Mechanism for intensity induced chimera states in globally coupled oscillators

V. K. Chandrasekar, R. Gopal, A. Venkatesan, and M. Lakshmanan

1Centre for Nonlinear Science & Engineering, School of Electrical & Electronics Engineering, SASTRA University Thanjavur-613401, Tamilnadu
2Centre for Nonlinear Dynamics, School of Physics, Bharathidasan University, Tiruchirappalli - 620 024, Tamilnada, India
3Department of Physics, Nehru Memorial College, Pathanampatti, Tiruchirappalli 621 007, India.

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We identify the mechanism behind the existence of intensity induced chimera states in globally coupled oscillators. We find that the effect of intensity in the system is to cause multistability by increasing the number of fixed points. This in turn increases the number of multistable attractors and we find that their stability is determined by the strength of coupling. This causes the coexistence of different collective states in the system depending upon the initial state. We demonstrate that intensity induced chimera states are generic to both periodic and chaotic systems. We have discussed possible applications of our results to real world systems like the brain and spin torque nano oscillators.

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Dynamics of globally coupled oscillators continue to be a highly active topic of research for the past four decades or so because of its applicability to complex systems [1,3]. In particular, chimera is a dynamical state where synchronized and desynchronized oscillators coexist in coupled oscillator systems [4]. Ranging from chemical reactions [5], laser arrays, and nano-oscillators to the cognitive behaviour of the brain [6], chimera states play a crucial role in explaining various important dynamical behaviour involved in these real world systems [7].

For instance, in the case of epileptic seizures, certain regions of the brain remain highly synchronized while the remaining regions are desynchronized [8]. In the case of Parkinson’s disease, synchronized activity is absent in certain regions of the brain [9] due to damaged or lost cells. In many mammals unihemispherical sleep is a phenomenon where only one hemisphere of the brain shows sleep activity and the sleeping side of the brain exhibits highly synchronized activity while the awake side shows desynchronized activity [10]. Chimera states have also been identified modeled in mechanical oscillator networks [11], formation of ocular dominance stripes [12], in social systems, during ventricular fibrillation [13], and so on [7]. Very recently distinct measures to characterize these state have also been introduced [14].

Earlier studies indicated that coupled oscillator systems should have weak, and nonlocal coupling in order to have chimera behaviour. Even though considerable promising results have been obtained both experimentally [1,11,15] and theoretically [16,17], the above two essential ingredients posed a restriction on the occurrence of chimera states. Interestingly, recently chimera states have been found to exist in globally coupled systems as well. The occurrence of chimera states is found to be mediated by the amplitude in the coupling in complex Ginzburg-Landau systems [18]. The existence of chimera states due to delay induced multistability have also been recently reported [19].

In this Letter, we explore and bring out explicitly the exact reason behind the occurrence of chimera states in globally coupled systems without the inclusion of weak, and nonlocal coupling. We find that the introduction of intensity dependent interaction increases the number of fixed points which in turn increases the number of multistable attractors in the system whose stability is determined by the strength of the coupling between the individual units. On this reasoning we find that the system exhibits a multistable behaviour where it follows a path either to synchronized state or to a chimera state depending upon the initial state.

In order to exemplify our findings, we consider a system of globally coupled Rössler oscillators

\[
\begin{align*}
\dot{x}_i &= -\omega_0(1 - a x_i^2)y_i - z_i + \epsilon(X - x_i) \\
\dot{y}_i &= \omega_0(1 - a y_i^2)x_i + ay_i + \epsilon(Y - y_i) \\
\dot{z}_i &= b + (x_i - c)z_i + \epsilon(Z - z_i), \quad i = 1, ..., N
\end{align*}
\]

where \(\{X, Y, Z\} = \frac{1}{N} \sum_{i=1}^{N} \{x_i, y_i, z_i\}\) and \(-\alpha r^2 = -\alpha(x_i^2 + y_i^2)\) represents the intensity dependent modification of the frequency. The parameter \(\alpha\) represents the strength of intensity and \(\epsilon\) is the coupling constant.

To begin with, we demonstrate the need for the introduction of intensity dependent frequency term in the system for the chimera state to exist. We consider \(\alpha = 0\) (absence of intensity) to start with, and we have plotted the strength of incoherence \(S\) after the removal of discontinuity [13] in Fig. 1 against the coupling strength \(\epsilon\) for \(a = 0.2, b = 1.7\) and \(c = 5.70\) (the individual units of the array are in the periodic regime for these parameters). We find that the system transits from a desynchronized state, characterized by \(S = 1\), to a synchronized/cluster state \((S = 0)\) upon increasing the coupling strength (represented by dotted line in Fig 1(a)).

