A Simplified Coding Scheme for the Broadcast Channel with Complementary Receiver Side Information under Individual Secrecy Constraints

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Abstract—This paper simplifies an existing coding scheme for the two-receiver discrete memoryless broadcast channel with complementary receiver side information where there is a passive eavesdropper and individual secrecy is required. The existing coding scheme is simplified in two steps by replacing Wyner’s secrecy coding with Carleial-Hellman’s secrecy coding as well as removing additional randomness in the common satellite codeword. This simplified scheme retains the existing achievable individual secrecy rate region. Not least, the simplified scheme requires fewer message splits and fewer random components.

Index Terms—Broadcast channel, individual secrecy, physical layer security, receiver side information.

I. INTRODUCTION

A. Background

The broadcast channel is a common model for noisy one-to-many communications, in which a sender wishes to reliably transmit independent messages to multiple receivers. Unfortunately, the broadcast nature of such communication model makes it susceptible to passive eavesdroppers present in the communication range. In order to ensure the secrecy of sensitive information transmitted over the channel, it is essential to look into the problem of secure communication over broadcast channels.

The earliest study of information theoretic secrecy dates back to Shannon’s paper in 1949 [1]. In the paper, Shannon proposed a secure communication strategy by having the transmitter and legitimate receiver share a secret key, which is unknown to the eavesdropper. This secret key approach (also known as one-time pad) which achieves perfect secrecy, i.e., zero information leakage, requires the secret key to be at least as large as the size of the message and is independent of the message.

Shannon’s result motivated future works on information theoretic secrecy. For instance, Wyner introduced the wiretap channel in [2]. The wiretapping coding scheme involves multi-coding and randomized encoding. The principle behind this coding scheme is to provide sufficient randomness to the codeword in order to protect the message from the eavesdropper. Secret communication is viable through this scheme if the channel to the eavesdropper is a degraded version of the channel to the legitimate receiver. Wyner’s wiretap coding was extended by Csiszar and Körner to the case of a general discrete memoryless broadcast channel with common and private messages [3]. Not least, Chia and El Gamal also applied Wyner’s wiretap coding to cases of broadcast channel with two receivers and an eavesdropper as well as broadcast channel with one receiver and two eavesdroppers [4]. In these works [3], [4], the transmitter will send a common message to all legitimate receivers and eavesdroppers, while keeping a private message secured from eavesdroppers.

The limitations of Wyner’s wiretap coding prompted academics to look into various secret communication strategies that utilize the resources available under different channel settings. Listing down a few, we have the source model [5] and channel model [6] for secret key agreement, usage of channel state information [7], [8], usage of feedback [9]–[11], usage of correlated sources [12], [13] and usage of receiver side information [14]–[16].

For broadcast channels where two messages need to be protected, two secrecy notions have been defined: joint secrecy and individual secrecy [15], [16]. For joint secrecy, the joint information leakage from both messages to the eavesdropper is made vanishing. On the other hand, individual secrecy requires the individual information leakage from each message to the eavesdropper to be vanishing. From a design perspective, joint secrecy guarantees a higher degree of secrecy compared to the individual secrecy since it keeps the eavesdropper completely ignorant even when a function of both confidential messages is received. However, joint secrecy may be difficult or impossible to fulfill, especially when the channel to at least one legitimate receiver is more noisy than the channel to the eavesdropper. This makes individual secrecy a more practical secrecy notion which ensures acceptable security strength without compromising transmission gains [17]–[19].

In this paper, we study the two-receiver discrete memoryless broadcast channel with complementary receiver side information and with an eavesdropper under individual secrecy constraints. The term complementary receiver side information means that the receivers know a priori, as side information, all messages that they do not need to decode. This problem was previously studied by Chen, Koyluoglu and Sezgin [17]. Chen et al. proposed a coding scheme which is constructed from one-time pad signals [1], superposition coding [20], Wyner’s secrecy coding [2] and Marton’s coding [21]. Although the coding scheme achieves an individual secrecy rate region that is tight for some special cases, we notice that the scheme did not utilize the Carleial-Hellman’s secrecy coding [22].
which allows each message to act as a random component which ensures individual secrecy of each another. Additional random component is still present in protecting specific message segments. As a result, this paper aims to simplify the existing coding scheme for the two-receiver broadcast channel with complementary receiver side information and with an eavesdropper under the context of individual secrecy.

