Tachyon Condensates and String Theoretic Inflation

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Cosmological solutions of the beta function equations for the background fields of the closed bosonic string are investigated at the one-loop level. Following recent work of Kostelecky and Perry, it is assumed that the spatial sections of the space-time are conformally flat. Working in the sigma-model frame, the non-trivial tachyon potential is utilized to determine solutions with sufficient inflation to solve the smoothness and flatness problems. The graceful exit and density perturbation constraints can also be successfully implemented.

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1. Introduction

For some time, it has been hoped that string theory would answer a number of the central questions concerning low energy phenomena. Initially, the focus was the low energy gauge group, the number of generations, etc. In part due to the increasing numbers of available ground states of string theory (in conformal field theory, or in for example, the free fermion approach), more recently attention has shifted to areas in which gravitational dynamics plays a key role. For example, string-theoretic and string inspired models have played a significant role in the recent study of information loss and Hawking radiation in two dimensions (see [1] for a recent review).

Another potential application of string theoretic methods lies in the physics of the early universe. It is by now well known that string theory contains two principal differences from general relativity. The first is the $\alpha'$ expansion of the equations of motion, which has Einstein’s equations as the leading term. The second is the particular low energy spectrum of string theory, which differs from more conventional models in the presence of the dilaton, Kalb-Ramond field, and tachyon (for bosonic strings). As for the former, the $\alpha'$ corrections should only be important for times of order $t_{PL}$ or less, and furthermore, $R^2$ gravity can and has been studied without the input of string theory.

As for the latter, the approach generally taken is to consider strings propagating in a non-trivial background and demand conformal invariance. This amounts to demanding vanishing beta functions, which in turn can be interpreted as the field equations of an Einstein-like theory. Generally, these equations (and hence the corresponding field theory) are truncated at leading order in $\alpha'$. It is presumed that any solution of the field equations corresponds to a well-defined ground state of string theory, i.e., a conformal field theory, but there is no proof of such a conjecture. The alternative approach is to find an exact conformal field theory which has the space-time interpretation of a cosmological model. This approach has yielded some success recently through the use of non-compact Wess-Zumino-Witten models, [2,3,4,6,7,8,9], but these solutions are generally not maximally symmetric and often have singularities. Furthermore a systematic approach for finding cosmologically relevant solutions does not seem to be available.

In the context of inflationary cosmology, for a number of reasons the focus in the past has been on the contribution of the dilaton [10,11,12,13,14,15]. One reason is that the dilaton appears in the action is a way similar to a Jordan-Brans-Dicke (JBD) field [16,17]. It should be noted, however, as Campbell et al have discussed [12], that the
dilaton cannot be identified with the standard Brans-Dicke field for any value of the JBD parameter $\omega$. It was also shown that including only the dilaton and the metric did not yield an adequate inflationary model. A second reason is the relation of the dilaton to supersymmetry breaking. Unfortunately, it is understood that the dilaton potential is flat to all orders in string perturbation theory, and as usual, the form of the nonperturbative potential is completely unknown, making it very difficult to make definitive statements about dilaton induced inflation.

Considerably less attention has been paid to the contributions of the tachyon. In [13], the tachyon was considered, but only in the context of its propagation in the so-called linear dilaton background (see Section 2). Another exception is its use in two dimensions [18], where the tachyon is actually massless. However, as has been stressed recently by Kostelecky and Perry [19], the tachyon mass squared is negative in $d > 2$ because one is expanding about a vanishing expectation value. Using the leading terms in the tachyon potential [20], they were able to show that the tachyon in four dimensions has positive mass squared when expanded about the minimum of its potential.

Kostelecky and Perry were primarily interested in the tachyon mass and the nature of singularities in the metric under the freedom of making conformal transformations using the dilaton. Their analysis was done primarily in the "sigma-model" frame (see Section 2), whereas most inflationary models are analyzed in the Einstein frame. By assuming a constant tachyon background they were able to solve for the metric and dilaton, assuming a $k = 0$ Friedman-Robertson-Walker metric.

