Observation of Induced Chern-Simons Term in P- and T- violating Superconductors

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We show in this letter that P- and T-odd terms of the electromagnetic potentials (P; parity, T; time reversal) are induced and lead an unusual phenomenon in P- and T-violating superconductors discovered recently. The Ginzburg-Landau action includes this term in the gauge invariant manner. The coefficient of the term is nearly topological invariant with a small correction and equals to the fine structure constant approximately. Unusual “Hall effect” without external magnetic field could be observed by SQUID.

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Recently the p-wave and the d-wave superconductors which violate parity (P) and time-reversal symmetry (T) by a condensation of P- and T-odd charged order seem to be discovered [11]. We show in this letter that an unusual P- and T-violating phenomenon occurs in these superconductors. A P- and T-odd term of the electromagnetic potentials that has been ignored so far [10] appears in a Ginzburg-Landau effective action. This term agrees with the Chern-Simons term in the static limit and its coefficient is unquantized due to the breaking of U(1) symmetry. Physical implications of this term shall be discussed also.

The Chern-Simons term is a P- and T-odd bi-linear form of the electromagnetic potentials and has one derivative. Hence this term is the lowest dimensional gauge invariant object and plays important roles in low energy and long distance physics of the P- and T-violating U(1) invariant systems such as massive Dirac theory in 2+1 dimensions [13] and the Quantum Hall system, where many exciting phenomena [6–8] have been found. The structure and properties of induced Chern-Simons term have been known well in U(1) invariant systems [1,10], but is not known in superconductors. A purpose of the present paper is to show an occurrence of P- and T- odd bi-linear form of the electromagnetic potentials and to analyze its structure, in the present superconductors. This term leads an “Hall effect” without external magnetic field. We estimate its signal and find the magnitude to be large enough for SQUID. Throughout our discussion we use the natural unit (h = c = 1) and g_{\mu\nu} = (1, -1, -1, -1).

We first obtain the Ginzburg-Landau action of general P- and T-violating superconductors. A generalized BCS Hamiltonian in the Bogoliubov-Nambu representation [11] is given by

\[ H_{\text{GBCS}} = \int d^2x \Psi^\dagger(x) (\varepsilon(p + eA_\mu) + eA_0) \tau_3 \Psi(x) + \int d^2xd^2y (\Psi^\dagger(x) \tau_+ \Psi(y))_{s1,s2} \times \]

\[ V_{s1s2;s3s4}(x,y)(\Psi(y)^\dagger \tau_- \Psi(x))_{s3s4}, \]

\[ \Psi_\alpha(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_\alpha(x) \\ \psi_\alpha^\dagger(x) \end{pmatrix}, \]

here an “isospin \( \alpha \) stands for particle and anti-particle, and subscript \( s \) stands for real spin. The \( \tau_\alpha \) (\( \alpha = 1, 2, 3 \)) are the 2x2 Pauli matrices with isospin indices and \( \tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2) \), therefore, \( (\Psi(x)^\dagger \tau_- \Psi(y))_{s1s2} = \psi_s(x)\psi_{s2}(y) \). In this representation, U(1) gauge transformation is written as

\[ \Psi \rightarrow e^{ie\tau_3 \xi} \Psi, \Psi^\dagger \rightarrow \Psi^\dagger e^{-ie\tau_3 \xi}, A_\mu \rightarrow A_\mu + \partial_\mu \xi. \]

We compute the gauge invariant Ginzburg-Landau effective action based on a path integral method of Ref. [12]. The generating functional is defined as

\[ Z[A_\mu] = \int D\Psi^\dagger D\Psi e^{iS[\Psi, \Psi^\dagger, A_\mu]}, \]

\[ S = \int d^3x \Psi^\dagger \partial_\mu \Psi - \int dt H_{\text{GBCS}}. \]

We rewrite Eq. (5) with a spatially bi-local auxiliary field \( \Phi \) via Stratonovich-Hubbard transformation. \( Z[A_\mu] \) is written as

\[ Z[A_\mu] = \frac{1}{N} \int D\Phi^\dagger D\Phi e^{iS + i\Delta S}, \]

\[ N = \int D\Phi^\dagger D\Phi e^{i\Delta S}, \]

\[ \Delta S = \int d^3x d^3x' (\Phi^\dagger(x, x') - \Psi^\dagger(x) \tau_+ \Psi(x')) \times V(x, x') (\Phi(x', x) - \Psi^\dagger(x') \tau_- \Psi(x)), \]

with appropriate spin indices.

