Structural transitions and dynamical regimes for directional locking of vortices and colloids driven over periodic substrates

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Abstract

We examine collective dynamical locking effects for superconducting vortices and colloids interacting with square and triangular substrate arrays under a slowly rotated dc drive. A rich variety of lattice configurations associated with a series of steps in the velocity–force curves occur during the locking transitions. These include triangular, square, smectic, and disordered particle arrangements that can be identified using the structure factor. We show that the step widths vary with the ratio of the number of particles to the number of pinning sites. Unlike a static system, where matching effects occur at simple integer commensuration ratios, we find dynamical commensuration effects that arise when an integer number of particle chains flow between rows of pins. We identify two distinct types of locking as a function of substrate strength, distinguished by whether the particles flow along or between the pinning rows.

1. Introduction

Directional or kinetic locking of driven particles interacting with periodic substrates has been studied for superconducting vortices as well as colloidal particles [1–5]. Here, some or all of the particles move along a symmetry direction of the substrate instead of in the direction of the applied drive. None of the numerous works on this subject have analyzed in detail the rich variety of structural transitions between different moving lattice symmetries produced by collective effects, the associated transport signature changes, or the effect of substrate strength on collective flow. In [1], vortices were driven over a square pinning array under a fixed dc drive $F_D$ while a perpendicular drive $F_y$ was gradually increased to rotate the driving direction. As the motion locked to different symmetry directions, the $y$-component of the response formed a devil’s staircase, with steps at $F_y/F_D = n/m$, with integer $n, m$. Similar locking was later predicted for classical charged particles moving over a two-dimensional (2D) periodic potential [2], while the locking of colloids driven over an optical substrate was proposed as a novel fractionation method for separating colloidal species in [3, 4]. Symmetry locking for particles or nanoparticles moving on periodic substrates has since been observed in numerous experiments and simulations [5–20], but most of these studies involved noninteracting particles and did not consider collective effects. In very recent work [21], structural transitions of colloids driven over patterned substrates has been reported.

Symmetry locking of vortices has recently been observed, both in numerical simulations [22, 23] and experiments [22, 24–26], and occurs even on a random substrate provided that the vortex–vortex interactions are strong enough to overcome the substrate and produce a triangular vortex lattice [27, 28]. On the locking steps, the transverse depinning threshold for motion perpendicular to the lattice symmetry directions is enhanced. Vortices and colloids driven over quasicrystalline arrays show directional locking effects even though the substrate has no long-range translational order [29].
In this work we consider collective effects in symmetry locking as the substrate strength and filling fraction is varied. Reference [1] considered only fillings near $B/B_0 = 1.0$, where $B_0$ is the field at which the number of vortices equals the number of pinning sites. We show that, as the filling changes, dynamical commensurability effects produce oscillations in the strength of the locking effects. Reference [1] also considered only relatively weak pinning, but here we show that there is a distinct strong pinning regime that arises when some vortices become permanently pinned. The remaining interstitially confined vortices depin at a low threshold and can exhibit dynamical locking, but for some driving directions they cause a portion of the vortices at the pinning sites to depin, altering or destroying the locking steps.

An understanding of collective effects is important for applications of directional locking such as sorting of different colloid species. If the collective interactions cause the particles to form a crystalline state, this could destroy the separation effect. Additionally, we show that collective effects produce several regimes of chaotic or disordered flow, where separation would become difficult or impossible. We find that the regions of maximum random flow appear near transitions between two ordered flow regimes. Superconducting vortices fall naturally into the strongly interacting regime, so it is important to understand features of collective locking as well as to identify which magnetic fields would maximize the locking effects.

