Opportunist Cooperation in Cognitive Femtocell Networks

Rahul Urgaonkar, Michael J. Neely

Abstract—We investigate opportunistic cooperation between unlicensed secondary users and legacy primary users in a cognitive radio network. Specifically, we consider a model of a cognitive network where a secondary user can cooperatively transmit with the primary user in order to improve the latter’s effective transmission rate. In return, the secondary user gets more opportunities for transmitting its own data when the primary user is idle. This kind of interaction between the primary and secondary users is different from the traditional dynamic spectrum access model in which the secondary users try to avoid interfering with the primary users while seeking transmission opportunities on vacant primary channels. In our model, the secondary users need to balance the desire to cooperate more (to create more transmission opportunities) with the need for maintaining sufficient energy levels for their own transmissions. Such a model is applicable in the emerging area of cognitive femtocell networks. We formulate the problem of maximizing the secondary user throughput subject to a time average power constraint under these settings. This is a constrained Markov Decision Problem and conventional solution techniques based on dynamic programming require either extensive knowledge of the system dynamics or learning based approaches that suffer from large convergence times. However, using the technique of Lyapunov optimization, we design a novel greedy and online control algorithm that overcomes these challenges and is provably optimal.

Index Terms—Resource Allocation, Opportunistic Cooperation, Cognitive Radio, Femtocell Networks, Optimal Control

I. INTRODUCTION

Much prior work on resource allocation in cognitive radio networks has focused on the dynamic spectrum access model [1], [2] in which the secondary users seek transmission opportunities for their packets on vacant primary channels in frequency, time, or space. Under this model, the primary users are assumed to be oblivious of the presence of the secondary users and transmit whenever they have data to send. Secondly, a collision model is assumed for the physical layer in which if a secondary user transmits on a busy primary channel, then there is a collision and both packets are lost. We considered a similar model in our prior work [3] where the objective was to design an opportunistic scheduling policy for the secondary users that maximizes their throughput utility while providing tight reliability guarantees on the maximum number of collisions suffered by a primary user over any given time interval. We note that this formulation does not consider the possibility of any cooperation between the primary and secondary users. Further, it assumes that the secondary user activity does not affect the primary user channel occupancy process.

There is a growing body of work that investigates alternate models for the interaction between the primary and secondary users in a cognitive radio network. In particular, the idea of cooperation at the physical layer has been considered from an information-theoretic perspective in many works (see [4] and the references therein). These are motivated by the work on the classical interference and relay channels [5]–[8]. The main idea in these works is that the resources of the secondary user can be utilized to improve the performance of the primary transmissions. In return, the secondary user can obtain more transmission opportunities for its own data when the primary channel is idle.

These works mainly treat the problem from a physical layer/information-theoretic perspective and do not consider upper layer issues such as queuing dynamics, higher priority for primary user, etc. Recent work that addresses some of these issues includes [9]–[13]. Specifically, [9] considers the scenario where the secondary user acts as a relay for those packets of the primary user that it receives successfully but which are not received by the primary destination. It derives the stable throughput of the secondary user under this model. [10], [11] use a Stackelberg game framework to study spectrum leasing strategies in cooperative cognitive radio networks where the primary users lease a portion of their licensed spectrum to secondary users in return for cooperative relaying. [12], [13] study and compare different physical layer strategies for relaying in such cognitive cooperative systems. An important consequence of this interaction between the primary and secondary users is that the secondary user activity can now potentially influence the primary user channel occupancy process. However, there has been little work in studying this scenario. Exceptions include the work in [14] that considers a two-user setting where collisions caused by the opportunistic transmissions of the secondary user result in retransmissions by the primary user.

In this paper, we study the problem of opportunistic cooperation in cognitive networks from a network utility maximization perspective, specifically taking into account the above mentioned higher-layer aspects. To motivate the problem and illustrate the design issues involved, we first consider a simple network consisting of one primary and one secondary user and their respective access points in Sec. II. This can model

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a practical scenario of recent interest, namely a cognitive femtocell [15], [16], as discussed in Sec. III. We assume that the secondary user can cooperatively transmit with the primary user to increase its transmission success probability. In return, the secondary user can get more opportunities for transmitting its own data when the primary user is idle. We formulate the problem of maximizing the secondary user throughput subject to time average power constraints in Sec. IV.

Unlike most of the prior work on resource allocation in cognitive radio networks, the evolution of the system state for this problem depends on the control actions taken by the secondary user. Here, the system state refers to the channel occupancy state of the primary user. Because of this dependence, this problem becomes a constrained Markov Decision Problem (MDP) and the greedy “drift-plus-penalty” minimization technique of Lyapunov optimization [17] that we used in [3] is no longer optimal. Such problems are typically tackled using Markov Decision Theory and Dynamic Programming [23], [24]. For example, [14] uses these tools to derive structural results on optimal channel access strategies in a similar two-user setting where collisions caused by the opportunistic transmissions of the secondary user cause the primary user retransmit its packets. However, this approach requires either extensive knowledge of the dynamics of the underlying network state (such as state transition probabilities) or learning based approaches that suffer from large convergence times.

Instead, in Sec. III, we use the recently developed framework of maximizing the ratio of the expected total reward over the expected length of a renewal frame [19]–[21] to design a control algorithm. This framework extends the classical Lyapunov optimization method [17] to tackle a more general class of MDP problems where the system evolves over renewals and where the length of a renewal frame can be affected by the control decisions during that period. The resulting solution has the following structure: Rather than minimizing a “drift-plus-penalty” term every slot, it minimizes a “drift-plus-penalty ratio” over each renewal frame. This can be achieved by solving a sequence of unconstrained stochastic shortest path (SSP) problems and implementing the solution over every renewal frame.

While solving such SSP problems can be simpler than the original constrained MDP, it may still require knowledge of the dynamics of the underlying network state. Learning based techniques for solving such problems by sampling from the past observations have been considered in [18]. However, these may suffer from large convergence times. Remarkably, in Sec. IV, we show that for our problem, the “drift-plus-penalty ratio” method results in an online control algorithm that does not require any knowledge of the network dynamics or explicit learning, yet is optimal. In this respect, it is similar to the traditional greedy “drift-plus-penalty” minimizing algorithms of [17]. We then extend the basic model to incorporate multiple secondary users as well as time-varying channels in Sec. V. Finally, we present simulation results in Sec. VI.

II. BASIC MODEL

We consider a network with one primary user (PU), one secondary user (SU) and their respective base stations (BS). The primary user is the licensed owner of the channel while the secondary user tries to send its own data opportunistically when the channel is not being used by the primary user. This model can capture a femtocell scenario where the primary user is a legacy mobile user that communicates with the macro base station over licensed spectrum (Fig. 1). The secondary user is the femtocell user that does not have any licensed spectrum of its own and tries to send data opportunistically to the femtocell base station over any vacant licensed spectrum. Similar models of cooperative cognitive radio networks have been considered in [9]–[13]. This can also model a single server queueing system with two classes of arrivals where one class has a strictly higher priority over the other class.

We consider a time-slotted model. We assume that the system operates over a frame-based structure. Specifically, the timeline can be divided into successive non-overlapping frames of duration $T[k]$ slots where $k \in \{1, 2, 3, \ldots \}$ represents the frame number (see Fig. 2). The start time of frame $k$ is denoted by $t_k$ with $t_1 = 0$. The length of frame $k$ is given by $T[k] = t_{k+1} - t_k$. For each $k$, the frame length $T[k]$ is a random function of the control decisions taken during that frame. Each frame can be further divided into two periods: PU Idle and PU Busy. The “PU Idle” period corresponds to the slots when the primary user does not have any packet to send to its base station and is idle. The “PU Busy” period corresponds to the slots when the primary user is transmitting its packets to its base station over the licensed spectrum. As shown in Fig. 2, every frame starts with the “PU Idle” period which is followed by the “PU Busy” period and ends when the primary user becomes idle again. In the basic model, we assume that the primary user receives new packets every slot according to an i.i.d. Bernoulli arrival process $A_{pu}(t)$ with rate $\lambda_{pu}$ packets/slot. This means that the length of the “PU Idle” period of any frame is a geometric random variable with parameter $\lambda_{pu}$. However, the length of the “PU Busy” period depends on the secondary user control decisions as discussed below.

In any slot $t$, if the primary user has a non-zero queue backlog, it transmits one packet to its base station. We assume that the transmission of each packet takes one slot. If the transmission is successful, the packet is removed from the primary user queue. However, if the transmission fails, the
packet is retained in the queue for future retransmissions. The secondary user cannot transmit its packets when the channel is being used by the primary user. It can transmit its packets only during the “PU Idle” period of the frame and must stop its transmission whenever the primary user becomes active again. However, the secondary user can transmit cooperatively with the primary user in the “PU Busy” period to increase its transmission success probability. This has the effect of decreasing the expected length of the “PU Busy” period. In order to cooperate, the secondary user must allocate its power resources to help relay the primary user packet. This cooperation can take place in several ways depending on the cooperative protocol being used (see [12] for some examples). In this simple model, these details are captured by the resulting probability of successful transmission.

The reason why the secondary user may want to cooperate is because it can potentially increase the number of time slots in the future in which the primary user does not have any data to send as compared to a non-cooperative strategy. This can create more opportunities for the secondary user to transmit its own packets. However, note that the trivial strategy of cooperating whenever possible may lead to a scenario where the secondary user does not have enough power for its own data transmission. Thus, the secondary user needs to decide whether it should cooperate or not considering these two opposing factors.

The probability of a successful primary transmission depends on the control actions such as power allocation and cooperative transmission decisions by the secondary user. This is discussed in detail in the next section. In this model, we assume that the network controller cannot control the primary user actions. However, it can control the secondary user decisions on cooperation and the associated power allocation.

