The $B \to \pi K$ Puzzle: A Status Report

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We discuss the theoretical interpretation of the $B \to \pi K$ system in the light of new data. Using the branching ratio and direct CP asymmetry of $B^0_d \to \pi^- K^+$, the picture of the direct CP violation in $B_d^0 \to \pi^+ \pi^-$ could be clarified: we predict $\mathcal{A}^{\text{dir}}_{\text{CP}}(B_d \to \pi^+ \pi^-) = -0.24 \pm 0.04$, which favours the BaBar measurement, and extract $\gamma = (70.0^{+5.3}_{-4.5})^\circ$, in agreement with the Standard-Model fits of the unitarity triangle. All $B \to \pi K$ modes with colour-suppressed electroweak penguin contributions are found in excellent agreement with the Standard Model. The data for the ratios $R_{\pi,0}$ of the charged and neutral $B \to \pi K$ branching ratios, which are sizeably affected by electroweak penguin contributions, have moved quite a bit towards the Standard-Model predictions, which are almost unchanged, thereby reducing the “$B \to \pi K$ puzzle”. On the other hand, the mixing-induced CP violation of $B^0_d \to \pi^0 K_S$ still looks puzzling and could be accommodated through a modified electroweak penguin sector with a large CP-violating new-physics phase, while the observed non-vanishing difference between the direct CP asymmetries of $B^+ \to \pi^0 K^\pm$ and $B_d \to \pi^+ K^\pm$ seems to be caused by hadronic and not by new physics.

I. INTRODUCTION

For more than a decade, the system of the $B \to \pi K$ decays is an outstanding topic in heavy-flavour physics (for a review, see [1]). Thanks to the $B$ factories, we could obtain valuable insights into these decays, raising the possibility of having a modified electroweak (EW) penguin sector through the impact of new physics (NP). The following discussion follows closely the strategy developed in [2], and explores the picture after the experimental updates that were reported in the summer of 2006 [3]. The corresponding working assumptions for the treatment of the hadronic sector can be summarised as follows:

i) SU(3) flavour symmetry: however, SU(3)-breaking corrections are included through ratios of decay constants and form factors whenever they arise, and the sensitivity of the numerical results on non-factorizable SU(3)-breaking effects is explored.

ii) Neglect of the penguin annihilation and exchange topologies: these contributions can be probed and controlled through the $B_d \to K^+ K^-$, $B_s \to \pi^+ \pi^-$ system, which can be fully exploited at LHCb.

All consistency checks which can be performed with the current data support these working assumptions and do not indicate any anomalous behaviour. Concerning the treatment of NP, we assume – although we are basically performing a Standard-Model (SM) analysis – that it manifests itself only in the electroweak (EW) penguin sector. Such a kind of physics beyond the SM can be accommodated, e.g., in SUSY, and models with extra $Z'$ bosons and extra dimension scenarios. The topic of having NP in the EW penguin sector of $B \to \pi K$ decays has received a lot of attention in the literature (see, e.g., [4]).

In the following discussion [2, 3], we use the notation

$$\Gamma(B^0_d(t) \to f) - \Gamma(B^0_d(t) \to \bar{f}) \over \Gamma(B^0_d(t) \to f) + \Gamma(B^0_d(t) \to \bar{f}) = \mathcal{A}^{\text{dir}}_{\text{CP}} \cos(\Delta M_d t) + \mathcal{A}^{\text{mix}}_{\text{CP}} \sin(\Delta M_d t),$$

(1)

where $\mathcal{A}^{\text{dir}}_{\text{CP}}$ and $\mathcal{A}^{\text{mix}}_{\text{CP}}$ denote the “direct” and “mixing-induced” CP-violating observables, respectively [2]; a sign convention similar to that of [1] will also be used for self-tagging $B_d$ and charged $B$ decays.