2. Simulation

We perform molecular dynamics simulations of a 2D system with periodic boundary conditions. We consider a sample of size $L_x \times L_y$ with $L_x = 24\lambda$, where length is measured in units of the London penetration depth $\lambda$. For square pinning arrays, $L_y = L_x$, and for triangular arrays, $L_y = 1.097 L_x$. The motion of an individual vortex evolves according to the overdamped equation

$$\eta \frac{d \mathbf{R}_i}{dt} = F_{\text{int}}^i + F_{\text{ext}}^i + F_{\text{v}}^i,$$

Here, $\mathbf{R}_i$ is the location of vortex $i$, $\eta = \frac{\Phi_0^2 d}{2\pi \xi^2 \rho N}$, $d$ is the sample thickness, $\xi$ is the superconducting coherence length, $\rho N$ is the normal state resistivity, and $\Phi_0 = h/2e$ is the flux quantum. The vortex–vortex interaction force is given by

$$F_{\text{int}}^i = \sum_{j \neq i} f_0 K_1(R_{ij}/\lambda)\dot{\mathbf{R}}_{ij},$$

where $K_1$ is the modified Bessel function, $f_0 = \Phi_0^2 / (2\pi \mu_0 \lambda^2)$, $R_{ij} = |\mathbf{R}_i - \mathbf{R}_j|$, and $|\mathbf{R}_i - \mathbf{R}_j| = (\mathbf{R}_i - \mathbf{R}_j)/R_{ij}$. The periodic substrate is modeled as $N_p$ pinning sites of radius $R_p$ and maximum strength $F_p$, with

$$F_{\text{p}}^i = \sum_{k=1}^{N_p} (F_{\text{p}} R_{ik}^{(p)}/R_p)\Theta((R_p - R_{ik}^{(p)})/\lambda)\dot{R}_k^{(p)}.$$  

Here, $\Theta$ is the Heaviside step function, $R_{ik}^{(p)}$ is the location of pinning site $k$, $R_{ik}^{(p)} = (\mathbf{R}_i - \mathbf{R}_k^{(p)})/R_{ik}$, and $\dot{R}_k^{(p)} = (\mathbf{R}_k^{(p)} - \dot{\mathbf{R}}_k^{(p)})/R_{ik}^{(p)}$. Unless otherwise noted, $R_p = 0.3\lambda$, and the pinning density is $n_p = 0.279/\lambda^2$. As an example of physical units, for the Al films considered in [44] with $T_c = 1.313$ K, $\rho N = 1.7 \times 10^{-8}$ $\Omega$ m, $d = 50$ nm, $\xi(0) = 145$ nm, and $\lambda(0) = 120$ nm, at $T = 1.278$ K we have $\lambda = 735$ nm, $\eta = 2.54 \times 10^{-18}$ N s m$^{-1}$, and $f_0 = 1.37 \times 10^{-6}$ N m$^{-1}$.

The initial vortex configurations are obtained by simulated annealing. We then apply an external force representing the Lorentz force from a current at an angle that changes very slowly, $F_{\text{ext}} = A \sin(\theta(t)) \mathbf{x} + A \cos(\theta(t)) \mathbf{y}$. Here $A = 2.0$, $\theta(t) = \delta(t)/|\delta(t)|$, $\delta = 0.0228^\circ$, and $\delta t = 10000$ simulation time steps. We measure the velocity response $\langle v_x \rangle = N^{-1} \sum_{i=1}^{N} v_x \cdot \mathbf{R}_i$, and $\langle v_y \rangle = N^{-1} \sum_{i=1}^{N} v_y \cdot \mathbf{R}_i$, averaged over only the final 4000 time steps for each value of $\theta$ in order to avoid any transient effects. We also measure the structure factor for the vortex positions, $S(k) = (1/L^2) \sum_{i,j} \exp(ik \cdot (\mathbf{R}_i - \mathbf{R}_j))$, that could be compared with scattering data.

For the colloidal simulations we use particle–particle interactions of the Yukawa form $F_{\text{p}} = -\sum_{i \neq j} \nabla \phi \cdot \nabla \phi$, with $\phi(R_{ij}) = q^2(E_0/R_{ij}) \exp(-\epsilon R_{ij})$, where $E_0 = Z^2/4\pi \epsilon \epsilon_0$, $\epsilon$ is the solvent dielectric constant, $Z^*$ is the unit effective charge of each colloid, $q$ is the dimensionless colloid charge, and $1/\epsilon = a_0/4$ is the screening length. The strength of the colloid–colloid interaction force can be altered by varying $q^2$.

