Abstract—This paper discusses the topic of uncertainty quantification in prognostics and explains the importance of accurately estimating such uncertainty in order to aid risk-informed operational decision-making. Since prognostics deals with predicting the future behavior of engineering systems, it is impossible to accurately predict the future response, and therefore, it is necessary to compute the uncertainty associated with such prediction. This paper discusses the various sources of uncertainty that influence prognostics and explains that the problem of quantifying their combined effect on prognostics can be posed as an uncertainty propagation problem. Different types of uncertainty quantification methods – sampling methods and analytical methods – are reviewed and their applicability to prognostics is investigated. The practical challenges involved in applying these methods for online health monitoring purposes are discussed. Finally, the various concepts presented in this paper are illustrated using a numerical example.

Uncertainty in Prognostics: Computational Methods and Practical Challenges

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1. INTRODUCTION

Complex engineering systems are being used for time-critical, safety-critical, and cost-critical missions, and recent research has been advocating the use of an onboard health management system to monitor the performance of such engineering systems. An accurate health management system directly aids in diagnosis and prognosis, and eventually guides operational decision-making. Diagnosis consists of fault detection, isolation, and estimation, while prognosis deals with predicting possible failures in the future and estimating the remaining useful life of these systems.

Uncertainty assessment and management are important aspects of health management, due to the presence of several unknown factors that affect the operations of the system of interest. Therefore, it is not only important to develop robust algorithms for diagnosis and prognosis, i.e., accurately perform diagnosis and prognosis in the presence of uncertainty, but also important to quantify the amount of confidence in the results of diagnosis and prognosis. This can be accomplished by quantifying the uncertainty in fault diagnosis and prognosis. It is also necessary to perform such uncertainty quantification (UQ) online so as to enable in-flight decision-making capabilities. Sankararaman and Mahadevan [1], [2] developed statistical (both frequentist and Bayesian) approaches to quantify the uncertainty in the three steps of diagnosis (detection, isolation, estimation) in an online health monitoring framework.

Recent research efforts in the domain of health monitoring have focused on prognostics and condition-based maintenance. Existing methods for quantifying uncertainty in prognostics and remaining useful life prediction can be broadly classified as being applicable to two different types of situations: offline prognostics and online prognostics. Methods for offline prognostics are based on rigorous testing before and/or after operating an engineering system, whereas methods for online prognostics are based on monitoring the performance of the engineering system during operation. For example, there are several research articles which discuss uncertainty quantification in crack growth analysis [3], [4], structural damage prognosis [5], [6], electronics [7], and mechanical bearings [8], primarily in the context of offline testing.

The offline testing approach may be applicable to smaller components when it may be affordable to run several such components to failure, and it may not be practically feasible to extend this approach to large scale systems. Further, the estimation of remaining useful life is more significant in an online health monitoring context where the performance of a system under operation needs to be monitored and its remaining useful life needs to be calculated. Engel et al. [9] discuss several issues involved in the estimation of remaining useful life in online prognostics and health monitoring. Though some of the initial studies on remaining useful life prediction lacked uncertainty measures [10], researchers have recently started investigating the impact of uncertainty on estimating the remaining useful life. For example, there have been several efforts to quantify the uncertainty in remaining useful life of batteries [11] and pneumatic valves [12] in the context of online health monitoring. Different types of sampling techniques [13] and analytical methods [14] have been proposed to predict the uncertainty in the remaining useful life.

Though several computational methods for tackling uncertainty are being developed, there are several issues and challenges that need to be addressed before such methods can be applied to practical problems. These challenges need to be identified and addressed by rigorous research before such methods can be applied to practical problems. There
are also several shortcomings regarding the understanding and interpretation of uncertainty in prognostics [15], and it is also necessary to clarify such issues. In this regard, the various sources of uncertainty that affect prognostics need to be identified and it is also necessary to quantify the combined effect of such sources of uncertainty on prognostics. This paper focuses on these issues, discusses several computational methods that may be suitable for uncertainty quantification in prognostics, and identifies some of the challenges involved in prognostics of practical engineering applications.

