Radiative Decays of the $X(3872)$ in the Charmonium-Molecule Hybrid Picture

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The $X(3872)$ radiative decay is discussed by employing a two-meson multichannel model with the charmonium components, which explains many of the other observed features of $X(3872)$. We have found that the ratio of the branching fractions of the $X(3872)$ radiative decays, $R_\gamma = \frac{Br(X(3872) \rightarrow \psi(2S)\gamma)}{Br(X(3872) \rightarrow J/\psi\gamma)}$, is 1.1 \(-\) 3.4 if one assumes that the decay occurs only from the charmonium components. The invariant mass spectra of $c\bar{c}(2P) \rightarrow J/\psi\gamma$ and $c\bar{c}(2P) \rightarrow \psi(2S)\gamma$ are also investigated. It is found that an enhancement appears at around $E \sim 3960$ MeV in the $\psi(2S)\gamma$ spectrum but not in the $J/\psi\gamma$ spectrum. This corresponds to the missing $c\bar{c}(2P)$ pole which was predicted by a quark model but has not been observed due to the coupling to the $D\bar{D}$ states. We argue that the fluctuation seen in the experimental $\psi(2S)\gamma$ spectrum reported by LHCb may correspond to this $c\bar{c}(2P)$ pole.

KEYWORDS: $X(3872)$, radiative decay, exotic hadron

1. Introduction

The $X(3872)$ peak has been found first by Belle [1] in the $J/\psi\pi\pi K$ observation from the $B$ decay, which was confirmed by various experiments [2]. The mass of $X(3872)$ is found to be $3871.69 \pm 0.17$ MeV, which is very close to or even corresponds to the $D^0\bar{D}^0$ threshold, $3871.80 \pm 0.12$ MeV, within the experimental errors. The spin parity is found to be $J^{PC}=1^{++}$. Some of this $X(3872)$’s features cannot be understood by a simple $q\bar{q}$ meson picture. For example, the observed total width is $<1.2$ MeV, which is very small for an excited state. A large isospin symmetry breaking is found in the decay: the decay fraction of $X(3872)$ into $\pi^+\pi^- J/\psi$ is comparable to that into $\pi^\pi J/\psi$ as

\[
\frac{Br(X \rightarrow \pi^+\pi^- J/\psi)}{Br(X \rightarrow \pi^\pi J/\psi)} = 1.0 \pm 0.4 \pm 0.3 \quad \text{(Belle)}
\]

\[
= 0.8 \pm 0.3 \quad \text{(BaBar)}. \tag{1}
\]

These two features can be nicely explained by a two-meson model with a charmonium component; i.e., the charmonium-molecule hybrid model [3, 4]. The size of the $c\bar{c}$ component is small, but plays an important role. It produces a short range attraction for the isospin 0 channel, whereas the threshold mass difference prefers the maximal isospin breaking at the long range. It is found that the $X(3872)$ can be a shallow bound state, which is very large in size, or a $S$-wave virtual state.

The absolute value of the $X(3872)$ radiative decay has not been reported yet, but the relative width is reported as

\[
R_\gamma = \frac{Br(X \rightarrow \psi(2S)\gamma)}{Br(X \rightarrow J/\psi\gamma)} = 3.4 \pm 1.4 \quad (3.5\sigma) \quad \text{(BaBar[5])} \tag{2}
\]
Table I. The factor $\langle \psi | r | \psi(c(nP)) \rangle$ calculated by the quark model (QM) as well as the harmonic oscillator wave function with the size parameter $b$ (H.O.) are shown. The decay width $\Gamma$ of $c\bar{c}(nP)$ to $\psi$ by the quark model wave function are also listed with the experimental value in parentheses.

| $\langle J/\psi | r | \psi(1P) \rangle$ | $\langle J/\psi | r | \psi(2P) \rangle$ | $\langle \psi(2S) | r | \psi(1P) \rangle$ | $\langle \psi(2S) | r | \psi(2P) \rangle$ |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| QM [fm]                         | 0.33                            | 0.04                            | -0.41                           | 0.52                            |
| H.O.                            | $\sqrt{\frac{1}{2}}b$          | 0                               | $-b$                            | $\sqrt{\frac{1}{2}}b$          |
| $\Gamma(c\bar{c}(1P) \to J/\psi)$ | $\Gamma(c\bar{c}(2P) \to J/\psi)$ | $\Gamma(c\bar{c}(1P) \to \psi(2S))$ | $\Gamma(c\bar{c}(2P) \to \psi(2S))$ |
| QM [keV]                        | 207 (285)                       | 21                              | -                               | 157                             |

