Zero-Temperature Dynamics of Ising Spin Systems
Following a Deep Quench: Results and Open Problems

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Abstract

We consider zero-temperature, stochastic Ising models $\sigma^t$ with nearest-neighbor interactions and an initial spin configuration $\sigma^0$ chosen from a symmetric Bernoulli distribution (corresponding physically to a deep quench). Whether $\sigma^\infty$ exists, i.e., whether each spin flips only finitely many times as $t \to \infty$ (for almost every $\sigma^0$ and realization of the dynamics), or if not, whether every spin — or only a fraction strictly less than one — flips infinitely often, depends on the nature of the couplings, the dimension, and the lattice type. We review results, examine open questions, and discuss related topics.

1 Introduction

The behavior of different kinds of magnetic systems following a deep quench comprises a central topic in the study of their nonequilibrium dynamics. Rigorous and nonrigorous results have been obtained on different questions arising naturally in this context: the formation of domains, their subsequent evolution, spatial and temporal scaling properties, and related questions (for a review, see Ref. [1]); the persistence properties at zero and positive temperature [2, 3, 4, 5, 6, 7]; the observed aging phenomena in both disordered and ordered systems (see, e.g., [8, 9, 10, 11]); and many others.

In this paper we will summarize results on a very basic question, whose answer is not only relevant to the questions mentioned above but often naturally precedes them. Put informally, consider a quench from infinite temperature to zero temperature, and let the system then evolve using standard Glauber dynamics. Will the spin configuration eventually settle down

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to a final state, or will it continue to evolve forever (and if so, in what sense)? Here we will concern ourselves mostly with this question of approach to a final state in different Ising spin models, and will not address, except briefly in the last section, the properties of such final states. We now state the problem more precisely.

Consider the stochastic process \( \sigma^t = \sigma^t(\omega) \) corresponding to the zero-temperature limit of Glauber dynamics for an Ising model with Hamiltonian,

\[
\mathcal{H} = - \sum_{\{x,y\}, \|x-y\|=1} J_{x,y} \sigma_x \sigma_y .
\]  

(1)

Here \( \| \cdot \| \) denotes Euclidean length. For now \( \sigma^t \) takes values in \( S = \{-1, +1\}^Z \), the space of (infinite-volume) spin configurations on the \( d \)-dimensional hypercubic lattice, but later (see Subsec. 4.1) we will examine other types of lattices as well. The initial spin configuration \( \sigma^0 \) is chosen from a symmetric Bernoulli product measure (denoted \( P_{\sigma^0} \)), corresponding physically to a deep quench. If the spin model is disordered, then the transition rates depend on a realization \( J \) of the (i.i.d., unless otherwise specified) random couplings \( J_{x,y} \), with (common, unless otherwise specified) distribution \( \mu \) on \( R \). The (continuous time) dynamics is given by independent (rate 1) Poisson processes at each \( x \) corresponding to those times \( t \) [think of these as clock rings at \( x \)] when a spin flip (\( \sigma^t_{x} + 0_x = -\sigma^t_{x} - 0_x \)) is considered. If the resulting change in energy is negative (or zero or positive), then the flip is done with probability 1 (or 1/2 [determined, say, by a fair coin toss], or 0). We denote by \( P_\omega \) the probability distribution on the realizations \( \omega \) of the dynamics. For most of this paper we consider only this single-spin-flip dynamics, but in the last section will briefly discuss multi-spin-flip dynamics. The joint distribution of \( J, \sigma^0, \) and \( \omega \) will be denoted \( P \).

