Path integrals in configuration space and the emergence of
classical behavior for closed systems

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April 14, 2015

Abstract
Traditionally, the field of quantum foundations has been preoccupied with different forms of the
question “How can an observer be in a state of quantum superposition?”. In this paper, I approach
this question by exploring a timeless interpretation of quantum mechanics of closed systems, solely
in terms of path integrals in non-relativistic timeless configuration space. What prompts a fresh look
at the foundational problems in this context, is the advent of multiple gravitational models in which
Lorentz symmetry is only emergent. In this setting, I propose a new understanding of records as
certain relations between two configurations, the recorded one and the record-holding one. These
relations are formalized through a factorization of the amplitude kernel, which forbids unwanted
‘recoherence’ of branches. On this basis, I show that in simple cases the Born rule emerges from
counting the relative density of observers with the same records. Furthermore, unlike what occurs
in consistent histories, in this context there is indeed a preferred notion of coarse-grainings: those
centered around piece-wise classical paths in configuration space. Thus, this new understanding
claims to resolve aspects of the measurement problem which are still deemed controversial in the
standard approaches.

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1 Introduction

1.1 A summary of main results

Before I begin, let me state more clearly, but still in words, the main claims and assumptions made in this paper. First, I take the entire time-independent configuration space to exist fundamentally and independently of dynamics – a point of view extensively argued for by Barbour in [1]. Amplitudes associated to configurations are defined through the path integral kernel. In this kernel, the completely degenerate configuration is taken to be the initial point, for definiteness. This uniquely defines the ‘wave-function of the Universe’ in this timeless context.

Relations between instantaneous observables only make sense if such observables all have the same records. Loosely, a configuration has a record of another if it holds, encoded in itself, the ‘prior existence’ of the recorded configuration. Since I am working in a timeless scenario, this is formalized by defining a recorded configurations as being a necessary intermediate point for the relevant coarse-grainings (bundles of paths interpolating between the initial configuration and the record-holding one).

I show that in the semi-classical case, in instances in which the relevant coarse-grainings have decohered, one can explicitly compare the relative density of configurations with the same records. In this simple case I show that this comparison approximates the Born rule for the amplitudes associated to these configurations, thus at least partially tying the meaning of the wave-function to a density distribution in configuration space induced by the dynamics. Using a relative density of observers is necessary for I do not define a kinetic inner product in the state space, but only a natural dynamical one that is closely associated to the decoherence functional.
I propose that there are naturally preferred coarse-grainings in this context: those centered around 
classical paths (or piece-wise classical, which are always defined). This could potentially resolve the 
‘sset selection’ problem of consistent histories. Lastly, I also show how partial fields in different regions 
decouple and entangle with each other, and how one can define a notion of a clock as a particular type 
of entangled subsystem (which recovers the standard non-relativistic path integral).

1.2 Attitudes towards foundational problems in quantum mechanics

“So irrelevant is the philosophy of quantum mechanics to its use, that one begins to suspect 
that all the deep questions about the meaning of measurement are really empty, forced on us 
by our language, a language that evolved in a world governed very nearly by classical physics. 
But I admit to some discomfort in working all my life in a theoretical framework that no one 
fully understands.” - Steven Weinberg [2]

It is hard to disagree that Weinberg’s pragmatic assessment reflects the majority opinion in the physics 
community. In everyday work, it is undeniable that most of us take a ‘shut up and calculate’ approach 
to quantum mechanics, a characterization due to David Mermim. Even the discomforting feeling that 
Weinberg alludes to is not there to be suppressed most of the times.

The argument that our conception of quantum mechanics is limited by intuition coming from everyday 
experience and language is a recurring one in informal discussions. Although there is nothing illogical 
about this line of reasoning, for many researchers in the foundational issues camp it would be defeatist to 
accept it at face value. After all, both our language and our intuition can change dramatically with time. 
A well-known example is that of the concept of ‘fields’: it is now grasped by the educated layperson, but 
more than a generation of physicists struggled to take it on board. It took time for our representations 
of the world and our language to evolve and encompass the notion, for which we can now say we have 
even an intuitive understanding.

Perhaps not enough time has yet passed to accommodate our intuition towards quantum mechanics, 
or perhaps there is something inherently ill-defined with our present understanding. For whichever 
reason, echoing Weinberg, many researchers deem it premature to state that the foundational issues 
with quantum mechanics are resolved. Feynman phrases some of his veiled concerns about this exact 
issue in his characteristic way [3]

“[...]we have always had a great deal of difficulty understanding the world view that quantum 
mechanics represents. At least I do, because I’m an old enough man that I haven’t got to the 
point that this stuff is obvious to me. Okay, I still get nervous with it ... you know how it 
always is, every new idea, it takes a generation or two until it becomes obvious that there’s 
no real problem. I cannot define the real problem, therefore I suspect there’s no real problem, 
but I’m not sure there’s no real problem.”

In the minds of many, the ‘real problem’ is the standing of realism in quantum mechanics, and, contrary 
to Feynman’s expectations, it is safe to say that two generations have not been enough to abolish this 
concern. In the words of Valentini [4]

“[a statistical or epistemic] interpretation does not solve the true measurement problem, which 
is the problem of what happens to macroscopic realism at microscopic scales. In quantum 
physics, we have definite states of reality at the macroscopic level but not at the microscopic 
level. There is no precisely defined boundary between these two domains. Therefore, standard 
quantum theory is fundamentally ill-defined.”

In this paper I would like to revisit some of the foundational issues of quantum mechanics in light of 
recent theoretical advances that allow a description of gravity for which Lorentz invariance is an emerging 
feature, as opposed to fundamental tenet (e.g. Hořava-Lifschitz [5], Einstein-Aether [6], shape dynamics 
[7], trace dynamics [8] etc). In particular, I would like to examine the conceptual picture that one obtains 
by considering configuration space as the fundamental ontological entity – a ‘beable’ in the foundational 
quantum mechanics lingo.

Although this feature resembles one of Bohmian mechanics, the interpretation put forward here 
requires aspects of both Everettian and Bohmian approaches. Everettian, in that the a priori existence 
of configuration space implies it is not merely a stage on top of which dynamics of a Universe unfolds. 
All configurations are taken to exist timeless, independently and on equal footing, as in Barbour’s 
picture of Platonia [1] (which is a disguised realization of many worlds). The Bohmian aspect enters the
description in that it is positions, or configurations, which exist in this manner. They are the preferred beables that constitute the underlying reality of the Universe. In the same manner, this interpretation has contextual and non-contextual elements. Contextual in that all results of observations are intrinsic to the given points in configuration space where they are obtained – a type of Bayesian post-selection – and non-contextual in that an underlying reality of the entire Universe exists in the totality of configuration space.

It is the merging of many-worlds with the preferred existence of configurations, which is only possible for theories with preferred notions of simultaneity (a preference which brings into the theory aspects of non-locality) that, with the help of decoherence, may dissolve issues related to ‘the quantum to classical transition’. In this paper I will focus on the quantum mechanics of a closed system, where the amplitude distributions are given by path integrals in timeless configuration space. In the present interpretation, arbitrary quantum states are secondary to configurations and transition amplitudes between them. There are no superpositions of configurations, only interference (or lack thereof) between the paths interpolating between them.

The three axioms that the present work is based on are the following:

1. **A closed Universe.** This will consist of a closed spatial manifold \( M \) (i.e. compact without boundary) endowed with an \( n \)-tuple of fields (for instance, the spatial metric field, the spatial electromagnetic potential, etc). Any instantaneous observer in the Universe, as well as any clock, has to somehow be represented by these fields. I will sometimes accord to standard nomenclature and call \( M \) the physical space, although in this interpretation configuration space is also a real entity.

2. **The configuration space \( \mathcal{M} \) for the \( n \)-tuple of fields.** This is the infinite-dimensional space of all possible field configurations over \( M \) – it is our setting of what could be loosely called a ‘quantum multiverse’, as in the Many-Worlds setting. Each configuration – the instantaneous field content of the entire Universe – is represented by a point of \( \mathcal{M} \). Each point of \( \mathcal{M} \) can be thus seen as a “snapshot” of a possible Universe (clocks inside the Universe included). This point of view eliminates, at least at a kinematic level, the distinction between different instants of time and “alternative universes”, similarly to the picture proposed by Barbour [1].

3. **The path integral kernel \( K[\phi_1, \phi_2] \).** Given a ‘in’ configuration \( \phi_1 \), the (timeless) transition amplitude (or propagator) to the ‘out’ configuration \( \phi_2 \) is given by a timeless Feynman path integral in configuration space:

\[
K[\phi_1, \phi_2] = \int_{\gamma \in \text{paths}(\phi_1, \phi_2)} D\gamma \exp \left[ iS[\gamma]/\hbar \right]
\]  

Here the action \( S[\gamma] \) should be invariant at least under 3-diffeomorphisms and global reparametrizations of \( \lambda \) (with fixed end-points).

These three relatively innocuous principles contain many novel features in the interpretation of quantum mechanics, using them, I will introduce the necessary tools for the interpretation developed here.

First, I will define ‘record configuration’ and ‘record-holding configuration’ by a relationship between the two. This definition makes sure the amplitude between any two configurations not connected by this relationship gives only superfluous information. With this view, there is no need to ever “update the wavefunction” of the Universe at each “measurement”: the initial configurations are effectively embedded in the final configurations themselves. You and I are in a single configuration, inside an instant, which encodes elements of other instants. An instant is not in time, so to say, time is in the instant. After selecting an initial configuration for the Universe - the completely degenerate one , for definiteness – this means that there exists a unique (necessarily “time-independent”) wave-function of the Universe.

The definition of records is such that in their presence it is impossible for histories not arising from a record to “recohere” onto any of the record-holding configurations. This avoids any relevant recoherence, seen as an issue of the Everettian+ decoherence approach [9]. With this definition of records in place, the job of physicists is interpreted as counting the relative frequency of configurations holding the same records, a completely Bayesian point of view. Since the entirety of configuration space exists fundamentally, a frequentist approach to probabilities in this form seems relatively unproblematic.

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1To obtain the correct number of degrees of freedom for gravity, one would require an extra scalar symmetry generator. For instance, in shape dynamics this is taken to be that of spatial Weyl transformations [7].
By taking this formalism to heart and interpreting it with what I feel is minimal extra baggage, I arrive at a picture in certain ways similar to consistent histories (see [10] for a review and [11] for an application to space-times). After a definition of coarse-grainings suitable to the formulation in terms of path integrals in configuration space, I approach the issue of the “set selection” problem of consistent histories, the global version of the “preferred basis” problem.

In this formalism, the problem is diluted: measurements in different basis (different ‘realms’ or ‘frameworks’) most often occur in different regions of configuration space, and thus have processes represented by completely different coarse-grainings belonging to different regions of configuration space. However we should note that we are not projecting some underlying quantum state of the Universe onto a given basis. We are measuring the relative frequency of configurations with the same memories (records), and only have access to ‘the quantum state of the Universe’ – whatever we might mean by that – over those configurations.

I am reluctant to say that this interpretation of quantum physics is “non-realist”. It is realist: it posits that each and every possible instant of the Universe is completely real. They all co-exist in ‘Platonia’, or the configuration space of the physical fields. It is also contextual, as different experimental setups belong to different parts of configuration space. As Griffiths has pointed out [12]: “The central issue is that the principle of unicity, the idea that there is a unique single true description of the world, is incompatible with our current understanding of quantum mechanics[...]'", and this is naturally realized here, without contradiction to (our notion of) realism.

