Joint Relay Selection and Power Control to Maximize Sum-Rate in Multi-Hop Networks

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Abstract—Focusing on the joint relay selection (RS) and power control (PC) problem with a view to maximizing the sum-rate, we propose a novel sub-optimal algorithm that iterates between RS and PC. The RS is performed by maximizing the minimum signal-to-interference-plus-noise-ratio (as opposed to maximizing the sum-rate) and the PC is performed using a successive convex approximation. By comparing the proposed algorithm with existing solutions via extensive simulations, we show that the proposed algorithm results in significant sum-rate gains. Finally, we analyze the two-user multi-hop network and show that optimum transmit power of at least two transmitting nodes can be found using binary power allocation.

Index Terms—Achievable sum-rate maximization, relay selection, power control, multi-user, multi-hop.

I. INTRODUCTION

Relay networks have been comprehensively studied in the literature. However, most existing works focus on single or dual-user networks and/or dual-hop relaying [1] and only limited attention has been paid to general multi-user multi-hop networks. More specifically, to the best of our knowledge there exists no work that maximizes the sum-rate in a general relay network. Such general networks play an important role in applications such as ad-hoc sensor networks, unmanned aerial vehicle (UAV) communication and vehicle-to-vehicle (V2V) communication [2]. In this paper, we focus on general relay networks and fill the important gap of joint relay selection (RS) and power control (PC) that aims to maximize the sum-rate.

First let us consider the RS problem. Due to the inherent interference and the competition among multiple users, the RS problem becomes extremely challenging when several source-destination (S-D) pairs are involved. In [3], a sub-optimal decentralized RS strategy is proposed to maximize the minimum signal-to-interference-plus-noise-ratio (SINR) of such a network, while in [4], an efficient algorithm based on dynamic programming is proposed to obtain the optimal RS. Work on the multi-user RS problem with focus on maximizing the overall network sum-rate in decode-and-forward (DF) relay networks is limited to dual-hop relay networks. In [5], orthogonal channels are adopted to remove interference between the S-D pairs thus simplifying the RS problem to an assignment problem. On the other hand, in [6], the interference in the second hop has been estimated to reformulate the RS problem as an updated assignment problem. While recent works focus on machine learning techniques to perform RS [7], they are limited to single-user networks.

Next, let us consider the PC problem that focus on maximizing the achievable sum-rate. The PC of a relay network with amplify-and-forward (AF) relaying is considered in [8] by using the successive convex approximation known as geometric programming, which is accurate in the high SNR regime. Taking a different approach, a distributed PC strategy is proposed in [9] for a multi-user dual-hop relay network that consists of a single relay node. Recently, the concept of SNR matching is considered as the optimum PC when multiple S-D pairs do not create interference to each other. This can be achieved due to the orthogonal transmissions between different S-D pairs [10]. Transmit PC in the presence of interference is considered in [11], where the interference is approximated by a lower bound to reformulate the PC problem as a concave optimization problem for a given relay assignment. Taking a different approach, the optimum PC for two-user dual-hop relay networks is derived analytically in [6].

While existing works focus on either the RS problem or the PC problem in multi-user, multi-hop relay networks, as far as we are aware there have been no work that considers the joint RS and PC problem in this general relay network. Given the dependency between RS and PC, the joint optimization is important to gain the optimum sum-rate performance. In this paper, we consider a multi-hop DF relay network with multiple S-D pairs and analyze the joint RS and PC problem with the aim of optimizing the achievable sum-rate. For this joint optimization problem, we present a sub-optimal algorithm that iteratively utilizes a dynamic programming based max-min RS and a successive convex approximation based PC. Further, we prove that the PC of any two-user relay network can be solved analytically where the optimum transmit power of at least two transmitting nodes can be found using binary power allocation.

II. OPTIMIZATION PROBLEM FORMULATION

We study a multi-user network with $N$ S-D pairs similar to [4]. The information transmitted by the source nodes are carried over to the corresponding destination nodes via a multi-hop relay network composed of $L \geq 2$ hops as illustrated in Fig. 1 where each hop consists of $M$ DF relays such that $M \geq N$. Similar to [3], [4], we consider that a given relay node can support only one S-D pair and any given S-D pair is supported by only one relay node in each hop. This is motivated by the benefits in terms of minimization of network power consumption, processing complexities in relay nodes and the relay synchronization requirements. Therefore,
N relays are selected in each hop from M relays and the relay node chosen in hop l for S-D pair i is denoted as $r_{i,l}$.

