The Properties of $D_{s1}^*(2700)^+$

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Abstract

The new particle $D_{s1}^*(2700)^+$ has stimulated many attentions. There are different assignments of its inherent properties. It may be a $2^3S_1$, $1^3D_1$ or the mixture of $2^3S_1 - 1^3D_1$ $c\bar{s}$ $1^-$ state. By considering its mass, decay modes, full width, production rate, and comparing with current experimental data, we point out that there is another more reasonable assignment: $D_{s1}^*(2700)^+$ could be identified as two resonances, one of which is a $2^3S_1$ state, another is a $1^3D_1$ state, and both are $c\bar{s}$ $1^-$ states. The two states have very close masses, which are around 2700 MeV, and both have broad decay widths. So in experiments, the overlapping of $DK$ or $D^*K$ invariant mass distribution coming from their decays is found, but the current experiments could not distinguish these two resonances and reported one particle.

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I. INTRODUCTION

In 2006, BABAR Collaboration first reported a broad structure $D_{sJ}^+(2690)$ with mass $2688 \pm 4 \pm 3$ MeV and full width $\Gamma = 112 \pm 7 \pm 36$ MeV \cite{1}, almost at the same time, Belle Collaboration also reported a new strange charmed $1^-$ state $D_{s1}^+(2700)$, with mass $M = 2715 \pm 11^{+11}_{-14}$ MeV and width $\Gamma = 115 \pm 20^{+36}_{-30}$ MeV \cite{2}, later modified to $M = 2708 \pm 9^{+11}_{-10}$ MeV and $\Gamma = 108 \pm 23^{+36}_{-32}$ MeV \cite{3}. In 2009, BABAR showed their new observation of $D_{s1}^+(2700)$ \cite{4}: $M = 2710 \pm 2^{+12}_{-7}$ MeV and a broad full width $\Gamma = 149 \pm 7^{+39}_{-52}$ MeV. Recently, LHCb Collaboration confirmed the existence of the $D_{s1}^+(2700)$ and measured its mass and width to be $M = 2709.2 \pm 1.9 \pm 4.5$ MeV, $\Gamma = 115.8 \pm 7.3 \pm 12.1$ MeV \cite{5}.

Based on different models, interested theorists gave different assignments of $D_{s1}^+(2700)$. In the works of Refs. \cite{6,7}, authors consider the possibility of it to be a multiquark configuration, but their conclusions are negative. Most authors believe that it is a conventional $c\bar{s}$ state. For example, in Refs. \cite{7–12}, the first radial excited $2^3S_1$ $(2S)$ state is favored, while in Ref. \cite{13}, the orbitally excited $1^3D_1$ $(1D)$ state is expected, and others \cite{14–16} believe it is most likely a mixture of $2^3S_1 − 1^3D_1$.

Although its existence has been confirmed by three different experiments through different production mechanism, and the data of its mass and full width agree with each other’s within their error bars, $D_{s1}^+(2700)$ is not currently reported in the summary table of the Particle Data Group. This shows that more careful experimental researches and theoretical studies are still needed. In this paper, we give another assignment of $D_{s1}^+(2700)$. We claim that the current detected $D_{s1}^+(2700)$ is not a single state, but could be two overlapping states. One of them is the $2S$ dominant state with a little $D$ wave mixed in it. Throughout the whole paper, we use the symbol $D_{s1}^{++}(2S)$ to label this $2S$ dominant state, and since there may be confusion, it will be also called low-mass state. The other is $1D$ dominant state with a little $S$ wave mixed in it, we use the symbol $D_{s1}^{++}(1D)$ for it, or high-mass state if there is confusion. Because the invariant mass distributions describing these two resonances overlap together, it is equal to say the overlapping states are mixtures of pure $2S$ state and pure $1D$ state.

