Applying regression analysis to optimize the length of components of a six bar mechanism

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Abstract. A mechanism can be defined as several rigid bodies connected to produce constrained relative motion between them and transmit forces and couples from the source of input power to result in motion. The movement of a body is studied using geometry so the links are considered to be rigid. The connections between links are modeled being ideal movement, pure rotation or sliding. The design of mechanism presupposes knowledge of the direction of motion, velocity, accelerations and the forces of each component at various positions during the operating cycle. The necessary informations can be obtained from the kinematic analysis of the motion of the mechanism. Because the purpose of a planar mechanism is to move one of the links or a point of an element through a specific motion, it is important to verify that the mechanism performs the established function. This paper presents the problem of modelling and analyzing the six bar linkages mechanism using specialized software. Regression analysis will be performed to find the optimum domain lengths of the parameters bars taking into account the angles of the mechanism to ensure the stability of the entire equipment. The results of the model were used to correct and optimize the design of the product. Effectiveness of regression models has been determined by analyzing correlation coefficient and by comparing with the experimental results. It has been concluded that regression modelling can be used to predict the datas accurately.

1. Introduction
Design engineers are often faced with the task of creating a linkage that generates an irregular motion. After a linkage is designed, it must be verified that it moves according to its motion design specifications. For mechanism design, the desired motion is known and the task is to determine the type of mechanism along with the required forces and torques [1]. Many studies have been conducted on almost all types of mechanisms and analysis methods. Ting et al. [12] studied the rotatability law for N-bar kinematic chains by a traditional method. Hang et al. [4] presented the decoupling conditions of spherical parallel mechanisms. Wolf et al. [13] analyzed the singularities of a three degree of freedom spatial Cassino Parallel manipulators. Researches on the relations between the global motion characteristics and the singularity of mechanisms are few. Gao et al. [3] introduced the concept of solution space to the mechanisms evolution analysis, so the workspace of parallel planar manipulators was obtained. Zhao et al. [14] explored with convex division method the classification of single-loop planar and spherical mechanisms. Kamat et al [6] synthesized an adjustable planar 4-bar mechanism for different angles, adjusting the length of different links to obtain different paths. Hwang
and Wang [5] presented a synthesis technique for the planar Watt-6 bar mechanism with a coupler point passing through 3 and 4 acceleration poles.

The kinematic analysis is the technique used that aims to determine the linear, and angular positions, trajectories, velocities and accelerations of some characteristic points, respectively of the elements from a mechanism, during a geometric cycle of movement, when both the structure and geometry of the mechanism are known, as well as the movement of the leading element, but without considering the forces that cause the movement. The geometric cycle of the movement of a mechanism represents the minimum period of time after which the elements of the mechanism reach the same positions.

2. 6-bar mechanism
The simplest closed form linkage is a 4-bar, which has three moving links plus one fixed link and four pinned joints. The first step in designing a mechanical linkage is to draw the kinematic diagram. The kinematic diagram is made up of nodes and straight lines and is used for design and analysis purposes and to show the relative motion between links. In a mechanism, there may be several loops, and for each loop it can be written a closing equation in vector form or complex numbers. To have an open chain with no loop, a link must disconnect two other links. When disconnecting the link, one must ensure that all elements are held in an open chain. A 6-bar linkage can be put together in two different configurations, the Watt 6-bar and the Stephenson 6-bar, as shown in the following figure, (figure 1).

![Figure 1. 2 configurations of 6-Bar linkage [10].](image)

The difference between these configurations is that the Watt 6-bar linkage has the two ternary links connected together, whereas the Stephenson 6-bar linkage has the two ternary link separated by a binary link. A 6-bar linkage is typically made up of two 4-bar linkages or a 4-bar linkage and a slider-crank mechanism. Therefore, it can be defined by analyzing the two separate linkages, using the procedures for 4-bar linkages and slider-crank mechanisms [9].

3. Mechanism analysis
The basic method of mechanism analysis is dividing the structural parameter space to obtain mechanism dimensional type with different performance characteristics, or dividing the motion parameter space to obtain the range of motion parameters with significantly different performance distribution characteristics [10].

The linkage was sketched, the sizes and orientation of the known links were adjusted, then request the position and orientation of the unknown links and points were determined. For this paper, all links are assumed rigid bodies. In order to develop a method of synthesizing a six bar mechanism it is important to go through its positional analysis to see how the joint angles affect one another, as shown in figure 2.
Figure 2. The analyzed mechanism.

With this lengths, Matlab software is used to produce the code for the analysis, synthesis and simulation of the 6 bar mechanism that is defined through the following equations, (equation (1) - (equation (11))).

\[
P_2 = A \cdot [\cos(\theta) ; \sin(\theta)]
\]

\[
E = \sqrt{A^2 + D^2 - 2 \cdot A \cdot D \cdot \cos(\theta)}
\]

\[
\alpha = \sin(A \cdot \sin(\theta)) / E
\]

\[
\beta = \cos((E^2 + C^2 - B^2) / (2 \cdot E \cdot C))
\]

\[
P_3 = [D - C \cdot \cos(\alpha + \beta) ; C \cdot \sin(\alpha + \beta)]
\]

\[
\theta_1 = \pi - (\alpha + \beta)
\]

\[
E_1 = \sqrt{C^2 + D_1^2 - 2 \cdot C \cdot D_1 \cdot \cos(\theta_1)}
\]

\[
\alpha_1 = \sin(C \cdot \sin(\alpha + \beta) / E_1)
\]

\[
\beta_1 = \cos((E_1^2 + C_1^2 - B_1^2) / (2 \cdot E_1 \cdot C_1))
\]

\[
\theta_2 = \alpha_1 + \beta_1
\]

\[
P_5 = [D + D_1 - C_1 \cdot \cos(\theta_2) ; C_1 \cdot \sin(\theta_2)]
\]

The mechanism was analyzed using the program developed in Matlab, with lengths of the mechanism ranging between 100 and 300 mm, with a variation of 20 mm.