Similar to [12], only the intra-hop interference caused by N transmissions in the same hop is considered where as inter-hop interference is neglected due to scheduling of transmissions. Further, the channel gain between transmitter $k$ and receiver $j$ in hop $l$ is modeled as a random variable denoted by $h[k,j,l]$. When we consider the communication in hop $l$, the received signal at node $j$ comprised of $N$ signals transmitted from the relay nodes chosen by $N$ S-D pairs and can be expressed as,

$$\begin{align*}
g[j,l] &= \sum_{i=1}^{N} h[r_{i,l-1},j,l] x[r_{i,l-1},l-1] + n[j,l],
\end{align*}$$

where $x[r_{i,l-1},l-1]$ is the transmitted data symbol from relay node $r_{i,l-1}$ in hop $l-1$. $E[|x[r_{i,l-1},l-1]|^2] = P[r_{i,l-1},l-1]$ with $P[r_{i,l-1},l-1]$ denoting the transmit power of node $r_{i,l-1}$ in hop $l-1$ and $n[j,l]$ is the additive white Gaussian noise (AWGN) at the receiving node $j$ in hop $l$ with mean zero and variance $\sigma^2$.

In DF relay networks, the minimum SINR across all the hops for a given S-D pair determines its received end-to-end SINR. Thus, for a given relay assignment and power allocation the achievable rate for S-D pair $i$ can be written as,

$$R_i = \log_2 \left(1 + \min_{l \in \{1, \ldots, L\}} \{\gamma[i,l]\} \right),$$

where $\gamma[i,l]$ denotes the received SINR for S-D pair $i$ in hop $l$ given by,

$$\begin{align*}
\gamma[i,l] &= \frac{P[r_{i,l-1},l-1]|h[r_{i,l-1},r_{i,l},l]|^2}{\sigma^2 + \sum_{j \neq i}^{N} P[r_{j,l-1},l-1]|h[r_{j,l-1},r_{i,l},l]|^2},
\end{align*}$$

with $r_{i,0} = r_{i,L} = i$. Next, we consider joint RS and PC that aims to maximize the achievable sum-rate and formulate the optimization problem as,

$$\begin{align*}
\max_{r_{i,j}, P[r_{i,j}]} \sum_{i=1}^{N} \log_2 \left(1 + \min_{l \in \{1, \ldots, L\}} \{\gamma[i,l]\} \right) \\
\text{s.t} \quad 0 \leq P[r_{i,j},l] \leq P, \forall i, l, \\
r_{i,j} \neq r_{j,i}, \forall i \neq j, \\
r_{i,j} \in \{1, 2, \ldots, M\} \quad \forall i, l,
\end{align*}$$

where $P$ is the maximum transmit power for each transmission. Please note that by restricting $r_{i,j}$ to a scalar, we ensure that S-D pair $i$ is only supported by one relay node in hop $l$. While additional constraints on $\gamma[i,l]$ can be included to guarantee system QoS, in this work we focus only on the sum-rate maximization. The optimization problem in (4) is non-convex. Due to the integer nature of $r_{i,j}$ and the existence of multiple hops combined with non-convex nature makes solving (4) an exceptionally challenging problem for a general multi-hop relay network with multiple users. As such, we proceed with two steps to solve this optimization problem.

First, we consider a given power allocation $P[r_{i,j},l] \forall i, l$ where $l \in \{0, \ldots, L-1\}$ and write the RS problem as,

$$\begin{align*}
\max_{r_{i,j}} \sum_{i=1}^{N} \log_2 \left(1 + \min_{l \in \{1, \ldots, L\}} \{\gamma[i,l]\} \right) \\
\text{s.t} \quad r_{i,j} \neq r_{j,i}, \forall l, i \neq j, \\
r_{i,j} \in \{1, \ldots, M\}, \forall l, i.
\end{align*}$$

Next, we consider a given relay assignment $[r_{1,l}, \ldots, r_{N,l}] \forall l$, where $l \in \{1, \ldots, L-1\}$ and write the PC problem as,

$$\begin{align*}
\max_{P[r_{i,j}]} \sum_{i=1}^{N} \log_2 \left(1 + \min_{l \in \{1, \ldots, L\}} \{\gamma[i,l]\} \right) \\
\text{s.t} \quad 0 \leq P[r_{i,j},l] \leq P, \forall i, l.
\end{align*}$$

We note that solving individual optimization problems in (5) and (6) separately, is still tremendously hard for a general multi-hop relay network with multiple users [8].

