Kerr-AdS Bubble Spacetimes and Time-Dependent
AdS/CFT Correspondence

A.M. Ghezelbash\textsuperscript{1} and R. B. Mann \textsuperscript{2}
Department of Physics, University of Waterloo,
Waterloo, Ontario N2L 3G1, Canada

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Abstract

We compute the boundary stress-energies of time-dependent asymptotically AdS spacetimes in 5 and 7 dimensions, and find that their traces are equal to the respective 4 and 6 dimensional field-theoretic trace anomalies. This provides good supporting evidence in favour of the AdS/CFT correspondence in time-dependent backgrounds.

\textsuperscript{1}EMail: amasoud@sciborg.uwaterloo.ca
\textsuperscript{2}EMail: mann@avatar.uwaterloo.ca
1 Introduction

An essential ingredient in constructing a theory of quantum gravity is to understand its behaviour in time-dependent settings. A recent promising approach to this end is to construct simple time-dependent solutions that provide (at least to leading order) consistent time-dependent backgrounds for string theory. This has been carried out in asymptotically flat spacetimes, and recently extended to include the asymptotically anti de Sitter (AdS) case [1, 2, 3, 4, 5]. This latter situation is of interest since the AdS/CFT correspondence conjecture could be employed to relate the time-dependence to the behaviour of the non-perturbative field theory dual. In the context of string theory this has recently led to intensive investigation of asymptotically de Sitter spacetimes to see what the prospects are for developing a de Sitter/CFT correspondence [6].

In this paper, we study in more detail the higher dimensional bubble spacetimes derived from analytic continuation of odd-dimensional Kerr-AdS spacetimes [7, 8, 9]. We find that the relationship between the field-theoretic trace anomaly [10] and the asymptotic boundary stress-energy is in agreement with the AdS/CFT prediction, demonstrating evidence for the correspondence for time-dependent backgrounds that are asymptotically AdS.

Although the relationship between the boundary stress-energy tensor and the trace anomaly have been previously studied [10, 11], these results cannot be directly applied to the time-dependent case due to subtleties in these settings. As an example of these subtleties, in ref. [12] the authors studied the time-dependent metric obtained by analytic continuation of the non-abelian T-dual of the Schwarzschild-AdS black hole. The result of the calculation shows that the trace of the boundary stress-energy tensor is different from the well known general trace anomaly. To solve this discrepancy, the authors considered the dualized model in the time direction and found that time-dependent AdS/CFT holds only on a special subspace of the whole spacetime. These observations indicate that some specific modifications may be needed in the application of the AdS/CFT correspondence to time-dependent backgrounds.

Motivated by the above, in this paper we study the AdS/CFT correspondence applied to time-dependent bubble spacetimes in five and seven dimensions. Seven is the smallest dimensionality with acceptable bubble structure (no signature changes or ergoregions [5]) in which the correspondence can be non-trivially tested. We compute the boundary stress-energies associated for such seven-dimensional bubble solutions and show that their traces are proportional to the Euler densities of the six-dimensional field theory, in accord with the AdS/CFT correspondence for a time-dependent setting. We also consider a double analytic continuation of five-dimensional Kerr-AdS spacetime. Although this bubble spacetime has an ergoregion (raising difficult issues as to its interpretation as a background) we again find an equivalence between the trace of the boundary stress-energy tensor and four-dimensional Euler density. The equivalence shows that the AdS/CFT correspondence conjecture holds even though the background metric changes sign. We extend our considerations to double analytic continuation of five-dimensional Kerr-AdS spacetime with two rotation parameters, again finding an equivalence between the trace of the stress-energy tensor and four-dimensional Euler density despite the presence of an ergoregion. Higher dimensional Kerr-AdS spacetimes have two acceptable double analytic continuations [2]. One involves the continuation of one of the coordinates in the $d\Omega_{d-4}$ part of the $d$-dimensional Kerr-AdS metric,
yielding

