An Empirical Correlations to Predict Shear Wave Velocity at Southern Iraq Oilfield

Raed H. Allawi¹*, Mohammed S Al-Jawad²

¹Thi-Qar Oil Company, Dhi-Qar, Iraq,
²University of Baghdad, Baghdad, Iraq

¹*Corresponding Author E-mail: r.allawi1308d@coeng.uobaghdad.edu.iq
²mjawad@coeng.uobaghdad.edu.iq

6th Iraq Oil and Gas Conference, 29-30/11/2021

Abstract

Geomechanical studies are very important in the development stages of oil fields to solve many problems such as wellbore instability and sand production. However, this study is not complete without the availability of mechanical properties of rocks. These properties estimate from petrophysical logs based on the compressional and shear wave velocities. But the shear wave is often missing from most wells, and the reason might be cost-saving. Therefore, this study aims to find correlations to predict the shear wave velocity of the Mishrif reservoir.

The empirical equations are formed using log data of six wells drilled in the southern Iraq oilfield. The Statistical Package for the Social Sciences (SPSS) software was relied on to find the empirical correlations. Eleven empirical equations have been obtained, but the best are three equations: linear, quadratic, and cubic because they give the highest value of $R^2 = 0.924$. Also, these three equations (linear, quadratic, and cubic) have been tested for sensitivity, and the most stable equation was quadratic. Moreover, the equations examined using four wells measured shear wave velocity, and the results were very reliable. Finally, the equations were tested by estimate the shear wave in wells where there are no measured data, then calculating the mechanical rock properties, predicting wellbore instability, and comparing the breakout with the caliper, and the result was excellent. This study is an excellent solution for shear wave estimation in wells where there are no measurements.

Keywords: Shear wave, Mechanical properties, Spss, Compressional wave, Geomechanical.
1. Introduction

In geomechanical studies, dynamic mechanical rock properties are calculated by using wave velocity (compressional Vp and shear Vs) [1, 2]. According to previous studies, for reliable calculations of formations' mechanical properties, the shear wave data must be available [3-7]. However, in practice, well log measurements are often only compressional sonic and sonic shear is not included due to the additional cost. Thus, some relationships are used to estimate the shear wave data from other available data. Shear wave and Poisson's ratio and in reservoir rocks were estimated by Wantland [8]. However, in practice, the Poisson's ratio varies widely; hence, the estimated shear wave data are unreliable [9].

The compressional and shear were measured using pulsation transmit techniques in the lab, which used to calculate the rocks elastic properties [10-15]. However, the data measured in the laboratory for Vs is relatively less compared to Vp [16]. It is difficult to measure Vs at low pressures because the Vs transmission during the sample needs strong sticking between the transducers and sample. If laboratory measurements are available, they remain specific because they cannot be generalized to the reservoir due to the different rocks and their heterogeneity. Many empirical correlations have been used to determine the Vs from the physical parameters of rocks [3, 17-20]. Despite this, these correlations formed in specific areas and specific formations, and therefore their use in other areas may give inaccurate results.

This field, targeted for the study, has 115 wells, but only ten wells have share wave measurements. Six wells were used to derive the correlations, and the other four wells were used to check the validity of those correlations. In this study, the Statistical Package for the Social Sciences (SPSS) software was relied on to find the empirical correlations based on a compressional wave in the Mishriff reservoir.

2. X Field Description

X oilfield is one of the important fields located in southern Iraq. The generalized lithostratigraphic column showing the formations intersected by the wells of X field is shown in Figure (1). The reservoir section currently being developed is the Mishriff
Limestone, with potential reservoir sections in deeper formations such as the Yamama Limestone.

3. **Previous empirical correlations for estimating the shear wave**

Previous studies have shown that the specific velocities of the shear-wave depend on the type of material, state (compaction and strength), and structural loading conditions, thus producing a difference in velocities [21]. Although the shear wave velocity determination on rock samples in the lab is only a small sample, it is important to overcome the Vs' difficulties or make its approximate value. It is worth noting that the laboratory measurements differ from the values that occurred in situ because the properties of the rocks show an environmental dependence, especially concerning stress. Thus, finding a correlation to calculate the Vs and at the same time reduce the cost of Vs measurements.

In the same context, several empirical equations suggested calculating the shear wave based on the compressional wave.

