Macroscopic Modelling of Pedestrian Flows Based on Conservation Law

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Abstract. Overcrowded of sidewalks can reduce the level of satisfaction of pedestrians. The large crowd on the sidewalks results in slowing down time of the pedestrian to arrive in their destination. This study focuses on pedestrian flows modelling using the macroscopic model. Numerical approximation of the macroscopic model is formulated as scalar hyperbolic conservation laws. The Lax-Wendroff scheme is used to discretize the equation of conservation laws. The simulation results show that the numerical approximation in term of density confirms the exact solution. In this simulation, the velocity function is obtained by curve fitting of observation data using linear regression method. The observation data, which are consist of velocity-density relation, are obtained from observation of pedestrian flows. The study case of this research conducts on the sidewalks of Braga Street, Bandung, Indonesia. There are two velocity functions used in the simulation, i.e. \( c_1 = 0.58206 - 0.94476\rho \) and \( c_2 = 0.59926 - 0.78052\rho \), respectively. In performing the velocity function \( c_2 \), the pedestrian leader position is approximately 1 meter in front of the pedestrian leader using the velocity function \( c_1 \) at final time \( T = 20 \) seconds and \( T = 30 \) seconds. Overall, the numerical experiment shows that the pedestrian leader using the velocity functions \( c_2 \) is faster than by using \( c_1 \).

1. Introduction

Pedestrian is a person who performs activities on foot as a means of transportation with a certain distance. Pedestrian ways are often called the sidewalk. It is an infrastructure used by the pedestrian as a connector of the centre of urban activity [1]. Crowds in the city make the sidewalk overcrowded, thus making pedestrians become uncomfortable and can slow them down to arrive at their destination. Congestion on the sidewalk often occurs mainly on weekends or during holidays and before feast days. An increasing population that is not comparable to infrastructure too can cause overcrowded on the sidewalk.

Problems such as pedestrian density can be described with numerical models; one of them is the macroscopic model. Partial differential equations construct this model is used to modelling traffic flow that occurs on the road, which involves parameters such as velocity, density and traffic flow. The macroscopic models are also often referred to as the LWR models [2, 3]. In previous study [4], The LWR model applied then model discretization using the finite volume method to simulate traffic flow via fluid dynamics model for a single road with a traffic light. Several other numerical methods used as references [5, 6, 7]. In this research, the Lax-Wendroff
scheme used to help discretize models, as in [6], only in this research, this is used to pedestrian flows modelling. This method is the second on the time and space approximation scheme [6, 8].

In this study, a pedestrian flows modelling using the LWR model will be elaborated. Mathematically this model is described as scalar hyperbolic conservation laws, can be rewritten as a transport equation. In the transport equation, there is a velocity variable, that is depicted as velocity function [6]. Velocity function is obtained from the observation data approach, using a linear method. Observation data consisted of pedestrian velocity-density relations, which is obtained from direct observation on the sidewalk of Braga Street, Bandung, Indonesia. The Author conduct experiment to analyze the impact of the density of pedestrian flows. The frequency has an influence on the estimated distance to be travelled.

This study organized as follows: In Section 2, numerical models and schemes for pedestrian flow modelling presented. In Section 3, explain about numerical approaches and model. Furthermore, the conclusion is present in Section 4.

2. Numerical Model And Scheme

2.1. Macroscopic Model

The macroscopic model is one of traffic flow models. The macroscopic models are also often referred to as the LWR models [2, 3]. Mathematically this model is described as scalar hyperbolic conservation laws [4]. The equation is explained below:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (q)}{\partial x} = 0 \quad (1)$$

where, \( \rho \) as the density of pedestrians, \( q \) denote the pedestrian’s flux, \( x \) denote the position and \( t \) the time variable. Here, the assumption that pedestrian flux is a function of density as follows:

$$q = \rho v, \quad (2)$$

where \( v \) as average velocity. In actual observations, the average velocity in the control volume depends on the density. As shown in [9], Velocity depends on the vehicle density on the road. In this study, velocity depends on pedestrian density on the sidewalk, and the formula as follows:

$$v = v(\rho), \quad (3)$$

where \( v(\rho) \) is velocity-density relation. And this is the new model of equation (1):

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v(\rho))}{\partial x} = 0 \quad (4)$$

