Graphing Calculator Programs for Instructional Data Diagnostics and Statistical Inference

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Abstract

A number of programs written for the TI-83 Plus calculator are demonstrated in this article to illustrate this graphing calculator's surprisingly advanced statistical capabilities. Examples include residual plots for analysis of variance, pairwise comparison in one-factor experimental design, statistical inference for simple linear regression and confidence intervals for contrasts used in experimental design. These and a number of other programs are available for download. Advances in graphing calculator statistical programs, such as those described in this article, allow instructors and students in introductory applied graduate level statistics courses to perform sophisticated statistical data diagnostic and inference procedures during class time in an ordinary class room.

1. Introduction

The one thing that has always bothered me about using computer-based statistical software packages is the disconnect between the classroom lecture and the application of the computer software to the material being taught. Based on my experiences, unless the instructor and student are both in a computer lab at the same time, any discussion involving the use of computer software is awkward at best. I believe hand held calculators to be superior in a classroom setting to computer-based software in giving students a better immediate understanding of how to analyze data.

I was able to teach an introductory graduate statistics course for biologists, based on the text by Rao (1998), relying almost exclusively on graphing calculators without having to resort to the use of computer-based statistical software packages such as SAS, SPSS or S-Plus or Minitab, or spreadsheets such as Excel. The calculator programs given here are essentially a step above the calculator programs which support introductory undergraduate statistics courses, such as the *TI-83 Graphing Calculator Manual* (Neal 2000) used to support *The Basic Practice of Statistics* (Moore 1999) or the *Technology Guide* used to support Brase and Brase's *Understandable Statistics* (see Brase and Brase 1999a; 1999b).

For the first ten years of my sixteen year teaching career, I used a variety of statistical software packages such as SAS and Minitab in teaching introductory statistics courses. In fact, the initial use of the TI-83 was driven essentially by necessity - Purdue University North Central did not have easy access to SAS until recently. The TI-83 was to fill in the gap until SAS arrived, but because I found the calculator so advanced, it has more or less replaced SAS, even though SAS is now readily available at Purdue University North Central. I do now use SAS, but only in a secondary role to the TI-83 calculator in the introductory graduate statistics course I teach.

This paper is intended for college faculty who teach introductory graduate courses in statistics, particularly for those who teach students in non-mathematical, non-statistical majors. This paper may also be of interest to college faculty and
Advanced Placement high school teachers who teach introductory undergraduate courses in statistics. In time, the graduate level statistical topics covered in this paper may well be taught at an introductory undergraduate level, in part because of the powerful instructional presentation features of the TI-83 calculator and other similar technologies.

The last ten years have seen a steady improvement in the complexity in the kinds of statistical problems that scientific and graphing calculators have been able to handle. Early versions of graphing calculators were really no more than glorified scientific calculators able to calculate various summary statistics such as the average, median, standard deviation, and percentiles with some simple graphing capabilities. More recent versions are now able to not only perform various statistical inference procedures such as z-tests, 2-proportion z-tests and analysis of variance (ANOVA) procedures, but also can produce a variety of statistical plots such as scatter plots, histograms and box plots (see Morgan 1997, for example).

Probably the most important change in graphing calculators has been their greatly expanded programming capabilities (see Kelly 1998, for example). “Flash-memory” has been introduced to graphing calculators, allowing these calculators to be electronically improved with maintenance and feature upgrades much like the operating system of a desk top computer can be upgraded. Various cables allow programs to be easily transferred from one calculator to another and to and from computers. This latter improvement now allows distance learning students to copy graphing calculator programs from the Internet, and so allow them to better participate in statistics courses that incorporate such programs. Graphing calculator programs stored on the Internet may also provide a viable alternative for those distance education students who do not have ready access to sophisticated statistical computer packages.