There are several reasons resulting in this overlapping assignment of two states, we will show these in different aspects. In section 2, we give the masses and wave functions of $1^− c\bar{s}$ states; in section 3, we show the decay modes and relative branching ratios; the production rates of $2S$ and $1D$ states in $B$ decays is given in section 4; finally we do some discussions and make a conclusion in the last section.
II. MASSES AND WAVE FUNCTIONS

Theoretically, the \( c \bar{s} \) \( 2S \) and \( 1D \) states have similar masses, and the same quantum number \( J^P = 1^- \), so they mix together, and become other two different states. There are many examples, such as \( \Psi(2S) \) and \( \Psi(3770) \). \( \Psi(3770) \) is a \( 1D \) dominant state but mixed with some contribution from \( 2S \) wave, famous as the \( 2S - 1D \) mixing state (there may be some contributions from \( 1S \) and \( 3S \) states, see Ref. [17] for example), while \( \Psi(2S) \) is the orthogonal partner of \( \Psi(3770) \), and it is \( 2S \) dominant state with a little \( D \) wave mixed in it. After mixing, two pure \( 2S \) and \( 1D \) states change into another two mixing states:

\[
|D^+_1(\text{low})\rangle = \cos \theta \ |2^3S_1\rangle - \sin \theta \ |1^3D_1\rangle,
\]

\[
|D^+_1(\text{high})\rangle = \sin \theta \ |2^3S_1\rangle + \cos \theta \ |1^3D_1\rangle,
\]

where the mixing angle \( \theta < \pi \), and because of \( S \) wave dominant, \( D^+_1(\text{low}) \) is the low-mass mixing state, while \( D^+_1(\text{high}) \) is \( D \) wave dominant high-mass state.

The method above of mixing seems sort of artificial, so we do not choose this method. Instead, we believe that the reason of mixing is \( 2S \) and \( 1D \) states have the same quantum number of \( J^P = 1^- \), and the forms of their wave functions are similar for the same reason. When we choose a relativistic method based on quantum field theory to deal with the bound state problem, the \( 2S \) and \( 1D \) states will be obtained, and the mixing should be exist automatically, not like man-made by hand. The Bethe-Salpeter equation [18] is such a relativistic method to describe a bound state, by solving it, the eigenvalue and relativistic wave function for a bound state will be obtained. So we choose the ordinary Cornell potential, and solve the exact instantaneous Bethe-Salpeter equations (or the Salpeter equations [19]) for \( 1^- \) states. As expected, \( 2S \) dominant state mixed with a little \( D \) wave component and \( 1D \) dominant state mixed with a little \( S \) wave component are obtained. We will outline the key point in this section.

The general form of a relativistic wave function for a vector meson (\( 2S \) or \( 1D \), or their mixing states) can be written as 16 terms constructed by \( P \), \( q \), \( \epsilon \) and the gamma matrices. Because we make the instantaneous approximation of the BS method, the 8 terms with \( P \cdot q_\perp \) vanished. So the general form for the relativistic Salpeter wave function which has the quantum number of \( J^P = 1^- \) for a vector state can be written as [20, 21]:

\[
\varphi^\Lambda_1^\Lambda(q_\perp) = q_\perp \cdot \epsilon^\Lambda_\perp \left[ f_1(q_\perp) + \frac{P}{M} f_2(q_\perp) + \frac{q_\perp}{M} f_3(q_\perp) + \frac{P q_\perp}{M^2} f_4(q_\perp) \right] + M \varphi^\Lambda_1^\Lambda f_5(q_\perp)
\]
in Figure 1-3. The results show that, each state has four different wave functions, their masses are 2669 MeV and 2737 MeV.

One can see that our predictions for the mass of the vector states appear in seemingly. But if we write them in spherical polar coordinates, there are wave components mixed in. So the third state is a \( D^* \) wave dominant state, 1\( ^- \)S wave dominant, while the \( D \) wave components, \( q^2 f_3/M^2 \) and \( q^2 f_4/M^2 \) (\( f_3 \) and \( f_4 \) always show up followed by \( q^2/M^2 \)) are neglected. So we conclude that the first one with mass 2112 MeV is the ground state \( D^*_s \), and the second one with a node structure is the 2\( S \) dominant state. For the third state, see figure 3, \( q^2 f_3/M^2 \), \( q^2 f_4/M^2 \) and \( f_5, f_6 \) are all sizable, and they look all like \( D \) waves on seemingly. But if we write them in spherical polar coordinates, there are sizable \( S \) wave components mixed in \( D \) wave (the \( Y_{20} \) term of \( B_\lambda \) in Eq. (21) in Ref. [20]), so the third state is a \( D \) wave dominant state.

