Understanding CP violation in lattice QCD

P. Mitra
Saha Institute of Nuclear Physics,
1/AF Bidhannagar,
Calcutta 700064, India

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Abstract

It is pointed out that any CP violation which may be found in lattice QCD with a chiral phase in the fermion mass term but without an explicit theta term cannot be relevant for the continuum theory. CP is classically conserved in the corresponding continuum theory and is non-anomalous.

1 Introduction

The strong interactions per se do not violate parity, but the Lagrangian of QCD may contain a $\theta \text{tr} F \tilde{F}$ term which violates parity and CP. Further, the quark mass term $\bar{\psi}m e^{i\gamma_5 \theta'} \psi$ has an unknown chiral phase $\theta'$ which also appears to violate these. However, there is no experimental evidence of such violations. It is difficult to estimate directly the effect of the gluonic $\theta$ term, but calculations and proposals are often made with the $\theta'$ term, mainly in the continuum and sometimes on the lattice.

This $\theta'$-term appears to violate CP classically [1], but it is known [2] that a classically and perturbatively conserved parity can be defined in the presence of this term: parity and CP violation can then occur either if there is a $\theta$ term, which explicitly breaks parity, or if induced by nontrivial topology

*mitra@tnp.saha.ernet.in
of gauge fields in functional integration, i.e., if the symmetries develop non-perturbative anomalies \[2, 3, 4\]. A chiral transformation with an associated anomaly does enter the definition of the conserved parity, so it may not be surprising if the CP symmetry is afflicted with an anomaly upon quantization. To understand whether the short distance singularities of the quantum field theory have such an effect, it is necessary to use a regularization.

A symmetry in a classical field theory can develop an anomaly in the quantized theory if it is not possible to find any regularization to preserve it. It is, of course, not difficult to find regularizations which violate a given symmetry. To prove that a classical symmetry is preserved in the quantum theory, it is sufficient to find one regularization which preserves the symmetry, whereas for the existence of an anomaly, it is necessary that all regularizations break it. This principle will be applied here to the question of parity or CP violation in the presence of the \(\theta'\) term.

A generalized Pauli-Villars regularization has been used \[5\], but it involves the introduction of unphysical regulator species. A lattice regularization can be used to avoid this complication and nonperturbative anomalies can be sought to be detected through the lattice. Lattice calculations of CP violation in QCD have in fact been considered in the literature \[3\]. In this note we shall therefore investigate this class of regularizations. This will confirm that the classically conserved CP does not develop an anomaly, so that any CP violation observed in lattice QCD with just a chiral phase \(\theta'\) in the mass term must be an artefact and will not be relevant for continuum QCD. This means that not only quenched lattice calculations with the \(\theta'\) term but even dynamical calculations with it \[3\] have nothing to do with continuum QCD. The \(\theta\) term must be used for any significant result.

2 Parity symmetry

The spinor part of the single flavour continuum QCD euclidean action reads

\[
S = \int d^4x \bar{\psi} [\gamma^\mu \Delta_\mu - m e^{i\theta'\gamma_5}] \psi, \tag{1}
\]

where \(\Delta_\mu\) stands for the covariant derivative. When \(\theta'\) is non-zero, this breaks the standard parity transformation in which \(A_0\) transforms as a true scalar and \(A_i\) as a polar vector and the fermion field transforms as

\[
\bar{\psi}(x_0, x_i) \rightarrow \bar{\psi}(x_0, -x_i) \gamma_0
\]
\[ \psi(x_0, x_i) \rightarrow \gamma_0 \psi(x_0, -x_i). \]  

As mentioned above, there is a parity transformation \cite{footnote} that leaves \( S \) invariant. This involves the usual transformation for gauge fields but that for fermions is altered to include a chiral rotation proportional to \( \theta' \):

\[
\bar{\psi}(x_0, x_i) \rightarrow \bar{\psi}(x_0, -x_i) e^{i\theta' \gamma_5} \gamma_0 \\
\psi(x_0, x_i) \rightarrow \gamma_0 e^{i\theta' \gamma_5} \psi(x_0, -x_i). \tag{3}
\]

This \( \theta' \)-dependent parity is easily seen to leave the action, with exactly the same chiral phase \( \theta' \) in the mass term, invariant, except for any \( \theta \) term in the pure gauge piece of the action, which of course changes sign.

This is not enough to guarantee that \( \theta' \) does not violate CP in the quantized theory, as the altered parity transformation contains a chiral transformation and may consequently be anomalous. To investigate this matter, we examine the theory regularized by introduction of the lattice.

