A potential including Heaviside function in 1+1 dimensional hydrodynamics by Landau

Its basic properties and application to data at RHIC energies

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Abstract

In 1+1 dimensional hydrodynamics originally proposed by Landau, we derive a new potential and distribution function including Heaviside function and investigate its mathematical and physical properties. Using the original distribution derived by Landau, a distribution function found by Srivastava et al., our distribution function, and the Gaussian distribution proposed by Carruthers et al., we analyze the data of the rapidity distribution on charged pions and K mesons at RHIC energies (\(\sqrt{s_{NN}} = 62.4\) GeV and 200 GeV). Three distributions derived from the hydrodynamics show almost the same chi-squared values provided the CERN MINUIT is used. We know that our calculations of hadron’s distribution do not strongly depend on the range of integration of fluid rapidity, contrary to that of Srivastava et al. Finally the roles of the Heaviside function in concrete analyses of data are investigated.

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1 Introduction

Recently authors of ref. [1, 2] have mentioned a paper by Srivastava et al. [3] as one of boost non-invariant solutions in 1+1 dimensional hydrodynamics proposed by Landau [4, 5, 6]. It is well known that the boost invariant distribution function have been proposed by Hwa [7] and independently Bjorken [8]. It is also worthwhile to notice recently interesting papers on 1+1 dimensional hydrodynamics published in refs. [9] [10].

In the present study, in particular we are interested in the paper by Srivastava et al., because their potential and the distribution function are similar to that calculated by Landau and coauthors. Our purpose is to obtain a new solution by use of different boundary conditions in the same partially differential equation obtained by Landau. (See eq. (3) below)

The hydrodynamics is briefly described in the following: The energy density \(e\), the pressure \(p\), the four velocity \(u^\mu\), and the entropy density \(s\) are necessary physical quantities. Then the conservation law of the energy-momentum and that of the entropy are given as

\[
\frac{\partial T^{\mu\nu}}{\partial x^\mu} = 0, \quad \frac{\partial (su^\mu)}{\partial x^\mu} = 0,
\]
where $T^{\mu\nu} = (e + p)u^\mu u^\nu - pg^{\mu\nu}$. Introducing the temperature of the fluid $T$, the equation of state $p = c_s^2e$ with the velocity of sound $c_s$, the thermodynamical relations $de = Tds$, and $e + p = Ts$, we have the relation $dp = sdT$. Moreover, introducing the rapidity of fluid $y$ and $x^\mu = (t, x) = (\tau \cosh \eta, \tau \sinh \eta)$ with $\eta = \tanh^{-1}(x/t)$, 1 + 1 dimensional hydrodynamics in the perfect fluid is described as follows

$$\frac{\partial}{\partial t}(T \sinh y) + \frac{\partial}{\partial x}(T \cosh y) = 0,$$

(2)

$$\frac{\partial T}{\partial t} + c_s^2 \frac{\partial y}{\partial x} \cosh y + \left( \frac{\partial T}{\partial x} + c_s^2 \frac{\partial y}{\partial t} \right) \sinh y = 0.$$

(3)

Utilizing the Legendre transformation $d\chi = d(\phi + tT \cosh y - xT \sinh y)$ from the variables $(t, x)$ to $(T, y)$ with potentials $\phi$ and $\chi$, we obtain the following expressions:

$$t = \frac{\partial \chi}{\partial T} \cosh y - \frac{1}{T} \frac{\partial \chi}{\partial y} \sinh y,$$

(4)

$$x = \frac{\partial \chi}{\partial T} \sinh y - \frac{1}{T} \frac{\partial \chi}{\partial y} \cosh y.$$

(5)

Moreover, we have the following relation between $y$ and $\eta$:

$$\tau^2 = t^2 - x^2 = \left( \frac{\partial \chi}{\partial T} \right)^2 - \left( \frac{1}{T} \frac{\partial \chi}{\partial y} \right)^2,$$

