A Comprehensive Study on the Variation Phenomena of AO 0235+164

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Abstract

The variation mechanism of blazars is a long-standing open question. Observations of polarization can provide us with more information to constrain models. In this work, we collect long-term multiwavelength data on AO 0235 +164, and analyse the correlations between them by using the local cross-correlation function. We find that both γ-ray and the optical V-band light curves are correlated with the radio light curve beyond the 3σ significance level. The regions emitting the γ-ray and optical radiation coincide within errors, and are located 6.6±0.6 pc upstream of the core region of 15 GHz, which is beyond the broad-line region. The color index shows the redder-when-brighter trend in the low flux state, but turns to the bluer-when-brighter trend in the high flux state, while the γ-ray spectral index always shows the softer-when-brighter trend. We propose that such complex variation trends can be explained by the increasing jet component with two constant components. The optical polarization degree (PD) flares and optical flux increases are not synchronous. It seems that one flux peak is sandwiched by two PD peaks, which have inverse rotation trajectories in the qu plane. The helical jet model can schematically show these characteristics of polarization with fine-tuned parameters. The change in viewing angle is suggested to be the primary variable that leads to all these variations, although other possibilities such as the shock-in-jet model or the hadronic model are not excluded completely.

Unified Astronomy Thesaurus concepts: Radio jets (1347); Relativistic jets (1390); Active galactic nuclei (16); Blazars (164)

1. Introduction

Blazars are a subclass of active galactic nuclei (AGNs) that have relativistic jets directed toward the Earth (Urry & Padovani 1995). The most remarkable feature of blazars is their violent variability in almost all wavelengths. The spectral energy distribution (SED) of blazars exhibits two bumps, which are interpreted as the synchrotron and inverse Compton scattering processes in the leptonic model (Königl 1981; Sikora et al. 1994, 2009; Celotti & Ghisellini 2008), or the synchrotron radiation of electrons and protons in the hadronic model (Mücke et al. 2003; Böttcher et al. 2013). Apart from several nearby sources, blazars can only be identified as point-like objects in the optical and γ-ray bands. The location of the optical and γ-ray emitting regions cannot be resolved directly. Analysis of the time lag between time series of multiple frequencies can help to answer this question (Kudryavtseva et al. 2011; Max-Moerbeck et al. 2014a). The variation in color behavior is one popular method to investigate the variation mechanism. Recently, it was suggested that the correlation between the polarization degree (PD) and optical fluxes can also help to determine the primary variation mechanism (Shao et al. 2019). Besides, the Stokes parameter can provide us with twice as much information as the flux, in principle, which can provide further evidence to diagnose the variation mechanism. Thus, a comprehensive study of various aspects of variation phenomena can help us to understand the essential nature of blazars.

Blazar AO 0235+164 (IAU name J0238+1636) was identified as a BL Lac object (Spinrad & Smith 1975). Its J2000 coordinates are R.A. = 02:38:38.9, decl. = +16:36:59 (Johnston et al. 1995), and the redshift is z = 0.94 (Cohen et al. 1987). Historically, this target has been investigated from various aspects. It has been described as a compact and violently variable source (Davis & Wolfe 1982). In 1975, its optical brightness increased a hundred-fold in a few weeks (Ledden et al. 1976; Rieke et al. 1976). The target showed strong γ-ray flares in 2008 September and October (Corbel & Reyes 2008; Foschini et al. 2008), as well as in 2015 August (Ciprini 2015). A good review of the monitoring of this target was given by Ackermann et al. (2012). Its long-term light curves in the optical and radio have been monitored, and the periodicity has been discussed (Jenkins 1996; Villata & Raiteri 1999; Raiteri et al. 2001; Ostorero et al. 2004; Liu et al. 2006; Wang 2014). Raiteri et al. (2001) revealed that the outbursts of this source indicate a period of ∼5.7 yr. Ostorero et al. (2004) found that the periodicity of the main optical and radio outbursts and the SED can be well explained by the helical jet model (Villata & Raiteri 1999). It was also proposed that the periodicity may be caused by the oscillatory accretion of the supermassive black hole binary (Liu et al. 2006). The correlations between different wavelengths were widely discussed (Ledden et al. 1976; MacLeod et al. 1976; Kraus et al. 1999; Raiteri et al. 2001; Chen et al. 2002; Agudo 2013; Volvach et al. 2015; Fan et al. 2017; Hagen-Thorn et al. 2018; Kutkin et al. 2018). Using discrete correlation function (DCF) analysis, Chen et al. (2002) found that short wavelengths vary before long ones. Agudo (2013) proposed that the region emitting high-energy photons can be inferred from the relative position of the features in very long baseline interferometry (VLBI) images. Fan et al. (2017) indicated that the optical variation leads the radio by 23 ± 13 days. Hagen-Thorn et al. (2018) presented multiwavelength monitoring results from 2007 to 2015, and found that there was no time lag between the optical and γ-rays, indicating that the optical and γ-rays were radiated from the same region. Using both the time lags and
size measurements of the radio core, Kutkin et al. (2018) derived the distance between the radio core and the jet apex, estimated the jet parameters, and indicated that the outflow is bent.

The variation of polarization and its correlation with optical flux can constrain the variation mechanism. Cellone et al. (2007) found that total flux of the target is correlated neither with the color nor with the PD. A model of transverse shock propagation along the jet was proposed to explain the correlation between the flux and PD in the outburst during 2006 December (Hagen-Thorn et al. 2008). Due to the correspondence between a series of optical PD flares and a pronounced maximum polarization of a superluminal feature at 7 mm, Agudo et al. (2011) suggested that the degree of ordering of the magnetic field fluctuates rapidly. Cellone et al. (2007) found a trend in which the color turns redder when the PD is higher. Sasada et al. (2011) reported that the short flares in 2008 and 2009 show a bluer-when-brighter (BWB) trend. They also found a significant positive correlation between the amplitudes of the flux density and the PD in short flares. Ackermann et al. (2012) found that strong γ-ray flares accompany the increase in optical PD. Recently, Shao et al. (2019) found that there is a significant correlation between PD and optical flux density for PKS 1502+106, which can be explained if the variation is due to the change in viewing angle. This provides us with more clues to the variation mechanism for AO 0235+164.

