Cold and Hot Dark Matter from a Single Nonthermal Relic

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The origin of dark matter in the universe may be scalar particles produced by amplification of quantum fluctuations during a period of dilaton-driven inflation. We show, for the first time, that a single species of particles, depending on its mass and interactions, can be a source of both cold and hot dark matter simultaneously. Detection of such weakly interacting particles with masses below a fraction of an eV presents a new challenge for dark matter searches.

It is widely accepted that a substantial fraction of the total energy density in the universe is in the form of dark matter (DM). The composition and amount of DM is not known, but many hypothetical particles were proposed as candidates: WIMPs, axions, LSP’s, massive neutrinos and more. DM is classified into two types according to the velocities of particles at the beginning of the epoch of structure formation when the temperature of the universe was about 1 eV, cold dark matter (CDM) if the particles are non-relativistic and hot dark matter (HDM) if they are relativistic. The prevailing wisdom is that CDM and HDM originate from different species of particles, for example, axions which weigh a fraction of an eV for CDM, and a few eV neutrinos for HDM. DM particles are traditionally assumed to have a thermal distribution of velocities (see, however, [4]). Since a thermal distribution is sharply peaked around a single velocity, if that velocity is relativistic, the amount of non-relativistic particles is extremely small and vice versa.
We show, for the first time, that weakly interacting nonthermal relics produced by amplification of quantum fluctuations during a period of dilaton-driven inflation can have an energy spectrum with two peaks, such that at the beginning of the epoch of structure formation some fraction of the particles are relativistic and some fraction are non-relativistic. The ratio and magnitude of energy densities in relativistic and nonrelativistic particles are determined by the mass of the particles and by the cosmological parameters of the model. The basic physics behind the appearance of a twin peak spectrum is the existence of two scales in the problem: the Planck scale, redshifted, and the mass. Our models suggest the possibility that DM in the universe might be composed of weakly interacting nonthermal relics and that it might be a mixture of CDM and HDM if the mass of the particles is lighter than a fraction of an eV. It is argued [4] that CDM+HDM models could be just what is needed to explain the data on CMB anisotropies [5]. Our models can provide the desired composition from a single source, as well as predict other observable consequences.

We consider particle production in models of string cosmology which realize the pre-big-bang scenario [6,7]. In this scenario the evolution of the universe starts from a state of very small curvature and coupling and then undergoes a long phase of dilaton-driven kinetic inflation and at some later time joins smoothly standard radiation dominated cosmological evolution, thus giving rise to a singularity free inflationary cosmology. Particles are produced during the period of dilaton-driven inflation by the standard mechanism of amplification of quantum fluctuations [8].

In the simplified model of background evolution we adopt, the evolution of the universe is divided into four distinct phases with specific (conformal) time dependence of the scale factor of the universe $a(\eta)$ and the dilaton $\phi(\eta)$. The first phase is a long dilaton-driven inflationary phase, the second phase is a high curvature string phase of otherwise unknown properties, followed by ordinary Friedman-Robertson-Walker (FRW) radiation dominated (RD) evolution and then a standard FRW matter dominated (MD) evolution. We assume throughout an isotropic and homogeneous four dimensional flat universe, described by a FRW metric. All other scalar fields are assumed to have a trivial vacuum expectation value.
during the inflationary phase.

