On the origin and the detection of characteristic axion wiggles in photon spectra

M. Kachelrieß and J. Tjemsland

Institutt for fysikk, NTNU, Trondheim, Norway

Abstract. Photons propagating in an external magnetic field may oscillate into axions or axion-like particles (ALPs). Such oscillations will lead to characteristic features in the energy spectrum of high-energy photons from astrophysical sources that can be used to probe the existence of ALPs. In this work, we revisit the signatures of these oscillations and stress the importance of a proper treatment of turbulent magnetic fields. We implement axions into ELMAG, a standard tool for modelling in a Monte Carlo framework the propagation of gamma-rays in the Universe, complementing thereby the usual description of photon-ALP oscillations with a Monte Carlo treatment of high-energy photon propagation and interactions. We also propose an alternative method of detecting axions through the discrete power spectrum using as observable the energy dependence of wiggles in the photon spectra.

Keywords: axion, axion-like particles, photon-ALP oscillation, turbulent magnetic fields
1 Introduction

The Standard Model of particle physics has attained immense success over the years. Yet, it has several theoretical shortcomings such as the strong CP problem. Moreover, there are some experimental indications for its incompleteness, with the absence of a suitable dark matter candidate as the most striking one. As a solution to the strong CP problem, Peccei and Quinn [1, 2] postulated in 1977 the existence of an additional U(1) symmetry that is spontaneously broken, thereby giving rise to a Nambu-Goldstone boson—the axion \( a \). The two-gluon-axion vertex introduced to solve the strong CP problem induces a small axion mass through pion mixing, \( m_a f_a \approx m_\pi f_\pi \), degrading the axion to a pseudo-Goldstone boson [3, 4]. Intriguingly, this promotes the axion into a suitable cold dark matter candidate despite its small mass if it is produced through, e.g., the so-called misalignment mechanism [5–8]. Other light pseudo-scalars bosons which have the same characteristic two-photon coupling as the axion, \( \mathcal{L} = -\frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma} E \cdot B \), are collectively known as axion-like particles (ALPs). In the case of the QCD axion, the two-photon vertex is inherited from the two-gluon vertex, thus fixing the relation of the mass and decay constant as \( |g_{a\gamma}| \text{GeV} \approx 10^{-16} m_a / \mu \text{eV} \) up to a \( \mathcal{O}(1) \) constant [9] (see Ref. [10] for a recent review on axion models). While ALPs do not solve the strong CP problem, they are theoretically well motivated as they arise naturally in string theories and other extensions of the Standard Model [11, 12].

Most axion and ALP searches are based on their two-photon coupling, see e.g. Refs. [13, 14] for recent reviews. Such a coupling leads to a conversion between photons and axions in the presence of an external magnetic field. This phenomenon has been utilised in, e.g., the ADMX haloscope [15] and the CAST helioscope [16] experiments.
which aim to reconvert respectively DM and solar axions in the fields of strong magnets. The most extensive limits on the coupling at sub-eV masses, $g_{a\gamma} < 6.6 \times 10^{-11}$, are currently set by CAST ($m_a \lesssim $ eV) [16] and by studying the lifetime of stars in the horizontal branch ($m_a \lesssim $ keV) [17]. The planned “shine light through a wall” experiment ALPS-II [18] and solar axion experiment IAXO [19] are expected to improve these limits significantly. At present, however, the possible mass of the QCD axion is practically unconstrained at sub-eV masses. An exception is the ADMX haloscope which excludes some parts of the QCD axion parameter space around few $\mu$eV under the condition that axions account for the observed dark matter. The planned ABRACADABRA [20] experiment is expected to improve limits on axionic dark matter immensely, while the IAXO experiments will probe QCD axions in the 1 meV–1 eV mass range [19].

The leading limits on $g_{a\gamma}$ for ALPs with $m_a \lesssim 10^{-6}$ eV, $g_{a\gamma} \lesssim (10^{-13}–10^{-11})$ GeV$^{-1}$, are currently set by astrophysical observations which utilise that the two-photon coupling leads to a number of interesting changes in gamma-ray spectra. Most notably, photon-ALP oscillations may effectively increase the mean-free path of photons in the extragalactic background light (EBL), since ALPs travel practically without any interactions [21]. Additionally, photon-axion oscillations may lead to characteristic features in the spectra of astrophysical sources of high-energy photons. The most stringent limits on $g_{a\gamma}$ for sub-$\mu$eV masses rely on the non-observation of such spectral signatures and are derived from, e.g., gamma-ray observations by HESS [22], Fermi-LAT [23], of SN1987A [24], and X-ray observations of Betelgeuse [25], the active galactive nuclei NGC1275 [26], the cluster-hosted quasar H1821+643 [27] and super star clusters [28]. The former two focus on “irregularities” induced by photon-axion oscillations, while the latter three focus on the production of axions through the Primakoff effect. A significant improvement is expected from the upcoming Cherenkov Telescope Array (CTA) [29]. The interpretation of such results depend, however, significantly on the treatment of the magnetic fields in and around the source as well as in the extragalactic space [30–33]. This applies in particular for the turbulent component of the magnetic field, for which often only oversimplified models are used. Moreover, one should note that large scale spectral features can also be produced by a number of astrophysical effects, including e.g. an electron-beam driven pair cascade [34] or even cascades from primary gamma-rays or nuclei [35]. It is therefore important to identify the features characteristic for axion-photon oscillations.

In this work, we study the effect of photon-axion oscillations in a Monte-Carlo framework based on the ELMAG$^2$ code [36, 37], which is a Monte Carlo program made to simulate electromagnetic cascades initiated by high-energy photons interacting with the extragalactic background light. The use of ELMAG allows us to include properly the interplay of cascading and oscillations, and in addition ELMAG provides tools to generate turbulent magnetic fields. We consider for concreteness only a turbulent extragalactic magnetic field, but ELMAG can use any magnetic field as input. We discuss the characteristic signatures expected in the photon spectra from distant gamma-ray

\footnote{We will from now on refer to both axions and ALPs simply as ‘axions’.

