SUSY dark matter*

—A Collider Physicist’s Perspective—

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Abstract. A short tour of supersymmetric dark matter and its connection to collider physics.

1. Quest for the dark matter

Recently the existence of dark matter has been confirmed through the CMB measurements [1]. Because the baryon density of the universe $\Omega_b$ is only 1/6 of the matter density of the universe $\Omega_m h^2 = 0.27 \pm 0.04$, the dark matter should be explained by the physics beyond standard model (SM). There are numerous on-going and planned dark matter search experiments which aim to obtain direct evidence of new particles that constitute the dark matter in the universe.

On the other hand, high energy colliders, such as Large Hadron Collider at CERN (LHC) or proposed Linear $e^+e^-$ colliders (LC), would significantly extend our ability to explore new physics at the TeV scale. The two approaches—collider experiments and dark matter searches—will open a new regime of the particle physics in this century.

Supersymmetry (SUSY) is one of the favorable candidates for new physics at the TeV scale. This is the only non-trivial extension of the Poincare algebra, and it solves the hierarchy problem of the SM. The minimal supersymmetric extension of standard model (MSSM) is going to be explored up to a few TeV by LHC, and current and future dark matter searches also have sensitivity to supersymmetric dark matter with the mass up to the TeV scale.

In the first part of my talk, I summarize SUSY dark matter searches. Once a dark matter signal is observed, the data can be used to study the density profile of our galaxy, which is not fully understood right now. In the latter half of my talk, I discuss the SUSY searches at colliders and more involved studies to measure the masses and interactions of the sparticles. Through the measurements, the thermal relic density and reaction of the SUSY dark matter would be constrained severely. These will be solid bases to discuss astrophysics and cosmology involving SUSY dark matter.

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*a Japanese rescue operation : -)

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2. From the Universe

2.1 Direct detections and local dark matter velocity distribution

For cosmologists, dark matter is important because it has created the structure of the universe through its gravitational interaction. Particle physicists see the same dark matter differently. They view the dark matter as weakly interacting particles, and search for them through their interactions.

In the MSSM, the lightest supersymmetric particle (LSP) is stable if R parity is conserved, and it is a candidate for the dark matter in the universe. The LSP may be the lightest neutralino \( \tilde{\chi}_1^0 \). The \( \tilde{\chi}_1^0 \) is a mixture of the superpartners of the gauge bosons (\( \tilde{B} \) and \( \tilde{W} \)) and superpartners of the Higgs bosons (\( \tilde{H}_1 \) and \( \tilde{H}_2 \)),

\[
\chi(\equiv \tilde{\chi}_1^0) = N_{\tilde{B}} \tilde{B} + N_{\tilde{W}} \tilde{W} + N_{\tilde{H}_1} \tilde{H}_1 + N_{\tilde{H}_2} \tilde{H}_2.
\] (1)

The masses and mixings of the neutralinos \( \tilde{\chi}_i^0 \) \((i = 1, .., 4)\) are determined by \( M_1 \) (Bino mass) \( M_2 \) (Wino mass parameter) and Higgsino mass parameter \( \mu \), and they also have weak dependence on the ratio of the vacuum expectation value of the Higgs bosons \( \tan \beta \).

The mass matrix is given as follows,

\[
M_N = \begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_\omega & m_Z s_\beta s_\omega \\
0 & M_2 & m_Z c_\beta c_\omega & -m_Z s_\beta c_\omega \\
-m_Z c_\beta s_\omega & m_Z c_\beta c_\omega & 0 & -\mu \\
m_Z s_\beta s_\omega & -m_Z s_\beta c_\omega & -\mu & 0
\end{pmatrix},
\] (2)

The \( \chi \) may be directly searched for through \( \tilde{\chi} N \) scattering mediated by a higgs boson exchange. On the other hand, the superpartner of the graviton (gravitino \( \tilde{\psi}_{3/2} \)) is the LSP in the gauge mediation model. The interaction of gravitino dark matter is too small to obtain direct evidence for its existence.

