Implication of giant photon bunching on quantum phase transition in the dissipative anisotropic quantum Rabi model

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We investigate the quantum phase transition in the dissipative anisotropic quantum Rabi model in the framework of quantum dressed master equation. From perspectives of both numerical and analytical analysis, we unravel the implication of the giant photon-bunching feature on the first-order quantum phase transition. The observed two-photon statistics can be well described analytically within a few lowest eigenstates at the low temperature. Moreover, such significant photon-bunching peak is generally exhibited at the deep-strong qubit-photon coupling, which is however lacking in the dissipative isotropic quantum Rabi model. Therefore, we suggest that the photon-bunching measurement is helpful to characterize the first-order QPT of the qubit-photon hybrid systems.

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I. INTRODUCTION

Deep understanding and smart manipulation of quantum light-matter interaction becomes a challenging inter-disciplinary frontier, which is scientifically important and practically demanding in the quantum community\textsuperscript{[1,2]}. One representative paradigm to theoretically describe quantum light-matter interaction is the qubit-photon coupled quantum system\textsuperscript{[3,4]}, i.e., a multitasking platform for complementary functionalities with different physical components (e.g., qubit and optical resonator), which has been extensively investigated in diverse fields, ranging from quantum optics\textsuperscript{[5,6]}, quantum information\textsuperscript{[7,8]}, quantum phase transition (QPT)\textsuperscript{[9,13]}, to quantum transport\textsuperscript{[14,17]}. The experimental realizations of the ultrastrong qubit-photon coupling\textsuperscript{[15,20]} stimulate the researchers to explore uncharted regimes of qubit-photon quantum systems.

The quantum Rabi model (QRM)\textsuperscript{[21-25]} is widely considered as the generic model to characterize qubit-photon systems, which is composed of one qubit interacting with a single-mode radiation field. Traditionally, the qubit-photon coupling in the quantum electrodynamics platforms, e.g., natural atoms in resonant cavity, is rather weak\textsuperscript{[26]}, which reduces the QRM to the Jaynes-Cummings model\textsuperscript{[27]} (JCM) by performing the rotating-wave approximation. However, along with the dramatic advances of solid-state quantum platforms, such as circuit quantum electrodynamics systems\textsuperscript{[28,31]}, the intriguing ultrastrong interaction has attracted tremendous attention\textsuperscript{[5,7]}, where the qubit-photon coupling strength is comparable to the individual components energies. This directly invalidates the rotating-wave approximation. Consequently, the counter-rotating-wave (CRW) terms produce a series of exciting physical phenomena, e.g., parity symmetry restricted photon-wavepacket propagation\textsuperscript{[32]}, vacuum Rabi splitting\textsuperscript{[33]}, one photon simultaneously exciting two atoms\textsuperscript{[34]}, and multiphoton sidebands transition\textsuperscript{[35]}. Aimed to bridge the gap between the JCM and QRM, the anisotropic quantum Rabi model (AQRM) was proposed\textsuperscript{[36,37]}, where the strengths of the rotating-wave (RW) and CRW terms are relaxed to be independent. In the weak CRW terms limit, the AQRM is shifted to the JCM, whereas the AQRM becomes QRM with identical coupling strengths for both RW and CRW terms. The AQRM nowadays can be realized by various quantum platforms, such as inductively coupled circuit quantum electrodynamics systems\textsuperscript{[38,40]}, two-dimensional electron gas with spin-orbit coupling\textsuperscript{[41]}, and a two-level qubit exchanging spin with one anisotropic ferromagnet\textsuperscript{[42]}. By exploring the energy spectrum of the AQRM, the first-order QPT is observed when the CRW terms are weaker than RW terms\textsuperscript{[37]}, which is, in sharp contrast, absent in the QRM. Very recently, Chen et al. found that the AQRM undergoes infinitely many first-order QPT by continuously increasing the qubit-photon coupling strength\textsuperscript{[11]}.