The rest of this paper is organized as follows. Section 2 discusses the topic of uncertainty in prognostics in detail. The various sources of uncertainty in prognostics are explained and the interpretation of uncertainty is also discussed. It is also elucidated that the problem of quantifying uncertainty in prognostics can be viewed as an uncertainty propagation problem. Section 3 discusses different types of computational methods that may be used to quantify uncertainty in prognostics, and Section 4 outlines some of the challenges involved in applying such methods in online health monitoring conditions. Section 5 illustrates some of the concepts presented in this paper through a numerical example, and Section 6 presents the conclusions of this paper.

2. Uncertainty in Prognostics

The presence of uncertainty has a significant impact on prognostics and the remaining useful life prediction. When the state estimates, future loading conditions, operating conditions, etc. are uncertain, the future states and the remaining useful life also become uncertain. While non-probabilistic methods [16] such as fuzzy logic, possibility theory, Dempster-Shafer theory, evidence theory, etc. may have been used for the treatment of uncertainty, probabilistic methods have predominantly used for uncertainty representation in prognostics [17], [18], [19]. Further, probabilistic approaches are contextually meaningful for uncertainty representation and quantification since they are consistent with decision-theory analysis. This section discusses the topic of uncertainty in prognostics in detail. First, the various sources of uncertainty in prognostics are outlined and their interpretation is discussed. Then, the importance of accurately quantifying the combined effect of the different sources of uncertainty on prognostics is emphasized.

Sources of Uncertainty

It has been conventional to classify the different sources of uncertainty into aleatory (physical variability) and epistemic (lack of knowledge), where epistemic uncertainty consists of data uncertainty and model uncertainty [20]. However, in the case of condition-based monitoring, there is only one particular system being monitored, and not multiple realizations of a population, and therefore, it is not meaningful to discuss variability. For example, the system is in a particular state at any time instant and there is nothing variable about it. Variability would need to be accounted for only in the case of reliability-testing methods where multiple components/systems are tested. Therefore, this paper classifies the different sources of uncertainty in prognostics into the following categories:

1. Modeling uncertainty: Predicting the future is the most important aspect of prognostics, and typically, a physics-based model or a data-driven model is used for predicting future behavior. This model is usually represented using state-space equations. Modeling uncertainty represents the difference between the predicted response and the true response (which can neither be known nor measured accurately), and comprises of several parts such as model parameters, model form, process noise, etc.

2. Present uncertainty: The first step of prognostics is to estimate the condition/state of the component/system at any time instant. Output data (usually collected through sensors) is used to estimate the state and many filtering approaches are able to estimate the state, and calculate the uncertainty associated with the state estimate.

3. Future uncertainty: The most important source of uncertainty in the context of prognostics is due to the fact that the future is unknown, i.e. both the loading and operating conditions are not known precisely. The future behavior needs to be estimated using a model; the usage of a model imparts additional uncertainty as explained earlier.

The goal in prognostics is to individually characterize each of the above uncertainties and then quantify their combined effect on prognostics. If it is desired to predict the how long it may be possible to operate the system at hand, then the uncertainty in such “remaining useful life” can be quantified and expressed in terms of a probability distribution.

Interpreting Uncertainty

While the mathematical axioms and theorems of probability have been well-established the literature, there is considerable disagreement among researchers on the interpretation of probability. There are two major interpretations based on physical and subjective probabilities, respectively. Physical probabilities [21], also referred to objective or frequentist probabilities, are related to random physical systems such as rolling dice, tossing coins, roulette wheels, etc. Each trial of the experiment leads to an event (which is a subset of the sample space), and in the long run of repeated trials, each event tends to occur at a persistent rate, and this rate is referred to as the relative frequency. These relative frequencies are expressed and explained in terms of physical probabilities. Thus, physical probabilities are defined only in the context of random experiments. On the other hand, subjective probabilities [22] can be assigned to any “statement”. It is not necessary that the concerned statement is in regard to an event that is a possible outcome of a random experiment. In fact, subjective probabilities can be assigned even in the absence of random experiments. The Bayesian methodology is based on subjective probabilities that are simply considered to be degrees of belief and quantify the extent to which the “statement” is supported by existing knowledge and available evidence. In this approach, even deterministic quantities can be represented using probability distributions that reflect the subjective degree of the analyst’s belief regarding such quantities.