$^2$The code used in this work will be made publicly available in a future release of ELMAG.}
sources, and using domain-like and Gaussian turbulent fields as examples, show that the predicted signatures depend rather strongly on the chosen magnetic field model. As a result, we argue that while the application of domain-like magnetic fields may be tenable for some quantitative discussions, it should be abandoned in qualitative studies. Finally, we propose the use of the discrete power spectrum to detect photon-axion oscillations in upcoming gamma-ray experiments such as CTA. This method directly uses the expected characteristic signatures as observable, namely the energy-dependent wiggles in the photon spectra induced by photon-axion oscillations. We show that these signatures can in principle be used to infer information about the magnetic field environment. While we focus oscillatory features in photon spectra, which we call axion wiggles, we comment also on the effect of photon-axion oscillations on the opacity of the Universe. In particular, we show that the apparent size of this effect depends strongly on the approximation used for the turbulent magnetic field.

This paper is structured as follows: In section 2, we present the numerical treatment of photon-axion oscillations and our Monte Carlo implementation. Next, we discuss in section 3 the treatment of turbulent magnetic fields. Thereafter, in section 4, we discuss the characteristic oscillatory features produced by photon-axion oscillations in the photon spectra and their dependence on the modelling of the magnetic field, followed by examples in section 5. In section 6, we present the suggestion to use the energy dependence of the oscillatory features as observable in the detection of axions. Finally, we comment on the importance of a proper treatment of the magnetic fields when considering the opacity of the Universe, before we conclude in section 8.

2 Numerical implementation of photon-axion oscillations

The physics underlying photon-axion oscillations is discussed clearly in the classic paper by Raffelt and Stodolsky [38]. Here, we only highlight the main features of the Lagrangian

\[
\mathcal{L} = \mathcal{L}_{\text{aa}} + \mathcal{L}_{a\gamma} + \mathcal{L}_{\gamma\gamma} = \frac{1}{2} \partial^\mu a a_\mu - \frac{1}{2} m_a^2 a^2 - \frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\alpha^2}{90 m_e^4} \left[ (F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right].
\]

(2.1)

The first two terms describe the axion \(a\) as a free scalar field with mass \(m_a\), while the third term includes the interaction of axions with photons, which in the presence of an external magnetic field will result in axion-photon oscillations. The last term is the Euler-Heisenberg effective Lagrangian that takes into account vacuum polarisation effects below the creation threshold of electron-positron pairs. In particular, this term leads to a refractive index for photons in an external electromagnetic field which influences the propagation and oscillation of axions and photons. Following the

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\(^3\)We use rationalised natural units with \(\hbar = c = 1\) and \(\alpha = e^2/4\pi \simeq 1/137\). Then the critical magnetic field is given by \(B_{cr} = m_e^2/e \simeq 4.414 \times 10^{13}\) G.
usual convention of the wave vector \( \mathbf{k} \) pointing in the direction of the photon electric field, the refractive indices of the photon in the longitudinal (\( || \)) and transverse (\( \perp \)) directions are given by

\[
n_{\perp} = 1 + \frac{4}{7} \xi \quad \text{and} \quad n_{||} = 1 + \frac{7}{2} \xi
\]  

(2.2)

with \( \xi \equiv (\alpha/45\pi)(B_{\perp}/B_{\text{cr}})^2 \).

The Lagrangian (2.1) leads after linearisation to the equation of motion

\[
(E + \mathcal{M} - i \partial_z) \phi(z) = 0,
\]  

(2.3)

where we denote the energy of photons and axions by \( E \), have chosen the \( z \)-axis as propagation direction and have introduced the wave function

\[
\phi(z) = \begin{pmatrix} A_{\perp} \\ A_{||} \\ a \end{pmatrix}.
\]  

(2.4)

The mixing matrix is given by

\[
\mathcal{M} = \begin{pmatrix} \Delta_{\perp} & 0 & 0 \\ 0 & \Delta_{||} & \Delta_{a||} \\ 0 & \Delta_{a||} & \Delta_{a} \end{pmatrix},
\]  

(2.5)

where \( \Delta_{||,\perp} = (n_{||,\perp} - 1)E \) and \( \Delta_{a} = -m_{a}^2/(2E) \). The two polarisation states of the photon are given as linear polarisation states perpendicular and parallel to the transverse magnetic field at a given position. The off-diagonal terms lead to photon-axion mixing in the presence of an external magnetic field,

\[
\Delta_{a||} = \frac{g_{a\gamma}}{2} B_{\perp}.
\]  

(2.6)

In general, the diagonal terms \( \Delta_{||,\perp} \) in Eq. (2.3) should include the total refractive index of the photon. Other contributions describing the photon dispersive effects of the medium and the EBL, as well as the chosen numerical values, will be discussed in section 4.

It is useful to consider the propagation through a homogeneous magnetic field to obtain an understanding of the problem. In this case, Eq. (2.3) simplifies to

\[
\left[ E - i \partial_z + \begin{pmatrix} \Delta_{||} & \Delta_{a||} \\ \Delta_{a||} & \Delta_{a} \end{pmatrix} \right] \begin{pmatrix} A_{||} \\ a \end{pmatrix} = 0.
\]  

(2.7)

The photon conversion probability then becomes

\[
P_s(\gamma \to a) = |\langle A_{||}(0) | a(s) \rangle|^2 = \left( \Delta_{a||} s \right)^2 \frac{\sin^2(\Delta_{\text{osc}} s/2)}{(\Delta_{\text{osc}} s/2)^2}
\]  

(2.8)
\[
\Delta_{\text{osc}}^2 = (\Delta_\parallel - \Delta_a)^2 + 4\Delta_{a\parallel}^2.
\]  
(2.9)

Similarly, the oscillation length in any sufficiently smooth environment is given by

\[ L_{\text{osc}} \approx 2\pi/\Delta_{\text{osc}}. \]

Thus, one can see that the oscillation length, the correlation length \( L_c \) of the magnetic field and the mixing strength \( 2\pi/\Delta_{a\parallel} \) are the main parameters determining the effects of photon-axion oscillations. For example, when \( \Delta_{\text{osc}} \approx 2\Delta_{a\parallel} \), we enter the strong mixing regime where Eq. (2.8) gives \( P_s(\gamma \to a) = \sin^2(\Delta_{a\parallel}s) \).