Nonetheless, even after this dilution there might still be degeneracy of selection of the coarse-grained histories representing a given process. This remaining ambiguity can also be done away with a preferred form of coarse-grainings arising naturally in this non-relativistic context. These are what we call piece-wise extremal coarse-grainings (PEC) and are made up of coarse-grained histories in configuration space which in a sense are “as classical as possible”. I will describe how to construct such sets in more generality and the semi-classical expansion method due to Cecille DeWitt – which can be used to extend the use of such coarse-grainings beyond the usual WKB approximation.

1.3 Roadmap

In section 2 I will first properly define the stage on which the present work takes place: the configuration space of a closed system. I show how limited knowledge of the configuration of the whole Universe gives rise to many copies of the observer, which gets to be represented in configuration space as a submanifold. Then I will present the semi-classical expansion as applied to the present context as a way of extending the use of classical paths beyond the usual WKB approximation. For multiple reasons, dynamical systems will usually require us to extend the use of such classical paths to the piece-wise extremal ones, which we do at the end of the section, in 2.2.3. In section 3 I introduce the present construction of coarse-grained histories. Decoherence is based on ‘fading’ of coarse-grained histories representing a given process. Then I introduce coarse-grainings centered on piece-wise extremal curves (PECs) and define in which sense such an object can be minimal, giving rise to minimal PECs. In the next subsection I define records and partial records (the latter are records of only parts of the configuration) and their relationship to the record-holding configurations. I show that relative amplitudes of configurations holding the same records do not require any more information about an initial state. In the next subsection, 4.2, in attempting to define quantum states related to the configurations, I define a plausible initial state over the completely degenerate configuration. From this initial state, the amplitude kernel defines the state related to any other configuration. I then define an inner product between these states, which incorporates the notions of decoherence introduced before. This inner product has many interesting properties, for instance, the Born rule can be represented as the inner product between a record and its record holding configuration, and it brings in the desired properties of decoherence among coarse-grained paths to the quantum state level. I then show that the frequentist approach for this setting naturally coincides with the Born rule obtained in this way. Next, in section 5, I discuss entanglement in the semi-classical context. Using a generic example for a system entangled with an environment, I use the previous methods to show how decoherence ensues, and how one can count the number of observers of such decohered branches, leading also to the Born rule obtained before. Lastly, in section 5.3, I draw a parallel between decoherence and classical chaotic behavior, and in section 6, I conclude.
2 Path integrals in configuration space

2.1 Configuration space

“Indeed, all measurements of quantum mechanical systems could be made to reduce eventually to position and time measurements (e.g., the position of a needle on a meter or time of flight of a particle). Because of this possibility a theory formulated in terms of position measurements is complete enough to describe all phenomena.” - Feynman and Hibbs [13]

I would add that even the reading of a clock is abstracted from a position measurement: we often identify it with a given configuration of that common contraption on the wall (or on our wrists) we are all familiar with. These facts are argued for extensively in favor of a representation of physics in configuration space in Barbour’s ‘The End of Time’ [1]. The underlying issue here can be traced back to Parmenides, and revolves around whether each subsequent moment is brought into existence with the passage of time. Following Barbour’s view, the answer is no: our notion of classical time is emergent from the path integral and the presence of records, which I will discuss in section 3.2.

Further, there are good reasons to assigning preferred status to a position representation of quantum mechanics. Here I will assume that configuration space is the ontological basis, or stage, of physics. In this interpretation the stage and actors are one and the same, parts of the configurations. The director of the play is the path integral.

Configuration space, which I will denote here by $M$, is the set of all possible field states over a given (in our case finite-dimensional, closed) manifold $M$. Each point of configuration space is indivisible, it is a “snapshot” of the whole Universe. There is a type of reciprocity between physical space and configuration space which can be described as follows. Whereas fixing the entire field configuration (non-locally on $M$) defines a point in the configuration space $M$, fixing only a partial field configuration on a subset of $M$ determines only a submanifold of $M$. These submanifolds are formed by all of the configurations which have that same fixed field, let’s say $\phi_O$ defined on $O \subset M$, i.e. those fields $\phi$ which coincide with $\phi_O$ on $O$ but are arbitrary elsewhere:

$$M_{\phi_O} := \{ \phi \in M \mid \phi|_O = \phi_O \}$$

Observers as submanifolds in configuration space Here I consider observers to be equal to brain configurations. There are no subjective overtones attributed to an observer – they are merely partial states of the fields and thus represented by such submanifolds as described above. Each point in this submanifold contains a copy of the observer’s part of the field, but differs elsewhere (since these points differ only in that part of the field configuration which does not contain the observer). This setting for quantum mechanics – essentially a (non-relativistic) version of Many-Worlds with a preferred basis – in this manner justifies the frequentist approach to probabilities: the intrinsic volume of the surface is basically a specific way to count (a continuous number of) copies of the observer $\phi_O$, as we will see in section 5.2.1.

2.2 The semi-classical expansion

One of the themes of this work is that for non-relativistic systems one can apply techniques developed for particle path integrals to path integrals in configuration space. Thus I start with an introduction of the semi-classical methods I will transfer to the full field configuration space setting.

2.2.1 The first order approximation

I will here mostly study transition amplitudes between configurations that have at least one extremal (classical) path interpolating between them, and only briefly touch on the more general case in the next subsection. In the context of path integrals in configuration space, that means I will be in the semi-classical approximation. I now assume that the following semi-classical approximation for the path

There is a matter of the choice of configuration space. The way I make this choice here is relatively arbitrary, and can be in principle argued against. For gravity, I will choose the space of 3-metrics, since the use of e.g. extrinsic curvatures requires time derivatives. See the conclusions for further discussion.

To be more careful, I should only ascribe to them the title of ‘subsets’, not submanifolds. However, under reasonable assumptions, as we will see in the next section, they indeed form submanifolds.
introduction in configuration space, (1), for (locally) extremal paths $\gamma_i$ between an initial and a final field configuration, denoting the on-shell action for these paths as $S_{\gamma_i}$ can be written:

$$K_i[\phi_i, \phi_f] = A \sum_{\gamma_i} (\Delta_{\gamma_i})^{1/2} \exp \left( iS_{\gamma_i}[\phi_i, \phi_f] / \hbar \right)$$

(2)

where $A$ is a normalization factor (independent of the initial and final configurations), I used square brackets to denote functional dependence of the on-shell action, and the the Van Vleck determinant is now defined as

$$\Delta_{\gamma_i} := \det \left( \frac{\delta^2 S_{\gamma_i}[\phi_i, \phi_f]}{\delta \phi_i(x) \delta \phi_f(y)} \right) = \det \left( \frac{\delta \pi_f[\phi_i; x]}{\delta \phi_i(y)} \right)$$

(3)

where we used DeWitt’s mixed functional/local dependence notation $[\phi_i; x]$, and the on-shell momenta is defined as

$$\pi_f[\phi_i; x] := \frac{\delta S_{\gamma_i}[\phi_i, \phi_f]}{\delta \phi_f(x)}$$

I will not discuss the technical difficulty in actually calculating the on-shell action or solving the Hamilton-Jacobi equation.

It should be noted that equation (2) is valid up to the point where the first eigenvalue of $\frac{\delta^2 S_{\gamma_i}}{\delta \phi_i \delta \phi_f}$ reaches a zero, which is a focal point of the classical paths. At such points the second order approximation momentarily breaks down. However, the second order approximation becomes again valid afterwards, acquiring the phase factor known as the Maslov index $\nu$, which is basically the Morse index (given by the signature of the Hessian) of the action. I will not need this extra degree of detail for the heuristic applications of the semi-classical approximations in the remaining of the paper.

Assuming that there exists at least one locally extremal (classical) path between the two configurations:

$$|K_{cl}|^2 = A \sum_{\gamma_i} \Delta_{\gamma_i} + 2 \sum_{\gamma_i \neq \gamma_0} |\Delta_{\gamma_i} \Delta_{\gamma_0}|^{1/2} \cos \left( \frac{S_{\gamma_0} - S_{\gamma_0}}{\hbar} \right)$$

(4)

where $A$ is a normalization factor.

I will also assume that one can write the usual semi-classical composition equation. Before a focusing point, we can derive, for the finite number of extremal paths $\gamma$ between $\phi_i$ and $\phi_f$:

$$K_{cl}(\phi_i, \phi_f) = \int \mathcal{D}\phi K_{cl}(\phi_i, \phi)K_{cl}(\phi, \phi_f) = \sum_{\gamma} K_{cl}(\phi_i, \phi_m)K_{cl}(\phi_m, \phi_f)$$

(5)

for a $\phi_m^\gamma$ for each path. This can be shown by the additivity of the integral, and the composition law for the Van-Vleck determinant derived from further differentiation of the following stationarity condition at an intermediary field configuration:

$$- \frac{\delta S_{\gamma_i}}{\delta \phi_m(x')}|_{\phi_m = \phi_m^\gamma} + \frac{\delta S_{\gamma_i}}{\delta \phi_m(x')}|_{\phi_m = \phi_m^\gamma} = 0$$

(6)

setting $\phi_m$ to be along the extremal path $\phi_m = \phi_m^\gamma$. This equation also establishes the continuity of the intermediary momentum as an extremizing condition. It is these two composition properties which ensure that the leading order of the multiple sum (implicit in each kernel) in the rhs of (5) reduce to a single sum on the lhs.

Equations (2)–(5), which are the timeless version of the finite-dimensional formulas, should not only be valid for an infinite number of degrees of freedom (and indeed all I require is their extension to a similarly non-relativistic setting, just now infinite-dimensional) but also for a ‘field history space’ (a proof of these statements can be obtained from [14], chapters 4.2, 15.2 and Bryce DeWitt’s contribution, 18.3), and particularly in [15].

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4In the usual quantum mechanics setting, if the energy of the particle is lower than a potential barrier, then the fixed energy transition amplitude from one side to the other is exponentially decaying in the phenomenon known as tunneling. Even in this case however, one can model the transition using imaginary time, or an Euclidean version of the path integral. In QFT for instance, a transition between different local minima of the potential (different vacua), is a semi-classical tunneling represented as an instanton: a classical solution to the Euclidean action equations of motion.

5In the standard particle path integral scenario, the configuration $\phi$ would be replaced by a position and a time. In that context, equation (5) means that the $\phi_m^\gamma$ are chosen at an intermediary time. One could integrate over the entire time interval, but that would only multiply the normalization constant by the time interval, $A \to A T$. 7
I will again interpret (3) as an indicator of the spread of classical trajectories in configuration space. That is, the Van Vleck determinant measures how the dynamics expand or contract paths along configuration space, i.e. how “dense” the classical paths are. As in the particle case, I assume that the classical field history maps an infinitesimal configuration space volume $\delta \phi_i$ to an infinitesimal configuration space volume $\delta \phi_f$, and the Van Vleck determinant gives a ratio of these volumes. I do not concern myself with the normalization factor $A$ and the effect of the Maslov index (which emerges after focusing points only). The field determinant contained in the Van Vleck factor is the only element that would require regularization. It can be said to contain 1-loop effects, albeit in a different, “field-history” sense.

The most appropriate way to the present formalism of extending equation (2) to higher order of approximations was described in [17], and is given in the the next subsection.

2.2.2 The DeWitt semi-classical expansion

Here I will very briefly report on some results of Cecille deWitt in [17], concerning a higher order semi-classical expansion around extremal paths. In that paper, the problem of computing the path integral is re-expressed completely in terms of classical paths and variations around it. The rationale is to expand the action in higher variations around a segmented classical path, obtaining arbitrarily high orders of approximation to the full path integral. Although the case studied in [17] is for finite-dimensional manifolds, the work was extended to infinite-dimensional Banach spaces in [15].

