HERA high $Q^2$ events as indications of excited leptons with weak isotopic spin $3/2$

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The H1 and ZEUS anomalous events are interpreted as being due to the production and the decay of excited leptons $E$, which correspond to spin 1/2 resonances of the first generation lepton doublet $(\nu_e, e)$ with $W$ triplet. This assumption is supported by considering of Bethe-Salpeter equation in the ladder approximation with anomalous triple gauge boson vertex. The solution with weak isospin $I = 3/2$ is shown to exist for zero mass state, that means $M_E$ is small in comparison with $TeV$ mass scale. The coupling of $E$ with leptons and $W$ is defined by the normalization condition. Calculation of the $E$ width and the production cross-sections agrees with HERA data for value of the triple $W$ coupling constant $\lambda \simeq 0.5$. Isotopic relations for different channels are presented as a tool for checking the interpretation.

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Recent results of H1 [1] and ZEUS [2] collaborations at HERA cause a number of interpretations being proposed, which mostly deal with leptoquark possibilities. In the present note I would discuss an excited lepton as a suitable object, which could manifest itself in data [1, 2]. The excited lepton is understood as a resonant state of the electron (or the electron neutrino) and $W$. The doublet $\psi_{eL} = (\nu_{eL}, e_L)$ and the $W$ triplet have weak isotopic spin, respectively, 1/2 and 1. Thus weak isotopic spin of their resonance may be either 3/2 or 1/2. We shall see, that from the point of view of both the theoretical arguments and the experimental data, prescription $I = 3/2$ is the most favorable. This state $E$ is to have large mass $M \simeq 200 \, GeV$.

Let us start with theoretical arguments. We consider the variant of the EW theory, in which the symmetry breaking is due to a self-consistent appearance of additional
triple gauge boson vertex in the region of small momenta [3, 4], which can be described by the following effective term

\[
\Delta L = \frac{\lambda g}{6 W^2} \epsilon^{abc} W^a_{\mu\nu} W^b_{\nu\rho} W^c_{\rho\mu}.
\] (1)

Note, that this term is currently considered [5, 6] among others in line of phenomenological analysis of possible gauge boson interactions. Interaction (1) defines a vertex in the momentum space, which according to the approach acts in the region \( p_i^2 < \Lambda^2 \), where \( p_i \) are three momenta of the vertex legs and the effective cut-off \( \Lambda \) is of the order of magnitude of few \( TeV \) [7, 8]. The vertex has the form

\[
\Gamma^{abc}_{\mu\nu\rho}(p, q, k) = \epsilon^{abc} \frac{\lambda g}{M_W^2} \Gamma^{\mu\nu\rho}(p, q, k);
\]

\[
\Gamma^{\mu\nu\rho}(p, q, k) = g^{\mu\nu}(p \rho(qk) - q\rho(pk)) + g^{\nu\rho}(q\mu(pk) - k\mu(pq)) + g^{\rho\mu}(k\nu(qk) - q\nu(pk)) + k_{\mu}p_{\rho} - q_{\mu}k_{\rho}p_{\rho}.
\] (2)

Let us now consider the Bethe-Salpeter (BS) equation for the system consisting of \( \Psi_{eL} \) and \( W \). Mass parameters \( M_W, M \) turn out to be much less than both \( \Lambda, M_W \sqrt{4\pi/\lambda \alpha_w} \), the last one, as we shall see, is the intrinsic dimensional parameter of the equation. So it looks reasonable to consider at the approximation \( M_W = M = 0 \). In this case we have just \( W^0 \) as the neutral component and it does not matter that it consists of \( Z \) and \( \gamma \). We consider the BS equation in the ladder approximation in which the upper vertex in the kernel is just expression (2) and the lower one is the usual \( \frac{3}{2} \bar{\psi}_L \gamma^a \tau^a \psi_L W^a_\rho \) term. Let us write down the equation for \( I = 3/2 \) state

\[
\phi_{\mu}(p) = \frac{i \lambda g^2}{4(2\pi)^4 M_W^2} \int \frac{d^4q}{(q^2)^2(p-q)^2} \Gamma^{\mu\nu\rho}(p, -q, q - p) \phi_{\nu}(q) \hat{\gamma}_{\nu}(1 + \gamma_5);
\] (3)

