An example and experience of a mathematical modelling activity: the hip roof problem

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Abstract. There is a growing body of literature that recognizes the importance of mathematical modelling which deal with real world problems in mathematics school. The present study set out to investigate potential uses of the Hip Roof problem to introduce a mathematical modelling activity to Indonesian students. This study was exploratory and interpretative in nature. The Hip Roof problem was originally developed by the researcher and was assigned to six groups of undergraduate students enrolled at one public university in Jakarta, Indonesia. Test administration and interview were conducted to describe their work. The works were analyzed descriptively. The result indicates that the problem is feasible with undergraduate students and the participants of the study appreciate the problem included in school mathematics to enhance their ability to implement mathematics in their real life. In solving the problem, the participants used their personal mathematical knowledge.

1. Introduction
Researcher and practitioners in the field mathematics education strive to come up with improvement in mathematics teaching and learning. The old-fashioned way of teaching is no longer be considered as it only emphasizes on the repetition of similar tasks, closed-ended problems, and memorization of formulas and procedures. Moreover, there is still tendency to use simple mathematical task that is ready for direct calculation regardless challenge, complexity, as well as real world context. Students, on the contrary, have to be provided an opportunity to construct actively their own knowledge and learn meaningfully through coming to grips with real world problems.

The issue of mathematical modelling for mathematics school has received considerable critical attention. It has been considered as one of effective means for developing students ability in coping with unfamiliar situation through flexible and creative thinking and in dealing with real world problem [1]. Haines and Crouch [2] further described mathematical modelling as a cyclical process which consists of translating real world problems into mathematical expression, solving in a symbolic system, and testing back the obtained solution in its real world context. In brief, Integration of real situation in mathematics learning in terms of word problems is an important aspect of mathematical modelling.

This has been discussed by a considerable number of researcher in literature that attaching modelling in mathematics teaching and learning process is essential for helping students improve their analytical thinking and problem solving abilities [3]. In addition, mathematical modelling assists students in comprehending the real world, facilitates mathematical learning, establishes different kind of mathematical ability and accurate attitudes, and contributes adequate cornerstone for the framework of mathematics. More importantly, through modelling students could learning mathematics meaningfully [4], develop metacognitive and critical reasoning [5], support internal and external reflections [6].
A challenging problem which emerges in this domain is the implementation of mathematical modelling approach in mathematics school as it creates various challenges for students and teachers. Students encountered significant obstacle when attempting to cope with mathematical modelling problems as they lack of prior experience of mathematical modelling or dealing with realistic word problems [7]. Furthermore, as there are inadequate resources of modelling task, teachers who intend to accommodate modelling activity into their teaching process need to design it [3].

There are six design principles for establishing a modelling activity, to wit: model construction, reality principle, self-assessment, model documentation, and construct share ability and re-usability, and effective prototype [8]. The first principle refers to that the process of obtaining the solution involves construction of procedure, clear description, or confirmed prediction. The second principle ensures that meaningful activity is applied by considering students distinct level of mathematical ability and previous knowledge. The third principles lead students to evaluate their ways of thinking. The fourth principle requires students to document their thinking process while completing the modeling activity. The fifth principle ensures that students generate solutions which are share-able and re-use-able by others. According to Erbas et. al. [3], there are several main characteristics of modelling problems which differs from problem solving, such as multiple cycles, multi-interpretations, authentic real-world situation, students’ work in developing and evaluating certain mathematical ideas and structure, cooperative and collaborative work emphasized, multidiscipline, generating mathematical description, as well as open-ended. Developing model is also critical aspect in modelling problems. To develop a model, students are required to comprehend the structure between quantities and variables attributed to the context of the problem. The developed model then is required to be evaluated whether it will lead to the solution of the problem [9].

There have been numerous studies to investigate students’ activity in solving mathematical modelling task. For example, Årlebäck and Bergsten [10] conducted a study to investigate the use of the Fermi task to introduce mathematical modelling activity to students in upper secondary school level in Sweden. They found that in the solving process, students applied their own extra-mathematical knowledge creatively, socially, and carefully. In Indonesian context, a study conducted by Edo et al [11] revealed various students difficulties in dealing with mathematical modelling problems such as defining real-life situation mathematically, determining mathematical structure, and relating the solution to the context of the real-world problem. In addition, a study involving Indonesian pre-service teachers was conducted to build their awareness of mathematical modelling by using the task of re-redesigning a Parking Lot [12].