The technical steps that make it possible are, very roughly, i) the path integration becomes an integral over the space of vector fields along the stationary path which vanish at the endpoints, $X$ ii) the measure on the space of these vector fields admits a projection onto the space of Jacobi vector fields at the endpoints, $\mathcal{D}(u)$ iii) one can find a normalized Gaussian pseudo-measure using cylindrical functions on this space, iv) one can induce a topology on the space of paths by the use of broken Jacobi fields v) the infinite limit of the path integral obtained with this projected measure at each segment yields the standard path integral. For a little more information, and a poor substitute to the original paper, see the appendix B.

$$K(\phi_i, \phi_f) = \Delta_i^{1/2} e^{iS(\gamma)/\hbar} \sum_{n=0}^{\infty} (i\hbar)^n A_n$$

(7)

where $A_0 = 1$ and the next orders depend on further variations around the classical action. Equation (7) is a most useful result for us in this paper. If it suitably generalizes to the infinite-dimensional case, it may provide a way to maintain preferred coarse-grainings centered on extremal paths and yet obtain higher order of expansions in $\hbar$ beyond the usual semi-classical approximation.

2.2.3 The piece-wise semi-classical approximation

There is still one more issue that can arise in the preparatory results before we move on to our constructions of preferred coarse-grainings. Namely, there might be cases in which there are no extremal paths between two given configurations $\phi_i$ and $\phi_f$. This failure can be most easily visualized by connecting the least action principle to the existence of a corresponding metric and geodesics over the same space, in which case the manifold with the induced metric becomes smooth-geodesically incomplete, as discussed at length in [18].

On the other hand, even for non-geodesically complete Riemannian manifolds, if two of its points are in the same connected component they can be connected by paths which piece-wise are geodesics. In

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6 The interpretation of this fact is quite ubiquitous, for instance, the Raychaudhuri equation – which describes the spreading of geodesic congruences – has a compact formulation in terms of the Van Vleck determinant [16]. The same is true of the Jacobi fields used to perform the semi-classical expansion of the path integral [17].

7 For instance, the 1-loop contribution to the effective action is given by

$$\frac{i}{2} \log(\det[i\gamma^{\mu \nu} \mathcal{L}_{\phi_i}])$$

where here $\mathcal{L}_{\phi_i}$ is the classical spacetime Lagrangian, and $\phi_i$ = $\langle \phi \rangle$, around which the expansion is done. There are many differences to the essentially $3 + 1$ formalism presented here, which uses Hamilton’s principal function in spatial fields configuration space.

8 The reasons for this incompleteness are many, and discussed more thoroughly as applied to our case in appendix C. The easiest way in which we can envisage this property even in the finite-dimensional case are for non-smooth potentials (such as Dirac delta barriers, or mirrors) in configuration space whose associated symplectic flows have “corners” [18] (or here called vertices).
the metric case, the space of all paths on \( \mathcal{M} \) can be densely covered by the (inductive limit in \( N \) of the) space of piecewise geodesic segments:

\[
\Gamma_{pc, \text{geod}}^N(\phi_i, \phi_f) = \left\{ \prod_{i=0}^N \gamma_i : [0, 1] \rightarrow \mathcal{M} \text{ is geodesic } \forall i, \gamma_0(0) = \phi_i, \gamma_N(1) = \phi_f \right\}
\]

Let us assume that this is also the case here – mutatis mutandis for configuration space (endowed with a Lagrangian density) instead of a Riemannian manifold, and extremal paths wrt the action we have on configuration space instead of geodesics. That is, I assume that any two points in \( \mathcal{M} \) can be connected by a segmented classical path (i.e. extremizing the given action functional), and define:

\[
\Gamma_{pc, \text{extremal}}^N(\phi_i, \phi_f) = \left\{ \prod_{i=0}^N \gamma_i : [0, 1] \rightarrow \mathcal{M} \text{ is extremal } \forall i, \gamma_0(0) = \phi_i, \gamma_N(1) = \phi_f \right\}
\]

The piece-wise approximation used here can be roughly seen as the inverse procedure as the Feynman - Dirac standard one of obtaining the path integral, using the decomposition of unity at successive time intervals. This approach to the Feynman path integral is known as the “time slicing approximation with piece-wise classical paths”, and it is extensively used in the literature not only for a formal definition of the path integral (see [Fujiiwara] [19] for a rigorous treatment), but also as a practical method of computation. In the simplest case, it consists in splitting the usual particle path integral time interval into regular \( N + 1 \) sub-intervals \( \delta t \), the amplitude kernel becomes

\[
K((q_1, 0), (q_f, \tau)) = \int \prod_{i=1}^N \langle \langle q_f, \tau \rangle(q_0, N\delta\tau) \cdots \langle \langle q_1, \delta\tau \rangle(q_i, 0) \rangle
\]

One obtains the path integral by taking the infinite limit \( N \rightarrow \infty \). The proposal is to take the time intervals small enough so that one can always find a classical path between the intermediary points and approximate each segment semi-classically. Here the proposal is analogous to the particle quantum mechanics case, but in the absence of time, we obtain

**Proposition 1 (The piece-wise expansion)** Given any two configurations \( \phi_i \) and \( \phi_f \) (not necessarily connected by an extremal path), there is a number \( N \) such that

\[
K(\phi_i, \phi_f) = \int \prod_{j=1}^N D\phi_j K(\phi_i, \phi_1) \cdots K(\phi_N, \phi_f)
\]

where extremal paths exist between \( \phi_j \) and \( \phi_{j+1} \) and thus \( K(\phi_j, \phi_{j+1}) \) is given to any order in \( \hbar \) by equation (7), and to first order in \( \hbar \) by (2).

If there is a unique extremal path between a given \( \phi_{j-1} \) and \( \phi_{j+1} \), one can use equation (5) to ensure that \( \phi_j \) is in that segment (thereby collapsing this subdivision). The approximation holds true in the trivial case when \( \phi_i \) and \( \phi_f \) are classically connected, by equation (7), and in the limit \( N \rightarrow \infty \) (as then we can approximate each segment by the Lagrangian, as in the infinite limit of (9)). Proposition 1 will form the basis of the PECs and minimal PECs in the next section.

### 3 Decoherence

In the present formulation, which is in its essence a form of Many-Worlds, decoherence properties are recovered dynamically, as lack of interference between different families of paths. However, I should stress that nothing “splits” at any point in the calculations. Configuration space exists fundamentally, and in it, myriad copies of each partial field configuration, and thus myriad copies of each observer, each of which has equal claim on “being real”.

I aim to show that the interpretation of quantum mechanics espoused in this work explains the lack of observed superpositions in a manner that precedes decoherence. Empirically, it is only through interference effects that we can observe superpositions of states in the first place. The formalism here naturally explains this: interference is not a sign of superposition, in a sense it is the superposition.

Decoherence is still indeed the explanation for the absence of interference, but it is not necessary in order to abolish the existence of macroscopic (or microscopic) superpositions. The fact that we cannot
observe interference of cats in double-slit experiments is indeed the effect of decoherence, and although decoherence does play a major role in suppressing interference, it does not have to be invoked in order to explain away the absence of simultaneously dead and alive cats. Their ‘simultaneous presence’ would also be absent for the hypothetical microscopic kitten, even if \(|\langle\text{alive}|\text{dead}\rangle \neq 0\). That is essentially because the concept of ‘simultaneous presence’ is identical to ‘contained in a single configuration’.

In the standard decoherence point of view, one would have thought of the Schroedinger cat gedanken-experiment as a unitary evolution selecting branches that through their interaction with the environment have become effectively orthogonal. To be more precise, one would have setup an initial state of the system \(|\psi_0\rangle\) with the box and radioactive element, and poison, and cat. After a while, the initial state has evolved into orthogonal branches, represented by the density matrix \(\rho = c_1^2|\psi_1\rangle\langle\psi_1| + c_2^2|\psi_2\rangle\langle\psi_2|\).

In the present work we have configurations which contain a dead cat, and configurations which contain one that is alive, and both kinds contain some record of the setup of the experiment. What we don’t have are coarse-grainings which 1) leave the ‘initial’ configuration \(\phi_i\) representing the setup of the experiment, 2) go through configurations where the cat is exclusively dead (or alive), 3) reach a final configuration where the cat is alive (resp. dead), \(\phi_f\) and, crucially, 4) have a relevant contribution to the overall amplitude \(K(\phi_i,\phi_f)\). Let us now make this more explicit.

### 3.1 Coarse-grained histories

Translated to this context fine-grained histories are just paths in configuration space. Coarse grained histories are partitions of all the fine grained histories from an initial submanifold of \(\mathcal{M}\) to a final one into bundles of paths, with certain characteristics. A fine-grained history for the present configuration space approach is given by a piecewise smooth curve in configuration space \(\gamma : I \rightarrow \mathcal{M}\). I would like to define coarse–grained histories so that each element be in some sense local in configuration space. That is, I would like the coarse-grainings to be made up of bundles of fine-grained paths. The purpose of this seemingly innocuous restriction is that it permits the use of variational (e.g. steepest descent) methods for elements of the coarse-grained histories. Clearly, these assumptions are already present in the usual semi-classical approximations (see section 5.1), but not in the context of coarse grained histories, which is why I now devote a couple of definitions to its straightforward adaptation to that context.

For this purpose I will loosely employ the notion of bundles:

**Definition 1 (Bundles of paths)** A bundle \(\nu, \gamma \in \Gamma\) are those elements of \(\Gamma\) entirely contained in some open neighborhood of \(\gamma \in \mathcal{M}\).

If one were to impose a condition on paths not intersecting each other, but still filling an open region of \(\mathcal{M}\) around the path \(\gamma\), one would have a congruence of paths around \(\gamma\), which can be a heuristic substitute for the notion of bundle I propose here.\(^9\)

Defining coarse–grained histories between two configurations \(\phi_i\) and \(\phi_f\), requires us to first define a collection of subsets of \(\Gamma(\phi_i,\phi_f)\), \(\{C_\alpha, \alpha \in A\}\), where each path \(\gamma \in C_\alpha\) has a bundle \(\nu, \gamma \in C_\alpha\). The amplitude kernel for \(C_\alpha\) is:

\[
K_\alpha := \int_{C_\alpha} D\gamma(t)e^{-iS[\gamma(t)]/\hbar} \quad (11)
\]

I then define:

**Definition 2 (Coarse-graining)** A set of coarse–grained histories \(\{C_\alpha \subset \Gamma(\phi_i,\phi_f), \alpha \in I\}\) between two configurations \(\phi_i\) and \(\phi_f\) will be said to be exclusive and exhaustive to order \(\epsilon\) if i) \(C_\alpha \cap C_{\alpha'} = \emptyset\) for \(\alpha \neq \alpha'\) (exclusive) ii) each element \(\gamma\) in \(C_\alpha\) has a bundle \(\nu, \gamma\) and iii) such that

\[
K(\phi_i,\phi_f) = \sum_\alpha K_\alpha + O(\epsilon) = \sum_\alpha \int_{C_\alpha} D\gamma(t)e^{iS[\gamma(t)]/\hbar} + O(\epsilon) \quad (12)
\]

i.e. the total transition amplitude from \(\phi_i\) to \(\phi_f\) is approximated to order \(\epsilon\) (in the absolute value sense) by taking the limited path integral of paths contained in the coarse-grained histories (exhaustive).

We can say that if we extend the coarse-graining above to \(I' \supset I\), the amplitude contribution of any bundle not contained in any of the original \(C_\alpha\) in definition 2 is negligible, or fades, to order \(\epsilon\). I.e., for \(\gamma \in C_\alpha, \alpha \in I' - I\), then \(|K_\alpha| \leq \epsilon\).