\section{2.1 Wilson regularization of fermions}

The standard lattice regularization for fermions with the Wilson prescription is

\[ S_W = a^4 \sum_x \bar{\psi} \left[ \gamma^\mu \frac{D_\mu + D^*_\mu}{2} - a D^*_\mu D_\mu - m e^{i\theta' \gamma_5} \right] \psi. \tag{4} \]

Here, \( a \) is the lattice spacing, \( D_\mu \) the forward covariant difference operator on the lattice and \( D^*_\mu \) the backward covariant difference operator. The fermion action is understood to be supplemented with the gauge field action which involves the standard plaquette contribution together with a \( \theta \) term, for which one may use one of the available forms like \cite{footnote}:

\[ S_g = \sum_{\text{plaquettes}} S_{\text{plaquette}} + i\theta Q. \tag{5} \]

The fermionic part \( S_W \) of the action is not invariant under the lattice version of \( (3) \), with the link variables transformed in the usual way, and one may think that the breaking of this classical symmetry of the unregularized theory by a regularization is an anomaly. Unlike the popular continuum approach to anomalies, this breaking does not arise from a regularization of the measure, but the lattice provides a regularization of the action, and
the chiral anomaly is known to manifest itself in the Wilson formulation of fermions as an explicit breaking of the chiral symmetry of the action.

However, this is not the whole story. Before suspecting that the classical parity symmetry develops an anomaly, one has to check if this symmetry is broken in all regularizations. As recognized in \[7, 8\], the $D^* D$ term in the lattice action can be assigned a chiral phase:

$$S_{\theta''} = a^4 \sum_x \bar{\psi} \left[ \gamma_\mu \frac{D_\mu + D^*_\mu}{2} - aD^*_\mu D_\mu e^{i\theta''(\gamma_5 - me^{i\theta'(\gamma_5)}]} \psi. \right. \quad (6)$$

Thus within the Wilson formulation of lattice fermions, there is a whole class of regularizations parametrized by $\theta''$.

2.1.1 The symmetric choice

Let us see what happens in the special case $\theta'' = \theta'$. In this case, the lattice implementation of (3), with the link variables transformed in the usual way, leaves the fermionic action (3), invariant, i.e., this parity is not broken by the $\theta'$ term in the quantum theory. This is a very special, non-generic regularization, but the existence of just one parity conserving regularization means that the transformation (3) is not anomalous. One can take the limit $a \to 0$ with $\theta'' = \theta'$ and the parity continues to be conserved in the continuum limit. The same is true of CP. This is the result announced: CP is neither explicitly broken nor anomalous if $\theta' \neq 0$. Of course, the symmetry is explicitly broken if $\theta \neq 0$ in $S_g$.

2.1.2 Other choices

It may be of interest to see what happens if one deliberately chooses a regularization with $\theta'' \neq \theta'$. It is straightforward to see that the term with the phase $\theta''$ is replaced, under (3), by a term with a phase $2\theta' - \theta''$. As a regularization which preserves the symmetry is available, this breaking by deliberate choice is an artefact rather than an anomaly. The explicit breaking in this class of regularizations in the fermionic piece of the action is governed by the effective CP violating parameter

$$\tilde{\theta}_f = \theta' - \theta''. \quad (7)$$

In addition, as before, the $\theta$ term in $S_g$ also violates CP. Thus there are two CP violating parameters on the lattice, $\tilde{\theta}_f$ and $\theta$. They cannot be transformed
into each other through chiral rotations on the lattice because the measure in this regularized theory is trivial. To first order, one may split any measure of CP violation like the electric dipole moment of the neutron in the form

$$\Delta(a) = \Delta_f(a) \bar{\theta} + \Delta_g(a) \theta,$$

where the $a$-dependent coefficients on the right hand side may be different for the fermion and gauge sectors. The behaviour of these two parameters in the continuum limit is of interest. It is possible to make a general statement without detailed calculations. In the limit $a \to 0$, where the regularization is removed, a dependence on the regularization parameter $\theta''$ must not survive:

$$\frac{\partial \Delta(0)}{\partial \theta''} = 0.$$  \hspace{1cm} (9)

Without calculating $\Delta(a)$, one can then argue that

$$\frac{\partial \bar{\theta}_f}{\partial \theta''} \Delta_f(0) = -\Delta_f(0) = 0.$$  \hspace{1cm} (10)

The fact that any CP violation from the quark sector has to depend on $\theta' - \theta''$ thus requires the $\theta'$ dependence to vanish in the continuum. The only continuum contribution that can survive is

$$\Delta(0) = \Delta_g(0) \theta,$$

showing that only $\theta$ can be a physical parameter, $\theta'$ is not. It is easy to generalize this to higher order effects. Any CP violation in the continuum theory can consequently come only from $\theta$. The above arguments cannot indicate whether the $\theta$ dependence is non-zero or not, so one can say that the vacuum angle, if nonzero, may lead to CP violation, but the phase in the fermion mass term has no such effect.

