Robust Fuzzy C-Means Cluster Algorithm through Energy Minimization for Image Segmentation

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Abstract

Background: The Fuzzy c-means (FCMCA) cluster algorithm with spatial information is adopted for image segmentation. In the direction of acceptable segmentation concert on noisy images, the anticipated technique exemplifies the foreign spatial evidence derived from the image and also inherits appropriateness which correspondingly reflects on the universal fuzzy fitness and fuzzy isolation among the clusters.

Methods: Segmentation combines two regions firstly, the physical dimension of the image and contextual data through energy reduction function. Secondly, since the kernel metric value is merged with fuzziness of the energy level, the dynamic delineation progresses is steadily deprived of the reinitialization progress for the level set process. Afterwards generating the bunch of non-conquered clarifications, the concluding clustering elucidation is preferred through Cluster Validity Index (CVI) by consuming the foreign spatial evidence. Additionally, the total number of clusters incorporates the actual oblique mutable string length scheme to encrypt the cluster groups in terms of grouped chromosomes spontaneously.

Findings: This novel fuzzy and nonlinear type of energy functionality brands the modernizing of region group’s added strength against the noise and edge of the image. The projected method is undergone with image polluted through noise and likened with fuzzy c & k means, dual FCM cluster based approaches with predefined spatial data and dynamic string size is inherited by fuzzy clustering procedure.

Applications/Improvements: The investigational outcome demonstrates that the anticipated technique performs thriving in developing the sum of clusters and procurement in acceptable performance on noise in image segmentation process.

1. Introduction

The Image segmentation (IS) is predominantly essential with significant task in computer vision and artificial trending mechanisms. There are numerous approaches for image segmentation have been anticipated for example thresholding⁴, Image clustering⁵, Image edge finding and so on. Nevertheless, because of existence of noise and minimal contrast and pixels in images, outliers. Amongst these approaches, an entrenched set of procedures like active contour models (ACMs). It has dual merits over conventional approaches stated above. This could be attain improved accurateness in segmentation⁶ expressed as a minimalist through energy function that permits the amalgamation of diverse and beneficial image material⁷. Moreover ACM is initially recognized as a snake model projected by the author Kass et al.⁸. Through the growth of active contour practice, diverse distinctions of ACM’s consumed and anticipated for IS. Conferring to the variable necessities pouring the advancement, prevailing
2. Related Works

One of the greatest well-known and extensively used ACM’s could be approximately placed into two nominal classes: Region-based Mmodels (RBM)\textsuperscript{11-14} and Edge-based models (EBM)\textsuperscript{5,6,10}.

There are many investigators familiarized the three-dimensional evidence to derivative relative pixel in the specified image into the concluding objective function of FCM method\textsuperscript{28,31-36}. This is for the unique purpose of reducing the secrecy level of FCM to the noise ratio with real image. Ahmed et al.\textsuperscript{33} adapted the unbiased method of FCM by integrating nearest spatial locality factor and anticipate FCM approach by including spatial locality information (FCM-S). Consequently, Chen and Zhang\textsuperscript{28} proposed two alternatives of FCM Spatial\textsuperscript{1} and FCM Spatial\textsuperscript{2} to minimize the computational difficulty of FCM-S. Additionally, in ref.\textsuperscript{29,30}, author exploited a kernel-based detachment to standby the Euclidean detachment and offered the kernel versions of FCM Spatial\textsuperscript{1} and FCM Spatial\textsuperscript{2}. To facilitate hasten in the image segmentation procedure of FCM including spatial information, Szilagyi et al.\textsuperscript{14} produced a linearly-weighted quantity duplicate image and projected a superior fuzzy c-means clustering procedure (EnFCM). Subsequently the gray level histogram takes place on engendered quantity of image in place of the individual pixels. This is comparable to EnFCM, Cai et al.\textsuperscript{36} author Zhao et al.\textsuperscript{37} familiarized a foreign spatial evidence resultant from a large image m and build the spatial restriction tenure (FCM-NLS). Although the foreign spatial evidence of any individual pixel is attained by exploiting and comparable formation of the specified pixel\textsuperscript{31,32,38,39}.

