Three-Quark Light-Cone Amplitudes of The Proton
And Quark-Orbital-Motion Dependent Observables

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Abstract

We study the three-quark light-cone amplitudes of the proton including quarks’ transverse momenta. We classify these amplitudes using a newly-developed method in which light-cone wave functions are constructed from a class of light-cone matrix elements. We derive the constraints on the amplitudes from parity and time-reversal symmetries. We use the amplitudes to calculate the physical observables which vanish when the quark orbital angular momentum is absent. These include transverse-momentum dependent parton distributions $\Delta q_T(x, k_\perp)$, $q_T(x, k_\perp)$, $\delta q(x, k_\perp)$, and $\delta q_L(x, k_\perp)$, twist-three parton distributions $g_T(x)$ and $h_L(x)$, helicity-flip generalized parton distributions $E(x, \xi = 0, Q^2)$ and its associates, and the Pauli form factor $F_2(Q^2)$. 

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I. INTRODUCTION

In the simplest model of the proton, the valence quarks are moving in the s-orbit and the proton spin is constructed from the quark spins. This picture has been ruled out definitely by the EMC data [1] and the follow-up experiments, SMC, E142, E143, E152, E153, and HERMES [2]. One then expects that the quarks in the proton must have non-zero orbital angular momentum and/or the gluons must carry part of the proton spin. When the orbital angular momenta of quarks and gluons are none-zero, the nucleon is intrinsically deformed. An interesting question is how big is the deformation? or how much does the quark orbital angular momentum contribute to the spin of the nucleon? A few years ago one of us derived a sum rule, relating the total angular momentum carried by quarks to an integral over the relevant generalized parton distributions (GPDs) [3, 4]. The orbital angular momentum can be obtained by subtracting the measured quark helicity contribution. This observation has lead to very active studies of GPDs and their measurements in hard exclusive processes [5, 6, 7].

There are, in fact, many observables which are potentially sensitive to the orbital angular momentum of the quarks, although they do not directly measure the orbital angular momentum itself. For example, the Pauli form factor $F_2(Q^2)$ of the proton, which has been measured recently using the recoil polarization technique at JLab [8, 9], has long been recognized as an observable related to the light-cone amplitudes with non-zero orbital angular momentum [10, 11, 12]. The twist-three spin-dependent structure function $g_T(x) = g_1(x) + g_2(x)$ vanishes in the naive parton model, and its interpretation requires quarks’ orbital motion [13, 14, 15]. The generalized parton distributions such as $E(x, \xi, Q^2)$ were introduced to characterize quark orbital angular momentum in the first place [8, 10, 17]. And finally, the transverse-momentum dependent parton distributions [18, 19, 20, 21, 22, 23] contain rich and direct information about the quark orbital motion [24, 25, 26, 27]. A natural question is can one correlate all these observables in terms of light-cone wave functions in a minimally-model-dependent way? This paper is an attempt in this direction.

Our approach starts with the light-cone expansion of the proton state truncated to the minimum Fock sector with just three valence quarks [28, 29]. Unlike the light-cone amplitudes studied in the exclusive processes [30, 31, 32, 33, 34], we keep the full dependence on the transverse momenta of the quarks. Some previous studies in this direction can found in Refs. [35, 36, 37]. Clearly, this truncation is ideal: we do not have the gluon and sea quark degrees of freedom; we do not have manifest rotational symmetry and gauge invariance. However, the truncation can be improved systematically by including higher Fock components, for example, adding one or more pair of sea quarks and/or one or more gluons, etc. Therefore, in this study we will ignore certain artifacts arising from this specific truncation.

Based on our work here, one can go on to parameterize the three-quark amplitudes and fit to the experimental data. The amplitudes thus determined are phenomenological, and in principle cannot be compared with those solved from QCD directly. However, a comparison helps to determine the importance of the three quark amplitudes vs. multi-particle Fock amplitudes. One may also consider the phenomenological amplitudes as the effective proton wave function after integrating out the gluon and sea quark degrees of freedom, although the integrations are hard to implement in practice.

By committing ourselves to the light-cone amplitudes, we are also committing to the light-cone gauges $A^+ = 0$ [38]. One can in principle add light-cone links between the quark fields to make the light-cone amplitudes gauge-invariant, but we do not find a compelling
reason to do this. One subtlety about the light-cone gauge is that it requires additional
gauge fixing \cite{39,40,41}. Depending on whether the additional gauge condition is time-
reversal invariant or not, the wave function amplitudes are real or fully complex. In the
latter case, the final state interaction effects are contained in the amplitudes \cite{27,42,43}.

Our plan of the presentation is as follows. We start in Sec. II by writing down the matrix
elements of three-quark light-cone operators between proton state and the QCD vacuum,
which serve to define a complete set of light-cone amplitudes within the truncation. We
simplify these matrix elements using color, flavor, spin, and discrete symmetries. We then
invert them in Sec. III to find the three-quark light-cone wave functions. As we have
explained in Ref. \cite{44}, there are many advantages in this construction. One is that the cut-
off dependence of the light-cone amplitudes can be studied directly from the matrix elements.
We derived the scale evolution equation of the individual Fock contributions to the parton
densities in \cite{44}. Another is that these matrix elements may be calculated in the nucleon
models or lattice QCD: it is not necessary to construct a model of the nucleon directly in the
light-cone frame. Finally, the large-momentum behavior of the nucleon amplitudes can be
determined from Bethe-Salpeter-like equations governing the matrix elements. In Sec. IV,
we calculate a number of physical observables which are directly linked to the amplitudes
with non-zero quark orbital angular momentum.

II. THREE-QUARK LIGHT-CONE MATRIX ELEMENTS

In this section, we are interested in classifying the matrix elements of three-quark oper-
ators between the proton and QCD vacuum. Since much of the discussion depends on the
use of light-cone coordinates and light-cone gauge, we start by reminding the reader some
salient features of the light-cone technology, and a more detailed exposition can be found in
\cite{28,29}.

The light-cone time \( x^+ \) and coordinate \( x^- \) are defined as \( x^\pm = 1/\sqrt{2}(x^0 \pm x^3) \). Likewise
we define Dirac matrices \( \gamma^\pm = 1/\sqrt{2}(\gamma^0 \pm \gamma^3) \). The projection operators for Dirac fields are
defined as \( P_{\pm} = (1/2)\gamma^\mp \gamma^\pm \). Any Dirac field \( \psi \) can be decomposed into \( \psi = \psi_+ + \psi_- \) with
\( \psi_\pm = P_{\pm}\psi \). \( \psi_+ \) is a dynamical degrees of freedom and has the canonical expansion,

\[
\psi_+(\xi^+, \xi^-, \xi_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} \sum_\lambda \left[ b_\lambda(k)u(k\lambda)e^{-i(k^+\xi^- - \vec{k}\cdot\vec{\xi}_\perp)} + d_\lambda^\dagger(k)v(k\lambda)e^{i(k^+\xi^- - \vec{k}\cdot\vec{\xi}_\perp)} \right].
\]  

Likewise, for the gluon fields in the light-cone gauge \( A^+ = 0 \), \( A_\perp \) is dynamical and has the expansion,

\[
A_\perp(\xi^+, \xi^-, \xi_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} \sum_\lambda \left[ a_\lambda(k)\epsilon(k\lambda)e^{-i(k^+\xi^- - \vec{k}\cdot\vec{\xi}_\perp)} + a_\lambda^\dagger(k)\epsilon^*(k\lambda)e^{i(k^+\xi^- - \vec{k}\cdot\vec{\xi}_\perp)} \right].
\]  

\( \psi_- \) and \( A^- \) are dependent variables.

The key observation in Ref. \cite{44} is that the light-cone Fock expansion of a hadron state
is completely defined by the matrix elements of a special class equal-light-cone-time quark-
gluon operators between the QCD vacuum and the hadron. These operators are specified
as follows: Take the + component of the Dirac field \( \psi^+ \) and the + \( \perp \)-component of the gauge field \( F^+\perp \). [We sometimes label the \( \perp \) components with index \( i = 1, 2 \).] Assume all these fields are at light-cone time \( x^+ = 0 \), but otherwise with arbitrary dependence on other spacetime coordinates. Products of these fields with the right quantum numbers (spin, flavor, and color) define a set of operator basis. [This has been done in the past for light-cone amplitudes in which all fields are separated along the light-cone, see for example [30].] Clearly, these operators are not gauge-invariant although one can gauge-invariantize them by inserting light-cone links extending from the locations of the fields to infinity for every field. Since all fields are at different spacetime points in the transverse direction, the operators do not require additional renormalization subtractions apart from the usual wave function renormalization. The matrix elements of all these operators between the hadron state and the QCD vacuum contain complete information about the hadron wave function.

In the following two subsections, we will use the above method to study the matrix elements of operators constructed out of three quark fields between proton and the QCD vacuum. We discuss separately the cases for the two up-quarks coupling to helicity-zero and one.

