Modelling the Nonlinear High-Frequency Response of a Short Josephson Junction under Two-Frequency Irradiation

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The nonlinear response of a short Josephson Junction (JJ), being irradiated simultaneously with two high-frequency signals, has been studied in the framework of the nonlinear Resistively-Shunted Junction (RSJ) Model. One of the signals, hereafter referred to as “probe signal”, has a small amplitude \( I_{pr} < I_c \) (\( I_c \) is the critical current of the JJ) and frequency \( f_{pr} \), and is used to monitor the response of the junction to the other high-power signal with amplitude \( I_{pm} \) and frequency \( f_{pm} \), hereafter referred to as “pump signal”. Varying the frequency ratio \( f_{pm}/f_{pr} \) from 0.5 to 100, and the current amplitude of the probe signal from 0.01 to 0.9 of \( I_c \), we found that the dependence of the joint impedance at the frequency \( f_{pr} \), \( Z_{pr}^{pm} \), versus \( f_{pm} \) preserves its general features, independent of \( f_{pm}/f_{pr} \) and \( I_{pm}/I_c \) values. At the same time, some particular features, like negative values of \( \text{Re}(Z_{pr}^{pm}) \) and “fine” structure of the steps in \( Z_{pr}^{pm}(I_{pm}) \) are observed for \( f_{pm}/f_{pr} < 1 \) and for particular values of \( I_{pm}/I_c \). In general, the behavior of \( Z_{pr}^{pm}(I_{pm}) \) is rather different from that predicted by the nonlinear RSJ model for a short JJ in the regime of single-frequency irradiation, when one and the same signal plays the roles of the pump and the probe signals simultaneously. Possible applications of the model are briefly discussed.

I. INTRODUCTION

The high-frequency nonlinear response of a Josephson Junction (JJ) is of significant interest because JJs are necessary units of almost all active microwave and millimeter (mm) wave devices, and because weak links, which are likely to be present even in the highest quality samples of high-\( T_c \) superconductors (HTS), can be modelled as JJs. The RSJ model is often used to simulate the characteristics of JJ-based devices, and was shown to give a good agreement with experimental data on point-contacts and microbridges in the microwave and mm wave ranges, where the capacitance of the junction can be neglected.

In the present paper we report simulation of the surface impedance of a JJ at the frequency of the low amplitude signal, hereafter referred to as “high-frequency response”, as a function of the current amplitude of the other elevated-power high-frequency signal. This case can be considered as a model of a microwave-biased electromagnetic radiation detector, which has been shown to have an improved sensitivity and noise figures when compared with dc-biased detectors. The other possible implication of the model is modeling the performance of microwave parametric amplifiers and Josephson mixers. In addition, the above model can also describe the nonlinear microwave response of superconducting weak links, which is often investigated using the so-called “pump-probe” technique. This technique is of a particular interest because it allows one to modulate or to pulse the powerful microwave signal, whilst measuring the surface impedance of the sample with the help of the other low-amplitude continuous wave microwave signal at a different frequency. In such a way, the pump-probe method avoids substantial heating effects and allows the study of intrinsic nonlinear phenomena in superconductors. However, using the pump-probe technique, one has to know how the nonlinear surface impedance, measured at the pump frequency, relates to that measured at the frequency of the probe signal, with respect to which the superconductor is in the linear regime. Although the model proposed in this paper cannot be considered as a model of the nonlinear response of HTS, an extension of the model to the case of a 2D JJ array with a random distribution of \( I_c \) and \( R_n \)-products (as recently proposed by Herd et al. for the single-frequency case) would allow the description of experiments on HTS thin films in the pump-probe regime.

II. NUMERICAL SIMULATION

In the case of two signals applied to a short JJ, the sine-Gordon Equation (which describes time dependence of the order parameter phase difference \( \varphi \) across the junction) in dimensionless form can be written as follows:

\[
\frac{d\varphi}{d\tau} = i_{pm} \sin(\Omega_{pm}\tau) + i_{pr} \sin(\Omega_{pr}\tau) - \sin \varphi, \quad (1)
\]

where \( \tau = \beta t \), \( i_{pm} = I_{pm}/I_c \), \( i_{pr} = I_{pr}/I_c \), \( I_{pm} \) and \( I_{pr} \) current amplitudes of the pump and probe signals respectively, \( \omega_{pm} = 2\pi f_{pm} \) and \( \omega_{pr} = 2\pi f_{pr} \) the corresponding circular frequencies, \( I_c \) the critical current of the junction, \( \beta = 2\pi R_n I_c / \hbar \), \( \Omega_{pm} = \omega_{pm} / \beta \), \( \Omega_{pr} = \omega_{pr} / \beta \), and \( R_n \) surface resistance of the junction in the normal state. Equation (1) does not take into account the effect of the junction capacitance, which can be neglected at microwave frequencies.