3. The Chan_Vese (CV) Model

The Chan–Vese (CV) model\textsuperscript{19} is articulated through minimizing an energy function by the Mumford–Shah (MS) model. This model fabricates agility against noise factor. The fundamental notion of MS model is to discover an original image \( I_{\text{On}} \) using to approximated image \( I_{\text{App}} \). In this formation the edge \( E \) which fragments the original host image into non-overlying sections of given image \( \Gamma \). The basic energy level function of MS model\textsuperscript{15} can be well-defined by Equation 1.

\[
MS(I, E) = \int (I_{\text{On}}(x) - I_{\text{App}}(x)) \, dx
+ \int_{\Gamma/E} \left( \nabla \cdot I_{\text{App}}(x) \right)^2 \, dx + o.o.Len(E) = - - - - - - 1
\]

Such that \( \nabla \) is known as gradient operator. In this equation the first segment defines the mean square data
term subsequently the second segment shows normalizing term which tends to deploy smooth portions. The final segment modernize the edge set $E$ to be normalized. In some special case, the intensities in the estimated image $I$ will remain constant. This occasion is known as the cartoon limit by which the second segment fulfills

$$
\int \left| \nabla I_{app}(x) \right|^2 dx = 0.
$$

Predominantly, in the state of two stage segmentation, the image $\Gamma$ could be diverse into dual mirrored regions: inside $(E)$ and outside $(\overline{E})$ represented with single constant. However the other two constants $e_i$ and $e_o$ to characterize the intensity averages of $I_{ori}$ both inside and outside. Henceforth, the energy formation of CV model is expressed by Equation 2,

$$
CV(e_i, e_o, E) = \omega \cdot Len(E) + \Delta_i \int_{e_i} |I_{ori}(x) - e_i|^2 dx
$$

$$
\Delta_2 \int_{e_o} |I_{ori}(x) - e_o|^2 dx - - - - - - - - - - 2
$$

Here $\Delta_1$ and $\Delta_2$ are constant parameters and typically both will set to 1. The level set methodology were introduced to minimize the energy function in Equation 2, and the edge $E$ will get evolved, so that it is denoted as the zero level significance of the LSF $(\psi)$ consequently it is represented as,

$$
\begin{align*}
E &= (x \in \Gamma : \psi(x) = 0) \\
e_i &= (x \in \Gamma : \psi(x) > 0) \\
e_o &= (x \in \Gamma : \psi(x) < 0)
\end{align*}
$$

Thus, the energy formation could be stated in addition with Equation 3

$$
CV = (e_i, e_o, E) = \omega \cdot \int_{\Gamma} \gamma(\psi(x)) |\nabla \psi(x)| dx
$$

$$
+ \Delta_1 \int_{\Gamma} |I_{ori}(x) - e_i|^2 K(\psi(x)) dx
$$

$$
+ \Delta_2 \int_{\Gamma} |I_{ori}(x) - e_o|^2 (1 - K(\psi(x))) dx - - - - - - - - - - 4
$$

And the Heaviside $K$ and Dirac $\gamma$ could be expressed through Equation 4

$$
K(x) = \begin{cases} 
1 & x \geq 0 \\
0 & x < 0
\end{cases}
$$

$$
\lambda(x) = \frac{d}{dx}(K(x)) - - - - - - - - - - 5
$$

The energy level formation in equation 5 is reduced by calculating related EL and steepest procedure to modernize the LSF. Based on implantation of region data, the CV model is become more agile against noise and very effective in perceiving weak limitations which are certainly sharp by gradient parameter.

### 4. Fuzzy Dynamic Delineation Model (FDDM)

The FDDM model is region-based prototype model which syndicates the fuzzy group sets with the dynamic delineation formation. This model is slightly vary with CV model, the progressing delineation is indirectly significant as the 0.5 zero level set of the LSF $F_M$, by which it is represented in equation 5.

$$
\begin{align*}
E &= (x \in \Gamma : F_M(x) = 0.5) \\
e_i &= (x \in \Gamma : F_M(x) > 0.5) \\
e_o &= (x \in \Gamma : F_M(x) < 0.5)
\end{align*}
$$

$$
F_M(E, e_i, e_o, F_M) = \omega \cdot Len(E) + \Delta_1 \int_{\Gamma} (F_M(x)) (I_{ori}(x) - e_i)^2 dx
$$

$$
+ \Delta_2 \int_{\Gamma} (1 - F_M(x)) (I_{ori}(x) - e_o)^2 dx - - - - - - - - - - 7
$$

The function $F_M$ is referred as fuzzy membership function. $F_M(x)$ describes the membership constant with the pixel $x$ fits into $e_i$ and $1 - F_M(x)$ signifies the membership constant with pixel $x$ fits to $e_o$. The member function $t$ typically dual with the weighting proponent on both fuzzy membership. The $e_i$ & $e_o$ are mid models of the origin image inside $E$ and outside $E$. With the constant factor $F_M$, the reduction of energy functions in equation 2 based on $I_{ori}$ & $F_M$.

