THERMAL FIELD THEORY AND INFINITE STATISTICS

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ABSTRACT

We construct a quantum thermal field theory for scalar particles in the case of infinite statistics. The extension is provided by working out the Fock space realization of a “quantum algebra”, and by identifying the hamiltonian as the energy operator. We examine the perturbative behavior of this theory and in particular the possible extension of the KLN theorem, and argue that it appears as a stable structure in a quantum field theory context.
(revised version)
1. Introduction

Quantum Field Theories (QFT’s) at finite temperature and/or densities have been studied extensively during the past few years, and there is no need to recall how crucial they are for our understanding of microphysics and cosmology. However, intimately related to these physical aspects, there are many important theoretical questions which are not fully understood. For example, even the very existence of perturbative regimes for those theories is not entirely obvious. Partially based on $C^*$-algebra analysis, some authors have developed convincing arguments in favor of an inherent non perturbativeness of QFT’s at finite temperature [1]. Feynman rules can be devised though, both in real and imaginary time formalisms [2], and the resulting perturbative series, however formal, are certainly worth studying. Not surprisingly, one then learns that some convenient resummations of the original series are sometimes necessary in order that the perturbative calculations make sense [3].

Setting aside the question of the stability of the perturbative expansions at finite temperature [4], any such expansion has to cope with the problem of Infra-Red (IR) singularities. At $T = 0$, the infra-red character of the QFT’s perturbative series is admittedly under control: famous results such as the Kinoshita-Lee-Nauenberg (KLN) theorem, the Poggio-Quinn theorem or the Bloch-Nordsieck mechanism [5] set the perturbative approach on a sound basis, at least in this respect. Finite $T$-extensions of those results have been looked for in the past four years and are still the subject of current investigation [6].

The KLN theorem in particular, has been proven to hold at first [7] and second [8] non-trivial orders of finite temperature perturbation theory, by several authors. In the scalar case, an iterative proof for the cancellation of the most IR singularities at all orders, has been proposed [9].

Along with these calculational proofs, a remarkable feature appeared. Indeed, the cancellation of sin-
regularities appears to be due to “the miraculous properties” of the Bose and Fermi statistics. The point was stressed by some authors \cite{7,10}, and trials performed with different statistical distributions were shown to spoil completely the IR-finite character of the perturbative series \cite{11}.

On the other hand there is an alternative opinion about this issue. The overall cancellation of the IR singularities would just be the reflection of a deeper coherence of the theory. Unitarity is usually thought of, or some of its involved formulations \cite{12}. But unitarity is a general principle, and it is not always easy to handle it in the course of practical calculations. Of course, at $T = 0$, the Cutkosky rules can be regarded as an explicit expression for it \cite{13}. But such an operational expression is precisely lacking at non zero $T$ where, in spite of the “cutting rules” generalized by Kobes and Semenoff \cite{14}, one cannot define asymptotic states, and thereby escape the ambiguities of a Feynman amplitude calculation \cite{15}. Such a situation is likely to preclude the existence of any general argument which would decide which kind of a mechanism is responsible for the IR-singularities cancellation, and what is its relation to conventional Bose and Fermi statistics.

In 1953, Green observed that statistics others than Bose and Fermi could be envisaged within the axiomatic context of QFT’s \cite{16}. This observation has been at the origin of a rapid development of the so-called “parastatistical fields” \cite{17}. These fields are quantized according to commutation relations which are trilinear in the creation and annihilation operators, and are characterized by an integer $p$, the order of the parastatistical field. This number basically corresponds to the number of quanta in a given symmetric or anti-symmetric state.

The situation referred to as “infinite statistics” corresponds to the case where the number $p$ is left as a free parameter without restriction, and turns out to be realized by the so-called “quantum” or “deformed” algebras. Our calculations will be performed in this framework.
The paper is organized as follows. In Section 2, we first recall the basics of “deformed quantizations” in some detail. Then we examine their compatibility with the subsequent constraints brought about by the context of covariant statistical mechanics. In section 3, we derive the expression for the contour propagator of a “$q$–deformed” scalar field. This is readily used to set up the finite $T$-perturbation theory of both imaginary and real time formalisms. Section 4 exhibits a direct use of these rules. A one-loop calculation is exhibited, and the validity of KLN theorem is examined in the case of a scalar field obeying infinite statistics. Our comments and conclusions are presented in Section 5.
2. Deformed algebras

2.1 Deformations

Finite $T$ formalisms rely on a formal analogy between imaginary time and inverse temperature. Only the time variable is involved in this implementation of temperature, so that we can first ignore the spatial degrees of freedom and focus on simpler quantum statistical mechanics. We therefore consider the one-dimensional harmonic oscillator. Since a free field is nothing but a superposition of independent frequencies, generalization to QFT will be straightforwardly obtained in the next section.

Let us now introduce the deformed algebras. For reasons to be discussed shortly, we choose to follow the basics of Macfarlane’s presentation [18]. We thus assume the existence of a Hilbert space $\mathcal{H}$ spanned by the vector basis $\{ |n >; n \in \mathbb{N} \}$, and endowed with the scalar product $< m|n > = \delta_{mn}, \forall m, n$. Acting upon $\mathcal{H}$, we define a set of three linear operators $a, a^+ \text{ and } N$ such that

$$a|0 > = 0, \quad a^+|n > = \sqrt{n+1}|n+1 >, \quad a|n > = \sqrt{n}|n-1 >, \quad N|n > = n|n > \quad (2.1)$$

These operators will be taken in the Heisenberg picture. In what follows, the bracketed quantities will stand for either

$$[r] = \frac{1-q^r}{1-q} \quad (2.2)$$

or

$$[r] = \frac{q^r-q^{-r}}{q-q^{-1}} \quad (2.3)$$

Indeed we will be mostly involved in the second case (2.3). Over the representation $\mathcal{H}$, the deformation parameter $q$ can be chosen so that the operators $a, a^+$ and $N$ satisfy

$$a^\dagger = a^+, \quad [N, a^+] = a^+, \quad [N, a] = -a \quad (2.4)$$
where the symbol $\dagger$ refers to the adjoint. Also, by formally extending to operators the above brackets postulated for scalars, one gets

$$a^+ a = [N] , \quad aa^+ = [N + 1]$$

(2.5)

