Measurement–based control of a mechanical oscillator at its thermal decoherence rate

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In real-time quantum feedback protocols¹, the record of a continuous measurement is used to stabilize a desired quantum state. Recent years have seen successful applications of these protocols in a variety of well-isolated micro-systems, including microwave photons⁴ and superconducting qubits⁵. However, stabilizing the quantum state of a tangibly massive object, such as a mechanical oscillator, remains very challenging: the main obstacle is environmental decoherence, which places stringent requirements on the timescale in which the state must be measured. Here we describe a position sensor that is capable of resolving the zero-point motion of a solid-state, 4.3-megahertz nanomechanical oscillator in the timescale of its thermal decoherence, a basic requirement for real-time (Markovian) quantum feedback control tasks, such as ground-state preparation. The sensor is based on evanescent optomechanical coupling to a high-Q microcavity⁶, and achieves an imprecision four orders of magnitude below that at the standard quantum limit for a weak continuous position measurement⁶—a 100-fold improvement over previous reports⁷–⁹—while maintaining an imprecision-back-action product that is within a factor of five of the Heisenberg uncertainty limit. As a demonstration of its utility, we use the measurement as an error signal with which to feedback cool the oscillator. Using radiation pressure as an actuator, the oscillator is cold damped¹⁰ with high efficiency: from a cryogenic-bath temperature of 4.4 kelvin to an effective value of 1.1 ± 0.1 millikelvin, corresponding to a mean phonon number of 5.3 ± 0.6 (that is, a ground-state probability of 16 per cent). Our results set a new benchmark for the performance of a linear position sensor, and signal the emergence of mechanical oscillators as practical subjects for measurement-based quantum control.

Feedback control of mechanical oscillation is found in many applications, ranging from stabilizing the approximately-kilogram mass in a gravity-wave interferometer¹¹ to trapping/cooling of atomic¹² and sub-atomic particles¹³. A basic approach to feedback control uses a sensor to track the position of the oscillator and an actuator to convert the measurement record into a continuous and prompt (‘real-time’) feedback force. Recently, the quantum limits of continuous feedback² have been explored, enabling demonstrations such as microwave Fock-state generation and persistent Rabi oscillations in a superconducting qubit. These protocols employ a ‘weak measurement’ capable of tracking a quantum state as rapidly as it decoheres due to measurement back-action.¹⁴ For mechanical oscillators, ideal weak position measurements¹⁵ have been available since the advent of the laser, in the context of shot-noise-limited interferometry¹⁶. Only recently, however, with the confluence of low-loss, cryogenic micromechanics and on-chip, integrated photonics (as explored in contemporary cavity optomechanics¹⁷), has it been feasible to consider their application to quantum feedback protocols¹⁷,¹⁸. The main challenge is thermal noise, which places stringent requirements on the timescale in which the measurement must take place.

Feedback cooling¹⁰,¹¹,¹²,¹⁷,¹⁹,²⁰ is a well-studied control protocol that illustrates both the utility and the challenge of quantum feedback applied to mechanical systems. In feedback cooling protocols, a mechanical oscillator undergoing thermal Brownian motion is steered towards its ground state by minimizing a measurement of its displacement, \( S_x \) (here expressed as a power spectral density) evaluated at the mechanical oscillator frequency, \( \Omega_m \). A conventional strategy is to apply a feedback force proportional to the time derivative of the measurement (which estimates the velocity of the oscillator). The resultant damping acts to reduce (cool) the motion of the oscillator until it coincides with the measurement imprecision, \( S_{x_{imp}} \). Cooling the oscillator until it spends most of the time in the ground state (a mean phonon occupancy of \( n_{th} < 1 \) is possible if \( S_{x_{imp}} \) remains lower than the zero-point fluctuations of the damped oscillator, that is, if \( S_{x_{imp}} \leq S_x^{th} / n_{th} \) (see Supplementary Information), where \( S_x^{th} \) is the spectral density of intrinsic (absent feedback) zero-point fluctuations and \( n_{th} \) is the phonon occupancy of the thermal bath. \( S_x^{th} = 4 x_{zp}^2 / \Gamma_m \) is proportional to the ratio of the ground-state variance \( x_{zp}^2 = h / (2 m \Omega_m) \) and the intrinsic mechanical damping rate \( \Gamma_m \), where \( h \) is the reduced Planck’s constant and \( m \) is the effective mass of the oscillator.) In the frequency domain, this condition on \( S_{x_{imp}} \) amounts to resolving the intrinsic thermal displacement \( S_x^{th} = 2 n_{th} S_x^{tot} \) with a signal-to-noise ratio of \( S_x / S_{x_{imp}} \geq 2 n_{th}^{2} / \Gamma_m^{2} \); in the time domain, it corresponds to resolving the zero-point motion at a characteristic ‘measurement’ rate:¹⁰