$$
\begin{align*}
e_i &= \frac{\int_{\Gamma} (F_M(x)) I_{ori}(x) dx}{\int_{\Gamma} (F_M(x))^2 dx} - - - - - - - - - - 8
\end{align*}
$$

$$
\begin{align*}
e_o &= \frac{\int_{\Gamma} (1 - F_M(x)) I_{ori}(x) dx}{\int_{\Gamma} (1 - F_M(x))^2 dx} - - - - - - - - - - 9
\end{align*}
$$
The $e_i$ & $e_o$ are stable and the length term sustainability, we minimize the energy formation in equation 7 with respect to $F_m$, without dropping the generalization, the equation used to update $F_m$ is defined by equation 10.

$$F_m = \frac{1}{1 + \left( \frac{\Delta_i(I_{ori}(x) - e_i)^2}{\Delta_e(I_{ori}(x) - e_o)^2} \right)^{-10}}$$

Precisely, for a particular pixel $x_i$, to calculate the novel fuzzy membership function using equation 10, there should be stability in understanding the change which is triggered by the particular alteration with fuzzy membership on the pixel $x_i$. Subsequently if there is any change in $F_m$ then it makes $F_m$ convert into smaller ($\Delta F_E < 0$), finally the new fuzzy membership will be fabricated. In case of no substantial change on old formation is taken for consideration. Moreover, conforming to the updating principle of $F_m$, pixels background might be simply tagged as like pixels which owned by object region space, only if current intensities are identical with the object region.

### A. Energy Level Construction

In these models designated above, the space $(l_{ori} - E)^2$ and $C = (e_i, e_o)$ is to paradigm image data in terms of energy level formulation. This cadenced process tends to lead non-robust image segmentation outcome of given images with stipulated noise. As formulated in equation (8) and (9) is for respective pixel $x_i$, it is perceptible that the modernizing of average patterns can be inclined by the original intensity ratio $I_{ori}(x_i)$ when it is outline scope. In order to minimize the consequence of noise and outline scope for a strong breakdown, we substitute the $L_2$ normalization along with nonlinear form of space constant.

The kernel scheme is extensively realistic in fabricating a nonlinear structure of a linear procedure. Provided this scheme could also suit an inactive delineation procedure. A mutual kernel distance metric constant could be nominated by a defined rule clarified in the upcoming sections. The defined $K_m(\alpha_1, \alpha_2)$ is corresponding to 1. Formerly we paradigm the nonlinear space distance via equation (13) to substitute through Euclidean distance variations.

$$||\Theta(\alpha_1) - \Theta(\alpha_2)||^2 = (\Theta(\alpha_1) - \Theta(\alpha_2))^T(\Theta(\alpha_1) - \Theta(\alpha_2))$$

$$= K_m(\alpha_1, \alpha_2) + K_m(\alpha_2, \alpha_2) - 2K_m(\alpha_1, \alpha_2)$$

For ease simplification, we typically utilized the kernel distance variations as $\exists_{EK} = 1 - K_m(\alpha_1, \alpha_2)$ to exchange the Euclidean distance $\exists_{EU} = ||\alpha_1 - \alpha_2||^2$. Henceforth the nonlinear energy process integrating fuzzy sets that articulated in equation (14) with the quasi level set previously illustrated in Equation (6).

$$F_g(E, e_i, e_o, F_m) = \alpha_1 Len(E) + \Delta_1\int_t (F_m(x))(1 - K_m(I_{ori}(x) - e_i))dx$$

$$+ \Delta_2\int_t (1 - F_m(x))(1 - K_m(I_{ori}(x) - e_o))dx$$

However the delineation is implicitly characterized as the 0.5 level through set of $F_m$ and consequently, we consume following Equation (15).

$$Len(E) = \int_t \gamma(K(F_m(x) - 0.5))dx$$

$$= \int_t \gamma(F_m(x - 0.5))|\nabla F_m(x - 0.5)|\ |dx$$

Such that the process $K$ and $\gamma$ are described already in equation (4). By Combining equation (14) and Equation (15), the projected energy function is articulated by equation (16).