$$ds^2 = \frac{\Delta_r}{\rho^2} \{d\chi + \frac{a}{2}(1 - X^2) d\phi\}^2 + (1 - X^2) \frac{\Delta_r}{\rho^2} \{a \ell d\chi - \frac{2 \ell^2}{\Xi} \ell^2 d\phi\}^2$$

$$+ \frac{\rho^2}{\Delta_r} d\tau^2 + \frac{\rho^2}{(1 - X^2) \Delta_r} dX^2 + \ell^2 r^2 X^2 \{-d\tau^2 + \cosh^2 \tau d\Omega^2_{d-5}\}$$

where

$$\Delta_r = (r^2 \ell^2 - a^2)(1 + r^2) - \frac{2GM}{(\ell \theta)^d - a}$$

$$\Delta_X = 1 + \frac{a^2 X^2}{\ell^2} \ , \quad \Xi = 1 + \frac{a^2}{\ell^2}$$

(2)

The bubble metric (1) has a time dependent \((d - 4)\)-dimensional de Sitter space provided \(d \geq 6\). Since only a coordinate of the \(S^{d-4}\) is continued there is no Milne phase in the bubble evolution.

The other bubble metric can be obtained from continuation of the one angular coordinate of the spherical section embedded in a \((d - 4)\)-dimensional hyperbolic space. The bubble metric is

$$ds^2 = \frac{\Delta_r}{\rho^2} \{d\chi - \frac{a}{2}(X^2 - 1) d\phi\}^2 + (X^2 - 1) \frac{\Delta_r}{\rho^2} \{a \ell d\chi - \frac{2 \ell^2}{\Xi} \ell^2 d\phi\}^2$$

$$+ \frac{\rho^2}{\Delta_r} d\tau^2 + \frac{\rho^2}{(X^2 - 1) \Delta_r} dX^2 + \ell^2 r^2 X^2 \{d\psi^2 + \sinh^2 \psi (-d\tau^2 + \cosh^2 \tau d\Omega^2_{d-6})\}$$

where

$$\Delta_r = (r^2 \ell^2 - a^2)(r^2 - 1) - \frac{2GM}{(\ell \theta)^d - a}$$

$$\Delta_X = 1 - \frac{a^2 X^2}{\ell^2} \ , \quad \Xi = 1 - \frac{a^2}{\ell^2}$$

(4)

The bubble metric (3) again has a time dependent \((d - 4)\)-dimensional de Sitter space, in this case provided \(d \geq 7\). As with the metric (1) there is no Milne phase because only an angular coordinate is analytically continued.

2 Boundary stress-energy tensor and trace anomaly for 7-dimensional Kerr-AdS bubbles

We compute the boundary stress-energy tensor for the bubble solutions (1) and (3) for \(d = 7\) using the well-known counterterm method [13]. The results are

$$8\pi GT_\tau = \frac{-1}{80 \ell^2 r^2 \sigma} (\pm 5 \ell^6 - 80 \ell^2 GM \pm 46a^4 X^2 \ell^2 + 51a^6 X^4 + 3a^2 \ell^4 X^2 - 55a^6 X^6 \mp 51a^4 \ell^4 X^2 \mp 5a^2 \ell^4 X^2 \mp 5a^4 \ell^2 - 5a^6) + O(\frac{1}{r^6})$$

$$8\pi GT_\phi = \frac{-1}{80 \ell^2 r^2 \sigma} (\pm 5 \ell^6 + 101a^4 \ell^2 - a^2 \ell^4 - 189a^6 X^2 + 231a^6 X^4 - 55a^6 X^6 - 80 \ell^2 GM \pm 168a^4 \ell^2 X^2 \pm 3a^2 \ell^4 X^2 \mp 51a^4 \ell^2 X^2 \pm 25a^6) + O(\frac{1}{r^6})$$

