New light on cavity QED with ultracold atoms

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Abstract. We give an overview on quantum electrodynamics (QED) with atoms located near a planar surface. Starting from basic concepts like electric and magnetic dipole transitions and electromagnetic near fields, we explain some properties of magnetic fields that are relevant for ultracold atoms in hybrid electromagnetic-surface traps. In particular, we discuss next order corrections to the atom-surface interaction potential due to the magnetic moment.

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1. Introduction
Quantum electrodynamics (QED) is one of the most precise physical theories today. A number of intriguing effects have been predicted as a consequence of field quantization and the corresponding electromagnetic vacuum fluctuations. Regarding the Lamb shift of the hydrogen atom and the anomalous magnetic moment of the electron, for example, extremely accurate results have been obtained both in theory and experiment.

The paradigm of cavity QED is that vacuum fluctuations can be manipulated by imposing boundary conditions on the field [1]. This gives geometry-dependent energy shifts (for example the Van der Waals interaction between polarizable objects) and may also enhance single photon effects. The development of cavity QED is partly directed towards realizing simple model systems where the relevant field modes are reduced to a few, using for example high-$Q$ cavities [2, 3]. This permits the exploration of the quantized nature of the field modes and their coupling to individual atoms. Subsequently, a high level of control over the time evolution of these systems can be achieved, opening up the possibility of encoding quantum information in them and to process it in a coherent way. Another bundle of activities is focused on the modified vacuum-induced atomic level shifts and decay rates in the vicinity of macroscopic objects [4, 5, 6]. In the context of “atom chips” [7, 8, 9], hybrid structures that combine electromagnetic trapping fields with macroscopic surfaces, structured on the micron scale or below, one is confronted with the unique possibility that both branches of cavity QED merge. On the one hand, few field modes can be selected by integrating micro-optical components onto the chip, and quantum information processing by manipulating and detecting single atoms is within reach of state-of-the-art experiments [10]. Atom-surface interactions mediated by the quantized electromagnetic field, on the other hand, play a crucial role for the shape and stability of the trapping potentials [11, 12]. One of the main tools that emerges
in this context is the propagation dynamics of atomic de Broglie waves: at the ultralow temperatures that are reached with Bose-Einstein condensates tiny potentials due to the surface become measurable because they significantly change the wave functions for the atomic centre-of-mass.

In Sec. 2 of this paper, we review the phenomenology of atom-surface interactions, and summarize the properties of atoms and surfaces relevant in the context of ultracold matter wave manipulation. We consider in particular magnetic fields and their vacuum fluctuations in the vicinity of macroscopic structures [13, 14, 15]. This is motivated by recent experimental confirmations that resonant spin flip transitions driven by thermal near field fluctuations kick atoms out of magnetic traps [16, 17, 18]. We recall that close to metallic surfaces, the magnetic energy density can be orders of magnitude larger than the electric one [19]. This has led us to investigate the magnetic dipole contribution to the atom-wall interaction (called Van Der Waals-London or Casimir-Polder potential). As outlined in Sec. 3 we find that this contribution is small compared to the well-known electric dipole coupling, mainly because of the large difference between the electric and magnetic polarizabilities, but it is repulsive. We speculate that this correction may be measurable in the context of quantum reflection of ultraslow atoms from the attractive atom-surface potential because this process sensitively depends on the strength of the atom-wall interaction [20, 21, 22].

2. Atom-field coupling

2.1. Electric and magnetic dipoles

An atomic system with its nucleus surrounded by bound electrons, is a globally neutral system and for many purposes, it is a good approximation to characterize its interaction with the electromagnetic field in terms of its lowest multipole moments [23, 24]. Usually, the electric dipole moment gives the dominant contribution, followed by the one from the magnetic dipole. This is because of the very different magnitudes of the dipole matrix elements and of the Bohr frequencies of the associated transitions (E1 and M1) between atomic levels.

