P-VORTICES, NEXUSES AND EFFECTS OF GRIBOV COPIES IN THE CENTER GAUGES

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We perform the careful study of the Gribov copies problem in $SU(2)$ lattice gauge theory for maximal direct and maximal indirect center projections. We find that this problem is much more severe than it was thought before. The projected string tension is not in agreement with the physical string tension. We also show that the particle–like objects, nexuses, might be important for the confinement dynamics.

1 Introduction

The old idea about the role of the center vortices in confinement phenomena has been revived recently with the use of lattice regularization. Both gauge invariant and gauge dependent approaches are developed. The gauge dependent studies were done in several center gauges. The center gauge leaves intact center group local gauge invariance. It is believed that gauge dependent P-vortices defined on the lattice plaquettes are able to locate thick gauge invariant center vortices and thus provide the essential evidence for the center vortex picture of confinement. So far 3 different center gauges have been used in practical computations: the indirect maximal center (IMC) gauge, the direct maximal center (DMC) gauge, and the Laplacian center gauge. The first two suffer from Gribov copies problem. Many results supporting the important role of P-vortices are obtained in these two gauges but the problem of Gribov copies effects, we are addressing here, has not been studied properly. We also investigate properties of recently introduced new objects called nexuses or center monopoles. One can define nexus in $SU(N)$ gauge theory as a particle-like object formed by $N$ center vortices meeting at the center, with the zero (mod $N$) net flux. We use P–vortices in the center projection to define nexuses in $SU(2)$ lattice gauge theory.

In we have found that in DMC gauge the projected string tension is much lower than the physical one which contradicts the earlier claims. We also confirmed the observation made in: there are gauge copies which correspond

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to higher maxima of the gauge fixing functional $F$ (see below for definition) than usually obtained and at the same time these new gauge copies produce P-vortices evidently with no center vortex finding ability. These results are discussed in section 2. In section 3 we present our results for IMC gauge. Results of sections 2 and 3 were obtained with the relaxation - overrelaxation (RO) algorithm. In section 4 we present our preliminary results obtained with more effective algorithm, simulated annealing (SA). We show that the use of this algorithm permits to obtain higher maxima and thus to solve the puzzle imposed in [10]. Moreover SA algorithm gives the lowest value of the string tension. We also discuss the finite volume effects [11] for the projected string tension.

2 Direct maximal center gauge

The DMC gauge is defined by the maximization of the following functional:

$$\mathcal{F}(U) = \frac{1}{4V} \sum_{n,\mu} \left( \frac{1}{2} \text{Tr} U_{n,\mu} \right)^2 = \frac{1}{4V} \sum_{n,\mu} \frac{1}{4} (\text{Tr}_{adj} U_{n,\mu} + 1),$$

with respect to local gauge transformations, $U_{n,\mu}$ is the lattice gauge field, $V$ is the lattice volume. DMC gauge condition fixes the gauge up to $Z(2)$ gauge transformation. The fixed configuration can be decomposed into $Z(2)$ and coset parts: $U_{n,\mu} = Z_{n,\mu} V_{n,\mu}$, where $Z_{n,\mu} = \text{sign} \text{Tr} U_{n,\mu}$. Plaquettes constructed from $Z_{n,\mu}$ field have values $\pm 1$. Those of them taking values $-1$ compose the so called P-vortices. P-vortices form closed surfaces in 4D space. Some evidence has been collected, that P-vortices in the center gauges can serve to locate gauge invariant center vortices. In ref. [4] the projected Wilson loops, $W_{Z(2)}$, are computed via linking number of the static quarks trajectories and P-vortices. It was found that the string tension, $\sigma_{Z(2)}$, obtained from $W_{Z(2)}$ is very close to the physical string tension $\sigma_{SU(2)}$. This fact has been called center dominance. Another important observation was that the density of P-vortices scales in agreement with asymptotic scaling [4, 12]. We inspect these statements using careful gauge fixing procedure.

