Disformal quintessence

Tomi S. Koivisto
Institute for Theoretical Physics, University of Heidelberg, 69120 Heidelberg, Germany
(Dated: November 12, 2008)

A canonic scalar field minimally coupled to a disformal metric generated by the field itself is considered. Causality and stability conditions are derived for such a field. Cosmolological effects are studied and it is shown that the disformal modification could viably trigger an acceleration after a scaling matter era, thus possibly alleviating the coincidence problem.

I. INTRODUCTION

Quantum effects in curved spacetime are known to introduce geometric corrections to the Einstein-Hilbert action. Thus a de Sitter era for the early universe and the resolution of the initial singularity were predicted from the semiclassical considerations of the spacetime itself. In the past few years, with the advent of the dark energy paradigm, the possibility has been contemplated that the present acceleration of the universe could also be derived from a more fundamental gravity theory superseding general relativity at large scales. This attractive prospect of unified inflation and dark energy (with perhaps dark matter) without ad hoc exotic matter sources has received a lot of attention. The prototype models are the nonlinear or f(R) gravity among more general scalar-tensor theories. As well known, these models are related to coupled scalar field matter by conformal transformation.

One may ask whether relations of different forms could be considered as a consistent framework for gravity theories and their cosmological applications. It has been argued that a general Finslerian relation between matter and gravity geometries is restricted, by requiring causality, validity weak equivalence principle and in particular covariance in its strictest sense, to a certain generalisation of the conformal relation. Let us therefore consider the class of the so called disformal transformations, which may depend on a scalar field φ,

\[ \tilde{g}_{\alpha\beta} = A(\phi)g_{\alpha\beta} + B(\phi)\phi,\alpha \phi,\beta. \]  

If \( B = 0 \), the relation reduces to the conformal transformation which preserves the angles and the light cones. Any nonzero \( B \) then causes a disformal modification which results in difference of the causal structures of the two Riemannian geometries. This feature has been exploited to construct variable speed of light cosmology to cope without a usual inflation. On the other hand, an interestingly short inflation model with \( A = -B = 1 \), has been considered in detail. The disformal property is crucial in producing lensing phenomena in relativistic MOND models. It appears also within the so called Palatini formalism if the action includes Ricci tensor squared terms.

The main aim of the present paper is to further explore the cosmological possibilities of the relation and in particular see how the disformal property could be used to unify dark energy within this framework. Previously, most authors couple matter to the disformal metric, while gravity is given by the Einstein-Hilbert action of \( g_{\mu\nu} \). The cosmological expansion is then effectively sourced by matter, whose energy density \( \rho \) and the pressure \( p \) are given by (dot denoting the time derivative)

\[ \rho = \left( 1 - \frac{B}{A} \phi^2 \right)^{-\frac{1}{2}} \dot{\rho}, \quad p = \left( 1 - \frac{B}{A} \phi^2 \right)^{\frac{1}{2}} \dot{\rho}. \]  

In usual coupled quintessence scenario, the cosmon field \( \phi \) has a conformal coupling to matter. The field should become energetically dominating recently to drive the acceleration. Now, we could consider a disformal extension of the usual coupled quintessence scenarios based on purely conformal (dilatation) symmetry considerations. Then we would consider dark matter living in the metric disformed by a nonzero \( B \). One notes that if the "bare" pressure vanishes, the coupling does not generate effective pressure. Then, as \( B/A \) grows, the physically effective energy density just dilutes faster, and one does not expect new effects if the \( \rho \) was not dynamically significant already earlier. Moreover, we have checked that viewing cosmology as a dynamical system, one does not find new fixed points by taking into account \( B/A \); all fixed points require \( B = 0 \). Thus a disformal coupling between dark matter and dark energy does not seem helpful to the coincidence problem.

