Fano interference effect on the transition spectrum of single electron transistors

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(October 25, 2018)

We theoretically study the intraband transition spectrum of single electron transistors (SETs) composed of individual self-assembled quantum dots. The polarization of SETs is obtained by using the nonequilibrium Green’s function technique and the Anderson model with three energy levels. Owing to nonradiative coupling between the two excited states through the continuum of electrodes, the Fano interference effect significantly influences the peak position and intensity of infrared wavelength single-photon spectrum.

I. INTRODUCTION

The quantum dot (QD) system has many potential applications in electronic devices, including quantum dot lasers and infrared detectors. Recently, the spontaneous emission spectrum of QDs embedded in a semiconductor p-n junction has been proposed as a generator of single photons, which is important in the application of quantum cryptography. The antibunching feature of a single photon source was demonstrated by optical and electrical emission spectrum of QDs embedded in a semiconductor. Owing to Coulomb blockade effect and Pauli principle, the SETs can be utilized not only as a single photon emitter, but also a single photon detector.

In some applications of quantum communications, a long-wavelength single photon near a 10 μm infrared wavelength is useful owing to its advantage of high transmission in the atmosphere. Note that a long wavelength in-

II. HAMILTONIAN OF SYSTEM

We start with the Anderson model with three energy levels to describe the studied system

\[ H_{\text{new}} = \sum_{k,\sigma,\alpha} c_{k,\sigma,\alpha}^\dagger c_{k,\sigma,\alpha} + \sum_{\sigma,\ell=1,2,3} E_{\ell} d_{\ell,\sigma}^\dagger d_{\ell,\sigma} + \sum_{k,\sigma,\alpha,\ell} V_{k,\sigma,\alpha,\ell} d_{\ell,\sigma}^\dagger c_{k,\sigma,\alpha} \]

where \( c_{k,\sigma,\alpha}^\dagger \) and \( d_{\ell,\sigma}^\dagger \) are the electron creation operators in the electrodes and QDs. The first term describes the left and right electrodes via index \( \alpha = L, R \). The second term describes the electronic states of QDs. We consider the situation where the QD contains three bound levels (\( \ell = 1, 2, 3 \)). The third and fourth terms describe the coupling between the QD states and the two electrodes. The last two terms describe the interaction of the QD electrons with electromagnetic field. Here \( \lambda = -\mu, \mathcal{E} \) is the
These equations give rise to significant contribution to the phonon-assisted energy conservation between discrete QD levels, and therefore the phonon bottleneck effect (inability to satisfy the energy PB due to electron-phonon interaction via the complex self-energy interaction). In this study we phenomenologically include the phonon bottleneck effect even though the hopping terms are time dependent, in which the energy and time dependence of the coupling are factorized. This factorization leads to time-independent decay rates of the transformed Hamiltonian by

\[ H_{\text{new}} = \sum_{k,\sigma,\ell} \epsilon_{k} c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_{\ell=1,2,3} \epsilon_{\ell} d_{\ell,\sigma}^\dagger d_{\ell,\sigma} \]

where the renormalized energy levels of QD are \( \epsilon_{1} = E_1 + \omega/2 \) and \( \epsilon_{\ell=2,3} = E_\ell - \omega/2 \). We see that the time-dependent phase in the inter-level Hamiltonian vanishes. However, the hopping terms are time dependent, \( v_{k,1}(t) = \gamma_{1}(t) e^{i\omega t/2} \) and \( v_{k,i=2,3}(t) = \gamma_{i}(t) e^{i\omega t/2} \), in which the energy and time dependence of the coupling are factorized. This factorization leads to time-independent tunneling rates. Even though the effect of electron correlation is significant in small semiconductor QDs, we can ignore the particle Coulomb interaction if the applied voltage is not sufficient to overcome the charging energies of QDs. Such condition is required throughout this article. According to Eq. (2), two excited states are indirectly coupled via the continuum of electrodes. This coupling can be enhanced by including electron-phonon interaction. In this study we phenomenologically include electron-phonon interaction via the complex self-energy of \( \Delta_{2,3} = \Delta_2 + i\gamma_{2}G_{22}^{\ell} \), which was considered to study double quantum wells in Ref. [13]. \( \Gamma_{2d} \) and \( \Gamma_{3d} \) denote the decay rates of \( E_2 \) and \( E_3 \). \( \Delta_{23} \) is the coupling strength between \( E_2 \) and \( E_3 \). Note that although the electron-phonon scattering rate can be small or vanishing in QDs due to the phonon bottleneck effect (inability to satisfy the energy conservation between discrete QD levels), it can still give rise to significant contribution to the phonon-assisted tunneling, since the energy conservation can be met by electron jumping between the QD and the leads.

