Trispectrum from ghost inflation

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Abstract. Ghost inflation predicts almost scale-invariant primordial cosmological perturbations with relatively large non-Gaussianity. The bispectrum is known to have a large contribution at the wavenumbers forming an equilateral triangle and the corresponding nonlinear parameter $f_{\text{equil}}^{\text{NL}}$ is typically of order $O(10^2)$. In this paper we calculate trispectrum from ghost inflation and show that the corresponding nonlinear parameter $\tau_{\text{NL}}$ is typically of order $O(10^4)$. We investigate the shape dependence of the trispectrum and see that it has some features different from DBI inflation. Therefore, our result may be useful as a template to distinguish ghost inflation from other models of inflation by future experiments.

Keywords: modified gravity, non-gaussianity, inflation, cosmological perturbation theory

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1 Introduction

Almost scale-invariant primordial cosmological perturbations predicted by inflation fits observational data very well [1]. While this is certainly a great success of the general idea of inflation, there still remain many unanswered important questions about inflation.

One of those important questions is how to distinguish different models of inflation observationally. There are many models of inflation which are consistent with observational data. It is often thought that tensor mode fluctuations [2] and non-Gaussianity [3–15] will be useful to distinguish some of them. Non-Gaussianity is the main subject of the present paper.

While single-field, simple slow-roll inflation predicts negligibly small non-Gaussianity [3], it is well known that there are many ways to generate non-Gaussianities large enough to be detected by near future experiments. They can be categorized into three by epochs in which non-Gaussianities are generated:

(i) super-horizon,

(ii) horizon-crossing and

(iii) sub-horizon epochs.

Each of these three types has a bispectrum with characteristic dependence on shapes of triangle formed by wave-vectors. The bispectrum for each type has a large contribution at

(i) squeezed- [4, 5],

(ii) equilateral- [5–7] and
(iii) folded-triangle [8], respectively.

Among them, our interest in the present paper is on the type (ii), in which large non-Gaussianity is typically due to higher derivative terms whose importance is enhanced by smallness of the sound speed. Concrete examples of this type includes k-inflation [5], DBI inflation [5, 6] and ghost inflation [7, 16, 17, 20].

While bispectrum is the leading deviation from Gaussian statistics and thus a useful tool to distinguish some models of inflation from others, trispectrum can also provide additional information about inflation [9–14]. In this paper we shall calculate trispectrum from ghost inflation, hoping to find ways to distinguish ghost inflation from other inflationary models which predict similar bispectra. We shall find that the shape-dependence of trispectrum in ghost inflation shows some difference from that in DBI inflation [10–14]. Therefore, the result of the present paper may be useful as a template to distinguish different models of inflation by future experiments.

The rest of this paper is organized as follows. In section 2 we review the ghost inflation and show the powerspectrum and the bispectrum in this model. In section 3 we calculate the trispectrum in the ghost inflation. In section 4 is devoted to a summary of this paper and discussion. In appendix A we give a brief review of the in-in formalism. In appendix B we present the method to construct the interaction Hamiltonian. In appendix C we show the details of the calculations.

2 Review of ghost inflation

In this section we briefly review the ghost inflation [7], which is an inflationary model in the ghost condensation [16, 17].

2.1 The model

The ghost condensation is the simplest Higgs phase for gravity and modifies gravity in the infrared. This can be realized if the derivative of a scalar field obtains a constant, timelike vacuum expectation value in a maximally symmetric spacetime, either Minkowski or de Sitter spacetime. By choosing the time coordinate properly, the vacuum expectation value of the scalar field can be written in the form

$$\langle \phi \rangle = M^2 t,$$  

(2.1)

where $M$ is a constant. The background energy momentum tensor of the scalar field is identical to that of a cosmological constant.

Ghost inflation is a model of inflation driven by the scalar field $\phi$ responsible for ghost condensation. This situation can be realized, for instance, a la hybrid inflation. We introduce another field $\chi$, which is assumed to be massive for $\phi$ smaller than some critical value $\phi_c$. At the critical point $\phi = \phi_c$, $\chi$ becomes massless and then becomes tachyonic for $\phi > \phi_c$ so that inflation ends as in hybrid inflation. An important point of this model is that the end of inflation depends only on the value of $\phi$.

In order to make predictions for the CMB anisotropy and compare with observational data, we need to consider perturbations around the background. Metric perturbations $h_{\mu\nu}$ are defined as

$$ds^2 = (-1 + h_{tt})dt^2 + 2h_{ti}dt dx^i + (a^2 \delta_{ij} + h_{ij})dx^i dx^j,$$  

(2.2)
where $a = e^{Ht}$. The scalar field is also decomposed into the background and perturbation as

$$\phi = M^2 t + \pi.$$  \hfill (2.3)

The perturbation $\pi$ is the Nambu-Goldstone boson associated with ghost condensate, i.e. spontaneous breaking of the time reparametrization symmetry, and thus we call it ghost-term [17]. Under the general infinitesimal diffeomorphism, $x^\mu \rightarrow x^\mu + \xi^\mu$, the perturbations $\pi$ and $h_{\mu \nu}$ are transformed as

$$\begin{align*}
\pi & \rightarrow \pi + M^2 \xi_t,
 h_{tt} & \rightarrow h_{tt} - 2 \partial_t \xi_t,
 h_{t i} & \rightarrow h_{t i} - \partial_i \xi_t - a^2 \partial_i (a^{-2} \xi_t),
 h_{i j} & \rightarrow h_{i j} + 2 H \xi_i a^2 \delta_{i j} - \partial_i \xi_j - \partial_j \xi_i,
\end{align*}$$  \hfill (2.4)

where the indices are raised and lowered by using the background metric.

Let us now construct the quadratic action for $\pi$ in de Sitter background. We can do this in a systematic way [16]. We begin with the unitary gauge, i.e. $\pi = 0$ so that $\phi = M^2 t$.

This gauge condition still leaves residual gauge freedom corresponding to the time-dependent spatial diffeomorphism:

$$t \rightarrow t, \quad x^i \rightarrow x^i + \xi^i(t, x).$$  \hfill (2.5)

Under this transformation, the gravitational perturbations $h_{\mu \nu}$ are transformed as

$$
\begin{align*}
h_{tt} & \rightarrow h_{tt} - 2 M^{-2} \partial_t \pi,
 h_{t i} & \rightarrow h_{t i} - a^2 \partial_i (a^{-2} \pi),
 h_{i j} & \rightarrow h_{i j} - \partial_i \pi_j - \partial_j \pi_i.
\end{align*}$$  \hfill (2.6)

Since the quadratic action must be invariant under this transformation, it should be constructed from the following three terms: $\int dt d^3 x \ a^3 h_{tt}^2$, $\int dt d^3 x \ a^3 K^{ij} K_{ij}$ and $\int dt d^3 x \ a^3 K^2$,

$$K_{ij} \equiv \frac{1}{2} \left[ a^2 \partial_i (a^{-2} h_{ij}) - \partial_i h_{ij} - \partial_j h_{ii} \right],$$  \hfill (2.7)

$$K^{ij} = a^{-4} \delta^{ik} \delta^{jl} K_{kl} \quad \text{and} \quad K = a^{-2} \delta^{ij} K_{ij}.$$  \hfill (2.8)

We can then obtain the quadratic action in general gauge by undoing the unitary gauge. This is achieved by performing the spontaneously broken diffeomorphism $\xi_t = M^{-2} \pi$, under which

$$
\begin{align*}
h_{tt} & \rightarrow h_{tt} - 2 M^{-2} \partial_t \pi, \\
K_{ij} & \rightarrow K_{ij} + M^{-2} (H a^2 \partial_i \partial_j \pi + \partial_i \partial_j \pi).
\end{align*}$$  \hfill (2.9)

After taking the decoupling limit $M/M_{Pl} \rightarrow 0$ and thus dropping $h_{\mu \nu}$ in the quadratic action, we obtain the leading quadratic action for $\pi$,

$$S_2 = \int dt d^3 x \ a^3 \left[ \frac{1}{2} (\partial_t \pi)^2 - \frac{\alpha_1}{2 M^2} \left( \frac{\vec{\nabla}^2}{a^2 \pi} + 3 H \partial_t \pi \right)^2 \right] - \frac{\alpha_2}{2 M^2} \left( \frac{\vec{\nabla} i \vec{\nabla} j}{a^2 \pi} + H \delta^{ij} \partial_t \pi \right) \left( \frac{\vec{\nabla} i \vec{\nabla} j}{a^2 \pi} + H \delta^{ij} \partial_t \pi \right),$$  \hfill (2.10)

where $\alpha_1$ and $\alpha_2$ are constants of order unity and we have normalized $\pi$. Here, $\vec{\nabla}^i = \delta^{ij} \vec{\nabla} j$ and $\vec{\nabla}^2 = \vec{\nabla} i \vec{\nabla} i$. For $H^2/M^2 \ll 1$, as we shall justify soon, we can drop the terms depending on $H \partial_t \pi$ and thus the quadratic action is reduced to

$$S_2 = \int dt d^3 x \ a^3 \left[ \frac{1}{2} (\partial_t \pi)^2 - \frac{\alpha}{2 M^2} \left( \frac{\vec{\nabla}^2}{a^2 \pi} \right)^2 \right],$$  \hfill (2.11)
where \( \alpha = \alpha_1 + \alpha_2 \). Therefore, there is no \((\vec{\nabla}\pi)^2\) term in the quadratic action for \( \pi \) and the lowest spatial derivative term is \( a^{-4}(\vec{\nabla}^2\pi)^2 \). In particular, the dispersion relation for the ghostone \( \pi \) is

\[
\omega^2 = \frac{\alpha}{M^2 a^4} k^4.
\] (2.12)

This implies that the fluctuation of \( \pi \) oscillates in the regime \( k^2/M \gg Ha \) while it freezes for \( k^2/M \ll Ha \). Moreover, because of the shift symmetry, interaction terms in the \( \pi \) action always include derivatives and thus are suppressed outside the sound horizon, i.e. for \( k^2/M \ll Ha \). Therefore, it is interactions during the epoch around the sound horizon crossing \( k^2/M \sim Ha \) that essentially determines the bispectrum and higher-order correlation functions of cosmological perturbations. Near the sound horizon crossing, we have \( \omega \sim (k/a)^2/M \sim H \) and thus \( (k/a)^2 \sim MH \gg H^2 \sim H\omega \) if \( H/M \ll 1 \). We shall see below that the COBE normalization for curvature perturbations fixes \( H/M \) to a small value (2.25). This is the reason why in (2.10), \( H \partial_t \pi \) can be neglected compared with \( a^{-2} \vec{\nabla}^2 \pi \).

In order to see the leading cubic and quartic interactions, we should identify the scaling dimensions of time coordinate \( t \), spatial coordinates \( x^i \) and the ghostone \( \pi \). Suppose that energy \( E \) is scaled by a factor of \( s \) as \( E \to sE \) and that time \( t \) is scaled as \( t \to s^{-1/2} t \). Since we know from the quadratic effective action for \( \pi \) (2.11) that the dispersion relation is \( \omega^2 \propto k^4 \), we have to scale \( k \) as \( k \to s^{1/2} k \) or \( x \) as \( x \to s^{-1/2} x \). Then, by demanding that the action is invariant under scaling, the scaling dimension of \( \pi \) is determined to be 1/4: \( \pi \to s^{1/4} \pi \). With the use of these scaling dimensions, we can identify the leading cubic and quartic operators.

When we introduced (2.1) as a consistent background, we implicitly assumed the shift symmetry, i.e. the invariance of the theory under a constant shift of the scalar field \( \phi \to \phi + \text{const} \). Because of the shift symmetry, the fluctuation \( \pi \) appears always with derivatives. We further assume that the theory is invariant under the \( Z_2 \) transformation \( \phi \to -\phi \).

This corresponds to the simultaneous change of the signs of \( t \) and \( \pi \):

\[
t \to -t \quad \text{and} \quad \pi \to -\pi.
\] (2.13)

For example, \( \partial_t \pi \vec{\nabla}^2 \pi \) (without being multiplied by \( H \)) is forbidden by this symmetry.