B. Contributions

Our contribution in this paper is the simplification of the existing coding scheme for the two-receiver discrete memoryless broadcast channel with complementary receiver side information and with a passive eavesdropper. This scheme simplification was performed in two steps. In the first step, simplification was done by replacing Wyner’s secrecy coding with Carleial-Hellman’s secrecy coding. This essentially helps us reduce the number of message splits. In the second step, we simplified the initial scheme further by dropping a random component. The final simplified coding scheme is shown to achieve the same rate region as the existing coding scheme.

C. Paper Organization

The entire paper will be organized as follows. Section II will focus on the system model. Section III will provide an overview of the related works that motivate this paper. Section IV will provide the main results on the coding scheme simplification. Next, Section V will include additional discussions relevant to the results obtained. Lastly, Section VI will conclude the paper.

II. SYSTEM MODEL

In this paper, we will denote random variables by uppercase letters, their corresponding realizations by lowercase letters and their corresponding sets by calligraphic letters. A \((j-i+1)\)-sequence of random variables will be denoted by \(X_i^j = (X_i, \ldots, X_j)\) for \(1 \leq i \leq j\). Whenever \(i = 1\), the subscript will be dropped, resulting in \(X^j = (X_1, \ldots, X_j)\). \(\mathbb{R}^d\) represents the \(d\)-dimensional real Euclidean space and \(\mathbb{R}_+^d\) represents the \(d\)-dimensional non-negative real Euclidean space. \(\mathcal{R}\) will be used to represent a subset of \(\mathbb{R}^d\), and the convex hull of \(\mathcal{R}\) will be denoted as \(\text{co}(\mathcal{R})\). Meanwhile, \([a:b]\) refers to a set of natural numbers between and including \(a\) and \(b\), for \(a \leq b\). Lastly, the operator \(\times\) denotes the Cartesian product.

The focus of this paper will be on the scheme simplification for the two-receiver discrete memoryless broadcast channel with complementary receiver side information and with a passive eavesdropper (hereafter referred as the broadcast channel with complementary side information). The system model for this case is illustrated in Fig. 1. In this model, we define \((M_1, M_2)\) as the source messages, \(M_i\) as the message requested by legitimate receiver \(i\) and \(M_{\bar{i}}\) as the message known a priori (i.e. receiver side information), for \(i = 1, 2\). Let \(X\) denote the channel input from the sender, while \(Y_i\) and \(Z\) denote the channel output to receiver \(i\) and the eavesdropper respectively. In \(n\) channel uses, \(X^n\) represents the transmitted codeword, \(Y^n_i\) represents the signal received by legitimate receiver \(i\) and \(Z^n\) represents the signal received by the eavesdropper. The memoryless nature of the channel also implies that

\[
p(y^n_1, y^n_2, z^n|x^n) = \prod_{i=1}^n p(y_{1i}, y_{2i}, z_i|x_i) \tag{1}
\]

In this case, the transmitter will be sending messages \(m_1\) and \(m_2\) to legitimate receiver 1 and 2, respectively through the channel \(p(y_1, y_2, z|x)\). Both legitimate receivers have complementary receiver side information to aid them in decoding the transmitted messages. In other words, receiver 1 will be having \(m_2\) as side information, whereas receiver 2 will be having \(m_1\) as side information.

Definition 1: A \((2^nR_1, 2^nR_2, n)\) secrecy code for the two-receiver discrete memoryless broadcast channel with complementary side information consists of:

- two message sets, \(\{M_1 = [1: 2^nR_1]\}\) and \(\{M_2 = [1: 2^nR_2]\}\);
- an encoding function, \(f : M_1 \times M_2 \rightarrow X^n\), such that \(X^n = f(M_1, M_2)\); and
- two decoding functions, \(g_i : Y^n_i \times M_i \rightarrow \hat{M}_i, \text{ such that } \hat{M}_i = g_i(Y^n_i, M_i)\), where \(i \triangleq (i \mod 2) + 1\), for \(i = 1, 2\).

Both messages, \(M_1\) and \(M_2\) are assumed to be uniformly distributed over their respective message set. Hence, we have \(R_i = \frac{1}{n}H(M_i)\), for \(i = 1, 2\). Meanwhile the individual information leakage rate associated with the \((2^nR_1, 2^nR_2, n)\) secrecy code is defined as \(R_{e,i}^{(n)} = \frac{1}{n}I(M_i; Z^n)\), for \(i = 1, 2\).