A. Control Decisions and Queueing Dynamics

Let \( Q_{pu}(t), Q_{su}(t) \in \{0, 1, 2, \ldots \} \) represent the primary and secondary user queues respectively in slot \( t \). New packets arrive at the secondary user according to an i.i.d. process \( A_{su}(t) \) of rate \( \lambda_{su} \) packets/slot respectively. We assume that there exists a finite constant \( A_{max} \) such that \( A_{su}(t) \leq A_{max} \) for all \( t \). Every slot, an admission control decision determines \( R_{su}(t) \), the number of new packets to admit into the secondary user queue. Further, every slot, depending on whether the primary user is busy or idle, resource allocation decisions are made as follows. When \( Q_{pu}(t) > 0 \), this represents the secondary user decision on cooperative transmission and the corresponding power allocation \( P_{su}(t) \). When \( Q_{pu}(t) = 0 \), this corresponds to the secondary user decision on its own transmission and the corresponding power allocation \( P_{su}(t) \). We assume that in each slot, the secondary user can choose its power allocation \( P_{su}(t) \) from a set \( \mathcal{P} \) of possible options. Further, this power allocation is subject to a long-term average power constraint \( P_{avg} \) and an instantaneous peak power constraint \( P_{max} \). For example, \( \mathcal{P} \) may contain only two options \( \{0, P_{max}\} \) which represents “Remain Idle” and “Cooperate/Transmit at Full Power”. As another example, \( \mathcal{P} = [0, P_{max}] \) such that \( P_{su}(t) \) can take any value between 0 and \( P_{max} \).

Suppose the primary user is active in slot \( t \) and the secondary user allocates power \( P(t) \) for cooperative transmission. Then the random success/failure outcome of the primary transmission is given by an indicator variable \( \mu_{pu}(P(t)) \) and the success probability is given by \( \phi(P(t)) = \mathbb{E}\{\mu_{pu}(P(t))\} \). The function \( \phi(P) \) is known to the network controller and is assumed to be non-decreasing in \( P \). However, the value of the random outcome \( \mu_{pu}(P(t)) \) may not be known beforehand. Note that setting \( P(t) = 0 \) corresponds to a non-cooperative transmission and the success probability for this case becomes \( \phi(0) \) and we denote this by \( \phi_{nc} \). Likewise, we denote \( \phi(P_{max}) \) by \( \phi_{c} \). Thus, \( \phi_{nc} \leq \phi(P(t)) \leq \phi_{c} \) for all \( P(t) \in \mathcal{P} \).

We assume that \( \lambda_{pu} \) is such that it can be supported even when the secondary user never cooperates, i.e., \( \lambda_{pu} < \phi_{nc} \). This means that the primary user queue is stable even if there is no cooperation. Further, for all \( k \), the frame length \( T[k] \geq 1 \) and there exist finite constants \( T_{min}, T_{max} \) such that under all control policies, we have:

\[
1 \leq T_{min} \leq \mathbb{E}\{T[k]\} \leq T_{max}
\]

Specifically, \( T_{min} \) can be chosen to be the expected frame length when the secondary user always cooperates with full power while \( T_{max} \) can be chosen to be the expected frame length when the secondary user never cooperates. Using Little’s Theorem, we have that:

\[
\frac{T_{min}}{T_{min} + 1/\lambda_{pu}} = \frac{\lambda_{pu}}{\phi_{c}}
\]

Similarly, we have:

\[
\frac{T_{max}}{T_{max} + 1/\lambda_{pu}} = \frac{\lambda_{pu}}{\phi_{nc}}
\]

Using these, we have:

\[
T_{min} \triangleq \frac{\phi_{c}}{(\phi_{c} - \lambda_{pu})\lambda_{pu}}, \quad T_{max} \triangleq \frac{\phi_{nc}}{(\phi_{nc} - \lambda_{pu})\lambda_{pu}}
\]

Finally, there exists a finite constant \( D \) such that the expectation of the second moment of a frame size, \( \mathbb{E}\{T^2[k]\} \), satisfies the following for all \( k \), regardless of the policy:

\[
\mathbb{E}\{T^2[k]\} \leq D
\]

This follows from the assumption that the primary user queue is stable even if there is no cooperation. In Appendix C, we exactly compute such a \( D \) that satisfies \( \mathbb{E}\{T^2[k]\} \leq D \).

When the primary user is idle in slot \( t \) and the secondary user allocates power \( P(t) \) for its own transmission, it gets a service rate given by \( \mu_{su}(P(t)) \). This can represent the success...
probability of a secondary transmission with a Bernoulli service process. This can also be used to model more general service processes. We assume that there exists a finite constant \( \mu_{\max} \) such that \( \mu_{\text{su}}(P) \leq \mu_{\max} \) for all \( P \in \mathcal{P} \).

Given these control decisions, the primary and secondary user queues evolve as follows:

\[
Q_{\text{pu}}(t + 1) = \max[Q_{\text{pu}}(t) - \mu_{\text{pu}}(P(t)), 0] + A_{\text{pu}}(t) \quad (3)
\]

\[
Q_{\text{su}}(t + 1) = \max[Q_{\text{su}}(t) - \mu_{\text{su}}(P(t)), 0] + R_{\text{su}}(t) \quad (4)
\]

where \( R_{\text{su}}(t) \leq A_{\text{su}}(t) \).

**B. Control Objective**

Consider any control algorithm that makes admission control decision \( R_{\text{su}}(t) \) and power allocation \( P(t) \) every slot subject to the constraints described in Sec. II-A. Note that if the primary queue backlog \( Q_{\text{pu}}(t) > 0 \), then this power is used for cooperative transmission with the primary user. If \( Q_{\text{pu}}(t) = 0 \), then this power is used for the secondary user’s own transmission. Define the following time-averages under this algorithm:

\[
\overline{R}_{\text{su}} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{R_{\text{su}}(\tau)\}
\]

\[
\overline{\mu}_{\text{su}} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P(\tau)\}
\]

\[
\overline{P}_{\text{su}} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\mu_{\text{su}}(P(\tau))\}
\]

where the expectations above are with respect to the potential randomness of the control algorithm. Assuming for the time being that these limits exist, our goal is to design a joint admission control and power allocation policy that maximizes the throughput of the secondary user subject to its average and peak power constraints and the scheduling constraints imposed by the basic model. Formally, this can be stated as a stochastic optimization problem as follows:

Maximize: \( \overline{P}_{\text{su}} \)

Subject to: \( 0 \leq R_{\text{su}}(t) \leq A_{\text{su}}(t) \forall t \)

\( P(t) \in \mathcal{P} \forall t \)

\( \overline{R}_{\text{su}} \leq \overline{\mu}_{\text{su}} \)

\( \overline{P}_{\text{su}} \leq \overline{P}_{\text{avg}} \quad (5) \)

It will be useful to define the primary queue backlog \( Q_{\text{pu}}(t) \) as the “state” for this control problem. This is because the state of this queue (being zero or nonzero) affects the control options as described before. Note that the control decisions on cooperation affect the dynamics of this queue. Therefore, problem (5) is an instance of a constrained Markov decision problem [24]. It is well known that in order to obtain an optimal control policy, it is sufficient to consider only the class of stationary, randomized policies that take control actions only as a function of the current system state (and independent of past history). A general control policy in this class is characterized by a stationary probability distribution over the control action set for each system state. Let \( \nu^* \) denote the optimal value of the objective in (5). Then using standard results on constrained Markov Decision problems [24]–[26], we have the following:

**Lemma 1:** (Optimal Stationary, Randomized Policy): There exists a stationary, randomized policy \( \overline{\nu}_{\text{su}}^* \) that takes control decisions \( R_{\text{su}}^\text{stat}(t), P_{\text{su}}^\text{stat}(t) \) every slot purely as a (possibly randomized) function of the current state \( Q_{\text{pu}}(t) \) while satisfying the constraints \( R_{\text{su}}^\text{stat}(t) \leq A_{\text{su}}(t), P_{\text{su}}^\text{stat}(t) \in \mathcal{P} \) for all \( t \) and provides the following guarantees:

\[
\overline{R}_{\text{su}}^\text{stat} = \nu^*
\]

\[
\overline{R}_{\text{su}}^\text{stat} \leq \overline{P}_{\text{su}}^\text{stat}
\]

\[
\overline{P}_{\text{su}}^\text{stat} \leq \overline{P}_{\text{avg}}
\]

where \( \overline{R}_{\text{su}}^\text{stat}, \overline{P}_{\text{su}}^\text{stat}, \overline{P}_{\text{avg}}^\text{stat} \) denote the time-averages under this policy.

We note that the conventional techniques to solve (5) that are based on dynamic programming [23] require either extensive knowledge of the system dynamics or learning based approaches that suffer from large convergence times. Motivated by the recently developed extension to the technique of Lyapunov optimization in [19]–[21], we take an different approach to this problem in the next section.

**III. SOLUTION USING THE "DRIFT-PLUS-PENALTY" RATIO METHOD**

Recall that the start of the \( k \)-th frame, \( t_k \), is defined as the first slot when the primary user becomes idle after the “PU Busy” period of the \( (k - 1) \)-th frame. Let \( Q_{\text{su}}(t_k) \) denote the secondary user queue backlog at time \( t_k \). Also let \( P(t) \) be the power expenditure incurred by the secondary user in slot \( t \). For notational convenience, in the following we will denote \( \mu_{\text{su}}(P(t)) \) by \( \mu_{\text{su}}(t) \) noting the dependence on \( P(t) \) is implicit. Then the queueing dynamics of \( Q_{\text{su}}(t_k) \) satisfies the following:

\[
Q_{\text{su}}(t_{k+1}) = \max[Q_{\text{su}}(t_k) - \sum_{t=t_k}^{t_{k+1}-1} \mu_{\text{su}}(t), 0] + \sum_{t=t_k}^{t_{k+1}-1} R_{\text{su}}(t) \quad (9)
\]

where \( R_{\text{su}}(t) \) denotes the number of new packets admitted in slot \( t \) and \( t_{k+1} \) denotes the start of the \( (k + 1) \)-th frame. The above expression has an inequality because it may be possible to serve the packets admitted in the \( k \)-th frame during that frame itself.