Let us now seek the leading cubic operator. The shift symmetry tells us that a cubic term has at least three derivatives. Because of the \( Z_2 \)-symmetry (2.13), a cubic term should have one or three time derivatives. Combining these with the fact that the scaling dimension of spatial derivative is lower than that of time derivative, we conclude that the leading cubic operator is

\[
S_3 = -\frac{\beta}{2M^2} \int dt d^3x \ a^3 \partial_t \pi \frac{(\vec{\nabla} \pi)^2}{a^2},
\] (2.14)

where \( \beta \) is a constant of unity. This operator has the scaling dimension 1/4. Similarly, the leading quartic operator is

\[
S_4 = -\frac{\gamma}{8M^4} \int dt d^3x \ a^3 \frac{(\vec{\nabla} \pi)^4}{a^4},
\] (2.15)

where \( \gamma \) is a constant of order unity, and has the scaling dimension 1/2.
2.2 Powerspectrum

In this subsection we review the power spectrum of the curvature perturbation generated by ghost inflation.

The scale $M$, which plays the role of ultraviolet cutoff of the effective field theory, is constrained by various observations. The strongest constraint is from the twinkling by lensing which bounds $M \lesssim 100 \text{GeV}$ [17].\(^1\) Since this scale is much lower than the Planck scale $M_{\text{Pl}}$, decoupling limit is a good approximation. Therefore, in order to calculate the power spectrum of ghostone $\pi$ we can simply study the $\pi$ action without coupling to gravity. We shall later relate $\pi$ to the curvature perturbation $\zeta$.

The equation of motion for $\pi$ in the linearized level is

$$u_k'' - \frac{2}{\eta^2} u_k + \frac{\alpha k^4 H^2 \eta^2}{M^2} u_k = 0,$$

(2.16)

where $u_k = a \pi_k$, $k$ is a comoving wavevector and $k = |k|$. Here, we have introduced the conformal time

$$\eta = -\frac{H}{a}$$

(2.17)

so that $dt = ad\eta$, and a prime represents derivative with respect to $\eta$.

We quantize $u$ as usual

$$u_k(t) = w_k(t)a_k + w_k^\dagger(t)a_k^\dagger,$$

(2.18)

where $a_k$ and $a_k^\dagger$ are annihilation and creation operators satisfying $[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k - k')$. Choosing $w_k(t)$ so that it corresponds to the correct mode function in flat spacetime for very short wavelength, we have

$$w_k(\eta) = \sqrt{\frac{\pi}{8}} (-\eta)^{1/2} H^{(1)}_3(q\eta^2),$$

(2.19)

where

$$q \equiv \sqrt{\frac{\alpha H k^2}{2M}}.$$  

(2.20)

The observed fluctuations in the CMB are generated quantum mechanically and stretched by the exponential expansion during the inflationary stage of the universe. Their wavelengths exceed the Hubble horizon and, at the end of the inflation, are much longer than the Hubble horizon scale. This means

$$k|\eta_e| \ll 1,$$

(2.21)

where $k$ is the wavenumber of the mode of interest and $\eta_e$ is the conformal time at the end of inflation. Therefore, it is a good approximation to take the limit $\eta_e \to 0$.

The power spectrum of the ghostone $\pi$ is obtained as

$$P_\pi = \frac{k^3}{2\pi^2} \left| \frac{w_k}{a} \right|^2 \bigg|_{\eta \to 0} = \frac{H^{1/2} M^{3/2}}{\pi (\Gamma(1/4))^2 \alpha^{3/4}}.$$  

(2.22)

\(^1\)It is known that the ghostone behaves like dark matter. If we suppose that the ghostone is responsible for all dark matter then the structure formation gives a lower bound on $M$: $M \gtrsim 10\text{eV}$ [18].
The gauge invariant curvature perturbation $\zeta$ \cite{19} is related to $\pi$ as

$$\zeta = -\frac{H}{\phi} \pi = -\frac{H}{M^2} \pi.$$  \hspace{1cm} (2.23)

Therefore, we obtain the power spectrum of the primordial curvature perturbation as \cite{7}

$$P_\zeta = \left( -\frac{H}{M^2} \right)^2 P_\pi = \frac{1}{\pi (\Gamma(1/4))^2 \alpha^{3/4}} \left( \frac{H}{M} \right)^{5/2}.$$  \hspace{1cm} (2.24)

The COBE normalization sets $P_\zeta^{1/2} \simeq 4.8 \times 10^{-5}$, and thus

$$\frac{H}{M} \simeq (1.6 \times 10^{-3}) \times \alpha^{3/10} \ll 1.$$  \hspace{1cm} (2.25)

### 2.3 Bispectrum

Let us now review the bispectrum of the curvature perturbation generated by ghost inflation.

As in the previous subsection, we calculate a correlation function for $\pi$ in the decoupling limit and then relate the ghostone $\pi$ to the curvature perturbation $\zeta$. For the calculation of the bispectrum, nonlinearity in the relation between $\pi$ and $\zeta$, if any, should be taken into account. Fortunately, in our model of ghost inflation the simple linear relation (2.23) is exact and there is no nonlinearity involved in the relation between $\pi$ and $\zeta$, essentially because the background $\phi$ is linear in $t$. Therefore, there are no additional nonlinear effects due to transformation of $\pi$ to $\zeta$, and the bispectrum of $\zeta$ is simply proportional to that of $\pi$.

As we have already seen in subsection 2.1, the leading cubic interaction in the $\pi$ action is

$$-\frac{\beta}{2M^3} \int dt d^3x \ a^3 \partial_t \pi \left( \vec{\nabla} \pi \right)^2 \frac{a}{a_2}.$$  \hspace{1cm} (2.26)

The corresponding operator in the Hamiltonian density is

$$\mathcal{H}_{int,3} = \frac{a^3 \beta}{2M^3} \partial_t \pi \left( \vec{\nabla} \pi \right)^2 \frac{a}{a_2}.$$  \hspace{1cm} (2.27)

Using the in-in formalism, we can calculate the tree-level bispectrum of ghostone $\pi$, $\langle \pi \pi \pi \rangle$, as

\begin{align*}
\langle \pi_{k_1}(t)\pi_{k_2}(t)\pi_{k_3}(t) \rangle \bigg|_{t \to \infty} \\
&= -i \int_{-\infty}^{t} dt' \left\langle \left[ \pi_{k_1}(t)\pi_{k_2}(t)\pi_{k_3}(t), \int d^3x \mathcal{H}_{int,3}(t') \right] \right\rangle \bigg|_{t \to \infty} \\
&= -\frac{4\sqrt{2\pi^{3/2}}HM^2\beta}{\Gamma(1/4)^3 \alpha^2} (2\pi^3) \delta^3 \left( \sum_i k_i \right) \prod_i \frac{1}{k_i} \\
&\quad \times \Re \left[ \int_{-\infty}^{0} d\eta \eta^{-1} F(\eta) F \left( \frac{k_2}{k_1} \eta \right) F \left( \frac{k_3}{k_1} \eta \right) k_3 (k_1 \cdot k_2) + \text{symm.} \right], \hspace{1cm} (2.28)
\end{align*}

where

$$F(\eta) = \sqrt{\frac{\pi}{8}} (\eta^{3/2} H_{3/4}^{(1)} (\eta^2 / 2)).$$  \hspace{1cm} (2.29)
Translating this into the bispectrum of $\zeta$, we obtain

$$
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = -\frac{4\sqrt{2\pi^{3/2}} \beta}{(\Gamma(1/4))^3 \alpha^2} \left( \frac{H}{M} \right)^4 (2\pi^3) \delta^3 \left( \sum_i k_i \right) \prod_i k_i^{-3} \times \Re \left[ \int_{-\infty}^{0} d\eta \eta^{-1} F(\eta) F\left( \frac{k_2}{k_1} \right) F\left( \frac{k_3}{k_1} \right) k_3 (k_1 \cdot k_2) + \text{symm.} \right] .
$$

(2.30)

It is known that the bispectrum from ghost inflation has a large contribution at equilateral triangles formed by three vectors $k_1$, $k_2$ and $k_3 = -k_1 - k_2$ [20]. The nonlinear parameter for equilateral configuration $f_{\text{NL}}^{\text{equil}}$ is defined as [1, 21]

$$
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle |_{k_1 = k_2 = k_3 = k} = (2\pi)^3 \delta^3 \left( \sum_i k_i \right) \cdot \frac{6}{5} f_{\text{NL}}^{\text{equil}} \cdot 3 \left( \frac{2\pi^2 P_{\zeta}}{k^3} \right)^2 .
$$

(2.31)

The value corresponding to the result (2.30) is [7]

$$
f_{\text{NL}}^{\text{equil}} \simeq 85 \cdot \beta \cdot \alpha^{-4/5} .
$$

(2.32)

The WMAP 7-year data bounds $f_{\text{NL}}^{\text{equil}}$ [1] as

$$
f_{\text{NL}}^{\text{equil}} = 26 \pm 140(68\% \text{ CL}).
$$

(2.33)

It is natural to take $\alpha \sim \beta \sim 1$ as they are dimensionless parameters. In this case, the ghost inflation is not excluded by the WMAP 7-year constraint but can be detected in the near future by, say, the PLANCK satellite.

3 Trispectrum

In this section we show the four-point function of late-time curvature perturbation $\zeta$ from ghost inflation. Details of calculations are presented in appendix C.

As we have already seen in subsection 2.1, the leading quartic interaction in the $\pi$ action is

$$
-\frac{\gamma}{8M^4} \int dt d^3 x \; a^3 \left( \vec{\nabla} \pi \right)^4 a^4 ,
$$

(3.1)

As shown in appendix B, the corresponding operator in the Hamiltonian density is

$$
\mathcal{H}_{\text{int},A} = \frac{\tilde{\gamma}}{8M^4} a^{-1} (\vec{\nabla} \pi)^4 , \quad \tilde{\gamma} = \gamma + 2\beta^2 ,
$$

(3.2)

where $\beta$ is the dimensionless coefficient of the leading cubic interaction.

There are two contributions to the four-point function: a contact term contribution and a scalar exchange contribution. According to the in-in formalism (See appendix A), the leading contributions to the four-point function are written as

$$
\langle \pi_{k_1}(t) \pi_{k_2}(t) \pi_{k_3}(t) \pi_{k_4}(t) \rangle \\
\equiv i \int_{-\infty}^{t} dt' \left\langle \left[ \int d^3 x' \mathcal{H}_{\text{int},A}(t', x'), \pi_{k_1}(t) \pi_{k_2}(t) \pi_{k_3}(t) \pi_{k_4}(t) \right] \right\rangle
$$

- 7 -
The first term in the right hand side of eq. (3.3) is called the contact term contribution and described in figure 1 as a four-point vertex. The second term is called the scalar exchange contribution whose diagram is shown in figure 2. This includes two three-point vertices. We calculate these contributions of the four-point function separately.

