Dissipative structures of quantized vortices in a coherently pumped polariton superfluid

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Steady states reached in a coherently pumped exciton–polariton superfluid are investigated. As the pump parameter is changed, the translational symmetry of the uniform system is spontaneously broken, and various steady patterns of quantized vortices are formed. This is peculiar to nonequilibrium dissipative systems with continuous pumping.

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I. INTRODUCTION

Quantum degenerate exciton–polaritons\textsuperscript{1–3} in a semiconductor microcavity have recently attracted growing interest. Such a system consists of quantum wells sandwiched between two distributed Bragg reflectors, where excitons in the quantum wells are coupled with photons confined in the reflectors. The coupling of the excitons with photons makes the exciton–polariton mass much smaller than the electron mass, which enables the system to maintain long-range coherence. In recent years, experimental studies on this system have shown rapid progress, for example, Bogoliubov excitations\textsuperscript{4} and quantized vortices\textsuperscript{5} have been observed, and various quantum fluid dynamics have been demonstrated\textsuperscript{6–8}.

In the present paper, we focus on a system in which polaritons are continuously pumped by near-resonant laser beams.\textsuperscript{9–14} The loss of polaritons with a short lifetime is balanced by the continuous pumping, thereby realizing a nonequilibrium open system.

Homogeneous steady states far from equilibrium can spontaneously break the translational symmetry, and they can exhibit patterns or structures, called dissipative structures\textsuperscript{15}. This kind of structure formation occurs in a variety of systems, such as Rayleigh–Bénard convection cells\textsuperscript{16,17} in a fluid heated from below, and Belousov–Zhabotinsky patterns\textsuperscript{18} and Turing patterns\textsuperscript{19,20} in chemical reaction–diffusion systems. However, few studies have been performed on dissipative structures in quantum mechanical systems\textsuperscript{19,20}. The polariton superfluid in a microcavity is a suitable system for realizing quantum dissipative structures, since polaritons can be injected into the system continuously and coherently with relative ease.\textsuperscript{1} The continuous injection and dissipation of polaritons is expected to drive the formation of quantum dissipative structures.

In the present paper, we show that various steady patterns of quantized vortices are formed in an exciton–polariton superfluid in a semiconductor microcavity pumped by near-resonant continuous-wave laser beams. We consider a situation in which two spatial regions are pumped and a polariton superfluid flows into the interspace between the two pumped regions, as illustrated in Fig. I (a). When the interval $d$ between the two regions is small, the steady state has continuous translational symmetry (with respect to $x$ in Fig. I (a)). When $d$ exceeds a critical value, an instability sets in and the continuous translational symmetry is spontaneously broken. The system develops into another steady state, in which quantized vortices are aligned periodically. These patterns of quantized vortices are spontaneously organized in nonequilibrium steady states and are peculiar to the open system with loss and pump. We also show that the pattern depends on $d$ and the phase difference between the two pumped regions. In finite systems, vortices escape from the edges of the pumped regions, which can be plugged by an appropriate potential.

This paper is organized as follows. Section II provides the formulation of the problem. Section III numerically demonstrates the formation of various patterns. Section IV presents the conclusions to this study.

II. FORMULATION OF THE PROBLEM

We used the mean-field theory to study the dynamics of an exciton–polariton superfluid. The energy of a photon in a quasi-two-dimensional microcavity can be ap-
approximated as $\hbar \omega_C^0 + p^2/(2M_C)$, where $p$ is the in-plane momentum and $M_C$ is the effective mass of a photon. The effective mass of an exciton is much larger than $M_C$, and we approximated the energy of an exciton as a constant, $\hbar \omega_X^0$. The excitons in the quantum wells and the photons in the microcavity couple with the Rabi frequency $\Omega_R$. The macroscopic wave functions for the photons $\psi_C$ and the excitons $\psi_X$ are assumed to obey the coupled nonlinear Schrödinger equations given by \cite{21}.

