Domain wall of the totally asymmetric exclusion process without particle number conservation

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Abstract

In this research, the totally asymmetric exclusion process without particle number conservation is discussed. Based on the mean field approximation and the Rankine-Hugoniot condition, the necessary and sufficient conditions of the existence of the domain wall have been obtained. Moreover, the properties of the domain wall, including the location and height, have been studied theoretically. All the theoretical results are demonstrated by the computer simulations.

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1 Introduction

One-dimensional driven diffusion system is a very interesting research topic in recent years. They were shown to exhibit boundary induced phase transitions [15], and phase separation [9] [21] [5]. In Ref. [17], the effect of a single detachment site in the bulk of an asymmetric simple exclusion process (ASEP) was studied. In Refs [21] [25], the interplay of the simplest one-dimensional driven model, the totally asymmetric exclusion process (TASEP) with local absorption/desorption kinetics of single particles acting at all sites, termed “Langmuir kinetics” (LK) was considered. These models were inspired by the dynamics of motor proteins [1] [2] [13] [27] [12], which move along cytoskeletal filaments in a certain preferred direction while detachment and attachment can also occur between the cytoplasm and the filament, and, in a

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very different setting, by dynamics of limit orders in a stock exchange market. Being an equilibrium process, LK is well understood, while the combined process of TASEP and LK showed the new feature of a localized domain wall in the density profile of the stationary state [21].

The TASEP is defined on a one-dimensional lattice of size $N$. Each site can either be empty or occupied by one particle. In the bulk, particles can hop from site $i$ to site $i+1$ with unit rate, provided the target site is empty. At site 1, particles can enter the lattice from a reservoir with density $\alpha$, provided the site is empty. They can leave the system from site $N$ into a reservoir of density $\beta$ with rate $1 - 1/\beta$. Thus in the interior of the lattice, the particle number is a conserved quantity. The phase diagram and steady states of the TASEP as a function of the boundary rates are known exactly [16] [21] [6]. Furthermore, a theory of boundary induced phase transitions exists, which explains the phase diagram quantitatively in terms of the dynamics of domain wall [14]. In the field of TASEP, Joel L. Lebowitz et al have done many excellent research and obtained much useful theoretical results [8, 7, 3].

Similar as in [25], in this research we equip the system with the additional feature of local particle creation at empty sites with rate $\omega_a$ and annihilation with rate $\omega_d$. In the thermodynamic limit $N \to \infty$, the case of the local rates being of the order of $1/N$ is the most interesting one [22]. It turns out that the presence of the kinetic rates significantly change the picture of TASEP, producing a completely reorganized phase diagram. In [21], the authors showed by computer simulations and mean-field arguments that, in this nonconserved dynamics, one can have phase coexistence where low and high density phases are separated by stable discontinuities (domain wall) in the density profile. Recently, this dynamics was also studied theoretically in [18] [19].

Up to now, the properties of the totally asymmetric exclusion process without particle number conservation have not been well studied theoretically, though many of which have been found in the computer simulations. It is no doubt that some particular properties of this process are very difficult to be found only by computer simulations, and the right theoretical analysis should be given to the full understanding of which. In this research, the necessary and sufficient conditions of the existence of domain wall and the properties of the domain wall, for this totally asymmetric exclusion process without particle number conservation, will be theoretically discussed. Two basic questions will be answered theoretically: (1) when and where does the domain wall exist? (2) How do the location and height of the domain wall change as the parameters change?

This paper is organized as following. In the next section, the mathematical model and some basic results of this process are introduced. The necessary and sufficient conditions of the existence of the domain wall will be given in section 3, then the
properties of the domain wall, including the location and the height, will be discussed theoretically in section 4. Finally, some concluding remarks are given in the last section.