3.1 Contact term contribution

Let us consider the contact term contribution to the four-point function. The contact term contribution to the four-point function is defined as

\[
\langle \pi_{k_1}(t)\pi_{k_2}(t)\pi_{k_3}(t)\pi_{k_4}(t) \rangle_{cc} = i \int_{-\infty}^{t} dt' \left\langle \left[ \int d^3x' \left( \frac{\hat{\gamma}}{8\bar{M}^2} \int d^3x \left( \nabla \pi(t', x') \right)^2, a(t') \pi_{k_1}(t') \pi_{k_2}(t') \pi_{k_3}(t') \pi_{k_4}(t') \right) \right] \right\rangle. \tag{3.4}
\]

which depends on \( \hat{\gamma} \). This parameter \( \hat{\gamma} \) does not affect the leading-order bispectrum and thus the four-point function provides independent information about the effective action.
We define contact term contributions to trispectra $T_{\pi,cc}(\eta, k_1, k_2, k_3, k_4)$ and $T_{\zeta,cc}(\eta, k_1, k_2, k_3, k_4)$ as

$$\langle \pi_{k_1}(t)\pi_{k_2}(t)\pi_{k_3}(t)\pi_{k_4}(t) \rangle_{cc} = (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4)T_{\pi,cc}(\eta, k_1, k_2, k_3, k_4),$$

$$\langle \zeta_{k_1}(t)\zeta_{k_2}(t)\zeta_{k_3}(t)\zeta_{k_4}(t) \rangle_{cc} = (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4)T_{\zeta,cc}(\eta, k_1, k_2, k_3, k_4),$$

where $\eta$ is comoving time. As mentioned at the beginning of subsection 2.3, the simple linear relation (2.23) holds and there is no nonlinearity involved in the relation between $\pi$ and $\zeta$. Therefore, we have the following simple relation between two trispectra.

$$T_{\zeta,cc}(\eta, k_1, k_2, k_3, k_4) = \left( \frac{H}{M^2} \right)^4 T_{\pi,cc}(\eta, k_1, k_2, k_3, k_4).$$

As shown in appendix C and presented as (C.9), the contact term contribution to the trispectrum of curvature perturbation is

$$T_{\zeta,cc}(\eta = 0, k_1, k_2, k_3, k_4) = \frac{\chi}{2^6} \left( \frac{H}{M} \right)^{12} \left( \frac{\pi}{\Gamma(1/4)} \right)^4 (q_1 q_2 q_3 q_4)^{-3/4} \left( (k_1 \cdot k_2)(k_3 \cdot k_4) + (23 \text{ terms}) \right) \times \Re \left\{ i \int_{-\infty}^{0} \eta' \left( (-\eta')^{3/2} H_{3/4}^{(1)}(q_1 \eta')^2 \right) \left( (-\eta')^{3/2} H_{3/4}^{(1)}(q_2 \eta')^2 \right) \right. \times \left. \left( (-\eta')^{3/2} H_{3/4}^{(1)}(q_3 \eta')^2 \right) \left( (-\eta')^{3/2} H_{3/4}^{(1)}(q_4 \eta')^2 \right) \right\}.$$

where

$$q_i \equiv \sqrt{\alpha} H k_i^2 / 2M.$$  

The trispectrum depends on four 3-momenta $k_i$ $(i = 1, 2, 3, 4)$ satisfying the constraint $\sum_i k_i = 0$ representing the momentum conservation. Assuming homogeneity and isotropy of the background, the independent variables are four amplitudes of the momenta, $k_i = |k_i|$, and two angles between momenta, $k_1 \cdot k_2 / (k_1 k_2)$ and $k_1 \cdot k_3 / (k_1 k_3)$. Although it is ideal to investigate the dependence of the trispectrum on all six parameters, it is somehow complicated. Fortunately, for the purpose of showing some differences between ghost inflation and other models of inflation such as DBI inflation, it is sufficient to investigate the dependence on a subset of six parameters. Following previous works [10, 12] on the trispectrum from DBI inflation, we consider the equilateral case where

$$k_1 = k_2 = k_3 = k_4 = k,$$

and the remaining independent variables are the two angles described above. We expect that the equilateral shapes give large contributions to the trispectrum as in the case of the bispectrum. As shown in (C.19), the equilateral trispectrum of the curvature perturbation is

$$T_{\zeta,cc}(k, C_2, C_3, C_4) = -2.215 \times 10^{-17} \times \left( \frac{(P_C(k))^{1/2}}{4.8 \times 10^{-5}} \right)^{22/5} \frac{\chi}{\alpha^{8/5}} \left( \sum_{i=2,3,4} C_i^2 \right) k^{-9}.$$
Figure 3. This plot shows the dependence of $T_{\zeta, \zeta}(k, C_2, C_3, C_4)$ on $C_2$ and $C_3$, where $C_4 = -1 - C_2 - C_3$. The vertical axis is rescaled by the value at $C_2 = C_3 = -1/3$.

where

$$\frac{k_1 \cdot k_2}{k^2} = \frac{k_3 \cdot k_4}{k^2} = C_2, \quad \frac{k_1 \cdot k_3}{k^2} = \frac{k_2 \cdot k_4}{k^2} = C_3, \quad \frac{k_1 \cdot k_4}{k^2} = \frac{k_2 \cdot k_3}{k^2} = C_4,$$  

and $C_i (i = 2, 3, 4)$ are constrained by the momentum conservation as

$$1 + \sum_{i=2,3,4} C_i = 0.$$  

Following ref. [12], we define the nonlinear parameter $\tau_{\zeta N L}^{cc}$ as

$$\tau_{\zeta N L}^{cc}(k_1, k_2, k_3, k_4) = T_{\zeta, \zeta}(k_1, k_2, k_3, k_4) (2\pi^2 P_{\zeta}(k_0))^3 \prod_{i=1}^4 k_i^3$$

$$\times \left[ \left(k_i^3 k_j^3 + k_i^3 k_k^3 + k_i^3 k_l^3 + k_j^3 k_k^3 + k_j^3 k_l^3 + k_k^3 k_l^3 \right) \sum_{i=2,3,4} C_i \right]^{-1},$$  

where $k_0$ is the wavelength corresponding to the present Hubble horizon size and $k_{ij} = |k_i + k_j|$. Since $P_{\zeta}(k)$ is almost scale invariant, we do not distinguish between $P_{\zeta}(k)$ and $P_{\zeta}(k_0)$ hereafter. In the equilateral case (3.10), $\tau_{\zeta N L}^{cc}$ is

$$\tau_{\zeta N L}^{cc}(k, C_2, C_3, C_4) \approx -1.665 \times 10^5 \times \frac{\gamma}{\alpha^{8/5}} \left( \frac{P_{\zeta}(k)^{1/2}}{4.8 \times 10^{-5}} \right)^{5/8} \frac{\sum_{i=2,3,4} C_i^2}{\sum_{i=2,3,4} (1 + C_i)^{-3/2}}.$$
Figure 4. This plot shows the dependence of $\tau_{NL}^{cc}(k,C_2,C_3,C_4)$ on $C_2$ and $C_3$, where $C_4 = -1 - C_2 - C_3$. The vertical axis is rescaled by the value at $C_2 = C_3 = -1/3$.

\begin{equation}
W e \text{ show the dependence of } \tau_{NL}^{cc} \text{ on the two angles } C_2 \text{ and } C_3 \text{ in figure 4. In the most symmetric case where } C_2 = C_3 = C_4 = \frac{-1}{3}, \tau_{NL}^{cc} \text{ becomes }
\tau_{NL}^{cc}(k,C_2 = -1/3,C_3 = -1/3,C_4 = -1/3) = -1.007 \times 10^4 \times \frac{\tilde{\gamma}}{0.8/5} \left( \frac{P_{\gamma}(k)^{1/2}}{1.8 \times 10^{-3}} \right)^{-8/5} \tag{3.17}
\end{equation}

3.2 Scalar exchange contribution

In this subsection we present the scalar exchange contribution to the trispectrum. The scalar exchange contribution, corresponding to the diagram shown in figure 2, is defined as

\begin{equation}
\langle \pi_{k_1} \pi_{k_2} \pi_{k_3} \pi_{k_4} \rangle_{se} \equiv - \int_{-\infty}^{t} dt_2 \int_{-\infty}^{t_1} dt_1 \left[ \int d^3 x_1 \, \frac{\beta a(t_1)}{2M^2} \left( \frac{d}{dt_1} \pi(t_1,x_1) \right) \left( \nabla \pi(t_1,x_1) \right)^2 , \right.
\left. \left[ \int d^3 x_2 \, \frac{\beta a(t_2)}{2M^2} \left( \frac{d}{dt_2} \pi(t_2,x_2) \right) \left( \nabla \pi(t_2,x_2) \right)^2 , \pi_{k_1} \pi_{k_2} \pi_{k_3} \pi_{k_4} \right] \right].
\end{equation}

(3.18)

If the bispectrum (2.30) from ghost inflation is observed then it determines $\beta$ and thus the scalar exchange contribution to the trispectrum. Then the full trispectrum from ghost inflation, which is the sum of this contribution and the contact term contribution described in the previous subsection, is fully specified by the other parameter $\tilde{\gamma}$ (or equivalently $\gamma$).

Trispectra $T_{\pi,se}(\eta,k_1,k_2,k_3,k_4)$ and $T_{\zeta,se}(\eta,k_1,k_2,k_3,k_4)$ are defined in the same manner as in the case of the contact term contribution,

\begin{equation}
\langle \pi_{k_1}(t) \pi_{k_2}(t) \pi_{k_3}(t) \pi_{k_4}(t) \rangle_{se} \equiv (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4) T_{\pi,se}(\eta,k_1,k_2,k_3,k_4).
\end{equation}

(3.19)

\begin{equation}
\langle \zeta_{k_1}(t) \zeta_{k_2}(t) \zeta_{k_3}(t) \zeta_{k_4}(t) \rangle_{se} \equiv (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4) T_{\zeta,se}(\eta,k_1,k_2,k_3,k_4).
\end{equation}

(3.20)
and are related to each other as

\[
T_{\zeta,se}(\eta, k_1, k_2, k_3, k_4) = \left( \frac{H}{M^2} \right)^4 T_{\pi,se}(\eta, k_1, k_2, k_3, k_4).
\]

As in the calculations of other quantities, we take the limit \( \eta \to 0 \). Then, as calculated in appendix C and presented in (C.23), we obtain

\[
T_{\zeta,se}(\eta = 0, k_1, k_2, k_3, k_4) = \frac{\beta^2 \alpha}{4} \left( \frac{H}{M} \right)^{14} \left( \frac{\pi}{8} \right)^3 (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4)
\]

\[
\left\{ - \int_{-\infty}^{\eta} d\eta_1 \int_{-\infty}^{\eta_2} d\eta_2 \left\{ \left( [k_1 \cdot k_2](k_1^2 + k_2^2 + 2k_1 \cdot k_2)((-\eta_1)^{3/2} H_{3/4}^{(1)}(q_1 \eta_1^2))
\right.
\right.
\]

\[
((-\eta_1)^{3/2} H_{3/4}^{(1)}(q_2 \eta_1^2)) ((-\eta_1)^{3/2} H_{1/4}^{(1)}(q_1 \eta_1^2))
\]

\[
-2(k_1^2 + k_1 \cdot k_2)k_2^2 ((-\eta_1)^{3/2} H_{3/4}^{(1)}(q_1 \eta_1^2))
\]

\[
((-\eta_1)^{3/2} H_{1/4}^{(1)}(q_2 \eta_1^2)) ((-\eta_1)^{3/2} H_{3/4}^{(1)}(q_1 \eta_1^2))
\]

\[
\times \left( [k_3 \cdot k_4](k_3^2 + k_4^2 + 2k_3 \cdot k_4)((-\eta_2)^{3/2} H_{3/4}^{(1)}(q_3 \eta_2^2))
\right.
\]

\[
((-\eta_2)^{3/2} H_{1/4}^{(1)}(q_4 \eta_2^2)) ((-\eta_2)^{3/2} H_{3/4}^{(2)}(q_4 \eta_2^2))
\]

\[
-2(k_3^2 + k_3 \cdot k_4)k_4^2 ((-\eta_2)^{3/2} H_{3/4}^{(1)}(q_4 \eta_2^2))
\]

\[
((-\eta_2)^{3/2} H_{1/4}^{(1)}(q_3 \eta_2^2)) ((-\eta_2)^{3/2} H_{3/4}^{(2)}(q_3 \eta_2^2))
\]

\[
\times \left( (-\eta w_{-k_1}^*(\eta)) (-\eta w_{-k_2}^*(\eta)) (-\eta w_{-k_3}^*(\eta)) (-\eta w_{-k_4}^*(\eta))
\right.
\]

\[
\text{+23 permutational terms} \} + c.c.
\]

\[
+ \left( \int_{-\infty}^{\eta} d\eta_1 \left( [k_1 \cdot k_2](k_1^2 + k_2^2 + 2k_1 \cdot k_2)((-\eta_1)^{3/2} H_{3/4}^{(1)}(q_1 \eta_1^2))
\right.
\]

