The event-by-event $p_T$–fluctuations in proton-proton and central Pb-Pb collisions, which have
been experimentally studied by means of the so-called $\Phi$–measure, are analyzed. The contribution
due to the correlation which couples the average $p_T$ to the event multiplicity is computed. The
correlation appears to be far too weak to explain the preliminary experimental value of $\Phi(\langle p_T \rangle)$ in
p-p interactions. The significance of the result is discussed.

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I. INTRODUCTION

The transverse momentum fluctuations in proton-proton and central Pb-Pb collisions at 158 GeV per nucleon have
been recently measured [1] on event-by-event basis. To eliminate trivial 'geometrical' fluctuations due to the impact
parameter variation the so-called $\Phi$–measure [2] has been used. $\Phi$ is constructed is such a way that it is exactly the
same for nucleon-nucleon (N-N) and nucleus-nucleus (A-A) collisions if the A-A collision is a simple superposition of
N-N interactions. Consequently, $\Phi$ is independent of the centrality of A-A collision in such a case. On the other hand,
$\Phi$ equals zero when the inter-particle correlations are entirely absent. The critical analysis of the $\Phi$–measure can be
found in [3,4]. For the central Pb-Pb collisions, the value of $\Phi(\langle p_T \rangle)$ measured in the laboratory rapidity window (4.0,
5.5) equals $4.6 \pm 1.5$ MeV [1]. The preliminary result for proton-proton interactions in the same acceptance is $5 \pm 1$
MeV [1]. Although the two values are close to each other the mechanisms behind them seem to be very different. It
has been shown [1] that the correlations, which are of the short range in the momentum space as those due to the
Bose-Einstein statistics, are responsible for the positive value of $\Phi(\langle p_T \rangle)$ in the central Pb-Pb collisions. When the
short range correlations are excluded $\Phi(\langle p_T \rangle)$ is reduced to $0.6 \pm 1$ MeV [1]. Our calculations have indeed demonstrated
[5,6] that the effect of Bose statistics of pions modified by the hadron resonances fully explains the observed $\Phi(\langle p_T \rangle)$
in the central Pb-Pb collisions. On the other hand, the short range correlations have been experimentally shown [1] to provide a negligible contribution to the $p_T$–fluctuations in the p-p interactions. Thus, the data suggest that
the dynamical long range correlations are reduced in the central Pb-Pb collisions (when compared to p-p) with the
short range due to the Bose statistics being amplified. The former feature is a natural consequence of the system
evolution towards the thermodynamic equilibrium. The amplification of the quantum statistics effect results from
the increased particle population in the final state phase-space. Since various dynamical correlations contribute to
$\Phi(\langle p_T \rangle)$ the question emerges what is the dynamical correlation in the nucleon-nucleon interactions which appear to be
washed out in the central nucleus-nucleus collisions. The aim of this paper is to discuss the question.

The average transverse momentum $\langle p_T \rangle$, which is measured at a given multiplicity $N$, is known to depend on $N$
in proton-proton collisions [1]. The correlation is negative for the collision energies below, say, $\sqrt{s} = 50$ GeV and
positive for the higher energies. At the beam energy of 205 GeV ($\sqrt{s} = 19.7$ GeV), which is very close to that of the
NA49 measurement [1], $\langle p_T \rangle$ significantly decreases with $N$ [1]. The data are shown in Fig. 1, where we have merged
the results for positive and negative particles. As already discussed in [1], the correlation which couples $\langle p_T \rangle$ to $N$
leads to $\Phi(\langle p_T \rangle) > 0$. Here, we compute $\Phi(\langle p_T \rangle)$ as a function of the correlation strength. Analytical and numerical
results are presented. The effect of the finite acceptance is studied and comparison with the experimental data is performed.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The average transverse momentum as a function of charged particle multiplicity in non-diffractive p-p collisions at 205 GeV. The data are taken from Ref. [8]. The line corresponds to $\langle N \rangle = 6.56$, $T = 167$ MeV and $\Delta T = 1.25$ MeV, see the text for the parameter’s meaning.}
\end{figure}

\section{II. ANALYTICAL CALCULATION}

Let us first introduce the $\Phi$–measure. One defines the single-particle variable $z = x - \overline{x}$ with the overline denoting averaging over a single particle inclusive distribution. Here, we identify $x$ with the particle transverse momentum. The event variable $Z$, which is a multiparticle analog of $z$, is defined as $Z = \sum_{i=1}^{N} (x_i - \overline{x})$, where the summation runs over particles from a given event. By construction, $\langle Z \rangle = 0$ where $\langle \ldots \rangle$ represents averaging over events. Finally, the $\Phi$–measure is defined in the following way

$$\Phi \equiv \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle} - \langle z^2 \rangle}.$$  