The CDMS II is an exciting dark matter search experiment in the forthcoming few years. The CDMS II aims to find a dark matter particle with the cross section \( \sigma(N_\chi \rightarrow N\tilde{\chi}) > 10^{-8} \text{pb} \) (or \( 3 \times 10^{-4}/(\text{kg} \cdot \text{keV} \cdot \text{day}) \)), while the current limit is around \( 10^{-6} \text{pb} \). The Cryogenic detector measures both phonon and ionization, therefore it actively discriminates the background caused by electrons or photons. The remaining neutron background at the Soudan mine (the depth of 2090 mwe) is significantly lower than for the previous CDMS experiment [2] at a shallow site. As we have already heard in Roskowski’s talk [3], CDMS II cuts into the significant region of the MSSM parameter space which is not excluded by current experimental constraints. In addition, there is a hope that the dark matter signal rate is very close to the current limit, because of the claimed “evidence” of the dark matter at DAMA [4]. It is therefore tempting to think about the implication of the dark matter signal when we have O(100) events at such high-tech detectors.

Counting rate of the dark matter depends on the velocity distribution. The standard assumption is virialized dark matter. The velocity distribution is Gaussian and the average velocity is zero. Therefore, the average dark matter velocity is roughly equal to the earth velocity \( \sim 220 \text{ km/s} \). Because of the earth motion around the sun, the dark matter signal modulates annually. The modulation is important to extract the fraction of the counting rate that comes from the dark matter scattering off the nuclei.
Currently we do not have any direct evidence of the dark matter velocity distribution in our galaxy. Dark matter in our galaxy may be co-rotating or counter-rotating around the center of the galaxy, and the dark matter flux significantly changes depending on that. After the discovery of the dark matter, the dark matter velocity distribution would be obtained experimentally by studying the energy deposit in the detector [5].

An even more exciting possibility was pointed out by Sikivie et al [6]. They assume a non-virialized dark halo in which the collisionless dark matter particles falling into the galaxy oscillate in and out many times. The expected phase of the annual modulation of a dark matter signal in direct detection experiments is opposite to the one usually expected [7].

2.2 Indirect detections and the dark matter profile in the galaxy

Exotic cosmic rays such as \( \gamma \), \( \bar{p} \) and \( e^+ \) are produced by the neutralino dark matter pair annihilation in the galaxy. They may be observed by space or balloon-borne experiments, and are called indirect detections of the dark matter.

Note that one needs to know the density profile of the dark matter in our galaxy to estimate the signal flux. A density profile of dark matter may be parameterized as follows;

\[
\rho(r) \propto \frac{1}{(r/a)^\alpha [1 + (r/a)^\alpha]^{(\beta-\gamma)/\alpha}}.
\]

(3)

Here \( r \) is the radial distance from the center of the galaxy and \( \alpha, \beta \) and \( \gamma \) are free parameters to be fixed.

The modified isothermal distribution is parameterized as \( (\alpha, \beta, \gamma) = (2, 3, 0) \), which is consistent with visible star distributions near the center of the galaxy. Recently singular density profiles are proposed and the implications are discussed. The singular distributions are motivated by \( N \) body numerical simulations and popular parameterizations are those of Navarro Frenk White (1996) [8] \( (\alpha, \beta, \gamma) = (1, 3, 1) \) and Moore (1999) [9] \( (\alpha, \beta, \gamma) = (1.5, 3, 1.5) \). The simulations have been significantly improved in recent years. The current best simulations contains typically \( O(10^7) \) points, compared with \( N = 400 \) in ’70. A better ‘simulation’ with large \( N \) tends to predicts a more singular profile.

The dark matter pair annihilation rate is proportional to \( \rho^2 \). A model with a singular density profile at the center of our galaxy predicts a high dark matter pair annihilation rate. For example, the \( \gamma \) ray flux from the process \( \tilde{\chi} \tilde{\chi} \rightarrow \gamma \tilde{X} \) is proportional to the \( \rho^2 \) integrated along the line of sight,

\[
\text{(signal rate)} \propto J(\psi) = 1 \frac{1}{8.5\text{kpc}} \left( \frac{1}{0.3\text{GeV/cm}^3} \right)^2 \int_{\text{line of sight}} \rho^2(l) dl(\psi).
\]

(4)

Note for Moore’s density profile, the integration of \( \rho^2 \) from \( r = 0 \) is infinite, therefore all dark matter particles in the region \( r < r_{\text{cut}} \) has been pair annihilated already [10]. To be quantitative

\[
\bar{J}(\Delta \Omega) \equiv \frac{1}{\Delta \Omega} \int_{\Delta \Omega} J(\psi) d\Omega
\]

(5)
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is $500$ for the NFW profile and $10^5$ for Moore’s profile for a typical angular resolution $\Delta \Omega = 10^{-3}$ sr. It was also pointed out that the density profile not only has a cusp at the center, but it is also clumpy. The map of $J$ shows numerous spots in the numerical simulations.