We describe now how the photon-axion equation of motion (2.3) is implemented into ELMAG \([36, 37]\). For convenience, we will refer to the superposition of a photon and an axion as a ‘phaxion’. The probability that the phaxion interacts with the EBL at position \( s \) is \( dP = P_s(\gamma \to \gamma)\sigma_{\text{pair}}(s)\,ds \) with \( P_s(\gamma \to \gamma) = 1 - P_s(\gamma \to a) \). This is equivalent to checking first whether a photon would interact at that position and next to account for the probability that a phaxion would be measured as a photon. This motivates the following numerical scheme which takes into account the absorption of photons:

1. Start with a pure photon with an energy drawn from the distribution for the injection energy. For an unpolarised gamma-ray source, choose randomly a linear polarisation state.
2. Draw the interaction length of a phaxion, \( \lambda \), at the current position in accordance to the mean free path length a photon.
3. Propagate the phaxion from \( s \) to \( s + \lambda \) according to the phaxion equation of motion.
4. If \( P_{\gamma\gamma} > r \) for a random number \( r \) chosen from a uniform distribution \( r \in [0, 1] \), the phaxion wave function collapses into a photon and the photon undergoes pair production in interaction with the EBL. If not, go to point 2.
5. Treat the electromagnetic cascade that arises, and for each photon go to point 2.

The Monte Carlo treatment of the photon-axion oscillations implemented in this work has several advantages compared to conventional (semi-)analytical approaches (see e.g. Ref. [39] and references therein) and the procedure in Ref. [40], at the cost of being computationally more demanding. First, the implementation of realistic magnetic fields and additional effects like an inhomogeneous electron density is trivial. Second, photon absorption can be considered on an event-by-event basis and the resulting electromagnetic cascade can be accounted for. Third, polarisation effects are by default included. Finally, this method allows to include the effect of photon-axion oscillations into other studies of electromagnetic cascades. For example, a potential increase in the size of gamma-ray halos around astrophysical sources because of the increased mean-free path of photon could be studied using ELMAG in a straight-forward way, see e.g. Ref. [41] for a recent review on the subject.
3 Turbulent magnetic fields

We describe in this work turbulent magnetic fields as isotropic, divergence-free Gaussian random fields with zero mean, rms value $B_{\text{rms}}$ and zero helicity. The algorithm implemented in ELMAG for the generation of such fields is based on the method suggested in Refs. [42, 43] and described in Ref. [37]. In this approach, the turbulent magnetic field is modelled as a superposition of $n$ left- and right-circular polarised Fourier modes. The spectrum of the modes is chosen as a power-law

$$B_j = B_{\text{min}}(k_j/k_{\text{min}})^{-\gamma/2}$$

between $k_{\text{min}}$ and $k_{\text{max}}$, corresponding to the largest $L_{\text{max}} = 2\pi/k_{\text{min}}$ and smallest scales $L_{\text{min}} = 2\pi/k_{\text{max}}$, respectively. The quantity $B_{\text{min}}$ is fixed by normalising the total field strength to $B_{\text{rms}}$. The coherence length is in turn connected to $L_{\text{max}}$ and $L_{\text{min}}$ as

$$L_c = \frac{L_{\text{max}}}{2} \frac{\gamma - 1}{\gamma} \frac{1 - (L_{\text{min}}/L_{\text{max}})^\gamma}{1 - (L_{\text{min}}/L_{\text{max}})^\gamma} \approx \frac{L_{\text{max}}}{2} \frac{\gamma - 1}{\gamma},$$

the last equality being valid for $L_{\text{min}} \ll L_{\text{max}}$. For definiteness, we will consider a time-independent $B_{\text{rms}}$ and a Kolmogorov spectrum with $\gamma = 5/3$ for which $L_c \simeq L_{\text{max}}/5$.

In the literature, a so-called domain-like magnetic field has often been used to describe photon-axion oscillations in turbulent astrophysical magnetic fields, see e.g. [26, 27, 39, 40, 44–50]. In this approach, the magnetic field along the line of sight is divided into patches with size equal to the coherence length $L_c$ of the field. The field in each patch is assumed to be homogeneous, while its direction is chosen randomly. Such an approximation certainly breaks down when the oscillation length becomes smaller than the coherence length, $L_{\text{osc}} < \sim L_c$: In this case, oscillations probe the magnetic field structure on scales smaller than the domain size, which are neglected in this simple model. Thus, a more realistic description of the turbulent magnetic field, including fluctuations on various scales, should be used.

In the case of a large oscillation length, $L_{\text{osc}} \gtrsim L_c$, on the other hand, one may expect the approximation of domains to be valid since the power of the turbulent magnetic field is contained mainly in its large-scale modes for $\gamma > 1$. However, also in this limit the large-scale fluctuations of the turbulent field will lead to important differences in the photon conversion relative to the domain-like case. To visualise these effects, the cumulative distribution of the magnetic field, i.e. the fraction $f(>B)$ of volume filled with a magnetic field stronger than $B$, is compared for the two models in Fig. 1. Since the fluctuations of the turbulent magnetic field are spread over a large range of scales, the variance of the perpendicular field is greater and extends to larger field strengths than in the domain-like case. Considering single field realisations—as should be done for the study of single sources—in the domain-like case thus underestimates the expected “cosmic variance”.