$$8\pi GT_\chi = \frac{-1}{80 \ell^2 r^2 \sigma} (\pm 5 \ell^6 - 5a^6 + 5a^2 \ell^4 \pm 5a^4 \ell^2 - 80 \ell^2 GM \pm 66a^4 \ell^2 X^2 \mp 3a^2 \ell^4 X^2 + 45a^6 X^4 - 25a^6 X^6 - 3a^6 X^2 \mp 45a^4 \ell^2 X^2) + O(\frac{1}{r^6})$$

$$8\pi GT'_\chi = \frac{3}{40 \ell^2 r^2 \sigma} a(-\ell^2 \mp a^2)(30a^4 X^4 \pm 32a^2 \ell^4 X^2 - 32a^4 X^2 \mp 22a^2 \ell^2 + 5 \ell^4 + 5a^4) + O(\frac{1}{r^6})$$

$$8\pi GT'_\lambda = 8\pi GT'_\phi = 8\pi GT'_r$$

(5)

where the upper signs refer to the bubble (1) and the lower signs to (3). From this relations, after rescaling with an appropriate power of conformal factor, (the boundary metric diverges due to an
infinite conformal factor and so by rescaling of the boundary metric, the CFT lives on a boundary without any divergences), we obtain

\[ \tilde{T}_\mu{}^\mu = \frac{-3a^2}{32\pi G\ell^7}(8a^4X^4 - 5a^4X^6 + 2\ell^4 \mp 2a^2\ell^2 \mp 8a^2X^4\ell^2 \pm 9a^2\ell^2X^2 - 3a^4X^2 - 3\ell^4X^2) + O\left(\frac{1}{r^2}\right) \] (6)

for the respective traces of the boundary stress-energies.

Alternatively the trace anomaly in the stress-energy tensor of a classically Weyl-invariant even-dimensional quantum field theory has the structure [10, 14]

\[ \mathcal{A} = \mathcal{A} + \mathcal{B} + \mathcal{D} \] (7)

where \( \mathcal{A} \) is proportional to the even-dimensional Euler density, \( \mathcal{B} \) is a sum of independent Weyl invariants that contain the Weyl tensor and its derivatives, and \( \mathcal{D} \) is a total derivative term of a covariant expression that can be removed by adding suitable local counterterms. Since the six-dimensional boundary metrics of the bubble spaces (1) and (3) are conformally flat, the Weyl invariant terms vanish. The only non-vanishing term in (7) is then the Euler density, which in six dimensions is given by [10],

\[ E_6 = \frac{1}{6912}(K_1 - 12K_2 + 3K_3 + 16K_4 - 24K_5 - 24K_6 + 4K_7 + 8K_8) \] (8)

in terms of metric invariants

\[
\begin{align*}
K_1 &= R^3, \\
K_2 &= RR_{\alpha\beta}R_{\alpha\beta}, \\
K_3 &= RR_{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}, \\
K_4 &= R_{\alpha\beta}^\gamma R_{\gamma\beta}^\alpha, \\
K_5 &= R_{\alpha\beta} R_{\alpha\gamma}^\delta R_{\gamma\delta}^\alpha, \\
K_6 &= R_{\alpha\beta} R_{\alpha\gamma}^\delta R_{\gamma\delta} R_{\gamma\delta}, \\
K_7 &= R_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}, \\
K_8 &= R_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}
\end{align*}
\] (9)

where \( R_{\alpha\beta\gamma\delta} \) is the curvature tensor of the boundary metric. Inserting (9) into (8) we find that for rescaled boundary metric, after a lengthy calculation

\[ E_6 = \frac{a^2}{24\ell^2}(-8a^4X^4 + 5a^4X^6 - 2\ell^4 \pm 2a^2\ell^2 \pm 8a^2X^4\ell^2 \mp 9a^2\ell^2X^2 + 3a^4X^2 + 3\ell^4X^2) + O\left(\frac{1}{r^2}\right) \] (10)

for the respective bubbles (1) and (3) in the large-\( r \) limit.