The $\gamma$ energy distribution from the dark matter pair annihilation consisted of two components. A “continuum component” comes from neutralino pair annihilation into $ff, WW, ZZ$ and so on. The $\gamma$ energy distribution is terminated at the kinematical limit $E_\gamma = m_{\tilde{\chi}}$.

The other component comes from the neutralino pair annihilation into $\gamma Z$ and $\gamma \gamma$. Because a neutralino does not have a direct coupling to a photon, these processes are radiative, and suppressed. Energy of the photons is monochromatic $E_\gamma = m_{\tilde{\chi}}$. Diffuse gamma ray background is observed by EGRET [11]. Therefore the continuum signal must be observed as a kink structure on the background spectrum. The monochromatic gamma signal is robust, because it cannot have any astrophysical origin.

EGRET covers $E_\gamma < 30$ GeV. It has a photon angular resolution about 1.3 degree at 1 GeV and 0.4 degree at 10 GeV. Sensitivity of the EGRET data to the gamma ray flux from the center of the galaxy is $\sim 10^{-8}$ cm$^{-2}$ sec$^{-1}$ based on the operation time looking into the center of our galaxy. A future $\gamma$ ray observation will be carried out by GLAST [12] (Gamma-ray Large Area Space Telescope) from 2005. It has a sensitivity to the photon $0.02$ GeV $< E_\gamma < 300$ GeV, with the energy resolution $\Delta E_\gamma \sim 10\%$. It has a significantly better point source sensitivity compared with EGRET, $\Phi > 10^{-10}$ cm$^{-2}$s$^{-1}$ for 1 GeV $< E < 300$ GeV. “Ground Based” ACTs (Airshower Cherenkov Telescopes) cover photons with $O(10)$ GeV $< E_\gamma < 10$ TeV. Future experiments such as VERITAS and HESS are planning to have a sensitivity to the photon flux up to $\Phi = 10^{-13}$ cm$^{-2}$s$^{-1}$ for $E_{th} > 1$ TeV and $\Phi > 5 \times 10^{-14}$ cm$^{-2}$s$^{-1}$ for 10 TeV.

Upper limit of the gamma ray flux from the dark matter annihilation comes from the continuum component. A typical dark matter pair annihilation produces $O(10-40)$ low energy photons for $m_{\tilde{\chi}} = 500$ GeV. The observed photon flux from the center of the galaxy set a limit on dark matter. EGRET has a sensitivity to the $\gamma$’s from the SUSY dark matter pair annihilation for $m_{\tilde{\chi}} < 500$ GeV for Moore’s density profile. (See a recent improved analysis [13] which takes into account the $E_\gamma$ dependence of the point source sensitivity.)

The gamma line is a robust dark matter signal if it is observed. The signal flux may be detectable for a wino-like (expected for the anomaly mediated SUSY breaking model) or a Higgsino-like neutralino. Note that the pair annihilation cross section into two $\gamma$s in the wino- or higgsino-limit is expressed as

$$\sigma(\tilde{\chi}\tilde{\chi} \rightarrow 2\gamma) \sim \frac{\alpha^2 \alpha_s^2}{m_{W}^2}.$$  

(6)

It does not have the usual $1/m_{\tilde{\chi}}^2$ dependence. This is because the mass of chargino in the loop satisfies $m_{\tilde{\chi}_1^\pm} \sim m_{\tilde{\chi}}$ for this case. The large enhancement also means that the perturbative calculations break down and all order resummations are required to obtain the cross section. Recently we calculated all order QED effect and 2 loop $W$ and $Z$ exchange effect [14], and the scale where perturbative approach breaks down is determined. Summation of ladder exchange sometimes leads a huge enhancement of the cross section, and we are studying all order corrections involving $W$ and $Z$ boson exchanges now.

Anti-particle searches are also increasing their sensitivity significantly. Here the neutralino dark matter signal is anti-protons or positrons produced by the pair annihilations.
The annihilations occur dominantly at the center of our galaxy, and the produced antiprotons propagate to our solar system without too much loss. Background secondary antiproton flux is small in low momentum region. Antiprotons coming from outside our solar system may be observed at the solar minimum.