In hydrogen-like atoms, for example, E1 transitions connect the ground state $|g\rangle$ to states $|e\rangle$ in the electronically excited manifold with frequencies $\omega_{ge} / 2\pi$ in the visible range ($10^{15}$ Hz) and matrix elements of the order of the Bohr radius, $\langle e | d_i | g \rangle \sim e a_0$. M1 transitions occur among the magnetic and hyperfine components $|m\rangle$ of the ground state manifold, with transition frequencies $\omega_{gm} / 2\pi$ in the MHz (for typical static fields) or GHz (hyperfine splitting) range and matrix elements of the order of the Bohr magneton, $\langle m | \mu_i | g \rangle \sim \mu_B = e \hbar / 2m$. Hence the dimensionless ratio $\mu_i / c d_i \sim \alpha_{fs}$ is of the order of the fine structure constant.

2.2. Atom-surface interaction

Atom-wall interactions are usually computed in second order perturbation theory [25, 26]. Typically, the magnetic contribution is neglected with the argument that this dipole moment is much smaller. However, in the perturbation series, one has to compare terms of order $d^2 / (\omega + \omega_{ge})$ and $\mu^2 / (\omega + \omega_{gm})$ where $\omega$ is the frequency of a virtual photon. The magnetic contribution may thus gain in magnitude because of the much smaller transition frequency. In the limit $\omega_{gm} \to 0$, the divergence leads to the appearance of a permanent magnetic dipole moment, $\langle g | \mu_i | g \rangle \neq 0$. This is forbidden for the electric dipole moment if parity is conserved, so that under these conditions the electric dipole is an induced one.

Following the approach of Wylie and Sipe [26, 27], the magnetic contribution to the atom-surface interaction potential can be written in the following form

$$ U_m(r) = -\hbar \text{Im} \text{Tr} \int_0^\infty \frac{d\omega}{2\pi} \beta(\omega) \cdot \mathcal{H}_S(r, r; \omega) $$

(1)

The electric quadrupole interaction is, in free space, of the same order as the magnetic dipole coupling. This changes close to a metallic surface where magnetic fields are dominant (see Section 2.3), so as a first step, we neglect the electric quadrupole.
\[ \beta_{ij}(\omega) = \sum_m \frac{2\omega_{mg}}{\hbar(\omega_{mg}^2 - \omega^2)} \langle g|\mu_i|m \rangle \langle m|\mu_j|g \rangle \]  

(2)

where \( \beta \) is the magnetic polarizability tensor and the quantum number \( m \) labels atomic states connected to the ground state \( |g\rangle \) by magnetic dipole transitions. The scattered part of the magnetic Green tensor, \( \mathcal{H}_S \), describes the part of the field of a magnetic point dipole at \( \mathbf{r} \) that is due to reflections from the surface. The more familiar electric energy shift \( U_e \) is obtained by an analogous expression, using the (scattered) electric Green tensor \( \mathcal{G}_S \) (i.e., the magnetic field \( \mathbf{E}(\mathbf{r}) \) radiated by an electric dipole) and the electric polarizability tensor \( \alpha \).

2.3. Electric and magnetic near fields

The energy shift Eq. (1) involves the local magnetic density of states (LMDOS) that we define here as \( \rho_m(\mathbf{r}; \omega) \equiv \text{Im} \text{Tr} \mathcal{H}_S(\mathbf{r}, \mathbf{r}; \omega) \). It is in fact the difference in LMDOS compared to free space, due to the presence of the surface. It turns out that at sub-wavelength distance from a metallic surface, both the LMDOS and its electric counterpart (LEDOS) are orders of magnitude larger than in free space. This is because one probes the non-propagating fields ‘leaking’ out of the surface that are attached to the continuum of electronic and phononic states inside the material [19].

We plot in Figure 1 the LMDOS and LEDOS as a function of distance from a metallic half-space. The LDOS have been normalized to their free-space counterparts

\[ \rho_{m,\text{vac}}(\omega) = \frac{\mu_0 \omega^3}{2\pi c^3}, \quad \rho_{e,\text{vac}}(\omega) = \frac{\omega^3}{2\pi \varepsilon_0 c^3}, \]  

(3)

and we see that at sub-wavelength distances from the metallic surface, the field modes have a dominantly magnetic character [19]. This could have been expected from the observation that excitations inside a metal are mainly associated to electric currents. A characteristic distance where the asymmetry in favor of the magnetic energy density peaks, is the penetration or skin depth \( \delta_\omega \). It is defined in terms of the low-frequency limit of the metal dielectric function, \( \varepsilon(\omega) = i\sigma_0/(\varepsilon_0\omega) \) (where \( \sigma_0 \) is the low-frequency conductivity), as

\[ \delta_\omega = \frac{\lambda}{\text{Im}\sqrt{\varepsilon}} = \frac{\sqrt{2}}{\sqrt{\mu_0\sigma_0\omega}} \]  