Regression analysis is the most common and used statistical method in the relation of dependent and independent variables. Regression analysis is divided into two parts, either linear or nonlinear. In linear regression, linear independent variables are used to model the data, in addition to the unknown model variables that are predicted from the data. Whereas for nonlinear regression, a function which is a nonlinear set of model parameters is used to model the data. One or more independent variables are used in this type of regression. Three models that are applied to predict the Vs have been selected as follows:
3.1 Castagna correlation

This correlation is the most common for predicting wave shear velocity. This correlation is built to calculate the shear wave velocity in limestone, sandstone, dolomite, and shale formations by Castagna [17]. This equation is presented as follows:

\[ V_S = -0.05509V_p^2 + 1.0168V_p - 1.0305 \]  

(1)

3.2 Brocher correlation

Broscher proposed a nonlinear equation to include a wide range of rock formations such as unconsolidated sediments, highly compact metamorphic rocks, non-welded volcanic tuffs, and igneous rocks [18, 19]. This equation is showed as follows:

\[ V_S = 0.7858 - 1.2344V_p + 0.7949V_p^2 - 0.1238V_p^3 + 0.006V_p^4 \]  

(2)
3.3 Carroll correlation
Carroll proposed an empirical equation based on the Poisson ratio between 0.22 and 0.28 for any rocks or, in other words, and the equation depends on the ratio of Vp/Vs; it must be between 1.61 and 1.85, and the unit of compression and shear is km/s. as showed: 

\[ V_S = 1.09913326V_P^{0.9238115336} \]  

(3)

4. Results and Discussion
4.1 New empirical correlations
This field contains 115 wells, but the wells with shear wave velocity measurements are only ten wells. Thus, six wells (GA-4, GA-D77, GA-J48, GA-J88, GA-Q41, and GA-Q44) were used to form the correlations, while the other four wells were used to check the accuracy of these correlations. The field measurements 27963 of compressional wave velocity are used to form these correlations. The nonlinear regression, a function that is a nonlinear set of model parameters, is used to model the data, which is applied in this study. The Statistical Package for the Social Sciences (SPSS) software was used, and the results were to obtain eleven equations, as present in Figure (2).

![Fig. (2) All correlations](image)
The parameters estimate (constants) for the obtained empirical equations are shown in Table (1). The R-square, which is defined as a statistical indicator for how close the data are in the fitted regression line, was used to predict the confidence of this empirical equation. The R-square is significantly high for the correlations obtained from this study (0.822-0.924), which is considered an excellent value. The R-square for the empirical correlation throughout ANOVA can be calculated via Eq. 4:

$$ R - square = 1 - \frac{\text{Sum of Squares (Residual)}}{\text{Sum of Squares (Corrected)}} $$

(4)

### Table (1) Model Summary and Parameter Estimates

| Dependent Variable: DTS | Model Summary | Parameter Estimates |
|-------------------------|---------------|---------------------|
| Equation                | R Square | F   | df1 | df2 | Sig. | Constant | b1   | b2   | b3    |
| Linear                  | 0.924    | 340202.400 | 1   | 27961 | 0.000 | 12.964   | 1.822 |
| Logarithmic             | 0.909    | 279966.132 | 1   | 27961 | 0.000 | -460.158 | 141.655 |
| Inverse                 | 0.822    | 129507.765 | 1   | 27961 | 0.000 | 282.946  | -9773.199 |
| Quadratic               | 0.924    | 170239.654 | 2   | 27960 | 0.000 | 8.816    | 1.927  | -0.001 |
| Cubic                   | 0.924    | 113745.140 | 3   | 27959 | 0.000 | -11.567  | 2.663  | -0.009 | 3.203E-5 |
| Compound                | 0.898    | 246253.059 | 1   | 27961 | 0.000 | 61.887   | 1.012  |
| Power                   | 0.914    | 295952.076 | 1   | 27961 | 0.000 | 2.844    | 0.918  |
| S                       | 0.862    | 175196.717 | 1   | 27961 | 0.000 | 5.879    | -64.691 |
| Growth                  | 0.898    | 246253.059 | 1   | 27961 | 0.000 | 4.125    | 0.012  |
| Exponential             | 0.898    | 246253.059 | 1   | 27961 | 0.000 | 61.887   | 0.012  |
| Logistic                | 0.898    | 246253.059 | 1   | 27961 | 0.000 | 0.016    | 0.988  |

The independent variable is DTC.