Now for the postulated hermiticity of $N$ to be consistent with properties (2.4) and (2.5) on the one hand, and the definition of the brackets (2.2) and (2.3) on the other hand, one must choose the parameter $q$ a real number or a pure phase only. The same consistency argument imposes that $q$ be real valued in the definition (2.2). Then one immediately verifies that the operators $a$, $a^+$ and $N$ satisfy

$$aa^+ - qa^+ a = 1$$

(2.6)

in the case of definition (2.2), and

$$aa^+ - qa^+ a = q^{-N} \text{ and/or } aa^+ - q^{-1} a^+ a = q^N$$

(2.7)

in the case of definition (2.3). That is, one gets two deformations, $q$-parametrized, of the familiar commutation algebras. Eventually, the and/or of (2.7) displays the symmetry of this latter deformation under the exchange $q \leftrightarrow q^{-1}$.

Some remarks are in order.

(i) At first sight, this construction may appear to be a somewhat formal and indirect way to introduce the deformations. However, it has the advantage of embedding the deformed algebras in their Fock space realization from the onset; this is certainly of interest when one tries to implement them as QFT’s. Moreover, being a construction, it automatically enjoys self-consistency, provided it is not an empty one. It is not. An explicit coordinate-space realization of the $q$-deformed harmonic oscillator has been proposed [18] in the case of definition (2.3). The operators $a$, $a^+$ and $N$ are referred to as the “$q$-deformed” annihilation, creation,
and number operators respectively. Note also that the hermitic conjugation of operators $a$ and $a^+$, eq.(2.4), ensures the unitarity of the representation.

(ii) The limit $q = 1$ reproduces the original bosonic quantification for both deformations. This is not so at the limit $q = -1$, which is not fermionic in the second case (2.7). For this reason, an alternate deformation is sometimes considered [19], which continuously extrapolates between both statistics

$$aa^+ - qa^+ a = q^{-2N}$$

We will not consider it though, for we found that its Fock space realization encounters serious contradictions. We will therefore exclude the case $q = -1$ and consider deformations about the bosonic statistics only [20].

(iii) The Hilbert space $\mathcal{H}$ is positive definite by construction

$$\forall n \in \mathbb{N}, \ |n> = \frac{(a^+)^n}{\sqrt{[n]!}} |0>, \ [n]! = [n] [n-1] ... [1], \ |||n>||^2 = 1$$

When $q$ is a pure phase, i.e. $q = e^{i\theta}$, one can parametrize $\theta = \pi r$ with $r$ some number. If $r$ is a rational number ($r = k/p$), then the Hilbert space is spanned by $p$ states ($|j>, j = 0, 1, ..., p-1$) [21]. If $r$ is not a rational number, there is an infinite number of such states, as in the ordinary case, but there exists an integer $n$ such that $[n] < 0$ implying that $a$ and $a^+$ are no longer hermitian conjugates (this is easily understood by inspection of eq.(2.3)). Fortunately, most formulae turn out to be independent of this $\theta$-parametrization and so are the results of the next sections.

(iv) As compared to the ordinary Bose case, a very distinctive feature of the deformations is worth emphasizing. Considering a system of several degrees of freedom denoted by indices $i, j$ (continuous or discrete), the quantizations (2.6) and (2.7) are not a priori completed by such relations as

$$[a_i, a_j] = [a_i^+, a_j^+] = 0$$
In effect, their would be generalization

\[ a_i a_j - qa_j a_i = a_i^+ a_j^+ - qa_j^+ a_i^+ = 0 \]  

(2.11)
is easily proven to hold only if \( q^2 = 1 \), that is in the standard cases. But, as in these standard cases, such relations are not necessary to define operator’s normal ordering, Wick’s theorem and the calculation of matrix elements. If we choose not to impose the set of relations (2.10), the unusual consequence is that the \( n! \) permutations of an \( n \)-identical particles state, now become \( n! \) linearly independent states [22].

The “\( q \)-deformed harmonic oscillator” is accordingly defined in terms of the above deformed operators. Within familiar conventions for the constants, one has

\[ H = \frac{1}{2m} P^2 + \frac{1}{2} m \omega^2 X^2 \]  

(2.12)

with

\[ X = \sqrt{\frac{\hbar}{2m \omega}} (a + a^+) \quad \text{and} \quad P = i \sqrt{\frac{\hbar m \omega}{2}} (a - a^+) \]  

(2.13)

and finds

\[ H = \frac{1}{2} \hbar \omega ([N] + [N + 1]) \]  

(2.14)

In terms of \( a \) and \( a^+ \), the number operator \( N \) can be given the formal expressions

\[ N = \frac{\ln ([a, a^+]_q)}{\ln q} \quad \text{and} \quad N = -\frac{\ln ([a, a^+]_q)}{\ln q} \]  

(2.15)

for the deformations (2.6) and (2.7) respectively, whereas in the second case we have introduced the “\( q \)-mutator”

\[ [a, a^+]_q \equiv aa^+ - qa^+ a \]  

(2.16)

Both formal expressions reduce to the usual number operator \( N = a^+ a \) in the limit \( q = 1 \), and can be defined as infinite series expansions in terms of deformed operators \( a^+ \) and \( a \) [19],[22].
2.2 Thermalization

At thermal equilibrium, statistical averages typically involve quantities of the kind

$$\mathcal{N} \, \text{Tr} \left( e^{-\beta H} \, T \, \text{Pol}\{X(t)\} \right)$$  \hspace{1cm} (2.17)

where $\mathcal{N}$ is some relevant normalization, $\beta$ the inverse temperature, $\text{Pol}\{X(t)\}$ some polynomial in the position operators and $T$ a prescription of time ordering. The point is that a trace must be taken over a complete set of states. For the calculations to be meaningful, it is therefore crucial that the cyclicity of the trace be consistent with the underlying algebraic structure.