\(^9\)This is not usually required in a normal path integral approach because there is no partition of path space, and thus the condition is automatically satisfied for the entire space of paths.
3.1.1 Fading

**Usual consistent histories** In the usual consistent histories framework (see [10] for a brief review), one has a set of projection operators \( \{ P_{\alpha_k}(t_k) \} \), where \( k \) denotes the alternatives available at time \( t_k \), and \( \alpha_k \) indexes the particular alternative in the set, obeying

\[
\sum_{\alpha_k} P_{\alpha_k}(t_k) = 1, \quad P_{\alpha_k}(t_k) \cdot P_{\alpha'_k}(t_k) = \delta_{\alpha_k\alpha'_k} P_{\alpha_k}(t_k)
\]  

(13)

Then the coarse-grained histories are chains of such operators:

\[
C_\alpha = P_{\alpha_n}(t_n) \cdots P_{\alpha_1}(t_1)
\]

In the Heisenberg picture then one obtains from this:

\[
P_{\alpha_n}(t_n) = e^{iH t_n} P_\alpha e^{-iH t_n}
\]

and the total state is decomposed into the \( C_\alpha \) ‘branches’:

\[
|\psi\rangle = \sum_{\alpha} C_\alpha |\psi\rangle
\]

In our case, we have not subdivided our coarse-grainings into time ordered segments, since that becomes awkward without a natural division of configuration space into co-dimension 1 hypersurfaces (such as would be given by a notion of intrinsic global time in the present framework). We have also not yet defined branches obtained from decoherence, let’s turn to that now.

**Fading of coarse-grainings** A general theory of environmental decoherence has two main consequences, which deal with two distinct issues associated to the measurement problem. The first is the suppression of superpositions between certain distinct states of the system, which decoherence is able to do maintaining a unitary evolution for the totality of observed system and environment. The second main effect of environmental decoherence is the selection of preferred states, named “pointer states” that were robust under system-environment interactions.

In the path integral in configuration space point of view, there are also two separate mechanisms that play distinct roles in their confrontation of the measurement problem. The first mechanism, which I will discuss below, is here called “fading”, and it is nothing more than a means of ascertaining histories which fail to interfere with each other. The second, which is the notion of records, deals with a more precise definition of a concept akin to “robust under system-environment interactions”.

Although the formal definition of decoherence in this work does not exactly match that of the consistent histories formalism, they are closely related. The standard non-relativistic decoherence functional for the configuration space of gravity for the coarse-grained sets \( \alpha_1, \alpha_2 \), can be written as:

\[
D(\alpha_1, \alpha_2) = \int_{\gamma_1 \in \alpha_1, \gamma_2 \in \alpha_2} D\gamma_1(t) D\gamma_2(t) e^{i(S[\gamma_1(t)] - S[\gamma_2(t)])/\hbar} \rho(\gamma_1(0), \gamma_2(0))
\]

(14)

for a given density matrix \( \rho \). The condition for a coarse-graining to be deemed consistent, i.e. for it to yield classical probabilities, is for the decoherence functional (14) to vanish.

Alternatively, I could also write the decoherence functional for a pure state,

\[
D(\alpha_1, \alpha_2) = \langle \psi_{\alpha_1}(\phi_f)|\psi_{\alpha_2}(\phi_f) \rangle
\]

(15)

for paths in the coarse grained histories converging to \( \phi_f \), where \( \psi_\alpha \) is given:

\[
|\psi_\alpha(\phi_f)\rangle := \int C_\alpha D\gamma \exp [iS[\gamma]/\hbar]|\psi\rangle
\]

(16)

The state \( |\psi_\alpha(\phi_f)\rangle \) is the complex number corresponding to the value of a wavefunction on the point \( \phi_f \), which in my case is the endpoint of the fine-grained histories in each of the \( C_\alpha \).

The first thing that I would like to note from the usual decoherence formulas is that a sufficient condition for \( D(\alpha, \alpha') \approx 0 \) is that the individual contribution from either coarse graining set is approximately zero. That is easily seen from (15), if \( \| |\psi_\alpha\rangle \|^2 \to 0 \) then \( D(\alpha, \alpha') \to 0 \) as well. This is also easily seen
from (14) if the density matrix is proportional to the identity, i.e. when the integrals in (14) decouple. Taking the decoherence functional for paths starting at a given point \( \phi_i \) and ending at \( \phi_f \):

\[
D(\alpha, \alpha') = \int_{\alpha} \mathcal{D}\gamma(t) e^{iS(\gamma(t))}/\hbar \int_{\alpha'} \mathcal{D}\gamma'(t) e^{-iS(\gamma'(t))}/\hbar = K_\alpha K_{\alpha'}
\]

(17)

which vanishes if the absolute value of either \( K_\alpha \) or \( K_{\alpha'} \) vanishes. In either case, there is no significant interference.

I will say a given coarse-graining \( C_\alpha \) fades, if its contribution to the total transition amplitude is negligible. This is not exactly the same concept as the original one of decoherence given by the decoherence functional (15). Two sets \( C_\alpha, C_{\alpha'} \) could still yield complex numbers in the final configuration which are nearly orthogonal (or whose real or imaginary parts vanish, depending on the definition).\(^1\)\(^2\) However, for my purposes the stronger concept of fading is more physically significant than that of yielding orthogonal complex amplitudes. I will employ the concept of fading to approximation schemes and to an ordering of the effects of decoherence and interference. We define relative fading, or strong decoherence, as

**Definition 3 (Fading)** One element, \( C_\alpha \subset \Gamma(\phi_i, \phi_f) \), of a coarse-graining set will be said to fade, or strongly decohere,\(^11\) to order \( \epsilon \) wrt another element \( C_{\alpha'} \subset \Gamma(\phi_i, \phi_f) \) if

\[
\frac{|K_\alpha|}{|K_{\alpha'}|} \approx \epsilon.
\]

where \( |f|^2 := f \bar{f} \).

This allows us to order the amount of interference. For example, strong interference occurs between coarse-grained sets that have relative fading of order one. If all elements of a coarse-graining \( C_\alpha \subset \Gamma(\phi_i, \phi_f) \) decohere, or lack interference up to order \( \epsilon \), we will call these elements branches of the given coarse-graining.

In most circumstances explored in this paper, I will assume that there exists at least one extremal field history for the given action, between \( \phi_i \) and \( \phi_f \).\(^12\) This is equivalent to saying that all of the non-fading coarse-graining sets of paths \( K_\alpha \) are given by the contribution from the bundle of paths around some extremal path \( \gamma_\alpha \):

\[
K_\alpha(\phi_i, \phi_f) = (\Delta_{\gamma_\alpha})^{1/2} e^{iS_{\gamma_\alpha}}
\]

(18)

where \( \gamma_\alpha \) is a classical field history, between \( \phi_i \) and \( \phi_f \).

When (18) holds for all the non-fading coarse-grainings, the path integral reduces to the semi-classical approximation (2), one can use (4), and the definition 3 of relative fading to order \( \epsilon \) becomes:

\[
\frac{|K_\alpha|}{|K_{\alpha'}|} = \left( \frac{|\Delta_\alpha|}{|\Delta_{\alpha'}|} \right)^{1/2} \approx \epsilon
\]

As we will see, in this approximation this means that the (relative) probability of the final configuration (relative to some other configuration with the same records) comes almost entirely from \( K_\alpha \). Note that I have not defined classical histories as a consequence of fading. Fading sets of histories imply that in principle the rules of probability can be applied to the coarse-grained sets, i.e. that interference among the distinct coarse-grained histories doesn’t occur, but it does not by itself imply that these sets correspond to classical histories, i.e. that the contribution of each coarse grained set approximately corresponds to that of a path that is extremal (obeys the classical equations of motion). In fact, in [20], a particular example was studied in which the decoherent histories are explicitly non-classical.

In sum, decoherence in this context is formulated in terms of paths interpolating between an initial point \( \phi_i \) and a final one \( \phi_f \). Two sets \( C_\alpha \) and \( C_\beta \) approximately decohere if the contribution of \( C_\alpha \) is negligible in comparison to \( C_\beta \). Thus, as in the consistent histories formalism – to which the present formalism is intimately related – one can recover classical *probabilities* associated to a coarse-grained set

\(^1\)Later, in section 3.2 we will discuss an inner product that better incorporates the present notion of orthogonality and has a closer connection to (17).

\(^2\)The ‘strongly’ here comes from the fact that we are taking the smallness of the absolute value of the transition amplitude as the criterion. This is a sufficient but not necessary condition for the usual concept of decoherence as defined in consistent histories.

\(^11\)For the standard quantum theorectic case with time, tunneling occurs for the constant energy kernel (the Green’s function for the given process), when a given potential barrier is higher than the energy. Even the tunneling case can be obtained from classical paths with Euclidean action (or imaginary time).
of paths if they have decohered with respect to each other, in which case we have called the elements of these coarse-graining, *branches*.

The usual criticism raised against claims that the standard formulation of consistent histories solves the measurement problems of quantum mechanics, is that one must still choose the coarse-grained sets, a choice which can be immensely arbitrary (and even inconsistent). Already at this stage in the present formulation this problem is diluted, since different basis, let’s say ‘vertical spine’ vs ‘horizontal spine’ detectors correspond to different points in configuration space, and accordingly will be the outcomes of processes associated to different coarse-grainings. More dilutions are to come, until the problem is reduced to homeopathic doses.

### 3.1.2 A piecewise approach

As discussed in section 2.2.3 and appendix C, unlike the case for complete Riemannian manifolds – where two given points can always be connected by an extremal path (a geodesic) – it is not always the case that two configurations can be connected by a path that extremizes the action. Any non-smoothness, for example a non-smooth potential for particle dynamics, can imply two given points are not connected by a smooth extremal path [18]. However they will be connected by extremal paths with corners.

If we want to extend the notion of coarse-grained histories that are “as classical as possible”, we would then adapt coarse-grained histories to be piecewise centered on smooth extremal paths as well. Before we can do this in full, we must determine what sorts of paths will be included in each coarse-graining seeded around an extremal path.

One way to define the width of the coarse grainings around an extremal path would be to demand that it is wide enough so that the total amplitude is approximated to the order of some formal parameter $\epsilon$ (usually taken to be $\hbar$). Up to order $\hbar$ the amplitude is usually concentrated on a tight bundle around each classical path, which renders the definition of a width to each element of the coarse-graining not strictly necessary in most cases. We discuss the precise way in which to determine this width in appendix C.

**Definition 4 (Extremal coarse-graining)** An extremal coarse-graining is a coarse-graining $\{C_\alpha, \alpha \in I\}$ such that each element $C_\alpha$ consists of bundles of paths $\nu_i^{\alpha}$ (as in definition 1) in a tubular neighborhood of $\gamma_\alpha$ with minimal width $\rho$ (determined by the exponential map as applied to the normal bundle to the path, see appendix C), such that the coarse-graining is still exhaustive to the given order.

This is analogous to the so-called “quasi-classical” coarse-grainings [12].