As for the \( S \) wave dominant states, 1\( S \) and 2\( S \) states (see Figures 1-2), we have the rough relations \( f_5 = -f_6 \) and \( f_3 = -f_4 \). If we delete the negligible terms \( f_3 \) and \( f_4 \) in Eq. (2), then the vector wave function is changed to the non-relativistic case \( \varphi_{f_1}^\lambda(q) = (M + P)\varphi_{f_1}^\lambda f_5(q) \).

But if we solve the Salpeter equation with this non-relativistic wave function form as input, as expected, we only obtain the 1\( S \), 2\( S \), 3\( S \) states, \( \text{et al.} \), and no \( D \) wave states appear in

\[
+ \varphi_{f_1}^\lambda P f_6(q) + (q \cdot \varphi_{f_1}^\lambda - q \cdot \varphi_{f_1}^\lambda) f_7(q) + \frac{1}{M} (P \varphi_{f_1}^\lambda q - P q \cdot \varphi_{f_1}^\lambda) f_8(q),
\]

where the \( P, q \) and \( \varphi_{f_1}^\lambda \) are the momentum, relative inner momentum and polarization vector of the vector meson, respectively; \( f_i(q) \) is a function of \(-q^2 \), and we have used the notation \( q^\mu \equiv q - (P \cdot q/M^2)P^\mu \) (which is \((0, \vec{q})\) in the center of mass system).

In the method of instantaneous BS equation, the 8 wave functions \( f_i \) are not independent. The constrain equations [21] result in the relations

\[
f_1(q) = \frac{[q^2 f_3(q) + M^2 f_5(q)](m_1 m_2 - \omega_1 \omega_2 + q^2)}{M (m_1 + m_2) q^2}, \quad f_7(q) = \frac{f_5(q M(-m_1 m_2 + \omega_1 \omega_2 + q^2)}{(m_1 - m_2) q^2},
\]

\[
f_2(q) = \frac{[-q^2 f_4(q) + M^2 f_6(q)](m_1 m_2 - \omega_1 \omega_2)}{M (\omega_1 + \omega_2) q^2}, \quad f_8(q) = \frac{f_6(q M(m_1 m_2 - \omega_2)}{(\omega_1 - \omega_2) q^2}.
\]

With this form of wave functions, we solved the full Salpeter equation for 1\( ^- \) states numerically. For strange quark, we choose the constitute quark mass \( m_s = 500 \text{ MeV} \); for the parameter \( V_0 \), we give it by fitting the ground state mass \( M(D^*(2112)) = 2112 \text{ MeV} \); for other parameters, we choose the same values as given in Ref. (22). The mass spectra of \( c\bar{s} \) and \( c\bar{c} \) systems are shown in Table I, where we also show the experimental data of \( c\bar{c} \) system from Particle Data Group [23]. One can see that our predictions for the mass of \( c\bar{c} \) 1\( ^- \) system fit the data very well. For \( c\bar{s} \) vector states, we found there are two states around 2700 MeV, their masses are 2669 MeV and 2737 MeV.

To see the nature of these states, we draw the wave functions of the first three states in Figure 1-3. The results show that, each state has four different wave functions, \( f_3, f_4, f_5 \) and \( f_6 \). For the first two states, the numerical values of \( S \) wave functions \( f_5 \) and \( f_6 \) are dominant, while the \( D \) wave components, \( q^2 f_3/M^2 \) and \( q^2 f_4/M^2 \) (\( f_3 \) and \( f_4 \) always show up followed by \( q^2/M^2 \)) are neglected. So we conclude that the first one with mass 2112 MeV is the ground state \( D^*_s \), and the second one with a node structure is the 2\( S \) dominant state. For the third state, see figure 3, \( q^2 f_3/M^2 \), \( q^2 f_4/M^2 \) and \( f_5, f_6 \) are all sizable, and they look all like \( D \) waves on seemingly. But if we write them in spherical polar coordinates, there are sizable \( S \) wave components mixed in \( D \) wave (the \( Y_{20} \) term of \( B_\lambda \) in Eq. (21) in Ref. [20]), so the third state is a \( D \) wave dominant state.
TABLE I: Our predictions for the masses (in unit of MeV) for the $1^{-} \bar{c}c$ and $c\bar{s}$ states, where the ground state masses of $M(D^{*}_{s}) = 2112.0$ MeV and $M(J/\Psi) = 3096.9$ MeV are input.