### A. Two Up-Quarks Coupling To Helicity-Zero

When the two up-quarks are coupled to helicity-0, we define the following two matrix elements,

\[
\langle 0 | u_+^T a (\xi_1) C \gamma^+ u_+^b (\xi_2) d_+^c (\xi_3) \epsilon^{abc} | P \rangle = \phi^{(1)} (1, 2, 3) \gamma_5 U_+ + (i \partial_1^i \phi^{(2)} (1, 2, 3) + i \partial_2^i \phi^{(2)} (2, 1, 3)) \gamma_5 \gamma^i U_+ + \tilde{\partial}_1^i \partial_2^i \phi^{(3)} (1, 2, 3) U_+ ,
\]

\[
\langle 0 | u_+^T a (\xi_1) C \gamma^+ \gamma_5 u_+^b (\xi_2) d_+^c (\xi_3) \epsilon^{abc} | p \rangle = \phi^{(4)} (1, 2, 3) U_+ + (i \partial_1^i \phi^{(5)} (1, 2, 3) - i \partial_2^i \phi^{(5)} (2, 1, 3)) \gamma^i U_+ + \tilde{\partial}_1^i \partial_2^i \gamma_5 \phi^{(6)} (1, 2, 3) U_+ ,
\]

where \( a, b, c = (1, 2, 3) \) are color indices of the quarks, \( C \) is the charge conjugation matrix \( C = i \gamma^2 \gamma^0 \), and \( | P \rangle \) is the proton state with momentum \( P^\mu \) and Dirac spinor \( U_+(P) \). The arguments 1, 2, and 3 in the scalar functions \( \phi^{(i)} \) stand for \( (\xi_1^-, \xi_1^\perp) \), \( (\xi_2^-, \xi_2^\perp) \), and \( (\xi_3^-, \xi_3^\perp) \), and the functional dependence on transverse coordinates are of type \( \xi_{1\perp} \cdot \xi_{2\perp} \) only. The up and down quark fields are represented by \( u \) and \( d \), respectively. The index \( i \) goes over the transverse coordinates 1, 2. In the above equations, we have used the symmetry relations (\( T \) stands for transpose)

\[
u_+^T a (\xi_1) C \gamma^+ u_+^b (\xi_2) \epsilon^{abc} = u_+^T a (\xi_2) C \gamma^+ u_+^b (\xi_1) \epsilon^{abc} ,
\]

\[
u_+^T a (\xi_1) C \gamma^+ \gamma_5 u_+^b (\xi_2) \epsilon^{abc} = - u_+^T a (\xi_2) C \gamma^+ \gamma_5 u_+^b (\xi_1) \epsilon^{abc} ,
\]

which imply

\[
\phi^{(1)} (1, 2, 3) = \phi^{(1)} (2, 1, 3) , \quad \phi^{(6)} (1, 2, 3) = \phi^{(6)} (2, 1, 3) ,
\]

\[
\phi^{(3)} (1, 2, 3) = - \phi^{(3)} (2, 1, 3) , \quad \phi^{(4)} (1, 2, 3) = - \phi^{(4)} (2, 1, 3) .
\]
The matrix elements have an overall dependence on $\xi_1 + \xi_2 + \xi_3$ through a phase factor $e^{i(\xi_1 + \xi_2 + \xi_3) P/3}$, which we assumed has been factorized implicitly. The remaining dependence is on the differences of the coordinates, for example, $\xi_1 - \xi_2$ and $\xi_2 - \xi_3$, because of the translational invariance. Therefore, the partial derivative $\partial_3^i$ is not independent of $\partial_1^i$ and $\partial_2^i$, and has been omitted. We have not included the structure $\tilde{\partial}^i \gamma^j$ in these equations because it is the same as $\tilde{\partial}^i \gamma^j \gamma_5$ when acting on $U_+$ using $\gamma^i \gamma^+ \gamma^- U_+ = 0$. We use the shorthand $\tilde{\partial}^i = \epsilon^{i3j} \partial^j$ ($\epsilon^{123} = 1$). Similarly the structure $\tilde{\partial}_1^i \tilde{\partial}_2^j \gamma^\gamma U_+$ can be reduced to the existing ones.

We can simplify the above equations by adding them and applying the chiral projection operators $P_{L,R} = (1 \mp \gamma_5)/2$

$$2\langle 0 | u_{+R}^{T a}(\xi_1) C \gamma^+ u_{+L}^b(\xi_2) d_{+R}^c(\xi_3) \epsilon^{abc} | P \rangle$$

$$= \left[ \phi^{(1)}(1,2,3) - \phi^{(4)}(1,2,3) \right] U_{+R} + \tilde{\partial}_1^i \partial_2^j \left[ \phi^{(3)}(1,2,3) - \phi^{(6)}(1,2,3) \right] U_{+R}$$

$$+ [i \tilde{\partial}_1^i \left( \phi^{(2)}(1,2,3) - \phi^{(5)}(1,2,3) \right) + i \tilde{\partial}_2^j \left( \phi^{(6)}(1,2,3) + \phi^{(5)}(2,1,3) \right)] \gamma^i U_{+L}.$$ 

(7)

The two-up quarks have been paired to helicity-zero, and the remaining down quark is projected to be right-handed. We can further simplify the above equation by introducing the new amplitudes,

$$\langle 0 | u_{+R}^{T a}(\xi_1) C \gamma^+ u_{+L}^b(\xi_2) d_{+R}^c(\xi_3) \epsilon^{abc}/\sqrt{6} | P \rangle$$

$$= \psi^{(1)}(1,2,3) U_{+R} + \epsilon^{ij} \tilde{\partial}_1^i \psi^{(2)}(1,2,3) U_{+R} + \left[ i \tilde{\partial}_1^i \psi^{(3)}(1,2,3) + i \tilde{\partial}_2^j \psi^{(4)}(1,2,3) \right] \gamma^i U_{+L}.$$ 

(8)

where $\psi$'s have no specific symmetry properties because the two up quarks are not in the same helicity state. If the nucleon is right-handed, we split the above equation into two

$$\langle 0 | u_{+R}^{T a}(\xi_1) C \gamma^+ u_{+L}^b(\xi_2) d_{+R}^c(\xi_3) \epsilon^{abc}/\sqrt{6} | P \rangle = \left[ \psi^{(1)}(1,2,3) + i \epsilon^{ij} \tilde{\partial}_1^i \psi^{(2)}(1,2,3) \right] U_{+R}.$$ 

(9)

$$\langle 0 | u_{+R}^{T a}(\xi_1) C \gamma^+ u_{+L}^b(\xi_2) d_{+R}^c(\xi_3) \epsilon^{abc}/\sqrt{6} | P \rangle = \left[ i \tilde{\partial}_1^i \psi^{(3)}(1,2,3) + i \tilde{\partial}_2^j \psi^{(4)}(1,2,3) \right] \gamma^i U_{+R}.$$ 

(10)

In our convention $\partial^i = -\partial/\partial x^i$ and $\gamma^i$ is that of Bjorken and Drell [13].

B. Two Up-Quarks Coupling To Helicity-One

When the two up quarks are coupled to helicity-1, we define the matrix element,

$$\langle 0 | u_{+R}^{T a}(\xi_1) C i \sigma^{ij} u_{+L}^b(\xi_2) d_{+R}^c(\xi_3) \epsilon^{abc}/\sqrt{6} | P \rangle$$

$$= \phi^{(7)}(1,2,3) \gamma_5 U_{+} + (i \tilde{\partial}_1^i \phi^{(8)}(1,2,3) + i \tilde{\partial}_2^j \phi^{(8)}(2,1,3)) \gamma_5 U_{+}$$

$$+ \left( \tilde{\partial}_1^i \phi^{(9)}(1,2,3) + \tilde{\partial}_2^j \phi^{(9)}(2,1,3) \right) U_{+}$$

$$+ \left[ i \tilde{\partial}_1^i \tilde{\partial}_2^j \phi^{(10)}(1,2,3) + i \tilde{\partial}_2^j \tilde{\partial}_1^i \phi^{(10)}(2,1,3) \right] \gamma^i \gamma_5 U_{+}. \quad \text{(11)}$$

where the parentheses on a pair of indices indicate symmetrization and subtraction of the trace. The symmetry between two up-quarks yields

$$\phi^{(7)}(1,2,3) = \phi^{(7)}(2,1,3), \quad \phi^{(11)}(1,2,3) = \phi^{(11)}(2,1,3), \quad \phi^{(12)}(1,2,3) = -\phi^{(12)}(2,1,3). \quad \text{(12)}$$
When inserting an additional $\gamma_5$ between the two up quark fields, and using $\sigma^{ij}\gamma_5 = -i\epsilon^{ij}\sigma^{ij}$, we obtain a similar expression,

$$
\langle 0|u_+^{aT}(\xi_1)Ci\sigma^{ij}\gamma_5u_+^{b}(\xi_2)d^c_{+L}(\xi_3)e^{abc}/\sqrt{6}|P \uparrow \rangle = \phi^{(7)}(1, 2, 3)\gamma^iU_+ + \left(\partial^i_1\phi^{(8)}(1, 2, 3) + \partial^i_2\phi^{(8)}(2, 1, 3)\right)\gamma_5U_+ + (i\partial^i_1\phi^{(9)}(1, 2, 3) + i\partial^i_2\phi^{(9)}(2, 1, 3))U_+ - i \left[i\gamma^j(i\gamma^j)\phi^{(10)}(1, 2, 3) + i\gamma^j(i\gamma^j)\phi^{(10)}(2, 1, 3)\right] + i\gamma^j(i\gamma^j)\phi^{(11)}(1, 2, 3) + i(\partial^i_1\partial^j_2 - \partial^j_2\partial^i_1)\phi^{(12)}(1, 2, 3)\gamma^j\gamma_5U_+ ,
$$

(13)

where we have used $\gamma^i = -i\epsilon^{ij}\gamma^j\gamma_5$, valid when acting on $U_+$.

