MAGNETAR FIELD EVOLUTION AND CRUSTAL PLASTICITY

S. K. LANDER
Mathematical Sciences and STAG Research Centre, University of Southampton, Southampton SO17 1BJ, UK; skl@soton.ac.uk
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ABSTRACT

The activity of magnetars is believed to be powered by colossal magnetic energy reservoirs. We sketch an evolutionary picture in which internal field evolution in magnetars generates a twisted corona, from which energy may be released suddenly in a single giant flare, or more gradually through smaller outbursts and persistent emission. Given the ages of magnetars and the energy of their giant flares, we suggest that their evolution is driven by a novel mechanism: magnetic flux transport/decay due to persistent plastic flow in the crust, which would invalidate the common assumption that the crustal lattice is static and evolves only under Hall drift and Ohmic decay. We estimate the field strength required to induce plastic flow as a function of crustal depth, and the viscosity of the plastic phase. The star’s superconducting core may also play a role in magnetar field evolution, depending on the star’s spindown history and how rotational vortices and magnetic fluxtubes interact.

Key words: dense matter – magnetic fields – stars: flare – stars: magnetars – stars: neutron

1. INTRODUCTION

Some of the most intriguing neutron stars (NSs) are the magnetars: highly magnetized objects whose surface fields are inferred to be in excess of $10^{14}$ G in some cases and whose interior fields may reach $10^{16}$ G (Thompson & Duncan 1995; Turolla et al. 2015). In contrast with many older, more predictable NSs, magnetars are volatile, alternating between quiescent states and highly energetic bursts and flares. Their most spectacular events are the giant flares, releasing over $\sim 10^{46}$ erg of energy in a very brief flash and decaying X-ray tail. Three of these frustratingly rare events have been seen to date, from three different magnetars—giving little idea of their event rate.

As with much of magnetar physics, there is a broadly accepted qualitative explanation for giant flares, but filling in the details is challenging. Storing and releasing the requisite amount of energy is believed to begin with internal magnetic-field evolution building stresses in the magnetar’s crust. Here, we try to produce a quantitative description of this and are led to one of two possible scenarios for the star’s magnetic-field evolution: it is either driven by persistent plastic flow in the crust, or rotational vortices dragging out superconducting fluxtubes from the core as the star spins down.

1.1. From Flare to Corona to Interior

The initial rise timescale for giant flares is 1 ms, suggestive of some explosive reconnection process in the charge-filled corona surrounding the star (Lyutikov 2003; Huang & Yu 2014; Elenbaas et al. 2016), in analogy with solar flares (Masada et al. 2010). For the timescale to be sufficiently short, this reconnection must occur within a few stellar radii ($\sim 10$ km) of the surface. Direct release of energy from the crust is unlikely: the shortest characteristic timescale is $\sim 0.2$ s, from shear-wave propagation—and the release of energy may be slower still (Lyutikov 2006; Link 2014).

The coronal-reconnection scenario for giant flares requires a huge amount of magnetic energy to be stored in strongly twisted exterior field lines. The long periods of magnetars, $\sim 1$–$10$ s, mean that the rotation-powered mechanism invoked for pulsar magnetospheres would only twist a small region of field lines near the polar cap (Glampedakis et al. 2014). Instead, shearing of the crust could move magnetoospheric footpoints (Thompson et al. 2002), inducing a twist and forming an equatorial lobe of current, with charges stripped off the surface (Beloborodov 2009).

This corona, in turn, requires a mechanism for build-up of crustal stresses. If the star were born rotating rapidly, its crust would freeze into an oblate spheroid shape, and the star’s later spindown would induce stresses as the preferred shape of the star became more spherical (Ruderman 1969)—but this axisymmetric process would displace footpoints vertically, without inducing any coronal twist. The only credible candidate to generate the required azimuthal crustal motion is the star’s magnetic field. Magnetar-corona simulations show how crustal motion (albeit added by hand) produces exterior twist (Parfrey et al. 2013), potentially leading to an oceanic instability (Wolfson 1995) and flare. The build-up of stresses due to a changing global equilibrium can produce the right kind of crustal displacement (Lander et al. 2015), and power giant flares, but this has not been verified with full core-crust field evolutions. On the other hand, magneto-thermal evolutions of the crust show how the crustal field can evolve to become locally intense and fail in small events (Pons & Rea 2012). While this provides a credible explanation of the phenomena of magnetar bursts and their persistent luminosity, it is less clear whether it can be applied to giant flares, with their shorter rise timescales and greater energy release.