The problem of the DMC gauge fixing is that the functional $\mathcal{F}(U)$ has many local maxima. This is the analogue of the Gribov problem in continuum gauge theories [4]. We call configurations corresponding to different local maxima Gribov copies. To perform unambiguous computations one must fix the gauge completely, i.e. to find the global maxima [14]. It is impossible to do it numerically, and we generate a large number $N_{cop}$ of local maxima, calculate observables using configuration corresponding to the highest maximum and extrapolate results to $N_{cop} \to \infty$ limit. The local maxima are produced by applying the RO algorithm to random gauge copies of the original configuration.
Table 1: The comparison of $\sigma_{Z(2)}$, $\sigma_{SU(2)}$ and $\rho$ for DMC gauge, RO gauge fixing procedure.

| $N_{\text{cop}}$ | $\frac{\sigma_{Z(2)}}{\sigma_{SU(2)}}$ | $\frac{2\rho}{\sigma_{SU(2)}a^2}$ |
|------------------|--------------------------------------|----------------------------------|
|                  | $\beta = 2.3$                       | $\beta = 2.4$                   | $\beta = 2.5$                     | $\beta = 2.3$ | $\beta = 2.4$ | $\beta = 2.5$ |
| 3                | 0.94(2)                             | 0.93(2)                         | 0.98(2)                          | 1.30(1)       | 1.51(1)       | 1.74(1)       |
| 20               | 0.87(2)                             | 0.80(2)                         | 0.83(3)                          | 1.27(1)       | 1.42(1)       | 1.61(2)       |
| $\infty$        | 0.82(3)                             | 0.71(3)                         | 0.71(3)                          | 1.24(1)       | 1.33(2)       | 1.49(2)       |

Our computations have been performed on lattices $L^4 = 12^4$ at $\beta = 2.3, 2.4$ and $L^4 = 16^4$ at $\beta = 2.5$. At $\beta = 2.3, 2.4$ ($\beta = 2.5$) we use 100 (50) statistically independent gauge field configurations. In Table 1 we show the ratio of string tensions $\frac{\sigma_{Z(2)}}{\sigma_{SU(2)}}$. $\sigma_{Z(2)}$ is computed from the Creutz ratio $\chi(I)$: $3 \leq I \leq 4$ on $12^4$ lattice, and $3 \leq I \leq 6$ on $16^4$ lattice. For $N_{\text{cop}} = 3$ (the number of gauge copies used in $\chi(I)$) $\sigma_{Z(2)}$ is close to $\sigma_{SU(2)}$. But it becomes significantly lower for large $N_{\text{cop}}$. Thus RO gauge fixing gives the strong dependence of $\sigma_{Z(2)}$ on $N_{\text{cop}}$. In the limit $N_{\text{cop}} \to \infty$ $\sigma_{Z(2)}$ is 20-30% lower than $\sigma_{SU(2)}$.

In Table 1 we also show the ratio $\frac{2\rho}{\sigma_{SU(2)}a^2}$ ($\rho$ is the density of P-vortices). As it is claimed in ref. 12, $2\rho$ coincides with the dimensionless string tension, $\sigma_{SU(2)}a^2$ if plaquettes carrying P-vortices are uncorrelated. Our results in Table 1 show that the density of P-vortices does not satisfy this relation. We have found that for $N_{\text{cop}} = 3$ $\rho$ is in a good agreement with asymptotic scaling, as it was observed before. But for $N_{\text{cop}} \to \infty$ $\rho$ deviates from the two loop asymptotic scaling formula and its dependence on $\beta$ becomes similar to that of $\sigma_{SU(2)}a^2$.

We also performed computations using the modified gauge fixing procedure (LRO) suggested in ref. 10: every copy has been first fixed to Landau gauge, and then the RO algorithm for DMC gauge has been applied. With this procedure we found the local maximum higher than local maxima of RO procedure for any $N_{\text{cop}} \in [1; 20]$ and in the limit $N_{\text{cop}} \to \infty$ (see Table 2). We confirm that RO and LRO gauge fixing procedures generate two classes of gauge equivalent copies with different properties: $\sigma_{Z(2)}$ is zero for local maxima produced with LRO and nonzero for those produced with RO. The density of P-vortices is essentially lower for LRO. Then the idea of the complete gauge fixing based on the choice of the global maximum would force us to choose the LRO local maxima. Thus the vortex finding property is completely lost. In chapter 4 we demonstrate that SA algorithm solves this problem.