We interchange the integration variables and integrate the Fermion fields at a stationary value of \( \Phi(x, x') \), \( \Phi^0(x, x') \),

\[ \Phi^0(x, x') = \langle \Psi^\dagger(x) \tau_- \Psi(x') \rangle. \]

From Eq. (5) and Eq. (6) U(1) gauge symmetry is broken, when \( \Phi^0 \) has non zero value. The gap function is related with \( \Phi^0 \) as
\[ \Delta_{S1s2}(x,x') = V_{s1s2;3s4}(x,x') \Phi^0_{3s4}(x',x). \] (6)

For the variables \( \Phi \) and \( \Phi^\dagger \), we consider phase degrees of freedom around \( \Phi^0 \) (Goldstone mode, which restores the gauge invariance), which is most important in the low energy region, and neglect fluctuations of other degrees of freedom. We write \( \Phi \) as a product of \( \Phi^0 \) part and phase part as follows; \( \Phi(x,x') = e^{-i\Phi(x,x')}e^{-i\Phi^0(x,x')} \), and transform Fermion fields to the corresponding gauge-invariant field \( \tilde{\Psi}(x) \) and \( \tilde{\Psi}(x) \) in the path integral as \( \tilde{\Psi} = e^{iS_\sigma}\Psi, \tilde{\Psi}^\dagger = \Psi e^{-iS_\sigma} \). This transformation does not change the path integral measure (i.e. \( D\tilde{\Psi}^\dagger D\tilde{\Psi} = D\Psi^\dagger D\Psi \)), the gauge field is transformed as \( A_\mu \rightarrow A_\mu + \frac{\epsilon}{c} \partial_\mu \theta \), and the generating functional is written by,

\[ Z[A_\mu] = \frac{1}{\mathcal{N}} \int D\theta D\tilde{\Psi}^\dagger D\tilde{\Psi} e^{i(S + \Delta S)[\tilde{\Psi}, A_\mu + \frac{\epsilon}{c} \partial_\mu \theta]}, \] (7)

where

\[ S + \Delta S = \int d^4x \left[ \tilde{\Psi}^\dagger (i\partial_\mu - eA_\mu(x)) \tilde{\Psi} - \mathcal{H}[^4A] \right], \] (8)

\[ \mathcal{H}[^4A] = \tilde{\Psi}^\dagger \left( \frac{\epsilon(p + eA)}{\Delta(p)} - \frac{\Delta(p)}{\epsilon(p - eA)} \right) \tilde{\Psi}, \]

\[ A'_\mu = A_\mu + \frac{\epsilon}{c} \partial_\mu \theta. \]

This action is gauge invariant under the transformation \( A_\mu \rightarrow A_\mu + \partial_\mu \xi, \theta \rightarrow \theta + e\xi, \) and \( \tilde{\Psi} \rightarrow \tilde{\Psi} \). Fermion propagator obtained from the action \( S + \Delta S \) is written as

\[ G(p) = \left( p_0 - \mathcal{H}[\Lambda = 0] \right)^{-1} = \left( p_0 - \tilde{\tau} \cdot \tilde{g}(p) \right)^{-1}, \] (9)

\( \tilde{g}(p) \) is defined as \( \tilde{g}(p) = \left( \text{Re}\Delta(p), -\text{Im}\Delta(p), \epsilon(p) \right)^T \).

After Fermion field is integrated in Eq. (9), we obtain a gauge invariant effective action of electromagnetic potentials and Goldstone mode as

\[ S^{(l)}_{\text{eff.}} = \int d^4x \frac{1}{2} \frac{m^2}{c_s^2} A_0^2 - m^2 A^2 + \frac{\sigma_{xy}}{2} \varepsilon_{0ij}(A_0 \partial_i A_j + A_i \partial_j A_0) + \frac{1}{2} \{(\partial_\mu \theta)^2 - c_s^2 (\partial_\mu \theta)^2 \} + \frac{m}{c_s} A_0 \partial_\mu \theta - m c_s \tilde{A} \cdot \tilde{\theta} + \frac{\sigma_{xy} c_s}{m} (\tilde{\theta} \times \tilde{A}) (\partial_\mu \theta) + O(\epsilon^3). \] (10)