This leads to the obvious question - is one particular interpretation more suitable to prognostics? In general, both interpretations may be suitable. However, in the particular context of condition-based monitoring or online health monitoring, there is only one system which is being monitored, and hence, at any time instant, there is no “physical randomness” associated with the system (from a frequentist point of view). Therefore, any quantity associated with a system, even though it may be uncertain, cannot be represented using a probability distribution, following the frequentist interpretation of probability. Nevertheless, system state estimation during health monitoring is commonly performed using particle filters and Kalman filters, and these approaches compute
probability distributions for the state variables; therefore, the only possible explanation for such calculation is that the subjective (Bayesian) approach is being inherently used for uncertainty quantification. Such filtering approaches are known as “Bayesian tracking” methods not only because they use Bayes theorem, but also fall within the realm of subjective probability. This implies that the uncertainty estimated through the aforementioned filtering algorithms are simply reflective of the analyst’s degree of belief, and not related to actual physical probabilities.

Computing Uncertainty in Prognostics

Consider a generic time-instant \( t_p \) at which prognostics needs to be performed. Prognostics, typically, consists of three steps, as shown in Fig. 1. The first step consists of estimation, where the health of the system at time \( t_p \) is estimated using Bayesian tracking techniques such as particle filtering, Kalman filtering, etc.

The second and third steps consist of prediction; future health states are forecasted and the time at which the system becomes unusable is determined. This time instant is referred to as the end-of-life (EOL) of the system; obviously, the EOL is a function of \( t_p \) and hence represented as \( \text{EOL}(t_p) \). The difference between \( \text{EOL} \) and \( t_p \) is indicative of the remaining time during which the system is usable and therefore, this difference is defined as the remaining useful life (RUL). The prediction of remaining useful life is extremely important in the context of prognostics and health monitoring. It is not only necessary to verify whether the mission goal(s) can be accomplished but also important to aid in online decision-making activities such as fault mitigation, mission replanning, etc. Bo Sun et al. [23] discuss the benefits of prognostics and explain how the calculation of RUL is important for technical health determination and life extension [24]. Since the prediction of RUL is critical to operations and decision-making, it is imperative that the RUL be estimated accurately. The various sources of uncertainty described in the beginning of this section affect the prediction of RUL and therefore, one important goal in prognostics is to quantify the effect of these sources of uncertainty on RUL prediction and thereby, compute the probability distribution of the RUL prediction.

In order to perform prediction at time \( t_p \), the state estimate (denoted by \( \hat{x}(t_p) \)) provided by the Bayesian tracking procedure is used along with a prediction model, that can be generically denoted by:

\[
\dot{x}(t) = h(x(t), u(t), w(t))
\]  

(1)

In the above equation, \( u(t) \) and \( w(t) \) denote the loading and model error (process noise) at any generic time instant \( t \), and \( h \) is the functional model that represents the state evolution. This model may be developed using first-principle physics or using data-driven approaches. Using Eq. 1, it is possible to forecast the state value at any generic future time-instant \( t \), based on the state value at time-instant \( t_p \).

The third and final step of RUL computation is complicated from an analytical point of view. It is first necessary to define the end of life using a threshold function. This threshold function is evaluated at a any generic time \( t \); its output at any time instant is binary, indicating whether failure has occurred or not. For example, when the charge in a battery decreases below a threshold limit, failure is said to have occurred.