A chimera state \((0 < S < 1)\) does not exist in the system in this case. Now, let us introduce intensity dependent
interaction in the system by setting $\alpha = 0.02$. The system takes a chimera route to the synchronized state from the desynchronized state upon increasing the coupling strength as shown in Fig. 1(a) as dashed line. Similarly chimera states emerge for $\alpha = 0.04$ (dashed dotted line) and $\alpha = 0.1$ (solid line).

In Fig. 1(b) the time evolution of the oscillators and in panel (c) the time averaged frequency of the oscillators are plotted for $\alpha = 0.02$ and $\epsilon = 0.07$. We clearly see the coexistence of synchronized and desynchronized oscillators. Fig. 1(d) demonstrates the absence of chimera states when $\alpha = 0$, where only the synchronized state exists. Hence we name this behaviour as intensity induced chimera.

In order to confirm the ubiquitous nature of the intensity induced chimeras we next consider a system of limit cycle exhibiting van der Pol oscillators coupled in a global fashion

$$\ddot{x}_i = b(1-x_i^2)x_i - (\omega_0^2 + \alpha_1 x_i^2 + \alpha_2 x_i^4)x_i + \epsilon(\dot{X} - \dot{x}_i) + \eta(X - x_i), \quad i = 1, \ldots, N$$

(2)

where $\alpha_i$, $i = 1, 2$, are the intensity parameters. Let us refer to Fig. 2(a) where we have plotted the snapshot of the state variables $x_i$ for two different initial states IS1 (black) and IS2 (grey). In IS1 the initial state of the oscillators are randomly distributed in $[-1,1]$ whereas in IS2 the initial state of the oscillators are chosen to be close to the synchronized state. For IS1 the system exhibits a chimera state whereas for IS2 we see only a synchronized state. Once again this confirms the existence of intensity induced chimera in the system. However since the onset of chimera depends on the initial state, the state is multistable.

We have also confirmed that the existence of chimera is solely induced by the intensity dependent term even when the individual units are in the chaotic regime in the Rössler system as shown in Fig. 2(b). One can note that this chimera state also coexists with the synchronized state leading to multistability in the system. The corresponding time averaged frequencies for IS1 for both the systems are plotted in the insets of Fig. 2.

In the chimera state the synchronized $(x_i)$ and desynchronized $(x_k)$ oscillators are represented by

$$\ddot{x}(l,k) = b(1-x(l,k)^2)x(l,k) - (\omega_0^2 + \omega_1 x(l,k)^2 + \omega_2 x(l,k)^4)x(l,k)$$

$$\times x(l,k) + \epsilon(\dot{X} - \dot{x}(l,k)) + \eta(X - x(l,k)), \quad l = 1, \ldots, d, k = 1, \ldots, s, s+d = N$$

(3)

$$\ddot{x}(s+d) = A(s,d)(\tau)e^{i\omega_0 t_0} + A(s,d)(\tau)^*e^{-i\omega_0 t_0},$$

where $A(s,d)(\tau)$ represents the amplitude of the synchronized and desynchronized oscillators and $t = t_0 + \tau; \tau$ and $\tau$ represent the fast and slow time scales, respectively. Thus the dynamics of each desynchronized oscillator is represented by

$$\frac{dA}{d\tau} = (b + i\Omega)A - (b + i(\beta + \gamma |A|^2))|A|^2A$$

$$- (\epsilon + i\zeta)(A - 1), \quad (4)$$

FIG. 1: (Color online) (a) The strength of incoherence $S$ is plotted against coupling strength $\epsilon$ for different values of intensity $\alpha$ for system (1). (b) Time evolution of $x_i$ for the chimera state is plotted for $\alpha = 0.02$, $\epsilon = 0.07$ and $N = 500$. Thick black line represents synchronized oscillations in the chimera state and the red/grey lines represent desynchronized oscillations. (c) The averaged frequencies $\Omega_i = <\omega_i>$ of the chimera state are shown. (d) Synchronized behaviour of the system for $\alpha = 0$. 

FIG. 2: (Color online) Snapshots of $x_i$ are plotted for two different initial states IS1 (black) and IS2 (grey), where the system exhibits chimera and synchronized states, respectively. (a) van der Pol system (2) with the parameters $\alpha_1 = 2.18$, $\alpha_2 = 2.15$, $b = 1$, $\epsilon = 0.62$ and $\eta = 0.10$. (b) Rössler system (1) with the parameters $\alpha = 0.02$, $\epsilon = 0.13$, $a = 0.42$, $b = 2$ and $c = 4$. Insets show the corresponding time averaged frequencies for IS1 (chimera state).
where $A = A_0 e^{i \delta r}$ and $A_s = e^{-i \delta r}$. Here $\Omega = \beta + \gamma$, $\beta = 3 \alpha_1 / \omega_0$, $\gamma = 5 \alpha_2 / \omega_0$ and $\zeta = - \eta / \omega_0$. Now the stability of the desynchronized state $A$ determines the stability of the chimera state. Eq. (4) has five fixed points for $\gamma \neq 0$ and three fixed points for $\gamma = 0$. The specific fixed point $A = 1$ is stable and represents the completely synchronized state of the system. The stability of the other fixed points determine the existence of chimera state.