To discuss inflation, it is necessary to go beyond the assumption of a constant tachyon, as it is by now well known that the dynamics of the inflaton are crucial to any successful model. That is what will be done in this paper. Of course, the equations of motion cannot then be solved exactly, but that is not necessary. Techniques now familiar in inflation can be used to evaluate the essential quantities, such as the number of e-foldings of the universe during inflation. Various additional constraints must also be satisfied in order to solve the smoothness, graceful exit, and density perturbation problems. Similar models have been investigated in the Einstein frame in [21].

Section 2 will briefly review the background field equations and the tachyon potential. The constant tachyon solutions of [19] will also be displayed. In Section 3, the inflationary scenario will be discussed, and in Section 4 the various phenomenological constraints will be considered, followed by some additional considerations in Section 5.
To leading order, the sub-Planckian energy physics of string theory can be well described by propagation in the set of condensates of its massless fields, namely the graviton $g_{\mu\nu}$, antisymmetric tensor $B_{\mu\nu}$, dilaton $\phi$, and tachyon $T$. For a world sheet $\Sigma$ with metric $\gamma_{ab}$ and local coordinates $\xi^a$, $a = 1, 2$, the location of the string is described by spacetime coordinates $X^\mu(\xi^a)$, and the action is

$$I = -\frac{1}{2\pi\alpha'} \int d^2\xi \sqrt{\gamma} \left( \frac{1}{2} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu} + \frac{1}{2} \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu} + \frac{1}{2} \alpha' T - \frac{1}{4} \alpha' R^{(2)}(\phi) \right),$$

(2.1)

where $R^{(2)}$ is the Ricci curvature of the world sheet. It is this curvature term which implies that the string coupling constant is $\hat{g} = \exp(\phi/2)$. As it stands, (2.1) is not conformally invariant, as conformal invariance is obviously broken by the tachyon and dilaton terms. It can be restored quantum mechanically by requiring that the beta functions for $g$, $B$, $T$ and $\phi$ vanish. To one loop, these equations are

$$R_{\mu\nu} = \nabla_\mu \nabla_\nu \phi + \nabla_\mu T \nabla_\nu T + \frac{1}{4} H_{\mu\nu\rho} H^{\mu\nu\rho}$$

(2.2)

for gravity

$$\nabla_\lambda H^{\lambda\mu\nu} + (\nabla_\lambda \phi) H^{\lambda\mu\nu} = 0$$

(2.3)

for the Kalb-Ramond field

$$\nabla^2 T + \nabla_\mu \phi \nabla^\mu T = V'(T)$$

(2.4)

for the tachyon, and

$$\dot{c} = R - (\partial \phi)^2 - 2\partial^2 \phi - (\partial T)^2 - 2V(T) - \frac{1}{12} H^2$$

(2.5)

for the dilaton. Here $H_{\mu\nu\rho} = 3\partial_\lambda [B_{\mu\nu}]$ and $V' = \frac{\partial V}{\partial T}$, where $V$ is the tachyon potential which shall be discussed shortly. Also, $\dot{c} = 2(d - 26)/3\alpha'$, which implicitly includes contributions from the conformal ghosts.

These equations, in turn, can be derived from the following space-time action

$$I_S = -\frac{1}{2\kappa^2} \int d^d x \sqrt{-g} e^\phi (\dot{c} - R - (\partial \phi)^2 + (\partial T)^2 + 2V(T) + \frac{1}{12} H^2),$$

(2.6)

where $2\kappa^2 = 8\pi G_N$ for $d = 4$. This is similar to a JBD theory with $e^\phi$ as the Brans-Dicke field. This form of the action is commonly known as the sigma-model frame because of its direct connection to the sigma model action in (2.1). By rescaling

$$g_{\mu\nu} \rightarrow e^{-2\phi/(d-2)} g_{\mu\nu}$$

(2.7)
the action can be put in the form
\[
I_E = -\frac{1}{2\kappa^2} \int d^d x \sqrt{g} ((\dot{c} + 2V(T))e^{-2\phi/(d-2)} - R - \frac{1}{d-2}(\partial\phi)^2 + (\partial T)^2 + \frac{1}{12}e^{4\phi/(d-2)}H^2).
\]
(2.8)

As stressed by Kostelecky and Perry, conformal invariance should guarantee that one frame is as good as another, and the analysis below will generally be conducted in the sigma-model frame. In addition, there is of course a Bianchi identity among the field equations. One can therefore assume that (2.7) is automatically satisfied if (2.2)-(2.4) are satisfied.