(4)

where \( \lambda = c/\omega \). Using the electric and magnetic Green tensors worked out in Refs. [13, 14, 15, 28], one can derive the following asymptotics above a metallic half-space:

\[ \frac{\rho_m(z; \omega)}{\rho_{m,\text{vac}}(\omega)} = \frac{\lambda^3}{4z^2\delta_\omega^2} \begin{cases} \frac{1}{2} & z \ll \delta_\omega \\ \frac{3z^3/(3z^3)}{2\delta_\omega} & \delta_\omega \ll z \ll (\lambda^2\delta_\omega)^{1/4} \end{cases} \]  

(5)

\[ \frac{\rho_e(z; \omega)}{\rho_{e,\text{vac}}(\omega)} = \frac{\lambda^2\delta_\omega^2}{4z^3} \begin{cases} \frac{1}{3z/2\delta_\omega} & z \ll \delta_\omega \\ \frac{3z^2}{2\delta_\omega} & \delta_\omega \ll z \ll (\lambda\delta_\omega)^{1/2} \end{cases} \]  

(6)

that are plotted as dashed and dotted lines in Fig. 1. We have found that, as outlined in the next section, for the magnetic energy shift as well, the skin depth provides a characteristic scale, provided one evaluates it at the frequency \( \omega_{mg} \) of the dominant magnetic dipole transition.

3. Magnetic Van der Waals shift

The energy shift (1) can also be written as an integral over imaginary frequencies [26]

\[ U_m(\mathbf{r}) = -\hbar \text{Tr} \int_0^\infty \frac{d\xi}{2\pi} \beta(i\xi) \cdot \mathcal{H}_S(\mathbf{r}, \mathbf{r}; i\xi) \]  

(7)
Figure 1. Local magnetic and electric densities of states (LMDOS and LEDOS) vs. distance from a metallic surface. The normalization is with respect to free space, Eq.(3).

chosen parameters: frequency $\omega/2\pi = 1.6$ GHz (similar to Fig.2) ($\lambda = 18.8$ cm), sodium surface with conductivity $\sigma_0 = 2.1 \times 10^7$ $(\Omega m)^{-1}$ and skin depth $\delta_\omega = 2.75 \mu m$. Dashed [dotted] lines: asymptotics Eqn.(5) [Eqn.(6)].

since both $\beta(\omega)$ and $\mathcal{H}_S(r, r; \omega)$ are response functions and therefore have no singularities in the upper right quadrant of the complex $\omega$-plane. This form is well suited for numerical computation because the integrand is a real, non-oscillatory function that decays to zero for large $\xi$. For a given excited state $|m\rangle$ in Eqn.(2), the transition frequency naturally defines a relevant domain $\xi \sim 0...\omega_{mg}$ along the $\xi$-axis that contributes to the energy shift. The behavior of the energy shift is determined by the behaviour of the reflected magnetic Green tensor in this domain.

3.1. Asymptotic expressions

We focus here on the limit that the distance $z$ is much shorter than the transition wavelength $\lambda_{mg} \equiv 2\pi c/\omega_{mg}$ associated with the magnetic dipole transition. This is a very good approximation for transitions between Zeeman or hyperfine states ($\lambda_{mg} \sim 1 \text{ cm} ... 100 \text{ m}$).

When we perform an asymptotic analysis we find two different power laws in the short-distance ($z \ll \delta_{mg} \ll \lambda_{mg}$) and medium-distance regimes ($\delta_{mg} \ll z \ll \lambda_{mg}$). To leading order we get

$$U_m(r) = \sum_m |\tilde{\mu}_{mg}|^2 \mu_0 \int \frac{z^2}{8\pi^2 z^3} \ln(\frac{\delta_{mg}}{z}) \text{ for } z \ll \delta_{mg} \ll \lambda_{mg},$$

$$= \frac{z^2 \pi}{8 \delta_{mg}^2} \text{ for } \delta_{mg} \ll z \ll \lambda_{mg},$$

where $|\tilde{\mu}_{mg}|^2 = 2(|m|\mu_z|g\rangle|^2 + |m|\mu_x|g\rangle|^2 + |m|\mu_y|g\rangle|^2$. Observe that the energy shifts in Eqn.(8) are positive, meaning that the atom is repelled from the metallic surface. Their behaviour at short distance could have been guessed from the $1/z$ asymptotics of the LMDOS in Eqn. (5), apart from the intriguing logarithmic correction.

