Research Article

A Simplified Capillary Bundle Model for CO₂-Alternating-Water Injection Using an Equivalent Resistance Method

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Received 26 June 2020; Revised 26 September 2020; Accepted 29 October 2020; Published 29 November 2020

Academic Editor: Micòl Mastrocicco

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CO₂-alternating-water injection is an effective way of enhancing recovery for low-permeability oil reservoirs. The injection process is one of the essential issues that are facing severe challenges because of the low permeability and poor pore space connectivity. Previous researchers mentioned that water injection ability could be decreased by around 20% after the CO₂-flooding; hence, it is necessary to quantify the water injectivity variation during an alternated injection process. In this paper, a CO₂ convection-diffusion model is established based on the seepage law of CO₂. The relationship between the width of miscible flooding and injection time is defined. Besides, an equivalent resistance method is introduced for developing a capillary bundle model for featuring an unequal diameter for CO₂ water vapor alternate flooding. CO₂-oil and CO₂-water interactions are analyzed using the new model. The effects of oil viscosity, pore throat ratio, CO₂ slug size, and equivalent permeability of the capillary bundle on water injection are analyzed. The result indicates that water injection ability increases with the rise of CO₂ slug size and equivalent permeability of the capillary bundle and decreases with the increase of viscosity and pore throat ratio.

1. Introduction

The CO₂-alternating-water injection method, used in the low-permeability reservoirs, has the advantages of both CO₂ flooding and water flooding. It represents a promising method for improving the recovery of oil in the future. In 1957, Mobil Company has performed a pilot CO₂-alternating-water injection test in North Pembina of Alberta, and then, several tests were conducted in Canada and the United States. In the United States, many tests have been performed because of the sufficient CO₂ sources [1, 2], whereas in China, the first pilot field application was started by CNOOC in Qikou oilfield. The test showed that during the middle-late stage of the nonhomogeneous reservoir, the CO₂-alternating-water injection could not only block the advantage flow channel but also contribute to the optimization of water injection [2]. Although its significant benefits, this method still has several problems. One serious problem is the decreasing of injection ability after CO₂-alternating-water has been injected into the formation [3, 4]. Therefore, it is significantly important to study the variation of injection ability with capillary bundle model during CO₂-alternating-water flooding process. The CO₂-alternating-water injection in the low-permeability reservoirs is a multiphase flow process. Early researchers have studied the seepage and flooding mechanism of multiphase fluid in the porous medium. Dong et al. presented the results of immiscible WAG injection in a water-wet micromodel and found that in a gas/oil/water system, stable oil layers were formed between the gas and water phases [5]. Due to gas bubbles were always surrounded by oil layers, direct
gas/water displacements were not observed. Meanwhile, he found that the oil recovery declined greatly with WAG cycles. Wang and Dong developed an interacting triangular tube bundle model based on capillaries of equilateral triangle cross sections [6]. The relationship between the residual oil saturation and the complete capillary number (CA) was investigated, and the effects of the tube size, tube size distribution, and viscosity ratio on the magnitude of entrapment were also studied using this tube bundle model. Piri et al. presented both the stagnant film model and the moving film model to study the energy loss problem when the fluid flows in the capillary [7]. They considered the interfacial pressure drop and the frictional pressure drop with the tube wall, and the influence of the static and fluid boundary layer on the fluid seepage. As microscopic flow mechanics gradually became the focus of investigation, some researchers began to pay attention to the complex seepage mechanism of pore space in porous media. Some of those researchers have been focusing on the alternate injection of water and gas by pore structure by establishing a pore network model as a simulation platform. Piri developed a quasi-static pore network model by fully considering the seepage characteristics of oil, gas, and water three-phase flow in the pore throat [8]. The variation of saturation and relative permeability of each phase in the process of water seepage are compared with the existing experiment, and the data is well fitted. Yang et al. simulated the WAG displacement process in different wettable porous media from a microscopic point of view and analyzed the oil, gas, and water saturation and distribution during WAG displacement [9]. The changed law of position and displacement characteristics explains the microscopic mechanism of three-phase seepage in porous media. Helland pointed out that a bundle-of-triangular-tube model can reproduce the main characteristics of mixed-wet capillary pressure curves with hysteretic scanning loops [10].

The displacement core test could be used to study the effect on the injection of CO₂-alternating-water process. Jinan et al. designed an experimental unit for long core displacement by considering the necessity of similarities upon core length, the complex mechanism, and periodic physical-chemical reaction during multicontact miscible phases [11]. Zhao et al. analyzed the affecting factors of CO₂ flooding, which include permeability, the relative density of crude oil, layer heterogeneity, interlayer heterogeneity, and layer fluid channeling by establishing the component numerical simulation model with Eclipse software [12, 13]. They concluded that it is appropriate to carry out CO₂ flooding for homogeneous and lightweight reservoirs in low-permeability oil fields. Zahoor et al. mentioned that proper water-alternating-gas process design and implementation requires better knowledge of wettability and wettability variations in particular [14]. Wettability, in addition to influencing flow parameters, strongly affects other design parameters such as the volumes of water and gas required for injection, well spacing, etc. Some other researchers also used experimental methods to study the increasing difficulty in injection. Prieditis et al. and Kamath et al. conducted displacement tests on carbonate and limestone cores and analyzed the factors that cause a decrease of injectivity [15, 16]. Prieditis et al. also performed a field test based on laboratory work and used a simple model to predict the injectivity, but good results were not achieved [15]. Roper et al. used a compositional model to simulate and interpret an injectivity test conducted in the Mabee Field in the San Andres Formation, Martin County, Texas [17, 18]. They mentioned that the validation of compositional simulation as a means for interpreting field tests and developing improved predictions of reservoir injectivity performance and geostatistical techniques could be used successfully to characterize high heterogeneity in carbonate reservoirs for injectivity calculations. Yang et al. used core samples collected from tight formations to conduct a series of water-alternating-CO₂ flooding experiments with different water-alternating-CO₂ ratios and slug sizes [19]. They found that fluid injectivity is strongly dependent on slug size, water-alternating-CO₂ ratio, and cycle time.