The stability nature of the fixed points is shown in Fig. 3(a) where we have plotted the bifurcation diagram in the $\epsilon$ vs $\beta$ space. Dot dashed, dotted and dashed lines represent the saddle node, Hopf and saddle connection bifurcation boundaries, respectively. Solid line represents the boundary above which other fixed points exist and is defined by $\epsilon_f = -(b^3 + b\beta(2 - \zeta) - \frac{(b^2 + \beta^2)(b^2 + \beta^2 - 4\beta\zeta)^2}{(2\beta^2)}$. The Hopf and saddle node curves are obtained by using $\epsilon_H = -2b^3 + 2b\beta\zeta + (b^2(5b^3 + \beta^2(\beta - 4\zeta) - 2(\beta^2 - 6\zeta - 2\zeta^2))^2)/(b^2 + \beta^2)$ and $\epsilon_{SN} = (b^2 + b\beta(\beta - 4\zeta) - 2(b^2 + \beta^2)(\beta - \zeta)^2)/(b^2 - 3\beta^2)$ for $\gamma = 0$. The saddle connection curve is obtained numerically by solving Eq. (4). In regions I and IV the system settles down in the synchronized state whereas in regions II and III the system exhibits clusters and chimera states, respectively.

The phenomenon of intensity induced chimera can be better understood from the perspective of $(\epsilon-\beta)$ space by looking at two transitions A and B in Fig. 3(a). Transition A denotes the case where intensity is very low in the system and one can clearly see a synchronization-synchronization transition for increasing coupling strength (in fact one cannot name this a transition, but we want to emphasize the absence of other states). On the other hand, in the case of transition B for a higher strength of intensity ($\beta = 5$) the system takes a synchronization-cluster-chimera-synchronization transition route which is similar to the swing-by mechanism addressed by Daido and Nakanishi [20]. However, in our case we find the existence of chimera state in addition to cluster states while the system swings by. The corresponding numerical behaviour of transition B is shown in the insets where we have plotted the snapshots of oscillator states in each region by solving Eq. (2).

The swing-by mechanism induced in system (2) is illustrated using the strength of incoherence before and after the removal of discontinuity [14], $S_0$ and $S$, respectively in Figs. 3(b) and 3(c). The existence of synchronized, cluster, chimera and desynchronized states are represented by $S$ and $S_0$, $(S, S_0) = (1, 1)$ represents the desynchronized state, $(S, S_0) = (0, 0)$ represents synchronized state. Further, $S = 0, 0 < S_0 < 1$ and $0 < S < 1, S_0 = 0$ represent cluster and chimera states, respectively [14].

In Fig. 3(b) we have plotted $S$ for different values of intensity $\alpha_1$. For $\alpha_1 = 5$ and 7 we clearly see the existence of chimera states sandwiched between two synchronized/cluster states ($S = 0$). However for a lower intensity, $\alpha_1 = 2$, the chimera state is absent and the system exhibits only the synchronized state as denoted by route A shown in Fig 3(a). In Fig. 3(c) we have plotted both $S$ and $S_0$ for $\alpha_1 = 10$ and we can clearly see the existence of cluster states in region II where there is mismatch between $S$ and $S_0$ followed by chimera states in region III. This is a realization of route B indicated in Fig. 3(a).

If we delve into the reasoning behind the occurrence of chimeras, one can deduce from the above analytical findings that the system acquires multistability upon introduction of intensity dependent interaction and sufficiently strong coupling. The effect of intensity in the system is to increase the number of fixed points whereas the stability of these fixed points is essentially determined by the coupling strength.

Let us validate these claims for chaotic oscillators: If we consider the Rössler system, and follow a similar analysis as in the case of periodic van der Pol oscillators, the dynamics of the desynchronized oscillators is given by

\[
\begin{align*}
\dot{x}_d &= -\omega_0(1 - \alpha r_3^2)y_d - z_d + \epsilon(x_s - x_d) \\
\dot{y}_d &= \omega_0(1 - \alpha r_3^2)x_d + ay_d + \epsilon(y_s - y_d) \\
\dot{z}_d &= b + (x_d - c)z_d + \epsilon(z_s - z_d),
\end{align*}
\]