2 Mean field approximation

The equations of the bulk dynamics of the totally asymmetric exclusive process with particle creation and annihilation are the following \[9,21\]

\[
\frac{dn_i}{dt} = n_{i-1}(1 - n_i) - n_i(1 - n_{i+1}) + \omega_a(1 - n_i) - \omega_d n_i \quad 2 \leq i \leq N - 1
\]  

(1)

while at the boundaries

\[
\begin{align*}
\frac{dn_1}{dt} &= \alpha(1 - n_1) - n_1(1 - n_2) \\
\frac{dn_N}{dt} &= n_{N-1}(1 - n_N) - \beta n_N 
\end{align*}
\]

(2)

where the occupation numbers \(n_i = 1\) for a site occupied by a particle and \(n_i = 0\) for an empty site. For fixed total length \(L = 1\) and \(N \to \infty\) one gets the differential equation for the average profile in the stationary state \[21\] \[14\]

\[
(2 \rho(x) - 1)\rho'(x) = (\Omega_a + \Omega_d)\rho(x) - \Omega_a \quad 0 < x < 1
\]

(3)

where the reduced rates \(\Omega_a = N\omega_a, \Omega_d = N\omega_d\) (because of the particle-hole symmetry, we restrict the discussion to the case \(\Omega_a \geq \Omega_d \[21\]). In the following, let \(\rho_{l\alpha}\) and \(\rho_{r\beta}\) (or \(\rho_l\) and \(\rho_r\) for simplicity) be the solutions of Eq. (3) with boundary conditions \(\rho_{l\alpha}(0) = \alpha\) and \(\rho_{r\beta}(1) = 1 - \beta\) respectively. It is to say, \(\rho_{l\alpha}\) and \(\rho_{r\beta}\) satisfy the following two equations respectively (see \[10\])

\[
\begin{align*}
\frac{2(\rho_{l\alpha} - \alpha)}{(K + 1)\Omega_d} + \frac{K - 1}{(K + 1)^2\Omega_d} \ln \left| \frac{K - (K + 1)\rho_{l\alpha}}{K - (K + 1)\alpha} \right| &= x \\
\frac{2(1 - \beta - \rho_{r\beta})}{(K + 1)\Omega_d} + \frac{K - 1}{(K + 1)^2\Omega_d} \ln \left| \frac{K - (K + 1)(1 - \beta)}{K - (K + 1)\rho_{r\beta}} \right| &= 1 - x
\end{align*}
\]

(4)

(5)

where \(K := \frac{\Omega_a}{\Omega_d} \geq 1\).

3 The existence of the domain wall

Due to the Rankine-Hugoniot condition \[4\], at the location \(x_S\) of the domain wall of Eqn. (3),

\[
(\rho_+^2 - \rho_-) = (\rho_-^2 - \rho_-)
\]

(6)
should be satisfied, where \( \rho_- = \lim_{x \to x_S^-} \rho(x) \), \( \rho_+ = \lim_{x \to x_S^+} \rho(x) \). It can be easily found that the condition (6) can be simplified as

\[
\rho_- + \rho_+ = 1 \tag{7}
\]

Therefore, to the Eqn. (3) with boundary conditions \( \rho(0) = \alpha \) and \( \rho(1) = 1 - \beta \), there exists domain wall in the interval \((0, 1)\) if and only if there exists a location \(0 < x_S < 1\) at which \( \rho_- + \rho_+ = 1 \), moreover,

\[
\rho(x) = \begin{cases} 
\rho_l(x) & 0 \leq x < x_S \\
\rho_r(x) & x_S < x \leq 1 
\end{cases} \tag{8}
\]

From the conditions (7) (8) and the Eqns. (4) (5), it is not difficult to obtain the following theoretical results:

(I) For \( 0 \leq \alpha \leq 0.5 \) and \( 0 \leq 1 - \beta \leq \frac{K}{K+1} \), the necessary and sufficient conditions of the existence of domain wall are (see Fig. 1 (left))

\[
\rho_{l\alpha}^{-1}(\gamma) \leq 1 \quad \text{and} \quad \rho_{r\beta}^{-1}(1 - \alpha) \leq 0 \quad \text{with} \quad \gamma = \min(0.5, \beta) \tag{9}
\]