Incorporating this definition into the piece-wise extremal context prompts me to formulate an alternative approach to singling out a preferred form of coarse-grainings which should always be available, the piecewise extremal one:

**Definition 5 (Piecewise extremal coarse-grainings (PECS))** A piecewise extremal coarse-graining (PEC) is a (exclusive, $\epsilon$-exhaustive) coarse-graining, where each element of the coarse graining set, $C_\alpha$, is composed of extremal “segments”: 

$$\{C_\alpha = \nu_1^{\alpha} \cdot \nu_2^{\alpha} \cdots \nu_N^{\alpha} \mid \gamma_1^{\alpha} \cdot \gamma_2^{\alpha} \cdots \gamma_N^{\alpha} \in \Gamma_N(\phi_i, \phi_f), \alpha \in I\}$$

(19)

where $\nu_i^{\alpha}$ is a tubular neighborhood of extremal segments $\gamma_i^{\alpha}$ with minimal width $\rho$ (determined by the exponential map as applied to the normal bundle to the path), such that the coarse-graining $\{C_\alpha, \alpha \in I\}$ is still exhaustive to order $\epsilon$. The endpoints of each segment will be called the vertices of the PEC. From Proposition 1, PECs can be used to arbitrarily approximate any transition amplitude.

For example, for very light systems (where the on-shell action for the paths is not much greater than $\hbar$), this radius $\rho$ can indeed be very large, since the transition amplitude from one branch to the other may be quite large.

In any case, definition 5 can better accommodate the analogy with the usual formulation of consistent histories, for which projectors $P_\alpha$ there would be equivalent to the vertices of the PEC here. The use of PECs for replacing projectors in the consistent histories framework coincides with the use of the distributional way in which one uses projection operators. For instance, a characteristic function $f_\xi$ for a given region $\xi$ (at time $t$) or distributional interactions as for example a reflective barrier.

We can – and should, for the more realistic case – enlarge the definition of projectors (or vertices) to include regions in configuration space, as opposed to just points in configuration space; this will be spelled out in the definition of partial records below (section 3.2.1).
Set selection and PECS  In the canonical setting of consistent histories, the crucial criterion for a set of projectors to yield probabilities is for each “branch” to be orthogonal to the others, the so-called ‘consistency condition’. Even so, there are uncountably infinite ways to choose consistent sets of projectors, and no objective principle to guide this choice. Once one has a certain splitting of the entire system under consideration into an ‘environment+system+apparatus’ tensorial product, one can use (generalizations of) the tri-decomposition uniqueness theorem, which states that for an orthogonal basis for each subsystem there is a unique choice of branches. Here I will impose a preferred selection principle for the coarse-grainings, which will be by definition compatible with the the tri-decomposition theorem once we have introduced a suitable inner product.

Let me first define the order of a PEC as the largest number of segments of any element of the PEC. From (19):

$$\mathcal{O}(I) := \max_{\alpha \in I} N_\alpha$$

(20)

A minimal-$\epsilon$ PEC is one that has the minimal order and is still exhaustive to order $\epsilon$:

$$\{ C_\alpha \in \Gamma^{N_\alpha} (\phi_i, \phi_f), \alpha \in I_{\text{min}} \mid \mathcal{O}(I_{\text{min}}) = \inf_{I \in \text{PEC}(\phi_i, \phi_f)} \mathcal{O}(I) \}$$

(21)

where PEC($\phi_i, \phi_f$) parametrizes the space of all possible piecewise extremal coarse-grainings between the two configurations. Apart from the formalities, a minimal PEC, if it exists, can be seen as just that minimal set of segments of extremal paths such that its corresponding PEC yields the approximate amplitude for the given process. In that way, I propose that the existence of a unique minimal PEC yields the preferred coarse-graining for any given transition amplitude between a record and a final configuration.

Uniqueness of the minimal PEC should be related to a unique solution of the variational problem of the transition amplitude wrt the vertices of the minimal PEC. For instance, if we focus on the space of piecewise geodesics with two segments, between $v_{i-1}$ and $v_{i+1}$, one has a well-defined calculus of variations where the position of the intermediary vertex $v_i$ is the variation parameter, and it makes sense to say that a certain $v_i^*$ is extremal. A simple, and yet non-trivial example in 2D Euclidean geometry is to consider two points (without loss of generality), $p_1 = (x_1,0)$ and $p_2 = (x_2,0)$ and remove from $\mathbb{R}^2$ an open line segment $L = (0,] - a, a[)$ between $p_3 = (0,0)$ and $p_4 = (0,a)$, so as to make the manifold geodesically incomplete. The variational principle yields the two piecewise straight segments $\gamma_1 = \frac{p_1 p_3}{2} \cdot \frac{p_3 p_2}{2}$ and $\gamma_2 = \frac{p_1 p_4}{2} \cdot \frac{p_4 p_2}{2}$.

A non-trivial example of a minimal PEC is of course the semi-classical approximation when extremal paths (without segmentation) exist between $\phi_i$ and $\phi$, (and $\phi$ is not a focusing point), when it is given implicitly by (2), i.e. each extremal path seeds one of the coarse-grained sets (and indeed variation on an intermediary vertex will project it back to the extremal curve). However, this is the only case in which we use minimal PECS quantitatively in this paper. A more quantitative treatment of this approach using the expansion (7) is possible but will not be explored here, in the cases explicitly discussed here we have a minimal PEC of order 1.

I can then define elements $\phi^k(t)$ of the semi-classical branches $\gamma^k$ from $\phi_i$ of order 1 as the configurations whose unique extremal path connecting it to $\phi_i$ is $\gamma^k$. In other words, the amplitude for a configuration of the $k$-th branch $\phi \in \gamma^k$ is given by $K(\phi_i, \phi) = \Delta^{1/2} e^{iS[\gamma^k]/\hbar}$ (where we have chosen a segment of $\gamma^k$ interpolating between $\phi_i$ and $\phi$), to first order presenting no interference with coarse-grainings seeded on other extremal paths (e.g. elements of the minimal PEC of order one). Each such isolated extremal path is a semi-classical branch of order 1, and we can extend the definition to order $N$ by replacing ‘unique extremal path’ by ‘unique element of a minimal PEC of order $N$’. In other words,

Definition 6 (Semi-classical branches) Given an initial state $\phi_i$, a semi-classical branch of order $N$ emanating from $\phi_i$, is composed of those elements $\phi$, for which the associated minimal PEC of order $N$ has a single element (piece-wise extremal path) connecting $\phi_i$ and $\phi$. If that minimal PEC is an extension of a branch connecting $\phi_i$ and a different configuration $\phi'$, we say that $\phi$ and $\phi'$ belong to the same branch at different (parameter) times.

3.2 Records

Let us attempt to describe the usual experimental setup from a timeless configuration space point of view (for a much more thorough justification of this view, see [1]). The usual laboratory experiment can be generally described as a two step process: i) first defining the experimental setting, call it $S$, and then ii) investigating outcomes after a certain time $t$, as measured by a clock in the laboratory. In
fact, what we do have access to right before we examine the results of an experiment are the memories, or records, of the setup of the experiment. This is what Carroll and Sebens call the ‘post-measurement, pre-observation’ state of affairs. These memories are somehow encoded in the present configuration, in the form of a photograph, a specific neural circuit, a rock formation, pencil markings on a book, etc.

In the point of view espoused here – i.e. that the totality of configuration space exists fundamentally instead of any particular configurations – there is a weak sense in which all measurements are of the pre-selection and post-selection sort. Each copy of the experimenter in a particular record-holding subset of \( \mathcal{M} \) (with the given record, say \( \phi_r \), i.e. belonging to the subset \( \mathcal{M}_r(\epsilon) \) of configuration points which have that record) will find itself in a specific configuration \( \phi \in \mathcal{M}_r(\epsilon) \). The pre-selection is the selection of the manifold \( \mathcal{M}_r(\epsilon) \), the post-selection is the finding out of where in that set your own configuration is (which is the one that should be included in the ‘out state’ of the post-selection).

This motivates my attempt to formalize a non-circular, meaningful notion of records in a strictly quantum mechanical and timeless manner in configuration space. I will demand that when a configuration \( \phi \) has a full quantum record of a configuration \( \phi_i \), it also obeys the following criterion: for a given ‘in’ configuration \( \phi_i \), the bundles of paths that actually contribute to the transition amplitude between the initial configuration and \( \phi \) – which is the record-holding configuration – have to pass through the ‘recorded’ configuration \( \phi_r \) (i.e. the configuration which \( \phi_f \) holds a record of). This is meant to give the intuitive notion that \( \phi_i \)’s “happening” is encoded in the state \( \phi_f \), i.e. that no ‘branches’ of the wavefunction contributing to the amplitude of \( \phi_f \) can ‘skip’ \( \phi_r \). It is clear that we at least approximately believe this to be true of what are usually referred to as ‘records’: we normally do not think that records of previous happenings spring into existence at the present moment, such as in the occurrence of Boltzmann brains.

If all paths contributing to the amplitude necessarily pass through \( \phi_r \), then from the composition identity, the approximate total amplitude will factorize:

\[
K(\phi_i, \phi_f) = \int_{\mathcal{M}} D\phi' K(\phi_i, \phi_j) K(\phi_j, \phi_f) \approx K(\phi_i, \phi_r) K(\phi_r, \phi_f)
\]

Although in my view this is only a property of configurations holding records, I will use it as its characterizing property:

**Definition 7 (Full record)**: Given an initial configuration \( \phi_i \), if a field configuration \( \phi_f \) contains a full quantum record of a field configuration \( \phi_r \) to order \( \epsilon \), then the transition amplitude can be decomposed as:

\[
K(\phi_i, \phi_f) \approx K(\phi_i, \phi_r) K(\phi_r, \phi_f)
\]

where the approximation is valid up to terms of the order of the formal parameter \( \epsilon \). I will use parenthesis to denote the record-holding manifolds: \( \phi_f \in \mathcal{M}_r(\epsilon) \).

Of course paths do interpolate between \( \phi_i \) and a given \( \phi \in \mathcal{M}_r(\epsilon) \) without passing through \( \phi_r \), it is just that the bundles of paths constituting coarse-grainings centered around such paths do not contribute considerably to the total amplitude. For any exclusive, exhaustive to order \( \epsilon \), coarse-graining \( \{C_\alpha, \alpha \in I'\} \) for \( \Gamma(\phi_i, \phi) \) between \( \phi_i \) and \( \phi \in \mathcal{M}_r(\epsilon) \), all non-fading elements \( \{C_\alpha, \alpha \in I'\} \) are such that their fine grained histories pass through \( \phi_r \). For the semi-classical approximation it can be seen directly from equation (5) that this criterion implies that one is able to decompose the semi-classical transition amplitude as \( K_{\alpha}(\phi_i, \phi_f) \approx K_{\alpha}(\phi_i, \phi_r) K_{\alpha}(\phi_r, \phi_f) \).

This definition of a full quantum record is special in exactly the fact that it expresses some sort of independence from the initial point of the amplitude kernel. It both gives us valid initial conditions (all we need to know) in an experimental setting, and disallows interference between paths that don’t go through \( \phi_r \). That is, once a configuration has a record, the setting of a god-given initial configuration of the Universe, \( \phi_i \), becomes irrelevant, for no contribution of bundles of paths can ‘re-cohere’ to \( \phi_f \) if they have not gone through \( \phi_r \). This means that we can ignore the updating of the wavefunction – or wave-function collapse – when considering the amplitude of a given process to \( \phi \in \mathcal{M}_r(\epsilon) \).