4. Regression analysis

Values of the mechanism lengths were used to perform a regression analysis in order to optimize the parameters of the equipment. Selected factors of experiment have changed in three levels of values, defined by an orthogonal array. Taking into account the configuration of the mechanism, we have chosen those dimensions of the elements for which the value of the output angle is between 0 and 10 degrees, and are presented in the following table, (table 1).
Table 1. Three level design parameters.

|   | A  | C  | D  | D1 |
|---|----|----|----|----|
| 1 | 1  | 1  | 1  | 1  |
| 1 | 2  | 2  | 2  |    |
| 1 | 3  | 3  | 3  |    |
| 2 | 1  | 2  | 3  |    |
| 2 | 2  | 3  | 1  |    |
| 2 | 3  | 1  | 2  |    |
| 3 | 1  | 3  | 2  |    |
| 3 | 2  | 1  | 3  |    |
| 3 | 3  | 2  | 1  |    |

The regression technique is used to adjust, as much as possible, the data observed at a given theoretical curve. It will be considered a random variable denoted by Y, which will be highlighted in the regression analysis. This variable, also called the dependent variable, will be the one whose values will be estimated based on multiple linear regression analysis, starting from the predictive factor values. This value is represented by the $\theta_3$ angle, the variable that influences the most the stability of the equipment.

In multiple linear regression model, the method of least squares is used to estimate the regression coefficients. They illustrate the unrelated contributions of each independent variable towards predicting the dependent variable. Regression coefficients represent the mean change in the response variable for one unit of change in the predictor variable while holding other predictors in the model constant. Unlike the simple linear regression, there must be inferences made about the degree of interaction or correlation between each of the independent variables. The values determined from the regression analysis are: the correlation coefficient 0.914 that highlights as percentage how the independent variables define the dependent one, for the multiple regression the coefficient that is taken into consideration is Adjusted $R^2 = 0.6735$ that means that the independent variables express two-thirds of the variance of the dependent variable. Adjusted $R^2$ indicates how well terms fit a curve or line, but adjusts for the number of terms in a model. If you add more useless variables to a model, adjusted $R^2$ will decrease.

The particular values for each predictive variable will be introduced into the determined equation. The relationship between the Y response variable and predictive variables X is linear. The effects of each variable are independent of the others. The model that is generated for 4 independent variables and 1 dependent variable was established based on the regression analysis for predicting the output variable represented by the angle and is shown below, (equation (12)).

$$Y = 0.535 + (-0.00093) \cdot x_1 + (0.019) \cdot x_2 + (0.004960) \cdot x_3 + (0.0011) \cdot x_4$$  \hspace{1cm} (12)

The value of the significance F 0.071 is greater than alpha value that is 0.05, so the model can’t be rejected. For example, variable 1 which corresponds with $l_1$ has the P value 0.92 that is much greater than the limit of significance alpha 0.05. Variable 2 which correspond with $l_2$ has the P value smaller than the value 0.05.

The strongest prediction for the response variable is chosen. For this we consider the simple linear regression between the dependent variable and each predictor in part, by choosing the variable for which the significance level p is the lowest. Then it is chosen from the remaining independent variables, which has the highest correlation between the measured values and the actual values with the model residue from the previous step. The previous step is repeated until the removing of a new variable becomes insignificant, for example the level of significance meets the coefficient of the
correlation with residues, that means the P value is close to 0.05. So, the obtained model is shown below, (equation (13):

\[ Y = 2.12 + 0.019 \times x_1 \]  

(13)

With this method, it can be established the order of variables influence and be excluded those without significant influence. Also it can be verified the accuracy of the determined mathematical model and this can be perfected in order to increase the precision.

A LabView virtual tool shown in figure 3 is used to obtain the simple regression analysis and it can represent graphically: the coordinates affected by errors of a set of experimental points on the theoretical line; the theoretical line on which the ideal points would be located; the regression line that passes through the experimental points. Then it is finded a curve that approximates the set of points by minimizing the sum of squares of point deviations from that curve.

![Figure 3. LabView Block Diagram.](image)

The \( \theta_3 \) angle which results from the model is shown in table 2. Experimental data were put into the model and the angle is determined.

| Regression Values [°] | Experimental Values [°] | Standard deviation |
|-----------------------|-------------------------|--------------------|
| 5.69                  | 5.66                    | 0.024              |
| 6.96                  | 6.43                    | 0.374              |
| 8.22                  | 8.19                    | 0.024              |
| 6.03                  | 6.15                    | 0.084              |
| 7.13                  | 7.57                    | 0.311              |
| 7.65                  | 8.29                    | 0.452              |
| 6.2                   | 6.24                    | 0.028              |
| 6.72                  | 6.56                    | 0.113              |
| 7.82                  | 7.34                    | 0.339              |

The comparison of the result between theoretical value of the output angle with the one given by mathematical regression is very close and deviation showing is very small.

5. Conclusions
The statistical control that regression provides is important because it isolates the role of one variable from all of the others in the model. Models that involve more than two independent variables are more complex in structure but can still be analyzed using multiple linear regression techniques. Software
programme was used to solve a set of equations and give the optimal dimensions of the mechanism. In experiments conducted, the independent variables are varied simultaneously taking a limited number of values in the considered domain. The method enables the highlighting of existing interactions between independent variables contributing to a more accurate determination of the overall optimum model.

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