3. Structural transitions and directional locking

On a triangular pinning substrate, locking occurs at driving angles $\theta = \tan^{-1}(\sqrt{3}m/(2n + 1))$ when the distance a vortex travels between consecutive pinning sites is minimized. The dominant locking angles fall at $\theta = 0^\circ$ for $m = 0, n = 0$; $\theta = 60^\circ$ for $m = 1, n = 0$; and $\theta = 30^\circ$ for $m = 1, n = 1$. We use the notation $(m, n)$ to denote the locking regions. In figure 1(a) we plot $V_x$ and $V_z$ versus $\theta$ for a sample with $F_p = 1.85$ at $B/B_0 = 1.5$. In the absence of pinning, the velocities would follow a smooth sinusoidal curve. Figure 1(a) shows that, instead, both $V_x$ and $V_z$ pass through a series of pronounced steps and jumps. For $\theta < 10^\circ$, the motion is locked in the $x$-direction. The value of the $y$ component of $\mathbf{F}_{\text{ext}}$ at the end of the (0, 0) step is the critical transverse depinning force $F_{\text{Tr}}$. A large (1, 1) locking step is centered at $\theta = 30^\circ$, and a second strong step in both $V_x$ and $V_z$ occurs at $\theta = 60^\circ$ at the (1, 0) locking. In figure 1(b) we plot the fraction of sixfold coordinated particles $P_6$ as a function of $\theta$. Here, $P_6 = N^{-1} \sum_{i=1}^{N} \delta(z_i - 6)$, where the coordination number $z_i$ of each particle is obtained from a Voronoi construction. On the (0, 0) step, $P_6 = 0.8$, indicating that some dislocations are present. Along the (1, 1) step, $P_6 = 0.9$, but $P_6$ dips at the start and the end of the step. In general, the vortex lattice shows abrupt changes in ordering at either end of a step.

To better characterize the different vortex structures in the locking regimes, in figure 1(c) we plot the vortex positions on the (0, 0) step and in figure 1(d) we show the corresponding $S(k)$. Here the particles are moving in one-dimensional (1D) channels along the pinning rows in the $x$-direction. $S(k)$ has smectic features with well-spaced peaks along the $k_y$-axis, indicating that periodic spacing of the particles along $y$ is induced by the pinning, while particles in adjacent rows slip past each other. Similar smectic ordering was observed for vortices moving over random pinning arrays [30–33] or vortices moving at finite temperature over periodic pinning arrays [34, 35], and is associated with a transverse depinning barrier [30, 31, 36]. In figure 1(c), the vortices are confined to move only along the pinning rows. Since $B/B_0 = 1.5$, this...
compresses the vortex lattice spacing along \(x\), producing an anisotropic vortex–vortex interaction. Some moving channels contain more vortices than others, creating dislocations in the vortex lattice. Along the (1, 1) step, the system has a smectic ordering similar to that shown in figures 1(c), (d) but tilted by 30\(^\circ\) with respect to the \(x\) axis. On the (1, 2) locking step, figures 1(e), (f) show that the vortex lattice has square symmetry with disordered regions that produce some smearing in \(S(k)\). The \(P_6\) measurement cannot detect square ordering, so in figure 1(b) \(P_6\) shows large fluctuations on the (1, 2) step even though the \(S(k)\) measurement indicates a consistent square ordering throughout the step. In figures 1(g), (h) we show the vortex positions and \(S(k)\) for \(\theta = 36^\circ\), a non-step region with \(P_6 \approx 0.95\) where the vortex channeling effect is lost and where for weak pinning the vortices revert to a mildly disordered triangular lattice. In figures 1(i), (j) at \(\theta = 60^\circ\) on the (1, 0) step, the particles form 1D channels and move along the pinning rows in a rotated version of the ordering found on the (0, 0) step. Some dislocations in the vortex lattice produce smectic ordering. At the dip in \(P_6\) at the end of the (1, 0) step, figures 1(k), (l) show that the vortices form an anisotropic liquid-like structure. Another locking region, centered at \(\theta = 90^\circ\), has \(V_y = 0.0\) and a cusp feature in \(V_y\). Along this step, \(P_6 \approx 0.78\), and in figures 1(m), (n) a distorted square lattice appears. As \(\theta\) increases above \(\theta = 60^\circ\), the system cycles back through the same structural transitions and steps.