To begin with, assume that there are no uncertainties, and all quantities (initial state \( x_p \), and loading and model errors at all times) are deterministically known. Starting with the state value of \( x_p \), the state value can be continuously forecasted until the first time instant when the end of life (as determined using the threshold function) is attained; this time instant corresponds to the end of life, and is denoted as \( t_{EOL} \). It can be easily seen that \( t_{EOL} \), when calculated at time \( t_p \), is a function of:

1. State value at the time at which prediction needs to be made, i.e., at \( t_p \). These state values are denoted by \( x(t_p) \).
2. Loading values continuously from time \( t_p \) until \( t_{EOL} \); the operating conditions, if known, may also be included. Let \( u \) denote this vector.
3. Model error values from time \( t_p \) until \( t_{EOL} \); let \( w \) denote this vector.

The evaluation of end-of-life can be graphically represented as shown in Fig. 2, and expressed mathematically as:

\[
\text{EOL}(t_p) = \Psi(x(t_p), u, w)
\]  

(2)

Then, the RUL at time \( t_p \) is calculated as:

\[
RUL(t_p) = \text{EOL}(t_p) - t_p
\]  

(3)

Note that \( t_p \) is deterministic, since the time at which prediction needs to be performed is known. In fact, the above two equations can be combined, and the RUL can be directly expressed mathematically as:

\[
RUL(t_p) = G(x(t_p), u, w)
\]  

(4)

Thus, using Eq. 4, for every value of \( x(t_p), u \), and \( w \), the value of RUL can be computed. However, the values of these variables are uncertain, and only probability distributions may be available for them. Since these variables are uncertain, RUL is also uncertain, and hence, the goal would be to compute the probability distribution of RUL. This probability distribution of RUL can be computed by “propagating” the uncertainty in \( x(t_p), u \), and \( w \) through \( G \) in Eq. 4. Hence, the estimation of RUL in prognostics is simply an uncertainty propagation problem, and well-established statistical tools for uncertainty propagation may be investigated for this purpose. The following section is devoted to this topic, and discusses the relevance of well-known uncertainty propagation methods in prognostics and health monitoring.

3. Computational Methods

This section reviews different types of computational methods that are suitable for uncertainty propagation and investigates their applicability to quantifying uncertainty in prognostics. Recall that the goal in prognostics to propagate the uncertainty in \( x(t_p), u \), \( w \) through \( G \) in Eq. 4 and thereby compute the uncertainty in the RUL at a given prediction time \( t_p \). Researchers in the areas of non-deterministic methods and uncertainty quantification techniques have developed different types of statistical methods for uncertainty propagation during the past 30 years. The most general case of uncertainty propagation considers the mathematical function given by:

\[
Y = G(X_1, X_2,...X_n)
\]  

(5)

It is clear that the above equation is very similar to Eq. 4 in Section 2. Here, there are \( n \) inputs given by \( X_i \) \((i = 1 \text{ to } n)\), and the uncertainty in each input is given by the probability density function (PDF) \( f_{X_i}(x_i) \) or the cumulative distribution function (CDF) \( F_{X_i}(x_i) \). The joint PDF of all inputs is
denoted as $f_X(x)$. The goal in uncertainty propagation is to compute the uncertainty in $Y$, either in terms of the PDF $f_Y(y)$ or CDF $F_Y(y)$. The entire CDF $f_Y(y)$ can be calculated as:

$$F_Y(y) = \int_{g(X) \leq y} f_X(x) dx$$  \hspace{1cm} (6)

It is harder to write a similar expression for PDF calculation, although the following equation attempts to.

$$f_Y(y) = \int f_Y(y|X)f_X(x)dx$$  \hspace{1cm} (7)

In Eq. 7, the domain of integration is such that $f_X(x) \neq 0$. Note that Eq. 7 is not very meaningful because $y$ is single-valued given $x$, and hence $f_Y(y|x)$ is nothing but a Dirac delta function. Alternatively, the PDF can be calculated by differentiating the CDF, as:

$$f_Y(y) = \frac{dF_Y(y)}{dy}$$  \hspace{1cm} (8)

The different methods which have been used by researchers for uncertainty quantification aim at solving the above equations in mathematically intelligent ways. These methods can be classified into two types – sampling-based and analytical methods; while some may calculate the CDF of $Y$, other methods directly generate samples from the probability distribution of $Y$.