In reality, the qubit-photon quantum system inevitably interacts with the environment\textsuperscript{[43]}. The system-bath interaction induced quantum dissipation will dramatically renew both transient and steady-state behaviors of quantum systems, e.g., JCM and QRM, which results in dissipative QPT\textsuperscript{[44,48]}. While from the photon statistics perspective, quantum dissipation enables the reasonable measurements of the nonclassicality\textsuperscript{[49]}. Two-photon correlation function is widely considered as the main observable to characterize the nonclassicality of output photons, which is originally introduced by Glauber to investigate the optical coherence of qun-
tum theory. However, due to the realizations of ultrastrong qubit-photon coupling, the system-bath interaction should be properly treated in the eigenstate framework of the whole qubit-photon quantum systems. In particular, Ridolfo et al. recently proposed a modified definition of the two-photon correlation function to characterize steady-state photon statistics in the dissipative QRM, based on the quantum dressed master equation (DME). Therefore, we naturally raise the question: Could the nonclassicality of photons of the AQRM coupled to the thermal baths at low temperature keeps the trace of the first-order QPT in the closed AQRM?

In this work, we apply the modified expression of the two-photon correlation function in Ref. to investigate quantum phase transition in the dissipative AQRM, where the dissipative dynamics is characterized by the DME. We will focus on the effect of the anisotropic qubit-photon interaction on two-photon statistics in the deep-strong coupling regime. We also reveal the implication of the multiple first-order QPTs with the observed sharp photon-bunching feature and the implication on the first-order QPT in the closed AQRM. We will focus on the effect of the anisotropic qubit-photon interaction on two-photon statistics in the deep-strong coupling regime. We also reveal the implication of the multiple first-order QPTs with the observed sharp photon-bunching feature and the implication on the first-order QPT in the closed AQRM.

In Sec. III, we analyze the emerging sharp photon-bunching behavior in the analytical sense. The paper is organized as follows: In Sec. II, we give a brief introduction to the anisotropic quantum Rabi model, the dressed master equation and two-photon correlation function. In Sec. III, we analyze the emerging sharp photon-bunching feature and the implication on the first-order quantum phase transition. We give a conclusion in the last section.

II. MODEL AND METHOD

A. Anisotropic quantum Rabi model

The Hamiltonian of AQRM is described as

\[ H_{AQRM} = \omega_0 a^\dagger a + \frac{\Delta}{2} \sigma_z + g \left[ (a \sigma_+ + a^\dagger \sigma_-) + r(a \sigma_- + a^\dagger \sigma_+) \right], \]

where \( a^\dagger \) (\( a \)) creates (annihilates) a photon in a cavity with the frequency \( \omega_0 \), \( \Delta \) is the energy splitting of qubit, \( \sigma_{\pm} = \frac{1}{2}(\sigma_x \pm i \sigma_y) \) excites (relaxes) the qubit from the state \( |0(1)\rangle \) to \( |1(0)\rangle \), with \( \sigma_{x,y,z} \) the Pauli operators and \( |0(1)\rangle \) the qubit ground (excited) state, \( g \) is the qubit-photon coupling strength, and \( r \) is the anisotropic parameter.

When \( r = 1 \), the AQRM is reduced to QRM. Furthermore, as the qubit-cavity coupling becomes weak, the CRW terms \( (a \sigma_- + a^\dagger \sigma_+) \) become negligible. Under the rotating-wave approximation, the AQRM is reduced to the JCM, which is identical with the \( r = 0 \) limit. Here, the AQRM ties the QRM and the JCM to investigate the nontrivial influence of CRW terms. Moreover, the AQRM possesses the same symmetry as the QRM, i.e., \( \mathbb{Z}_2 \) symmetry. Specifically, the eigenstate \( |\phi_n\rangle \) of the system Hamiltonian \( H_{AQRM} \) satisfies \(-\sigma_x e^{i\pi a^\dagger a} |\phi_n\rangle = \Pi |\phi_n\rangle\), with the parity eigenvalue \( \Pi = \pm 1 \). Hence, we are able to classify the eigenstates in the even and odd parity subspaces, which correspond to \( \Pi = 1 \) and \( \Pi = -1 \), respectively.

Due to the tremendous advances of the quantum technology, the AQRM with ultrastrong qubit-photon coupling can be realized in superconducting circuits, e.g., based on two inductively interacting superconducting quantum interference devices (SQUIDs). Specifically, a primary SQUID with the comparative large loop generates an electromagnetic field, which is inductively coupled to the second SQUID qubit. In the limit of ignored capacitive interaction between these two SQUIDs, the anisotropic qubit-photon coupling can be steadily realized and operated by including both inductance of the circuit and the mutual inductance.

