Fission Transient Times
from Fission Probabilities
of Neighboring Isotopes

K.X. Jing, L. Phair, L.G. Moretto,
Th. Rubenh, L. Beaulieu, T.S. Fan,
and G.J. Wozniak

Nuclear Science Division

June 2000
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof, or The Regents of the University of California.

Ernest Orlando Lawrence Berkeley National Laboratory
is an equal opportunity employer.
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
Fission Transient Times from Fission
Probabilities of Neighboring Isotopes

K.X. Jing, L. Phair, L.G. Moretto, Th. Rubehn,
L. Beaulieu, T.S. Fan, and G.J. Wozniak

Nuclear Science Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

June 2000

This work was supported by the Director, Office of Science, Office of High Energy & Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.
Fission Transient Times from Fission Probabilities of Neighboring Isotopes

K. X. Jing\(^1\), L. Phair, L. G. Moretto, Th. Rubehn\(^2\), L. Beaulieu\(^3\), T. S. Fan\(^4\), and G. J. Wozniak

Nuclear Science Division, Lawrence Berkeley National Laboratory,
University of California, Berkeley, California 94720

(June 27, 2000)

We present a new and straightforward method to estimate the fission transient time by utilizing the cumulative fission probabilities of neighboring isotopes. The fission probabilities were determined as the ratio of the measured fission cross sections to the Bass Model fusion cross sections. For five neighboring \(^{185-189}\)Os compound nuclei produced in \(^{3}\)He/\(^{4}\)He-induced reactions on separated isotope W targets, the transient time \(\tau_D\) is estimated to be smaller than \(25 \times 10^{-21}\) seconds, and the most likely value of \(\tau_D\) is about \(10 \times 10^{-21}\) seconds.

The evolution of a fissioning nucleus from an assumed spherical shape towards the fission saddle, and eventually to the scission point, has been studied extensively [1-3]. If the transient time \((\tau_D)\) that a nucleus takes to evolve from a ground state shape to the saddle point is longer than the characteristic time for compound nucleus decay \((\tau_{CN})\), then the fission probability is expected to be suppressed, and additional particles can be emitted as compared to those predicted by the standard theory. If on the other hand the transient time is short compared to \(\tau_{CN}\), then the stationary Kramers current [4] (i.e., the transition state fission rate) is expected.

This transient time effect has been advocated as an explanation for the large number of prescission neutrons [5-7], charged particles [8,9], and electric dipole \(\gamma\) rays [2,10] observed in relatively heavy fissioning systems. Fission time scales as long as \(\sim 10^{-19}\) seconds have been inferred for fission from the observed prescission particles [1,11,12], although other recent works [13-16] indicate a shorter time scale.

Prescission particles can be emitted either before the system reaches the fission saddle, or during the descent from saddle to scission. Therefore, the fission time inferred from prescission particle emission is the sum of the transient time discussed above and of the time required for the nucleus to descend from saddle to scission. It is important to distinguish between presaddle and postsaddle times since postsaddle times do not affect the fission probability. Efforts have been made to separate the presaddle and postsaddle time components by examining the differences in the mean kinetic energy of charged particles emitted pre- and postsaddle [17]. The separation of presaddle and postsaddle particle emissions is, however, fraught with difficulties and ambiguities.

The transient time has a strong and direct effect on the fission probability. Consequently, its magnitude may be determined more reliably from fission probabilities [18] rather than from indirect methods such as particle/photon emission.

In the following, we illustrate a new method to estimate the transient time, based upon high precision fission probabilities of several neighboring isotopes. This approach is based on the fact that, except for a factor accounting for the transient time effects, the second chance fission probability of a nucleus \((Z,A)\) appears as the first chance fission probability of the neighboring nucleus \((Z,A-1)\), whose second chance fission probability is in turn the first chance fission probability of its neighbor \((Z,A-2)\), and so on. This novel approach, which does not involve any consideration beyond the fission saddle, automatically bypasses the difficulties associated with the separation of the presaddle and postsaddle particle emissions.

Assuming a step function for the transient time effects, the fission decay width can be written as

\[
\Gamma_f = \Gamma_f^{(0)} \int_{0}^{\infty} \xi(t) \frac{N(t)}{N_0} \lambda_{CN} dt = \Gamma_f^{(0)} \int_{\tau_D}^{\infty} \frac{N(t)}{N_0} \frac{dt}{\tau_{CN}},
\]

where \(\xi(t) = 0 (t < \tau_D)\) and \(\xi(t) = 1 (t \geq \tau_D); \tau_D\) is the fission transient time; \(N(t)\) is the number of remaining compound nuclei at time \(t\) (starting with \(N_0\) compound nuclei at \(t=0\); \(\lambda_{CN}\) is the total decay constant of the compound nucleus, and \(\tau_{CN} = 1/\lambda_{CN}\) is the compound nucleus lifetime; \(\Gamma_f^{(0)}\) denotes the transition-state fission width, or the fission width expected when no transient time effects are present. In the following we will use \(\lambda's\) (\(\tau's\)) to refer implicitly to the corresponding decay (time) constants without transient time effects since the effects have been taken care of by the step function \(\xi(t)\).