On the other hand, positrons loose their energy quickly by synchrotron radiation and inverse Compton scattering in the Universe, therefore, they are sensitive to local clumps nearby our solar system. Recently the HEAT positron data is interpreted as a dark matter signal and implication is discussed by several groups [15].

The balloon-borne experiment BESS [16] has been looking into low energy anti-protons in Canada. Duration of each flight was typically a few days and the data is statistically limited. The experiment is going to move to Antarctica, BESS-Polar. They expect to fly in Jan 2004 and Feb 2006 with significantly improved flight time $\sim 20$ days. The second flight is planned at the solar minimum. The detector is sensitive down to the antiproton with $E_{\text{kin}} \sim 100$ MeV. Another anti-matter search will be carried out by space-based AMS-02. This is an experiment at International Space Station (ISS) and it was planned to start from March 2004.\(^1\) The threshold of anti-proton kinetic energy is higher, $E_{\text{kin}} > 300$ MeV, but long term operation for 3 to 5 years is possible. The statistics of positrons will be increased by a factor of 10 from HEAT experiment.

SUSY dark matter search is not the unique target of these experiments. PBH or domain wall produce large antimatter signals. Also it is only very recently pointed out that solar modulation is charge dependent [17]. BESS confirmed the predicted quick increase of the $\bar{p}/p$ ratio at the last solar maximum. In next few years, the solar modulation is expected to be time dependent, and it is interesting to continue the observation of the $\bar{p}/p$ ratio.

### 2.3 Solutions of the cusp problem

There is a discussion that the number of the observed dwarf galaxies around the Milky Way halo is not consistent with the $N$ body simulations. The discrepancy may be the problem of numerical simulations. These simulations go through complicated procedures such as interfacing a large scale simulation to a small scale one. In addition, effects of radiations have not been taken into account. However, the contradiction has been discussed from various directions recently.

The cusp problem may be solved by introducing a warm dark matter with a free streaming scale $R_f \sim 0.1$ Mpc. The warm dark matter washes out small scale density perturbations, so that cusps of the galaxy do not have time to develop. For the gravitino dark matter, the required free streaming scale corresponds to the gravitino mass around $m_{3/2} \sim 1$ keV,

$$R_f = 0.2(\Omega_W h^2)^{1/3} \left( \frac{m_{3/2}}{\text{keV}} \right)^{-4/3} \text{Mpc.}$$

(7)

On the other hand, the thermal relic density of O(1) keV gravitino is too large,

$$\Omega_{3/2} h^2 = 0.5 m_{3/2}(\text{keV}).$$

(8)

\(^1\)The experiment might be delayed due to the recent tragic space shuttle accident.
Therefore some entropy production to dilute the gravitino dark matter is required. A neutralino dark matter also could be a warm dark matter if it is produced from topological defects [18] or moduli (or heavy gravitino) decays. However, because the scattering of the neutralino with the medium reduces the energy, one needs 10 TeV initial neutralino energy at very late time $T_I < 5\text{MeV}$ [19]. However, we need the density perturbation at the small scale to have sufficiently early re-ionization of the universe. The constraint from Lyman-$\alpha$ forest $z_{\text{r.i.}} > 3$ requires $m_{3/2} > 750$ eV for gravitino dark matter [20], while the recent WMAP data [1] push back the re-ionization period significantly $z_{\text{r.i.}} = 20^{+10}_{-9}$. Another solution is found in [21], where they propose an inflation model with suppressed density perturbation at small scale.

3. On the ground

3.1 $\Omega$ and MSSM parameter measurement

We have seen many experimental and theoretical aspects related to SUSY dark matter searches in the previous section. On the other hand, the particle of which dark matter consists can be produced and studied at collider experiments. Large Hadron collider (LHC) is a $pp$ collider at $\sqrt{s} = 14$ TeV. Experiments are expected to start in 2007. Significant SUSY parameter space would be covered within a year of operations. In addition to that, $e^+e^-$ colliders at $\sqrt{s} = 500$ GeV to 1 TeV are proposed by DESY, KEK and SLAC [22–24]. The LC will be a powerful tool to determine sparticle interactions.