Combining Eqs. (11) and (13), we obtain the matrix elements of the quark fields in definite chirality states. For example, when the two up-quarks are right-handed, and the down quark left-handed, the total helicity of the quarks is 1/2. In this case, we have

$$
\langle 0|u_+^{Ta}(\xi_1)Ci\sigma^{ij}u_+^{b}(\xi_2)d^c_{+L}(\xi_3)e^{abc}/\sqrt{6}|P \uparrow \rangle = \left[\phi^{(7)}(1, 2, 3) + i(i\partial^i_1i\bar{\partial}_2)\phi^{(12)}(1, 2, 3)\right]\gamma^iU_+ ,
$$

(14)

where we have used the relations $\gamma^jU_+ = -i\gamma^jU_+ + \partial^{(j}\partial^{i)}\gamma^jU_+ = 0$ with $\partial^i_1 = -\partial^i_1 + i\epsilon^{ij}\partial^j_1$. The isospin symmetry imposes the following constraint,

$$
\langle 0|u_+^{Ta}(\xi_1)Ci\sigma^{ij}u_+^{b}(\xi_2)d^c_{+L}(\xi_3)e^{abc}|P \uparrow \rangle
+ \langle 0|d_+^{Ta}(\xi_1)Ci\sigma^{ij}u_+^{b}(\xi_2)u^c_{+L}(\xi_3)e^{abc}|P \uparrow \rangle
+ \langle 0|u_+^{Ta}(\xi_1)Ci\sigma^{ij}u_+^{b}(\xi_2)u^c_{+L}(\xi_3)e^{abc}|P \uparrow \rangle = 0 .
$$

(15)

Using Fierz identity, one can show

$$
u_+^{Ta}(\xi_1)Ci\sigma^{ij}d^b_{+R}(\xi_2)u^c_{+L}(\xi_3)e^{abc} = -u_+^{Ta}(\xi_1)C\gamma^j d^b_{+R}(\xi_2)\gamma^i u^c_{+L}(\xi_3)e^{abc} ,
$$

$$
d_+^{Ta}(\xi_1)Ci\sigma^{ij}u^b_{+R}(\xi_2)u^c_{+L}(\xi_3)e^{abc} = -u_+^{Ta}(\xi_1)C\gamma^j d^b_{+R}(\xi_2)\gamma^i u^c_{+L}(\xi_1)e^{abc} ,
$$

(16)

which means that $\phi^{(7,12)}$ are not independent amplitudes,

$$
\phi^{(7)}(1, 2, 3) = \psi^{(1)}(1, 3, 2) + \psi^{(1)}(2, 3, 1) ,
$$

$$
\phi^{(12)}(1, 2, 3) = \psi^{(2)}(1, 3, 2) - \psi^{(2)}(2, 3, 1) .
$$

(17)

Hence we write,

$$
\langle 0|u_+^{Ta}(\xi_1)Ci\sigma^{ij}u_+^{b}(\xi_2)d^c_{+L}(\xi_3)e^{abc}/\sqrt{6}|P \uparrow \rangle
= \left[\psi^{(1)}(1, 3, 2) + \psi^{(1)}(2, 3, 1) + i(i\partial^i_1i\bar{\partial}_2)\left(\psi^{(2)}(1, 3, 2) - \psi^{(2)}(2, 3, 1)\right)\right]\gamma^iU_+ ,
$$

(18)

which contains no new amplitude.

If all quarks are right-handed, the total helicity of the quarks is 3/2. We need one unit of orbital angular momentum (projection on the z-direction) to construct the helicity of the proton. We write,

$$
2\langle 0|u_+^{Ta}(\xi_1)Ci\sigma^{ij}u_+^{b}(\xi_2)d^c_{+R}(\xi_3)e^{abc}/\sqrt{6}|P \uparrow \rangle
= \left[i\partial^i_1\phi^{(8+9)}(1, 2, 3) + i\partial^i_2\phi^{(8+9)}(2, 1, 3)\right]U_+ ,
$$

(19)
where $\phi^{(8+9)} = \phi^{(8)} + \phi^{(9)}$. The isospin symmetry leads to the following relation

$$\phi^{(8+9)}(1, 2, 3) + \phi^{(8+9)}(1, 3, 2) - \phi^{(8+9)}(3, 1, 2) - \phi^{(8+9)}(3, 2, 1) = 0.$$  

(20)

If we define

$$\psi^{(8+9)}(1, 2, 3) = \psi^{(5)}(1, 2, 3) - \psi^{(5)}(1, 3, 2),$$

(21)

the above constraint is solved. Therefore,

$$\langle 0 | u_{1+R}^T(\xi_1) C i \sigma^{ij} u_{1+R}^b(\xi_2) d_{1+R}^j(\xi_3) \frac{\epsilon^{abc}}{\sqrt{6}} | P \uparrow \rangle$$

$$= \left[ i \partial^{j-} \psi^{(5)}(1, 2, 3) + i \partial^{j-} \psi^{(5)}(2, 1, 3) \right] U_{1+R},$$

(22)

which defines a new amplitude $\psi^{(5)}$.

When the two up quarks are left-handed and the down quark right-handed, the total quark helicity is $-1/2$. We again need one unit of orbital angular momentum to construct the proton helicity. The orbital angular momentum can either be on the first or second particle,

$$2 \langle 0 | u_{1+L}^T(\xi_1) C i \sigma^{ij} u_{1+L}^b(\xi_2) d_{1+R}^j(\xi_3) \frac{\epsilon^{abc}}{\sqrt{6}} | P \uparrow \rangle$$

$$= \left[ i \partial^{j+} \phi^{(8-9)}(1, 2, 3) + i \partial^{j+} \phi^{(8-9)}(2, 1, 3) \right] U_{1+R},$$

(23)

where $\partial^{j+} = \partial^{j} + i \epsilon^{ij} \partial^{i}$ and so on. Using the constraint from the isospin symmetry, one can express $\phi^{(8-9)}$ in terms of $\psi^{(3)}$ and $\psi^{(4)}$,

$$\phi^{(8-9)}(1, 2, 3)/2 = \psi^{(4)}(3, 1, 2) - \psi^{(3)}(3, 1, 2) - \psi^{(3)}(3, 2, 1).$$

(24)

Hence we write

$$\langle 0 | u_{1+L}^T(\xi_1) C i \sigma^{ij} u_{1+R}^b(\xi_2) d_{1+R}^j(\xi_3) \frac{\epsilon^{abc}}{\sqrt{6}} | P \uparrow \rangle$$

$$= \left[ i \partial^{j+} \psi^{(4)}(3, 1, 2) - \psi^{(3)}(3, 1, 2) - \psi^{(3)}(3, 2, 1) \right] + (1 \leftrightarrow 2) U_{1+R},$$

(25)

which does not contain any new amplitude.