2. Graphing Calculator Programs

Four example TI-83 Plus programs and their outputs are demonstrated below. The programs include:

1. residual plots for analysis of variance
2. pairwise comparison in one-factor experimental design
3. statistical inference for simple linear regression
4. confidence intervals for contrasts used in one-factor experimental design

These programs are written for a graduate level introductory course in statistics. Although they could be used to support topics taught at an undergraduate level, they are intended to support the same topics presented in greater depth. These and other programs are alphabetically listed later on in this paper and are available for download at:

ww2.amstat.org/publications/jse/kuhn/ti83.html

2.1 Residual Plots For Analysis of Variance

The program QQPLTANV creates a q-q plot to check for normality, and the program EVPPLLOT creates an e versus p plot to check for constant variance where both assumptions would be necessary to proceed with an analysis of variance. These programs could be used after a discussion on how to conduct an analysis of variance, to clarify when it would be possible to conduct such a statistical procedure.

In this example, the QQPLTANV and EVPPLLOT programs are used on the oxygen consumption rate of mice for various humidity levels data. The data are provided in Table 1.

Table 1. Oxygen Consumption Of Mice For Various Humidity Levels.

| Humidity Level | 5% | 10% | 15% | 20% | 25% |
|----------------|----|-----|-----|-----|-----|
| 5%             | 7  | 12  | 14  | 19  | 7   |
| 7              | 17 | 18  | 25  | 10  |

2}
Type the five sets of six data points, corresponding to each of the five humidity levels, into the first five lists of the calculator, \( L_1, \ldots, L_5 \).

1. Press the program key, PRGM, choose the QQPLTANV program, followed by pressing ENTER, then press 5, where "5" is the number of treatments, followed by ENTER.

2. In a similar way, enter PRGM EVPPLT ENTER 5 ENTER.

The two programs produce the two residual plots given in Figure 1 below: the \( q-q \) plot (on the left, which indicates normality) and \( e \) versus \( p \) plot (which indicates constant variance).

![Figure 1. Residual Plots for Analysis of Variance.](image)

### 2.2 Pairwise Comparison In One-Factor Experimental Design

The program PAIRWISE performs four types of least significance difference (LSD) for pairwise comparisons in means tests: Fisher (F), Bonferroni (B), Scheffe (S) and Tukey (T). This program would be used if an overall analysis of variance test was found to be significant in order to clarify how the various means differed from one another.

For the data provided in Table 1, the program PAIRWISE performs both Fisher's LSD test and Bonferroni's LSD test to decide if there are any significant pairwise differences in the mean oxygen rate of consumption for the \( i \)-th humidity and the mean oxygen rate of consumption for the \( j \)-th humidity, for \( i, j = 1, \ldots, 5 \), at a level of significance of \( \alpha = 0.05 \).

The inputs required for the Fisher and Bonferroni LSD pairwise tests using the PAIRWISE program appear on the calculator screen displayed in Figure 2.
There are $T = 5$ treatments, and the value of the studentized range is $Q = 4.17$ (although in this case it is not necessary to know this value, since Tukey's test is not chosen), the level of significance is $\alpha = 0.05$ and \{1,1,0,0\} for \{F,B,S,T\}? means the first two (Fisher and Bonferroni) of the four possible least significant tests are chosen. The code "1" means "yes, do the test" and "0" means "do not do the test."

The results for the Fisher LSD pairwise tests are displayed in two different but complementary ways in Figure 3. The two different types of displays show all pairwise treatment means are significantly different from one another, except for the (5%, 25%) treatment pair and the (10%, 15%) treatment pair.

In the first two panels of Figure 3, $I$ is the $i$-th ordered mean oxygen consumption observation, $J$ is the $j$-th ordered observation, $MI - MJ$ is the difference in the means, LSD is Fisher's least significant difference and $S?$ represents whether the observed difference in means is significant or not significant (that is, larger than the Fisher LSD or not).