The variation of blazars is difficult to understand because there are many emission components and variable factors in AGNs. A comprehensive analysis of various phenomena of variation is necessary to pin down the variables. In this paper, we collect the long-term data on AO 0235+164, including the γ-ray, optical, and radio, as well as the spectra and polarization. We analyze the correlation between different bands to locate the emitting regions. We study the variable behaviors of color and spectral index of γ-rays to figure out the primary variation mechanism. The optical polarization data, including the time series of PD and trajectories of (q, u), constrain the variation mechanism in a further step. Our main conclusion is that the helical jet model can roughly explain all these variation phenomena of AO 0235+164, and the viewing angle is the primary variable. The shock-in-jet model is not excluded completely. This paper is organized as follows. In Section 2, nearly nine years of archival data are collected and reduced. In Section 3, the analysis of the local cross-correlation function (LCCF) and time lag is presented. Then, the location of the γ-ray and optical emitting regions is derived. In Section 4, the spectral index behaviors and variation of polarization are manifested. The helical jet model is simulated to explain the observational phenomena. Finally, our conclusion is given in Section 5.

2. Data Collection and Reduction

In this work, we collected multiwavelength data on AO 0235+164 from the blazar monitoring programs, which are retrieved from the Fermi Science Support Center (FSSC), the Steward Observatory,2 the Small and Moderate Aperture Research Telescope System (SMARTS),3 as well as the Owen Valley Radio Observatory (OVRO)4 (Atwood et al. 2009; Smith et al. 2009; Richards et al. 2011; Bonning et al. 2012).

Fermi LAT data. The Fermi Large Area Telescope (LAT) data on AO 0235+164, which span nearly nine years from 2008 August 4 to 2017 June 30, are downloaded. The energy of γ-ray photons is selected to be in the range 0.1–300 GeV. The region of interest (ROI) centered on the target has a radius of 15°. The Fermi tools (version 1.0.1) was installed within the Conda package manager. We use the unbinned likelihood method to reduce the γ-ray data (Abdo et al. 2009). In the standard pipeline, the XML model files are generated by using make3FGLxml.py. The instrument response function is P8R2_SOURCE_V6. The Galactic diffusion background template model GLL_IEM_V06.fit and the extragalactic isotropic dispersion background iso_P8R2_SOURCE_V6_v06.txt are accounted for. The target AO 0235 is named as “3FGL J0238.6+1636” in the 3FGL catalog, which has a log-parabola spectrum \( dN/dE \propto (E/E_0)^{-(\alpha + \beta \log(E/E_0)}) \) (Acero et al. 2015). Parameters of point sources within a radius of 5° from the center of the ROI were left free, while the others were fixed according to the values given in the 3FGL catalog. To obtain the spectral indices of γ-rays, we divide the energy range 0.1–219 GeV into seven logarithmically equal bins and set the time bin as 4 days. We perform the SED fitting by using the γ-ray fluxes of different energy bins in one time bin. To obtain high-quality spectral index data, we set the threshold of the test statistics (TS) value to be 10. Fluxes with TS less than 10 will be ignored in the SED fitting. The fitting function log \( F(E) = \alpha \log E + A_0 \) is adopted. Thus, one obtains the γ-ray spectral indices of power-law spectra. The variation behavior of spectra will be studied by pairing the simultaneous flux and spectral index.

Photometry and polarization data. The optical photometry and polarization data are taken from the Steward Observatory (SO), which runs the ground-based monitoring program to support the Fermi LAT program (Smith et al. 2009). We collect the V-band and R-band photometry data from 2008 October 4 to 2018 February 12. Considering the lunar phase and weather conditions, as well as other difficult times, the observing cadence of the data is uneven. The PD and the Stokes parameters \( (q = Q/I \text{ and } u = U/I) \) observed by the SPOL polarimeter are also determined, and provide more constraints for the variation models. The interstellar polarization is not considered in the calibration of \( q \) and \( u \). We also collect the optical and near-infrared data from SMARTS (Bonning et al. 2012). This monitoring project uses the 1.3 m telescope, which was mounted with the BVRI and JHK filters. To study the color index behavior, we consider only data in the B, V, and J bands in this work. The time interval of the SMARTS data is from 2008 February 5 to 2015 August 31. The intercalibration of V-band data from both SMARTS and Steward is performed by using the same comparison star in the finding charts. We combine them to produce the V-band light curve.

Radio 15 GHz data. The 15 GHz radio flux data were obtained from OVRO 40 m monitoring program (Richards et al. 2011). In this work, the calibrated data span from 2008 June 6 to 2017 November 14, and 569 data points are present in this period. Figure 1 presents the γ-rays in the energy range 0.3–0.9 GeV, optical V-band, radio 15 GHz, V–J color index, and PD light curves.

https://fermi.gsfc.nasa.gov/ssc
2 http://james.as.arizona.edu/~psmith/Fermi
3 http://www.astro.yale.edu/smarts/glast/
4 http://www.astro.caltech.edu/ovroblazars/
3. Locations of Emitting Regions

3.1. Time Lags

The correlations of multiwavelength light curves are important for revealing the emission mechanism of blazars. Two methods are often used to calculate the correlation between two time series with uneven samplings, i.e., the DCF (Edelson & Krolik 1988) and the LCCF (Welsh 1999). By comparing the performance of the two methods, Max-Moerbeck et al. (2014b) proposed that the LCCF method can manifest physical signals more effectively than the DCF method. In addition, the coefficients of the DCF can exceed the range $[-1, 1]$, making it inapplicable for the standard statistical test (White & Peterson 1994). The coefficients of the LCCF are in the range $[-1, 1]$, which is a good property for estimation of the significance level. Thus, we use the LCCF to calculate the correlations in this work. In order to estimate the significance levels of signals, the Monte Carlo (MC) simulation is used (Shao et al. 2019). First, $10^4$ artificial radio light curves are simulated by using the Timmer–König algorithm (TK95, Timmer & König 1995). For AO 0235+164, the slope of the power spectral density for the simulated radio light curve is set to be $\beta = 2.3$, referring to Max-Moerbeck et al. (2014a). Each simulated light curve contains 10,000 data points and the time bin is 1 day. Since the observed radio light curve is unevenly sampled (US), we extract a subset time series that has the exact same samplings as the observed one. Second, LCCFs between the simulated US time series and the observed $\gamma$-ray (or optical V-band) light curve are calculated at each lag bin. Then, the $1\sigma$, $2\sigma$, and $3\sigma$ significance levels corresponding to chance probabilities of 68.26%, 95.45%, and 99.73% can be obtained. Having the significance levels, the signals of lag and the estimation of its $1\sigma$ standard deviation are based on the model-independent Monte Carlo method proposed by Peterson et al. (1998). This method considers both the flux randomization (FR) and the random subset selection (RSS) processes (Peterson et al. 1998; Larsson 2012). Two kinds of time lags can be calculated, $\tau_p$ and $\tau_c$. $\tau_p$ is the lag for the highest peak of LCCF, while $\tau_c$ is the centroid lag defined as $\tau_c = \sum_{i} \tau C_i / \sum_{i} C_i$, where $C_i$ is the correlation coefficient satisfying $C_i > 0.8LCCF(\tau_i)$ (Shao et al. 2019). We repeated this $10^4$ times to obtain the distribution of $\tau_p$ and $\tau_c$. The error of time lag is taken as the $1\sigma$ standard deviation.