(II) For \( 0 \leq \alpha \leq 0.5 \) and \( \frac{K}{K+1} \leq 1 - \beta \leq 1 \), the necessary and sufficient conditions of the existence of domain wall are (see Fig. 1 (right))

\[
\rho_{l\alpha}^{-1}(\beta) \leq 1 \quad \text{and} \quad \rho_{r\beta}^{-1}(1 - \alpha) \geq 0 \tag{10}
\]

(III) For \( 0.5 \leq \alpha \leq 1 \), there no domain wall exists in \((0, 1)\) (for details, see [26]).
Figure 2: The relationship between the location of the domain wall and the parameter $\Omega_d$, for $0 \leq \alpha \leq 0.5, \frac{1}{K+1} \leq \beta \leq 0.5$ (left). $x_S(\Omega_d)$ is monotonously decreased as a function of $\Omega_d$ for $0 \leq \alpha \leq 0.5, \frac{1}{K+1} \leq \beta \leq 0.5$ (right).

4 The properties of the domain wall

In this section, we will discuss the properties of the location $x_S$ and the height $2\epsilon$ of the domain wall, which can be regarded as functions of the parameters $K, \Omega_d, \alpha, \beta$. Where $2\epsilon = |\rho_+ - \rho_-| = |2\rho_+ - 1|$. In the following, we assume $0 \leq \alpha \leq 0.5$ (which is the necessary condition of the existence of domain wall), $0 \leq \beta \leq 0.5$ (if $\beta > 0.5$ and the domain wall exists, it is equivalent to the case in which $\beta = 0.5$, see [26]), $\Omega_d \geq 0$ and $K \geq 3$. For the sake of the convenience, we define the following functions

\[
A = A(\epsilon, K) := 2\epsilon + 2K\epsilon - K + 1 \quad B = B(\epsilon, K) := 2\epsilon + 2K\epsilon + K - 1
\]
\[
C = C(K, \beta) := K\beta + \beta - 1 \quad D = D(K, \alpha) := K\alpha + \alpha - K
\]
\[
E = E(\epsilon, \alpha) := 1 - 2\alpha - 2\epsilon \quad F = F(\epsilon, \beta) := 1 - 2\beta - 2\epsilon
\]

(11)

4.1 The Properties of the Location of the domain wall

From Eqns. (4) and (5), we can get the following theoretical results:

(a) \[
\frac{\partial x_S}{\partial \Omega_d} = \frac{A(\epsilon, K) - 4\epsilon x_S(K + 1)}{4\epsilon \Omega_d(K + 1)} = \frac{2\epsilon(K + 1)(1 - 2x_S) - (K - 1)}{4\epsilon \Omega_d(K + 1)}
\]

(12)

For $\frac{1}{K+1} \leq \beta \leq 0.5$, i.e. $0.5 \leq 1 - \beta \leq \frac{K}{K+1}$, it can be easily proved that $2\epsilon \leq \frac{K-1}{K+1}$, which implies $A(\epsilon, K) \leq 0$, so $\frac{\partial x_S}{\partial \Omega_d} < 0$. It is to say that the location $x_S$ of the domain wall is monotonously decreased as a function of the parameter $\Omega_d$ (the corresponding computer simulations are plotted in Figure 2). For $0 \leq \beta < \frac{1}{K+1}$, i.e. $\frac{K}{K+1} < 1 - \beta \leq 1$, \[
\frac{\partial x_S}{\partial \Omega_d} = \frac{2\epsilon(K + 1)(1 - 2x_S) - (K - 1)}{4\epsilon \Omega_d(K + 1)}
\]