During the dilaton-driven inflationary phase \( \eta < \eta_s \), both scale factor and coupling \( e^\phi \) are growing as powers \( a(\eta) = a_s \left( \frac{\eta}{\eta_s} \right)^\alpha \) and \( e^{\phi(\eta)} = e^{\phi_s} \left( \frac{\eta}{\eta_s} \right)^\beta \), where \( \alpha \) and \( \beta \) are negative. The dilaton-driven phase is expected to last until curvatures reach the string scale and the background solution starts to deviate substantially from the lowest order solution. For ideas about how this may come about see [9]. The string phase lasts while \( \eta_s < \eta < \eta_1 \). We assume that curvature stays high during the string phase. As in [10], we assume that the string phase ends when curvature reaches the string scale \( M_s \). We parametrize our ignorance about the string phase background, as in [11], by the ratios of the scale factor and the string coupling \( g(\eta) = e^{\phi(\eta)/2} \), at the beginning and end of the string phase \( z_S = a_1/a_S \) and \( g_1/g_S \), where \( g_1 = e^{\phi_1/2} \) and \( g_S = e^{\phi_S}/2 \), where \( a_S = a(\eta_s) \) and \( \phi_S = \phi(\eta_s) \). We take the parameters to be in a range we consider reasonable. For example, \( z_S \) could be in the range \( 1 < z_S < e^{45} \sim 10^{20} \), to allow a large part of the observed universe to originate in the dilaton-driven phase, and \( g_1/g_S > 1 \), assuming that the coupling continues to increase during the string phase and \( 10^{-3} \lesssim g_1 \lesssim 10^{-1} \) to agree with the expected range of string mass (see e.g. [10]). Some other useful quantities that we will need are \( \omega_1 \), the frequency today, corresponding to the end of the string phase, estimated in [10] to be \( \omega_1 \sim 10^{10} \text{Hz} \), and the frequency \( \omega_S = \omega_1/z_S \), the frequency today corresponding to the end of the dilaton-driven phase. In the RD phase and MD phase are assumed to follow the string phase, the dilaton is taken to be strictly constant, frozen at its value today.

We have computed the spectrum of produced particles for the models described previously [12]. We have solved a linear perturbation equation, \( \chi'' + \left( k^2 + M^2 a^2 - \frac{s''}{s} \right) \chi_k = 0 \) where \( s(\eta) \equiv a(\eta)^{m} e^{\phi(\eta)/2} = a_s^{m} e^{\phi_s/2} \left( \frac{\eta}{\eta_s} \right)^{1/2-n_s} \), imposing initial conditions corresponding to normalized vacuum fluctuations. The parameter \( m \) depends on the spin of the particle and \( l \) depends on its coupling to the dilaton. The perturbation first “exit the horizon” when \( k\eta \sim 1 \), when curvatures become larger than their wavelength, then they are “frozen” outside the horizon when \( k\eta < 1 \), and then “reenter the horizon” at \( \eta_{re} \). A duality symmetry [13] exchanging the perturbation and its conjugate momentum can be used to follow the
evolution of the perturbation equations for times at which the background evolution is not

known precisely. To read off the spectrum for a more general case of background evolution

that we consider here, the only required substitution in the results of [12] is

\[ n_s = \frac{1}{2} - \alpha m - \frac{\beta}{2} l. \]

(see also [14]). Similar calculations have been performed by several groups and the results

agree [15–17], whenever a comparison was possible.

We will consider weakly interacting scalar particles, abundant in string theory and supergravity. For scalar fields \( m = 1 \), and we will consider for concreteness the following values for \( l, l = -1, 0, 1 \) corresponding, respectively, to moduli (including the dilaton), Ramond-Ramond axions, and Neveu-Schwartz axions. We will assume that the produced particles interact so weakly, that their interactions and decay are not sufficient to alter the primordial spectrum substantially. The particles we have in mind have typically gravitational strength interactions, which is definitely weak enough to satisfy our assumption, and masses below a fraction of an eV.

A typical spectrum of a light scalar may be divided, at a given time, into three frequency

regions: i) The massless region, \( \omega_S > \omega > M \). In this region particles are relativistic. ii) The “false” massive region, \( M > \omega > \omega_m \), where \( \omega_m = \omega_1 (M/M_s)^{1/2} \). In this region particles are nonrelativistic, but have reentered the horizon as relativistic modes. iii) The “real” massive region \( \omega_m > \omega \). In this region particles are non-relativistic, and have reentered the horizon as non-relativistic modes. Note that physical frequencies redshift as the universe expands, and therefore boundaries of regions change in time.