Besides the experimental work, early researchers have done some theoretical modeling work. Li et al. derived the injectivity equation for water-alternating-gas injection based on similarity theory [20]. Zhou et al. analyzed the effects of injection pressure, injection speed, slug size, and gas-water slug ratio on oil displacement efficiency [21]. Yan et al. used a numerical simulation method to study the best injection parameters for CO₂ gas water alternative drive [22]. Hu substituted the permeability of the capillary model into low-permeability equation and used superposition method to get total flow rate and threshold pressure gradient of the rock [23]. This model gives a theoretical description of low-permeability seepage characteristics. Several researchers have studied the injection capacity of different reservoirs and found that the injection volume, injection velocity, formation temperature, formation pressure segment size, and proportion have an impact on the final oil displacement efficiency and recovery factor [24–29]. Pizarro and Lake considered the influence of heterogeneity and autocorrelation of reservoir permeability distribution on the injectivity [30]. This model gives insight into why injectivity calculated from a core permeability average is frequently different from the injectivity manifested by the well in question. Yang et al. performed numerical simulations to history-match the experimental measurements and conduct sensitivity analysis on operational parameters and achieved good matching [19].

Overall, researchers have done relatively thorough research on the decreasing injectivity happens in low-permeability reservoir during CO₂ injection. However, due to the complexity of flow behavior of water-oil-gas, just a few researchers have explained the abnormal variance of injectivity during water-alternating-CO₂ injection. In this paper, a new seepage diffusion equation is established by considering the CO₂ diffusion and seepage in the reservoir. It could effectively reflect the variance of CO₂ injectivity in water-alternating-CO₂ injection process. The distribution of oil, CO₂, and water in the capillary model with different diameters has been analyzed, and the influence of geology and fluid properties on injectivity has been provided.

2. CO₂ Flooding Dynamics considering Mass Transfer

2.1. Mathematical Modeling. Assuming that the CO₂ flooding process in the capillary model is a piston, the molecular
diffusion, convective diffusion, and viscosity difference diffusion due to the difference of viscosity between CO₂ and crude oil will occur at the interface between CO₂ and crude oil. From a thermodynamic point of view, diffusion coefficients are functions of pressure, temperature, and composition, and driven by chemical potential difference; however, the proposed model is complex and hard to use [31]. Therefore, in this paper, the classical Fick’s law is utilized. According to the classical Fick’s law, molecular diffusion is mainly caused by the change of CO₂ concentration in crude oil, and the diffusion coefficients are assumed to be constant. Convective diffusion is mainly affected by the complexity of the internal channels of the pores. The diffusion under the viscosity gradient is related to the difference of viscosity between CO₂ and crude oil, and it is also affected by molecular diffusion and convection-diffusion [32]. The diffusion equation is derived from mass transfer equations by considering the balance of matter into and out of the capillary unit. To derive the diffusion equation of CO₂ in the capillary, here, a πR²dx flowing unit of the capillary tube is studied (Figure 1).

According to the principle of material balance, the influent mass = effluent mass + amount of substance accumulated in the unit, which is as follows:

\[
\frac{\partial C}{\partial t} = -\frac{\partial \nu_{D}}{\partial x} - \omega \frac{\partial C}{\partial x}, \tag{1}
\]

According to the diffusion law,

\[
\nu_{D} = -D \frac{\partial D}{\partial x}. \tag{2}
\]

Substituting Equation (2) into Equation (1), the material diffusion equation of the flow unit in the capillary is obtained.

\[
\frac{\partial C}{\partial t} + \omega \frac{\partial C}{\partial x} = \frac{\partial}{\partial x} \left( D \frac{\partial C}{\partial x} \right), \tag{3}
\]

where \(C\) is a unit concentration of CO₂ after a mixed phase of crude oil and CO₂, \(\%\); \(D\) is a comprehensive diffusion coefficient, \(\text{cm}^2/\text{s}\); \(\omega\) is the flow velocity of the fluid in the capillary, \(\text{cm/s}\). Here, the comprehensive diffusion coefficient \(D\) can be understood as a composite coefficient which not only considers the molecular diffusion and convection-diffusion of the single-phase fluid in the capillary but also considers the viscosity difference between the displacement and the displaced solution.

For the method of determining the diffusion coefficient, based on the Stokes-Einstein relation, Kooijman developed a correlation and used UNIFAC parameters to correct for particle roundness and size. Hayduk and Minhas developed a set of correlations, specific to certain types of mixtures. Sigmund developed a correlation for high-pressure gas and liquid binary mixtures with the expression that related the reduced density-diffusion coefficient product to a third-degree polynomial function of reduced density. For nonideal binary mixtures, Riaz and Whiston developed a correlation to predict gas and liquid diffusion coefficient. Currently, the most widely used correlation to calculate diffusion coefficient

is presented by Wilke and Chang and is the function of viscosity, which is used in this paper [33].

The results of a displacement experiment in which a liquid is miscible with another liquid in the presence of differential pressure indicate that the best approximation of the overall diffusion coefficient is as follows:

\[
D = D_{E} \left( 1 + K_{\mu} \nabla \mu_{m} \right),
\]

\[
D_{E} = D_{m} + D_{\mu}, \tag{4}
\]

\[
D_{\mu} = K_{\omega} \omega, \tag{5}
\]

where \(D_{m}\) is the molecular diffusion coefficient, \(\text{cm}^2/\text{s}\); \(D_{\mu}\) is the convection-diffusion coefficient of single-phase fluid, \(\text{cm}^2/\text{s}\); \(\mu_{m}\) is the viscosity of two liquid mixtures, \(\text{mPa-s}\); \(K_{\omega}\) and \(K_{\mu}\) are the experimental coefficients which consider the experimental coefficients of single-phase fluid convection and different viscosity diffusion, with the unit of cm and \(\text{cm}/(\text{MPa-s})\).

From Equation (4),

\[
D = D_{m} + D_{\mu} + (D_{m} + D_{\mu}) K_{\mu} \nabla \mu_{m}. \tag{6}
\]

Meanwhile,

\[
D_{\mu} = (D_{m} + D_{\mu}) K_{\mu} \nabla \mu_{m}. \tag{7}
\]

\(D_{m}\) is the diffusion coefficient of viscosity difference between displacement fluid and displaced fluid. Equation (6) shows that the comprehensive diffusion coefficient is the sum of the molecular diffusion coefficient, the convection-diffusion coefficient, and the diffusion coefficient of the viscosity difference between the displacement fluid and the displaced fluid, which is a mixing coefficient.

2.2. Solution Strategy. In miscible region, we assume that there is only a single phase, which is the mixture of oil and gas phase. We introduced a new variable, \(\tau\), which is the mixture of oil and gas phase. We introduced a new variable, \(\xi = x - \omega \phi\), into Equation (3). Then, Equation (3) becomes the standard diffusion equation.

\[
\frac{\partial C}{\partial \tau} = \frac{\partial}{\partial \xi} \left( D \frac{\partial C}{\partial \xi} \right), \tag{8}
\]

where \(\xi\) is called the self-modulation variable which changes with time and space.
The boundary conditions are as follows:

\[ C(-\lambda, t) = 1, \]  
\[ C(\lambda, t) = 0, \]  
\[ C(0, t) = 0.5, \]  
where \( \pm \lambda \) is the miscible half-width, cm.