\[
((-\eta_1)^{3/2} H_{3/4}^{(1)}(q_2 \eta_1^2)) ((-\eta_1)^{3/2} H_{1/4}^{(1)}(q_1 \eta_1^2))
\]

\[
-2(k_1^2 + k_1 \cdot k_2)k_2^2 ((-\eta_1)^{3/2} H_{3/4}^{(1)}(q_1 \eta_1^2))
\]

\[
((-\eta_1)^{3/2} H_{1/4}^{(1)}(q_2 \eta_1^2)) ((-\eta_1)^{3/2} H_{3/4}^{(1)}(q_1 \eta_1^2))
\]

\[
\times \left( [k_3 \cdot k_4](k_3^2 + k_4^2 + 2k_3 \cdot k_4)((-\eta_2)^{3/2} H_{3/4}^{(1)}(q_3 \eta_2^2))
\right.
\]

\[
((-\eta_2)^{3/2} H_{1/4}^{(1)}(q_4 \eta_2^2)) ((-\eta_2)^{3/2} H_{3/4}^{(2)}(q_4 \eta_2^2))
\]

\[
-2(k_3^2 + k_3 \cdot k_4)k_4^2 ((-\eta_2)^{3/2} H_{3/4}^{(1)}(q_4 \eta_2^2))
\]

\[
((-\eta_2)^{3/2} H_{1/4}^{(1)}(q_3 \eta_2^2)) ((-\eta_2)^{3/2} H_{3/4}^{(2)}(q_3 \eta_2^2))
\]

\[
\times \left( (-\eta w_{-k_1}^*(\eta)) (-\eta w_{-k_2}^*(\eta)) (-\eta w_{-k_3}^*(\eta)) (-\eta w_{-k_4}^*(\eta))
\right.
\]

\[
\text{+23 permutational terms} \} \right), \tag{3.22}
\]
where \( q_i \) is defined by eq. (3.9) and

\[
q_{ij} \equiv \frac{\sqrt{\alpha H (k_1 + k_j)^2}}{2M}.
\]

(3.23)

The scalar exchange contribution to the trispectrum \( T_{\zeta,se}(\eta = 0, k_1, k_2, k_3, k_4) \) depends on six parameters. As in the case of the contact term contribution, we consider the equilateral configurations by setting the amplitudes of all momenta to be the same (see eq. (3.10)). Then the remaining independent variables are \( k_1 \cdot k_2 / (k_1 k_2) \) and \( k_1 \cdot k_3 / (k_1 k_3) \). As shown in eq. (C.30), for equilateral configurations the scalar exchange contribution is reduced to

\[
T_{\zeta,se}(k, C_2, C_3, C_4) = 1.190 \times 10^{-16} \left( \frac{(P_\zeta(k))^{1/2}}{4.8 \times 10^{-5}} \right)^{22/5} \frac{\beta^2}{\alpha^{3/5}} k^{-9} \sum_{i=2,3,4} [A(C_i) + B(C_i)],
\]

(3.24)

\[
A(C) = \sqrt{2\pi (1 + C)^2} \int_0^\infty dy_2 \int_{y_2}^\infty dy_1 y_1^{9/2} y_2^{9/2}
\times \left\{ C \left( K_{3/4}(y_1^2) \right)^2 K_{1/4} (2(1+C)y_1^2) - K_{3/4}(y_1^2) K_{1/4}(y_1^2) K_{3/4} (2(1+C)y_1^2) \right\}
\times \left\{ C \left( K_{3/4}(y_2^2) \right)^2 I_{1/4} (2(1+C)y_2^2) + K_{3/4}(y_2^2) K_{1/4}(y_2^2) I_{3/4} (2(1+C)y_2^2) \right\},
\]

(3.25)
and figure 12 shows that the scalar exchange contribution to the

$\trispectrum$ has the largest value

where $C_i$ ($i = 2, 3, 4$) are defined by eqs. (3.12) and constrained by momentum conservation as (3.13). The scalar exchange contribution $T_{\zeta,se}(k, C_2, C_3, C_4)$ is decomposed into six parts: those represented by $A(C_i)$ and $B(C_i)$ ($i = 2, 3, 4$). The functions $A(C)$ and $B(C)$ defined above are shown in figure 5 and figure 6, respectively. Figure 7 and figure 8 show the sums $A(C_2) + A(C_3) + A(C_4)$ and $B(C_2) + B(C_3) + C(C_4)$, respectively, with $C_4 = -1 - C_2 - C_3$ as functions of $C_2$ and $C_3$.

Figure 9 is the plot of the total scalar exchange contribution $T_{\zeta,se}(k, C_2, C_3, C_4)$, which is the sum of the six parts. In the most symmetric case where $C_2 = C_3 = C_4 = -1/3$, the trispectrum has the largest value

$$T_{\zeta,se}(k, C_2 = -\frac{1}{3}, C_3 = \frac{1}{3}, C_4 = -\frac{1}{3}) = 2.561 \times 10^{-17} \left( \frac{P_\zeta(k)}{4.8 \times 10^{-5}} \right)^{22/5} \frac{\beta^2}{\alpha^{8/5}} k^{-9}. \quad (3.27)$$

This is always positive. Actually, figure 9 shows that the scalar exchange contribution to the equilateral trispectrum $T_{\zeta,se}(k, C_2, C_3, C_4)$ is positive in all the parameter region.

Following ref. [12], we define the nonlinear parameter $\tau^se_{NL}$ as

$$\tau^se_{NL}(k_1, k_2, k_3, k_4) = \frac{T_{\zeta,se}(k_1, k_2, k_3, k_4)}{(2\pi^2 P_\zeta(k))^2} \prod_{i=1}^{4} k_i^3 \times \left[ (k_1^2 k_2^2 + k_3^2 k_4^2) (k_{13}^{-3} + k_{14}^{-3}) + (k_1^2 k_3^2 + k_2^2 k_4^2) (k_{12}^{-3} + k_{14}^{-3}) + (k_1^2 k_2^2 + k_3^2 k_4^2) (k_{12}^{-3} + k_{13}^{-3}) \right]^{-1}. \quad (3.28)$$
Figure 8. The plot of the sum \( B(C_2) + B(C_3) + B(C_4) \) as a function of \( C_2 \) and \( C_3 \) where \( C_4 = -1 - C_2 - C_3 \). The vertical axis is rescaled by the value at \( C_2 = C_3 = -1/3 \).

Figure 9. This plot shows the dependence of \( T_{\chi,se}(k, C_2, C_3, C_4) \) on \( C_2 \) and \( C_3 \), where \( C_4 = -1 - C_2 - C_3 \). The vertical axis is rescaled by the value at \( C_2 = C_3 = -1/3 \).

As shown in eq. (C.32), \( \tau_{\chi,se}^{\text{NL}} \) for the equilateral configurations is

\[
\tau_{\chi,se}^{\text{NL}}(k, C_2, C_3, C_4) \simeq 8.945 \times 10^5 \times \frac{\beta^2}{\alpha^{8/5}} \left( \frac{(P_\chi(k))^{1/2}}{4.8 \times 10^{-5}} \right)^{-8/5} \sum_{i=2,3,4} \frac{[A(C_i) + B(C_i)]}{\sum_{i=2,3,4}(1 + C_i)^{-3/2}}.
\]

Figure 10 shows the plot of the shape of \( \tau_{\chi,se}^{\text{NL}} \).

In the most symmetric case where \( C_2 = C_3 = C_4 = -1/3 \), \( \tau_{\chi,se}^{\text{NL}} \) becomes

\[
\tau_{\chi,se}^{\text{NL}}(k, C_2 = -1/3, C_3 = -1/3, C_4 = -1/3) \simeq 3.494 \times 10^4 \times \frac{\beta^2}{\alpha^{8/5}} \left( \frac{(P_\chi(k))^2}{4.8 \times 10^{-5}} \right)^{-8/5}.
\]
4 Summary and discussion

In this work we have calculated and investigated the trispectrum of curvature perturbation generated during ghost inflation.

The analysis of scaling dimensions of operators makes it possible to identify the leading diagrams contributing to the trispectrum in ghost inflation. Actually, there are two leading-order contributions. One is represented by a diagram with one four-point vertex. This contribution is called contact term contribution. The other is represented by a diagram with two three-point vertices and called scalar exchange contribution. We have analyzed these two contributions separately.

We have obtained general expressions for the two contributions as functions of six independent parameters. The six parameters are amplitudes of four 3-momenta and two angles between momenta. (Note that the sum of four 3-momenta must vanish because of the momentum conservation.)

From the known result for the bispectrum [20], it is expected that the equilateral configuration is most important also for the trispectrum. In order to calculate the concrete values, we have focused on the equilateral case where all momenta has the same amplitude and where there remain two independent angular parameters as well as an overall amplitude of 3-momenta. Then we have calculated the non-linear parameters $\tau_{\NL}^{cc}$ and $\tau_{\NL}^{se}$ for the contact term contribution and the scalar exchange contribution, respectively.

The scalar exchange contribution is comparable to the contact term contribution. We can see from eq. (3.14) and eq. (3.27) that the coefficient of the trispectrum from the scalar exchange contribution is only about 20 times larger than that from the contact term contribution at the most symmetric point $C_2 = C_3 = C_4 = -1/3$. While from the viewpoint of the effective field theory both $\beta$ and $\tilde{\gamma}$ are expected to be order unity, we do not know their actual values. For this reason we do not consider the difference of factor "20" so large. Moreover, at the most symmetric point the trispectrum from the scalar exchange contribution has the largest value while the trispectrum from the contact term contribution has the smallest value. Therefore, in regions away from the most symmetric point, both contributions are important even if we take the numerical factor "20" at face value.
In the case of local-type non-Gaussianity, it was forecasted that PLANCK will give the constraint $|\tau_{NL}| \sim 560[22]$. In the present paper we have shown that $\tau_{NL}^{cc}$ and $\tau_{NL}^{se}$ are typically of order $O(10^4)$. Therefore, the trispectrum from ghost inflation is probably detectable by PLANCK. Note, however, that the meaning of the non-linear parameter $\tau_{NL}$ is different for different types of non-Gaussianities. (The same is true for the non-linear parameter $f_{NL}$ of bispectra.) Therefore, as a future work, it is important to investigate detectability of the trispectrum predicted by ghost inflation in more detail.

Now let us compare our results with the trispectrum from DBI inflation calculated in refs. [12, 13]. The overall behaviors of trispectra are indeed similar. The trispectrum from ghost inflation has a peak at the equilateral configurations, i.e. when all four 3-momenta have the same amplitude, because non-Gaussianity is mainly generated in the horizon-crossing epoch. This feature of trispectrum is shared with DBI inflation. Moreover, the dependence of the equilateral trispectrum on the angular variables $C_2$ and $C_3$ also has similarities. In both DBI inflation and ghost inflation, the scalar exchange contribution has the maximum value at the most symmetric point $C_2 = C_3 = C_4 = -1/3$, and the absolute value of the contact term contribution becomes minimum at that point. The contact term contribution to the equilateral trispectrum has similar dependence on $C_2$ and $C_3$ in the two models of inflation. This can be easily seen by comparing figure 3 in the present paper and the right figure of figure 1 in ref. [13].

There are also some differences between the trispectrum from DBI inflation and that from ghost inflation. In DBI inflation, the value of the equilateral trispectrum is almost constant except for the edge region near the boundaries defined by $C_i = -1$ ($i = 2, 3, 4$). This feature can be seen in, e.g., the left figure of figure 1 of ref. [13]. The trispectrum rapidly decreases near the boundaries $C_i = -1$ and the plateau looks like a triangle. On the other hand, as we can see in figure 9 of the present paper, in ghost inflation the value of the equilateral trispectrum smoothly decreases towards the boundaries $C_i = -1$ ($i = 2, 3, 4$). As a result the shape of the plateau looks different from that in DBI inflation. While we can in principle distinguish ghost inflation from DBI inflation by the angular dependence of equilateral trispectra, dependence on other parameters may also give useful information. For instance, instead of fixing the ratios among lengths of wave vectors and looking at angular dependence, we can fix the angles as $k_1 \cdot k_2 = k_1 \cdot k_3 = k_2 \cdot k_3 = 0$ and plot the trispectra as functions of the ratios $k_2/k_1$ and $k_3/k_1$.

