Model of phase synchronization system operating under conditions of violent ionospheric disturbances

V A Tsimbal, S N Shimanov, D V Mokrinskiy, V E Toiskin
MGEI Institute of Engineering Physics, Udarniy Bolshoi per., 1-a, Serpukhov town, 142210, Moscow region, Russia
E-mail dmitrimokrinski1991@mail.ru

Abstract. A model of the phase synchronization system based on the circuit of digital phase-locked-loop frequency control, adaptive equalizer, and a system of frequency-matched filters is proposed. This model provides the required value of the bit error probability at the output of the digital demodulator under the influence of strong ionospheric disturbances in the decameter communication channel.

Keywords: synchronization, phase-locked loop frequency, adaptive equalizer, transfer function, loop filter, frequency decay.

1. Introduction
It is known that the decametric channel (DCW) is characterized by a complex of negative effects that significantly complicate the qualitative detection of the received signal. One of the factors (along with additive and multiplicative noise [1,2]) that significantly affect the quality of reception is the presence of a low frequency in the communication channel. Frequency decay can be associated with the movement of the transmitter and receiver relative to each other as well as with the uniform rise or fall of the reflecting region of the ionosphere for a limited time interval. In the second case, there will be a classic Doppler effect, i.e. a change in the frequency of the received signal. According to P. Grin and M. P. Dolukhanov [3,4], a value of the maximum decay of the DCW frequency of the communication line during strong ionospheric disturbances can reach 100 Hz. Currently, in order to compensate for frequency loss in the digital communication systems the adapted classical methods are proposed based on the implementation of digital (software-implemented) circuit of the phase-locked loop (PLL), adaptive equalizers (AE), and a set of frequency-matched filters [5-9]. However, the problem of frequency loss compensation in the presence of fading in the communication channel is not sufficiently studied. One of the main problems of such systems is the presence of circuit synchronization failures with significant frequency deviations [10,11]. The long process of restoring synchronism leads to irremediable reception errors, which negatively affects the noise immunity of the communication as a whole. Thus, an urgent task is to develop a model of the received signal phase synchronization system that can meet the specified requirements for noise immunity under the influence of strong ionospheric disturbances.

2. Formal statement of the problem
Source data:
1. The structure of the DCW communication protocol, a feature of which is the sequential alternation of information sequences and sequences for learning AE:

\[ F_r(\left(f_{r_n}, f_{r_m}\right)), \]  

where \( f_{r_n} \) is the length of information sequence; \( f_{r_m} \) is the length of AE learning sequence.

2. Parameters of the adaptive filtering subsystem:

\[ F_a(\left(F_{a_type}, F_{a_n}, F_{a_m}\right)) \]

where \( F_{a_type} \) is the type of adaptive filtering algorithm; \( F_{a_n}, F_{a_m} \) is the number of forward and reverse branches of the AE (determine the order of the AE).

3. The PLL circuit parameters:

\[ F_{PLL}(K_d, F_{Filt.Order}, G_1, G_2, K_{NCO}) \]

where \( K_d \) is the gain of the phase detector (FD); \( F_{Filt.Order} \) is the order of the loop filter; \( G_1, G_2 \) is the proportional and integral gain of the loop filter; \( K_{NCO} \) is the gain of the voltage-controlled generator (VCO).

4. Type of signal modulation: \( F_m \).

5. Parameters of communication channel:

\[ F_c = \{F_{C_{AWGN}}, F_{C_{Fad.type}}, I, W_{ci}, \theta, T_{ci}, \Omega T, \Omega^2 T, f_c, i = 1, I\} \]

where \( F_{C_{AWGN}} \) is the parameter of additive noise (value of the ration signal-noise at the demodulator input; \( F_{C_{Fad.type}} \) is a fading type; \( I \) is the number of rays distributions; \( W_{ci}, T_{ci} \) are beam power and delay relative to the principal one; \( \theta \) is a jump phase value; \( \Omega T \) is a frequency jump value, \( \Omega^2 T \) is a value of frequency drift rate; \( f_c \) is a value of maximum frequency decay.

6. Sample rate of the received signal: \( f_D \).

7. Number of frequency-matched filters: \( N \).

It is necessary to justify the structure of the phase synchronization system \( (F_{PLL}, N) \) that provides the required noise immunity of communication, with the specified \( F_r, F_a, F_m, F_c, f_D \).