\footnote{The data for $\sigma_{SU(2)}$ are taken from ref. 13}
Table 2: $< F_{\text{max}} >$ for DMC gauge obtained with various algorithms

| Algorithm | $\beta = 2.3$ | $\beta = 2.4$ | $\beta = 2.5$ |
|-----------|--------------|--------------|--------------|
| RO($N_{\text{cop}} = \infty$) | 0.7552(1) | 0.7764(2) | 0.7943(2) |
| LRO($N_{\text{cop}} = \infty$) | 0.7564(1) | 0.7759(3) | 0.7955(2) |
| SA($N_{\text{cop}} = 3$) | 0.7588(1) | 0.7770(2) | 0.7970(2) |

2.1 Nexuses.

We also investigate the properties of the point like objects, called nexuses. On the 4D lattice we have the conserved currents of nexuses, defined after the center projection. First we calculate the phase, $s_l$, of the $Z(2)$ link variable: $Z_l = \exp(i\pi s_l)$, $s_l = 0, 1$. Then we define the plaquette variable $\sigma_P = ds \mod 2$, $(\sigma_P = 0, 1)$. The nexus current (or center monopole current) is then defined as $*j = \frac{1}{2} 5^*\sigma_P$. These currents live on the surface of the P-vortex (on the dual 4D lattice) and P-vortex flux goes through positive and negative nexuses in alternate order. The important characteristic of the cluster of currents is the condensate, $C$, defined as the percolation probability. As it is shown in ref. the condensate $C$ of the nexus currents is the order parameter for the confinement – deconfinement phase transition. We found that $C$ is nonzero for the gauge copies obtained via RO procedure (when the projected Wilson loops have the area law). On the other hand $C$ is zero (in the thermodynamic limit $L \to \infty$) for gauge copies obtained using LRO procedure (when the projected Wilson loops have no area law). It is interesting that for RO procedure $C$ seems to scale as the physical quantity with the dimension $(mass)^4$. This is illustrated in Fig.1, where we plot the $\beta$-dependence of the ratio $C/(\sigma_{SU(2)}a^2)^2$. Thus these new objects might be important degrees of freedom for the description of the nonperturbative effects.

3 Indirect maximal center gauge

The gauge fixing condition for IMC gauge consists of two steps:

Maximal abelian gauge ($SU(2) \to U(1)$) is fixed by solving maximization problem:

$$\max_{\{g\}} \{ F_1(U^g) \}, \quad g \in SU(2)/U(1); \quad F_1(U) = \frac{1}{8V} \sum_{n,\mu} \text{Tr} (U_{n,\mu} \sigma_3 U_{n,\mu}^1 \sigma_3)$$

and $U(1)$ field is extracted: $U_{n,\mu} = V_{n,\mu} u_{n,\mu}$, $u_{n,\mu} \in U(1)$. 

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Then $Z(2)$ gauge is fixed by maximizing

$$\max_{\{\omega\}} \{F_2(u^\omega)\}; \quad \omega \in U(1)/Z(2), \quad F_2(u) = \frac{1}{4V} \sum_{n,\mu} (\text{Re}u_{n,\mu})^2.$$ 

The $Z(2)$ field is defined as: $u_{n,\mu} = Z_{n,\mu} v_{n,\mu}$.

In this section we discuss our results obtained with two gauge fixing procedures:

- RO procedure: RO algorithm at both steps;
- LRO procedure: LRO algorithm at the 1st step and RO algorithm at the 2nd one;

Our results for the ratios $\sigma_{Z(2)}/\sigma_{SU(2)}$ and $2\rho/\sigma_{SU(2)}a^2$ obtained with RO procedure are presented in the Table 3. These results are in good qualitative and even quantitative agreement with the results obtained in DMC gauge. The LRO procedure (as in DMC gauge case) corresponds to $\sigma_{Z(2)} = 0$. Another important observation is that at the first stage (MA gauge) LRO procedure local maxima average $<F_{1,max}>$ is higher than those of RO procedure (see Table 4). Thus the problem discussed in the previous section is also relevant for IMC gauge.
Table 3: The comparison of $\sigma_{Z(2)}$, $\sigma_{SU(2)}$ and $\rho$ for IMC gauge, RO gauge fixing procedure.

| $N_{cop}$ | $\beta = 2.3$ | $\beta = 2.4$ | $\beta = 2.5$ | $\beta = 2.3$ | $\beta = 2.4$ | $\beta = 2.5$ |
|-----------|---------------|---------------|---------------|---------------|---------------|---------------|
| (1,1)     | 0.89(3)       | 0.81(2)       | 0.86(2)       | 1.44(1)       | 1.69(1)       | 1.97(2)       |
| (20,10)   | 0.81(4)       | 0.69(3)       | 0.72(3)       | 1.40(1)       | 1.63(1)       | 1.83(2)       |
| ($\infty$,\$\infty$) | 0.81(4)       | 0.69(3)       | 0.72(3)       | 1.40(1)       | 1.63(1)       | 1.83(2)       |