II. THE STARTING POINT: $B \to \pi \pi$

We have seen interesting progress in the exploration of CP violation in $B^0 \to \pi^+ \pi^-$. In the SM, the decay amplitude of this decay can be written as follows [3]:

$$A(B^0 \to \pi^+ \pi^-) = - |\bar{T}| e^{i\delta} [e^{-i\gamma} - e^{i\theta}],$$

(2)

where the $\bar{T}$ amplitude is governed by the colour-allowed tree topologies, and the CP-conserving hadronic parameter $e^{i\delta}$ describes, sloppily speaking, the ratio of penguin to tree contributions. There is now – for the first time – a nice agreement between the BaBar and Belle measurements of the mixing-induced CP asymmetry:

$$\mathcal{A}^{\text{mix}}_{\text{CP}}(B_d \to \pi^+ \pi^-) = \{ 0.53 \pm 0.14 \pm 0.02 \text{ (BaBar)}, 0.61 \pm 0.10 \pm 0.04 \text{ (Belle)} \},$$

(3)

which yields the average of $\mathcal{A}^{\text{mix}}_{\text{CP}}(B_d \to \pi^+ \pi^-) = 0.59 \pm 0.09 [4]$. On the other hand, the picture of direct CP violation is still not settled:

$$\mathcal{A}^{\text{dir}}_{\text{CP}}(B_d \to \pi^+ \pi^-) = \{ -0.16 \pm 0.11 \pm 0.03 \text{ (BaBar)}, -0.55 \pm 0.08 \pm 0.05 \text{ (Belle)} \}.$$ 

(4)

This unsatisfactory situation can be resolved with the help of the $B^0_d \to \pi^0 K^+$ mode, which is governed by
QCD penguin contributions (this feature holds for all $B \to \pi K$ decays). Direct CP violation in this channel is now experimentally well established, with a nice agreement between the BaBar, Belle and CDF results, yielding an average of $A_{\text{dir}}(B_d \to \pi^+ K^-) = 0.095 \pm 0.013$ [4]. In the SM, the $B_d \to \pi^- K^+$ decay amplitude can be written as follows:

$$A(B_d^0 \to \pi^- K^+) = P' [1 - re^{i\gamma}].$$

Using the $SU(3)$ flavour symmetry and the dynamical assumptions specified in Section I, we obtain

$$re^{i\gamma} = \frac{c}{d} e^{i(\pi - \theta)}$$

with $c \equiv \lambda^2/(1 - \lambda^2) = 0.05$, implying the relation [3]:

$$H_{\text{BR}} \equiv \frac{1}{c} \left( \frac{f_K}{f_\pi} \right)^2 \left[ \frac{\text{BR}(B_d \to \pi^+ \pi^-)}{\text{BR}(B_d \to \pi^+ K^+)} \right]$$

$$= -1 \left( \frac{A_{\text{dir}}(B_d \to \pi^+ K^+)}{A_{\text{dir}}(B_d \to \pi^+ \pi^-)} \right).$$

(7)

Since the CP-averaged branching ratios and the direct CP asymmetry $A_{\text{dir}}(B_d \to \pi^+ K^+)$ are well measured, we may use this relation to predict the following value:

$$A_{\text{dir}}(B_d \to \pi^+ \pi^-) = -0.24 \pm 0.04,$$

(8)

which favours the BaBar result in [3]. Since we can express $H_{\text{BR}}$, $A_{\text{dir}}(B_d \to \pi^+ K^+)$ and $A_{\text{mix}}(B_d \to \pi^+ \pi^-)$ in terms of $\gamma$ and $d$, $\theta$, these parameters can be extracted from the data:

$$\gamma = \left(70.0^{+3.8}_{-4.3}\right)^\circ, \quad d = 0.46 \pm 0.02, \quad \theta = (155 \pm 4)^\circ.$$  

(9)

The value of $\gamma$ is in agreement with the SM fits of the unitarity triangle, and will be used for the remainder of this analysis.

Applying the isospin symmetry, we may write

$$\sqrt{2}A(B_d^0 \to \pi^0 \pi^0) = |P|e^{i\gamma}[1 + (x/d)e^{i(\Delta - \theta)}]$$

$$\sqrt{2}A(B^+ \to \pi^0 \pi^0) = -|T|e^{i\gamma}[1 + x e^{i\Delta}],$$

(10)

where the hadronic parameter $x e^{i\Delta}$ denotes the ratio of “colour-suppressed” to “colour-allowed tree” amplitudes. The experimental values of the ratios of the CP-averaged $B \to \pi \pi$ branching ratios allow an extraction of this quantity, with the following result:

$$x = 0.92^{+0.08}_{-0.09}, \quad \Delta = (-50^{+11}_{-12})^\circ.$$  

(11)

