Chiral magnetohydrodynamics for heavy-ion collisions

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Abstract

The chiral magnetic effect (CME) is a macroscopic transport effect resulting from the chiral anomaly. We review the recent progress in theoretical understanding the properties of chiral plasmas, in which the CME and other anomaly-induced transports take place. In particular, the nontrivial interplay of anomalous currents and dynamical electromagnetic fields is discussed. We also review the theoretical status of the modeling of anomalous transport effects in heavy-ion collisions.

Keywords: Chiral Magnetic Effect, Chiral Vortical Effect, Magnetohydrodynamics, Heavy Ion Collisions

1. Introduction

Macroscopic transport effects arising from the chiral anomaly have been attracting much attention in recent years. For example, magnetic fields generate dissipationless electric currents when they are applied to chirally imbalanced media, in which the numbers of left-handed and right-handed fermions are different. This is the chiral magnetic effect (CME) [1, 2]. Theoretically, the existence of CME can be derived by a number of ways such as the perturbation theory, lattice QCD & QED simulations [3, 4], and holography [5]. Moreover, the CME and other chiral transport effects constitute an integral part of relativistic hydrodynamics. Those effects are not only allowed but required from the consistency with the second law of thermodynamics [6]. Recently, the first experimental observation of CME using a Dirac semimetal is reported [7]. Heavy-ion collisions offer the opportunity to observe anomalous transport effects as well.

In this talk, we would like to discuss the interplay of anomalous chiral effects and dynamical electromagnetic fields. The CME currents are generated by applied magnetic fields. The currents in turn produce electromagnetic fields, that affect the configuration of the electromagnetic fields. We are interested in the fate of such coupled systems of chiral media and electromagnetic fields. For the complete understanding of various phenomena in such systems, one needs a consistent framework to describe both the chiral plasma and the electromagnetic fields. Chiral magnetohydrodynamics (MHD) is such a theory. An ordinary MHD is a low-energy theory for electrically conducting fluids. It can describe the time evolution of the coupled system of the conducting fluids and electromagnetic fields in a consistent way. In chiral MHD, the fluid is a chiral one, which includes the anomalous chiral effects like CME as a medium response. It has been pointed out that the chiral plasma develops an instability [8, 9]. Chiral MHD can answer the eventual fate of
the instability. Description in terms of chiral MHD is appropriate and important not only for the heavy-ion collisions, but also for early Universe before electroweak phase transition. For example, the interplay of chiral fermions with dynamical gauge fields leads to a formation of large-scale magnetic fields like we see in the current Universe.

In this contribution, we review the recent theoretical progress understanding the nature of chiral plasmas. We also review recent hydrodynamic attempts at describing CME in heavy-ion collisions.

2. Anomalous chiral effects and dynamical electromagnetic fields

2.1. Chiral anomaly and the topology of magnetic fields

Let us start by explaining a relation of the topology of magnetic fields and fermions. The conservation of the axial current $j^A_\mu$ is broken by a quantum effect, the extent which is quantified by the anomaly equation,

$$\partial_\mu j^A_\mu = C_A E \cdot B,$$

(1)

where $E$ and $B$ are electric and magnetic fields. After a spatial integration, the anomaly equation (1) takes the following form,

$$\frac{d}{dt} \left[ \mathcal{H} + \mathcal{H}_F \right] = 0,$$

(2)

where we have defined

$$\mathcal{H} \equiv \int d^3x A \cdot B, \quad \mathcal{H}_F \equiv \frac{2}{C_A} \int d^3x n_A.$$

(3)

Here $A$ is the vector potential, and $n_A$ is the axial charge density. We have introduced two helicities: $\mathcal{H}$ is so-called magnetic helicity, and $\mathcal{H}_F$ is the total fermionic helicity. Equation (2) tells us that the total “chirality” is a constant, although it can be stored either in magnetic fields or in fermions. When the chirality is stored in the fields, the field takes a topologically nontrivial form. Indeed, it is well known that the magnetic helicity is a measure of the topology of magnetic fields. When the system is made of magnetic flux tubes, the magnetic helicity can be written in terms of topological invariants as

$$\mathcal{H} = \sum_i S_i \phi^2_i + 2 \sum_{i,j} L_{ij} \phi_i \phi_j,$$

(4)

where $\phi_i$ is the magnetic flux of the $i$–th flux tube, $S_i$ is the Călugăreanu-White self-linking number, and $L_{ij}$ is the Gauss linking number [10, 11, 12]. The integrated anomaly equation (2) tells us that fermions can change the magnetic helicity, hence the topology of $B$ fields.

