Low Complexity Adaptation for Reconfigurable Intelligent Surface-Based MIMO Systems

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Abstract—Reconfigurable intelligent surface (RIS)-based transmission technology offers a promising solution to enhance wireless communication performance cost-effectively through properly adjusting the parameters of a large number of passive reflecting elements. This letter proposes a cosine similarity theorem-based low-complexity algorithm for adapting the phase shifts of an RIS that assists a multiple-input multiple-output (MIMO) transmission system. A semi-analytical probabilistic approach is developed to derive the theoretical average error probability (ABEP) of the system. Furthermore, the validity of the theoretical analysis is supported through extensive computer simulations.

Index Terms—Reconfigurable intelligent surfaces (RISs), multiple-input multiple-output (MIMO), cosine similarity theorem.

I. INTRODUCTION

The fifth generation (5G) wireless communication technology promises an explosive growth on data rate, massive connectivity and latency performance. To achieve these goals, various transmission technologies have been developed in recent years. Massive multiple-input multiple-output (MIMO) and millimeter wave (mmWave) communication systems are considered as some of the prominent candidates among these technologies. On the other hand, to meet this challenge beyond 5G requirements, utilization of an increasing number of multi-antenna systems has raised strong concerns about the energy efficiency and hardware cost of large-scale MIMO systems.

Recently, reconfigurable intelligent surface (RIS)-assisted communication technology has been considered as a promising solution to overcome the energy efficiency related issues of future wireless networks [1]–[3]. An RIS is a planar metasurface that consists of a number of low-cost passive reflecting elements, each of which smartly induces an independent phase shift to modify the propagation environment in more favorable way for the communication performance [4].

The unprecedented potential of RISs on the signal quality of a communication system has led researchers to largely consider the RIS technology in various frontiers. In one of the early studies [3], the error performance of an RIS aided single-input single-output (SISO) system is investigated through a mathematical framework. Also, in [5], to improve the spectral efficiency, an RIS aided index modulation (IM) system has been proposed. Later, RISs are considered for multiuser systems to enhance the energy efficiency [6] and to maximize the signal-interference-noise ratio (SINR) [2], [7], [8]. Even more recently, for RIS-assisted multi-antenna systems, new path loss models [9], [10] have been developed.

Notably, most of the existing RIS-aided designs developed for multiple-input single-output (MISO) [2], [6], [7], [11] and MIMO systems [1], [8] exploit computationally intensive and complex algorithms. Specifically, no single study exists performing a statistical analysis on the theoretical bit error rate (BER) performance of RIS-aided multi-antenna systems.

In this letter, an efficient low-complexity algorithm, which is based on cosine similarity theorem [12] and adapts the phase shift of each reflecting elements, is proposed for RIS-aided MIMO (MIMO-MIMO) and RIS-aided spatial modulation (RIS-SM) schemes in which the transmission principles of classical MIMO and classical SM [5], [13] are considered, respectively. Moreover, a semi-analytical probabilistic model is developed to derive the average bit error probability (ABEP) of the proposed RIS-MIMO and RIS-SM schemes. Furthermore, through computer simulations the validity of our semi-analytic model is approved.

The rest of this letter is structured as follows. In Section II, the system model of the proposed RIS-aided MIMO transmission systems is presented. The theoretical performance analysis of the system is carried out in Section III. Computer simulation results are described in Section IV and the paper is concluded in Section V.

Notation: Bold lowercase and uppercase letters are used for vectors and matrices, respectively. $(\cdot)^T$, $(\cdot)^H$ and $||\cdot||$ stand for transposition, Hermitian transposition and Euclidean/Frobenius norm operators, respectively. vec$(\cdot)$ denotes vectorization operator and det$(\cdot)$ symbolizes determinant of a matrix. The Kronecker product and Euclidean inner product of two vectors are denoted by $\otimes$ and $<\cdot,\cdot>$, respectively. $\mathbb{C}^{a \times b}$ represents the set of $a \times b$ dimensional matrices while diag$(\cdot)$ symbolizes a square diagonal matrix. $CN(\mu,\sigma^2)$ denotes the complex Gaussian distribution of a random variable with $\mu$ mean and $\sigma^2$ variance. $\mathcal{O}(\cdot)$ and $P(\cdot)$ stand for the big $\mathcal{O}$ notation and probability of an event, respectively. $I_n$ denotes $n \times n$ identity matrix while $1$ represents all-ones column vector.
SM schemes are introduced. In the proposed systems, the transmitter and the receiver are assumed to be equipped with $T_x$ and $R_x$ antennas, respectively, as shown in Fig. 1. In addition, an RIS with $N$ passive reflecting elements is used to improve the communication performance by appropriately adjusting phase shift of each reflecting element.