In Figure 2, LCCFs of the $\gamma$-ray and optical $V$-band versus radio 15 GHz are plotted in the left and right panels, respectively. We use the $\gamma$-ray light curve in the energy interval 0.3–0.9 GeV to do the LCCF calculation. This light curve has the TS threshold of 10, and 68% of data points are ignored. Large gaps appear in the reduced $\gamma$-ray light curve if the TS threshold is taken to be 0, only 14% of data are ignored. The lag result is almost invariant under different TS thresholds, but the fluctuation of the estimation of significance is reduced with a smaller TS threshold. The lag range is $[-500, 500]$, and the lag bin is 5 days. In the left panel of Figure 2, the peak of LCCF between the $\gamma$-ray and radio light curves is beyond the $3\sigma$ significance level. The $\gamma$-ray leads the radio with $\tau_p = -45.3^{+14.9}_{-15.0}$ and $\tau_c = -45.4^{+7.3}_{-14.4}$ days. Max-Moerbeck et al. (2014a) found that the $\gamma$-ray leads radio by 150 ± 8 days with 99.99% significance. However, there are three peaks in the plot of the LCCF (see the top right panel in Figure 1 in the reference). Among them, the most significant one is located at $-30$ days, which roughly agrees with our result. The flares in the light curve can significantly affect the correlation results. In their work, only four years of $\gamma$-ray data have been used, which

![Figure 1](image-url)
Figure 2. Plots of LCCF results of γ-ray (TS > 10) vs. radio (15 GHz) (left panel) and optical V-band flux vs. radio (right panel). Black dots with error bars denote correlation values. The green dashed–dotted, red dotted, and blue dashed lines denote the 1σ, 2σ, and 3σ significance levels, respectively. The negative lag indicates that the former leads the latter.

Table 1

| Time Lags | γ-Ray vs. Radio | V-band vs. Radio | γ-Ray vs. V-band |
|-----------|----------------|-----------------|-----------------|
| \( \tau_p \) (days) | \(-45.3^{+15.9}_{-5.0} \) | \(-0.5^{+0.4}_{-0.4} \) | \(-5.5^{+16.0}_{-5.0} \) |
| \( \tau_c \) (days) | \(-45.4^{+7.3}_{-14.4} \) | \(-30.1^{+9.8}_{-13.1} \) | \(-15.1^{+4.7}_{-5.8} \) |

Note. Here \( \tau_p \) and \( \tau_c \) denote the peak and centroid time lags (in unit of days), respectively. The negative lags indicate that the former lead the latter.

contain only the flare state and quiescent state. Our γ-ray data cover more than eight years, and the active state (MJD 56800 to MJD 58000) has been recorded. The long duration and complex states will enhance the significance of our result. In the right panel of Figure 2, the LCCF between the V-band and radio light curves also shows one peak beyond the 3σ significance level. The optical V-band leads the radio by \( \tau_p = 0.5^{+5.0}_{-0.0} \) and \( \tau_c = 30.1^{+18.1}_{-9.8} \) days. Raiteri et al. (2001) indicated that the time lags between optical and radio vary for different outbursts. Fan et al. (2017) used DCF to show that the optical leads the radio by 23 ± 13 days. These results do not conflict with our result. In Table 1, one also notes that the \( \tau_c \) results are consistent for the three relative time lags, while \( \tau_p \) results are inconsistent. The difference between \( \tau_p \) and \( \tau_c \) may be because the gap in observations in the optical and radio monitoring programs can lead to deformation of the LCCF to some extent (Shao et al. 2019). It is also possible that the propagation of disturbance from optical to radio varies each time, which leads to the uncertainty in the lag.

In Figure 3, the LCCF between γ-rays and V-band is plotted. In the MC simulation of significance levels, we use the TK95 algorithm and set \( \beta = 1.0 \) (referred to Max-Moerbeck et al. 2014a) to produce 10^{5} artificial γ-ray light curves. Each curve contains 10,000 days with 1 day time bin. To mimic the reduced γ-ray light curve, we rebin the artificial curves with a time interval of 4 days, and take exactly the same samplings with the γ-ray light curve with TS > 10. The FR/RSS procedures yield \( \tau_p = -5.5^{+15.9}_{-5.0} \) and \( \tau_c = -15.1^{+4.7}_{-5.8} \) days, respectively. From the result of \( \tau_c \), it seems that the γ-ray emission leads the V-band. However, the γ-ray and optical emissions are possibly simultaneous within the uncertainty of \( \tau_p \). Hagen-Thorn et al. (2018) claimed that there is no time delay between the γ-ray and optical light curves, but no correlation analysis was performed to exhibit that. With the current precision of samplings, the optical emitting region is probably the same as the γ-ray emitting region. More intensive samplings help to improve the precision of LCCF results.