When all quarks are left-handed, the total quark helicity is $-3/2$; two units of orbital angular momentum are need to construct the nucleon helicity. Taking the appropriate projection of the above equations, we get

$$2 \langle 0 | u_{1+L}^T(\xi_1) C i \sigma^{ij} u_{1+L}^b(\xi_2) d_{1+L}^j(\xi_3) \frac{\epsilon^{abc}}{\sqrt{6}} | P \uparrow \rangle$$

$$= \left[ i \partial^{j+} \phi^{(10)}(1, 2, 3) + i \partial^{j+} \phi^{(10)}(2, 1, 3) + i \partial^{j+} \phi^{(11)}(1, 2, 3) \right] \gamma^j U_{1+R},$$

(26)

where $\phi^{(10)}$ describes the state in which the same quark carries two units of angular momentum, and $\phi^{(11)}$ describes each up quark carrying one unit of angular momentum. The constraint from the isospin symmetry relates $\phi^{(11)}$ to $\phi^{(10)}$

$$\phi^{(11)}(1, 2, 3) = \phi^{(10)}(1, 2, 3) + \phi^{(10)}(2, 1, 3) - \phi^{(10)}(1, 3, 2) - \phi^{(10)}(2, 3, 1).$$

(27)
Following our convention, we rename $\phi^{(10)}(1,2,3)/2$ to $\psi^{(6)}(1, 2, 3)$, and Eq. (26) becomes

$$
\langle 0 | u^T_{+L}(\xi_1) C i \sigma^+ i u^L_{+L}(\xi_2) d^c_{+L}(\xi_3) \epsilon^{abc} \gamma^5 | P \uparrow \rangle =
$$

$$
= \left[ i \partial_1^{(i+1}) \psi^{(6)}(1, 2, 3) + i \partial_2^{(i+1}) \psi^{(6)}(2, 1, 3)
+ i \partial_1^{(i+1}) \psi^{(6)}(1, 2, 3) + \psi^{(6)}(2, 1, 3) - \psi^{(6)}(1, 3, 2) - \psi^{(6)}(2, 3, 1) \right] \gamma^5 U_{+R} ,
$$

which contains the new amplitude $\psi^{(6)}$.

### III. The Three-Quark Light-Cone Wave Function of the Proton

The matrix elements considered in the previous section can be calculated in models such as the MIT bag or QCD sum rules or lattice field theory. Once they are known, one can invert the equations to get the light-cone wave function of the proton in the minimal Fock sector. In this section, we work out the light-cone wave function in terms of these matrix elements. The expression allows us to calculate the physical observables of the proton in the next section.

Consider first the case in which all quarks are coupled to helicity $1/2$. Define the measure for the quark momentum integrations,

$$
d[1]d[2]d[3] = \sqrt{2} \frac{dx_1 dx_2 dx_3}{\sqrt{2x_1 2x_2 2x_3}} \frac{d^2 k_{1\perp} d^2 k_{2\perp} d^2 k_{3\perp}}{(2\pi)^3}
\times 2\pi \delta(1-x_1-x_2-x_3)(2\pi)^2 \delta^{(2)}(k_{1\perp} + k_{2\perp} + k_{3\perp}) .
$$

where $x_i$ are the fraction of the nucleon momentum carried by the quarks, and $k_{i\perp}$ are their transverse momenta. We introduce a Fock component for a right-handed proton,

$$
|P \uparrow\rangle_{1/2} = \int d[1]d[2]d[3] \left( \bar{\psi}^{(1)}(1, 2, 3) + i(k_1^x k_2^y - k_1^y k_2^x) \bar{\psi}^{(2)}(1, 2, 3) \right)
\times \frac{\epsilon^{abc}}{\sqrt{6}} u^\dagger_{\alpha L}(1) \left( u^\dagger_{b_1}(2) d^c_{\gamma L}(3) - d^c_{\gamma L}(2) u^\dagger_{b_3}(3) \right) |0\rangle ,
$$

where $\bar{\psi}^{(1,2)}$ are functions of quark momenta with argument 1 represents $x_1$ and $k_{1\perp}$ and so on. The dependence on the transverse momenta is of form $k_{i\perp} \cdot k_{j\perp}$ only. The $\bar{\psi}^{(2)}$ amplitude has a pre-factor $k_1^x k_2^y - k_1^y k_2^x$, which signals the contributions from quarks with non-zero orbital angular momentum, although all magnetic quantum numbers sum to zero. Using the above state, the matrix elements shown in Eqs. (9) and (18) can be calculated. We find the relation between the wave function amplitudes and the matrix elements

$$
\psi^{(\alpha)}(\xi_1, \xi_2, \xi_3) = \int d[1]d[2]d[3] \sqrt{x_1 2x_2 2x_3} e^{-i(x_1 P^+ \xi_1 - k_{1\perp} \cdot \xi_{1\perp})}
\times e^{-i(x_2 P^+ \xi_2 - k_{2\perp} \cdot \xi_{2\perp})} e^{-i(x_3 P^+ \xi_3 - k_{3\perp} \cdot \xi_{3\perp})} \bar{\psi}^{(\alpha)}(1, 2, 3) ,
$$

where $\alpha = 1, 2$. 
Turn to the case where the quark helicities sum to $-1/2$. The corresponding Fock component can be written as

$$
|P \uparrow\rangle_{-1/2} = \int d[1]d[2]d[3] \left( (k_1^x + i k_1^y) \tilde{\psi}^{(3)}(1, 2, 3) + (k_2^x + i k_2^y) \tilde{\psi}^{(4)}(1, 2, 3) \right) \\
\times \frac{\epsilon^{abc}}{\sqrt{6}} (u_{a\uparrow}^\dag(1) u_{b\downarrow}^\dag(2) d_{c\uparrow}^\dag(3) - d_{b\uparrow}^\dag(1) u_{a\downarrow}^\dag(2) u_{c\downarrow}^\dag(3)) |0\rangle \quad .
$$

(32)

The matrix elements in Eqs. (11) and (25) can be calculated, and the resulting $\tilde{\psi}^{(3,4)}$ are related to the above amplitudes in the same way as in Eq. (31). One might suspect if additional independent amplitudes can be constructed by adding terms with structure $k_1^x k_2^y - k_1^y k_2^x$. A careful examination indicates that they can be reduced to the already existing ones.

When the quark helicity is added to $3/2$, the Fock component is

$$
|P \uparrow\rangle_{3/2} = \int d[1]d[2]d[3] (-k_1^x + i k_1^y) \tilde{\psi}^{(5)}(1, 2, 3) \\
\times \frac{\epsilon^{abc}}{\sqrt{6}} u_{a\downarrow}^\dag(1) \left( u_{b\uparrow}^\dag(2) d_{c\uparrow}^\dag(3) - d_{b\uparrow}^\dag(2) u_{c\uparrow}^\dag(3) \right) |0\rangle \quad .
$$

(33)

Calculating the matrix element in Eq. (22), we find $\tilde{\psi}^{(5)}(\xi_1, \xi_2, \xi_3)$ is the Fourier transformation of $\tilde{\psi}^{(5)}(k_1, k_2, k_3)$.

Finally, we consider the case when the quark helicity adds to $-3/2$, the Fock component is

$$
|P \downarrow\rangle_{-3/2} = \int d[1]d[2]d[3] (k_1^x + i k_1^y) (k_2^x + i k_2^y) \tilde{\psi}^{(6)}(1, 2, 3) \\
\times \frac{\epsilon^{abc}}{\sqrt{6}} u_{a\uparrow}^\dag(1) \left( d_{b\downarrow}^\dag(2) u_{c\downarrow}^\dag(3) - u_{b\downarrow}^\dag(2) d_{c\downarrow}^\dag(3) \right) |0\rangle \quad .
$$

(34)

Using this, we calculate the matrix elements in Eq. (28) and find $\tilde{\psi}^{(6)}(\xi_1, \xi_2, \xi_3)$ is just the Fourier transformation of $\tilde{\psi}^{(6)}(k_1, k_2, k_3)$.

The complete three-quark light-cone Fock expansion of the proton has the following form,

$$
|P \uparrow\rangle = |P \uparrow\rangle_{-3/2} + |P \uparrow\rangle_{-1/2} + |P \uparrow\rangle_{1/2} + |P \uparrow\rangle_{3/2} \quad .
$$

(35)

Many interesting proton observables can be calculated using the above wave function as we will show in the next section.

Using the results in the previous section, one can also construct a proton state with the negative helicity $|P \downarrow\rangle$. All the wave function amplitudes are the same, except that the quark helicities are flipped, $k^x \pm ik^y$ becomes $k^x \mp ik^y$, and some signs are added.

$$
|P \downarrow\rangle_{-1/2} = \int d[1]d[2]d[3] \left( -\tilde{\psi}^{(1)}(1, 2, 3) + i(k_1^x k_2^y - k_1^y k_2^x) \tilde{\psi}^{(2)}(1, 2, 3) \right) \\
\times \frac{\epsilon^{abc}}{\sqrt{6}} u_{a\downarrow}^\dag(1) \left( u_{b\uparrow}^\dag(2) d_{c\downarrow}^\dag(3) - d_{b\uparrow}^\dag(2) u_{c\downarrow}^\dag(3) \right) |0\rangle \quad ,
$$

(36)

$$
|P \downarrow\rangle_{1/2} = \int d[1]d[2]d[3] \left( (k_1^x - i k_1^y) \tilde{\psi}^{(3)}(1, 2, 3) + (k_2^x - i k_2^y) \tilde{\psi}^{(4)}(1, 2, 3) \right) \\
\times \frac{\epsilon^{abc}}{\sqrt{6}} \left( u_{a\downarrow}^\dag(1) u_{b\uparrow}^\dag(2) d_{c\uparrow}^\dag(3) - d_{a\downarrow}^\dag(1) u_{b\uparrow}^\dag(2) u_{c\uparrow}^\dag(3) \right) |0\rangle \quad ,
$$

(37)
The same expressions can be obtained from Jacob and Wick’s method \[46\],
\[
(-1)^{s-\lambda}|P-\lambda\rangle = \hat{Y}|P\lambda\rangle ,
\]
where \(\hat{Y}\) is the parity operation followed by a 180° rotation in the \(y\)-direction, and \(s\) is the spin and \(\lambda\) the helicity.