The probability of error for the secrecy code at each receiver \(i\) is defined as \(P_{e,i}^{(n)} = P(\hat{M}_i \neq M_i)\), for \(i = 1, 2\). A rate pair \((R_1, R_2)\) is said to be achievable if there exist a sequence of \((2^nR_1, 2^nR_2, n)\) codes such that

\[
P_{e,1}^{(n)} \leq \epsilon_n, \text{ for } i = 1, 2 \tag{2}
\]

\[
R_{e,2}^{(n)} \leq \tau_n, \text{ for } i = 1, 2 \tag{3}
\]

\[
\lim_{n \rightarrow \infty} \epsilon_n = 0 \quad \text{and} \quad \lim_{n \rightarrow \infty} \tau_n = 0 \tag{4}
\]

III. RELATED WORKS

The problem in this paper is motivated by the discussions of Chen et al. [17]. Chen et al. proposed a coding scheme for the broadcast channel with complementary side information under the context of individual secrecy. As illustrated in Fig. 2a, this coding scheme involves splitting each of \(m_1\) and \(m_2\) into four segments. These message segments then make up the coding scheme in Fig. 2b.
In this scheme, the cloud center codeword \( U^n \) comprises a one-time pad signal [1]. This secret key is formed by XORing the first segment of each message. The utilization of a one-time pad signal is possible since the legitimate receivers can utilize their receiver side information to unveil the original message.

A main contribution in the coding scheme proposed by Chen et al. [17] is a common satellite codeword \( V^n \) formed and combined with \( U^n \) using superposition coding [20]. In the \( V^n \) codeword layer, Wyner’s secrecy coding [2] is implemented to protect the message segments. However, apart from the regular randomness \( r \), the authors presented an interesting idea of providing additional randomness through another one-time pad signal.

Not least, once again, the private satellite codewords, \( V_{i}^{n} \) and \( V_{j}^{n} \), which are responsible of carrying the remaining message segments to their respective legitimate receivers are secured with Wyner’s secrecy coding [2]. All the four codeword layers, \( U^n \), \( V^n \), \( V_{i}^{n} \), and \( V_{j}^{n} \), are finally combined under the Marton’s coding [21] framework to achieve an improved rate region. This coding scheme has the achievability conditions listed in Theorem [1]. After the Fourier-Motzkin procedure [20, Appendix D], the achievable individual secrecy rate region, \( R_{s}^{n} \), is obtained as in Corollary [1].

Theorem 1 (Chen et al. [17]): The following individual secrecy rate region is achievable for the two-receiver discrete memoryless broadcast channel with complementary receiver side information and with a passive eavesdropper:

\[
\begin{align*}
R_{1} &= R_{a} + R_{b} + R_{i} + R_{d}, \\
R_{2} &= R_{a} + R_{b} + R_{d} + R_{2d}, \\
R_{c1} + R_{c2} &> I(V_{1};V_{2}|U,V), \\
R_{1} + R_{c} + R_{i} + R_{c1} < I(U,V_{1};Y_{1}), \\
R_{1d} + R_{c1} + R_{c2} < I(V_{1};Y_{1}|V), \\
R_{c1} + R_{c1} &> I(V_{1};Z|V), \\
R_{c2} + R_{c2} &> I(V_{2};Z|V), \\
R_{a} + R_{r} + R_{b} + R_{d} + R_{2d} &> I(U,V_{2};Y_{2}), \\
R_{a} + R_{r} + R_{b} + R_{2d} + R_{2d} &> I(V_{2};Z|V), \\
R_{a} + R_{r} + R_{b} + R_{d} + R_{2d} &> I(V_{2};Y_{2}|V), \\
R_{a} + R_{r} + R_{b} + R_{d} + R_{2d} &> I(V_{1};Z|V), \\
R_{a} + R_{r} + R_{b} + R_{d} + R_{2d} &> I(V_{1};Y_{1}|V).
\end{align*}
\]

(5)

Corollary 1: The achievable individual secrecy rate region in Theorem [1] can be written as follows:

\[
\begin{align*}
R_{1} &< I(U;Y_{1}) + I(V,V_{1};Y_{1}|U) \\
- I(V,V_{1};Z|U) + I(V;Z|U), \\
R_{1} - R_{2} &< I(V_{1};V_{1}|Y_{1}) \\
- I(V_{1};V_{1};Z|U), \\
R_{2} &< I(U;Y_{2}) + I(V_{2};Y_{2}|U) \\
- I(V_{2};V_{2};Z|U) + I(V;Z|U), \\
R_{2} - R_{1} &< I(V_{2};Y_{2}|U) \\
- I(V_{2};Z|U).
\end{align*}
\]

(6)

IV. PROPOSED SIMPLIFICATION FOR THE TWO-RECEIVER BROADCAST CHANNEL WITH COMPLEMENTARY SIDE INFORMATION

The coding scheme simplification for the two-receiver broadcast channel with complementary side information was carried out in two steps.