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4Since the distribution of the magnetic fields for the Gaussian turbulent field and domain-like turbulence only depends on $B_{\text{rms}}$, Fig. 1 remains unchanged if the coherence length $L_c$ or the slope $\gamma$ is changed.
The effects discussed above are clearly visible in Fig. 2, where we plot the photon survival probability as a function of the oscillation length for a propagation distance equal to twelve coherence lengths choosing the parameters\(^5\) of the extragalactic field as \(B_{\text{rms}} = B_{\text{tot}} = 10^{-12}\) G and \(L_c = 2\) Mpc, while the axion parameters were set to \(m_a = 10^{-10}\) eV, \(g_{a\gamma} = 10^{-16}\) eV\(^{-1}\). The average photon survival probability from 100 realisations for a turbulent (blue solid) and a domain-like field (red solid) are shown. The opaqueness of the blue lines indicates the number of modes included; \(n_k = \{2,10,20,50,100\}\) from the lightest to the darkest line. The result for a single realisation of the magnetic field is shown in dashed lines for comparison. The general energy dependence of the survival probability is similar for the two magnetic field models: Below a given threshold energy determined by \(\Delta_{\text{osc}} \sim \Delta_{a\gamma}\), the survival probability is close to unity. Above the threshold energy, we are in the strong mixing regime. There are, however, clear differences in the detailed behaviour. Most notably, the oscillations for a turbulent field are smoothed out by the variation in the transverse magnetic field strength compared to the domain-like case. Furthermore, the survival probabilities predicted using the two magnetic field models differ even in the case when \(L_c > L_{\text{osc}}\), as expected from the discussion in the previous paragraph. Meanwhile, the

\[\text{We set } L_{\text{min}} = 0.01\ \text{Mpc and } L_{\text{max}} = 5\ \text{Mpc.}\]
The average photon survival probability after 12 coherence lengths as a function of the oscillation length using 100 realisations is shown in for Gaussian (blue solid lines) and domain-like (red dashed dotted lines) turbulence. The oscillation length is computed by setting $B_{\perp} \simeq B$, and the parameters $m_\alpha = 10^{-10} \text{eV}$, $g_{\alpha\gamma} = 10^{-16} \text{eV}^{-1}$, $B_{\text{rms}} = B_{\text{tot}} = 10^{-12} \text{G}$ and $L_c = 2 \text{Mpc}$ are used. The results for a single realisation are shown in dashed lines and dotted lines. The opaqueness of the blue lines indicate the number of modes $n_k$ included in the generation of the Gaussian turbulence as described in the text; $n_k = \{2, 10, 20, 50, 100\}$ from lightest to darkest.

The change induced by the increase of the number of modes\textsuperscript{6} shows the importance of an accurate description of magnetic field also on small-scales. It is interesting to note that Ref. [39] introduced a linear interpolation between the magnetic fields in the domain-like approach in order to resolve the discontinuities in the domain-like magnetic fields. While this approach leads to a slight smoothening of the peaks observed in Fig. 2 for the domain-like magnetic field, other effects like the variation for different realisations and the shift in the threshold energy are not reproduced in this approach.

\textsuperscript{6}The normalisation is kept constant while additional modes are added towards smaller scales. Therefore, the total energy stored in the magnetic field increases slightly with increasing number of included modes in this example.
4 Parameter space of photon-axion oscillations

Photon-axion oscillations are essentially determined by the axion parameters ($m_a$ and $g_{a\gamma\gamma}$) and the refractive indices induced by the magnetic field, the medium and the EBL. In addition, the propagation distance and the photon energy enter the problem. The effect of the magnetic field via the QED vacuum polarisation, $\Delta_{QED}^{\parallel,\perp}$, was already discussed in section 2. Among the medium effects, we neglect the Faraday contribution as the random direction of the turbulent magnetic field averages out its effect, as well as the Cotton-Mouton effect. Then the effective mass of the photon in a plasma,

$$m_{pl} \simeq \omega_{pl} = \sqrt{\frac{4\pi\alpha n_e}{m_e}} \simeq 0.0371 \left(\frac{n_e}{1\text{ cm}^{-3}}\right)^{1/2} \text{neV},$$

leads to the only additional contribution induced by the medium, $\Delta_{pl}^{\parallel,\perp} = -m_{pl}^2/(2E)$ [38]. In addition, the EBL and starlight may have profound effects on the refractive index at large energies, as first realised in Ref. [51]. The isotropic EBL influences the two polarisation states equally, and its contribution is given by

$$\Delta_{EBL} \simeq \Delta_{CMB} \simeq 0.522 \cdot 10^{-42} E.$$ (4.2)

This approximation is valid below the pair creation threshold $E_{th,CMB} \simeq 400\text{ TeV}$ on CMB photons, which are dominating the contribution of the EBL to the photon refractive index. In summary, we take into account the most important additional effects on the photon refractive index by using $\Delta_{\parallel,\perp} = \Delta_{QED}^{\parallel,\perp} + \Delta_{CMB} + \Delta_{pl}$ in the mixing matrix (2.5).

For the ease of comparison and identification of scales in different astrophysical environments, we will consider in the following as numerical values for these free parameters

$$\Delta_{QED}^{\parallel} = 1.5 \times 10^{-9} \text{ Mpc}^{-1} \left(\frac{E_{10^{11}\text{ eV}}}{10^{11}\text{ eV}}\right)^2 \left(\frac{B_{10^{-9}\text{ G}}}{10^{-9}\text{ G}}\right)^2;$$

$$\Delta_{pl}^{\parallel} = -1.1 \times 10^{-10} \text{ Mpc}^{-1} \left(\frac{n_e}{10^{-7}\text{ cm}^{-3}}\right) \left(\frac{E_{10^{11}\text{ eV}}}{10^{11}\text{ eV}}\right)^{-1},$$

$$\Delta_{CMB}^{\parallel} = 8.2 \times 10^{-3} \text{ Mpc}^{-1} \left(\frac{E_{10^{11}\text{ eV}}}{10^{11}\text{ eV}}\right),$$

$$\Delta_a^{\parallel} = -7.8 \times 10^{-3} \text{ Mpc}^{-1} \left(\frac{m_a}{10^{-10}\text{ eV}}\right)^2 \left(\frac{E_{10^{11}\text{ eV}}}{10^{11}\text{ eV}}\right)^{-1},$$

$$\Delta_{a\parallel} = 1.5 \times 10^{-2} \text{ Mpc}^{-1} \left(\frac{B_{10^{-9}\text{ G}}}{10^{-9}\text{ G}}\right) \left(\frac{g_{a\gamma}}{10^{-20}}\right).$$

(4.3)

We note that the value of the extragalactic magnetic field—which is often used in the literature—is close to the limits derived in Refs. [52, 53].