The electrically driven transition spectrum of individual SETs can be calculated by using the Keldysh-Green's function method[14,15]. The lesser Green's function \( G_{1,2}^{\ell}(t_1, t_2) = -i \langle \langle \phi_1 \rvert d_{1,\sigma}^\dagger(t_2) d_{1,\sigma}(t_1) \rangle \rangle \), which describes the correlation of electron in the energy level \( \epsilon_1 \) and \( \epsilon_2 \) at time \( t_1 \) and \( t_2 \), is introduced to calculate the polarization. Using the method of equation of motion, the stationary solution is given by

\[ \epsilon_{2} - \epsilon_{1} + i(\Gamma_{1} + \Gamma_{2}) G_{1,2}^{\ell}(\epsilon) = -i\lambda_{1,2} G_{1,1}^{\ell}(\epsilon) - G_{2,2}^{\ell}(\epsilon) + \Sigma_{1}^{\ell} G_{1,2}^{\ell}(\epsilon + \frac{\omega}{2}) \]

\[ \} \]

In quasi-equilibrium the intra-level lesser self energies of Eq.(3) are \( \Sigma_{1}^{\ell} = \Gamma_{1}(f_{L}(\epsilon) + f_{R}(\epsilon)) \), where \( f_{L}(\epsilon) \) and \( f_{R}(\epsilon) \) are the Fermi distribution function of the left and right electrodes. The chemical potential difference between these two leads is related to the applied bias \( \mu_{L} - \mu_{R} = 2eV_{a} \). \( \Gamma_{\ell=\ell,R} = \sum_{k} |V_{k,\ell}\alpha|^{2} \delta(\epsilon - \epsilon_{k}) \) denote the tunneling rates from the QD to the left and right electrodes, respectively. For simplicity, we consider \( \Gamma_{1,L} = \Gamma_{1,R} = \Gamma_{1} \). It is very difficult to fully include the tunneling rate as a function of energy and momentum, and we assume that these tunneling rates are energy and bias independent even though \( \Gamma_{1}(\Gamma_{2}) \) can be calculated with a reliable method[16].

The lesser Green’s functions \( G_{1,1}^{\ell}(\epsilon) \) and \( G_{2,2}^{\ell}(\epsilon) \) of Eq. (3) denote the electron occupation numbers, which are determined by the spectrum functions \( G_{1,1}^{\ell}(\epsilon) = f_{L}(\epsilon) A_{1}(\epsilon + \frac{\omega}{2}) \) and \( G_{2,2}^{\ell}(\epsilon) = f_{R}(\epsilon) A_{2}(\epsilon - \frac{\omega}{2}) \). The quasi-equilibrium distribution function of QD is expressed by \( f_{\ell}(\epsilon) = (\Gamma_{\ell} f_{L}(\epsilon) + \Gamma_{R} f_{R}(\epsilon))/(\Gamma_{L} + \Gamma_{R}) \). Owing to the weak electron-phonon coupling, we make the approximations of the spectrum functions \( A_{1,2}(\epsilon) \approx \pm 2i \mu G_{1,1}(\epsilon) \), which are the imaginary part of Green’s functions, where the retarded and advanced Green functions are \( G_{1,0}^{\ell}(\epsilon) = 1/(\epsilon - \epsilon_{1} + i\gamma_{1}) \), and \( G_{1,0}^{\ell}(\epsilon) = 1/(\epsilon - \epsilon_{2} + i\gamma_{2}) \). Such approximation implies that the electron occupation numbers are mainly contributed from the tunneling process. In Eq.(3) the interlevel Green’s function is given by \( G_{1,2}(\epsilon) = \lambda_{1,2} G_{1,1}(\epsilon) G_{2,2}(\epsilon) \). It is worthy noting that we ignored the term of \( \lambda_{1,2} G_{1,2}(\epsilon) \) in Eq. (3), where \( G_{1,2}^{\ell}(\epsilon) = \kappa_{2,3}[f_{L}(\epsilon) A_{2}(\epsilon + \frac{\omega}{2}) G_{2,0}(\epsilon + \frac{\omega}{2}) - f_{R}(\epsilon) A_{3}(\epsilon) G_{2,0}(\epsilon) \] which can be readly proved as a negligible term.