A natural question in both the disordered and non-disordered models is whether \( \sigma^t \) has a limit (with \( P \)-probability one) as \( t \to \infty \) or equivalently whether for every \( x \), \( \sigma^t_x \) flips only finitely many times. More generally, one may call such an \( x \) an \( F \)-site (\( F \) for finite) and otherwise an \( I \)-site (\( I \) for infinite). By translation-ergodicity, the collection of \( F \)-sites (resp., \( I \)-sites) has (with \( P \)-probability one) a well-defined non-random spatial density \( \rho_F \) (resp., \( \rho_I \)). The densities \( \rho_F \) and \( \rho_I \) depend only on \( d \), \( \mu \), and possibly lattice type, and of course satisfy \( \rho_F + \rho_I = 1 \). We characterize the triplet \((d, \mu, \text{lattice type})\) as being type \( \mathcal{F} \) or \( \mathcal{I} \) or \( \mathcal{M} \) (for mixed) according to whether \( \rho_F = 1 \) or \( \rho_I = 1 \) or \( 0 < \rho_F, \rho_I < 1 \).

In the remainder of the paper, we review results for different \( d \), different lattices, and a number of important special cases of \( \mu \). In Sec. 2, we present results on one-dimensional chains for both homogeneous and disordered systems. In Sec. 3 we consider the homogeneous ferromagnet (or antiferromagnet) on the square lattice \( Z^2 \) and show it is type \( \mathcal{I} \). In Sec. 4 we review models with continuous disorder, and discuss a theorem whose consequence is that most such systems of interest, including ordinary random ferromagnets and the Edwards-Anderson (EA) spin glass [12], are type \( \mathcal{F} \) for any \( d \) and lattice type. We also show why this theorem implies that homogeneous ferro- and antiferromagnets, on lattices (in any \( d \)) where each site has an odd number of neighbors, are now type \( \mathcal{F} \). In Sec. 5, we discuss \( \pm J \) spin glasses on \( Z^2 \) and other models with noncontinuous disorder on \( Z^d \) that are type \( \mathcal{M} \). In
Sec. 6 we summarize our findings and list a number of open problems. We also discuss there several situations not discussed in the bulk of the paper, including positive temperature, multi-spin-flip dynamics and persistence.

2 One-dimensional models

In one dimension the analysis is particularly simple, and it is not hard to show that: a) when the couplings all have the same magnitude (regardless of sign, since in one dimension all such models can be gauge-transformed to the uniform ferromagnet), the model is type \( I \); b) disordered models with \( \mu \) continuous are type \( F \); and c) models in which the couplings can (with positive probability) take on two or more discrete magnitudes are type \( M \). A proof of the first two claims may be found in \[13\], and a proof of c) in \[14\]. Here we summarize the proof of b); the proofs of a) and c) are simplifications of the corresponding \( d = 2 \) proofs given in Secs. 3 and 4, respectively, and so will not be given separately. Modified arguments can be used to examine the dynamical behavior for \( \mu \)'s with both a continuous part and a single discrete magnitude; see \[14\] for details.

A sketch of the proof of b) is as follows. Consider a site \( x \) such that \( |J_{x,x+1}| \) strictly exceeds the two neighboring coupling magnitudes; because \( \mu \) is continuous, such sites occur with positive density. It is clear that once \( \sigma_x \sigma_{x+1} = \text{sgn}(J_{x,x+1}) \) (either initially in \( \sigma^0 \), or through a subsequent flip of one of the two spins as determined by \( \omega \)), neither spin will flip again, demonstrating already that \( \rho_F > 0 \). By translation-ergodicity of \( P \), there will be (with \( P \)-probability one) a doubly infinite sequence of such sites \( x_n \) (with positive density).

Consider now the interval \( \{x_{n-1} + 1, x_{n-1} + 2, \ldots, x_n\} \). By the preceding argument, there will be some time after which, both \( \sigma_{x_{n-1}+1} \) and \( \sigma_{x_n} \) cease to flip. After that time we have a Markov process (restricted to the interval) with a finite state space. Because \( \mu \) is continuous, each flip within the interval will strictly lower the energy, which (for fixed \( J \)) is bounded below, by some minimal amount. The process must therefore eventually reach an absorbing state in which all spins have stopped flipping. Because this argument applies to every such interval, a continuous \( \mu \) in one dimension is type \( F \).