The thermal relic density of the neutralino dark matter is calculated by the following equation,

$$\Omega_{\tilde{\chi}} h^2 \simeq 1.07 \times 10^9 \frac{x_f \text{GeV}^{-1}}{\sqrt{g_* m_{pl}(a + 3b/x_f)}}$$

(9)

Here $x_f = m/T_F$ and $T_F$ is the temperature where the neutralino decouples from the thermal equilibrium. The parameters $a$ and $b$ are related to the total lightest neutralino pair annihilation cross section at low energy which is expressed by the expansion in the relative velocity $v$, $\langle \sigma v \rangle = a + b v^2$.

$\Omega_{\tilde{\chi}} h^2$ depends on the sparticle mass spectrum. SUSY parameter space is strongly constrained if $\Omega_m \sim \Omega_{\tilde{\chi}} h^2$ is assumed. The dark matter constraint is studied extensively in the MSUGRA. The MSUGRA model is parameterized by scalar mass $m_0$, gaugino mass $M_0$, trilinear coupling $A_0$ and $\tan \beta$ at the GUT scale and SUSY mass spectrum at weak scale can be calculated from these parameters by solving the SUSY renormalization group equations.

The neutralino pair annihilation process is controlled by the following weak scale parameters at the low energy scale.

1. The lightest neutralino and slepton masses $m_{\tilde{\chi}}$, $m_{\tilde{l}}$

   In the minimal supergravity model, $\tilde{l}$ is lighter than $\tilde{q}$, and the LSP is $\tilde{B}$-like unless $m_0 \gg M_0$. Therefore the neutralino pair annihilation into leptons through $t$-channel exchange of a slepton is likely a dominant process. $\Omega_{\tilde{\chi}} h^2 \propto m_{\tilde{l}}^4/m_{\tilde{\chi}}^2$. 

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2. The Higgsino component of the lightest neutralino.

The amplitude of the neutralino pair annihilation through $s$-channel exchange of a Higgs boson is proportional to $N_B^2 N_{B(W)}$. Also, $\sigma(\tilde{\chi}\tilde{\chi} \rightarrow WW)$ depends on the coupling to a $W$ boson which is proportional to $N_B^2$. Therefore a large higgsino component leads to small $\Omega_\chi^{th}$.

3. Higgs bosons with the mass close to $2m_\chi$

If a neutralino pair annihilation hits the $s$-channel pole, the pair annihilation cross section becomes very large. The amplitude $M$ is proportional to $1/(4m_\chi^2 - m_P^2)$ in that case. This happens for large $\tan \beta$ in MSUGRA.

4. Nature of sparticles with the masses close to $m_\chi$

If $m_\chi$ and the next lightest sparticle mass (NLSP) is degenerate, the co-annihilation of the LSP and the NLSP cannot be neglected at the time of the decoupling. The relevant co-annihilation processes discussed in the literature are $\tilde{\chi}\tilde{\chi} \rightarrow VV'$, $\tilde{\chi}\tilde{\tau} \rightarrow \tau\gamma$ and so on. Such co-annihilation processes can be $O(100)$ faster than the pair annihilation process, and reduce the relic density significantly.

The relevant masses and mixings need to be constrained precisely to calculate the thermal relic density of the LSP. If this can be done, we can compare $\Omega_\chi^{th}$ with $\Omega_m$. They need not to be the same, because the calculation of the thermal relic density assumes the neutralino was once in thermal equilibrium and there was no entropy production after the decoupling. For example $\Omega_m < \Omega_\chi^{th}$ if there are late decaying particles. On the other hand, the neutralino dark matter may be produced from the heavy particle (such as gravitino or moduli) decays after the neutralino decoupling [25], in that case, $\Omega_m > \Omega_\chi^{th}$. Finally the LSP dark matter may co-exist with other stable particles such as axion, then $\Omega_m > \Omega_\chi^{th}$.

The potential of the future collider experiments to determine SUSY parameters has been studied extensively. The main motivation is to understand the origin of supersymmetry breaking. The SUSY breaking in the MSSM sector must originate from the SUSY breaking in a “hidden sector”. Sparticle mass spectrum contains information on the SUSY breaking and mediation mechanism. The mechanism may involve the gravitational interaction, geometry of the higher dimensional space or new interactions, namely the physics at much higher than TeV scale. Here we use techniques to determine the SUSY breaking parameters in order to estimate our knowledge about $\Omega_\chi^{th}$ in future.