B. Quantum dressed master equation

Practically, the quantum system inevitably interacts with the environment. To describe the influence of thermal baths on the AQRM, i.e., the qubit and the photon field individually coupled to two bosonic thermal baths, the total Hamiltonian is expressed as

\[ H_{total} = H_{AQRM} + H_B + V, \]

where the first term is just the AQRM Hamiltonian. Two bosonic thermal baths are described as \( H_B = \sum_k \omega_k b_k^\dagger b_k \), where \( b_k(a_k) \) is the creation (annihilation) operator of bosons with the frequency \( \omega_k \) in the \( k \)th bath. The interaction between the AQRM and two bosonic thermal baths is denoted as \( V = V_q + V_c \), where two interaction terms associated with the qubit and the photon field are

\[ V_q = \sum_k \lambda_{q,k} (b_{q,k} + b_{-q,k}^\dagger) \sigma_x, \]

\[ V_c = \sum_k \lambda_{c,k} (b_{c,k} + b_{-c,k}^\dagger) (a + a^\dagger), \]

with \( \lambda_{q,k} \) (\( \lambda_{c,k} \)) the coupling strength between the qubit (photon field) and the corresponding bosonic thermal bath. Generally, the system-bath interaction can be characterized as the spectral function \( \gamma_{q(c)}(\omega) = 2\pi \sum_k |\lambda_{q(c),k}|^2 \delta(\omega - \omega_k) \). In this work, we apply the Ohmic spectrum case to quantify the system-bath interaction, i.e., \( \gamma_q(\omega) = \alpha_q \omega \exp(-\omega/\omega_c)/\Delta \) and \( \gamma_c(\omega) = \alpha_c \omega \exp(-\omega/\omega_c)/\omega_0 \), where \( \alpha_q(\omega) \) is the system-bath coupling strength, and \( \omega_c \) is the cutoff frequency of bosonic thermal bath.

Under the assumption that the system-bath interaction is weak, we can separately perturb both \( V_c \) and \( V_q \) to obtain the DME by including Born-Markov approximation, which may be proper to handle long-time dissipative dynamics at ultrastrong and deep-strong qubit-photon couplings. Indeed, as the interaction between the qubit and photons goes beyond...
strong, the light-matter system should be treated as a whole \[3, 6\]. Accordingly, the system-bath interactions \[23-32\] should be described under the eigenmodes of the AQRM, i.e., $V_q = \sum_{k,m,n} \lambda_{q,k}(b_{q,k} + b_{q,k}^\dagger)P_{nm}$ and $V_c = \sum_{k,m,n} \lambda_{c,k}(b_{c,k} + b_{c,k}^\dagger)P_{nm}^c$, with the eigenmode projectors $P_{nm}^Q$ and $P_{nm}^C$, the eigenstate of the AQRM $|\phi_{nm(n)}\rangle$. This ensures that zero-temperature thermal baths will drive the quantum system to the ground state. Finally, the DME is described as

$$
\frac{\partial \rho_s}{\partial t} = -i[H_{\text{AQRM}}, \rho_s] + \sum_{j,k>j} \{ \gamma_{u,j}^{\dagger} [1 + n_u(\Delta_{k,j})] D[|\phi_j\rangle\langle\phi_k|, \rho_s] + \gamma_{u,j} n_u(\Delta_{k,j}) D[|\phi_j\rangle\langle\phi_j|, \rho_s] \},
$$

(4)

where the dissipator is $D[O, \rho_s] = \frac{1}{2}(2O\rho_s O^\dagger - O^\dagger O\rho_s - \rho_s O^\dagger O)$. $\rho_s$ is the reduced density matrix of the AQRM, $\Delta_{k,j} = E_k - E_j$ is the energy gap between two eigenstates (i.e., $|\phi_k\rangle$ and $|\phi_j\rangle$), $n_u(\Delta_{k,j}) = 1/\exp(\Delta_{k,j}/k_BT_u) - 1$ is the Bose-Einstein distribution function, with $k_B$ the Boltzmann constant and $T_u$ the temperature of the $\nu$-th thermal bath, and the transition rates $\Gamma_{u,j}^{\dagger}$ and $\Gamma_{u,j}$ are given by

$$
\Gamma_{u,j}^{\dagger} = \alpha_u^{\nu} \frac{\Delta_{k,j}}{\omega_0} e^{\frac{\Delta_{k,j}}{\omega_0}} |\langle\phi_j|(|\sigma_- + \sigma_+)|\phi_k\rangle|^2, \quad \quad \quad (5a)
$$

$$
\Gamma_{u,j} = \alpha_u^{\nu} \frac{\Delta_{k,j}}{\omega_0} e^{\frac{\Delta_{k,j}}{\omega_0}} |\langle\phi_j|(|a + a^\dagger)|\phi_k\rangle|^2. \quad \quad \quad (5b)
$$

Note that only $\sigma_\pm$ and linear photon operator appear in these expressions, the transitions are constrained between the eigenstates with different parities. Based on Eq. (4), the steady state of AQRM becomes the ground state under the condition of $T_u = T_c = 0$.