Now consider a decay chain starting from the compound nucleus \((Z,A)\) with excitation energy \(E:\)

\[
\begin{array}{c}
\text{Z} \quad n \quad \text{Z} \quad n \quad \text{Z} \quad n \quad \ldots
\end{array}
\]

where \(\Delta E_i\) (\(i=1,2,\ldots\)) denotes the average energy loss by evaporation of the \(i\)th neutron. \(\Delta E_i\) can be estimated as \((B_n + 2T)\), where \(T_i\) is the temperature of the residual nucleus after \(i\)th neutron emission and \(B_n\) is the corresponding neutron separation energy. Let \(\lambda_i^{(t)}\),
\( \lambda_j^{(i)}, \lambda_{CN}^{(i)} = \lambda_n^{(i)} + \lambda_f^{(i)} \) (\( i = 0, 1, 2, \cdots \)) be the neutron, fission and total decay constants of the nucleus \((Z, A - i, E - \sum_j \Delta E_j)\), respectively. The inverse of these decay constants defines the corresponding characteristic times: \( \tau_n^{(i)}, \tau_f^{(i)}, \tau_{CN}^{(i)} \). Let \( N_0(t), N_1(t), N_2(t), \cdots, N_i(t), \cdots \), be the numbers of nuclei \((Z, A)\), \((Z, A - 1)\), \((Z, A - 2)\), \cdots, \((Z, A - i)\), \cdots, respectively, at time \( t \) (starting with \( N_0(0) = N_0, N_1(0) = 0, N_2(0) = 0, \cdots \)). Given a transient time \( \tau_D \) and assuming a step function for the transient time effects, the number of nuclei \((Z, A - i)\) must satisfy the balance equations:

\[
\frac{dN_i(t)}{dt} = \lambda_n^{(i-1)} N_{i-1}(t) - \lambda_n^{(i)} N_i(t), \quad (t \leq \tau_D) \tag{2}
\]

\[
\frac{dN_i(t)}{dt} = \lambda_n^{(i-1)} N_{i-1}(t) - \lambda_{CN}^{(i)} N_i(t), \quad (t \geq \tau_D) \tag{3}
\]

where \( \lambda_n^{(i-1)} \) and \( \lambda_n^{(i)} \) are the neutron decay constants of the nuclei \((Z, A - i + 1)\) and \((Z, A - i)\), respectively; \( \lambda_{CN}^{(i)} \) is the total decay constant of the nucleus \((Z, A - i)\), and \( \lambda_{CN}^{(i)} = \lambda_n^{(i)} + \lambda_f^{(i)} \) with \( \lambda_f^{(i)} \) being the fission decay constant. The solution of the above equations is straightforward [19]:

\[
N_i(t) - N_0 = \sum_{j=0}^{i} a_{i,j} \exp(-\lambda_n^{(j)} t), \quad (t \leq \tau_D) \tag{4}
\]

\[
\begin{align*}
N_i(t) - N_0 &= \sum_{j=0}^{i} b_{i,j} \exp(-\lambda_{CN}^{(j)} t), \quad (t \geq \tau_D) \\
b_{i,j} &= \frac{\lambda_{CN}^{(i-1)} - 1}{\lambda_n^{(i)} - \lambda_{CN}^{(i)}}, \quad j = 0, 1, 2, \cdots, i - 1, \\
b_{i,i} &= \sum_{j=0}^{i-1} a_{i,j}, \\
a_{0,0} &= 1.0; \\
b_{0,0} &= \exp(\lambda_{CN}^{(i)} \tau_D) \left[ \frac{N_i(\tau_D)}{N_0} - \sum_{j=0}^{i-1} b_{i,j} \exp(-\lambda_{CN}^{(j)} \tau_D) \right], \\
b_{i,i} &= \exp(\lambda_{CN}^{(i)} \tau_D) \left[ \frac{N_i(\tau_D)}{N_0} - \sum_{j=0}^{i-1} b_{i,j} \exp(-\lambda_{CN}^{(j)} \tau_D) \right] \exp(\lambda_{CN}^{(i)} \tau_D).
\end{align*}
\]

This solution, written above also, provides the algorithm to follow the decay chain until all the excitation energy is exhausted.

