Estimating the Time Parameters of a Free-form Radio Pulse with Unknown Initial Phase

Oleg V. Chernoyarov1,2,3,4,* , Alexandra V. Salnikova1,2,3, Yury E. Korchagin5 and Alexander A. Makarov1,2

1National Research University “Moscow Power Engineering Institute”, 14, Krasnokazarmennaya st., 111250 Moscow, Russia
2National Research Tomsk State University, 36, Lenin Avenue, 634050 Tomsk, Russia
3University of Zilina, Univerzitna 8215/1, 010 26 Zilina, Slovakia
4Maikop State Technological University, 191, Pervomayskaya st., 385000 Maikop, Russia
5Voronezh State University, 1, Universitetskaya sq., 394018 Voronezh, Russia

*Corresponding author

Abstract—This paper focuses on synthesis and analysis of the algorithms for estimating the time of appearance and the duration of a narrow-band radio signal with arbitrary-function envelope and unknown initial phase, such algorithms provide overcoming the prior parametric uncertainty in different ways. We tested performance of quasi-likelihood, maximum likelihood and quasi-optimal estimation algorithms. One of our objectives was to find the closed analytical expressions for biases, variances and correlation coefficients of the estimates of time of appearance and duration that their accuracy would increase with signal-to-noise ratio. We also determined the influence of prior ignorance of the initial signal phase on the quality of the introduced estimates of time of appearance and duration of a radio signal.

Keywords—narrow-band radio signal; time of appearance; signal duration; unknown initial phase; quasi-likelihood estimate; maximum likelihood estimate; quasi-optimal reception.

I. INTRODUCTION

Receiving a signal with unknown time of appearance and duration is a common task, it is of great interest for the location and communication theory, seismology, radio astronomy, etc. In papers [1-4], on the basis of the maximum likelihood (ML) method the estimates of time of appearance and duration were synthesized of a video pulse of rectangular and arbitrary form. There was presupposed there that such signal appears and disappears spasmodically. In [1], the quasi-likelihood (QL) algorithm was presented that measures time of appearance and duration of the high-frequency signal with rectangular and arbitrary form. In paper [4], synthesis and analysis of the estimates of time of appearance and duration were carried out when receiving a video pulse of arbitrary form with unknown amplitude. The influence was studied of the prior amplitude ignorance on the accuracy of the estimated time parameters. However, in many practical applications, we do not know the initial phase of the high-frequency (radio) signals due to the unpredictable time shift during their propagation. In [5], QL, ML and quasi-optimal algorithms were studied for estimating the time of appearance and the duration of the high-frequency signal with rectangular envelope and unknown initial phase. Below the results [5] are generalized for the case of receiving the radio pulse with a free-form envelope.

II. THE PROBLEM STATEMENT

We now start with determining mathematically the high-frequency signal with envelope that can be described by the continuous function \( f(t) \) in the following way:

\[
s(t, \varphi_0, \lambda_0, \tau_0) = a(t) I\left(t - \lambda_0, \tau_0 \right) \cos(\omega t - \varphi_0). \tag{1}
\]

Here \( I(x) \) is the unit duration indicator: \( I(x) = 1 \), if \( |x| \leq 1/2 \), and \( I(x) = 0 \), if \( |x| > 1/2 \), and we have \( a \) for the amplitude, \( \omega \) – for the carrier frequency and \( \varphi_0 \) – for the initial phase, while \( \lambda_0 \) and \( \tau_0 \) are the signal time of appearance and duration, respectively. We presuppose that the signal initial phase, time of appearance and duration are a priori unknown and that they possess the values from the following prior intervals:

\[
\varphi_0 \in \left[0, 2\pi \right], \quad \lambda_0 \in \left[-\Lambda_0/2, \Lambda_0/2 \right], \quad \tau_0 \in \left[T_{\min}, T_{\max} \right].
\]

In order to prevent the signal disappearance before it is located, we require the fulfillment of the inequality \( \Lambda_0 \leq T_{\min} \). We would also presuppose that \( f(0) \neq 0 \), \( i = 1, 2 \), where \( \theta_{01} = \lambda_0 - \tau_0/2 \) and \( \theta_{02} = \lambda_0 + \tau_0/2 \) are the rising and trailing signal edges, respectively, so the signal (1) appears and disappears spasmodically, i.e. it is discontinuous [6, 7].