In the third panel of Figure 3 UNOR is the original unordered set of treatment means, (5%, 10%, 15%, 20%, 25%), and ORD is the ordered set of treatment means, (5%, 25%, 10%, 15%, 20%). In particular, notice that the 25% humidity level mean, 11.3, is the second smallest mean, after the 5% mean, 9.8. In this last section, a column (or line or string) of "I"s indicate which treatments means are not significantly different from one another. For example, a column of "I"s ties the (9.8, 11.3) pair, or, equivalently, the (5%, 25%) pair, together and so this treatment pair mean difference is not significant.

The output for the Bonferroni LSD pairwise tests is displayed in Figure 4 shows that four treatment pair mean differences are not significant: (9.8, 11.3), (11.3, 15.5), (15.5, 17.5) and (17.5, 22). Notice that there are more nonsignificant pairwise means in the conservative Bonferroni case than in the previous Fisher case because the LSD cutoff level for significance in the Bonferroni case, 4.7, is larger than the LSD cutoff level for significance in the Fisher case, 3.1.
2.3 Statistical Inference in Simple Linear Regression

The program REGINF performs statistical inference for the slope, intercept, and expected response in simple linear regression analyses. This program could be used after a discussion on how to calculate a simple linear regression line.

In this example, the program REGINF performs three statistical inference procedures for the expected reading ability response, $\mu(x)$, all for the reading ability versus level of illumination data given below. The program tests if the expected reading ability is $\mu(x) = 75$ at a level of illumination of $x = 1.5$, it calculates a confidence interval for $\mu(x)$, and it calculates a prediction interval with $m = 1$ future value.

Table 2. Reading Ability Versus Level Of Illumination.

| Illumination Level (x) | Reading Ability (y) |
|------------------------|---------------------|
| 1                      | 70                  |
| 2                      | 70                  |
| 3                      | 75                  |
| 4                      | 88                  |
| 5                      | 91                  |
| 6                      | 94                  |
| 7                      | 100                 |
| 8                      | 92                  |
| 9                      | 90                  |
| 10                     | 85                  |

Enter the data for illumination levels and reading ability levels into $L_1$ and $L_2$, respectively, then execute the following instructions on the calculator:

1. Enter PRGM REGINF ENTER.

2. Type \{0,0,1,1\} for \{B0,B1,MN,PI\}?$, which directs the calculator not perform statistical inference for either the intercept, $\beta_0(B0)$, or slope, $\beta_1(B1)$, but do perform statistical inference (test and determine a confidence interval) for the mean $\mu(x)(MN)$ and do calculate a prediction interval (PI) for the mean $\mu(x)$.

3. Type 75, the null expected reading ability response hypothesis, in response to "$\mu(x)=$."
4. Press 1, to calculate the prediction interval for one (as opposed to two or more) future sample value(s), in response to "M = ."

5. Type 1.5, to test the expected reading ability response at an illumination level of 1.5, in response to "X = ."

6. Type the level of significance, 0.05, for "\( \alpha \) = ."

The output is given in Figure 5. The test fails to reject the null mean expected reading ability response at \( \mu(x) = 75 \) because of a large \( p \)-value of 0.85. The confidence interval and prediction intervals are (66.04, 85.61) and (55.3, 96.36), respectively.

![Figure 5. Inference for \( \mu(x) \).](image)

The value ESTM is \( \hat{\mu}(75) \), the estimated expected mean reading ability using a simple linear regression at \( x = 1.5 \), SE is the standard error (in both a confidence interval and a prediction interval situation, respectively), TCRT is the critical value of the \( t \)-distribution at 2.5% (half of 5%, since it is a two-tailed test), TOBS is the observed value of the test statistic, PVAL is the \( p \)-value, CI is the confidence interval, and PI is the prediction interval.

### 2.4 Confidence Intervals For Contrasts Used In One-Factor Experimental Design

Questions from statistics texts often require the student to work not from a data set but from a summary of data. The program CSTCISTA determines confidence intervals for contrasts used in one-factor experimental design when the data have been summarized (another program, CSTCI, determines the confidence intervals from the dataset). This program would probably be used after a discussion on analysis of variance to clarify in what (not necessarily pairwise) ways the various means differed from one another.

