Estimation of internal heat transfer coefficients and detection of rib positions in gas turbine blades from transient surface temperature measurements

P Heidrich¹, J v Wolfersdorf¹, S Schmidt¹, M Schnieder²

¹ Institute of Aerospace Thermodynamics, Universität Stuttgart, Pfaffenwaldring 31, 70569 Stuttgart, Germany.
² ALSTOM (Schweiz) AG, Brown Boveri Strasse 7, 5401 Baden, Switzerland

Email: peter.heidrich@itlr.uni-stuttgart.de

Abstract. This paper describes a non-invasive, non-destructive, transient inverse measurement technique that allows one to determine internal heat transfer coefficients and rib positions of real gas turbine blades from outer surface temperature measurements after a sudden flow heating. The determination of internal heat transfer coefficients is important during the design process to adjust local heat transfer to spatial thermal load. The detection of rib positions is important during production to fulfill design and quality requirements. For the analysis the one-dimensional transient heat transfer problem inside of the turbine blade’s wall was solved. This solution was combined with the Levenberg-Marquardt method to estimate the unknown boundary condition by an inverse technique. The method was tested with artificial data to determine uncertainties with positive results. Then experimental testing with a reference model was carried out. Based on the results, it is concluded that the presented inverse technique could be used to determine internal heat transfer coefficients and to detect rib positions of real turbine blades.

1. Introduction

During the last decades gas turbine engines became more and more important in electricity industry. They were used as additional generator sets during peak hours as well as to compensate fluctuations due to renewable energy sources. And, because of their high energy efficiency gas turbine engines were used more often as main power units.

Thus, gas turbine industry searches for ways to increase turbine inlet temperatures to enhance specific power output and thermal efficiency. But, this leads to an increase in turbine blade temperatures, too. Therefore, the properties of temperature-resistant materials were improved and active turbine blade cooling was necessary. For this purpose, turbine blades were firstly manufactured with radial cavities. Cooling air taken from the engine’s compressor section flowed through these cavities and cooled the turbine blades convectively. Over the years the cavities were developed further into enhanced cooling configurations [1, 2].

To use the cooling air as efficiently as possible and to avoid additional losses, local heat transfer coefficients must be adjusted to the spatial thermal load of turbine blades. Commonly used experimental techniques are the transient liquid crystal technique, and heater foils combined with thermocouples, among others [3 – 5]. Mostly, this experimental work is carried out with simplified
models of significant parts of the blades [6, 7]. On the other hand rib positions of real turbine blades must be detected after the casting process to ensure design and quality requirements.

Since, none of the experimental techniques mentioned were appropriate for use with real turbine blades, there is need for the development of a non-invasive, non-destructive, transient inverse technique. To allow testing of real turbine blades temperature measurements can be based on infrared thermography. For this study an experimental setup similar to those described in [8 – 10] was built. In addition to [10], where a lumped capacity model was used, the transient heat conduction within the wall itself was taken into account. The one-dimensional forward problem with two convective boundary conditions was solved. The Levenberg-Marquardt algorithm was chosen as the best fitting optimization method. This was verified with artificial data including random noise with positive results. Experimental data was measured for a rectangular reference model made of stainless steel with 90° ribs. Spatial distributions of local heat transfer coefficients were evaluated and rib positions were detected. It is concluded that the presented measurement technique could be used to quantitatively determine internal heat transfer coefficients and to locate rib positions of real turbine blades.

2. Forward problem

The transient heat transfer problem in a turbine blade wall is described by the heat conduction equation. Because length and width are much larger than the thickness d, a one-dimensional heat flux is assumed. Supposing the initial temperature distribution inside of the wall is known as well as the two boundary conditions. Then the temperature distribution inside of the wall is given by

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]  

(1)

Assuming a uniform temperature \( T_\infty \) of ambient air and reference model at the beginning and a sudden step of the air temperature to \( T_{air} \) yields the following initial and boundary conditions:

\[ t = 0: \quad T(x,0) = T_\infty, \quad x = 0: \quad -k \frac{\partial T}{\partial x}_{x=0} = h_{air}(T_{air} - T_i), \quad x = d: \quad -k \frac{\partial T}{\partial x}_{x=d} = h_{\infty}(T_\infty - T_i) \]  

