Nuclear suppression of azimuthal asymmetries in semi-inclusive deep inelastic scattering off polarized targets

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We extend the study of nuclear dependence of the transverse momentum dependent parton distribution functions and azimuthal asymmetries to semi-inclusive deep inelastic scattering (SIDIS) off polarized nuclear targets. We show that azimuthal asymmetries are suppressed for SIDIS off a polarized nuclear target relative to that off a polarized nucleon due to multiple scattering inside the nucleus. Using the value of transport parameter inside large nuclei extracted from jet quenching analyses in SIDIS off nuclear targets, we also present a numerical estimate of the nuclear suppression of the azimuthal asymmetry that might be useful to guide the future experimental studies of SIDIS off polarized nuclear targets.

Introduction. Azimuthal asymmetries in semi-inclusive deep-inelastic lepton-nucleon scattering (SIDIS) are sensitive probes of the transverse momentum dependent (TMD) parton distributions and/or correlation functions in a nucleon. In SIDIS off nuclear targets, multiple parton scattering off different nucleons inside the nucleus can lead to many interesting phenomena such as suppression of leading hadrons [1, 2], transverse momentum broadening [3, 4] and nuclear modification of the azimuthal asymmetries [5, 6]. In a recent publication [6], we calculated azimuthal asymmetries in semi-inclusive DIS process $e^- + N$ or $A \rightarrow e^- + q + X$ for both polarized and unpolarized targets which could be a nucleon or nucleus. We also studied the nuclear dependence of the TMD parton distributions and correlation functions and azimuthal asymmetries in reactions with unpolarized target nuclei. Since future experiments at the Jefferson Laboratory for SIDIS with polarized nuclear targets are becoming possible [7], it is of great interest to extend our previous study and provide some numerical estimates of the predicted nuclear suppression of the azimuthal asymmetry for polarized nuclear targets. In this Brief Report we provide an addendum to [6] and extend the study of the nuclear dependence to reactions with polarized targets. We also present numerical estimates of the nuclear dependence of the azimuthal asymmetry.

Cross sections and azimuthal asymmetries. In the semi-inclusive process $e^- (l, s_i) + N(p, s) \rightarrow e^- (l') + q(k') + X$, where variables in the brackets denote the momenta and polarizations, the cross section and the azimuthal asymmetry can be obtained as functions of the TMD parton distributions and/or correlation functions [6, 8],

$$\frac{d\sigma}{dx_B dy d^2 k_T} = \frac{2\alpha_s^2 e_q^2}{Q^2 y} (W_{UU} + \lambda W_{UL} + s_W W_{UT}) + \lambda W_{LU} + \lambda \eta W_{LL} + \lambda \eta s_W W_{LT}),$$

where $W_{s,W}$’s represent contributions in the different polarization cases and are given by,

$$W_{UU} = A(y) f_1 - \frac{2x_B|k_{1\perp}|}{Q} B(y) f_1^+ \cos \phi,$$

$$W_{UL} = -\frac{2x_B|k_{1\perp}|}{Q} B(y) f_1^+ \sin \phi,$$

$$W_{LU} = -\frac{2x_B|k_{1\perp}|}{Q} D(y) g_1 \sin \phi,$$

$$W_{LL} = C(y) g_{1L} - \frac{2x_B|k_{1\perp}|}{Q} D(y) g_1^+ \cos \phi,$$

$$W_{LT} = \frac{|k_{1\perp}|}{M} C(y) g_{1LT} \cos (\phi - \phi_s),$$

$$-\frac{2x_B M}{Q} D(y) g_1^+ \cos \phi,$$

where

$$A(y) = 1 + (1 - y)^2, \quad B(y) = 2(2 - y)(\sqrt{1 - y}) - y,$$

$$C(y) = y(2 - y), \quad D(y) = 2\sqrt{1 - y},$$

$$\cos \phi = \mathbf{\tilde{l}}_T \cdot \mathbf{\tilde{k}}_{1\perp} / ||\mathbf{\tilde{l}}_T|| ||\mathbf{\tilde{k}}_{1\perp}||, \quad \sin \phi = (\mathbf{\tilde{l}}_T \times \mathbf{\tilde{k}}_{1\perp}) \cdot \mathbf{\tilde{e}}_z / ||\mathbf{\tilde{l}}_T|| ||\mathbf{\tilde{k}}_{1\perp}||,$$

$$\cos \phi_s = (\mathbf{\tilde{l}}_{1\perp} \times \mathbf{\tilde{k}}_{1\perp} / ||\mathbf{\tilde{l}}_{1\perp}|| ||\mathbf{\tilde{k}}_{1\perp}||), \quad \sin \phi_s = (\mathbf{\tilde{l}}_{1\perp} \times \mathbf{\tilde{k}}_{1\perp} / ||\mathbf{\tilde{l}}_{1\perp}|| ||\mathbf{\tilde{k}}_{1\perp}||),$$

where $\mathbf{\tilde{l}}_T$ and $\mathbf{\tilde{k}}_{1\perp}$ are Bjorken-$L$ and Bjorken-$T$ momentum carried by the struck quark.