2.2. Inverse cascade

Let us find how the fermions affect the $B$ field topology. Coupled system of chiral matter and electromagnetism have been studied using the Maxwell-Chern-Simons theory [13, 14, 15, 16, 17]. The total helicity is conserved and this constrains the dynamics of the system. Remarkably, such systems exhibit the so-called “inverse cascade,” in which the energy is transferred from smaller to larger scales. This leads to large structures of magnetic fields as the system evolves. It also turns out that the evolution is self-similar [16], although the existence of such solution might depend on the choice of equation of state [18].

This discussion can be extended to include the degrees of freedom of the fluid. Fluid can also share the helicity in the form of fluid helicity, $\int d^3x \mathbf{v} \cdot \mathbf{\Omega}$, where $\mathbf{\Omega} \equiv \nabla \times \mathbf{v}$ is the vorticity of the fluid. The turbulent spectrum in chiral MHD is discussed [19, 20] and self-similar inverse cascade remains.
2.3. Quantized CME

An explicit formula that connects the change in the topology of field and CME current has been derived recently \cite{21},
\[ \sum_i \oint_{C_i} \Delta J \cdot dx = - \frac{e^3}{2\pi^2} \Delta \mathcal{H}, \]
where \( \Delta J \) is the generated CME current, \( C_i \) are the trajectories of the magnetic flux tubes, \( \Delta \mathcal{H} \) is the change in magnetic helicity, and \( e \) is the electric charge. Equation (5) indicates that a change in the topology of the magnetic fields (\( \Delta \mathcal{H} \)) necessarily results in the generation of a CME current (see Fig. 1 as a sketch). This relation can be extended to include vortices \cite{22}. Namely, the reconnections of magnetic fields and vortices can also lead to the generation of CME currents in a chiral fluid.

2.4. Equations of motion of (chiral) MHD

Let us turn to the dynamical description of a chiral plasma. A low-energy theory of a conducting fluid and electromagnetic fields is MHD, equations of motions of which are given by
\[ \partial_\mu T^{\mu\nu}_{\text{tot}} = 0, \quad \partial_\mu \tilde{F}^{\mu\nu} = 0, \]
where \( T^{\mu\nu}_{\text{tot}} \) is the energy-momentum tensor of whole system, and the latter equation is the Bianchi identity. Ideal MHD is characterized by the constitutive relation for the electric field, \( E^\mu(0) = 0 \). This corresponds to the limit of large conductivity. \( E^\mu \) is the electric field in the frame of the fluid element, so the observer on the fluid element does not feel any electric field.

In ideal MHD, the magnetic helicity is conserved and the topology of field is unchanged, which can be seen as follows. The magnetic helicity is the volume integral of the zero-th component of the Chern-Simons current \( h^\mu_B = \tilde{F}^{\mu\nu} A_\nu \). The divergence of \( h^\mu_B \) reads
\[ \partial_\mu h^\mu_B = 8 E^\mu B^\mu, \]
and since \( E^\mu = 0 \) in ideal MHD, \( h^\mu_B \) is conserved in this limit. Although anomalous chiral effects are dissipationless, they do not appear in ideal MHD.

Anomalous effects enter if one considers the contribution from finite conductivity \( \sigma \). The correction from resistivity to the electric field reads
\[ E^\mu_{(1)} = \lambda e^{\text{proj}} \partial_\alpha [u_\nu B_\nu] - \epsilon_B B^\mu - \epsilon_\omega \omega^\mu, \]
where \( \lambda \equiv 1/\sigma \) is the resistivity, \( \epsilon_B = \sigma_B/\sigma \), and \( \epsilon_\omega = \sigma_\omega/\sigma \). Coefficients \( \sigma_B \) and \( \sigma_\omega \) are chiral magnetic/vortical conductivities. The latter two terms in Eq. (8) are anomalous effects.