(6)

$$y = \tanh^{-1} \left( \frac{x}{t} \right) + \tanh^{-1} \left( \frac{1}{T} \frac{\partial \chi}{\partial y} \right) = \eta + \Delta.$$  

(7)

From the above equations and relations, we obtain the following partial differential equation of the potential $\chi$ in 1 + 1 dimensional hydrodynamics,

$$\frac{\partial^2 \chi}{\partial y^2} = c_s^2 \frac{\partial^2 \chi}{\partial \omega^2} + (1 - c_s^2) \frac{\partial \chi}{\partial \omega},$$

(8)

where $\omega = -\ln(T/T_0)$ (the logarithmic temperature). To look for a new solution in eq. (8) is our aim, as mentioned before.

As the solution of eq. (8), we have known the following two solutions [4, 5]. (Notice that two different initial conditions are used.) Utilizing the method of Green function [11, 12], we obtain a new potential including the Heaviside function for the differential equation in 1+1 dimensional hydrodynamics.

On the other hand, the rapidity distributions of the charged mesons at RHIC energies ($\sqrt{s_{NN}} = 62.4$ GeV [13] and 200 GeV [14]) have been reported by BRAHMS Collaboration. We are going to analyze them by several formulae by Landau, Srivastava et al., and ours.

In the second sec. we briefly explain the formula by Landau and the boost non-invariant formula calculated by Srivastava et al. In the third sec. we present the new formula including the Heaviside function. In the 4th one we present analyses of the data at RHIC energies by three formulae and a formula proposed by Carruthers et al. [15] in addition to them. In the final one the concluding remarks are presented.

2 Solutions of Potential (eq. (8)) by Landau and Srivastava et al.

We mention the solution by Landau and that by Srivastava et al. An original formula by Landau is obtained by the following initial condition,

$$\left. \frac{\partial \chi}{\partial y} \right|_{y=0} = T_0 e^{\omega}, \quad \chi(y = \omega/c_s) = 0,$$

(9)
where \( l \) denotes the length of colliding matter.

The potential is expressed by the integral formula
\[
\chi \sim e^{-\omega} \int_{y_c}^{\omega} e^{(1+\beta)\omega'} I_0 \left( \beta \sqrt{\omega'^2 - c_s^2 y^2} \right) d\omega',
\]
where \( \beta = (1 - c_s^2)/2c_s^2 \). The entropy distribution is calculated as follows,
\[
\frac{dS}{dy} \sim \frac{dN}{dy} \sim \text{const} \frac{\partial}{\partial \omega} \left( \chi + \frac{\partial \chi}{\partial \omega} \right) \bigg|_{\omega = \omega_f},
\]
where \( \omega_f = -\ln(T_f/T_0) \). Using (11), we have the rapidity distribution
\[
\frac{dN}{dy} \sim e^{-\beta \omega_f} \left[ I_0(p) + \frac{\beta \omega_f}{p} I_1(p) \right],
\]
where \( p = \beta \sqrt{\omega^2 - c_s^2 y^2} \). \( I_0 \) and \( I_1 \) are the modified Bessel functions of the 1st order and 2nd one, respectively.