(2)

where \( T_i \) is the inner surface temperature, \( T_\infty \) is the outer surface temperature, \( h_{air} \) and \( h_{\infty} \) are the inner and outer heat transfer coefficients respectively and \( k \) is the thermal conductivity of the wall material. Assuming material properties are constant within the temperature range investigated and defining the following dimensionless variables

\[ \Theta = \frac{T(x,t) - T_{air}}{T_\infty - T_{air}}, \quad Bi = \frac{h d}{k}, \quad \tilde{x} = \frac{x}{d}, \quad \tilde{t} = \frac{\alpha t}{d^2} \]  

(3)

results for equation (1) in the following, dimensionless form:

\[ \frac{\partial \Theta}{\partial \tilde{t}} = \alpha \frac{\partial^2 \Theta}{\partial \tilde{x}^2} \]  

(4)

Employing these variables to the initial and boundary conditions as given by (2) results in:

\[ \tilde{t} = 0: \quad \Theta(\tilde{x},0) = 1, \quad \tilde{x} = 0: \quad Bi_{air} \Theta(0,\tilde{t}) - \frac{\partial \Theta}{\partial \tilde{x}}\bigg|_{\tilde{x}=0} = 0, \quad \tilde{x} = 1: \quad Bi_{\infty} \Theta(1,\tilde{t}) + \frac{\partial \Theta}{\partial \tilde{x}}\bigg|_{\tilde{x}=1} = Bi_{\infty} \]  

(5)

The solution of the inhomogeneous partial differential equation given by (4) taking (5) into account is derived by splitting the dimensionless temperature distribution into a steady-state solution and a transient solution. The steady state solution is given by

\[ \Theta_s(\tilde{x}) = \frac{Bi_{\infty}(1 + Bi_{air} \tilde{x})}{Bi_{air}(Bi_{\infty} + 1) + Bi_{\infty}} \]  

(6)
where $B_{air}$ is the Biot number at the inner surface of the model while $B_{∞}$ is the Biot number at the outer surface of the model. The transient solution is obtained by the method of separation of variables and given by

$$\Theta(x, \tau) = \sum_{n=1}^{∞} C_n \left[ \cos(\lambda_n x) + \frac{B_{air}}{\lambda_n} \sin(\lambda_n x) \right] \exp(-\lambda_n \tau)$$  

(7)

The eigenvalues of the transient solution have to be calculated from their characteristic equation

$$(B_{∞} + B_{air}) \cos(\lambda_n) + \left( \frac{B_{∞}}{\lambda_n} - \lambda_n \right) \sin(\lambda_n) = 0$$  

(8)

If Biot numbers are considerably smaller than 1, not more than 5 eigenvalues are necessary. With the material properties of stainless steel and the range of expected heat transfer coefficients this requirement is fulfilled. The constants $C_n$ in (7) are given by

$$C_n = \frac{A}{\lambda_n} \left[ \frac{\sin(\lambda_n) + B_{air} (1 - \cos(\lambda_n))}{\lambda_n} \right] - B \left[ \frac{\cos(\lambda_n)}{\lambda_n} - \frac{1}{\lambda_n} \sin(\lambda_n) + \frac{B_{air} (\sin(\lambda_n) - \cos(\lambda_n))}{\lambda_n} \right]$$

$$\frac{1}{2} \left[ 1 + \left( \frac{B_{air}}{\lambda_n} \right)^2 \right] + \frac{\sin(2\lambda_n)}{4\lambda_n} \left[ 1 - \left( \frac{B_{air}}{\lambda_n} \right)^2 \right] + \frac{B_{air}}{\lambda_n} \sin^2(\lambda_n)$$

(9)

with the abbreviations $A$ and $B$ defined as

$$A = 1 - \frac{B_{∞}}{B_{air}(1 + B_{∞}) + B_{∞}}$$

$$B = \frac{B_{∞}}{B_{air}(1 + B_{∞}) + B_{∞}}$$

(10)