Following [6, 7], we approximate interferences and inaccuracy recording by Gaussian white noise \( n(t) \) with the one-sided spectral density \( N_0 \). With the realization

\[
\xi(t) = s(t, \varphi_0, \lambda_0, \tau_0) + n(t)
\]
observed over the time interval $[-T/2, T/2]$, there can be

generated the joint estimates of the signal (1) time of
appearance and duration.

When the initial phase of the signal (1) is a priori known,
we can apply the ML estimation algorithm [6]. Such a
measurer finds the estimates of time of appearance and
duration as the coordinates of the position of the absolute
(greatest) maximum of the FLR log, i.e. the logarithm of the
functional of the likelihood ratio [6, 7]. In case initial phase,
time of appearance and duration are unknown, the FLR log
depends on the three current values $\varphi, \lambda, \tau$ of the unknown
parameters $\varphi_0, \lambda_0, \tau_0$ and, according to [6, 7], it can be
presented in the form of

$$L(\varphi, \lambda, \tau) = \frac{2a}{N_0} \int_{-\varphi/2}^{\varphi/2} \xi(t)f(t)\cos(\omega t - \varphi)dt - \frac{a^2}{N_0} \int_{-\varphi/2}^{\varphi/2} f^2(t)\cos^2(\omega t - \varphi)dt.$$  

(2)

III. THE QUASI-LIKELIHOOD ESTIMATION ALGORITHM

In order to overcome the prior uncertainty concerning the
initial phase, we can apply QL estimation algorithm [8].
According to it, the FLR log is formed for some expected value
$\varphi^*$ of the initial phase $\varphi_0$ and for all the possible values of
time of appearance and duration:

$$L' (\lambda, \tau) = L(\varphi^*, \lambda, \tau).$$  

(3)

Then QL estimates of time of appearance and duration are
determined as the coordinates of the position of the absolute
maximum of the random field (3):

$$(\lambda^*, \tau^*) = \arg \sup_L L'(\lambda, \tau).$$  

(4)

Similarly to [2-5], we introduce the new variables

$$\theta_i = \lambda - \tau/2, \quad \theta_2 = \lambda + \tau/2$$  

(5)

that are the current values of rising and trailing edges of the
pulse (1), respectively. And we designate their definitional
domain as $\Theta$. It can be seen that the linear transformations (5)
are one-to-one. Therefore, if we determine the characteristics of
QL estimates of the positions of rising and trailing edges $\theta_i^*$
and $\theta_2^*$, then the characteristics of QL estimates of time of
appearance and duration

$$\lambda^* = \left(\theta_2^* + \theta_1^*\right)/2, \quad \tau^* = \theta_2^* - \theta_1^*.$$  

(6)

can be easily found.

In variables $\theta_1$ and $\theta_2$, the FLR log (2) can be presented as

$$L_1(\theta_1, \theta_2) = L_0(\theta_1) + L_2(\theta_2),$$  

(7)

where

$$L_j(\theta_j) = \left(-1\right)^j \frac{2a}{N_0} \int_0^{\theta_j} \xi(t)f(t)\cos(\omega t - \varphi^*)dt + \left(-1\right)^j \frac{a^2}{N_0} \int_0^{\theta_j} f^2(t)\cos^2(\omega t - \varphi^*)dt, \quad j = 1, 2,$$  

(8)

while $\theta$ is the arbitrary fixed point from the interval

$[-(T_1 - \Lambda_0)/2, (T_1 - \Lambda_0)/2]$. The QL estimates $\theta_1^*$
and $\theta_2^*$ are determined as the coordinates of the position of the absolute
maximum of the random field (7) under $(\theta_1, \theta_2) \in \Theta$. To
simplify the implementation of the estimation algorithm as well
as the calculation of the characteristics of the estimates, by
analogy with [2-5], we now expand the prior range
$\Theta_0$ to the square of the minimum area $\Theta$, that has sides parallel to the
axes $\theta_1, \theta_2$ and includes the range $\Theta$. We can easily see that the
range $\Theta_0$ is set by the inequalities