We can compare transition amplitudes in the following way, given \( \phi_1, \phi_2 \in \mathcal{M}_r(\epsilon) \), for any initial \( \phi_i \):

\[
\frac{K(\phi_i, \phi_1)}{K(\phi_i, \phi_2)} \approx \frac{K(\phi_i, \phi_r) K(\phi_r, \phi_1)}{K(\phi_i, \phi_r) K(\phi_r, \phi_2)} = \frac{K(\phi_r, \phi_1)}{K(\phi_r, \phi_2)}
\]

This ratio gets rid of a common quantity to both amplitudes.\(^{13}\)

\(^{13}\)Analogously to the usual definition of expectation values,

\[
\langle F \rangle = Z^{-1} \int D\gamma F(\phi) e^{S[\gamma]/\hbar}, \ Z := \int D\gamma e^{S[\gamma]/\hbar}
\]
In most experimental settings then, as I am arguing we can only take ratios of possible outcomes, equation (23) justifies the practical use of the record configuration \( \phi_r \) as the effective initial point in the kernel. One should note that even in standard quantum mechanics, one can set up a system so that the history of the system prior to those initial conditions being set is irrelevant. To sum up, the present notion of records makes that assumption clear in the present context: it renders irrelevant anything ‘before’ the record (in parameter time) and makes sure the relevant, non-fading coarse-grained histories which do not pass through \( \phi_r \) do not re-cohere.

3.2.1 Partial records

The definition of a full record is clearly very restrictive and unrealistic. As mentioned earlier, we usually only have access to partial field configurations, which hold records of other partial field configurations. I can similarly define (the more realistic case of) partial records: A field configuration \( \phi_f \) contains a partial record of a field configuration \( \phi_O^r \), if for any other field configuration \( \phi_i \), and weakly exclusive coarse-grained sets for \( \Gamma(\phi_i, \phi_f) \), all non-fading paths pass through the submanifold defined by the partial field configuration \( \phi_O^r \), i.e. pass through the recorded submanifold \( \mathcal{M}_r = \{ \phi_r \mid \phi_r \in \phi_O^r \} \). That is

\[
K(\phi_i, \phi_f) \approx \sum_{\phi_r \in \mathcal{M}_r} K(\phi_i, \phi_r) K(\phi_r, \phi_f)
\]

(24)

In this paper, I assume that different initial conditions for the usual quantum mechanical wavefunction can be represented by different partial records. This is not a small assumption, and it needs to be further investigated and justified.

There are similarities between partial records and the projectors \( P_a \) in the consistent histories framework. One could indeed generalize the concept of partial records to be defined by disconnected regions in configuration space, which would then encompass the notion of projectors:

\[
K(\phi_i, \phi_f) \approx \sum_a \sum_{\phi^a_k \in \mathcal{U}_a} K(\phi_i, \phi^a_k) K(\phi^a_k, \phi_f)
\]

(25)

where \( \mathcal{U}_a \subset \mathcal{M} \) are the regions in configuration space. Usually in consistent histories the coarse-grained histories are labeled by such types of sets. It seems very likely that minimal PECs, given in section 3.1.2, exist only when vertices lie on projected regions as per (25), but I leave this conjecture for further study.

3.2.2 Relations between observables

In the introduction, I mentioned that relations between instantaneous observables only make sense if such observables all have the same records.

First, I should make clear that I am calling observables the usual classical observables. For instance, invariants under spatial diffeomorphisms and spatial Weyl transformations (and also invariant under gauge symmetries we might have for other fields). These are relational quantities, they do not require an observer, the observers are part of the configuration, and the observables which we consider as important are usually relations between the configuration of our brains and of the outside world. If one wants to be really rigorous, these would be relations just between different parts of our brains. But for most configurations where these relations are obtained – most in the sense of it being where most of the volume of configurations is – they are actually representative of the corresponding outside world, i.e. they also belong to a configuration where the approximate state of affairs is as represented inside our brains, and there exists a classical path between such configurations and their records. In very few configurations, this internal representation of the outside state of affairs do not hold.

In fact, one could indeed compare relations between observables that don’t have the same records, contrary to what I said above, but these comparisons wouldn’t be very informative. In other words, suppose you were comparing two state of affairs \( \phi_1 \) and \( \phi_2 \) with different records, \( \phi^r_1 \) and \( \phi^r_2 \). Since the factorization of the amplitude is different for the two, you wouldn’t get the cancellation of \( K(p, \phi^r_1) \) and \( K(p, \phi^r_2) \), as in (23). This makes the comparison very unrefined, since under normal circumstances (records much closer to the final configuration than to the initial completely degenerate one) \( K(\phi_1, \phi^r_2) \ll K(\phi_2, \phi_2) \).

The “records” terminology is also perhaps not ideal. It is defined by a property I expect of records, but it doesn’t necessarily correspond to things we would usually call records. For instance, the choice of

\[\text{To be more precise with the notation, I could have called this recorded submanifold, } \mathcal{M}_r^O \text{ and the record-holding one } \mathcal{M}_{(r,O)} .\]
vacuum (or what the completely degenerate initial state is) has the characteristics of a record, a record of all configurations. Perhaps the best way to think about records is to think of projectors in the consistent histories framework, the difference being that a record is a projector that is identical for all the different elements of the coarse-graining under consideration.

4 The space of quantum states

4.1 A structureless initial state

Ideally, one would like to have the definition of records, Definition 7 to be valid for any initial condition of the Universe, since in principle a record should be a property of a configuration itself. Namely, it should be a property solely of the configuration that has the given record – the record-holding configuration. However, it is not clear if there are any non-trivial examples of this extended version of the definition, which is likely too strong to ever implement.

Alternatively, I have used the existence of some god-given initial configuration $\phi_i$, in terms of which records are defined. This plays an analogous role to the vacuum to be considered in the theory, and there is a very natural candidate for such a configuration: the completely degenerate one, which exists for vector and tensor fields (sections of vector bundles in general). For gravity, this would correspond to the completely degenerate (or “zero metric”), which is structureless and lies at the tip of the cone that forms $\text{Riem}(M)$. Let us call this configuration $p$, standing for “point”, and define it to be a record of all configurations.

I postulate a 1-dimensional Hilbert space over $p$, $\{\mathbb{H}_p, \| \cdot \| \}$, and define a state $|p\rangle$ with unit norm $\|p\|^2 = \langle p|p \rangle = 1$, where I have denoted an element of the dual space $\langle p \rangle \in \mathbb{H}_p^*$ obtained by the metric isomorphism. I could then do more than defining the propagation kernel in this formalism, I could define “preferred” states by

$$|\phi\rangle := \hat{K}(p, \phi)|p\rangle$$

where $\hat{K}(p, \phi)$ (if non-zero) defines an isomorphism $\hat{K}(p, \phi) : \mathbb{H}_p \rightarrow \mathbb{H}_\phi$ between the two 1-dimensional Hilbert spaces, by the propagation of the state $|p\rangle$. Note that we have not defined a Hilbert space over the totality of $\text{Riem}(M)$, but a one-dimensional one over each configuration instead.

This prescription yields a single quantum state for the entirety of configuration space, nothing “evolves”. The amplitude for each configuration can in this sense be seen as basically a tunneling from the totally degenerate configuration to any other, in a ‘Boltzmann’s configuration’ type of scenario. Of course, the path integral does focus around certain paths, which can mimic a notion of evolution.

4.2 The inner product

4.2.1 An inner product for each $\mathbb{H}_\phi$

Moving on, one then selects the induced inner product in each $\mathbb{H}_\phi$, in which case we define

$$\langle \phi|\phi \rangle = K(p, \phi)K(p, \phi)$$

which characterizes $\mathbb{H}_\phi$ and the map $\hat{K}(p, \phi)$ up to a phase rotation. The transition amplitudes $K(\phi_r, \phi)$ effectively emerge for states whose corresponding configuration possess records, by definition 7 and (27):

$$\langle \phi|\phi \rangle = \|\phi_r\|^2 K(\phi_r, \phi)K(\phi_r, \phi)$$

There are two points that I need to call attention to here: i) it is not true that the state $a|\phi\rangle$ will in general correspond to the configuration $\phi = a\phi$, i.e. to the isomorphism $\hat{K}(p, a\phi)$, and ii) the norm in (27) does not necessarily have to do with any type of probability yet. I will show that there is a relation in section 5.2.1.

Since this is a timeless picture, in the general case unitarity can only be encoded in the relation:

$$\int D\phi \langle \phi|\phi \rangle = \int D\phi K(p, \phi)K(p, \phi) = 1$$

There is some relation between this assumption and the no-boundary formulation, which links it to a definition of the Hartle-Hawking vacuum. In Loop Quantum Gravity this would be akin to the Ashtekar-Lewandowski vacuum [21].
where $D\phi$ is a classical measure in configuration space. Unitarity in this simplified sense is guaranteed then if $\hat{K}(\phi, p) : H\phi \to H\phi$ is defined as $\hat{K}(p, \phi)$, where a $\pi/2$ phase in the actions of the paths is obtained upon reflection of the paths at $\phi$:

$$\int D\phi \langle \phi | \phi \rangle = \int D\phi K(p, \phi)K(p, \phi) = \int D\phi K(\phi, p)K(p, \phi) = K(p, p) = 1$$

and indeed matches the definition of a unitary operator in that the field operator inverse of the amplitude kernel $K(p, \phi)$ is its complex conjugate.

### 4.2.2 An inner product between different configurations.

So far we have not attributed any overall Hilbert structure to the entirety of configuration space, only over individual configurations. Through the definition of an initial state, one removes the choice of quantum state over each configuration: this choice is given by (26). In particular, strictly speaking one does not need to define $\langle \phi_1 | \phi_2 \rangle$ for $\phi_1 \neq \phi_2$, i.e. transition amplitudes between arbitrary quantum states.

However, thinking of the amplitude propagators as morphisms in between these Hilbert spaces, the inner product could be naturally defined for elements that form loops in this space. The use of the initial state here is not unlike the use of an initial (or timeless) state in the Heisenberg representation

$$\langle \phi \rangle = \sum_{\psi} \langle \psi \rangle K(\phi, \psi)$$

which in fact gives us the the most natural form compatible with (26) and (27):

$$\langle \phi | \psi \rangle = K(\phi, \psi)K(\psi, p)p$$

(30) clearly reduces to (27) for $\phi \to \psi$, assuming $K(\phi, \phi) = 1$ (and can be (sesqui-)linearly extended). It is easy to check that we can have a completeness relation (a decomposition of the identity), i.e. for any dual state $\langle v |$, and normalized $|\phi \rangle$:

$$\int D\phi v | \phi \rangle = \int D\phi K(p, v)K(\psi, \phi)K(\phi, p)K(\phi, \psi)K(\psi, p) = \int D\phi K(p, v)K(\phi, p)K(\psi, p) = \langle v | \psi \rangle$$

(31) and that the decomposition is a projection, in that it is idempotent:

$$\int D\phi D\phi' | \phi \rangle | \phi' \rangle = \int D\phi D\phi' | \phi' \rangle K(p, \phi)K(\phi, \psi)K(\psi, p) = \int D\phi D\phi' | \phi \rangle K(p, \phi)K(\phi, \phi)$$

It is easy to see that $\langle \phi, | \phi \rangle = \| \phi \|^2$, for the inner product between a record and the record-holding configuration, an expected result since all non-fading coarse grainings reaching $\phi$ pass through $\phi_\psi$. This has the nice side effect of equating the Born rule with the inner product between record and record holding configuration (of course, we have not yet derived nor postulated the Born rule).

This is already indicative of the advantage of this inner product: it makes it explicit that PECs in configurations space that have decohered have no non-trivial projection on each other, as we shall see in subsection 4.3 below.

**Relation to the standard equal time inner product.** One might worry that this inner product need not recover the standard equal time inner product for quantum mechanics. It is not true however that the equal time inner product knows nothing of the amplitude kernel; it can be seen as an initial condition for the amplitude kernel when defined with a time variable. It is easy to show that at least for

---

16Not will not discuss the difficulties in the construction of this classical measure, other than to say that it should be invariant under 3-diffeomorphisms and any other instantaneous symmetries of the action, and that in three dimensions there are no anomalies for the conformal diffeomorphisms. For a more technical construction of this measure, see [22].