The correlation $\langle p_T \rangle$ vs. $N$ is introduced to our model calculations through the multiplicity dependent temperature or slope parameter of $p_T$–distribution. Specifically, the single particle transverse momentum distribution in the events of multiplicity $N$ is chosen in the form suggested by the thermal model i.e.

$$P_{(N)}(p_T) \sim p_T \exp \left[ - \frac{\sqrt{m^2 + p_T^2}}{T_N} \right],$$

where $m$ is the particle mass while $T_N$ is the multiplicity dependent temperature defined as

$$T_N = T + \Delta T (\langle N \rangle - N)$$

with $\Delta T$ controlling the correlation strength. For $N = \langle N \rangle$ one gets $T_N = T$. The inclusive transverse momentum distribution, which determines $z^2 = \overline{p_T^2} - \overline{p_T^2}$, reads

$$P_{\text{incl}}(p_T) = \frac{1}{\langle N \rangle} \sum_{N} \mathcal{P}_N N P_{(N)}(p_T),$$

where $\mathcal{P}_N$ is the multiplicity distribution.
The \( N \)-particle transverse momentum distribution in the events of multiplicity \( N \) is assumed to be the \( N \)-product of \( P_{(N)}(p_T) \). Therefore, all inter-particle correlations different than \( \langle p_T \rangle \) vs. \( N \) are entirely neglected. Then, one easily finds

\[
\langle Z^2 \rangle = \sum_{N} P_N \int_0^\infty dp_T^1 \ldots \int_0^\infty dp_T^N \left( p_T^1 + \ldots + p_T^N - Np_T^N \right)^2 P_{(N)}(p_T^1) \ldots P_{(N)}(p_T^N) .
\]

Assuming that the particles are massless and the correlation is weak i.e. \( T \gg \Delta T((N^2) - \langle N^2 \rangle)^{1/2} \) the calculation of \( \Phi \) can be performed analytically. The results read:

\[
\frac{\langle Z^2 \rangle}{\langle N \rangle} = 2T^2 - 4T\frac{\Delta T}{\langle N \rangle} (\langle N^2 \rangle - \langle N \rangle^2) \\
+ 2\frac{\Delta T^2}{\langle N \rangle^3} (2\langle N^4 \rangle\langle N \rangle^2 - 4\langle N^3 \rangle\langle N \rangle^2 + \langle N^3 \rangle\langle N \rangle^2 - 2\langle N^2 \rangle^2 + 2\langle N^2 \rangle^3 + \langle N \rangle^5) ,
\]

\[
\overline{z^2} = 2T^2 - 4T\frac{\Delta T}{\langle N \rangle} (\langle N^2 \rangle - \langle N \rangle^2) \\
+ 2\frac{\Delta T^2}{\langle N \rangle^3} (3\langle N^3 \rangle\langle N \rangle - 2\langle N^2 \rangle\langle N \rangle^2 + \langle N \rangle^4 - 2\langle N^2 \rangle^2) ,
\]

\[
\Phi(p_T) = \sqrt{2} \frac{\Delta T^2}{T^\langle N \rangle^\langle N \rangle^\langle N \rangle^\langle N \rangle^\langle N \rangle^\langle N \rangle} (\langle N^4 \rangle\langle N \rangle^2 - 2\langle N^3 \rangle\langle N \rangle^2 + \langle N^3 \rangle\langle N \rangle^2 + \langle N^2 \rangle^3 + \langle N^2 \rangle^2 + \langle N^2 \rangle) , \tag{2}
\]

where terms of the third and higher powers of \( \Delta T \) have been neglected. One observes that the lowest non-vanishing contribution to \( \Phi \) is of the second order in \( \Delta T \). The above formulas are much simplified for the Poisson multiplicity distribution. Then, one finds

\[
\frac{\langle Z^2 \rangle}{\langle N \rangle} = 2T^2 - 4T\Delta T + 2\Delta T^2 (2\langle N \rangle^2 + 5\langle N \rangle + 1) ,
\]

\[
\overline{z^2} = 2T^2 - 4T\Delta T + 2\Delta T^2 (3\langle N \rangle + 1) ,
\]

\[
\Phi(p_T) = \sqrt{2} \frac{\Delta T^2}{T^\langle N \rangle^\langle N \rangle} (\langle N^2 \rangle + \langle N \rangle) .
\]