1.2. Energy Budget and Storage

The relevant region for energy storage is whatever portion of the interior stellar magnetic field can be tapped for giant flare energy; a deeply buried core field may be irrelevant over magnetar lifetimes. The most energetic giant flare, from SGR 1806–20, was estimated to be $3 \times 10^{46}$ erg (Hurley et al. 2005); the other two giant flares, from SGR 1900+14 (Feroci et al. 2001) and SGR 0526–66 (Mazets et al. 1979), were both $\sim 10^{45}$ erg. In order to have a mechanism that can credibly provide enough energy to power any giant flare, we will adopt a magnetar model harboring 10 times the energy of the largest one observed: $3 \times 10^{47}$ erg. Note that the more detailed discussion in Thompson & Duncan (2001) suggests a
magnetar energy budget \( \gtrsim 10^{47} \) erg; this is probably a conservative value since the paper predated the largest giant flare.

2. CRUST-ONLY FIELD EVOLUTION IMPLIES PLASTIC FLOW

Evolution of the field in an NS core is not fully understood and may not be fast enough to explain magnetar activity (see Section 3). It is thus natural to begin by examining crustal evolution and to assume that the core does not contribute to the energy powering giant flares. A magnetic field \( B \) confined to the crust, with the \( 3 \times 10^{47} \) erg of energy we require, must have the following average strength \( \langle B \rangle \):

\[
3 \times 10^{47} \text{erg} \lesssim \int \frac{B^2}{8\pi} dV \Rightarrow \langle B \rangle \gtrsim 3 \times 10^{15} \text{G}. \tag{1}
\]

Since models of crustal fields harbor a maximum \( B \) that is a few times \( \langle B \rangle \) (Gourgouliatos et al. 2013), we may expect \( B \sim 10^{16} \) G locally. If we had instead allowed the field to extend throughout the star, \( \langle B \rangle \) would (roughly) halve. Although dipole-field estimates from spindown can be unreliable (Younes et al. 2015), this suggests that values around \( 10^{15} \) G are credible.

A field confined to the crust will not satisfy global hydromagnetic equilibrium and must be supported by elastic stresses. Indeed, the estimate above is for a field that is stressing the crust since we eventually require this magnetic energy to be released into the corona. However, such a strong field will stress the crust beyond its elastic limit (Thompson & Duncan 1995; Perna & Pons 2011; Lander et al. 2015). Therefore, if we assume the core’s field does not contribute to giant flares, then the crust must undergo magnetically induced plastic flow (Jones 2006).

2.1. Equations of Motion for Magnetically Driven Plastic Flow

We assume that an NS crust responds in an elastic, reversible manner to stresses below some yield value \( \tau_\text{el} \) and plastically above it. Many terrestrial media can also be approximated as having elastic or plastic responses, depending on the magnitude of the applied stress; they include toothpaste, crude oil, mud, and concrete.

Let us first consider a magnetized crust below \( \tau_\text{el} \). \( B \) need not be in a global fluid equilibrium with the star since the crust can absorb magnetic stresses:

\[
0 = -\nabla P - \rho \nabla \Phi + \frac{1}{4\pi} (\nabla \times B) \times B + \nabla \cdot \tau, \tag{2}
\]

where \( P \) is pressure, \( \rho \) mass density, \( \Phi \) the gravitational potential, and \( \tau \) the elastic stress tensor. We will think of the crust reaching \( \tau_\text{el} \) for some Lorentz force \( (\nabla \times B_{\text{el}}) \times B_{\text{el}} \), which defines the “yield magnetic field” \( B_{\text{el}} \). For \( B > B_{\text{el}} \) the crust’s response to the force

\[
(\nabla \times \Delta B) \times \Delta B \equiv (\nabla \times B) \times B - (\nabla \times B_{\text{el}}) \times B_{\text{el}} > 0
\]

will be plastic, unless the crust is in hydromagnetic equilibrium (for which \( \tau = 0 \)). Equation (3) defines the field \( \Delta B \) that sources the plastic flow; note that in general \( \Delta B \neq B - B_{\text{el}} \).