If we look into actual values, we find an important difference. (Note that those figures mentioned above are normalized by the values at $C_2 = C_3 = C_4 = -1/3$ and thus do not tell the actual values of trispectra.) The sign and magnitude of the contact term contribution depend on the sign and magnitude of the quartic coupling constant. (This is in contrast to the scalar exchange contribution, whose sign does not depend on the sign of the cubic coupling constant.) In DBI inflation, the quartic coupling constant is determined by the sound speed. As a result, the equilateral trispectrum from DBI inflation has a positive value at the most symmetric configuration $C_2 = C_3 = C_4 = -1/3$. On the other hand, in ghost inflation the dimensionless quartic coupling constant is an arbitrary parameter of order unity and thus the equilateral trispectrum at $C_2 = C_3 = C_4 = -1/3$ can be either positive or negative. Therefore, if equilateral-type non-Gaussianity is detected either by bispectrum or by trispectrum and if the negative equilateral trispectrum at the most symmetric point is observed, then it will support the ghost inflation scenario.
Note added. While we were preparing the present paper, ref. [25] appeared on the arXiv. Before that, we had finished all calculations and presented our results at the workshop "The non-Gaussian universe" at Yukawa Institute for Theoretical Physics on March 25, 2010. The presentation file has been available from the workshop website at http://www2.yukawa.kyoto-u.ac.jp/~nlg/2010_3/program.htm since March 27, 2010.

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A Review of in-in formalism

In Heisenberg picture, the expectation value of an observable is

\[ \langle \Omega | Q(t) | \Omega \rangle, \quad (A.1) \]

where \( Q(t) \) is a Heisenberg operator corresponding to the observable and \( | \Omega \rangle \) is the vacuum state in the initial time. A Heisenberg operator is sandwiched between initial vacuum states [23].

The time evolution of the Heisenberg operator is

\[ Q(t) = e^{iH(t-t_0)}Q(t_0)e^{-iH(t-t_0)}, \quad (A.2) \]

where \( H \) is Hamiltonian. On the other hand, an operator in interaction picture evolves by the Hamiltonian \( H_{\text{free}} \) of the free theory as

\[ Q_I(t) = e^{iH_{\text{free}}(t-t_0)}Q(t_0)e^{-iH_{\text{free}}(t-t_0)}, \quad (A.3) \]

The relation between these operators is

\[ Q(t) = [U(t,t_0)]^\dagger Q_I(t)U(t,t_0), \quad (A.4) \]

where

\[ U(t,t') \equiv e^{iH_{\text{free}}(t-t')}e^{-iH(t-t')} = T \left\{ \exp \left[ -i \int_{t'}^t dt'' H_I(t'') \right] \right\} \quad (t \geq t'), \quad (A.5) \]

\[ H_I = H - H_{\text{free}}, \quad (A.6) \]

and \( T \) indicates time-ordered products.
Let us now consider the time evolution of the vacuum state. First, we evolve the vacuum state $|0\rangle_{\text{free}}$ of $H_{\text{free}}$ by the total Hamiltonian $H$:

$$e^{-iH(t_0-t_m)}|0\rangle_{\text{free}} = e^{-iE_0(t_0-t_m)}|\Omega\rangle_{\text{free}} + \sum_n e^{-iE_n(t_0-t_m)}|n\rangle \langle n|_{\text{free}}$$

$$= e^{-iE_0(t_0-t_m)}|\Omega\rangle \quad (t_m \rightarrow -\infty(1-i\epsilon)), \quad \text{(A.7)}$$

where $|n\rangle$ ($n = 1, 2, \ldots$) represent excited eigenstates of $H$ and

$$H|\Omega\rangle = E_0|\Omega\rangle, \quad H|n\rangle = E_n|n\rangle. \quad \text{(A.8)}$$

Thus,

$$|\Omega\rangle = C U(t_0, -\infty) |0\rangle_{\text{free}}, \quad \text{(A.9)}$$

where $C$ is a constant factor. Similarly, we have

$$\langle \Omega| = C^\ast \langle 0| U(t_0, -\infty)^\dagger. \quad \text{(A.10)}$$

By taking the inner product $\langle \Omega|\Omega\rangle$, we can determine $CC^\ast$ as

$$1 = \langle \Omega|\Omega\rangle = CC^\ast. \quad \text{(A.11)}$$

Eventually, we have

$$\langle \Omega|Q(t)|\Omega\rangle_{\text{free}} = \langle 0| U(t_0, -\infty)^\dagger Q(t) U(t_0, -\infty) |0\rangle_{\text{free}}$$

$$= \langle 0| U(t, -\infty)^\dagger \sum_{N=0}^{\infty} \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \cdots \int_{-\infty}^{t_N} dt_N \cdot H_f(t_1), H_f(t_2), \ldots |0\rangle_{\text{free}}.$$  \quad \text{(A.12)}$$

B Interaction term of Hamiltonian

If time derivatives of fields are included in interaction terms in a Lagrangian, the interaction terms in the Hamiltonian are not simply the minus the corresponding terms in the Lagrangian. The time evolution of a quantum state is driven by a Hamiltonian (See appendix A) but a theory is usually defined in terms of a Lagrangian. Thus, it is important to understand the difference between interaction terms in a Lagrangian and those in a Hamiltonian. In this appendix, we consider this issue and obtain a perturbative expression for the difference. In Ref [11], this problem was investigated for a system with one particle. This appendix extends it to a multi particle system.

We consider the case where the Lagrangian is of the following form:

$$L(q^a, \dot{q}^a, t) = L_{\text{free}} + L_{\text{int}}(q^a, \dot{q}^a, t), \quad \text{(B.1)}$$

$$L_{\text{free}} \equiv \frac{1}{2} g_{ab}(t) \dot{q}^a \dot{q}^b - \frac{1}{2} h_{ab}(t) q^a q^b + A_{ab}(t) q^a \dot{q}^b, \quad \text{(B.2)}$$

where $g_{ab}$, $h_{ab}$ and $A_{ab}$ are independent of $q^a$ (but can depend on time) and $L_{\text{int}}(q^a, \dot{q}^a, t)$ represents terms of third or higher order in $q^a$ and $\dot{q}^a$. For simplicity, we consider the case
where there is no constraint. In this case the inverse matrix of \( g_{ab}(t) \) exists and we denote it by \( g^{ab}(t) \). The canonical momenta are
\[
p_a = \frac{\partial L}{\partial \dot{q}^a} = g_{ab}(t)\dot{q}^b + A_{ab}(t)q^b + \frac{\partial L_{\text{int}}}{\partial \dot{q}^a}(q^a, \dot{q}^a(q^b, p_b, t), t), \tag{B.3}
\]
and the Hamiltonian is
\[
H(t) = p_a \dot{q}^a(q^c, p_c, t) - L(q^a, \dot{q}^a(q^c, p_c, t), t) = p_a \dot{q}^a(q^c, p_c, t) - \frac{1}{2} g_{ab}(t)\dot{q}^a \dot{q}^b + \frac{1}{2} h_{ab}(t)q^a q^b - L_{\text{int}}(q^a, \dot{q}^a(q^c, p_c, t), t)
\]
\[
= \frac{1}{2}g^{ab}(t)(p_a - A_{ac}(t)q^c)(p_b - A_{bd}(t)q^d) + \frac{1}{2} h_{ab}(t)q^a q^b - L_{\text{int}}(q^a, \dot{q}^a(q^b, p_b, t), t)
\]
\[
- \frac{1}{2}g^{ab}(t)\frac{\partial L_{\text{int}}}{\partial q^a}(q^c, \dot{q}^c(q^d, p_d, t), t)\frac{\partial L_{\text{int}}}{\partial q^b}(q^c, \dot{q}^c(q^d, p_d, t), t), \tag{B.4}
\]
where \( \dot{q}^a(q^b, p_b, t) \) is defined by solving eq. (B.3) with respect to \( \dot{q}^a \).

Now, we quantize the theory. What we have to do is to promote the variables \( q^a \) and \( p_a \) to the corresponding operators \( Q^a \) and \( P_a \). Then we have the standard commutation relations,
\[
[Q^a, P_b] = i\delta^a_b, \quad [Q^a, Q^b] = [P_a, P_b] = 0. \tag{B.5}
\]
A complete set of quantum states is given by the eigenstates of \( Q^a \) or \( P_a \) which satisfy, respectively,
\[
Q^a |q\rangle = q |q\rangle, \quad P_a |p\rangle = p |p\rangle, \tag{B.6}
\]
and the inner products of them are
\[
\langle q'|q\rangle = \prod_a \delta_{q'^a q^a}, \quad \langle p'|p\rangle = \prod_a \delta_{p'^a p_a}, \quad \langle q|p\rangle = \prod_a \frac{1}{\sqrt{2\pi}} \exp(iq^a p_a). \tag{B.7}
\]

In order to use the in-in formalism reviewed in appendix A, we should adopt the interaction representation. The time-evolution equation of the operators \( Q^a_t \) and \( P_{t,a} \) is a linear differential equation, where the subscript “I” implies that these operators are in the interaction representation. In the interaction representation, an operator \( O_I \) depends on time as
\[
O_I(t) = \left[ T \left\{ \exp(i \int_{t_0}^t H_{\text{free}}(t')dt') \right\} \right] O(t_0) \left[ T \left\{ \exp(-i \int_{t_0}^t H_{\text{free}}(t')dt') \right\} \right], \tag{B.8}
\]
where
\[
H_{\text{free}}(t) = -\frac{1}{2}g^{ab}(t)(p_a - A_{ac}(t)q^c)(p_b - A_{bd}(t)q^d) - \frac{1}{2} h_{ab}(t)q^a q^b.
\]
Thus, the time derivative of the operator \( O_I(t) \) is
\[
\dot{O}_I(t) = i \left[ T \left\{ \exp(i \int_{t_0}^t H_{\text{free}}(t')dt') \right\} \right] H_{\text{free}}(t) O(t_0) \left[ T \left\{ \exp(-i \int_{t_0}^t H_{\text{free}}(t')dt') \right\} \right]
\]
\[
- i \left[ T \left\{ \exp(i \int_{t_0}^t H_{\text{free}}(t')dt') \right\} \right] O(t_0) H_{\text{free}}(t) \left[ T \left\{ \exp(-i \int_{t_0}^t H_{\text{free}}(t')dt') \right\} \right]
\]
\[
= -i \{ O_I(t), H_{\text{free},I}(t) \} \tag{B.10}
\]
The commutation relations \((B.5)\) lead to the same commutation relations for \(Q_I^a\) and \(P_{I,a}\) as
\[
\{Q_I^a(t), P_{I,b}(t)\} = i\delta_{ab}^a, \quad [Q_I^a(t), Q_I^b(t)] = [P_{I,a}(t), P_{I,b}(t)] = 0,
\]
and their time derivatives are
\[
\begin{align*}
\dot{P}_{I,a}(t) &= -h_{ab}(t)Q_I^b(t), \\
\dot{Q}_I^a(t) &= g^{ab}(t)(P_{I,b}(t) - A_{bc}(t)Q_I^c(t)).
\end{align*}
\]

Since these equations are the same as those for free particles, we can easily represent \(Q_I^a\) and \(P_{I,a}\) by using the creation and annihilation operators as in the case of free particles.