A simple solution to the field equation is the so-called linear dilaton state, with the flat metric, vanishing Kalb-Ramond field and a dilaton background of
\[
\phi = \mu t, \quad \mu^2 = \frac{2(d - 26)}{3\alpha'}.
\]
(2.9)

Ignoring cubic and higher terms in the tachyon potential (i.e., expanding about \(T = 0\)), the tachyon must then satisfy
\[
\nabla^2 T + \mu \partial_t T = -\frac{4}{\alpha'} T
\]
(2.10)
from which, after shifting \(T\) by a constant to eliminate the first derivative term, one can read off the usual mass relation \(m^2 = \frac{2-d}{6\alpha'}\). The idea explored by Kostelecky and Perry is that the tachyon potential may have an alternative extremum \(T_0\) with positive \(V''(T_0)\), thus leading to a positive mass and avoiding the usual instability problems. Since \(V(T_0)\) is likely to be non-vanishing, the resulting solutions are likely to behave as de Sitter or anti-de Sitter space, so they assumed a \(k=0\) Friedman-Robertson-Walker (FRW) metric
\[
ds^2 = -dt^2 + a(t)^2(dx_1^2 + \cdots dx_{d-1}^2).
\]
(2.11)

Flat spatial sections are favored, since the false vacuum \(T = 0\) can then evolve into the true vacuum without any topology change. With this assumption, the field equations can be written
\[
(d - 1)\frac{\ddot{a}}{a} + \ddot{\phi} + \dot{T}^2 = 0
\]
(2.12)
\[
a\ddot{a} + (d - 2)\dot{a}^2 + a\dot{a}\dot{\phi} = 0
\]
(2.13)
\[
\ddot{T} + (d - 1)\frac{\dot{a}}{a}\dot{T} + \dot{\phi}\dot{T} + V'(T) = 0
\]
(2.14)
\[ \dot{c} = \ddot{\phi}^2 + \ddot{\phi} + (d - 1) \frac{\dot{a}}{a} \dot{\phi} - 2V(T) \]  
(2.15)

where (2.15) has already been used to simplify (2.12).

The solutions of Kostelecky and Perry are found by letting \( T = T_0 \) where \( V'(T_0) = 0 \), i.e., the tachyon sits at its minimum. Letting \( h = \ln(\sqrt{\beta} \dot{a}/a) \), \( \phi \) can be eliminated from the remaining equations, which then reduce to

\[ \beta \ddot{h} = (d - 1)e^{2h}. \]  
(2.16)

Note that the Hubble parameter \( H = \dot{a}/a = e^h/\sqrt{\beta} \). Kostelecky and Perry chose \( \beta = \alpha' \), which is the natural unit of time, but that is obviously not required, since \( \alpha' \) does not appear in (2.12)-(2.15). In fact, it will be essential in what follows that \( \beta \) be different from \( \alpha' \) (see the next section). (2.16) can be integrated once to yield

\[ \dot{h}^2 = \frac{d - 1}{\beta} e^{2h} + k \]  
(2.17)

where \( k \) is an integration constant. For \( k = 0 \) the solution is

\[ a = a_0 t^{1/\lambda}, \quad \phi = \phi_0 + (1 - \lambda)\ln(t/\sqrt{\beta}) \]  
(2.18)

where \( \lambda = \pm (d - 1)^{1/2} \) (only the positive square root will be needed here). For \( k > 0 \) the solution is

\[ a(t) = a_0 (\tanh(\lambda t \sqrt{k}/2))^{1/\lambda} \]  
(2.19)

\[ \phi(t) = \phi_0 + (1 + \lambda)\ln(\cosh(\lambda \sqrt{k} t/2)) + (1 - \lambda)\ln(\sinh(\lambda \sqrt{k} t/2)). \]  
(2.20)

Note that the two solutions are equivalent in the \( t \to 0 \) limit. For \( k < 0 \), solutions similar to (2.19) exist, with \( \tanh \) replaced by \( \tan \), etc, but there, the solution diverges in a finite amount of time, which is assumed to be unphysical.