On the other hand, Srivastava et al. have assumed that \( \chi = \chi_1 \exp(\beta \omega) \) and obtained the following equation
\[
\frac{\partial^2 \chi_1}{\partial y^2} = c_s^2 \frac{\partial^2 \chi_1}{\partial \omega^2} - c_s^2 \beta^2 \chi_1. \quad (13)
\]
Using a new variables with \( p = \beta \sqrt{\omega^2 - c_s^2 y^2} \),
\[
q = \tanh^{-1} \left( \frac{c_s y}{\omega} \right)
\]
they obtain the following equation which satisfies the Bessel functions,
\[
\frac{1}{p^2} \frac{\partial^2 \chi_1}{\partial q^2} - \frac{\partial^2 \chi_1}{\partial p^2} - \frac{1}{p} \frac{\partial \chi_1}{\partial p} + \chi_1 = 0. \quad (15)
\]
Adopting the condition \( \frac{\partial \chi_1}{\partial q} = 0 \), they have obtained the following solution,
\[
\chi \sim \begin{cases} 
  e^{\beta \omega} I_0(\beta \sqrt{\omega^2 - c_s^2 y^2}) & \text{for } \omega^2 > c_s^2 y^2, \\
  e^{\beta \omega} J_0(\beta \sqrt{c_s^2 y^2 - \omega^2}) & \text{for } \omega^2 < c_s^2 y^2.
\end{cases} \quad (16)
\]
To show some mathematical properties of two solutions, we present their contour maps in fig. 1 (a) and (b).

The entropy distribution related to eq. (16) is calculated as
\[
\frac{dS}{dy} \sim \left[ \beta(\beta + 1) I_0(p) + \frac{\beta^2 (p^2 + p^2 (2\beta + 1) \omega - \beta^2 \omega^2) I_0'(p)}{p^3} + \frac{\beta^4 \omega^2 I_0''(p)}{p^2} \right] \quad (17)
\]
for \( \omega^2 > c_s^2 y^2 \). For \( \omega^2 < c_s^2 y^2 \) we have
\[
\frac{dS}{dy} \sim \left[ \beta(\beta + 1) J_0(\bar{p}) + \frac{\beta^2 (\bar{p}^2 + \bar{p}^2 (2\beta + 1) \omega - \beta^2 \omega^2) J_0'(\bar{p})}{\bar{p}^3} + \frac{\beta^4 \omega^2 J_0''(\bar{p})}{\bar{p}^2} \right] \quad (18)
\]
where \( \bar{p} = \beta \sqrt{c_s^2 y^2 - \omega^2} \). Authors of refs. [16, 17, 18] have applied eqs. (12), (17) and (18) to analyses of data.
Figure 1: Distributions and contour maps of $\chi$ of eqs. (10) and (16). (a) Distribution of eq. (10) with $c_s^2 = 1/3$. (b) Contour map of fig. (a). (c) Distribution of eq. (16) with $c_s^2 = 1/3$. (d) Contour map of fig. (c).
3 Solution including the Heaviside function

Hereafter, to consider a new solution, we use the method of Green functions \[11, 12\]

\[
\frac{\partial^2 \chi}{\partial \omega^2} + \left( \frac{1}{c_s^2} - 1 \right) \frac{\partial \chi}{\partial \omega} - \frac{1}{c_s^2} \frac{\partial^2 \chi}{\partial y^2} = \frac{1}{c_s^2} \delta(\omega - \omega_0)\delta(y - y_0). \quad (\omega_0, \ y_0 \to 0) \tag{19}
\]

Using the function \( \chi = \chi_1 e^{\beta \omega} \), we choose the following initial conditions in the \((\omega, y)\) space, because we can consider the initial condition at \( \omega = 0 \),

\[
\left. \frac{\partial \chi_1}{\partial \omega} \right|_{\omega=0} = G(y) = \delta(y), \quad \chi_1(\omega = 0) = g(y) = 0. \tag{20}
\]

Since the hydro-system in collisions develops from \( \omega = 0 \) to \( \omega = \omega_f = \ln(T_0/T_f) \), we have the following solution,

\[
\chi_1(y, \omega) = \frac{1}{2} \{ g(y + \omega/c_s) + g(y - \omega/c_s) \} + \frac{1}{2} \int_{-\omega/c_s}^{\omega/c_s} \left\{ c_s G(z + y) I_0(\beta \sqrt{\omega^2 - c_s^2 z^2}) + \beta^2 c_s \omega g(z + y) I_1(\beta \sqrt{\omega^2 - c_s^2 z^2}) \right\} dz. \tag{21}
\]

This set of initial conditions suggests us that the present approach mainly describes the collision in the central region.