The TMD parton distributions and/or correlation functions are defined via the decomposition of the correlation matrix element that is given by,

$$\Phi_{a}^{(0)}(X, k_{1\perp}) = \int \frac{p+dy d^2 y d^3 p_{LT}}{(2\pi)^3} e_{LT}^{y^a} e_{LT}^{y_B}(\mathbf{\tilde{l}}_T \cdot \mathbf{\tilde{e}}_L) N \langle \Phi_{a}^{(0)}(0; 0; y) \psi(y) | N \rangle / 2 + \ldots,$$

where $\Phi_{a}^{(0)}$ and $\Phi_{a}^{(0)}$ are decomposed as,

$$\Phi_{a}^{(0)} = \left( f_1 - e_{1\perp} f_1^+ \right) p_a + f_1^+ k_{1a} - f_T M e_{\perp a} k_{1\perp}$$

$$f_T^+(k_{1\perp} k_{1\perp} - \frac{1}{2} k_{1\perp}^2 d_{a\perp}) e_{1\perp} s_{1\perp} - A f_1^+ e_{\perp a} k_{1\perp} + \ldots$$
These asymmetries all depend on the TMD parton distribution contains information of the interaction between the struck quark and the remnant of the target. In a nucleus target, such interaction can occur between the struck quark and multiple nucleons inside the nucleus. Such multiple interaction will lead to nuclear broadening of the TMD quark distribution. Under the “maximal two-gluon” or random-walk approximation, the TMD quark distribution \( \Phi^q_{\alpha}(x, k_{\perp}) \) in nucleus,

\[
\Phi^q_{\alpha}(x, k_{\perp}) \equiv \int \frac{p^+ dp^+ dp^+_{\perp}}{(2\pi)^3} \langle A | \bar{\psi}(0) \Gamma_{\alpha} \mathcal{L}(0; y) \psi(y) | A \rangle,
\]

(23)
can be given by a convolution of the corresponding TMD quark distribution \( \Phi^q_{\alpha}(x, k_{\perp}) \) in a nucleon and a Gaussian broadening [9],

\[
\Phi^q_{\alpha}(x, k_{\perp}) \approx \frac{A}{\pi \Delta_{2F}} \int d^2 \xi_{\perp} e^{-(\xi_{\perp} \cdot \xi_{\perp})/\Delta_{2F}} \rho^N_{\alpha}(x, \xi_{\perp}),
\]

(24)
where \( \Gamma_{\alpha} \) is any gamma matrix. The broadening width \( \Delta_{2F} \), representing the total transverse momentum broadening squared, is given by,

\[
\Delta_{2F} = \int d^2 \xi_{\perp} \hat{q}_F(\xi_{\perp}),
\]

(25)
where the quark transport parameter,

\[
\hat{q}_F(\xi_{\perp}) = \frac{2\pi^2 \alpha_s}{N_c} \rho^N_{\alpha}(x, \xi_{\perp})[x f^N_{\alpha}(x)]_{x=0},
\]

(26)
is the effective transverse momentum broadening squared per unit distance for a fundamental quark which is proportional to the nucleon number density \( \rho^N_{\alpha}(x, \xi_{\perp}) \) and the gluon distribution \( f^N_{\alpha}(x) \) per nucleon.

The above relationship between the TMD quark distributions inside a nucleus and nucleon applies to both polarized and unpolarized targets. One can derive in particular [5, 6],

\[
f_1^A(x, k_{\perp}) \approx \frac{A}{\pi \Delta_{2F}} \int d^2 \xi_{\perp} e^{-(\xi_{\perp} \cdot \xi_{\perp})/\Delta_{2F}} f_1^N(x, \xi_{\perp}),
\]

(27)
\[
k_{\perp}^2 f_{1A}^N(x, k_{\perp}) \approx \frac{A}{\pi \Delta_{2F}} \int d^2 \xi_{\perp} e^{-(\xi_{\perp} \cdot \xi_{\perp})/\Delta_{2F}} f_{1N}(x, \xi_{\perp}),
\]