In order to meet the time average power constraint, we make use of a virtual power queue \( X_{\text{su}}(t_k) \) [22] which evolves over frames as follows:

\[
X_{\text{su}}(t_{k+1}) = \max[X_{\text{su}}(t_k) - T[k]P_{\text{avg}} + \sum_{t=t_k}^{t_{k+1}-1} P(t), 0] \quad (10)
\]

where \( T[k] = t_{k+1} - t_k \) is the length of the \( k \)-th frame. Recall that \( T[k] \) is a (random) function of the control decisions taken during the \( k \)-th frame.
In order to construct an optimal dynamic control policy, we use the technique of [19]–[21] where a ratio of “drift-plus-penalty” is maximized over every frame. Specifically, let $Q(t_k) = (Q_{su}(t_k), X_{su}(t_k))$ denote the queueing state of the system at the start of the $k$th frame. As a measure of the congestion in the system, we use a Lyapunov function $L(Q(t_k)) = \frac{1}{2}Q_{su}^2(t_k) + X_{su}^2(t_k)$. Define the drift $\Delta(t_k)$ as the conditional expected change in $L(Q(t_k))$ over the frame $k$:

$$\Delta(t_k) \triangleq \mathbb{E} \{L(Q(t_{k+1})) - L(Q(t_k)) \mid Q(t_k)\}$$

(11)

Then, using (9) and (10), we can bound $\Delta(t_k)$ as follows:

$$\Delta(t_k) \leq B - Q_{su}(t_k)\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} [\mu_{su}(t) - R_{su}(t)]Q(t_k) \right\} - X_{su}(t_k)\mathbb{E} \left\{ T[k]P_{avg} - \sum_{t=t_k}^{t_{k+1}-1} P(t)Q(t_k) \right\}$$

(12)

where $B$ is a finite constant that satisfies the following for all $k$ and $Q(t_k)$ under any control algorithm:

$$B \geq \frac{1}{2}\mathbb{E} \left\{ \left( \sum_{t=t_k}^{t_{k+1}-1} \mu_{su}(t) \right)^2 + \left( \sum_{t=t_k}^{t_{k+1}-1} R_{su}(t) \right)^2 + \left( \sum_{t=t_k}^{t_{k+1}-1} P(t) - T[k]P_{avg} \right)^2 \right\}$$

(13)

Using the fact that $\mu_{su}(t) \leq \mu_{max}$, $P(t) \leq P_{max}$ for all $t$, and using the fact (2), it follows that choosing $B$ as follows satisfies the above:

$$B = \frac{D[\mu_{max}^2 + A_{max}^2 + (P_{max} - P_{avg})^2]}{2}$$

(14)

Adding a penalty term $-V\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} R_{su}(t)Q(t_k) \right\}$ (where $V > 0$ is a control parameter that affects a utility-delay trade-off as shown in Theorem 1) to both sides and rearranging yields:

$$\Delta(t_k) - V\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} R_{su}(t)Q(t_k) \right\} \leq B + (Q_{su}(t_k) - V)\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} R_{su}(t)Q(t_k) \right\} - X_{su}(t_k)\mathbb{E} \left\{ T[k]P_{avg}Q(t_k) \right\} - \mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} Q_{su}(t)\mu_{su}(t) - X_{su}(t)P(t) \right\}Q(t_k)$$

(15)

Minimizing the ratio of an upper bound on the right hand side of the above expression and the expected frame length over all control options leads to the following Frame-Based-Drift-Plus-Penalty-Algorithm. In each frame $k \in \{1, 2, 3, \ldots\}$, do the following:

1) Admission Control: For all $t \in \{t_k, t_k+1, \ldots, t_{k+1}-1\}$, choose $R_{su}(t)$ as follows:

$$R_{su}(t) = \begin{cases} A_{su}(t) & \text{if } Q_{su}(t) \leq V \\ 0 & \text{else} \end{cases}$$

(16)

2) Resource Allocation: Choose a policy that maximizes the following ratio:

$$\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} \left( Q_{su}(t)\mu_{su}(t) - X_{su}(t)P(t) \right)Q(t_k) \right\} \mathbb{E} \left\{ T[k]Q(t_k) \right\}$$

(17)

Specifically, every slot $t$ of the frame, the policy observes the queue values $Q_{su}(t_k)$ and $X_{su}(t_k)$ at the beginning of the frame and selects a secondary user power $P(t)$ subject to the constraint $P(t) \in \mathcal{P}$ and the constraint on transmitting own data vs. cooperation depending on whether slot $t$ is in the “PU Idle” or “PU Busy” period of the frame. This is done in such a way that the above frame-based ratio of expectations is maximized. Recall that the frame size $T[k]$ is influenced by the policy through the success probabilities that are determined by secondary user power selections. Further recall that these success probabilities are different during the “PU Idle” and “PU Busy” periods of the frame. An explicit policy that maximizes this expectation is given in the next section.

3) Queue Update: After implementing this policy, update the queues as in (4) and (10).

From the above, it can be seen that the admission control part (15) is a simple threshold-based decision that does not require any knowledge of the arrival rates $\lambda_{su}$ or $\lambda_{pu}$. In the next section, we present an explicit solution to the maximizing policy for the resource allocation in (16) and show that, remarkably, it also does not require knowledge of $\lambda_{su}$ or $\lambda_{pu}$ and can be computed easily. We will then analyze the performance of the Frame-Based-Drift-Plus-Penalty-Algorithm in Sec. [V].

IV. THE MAXIMIZING POLICY OF (16)

The policy that maximizes (16) uses only two numbers that we call $P^*_0$ and $P^*_1$, defined as follows. $P^*_0$ is given by the solution to the following optimization problem:

Maximize: $Q_{su}(t_k)\mu_{su}(P_0) - X_{su}(t_k)P_0$
Subject to: $P_0 \in \mathcal{P}$

(18)

Let $\theta^* = Q_{su}(t_k)\mu_{su}(P^*_0) - X_{su}(t_k)P^*_0$ denote the value of the objective of (17) under the optimal solution. Then, $P^*_1$ is given by the solution to the following optimization problem:

Minimize: $\frac{\theta^* + X_{su}(t_k)P_1}{\phi(P_1)}$
Subject to: $P_1 \in \mathcal{P}$

(19)

Note that both (17) and (18) are simple optimization problems in a single variable and can be solved efficiently. Given $P^*_0$ and $P^*_1$, on every slot $t$ of frame $k$, the policy that maximizes (16) chooses power $P(t)$ as follows:

$$P(t) = \begin{cases} P^*_0 & \text{if } Q_{pu}(t) = 0 \\ P^*_1 & \text{if } Q_{pu}(t) > 0 \end{cases}$$

(20)

That is, the secondary user uses the constant power $P^*_0$ for its own transmission during the “PU Idle” period of the frame, and uses constant power $P^*_1$ for cooperative transmission.
A. Proof Details

Recall that the Frame-Based-Drift-Plus-Penalty-Algorithm chooses a policy that maximizes the following ratio over every frame $k \in \{1, 2, 3, \ldots\}$

$$
\max_{P \in \mathcal{P}} \frac{\mathbb{E} \left\{ \sum_{t=k+1}^{r_k} (Q_{su}(t_k) P_{su}(t) - X_{su}(t_k) P(t)) \right\} | Q(t_k)}{\mathbb{E} \{T[k] | Q(t_k)\}}
$$

subject to the constraints described in Sec. III Here we examine how to solve (20) in detail. First, define the state $r \in \mathcal{R}$ denoted by the class of stationary, randomized policies where every policy $r \in \mathcal{R}$ chooses a power allocation $P_i(r) \in \mathcal{P}$ in each state $i$ according to a stationary distribution. It can be shown that it is sufficient to only consider policies in $\mathcal{R}$ to maximize (20). Now suppose a policy $r \in \mathcal{R}$ is implemented on a recurrent system with fixed $Q_{su}(t_k)$ and $X_{su}(t_k)$ and with the same state dynamics as our model. Note that $\mu_{su}(t) = 0$ for all $t$ when the state $i \geq 1$. Then, by basic renewal theory [27], we have that maximizing the ratio in (20) is equivalent to the following optimization problem:

Maximize: $Q_{su}(t_k) \mathbb{E} \{ \mu_{su}(P_0(r)) \} \pi_0(r)$

subject to: $r \in \mathcal{R}$

where $\pi_i(r)$ is the resulting steady-state probability of being in state $i$ in the recurrent system under the stationary, randomized policy $r$ and where the expectations above are with respect to $r$. Note that well-defined steady-state probabilities $\pi_i(r)$ exist for all $r \in \mathcal{R}$ because we have assumed that $\lambda_{pu} < \phi_{nc}$ so that even if no cooperation is used, the primary queue is stable and the system is recurrent. Thus, solving (20) is equivalent to solving the unconstrained time average maximization problem (21) over the class of stationary, randomized policies. Note that (21) is an infinite horizon Markov decision problem (MDP) over the state space $i \in \{0, 1, 2, \ldots\}$. We study this problem in the following.

Consider the optimal stationary, randomized policy that maximizes the objective in (21). Let $\chi_i$ denote the probability during all slots of the “PU busy” period of the frame. Note that $P_0^*$ and $P_1^*$ can be computed easily based on the weights $Q_{su}(t_k), X_{su}(t_k)$ associated with frame $k$, and do not require knowledge of the arrival rates $\lambda_{su}, \lambda_{pu}$.