Even though studies of mathematical modelling have been conducted by many researchers, the implementation in Indonesia is still insufficiently explored. There has been less previous evidence for explaining in detail about Indonesian students’ performance within modelling situation. Therefore, the aim of this work is to design an authentic mathematical modelling problem for Indonesian students and investigate their ability to deal with mathematical modelling task. This study also aims to contribute to this growing area of research by providing an example of an authentic mathematical modelling problems and Indonesian students performance on getting to grips with the modelling task.

2. Method
There were 18 participants in this study. They were undergraduate students who enrolled at one public university in Jakarta. They were selected conveniently. This study used different data collection methods such as test administration, classroom observation, and interview to elicit further data.

The authentic instruments used in this study is presented and it became the main feature of the study. The main criteria to ensure that the problem classified into modeling problem was that it took real world problem into account. Students are most probably familiar to the forms of roof, and they were expected to investigate various types of roof based on mathematical concepts. The others criteria also can be considered such as assumptions, mathematics connection, different strategies as well as technology as those criteria are employed to reach correct solution. The researchers considered that the situation appeared in the problem was familiar and interesting for Indonesian students in which it was
in line with the main characteristics of modelling problems. The instrument is The Hip Roof Problem as shown in figure 1. The content and face validity of the problem were assessed by two experts in mathematics education.

Before the implementation, lesson plan was prepared to design class situation and manage participants activities so that two lesson hours could be used effectively. To prepare it, we considered all possibilities that would occur during implementation. There were several important things that we attended to in preparing lesson plan such as time allocation, objectives of the lesson, material and tools, opening class, grouping participants, explanation of to-do’s to participants, participants difficulties, grading and evaluation as well.

![THE HIP ROOF PROBLEM](image)

**Figure 1.** The Hip Roof Problem

In the wake of implementation, considering participants questions and their difficulties to grasp the idea of the problem, we thought that the problem should be revised. We revised about information by adding clear information. For instance, the height of the house is 5 meters. Since most of participants were confused in this part. Also, to simplify the calculation, the length of house becomes 18 meters and the width of the house becomes 14 meters. There are several changes lesson plan. For instance, at the first time we didn’t take differentiation and adjusting the length of the top as strategies to reach the solution.

Participants works, observation, and interview were analyzed in a descriptive way. All the collected data were used and correlated each other to gain deep and complete comprehension with respect to participants experience in mathematical modelling activities.

### 3. Result and Discussion

This is an important finding in the understanding of the use of the Hip Roof Problem to introduce a mathematical modelling activity to Indonesian students. In order to gain comprehensive finding of the study, we tried to describe by highlighting four sections, namely, drawing strategies, the modelling process and solution, group and classroom discussion, and participants perspectives.
3.1. Drawing strategies
The role of drawing in mathematical modelling activities is crucial. In this study, we found that participants did not encounter difficulties in generating a sketch representing the situation in the problem. The figure 2 show two examples of participants efforts in making a drawing. The difference between the two figures could describe the difference of participants in understanding of the problem and the process of tackling it.

![Figure 2. Two examples of participants drawing](image)

According to Rellensmann et all [13], there is positive relation between strategic knowledge about drawing and students’ modelling performance. The drawing, in a fundamental sense, could explicate the process and the product of creating an interpretation that conform to the objects and relations defined in the problem. In addition, attaching detail attribution in the drawing might lead to satisfactory result. It is supported by Edens [14] in which students problem solving achievement are affected by the level of spatial ability and use of schematic drawings.

3.2. The modelling process and solution
The problem is quite interesting and simple. So as to cope with it, several mathematical concepts were considered such as trigonometry, surface area, differentiation as well as the concept of three-dimensional shapes. There are several obstacles that students faced in copying with the mathematical modelling problem. The first obstacle is that several participants use incorrect assumption. Occasionally, there is not enough information presented in a problem, yet to reach the solution it is necessary to get it and any efforts have been made and it didn’t work at all. Therefore, assumption can be taken into account. Students can take assumption as long as it makes sense and doesn’t contradict to the mathematical concepts and principles. Assumption is considered as one of four aspects typically needed when dealing with mathematical modelling activities [10]. Students might try to name the points and try to find the length of each side by making assumptions. Based on participants works, assumption used was not directed to the best answer.