III. JOINT RELAY SELECTION AND POWER CONTROL

In this section we separately consider the optimization problems in (5) and (6) and then combine the proposed solutions to provide a new joint solution by proposing an iterative algorithm that can be implemented at a central node using global channel state information.

A. Relay Selection

First, we note that when the objective of the RS is to maximize the minimum SINR, the dynamic programming based max-min RS strategy in [4] provides the optimal RS. However, when we consider the sum-rate maximization, the final effective sum-rate needs to be computed based on the received end-to-end SINR of each S-D pair and thus, relies on the combination of $N$ SINR values instead of the received SINR of each S-D pair in each hop or the sum-rate in each hop. As a result, there exists no known RS strategy to find the optimum relay assignment when the objective of the RS is the maximization of the sum-rate.

Therefore, we consider several existing RS strategies and conduct a comprehensive simulation to compare their performance when the objective is to optimize the sum-rate. We consider four sub-optimal RS strategies that were considered in [13] with the aim to maximize the sum-rate including hop-by-hop RS, ad-hoc RS, block-by-block RS and sliding window based RS as well as the dynamic programming based max-min RS strategy proposed in [4] with the aim to maximize the minimum end-to-end SINR among all S-D pairs. Under hop-by-hop RS, the RS in each hop is performed independently such that the sum-rate of a given hop is maximized when the signals are transmitted from the relays selected in the previous hop. Under ad-hoc RS, the hop-by-hop RS is extended by...
combining the last two hops together to achieve full diversity. Under block-by-block RS, \( L \) hops are divided into non-overlapping blocks of hops and the relays are selected such that the sum-rate of each block is maximized. Under sliding window based RS, a sliding window across the hops are considered to determine the RS of the first hop in the window such that the sum-rate in that window is maximized. Next, the window is slid by one hop to determine the RS of the next hop. This is continued until the last window where the RS is determined for all the hops in the window. Since the RS is performed by considering segments of hops at a time, these four RS strategies are known to be sub-optimal. Under the max-min RS, the multi-hop relay network with multiple users is mapped into a trellis diagram and the RS problem is mapped to finding the trellis path that maximizes bottleneck branch weight. As a result, the optimal trellis path provides the relay assignment with the maximum value of the minimum SINR of the relay network. Even though this RS strategy considers all the hops in the network, we note that this relay assignment might not maximizes the sum-rate due to the consideration of a different objective function.

**Example:** Consider a multi-user, multi-hop relay network where the channels between nodes follow a Rayleigh distribution with zero mean and unit variance. For such a network, the gain in the sum-rate obtained based on different RS strategies compared to that of the hop-by-hop RS is given in Table I. We consider \( N = 2, M = 2, 3, 4 \) and \( L = 4, 6, 8, 10, 12 \) and the average received SNR of 10 dB with the block/window size of 4 under block-by-block RS and sliding window based RS. In order to maintain full diversity gain, we only consider block-by-block RS when \( L \) can be fully divided by 4. This introduces one limitation of block-by-block RS, where the block size needs to be selected depending on the number of hops \( L \). From the table, we observe that when \( L = 4 \), both block-by-block RS and sliding window based RS have the same sum-rate gains. When \( L = 4 \), both these RS strategies are equivalent to the optimum RS which considers all four hops together. Thus, they result in same sum-rate. On the other hand, with increasing \( L \), the sum-rate gain of ad-hoc RS, block-by-block RS and sliding window based RS decreases for a given \( M \) where as that of max-min RS increases outperforming all other RS strategies. This can be explained by the fact that max-min RS considers the channel gains of all \( L \) hops while the other three RS strategies are unaware of the future channel gains when making the RS decision. Therefore, the performance of other RS strategies deteriorate with increasing \( L \). As such, even though the objective function is different, the consideration of all hops improves the sum-rate obtained with max-min RS. Similarly, when \( M = 2 \), max-min RS has the lowest sum-rate gain for any given \( L \). However, with increasing \( M \), it outperforms all other RS strategies. In addition, we can also observe that with increasing \( M \), the sum-rate gain of all four RS strategies increases for a given \( L \). This can be explained by the improved diversity introduced by increasing \( M \).