Then we have the following solution

\[
\chi \sim e^{\beta \omega} I_0 \left( \beta \sqrt{\omega^2 - c_s^2 y^2} \right) H(\varepsilon, \ Q = \omega - c_s |y|), \tag{22}
\]

where \( H(\varepsilon, Q) \) is the Heaviside function defined as

\[
H(\varepsilon, Q) = \frac{1}{e^{-\varepsilon Q} + 1}. \tag{23}
\]

To show some mathematical properties of the above solution, we present its contour map in fig. 2. Moreover, we show diagrams between \((t, x)\) for eq. \((15)\) and \((22)\) in fig. 3 (a), (b). In fig. 3 (a) we confirm calculations in ref. [3], and in fig. 3 (b) our calculation for eq. (22) is presented. The following expression is the entropy distribution,

\[
\frac{dN}{dy} \sim \frac{dS}{dy} \sim \left[ \beta(\beta + 1) I_0(p) + \frac{\beta^2 (p^2 + p^2 (2\beta + 1) \omega - \beta^2 \omega^2) I_0(p)}{p^2} + \frac{\beta^4 \omega^2 I_0''(p)}{p^2} \right] H(\varepsilon, \ Q) + \left[ (2\beta + 1) I_0(p) + \frac{2\beta^2 I_0'(p)}{p} \right] H'(\varepsilon, \ Q) + I_0(p) H''(\varepsilon, \ Q). \tag{24}
\]

When \( p \) is imaginary, the modified Bessel function should be replaced by the Bessel functions. The typical behavior of eq. (24) is shown in fig. 4.

To analyze the measured distribution of hadrons, the following calculation is necessary,

\[
\frac{dN}{dy_h} = c \int_{y_0}^{\infty} dy \int_{0}^{\infty} dp_T p_T \frac{m_T \cosh(y_h - y)}{\exp\left\{ [m_T \cosh(y_h - y) - \mu_B]/T \right\} + \delta} \frac{dN}{dy}, \tag{25}
\]

where \( y_h, p_T, m_T = \sqrt{p_T^2 + m_B^2} \) and \( \mu_A \) are the rapidity of hadron, the transverse momentum of hadron and the baryonic chemical potential of hadron A, respectively. For mesons, \( \delta = -1, \ \mu_B = 0 \) and for baryons \( \delta = 1, \ \mu_B = \mu \). As you see fig. 4, due to the derivative of the Heaviside function, there are small-finite contributions outside of the asymptotic lines \( \omega = \pm c_s |y| \).
Figure 2: (a) Distribution and contour map of eq. (22). (b) Behaviors distinguished from that of fig. 1 are observed along the lines $\omega = \pm yc_s$.

Figure 3: Trajectories of $t = \sqrt{\tau^2 + x^2}$ ($\tau = 1$ fm). (a) Boost invariant solution by Hwa and Bjorken (dashed lines) and boost non-invariant one by Srivastava et al. (solid line). (b) Boost invariant solution (dashed lines) and our boost non-invariant one by us (solid line). Finite difference method is used.
4 Analyses of data by several formulæ

Since BRAHMS Collaboration has reported the rapidity distributions of charged $\pi$ mesons and K mesons at RHIC $\sqrt{s_{NN}} = 62.4$ GeV and 200 GeV, we can analyze them by three formulæ mentioned above. The freeze-out temperature and chemical potential are cited from ref. [19]. $T_f = 0.1605$ (200 GeV and 62.4 GeV) and $\mu = 0$. See also ref. [20]. Moreover, we can analyze the data by the Gaussian distribution proposed by Carruthers and Duong-Van [15].

$$\frac{dN}{dy_{th}} = \frac{c}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{y^2_{th}}{2\sigma^2} \right], \quad \sigma^2 = \ln \left( \frac{\sqrt{s_{NN}}}{2m_p} \right)$$  \hspace{1cm} (26)

Our results by eq. (25) in which $c^2_s$ is assumed as a free parameter are presented in figs. 5 and 6. Notice that we examine different cutoff values for $y_0$ in calculations of eq. (25). Analyses of data by eq. (26) are cited in table 6 and shown in figs. 5 and 6.

