The GSI anomaly

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Abstract. Recently, an experiment at GSI Darmstadt has observed oscillating decay rates of heavy ions. Several controversial attempts have been made to explain this effect in terms of neutrino mixing. We briefly describe the experimental results, give an overview of the literature, and show that the effect cannot be due to neutrino mixing. If the effect survives, it could, however, be explained by hypothetical internal excitations of the mother ions ($\sim 10^{-15}$ eV).

The accelerator facility at GSI Darmstadt can produce monoisotopic beams of highly ionized heavy atoms and store them for extended periods of time in the Experimental Storage Ring. There, an experiment has been performed in which electron capture (EC) decays of hydrogen-like $^{140}$Pr$^{58+}$ and $^{142}$Pm$^{60+}$ ions have been studied using time resolved Schottky mass spectrometry [1]. This technique allows to detect changes of the ions’ revolution frequencies which occur upon EC decay. For a small number of stored ions ($\leq 3$), Schottky mass spectrometry allows for a measurement of the individual decay times. Repeated measurements have led to the distributions shown in Fig. 1. On top of the expected exponential behavior, there is a superimposed oscillation with a period of $T \sim 7$ s. The experiment avoids many systematical errors because it provides a quasi-continuous monitoring of the ions. Statistical fluctuations are excluded as the origin of the effect at the 99% confidence level [1].

Several authors have attempted to relate the anomaly to neutrino mixing [2, 3, 4, 5, 6], but these attempts have been refuted [7, 8, 9]. In the following, we will first argue why the GSI experiment is for general reasons distinct from a neutrino oscillation experiment, and then present a careful treatment of the problem using the density matrix formalism and the wave packet technique. We will show that the GSI anomaly cannot originate from neutrino mixing, but could be explained by hypothetical internal excitations of the mother ions ($\sim 10^{-15}$ eV). In the end, we will give a critical overview of the existing literature on the subject.

Let us start with a general argument why the GSI anomaly is distinct from neutrino oscillations, a fact which is not always correctly reported in the literature [10]. Firstly, a Feynman diagram for neutrino oscillations, e.g. for the $\nu_e$ to $\nu_e$ survival channel, has the following form:

![Feynman diagram](image)

Important are three different ingredients: The production process of the neutrino, which is a weak interaction, ensures that the neutrino is produced in a flavour eigenstate ($\nu_e$ in our
example). The propagation of the neutrino has to be described in terms of mass eigenstates because only those have definite momenta (since the mass of a particle is explicitly included in its propagator). The flavour eigenstate is a superposition of mass eigenstates, $|\nu_e\rangle = \sum_i U_{ei}^* |\nu_i\rangle$, where $U_{ei}$ are the corresponding elements of the leptonic mixing matrix. And finally, the detection process involves another weak interaction, where the flavour of the neutrino is measured again. Oscillations arise because the propagation amplitudes of the three mass eigenstates are different, and hence their mixture after some propagation distance is not the same as at the time of production. The oscillation amplitude is given by a coherent sum,

$$A_{ee} = \sum_i |U_{ei}|^2 e^{-ip_{\mu} x_{\mu}},$$

from which the standard quantum mechanical oscillation formula is easily obtained.