In general, photon-axion oscillations depend both on the magnetic field strength and the plasma density. However, we can eliminate one of the two variables using the
conservation of magnetic flux in a plasma. Then the magnetic field lines are frozen to the fluid elements and, neglecting dissipation and dynamo effects, we can employ the simple scaling relation

\[ n_e \simeq n_{e,0} \left( \frac{B}{B_0} \right)^{\eta}, \]  

with \( \eta = 3/2 \) for isotropic volume changes. For concreteness, we set as reference \( B_0 = \mu G \) and \( n_{e,0} = 0.02 \text{ cm}^{-3} \) which is suitable for the Milky Way\footnote{Although we focus on extragalactic environments in this work, the galactic environment is put as reference since plasma effects are negligible for extragalactic propagation, as we soon will see. However, for magnetic fields \( B \sim (10^{-9}–10^{-10}) \text{G} \) one obtains \( n_e \sim (6 \times 10^{-7}–2 \times 10^{-8}) \text{ cm}^{-3} \), suitable for extragalactic space.} [54]. Although this scaling should not be considered a general rule, it is sufficient for the purposes in this paper.

From the homogeneous solution (2.8), one can further conclude that the photon conversion probability will be governed by the relative ratios of \( \Delta_{\text{osc}}^{-1}, \Delta_{a||}^{-1}, L_c \) and the distance \( s \) travelled. That is, in order to have a significant conversion of photons, one must have a sizeable amount of oscillations \( (s\Delta_{\text{osc}} \gtrsim 1) \) and a sizeable mixing \( (\Delta_{a||} \sim \Delta_{\text{osc}}) \). The coherence length, meanwhile, determines the intrinsic behaviour of the conversion probability: If \( L_c \gg 2\pi/\Delta_{\text{osc}} \) the conversion probability “probes” the magnetic field with several oscillations per coherence length and the photon state parallel to the transverse magnetic field is completely mixed with the axion for each coherence length. If \( L_c \ll 2\pi/\Delta_{\text{osc}} \), on the other hand, the magnetic field changes quickly so that the mixing slowly converges. Observationally, one can measure the energy spectrum of single gamma-ray sources, which means that one can probe the energy dependence of the photon-axion oscillation probability. The only energy dependence of the characteristic parameters lies in \( \Delta_{\text{osc}} \) and its generic behaviour is the same for all astrophysical environments (see also Ref. [39] for a similar discussion):

1. \( \Delta_{\text{osc}} \sim E^{-1} \) at low energies. Here, the oscillation length is determined by the effective photon mass or the axion mass, depending on the magnetic field strength and the axion parameters.

2. \( \Delta_{\text{osc}} \sim E^0 \) at intermediate energies. This is the strong mixing regime where the oscillation length is determined by the mixing term, \( \Delta_{a||} \sim 2\Delta_{a||} \).

3. \( \Delta_{\text{osc}} \sim E^1 \) at large energies. The oscillation length is here determined by either the CMB or the vacuum polarisation depending on the magnetic field strength.

The transitions between these regimes occur around the energies \( E_{\text{min}} \) and \( E_{\text{max}} \) defined by \( 4\Delta_{a||} = (\Delta_{a||} - \Delta_a)^2 \). Depending on the treatment of the magnetic fields, the oscillation probability in the transition region vary. For instance, the larger variance in \( B \) seen in Fig. 1 for a turbulent field will lead to a larger variance in the threshold energies \( E_{\text{min}} \) and \( E_{\text{max}} \). This will effectively reduce or even cancel oscillations close to the thresholds upon averaging and shift the threshold energies, as already seen in Fig. 2. In section 6, we will discuss how these generic energy dependence can be used as an observable in the search for axion-photon oscillations in astrophysical environments.
The discussions above can be summarised in the energy-magnetic field plane, as shown in Fig. 3 choosing the axion parameters\(^8\) as in Eq. (4.3). This plot is essentially a representation of \(\Delta_{\text{osc}}\) as function of energy and the transverse magnetic field strength. The red lines indicate the value of \(\Delta_{\text{osc}}\) in terms of \(\Delta_{a\parallel}\), and the brown lines its value. The transverse magnetic field value and its 1\(\sigma\) distribution (from Fig. 1) is shown for three field strengths indicating the upper limit for extragalactic fields, a typical value for the Galactic magnetic field and for field close to sources. The three regions, \(\Delta_{\text{osc}} \sim E^{-1}\), \(\Delta_{\text{osc}} \sim E^0\) and \(\Delta_{\text{osc}} \sim E^1\), are indicated in the figure. Furthermore, the parameter space is divided into five regions depending on the dominant contributions

\(^8\)Note that a change in the magnetic field strength is equivalent to the inverse change of the coupling strength (with the exception of the contribution from the plasma), since the product of these two quantities determines the oscillation amplitude.
to $\Delta_{\text{osc}}$: axion mass (upper left), plasma (lower left), mixing (middle), CMB (upper right) and QED (lower right). For very weak magnetic field strengths, there is no strong mixing regime and so the transition is marked in yellow. Likewise, the transition from a CMB to a QED dominated refractive index is shown in green. As an example of how this plot can be used, consider the extragalactic magnetic field: At low energies, the oscillations will be governed by the axion mass term $\Delta_a$. In the energy range $E = 10^9$–$10^{12}$ eV, we are in the strong mixing regime where there are no energy-dependent oscillations. The exact energy of this transition will vary by around an order of magnitude for different realisations of the Gaussian turbulence. The energy oscillations will occur close to, but outside, the strong mixing regime, and their strength depends on $\Delta_{\text{osc}}/\Delta_a$. These features will be shown explicitly with an example in the next section. A short discussion on how Fig. 3 changes with $m_a$ is given in the appendix.