Also, it is assumed that there is no CO\(_2\) inflow or outflow at the boundary of the miscible zone.

\[ \frac{\partial C}{\partial x} \bigg|_{x=\pm \lambda} = 0. \]  

The viscosity of the miscible zone \( \mu_m \) is also given in Figure 2 and is considered to be linear, so the viscosity of the miscible zone is as follows:

\[ \nabla \mu_m = \frac{\mu_2 - \mu_1}{2\lambda}. \]  

Assuming \( \beta = K_p(\mu_2 - \mu_1)/2 \), then \( K_p \nabla \mu_m = \beta/\lambda \). Equation (7) becomes the following:

\[ \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left[ D_x \left( 1 + \frac{\beta}{\lambda} \right) \frac{\partial C}{\partial x} \right]. \]  

After solving Equation (13), the differential form solution can be obtained as follows:

\[ \frac{d\lambda}{dt} = 4D_x \frac{1}{\Lambda} \left( 1 + \frac{\beta}{\lambda} \right). \]

Thus, the corresponding "integral form solution" is given as follows:

\[ \frac{\lambda^2}{2} - \beta \lambda + \beta^2 \ln \left( \frac{\lambda + \beta}{\beta} \right) = 4D_x t. \]  

If the full length of the mixed-phase zone is \( \Lambda = 2\lambda \), Equation (15) can be transformed as follows:

\[ \frac{\Lambda^2}{4} - \beta \Lambda + 2\beta^2 \ln \left( \frac{\Lambda + 2\beta}{2\beta} \right) = 8D_x t. \]  

Equations (15) and (16) can be used to calculate the half-width \( \lambda \) or width \( \Lambda \) of the miscible region at any given time \( t \), and \( \lambda = \lambda(t) \) and \( \Lambda = \Lambda(t) \) can be obtained by the iterative method that can be used to solve.

3. Unequal Diameter Capillary Beam Model for CO\(_2\)-Alternating-Gas Flooding

3.1. Physical Model. It is assumed that the capillary beam model consists of \( n \) capillaries with different capillary radii. Since in real oil reservoir, for any one flowing path, there are pores and throats; therefore, to reflect this pattern, for each capillary, the radius is not equal everywhere along the axis, and there is a throat. For the \( i \)th capillary, the pore throat structure is as follows in Figure 3.

The capillary length is \( L \), the throat position is \( L_f \), and the capillary radius is \( R_p \), and the radius at the throat is as follows:

\[ R(x) = \left( \frac{R_p + R_i}{2} \right) + \left( \frac{R_p - R_i}{2} \right) \cos \left( \frac{2\pi x}{L_f} \right). \]

where \( R_i \) is the throat radius, \( \mu m \); \( R(x) \) is the radius of the junction of the pore and the throat, \( \mu m \); \( x \) is between \( 0 \sim L_f \), cm; the total volume of the throat is as follows:

\[ V = \pi \int_{0}^{L_f} R^2(x)dx. \]

According to the definition of alternate driving of water and gas, the injection process can be divided into two stages, namely, CO\(_2\) injection and water injection. Because the throat is arbitrarily distributed for the same capillary, the throat position is various for different capillaries, and the fluid distribution in the capillary is complicated because of the difference in CO\(_2\) injection time.

In the process of CO\(_2\) injection, six distribution patterns exist in the crude oil, mixed-phase zone, and CO\(_2\) in the capillary tube, and in the process of water flooding process, there are three fluid distribution patterns. Schematics and descriptions for those fluid distribution patterns are shown in Table 1.

3.2. CO\(_2\) Flooding Seepage Mode and Mathematical Description. To facilitate the solution and analysis of the mathematical model, CO\(_2\)/water is injected by constant pressure. The injection pressure of each capillary is equal, and the pressure difference between the injection end and the production end is \( \Delta P \). Introducing the seepage resistance coefficient to describe the magnitude of the seepage resistance and using the equivalent resistance model to solve the capillary beam injection flow rate are as follows [34]:

3.2.1. Fluid Distribution Pattern I. For the \( i \)th capillary, at the beginning of time \( j (j = 1, 2, 3, \ldots, N_{ij}) \), when \( x_{gi}^{j-1} + x_{gi}^{j-1} < L_f \), the seepage resistance in the CO\(_2\) zone is as follows:

\[ W_{gi} = \frac{8\mu_g x_{gi}^{j-1}}{\pi R_i^{4}}, \]

where \( x_{gi}^{j-1} = 0 \) is the length of CO\(_2\) region at the \( j \) moment for the \( i \)th capillary, cm; for the beginning, \( x_{gi}^{0} = 0 \). \( \mu_g \) is the CO\(_2\) viscosity, mPa·s; \( R_p \) is the capillary radius, \( \mu m \).
The seepage resistance of the miscible zone is as follows:

\[ W_{ji} = \frac{8\mu_0x_{ji}^{-1}}{\pi R_{pi}^4} + \frac{1}{\mu_0} \int_0^{x_{ji}} \frac{dx}{G(x)}, \quad (20) \]

where \( x_{ji}^{-1} \) is the miscible length at the moment of \( j-1 \), cm; for the beginning, \( x_{j0} = 0 \); \( \mu_0 \) is the miscible fluid viscosity, mPa.s.

The seepage resistance in the crude oil zone is as follows:

\[ W_{oi} = \frac{8\mu_0(L - x_{oi}^{-1} - x_{oi}^{-1} - L_{oi})}{\pi R_{pi}^4} + \frac{1}{\mu_0} \int_0^{x_{oi}} \frac{dx}{G(x)}, \quad (21) \]

where \( G \) is the conductivity of a fluid per unit length, \( \mu m^4 \).

\[ G(x) = \frac{\pi}{128} \left( \frac{\sqrt{A_i}}{\pi} + R(x) \right)^4 = \frac{\pi}{8} R^4(x). \quad (22) \]

In the case of a constant pressure difference, the injection flow rate is as follows:

\[ q_{ji} = \frac{\Delta P + P_{co-g}}{W_{eg}^j}, \quad (23) \]

where \( W_{eg}^j = W_{gi}^j + W_{li}^j + W_{oi}^j \), because there is no interfacial tension between CO\(_2\) and crude oil, \( P_{co-g} = 0 \).