Table 1: Analysis of charged pions and K mesons rapidity distributions by means of eqs. (12) and (25) with $y_0 = \omega/\sqrt{c^2_s}$. (Distribution by Landau)

| Data            | $\omega$   | $c^2_s$  | $c$          | $\chi^2$/n.d.f |
|-----------------|------------|----------|--------------|----------------|
| $\pi^-$ (200 GeV) | 1.77±0.03  | 0.194±0.016 | 1230±426  | 18/11          |
| $\pi^+$ (200 GeV) | 1.84±0.02  | 0.135±0.001 | 169±12    | 10/11          |
| $K^-$ (200 GeV)   | 1.58±0.05  | 0.179±0.024 | 691±371   | 6.3/9          |
| $K^+$ (200 GeV)   | 2.49±0.02  | 0.066±0.001 | 0.0013±0.0002 | 1.8/9       |
| $\pi^-$ (62.4 GeV)| 1.61±0.01  | 0.333±0.059 | 4860±92   | 13/7           |
| $\pi^+$ (62.4 GeV)| 1.49±0.16  | 0.295±0.074 | 4250±1580 | 15/7           |
| $K^-$ (62.4 GeV)  | 1.22±0.23  | 0.189±0.107 | 1040±1250 | 1.3/5          |
| $K^+$ (62.4 GeV)  | 1.40±0.08  | 0.178±0.043 | 724±503   | 0.44/5         |

5 Concluding remarks

We summarize the present studies as follows:

1) We have investigated the new solution including the Heaviside function in 1+1 dimensional
Table 2: Analysis of charged pions and K mesons rapidity distributions by means of eqs. (17), (18) and (25) with $y_0 = \omega/\sqrt{s^2 + \delta}$ ($\delta = 2$). (Distribution by Srivastava et al.)

| data       | $\omega$  | $c_0^2$  | $c$      | $\chi^2$/n.d.f |
|------------|-----------|----------|----------|----------------|
| $\pi^-$ (200 GeV) | 1.09±0.15 | 0.246±0.024 | 1140±659 | 17/11          |
| $\pi^+$ (200 GeV) | 1.73±0.02  | 0.128±0.001 | 6.52±0.46 | 10/11          |
| $K^-$ (200 GeV) | 0.90±0.20  | 0.236±0.032 | 657±500  | 6.4/9          |
| $K^+$ (200 GeV) | 2.24±0.04  | 0.102±0.001 | 0.0524±0.0090 | 1.8/9     |
| $\pi^-$ (62.4 GeV) | 0.48±0.02  | 0.333±0.033  | 4120±77  | 14/7          |
| $\pi^+$ (62.4 GeV) | 0.43±0.02  | 0.333±0.245  | 4200±211 | 15/7          |
| $K^-$ (62.4 GeV) | 0.41±0.13  | 0.244±0.098  | 830±1100 | 1.3/5         |
| $K^+$ (62.4 GeV) | 0.66±0.12  | 0.240±0.039  | 703±464  | 0.42/5        |

Table 3: Analysis of charged pions and K mesons rapidity distributions by means of eqs. (17), (18) and (25) with $y_0 = \ln(\sqrt{s_{NN}/m_p})$. (Distribution by Srivastava et al.)