Comparing our result (6) to the Euler density (10) of the boundary theory we find

\[ \tilde{T}_\mu{}^\mu = \alpha E_6 \] (11)

where \( \alpha = \frac{9\ell^5}{4\pi G} = \frac{12N^3}{\pi^2} \). In the last equation, we employed the relation \( G = \frac{3\pi^2\ell^5}{16N^3} \) for the seven-dimensional Newton’s constant. Note that our definition of the Euler density (8) is different by a factor of \( \frac{-1}{55296} \) with the definition of the Euler density considered in [14] which we denote it by \( \hat{E}_6 \). In other words, \( E_6 = \frac{1}{55296} \hat{E}_6 \), and so from the relation (11), we have

\[ \tilde{T}_\mu{}^\mu = -\frac{N^3}{4608\pi^3} \hat{E}_6 = \frac{4N^3}{(4\pi)^37!} \left( -\frac{35}{2} \hat{E}_6 \right) \] (12)

which is in good agreement with what is predicted by AdS/CFT correspondence. The conformal anomaly of the free 6-dimensional chiral (2,0) tensor multiplet has been calculated in [14] and the
result is similar to (12), up to a well known factor of $4/7$. The reason of this difference is due to
the fact that the coefficient in front of the Euler density in six dimensions is related to the 4-point
function which is not governed by a non-renormalization theorem.

We recall here some points about the field-theoretic calculation of the trace anomaly in the
time-dependent metrics. The trace anomaly is related to the logarithmically divergent part of
the effective action $\Gamma = \frac{1}{2} \log \det \Delta$ of any Euclidean field theory defined over a $d$–dimensional
compact Riemannian manifold $M$, where $\Delta$ is the Laplace operator. After using the Seely-DeWitt
asymptotic expansion $[15]$, the logarithmically divergent part of the effective action takes the form

$$\Gamma_{\infty} = -\frac{1}{2} \log \frac{L^2}{\mu^2} \int_M d^d x \sqrt{g} b_d$$

where $L$ is an UV cut-off and then the trace anomaly $T^\mu_\mu$ is simply equal to $b_d$, the Seely coefficient
[14] of the non-singular term in the Seely-DeWitt asymptotic expansion of the operator $\Delta$.

The above field theoretic trace anomaly calculation is valid if we switch to a time-independent
Lorentzian field theory by performing a Wick rotation on Euclidean field theory. For time-
dependent Lorentzian field theory (which at least locally could change to a static form by some
Wick rotations), the trace anomaly calculation also is valid since in this case, after applying the
proper Wick rotations, we get an Euclidean field theory. As an example, in the next section, the
boundary Euclidean field theory for a bubble spacetime after Wick rotation is considered.

The calculation of $b_d$ shows that the general form of the trace anomaly for an even $d$-dimensional
field theory is in the form of (7).

3 Boundary stress-energy tensor and trace anomaly for
5-dimensional Kerr-AdS bubbles with one and two ro-
tational parameters

We now extend these considerations to Kerr-AdS bubbles in five dimensions. The time dependent
five-dimensional metric is given by

$$ds^2 = \frac{\Delta_r}{\rho^2} \{ d\chi + \frac{a}{\Xi}(1 + X^2)d\phi \}^2 + (1 + X^2) \frac{\Delta_X}{\rho^2} \{ ad\chi - \frac{r^2 - a^2}{\Xi} d\phi \}^2$$

$$+ \rho^2 d\tau^2 - \frac{\rho^2}{(1 + X^2)\Delta_X} dX^2 + r^2 X^2 d\lambda^2$$

where

$$\Delta_r = (r^2 - a^2)(1 + r^2/\ell^2) - 2GM$$

$$\Delta_X = 1 + \frac{a^2 X^2}{\ell^2} , \quad \Xi = 1 + \frac{a^2}{\ell^2}$$

These bubbles have ergoregions, unlike their seven-dimensional counterparts. For (14) the condi-
tion

