Phase-kicked control of counter-rotating interactions in the quantum Rabi model

Jin-Feng Huang\textsuperscript{1} and C. K. Law\textsuperscript{1}

\textsuperscript{1}Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Shatin, Hong Kong Special Administrative Region, People’s Republic of China

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We present an interaction scheme to control counter-rotating terms in the quantum Rabi model. We show that by applying a sequence of \(\pi/2\) phase kicks to a two-level atom and a single mode quantized field, the natural dynamics of the Rabi model can be interrupted in a way that counter-rotating transitions can be significantly enhanced. This is achieved by a suitable timing of the phase kicks determined by a phase matching condition. If the time between successive kicks is sufficiently short, our scheme is turned into a dynamical decoupling problem in which the effects of counter-rotating terms can be strongly suppressed under ultrastrong coupling.

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I. INTRODUCTION

In this paper we investigate a mechanism of controlling virtual processes occurring in a two-level atom strongly interacting with a single-mode quantized electromagnetic field. Virtual processes here refer to excitation-number non-conserving processes due to counter-rotating terms in the interaction Hamiltonian. Such processes are generally fast oscillatory with feeble amplitudes and therefore the employment of rotating wave approximation (RWA) is justified when studying the quantum dynamics under the framework of the Jaynes-Cummings (JC) model \cite{1, 2}. In doing so, however, interesting physics of virtual processes is lost. To go beyond the JC model, recent investigations have begun to turn to the ultrastrong coupling regime in which the vacuum Rabi frequency is comparable to the atomic transition frequency and the field frequency. In such a regime, RWA is no longer valid and counter-rotating terms can give rise to novel features such as quantum integrability \cite{3}, asymmetry of vacuum Rabi-splitting \cite{4}, non-classical photon statistics \cite{5, 6}, superradiance transition \cite{7}, and the spontaneous release of virtual photons \cite{8, 9}. In addition, counter-rotating terms can modify the collapse and revival dynamics \cite{10}, and quantum Zeno and anti-Zeno effects \cite{11, 12}.

We note that the realization of an ultrastrong coupling in natural systems is still a challenge \cite{13}, although recent experiments have demonstrated ultrastrong coupling in artificial atomic systems with the vacuum Rabi frequency being a moderate fraction of the field frequency \cite{14–18}. Therefore an interesting question is how to enhance the counter-rotating terms or the corresponding virtual transitions without increasing the coupling strength to the ultrastrong coupling regime. One possible solution is to use a fast modulation of the coupling strength such that counter-rotating terms are converted into co-rotating ones. This strategy has been considered in Ref. \cite{19} to generate quantum vacuum radiation, and it requires the coupling strength to oscillate at a frequency which is about twice the cavity field frequency.

In this paper we indicate an alternative approach based on a sequence of phase kicks to the system. Each phase kick corresponds to a unitary operation determined by the square root of the parity operator. As we shall see below, by repeatedly applying phase kicks to the system with a suitable time separation between successive kicks, we can strongly enhance a certain virtual transition for systems with an interaction strength far below the ultrastrong coupling regime. We point out that the enhanced virtual transition is induced by matching the phase of quantum evolution rather than the traditional energy-level resonance, and so a high frequency modulation of coupling strength is not required.

Interestingly, if the time between adjacent kicks is sufficiently short, counter-rotating terms can be significantly suppressed even in the ultrastrong coupling regime. In other words, states of different excitation numbers are dynamically decoupled. We note that dynamical decoupling have been studied extensively in the context of decoherence control \cite{20}. Our scheme here generalizes the concept by Vitali and Tombesi \cite{21, 22} in order to suppress unwanted transitions by counter-rotating terms. The problem is relevant to systems in the ultrastrong coupling regime in which the Rabi oscillations are distorted appreciably. With our scheme, we can restore the JC dynamics effectively.

