Kinematic and Dynamic Characteristics in a Crank-rocker Four-bar Linkage

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Abstract. Planar four-bar linkage is widely used in mechanical engineering. The degree of freedom of the linkage is one, and different organizations of the lengths of the four bars could distinguish it in several different types. The one is the crank-rocker mechanism. The parameters decided by the lengths of the bars are the wiping angle, transmission angle, and the quick-return ratio, which were analyzed in this report. Then the three-dimensional model and the physical model were built. In addition, the kinematic characteristics of the prototype were deduced by simulation, and some experimental works researched dynamic characteristics. Finally, the conclusions about the linkage were made.

1. Introduction

The four-bar linkage mechanism plays an important role in mechanical engineering and civil engineering by saving much room and offering great convenience. A four-bar linkage mechanism is preferred for its excellent performance in load bearing with simple structure. The kinematic relationship between any two components can be derived according to the geometric closure and particular motion laws [1]. The four-bar linkage mechanism can be further used in such fields as robotics, the automobile industry, and artificial intelligence (AI).

Judging from the current application in engineering, the crank-rocker four-bar linkage mechanism is practical and most widely used. For example, Wahit, M. A. A. et al. [2] designed a three-dimensional printed robotic hand as part of artificial limbs by using four-bar linkages. By observing that birds and insects can demonstrate the complicated three-dimensional wing moments, McDonald, M. et al. [3] found the special wing movement mode can create favourable aerodynamics. Drawing upon biomorphic sphere four-bar linkage with good aeromechanics, they designed Flapping-Wing Micro Air-Vehicle. With the increase of senior people, a new auxiliary seat in a sitting structure came into existence. Conventional auxiliary structure primarily relies on the actuator’s power to produce adequate drive
efficiency, which can lead to the overload of the actuator and high relevant costs. On the contrary, a seating system consisting of three four-bar linkage mechanisms created by Jeong, H. et al. [4] can accomplish vertical and rotational displacement and enhance stability. Noh, J. et al. [5] developed wearable exoskeleton robots with the design which is based on a four-bar linkage mechanism structure consisting of passive and active actuators. Though expensive, the creation can help a person with a maximum weight of 85 kg to walk in any position without consuming power. To fulfil a higher load bearing requirement, more connecting bars need to be attached to the four-bar linkage mechanisms, which calls for more installation space and expenses on material costs. Moreover, the nonlinear dynamic behaviour of the four-bar linkage mechanism can lead to management difficulty. An AI system developed by Rodríguez-Molina, A. et al. [6] gives rise to the need for small and medium-sized four-bar linkage mechanisms, which can optimize the structure.

In addition, the dynamic issues and related analysis of the four-linkage mechanism is another research concentration. Tang, Y. et al. [7] find that the inevitable clearances in the four-link mechanisms introduce the nonlinear dynamic behaviors. The clearance size and the relationship between the clearance mechanism and the non-clearance mechanism were paid much attention to. In some cases, periodic response and subharmonic response phenomena may appear. There is also research about the effect of the static balance method on the dynamic performance of the closed chain mechanism regarding the electrodynamics characteristics in the balanced closed-loop mechanism. Martini, A. et al. [8] find that the four-bar linkage mechanism's natural frequency and mode shape are estimated through modal analysis, and the dynamic performance of the four-bar linkage mechanism under different working states can be analyzed. Kinematic synthesis and dynamic design are two important aspects in the development of the four-bar linkage mechanism. However, they are often regarded as two rather irrelevant stages in the traditional design process. Yan, H. S. et al. [9] proposed a new design method of a four-bar linkage mechanism that met the requirements of motion design and realized dynamic balance. By designing a reasonable velocity trajectory, a disk counterweight of moving linkage, and specified linkage dimensions, the ideal output motion characteristics could be obtained.

In this paper, some basic rules in the design of the crank-rocker four-bar linkage mechanism are presented, and the prototype of the linkage is constructed. Finally, the kinematic and dynamic analysis of the linkage is proposed.

2. Theoretical Analysis of Kinematic and Dynamic Characteristics in Crank-Rocker

The model of crank-rocker four-bar linkage can be shown in Figure 1. below.

![Figure 1. Crank-rocker four-bar linkage.](image)

According to the Kutzbach criterion, the degree of freedom (DOF) of the crank-rocker mechanism can be calculated as [10]
Therefore, the DOF of the linkage is 1.

