High Efficiency of Gamma-Ray Bursts
Revisited

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Abstract

Using the conservation of energy and momentum during collisions of any two shells, we consider the efficiency of gamma-ray bursts by assuming that the ejecta from the central engine are equally massive and have the same Lorentz factors. We calculate the efficiency and the final Lorentz factor of the merged whole shell for different initial diversities of Lorentz factors and for different microscopic radiative efficiency. As a result, a common high efficiency in the range of 0.1 to 0.9 is considerable, and a very high value near 100\% is also reachable if the diversity of the Lorentz factors is large enough.

Key words: gamma rays: bursts, theory

1 Introduction

The efficiency of a gamma-ray burst (GRB) $\eta$ has aroused great interests in the theoretical studies (Meszárós et al., 1993; Xu & Dai, 2004), because it firmly links with the total energy of the central engine ejected. The energy of the gamma rays is almost standard energy with about $1.3 \times 10^{51}$ erg (Bloom, Djorgovski & Kulkarni, 2001). Thus, if the efficiency is too low, the total ejected energy should be too large and incredible. If it is too high, however, the required large velocity diversity of the ejected shells is far from understanding. Many previous works calculated the efficiency, but the results are rather different. Kumar (1999) drew the conclusion that the efficiency can only be around 1\%, while Beloborodov (2000) argued that it can reach about 100\%. Considering different models for

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the velocity diversities, Kobayashi & Sari (2001); Guetta, Spada & Waxman (2001) found that the results are highly model-dependent, and the efficiency varies in the range of 0.1 to 0.9.

The efficiency can also be obtained by the model fitting for individual bursts (e.g. Freedman & Waxman, 2001; Panaitescu & Kumar, 2002). They found that the $\eta$ varies for different bursts, and some bursts have a very high value about 0.9 (Freedman & Waxman, 2001). Some statistics were carried out for the efficiency by Lloyd-Ronning & Zhang (2004); Eichler & Jontof-Hutter (2005), who found that it may be related to some energies such as the peak energy or the total energy of gamma-rays.

Recently, the shallow decay phase of early X-ray afterglows (Nousek et al., 2006) appeared in many bursts detected by Swift. This phenomenon is generally interpreted as post-burst energy injection. This model enhances the efficiency enormously (Eichler & Jontof-Hutter, 2005). Granot, Königal & Piran (2006) suggested alternate models (e.g., two-component jet model) to avoid this paradox, while by considering the reverse Compton scattering effect, Fan & Piran (2006) found that the efficiency may not be too large even with the energy injection.

In the dynamics, the essential problem is the diversity of the velocities (or Lorentz factors) of sub-shells ejected from the central engine. It is not natural for the central engine to eject shells with very high diversity. Enlightened by the equal energetic ejecta coming from a differentially rotating pulsar (Dai et al., 2006), we suggest that the ejected shells from the central engine are all equal, and the burst itself and the followed X-ray flares (Nousek et al., 2006) all originate from collisions between ejected shells and a slow proceeding shell at the front, which may originally be the envelope of the massive progenitor and be accelerated by the rapid overtaken shells. This model has been used to successfully fit the multi-band observations of the burst GRB 050904 (Zou, Dai & Xu, 2006). In this model, the initial Lorentz factor of the envelope may be unity at the beginning. In this paper, we consider the dynamical evolution of the envelope and the collisions with the upcoming ejected shells, and calculate the emission efficiency, which is taken as the efficiency of GRBs. In this model, the emitted energy of photons can be much greater than the final kinetic energy. We give the dynamics in section §2, calculate the efficiency numerically in §3, and summarize our results in §4.

2 Analysis

We consider one collision between two shells with mass $m_1$ and $m_2$, Lorentz factor $\gamma_1$ and $\gamma_2$ respectively. They merge into one whole shell with mass $m$
(where the materials may be hot) and Lorentz factor $\gamma$. We assume that a fraction $\epsilon$ of the internal energy will convert into photons. This process should obey the conservation of energy and momentum:

\begin{align}
\gamma_1 m_1c^2 + \gamma_2 m_2c^2 &= \gamma mc^2 + E_\gamma, \\
\gamma_1 m_1 \beta_1 c + \gamma_2 m_2 \beta_2 c &= \gamma m_\beta c + E_\beta / c,
\end{align}

where $\beta$ is the velocity in units of $c$ (corresponding to the Lorentz factor $\gamma$), $E_\gamma = \gamma \epsilon E$ is radiated as photons and $(1 - \epsilon)E$ remains in the merged shell, while $E$ is the additional internal energy produced in this collision in the comoving frame of the merged shell. The mass of the merged shell is $m = m_1 + m_2 + (1 - \epsilon)E/c^2$. Define $\epsilon$ as the radiation efficiency, which should be differentiated with the total efficiency of the gamma-ray burst $\eta$. For one collision, the efficiency is $\eta = E_\gamma / (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2)$. For $M$ shells with $N$ collisions, the total efficiency is then

$$\eta = \frac{\sum_{i=1}^{N} E_{\gamma,(i)}}{\sum_{i=1}^{M} \gamma_{(i)} m_{(i)} c^2}.$$  

