The magnetic field dependence of the threshold electric field in unconventional charge density waves

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Many experiments suggest that the unidentified low temperature phase (LTP) of \( \alpha-(BEDT-TTF)_2KHg(SCN)_4 \) is most likely unconventional charge density wave (UCDW). To further this identification we present our theoretical study of the threshold electric field of UCDW in a magnetic field. The magnetic field-temperature phase diagram is very similar to those in a d-wave superconductor. We find a rather strong field dependence of the threshold electric field, which should be readily accessible experimentally.

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I. INTRODUCTION

In recent papers two of us have studied the thermodynamics and the optical conductivity of unconventional spin density wave (USDW) and unconventional charge density wave (UCDW)\cite{1,2}. Unlike conventional SDW or CDW, we define USDW and UCDW as the SDW and CDW where the order parameter \( \Delta(\mathbf{k}) \) depends on the quasi-particle momentum \( \mathbf{k} \). As to the initial motivation, the presence of many unconventional superconductors is an adequate justification\cite{3,4}. Indeed the thermodynamics of USDW or UCDW are identical to the one of d-wave superconductor\cite{5}.

On the other hand the low temperature phase (LTP) of \( \alpha-(BEDT-TTF)_2KHg(SCN)_4 \) has not been understood until now. The LTP does not exhibit X-ray or NMR signals characteristic to conventional CDW or SDW\cite{6,7}, and this property is naturally born out by the UDW model\cite{1,2}. The response of the LTP in a magnetic field suggests that it is not SDW but more likely a kind of CDW\cite{8,9}. Indeed the phase diagram of the LTP in a magnetic field is rather similar to the one of a d-wave superconductor in the presence of the Pauli paramagnetism\cite{10}.

In the presence of magnetic field (say parallel to the conduction chain in order to avoid the orbital effect) the LTP splits into two regimes: the low field regime, where \( \Delta(\mathbf{k},\mathbf{r}) \) is constant spatially and the high field regime where \( \Delta(\mathbf{k},\mathbf{r}) \) varies periodically in space\cite{11,12,13}. In superconductors the latter regime is called Fulde-Ferrell-Larkin-Ovchinikov (FFLO) state\cite{14,15}.

Very recently the threshold electric field associated with the sliding motion of CDW or SDW of \( \alpha-(BEDT-TTF)_2KHg(SCN)_4 \) has been reported\cite{16}. The temperature dependence of the threshold electric field is very different from the ones we know for conventional CDW like \( NbSe_3 \) and conventional SDW like \( (TMTSF)_2PF_6 \), though somewhat closer to the one in SDW\cite{17}.

We have shown recently\cite{18} that UDW (either charge or spin) gives the temperature dependent threshold electric field, which is very close to the observation. Though the agreement is not perfect, we believe that the discrepancy is due to imperfect nesting what we neglected for simplicity.

The object of the present paper is to extend the earlier analysis in the presence of a magnetic field. As in earlier works\cite{8,9,11,12,13} on the related subjects we focus on the Zeeman splitting (or the Pauli paramagnetic effect) due to the external magnetic field. Also for simplicity we limit ourselves to the regime where \( \Delta(\mathbf{k},\mathbf{r}) \) is spatially uniform. Then the related thermodynamic quantities have been worked out already in\cite{12,13}. Therefore the model used in\cite{12,13} can be readily extended. We find that the threshold electric field depends on the external magnetic field as well as on the temperature.

Therefore we believe that the predicted field dependence should be readily accessible experimentally. The \( \alpha-(ET)_2 \) salts can be put into two groups: one superconducting and another with this mysterious LTP. \( \alpha-(ET)_2MHg(SCN)_4 \) with \( M=K, Tl \) and \( Rb \) belong to the second group. Hence we suggest that our theory applies to the LTP in the second group.
II. PHASE DIAGRAM AND DENSITY OF STATES

The phase diagram is the same as the one in a d-wave superconductor without the FFLO state. At \( T = 0 \) a first order transition occurs to the normal state at \( h = 0.56 \Delta_0 \), where \( \Delta_0 \) is the zero field zero temperature order parameter and \( h = \mu_B H \). The value of the gap is 0.92\( \Delta_0 \) at the transition point. With decreasing field, the transition occurs at \( h = 0.41 \Delta_0 \), and the gap jumps from zero to 0.97\( \Delta_0 \). For \( T < 0.56T_c \) (\( T_c \) is the transition temperature at \( h = 0 \)) the transition remains first order, and hysteresis is observable somewhere between 0.41 < \( h/\Delta_0 \) < 0.56.