(13)
Figure 3: The relationship between the location of the domain wall and the parameter $\Omega_d$. $0 \leq \alpha \leq 0.5, 0 \leq \beta < \frac{1}{K+1}$ (left). $x_S(\Omega_d)$ is monotonously decreased as a function of $\Omega_d$ for $0 \leq \alpha \leq 0.5, 0 \leq \beta < \frac{1}{K+1}$ (right). In all the computer simulations, $2\epsilon(K+1)(1-2x_S) \leq (K-1)$ is satisfied.

if $2\epsilon(K+1)(1-2x_S) \leq (K-1)$, the location $x_S$ of the domain wall is also decreased as the parameter $\Omega_d$ increases (Figure 3). Otherwise, the location $x_S$ of the domain wall is increased as the parameter $\Omega_d$ increases (Figure 4).

(b) \[
\frac{\partial x_S}{\partial K} = -\frac{1}{4(K+1)\epsilon} \left[ \frac{K-3}{K^2-1} \left[ (A+B)x_S - A \right] \right.
\]
\[
+ \left. \frac{(K^2-1)(2\alpha-1) + 4\epsilon(K+1)DCE - [4\epsilon(K+1)C + 2(K+1)(1-2\beta)]DF}{\Omega_d(K+1)^2(K-1)CD} \right]
\]

For $0 \leq \alpha \leq 0.5, \frac{1}{K+1} \leq \beta \leq 0.5$, it can be verified that

\[
A \leq 0 \quad A+B > 0 \quad C \geq 0 \quad D \leq 0 \quad E \geq 0 \quad F \leq 0
\]

which imply

\[
\frac{\partial x_S}{\partial K} \leq 0 \quad \text{for} \quad K \geq 3
\]

i.e. the location $x_S$ of the domain wall is monotonously decreased as a function of the parameter $K$ for $K \geq 3, 0 \leq \alpha \leq 0.5, \frac{1}{K+1} \leq \beta \leq 0.5$ (Figure 5). At the same time, Eq. (14) can be reformulated as

\[
\frac{\partial x_S}{\partial K} = -\frac{1}{4(K+1)\epsilon \Omega_d(K+1)^2(K-1)CD} \left[ (K-3)\Omega_d[2\epsilon(K+1)(2x_S - 1) + (K-1)] \right.
\]
\[
\left. + 8\epsilon(\beta - \alpha) + (K^2-1)(2\alpha-1)CE + (K-1)(2\beta-1)DF \right]
\]

For $0 \leq \beta < \frac{1}{K+1}$, it is easy to verify

\[
C \leq 0 \quad D \leq 0 \quad E \geq 0 \quad F \geq 0
\]
Figure 4: The relationship between the location of the domain wall and the parameter Ωd for 0 ≤ α ≤ 0.5, 0 ≤ β < \frac{1}{K+1}: monotonously increased as a function of the parameter Ωd when Ωd is small, then monotonously decreased when Ωd is large enough (left). The corresponding figure of the function xS(Ωd). (right). At the critical point, 2ε(K + 1)(1 − 2xS) = (K − 1).

Figure 5: For 0 ≤ α ≤ 0.5, \frac{1}{K+1} ≤ β ≤ 0.5 the location xS of the domain wall is decreased as the parameter K increases (left). The figure of the function xS(K) for 0 ≤ α ≤ 0.5, \frac{1}{K+1} ≤ β ≤ 0.5 (right).
Figure 6: For $0 \leq \alpha \leq 0.5, 0 \leq \beta < \frac{1}{K+1}, 2\epsilon(K + 1)(1 - 2x_S) \leq (K - 1)$ the location $x_S$ of the domain wall is decreased as the parameter $K$ increases (left). The figure of the function $x_S(K)$ for $0 \leq \alpha \leq 0.5, 0 \leq \beta < \frac{1}{K+1}, 2\epsilon(K + 1)(1 - 2x_S) \leq (K - 1)$ (right).