A. Simplification Step 1

For the first simplification step, we performed rate splitting to break \( M_{ic} \), \( i = 1, 2 \), into three segments, namely \( M_{ia} \) at rate \( R_{ia} \), \( M_{ib} \) at rate \( R_{ib} \) and \( M_{ic} \) at rate \( R_{ic} \), i.e. \( M_{i} = [M_{ia}, M_{ib}, M_{ic}] \) where \( m_{ia} \in [1 : 2^{R_{ia}}], m_{ib} \in [1 : 2^{R_{ib}}], m_{ic} \in [1 : 2^{R_{ic}}] \) are independent and \( R_{i} = R_{ia} + R_{ib} + R_{ic} \). This splitting is shown in Fig. 3a. In line with the existing coding scheme, our proposed simplified coding scheme 1 is then constructed from a combination of a one-time pad signal [1], superposition coding [20], Wyner’s secrecy coding [2] and Marton’s coding [21] as illustrated in Fig. 3b.

Our simplified coding scheme 1 differs from the existing coding scheme proposed in [17] as we bring in Carleial-Hellman’s secrecy coding [22] to replace Wyner’s secrecy coding [2] in the common satellite codeword \( V^n \). Through this approach, the message segments, \( R_{ib} \) and \( R_{2d} \), are now responsible of ensuring the individual secrecy of each another with the aid of an additional randomness, \( r \).

The gap in individual secrecy analysis due to the replacement of Wyner’s secrecy coding [2] with Carleial-Hellman’s secrecy coding [22] is closed through the arguments in Lemma [1] and Lemma [2]. The achievable individual secrecy rate region of the simplified coding scheme 1 is then presented in Theorem [2] and compared through Corollary [2]. We will also use \((xx,yy)\) to denote the \( yy \) constraint in equation \( xx \).

Lemma 1: The existing coding scheme by Chen et al. [17] for the two-receiver discrete memoryless broadcast channel with complementary receiver side information and with a
passive eavesdropper also achieves an alternate individual secrecy rate region, $R^{2o}$, stated below:

$$
(R_1, R_2) \in \mathbb{R}^2_+
$$

$$
\begin{align*}
R_1 &= R_a + R_b + R_{1c} + R_{1d}, \\
R_2 &= R_a + R_b + R_{2c} + R_{2d}, \\
R_{c1} + R_{c2} &> I(V_1; V_2 | U, V), \\
R_1 + R_{c1} + R_{c2} &< I(U, V, V_1; V_1'), \\
R_1 - R_{c1} &< I(U, V_1; V_1'), \\
R_{1d} + R_{c1} + R_{c1} &< I(V_1; V_1 | V), \\
R_2 + R_{c1} + R_{c2} &< I(U, V, V_1; V_2'), \\
R_2 - R_{c2} &< I(U, V_2; V_2'), \\
R_2 &< R_{c2} + R_{c2} + R_{c2} + R_{c2} < I(V, V_2; V_2') \cup U, \\
R_{2d} &< R_{c2} + R_{c2} + R_{c2} < I(V, V_2; V_2'), \\
R_{1c} &< I(V_1; Z; V), \\
R_{c2} &< I(V_2; Z; V), \\
R_{b} &< R_{c2} + R_{c2} + R_{c2} + R_{c2} > I(V; Z; V), \\
R_{c1} &< I(V_1; Z; V) + I(V_2; Z; V) - I(V_1, V_2; Z; V), \\
R_1 &> 0, R_0 > 0, R_0 > 0, R_{1c} > 0, \\
R_{1d} &> 0, R_0 > 0, R_{2c} > 0, R_{2d} > 0, \\
R_r &> 0, R_r > 0, R_r > 0, R_r > 0, R_{c2} > 0
\end{align*}
$$