Instead, we then consider the case that only the field itself is coupled to the disformal metric. In the simplest case, we have the Einstein-Hilbert action for gravity coupled minimally to the matter sector and a canonical scalar field coupled to the barred metric. This seems to be a minimal set-up employing the relation, in the sense that all other matter than \( \phi \) is minimally coupled to standard general relativity. The only unusual feature is then an effective self-coupling of the field \( \phi \). Explicitly, we write

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R + \mathcal{L}_m \right) \]
where \( \mathcal{L}_m \) is the Lagrangian density for matter, and \( \kappa^2 = 8\pi G. \) In the following, we denote

\[
I = g^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta}.
\]

The scalar field has a canonical energy momentum tensor in the disformal metric and is covariantly conserved with respect to this barred metric, \( \nabla^{\mu} \mathcal{T}^{(\phi)}_{\mu\nu} = 0. \) However, the physical metric is \( g_{\mu\nu}, \) and to avoid confusion, we rather write everything in terms of this unbarred metric and its Levi-Civita connection. The field couples also, through the relation \( I \) to the physical metric. One may thus associate an effective energy momentum tensor with the field. It can be shown to have the form

\[
T^{(\phi)}_{\mu\nu} = \frac{1}{\sqrt{A(A+IB)}} \left[ \left( \frac{2A+IB}{2(A+IB)} + BV \right) \phi_{,\mu} \phi_{,\nu} - \frac{1}{2} I + (A+IB)V \right] g_{\mu\nu}.
\]

This tensor is also covariantly conserved, but with respect to the unbarred metric, consistently with the generalized Bianchi identity \( \Box \phi = 0. \) It is easy to see that this energy-momentum tensor can be put in the perfect-fluid form

\[
\rho = p = \frac{1}{A}(\phi^2) + \frac{1}{2} I + (A+IB)V,
\]

where distances and time intervals are measured with respect to the \( "\)physical", unbarred metric, the disformal field \( \phi \) begins to look frozen when \( B \) is large enough. This can be seen from the formulas (7,8), where the kinetic terms are suppressed by the square of the ratio of peculiar times as measured using the different metrics. However, the feature will always disappear when the field is a constant, and is thus a purely dynamical effect requiring some rolling of the scalar field.

This suggests the following cosmological application. The cosmic field is well known to have so called tracking property, which guarantees it exhibits a constant ratio of the total energy density of the universe regardless of the initial conditions \( \Phi \). Hence, one may have a scalar field present during the whole evolution of the universe without fine-tuning. However, the field cannot explain the observed acceleration if it stays on the attractor. To toss the field off the attractor, one may reshape the potential \( \Phi \). A theoretically motivated possibility is to couple the field nonminimally to matter \( \Phi \), but this is known to lead to an instability at the linear level \( \Phi \) and possible problems with quantum loop corrections \( \Phi \). However, considering neutrino coupling allows to link the acceleration scale with the neutrino mass \( \Phi \). Mechanisms based on nonminimal gravity couplings are employed in extended quintessence \( \Phi \), and in more general models with nonlinear functions of curvature invariants \( \Phi \).

Here we instead let the disformal effect freeze the scalar field. Thus we consider the possibility that the lapse distortion \( B \) redshifts the kinetic energy away, thus stopping the field and triggering a potential dominated era. Clearly, the potential of the field then provides an effective cosmological constant and the universe evolves into
future de Sitter stage. To look into this in more detail, let us specialize to the simple case

$$A = 1, \quad B = B_0 e^{\beta \kappa \phi}, \quad V = V_0 e^{\lambda \kappa \phi}. \quad (12)$$

We set the purely conformal factor to unity to focus on the novel features. We choose the exponential forms for the disformal factor and the potential motivated by high-energy physics considerations and convenience. Now, assuming initially $B \ll 1$, the tracking stage is characterized by

$$\kappa \phi = \frac{3(1 + w_m)}{\lambda} + \text{constant}. \quad (13)$$

Then the field mimics the background component with the equation of state $w_m$. By Eq. (9), the Hubble parameter drops as $H^2 \sim a^{-3(1+w_m)}$. The disformal factor grows as $B \sim a^{3(1+w_m)/\beta}$. Therefore, the importance of the term $B \dot{\phi}^2$ is growing, and the tracking regime will eventually be interrupted iff $\beta > \lambda$. A similar condition exists for Gauss-Bonnet dark energy proposed in Ref. [40]: iff the slope of the Gauss-Bonnet coupling is steeper than that of the potential, dark energy domination occurs. Numerical examples of the evolution in the present models are shown in Figure 1. There, as usual $\Omega_m = \kappa^2 p_m/(3H^2)$ is the fractional matter density, and $w_{eff}$ is the effective pressure per density of the total matter content. The quantity $c_\phi^2$ is discussed in the next section.

