SUSY induced CP violation in $e^+e^- \to t \bar{t}$

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The CP violating electric and weak dipole moment form factors of the top quark, $d^{\gamma}(s)$ and $d^{Z}(s)$, appear in the process $e^+e^- \to t \bar{t}$. We present a complete analysis of these dipole moment form factors within the Minimal Supersymmetric Standard Model with complex parameters. We include gluino, chargino, and neutralino exchange in the loop of the $\gamma t \bar{t}$ and $Z t \bar{t}$ vertex. We give analytic expressions and present numerical results for the asymmetries that measure these form factors.

1 Introduction

The large mass of the top quark allows one to probe physics at a high energy scale, where new physics might show up. In the last years a number of papers considered CP observables in top quark production (and decays) as tests for new physics. In $e^+e^-$ annihilation these effects are due to the weak and electric dipole moment form factors $d^{\gamma}(s)$ and $d^{Z}(s)$ of the top quark. In general, the $\gamma t \bar{t}$ and $Z t \bar{t}$ vertices $V^{\gamma}_{t}$ and $V^{Z}_{t}$ including the CP formfactors are

$$e(V^{\gamma}_{t})_{\mu} = e(\frac{2}{3} \gamma_{\mu} - id^{\gamma}(s)(P_{\mu}/m_{t})\gamma_{5})$$

$$gZ(V^{Z}_{t})_{\mu} = gZ(\gamma_{\mu}(g_{V} + g_{A}\gamma_{5}) - id^{Z}(s)(P_{\mu}/m_{t})\gamma_{5})$$

where $P_{\mu} = p_{\mu} - p_{\mu}$, $g_{V} = (1/2) - (4/3)\sin^{2}\Theta_{W}$, $g_{A} = -(1/2)$, $g_{Z} = g/(2\cos\Theta_{W})$, and $g = e/\sin\Theta_{W}$ with $e$ the electromagnetic coupling constant and $\Theta_{W}$ the Weinberg angle.

In the Standard Model (SM) CP can appear only through the phase in the CKM–matrix. The dipole moment form factors $d^{\gamma}(s)$ and $d^{Z}(s)$ for quarks are at least two–loop order effects and hence very small. In the Minimal Supersymmetric Standard Model (MSSM) with complex parameters additional complex couplings are possible leading to CP within one generation at one–loop level. If the masses of SUSY particles are not very much higher than the mass of the top, one expects SUSY radiative corrections to induce non–negligible contributions to $d^{\gamma}(s)$ and $d^{Z}(s)$. They are calculated in. Although the gluino contribution is proportional to the strong coupling $\alpha_{s}$, the chargino contribution, which is proportional to $\alpha_{w}(=g^{2}/(4\pi))$ can be equally important (see also). This is due to threshold enhancements and the large Yukawa

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couplings: \( Y_t = m_t / (\sqrt{2} m_W \sin \beta) \) and \( Y_b = m_b / (\sqrt{2} m_W \cos \beta) \). In general the neutralino contribution turns out to be smaller. However, there are cases where it is important.

2 Complex couplings in the MSSM

In the MSSM the higgsino mass parameter \( \mu \) and the trilinear scalar coupling parameters \( A_t \) and \( A_b \) can be complex and thus provide \( \mathcal{CP} \) phases. Assuming unification there are constraints on the phases from the measurement of the electric dipole moment (EDM) of the neutron. Being less restrictive we only assume unification of the gauge couplings: \( m_\beta = (\alpha_3 / \alpha_2) M \approx 3M \) and \( M' = \frac{5}{3} \tan^2 \theta_W M \). The calculation of the dipole moment formfactors requires the diagonalization of the squark, chargino, and neutralino mass matrices. We use the singular value decomposition to diagonalize the complex neutralino and chargino mass matrices.

The gluino contribution has been calculated in \( \mathcal{CP} \). The chargino and neutralino contributions depend on the gaugino and higgsino couplings, as well as on the squark mixing angles and phases. Quite generally, the terms proportional to \( Y_t \) are important. We have also included the terms proportional to the bottom Yukawa coupling \( Y_b \) which cannot be neglected for large \( \tan \beta \).

The loop integrations for \( d^\gamma(s) \) and \( d^Z(s) \) are done with the Passarino–Veltman three point functions \( C_0, C_i, \) and \( C_{ii} \) (\( i = 1, 2 \)).

More details about the calculation of \( d^\gamma(s) \) and \( d^Z(s) \) can be found in \( \mathcal{CP} \).

3 Asymmetries

With the formfactors \( d^\gamma(s) \) and \( d^Z(s) \) we can define asymmetries that measure the \( t \)-polarization. The first asymmetry we consider is the energy asymmetry in \( b \) or \( \bar{b} \)

\[
A_{b(\bar{b})}^\gamma(s) = \frac{\#(E_{(b)}) > E_0 - \#(E_{b(\bar{b})}) < E_0}{\#(E_{(b)}) > E_0 + \#(E_{b(\bar{b})}) < E_0},
\]

where \( E_0 = \sqrt{s} (m_1^2 - m_\nu^2) / (4m_1^2) \) is the average energy of the \( b \) or \( \bar{b} \). The corresponding \( \mathcal{CP} \) quantity (Figure 1) is

\[
R(s) = A_{b(\bar{b})}^\gamma(s) - A_{\bar{b}(b)}^\gamma(s) = -8 \alpha_b \beta_t \Im [H_1(N_t) / N_t]
\]

where \( N_t \) is a normalization factor proportional to the total cross section, \( \beta = \sqrt{1 - 4m_1^2 / s} \), \( \alpha_b = (m_1^2 - 2m_\nu^2) / (m_1^2 + 2m_\nu^2) \),

\[
H_1 = (1 - P \bar{P}) H_1 + (P - \bar{P}) H_1^t,
\]

\[
H_1^t = \left( \frac{2}{3} - c_V g_V h_Z \right) d^\gamma(s) - \left( \frac{2}{3} c_V h_Z - c_V^2 + c_A^2 \right) g_V h_Z^2 d^Z(s),
\]

\[
H_1^z = -c_A g_V h_Z d^\gamma(s) - \left( \frac{2}{3} c_A h_Z - 2c_V c_A g_V h_Z^2 \right) d^Z(s),
\]

