Energy-level statistics and localization of 2d electrons in random magnetic fields

M. Batsch\textsuperscript{a,b}, L. Schweitzer\textsuperscript{a,*}, B. Kramer\textsuperscript{b}
\textsuperscript{a}Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, Germany
\textsuperscript{b}I. Institut für Theoretische Physik, Universität Hamburg, Jungiusstraße 9, D-20355 Hamburg, Germany

Abstract

Using the method of energy-level statistics, the localization properties of electrons moving in two dimensions in the presence of a perpendicular random magnetic field and additional random disorder potentials are investigated. For this model, extended states have recently been proposed to exist in the middle of the band. In contrast, from our calculations of the large-$s$ behavior of the nearest neighbor level spacing distribution $P(s)$ and from a finite size scaling analysis we find only localized states in the suggested energy and disorder range.

1 Introduction

Many investigations, both analytical and numerical, have been undertaken to elucidate the localization problem of non-interacting electrons in random magnetic fields (RMF) (see e.g.,\textsuperscript{1, 2, 3, 4}). The RMF model has been proposed for studying the quantum Hall effect with Landau level filling $\nu = 1/2$ where the system of quasi-particles may be viewed as a Fermi liquid like metal. In a mean field approximation these composite fermions\textsuperscript{5, 6, 7} are subject to a fluctuating local magnetic field with zero mean.

However, up to now, no satisfactory agreement has been reached on the question, whether or not a metal-insulator transition exists in the thermodynamic limit. In recent numerical studies\textsuperscript{8, 9}, current carrying states have been reported for a RMF model including additional random disorder potentials. The authors of Ref.\textsuperscript{8} report that a metal-insulator transition has been detected in the band center at a critical disorder $W_c = 3.0 \pm 0.2$ by making use of the topological properties of the wave functions.

The method of calculating the topological quantum number (Chern number) of the one-electron wave functions, which has proven to be a useful tool in the integer quantum Hall effect\textsuperscript{9}, considerably restricts the achievable system size, $L/a < 20$. On the other hand, using the method of energy-level statistics\textsuperscript{10, 11, 12, 13}, system sizes of more than a factor of 10 larger are accessible. In this method, the knowledge of the eigenfunctions is not necessary, because the information about localized, critical or extended states is gained from the spectral correlations. Therefore, we report here on an independent check of whether the extended states found in Ref.\textsuperscript{8} can really be expected in the thermodynamic limit or if their result must be attributed to finite size effects.

2 Model and Method

A one-band tight-binding model is used to describe non-interacting particles moving in two dimensions in the presence of spatially fluctuating magnetic fields.

$$H = \sum_{<k\neq l>} V \exp(i\phi_{kl})|k\rangle\langle l| + \sum_k \varepsilon_k |k\rangle\langle l|$$ (1)

The $|k\rangle$ are a complete set of lattice vectors connected with the sites of a square lattice with
over nearest neighbors only. The phases are distributed according to uniform distribution, $p(s) = 2\pi^{-1}$ for $s = h_0 \pi \leq \Phi_k \leq h_0 \pi$ with probability density $p(\Phi_k) = 1/(2h_0)$. The strength of the flux was taken to be $h_0 = 0.5$.

The diagonal disorder potentials $\varepsilon_k$ are chosen at random from an interval $[-W, W]$ with uniform distribution, $p(\varepsilon_k) = 1/(2W)$. We considered system sizes $L/a$ in the range 20 to 200 and disorder values $W/V$ between 1.5 and 3.5. The energy eigenvalues within a small range $(|E|/V < 0.3)$ in the middle of the tight-binding band are calculated using a Lanczos algorithm. For each pair of parameters $(L,W)$ about 250 000 eigenvalues are accumulated taking different realizations of the disorder potentials. We carefully checked that the results were independent of the width of the energy interval chosen.

From the unfolded energy eigenvalues the nearest neighbor level spacing distribution $P(s)$ is calculated, where $s = |E_{i+1} - E_i|/\Delta$ and $\Delta$ is the mean level spacing. For infinite systems it is known from random matrix theory (RMT) \cite{14, 15} that in the metallic regime $P(s)$ is close to the Wigner surmise, $P(s) = A_2 s^2 \exp(-B_2 s^2)$, where $\beta$ reflects the symmetry of the system: orthogonal ($\beta = 1$), unitary ($\beta = 2$), and symplectic ($\beta = 4$). In the insulating case the spacings of the uncorrelated eigenvalues are well described by the Poisson probability density distribution $P(s) = \exp(-s)$.

