Study on Error Correction Model of Inertia and Star Compact Combination

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Abstract. According to the existing inertial and starlight integrated navigation affected by the horizontal reference error, an inertial and star compactly combined error correction model is studied. The state space model and the measurement model in the compact combination mode are derived through theoretical deduction. The new model taking into account the attitude error and position error of the inertial solution, the influence of the reference error of the inertial navigation system on the calculation result of the related parameters of the integrated navigation can be effectively suppressed, and the precision of the integrated navigation can be improved. In order to verify the correctness of the research theory, a set of simulation experiments was designed. The results show that under the set carrier simulation trajectory, the compact combination model studied has better error correction effect on speed, position and attitude parameters than the loose combination method. Parameters in the convergence rate, fluctuation range, and accuracy reflect better advantages. The navigation position accuracy under simulation conditions can reach within 300m.

1. Introduction
In order to restrain the influence of accumulated errors on the calculation parameters of INS, integrated methods are often used to improve the performance of navigation system, such as the combination of satellite and INS, the combination of starlight and INS, etc. The latter has strong autonomy and anti-jamming ability [1-3]. The core of starlight and inertial navigation system is to use the output of starlight to correct the relevant parameters of inertial navigation. How to achieve the combination correction, we need to combine the combination of inertia and starlight, the establishment of the state space model of the relevant parameters, the establishment of the measurement model and the selection of the filter estimation method to achieve [2-6]. There are two main types of existing star and inertial navigation systems: one is to provide the horizontal reference through inertial navigation, and get the position information by combining the star output, which is called the simple combination method; the other is to use the output information of star position under the horizontal reference of inertial navigation as the reference standard, and use the filtering estimation method to correct the error of the inertial navigation solution parameters, so as to achieve the high integrated navigation. The output of precision parameters is also called loose combination method [7-9]. It can be seen that the above two combination methods adopt the horizontal reference of inertial navigation output. Because of the drift error of the inertial sensor itself, the reference
error will be produced, and the error will be transmitted to the relevant calculation parameters [10, 11]. On the basis of the above, this paper studies an error correction model of inertial plus starlight tight combination. The model takes into account the horizontal reference error provided by inertial navigation. In theory, the combined model can reduce the influence of horizontal reference error on the calculation results, and its accuracy is better than that of loose combination.

2. State Space Model

As \( X = [\phi_e, \phi_n, \phi_v, \delta V_e, \delta V_n, \delta V_v, \delta L, \delta \delta h, e_x, e_y, \rho_x, \rho_y, \rho_z] \), according to the corresponding parameter error model, the state space equation is established as follows:

\[
\dot{X}(t) = F(t)X(t) + G(t)W(t)
\]  

where \( X(t) \) is a state variable in \( \mathbb{R}^{15} \) and \( F(t) \) is a system error matrix in \( \mathbb{R}^{15 \times 15} \), which can be derived from the established error model. It is a system noise vector in \( \mathbb{R}^6 \) and a system noise driving matrix in \( \mathbb{R}^{15 \times 6} \). \( E, N, U \) represent the three axes of the east, north and sky respectively. \( \phi_e, \phi_n, \phi_v \) are attitude error angle, velocity error \( \delta V_e, \delta V_n, \delta V_v \), position error \( \delta L, \delta \delta h \), gyro Random Drift \( e_x, e_y, e_z \) and accelerometer error \( \rho_x, \rho_y, \rho_z \).

The system noise vector is as follows:

\[
W(t) = \begin{bmatrix} \omega_{x} & \omega_{y} & \omega_{z} & \omega_{px} & \omega_{py} & \omega_{pz} \end{bmatrix}
\]  

Among them, the gyro white noise term \( \omega_{x}, \omega_{y}, \omega_{z} \) and the accelerometer white noise term \( \omega_{px}, \omega_{py}, \omega_{pz} \) are represented.

The noise driving matrix \( G(t) \) of the system is a \( 15 \times 6 \) matrix, which can be expressed as:

\[
G(t) = \begin{bmatrix} C_x^\omega & 0_{3 \times 3} \\ 0_{3 \times 3} & C_x^\omega \\ 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}
\]  

3. Tight Combination Measurement Model

In order to reduce the influence of inertial attitude error and position error on the result of star light calculation, it is necessary to establish a comprehensive model considering the influence of inertial attitude error and position error.