The second obstacle is misunderstanding of the problem. At first, most of them had difficulty when trying to imagine the picture of the problem. They are confused how to solve the problem. This difficulty might not lead them to process their strategies planned. It was supported by Dixon and Moore [15] who found that comprehension of intuitive principle of the problem was required to develop a mathematics strategy. For instance, they were confused with the height of house, what the problem intends to, and the mean of overhang level. Finally, the third obstacle is participants miscalculation. It was obviously seen when students asked questions in effort to comprehend the problem. Some participants performed calculation carelessly so that many mistakes they made. The following are two example of participants
Based on the Figure 1., the group decision was true. They made assumption that the length of KL was 15 meter. The procedure made sense. Meanwhile, Figure 2. indicates that the group made correct decision, yet they performed calculation carelessly so that many mistakes they made. This result ties well with previous studies wherein students encountered various obstacles in dealing with mathematical modelling problems such as having puzzlement in accepting or understanding modelling tasks, lack of pre-matematisasi and mathematization, and inadequate employed anticipation, and problems in their mathematical work [16].

3.3. Group and classroom observation

During the implementation, there were several difficulties participants faced while trying to read and understand the problem. There were several questions that they asked during the lesson such as: What does the height of 5 meters mean? What is the height of the roof? What does the overhang level mean? We tried to provide necessary guidance and clear explanation to assist them in dealing with it. For instance, to answer the first question, we showed what the mean of the height of house by writing a sign in their worksheet. For the second and the third question, we showed the notes and the figure that I have drawn in the worksheet. They also had difficulty when trying to calculate and imagine the picture. They didn’t know how to do, even they understood the problem. Some of them predicted that pyramid hip roof was the solution. However, they were still in confusion. To deal with student’s difficulties, we tried to give them main keys such as considering the length of KL and the measure of angle KEM. It seems that they had inadequate mathematical reading competence. Leiss [17] claimed that students success in coping with the modelling problem might be affected by their mathematical reading and intra-mathematical ability.

They then tried to address the problem by means of different solution strategies and ways of thinking. Some of them considered the length of KL so they took assumption about it. They took arbitrary number more than zero (if KL equal to zero, so it would be pyramid hip roof). Some of them made assumption that the length of EM equals to the length of MF. Even this assumption led to the correct answer, however it didn’t make sense.
Discussion in the group also play important role in reaching intended solution as students shared their own thinking and negotiated with other members. Through deriving students understanding or ideas, discussion contributes to generate model of the real context [18]. Besides, the teacher’s role in mathematical modeling activity is as facilitator who guide and give necessary help for students or participants reach the intended solution. During implementation, at the beginning, we asked them whether they understand or not by encouraging them to explain by using their own words what the problem ask. Afterwards, we tried to walk around classroom to check whether they could overcome the problem. They asked us to ensure that the main key of the problem was the length of KL. Some of them made assumptions by taking the length of KL as x, whereas the others tried to take the length of KL as arbitrary numbers more than zero. Some of them could reach the correct answer.

Even though teacher doesn’t dominate the classroom situation since he/she is as a facilitator and the instruction tends to be student-centered instruction, teacher has to prepare the lesson plan as well as possible. The points that a teacher needs to attend in preparing lesson plan are time allocation and clarity of the problem. Besides, during implementation teacher have to walk around the class to check whether students can deal with the problem and provide help if necessary. It is in line with the result of study conducted by Leiss [17] in which teachers support might facilitate students to construct situation model.

3.4. Students perspectives

Students were impressive to the use of modeling activities in mathematics instruction, since it is the most effective technique to introduce mathematics by means of real-world problem. Students were interested in solving the problems as it challenged and encouraged them to connect multiple concepts. For example, several participants expressed, “I think that mathematical modelling activities is interesting for us as it connects mathematics with real situation”. In other words, they argue that they were impressive to the use of modeling activities in mathematics instruction, since it is the most effective technique to introduce mathematics by means of real-world problem. It seems that they were interested in solving the problems as it challenged and encouraged them to connect multiple concepts. This result ties well with a previous study conducted by Kaiser et.al who found that majority of students took part in their study appreciated mathematical modelling problem as it might improve their ability to connect school mathematics and their real life [19].

4. Conclusion

Returning to the research question, the study tried to study potential uses of the Hip Roof problem to introduce a mathematical modelling activity to Indonesian students. The carried out descriptive analysis leads to the following conclusions: (1) Drawing plays a substantial role in dealing with mathematical modelling problem; (2) there are several obstacles found such as incorrect assumption, miscalculation, and misunderstanding; (3) discussion and teachers intervention help students in modelling real situation. Therefore, this study indicates that small group work on the Hip Roof problems may offer a good and probably beneficial opportunity to introduce mathematical modelling activity for students at secondary school level.

Due to practical constraints, this paper cannot provide a comprehensive review of participants various strategis in dealing with the modelling problem. Therefore, future studies on the current topic are therefore recommended in order to elucidate students various strategies in coping with mathematical modelling problems.

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