### A. Constraints From Time-Reversal And Parity

Consider the proton state \(|P\uparrow\rangle\). Under parity, the 3-components of the proton momentum change the sign: \((P^0, \vec{P})\) becomes \((-P^0, -\vec{P})\). Under time-reversal, the momentum changes in the same way. Therefore under combined time-reversal and parity, the momentum of the proton does not change. Neither do the quark momenta. On the other hand, helicity changes sign under the combined parity and time reversal, and hence the proton state \(|P\uparrow\rangle\) becomes \(|P\downarrow\rangle\). Similar changes occur for the quark states.

Under time-reversal, one must replace C-numbers with their complex conjugates. Thus under the combined time reversal and parity, a positive-helicity proton state becomes a negative helicity one \(|P\downarrow\rangle\) with wave function amplitudes complex-conjugated, quark helicity flipped, and \(k^x \pm ik^y\) becoming \(k^x \mp ik^y\). Moreover, there is a sign change of \((-1)^{s-\lambda}\) from time-reversal. The resulting state \(|P\downarrow\rangle\) is exactly the same as that in Eqs. (36-39), except all amplitudes are complex conjugated. Hence if time-reversal symmetry applies, all the wave function amplitudes must be real,
\[
\tilde{\psi}^{(i)*} = \tilde{\psi}^{(i)} , \quad i = 1, 2, 3, 4, 5, 6 ,
\]
which is true in many model calculations.

The above result, however, is only correct if the gauge condition is also invariant under the discrete symmetries. In light-cone gauge, \(A^+ = 0\) does not fix the gauge completely, additional boundary conditions for the gauge field must be specified [38, 40, 41]. This additional gauge fixing might not be invariant under the combined parity and time-reversal. For example, the advanced and retarded boundary conditions transform into each other under time-reversal; however the principal value prescription is self-conjugating. In the former case, the wave function amplitudes are no longer constrained by the above condition. Therefore, although the final-state-interaction gauge links vanish in the momentum-dependent parton distributions in the light-cone gauge, the imaginary part of the wave function amplitudes can reproduce the final state interaction effects [24, 25, 26, 27]. In the following discussion, we use the advanced boundary condition and hence the imaginary parts of the amplitudes do not vanish.
IV. OBSERVABLES DEPENDING ON QUARK ORBITAL ANGULAR MOMENTUM

As we have explained in Introduction, the quark orbital angular momentum in the proton is certainly non-zero. To determine it experimentally, one has to measure the relevant generalized parton distributions [3]. On the other hand, there are many other nucleon observables which are sensitive to the quark orbital angular momentum, although they do not measure it directly. The most familiar example is the Pauli form factor $F_2(Q^2)$ of the nucleon. In light-cone quantization, $F_2(Q^2)$ is an helicity-flip observable which depends on the interference between the wave function amplitudes differing in one unit of orbital angular momentum [10, 11].

We have to caution, however, that when the gauge fields are taken into account explicitly, some orbital angular momentum effects discussed here cannot be cleanly separated from the gluon contributions. In fact, the quark angular momentum

$$L_q = \int d^3x \bar{\psi} \vec{x} \times (-i\vec{D})\psi,$$

(42)
does contain the gluon contribution through the gauge potential in the covariant derivative. In the example of the electron magnetic moment in quantum electrodynamics, the photon Fock component plays an important role [11]. Hence, some results in this section can only make sense if considered as an effective description after integrating out the gluon degrees of freedom.

In the quark models, the need for a quark orbital angular momentum can sometimes be avoided by large constituent quark masses. The quarks can have significant spin flips through the mass term. Then some observables discussed in this section are directly proportional to the constituent masses [17]. In QCD, because the light flavors have negligible masses, the quark spin flips cannot occur and the orbital angular momentum and gluon angular momentum are thereby essential.

We start by calculating the hadron helicity-flip transverse-momentum dependent parton distributions. The twist-three distributions are found to be simply the integrals over the appropriate transverse-momentum dependent distributions. After this, we calculate the generalized parton distribution and electromagnetic form factors, with $E(x, \xi, Q^2)$ and $F_2(Q^2)$ as examples.

A. Transverse-Momentum Dependent Parton Distributions

The transverse momentum dependent parton distributions were first introduced by Collins and Soper to describe Drell-Yan production [18, 48] and later by Sivers to describe the single spin asymmetry in hadron-hadron scattering [19, 21]. A classification of the leading distributions in terms of spin and chirality can be found in Ref. [22, 23]. Recently, there has been much interest in measuring these distributions in the semi-inclusive deep-inelastic scattering [22, 23, 24, 49, 50, 51, 52, 53, 54].

Let us first recapitulate the classification, and in the process, we introduce some new notations which we believe are easy to systematize. For an unpolarized nucleon target, one can introduce the unpolarized quark distribution $f_1(x, k_\perp) \equiv q(x, k_\perp)$ and time-reversal odd transversely-polarized quark distribution $h_1^+(x, k_\perp) \equiv \delta q(x, k_\perp)$ arising from the final-state or initial-state interactions, where $x$ and $k_\perp$ are the longitudinal momentum fraction and...
The distributions we are interested in can be obtained from the vector of the nucleon normalized to one. For a longitudinally-polarized nucleon, one introduces a longitudinally-polarized quark distribution \( g_{1L}(x, k_L) \equiv \Delta q_L(x, k_L) \) and a transversely-polarized distribution \( h_{1T}^T(x, k_L) \equiv \delta q_T(x, k_L) \). Finally, for a transversely-polarized nucleon, one introduces a spin-independent distribution \( f_{1T}^T(x, k_L) \equiv q_T(x, k_L) \) arising from final and/or initial state interactions, and a longitudinally-polarized quark polarization \( g_{1L}(x, k_L) = \Delta q_T(x, k_L) \), a symmetrical transversely-polarized quark distribution \( h_{1T}^T(x, k_L) = \delta q_T(x, k_L) \) and an asymmetric transversely-polarized quark distribution \( h_{1T}^T(x, k_L) = \delta q_T(x, k_L) \).

Out of the eight distributions, four of them are related to the nucleon helicity flip and are the main interest in this subsection: unpolarized and longitudinally-polarized quark distributions in a transversely-polarized target, \( q_T(x, k_L) \) and \( \Delta q_T(x, k_L) \), and transversely-polarized quark distributions in an unpolarized and longitudinally-polarized nucleon, \( \delta q(x, k_L) \) and \( \delta q_L(x, k_L) \). Two of them, \( q_T(x, k_L) \) and \( \delta q(x, k_L) \), are non-zero only when there are initial and final state interactions.

Unlike the usual Feynman parton distributions, there are some important subtleties about the transverse-momentum dependent parton distributions. First is gauge invariance and universality. Each of these distributions can be gauge-invariantized in different ways depending on choices of gauge links and the results are different. Which one appears in a particular process requires a careful study. In the Drell-Yan process studied by Collins and Soper [18], the distributions are defined in axial gauges. In deep-inelastic scattering, it has been shown that the gauge links are in the future direction along the light-cone. If the light-cone gauge is used, an additional gauge link at spatial infinity is required to render the distributions independent of additional gauge fixing [13]. Different versions of the distributions appear in different hard processes imply that the universality of these distributions are lost. For example, the transverse-momentum dependent parton distributions measured in DIS cannot be used to make predictions of the cross sections for Drell-Yan. For processes such as jet production in hadron-hadron scattering, it is not yet clear which version of the distributions is relevant.

Another issue is infrared divergences. The distributions involving light-cone gauge links have infrared divergences, which are cancelled after integrating out parton transverse momentum. Physically, the infrared divergences can be understood in the following way. The transverse-momentum dependent distributions can be measured in DIS through single-jet production. However, the single-jet cross section is not well-defined because the vertex correction has infrared divergence. The divergence is cancelled by the soft gluon radiation. Since a hard gluon radiation is considered as a two-jet event, one must introduce an infrared cut-off to separate the single from double jet event. In the following discussion, we implicitly assume the infrared cut-off is there when needed.