Generally, the solution of (1) is not necessarily periodic with time. Only in the case when the ratio \( \Omega_{pm}/\Omega_{pr} \) is an integer is the solution of (1) periodical with a period equal to the least common multiple (LCM) of the pump and the probe signal periods. If we expand the phase derivative \( \dot{\varphi} \) into a Fourier series with respect to time over the LCM period of the two signals, we obtain coefficients which couple the voltage \( \sim \dot{\varphi} \) to the current \( I_{pr} \). If then we single out the series’ terms at the probe frequency, we obtain the surface impedance at the relevant frequency as follows

\[
Z_s = R_s + jX_s = \lim_{n \to \infty} \frac{\Omega R_n}{\pi I_{pr} R} \int_0^{2\pi n/\Omega} \dot{\varphi}(\tau) \exp(j \Omega_{pr} \tau) d\tau, \quad (2)
\]
where $\Omega = 2\pi/T$, and $T$ is the LCM of the pump and probe signal periods. Because one can assume different initial conditions for (1), its solution is not strictly periodic with $T$, and hence in (2) integration over a few periods is required to get an appropriate convergency of the results.

The parameters used for simulation are as follows: $R_n = 10^{-3} \Omega, I_c = 0.5 \, \text{A}, f_{pm} = 3.6 \cdot 10^{10} \, \text{Hz}, f_{pr} = (0.036-7.2) \cdot 10^{10} \, \text{Hz}, \Omega_{pm}/\Omega_{pr} = 2$, and the probe current $i_{pr}$ is that given in each of the figures.

Results of the simulation for different ratios of $\Omega_{pm}/\Omega_{pr}$ varied from 0.5 to 100, and for different values of the probe current amplitude $I_{pm} = I_{pm}/I_c$, simulated within the framework of the RSJ model for the two-frequency case. Here, the frequency ratio $\Omega_{pm}/\Omega_{pr} = 2$, and the probe current $i_{pr}$ is that given in each of the figures.

FIG. 1. Normalised surface resistance $R_s/R_n$ (solid line) and surface reactance $X_s/R_n$ (broken line) for a short Josephson junction, as a function of normalised pump current amplitude $i_{pm} = I_{pm}/I_c$, simulated within the framework of the RSJ model for the two-frequency case. Here, the frequency ratio $\Omega_{pm}/\Omega_{pr} = 2$, and the probe current $i_{pr}$ is that given in each of the figures.

FIG. 2. $R_s/R_n$ and $X_s/R_n$ for a short Josephson junction, as a function of $i_{pm} = I_{pm}/I_c$, simulated within the framework of the RSJ model for the two-frequency case. Here, the frequency ratio $\Omega_{pm}/\Omega_{pr} = 10$, and the probe current $i_{pr}$ is rather different from that expected for the single-frequency situation. In the latter case, the surface resistance $R_s$ has a staircase-like form, starting to increase rapidly for $i_{pm} > 1$, and gradually approaching the $R_n$ value as $i_{pm} \to \infty$. With regards to the surface reactance $X_s$, for the single-frequency case it oscillates around zero with amplitude decaying with increased $i_{pm}$ (see, e.g., Fig. 2b). However, in the two-frequency regime, no obvious decay of the peaks amplitude in $X_s$ up to $i_{pm} = 4$ is observed. In addition, every oscillation peak seen in $X_s(i_{pm})$ in the single-frequency regime translates into a peak with a complicated structure, containing many upward and downward minor peaks of smaller amplitude. With increased $i_{pr}$ the major peaks in $X_s^{\text{pr}}(i_{pm})$ broaden, and more complicated “fine” structure of minor peaks develops (see Fig. 2b, Fig. 3b and Fig. 4b). As far as $R_s^{\text{pr}}$ is concerned, an increase of $i_{pm}$ leads to the appearance of steps in $R_s^{\text{pr}}(i_{pm})$, similar to those seen in $R_s(i_{pm})$ for the single-frequency regime. The higher $i_{pr}$, the more steps are observed.
in $R_{f}^{pr}(i_{pm})$, before it levels off and starts to oscillate near some average value around unity for $i_{pm} \gg 1$.

