Numerical Models of Newtonian and Post-Newtonian Binary Neutron Star Mergers

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Abstract. This article describes a comparison of two calculations of the merger of a binary neutron star (NS) system which is initially within the tidal instability as described by Rasio & Shapiro [1, 2]. The same initial data is used with one simulation involving a purely Newtonian evolution of the Euler equations for compressible fluids and another similarly evolved except there is an inclusion of a gravitational radiation reaction (GRR) term at the 2.5 Post-Newtonian (PN) order as prescribed by Blanchet, Damour, & Schäfer [3]. The inclusion of GRR is to allow an approximation of the full relativistic effect which forces the inspiral of binary systems. The initial data is identical at the start of each evolution and only the inclusion of the 2.5PN term differs in the evolutions. We chose two co-rotating, $\Gamma=2$ polytropic stars with masses $1.4M_\odot$, radii $R_*=9.56\text{km}$ and central densities of $2.5 \times 10^{15}\text{g/cm}^3$. The initial binary separation is $2.9R_*$, inside the region where the dynamical tidal instability has been shown to cause the merger of the two stars. Comparisons between measured quantities will indicate a substantial difference between the two evolutions. It will be shown that the effect of GRR on the dynamics of the merger is significant.

1. Introduction

We study the evolution of binary NS systems as it represents one of the most interesting astrophysical systems of the known universe. Since the discovery of the first binary pulsar system in 1974 by Hulse & Taylor [4] and the subsequent measurement of the orbital period decrease as predicted by general relativity it is has been thought that the energy loss from these systems, in the form of gravitational waves (GWs), could be measured. Such measurements are expected to begin early in the next century when several GW observatories or detectors, such as the US-based LIGO experiment [5], are to come online in their advanced stages. These systems also represent interesting laboratories for
nuclear physics. Most recently, the site of \( \gamma \)-ray bursts has been suggested as binary NS mergers[8, 9]. See Calder, Swesty & Wang 1988 for discussion [10].

2. Method of simulation

2.1. Numerical hydrodynamics and self gravity

We assume the neutron star can be described by a compressible fluid which satisfies the Euler equations, plus the solution to Poisson’s equation for self-gravity and the 2.5PN GRR potential \( \Psi \) which enters into the momentum equation (see [3, 10] for a discussion of the the Euler equations with self-gravity). The method is similar to the one employed in the ZEUS codes [11]. Usage of the 2.5PN GRR potential is consistent with [3] for hydrodynamic evolution up to 2.5PN order.

2.2. 2.5PN gravitational radiation reaction potential \( \Psi \) & boundary conditions

Solving for the 2.5PN GRR potential \( \Psi \) at any time during the evolution requires a solution of a Poisson equation of the form

\[
\nabla^2 R = 4\pi G D^{ij} x_i \frac{\partial \rho}{\partial x_j} \tag{1}
\]

where \( D^{ij} = \frac{d^3 \hat{T}^{ij}}{dt^3} \) is third time-derivative of the symmetric, trace-free mass quadrupole tensor, and the 2.5PN GRR potential is defined as

\[
\Psi = \frac{2}{5} \frac{G}{c^2} \left( R - D^{ij} x_i \frac{\partial \Phi}{\partial x_j} \right) \tag{2}
\]

We utilize a full multi-grid W-cycle algorithm to calculate the solution of (1). To define the boundary values of \( R \) we utilize a Riemann sum decomposition similar to what is described in [10] for the Newtonian potential. The solution to (1) will be of the form,

\[
R = -G \int \frac{D^{ij} x_i' \frac{\partial \rho}{\partial x_j'}}{|\vec{r} - \vec{r}'|} d\vec{r}' \tag{3}
\]

where \( \vec{r}' \) is the distance from the mass element to the boundary zone (see [10] for discussion). We have found that \( R=0 \) boundary conditions are sufficient.

3. Simulations

3.1. Technical concerns: Evolution time scales

The high densities present in neutron stars implies the adiabatic sound speed \( C_s \) will be very large. The Courant-Friedrichs-Lewy (CFL) condition defines the minimum time step and since it is dependent on the inverse of \( C_s \) one expects evolutions on physical time scales (milliseconds) to require many thousands of numerical time steps. Explicitly, the CFL condition may be written as

\[
\Delta t = \alpha (\Delta x/C_s) \tag{4}
\]

where \( \alpha \) is the Courant number. The time step may also be reduced if high velocities are present or if there is dissipation [11]. A suite of code tests, spatial and temporal resolution tests, and questions of stability have been addressed [10].
3.2. Initial data and atmosphere