\[
\Gamma_{meas} \equiv \frac{x_{zp}^2}{2 S_{x_{imp}}^{tot}} \geq \frac{\Gamma_m}{8} \tag{1}
\]

where \( \Gamma_m \approx \Gamma_{pp} n_{th} \) is the thermal decoherence rate of the oscillator. Equation (1) implies the ability to resolve a displacement of \( x_{zp} \) in the timescale over which a single phonon enters from the thermal bath, \( \Gamma_m^{-1} \). Satisfying this requirement is a technically daunting challenge: because of the small \( x_{zp} \) and large \( \Gamma_m \), of typical engineered mechanical oscillators. As a consequence, despite the success of autonomous feedback¹⁰,¹¹,¹²,¹⁹, ground-state cooling using measurement-based feedback has yet to be demonstrated.

An additional, fundamental caveat compounds the challenge of ground-state cooling and hints at the underlying virtue of quantum feedback: Heisenberg’s uncertainty principle predicts that a weak \( (\Gamma_{meas} \ll \Omega_m) \) continuous position measurement with an imprecision of \( S_x^{th} / 2 \) will produce a stochastic ‘back-action’ force that disturbs the position of the oscillator by at least the same amount. By inference, a thermal-phonon-equivalent imprecision of \( n_{imp} \approx S_{x_{imp}}^{tot} / (2 S_x^{tot}) \) results in an effective increase of the thermal bath occupation by \( n_{th} \approx 1 / (16 n_{imp}) \) (see Supplementary Information). This penalty would appear to prohibit ground-state cooling, as it entails substantially heating the oscillator to achieve the necessary measurement precision. Remarkably, however, feedback counteracts back-action⁴, so that a phonon occupancy of \( n_{th} \approx 2 \sqrt{\Gamma_{meas} (n_{th} + n_{imp})} - 1 / 2 < 1 \) (see Supplementary Information) can still be achieved.²⁰ The limiting case of \( n_{th} \to 0 \) is approached when the measurement record is dominated by back-action-induced fluctuations. This occurs when the measurement is maximally efficient,¹⁴ that is, when the measurement rate, \( \Gamma_{meas} \equiv \Gamma_m / (16 n_{imp}) \), approaches the effective thermal decoherence rate, \( \Gamma_{tot} \approx (n_{th} + n_{imp}) / \Gamma_m \). To meet this condition for a typical

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engineered oscillator, a linear position sensor must achieve an imprecision far (about \( n_{\text{ba}} \) times) below the natural scale set by the ‘standard quantum limit’ (SQL)\(^6\) (\( n_{\text{imp}} = n_{\text{ba}} = 1/4 \)), while maintaining back action near the uncertainty limit: \( 4 \sqrt{n_{\text{imp}}n_{\text{ba}}} \geq 1 \).