After substituting the constants from Table 1 into all standard equations, the empirical equations will be as follows:

1. Linear equation \( DTS = 12.964 + 1.822 \text{ DTC} \)  
2. Logarithmic equation \( DTS = -460.158 + 141.655 \ln \text{ DTC} \)
3. Inverse equation  \[ DTS = 282.946 - \frac{977.3199}{DTC} \]  

4. Quadratic equation  \[ DTS = 8.816 + 1.927DTC - 0.001DTC^2 \]  

5. Cubic equation  \[ DTS = -11.567 + 2.663DTC - 0.009DTC^2 - 0.00003203DTC^3 \]  

6. Compound equation  \[ DTS = 61.887 \times 1.012^{DTC} \]  

7. Power equation  \[ DTS = 2.844 DTC^{0.918} \]  

8. S-curve equation  \[ DTS = e^{(5.879 + \frac{-64.691}{DTC})} \]  

9. Growth equation  \[ DTS = e^{(4.125 + 0.012 \times DTC)} \]  

10. Exponential equation  \[ DTS = 61.887 e^{0.012DTC} \]  

11. Logistic equation  \[ DTS = \frac{1}{u + 0.016 \times 0.988^{DTC}} \]  

4.2 Statistical parameters of correlations

The three best equations (Linear, Quadratic, and Cubic equation) that give the highest R-square value of 0.924 were selected. Then the sensitivity of these three equations was tested using Root Mean Square Error (RMSE), as presented in equation 16.

\[
AAPE = 100 \times \frac{1}{n} \sum_{i=1}^{n} \left| \frac{P_m - P_e}{P_m} \right| \]  

\[
MSE = \frac{1}{n} \sum_{i=1}^{n} [P_m - P_e]^2 \]  

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} [P_m - P_e]^2} \]  

Absolute Percent Error (AAPE), Mean Square Error (MSE), and Absolute Root MSE (ARMSE) were used to judge the accuracy of equations. Therefore, the quadratic equation produced a low value of statistical parameters, as shown in Table (2). Furthermore, statistical parameters were applied to other global correlations, as shown in Table (3).

| Equation  | AAPE | MSE  | RMSE |
|-----------|------|------|------|
| Linear    | 4.3  | 60.9 | 7.8  |
| Quadratic | 3.6  | 46.2 | 6.8  |
| Cubic     | 16.4 | 829  | 28.8 |
Table (3) Statistical parameters of global correlations

| correlation | AAPE | MSE   | RMSE |
|------------|------|-------|------|
| Castagna   | 9.5  | 134   | 11.6 |
| Brocher    | 11.2 | 143.2 | 12   |
| Carroll    | 52.5 | 2652.8| 51.5 |

4.3 Examination of correlation to predict shear wave velocity

The second stage is to test the accuracy of those equations. Therefore, those equations were used to calculate the shear wave velocity of the four wells (GA-5, GA-D85, GA-J40, and GA-Q45) with shear wave measurements, and they were not forming those equations. The quadratic correlation produced a higher R-square value than other correlations, as shown in Table (4) and Figures. (3, 4, 5, and 6).

Table (4) R-square of Examination wells

| Well No.  | R-square |
|-----------|----------|
| GA-5      | 0.847    |
| GA-D85    | 0.901    |
| GA-J40    | 0.916    |
| GA-Q45    | 0.896    |

Fig. (3) DTSmeasured vs. DTScorrelation for GA-5
Fig. (4) DTSmeasured vs. DTScorrelation for GA-D85

Fig. (5) DTSmeasured vs. DTScorrelation for GA-J40
4.4 Wellbore instability test

In the final validation, the shear wave velocity was calculated for the well GA-F23, which did not have shear wave measurement. Then calculate the mechanical rock properties and predict the wellbore instability of the well and compare breakout with the actual breakout from caliper log. The results showed high agreement between the predicted breakout and the breakout measured by caliper log, as presented in Figure (7).

Fig. (6) DTSmeasured vs. DTScorrelation for GA-Q45
Fig. (7) Wellbore instability of GA-F23
5. **Conclusion**

This study proposed empirical equations to predict shear wave velocity in X oilfield in southern Iraq. Also, the suggested equation was validated by more than 20000 shear wave measurements from the same X oilfield. The main conclusions from this study are as follows:

- This study showed that the proposed empirical equations are reliable in estimating the shear wave in the X oilfield.
- Eleven equations calculate the shear wave, and three equations (linear, quadratic, and cubic) are the best according to the R-square value of 0.924.
- The results showed that the quadratic equation has a low value of statistical parameters more than the other equations (linear and cubic).
- The quadratic equation has been used in other wells that have measurements, data of the shear wave. The results were excellent due to the high R-square value.
- Also, the equation showed excellent results when used to predict the wellbore instability of wells that do not have shear velocity measurements.