The achievability conditions of $R^{2o}$ are obtained based on the existing coding scheme for the two-receiver broadcast channel with complementary receiver side information \[\text{(7)}\]. However, the achievability conditions of $R^{2o'}$ differ slightly from the achievable conditions of $R^{2o}$ in Theorem 1, i.e., the constraints \((7.a) and (7.b)\) have replaced the constraint \((5.a)\). This is a result of a difference in individual secrecy analysis for the $V^n$ codewords. In Theorem 1 both the message segments, $R_{1c}$ and $R_{2c}$, are secured by the XOR-ed message segments, $R_{1b} \oplus R_{2b}$, and additional randomness $r$. Meanwhile, in Lemma 1 we apply the concept of Carleial-Hellman’s secrecy coding and treat the respective message segments, $R_{1c}$ or $R_{2c}$, as additional randomness in ensuring the individual secrecy of their counterparts. This approach results in the looser constraints, \((7.a) and (7.b)\). Although $R^{2o}$ seems to have stricter achievability conditions compared to $R^{2o'}$, we now show that the two regions are in fact the same.

Lemma 2: $R^{2o'} = R^{2o}$

**Proof of Lemma 2** Applying the Fourier-Motzkin procedure to eliminate the terms $R_{1c}, R_b, R_{1c}, R_{1d}, R_{2c}, R_{2d}, R_r, R_{r1}, R_{r2}, R_{c1}$ and $R_{c2}$ in \[\text{(7)}\], we obtain the individual
secrecy rate region, $\mathcal{R}^{2\alpha}$, as follows:

$$
\mathcal{R}^{2\alpha} = \begin{cases} 
R_1 < I(U; Y_1) + I(V, V_1; Y_2 | U) \\
- I(V, V_1; Z | U) + I(V; Z | U), \\
R_1 - R_2 < I(V, V_1; Y_1 | U) \\
- I(V, V_1; Z | U), \\
R_2 < I(U; Y_2) + I(V, V_2; Y_2 | U) \\
- I(V, V_2; Z | U) + I(V; Z | U), \\
R_2 - R_1 < I(V, V_2; Y_2 | U) \\
- I(V, V_2; Z | U) \\
\text{over all } p(u)p(v|u)p(v_1,v_2|v)
\end{cases}
$$

Note that the achievable individual secrecy rate region, $\mathcal{R}^{2\alpha}$, is eventually the same as the achievable individual secrecy rate region, $\mathcal{R}^{2\alpha}$, in Corollary 1. Hence, the relationship $\mathcal{R}^{2\alpha} = \mathcal{R}^{2\alpha}$.

**Theorem 2:** The simplified coding scheme 1 for the two-receiver discrete memoryless broadcast channel with complementary receiver side information and with a passive eavesdropper achieves an individual secrecy rate region, $\mathcal{R}^{2\alpha}$, as follows:

$$
\mathcal{R}^{2\alpha} = \begin{cases} 
R_1 < I(U; Y_1) + I(V, V_1; Y_2 | U) \\
- I(V, V_1; Z | U) + I(V; Z | U), \\
R_1 - R_2 < I(V, V_1; Y_1 | U) \\
- I(V, V_1; Z | U), \\
R_2 < I(U; Y_2) + I(V, V_2; Y_2 | U) \\
- I(V, V_2; Z | U) + I(V; Z | U), \\
R_2 - R_1 < I(V, V_2; Y_2 | U) \\
- I(V, V_2; Z | U) \\
\text{over all } p(u)p(v|u)p(v_1,v_2|v)
\end{cases}
$$

\text{Proof of Theorem 2} The proposed simplified coding scheme 1 in Fig. 3b can be readily obtained from the existing coding scheme suggested by Chen et al. [17] in Fig. 2b through the following changes:

- Setting $m_{1b} = m_{2b} = \phi$,
- Replacing $m_{1c}$ with $m_{1b}$,
- Replacing $m_{1d}$ with $m_{1c}$,
- Replacing $m_{2c}$ with $m_{2b}$,
- Replacing $m_{2d}$ with $m_{2c}$ in the existing coding scheme.

With these changes, $R_0 = 0$, $R_{1b}$ is replaced by $R_{1b}$, $R_{1d}$ is replaced by $R_{1c}$, $R_{2c}$ is replaced by $R_{2b}$ and $R_{2d}$ is replaced by $R_{2c}$, allowing us to rewrite the achievability conditions in

Applying the Fourier-Motzkin procedure to eliminate the terms $R_{2a}$, $R_{1b}$, $R_{1c}$, $R_{2b}$, $R_{2c}$, $R_r$, $R_1$, $R_2$, $R_{r1}$ and $R_{r2}$ in (10), the individual secrecy rate region, $\mathcal{R}^{2\alpha}$, in Theorem 2 is obtained.