4. Density dependence and dynamic commensuration effects

In figure 2(a) we plot \(V_y\) for different vortex densities \(B/B_0\). The number of resolvable steps is high for low \(B/B_0\) and low for high \(B/B_0\). Different steps respond differently to changes in \(\theta\). For example, for \(B/B_0 < 2.0\) in figure 2(a), the (0, 0) and (1, 0) steps are present, but at \(B/B_0 = 2.0\), these steps are lost. In contrast, the step at (1, 1) is present at all the values of \(B/B_0\). The (0, 0) and (1, 0) steps reappear for \(B/B_0 \geq 2.78\) and grow in width with increasing vortex density up to \(B/B_0 = 4.07\). Figure 2(b) shows a blowup of the region around the (0, 0) step, indicating that at \(B/B_0 = 2.0\) the width of the (0, 0) step drops to zero but that at \(B/B_0 = 4.07\) the step reappears, producing a crossing in the \(V_y\) versus \(\theta\) curves. In figure 2(c) we plot the width \(F^{Tr}_c = A \cos(\theta)\) of the (0, 0) step versus \(B/B_0\). For low \(B/B_0\), the vortex–vortex interactions are weak, so the system is in the single particle limit and \(F^{Tr}_c\) is large. As \(B/B_0\) increases, \(F^{Tr}_c\) decreases to a local minimum at \(B/B_0 = 0.75\) before increasing and decreasing once again. This non-monotonic behavior of \(F^{Tr}_c\) contrasts with the critical depinning force observed in a system with random pinning, which monotonically decreases to a saturation level with increasing vortex density. The maxima and minima in \(F^{Tr}_c\) generally do not fall at integer multiples of \(B/B_0\), where static matching effects are expected to occur [37–40]. This is because the dynamical ordering that occurs just below a transverse depinning transition differs.
Figure 2. (a) Vortex velocities \( V_c \) versus \( \theta \) for a triangular pinning array with \( F_p = 1.85 \) at \( B/B_\phi = 0.852 \) (black), 1.26 (red), 1.5 (light green), 2.0 (blue), 2.78 (purple), and 4.07 (dark green), from bottom right to top right. The velocities are normalized by \( B_\phi \). (b) A blowup of panel (a) in the region near the \((0,0)\) step, showing that the width of the \((0,0)\) step decreases to zero at \( B/B_\phi = 2.0 \) and increases again for \( B/B_\phi = 4.07 \). (c) The width \( F_c^{\text{Tr}} = A \cos(\theta) \) of the \((0,0)\) step versus \( B/B_\phi \) for the same system. A series of minima and maxima appear that do not coincide with the commensuration effects expected at integer multiples of \( B/B_\phi \). (d)--(g) The positions of the pinning sites (large open circles) and vortices (small filled circles) in a portion of the sample along the \((0,0)\) step. (d) \( B/B_\phi = 1.5 \). The dashed lines indicate the motion of the vortices in 1D channels aligned with the pinning rows. (e) \( B/B_\phi = 2.037 \), a filling at which \( F_c^{\text{Tr}} = 0 \) in panel (c). Here the 1D channel structure shown in panel (d) is lost. (f) \( B/B_\phi = 4.07 \), where there are 1D channels of motion both along and between the pinning sites, as indicated by the dashed lines. (g) \( B/B_\phi = 5.93 \), where \( F_c^{\text{Tr}} \) is small. The 1D channel structures are lost.

from the static structure that forms just below the longitudinal depinning transition and has a distinct field dependence. Smectic ordering appears close to the peak in \( F_c^{\text{Tr}} \) at \( B/B_\phi = 1.5 \), as shown in figures 1(c), (d), as well as within the broad peak in \( F_c^{\text{Tr}} \) centered around \( B/B_\phi = 4 \). In the regions where \( F_c^{\text{Tr}} \) drops to zero, near \( B/B_\phi = 2.0 \) and above \( B/B_\phi = 5.9 \), the lattice is disordered and \( S(k) \) has an anisotropic ring-like structure.