Our proof that the above decisions maximize (16) has the following parts: First, we show that the decisions that maximize the ratio of expectations in (16) are the same as the optimal decisions in an equivalent infinite horizon Markov decision problem (MDP). Next, we show that the solution to the infinite horizon MDP uses fixed power $P_i$ for each queue state $Q_{pu}(t) = i$ for $i \in \{0, 1, 2, \ldots\}$. Then, we show that $P_i$ are the same for all $i \geq 1$. Finally, we show that the optimal powers $P^*_0$ and $P^*_1$ are given as above. The detailed proof is in the next section.
Theorem, we have \( \pi \) in any state

The effective probability of a successful primary transmission (idle) is the value of the objective of (29) under the optimal solution. Let (28) and let \( \pi \) for all states \( i \). Also, the average power incurred in cooperative transmissions under this alternate policy is given by:

\[
T = \sum_{k \geq 1} \pi_k \mathbb{E}(P') = \sum_{k \geq 1} \pi_k \left( \sum_{i \geq 1} \mathbb{E}(X_i, \{P_i\}) \sum_{j \geq 1} \pi_j \right)
\]

Further, the expectations can be simplified as follows:

\[
\mathbb{E}(X_i, \{P_i\}) = \mathbb{E}(X_i) = \frac{\lambda_{pu}}{\sum_{j \geq 1} \pi_j}
\]

This is the same as (17). Let \( \pi \) for all states \( i \) where 

Thus, if we choose \( \chi' = \chi_0 \) in state \( i = 0 \) and choose \( \chi' \) as defined in (24) in all other states, it can be seen that the alternate policy achieves the same time average value of the objective \( (27) \) as the optimal policy. This implies that to maximize \( (21) \), it is sufficient to optimize over the class of stationary policies that use the same distribution for choosing \( P_i \) for all states \( i \). Then for all \( i > 1 \), we have that \( \mathbb{E}(P_i(r)) = \mathbb{E}(P_i(r)) \) for all \( r \). Using this and the fact that \( 1 - \pi_0(r) = \sum_{i \geq 1} \pi_i(r) \), (21) can be simplified as follows:

Maximize: \[ Q_{su}(t_k) \mathbb{E}(\mu_{su}(P_0(r))) - X_{su}(t_k) \mathbb{E}(P_0(r)) \pi_0(r) - X_{su}(t_k) \mathbb{E}(P_1(r)) (1 - \pi_0(r)) \]

Subject to: \( r \in \mathcal{R}' \) \hspace{1cm} (28)

where \( \pi_0(r) \) is the resulting steady-state probability of being in state 0 and where \( \mathbb{E}(P_i(r)) \) is the average power incurred in cooperative transmission in state \( i = 1 \) (same for all states \( i \geq 1 \)). Next, note that the control decisions taken by the secondary user in state \( i = 0 \) do not affect the length of the frame and therefore \( \pi_0(r) \). Further, the expectations can be removed. Therefore the first term in the problem above can be maximized separately as follows:

Maximize: \[ Q_{su}(t_k) \mathbb{E}(\mu_{su}(P_0(r))) - X_{su}(t_k) \mathbb{E}(P_0(r)) \]

Subject to: \( P_0 \in \mathcal{P} \) \hspace{1cm} (29)

This is the same as (17). Let \( P_0 \) denote the optimal solution to (29) and let \( \theta^* = Q_{su}(t_k) \mathbb{E}(\mu_{su}(P_0(r))) - X_{su}(t_k) P_0 \) denote the value of the objective of (29) under the optimal solution. Note that we must have that \( \theta^* \geq 0 \) because the value of the objective when the secondary user chooses \( P_0 = 0 \) (i.e., stays idle) is 0. Then, (28) can be written as:

Maximize: \[ \theta^* \pi_0(r) - X_{su}(t_k) \mathbb{E}(P_1(r)) (1 - \pi_0(r)) \]

Subject to: \( r \in \mathcal{R}' \) \hspace{1cm} (30)

The effective probability of a successful primary transmission in any state \( i \geq 1 \) is given by \( \mathbb{E}(\phi(P_1(r))) \). Using Little’s Theorem, we have \( \pi_0(r) = 1 - \frac{\lambda_{pu}}{\mathbb{E}(\phi(P_1(r)))} \). Using this and rearranging the objective in (30) and ignoring the constant terms, we have the following equivalent problem:

Minimize: \[ \theta^* + X_{su}(t_k) \mathbb{E}(P_1(r)) \]

Subject to: \( r \in \mathcal{R}' \) \hspace{1cm} (31)

It can be shown that it is sufficient to consider only deterministic power allocations to solve (31) (see, for example, Section 7.3.2). This yields the following problem:

Minimize: \[ \theta^* + X_{su}(t_k) P_1 \]

Subject to: \( P_1 \in \mathcal{P} \) \hspace{1cm} (32)

This is the same as (13). Note that solving this problem does not require knowledge of \( \lambda_{pu} \) or \( \lambda_{su} \) and can be solved easily for general power allocation options \( \mathcal{P} \). We present an example that admits a particularly simple solution to this problem.

Suppose \( \mathcal{P} = \{0, P_{max}\} \) so that the secondary user can either cooperate with full power \( P_{max} \) or not cooperate (with power expenditure 0) with the primary user. Then, the optimal solution to (32) can be calculated by comparing the value of its objective for \( P_1 \in \{0, P_{max}\} \). This yields the following simple threshold-based rule:

\[
P_1^* = \begin{cases} 0 & \text{if } X_{su}(t_k) \geq \frac{\theta^*(\phi_0 - \phi_{nc})}{P_{max} \phi_{nc}} \\ P_{max} & \text{else} \end{cases}
\]

(33)

We also note that this threshold can be computed without any knowledge of the input rates \( \lambda_{pu}, \lambda_{su} \).

To summarize, the overall solution to (16) is given by the pair \( (P_0^*, P_1^*) \) where \( P_0^* \) denotes the power allocation used by the secondary user for its own transmission when the primary user is idle and \( P_1^* \) denotes the power used by the secondary user for cooperative transmission. Note that these values remain fixed for the entire duration of frame \( k \). However, these can change from one frame to another depending on the values of the queues \( Q_{su}(t_k), X_{su}(t_k) \). The computation of \( (P_0^*, P_1^*) \) can be carried out using a two-step process as follows:

1) First, compute \( P_0^* \) by solving problem (29). Let \( \theta^* \) be the value of the objective of (29) under the optimal solution \( P_0^* \).
2) Then compute \( P_1^* \) by solving problem (32).

It is interesting to note that in order to implement this algorithm, the secondary user does not require knowledge of the current queue backlog value of the primary user. Rather, it only needs to know the values of its own queues and whether the current slot is in the “PU Idle” or “PU Busy” part of the frame. This is quite different from the conventional solution to the MDP (5) which is typically a different randomized policy for each value of the state (i.e., the primary queue backlog).

V. PERFORMANCE ANALYSIS

To analyze the performance of the Frame-Based-Drift-Plus-Penalty-Algorithm, we compare its Lyapunov drift with that of the optimal stationary, randomized policy \( STAT \) of Lemma 1. First, note that by basic renewal theory (27), the
performance guarantees provided by STAT hold over every frame \( k \in \{1, 2, 3, \ldots\} \). Specifically, let \( t_k \) be the start of the \( k \text{th} \) frame. Suppose STAT is implemented over this frame. Then the following hold:

\[
\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} P_{su}^{stat}(t) \right\} = \mathbb{E} \left\{ \hat{T}[k] \right\} t^* \tag{34}
\]

\[
\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} P_{su}^{stat}(t) \right\} \leq \mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} \mu_{su}^{stat}(t) \right\} \tag{35}
\]

\[
\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} P_{su}^{stat}(t) \right\} \leq \mathbb{E} \left\{ \hat{T}[k] \right\} P_{avg} \tag{36}
\]

where \( \hat{t}_{k+1} \) and \( \hat{T}[k] \) denote the start of the \((k+1)\text{th}\) frame and the length of the \(k\text{th}\) frame, respectively, under the policy STAT. Similarly, \( R_{su}^{stat}(t), P_{su}^{stat}(t), \mu_{su}^{stat}(t) \) denote the resource allocation decisions under STAT.

Next, we define an alternate control algorithm ALT that will be useful in analyzing the performance of the Frame-Based-Drift-Plus-Penalty-Algorithm.

**Algorithm ALT:** In each frame \( k \in \{1, 2, 3, \ldots\} \), do the following:

1. **Admission Control:** For all \( t \in \{t_k, t_k + 1, \ldots, t_{k+1}-1\} \), choose \( R_{su}(t) \) as follows:

   \[
   R_{su}(t) = \begin{cases} 
   A_{su}(t) & \text{if } Q_{su}(t_k) \leq V \\
   0 & \text{else}
   \end{cases} \tag{37}
   \]

2. **Resource Allocation:** Choose a policy that maximizes the following ratio:

   \[
   \mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}(t_k)\mu_{su}(t) - X_{su}(t_k)P(t)) | Q(t_k) \right\} \mathbb{E} \left\{ \hat{T}[k] | Q(t_k) \right\} \tag{38}
   \]

3. **Queue Update:** After implementing this policy, update the queues as in \( (9), (10) \).

By comparing with the Frame-Based-Drift-Plus-Penalty-Algorithm, it can be seen that this algorithm differs only in the admission control part while the resource allocation decisions are exactly the same. Specifically, under ALT, the queue backlog \( Q_{su}(t_k) \) at the start of the \( k\text{th}\) frame is used for making admission control decisions for the entire duration of that frame. However, under the Frame-Based-Drift-Plus-Penalty-Algorithm, the queue backlog \( Q_{su}(t) \) at the start of each slot is used for making admission control decisions. Note that since the length of the frame depends only on the resource allocation decisions and they are the same under the two algorithms, it follows that implementing them with the same starting backlog \( Q(t_k) \) yields the same frame lengths.

The following lemma compares the value of the second term in the Lyapunov drift bound \( (14) \) that corresponds to the admission control decisions under these two algorithms.