For short, we hereafter set $\hbar = 1$ everywhere. For operators in the Heisenberg picture, the equation of motion reads

$$\frac{dA}{dt} = -i[A, H]$$  \hspace{1cm} (2.18)

and it is important to realize that the cyclicity of the trace precludes any $q$-extension of the above equation, where for instance the usual right hand side commutator would be replaced by the corresponding $q$-mutator $[A, H]_q$.

The Hamiltonian usually dealt with is given by (2.14) [23,24]. As our Hamiltonian though, we will rather take

$$H = \omega(N + C^{te})$$  \hspace{1cm} (2.19)

with $N$ the deformed number operator of (2.15), and thereby identify the Hamiltonian with the energy operator. One recovers the well known relations

$$[H, a] = -\omega a \quad \text{and} \quad [H, a^+] = \omega a^+$$  \hspace{1cm} (2.20)
and two properties follow by construction: the positive definiteness of the energy eigenvalues for all \( n \) and \( q \)-values, and, most importantly, the possibility of well defined statistical averages. Considering for example the two-point function \( G(t) \equiv \mathcal{N} \text{Tr} \left( e^{-\beta H} T X(t) X(0) \right) \), we have

\[
G^+(t - i\beta) = G^-(t)
\]

(2.21)

where \( G^\pm \) refers to the usual decomposition

\[
G(t) = \Theta(t)G^+(t) + \Theta(-t)G^-(t)
\]

(2.22)

that is, the Kubo-Martin-Schwinger (KMS) conditions [2] are satisfied by the two-point function at thermal equilibrium. One can check easily that these two crucial properties are not clearly at issue if the Hamiltonian (2.14) is used instead. Note that the limiting value of the constant appearing in (2.19) is known only when \( q \to 1 \). However this latter non-determination will never show up in the subsequent analysis, so that we can either take \( C_{\text{te}} = 1/2 \) or simply drop it. Note also that this choice of \( H \) can be compared with the ones performed in parastatistics [25] and is in full agreement with the traditional choice of an Hamiltonian for a fermionic oscillator [26].

### 2.3 Deformed statistics

It is now an easy task to calculate the deformed Bose-Einstein distributions. Considering the deformation (2.7), one calculates the thermal averages of both sides, that is

\[
< aa^+ >_\beta - q < a^+ a >_\beta = < q^-^N >_\beta
\]

(2.23)

Within obvious notations one has

\[
< q^-^N >_\beta = \frac{1}{Z} \sum < n\lvert e^{-\beta H} e^{-iN\theta} \rvert n > = \frac{1}{Z} e^{-\beta \omega/2} \frac{1}{1 - q^* e^{-\beta \omega}}
\]

(2.24)
By evaluating the partition function $Z$ we eventually get

$$< q^{-N} >_\beta = \frac{e^{\beta \omega} - 1}{e^{\beta \omega} - q^*}$$

(2.25)

Then by using the commutators (2.20) and the cyclicity of the trace, one derives

$$< q^{-N} >_\beta = (e^{\beta \omega} - q) < a^+ a >_\beta$$

(2.26)

and eventually

$$< a^+ a >_\beta = \frac{e^{\beta \omega} - 1}{e^{2\beta \omega} - 2\cos \theta e^{\beta \omega} + 1}$$

(2.27)

that is, a real quantity for all $\theta$. Following the same steps, the first algebra leads to a simpler expression

$$< a^+ a >_\beta = \frac{1}{e^{\beta \omega} - q}$$

(2.28)

Then we can analyze the generalization to free quantum field theory.

3. Perturbation Theory

3.1 Locality

We now consider the extension to free scalar field theory, and begin by briefly reviewing the possibility of a local realization of the deformations. Locality is presumably one of the most tricky points when one wishes to implement the deformations as (local) QFT’s. This axiom is referred to as the basic requirement that observables be pointlike functionals of the fields, and commute at spacelike separations [27]. Indeed, a general theorem [27] prevents any field theory with statistics other than Fermi/Bose or parafermi/parabose, from being realized as local QFT’s (the latter “para”-alternative being restricted to $D \geq 3$ dimensional spaces). Even though, TCP theorem has been shown to hold true for “$q$-deformed” free fields [22].
When $q$-deformed fields are considered, locality considerations are usually made on the basis of $q$-mutators, which then play the role of ordinary commutators. This generalization can be thought of as being natural, and is intended to take advantage of the $q$-mutators

$$a(k)a^+(p) - qa^+(p)a(k) = (2\pi)^3 2\omega_k q^{-N_k} \delta^{(3)}(k - p)$$  \hspace{1cm} (3.1)

But, considering such quantities as $[\phi(x), \phi(y)]_q$, implicitly supposes that the underlying space-time manifold is endowed with some non commutative geometrical properties [28]. Whether it exists or not any compelling reason to consider such situations, falls beyond the scope of the present analysis. Indeed, one can check that local $q$-mutativity is not satisfied with the deformations (2.6) and (2.7), and we will restrict ourselves to ordinary locality considerations.

Take the free hermitian scalar field $\phi(x)$ in a four dimensional space-time

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} (a(k)e^{-ik.x} + a^+(k)e^{ik.x})$$  \hspace{1cm} (3.2)

where the Fourier components are quantized according to the deformation (3.1), and act linearly on the Fock space of states obtained by the standard procedure (smeared polynomials of $a^+(k)$ acting upon the vacuum [22]).