|       | 1S     | 2S    | 1D    | 3S    | 2D    |
|-------|--------|-------|-------|-------|-------|
| Th($\bar{c}c$) | 3096.9 (input) | 3688.1 | 3778.9 | 4056.8 | 4110.7 |
| Ex($\bar{c}c$)  | 3096.916 | 3686.093 | 3772.92 | 4040 | 4159 |
| Th($c\bar{s}$)  | 2112.0 (input) | 2669.0 | 2737.3 | 2994.3 | 3033.2 |

FIG. 1: Wave functions for the ground state $D^{*}_{s}(1S)$.

this non-relativistic case. In this case, the $D$ wave state has to be dealt with another wave function form as input and the mixing should be treated as the method shown in Eq. (1).

Many authors have studied the mass spectra of $c\bar{s}$ 1$^{-}$ states using different models. We list some of them which have both the masses of $2S$ and $1D$ in Table III. Except our results, all of the predictions are pure $2S$ and $1D$ states, and the mass shift $\Delta M = M(1^{3}D_{1}) - M(2^{3}S_{1})$ between $2S$ and $1D$ is also shown in Table III, the values of mass shifts lie in the region from 62 MeV to 170 MeV. We notice that in the mass region of 2600 $\sim$ 3000 MeV which has been scanned by experiments, currently there is no $1^{-}$ candidate except $D^{*}_{s1}(2700)^{+}$. The $1^{-}$ candidate $D^{+}_{sJ}(2632)$ observed by SELEX Collaboration [30] has not been confirmed by BABAR [31], CLEO and FOCUS [32], and its mass of 2632 MeV seems a little small as a
FIG. 2: Wave functions for the first excited state $D_s^*(2S)$.

TABLE II: Mass predictions for $D_s^{*+}(2S)$ and $D_s^{*+}(1D)$ vector states in unit of MeV, where $\Delta M = M(1^3D_1) - M(2^3S_1)$.

|          | ours | Godfrey [24] | Zeng [25] | Lahde [26] | Pierro [27] | Close [8] | Li [14] | Matsuki [28] | Nowak [29] |
|----------|------|--------------|-----------|------------|-------------|-----------|--------|--------------|------------|
| $2^3S_1$ | 2669 | 2730         | 2730      | 2722       | 2806        | 2711      | 2653   | 2755         | 2632       |
| $1^3D_1$ | 2737 | 2900         | 2900      | 2820       | 2845        | 2913      | 2784   | 2775         | 2817       |
| $\Delta M$ | 68   | 170          | 90        | 123        | 107         | 73        | 122    | 62           | 88         |

2S or 1D state (see Table II).

III. DECAY MODES AND RELATIVE RATIO

Both 2S and 1D $c\bar{s} \, 1^-$ vector states around 2700 MeV (or higher) can decay to $D^0K^+$, $D^+K^0$, $D^{*0}K^+$, $D^{*+}K^0$, $D^+_s\eta$ and $D^{*+}\eta$. Comparing with other possible decay channels, these six have dominant branching ratios because they are all OZI allowed strong decays. So one can use them to estimate the full widths of 2S and 1D.

In a previous Letter [33], we have already calculated the pure $2^3S_1$ or $1^3D_1 \, c\bar{s} \, D_s(2632)$ state decaying to $D^0K^+$, $D^+K^0$ and $D^+_s\eta$ [33], where we used the reduction formula, PCAC.
relation and low energy theorem, thus the transition \( S \)-matrix is given as a formula involving the light meson decay constant and the corresponding transition matrix element between two heavy mesons. The transition matrix element is written as an overlapping integral of the relevant wave functions, which are obtained numerically by solving the Salpeter equation with further non-relativistic approximation. In this paper, we re-calculate the OZI allowed decay modes but with full wave functions as input, and use the new predicted masses of \( 2S \) and \( 1D \) in Table I as input, so three more channels are opened, and the results are shown in Table III, where the errors in our results are obtained by varying all the input parameters simultaneously within \( \pm 5\% \). We do not repeat the calculation here, interested reader can find the details in Ref. [33].