3.2. Location of Emitting Regions

The time lag between γ-ray and radio can be well explained by the jet model, i.e., the disturbance propagates along the jet, and γ-rays and radio are radiated in the upstream and downstream regions, respectively. The distance between regions emitting different bands is given by (Kudryavtseva et al. 2011; Max-Moerbeck et al. 2014a)

\[
\Delta D = \frac{\beta_{\text{app}} c \Delta T}{(1+z) \sin \theta},
\]

where \( \beta_{\text{app}} \) is the apparent velocity in the observer frame, \( c \) is the speed of light, and \( \Delta T \) is the time lag (\( \tau_p \) or \( \tau_c \)) between different bands. For this target, the redshift is \( z = 0.94 \) (Cohen et al. 1987).
Table 2: Relative Distances

| Distance | γ-Ray vs. Radio | V-band vs. Radio | γ-Ray vs. V-band |
|----------|----------------|-----------------|------------------|
| $D_\parallel$ (pc) | 6.6±0.1 | 0.1±0.7 | 0.8±0.2 |
| $D_\perp$ (pc) | 6.6±0.1 | 4.4±0.6 | 2.2±0.3 |

Note. $D_\parallel$ and $D_\perp$ (in units of pc) are distances derived according to $\tau_\parallel$ and $\tau_\perp$ in Table 1, respectively.

Lister et al. (2009) stated that $\beta_{\text{app}}$ was not available due to the poor resolution of VLBI images at 15 GHz. One can take advantage of the VLBI 43 GHz data to estimate $\beta_{\text{app}}$. Jorstad et al. (2017) presented the apparent velocities $\beta_{\text{app}}$ of three jet components of AO 0235+164, which are 26.27 ± 1.67, 13.39 ± 1.47, and 3.05 ± 0.31, respectively, in units of $c$. Kutkin et al. (2018) incorporated the birth time, date-resolved feature size, and distance from the core to conclude that the true physical component indicates an apparent velocity $\beta_{\text{app}} = 10$. For the viewing angle $\theta$, Hovatta et al. (2009) derived that $\theta = 0.5\alpha$ based on $\beta_{\text{app}} = 2$. We take $\beta_{\text{app}} = 10$ to calculate the distance, and derive the viewing angle to be $\theta = 1.7\alpha$.

The relative distances between emitting regions of different bands, i.e., $D_\parallel$ and $D_\perp$, are calculated according to Equation (1) by inserting $\tau_\parallel$ and $\tau_\perp$, respectively. The results are summarized in Table 2. The γ-ray emitting region is located at a distance of about 7 pc upstream of the core region of 15 GHz. There are large uncertainties in the time lag between $V$-band and radio due to unevenly sampled data. We tend to adopt the more significant result between γ-ray and $V$-band to determine the optical emitting region, i.e., the γ-ray and optical emitting regions coincide within the error.

To obtain the locations of emitting regions, we need to determine the distance between the core region of 15 GHz and the jet base, which is denoted as $r_{\text{core}}$. Two methods can be applied to derive $r_{\text{core}}$. The first method considers the geometrical relation for a conical jet, which is given as (Hirotani 2005)

$$r_{\text{core}} = \frac{r_\perp}{\varphi} = \frac{0.5\theta_d d_L}{(1 + z)^2 \varphi}, \quad (2)$$

where $r_\perp$ is the transverse size of the jet at the core position, $\theta_d$ is the angular diameter at a certain frequency, and $\varphi$ is the half-opening angle of the jet. Following Kutkin et al. (2018), the half-opening angle is $\varphi \approx 1.7\alpha$, and the core size of 15 GHz is $\theta_d \approx 0.125$ mas. The redshift $z = 0.94$ indicates that the luminosity distance of the target is $d_L = 6142$ Mpc (Cohen et al. 1987). With these parameters, $r_{\text{core}}$, 15 GHz, is derived to be 29 pc. Core-shift measurement is the second commonly used method to derive $r_{\text{core}}$ (Königl 1981; Lobanov 1998; Hirotani 2005). Considering the opacity, the positions of photospheres will shift systematically toward the downstream of the jet as the frequency decreases, which can be observed by VLBI images at millisecond resolution. $r_{\text{core}}$ at the frequency $\nu$ can be derived as (Lobanov 1998; Hirotani 2005)

$$r_{\text{core}}(\nu) = \frac{\Omega_m}{\sin \theta} \frac{\nu^{-1/k}}{k},$$

$$\Omega_m = 4.85 \times 10^{-9} \Delta r_{\text{rel}} d_L \left( \frac{\nu_1^{1/k} \nu_2^{1/k}}{\nu_2^{1/k} - \nu_1^{1/k}} \right) \text{pc GHz.} \quad (3)$$

where $\Omega_m$ is the core position offset. Kutkin et al. (2018) presented $\Delta r_{15 \text{GHz}}$, $5 \text{GHz} = 0.1$ mas and $k = 1.25$. With these parameters, one obtains $\Omega_m = 4.9$ pc GHz and $r_{\text{core}}$, 15 GHz $\approx 18.9$ pc. Based on the geometrical method, Max-Moerbeck et al. (2014a) have estimated $r_{\text{core}}$, 15 GHz $\geq 23 \pm 6$ pc by using the averaged core angular size of multiple epochs. Our results for $r_{\text{core}}$, 15 GHz are consistent with the constraint given by Max-Moerbeck et al. (2014a). Taking $r_{\text{core}}$, 15 GHz $\approx 18.9$ pc, the location of the γ-ray and optical emitting region is about 12 pc away from the jet base, which is far beyond the broad-line region (BLR) region.

With the known $\Omega_m$, the magnetic field strength and electron number density at 1 pc can be derived as (Hirotani 2005; O’Sullivan & Gabuzda 2009; Kutkin et al. 2018; Jiang et al. 2020)

$$B_1 \approx 0.025 \left[ \frac{\sigma_{\text{el}} \Omega_m^{3/2} (1 + z)^2}{\varphi \sin^{3k-1} \theta_d \delta^2} \right]^{1/4}, \quad (4)$$

$$N_1 \approx 3.3 \left[ \frac{\sigma_{\text{el}} \Omega_m^{3/2} (1 + z)^2}{\varphi \sin^{3k-1} \delta^2} \right]^{1/2}, \quad (5)$$

where $\gamma_{\text{min}}$ is the minimum Lorentz factor of radiative electrons, and $\sigma_{\text{el}}$ is the ratio of the magnetic field energy density to the nonthermal particle energy density. Setting $\sigma_{\text{el}} = 1$ and $\Omega_m = 4.9$ pc GHz, one obtains two parameters directly, i.e., $B_1 \approx 0.97$ G and $N_1 \approx 5012 \gamma_{\text{min}}^{-1}$ cm$^{-3}$. Considering the distribution of matter and fields in the jet, the magnetic field strength and number density of electrons follow the scaling laws $B = B_1 (r/r_1)^{-m}$ and $N = N_1 (r/r_1)^{-n}$. Using both the core-shift measure and time lags for radio frequencies, Kutkin et al. (2018) obtained that the power-law indices are $m = 1.2 \pm 0.1$ and $n = 2.4 \pm 0.2$. Based on these parameters, the magnetic field and electron number density in the optical emission region are calculated to be 0.05 G and $12\gamma_{\text{min}}^{-1}$ cm$^{-3}$, respectively.