An interesting region in parameter space is where the plasma contribution cancels (on average) the axion contribution, $|\Delta_a| = |\Delta_{pl}|$. According to the previous discussions, the strong mixing regime will then extend to arbitrary low energies, potentially reaching the CMB. The homogeneity of the CMB can in principle thus be used to set stringent bounds on $g_a\gamma$ for specific $m_a$. Interestingly, from Fig. 3 one notes that this “resonance” transition will always occur in the passage from a typical gamma-ray source to its surrounding with weaker magnetic fields. It should be noted, however, that the resonance region should be comparable to the oscillation length in order to have detectable effects. Such a resonance transition in the early Universe has previously been discussed in Ref. [55], where the homogeneity of the CMB was used to set limits on the axion parameters.

5 Characteristic axion signatures

In order to apply the concepts discussed in the previous sections, we consider now two concrete, albeit over-simplified, examples. A more realistic example will be considered in the next section, wherein we introduce a method of detecting the characteristic wiggles in the photon spectrum from a physical source.

We consider a photon source at a distance of 5 Mpc for the parameters given in Eq. (4.3). The resulting photon survival probabilities using different models for the magnetic field are shown in Fig. 4 as a function of the photon energy. Averaging over 100 realisations, we see the same characteristics as before: at low and high energies, the photon survival probability is close to unity, while there is a strong mixing regime at intermediate energies. However, there are even after averaging clear differences in the results caused by the treatment of the magnetic field. Importantly, the variation in the survival probability between realisations is much larger using a turbulent field than in the domain-like treatment, in accordance to the variation in the magnetic field itself shown in Fig. 1. Looking at single realisations, one can clearly see the effect of the energy-dependent oscillation length. That is, the energy spectrum will have wiggles with a wave number scaling with $\Delta_{\text{osc}}$, where the energy dependence of $\Delta_{\text{osc}}$ at low energies is $\Delta_{\text{osc}} \sim E^{-1}$, then $\Delta_{\text{osc}} \sim \text{const.}$ and finally at high-energies $\Delta_{\text{osc}} \sim E^1$. We note in particular that even though the oscillations on average cancel in a turbulent
Figure 4. The photon survival probability for polarised photons propagating through five coherence lengths is plotted as a function of energy. The coherence length is set to $L_c = 1$ Mpc, and the axion parameters and extragalactic magnetic field parameters in Eq. (4.3) are used. The average of 100 realisations of the magnetic field is plotted for Gaussian (blue solid) and domain-like turbulence (red dashed dotted), with their corresponding standard deviations indicated by the shaded regions. The results from a single realisation is shown in dashed and dotted lines, respectively. To better visualise the oscillations outside the strong mixing regime, a portion of the plot is enlarged.

magnetic field, the oscillations in a single realisation—which is the relevant case for observations—may be huge.

Next, we consider in Fig. 5 the same set-up but for a fixed energy, $E = 10^{11}$ eV, as function of the distance for $10^4$ realisations of the magnetic field. The average photon conversion probability increases, as expected, slowly towards $1/3$. Again, the relative variation for different realisations of the turbulent field is much larger than for the domain-like case, although the average values are similar. Interestingly, the survival probability for a single magnetic field realisation can vary almost over all the allowed range of values. This implies that the oscillation probability for a specific source can deviate strongly from the average, as already discussed in Refs. [30, 56]. Moreover, the large “cosmic variance” prevents to define a characteristic signal which can be probed as function of the source distance.

An interesting observation is that the convergence time for the turbulent field is much larger than for the domain-like one at large distances. At short distances, the
Figure 5. The average photon survival probability for polarised photons is plotted as a function of distance for $10^4$ realisations of the magnetic fields and a fixed energy, $E = 10^{11}$ eV. The same parameters and color scheme as in Fig. 4 are used. The solid lines indicate the averaged results, while the dashed lines indicate their $1\sigma$ variation. Furthermore, the results from two single realisations are shown in dashed dotted (Gaussian) and dotted (domain-like) lines for visualisation. Finally, the analytical approximation (Eq. (5.1)) is shown in yellow for comparison.

Conversion probability is slightly larger for the turbulent field, cf. also Fig. 4. This can be understood by the following argument. The average conversion probability for a photon in a domain-like turbulence with constant transverse magnetic field, $B_\perp$, is $[44, 57]$

$$P_{\gamma \rightarrow a} = \frac{1}{3} \left(1 - e^{-r/L_\tau}\right),$$  

(5.1)

where $L_\tau = 2L_c/3P$ and $P$ is given by Eq. (2.8). In Fig. 5, the strong mixing regime was considered, in which case $P$ depends on the magnetic field as $P = \sin^2(B_{\perp} g_\alpha L_c)/4 \sim B_\perp^2 + \mathcal{O}(B_\perp^4)$. The decay rate will thus be dominated by $\langle B_\perp^2 \rangle$, which is larger for domain-like than for a turbulent field. For the domain-like turbulence we have $\langle B_\perp^2 \rangle = 0.67B_{\text{tot}}$. Because of the small variation in $B_\perp$, the analytical approximation (5.1) reproduces well the numerical results for the domain-like case. A turbulent field has, on the other hand, a larger variation in the transverse magnetic field. Moreover, the coherence length characterizes only on average the “typical” size of turbulent domains. Both effects lead to a distance dependence that differs significantly from Eq. (5.1).
6 Direct detection of photon-axion oscillations

Although the wiggles after averaging over realisations of the turbulent field tend to disappear, the oscillatory behaviour in single realisations may be large. Moreover, the detection of these wiggles is made more difficult by the finite energy resolution of realistic experiments. Therefore, it is common to use either the increased photon survival probability at large energies (i.e. the opacity of the Universe) [21, 46–48, 58], the presence of large scale excesses in photon spectra [59–61] or the presence of irregularities in photon spectra (i.e. the variance in residuals) [45] to probe the existence of axion-photon oscillations.