(28)
\[
k_{\perp}^2 g_{1A}^N(x, k_{\perp}) \approx \frac{A}{\pi \Delta_{2F}} \int d^2 \xi_{\perp} e^{-(\xi_{\perp} \cdot \xi_{\perp})/\Delta_{2F}} g_{1N}(x, \xi_{\perp}).
\]

(29)

To numerically estimate effects of the nuclear dependence of the TMD quark distribution, we take a Gaussian ansatz [5, 6] for TMD quark distributions in a nucleon,

\[
f_1^N(x, \xi_{\perp}) = \frac{1}{\alpha \beta} f_1^N(x) e^{-\xi_{\perp}^2/\alpha \beta},
\]

(28)
\[
f_{1A}^N(x, \xi_{\perp}) = \frac{1}{\alpha' \beta'} f_{1N}(x) e^{-\xi_{\perp}^2/\alpha' \beta'},
\]

(29)
\[
g_{1A}^N(x, \xi_{\perp}) = \frac{1}{\gamma} g_{1N}(x) e^{-\xi_{\perp}^2/\gamma},
\]

(30)

One can obtain the TMD quark distributions in a nucleus,

\[
f_1^A(x, k_{\perp}) \approx \frac{A}{\pi \sigma_A} f_1^N(x) e^{-k_{\perp}^2/\pi \sigma_A},
\]

(31)
\[
f_{1A}(x, k_{\perp}) \approx \frac{A}{\pi \sigma_A} f_{1N}(x) e^{-k_{\perp}^2/\pi \sigma_A},
\]

(32)
\[
g_{1A}(x, k_{\perp}) \approx \frac{A}{\pi \gamma_A} g_{1N}(x) e^{-k_{\perp}^2/\pi \gamma_A},
\]

(33)

Nuclear dependence. In SIDIS, the TMD quark distribution contains information of the interaction between the struck quark and the remnant of the target. In a nucleus target, such interaction can occur between the struck quark and multiple nucleons inside the nucleus. Such multiple interaction will
where \( \alpha_A = \alpha + \Delta_{2F}, \beta_A = \beta + \Delta_{2F}, \gamma_A = \gamma + \Delta_{2F} \). We see that they all have a \( k_1 \)-broadening with the same width \( \Delta_{2F} \). We also note that the twist-3 quark correlation functions \( f^{A}(x, k_1) \) and \( g^{A}(x, k_1) \) have an extra suppression factor \( \beta/\beta_A \) or \( \gamma/\gamma_A \).

The above approximation of TMD parton distributions and nuclear broadening can also be extended to SIDIS off polarized targets. Here, we consider a nucleus with atomic number \( A \) and spin \( J_A \) and study the \( A \)-dependence of the spin related TMD parton distributions and correlation functions. As a rough approximation, we assume that each nucleon has an equal polarization \( J_A/A \) inside a nucleus. In this case, the TMD quark distribution functions for a longitudinally polarized nucleus can be written similarly as,

\[
g^{A}_{1L}(x, k_1) \approx \frac{2J_A}{\pi\Delta_{2F}} \int d^2\ell \, e^{-i\vec{k}_1 \cdot \vec{\ell}/\Delta_{2F}} g^{N}_{1L}(x, \ell_1),
\]

\[
k^{A}_{1L}(x, k_1) \approx \frac{2J_A}{\pi\Delta_{2F}} \int d^2\ell \, e^{-i\vec{k}_2 \cdot \vec{\ell}/\Delta_{2F}} (k_1 \cdot \ell_1) f^{N}_{1L}(x, \ell_1),
\]

\[
k^{A}_{2L}(x, k_1) \approx \frac{2J_A}{\pi\Delta_{2F}} \int d^2\ell \, e^{-i\vec{k}_2 \cdot \vec{\ell}/\Delta_{2F}} (k_1 \cdot \ell_1) g^{N}_{2L}(x, \ell_1).
\]

These results are very similar as those for the unpolarized TMD distribution functions. The only difference is that the overall multiplicative factor \( A \) is now replaced by \( 2J_A \). If we take the Gaussian ansatz for the TMD quark distribution functions with parameters \( \alpha_x^T, \beta_x^T, \) and \( \gamma_x^T \) for the longitudinal polarization, we obtain,

\[
g^{A}_{1L}(x, k_1) \approx \frac{2J_A}{\pi \alpha_x^T} g^{N}_{1L}(x) e^{-\vec{k}_1^T / \alpha_x^T},
\]

\[
f^{A}_{1L}(x, k_1) \approx \frac{2J_A}{\pi \beta_x^T} f^{N}_{1L}(x) e^{-\vec{k}_2^T / \beta_x^T},
\]

\[
g^{A}_{2L}(x, k_1) \approx \frac{2J_A}{\pi \gamma_x^T} g^{N}_{2L}(x) e^{-\vec{k}_2^T / \gamma_x^T}.
\]