One can see that, both \( D_{s}^{*+}(2S) \) and \( D_{s}^{*+}(1D) \) have broad full widths, since they have at least 6 OZI allowed strong decay channels. The full width of \( D_{s}^{*+}(1D) \) is much broader than that of \( D_{s}^{*+}(2S) \), which is caused by two reasons. One is the node structure of \( 2S \) wave function (see Figure 2): the \( 2S \) wave function before the node provides positive contribution to the width, while after the node it gives negative contribution. The \( D_{s}^{*+}(1D) \) state does not suffer from the node structure since it’s wave function has no node structure as a \( 1D \) state. The other reason is that the higher mass state \( D_{s}^{*+}(1D) \) has larger phase space. Thus the full width of \( D_{s}^{*+}(1D) \) is broader than that of \( D_{s}^{*+}(2S) \).
TABLE III: Decay widths of 2S dominant and 1D dominant states in unit of MeV, and the last column is the summed width of these decay channels.

|          | $D^0K^+$ | $D^+K^0$ | $D^{*0}K^+$ | $D^{*+}K^0$ | $D_s^+\eta$ | $D_s^{*+}\eta$ | total widths |
|----------|----------|----------|-------------|-------------|-------------|----------------|-------------|
| $D_s^{*+}(2S)$ | $8.9 \pm 1.2$ | $8.7 \pm 1.2$ | $12.2 \pm 1.7$ | $11.6 \pm 1.7$ | $11.1 \pm 1.8$ | $3.9 \pm 0.3$ | $4.9 \pm 0.05$ | $46.4 \pm 6.2$ |
| $D_s^{*+}(1D)$ | $23.3 \pm 3.2$ | $21.5 \pm 3.1$ | $12.7 \pm 1.9$ | $11.1 \pm 1.8$ | $3.9 \pm 0.3$ | $4.9 \pm 0.05$ | $73.0 \pm 10.4$ |

TABLE IV: Decay widths (MeV) and their ratios with 2S and 1D assignments of $D_s^*(2700)^+$ as well as the full width (MeV), our results are from the cases of 2S dominant and 1D dominant assignments.

|          | $D_s^*(2700)^+$ | $DK$ | $D^*K$ | $Br(D_s^*(2700)^+\rightarrow D^*K)$ | $Br(D_s^*(2700)^+\rightarrow DK)$ | Full width |
|----------|-----------------|------|--------|----------------------------------|----------------------------------|------------|
| ours     | 2S              | 17.6 | 23.8   | 1.35                             | 46.4                             |
| Close [8] | 2S             | 22   | 78     | 3.55                             | 103.8                            |
| Zhang [13] | 2S             | 3.2  | 27.2   | 8.5                              | 32.0                             |
| Colangelo [9] | 2S         | 11   | 18.1   | 1.65                             | 31.0                             |
| Zhong [16]  | 2S             | 4.4  | 34.9   | 7.9                              | 41.4                             |
| ours       | 1D             | 44.8 | 23.8   | 0.53                             | 73.0                             |
| Zhang [13]  | 1D             | 49.4 | 13.2   | 0.27                             | 73.0                             |
| Colangelo [9] | 1D         |      |        | 0.043                            |                                  |
| Zhong [16]  | 1D             | 148.6| 36.3   | 0.24                             | 200.0                            |
| Li [15]     | 1D             | 86.8 | 37.2   | 0.43                             | 138.2                            |

BABAR recently measured the ratios of branching fractions [4]:

$$\frac{Br(D_s^*(2700)^+\rightarrow D^*K)}{Br(D_s^*(2700)^+\rightarrow DK)} = 0.91 \pm 0.13 \pm 0.12. \quad (3)$$