Complementing these numbers with those in [3], the following predictions can be made in the SM:

$$A_{\text{dir}}(B_d \to \pi^0 \pi^0) = -0.40^{+0.10}_{-0.21},$$

$$A_{\text{mix}}(B_d \to \pi^0 \pi^0) = -0.71^{+0.16}_{-0.17},$$

(12)

(13)

which offer the exciting perspective of observing large CP violation in the $B_d^0 \to \pi^0 \pi^0$ channel. So far, only data for the direct CP asymmetry are available from the BaBar and Belle collaborations, with the average of $A_{\text{dir}}(B_d \to \pi^0 \pi^0) = -0.36^{+0.33}_{-0.31}$, which is – note the signs – in remarkable agreement with [12], giving us further confidence in our analysis.

III. THE MAIN TARGET: $B \to \pi K$

The $B \to \pi K$ decays are dominated by QCD penguin topologies, and can be divided into two classes, depending on the impact of EW penguins:

- The EW penguins are colour-suppressed, leading to tiny contributions: $B_d^0 \to \pi^- K^+, \ B^+ \to \pi^+ K^0$.
- The EW penguins are colour-allowed, leading to sizeable effects: $B_d^0 \to \pi^0 K^0, \ B^+ \to \pi^0 K^+$.

A. Observables with Tiny EW Penguin Effects

Let us first have a closer look at the $B \to \pi K$ observables with a tiny impact of the EW penguins. For the determination of $\gamma$ discussed above, we have already used the CP-averaged branching ratio and the direct CP asymmetry of $B_d^0 \to \pi^- K^+$, yielding a value of $\gamma$ in excellent agreement with the SM fits of the unitarity triangle. Another decay with colour-suppressed EW penguins is at our disposal, with the following amplitude:

$$A(B^+ \to \pi^+ K^0) = -P' [1 + \rho e^{i\theta_0}e^{i\gamma}],$$

(14)

where the doubly Cabibbo-suppressed parameter $\rho e^{i\theta_0}$ is usually neglected, implying vanishing direct CP violation. This feature is nicely supported by the experimental average $A_{\text{dir}}(B^\pm \to \pi^\pm K) = 0.009 \pm 0.025$ [4].

Finally, the working assumptions specified in Section I allow us to predict the following ratio:

$$R \equiv \frac{\text{BR}(B_d \to \pi^+ K^+)}{\text{BR}(B^\pm \to \pi^\pm K)}$$

$$\equiv 0.942 \pm 0.012 \exp(0.93 \pm 0.05).$$

(15)

Consequently, we obtain an excellent agreement with the SM, and no anomalous value of $\rho$ is indicated, 1 thereby ruling out toy models of final-state interaction effects that were discussed several years ago.

The strategy developed in [3] allows also the prediction of the observables of the $B_s \to K^+ K^-$ decay, where the impact of EW penguins is tiny (colour-suppressed) as well. In the SM, the corresponding CP asymmetries are predicted as follows:

$$A_{\text{dir}}(B_s \to K^+ K^-) = 0.093 \pm 0.015$$

$$A_{\text{mix}}(B_s \to K^+ K^-) = -0.234^{+0.017}_{-0.014}.$$  

(16)

(17)

In contrast to the CP asymmetries, an $SU(3)$-breaking form-factor ratio enters the prediction of the CP-averaged branching ratio. Using the result of a recent

1 This picture of $\rho_0$ follows also from $B^\pm \to K^\pm K$ decays [3].
QCD sum-rule calculation \( [3] \) yields

\[
\text{BR}(B_s \to K^+ K^-) = \left\{ \begin{array}{l}
(27.9^{+7.1}_{-5.0}) \times 10^{-6} \ [B \to \pi \pi] \\
(28.1^{+4.9}_{-5.1}) \times 10^{-6} \ [B \to \pi K].
\end{array} \right.
\]

(18)

As indicated, there are two options for the prediction of this branching ratio, using either \( B \to \pi \pi \) or \( B \to \pi K \), which are in remarkable agreement with each other. The \( B_s \to K^+ K^- \) channel has recently been observed at CDF, with the following branching ratio \( [7] \):

\[
\text{BR}(B_s \to K^+ K^-) = (24.4 \pm 1.4 \pm 4.6) \times 10^{-6}. \quad (19)
\]

Within the uncertainties, \( [18] \) is in nice agreement with \( [19] \), which is another support of the assumptions listed in Section I. The \( B_s \to K^+ K^- \), \( B_d \to \pi^+ \pi^- \) system offers a powerful \( U \)-spin strategy for the extraction of \( \gamma \) at LHCb \( [2, 10] \), the predictions and hadronic parameters given above are useful for further experimental studies to prepare the real data taking at the LHC.