3 Homogeneous ferromagnet on \( Z^2 \)

In \[13\] it was shown that the homogeneous ferromagnet on \( Z^2 \) is type \( I \); we will sketch the argument here. Essentially the same argument holds for the homogeneous antiferromagnet on \( Z^2 \). In this section \( P \) refers to the joint distribution of \( \sigma^0 \) and \( \omega \).

To begin, we note that two possible absorbing states are the two uniform spin configurations: \( \sigma_x \equiv +1 \) for every \( x \) and \( \sigma_x \equiv -1 \) for every \( x \). Because of global spin-flip symmetry, these outcomes must have equal probability \( p (\leq 1/2) \). But because of translation-invariance and translation-ergodicity of \( P \), \( p \) must be either 0 or 1; therefore, \( p = 0 \). The only other absorbing states are those with one or more parallel domain walls separating strips of uniform +1 and −1 spin configurations; if the state is to be absorbing, then all these domain walls...
must be parallel to either the $x$- or $y$-axis. Using the argument above, but with spin-flip symmetry replaced by invariance with respect to rotations by $\pi/2$, we conclude similarly that each of these two sets of outcomes must also have zero probability. This shows at least that $\rho_\mathcal{F} > 0$.

To show that the model is type $\mathcal{I}$, we need to show that every spin flips infinitely often. By translation-invariance and spin-flip symmetry, if $\rho_\mathcal{F} > 0$, then the following must occur with positive $P$-probability: for some $x = (x_1, x_2)$, $y = (x'_1, x_2)$ and $z = (x_1, x'_2)$ with $x_1 < x'_1$ and $x_2 < x'_2$ and for some $t'$, $\sigma_x^t = +1$ and $\sigma_y^t = -1$ and $\sigma_z^t = -1$ for all $t \geq t'$. But this would require that

$$\inf_{\sigma \in S''} P_\omega(\sigma^{t+1} \notin S''|\sigma^t = \sigma) = 0,$$

where $S''$ is the set of spin configurations on $\mathbb{Z}^2$ with the values $+1, -1, -1$ at the sites $x, y, z$.

But this is not so as can be seen from the following argument. Let us define $w = (x'_1, x'_2)$ and $\mathcal{R}$ to be the rectangle with corners at $x, y, z$ and $w$. Any spin configuration in $S''$ must have a domain wall that is contained within $\mathcal{R}$ and that either (a) connects the $[x, z]$ segment to the $[x, y]$ segment or else (b) connects the $[x, y]$ segment to the $[y, w]$ segment or else (c) connects the $[x, z]$ segment to the $[z, w]$ segment. In case (a), there is some sequence of clock rings (and absence of rings) and coin toss outcomes within $\mathcal{R}$ during the time interval $[t, t+1]$ (that occurs with $P_\omega$-probability bounded away from zero) that will move the domain wall towards the Southwest so that $\sigma_x^{t+1} = -1$. Similarly, in cases (b) or (c), the domain wall can move to the Southeast or to the Northwest so that $\sigma_y^{t+1} = +1$ or $\sigma_z^{t+1} = +1$.

## 4 Models with continuous disorder

A central result is that in any dimension (and on any lattice), models with continuous disorder distribution $\mu$ of finite mean are type $\mathcal{F}$. These models include EA spin glasses with a Gaussian $\mu$ and random ferromagnets with a uniform $\mu$. The idea behind the proof was already used in Sec. 2 as part of the proof that one-dimensional models with continuous $\mu$ are type $\mathcal{F}$. Here we sketch the more general proof; for further details, see [13].

Let $\sigma_x^t$ be the value of $\sigma_x$ at time $t$ for fixed $\omega$, $\sigma^0$ and $\mathcal{J}$. Let

$$E(t) = -(1/2) \sum_{y: ||x-y||=1} J_{xy} \sigma_x^t \sigma_y^t,$$

where the bar indicates an average over $P$. By translation-ergodicity of $P$, and using the assumption that $|J_{xy}| < \infty$, it follows that $E(t)$ exists, is independent of $x$, and equals the energy density (i.e., the average energy per site) at time $t$ in almost every realization of $\mathcal{J}$, $\sigma^0$, and $\omega$.