If

$$2\epsilon(K+1)(1-2x_S) \leq (K-1)+ \frac{8\epsilon(\beta - \alpha) + (K^2 - 1)(2\alpha - 1)CE + (K - 1)(2\beta - 1)DF}{K - 3}$$

then $\frac{\partial x_S}{\partial K} \leq 0$, the location $x_S$ of the domain wall is decreased as the parameter $K(\geq 3)$ increases (Figure 6). Since

$$\frac{(K^2 - 1)(2\alpha - 1)CE + (K - 1)(2\beta - 1)DF}{K - 3} \geq 0 \quad 0 \leq \alpha, \beta \leq \frac{1}{K+1} \quad (20)$$

the breakdown of the inequality (19) is difficult to be found in the computer simulations.

(c)

$$\frac{\partial x_S}{\partial \alpha} = \frac{(1 - 2\alpha)B}{4\epsilon\Omega_d(K + 1)D} \leq 0 \quad \text{for} \ \forall 0 \leq \alpha, \beta \leq 0.5 \quad (21)$$

so the location $x_S$ of the domain wall is monotonously decreased as a function of the parameter $\alpha$ for $\forall \ K \geq 1, 0 \leq \alpha \leq 0.5, 0 \leq \beta \leq 0.5$ (the results of the computer simulations are plotted in Figure 7, Figure 8)

(d)

$$\frac{\partial x_S}{\partial \beta} = \frac{(2\beta - 1)A}{4\epsilon\Omega_d(K + 1)C} \geq 0 \quad \text{for} \ \forall 0 \leq \alpha, \beta \leq 0.5 \quad (22)$$

in fact, $A \leq 0, C \geq 0$ if $\frac{1}{K+1} \leq \beta \leq 0.5$; and $A \geq 0, C \leq 0$ if $\frac{1}{K+1} \leq \beta \leq 0.5$. So the location $x_S$ of the domain wall is monotonously increased as a function of the parameter $\beta$ for $\forall \ K \geq 1, 0 \leq \alpha \leq 0.5, 0 \leq \beta \leq 0.5$ (Figure 9, Figure 10).
Figure 7: The location $x_S$ of the domain wall is decreased as the increase of the parameter $\alpha$ for $\frac{1}{K+1} \leq \beta \leq 0.5$ (left). The figure of the function $x_S(\alpha)$ for $\frac{1}{K+1} \leq \beta \leq 0.5$ (right).

Figure 8: The location $x_S$ of the domain wall is decreased as the increase of the parameter $\alpha$ for $0 \leq \beta < \frac{1}{K+1}$ (left). The figure of the function $x_S(\alpha)$ for $0 \leq \beta < \frac{1}{K+1}$ (right).

Figure 9: The location $x_S$ of the domain wall is increased as the increase of the parameter $\beta$ for $\frac{1}{K+1} \leq \beta \leq 0.5$ (left). The figure of the function $x_S(\beta)$ for $\frac{1}{K+1} \leq \beta \leq 0.5$ (right).
Figure 10: The location \( x_S \) of the domain wall is increased as the increase of the parameter \( \beta \) for \( 0 \leq \beta \leq \frac{1}{K+1} \) (left). The figure of the function \( x_S(\beta) \) for \( 0 \leq \beta \leq \frac{1}{K+1} \) (right).

### 4.2 The Properties of the Height of the Domain Wall

In view of the Eqns. (4) (5) and the definition of the height of the domain wall \( 2\epsilon \), using the chain rule of the derivative, we can get the following theoretical results:

(a) \[
\frac{\partial \epsilon}{\partial \Omega_d} = -\frac{AB}{16(K+1)\epsilon^2} \left\{ \begin{array}{ll}
\geq 0 & \text{for } \frac{1}{K+1} \leq \beta \leq 0.5 \\
\leq 0 & \text{for } 0 \leq \beta \leq \frac{1}{K+1}
\end{array} \right. \tag{23}
\]