**Sampling-based Methods**

The most intuitive method for uncertainty propagation is to make use of Monte Carlo simulation (MCS). The basic underlying concept of Monte Carlo simulation is to generate a pseudo-random number which is uniformly distributed on the interval $[0, 1]$; then the CDF of $X$ is inverted to generate the corresponding realization of $X$. Following this procedure, several random realizations of $X$ are generated, and the corresponding random realizations of $Y$ are computed. Then the CDF $F_Y(y)$ is calculated as the proportion of the number of realizations where the output realization is less than a particular $y_e$. The generation of each realization requires one evaluation/simulation of $G$. Several thousands of realizations may often be needed to calculate the entire CDF, especially for very high/low values of $y$. Error estimates for the CDF, in terms of the number of simulations, are available in the literature [25]. Alternatively, the entire PDF $f_Y(y)$ can be computed based on the available samples of $Y$, using kernel density estimation [26].

There are several variations of the basic Monte Carlo algorithm which are used by several researchers [27], [28]. Some of these approaches are listed below:

1. **Importance Sampling:** This algorithm does not generate random realizations of $X$ from the original distribution. Instead, random realizations are generated from a proposal density function, statistics of $Y$ are estimated and then corrected based on the original density values and proposal density values.
2. Stratified Sampling: In this sampling approach, the overall domain of $X$ is divided into multiple sub-domains and samples are drawn from each sub-domain independently. The process of dividing the overall domain into multiple sub-domains is referred to as stratification. This method is applicable when subpopulations within the overall population are significantly different.

3. Latin Hypercube Sampling: This is a sampling method commonly used in design of computer experiments. When sampling a function of $N$ variables, the range of each variable is divided into $M$ equally probable intervals, thereby forming a rectangular grid. Then, sample positions are chosen such that there is exactly one sample in each row and exactly one sample in each column of this grid. Each resultant sample is then used to compute a corresponding realization of $Y$, and thereby the PDF $f_Y(y)$ can be calculated.

4. Unscented Transform Sampling: Unscented transform sampling [13] is a sampling approach which focuses on estimating the mean and variance of $Y$ accurately, instead of the entire probability distribution of $Y$. Certain pre-determined sigma points are selected in the $X-$space and these sigma points are used to generate corresponding realizations of $Y$. Using weighted averaging principles, the mean and variance of $Y$ are calculated.

Analytical Methods

A new class of methods was developed by reliability engineers in order to facilitate efficient, quick but approximate calculation of the CDF $F_Y(y)$; the focus is not on the calculation of the entire CDF function but only to evaluate the CDF at a particular value ($y_c$) of the output, i.e. $F_Y(Y = y_c)$. The basic concept is to “linearize” the model $G$ so that the output $Y$ can be expressed as a linear combination of the random variables. Further, the random variables are transformed into uncorrelated standard normal space and hence, the output $Y$ is also a normal variable (since the linear combination of normal variables is normal). Therefore, the CDF value $F_Y(Y = y_c)$ can be computed using the standard normal distribution function. The transformation of random variables $X$ into uncorrelated standard normal space ($U$) is denoted by $U = T(X)$, and the details of the transformation can be found in Haldar and Mahadevan [27].