In this work, we perform the numerical diagonalization in each parity subspace of the AQRM to obtain the eigenstates and eigen-energies needed in the DME. One can note that the very high excited states are actually not involved. Even in the deep-strong coupling regime considered below, the convergence for both the eigenstates and eigen-energies employed in the calculation can be achieved very well within the truncated photon number around two hundred in the Fock-state basis.

C. Two-photon correlation function

In quantum optics, the two-photon correlation function is widely considered as one powerful utility to measure the nonclassical radiation of photons, which also reflects the intrinsic correlation properties of quantum materials in light-matter interacting systems \[1, 53, 52\]. The two-photon correlation function is defined as \[41, 54\]

$$
G_2(\tau) = \frac{\langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t) \rangle}{\langle a^\dagger(t)a(t) \rangle^2}.
$$

(6)

where $\langle \cdots \rangle$ stands for the mean value under the $t$-time reduced system density matrix. When the qubit-cavity coupling of the AQRM is sufficient weak, the output field is approximately proportional to the intra-cavity field. Thus, the photon statistic of the output field at the steady state can be properly described by Eq. (4). However, such definition may break down in the ultrastrong qubit-photon coupling regime \[53\].

To characterize the two-photon statistics at strong qubit-photon coupling, Ridolfo et al. derived a modified input-output relation in the dressed-state basis, i.e., $A_{out} = A_{in} - i\sqrt{\gamma}X^+$ \[52\], where $A_{in(out)}$ denotes the input (output) radiation field operator, $\kappa$ is the loss rate of the photons via the interaction with the external detection modes, and the detection operator $X^+$ in the dressed-state basis is

$$
X^+ = -i \sum_{j,k>j} \Delta_{k,j} X_{j,k} |j\rangle |\phi_j\rangle,
$$

(7)

with $X_{j,k} = |\phi_j\rangle (a + a^\dagger) |\phi_k\rangle$ and $X^- = (X^+)^\dagger$. This treatment naturally overcomes the inconsistency that finite photon current unphysically occurs at the ground state, i.e., $X^+ |\phi_0\rangle = 0$. By setting the input field as vacuum, the output photon flux emitted by the optical resonator is expressed as $I_{out} = \kappa \langle X^-(t)X^+(t) \rangle$. Similarly, the output delayed coincidence rate is proportional to the two-photon correlation term $\langle X^-(t)X^-(t+\tau)X^+(t+\tau) \rangle$. Hence, the two-photon correlation function is redefined as

$$
G_2(\tau) = \frac{\langle X^-(t)X^-(t+\tau)X^+(t+\tau)X^+(t) \rangle}{\langle X^-(t)X^+(t) \rangle^2},
$$

(8)

where $\langle A \rangle = \text{Tr}[\rho_s(t)A]$ stands for the mean value at the reduced density matrix $\rho_s(t)$. Consequently, the two-photon correlation function has been applied broadly to characterize the two-photon statistics \[52, 53\]. While for the representative zero-time delay case at the steady state, $G_2(\tau)$ is reduced to

$$
G_2(0) = \frac{\langle X^-X^-X^+X^+ \rangle_{ss}}{\langle X^-X^+ \rangle_{ss}^2},
$$

(9)

where $\langle \cdots \rangle_{ss}$ denotes the expectation value at the steady state. It should be emphasized that the influence of the two-photon correlation measurement on the steady state is negligible due to much weaker coupling assumption compared to the system-environment interactions. As is well known, the bunching and antibunching features of photons are two representative non-classical phenomena of light \[1, 53, 52\]. The bunching behavior is characterized as $G_2(0) > 1$, which shows the super-Poisson distribution. $G_2(0) < 1$ denotes antibunching behavior of photons, corresponding to the sub-Poisson distribution. $G_2(0) = 2$ in the photon thermal field, whereas $G_2(0) = 1$ in the photon coherent state.
Xie et al. observed the first level crossing in the AQR at $g_c^{(1)} = \sqrt{\omega_0\Delta/(1-r^2)}$ using the Bargmann representation [32]. At the level crossing, the parity of the ground state changes sign and the system undergoes a first-order QPT. Within the Bogoliubov operator approach, Chen et al. [11] further revealed that the ground state and the first excited state can cross several times, indicating multiple first-order QPTs at $g_c^n$ with $n$ a positive integer. Recently, Fink et al. experimentally characterized the first-order QPT by measuring two-photon correlation function in the driven-dissipative Bose-Hubbard system [63]. Then, it is interesting to see whether the first-order QPTs in the AQR under the quantum dissipation can also be signified by the two-photon correlation function.