The time evolution of a state \(|\psi(t)\rangle_I\) is represented as
\[
i\frac{d}{dt} |\psi(t)\rangle_I = H_{int,I}(t) |\psi(t)\rangle_I,
\]
where
\[
H_{int,I}(t) \equiv \left[ \bar{T} \left\{ \exp(i \int_0^t H_{free}(t')dt') \right\} \right] H_{int}(Q^a, P_a, t) \left[ T \left\{ \exp(-i \int_0^t H_{free}(t')dt') \right\} \right]
\]
and
\[
H_{int}(Q^a, P_a, t) = H(Q^a, P_a, t) - H_{free}(Q^a, P_a, t)
\]
\[
= -L_{int}(Q^a, \dot{Q}^a(Q^b, P_b), t) - \frac{1}{2} g^{ab}(t) \frac{\partial L_{int}}{\partial \dot{q}^b}(Q'^b, \dot{q}'(Q'^d, P_d), t) \frac{\partial L_{int}}{\partial q^b}(Q'^c, \dot{q}'(Q'^d, P_d), t).
\]

Solving eq. \((B.3)\) with respect to \(\dot{q}^a\) iteratively, we obtain
\[
\dot{q}^a(Q^b, P_b) = g^{ab}(t) \left( (P_b - A_{bc}(t)Q^c) - \frac{\partial L_{int}}{\partial \dot{q}^b}(Q'^c, \dot{q}'(Q'^d, P_d), t) \right)
\]
\[
= \dot{Q}^a - g^{ab}(t) \frac{\partial L_{int}}{\partial \dot{q}^b}(Q'^c, \dot{q}'(Q'^d, P_d), t) + (\text{more than third order in } Q^a \text{ and } P_a),
\]

where we have used eq. \((B.13)\). Substituting this to eq. \((B.16)\), we can see the difference between interaction terms in the Hamiltonian and those in the Lagrangian:
\[
H_{int}(Q_I^a, P_{I,a}, t) = -L_{int}(Q_I^a, \dot{Q}_I^a, \dot{Q}_I^a, t) + \frac{1}{2} g^{ab}(t) \frac{\partial L_{int}}{\partial \dot{q}^a}(Q_I^b, \dot{Q}_I^b, t) \frac{\partial L_{int}}{\partial \dot{q}^a}(Q_I^b, \dot{Q}_I^b, t)
\]
\[
+ (\text{more than fifth order in } Q^a \text{ and } P_a).
\]

C Detailed calculations

In this appendix we will show the concrete calculations of the trispectrum from ghost inflation.
C.1 Contact term contribution

The contact term contribution to the four-point function can be written as

\[
\langle \pi_1(t) \pi_2(t) \pi_3(t) \pi_4(t) \rangle_{cc} \\
\equiv i \int_{-\infty}^{t} dt' \left\langle \frac{\gamma}{8M^4} \int d^3 x' \frac{\nabla \pi(t', x')}{a(t')} \pi_1(t) \pi_2(t) \pi_3(t) \pi_4(t) \right\rangle. \tag{C.1}
\]

Transforming variables as

\[
dt' = a(\eta')d\eta, \quad \pi(t') \equiv \frac{u(\eta')}{a(\eta')}, \quad a(\eta) = -\frac{1}{H\eta}, \tag{C.2}
\]

we obtain

\[
\langle \pi_1(t) \pi_2(t) \pi_3(t) \pi_4(t) \rangle_{cc} \\
= i \int_{-\infty}^{\eta} d\eta' \frac{\gamma H^2}{8M^4} \left\langle \frac{\gamma}{8M^4} \int d^3 x' (-\nabla \eta' u(\eta', x'))^4 \right. \\
\left. \times \left( -\eta u_{k_1}(\eta) \right) \left( -\eta u_{k_2}(\eta) \right) \left( -\eta u_{k_3}(\eta) \right) \left( -\eta u_{k_4}(\eta) \right) \right\rangle \\
= \frac{\gamma H^2}{8M^4} \int_{-\infty}^{\eta} d\eta' \int d^3 k_1' d^3 k_2' d^3 k_3' d^3 k_4' \left( -k_1' \cdot k_2' - k_2' \cdot k_3' - k_3' \cdot k_4' \right) \\
\times \left\langle \left( -\eta u_{k_1}(\eta) \right) \left( -\eta u_{k_2}(\eta) \right) \left( -\eta u_{k_3}(\eta) \right) \left( -\eta u_{k_4}(\eta) \right) \right\rangle, \tag{C.3}
\]

where \( u_k(\eta) \) is written in terms of mode functions \( w_k(\eta), w^*_k(\eta) \) and operators \( a_k \) and \( a^\dagger_k \) as

\[
u_k(\eta) = w_k(\eta)a_k + w^*_k(\eta)a^\dagger_k, \tag{C.4}
\]

\[
[a_k, a^\dagger_{k'}] = (2\pi)^3 \delta^3(k - k'). \tag{C.5}
\]

Substituting eq. (C.4) into eq. (C.3), we have

\[
\langle \pi_1(t) \pi_2(t) \pi_3(t) \pi_4(t) \rangle_{cc} \\
= i \frac{\gamma H^2}{8M^4} \int_{-\infty}^{\eta} d\eta' (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4) \left\{ (k_1 \cdot k_2)(k_3 \cdot k_4) + (23 \text{ terms}) \right\} \\
\times \left\{ (-\eta' w_{-k_1}(\eta'))(-\eta' w_{-k_2}(\eta'))(-\eta' w_{-k_3}(\eta'))(-\eta' w_{-k_4}(\eta')) \right. \\
\left. \left( -\eta w^*_{-k_1}(\eta)(-\eta w^*_{-k_2}(\eta))(-\eta w^*_{-k_3}(\eta))(-\eta w^*_{-k_4}(\eta)) \right) \right\} - c.c. \tag{C.6}
\]

The contact term contribution to the trispectrum \( T^\eta_{\pi,cc}(k_1, k_2, k_3, k_4) \) is

\[
T^\eta_{\pi,cc}(k_1, k_2, k_3, k_4) \\
= \frac{\gamma H^2}{8M^4} 2\Re \left\{ i \left( (k_1 \cdot k_2)(k_3 \cdot k_4) + (23 \text{ terms}) \right) \right\}.
\]

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where 
\[
\begin{align*}
\pi,cc & \equiv \int d\eta' (-\eta' w_{-k_1}(\eta')(-\eta' w_{-k_2}(\eta'))(-\eta' w_{-k_3}(\eta'))(-\eta' w_{-k_4}(\eta')) \Bigg] .
\end{align*}
\]
(C.7)

We are interested in the case where the physical length scales of all arguments \( k_i \) \( (i = 1, 2, 3, 4) \) of the trispectrum are larger than the Hubble scale of de Sitter spacetime at the end of inflation, which means

\[
k_i \eta \ll 1.
\]
(C.8)

Therefore, we approximate the trispectrum at the end of inflation as \( T_{\pi,cc}^{(n) \rightarrow 0}(k_1, k_2, k_3, k_4) \). Then, we have

\[
T_{\pi,cc}(k_1, k_2, k_3, k_4) \equiv T_{\pi,cc}^{(n) \rightarrow 0}(k_1, k_2, k_3, k_4)
\]

\[
= \frac{\hat{\gamma} H^8}{4 M^4} \Re \Bigg\{ i \int_{-\infty}^{\eta} d\eta' \frac{\pi}{\Gamma(1/4)} \frac{\eta^2}{q_1 q_2 q_3} \left( (\eta')^{3/2} H_{3/4}^{(1)}(q_1 \eta^2) \left( (\eta')^{3/2} H_{3/4}^{(1)}(q_2 \eta^2) \right) + (23 \text{ terms}) \right) \Bigg\}
\]

\[
= \frac{\hat{\gamma} H^8}{20 M^4} \left( \frac{\pi}{\Gamma(1/4)} \right)^4 \frac{\eta^2}{q_1 q_2 q_3} \left( (\eta')^{3/2} H_{3/4}^{(1)}(q_1 \eta^2) \left( (\eta')^{3/2} H_{3/4}^{(1)}(q_2 \eta^2) \right) + (23 \text{ terms}) \right)
\]

where \( q_i \) is defined in eq. (3.9) and we have used

\[
\eta w_{-k_i}(\eta) = -\sqrt{\frac{\pi}{8}} (\eta')^{3/2} H_{3/4}^{(1)}(q_i \eta^2),
\]
(C.10)

\[
\eta w_{-k_i}(\eta) \mid \eta = 0 = -i \sqrt{\frac{\pi}{8}} \left( \frac{q_i}{2} \right)^{3/4} \frac{1}{\Gamma(1/4)}.
\]
(C.11)

Transforming variable as

\[
- \sqrt{q} \eta = x \quad (q \equiv q_1 = q_2 = q_3 = q_4),
\]
(C.12)

the contact term contribution to the equilateral trispectrum is

\[
T_{\pi,cc}(k, C_2, C_3, C_4)
\]

\[
= \frac{\hat{\gamma} H^8}{20 M^4} \left( \frac{\pi}{\Gamma(1/4)} \right)^4 q^{-3} \left( 8 \sum_{i=2,3,4} C_i^2 \right) k^4
\]

\[\text{− 23 −}\]
\[ \times \Re \left\{ i \int_{0}^{\infty} \left( \frac{dx}{\sqrt{q}} \right) \left( \left( \frac{x}{\sqrt{q}} \right)^{3/2} H_{3/4}^{(1)}(x^2) \right)^4 \right\} \]

\[ = \sqrt{2} \frac{\gamma}{\alpha^{13/4}} H^4 \left( \frac{H}{M} \right)^{-\frac{1}{2}} \left( \frac{\pi}{\Gamma(1/4)} \right)^4 \left( \sum_{i=2,3,4} C_i^2 \right) k^{-9} \times \Re \left\{ i \int_{0}^{\infty} dx \left( x^{3/2} H_{3/4}^{(1)}(x^2) \right)^4 \right\} , \] (C.13)

where \( C_i \) is defined in eq. (3.12) and \( C_i \)s are constrained by the momentum conservation as shown in (3.13).

By using the relation (2.23) between \( \zeta \) and \( \pi \), we obtain the contact term contribution to the trispectrum of curvature perturbation as

\[ T_{\zeta,cc}(k, C_2, C_3, C_4) = \left( \frac{H}{M} \right)^4 \left( \frac{T_{\pi,cc}(k, C_2, C_3, C_4)}{M^4} \right) \]

\[ = \sqrt{2} \frac{\gamma}{\alpha^{13/4}} \left( \frac{H}{M} \right)^{11/2} \left( \frac{\pi}{\Gamma(1/4)} \right)^4 \left( \sum_{i=2,3,4} C_i^2 \right) k^{-9} \times \Re \left\{ i \int_{0}^{\infty} dx \left( x^{3/2} H_{3/4}^{(1)}(x^2) \right)^4 \right\} . \] (C.14)

In order to tame the asymptotic behavior of the integrand at \( x \to \infty \), we alter the integral route. The asymptotic expansion of Hunkel function is

\[ H_{\nu}^{(1)}(z) \to \sqrt{\frac{2}{\pi z}} \exp \left[ i \left( z - \frac{2\nu + 1}{4} \pi \right) \right] \quad (z \to \infty). \] (C.15)

Thus, displacing the integral route from the real axis to an angle of \( \pi/4 \) does not change the value of integral. This change of the integral route is consistent with the \( i\epsilon \) prescription in the in-in formalism and picks up the correct vacuum. (See (A.7).) We define this axis as \( y \)-axis, which means

\[ x = \exp \left( \frac{\pi i}{4} \right) y. \] (C.16)

Under this transformation of variable, we have

\[ x^{3/2} H_{3/4}^{(1)}(x^2) = \frac{2}{i \pi} y^{3/2} K_{3/4}(y^2). \] (C.17)

The expression (2.24) for the power spectrum \( P_{\zeta}(k) \) can be inverted to express \( H/M \) in terms of the power spectrum as

\[ \frac{H}{M} = \left( \pi \left( \Gamma(1/4) \right)^2 \alpha^{3/4} P_{\zeta}(k) \right)^{2/5}. \] (C.18)

Hence, we obtain

\[ T_{\zeta,cc}(k, C_2, C_3, C_4) = \left( 16 \sqrt{2}\pi^{11/5} \left( \Gamma(1/4) \right)^{2/5} \Re \left\{ i \int_{0}^{\infty} dy \exp \left( \frac{\pi i}{4} \right) \left( y^{3/2} K_{3/4}(y^2) \right)^4 \right\} \right) \]