B. Observables with Sizeable EW Penguin Effects

The following ratios are key quantities for an analysis of the \( B \to \pi K \) system:

\[
R_c \equiv 2 \frac{\text{BR}(B_s \to \pi^0 K^0)}{\text{BR}(B_s \to \pi\pi)} \exp 1.11 \pm 0.07 \quad (20)
\]

\[
R_n \equiv 2 \frac{\text{BR}(B_d \to \pi^0 K^0)}{\text{BR}(B_d \to \pi^+ \pi^-)} \exp 0.99 \pm 0.07. \quad (21)
\]

The EW penguins, which provide an interesting avenue for NP to manifest \( [11] \), enter here in colour-allowed form through the modes involving neutral pions, and are theoretically described by two parameters: \( q \), which measures the “strength” of the EW penguin with respect to the tree contributions, and a CP-violating phase \( \phi \). In the SM, the \( SU(3) \) flavour symmetry allows a prediction of \( q = 0.60 \) \( [12] \), and \( \phi \) vanishes.

If we look at Fig. 1 showing the time evolution of the experimental values of \( R_c \) and \( R_n \), we observe that the central values have significantly moved up (partly due to radiative corrections affecting final states with charged particles \( [12] \)), while the errors were only marginally reduced. In Fig. 2 we show the situation in the plane of the observables \( R_n \) and \( R_c \); the contours correspond to different values of \( q \), and are parametrized through the phase \( \phi \). We see that the SM prediction (on the right-hand side) is very stable in time, having now significantly reduced errors. On the other hand, the \( B \)-factory data have moved quite a bit towards the SM. Converting the experimental values of \( R_n \) and \( R_c \) into \( q \) and \( \phi \) yields

\[
q = 0.65_{-0.39}^{+0.39}, \quad \phi = -52_{-50}^{+21}\pi. \quad (22)
\]

A similar trend – see, in particular, the time evolution of \( (\sin 2\beta) \phi_{K_S} ^\pm \) – is also present in the analysis of CP violation in \( b \to s \) penguin-dominated decays \( [6] \).

Let us now have a closer look at the CP asymmetries of the \( B_d^0 \to \pi^0 K_S \) and \( B^\pm \to \pi^0 K^\pm \) channels, which have received a lot of attention and can also be analysed in the strategy of \( [2] \). As can be seen in Fig. 3 SM predictions for the CP-violating observables of \( B_d^0 \to \pi^0 K_S \) are obtained that are much sharper than the current \( B \)-factory data. In particular \( A_{CP}^{\text{mix}}(B_d \to \pi^0 K_S) \) offers a very interesting quantity. We also see that the experimental central values can be reached for large positive values of \( \phi \). Concerning direct CP violation in \( B^\pm \to \pi^0 K^\pm \), the
following SM prediction arises:

\[ \mathcal{A}_{\text{dir}}^{\text{CP}}(B^\pm \to \pi^0 K^\pm) = -0.001^{+0.049}_{-0.041}, \]  

which differs from zero at the 4.7\,\sigma level, is likely to be generated through hadronic effects, i.e. not through the

experimental value of \( \sin(2\beta) \). Consequently, the non-vanishing experimental
data, this feature turns interestingly out to be almost independent of NP. Consequently, the non-vanishing experimental value of

\[ \Delta A \equiv \mathcal{A}_{\text{CP}}(B^\pm \to \pi^0 K^\pm) - \mathcal{A}_{\text{CP}}(B_d \to \pi^\mp K^\pm) \]  

\[ \exp = -0.140 \pm 0.030, \]  

which differs from zero at the 4.7\,\sigma level, is likely to be generated through hadronic effects, i.e. not through the presence of NP.

