Discrete and continuous models in calculating the bearing capacity of soil massifs

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Abstract. The article considers the continuous and discrete models of the soil medium using the calculation examples under the action of a strip load with a flat load. The options for calculating both slopes and a soil foundation for the bases in natural, artificial and composite environments are considered. A theoretical method for calculating the bearing capacity of the anisotropic soil base’s composite medium in shear resistance is given. The formulas and graphs of the compared ground conditions’ calculation results are given and their quantitative strength assessment compared with the natural environment is given.

1. Introduction

Models of continuous media (linearly elastic body, nonlinear elastic body, rigid plastic body, etc. [1, 2, 3]) are widely used in the calculation of building structures, foundations, bases because they allow the mathematical analysis methods’ use, in particular, the differential equations. At the same time, building material can be discrete (non-rocky soil, reinforced concrete, etc.).

The digital computers development has caused an inverse tendency to reduce the continuous tasks to the discrete tasks. Continuous problems of the linear and nonlinear theory of elasticity are discretized by the finite element method (FEM). These problems are ultimately reduced to the algebraic systems of linear equations. Various software systems with the help of FEM successfully calculate the building structures for deformations (according to SLS). However, when calculating the bearing capacity (according to ULS), theoretical difficulties appear.

The linear and nonlinear elastic models used in the software systems cannot describe the destruction of the body and, therefore, indicate the ultimate force. The problem is not solved by using the models of bodies described by the deformation theory of plasticity or the theory of plastic flow with hardening. And in this case, the question of the destruction possibility remains open, and the model of the hardening body itself does not contain an element that allows us to pose the question on destruction. The search for destructive force is possible within the framework of the ideal plasticity theory model, used directly or in the form of a yield surface in models with hardening.

The problem of ULS plastic systems’ calculation, including the soil massifs, reinforced by geosynthetics [4, 5, 6, 7, 8], can be solved by choosing a continuous rigidly plastic model anisotropic with respect to the medium shear resistance.

Three factors contribute to this.

First, there are the analytical solutions to the ultimate band load problem with a load on an anisotropic in terms of shear resistance soil foundation [9, 10].
Second, there is a technique for modeling a discrete medium (sand gravel mixture periodically shifted by geosynthetics) with a continuous rigid plastic medium anisotropic in shear resistance [11].

The third condition for successful calculation is the application of the limit analysis of plastic systems.

2. Let us consider the research results [10].

\[ \sigma_3 = -C(\theta) + A(\theta)\sigma_I \] (1)

The solution to the problem of finding the maximum load is reduced to solving a nonlinear differential equation of the first order:

\[ g'(\theta) = \frac{C'(\theta) - A'(\theta)g(\theta) \pm \sqrt{(A'(\theta)g(\theta) - C'(\theta))^2 + 4A(\theta)[(A(\theta) - 1)g(\theta) - C(\theta)]^2}}{2A(\theta)} \] (2)

Let the function \( g(\theta) \) be determined from the equation (2), and the initial condition to it can be determined from the continuity of stresses at the boundary zones III and II: \( g(\theta) = -q \). Then the ultimate pressure is determined by the formula

\[ P = C\left(\frac{\pi}{2}\right) - g\left(\frac{\pi}{2}\right) \cdot A\left(\frac{\pi}{2}\right). \] (3)

It is clear that in the general case of arbitrary functions \( A(\theta) \), \( C(\theta) \) the equation (2) has to be solved by numerical methods. However, in a number of special cases, the ultimate force (3) can be represented in a closed analytical form. Let us consider some of them.

If we assume that the soil foundation is composed of loose soils, i.e. \( C(\theta) = 0 \) then the formula (3) goes into formula (4).

\[ P = qA\left(\frac{\pi}{2}\right)e^{\frac{\pi}{2}} \int \frac{A(\theta)}{\sqrt{\ln[A(\theta)]^2 + [A(\theta) - 1]^2}} d\theta. \] (4)