17Note also the similarity between this and the standard decoherence functional (17), for $K \to K_\alpha$ and $\phi = \psi$, i.e. taking different coarse-grained sets but the same final state.
the non-relativistic particle this is the case. Since different instants here can be seen merely as different points in configuration space, “equal time” should be equivalent to the limit:

$$\lim_{T \to 0} \langle x_1, 0 | x_2, T \rangle = \lim_{T \to 0} K(x_1 - x_2, T) = \lim_{T \to 0} e^{-i(x-y)^2/T} = \delta(x, y)$$

which it is. More generally, if the amplitude can be written in Hamiltonian form as

$$\langle (\phi_1, t_1) | (\phi_2, t_2) \rangle = \langle \phi_1 | e^{iH\Delta T} | \phi_2 \rangle$$

it will recover the equal time inner product for $\Delta T \to 0$.

“Unphysical” states One should note that states of the form (26) are a basis for the linear vector space structure over configuration space (which I have not imbued with a full Hilbert space structure), but of course not all states are of the form (26). One could compare this with the notion of “ontic” states [23]. In other words, states of the form (26) would correspond to real physical (ontic) $\phi$.

That is, in general there is no configuration $\phi_{12}$ such that a linear combination of two basis elements gives

$$a|\phi_1 + b|\phi_2 = |\phi_{12}\rangle$$

For this to hold, for any configuration $v$, one would require that for the corresponding state $\langle v |$, we would obtain $a\langle v | \phi_1 + b\langle v | \phi_2 = \langle v | \phi_{12}\rangle$, which would imply that:

$$aK(\cdot, \phi_1)K(\phi_1, p) + bK(\cdot, \phi_2)K(\phi_2, p) = K(\cdot, \phi_{12})K(\phi_{12}, p)$$

which is basically the condition that the configurations $\phi_1$ and $\phi_2$ are equivalent projectors to the configuration $\phi_{12}$ in reaching any given configuration. We can use the left hand side of (33) to define the state $|\phi_{12}\rangle$, but it doesn’t generically correspond to some configuration $\phi_{12}$ as in the rhs of (33), and it is useful to keep this in mind.\footnote{In fact, all the statements utilizing the inner product (such as the decomposition of unity), have been restricted to states of the form (26), but could have been extended to states defined by the lhs of (32), since those of (26) form a (possibly overcomplete) basis for the entire vector space.}

Only when such a configuration $\phi_{12}$ exists could one say that one had a “superposition” of configurations, but clearly, even here the word has been emptied of any Bayesian meaning. Indeed, it represents only that which is contained in the lhs of (33): an inherent degeneracy in the way the propagation can take place. Nonetheless, one should note that indeed there might exist subsets of $\mathcal{M}$ which form a vector space in the stronger sense of $|\phi_{12}\rangle$ as in (33).

4.3 Decoherence and the inner product

One last thing to consider is a comparison of the notion of orthogonality of states in the sense of the present inner product (30) and semi-classical decoherence through fading of elements of the coarse-graining, as presented in Definition 3. The two notions refer to different objects: one is orthogonality between states and the other between coarse-grained histories. Nonetheless, with the use of PECs and records, one can connect the two notions.

Suppose $\phi_1, \phi_2 \in \mathcal{M}_{(r)}$. Now, let us look at the minimal PECs between $\phi_1$ and $\phi_2$. Suppose that one of the elements of the minimal PEC for $K(\phi_r, \phi_1)$ goes through $\phi_2$. For simplicity, suppose $\phi_2$ lies along the $k$-th extremal path between $\phi_r$ and $\phi_1$ and suppose furthermore, without loss of generality that $||\phi_r|| = 1$, then from (2)

$$\langle \phi_1 | \phi_2 \rangle \approx \Delta_k + \sum_{i \neq k} \Delta_i^{1/2} \Delta_k^{1/2} e^{(S[\gamma_i] - S[\gamma_k])/\hbar}$$

Which still allows the inner product to be of the same order of magnitude as the norm of the states (if we suppose for example that the segment of $\gamma_k$ between $\phi_r$ and $\phi_2$ is the unique extremal one, then $||\phi_2|| \approx \Delta_k$).

In more generality, for $K(\phi_1, \phi_2) \sim \epsilon$, then $\langle \phi_1 | \phi_2 \rangle \leq \epsilon$, meaning that if exhaustive coarse-grainings between $\phi_1$ and $\phi_2$ are fading to order $\epsilon$ (relative to $||\phi_1||$), then their inner product is bounded to that order, which converges with our concept of decoherence.

If $\phi_2$ is not in the minimal PEC for $K(\phi_r, \phi_1)$, then all we can say is that $\langle \phi_1 | \phi_2 \rangle$ is bounded by $\rho$, the minimum radius of the PEC (see definition 5). We should note that this radius can still be quite large for light systems.
For instance, let us take a two slit experiment with an electron beam and consider interactions of gravitons with each electron path. Without amplification, the difference in the gravitational field between electron taking path 1 and electron taking path 2 is very small. Thus the amplitude at any given parameter time for the two paths, $K(g_1(t), g_2(t))$ would be close to 1. That is, the dynamical inner product between the two could be close to one and indistinguishable on the basis of the gravitational interaction.

**The geometric analogy**  From (3), we have the following:

$$\Delta_{\gamma_0}(\phi_i, \phi_f) := \det \left( \frac{\delta^2 S_{\gamma_0}}{\delta \phi_i(x) \delta \phi_f(y)} \right) \left( \frac{\delta \phi_f}{\delta \pi^i_f} \right)^{-1}$$

which is the inverse of the sensitivity of the given final configuration with respect to the initial momentum. The more sensitive the final configuration is wrt the initial momentum, the more diluted the trajectories in the bundle around the classical path will become (see the coarse-graining set definition 2), and the smaller the weight from this trajectory in the transition amplitude.

In an analogy with geodesics in Riemannian manifolds (see appendix C), extremal paths for generic interacting systems with more than 3 degrees of freedom can related to geodesics in almost hyperbolic metrics (i.e. a manifold with Ricci curvature with negative eigenvalues almost everywhere). In this analogy, the on-shell action is given by geodesic distance, and we can use the Rauch comparison theorem, which roughly states that geodesics in regions of lower curvature diverge quicker than in regions of higher curvature. Following this analogy, the farther two points are, the more sensitive the final configurations are to the initial conditions, and the smaller the Van Vleck determinant. Thus, if the effective dynamics is hyperbolic, for far away points (in terms of the on-shell action) the Van Vleck becomes quite small, which in turn makes the overall semi-classical transition amplitude obtained very small.

## 5 Entanglement

In standard QFT, entanglement is a robust feature, and embodies strong non-local effects.\(^\text{19}\)

But a more pedestrian approach already captures the principal features of non-locality through entanglement. As Schrödinger pointed out in response to the original EPR paper, non-locality is merely a consequence of the non-factorizability (or entanglement) of the two-particle wavefunction:

$$\psi(x_1, x_2) \neq \xi(x_1)\xi(x_2)$$

In the present interpretation this is straightforward. A given path in configuration space contains the configuration of both particles. Their individual state will thus be correlated, but the amplitude distribution in configuration space will still be given by the path integral kernel, which is not explainable by a hidden variable interpretation.

Let us then describe entanglement in the semi-classical path integral representation. In the most general semi-classical non-decoupling case, a classical field history $\phi(t)$ would depend on the field on the entire manifold $M$, and thus could be highly entangled in the sense of equation (35). That is, the classical curve $\phi(t)$ does not decouple completely region $O$ and the remainder. The two regions interact and influence the dynamics of each other. Even if after a while there is a separation of the systems $O$ and $O'$, so that they are no longer in interaction, and we can distinguish the numerical contribution to the action coming from each region, the fact that each region’s extremal field histories will be particular to the other’s means that we cannot add the contribution of field histories over $O$ independently from field histories over $O'$. This implies that $K^{O \cup O'} \neq K^O \cup K^{O'}$, which is the analogous statement to (35). Even if one can find boundary conditions for the fields at $\partial O$ which stay reasonably fixed during semi-classical evolution,\(^\text{20}\) this entanglement is due to initial conditions for the paths. In the usual nomenclature, this means that even though we would be able to write the entire Hilbert space as a tensor product, for example the bi-partite $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, general states are entangled and not product states, $|\psi\rangle \neq |\psi_1\rangle|\psi_2\rangle$.

---

\(^\text{19}\) The best illustration I know of this is given by the Reeh-Schlieder theorem [24], which shows that for different regions $A$ and $B$, one can, by acting on the vacuum with operators located on $A$, create a set of states that is dense on the full Hilbert space! This is possible because of the highly entangled nature of the vacuum in QFT, if the field theory degrees of freedom were in a product state, this would of course be impossible.

\(^\text{20}\) This is specially difficult with gravity, since one cannot perfectly isolate systems which are not infinitely separated. In more generality, as the fields approach $\partial O$, their correlations become less and less isolated, usually demanding some kind of cuttoff or regularization in calculations, e.g. of entanglement entropy.
5.1 Semi-classical decoupling and relative transition amplitudes

I will start by describing a highly decoupled case, and then move on to a more entangled scenario, but for which we can still separate the systems, i.e. to a more general definition of the multi-partite systems. The decoupling can be achieved when our laboratory in $O$ is effectively dynamically disconnected from the rest of the Universe, $M - O$, in the sense that the classical evolution of fields is decoupled from one set to the other. The definitions and results here can be extended from classical history – which is a minimal PEC of order 1 – to a minimal PEC of more general order.

5.1.1 Semi-classical decoupling.

Suppose we are given an initial laboratory field $\phi_\omega$ on $O$ with boundary conditions $\partial \phi_\omega$ on $\partial O$, such that $\phi_\omega = \phi|O$, and an initial velocity $\dot{\phi}_\omega = \phi|O$. Dynamical decoupling here means that I can evolve the field in $O$ and $M - O$ independently, for a given set of initial conditions, without altering the boundary conditions. Assuming that the action defines dynamics with a well-posed Cauchy problem, in standard contexts this means that the Cauchy problem for the entire region factorizes. Appropriately to the present context, I will define decoupling for field histories between two configurations (as opposed to for an initial value problem):

**Definition 8** A semi-classical decoupling for two regions $O$ and $M - O$ in the semi-classical transition amplitude $K_\omega(\phi^r, \phi^f)$ occurs if any classical field history over $O$ between $\phi_\omega^r$ and $\phi_\omega^f$ and, independently any classical field history over $M - O$ between $\phi^r_{M-O}$ and $\phi^f_{M-O}$, can be glued together to make up a classical field history of the total field and furthermore the totality of fields over $M$ constructed in this manner account for all the classical field histories, between the two complete configurations $\phi^r$ and $\phi^f$.

In usual nomenclature, this means that semi-classically the end states would be describable by products; $|\psi\rangle = |\psi_1\rangle|\psi_2\rangle$. I should note however, that it is very far from trivial to separate fields in this manner, especially for gravitational fields, since one cannot “isolate the field”, as mentioned above. Nonetheless it is uncontroversial that to some degree we are able to do this in the laboratory. Appendix D discusses how to technically make the decoupling more precise in the context we are exploring here.

5.1.2 Relative transition amplitudes.

In any case, for a semi-classically decoupled system, a complete classical field is an independent sum $\phi^r_M = \phi^r_O \oplus \phi^r_{M-O}$, where I have heuristically used the direct sum sign $\oplus$ to signify a sum of tensor fields with complementary supports on $M$ (see appendix D) and the action, an extensive quantity, on-shell becomes an independent sum of the on-shell actions over $O$ and $M - O$. That is, even though $S[\phi^r_O + \phi^r_{M-O}] \neq S[\phi^r_O] + S[\phi^r_{M-O}]$ (i.e. it is not a linear functional, and thus fields over the same region of space do not decouple) it is still true that $S[\phi^r_O \oplus \phi^r_{M-O}] \approx S[\phi^r_O] + S[\phi^r_{M-O}]$. I.e. it is extensive (ignoring issues with the boundary contribution from $\partial O$).