However, in the GSI experiment, the situation is completely different, since an electron neutrino is produced, but there is no second flavour measurement. The neutrino escapes undetected, as shown in the following Feynman diagram:

```
  undetected neutrino
  mass eigenstate

  mother ion  \(\nu_i\)

  daughter ion
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Since Feynman diagrams describe transitions between states of definite energy and momentum, the final state neutrino must be a mass eigenstate $\nu_i$ due to energy-momentum conservation. In a hypothetical situation where the energies and momenta of the mother and daughter ions are measured with infinite precision, it would even be possible to tell from kinematics which neutrino mass eigenstate has been produced. Only this mass eigenstate, i.e. only one Feynman diagram, would contribute and the rate would be proportional to $|U_{ei}|^2$, but there would be no oscillations. However, this is far from the real kinematical situation in the GSI setup.

Realistic energy and momentum uncertainties imply that it is not possible to tell which mass eigenstate was produced, so that all of them have to be taken into account, and must be treated as distinct final states. They contribute to the total rate as an incoherent sum of the sub-amplitudes, since each mass eigenstate lives in its own Hilbert space. The total rate is therefore proportional to $\sum_i |U_{ei}|^2 = 1$, implying that in principle there cannot be any mixing effects. Of course, this cannot change in a quantum mechanical approximation of field theory.
Let us now discuss this in more detail, using the density matrix formalism for a proper theoretical treatment of the detection process. We have to take into account that the GSI detector is sensitive to the daughter ion, but not to the neutrino. In the density matrix formalism, the probability for detecting a particular state \(|n\rangle\) in an experiment is given by \(\text{tr}(\hat{P}\rho)\), where \(\rho\) is the density matrix of the system, and \(\hat{P}\) is the projection operator onto \(|n\rangle\). If one particle is not observed, the projection is not onto one particular state, but rather onto a set of states \(|\ldots n,m \rangle: n \text{ fixed}, m = 0 \ldots \infty\rangle\), where \(n\) denotes the degrees of freedom of the observed particles, and \(m\) stands for those of the unobserved particle. In the GSI experiment, the detection of a daughter state \(|\psi_{D,k}\rangle\) is thus described by the operator

\[
\hat{P}^{(k)} = \sum_{j=1}^{3} \int d^3 p_\nu \langle \psi_{D,k}; \nu_j, p_\nu \rangle \langle \psi_{D,k}; \nu_j, p_\nu \rangle.
\]  

(2)

The sum and the integral run over a complete set of neutrino mass eigenstates \(|\nu_j\rangle\) with momenta \(p_\nu\). With the density matrix for the time-evolved mother state \(|\psi_M\rangle\), given by \(\rho = |\psi_M\rangle \langle \psi_M|\), the probability for the observation of \(|\psi_{D,k}\rangle\) becomes

\[
\mathcal{P}_k = \text{tr} \left[ \hat{P}^{(k)} \rho \right] = \sum_{j=1}^{3} \int d^3 p_\nu \left| \langle \psi_{D,k}; \nu_j, p_\nu | \psi_M \rangle \right|^2.
\]  

(3)

We see that the sum over neutrino states is incoherent. Therefore, if \(\mathcal{P}_k\) contains oscillatory interference terms, they cannot be due to neutrino mixing, and would occur also in a hypothetical model with only one neutrino flavour. All attempts to explain the GSI anomaly in terms of neutrino mixing are thus foredoomed.

It is, however, imaginable that different components of the mother wave packet \(|\psi_M\rangle\) acquire relative phase differences during propagation. If several such components could decay into the same daughter state \(|\psi_{D,k}; \nu_j, p_\nu \rangle\), they would induce interference terms in \(\mathcal{P}_k\). To see under which conditions such a mechanism could explain the GSI anomaly, we will now compute the matrix element \(\langle \psi_{D,k}; \nu_j, p_\nu | \psi_M \rangle\) in the wave packet formalism (ref. \[\text{[11]}\] and references therein).