Note that when substituting into the formula (4)
$A(\theta) \equiv A = \text{const}$,

it will go into the well-known formula of the loose base bearing capacity

$$P = Ae^{\frac{\pi(A-1)}{2\sqrt{A}}} q.$$

We assume that the soil base is composed of the cohesive soils with negligible internal friction. In this case, the strength condition is written as:

$$\sigma_3 = -C(\theta) + \sigma_I$$

This implies the bearing capacity formula:

$$P = \frac{C\left(\frac{\pi}{2}\right) + C(0)}{2} + \frac{\pi}{2} \int_0^\theta \sqrt{\left[C'(\theta)\right]^2 + 4\left[C(\theta)\right]^2} \, d\theta + q. \quad (5)$$

It is necessary to take into consideration the fact that when substituting $C(\theta) = 2cT$ into the formula (5), we obtain the well-known formula of the purely connected medium bearing capacity:

$$P = q + (2 + \pi)c_T.$$

The formulas (3), (4), (5) determine the bearing capacity of a weightless soil base and, therefore, are a lower estimate of the real bearing capacity of the base.

It is possible to take into account the specific gravity of the soil, thereby increasing the lower estimate, using the approximate technique proposed by V.V. Sokolovsky [12] and generalized to the condition (1) in [10].

3. Let us consider a modeling technique for a discrete medium (a sand gravel mixture periodically shifted by geosynthetics) with a continuous rigid plastic medium anisotropic in shear resistance [11].

We consider the arbitrary representative (representative) volume of the composite base (Figure 2), which is in the ultimate state. If this volume is located in the active extreme stress state zone III (Fig. 1), then the layers’ presence of geosynthetics perpendicular to the first main direction will not lead to a significant hardening of this soil volume (Figure 2a). If this volume is located in the zone of passive ultimate stress state I (Figure 1), then the presence of the geosynthetics parallel layers to the first main direction will significantly strengthen this soil volume depending on the value of the geosynthetics tensile strength (Figure 2b).
Let an arbitrary representative volume of the base, the faces of which coincide with the main sites, be in the region of a centered wave II. The angle between the first main direction and the layers of geosynthetics is equal to π/2 and varies from π/2 till 0 by the region II.

The question of increasing the soil medium volume strength, strengthened by the inclined geosynthetics layers, can be investigated experimentally.

In [11], a calculation hypothesis on the linear dependence of the strength characteristics of A and C on the angle 0 was put forward. In this case, the strength characteristics in the region III are accepted as for soil without geosynthetics. And in area I we add the "equivalent" adhesion to the soil adhesion, calculated as the ratio of the geosynthetics breaking force to the corresponding site.

We suppose, for example, a base made of a sand-gravel mixture that does not have adhesion but has an angle of internal friction φ = 30°, through each meter is shifted by horizontal layers of stabitex. Breaking force of a meter band of stabitex 80 kN. Therefore, in this case, the “equivalent” cohesion appears in the region I $c_e = 80 \text{kN/m}^2$. Let us say that the load $q = 20 \text{kN/m}^2$. We assume that the introduction of geosynthetics does not increase the angle internal friction and therefore parameter A equal to 3.

Hypothesis of a linear dependence of strength characteristics $A(\theta), B(\theta)$ from the corner $\theta$ requires the experimental and theoretical research with further possible correction.

4. Let us consider some problems of the discrete plastic systems’ bearing capacity, the solution of which is facilitated by the transition to a continuous model.

2. The problem of the ultimate band load on an anisotropic shear resistance ground foundation.

This task is presented in paragraph 2 of this text. Its solution can be used: a) when specifying the formula for the ultimate resistance of the foundation soils in the building rules; b) when determining the ultimate load on the composite base in the sand gravel mixture form, shifted by the geosynthetics horizontal layers.

Paragraph 2 and in [2] studies the unreviewed special case of an anisotropic soil shear resistance (Figure 4) with a strength condition

$$\sigma_3 = -C(\theta) + A \sigma_1$$

(6)

In condition (6), we take the parameter $A = 3$, and a function in the form $C(\theta) = \frac{2\theta}{\pi} C_0$.

Then equation (2) will be simplified and written in the form:

$$6g'(\theta) = \frac{2C_0}{\pi} - \sqrt{\left(\frac{2C_0}{\pi}\right)^2 + 12g(\theta) - \frac{2C_0}{\pi}}.$$  

(7)

The first-order differential equation (7) with respect to the unknown functions $g(\theta)$ is an equation with separable variables. Its general integral is written as

$$\theta = \frac{6dg}{k - \sqrt{k^2 + 12g(k)}} + D,$$

(8)

where $k = \frac{2\theta}{\pi}$, and D is the arbitrary integration constant.