For \( I = 1/2 \) state the coefficient afore the integral has extra factor \((-2\). Now we decompose BS wave function in terms of Dirac matrix structures, taking into account conditions of gauge invariance

\[
\phi_{\mu}(p) = \sigma_{\mu\nu} p_{\nu}(1 + \gamma_5) F_1(p^2) + (p^2 \gamma_\mu - \hat{p} p_\mu)(1 + \gamma_5) F_2(p^2);
\] (4)

After usual trace calculations and the Wick rotation we obtain identical equations for both functions \( F_i \) \((i = 1, 2)\)

\[
F_i(p^2) = \frac{2 \lambda g^2}{3(2\pi)^4 M_W^2} \int \frac{(p^2 q^2 - (pq)^2)}{p^2 q^2 (p-q)^2} F_i(q^2) \; d^4q;
\] (5)

where now \( p \) and \( q \) are Euclidean momenta. We can obtain an explicit solution of equation (3) following well-known old method [9]. Namely, after the angular integration we get from (3)

\[
F_i(x) = \frac{\lambda g^2}{96 \pi^2 M_W^2} \left( - \frac{1}{x^2} \right) \int_0^x y^2 F_i(y) dy + \frac{3}{x} \int_x^\infty y F_i(y) dy + 3 \int_x^\infty F_i(y) dy - x \int_x^\infty \frac{F_i(y)}{y} dy;
\] (6)
where \( x = p^2, \) \( y = q^2. \) Integral equation (6) is equivalent to the following differential one
\[
\left( x \frac{d}{dx} + 2 \right) \left( x \frac{d}{dx} + 1 \right) \left( x \frac{d}{dx} - 1 \right) F_i(x) - \frac{\lambda g^2}{16\pi^2 M_W^2} x F_i(x) = 0;
\]
provided integrals in equation (3) converge at infinity and \( F(0) < \infty. \) The solution of this boundary problem is expressed in terms of Meyer functions \[[10]. \] Namely, the solution of equation (7), which satisfies the boundary conditions, reads
\[
F_i(x) = C_i \cdot G_{04}^{20}(hx|1, 0, -1, -2); \quad h = \frac{\lambda g^2}{16\pi^2 M_W^2}.
\]

The behavior of this function at boundary points is the following
\[
G_{04}^{20}(0|1, 0, -1, -2) = \frac{1}{2};
\]
\[
G_{04}^{20}(x|1, 0, -1, -2) \sim x^{-7/8} \sin(4x^{1/4} + \delta), \quad x \to \infty.
\]
From here we immediately see, that the solution exists only for \( \lambda > 0; \) this is, so to say, the attraction condition. For the other sign asymptotic formula (8) gives exponential increase at infinity. Hence for the sign of \( \lambda, \) which provides the existence of \( I = 3/2 \) state, a state with \( I = 1/2 \) does not exist.

