Warped Compactification with an Abelian Gauge Theory

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Abstract

We investigate warped compactification with an abelian gauge theory in six dimensions. The vanishing cosmological constant in four dimensions can generically be realized with a regular metric even in a 3-brane background without fine tuning of couplings.
1 Introduction

The smallness of the cosmological constant poses a severe problem [1] on our natural understanding of an effective field theory description of the universe. The problem is two-fold: one is the apparent absence (or cancellation) of the contributions from the standard model dynamics including gravity to the vacuum energy; the other is to choose the vanishing (or tiny) value itself among possible values of the cosmological constant even if we can choose it.

Warped compactification of a higher-dimensional theory is an attempt to achieve the four-dimensional vanishing cosmological constant without fine tuning of couplings [2]. It results in a degenerate metric in six-dimensional pure gravity [2]. Six-dimensional warped compactification with an abelian gauge theory was investigated in Ref. [3] and that with a 3-brane in Ref. [4].

In this paper, we consider warped compactification with an abelian gauge theory in a 3-brane background. The vanishing four-dimensional cosmological constant can generically be realized with regular and compact extra dimensions in contrast to the case of pure gravity without an abelian gauge theory.

2 The Model

Let us consider six-dimensional gravity coupled to an abelian gauge theory. The action with a 3-brane is given by [3]

\[
S = \int d^6x \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{4} F_{MN} F^{MN} - \Lambda \right) - \int d^4x \sqrt{-g_4} \lambda,
\]

where \( g = \det g_{MN}, \) \( g_4 = \det g_{\mu\nu}, \lambda > 0, \) the six-dimensional gravitational scale is set to unity, and the brane is located at the origin in the extra two dimensions. Here \( M \) and \( N \) denote six-dimensional indices, \( \mu \) and \( \nu \) denote four-dimensional ones, and \( g_{\mu\nu} \) is the induced metric on the brane. Then the Lagrangian in six dimensions is given by

\[
\mathcal{L} = \sqrt{-g} \left( \frac{1}{2} R - \frac{1}{4} F_{MN} F^{MN} - \Lambda \right) - \sqrt{-g_4} \frac{\lambda}{2\pi \epsilon} \Theta(\epsilon - r),
\]

where \( \Theta \) denotes a step function and \( \epsilon = +0. \) Here we have adopted the polar coordinates \( (r, \theta) \) for the extra two dimensions \( (0 \leq r, \ 0 \leq \theta < 2\pi). \)
The equations of motion are obtained as
\[
R^{MN} - \frac{1}{2} g^{MN} R = F^M_L F^{NL} - \left( \frac{1}{4} F_{KL} F^{KL} + \Lambda \right) g^{MN} - \sqrt{\frac{g}{\delta}} \delta^M_\mu \delta^N_\nu \frac{\lambda}{2 \pi \epsilon} \Theta(\epsilon - r),
\]
\[
\partial_M (\sqrt{-g} F^{MN}) = 0.
\]
In the following sections, we solve these equations based on an ansatz of four-dimensionality after compactification.

3 Warped Compactification

In order to obtain four-dimensional spacetime from six dimensions, we compactify the extra two dimensions. Under an assumption of rotational symmetry in the extra dimensions (θ-independence or orbifolding by \( S^1 \)), the warped metric is given by
\[
ds^2 = \sigma(r) \bar{g}_{\mu\nu} dx^\mu dx^\nu - dr^2 - \rho(r) d\theta^2
\]
and the background gauge field is given by
\[
A_\mu = A_r = 0, \quad A_\theta = a(r),
\]
where \( \bar{g}_{\mu\nu} \) denotes the four-dimensional metric independent of \((r, \theta)\).

With the aid of Eq. (3), we obtain
\[
F_{r\theta} = \frac{B}{\sigma^2 \sqrt{\rho}},
\]
where \( B \) is an integration constant taking continuous values. Then the Einstein equations are reduced to
\[
\begin{aligned}
&\frac{3}{2} \sigma'' + \frac{3}{4} \sigma' \rho' - \frac{1}{4} \rho^2 + \frac{1}{2} \rho'' = -\frac{B^2}{2 \sigma^4} - \Lambda - \frac{\lambda}{2 \pi \epsilon \sqrt{\rho}} \Theta(\epsilon - r) + \frac{\Lambda_4}{\sigma}, \\
&\frac{3}{2} \sigma'^2 + \frac{\sigma'}{\sigma} \rho' = \frac{B^2}{2 \sigma^4} - \Lambda + \frac{2 \Lambda_4}{\sigma}, \\
&\frac{3}{2} \sigma'' + \frac{\sigma'}{\sigma} \rho'' = \frac{B^2}{2 \sigma^4} - \Lambda + \frac{2 \Lambda_4}{\sigma},
\end{aligned}
\]
\( ^1 \) The freedom to choose a value of one continuous variable \( B \) eventually leads to that of the effective four-dimensional cosmological constant \( \Lambda_4 \).
where $\Lambda_4$ denotes the four-dimensional cosmological constant for the metric $\bar{g}_{\mu\nu}$ introduced as an integration constant and the prime denotes a derivative with respect to $r$.