III. FLUCTUATIONS IN THE FIELD

The propagation of the perturbations of the field is characterized by the sound speed squared, $c_\phi^2 = \delta p/\delta \rho |_{T_\text{r}=0}[12]$. This is an important determinant of the physical properties of the effective fluid; one requires $c_\phi^2 < 1$ in order to eliminate the possibility of superluminal information exchange via excitations of the field, and in addition one sees that if $c_\phi^2 < 0$ the perturbations are unstable and may blow up too fast. The sound speed by definition depends on the background. Here we consider the FRW background as it includes Minkowski and de Sitter as particular limits. The sound speed is evaluated as the ratio of pressure and density perturbation in the comoving frame, and therefore no gauge ambiguity arises. Consider the so called total matter gauge, where the metric perturbations are parameterised by two longitudinal scalar potentials and the matter velocity vanishes. As we argued above, even the nonstandard scalar field does not generate anisotropic stresses, and its velocity field is proportional to the field perturbation. It follows that the mentioned longitudinal metric potentials are equal and the field $\phi$ is smooth in this frame. With these observations, it becomes straightforward to obtain from (12) that

$$c_\phi^2 = \frac{\left(-\dot{\phi}^2 B + 2ABV - 2\dot{\phi}^2 B^2 V + 2A\right)}{\dot{\phi}^2 B + 2ABV - 2\dot{\phi}^2 B^2 V + 2A} \quad (14)$$

One notes that when $B$ vanishes the sound speed squared becomes identically unity. It is a well known property of canonic fields that their perturbations always propagate with the speed of light. When $B$ is not identically zero, one gets nontrivial constraints on the model by requiring causality and stability.

In the scenario [12] the sound speed deviates from unity at the present and during the matter dominated epoch when the disformal effect is freezing the field. The sound speed decreases to a smaller positive value and then evolves to back to unity. Two examples of this evolution are shown in Figure 1. This confirms that the model is free of instabilities and that causality violations do not
IV. CONCLUSION

Modified gravity could unify inflation and the dark sector in cosmology. The standard frameworks for this pursuit, the so-called nonlinear gravity as a particular class of scalar-tensor theories, and quintessence models are connected by conformal transformation. In this article we investigated the possibility to use the disformal generalisation \[ f(R) \] as the fundamental relation between the two metrics. For the purpose of constructing a minimal setup where this is possible, we considered the case where only the scalar field itself lives in the disformally related metric. Then all usual matter respects the equivalence principle. The disformal effect, a new type of effective self-interaction, is only seen when the field is dynamical.

This can have interesting consequences for cosmological, rolling scalar fields. In particular, the disformal modification can freeze a tracking quintessence field in such a way that an accelerating de Sitter era follows a scaling matter era. For an exponential potential with the slope \( \lambda \) and an exponential disformal factor with the slope \( \beta \) the necessary condition for acceleration becomes very simply \( \beta > \lambda \). The relative scale of the potential and the disformal factor is then determined by requiring the observed relative abundance of matter today. This mechanism resembles the scenario where the string-motivated exponential Gauss-Bonnet coupling of the Nojiri-Odintsov-Sasaki modulus triggers the acceleration.

To determine the causality and stability of the propagating perturbations in the field, we derived an expression for the sound speed squared \[ c_s^2 \]. Requiring consistency limits the possible solutions. These nontrivial conditions are generically satisfied by the scenarios described above. This seems, together with the previous considerations of gravitational models of inflation and dark matter mentioned in the introduction, to suggest that a subtle modification of the relation of matter and dark matter could also have a role in the resolution of present cosmological problems.
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