In this region, the normal state becomes local minimum of the free energy, and depending on the direction of the change of the external field, the first order transition occurs at smaller field approaching from the normal state than quitting the DW phase with increasing field. By exceeding \( T \approx 0.56T_c \), the transition becomes second order at the bicritical point (\( h \approx 0.51 \Delta_0 \), \( T \approx 0.56T_c \)). It is worth noting that the phase diagram is modified at \( T < 0.56T_c \) and \( h \sim 0.51 \Delta_0 \) because of the possibility of the FFLO regime what we excluded here for simplicity. The order parameter as a function of \( T \) and \( h \) is shown in Fig. 1.

![FIG. 1: Stereograph of the order parameter in the reduced temperature and field plane. The dotted line denotes the metastability line above which the normal state becomes local minimum of the free energy.](image)

The quasiparticle density of states is obtained as:\[ N(E) = \frac{1}{2}(\rho(E + h) + \rho(E - h)), \] (1)

where \( \rho(E) \) is the density of states in the absence of magnetic field, and is given by\[ \rho(E)/\rho_0(0) = (2|E|/\pi|\Delta|)K(|E|/|\Delta|) \] if \(|E| < |\Delta|\), and \( \rho(E)/\rho_0(0) = (2/\pi)K(|\Delta|/|E|) \) if \(|E| > |\Delta|\). \( K(z) \) is the complete elliptic integral of the first kind. As \( h \) increases, the valley at the Fermi surface is filled in. Also the divergent peaks at \( \pm \Delta \) split into four new peaks at \( \pm \Delta \pm h \). Interestingly, at \( h = \Delta \) the density of states is divergent at the Fermi surface. These properties can be seen in Fig. 2.

III. PHASE HAMILTONIAN AND THE THRESHOLD ELECTRIC FIELD

It is the most convenient to formulate the threshold electric field in terms of the phase Hamiltonian which is given as:\[ H(\Phi) = \int d^3 r \left\{ \frac{1}{4}N_0 f \left[ v_F^2 \left( \frac{\partial \Phi}{\partial x} \right)^2 + v_b^2 \left( \frac{\partial \Phi}{\partial y} \right)^2 + v_c^2 \left( \frac{\partial \Phi}{\partial z} \right)^2 + \left( \frac{\partial \Phi}{\partial t} \right)^2 - 4v_F eE \Phi \right] + V_{imp}(\Phi) \right\} \] (2)

where \( N_0 \) is the density of states in the normal state at the Fermi surface per spin, \( f = \rho_s(T,h)/\rho_s(0,0) \) where \( \rho_s(T,h) \) is the condensate density and \( E \) is an electric field applied in the \( x \) direction. Here \( v_F \), \( v_b \) and \( v_c \) are the
characteristic velocities of the quasi-one dimensional electron system in the three spatial directions. For UDW the condensate density is the same as the superfluid density in d-wave superconductors and is shown in Fig. 3.

We may think of Eq. (2) as a natural extension of the Fukuyama-Lee-Rice Hamiltonian for UCDW and for $T \neq 0$, $H \neq 0$ and for three spatial directions.
The pinning potential is obtained as

\[
V_{\text{imp}}(\Phi) = -\frac{8V_0V_yN_z}{\pi} \sum_j \cos(2(QR_j + \Phi(R_j))) \Delta(T, h) \times
\int_0^1 \frac{1}{2} \left( \tanh \frac{\beta(\Delta(T, h)x + h)}{2} + \tanh \frac{\beta(\Delta(T, h)x - h)}{2} \right) E(\sqrt{1 - x^2})(K(x) - E(x))dx,
\]

where \(R_j\) is an impurity site, \(K(z)\) and \(E(z)\) are the complete elliptic integrals of the first and second kind, respectively.

In obtaining Eq. 3 we assumed a nonlocal impurity potential \(U(Q, q) = V_0 + \sum_{i=y,z} V_i \cos(q_i \delta_i)\).