The velocity-density relation is \( v(\rho) \) obtained using the linear regression method. There are two types of linear regression; one of them is a simple linear regression used in this study. Simple linear regression is a statistical method that works to test on how far the causal relation [10] between causal variables for the effect variable, see [11, 12] for more details. The data used in order to find the velocity density function is the observation data. The following equation is an example of the simple linear regression used in this research:

$$v(\rho) = a + b\rho, \quad (5)$$

where \( a \) as constant, \( b \) as the regression coefficient and \( \rho \) as the causal variable. The equation (4) also can be rewritten as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v(\rho))}{\partial \rho} \frac{\partial \rho}{\partial x} = 0 \quad (6)$$

or also can rewritten as a transport equation:

$$\frac{\partial \rho}{\partial t} + c \frac{\partial \rho}{\partial x} = 0, \quad (7)$$

where \( c = \frac{\partial \rho v(\rho)}{\partial \rho} \) is a velocity function that depends on density.
2.2. Lax-Wendroff Scheme

Lax-Wendroff named after Peter Lax and Burton Wendroff [13]. Lax-Wendroff is a numerical method for solving hyperbolic partial differential equations in a finite domain. This method is one of the numeric schemes used to approach the solution of the transport equation (7). This method is also a numerical method for hyperbolic differential equation solutions which are more accurate in space and time [14]. The Lax-Wendroff equation developed from Taylor’s second-order expansion equation which has been substituted by equation (6). Then the following is the Lax-Wendroff equation:

$$\rho_{i}^{n+1} = \rho_{i}^{n} - \frac{c \Delta t}{2 \Delta x} (\rho_{i+1}^{n} - \rho_{i-1}^{n}) + \frac{c^{2} \Delta t^{2}}{2 \Delta x^{2}} (\rho_{i+1}^{n} - 2\rho_{i}^{n} + \rho_{i-1}^{n}),$$  \hspace{1cm} (8)

where $\Delta t$ and $\Delta x$ represents space and time. The completion of transport Equation (7) shown in the Algorithm 1. It is described as the algorithm of the lax-Wendroff scheme.

Algorithm 1

1: procedure (Tfinal, $\Delta x$, $\Delta t$)
2: Start
3: Initialize the density $\rho_{i}^{0}$
4: Calculate the velocity $c = \frac{\partial pv(\rho)}{\partial \rho}$
5: Calculate the next time density (8)
6: Back to step 5 until the final time step
7: End
8: end procedure

3. Result and Discussions

3.1. Observation Procedure

The data in this study is obtained from direct observation on the one-way sidewalk of Jalan Braga, Bandung, Indonesia, with length 8 meters and width 2 meters. The data are taken from the observation area shown in Figure 1.

![Figure 1: Observation domain](image)

From the observation data, the first thing to do is calculating the velocity average of the pedestrian in the domain. Furthermore, the second thing to do is to calculate the average pedestrian density in the domain.

3.2. Velocity Function

Observation data applied to find out the velocity function; observation data consist of average velocity and density pedestrian in the domain of observation. Based on the equation (7), velocity function $c = \frac{\partial pv(\rho)}{\partial \rho}$ it can be seen that there is a velocity-density relations obtained using
simple linear regression based on observation data. Two velocity-density relations used for the velocity function in pedestrian flow modelling—the velocity-density data obtained using simple linear regression, as shown in Figure 2.

![Figure 2: Velocity-density data for velocity functions c1 and c2.](image)

Based on Figure 2, velocity-density relations results are used for velocity function $c_1$ and $c_2$. The following equation as follows:

$$c_1 = 0.58206 - 0.94476\rho$$

$$c_2 = 0.59926 - 0.78052\rho$$

### 3.3. Test Case

The purpose of this test is to obtain a degree of convergence from the results of the lax-Wendroff scheme with an exact solution, which also shows the reliability of the Lax-Wendroff scheme. Here, assume the length of domain is $L = 10$ meters with final time $T = 1.5$ seconds and $c = 2$. The initial conditions of this test case as follows:

$$\rho(x, 0) = \begin{cases} 1, & \text{if } 1 \leq x \leq 3 \\ 0, & \text{otherwise.} \end{cases}$$

The density of pedestrians for Test Case using the Lax-Wendroff scheme against the exact solution shown in Figure 3. As can be seen from the figure, the Lax-Wendroff scheme gives approximation nicely to the exact solution. Can be seen clearly that the velocity of pedestrian depends on the pedestrian density on the sidewalk. To assess the reliability of the lax Wendroff scheme, we compare the numerical solution of the lax Wendorff scheme with the exact solution and then produce an error value. Root Mean Square Error (RMSE) is often used to measure the difference between value predicted by a model or an estimator, and its observed value [9]. The error values for Test Case recorded in Table 1. In this table, the errors of the Test Case for Lax-Wendroff scheme using $\Delta x = 0.071$ are lower with the error values is 0.0937335. Therefore the model using $\Delta x = 0.071$ is better than others because the lower the error value, the better the resulting model.