$$F_g(e_i, e_o, F_m) = \alpha_1\int_t \gamma(F_m(x - 0.5))|\nabla F_m(x - 0.5)|\ dx$$

$$+ \Delta_1\int_t (F_m(x))(1 - K_m(I_{ori}(x) - e_i))dx$$

$$+ \Delta_2\int_t (1 - F_m(x))(1 - K_m(I_{ori}(x) - e_o))dx$$

The Gaussian radial basis function (GRBF) is a broadly applied in kernel process and the customized version of GRBF is implemented this presented work is affirmed by equation (12).

$$K_m = \exp\left(-\frac{(\alpha_1 - \alpha_2)^2}{\sigma}\right)$$

The straight minimization method used in FDDDM process is not appropriate for this nonlinear energy functionality (16). It could be demonstrated through particular modification of fuzzy membership function $F_M$ in the defined pixel $x_i$, the alteration of $F_E$ is a nonlinear alteration connected with the previous $F_E$. In subsequent section we will deliver the minimization practice for the anticipated energy functionality.

B. Energy Level Minimization

The functional value of $F_M$ is much varied when it corresponds minimum value of $F_E$ described in Equation (16). Henceforth the deployment of sharpest linear scheme is engaged to diminish the anticipated energy function on regards with $F_M$, $e_t$ & $e_O$. Initially, the fuzzy membership functional $F_M$ value is remain constant, then we reduce the quantity of $F_E$ through equation (16) based on $e_t$ & $e_O$. Finally it is feasible to acquire the modernizing equations of $e_t$ & $e_O$ expressed by the equations (17 and 18).

$$e_t = \int (F_M(x))_t K_M(I_{ori}(x), e_t) I_{ori}(x) dx$$

$$e_O = \int (1-F_M(x))_t K_M(I_{ori}(x), e_O) I_{ori}(x) dx$$

It is perceptible that the section patterns $e_t$ & $e_O$ stagnant in the distance after modification. The distinctive that suggested energy origination is strong with noise and outline scope which can also attain by instinctive elucidation by equations (17) and (18). Each intensity $I_{ori}(x)$ is capable with an added value of $K_M(I_{ori}(x), \alpha_1 + \alpha_2)$ that possess the detachment by the original intensity $I_{ori}(x)$ to typical prototype models. If the distance of the region prototype models is very extreme level in terms of their intensity and additional value of the related intensity is very minor. Therefore, for each single pixel $x_i$ with the intensity $I_{ori}(x)$ is an outlined and isolated from the region prototype models, the inspiration employed by $I_{ori}(x)$ on the informing of region prototype models can be inhibited by the added value of $K_M(I_{ori}(x), \alpha)$ in this given pixel. Formerly, when $e_t$ & $e_O$ are constant, we demise $F_E$ in equation (16) based on $F_M$. Now the EL equalities are engaged to build the undesirable gradient flow articulated as,

$$-\frac{\partial F_E}{\partial F_M} = \alpha\gamma(F_M(x) - 0.5)D_f \left( \frac{\gamma(F_M(x) - 0.5)}{\gamma(F_M(x) - 0.5)} \right)$$

$$-\Delta t. t(F_M(x))^{t-1}(1-K_M(I_{ori}(x), e_t))$$

$$+\Delta t. t(1-F_M(x))^{t-1}(1-K_M(I_{ori}(x), e_O))$$

$$F_M(0, x) = F_{M-ori}(x) \in \gamma$$

Where, $F_{M-ori}(x)$ is the preliminary LSF and the Neumann boundary condition nominated to offer flexibility on implementation.

C. Classical FCM Algorithm

The FCMCA algorithm is process which permits a data origin point to fit single or multiple group clusters together. Consider an image represented as $A = \{a, a, a, \ldots, a\}$ with $x$ number of pixels, such that $a$ signifies the gray value of corresponding $l^th$ pixel. The core function of formal FCMCA algorithm is,

$$M = \sum_{N=1}^{x} \sum_{i=1}^{N} \int_{nl} \sum_{m} p \| a_i - q_n \|^2$$

Such that $q_n (1 \leq n \leq N)$ symbolizes the pixelated particles of the $n^{th}$ cluster and consequently $q_{nl} (1 \leq n \leq N, 1 \leq l \leq x)$ denotes the membership and association degree functional value of the $l^{th}$ pixel which fit into the $n^{th}$ cluster. Furthermore, $p_{nl}$ desires to fulfill the subsequent limitations,