The indicator of noise immunity as a manifestation intensity of this property, we assume the probability of a bit error at the output of the demodulator (BER) of the radio receiving digital device \( E_b \).

3. Mathematical model

To analyze the functioning and stability features of the system under study, the mathematical apparatus of z-transformations was used. A linearized model of a typical PLL circuit in the approximation of small phase angles is shown in figure 1. This system is described by a transfer function (TF) of the following type [12,13]:

\[ H_c(z) = \frac{\phi(z)}{\psi(z)} = \frac{\sqrt{2PK_d}N(z)\sqrt{2PK_d}N(z)}{1 + 2PK_dN(z)F(z)}, \]  

where \( N(z) \) is the VCO TF; \( F(z) \) is the loop filter TF; \( 2P \) is a square of the amplitude of the received signal; \( \phi(z) \) and \( \psi(z) \) are the z-images \( \phi(n) \) and \( \psi(n) \) (of input readouts and ones from VCO output).
In the course of the researches carried out to substantiate the structure of the phase synchronization system \((F_{PLL})\) an option was offered based on the PLL and AE included in the synchronization circuit (figure 2). Taking into account that the AE is a linear system at each fixed time, the AE was represented as a TF linear adder \([14]\):

\[
H_{AE}(z) = \frac{1 + \omega_0 z^{-1}}{1 - \omega_1 z^{-1}} = \frac{z + \omega_0}{z - \omega_1},
\]

(6)

where \(\omega_0, \omega_1\) are the weight coefficients of the linear adder.

Considering the known expressions \([13]\) for describing the VCO TF (7), the TF of the loop filter of the first \((F_1)\) and second \((F_2)\) order (8) and the expression for the TF by mistake (9):

\[
N(z) = \frac{K_{NCO} z^{-1}}{1 - z^{-1}} = \frac{K_{NCO}}{z - 1},
\]

(7)

\[
F_1(z) = G_1, \quad F_2(z) = G_1 + \frac{G_2}{1 - z^{-1}},
\]

(8)

\[
H_{e}(z) = 1 - H_{e}(z);
\]

(9)

the following expressions were obtained:

\[
H_1 = \frac{z - 1}{z - 1 + K_1 \left(\frac{z + \omega_0}{z - \omega_1}\right) G_1}, \quad H_2 = \frac{(z - 1)^2}{(z - 1)^2 + K_1 \left(\frac{z + \omega_0}{z - \omega_1}\right) (G_1 (z - 1) + G_2 z)}
\]

(10)

where \(H_1\) is a TF of the first-order circuit; \(H_2\) is a TF of the second-order circuit; \(K_1 = \sqrt{2} PK_d K_{NCO}\).

The pulse characteristics of TF (10) under the influence of typical disturbances (in the form of a phase jump \(H_{e1}\), a frequency jump \(H_{e2}\) and a linear frequency decay \(H_{e3}\) \([13]\))
\[
H_{c1} = \left( \frac{\theta z}{z-1} \right), \quad H_{c2} = \frac{\Omega T z}{(z-1)^2}, \quad H_{c3} = \frac{\Omega^2 T z(z+1)}{2(z-1)^3}.
\]  
(11)

are shown in figure 3. Thus: for \(H_1\), the qualitative characteristics of a typical first-order PLL are preserved (first-order astatism); for \(H_2\), the circuit is unstable when exposed to disturbances in the form of a frequency jump and linear frequency decay. Variation of initial parameters gives stable solutions only at abnormally low values of \(G_2\) \((G_2 < 0.01)\). Based on this the choice of the loop filter order is justified: \(F_{\text{Filt, Order}} = 1\)

Consider the behavior of the system with a fixed frequency shift when the overall gain of the loop displaces \(K = K_c G_1 = \sqrt{2P} K_c K_{\text{NCO}} G_1\). The system response for frequency jumps of 1 and 10 Hz is shown in figure 4.

The analysis shows that for each frequency shift value, there is a quasi-optimal value of the total loop gain \((K)\), which minimizes the phase error \(\Delta \phi\) at the output of the system. Based on this, it is proposed to minimize the phase error by adjusting the overall gain of the loop. It is physically feasible \([12,13]\) to use \(G_1\) as a control parameter.