**Lemma 2:** Let \( R_{su}^{fab}(t) \) and \( R_{su}^{stat}(t) \) denote the admission control decisions made by the Frame-Based-Drift-Plus-Penalty-Algorithm and the ALT algorithm respectively for all \( t \in \{t_k, t_k + 1, \ldots, t_{k+1}-1\} \). Then we have:

\[
\mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}(t_k) - V)R_{su}^{stat}(t) | Q(t_k) \right\} \geq \mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}(t_k) - V)R_{su}^{fab}(t) | Q(t_k) \right\} - C \tag{39}
\]

where \( C = \frac{D(A_{max} + \mu_{max})A_{max}}{2} \) is a constant that does not depend on \( V \).

**Proof:** See Appendix A.

We are now ready to characterize the performance of the Frame-Based-Drift-Plus-Penalty-Algorithm.

**Theorem 1:** (Performance Theorem) Suppose the Frame-Based-Drift-Plus-Penalty-Algorithm is implemented over all frames \( k \in \{1, 2, 3, \ldots\} \) with initial condition \( Q_{su}(0) = 0, X_{su}(0) = 0 \) and with a control parameter \( V > 0 \). Let \( \mu_{su}^{ab}(t), P_{su}^{ab}(t) \) denote the resource allocation decisions under this algorithm. Then, we have:

1. The secondary user queue backlog \( Q_{su}(t) \) is upper bounded for all \( t \):

   \[
   Q_{su}(t) \leq Q_{max} \frac{A_{max}}{V} + V \tag{40}
   \]

2. The virtual power queue \( X_{su}(t_k) \) is mean rate stable, i.e.,

   \[
   \lim_{K \to \infty} \frac{\mathbb{E} \left\{ X_{su}(t_K) \right\}}{K} = 0 \tag{41}
   \]

Further, we have:

\[
\limsup_{K \to \infty} \left( \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} (P_{su}^{ab}(t) - P_{avg}) \right\} \right) \leq 0 \tag{42}
\]

\[
\limsup_{K \to \infty} \left( \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \hat{T}[k] \right\} \right) \leq P_{avg} \tag{43}
\]

3. The time-average secondary user throughput (defined over frames) satisfies the following bound for all \( K > 0 \):

   \[
   \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} P_{su}^{ab}(t) \right\} \geq \frac{t^*}{V} - B + C \tag{44}
   \]

where \( B = \frac{D(A_{max}^2 + \mu_{max}^2) + (P_{max} - P_{avg})^2}{2D(A_{max} + \mu_{max})A_{max}} \) and \( C = \frac{D(A_{max} + \mu_{max})A_{max}}{2} \).

**Theorem 1** shows that the time-average secondary user throughput can be pushed to within \( O(1/V) \) of the optimal value with a trade-off in the worst case queue backlog. By Little’s Theorem, this leads to an \( O(1/V) \) utility-delay tradeoff.

**Proof:** Part (1): We argue by induction. First, note that \( (40) \) holds for \( t = 0 \). Next, suppose \( Q_{su}(t) \leq Q_{max} \) for some \( t > 0 \). We will show that \( Q_{su}(t + 1) \leq Q_{max} \). We have two cases. First, suppose \( Q_{su}(t) \leq V \). Then, by \( (9) \), the maximum that \( Q_{su}(t) \) can increase is \( A_{max} \) so that \( Q_{su}(t+1) \leq A_{max} + V = Q_{max} \). Next, suppose \( Q_{su}(t) > V \). Then, the admission control decision \( (15) \) chooses \( R_{su}(t) = 0 \). Thus, by \( (9) \), we have that \( Q_{su}(t+1) \leq Q_{su}(t) \leq Q_{max} \) for this case as well. Combining these two cases proves the bound \( (40) \).

Parts (2) and (3): See Appendix B.
VI. EXTENSIONS TO BASIC MODEL

We consider two extensions to the basic model of Sec. [II]

A. Multiple Secondary Users

Consider the scenario with one primary user as before, but with \( N > 1 \) secondary users. The primary user channel occupancy process evolves as before where the secondary users can transmit their own data only when the primary user is idle. However, they may cooperatively transmit with the primary user to increase its transmission success probability. In general, multiple secondary users may cooperatively transmit with the primary in one timeslot. However, for simplicity, here we assume that at most one secondary user can take part in a cooperative transmission per slot. Further, we also assume that at most one secondary user can transmit its data when the primary user is idle.

Our formulation can be easily extended to this scenario. Let \( \mathcal{P}_i \) denote the set of power allocation options for secondary user \( i \). Suppose each secondary user \( i \) is subject to average and peak power constraints \( P_{\text{avg},i} \) and \( P_{\text{max},i} \) respectively. Also, let \( \phi_i(P) \) denote the success probability of the primary transmission when secondary user \( i \) spends power \( P \) in cooperative transmission. Now consider the objective of maximizing the sum total throughput of the secondary users subject to each user’s average and peak power constraints and the scheduling constraints of the model. In order to apply the “drift-plus-penalty” ratio method, we use the following queues:

\[
Q_i(t_{k+1}) \leq \max(Q_i(t) - \sum_{t=t_k}^{t_{k+1}-1} \mu_i(t), 0) + \sum_{t=t_k}^{t_{k+1}-1} R_i(t)
\]

\[
X_i(t_{k+1}) = \max(X_i(t_k) - T[k] P_{\text{avg},i} + \sum_{t=t_k}^{t_{k+1}-1} P_i(t), 0)
\]

(45)

(46)

where \( Q_i(t_k) \) is the queue backlog of secondary user \( i \) at the beginning of the \( k \)th frame, \( \mu_i(t) \) is the service rate of secondary user \( i \) in slot \( t \), \( R_i(t) \) and \( P_i(t) \) denote the number of new packets admitted and the power expenditure incurred by the secondary user \( i \) in slot \( t \). Finally, \( t_{k+1} \) denotes the start of the \( (k+1) \)th frame and \( T[k] = t_{k+1} - t_k \) is the length of the \( k \)th frame as before.

Let \( Q(t_k) = (Q_1(t_k), \ldots, Q_N(t_k), X_1(t_k), \ldots, X_N(t_k)) \) denote the queueing state of the system at the start of the \( k \)th frame. Using a Lyapunov function \( L(Q(t_k)) = \sum_{i=1}^{N} Q_i(t_k)^2 + \sum_{i=1}^{N} X_i^2(t_k) \) and following the steps in Sec. [III] yields the following Multi-User Frame-Based-Drift-Plus-Penalty-Algorithm. In each frame \( k \in \{1, 2, 3, \ldots \} \), do the following:

1) Admission Control: For all \( t \in \{t_k, t_k+1, \ldots, t_{k+1}-1\} \), for each secondary user \( i \in \{1, 2, \ldots, N\} \), choose \( R_i(t) \) as follows:

\[
R_i(t) = \begin{cases} A_i(t) & \text{if } Q_i(t) \leq V \\ 0 & \text{else} \end{cases}
\]

(47)

where \( A_i(t) \) is the number of new arrivals to secondary user \( i \) in slot \( t \).

2) Resource Allocation: Choose a policy that maximizes the following ratio:

\[
\sum_{i=1}^{N} \mathbb{E} \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_i(t_k) \mu_i(t) - X_i(t_k) P_i(t)) Q(t_k) \right\} \mathbb{E} \{T[k]|Q(t_k)\}
\]

(48)

3) Queue Update: After implementing this policy, update the queues as in (45) and (46).

Similar to the basic model, this algorithm can be implemented without any knowledge of the arrival rates \( \lambda_i \) or \( \lambda_{pu} \). Further, using the techniques developed in Sec. [IV], it can be shown that the solution to (48) can be computed in two steps as follows. First, we solve the following problem for each \( i \in \{1, 2, \ldots, N\} \):

Minimize: \( \phi_i(P) \)

Subject to: \( P \in \mathcal{P}_i \)

(49)

Let \( P_0^* \) denote the optimal solution to (49) achieved by user \( i^* \) and let \( \theta^* \) denote the optimal objective value. This means user \( i^* \) transmits on all idle slots of frame \( k \) with power \( P_0^* \). Next, to determine the optimal cooperative transmission strategy, we solve the following problem for each \( i \in \{1, 2, \ldots, N\} \):

Minimize: \( \theta^* + X_i(t_k) P \)

Subject to: \( P \in \mathcal{P}_i \)

(50)

Let \( P_i^* \) denote the optimal solution to (50) achieved by user \( j^* \). This means user \( j^* \) cooperatively transmits on all busy slots of frame \( k \) with power \( P_i^* \).

B. Fading Channels

Next, suppose there is an additional channel fading process \( S(t) \) that takes values from a finite set \( \mathcal{S} \) in an i.i.d. fashion every slot. We assume that in every slot, \( \text{Prob}[S(t) = s] = q_s \) for all \( s \in \mathcal{S} \). The success probability with cooperative transmission now is a function of both the power allocation and the fading state in that slot. Specifically, suppose the primary user is active in slot \( t \) and the secondary user allocates power \( P(t) \) for cooperative transmission. Also suppose \( S(t) = s \). Then the random success/failure outcome of the primary transmission is given by an indicator variable \( \mu_{pu}(P(t), s) \) and the success probability is given by \( \phi_s(P(t)) = \mathbb{E}\{\mu_{pu}(P(t), s)\} \). The function \( \phi_s(P(t)) \) is known to the network controller for all \( s \in \mathcal{S} \) and is assumed to be non-decreasing in \( P \) for each \( s \in \mathcal{S} \). For simplicity, we assume that the secondary user transmission rate \( \mu_{su}(t) \) depends only on \( P(t) \).

By applying the “drift-plus-penalty” ratio method to this extended model, we get the following control algorithm. The admission control remains the same as (15). The resource allocation part involves maximizing the ratio in (16). Using the same arguments as before in Sec. [IV] it can be shown that maximizing this ratio is equivalent to the following...
where $\pi_i(r)$ is the resulting steady-state probability of being in state $(i,s)$ in the recurrent system under the stationary, randomized policy $r$ and where the expectations above are with respect to $r$. We study this problem in the following.

