Abstract—Recently, the \(k\)-induction algorithm has proven to be a successful approach for both finding bugs and proving correctness in program verification. However, since the algorithm is an incremental approach, it might waste resources trying to prove incorrect programs. In this paper, we suggest an extension to the \(k\)-induction algorithm, which uses the counterexample produced from over-approximating the loops occurring in the program, in order to shorten the number of steps required to find bugs. We show that our approach can substantially reduce the number of steps to find the counterexample.

I. INTRODUCTION

Embedded systems are used in a variety of applications, ranging from nuclear plants and automotive systems to entertainment and games [1]. This ubiquity drives a need to test and validate a system before releasing it to the market, in order to protect against system failures. Even subtle system bugs can have drastic consequences, such as the recent Heartbleed bug on OpenSSH, which might have leaked private information from several servers [2].

One promising technique to validate a system is called bounded model checking (BMC) [3]. The basic idea of BMC is to check the negation of a property at a given depth: given a transition system \(M\), a property \(\phi\), and a bound \(k\), BMC unrolls the system \(k\) times and generates verification conditions (VC) \(\psi\), such that \(\psi\) is satisfiable if and only if \(\phi\) has a counterexample of depth \(k\) or less. BMC tools based on Boolean Satisfiability (SAT) or Satisfiability Module Theories (SMT) have been applied on the verification of both sequential and parallel programs [4], [5], [6]. However, BMC tools are aimed to find bugs; they cannot prove correctness, unless the bound \(k\) safely reaches all program states.

Despite the fact that BMC cannot prove correctness by itself, there are algorithms that use BMC as a “component” to prove correctness. In particular, the \(k\)-induction algorithm is an incremental BMC algorithm that aims to find bugs and prove correctness using an ever increasing number of unwindings. In this paper, we propose an extension to the \(k\)-induction algorithm; the main original contributions of this paper are:

- We describe an improved \(k\)-induction algorithm, and the result of the application of the technique in a number of public available benchmarks. Our improved \(k\)-induction, implemented in ESBMC [5], outperforms all others tools that use \(k\)-induction to verify programs (Section [II]).
- We propose a novel extension to the \(k\)-induction algorithm, that use the information from the counterexample generated by the inductive step, which is an overapproximation of the loops occurring in the program. The proposed extension changes the original program, during the verification, in such way that the number of iterations required by the base case to find a bug is potentially cut in half (Section [III]).

II. THE \(k\)-INDUCTION ALGORITHM

The first version of the \(k\)-induction algorithm was proposed by Niklas Eén [7], which applies BMC to find bugs and prove correctness. Unless the bound \(k\) is appropriate to reach the completeness threshold (i.e., a value that will fully unroll all loops occurring in the program), which is often impractically large [8], BMC tools cannot prove correctness. For instance, consider the simple program shown in Figure 1 the loop in line 2 runs an unknown number of times, depending on the initial nondeterministic value of \(x\), however, the assertion in line 3 always hold. BMC tools as CBMC [4], ESBMC [5] or LLBMC [9] typically fail to verify programs such class of programs. Soundness requires the insertion of the unwinding assertion, after each loop occurring in the program, as shown in Figure 2 line 5. The verification using a BMC tool of this program will fail the unwinding assertion if \(k\) is too small (i.e., \(k < 2^{12}\) in 32-bit and 64-bit architectures).

```c
unsigned int x;
while (x>0) x--;
assert (x==0);
```

Fig. 1. Simple unbounded loop program.

In mathematics, one usually approaches such unbounded problems using proof by induction. The \(k\)-induction variant has been successfully combined with continuously-refined invariants [10], was used to prove that C programs do not contain data races [11], or that design time constraints are respected [7]. The \(k\)-induction is an well-established technique in hardware verification, where it is applied due to the monolithic transition relation present in hardware designs [12], [13].
We implement an improved version of the k-induction algorithm [14] that consists of three steps, and builds on top of an incremental BMC [15], to iteratively increase the depth of loop unrollings while checking for either a property violation or correctness.

The three steps of the algorithm are the base case, the forward condition and the inductive step. For each value of k, our k-induction algorithm can be formulated as:

\[ -B(k) \rightarrow \text{program contains bug} \]
\[ B(k) \land F(k) \rightarrow \text{program is correct} \]
\[ B(k) \land I(k) \rightarrow \text{program is correct} \]

where \( B(k) \) is the base case, \( F(k) \) is the forward condition and \( I(k) \) is the inductive step; each function is a BMC call and returns false if a property violation is reachable in k steps.

If the base case finds a property violation, then it is a real bug, which is reported by presenting the set of assignments (the counterexample) that leads to the property violation. If no bug is found by the base case, then the other steps aim to prove correctness. The forward condition checks if the current k is the completeness threshold [8], while the inductive step checks if all properties hold inductively; any other result will lead to an increment of k and the algorithm performs the checks again.