Now we proceed to the definition of constants \( C_i. \) For the purpose we use the following one-loop normalization condition, where the outer legs correspond to exited lepton \( E \)
\[
\frac{i}{(2\pi)^4} \int \phi_{\mu}(q,p) \frac{\hat{p} - \hat{q}}{q^2(p-q)^2} \bar{\phi}_\mu(q,p) \, d^4q = \hat{p}f_1(p^2) - f_2(p^2);
\]
\[
f_1(0) = 1; \quad f_2(0) = M;
\]
where \( \bar{\phi} \) means the Dirac conjugated quantity. In condition (10) we need vertex function not only for \( p = 0 \) as in (4), but also for small nonzero \( p. \) Let us write the expression \((k = p-q)\)
\[
\phi_{\mu}(q,p) = \left( \sigma_{\mu\nu} q_\nu F_1(q^2) + (k^2 - p^2)\gamma_{\mu} - (k_\mu + p_\mu)(\hat{k} - \hat{p}) \right) F_2(q^2) \right) \right) (1 + \gamma_5); \]
Here we take into account the exact Lorentz-Dirac gauge invariant structures and assume, that form-factors \( F_i \) depend on the W-boson momentum squared \( q^2. \) Now from expressions \((10), (11)\) we obtain in Euclidean space
\[
\frac{3}{16\pi^2} \int_0^\infty F_1^2(x)dx \ (1 - \gamma_5) + \frac{3}{4\pi^2} \int_0^\infty F_2^2(x)dx \ (1 + \gamma_5) = 1;
\]
\[
M = 0.
\]
The second line here confirms the consistency of the approach. Both terms in the first line of normalization relation (12) have to be equal to $1/2$. According to asymptotic formula (9) the first integral converges, while the second one diverges. However, we are to remember, that the basic vertex (2) has cut-off $\Lambda$. In previous calculations [3, 7, 8] we have used Pauli-Villars type of formfactor on each gauge boson leg. Following this line let us define the following integrals

$$I_1(\Lambda) = \int_0^\infty \left( G_{04}^{20}(hx|1, 0, -1, -2) \right)^2 \frac{\Lambda^4}{(x + \Lambda^2)^2} dx; \quad (13)$$

$$I_2(\Lambda) = \int_0^\infty \left( G_{04}^{20}(hx|1, 0, -1, -2) \right)^2 \frac{\Lambda^4}{(x + \Lambda^2)^2} x dx.$$

Let us take for estimates $\Lambda = 5\, TeV, \lambda = 0.5$, then a numerical integration gives

$$I_1(5\, TeV) = 0.212 \frac{h}{\hbar}; \quad I_2(5\, TeV) = 0.233 \frac{\lambda^2}{\hbar^2}. \quad (14)$$

Note that $I_1(\infty) = 1/3\hbar$. Constants $C_i$ are calculated with the use of values (14), that gives

$$|C_1| = \sqrt{\frac{8\pi^2 h}{3I_1(\Lambda)}} = 0.887 \frac{\sqrt{\lambda g}}{M_W}; \quad (15)$$

$$|C_2| = \sqrt{\frac{2\pi^2 h^2}{3I_2(\Lambda)}} = 0.443 \frac{\lambda \alpha_w}{M_W^2}.$$

Thus we conclude, that in our approach we show the existence of $I = 3/2$ lepton-W state, which is light in comparison with few $TeV$ scale.

Now let us proceed to a phenomenology. Relying on the above results, we assume that there exists spin $1/2$ heavy lepton $E$ with mass $M \simeq 200\, GeV$, corresponding to weak isotopic spin $I = 3/2$, that means multiplets: $(E^+, E^0, E^-, E^{--})$ and $(\bar{E}^{++}, \bar{E}^+, \bar{E}^0, \bar{E}^-)$. From (13) we see, that for an energy scale of few hundreds $GeV$ $C_2$ gives much smaller contribution than $C_1$ and so we may restrict ourselves by the following vertex of $(ElW)$ interaction (mind also Eqs. (9), (11))

$$V_\mu(k) = i \frac{\xi \sqrt{\lambda g}}{2M_W} \delta^{3/2} I_z \delta^{1/2} I_{W,z} > \sigma_{\mu\nu} k_\nu (1 + \gamma_5); \quad (16)$$

where usual Clebsch-Gordan coefficient is entering, $I = 1/2$ lepton means $(\nu_e, e)$ pair, $k_\nu$ is the $W$ momentum and $\xi$ is a dimensionless coefficient of order of unity ($\xi = 0.887$ from Eq. (14)). Vertex (16) describes an interaction of a left lepton with a right $E$.