Next, we wish to make a depth-dependent estimate of \( B_{\text{el}} \), rather than using the standard assumption that \( B_{\text{el}} \sim 10^{15} \) G. As in Lander et al. (2015), we make fits to NS equation-of-state parameters from Douchin & Haensel (2001) and a magnetar temperature profile from Kaminker et al. (2009). Using these, we calculate \( \tau_\text{el} \) from the formula of Chugunov & Horowitz (2010). Now, Equation (2) evaluated at the yield stress shows us that a natural definition for \( B_{\text{el}} \) is

\[
B_{\text{el}} \equiv 4\pi \tau_{\text{el}}. \tag{4}
\]

Plotting this in Figure 1, we see that \( B_{\text{el}} \) may be approximated by the linear relation

\[
B_{\text{el}} \approx 1.8 \times 10^{16} \left(1 - \frac{r}{R_*}\right) \text{G}, \tag{5}
\]

where \( r \) is the radial coordinate and \( R_* \) the surface radius.

Beyond the elastic yield stress, the crust must be in motion, with some velocity \( v \). Since we expect this flow to be approximately incompressible, we take \( \nabla \cdot v = 0 \). General models of viscoplastic flow are based on a relation between two tensorial quantities: the rate of strain \( \dot{\varepsilon} \) and the stress. Let us however assume that the crust is under simple shear stress, for which each tensor has a single independent component, \( \dot{\varepsilon} \) and \( \tau \), and the problem reduces to a scalar one. We may now adopt the Bingham model to relate \( \dot{\varepsilon} \) and \( \tau \) when \( \tau > \tau_{\text{el}} \):

\[
\dot{\varepsilon} = \frac{1}{2\nu}(\tau - \tau_{\text{el}})H(\tau - \tau_{\text{el}}), \tag{6}
\]

where \( \nu \) is the dynamical viscosity of the plastic flow and \( H(\cdot) \) the Heaviside function. Under Equation (6), one can derive an equation of motion valid above \( \tau_{\text{el}} \) (Prager 1961); for our magnetar crust model this is

\[
\rho \ddot{v} + \rho (v \cdot \nabla)v = -\nabla P - \rho \nabla \Phi + \nu \nabla^2 v + \frac{1}{4\pi} (\nabla \times B) \times B + \nabla \cdot \tau, \tag{7}
\]

The fluid terms in this equation have the same form as in the standard Navier-Stokes equation for a viscous medium. In general, however, viscoplastic dynamics can be richer than those of viscous fluids—if, for example, the medium yields in a more complicated manner than through simple shearing, a
tensor generalization of Equation (6) is required. This in turn results in the appearance of a new, uniquely plastic force term in the equation of motion, \( \tau_{pl} \nabla \cdot (\varepsilon ||\varepsilon||) \), where || : || is a tensor norm (Prager 1961). This may be important for more sophisticated modeling of NS crustal failure.

Returning to Equation (7), we anticipate the plastic flow to be slow and steady, and hence neglect the advective term \((n \cdot \nabla)\nu\) for being quadratic in \( \nu \) and the acceleration term \( \dot{\nu} = 0 \). The resulting viscous crust motion is Stokes flow. Comparing this limiting case of Equation (7) with (2), and dropping the tiny differences between the hydrostatic terms \( \nabla P + \rho \nabla \Phi \) in the elastic and plastic regimes, we arrive at the intuitive result that the unbalanced piece of the Lorentz force sources a viscous flow:

\[
\nu \nabla^2 \nu = -\frac{1}{4\pi} (\nabla \times \Delta B) \times \Delta B, \nabla \cdot \nu = 0. \tag{8}
\]

If this unbalanced Lorentz force is curl-free, Equation (8) takes an extremely compact form. Since \( \nabla \cdot \nu = 0 \), we may write \( \nu = \nabla \times \psi \) for some potential \( \psi \). This \( \psi \) is only fixed up to transformations of the form \( \psi \rightarrow \psi + \nabla \phi \), a freedom which allows us to choose \( \nabla \cdot \psi = 0 \) (the Coulomb gauge). Now taking the curl of Equation (8) and using these results, together with various vector identities (including those for a triple curl), we find that the flow is governed by the biharmonic equation:

\[
\nabla^2 \psi = 0. \tag{9}
\]