$$\sum_{n=1}^{N} p_{nl} = 1, \quad p_{nl} \in [0,1],$$

$$0 \leq \sum_{i=1}^{x} p_{nl} \leq x$$
In equation (1), represents the Euclidean normalization and the factor $em$ ($em > 1$) is act as weight proponent which concludes the overall mass of fuzziness of resultant panel, by Equation (1), the modernize equalities of association degree function $p_{nl}$ which includes the cluster pixilated particles $q_{n}$ represents by,

$$\sum_{l=1}^{N} (\|a_l - q_n\|^2 / \|a_l - q_m\|^2)^{1/(r-1)}$$

$$q_n = \sum_{l=1}^{N} p_{nl} a_l$$

FCM cluster algorithms including spatial information

D. Spatial Information

To decrease FCMCA's level of sensitivity over noise and other factors in the given image region, there are numerous improved FCMCA algorithms exploits the spatial information resulting by the district window nearby each pixel elements. This type of spatial information is specifies native spatial data. The local spatial information consumes had many manifestation procedures. The mean spatial information of the $l^{th}$ pixel can be defined in the subsequent equation. The native local information includes other information like mean spatial information and median spatial information.

$$\psi_l = \frac{1}{|NG_l|} \sum_{p \in NG_l} a_p$$

Such that the set of neighborhood pixels symbolized by $NG_l$ in defined window which focus the pixilated particles at the $l^{th}$ pixel. The cardinality value represents through $|NG_l|$. Likewise, for the $l^{th}$ pixel the median spatial information is articulated by,$$NG_l = Median\{a_p\}, p \in NG_l$$

 Apparently, numerous pixel values holding a relative formation of anhost image. By exploiting the pixels with a comparable distinct neighborhood formation. This type of configuration is called as foreign spatial evidence. For the given $l^{th}$ pixel, the foreign native spatial information $\bar{a}$ is figured via,

$$\bar{a}_l = \sum_{m \in T_m} T_m a_m$$

In above presented form, $T^s_l$ signifies a $s \times s$ search exploration window positioned at the pixilated particles $l^{th}$ pixel. The foreign spatial value of $l^{th}$ pixel value is calculated by exploiting the given pixels in this frame. $T_{im}$ ($m \notin T^s_l$) is the weightage among the $l^{th}$ and $m^{th}$ pixels, has to fulfill $0 \leq T_{im} \leq 1$ and $\sum_{m \in T^s_l} T_{im} = 1$. The factual weight $T_{im}$ is well-defined by

$$T_{im} = \frac{1}{DW_i} \exp(-\|a(NC_l) - a(NC_m)\|^2 / F^2)$$

In this equation the constant $F$ denotes the filtering degree factor which restricts the deterioration of the weight functional value of $T_{im}$, subsequently $DW_i = \sum_{l \in T^s_l} \exp(-\|a(NC_l) - a(NC_m)\|^2 / F^2)$ is the normalizing factor. Considerably, this weight value $T_{im}$ depends on the resemblance among the $l^{th}$ and $m^{th}$ pixels. The significance resemblance is calculated through Gaussian weighted Euclidean distance function $\|a(NC_l) - a(NC_m)\|^2$, such that $\tau (\tau > 0)$ signifies the standard aberration of Gaussian kernel. $a(NC_l)$ is the gray level value vectors within the defined $s \times s$ right-angled neighborhood pixels $NC_l$ pixilated particles at the $l^{th}$ pixel. This is can be formed through equation (8) by which all pixels with a related gray level values of neighboring $l^{th}$ pixel with native larger weights.

E. Merging FCM Cluster Algorithms and Spatial Information

The FCMCA algorithm initially consumes the spatial evident information for every respective pixel value to describe a spatial limitation. The modernized native objective formation will be articulated as

$$M = \sum_{x=1}^{N} \sum_{i=1}^{r} p_{ai} ||a_i - q_n||^2 + \rho \sum_{x=1}^{N} ||\bar{a}_i - q_n||^2$$

$$= \sum_{x=1}^{N} \sum_{i=1}^{r} p_{ai} ||a_i - q_n||^2 + \rho \|\bar{a}_i - q_n\|^2$$

Such $\bar{a}_i$ defines the local spatial evident information otherwise the foreign spatial facts of the $l^{th}$ pixel. In succeeding formula in equation (29) is spatial evident limitation tenue and factual parameter $\rho$ restricts the forfeit consequence. The reduction of equation (29) by Lagrange multiplier scheme produces the subsequent membership
degree parameter. The updated cluster pixelated particles equations can be expressed as,

\[
p_n = \frac{1}{\sum_{i=1}^{x} (||a_i - q_n||^2 + \rho ||P_i - q_n||^2) / (||a_i - q_n||^2 + \rho ||P_i - q_n||^2)^{(1-\mu)}}
\]