It can be verified that \( A \leq 0, \ B \geq 0 \) for \( \frac{1}{K+1} \leq \beta \leq 0.5 \); and \( A \geq 0, \ B \geq 0 \) for \( 0 \leq \beta \leq \frac{1}{K+1} \). So the height \( 2\epsilon \) of the domain wall is monotonously increased as a function of the parameter \( \Omega_d \) for \( \frac{1}{K+1} \leq \beta \leq 0.5 \), and monotonously decreased as a function of the parameter \( \Omega_d \) for \( 0 \leq \beta \leq \frac{1}{K+1} \) (Figure 2 (left), Figure 3 (left), Figure 4 (left), Figure 11).

(b) \[
\frac{\partial \epsilon}{\partial K} = -\frac{1}{16(K+1)\epsilon^2} \left\{ \begin{array}{l}
\frac{[(K-1)^2 + 2BD]ACE + [(K-1)^2 + 2AC]BDF}{(K+1)^2(K-1)CD} \\
+ \frac{(K-3)AB\Omega_d}{K^2-1} \end{array} \right\} \tag{24}
\]

where
\[
(K-1)^2 + 2BD = (K^2-1)(2\alpha - 1) + 4\epsilon(K+1)D
\]
\[
(K-1)^2 + 2AC = 4\epsilon(K+1)C + (K-1)(1 - 2\beta)
\]

From (15) and (24), one can know
\[
\frac{\partial \epsilon}{\partial K} \geq 0 \quad \text{for } K \geq 3, \quad \frac{1}{K+1} \leq \beta \leq 0.5, \quad 0 \leq \alpha \leq 0.5 \tag{26}
\]
Figure 11: The figure of the function $2\epsilon(\Omega_d)$ for $\frac{1}{K+1} \leq \beta \leq 0.5$ (left). The figure of the function $2\epsilon(\Omega_d)$ for $0 \leq \beta \leq \frac{1}{K+1}$ (right).

Figure 12: The figure of the function $2\epsilon(K)$ for $\frac{1}{K+1} \leq \beta \leq 0.5$ (left). The height $2\epsilon$ of the domain wall is not a monotone function of the parameter $K$ for $1 < K \leq 3$, $0 \leq \beta < \frac{1}{K+1}$, $0 \leq \alpha \leq 0.5$ (right).

so the height $2\epsilon$ of the domain wall is monotonously increased as a function of the parameter $K$ for $K \geq 3$, $\frac{1}{K+1} \leq \beta \leq 0.5$, $0 \leq \alpha \leq 0.5$ (Figure 5 (left), Figure 12 (left)). However, for $0 \leq \beta < \frac{1}{K+1}$, the height $2\epsilon$ of the domain wall is not a monotone function of parameter $K$ (Figure 12 (right)).

(c) $\frac{\partial \epsilon}{\partial \alpha} = \frac{(1 - 2\alpha)AB}{16(K+1)^2e^2D} \begin{cases} 
  \geq 0 & \frac{1}{K+1} \leq \beta \leq 0.5 \\
  \leq 0 & 0 \leq \beta \leq \frac{1}{K+1} 
\end{cases}$ (27)

so the height $2\epsilon$ of the domain wall is monotonously increased as a function of the parameter $\alpha$ for $\frac{1}{K+1} \leq \beta \leq 0.5$, and monotonously decreased as a function of the parameter $\alpha$ for $0 \leq \beta \leq \frac{1}{K+1}$ (Figure 7 (left), Figure 8 (left), Figure 13).
Figure 13: The figure of the function $2\epsilon(\alpha)$ for $\frac{1}{K+1} \leq \beta \leq 0.5$ (left). The figure of the function $2\epsilon(\alpha)$ for $0 \leq \beta \leq \frac{1}{K+1}$ (right).