2.2. Timescale for Plastic-flow-induced Field Evolution

The overwhelming majority of studies of magnetized NS crusts assume electron magnetohydrodynamics: the crustal ion lattice is strictly static, magnetic forces are balanced by elastic stresses (rendering the equation of motion irrelevant), and only the electrons are mobile (see, e.g., Gourgouliatos & Cumming 2014; Wood & Hollerbach 2015). Field evolution is then governed by the interplay of two secular terms (Cumming et al. 2004): the conservative Hall drift and Ohmic decay (the second and third terms, respectively, in Equation (10)). One situation where this is clearly no longer valid is for \( \tau > \tau_{el} \), when we need to add a third term involving the plastic-flow velocity \( \nu \):

\[
\frac{\partial B}{\partial t} = \nabla \times (\nu \times B) - \nabla \times \left[ \frac{(\nabla \times B) \times B}{4\pi \rho_e} \right] - \nabla \times \left[ \frac{\nabla \times B}{4\pi \sigma_0} \right]. \tag{10}\]

where \( \rho_e \) is charge density and \( \sigma_0 \) electrical conductivity. Next, we check when this plastic-flow term may dominate the crust’s evolution. Dimensional analysis of Equation (8) shows that

\[
\nu \sim \frac{L_{pl}^2 (\Delta B)^2}{L_B \nu}, \quad \text{when } B > B_{el}. \tag{11}\]

The plastic viscosity \( \nu \) is an unknown crustal parameter, whose calculation might require molecular-dynamics simulations, but we may make an estimate by arguing from dimensional analysis that \( \nu \sim t_{\text{char}} \tau_{pl} \) for some characteristic plastic timescale \( t_{\text{char}} \). We already know \( \tau_{pl} \), from the calculation described before (4). For \( t_{\text{char}} \), let us demand it be short enough to allow for a persistent corona. Since coronal dissipation occurs over \( \sim 1-10 \) years (Beloborodov & Thompson 2007), we need to take \( t_{\text{char}} \) \( = 10 \) years to allow sufficiently fast regeneration of the exterior current. \( \nu \) will increase with crustal depth and decrease with temperature, but rather than exploring these dependences here we will simply evaluate \( t_{\text{char}} \) at the crust–core boundary for our (hot) magnetar model and use this as a reference value: \( \nu \sim 10^{38} \) poise. If this estimate is reliable, then the \( \nu \lesssim 10^{35} \) poise condition for thermo-plastic instability (Beloborodov & Levin 2014) would only be attained in the outer crust. For comparison, a famously viscous terrestrial material is pitch; in an experiment begun in 1930, nine drops of pitch have fallen to date, yielding a viscosity estimate of \( 2 \times 10^9 \) poise (Edgeworth et al. 1984).

Using our viscosity estimate, we may compare \( t_{pl} \) with the timescale \( t_{\text{Hall}} \) for Hall drift (which is faster than Ohmic decay for high \( B \)), using the results of Cumming et al. (2004). We forsee three regimes:

1. If \( B \gtrsim 10^{15} \) G, plastic flow always dominates, with a characteristic timescale \( \lesssim 3L_B/L_{pl} \) years.
2. In the range \( B \sim (2-7) \times 10^{14} \) G, \( B_{el} \) is exceeded for crustal depths \( \lesssim 100-400 \) m, and \( t_{pl} \sim t_{\text{Hall}} \sim 10-100 \) years there, so the two field-evolution
mechanisms compete. Deeper into the crust there is no plastic flow.

3. If \( B \lesssim 10^{13} \) G, plastic flow shuts off essentially everywhere.

2.3. A Toy Model

We now consider a simple illustrative example of magnetically induced plastic flow that can be solved analytically. Taking a segment of the crust and neglecting its curvature, we are left with a Cartesian slab of material, as shown in Figure 2: the \( x \) and \( z \) coordinates represent azimuthal and radial directions in the crust, respectively.