Consider $H \in \mathbb{C}^{N \times T_x}$ and $G \in \mathbb{C}^{N \times R_x}$ as the matrices of uncorrelated Rayleigh fading channel from the transmitter to the RIS, and from the RIS to the receiver, respectively, whose elements follow $CN(0, 1)$ distribution. On the other hand, $\Phi \in \mathbb{C}^{N \times N}$ stands for the matrix of RIS reflection coefficients with $\Phi = \text{diag}\{e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_N}\}$, where $\phi_i \in [-\pi, \pi]$ is phase shift of the $i$th reflecting element, for $i \in \{1, 2, \ldots, N\}$. Therefore, the composite MIMO channel matrix $C \in \mathbb{C}^{R_x \times T_x}$ from the transmitter to the receiver becomes $C = G^H \Phi H$. In the proposed systems, perfect channel state information (CSI) of all channels are available at all nodes and quasi-static block fading channels are assumed.

### A. Proposed Algorithm

In this subsection, we develop a low-complexity algorithm to maximize the average received signal-noise ratio (SNR) of the RIS aided MIMO systems, which results in maximizing the overall channel gain of the system by arranging the phase shift of each reflecting element. Then, using $C = G^H \Phi H = \sum_{i=1}^{N} g_i^H e^{j\phi_i} h_i$, our problem is formulated as

$$\max_{\phi_i} \|C\| = \max_{\phi_i} \|\sum_{i=1}^{N} g_i^H e^{j\phi_i} h_i\| \quad \text{s.t.} \quad |e^{j\phi_i}| = 1$$

(1)

where $g_i^H$ and $h_i$ stand for the $i$th column and $i$th row of $G^H$ and $H$, respectively. Although the maximization problem in (1) is non-convex due to the constraint $\phi_i \in [-\pi, \pi]$, the achievable channel gain can be upper bounded as

$$\|C\| = \|\sum_{i=1}^{N} g_i^H e^{j\phi_i} h_i\| \leq \sum_{i=1}^{N} \|g_i^H\| \|h_i\|.$$  

(2)

Then, exploiting (2), the maximum achievable gain of the component at the $k$th row and the $l$th column of $C$, shown by $c_{k,l}$, can be given as

$$c_{k,l} = \|\sum_{i=1}^{N} g_k i e^{j\phi_i} h_{i,l}\| \leq \sum_{i=1}^{N} \|g_k i\| |h_{i,l}|.$$  

(3)

where $g_k i$ and $h_{i,l}$ are the $k$th and $l$th components of the channel vectors $g_i^H$ and $h_i$, respectively, for $k \in \{1, 2, \ldots, R_x\}$ and $l \in \{1, 2, \ldots, T_x\}$. It is quite obvious that there exists an optimum $\Phi$ matrix that satisfies (3) with an equality to achieve the maximum channel gain. However, since multi-antenna system is considered at both transmitter and receiver, each reflecting element affects multiple channel coefficients at each coherence interval. Therefore, it is not easy to find an optimal solution for this in a computationally efficient and robust manner. Due to this limitation, we develop an efficient suboptimal solution using the cosine similarity theorem [12], as

$$\cos(u, v) = \frac{\langle u, v \rangle}{||u|| \cdot ||v||}.$$  

(4)

In the proposed algorithm, to maximize the individual gain of each $c_{k,l}$ component (3), the phase shift $\phi_i$ is adjusted to approximate the complex channel vectors $h_i$ and $g_i^H$ to their component-wise absolute vectors $\tilde{h}_i = |h_{i,1}|, |h_{i,2}|, \ldots, |h_{i,T_x}|$ and $\tilde{g}_i^H = |g_{i,1}|, |g_{i,2}|, \ldots, |g_{i,R_x}|$ respectively. Therefore, in Algorithm 1, real $\phi^h_i$ and $\phi^g_i$ angles are calculated in order to measure the cosine similarity between the vectors $h_i$ and $\tilde{h}_i$ and $g_i^H$ and $\tilde{g}_i^H$, respectively. Then, for the $i$th reflecting element, the overall phase shift $\phi_i$ is determined as $\phi_i = - (\phi^h_i + \phi^g_i)$ and the overall reflection matrix $\Phi$ is obtained accordingly.