Consider the optimal stationary, randomized policy that maximizes the objective in (51). Let $\chi_i,s$ denote the probability distribution over $P$ that is used by this policy to choose a control action $P_{i,s}$ in state $(i,s)$. Let $\mu_{i,s} = \mathbb{E}_{\chi_i,s} \{ \phi_s(P_{i,s}) \}$ denote the resulting effective probability of successful primary transmission in state $(i,s)$ where $i \geq 1$. Since the system is stable under any stationary policy, total incoming rate = total outgoing rate. Thus, we get:

$$\lambda_{pu} = \sum_{i \geq 1} \sum_{s \in S} \pi_i,s \mu_{i,s}$$  \hspace{1cm} (52)

where $\pi_i,s$ denotes the steady-state probability of being in state $(i,s)$ under this policy. Note that the system is stable and has a well-defined steady-state distribution. The average power incurred in cooperative transmissions under this policy is given by:

$$P = \sum_{i \geq 1} \sum_{s \in S} \pi_i,s \mathbb{E}_{\chi_i,s} \{ P_{i,s} \}$$  \hspace{1cm} (53)

Now consider an alternate stationary policy that, for each $s \in S$, uses the following fixed distribution $\chi'_s$ for choosing control action $P'_s$ in all states $(i,s)$ where $i \geq 1$:

$$\chi'_s \triangleq \begin{cases} 
\chi_{1,s} & \text{with probability } \sum_{j \geq 1} \pi_j,s \\
\chi_{2,s} & \text{with probability } \sum_{j \geq 1} \pi_j,s \\
\vdots & \vdots \\
\chi_{s,s} & \text{with probability } \sum_{j \geq 1} \pi_j,s 
\end{cases}$$  \hspace{1cm} (54)

For each $s \in S$, let $\mu'_s$ denote the resulting effective probability of a successful primary transmission in any state $(i,s)$ where $i \geq 1$ under this policy. Note that this is same for all states $(i,s)$ where $i \geq 1$ by the definition (54). Then, we have that:

$$\mu'_s = \sum_{i \geq 1} \mu_{i,s} \pi_i,s \sum_{j \geq 1} \pi_j,s$$  \hspace{1cm} (55)

Let $\pi'_{i,s}$ denote the steady-state probability of being in state $(i,s)$ under this alternate policy. Since the system is stable under any stationary policy, total incoming rate = total outgoing rate. Thus, we get:

$$\lambda_{pu} = \sum_{s \in S} \sum_{k \geq 1} \pi'_{k,s} \mu'_s = \sum_{s \in S} \mu'_s \left( \sum_{k \geq 1} \pi'_{k,s} \right)$$

$$= \sum_{s \in S} \left[ \sum_{i \geq 1} \mu_{i,s} \pi_{i,s} \right] \left( \sum_{k \geq 1} \pi'_{k,s} \right)$$  \hspace{1cm} (56)

where we used (55) in the last step. Since $S(t)$ is i.i.d., for any $s_1,s_2 \in S$, we have that:

$$\pi_0 q_{s_1} + \sum_{j \geq 1} \pi_{j,s_1} = q_{s_1}, \hspace{1cm} \pi_0 q_{s_2} + \sum_{j \geq 1} \pi_{j,s_2} = q_{s_2}$$

Similarly, we have:

$$\pi'_0 q_{s_1} + \sum_{j \geq 1} \pi'_{j,s_1} = q_{s_1}, \hspace{1cm} \pi'_0 q_{s_2} + \sum_{j \geq 1} \pi'_{j,s_2} = q_{s_2}$$

Using this, for any $s_1,s_2 \in S$, we have:

$$\sum_{j \geq 1} \pi_{j,s_1} = \sum_{j \geq 1} \pi'_{j,s_2} \hspace{1cm} (57)$$

Using this in (56), we have for each $s \in S$:

$$\lambda_{pu} = \sum_{s \in S} \left[ \sum_{i \geq 1} \mu_{i,s} \pi_{i,s} \right] \left( \sum_{k \geq 1} \pi'_{k,s} \right) \sum_{j \geq 1} \pi_{j,s} = \lambda_{pu} \sum_{s \in S} \pi_{i,s} \sum_{j \geq 1} \pi_{j,s}$$  \hspace{1cm} (58)

where we used (52) in the last step. This implies that $\sum_{i \geq 1} \pi'_{k,s} = \sum_{j \geq 1} \pi_{j,s}$ for every $s \in S$ and therefore $\pi'_0 = \pi_0$. Also, the average power incurred in cooperative transmissions under this alternate policy is given by:

$$P' = \sum_{k \geq 1} \sum_{s \in S} \pi'_{k,s} \mathbb{E}_{\chi'_s} \{ P'_s \}$$

$$= \sum_{k \geq 1} \sum_{s \in S} \pi'_{k,s} \left( \sum_{i \geq 1} \mathbb{E}_{\chi_i,s} \{ P_{i,s} \} \sum_{j \geq 1} \pi_{j,s} \right)$$

$$= \sum_{s \in S} \sum_{i \geq 1} \mathbb{E}_{\chi_i,s} \{ P_{i,s} \} \pi_{i,s} = P$$  \hspace{1cm} (59)

where we used the fact that $\sum_{k \geq 1} \pi'_{k,s} = \sum_{j \geq 1} \pi_{j,s}$ for all $s$. Thus, if we choose $\chi' = \chi_0$ in state $i = 0$ and choose $\chi'_s$ as defined in (54) in all states $(i,s)$ where $i \geq 1$, it can be seen that the alternate policy achieves the same time average value of the objective (51) as the optimal policy. This implies that to maximize (51), it is sufficient to optimize over the class of stationary policies that, for each $s \in S$, use the same distribution for choosing $P_{i,s}$ for all states $(i,s)$ where $i \geq 1$. Denote this class by $\mathcal{R}'$. Using this and the fact that $\sum_{i \geq 1} \pi_{i,s}(r) = (1 - \pi_0(r))q_s$ for all $s$, (51) can be simplified as follows:

Maximize: $[Q_{su}(t_k)\mathbb{E}\{\mu_{su}(P_0(r))\} - X_{su}(t_k)\mathbb{E}\{P_0(r)\}]\pi_0(r)$

$$- \sum_{s \in S} \sum_{i \geq 1} \mathbb{E}\{P_{i,s}(r)\} (1 - \pi_0(r))q_s$$  \hspace{1cm} (60)

Subject to: $r \in \mathcal{R}'$

where $\pi_0(r)$ is the resulting steady-state probability of being in state 0 and where $\mathbb{E}\{P_{i,s}(r)\}$ is the average power incurred in cooperative transmission in any state $(i,s)$ with $i \geq 1$. Using the same arguments as before, the solution to (60) can be obtained in two steps as follows. We first compute the solution to (29) as before. Denoting its optimal value by $\theta^*$, (60) can be written as:

Maximize: $\theta^* \pi_0(r) - X_{su}(t_k) \sum_{s \in S} \mathbb{E}\{P_{i,s}(r)\} (1 - \pi_0(r))q_s$

Subject to: $r \in \mathcal{R}'$
It can be shown that it is sufficient to consider only deterministic power allocations to solve (62) (see, for example, Section 7.3.2). This yields the following problem:

Maximize: \(-\theta^* - X_{su}(t) \sum_{s \in S} q_s E\{P_s(r)\}\)

Subject to: \(r \in \mathcal{R}'\) \hspace{1cm} (62)

Note that solving this problem does not require knowledge of \(\lambda_{su}\) or \(\lambda_{pu}\) and can be solved efficiently for general power allocation options \(\mathcal{P}\).

VII. SIMULATIONS

In this section, we evaluate the performance of the Frame-Based-Drift-Plus-Penalty-Algorithm using simulations. We consider the network model as discussed in Sec. IID with one primary and one secondary user. The set \(\mathcal{P}\) consists of only two options \(\{0, P_{max}\}\). We assume that \(P_{avg} = 0.5\) and \(P_{max} = 1\). We set \(\phi_{nc} = 0.6\) and \(\phi_{c} = 0.8\). For simplicity, we assume that \(\mu_{su}(P_{max}) = 1\).

In the first set of simulations, we fix the input rates \(\lambda_{pu} = \lambda_{su} = 0.5\) packets/slot. For these parameters, we can compute the optimal offline solution by linear programming. This yields the maximum secondary user throughput as 0.25 packets/slot. We now simulate the Frame-Based-Drift-Plus-Penalty-Algorithm for different values of the control parameter \(V\) over 1000 frames. In Fig. 4, we plot the average throughput achieved by the secondary user over this period. It can be seen that the average throughput increases with \(V\) and converges to the optimal value 0.25 packets/slot, with the difference exhibiting a \(O(1/V)\) behavior as predicted by Theorem I.

In Fig. 5, we plot the average queue backlog of the secondary user over this period. It can be see that the average queue backlog grows linearly in \(V\), again as predicted by Theorem I.

Also, for all \(V\), the average secondary user power consumption over this period was found not to exceed \(P_{avg} = 0.5\) units/slot.

For comparison, we also simulate three alternate algorithms. In the first algorithm “No Cooperation”, the secondary user never cooperates with the primary user and only attempts to maximize its throughput over the resulting idle periods. The secondary user throughput under this algorithm was found to be 0.166 packets/slot as shown in Fig. 4. Note that using Little’s Theorem, the resulting fraction of time the primary user is idle is \(1 - \lambda_{pu}/\phi_{nc} = 1 - 0.5/0.6 = 0.166\). This limits the maximum secondary user throughput under the “No Cooperation” case to 0.166 packets/slot.