Let us begin with $E$ width. It reads

$$\Gamma_E = \frac{\xi^2 \pi \lambda \alpha_w (M^2 - M_W^2)^2 (3M^2 - 2M_W^2)}{8 M_W^3 M^3}. \quad (17)$$
The next quantity to calculate is the differential cross-section of reaction $e^+ + u \rightarrow \bar{E}^{++} + d$

$$\frac{d\sigma}{dt} = \frac{4\pi^2 \xi \lambda \alpha^2_w}{M_W^2} \frac{(s - M^2)(s + t - M^2)(-t)}{s^2(t - M_W^2)^2}; \quad (18)$$

where $M^2 - s < t < 0$. For the total cross-section we have

$$\sigma(s) = \int_{m^2 - s}^0 \frac{d\sigma}{dt} dt =$$

$$= \frac{4\pi^2 \xi \lambda \alpha^2_w}{M_W^2} \frac{s - M^2}{s^2} \left( \left(s + 2M_W^2 - M^2\right) \ln\left(\frac{s + M_W^2 - M^2}{M_W^2}\right) + 2M^2 - 2s \right). \quad (19)$$

For the production of 200 GeV particle in $\sqrt{s} = 300 GeV$ $eP$ collisions, as it is in experiments [1, 2], large $x$’s are needed. Bearing in mind, that $u(x)$ in proton is dominant for large $x$, we use just $u(x)$ to estimate the cross-section in reaction $e^+ P \rightarrow \bar{E}^{++} + X$. Thus we have for this process

$$\sigma(e^+ P \rightarrow \bar{E}^{++}) = \int_{x_0}^1 u(x)\sigma(xs)dx; \quad x_0 = \frac{M^2}{s}. \quad (20)$$

Let us take the simplified expression of structure function $u(x)$ from presentation [11]. From differential cross-section (18) we see, that in our case the maximum of $Q^2 \equiv -t$ distribution is achieved at $Q^2 \simeq (60 \div 70 GeV)^2$. Taking value $Q^2 = (70 GeV)^2$ and neglecting terms with higher powers of $(1 - x)$, we have

$$u(x) = 3.2 \cdot (1 - x)^{3.7}. \quad \text{(18)}$$

With this function we obtain the following expression for the total cross-section at $\sqrt{s} = 300 GeV$, which depends on $m = M/GeV$

$$\sigma_0(m) = \xi^2 \lambda \cdot 2.76 \cdot 10^{-33} I(m) \text{ cm}^2; \quad (21)$$

$$I(200) = 0.460 \cdot 10^{-3}; \quad I(205) = 0.318 \cdot 10^{-3}; \quad I(210) = 0.217 \cdot 10^{-3}. \quad (21)$$

Let us now consider the isotopic relations. Taking into account values of Clebsch-Gordan coefficients and the usual EW interaction we have the following values for different cross-sections in terms of $\sigma_0$ provided only reactions on $u$-quark are taken into account

$$e^+ P \rightarrow \bar{E}^{++} + X; \quad \sigma = \sigma_0(m); \quad (22)$$
$$e^+ P \rightarrow \bar{E}^+ + X; \quad \sigma = \frac{1}{3}\sigma_0(m); \quad (22)$$
$$e^- P \rightarrow E^0 + X; \quad \sigma = \frac{1}{3}\sigma_0(m); \quad (22)$$
$$e^- P \rightarrow E^- + X; \quad \sigma = \frac{1}{3}\sigma_0(m). \quad (22)$$
Now let us consider the necessary decay channels, which we write below with the corresponding BR in brackets

$$
\bar{\nu}_e \rightarrow e^+ + W^+ \; (100\%); \\
\bar{\nu}_e \rightarrow e^+ + W^0 \; (66.7\%), \; \bar{\nu}_e \rightarrow \bar{\nu}_e + W^+ \; (33.3\%); \\
E^0 \rightarrow e^- + W^+ \; (33.3\%), \; E^0 \rightarrow \nu_e + W^0 \; (66.7\%); \\
E^- \rightarrow e^- + W^0 \; (66.7\%), \; E^- \rightarrow \nu_e + W^- \; (33.3\%).
$$