During the \( \Delta t \) time, it is assumed that the fluid seepage velocity in the capillary remains constant.

\[ W_i^j = \frac{q_{ji}^j}{\pi R_{pi}^4}. \quad (24) \]

The convection-diffusion coefficient between CO\(_2\) and crude oil is as follows:

\[ D_{fi}^j = D_m + K_w W_{gi}^j. \quad (25) \]

According to the derivation in Section 2.2, the relationship for the length of miscible band at time \( j \) is obtained, and the relevant parameters are substituted. Using the Newton iteration method, the length of miscible band at the end of time \( j \) can be obtained. Since in Equation (1), \( C \) is defined as the unit concentration of CO\(_2\) in the mixture phase; it also can be seen as the volume ratio of CO\(_2\) in the unit bulk volume approximately, as shown in Figure 2. Therefore, assuming the volume of dissolved CO\(_2\) in the miscible zone occupies 1/2 of its total volume, the amount of CO\(_2\) dissolved in the mixed phase is as follows:

\[ V_{mi}^j = \frac{1}{2} \pi R_{pi}^2 x_{li}^j. \quad (26) \]

The length of the CO\(_2\) zone formed at the beginning of the time \( j+1 \) is as follows:

\[ x_{gi}^j = \frac{V_{mi}^j}{\pi R_{pi}^2} = x_{ji}^{j-1} + \frac{q_{gi}^j \Delta t - V_{mi}^j}{\pi R_{pi}^2}. \quad (27) \]
The total amount of CO₂ injected into the bundle of tubes is as follows:

\[ Q_j = \sum_{i=1}^{n} q_{ji}. \]  \( \text{(28)} \)

If the calculated sum of the CO₂ zone \( x_{gi} \) and the miscible zone \( x_{ki} \) is less than \( L_{ci} \), iterative calculation of \( j + 1 \) time can be performed according to the above steps.

### 3.2.2. Fluid Distribution Pattern II

At the beginning of \( j_n \) moment, when the sum of the obtained CO₂ zone and the miscible zone length is greater than \( L_{ci} \), but the volume of the miscible belt is smaller than the volume of the throat, that is \( x_{gi}^{j_n-1} + x_{ki}^{j_n-1} \geq L_{ci} \), \( x_{gi}^{j_n-1} \leq L_{ci} - 2V_{mis} < V_{ti} \). At the end of the \( j_n \) moment, the length of the miscible belt in the throat \( L_{ki}^k \) \( (k = 1, k = k + 1) \) is as follows:

\[
\left( x_{gi}^{j_n-1} + x_{ki}^{j_n-1} - L_{ci} \right) R_{pi}^2 = \int_{L_{ki}^k}^{t_{ki}^k} R^2(x) dx. \]  \( \text{(29)} \)

The Lobatto numerical integral formula can be used to solve \( t_{ki}^k \); \( x_{ki}^{j_n-1} \) is the equivalent length of the miscible band, cm; the actual width of the miscible belt is \( x_{ki}^{j_n-1} \). Meanwhile, the actual length of the crude oil zone is \( x_{oi}^{j_n-1} = L - L_{ci} - L_{ki}^k \).

A table for fluid distribution pattern during CO₂ and water flooding.

### Table 1: Schematics for fluid distribution pattern during CO₂ and water flooding.

| Process Number | Fluid distribution pattern | Description |
|----------------|---------------------------|-------------|
| I              |                           | The front edge of the mixed-phase zone does not enter into the throat; the fluid in the throat is crude oil. |
| II             |                           | The front of the CO₂ flooding is not entering the throat, but the right end of the miscible belt has entered the throat and occupied a part of the throat. |
| III            |                           | The front of the CO₂ flooding is not entering the throat, but the right end of the miscible belt has entered the throat and occupied a part of the throat. |
| IV             |                           | The leading edge of the CO₂ displacement has not entered the throat, the volume of the mixed phase is larger than the volume of the throat, and the right end has passed through the throat. |
| V              |                           | The leading edge of the CO₂ displacement enters into the throat, and the right end of the miscible zone passes through the throat. |
| VI             |                           | Water leading edge does not enter the throat. |
| VII            |                           | Water leading edge enters the throat. |
| VIII           |                           | Water leading edge passes through the throat. |

The seepage resistance of the miscible zone includes two parts, one is the seepage resistance in the capillary at the left end of the throat and the other is the percolation resistance in the throat. The expression is as follows:

\[
W_{ki}^{j_n-1} = \frac{8 \mu_0 (L_{ci} - x_{gi}^{j_n-1})}{\pi R_{pi}^2} + \mu_0 \int_{L_{ki}^k}^{t_{ki}^k} dx G(x). \]  \( \text{(30)} \)

The percolation resistance of crude oil and CO₂ in the capillary is as follows:

\[
W_{oi}^{j_n} = \mu_0 \int_{L_{oi}^{j_n}}^{dx G(x)} + \frac{8 \mu_0 (L - L_{ci} - L_{ki})}{\pi R_{pi}^2}, \]  \( \text{(31)} \)
\[ W^{j_{gi}}_{\text{GL}} = \frac{8\mu_o x_{gi}^{j-1}}{\pi R_{pi}^4}. \] (32)

At the end of \( j_n \) moment, the injection amount of the \( i \)th capillary and the total injection amount of the capillary beam CO\(_2\) can be calculated by Equations (23) and (28). At this time, the leading edge of CO\(_2\) displacement is still outside the throat. Using Equations (27) and (26), the length of the CO\(_2\) zone, \( x_{gi}^{jn} \), and the length of the equivalent miscible zone CO\(_2\), \( x_{\text{Li}}^{jn} \), at the end of \( j_n \) can be obtained (regardless of the length of the throat).

### 3.2.3. Fluid Distribution Pattern III

At the beginning of \( j_{n+1} \) moment, when the sum of the lengths of the CO\(_2\) region and the miscible region is greater than \( L_{ci} \), that is, \( x_{gi}^{jn+1} \geq L_{ci} \), \( 2V_{\text{min}}^{jn+1} > V_m \), it indicates that at the beginning of \( j_{n+1} \), the right end of the miscible band has begun to pass through the throat. The length of each zone and the seepage resistances are shown in Equations (30)–(32). The same iterative calculation procedure as in mode (2) is used to obtain the capillary tube injection flow rate.