The complete solution is given by superposition of (6) and (7) to

$$\Theta(x, \tau) = \Theta(x) + \Theta(x, \tau) = \frac{B_{∞} (1 + B_{air} x)}{B_{air}(1 + B_{∞}) + B_{∞}} + \sum_{n=1}^{∞} C_n \left[ \cos(\lambda_n x) + \frac{B_{air}}{\lambda_n} \sin(\lambda_n x) \right] \exp(-\lambda_n \tau)$$

(11)

### 3. Inverse problem

If the initial temperature distribution of the turbine blade and the spatial distributions of heat transfer coefficients are known, the problem is well-posed and a unique solution can be obtained. In this study the initial temperature distribution and the outer convective boundary condition are known. The inner convective boundary condition is unknown. But, the transient temperature history of the outer surface is known as well. Hence requirements of an inverse problem are achieved. The temperature distribution inside of the wall as a function of time is still given by equation (1). However, the initial and boundary conditions are now given by

$$t = 0: \quad T(x, t) = T_v, \quad x = d: \quad -k \frac{\partial T}{\partial x} = h_v (T_v - T_x), \quad x = d: \quad T(d, t) = T_{mess, j} + \epsilon_j$$

(12)

where $t_j$ are discrete times at which the temperature values $T_{mess, j}$ are measured with random error $\epsilon_j$. The objective is to estimate the spatial distribution of internal heat transfer coefficients.

As optimization methods the steepest descent method, the conjugate gradient method, and the Levenberg-Marquardt method [11, 12] were taken into account. A comparison regarding performance on the actual hardware (Pentium 4-CPU, 3.20 GHz, 2 GB RAM) resulted in the Levenberg-Marquardt method as best fitting choice.

### 4. Uncertainty analysis

To prove accuracy of the method an uncertainty analysis was performed. For this the transient temperature history of the outer surface was calculated from (11) with parameters as listed in table 1.
The span of time was set to 600 s. Material properties, wall thickness and heat transfer coefficient on the outside of the model were varied by ± 0.5 %, ± 1.0 %, ± 2.0 %, ± 5.0 %, and ± 10.0 %. Air temperature inside of the model and ambient air temperature were varied with disturbances from -5 K to +5 K. The variations were used as input data to determine the influence of flawed parameters on the unknown heat transfer coefficients.

### Table 1. Parameters and values used in forward problem

| Parameter                                      | Unit       | Value  |
|------------------------------------------------|------------|--------|
| Heat transfer coefficient inside of model       | W/(m² K)   | 250    |
| Heat transfer coefficient outside of model      | W/(m² K)   | 7.0    |
| Air temperature inside of model after sudden step | K          | 333.15 |
| Ambient temperature                             | K          | 293.15 |
| Wall thickness                                  | m          | 0.01   |
| Thermal conductivity (stainless steel)          | W/(m K)    | 15     |
| Density (stainless steel)                       | kg/m³      | 7900   |
| Specific heat capacity (stainless steel)        | J/(kg K)   | 500    |

Figure 1 shows the influence of material properties, wall thickness and heat transfer coefficient at the outer surface. Thermal conductivity of stainless steel has a negligible influence of less then 1 %. Density and specific heat capacity have the identical influence, because they are used only in the thermal diffusivity as product. Here the parameter variation of up to ± 10.0 % results in deviations of same magnitude. The influence of wall thickness has the same characteristic and magnitude as density or specific heat capacity. The heat transfer coefficient at the outer surface again has a negligible influence of less then 1 %. As material properties are known within a range of ± 5.0 % at worst case, their influence on the results is relatively small. The wall thickness of the reference model is known in the range of ± 1.0 %. So this influence is also small. As heat transfer on the outside occurs as free convection, the heat transfer coefficient could vary from about 5 to 15 W/(m² K) along the model’s height. This is a wider range then covered by the variation. But correlations to describe the height dependency of free convection are known. Therefore, if necessary, height dependent heat transfer coefficients on the outer surface must be implemented into the evaluation procedure.