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The scalar exchange contribution to the four point function is defined as

\[ \langle \pi_{i-} \pi_{\tau} k_{NL} \rangle \]

\[ \text{C.2 Scalar exchange contribution} \]

\[ \langle \pi_{i-} \pi_{\tau} k_{NL} \rangle = 1 \times (P_{\zeta}(k))^{11/5} \frac{\tilde{\gamma}}{\alpha^{8/5}} \left( \sum_{i=2,3,4} C_i^2 \right) k^{-9} \]

\[ \simeq -2.215 \times 10^{-17} \times \left( \frac{(P_{\zeta}(k))^{1/2}}{4.8 \times 10^{-5}} \right)^{22/5} \frac{\tilde{\gamma}}{\alpha^{8/5}} \left( \sum_{i=2,3,4} C_i^2 \right) k^{-9}. \quad (C.19) \]

In the equilateral case where eq. (3.10) is satisfied, \( \tau_{NL}^{cc} \) becomes

\[ \tau_{NL}^{cc}(k, C_2, C_3, C_4) = \frac{k^9}{\sqrt{2}} \left[ \sum_{i=2,3,4} (1 + C_i)^{-3/2} \right]^{-1} \frac{T_{\zeta,cc}(k)}{(2\pi^2 P_{\zeta}(k))^3} \]

\[ = \frac{1}{8\pi^2 (\Gamma(1/4))^4 \alpha^{13/4}} \frac{\tilde{\gamma}}{(H/M)^{11/2}} \left( P_{\zeta}(k) \right)^{-3} \Re \left\{ i \int_0^\infty dx \left( x^{3/2} H_{3/4}^1(x^2) \right)^4 \right\} \times \frac{\sum_{i=2,3,4} C_i^2}{\sum_{i=2,3,4} (1 + C_i)^{-3/2}}. \quad (C.20) \]

Substituting eq. (C.18) into eq. (C.20), we obtain

\[ \tau_{NL}^{cc}(k, C_2, C_3, C_4) = \frac{\tilde{\gamma}}{\alpha^{8/5}} \left( P_{\zeta}(k) \right)^{-4/5} \left( 2\pi^{-19/5} (\Gamma(1/4))^{2/5} \Re \left\{ i \int_0^\infty dy \exp \left( \frac{\tilde{\gamma}}{4} i \right) \left( y^{3/2} K_{3/4}(y^2) \right)^4 \right\} \right) \times \frac{\sum_{i=2,3,4} C_i^2}{\sum_{i=2,3,4} (1 + C_i)^{-3/2}} \]

\[ \simeq -1.665 \times 10^5 \times \frac{\tilde{\gamma}}{\alpha^{8/5}} \left( \frac{(P_{\zeta}(k))^{1/2}}{4.8 \times 10^{-5}} \right)^{-8/5} \frac{\sum_{i=2,3,4} C_i^2}{\sum_{i=2,3,4} (1 + C_i)^{-3/2}}. \quad (C.21) \]

C.2 Scalar exchange contribution

The scalar exchange contribution to the four point function is defined as

\[ \langle \pi_{k_1} \pi_{k_2} \pi_{k_3} \pi_{k_4} \rangle_{se} = - \int_{-\infty}^{t} dt_2 \int_{-\infty}^{t_2} dt_1 \left( \left[ \int d^3 x_1 \frac{\beta a(t_1)}{2M^2} \left( \frac{d}{dt_1} \pi(t_1, x_1) \right) (\nabla \pi(t_1, x_1))^2 \right] \left[ \int d^3 x_2 \frac{\beta a(t_2)}{2M^2} \left( \frac{d}{dt_2} \pi(t_2, x_2) \right) (\nabla \pi(t_2, x_2))^2 \right] \pi_{k_1} \pi_{k_2} \pi_{k_3} \pi_{k_4} \right) \]

\[ = - \int_{-\infty}^{t} dt_2 \int_{-\infty}^{t_2} dt_1 \left\{ \left[ \int d^3 x_1 \frac{\beta a(t_1)}{2M^2} \left( \frac{d}{dt_1} \pi(t_1, x_1) \right) (\nabla \pi(t_1, x_1))^2 \right] \pi_{k_1} \pi_{k_2} \pi_{k_3} \pi_{k_4} \right\} \]

\[ + \left[ \int d^3 x_2 \frac{\beta a(t_2)}{2M^2} \left( \frac{d}{dt_2} \pi(t_2, x_2) \right) (\nabla \pi(t_2, x_2))^2 \right] \pi_{k_1} \pi_{k_2} \pi_{k_3} \pi_{k_4} \]

\[ \left( \int d^3 x_1 \frac{\beta a(t_1)}{2M^2} \left( \frac{d}{dt_1} \pi(t_1, x_1) \right) (\nabla \pi(t_1, x_1))^2 \right) \pi_{k_1} \pi_{k_2} \pi_{k_3} \pi_{k_4} \]
\begin{align}
&+ \int_{-\infty}^t dt_2 \int_{-\infty}^t dt_1 \left( \left( \int d^3x_1 \frac{\beta a(t_1)}{2M^2} \left( \frac{d}{dt_1} \pi(t_1, x_1) \right) (\nabla \pi(t_1, x_1))^2 \right) \right. \\
&\left. \pi_k \pi_k \pi_k \pi_k \left( \int d^3x_2 \frac{\beta a(t_2)}{2M^2} \left( \frac{d}{dt_2} \pi(t_2, x_2) \right) (\nabla \pi(t_2, x_2))^2 \right) \right). \tag{C.22}
\end{align}

Using eqs. (C.2), (C.4) and (C.5), we can transform this as

\begin{align}
\langle \pi_{k_1} \pi_{k_2} \pi_{k_3} \pi_{k_4} \rangle_{se} & = \frac{\beta^2}{4M^4} \left\{ - \int_{-\infty}^0 a(\eta_2) d\eta_2 \int_{-\infty}^{\eta_2} a(\eta_1) d\eta_1 \left( \int d^3x_1 \left( \frac{d}{d\eta_1} u(\eta_1, x_1) \right) \right) \\
&\times \left( \frac{\nabla u(\eta_1, x_1)}{a(\eta_1)} \right)^2 \right. \left( \int d^3x_2 \left( \frac{d}{d\eta_2} u(\eta_2, x_2) \right) \right) \left( \frac{\nabla u(\eta_2, x_2)}{a(\eta_2)} \right)^2 + \text{h.c.} \\
&\left. + \int_{-\infty}^0 a(\eta_2) d\eta_2 \int_{-\infty}^{\eta_2} a(\eta_1) d\eta_1 \left( \int d^3x_1 \left( \frac{d}{d\eta_1} u(\eta_1, x_1) \right) \right) \left( \frac{\nabla u(\eta_1, x_1)}{a(\eta_1)} \right)^2 \right. \left( \int d^3x_2 \left( \frac{d}{d\eta_2} u(\eta_2, x_2) \right) \right) \left( \frac{\nabla u(\eta_2, x_2)}{a(\eta_2)} \right)^2 \right\} \left\{ - \int_{-\infty}^\eta d\eta_2 \int_{-\infty}^{\eta_2} d\eta_1 \right.
\end{align}