Different spectral shapes may result depending on parameters. For the interesting case,

the spectrum increases with \( \omega \) in the massless region, decreases in the “false” massive region

and increases in the “real” massive region. In this case, the energy density in relativistic

particles \( \Omega_{REL} \approx \frac{d\Omega}{d\ln w} (\omega = \omega_S) \), and that of nonrelativistic particles is given by \( \Omega_{NR} \approx \frac{d\Omega}{d\ln w} (\omega = \omega_m) \). For this case the spectrum takes the following approximate form at the beginning of the epoch of structure formation \( \eta_{eq} \).
\[
\frac{d\Omega}{d\ln\omega} = \begin{cases} 
N g_1^2 \left( \frac{g_s}{g_1} \right)^{-2l} \left( \frac{\omega}{\omega_S} \right)^2 & \text{if } \omega > \omega > M, \\
N g_1^2 \left( \frac{g_s}{g_1} \right)^{-2l} \frac{M}{\omega_S} \left( \frac{\omega}{\omega_S} \right)^{x-1} & \text{if } M > \omega > \omega_m, \\
N g_1^2 \left( \frac{g_s}{g_1} \right)^{-2l} \frac{M}{\omega_1} \left( \frac{\omega}{M} \right)^{-1/2} \left( \frac{\omega}{\omega_S} \right)^x & \text{if } \omega_m > \omega,
\end{cases}
\]

where \( x \equiv 2 + 2\alpha + l\beta \), and \( N \) is a numerical factor, estimated in \([12]\), which we will set to unity in what follows.

We have ignored, so far, particles produced during the string phase, since that phase is at the moment less well understood. If the spectrum decreases there, then our approximations remain valid. If, however, the spectrum increases there, then a good approximate relation to use would be \( \Omega_{REL} \simeq \frac{d\Omega}{d\ln\omega} (\omega = \omega_1) \). Otherwise more parameters describing the string phase should be added. Since this is not relevant to our main point, we choose not to do so.

We impose constraints on the spectrum, and show that it is possible to satisfy all of them by giving a specific example. First we require that the desired spectral shape is obtained

\[ 0 < x < 1. \quad (2) \]

Then we require that some relativistic DM particles and some nonrelativistic DM particles exist at \( \eta_{eq} \),

\[ M < \omega_S(\eta_{eq}), \quad \omega_m > \omega_{eq}, \quad (3) \]

where \( \omega_{eq} = \frac{1}{a^2(\eta_{eq})} \frac{da}{d\eta}(\eta_{eq}) \). We also require that the energy density in produced particles is the main source of energy density in the universe and that approximately equal amounts exists (later we will see that this condition can be significantly relaxed).

\[ \Omega_{massive}(t_{eq}) \simeq 1, \quad \Omega_{massless}(t_{eq}) \simeq 1. \quad (4) \]

Assuming that the level of fluctuations at very large scales is well below the CMB level (as we show later this must be the case), we require

\[ \frac{d\Omega}{d\ln\omega} (\omega = \omega_0) < 10^{-5}. \quad (5) \]

A typical interesting spectrum is shown in Figure II.
FIG. 1. A typical twin peaked spectrum. The dashed lines represents two possible string phase spectra.

The constraints on the spectral parameters can be translated into constraints on cosmological parameters. The range of powers $\alpha$ and $\beta$ allowed by condition (2) is summarized in Table I.

| $l$      | $\alpha$         | $\beta$         |
|----------|------------------|------------------|
| $l = -1$ | $\alpha < 0$     | $\beta < 0$     |
| $l = 0$  | $-1 < \alpha < -\frac{1}{2}$ | $\beta < 0$     |
| $l = 1$  | $\alpha < 0$     | $-2 < \beta < 0$ |
| $\frac{\beta}{2} - 1 < \alpha < \frac{\beta}{2} - \frac{1}{2}$ | $\frac{\beta}{2} - 1 < \alpha < \frac{\beta}{2} - \frac{1}{2}$ |

Condition (3) leads to

$$M < \omega_1 (\eta_{eq}) z_S^{-1}, \quad M > M_s \left( \frac{\omega_{eq}}{\omega_1} \right)^2.$$  \hspace{1cm} (6)

Condition (4) leads to

$$g_1^2 \left( \frac{g_1}{g_S} \right)^{2l} \simeq 1,$$  \hspace{1cm} (7)
and
\[ g_1^2 \left( \frac{g_1}{g_s} \right)^{2l} \frac{M}{\omega_1} \left( \frac{\omega_m}{\omega_S} \right)^x \left( \frac{M_s}{M} \right)^{1/2} \simeq 1. \tag{8} \]