| data       | $\omega$  | $c_0^2$  | $c$      | $\chi^2$/n.d.f |
|------------|-----------|----------|----------|----------------|
| $\pi^-$ (200 GeV) | 0.87±0.68  | 0.268±0.065 | 2030±2910 | 17/11          |
| $\pi^+$ (200 GeV) | 1.73±0.02  | 0.128±0.001 | 6.51±0.46 | 10/11          |
| $K^-$ (200 GeV) | 0.69±1.59  | 0.255±0.259 | 1100±5110 | 6.5/9          |
| $K^+$ (200 GeV) | 2.24±0.04  | 0.102±0.001 | 0.0542±0.0093 | 1.8/9     |
| $\pi^-$ (62.4 GeV) | 4.12±10^{-7}±0.35 | 0.265±0.002  | 3470±62  | 21/7          |
| $\pi^+$ (62.4 GeV) | 1.40±10^{-6}±0.60 | 0.259±0.002  | 3230±59  | 18/7          |
| $K^-$ (62.4 GeV) | 4.55±10^{-5}±9.84 | 0.247±0.011  | 1330±209 | 1.4/5         |
| $K^+$ (62.4 GeV) | 1.17±10^{-6}±6.47 | 0.283±0.008  | 2120±134 | 0.56/5        |

Table 4: Analysis of charged pions and K mesons rapidity distributions by means of eqs. (24) and (25) with $y_0 = \omega/\sqrt{c_0^2 + \delta}$ ($\delta = 2$). (Our distribution)

| data       | $\omega$  | $c_0^2$  | $c$      | $\xi$      | $\chi^2$/n.d.f |
|------------|-----------|----------|----------|------------|----------------|
| $\pi^-$ (200 GeV) | 1.55±0.15  | 0.276±0.052 | 1120±1190 | 1.71±0.44 | 16/10          |
| $\pi^+$ (200 GeV) | 1.86±0.02  | 0.112±0.001 | 1.35±0.09 | 3.35±0.80 | 10/10          |
| $K^-$ (200 GeV) | 1.40±0.14  | 0.165±0.010 | 50.9±18.6  | 6.15±7.66  | 6.6/8          |
| $K^+$ (200 GeV) | 2.28±0.04  | 0.105±0.001 | 0.064±0.010 | 2.50±1.61 | 1.9/8          |
| $\pi^-$ (62.4 GeV) | 0.86±0.02  | 0.333±0.331 | 2870±80  | 3.22±0.43 | 8.5/6          |
| $\pi^+$ (62.4 GeV) | 0.77±0.03  | 0.333±0.223 | 3100±140  | 2.52±0.33 | 16/6          |
| $K^-$ (62.4 GeV) | 0.68±0.09  | 0.333±0.267 | 1570±220  | 2.12±0.74 | 1.2/4          |
| $K^+$ (62.4 GeV) | 1.16±0.18  | 0.217±0.132 | 241±822  | 3.66±3.86 | 0.40/4        |
Table 5: Analysis of charged pions and K mesons rapidity distributions by means of eqs. (24) and (25) with $y_0 = \ln(\sqrt{s_{NN}/m_p})$. (Our distribution)

|                  | data         | $\omega$   | $c^2$  | $c$     | $\varepsilon$ | $\chi^2$/n.d.f |
|------------------|--------------|------------|--------|---------|---------------|----------------|
| $\pi^-$ (200 GeV)| 1.55±0.09    | 0.277±0.030| 1150±690| 1.70±0.32| 16/10         |
| $\pi^+$ (200 GeV)| 1.86±0.02    | 0.113±0.001| 1.35±0.09| 3.35±0.80| 10/10         |
| $K^-$ (200 GeV)  | 1.37±0.16    | 0.167±0.007| 58.9±28.3| 7.31±6.82| 6.5/8         |
| $K^+$ (200 GeV)  | 2.28±0.04    | 0.105±0.001| 0.0639±0.0098| 2.50±1.61| 1.9/8         |
| $\pi^-$ (62.4 GeV)| 0.90±0.08    | 0.306±0.047| 2150±1210| 3.86±1.35| 8.6/6         |
| $\pi^+$ (62.4 GeV)| 0.93±0.02    | 0.227±0.026| 741±334  | 6.31±3.12| 14/6          |
| $K^-$ (62.4 GeV)  | 0.89±0.10    | 0.215±0.076| 291±439  | 4.95±3.73| 1.1/4         |
| $K^+$ (62.4 GeV)  | 1.16±0.18    | 0.217±0.246| 243±857  | 3.65±3.93| 0.40/4        |