Thus far, no assumptions concerning the form of the tachyon potential have been made. This has been evaluated in a number of references, see for example [13] and [20], in string field theory. These calculations are perturbative in the tachyon, but sum all orders of the loop expansion. The result is

\[ \dot{V}(\dot{T}) = -\frac{2}{\alpha'} \dot{T}^2 + \frac{g_3}{3!} \dot{T}^3 + \cdots. \]  
(2.21)
where $g$ is the tree-level three-tachyon coupling defined at zero momentum. Now, in (2.21), $\tilde{T}$ has mass dimensions 12 and the tachyon potential has mass dimension 26, as appropriate for the string field theory calculation, which is conducted in the critical dimension $d = 26$. Kostelecky and Perry chose to work with a dimensionless tachyon and a tachyon potential of mass dimension 2 by rescaling

$$\tilde{T} = \kappa g \alpha' \hat{T}, \quad V(\tilde{T}) = \kappa^2 g^2 \alpha'^2 \hat{V}(\hat{T})$$

(2.22)

$$\tilde{V}(\tilde{T}) = -\frac{2}{\alpha'} \tilde{T}^2 + \frac{1}{3! \kappa \alpha'} \tilde{T}^3 + \cdots,$$

(2.23)

where $\kappa$ is a dimensionless constant. In the cosmological context, it seems more natural to choose a dimension one tachyon and dimension four potential, so instead the rescaling

$$T = \kappa g \alpha'^{1/2} \hat{T}, \quad V(T) = \kappa^2 g^2 \alpha' \hat{V}(\hat{T})$$

(2.24)

$$V(T) = -\frac{2}{\alpha'} T^2 + \frac{1}{3! \kappa \sqrt{\alpha'}} T^3$$

(2.25)

will be used here.†

The freedom to choose $\kappa$ reflects the fact that only the $T^2$ term in the potential is universal. The original calculations [13, 20] indicate that $\kappa = 1/4$, but the result is dependent on the regularization procedure. It is presumed that varying $\kappa$ would generally induce higher derivative terms [22]. Since such terms are generally unimportant during inflationary epochs, it seems plausible to leave $\kappa$ as a free parameter for now.

3. The Inflationary Scenario

It is clear that the solution in equation (2.18) is unsatisfactory for inflation, as it is well known [23] that power law inflation $a \sim t^p$ requires that the power $p$ be of order 10. Instead, one should proceed as in more standard inflationary models [24, 25, 26]. There, one assumes that the inflaton, here the tachyon, undergoes a “slow roll” towards its true minimum. It is during this slow roll that the universe inflates.

First, it is useful to eliminate $\phi$ from (2.12) and (2.14) by using (2.13). This yields

$$\tilde{T}^2 = \bar{h} - (d - 1) e^{2h} / \beta$$

(3.1)

† Note: This choice of rescaling induces various factors of $8\pi G_N$ which will be suppressed below.
The physical picture is to suppose that for small $t$, the tachyon is near the false vacuum of zero, and that the metric and dilaton have the form as in say (2.18). For small $t$, $\dot{h} \ll 0$, so that from (3.2) it is clear that the tachyon is overdamped and deviations from $T = 0$ strongly suppressed. As time goes on $|\dot{h}|$ decreases, and presumably a quantum fluctuation starts the tachyon rolling towards its true minimum.

The standard procedure is to then assume that both the $\ddot{T}$ term in (3.2) and the $T^2$ term in (3.1) are small. However, one only ignores $\dot{T}^2$ relative to $V(T)$ during the slow roll, but in contrast to the usual case of minimally coupled scalars, $V(T)$ does not appear in (3.1), so this term must be kept (otherwise, (3.1) is the same as (2.16) and its solutions known!). The question as to whether $\ddot{T}$ is small will be considered shortly, but to the extent that it is small, $T$ can be approximated as a linear function of $t$, i.e.,

$$T(t) = C t + T_0,$$

while the tachyon equation is

$$\dot{h}\dot{T} = V'(T),$$

where $C$ and $T_0$ are constants which will be estimated below. Since then $\dot{T} = C$, (3.1) can still be integrated once, yielding

$$\dot{h}^2 - (d - 1)e^{2h}/\beta - 2C^2 h = k_2$$

where $k_2$ is again an integration constant. Equation (3.5) does not appear to be solveable in closed form. As a somewhat crude approximation, let us assume that the exponential in (3.5) can be approximated by the first two terms in its power series. Then (3.5) is approximately

$$\dot{h}^2 = C_1 + C_2 h$$

\begin{align*}
C_1 &= (d - 1)/\beta + k_2, \\
C_2 &= 2(d - 1)/\beta + 2C^2.
\end{align*}