The length of the miscible zone formed at the right end of the throat is \( x_{\text{Li}}^{jm} \) \((m = 1, m = m + 1)\), and the expression is as follows:

\[ x_{\text{Li}}^{m} = \left( x_{gi}^{jm} + x_{\text{Li}}^{jm} - L_{ci} \right) - \left[ \int_0^{L_{ti}} R^2(x) dx \right]. \] (33)

The actual miscible band width is \( x_{\text{Li}}^{jm} = L_{ci} - x_{gi}^{jm} + L_{ti} + x_{\text{Li}}^{jm} \). \( x_{\text{Li}}^{jm} \) is the length of the equivalent miscible region at the end of \( j_{n+1} \) moment. The expression of seepage resistance in the miscible zone is as follows:

\[ W_{\text{Li}}^{jm+1} = \frac{8\mu_o (L_{ci} - x_{gi}^{jm} + x_{\text{Li}}^{jm})}{\pi R_{pi}^4} + \mu_o \int_0^{L_{ti}} G(x) dx. \] (34)

The seepage resistance of the CO\(_2\) zone and the crude oil zone in the capillary is as follows:

\[ W_{gi}^{jm+1} = \frac{8\mu_o x_{gi}^{jm}}{\pi R_{pi}^4}, \] (35)

\[ W_{o,j}^{jm+1} = \frac{8\mu_o (L - L_{ci} - L_{ti} + x_{gi}^{jm})}{\pi R_{pi}^4}. \] (36)

At this time, CO\(_2\) flooding interface is still outside the throat. Using Equations (27) and (26), the length of the CO\(_2\) zone at the end of \( j_{n+1} \) is \( x_{gi}^{jm+1} \), and the length of the equivalent miscible zone is \( x_{\text{Li}}^{jm+1} \). At this time, it is necessary to judge the relative size of \( x_{gi}^{jm+1} \) and \( L_{ci} \). If \( x_{gi}^{jm+1} \leq L_{ci} \), the iterative calculation can be continued according to the steps in the distribution pattern II.

### 3.2.4. Fluid Distribution Pattern IV

Let CO\(_2\) entering length is \( L_{gi} \) \((a = 1, a = a + 1)\), and the expression is as follows:

\[ x_{gi}^{jn+1} = \frac{\int_0^{L_{gi}} R^2(x) dx}{G(x)}. \] (37)

where \( x_{gi}^{jn} \) is the equivalent length of the CO\(_2\) zone, cm; actual length is \( x_{gi}^{jn} = L_{ci} + L_{gi} \).

It is assumed that after the CO\(_2\) flooding leading edge enters the throat, CO\(_2\) is no longer dissolved in the crude oil, and the equivalent length of the CO\(_2\) zone remains constant. At the end of \( j_m \), the seepage resistance in the CO\(_2\) zone is as follows:

\[ W_{gi}^{jm+1} = \frac{8\mu_o L_{ci}}{\pi R_{pi}^4} + \mu_o \int_0^{L_{ti}} G(x) dx. \] (38)

Since the volume of the miscible zone is smaller than the throat volume, the actual length of the miscible zone in the throat at the end of \( j_m \) is \( x_{\text{Li}}^{jm} = L_{\text{Li}}^{k} - L_{gi}^{a} \) \((k \text{ has been accumulated})\), where \( L_{\text{Li}}^{k} \) can be defined by the following:

\[ R_{pi}^{2}x_{gi}^{jm} = \int_0^{L_{gi}} R^2(x) dx. \] (39)

where \( x_{gi}^{jm} \) is the end of \( j_m \), the equivalent length of the miscible band, cm.

Since CO\(_2\) does not dissolve into the crude oil when the CO\(_2\) flooding leading edge is in the throat, the equivalent length of the miscible zone remains constant until the CO\(_2\) flooding leading edge passes through the throat. The seepage resistance of the miscible zone is as follows:

\[ W_{\text{Li}}^{jm+1} = \frac{\mu_o \int_0^{L_{ti}} G(x) dx}{L_{gi}}. \] (40)

The actual length of oil zone is \( x_{\text{Li}}^{jm} = L - L_{ci} - L_{ti} + (L_{ti} - L_{\text{Li}}^{k}) \). The seepage resistance in the crude oil zone is as follows:

\[ W_{o,j}^{jm+1} = \frac{8\mu_o (L - L_{ci} - L_{ti})}{\pi R_{pi}^4} + \mu_o \int_0^{L_{ti}} G(x) dx. \] (41)

At the end of \( j_m \), the length of the formed CO\(_2\) zone is as follows:

\[ x_{gi}^{jm+1} + q_{gi}^{jm+1} \Delta t. \] (42)

The amount of CO\(_2\) injected in the \( i \)th capillary and the total injection amount of the capillary bundle can be calculated by Equations (23) and (28).
3.2.5. Fluid Distribution Pattern V. At this time, the length of the miscible band formed in the capillary at the right end of the throat is \( x_{\text{r}, b}^b \) (\( b = 1, b = b + 1 \)).

\[
x_{\text{r}, b}^b = x_{\text{r}, b}^j - \frac{\left[ \int_{\text{g}}^j R^2(x)dx \right]}{R_{pi}^2}.
\]  

(43)

The actual width of the miscible band is \( x_{\text{r}, b}^j = L_{\text{r}, b} - L_{\text{r}, b} \). At the beginning of \( j_{i+1}, \) the seepage resistance of the miscible zone is as follows:

\[
W_{i, j_{i+1}} = \frac{8\mu_o L_{\text{r}, b} x_{\text{r}, b}^j}{\pi R_{pi}^4} + \mu_o \int_{\text{g}}^j \frac{dx}{L_{\text{r}, b}^2 G(x)}.
\]  

(44)

The actual length of oil zone is \( x_{\text{o}, b}^j = L_{\text{o}} - L_{\text{o}} - L_{\text{o}} + x_{\text{r}, b}^b \), and the seepage resistance in the crude oil zone is as follows:

\[
W_{i, j_{i+1}} = \frac{8\mu_o (L - L_{\text{o}} - L_{\text{o}} - x_{\text{r}, b}^b)}{\pi R_{pi}^4}.
\]  

(45)

The seepage resistance in the CO2 zone is shown in Equation (38). The CO2 injection amount in the ith capillary and the total injection amount in the capillary bundle can be obtained by Equations (23) and (28). The equivalent length of the miscible zone is constant, and the length of the formed CO2 zone can be obtained by Equation (42).