We first investigate the influence of the anisotropic qubit-photon coupling on the two-photon correlation function $G_2(0)$ at the steady state in terms of Eq. (9) at low temperature $(k_B T_c = k_B T_a = 0.07\omega_0)$. In Fig. 1(a), we plot $G_2(0)$ as a function of $g/\omega$ and $r$ in a three-dimensional view. To be clearer, we also present the contour of $G_2(0)$ with antibunching and bunching area in Fig. 1(b). Surprisingly, there exists a significant photon-bunching peak in the middle of Fig. 1(a) within the anisotropic parameter range $0 < r < 0.8$. In particular, the behavior of the first critical point $g_c^{(1)}$ by tuning the anisotropic parameter $r$ agrees well with the narrow photon-bunching regime, as indicated by the white dashed line in Fig. 1(b). Moreover, the divergent trend of the critical point $g_c^{(1)}$ when the anisotropic parameter $r$ is approaching unit showed by Chen et al. [11] is also matched with the vanishing of narrow photon-bunching peak near isotropic situation.

Next, we derive an approximate expression of the two-photon correlation function at the steady state analytically. In the low temperature regime, the general relation of steady-state populations becomes $P_0 \gg P_1 \gg P_2 \gg \cdots$, under the assumption of finite energy-level spacing $\Delta_{k\ell} \gg k_B T$, with $E_k > E_j$ and $H_{AQR} \langle \phi_k \rangle = E_k \langle \phi_k \rangle$. For simplicity, the Hilbert space of AQRM is truncated to the subspace spanned by four lowest eigenstates of the AQRM, i.e., the ground state $|\phi_0\rangle$, and three low exited states $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$. Accordingly, the one-photon correlation term is estimated as $\langle X^{-}X^{+}\rangle_{ss} = \Delta_0^2 \langle X_0 \rangle_{ss}^2 P_0$, which is contributed by the main transition $|\phi_0\rangle \rightarrow |\phi_1\rangle$.

And the two-photon correlation term is given by $\langle X^{-}X^{-}X^{+}X^{+}\rangle_{ss} = \sum_{j<k<l\leq 3} \Delta_{jk}^2 \Delta_{kl}^2 \langle X_{jk} X_{kl}\rangle_{ss}^2 P_l$, which is generally composed by four cooperative transitions, i.e., $|\phi_0\rangle \rightarrow |\phi_1\rangle \rightarrow |\phi_2\rangle$, $|\phi_0\rangle \rightarrow |\phi_1\rangle \rightarrow |\phi_3\rangle$, $|\phi_0\rangle \rightarrow |\phi_2\rangle \rightarrow |\phi_3\rangle$, and $|\phi_1\rangle \rightarrow |\phi_2\rangle \rightarrow |\phi_3\rangle$. Consequently,
the two-photon correlation function is approximated as
\[
G_2(0) \approx \frac{1}{\Delta^2_{1,0}|X_{0,1}|^2 P_3^2} \left\{ \Delta^2_{1,0}|X_{0,1}|^2 \Delta^2_{2,1}|X_{1,2}|^2 P_2 + \left[ (\Delta^2_{1,0}|X_{0,2}|^2 + \Delta^2_{2,1}|X_{1,2}|^2)^2 \Delta^2_{3,2}|X_{2,3}|^2 + \Delta^2_{1,0}|X_{0,1}|^2 \Delta^2_{2,1}|X_{1,2}|^2 P_3 \right] \right\},
\]
(10)
which tightly relies on the transition coefficient \(X_{kj}\) between two eigenstates |\(\phi_k\rangle\) and |\(\phi_j\rangle\). Note that \(X_{kj} = 0\) if |\(\phi_k\rangle\) and |\(\phi_j\rangle\) are of the same parity, providing the selection rule of the correlation measurement induced eigenstate transition in the transition blockade |\(\phi_k\rangle \rightarrow |\phi_j\rangle\).