Let $c_e = 80 \text{kN/m}^2$, then $C(\pi/2) = C_0 = 273.6 \text{kN/m}^2$. And the load $q = 20 \text{kN/m}^2$.

According to the graph in Figure 5 at $\theta = \pi/2$ we find $g = -450 \text{kN/m}^2$. Then we can find the formula (3)
Р = 273,6+450-3 = 1624 kN/m².

Figure 4. Ultimate band load on the composite base

Figure 5. Dependency graph θ from g, given by the formula (8) for cₑ = 80 kN/m²

3. An increase in the bearing capacity of base soils by replacing them with a sand-gravel mixture shifted by horizontal layers of geosynthetics

5. Let, for example, the cohesive soils-loams that have an internal friction angle serve as the initial base soils under the strip foundations of the building φ = 13,8° and the specific grip c = 0,014 MPa.

We suppose that the foundation soil depth is h = 3,0 m, where h is the depth of the foundation soil, and the specific gravity of the loamy soil adopted in the problem γ = 1,69 g/cm³ or γ = 16,57 kN/m³. Then the load q = γh will make q = 3,0 m*16,57 kN/m³ = 49,71 kN/m² or 0,0497 MPa.

The ultimate pressure on the base soil is determined by the well-known Coulomb-Mohr formula:

\[ p^2 := A \cdot e^{\frac{-\pi(A-I)}{2\sqrt{A}}} \left( q + \frac{C}{A-I} \right) \frac{C}{A-I} \] (9)

where strength characteristics will be the angle θ functions between the first main direction of the stress tensor and the axis OX (Fig.1), therefore, subjected to experimental determination of the function A=A(θ), C=C(θ):

\[ C := \frac{2c \cos(\varphi)}{1 - \sin(\varphi)} \quad A := \frac{1 + \sin(\varphi)}{1 - \sin(\varphi)}, \]

where C = 0,036 MPa and A = 1,627.

As a result, we obtain that the ultimate pressure on the soil under consideration will be 0,316 MPa or 32,2 T/m², after which it will be destroyed.
6. Now we will consider the bearing capacity of loose soil (LSC), constructed instead of the existing clay method of excavating the initial and backfilling the next to a depth equal to 1.5 of the foundation base width, since the stresses arising below this level are negligibly small.

When used LSC as base, then \( C(\theta) = 0 \), since incoherent soil is used. The angle of internal friction for LSC is \( \varphi = 31^\circ \). The load will be \( q = 3.0 \text{ m}^2 \times 20.0 \text{ kN/m}^3 = 60.0 \text{ kN/m}^2 \) or 0.06 MPa.

Next, we obtain the following strength characteristics \( C = 0 \text{ MPa} \) and \( A = 3 \). For this problem, a special case of the anisotropic in terms of the soil base shear resistance (Fig. 4) is valid with the condition (6).

In the condition (6), we take the parameter \( A = 3 \), and a function in the form \( C(\theta) = \frac{2\theta}{\pi} C_0 \).

Then equation (2) will be simplified and written in the form (7). Let the grip \( c = 0 \text{ kN/m}^2 \), then \( C(\pi/2) = C_0 = 0 \text{ kN/m}^2 \). Then the load \( q = \gamma h \) will make \( q = 3.0 \text{ m}^2 \times 20.0 \text{ kN/m}^3 = 60.0 \text{ kN/m}^2 \) or 0.06 MPa.

According to the graph in Fig. 6 at \( \theta = \pi/2 \) we find \( g = -120 \text{ kN/m}^2 \).

\[ \text{Figure 6. Dependency graph } \theta \text{ from } g, \text{ given by the formula (8) at } c = 0 \text{ kN/m}^2 \]

Then by the formula (3):

\[ P = 0 + 120 \times 3 = 360 \text{ kN/m}^2. \]

The maximum pressure on the soil from LSC shifted by the horizontal layers of geosynthetics will be 0.36 MPa or 36.7 T/m², after which the destruction of the soil will occur. However, the bearing capacity of LSC is slightly higher than the previously considered loamy soil with certain characteristics due to the particles’ adhesion lack in incoherent soil.