It can be inferred that during the crank AB and coupler BC are aligned twice in one turning process, as shown in Figure 2., the rocker CD finishes its rotate stroke within a certain angle, which can be deduced as

\[
\delta = \varphi - \varphi'.
\]

(2)

Moreover, the angle and can be calculated by using the cosine theorem as

\[
\cos\varphi = \frac{AD^2 + CD^2 - (BC + AB)^2}{2 \cdot AD \cdot CD},
\]

(3)

\[
\cos\varphi' = \frac{AD^2 + CD^2 - (BC - AB)^2}{2 \cdot AD \cdot CD}.
\]

(4)

Figure 2. Crank AB and coupler align.

As shown in Figure 3., we can define pressure angle and transmission angle . The equals to the angle between the coupler and the rocker, and the equals to \( \pi / 2 \) minus .

Figure 3. Definitions of pressure angle and transmission angle.

In addition, the overall force transmitted from the crank to the rocker by the coupler could be divided into two components and , which are perpendicular to each other, and the magnitude of these two forces can be derived as

\[
F_{\text{max}} = F_{\text{all}} \cdot \sin \gamma,
\]

(5)

\[
F_{\text{press}} = F_{\text{all}} \cdot \cos \gamma.
\]

(6)
The reason why we can define the name of the two components is that the effect of the is to transmit the motion from the crank to the rocker. However, the effect of is to load a press force along the rocker bar, which makes little contribution to the motion of the rocker. It is obvious that when the transmission angle becomes larger, turns larger while turns smaller.

The former researchers have noted that the minimum transmission angle shows when the crank is parallel to the frame bar. As shown in Fig. 4., there are two probably minimum angles, and the angle is the actual one. To ensure the four-bar linkage moves well, should be larger than at least empirically. However, if the linkage moves very slowly, the could be slightly smaller.

By using the cosine theorem, the can be calculated as

\[ \cos \gamma_{\text{min}} = \frac{BC^2 + CD^2 - BD^2}{2 \cdot BC \cdot CD}. \] (7)

Therefore, to make the crank-rocker four-bar linkage move smoothly, we need to design the lengths of bars properly by increasing the transmission angle in the whole turning process.

When the crank turns around, there will be two particular positions for the crank and the rocker. As illustrated in Fig. 5., assuming that the crank turns clockwise, when it stays at the position AB, the rocker stays at a limitation position CD. While the crank stays at AB', the rocker stays at another limitation position C'D, with the rotation angle by the crank. Between these two positions, the angle is the whole turning stroke of the rocker.

However, when crank AB' turns continually in a clockwise direction, it will be at the position AB once again after the rotation angle. In this process, the rocker will turn clockwise, finishing another whole stroke angle clockwise.

As analyzed above, the rocker will finish its stroke twice in two opposite directions in one turn of the crank. But if the crank turns in a constant rotation speed, it is obvious that the time consumed by two strokes is not equal because the rotation angle of the crank varies. And the ratio of the time consumed by the two processes can be shown as
ratio = \frac{t_{anti-clockwise}}{t_{clockwise}} = \frac{\pi - \theta}{\pi + \theta}.

(8)

In other words, the anti-clockwise stroke of the rocker will be finished quicker than the one in the clockwise direction. This character of the crank-rocker mechanism is called quick-return.

By using the cosine theorem

\cos \angle C'AD = \frac{(C'B-AB)^2 + AD^2 - C'D^2}{2 \cdot (C'B-AB) \cdot AD},

(9)

\cos \angle CAD = \frac{(AB + BC)^2 + AD^2 - CD^2}{2 \cdot (AB + BC) \cdot AD}.

(10)

The angle \theta can be calculated by

\theta = \angle C'AD - \angle CAD.

(11)

3. A Case study of the Crank-Rocker Linkage

3.1. Size Design and the Quick-Return Ratio of the Linkage

To simulate a design process of the crank-rocker linkage, there could be some requirements settled in advance. In this case study

\varphi = 90^\circ,

\varphi' = 60^\circ,

\gamma_{min} = 40^\circ.

To make the designing process more flexible and ensure the size of the linkage is constrained in a particular range, such as the lengths of the bars will not be larger than several hundred millimeters. In this case, another initial condition could be settled as

\[ AB = 35mm \]

Then, there are four conditions given as above, and there are also four bars in the linkage, so the equations can be constructed as

\[ \cos \varphi = \frac{AD^2 + CD^2 - (BC + AB)^2}{2 \cdot AD \cdot CD} = \cos 90^\circ = 0, \]

\[ \cos \varphi' = \frac{AD^2 + CD^2 - (BC - AB)^2}{2 \cdot AD \cdot CD} = \cos 60^\circ = \frac{1}{2}, \]

\[ \cos \gamma_{min} = \frac{BC^2 + CD^2 - BD^2}{2 \cdot BC \cdot CD} = \cos 40^\circ = 0.766, \]

\[ AB = 35mm. \]