In the collider experiments, the LSP is not directly visible because it is neutral and stable. The nature of the LSP would be studied by looking into production of heavier sparticles which decays into the LSP.

3.2 LHC SUSY studies

Squarks $\tilde{q}$ and gluinos $\tilde{g}$ are produced in the high energy $pp$ collisions at LHC with large cross sections. They would further decay into jets(+leptons) + LSPs, and LSPs escape from detection. Isolation of events from a single cascade decay chain plays a key role for sparticle mass determinations, because the decay distributions depend on the sparticle masses involved in the decay processes. For example, events with high $p_T$ jets and opposite sign and same flavor leptons may be from the decay cascades $\tilde{q} \rightarrow j\tilde{\chi}_2^0 \rightarrow \tilde{j}l \rightarrow \tilde{\chi}_1^0 jll$. 

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The $m_W$ distribution is expressed as $d \Gamma / dm_W \propto m_W$ as can be seen in Fig. 1. The end points of the $m_W$ distribution is a function of $m_{\tilde{\chi}}$, $m_{\tilde{\chi}^0}$, and $m_t$ and can be measured with an error less than 1 GeV. The measurement of the end point of the $m_{\tilde{\chi}}$ and $m_t$ distribution for a selected jet and the leptons constrain $m_{\tilde{\chi}}$, $m_{\tilde{\chi}^0}$, and $m_t$ [26]. The errors on slepton mass and neutralino mass are found to be 10% in ATLAS TDR study [27] with strongly correlated error. See Fig. 1(right). This corresponds to $\Delta(\sigmav) / \langle(\sigmav) \rangle \sim 20\%$.

LHC also has a sensitivity to the higgsino and gaugino mixing of the LSP. A squark can decay into the heaviest neutralino $\tilde{q}_L \rightarrow \tilde{\chi}^0_4$ followed by $\tilde{\chi}^0_4 \rightarrow \tilde{l}_R$ with $O(5) \text{GeV}$ error. If $M_1 < M_2 < \mu$ as in MSUGRA, the end point gives us information on the $\mu$ parameter, and it strongly constrains the higgsino component of the lightest neutralino [28].

![Figure 1](image-url) The $e^+e^-\mu^+\mu^- - e^+\mu^-\mu^-$ mass distribution for LHC minimal supergravity Point 5 with $\tilde{\chi}^0_2 \rightarrow \tilde{l} \tilde{l} \tilde{\chi}^0_1 t^-$ [27]. Right: Scatter plot of reconstructed values of the $\tilde{l}_R$ and $\tilde{\chi}^0_1$ masses for LHC point 5 (S5) and for a different model (O1) using the decay chain $\tilde{q}_L \rightarrow \tilde{\chi}^0_2 q \rightarrow \tilde{l}_R l q \rightarrow \tilde{\chi}^0_1 l l q$ [29].

It is difficult to access all of the MSSM parameters by the LHC alone. However, if we assume MSUGRA, the parameters $m_0, M_0, A_0$ and $\tan\beta$ are precisely determined. The errors could be around 1% for $m_0$ and $M_0$ from slepton and ino mass measurements. $\tan\beta$ is determined through the $\tilde{\tau}$ mass and Higgs sector. $A_0$ measurement is rather difficult, because it does not affect the low energy spectrum except stop masses. We recently showed that measurement of the decay distribution of $\tilde{g} \rightarrow (\tilde{t}\tilde{t} \text{ or } \tilde{b}\tilde{b}) \rightarrow t\tilde{\chi}^+ \tilde{\chi}^0$ constrain $A_0$ with reasonable accuracy [30]. Within the MSUGRA assumption, the $\Omega_{\tilde{\chi}^0}$ may be determined within a few percent accuracy.

### 3.3 Future $e^+e^-$ linear colliders

Building a LC at $\sqrt{s} = 500$ GeV to 1 TeV has been proposed and the physics at the LC has been studied world-wide. Sparticles with masses lighter than 250 GeV are accessible at the first stage of the LC experiments. The mass reach is significantly lower than that of LHC, where squark and gluino with $m_{\tilde{q}}, m_{\tilde{g}} < 2 \text{ TeV}$ can be discovered. However in the model with the universal gaugino mass at the GUT scale, such as MSUGRA, charginos and neutralinos are significantly lighter than gluino, $M_1 : M_2 : M_3(= m_{\tilde{g}}) = 0.4 : 0.8 : 2.7$. 