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1 The formulation of MHD has recently been revisited in Refs. \cite{23,24}.
2.5. Linear excitations in chiral MHD

In idea MHD, the magnetic field is “frozen in” to the fluid. The fluid is pierced by magnetic fields and they move together. Because of the tension of the magnetic field and the moment of inertia of the fluid, oscillatory motion of the magnetic field line happens, and it propagates along the magnetic field. This is the Alfvén wave. The nature of the Alfvén wave is affected by anomalous chiral effects. If we take the wave vector parallel to \( \mathbf{B} \), the dispersion relation reads

\[
\omega = \pm v_A k_\parallel - \frac{i}{2} \left[ (\bar{\eta} + \lambda) k_\parallel^2 - s \epsilon B k_3 \right],
\]

where the Alfvén velocity \( v_A \) is defined by \( v_A^2 \equiv B^2/(e + p + B^2) \), \( \bar{\eta} \equiv \eta/(e + p + B^2) \) is a normalized shear viscosity, and \( s \) is the helicity of the wave (there are left-handed and right-handed Alfvén waves). Because of the contribution proportional to \( \sigma_B \), helicity-dependent instability appears. For example, if \( \sigma_B > 0 \), the positive helicity modes with \( k < k_c \) is unstable, where

\[
k_c = \frac{\sigma_B}{1 + \bar{\eta} \sigma}.
\]

This instability generates helical flows in the presence of chirality imbalance, hence is a mechanism to transfer helicity from fermions to fluid flow \[25\].

3. Anomalous hydrodynamic modeling of heavy-ion collisions

3.1. Chirality production and CME in the glasma

Let us turn to the anomalous chiral effects in heavy-ion collisions. Shortly after the collisions, the matter is in a nonequilibrium state called glasma, which consists of highly occupied gluons. Then, fermions are created from those fields and the system will reach the local equilibrium state described by hydrodynamics. In the context of the CME search, the glasma dynamics is important, because chromo \( \mathbf{E} \cdot \mathbf{B} \) creates chirality imbalance, that is necessary for CME current to be generated \[26\]. The amount of axial charge in the initial stage of hydrodynamics strongly affects the value of the final observable, and its quantitative estimation is important for the experimental detection of CME.

The chirality generation from nonequilibrium color fields has been studied via real-time classical lattice simulations. In Ref. \[27\], the sphaleron rate is measured in a nonequilibrium non-Abelian plasma, and it is found to be enhanced compared to the equilibrium values. This means that a glasma can more efficiently produce chirality imbalance than an equilibrium plasma.

![Fig. 2. Time evolution of the axial and vector charge densities. Taken from Ref. \[28\].](image-url)

Furthermore, CME itself can also be happening in non-equilibrium states of glasma and fermions. Indeed, CME is simulated in real-time lattice simulations, in which \( U(1) \) magnetic fields are applied in addition to color fields \[28\]. Figure 2 shows the time evolution of axial and vector charge densities. A sphaleron transition creates axial charges, and later CME current develops in the direction of the applied magnetic field. Since magnetic fields are stronger at earlier times, the contribution of the CME current in glasmas
can be important for experimental search of CME. Those pre-hydro CME currents should enter in the initial conditions of the subsequent anomalous hydrodynamic stage. In those works, the backreaction from the generated fermions to the fields is not included, but it can also be incorporated. In Ref. [30], such a study is performed in the case of an Abelian plasma.

### 3.2. Anomalous hydrodynamic calculations

To reach a decisive conclusion about the existence of anomalous chiral effects in heavy-ion collisions [31], we need a tool to describe this phenomena quantitatively. For this purpose, hydrodynamic models with anomaly-induced transports have been developed [32, 33]. In Ref. [34], charge transport from anomalous currents are studied on a background solution of second order viscous fluid in 2+1 dimensions (VISHNew). The authors incorporated the effects of resonance decays, which contribute as a background effect, and simulations are performed on an event-by-event basis. Figure 3 shows the centrality dependence of the so-called \( H \) correlation [35], which shows a very similar trend with the STAR data.