Since the model $G$ is non-linear, the calculated CDF value depends on the location of “linearization”. This linearization is done at the so-called most probable point (MPP) which is the shortest distance from origin to the limit state, calculated in the $U-$space. Then, the CDF is calculated as $F_Y(y_c) = \Phi(-\beta)$, where $\Phi$ denotes the standard normal CDF function, and $\beta$ denotes the aforementioned shortest distance. The MPP and the shortest distance are estimated through a gradient-based optimization procedure. This optimization is solved using the well-known Rackwitz-Fiessler algorithm [29], which is in turn based on repeated linear approximation of the non-linear constraint $G(x) - y_c = 0$. This method is popularly known as the first-order reliability method (FORM). There are also several second order reliability methods (SORM) based on the quadratic approximation of the limit state [27], [30], [31], [32].

The entire CDF can be calculated using repeated FORM analyses by considering different values of $y_c$; for example, if FORM is performed at 10 different values of $y_c$, the corresponding CDF values are calculated, and an interpolation scheme can be used to calculate the entire CDF, which can be differentiated to obtain the PDF. This approach is difficult because it is almost impossible to choose such multiple values of $y_c$ because the range (i.e. extent of uncertainty) of $Y$ is unknown. This difficulty is overcome by the use of an inverse FORM method [33], [14] where multiple CDF values are chosen and the corresponding values of $y_c$ are calculated. This approach is simpler because it is easier to choose multiple CDF values since the range of CDF is known to be $[0, 1]$.

Discussion

Since sampling-based methods may require several thousands of “samples” or “particles” in order to accurately calculate the PDF or CDF, they are time consuming and hence, may not be suitable in the context of online prognostics and decision-making. Further, in general, sampling-based methods (other than the unscented transform sampling approach) are not “deterministic methods”; in other words, every time a sampling-based algorithm is executed, it may result in a slightly different PDF or CDF. The ability to produce a deterministic solution is, sometimes, an important criterion for existing verification, validation, and certification protocols in the aerospace domain.

On the other hand, analytical methods are not only computationally cheaper but also usually deterministic; in other words, they produce the same PDF or CDF every time the algorithm is executed. However, these analytical methods are still based on approximations, and not readily suitable to account for all types of uncertainty in prognosis. For example, consider the FORM method, which is solved using gradient-based optimization equations. If $t_{EOL} >> t_{y_e}$, then the number of elements in $u$ and $w$ may be of the order of a few hundreds or thousands, and hence, it is necessary to compute hundreds or thousands of derivatives of Eq. 5. In that case, the computational efficiency of the analytical approach is as good (or as bad) as sampling-based approaches. It is clear from the above discussion that, though uncertainty propagation methods may be available in the literature, it is challenging to make direct use of them for prognostics.

In addition to the above described methods, researchers have also advocated the use of surrogate models for uncertainty propagation. These surrogate models approximate the function “$G$” using different types of basis functions such as radial basis [34], Gaussian basis [35], Hermite polynomials [36], etc. These surrogate models are inexpensive to evaluate and therefore, facilitate efficient uncertainty propagation. Future research will investigate the use of such surrogate models for uncertainty quantification in prognostics.
4. CHALLENGES IN PROGNOSTICS

There are several challenges in using different types of uncertainty quantification methods for prognostics, health management, and decision-making. It is not only important to understand these challenges but also necessary to understand the requirements of PHM systems in order to integrate efficient uncertainty quantification along with prognostics and aid risk-informed decision-making. Some of the issues involved in such integration are outlined below:

1. Timely Calculations: An uncertainty quantification methodology for prognostics needs to be computationally feasible for implementation in online health monitoring. This requires quick calculations, while uncertainty quantification methods have been traditionally known to be time-consuming and computationally intensive.

2. Uncertainty Characterization: In many practical applications, it is even difficult to assess each individual source of uncertainty. For example, the future loading uncertainty, which is an important contributor of uncertainty to prognostics, is highly uncertain and it may not even be possible to characterize this uncertainty. The model that is used for prediction may also be uncertain; it may be difficult to estimate the future values of the model parameters and the model errors in advance.