$$\Delta_X < 0$$

defines the ergoregion.
Hence (14) describes a well defined bubble metric provided \(|X| < X_{cr}\) where \(X_{cr} = \ell/a\). While this might render them unsuitable as pure bubble metrics [2], we find that trace anomaly in of the boundary stress-energy tensor (7) is related to the four-dimensional Euler density of the boundary metric of the bubble (14), in accord with the AdS/CFT prediction. Specifically we find that the large-\(r\) boundary stress-energy tensor components for the bubble spacetime (14) are

\[
\begin{align*}
8\pi G T_{\phi}^\phi &= -\frac{a^2}{8\ell} \{ (7X^2 + 10)X^2a^4 - 2a^2\ell^2(X^2 + 1) - \ell^2(\ell^2 + 8GM) + 3a^4 \} + O\left(\frac{1}{r^2}\right) \\
8\pi G T_{X}^X &= -\frac{1}{8\ell} \{ (3X^2 + 2)X^2a^4 - 2a^2\ell^2(X^2 + 1) - \ell^2(\ell^2 + 8GM) - a^4 \} + O\left(\frac{1}{r^2}\right) \\
8\pi G T_{\lambda}^\lambda &= -\frac{1}{8\ell} \{ (7X^2 + 2)X^2a^4 - 2a^2\ell^2(5X^2 + 1) + 3\ell^2(\ell^2 + 8GM) - a^4 \} + O\left(\frac{1}{r^2}\right) \\
8\pi G T_{\phi}^X &= -\frac{1}{8\ell} a\ell^2(2a^2X^2 - \ell^2 + a^2) + O\left(\frac{1}{r^2}\right)
\end{align*}
\]

and so its trace after rescaling is

\[
\tilde{T}_\mu^\mu = \frac{a^2}{8\pi G \ell} \{ a^2X^2(3X^2 + 2) - \ell^2(1 + 2X^2) \} + O\left(\frac{1}{r^2}\right)
\]

The trace anomaly (7) of the four-dimensional boundary of the bubble spacetime has the same form of (7), where \(A\) is proportional to the Euler density in four dimensions. The \(B\) term vanishes since the boundary metric is conformally flat. The Euler density is given by the relation [10]

\[
E_4 = \frac{1}{64} \left( \frac{2}{3} \mathcal{R}^2 - 2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} \right)
\]

where \(\mathcal{R}_{\mu\nu}\) and \(\mathcal{R}\) are the Ricci tensor and Ricci scalar of the boundary spacetime. For the boundary metric of the bubble (14), relation (19) yields,

\[
E_4 = \frac{a^2}{4\ell^4} \{ a^2X^2(3X^2 + 2) - \ell^2(1 + 2X^2) \} + O\left(\frac{1}{r^2}\right)
\]

Comparing (18) with (20), we find

\[
\tilde{T}_\mu^\mu = \beta E_4
\]

where \(\beta = -\frac{\ell^3}{2\pi G} = -\frac{N^2}{r^2}\), in good agreement with the prediction of AdS/CFT correspondence. In the above calculation, we used the standard relation between the four-dimensional Newton’s constant and \(N\), which is given by \(2N^2G = \pi\ell^3\). The bubble metric (14), after the following coordinate transformations:

\[
\begin{align*}
\Xi y^2 \cosh^2 \Theta &= (r^2 - a^2)(1 + X^2) \\
y \sinh \Theta &= rX \\
\Phi &= \phi + a\chi/\ell^2
\end{align*}
\]

and a rescaling by the factor \(\frac{\ell^2}{y^2}\), at large \(y\), takes the form

\[
ds_6^2 = d\chi^2 + \ell^2(-d\Theta^2 + \cosh^2 \Theta d\Phi^2 + \sinh^2 \Theta d\lambda^2)
\]

for \(y \to \infty\), which is a time dependent Einstein universe. We note all dependence on the rotational parameter disappears in boundary metric when we go on the hypersurface of constant large \(y\).
This observation is in agreement with the result on the boundary of Kerr-AdS metric [7].