Coupling micromechanical oscillators to optical cavities has emerged as a promising way to meet the above requirements. Transduction in such ‘cavity-optomechanical’ systems\(^6\) arises from a parametric coupling, \( G = \tilde{e} \tilde{\Omega}/\tilde{g} \), between the position of the oscillator and the resonance frequency \( \omega_0 \) of a cavity. For broadband sensing, characterized by a cavity decay rate \( \kappa \gg \Omega_m \), a resonant laser field passing through the cavity acquires a phase shift of \( 2G\delta x/\kappa \); this can be resolved in a homodyne interferometer with a quantum-noise-limited imprecision of \( S_{\text{imp}} = (8G^2n_{\text{ba}}\eta/\kappa)^{-1} \), where \( n_{\text{ba}} \) is the mean intracavity photon number and \( \eta \in [0,1] \) is the effective photon collection efficiency (see Supplementary Information). The associated measurement rate is \( I_{\text{meas}} = 4G_0^2n_{\text{ba}}\eta/\kappa \equiv T_mC_0 n_{\text{ba}}\eta \), where \( g_0 = G_0 \) is the vacuum optomechanical coupling rate and \( C_0 \equiv 4G_0^2/\kappa T_m \) is the ‘single-photon cooperativity’\(^5,16\), which characterizes the per-photon measurement rate. To achieve efficient measurements, contemporary cavity-optomechanical systems combine the state-of-the-art in high-\( Q \) nanomechanics and microphotronics\(^{25,26}\). As a consequence, imprecision below that at the SQL\(^7–9\), as well as quantum back-action (that is, radiation pressure shot noise\(^{15}\))\(^{25,26}\) has recently been observed. In none of these experiments, however, was \( I_{\text{meas}} = I_{\text{th}} \) demonstrated at the photodetector, owing to a combination of large thermal occupation, extraneous imprecision, optical loss and dynamic instabilities. (An electromechanical device operating deep in the ‘good-cavity’ limit, \( \kappa \ll \Omega_m \), has recently achieved \( I_{\text{meas}} > I_{\text{th}} \) (ref. 22); however, the use of a far-off-resonant probe resulted in strong dynamic coupling of the optical and mechanical mode.)

Our system addresses these challenges using near-field optomechanical coupling\(^6\), a paradigm for combining mechanical and optical resonators with differing material and geometry. Building on earlier work\(^5,27\), we couple a mechanical oscillator with an exceptionally

![Image](image_url)

**Figure 1** | Measuring and controlling the position of a nanomechanical beam using a near-field optomechanical transducer. a, Whispering gallery modes of a SiO\(_2\) microdisk are excited using a tapered optical fibre driven by a pair of tunable diode lasers (sensor and feedback). Displacement of a Si\(_3\)N\(_4\) nanomechanical beam, which samples the evanescent mode volume of the microdisk, is recorded in the phase of the transmitted sensor field using a balanced homodyne detector (LO, local oscillator field). Radiation-pressure feedback is applied by modulating the amplitude of the feedback laser with an electronically processed (delayed, band-pass-filtered and amplified) copy of the homodyne photocurrent (AM, amplitude modulation). Sample and fibre are both embedded in a \(^3\)He cryostat operating at 4.4 K. b, Above, finite element model of the optical mode (colour scale shows normalized field amplitude), shown in vertical cross-section. Wedged disk and beam (separated by 50 nm) are outlined in white. Optomechanical coupling is proportional to the intensity gradient at the position of the beam. Below, scanning electron micrograph of the device. False-colouring indicates material: blue, SiO\(_2\) (microdisk); red, Si\(_3\)N\(_4\) (nanobeam); grey, Si (wafer substrate). c, Thermal displacement of the fundamental beam mode, expressed as a power spectral density of cavity frequency noise, \( S_{\Omega}^N \) (where \( \Omega \) is the angular Fourier frequency). Various measurement strengths (different \( n_{\text{tot}} \)) are shown. In units scaled to the zero temperature spectral density, \( S_{\Omega}^N \), the peak and background correspond to noise quanta \( n_{\text{tot}} \) and \( n_{\text{imp}} \), respectively (see text). d, Broadband (shot-noise-subtracted) homodyne signal expressed as apparent cavity-frequency noise; the spectral density is divided by \( 2\pi \) because \( \omega \) is an angular frequency. Solid red line corresponds to measurement data; dashed blue, green, and black lines correspond to estimated contributions from nanobeam thermomechanical, microdisk thermomechanical, and microdisk thermorefractive noise, respectively. Coloured bands denote the imprecision required for \( I_{\text{meas}} = I_{\text{th}} \) at different bath temperatures \( T \); as labelled: \( S_{\Omega,\text{imp}}^N = g_0^2kQ_{\text{imp}}/(2k_BT) \), assuming \( g_0^2 \ll 1/\Omega \) and \( Q_{\text{imp}} = 7.6 \times 10^6 \). Noise peaks arising from microdisk and nanobeam thermal motion are labelled by numerical models of the corresponding mode shapes (not to scale).
high Q/(mass) ratio and low optical absorption—a high-stress Si3N4 nanomechanical beam—to an optical cavity with a high Q/(mode volume) ratio and low optical nonlinearity—a SiO2 microdisk—by localizing a portion of the beam within the evanescent volume of one of the whispering gallery modes of the microdisk (Fig. 1b). The system is integrated onto a silicon chip, allowing for robust cryogenic operation. In contrast to earlier work, we employ a fabrication technique (see Supplementary Information) that substantially reduces the distance between the mechanical and optical element, enabling separation of the Si3N4 and SiO2 surfaces by 10–100 nm. In conjunction with cryogenic operation, we realize an enhancement of $\gamma_{\text{imp, shot}}$ by nearly five orders of magnitude compared to the device reported in ref. 27, while reducing the thermal bath occupation by two orders of magnitude.