Similarly, in the transversely polarized case, we obtain,

\[
e^{k_1^T}_{\perp} f^{A}_{1T}(x, k_1) \approx \frac{2J_A}{\pi \alpha_x^T} \int d^2\ell \, e^{-i\vec{k}_1 \cdot \vec{\ell}} e^{-\vec{k}_2 \cdot \ell_{T}}/\Delta_{2F}
\]

\[
\times e^{\vec{k}_1^T / \alpha_x^T} f^{N}_{1T}(x, \ell_1),
\]

\[
(k_1 \cdot s_1)g^{A}_{1T}(x, k_1) \approx \frac{2J_A}{\pi \alpha_x^T} \int d^2\ell \, e^{-i\vec{k}_1 \cdot \vec{\ell}} e^{-\vec{k}_2 \cdot \ell_{T}}/\Delta_{2F}
\]

\[
\times (k_1 \cdot s_1)g^{N}_{1T}(x, \ell_1),
\]

\[
e^{k_1^T}_{\perp} (k_1 \cdot s_1)g^{A}_{1T}(x, k_1) \approx \frac{2J_A}{\pi \alpha_x^T} \int d^2\ell \, e^{-i\vec{k}_1 \cdot \vec{\ell}} e^{-\vec{k}_2 \cdot \ell_{T}}/\Delta_{2F}
\]

\[
\times \left[ e^{k_1^T}_{\perp} (k_1 \cdot s_1)g^{N}_{1T}(x, \ell_1) - (k_1 \cdot s_1)g^{N}_{1T}(x, \ell_1) \frac{k_1^T}{M^2} f^{N}_{1T}(x, \ell_1) \right],
\]

\[
\text{After integrating over } |\vec{k}_1|, \text{ we have,}
\]

\[
\frac{\langle \cos(\phi) \rangle_{UU}^A}{\langle \cos(\phi) \rangle_{UU}^N} \approx \frac{\langle \sin(\phi) \rangle_{UU}^A}{\langle \sin(\phi) \rangle_{UU}^N} \approx \frac{\alpha}{\alpha + \Delta_{2F}}.
\]

For reactions with polarized targets, the two leading twist azimuthal asymmetries in Eqs. (13) and (14) are given by,

\[
\frac{\langle \cos(\phi - \phi_1) \rangle_{UU}^A}{\langle \cos(\phi - \phi_1) \rangle_{UU}^N} \approx \frac{2J_A \alpha_A}{\alpha} \left( \frac{\gamma^2}{\alpha} e^{-\left( \frac{\phi - \phi_1}{\Delta_{2F}} \right)^2} \right),
\]

\[
\frac{\langle \cos(\phi - \phi_1) \rangle_{UU}^A}{\langle \cos(\phi - \phi_1) \rangle_{UU}^N} \approx \frac{2J_A \alpha_A}{\alpha} \left( \frac{\gamma^2}{\alpha} e^{-\left( \frac{\phi - \phi_1}{\Delta_{2F}} \right)^2} \right).
\]
The quark transport parameter is proportional to the nuclear proximity, it is taken as the quark's transverse momentum distribution in a nucleon. Approximated by Eq. (56), can be obtained. 

\[
\frac{\langle \sin \phi \rangle_{UL}^{CA}}{\langle \sin \phi \rangle_{UL}^{N}} \approx \frac{2J_A}{A} \frac{\alpha}{\beta_A^2} e^{\left(\frac{1}{\alpha} - \frac{1}{\beta_A} + \frac{1}{\alpha} - \frac{1}{\beta_A}\right)\xi^2},
\]

(56)

\[
\frac{\langle \sin \phi \rangle_{UL}^{CA}}{\langle \cos \phi \rangle_{UL}^{CA}} \approx \frac{2J_A}{A} \frac{\alpha}{\beta_A^2} e^{\left(\frac{1}{\alpha} - \frac{1}{\beta_A} + \frac{1}{\alpha} - \frac{1}{\beta_A}\right)\xi^2},
\]