Assume a Lorentz force that stresses the crust beyond \( \tau_d \) in some band \( -\gamma_0 < y < \gamma_0 \). To produce a constant, non-radial force, we choose

\[
\Delta B = b \sqrt{\frac{\nu}{\epsilon_z}}
\]

where \( b \) is a constant. Note that \( \Delta B \) is divergence-free, and its corresponding force curl-free, as required. Our assumptions, however, force us to have a depth-independent \( \nu \); the intention is to relax this restriction in future work. Now, returning to Equation (8),

\[
\nabla^2 v = (\nabla^2 \nu_z) \mathbf{e}_z + (\nabla^2 \nu_y) \mathbf{e}_y = \frac{b^2}{8\pi\nu} \mathbf{e}_z.
\]  

(14)

Here, \( \nu_z = 0 \) to ensure incompressibility of the flow. Equation (14) implies \( \nabla^2 \nu_z = 0 \); we will satisfy this by taking \( \nu_z = 0 \) (this is not actually a simplification; one can show that keeping a \( \nu_z \) term is incompatible with confining the flow into a channel \( -\gamma_0 < y < \gamma_0 \)).

In terms of the velocity potential, \( v = e_z \partial \psi / \partial y - e_y \partial \psi / \partial x \), so \( \nu_z = 0 \) implies \( \psi = \psi(y) \), and Equation (9) becomes simply \( d^2 \psi / dy^2 = 0 \). Integrating once and comparing with Equation (14) gives

\[
\frac{d^3 \psi}{dy^3} = \frac{b^2}{8\pi\nu}.
\]

(15)

Integrating twice more and imposing the boundary conditions that the flow must stop when there is no unbalanced Lorentz force, i.e., \( v_z(-\gamma_0) = v_z(\gamma_0) = 0 \), we find that the plastic-flow velocity is

\[
v = \frac{b^2}{16\pi\nu} (y^2 - \gamma^2(z)) \mathbf{e}_z.
\]

(16)

The evolution of a crustal field \( \mathbf{B} = B_x(x, y, t) \mathbf{e}_x + B_y(x, y, t) \mathbf{e}_y + B_z(x, y, t) \mathbf{e}_z \) under this flow is given by \( \nabla \times (\mathbf{v} \times \mathbf{B}) \), i.e.,

\[
\frac{\partial \mathbf{B}}{\partial t} = \frac{b^2}{16\pi\nu} \left\{ \frac{\partial}{\partial y} \left[ (y^2 - \gamma^2(z)) B_z \right] \mathbf{e}_x + \frac{\partial}{\partial x} \left[ (y^2 - \gamma^2(z)) B_x \right] \mathbf{e}_y + \frac{\partial}{\partial x} \left[ (y^2 - \gamma^2(z)) B_z \right] \mathbf{e}_y \right\}.
\]

(17)

By separation of variables, writing each field component in the form \( X(x)Y(y)T(t) \), we find that we require \( B_x = B_y = (y^2) \) and \( B_z = B_z(y) \) for consistency, so that only \( B_z \) is time-varying,

with

\[
B_z = \frac{b^2}{16\pi\nu} \frac{d}{dy} \left[ (y^2 - \gamma^2(z)) B_z \right].
\]

(18)

This toy model suggests that a magnetic field with an unbalanced radial component induces an azimuthal plastic flow, whose velocity depends (through \( b \)) on how much one exceeds \( \tau_d \); field evolution along this flow provides an ever-increasing twist to the corona. More realistically, the twist would be relieved gradually through viscoplastic or Ohmic dissipation, or suddenly in coronal flare events.

3. CORE-FIELD EVOLUTION DRIVEN BY STELLAR SPINDOWN

We may be able to avoid persistent plastic crustal flow if the core field contributes to the magnetar’s usable energy reservoir, so next we review core-field evolution mechanisms to identify any that may be sufficiently rapid. We report timescales for a field lengthscale \( L_B = 1 \) km and the crust–core boundary density \( 1.28 \times 10^{14} \) g cm\(^{-3} \), and normalize to a temperature \( T_8 = T/(10^8 \) K) and field strength \( B_{15} \equiv B/10^{15} \) G.