This argument fits in with the result reached above that the parity which is classically conserved in the presence of $\theta'$ is not anomalous. The regularization which is needed on the lattice to preserve this symmetry has $\theta'' = \theta'$, i.e., $\bar{\theta}_f = 0$. The violation of the symmetry for any other value of this parameter is a regularization artefact, because one symmetric choice of the regularization parameter $\theta''$ is available. It is well known that the breaking of rotation invariance on the cubical lattice is not an anomaly but a regularization artefact that can be avoided by using random lattices or continuum regularizations and in fact goes away in the continuum limit. In the same way, parity and CP are violated by $\theta'$ only in regularizations with a different value of $\theta''$ and the violation does not survive in the continuum limit.
2.2 Ginsparg-Wilson formulation

So far we have considered Wilson fermions. All this can be easily generalized to more general lattice actions, including the theoretically popular ones satisfying the Ginsparg-Wilson relation [9],

\[ S^{\theta''}_{GW} = a^4 \sum_x \bar{\psi} \left[ e^{i\theta'' \gamma_5} D e^{-i\theta'' \gamma_5} - m e^{i\theta' \gamma_5} \right] \psi, \]  

(12)

where \( D \) stands for the generalized lattice Dirac operator, which does not anticommute with \( \gamma_5 \). It is to be noted that the phase \( \theta'' \) has to be introduced through an ordinary chiral transformation, under which \( D \) is not invariant. If \( \theta'' = \theta' \), this action is again invariant under the lattice version of (3), so that this parity symmetry and CP are seen to be anomaly-free. The measure does not break the symmetry as the chiral transformation involved in (3) is only a normal chiral transformation, not involving \( D \): for such transformations, the measure changes trivially, i.e., there is no Jacobian factor different from unity. When \( \theta'' \neq \theta' \), the transformation (3) alters \( \theta' \) to \( 2\theta'' - \theta' \), and the measure changes trivially, so that the parity violating parameter is again \( \theta' - \theta'' \equiv \bar{\theta}_f \). As before, this violation is an artefact and has to disappear in the continuum limit.

3 Conclusion

The chiral phase \( \theta' \) in the quark mass term cannot be held responsible for CP violation, as demonstrated clearly with a lattice regularization. Lattice computations with the \( \theta \) term, if possible, could be of use in giving an indication of the strength of CP violation resulting from this term. Nonzero CP violation in QCD, if ever detected in experiments, can be accommodated only through a nonzero \( \theta \) term.

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References

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Appendix: Discussion of some experts’ comments

Excerpts from comment:
“It is not clear to me what is the question to which the paper tries to give an answer. I thought that the concepts of anomaly and lattice artifacts were understood a long time ago (if indeed this is the subject [sic] of the paper).”

It would be good if the subjects were understood by everybody!

Excerpts from comment:
“The action he considers is the most general one under the standard parity but not under the new parity definition. A mass term with a complex phase (the same which appears in the symmetry transformation) is an eigenstate of the new parity transformation (+1). Therefore a general parity violating action ... would be to add a combination of two terms $\bar{\psi} m e^{i\theta} \gamma_5 \psi$ and $\bar{\psi} m \gamma_5 e^{i\theta} \gamma_5 \psi$ without changing the phase in the Wilson term.... This is not defining a different regularization but just adding the same content to the theory as in the standard case.”

As this expert knows, it is elementary to rewrite this mass term in the standard form with a change of parameters. Correspondingly, the parity symmetry gets altered, and the phase in the Wilson term too must be altered to preserve this symmetry.

Excerpts from comments on a related continuum article:
“For a clear discussion consider a lattice regularization (as the author did ...) where, for finite ‘a’, everything is explicitly defined. As the author says, one can introduce gauge field dependent phases in the measure in a QCD like theory. One might consider this as an extra contribution to the action.

The action should be local, gauge invariant and Euclidean rotation (Lorentz) invariant. These constraints allow a nonzero theta term and also a nonzero phase in the fermion mass matrix. Due to the anomaly, these phases are connected, and we can put all the phases in theta.

...There is no CP anomaly here which would prevent us to define a CP symmetric QCD. On that point the author is correct...”
While these remarks tend to agree with this article, a clarification is needed here. The phases are indeed “connected” in the continuum, if there is no regularization and if the usual measure is used. But “for a clear discussion” without going into the symmetry of the measure, one has to remain on the lattice. Then these phases are no longer connected because Jacobians are trivial for ordinary chiral rotations. The phases of the mass term and the regulator term are of course connected, but not because of any anomaly.