Another approach for describing heavy-ion collisions is chiral kinetic theory [36, 37, 38, 39]. Examples for such calculations have been reported [40, 41] in this Quark Matter.

### 3.3. MHD and magnetic fields

In the search of CME in heavy-ion collisions, one of the biggest uncertainties is the strength and life time of magnetic fields. The MHD-type description is also useful in investigating electromagnetic properties of the plasma [42, 43]. Recently, MHD simulations for heavy-ion collisions are performed [44, 45] and the effects on observables like \( v_2 \) is discussed. In Fig. 4, the values of magnetic fields are plotted as a function of Bjorken time [44]. Compared to vacuum evolution (dotted line), the result from MHD simulation (solid line) shows slower decay as an effect of the medium. The initial \( B \) field for the MHD calculation (which starts from \( \tau = 0.4 \text{fm} \)) is given by the solution of Maxwell equations at finite conductivity [46].

In most of the calculations so far, the sources of electromagnetic fields are treated as classical ones. The importance of quantum effects in the estimation of magnetic fields is pointed out in Ref. [47]. The authors treated the sources as wave packets satisfying the Dirac equation, and the obtained field configurations turned out to show different behavior from classical treatment.

### 3.4. Vorticity

Formation of vortices in heavy-ion collisions is attracting renewed attentions since the report of finite \( \Lambda \) polarization from STAR [48]. The fluids formed after collisions naturally have global vortical structure pointing in the direction of the angular momentum. In addition, in event-by-event initial conditions of fluid calculated from transport models (HIJING/UrQMD), more complex vortex structures are found [49, 50, 51, 52]. Since the life-time and magnitude of vorticities are less uncertain compared to magnetic fields,
detection of CVE in heavy-ion collisions can be more feasible than CME. Since vorticities are larger at lower collisions energies, the search for CVE can benefit from the Beam Energy Scan II at RHIC. Event-by-event anomalous hydrodynamic analysis of γ correlation has been reported [53] in this conference.

3.5. Isobaric collisions

γ correlation can be contaminated with background effects, such as transverse momentum conservation [54], charge conservation [55] and cluster particle correlations [56]. Those effects are “flow driven” because their contributions are proportional to \( v_2 \). In order to identify the contributions from anomalous transport, RHIC is planning to perform the collisions of isobars using \(^{96}\text{Ru} + ^{96}\text{Ru}\) and \(^{96}\text{Zr} + ^{96}\text{Zr}\) in 2018. Since those isobars have the same mass number, the geometry of the collisions of Ru + Ru and Zr + Zr are the same. But the numbers of protons are different and the strength of the magnetic fields can be varied without changing the flow. In Ref. [57], it is shown that the two types of collisions can indeed give rise to sizable (\(~20\%\) difference in the observables. Figure 5 shows \( \langle B^2 \cos [2(\Psi_B - \Psi_{RP})] \rangle \) as a function of centrality for the two types of collisions. The quantity \( \langle B^2 \cos [2(\Psi_B - \Psi_{RP})] \rangle \) is a good measure for the anomalous contribution for the following reasons: γ correlation from anomalous transport should scale as \( |B|^2 \). Since γ quantifies charge separation in the out-of-plane direction, if the direction of \( B \) field (\( \Psi_B \)) is decorrelated with \( \Psi_{RP} \), the signal should vanish. The quantity \( \langle B^2 \cos [2(\Psi_B - \Psi_{RP})] \rangle \) captures this.

4. Summary

In summary, the interplay of chiral fluids and dynamical electromagnetic fields leads to a rich variety of phenomena. The fermions affect the topological configuration of magnetic fields and fluid velocities. There are ongoing efforts for more sophisticated description of anomalous chiral effects aiming at the detection of those effects in heavy-ion collisions.
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