kind of dark energy is the vacuum energy (or cosmological constant), with $w_Q = -1$ independent of time. However, any plausible scalar field from fundamental theories has a vacuum expectation value that would close the universe by tens of orders of magnitude. More general scalar fields, termed quintessence, may be spatially inhomogeneous, with $w_Q \neq -1$ and varying in time. A number of strategies have been proposed to investigate the nature of dark energy: supernovae observations at multiple redshifts, weak lensing, cluster counting, redshift surveys, Lyman-α forest, CMB anisotropy, as well as combinations of these methods (see e.g. Kujat et al. 2002 and refs. therein).

In this work we investigate the ability of space-based high-resolution CMB anisotropy observations to constrain the dark energy parameters. We focus on the currently underway MAP (http://map.gsfc.nasa.gov) satellite mission and on the forthcoming Planck Surveyor (http://astro.estec.esa.nl/Planck) and produce projections for the model parameter errors attainable by these experiments. We model the dark energy component as a minimally coupled quintessence field following tracking trajectories for an inverse power law potential (Ratra & Peebles 1987; Wetterich 1988). Such a field is in general characterized by two main features: (i) spatial inhomogeneities are on scales comparable to the horizon (whereas in non-minimally coupled theories they can generally appear on all scales) (Perrotta & Baccigalupi 2002)); (ii) the equation of state does not vary in time during the tracking regime, when quintessence is subdominant: in this regime the equation of state is fixed by the attractor solution of the Klein Gordon equation. The equation of state can, however, change significantly at low redshifts, when quintessence is no longer subdominant, and in this case it tends to a cosmological constant behavior, thus having a present equation of state generally smaller than during the tracking regime.

2. METHOD

To quantify how well MAP and Planck can estimate the dark energy parameters, we followed a Fisher information matrix approach (Fisher 1935; see also Tegmark, Taylor & Heavens 1997). The Fisher information matrix is defined as the expectation value

$$F_{ij} \equiv -\left\langle \frac{\partial^2 \log L}{\partial \theta_i \partial \theta_j} \right\rangle$$

where $L$ is the likelihood of the data and $\theta_i$ are the model parameters. The diagonal elements of the inverse Fisher matrix give the minimum variance of model parameters estimated from a given dataset, assuming an underlying fiducial "target" model. This approach relies on the assumption that the likelihood is well approximated by a Gaussian around its peak.

The Fisher matrix depends on the cosmological model and parameters adopted, as well as on the covariance matrix of the data, which in turn depends on the experimental setup. The MAP and Planck experimental parameters assumed in our analysis are summarized in Table 1. We included in the analysis three frequency channels from MAP, three from Planck/LFI and four from Planck/HFI. We fully took into account Planck polarization capabilities. For both experiments, we conservatively assumed that foreground contamination from the galactic plane leaves an observed symmetric strip of $\pm20^\circ$ around the galactic equator, so that the observed fraction of the sky is $f_{\text{sky}} = 0.66$.

We considered inflationary adiabatic cosmological models with 8 free parameters: the present-day dark energy density $\Omega_Q$ and equation of state $w_Q$; the total energy density of the universe $\Omega_0$; the physical baryon and cold dark matter masses $\omega_b \equiv \Omega_b h^2$ and $\omega_cdm \equiv \Omega_{cdm} h^2$ (here $h$ is the present value of the Hubble parameter in units of $100 \text{ km/s/Mpc}$); the primordial spectral index of scalar perturbations $n_s$; the overall amplitude of the CMB power spectrum in units of the COBE normalized $C_{10}$ multipole; the ratio $R$ between the tensor and scalar contribution to the CMB quadrupole. We used a single field inflation consistency relations between the tensor amplitude and
neutrino contribution and reionization optical depth. Note that \( \Omega = \Omega_M + \Omega_Q \), where \( \Omega_M = \Omega_b + \Omega_{cdm} \) is the total matter density: therefore \( h \) is a dependent parameter, determined by the constraint: \( h = [\Omega_b + \Omega_{cdm}]/(\Omega - \Omega_Q)^{1/2} \). The choice of \( \Omega, \omega_b \) and \( \omega_{cdm} \) as free parameters (rather than, for example, \( \Omega_b, \Omega_{cdm} \) and \( h \)) has become usual practice in this sort of analysis. In fact, the combinations \( \omega_b \) and \( \omega_{cdm} \) directly govern the physics of acoustic oscillations which defines the CMB anisotropy pattern, while \( \Omega \) fixes the geometry of the universe and then the angular size of characteristic features on the CMB. For this reason, \( \Omega, \omega_b \) and \( \omega_{cdm} \) are much better constrained by the CMB than other combinations of parameters, and are therefore a more suitable choice in a Fisher matrix analysis (Efstathiou & Bond 1999).

We chose our target to be the flat quintessence model which best fits the currently available CMB data (Baccigalupi et al. 2002). The parameters of this model are summarized in Table 2. To quantify variations around the target model we numerically computed two-sided derivatives of the theoretical CMB angular power spectrum with respect to the parameters, using a step size which was roughly 5\% of the target parameter value. The variation of the total energy density was taken into account through its effect on the angular diameter distance, which, for given \( \omega_b \) and \( \omega_{cdm} \) just results in a shift in multipole space of the CMB angular power spectrum.