Considering the commutator of two free field operators at space-like separation, one finds, as expected in view of the theorem cited above

$$[\phi(x), \phi(y)]_{x_0 = y_0} \neq 0$$  \hspace{1cm} (3.3)

However, the thermal average of the commutator reads

$$\langle [\phi(x), \phi(y)] \rangle_\beta = \int \frac{d^4k}{(2\pi)^4} e^{-ik.(x-y)} 2\pi\epsilon(k_0) \delta(k^2 - m^2) n_\beta(\omega_k)(e^{\beta|k_0|} - 1)$$  \hspace{1cm} (3.4)
where $\epsilon(k_0)$ is the usual sign distribution, and where we have introduced the notation

$$n_\theta(\omega_k) \equiv <a_k^+ a_k >_\beta = \frac{e^{\beta\omega_k} - 1}{e^{2\beta\omega_k} - 2 \cos \theta e^{\beta\omega_k} + 1} \quad (3.5)$$

At space-like separations one gets

$$< [\phi(x), \phi(y)] >_\beta |_{x_0 = y_0} = 0 \quad (3.6)$$

that is, local commutativity is restored by taking the statistical average, which is of course pretty remarkable and unexpected a result. This important issue is certainly of interest in order to get a reliable perturbation theory, as we will observe in the following.
3.2 The contour propagator

Let $G_C(x - y)$ be the contour propagator

$$G_C(x - y) = \langle T_C \phi(x)\phi(y) \rangle_\beta$$  \hspace{1cm} (3.7)

where the mean value refers to the thermal average (2.17), whereas the prescription $T_C$ orders the field operators along some given contour $C$ of the time-complex-plane. Defining the $G^\pm$ components by

$$G_C(x - y) = \Theta_C(x_0 - y_0) \ G^+(x - y) + \Theta_C(y_0 - x_0) \ G^-(x - y)$$  \hspace{1cm} (3.8)

one has (see (2.21))

$$G^+(x - y - i\beta) = G^- (x - y)$$  \hspace{1cm} (3.9)

that is the KMS condition. Taking equation (3.2) into account, $G_C$ can be explicitly written as

$$G_C(x - y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik(x-y)}}{2\omega_k} \left\{ \Theta_C(x_0 - y_0) \left( < aa^+ >_\beta e^{-ik(x-y)} + < a^+a >_\beta e^{ik(x-y)} \right) + \Theta_C(y_0 - x_0) \left( < a^+a >_\beta e^{-ik(x-y)} + < aa^+ >_\beta e^{ik(x-y)} \right) \right\}$$ \hspace{1cm} (3.10)

Once a contour is given, it is just a book-keeping device to deduce the corresponding set of Feynman rules.

1) The real time formalism

We will consider the oriented contour which is usually choosen, depicted in Fig.1, and basically composed of the real axis and of its $-i\sigma$-shifted part, counter-oriented [2]. The parameter $\sigma$ is taken in the range $0 \leq \sigma \leq \beta$. The real time free propagator is a two by two matrix with components $G^{rs}$, $\{r, s\} = 1, 2$.

By using (3.10), one gets for temporal arguments lying on the $C_1$ part of $C$

$$G^{11}(x - y) = \int \frac{d^4k}{(2\pi)^4} \ e^{-ik(x-y)} \left\{ < aa^+ >_\beta \ \Delta(k) + < a^+a >_\beta \ \Delta^*(k) \right\}$$ \hspace{1cm} (3.11)
where $\Delta(k)$ is the usual Feynman propagator

$$
\Delta(k) = \frac{i}{k^2 - m^2 + i\epsilon} \tag{3.12}
$$

Then, for the $C_2$ part, one has

$$
G^{22}(k) = (G^{11}(k))^* = <a a^+ > \Delta^*(k) + < a^+ a > \Delta(k) \tag{3.13}
$$

Non-diagonal components are those for which $x_0$ belongs to $C_1$, $y_0 - i\sigma$ to $C_2$, and vice versa

$$
G^{12}(x - y) = \int \frac{d^4k}{(2\pi)^3 2\omega_k} \left\{n_\theta(\omega)e^{\omega\sigma}e^{-ik(x-y)} + e^{i\omega\sigma}n_\theta(\omega)e^{-\omega\sigma}e^{ik(x-y)} \right\} = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \left\{2\pi\delta(k^2 - m^2)e^{\sigma k_0}n_\theta(\omega) (\Theta(k_0)e^{i\omega} + \Theta(-k_0)) \right\} \tag{3.14}
$$

where we have used the relations (2.23) and (2.26) in order to relate $<a a^+ >$ to $< a^+ a >$. A similar expression can be derived for the $G^{21}$ component. In momentum space, the result can be summarized as

$$
G^{12}(k) = 2\pi\delta(k^2 - m^2)n_\theta(\omega)e^{\sigma k_0} (\Theta(k_0) + \Theta(-k_0)e^{\beta\omega}) \tag{3.15}
$$

and likewise

$$
G^{21}(k) = 2\pi\delta(k^2 - m^2)n_\theta(\omega)e^{-\sigma k_0} (\Theta(-k_0) + \Theta(k_0)e^{\beta\omega}) \tag{3.16}
$$

Thanks to energy conservation, the calculation of Green’s functions is independent of $\sigma$ [2], and a convenient choice is the one performed in Thermo Field Dynamics, which sets $\sigma = \beta/2$. One finds

$$
G^{12}(k) = G^{21}(k) = 2\pi\delta(k^2 - m^2)e^{\beta\omega/2}n_\theta(\omega) \tag{3.17}
$$

completed by

$$
G^{11}(k) = (G^{22}(k))^* = n_\theta(\omega) (\Delta(k)e^{i\omega} + \Delta^*(k)) \tag{3.18}
$$
The two previous equations set up the Feynman rules for a free scalar $q$–deformed field, assuming that the bare vertices remain unaltered by the deformation, that is

$$(-ig) \text{ for type 1 vertices, and } (+ig) \text{ for type 2 vertices}$$  \hspace{1cm} (3.19)

and no couplings between the different types of fields ($g$ stands for the bare coupling constant).