Table 6: Analysis of charged pions and K mesons rapidity distributions by means of eq. (26). (Distribution by Carruthers et al.)

|                  | data         | $\sigma$  | $c$     | $\chi^2$/n.d.f |
|------------------|--------------|-----------|---------|----------------|
| $\pi^-$ (200 GeV)| 2.16         | 1626±8    | 101/13  |
| $\pi^+$ (200 GeV)| 2.16         | 1621±8    | 45/13   |
| $K^-$ (200 GeV)  | 2.16         | 244±3     | 9.2/11  |
| $K^+$ (200 GeV)  | 2.16         | 264±3     | 24/11   |
| $\pi^-$ (62.4 GeV)| 1.87         | 1013±11   | 82/9    |
| $\pi^+$ (62.4 GeV)| 1.87         | 986±10    | 117/9   |
| $K^-$ (62.4 GeV)  | 1.87         | 130±4     | 14/7    |
| $K^+$ (62.4 GeV)  | 1.87         | 161±4     | 4.2/7   |
Figure 5: Comparison of pions and K mesons rapidity distribution and several formulae at $\sqrt{s_{NN}} = 200$ GeV. Results by Srivastava and ours in tables 2 and 4 are used.

Figure 6: Comparison of pions and K mesons rapidity distribution and several formulae at $\sqrt{s_{NN}} = 62.4$ GeV. Results by Srivastava and ours in tables 2 and 4 are used.
hydrodynamics. The Heaviside function is phenomenologically expressed by eq. (22). Its typical behaviors are shown in fig. 4. The concrete potential and the entropy distribution are presented in fig. 7. Moreover, three concrete components of our distribution are shown in fig. 8. Of course, the second and third ones depend on the values of $\varepsilon$’s.

2) Three distributions by Landau, Srivastava et al, and ours show almost the same minimum chi-squared as CERN MINUIT is used. However, values of $\omega$’s are different, provided that eq. (24) are applied to the data. Eq. (25) by Srivastava et al. strongly depends on cutoffs values $y_0$'s at 62.4 GeV. In other words, for values of cutoff no principle exists.

3) On the other hand, our distribution function, due to the Heaviside function, does not strongly depend on cutoffs at both energies. It is interesting that three sound velocities $c_s$’s at 62.4 GeV are 0.333, and decrease at 200 GeV in table 4. Among the Gaussian distribution and distributions based on 1+1 dimensional hydrodynamics there are significant differences as shown in figs. 5 and 6.

4) In conclusion, it can be added that the temperatures in collisions are estimated as $\omega$ (200 GeV) > $\omega$ (62.4 GeV): $T_0 = 0.7$ GeV (200 GeV) and $T_0 = 0.4$ GeV (62.4 GeV), respectively, provided that the explicit transition (hadrons $\leftrightarrow$ QCD) (See [17]) was not taken into account. In a future we are going to investigate the role of the phase transition between hadrons and QCD in this scheme [17] and problem of the fluctuation [21].

Figure 7: (a) Potential of fluid (dashed line) changes on the line of $\omega_f = 1.55$. $\varepsilon_s^2 = 0.277$ and $\varepsilon = 1.7$ (200 GeV, $\pi^-$). (b) Entropy distribution of fluid.

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Figure 8: Three components of our distributions on charged pions and K mesons at $\sqrt{s_{NN}} = 200$ GeV. The first term, second one, and third one of eq. (24) are named $F_1(\varepsilon, Q)$, $F_2(\varepsilon, Q)$, $F_3(\varepsilon, Q)$, respectively. Results in table 4 are used.

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