**Corollary 2:** $\mathcal{R}^{2\alpha} = \mathcal{R}^{2\alpha} = \mathcal{R}^{2\alpha}$.

\text{Proof of Corollary 2} Comparing (8) and (9), we notice that both $\mathcal{R}^{2\alpha}$ and $\mathcal{R}^{2\alpha}$ have the same achievable individual secrecy rate region, hence the relationship $\mathcal{R}^{2\alpha} = \mathcal{R}^{2\alpha}$.

Combining this with $\mathcal{R}^{2\alpha} = \mathcal{R}^{2\alpha}$ in Lemma 2, we have $\mathcal{R}^{2\alpha} = \mathcal{R}^{2\alpha} = \mathcal{R}^{2\alpha}$.

**B. Simplification Step 2**

Recall that Carleial-Hellman’s secrecy coding [22] allows each message to act as a random component which ensures individual secrecy of each another. Intuitively, additional random component should not be necessary in securing messages since each message has provided sufficient randomness. With this concept in mind, we proceed with the removal of the
randomness $r$ from the $V^n$ codeword of the previous simplified coding scheme 1. The final simplified coding scheme 2 is illustrated in Fig. 3. The achievable individual secrecy rate region of this simplified coding scheme 2 is shown in Theorem 3 and rate region comparison is done through the proofs for Theorem 4 and Theorem 5 below.

**Theorem 3:** Consider

$$\mathcal{R}^{2b}_{0} \triangleq (R_1, R_2) \in \mathbb{R}^2_+$$

and

$$\mathcal{R}^{2b}_{1} \triangleq \mathcal{R}^{2b}_{0} \text{ without constraints (11.a) and (11.c)}$$

but with constraint $|\mathcal{V}| \leq 1$. (12)

The simplified coding scheme 2 for the two-receiver discrete memoryless broadcast channel with complementary receiver side information and with a passive eavesdropper achieves an individual secrecy rate region, $\mathcal{R}^{2b}$, as follows:

$$\mathcal{R}^{2b} \triangleq \mathcal{R}^{2b}_{0} \cup \mathcal{R}^{2b}_{1}. \quad \text{(13)}$$

**Proof of Theorem 3** The proposed simplified coding scheme 2 is a special case of the proposed simplified coding scheme 1 by setting $r = \phi$. Setting $r = \phi$ forces the rate $R_r = 0$, hence, we rewrite the achievable individual secrecy rate region in (10) as

$$\begin{align*}
R_1 & = R_a + R_{1b} + R_{1c}, \\
R_2 & = R_b + R_{2b} + R_{2c}, \\
R_{1b} + R_{2c} & > I(V_1; V_2 | U, V), \\
R_1 + R_{1c} + R_{1a} & < I(V; V_1; Y_1 | U), \\
R_1 - R_a + R_{1c} + R_{1a} & < I(V; V_1; Y_1 | U), \\
R_{1c} + R_{1a} + R_{1c} & < I(V_1; Y_1 | U), \\
R_2 + R_{2b} + R_{2c} & < I(V; V_2; Y_2), \\
R_2 - R_a + R_{2b} + R_{2c} & < I(V; V_2; Z | U), \\
R_{2b} + R_{2c} & < I(V; V_2; Y_2 | V), \\
R_{2c} + R_{2c} & < I(V; V_2; Z | V), \\
R_2 - R_a + R_{2c} + R_{2c} & < I(V, V_2; Y_1 | V), \\
R_2 & < I(V, V_2; Z | U), \\
R_{1b} & > I(V; Z | U), \\
R_{1c} & = 1 \\
R_{2c} + R_{2c} & > 0, \quad R_{1b} > 0, \quad R_{1c} > 0, \\
R_2 & > 0, \quad R_{2c} > 0, \quad R_{1c} > 0, \quad R_{2c} > 0, \\
R_{1b} & > 0, \quad R_{1c} > 0, \quad R_{2c} > 0 \\
\end{align*}$$

Applying the Fourier-Motzkin procedure to eliminate the terms $R_a, R_{1b}, R_{1c}, R_{2b}, R_{2c}, R_1, R_2, R_{1c}$ and $R_{2c}$ in (14), the region $\mathcal{R}^{2b}_{0}$ is obtained. Note that when $|\mathcal{V}| \leq 1$, we drop the $V^n$ common satellite codeword. This simplifies the message splitting (i.e., $m_{ib} = \phi, R_{ib} = 0$, $i = 1, 2$) as well as removes the constraints (11.a) and (11.c), hence giving us the region $\mathcal{R}^{2b}_{1}$. Taking the union of $\mathcal{R}^{2b}_{0}$ and $\mathcal{R}^{2b}_{1}$ will then give us the individual secrecy rate region, $\mathcal{R}^{2b}$, in Theorem 5.