We describe the states \(|\psi_M\rangle\) and \(|\psi_D\rangle\) of the mother and daughter ions by Gaussian wave packets

\[
\psi_A(x, t) = \left( \frac{2\pi}{\sigma^2_A} \right)^{3/4} \int \frac{d^3 p_A}{(2\pi)^3 \sqrt{2E_A}} \exp \left[ -\frac{(p_A - p_{0A})^2}{4\sigma^2_A} - iE_A(t - t_A) + ip_A(x - x_A) \right],
\]  

(4)

where \(A = M, D\). In our notation, \(p_{0M}\) and \(p_{0D}\) are the central momenta of the wave packets, and \(\sigma_M, \sigma_D\) are their momentum space widths. \(E_M\) and \(E_D\) are related to \(p_M\) and \(p_D\) by the relativistic energy-momentum relation. \(|\psi_M\rangle\) is defined such that, at the injection time \(t_M\), its peak is located at \(x_M\). Similarly, at the detection time \(t_D\), the peak of \(|\psi_D\rangle\) is located at \(x_D\). The coordinate space Feynman rules yield for the amplitude of the decay into \(|\psi_D; \nu_j, p_\nu \rangle\):

\[
i \langle \psi_D; \nu_j, p_\nu | \psi_M \rangle = \int \frac{d^3 p_M}{(2\pi)^3 \sqrt{2E_M}} \int \frac{d^3 p_D}{(2\pi)^3 \sqrt{2E_D}} \int d^3 x dt \left( \frac{2\pi}{\sigma_M \sigma_D} \right)^{3/2} U_{ej, \mathcal{M}^EC_j}(E_M, E_D, E_\nu) \cdot \exp \left[ -\frac{(p_D - p_{0D})^2}{4\sigma^2_D} + iE_D(t - t_D) + ip_D(x - x_D) \right] \cdot \exp \left[ -\frac{(p_M - p_{0M})^2}{4\sigma^2_M} - iE_M(t - t_M) + ip_M(x - x_M) \right] \exp \left[ i\nu_j t - ip_\nu x \right],
\]  

(5)

with \(\mathcal{M}^EC\) being the transition amplitude between plane wave states, as computed in \[\text{[12]}\] [13]. To evaluate this expression, we are first going to compute the \(p_M\) integral

\[
\int \frac{d^3 p_M}{(2\pi)^3 \sqrt{2E_M}} \exp \left[ -\frac{(p_M - p_{0M})^2}{4\sigma^2_M} - iE_M(t - t_M) + ip_M(x - x_M) \right].
\]  

(6)
We expand $E_M = (p_M^2 + m_M^2)^{1/2}$ in the exponent up to first order in $p_M - p_{0M}$ (this approximation neglects wave packet spreading, as has been shown in [11]). The prefactor $1/\sqrt{2E_M}$ is assumed to vary slowly over the width of the wave packet, and will therefore be replaced by its value at $p_{0M}$, which we denote by $1/\sqrt{2E_{0M}}$. Similarly, we also neglect the energy dependence of the matrix element $\mathcal{M}^{EC}_j$. We then have to evaluate

$$
\exp \left[ -i E_{0M} (t - t_M) + i p_{0M} (x - x_M) \right] \int \frac{d^3 p_M}{(2\pi)^3 \sqrt{2E_{0M}}} \exp \left[ - \frac{\left( p_M - p_{0M} \right)^2}{4\sigma_M^2} \right] \cdot \exp \left[ i (p_M - p_{0M}) (x - x_M) - i v_{0M} (t - t_M) \right],
$$

(7)

where $v_{0M} = p_{0M}/E_{0M}$ is the group velocity of the wave packet. The result is

$$
\exp \left[ -i E_{0M} (t - t_M) + i p_{0M} (x - x_M) \right] \left( \frac{\sigma_M^2}{2\pi} \right)^{3/2} \frac{2}{\sqrt{E_{0M}}} \exp \left[ - \frac{(x - x_M - v_{0M} (t - t_M))^2 \sigma_M^2}{2} \right].
$$

(8)

After evaluation of the $p_D$ integral in a completely analogous way, eq. (5) becomes

$$
i \langle \psi_D; \nu_j, \nu_D | \psi_M \rangle = \frac{4}{\sqrt{E_{0M} E_{0D}}} \left( \frac{\sigma_M \sigma_D}{2\pi} \right)^{3/2} \int d^3 x \int d^3 t \ U_{\nu j} \mathcal{M}^{EC}_{\nu j} (E_{0M}, E_{0D}, E_{\nu j})
\cdot \exp \left[ - i E_{0M} (t - t_M) + i p_{0M} (x - x_M) - (x - x_M - v_{0M} (t - t_M))^2 \sigma_M^2 \right]
\cdot \exp \left[ - i E_{0D} (t - t_D) + i p_{0D} (x - x_D) - (x - x_D - v_{0D} (t - t_D))^2 \sigma_D^2 \right] \exp \left[ i E_{\nu j} t - i p_{\nu} x \right].
$$

(9)

We see that the $x$-integral, as well as the subsequent $t$-integral, are Gaussian; a straightforward calculation thus leads to

$$
i \langle \psi_D; \nu_j, \nu_D | \psi_M \rangle = \sqrt{\frac{2\sigma_D \sigma_M}{\pi (v_{0D} - v_{0M})^2 E_{0M} E_{0D}}} \exp \left[ - f_j \right] \exp \left[ i \phi_j \right] U_{\nu j} \mathcal{M}^{EC}_{\nu j} (E_{0j}, E_{0D}, E_{\nu j}),