The data from these three experiments seem not to be inconsistent, though BaBar and LHCb prefer a larger value whereas Belle prefers a smaller one. It is important to investigate the above ratio in order to understand the nature of the $X(3872)$, the most well-investigated exotic meson. The theoretical works have been done with an assumption that the $X(3872)$ is a bound state [8–11]. In this work, we study the radiative decays of the $X(3872)$ with the above hybrid model. Our approach here enables us to deal with the $X(3872)$ as a resonance and to produce the mass spectrum of $J/\psi\gamma$ and $\psi(2S)\gamma$.

2. Model

In the present model, the $X(3872)$ consists of $D^0\bar{D}^0$, $D^+D^\ast$, $J/\psi\omega$, $J/\psi\rho$, and the $c\bar{c}(1P)$ ($\chi_{c1}(1P)$) and $c\bar{c}(2P)$ components. The radiative decay can occur from each of these components. Here we assume that the decay occurs only from the $c\bar{c}(nP)$ components and neglect the other ones as a first step [8]. The present results, especially the absolute values, may change when the decay from the two-meson components are included. We, however, consider that the characteristic feature found in the decay energy spectrum will probably remain unchanged if the other decay modes are included.

The the $E1$ transition from the $c\bar{c}$ components in the bound $X(3872)$ to the final $\psi$, which stands for $J/\psi$ or $\psi(2S)$, can be written as

$$
\Gamma(X(3872) \to \psi + \gamma) = \frac{4}{9}Q_c^2 E_\gamma \left| \frac{\omega_\gamma E_\gamma}{M_X} Z_{c(1P)} \langle \psi | r | \psi(1P) \rangle + Z_{c(2P)} \langle \psi | r | \psi(2P) \rangle \right|^2
$$

where $Q_c$ is the electric charge of the charm quark, $\alpha$ is the fine-structure constant, $\omega_\gamma$ and $E_\gamma$ is the energy of the final $\gamma$ and $\psi$, respectively, $M_X$ is the $X(3872)$ mass, $Z_{c(1P)}^2$, $Z_{c(2P)}^2$, is the probability of each $c\bar{c}(nP)$ component in the $X(3872)$.

The transfer matrix element $\langle \psi | r | \psi(c(nP)) \rangle$ is calculated by using the quark model wave function [12]. Their values and the corresponding radiative decay widths are listed in Table I together with the matrix elements evaluated by the harmonic oscillator wave function. The width for the $n = 1$ and $\psi = J/\psi$ case can be calibrated from the observed $\chi_{c1}(1P)$ radiative decay width, which is shown in Table I in parentheses. Note that the matrix element between the $c\bar{c}(2P)$ and $J/\psi$ is very small; it is zero if the harmonic oscillator wave function is employed. This means that the radiative decay to the $J/\psi\gamma$ mode (and also the ratio $R_c$) is sensitive to the size of the $\chi_{c1}(1P)$ component in the $X(3872)$, which is very small. Also, in order to see the $c\bar{c}(2P)$, one has to look into the final $\psi(2S)\gamma$ decay mode or into the difference between the final $\psi(2S)\gamma$ and $J/\psi\gamma$ decay modes.

The relative phase of the $c\bar{c}(1P)$ and the $c\bar{c}(2P)$ components in the $X(3872)$ wave function has not been determined yet. We calculated three extreme cases: there is no $c\bar{c}(1P)$ component (case A00), the decay from the two charmonia is constructive (case A01), and destructive (case A10). All the
Table II. The probability of the $c\bar{c}$ components in the $X(3872)$, the radiative decay width $\Gamma$ in keV, and the ratio $R_\gamma$ are shown for each parameter set, A00, A01, and A10.