II. THE INTERACTION SCHEME

To begin with we consider a two-level atom interacting with a single-mode cavity field. The system is modelled by the quantum Rabi model with the Hamiltonian (\(\hbar = 1\)) \cite{23}:

\[
\mathcal{H}_R = \mathcal{H}_{JC} + \mathcal{H}_V,
\]

where \(\mathcal{H}_{JC}\) is the JC Hamiltonian, and \(\mathcal{H}_V\) contains counter-rotating terms:

\[
\mathcal{H}_{JC} = \frac{\omega_0}{2} \sigma_z + \omega_0 a^\dagger a + \lambda_0 (a \sigma_+ + a^\dagger \sigma_-),
\]

\[
\mathcal{H}_V = \lambda_0 (a^\dagger \sigma_+ + a \sigma_-).
\]
Here, $a$ and $a^\dagger$ are annihilation and creation operators of the field mode, and the two-level atom has a ground state $|g\rangle$ and an excited state $|e\rangle$ so that $\sigma_z \equiv |e\rangle \langle e| - |g\rangle \langle g|$ is defined as usual. The $\omega_0$ and $\omega_c$ are atomic and cavity field frequencies respectively, and $\lambda_0$ denotes the atom-cavity interaction strength which is half the vacuum Rabi frequency. In this paper we shall focus on the near resonance interaction strength which is half the vacuum Rabi frequency. We assume that the ground state of $H_{JC}$ is denoted by $|g,0\rangle$ which refers to a ground state atom in the vacuum field. For later purpose, we let $|n,s\rangle$ be excited states of $H_{JC}$, i.e., $H_{JC} |n,s\rangle = \epsilon_{n,s} |n,s\rangle$ ($n = 1, 2, 3, \ldots$, $s = \pm$) with $n$ being the excitation number (eigenvalues of $a^\dagger a + |e\rangle \langle e|$), and $s = + (-)$ labelling the higher (lower) energy level of the doublet for a given $n$.

Our strategy to control the effects of $H_V$ is to introduce a sequence of phase kicks to the system (Fig. 1). Each phase kick is described by the unitary operator $P$:

$$P = \exp \left[ -i\pi (a^\dagger a + \sigma_z/2)/2 \right],$$

which is realized in a duration $\tau_P$. We assume that the system evolution during each phase kick is entirely due to $P$. Therefore the evolution operator of the system after $N$ phase kicks is given by:

$$U(NT) = P e^{-iH_R\tau_I}_N P e^{-iH_R\tau_I}= (P e^{-iH_R\tau_I})^N,$$

where $T = \tau_I + \tau_P$, and $\tau_I$ denotes duration of evolution under $H_R$.

III. ENHANCEMENT OF COUNTER-ROTATING TRANSITIONS

We consider the regime $\lambda_0/\omega_c \ll 1$ in which the ground state of $H_R$ is well approximated by $|g,0\rangle$ under RWA. However, by applying the sequence of phase kicks with $\tau_I$ properly chosen, we find that a system initially prepared in $|g,0\rangle$ is no longer stable but to evolve to higher excited states of $H_{JC}$. Note that the $P$ operator alone does not change the photon number and atomic excitations. An example is given in Fig. 2 in which the time-dependent state $|\psi(t)\rangle = U(t)|g,0\rangle$ is calculated numerically, and the figure shows the time-dependence of occupation probabilities of $|g,0\rangle$ and $|2,-\rangle$. We see that the system can make a transition to $|2,-\rangle$ almost completely after some time, and this is impossible without applying the phase kicks.

In order to visualize the dynamics of the phase-kicked system, we have plotted Fig. 2 in continuous time (not only the discrete time $NT$). A detailed picture is displayed in the lower figure. We see that during each time interval when the system is governed by $U_R$, the population of $|2,-\rangle$ is fast oscillatory about a constant mean value. However, the action of $P$ can interrupt the phase of the oscillations in the way that the population of $|2,-\rangle$ climbs up after each kick. This feature continues until the probability of $p_{2,-}$ is almost one.

The timings of phase kicks are important to obtain the enhanced transition. In Fig. 3, we plot the maximum probability $p_{2,\pm}^{\text{Max}}$ for the states $|2,\pm\rangle$ attainable as a function of $\tau_I$. The sharp peaks in Fig. 3 indicate that $p_{2,\pm}^{\text{Max}}$ are significant only at certain values of $\tau_I$ located at $\tau_I \approx (2m+1)\pi/\Delta_\pm$ ($m = 0, 1, 2, \ldots$), where $\Delta_\pm = \epsilon_{2,\pm} - \epsilon_0$ ($s = \pm$) and $\epsilon_0 = -\omega_0/2$ is the ground state energy of $H_{JC}$. These ‘resonance’ values of $\tau_I$ correspond to a phase matching condition that $P$ is switched on at the moment when $p_{2,\pm}$ reaches a turning (maximal) point, as illustrated in the lower figure of Fig. 2.