Here we propose instead to use the energy-dependent frequency of the wiggles themselves as observable. A similar concept has previously been used to study the Earth-matter effect on neutrino oscillations [62–65] by considering the windowed power spectrum

\[ G(k) = |g(k)|^2 = \left| \int_{\eta_{\text{min}}}^{\eta_{\text{max}}} d\eta \, q(\eta) e^{i\eta k} \right|^2, \]  

(6.1)

where \( \eta \) is a function of energy and \( q(\eta) \) is the observed neutrino flux from a hypothetical source. The oscillations are energy dependent and the strongest close to the strong-mixing regime. Thus, the window \((\eta_{\text{min}}, \eta_{\text{max}})\) should be chosen such that it includes the oscillations that can be resolved, while excluding the strong-mixing regime to remove noise. For large energies, the oscillation length \( L_{\text{osc}} \sim 2\pi/\Delta_{\text{osc}} \) scales as \( E \), while at low energies it scales with \( E^{-1} \). Therefore, we expect clear peaks in the power spectrum with \( \eta \sim E^{-1} \) or \( \eta \sim E^{1} \). The windowing has unfortunately a major drawback that must be handled with care: It induces a low-frequency peak which may interfere with the signal peaks (see Eq. (6.1) with \( q = 1 \)),

\[ G_{\text{window}} = 2 \frac{1 - \cos k \Delta \eta}{k^2}. \]  

(6.2)

According to Eq. (6.2), the power spectrum converges towards one as \( k^{-2} \). This method shows the importance of a proper treatment of the magnetic field: As discussed in the previous sections, the characteristic signatures induced by photon-axion oscillations in more realistic magnetic field models are expected to have a larger cosmic variance and thus to be harder to detect than in simplified models.

For practical purposes, it is more relevant to consider the discrete power spectrum

\[ G_N(k) = N \left| \frac{1}{N} \sum_{\text{events}} e^{i\eta k} \right|^2, \]  

(6.3)

where the sum goes over detected events. Choosing the correct energy scale \( \eta \) and an optimal window \((\eta_{\text{min}}, \eta_{\text{max}})\), one may observe a peak that exceeds the expected background. The location of the signal peak depends on the periodicity of the wiggles and the chosen window. For example, in a homogeneous magnetic field the photon survival probability depends on the oscillation length via Eq. (2.8). With \( \eta = \Delta_{\text{osc}}(E) \) the signal peak will be located around the distance travelled, \( k_{\text{peak}} = s \). We can,
however, look for the generic features discussed in section 4: One expects peaks in two
different window regions and having different energy dependencies, one at low energies
with $\eta \sim E^{-1}$ and one with $\eta \sim E$ at larger energies. A similar concept was introduced
in Ref. [66]. There, the idea was to search for sinusoidal axion signatures in photon
spectra by analysing the Fourier-transformed data or performing a sinusoidal fit.

To exemplify the suggestion of using the power spectrum to detect photon axion
oscillations, we plot in Fig. 6 the photon distribution (first row) and the discrete power
spectrum (second row) for two realisations of the turbulent magnetic field with the
parameters given in Eq. (4.3) for a source at redshift $z = 0.02$ and a turbulent field
with $B_{\text{rms}} = 5$ nG and $L_{\text{max}} = 10$ Mpc. In addition, we plot in the last row the
power spectrum multiplied by $k^2$ to better distinguish the peak from the background.
The opaqueness of the lines indicates the number of photons used in the analysis. The
signal peak is clearly visible by eye, and becomes visible already for $O(1000)$ photons in
these examples. A proper analysis may therefore yield a significant improvement in the
sensitivity compared to previously suggested approaches. Furthermore, this problem
may be a well-suited for machine learning that potentially can make the method even
more sensitive.

Finally, we would like to comment on the interesting discussions recently given in
Ref. [67]. Here, it was shown that to first order in the coupling, Eq. (2.3) can be solved
in the interaction picture outside the strong mixing regime to obtain the polarised
photon-conversion probability

$$P_{\gamma \rightarrow a}(\eta) = \left| -i \int_0^{s_{\text{max}}} ds' \Delta_a((s') e^{-i\eta s'}) \right|^2 = \frac{g_{a\gamma}^2}{4} \left| \int_0^\infty ds' B_i(s') e^{i\eta s'} \right|^2 = \frac{g_{a\gamma}^2}{4} |\mathcal{F}[B(s)](\eta)|^2;$$

which in turn can be connected to the auto-correlation function of the magnetic field.
Interestingly, photon-axion oscillations are therefore (within these assumptions) deter-
mined by the auto-correlation function. Since the autocorrelation function contains
less information than the magnetic field, different magnetic fields may share the same
autocorrelation function. This means that the framework introduced in Ref. [67] al-
lows for a more efficient scanning over magnetic field configurations in axion searches.
There are, however, a few complications that must be considered: The description is
not valid close to the strong mixing regime where the oscillations are the strongest, it
does not apply to the full energy range, and one still need to make assumptions on
the magnetic fields. Furthermore, the photon conversion probability predicted from
the autocorrelation function is highly sensitive to small changes in the autocorrelation
function (see e.g. Fig. 3 in Ref. [67]). The most suitable approach to photon-axion
oscillations is therefore in our opinion to look for generic features of photon-axion os-
cillations in photon spectra without considering specific magnetic field environments,
and from that infer information of the magnetic fields. In fact, computing the power
spectrum of the conversion probability, it follows that

$$G(k) = \frac{g_{a\gamma}}{4} \int_0^{s_{\text{max}}} dz B(z) B^*(z - k) = \frac{g_{a\gamma}}{4} \int_{-\infty}^\infty dk \, B(k) B^*(k) e^{i\eta k},$$

\footnote{The integration limit has been extended to infinity by assuming that $B_i$ vanishes at $s_{\text{max}}$.}
Figure 6. The photon spectrum with the number of photons per bin (first row), photon power spectrum (second row), and the photon spectrum multiplied by $k^2$ from a source at redshift $z = 0.02$ influenced by a Gaussian turbulence are shown for two realisations. The parameters in Eq. (4.3) are used.

which means that if axions are detected, one can in theory use the power spectrum of
the oscillations, as shown e.g. in Fig. 6, to directly infer information about the magnetic field. One of the main advantages of using the discrete power spectrum compared to standard approaches of using fit residuals (such as mentioned in Ref. [67]), is that no information on the microstructure of the axion wiggles is lost by the binning of the data.