Performing, finally, a fit to \( R_c \) and the CP asymmetries of \( B^0_d \to \pi^0 K_S \) yields

\[ q = 1.7^{+0.5}_{-1.3}, \quad \phi = + (73^{+6}_{-18})^\circ. \]  

Interestingly, these parameters – in particular the large positive phase - would also allow us to accommodate the experimental values of \( \sin(2\beta) \) and the CP asymmetries of other \( b \to s \) penguin modes with central values smaller than \( \sin(2\beta) \). The large value of \( q \) would be excluded by constraints from rare decays in simple scenarios where NP enters only through \( Z \) penguins, but could still be accommodated in other scenarios, e.g. in models with leptophobic \( Z^\prime \) bosons.

IV. CONCLUSIONS

The strategy developed in \cite{2} continues to provide a powerful tool for the theoretical interpretation of the \( B \to \pi\pi, \pi K \) data \cite{3}. Thanks to the progress at the \( B \) factories, new data could be used where the BaBar and Belle collaborations are in full agreement with each other. However, the corresponding SM predictions are very stable, with almost unchanged central values since the original analysis of 2003, and significantly reduced errors.

Using the branching ratio and direct CP asymmetry of the \( B^0_\ell \to \pi^+ K^- \) channel, the picture of direct CP violation in \( B^0_\ell \to \pi^+ \pi^- \) could be clarified, with the prediction of \( \mathcal{A}_{\text{CP}}(B_d \to \pi^\mp \pi^\pm) = -0.24 \pm 0.04 \), which favours the BaBar result, and the extraction of \( \gamma = (70.0^{+3.9}_{-4.3})^\circ \), which is in agreement with the SM fits of the unitarity triangle.

The current status of the \( B \to \pi K \) system can be summarized as follows:

- All modes with colour-suppressed EW penguins are in excellent agreement with the SM.
- The data for the \( R_{c} \) have moved quite a bit towards the SM predictions, which are almost unchanged, thereby reducing the "\( B \to \pi K \) puzzle" for the CP-averaged branching ratios.
- The non-zero experimental value of \( \Delta A \) seems to be caused by hadronic and not by NP effects.
- On the other hand, the mixing-induced CP violation in \( B^0_\ell \to \pi^0 K_S \) still looks puzzling, and can straightforwardly be accommodated through a modified EW penguin sector with a large, positive value of the CP-violating NP phase \( \phi \).

Unfortunately, we still cannot draw definite conclusions about the presence of NP in the \( B \to \pi K \) system (and other \( b \to s \) penguin decays, such as \( B^0_d \to \phi K_S \)). It will be interesting to keep track of the picture of these decays once the data improve further.

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[1] R. Fleischer, J. Phys. G 32, R71 (2006).
[2] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Phys. Rev. Lett. 92, 101804 (2004); Nucl. Phys. B 697, 133 (2004); Eur. Phys. J. C 45, 701 (2006).
[3] R. Fleischer, S. Recksiegel and F. Schwab, hep-ph/0502275.
[4] T. Yoshikawa, Phys. Rev. D 68, 054023 (2003); M. Gronau and J. L. Rosner, Phys. Lett. B 572, 43 (2003); M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003); V. Barger, C. W. Chiang, P. Langacker and H. S. Lee, Phys. Lett. B 598, 218 (2004); Y. L. Wu and Y. F. Zhou, Phys. Rev. D 72, 034037 (2005).
[5] R. Fleischer, Phys. Lett. B 459, 306 (1999); Eur. Phys. J. C 16, 87 (2000).
[6] The Heavy Flavour Averaging Group (HFAG), http://www.slac.stanford.edu/xorg/hfag/.
[7] G. Punzi, talk at this workshop.
[8] R. Fleischer and S. Recksiegel, Eur. Phys. J. C 38, 251 (2004); Phys. Rev. D 71, 051501 (2005).
[9] A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D 70, 094001 (2004).
[10] J. Nardulli, talk at this workshop.
[11] R. Fleischer and T. Mannel, hep-ph/9705261, Y. Grossman, M. Neubert and A. L. Kagan, JHEP 9910, 029 (1999).
[12] M. Neubert and J. L. Rosner, Phys. Rev. Lett. 81, 5076 (1998).
[13] E. Baracchini and G. Isidori, Phys. Lett. B 633, 309 (2006).