$$0_{\min} \leq \theta_i \leq 0_{\max}, \quad i = 1, 2,$$

$$0_{\min} = -\beta, \quad 0_{\max} = -\alpha, \quad 0_{2\min} = \alpha, \quad 0_{2\max} = \beta,$$

$$\alpha = (T_{\min} - \Lambda_0)/2, \quad \beta = (T_{\max} - \Lambda_0)/2.$$  

As the expressions from (8) include the integrals from
Gaussian white noise in nonoverlapping intervals, the processes
$L_1(\theta_1)$ and $L_2(\theta_2)$ are statistically independent Gaussian
random ones. Thus, while determining the position of the absolute
maximum of the random field $L'(\theta_1, \theta_2)$ by variables
$\theta_1$ and $\theta_2$, we get two separate positions of the maxima of the
random processes (8):

$$\theta_i^* = \arg \sup L_i(\theta_i), \quad i = 1, 2, \quad (\theta_1, \theta_2) \in \Theta_0.$$  

(9)

In paper [9], the probability densities are found of the
estimates (9) in the form of

$$w_i^*(\theta_i | \theta_j) = w_0^* (\theta_i | \theta_j) \left| dl_j/d\theta_j \right|,$$  

where
where the probability densities of the random variables \( l_j \) related to the estimates (9) by one-to-one transformations \( l_j = (-1)^j Q(0,0_j) \), \( j \in [I_{min}/I_{max}] \). Here \( I_{min} = (-1)^j Q(0,0_{jmin}) \), \( I_{max} = (-1)^j Q(0,0_{jmax}) \), \( 0_{j} = (-1)^j Q(0,0_{j0}) \),

\[
\mathcal{Q}(0_1,0_2) = \frac{\alpha_0^2}{\nu_0} \int_0^{\pi/2} f^2(t) \, dt \tag{10}
\]

is the output power signal-to-noise ratio (SNR) for the ML receiver, \( \nu_0 = \lambda_0 + (-1)^j \tau_0/2 \), \( j = 1, 2 \), \( R = 2 \cos \Delta \phi - 1 \), \( \Delta \phi = (\varphi^* - \varphi_0) \) is the initial phase detuning of the received signal in the QL measurer,

\[
\Psi(y_1,y_2,y_3) = \frac{1}{2^{11/2} \pi^{3/2}} \left[ \frac{1}{\sqrt{2}} \exp\left(-\frac{y_1^2+y_2^2}{4} \right) \right] \times
\]

\[
\times \exp\left(-\frac{(x+y)^2}{4y} \right) \left[ \frac{1}{\sqrt{2y}} \exp\left(-\frac{y_3x+y_3y}{2y} \right) \right] \, dx.
\]

In [9], the asymptotic expressions are also found for biases and variances of QL estimates of the positions of rising and trailing edges (9)

\[
B[0^*_j,0_{j0}] = \langle 0^* - 0_0 \rangle = (-1)^j 2T_{max} P_{\theta}(R)/\rho^2_j,
\]

\[
V[0^*_j,0_{j0}] = \langle (0^* - 0_0)^2 \rangle = 8T_{max}^2 P_{\theta}(R)/\rho^4_j,
\]

where \( \rho^2_j = 2f^2(0_{j0})T_{max}/\nu_0 \), \( P_{\theta}(R) = (R^2 - 1)/R^2 \), \( P_{\theta}(R) = (R^2(2R^2 + 6R + 5) + 5R^2 + 6R + 2)/R^4(R + 1)^3 \).