Starlight vector altitude angle and azimuth angle in geographic horizontal coordinate system are taken as observational variables. By comparing the observational and computational quantities, the error of star vector altitude angle \( H_p \) and azimuth angle \( A_p \) can be obtained. This error can be used as measurement information to establish measurement model. According to the above ideas, the observation quantity of star vector under high-precision horizontal datum is set as altitude angle \( H_c \) and azimuth angle \( A_c \), and the star vector under horizontal datum is calculated to calculate altitude angle and azimuth angle. Under strapdown installation, the horizontal datum of observation is provided by inertial attitude, and the horizontal datum of calculation is provided by inertial position. Because of the influence of gyro and accelerometer output drift on the attitude and position of inertial
calculation, there are errors between the horizontal datum of observation and the horizontal datum of calculation. Therefore, it is necessary to establish a comprehensive consideration of the attitude error sum of inertial calculation. Comprehensive measurement model of position error.

Taking the difference between the observation and the calculation as the measurement quantity, we can get the result.

\[
\begin{align*}
\Delta H &= H_p - H_e \\
\Delta A &= A_p - A_c
\end{align*}
\]  

(4)

Assuming the altitude angle \(H_b\) and azimuth angle \(A_b\) of the star vector in the carrier system, the two values can be read directly, and the observation altitude angle \(H_p\) and azimuth angle \(A_p\) can be calculated by combining the inertia to solve the horizontal datum \(C_p\). After recognizing the navigation star, the declination \(\delta\) and Green's time angle \(G_t\) of the navigation star can be obtained by the navigation ephemeris, and the longitude \(\lambda_c\) and latitude \(L_c\) of the position information of the carrier can be obtained by the inertia solution. It is possible to calculate the altitude angle \(H_c\).

\[
H_c = \begin{cases} 
\arcsin(\sin L_c \cdot \sin \delta + \cos L_c \cdot \cos \delta \cdot \cos(t_g + \lambda_c)) & t_g + \lambda_c < 180^\circ \\
\arcsin(\sin L_c \cdot \sin \delta + \cos L_c \cdot \cos \delta \cdot \cos(t_g + \lambda_c - 360^\circ)) & t_g + \lambda_c > 180^\circ
\end{cases}
\]

(5)

Set up

\[
t_c = \begin{cases} 
t_g + \lambda_c & t_g + \lambda_c < 180^\circ \\
t_g + \lambda_c - 360^\circ & t_g + \lambda_c > 180^\circ
\end{cases}
\]

(6)

Corresponding calculation azimuth \(A_c\).

\[
A_c = \begin{cases} 
\arccos \left(\frac{\sin \delta - \sin H_c \cdot \sin L_c}{\cos H_c \cdot \cos L_c}\right) & t_c < 0^\circ \\
360^\circ - \arccos \left(\frac{\sin \delta - \sin H_c \cdot \sin L_c}{\cos H_c \cdot \cos L_c}\right) & t_c > 0^\circ
\end{cases}
\]

(7)

Starlight direction vectors \(X_p\) can be obtained from observations \(H_p\) and \(A_p\).

Starlight direction vectors \(X_c\) can be obtained from the computation \(H_c\) and \(A_c\) are expressed as follows.

\[
X_p = \left[\cos H_p \sin A_p \cos H_p \cos A_p \sin H_p\right]^T
\]

(8)

\[
X_c = \left[\cos H_c \sin A_c \cos H_c \cos A_c \sin H_c\right]^T
\]

(9)

\[
X_p = C_p \cdot C_c \cdot X_c
\]

(10)

If the error angle of the relative navigation system of the platform system caused by the attitude error of inertia solution is \(\varphi\) (a small amount \(\varphi\)), then there is
When the position error of inertial solution is small, the matrix relation between navigation system and calculation system caused by position error is as follows.

\[
C_p = \begin{pmatrix}
1 & \phi_e & -\phi_N \\
-\phi_e & 1 & \phi_N \\
\phi_N & -\phi_e & 1
\end{pmatrix}
\]

(11)

Then the relationship between the observed star light vector and the calculated star light vector can be expanded and the result can be obtained.

\[
C_p = \begin{pmatrix}
1 & \delta\lambda \sin L & -\delta\lambda \cos L \\
-\delta\sin L & 1 & -\delta L \\
\delta\lambda \cos L & \delta L & 1
\end{pmatrix}
\]

(12)

Considering the relationship between measurement, observation and calculation, if the measurement is regarded as a small quantity, then there are

\[
\cos \Delta A = \cos(\Delta L) \approx \cos A_c - \Delta A \sin A_c
\]