In figure 2 the influence of temperature deviations on the unknown heat transfer coefficient are displayed. Here the influence is much larger. An error of ± 2.0 K in the air temperature inside of the model results in a deviation of up to ± 15 %. Therefore, air temperatures inside of the model have to be measured as accurately as possible. The influence of the ambient air temperature is not as high as the one of the air temperature inside of the model. But still an error of ± 2.0 K results in a deviation of about ± 6 %. So, ambient temperature has to be measured during testing and, if necessary, it has to be implemented as time dependent temperature in the evaluation procedure.

Next was to analyze the influence of measurement errors in the surface temperature. Again the transient temperature history of the outer surface was calculated from (11) with parameters as listed in table 1. Then random noise of ± 1 K, ± 2 K, ± 3 K, ± 4 K, and ± 5 K was added. The noisy data was used as input data for the inverse method and internal heat transfer coefficients were estimated. To get statistic proof, a sufficient number of simulations were done for each error level.

Figure 3 shows representative data for an error level of ± 3 K. The green circles represent the smooth forward data calculated with an internal heat transfer coefficient of 250 W/(m² K). It is covered nearly perfect by the blue line, which represents the data calculated with the estimated heat transfer coefficient of about 252 W/(m² K). The red crosses represent the noisy data, which was used as input to the inverse method. In general data for all error levels look alike.

In figure 4 the results of the statistic analysis are shown. The average values of the estimated internal heat transfer coefficients are for all error levels close to the value of the smooth data (250 W/(m² K)). However, standard deviations increase with increasing error level. For an error level of ± 1...
The result is about $\sigma = 0.9 \text{ W/(m² K)}$ and for an error level of $\pm 5 \text{ K}$ it is about $\sigma = 3.2 \text{ W/(m² K)}$. Regarding that 99.7% of each Gaussian distribution is within the range of $\pm 3 \sigma$, this means, that relative errors of the estimated heat transfer coefficients are within a range of $\pm 1.1\%$ up to $\pm 3.9\%$ for increasing measurement error levels.

The uncertainty analysis has shown that influences of material properties are negligible, if they are known within the usual range of $\pm 5\%$ at worst case. The same is valid for the wall thickness. The influence of the outer heat transfer coefficient is negligible, too. Due to their large influence, both air temperatures, inside of the model and at the ambience, have to be measured as accurate as possible. Influences of the surface temperatures are of minor importance as long as they are Gaussian distributed. But, with additional systematic errors, that distort the characteristic of the transient temperature history, this is no longer valid.

5. Sensitivity analysis
Next was to analyze the sensitivity of the outer surface temperature on the unknown heat transfer coefficient $h_{\text{air}}$ over measuring time. The sensitivity coefficient is defined by

![Figure 1. Influence of parameter variation on accuracy of unknown heat transfer coefficients](image1)

![Figure 2. Influence of temperature deviation on accuracy of unknown heat transfer coefficients](image2)

![Figure 3. Data of transient surface temperature, noisy artificial data and inverse method solution at an error level of $\pm 3 \text{ K}$](image3)

![Figure 4. Influence of measurement error level on average value and standard deviation of heat transfer results](image4)
The numerically calculated results of the heat transfer’s sensitivity coefficient over time are shown in figure 5. In the time span from about 90 to 240 s there is maximum sensitivity of the surface temperature to the unknown heat transfer coefficient. Thus, this time span should be used in any case for estimation of the unknown heat transfer coefficients. On one hand this meets quite well the requirements of the used thermographic system for recalibration of the infrared sensor due to thermal drift. On the other hand all data measured within the chosen calibration range of the infrared sensor could be used for the evaluation procedure.

6. Experimental results

Measurements of outer surface temperatures and the estimation of internal heat transfer coefficients were carried out with a rectangular reference model. The model dimensions were length \( L = 600 \text{ mm} \), width \( W = 100 \text{ mm} \), and height \( H = 25 \text{ mm} \). So, the aspect ratio of the model was \( H/W = 1:4 \). The width of the model was ribbed on both sides. The ribs had square cross section and the rib height was chosen to 3.1 mm. This resulted in a rib height to hydraulic diameter ratio of \( e/D_h = 0.078 \). The pitch to rib height ratio was chosen to \( p/e = 10 \). The ribs were designed as parallel ribs with a rib angle of 90°.