By the same token, the cut propagators $G^\pm$ which are conveniently used for the calculation of imaginary parts of Green’s functions [13,14], are easily derived by setting $\sigma = 0$ in (3.15) and (3.16), [10]. One finds

$$G^-(k) = G^{12}(k)|_{\sigma=0} = \left(\Theta(k_0) + \Theta(-k_0)e^{\beta\omega}\right)n_\theta(\omega) \frac{2\pi\delta(k^2 - m^2)}{n_B(\omega)}  \hspace{1cm} (3.20)$$

$$G^+(k) = G^{21}(k)|_{\sigma=0} = \left(\Theta(-k_0) + \Theta(k_0)e^{\beta\omega}\right)n_\theta(\omega) \frac{2\pi\delta(k^2 - m^2)}{n_B(\omega)}  \hspace{1cm} (3.21)$$

Each of these involved expressions can be shown to reduce to the well-known ones in the limit of no deformation, $\theta = 0$. For later purposes we here give these limits for the two previous equations

$$\lim_{\theta=0} G^\pm(k) = \left(\Theta(\mp k_0) + \Theta(\pm k_0)e^{\beta\omega}\right)n_B(\omega) \frac{2\pi\delta(k^2 - m^2)}{n_B(\omega)}$$

$$= 2\pi \left(\Theta(\mp k_0) + n_B(\omega)\right)\delta(k^2 - m^2)  \hspace{1cm} (3.22)$$

where the first expression of the RHS of (3.22) will consequently be considered as more fundamental than the second customary one, which is specific to the pure bosonic case only. This observation will play a central role in the next section. Another way to understand this, is to note that a direct replacement of $n_B$ by $n_\theta$ in the usual real-time Feynman rules would yield an incorrect result. We will similarly make use of the limit of (3.18)

$$lim_{\theta=0} G^{11}(k) = n_B(\omega)\left(e^{\beta\omega}\Delta(k) + \Delta^*(k)\right)$$

$$= ((1 + n_B(\omega))\Delta(k) + n_B(\omega)\Delta^*(k))  \hspace{1cm} (3.23)$$
Now let us come to the following striking feature: considering the zero temperature limit of the above Feynman rules, one gets the usual $T = 0$ limit of an undeformed theory, that is

$$\lim_{T \to 0} G^{11}(k) = \Delta(k), \quad \lim_{T \to 0} G^{22}(k) = \Delta^*(k) \quad (3.24)$$

and likewise, for the cut propagators

$$\lim_{T \to 0} G^\pm(k) = 2\pi \Theta(\pm k_0)\delta(k^2 - m^2) \quad (3.25)$$

Eventually,

$$\lim_{T \to 0} G^{12}(k) = \lim_{T \to 0} G^{21}(k) = 0, \forall \theta \quad (3.26)$$

All the effects of the deformation have disappeared in the limit of a zero temperature. This can be readily realized by taking the infinite $\beta$ limit in an expression such as (3.10), where only the vacuum state is seen to survive. This rather peculiar feature has recently been noticed [29]. One may note, however, that the deformation is manifest at the 4-point function level [22].

The set of Feynman rules explicitied above define a well behaved perturbation theory for free fields that we shall use in the following. Because of equations (3.24)-(3.26), the ultra-violet sector of this theory is unaltered by the deformation. This makes it possible to renormalize the theory at $T = 0$, as is usually done in the undeformed case. Note that the infra-red sector is modified instead. In effect, the customary IR behavior of the Bose distribution

$$n_B(\omega) \sim \frac{1}{\beta \omega}, \quad \beta \omega \ll 1 \quad (3.27)$$

is now replaced by

$$n_\theta(\omega) \sim \frac{\beta \omega}{2(1 - \cos \theta)}, \quad \beta \omega \ll 1 \quad (3.28)$$
Thus the change is drastic: note the gap of two powers of $(\beta \omega)$ between (3.27) and (3.28), as soon as the deformation is switched on. Had we used the deformation (2.6) instead, a gap of one power of $\beta \omega$ had been obtained.

For the sake of completeness, we next give the free deformed propagator in the imaginary time formalism.

2) The imaginary time formalism

This formalism is generated by the simplest choice of a contour, namely a straight line stretching from $t = 0$ to $t = -i \beta$ on the imaginary time axis. The $T_C$ ordering reduces to the so-called temperature ordering $T_\tau$, where the variable $\tau = it$ can be restricted to the interval $-\beta \leq \tau \leq \beta$, in view of periodicity.

Taking relations (2.23) and (2.26) into account, one straightforwardly derives the imaginary time deformed propagator in the popular form

$$\Delta_\theta(\tau, k) = \frac{1}{2\omega} \left( e^{\omega (\beta - |\tau|)} + e^{\omega |\tau|} \right) n_\theta(\omega) \quad (3.29)$$

This propagator can be checked to verify the periodicity conditions induced by the KMS condition, that is, one has $\Delta_\theta(\tau - \beta, k) = \Delta_\theta(\tau, k)$ for $0 \leq \tau \leq \beta$, and $\Delta_\theta(\tau + \beta, k) = \Delta_\theta(\tau, k)$, for $-\beta \leq \tau \leq 0$.

4. The KLN theorem

We now come to our original goal. As advertised in the introduction, the thermal extension of the KLN theorem seems due to some “miraculous properties” of the Bose and Fermi distributions. In effect, for a large variety of three body processes (where a particle with energy $y$ splits into two others with energy $x$ and $y - x$), relations such as [7]

$$n_B(x) \ n_B(y - x) = n_B(y) \ (1 + n_B(x) + n_B(y - x)) \quad (4.1)$$
or in QED and QCD, such as [7,10]

\[ n_B(x) n_F(y - x) = n_F(y) \left( 1 + n_B(x) - n_F(y - x) \right) \] (4.2)

have been recognized to be at the origin of the cancellation of IR (including collinear) singularities. Thanks to these relations, real singular contributions add up to match singular virtual ones. Restricting ourselves to the bosonic case, it can be shown that all these “miraculous properties” derive from the only basic relation

\[ n_B(x + y) \left( 1 + n_B(x) \right) \left( 1 + n_B(y) \right) = \left( 1 + n_B(x + y) \right) n_B(x) n_B(y) \] (4.3)

which will be interpreted shortly. Now, relation (4.3) is not satisfied by the deformed statistical distribution \( n_\theta \), so that one could think that the cancellation of IR singularities is jeopardized. But indeed, (4.3) turns out to be specific to the case of zero deformation, *i.e.* the pure bosonic case, and this explains why trials performed with different distributions were doomed to failure.