We evaluated our improved k-induction, implemented in ESBMC [5], against both CPAChecker [16] and 2LS [17] on the 2017 International Competition on Software Verification (SV-COMP) [18], both tools competed using the k-induction algorithm. ESBMC scored 4335 in 120000 seconds, more than both CPAChecker and 2LS, that scored 1963 in 82000 seconds and -1204 in 93000 seconds, respectively. The results are publicly available at https://sv-comp.sosy-lab.org/2017/results/results-verified/.

### III. Extending the k-induction Algorithm

One of the limitations of the k-induction algorithm is that resources can be wasted. For instance, when verifying a correct program, the base case will be executed at least once, as the step is required for soundness. When verifying programs that contain infinite loops, the unwinding assertion added by the forward condition will always fail (e.g., an infinite loop can never be fully unrolled) and the inductive step will always find a property violation when verifying programs with bugs.

The inductive step is the most computationally expensive of all the k-induction algorithm steps; it is an overapproximation, forcing the SMT solver to find a set of assignments in a greater state space than the original program [14]. Currently our k-induction algorithm ignores the counterexample generated by each step and only reasons about the verification result, even if the property violation is reachable for greater values of k.

```c
unsigned int x;
if (x>0)
    x--; // k copies
...
assert (! (x>0)); // unwinding assertion
assert (x==0);
```

![Fig. 2. Finite k unw windings done by BMC.](https://example.com/fig2.png)

```
int main(void)
{
    unsigned int a = 1;
    while (1)
    {
        if (a == 6) assert (0);
        a = a + 1;
    }
}
```

![Fig. 3. Simple counter that fails when it reaches 6.](https://example.com/fig3.png)

![Fig. 4. CFG of the Program in Figure 3.](https://example.com/fig4.png)

Consider the program shown in Figure 3 and its control-flow graph (CFG) representation in Figure 4. The CFG is a directed graph that represents an state transition system. In a CFG, an state \( s \in S \) is a tuple \( \langle pc, v_0, \ldots, v_n \rangle \), where \( pc \) is the program counter and \( v_0, \ldots, v_n \in V \) are the values of all program variables. A transition \( t \in T \) is a guarded assignment \( \langle \gamma, x := e \rangle \), where \( \gamma \) is a predicate over the program variables and \( e \) is an expression assigned to \( x \). The guard may be omitted if it is true and the assignment may be omitted if the edge is a conditional jump. Also, \( \xi \) is the set of error states, where an error state \( \epsilon \in \xi \) represents a bug in a program. We define a path \( \pi = \langle s_0, s_1, \ldots, s_k \rangle \) of length k as a sequence of states between two states \( s \in S \). A counterexample is a path from an initial state \( s \in S \) to one error state \( \epsilon \in \xi \).

In our running example, when applying the k-induction algorithm, it requires 6 iterations to reach the assertion failure. This means that the base case will be called 6 times (\( k = \lfloor 1.6 \rfloor \)), and the forward condition and the inductive step will be called 5 times each (\( k = \lfloor 1.5 \rfloor \)). The base case will produce the following counterexample for \( k = 6 \):

- LINE 3: a = 1
- LINE 7: a = 2
- LINE 7: a = 3
- LINE 7: a = 4
- LINE 7: a = 5
- LINE 7: a = 6
- LINE 6: assertion 0
which is a set of assignments, a path in the CFG that leads to a property violation, i.e., an assertion failure. Now consider the counterexample generated by the inductive step for $k = 1$

\begin{align*}
\text{LINE 7: } & a = 6 \\
\text{LINE 6: } & \text{assertion 0}
\end{align*}

$k = 2$

\begin{align*}
\text{LINE 7: } & a = 5 \\
\text{LINE 7: } & a = 6 \\
\text{LINE 6: } & \text{assertion 0}
\end{align*}

and $k = 3$

\begin{align*}
\text{LINE 7: } & a = 4 \\
\text{LINE 7: } & a = 5 \\
\text{LINE 7: } & a = 6 \\
\text{LINE 6: } & \text{assertion 0}
\end{align*}

For $k = 1$, the property violation is reachable when $a == 6$. In this case, the inductive step can be interpreted as the following: “is there any path of size 1 that reaches an error state?”. Furthermore, each $k$ increment extends the set of assignments back in the path to the initial state, e.g., $k = 2$ can be interpreted as: “is there any path of size 2 that reaches an error state?”, and so forth.