We study a system consisting of a 65 $\mu$m $\times$ 400 nm $\times$ 70 nm (effective mass $m = 2.9$ pg) nanobeam placed approximately 50 nm from the surface of a microdisk with a diameter of 30 $\mu$m. The microdisk is optically probed using a low-loss (approximately 6%) fibre taper that is pumped by a tunable diode laser. Displacement of the nanobeam is observed in the phase of the transmitted cavity field using a vacuum optomechanical coupling rate between the oscillator and the sensor mode is $\gamma_{\text{imp}} = \gamma_{\text{imp, shot}} + \gamma_{\text{imp, ex}}$, as shown in the main text. The performance of the sensor is limited by optical loss, photothermal and radiation-pressure instabilities, and extraneous sources of measurement back-action. We investigate these constraints by recording the measurement imprecision, $\eta_{\text{imp}}$, and the effective bath occupation, $n_{\text{tot}} = n_{\text{th}} + n_{\text{ba}}$, as a function of intracavity photon number, and comparing their product to the uncertainty-limited value, $\sqrt{\Delta n_{\text{tot}}} \approx 1$ (Fig. 2). Two considerations are crucial. First, to

\begin{equation}
\Gamma_{\text{meas}} = \frac{\gamma_{\text{imp, shot}}^2}{S_{\text{imp, shot}}} + \gamma_{\text{imp, ex}} = \frac{\gamma_{\text{tot}}/16}{n_{\text{tot}} + \eta_{\text{imp}}^2}
\end{equation}