7 Opacity of the Universe

The focus in this work has been on the origin and the characterisation of axion wiggles in photon spectra, their detection and the effect of the magnetic fields. However, there is an additional important signature of photon-axion oscillations that can be used to probe the existence of axions: Since axions propagate practically without any interactions, there will be an increased photon survival probability at large energies, thus decreasing the opacity of the Universe. In Ref. [47], the difference in the apparent opacity using a turbulent and a domain-like magnetic field was already considered.

In order to strengthen our message that one should refrain from using domain-like turbulence in quantitative studies on axion oscillations, we plot in Fig. 7 the normalised flux from a source with the injection spectrum \( \frac{dN}{dE} \propto E^{-1.2} \) at a distance \( z = 1 \) for the magnetic field and axion parameters given in Eq. (4.3). The flux obtained averaged over many realisations of the magnetic field is shown as a solid (dashed) line for a Gaussian turbulent (domain-like) field, with the shaded regions corresponding to the \( 1\sigma \) variance between single realisations. In order to increase the statistics at high energies, we take into account the photon absorption by including a complex term \( i\lambda \) in the equation of motion that describes the mean free path length of the photon\(^{10}\), and update the mean free path length at redshift increments \( \Delta z = 10^{-3} \). In other words, the photon attenuation is treated as a continuous change in the photon survival probability, in contrast to the Monte Carlo treatment of the electromagnetic cascade that is considered elsewhere in this work. As a basis for comparison and to check our numerical calculations, we plot also the spectrum obtained for a single realisation of the magnetic field using a Gaussian turbulent field with continuous attenuation (red squares) and the Monte Carlo treatment of the electromagnetic cascade (green circles). Furthermore, the spectrum obtained with the electromagnetic cascade without axions, i.e. using \texttt{ELMAG} without photon-axion oscillations, is shown (blue diamonds). The error bars are computed as the statistical \( 1\sigma \) Poisson uncertainty of the counts in a given bin, and reflect thus only the statistical uncertainty of the Monte Carlo run\(^{11}\).

It is clear from Fig. 7 that photon axion oscillations lead to a decreased opacity of the Universe. However, with the parameters considered here, the difference between the two treatments of the magnetic field leads to a significant difference in the predicted average flux which is increasing with energy: At \( \bar{E} \approx 1 \text{ TeV} \) the difference is around

\(^{10}\)This implies that only prompt photons are considered. However, the effect of cascade photons is negligible since the spectra are dominated by prompt photons.

\(^{11}\)The number of injected photons in each energy bin is uniformly distributed. In the simulation with the continuous attenuation, all injected photons contribute to the statistics by a weight corresponding to the photon survival probability, making the error bars energy independent.
Figure 7. The normalised diffuse photon flux is plotted as a function of energy for a source with a power-law index $\alpha = -1.2$ at redshift $z = 1$. The magnetic field and axion parameters in Eq. 4.3 are used. The average over many realisations of the magnetic field is shown for Gaussian (solid) and domain-like (dashed) as black lines. In order to increase the statistics at large energies, the photon absorption is taken into account by including an attenuation determined by the photon survival probability. Furthermore, the flux obtained for a single realisation is shown by red squares, while the flux obtained using the same magnetic field and the standard Monte Carlo treatment of the photon absorption as implemented in ELMAG is indicated by green circles. The errorbars indicate the $1\sigma$ Poisson uncertainty. Finally, the flux predicted by ELMAG without any axions is shown in blue diamonds to visualise the effect that the axions have on the flux.

A factor 20. This result can be understood from Fig. 5: The conversion probability of photons into axions obtained using a domain-like field is on average larger than employing a Gaussian turbulent field at large distances.

The example considered in Fig. 7 further demonstrates the importance of a proper treatment of magnetic fields in the study of photon-axion oscillations. While the effect on the opacity on average is much less prominent for the Gaussian turbulence than for the domain-like approximation, there is still a significant variation between single realisations in both cases. The variation is however noticeably larger for the Gaussian turbulence than for the domain-like approximation.
8 Conclusion

In this work, we have studied photon-axion oscillations in a Monte-Carlo framework based on the ELMAG program. We have argued that the use of statistically averages over magnetic field configurations is misleading and should be abandoned in the search for signatures from axion-photon oscillation in the spectra of single sources. Moreover, we have shown that the predicted signatures—axion wiggles in photon spectra and the decreased opacity of the Universe—depend strongly on the chosen magnetic field models. Therefore, over-simplified magnetic field models as the domain-like field should be used with care for quantitative predictions.

We have discussed mainly those characteristic signatures of axion-photon oscillations which are independent of the concrete astrophysical environment. In particular, the oscillation length will scale as $E^1$ below the threshold energy $E_{\text{min}}$ and as $E^{-1}$ above the threshold energy $E_{\text{max}}$, while it will be constant at intermediate energies. We have proposed to use this energy dependence as an observable in the search for photon-axion oscillations using the discrete power spectrum. This method can in principle be also used to infer information about the magnetic field environment from the observation of axion wiggles.

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A Varying axion mass

For completeness, we plot in Fig. 8 the transition energies from Fig. 3 varying the axion mass and plasma density. The high energy transition to the strong mixing regime (black line) is independent of the axion mass and plasma density, while the low energy transition is decreasing for a decreasing axion mass and plasma density. Interestingly, this means that a weak magnetic field only will produce axion wiggles in photon spectra if the axion mass is sufficiently small. For example, if \( m_a \sim 10^{-12} \text{eV} \), the magnetic field strength must be stronger than \( \sim 10^{-11} \text{G} \) to produce wiggles. On the other hand, if \( m_a \gtrsim 10^{-6} \text{eV} \), the strong-mixing regime disappears and therefore no significant axion wiggles are produced.

![Graph showing transition energies for different axion masses](image)

**Figure 8.** The transition energies in Fig. 3 is replotted for different axion masses. In addition, the effect of decreasing the reference electron density to \( n_{e,0} = 0.002 \text{cm}^{-3} \) is shown by dashed lines.