From these equations, one can find that the evolution depends very sensitively on the collision history. For example, if all the rapid shells are merged first, which cannot produce many photons for a low diversity of velocities, and then, the whole merged shell overtakes a slowly proceeding shell, then the final efficiency will be very small, provided that the slow shell has much less mass than the whole fast shell. A more efficient strategy is that faster shells catch up with slower ones one by one. In this model, all the nearly equal energetic and massive shells colliding with the envelope of the progenitor (like Zou, Dai & Xu, 2006), it just satisfies the efficient strategy. We can expect that a high efficiency appears in this scenario. As all the shells are colliding with the sole foregoing one, we obtain $M = N + 1$. However, to get the final efficiency, Eqs. (1) and (2) should be used in each collision, numerical simulations must be performed.

### 3 Calculations

We assume: $\gamma_1$ and $m_1$ are the initial Lorentz factor and mass of the slow shell in the front; $\gamma_2$ and $m_2$ are the Lorentz factor and mass of the ejected shells from the central engine, which are taken as constant in the model; $\gamma$ is Lorentz factor of the accelerating merged shell, which is originally $\gamma_1$, and afterwards, mixed with the ejected shells. We suppose that the ejected shell with mass $m_2$ and Lorentz factor $\gamma_2$ collides with the envelope. They merge into one shell proceeding with Lorentz factor $\gamma$, and then another ejected shell with same
mass $m_2$ and Lorentz factor $\gamma_2$ collides with the merged shell, which proceeds with a new Lorentz factor $\gamma$, and so on.

Fig. 1 shows the efficiency varying with the total mass of ejected shells. The efficiency increases with $(\Sigma m_2)/m_1$ first, because when the $m_1 > \Sigma m_2$, from Eq. 3, the denominator can be regarded as constant approximately, while the numerator increase with the number of collisions. With the increase of $(\Sigma m_2)/m_1$, the Lorentz factor of merged shell $\gamma$ approach the Lorentz factor of the faster shells $\gamma_2$ more and more closely, then the collisions produce less and less emissions, therefore, the efficiency decreases. Another clear tendency is that, for larger value of $(\Sigma m_2)/m_1$, the collisions are less efficient but with higher final Lorentz factor, and for smaller value of $(\Sigma m_2)/m_1$, the collisions are more efficient but with lower Lorentz factor. For the case $\gamma_2 = 1000$, the efficiency is greater than 0.95, even though the total ejected shells is 100 times more massive than the envelope. At the same time, one should consider the final Lorentz factor to be reasonable. As this final Lorentz factor is the initial Lorentz factor for the afterglow, the value of final Lorentz factor should be in the range of tens to hundreds. Fig. 2 shows the final Lorentz factor corresponding to the cases shown in Fig. 1. It shows, for these values of $\gamma_2$, $(\Sigma m_2)/m_1$ should not be less than 20, and a value larger than 100 is also reasonable. For larger $\gamma_2$, the efficiency and the final Lorentz factor become larger. But a very large $\gamma_2$ may be difficult to occur from the central engine.

These figures may have one implication about the evolution history of the merged shell. The merged shell is initially the envelope at rest. It is accelerated with the accumulation of ejected fast shells. The efficiency increases before the $(\Sigma m_2)$ is less than several times $m_1$, and then it decreases.

It is possible that the radius of the envelope is too small, and the number density of electrons is too large, which prevents the gamma-rays to radiate because of a high optical depth. Therefore, the envelope may be accelerated by the collisions but few photons radiate at first. With the collision radius increasing, the merged shell becomes optically thin. We can simply consider this optical thin shell as the initial foregoing shell, with mass $m_1$ and $\gamma_1 > 1$. We calculate the efficiency and the final Lorentz factor for different $\gamma_1$, while $\epsilon$ and $\gamma_2$ are constant, which are shown in Figs. 3 and 4. Generally, the efficiency is less and the final Lorentz factor is larger than those in the case of $\gamma_1 = 1$, which are also plotted in Figs. 3 and 4 as solid lines. For the case $\gamma_1 = 300$, the efficiency is too small and the final Lorentz factor is too large, which should be ruled out for a normal gamma-ray burst. One may select a set of eclectic parameters for a real burst.