By means of Eq. (8) and (9), we obtain

$$z'' = -\frac{\partial V}{\partial z}; \quad V(z) = \frac{25}{96}B^2z^{-\frac{4}{5}} + \frac{5}{16}\Lambda z^2 - \frac{25}{24}\Lambda_4 z^\frac{4}{5},$$

(10)

$$\sigma = z^{\frac{4}{5}}, \quad \rho = C^{-2}z^\frac{2}{5}z^{-6/5},$$

(11)

where $C$ is an integration constant. Note that Eq. (10) looks like an equation of motion for a particle with the position $z(r)$ at the time $r$ in a potential $V(z)$ [2, 3].

4 The Solutions

Let us seek regular metrics with $\Lambda_4 = 0$. From the Einstein equations in the previous section, boundary conditions at $r = \epsilon$ are given by [2, 3, 4]

$$\left(\sqrt{\rho}\right)'|_{r=0} = -\frac{\lambda}{2\pi}, \quad (\sqrt{\rho})'(0) = 1, \quad \rho(\epsilon) = 0.$$  

(12)

Namely, the extra dimensions are conical around the brane with a deficit angle $\lambda$, which is regular in the presence of the brane. Owing to Eq. (11), this implies

$$z'(\epsilon) = 0, \quad z''(\epsilon) = C \left(1 - \frac{\lambda}{2\pi}\right).$$

(13)

Here we have taken $z(\epsilon) = 1$ without loss of generality. Then Eq. (10) leads to

$$C \left(1 - \frac{\lambda}{2\pi}\right) = z''(\epsilon) = -\frac{\partial V}{\partial z}(z(\epsilon)) = \frac{5}{16}B^2 - \frac{5}{8}\Lambda.$$  

(14)

For $\Lambda > 0$ and $\lambda < 2\pi$, we have desired solutions of Eq. (10), which are half periods of oscillations in $z(r)$ between two values $1 = z(\epsilon)$ and $\bar{z} = z(\bar{r})$ with $z'(\bar{r}) = 0$ given by

$$\frac{25}{96}B^2 + \frac{5}{16}\Lambda = V(z(\epsilon)) = V(z(\bar{r})) = \frac{25}{96}B^2\bar{z}^{-\frac{4}{5}} + \frac{5}{16}\Lambda\bar{z}^2.$$  

(15)

Imposing a condition $(\sqrt{\rho})'(\bar{r}) = -1$ for a regular metric (without a conical singularity) at $r = \bar{r}$, we obtain

$$-C\bar{z}^{\frac{4}{5}} = z''(\bar{r}) = -\frac{\partial V}{\partial z}(z(\bar{r})) = \frac{5}{16}B^2\bar{z}^{-\frac{4}{5}} - \frac{5}{8}\Lambda\bar{z}.$$  

(16)

The constants $B$, $C$ and $\bar{z}$ are determined by Eq. (14), (15) and (16). This result indicates that we indeed have compact extra dimensions of sphere topology with a completely regular metric in six dimensions.
5 Conclusion

We have considered warped compactification Eq.(4) and (5) with an abelian gauge theory in a 3-brane background Eq.(1) or (2). The vanishing cosmological constant $\Lambda_4 = 0$ can generically be realized with a regular metric determined by Eq.(14), (15) and (16) with Eq.(10) and (11). This is achieved without fine tuning of Lagrangian parameters.

We have only obtained the backgrounds with $\Lambda_4 = 0$, just for simplicity; the backgrounds with $\Lambda_4 \neq 0$ are also possible. The four-dimensional cosmological constant $\Lambda_4$ is an integration constant in the present setup. This diminishes a half of the cosmological constant problem stated in the Introduction: the desired backgrounds with $\Lambda_4 = 0$ exist for positive couplings $\Lambda$ and $\lambda(<2\pi)$, which the standard model dynamics directly affect. The remaining half is to select the vanishing (or tiny) value $\Lambda_4 = 0$.

We note that inclusion of another 3-brane at $r = \tilde{r}$ is straightforward. The selection of $\Lambda_4 = 0$ might be possible, for instance, by a sector on a brane other than the brane at the origin. For that purpose, we should also consider the case for a negative bulk cosmological constant, which will be investigated elsewhere.

Acknowledgments

We would like to thank T. Yanagida for careful reading of the manuscript.
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