This can now be integrated, and the solution is

$$h(t) = h_0 \pm t\sqrt{C_2 h_0 + C_1 + C_2 t^2/4},$$

where $t = 0$ has been taken as the start of the inflationary epoch and $h_0$ is the initial condition on $h$. It is clear from (3.4), since $\dot{T} > 0$, $V'(T) < 0$, that $\dot{h}$ must be less than
zero. This necessitates taking the minus sign in (3.8), and more importantly implies that the inflationary period (or rather, the period during which the above approximations are valid) must end when $\dot{h} \sim 0$. This occurs when

$$ t = t_0 = 2\sqrt{h_0 + C_1/C_2} = 2\dot{h}_0/C_2. \quad (3.9) $$

At this point, given $h(t)$, the number of e-foldings through which the universe passes during the inflationary epoch can be evaluated. This is given by

$$ N = \int_0^{t_0} Hdt = \frac{1}{\sqrt{\beta}} \int_0^{t_0} e^h $$

which, after some simple manipulations, can be expressed as

$$ N = 4e^{-C_1/C_2} \sqrt{\beta C_2} \int_0^{\sqrt{h_0+C_1/C_2}} e^{x^2} dx. \quad (3.11) $$

In general, $\beta C_2$ is of order one, and some numerical evaluation then implies that for $C_1 \sim C_2$, that a value of $h_0$ of about 5 is sufficient for the 50-60 e-folds which are necessary to solve the flatness and smoothness problems. Of course, $h_0 = 5$ means that the approximation made in (3.6) is not well justified. But for $C_1 \sim C_2$, it can be demonstrated numerically that the associated errors are not too large, and the qualitative picture is unchanged. In any case, it is clear that the exponential factor tends to make $h$ larger (for fixed $t$), which in turn make $e^h$ much larger which therefore leads to an even larger expansion of the universe (even thought the duration of the expansion is slightly smaller).

When $\dot{h} > 0$, the $\ddot{T}$ term cannot be neglected, and the tachyon accelerates towards its true minimum. It will then oscillate about this minimum, thereby reheating the universe. To discuss this epoch, consider again equation (2.14) in $d = 4$. Then $V'(T)$ can be set to zero, since the tachyon is near its minimum. Therefore, in this region, (2.14) has the standard form of a scalar wave equation in an expanding universe with decay constant equal to $\dot{\phi}$. There is one difference, however. From the action $I_S$ in equation (2.6), one easily determines that the energy momentum tensor for the tachyon has an additional factor of $e^\phi$, i.e., its energy density and pressure are given by

$$ \rho_T = e^\phi(\dot{T}^2/2 + V(T)) \quad (3.12) $$

$$ p_T = e^\phi(\dot{T}^2/2 - V(T)) \quad (3.13) $$
After multiplying (2.14) by $e^{\phi T}$, it can be rewritten as

$$\dot{\rho}_T + 3H\rho_T = -\dot{\phi}\rho_T,$$  \hspace{1cm} (3.14)

where, as usual, the virial theorem for the harmonic oscillator has been used, and the potential term in (3.12) ignored (since it is roughly constant during the oscillations about the minimum). Then (3.14) can be integrated to yield

$$\rho_T = M^4 (a/a_0)^{-3} e^{\phi_0 - \phi},$$  \hspace{1cm} (3.15)

where $M$ is a constant of integration. For example, if it is assumed that the solution is roughly similar to the constant $T$ solution (2.19), where $\phi_0 - \phi$ varies as $-\lambda \sqrt{k} t$ for large $t$, then the tachyon energy density decays in the usual fashion. In particular, for sufficiently large $k$, the decay and therefore the reheating of the universe is quite rapid, and the reheat temperature on general grounds is basically equal to $M$, i.e., the vacuum energy is converted directly into radiation. More generally, during the oscillation epoch, the maximum temperature is roughly given by [27]

$$T_{RH} \sim T_{\text{max}} \sim M^{1/2} (\lambda \sqrt{k} m_{PL})^{1/4}. \hspace{1cm} (3.16)$$