The parameters \( m \) and \( c_s \) are determined as \( m = \frac{\langle \varepsilon \rangle}{m_e^2} \) \( c_s = \frac{\langle \varepsilon \rangle}{m_e^2} \) \( c_s = \frac{\langle \varepsilon \rangle}{m_e^2} \), where \( m_e, \rho_e \) and \( \nu_F \) show mass, number density and Fermi velocity of electrons. \( \lambda = m^{-1} \) shows the penetration depth for the magnetic field in the usual case (i.e. \( \sigma_{xy} = 0 \)). In Eq. (10) we find that a P- and T-odd term, \( \sigma_{xy} \varepsilon_{ij} A_0 \partial_i A_j \) is induced in the density-current correlation function \( \pi_{ij}(q) \) at Fermion 1-loop level. The coefficient \( \sigma_{xy} \) is written as

\[ \sigma_{xy} = \frac{1}{2!} \varepsilon_{ij} \partial \pi_{ij}(q) |_{q=0} = \frac{e^2}{2!} \varepsilon_{ij} \int \frac{d^4p}{(2\pi)^3} \text{Tr} \left[ \gamma_0 (p,p) \partial_i G(p) \gamma_j (p,p) G(p) \right], \] (11)

where \( \partial_i = \partial_i / \partial p^i \) and \( \gamma_\mu(p,p) = (\tau_3, \frac{\partial \gamma_{ij}(p)}{\partial p^i}) \) is the vertex part obtained from Eq. (8). \( \gamma_\mu \) is related with the “bare” propagator \( G_0(p) = (p_0 - \tau_3 g_3)^{-1} \) as

\[ \gamma_\mu(p,p) = \tau_3 \partial_\mu G_0^{-1}(p), \] (12)

We should note that \( \gamma_\mu \) is connected with \( G_0^{-1} \) not with \( G^{-1} \), and there is \( \tau_3 \) in the r.h.s. of Eq. (12). By substituting Eq. (12) into Eq. (11), \( \sigma_{xy} \) is written as

\[ \sigma_{xy} = \frac{e^2 \varepsilon_{ij}}{8} \int \frac{d^4p}{(2\pi)^3} \text{Tr} \left[ \tau_3 \partial_0 G_0^{-1} \partial_i G_0^{-1} \tau_3 \partial_j G_0^{-1} G \right] \]

\[ = \frac{e^2}{8} \int \frac{d^4p}{(2\pi)^3} \frac{\text{tr}[(\tilde{g} \times \tilde{g}) - g_3(\tilde{g} \times \tilde{g})]}{\text{tr}[\tilde{g} \cdot \tilde{g}^2]}, \] (13)

here “tr” means the trace about spin indices. The first term in Eq. (14) is a topological invariant \( \chi \), the second term is not a topological invariant. The existence of the second term comes from the fact that, because of the spontaneous breakdown of the \( U(1) \) gauge symmetry, \( \gamma_\mu \) in Eq. (12) is written by \( G_0 \), but not by \( G \), and there is the matrix \( \tau_3 \). The co-existence of the “bare” propagator \( \cdot \) (“bare” vertex) and “dressed” propagator in Eq. (13) seems to contradict with the gauge invariance, but in fact it does not because Goldstone mode guarantees the gauge invariance.

The action Eq. (14) is gauge invariant. Hence we have the manifestly gauge invariant P- and T- odd term which agrees with the Chern-Simons term in static limit,

\[ \int d^3x \frac{\sigma_{xy}}{2} \varepsilon_{ij} (A_0^T \partial_i A_j + A_i \partial_j A_0^T), \] (15)

after Goldstone mode is integrated out. \( A_0^T \) is the transversal component of \( A_0 \) written as \( A_0^T = A_0 - \frac{\partial_\mu}{m} (\partial_\mu A_0 - c_s^2 \partial \cdot A_0) \). Eqs. (10), (13), (14), (15) are our first results.

Now, we study particular examples of P- and T-violating superconductors. The first example is \( d_{x^2-y^2} + i d_{xy} \) order \( \overline{\text{B}} \). The \( i d_{xy} \) component violates P and T, and the parameter \( \epsilon \) shows the magnitude of the P- and T-violation. Therefore \( \epsilon \) corresponds to the mass of the fermion in the 2+1 dimensional Dirac QED \( \overline{\text{B}} \). We use the square lattice model with half-filling. In this case,

\[ \tilde{g} = \begin{pmatrix} i \sigma_3 \Delta (\cos(p_x b) - \cos(p_y b)) & -2i \sigma_2 \Delta \sin(p_x b) \sin(p_y b) & -2i \sigma_1 \cos(p_x b) + \cos(p_y b) \\ -2i \sigma_2 \Delta \sin(p_x b) \sin(p_y b) & i \sigma_3 \Delta (\cos(p_x b) - \cos(p_y b)) & -2i \sigma_1 \cos(p_x b) + \cos(p_y b) \\ -2i \sigma_1 \cos(p_x b) + \cos(p_y b) & -2i \sigma_1 \cos(p_x b) + \cos(p_y b) & i \sigma_3 \Delta (\cos(p_x b) - \cos(p_y b)) \end{pmatrix}, \]