We now use the wave function of the previous section to calculate the helicity-flip parton distributions. First consider the case of a transversely polarized target. Introduce the quark density matrix,

\[
\mathcal{M} = p^+ \int \frac{d\xi \cdot d^2 \xi_L}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{\xi} - \mathbf{k} \cdot \mathbf{\xi}_L} \langle \psi(0) L_\xi \psi(\xi) | PS \rangle ,
\]

where \( L_\xi \) is the gauge link along the light-cone in the covariant gauges, \( S^\mu \) is the polarization vector of the nucleon normalized to \( S_\mu S^\mu = -1 \), \( p^\mu \) is a light-cone vector such that \( p^- = 0 \). The distributions we are interested in can be obtained from \( \mathcal{M} \) through the expansion [22],

\[
\mathcal{M} = \frac{1}{2M} \left[ q_T(x, k_L) \epsilon^{\mu\nu\alpha\beta} \gamma_\mu P_\nu k_\alpha S_\beta + \Delta q_T(x, k_L) \gamma_5 \not{q} (\mathbf{k}_L \cdot \mathbf{S}_L) + \ldots \right] ,
\]

\[\text{(43)}\]

\[\text{(44)}\]
where $M$ is the nucleon mass. Inverting the above equation, we obtain

$$q_T(x, k) = -rac{M}{2e^{i/k} S_i} \int \frac{d\xi - d^2 \xi}{(2\pi)^3} e^{i(k + \xi - \vec{k}_\perp \vec{\xi}_\perp)} \langle PS_\perp | \overline{\psi}(0) L^\dagger \gamma^+ L_\xi | PS_\perp \rangle,$$

$$\Delta q_T(x, k) = \frac{M}{2k_i S_i} \int \frac{d\xi - d^2 \xi}{(2\pi)^3} e^{i(k + \xi - \vec{k}_\perp \vec{\xi}_\perp)} \langle PS_\perp | \overline{\psi}(0) L^\dagger \gamma^+ \gamma_5 L_\xi | PS_\perp \rangle.$$ (45)

where the transversely-polarized nucleon in the $x$ direction is related to the helicity states by $|S_x\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$.

If these distributions are calculated in the light-cone gauge with the advanced boundary condition for the gauge potential, the gauge links can be ignored, but the wave function amplitudes are complex [13]. Express the quark fields in term of Fock operators ($x > 0$),

$$q_T(x, k) = -\frac{M}{2e^{i/k} S_i} \frac{1}{4x(2\pi)^3 V_3} \langle PS_\perp | \sum_\lambda q^\dagger_\lambda(q_\lambda(k)|PS_\perp\rangle,$$

$$\Delta q_T(x, k) = \frac{M}{2k_i S_i} \frac{1}{4x(2\pi)^3 V_3} \langle PS_\perp | \sum_\lambda (-1)^{(1/2 - \lambda)} q^\dagger_\lambda(q_\lambda(k)|PS_\perp\rangle,$$ (46)

where $V_3$ is the 3-space volume. Using the above and the proton wave function from the previous section, we obtain

$$\Delta q_T(x, k) = \frac{M}{k^2} \int [1][2][3] \sqrt{x_1 x_2 x_3} \text{Re}[F_q],$$

$$q_T(x, k) = \frac{M}{k^2} \int [1][2][3] \sqrt{x_1 x_2 x_3} \text{Im}[F_q].$$ (47)

The functions $F_q$ for the u-quark is

$$F_u = 2 \left\{ \delta^{(3)}(k - k_1) \tilde{\psi}_{j}^{(1,2)*}(1,2,3) \tilde{\psi}_{j}^{(3,4)}(1,2,3) - \delta^{(3)}(k - k_2) \tilde{\psi}_{j}^{(1,2)}(1,2,3) \tilde{\psi}_{j}^{(3,4)*}(2,1,3) + (\delta^{(3)}(k - k_1) + \delta^{(3)}(k - k_3)) \tilde{\psi}_{j}^{(1,2)*}(1,2,3)(\tilde{\psi}_{j}^{(3,4)}(2,1,3) + \tilde{\psi}_{j}^{(3,4)}(2,3,1)) + (\delta^{(3)}(k - k_1) + \delta^{(3)}(k - k_2)) \tilde{\psi}^{(5-)*}(1,2,3) \tilde{\psi}^{(6+)}(1,2,3) \right\},$$ (48)

where

$$\delta^{(3)}(k - k_i) = \delta(x - x_i)\delta^{(2)}(\vec{k}_\perp - \vec{k}_i),$$

$$\tilde{\psi}_{j}^{(1,2)}(1,2,3) = \tilde{\psi}_{j}^{(1)}(1,2,3)k^j - (k^1 k_2^2 - k^2 k_1^2)\tilde{\psi}_{j}^{(2)}(1,2,3)k^i e^{ij},$$

$$\tilde{\psi}_{j}^{(3,4)}(1,2,3) = k^i \tilde{\psi}_{j}^{(3)}(1,2,3) + k^j \tilde{\psi}_{j}^{(4)}(1,2,3),$$

$$\tilde{\psi}^{(5-)}(1,2,3) = \tilde{\psi}^{(5)}(1,2,3) - \tilde{\psi}^{(5)}(1,3,2),$$

$$\tilde{\psi}^{(6+)}(1,2,3) = -\vec{k}_{\perp 1} \cdot \vec{k}_{\perp 1} \tilde{\psi}^{(6)}(1,2,3) + (\vec{k}_{\perp 2} \cdot \vec{k}_{\perp 2} \cdot \vec{k}_{\perp} + \vec{k}_{\perp 2} \cdot \vec{k}_{\perp} \tilde{\psi}^{(6)}(2,1,3),$$ (49)

For the d-quark, on the other hand, we obtain

$$F_d = 2 \left\{ \delta^{(3)}(k - k_3) \tilde{\psi}_{j}^{(1,2)*}(1,2,3) \tilde{\psi}_{j}^{(3,4)}(1,2,3) - \delta^{(3)}(k - k_2) \tilde{\psi}_{j}^{(1,2)}(1,2,3)(\tilde{\psi}_{j}^{(3,4)*}(2,1,3) + \tilde{\psi}_{j}^{(3,4)}(2,3,1)) + \delta^{(3)}(k - k_3) \tilde{\psi}^{(5-)*}(1,2,3) \tilde{\psi}^{(6+)}(1,2,3) \right\}. $$ (50)
If all the wave function amplitudes are real, \(q_T(x, k_\perp)\) vanishes identically. \(q_T(x, k_\perp)\) was first introduced by Sivers [14, 21] and it characterizes the azimuthal asymmetry in the quark transverse momentum distribution when the nucleon is transversely polarized. There is much interest in this distribution in interpreting the single spin asymmetry measured in electron-proton deep inelastic scattering [24, 49, 50, 51, 52, 53, 54].

The other two helicity-flip distributions can be obtained from the quark density matrix \(M\) through the following projection [22]

\[
\mathcal{M} = \frac{1}{2M} \left[ \delta q(x, k_\perp) \sigma^{\mu\nu} k_{\mu} p_{\nu} + \delta q_L(x, k_\perp) i \sigma^{\mu\nu} \gamma_5 p_{\mu} k_{\nu} (S \cdot n) + \ldots \right] .
\] (51)

The transversely-polarized quark distribution in an unpolarized nucleon \(\delta q(x, k_\perp)\) is novel, and has the similar physical origin as that of the experimental phenomenon where a hyperon produced in unpolarized elastic proton-proton scattering is polarized in the direction perpendicular to the production plane [53, 54]. Invert the above equation, we arrive at

\[
\begin{align*}
\delta q(x, k_\perp) &= -\frac{M}{2k_i} \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(k^+ \xi^- - k_\perp \cdot \xi_\perp)} \langle P|\bar{\psi}(0) L^\dagger_0 \sigma^{ij} L^\dagger_\xi \psi(\xi)|P \rangle , \\
\delta q_L(x, k_\perp) &= \frac{M}{2\epsilon^2 k_j} \int \frac{d\xi^- d^2 \xi_\perp}{(2\pi)^3} e^{i(k^+ \xi^- - k_\perp \cdot \xi_\perp)} \langle PS||\bar{\psi}(0) L^\dagger_0 \sigma^{ij} L^\dagger_\xi \psi(\xi)|PS|| \rangle .
\end{align*}
\] (52)

where \(i\) can take either \(x\) or \(y\).