### 3. RESULTS

The main effect of a dark energy component on the CMB anisotropy pattern is purely geometric. Varying the dark energy equation of state changes the angular diameter distance (by changing the expansion rate of the universe), resulting in a shift of features in the angular power spectrum of the CMB towards larger angular scales (lower multipoles) as \( \omega_Q \) gets larger than \(-1\). Varying the total energy density of the universe, \( \Omega \), also changes the angular diameter distance, because of the geodesic deviation of CMB photons from recombination to the present. We thus expect to observe a degeneracy between \( \omega_Q \) and \( \Omega \), i.e. a variation in the angular diameter distance due to \( \omega_Q \) can be compensated by an opposite variation due to \( \Omega \). This is just an aspect of the well known geometrical degeneracy (Bond, Efstathiou & Tegmark 1997; Zaldarriaga, Spergel & Seljak, 1997) inherent to any CMB anisotropy measurement. The degeneracy is not exact because different values of \( \omega_Q \) result in a different integrated Sachs-Wolfe contribution at large angular scales: however, this is precisely where the cosmic variance uncertainty on the CMB angular power spectrum is larger. The amount of degeneracy can be quantified by investigating the covariance between \( \omega_Q \) and \( \Omega \), obtained from the 2x2 submatrix of \( F^{-1} \) corresponding to this pair of parameters. In Figure 1 we show the 68\% confidence level constraint in the \( (\Omega, \omega_Q) \) plane obtained with this method. Clearly, varying \( \omega_Q \) even by a considerable amount has a much weaker effect than varying \( \Omega \). As a result, the dark energy equation of state is poorly determined by CMB observations, even though the high sensitivity achievable by Planck allows one to put an upper limit to \( \omega_Q \). On the other hand, the determination of \( \Omega \) is not very much affected by variations in the dark energy equation of state, because of the much stronger dependence of the angular diameter distance on \( \Omega \). Note that these results do not change when CMB polarization is included in the analysis.

Constraints in the space of dark energy parameters \( (\Omega_Q, \omega_Q) \) obtained with the same technique are shown in Figure 2. Again, there exists a strong degeneracy between the two parameters: due to this, MAP is basically unable to distinguish our target model from a cosmological constant case \( \omega_Q = -1 \). The situation improves when a flat universe \( (\Omega = 1) \) is assumed. This additional constraint partially breaks the degeneracy, reducing the allowed region in the dark energy parameter space. The improvement is dramatic for Planck: the confidence level contours

### Table 1

| Parameter     | MAP      | Planck/LFI | Planck/HFI |
|---------------|----------|------------|------------|
| Center frequency (GHz) | 40 60 90 | 44 70 100 | 100 143 217 |
| Angular resolution (FWHM, arcmin.) | 31.8 21 13.8 | 24 14 10 | 9.2 7.1 5.0 |
| \( \sigma^T \) per pixel \( (\times 10^{-6}) \) | 4.1 9.4 21.8 | 2.7 4.7 6.6 | 2.0 2.2 4.8 |
| \( \sigma^P \) per pixel \( (\times 10^{-6}) \) | ... ... ... | 3.9 6.7 9.3 | 4.2 9.8 29.8 |

Note. — Sensitivities to temperature and polarization, \( \sigma^T \) and \( \sigma^P \), are relative to the average CMB temperature (2.73 K). A pixel is a square whose side is the FWHM extent of the beam.

### Table 2

| Parameter | Value |
|-----------|-------|
| \( \Omega_Q \) | 0.7 |
| \( \Omega \) | 1 |
| \( \omega_b \) | 0.022 |
| \( \omega_{cdm} \) | 0.145 |
| \( n_s \) | 1 |
| \( R \) | 0.1 |
| \( h \) | 0.746 |

Note. — \( C_\Omega \) is normalized to COBE.
get closed, enabling accurate determination of the dark energy parameters.

Our results are summarized in Table 3, where we show the projected 1σ error bars on each parameter of our model when marginalizing the others. As it is well known, some cosmological parameters are constrained to very high accuracy by the CMB: notably, the baryon and cold dark matter physical densities, which control the relative peak heights in the angular power spectrum, can be measured to better than percent accuracy. This result does not change considerably due to geometrical effects, and is therefore still valid when different dark energy equations of state are considered. Even in the presence of a strong geometrical degeneracy, Planck is able to set interesting constraints on the dark energy equation of state: our target model \( w_0 = -0.8 \) can be distinguished at 1σ from the cosmological constant case \( w_0 = -1 \) using Planck measurements. Imposing the flat universe constraint \( \Omega = 1 \) results in a decrease of roughly a factor of 2 in the error bars for the dark energy parameters \( w_0 \) and \( \Omega_Q \).