We listed theoretical predictions of these two decay widths and their ratios with 2S and 1D assignments of $D_s^*(2700)^+$ in Table IV. One can see that in the current existing theoretical predictions, except the Colangelo’s result, all the ratios of 2S assignments are larger than 1.3, and all the ratios of 1D assignments are smaller than 0.5, so neither the pure 2S nor pure 1D assignments of $D_s^*(2700)^+$ consist with BABAR’s data. This data favor sizable 2S component and sizable 1D component in $D_s^*(2700)^+$, so only the assignments of two overlapping states or mixing state can match the experimental data.
IV. PRODUCTION IN BELLE

The Belle Collaboration [3] have detected the production of $D_{s1}^*(2700)^+$ which is produced in $B^+$ exclusive decay, and the branching fraction is:

$$Br(B^+ \rightarrow \bar{D}^0 D_{s1}^*(2700)^+) \times Br(D_{s1}^*(2700)^+ \rightarrow D^0 K^+) = (1.13^{+0.26}_{-0.36}) \times 10^{-3}. \quad (4)$$

In Ref. [12], we calculated the production rate of $D_{s1}^*(2700)^+$ in $B^+$ strong decay with two assignments, $D_{s1}^*(2700)^+$ is a $2S$ dominant state $D_{s}^{*+}(2S)$ which is mixed with a little bit of $D$ wave component, or it is a $1D$ dominant state $D_{s}^{*+}(1D)$ with small $S$ wave mixed in it. With these assignments, we obtained

$$Br(B^+ \rightarrow \bar{D}^0 D_{s}^{*+}(2S)) = (0.72 \pm 0.12)\%, \quad (5)$$

$$Br(B^+ \rightarrow \bar{D}^0 D_{s}^{*+}(1D)) = (0.027 \pm 0.007)\%. \quad (6)$$

These results are exactly proportional to the square of decay constant of $D_{s}^{*+}(2S)$ or $D_{s}^{*+}(1D)$, so the results are very sensitive to the values of decay constants. We know that usually the decay constant of $2S$ state is much larger than that of $1D$ state. With further calculations we gave the product of ratios [12]

$$Br(B^+ \rightarrow \bar{D}^0 D_{s}^{*+}(2S)) \times Br(D_{s}^{*+}(2S) \rightarrow D^0 K^+) = (1.4 \pm 0.5) \times 10^{-3} \quad (7)$$

and

$$Br(B^+ \rightarrow \bar{D}^0 D_{s}^{*+}(1D)) \times Br(D_{s}^{*+}(1D) \rightarrow D^0 K^+) = (0.9 \pm 0.3) \times 10^{-4}. \quad (8)$$

The former is consistent with Belle’s data, and the later is about one order smaller. These results show that the production of $2S$ state $D_{s}^{*+}(2S)$ is favored, while $1D$ state $D_{s}^{*+}(1D)$ is suppressed in the experiment of Belle.

The new state $D_{s1}^*(2700)^+$ detected by Belle is based on an analysis of $B \bar{B}$ events collected at the $\Upsilon(4S)$ resonance. While in the experiment of BABAR, their analysis of $D_{s1}^*(2700)^+$ is based on data sample recorded at the $\Upsilon(4S)$ resonance and 40 MeV below the resonance, and the background from $e^+e^- \rightarrow B \bar{B}$ events is removed by requiring the center of mass momentum $p^*$ of the $DK$ or $D^*K$ system to be greater than 3.3 GeV. Unlike Belle or BABAR, LHCb Collaboration try to made sure that the candidates of $D_{s1}^*(2700)^+$ are produced in the primary $pp$ interaction, and reduces the contribution from particles originating from $b$–hadron decays. It is not clear to us whether the $1D$ state $D_{s}^{*+}(1D)$ is suppressed
or not in the experiments of BABAR and LHCb, but according to the broad full width
\[ \Gamma = 149 \pm 7^{+30}_{-52} \text{ MeV} \] of \( D_{s1}^{*}(2700)^{+} \) and the ratio shown in Eq. (3) detected by BABAR, it
seems that there is no suppression of 1D state \( D_{s}^{+(1D)} \). We also point out that it is crucial
to detect the ratio
\[ \frac{\text{Br}(D_{s1}^{*}(2700)^{+} \to D^{*}K)}{\text{Br}(D_{s1}^{*}(2700)^{+} \to DK)} \]
in Belle and LHCb to see if the 1D state is suppressed or not.