In studies of vortex matter confined to narrow channels, the critical depinning force oscillates as a function of vortex density, depending on how many rows of vortices can fit inside the channel [41, 42]. More recent studies of 2D periodic pinning arrays show a dynamical commensuration effect that occurs in the limit where the vortices at the pinning sites remain pinned but the number of interstitial vortices increases with \( B/B_\phi \) [43]. The transverse depinning force \( F_c^{\text{Tr}} \) passes through local maxima as a function of \( B/B_\phi \) whenever an integer number \( n \) of vortex rows fit between the pinned vortices and the vortex trajectories are highly ordered. Local minima of \( F_c^{\text{Tr}} \) appear when a buckling instability of the rows occurs and produces a disordered moving vortex structure. The oscillations in \( F_c^{\text{Tr}} \) shown in figure 2(c) are very similar in nature to this effect; however, a key difference is that there are no pinned vortices in the rotating drive system. Along the \((0,0)\) step, some of the vortices slide over rows of pinning sites and the rest slide through the interstitial regions. When an integer number of interstitial sliding rows fits between adjacent rows of pinning sites, the vortex lattice has smectic structure and there is a local maximum in \( F_c^{\text{Tr}} \). At other values of \( B/B_\phi \), where an integer number of interstitial sliding rows is unable to form, the vortex lattice is more disordered and \( F_c^{\text{Tr}} \) is low or zero.

We illustrate the effect of commensurate or incommensurate numbers of rows of flowing vortices in figures 2(d)–(g) at different values of \( B/B_\phi \) on the \((0,0)\) step. Near the local maximum in \( F_c^{\text{Tr}} \) at \( B/B_\phi = 1.5 \), figure 2(d) indicates that the vortices flow only in 1D channels along the pinning rows. At \( B/B_\phi = 2.037 \) in figure 2(e), the vortices are no longer aligned with the pinning rows and some vortices flow through the interstitial regions. Near another local maximum in \( F_c^{\text{Tr}} \) at \( B/B_\phi = 4.07 \), figure 2(f) shows that all of the vortices are flowing in 1D rows, with half of the rows passing through pinning sites and the other half passing through interstitial regions. At \( B/B_\phi = 5.93 \), near a local minimum in \( F_c^{\text{Tr}} \), figure 2(g) illustrates that the 1D channel structure is lost. We expect that for even higher values of \( B/B_\phi \), additional local maxima in \( F_c^{\text{Tr}} \) will occur for fields at which two, three, or higher integer numbers of rows of vortices can be accommodated in the interstitial regions.

The same general behavior found for the triangular pinning array also occurs for vortices moving over square pinning arrays. Locking steps appear when \( \theta = \tan^{-1}(m/n) \), with \( m \) and \( n \) integers. The most prominent steps fall at \((0,0)\) for \( \theta = 0^\circ \) and \((1,1)\) for \( \theta = 45^\circ \). One distinction is that, for square pinning, there is a much larger number of locking steps where the vortices form a square moving lattice structure. For superconducting systems, the pinning lattice constant \( a_p \) and pin radius \( R_p \) are important parameters, since smaller structures are more difficult to fabricate. To investigate the robustness of the dynamical locking effects, we consider a system with a square pinning array with \( F_p = 1.85 \) and \( B/B_\phi = 1.0 \) at fixed \( R_p = 0.35\lambda \) and varied \( a_p \). In figure 3(a), the directional locking effects in \( V_c \) versus \( \theta \) are much stronger for smaller \( a_p \), as indicated by the highlighted \((0,1), (1,1), \) and \((1,0)\) steps. Strong directional locking
effects can still be achieved even for larger $a_p$ provided that $R_p$ is sufficiently large, as illustrated in figure 3(b) for $a_p/\lambda = 3.428$ and in figure 3(c) for $a_p/\lambda = 4.8$, where the locking steps become more pronounced as $R_p$ increases. These results indicate that directional locking is strongest for large, dense pinning sites, but that in arrays with larger lattice spacing, locking effects can still be observed at large pinning radii.