According to (5), (6), biases and variances of the estimates (4), as well as the correlation coefficient of the errors of estimates, can be expressed through biases and variances of QL estimates of the moments of appearance and disappearance in the following way:

\[
B[\lambda^*,\tau_0] = \left\{ \begin{array}{ll}
B[0^*_1,0_{10}]/2 = T_{max} P_{\theta}(R)/\rho^2_1, \\
B[0^*_2,0_{20}]/2 = 2T_{max}^2 P_{\theta}(R)/\rho^4_2,
\end{array} \right.
\]

\[
V[\lambda^*,\tau_0] = \left\{ \begin{array}{ll}
V[0^*_1,0_{10}] = 8T_{max}^2 P_{\theta}(R)/\rho^4_1, \\
V[0^*_2,0_{20}] = 8T_{max}^2 P_{\theta}(R)/\rho^4_2,
\end{array} \right.
\]

Then we introduce the normalized biases

\[
b_{\lambda}(\Delta \phi) = B[\lambda^*,\tau_0]/\sqrt{V[\lambda^*,\tau_0]}, \quad b_{\tau}(\Delta \phi) = B[\tau^*,\tau_0]/\sqrt{V[\tau^*,\tau_0]}
\]

and the normalized variances

\[
v = v_{\lambda} = V[\lambda^*,\tau_0]/V[0^*_0], \quad v_{\tau} = V[\tau^*,\tau_0]/V[0^*_0].
\]
The value of $v$ shows how many times the variances of QL estimates of time of appearance and duration are greater than the variances of ML estimates of the same parameters of the signal with a priori known initial phase. In other words, the value of $v$ characterizes the loss in accuracy that QL estimation algorithm (4) demonstrates in comparison with the corresponding ML estimation algorithm.

In Fig. 1, the dependences are presented of the normalized estimate of time of appearance $b_\lambda$ on the detuning by the initial phase $\Delta\varphi$ under various $q$ (12). Curve 1 is calculated for the value $q = 10$, curve 2 for $q = 1.25$, curve 3 for $q = 0.5$, curve 4 for $q = 0.8$, curve 5 for $q = 0.1$. In Fig. 2, the dependences are drawn of the normalized bias of the estimate of duration $b_\tau$ on the detuning by the initial phase $\Delta\varphi$ under various values of the parameter $q$. Solid line is calculated for $q = 0.1$ and $q = 10$, dashed line – for $q = 0.5$ and $q = 2$, dash-dotted line – for $q = 0.8$ and $q = 1.25$.

In Fig. 3, the matching dependences are plotted of the normalized variances of the estimates of time of appearance $v_\lambda$ and duration $v_\tau$ on the values $\Delta\varphi$ under $q = 1$. When the value of the parameter $q$ is changing, curve in Fig. 3 transforms insignificantly.

As we can see from Figs. 1-3, biases of the estimates of time of appearance $b_\lambda$ and duration $b_\tau$ strongly depend on the value of the parameter $q$. If detuning by the initial phase is absent ($\Delta\varphi = 0$), then the QL estimates of the time parameters the signal (1) have zero biases while their variances coincide with the variances of the corresponding ML estimates. The presence of detuning by the initial phase results in both biases of the estimates of time of appearance and duration and increase of their variances by ten times.

**IV. MAXIMUM LIKELIHOOD ESTIMATION ALGORITHM**

In order to improve the accuracy of measuring the time of appearance and the duration, we can apply ML estimation algorithm. In this case, the unknown initial phase in the expression (2) is substituted by its ML estimate $\varphi_m$. It is equivalent to the maximization of the FLR log (2) by initial phase:

$$L(\lambda, \tau) = L(\varphi_m, \lambda, \tau) = \max_\varphi L(\varphi, \lambda, \tau). \quad (14)$$

Then, ML estimates of time of appearance and duration are determined as the coordinates of the position of the absolute maximum of decision statistics (14):

$$\left(\lambda_m, \tau_m\right) = \arg\max L(\lambda, \tau). \quad (15)$$

The maximization of the FLR log (2) by the variable $\varphi$ can be implemented analytically. For this purpose, we substitute the
signal (1) into the expression (2) and present the FLR log in the form of

\[ L(\phi, \lambda, \tau) = X(\lambda, \tau) \cos \phi + Y(\lambda, \tau) \sin \phi - \frac{a^2}{2N_0} \int f^2(t) dt \]  

(16)