3. Uncertainty Propagation: After each source of uncertainty has been characterized, it is not straightforward to compute their combined effect on prognostics and the remainder of useful life prediction. This computation must be the result of rigorous uncertainty propagation, resulting in the entire probability distribution of remaining useful life prediction.

4. Capturing Distribution Properties: Sometimes, the probability distribution of remaining useful life in prognostics may be multi-modal and the uncertainty quantification methodology needs to be able to accurately capture such distributions.

5. Accuracy: The uncertainty quantification method needs to be accurate, i.e., the entire probability distribution of \( X \) needs to be correctly accounted for, and the functional relationship defined by \( G \) in Fig. 2. Some methods use only a few statistics (usually, mean and variance) of \( X \) and some methods make approximations (say for example, linear) of \( G \). It is important to correctly propagate the uncertainty to compute the entire probability distribution of RUL, without making significant assumptions regarding the distribution types and functional shapes.

6. Uncertainty Bounds: While it is important to be able to calculate the entire probability distribution of RUL, it is also important to be able to quickly obtain bounds on RUL which can be useful for online decision-making.

7. Deterministic Calculations: Existing verification, validation, and certification protocols require algorithms to produce deterministic, i.e., repeatable calculations. Several sampling-based methods do produce different (albeit, only slightly if implemented well) results on repetition.

Each uncertainty quantification method may address one or more of the above issues, and therefore, it may even be necessary to resort to different methods to achieve different goals. Future research needs to continue this investigation, analyze different types of uncertainty quantification methods and study their applicability to prognostics.

5. NUMERICAL EXAMPLE

In order to illustrate the importance of uncertainty quantification in prognostics and online health monitoring, this section considers the health management of a lithium-ion battery that is used to power an unmanned aerial vehicle [37]. This unmanned aerial vehicle is being used as a test-bed for prognostics and decision-making at NASA Langley and NASA Ames Research Centers.

Description of the Model

The lithium-ion battery is modeled as an electrical circuit equivalent, as shown in Fig. 4. In this circuit, the large capacitance \( C_b \) holds the charge \( q_b \) of the battery. The nonlinear \( C_b \) captures the open-circuit potential and concentration overpotential. The \( R_{sp}, C_{sp} \) pair captures the major nonlinear voltage drop due to surface overpotential, \( R_s \) captures the so-called Ohmic drop, and \( R_p \) models the parasitic resistance that accounts for self-discharge. This empirical battery model is sufficient to capture the major dynamics of the battery, but ignores temperature effects and other minor battery processes.

The complete details of the battery model along with the governing state-space equations and numerical values can be found in the earlier publications [14], [38] that focused on using the first-order reliability method for battery prognostics. This paper focuses on the aspects of uncertainty and it is of interest to predict the end-of-discharge, which is defined to occur when the battery voltage is less than a specified threshold \( V_{EOD} \). The remaining useful life (RUL) of the battery is indicative of the time until end-of-discharge. The goal is to estimate the entire probability distribution of end-of-discharge that can then be used to calculate the RUL.

Sources of Uncertainty

The different sources of uncertainty considered in this case study are listed below:

1. Loading Uncertainty: In order to illustrate the importance of accurate uncertainty quantification, two different loading cases are considered. First, a constant amplitude loading is considered and the constant amplitude is chosen to be random; the constant amplitude (in Amps) is considered to be normally distributed \( N(35, 5) \), and this distribution is truncated at a specified lower bound (5.0) and upper bound (80). Second, a variable amplitude loading scenario is considered with six segments; within each segment, the amplitude is considered constant. The duration \( \bar{T} \) and the amplitude (current, \( I \)) of each segment are considered to be random; therefore, there are 12 random variables each of which are assumed to follow truncated normal distributions. The statistics, as calculated by Saha et al. [37], are provided in Table 1.
Table 1. Variable Amplitude Loading: Statistics

| Segment   | \(T_1\) | \(T_2\) | \(T_3\) | \(T_4\) | \(T_5\) | \(T_6\) | \(T_7\) | \(T_8\) |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|
| Take-off  | 80      | 70      | 100     | 60      | 10      | 50      | 75      |
| Climb     | 30      | 5       | 22      | 40      | 10      | 90      | 140     |
| Cruise    | 15      | 3       | 10      | 22      | 90      | 10      | 70      | 115     |
| Turn      | 35      | 5       | 25      | 47      | 120     | 10      | 100     | 145     |
| Glide     | 5       | 1       | 2       | 8       | 90      | 10      | 75      | 120     |
| Landing   | 40      | 5       | 30      | 53      | 60      | 10      | 40      | 80      |