4. Variation Analysis

The variation phenomena of AO 0235+164 are rich. Combined with the color index behavior, the variation of polarization can help us to further constrain the emission mechanism.

4.1. Variation of Spectral Index

The color behavior gives us the most direct clues to analyze variation. Figure 4 is a diagram of $V - R$ color versus $V$-band magnitude. In epoch II, the variation of color has a weak redder-when-brighter (RWB) trend (with Pearson’s $r_{\text{II}} = -0.60$). In epochs I and III, no obvious trends are found ($r_{\text{I}} = -0.02$, $r_{\text{III}} = 0.04$). In Figure 5, the $B - J$ versus $J$-band magnitude is plotted. An obvious RWB trend is evident in the quiescent state. For the flare state (epoch I), the red triangles indicate a weak BWB trend. For the active state, the points are scattered, and the plateau pattern is possibly due to the change of trends. A similar plot was also presented by Bonning et al. (2012) with 825 days of data. Complex color behavior has been found for CTA 102 (see Figure 3 in Raiteri et al. 2017), which is a flat-spectrum radio quasar (FSRQ) object. AO 0235+164 was classified as a BL Lac object in Stein et al. (1976).
However, for this target, Cohen et al. (1987) and Raiteri et al. (2007) indicated the broad emission line of Mg II with FWHM 3100 and 3500 km s$^{-1}$, respectively. In Figure 6, we plot the spectra observed by SO. It is evident that the broad line of Mg II with FWHM 3260 km s$^{-1}$ appears in the quiescent state and disappears in the flare and active states. Chen & Jiang (2001) investigated the spectral variability of this source during two active phases, and revealed that AO 0235+164 is much more like an FSRQ object in many aspects. Adopting the quasar composite SED from Elvis et al. (1994), the numerical fitting of the broadband SED indicates that the accretion disk component is 100 times less bright than the jet component (Ackermann et al. 2012; Rainò et al. 2013).

The spectra in Figure 6 also present us with an explanation for the complex variation trend of this target. The observed flux contains both the disk component and the jet component. In the plot, it is evident that the spectral slopes are always negative. So, the jet component dominates over the disk one even in the quiescent state. The peak frequency of the synchrotron component should be less than $10^{14.6}$ Hz. As the jet component increases, an RWB trend will be produced. This mechanism has been suggested to explain the RWB trend for 3C 454.3 by Villata et al. (2006). When the $J$-band magnitude is less than 15, the $B - J$ color shows an evident BWB trend. In Figure 6, the two lines at MJD 54748 and 54749 show the redder-when-fainter trend (BWB trend in statistics) in the decaying phase of the flux.

Two mechanisms can account for the BWB trend, i.e., the geometrical model (Raiteri et al. 2006, 2017; Villata et al. 2009) and the shock-in-jet model (Kirk et al. 1998). For this target, many studies investigated the periodicity (Jenkins 1996; Raiteri et al. 2001; Ostorero et al. 2004; Liu et al. 2006; Wang 2014; Vol`vach et al. 2015; Fan et al. 2017). The helical path-in-jet model was proposed to explain the periodicity (Ostorero et al. 2004; Raiteri et al. 2006; Villata et al. 2009; Vol`vach et al. 2015). The viewing angle between the moving direction and our line of sight varies along a helical trajectory, which results in variation of the Doppler factor. For a decreasing viewing angle, the peak frequency ($\nu_p \propto \delta$) will shift to higher frequency in the observing frame, and the spectral index of the observing frequency will increase from negative (when $\nu_p < \nu_{obs}$) to positive (when $\nu_p > \nu_{obs}$). The flux also varies as $F_{\nu} \propto \delta^{\alpha} F_{\nu}^*$. This will produce a BWB trend naturally. For the shock-in-jet model, Kirk et al. (1998) found that the variation shows the BWB trend when the electronic cooling timescale is longer than the accelerating timescale, and shows the RWB trend when the two timescales are approximately the same. The shock-in-jet model cannot be ruled out by the current analysis.

The spectral index of the $\gamma$-ray emission is plotted versus its flux in Figure 7. In all three epochs, the spectral index decreases as the flux increases. Pearson’s $r$ for epochs I, II, and III is $-0.42$, $-0.20$, and $-0.27$, respectively. This is the softer-when-brighter (SWB) trend. Ackermann et al. (2012) also found this trend in the fitting of the SED. In Figure 8, we also plot the correlation between $\gamma$-ray and optical $V$-band fluxes (log–log diagram). There is a strong linear correlation with slope $1.15 \pm 0.12$. This slope can help us further constrain the radiation process of $\gamma$-rays and the variation mechanism according to the theory (see Table 3 in Shao et al. 2019). For the synchrotron self-Compton (SSC) process, the predicted slopes for the variables $N$ (number density), $B$ (magnetic field), and $\delta$ (Doppler factor) are 2, 1, and [0.49, 1.11], respectively. For the external Compton (EC) process, the predicted slopes for the variables $N$ and $\delta$ are 1 and [0.68, 1.57], respectively. The case of EC with variable $B$ can be excluded, since no
correlation can be observed for this case. The SSC process is not favored, since the output energy of γ-rays should be smaller than that of optical emission in that process. Ackermann et al. (2012) found that EC process with seed photons from a dust torus can fit the SED well, and EC with seed photons from the BLR cannot reproduce the observed GeV γ-rays. We obtained that the emitting region of γ-rays is far beyond the BLR, which agrees with the scenario of EC with seed photons from the torus.