Inserting the plane wave expansion for the quark fields, we get

\[
\begin{align*}
\delta q(x, k_\perp) &= -\frac{M}{k^+} \frac{1}{4 \times (2\pi)^3 V_3} i \langle P|q^\dagger_i(k) q_i(k) - q^\dagger_4(k) q_4(k)|P \rangle , \\
\delta q_L(x, k_\perp) &= \frac{M}{\epsilon^2 k_j} \frac{1}{4 \times (2\pi)^3 V_3} i \langle PS||q^\dagger_i(k) q_i(k) - q^\dagger_4(k) q_4(k)|PS|| \rangle .
\end{align*}
\] (53)

Using the above and the proton wave function from the previous section, we obtain,

\[
\begin{align*}
\delta q_L(x, k_\perp) &= \frac{M}{k^2_\perp} \int d[1] d[2] d[3] \sqrt{x_1 x_2 x_3} \text{Re}[H_q] , \\
\delta q(x, k_\perp) &= \frac{M}{k^2_\perp} \int d[1] d[2] d[3] \sqrt{x_1 x_2 x_3} \text{Im}[H_q] .
\end{align*}
\] (54)

The function \(H_q\) is

\[
H_u = 2 \left\{ -\delta^{(3)}(k - k_1) \bar{\psi}^{(1,2)^\dagger}_j(1, 2, 3) (\bar{\psi}^{(3,4)*}_j(3, 1, 2) + \bar{\psi}^{(3,4)*}_j(3, 2, 1)) \\
-\delta^{(3)}(k - k_2) (\bar{\psi}^{(1,2)*}_j(1, 3, 2) + \bar{\psi}^{(1,2)*}_j(2, 3, 1)) \bar{\psi}^{(3,4)*}_j(1, 2, 3) \\
-\delta^{(3)}(k - k_3) (\bar{\psi}^{(1,2)*}_j(1, 2, 3) (k^\dagger_1 \bar{\psi}^{(5-)}_j(2, 1, 3) + k^\dagger_2 \bar{\psi}^{(5-)}_j(1, 2, 3)) \\
+\delta^{(3)}(k - k_1) (\bar{\psi}^{(6+)*}_j(1, 2, 3) \bar{\psi}^{(3)}(1, 2, 3) + \bar{\psi}^{(6+)*}_j(2, 1, 3) \bar{\psi}^{(4)}(1, 2, 3)) \right\} ,
\] (55)

where \(\bar{\psi}^{(1,2)^\dagger}_j(1, 2, 3) = \bar{\psi}^{(1)}(1, 2, 3) k^j + (k^1_1 k^2_2 - k^1_2 k^2_1) \bar{\psi}^{(2)}(1, 2, 3) k^i e^{ij}\). For the down quark, \(H_d\) is

\[
H_d = 2 \left\{ \delta^{(3)}(k - k_3) \bar{\psi}^{(1,2)^\dagger}_j(1, 2, 3) \bar{\psi}^{(3,4)*}_j(1, 2, 3) \\
-\delta^{(3)}(k - k_2) (\bar{\psi}^{(1,2)*}_j(1, 2, 3) (k^\dagger_1 \bar{\psi}^{(5-)}_j(1, 2, 3) + k^\dagger_2 \bar{\psi}^{(5-)}_j(3, 2, 1)) \\
+\delta^{(3)}(k - k_1) (\bar{\psi}^{(6+)*}_j(1, 2, 3) \bar{\psi}^{(3)}(1, 2, 3) - \bar{\psi}^{(6+)*}_j(2, 3, 1) \bar{\psi}^{(4)}(1, 2, 3)) \right\} ,
\] (56)
where
\[
\tilde{\psi}^{(6+)}(1, 2, 3) = (k_{21}^2 k_{32} \cdot k_{12} + k_{31}^2 k_{22} \cdot k_{12})(\tilde{\psi}^{(6)}(2, 1, 3) + \tilde{\psi}^{(6)}(3, 1, 2)) \\
+ k_{12}^2 k_{23} \cdot k_{13} \tilde{\psi}^{(6)}(2, 3, 1) + k_{12}^2 k_{32} \cdot k_{13} \tilde{\psi}^{(6)}(3, 2, 1) .
\] (57)

If all amplitudes are real, \( \delta q(x, k_{\perp}) \) vanishes identically.

**B. Twist-Three Parton Distributions**

It has been known for many years that the helicity flip phenomena in inclusive scattering are of higher twist effects (twist-three, to be exact). For example, the single spin asymmetry measured in pion production in polarized baryon (proton, hyperons) and unpolarized target scattering arises from hadron helicity flip \[^{57}\]. The asymmetry is expected to vanish asymptotically like \( 1/k_{\perp} \), where \( k_{\perp} \) is the pion transverse momentum \[^{58}\]. The other example is the \( g_2 \) structure function measured in inclusive DIS with transversely polarized nucleon targets. The structure function arises from the interference between photon scatterings with longitudinal and transverse polarizations and involves the helicity flip of the nucleon target because of the angular momentum conservation \[^{59}\]. The associated asymmetry goes like \( 1/Q \) where \( Q \) is the virtual photon mass.

In both examples, the baryon helicity flip must be reflected in the hard scattering subprocesses. However, if the quark transverse momentum is neglected as one usually does in the Feynman parton model, the helicity flip cannot be managed through a massless quark; this has been considered as one of the difficulties in perturbative QCD \[^{15, 60}\]. It turns out that the leading contribution to the baryon helicity flip comes from the quark orbital angular momentum and associated transversely-polarized gluons \[^{58, 61}\]. This can be seen from the following light-cone expression for the \( g_2(x) \) structure function \[^{59}\]

\[
g_T(x) = \frac{-1}{4xM} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma_5 \gamma_\perp \bar{\sigma} P_D(\lambda n) \psi(\lambda n) | PS \rangle + \text{h.c.} .
\] (58)

In our approximation, the gluon potential in the covariant derivative is neglected. Using the three-quark wave function, we find,

\[
g_T(x) = \frac{1}{2xM^2} \int k^2_\perp \Delta q_T(x, k_\perp) d^2k_\perp .
\] (59)

In fact, the above result holds for any nucleon wave function so long as the gluon potential in the covariant derivative can be ignored \[^{23}\]. A word of caution, however, is appropriate.

When the gauge potential is neglected, \( g_T(x) \) is no longer gauge invariant. How it is possible then to express \( g_T(x) \) in terms of the gauge-invariant \( \Delta q_T(x, k_\perp) \)? The answer is that the above relation is only true in light-cone gauge with the advanced prescription for the light-cone singularity. In any other gauge, the two have no relation.

It is interesting to note that the rotational invariance imposes the following condition on \( g_T(x) \) \[^{17}\]

\[
\int_0^1 dx g_T(x) = \int_0^1 dx g_1(x) .
\] (60)

Therefore, when the gluons are neglected, the rotational symmetry demands the quarks have non-vanishing orbital angular momentum. This is indeed true for a massless quark in the
MIT bag model. However, it is possible that $g_T(x)$ may have a delta function at $x = 0$ \[62\]. In this case, the above is useless in constraining $g_T(x)$ at any non-zero $x$.

Similar analysis can be carried out for the twist-three distribution $h_L(x)$ which has the following expression,

$$h_L(x) = -\frac{1}{4xM} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp |\psi(0) \not\gamma_5 i D_\perp(\lambda n) \psi(\lambda n) |PS_\perp \rangle + \text{h.c.}. \quad (61)$$

Using the proton wave function, we find

$$h_L(x) = -\frac{1}{xM^2} \int k_\perp^2 \delta q_L(x, k_\perp) d^2k_\perp, \quad (62)$$

which is again consistent with \[23\] apart from a factor of 2. The twist-three distribution $h_L(x)$ can be measured in Drell-Yan scattering with the transversely-polarized protons scattering on longitudinally-polarized protons \[59\].

C. Generalized Parton Distributions

Generalized parton distributions (GPDs) were originally introduced to describe the angular momentum structure of the nucleon \[3, 63\]. D. Mueller et al. encountered the same distributions earlier in searching for an observable whose scale evolution interpolates between those of the Feynman parton distributions and the quark distribution amplitudes in mesons \[4\]. Radyushkin defined the so-called double distributions which contain essentially the same information \[64\]. The new distributions contain all the form factors of the twist-two operators \[5\] and, in a special kinematic limit, reduce to the Feynman distributions. In this subsection, we calculate the helicity-flip GPDs using the three-quark wave function of the proton.

At the leading twist, there are eight GPDs for each quark flavor \[65, 66\], out of which four involve the nucleon helicity flip. In this paper we consider only $E(x, \xi, Q^2)$ which is directly relevant for the spin structure of the nucleon \[16\]. Extending the calculation to other three is straightforward and will not be presented here. The definition of $E(x, \xi, Q^2)$ follows from \[63\]

$$\frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P' \right| \bar{\psi}_q \left( -\frac{\lambda}{2} n \right) \not\gamma_P e^{-ig} \int_{-\lambda/2}^{\lambda/2} da \, A(\alpha n) \psi_q \left( \frac{\lambda}{2} n \right) \left| P \right\rangle = H_q(x, \xi, Q^2) \frac{1}{2} \overline{U}(P') \not\gamma U(P) + E_q(x, \xi, Q^2) \frac{1}{2} \overline{U}(P') i\sigma^{\mu\nu} n_\mu \Delta_* U(P), \quad (63)$$

where the explanation of various notations can be found in the original references.