The metric (23) has a positive Ricci scalar, which describes locally a (2 + 1)-dimensional dS space times a circle. Locally, the boundary metric (23) could be written as

\[ ds_b^2 = d\chi^2 + \ell^2 \{ -d\Theta^2 + \cosh^2 \Theta (d\alpha^2 + \sin^2 \alpha d\beta^2) \} \]

which by the Wick rotation \( \Theta \rightarrow i\Omega \) changes to the following static metric:

\[ ds_b^2 = d\chi^2 + \ell^2 \{ d\Omega^2 + \cos^2 \Omega (d\alpha^2 + \sin^2 \alpha d\beta^2) \} \]

and so the field theoretic trace anomaly calculation could be done on this Euclidean space.

For the dual field theory on the boundary, since the metric (23), is conformally flat we can use the following relation for the stress tensor [16]:

\[ \langle T_{\mu}^{\nu} \rangle = -\frac{1}{16\pi^2} (A^{(1)} H_{\mu}^{\nu} + B^{(3)} H_{\mu}^{\nu} + \tilde{T}_{\mu}^{\nu}) \]

where \( (1)^\mu_{\nu} \) and \( (3)^\mu_{\nu} \) are conserved quantities constructed from the curvatures via

\[ (1)^\mu_{\nu} = 2R_{\rho\nu\mu} - 2g_{\rho\nu} \Box R - \frac{1}{2} \eta_{\rho\nu} R^2 + 2RR_{\rho\nu} \]

\[ (3)^\mu_{\nu} = \frac{1}{12} R^2 g_{\rho\nu} - \tilde{R}^{\rho\sigma} R_{\rho\mu\sigma\nu} \]

and \( \tilde{T}_{\mu}^{\nu} \) is the traceless part and the constants \( A \) and \( B \) are given by: \( A = 0 \) and \( B \simeq \frac{\pi \ell^3}{4G^5} \). Using the boundary metric (23), we calculate the functions \( (1)^\mu_{\nu} \) and \( (3)^\mu_{\nu} \) and find that the first term of (26) reproduces some parts of stress tensor (17) which do not depend on bubble parameters \( a \) and \( M \).

Similar results hold for bubbles obtained by double analytical continuation of the two-rotation-parameter five-dimensional Kerr-AdS spacetime. The bubble metric after continuation is given by

\[ ds^2 = \frac{\Delta_r}{\rho^2} \left\{ d\chi + \frac{\rho^2}{\Xi_a} (1 + X^2) d\phi - \frac{b}{\Xi_b} X^2 d\lambda \right\}^2 + (1 + X^2) \frac{\Delta_r}{\rho^2} \left\{ a d\chi - \frac{r^2 - a^2}{\Xi_a} d\phi \right\}^2 + \frac{\rho^2}{\Delta_r} dr^2 \]

\[ + X^2 \frac{\Delta_r}{\rho^2} \left\{ -b d\chi - \frac{r^2 - b^2}{\Xi_b} d\lambda \right\}^2 - \frac{\rho^2}{(1 + X^2) \Delta_X} dX^2 + \frac{1}{\rho^2} \left\{ abd\chi - b(1 + X^2) \frac{r^2 - a^2}{\Xi_a} d\phi - a X^2 \frac{r^2 + b^2}{\Xi_b} d\lambda \right\}^2 \]

where

\[ \Delta_r = (r^2 - a^2)(r^2 + b^2)(1 + r^2/\ell^2) - 2GM \]

\[ \Delta_X = 1 - \frac{a^2 X^2}{\ell^2} - \frac{b^2 (1 + X^2)}{\ell^2} \]

\[ \Xi_a = 1 + \frac{a^2}{\ell^2} \]

\[ \Xi_b = 1 - \frac{b^2}{\ell^2} \]

It also contains an ergoregion \( \Delta_X < 0 \); hence (14) describes a well defined bubble when \( |X| < X_{cr} \).