To solve the quaternion quadratic equations, the simulation software could be utilized [11]. According to the results, the lengths approximately are

\[ AB = 35mm, \]

\[ BC = 204mm, \]

\[ CD = 171mm, \]

\[ AD = 167mm. \]
Then, the quick-return ratio can be deduced as
\[
\cos \angle C'AD = \frac{(C'B' - AB)^2 + AD^2 - C'D^2}{2 \cdot (C'B' - AB) \cdot AD} = 0.4820
\]
\[
\cos \angle CAD = \frac{(AB + BC)^2 + AD^2 - CD^2}{2 \cdot (AB + BC) \cdot AD} = 0.6986
\]
Therefore,
\[
\angle C'AD = 61.19^\circ, \\
\angle CAD = 45.69^\circ, \\
\theta = \angle C'AD - \angle CAD = 15.5^\circ,
\]
\[
\text{ratio} = \frac{t_{\text{anti-clockwise}}}{t_{\text{clockwise}}} = \frac{\pi - \theta}{\pi + \theta} \times 100\% = 84\%.
\]
In this crank-rocker linkage, when the crank rotates at a constant speed, the time consumed by return stroke will be less than 85% of the other stroke. However, it could be inferred that too small a quick-return ratio will introduce instability and even shock and vibration when the crank turns fast, which could cause damage to the whole mechanism [12].

3.2. Construction of the Prototype
On account of the lengths of the four bars designed before are somewhat too long, another revised plan could be implemented. The main shape parameters of the revised crank-rocker mechanism are decided as Figure 6., in which p1 equals 35mm, p2 equals p3 equals 120mm, p4 equals 138mm.

![Figure 6. The size parameters of the crank-rocker.](image)

The parameters and characteristics of this revised model could be exposed by using the same method above, those are
\[
\varphi = 73.48^\circ, \\
\varphi' = 37.66^\circ, \\
\gamma_{\text{min}} = 50.30^\circ, \\
\text{ratio} = 0.878.
\]
Indeed, there are some slight differences between the revised model and the former one. However, the transmission angle is much larger, which will ensure the mechanism turns in a smoother way, and the prototype could be more feasible to be built.
As shown in Figure 7., the three-dimensional model of the four-bar linkage is built.
3.3. Kinematic characteristics of the prototype by simulation

The relationship between the input angle $\theta_1$ and the output angle $\theta_2$ could be analysed by simulation. According to the geometric conditions of the crank-rocker linkage, the relationship could be generated as Figure 9.

Figure 9. The relationship between input and output.

Figure 9. describes the kinematic characteristics deduced in the former chapter, as the limitation angles of the output are 37.66° and 73.48°.

Moreover, it is easy to detect that the curve is not in the sinusoidal regulation because the input angle consumed to reach the peak does not equal that to reach the trough, and the former is slightly smaller. Although the quick-return ratio of the mechanism is approximate 90%, which seems to be not too small, the stability of the mechanism in high-speed conditions still needs more attention to be paid.
3.4. Dynamic characteristics analysis by Simulation and experiments

Structural analysis by finite element method (FEM) can be used to solve the displacement, stress, and strain of the structure generated by the external load. The joint pins in the four-bar linkage would suffer from shocks and vibrations under fast revolution speed. To simulate the generated strain and stress, FEM could be utilized. In this case, the joint pin connecting the frame and the output rocker is the item under observation. The three-dimensional model of the joint pin is illustrated in Figure 10.

![Figure 10. 3D model of the pin.](image1)

![Figure 11. The meshing of the pin.](image2)

In the meshing process, the regular tetrahedral element with an edge length of 2 mm is used. The meshing result is illustrated in Figure 11. The constrain condition is that one end of the pin is clamped rigidly, and the other end is free, as the actual condition in the linkage. Then, the external load is imposed according to the dynamic simulation results.

The equivalent elastic strain and stress distribution is illustrated in Figure 12 and Figure 13. The position where the maximum strain and stress occurs is at the upper and lower surface of fixed support of that pin because, under the varying external load, the upper and the lower side of the neutral plane is always under tension or compression.

![Figure 12. Distribution of equivalent elastic strain.](image3)

![Figure 13. Distribution of equivalent elastic stress.](image4)

In addition, to observe the changing pattern of the strain under different conditions, more simulation researches can be done by imposing different external loads with different amplitudes.

The actual time period of the load is 2 seconds. Therefore the frequency is 0.5 Hz. As for the amplitude, a series of values range from 10 N to 30 N are adopted. Under these different conditions, the maximum strains on the pin are tested. The results expressed in percentages are shown in Figure 14.

![Figure 14. The variation pattern of relative maximum strain on the joint pin.](image5)
The dynamic characteristics of the mechanism could be researched in such an aspect that the strain the joint pins suffered [14]. Normally, the deployable structures will not introduce any strain and stress into the mechanism. However, when the mechanism moves at high speed, the strain also could be detected, especially on the surface of the joint parts.