4. DISCUSSION

In this Letter we quantified the capability of MAP and Planck to constrain dark energy cosmologies. The issue of the importance of CMB measurements for dark energy has recently received considerable attention and different interpretations: for example, Baccigalupi et al. (2002) have shown that with a strong prior on the Hubble constant one could constrain the dark energy at the level of 10% even using present CMB data, also obtaining an indication that \( w_0 \approx -0.8 \). On the other hand, Bean & Menci (2002) have shown that when that prior is relaxed, our knowledge of dark energy from CMB data only is still poor.

For the background cosmology, we evaluated the attainable precision of measurements of the dark energy abundance and equation of state, the amount of baryons and cold dark matter, the Hubble constant and the cosmological curvature; for the early universe, we considered the perturbation normalization, the scalar spectral index, and the ratio between tensor and scalar perturbations, by only assuming single field inflation consistency relations between the tensor amplitude and spectral index. We found that the performance of Planck is a factor 4 to 6 better than MAP in all cases. For the tensor to scalar ratio the inclusion of polarization for Planck allows to gain a factor 2 in accuracy, as expected since polarization is directly sensible to tensor perturbations.

As for the dark energy, we confirmed expected parameter degeneracies: indeed, the main effect of \( w_0 > -1 \) on the CMB is to decrease the distance to last scattering; however, such effect can be mimicked by a closed geometry (i.e. \( \Omega > 1 \)). Inclusion of polarization information does not help breaking these degeneracies. On the other hand, despite such drawbacks, our main result is that the capability of Planck to constrain the dark energy is quite good; this statement holds in particular when we compare the Planck forecasted constraints on the dark energy parameters with those from dedicated experiments such as SNAP (http://snap.lbl.gov). The latter is expected to reach a precision of a few percent in both dark energy abundance and equation of state. According to our results, Planck should perform at the level of a few percent accuracy on the dark energy abundance, but of only about 20% on the equation of state, mainly because of the degeneracy with the spatial curvature; if the latter is set to zero (i.e. if \( \Omega = 1 \)), the equation of state can be determined at the level of 10%. We also recall that, as it is well known, contours in the \((\Omega_Q, w_0)\) plane obtained from supernovae measurements are roughly orthogonal to those obtained from the CMB, thus maximizing the accuracy that can be achieved by combining the two methods of observation. Due to the precision attainable by Planck and SNAP, it is reasonable to expect that an accuracy of about 1% on the determination of the dark energy parameters will be obtained by complementing the results of the two experiments.

We point out that the results obtained here do not depend significantly on the target model assumed or on the quintessence details; indeed, we repeated the same analysis by adopting an effective dark energy model, having a constant equation of state, and another target model, with equation of state \( w_0 = -1 \), obtaining similar results.

We conclude that the CMB is one of the most sensitive observables to the main dark energy parameters, i.e. its abundance and equation of state. By combining CMB observations with those from dedicated experiments such as SNAP, we can expect to measure the equation of state at the level of percent. Finally we also recall that the use of CMB for any cosmological measurement, including dark energy, has the great advantage that CMB perturbations are linear and therefore relatively simple to describe. Any other observable from large scale structure needs to fully understand the non-linear structure formation in dark energy cosmology.

AB thanks Paolo Natoli for useful discussions.

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FIG. 2.— 68% confidence level constraints in the dark energy parameters space \((\Omega_Q, w_Q)\). Left: the shaded regions (lighter to darker) are obtained from MAP, Planck (temperature only) and Planck (temperature and polarization) respectively. Right: same contours, when we impose the flat universe constraint \((\Omega = 1)\).

**Table 3**

**Marginalized 1σ Errors**

| Parameter   | MAP          | Planck, T only | Planck, T and P |
|-------------|--------------|----------------|-----------------|
| \(\Delta w_Q\) | 0.81 (0.56)  | 0.17 (0.075)   | 0.14 (0.062)    |
| \(\Delta \Omega_Q\) | 0.22 (0.15) | 0.044 (0.020)  | 0.037 (0.016)   |
| \(\Delta \Omega\) | 0.0086 (· · ·) | 0.0017 (· · ·) | 0.0014 (· · ·)  |
| \(\Delta \omega_b\) | 0.00039 (0.00038) | 9.6 \times 10^{-5} (9.6 \times 10^{-5}) | 8.2 \times 10^{-5} (8.2 \times 10^{-5}) |
| \(\Delta \omega_{cdm}\) | 0.0041 (0.0040) | 0.0018 (0.0015) | 0.0012 (0.0011) |
| \(\Delta n_s\) | 0.011 (0.011) | 0.0042 (0.0034) | 0.0031 (0.0027) |
| \(\Delta C_{10}/C_{10}\) | 0.044 (0.044) | 0.041 (0.041)   | 0.020 (0.020)   |
| \(\Delta R\) | 0.071 (0.071) | 0.054 (0.053)   | 0.025 (0.024)   |
| \(\Delta h\) | 0.29 (0.20) | 0.061 (0.028) | 0.051 (0.022) |

Note. — Errors relative to the target model of Table 2. The number in parentheses were obtained by imposing the additional constraint \(\Omega = 1\).

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