We construct initial data for a close binary system because of the computational impracticality of evolving two stars from infinity. Initially, two non-rotating, $\Gamma=2$ polytropic stars with masses $1.4M_\odot$, radii $R_*=9.56$ km and central densities of $2.5 \times 10^{15} g/cm^3$ are placed on a grid with an initial binary separation of $2.9R_*$ between the star centers. The equation of state we chose is a polytropic one which involves a relationship between pressure $P$ and density $\rho$, only. It should be noted that the equation of state for the evolutions is not isentropic but is an ideal gas equation of state defined in terms of

$$P = (\Gamma - 1)E = (\Gamma - 1)e\rho$$

where $e$ is the specific internal energy. We prescribe a low density atmosphere, $\rho=1.0\times10^3 g/cm^3$, with a high internal energy. This prevents the atmosphere from falling onto the star surface and shocking over short time scales. We have found other treatments of the atmosphere, such as the one employed by Ruffert et al [12], to be problematic and erroneous. All the fluid elements in the stars and a nearby region of atmosphere are then set to co-rotate about the center of the grid with a frequency defined by Kepler’s third law for two point particles. The axis of rotation is the $z$-axis.

3.3. Results

For the two simulations we consider, the grid size is $129^3$ with a $dx=0.89$ km allowing for a resolution of $\approx 21$ zones across each star. The Newtonian evolution, simulation N2.9, is evolved for 3776 time steps with a total physical time of 5.40ms. Tidal distortion of the stars begins immediately. By one-eighth of an orbit the surfaces of the stars are touching. After one-half an orbit only the densest regions of the stars are not in contact. However, it is after this time during which the cores of the individual stars begin to fall inward toward the system center of mass that we find mass shedding and angular momentum transfer from the cores to the lower density regions. As these low density regions gain angular momentum spiral arms form carrying this material away from the coalescing object and off the evolution grid. By seven-eights of an orbit the densest parts of the stars are touching and the spiral arms are pronounced. Soon after the peak in the gravitational wave luminosity (GWL) occurs the magnitude of the GWL and the GWs $h_+$ and $h_\times$ falls-off dramatically, as depicted by $h_\times$ in figure 1. A differentially rotating object remains by one and a half orbits with the merged core rotating faster than the lower density disk surrounding it. The core is non-axisymmetric, as would be indicated by a non-zero waveform $h_\times$ (figure 1).

The PN2.9 simulation where GRR at 2.5PN order is included is evolved for 3694 time steps with a total physical time of 5.26ms. The results are qualitatively similar to the Newtonian simulation except that the merger occurs substantially earlier.

The simulations have some substantial differences in the gravitational waveforms, the time scales to merger, and the equilibrium conserved quantities. A phase difference develops in $h_\times$ after 0.8ms due to the inclusion of the GRR which accelerates the merger. By 1.5ms, a 90 degree phase difference can be seen in $h_\times$; the merger has occured a quarter orbit earlier. The mass loss that occurs is the means by which the pre-merger stars shed angular momentum to form the central object. Since GRR reduces the total angular momentum of the system throughout the evolution in PN2.9 we see a higher total
mass of the post-merger object or less mass shedding than in the N2.9 calculation. Both mass and angular momentum are less in the post-merger N2.9 object after the merger (figure 2). The inclusion of GRR alters the evolution of the system and the structure of the post-merger object. Exclusion of GRR clearly ignores an effect which significantly changes the evolution of a binary NS system. This effect is not overwhelmed by the Newtonian dynamical tidal instability of \[1,2\].

At 3.485 ms into the evolution and nearly 1.8 ms past the merger the core region can be defined as having a torus-like mass distribution (figure 3). Outward from the center to about 3 km the mass density rises along the \(x\)- and \(y\)-axes then falls off rapidly. As seen in figure 4, most of the mass in the system is contained within 27 km of the system center of mass. However, a significant fraction of the kinetic energy of the system lies outside this region (figure 5). The merged object is differentially rotating (figure 6) with the merged core rapidly rotating inside low density disk. The post-merger object appears to be nearly axisymmetric, and more so than in N2.9.

In conclusion, the effect of GRR on the evolution of the binary NS merger is significant. There is a substantial difference between the Newtonian and Newtonian+2.5PNRR evolutions. The final merged objects are different in structure and total angular momentum. The differing pre-merger inspiral rate produces a phase difference in the waveforms and the total radiated energy through GWs is somewhat different. Thus, it can be stated that the effect of the 2.5PNRR potential is significant in the late portion of the merger and in forming the final coalesced object. This further motivates the need for fully relativistic simulations.

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Figure 1. Gravitational waveform $h_\times$ is shown for 5.26ms as seen by an observer along the $z$-axis.

Figure 2. The mass for a similar time at the four density thresholds.

Figure 3. Mass density as a function of radius.

Figure 4. Total mass enclosed as a function of radius as a solid line while the dotted lines are factors of the Schwarzschild radius $R_s$.

Figure 5. Kinetic energy as a function of radius.

Figure 6. The norm of the fluid velocity as a function of the radius.