V. DISCUSSION AND CONCLUSION

It is known that there exist the 2S and 1D vector states, and their masses are usually close
to each other. In the case of \( c\bar{s} \) bound states, the mass predictions by different theoretical
models are listed in Table II, and we did find that most of the predicted mass shifts \( M(1D) - M(2S) \) are small. In Table III and Table IV, theoretical models also show that both 2S and 1D are broad states. Two states whose full widths are all broad with closed masses
means that we will find overlapping in \( DK \) or \( D^{*}K \) invariant mass distributions in the mass
regions of 2S and 1D \( c\bar{s} \) states. Some models, for example, Godfrey estimated a higher
mass of 1D state with large mass shift \( M(1D) - M(2S) \), but we argue that higher 1D mass
will result in broader full width. And currently, the scanned results in experiment show
that there is no other 1− state candidate in the mass region from 2600 MeV to 3000 MeV.
One may argue that the production of 1D state may be suppressed like in Belle, but the 3−
state of 2860 MeV is already found, there is no reason to believe that it is more difficult to
produce a 1D state than to produce a 3− state.

For the pure 2S state assignment of \( D_{s1}^{*}(2700)^{+} \), theoretically, except for Ref. [8], which
gives a large 2S full width of 103 MeV, all the predictions show that the pure 2S whose
full width ranges from 31 to 46 MeV is too narrow to fit data, and the ratios of \( D_{s1}^{*}(2700)^{+} \) decays to \( D^{*}K \) over those to \( DK \) can not match data except the result of Colangelo.
For a pure 1D assignment, though the predicted full widths can match the data, we still can
not explain the ratio of \( \frac{\text{Br}(D_{s1}^{*}(2700)^{+} \to D^{*}K)}{\text{Br}(D_{s1}^{*}(2700)^{+} \to DK)} \) detected by BABAR, and we can not explain why we
found the 1D state, but the 2S state is not found since the later is favored while the former
is suppressed in Belle. So the pure 2S and pure 1D assignment of \( D_{s1}^{*}(2700)^{+} \) can be ruled out.

The other possibility is that \( D_{s1}^{*}(2700)^{+} \) is a 2S − 1D mixing state. As shown in the BS
relativistic method, the reason of mixing is that they have the same quantum number of 1−,
and the way of mixing may not follow the the method shown in Eq. (1). Even we believe
that it is correct, i.e, two states 2S and 1D mix together, and after mixing, they turn into
another two mixing states. Experiment find one of them, the open question is, where is the orthogonal partner of this found mixing state? One may argue that the missing one may be far away from the region of 2700 MeV. But if it is true, then there is no reason to mix together.

So the most reasonable assignment is that \( D_{s1}^+(2700) \) is not one single state, but two overlapping states, \( D_{s1}^{+(2S)} \) and \( D_{s1}^{+(1D)} \), both of them are broad states and there is a narrow mass gap between them. So the \( DK \) or \( D^*K \) invariant mass distributions from their decays overlap together, and then one single state \( D_{s1}^*(2700)^+ \) with a broader full width is detected and reported in experiments. We point out that, with same quantum number \( J^P = 1^- \) and close masses, the two broad overlapping states can not be easily distinguished by angular analysis in experiments. By this two overlapping states assignment, the experimental data can be explained easily. If we set the possibilities of production of \( D_{s1}^{+(2S)} \) and \( D_{s1}^{+(1D)} \) are same, we roughly estimate that a state with mass \( M = 2703 \) MeV and full width \( \Gamma = 127.7\pm8.3 \) MeV will be detected by experiments, and the predicted ratio is \( \frac{Br(D_{s1}^+ \rightarrow D^*K)}{Br(D_{s1}^+ \rightarrow DK)} = 0.76^{+0.25}_{-0.19} \), all these values consist with the experimental data.

In summary, from a study of relativistic BS method, we find around mass region 2700 MeV, there are two \( c\bar{s} \) states, \( D_{s1}^{+(2S)} \) and \( D_{s1}^{+(1D)} \), which have the same quantum number \( J^P = 1^- \) and similar masses. Theoretical calculations show that both of them have broad full width, so they overlap together. This two states assignment of \( D_{s1}^*(2700)^+ \) can fit data very well.

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