3.2.6. Fluid Distribution Pattern VI. In the capillary, at the right end of the throat, the length of CO2 region formed by \( x_{\text{r}, g}^c \) (\( c = 1, c = c + 1 \)) is as follows:

\[
\frac{q_i^c \Delta t - \int_{\text{r}, g}^c \pi R^2(x)dx}{\pi R_{pi}^2} = x_{\text{r}, g}^c.
\]  

(46)

At this moment, the CO2 displacement front is outside the throat and CO2 will continue to diffuse into the crude oil. At the beginning of \( j_{i+1}, \) the actual length of the CO2 zone is \( x_{\text{r}, g}^j = L_{\text{r}, c} + L_{\text{r}, c} + x_{\text{r}, g}^c \). The equivalent length of the miscible band is \( x_{\text{r}, g}^j \), cm. Thereafter, the equivalent length of the miscible zone is equal to the actual length, and the actual length of the crude oil zone is \( x_{\text{o}, c}^j = L_{\text{o}, c} - x_{\text{r}, g}^b - x_{\text{r}, b}^c \).

At the beginning of \( j_{i+1}, \) the seepage resistance in the CO2 zone is as follows:

\[
W_{i, j_{i+1}} = \frac{8\mu_o (L_{\text{r}, c} + x_{\text{r}, g}^c)}{\pi R_{pi}^4} + \mu_o \int_{\text{g}}^j \frac{dx}{L_{\text{r}, c}^2 G(x)}.
\]  

(47)

The seepage resistances of the miscible zone and the crude oil zone are as follows:

\[
W_{i, j_{i+1}} = \frac{8\mu_o \lambda_{\text{r}, c} x_{\text{r}, g}^c}{\pi R_{pi}^4},
\]  

(48)

Knowing the magnitude of seepage resistance in each zone, the amount of CO2 injection in the ith capillary and capillary bundle can be calculated by using Equations (23) and (28). According to Equations (47)–(49), combined with the formula of the mixed-phase band in Section 2, the length of CO2 zone and the miscible zone can be obtained. Here, it should be noted that since the length of the miscible slug is constant when the leading edge of the CO2 displacement is assumed to be in the throat, the calculation of the miscible zone is performed after deducting the period of the CO2 displacement leading edge through the throat. In this way, according to the distribution pattern VI, the next iterative calculation is performed.

3.3. Water Flooding Seepage Mode and Mathematical Description. Suppose that the CO2 injection time is \( T_{i}, \) the time step has \( N_{tg} \) (\( N_{tg} = T_{i}/\Delta t \)). At the beginning of \( N_{tg} + 1, \) the water is injected with a constant pressure difference, \( \Delta P. \) Assuming that CO2 is not dissolved in water, there is a gas-water interface. There is a capillary force, the CO2-water interfacial tension is \( \sigma_{wg}, \) and the contact angle is \( \theta_{wg}. \) At the beginning of water injection, when the leading edge of CO2 flooding is at the left end of the throat, during the subsequent water injection process, there may be six distribution patterns in the CO2 flooding process, but on the other hand, due to water injection, the following three fluid distribution patterns appear.

3.3.1. Fluid Distribution Pattern VII. At this time, the seepage resistance of the water zone is as follows:

\[
W_{i, j_{i+1}} = \frac{8\mu_o \lambda_{\text{r}, c} x_{\text{r}, g}^c}{\pi R_{pi}^4}.
\]  

(49)

where \( x_{\text{r}, c}^{j_{i+1}} \) is the length of the water zone in the capillary at the beginning of \( N_{tg} + j, \) cm, \( x_{\text{r}, c}^0 = 0. \)

The pressure difference between the ends of the capillary bundle is \( \Delta P + P_{ci, i}, \) which is kept constant, and the flow rate of injected water is as follows:

\[
q_{w, i} = \frac{\Delta P + P_{ci, i}}{W_{N_{tg}, j_{i+1}}^c},
\]  

(51)

where \( W_{N_{tg}, j_{i+1}}^c = W_{w, i} + W_{w, j_{i+1}}^c + W_{w, j_{i+1}}^c + W_{w, j_{i+1}}^c + W_{w, j_{i+1}}^c + W_{w, j_{i+1}}^c, \) \( W_{w, j_{i+1}}^c, \) \( W_{w, j_{i+1}}^c, \) is the magnitude of seepage resistance in the CO2 zone, the miscible zone, and the crude oil zone at the beginning of \( N_{tg} + j. \)

The total amount of water injected is as follows:

\[
Q_{w, i+1} = \sum_{j=1}^{N_{tg}+1} q_{w, i}.
\]  

(52)
During this process, the water seepage velocity is the same as CO₂.

\[ W_{wi}^j = \frac{q_{wi}^j}{\pi R_{pi}^2} = W_{gi}^{N+1,j}. \]  

(53)

According to the principle of material balance, due to the constant total injection volume, CO₂ will continue to diffuse into the crude oil during the water injection process, the length of the CO₂ zone and the crude oil zone will decrease, the width of the miscible zone will increase, and the calculation formula of the miscible zone will be for the distribution mode. For the first, second, third, and sixth fluid distribution pattern, the formula for calculating the length of the CO₂ zone should be corrected to the following:

\[ j \Delta t < T_g \] Yes

Output capillary bundle gas injection rate \( Q_g \)

\[ j = 1 \]

Calculate \( q_{iur}^j \), obtain water injection flow rate of capillary bundle

Calculate water seepage velocity, miscible zone width \( w_{iur}^{N+1,j}, x_{iur}^{N+1,j} \)

Calculate the length of CO₂ and water zone \( x_{gi}^{N+1,j}, x_{wi}^{N+1,j} \)

\[ j = j + 1 \]

\[ j \Delta t < T_g + T_w \]

Yes

Output capillary bundle water injection rate \( Q_w \)

Calculate model injectivity

End

Figure 4: Flow diagram of solution.
where $\Delta V_{\text{mis}}^j$ is the amount of increase in the amount of CO$_2$ dissolved in the miscible zone, cm$^3$; $x_{\text{gi}}^{N_tg+j}$ and $x_{\text{gi}}^{N_tg+j+1}$ are the equivalent lengths of the CO$_2$ zone at the time of $N_{tg} + j$ and $N_{tg} + j + 1$, cm.

$$\Delta V_{\text{mis}}^j = x_{\text{gi}}^{N_tg+j+1} - x_{\text{gi}}^{N_tg+j} = \frac{1}{2} \pi R_{pi}^2 \left( x_{\text{gi}}^{N_tg+j+1} - x_{\text{gi}}^{N_tg+j+1} \right),$$  \hspace{1cm} (55)

where $x_{\text{gi}}^{N_tg+j}$ and $x_{\text{gi}}^{N_tg+j+1}$ are the equivalent lengths of the miscible band at the time of $N_{tg} + j$ and $N_{tg} + j + 1$. Since the diffusion of CO$_2$ in water is not considered, the length of water zone in the capillary is as follows:

$$x_{wij}^j = x_{wij}^{j-1} + \frac{q_{wi}^j \Delta t}{\pi R_{pi}^2},$$  \hspace{1cm} (56)