5. Substrate strength and different locking regimes

We next consider the effect of substrate strength on the locking regimes for a system with a triangular pinning lattice at fixed $B/B_{k_b} = 1.11$. We find that the step widths initially grow with increasing $F_p$, but for large $F_p$ the step widths diminish again and the nature of the ordered flow on the steps changes. In figure 4(a) we plot $V_\theta$ versus $\theta$ for $F_p = 2.35$, where only the $(0,0)$ and $(1,0)$ steps are present and the remaining steps have been replaced by a strongly fluctuating regime where structural transitions of the vortex lattice can occur. Near $F_p = 2.5$, a large portion of the vortex dynamics falls in the random fluctuating phase rather than on locking steps. In the strong pinning regime, the second type of locking step, associated with soliton-like pulses of moving vortices, appears. For $F_p \geq 3.9$ all the vortices are pinned for all $\theta$.

Figure 3. $V_\theta$ versus $\theta$ for a square pinning array with $F_p = 1.85$ at $B/B_{k_b} = 1.0$. (a) A sample with $R_p = 0.35$, for pinning lattice constants $a_p/\lambda = 1.414, 1.6, 2.0, 2.4, 2.67, 3.0, 3.428, 4.0,$ and $4.8$, from top to bottom. For increasing $a_p$, the locking effects are reduced. (b) A sample with $a_p/\lambda = 3.428$ and varied $R_p/\lambda = 1.9, 1.6, 1.3, 1.0, 0.7$, and $0.35$, from top to bottom. (c) A sample with $a_p/\lambda = 4.8$ and varied $R_p/\lambda$ with the same values as in panel (b). In (b) and (c), for larger $a_p$, the locking effects are significantly enhanced at larger $R_p$.

Figure 4. A sample with a triangular pinning array at $B/B_{k_b} = 1.11$. (a)–(d) $V_\theta$ versus $\theta$. (a) $F_p = 2.35$. There are only two locking regions separated by strongly fluctuating regions where some guided motion of the particles occurs. (b) $F_p = 2.5$. The locking regions are almost completely absent. (c) $F_p = 2.75$. Several locking regions reappear. (d) $F_p = 2.85$. There are a larger number of locking steps intermixed with randomly fluctuating regions. (e)–(g) Pinning site (large open circles) and vortex (small filled circles) positions as well as vortex trajectories (lines) over a fixed time interval in a portion of a sample on the $(1,1)$ locking step at $\theta = 30^\circ$. (e) In the weak pinning regime at $F_p = 1.75$, an ordered flow occurs with all the vortices moving along the $\theta = 30^\circ$ direction. (f) In the strong pinning regime at $F_p = 2.75$, a portion of the vortices are pinned and the flow occurs by a pulse motion or flowing kink. (g) At $F_p = 2.75$ for $\theta = 15^\circ$, a nonlocking fluctuating flow phase occurs. (h) The dynamic phase diagram of $\theta$ versus $F_p$. The regions in which the $(0,0)$, $(1,1)$, $(1,0)$, and $90^\circ$ locking steps appear are marked. The left side of the diagram at lower $F_p$ is the weak pinning regime where structural transitions of the vortex lattice can occur. Near $F_p = 2.5$, a large portion of the vortex dynamics falls in the random fluctuating phase rather than on locking steps. In the strong pinning regime, the second type of locking step, associated with soliton-like pulses of moving vortices, appears. For $F_p \geq 3.9$ all the vortices are pinned for all $\theta$. 

We next consider the effect of substrate strength on the locking regimes for a system with a triangular pinning lattice at fixed $B/B_{k_b} = 1.11$. We find that the step widths initially grow with increasing $F_p$, but for large $F_p$ the step widths diminish again and the nature of the ordered flow on the steps...
effects would exhibit a significant flux-flow regime and the system enters the random fluctuating phase. For \( \theta = 15^\circ \), a fluctuating flow phase occurs and there is no longer any 1D channeling of the vortex motion. In the strong pinning regime, along the locking steps the vortex lattice is largely triangular since most vortices are pinned, and the large scale structural transitions are lost. By conducting a series of simulations for varied \( F_p \), we map the transition between the strong and weak pinning regimes, as shown in figure 4(h), where we highlight the widths of the (0, 0), (1, 1), (0, 0), and \( \theta = 90^\circ \) locking steps. The (0, 0) step increases in width with increasing \( F_p \) up to \( F_p = 2.45 \), and then vanishes at the transition from the weak pinning locking regime to the random fluctuating phase. For \( F_p > 2.55 \) the (0, 0) step reappears in the strong pinning regime, and its width saturates for \( F_p > 3.0 \). The higher order steps show similar features.