Then following FLR, in the strong pinning limit the threshold electric field at \(T = 0\) is given by

\[
E_{ST}(0, h) = 2k_F e n_0 N_0 \frac{16 \Delta(0, h)}{\pi} \rho_s(0, h) \int_{h/\Delta_{00}}^1 E(\sqrt{1 - x^2})(K(x) - E(x))dx,
\]

and for general temperature it is obtained as

\[
\frac{E_{ST}^T(0, h)}{E_{ST}(0, 0)} = \frac{\rho_s(0, 0) \Delta(T, h)}{\rho_s(T, h) \Delta_{00}} \frac{1}{0.5925} \times
\int_0^1 \frac{1}{2} \left( \tanh \frac{\beta(\Delta(T, h)x + h)}{2} + \tanh \frac{\beta(\Delta(T, h)x - h)}{2} \right) E(\sqrt{1 - x^2})(K(x) - E(x))dx.
\]

In the \(h, T \ll \Delta_{00}\) range, for \(T > 0.56 T_{c0}\), along the second order phase boundary the threshold field reads as

\[
\frac{E_{ST}^T(0, h)}{E_{ST}(0, 0)} = -\frac{\text{Re}\Psi' \left( \frac{1}{2} + \frac{i h}{2 T} \right) T}{\text{Re}\Psi'' \left( \frac{1}{2} + \frac{i h}{2 T} \right) \Delta_{00}} \frac{\pi^3}{4 \times 0.5925}.
\]
which is divergent at the bicritical point (possibly tricritical with the FFLO state) due to the zero of \( \text{Re}\Psi'' \left( \frac{1}{2} + \frac{i h}{2\pi T} \right) \) at \( h/T \approx 1.91 \). As a result the threshold electric field close to the bicritical point is given by

\[
\frac{E^S_T(T, h)}{E^S_T(0, 0)} = \frac{1.12}{1.91 - h/t} \tag{9}
\]

from the second order phase boundary. We show the threshold electric field as a function of the temperature and the magnetic field in Fig. 4 in the strong pinning limit.

The weak-pinning limit is more appropriate for high quality crystals. Then we obtain for a three dimensional system

\[
E^W_T(T, h) = \left( \frac{E^S_T(T, h)}{E^S_T(0, 0)} \right)^4. \tag{10}
\]

\( E^W_T(T, h) \) is plotted as a function of temperature and magnetic field in Figs. 5 and 6, respectively.

\[
\begin{array}{c}
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\includegraphics[width=0.5\textwidth]{figure5.png}
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\]

FIG. 5: The threshold electric field in the weak pinning limit is plotted as a function of the reduced temperature for \( h/\Delta_{00} = 0, 0.1, 0.2, 0.3, 0.4, 0.45, 0.5, 0.52, 0.55 \) with endpoints from right to left. The circle represents the end of the \( h = 0.4\Delta_{00} \) curve, which is very close to the \( h = 0.3\Delta_{00} \) one. The dashed line accounts for the threshold field along the second order phase boundary while the dotted for the one along the first order phase boundary.

At small but increasing fields, the enhancement of the threshold electric field at the transition temperature relative to the \( T = 0 \) value becomes smaller due to the initial linear decrease of the condensate density versus \( h \) at \( T = 0 \).

At low temperature \( E^W_T(T, h) \) increases with \( h \) almost linearly. Further \( E^W_T(T, h) \) diverges for \( T \approx 0.56T_{c0} \) when the magnetic transition changes from second order to first order. Indeed the strong \( H \) dependence of the threshold electric field at \( T = 2.2\text{K} \) is described in \( 14 \) which appears to be consistent with the present result,though no detail is available. Unfortunately the present result does not apply for \( T < 0.56T_{c0} \) and \( h \gtrsim 0.51\Delta_{00} \) due to the presence of the FFLO regime. Nevertheless the present result can be tested in a wide range of the \( H - T \) phase diagram of the LTP in \( \alpha-(ET)_2 \) salts. The effect of the FFLO state and the related threshold electric field will be discussed elsewhere.

IV. CONCLUDING REMARKS

We have extended our earlier analysis of the threshold electric field in UCDW in the presence of magnetic field. The magnetic field is introduced as the Zeeman splitting. The phase diagram is found to be identical to the one in a d-wave superconductor\( 10 \) without the FFLO state. The present model predicts very strong \( H \) dependence of the threshold electric field, even divergent behaviour at the bicritical point, which should be readily accessible experimentally.

If such a strong \( H \) dependence of \( E_T \) is observed, this will surely strengthen our proposal that the LTP of \( \alpha-(ET)_2 \) salts should be UCDW.
FIG. 6: The threshold electric field in the weak pinning limit is plotted as a function of the magnetic field for $T/T_c = 0, 0.2, 0.3, 0.4, 0.6$ and $0.7$ with endpoints from right to left. The dashed line accounts for the threshold field along the second order phase boundary while the dotted for the one along the first order phase boundary.

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