Because every spin flip lowers the energy, $E(t)$ monotonically decreases in time (note that $E(0) = 0$) and has a finite limit $E(\infty) \geq -d|J_{xy}|$. Now choose any fixed number $\epsilon > 0$, and let $N_x^\epsilon$ be the number of spin flips (over all time) of the spin at $x$ that lower the energy by an amount $\epsilon$ or greater. Then $-\infty < E(\infty) \leq -\epsilon N_x^\epsilon$ so that for every $x$ and $\epsilon > 0$, $N_x^\epsilon$ is
finite. Let $\epsilon_x$ be the minimum energy (magnitude) change resulting from a flip of $\sigma_x$; then although $\epsilon_x$ varies (differently in each $J$) with $x$, it is sufficient that it is strictly positive.

It is implicit in the proof of this result that, even without the continuity assumption on the distribution of couplings, with $P$-probability one there can be only finitely many flips of any spin that cause a nonzero energy change. When $\mu$ is continuous, the probability of a "tie" in any sum or difference of a given spin’s nearest-neighbor coupling strengths (and therefore the probability of a spin flip costing zero energy) is zero, yielding the result that these systems are type $F$.

4.1 Homogeneous ferromagnets on lattices other than $Z^d$

It follows from the argument above that this type $F$ result applies also to homogeneous Ising spin systems on lattices with an odd number of nearest neighbors, so that ties in energy cannot occur. (It also applies to $\pm J$ models (as defined below) on these lattices.) Such lattices include the two-dimensional hexagonal (or honeycomb) lattice, and the double-layered cubic lattices $Z^d \times \{0, 1\}$ (i.e., a "ladder" when $d = 1$, two horizontal planes separated by unit vertical distance when $d = 2$, and so on).

4.2 Continuous disorder with infinite mean

What about models where the disorder is continuous but the mean is infinite? We can show that a restricted class of these models are type $F$; these are models where influence percolation \cite{15} does not occur. Such models include all one-dimensional models with continuous disorder (regardless of whether the mean is finite), and strongly and highly disordered models \cite{16, 17, 18}. We refer the reader to \cite{13} for a discussion of this situation.

5 $\pm J$ and related models

We now turn to models on $Z^d$ where

$$\mu = \alpha \delta J_1 + (1 - \alpha) \delta J_2,$$

with $0 < \alpha < 1$ and $J_1 \neq J_2$. When $J_1 = -J_2 \neq 0$, we call this a $\pm J$ model. (In much of the literature, $\pm J$ spin glasses refer to the specific case $\alpha = 1/2$.) These models have been analyzed in \cite{14}, along with modifications where, e.g., $\mu$ consists of both a continuous and a discrete part. Because such modified models appear to be of less interest, we note here only that these models are generally type $M$, and refer the reader to \cite{14} for details; for the remainder of this section, we confine ourselves to distributions of the form given in Eq. (4).

The main results of \cite{14} were to prove the following two assertions: a) models in which $J_1 \neq -J_2$ are type $M$ in any dimension, and b) $\pm J$ models in two dimensions are also type $M$. The first of these is much easier to prove, and we start with that case.

We sketch here a proof for the $d = 2$ case; the extension to other $d$ is straightforward. To show that $\rho_F > 0$ on $Z^2$, let $|J_1| < |J_2|$ and consider a plaquette all of whose edges
have coupling $J_2$, and all edges outside the plaquette but that connect to one of its corners have coupling $J_1$. With positive $P$-probability, such a coupling configuration will occur with the spins at the four corners initially (or eventually) all +1 (or all −1) if $J_2 > 0$ or else alternating in sign if $J_2 < 0$. These spins will thereafter never flip, proving that $\rho_F > 0$.