From the above comprehensive simulation example, we can observe that max-min RS strategy provides better sum-rate performance in relay networks with large \( M \) and \( L \). Since, the optimal RS can be found via exhaustive search for smaller relay networks, in this work we focus on the RS of larger multi-user, multi-hop relay networks. As such, we proceed to solve the RS problem in (5) using the dynamic programming based max-min RS strategy proposed in [4].

### B. Power Control

Next, we consider the PC problem for a given relay assignment which is non-convex in relation to \( P[r_{i,t}, l] \) [8]. Therefore, we use the successive convex approximation known as the tight lower bound approximation proposed in [14]. Using the variable transformations \( P[r_{i,t}, l] = e^{q[r_{i,t}, l]} \) and \( t[i] = \log \left( \min_{\{l \in \{1, \ldots, L\}} \{\gamma[i, l]\} \right) \), (6) can be reformulated as,

\[
\max_{q[r_{i,t}, l], \forall i,l} \frac{1}{\log(2)} \sum_{i=1}^{N} a_i t[i] + b_i
\]

s.t \( t[i] \leq q[r_{i,t}, l] + \log((h[r_{i,t}, r_{i,t+l+1}, l+1])^2) - \log\left( \sigma^2 + \sum_{j \neq i} e^{q[r_{j,t+1}, l]} |h[r_{j,t}, r_{j,t+l+1}, l+1]|^2 \right), \forall i, l, \)

\[
q[r_{i,t}, l] \leq \log(P), \forall i, l,
\]

where \( a_i, b_i \) are constants computed as,

\[
a_i = \frac{t_i}{1 + t_i}, \quad b_i = \log(1 + t_i) - \frac{t_i}{1 + t_i} \log(t_i),
\]

with \( t_i \) denoting the received end-to-end SINR of S-D pair \( i \), computed using the solution achieved via the previous iteration or the initial solution. As a result of the convex nature of the log-sum-exp terms the optimization problem in (7) is concave for a given relay assignment. Therefore, the coefficients \( a_i \) and \( b_i \) can be computed in each iteration using the results of the previous iteration. Then, approximated problem in (7) can be solved via a gradient decent algorithm or using any existing convex solver. Due to the monotonically improving objective function resulted from the tight lower bound approximation, the sequence always converges [14]. Therefore, for a given relay assignment, the approximated value of the optimum sum-rate can be found by solving (7) iteratively.

### Table I: Percentage of the sum-rate gain compared to hop-by-hop RS

|          | Sliding window RS | Block-by-block RS | Ad-hoc RS | Max-min RS |
|----------|------------------|------------------|-----------|------------|
| M=2, L=4 | 28.06            | 28.06            | 10.01     | 6.44       |
| M=2, L=6 | 34.01            | -                | 7.58      | 7.38       |
| M=2, L=8 | 37.52            | 23.07            | 7.14      | 7.03       |
| M=2, L=10| 36.76            | -                | 5.92      | 5.88       |
| M=2, L=12| 36.54            | 13.95            | 4.73      | 5.04       |
| M=3, L=4 | 52.48            | 52.48            | 21.30     | 30.74      |
| M=3, L=6 | 53.83            | -                | 19.02     | 45.39      |
| M=3, L=8 | 52.21            | 48.88            | 17.34     | 53.00      |
| M=3, L=10| 48.36            | -                | 15.69     | 58.65      |
| M=3, L=12| 47.34            | 42.05            | 14.20     | 65.58      |
| M=4, L=4 | 68.73            | 68.73            | 29.98     | 39.92      |
| M=4, L=6 | 63.15            | -                | 25.90     | 59.84      |
| M=4, L=8 | 62.07            | 39.60            | 25.39     | 68.95      |
| M=4, L=10| 59.55            | -                | 22.14     | 75.54      |
| M=4, L=12| 57.77            | 51.91            | 20.96     | 81.64      |
C. Proposed Joint Solution