For EC, both variables $N$ and $\delta$ can lead the slope in Figure 8. We need to use the variation trend to further constrain the variation mechanism. The interesting thing is that the optical and γ-ray fluxes are strongly correlated, but the BWB trend (optical) and the SWB trend (γ-rays) coexist. In the lepton models, the variation trends should be the same for V-band and γ-rays, since the radiative particles are the same population, and both bands have negative spectral indices. However, if the spectrum of radiative particles is evolved during the flare, a broken power law can be formed (Jiang et al. 2016). The particles radiating γ-ray photons and the particles radiating V-band photons in different energy ranges may have different variation trends. Such a process needs detailed numerical simulation and fine-tuning of parameters to verify, which does not seem natural enough to be true. The hadronic models may provide a way out. Böttcher et al. (2013) showed that the lepton model cannot fit the SED of AO 0235+164 well, since the spectrum of γ-rays for this target is hard, while the IR–optical–UV continuum has a very steep spectrum. Then, it was proposed that synchrotron emission from protons can produce the hard spectrum of γ-rays, while synchrotron emission from electrons will correspond to the IR–optical–UV continuum. The electrons and protons must be in the same emitting region, and they should be subject to similar shock accelerations. So, the hadronic model also needs fine-tuning to explain the reverse trends. Ackermann et al. (2012) found that the SED of low and high states can be reproduced by adjusting the viewing angle and keeping the spectrum of electrons. But the simulated SED of GeV γ-rays in the low flux state is not suitable to fit the observed hard spectrum. Similar to the RWB trend for the optical continuum, we propose that an invariant component with energy higher than 1 GeV, which is most probably produced by the EC with seed photons from the BLR (with energy 10 eV), can lead to the SWB trend. In the low flux state, this constant high-energy component will produce a hard spectrum of γ-rays. As the GeV γ-ray photons (relatively "soft") of the jet component dominate, the spectral indices of γ-rays will become smaller. This proposal can naturally explain the interesting puzzle for AO 0235+164.

### 4.2. Variation of Polarization

To further constrain the variation mechanism, we analyze the polarization data from SO. In Figure 9, we find that logPD has no linear relation with log $i F_\nu$. (Ikejiri et al. 2011; Sasada et al. 2011). Ikejiri et al. (2011) studied the rotations in the qu plane for a sample of blazars, in which AO 0235+164 was also studied. It was found that there is a linear correlation between PD and fluxes during the outburst period (epoch I). The light curve of the target is similar to that of PKS 1502+106, which also has three ordered epochs (the flare, quiescent, and active states in the Fermi-monitored period) (Shao et al. 2019). However, a significant linear relation between logPD and log $i F_\nu$ was found for PKS 1502+106. In the top panel of Figure 10, we segment the data on the V-band and polarization observed during Steward’s first observing cycle (from 2008 October 3 to 2009 October 26, which is in the flare state, see Figure 1) into six periods. Among them, four periods with continuous flare profiles are marked as (a), (b), (c), and (d), respectively. A double logarithmic diagram of PD versus V-band flux for the four segments is plotted in Figure 11. At different flux levels, the polarization varies significantly. It is evident that a superposition of these polarizations will produce an uncorrelated relation between PD and the flux. If

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**Figure 7.** The γ-ray spectral index vs. $F_\nu$. Pearson’s $r$ for epochs I, II, and III is $-0.42$, $-0.20$, and $-0.27$, respectively.

**Figure 8.** The log–log diagram of γ-rays vs. V-band fluxes. The blue dashed line is the linear fitting result with slope $1.15 \pm 0.12$.

**Figure 9.** Distributions of log PD vs. log $i F_\nu$ of the optical V-band.
there is a disk component, the combination of the jet and disk component will diminish the PD in principle. In the low flux state, the PD must be small. When the jet component dominates over the disk component, the PD will be high and oscillate. This can naturally explain why the PD is strongly correlated with the flux for PKS 1502+106. However, from the SED of AO 0235+164 (Ackermann et al. 2012), one knows that the jet component dominates over the disk component even at low flux levels. This leads to a large variation of PD at the low flux levels for our target. We conclude that the domination of the jet component can explain both the lack of correlation between PD and fluxes and the RWB trend of the color for our target.

The variation of PD in different flares provides us more information with which to infer the variation mechanism of blazars. In the following, we will study the variation of PD and the rotation of Stokes parameters in the $qu$ plane in detail. In the four bottom panels of Figure 10, we show the segmented light curves of V-band optical flux and PD. It is obvious that the peak times of the flux flares and the PD flares are not synchronous. In segment (a), the PD first rises and then declines during the increasing phase of the flux. We name the PD flare that appears in the rising phase of the flux as the former PD flare. For this segment, the decreasing phase of the flux is not monitored. There are two flux flares in segment (b). The first one has only the decreasing phase, accompanied...
by a complete PD flare, which is named as the latter PD flare. For the second one, the PD peak appears before the flux and can be classified as the former PD flare. In segment (c), the flux first rises while the PD decreases. Then the flux declines, and the PD first rises and then declines, which is ascribed to being a latter PD flare. In segment (d), as the flux decreases, the PD first rises and then declines, and can be identified as the latter PD flare. In summary, monotonic variation of the flux will correspond to one complete PD flare. If we assume that one flux flare is sandwiched between the former and latter PD flares, the variations of flux and PD can be understood in a unified manner. However, this assumption is still not qualified from the current data. Only in segment (b) are one and a half flux flares monitored, and the PD flares vary more quickly than the flux. Sasada et al. (2011) showed two complete flux flares during the same period for this target, and more non-simultaneous PD flares indeed were monitored (see Figure 1 in this article). There is a caveat that no one complete sandwiched pattern of the PD flares is confirmed from our data. The samplings and the time coverage of the current data are not good enough. Other correspondences between the PD flares and the flux flares are possible.