7. Therefore, in the following example, presented below, we consider the bearing capacity of loose soil (LSC), shifted by the geosynthetics horizontal layers.

Let us suppose, for example, a base made of a sand-gravel mixture that does not have adhesion but has an angle of internal friction \( \varphi = 31^\circ \) every 1.0 m shifted by the “stabitex” horizontal layers with a breaking load of 50 kN/m (strength specified by the manufacturer). Then an “equivalent” grip appears in the region I \( c_e = 50 \text{ kN/m}^2 \). Let us say that the load \( q = 20 \text{ kN/m}^2 \). We assume that the geosynthetics introduction does not increase the angle \textit{internal friction} and therefore the parameter \( A \) equal to 3.

For this problem, a special case of the anisotropic in terms of the soil base shear resistance (Figure 4) is valid with the condition (6).

In the condition (6), we take the parameter \( A = 3 \), we also find (7) and (8).

If accept \( c_e = 50 \text{ kN/m}^2 \), then \( C(\pi/2) = C_0 = 177 \text{ kN/m}^2 \). The load \( q = \gamma h \) will make \( q = 3.0 \text{ m}^2 \times 20.0 \text{ kN/m}^3 = 60.0 \text{ kN/m}^2 \) or 0.06 MPa.
According to the graph in Fig. 7 at $\theta = \pi/2$ we find $g = -336 \text{kN/m}^2$.

**Figure 7.** Dependency graph $\theta$ from $g$, given by the formula (8) for $c_e = 50 \text{kN/m}^2$

Then by the formula (3):

$$P = 177 + 336 \times 3 = 1185 \text{ kH/m}^2.$$

The maximum pressure on the soil from LSC, shifted by the geosynthetics horizontal layers, will be 1.185 MPa or 120.0 T/m$^2$, after which soil destruction will occur.

4. **Summary**

In this case, the bearing capacity of incoherent soil made of LSC shifted by the geosynthetics layers increased by 327% compared to the usual LSC layer and by 372% compared to the accepted loamy soil.

**References**

[1] Tsytovich N A, Ter-Martirosyan Z G 1981 Fundamentals of applied geomechanics in construction (High School, Moscow).

[2] Zaretsky Yu K 1989 Lectures on modern soil mechanics (Ed. RSU).

[3] Malyshev M V 1980 The strength of the soil and the stability of the foundations of structures (Stroyizdat, Moscow).

[4] Bugrov A K, Golubev A I 1993 Anisotropic soils and foundations of structures (Nedra, SPb.).

[5] School A V 1990 Anisotropy of the strength properties of loess soils and the calculation of the bearing capacity taking into account it (In the book: Loess subsidence soils as the foundation of buildings and structures, Prince 2. Part 2, Barnaul) pp. 212-217.

[6] Batugin S A 1988 Anisotropy of the rock mass (Nauka, Novosibirsk).

[7] Dyba V P 1995 Bearing capacity of an anisotropic shear resistance of a soil base loaded with a strip load with a load, *Novocherkassk: NSTU, 1995.9 Dep. at VINITI 07/19/95 2207-B95.*

[8] Shkola A V 1989 Calculation of the bearing capacity of anisotropic shear resistance of non-rocky foundations of hydraulic structures *Hydrotechnical construction 4* 22-24.

[9] Dyba V P 1995 Bearing capacity of an anisotropic shear resistance of a soil base loaded with a strip load with a load *Novocherkassk: NSTU, 1995. 9. Dep. in VINITI 19.07.95 2207*, release 95.

[10] Dyba V P 2008 Estimates of the bearing capacity of the foundations: monograph (Yuzh.-Ros. state tech. at-Novocherkassk: SRSTU).

[11] Dyba V P 2012 Bearing capacity of slopes reinforced with geotextile *Soil mechanics in geotechnics and foundation engineering: materials of the All-Russian Scientific and Technical Conference, Novocherkassk June 7-8, 2012, Yuzh.-Ros. state tech. University (NPI), Novocherkassk, SRSTU (NPI) 365-370.
[12] Sokolovsky V V 1952 About the approximate reception in the statics of granular medium *PMM* 16(2) 246-248.