Defining frequency detuning as \( \Delta_{1,2} = \epsilon_{2} - \epsilon_{1} + i(\Gamma_{1} + \Gamma_{2}) = 1/(\Gamma_{1,0}(\epsilon) - 1/\Gamma_{2,0}(\epsilon) \) and using \( 2\Gamma_{1} = -i(\frac{1}{\Gamma_{1,0}(\epsilon)} - \frac{1}{\Gamma_{2,0}(\epsilon)} \) \), we rewrite Eq. (3) as

\[ G_{1,1}^{\ell}(\epsilon) = \lambda_{1,2} f_{L}(\epsilon) A_{2,0}(\epsilon - \frac{\omega}{2}) G_{1,0}(\epsilon - \frac{\omega}{2}) \]

\[ + f_{L}(\epsilon) A_{1,0}(\epsilon + \frac{\omega}{2}) G_{2,0}(\epsilon + \frac{\omega}{2}) - \frac{\kappa_{2,3}}{\Delta_{1,2}} G_{1,1}^{\ell}(\epsilon), \]

\[ = P_{1}(\epsilon) - \frac{\kappa_{2,3}}{\Delta_{1,2}} G_{1,1}^{\ell}(\epsilon). \]

In the absence of \( \kappa_{2,3} \), \( G_{1,1}^{\ell}(\epsilon) \) describes the transition spectrum between the ground state \( E_{1} \) and the first excited state \( E_{2} \), which is viewed as the first line shape. The self energy \( \kappa_{2,3} \) leads \( G_{1,1}^{\ell}(\epsilon) \) to couple with Green’s functions \( G_{1,1}^{\ell}(\epsilon) \), which is expressed by

\[ G_{1,1}^{\ell}(\epsilon) = \lambda_{1,2} f_{L}(\epsilon) A_{2,0}(\epsilon - \frac{\omega}{2}) G_{1,0}(\epsilon - \frac{\omega}{2}), \]

\[ + f_{L}(\epsilon) A_{1,0}(\epsilon + \frac{\omega}{2}) G_{2,0}(\epsilon + \frac{\omega}{2}) - \frac{\kappa_{2,3}}{\Delta_{1,2}} G_{1,1}^{\ell}(\epsilon), \]

\[ = P_{2}(\epsilon) - \frac{\kappa_{2,3}}{\Delta_{1,2}} G_{1,1}^{\ell}(\epsilon), \]
where \( \Delta \omega_{13} = \epsilon_3 - \epsilon_1 + i(\Gamma_3 + \Gamma_3) = 1/G_{1,0}^r(\epsilon) - 1/G_{3,0}^r(\epsilon) \). \( P_2 \) describes the second line shape. This second line shape will interfere with the first line shape due to \( \kappa_{2,3} \). Solving Eqs. (4) and (5), we obtain the polarization consisted of two components \( X_{12}(\omega) = \int \frac{d\epsilon}{2\pi} \frac{\lambda_{1,2}(\epsilon)P_{1,2}(\epsilon)}{\Delta \omega_{1,2} - |\kappa_{2,3}|^2} \), and \( X_{13}(\omega) = \int \frac{d\epsilon}{2\pi} \frac{\lambda_{1,3}(\epsilon)P_{1,2}(\epsilon)}{\Delta \omega_{1,3} - |\kappa_{2,3}|^2} \).