From (22) and (23) we have total cross-sections for observable inclusive reactions

$$
\sigma^+_\pm \equiv \sigma(e^+ + P \rightarrow e^+ + X) = \frac{11}{9} \sigma_0(m); \\
\sigma^0_\pm \equiv \sigma(e^+ + P \rightarrow \bar{\nu}_e + X) = \frac{1}{9} \sigma_0(m); \\
\sigma^- \equiv \sigma(e^- + P \rightarrow e^- + X) = \frac{1}{3} \sigma_0(m); \\
\sigma^0^- \equiv \sigma(e^- + P \rightarrow \nu_e + X) = \frac{1}{3} \sigma_0(m).
$$

We conclude from (24), that the positron neutral current reaction is the most suitable one for studying the effect. This agrees with data [1, 2].

To compare absolute values of calculated quantities with the data one needs to define the value of $\lambda$. There are experimental limitations $|\lambda| \leq 0.5$ [12, 13]. On the other hand, theoretical considerations in the framework of EW symmetry breaking model [3, 4, 7, 8] lead to the conclusion, that $\lambda$ has to be of order of magnitude of few tenths for the model being meaningful. So in our approach it seems reasonable to consider $\lambda$ being close to the experimental limitation. Bearing in mind also our estimate (15) of $\xi$, let us take

$$\xi^2 \lambda = 0.4. \tag{25}$$

With this value we obtain quite satisfactory total cross-section for the effect under discussion. For example, for $M = 207 \text{ GeV}$ and value (25) we have

$$\sigma(e^+ + P \rightarrow e^+ + X) = 0.37 \text{ pb}; \\
\Gamma_E = 14.4 \text{ GeV}. \tag{26}$$

The experimental total cross-section from combined results [1, 2] is $\sigma = (0.32 \pm 0.10) \text{ pb}. The value of the width also fits data [1].

For possibility $\lambda < 0$, which corresponds to excited lepton with $I = 1/2$, we obtain the width to be twice expression (17) and instead of (24) cross-sections to be the following

$$\sigma^+_\pm = \frac{1}{9} \sigma_0(m), \; \sigma^0_\pm = \frac{2}{9} \sigma_0(m), \; \sigma^- = \sigma_0(m), \; \sigma^0^- = \frac{2}{3} \sigma_0(m).$$
These relations do not agree with data [1, 2]. So we have to rest with \( I = 3/2 \) option.

For checking the mechanism of the effect, relations (24) can be used. There is also one interesting possibility to check the interpretation with the use of photons in the final state. Indeed, \( W^0 \) contain \( \gamma \) with coefficient \( \sin^2 \theta_W \). We have from the previous relations

\[
\begin{align*}
\sigma_+^{\gamma} &\equiv \sigma(e^+ + P \to e^+ + \gamma + X) = 0.23 \cdot \frac{2}{9} \sigma_0(m) = 0.051 \sigma_0(m); \\
\sigma_0^{\gamma} &\equiv \sigma(e^+ + P \to \nu_e + \gamma + X) = 0; \\
\sigma_-^{\gamma} &\equiv \sigma(e^- + P \to e^- + \gamma + X) = 0.23 \cdot \frac{2}{9} \sigma_0(m) = 0.051 \sigma_0(m); \\
\sigma_-^{\gamma} &\equiv \sigma(e^- + P \to \nu_e + \gamma + X) = 0.23 \cdot \frac{1}{9} \sigma_0(m) = 0.026 \sigma_0(m); \\
\end{align*}
\]

Thus 4% of \( e^+ \to e^+ \) anomalous high \( Q^2 \) events and 15% of \( e^- \to e^- \) ones contain hard \( \gamma \), the mass of the \((e\gamma)\) system being that of \( E \). There is also a possibility to look for leptonic decays of \( W \). For example, 9% of \( e^+ \to e^+ \) events contain the second \( e^+ \), and the same is valid for an extra \( \mu^+ \).

Of course, the best way to check the approach as a whole is to look for anomalous triple \( W \) vertex (3) with \( \lambda \simeq 0.3 \div 0.5 \). There is a firm hope, that such check soon can be done at the forthcoming LEP200.

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