Given \( t = 5 \) treatments, with means \( \{2.5, 3.1, 4.5, 1.2, 6.5\} \), the sample sizes are \( \{5, 5, 5, 5, 5\} \), and standard deviation \( s = 0.45 \), this program calculates Fisher, Bonferroni, Scheffe, and Tukey confidence intervals for the contrast
\[
\theta = \left( \mu_1 + \mu_2 + \mu_3 + \mu_4 \right) / 4 - \mu_5
\]
The interval is assumed to be one of \( k = 4 \) confidence intervals, with studentized range statistic \( q = 4.17 \), and \( \alpha = 0.05 \).

The program returns the four 95% confidence intervals shown in Figure 6, where the FI represents Fisher, BF stands for Bonferroni, SH is Scheffe, and TK is Tukey.
3. Alphabetical Listing of TI-83 Plus Programs

An alphabetical list of all of the TI-83 Plus programs I created along with a brief description of each program is given in Table 3.

| Program                      | Description                               |
|------------------------------|-------------------------------------------|
| CCV                          | critical contrast value                   |
| CCVDISP (CCV)                | critical contrast value display           |
| CORR                         | Fisher correlation test/CI, data          |
| CORRSTAT                     | Fisher correlation test/CI, stats         |
| CORRZERO                     | exact correlation test/CI, data           |
| CORRZSTA                     | exact correlation test/CI, stats          |
| CSTCI                        | contrast CI, from data                    |
| CSTCISTA                     | contrast CI, from stats                   |
| CSTDISP (CSTCI, CSTCISTA)    | contrast CI display                       |
| EVPLOT                       | \( e \) versus \( p \) plot, ANOVA        |
| EVXPLT                       | \( e \) versus \( x \) plot, linear regression |
| EVYPLT                       | \( e \) versus \( y \) plot, linear regression |
| EVYPTQRG                     | \( e \) versus \( y \) plot, quadratic regression |
| HYPcdf                       | hypergeometric cdf                        |
| HYPPdf                       | hypergeometric pdf                        |
| INVC2                        | \( \chi^2 \) percentile                   |
| INVF                         | \( F \) percentile                        |
| INVT                         | \( t \) percentile                        |
| PAIRCMPR (PAIRWISE, PAIRSTAT)| pairwise contrast comparison              |
| PAIRPICT (PAIRWISE, PAIRSTAT)| pairwise contrast by stats                |
| PAIRSTAT                     | pairwise contrast by data                 |
| PAIRWISE                     | prediction interval by data               |
| PI                           | prediction interval display               |
| PIDISP (PI, PISTAT)          | prediction interval by stats              |
| PISTAT                       | q-q plot for ANOVA                        |
| QQPLTANV                     | q-q plot for quadratic regression         |
| QQPLTQRG                     | q-q plot for linear regression            |
| QQPLTREG                     | regression test \( \hat{\beta} \) by ANOVA|
| REGANOVA                     | regression inference calculation, \( \hat{\beta}_0, \hat{\beta}_a, \mu \) |
| REGCALC (REGINF)             | regression inference display, \( \hat{\beta}_a, \hat{\beta}_s, \mu \) |
| REGDISP (REGINF)             | regression inference, \( \hat{\beta}_a, \hat{\beta}_s, \mu \) |
Notice that "CCVDISP (CCV)," for example, means that the program CCVDISP is a **subroutine** of the program CCV. It is not possible to run the subroutine CCVDISP **directly**; subroutine CCVDISP is accessed indirectly when running program CCV.

### 4. Discussion

The main disadvantages of graphing calculators are that they are small and can be slow. Small screens restrict the amount of information that can be displayed at one time. Small keyboards mean programming or accessing information can be slow and difficult. Rather than the microseconds it might take a typical desk-top computer, it might take the calculator seconds, or tens of seconds (but not minutes) to complete some of the programs demonstrated above. Flash memory and cables that allow calculators to link to microcomputers help to alleviate some of these problems.