Two models can explain the observed variation of PD and flux for AO 0235+164. Nalewajko (2010) presented the polarization produced by the helical motion of the emitting blobs in the jet model. When a blob swings in our line of sight, the viewing angle $\xi$ between the velocity direction and the line of sight will decrease first. When $\xi$ reaches $1/\Gamma$, the PD will reach its peak. When $\xi$ approaches the minimum, the optical flux reaches the peak and the PD value decreases. When the blob continues to move along the helical trajectory, $\xi$ will increase and pass through $1/\Gamma$ again, leading to another peak in the PD light curve. Therefore, if the line of sight is inside the jet cone, an optical flux flare will be sandwiched between the former and latter PD flares. Another model that can produce such a sandwiched pattern of time series was proposed by Zhang et al. (2014, 2016). This model considers the synchrotron radiation of the proton, and studies the polarization and radiation signatures. The shock moves along the jet and sweeps the helical magnetic field. The flux observed in one sampling is composed of radiation from different regions in the jet; the averaged Stokes parameter will manifest the effective direction of magnetic fields. Enhanced particle injection without changing the topology of the magnetic field will produce two PD flares and one flux flare. The polarization angle (PA) rotation of the first PD flare is the inverse of the second one.

Rather than studying the PA rotation, we study the variation behavior of $(q, u)$, since the PA rotation has the $\pi - \pi$ ambiguity problem (Marscher et al. 2008; Kiehlmann et al. 2016), and $q$ and $u$ provide more information than the PA. In Figure 12, the trajectories of $(q, u)$ corresponding to the four segments are plotted. In segment (a), the rotation is mainly counterclockwise, except that the first two and last data points show the opposite trend. In segment (b), the rotation is first clockwise and then counterclockwise. The lines of segments (c) and (d) mainly show clockwise rotations. From the analysis above, the likely conclusion is that the former and latter PD flares correspond to the counterclockwise and clockwise rotations, respectively. There is a caveat that other rotation trends are possible due to the large uncertainty of $q$ and $u$. Such rotation is not likely to be caused by the random walk process. One also notes that all lines are in the range $q < 0$ and cross the line $u = 0$. The deviation from the origin of the $qu$ plane indicates that another constant polarization component exists. Sasada et al. (2011) also presented $qu$ rotations for this target in epoch I (see their Figure 3). Their monitoring shows that both the counterclockwise and clockwise rotations appear during the period of flux flares, which agrees with our results. For 3C 454.3, Sasada et al. (2010) found that a counterclockwise rotation in the $qu$ plane appears for the outburst in 2017, and clockwise rotations appear in the active state. Uemura et al. (2017) studied PKS 1749+096 and found the lag between PD and flux light curves. Rotations of $(q, u)$ were also observed during many flares. Moving shocks passing through the curved trajectories as well as the Doppler shift were proposed to explain the variation phenomena of polarization.

We will present the variations of polarization by using the helical jet model, which was proposed by Steffen et al. (1995) and investigated by Mohan et al. (2015) and Shablovinskaya & Afanasiev (2019). The helical jet model respects the conservation of momentum along the jet axis, of angular momentum, and of kinetic energy. The trajectory of the emitting blob is described by the cylindrical coordinates $(\rho, \psi, z)$ in the jet coordinate system, where $\rho$ is the distance from the blob to the jet axis, $\phi$ is the azimuth angle, and $z$ is the distance from the blob to the jet base. Suppose that the emitting blob is launched at a cylindrical distance $\rho_0$ from the jet base with an initial velocity $\beta = v/c$. The jet half-opening angle $\varphi$ is assumed to be constant. The coordinates and the velocities of the emitting blob are given as

$$\rho = f \sqrt{1 + \left(\frac{at + b}{f}\right)^2},$$

$$\dot{\rho} = \frac{a}{\rho}(at + b),$$

$$\dot{\phi} = \frac{af}{\rho^2 \sin \varphi},$$

$$\dot{z} = \frac{\rho - \rho_0}{\tan \varphi},$$
where the constants \( a, b, \) and \( f \) are given as \( a = \beta \sin \varphi, \)
\( b = \sqrt{\rho_0^2 - f^2}, \) and \( f = j / \beta \) (\( j \) is the normalized angular momentum in units of distance). In the observer system, the angle \( \xi \) between the instantaneous velocity direction and the line of sight is described by (Li et al. 2018)

\[
\cos \xi = \frac{\hat{\rho} \cos \phi \sin i - \rho \dot{\phi} \sin \phi \sin i + \dot{\xi} \cos i}{(\rho^2 + \rho^2/2 + \dot{\rho}^2)^{1/2}},
\]

where \( \hat{\rho} \) is the angle between the jet axis and the line of sight. The Doppler factor is \( \delta = 1 / [\Gamma(1 - \beta \cos \xi)], \) which changes with time because the angle \( \xi \) is time-dependent. Cav ATHorne & Cobb (1990) stated that \( \Pi (PD) \) depends on \( \xi, \) which can be described by the empirical relation \( \Pi \sim A \sin^n \xi', \) where \( n \) is a positive real number (Shao et al. 2019). \( \Pi \) reaches its maximum for \( \xi' = \pi / 2 \) and decreases to its minimum for \( \xi' = 0. \) Raiteri et al. (2013) suggested that \( n = 2. \) However, we have found that numerical integrations of Stokes parameters (presented in Lyutikov et al. 2005) are best fitted with the empirical relation for \( n = 3, \) i.e., \( \Pi \sim \Pi_{\text{max}} \sin^3 \xi'. \) \( \xi' \) can be obtained by the Lorentz transformation from \( \xi \) via \( \sin \xi' = \sin \xi / [\Gamma(1 - \beta \cos \xi)]. \) In the observer system, the electric vector position angle (EVPA) is given by

\[
\tan \chi = \frac{e_u}{e_w} = \frac{\cos \phi \cos i - \tan \varphi \sin i}{\sin \phi},
\]

where \( e_u \) and \( e_w \) are values projected onto the \((u, w)\) reference frame in the sky. The fractional Stokes parameters \( q = Q/I \)
and \( u = U/I \) can be expressed as \( q = \Pi \cos 2\chi \) and \( u = \Pi \sin 2\chi, \) respectively.