2. **State Uncertainty**: State estimation is addressed using a filtering technique that can continuously estimate the uncertainty in the state based on the available measurements. Though there are three state variables ((1) charge in \(C_b\); (2) charge in \(C_{sp}\); and (3) charge in \(C_s\)) in this problem, the charge in \(C_b\) is the most influential state variable in the context of predicting the end-of-discharge. The mean of this state variable is shown in Fig. 5 as a function of time, and its CoV (coefficient of variation, defined as the ratio of the standard deviation to the mean) is chosen to be equal to 0.1 for the purpose of illustration.

The model parameter values are considered to be constant, and the process noise is found to be insignificantly small in this example [39] and therefore, is considered to be absent. As mentioned earlier, the goal is to account for the above sources of uncertainty and compute the probability distribution of EOD.

**Results of Uncertainty Quantification**

First, consider the case of constant amplitude loading. The uncertainty in the end-of-discharge prediction is computed continuously as a function of time, and the corresponding PDFs are multiple time instants can be seen in Fig. 6 and Fig. 7.

As it can be seen clearly, the shape of the probability density function significantly changes near failure, from a bell shaped distribution to a triangular distribution. Computational approaches of uncertainty quantification need to be able to capture such transitions accurately. While the constant amplitude loading scenario illustrates a sudden change in shape of the probability distribution, the variable amplitude loading scenario illustrates the case where the distribution of EOD may be multi-modal, as shown in Fig. 8.

**Discussion**

The case study presented in this section illustrated the following points:

1. It is important not to arbitrarily assign statistical properties

(such as distribution type, mean, standard deviation, etc.) for the remaining useful life prediction.

2. The shape of the distribution of RUL may significantly change during the course of the operation of the engineering system.

3. The distribution of RUL may have multiple modes and it is important to accurately capture such modes so that this distribution may be useful for decision-making.

In summary, it is important to capture all the characteristics of the probability distribution of RUL (which is equivalent to the end-of-discharge in this case study), and this can be accomplished only by using accurate uncertainty quantification methodologies without making critical assumptions regarding the probability distribution of the RUL. The goal must be to accurately calculate the probability distribution of \(R\) by propagating the different sources of uncertainty through \(G\) as indicated in Fig. 2. While computationally extensive Monte Carlo sampling can achieve this goal with reasonable accuracy, it may not be suitable for online prognostics and health monitoring since Monte Carlo sampling is time-consuming. It is necessary to investigate whether other computational approaches (that were discussed earlier in Section 3) are suitable for this purpose. This investigation will be continued in the future and used to guide further research.

6. **Conclusion**

This paper discussed the importance of uncertainty in prognostics and described the various sources of uncertainty that
affect prognostics. Uncertainty in prognostics arises from state estimation, assessment of future loading, operating and environmental conditions, usage of an inaccurate prediction model, etc. It is important to quantify the combined effect of these different sources of uncertainty on prognostics by computing the probability distribution of the remaining useful life prediction. Such computation needs to be viewed as an uncertainty propagation problem and statistical uncertainty propagation methods may be investigated for this purpose. Some of the popular uncertainty propagation methods were discussed in this paper and their applicability to prognostics was investigated. There are several practical challenges that need to be addressed before such methods can be used for online health monitoring purposes. Some of these challenges were identified in this paper, thereby suggesting possible directions for further research.

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**Biography**

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