The temperature then decreases during the oscillation phase, and the temperature at the beginning of the radiation-dominated epoch is roughly [27]

$$T_{RH} \sim (m_{PL} \lambda \sqrt{k})^{1/2}. \hspace{1cm} (3.17)$$

Before considering the remaining inflationary constraints, it is appropriate at this point to determine the constant $C$ in (3.3). Consider the limit of small $t$ (as measured from the start of the inflationary epoch), for which (3.3) should be a good approximation. It will also be assumed that in this limit $T$ itself is small. Substituting (3.3) and (3.8) into (3.4), ignoring quadratic and higher order terms in $V'(T)$, and assuming that $C_1 \sim C_2$, one finds

$$C^2 = -\frac{d - 1}{\beta} + \sqrt{(\frac{d - 1}{\beta})^2 + \frac{T_0^2}{2\alpha r^2}}. \hspace{1cm} (3.18)$$

where $h_0 \sim 5$ has been assumed. The physical interpretation of (3.18) is relatively clear; smaller values of $T_0$ correspond to the tachyon starting very near its (false) vacuum, where the potential is quite flat so that the starting velocity is correspondingly small.
4. Inflationary Constraints

A number of issues remain to be answered before the scenario described above can be considered self-consistent. First, there were a number of approximations which must be justified for some non-vanishing range of parameters. The most important of these was the slow-roll approximation. The general condition which should satisfied for the slow-roll to be a good approximation is

$$|V''(T)| < 9H^2.$$  \hspace{1cm} (4.1)

Assuming that before inflation the metric is behaving as in (2.18), then for small $T$, (4.1) gives a condition on the starting time of the inflationary epoch, namely

$$t < \sqrt{\alpha'/2}.$$  \hspace{1cm} (4.2)

Now, since it was shown above that $h_0 > 5$ is necessary for sufficient inflationary expansion, the parameter $\beta$ can be determined. This implies

$$\beta/\alpha' > 10^4.$$  \hspace{1cm} (4.3)

Another possible phenomenological constraint is the graceful exit problem. Simply put, the graceful exit problem is the apparent conflict between adequate inflation, which requires a small bubble nucleation rate, and percolation, which requires a fast bubble nucleation rate so that the phase transition goes to completion. This has been considered in [28] for the general case of extended inflation [29], and applied to string theoretic models in [12]. The results of [28] imply that the nucleation rate $\bar{\lambda}$ varies as

$$\bar{\lambda} = \hat{A}\exp(2\phi - B_0e^\phi)$$ \hspace{1cm} (4.4)

where $\hat{A}$ and $B_0$ are positive constants which are computed in [28] but which do not depend on the inflationary dynamics. It normally suffices to show that the nucleation rate is a non-decreasing as a function of time. From (2.13) it is trivially shown that

$$\dot{\phi} = -\dot{h} - (d-1)e^h/\sqrt{\beta},$$ \hspace{1cm} (4.5)

which can be integrated using (3.10) to determine the change in the dilaton $\delta\phi = \phi(t_0) - \phi_0$ during inflation

$$\delta\phi = h_0 - h(t_0) - N(d-1) \sim -200,$$ \hspace{1cm} (4.6)
for \( N \sim 60 \). Therefore, the dominant factor in (4.4), namely the second term inside the exponential, leads to a non-decreasing nucleation rate and the phase transition should proceed to completion.

The last phenomenological condition that will be considered here is that of density perturbations. Of course, these cannot be too large in view of the homogeneity of the microwave background, nor too small in view of the recent COBE results. The now standard parameter which one must evaluate is \((\delta \rho / \rho)_H\) where \( \rho \) is the density in a particular wavenumber, and where \( H \) in the subscript refers to the time when that perturbation reenters the horizon [27]. Actually, the standard calculation is to compute a gauge invariant quantity which equals \( \delta \rho / (\rho + p) \) at horizon crossing, and which equals \((\delta \rho / \rho)_H\) up to a factor of order unity. This can then be computed when the perturbation leaves the horizon during inflation. One then takes \( \delta \rho = \delta T e^\phi \frac{\partial V(T)}{\partial T} \) and \( \delta T = H/(2\pi) \). The latter statement is calculated for a minimally coupled scalar. If the dilaton were constant during inflation, we could roughly speaking compensate for the tachyon-dilaton coupling by including an additional factor of \( e^{\phi/2} \). One then must use the equation of motion for the tachyon during inflation (it is at this point where the unusual dilaton couplings come into play), i.e., equation (3.4). One then has

\[
\delta \rho / \rho \sim e^{\phi/2} \frac{\dot{H}}{\dot{T}}. \tag{4.7}
\]