Let the communication channel has a linear frequency departure from 0 to \(f_c\). Divide the entire frequency decay range into \(N\) discrete intervals. In each interval \(f_{(m-1)c} \leq f_m < f_{mc};\ 1 \leq m \leq N\)

the system operates with \(G_{1m}\) selected in such a way that the total phase error of the selected interval \(\Delta \phi\) does not exceed the required value \(\Delta \phi < \Delta \phi_{\text{max}}\). In this case the system of equations describing the
functioning of this system has the form:

\[
\begin{cases}
0 \leq f_1 < f_{l1} \rightarrow H_{e1} = \left( \frac{\Omega T z}{(z-1)^2} \right) \left( \frac{z-1}{z-1 + K_1 \left( \frac{z + \omega_0}{z - \omega_1} \right) G_1} \right) \quad \Delta \varphi < \Delta \varphi_{\text{max}}; \\
f_{l1} \leq f_2 < f_{l2} \rightarrow H_{e2} = \left( \frac{\Omega T z}{(z-1)^2} \right) \left( \frac{z-1}{z-1 + K_1 \left( \frac{z + \omega_0}{z - \omega_1} \right) G_{12}} \right) \quad \Delta \varphi < \Delta \varphi_{\text{max}}; \\
\vdots \\
f_{(N-1)l} \leq f_N < f_c \rightarrow H_{eN} = \left( \frac{\Omega T z}{(z-1)^2} \right) \left( \frac{z-1}{z-1 + K_1 \left( \frac{z + \omega_0}{z - \omega_1} \right) G_{1N}} \right) \quad \Delta \varphi < \Delta \varphi_{\text{max}}.
\end{cases}
\] (13)

By adjusting \( G_i \), you can ensure that the phase error in the steady-state mode does not exceed the required value. As a result, we get a system that has a non-zero phase error at the output. The effect of the residual phase error on the final probability of the \( E_b \) bit error (\( \Delta \varphi \)) for different types of modulation is well studied and described [15,16].

![Figure 4. System response to frequency jump at different values of the overall circuit gain](image)

The resulting mathematical model has the following limitations:

Limit 1: the parameters of additive and multiplicative noise in the communication channel are not taken into account: \( F_{C_{\text{AWGN}}}, F_{C_{\text{Fad, type}}}, I, W_{ci}, T_{ci} \).

Limit 2: the type of modulation of the received signal is not taken into account: \( F_m \).

Limit 3: the quantitative characteristics of the AE are not taken into account: \( F_a \).

Limit 4: the specifics of the Protocol used are not taken into account: \( F_r \).

A special feature of the model is the need to adjust \( G_i \) depending on the instantaneous frequency decay in the \( f_{nc} \) communication channel, in this case, the \( f_{nc} \) search is performed using a set of matched filter blocks, each of which is consistent with the expected signal with a certain carrier frequency shift [1], while reducing the frequency uncertainty is limited by the number of matched filters in the system.
Thus, according to the formal statement of the problem, the mathematical model justifies the structure of the phase synchronization system ($F_{PLL}$). At the same time, in order to justify the number of agreed filters ($N$), avoid the specified restrictions, and in order to move from a qualitative analysis of the system's functioning to a quantitative assessment, it is necessary to use simulation tools.

4. Simulation model

Based on the mathematical model of the phase synchronization system (12), a simulation model was built in the visual-oriented programming environment Simulink (MatLab package), its general structure is shown in figure 5.

The transmitting side contains the following basic structural elements: random binary sequence generator, AE training sequence generator (ASG), package structure generator ($F_r$ is taken into account.), converter of binary sequence to numeric one, digital modulator ($F_m$ is taken into account.).

The communication line contains: the block of frequency-phase mismatch of the channel (implements the effect of typical disturbances $\theta$, $\Omega_T$, $\Omega^2 T_2$, $f_c$), the block of multiplicative noises ($F_{C_{Fad\_type}}$, $I$, $W_{ci}$, $T_{ci}$ are considered.), the block of additive noise generation ($F_{C_{AWGN}}$ is taken into account).

The receiving side consists of a phase synchronization system, training sequence generator, demodulator, redundancy elimination unit and message receiver.

The simplified block diagram of the phase synchronization system is shown in figure 6 and contains the following basic structural elements: AE (considers $F_a$), CBPE – a calculation block of phase error (is a part of PD (considers $K_d$)), BPF - bandpass filter (considers $F_{Filt\_Order}$, $G_1$, $G_2$), NCO – numerically controlled oscillator – digital analog of VCO (considers $K_{NCO}$).