Such that \(a_i\) represents the foreign native spatial evident information of defined \(l^h\) pixel value provided \(P_i\) is the controlled limitation. The pixelated particles \(s \{q_1, q_2, \ldots, q_N\}\) coded in a specified pixelated particles. The determined and the association member degree values \(p_{nl}, n = 1, 2, \ldots, N, l = 1, 2, \ldots, n\) are computed by, Equation (10).

\[
\sum_{l=1}^{x} p_{nl}^l (||a_i - q_n||)
\]

 Represents the variation of the \(n^th\) cluster, subsequently,

\[
\sum_{l=1}^{x} p_{nl}^l (||a_i - q_n|| + \rho ||a_i - q_n||)
\]

Could be deliberated as spatial weighted difference of the \(n^th\) cluster and represented as \(\tau(F_i) = \sum_{l=1}^{x} p_{nl}^l ||a_i - q_n||\) in the formation the fuzzy cardinality of defined \(n^th\) cluster has represented as \(x_l\). Hence, \(F_F\) is the overall fuzzy density with spatial data and be exemplified by \(F_F = \frac{\sum_{l=1}^{x} p_{nl}^l}{x_l}\)

If the cluster midpoint \(q_a\) is recognized as the midpoint of the given fuzzy set \(\{q_b \mid 1 \leq b \leq N, b \neq a\}\), the membership degree value of \(q_b\) to \(q_a\) is distinct as,

\[
\omega_{ab} = \frac{1}{\sum_{b=1, b \neq a}^{N} (||q_b - q_a||^2 + \rho q_b - q_a||^2)^{(1-\mu)}}
\]

\(F_S = \sum_{a=1}^{N} \omega_{ab} ||q_b - q_a||^2\)

Figure 1. Original host Image.
G. Final Selection by Boundary and Transformation Operation

In FCMCA, packed binary competition selection scheme is embraced to produce the copulating group of pixelated particles. In this selection system, formerly the pixelated particles are ordered depends on identical sorting with allocated massive distance parameter, the formation is accomplished by consuming a massive comparative operations. The crossover sector points could falls amongst two cluster midpoints. Subsequently the crossover process is accomplished stochastically through the crossover likelihood \( \text{Prob}_{cv} \) and requisite to confirm that descendant’s pixelated particles coded as a minimum of dual midpoints. This determination, the crossover operative scheme in equation (30) is incorporated in FCMCA. Presume the native pixelated particles \( \theta_1 \) and \( \theta_2 \) and coded \( N_1 \) and \( N_2 \) cluster midpoints, correspondingly. Consider \( \phi \) be the crossover fact in \( \theta_i \) and can be produced as

\[
\phi = \text{rand}(\mod N_i)
\]

Such that \( \text{rand} (\cdot) \) is functionality of rearranging an arbitrary integer among 0 and \( N \). Consider \( \phi_2 \) be the crossover midpoint of \( N_2 \) and it might differ in \( [L_B(\phi_2), U_B(\phi_2)] \), wherever \( L_B(\phi_2) \) and \( U_B(\phi_2) \) are the lower and upper boundaries of the series of \( \phi_2 \), correspondingly. \( L_B(\phi_2) \) and \( U_B(\phi_2) \) are well-defined as,

\[
L_B(\phi_2) = \min[2, \max[0, 2 - (N_1 - \phi_1)]]
\]

\[
U_B(\phi_2) = [N_2 - \max[0, (2 - \phi_1)]]
\]

Consider the case \( U_B(\phi_2) \geq L_B(\phi_2) \), \( \phi_2 \) is produced arbitrarily as a numeral among \( L_B(\phi_2) \) and \( U_B(\phi_2) \). Else \( \phi_2 = 0 \), this could be effortlessly confirmed. In case if crossover midpoints \( \phi_1 \) and \( \phi_2 \) are selected permitting to the above described rules, formerly the quantity of clusters in every off springs may not decrease to 2. The quantity with clusters sequence is assured on off-springs surpass value \( N_{\text{max}} \). Afterwards execution of crossover procedure, the transformation function is functional to pixelated particles. All cluster midpoint coded in a pixelated particles is transformed with the transformation probability \( \text{Prob}_{\text{trans}} \). However if the \( N^{th} \) cluster midpoint \( P_a \) requisite to be mutated, then the significance of the \( a^{th} \) dimension of given \( P_{N,a} \) \( (P_{N,a}) \) will converts \( P_{N,a} \pm 10\zeta' \), whereas \( \zeta' \) arbitrary sum with the series of \([0,1]\) having uniform distribution. This signs ‘+’ or ‘-‘ ensembles with equal probability.