Contrary to the case of $\Omega_{pm}/\Omega_{pr} > 1$, when the major peaks in $R_{f}^{pr}(i_{pm})$ are almost symmetrical with respect to a vertical line drawn through the middle of their width, in the case of $\Omega_{pm}/\Omega_{pr} < 1$ these peaks are obviously asymmetric (see Fig. 3). Another distinctive feature for this case are discontinuous double-peak structures (upward peak followed by downward peak) at the beginning and the end of every major peak. One further significant difference of $R_{f}^{pr}(i_{pm})$ in the case of $\Omega_{pm}/\Omega_{pr} < 1$, as compared to the case with $\Omega_{pm}/\Omega_{pr} > 1$, is the appearance of regions with negative values of $R_{f}^{pr}$. This means that under these particular conditions the JJ can contribute energy to the external circuit, i.e. it works as a generator. This effect was theoretically predicted and experimentally observed in JJs made of low-temperature superconductors, and was called “the effect of nondegenerated single-frequency parametric regeneration”. As the theoretical analysis showed, this phenomena can be realised in any parametric element, a reactive parameter of which can take negative values with changing time.

All other features of $R_{f}^{pr}(i_{pm})$ for the case of $\Omega_{pm}/\Omega_{pr} < 1$, such as the appearance of steps, an increase in their number, and a shift of the oscillatory part of the dependence to higher $i_{pm}$ with increased $i_{pr}$, are similar to those seen in the case of $\Omega_{pm}/\Omega_{pr} > 1$. As far as $X_{f}^{pr}(i_{pm})$ is concerned, features like asymmetry of the major peaks and discontinuous double-peak structures are observed, similar to those present in $R_{f}^{pr}(i_{pm})$.

A. Implications for applications

One of the possible applications of the two-frequency regime, simulated in this paper, is a microwave-biased JJ detector. An advantage of this regime is that the amplitude of the oscillation peaks (or, equivalently, the impedance steps)
can be made several times (up to a factor of 3-4) higher than the resistance of the junction in the normal-state, especially at a small probe current (see Fig. 1, Fig. 2, and Fig. 3). This should lead to enhanced sensitivity of the detector, as compared to the single-frequency regime. Despite that similar results (increased step heights) have been also obtained for a dc-biased JJ, a microwave-biased detector was shown to benefit from reduced noise temperature and enhanced responsivity as compared to the dc-biased one. Moreover, since the step amplitudes in the saturation regime ($i_{pm} \gg 1$) are almost independent of the pump current, such a detector will possess amplitude-independent sensitivity.

In the case when $\Omega_{pm}/\Omega_{pr} < 1$, the JJ can be used as a parametric amplifier operating in the single-frequency nondegenerate regime which, when compared to the self-pumped regime, was shown to give a reduced noise temperature and a narrower frequency response of the junction at the probe frequency, allowing one to use high quality factor resonators for matching the junction with the external circuits.

III. CONCLUSION

The numerical simulation performed by us has shown that the nonlinear high-frequency response $Z_{pr}^{s\nu}$ of a short JJ in the regime of two-frequency irradiation can be rather different from the surface impedance $Z_s$ measured in the single-frequency regime. Depending on the ratio of the pump to the probe frequencies, a number of new features in $Z_{pr}^{s\nu}$ ($i_{pm}$) are predicted. Among them are the absence of the steps in $R_{pr}^{s\nu}$ ($i_{pm}$) at low probe currents ($i_{pr} < 0.05$); persistent oscillations of $R_{pr}^{s\nu}$ ($i_{pm}$) around some average value which tends to unity with increased $i_{pm}$; a multiple-peak structure of $X_{pr}^{s\nu}$ ($i_{pm}$), which becomes more complicated with increased ratio $\Omega_{pm}/\Omega_{pr}$; appearance of regions with negative values of surface resistance in $R_{pr}^{s\nu}$ ($i_{pm}$) for the case of $\Omega_{pm}/\Omega_{pr} < 1$.

At the same time, there are some features which are similar to the those in the single-frequency regime. These are the appearance of steps in $R_{pr}^{s\nu}$ ($i_{pm}$) with increased $i_{pr}$, and the oscillation of $X_{pr}^{s\nu}$ ($i_{pm}$) around a zero-$X_{pr}^{s\nu}$ value.

The model presented here was shown to give a useful basis knowledge for an application of the JJ as a microwave-biased detector of electromagnetic radiation.

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