\( \sigma_i \) (i=1,2,3) are the Pauli matrices with spin indices and \( \sigma_{xy} \) becomes
\[
\sigma_{xy} = \frac{e^2}{4\pi |\epsilon|} + \mathcal{O}(\epsilon),
\]
(16)
\[
\mathcal{O}(\epsilon) \text{ term comes from the last term in the r.h.s. of Eq.}(14). \text{ The above result shows that the P- and T- odd term Eq.(15) has a seizable magnitude even if the id_{xy} component is extremely small. This behavior is due to the topological nature of the first term in Eq.(14) and it is also seen in the 2+1 dimensional Dirac QED. }
\]
Second example is \(\text{Sr}_2\text{RuO}_4\) [8]. Its cristal structure is tetragonal, therefore we use the square lattice model. The energy band structure have been investigated by experiments [14] and they have shown that there bands cross the Fermi energy. The proposed gap function is
\[
\Delta(p) = i\sigma_3 \sigma_2(\sin(p_x b) + i\sin(p_y b)).
\]
(17)
Considering these circumstances, \(\sigma_{xy}\) becomes
\[
\sigma_{xy} = \frac{e^2}{4\pi}(1 + \mathcal{O}((\Delta/2)^2 \times 10^{-2})).
\]
(18)
The last term comes from that in Eq.\(\text{(14)}\) and it is negligibly small if \(\Delta/2 \sim 1\).

Finally we study a physical implication of Eq.\(\text{(13)}. \) A Hall effect without magnetic field is discussed based on the Ginzburg-Landau effective action. Neglecting the space-time dependence of the order parameters, the action for the layered superconductor in which each layer is perpendicular to the z-axis is written in static case with \(\mathbf{\nabla} \cdot \mathbf{A} = 0\) gauge as
\[
S^G, L = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \sigma_{xy}^2 (A_0 \partial_n A_j + A_i \partial_j A_0) \right],
\]
(19)
\(n_p\) is a number of layers in the unit length along the z-axis [6]. Consider a cylindrical hole of radius \(a\) in the superconductor. We assume that a charged wire with charge density \(q\) at the center of the hole, as is given in Fig.1.

In the cylindrical coordinates, the equations for the gauge field in the spatial geometry of the above situation is written in the superconductor (\(i.e.\ a < r\)) as
\[
\frac{1}{r} \partial_r (r \partial_r A_0) = -j_0 = \left\{ \frac{m^2}{c_s} A_0 - n_p \sigma_{xy} \frac{1}{r} \partial_r (r A_0) \right\}
\]
\[+ n_p \sigma_{xy} \chi A_0 \delta(r - a),\]
\[
\partial_r \left( \frac{1}{r} \partial_r r A_0 \right) = -j_0 = \left\{ \frac{m^2}{c_s} A_0 - n_p \sigma_{xy} \partial_r A_0 \right\}
\]
\[+ n_p \sigma_{xy} \chi A_0 \delta(r - a),\]
\[
0 = j_r = -m^2 A_r.
\]
(20)
Considering an ambiguity at boundary, we discuss two cases, \(\chi = 0\) case and \(\chi = 1\) case. When \(\chi = 0\) the chiral edge mode on the boundary surface exists [15] and when \(\chi = 1\) it does not. The boundary current comes from the gauge non-invariance of the Chern-Simons term on the boundary. It is known that the chiral edge mode recover the gauge invariance on the boundary surface, so that the chiral edge current cancels the boundary current.

We find from Eqs.(20) that \(A_r = 0\) and \(A_0\) is mixed with \(A_\theta\) in the superconductor. In the equations for the gauge field in \(r < a\) the right hand sides of Eqs.\(\text{(20)}\) vanish and we have \(A_0 = -\frac{\sigma_{xy}}{2\pi} \ln \frac{a}{\pi} + C_0\), \(A_\theta = r C_\theta\), \(A_r = 0\). \(C_0, C_\theta\) are constants.