**Nomenclature**

\( V_S \) : shear wave velocities ft/us (foot to microsecond).
\( V_P \) : Compressional wave velocities ft/us (foot to microsecond).
\( u \) : the higher boundary value
CALS: Caliper log
BS: Bite size
MW: Mud weight
Reference

[1] Singh R, Kainthola A, Singh T. Estimation of elastic constant of rocks using an ANFIS approach. Applied Soft Computing. 2012;12(1):40-5.

[2] Allawi RH, Al-Jawad MS. Wellbore instability management using geomechanical modeling and wellbore stability analysis for Zubair shale formation in Southern Iraq. Journal of Petroleum Exploration and Production Technology. 2021:1-16.

[3] Ameen MS, Smart BG, Somerville JM, Hammilton S, Naji NA. Predicting rock mechanical properties of carbonates from wireline logs (A case study: Arab-D reservoir, Ghawar field, Saudi Arabia). Marine and Petroleum Geology. 2009;26(4):430-44.

[4] Boonen P, Bean C, Tepper R, Deady R, editors. Important Implications From A Comparison Of Lwd And Wireline Acoustic Data From A Gulf Of Mexico Well. SPWLA 39th Annual Logging Symposium; 1998: OnePetro.

[5] Eissa E, Kazi A. Relation between static and dynamic Young's moduli of rocks. International Journal of Rock Mechanics and Mining & Geomechanics Abstracts. 1988;25(6).

[6] Rasouli V, Pallikathekathil ZJ, Mawuli E. The influence of perturbed stresses near faults on drilling strategy: a case study in Blacktip field, North Australia. Journal of Petroleum Science and Engineering. 2011;76(1-2):37-50.

[7] Nader AF, AL-Saad HF, Jaber KD, Madhi A, Swadi A-M. Mathematical models describing the effect of overburden pressure on porosity and permeability of reservoir rocks of Upper shale member for South Rumelia and upper sandston (3 rd pay) member for Zubair oil field. Journal of Petroleum Research & Studies. 2021(31).

[8] Wantland D, Laroque G, Bollo M, Dickey D, Goodman R. Geophysical measurements of rock properties in situ. 1964.

[9] Carroll RD, editor The determination of the acoustic parameters of volcanic rocks from compressional velocity measurements. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts; 1969: Elsevier.

[10] Birch F. The velocity of compressional waves in rocks to 10 kilobars: Journal of Geophysical Research. 1960;65(4):1083-102.

[11] Christensen NI. Compressional wave velocities in possible mantle rocks to pressures of 30 kilobars. Journal of Geophysical Research. 1974;79(2):407-12.

[12] Kern H. P-and S-wave velocities in crustal and mantle rocks under the simultaneous action of high confining pressure and high temperature and the effect of the rock microstructure. 1982.
[13] Burlini L, Fountain DM. Seismic anisotropy of metapelites from the Ivrea-Verbano zone and Serie dei Laghi (northern Italy). Physics of the Earth and Planetary Interiors. 1993;78(3-4):301-17.

[14] Ji S, Salisbury MH. Shear-wave velocities, anisotropy and splitting in high-grade mylonites. Tectonophysics. 1993;221(3-4):453-73.

[15] Watanabe T, Kasami H, Ohshima S. Compressional and shear wave velocities of serpentinized peridotites up to 200 MPa. Earth, planets and space. 2007;59(4):233-44.

[16] Ji S, Wang Q, Xia B. Handbook of seismic properties of minerals, rocks and ores: Presses inter Polytechnique; 2002.

[17] Castagna JP, Backus MM. Offset-dependent reflectivity—Theory and practice of AVO analysis: Society of Exploration Geophysicists; 1993.

[18] Brocher TM. Empirical relations between elastic wavespeeds and density in the Earth's crust. Bulletin of the seismological Society of America. 2005;95(6):2081-92.

[19] Brocher TM. Key elements of regional seismic velocity models for long period ground motion simulations. Journal of Seismology. 2008;12(2):217-21.

[20] Yasar E, Erdogan Y. Correlating sound velocity with the density, compressive strength and Young's modulus of carbonate rocks. International Journal of Rock Mechanics and Mining Sciences. 2004;41(5):871-5.

[21] Sirles PC, Viksne A. Site-specific shear wave velocity determinations for geotechnical engineering applications. Geotechnical and enviromental geophysics. 1990;3:121-31.