$$

(10)

with

$$f_j = \frac{(E_j - p \nu)^2}{4\sigma_E^2} + \frac{p^2}{4\sigma_p^2} - \frac{\sigma_D^2 \sigma_M^2}{\sigma_p^2 (v_{0D} - v_{0M})^2} \left( (y_D - y_M) \times (v_{0D} - v_{0M}) \right)^2,
$$

(11)

$$\phi_j = \frac{(v_{0D} - v_{0M}) (y_D - y_M) (E_j - p \nu)}{(v_{0D} - v_{0M})^2} + \frac{y_D \sigma_D^2 + y_M \sigma_M^2}{\sigma_p^2}
+ i E_{0M} t_M - i p_{0M} x_M - i E_{0D} t_D + i p_{0D} x_D.
$$

(12)

Here, we use the notation

$$E_j = E_{0M} - E_{0D} - E_{\nu j}, \quad p = p_{0M} - p_{0D} - p_{\nu},$$

$$\sigma_E^2 = \frac{\sigma_D^2 \sigma_M^2 (v_{0D} - v_{0M})^2}{\sigma_D^2 + \sigma_M^2}, \quad \sigma_p^2 = \sigma_D^2 + \sigma_M^2,$$

$$v = \frac{v_{0D} \sigma_D^2 + v_{0M} \sigma_M^2}{\sigma_D^2 + \sigma_M^2}, \quad y_D, M = x_D, M - v_{0D,0M} t_D, M. $$

(13)
The group velocities of the wave packets are given by $v_{0D,0M} = p_{0D,0M}/E_{0D,0M}$. The real factor $\exp[-f_j]$ enforces sufficient overlap of the wave packets, but is non-oscillatory for Gaussian wave packets. The complex phase factor $\exp[i\phi_j]$ is oscillatory, but is irrelevant for the modulus of the matrix element appearing in eq. (8).

We will now construct a hypothetical situation in which the GSI oscillations can be explained by a quantum mechanical interference effect, namely by quantum beats of the mother ion. This possibility has been pointed out previously in [7, 8, 9]. Let us assume that the state of the mother ion is split into several sublevels $|\psi_M^{(n)}\rangle$, and that, for some reason, the production process creates the mother ion in a superposition

$$|\psi_M\rangle = \sum_n \alpha_n |\psi_M^{(n)}\rangle,$$

where the coefficients $\alpha_n$ have to fulfill the normalization condition $\sum_n |\alpha_n|^2 = 1$. With this modification, eq. (10) turns into

$$i \langle \psi_D; \nu_j, p_r | \psi_M \rangle \propto \sum_n \alpha_n \exp \left[ -f_j^{(n)} + i\phi_j^{(n)} \right],$$

where $f_j^{(n)}$ and $\phi_j^{(n)}$ are defined as in eqs. (11) and (12), but including an upper index $(n)$ for the quantities $E_{0M}$, $p_{0M}$, $v_{0M}$, $E_j$, $p$, $v$, and $y_M$. For simplicity, we have neglected the $n$-dependence of the normalization factors and of the matrix element. Typically, also the wave packet overlap factor $\exp[-f_j^{(n)}]$ will be almost independent of $n$, so we can safely omit it in the following, assuming it to be absorbed in the overall normalization constant.

Upon squaring $| \langle \psi_D; \nu_j, p_r | \psi_M \rangle |$, we now obtain interference terms proportional to $\exp[i(\phi_j^{(n)} - \phi_j^{(m)})]$. To simplify these, let us go to the rest frame of the daughter nucleus, in which $v_{0D} = 0$ and $p_{0D} = 0$. Moreover, we will choose $\sigma_D = \sigma_M \equiv \sigma$, and we will expand $\phi_j^{(n)} - \phi_j^{(m)}$ up to first order in the small quantities

$$\Delta E_{M0}^{(nm)} \equiv E_{M0}^{(n)} - E_{M0}^{(m)} \simeq \frac{\Delta m_{nm}^2}{2E_{M0}},$$

$$\Delta P_{M0}^{(nm)} \equiv P_{M0}^{(n)} - P_{M0}^{(m)} \simeq -(1 - \xi) \frac{\Delta m_{nm}^2 P_{M0}^{(m)}}{2|P_{M0}^{(m)}|^2}.$$  

Here, $\xi$ is a real parameter that is determined by the details of the production process. If we finally neglect terms of $O(\sigma/E_{M0}^{(n)})$, we find

$$| \langle \psi_D; \nu_j, p_r | \psi_M \rangle |^2 \propto \sum_{n,m} \alpha_n \alpha_m^* \exp \left[ -i(x_D - x_M) \left( \frac{\Delta m_{nm}^2 P_{M0}^{(m)}}{2|P_{M0}^{(m)}|^2} \right) \right].$$