With the solution \( N_i(t) \) \((i = 0, 1, 2, \cdots)\) in hand, the total fission probabilities \( P_f^{(i)} \) can be simply calculated as

\[
P_f^{(i)} = \sum_{i=0}^{\infty} P_f^{(i)}
\]

\[
P_f^{(i)} = \int_{\tau_D}^{\infty} \lambda_f^{(i)} \frac{N_i(t)}{N_0} \, dt = P_f(Z, A - i, E - \sum_j \Delta E_j)
\]

\[
\text{or} \sum_{j=1}^{\infty} \frac{b_{i,j} \lambda_{CN}^{(j)}}{\lambda_f^{(j)}} \exp(-\tau_D / \lambda_f^{(j)}), \tag{7}
\]

where \( \tau_D^{(i)} = 1/\lambda_f^{(i)} \), and \( P_f(Z, A - i, E - \sum_j \Delta E_j) = \lambda_f^{(i)} / \lambda_{CN}^{(i)} \) is the expected (theoretical) first chance fission probability for the compound nucleus \((Z, A - i)\) with excitation energy \( E - \sum_j \Delta E_j \) when no transient time effects are present.

The transition state fission width \( \Gamma_f^{(\infty)} \approx T_s \rho_s (E - B_f - E_f^p) \) \(2/\pi \rho_s (E - E_f^p)\),

\[
\Gamma_n \approx \frac{K T_n^2}{(E - B_n - E_f^p)} \frac{\rho_n (E - B_n - E_f^p) \exp(2\sqrt{\rho_s (E - B_n - E_f^p - \Delta E_c))}}{\rho_n (E - B_n - E_f^p) \exp(2\sqrt{\rho_s (E - B_n - E_f^p - \Delta E_c))}}, \tag{10}
\]

where \( \rho_s \) and \( \rho_n \) are the densities of the fuel and neutron states, respectively; \( K \) is the fission barrier; \( E_f^p \) and \( E_f^p \) are the rotational energies of the system at the saddle and at the ground state, respectively. The constant \( K = \frac{3}{2} m R^2 g^2 / \hbar^2 \) where \( m \) is the neutron mass, \( R \) the radius of the residual nucleus after neutron emission, and the spin degeneracy \( g = 2 \). Taking the simplest form for the level density \( \rho \propto \exp(2\sqrt{\rho_s E}) \), the level density at the saddle \( \rho_s \) and the level density at the ground state \( \rho_n \) can be expressed as [18, 20]:

\[
\rho_n (E - B_f - E_f^p) \propto \exp\left[ \left( \frac{2}{3} \sqrt{\rho_s (E - B_f - E_f^p - \Delta E_c))} \right) \right],
\]

where \( B_f \) is the fission barrier, \( \Delta E_c \) is the pairing condensation energy, and \( \Delta E_{shell} \) is the ground state shell effect of the daughter nucleus after neutron emission. For an even-even nucleus, \( \Delta E_c = (1/2) g \Delta \Omega_0^2 \), and for an odd \( A \) nucleus, \( \Delta E_c = (1/2) g \Delta \Omega_0^2 - \Delta_0 \), where \( \Delta_0 \) is the gap parameter and \( g \) is the doubly-degenerate single particle levels (\( g = (3/2) a \)) with \( a \) being the level density parameter either at the saddle \( \rho_s \), or at the ground state \( \rho_n \).

The total fission probabilities \( P_f^{(i)} \) \((Z, A, E)\) of different isotopes at different excitation energies can be determined as the ratio of the fission cross section \( \sigma_f \) to the fusion cross section \( \sigma_0 \).

We have recently measured with high precision the fission excitation functions of the neighboring compound osmium nuclei \( 182^{186}, 187^{188}, 189^{186} \text{Os} \) produced in \( 3^\text{He} \)-induced reactions on isotopically enriched tungsten targets \( 182^{183}, 184, 186\text{W} \) (see Fig. 1). The isotopic enrichments of the \( 182^{186}, 183^{186}, 186^{186} \text{W} \) targets were
all the available fission excitation functions for the compound nuclei $^{189}$Os, $^{188}$Os, $^{187}$Os, $^{186}$Os and $^{185}$Os with Eqs. 6 & 7. In order to reduce possible correlations between different parameters, we proceeded as follows. To extract the fission barriers $B_f$, we first fit the low energy (<70 MeV) portion of the fission excitation functions. In this fit, the fission barriers $B_f$ for the nuclei $^{189,188,187,186,185}$Os were taken as free parameters. The value of $\tau_D$ was set to zero since first chance fission is expected to dominate at low energies. Setting $\tau_D = 0$ reduces the formalism to the form with which the first chance fission probability is usually obtained. The ratio $a_f/a_n$ was assumed to be the same for all nuclei, but its value was let free in the fit. $a_n$ was assumed to be $A/8$ (MeV$^{-1}$), and the shell effects of all nuclei involved were taken to be the nominal values from Möller et al. [24].