Next, we combine the iterative PC solution with RS and propose Algorithm 1 that aims to maximize the sum-rate under the joint optimization. At the start of Algorithm 1, the optimum sum-rate, $R^*$, and all transmit powers are initialized to zero and $P$, respectively. In iteration $n$, we consider a given transmit power allocation and first solve the RS problem using the dynamic programming based max-min RS proposed in [4] and assign the chosen relay nodes to a matrix marked by $X$. $X$ is assigned to the optimum relay assignment matrix, $X^*$, if the resulting sum-rate, $R(n)$, is higher than $R^*$. Therefore, after the first iteration, the RS is only changed if a different RS resulted in a higher $R(n)$ with the updated transmit power allocation. Then we continue to solve the PC problem iteratively.

Algorithm 1: Proposed Joint Solution

```plaintext
1. $n = 1$, $m = 1$, $X^* \leftarrow \{\}$, $R^* \leftarrow 0$, $P[r_{i,l},l] \leftarrow P, \forall i,l$
2. while true do
3.   $[X, R(n)] \leftarrow$ solution to RS problem using [4]
4.   if $(R(n) - R^*)/R(n) > \epsilon_h$ then
5.     $r_{i,l} \leftarrow X(i,l), \forall i,l$
6.   end
7. end
8. while true do
9.   $Q(n) \leftarrow$ solution to problem (7)
10. if $Q(n) - Q(n-1) |/ Q(n) | < \epsilon_h$ then
11.   break
12. end
13. $m \leftarrow m+1$
14. end
15. $P[r_{i,l},l] = e^{q[r_{i,l},l], \forall i,l}
16. R(n) \leftarrow$ sum-rate for $r_{i,l}$ and $P[r_{i,l},l], \forall i,l$
17. if $(R(n) - R^*)/R(n) > \epsilon_h$ then
18.   $R^* \leftarrow R(n)$, $n \leftarrow n+1$, $m = 1$
19. else
20.   stop
21. end
22. end
```

In the $m^{th}$ iteration, the concave optimization problem in (7) is solved and the solution is assigned to $L \times N$ matrix $Q^{(m)} = [q[r_1,l], \ldots, q[N,l], l].$ Then a user defined threshold, $\epsilon_h$, is used to compare the calculated error. The objective function monotonically improves under the tight lower bound approximation and always converges [14]. Therefore, within the inner loop, the sum-rate increases at each iteration and $P[r_{i,l},l], \forall i,l$ is only changed at iteration $n$ if a different transmit power allocation provides a higher $R(n)$ under the new relay assignment. Thus, the sum-rate is monotonically improved in the $n^{th}$ iteration of the outer loop until it converges to a solution. Due to the non-convex nature of the optimization problem (4), we note there might exist multiple local peak points. As a result, the objective function would be converged to one of the local solutions in the end. Since, we cannot guarantee the convergence to the global optimum solution, the proposed algorithm is considered to be sub-optimal. In the implementation, the algorithm is considered to be converged to a local solution when the relative gap between $R(n)$ and $R^*$ is less than $\epsilon_h$.

IV. SPECIAL CASE OF TWO-USER NETWORK

In this section, we consider the special case of two-user networks where $N = 2$ and prove that for at least two transmitting nodes the binary power allocation is optimum. We start by presenting Lemma 1.

Lemma 1. An optimum power allocation that maximizes the sum-rate in a multi-hop relay network with two-users can be found such that the resulting SINRs for each user in all the hopes are equal.

Proof. Please refer to Appendix D.5 in [13].

Next, we construct the following theorem.

Theorem 1. Optimum power allocation for at least two transmitting nodes in a two-user multi-hop DF relay network can be obtained using binary power allocation. SINR matching1 can be used to obtain the optimum transmit power allocation of the rest of the nodes.