H. The Absolute Non-Dominated Solution Set

After evaluation of our FCMCA algorithm, it produces a group of unpredictable resolutions are attained. Through algorithmic perspective, all the attained answers were treated with equal importance however the single solution may treated as most accurate and validated properly. So as to accomplish the single solution the new parameter a cluster validity index (CVI) \( C_{\text{val}} \) has introduced the \( C_{\text{val}} \) is simplified as

\[
C_{\text{val}} = \left( \frac{1}{N} \times \frac{1}{\exists_{\text{EU}}} \times \exists_{\text{KE}} \right)
\]

Such that \( N \) represents the no. of cluster groups. \( \exists_{\text{EU}} \) & \( \exists_{\text{KE}} \) are exemplified as,

\[
\exists_{\text{EU}} = \sum_{N=1}^{N} \sum_{i=1}^{x} p_{nl} \| a_i - q_n \|
\]

\[
\exists_{\text{KE}} = \max_{a,b=1}^{N} \| q_b - q_a \|
\]

The cluster validity index \( C_{\text{val}} \) is a structure of three basic aspects, such as \( 1/N, \exists_{\text{EU}}, \exists_{\text{KE}} \). \( \exists_{\text{EU}} \) computes the fuzzy density of the data set and minimizes as by \( N \) maximizes. This is apparent that \( \exists_{\text{EU}} \) is persistent for all defined data set. The \( \exists_{\text{KE}} \) computes determine parting among any two cluster groups from all cluster groups. The upper constrained by the concentrated parting among any two root points in the given data set. When \( N \) is maximized, \( 1/N \) is minimized, \( \exists_{\text{EU}}/\exists_{\text{KE}} \) is maximized and \( \exists_{\text{KE}} \) is maximized. Therefore these three aspects are originate to compete critical equilibriums. If index \( C_{\text{val}} \) has larger value which suggests more dense and well-parted cluster groups and specifies improved resolution. More over, the non-local spatial data resultant from the image is familiarized into the index \( C_{\text{val}} \) and an innovative cluster validity index through spatial information \( C_{sl} \) is anticipated to choose the optimum resolution. \( C_{sl} \) is simplified as.
\[ C_{SI}(N) = \frac{1}{N} \times \frac{\exists_{opt}}{\exists_{opt-ke}} \times \exists_{opt-eu} \]

Such that the calculation of \( \exists_{opt-eu} \) is similar to \( \exists_{ke} \), & \( \exists_{opt-ke} \) is elaborated as

\[ \exists_{opt-ke} = \sum_{n=1}^{N} \sum_{l=1}^{x} p_{nl}(\|a_i - q_n\| + \rho \|a_i - q_n\|) \]

From the above equation the membership function \( p_{nl} \) is computed via Eq. (10), \( a_i \) is well-defined as the non-local spatial data of the \( l^{th} \) pixel. The \( \rho \) is the spatial controlled factor.

### 6. Experimental Results and Analysis

To validate the efficiency of FCMCA, certain artificial pictures are utilized in the experimentation and KM, MSFCA, FGFCM, FCM, MOVGA, FCM NLS are implemented as the proportional approaches. These approaches, FCMCA and MOVGA can inevitably progress the sum of cluster groups sets. The supplementary algorithms are accomplished under diverse quantity of cluster groups stretching from 2 to \( N_{\text{max}} \) and the final resolution with the maximum value of index \( C_{nl} \) (Eq.(19)) is deliberated beside with the equivalent quantity of cluster groups sets. In this proposed work, \( N_{\text{max}} \) is defined as 10 for all the experimentations. Furthermore, the cluster accuracy (CTA) and the Adjusted Rand Index (ADRI) are embraced to assess the robustness of proposed model.

With respect of all the defined methods, except the KM method hold the index value is set to 2. For all the methods the highest iteration number \( Max \), along with the stopping threshold value \( S_{Thres} \) are set to 350 and 10–6, correspondingly. On other hand the two basic parameters \( \Delta_s \) and \( \Delta_g \) of FGFCM method sets to 3 and 6 and these outcomes and investigation are described in the article. Furthermore from the outcome of FGFCM the neighbor window with the size of 3 x 3 has selected because of its optimality when compared with other methods.