\begin{align}
\hbar \frac{\partial \psi_C}{\partial t} &= \left( \hbar \omega_C^0 - \frac{\hbar^2}{2M_C} \nabla^2 \right) \psi_C + \hbar \Omega_R \psi_X + V(r) \psi_C \\
&\quad - i \hbar \gamma_C \psi_C + e^{i(k_p \cdot r - \omega_p t)} F(r), \quad (1a) \\
\hbar \frac{\partial \psi_X}{\partial t} &= \hbar \omega_X^0 \psi_X + \hbar \Omega_R \psi_C + g |\psi_X|^2 \psi_X - i \frac{\hbar \gamma_X}{2} \psi_X \quad (1b)
\end{align}

where $V(r)$ is the external potential for the photons, $g$ is an exciton–exciton interaction coefficient, and $\gamma_C(\omega_X)$ is the decay constant for the photons (excitons). The last term on the right-hand side of Eq. \((1a)\) describes the coherent pumping of photons by external laser beams, where $k_p$, $\omega_p$, and $F(r)$ are the in-plane wave number, frequency, and profile of the pumping, respectively.

Diagonalizing the first and second terms on the right-hand side of Eq. \((1)\) for the mode with wave number $k$, we obtain the energies of the upper and lower polaritons as

$$\omega_{\pm}(k) = \frac{1}{2} \left[ \omega_k^0 + \omega_C^0 + \omega_X^0 \pm \sqrt{(\omega_k^0 + \omega_C^0 - \omega_X^0)^2 + 4\Omega_R^2} \right],$$

where $\omega_k^0 = \hbar k^2/(2M_C)$. The dispersion relation in Eq. \((2)\) is depicted in Fig. \((1b)\). The difference between the pump and lower polariton frequencies is defined as

$$\delta = \omega_p - \omega_X(0).$$

For simplicity, in the following calculations, we restrict ourselves to the case of $\omega_k^0 = \omega_X^0 = 0$, $\gamma_C = \gamma_X \equiv \gamma$, and $k_p = 0$. The parameters are fixed as follows: $M_C = 2 \times 10^{-5} m_e$ with $m_e$ being the electron mass, $\hbar \Omega_R = 5 \text{ meV}$, $g = 0.01 \text{ meV}\mu^2$, $\gamma^{-1} = 10 \text{ ps}$, and $\delta = 0.76 \text{ meV}$.

**III. NUMERICAL RESULTS**

**A. Steady states with continuous translational symmetry**

We first consider an infinite system without an external potential ($V = 0$). We use a pump profile as illustrated in Fig. \((1a)\), given by

$$F(r) = \begin{cases} 
F_0, & (y \leq -d/2), \\
0, & (|y| < d/2), \\
F_0 e^{i \phi}, & (y \geq d/2),
\end{cases} \quad (4)$$

where $F_0 = 5.7 \text{ meV}$, $\phi$ is the phase difference between the two pump beams, and $d$ is the width of the interspace between the two pumped regions. For this value of $F_0$, the pumped regions are excited into the stable uppermost branch of the bistability curve \cite{21}.

In the limit of $d \rightarrow 0$ with $\phi = 0$, the whole system is uniformly pumped into the stable state $\psi_{C,X}(r,t) \propto e^{-i \omega_p t}$. When $d$ is increased, the density at $|y| \lesssim d/2$ decreases and the continuous translational symmetry with respect to $x$ is maintained if $d$ is not very large. We sought such steady states that is uniform in $x$ by solving Eq. \((1)\) using the Newton-Raphson method. We also checked the stability of the steady states by using Bogoliubov analysis. If there is a Bogoliubov frequency that has a positive imaginary part, the state is dynamically unstable. Figure \((2)\) shows the $d$-dependence of the steady
states. For $\phi = 0$, the steady states have even-parity density profiles with two minima, where the velocity is largest, as shown in the inset in Fig. 2(a) (the exciton wave function $\psi_X$ is always qualitatively the same as the photon wave function $\psi_C$, and thus in the following discussion, we will show only $\psi_C$). There are two steady states for each $d$; one is stable (red solid curve) and the other is dynamically unstable (green dashed curve). The dynamically unstable modes grow exponentially and break the continuous translational symmetry with respect to $x$. The density dips in the unstable state are deeper than those of the stable state. These minimum values of the density are shown in the main panels of Fig. 2. As $d$ is increased, the stable and unstable branches meet at $d = d_{\text{max}}$ and the steady states disappear for $d > d_{\text{max}}$, which is a manifestation of the saddle-node bifurcation. Figure 2(b) shows the case of $\phi = \pi$. The $d$-dependence is similar to that in Fig. 2(a) except that the density and velocity profiles are asymmetric, as shown in the inset of Fig. 2(b). There are two degenerate states, since Eq. (1) is symmetric with respect to $\phi = 0$. The steady state shown in Fig. 3(a) is uniform in $x$ and hence has even-parity $\psi_X$, which corresponds to the stable branch for $d < d_{\text{max}}$, and the flow velocity has an upper bound $v_{\text{max}}$. The loss of polaritons per unit time at $y > d/2$ is $\gamma \rho d$, where $\rho$ is the polariton density. The loss must be replenished by injection $\sim \rho v$, i.e., $\gamma \rho d \sim \rho v$. Thus, $d$ has an upper bound $d_{\text{max}} \sim v_{\text{max}}/\gamma$. We have numerically confirmed that $d_{\text{max}}$ is roughly proportional to $\gamma^{-1}$.