Proof. According to Lemma 1, SINRs for each user in all the hopes are equal when the sum-rate of the network is maximized. Therefore, the optimization problem in (6) can be re-expressed as,

$$\max_{P[r_{i,l},l], P[r_{2,l},l]} \left(1 + \gamma[1,1]\right)\left(1 + \gamma[2,1]\right)$$

s.t. $\gamma[1,1] = \gamma[1,l], \gamma[2,1] = \gamma[2,l], \forall l,$

$$0 \leq P[r_{1,l},l], P[r_{2,l},l] \leq P, \forall l, \tag{8}$$

where $\gamma[i,l]$ is a function of $r_{i,l}$ and $P[r_{i,l},l]$ as given in (3) with $i \in \{1,2\}. $ Due to double differentiability of the objective function and the equality constraints relative to $P[r_{1,l},l]$ and $P[r_{2,l},l], \forall l$, (8) can be re-written as an unconstrained optimization problem

$$\max_{\lambda_1^{(l)}, \lambda_2^{(l)}} \min_{P[r_{1,l},l], P[r_{2,l},l], l} \left[ -1 - \gamma[1,1] \gamma[2,1] \right.$$

$$\left. + \left( \sum_{l=1}^{L-1} \lambda_1^{(l)} - 1 \right) \gamma[1,1] + \left( \sum_{l=1}^{L-1} \lambda_2^{(l)} - 1 \right) \gamma[2,1] \right. - \left. \sum_{l=1}^{L-1} \left( \lambda_2^{(l)} \gamma[2,1] + 1 - \lambda_1^{(l)} \gamma[1,l] + 1 \right) \right]$$

s.t. $0 \leq P[r_{1,l},l], P[r_{2,l},l] \leq P, \forall l, \tag{9}$

using the Lagrangian dual where $\lambda_1^{(l)}, \lambda_2^{(l)}$ are the Lagrangian multipliers. Note that the objective function of (9) need to be minimized to achieve the maximum sum-rate. This is because in (9), we minimize the negative sum-rate. Hereafter, we use the variable $f$ to represent the objective function of (9) and note that $f$ is a variable of $P[r_{1,l},l]$ and $P[r_{2,l},l], \forall l$. Since the Lagrangian multipliers correspond to the equality constraints, $\lambda_1^{(l)}, \lambda_2^{(l)} \forall l$ can have any real value at

1Under SINR matching, the transmit power of each transmitting node in a given S-R-D path is selected such that the SINR of each receiving node in that path are equal.
the optimum solution of (9). Whilst not given here due to page limitations, we can show that \( f \) is not convex with respect to at least two variables. This can be done by either showing that the first derivative cannot be zero, which indicates that \( f \) is either a increasing or decreasing function. In case that the first derivative can be zero, it can be shown that the second derivative is either negative or zero which indicates that \( f \) is either a concave function or a increasing/decreasing function. Therefore, \( f \) is minimized at the corner points implying that the maximum sum-rate is obtained at zero or maximum transmit power for at least two of the variables out of \( P[r_{1,i},l] \) and \( P[r_{2,i},l] \) \( \forall l \). Therefore, we can conclude that at least for two transmitting nodes either zero power or maximum transmit power is optimum irrespective of the value of the Lagrangian multipliers. Finally, the values of other \( 2(L-1) \) transmit powers can be obtained by solving the \( 2(L-1) \) equality constraints in (8) which link all \( 2L \) transmit powers. This concludes the proof of Theorem 1.

Thus, for a two user network, the optimum solution can be analytically obtained for the PC problem in (6) by considering that the optimum power allocation for at least two transmitting nodes is zero or maximum power. As such, lines 8-15 in Algorithm 1 can be replaced with Theorem 1 when \( N = 2 \) to improve the performance with reduced complexity.

V. SIMULATION RESULTS

In this section, we present simulation results to illustrate the performance of our proposed joint RS and PC solution. We note that there exist no state-of-the art solutions that maximizes the sum-rate in a general relay network. As such, we compare the proposed scheme against two commonly used reference techniques, namely, the greedy RS and the random RS. Under the greedy RS, each user selects the best path from the set of available relay nodes, using the dynamic programming based optimal RS proposed in [12], according to a priority order. Under the random RS, each user randomly selects a relay path without any conflict. Under both these reference techniques, we consider PC via SINR matching.

The channel gain between any two nodes are computed based on both slow and fast fading. Slow fading in terms of path loss is considered by setting the distance of all \( L \) hops to 2 km with equal distance between hops and a path loss exponent 3.6. Rayleigh fading distribution with zero mean and unit variance is considered in terms of fast fading. For all the simulation examples, the user defined threshold, \( e_0 \), is fixed to \( 10^{-3} \). AWGN noise variance, \( \sigma^2 = kTB \), where \( k \) is the Boltzmann’s constant, \( T = 290 \) K is the ambient temperature and \( B = 200 \) kHz is the equivalent noise bandwidth.