(14)

\[
\sin \Delta A = \sin(\Delta L) \approx \sin A_c + \Delta A \cos A_c
\]

(15)

\[
\cos A_p = \cos(A_c + \Delta A) \approx \cos A_c - \Delta A \sin A_c
\]

(16)

\[
\sin A_p = \sin(A_c + \Delta A) \approx \sin A_c + \Delta A \cos A_c
\]

(17)

By synthesizing the above formulas, it can be concluded that

\[
\Delta h = h_i - h_c = \Delta h + M_V
\]

(20)

Assume observability \( Z = [\Delta H \quad \Delta A \quad \Delta h]^T \).

Then there are

\[
Z(t) = H(t)X(t) + V(t)
\]

(21)

Among
\[
H(t) = \begin{bmatrix}
\cos A_t & -\sin A_t & 0 & 0_{1,1} & -\cos A_t & -\cos L \sin A_t & 0 & 0_{1,1} \\
\tan H \sin A_t & \tan H \cos A_t & -1 & 0_{1,1} & -\tan H \sin A_t & \cos L \tan H \cos A_t & \sin L & 0_{1,1} \\
0 & 0 & 0 & 0_{1,1} & 0 & 0 & 1 & 0_{1,1}
\end{bmatrix}
\] (22)

\(V(t)\) is a vector consisting of three corresponding observed white noise.

4. Analysis of Simulation Experiment

Using the location information as the external information source, the State Planning of the carrier is as follows: under the initial condition, the three-axis attitude of the carrier relative to the navigation system is 0, the initial velocity of the carrier is 50 m/s, the first 400 s inner carrier climbs up, the pitch angle accelerates from 0 to 30 degrees, the pitch angle accelerates from 401 seconds to 600s to 1m/s², the pitch angle from 601s to 720s decelerates uniformly from 30 degrees to 0 degrees, and the pitch angle from 721 seconds to 920s- 0.5m/s² decelerated flight, 921s to 1720s pitch angle accelerated uniformly from 0 degree to 30 degree, roll angle accelerated uniformly from 0 degree to 10 degree, azimuth angle accelerated uniformly from 0 degree to 30 degree, pitch angle decelerated uniformly from 1721s to 1830s, roll angle decelerated uniformly from 10 degree to 0 degree, azimuth angle decelerated uniformly from 30 degree to 0 degree, and evenly decelerated to 170S. The trajectory of the vehicle is shown in Fig. 1. In the simulation process, the zero offset of accelerometer is 100ug, the zero offset of gyro is 0.01 degree/h, the white noise of starlight output 20° and the white noise of barometric altimeter is 100m. Comparing the calculated results with the theoretical values, the error output of loose and tight combination is obtained as shown in figs. 2, 3 and 4.

Figure 1. Simulates the trajectory change curve of the vehicle.

Figure 2. Variation curve of velocity error.
It can be seen from the figure that, compared with the gradual divergence of pure inertia solution error, the error change after loose combination converges gradually. In the steady-state part of the error change data, the error of each parameter is larger, especially the heading angle error. Comparing with the loose combination method, the speed, position and attitude errors of the tight combination correction method are greatly improved in the steady-state part. The fluctuation range is small and the convergence speed is fast. The position errors in the steady-state part can be controlled within 300m. The improvement effect of attitude error is more obvious, which shows better attitude output performance. Compared with the loose combination mode, the tight combination output has faster attitude convergence speed and smaller fluctuation range, and the attitude error of the steady-state part can reach the angular second order.

5. Conclusion
The main advantage of Inertial + Starlight Integrated Navigation is that it can modify the inertial navigation parameters by using more autonomous Starlight Navigation, which is suitable for situations with higher requirements for concealment. To make good use of this advantage, the main problem to be solved is to ensure the accuracy of the horizontal datum required for star navigation. Based on the study of simple combination and loose combination, this paper studies a tight combination error model considering the horizontal reference error of inertial navigation system. Through simulation experiments, the results of error correction of loose combination and tight combination are compared. The results show that the tight combination model studied in this paper improves the correction effect of speed, attitude and position errors obviously, and has the
advantages of convergence speed, fluctuation range and stabilization precision. The research of this model can provide theoretical reference and simulation basis for restraining the influence of the horizontal reference error of INS on the calculation parameters of INS and starlight integrated navigation and improving the precision of INS and starlight integrated navigation.

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