Let us present the concept of the proposed algorithm by an example. Consider an RIS-aided MIMO system, with $T_x = 2$, $R_x = 3$ ve $N = 2$ whose composite channel matrix $C = G^H \Phi H$ is constructed as

$$C = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix} \begin{bmatrix} e^{j\phi_1} & 0 \\ 0 & e^{j\phi_2} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}.$$  

(5)

which can be rewritten, in the form of $C = \sum_{i=1}^{N} g_i^H e^{j\phi_i} h_i$, as

$$C = \begin{bmatrix} g_{11} e^{j\phi_1} h_{11} + g_{12} e^{j\phi_2} h_{12} \\ g_{21} e^{j\phi_1} h_{21} + g_{22} e^{j\phi_2} h_{22} \end{bmatrix}.$$  

(6)

Since the phase shift $\phi_i$ only affects $g_i^H$ and $h_i$ vectors, instead of jointly adjusting all phases, each $\phi_i$ can be individually determined in a more computationally efficient manner. Therefore, in the proposed algorithm, considering (3), we singly determine each $\phi_i$ to improve the overall channel gain. For

### Algorithm 1 Cosine Similarity-based Low-Complexity Algorithm

**Input:** $h_i, g_i, h_i, g_i$  
**Output:** $\Phi$

1: for $i = 1 : N$ do  
2: $\phi^h_i = \arccos(\frac{\langle h_i, \tilde{h}_i \rangle}{||h_i|| \cdot ||\tilde{h}_i||})$  
3: $\phi^g_i = \arccos(\frac{\langle e^{j\phi^h_i} g_i^H, \tilde{g}_i^H \rangle}{||e^{j\phi^h_i} g_i^H|| \cdot ||\tilde{g}_i^H||})$  
4: $\phi_i = - (\phi^h_i + \phi^g_i)$  
5: end for  
6: $\Phi = \text{diag}(e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_N})$
this aim, $\phi_i$ is properly adjusted to make the channel vectors $h_i$ and $g_i^H$ approximate to their component-wise absolute vectors $h_i = [|h_{i,1}|, |h_{i,2}|]$, and $g_i^H = [|g_{i,1}|, |g_{i,2}|, |g_{i,3}|]^H$, respectively. Therefore, as given in Algorithm 1, the real angles $\phi_i^R$ and $\phi_i^B$, which respectively measure the similarities between the vectors $h_i$ and $h_i$, and the vectors $g_i^H$ and $g_i^H$, are calculated using the cosine similarity theorem [12]. Then, the effective phase shift of $i$th reflecting elements is set to $\phi_i = -(\phi_i^R + \phi_i^B)$. After $N$ repetitions are performed, the overall reflection matrix $\Phi$ is constructed.

The required complexity to perform this algorithm is $O(6N(T_x + R_x))$ in terms of real multiplications (RMs). It is worth noting that when a SISO system ($T_x = R_x = 1$) is considered, this algorithm satisfies (3) with an equality that achieves the optimum channel gain in [3].

\section*{B. RIS-MIMO Scheme}

In the proposed RIS-MIMO scheme with $T_x$ transmit antennas, $R_x$ receive antennas and $N$ reflecting elements, a classical MIMO transmission principle is applied and the reflection parameters are determined by performing the proposed low-complexity algorithm to improve the overall channel gain. Let $x = [x_1, x_2, \ldots, x_T]^T \in C^{T_x \times 1}$ be the transmitted signal vector of the RIS-MIMO scheme, where $x_j$ denotes an $M$-ary phase shift keying/quadrature amplitude modulation (PSK/QAM) symbol transmitted through $j$th transmit antenna, for $j \in \{1, 2, \ldots, T_x\}$. Then, the received signal vector $y \in C^{R_x \times 1}$ becomes

$$y = G^H \Phi H x + n$$

where $n \in C^{R_x \times 1}$ is the vector of additive white Gaussian noise samples whose elements are i.i.d. and follow $CN(0, N_0)$ distribution.