Figure 14: The figure of the function $2\epsilon(\beta)$ for $\frac{1}{K+1} \leq \beta \leq 0.5$ (left). The figure of the function $2\epsilon(\beta)$ for $0 \leq \beta \leq \frac{1}{K+1}$ (right).

(d) \[
\frac{\partial \epsilon}{\partial \beta} = \frac{(1 - 2\beta)AB}{16(K+1)e^2C} \leq 0 \quad \text{for } 0 \leq \alpha, \beta \leq 0.5
\] (28)

since $\frac{A}{C} \leq 0$ and $B \geq 0$ for $\forall 0 \leq \alpha, \beta \leq 0.5$. Therefore, the height $2\epsilon$ of the domain wall is monotonously decreased as a function of the $\beta$ for $\forall 0 \leq \beta \leq 0.5$ (Figure 9 (left), Figure 10 (left), Figure 14).

5 Conclusions

In this research, the totally asymmetric exclusion process without particle number conservation in large particle number limit have been studied theoretically. Two questions are answered completely: (1) when and where does the domain wall exist? (2) How do the location and height of the domain wall change as the parameters
$\alpha, \beta, K, \Omega_d$ change? (see Table 1, where $\uparrow$ ($\downarrow$) means the function is a monotonously increased (decreased) one, $\uparrow\downarrow$ ($\downarrow\uparrow$) means the function has an unique maximum (minimum) point, $\downarrow?$ means that the function might be monotonously decreased, but the proof is not completed in this research)

In summery, we have found: (1) For $0 \leq \alpha \leq 0.5, 0 \leq 1 - \beta \leq \frac{K}{K+1}$, there exists domain wall if and only if $\rho_{lo}^{-1}(\gamma) \leq 1$ and $\rho_{r\gamma}^{-1}(1 - \alpha) \leq 0$, where $\gamma = \min(0.5, \beta)$.

(2) For $0 \leq \alpha \leq 0.5, \frac{K}{K+1} \leq 1 - \beta \leq 1$, there exists domain wall if and only if $\rho_{l}^{-1}(\beta) \leq 1$ and $\rho_{r}^{-1}(1 - \alpha) \geq 0$. (3) For $0.5 \leq \alpha \leq 1$, the domain wall doesn’t exist. (4) The location $x_S$ of the domain wall is monotonously increased (decreased) as a function of the parameter $\beta$ ($\alpha$). The height $2\epsilon$ of the domain wall is monotonously decreased as a function of the parameter $\beta$. (5) For $\frac{1}{K+1} \leq \beta \leq 0.5, K \geq 3$, The location $x_S$ of the domain wall is monotonously decreased as a function of the parameters $\Omega_d$ and $K$; the height $2\epsilon$ of the domain wall is monotonously increased as a function of the parameters $\Omega_d$ and $K$. (6) For $0 \leq \beta \leq \frac{1}{K+1}, \omega$, the height $2\epsilon$ of the domain wall is monotonously decreased as a function of the parameters $\Omega_d$ and $\alpha$.

$$
\begin{array}{cccc}
0 \leq \alpha \leq 0.5 & \Omega_d & K (\geq 3) & \alpha \\
\frac{1}{K+1} \leq \beta \leq 0.5 & x_S \downarrow, 2\epsilon \uparrow & x_S \downarrow, 2\epsilon \uparrow & x_S \downarrow, 2\epsilon \uparrow \\
0 \leq \beta \leq \frac{1}{K+1} & x_S \uparrow, 2\epsilon \downarrow & x_S \uparrow, 2\epsilon \downarrow & x_S \uparrow, 2\epsilon \downarrow \\
\end{array}
$$

Table 1: The properties of the domain wall.

Recently, totally asymmetric exclusion processes with internal states and particle detachment and attachment, which serve as generic transport models in various context, have been introduced and extensively studied ([23] [11] [20]). Using the similar methods as in this research, these generalized models also can be studied theoretically, and the corresponding results will be given in the future.

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