where $S_{\text{imp, shot}}$ is the photocount shot noise referred to an apparent cavity resonance-frequency noise, $\eta_{\text{imp}} = \gamma_{\text{imp, shot}}/2S_{\text{imp, shot}}$ and $n_{\text{imp}}^2 = \gamma_{\text{imp, ex}}/2S_{\text{imp, ex}}$. Figure 1d shows the extraneous-noise floor of our sensor over a broad range of frequencies. We obtained this spectrum by subtracting shot noise from a measurement made with a large intracavity photon number, $n > 10^5$. (To mitigate dynamic instabilities, the measurement was conducted using approximately 10 mbar of gas pressure at an elevated temperature of 15.7 K.) High- and low-Q noise peaks correspond to thermal motion of the nanobeam and the microdisk, respectively. Near the fundamental noise peak, we observe an extraneous frequency noise of $S_{\text{imp, ex}} = (2\pi \times 30 \text{ Hz}/\sqrt{\text{Hz}})^2$, corresponding to a displacement imprecision of $S_{\text{imp, ex}} = (4.3 \times 10^{-17} \text{ m}/\text{Hz})^2$. We identify this imprecision as a combination of thermorefractive noise, diode laser frequency noise and off-resonant thermal motion of the in-plane nanobeam mode at 4.6 MHz. Owing to the large zero-point motion of the oscillator, $S_{\text{imp}} = 4\gamma_{\text{imp}}/\Gamma_{\text{tot}} = (2\pi \times 6.7 \text{ kHz}/\sqrt{\text{Hz}})$ (corresponding to $S_{\text{imp}}^2 = (9.5 \times 10^{-14} \text{ m}/\text{Hz})^2$), the thermal-phonon-equivalent magnitude of $S_{\text{imp, ex}}$ has an exceptionally low value of $n_{\text{imp}}^2 = 1.0 \times 10^{-5}$, and 44 dB below that at the SQL. The associated measurement rate, $\Gamma_{\text{tot}}/(16n_{\text{imp}})$, is equal to the thermal decoherence rate at an experimentally accessible temperature of 1.3 K, which implies that ground-state cooling ($\Gamma_{\text{meas}} > \Gamma_{\text{tot}}/8$) should be achievable at 10 K.

The performance of the sensor is limited by optical loss, photothermal and radiation-pressure instabilities, and extraneous sources of measurement back-action. We investigate these constraints by recording the measurement imprecision, $\eta_{\text{imp}}$, and the effective bath occupation, $n_{\text{tot}} = n_{\text{th}} + n_{\text{ba}}$, as a function of intracavity photon number, and comparing their product to the uncertainty-limited value, $\sqrt{\Delta n_{\text{tot}}} \approx 1$ (Fig. 2). Two considerations are crucial. First, to...
efficiently collect photons from the cavity, it is necessary to increase the taper-coupling efficiency to $k_{\text{ex}} \gtrsim k_0$, thereby increasing the cavity decay rate to $k = k_0 + k_{\text{ex}}$. We operate at a near-critically coupled ($k_{\text{ex}} = k_0$) value of $k \approx 2\times 0.91$ GHz, which reduces the single photon cooperativity to $C_0 \approx 0.31$ in exchange for an output coupling efficiency of $n_c = (k - k_0)/k \approx 0.52$. Second, to minimize $F_{\text{imp}}^\text{tot}$, it is necessary to maximize $n_c$, while avoiding dynamic instabilities. We accomplish this by damping the oscillator using radiation pressure feedback. Feedback is performed by modulating the intensity of the feedback laser with an electronically amplified and delayed copy of the homodyne photocurrent. A feedback phase of $-3\pi/2$ is chosen by tuning the total delay, $\tau \approx 172$ ns, such that the feedback-induced spring effect is minimized (see Supplementary Information). The resulting viscous radiation pressure gives rise to a well-known cooling effect (‘cold-damping’10–17), which reduces the phonon occupancy of the mechanical mode to a mean value of $n_m = n_{\text{tot}}/F_m/(F_m + F_0)$, where $F_0$ is the optically induced damping rate.