Eqs. (6) and (7) are the central results of this article. Based on Eqs. (6) and (7), we will investigate the Fano interference effect arising from \( \kappa_{2,3} \) on the absorption and emission spectrum, respectively.

### III. RESULTS AND DISCUSSIONS

To generate a single photon at the 10 \( \mu \text{m} \) wavelength, the QD with base length \( b \approx 14 \text{ nm} \) is sufficient to provide such photons (\( \omega = E_2 - E_1 \approx 124 \text{ meV} \)). To simulate the system, we also choose the Fermi energy \( E_F = 50 \text{ meV} \) which is 30 meV below \( E_1 \) at zero bias. In addition, we assume \( E_3 - E_2 = 6 \text{ meV} \). Therefore, the levels of QD are empty at zero bias. When the chemical potential of left electrode \( \mu_L \) (source) sweeps through the ground state and far away from the excited states (the energy level separation \( \Delta E = E_2 - E_1 \gg k_B T \)), Eqs. (6) and (7) can be rewritten as the following expression

\[
X_a = \mathcal{P}_1(\omega) \int \frac{d\epsilon}{2\pi} f_1^r(\epsilon) G_{2,0}^r(\epsilon + \omega/2) A_{1,0}(\epsilon + \omega/2) + \mathcal{P}_2(\omega) \int \frac{d\epsilon}{2\pi} f_1^r(\epsilon) G_{3,0}^r(\epsilon + \omega/2) A_{1,0}(\epsilon + \omega/2),
\]

where \( \mathcal{P}_1 = (|\lambda_{1,2}^2| \Delta \omega_{1,2} \Delta \omega_{1,3} - \lambda_{1,2} \lambda_{1,3} \kappa_{2,3} \Delta \omega_{1,2})/D \), \( \mathcal{P}_2 = (|\lambda_{1,3}^2| \Delta \omega_{1,2} \Delta \omega_{1,3} - \lambda_{1,2} \lambda_{1,3} \kappa_{2,3} \Delta \omega_{1,3})/D \) and \( D = \Delta \omega_{1,3} \Delta \omega_{1,2} - |\kappa_{2,3}|^2 \).

Owing to very small tunneling rate of \( \Gamma_1 \) for deep energy level, the expression of (8) at zero temperature has a simple form

\[
X_a = \frac{1}{2} \frac{|\lambda_{1,2}^2| \Delta \omega_{1,3} + |\lambda_{1,3}^2| \Delta \omega_{1,2} - 2 \lambda_{1,2} \lambda_{1,3} \kappa_{2,3} \lambda_{3,1}}{\Delta \omega_{1,3} \Delta \omega_{1,2} - |\kappa_{2,3}|^2}.
\]

The expression of Eq. (9) was also obtained for atomic systems to study electromagnetically induced transparency (EIT) \(^{17,18}\). Fig. 2 shows the imaginary part of \( X_a(\omega) \) for \( \kappa_{2,3} = 1 \text{ meV} \) and \( \Gamma_2 = \Gamma_3 = 1 \text{ meV} \). The solid line and dashed line denote, respectively, with and without \( \kappa_{2,3} \). We see that the Fano interference effect arising from \( \kappa_{2,3} \) significantly influences the absorption spectrum. It is destructive between two peaks, but constructive in the wings of the two absorption lines. This EIT effect can be used to efficiently modulate the light group velocity (or stop light) \(^{12,19} \) as a result of very small broadening of energy levels for QDs. To date, this quantum interference has not been reported in isolated QD system, whereas it was reported in the quantum well system \(^{20} \). Although Eq. (8) can provide further information about the absorption spectrum for the different applied voltages and temperatures, we will discuss these effects on the emission spectrum.