To show that $\rho_I > 0$, consider a configuration in which the couplings between some $w$ and its four neighbors $z_i$ are all $J_1$. Suppose also that each $z_i$ belongs to a plaquette satisfying the set of conditions described in the previous paragraph, with the spins at $z_1$ and $z_2$ equal to +1 and the spins at $z_3$ and $z_4$ equal to −1. This will ensure that the spin at $w$ flips infinitely often, and because such a situation will occur with positive $P$-probability, $\rho_I > 0$.

We now turn to a discussion of b). It is relatively easy to show that for two-dimensional $\pm J$ models, $\rho_I > 0$, and we will sketch the proof of that here. It takes considerably more work to show that $\rho_F > 0$, and so we will present here only the idea behind the proof, and refer the reader to [14] for details.

To show that $\rho_I > 0$, we consider a configuration of a $5 \times 5$ square of sites in the dual lattice $\mathbb{Z}^2* \equiv \mathbb{Z}^2 + (1/2, 1/2)$ in which exactly 9 of the 25 sites are frustrated (corresponding to plaquettes in $\mathbb{Z}^2$ with an odd number of negative couplings): the central site $w_c$; two Southeastern sites, $w_1 = w_c + (1, -1)$ and $w_2 = w_c + (2, -1)$; and six other sites obtained from $\{w_1, w_2\}$ by (multiple) $\pi/2$-rotations about $w_c$. Such a frustration configuration occurs with positive $P$-probability, so we are done if we can show that there always exists at least one $\mathbb{Z}^2$ site (among the 36 whose plaquettes form our $5 \times 5$ square) that has positive flip rate. We now proceed to do this.

We note first that any domain wall (i.e., a path of unsatisfied edges in $\mathbb{Z}^2*$ [connecting a pair of frustrated sites]) that is not straight implies a site in $\mathbb{Z}^2$ with positive flip rate; this is because there must exist a $\mathbb{Z}^2$ site with at least two unsatisfied edges. Now in our frustration configuration, there must be a domain wall starting from $w_c$. Either this domain wall already determines a positive flip rate site because it is not straight, or else it runs straight out of the square; in the latter case, by the invariance with respect to rotations by $\pi/2$, we may assume (without loss of generality) that the domain wall emanating from $w_c$ runs to the East and passes just above the (dual) edge joining the two Southeastern sites, $w_1$ and $w_2$. But then there must be another domain wall starting from $w_1$. Either these two domain walls together determine a positive flip rate site or else the second one runs from $w_1$ straight out of the square to the South. But then there must be a third domain wall starting from $w_2$, that (together with the previous two) will determine a positive flip rate site, no matter what direction it runs off to. Using as usual translation-invariance and translation-ergodicity of $P$, we conclude that $\rho_I > 0$.

A proof demonstrating that $\rho_F > 0$ is considerably more involved, as noted, but the general strategy is similar. We again consider an event involving the frustration configuration in a finite region of $\mathbb{Z}^2*$, and the spin configuration in a related region of $\mathbb{Z}^2$. One wants to show that at least one of these $\mathbb{Z}^2$ sites will eventually have flip rate zero and hence will flip only finitely many times, thus proving $\rho_F > 0$. This is done by proving that the domain wall geometry in $\mathbb{Z}^2*$ must eventually satisfy various constraints, in particular that certain contour events recur indefinitely with probability zero; otherwise, there would be infinitely
many energy-lowering flips in the fixed square with positive probability, violating the result presented in Sec. 4. For further details, see [14].

6 Summary and open problems

We have studied the dynamical evolution of several categories of Ising spin models, both ordered and disordered, following a deep quench (from infinite to zero temperature), in all dimensions and for different kinds of (regular) lattices. Our concern here has centered on the question of existence of a final state, given the usual zero-temperature Glauber dynamics and the Hamiltonian (1). It is interesting to consider other types of situations, but these will be left for the future. These include, for example, other kinds of spin models (Potts, XY, Heisenberg), initial spin configurations not chosen from the symmetric Bernoulli distribution, and others. Nevertheless, we feel that substantial progress has so far been made, and we review the results below, including a discussion of remaining open questions (for this type of model and situation).