\section*{C. RIS-SM Scheme}

In this subsection, the system model of the RIS-SM scheme, where an RIS-aided MIMO scheme applying the traditional SM transmission principle [13] is presented. Unlike traditional SM [13], in the proposed RIS-SM scheme, the transmitted signal quality is significantly improved by the RIS, whose each reflection parameter is arranged using the proposed low-complexity algorithm.

In RIS-SM, one of the $T_x$ transmit antenna is activated and a modulated symbol $s$ from the $M$-PSK constellation is transmitted through this active antenna. Since the remaining transmit antennas are deactivated, the signal vector is determined as $x = [0 \ldots 0 \ s \ 0 \ldots 0]^T$ and transmitted through the overall channel matrix $C$ as given in (7).

Note that in the proposed RIS-SM scheme, the knowledge of the active antenna index is not provided to the RIS.

At the receiver of both RIS-MIMO and RIS-SM schemes, to obtain the best BER performance, a maximum likelihood (ML) detector is considered, and the transmit RIS-SM and RIS-MIMO vectors are detected by considering all possible $x$ realizations as follows

$$\hat{x} = \text{arg min}_x \| y - C x \|^2. \quad (8)$$

\section*{III. PERFORMANCE ANALYSIS}

In this section, the theoretical BER performance of the proposed RIS-MIMO and RIS-SM schemes is evaluated by performing a semi-analytical probabilistic approach.

\subsection*{A. Numerical Analysis}

As given in Algorithm 1, since the proposed algorithm relates the phase shifts of the reflecting elements with the channel statistics, the reflection matrix $\Phi$ is directly correlated with the channel matrices $H$ and $G$. Therefore, the distribution of the composite channel matrix $C$ could not be derived through a fully analytic approach. As a result, we resort to a comprehensive numerical analysis in order to obtain the statistics of the channel matrix $C$.

As it is stated in the previous section, the elements of the channel matrices $H$ and $G$ are i.i.d. and follow $CN(0, 1)$ distribution. Our comprehensive numerical analysis, which performs $10^6$ Monte-Carlo trials for each $R_x \times T_x$ configuration, indicates that the elements of the composite $C$ matrix are complex Gaussian random variables with $CN(N\mu, N)$ distribution, where $\mu$ is numerically calculated, in terms of $T_x$ and $R_x$, as

$$\mu = \frac{1.8}{(1 + 2T_x)(1 + 2R_x)}. \quad (9)$$

For supporting the accuracy of this estimation, in Fig 2, is compared to the Monte Carlo simulations that perform at least $10^6$ trials for each $R_x \times T_x$ set-up. The results show that the estimated $\mu$ in (9) perfectly fit the computer simulations per $R_x \times T_x$.

Then, (9) is utilized to determine the ABEP of the system in the following subsection.

\subsection*{B. ABEP Analysis}

In this subsection, after obtaining a numerical approximation for the statistics of the channel matrix $C$, an upper bound expression for the ABEP of the proposed system is given as follows [14]:

$$P_c \leq \frac{1}{\kappa 2^n} \sum_x \sum_{\hat{x}} P(x \rightarrow \hat{x})e(x, \hat{x}) \quad (10)$$
where \( \kappa \) is the number of incoming information bits, \( P(x \to \hat{x}) \) is the unconditional pairwise error probability (PEP) and \( e(x, \hat{x}) \) is the number of error bits for the corresponding PEP event.

To obtain the PEP expression, first, the conditional PEP (CPEP) of the system is derived, using the Q-function, as follows

\[
P(x \to \hat{x}|z) = Q\left( \sqrt{\frac{\Omega}{2\sigma^2}} \right) \tag{11}
\]

where \( \Omega \) is given, for \( \Delta = (x - \hat{x})(x - \hat{x})^H \), as

\[
\Omega = ||C(x - \hat{x})||^2 = \text{vec}(C^H)(\Delta \otimes I_{R_z})\text{vec}(C^H). \tag{12}
\]

Therefore, considering \( Q(x) = \frac{1}{\pi} \int_{0}^{\pi/2} e^{-x^2/2\sin^2\theta} \, d\theta \), the CPEP (11) can be rewritten as

\[
P(x \to \hat{x}|z) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp\left( -\varphi \frac{\text{vec}(C^H)(\Delta \otimes I_{R_z})\text{vec}(C^H)}{4\sin^2\theta} \right) d\theta \tag{14}
\]

where \( \varphi = 1/N_0 \).