The proposed extension will change the program by appending assertions to the loop body, which will be checked by the base case. The assertion will verify whether the first state on the path to the error state is reachable; since an state is a set of values for all program variables, the condition asserted is represented by a conjunction of inequalities in the form

$$
\text{assert}(v_0 \neq v_{c0} \land v_1 \neq v_{c1} \land \ldots \land v_n \neq v_{cn}),
$$

where $v_0 \ldots v_n$ are the values of all variables, $v_{c0} \ldots v_{cn}$ are the values of all variables extracted from the inductive step counterexample and $n$ is the number of variables.

\begin{figure}[ht]
\centering
\includegraphics[width=0.5\textwidth]{unrolledCFG.png}
\caption{Unrolled CFG from Figure 4 and the “direction” of the verification for the base case and the inductive step.}
\end{figure}

Figure 6 shows the modified program from Figure 3 based on the counterexample from the inductive step, for $k = 2$; the program contains one variable so our extension only asserts one inequality. For $k = 2$, the first state reachable in the path to the error state is $a == 5$, as previously shown by the counterexample. For $k = 3$, the first reachable state is $a == 4$ and the program will be changed accordingly during the verification.

Essentially, this extension is a meet-in-the-middle bidirectional algorithm [19] to find bugs, with the base case starting from the initial state towards an error state and the inductive step starting from an error state towards the initial state. Given a path $\pi = (s_0, s_1, \ldots, \epsilon)$ of length $k$, our proposed extension requires $\lceil \frac{k}{2} \rceil + 1$ steps to find the counterexample, instead of $k$ steps required by the plain version of the $k$-induction algorithm.

IV. WORK IN PROGRESS

We are currently implementing the idea in our BMC tool, ESBMC [5]. The prototype extension is in early stages but it is already able to parse the counterexample generated by the inductive step. The next major steps are to append the assertion to the loop body during the base case verification, effectively implementing the idea, and evaluate the extension against current $k$-induction tools.

However, for the technique to be truly useful, we shall implement a counterexample cache. The cache will hold the counterexample from the inductive step and will merge with counterexample from the base case, otherwise the latter without the former will be incomplete.

We are also using DepthK [20], [21] to generate loop invariants, before applying our method. That way we avoid adding assertions from spurious counterexamples that will only make the verification slower.
Abstraction is the most important technique to handle the state explosion problem in verification [22]. However, abstraction is an overapproximation and might result in spurious counterexamples that need to be refined; algorithms as CEGAR (Counterexample-Guided Abstraction Refinement) check the feasibility of the counterexample and add constraints to the abstract model in the case of the counterexample being spurious. Here, we describe related approaches that use the counterexample-guided techniques to make BMC faster.

Gupta et al. [23] describe a counterexample-guided abstraction refinement technique for SAT-based BMC; the authors describe an algorithm that uses CEGAR and PBR (Proof-Based Refinement). Their algorithm is an incremental verification process in which a model checker runs until it finds a counterexample in the abstract model, that is checked against the concrete model using BMC. If the counterexample is spurious, it is added back to the abstract model as constraints. The PBR version of the algorithm eliminates all counterexamples of a given length in a single refinement step, while the CEGAR version only removes one at a time. The authors evaluated the CEGAR version against a number of benchmarks and the new algorithm presented faster results when compared to plain BMC. No results were presented for the PBR version of the algorithm.

Bjesse et al. [24] describe a method to guide a SAT-based BMC with information gathered from their counterexample-guided abstraction refinement technique, in synthesized industrial circuits from Verilog descriptions and environment constraints. Their algorithm uses Binary Decision Diagrams (BDDs) representing the shells generated backwards as intermediate targets, that can be traversed both forward and backward. Their results show that the approach can outperform stand-alone BMC and other abstraction information guided techniques and was able to find undiscovered failures.

Compared to these works, our proposal shares a common goal: to not waste information generated from an overapproximation step, whether it came from induction, in our case, or from BDDs or abstraction in theirs. The technique potentially leads to an improved BMC search for property violation and their results is evidence that we are in the proper direction.

VI. CONCLUSION

In this paper, our main contribution is a novel extension to the k-induction algorithm, to perform a meet-in-the-middle bidirectional counterexample search. The extension is currently under development in ESBMBC. We plan to evaluate the improvement over the SV-COMP benchmarks, where our plain k-induction algorithm already proved to be the state-of-art [25], if compared to other k-induction tools [25].

Our extension to the k-induction algorithm aims to shorten the number of steps required to find a bug in half, by using the information in the counterexample generated from an overapproximation, the inductive step. We expect our approach to be specially effective in reactive systems [26], e.g., event-condition-action systems.