3.3.2. Fluid Distribution Pattern VIII. Assuming at the beginning of $N_{tg} + j + 1$ moment (which is also at the end of $N_{tg} + j$ moment), the length of water zone is calculated as $x_{wij} > L_{ci}$. It means that at the end of $N_{tg} + j$, the injected water has entered the throat. The length needs to be corrected, and the actual water zone length is $x_{wij}^{j-1}$.

$$x_{wij}^j = x_{wij}^{j-1} = L_{ci} + L_{d},$$  \hspace{1cm} (57)

$$R_{pi}^2 \left( x_{wij}^{j-1} - L_{ci} \right) = \int_0^{L_{wij}} R^2(x) \, dx,$$  \hspace{1cm} (58)

where $x_{wij}$ is the equivalent length of the initial water zone at the time of $N_{tg} + j + 1$, cm. $L_{wij} \quad (d = 1, \, d + 1)$ is the extended length of the injected water in the throat.

In the throat, the radius of the water-CO$_2$ interface is

$$R = \frac{R_{p} + R_{ci}}{2} = \frac{R_{p} - R_{ci}}{2} \cos \left( \frac{2 \pi L_{wij}}{L_{ci}} \right),$$  \hspace{1cm} (59)

$$P_{ci} = 2 \sigma w \cos \frac{\theta + \gamma}{R_{ci}(x)},$$  \hspace{1cm} (60)

$$\gamma = \tan^{-1} \left( \frac{dR(x)}{dx} \right),$$  \hspace{1cm} (61)

$$\tan \gamma = \frac{dR(x)}{dx} = \frac{R_{ci}(L_{wij}) - R_{ci}(L_{wij})}{L_{wij} - L_{wij}},$$  \hspace{1cm} (62)

where $R_{ci}(L_{wij}) = R_{pi}$, $L_{wij} = 0$; $\gamma$ is the correction value of the contact angle.

At the beginning of $N_{tg} + j + 1$, the actual CO$_2$ equivalent length is $x_{gi}^{N_tg+j}$, where the extension length at the right end of the throat is $x_{ri}^{j+1}$.

$$x_{ri}^{j+1} = x_{gi}^{N_tg+j} - \frac{\int_0^{x_{gi}^{N_tg+j}} R(x) \, dx}{R_{pi}^2},$$  \hspace{1cm} (63)

The seepage resistances in the water area and CO$_2$ area are given in Equations (64) and (65).

$$W_{wij}^{N_tg+j} = \frac{8 \mu_{wi} L_{wij}}{\pi R_{pi}^2} + \mu_{wi} \int_0^{L_{wij}} \frac{x_{wij}^j \, dx}{G(x)},$$  \hspace{1cm} (64)

$$W_{wij}^{N_tg+j+1} = \frac{8 \mu_{gi} x_{wij}^{j+1}}{\pi R_{pi}^2} + \mu_{gi} \int_0^{x_{wij}^{j+1}} \frac{x_{wij}^j \, dx}{G(x)},$$  \hspace{1cm} (65)

At this time, the length of the crude oil zone is

$$L - L_{ci} - L_{wij} - x_{wij}^{j+1} - x_{gi}^{N_tg+j},$$  \hspace{1cm} (66)

and the resistance of the crude oil zone can be obtained by substituting Equation (49), and the seepage resistance of the mixed phase can be obtained by using Equation (48). By substituting the formula for seepage resistance in each zone into Equations (51) and (52), the water injection amount of the $i$th capillary and capillary bundle can be obtained. According to Equations (51)-(55), the length of the water zone and the CO$_2$ zone can be obtained, and the method of obtaining the miscible zone is the same as in previous cases.

3.3.3. Fluid Distribution Pattern IX. At the beginning of $N_{tg} + j + 1$, if the calculated $L_{wij}$ is greater than $L_{ci}$, the water-CO$_2$ interface has passed through the throat, and the length formed at the right end of throat is $x_{wij}^j \quad (e = 1, \, e + 1)$.

$$x_{wij}^j = x_{wij}^j = L_{wij} - \left[ \int_0^{x_{wij}} R(x) \, dx \right]/R_{pi}^2.$$

At the end of $N_{tg} + j$, the actual length of the water area is $x_{wij}^j = L_{ci} + L_{wij} + x_{wij}^j$. $x_{wij}^j$ is the equivalent length of water zone, the length of CO$_2$ zone is $x_{gi}^{N_tg+j}$, the length of miscible zone is $x_{gi}^{N_tg+j}$, and the length of the crude oil zone is $L - L_{ci} - L_{wij} - x_{wij}^{N_tg+j} - x_{gi}^{N_tg+j} - x_{wij}^j$. Water-CO$_2$ interfacial tension remains constant.

At the beginning of $N_{tg} + j + 1$, the seepage resistance for each zone is as follows:

$$W_{wij}^{N_tg+j+1} = \frac{8 \mu_{wi}(L_{wij} + x_{wij}^j)}{\pi R_{pi}^2} + \mu_{wi} \int_0^{L_{wij}} \frac{x_{wij}^j \, dx}{G(x)},$$  \hspace{1cm} (66)

$$W_{wij}^{N_tg+j+1} = \frac{8 \mu_{gi} x_{wij}^{j+1}}{\pi R_{pi}^2} + \mu_{gi} \int_0^{x_{wij}^{j+1}} \frac{x_{wij}^j \, dx}{G(x)},$$  \hspace{1cm} (67)

$$W_{wij}^{N_tg+j+1} = \frac{8 \mu_{gi} x_{wij}^{j+1}}{\pi R_{pi}^2} + \mu_{gi} \int_0^{x_{wij}^{j+1}} \frac{x_{wij}^j \, dx}{G(x)},$$  \hspace{1cm} (68)

$$W_{wij}^{N_tg+j+1} = \frac{8 \mu_{gi}(L - x_{wij}^j - x_{wij}^{j+1} - x_{gi}^{N_tg+j})}{\pi R_{pi}^2}.$$  \hspace{1cm} (69)
Substituting Equations (66)-(69) into Equations (51)-(55), the length of the water zone and the CO$_2$ zone can be obtained, and the parameters such as $\beta$ and $D_m$ are substituted into Equation (40) to obtain the width of the miscible zone.

### 4. Sensitivity Analysis

#### 4.1. Model Solving Process and Basic Parameter Settings

According to the above descriptions, the solution process can be divided into two steps, CO$_2$ injection and water injection, and the flow chart of solution is shown in Figure 4.