Table 4: $< F_{1,\text{max}} >$ for IMC gauge obtained with various algorithms

|          | $\beta = 2.3$ | $\beta = 2.4$ | $\beta = 2.5$ |
|----------|---------------|---------------|---------------|
| RO($N_{cop} = \infty$) | 0.7132(1)     | 0.7324(3)     | 0.7509(2)     |
| LRO($N_{cop} = \infty$) | 0.7134(1)     | 0.7331(2)     |               |
| SA($N_{cop} = \infty$)  |               | 0.7337(2)     |               |
|          | 0.7337(2)     |               |               |

4 Results of the SA algorithm.

The simulated annealing algorithm has been applied to the maximal abelian gauge fixing in $\mathbb{R}^4$ and its advantages in reducing the bias due to Gribov copies effects has been demonstrated. In this section we describe our preliminary results in fixing for both DMC and IMC gauges obtained with SA algorithm. For IMC gauge SA algorithm has been applied at the first step only, while RO algorithm has been applied at the 2nd step. The details of the SA algorithm for DMC gauge will be explained elsewhere.

Our first observation is that with SA algorithm it is possible to reach higher local maxima $< F_{\text{max}} >$ (for DMC gauge) and $< F_{1,\text{max}} >$ (at the first step of IMC gauge - MA gauge) than those reached with the LRO procedure. This can be seen from the Tables 2 and 4. Our second observation is that results obtained with these local maxima are qualitatively similar to those obtained with RO procedure, i.e. $\sigma_{Z(2)}$ computed on these configurations is nonzero. The values of $\sigma_{Z(2)}$ and $\rho$ are lower than those obtained with RO algorithm.

For SA algorithm applied to DMC gauge we generated only 3 local maxima per configuration, since this algorithm is rather time consuming. Thus we are not able to make extrapolation $N_{cop} \to \infty$. Nevertheless the example $N_{cop} = 3$ is instructive. Our results are $\frac{\sigma_{Z(2)}}{\sigma_{SU(2)}} = 0.69(2), 0.72(2), 0.49(2)$ at $\beta = 2.3, 2.4, 2.5$ correspondingly. Comparing with results in Table 1 one can see that already for $N_{cop} = 3$ SA values for $\sigma_{Z(2)}$ are lower than values obtained with RO procedure in $N_{cop} = \infty$ limit. For IMC gauge with SA algorithm...
at the first step we get $\frac{\sigma_{Z(2)}}{\sigma_{SU(2)}} = 0.73(4)$ at $\beta = 2.4$ (computations have been made at this $\beta$ only). This is in agreement with the result in Table 3 obtained with RO procedure. It is still possible that employing SA algorithm also at the second step of IMC gauge fixing will bring further decreasing of the projected string tension.

In it has been argued that low values for $\sigma_{Z2}$ reported in can be due to finite volume effects which spoil vortex finding property even on the lattices where finite volume effects are not visible in the gauge invariant observables. We made some computations for DMC gauge with RO procedure for lattices $10^4$ to $20^4$ and found out results in qualitative agreement with fig. 3 of ref. But the problem is still not settled since increasing of $\sigma_{Z2}$ with lattice volume can be due to lower value of local maxima found on larger lattices where finding higher maxima becomes too costly, while there is clear anticorrelation between $\sigma_{Z2}$ and the value of $\langle F_{max} \rangle$. Moreover our results with SA algorithm indicate that these finite volume effects must be much more pronounced if one takes the highest maxima available. We made computation in DMC gauge on $L = 16$ lattice at $\beta = 2.4$. We apply the SA algorithm with more updating sweeps than it was done for $L = 12$ computations at the same $\beta$. We obtained higher value $\langle F_{max} \rangle = 0.7784(1)$ and lower value $\frac{\sigma_{Z(2)}}{\sigma_{SU(2)}} = 0.64(2)$ than corresponding values given in Table 1. The question of the finite volume effects for IMC gauge has not been studied so far.

Conclusions

We conclude that DMC and IMC gauges suffer from strong Gribov copies effects and careful gauge fixing is necessary to make the bias caused by these effects reasonably small. This procedure is rather costly. Another problem of DMC and IMC gauges is that $\sigma_{Z(2)}$ is too low. It is not clear whether relation $\sigma_{Z(2)} = \sigma_{SU(2)}$ can be valid at large enough lattices. Even if this is the case, the size of these lattices is enormous. The alternative gauge, the Laplacian center gauge is then more favorable.

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