where $\{x, y, z\}_s$ are the state variables of the synchronized oscillators which are uncoupled from the desynchronized ones so that the dynamics of the former is not influenced by the latter. We have shown the existence of multistability in the system and the reasoning behind the occurrence of chimera in Fig. 4. In the absence of coupling ($\epsilon = 0$) and intensity dependent interaction ($\alpha = 0$) we have plotted the time series of system (5) in Fig 4(a) where all the oscillators in the system oscillate periodically with same amplitude and frequency but with different phases. Fig 4(b) shows the phase plot for the system and the black circle corresponds to the desynchronized state shown in Fig. 4 (a). We have taken $10^2$ different initial conditions and as we can see all of them are drawn towards the desynchronized periodic attractor $A_1$ (black circle). Now, if we introduce intensity in the system ($\alpha = 0.01$) the number of fixed points increase (not shown in the figure); however, only one stable desynchronized attractor $B_1$ exists as shown in Fig. 4 (b) red/grey circle. The corresponding time series of the system is shown in Fig. 4 (c).

Now if we consider the presence of coupling in the system, we find that the oscillators are completely synchronized as evident from the time series plotted in Fig. 4 (d) in the absence of intensity. The black circle in Fig. 4 (e) shows the existence of one synchronized attractor $A_1$ for this case. However in the presence of coupling if the intensity is also present, in addition to the desynchronized attractor $B_1$ we get a synchronized attractor $B_2$. The attractor $B_1$ in the case of absence of coupling (shown
FIG. 3: (Color online) (a) Analytical bifurcation diagram in $(\epsilon-\beta)$ space showing various dynamical regimes of system (2). Dot dashed, dotted and dashed lines represent the saddle node (SN), Hopf and saddle connection (SC) bifurcation boundaries, respectively. Solid line and black circle represent the existence of fixed points (EFP) and Takens-Bogdanov point (TB), respectively. Regions I and IV represent synchronization regime and regions II and III represent the cluster and chimera regimes, respectively. The insets show the snapshot of $x_i$ in the corresponding regions. Lines A and B represent two different synchronization routes taken by the system (explained in the text). (b), (c) The strengths of incoherence $S(\star)$ and $S_0(\circ)$ are plotted against $\epsilon$ for various $\alpha_1$. The parameter values are $b = 1$, $\zeta = 0$ and $\beta_2 = 0$.

in Fig. 4 (b)) represents the oscillators that are desynchronized only in the phase but have the same amplitude and frequency. On the other hand, the attractor $B_1$ in the presence of coupling (shown in Fig. 4 (e)) represents the oscillators that are not only desynchronized in phase but also have different amplitudes and frequencies. The coexistence of attractors $B_1$ and $B_2$ demonstrates the existence of chimera states in the system which is evident from the corresponding time series shown in Fig. 4 (f). The bold green/black line represents the synchronized group in the attractor $B_2$ and the desynchronized oscillators in attractor $B_1$ are shown in thin red/grey lines.

From the above we conclude that even though the presence of intensity dependent term in globally coupled system increases the number of fixed points, their stability is determined by the strength of coupling. Since there are more number of stable attractors in the system when coupling is present (in both the van der Pol and Rössler case), depending upon the initial state the system will settle down to a chimera state or a different collective state as demonstrated in Fig. 2 where for two different initial states we get chimera and synchronized states. If we choose the initial state of the system to be close to attractor $B_2$ we get a synchronized state in the system for increasing coupling strength. On the other hand, if we choose the initial state so that the oscillators are spread across close to different attractors (like Fig. 4(e)) we can either get a chimera state or a multi clustered state.

The above dynamical aspects resemble task specific synchronization/desynchronization in the brain [21, 22], where depending upon the state of the neuronal oscillations at the time of incoming tasks, the brain either chooses to accept a new task or ignores it and continues with the current task. Thus chimera states play a crucial role in the functioning of a normal brain where it is capable of taking up and processing multiple tasks at once (corresponding to multistability). On the other hand, in the case of pathological brain, due to the presence of damaged cells or due to the loss of normal brain cells [23], the coupling in the system is either too low or too high (as experienced by the individual neurons) so that the system swings to a mass pathological synchronization state corresponding to regions I or IV in Fig. 3. In the case of spin torque nano oscillators the problem of coupling a large number of oscillators in order to achieve coherent microwave power still remains open [24]. This is because introducing the coupling leads to an increase in the number of stable attractors in the system [25], and this causes the emergence of chimera like states (synchronized clusters and desynchronized oscillators coexist) [26].

Our results therefore help in identifying the underlying cause for the crucial dynamical phenomena that occur in real world systems. The results also assist in gaining a better understanding of those dynamical phenomena thereby possibly helping researchers to control the occurrence of them depending upon whether they are desirable (task specific synchronization in the brain, and synchronization of spin torque nano oscillators) or undesirable (pathological mass synchronization).
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