2
The forward–backward asymmetry is

$$A_{b(\bar{b})}^{FB}(s) = \frac{\#_{h(b)}(\cos \theta_{h(b)}>0) - \#_{h(b)}(\cos \theta_{h(b)}<0)}{\#_{h(b)}(\cos \theta_{h(b)}>0) + \#_{h(b)}(\cos \theta_{h(b)}<0)}$$

(8)

with $\cos \theta_{h(b)} = \hat{p}_x \cdot \hat{p}_{h(b)}$, where $\hat{p}_x$ is the direction of particle $x$.

Since $\text{CP}(A_{b}^{FB}) = -A_{\bar{b}}^{FB}$, the $\mathcal{O}F$B quantity is $\mathcal{O}^{FB} = A_{b}^{FB} + A_{\bar{b}}^{FB}$ (Figure 2)

$$= -12 \alpha_b \left( \frac{2}{1 - \beta_t^2} + \frac{1}{\beta_t} \ln \left[ \frac{1 - \beta_t}{1 + \beta_t} \right] \right) 3 m [H_2]/N_t$$

(9)

where

$$H_2 = (1 - P\bar{P}) H^2 + (P - \bar{P}) H^2_2$$

(10)

$$H^s_2 = -c_A g_A h_Z d^2(s) + 2 c_V c_A g_A h^2_Z d^2(s)$$

(11)

$$H^A_2 = -c_V g_A h_Z d^2(s) + (c^2_V + c^2_A) g_A h^2_Z d^2(s)$$

(12)

With the triple products $T_{b(\bar{b})} = \hat{p}_e \cdot (\hat{p}_t \times \hat{p}_{b(\bar{b})})$ one can define the asymmetry $A_{b(\bar{b})}^{T} = \frac{\#_{T_{b(\bar{b})}}>0}{\#_{T_{b(\bar{b})}}>0 + \#_{T_{b(\bar{b})}<0}}$. The $\mathcal{O}T$ quantity is the difference

$$\mathcal{O}T = A_{b}^{T} - A_{\bar{b}}^{T} = 3 \alpha_b \pi \frac{\sqrt{s}}{2 m_t} \Re \left[ D_1 \right]/N_t$$

(13)

where $D_1 = (1 - P\bar{P}) H^s_1 + (P - \bar{P}) H^A_1$.
For the numerical analysis we take $\sqrt{s} = 500$ GeV, $m_W = 80$ GeV, $m_t = 175$ GeV, $m_b = 5$ GeV, $\alpha_s = 0.1$, $\alpha_{em} = \frac{1}{137}$, and the following set of SUSY parameter values: $M = 230$ GeV, $|\mu| = 250$ GeV, $m_{\tilde{t}}(1,2) = 150(400)$ GeV, $m_{\tilde{b}}(1,2) = 270(280)$ GeV, \( \tan \beta = 2 \), $\varphi_\mu = \frac{4\pi}{3}$, $\theta_\ell = \frac{\pi}{9}$, $\varphi_\ell = \frac{\pi}{6}$, $\theta_b = \frac{\pi}{36}$, $\varphi_b = \frac{\pi}{3}$ and for the beam polarizations: $P = 0.8$, \( \bar{P} = -0.8 \).

In Figure 1 we show $R(s)$ and in Figure 2 we show $O^{FB}$ for the set of parameters given above. The $O^{FB}$ ratios $R(s)$ and $O^{FB}$ depending on $\Im m d(s)$ and $\Im m d^2(s)$, show spikes (see Fig. 1). They are due to threshold effects in the fermion pair production in the loop. These spikes only appear in the chargino and neutralino contributions.

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References

1. see references [1] and [2] of
2. A. Bartl, E. Christova, W. Majerotto, *Nucl.Phys.* B460 (1996) 235; erratum *Nucl.Phys.* B465 (1996) 365;
3. H. E. Haber, G. L. Kane, *Phys.Rep.* 177 (1985) 75
4. M. Dugan, B. Grinstein, L. Hall, *Nucl.Phys.* B255 (1985) 413; W. Bernreuther, M. Suzuki *Rev.Mod.Phys.* 63 (1991) 313; W. Hollik, J.I. Illana, S. Rigolin, D. Stöckinger, *hep-ph* / 9711322
5. A. Bartl, E. Christova, T. Gajdosik, W. Majerotto, HEPHY-PUB 669/97, UWThPh-1997-13, *hep-ph* / 9705245, to be published in *Nucl.Phys.* B
6. Y. Kizukuri and N. Oshimo, *Phys.Rev.* D45 (1992) 1806; *Phys.Rev.* D46 (1992) 3025
7. R. Garisto, J. D. Wells, *Phys.Rev.* D55 (1997) 1611
8. J. M. Ortega, *Matrix Theory* (Plenum Press, New York 1987)
9. G. Passarino and M. Veltman, *Nucl.Phys.* B160 (1979) 151; A. Denner, *Fortschritte der Physik* 41 (1993) 4, 307
10. E. Christova, D. Draganov, *hep-ph* / 9710225