The $G_i$ parameter of the bandpass filter is adjusted based on the $f_{nc}$ value using a set of matched filter blocks, the estimation is based on the highest value of the output signal (figure 7). The main requirement for this system is to design a set of filter blocks that cover the entire range of possible frequency shifts, as well as a decision-making block that implements control of the synchronization system parameters depending on the maximum output of one of the filters.
5. Simulation results

Figure 8 shows the results of simulation under the following initial conditions: structure of data transmission packet \( f_{\text{in}}=20 \) bits, \( f_{\text{in}}=20 \) bits; \( F_{a,\text{type}} - \) RLS (recurrent calculation of weight coefficients based on the criterion of minimum sum of error signal squares); \( F_{a,n}=20 \); \( F_{a,m}=20 \); \( K_d = 1 \); \( G_1 - \) adjustable parameter; \( G_2 = 0 \); \( K_{\text{NCO}} = 1 \); \( F_{\text{m}} - \) QPSK; \( T = 20 \) (c) - simulation time.

Characteristics of the communication channel:

Option 1 (blue curve on the diagram): \( I = 1 \); \( F_{C,\text{AWGN}} - \) signal / noise ratio (SNR) (1-30) (dB); \( F_{C,\text{Fad,\text{type}}} - \) amplitude distribution of the incoming ray according to the Rayleigh rating; disturbance parameters: \( f_c = 72 \) (Hz), \( \Omega^2 T = 4.4 \) (Hz/s).

Option 2 (red curve on the diagram): \( I = 2 \); \( F_{C,\text{AWGN}} - \) (1-30) (dB); \( W_{\text{c}} = - 3 \) (dB); \( T_c = 0.003 \) (s); \( F_{C,\text{Fad,\text{type}}} - \) amplitude distribution of the incoming rays according to the Rayleigh rating; disturbance parameters: \( f_c = 72 \) (Hz), \( \Omega^2 T = 4.4 \) (Hz/s).

The simulation was performed for the following configurations of frequency matched filters: \( N = 4 \) (corresponds to the maximum value of the phase error \( \Delta \phi = 14,7^\circ \)), \( N = 7 \) (corresponds to the value of the phase error \( \Delta \phi = 8,4^\circ \)), \( N = 14 \) (corresponds to the value of the phase error \( \Delta \phi = 4,1^\circ \)).

![Figure 8](image_url)
Based on the results of the simulation, the following conclusions can be drawn:

For option 1 - in the presence of additive and multiplicative interference (taking into account a single propagation beam), increasing the number of frequency-matched filters gives a significant gain in noise immunity. The difference in reception characteristics between \( N = 4 \) and \( N = 7 \), can range from 15% (SNR 10 (dB)) to 40% (SNR 30 (dB)). The further increase in \( N \) - does not lead to a significant improvement in the noise immunity index. The difference between \( N = 7 \) and \( N = 14 \) is on the average 7%.

For option 2 - in the presence of additive and multiplicative noise (taking into account two propagation beams), an increase in the number of matched filters \( (N) \) has a stronger effect on the quality of detection of the received signal. Thus, the difference in reception characteristics between \( N = 4 \) and \( N = 9 \), can range from 15% (SNR 10 (dB)) to 25% (SNR 30 (dB)). The further increase in \( N \) - leads to a significant improvement in the noise immunity index (in contrast with option 1). Thus, the difference in reception characteristics between \( N = 7 \) and \( N = 14 \), can range from 7% (SNR 10 (dB)) to 35% (SNR 30 (dB)).

6. Conclusion

Based on the developed mathematical model, a phase synchronization system (PLL) based on PLL and AE was proposed. The analysis of the obtained TF in the z-region revealed that the system under study has the first order of astaticism, which makes it necessary to adjust the proportional gain of the loop filter depending on the values of the instantaneous frequency decay in the communication channel. Using a set of frequency-matched filters as a control mechanism, it is possible to control the residual value of the phase error at the output of the synchronization system. Increase of the number of frequency-matched filters \( (N) \) reduces the residual phase error \( \Delta \phi \), which allows you to achieve the desired level of noise immunity. The most demanding to \( N \) are the communication channels, in which the effect of multipath propagation of the radio signal is more pronounced. The simulation results show that to ensure the required level of bit error probability for a radio channel model with two propagation beams, twice as many frequency-matched filters are required as compared to the single-propagation channel model.

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