Since \( \omega_m = \omega_1 (M/M_s)^{1/2} \) and \( \omega_S = \omega_1 z_S^{-1} \), we obtain by substituting (7) into (8)
\[ \left( \frac{M}{M_s} \right)^{(1+x)/2} \frac{M_s}{\omega_1} z_S^x \simeq 1 \]. Condition (3) leads to \( g_1^2 \left( \frac{g_1}{g_s} \right)^{2l} \frac{M}{\omega_1} \left( \frac{\omega_m}{\omega_S} \right)^x \left( \frac{M_s}{M} \right)^{1/2} < 10^{-5} \), which, using (4), leads to \( \left( \frac{M}{M_s} \right)^{1/2} \frac{M_s}{\omega_1} \left( \frac{\omega_m}{\omega_S} \right)^x z_S^x < 10^{-5} \). We therefore obtain the following two conditions,
\[ M > 10^{10/x} M_s \left( \frac{\omega_0}{\omega_1} \right)^2, \]
\[ z_S < 10^{-5(1+x)/x^2} \left( \frac{\omega_0}{\omega_1} \right)^{(1-x)/x} \left( \frac{M_s}{\omega_1} \right)^{-1/x} \], \tag{9} \]

which, using the following numerical values for parameters \( [2, 10] \), \( \omega_0 \sim h \times 10^{-30} \text{eV}, \omega_1 \sim 10^{-1} \text{eV}, M_s = g_1 \times 10^{28} \text{eV} \) (where \( 0.5 \leq h \leq 0.8 \) and \( 10^{-3} \lesssim g_1 \lesssim 10^{-1} \)), become
\[ M > h^2 g_1 \times 10^{10/x-30} \text{eV}, \]
\[ z_S < h^{-1/x-1} g_1^{-1/x} 10^{29-8/x-5/x^2}. \tag{10} \]

We now present a concrete example which serves to demonstrate that a reasonable range of parameters exists, in which all conditions are naturally satisfied. We look at axions \( (l = 1) \) with masses below .1 eV for a cosmological model described in [7]. In this model \( \alpha = -2/(d+n+3) = -1/6 \) and \( \beta = -4d/(d+n+3) = -1 \), for \( d = 3, n = 6 \), which is in the range specified in Table [1]. For this specific model \( x = 2/3 \), \( \Omega_{\text{REL}} \simeq g_1^2 \left( \frac{g_1}{g_s} \right)^2, \)
\( \Omega_{\text{NR}} \simeq g_1^2 \left( \frac{g_1}{g_s} \right)^2 \frac{M}{\omega_1} \left( \frac{\omega_m}{\omega_S} \right)^{-1/3}. \)

The ratio of relativistic to nonrelativistic energy densities is given by \( \frac{\Omega_{\text{REL}}}{\Omega_{\text{NR}}} = \frac{\omega_m}{\omega_S} \left( \frac{\omega_m}{\omega_S} \right)^{-1/3} \), and since \( \omega_m = \omega_1 (M/M_s)^{1/2} \) we obtain
\[ \frac{\Omega_{\text{REL}}}{\Omega_{\text{NR}}} = \omega_1 M_s^{-2/3} \left( \frac{M_s}{M} \right)^{5/6}. \]