Fig. 2 plots the behavior of the sum-rate against \( P \) with \( N = 2, M = 4,6 \) and \( L = 6 \). From the plot, it can be observed that the resulting sum-rate from the proposed solution increases slightly with \( M \) while the increment with both \( P \) and \( N \) is significant. When RS is performed by considering the interference, we can increase the overall network sum-rate with \( P \) by controlling the interference while improving the received SNR. As \( N \) increases, the number of summation terms in (4) increases. While the individual rate of each user decreases due to the extra interference, by controlling the interference, we can increase the overall network sum-rate. As \( M \) increases, the number of available relay combinations increases thus improving the probability of a user selecting a relay path with larger gain and lower interference increases. Since the RS in two reference techniques does not depend on the interference, both the received SNR and the interference increase with \( P \) resulting in a constant sum-rate. As such, the sum-rate of the greedy RS remains constant with \( P \) and increases with \( M-N \) given that it only depends on the number of available relay combinations. On the other hand, the random RS remains constant with respect to both \( P \) and \( N \) while increases with \( M \). As a result, the Algorithm 1 improves the sum-rate of multi-user relay networks in the presence of interference, in comparison to existing RS solutions.

Fig. 3 plots the behavior of the sum-rate against \( L \) with \( N = 2, M = 3 \) and \( P = 10 \) dBW. The optimum solution is obtained via exhaustive RS combined with Theorem 1 based optimum PC. From the plot, it can be observed that the optimum sum-rate and the sum-rate resulted from Algorithm 1 increases with \( L \) where as that of the two reference techniques slightly decreases with \( L \). For a given hop, the distance between transmitting and receiving nodes decreases with increasing \( L \). This increases the sum-rate due to the reduction in the path loss between two nodes. However, as the two reference techniques make their RS decision purely based on the SNR, they can end up selecting relay paths that create significant intra-hop interference to each other. As the reduction in path loss increases the interference caused by other transmissions in the same hop, the sum-rate slightly decreases with \( L \) for the greedy RS and random RS.

Fig. 4 plots the average computation time taken by Algorithm 1 and the reference techniques versus \( L \) when \( N = 2, M = 6 \) and \( P = 10 \) dBW. We have ignored the optimal solution in this example as it is computationally unfeasible for large \( M \) and \( L \). From the plot, it can be observed that the computation time of our proposed algorithm increases with \( L \). We also observe that compared to the computation time of our proposed algorithm, that of the two reference techniques are significantly smaller. As such, when we consider the sum-
rate performance and the complexity, a clear trade-off can be observed. As both RS strategies used in the greedy RS and Algorithm 1 have linear complexity relative to $L$, the difference associated with the computation time is due to the iterative approach considered in Algorithm 1. Therefore, we next analyze the complexity of the proposed algorithm in terms of the number of iterations for convergence.

Fig. 5 plots the total number of iterations including both inner and outer loops of Algorithm 1 versus $L$ when $N = 2, M = 6$ and $P = 10$ dBW. From the figure, we observe that as $L$ increases, the number of iterations increases as well. Since, the increment in the number of iterations is linear, we can conclude that the proposed algorithm has linear complexity with respect to the total number of iterations.

VI. CONCLUSION

We considered the joint RS and PC problem in a general multi-user, multi-hop relay network with a view to maximizing the sum-rate. A novel iterative solution is presented by combining a dynamic programming based RS that maximizes the minimum SINR as opposed to maximizing the sum-rate and a successive convex approximation based PC. The achievable sum-rate is optimized by our proposed algorithm by iteratively updating the relay assignment and power allocation. Further, we proved that for two-user networks, the optimal transmit power of at least two transmitting nodes can be found using binary power allocation. Transmit power of other nodes can be obtained by considering that the received SINR of each user is equal over all the hops. We note that single user decoding is considered in this work where the interference is treated as noise at each receiver. As such, the joint RS and PC in the presence of successive interference cancellation would be an interesting future extension.

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