In finite systems, when the correlation length exceeds the system size, one expects a hybrid \cite{10} of the Wigner and the Poisson form which eventually tends to the limiting true metallic case, if $W < W_c$, or to a complete insulating behavior, if $W > W_c$, as the system size is increased, $L \rightarrow \infty$. For disorder strength $W$ close to the critical value $W_c$ various new scale independent distributions have been discovered recently in several systems \cite{10, 11, 13, 16, 17} that exhibit a metal-insulator-transition or at least a critical point.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The large-s behavior of the nearest neighbor level spacing distribution $P(s)$ for different system sizes $L/a = 20$ (\ocircle), 40 (\odiamond), 100 (\odot), and 200 (\oplus). Disorder $W/V = 2.75$ and strength of the random flux $h_0 = 0.5$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The calculated $J_0(L,W)$ for various system sizes $L$ and disorder strength $W$ showing a behavior expected for localized states.}
\end{figure}

\section{Results and Discussion}

In the 2d random flux model with additional diagonal disorder potentials, we find for small $s$, $P(s) \sim s^2$, as expected for broken time reversal symmetry (unitary). Fig. 3 shows only the large-s behavior of $P(s)$ for a disorder strength

\begin{equation}
\text{Figure 1: The large-s behavior of the nearest neighbor level spacing distribution $P(s)$ for different system sizes $L/a = 20$ (\ocircle), 40 (\odiamond), 100 (\odot), and 200 (\oplus)$. Disorder $W/V = 2.75$ and strength of the random flux $h_0 = 0.5$.}
\end{equation}
Figure 3: The result of the finite size scaling analysis is a one-branch scaling function, $J_0(L/\xi)$. All states are localized for disorder potential strength $2.25 \leq W/V \leq 3.5$.

$W = 2.75V$, which is smaller than the proposed critical disorder $W_c = 3V$, and for system sizes $L/a = 20$, 40, 100, and 200. The RMT result for the Gaussian unitary ensemble (GUE) which holds in the metallic case and the Poisson result applicable for the uncorrelated eigenvalues of the localized states are also plotted. Increasing the system size shifts the curves away from the hybrid $P(s)$ towards the $\exp(-s)$ decay of the localized states. Hence, even for $W$ smaller than the suggested critical $W_c/V = 3$, all the states still tend to localization in the limit $L \to \infty$.

To be more quantitative, we have calculated the number $J_0(L, W) = \int_0^\infty Q(0, s) ds$ where $Q(n, s)$ is the probability that a given energy interval $s$ contains exactly $n$ eigenvalues, and $P(s) = d^2Q(0, s)/ds^2$. In the metallic regime, a value $J_0^M = 0.590$ is known for infinite systems from RMT while for localized states $J_0^L = 1$. In Fig. 3 $J_0(L, W)$ is shown for different system sizes and disorder strength. The values clearly increase with increasing $L/a$, at least for $W/V > 2.25$. This indicates localized states at the band center for this disorder range. A weak decrease of $J_0(L, W)$ is still found for smaller disorder strength, but no size dependence could be observed anymore, because in that case the localization length considerably exceeds our largest system size $L/a = 200$.

It is possible to use $J_0(L, W)$ also as a scaling variable, a procedure which was exploited recently [5] to determine the critical exponent of the correlation function at the metal-insulator-transition (MIT) in a 2d symplectic system. For the RMF model we have tested the validity of the single parameter hypothesis $J_0(L, W) = f(L/\xi(W))$ of a finite size scaling analysis which was performed for the data showing a size dependence ($W/V \geq 2.25$). The result is plotted in Fig. 4 where only a single branch of the scaling function can be seen which excludes the existence of a MIT for the disorder strengths investigated.

Figure 4: Disorder dependence of the correlation function $\xi(W)$ of the RMF (●) and of the 2d Anderson model (○) both normalized such that $\xi(W/V = 3.5) = 1$.

The disorder dependence of the scaling parameter $\xi(W)$ is shown in Fig. 4 in comparison with the result for a conventional 2d Anderson model without random magnetic fields. Since the prefactor $\xi_0$ was not determined in our calculation, both curves were set to 1 at $W/V = 3.5$. The faster increase of $\xi(W)$ in the RMF model indicates a weakening of the localization due to the random magnetic fields. The increase of the localization length is, however, much larger than the factor of 2 expected from RMT [19] for the breaking of the time reversal symmetry.
In conclusion, the localization properties of a two-dimensional electronic model containing random magnetic fields and additional disorder potentials have been investigated numerically. Taking the same parameters as in Ref. [8] for the strength of the random flux, $h_0 = 0.5$, and the disorder potential $W/V = 3$, but system sizes up to $L/a = 200$, our data definitely show complete localization of the electronic states. Even for smaller disorder no sign of a delocalized phase could be detected.

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