In the second algorithm, we consider the “Always Cooperate” case where the secondary user always cooperates with the primary user. For the example under consideration, this uses up all the secondary user power and thus, the secondary user achieves zero throughput.

In the third algorithm “Counter Based Policy”, a running average of the total secondary user power consumption so far is maintained. In each slot, the secondary user decides to transmit/cooperate only if this running average is smaller than \(P_{avg}\). The maximum secondary user throughput under this algorithm was found to be 0.137 packets/slot. This demonstrates that simply satisfying the average power constraint is not sufficient to achieve maximum throughput. For example, it may be the case that under the “Counter Based Policy”, the running average condition is usually satisfied when the primary user is busy. This causes the secondary user to cooperate. However, by the time the primary user next becomes idle, the running average exceeds \(P_{avg}\) so that the secondary user does not transmit its own data. In contrast, the Frame-Based-Drift-Plus-Penalty-Algorithm is able to find the opportune moments to cooperate/transmit optimally.

In the second set of simulations, we fix the input rate \(\lambda_{su} = 0.8\) packets/slot, \(V = 500\), and simulate the Frame-Based-Drift-Plus-Penalty-Algorithm over 1000 frames. At the start of the simulation, we set \(\lambda_{pu} = 0.4\) packets/slot. The values of the other parameters remain the same. However, during the course of the simulation, we change \(\lambda_{pu}\) to 0.2 packets/slot after the first 350 frames and then again to 0.55 packets/slot after the first 700 frames. In Figs. 6 and 7, we plot the running average (over 100 frames) of the secondary user throughput and the average power used for cooperation. These show that the Frame-Based-Drift-Plus-Penalty-Algorithm automatically adapts to the changes in \(\lambda_{pu}\). Further, it quickly approaches the optimal performance corresponding to the new \(\lambda_{pu}\) by
adaptively spending more or less power (as required) on cooperation. For example, when \( \lambda_{pu} \) reduces to 0.2 packets/slot after frame number 350, the fraction of time the primary is idle even with no cooperation is \( 1 - 0.2 / 0.6 = 0.66 \). With \( P_{avg} = 0.5 \), there is no need to cooperate anymore. This is precisely what the Frame-Based-Drift-Plus-Penalty-Algorithm does as shown in Fig. 7. Similarly, when \( \lambda_{pu} \) increases to 0.55 packets/slot after frame number 700, the Frame-Based-Drift-Plus-Penalty-Algorithm starts to spend more power on cooperative transmissions.

VIII. CONCLUSIONS

In this paper, we studied the problem of opportunistic cooperation in a cognitive femtocell network. Specifically, we considered the scenario where a secondary user can cooperatively transmit with the primary user to increase its transmission success probability. In return, the secondary user can get more opportunities for transmitting its own data when the primary user is idle. A key feature of this problem is that here, the evolution of the system state depends on the control actions taken by the secondary user. This dependence makes it a constrained Markov Decision Problem traditional solutions to which require either extensive knowledge of the system dynamics or learning based approaches that suffer from large convergence times. However, using the technique of Lyapunov optimization, we designed a novel greedy and online control algorithm that overcomes these challenges and is provably optimal.

APPENDIX A

PROOF OF LEMMA 2

Let \( Q_{su}^{fab}(t) \) denote the queue backlog value under the Frame-Based-Drift-Plus-Penalty-Algorithm for all \( t \in \{ t_k, t_k+1, \ldots, t_{k+1} - 1 \} \). Then, since the admission control decision (15) of the Frame-Based-Drift-Plus-Penalty-Algorithm minimizes the term \( (Q_{su}(t) - V)R_{su}(t) \) for all \( Q_{su}(t) \), we have:

\[
\begin{align*}
E & \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}^{fab}(t) - V)R_{su}^{alt}(t) | Q(t_k) \right\} \\
& \geq E \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}^{fab}(t) - V)R_{su}^{fab}(t) | Q(t_k) \right\} \\
& \geq \frac{DA_{max}^2}{2} \\
& \leq \frac{DA_{max}^2}{2}
\end{align*}
\]

Note that we are not implementing the admission control decisions of \( ALT \) in the left hand side of the above.

Next, we make use of the following sample path relations in (64) to prove (39). For all \( t \in \{ t_k, t_k+1, \ldots, t_{k+1} - 1 \} \), the following hold under any control algorithm:

\[
Q_{su}(t_k) \geq Q_{su}(t) - (t - t_k)A_{max} \quad (65)
\]

\[
Q_{su}(t_k) \leq Q_{su}(t) + (t - t_k)\mu_{max} \quad (66)
\]

(65) follows by noting that the maximum number of arrivals to the secondary user queue in the interval \( [t_k, t, t_{k+1}] \) is at most \( (t - t_k)A_{max} \). Similarly, (66) follows by noting that the maximum number of departures from the secondary user queue in the interval \( [t_k, t, t_{k+1}] \) is at most \( (t - t_k)\mu_{max} \).

Using (65) in the left hand side of (64) yields:

\[
E \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}^{fab}(t) - V)R_{su}^{alt}(t) | Q(t_k) \right\} \leq \frac{DA_{max}^2}{2} \]

Next, using (66) in the right hand side of (64) yields:

\[
E \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}^{fab}(t) - V)R_{su}^{fab}(t) | Q(t_k) \right\} \geq \frac{DA_{max}^2}{2}
\]
Again using the fact that $R_{su}^{fab}(t) \leq A_{\text{max}}$ and $\sum_{t=t_k}^{t_{k+1}-1} (t - t[k]) = \frac{T[k](T[k]-1)}{2}$, we get:

$$E \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}(t) - V) R_{su}^{fab}(t) | Q(t_k) \right\} \geq$$

Using (67) and (68) in (64), we have:

$$E \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}(t) - V) R_{su}^{alt}(t) | Q(t_k) \right\} \geq$$

$$E \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}(t) - V) R_{su}^{fab}(t) | Q(t_k) \right\} - C$$

**APPENDIX B**

**PROOF OF THEOREM 1 PARTS 2 AND 3**

We prove parts (2) and (3) of Theorem 1 using the technique of Lyapunov optimization. Using (14), a bound on the Lyapunov drift under the Frame-Based-Drift-Plus-Penalty-Algorithm is given by:

$$\Delta(t_k) - V E \left\{ \sum_{t=t_k}^{t_{k+1}-1} R_{su}^{fab}(t) | Q(t_k) \right\} \leq B + (Q_{su}(t_k) - V)$$

Next, note that under the ALT algorithm, we have:

$$E \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}(t) - V) P_{su}^{alt}(t) | Q(t_k) \right\}$$

To see this, we have two cases:

1) $Q_{su}(t_k) > V$: Then, $R_{su}^{alt}(t) = 0$ for all $t \in \{t_k, t_k+1, \ldots, t_{k+1} - 1\}$, so that the left hand side above is 0 while the right hand side is $\geq 0$. Hence, the inequality follows.

2) $Q_{su}(t_k) \leq V$: Then, $R_{su}^{alt}(t) = A_{su}(t)$ for all $t \in \{t_k, t_k+1, \ldots, t_{k+1} - 1\}$, so that the left hand side becomes $(Q_{su}(t_k) - V) \lambda_{su}$ while the right hand side cannot be smaller than $(Q_{su}(t_k) - V) \lambda_{su}$.

Combining these, we get:

$$(Q_{su}(t_k) - V) E \left\{ \sum_{t=t_k}^{t_{k+1}-1} R_{su}^{fab}(t) | Q(t_k) \right\} \leq C$$

Finally, since the resource allocation part of the Frame-Based-Drift-Plus-Penalty-Algorithm maximizes the ratio in (16), we have:

$$E \left\{ \sum_{t=t_k}^{t_{k+1}-1} (Q_{su}(t) \mu_{su}^{stat}(t) - X_{su}(t_k) P_{su}^{stat}(t)) | Q(t_k) \right\}$$

Using these in (69), we have:

$$\Delta(t_k) - V E \left\{ \sum_{t=t_k}^{t_{k+1}-1} R_{su}^{fab}(t) | Q(t_k) \right\} \leq B + C$$

To prove (41), we rearrange (70) to get:

$$\Delta(t_k) \leq B + C - V \nu^* E \{ T[k] | Q(t_k) \}$$

(41) now follows from Theorem 4.1 of [21]. Since $X_{su}(t_k)$ is mean rate stable, (42) follows from Theorem 2.5(b) of [21].
Summing over $k \in \{1, 2, \ldots, K\}$, dividing by $V$, and rearranging yields:

$$
\sum_{k=1}^{K} \frac{E\left\{ \sum_{i=t_k}^{t_{k+1} - 1} R_{a,b}^k(t) \right\}}{E\{T[k]\}} \geq u^* \frac{E\{T[k]\} - (B + C)K}{V} \frac{K}{E\{T[k]\}}
$$

where we used the fact that $E\{L(Q(t_{K+1}))\} \geq 0$ and $E\{L(Q(t))\} = 0$. From this, we have:

$$
\sum_{k=1}^{K} \frac{E\left\{ \sum_{i=t_k}^{t_{k+1} - 1} R_{a,b}^k(t) \right\}}{E\{T[k]\}} \geq \frac{E\{T[k]\} - (B + C)K}{V} \frac{K}{E\{T[k]\}} \geq u^* \frac{B + C}{V T_{min}}
$$

since $\sum_{k=1}^{K} E\{T[k]\} \geq KT_{min}$. This proves (44).

\section*{Appendix C

\textbf{Computing D}}

Here, we compute a finite $D$ that satisfies (2). First, note that $E\{T^2[k]\}$ would be maximum when the secondary user never cooperates. Next, let $I[k]$ and $B[k]$ denote the lengths of the primary user idle and busy periods, respectively, in the $k$th frame. Thus, we have $T[k] = I[k] + B[k]$.