The multiplicity distribution of charged particles produced in high energy proton-proton collisions is, of course, not poissonian. First of all, the number of charged particles is always even due to the charge conservation. The multiplicity distribution of positive (or negative) particles is also not poissonian - the dispersion does not grow as \( \sqrt{\langle N \rangle} \) but it follows the so-called Wróblewski formula \([10] \) i.e. the dispersion is the linear function of \( \langle N \rangle \). However, for the average multiplicities as low as those discussed here the Poisson distribution provides a reasonable approximation. Therefore, the multiplicity distribution, which is further used in our calculation, is poissonian for negative pions with the number of positive pions being exactly equal to that of negative charge. Thus, the charge conservation is satisfied in every event. For such a multiplicity distribution \( \Phi(p_T) \), which is given by approximate Eq. (2), reads

\[
\Phi(p_T) = 2\sqrt{2} \frac{\Delta T^2}{T} (\langle N \rangle^2 + 3\langle N \rangle) . \tag{3}
\]

**III. NUMERICAL SIMULATION**

The results from the previous section are instructive but the adopted approximations are very rough. So, let us present more realistic Monte Carlo calculations which can be confronted with the experimental data \([1] \). The proton-proton collisions are simulated event by event in the following way. For every event we first generate the multiplicity of negative particles from the Poisson distribution and then add the equal number of positive particles. The average multiplicity of negative particles has been taken as \( \langle N^- \rangle = 3.28 \) which is the experimentally observed negative multiplicity in non-diffractive proton-proton interactions at 205 GeV \([11] \). This is the collision energy corresponding
to the data from Fig. 1. Further, we attribute the transverse momentum from the distribution (1) to each particle assuming that all particles are pions. The numerical values of the temperature and correlation strength have been found fitting the data [8] shown in Fig. 1. We have got T = 167 ± 1.5 MeV and ΔT = 1.25 ± 0.25 for charged particle multiplicity ⟨N⟩ = 2⟨N⟩ = 6.56.

Due to the particle registration inefficiency and finite detector coverage of the final state phase-space, only a fraction of particles produced in high-energy collisions is usually observed in the experimental studies. As an input to our model calculations we have taken the ⟨pT⟩ − N correlation observed in the full phase-space. The rapidity, which determines the acceptance domain, is not considered at all. Therefore, there is no difference whether a particle is lost due to the limited acceptance or due to the tracking inefficiency. Our Monte Carlo simulation takes into account the two effects in such a way that each generated particle - positive or negative pion - is registered with the probability p and rejected with (1−p). Below, we further discuss the procedure in the context of NA49 data [1]. The results of our simulation are collected in Fig. 2 where Φ(pT) as a function of ΔT for several values of p is shown. We have checked that our simulation fully reproduces the results from [2], where only negative particles in the full acceptance have been studied.

As seen in Fig. 2, Φ(pT) grows quadratically with ΔT in agreement with the approximate equation (3). The increase of Φ(pT) with p also follows from Eq. (3). Indeed, when a particle is detected with the probability p the temperature dispersion effectively decreases and the multiplicity ⟨N⟩ should be replaced by p⟨N⟩ in (3). Consequently, Φ(pT) grows quadratically with the observed particle multiplicity and the correlation, which is easily observable in the full phase-space, is hardly seen in the acceptance as small as 20%. This behavior is very different than that found in Refs. [5,6] where Φ(pT) due to the Bose-Einstein correlations has been computed. Then, Φ(pT) is independent of the particle multiplicity.

The NA49 measurement of Φ(pT) in proton-proton collisions has been performed in the transverse momentum and pion rapidity intervals (0.005, 1.5) GeV and (4.0, 5.5), respectively [1]. Only about 20% of all produced particles have been observed. Our simulation gives Φ(pT) = 0.41 ± 0.07 MeV for the values of ΔT = 1.25 MeV and p = 0.2 which are adequate for the NA49 measurement. Thus, the theoretical result significantly underestimates the preliminary experimental one which is, as already noted, Φ(pT) = 5 ± 1 MeV [1].