6.1 Review of results

All results below are for Ising spin systems with Hamiltonian (1), and \( J, \sigma^0 \) and \( \omega \) chosen as discussed in Sec. 1. A given system may have three possible dynamical outcomes: it may be type \( I, F, \) or \( M \), whose meanings were given in Sec. 1.

Homogeneous ferromagnets and antiferromagnets: In one dimension and in two dimensions on \( Z^2 \), these are type \( I \). In any dimension on a lattice where each site has an odd number of nearest neighbors, they are type \( F \) [13].

Models with continuous disorder: These need to be further subdivided. The most important, and commonly studied, cases are those models where the coupling distribution has finite mean; these include ordinary (EA) spin glasses and random ferromagnets. In all dimensions (and for all lattices) these models are type \( F \) [13]. Another class of models that are type \( F \) [13] are those in which influence percolation [15] does not occur; these include all one-dimensional models with continuous disorder, and the strongly and highly disordered models of spin glasses and random ferromagnets [16, 17, 18].

±\( J \) models: In one dimension, these are type \( I \); in two dimensions, type \( M \) on the square lattice [14]. On a lattice where each site has an odd number of neighbors, they are type \( F \) in any dimension.

Models with other \( \mu \): If \( \mu \) is of the form \( \alpha \delta_J + \beta \delta_{-J} + \nu \) with \( J > 0, 0 < \alpha + \beta < 1 \), and \( \nu \) continuous and supported on \([-J, +J]\), then it is type \( F \) in one dimension [14]. Other examples are distributions of the form Eq. (1), where \( |J_1| \neq |J_2| \), distributions supported partially on a continuous interval and partially on atoms at discrete values, and so on. These are type \( M \) in all dimensions [14].
6.2 Open problems

For Ising spin systems with Hamiltonian (4), and \( J, \sigma^0 \) and \( \omega \) chosen as discussed in Sec. 4, the problems of the dynamical outcomes of the following situations remain open:

1) Homogeneous ferromagnets (and antiferromagnets) on the lattice \( \mathbb{Z}^d \) (or others except where each site has an odd number of neighbors) for \( d \geq 3 \). Numerical results [2] suggest that these should be type \( I \) for \( d = 3 \) and perhaps 4, but may possibly be type \( F \) (or \( M \)) for \( d \geq 5 \).

2) Models with continuous disorder in \( d > 1 \), where both the coupling distribution has infinite mean and influence percolation occurs. The proof [13] sketched in Sec. 4 already implies that \( \rho_F > 0 \) for these models, so they are either type \( F \) or \( M \). It seems reasonable to conjecture that they are type \( F \).

3) \( \pm J \) models on \( \mathbb{Z}^d \) (or other lattices except where each site has an odd number of neighbors) for \( d \geq 3 \). There is little we can say about these right now, except that it would be interesting to relate \( \pm J \) models to homogeneous ferromagnets on the same lattice. E.g., perhaps \( \rho_F(\pm J \text{ model}) \geq \rho_F(\text{homogeneous ferromagnet}) \). If this were the case, then demonstrating that homogeneous ferromagnets are type \( F \) on \( \mathbb{Z}^d \) for \( d \geq 5 \) would immediately resolve the question for \( \pm J \) spin glasses on those lattices.

6.3 Related topics

In this section we touch on several related topics. In particular, throughout this paper we have concerned ourselves solely with the question of convergence of \( \sigma_t \) (with \( P \)-probability one) to a final state, but have not examined the properties of this final state (when it exists) in the different models of interest. We also have not discussed rates of convergence to the final state. We will briefly discuss these issues, but first will consider positive temperature.

**Positive temperatures.** Here one deals with the behavior of the local order parameter rather than that of single spins. Construction of dynamical measures, analysis of their evolution, and relation to pure state structure are extensively discussed in Ref. [19]. Here we mention only a few relevant results.