Here the designations

\[ \left\{ \begin{array}{l}
X(\lambda, \tau) = \frac{a^2}{N_0} \int \xi(t)f(t)\cos(\omega t - \phi) dt \\
Y(\lambda, \tau) = \frac{a^2}{N_0} \int \xi(t)f(t)\sin(\omega t - \phi) dt
\end{array} \right. \]

are introduced and the integrals from the functions oscillating with double frequency are neglected. Now we find the derivative of the function (16) by \( \phi \), equate it with zero

\[ dL(\phi, \lambda, \tau)/d\phi = -X(\lambda, \tau) \sin \phi + Y(\lambda, \tau) \cos \phi = 0 \]

and then solve the obtained equation with regard to \( \phi \):

\[ \phi = \arctg[Y(\lambda, \tau)/X(\lambda, \tau)] \]  

By substituting the computed solution into (16) we get

\[ L(\lambda, \tau) = \sqrt{X^2(\lambda, \tau) + Y^2(\lambda, \tau)} - \frac{a^2}{2N_0} \int f^2(t) dt \]  

(17)

Thus, based on the expression (17), we can define the structure of ML measurer. The measurer should form the random field (17) and find the desired estimates as the coordinates of its absolute maximum. The decision statistics (17) cannot be practically implemented as the analog function of time of appearance and duration. Therefore, the measurer should produce the samples \( L_{\text{mg}} = L(\lambda, \tau) \) of the random field (17) for the discrete set of its arguments values. Here \( \Delta \lambda, \Delta \tau \) are the discretization steps sampled by time of appearance and duration, \( m = 1, 2, \ldots, n_1 \) and \( g = 1, 2, \ldots, n_2 \) are integers and \( n_1 \times n_2 \) is the number of measurable channels. It is obvious that the more accurate estimation of the time parameters we want to achieve, the greater number of channels the measurer design requires.

The random field (16) depends on the regular parameter \( \phi \) and two discontinuous parameters \( \lambda \) and \( \tau \). In [10, 11], it is stated that, in the conditions of high posterior accuracy, i.e. under big SNR (10), the characteristics of ML estimates of the discontinuous parameters appear to be asymptotically the same in both cases – when we know the value of the regular parameter and when we do not. Therefore, under big SNR, the variances of ML estimates of time of appearance and duration of a radio signal are the same both when the initial phase of the signal is unknown and when it is a priori known. Thus, the characteristics of ML estimates (15) under the unknown initial phase and big SNR are determined by the formulas (13).
\begin{align}
X_j(\theta_j) &= (-1)^j \frac{\alpha_0^2}{2N_0} \int_0^t \xi(t) f(t) \cos(\omega t) \, dt, \\
Y_j(\theta_j) &= (-1)^j \frac{\alpha_0^2}{2N_0} \int_0^t \xi(t) f(t) \sin(\omega t) \, dt. \tag{22}
\end{align}

The expressions (20)-(22) determine the structure of the measurer intended for producing the quasi-optimal estimates of time of appearance and duration

\[ \lambda^*_m = \left( \frac{\theta^*_m + \theta^*_m}{2} \right), \quad \tau^*_m = \theta^*_m - \theta^*_m. \tag{23} \]

As it can be seen, the application of the estimates makes it possible to simplify the measurer practical implementation significantly. Indeed, in order to implement ML estimation significantly, the multichannel measurer is required while the estimates (23) can be formed by means of the two-channel circuit.

The characteristics of the estimates (20) are found in [9]:

\[ B_0(0_0^*, 0_0^*) = 0, \quad V_0(0_0^*, 0_0^*) = 26T^2_{\text{max}}/\rho_0^2. \tag{24} \]

Then, from (24), for the asymptotic characteristics of the quasi-optimal estimates (23) of the signal (1) time of appearance and duration, we have

\begin{align}
B_0(\lambda^*_m, \tau_0^*) &= B_0(\theta^*_m, \tau_0) = 0, \\
V_0(\lambda^*_m, \tau_0^*) &= 13T^2_{\text{max}}/\rho_1^2 + \rho_2^4/\rho_1^2, \\
V_0(\tau^*_m, \lambda^*_m, \tau_0^*) &= 26T^2_{\text{max}}/\rho_1^2 + \rho_2^4/\rho_1^2. \tag{25} \end{align}

The expressions (25) coincide with the like expressions for biases and variances of ML estimates of time of appearance and duration, while the signal initial phase is a priori known, having been found in [3]. Therefore, the accuracy of the quasi-optimal estimates (23) coincides asymptotically with the accuracy of ML estimates when the signal initial phase is known. Thus, the dependences drawn in Figs 1-3 also demonstrate a gain in accuracy of the quasi-optimal estimates in comparison with QL estimates, and we can see that this gain may be significant.