Note from Eq. (10) that \(G_2(0)\) diverges as \(\Delta_{1,0} \rightarrow 0\). Combining with small contributions of other unimportant processes, the asymptotic trend of the energy level crossing between the ground state and the first excited state lifts up the two-photon correlation function, resulting in a peak structure in the \(G_2(0)\) curve around \(g/\omega_0 = 1.1\) in the dissipative AQRM, e.g., \(r = 0.5\). For comparison, there are no any first-order QPT, i.e., those two lowest energy levels never cross, and are only closer with the increasing coupling strength, thus the two-photon correlation function varies smoothly and does not show a peak structure. Hence, this assures us that the giant photon-bunching signal is an emerging phenomenon when switching from the isotropic dissipative QRM to anisotropic case.

Except the photon-bunching peak in the middle of Fig. 1 (a), it is also intriguing to find that the second critical coupling strength \(g_c^{(2)}\) for the second first-order QPT derived in Ref. [11] can also be roughly captured by the photon-bunching feature with \(r \lesssim 0.25\), as shown in Fig. 1 (b). To describe the implication of the photon bunching peak on the first-order QPTs, we refine the detection signal of second first-order QPT by modulating two bath temperatures in Fig. 2. It is clearly shown that with the decreasing temperature, the photon-bunching range monotonically shrinks to the positions of the second and the third first-order QPTs denoted by the dashed lines.

Then, we analytically connect photon-bunching feature with the first-order QPT of the AQRM. As the first excited state approaches the ground state, \(\Delta_{10}\) vanishes gradually. In the low temperature regime, two-photon correlation function Eq. (10) is approximated as
\[
G_2(0) \approx \frac{4(\Delta^2_{2,0}|X_{0,2}|^2 + \Delta^2_{2,1}|X_{1,2}|^2)\Delta^2_{3,2}|X_{2,3}|^2 P_3}{\Delta^4_{1,0}|X_{0,1}|^4},
\]
(11)
with \(P_3 \approx 1/2\). As the qubit-photon coupling strength \(g\) approaches \(g_c^{(n)}\), one-photon term \(\langle X^+X^+\rangle \approx \Delta^2_{2,0}|X_{0,1}|^2/2\) is strongly suppressed, which directly leads to the pronounced photon-bunching feature.

We should note that from Ref. [11] that the numerical resolution of the level crossings is very difficult, especially in the deep-strong coupling regime, so it is challenging to identify the multiple first-order QPTs using the energy spectra. However, the signal of the two-photon correlation function, which is actually dependent on both the energy spectra and the eigenstates, is pronounced and quite obvious. Since the two-photon correlation function can be measured experimentally [60, 63, 64], we thus propose an alternative way to detect the locations of the first-order QPTs in this work.

IV. CONCLUSION

To summarize, we apply the quantum dressed master equation to investigate first-order quantum phase transition via giant photon-bunching feature in the dissipative AQRM, which enables us to properly treat the strong qubit-photon coupling. The two-photon correlation function at the steady state can be reasonably obtained, and it generally shows a giant photon-bunching peak at deep-strong qubit-photon coupling, which is however unavailable in the dissipative QRM. Based on the numerical and analytical analysis, it is found that such photon-bunching peak is highly related with the emergence of the first-order quantum phase transition, which is characterized as the level crossing of the ground state and first-excited state. Since the two-photon correlation can be measured experimentally, we suggest that measuring photon bunching signal would characterize the first-order QPT of the qubit-photon systems. Finally, it should be noted that the present results are irrelevant with system-bath dissipation strengths, because weak interactions between the quantum system components, i.e., qubit and photons, and the corresponding thermal baths have been taken into account.

We should admit that some deep-strong qubit-photon coupling strengths in this work, e.g., \(g/\omega_0 = 2\), currently is experimentally unavailable based on the circuit quantum electrodynamics setups, for the largest reported qubit-photon interaction strength is \(g/\omega_0 \approx 1.34\) [20]. However, with the tremendous progress of the circuit quantum electrodynamics systems from strong [65] ultrastrong [15, 66], to deep-strong couplings [20], we believe that the quantum technology of the superconducting circuits [67] may break through this bottleneck in near future. As a consequence, our theoretical exploration of interesting quantum optical phenomena could be observed, which may provide insights for nonclassical photon statistics and the first-order quantum phase transition of the AQRM.

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