The plots of \( V_f \) versus \( F_p \) at fixed \( \theta \) shown in figure 5(a) for \( \theta = 90^\circ \) and 30° illustrate the crossover between the two pinning regimes. As \( F_p \) increases, \( V_f \) monotonically decreases until, at \( \theta = 90^\circ \), the system enters the strong pinning regime, in which \( V_f \) remains constant for increasing \( F_p \). In the weak pinning regime, \( V_f \) can be fitted to the functional form \( V_f - V_f^{\text{sat}} \propto (F_p - F_p^{\text{lock}})^{\alpha} \), where \( V_f^{\text{sat}} \) is the saturation value of the velocity in the strong pinning regime. In figure 5(c), we show this scaling with \( V_f \) normalized by \( \sin(\theta) \). Here, the \( \theta = 30^\circ \) and 90° curves collapse on each other and the solid line indicates a fit with \( \alpha = 1/3 \). The behavior of the \( V_f \) curves varies on different sets of locking steps, so a straightforward scaling such as that shown in figure 5(c) is not always possible. For example, in figure 5(b) we plot \( V_f \) versus \( F_p \) on the (1, 0) step as well as for a driving angle of \( \theta = 17^\circ \). In both cases, \( V_f \) decreases with increasing \( F_p \) for \( F_p < 2.5 \). Near \( F_p = 2.5 \), \( V_f \) for both driving angles passes through a local maximum when the system enters the random fluctuating flow regime. We have also studied the behavior of \( F_p^{\text{lock}} \) in the strong pinning regime as a function of \( B/B_0 \) and find features similar to those observed for the weak pinning regime. We find the same transition from a weak to a strong pinning regime for square pinning arrays.

An ideal system for exploring directional locking effects would exhibit a significant flux-flow regime and no multi-quantum vortex states, such as the recently studied thin-film Al samples with periodic pinning [44]. We expect that, in this system, the weak pinning regime where all the vortices are flowing could be explored at \( T = 1.278 \) K with a rotating dc drive of amplitude \( J = 10 \) kA cm\(^{-2} \) just above \( B/B_0 \), where steps of width 2–3 kA cm\(^{-2} \) should occur. The strong pinning regime where only incommensurate flow occurs should appear near \( J = 5 \) kA cm\(^{-2} \).

6. Colloidal particles moving over triangular and square substrates

Another question is how general our results are for other types of particle interaction, particularly colloidal particles, where there is currently considerable interest in kinetic locking effects. We have tested all of our major predictions for colloidal particles interacting with a screened Yukawa potential. In figure 5(d), we plot \( V_f \) versus \( \theta \) for colloidal particles on a square pinning array of strength \( F_p = 1.85 \) for varied colloid densities \( N_c/N_p \) at \( q^2 = 0.001 \). Here we find the same locking step features observed for the vortex system, with the same oscillations in the width of the first and higher order steps. At \( N_c/N_p = 1.82 \), \( F_p^{\text{lock}} \) drops to zero, but the first step reappears at \( N_c/N_p = 2.85 \), similar to what was observed for the vortex system. We also find the same behaviors for colloids on triangular pinning arrays.

The colloid–colloid interaction strength can be altered by changing the effective colloid charge \( q \). Figure 5(e) shows \( F_p^{\text{lock}} \) as a function of \( q^2 \) for colloids on a square pinning lattice from the system in figure 5(d). As \( q^2 \) increases, \( F_p^{\text{lock}} \) gradually decreases to zero. There is no clear transition between weak and strong pinning regimes when \( q^2 \) is varied, unlike the transition found in the vortex system for varied \( F_p \). This is because we are in the regime \( F_p < A \), so all the colloids are moving even when the particle–particle interaction strength is negligible. In order to observe the strong pinning regime discussed earlier, it is necessary for a portion of the particles to be pinned and for the motion to occur in the form of depinned incommensurations.