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Therefore the accessible parameter space for the LC at $\sqrt{s} \geq 1$ TeV is not significantly smaller compared with that for LHC.

Weakly interacting sparticles are dominantly produced at the LC. The background is less severe compared with LHC. Furthermore the dominant background such as $W^+W^-$ pair production can be controlled by a polarized electron beam. Precise measurements of the masses and cross sections are possible for all kinematically accessible sparticles with typical errors of $O(1)\%$ or less. The mass precisions through threshold scans are estimated typically as

$$\Delta m_{\tilde{\chi}_1^\pm}, \Delta m_{\tilde{\chi}} \sim 0.3\text{GeV},$$
$$\Delta m_{\tilde{f}} \sim \Delta m_{\tilde{\nu}} \sim 0.1\text{GeV},$$
$$\Delta m_{\tilde{t}} \sim 0.6\text{GeV}.$$  \hspace{1cm} (10)

![Figure 2. Contours of $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ (in fb) in the ($\mu, M_2$) plane for fixed $\tan\beta = 4$ and $\sqrt{s} = 500$ GeV. The cross-hatched region is excluded by current bounds, and charginos are kinematically inaccessible in the hatched region [31].](image)

The production cross sections of the sparticles also depend strongly on the MSSM parameters. For example, even if $M_1, M_2 \ll \mu$, and higgsino-like inos are not kinematically accessible at LC, the $\mu$ parameter would be constrained by measuring the the chargino pair production cross sections for a polarized electron beam. In Fig. 2 the $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ is shown in $\mu$ and $M_2$ plane. The cross section shows strong dependence on $\mu/M_2$ ratio. The clean environment at LC is also good to search for sparticles which are mass degenerate with the LSP. Finally combination of LHC and LC data will improve the SUSY parameter study significantly [32].

### 3.4 Gravitino and collider physics

As it was mentioned already, a light gravitino with mass $\sim O(1)$ keV provides an interesting solution for the cusp problem. At collider experiments, such a light gravitino cannot be produced directly, but it arises from the NLSP decay. When the lightest neutralino is the NLSP, the lifetime of the neutralino $c\tau$ is expressed by the gravitino mass $m_{3/2}$.

$$c\tau(\tilde{\chi} \to \psi_{3/2}) \sim 24m \left(\frac{100\text{GeV}}{m_{\chi}}\right)^5 \left(\frac{m_{3/2}}{1\text{keV}}\right)^2.$$  \hspace{1cm} (11)

The expression for the other NLSP sparticles are similar.
LHC is sensitive to the NLSP decays. If the NLSP is a charged particle, the measurements of the track of the charged NLSP improve the SUSY parameter measurement. On the other hand, if the NLSP is neutralino, the decay into $\gamma$ would be found for $c\tau < 1$km. The error of the NLSP life time measurement, or equivalently the error of the gravitino mass at LHC is not yet fully explored. For LC $c\tau$ between $O(10)\mu$m $< c\tau < O(10)$m can be measurement within 10% [33].

4. Outlook

We discussed two directions to study the lightest neutralino SUSY dark matter. Because the SUSY dark matter has significant weak interactions, it can be searched for through $\tilde{\chi}N$ scattering, or through the observation of exotic cosmic rays $\gamma$, $\bar{p}$ or $e^+$ arising from neutralino pair annihilation in the galaxy. Although the search will cover significant MSSM parameter space in coming 10 years, there is large uncertainty in the dark matter signal rate, because the dark matter density profile is not known. It is therefore difficult to interpret the observation, especially when searches appear to be negative.

On the other hand, the same dark matter particle will be produced at future colliders. The interaction of the lightest neutralino will be measured within reasonable errors if both LHC and LC are build. The measurement would be useful to estimate the thermal relic density of the neutralino in the universe, and lower limits on the cross sections relevant to the neutralino dark matter searches. These measurements will provide solid bases to understand the SUSY dark matter, the density profiles in our galaxy, and the thermal history of our universe. The situation is sketched in Fig. 3. The science of the dark matter will proceed through the interplay between particle physics and astrophysics, and it is a promising field even in the 21st century.

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