The pairing gap parameter $\Delta_0$ was chosen to be 0.85 MeV. The extracted fission barriers yield corresponding liquid-drop values ($\bar{B}_{\text{macro}} = B_f - \Delta$) which should vary smoothly with the mass number $A$ of the Os isotopes. The barrier values for isotopes lighter than $^{188}$Os can therefore be obtained by a linear extrapolation of the corresponding liquid-drop values of the extracted barriers for $^{185-189}$Os. The fission barriers extracted from this fit, the corresponding liquid-drop values, and their extrapolations for light Os isotopes are shown in Fig. 2.

We now let $\tau_D$ free, and fit the complete excitation functions with $\tau_D$ and $a_f/a_n$ as the only free parameters, using the fission barriers obtained above as the fixed parameters. In Fig. 1, we show the simultaneous fit for five neighboring osmium compound nuclei among which $^{186}$Os and $^{187}$Os were produced in both $^3$He- and $^4$He-induced reactions. All seven fission excitation functions are well reproduced with only two free parameters, and the value obtained for $\tau_D$ from this fit is $10(\pm1) \times 10^{-21}$ sec. This $\tau_D$ value is consistent with the conclusion reached from the universal scaling in fission probabilities [18,25], and also consistent with the recent $\tau_D$ values reported in [13-16].

The $a_f/a_n$ value given by the fit is 1.062. It is found that fits of comparable quality can be achieved for other $a_f/a_n$ values in a small range centered at 1.062 (see the $\chi^2$ values in the upper panel of Fig. 3). Higher estimates for fission probabilities resulting from a larger $a_f/a_n$ value seem to be (to a substantial extent) compensated by a larger value of $\tau_D$ (bottom panel of Fig. 3), and vice versa. This correlation between $a_f/a_n$ and $\tau_D$ values makes it difficult to obtain a unique value for $\tau_D$. A good fit can be obtained with a $\tau_D$ value as small as zero, but not with a $\tau_D$ value larger than $25 \times 10^{-21}$ sec, above which the fit not only requires an even larger $a_f/a_n$ value (>1.075), but also the $\chi^2$ of the fit become greater than twice the minimum value.

The fusion cross sections $\sigma_f$ (see Fig. 1), which were calculated with the Bass Model [23] and used to determine the total fission probability $P_f$ in the current analysis, are a source of uncertainty. Unfortunately, no significant experiments are available in the energy regime of

![FIG. 1. Measured fission excitation functions for five adjacent Os compound nuclei produced in the $^3$He($^3$He,)$^3$He($^3$He,)$^4$He($^4$He)-induced reactions on W targets. The fusion cross sections ($\times$) are the Bass Model predictions [23]. For each excitation function, the contributions from first, second, third, ... chance fission to the total fit (solid line) are shown. The $\tau_D$ value obtained from this simultaneous fit to seven excitation functions is $10 \times 10^{-21}$ sec, and $a_f/a_n$ is 1.062.](image-url)
interest to this work (>70 MeV), that could be used to judge the correctness of the Bass Model calculations. If the actual fusion cross sections are lower than the Bass predictions, which is likely [19,26], the resulting value for the transient time \( \tau_D \) will be smaller.

In summary, we have found a new and straightforward way to estimate the transient time of fissioning systems, by utilizing the cumulative fission probabilities of neighboring isotopes. For five Os isotopes, the fission transient time \( \tau_D \) is estimated to be smaller than \( 25 \times 10^{-21} \) sec, and the most likely value of \( \tau_D \) is about \( 10(\pm1) \times 10^{-21} \) sec. The quality of the fit for \( \tau_D=0 \) is such that no modification of the standard theory is demanded.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy, under Contract No. DE-AC03-76SF00098.

Present addresses:
1 Department of Physics, Sichuan University, Chengdu, China
2 Rubehn@t-online.de
3 beaulieu@phy.ulaval.ca
4 Department of Technical Physics, Peking University, Beijing, China

[1] D. Hilscher, et al., Ann. Phys. Fr. 17, 471 (1992).

**FIG. 2.** Os fission barriers (o or *) are plotted vs the mass number \( A \). The fission barriers for \( A=185-189 \) (*) were obtained by simultaneously fitting the low energy portion (<70 MeV) of the fission excitation functions shown in Fig. 1 (see text). For \( A < 185 \), the fission barriers (o) were obtained by a linear extrapolation (o) of the corresponding liquid-drop values \( B_f - \Delta_{\text{shell}} \) (filled diamond) for the extracted barriers \( B_f \) for \( A=185-189 \).

**FIG. 3.** Bottom panel: The \( r_D \) value extracted from the simultaneous fit with the \( a_f/a_n \) value fixed is plotted versus \( a_f/a_n \). Top panel: The \( \chi^2 \) value per degree of freedom (v) corresponding to the simultaneous fit with a fixed \( a_f/a_n \) value is plotted vs \( a_f/a_n \) value.