In the following, we drop $[k]$ from the notation for convenience. Using the independence of $I$ and $B$, we have:

$$
E\{T^2\} = E\{I^2\} + E\{B^2\} + 2E\{I\}E\{B\}
$$

We note that $I$ is a geometric r.v. with parameter $\lambda_{pu}$. Thus, $E\{I\} = 1/\lambda_{pu}$ and $E\{I^2\} = (2 - \lambda_{pu})/\lambda_{pu}^2$. To calculate $E\{B\}$, we apply Little’s Theorem to get:

$$
E\{I\} = \left(1 - \frac{\lambda_{pu}}{\phi_{nc}}\right) (E\{I\} + E\{B\})
$$

This yields $E\{B\} = 1/(\phi_{nc} - \lambda_{pu})$. To calculate $E\{B^2\}$, we use the observation that changing the service order of packets in the primary queue to preemptive LIFO does not change the length of the busy period $B$. However, with LIFO scheduling, $B$ now equals the duration that the first packet stays in the queue. Next, suppose there are $N$ packets that interrupt the service of the first packet. Let these be indexed as $\{1, 2, \ldots, N\}$. We can relate $B$ to the service time $X$ of the first packet and the durations for which all these other packets stay in the queue as follows:

$$
B = X + \sum_{i=1}^{N} B_i \tag{71}
$$

Here, $B_i$ denotes the duration for which packet $i$ stays in the queue. Using the memoryless property of the i.i.d. arrival process of the primary packets as well as the i.i.d. nature of the service times, it follows that all the r.v.’s $B_i$ are i.i.d. with the same distribution as $B$. Further, they are independent of $N$. Squaring (71) and taking expectations, we get:

$$
E\{B^2\} = E\{X^2\} + 2E\{X\}E\{N\}E\{B\} + E\left\{ \sum_{i=1}^{N} B_i \right\}^2 \tag{72}
$$

Note that $X$ is a geometric r.v. with parameter $\phi_{nc}$. Thus $E\{X\} = 1/\phi_{nc}$ and $E\{X^2\} = (2 - \phi_{nc})/\phi_{nc}^2$. Also, $E\{N\} = \lambda_{pu}E\{X\} = \lambda_{pu}/\phi_{nc}$. Using these in (72), we have:

$$
E\{B^2\} = \frac{(2 - \phi_{nc})}{\phi_{nc}^2} + \frac{2\lambda_{pu}}{\phi_{nc}(\phi_{nc} - \lambda_{pu})} + E\left\{ \sum_{i=1}^{N} B_i \right\}^2
$$

To calculate the last term, we have:

$$
E\left\{ \sum_{i=1}^{N} B_i \right\}^2 = E\left\{ \sum_{i=1}^{N} B_i^2 \right\} + 2E\left\{ \sum_{i\neq j} B_i B_j \right\}
$$

$$
= E\{N\}E\{B^2\} + 2(E\{B\})^2(E\{N^2\} - E\{N\})
$$

Note that given $X = x$, $N$ is a binomial r.v. with parameters $(x, \lambda_{pu})$. Thus, we have:

$$
E\{N^2\} = \sum_{x=1}^{\infty} E\{N^2|X = x\} Prob[X = x]
$$

$$
= \sum_{x=1}^{\infty} \left[(x\lambda_{pu})^2 + x\lambda_{pu}(1 - \lambda_{pu})\right](1 - \phi_{nc})^{x-1}\phi_{nc}
$$

$$
= \lambda_{pu}^2 \sum_{x=1}^{\infty} x^2\phi_{nc}(1 - \phi_{nc})^{x-1}
$$

$$
+ \lambda_{pu}(1 - \lambda_{pu}) \sum_{x=1}^{\infty} x\phi_{nc}(1 - \phi_{nc})^{x-1}
$$

$$
= \lambda_{pu}^2 \frac{(2 - \phi_{nc})}{\phi_{nc}^2} + \lambda_{pu}(1 - \lambda_{pu}) \frac{1}{\phi_{nc}}
$$

Using this, we have:

$$
E\left\{ \sum_{i=1}^{N} B_i^2 \right\}
$$

$$
= \frac{\lambda_{pu}}{\phi_{nc}} E\{B^2\} + 2 \left( \frac{1}{\phi_{nc} - \lambda_{pu}} \right)^2 (E\{N^2\} - E\{N\})
$$

$$
= \frac{\lambda_{pu}}{\phi_{nc}} E\{B^2\} + 2 \left( \frac{1}{\phi_{nc} - \lambda_{pu}} \right)^2 \left( \frac{2\lambda_{pu}(1 - \phi_{nc})}{\phi_{nc}} \right)
$$

Using this, we have:

$$
E\{B^2\} = \frac{(2 - \phi_{nc})}{\phi_{nc}^2} + \frac{2\lambda_{pu}}{\phi_{nc}(\phi_{nc} - \lambda_{pu})} + \frac{2\lambda_{pu}(1 - \phi_{nc})}{\phi_{nc}} + 2 \left( \frac{1}{\phi_{nc} - \lambda_{pu}} \right)^2 \left( \frac{2\lambda_{pu}(1 - \phi_{nc})}{\phi_{nc}} \right)
$$

Simplifying this yields:

$$
E\{B^2\} = \frac{(2 - \phi_{nc})}{\phi_{nc}(\phi_{nc} - \lambda_{pu})} + \frac{2\lambda_{pu}}{\phi_{nc}(\phi_{nc} - \lambda_{pu})^2} + \frac{4\lambda_{pu}^2(1 - \phi_{nc})}{\phi_{nc}(\phi_{nc} - \lambda_{pu})^3}
$$

\textbf{References}

[1] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty. NoXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey. Comput. Netw., 50:2127-2159, Sept. 2006.

[2] Q. Zhao and B. Sadler. A survey of dynamic spectrum access. IEEE Signal Processing Magazine, 24(3):79-89, May 2007.

[3] R. Urgaonkar and M. J. Neely. Opportunistic scheduling with reliability guarantees in cognitive radio networks. IEEE Trans. Mobile Computing, 8(6):766-777, June 2009.

[4] A. I. Goldsmith, S. A. Jafar, I. Maric, and S. Srivastava. Breaking spectrum gridlock with cognitive radios: An information theoretic perspective. Proc. of the IEEE, 97(5):894-914, May 2009.
[5] A. Carleial. Interference channels. *IEEE Trans. Inform. Theory*, 24(1):60-70, Jan. 1978.
[6] T. Han and K. Kobayashi. A new achievable rate region for the interference channel. *IEEE Trans. Inform. Theory*, 27(1):49-60, Jan. 1981.
[7] T. Cover and A. E. Gamal. Capacity theorems for the relay channel. *IEEE Trans. Inform. Theory*, 25(5):572-584, Sep. 1979.
[8] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. New York: John Wiley & Sons, Inc., 1991.
[9] O. Simeone, Y. Bar-Ness, and U. Spagnolini. Stable throughput of cognitive radios with and without relaying capability. *IEEE Trans. Communications*, 55(12):2351-2360, Dec. 2007.
[10] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz. Spectrum leasing to cooperating secondary ad hoc networks. *IEEE ISAC Special Issue on Cognitive Radio: Theory and Applications*, 26(1):203-213, Jan. 2008.
[11] J. Zhang and Q. Zhang. Stackelberg game for utility-based cooperative cognitive radio networks. *Proc. ACM MobiHoc*, May 2009.
[12] J. Krikidis, J. N. Laneman, J. Thompson, and S. McLaughlin. Protocol design and throughput analysis for multi-user cognitive cooperative systems. *IEEE Trans. Wireless Commun.*, 8(9):4740-4751, Sept. 2009.
[13] B. Rong, J. Krikidis, and A. Ephremides. Network-level cooperation with enhancements based on the physical layer. *IEEE Information Theory Workshop*, Cairo, Egypt, Jun. 2010.
[14] M. Levorato, U. Mitra, and M. Zorzi. Cognitive interference management in retransmission-based wireless networks. *Proc. 47th Allerton Conference on Communication, Control, and Computing*, Sept. 2009.
[15] G. Gur, S. Bayhan, and F. Alagoz. Cognitive femtocell networks: An overlay architecture for localized dynamic spectrum access. *IEEE Wireless Communications*, 17(4):62-70, Aug. 2010.
[16] J. Jin and B. Li. Cooperative resource management in cognitive wimax with femto cells. *Proc. IEEE INFOCOM*, March 2010.
[17] L. Georgiadis, M. J. Neely, and L. Tassiulas. Resource allocation and cross-layer control in wireless networks. *Foundations and Trends in Networking*, 1(1):1-149, 2006.
[18] M. J. Neely. Stochastic optimization for markov modulated networks with application to delay constrained wireless scheduling. *IEEE Conference on Decision and Control*, Dec. 2009.
[19] C. Li and M. J. Neely. Network utility maximization over partially observable markovian channels. *arXiv:1008.3421v1*, Aug. 2010.
[20] M. J. Neely. Dynamic optimization and learning for renewal systems. *Proc. Asilomar Conference*, Nov. 2010.
[21] M. J. Neely. *Stochastic Network Optimization with Application to Communication & Queueing Systems*. Morgan&Claypool, 2010.
[22] M. J. Neely. Energy optimal control for time varying wireless networks. *IEEE Trans. Inform. Theory*, 52(7):2915-2934, July 2006.
[23] D. P. Bertsekas. *Dynamic Programming and Optimal Control*, vols. 1 & 2, Belmont, MA: Athena Scientific, 2007.
[24] E. Altman. *Constrained Markov Decision Processes*. Boca Raton, FL: Chapman and Hall/CRC Press, 1999.
[25] M. L. Puterman. *Markov Decision Processes*. John Wiley & Sons, 2005.
[26] D. P. Bertsekas and J. N. Tsitsiklis. *Neuro-Dynamic Programming*. Belmont, MA: Athena Scientific, 1996.
[27] R. Gallager. *Discrete Stochastic Processes*. Kluwer Academic Publishers, Boston, 1996.