Assume the quadratic form expression in (13) is given in terms of a Hermitian matrix \( B = \Delta \otimes I_{R_z} \) and complex Gaussian vector \( z = \text{vec}(C^H) \) as \( \Omega = z^HBz \). Then, averaging (14) over the matrix \( C \) through the moment generating function (MGF) approach of this quadratic form (15) results in the following PEP expression

\[
P(x \to \hat{x}) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp\left( -\varphi \frac{\zeta^H(\frac{\varphi}{4\sin^2\theta}B(I + \frac{\varphi}{4\sin^2\theta}C_zB)^{-1}\zeta)}{\det(I + \frac{\varphi}{4\sin^2\theta}C_zB)} \right) d\theta \tag{15}
\]

where, given the numerically obtained channel statistics in the previous subsection, \( \zeta = \text{vec}(C^H) = \mu N_1 \) and \( C_z = N_1 \) are respectively defined as the mean vector and the covariance matrix for the Gaussian vector \( z = \text{vec}(C^H) \).

IV. SIMULATION RESULTS

In this section, the BER performance of the proposed RIS-MIMO and RIS-SM schemes are investigated through theoretical analysis and computer simulations. All results are performed as a function of transmitted signal energy to noise ratio \((E_s/N_0)\) and for a \(4 \times 4\) MIMO system and BPSK modulation.

In Fig. 3 for various \( N \) reflecting elements, the theoretical BER performance of RIS-MIMO and RIS-SM schemes are compared with the computer simulation results. It is obvious from this figure that the derived semi-analytic results perfectly match with the simulation results as \( N \) increases.

In Fig. 4 the BER performance of the RIS-MIMO and RIS-SM schemes using the proposed cosine similarity theorem-based algorithm and pseudoinverse (pinv)-based algorithm [8] is compared. The results show that compared to [8], the BER performance of both RIS-MIMO and RIS-SM schemes using the proposed algorithm, which requires significantly lower computational complexity, improves better as \( N \) increases.

In Fig. 5, the BER performance of RIS-MIMO scheme in case of the path loss effect [9].
As an illustration, the proposed algorithm with $O(6N(T_x + R_x))$ complexity performs 3072 RMs for $N = 64$, while the reference pinn algorithm [8] with $O(N^3)$ complexity performs 262144 RMs. This means, the proposed algorithm provides 98.8% reduction in computational complexity over the reference algorithm [8] for $N = 64$.

Fig. 5 demonstrates the BER performance of the proposed RIS-MIMO scheme when path loss effect [9] is considered. Denoting the distances from the transmitter to the RIS and from the RIS to the receiver as $d_1$ and $d_2$, respectively, the path loss $P_L$ of the overall system is calculated as [9]

$$P_L^{-1} = \frac{\lambda^4}{256\pi^2} \frac{1}{d_1^2 d_2^2}$$

(16)

where $\lambda$ is the wavelength at 2.4 GHz operating frequency. The simulations are performed for $d_1 = d_2 = 10$ m and $d_1 = 2$ m, $d_2 = 18$ m. It can be deduced from the results that compared to the performance in the absence of the path loss given in Fig. 4 the error performance of the RIS-MIMO scheme significantly degrade with the path loss effect. On the other hand, since the $P_L$ is proportional to $d_1^2 d_2^2$, the BER performance of the system fairly improves when the RIS is closer to the transmitter or the receiver ($d_1 = 2$ m and $d_2 = 18$ m).

V. CONCLUSION

In this letter, a low-complexity algorithm has been developed exploiting the cosine similarity theorem for RIS aided MIMO transmission schemes to enhance the overall path gain of the communication channel. Moreover, performing a semi-analytic approach, the ABEP of the system has been derived. Through theoretical analysis and computer simulations the BER performance of RIS aided MIMO schemes with and without the path loss effect has been investigated.

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