Measurements of the thermal motion of the oscillator are shown in Fig. 1c. We determine $n_{\text{tot}}$ and $n_{\text{imp}}$ by fitting each noise peak to a Lorentzian with a linewidth of $I_m + I_0 + I_0$ (which includes a minor contribution from dynamic back action, $I_0$; see Supplementary Information), a peak amplitude of $S_n(\Omega_m) = 2n_{\text{tot}}(I_m + I_0)F_m + I_0$, and an offset of $F_{\text{imp}}^\text{tot} = 2n_{\text{imp}}F_m$. For low input powers, $n_{\text{imp}} \ll n_0/C_0$, we observe that the effective bath occupation is dominated by thermalization to the cryostat, $n_{\text{tot}} = n_{\text{th}}$, and that imprecision scales as $n_{\text{imp}} = (16\xi C_0 n_{\text{imp}}^\text{tot})^{-1}$, where $\xi \approx 0.23$. $\xi$ represents the ideality of the measurement, and includes optical losses as well as reduction of optomechanical transduction due to cavity mode splitting (see Supplementary Information). When operating with the higher input power—ultimately limited by dynamic instability of higher-order beam modes—the lowest imprecision we observe is $n_{\text{imp}} = 2.7(0.2) \times 10^{-5}$, which corresponds to an imprecision of $39.7 \pm 0.3\%$ below that at the SQL (see Supplementary Information for the budget of uncertainties). The associated measurement rate, $I_m = 2\pi \times (13 \pm 1)\text{kHz}$, is a factor of 9.2 lower than the rate of thermal decoherence to the ambient 4.4 K bath, $I_0 = 2\pi \times 120\text{kHz}$. Notably, this value is within 15% of the requirement for ground-state cooling.

For large measurement strengths, quantum back-action26 should in principle exceed the ambient thermal force, scaling as $n_{\text{ba}} = C_0 n_c$ (see Supplementary Information). As shown in Fig. 2 (red data), our system deviates from this ideal behaviour, owing to excess back-action. The back-action manifests as an extraneous cooperativity, $C_0^n = 0.56$, and limits the fractional contribution of quantum back-action to $C_0/(C_0 + C_0^n) \approx 35\%$. Similar behaviour for higher-order mechanical modes suggests that photothermal heating is the cause of this excess heating, as does our observation that $C_0^n$ is markedly higher at lower cryostat temperatures, which is consistent with the reduction of thermal conductivity in amorphous glasses below 10 K (see Supplementary Information). Including extraneous back-action, we model the effective imprecision–back-action product (green curve in Fig. 2) as

$$4\sqrt{n_{\text{imp}}^\text{tot}} = \sqrt{\frac{1}{\xi} \left(1 + \frac{n_c}{n_{\text{imp}}^\text{tot}}\right) \left(1 + \frac{n_0}{C_0 n_c + C_0^c} \right)}$$

where $n_{\text{imp}}^\text{tot} = (16\xi C_0 n_{\text{imp}}^\text{tot})^{-1}$ is the photon number for which $n_{\text{imp}} = n_{\text{imp}}^\text{tot}$. Using $n_c = 5 \times 10^4 \ll n_{\text{imp}}^\text{tot}$, we observe a minimum value of $4\sqrt{n_{\text{imp}}^\text{tot}} \approx 5.0$, which corresponds to a measurement efficiency of $I_m/I_m^\text{tot} \approx 0.040$.

To illustrate the utility of this high efficiency, we increase the strength of the feedback used to cold-damp the oscillator in Fig. 1c. The limits of cold-damping are well-studied12–17,20. Ignoring back-action from the weakly driven ($n_c < 100$) feedback optical mode, the phonon occupancy of the cooled mechanical mode depends on the balance between its coupling to thermal, measurement and feedback reservoirs at rates $I_0$, $I_m^\text{tot} n_{\text{imp}}$ and $g_{\text{th}} I_m^\text{tot} n_{\text{imp}}$, respectively, where $g_{\text{th}} = I_0/I_m$ is the open loop feedback gain (see Supplementary Information):

$$n_m + \frac{1}{2} = \frac{1}{1 + g_{\text{th}}} n_{\text{tot}} + \frac{g_{\text{th}}^2}{1 + g_{\text{th}}} n_{\text{imp}} \geq 2\sqrt{n_{\text{imp}}^\text{tot}}$$

(here we assume $I_0 \ll I_m$ and $n_{\text{tot}} \gg 1$). The photon occupancy $n_m$ is minimized for an optimal gain of $g_{\text{th}} = \sqrt{n_{\text{imp}}^\text{tot}}$; which corresponds to suppressing $S_k$ to $S_k^\text{tot}$ (black curve in the inset of Fig. 3).