During water-alternating-gas flooding process, the injection capacity is defined as the ratio of injected CO$_2$/water rate to the pressure difference across the capillary bundle:

$$I = \frac{Q_{w,g}}{\Delta P_{w,g}},$$

where $I$ is the injection capacity, cm$^3$/MPa-s; $Q_{w,g}$ is the CO$_2$/water injection flow rate, cm$^3$/s; $\Delta P_{w,g}$ is the pressure difference across the capillary beam, MPa.

For the convenience of analysis, it is stipulated that during the injection of CO$_2$ and water injection, the pressure difference across the capillary bundle is equal. To quantitatively characterize the ability of the capillary tube to flow, the equivalent permeability is introduced, and the expression is as follows:

$$k = \frac{L}{8 \sum_{i=1}^{n} R_{pi}^2 \left( \left( \frac{1}{R_{pi}^4} \right) + \left( \int_{0}^{L} \left( \frac{1}{R_{pi}^4}(x) \right) dx \right) \right)},$$

According to the variables used in the model derivation process, the pore throat size of the reservoir in low-permeability reservoirs and the characteristics of fluid properties of water-alternating-CO$_2$ injection are investigated [35], and the basic parameters of the model are set, as shown in Table 2.

#### 4.2. Injection Capacity Analysis

To better reflect the change law of injectivity of CO$_2$, mixed-phase slug, and injection capacity, the throat is set closer to the injection end. Then, the influences of viscosity, pore throat ratio, CO$_2$ slug size, and equivalent permeability on the variance of CO$_2$-alternating-water capacity have been studied.

**Table 2: Parameters for CO$_2$ water-alternating-gas model.**

| Parameters                        | Value       | Parameters                        | Value       |
|-----------------------------------|-------------|-----------------------------------|-------------|
| Capillary radius ($\mu$m)         | 5–15        | Capillary length (cm)             | 30          |
| Water/CO$_2$ interfacial tension (mN/m) | 20          | Water/CO$_2$ contact angle (degree) | 60          |
| Crude oil viscosity (mPa·s)       | 5           | CO$_2$ viscosity (mPa·s)           | 0.05        |
| Water viscosity (mPa·s)           | 2           | $K_w$ (cm)                        | 0.001       |
| $K_s$ (cm/(Pa·s))                 | 0.001       | CO$_2$ injection time (s)          | 5           |
| $D_m$ (cm$^2$/s)                  | $1 \times 10^{-5}$ | Time interval (s)          | 0.1         |
| Injection time (s)                | 5           | Pore throat ratio                 | 3           |
| Pressure difference at both ends of the capillary bundle (MPa) | 1 | Distance between throat and inlet end (cm) | 2–3 |
| Throat length (cm)                | 1           | Number of capillaries             | 11          |
in the throat is large, after the water flooding front passes through throat, the rising rate of water injectivity will decrease with the increase of pore throat ratio.

4.2.3. CO₂ Slug Size. Because CO₂ is injected with constant pressure, the size of CO₂ slug cannot be changed by changing the CO₂ injection rate. However, CO₂ slugs with different sizes can be equivalently injected by setting different injection times. According to this treatment method, CO₂ injection time is set to be 4 s, 5 s, and 6 s, and then, different CO₂ slug size is obtained. After CO₂ injection, the changes of injectivity with time are shown in Figure 7.

![Figure 6: Injectivity change with time for different oil viscosity.](image1)

![Figure 7: Injectivity change with time for different CO₂ slug size.](image2)

The larger the CO₂ injection time is, the longer the CO₂ slug is formed, and the smaller the total seepage resistance of the fluid in the capillary bundle is, so the injection capacity increases with the injection time. The throat is closer to the injection end, and the injection capacity increases linearly before the front of the water drive enters the throat. When the front edge of the water drive enters the throat, the injectivity decreases sharply. When the water drive front passes through the throat, the injectivity will increase slowly.

4.2.4. Equivalent Permeability. Since the pore throat ratio between the capillary and the throat is constant, the capillary radius distribution range in the capillary bundle is changed, and the equivalent permeability under different capillary bundles can be obtained by Equation (71), which is:

\[ \frac{10^{-3} \mu m^2}{5 \times 10^{-3} \mu m^2, 18.5 \times 10^{-3} \mu m^2, 28.5 \times 10^{-3} \mu m^2}. \]  

The injectivity is calculated at different permeability, and the resulting injection capacity changes with time are shown in Figure 8.

As can be seen from Figure 8, the smaller the equivalent permeability (the smaller the range of the capillary bundle radius) is given, the smaller the injectivity will be. The reason is that for crude oil with the same viscosity, the smaller the equivalent permeability, the greater the resistance to be overcome by the fluid flow. Under the same injection pressure difference, the smaller the injection flow rate, the lower the injection capacity. This change law indicates that for low-permeability reservoirs, the injection capacity is lower due to the smaller pore throat, and the required injection pressure difference is greater at the same injection amount compared with the high-permeability reservoir. Under the same injection amount, the required water injection pressure difference is larger. If the fluid is injected with constant pressure, it is hard to inject water, which has an impact on the oil recovery for low-permeability reservoirs. Comparing Figure 5 to Figure 8, it can be found that the change of equivalent permeability has the greatest influence on the injection capacity,
which also indicates that in a practical pilot test for CO\textsubscript{2}-alternating-water injection, the influence of permeability on injectivity should be fully considered.

5. Conclusions

(1) Considering the seepage, mass transfer, and CO\textsubscript{2} diffusion during CO\textsubscript{2} flooding, a mathematical model of convection-diffusion is established and solved. The relationship between the width of the mixed-phase zone and the injection time is obtained.

(2) According to the difference of distribution forms for the unswept oil region, mixed-phase zone, CO\textsubscript{2} region, and water region in a capillary tube, six possible patterns are described during CO\textsubscript{2} displacement, and three distribution patterns are described during water flooding. Through the introduction of the equivalent resistance model, the injectivity for each possible distribution pattern is evaluated during water-alternating-gas displacement.

(3) Through parameter analysis, the injectivity decreases with the increase of crude oil viscosity and pore throat ratio. Oppositely, the injectivity increases with the equivalent permeability and CO\textsubscript{2} slug size. The equivalent permeability has the greatest influence on injection capacity compared with other factors; therefore, in practice, the influence of permeability on injectivity should be carefully considered.

Data Availability

The data used to support the findings of this study are available from the first author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study was supported by the National Science Foundation (51804328 and 51974348), the Fundamental Research Funds for the Central Universities (18CX02168A), and the European Union’s Horizon 2020 Research and Innovation Program (846775).

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