Having presented our numerical observations, we now provide a theory to explain the enhancement effect. Specifically we consider the case of enhancing the transition from $|g,0\rangle$ to $|2,\pm\rangle$ with the required $\tau_I$
suggested, the system is approx-
imated confined to the subspace formed by $|g,0\rangle$ and $|2,−\rangle$, therefore we shall discuss the dynamics in this subspace. The justification of such a two-level approximation will be seen later.

The evolution operator for each cycle $T$ can be ob-
tained approximately by using first-order time-dependent perturbation theory in which $H_V$ is treated as a pertur-
bation in the interaction picture defined by $H_{JC}$. To the first order of $\lambda_0/\omega_c$, we have

$$P e^{-iH_{R\tau_I}} \approx e^{i\theta}(I - iKNT),$$

where $\theta = \pi/4 - \epsilon_\tau_I$ is a constant phase, and $K$ is given by

$$K \simeq \frac{\phi}{T} [2, s] \langle 2, s | + \frac{2ig_s}{T} [2, s] \langle g, 0 | - \frac{2ig_s}{T} | g, 0 \rangle \langle 2, s |,$$  

(8)

with $g_s = \lambda_0 d_{2,s}/\Delta_s$, $d_{2,s} = \langle e, 1 | 2, s \rangle$, and $I$ being the identity operator. In deriving $K$ in Eq. (8), we have made the expansion $\exp(i\phi) \approx 1 + i\phi$. After $N$ phase kicks, the evolution operator in the two-level subspace is approximated by

$$U (NT) \approx e^{iN\theta} e^{-iKNT},$$

(9)

where the correction is $O (N\kappa^2T^2)$ which can be neglected if $N(g_s^2 + \phi^2) \ll 1$.

Now we see that $K$ in Eqs. (8-9) plays the role of an effective two-level Hamiltonian governing the system evolu-
tion after each kick. In particular, the terms $\phi/T$ and $g_s/T$ act as an effective detuning and Rabi frequency respec-
tively. Therefore, on one hand, $\phi = 0$ corre-
sponds to a resonance condition so that an initial $|g,0\rangle$ state can be completely transferred to $|2,−\rangle$ at the time $t = \pi T/(4g_{−1})$, and this agrees with the exact result ob-
tained by numerical calculations in Fig. 2; on the other hand, $\phi \gg g_−$ corresponds to a large detuning situ-
tion in which the phase kicks have almost no effects on the system. This explains the resonance curve in Fig. 3 be-
cause $\phi$ is related to $\tau_I$. In particular, the sharp resonance peaks in Fig. 3 are located at $\tau_I$ corresponding to $m = 0, 1, 2, \ldots$ by Eq. (6), and the peak width is $8\omega_c g_s/\Delta_s$.

The theoretical analysis above can be applied to the $|2, +\rangle$ transition by simply taking $s = +$ in Eqs. (6-
9), and this would yield a different set of resonance. Note that in Fig. 3, the peaks associated with the $|2, +\rangle$ transition (blue dashed curve) are well separated from those for the $|2, −\rangle$ transition (red curve), except for the $m = 0$ case. To understand this, let us choose $\tau_I$ at the exact resonance of the $|2, −\rangle$ transition, i.e., $\tau_I = (2m + 1)\pi/\Delta_−$, then this value of $\tau_I$ is deviated from the resonance peak of $|2, +\rangle$ of the same $m$ because $\Delta_+ \neq \Delta_−$. By Eq. (6) with $s = +$, the deviation is $\phi = (2m + 1)\pi(\Delta_+ - \Delta_-)/\Delta_-$. Such a $\phi$ acts like an effective detuning which increases with $m$. In fact, for the parameter used in Fig. 3, a large detuning condition $\phi \gg g_+$ is satisfied when $m > 1$, and hence the $|2, +\rangle$ transition is suppressed when $\tau_I$ is taken to be at the resonance peak of the $|2, −\rangle$ transition. This explains why peaks of different transitions in Fig. 3 are more separ-
ated for higher $m$’s, which also justifies our two-level treatment in Eq. (8). The case of $m = 0$ involves a par-
tial overlap of resonances, and it requires a multi-level treatment. However, the details of this special case will not be discussed here.