However, in the case of non zero deformation one has

\[ < a^+ a >_\beta (x + y) < aa^+ >_\beta (x) < aa^+ >_\beta (y) = < aa^+ >_\beta (x + y) < a^+ a >_\beta (x) < a^+ a >_\beta (y) \] (4.4)

that is, in view of (2.23) and (2.26)

\[ n_\theta(x + y) e^{\beta x} n_\theta(x) e^{\beta y} n_\theta(y) = e^{\beta(x+y)} n_\theta(x + y) n_\theta(x) n_\theta(y) \] (4.5)

By observing that \( e^{\beta x} n_B(x) = 1 + n_B(x) \) one realizes that the “miracle” (4.3) falls into a whole, much wider, family of similar \( \theta \)-indiced “miracles” (4.5). Indeed, the formal triviality of these relations reveals their conservation law character. In effect, by consistently associating the weight \( n_\theta(x) \) to an incoming particle, and the weight \( e^{\beta x} n_\theta(x) \) to an outgoing one, relations (4.3) and (4.5) are readily recognized as expressing the well-known micro-reversibility property of elementary processes. As such, they extend to many more
kinds of elementary processes (they are pictured in Fig.2 for three body ones). Also, they display the crucial role played by the KMS condition in this respect. In its turn, the KMS condition is rendered possible by choosing (2.19) as the Hamiltonian.

Property (4.5) is not sufficient to ensure that KLN theorem can be satisfied in the deformed case. In this latter situation, one also has to make sure that perturbative calculations develop analytic structures which are the same as the ones at play in the ordinary case of no deformation. We therefore turn to this task now.

Let us consider the kind of processes which have been studied till recently [7,8], that is the damping rate of a heavy photon or a Higgs particle in a thermal bath. We assume that the Higgs can decay into scalar particles which have self-cubic interactions. Since these are dynamical quantities, the real time formalism is more appropriate. At first non trivial order of perturbation theory, this rate is given by the imaginary part of the self-energy function of the particle. We restrict ourselves to the topology of Fig.3, as the cancellation of singularities is known to take place in each topology separately [5].

The best way to proceed consists in briefly recalling the structures involved in the standard situation. The thermal free propagator can be conveniently written as

\[ G(k) = \mathcal{U} \begin{pmatrix} \Delta(k) & 0 \\ 0 & \Delta^*(k) \end{pmatrix} \mathcal{U} \]

where \( \mathcal{U} \) stands for the unimodular matrix representation of a Bogoliubov transformation (not unitary!) [2]

\[ \mathcal{U} = \left( \begin{array}{cc} \sqrt{1 + n_B(k_0)} & \sqrt{n_B(k_0)} \\ \sqrt{n_B(k_0)} & \sqrt{1 + n_B(k_0)} \end{array} \right) \]  

Assuming a Schwinger-Dyson equation for the dressed propagator \( \mathcal{G}^{rs}(k) \)

\[ \mathcal{G}^{rs}(k) = G^{rs}(k) + G^{rr'}(k) \left( -i\Sigma^{r's'}(k) \right) \mathcal{G}^{s's}(k) \]  

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one easily derives the set of relations

\[(G^{rs}(k)) = U \begin{pmatrix} \hat{\Delta}(k) & 0 \\ 0 & \hat{\Delta}^*(k) \end{pmatrix} U \]  

(4.9)

with

\[\hat{\Delta}(k) = \frac{i}{k^2 - m^2 - \Sigma(k)} \]  

(4.10)

and where the self energy function \(\hat{\Sigma}(k)\) is related to the matrix self energy by the relation

\[(-i\Sigma(k)) = U^{-1} \begin{pmatrix} -i\hat{\Sigma}(k) & 0 \\ 0 & +i\hat{\Sigma}^*(k) \end{pmatrix} U^{-1} \]  

(4.11)

that is, for practical purposes

\[\hat{\Sigma}(k) = Re\Sigma_{11}^{11}(k) + i \tanh(\beta k_0/2)Im\Sigma_{11}^{11}(k) \]  

(4.12)

Actually the quantities \(\hat{\Sigma}(k)\) and \(\hat{\Delta}(k)\) are those related to the corresponding imaginary time ones, by analytic continuation. The cut propagator with one loop self-energy correction immediately follows [8]

\[G^+(k) = 2\epsilon^{\beta k_0/2} n_B(k_0) \sqrt{(\Theta(-k_0) + e^{\beta \omega}\Theta(k_0)) Re\left(-i\hat{\Sigma}(k) \Delta^2(k)\right)} \]  

(4.13)

In terms of the previously defined expressions, the relevant quantity is conveniently expressed as

\[\Gamma(q) = \int \frac{d^Dk}{(2\pi)^D} G^+(k) \, G^-(k - q) \]

\[= -2 \int \frac{d^Dk}{(2\pi)^D} G^-(k - q) \left(\Theta(-k_0) + e^{\beta \omega}\Theta(k_0)\right) n_B(k_0) Re\left(-i\hat{\Sigma}(k) \Delta^2(k)\right) \]  

(4.14)

In (4.14), \(q = (q, \vec{0})\), stands for the \(D\)-moment of the Higgs particle considered. The mass \(m\) (and the coupling constant \(g\) in \(\hat{\Sigma}\)) can be thought of as the parameters of the \(T = 0\) renormalization procedure, and as we have pointed out in Section 3.2, the same steps can be taken in the deformed case.
Now, the structure displayed by equation (4.14) has been proven to yield IR-singularity cancellation, either by direct computation [8], using different types of regularization, or by appealing to dispersion relations [9].