| model | $g_{c\bar{c}(1P)-D\bar{D}}$ | $Z_{c\bar{c}(1P)}^2$ | $Z_{c\bar{c}(2P)}^2$ | $\Gamma(X \rightarrow J/\psi)$ | $\Gamma(X \rightarrow \psi(2S))$ | $R_\gamma$ | $R_\gamma$ (spectrum) |
|-------|-----------------------------|-----------------|-----------------|----------------------------|----------------------------|---------|-------------------|
| A00   | 0                           | 0               | 0.036           | 0.6                       | 2.1                       | 3.6     | 3.4               |
| A01   | $\frac{1}{10}g_{c\bar{c}(2P)-D\bar{D}}$ | 0.001           | 0.036           | 1.1                       | 2.0                       | 1.8     | 1.9               |
| A10   | $-g_{c\bar{c}(2P)-D\bar{D}}$ | 0.011           | 0.060           | 6.1                       | 6.2                       | 1.0     | 1.1               |

The obtained widths are $\Gamma(X(3872) \rightarrow J/\psi) = 0.6\text{-}6.1$ keV and $\Gamma(X(3872) \rightarrow \psi(2S)) = 2.1\text{-}6.2$ keV when we assume that the $X(3872)$ is a bound state and use eq. (3). The ratio, $R_\gamma$, becomes 2.1-6.2, which is close to the BaBar and LHCb results. This, however, does not mean that our results exclude the Belle value because the ambiguity due to the unknown relative phase of the two $c\bar{c}$ components is very large and because the results may change when we introduce the radiative decay from the two-meson components.

The $J/\psi\gamma$ and the $\psi(2S)\gamma$ mass spectra are shown in Figure 1. Both of the final $J/\psi\gamma$ and $\psi(2S)\gamma$ decay have a peak at the energy which corresponds to the $X(3872)$ mass. We calculated the radiative decay width of the $X(3872)$ by integrating the strength of this peak up to the $D^*\bar{D}^*$ threshold, 3879.87 MeV. The ratio $R_\gamma$ is listed in Table II under the entry $R_\gamma$ (spectrum). The results are not very different from those with the bound state approach. In addition to the peak at the $X(3872)$ mass, there is an enhancement in the $\psi(2S)\gamma$ mass spectrum at around 3500-4000 MeV. This enhancement occurs because of the $c\bar{c}(2P)$ pole, which exists at $3959 - \frac{1}{2}72$ MeV for the A00 and A01 parameter sets while it moves to $3969 - \frac{1}{2}40$ MeV for the A10 parameter set. Since the $c\bar{c}(2P)$ state decays only weakly
to $J/\psi\gamma$, as seen in Table I, such a structure is not seen in the $J/\psi\gamma$ mass spectrum. This enhancement is considered to survive if we introduce the decay from the two-meson channels because the effects of the $c\bar{c}(2P)$ pole on the two-meson channels are an indirect one: $c\bar{c}(2P) \rightarrow DD^* \rightarrow \psi\rho \rightarrow \psi\gamma$; the $c\bar{c}(2P)$ contributes also to the final $J/\psi\gamma$ mode, but their peak structure will be much weaker. The existence of this $c\bar{c}(2P)$ pole has been predicted by quark models but has not been observed because it strongly couples to the open charm channels. We would like to point out that the fluctuation at around 3500-4000 MeV in the final $\psi(2S)\gamma$ mass spectrum (but not seen in the final $J/\psi\gamma$ mass spectrum) measured by LHCb [7] may correspond to this enhancement. If that is the case, it will give us a novel method to reveal heavy quarkonia embedded in the continuum.

To evaluate the radiative decay of the $X(3872)$, it is necessary to look into the decay from the two-meson components in addition to that from the $c\bar{c}$ components. Since the relative phase between $c\bar{c}(1P)$ and $c\bar{c}(2P)$ affects the results largely, this should be determined by considering a more fundamental theory. Also, the consistent picture among many exotic mesons should be constructed; looking into the radiative decay such as $Y(4260) \rightarrow X(3872)\gamma$ may help to understand the exotic mesons. It should also be interesting to know how the situation changes if the charm quark is replaced by the bottom quark.

![Fig. 1. The $c\bar{c}(2P) \rightarrow J/\psi\gamma$ and the $c\bar{c}(2P) \rightarrow \psi(2S)\gamma$ mass spectra calculated by the present work are shown. Figure (a) corresponds to the parameter set A00, (b) to A01, and (c) to A10.](image-url)

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