IV. SUPPRESSION OF COUNTER-ROTATING TRANSITIONS

In this section we show that the sequence of phase kicks can be used to suppress counter-rotating terms in the ultrastrong coupling regime if $\tau_I < 1/\omega_0$ is suf-
ciently short. This is different from the previous section in which the values of $\tau_I$ considered in Eq. (6) can be much longer than $1/\omega_0$ (or $1/\omega_c$). We begin by noting that the parity operator of the Rabi model is given by $\Pi = -\sigma_z(-1)^n a$ which commutes with $H_R$ [10]. By the fact that $\Pi = -iP^2$, we have $P = P^2 P^\dagger = i\Pi P^\dagger$, and so the evolution operator in Eq. (5) with an even $N$ is:

$$U (NT) = (i\Pi)^{N/2} e^{-i(H_{JC} + H_V)\tau_I} e^{-i(H_{JC} + H_V)\tau_I)^{N/2}},$$

since $[H_{R\Pi}, \Pi] = [\Pi, P] = [\Pi, P^\dagger] = 0$. Next by $P^\dagger H_R P = H_{JC} - H_V$, we have

$$U (NT) = (i\Pi)^{N/2} e^{-i(H_{JC} - H_V)\tau_I} e^{-i(H_{JC} + H_V)\tau_I)^{N/2}},$$

(10)

which indicates that the application of two successive phase kicks can alternatively change the sign of $H_V$. If $\tau_I$
is sufficiently small, we can make the following approximation up to the second order of $\tau_I$:

$$e^{-i(H_{JC} - H_V)\tau_I} e^{-i(H_{JC} + H_V)\tau_I} \approx e^{-i(H_{JC} + \hat{\varepsilon})2\tau_I},$$  

where $\hat{\varepsilon} = -i\tau_I [H_{JC}, H_V]/2$. Therefore, in the limit $\tau_I \to 0$ (but keeping time $NT$ fixed), we have $\hat{\varepsilon} \to 0$, and $U(NT) \to (i\Pi)^{NT/2}\exp(-iH_{JC} NT\tau_I)$, i.e., $H_V$ is completely suppressed. In this limit, the system is governed by an effective JC Hamiltonian $H'_{JC} = \tau_I H_{JC}/T$ since $NT$ is the total time.

To illustrate the suppression effect, we resort to numerical calculations and show our results in Fig. 4. We consider the system in the ultrastrong coupling regime so that the effects of counter-rotating terms are prominent. With an initial state given by $|g, 0\rangle$, Fig. 4 shows the time-dependence of the probability of $|g, 0\rangle$ with and without phase kicks. Because of the ultrastrong coupling, $p_{g,0}$ for the case without kicks is oscillatory and it is significantly deviated from one because of counter-rotating transitions. However, by applying the phase-kick sequence, we see that $p_{g,0}$ oscillation amplitudes are suppressed. In particular, $p_{g,0} \approx 1$ for the case with a smaller $\tau_I$ (red thick solid curve). This demonstrates that $|g, 0\rangle$ is dynamically decoupled from the counter-rotating transitions.

We also consider a different initial state given by $|e, 0\rangle$, Fig. 5(a) shows the evolution without phase kicks. We see that the Rabi oscillations between $|e, 0\rangle$ and $|g, 1\rangle$ are strongly modified by $H_V$. In particular, the sum of probabilities of $|e, 0\rangle$ and $|g, 1\rangle$ is not equal to one most of the time, which indicates that virtual transitions beyond $|e, 0\rangle$ and $|g, 1\rangle$ levels are significant. By using phase kicks [Fig. 5(b,c)], we see that sinusoidal Rabi oscillations are recovered. These oscillations agree well with the prediction by the effective JC Hamiltonian $H'_{JC}$ described above. In particular, a smaller $\tau_I$ gives a better agreement [Fig. 5(c)]. Note that the effective vacuum Rabi frequency is modified to $\tau_I\lambda_0/T$.