A normal-matter core with protons, neutrons, and electrons has three field-evolution mechanisms (Goldreich & Reisenberger 1992). Two of these—Ohmic decay (\( \sim 10^3 T_8^{-2} \) years) and Hall drift (\( \sim 5 \times 10^3 B_{15}^{-1} \) years)—are too slow for our purposes. The third, ambipolar diffusion, appears more promising: its solenoidal component could cause a buoyant rise of flux out of the core over \( \sim 10^7 T_8^2 B_{15}^{-1} \) years. However, the normal-fluid model is not applicable to any known NS; all have cooled sufficiently to harbor superfluid neutrons and type-II superconducting protons (Hou et al. 2012). Neutron superfluidity drastically alters the action of ambipolar diffusion: the solenoidal drift becomes too fast to be relevant, while the other drift component acts over \( \sim 10^{20} \) years.

In the superfluid–superconducting outer NS core, bulk stellar rotation is quantized into neutron vortices, and the magnetic field into fluxtubes, provided the magnetic field is below the value \( H_{c2} \sim 10^{16} \) G at which superconductivity is destroyed. Vortices and fluxtubes cannot generally move far without encountering one another. If the energy penalty associated with them cutting through each other is sufficiently large, they will instead “pin” together and their motion will be coupled (Sauls 1989; Link 2003; Gügercino & Alpar 2014). In particular, as the star spins down vortices will move outward and, in turn, drag fluxtubes with them (Ruderman et al. 1998). This could lead to core-field evolution over a timescale as short as \( \sim 10^4 \) years, though Jones (2006) and Glampedakis & Andersson (2011) have reached opposing conclusions about whether this mechanism would operate efficiently in magnetars. Even if it does though, the extent to which vortex–fluxtube “pinning” can induce magnetic stresses depends strongly on the
star’s spindown history; rapid core-field evolution in a magnetar would suggest that it had been born rapidly rotating.

4. DISCUSSION AND IMPLICATIONS

We have argued that magnetar field evolution—in particular, for the objects that have suffered giant flares—cannot be entirely driven by the conventional evolution mechanisms of Ohmic decay and Hall drift. Instead, if the evolution proceeds mainly in the crust, then magnetic stresses are likely to become large enough to induce evolution through persistent plastic flow. The role of core-field evolution for magnetars is less clear: it depends on whether vortices and fluxtubes “pin” to one another and relies on the protons forming a type-II superconductor, which occurs if \( B < H_{c2} \sim 10^{16} \text{G} \). This latter condition represents another uncertainty: not only are magnetar core fields unknown, but calculations of \( H_{c2} \) also vary over an order of magnitude (see, e.g., Glampedakis et al. 2011a; Sinha & Sedrakian 2015).

The two scenarios suggest different mechanisms for generating a magnetar corona and triggering a giant flare. With core evolution, the crust could respond elastically as stresses build, then fail rapidly; in this case, we anticipate the immediate formation of a transient corona, which may be dynamically unstable and result in a giant flare. Crustal evolution leads to a more permanent corona, sourced by shifting magnetospheric footpoints embedded in the plastically deforming crust, and a giant flare might represent an overturning instability. The presence of long-lived magnetar coronae is consistent with the persistently high spindown rate following the giant flare of SGR 1806-20 (Younes et al. 2015) and the X-ray spectra of a number of other magnetars (Weng 2015). Note that Lyutikov (2015) also briefly discusses plastic-flow-driven field evolution, arguing however that it is not necessary to explain magnetar activity; our focus is different, as we suggest plastic flow is inevitable in some circumstances, and that its effects are consistent with magnetar phenomena.

We suggest that plastic flow will dominate NS crustal field evolution for \( B \gtrsim 10^{15} \text{G} \), compete with Hall drift in the outer crust for \( B \sim 10^{14} \text{G} \), and probably be mostly irrelevant for \( B \lesssim 10^{13} \text{G} \). This suggests that it plays a key role for young magnetars, in particular. Plastic flow is an unusual field-evolution mechanism since it shuts down below a certain (density-dependent) field strength—meaning that there may be a genuine distinction in the underlying physics of some magnetars compared with other NSs and presenting an additional challenge to attempts (e.g., Vigano et al. 2013) to “unify” the different observational manifestations of NSs.

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