VI. CONCLUSION

We have synthesized the quasi-likelihood, maximum likelihood and quasi-optimal algorithms for estimating the time of appearance and the duration of the narrow-band radio signal with the unknown initial phase. We have found the asymptotically exact (with SNR increasing) expressions for the characteristics of the introduced algorithms. It was established that the maximum likelihood and quasi-likelihood algorithms are effective and provide the same performance in the conditions of the high posterior accuracy. However, hardware or software implementation of the maximum likelihood measurer is a much more complex task than the like implementation of the quasi-likelihood measurer. We have shown that the prior ignorance of the radio signal initial phase under high signal-to-noise ratios does not asymptotically influence the accuracy of the maximum likelihood and quasi-optimal estimates of the radio signal time of appearance and duration.

The obtained results allow us to make a choice between the considered measurers, depending on the available prior information as well as the requirements to the estimates accuracy and to the simplicity of the measurer practical implementation.

ACKNOWLEDGMENT

This study was financially supported by the Council on grants of the President of the Russian Federation (research project No. SP-834.2019.3).

REFERENCES

[1] O. V. Chernoyarov, Yu. A. Kutoyants, and B. I. Shakhitdin, “The new approach to the synthesis of single-channel consistent estimates of the time signal parameters,” in Proc. International Conference on Mechatronics, Manufacturing and Materials Engineering. Hong Kong, China, June 11-12, 2016, pp. 1-7.

[2] A. P. Trifonov, and V. K. Butyko, “Joint estimation of two parameters of an analog signal against a white-noise background,” Soviet Journal of Communications Technology & Electronics, vol. 35, no. 12, pp. 25-32, December 1990.

[3] A. P. Trifonov, and Yu. E. Korchagin, “Estimation of the time of appearance and duration of a signal,” Electromagnetic Waves and Electronic Systems (in Russian), vol. 17, no. 7, pp. 4-15, 2012.

[4] A. P. Trifonov, Yu. E. Korchagin, and P. A. Kondratovich, “Estimations of appearance time and duration of a signal with unknown amplitude,” Electromagnetic Waves and Electronic Systems (in Russian), vol. 17, no. 7, pp. 4-15, 2012.

[5] A. P. Trifonov, O. V. Chernoyarov, Yu. E. Korchagin, and S. V. Korolkov, “Estimating time of appearance and duration of the rectangular radio signal with the unknown initial phase,” in Proc. 4th International Conference on Manufacturing Engineering and Technology for Manufacturing Growth. Singapore, Singapore, May 7-8, 2017, pp. 87-94.

[6] E. I. Kulikov, A. P. Trifonov, Signal Parameter Estimation against Hindrances (in Russian). Moscow: Sovetskoe Radio, 1978.

[7] A. P. Trifonov, Yu. S. Shirnakov, Joint Discrimination of Signals and Estimation of Their Parameters against Background (in Russian). Moscow: Radio i Svyaz, 1986.

[8] V. I. Mudrov, V. L. Kushko, Methods of Measurements Processing. Quasi-Likelihood Estimates (in Russian). Moscow: Radio i Svyaz, 1983.

[9] A. P. Trifonov, Yu. E. Korchagin, M. V. Trifonov, and O. V. Chernoyarov, “Estimate of the moments of appearance and disappearance of a radio signal with unknown initial phase,” Walenia, vol. 22, no. 9, pp. 105-116, September 2015.

[10] A. P. Trifonov and V. K. Butyko, “Characteristics of joint estimations of signal parameters under partial breach of regularity conditions,” Radiotekhnika i Elektronika, vol. 36, no. 2, pp. 319-327, February 1991.

[11] O. V. Chernoyarov, Yu. A. Kutoyants, and A. P. Trifonov, “On misspecifications in regularity and properties of estimators,” Electronic Journal of Statistics, vol. 12, no. 1, pp. 80-106, January 2018.