To evaluate $E(x, \xi, Q^2)$, we choose a coordinate system in which the momentum transfer $q^\mu$ vanishes along the + direction,

$$q^+ = 0, \quad Q^2 = -q_\perp^2. \quad (64)$$

Then the initial and final nucleons have the same $P^+$ momenta. Their transverse momenta are equal in magnitude and opposite in direction (a Breit frame in the transverse direction). The energy-momentum conservation constrains $q^- = 0$. With these choices, we can only
calculate the distribution at the skewness variable $\xi = 0$. [For $\xi \neq 0$, one can use the coordinates in Ref. [67].]

Our calculation follows a similar formula for $F_2(Q^2)$ in Ref. [10],

$$-(q^x - iq^y) E(x, \xi = 0, Q^2) \frac{P^+}{M} = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P' \left| \bar{\psi} \left(-\frac{\lambda}{2} n\right) \gamma^+ \psi \left(\frac{\lambda}{2} n\right) \right| P \right\rangle. \quad (65)$$

For $x > 0$, the bilocal operator reduces to quark creation and annihilation operators in the momentum space, and for $x < 0$, it becomes antiquark creation and annihilation. The latter contribution vanishes in the valence approximation.

Using the proton wave function from the previous section, one finds

$$E_u(x, \xi = 0, Q^2) = \frac{2M}{-q^x + iq^y} \int d[1]d[2]d[3] \sqrt{x_1x_2x_3} \left\{ A^{(1,2)}(2)\phi^{(3,4)}(1, 2, 3) - A^{(3,4)}\phi^{(1,2)}(1, 2, 3) - A^{(5)}\phi^{(6)}(1, 2, 3) + A^{(6)}\phi^{(5)}(1, 2, 3) \right\}, \quad (66)$$

where

$$\begin{align*}
\phi^{(1,2)}(1, 2, 3) &= \tilde{\psi}^{(1)}(1, 2, 3) + i(k^x_1k^y_2 - k^y_1k^x_2)\tilde{\psi}^{(2)}(1, 2, 3), \\
\phi^{(1,2)'}(1, 2, 3) &= \tilde{\psi}^{(1)}(1, 2, 3) - i(k^x_1k^y_2 - k^y_1k^x_2)\tilde{\psi}^{(2)}(1, 2, 3), \\
\phi^{(3,4)}(1, 2, 3) &= k^-_1\tilde{\psi}^{(3)}(1, 2, 3) - k^-_2\tilde{\psi}^{(4)}(1, 2, 3), \\
\phi^{(5)}(1, 2, 3) &= k^+_2(\tilde{\psi}^{(5)}(1, 2, 3) - \tilde{\psi}^{(5)}(1, 3, 2)), \\
\phi^{(6)}(1, 2, 3) &= k^-_1k^-_3\tilde{\psi}^{(6)}(1, 2, 3) - k^-_1k^-_2\tilde{\psi}^{(6)}(1, 3, 2). \quad (67)
\end{align*}$$

And the functions $A$ are defined through

$$\begin{align*}
A^{(1,2)} &= \delta(x - x_1)(\phi^{(1,2)}(2', 1', 3') + \delta(x - x_2)(2\phi^{(1,2)}(2', 1', 3') + \phi^{(1,2)}(3', 1', 2')) \\
+ \delta(x - x_3)(\phi^{(1,2)}(2', 1', 3'') + \tilde{\psi}^{(1)}(3', 1', 2')), \\
A^{(3,4)} &= \delta(x - x_2)\phi^{(3,4)}(2', 1', 3') + \delta(x - x_1)(2\phi^{(3,4)}(2', 1', 3') + \phi^{(3,4)}(2', 3', 1')) \\
+ \delta(x - x_3)(\phi^{(3,4)}(2', 1', 3'') + \tilde{\psi}^{(3,4)}(2', 3', 1')), \\
A^{(5)} &= \delta(x - x_1)(\phi^{(5)}(1', 2', 3') + \phi^{(5)}(2', 1', 3')) \\
+ \delta(x - x_2)(\phi^{(5)}(1', 2', 3') + \phi^{(5)}(2', 1', 3')), \\
A^{(6)} &= \delta(x - x_1)(\phi^{(6)}(1', 2', 3') + \phi^{(6)}(2', 1', 3')) \\
+ \delta(x - x_2)(\phi^{(6)}(1', 2', 3') + \phi^{(6)}(2', 1', 3')), \quad (68)
\end{align*}$$

where the transverse coordinates are $i' = k_i - x_iq_\perp$ and $i'' = k_i + (1 - x_i)q_\perp$, $k_i^\pm = k_i^x \pm ik_i^y$. The result depends on the interference of the wave function amplitudes with different quark orbital angular momentum.

Similarly for the d-quark,

$$E_d(x, \xi = 0, Q^2) = \frac{2M}{-q^x + iq^y} \int d[1]d[2]d[3] \sqrt{x_1x_2x_3} \left\{ B^{(1,2)}\phi^{(3,4)}(1, 2, 3) - B^{(3,4)}\phi^{(1,2)}(1, 2, 3) - B^{(5)}\phi^{(6)}(1, 2, 3) + B^{(6)}\phi^{(5)}(1, 2, 3) \right\}, \quad (69)$$

where

$$\begin{align*}
B^{(1,2)} &= \delta(x - x_3)(\phi^{(1,2)}(2', 1', 3') + \delta(x - x_2)(\phi^{(1,2)}(2', 1', 3') + \phi^{(1,2)}(3', 1', 2')) \\
B^{(3,4)} &= \delta(x - x_3)(\phi^{(3,4)}(2', 1', 3') + \delta(x - x_2)(\phi^{(3,4)}(2', 1', 3') + \phi^{(3,4)}(2', 3', 1')), \\
B^{(5)} &= \delta(x - x_3)(\phi^{(5)}(1', 2', 3') + \phi^{(5)}(2', 1', 3')), \\
B^{(6)} &= \delta(x - x_3)(\phi^{(6)}(1', 2', 3') + \phi^{(6)}(2', 1', 3')). \quad (70)
\end{align*}$$
Although we have allowed $\vec{q}$ to be arbitrary in the above formulas, it is simpler to take it in the $x$ direction. With this choice, it is not difficult to see that the distributions are real.

**D. The Pauli Form Factor**

The $F_2$ form factor can be obtained from the sum rule [3],

$$F_2(Q^2) = \int dx [e_u E_u(x, \xi, Q^2) + e_d E_d(x, \xi, Q^2)],$$  \hspace{1cm} (71)

where we have neglected the strange quark contribution. Recently, the Pauli form factor of the proton has been extracted from the recoil polarization [8, 9], which differs significantly from the previous extraction. In the valence approximation, independent of $Q^2$, a non-zero $F_2(Q^2)$ is an indication of non-zero orbital angular momentum of the quarks in light-cone quantization. This point has been stressed in a number of recent papers which seek to explain the new Jlab data [68, 69].

In large-$Q^2$ limit, we can use the asymptotic behavior of amplitudes to derive the asymptotic behavior of $F_2(Q^2)$. The result will be related to the leading and twist-three light-cone amplitudes introduced by Braun et al. [31]. We leave this subject to a future publication.

Finally, we pointed out that the $B$-form factor of the gravitational current can be obtained from the second moment of $E(x, \xi, Q^2)$ [3] and is also helicity-flipping. In Ref. [12], the $B$ form factor is calculated for the electron in quantum electrodynamics, with explicit involvement of orbital angular momentum.

**V. SUMMARY AND CONCLUSION**

This paper is motivated by determining the components of the proton wave function with non-zero orbital angular momentum. This of course can only be done in a certain truncation scheme, because the field-theoretical nucleon has an infinite number of non-vanishing amplitudes and there is no finite set of experiments which can determine them completely.

We use the valence quark models as a guide to truncate the light-cone expansion of the proton wave function. We can view this either as a starting point for a systematic expansion or as an effective theory in which gluons and sea quarks have been “integrated out”.

We classify the light-cone amplitudes with three quarks, and find that six independent amplitudes are needed for a complete description: two with $L_z = 0$, three with $L_z = 1$, and one with $L_z = 2$. If the light-cone gauge fixing is invariant under combined parity and time-reversal, these amplitudes are real. Otherwise, they are complex. Apart from one of the $L_z = 0$ amplitudes, the other five contain nontrivial quark orbital motion.

We calculated a number of nuclear observables which depend at least linearly on the amplitudes with non-zero orbital angular momentum. Therefore, a non-vanishing result of the experimental measurement is an unambiguous indication of the quark orbital angular momentum, barring the neglected gluonic contributions mentioned above.

It is our hope that multiple observables of this type can be explained by a unified set of phenomenological amplitudes with nontrivial orbital angular momentum. If not, one may systematically go beyond the minimal Fock component by including, for example, the
...gluon contributions. Ultimately, we hope to have a semi-realistic picture about the intrinsic deformation of the proton from experimental data.

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