It should first be noted that the categorization into types \( I, F, \) and \( M \) needs to be modified and refined at positive temperature. Without going into detail, we will simplify matters here by dividing systems into those where on any finite lengthscale, the system equilibrates (into a pure state) after a finite time (depending on \( \sigma^0, \omega, \) the lengthscale, and \( J \) if relevant), in the sense that interfaces cease to move across the region after that time; and those where this local equilibration does not occur. The latter case we call local non-equilibration (LNE), of which there are two types. A precise definition requires the use of a dynamical measure; we refer the reader to [19] for details and to [20] where one of the types (Chaotic Time Dependence) is shown to occur in a \( d = 1 \) model with disordered rates.

A main result of [19] is that if only a single pair, or countably many pairs (including a countable infinity) of pure states exists (with fixed \( J \)), and these all have nonzero EA order
parameter \([12]\), then LNE occurs. A corollary is that if LNE does \textit{not} occur, and the limiting pure states have nonzero EA order parameter, then there must exist an \textit{uncountable} infinity of pure states, with almost every pair (as the realizations of the initial state and dynamics vary) having overlap zero.

One consequence of these results is that LNE (in a rough sense the positive temperature equivalent of type \(I\) behavior) occurs at positive temperature (with \(T < T_c\) in the \(2D\) uniform ferromagnet and (presumably) random Ising ferromagnets for \(d < 4\). Because the number and structure of pure states at positive temperature in Ising spin glasses is unknown for \(d \geq 3\) (and, from a rigorous point of view, unproved even for \(d = 2\)), occurrence of LNE there remains an open question.

\textit{Multi-spin dynamics.} In Ref. \([21]\) we examined ordinary spin glasses and random ferromagnets in arbitrary dimension and considered an extension of the zero-temperature single-spin flip Glauber dynamics in which rigid flips of all lattice animals (i.e., finite connected subsets of \(\mathbb{Z}^d\), not necessarily containing the origin) up to size \(M\) spins can occur. Of particular interest here is the limit \(M \to \infty\). For the dynamics to remain sensible, we need to choose the rates for \(K\)-spin lattice animal flips to decrease as \(K\) increases, so that the probability that any fixed spin considers a flip in a unit time interval remains of order one, uniformly in \(M\). A further requirement for the dynamics to be well-defined is that information not propagate arbitrarily fast throughout the lattice as \(M \to \infty\). Such a dynamics can be constructed, and generates infinite-volume ground states \([21]\).

\textit{Persistence.} A topic of current interest is the \(P\)-probability \(p(t)\) at time \(t\) that a spin has not yet flipped. For the homogeneous ferromagnetic Ising model on \(\mathbb{Z}^d\), this probability has been found to decay at large times as a power law \(p(t) \sim t^{-\theta(d)}\) \([2, 3, 4]\) for \(d < 4\). The “persistence” exponent \(\theta(d)\) is considered to be a new universal exponent governing nonequilibrium dynamics following a deep quench \([4]\). Our work \([7]\) shows that dependence on lattice type (e.g., square vs. hexagonal lattices for homogeneous ferromagnets) implies nonuniversality of this behavior, and moreover that the persistence phenomenon (which seems to require a system to be type \(I\)) is unstable to the introduction of randomness into the spin couplings. Moreover, in some simple systems it was shown that the decay \(p(t) - p(\infty)\) to the final state is \textit{exponential} as \(t \to \infty\).

\textit{Properties of the final state.} We turn finally to a discussion of the properties of the limiting state \(\sigma_\infty\) for ordinary spin glasses and random ferromagnets in any dimension. For the \(M\)-spin-flip dynamics (with \(M\) finite) discussed above, the final states are energetically stable up to a flip of any subset of \(M\) spins; we call these \(M\)-spin-flip stable states. A number of results were obtained in \([21]\); a central result is that there is an uncountable infinity of these metastable states, in any \(d\) and for any \(M\), and their overlap distribution is a delta-function at zero. For further results and discussion, we refer the reader to \([21]\).
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