The large-\( r \) boundary diagonal stress-energy tensor components for the metric (28) are

\[ 8\pi G T^{\phi}_{\phi} = \frac{1}{\text{str}^2} \left\{ -7X^4(a^2 + b^2)^2 + 2X^2 \left[ -5a^4 + \ell^2 (a^2 + b^2) - 6b^4 - 11a^2 b^2 \right] + 8GM \ell^2 - 8a^2 b^2 - 3a^4 - 4b^4 + 2a^2 \ell^2 + \ell^4 \right\} + O \left( \frac{1}{\ell^2} \right) \]

\[ 8\pi G T^{X}_{X} = \frac{1}{\text{str}^2} \left\{ -3X^4(a^2 + b^2)^2 + 2X^2 \left[ a^4 - \ell^4 (a^2 + b^2) - 2b^4 - 3a^2 b^2 \right] + 8GM \ell^2 + a^4 + 2a^2 \ell^2 + \ell^4 \right\} + O \left( \frac{1}{\ell^2} \right) \]

\[ 8\pi G T^{Y}_{Y} = \frac{1}{\text{str}^2} \left\{ -7X^4(a^2 + b^2)^2 - 2X^2 \left[ a^4 - 5\ell^2 (a^2 + b^2) + 6b^4 + 7a^2 b^2 \right] - 24GM \ell^2 - a^4 - 4b^4 + 2a^2 \ell^2 + 8b^2 \ell^2 - 3\ell^4 \right\} + O \left( \frac{1}{\ell^2} \right) \]

\[ 8\pi G T^{\lambda}_{\lambda} = \frac{1}{\text{str}^2} \left\{ -7X^4(a^2 + b^2)^2 + 2X^2 \left[ a^4 + \ell^2 (a^2 + b^2) - 2b^4 - 3a^2 b^2 \right] + 8GM \ell^2 + a^4 + 2a^2 \ell^2 + \ell^4 \right\} + O \left( \frac{1}{\ell^2} \right) \]
and the resultant trace after rescaling is
\[
\tilde{T}_{\mu} = \frac{1}{8\pi G\ell} \{ -3(a^2+b^2)^2X^4 + 2[-2b^4 - 3a^2b^2 - a^4 + (a^2 + b^2)\ell^2]X^2 + (a^2 + b^2)(\ell^2 - b^2) \} + O \left( \frac{1}{r^2} \right) \quad (31)
\]

The Euler density (19), after a lengthy calculation is given by
\[
E_4 = \frac{1}{4\ell^4} \{ 3(a^2 + b^2)^2X^4 - 2[-2b^4 - 3a^2b^2 - a^4 + (a^2 + b^2)\ell^2]X^2 - (a^2 + b^2)(\ell^2 - b^2) \} + O \left( \frac{1}{r^2} \right) \quad (32)
\]

Once again, comparing our result (31) with the Euler density of the boundary theory (32), we find
\[
\tilde{T}_{\mu} = \beta E_4 \quad (33)
\]

similar to the relation (21). The above result is in good agreement with the AdS/CFT correspondence.

4 Conclusions

We have demonstrated that the AdS/CFT correspondence conjecture gives reasonable results in time-dependent rotating bubble backgrounds: the predicted relationship between the field-theoretic trace anomaly and the asymptotic boundary stress-energy hold. This relationship is quite robust, being satisfied not only for the conventional bubble metrics (1) and (3) in \( d = 7 \), but also for the more complicated time-dependent metrics (14) and (28) with ergoregions. This work also shows the usefulness of the counterterm subtraction method applied to the time-dependent asymptotically AdS spacetimes. An interesting problem would be to see what happens to this prediction for the next-to-leading order terms. We leave this for future consideration.

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