In Figure 13, we schematically present the variations of PD, fluxes, and \((q, u)\) using the analytical equations in the helical jet model. The input parameters are set as \( i = 6^\circ, \varphi = 2^\circ, \Gamma = 14, \)
\( j = 0.04, \) and \( \rho_0 = 0.07 \) pc. The parameter \( \rho_0 \) controls the timescale. Setting \( \rho_0 = 0.07 \) pc leads to the flux flare appearing at \( t \approx 9000 \) days, which roughly agrees with the optical emitting regions being 8 pc away from the jet base. Since the viewing angle is larger than the jet opening angle, our line of...
sight is beyond the jet cone. From the left panel of Figure 13, it is evident that the two PD flares sandwich the flux flare as expected. We also check the angle $\xi$, which takes a minimal value $\sim 1.4^\circ$ for $t \approx 9000$ days. To study the trajectory of $(q, u)$, we add a constant vector component to $e_u$ and $e_v$, i.e., $e_u + 0.8$ and $e_v + 0.6$. The idea is that there should be a disk component to explain the RWB trend. Such a component can also affect the trajectories of $(q, u)$. The values of added components are set so that $u$ changes sign while $q$ remains negative, as illustrated in Figure 12. In the right panel of Figure 13, the trajectory of $(q, u)$ shows a counterclockwise rotation for the first PD flare, reflects between two PD flares, shows another counterclockwise rotation for the second flare, and finally exhibits a clockwise rotation at the end of the second PD flare. Li et al. (2018) studied the polarization of CTA 102 using the same helical jet model, and obtained similar PD flares and $qu$ rotations without flipping the rotation trends. We found that the reflection of the rotation trend appears when tan $\chi$ passes the singular point, which means that the PA rotation is also flipped. The shock-in-jet model with a helical magnetic field can also produce the PA variation with a sine profile (Zhang et al. 2016), which may also explain the $qu$ rotations. Manipulating with other sets of parameters, we found that no flip of trends appears, as illustrated by Li et al. (2018) in the study of CTA 102. Thus, the helical jet model can explain the trend for a flip of rotation with a fine-tuning of parameters.

Based on the above schematic illustrations, we conclude that the observed characteristics of polarization for AO 0235+164 can be explained roughly by the helical jet model. Combining with the analysis of color and the spectral index of $\gamma$-rays, the change in the viewing angle is the primary variable to explain the observed variation phenomena of AO 0235+164. The shock-in-jet model with helical magnetic field is an alternative. Better sampling of optical observations, especially the monitoring of polarization, is indispensable for diagnosing models to reveal the variation mechanism.

5. Conclusion

In this work, we collected nine years of light curves of $\gamma$-rays, the optical V-band, radio 15 GHz, and polarization. We analysed the correlation to study the optical and $\gamma$-ray emitting regions. We analyzed the color and spectral index behavior to investigate the variation mechanism. The behaviors of PD and $qu$ rotations enable us to further constrain the models. The principal conclusions are given as follows.

1. Based on the LCCF analysis, the optical V-band and $\gamma$-ray light curves are correlated with the radio 15 GHz with 3$\sigma$ significance. The $\gamma$-ray and optical emitting regions are roughly the same, and are located $6.6^{+1.7}_{-1.0}$ pc upstream of the core region of 15 GHz. The estimation of the radio core region predicts that the optical and $\gamma$-ray emitting regions are far away from the BLR.

2. For the variation of color, the target shows an RWB trend in the low flux state and a BWB trend in the high flux state, while the spectral index of $\gamma$-rays always shows the SWB trend. With contamination of the accretion disk, the increase in the dominant jet component will turn the RWB trend into the BWB trend in a unified manner. The SWB trend for $\gamma$-rays can be explained naturally by the existence of a constant higher energy (GeV) $\gamma$-ray component. The EC process with seed photons from the torus is favored to explain the correlation between optical and $\gamma$-ray fluxes. Broad emission lines are evident in the low flux states, which indicates that AO 0235+164 is an FSRQ source.

3. As a whole, the PDs are not correlated with the optical fluxes. This can be explained by the superposition of multiple flares and the weak disk component. It was found that the PD flares and flux flares are not synchronous. It seems that the trends of counterclockwise and clockwise rotation in the $qu$ plane correspond to the former and latter PD flares, respectively. The possibility that such a correspondence is an illusion due to uneven samplings and bad time coverage cannot be ruled out. Using the helical jet model, the variations of the flux, PD, and Stokes parameters $(q, u)$ for the target can be simulated systematically.

To sum up, AO 0235+164 is an FSRQ target, and the change in the viewing angle can explain its various phenomena of variation.

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References

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009, ApJS, 183, 46
Acero, F., Ackermann, M., Ajello, M., et al. 2015, ApJS, 218, 23
Ackermann, M., Ajello, M., Ballet, J., et al. 2012, ApJ, 751, 159
Agudo, I. 2013, EPJWC, 61, 04002
Agudo, I., Marscher, A., Jorstad, S., et al. 2011, ApJL, 735, L10
Atwood, W. B., Abdo, A. A., Ackermann, M., et al. 2009, ApJ, 697, 1071
Bonning, E., Urry, C. M., Bailyn, C., et al. 2012, ApJ, 756, 15
Böttcher, M., Reimer, A., Sweeney, K., & Prakash, A. 2013, ApJ, 768, 54
Cawthorne, T. V., & Cobb, W. K. 1990, ApJ, 350, 536
Cellone, S. A., Romero, G. E., Combi, J. A., & Martí, J. 2007, MNRAS, 381, L60
Celotti, A., & Ghisellini, G. 2008, MNRAS, 385, 283
Chen, Y. J., & Jiang, D. R. 2000, A&A, 376, 69
Chen, Y. J., Jiang, D. R., & Zhang, F. J. 2002, A&A, 26, 14
Ciprini, S. 2015, AtEl, 7975
Cohen, R. D., Smith, H. E., Junkkarinen, V. T., & Burbidge, E. M. 1987, ApJ, 318, 577
Corbel, S., & Reyes, L. C. 2008, AtEl, 1744
Davis, M. M., & Wolfe, A. M. 1982, in IAU Symp. 97, Extragalactic Radio Sources, ed. D. S. Heeschen & C. M. Wade (Dordrecht: Springer), 311
Edelson, R. A., & Krolik, J. H. 1988, ApJ, 333, 646
Elvis, M., Wilkes, B. J., McDowell, J. C., et al. 1994, ApJS, 95, 1
Fan, J. H., Kurtanidze, O., Liu, Y., et al. 2017, ApJ, 837, 45
Foschini, L., Longo, F., & Iafrate, G. 2008, AtEl, 1784
Hagen-Thorn, V. A., Larionov, V. M., Jorstad, S. G., et al. 2008, ApJ, 672, 40