Using the relation $x_D - x_M \simeq v_{M0}^{(m)} (t_D - t_M)$, which is a good approximation for sufficiently well localized wave packets, the phase factor can equivalently be written as $\exp[-i(t_D - t_M)\Delta m_{nm}^2/2E_{M0}^{(m)}]$. To explain the GSI oscillations with $T \simeq 7 \text{ s}$, one would require $\Delta m^2 \sim 2.2 \cdot 10^{-4} \text{ eV}^2$, which corresponds to $|m^{(2)} - m^{(1)}| \sim 8.4 \cdot 10^{-16} \text{ eV}$. As has been pointed out in [14], there is no known mechanism that could split up the ground state of the mother ion by such a small amount, nor a known reason why the production process should create a coherent superposition of the substrates.
Before concluding, let us discuss why our results disagree with those of several other authors, who have claimed that the GSI anomaly is a consequence of neutrino mixing. Ivanov, Reda, and Kienle [2] perform a calculation in which the amplitudes $A_j = A_j^{(140\text{Pr}^{58+} \rightarrow 140\text{Ce}^{58+} + \nu_j)}$, for $j = 1 \ldots 3$, receive different phase factors. The appearance of these phase factors can be traced back to the assumption of a finite domain for the time integral at the Feynman vertex, and to the assumption of momentum non-conservation. However, the authors sum the $A_j$ coherently, and thus obtain oscillatory interference terms in the decay rate. To match the observed oscillation period $T \sim 7$ s, a value of $\Delta m^2_{21} \sim 2.22(3) \cdot 10^{-4}\text{ eV}^2$ is required for the solar mass squared difference, in conflict with KamLAND results. In a later work [15], the authors relate this discrepancy to loop-induced Coulomb interactions of the neutrino. Moreover, they apply their formalism also to $\beta^+\text{-decays}$ [16]. The treatment of the detection process in Refs. [2, 15, 16] is in conflict with our results, which show that the sum over the $A_j$ should be incoherent rather than coherent. Similar arguments have been given previously by Giunti [7, 8], by Burkhardt et al. [17], and by Peshkin [9]. (Note that Ivanov et al. have replied to some of Giunti’s remarks in [14, 18].) Moreover, Giunti has shown another problem, namely that the decay rate computed in [2] does not reduce to the Standard Model result if the neutrino masses are set to zero [3].

Further explanation attempts for the GSI anomaly are due to Faber [4] and Lipkin [5, 6]. Both authors employ various kinematical arguments and assumptions, which in both cases yield relative phase differences for the $A_j$. Faber and Lipkin also sum the amplitudes coherently, in conflict with refs. [7, 8, 9, 17] and with our discussion.

Finally, Kleinert and Kienle propose an explanation of the GSI anomaly in terms of a “neutrino-pulsating vacuum” [3]. The authors interpret the $\nu_e$ emission in EC decay as the absorption of a negative energy $\bar{\nu}_e$ from the Dirac sea forming the vacuum. They assume these negative energy anti-neutrinos to undergo oscillations, and thus come to the conclusion that the rate of EC decay should oscillate as well. However, the Dirac sea contains all three neutrino flavours, so that, due to unitarity, its $\bar{\nu}_e$ charge remains constant over time. This remains true even if the local density of $\bar{\nu}_e$ neutrino states should be modified by the presence of an atomic nucleus, as alluded by Kleinert and Kienle.

Besides these theoretical works, two more experiments have been performed investigating EC decays of stopped $^{142}\text{Pm}$ and $^{180}\text{Re}$ atoms [19, 20] (see also [21]). None of these experiments has found any oscillatory signature, indicating that the anomaly, if physical, must be related to the differing features of the GSI setup.

In conclusion, we have shown that neutrino mixing cannot cause the oscillating electron capture decay rate of heavy ions that has been observed at GSI. If the oscillations were confirmed, one must think of new physics, and we have presented a possible, though exotic, mechanism which explains the anomaly by hypothetical internal excitations of the mother ion ($\sim 10^{-15}\text{ eV}$).

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