When the left lead supplies electrons into the excited states, light emission process occurs. Emission spectrum intensity is given by

\[
X_e(\omega) = \mathcal{P}_1(\omega) \int \frac{d\epsilon}{2\pi} f_1^r(\epsilon) A_{2,0}(\epsilon - \omega/2) G_{1,0}^r(\epsilon - \omega/2) + \mathcal{P}_2(\omega) \int \frac{d\epsilon}{2\pi} f_1^r(\epsilon) A_{3,0}(\epsilon - \omega/2) G_{1,0}^r(\epsilon - \omega/2).
\]

We show the imaginary part of \( X_e(\omega) \) for the applied voltage \( V_a = 165 \text{ mV} \) and temperature \( k_B T = 1 \text{ meV} \) in Fig. 3: the solid line and dashed line denote, respectively, \( \kappa_{2,3} = (-0.5 + i) \text{ meV} \) and \( \kappa_{2,3} = 0 \). Comparing to absorption spectrum shown in Fig. 2, we see two asymmetric lines resulting from the real part of \( \kappa_{2,3} \), which was ignored in the absorption spectrum. In addition to the destructive interference between two peaks, the constructive interference is also observed in the wings of these two peaks. This feature is the same as that of absorption spectrum. However, note that the intensity of \( L_1 \) is enhanced and slightly shifted from \( \omega = 124 \text{ meV} \). The intensity enhancement of \( L_1 \) implies that the number of single photons emitted from \( L_1 \) is enhanced. Therefore, we can take this advantage to the application of quantum cryptography.

It is significant to provide single-photon sources at room temperature for the development of quantum cryptography. Therefore, we attempt to understand temperature effect on the emission spectrum. We show the imaginary part of \( X_e(\omega) \) for different temperatures at applied voltage \( V_a = 165 \text{ mV} \) in Fig. 4: solid line \( (k_B T = 1 \text{ meV}) \), dashed line \( (k_B T = 2 \text{ meV}) \), dotted line \( (k_B T = 3 \text{ meV}) \) and dash-dotted line \( (k_B T = 4 \text{ meV}) \). When the applied voltage is \( V_a = 165 \text{ mV} \), the left electrode supplies maximum electron number into two resonant levels \( E_2 \) and \( E_3 \) at zero temperature. Consequently, \( L_1 \) and \( L_2 \) display the strongest intensities at the temperature of \( k_B T = 1 \text{ meV} \). The peaks \( L_1 \) and \( L_2 \) decline in heights, as the temperature increases. Therefore, high temperature diminishes the efficiency of photon emission. Besides, thermionic emission not included in this study could also seriously destroy photon emission efficiency at room temperature since the energy levels \( E_2 \) and \( E_3 \) are not very deep (See
IV. SUMMARY

In this study we have proposed to utilize the intraband transition of InAs/GaAs SETs to produce a single-photon source in the infrared wavelength range for the application of wireless quantum cryptography. The Anderson model with three energy levels is used to simulate the studied system. It is found that quantum interference effect leads two asymmetry line-shapes. It is destructive between two peaks, but constructive in the wings of two transition lines. The expression of Eqs. (5) and (6) can be readily extended to the case of double quantum dot system or quantum well system, where the bond and antibond states correspond, respectively, to $E_2$ and $E_3$. 

ACKNOWLEDGMENTS

This work was supported by National Science Council of Republic of China Contract No. NSC 94-2215-E-008-027.

Figure Captions

Fig. 1: The lowest three energy levels of a quantum dot (QD) as functions of the QD size $b$. $E_0$ and $E_p$ denote, respectively, the ground state and the first excited state.

Fig. 2: Imaginary part of $X_e(\omega)$ as functions of detuning frequency.

Fig. 3: Imaginary part of $X_e(\omega)$ as functions of detuning frequency at applied voltage $V_a = 165 \, \text{mV}$ and temperature $k_BT = 1 \, \text{meV}$. Solid line ($\kappa_{2,3} = (-0.5 + i) \, \text{meV}$) and dashed line ($\kappa_{2,3} = 0$).

Fig. 4: Imaginary part of $X_e(\omega)$ as functions of detuning frequency for different temperatures at applied voltage $V_a = 165 \, \text{mV}$ and $\kappa_{2,3} = (-0.5 + i) \, \text{meV}$.

Fig. 5: Imaginary part of $X_e(\omega)$ as functions of detuning frequency for different applied voltages at temperature $k_BT = 1 \, \text{meV}$ and $\kappa_{2,3} = (-0.5 + i) \, \text{meV}$.

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