![Figure 3](https://example.com/image3.png)

**Figure 3** Radiation-pressure feedback cooling to near the ground state. Red and blue points correspond to measurements of the phonon occupancy of the mechanical mode, $n_m$ (plus a phonon-equivalent zero-point energy of 1/2), and its component due to feedback of measurement noise, $n_{m,\text{fb}} = n_{\text{imp}}^\text{tot} F_m/(1 + g_{\text{th}})$, respectively, versus measured damping rate, $I_m = (1 + g_{\text{th}}) I_m^\text{tot}$. Red, blue and black dashed lines correspond to models of the components in equation (3): $n_m/(1 + g_{\text{th}})$, $n_{m,\text{fb}}$ and $n_m + 1/2$, respectively, using experimental parameters $I_m^\text{tot} = 2\pi \times 5.7\text{kHz}$, $n_{\text{tot}} = 2.4 \times 10^5$ and $n_{\text{imp}} = 2.9 \times 10^{-4}$. Inset, thermomechanical noise spectra for various feedback gain settings (represented by the differently coloured curves); Lorentzian fits to these spectra were used to infer the red and blue points. Error bars are due to systematic uncertainty in $n_{\text{tot}}$ (see Supplementary Information).
Notably, for $C_0^2 = 0$, $n_{\text{imp}} < 1$ requires $n_{\text{imp}} < 1/(2n_0)$. The results presented in Fig. 2 suggest that $n_0 \approx 2$ should be achievable.

Figure 3 shows the results of feedback cooling using a measurement with an imprecision far below that at the SQL. We emphasize that for this demonstration, imprecision was deliberately limited to $n_{\text{imp}} \approx 2.9 \times 10^{-4}$ to reduce uncertainties due to extraneous heating and the off-resonant tail of the thermal noise peak at 4.6 MHz. (The latter restriction limits the applicability of equation (3) to effective damping rates of $\Gamma_{\text{eff}} = (1 + 3n_0) \Gamma_m \leq 2\pi \times 200$ kHz.) Feedback gain was controlled by changing the magnitude of the electronic gain, leaving all other parameters (for example, laser power) fixed. The Markovianity condition $\tau \ll 2\pi/\Omega_0$ is satisfied by the feedback delay42. By fitting the closed-loop noise spectrum (Fig. 3, inset) to a standard Lorentzian noise-squashing model24 (see Supplementary Information), we estimate the phonon occupancy of the mechanical mode using $n_m + 0.5 \approx \Gamma_{\text{eff}} (S_m(\Omega_m) + S_{\text{imp}}^p) / (2S^p_{\text{imp}})$, where $S_{\text{imp}}^p$ denotes the off-resonant background. After accounting for extraneous back-action, we infer a minimum occupation of $n_m \approx 5.3 \pm 0.6$ (see Supplementary Information for budget of uncertainties) at an optimal damping rate of $\Gamma_{\text{eff}} \approx 2\pi \times 52$ kHz, which corresponds to a ground-state probability of $1/(1 + n_m) \approx 16\%$. This value agrees well with the prediction based on equation (3) and the data shown in Fig. 2. Notably, for larger feedback strengths, shot-noise ‘squashing’22 leads to an apparent reduction of $S_m$ even though $n_m$ physically increases. This discrepancy can be resolved with an out-of-loop measurement43, at the cost of reduced measurement efficiency.

Collectively, our results establish new benchmarks for the linear measurement and control of a mechanical oscillator. Using an optomechanical sensor with a readout imprecision that is nearly 40 dB below that at the SQL, we have shown that traditional radiation pressure cold-damping36 can be used to cool a nanomechanical oscillator to a mean phonon occupancy of approximately 5.3; this represents a 40-fold improvement over previous reports11,22,28, and invites comparison20,30 to the recent success of autonomous feedback (side-band) cooling21–22. Looking forward, high-efficiency optomechanical sensors may enable a variety of feedback applications such as back-action evasion40 and mechanical squeezing48.

Received 17 October 2014; accepted 3 June 2015.

Published online 10 August 2015.

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