The search window with size of \( s \times s \) and the patch with the size of \( e \times e \) and the filtering gradation bound \( F_{pl} \) for the non-local spatial evidence has clearly discussed in. By conferring to the investigational examination in, the factors \( s, e \) and \( F_{pl} \) are set to 22, 8 and 30. Subsequently the spatial evident factor \( \rho \) confines the forfeit penalty consequence and it that has been verified on a set stretching from 0 to 8 in. On other hand Trials in demonstrations that the segmentation outcomes below \( \rho = 6 \) are acceptable. At this time, the factor \( \rho \) in FCM NLS & FCMCA is fixed to 6. The maximum amount of generation \( G_{\text{max}} \) and the total population size \( P \) remain set to 100 to 50 in, in the same way it is set to 20 and 20 similarly 40 to 20 in. By widely seeing these comparative situations and algorithm difficulty, the maximum amount

![Figure 2.](image-url)
of generation $G_{\text{max}}$ and the total the population size $P$, with FCMCA and MOVGA are fixed to 40 and 20, correspondingly. In $\rho_0$, the crossover probability $Prob_c$ along with the mutation probability $Prob_m$ are fixed to 0.8 & 0.1, individually. Therefore, FCMCA & MOVGA approve the similar value fixing of the factors $Prob_c$ and $Prob_m$.

### 7. Experiments on Real Image

For this evaluation the real test image with dimension of $100 \times 100$ pixels (Figure 1) and it comprises 7 cluster groups with the equivalent gray values as 0, 34, 75, 107, 136, 172 and 217 (Figure 2). In this segment, we make use of this picture along with damaged noise parts along with diverse Gaussian Noise (GN) is to examine the attained amount of cluster groups and divided enactment of FCMCA and proportional procedures. To produce these tainted image sequences, the Gaussian white noise (GWN) with singular mean value of 0 and other normalized modification 0.0003, 0.002, 0.003, 0.005, 0.007, 0.013 and 0.018 is appended to this artificial image, correspondingly.

Figure 3a signifies the attained amount of cluster groups of these approaches on original host images with diverse noise level sequences, and Figures 3b and 3c correspondingly shows that CTA and ADRI bends of these six approaches on defined corrupted images. Figure 2d demonstrations that FCMCA can appropriately progress amount of cluster groups with all noise stages. The cluster value of KM, FCM NLS, FCM and FGFCM conforming to the maximum significance of index $C_{\text{val}}$ are accurate. Although the FCM, attained amount of cluster groups lesser noise level with mean of 0 and variance 0.007 is right, the equivalent CTA and ADRI standards are unacceptable, which are presented in Figure 2b and c. The FCMCA attains the maximum CTA and ADRI standards underneath to entire noise levels excluding the final noise level. Consequently, further down the most noise levels, FCMCA performs fine in developing the precise amount of cluster groups and attaining acceptable CTA and ADRI values. Figure 3 illustrates the corrupted artificial image with Gaussian white noise with zero mean value and standardized variance value 0.013. This could be originate from Figure 3a that merely FCMCA amongst all the approaches can perceive the amount of cluster groups on this noise corrupted picture. The equivalent segmentation outcomes of these approaches are offered in Figures 3b–g. These outcomes disclose that FCMCA is the finest amongst all the approaches even if the actual amount of cluster groups is assumed for the additional approaches.

### 8. Conclusion

The proposed FCMCA model for effective image segmentation is to improve agility and performance of conventional FCM clustering technique on noisy images, FCMCA commences effective foreign spatial evident information via the noisy image and transform into robustness. Furthermore, the foreign spatial evidence is also initiated into CVI through the best possible resolutions. The appropriateness and CVI through foreign spatial evidence could conquer the impact of noisy image
into the grouping concert. In this process a variable length string technique is emphasized. The evaluation outcomes on the images shows dynamism on FCMCA, as a result of the non-linearity of the energy fitness function, conventional scheme by proving EL equation is tend to reduce the energy consumption. This reflects in lower conjunction rapidity with our proposed system by comparing with KM, MSFCA, FGFCM, FCM, MOVGA and FCM NLS. It is recognized that the appropriateness is the critical important characteristic for FCM clustering algorithm. The performance of an algorithm is rectifies the superior influence level of noisy image; our prospect work comprises few additional effectual image spatial information hooked on to refine fitness functions. Additionally, our potential future investigation furthermore incorporate by utilizing two other factors edge and region which is consequent since the noisy host image to build final fitness calculations.

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