One wonders whether the discrepancy is not caused by our highly simplified procedure of taking into account the effect of finite acceptance. We first note that the rapidity coverage of the NA49 measurement which is (4.0, 5.5) in the laboratory translates into (1.1, 2.6) in the center of mass frame. Therefore, the observed pions are not far the very central rapidity region. Since most of pions originate from the domain the average characteristics of all pions and that of the central ones are expected to be similar to each other. Indeed, the data from [8] show that ⟨pT⟩ for
all pions and those from the central region are essentially the same. Therefore, the correlation strength parameter \( \Delta T = 1.25 \text{ MeV} \), which corresponds to the \( \langle p_T \rangle - N \) correlation averaged over full phase-space, seem to be applicable not only to all pions but to the central ones as well. One further notes that \( \langle p_T \rangle \) shown in [8] changes with the rapidity similarly for different \( N \). Consequently, the correlation \( \langle p_T \rangle \) \( \text{vs.} \) \( N \) only weakly varies with \( y \) and it is hard to expect that the correlation strength observed in the NA49 acceptance domain is significantly larger than that averaged over the whole phase-space. The data [8] suggests rather the opposite effect. Therefore, we conclude that our simplified procedure cannot distort the results dramatically and that the correlation \( \langle p_T \rangle \) \( \text{vs.} \) \( N \) does not explain the NA49 p-p preliminary data.

As already mentioned, \( \Phi \) is constructed is such a way that it is exactly the same for nucleon-nucleon and nucleus-nucleus collisions if the latter is a simple superposition of former ones. Therefore, \( \Phi(p_T) \) shown in Fig. 2 holds for nucleus-nucleus when all secondary interactions are neglected. Since our model neglects the Bose-Einstein correlations it can be compared with the NA49 data for the central Pb-Pb collisions when the short range correlations are excluded. In such a case, our result \( \Phi(p_T) = 0.41 \pm 0.07 \text{ MeV} \) for \( \Delta T = 1.25 \text{ MeV} \) and \( p = 0.2 \) appears to be compatible with the experimental value \( \Phi(p_T) = 0.6 \pm 1 \text{ MeV} \) [1]. The smallness of \( \Phi(p_T) \) reported by NA49 collaboration is then not surprising at all. As follows from Fig. 2 it is caused by the limited acceptance. However, it is premature to draw a conclusion about \( p_T \)–correlations in Pb-Pb collisions until the origin of \( \Phi(p_T) \) p-p interactions is not explained.

**IV. CONCLUDING REMARKS**

We have studied how the correlation, which couples the average \( p_T \) to the event multiplicity, influences the transverse momentum fluctuations observed by means of the \( \Phi \)–measure. The approximate analytical formula has been derived and then the numerical simulation has been performed. The effect of the finite detector acceptance has been taken into account. The procedure is highly simplified but it seems to be adequate for the NA49 data. It has been shown that the effect of the correlation \( \langle p_T \rangle \) \( \text{vs.} \) \( N \) is very weak if the particles from the small acceptance region are studied. Consequently, the correlation is far too weak to explain the preliminary experimental value of \( \Phi(p_T) \) in proton-proton collisions [1]. If the preliminary data is confirmed by the final analysis one should look for other sources of dynamical \( p_T \)–fluctuations. The effect of the conservation laws presumably plays no role in the acceptance as small as 20%. The decays of hadron resonances which strongly correlate the momenta of decay products might be important. However, the hadron resonances are present in proton-proton and nucleus-nucleus collisions as well. Consequently, the reported reduction of long range dynamical correlations in the central Pb-Pb collisions when compared to p-p [4] remains unexplained. The situation is much simpler if preliminary data on \( \Phi(p_T) \) in proton-proton collision [1] overestimates the real value. Then, \( \Phi(p_T) \) from p-p and central Pb-Pb are close to each other. There is also no conflict between our calculations and the experimental data. However, one cannot conclude that the long range correlations present in the proton-proton interactions are washed out in the central heavy-ion collisions. The problem obviously needs further experimental and theoretical studies. In particular, the data on proton-proton and nucleus-nucleus collisions from the enlarged acceptance are needed.

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[1] H. Appelshäuser et al., Phys. Lett. B 459, 679 (1999).
[2] M. Gaździcki and St. Mrówczyński, Z. Phys. C 54, 127 (1992).
[3] T.A. Trainor, hep-ph/0001148.
[4] O.V. Utyuzh, G. Wilk, and Z. Włodarczyk, hep-ph/0103158 to appear in Phys. Rev. C.
[5] St. Mrówczyński, Phys. Lett. B 439, 6 (1998).
[6] St. Mrówczyński, Phys. Lett. B 465, 8 (1999).
[7] V.V. Aivazyan et al., Phys. Lett. B 209, 103 (1988).
[8] T. Kafka et al., Phys. Rev. D 16, 1261 (1977).
[9] S. Barish et al., Phys. Rev. D 9, 2689 (1974).
[10] A. Wróblewski, Acta Phys. Pol. B 4, 857 (1973).