My experience indicates there are a number of advantages of using graphing calculators instead of computer-based statistical software:

1. Students get hands-on experience manipulating datasets along with the instructor, inside an ordinary class room setting and outside a computer lab.
2. Realistic datasets can be explored and analyzed in real-time, even in a course testing situation.
3. Instructors and teaching assistants can help students with homework assignments with realistic datasets away from a computer monitor.
4. The programming capabilities of the TI-83 Plus are surprisingly advanced. It is possible to write programs for this calculator that are even more advanced than those given in this paper.

There might be some concern that students who have had previous training on the TI-83 calculator have an advantage over those who have not had such previous training. The "official" calculator used at Purdue University North Central is the TI-83 calculator. That is, although students can use any calculator they wish, only the TI-83 calculator is used and supported by all instructors and tutors in the mathematics, statistics, and physics section (and to a lesser extent, in all of the other sections on campus) at Purdue University North Central. This has been a policy for a few years now. By the time students get to this introductory graduate biology course in statistics, they almost certainly will be familiar with the TI-83 calculator. Even those who somehow start this course without having used the TI-83 in a previous course tend to pick up on the calculator quickly because they can seek help from almost anyone else in the class or from the tutors who are all skilled in the use of the calculator. More than this, it seems to me that most students have had some experience with some type of Texas Instrument calculator in their high school careers.

I have not yet carried out a statistical study, but I am interested in knowing by how much a student's understanding (as measured by homework, quiz, and final exam scores) of the concepts in the course material is influenced by using either the calculator or using a statistical software package.

There is not as much of a difference between the calculator and the statistical software as might at first be imagined. For instance, consider the residual $e$ versus $p$ plot example above. On the calculator, as explained, the data are first typed into the five lists of the calculator and then the key strokes PRGM (ENTER), 5 (for the number of treatments) and then ENTER to display the plot. For the computer, using, say, the SAS computer package, the following code could be used to obtain the same graph:

```plaintext
DATA OXY_COMP;
  INPUT HUM $ OXY;
DATALINES;
  5 7
  5 7
```
and rest of data ... 
;
PROC GLM DATA=OXY_COMP;
CLASS HUM;
MODEL OXY = HUM;
PLOT OUT=PLT R=RESID;
RUN;
PROC PLOT DATA=PLT;
PLOT RESID*HUM / VREF=0;
RUN;

In this particular case, the calculator provides the same functionality as the computer software and is arguably easier to use.

The calculator does have a number of built-in programs for statistical (and related probability) analyses, grouped mostly under three menus: STAT PLOT, STAT, and 2nd DISTR. The STAT PLOT key gives six plots, including a scatter plot, line plot, histogram, box plots, and a q-q plot for checking normality. The STAT key is the most powerful key. It provides three menus: EDIT for entering, sorting, and editing data; a CALC menu which provides a variety of one- and two-variable summary statistics, as well as various regressions (linear, quadratic, and cubic, for example); and a TESTS menu which allows access to a variety of simple one and two sample tests, a simple linear regression test, and an ANOVA test. I do not believe any of the programs I have written duplicate any program functions already provided on the calculator. My programs either complement what is already built in to the calculator or provide more advanced statistical tests or displays.

Software for downloading and installing TI-83 programs through the Internet and directions for obtaining the special computer-to-calculator cable, can be obtained from the following Web location: education.ti.com.

Once instructors install a copy of the programs onto their calculators they can give their students the option of doing the same or they can simply pass these programs from their calculator to the students' calculator via the cable link that is provided with every new calculator.

5. Summary

Programs written for the TI-83 Plus calculator are described in this article to illustrate this graphing calculator's advanced statistical capabilities. I hope this article encourages other educators to not only recognize the value of using graphing calculators in applied graduate level statistics courses but also to create programs to continue to improve the statistical capabilities of these calculators.

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