In the case of deformed Feynman rules (3.17)-(3.21), there is no unimodular Bogoliubov matrix such as (4.7). Nevertheless one can still write

\[ G^\theta(k) = U^\theta \begin{pmatrix} \Delta(k) & 0 \\ 0 & \Delta^*(k) \end{pmatrix} U^\theta \]  

(4.15)

with \( U^\theta \) the invertible matrix

\[ U^\theta = \begin{pmatrix} \sqrt{\frac{e^{\beta\omega_n(k)}}{n^\theta(k)}} & \sqrt{\frac{n^\theta(k)}{e^{\beta\omega_n(k)}}} \end{pmatrix}, \quad \text{det } U^\theta = \frac{n^\theta(\omega)}{n_B(\omega)} = \frac{n^\theta(\omega)}{n^\theta=0(\omega)} \]  

(4.16)

and where we hereafter append a subscript \( \theta \) for the deformed quantities.

As they stand, the Feynman rules of the previous section can be used to calculate the one-loop dressed propagator \( G^\theta_{rs}(k) \), which is obtained by summing over the thermal indices, the one-loop self energy insertion

\[ G^\theta_{rs}(k) = G^\theta_{rs}(k) + G^\theta_{rs'}(k) \left(-i\Sigma^\theta_{rs'}(k)\right) G^\theta_{s's}(k) \]  

(4.17)

For the self energy matrix itself, the following relations are easily established at one-loop order (\( O(g^2) \))

\[ \Sigma^\theta_{22}(k) = -\left(\Sigma^\theta_{11}(k)\right)^* \]

\[ \Sigma^\theta_{12}(k) = \Sigma^\theta_{21}(k) \]  

(4.18)

\[ Im\Sigma^\theta_{11}(k) = i \cosh(\beta k_0/2)\Sigma^\theta_{12}(k) \]

and at least at this order, just correspond to the standard non deformed expressions with the distribution \( n^\theta \) replacing the standard one \( n_B \). Defining the self energy function \( \tilde{\Sigma}^\theta \) by the relation

\[ \tilde{\Sigma}^\theta(k) = \frac{n^\theta(\omega)}{n_B(\omega)} \left\{ \text{Re}\Sigma^\theta_{11}(k) + i \tanh(\beta k_0/2) \text{Im}\Sigma^\theta_{11}(k) \right\} \]  

(4.19)
the above set of relations leads to the matricial form

\[
(-i \Sigma_\theta(k)) = U_\theta^{-1} \begin{pmatrix}
-i \tilde{\Sigma}_\theta(k) & 0 \\
0 & +i \tilde{\Sigma}_\theta(k)
\end{pmatrix} U_\theta^{-1}
\]  

(4.20)

with \(U_\theta\) as defined in (4.16). Note that the previous remark does not extend to the function \(\tilde{\Sigma}_\theta\) which is not obtained out of (4.12), by the replacement of \(n_B\) by \(n_\theta\). In effect, the non unimodular character of \(U_\theta\) induces some overall rescaling function \((n_\theta/n_B)\). However this is a perfectly regular function of \(k\), which factors out the singular structures involved in the one-loop series. Accordingly, the cut propagator with one loop self energy insertion can be written as

\[
G^+_\theta(k) = 2e^{\beta(k_0+\omega)/2} n_\theta(\omega) \Re \left( -i \tilde{\Sigma}_\theta(k) \Delta^2(k) \right)
\]

\[
= 2 \left( \Theta(-k_0) + e^{\beta\omega} \Theta(k_0) \right) n_\theta(\omega) \Re \left( -i \tilde{\Sigma}_\theta(k) \Delta^2(k) \right)
\]

(4.21)

and the decay rate under consideration as

\[
\Gamma_\theta(q) = \int \frac{d^Dk}{(2\pi)^D} G^+_\theta(k) \ G^-_\theta(k-q)
\]

\[
= -2 \int \frac{d^Dk}{(2\pi)^D} G^-_\theta(k-q) \left( \Theta(-k_0) + e^{\beta\omega} \Theta(k_0) \right) n_\theta(k_0) \Re \left( -i \tilde{\Sigma}_\theta(k) \Delta^2(k) \right)
\]

(4.22)

It is here made clear that, for the replacement of (standard) \(n_B\) by (the deformed) \(n_\theta\), the structure of the deformed case as displayed in (4.22), is exactly the same as the one displayed in (4.14) in the ordinary case of no deformation. Then, because of this complete structural analogy and thanks to identity (4.5), we are now in a position to conclude that cancellation of one-loop IR singularities will take place with deformed statistical distributions if it so with standard ones. In other words, one-loop KLN theorem is satisfied with deformed statistical distributions.

For the sake of illustration, let us give some calculational details and remarks about the cancellation of IR singularities (here collinear only). We need the following expressions

\[
\Re G^{11}(k) = (e^{\beta\omega} + 1) \ n_\theta(\omega) \pi \delta(k^2)
\]

(4.23)
\[
Im G^{11}(k) = (e^{\beta \omega} - 1) n_\theta(\omega) \mathcal{P} \left( \frac{1}{k^2} \right) \tag{4.24}
\]

which are easily derived from (3.18) in the massless limit. The zero-temperature part is exactly the same as in the usual \((\phi^4)_6\) theory, and contains all of the ultra-violet singularities. The latter are disposed of by renormalization, and by analytic continuation from \(\epsilon < 0\) to \(\epsilon > 0\) [8]. For the finite temperature part, we find the leading term

\[
Re \Sigma^{11}_\theta(k) = m^2 \theta \frac{k^2 \omega}{K^3} \ln \frac{\omega - K}{\omega + K} \tag{4.25}
\]

where \(K = |\vec{k}|\), and where we have introduced the finite temperature mass

\[
m^2 \theta = \frac{g^2 T^2}{32\pi^3} \int_0^\infty x dx \left( (e^{2x} - 1) n^2_\theta(x) - 1 \right) \tag{4.26}
\]