**B. Symmetry breaking and pattern formations**

Figures 3(a)–3(c) show the stable steady states for $\phi = 0$. The steady state shown in Fig. 3(a) is uniform in $x$, which corresponds to the stable branch for $d < d_{\text{max}}$ in Fig. 2(a). As $d$ increases and exceeds $d_{\text{max}}$, the stable branch in Fig. 2(a) disappears. As a consequence, the continuous translational symmetry with respect to $x$ is spontaneously broken and quantized vortex pairs are created; these align periodically, as shown in Fig. 3(b). The vortex state shown in Fig. 3(b) is a stable steady state, in which vortex dipoles (clockwise and counterclockwise vortex pairs) are the building blocks of the pattern: two vortex dipoles form a pair that has parity symmetry with respect to $y$. Although a vortex dipole propagates at a constant velocity in a uniform system, the vortex dipoles in Fig. 3(b) remain at rest. This is because the vortex dipoles are prevented from propagating by the counterflow from the pumped regions, as shown in Fig. 3(e). The polaritons pumped in the region of $y > d/2 (y < -d/2)$
profiles of the steady state for $d = 24$ $\mu$m, where the phase difference between the two pumped regions is $\phi = \pi$. The unit of density is $350$ $\mu$m$^{-2}$. (b) The flow field in the dashed square in (a). See Supplemental Material for the dynamics in which $\phi$ is changed from 0 to $\pi$.

flow in the direction of $-y$ ($+y$) and sustain the vortex dipoles located in $y > 0$ ($y < 0$). These vortex dipoles would propagate in the $+y$ ($-y$) direction if they were in a uniform system without superflow.

As $d$ is increased further, the steady vortex structure in Fig. 3(b) makes a transition to another vortex structure, as shown in Fig. 3(c). The vortex pairs aligned along $y \simeq 0$ are not vortex dipoles but twin vortices in which two vortices have the same circulation. Such twin vortices with clockwise and counterclockwise circulations alternate along $y \simeq 0$, resulting in a structure similar to the vortex street reported in Ref. 23. Although both vortex pairs with the same circulation rotate around one another in a uniform system, the twin vortices in Fig. 3(c) are at rest. This can be understood from the Magnus force on the vortices, which is due to the density distribution in Fig. 3(c): a vortex with circulation $\kappa$ experiences an effective force proportional to $-\kappa \times \nabla \rho$, where $\rho$ is the polariton density. For example, in a uniform system, the twin vortices magnified in the right panel of Fig. 3(f) would rotate around one another in the counterclockwise direction. The density gradient in the directions of the red (light gray) arrows exerts forces in the directions of the blue (dark gray) arrows, which holds the rotation of the twin vortices.

The continuous translational symmetry with respect to $x$, as shown in Fig. 3(a), thus changes to discrete translational symmetry due to the vortex pattern formation, as shown in Figs. 3(b) and 3(c). For a larger $d$, there is no steady state, and the vortex pattern dynamically changes in a random manner, which has no symmetry (a snapshot of the dynamics is shown in Fig. 3(d)).