We solve the equations with boundary conditions. Namely \(A_\mu\) is continuous on the boundary and the electric field \(E_r(r)\) and the magnetic field \(B(r)\) satisfy,
\[
\lim_{\delta \to 0} [E_r(a + \delta) - E_r(a - \delta)] = -\frac{n_p \sigma_{xy} \chi}{2} A_\theta(a),
\]
\[
\lim_{\delta \to 0} [B(a + \delta) - B(a - \delta)] = -\frac{n_p \sigma_{xy} \chi}{2} A_0(a).
\]
(21)
Using the fact that \(c_s << 1\), the solution in the superconductor \((a < r)\) is
\[
A_0(r) \simeq \frac{E_0 c_s}{m} \frac{K_0 \left( \frac{m r}{c_s} \right) + K_1 \left( \frac{m r}{c_s} \right)}{K_1 \left( \frac{m a}{c_s} \right) + m(2K_1(\frac{ma}{c_s}) + maK_0(\frac{ma}{c_s}))},
\]
(22)
\[
A_\theta(r) \simeq \frac{E_0 c_s n_p \sigma_{xy}}{m} \frac{K_1 \left( \frac{2 m r}{c_s} \right) c_s}{K_1 \left( \frac{2 m a}{c_s} \right) \left( 2K_1(\frac{ma}{c_s}) + maK_0(\frac{ma}{c_s}) \right)}.
\]
(23)
\(K_\nu(z)\) is the modified Bessel function. From this solution the current \(j_\phi(r)\) is
\[
j_\phi(r) = \theta(r - a) E_0 n_p \sigma_{xy} \left[ -\frac{K_0 \left( \frac{m r}{c_s} \right)}{K_1 \left( \frac{m a}{c_s} \right)} \right]
\]
\[+ \frac{K_1(\frac{m r}{c_s}) c_s}{2K_1(\frac{ma}{c_s}) + maK_0(\frac{ma}{c_s})} \delta(r - a) E_0 n_p \sigma_{xy} \frac{c_s}{2m}\]
(23)
This shows a circulating current. Its magnitude is proportional to the electric field in the radial direction hence this corresponds to “Hall effect” \textit{without external magnetic field}. Occurs. The “Hall current” Eq.\(\text{(23)}\) exists in \(a < r < a + \lambda (\lambda = m^{-1})\). A magnetic field induced by this current is
\[
B(r) = -\int_r^\infty dr' j_\phi(r')
\]
\[=-\theta(r - a) E_0 n_p \sigma_{xy} c_s \frac{K_0 \left( \frac{m r}{c_s} \right)}{m K_1 \left( \frac{ma}{c_s} \right)} \left[ \frac{K_0(\frac{mr}{c_s})}{2K_1(\frac{ma}{c_s}) + maK_0(\frac{ma}{c_s})} \right]
\]
\[+ \theta(r - a) E_0 n_p \sigma_{xy} \left( 1 - \frac{\chi}{2} \right) \times c_s \frac{K_1(\frac{ma}{c_s})}{m(2K_1(\frac{ma}{c_s}) + maK_0(\frac{ma}{c_s})).\]
(24)
A magnetic field is uniform in the hole.

The magnetic flux in the hole could be observed by SQUID measurement. Assume the SQUID coil on the hole, and the radius of the hole \( a \sim 1 \text{mm} \) and that of the SQUID coil \( R \sim a/2 \). We take \( c_s \simeq 10^8 \text{cm/s} \), \( n_p \simeq 10^{-1} \text{A}^{-1} \), and \( \lambda \simeq 10^4 \text{A} \). These are typical values for high-\( T_c \) superconductors and \( Sr_2RuO_4 \). The magnetic flux in the coil is

\[
\Phi_{\text{flux}} \simeq -10^{-13} \times \left( 1 - \frac{2}{3} \right) \left( \frac{E_0}{100 \text{Volt/mm}} \right) \text{Wb}. \tag{25}
\]

In summary, we have shown that the P- and T-odd, which is equivalent to the Chern-Simons term in the static case, is induced in P- and T-violating superconductors. In these systems P, T and the U(1) gauge symmetry are spontaneously broken by one order parameter. We have discussed both spin-singlet pairing case and the spin-triplet unitary pairing case. The U(1) Goldstone mode guarantee the gauge invariance and the coefficient of the odd term, the “Hall conductance”, becomes almost topological invariant and approximately equals to the fine structure constant. The induced term should be included in the Ginzburg-Landau action for the P- and T-violating superconductors. The term causes non-trivial P- and T-violating electromagnetic phenomena, such as the Hall effect without an external magnetic field. The magnetic field induced by the Hall current could be observed by SQUID or \( \mu \text{SR} \).

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Caption

Fig. 1. Setup of our calculation. There is a charged wire with charge density \( q \) at the center of the cylindrical hole, and the superconductor (represented by diagonal lines). \( E_0(=\frac{q}{2\pi a}) \) is the electric field at the boundary.