There are four pins in the crank-rocker linkage, and to make the dynamic issue be illustrated more feasibly, the joint pin connecting the rocker and the frame could be chosen as the research object, which stays still in the whole process of motion.

The strain could be measured by the strain gauges as shown in Figure 15, in the Wheatstone bridge circuit. The output voltage of the Wheatstone bridge is proportional to the strain tested. The strain gauge was positioned like the way shown in Figure 16.

![Figure 15. Strain gauge](image1)

![Figure 16. The positioned strain gauge](image2)

The system diagram to test the strain could be shown in Figure 17, and the test system is shown in Figure 18.

![Figure 17. The test system diagram](image3)

![Figure 18. The test system](image4)

When the crank stays still, the strain is zero, with inevitable weak electronic noise detected. However, when it turns in a relatively high revolution speed in 30 rpm, the output strain can be detected as sharp peaks. These differences could be observed in the computer screen as Figure 19.

![Figure 19. The different strains tested in static and dynamic conditions.](image5)

Actually, on many occasions, with the use of the four-bar linkage, the dynamic characteristics are important to be researched, especially in conditions of heavy load and high speed.
4. Conclusions
The planar four-bar linkages could be distinguished in different types by varying the lengths of the bars. The Crank-rocker is one of the most widely used mechanisms. The DOF of the mechanism is one. The wiping angle, the transmission angle, and the quick-return ratio are important parameters. By constructing a particular model of the linkage, the parameters above were analysed. In addition, a three-dimensional model and a prototype were built. Finally, the kinematic and dynamic characteristics of the prototype were obtained.

References
[1] Acharyya, S. K., & Mandal, M. (2009). Performance of EAs for four-bar linkage synthesis. Mechanism and Machine Theory, 44(9), 1784-1794.
[2] Wahit, M. A. A., Ahmad, S. A., Marhaban, M. H., Wada, C., & Izhak, L. I. (2020). 3D Printed Robot Hand Structure Using Four-Bar Linkage Mechanism for Prosthetic Application. Sensors (Basel, Switzerland), 20(15).
[3] McDonald, M., & Agrawal, S. K. (2010). Design of a bio-inspired spherical four-bar mechanism for flapping-wing micro air-vehicle applications.
[4] Jeong, H., Guo, A., Wang, T., Jun, M., & Ohno, Y. (2019, July). Development of four-bar linkage mechanism on chair system for assisting sit-to-stand movement. In 2019 IEEE 4th International Conference on Advanced Robotics and Mechatronics (ICARM) (pp. 303-308). IEEE.
[5] Noh, J., Kwon, J., Yang, W., Oh, Y., & Bae, J. H. (2016, October). A 4-bar mechanism based for knee assist robotic exoskeleton using singular configuration. In IECON 2016-42nd Annual Conference of the IEEE Industrial Electronics Society (pp. 674-680). IEEE.
[6] Rodríguez-Molina, A., Villarreal-Cervantes, M. G., & Aldape-Pérez, M. (2020). Indirect adaptive control using the novel online hypervolume-based differential evolution for the four-bar mechanism. Mechatronics, 69, 102384.
[7] Tang, Y., Chang, Z., Dong, X., Hu, Y., & Yu, Z. (2013). Nonlinear dynamics and analysis of a four-bar linkage with clearance. Frontiers of Mechanical Engineering, 8(2), 160-168.
[8] Martini, A., Troncossi, M., Carricato, M., & Rivola, A. (2014). Elastodynamic behavior of balanced closed-loop mechanisms: numerical analysis of a four-bar linkage. Meccanica, 49(3), 601-614.
[9] Yan, H. S., & Soong, R. C. (2001). Kinematic and dynamic design of four-bar linkages by links counterweighing with variable input speed. Mechanism and machine theory, 36(9), 1051-1071. G. N. Sandor & A. G. Erdman. Advanced Mechanism Design V. 2: Analysis and Synthesis. Prentice-Hall, 1984.
[10] Sandor, G. N., & Erdman, A. G. (1984). Advanced Mechanism Design V. 2: Analysis and Synthesis. Prentice-Hall.
[11] Quarteroni, A., Saleri, F., & Gervasio, P. (2006). Scientific computing with MATLAB and Octave (Vol. 3). Berlin: Springer.
[12] Kelly, S. G. (1992). Fundamentals of mechanical vibrations.
[13] Shigley, J. E. (2006). Mechanical Engineering Design, Eight Edition.
[14] Lees, A. W. (2020). Vibration problems in machines: diagnosis and resolution. CRC Press.