From conditions (10) we obtain conditions on \( M \) and \( z_S \):
\[ M > h^2 g_1 \times 10^{-15} \text{eV}, \]
\[ 1 < z_S < 6 \times 10^5 h^{-5/2} g_1^{-3/2} \]. \tag{12}
Taking $10^{-10}$eV $< M < 10^{-2}$eV, for which condition (1) is comfortably satisfied, we observe that the ratio $\Omega_{\text{REL}} : \Omega_{\text{NR}}$ can vary in a range from well above unity to well below unity. For example, choosing $g_1 = 10^{-1}$ and $g_S = 10^{-2}$ if the axion’s mass is $10^{-10}$eV, and for $z_S \sim 2 \times 10^4$ we get $\Omega_{\text{REL}} : \Omega_{\text{NR}} = 1 : 1$ with both energy densities being near critical, and if we choose $g_1 = 10^{-1}$ and $g_S = 3 \times 10^{-2}$, making $\Omega_{\text{REL}} \simeq 0.1$, and if $z_S \sim 10^6$ we obtain $\Omega_{\text{REL}} : \Omega_{\text{NR}} = 1 : 10$, with $\Omega_{\text{NR}} \simeq 1$, and just for fun, the preferred ratio of 70% CDM and 20% HDM \[3\] is obtained for the same values of couplings, if $z_S \sim 2 \times 10^5$. Note that the ratio (11) depends on $z_S^{-2/3} M^{-5/6}$, so the previous examples correspond to a range of allowed values.

We now show that it is not possible to obtain HDM+CDM and produce the CMB fluctuations by the same field, accepting all conditions on the spectrum. If the spectral amplitude is indeed at the level of the observed CMB fluctuations of about $10^{-5}$, then the spectral slope is also constrained by the data \[3\]. By parametrizing the slope as $d\Omega/d\ln \omega \sim \omega^{(n-1)/2}$, we deduce that $n$ can be identified with the tilt parameter of the spectrum \[18\], which is constrained by the data to be in the range $0.8 < n < 1.4$, which corresponds to $-0.1 < x < 0.2$. We have required a positive slope, so the available range is just $0 < x < 0.2$.

Now it is a simple exercise to calculate the energy density at $\omega_m$ and see that it is much below unity, $d\Omega/d\ln \omega(\omega_m)/d\Omega/d\ln \omega(\omega_0) = \left( \frac{\omega_m}{\omega_0} \right)^x$. Since $\frac{\omega_m}{\omega_0} = \frac{\omega_f}{\omega_0} \left( \frac{M_{\text{f}}}{M_{\text{s}}} \right)^{1/2} < 10^{16}$, and since $x < 0.2$, we can estimate that this ratio cannot naturally be above $10^3$, making the energy density at $\omega_m$ much less than unity in most of parameter space. Perhaps, by tuning and forcing parameters into corners, it is possible to find an artificial example, but we will not pursue this possibility.

We further show that, in the range of parameters we are interested in, it is not possible that one scalar field provides the required CMB fluctuations and a different scalar field provides HDM+CDM in a single cosmology. Since the two fields must be different $l_f \neq l_{HC}$, where the subscript $f$ denotes a field that is supposed to produce the CMB anisotropy and the subscript $HC$ denotes the field that is supposed to produce HDM+CDM. We require $\Omega_{\text{REL}}^{HC} \simeq$
$g_1^2 \left( \frac{g_s}{g_1} \right)^{-2l_{HC}} \simeq 1$, and $\Omega_{REL}^f \simeq g_1^2 \left( \frac{g_s}{g_1} \right)^{-2l_f} \lesssim 1$. Therefore, $\frac{\Omega_{HC}}{\Omega_{REL}} = \left( \frac{g_s}{g_1} \right)^{2l_f - 2l_{HC}} \gtrsim 1$, so for $\frac{g_s}{g_1} < 1$ we have to have $l_f < l_{HC}$ and from $x_f = 2 + 2\alpha + l_f \beta$ and $x_{HC} = 2 + 2\alpha + l_{HC} \beta$ (recall that $\alpha < 0$ and $\beta < 0$) we obtain $x_f > x_{HC}$. The conclusion is that the only possibility (for values of $l$, $-1, 0, 1$) is $0 < x_{HC} < x_f < 0.2$. But then $\Omega_{HC}$ dominates also at the lowest frequency, in contradiction to our assumptions. Therefore, either CMB fluctuations are of different origin [18,19], or DM source is different.

If DM in the universe is indeed made of light particles with gravitational strength interactions its detection in current direct searches is extremely difficult, and will probably require new methods and ideas.

Finally, we note that the models we have described have predictions and consequences other than DM. Additional particles such as gravitons [11] get produced and provide additional possible experimental and observational signatures.

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