Note that all of the \(\theta\)-dependence is being kept in the overall constant \(m_\theta\). Indeed, (4.25) is nothing but the “Hard Thermal Loop” result for the self-energy [3]. These HTL are endowed with remarkable properties and it is very encouraging to see that the same structures do appear also in the case of infinite statistics. In (4.22) one can decompose

\[
Re \left( -i \tilde{\Sigma}_\theta(k) \Delta^2(k) \right) = -\pi \delta'(k^2) Re \tilde{\Sigma}_\theta(k) - \mathcal{P} \left( \frac{1}{(k^2)^2} \right) Im \tilde{\Sigma}_\theta(k) \tag{4.27}
\]

where the symbol \(\mathcal{P}\) stands for a principal value prescription. As usual, the potential singularities arise when expanding around the mass-shell. In (4.25), the momentum dependence is exactly the same as in the non-deformed case, and therefore the analytic properties of the self-energy at one loop order are identical to the one studied for example in [8]. The \(\ln(\omega - K)\) term in \(Re \tilde{\Sigma}_\theta(k)\), when dimensionally regularized, is responsible for a collinear singularity \((1/\epsilon)\) in the wave function renormalization constant. On the other hand, the imaginary part of \(\tilde{\Sigma}_\theta(k)\) which can be extracted from (4.25) with the analytic continuation \(\ln(\omega - K)/(\omega + K) \rightarrow -i\pi\), for space-like \(k^2\), together with the \(1/(k^2)^2\) principal part in (4.27) leads to the
same diverging behavior [8]. According to the analysis done in [9], the dispersive properties of the self-energy lead to the cancellation of this IR singularity (here a collinear one), which therefore verifies the KLN theorem at this order.

The same analysis can be carried through with the two-loop topology of ref.[8]. Though lengthy, one can check that the IR most singular terms ($O(1/\varepsilon^2)$, with $D = 6 + 2\varepsilon$) cancel out, following the same patterns as in the non deformed case. It is therefore tempting to speculate that our result is valid at any higher number of loops. However, this possible extension of our result would be somewhat formal and premature. One has to make sure that Feynman rules as defined in this paper effectively correspond to the perturbation theory allowing for the calculation of the thermal Green’s functions of eq.(2.17). Clearly, more analysis is needed, involving higher $n \geq 4$-point Green’s functions [30].

5. Conclusion

By working out the QFT representation of some “deformed” or “quantum” algebra, we have been able to generalize the usual bosonic Feynman rules of finite temperature QFT, to the case of so-called “infinite statistics”, in both real and imaginary-time formalisms. The Feynman rules derived in this paper, are those for a scalar neutral field in arbitrary space-time dimensions; they might be very different in other cases, the charged scalar one included. In agreement with a general theorem, the QFT representation of the deformation could not be made local at the operatorial level. But the remarkable point is that locality is restored by thermal averaging, which property let room for a well behaved perturbation theory, at least at the two-point function level.

Indeed, the resulting perturbation theory has been shown to display the same overall structure as the ordinary non deformed one. The ultra-violet sector is found to be totally unaffected by the deformation,
allowing for customary zero temperature renormalization algorithms, whereas, on the contrary, the IR sector is found to be deeply modified. Even though, the KLN theorem which rules the cancellation of IR perturbative singularities, has been shown to hold in case of deformations, at first non-trivial orders of perturbation theory, if it is satisfied in the corresponding non-deformed situations.

In particular, this one-loop verification of KLN has proven that the fine tuning mechanism responsible for IR singularity cancellation, is not due to some “miraculous properties” of the original Bose-Einstein statistical distribution, as one might have thought at first. The “miraculous properties” themselves have been seen to fall into a much wider family of similar properties, which all express the micro-reversibility property of elementary processes. In this respect, the KMS conditions have been recognized as playing a crucial role in allowing for micro-reversibility, and we think of this latter property as one of the basic conditions ensuring the validity of KLN.

The KLN theorem was originally proven in Quantum Mechanics and in QFT at zero temperature. Recently, enough evidence has been accumulated in favour of its validity at non-zero temperature and/or chemical potential too. Throughout this analysis, we have seen that the theorem can also accommodate deformed Bose statistics. One may therefore think of it as a pretty stable structure of the QFT context.

Our identification of the hamiltonian with the energy operator is worth emphasizing. This choice has been motivated by the merging constraints of the time evolution of operators in the Heisenberg picture and the cyclicity of the trace prescription. It is in contra-distinction with what is usually defined as the hamiltonian (eq.(2.14)), and which, we think, would lead to serious difficulties in defining any reliable thermal average. Exactly for the same reason, but at \( T = 0 \), it is not entirely obvious to us that this latter hamiltonian should be used in order to find some small departures from a pure bosonic behavior for atoms (or small violations of
the Pauli exclusion principle, as the same strategy is applied in that case too) [23,24]. On the other hand, if our choice is legitimate, we have seen that the effects of the deformation (the \( \theta \)-terms) comes in the thermal part of the Green’s functions only, and could therefore be difficult to observe. Nevertheless, they would come out as pure collective effects, quite in line with some of the ideas recently developed in [31].

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FIGURE CAPTIONS

Figure 1: Contour in the time complex-plane for the real time formalism.

Figure 2: Micro-reversibility (trilinear couplings). A symbolic representation of relations (4.5) (the diagrams stand for processes, not for Feynman amplitudes).

Figure 3: The damping rate $\Gamma_\theta(q)$ of a Higgs particle (dashed line) as given by the imaginary part of its one-loop (dark blob) corrected two-point function.