\begin{align}
&\left. \left( \frac{d}{d\eta_1} \right) \left( \frac{d}{d\eta_2} \right) \left( -\eta_1 w_{k_1+k_2}(\eta_1) \right) \right) \left( -\eta_1 w_{k_1+k_2}(\eta_1) \right) + 5 \text{ terms} \\
&+ \left\{ \frac{d}{d\eta_1} \right. \left( \frac{d}{d\eta_2} \right) \left( -\eta_1 w_{k_1+k_2}(\eta_1) \right) \left( -\eta_1 w_{k_1+k_2}(\eta_1) \right) + 11 \text{ terms} \right. \\
&\left. \left. \left( \frac{d}{d\eta_1} \right) \left( \frac{d}{d\eta_2} \right) \left( -\eta_1 w_{k_1+k_2}(\eta_1) \right) \right) \left( -\eta_1 w_{k_1+k_2}(\eta_1) \right) + 23 \text{ terms} \right. + \text{c.c.}
\end{align}
\[+ \int_{-\infty}^{\eta} \frac{d\eta_1}{\eta_1} \int_{-\infty}^{\eta_2} \frac{d\eta_2}{\eta_2} \left( \{ 4(k_1 \cdot k_2)(k_3 \cdot k_4)(-\eta_1 w_{-k_1}(\eta_1))(-\eta_1 w_{-k_2}(\eta_1)) \right. \]
\[\times \left( \frac{d}{d\eta_1}(-\eta_1 w_{|k_1+k_2|}(\eta_1)) \right) (-\eta w^*_{-k_1}(\eta)) (-\eta w^*_{-k_2}(\eta)) (-\eta w_{k_3}(\eta)) (-\eta w_{k_4}(\eta)) \]
\[\times \left( -\eta_2 w_{k_3}^*(\eta_2) \right) (-\eta_2 w_{k_4}^*(\eta_2)) \left( \frac{d}{d\eta_2}(-\eta_2 w_{|k_1+k_2|}^*(\eta_2)) \right) \]
\[\{ + 5 \text{ terms} \} \right) \]
\[+ \left\{ 4(-k_1^2 - k_1 \cdot k_2)(k_3 \cdot k_4)(-\eta_1 w_{-k_1}(\eta_1))(-\eta_1 w_{|k_1+k_2|}(\eta_1)) \right. \]
\[\times \left( -\eta w^*_{-k_1}(\eta) \right) (-\eta w^*_{-k_2}(\eta)) (-\eta w_{k_3}(\eta)) (-\eta w_{k_4}(\eta)) \]
\[\times \left( (-\eta_2 w_{k_3}^*(\eta_2)) (-\eta_2 w_{k_4}^*(\eta_2)) \right) \left( \frac{d}{d\eta_2}(-\eta_2 w_{|k_1+k_2|}^*(\eta_2)) \right) \]
\[\{ + 11 \text{ terms} \} \right) \]
\[+ \left\{ 4(k_1 \cdot k_2)(-k_3^2 - k_3 \cdot k_4)(-\eta_1 w_{-k_1}(\eta_1))(-\eta_1 w_{|k_1+k_2|}(\eta_1)) \right. \]
\[\times \left( -\eta w^*_{-k_1}(\eta) \right) (-\eta w^*_{-k_2}(\eta)) (-\eta w_{k_3}(\eta)) (-\eta w_{k_4}(\eta)) \]
\[\times \left( (-\eta_2 w_{k_3}^*(\eta_2)) (-\eta_2 w_{k_4}^*(\eta_2)) \right) \left( \frac{d}{d\eta_2}(-\eta_2 w_{|k_1+k_2|}^*(\eta_2)) \right) \]
\[\{ + 11 \text{ terms} \} \right) \]
\[+ \left\{ 4(-k_1^2 - k_1 \cdot k_2)(-k_3^2 - k_3 \cdot k_4)(-\eta_1 w_{-k_1}(\eta_1))(-\eta_1 w_{|k_1+k_2|}(\eta_1)) \right. \]
\[\times \left( -\eta w^*_{-k_1}(\eta) \right) (-\eta w^*_{-k_2}(\eta)) (-\eta w_{k_3}(\eta)) (-\eta w_{k_4}(\eta)) \]
\[\times \left( (-\eta_2 w_{k_3}^*(\eta_2)) (-\eta_2 w_{k_4}^*(\eta_2)) \right) \left( \frac{d}{d\eta_2}(-\eta_2 w_{|k_1+k_2|}^*(\eta_2)) \right) \]
\[\{ + 23 \text{ permutational terms} \} \} \right) \]
\[= \frac{\beta^2 H^8}{4 M^8} (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4) \left\{ - \int_{-\infty}^{\eta_1} \frac{d\eta_1}{\eta_1} \int_{-\infty}^{\eta_2} \frac{d\eta_2}{\eta_2} \int_{-\infty}^{\eta_2} \frac{d\eta_3}{\eta_3} \int_{-\infty}^{\eta_3} \frac{d\eta_4}{\eta_4} \right\} \]
\[\left\{ \left[ (k_1 \cdot k_2)(-\eta_1 w_{-k_1}(\eta_1))(-\eta_1 w_{-k_2}(\eta_1)) \right. \right. \]
\[\left. \times \left( \frac{d}{d\eta_1}(-\eta_1 w_{|k_1+k_2|}(\eta_1)) \right) \left( \frac{d}{d\eta_1}(-\eta_1 w_{|k_1+k_2|}(\eta_1)) \right) \right. \]
\[\times \left[ (k_3 \cdot k_4)(-\eta_2 w_{-k_3}(\eta_2))(-\eta_2 w_{-k_4}(\eta_2)) \right. \]
\[\left. \times \left( \frac{d}{d\eta_2}(-\eta_2 w_{|k_1+k_2|}(\eta_2)) \right) \left( \frac{d}{d\eta_2}(-\eta_2 w_{|k_1+k_2|}(\eta_2)) \right) \right. \]
\[\times \left[ (-\eta_3 w_{k_3}(\eta_3))(-\eta_3 w_{k_4}(\eta_3)) \right. \]
\[\left. \times \left( \frac{d}{d\eta_3}(-\eta_3 w_{|k_1+k_2|}(\eta_3)) \right) \left( \frac{d}{d\eta_3}(-\eta_3 w_{|k_1+k_2|}(\eta_3)) \right) \right. \]
\[\left. \}\right\} \]
\[+ 23 \text{ permutational terms} \} \} \}
\[+ \left\{ \left( \int_{-\infty}^{\eta_1} \frac{d\eta_1}{\eta_1} \left[ (k_1 \cdot k_2)(-\eta_1 w_{-k_1}(\eta_1))(-\eta_1 w_{-k_2}(\eta_1)) \right. \right. \]
\[\left. \times \left( \frac{d}{d\eta_1}(-\eta_1 w_{|k_1+k_2|}(\eta_1)) \right) \left( \frac{d}{d\eta_1}(-\eta_1 w_{|k_1+k_2|}(\eta_1)) \right) \right. \]
\[\times \left[ (k_3 \cdot k_4)(-\eta_2 w_{-k_3}(\eta_2))(-\eta_2 w_{-k_4}(\eta_2)) \right. \]
\[\left. \times \left( \frac{d}{d\eta_2}(-\eta_2 w_{|k_1+k_2|}(\eta_2)) \right) \left( \frac{d}{d\eta_2}(-\eta_2 w_{|k_1+k_2|}(\eta_2)) \right) \right. \]
\[\times \left[ (-\eta_3 w_{k_3}(\eta_3))(-\eta_3 w_{k_4}(\eta_3)) \right. \]
\[\left. \times \left( \frac{d}{d\eta_3}(-\eta_3 w_{|k_1+k_2|}(\eta_3)) \right) \left( \frac{d}{d\eta_3}(-\eta_3 w_{|k_1+k_2|}(\eta_3)) \right) \right. \]
\[\left. \}\right\} \}
\[+ 23 \text{ permutational terms} \} \]
\[
\left\{ \times (-\eta w^*_{-k_1}(\eta))(\eta w^*_{-k_2}(\eta))(-\eta w_k(\eta))(-\eta w_k^*(\eta)) \right\}
\]
\[
+ 23 \text{ permutational terms}
\]
\[
= \frac{\beta^2 \alpha H^{10}}{4 M^{10}} \left( \frac{2\pi}{k} \right)^3 (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4)
\]
\[
- \int_{-\infty}^{\eta_2} d\eta_1 \int_{-\infty}^{\eta_2} d\eta_2 \left\{ \{(k_1 \cdot k_2)(k_1^2 + k_2^2 + 2k_1 \cdot k_2)\left( (-\eta_1)^{3/2} H^{(1)}_{3/4}(q_1 \eta_1^2) \right) \right.
\]
\[
- 2(k_1^2 + k_1 \cdot k_2)k_2^2 \left( (-\eta_1)^{3/2} H^{(1)}_{3/4}(q_1 \eta_1^2) \right) \right)
\]
\[
\times \{(k_3 \cdot k_4)(k_3^2 + k_4^2 + 2k_3 \cdot k_4)\left( (-\eta_2)^{3/2} H^{(1)}_{3/4}(q_2 \eta_2^2) \right) \right.
\]
\[
- 2(k_3^2 + k_3 \cdot k_4)k_4^2 \left( (-\eta_2)^{3/2} H^{(1)}_{3/4}(q_2 \eta_2^2) \right) \right) \}
\]
\[
\times (-\eta w^*_{-k_1}(\eta))(-\eta w^*_{-k_2}(\eta))(-\eta w_k(\eta))(-\eta w_k^*(\eta))
\]
\[
+ 23 \text{ permutational terms} \right\} \] + c.c.
\]
\[
+ \left\{ \int_{-\infty}^{\eta} d\eta_1 [(k_1 \cdot k_2)(k_1^2 + k_2^2 + 2k_1 \cdot k_2)\left( (-\eta)^{3/2} H^{(1)}_{3/4}(q_1 \eta_1^2) \right) \right.
\]
\[
- 2(k_1^2 + k_1 \cdot k_2)k_2^2 \left( (-\eta)^{3/2} H^{(1)}_{3/4}(q_1 \eta_1^2) \right) \right)
\]
\[
\times \{(k_3 \cdot k_4)(k_3^2 + k_4^2 + 2k_3 \cdot k_4)\left( (-\eta)^{3/2} H^{(1)}_{3/4}(q_3 \eta_3^2) \right) \right.
\]
\[
- 2(k_3^2 + k_3 \cdot k_4)k_4^2 \left( (-\eta)^{3/2} H^{(1)}_{3/4}(q_3 \eta_3^2) \right) \right) \}
\]
\[
\times (-\eta w^*_{-k_1}(\eta))(-\eta w^*_{-k_2}(\eta))(-\eta w_k(\eta))(-\eta w_k^*(\eta))
\]
\[
+ 23 \text{ permutational terms} \right\}, \quad (C.23)
\]

where \( q_i \) and \( q_{ij} \) are defined in eq. (3.9) and eq. (3.23), respectively, and we have used the following identity:
\[
\frac{d}{d\eta} \left( (-\eta)^{3/2} H^{(i)}_{3/4}(q \eta^2) \right) = 2q \eta \left( (-\eta)^{3/2} H^{(i)}_{3/4}(q \eta^2) \right) \quad (i = 1, 2). \quad (C.24)
\]

Now let us take the limit \( \eta \to 0 \) and consider the equilateral case where the amplitudes
of all momenta are the same. Then, we have

\[
T_{\pi,se}(k, C_2, C_3, C_4) = \frac{2\beta^2 \alpha H^{10} k^8}{M^6} \left( \frac{\sqrt{\pi}}{2} \left( \frac{2}{q} \right)^{3/4} \Gamma(1/4)^{-1} \right)^4 \left( \frac{\pi}{8} \right)^3 q^{-11/2} \sum_{i=2,3,4} \left\{ 4(1 + C_i)^2 \left[ - \left( \int_0^\infty dx_2 \int_0^{x_2} dx_1 \times x_1^{9/2} \left( C_i H^{(1)}_{3/4}(x_1^2) H^{(1)}_{-1/4}(2(1 + C_i)x_1^2) - H^{(1)}_{3/4}(x_1^2) H^{(1)}_{-1/4}(2(1 + C_i)x_1^2) \right) \times x_2^{9/2} \left( C_i H^{(1)}_{3/4}(x_2^2) H^{(2)}_{-1/4}(2(1 + C_i)x_2^2) - H^{(1)}_{3/4}(x_2^2) H^{(1)}_{-1/4}(2(1 + C_i)x_2^2) \right) \right] + c.c. \right) \left[ \frac{x^{-1/2} H^{(1)}_{-1/4}(x^2)}{\pi^2 y^{-1/2} K_{-1/4}(y^2)} \right]^2 = \frac{27\sqrt{2}}{\pi} (\Gamma(1/4))^{-4} \beta^2 \alpha^{-13/4} H^4 \left( \frac{H}{M} \right)^{-\frac{2}{3}} k^{-9} \sum_{i=2,3,4} (A(C_i) + B(C_i)), \tag{C.25} \right.
\]

where \( C_i, A(C) \) and \( B(C) \) are defined in eq. (3.12) eq. (3.25) and eq. (3.26), respectively, and we have used eq. (C.17) and

\[
\begin{align*}
&x^{-1/2} H^{(1)}_{-1/4}(x^2) = \frac{2}{\pi^2} y^{-1/2} K_{-1/4}(y^2), \\
&x^{-1/2} H^{(2)}_{-1/4}(x^2) = \sqrt{2} y^{-1/2} \left( I_{-1/4}(y^2) - i I_{1/4}(y^2) \right), \\
&x^{3/2} H^{(2)}_{3/4}(x^2) = 2 y^{3/2} \left( - I_{3/4}(y^2) + i I_{-3/4}(y^2) \right). \tag{C.27}
\end{align*}
\]

Equations (C.28) and (C.27) are special cases of the following relation:

\[
H^{(2)}_\nu(iz) = \frac{ie^{-\frac{\pi}{2} i \nu}}{\sin \nu \pi} \left( -e^{2\nu \pi i} I_\nu(z) + I_{-\nu}(z) \right). \tag{C.29}
\]

The scalar exchange contribution to the four-point function of curvature perturbation is

\[
T_{\zeta,se}(k, C_2, C_3, C_4) = \frac{H^4}{M^4} \frac{T_{\pi,se}(k, C_2, C_3, C_4)}{M^4} = \frac{27\sqrt{2}}{\pi} (\Gamma(1/4))^{-4} \beta^2 \alpha^{-13/4} \left( \frac{H}{M} \right)^{11/2} k^{-9} \sum_{i=2,3,4} (A(C_i) + B(C_i)) = 2^{15/2} \pi^{6/5} (\Gamma(1/4))^{2/5} \left( {\cal P}_\zeta(k) \right)^{11/5} \beta^2 \alpha^{8/5} k^{-9} \sum_{i=2,3,4} (A(C_i) + B(C_i)) = 1.190 \times 10^{-16} \left( \frac{({\cal P}_\zeta(k))^{1/2}}{4.8 \times 10^{-5}} \right)^{22/5} \beta^2 \alpha^{8/5} k^{-9} \sum_{i=2,3,4} (A(C_i) + B(C_i)), \tag{C.30}
\]

\( \)
where we have used eq. (C.18) in the third equality. The corresponding nonlinear parameter
\( \tau_{NL}(k, C_2, C_3, C_4) \) is

\[
\tau_{NL}^{\text{se}}(k, C_2, C_3, C_4) = \frac{k^9}{\sqrt{2}} \left[ \sum_{i=2,3,4} (1 + C_i)^{-3/2} \right]^{-1} T_{\zeta, \text{se}}(k, C_2, C_3, C_4) \\
= 2^{4/7} \pi^7 (\Gamma(1/4))^4 \alpha^{15/4} \left( \frac{H}{M} \right)^{11/2} (P_\zeta(k))^{-3} \\
\times \sum_{i=2,3,4} (A(C_i) + B(C_i)) \left[ \sum_{i=2,3,4} (1 + C_i)^{-3/2} \right]^{-1}.
\]

Substituting eq. (C.18) to this expression, we obtain

\[
\tau_{NL}^{\text{se}}(k, C_2, C_3, C_4) = \frac{\beta^2}{\alpha^{8/5}} (P_\zeta(k))^{-4/5} \left( 2^{4/7} \pi^{-24/5} (\Gamma(1/4))^2/5 \right) \\
\times \left[ \sum_{i=2,3,4} (A(C_i) + B(C_i)) \right] \left[ \sum_{i=2,3,4} (1 + C_i)^{-3/2} \right]^{-1} \\
\simeq 8.945 \times 10^5 \times \frac{\beta^2}{\alpha^{8/5}} \left( \frac{(P_\zeta(k))^{1/2}}{4.8 \times 10^{-5}} \right)^{-8/5} \\
\times \left[ \sum_{i=2,3,4} (A(C_i) + B(C_i)) \right] \left[ \sum_{i=2,3,4} (1 + C_i)^{-3/2} \right]^{-1}.
\]

(C.31)

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