All sensor data like temperature, pressure, and mass flow rate were measured with multiplexers and a sampling rate of 1 Hz expect of the air temperature inside of the model. Here the sampling rate was chosen to be 5 Hz. Temperatures were measured with type K thermocouples. The air temperature inside of the model was measured at three positions (inlet, half way, outlet) on the centerline. From this data a one-dimensional temperature model was obtained by linear interpolation. The mass flow rate was measured with a vortex flow meter. The infrared images were taken with a thermographic system from InfraTec at a frame rate of 1 Hz. All data presented here was measured at a mass flow rate of 95 g/s and a temperature step of 40 K.

Figure 6 shows a characteristic subset of infrared images at 30 s, 60 s, 90 s, and 120 s. In the beginning at low surface temperatures like after 30 s, there is a weak pattern visible referring to the rib locations. With increasing surface temperatures this pattern disappears more and more until it is no longer visible with higher surface temperatures like after 120 s. The whole series of about 160 infrared images were converted to pixelwise surface temperature over time files, and synchronized with the remaining data from the multiplexers. Then, spatial distributions of heat transfer coefficients are evaluated with a MatLab routine.

The idea of rib detection is based on the typical pattern of heat transfer coefficients within each pitch. This pattern should be visible in the estimated spatial distribution of the heat transfer coefficients. Then rib positions are detectable.

Due to the large thermal conductivity of stainless steel, the much higher heat transfer coefficient on ribs compared to pitches [13], and as heat absorbing surfaces of each rib are three times higher than the emitting cross section, each rib can be seen as an additional heat source. This leads with the one-dimensional evaluation to virtual higher heat transfer coefficients at rib positions. Figure 7 shows the spatial distribution of heat transfer coefficients, a representative infrared image and the locations of rib positions. While figure 8 displays a detailed look on heat transfer results on the centerline in the region where nearly periodic flow conditions could be assumed. In both figures it is visible, that at each rib positions there is a maximum in the estimated heat transfer coefficient. Next was to fit the centerline heat transfer data with a sinusoidal curve as shown in figure 8 by the black line. The fitting equation was chosen to

\[
\begin{align*}
  X_{\text{num}}(t) &= \frac{\partial T_a(t)}{\partial h_{\text{air}}} \\
  h_{\text{air}}(x) &= c_1 + c_2 \sin(c_3 2\pi x + c_4)
\end{align*}
\]
For all rib patterns with regular rib spacing the latter could be determined directly from fitting constant \( c_1 \). In our case with a length of the evaluation area of 530 mm the resulting rib spacing is 30.3 mm. The real rib spacing was 31 mm. Hence, the difference is less than 3 %.

![Figure 5. Sensitivity coefficient of unknown heat transfer coefficient over time](image)

![Figure 6. Infrared images of the reference model with 90° ribs at a flow rate of 95 g/s an a temperature step of 40 K](image)

![Figure 7. Spatial distribution of heat transfer coefficients and detection of rib positions](image)

![Figure 8. Location of peaks in heat transfer coefficient on centerline in infrared data and TLC data](image)

7. Conclusion
A non-invasive, non-destructive, transient inverse measurement technique to determine internal heat transfer coefficients and to detect rib positions of real gas turbine blades was developed. For this purpose the one-dimensional forward problem was solved. Furthermore the Levenberg-Marquardt method was chosen as optimization method to solve the inverse heat transfer problem.

An uncertainty analysis has shown that the influence of material properties as long as they are known within the usual range of \( \pm 5 \% \) is negligible. The same is valid for the wall thickness as long as it is known within \( \pm 1.0 \% \), and for the outer heat transfer coefficient. The influence of air temperature inside of the model and ambient air temperature is not negligible. Therefore, temperatures have to be measured with large accuracy. The sensitivity analysis has shown that the sensitivity of the
unknown heat transfer coefficients has its maximum within the time span of normal measurements of less than 300 s. Heat transfer distributions were estimated from experimental temperature data. Then, rib positions were detected from these results. Rib spacing was determined from a sinusoidal curve fit to heat transfer results on centerline with less than 3 % error.

Finally it is concluded that the presented inverse measurement method could be used including further developments to determine quantitatively internal heat transfer coefficients and to detect rib positions of real turbine blades.

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