For perturbations which leave the horizon near the end of inflation (i.e., small scale perturbations), the right hand side of (4.7) is small because \( \dot{H} = H \dot{h} \sim 0 \). However, for scales which leave the horizon in the beginning of inflation, the Hubble parameter is large (since \( h_0 \sim 5 \)), and the result in (4.2) implies that \( \dot{h} \) is large as well. This can be used to put a constraint on the initial value of the dilaton \( \phi_0 \). Alternatively, this potential problem can be removed if one assumes that \( \sqrt{\alpha'} \gg t_{PL} \), but that is somewhat unnatural in the context of string theory.

Similar models have been discussed by Berkin et al [21] in the Einstein frame. In those models, because of the power law behavior of the scale factor, the constraint due to density perturbations are found to be somewhat weaker than in conventional inflation models. This suggests that a more complete treatment of the tachyon perturbations should not cause a serious problem.
5. Discussion

While a complete examination of the parameter space of tachyon-induced inflation has not been attempted here, it has been shown that cosmologically viable models do exist. In fact, it is gratifying that no dimensionless parameter of order $10^{-15}$ arose in the construction or in the constraints, in sharp contrast to most conventional inflationary models (though several parameters of order 100 have arisen).

One difference between the model presented here and most inflationary models is the use here of the “string” frame, often referred to as the Jordan frame in cosmology. When the dilaton is constant, the change between frames is quite trivial (merely corresponding to a linear rescaling of $t$), but when the dilaton is dynamical the correspondence between frames is often rather obscure. This is primarily due to the fact that the rescaling (2.7) also requires a redefinition of “time” to maintain the Friedman-Robertson-Walker form of the metric. This has recently been emphasized by Tseytlin [10], who showed that the scale factor $a_S$ and $a_E$ in the two frames are related by

$$\ln(a_E) = \ln(a_S) - \frac{2}{d-2}\phi. \quad (5.1)$$

For example, it is possible for the metric to be static in one frame and linearly rising in the other. In [23] it is argued that neither frame should be considered as more fundamental than the other. In fact, they make an even stronger suggestion, namely that singularities in the metric should be considered essentially benign as long as a frame can be chosen in which the singularity is not present (in addition to the usual way of avoiding singularities by diffeomorphisms when the singularity is merely a coordinate singularity). This criteria then determines the preferred choice of frame. While singularities are not the issue here, the same principle should apply, i.e., the frame can be chosen so as to fit the requirements of inflation. One should also consider the fact that physical scales, such as $\Lambda_{QCD}$, vary in time depending on the chosen frame, and the expansion of the universe should in principle be measured against those scales, and not just the string scale [12].

There are a number of possible extensions of the present work. For example, the antisymmetric tensor has thus far been ignored. This has been considered in previous work which did not include the tachyon, see for example [10,30,31,32,33,34,35]. The solution in [33] can in fact be related to a Wess-Zumino-Witten model conformal field theory, while other solutions have arisen in the context of Kaluza-Klein theories. In [10] it is claimed that the presence of the Kalb-Ramond field does not greatly change the behavior of the system.
As mentioned in [19], it would be quite interesting to consider solutions to the background field equations for the heterotic string, since that is a much more viable candidate for a grand unified model, though of course that would involve additional fields which would complicate the analysis. The effective action up to quartic order, has been computed for the heterotic string in [36].

A final important question concerns the necessity of adding higher order terms in the $\alpha'$ expansion. Indeed, if $\sqrt{\alpha'} \sim t_{P_L}$, then equation (4.2) would suggest the necessity of adding such terms. Such higher order terms would change the initial conditions for inflation, in particular the relationship between $h_0$ and $\dot{h}_0$ which were used implicitly in the last section. But the dynamics of the inflationary epoch itself, when such terms should be small, should remain unchanged.
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