3.2. Discussion

We have compared the asymptotic form of Eqn. (8) to a numerical calculation based on Eqn. (7). The results are shown in Fig.2 in a doubly logarithmic scale: one sees a cross over between power laws around the distance $\delta_{mg}$. In this region, the asymptotic result is wrong by at most a factor of $\approx 10$.

In Fig.3, we compare the electric and magnetic energy shifts for the sodium atom. The electric shift is always larger than the magnetic one so that the total atom-surface interaction remains attractive. In the distance range displayed here, the electric energy shift follows the retarded $1/z^4$ law [26]. The cross over between the asymptotics of Eq.(8) for the magnetic shift is clearly seen, with the (non-retarded) $1/z^3$ law prevailing at large distances. (This changes only for $z \gg \lambda_{mg}$ which is in the cm range.)
Figure 2. Magnetic contribution to the Van der Waals interaction between an atom and a metallic surface. The shift is plotted in units of $\mu_0 |\mu|^2 / (8\pi^2 z^3)$ vs. the scaled distance $z/\delta_{mg}$. The solid lines with the symbols give the exact expression (7) and the dashed lines the asymptotics (8). Light gray line and upper dashed lines: magnetic dipole perpendicular to the surface ($|\mu_\perp|^2 = |\mu|^2$) dark gray line: isotropic average over dipole orientations ($|\mu||^2 = \frac{2}{9}|\mu|^2$; $|\mu_\perp|^2 = \frac{1}{3}|\mu|^2$). Chosen parameters: single hyperfine transition (Na 23) $\omega_{mg} = 8.95 \times 10^{15}$ s$^{-1}$ and $\tau = 5.06 \times 10^{-15}$ s. The atomic polarizability is assumed isotropic with $\alpha_{ij}(0) = \delta_{ij} \varepsilon_0 (0.67 \text{ nm})^3$ and contains a single resonance at $\lambda_{eg} = 590 \text{ nm}$ (the arrow marks $\bar{\lambda}_{eg}$). The magnetic shift is computed with the parameters given in Fig.2, assuming an isotropic dipole moment of one Bohr magneton, $|\mu| = \mu_B$.

Figure 3. Electric and magnetic ground state energy shifts (in Joules) for the sodium atom in front of a sodium surface. For the electric shift, we modelled the surface by a Drude metal with parameters $\omega_p = 8.95 \times 10^{15}$ s$^{-1}$ and $\tau = 5.06 \times 10^{-15}$ s. The atomic polarizability is assumed isotropic with $\alpha_{ij}(0) = \delta_{ij} \varepsilon_0 (0.67 \text{ nm})^3$ and contains a single resonance at $\lambda_{eg} = 590 \text{ nm}$ (the arrow marks $\lambda_{eg}$). The magnetic shift is computed with the parameters given in Fig.2, assuming an isotropic dipole moment of one Bohr magneton, $|\mu| = \mu_B$.

4. Conclusions

In the vicinity of metallic surfaces, electromagnetic field fluctuations are dominantly magnetic. We have estimated the atom-surface interaction due to magnetic dipole transitions and found that it represents a small, repulsive correction. Similar results have been reported for the ‘dual’ situation of an electric dipole close to a magneto-dielectric surface [6]. We have identified a characteristic distance parameter, the skin depth at the magnetic transition frequency. Preliminary asymptotic expressions to leading order agree well with numerical calculations.

The magnetic energy shift increases relative to the electric one as the atom recedes from the surface. In the range of several micrometers (typical for the relevant skin depths), it gives a correction of a few percent that may be relevant for quantum reflection of ultracold atoms [20, 22]. However, a quantitative assessment should also take into account the finite temperature $T$ of the surface. We cannot yet definitely
exclude that this may enhance the magnetic dipole contribution. More details will be reported elsewhere.

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