**Theorem 4:** $\text{co}(\mathcal{R}^{2b}_{0} \cup \mathcal{R}^{2b}_{1}) = \mathcal{R}^{2a}$. (15)

**Proof of Theorem 4** Now, we show that by taking the convex hull of the union of $\mathcal{R}^{2b}_{0}$ and $\mathcal{R}^{2b}_{1}$, we will be able to achieve the region $\mathcal{R}^{2a}$. As a start, we plot the rate region $\mathcal{R}^{2a}$ as illustrated in Fig. 3. It can be observed that $\mathcal{R}^{2b}_{0}$ has two additional constraints, (11.a) and (11.c), compared to $\mathcal{R}^{2b}_{1}$. By comparing RHS of (11.a) and (11.c) to RHS of (11.b) and (11.d), $\mathcal{R}^{2b}_{0}$ can be plotted for the case $I(V; Z | U) \leq I(V, V_1; Y_1 | V) - I(V, V_2; Z | U)$ as shown in Fig. 5b and for the case $I(V; Z | U) > I(V, V_1; Y_1 | V) - I(V, V_2; Z | U)$ as shown in Fig. 5c for $i = 1, 2$. For all plots in Fig. 5 we have $a = (A - \epsilon, 0), b = (B - \epsilon, 0), c = (0, C - \epsilon), d = (0, D - \epsilon), e_1 = (E - \epsilon, 0), \text{ and } e_2 = (0, E - \epsilon)$, where $A = I(U; Y_1) + I(V; V_1; Y_1 | U) - I(V, V_1; Z | U) + I(V; Z | U), \quad B = I(V, V_1; Y_1 | U) - I(V, V_1; Z | U), \quad C = I(U; Y_1) + I(V, V_2; Y_2 | U) - I(V, V_2; Z | U) + I(V; Z | U), \quad D = I(V, V_2; Y_2 | U) - I(V, V_2; Z | U), \text{ and } E = I(V; Z | U)$ for some arbitrary small $\epsilon, \varepsilon > 0$. Note that $E \leq A$ and $B \leq C$ at all times. The proof of $E \leq A$ is as follows:

$$A = I(U; Y_1) + I(V; V_1; Y_1 | U) - I(V, V_1; Z | U) + I(V; Z | U) \geq I(U; Y_1) + I(V; Z | U)$$
\[ \geq I(V; Z|U) \]

where (a) follows from (9.a). The same proof applies to \( E \leq C \).

Referring to Fig. 5c when \( I(V; Z|U) \leq I(V, V_i; Y_i|U) - I(V, V_i; Z|U) \), \( i = 1, 2 \), the additional constraints, (11a) and (11b), results in the blue shaded excluded region, T1. Notice that the point \((0, 0) \in \mathcal{R}_1^{2\text{nb}} \), hence, by time-sharing, we take \( \text{co}(\mathcal{R}_1^{2\text{nb}}) \) to recover the excluded rate points in T1 and achieve \( \mathcal{R}_2^{2\text{na}} \).

Meanwhile, in Fig. 5c when \( I(V; Z|U) > I(V, V_i; Y_i|U) - I(V, V_i; Z|U) \), the additional constraints, (11a) and (11b), forms a hexagonal cut and results in the blue shaded excluded region, T2. In order to retrieve the excluded rate points in region T2, it is essential to recover both point \( b \) and point \( d \) in Fig. 5c. To recover point \( b \), from (12), we can set \( V' = U \), \( V'_1 = (V, V_1) \) and \( V'_2 = V = U \), allowing us to obtain a special case, \( \mathcal{R}_2^{2\text{nb}} \):

\[
\mathcal{R}_2^{2\text{nb}} = \left\{ (R_1, R_2) \in \mathbb{R}_+^2 \mid \begin{align*}
R_1 &< I(U; Y_1) + I(V, V_1; Y_1|U) - I(V, V_1; Z|U), \\
R_1 - R_2 &< I(V, V_1; Y_1|U) - I(V, V_1; Z|U), \\
R_2 &< I(U; Y_2), \\
R_2 &< R_1 \\
\text{over all } p(u)p(v|u)p(v_1, v_2|v) &\text{ subject to } I(V, V_1; Y_1|U) > I(V, V_1; Z|U). 
\end{align*} \right\}
\]