REFERENCES

[1] S. Heath, Embedded Systems Design. Oxford, United Kingdom: Newnes, 2003.
[2] Z. Durumeric, J. Kasten, D. Adrian, J. A. Halderman, M. Bailey, F. Li, N. Weaver, J. Amann, J. Beekman, M. Payer, and V. Paxson, “The matter of heartbleed,” in Proceedings of the 2014 Conference on Internet Measurement Conference, ser. IMC ’14, 2014, pp. 475–488.
[3] A. Biere, A. Cimatti, E. Clarke, and Y. Zhu, “Symbolic model checking without BDDs,” in TACAS. Springer-Verlag, 1999, pp. 193–207.
[4] E. Clarke, D. Kroening, and F. Lerda, “A tool for checking ANSI-C programs,” in TACAS, ser. LNCS, vol. 2988. Springer, 2004, pp. 168–176.
[5] L. C. Cordeiro, B. Fischer, and J. Marques-Silva, “SMT-based bounded model checking for embedded ANSI-C software,” IEEE Transactions on Software Engineering, vol. 38, no. 4, pp. 957–974, 2012.
[6] S. Qadeer and J. Rehof, “Context-bounded model checking of concurrent software,” in Proceedings of Tools and Algorithms for the Construction and Analysis of Systems. Redmond, EUA: Microsoft Research, 2005, pp. 93–107.
[7] N. Eén and N. Sörensson, “Temporal induction by incremental SAT solving,” Electronic Notes in Theoretical Computer Science, vol. 89, no. 4, pp. 543–560, 2003.
[8] D. Kroening, J. Ouaknine, O. Strichman, T. Wahl, and J. Worrell, “Linear completeness thresholds for bounded model checking,” in CAV, 2011, pp. 557–572.
[9] F. Merz, S. Falke, and C. Sinz, “LLBMC: Bounded model checking of C and C++ programs using a compiler IR,” in VSTTE, ser. LNCS, vol. 7152. Springer-Verlag, 2012, pp. 146–161.
[10] D. Beyer, M. Dangel, and P. Wendler, Combining k-Induction with Continuously-Refined Invariants. Springer International Publishing, 2015, pp. 622–640.
[11] A. Donaldson, D. Kroening, and P. Rümmer. “SCRATCH: a tool for automatic analysis of DMA races,” in PPoPP, 2011, pp. 311–312.
[12] D. Große, H. Le, and R. Drechsler, “Induction-based formal verification of SystemC TLM designs,” in MTV, 2009, pp. 101–106.
[13] M. Sheeran, S. Singh, and G. Stålmarck, “Checking safety properties using induction and a SAT-solver,” in FMCAD, 2000, pp. 108–125.
[14] M. Y. R. Gadelha, H. I. Ismail, and L. C. Cordeiro, “Handling loops in bounded model checking of C programs via k-induction,” STTT, vol. 19, no. 1, pp. 97–114, 2017.
[15] H. Günther and G. Weissbacher, “Incremental bounded model checking,” in Proceedings of the 2014 International SPIN Symposium on Model Checking of Software, ser. SPIN, 2014, pp. 40–47.
[16] D. Beyer and M. E. Keremoglu, “CPChecker: A tool for configurable software verification,” in CAV, ser. LNCS, vol. 6806, 2011, pp. 184–190.
[17] P. Schrammel, D. Kroening, M. Brain, R. Martins, T. Teige, and T. Biemmüller, “Incremental bounded model checking for embedded software (extended version),” CoRR, vol. abs/1409.5872, 2014.
[18] D. Beyer, “Software verification with validation of results (report on SV-COMP 2017),” in TACAS, ser. LNCS, vol. 10206, 2017, pp. 331–349.
[19] R. C. Holte, A. Felner, G. Sharon, and N. R. Sturtevant, “Bidirectional search that is guaranteed to meet in the middle,” in AAAI, 2016, pp. 3411–3417.
[20] W. Rocha, H. Rocha, H. Ismail, L. C. Cordeiro, and B. Fischer, “Depth: A k-induction verifier based on invariant inference for C programs - (competition contribution),” in TACAS, 2017, pp. 360–364.
[21] H. Rocha, H. Ismail, L. C. Cordeiro, and R. S. Barreto, “Model checking embedded C software using k-induction and invariants,” in SBESC, 2015, pp. 90–95.
[22] E. Clarke, O. Grumberg, S. Jha, Y. Lu, and H. Veith, Counterexample-Guided Abstraction Refinement, 2000, pp. 154–169.
[23] A. Gupta and O. Strichman, “Abstraction refinement for bounded model checking,” in CAV, ser. LNCS, vol. 3576, 2005, pp. 112–124.
[24] P. Bjesse and J. Kukula, “Using counter example guided abstraction refinement to find complex bugs,” in DATE, vol. 1, 2004, pp. 156–161.
[25] D. Beyer, “Software verification with validation of results (report on sv-comp 2017),” in TACAS, ser. LNCS, vol. 10206, 2017, pp. 331–349.
[26] L. Aceto, A. Ingólfsdóttir, K. G. Larsen, and J. Srba, Reactive Systems: Modelling, Specification and Verification. New York, NY, USA: Cambridge University Press, 2007.