(57)

\[
\frac{\langle \sin \phi \rangle_{UL}^{CA}}{\langle \sin \phi \rangle_{UL}^{N}} \approx \frac{2J_A}{A} \frac{\alpha}{\beta_A^2} e^{\left(\frac{1}{\alpha} - \frac{1}{\beta_A} + \frac{1}{\alpha} - \frac{1}{\beta_A}\right)\xi^2},
\]

(58)

If all the widths for the transverse momentum dependence are taken as the same, these ratios become equal and \( k_{\perp} \)-independent,

\[
\frac{\langle \sin \phi \rangle_{UL}^{CA}}{\langle \sin \phi \rangle_{UL}^{N}} \approx \frac{2J_A}{A} \frac{\alpha}{\alpha},
\]

(59)

\[
\frac{\langle \sin \phi \rangle_{UL}^{CA}}{\langle \cos \phi \rangle_{UL}^{N}} \approx \frac{2J_A}{A} \frac{\alpha}{\alpha},
\]

(60)

We see that the asymmetries in the case with a polarized nucleus target are in general much more suppressed by the dilution factor \( 2J_A/A \).

The situations for \( \langle \cos \phi \rangle_{LL} \) and \( \langle \cos \phi \rangle_{LT} \) are more complicated because of the competition between the two terms in the numerators and the denominators. No such simple results can be obtained.

**Numerical estimates.** As discussed in earlier, if the widths of the nucleon TMD parton distributions are taken as the same, i.e. \( \alpha = \beta = \gamma \) and they are the same for longitudinally or transversely polarized nucleon, all the asymmetries in the unpolarized case are suppressed by a factor \( \alpha/(\alpha + \Delta_{2F}) \) and those in reactions with polarized targets are further suppressed by the dilution factor \( 2J_A/A \). We see that the most important factor describing the nuclear dependence is \( \alpha/(\alpha + \Delta_{2F}) \) that we denote by a suppression factor \( f_s \) in the following.

The suppression factor \( f_s \) is determined by two parameters. The constant \( \alpha \) is the width of the Gaussian ansatz for the quark’s transverse momentum distribution in a nucleon. Approximately, it is taken as \( \alpha = 0.2 \sim 0.3 \) GeV\(^2\) (see e.g. [10]).

The other parameter is the nuclear broadening width \( \Delta_{2F} \) given by Eq. (25). We assume a hard-sphere nuclear distribution for \( \rho_N^A(\xi, b) \) with a normalization \( \int d\xi d^2b \rho_N^A(\xi, b) = A \). The quark broadening parameter is proportional to the nuclear density

\[
\tilde{q}_F(\xi, b) = \tilde{q}_0 \rho_N^A(\xi, b)/\rho_N^A(0, 0),
\]

(61)

where \( \tilde{q}_0 \) is the quark transport parameter at the center of a nucleus. Since the probability of \( \gamma' N \) interaction inside a nucleus is assumed to be proportional to \( \rho_N^A(\xi, b) \), the averaged transverse momentum broadening in DIS is then

\[
\Delta_{2F} = \frac{1}{A} \int_{-\infty}^{\infty} d\xi d^2b \int_{-\infty}^{\infty} d\xi \tilde{q}_F(\xi, b) \rho_N^A(\xi, b) = 3 \sqrt{2} \tilde{q}_0 r_0 A^{1/3}/4,
\]

(62)

where \( R_A = r_0 A^{1/3} \) is the nuclear radius with \( r_0 \approx 1.12 \) fm and the \( \sqrt{2} \) comes from the \( \xi \rightarrow \xi_z \) transformation. The nuclear suppression factor for the azimuthal asymmetry is then

\[
f_s \approx (1 + 3 \sqrt{2} \tilde{q}_0 A^{1/3}/4 \alpha)^{-1}.
\]

(63)

From analyses of jet suppression of leading hadrons in SIDIS of large nuclei due to multiple scattering, the quark transport parameter at the center of a large nucleus has been determined to be \( \tilde{q}_0 \approx 0.024 \pm 0.008 \) GeV\(^2\)/fm [11, 12]. Assuming \( \alpha = 0.25 \) GeV\(^2\), we have

\[
f_s \approx (1 + 0.114 A^{1/3})^{-1}.
\]

(64)

In Fig. 1, we plot \( f_s \) as a function of \( A \).

**Summary.** In summary, we have extended the study of nuclear dependence of azimuthal asymmetries in [6] to DISIS off polarized targets. The results show a further suppression factor \( 2J_A/A \) as compared to the unpolarized SIDIS. We also present a simple numerical estimate of the broadening width \( \Delta_{2F} \) and the nuclear suppression factor for azimuthal asymmetries that might be helpful in guiding future experiments.

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