(15)

From RHS of (15a), we see that point \( b \) is recoverable and lies in \( \mathcal{R}_2^{2\text{nb}} \). Likewise, to recover point \( d \), from (12), we can set \( V' = U, V'_1 = V = U \) and \( V'_2 = (V, V_2) \). These arguments show that \( b, d \in \mathcal{R}_2^{2\text{nb}} \). Once again, by time-sharing, we take \( \text{co}(\mathcal{R}_2^{2\text{nb}}) \) to achieve \( \mathcal{R}_2^{2\text{na}} \).

**Theorem 5:** \( \text{co}(\mathcal{R}_2^{2\text{nb}}) = \mathcal{R}_2^{2\text{nb}} \).

**Proof of Theorem 5** The regions, \( \mathcal{R}_2^{2\text{nb}} \) and \( \mathcal{R}_1^{2\text{nb}} \), can be convexified by introducing an independent time-sharing variable \( Q \). Alternatively, by defining \( U' = (Q, U) \), we naturally include the time-sharing variable and show that \( \mathcal{R}_0^{2\text{nb}} \) and \( \mathcal{R}_1^{2\text{nb}} \) are convex.

Note that all the points in \( \mathcal{R}_2^{2\text{nb}} \setminus \mathcal{R}_0^{2\text{nb}} \) are limit points of \( \mathcal{R}_0^{2\text{nb}} \). This is because \( \mathcal{R}_0^{2\text{nb}} \) covers all points in \( \mathcal{R}_2^{2\text{nb}} \), except for the limit points along the axis of \( R_1 \) and \( R_2 \). As a result, assuming that we pick point \( a \) from \( \mathcal{R}_2^{2\text{nb}} \) and limit point \( b \) from \( \mathcal{R}_1^{2\text{nb}} \), all internal points along the line joining \( a \) and \( b \) will always lie within \( \mathcal{R}_2^{2\text{nb}} \). Combined with the fact that \( \mathcal{R}_0^{2\text{nb}} \) and \( \mathcal{R}_1^{2\text{nb}} \) are convex, this argument allows us to prove that \( \mathcal{R}_1^{2\text{nb}} \cup \mathcal{R}_2^{2\text{nb}} \) is in fact convex as well.

Since \( \mathcal{R}_1^{2\text{nb}} \cup \mathcal{R}_2^{2\text{nb}} \) is convex, the convex hull, \( \text{co}(\mathcal{R}_2^{2\text{nb}}) = \mathcal{R}_1^{2\text{nb}} \).

**Corollary 3:** \( \mathcal{R}_2^{2\text{nb}} = \mathcal{R}_2^{2\text{na}} = \mathcal{R}_2^{2o} \).

**Proof of Corollary 3** Corollary 3 is a result of combining the relationships \( \text{co}(\mathcal{R}_2^{2\text{nb}}) = \mathcal{R}_2^{2\text{nb}} \) in Theorem 5 and \( \text{co}(\mathcal{R}_2^{2\text{nb}} \cup \mathcal{R}_1^{2\text{nb}}) = \mathcal{R}_2^{2\text{na}} \) in Theorem 4 and \( \mathcal{R}_2^{2\text{na}} = \mathcal{R}_2^{2o} = \mathcal{R}_2^{2o} \) in Corollary 2.

### V. DISCUSSIONS

For the case of the two-receiver broadcast channel with complementary receiver side information where there is a passive eavesdropper, our proposed scheme simplification shows that the usage of Carleial-Hellman’s secrecy coding [22] for the common satellite codewords achieves the same result as Wyner’s secrecy coding [2]. The usage of Carleial-Hellman’s secrecy coding [22] drops the usage of one-time pad signal for randomness, effectively reducing the number of message splits and simplifying the computation process. Not least, no additional random component is needed to ensure individual secrecy of message segments in the common satellite codeword since each of the message segments is capable of fulfilling this requirement.

### VI. CONCLUSION

In this work, we performed simplifications on the coding scheme proposed by Chen et al. for the two-receiver discrete memoryless broadcast channel with complementary receiver side information and with a passive eavesdropper [17]. The essence of these simplifications lies within the usage of Carleial-Hellman’s secrecy coding which removes the need of a one-time pad signal and additional randomness \( r \) to ensure individual secrecy of messages. This coding strategy helps reduce
the number of message splits and the number of random components.

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