Figure 4 shows the steady state for $\phi = \pi$, which corresponds to the case of Fig. 2(b). For the width $d = 24$ $\mu$m, there is no steady state that is uniform in $x$, and a vortex pattern emerges as shown in Fig. 4. The clockwise and counterclockwise vortices are aligned alternately along $y \simeq 0$, whose flow pattern (Fig. 4(b)) is similar to that in Fig. 3(f). For a classical incompressible inviscid fluid, such an arrangement of point vortices is unstable. (The only stable arrangement of this kind of vortex rows is the one found by von Kármán.) The vortex pattern in Fig. 4 is stabilized by the superflow from the pumped regions $|y| > d/2$ toward $y = 0$. The density distribution around the vortices may also be responsible for the stabilization.

The vortex patterns in Figs. 3(b), 3(c), and 3(d) are periodic in $x$, and their periods are $\simeq 28$ $\mu$m, 31 $\mu$m, and 34 $\mu$m, respectively. The periods of the vortex patterns are determined by the geometry of the system, and they are of the order of the interval $d$ between the pumped regions.

C. Finite systems

Thus far, we have considered only infinite systems, in which the steady states have continuous or discrete translational symmetry with respect to $x$. In this section, we
examine pumped regions of finite size, as follows:

\[
F(r) = \begin{cases} 
F_0 & (|x| \leq 60 \mu m, -40 \mu m \leq y \leq -d/2), \\
F_0 e^{i\phi} & (|x| \leq 60 \mu m, d/2 \leq y \leq 40 \mu m), \\
0 & \text{(otherwise)},
\end{cases}
\]

where \( F_0 = 5.7 \text{ meV} \) is the same as that in Figs. 3 and 4. Figures 5(a)–5(d) show the dynamics for the parameters \( \phi = \pi \) and \( d = 24 \mu m \), where the origin of time is chosen appropriately. For these values of \( d \) and \( \phi \), the infinite pumped system has the steady state with a vortex pattern, as shown in Fig. 4. In the finite system, however, the polaritons in the space between the pumped regions flow out from the edges at \( |x| = 60 \mu m \). As a consequence, vortices also flow out, as indicated by the arrows in Figs. 5(a)–5(d). As the vortices flow out, they separate from each other, and new vortex-antivortex pairs are created between them, as shown by the dashed circles in Figs. 5(c) and 5(d). Such separations and creations of vortices are repeated, and the system does not reach a steady state.

To stabilize the system and obtain steady states, we add a plug potential as

\[
V(r) = \begin{cases} 
V_0 & (50 \mu m \leq |x| \leq 60 \mu m, |y| \leq -d/2), \\
0 & \text{(otherwise)},
\end{cases}
\]

which prevents the polaritons from flowing out. Figures 5(e) and 5(f) show the steady states for \( \phi = \pi \) and \( \phi = 0 \), where the regions of \( V = V_0 \) are enclosed by the dashed squares. The clockwise and counterclockwise vortices alternately align in Fig. 5(e) and the vortex dipoles make pairs in Fig. 5(f), which are similar to Figs. 4 and 3(b), respectively. Thus, the steady states can also be obtained for finite systems by using a plug potential that confines polaritons to a finite region. The steady-state patterns are suitable for time-integrated imaging in experiments.

IV. CONCLUSIONS

We have investigated the steady states of an exciton–polariton superfluid coherently pumped into a semiconductor microcavity; this is a nonequilibrium open system. We considered the situation in which polaritons with \( k = 0 \) are pumped into two regions and flow into the space between them, as illustrated in Fig. 1. When the distance \( d \) between the two pumped regions is small, there exists a stable steady state with continuous translational symmetry with respect to \( x \), as shown in Fig. 1(a). As \( d \) is increased, this steady state disappears (Fig. 2), and the system goes into other steady states with periodic patterns of quantized vortices (Figs. 3(b), 3(c), and 4). Such patterns are specific to a nonequilibrium system with loss and pump, and they can be recognized as dissipative structures. We have shown that steady-state patterns can also be observed in a finite system with an appropriate potential (Fig. 5).

The structure formations shown in the present paper are peculiar to nonequilibrium steady states and should be distinguished from those in equilibrium systems, such as crystals. For example, the triangular lattice structure of quantized vortices in a rotating superfluid cannot be regarded as a dissipative structure, since it is the ground state in the rotating frame of reference.

Note added. Very recently, a preprint has appeared, which reports experimental and numerical observations of self-arranged vortex–antivortex pairs in a polariton condensate.

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