The microscopic radiative efficiency $\epsilon$ may not be 1 perfectly, i.e., all the internal energy cannot be transferred into radiation. But $\epsilon$ shouldn’t be too small neither, otherwise, the internal energy will accumulate more and more
with collisions going on, and then the very high proportion of internal energy will be emitted out definitely. The efficiency and the Lorentz factor as function of \((\Sigma m_2)/m_1\) for different \(\epsilon\) are plotted in Figs. 5 and 6, with \(\gamma_1 = 1\) and \(\gamma_2 = 1000\). For simplicity, we set \(\epsilon\) as constant. As a part of the internal energy is not radiated, the efficiency decreases greatly and the final Lorentz factor increases appreciably as \(\epsilon\) decreases. In Fig. 5, for smaller \(\epsilon\)s, it looks like the efficiency decreases with the increase of \((\Sigma m_2)/m_1\) directly. In fact, it is just that the stage of increasing stops for less \((\Sigma m_2)/m_1\). As for smaller \(\epsilon\), it means more produced internal energy is left in the merged shell. This makes the total mass of merged shell be comparable with \(m_1\) more early (see Eq. 3), and correspondingly, the decreasing stage comes more early.

In the calculations, we assume that \(m_2 = m_1/10\). But the testing calculations for different \(m_2\) show that the results do not depend on the particular value of \(m_2\), provided \(m_2 < m_1\). Therefore, the mass of ejected shell is not required to be equal for the results to be valid.

4 Conclusions

Considering the conservations of energy and momentum, we calculate the efficiency of a gamma-ray burst in different cases, assuming that the ejected shells from the central engine are equally energetic and massive and they all collide onto a slower shell proceeding in the front. A general conclusion is that for a large diversity of the low and high initial Lorentz factors, the efficiency of bursts is higher, which is consistent with the above analysis. We have detailedly considered the influences of the values of initial Lorentz factors and the microscopic radiative efficiency. The final Lorentz factor depends sensitively on the initial Lorentz factor of the slower shell, and the efficiency depends more sensitively on the values of the Lorentz factor of the ejected shells, while both are sensitive to the microscopic radiative efficiency. The scenario provided here is a possible solution for the very high efficiency obtained in the model fittings. Please note that because the envelopes only exist in collapsars, these calculations are only suitable for long duration bursts.

In this scenario, there are four free parameters: the initial Lorentz factor of the slow shell \(\gamma_1\) and of the fast shell \(\gamma_2\), the radiative microscopic efficiency \(\epsilon\), and the mass ratio of fast shells and the slow shell \((\Sigma m_2)/m_1\). These can make the final efficiency be adaptive for a relative large range, say \((0.1,0.9)\). On the other hand, these parameters will be restricted by other aspects of observations, e.g., the peak energy, the deductive Lorentz factor from the afterglow observations, and son on.

In the view of efficiency, this scenario is the most efficient one to produce
gamma-rays. In other scenarios, the efficiency is relatively low. For example, if all the faster shells merge first, which will not produce any photons because of the same Lorentz factors, and then the merged shell collides with the foregoing slow one, the emission will be very inefficient.

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Fig. 1. The efficiency of a gamma-ray burst as a function of total ejected mass over $m_1 \left( \Sigma m_2 / m_1 \right)$. The Lorentz factor of the foregoing slower shell is set as $\gamma_1 = 1$, and the radiative efficiency for each collision is $\epsilon = 1$. The different curves correspond to different Lorentz factor of the ejected fast shells, which are marked in the figure.
Fig. 2. The Lorentz factor of the merged shell as function of $(\Sigma m_2)/m_1$, with $\gamma_1 = 1$ and $\epsilon = 1$. Different curves are for the same cases as in Fig. 1.
Fig. 3. Same as Fig. 1 with parameters $\gamma_2 = 1000$, $\epsilon = 1$ and different $\gamma_1$. 
Fig. 4. Same as Fig. 2 with parameters $\gamma_2 = 1000$, $\epsilon = 1$ and different $\gamma_1$. 
Fig. 5. Same as Fig. 1 with parameters $\gamma_1 = 1$, $\gamma_2 = 1000$, and different $\epsilon$. 
Fig. 6. Same as Fig. 2 with parameters $\gamma_1 = 1$, $\gamma_2 = 1000$, and different $\epsilon$. 