### 3.4. Numerical Modelling

In this Section, a pedestrian flows modelling using the velocity functions $c_1$ (9) and $c_2$ (10) will be explained. In this, the domains $L = 20$ meters of modelling is used. The discrete time is
Figure 3: Lax-Wendroff scheme against exact solution of Test Case at final time $T = 1.5$ seconds using $\Delta x = 0.071$ and $\Delta x = 0.125$.

Table 1: RMSE value for test case

| $\Delta x$ | RMSE          |
|------------|---------------|
| 0.071      | 0.0937335     |
| 0.083      | 0.1008546     |
| 0.1        | 0.1093685     |
| 0.125      | 0.1221952     |

Figure 4: The density of pedestrian flow using velocity functions $c_1$ and $c_2$ at final time $T = 20$ seconds (left) and $T = 30$ seconds (right).

$\Delta t = 0.69$ and space is $\Delta x = 0.34$. Therefore the initial conditions of this model is given by $\rho(x,0) = 0.5$ if $1 \leq x \leq 3$ and otherwise $\rho(x,0) = 0$.

The initial appearance in this modelling shown in Figure 4. The initial conditions of the pedestrian flow modelling show the density of $x = 1$ meter decreases to zero at $x = 3$ meters, this condition illustrated that the sidewalks along interval $1 \leq x \leq 3$ meters filled with several pedestrians. However, the pedestrian reached the maximum at $x = 1$ meter and emptied at $x = 3$ meters.

Based on Figure 4(left) the pedestrian flows modelling with final time $T = 20$ seconds for the velocity functions $c_1$ and $c_2$ can be seen clearly. At $t = 19.6$ seconds, the pedestrian leader using the velocity function $c_1$ successfully reached the position $x = 16.5$ meters and congestion
starts at vulnerable distances around $4 \leq x \leq 16.5$ meters, while the pedestrian leader used the velocity function $c_2$ reached position $x = 17.5$ meters and congestion starts at vulnerable distances around $6 \leq x \leq 17.5$, this showed that $c_2$ is faster than $c_1$.

The pedestrian flows modelling continue until the final time $T = 30$ seconds, which can be seen in Figure 4(right). Results at $t = 29.4$ seconds shows that pedestrian leaders who used velocity function $c_1$ managed to reach the position $x = 22$ meters and congestion starts at vulnerable distances around $6 \leq x \leq 22$ meters, similar with the result at final time $T = 20$ seconds, where the pedestrian leader use the velocity function $c_1$ slower compared to pedestrian leaders who use velocity function $c_2$ that successfully reaches the position $x = 23$ meters and congestion starts at vulnerable distances around $8 \leq x \leq 23$ meters.

4. Conclusion
Pedestrian flows modelling using macroscopic model with Lax-Wendroff scheme have been presented. The Lax-Wendroff scheme gives nicely approximation to the exact solution. Table 1 is constructed to assess the reliability of the Lax-Wendroff scheme. The modelling operated by using two velocity functions, $c_1 = 0.58206 + 0.94476 \rho$ and $c_2 = 0.59926 + 0.78052 \rho$ respectively. The results shows that the pedestrian leader position using the velocity function $c_1$ reached the position at $x = 16.5$ meters while the pedestrian leader use velocity function $c_2$ to reach position $x = 17.5$ meters at $t = 19.6$ seconds, which can be seen in Figure 4(left). Meanwhile, at final time $T = 30$ seconds, each pedestrian leader use the velocity functions $c_1$ and $c_2$ reach position $x = 22$ and $x = 23$ meters, which can be seen in Figure 4(right). Same as before, the difference of the distance is also 1 meter. Pedestrian leader that used the velocity function $c_1$ is a little slower than pedestrian leader that use the velocity function $c_2$.

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