We point out that our dynamical decoupling scheme is a generalization of the parity-kick approach proposed by Vitali and Tombesi [21] in dealing with the decoherence-control problems, and related ideas have been discussed in different systems [24, 25]. In the case of our system, we exploit the fact that the Rabi Hamiltonian has a parity symmetry, i.e., $[H_R, \Pi] = 0$, and $P$ is effectively a parity kick on the atom+field system. Such a kick can change the sign of the coherence (off-diagonal elements of the density matrix) associated with the unwanted virtual transition, and so reversing the evolution due to $H_V$.

Finally, we discuss the influence of $\hat{\varepsilon}$ in Eq. (11) when $\tau_I$ is finitely small. Explicitly, $\hat{\varepsilon}$ is given by,

$$\hat{\varepsilon} = g_1 a^\dagger \sigma_+ + g_2 a^\dagger^2 \sigma_+ + \text{h.c.},$$  

where $g_1 = -i\tau_I (\omega_0 + \omega_e) \lambda_0/2$ and $g_2 = i\tau_I \lambda_0^2/2$ are defined. We can estimate the correction due to $\hat{\varepsilon}$ by using first-order perturbation theory, which gives the evolution
operator

\[ U(NT) \approx U_0 \left( 1 - i \int_0^{N\tau_I} e^{iH_{JC}t'} \xi e^{-iH_{JC}t'} dt' \right), \tag{13} \]

where \( U_0 = (i\Pi)^{N/2} \exp(-iH_{JC}N\tau_I) \). By comparing \( H_{JC} \) and \( \xi \), the correction term in the bracket can be neglected as long as \( (\omega_0 + \omega_c)\tau_I \ll 1 \) and \( \lambda_0\tau_I \ll 1 \). As an example, for the system evolving from \( |e, 0\rangle \), \( \xi \) can cause virtual transitions to higher states \( |3, \pm\rangle \). The probability of excitation-number non-conserving transitions is characterized by \( p_\varepsilon(t) = 1 - |(e, 0|\psi(t))|^2 - |(g, 1|\psi(t))|^2 \), where \( |\psi(t)\rangle \) is the system state. By Eq. (13), we find that \( p_\varepsilon \) is of order \( \tau_I^2\lambda_0^2 \) in the ultrastrong coupling regime. In Fig. 3, we plot \( p_\varepsilon(t) \) by using first-order perturbation theory Eq. (13) and exact numerical evolution operator in Eq. (5), and they have a good agreement. For the case \( \tau_I = \pi/(18\omega_c) \) considered in Fig. 6, \( p_\varepsilon(t) \) is oscillatory with an amplitude order of \( 10^{-3} \).

V. CONCLUSION

We have discovered a mechanism to control virtual transitions in the quantum Rabi model. On one hand, virtual transitions can be suppressed by using a sufficiently small \( \tau_I \). Since the dynamical decoupling only affects the counter-rotating terms, JC dynamics can be recovered for the systems in the ultrastrong coupling regime. On the other hand, virtual transitions can be amplified by applying phase kicks at suitable timings. Particularly, the enhanced virtual transitions can be exploited to probe different energy levels by tuning \( \tau_I \) as demonstrated in Fig. 3. It is worth noting that the mechanism in Sec. III can be employed as an excitation scheme for a general two-level system in which \( U_R \) and \( P \) can be realized by non-resonant classical pulses, and so a two-level system can be fully excited without using a resonant driving field.

Our investigation suggests that the ability to imprinting quantum phase changes with controllable timings could be a key to trigger interesting quantum transitions. For our system, the operation sequence in Eq. (5) can be achieved by switching on and off \( \lambda_0 \) with the duration \( \tau_I \) and \( \tau_P \) respectively. This is because when \( \lambda_0 = 0 \), \( H_R = \omega_0\sigma_z/2 + \omega_c a^\dagger a \) is a free Hamiltonian, and the corresponding free evolution operator can be made equal to \( P \) if \( \tau_P \) is suitably chosen. For example, in the resonance case of \( \omega_c = \omega_0 \), \( P \) can be achieved by switching off \( \lambda_0 \) in a duration \( \tau_P = (2m + 1/2)\pi/\omega_c \), where \( m \) is a non-negative integer. We note that experimental progress of switchable coupling for systems under ultrastrong coupling has been made \[14, 26, 27\], and there are also theoretical studies on the designs and effects of switchable coupling \[28–30\]. We hope our work would motivate further investigations of the subject in the future.

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