Recent experiments on optical light arrays trapping charged polystyrene spheres which interact via a Yukawa potential [21] provide an ideal system for testing the locking effects. Near the first incommensurate filling, the locking effects could be studied in the weak pinning regime, where all the particles flowed under a drive of 80 fN in [21]. At incommensurate fillings the strong pinning regime would correspond to 10–25 fN drives, where only incommensurate particles flow and the remaining particles are pinned.

7. Discussion

In table 1, we summarize our results and list the basic features of the weak, strong, and transitional pinning regimes. For vortices, the best candidate system to explore the directional locking effects would have a large flux-flow regime. Recent experiments on Al thin films with periodic pinning arrays showed a large flux-flow regime, and the features in the transport curves are in excellent agreement with those
predicted in simulations of vortices driven in one direction over periodic pinning arrays [44]. The different pinning regimes could be probed by varying the applied field and applied drive. Direct vortex imaging [37] would provide information on the individual vortex level for driving vortices along different angles with respect to the pinning axis. In the colloidal system, the most straightforward experimental realization would be strongly interacting charged colloids driven over a periodic substrate. Recent experiments at commensurate and incommensurate fillings for collectively interacting colloids obtained velocity–force curves and images of colloid motion for colloids driven in only one direction [21]. It should be straightforward to drive the system at different angles to explore the collective direction locking effects, while the strong and weak pinning regimes can be accessed by varying the laser power to change the substrate strength. An experimental realization of directional locking for colloids driven over quasicrystalline arrays [45] has already shown excellent agreement with predictions from simulations [29].

Table 1. Summary of strong and weak pinning locking regimes and the behaviors associated with them.

| Region       | Particle flow                     | Structure               | Transport     |
|--------------|-----------------------------------|-------------------------|---------------|
| Weak pinning | All particles flowing             | Dynamic lattices        | Steps         |
| Crossover    | Fraction of particles flowing     | Disordered guidance     | Effects       |
| Strong pinning | Only incommensurate particles flowing | Lattice symmetry of substrate | Steps, dips  |

In summary, we have shown that a rich variety of dynamical locking effects associated with structural transitions can occur for collectively interacting particles such as vortices and colloids moving over triangular and square substrate arrays. We identify two different locking regimes as a function of the substrate strength and of the ratio of particle density to substrate minima density that have not been previously studied. For weak substrates, at certain driving angles, all the particles flow along 1D channels through the pinning sites, generating a series of constant velocity steps on which the motion remains locked to a certain direction over a range of driving angles. As the pinning strength decreases, the width of the locking steps decreases and there are larger nonlocking regions in which the particle–particle interactions dominate and a triangular particle lattice forms.

In the weak substrate regime, the widths of the steps including the initial transverse depinning barrier pass through local minima and maxima as a function of the ratio of particle density to substrate minima density. In contrast to the longitudinal depinning threshold for periodic substrates, which shows peaks at commensurate fields, the local maxima for the width of the first locking step are not correlated with the 2D periodicity of the pinning array, but are instead related to a dynamical commensuration effect caused by the formation of 1D channels of moving particles. A local maximum in the width of the first locking step occurs when integer numbers of rows of moving vortices can fit in the interstitial areas between the pinning sites. For fillings at which the moving rows are unable to fit without buckling, the particle structure becomes disordered and the width of the first locking step is small or zero.
As a function of substrate strength for fixed particle density, we identify two distinct locking regimes: a weak pinning regime, where all the particles flow along the pinning sites, and a strong pinning regime, where the flow occurs by means of an incommensuration or a pulse passing through a background of pinned particles. Between these two regimes the locking steps are lost and are replaced by a strongly fluctuating regime where the particle motion does not lock to a particular direction. For a fixed driving angle, the average particle velocity drops sharply at the crossover between these two regimes, and the velocity saturates to a plateau value in the strong substrate limit. In the strong substrate regime the width of the first step displays commensurate peaks when the number of particles is a fractional matching ratio of the number of substrate minima, in addition to showing peaks at the incommensurate fields as in the weak pinning regime. We expect these effects to be relevant to a wide class of collectively interacting particles moving over periodic substrates.

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