Assuming that for decoupled regions $O_j$ and $\{\phi_j\}$ the on-shell variation obeys $\delta S[\phi_j] = 0$ for initial and final configurations of regions $k \neq j$, the on-shell Hessian matrix diagonalizes and thus the Van Vleck determinant becomes $\Delta = \prod_i \Delta_i$. In this semi-classical approximation I can then write (for notational simplicity now in reduced configuration space, thus dropping the action of the $h^r_i$’s present in (56)):

$$K(\phi_r, \phi_f) \approx K^r_O(\phi_r|O, \phi_f|O) \times K^M_{M-O}(\phi_r|M-O, \phi_f|M-O)$$

where $K^O$ (or $K^M_{M-O}$) refers to the path integral in the configuration space of fields over $O$ (resp. $M - O$) as above.

Now, as with equation (23), we would like to know whether we can calculate relative transition amplitudes locally. Let’s say we would like to calculate the relative amplitude to reach the different final states $\phi_{f|O}$ for $f = 1, 2$, from $\phi_{r|O}$. As demanded by the global, non-local nature of the formalism, we

\[\text{[21]}\] A speculative comment: isolating the gravitational field of different finite regions $O$ and $M - O$ would require one to have degenerate metrics in some boundary regions $\partial O$. But degenerate metrics effectively change the metric topology of physical space. Thus one could perhaps see here the appearance of “geometry from entanglement”, in that the less entangled two regions of space become, the more they become separate metrically, finally pinching off when the metric of the two regions reaches a product state. This pinching off through lack of entanglement is precisely what happens in the Maldacena model of wormholes [25].
start with the relative transition amplitudes for the entire Universe:

\[
\frac{K(\phi_1, \phi_2)}{K(\phi_1, \phi_2')} \approx \sum_{\phi_r \in \mathcal{M}_r} K(\phi_1, \phi_r)K(\phi_r, \phi_2) \approx \sum_{\phi_r \in \mathcal{M}_r} K(\phi_1, \phi_r)K(\phi_r, \phi_2') = K_0(\phi_1, \phi_2')
\]

The first approximation is that of partial records, (24), the second is from semi-classical decoupling (36), \(\mathcal{M}_r\) is comprised of all the fields on \(M - O\), which I called \(\mathcal{M}_{M-O}\) in the third equality, and in the last line I used the fact that by semi-classical decoupling one can always set \(\phi^1_{M-O} = \phi^{1}_{M-O}\). That is, given \(\phi^1 = \phi^1_{O} \oplus \phi^1_{M-O}\) in the heuristic notation given above, one calculates the relative transition amplitude to \(\phi^2 = \phi^2_{O} \oplus \phi^2_{M-O}\) where \(\phi^2_{O}\) is a general field configuration on \(O\).

This localization of relative transition amplitudes to subsystems (under the above assumptions), completely avoids the difficult questions of assigning meaning to an initial configuration in time (besides the recorded one), but is only partially agnostic with respect to the rest of the Universe, since it requires the two final configurations on the rest of the Universe to be the same, i.e. \(\phi_{M-O}^2 = \phi_{M-O}^1\), (which we deemed justified in light of semi-classical decoupling). Of course, if we can be completely agnostic with respect to the rest of the Universe \(M - O\), we in principle start and end up in any of the configurations over \(M - O\), which amounts to taking the sum over all the states \(\phi_{M-O}^1\) and \(\phi_{M-O}^2\), in which case no extra assumption is necessary to obtain the final result of (37), which makes the choice effectively irrelevant.

In any case, this definition of records and their relation to record-holding configurations gives a Bayesian spin to the present approach, which is non-subjective in the sense that it is concerned solely with correlations between partial field configurations.

### 5.2 Semi-classical entanglement

Now I move on to consider cases with more entanglement. Each contribution of a field history for \(O\) can be accompanied by a family of field histories for \(O'\). Supposing that the region \(O \cup O'\) semi-classically decouples from the remaining \(M-O\) in terms of the classical actions, let \(\gamma\) be the curve traced out by \(\phi(t)\) between \(\phi\) and \(\phi_f\). Then \(S[\gamma] = S[\gamma_{\partial O}] + S[\gamma_{\partial O'}]\), and the evolution of the system, i.e. the classical field histories, in \(O \cup O'\) are independent from the remaining \(M - O \cup O'\).

Focusing on the entangled \(O \cup O'\), if the systems become separate, so that the numerical contribution to the action can be distinguished, \(S[\gamma_{\partial O}] = S[\gamma_{\partial O}] + S[\gamma_{\partial O'}]\), the point is that two partial trajectories leaving the joint record configuration are part of the same classical trajectory of the joint system, and cannot be summed independently, which means that their contributions to the amplitude propagator cannot be summed independently. I.e. independent classical trajectories \(\phi_{O}^1\) in \(O\) and \(\phi_{O'}^1\) in \(O'\), starting at \(\phi_{O'}^O\) may not jointly form a classical trajectory of the field in \(O \cup O'\) due to initial conditions, even if from spatial separation we are allowed to numerically split the contributions to the joint action as a sum of the partial actions.

For example, in the case where there is a unique extremal curve restricting to \(O'\) for each extremal curve restricted to \(O\) (e.g. a Bell pair):

\[
K^{O \cup O'}_n(\phi_{O}^1 \cup \phi_{O'}^1, \phi_{O} \cup \phi_{O'}) = A \sum_j (\Delta j)^{1/2}e^{is[\gamma_{\partial O}')}e^{is[\gamma_{\partial O']})
\]

\[
\neq A \sum_j (\Delta j)^{1/2}e^{is[\gamma_{\partial O}']} \sum_{\gamma'} (\Delta j)^{1/2}e^{is[\gamma_{\partial O}']} = K_0(\phi_{O}^1, \phi_{O})K_0(\phi_{O'}^1, \phi_{O'})
\]

In this example, the partial classical field histories – which are restrictions of a joint classical field history – are perfectly correlated to form each joint classical field history. Of course, not all possible choices of \(\phi_{O'}\) and \(\phi_{O}\) will be connected to the records by extremal paths, and fixing either part of the partial record holding configuration will limit the possible array of the other part. In other words, the distributional density of states on \(\mathcal{M}_{(r, O \cup O')}\) is (semi-classically) peaked on some ‘diagonal’ submanifold of \(\mathcal{M}_{(r, O)} \times \mathcal{M}_{(r, O')}\).

Equation (38) is the case for a 1-1 correlation, i.e. analogous to ‘joint coarse grained histories’ of the form \(C_{\alpha} = C_{\alpha}^O \cup C_{\alpha}^{O'}\). The more general case can be written as:

\[
K^{O \cup O'}_n(\phi_{O}^1 \cup \phi_{O'}^1, \phi_{O} \cup \phi_{O'}) = A \sum_{j \in J} \sum_{k \in I(j)} (\Delta j)^{1/2}e^{is[\gamma_{\partial O}']}e^{is[\gamma_{\partial O']})
\]

(39)
where \( I(j) \) is some set of classical field histories between \( \phi_O' \) and \( \phi_O \), compatible (i.e. form a joint classical field history) with the j-th classical field history between \( \phi_O' \) and \( \phi_O \).

5.2.1 Counting decohered coarse-grainings and the Born rule: an example

The standard setup (in new clothing) Let us look at a standard example of decoherence in the usual quantum mechanics setting, for comparison. Suppose the system under observation consists of a single qubit initially in a state \( \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \). After some interaction with a measurement apparatus – a complex system whose role is to amplify the results of any interaction with the system under observation and whose initial state we set to \( |A_o\rangle \) – the “environment” – degrees of freedom not under experimental control but which still are included in the local configuration of the system, such as a clock – and finally the observer (initially \( |O_o\rangle \), become successively entangled with the qubit states. Let \( |\omega_o\rangle \) be the initial environment state, then, following the presentation in [26], the entire measurement process is laid out as (omitting the normalization):

\[
|\psi\rangle = |O_o\rangle|\text{Sys}_o\rangle|A_o\rangle|\omega_o\rangle
\]

\[
\rightarrow |O_o\rangle|\uparrow\rangle|A_o\rangle|\omega_o\rangle + |O_o\rangle|\downarrow\rangle|A_o\rangle|\omega_o\rangle
\]

\[
\rightarrow |O_o\rangle|\uparrow\rangle|A^1_o\rangle|\omega_o\rangle + |O_o\rangle|\downarrow\rangle|A^2_o\rangle|\omega_o\rangle
\]

\[
\rightarrow |O_o\rangle|\uparrow\rangle|A^1_o|\omega^1_o\rangle + |O_o\rangle|\downarrow\rangle|A^2_o|\omega^2_o\rangle
\]

\[
= |\psi_1\rangle + |\psi_2\rangle
\]

The process, in the language employed here, can be read as follows: the original states of the subsystem + apparatus + environment + observer is given in the first line. The subsystem is prepared in such a way that a Bell pair is to be produced. This plays the role of the initial “record” configuration \( \phi_r = (O_o, \text{Sys}_o, A_o, \omega_o) \), which effectively means that we only want to consider paths that had this configuration as an initial point, i.e. final configurations must lie in \( \mathcal{M}_{(\psi)} \). There are of course infinitely many classical field histories coming out of this initial point, one for each Bell pair, however, we only want to consider those that will reach a specific final configuration containing the partial fields characterized by, e.g., spin up and spin down in the z axis, which we will denote by \( \uparrow \) and \( \downarrow \). This determines two sets of possible initial directions of the spin system for the initial data of extremal curves.

In the second line, in the semi-classical approximation employed here, we have two paths diverging from \( \phi_r \), let’s say \( \gamma_1(t) \) and \( \gamma_2(t) \), where \( t \) is the curve parameter. Note that all of the other parts of the fields that make up the configuration have ‘a copy’ for each configuration in the curves \( \gamma_{1,2}(t) \). For each subsequent line, in order, the field corresponding to the subsystem interacts first with the apparatus, which then influences the environment, which finally entangles itself with the observer. We can separate the curve parameters into four intervals, corresponding to the four intermediary lines of (40). At every step, I am simply describing two different curves of classical field histories, which diverge more and more in configuration space. Due to the complexity of the environmental components of the field, this divergence occurs very quickly and an amplification of the initial difference between the configurations ensues. After a very short time, \( t, t' > \epsilon, \gamma_1(t) \cap \gamma_2(t') = \emptyset \), the two classical field histories will no longer cross, and thus no longer interfere. I.e., to reach the same final configuration one of the histories must now fade and the two results have branched in the sense of section 3.1.1.

Probabilities in the branching case We already have good indications of how the Born rule is related to the inner product between a record and its configuration (obtained in section 4.2), let us now cement that relation by showing how the Born rule arises in the branching case. In the usual density operator description, after the second line of (40) we would already have the diagonalization of the reduced density matrix, but we have not yet related the diagonal coefficients to probabilities. We need to ‘count’ the strongly decohered configurations, i.e. to count the density of paths around the ‘branches’. Even if the relative transition amplitudes do not entirely decouple, if the semi-classical kernel has branched we can define relative probabilities of these branches. The Van Vleck determinant gives us a straightforward way of measuring a “density of observers”. It is the appropriate way to ‘count’ the strongly decohered configurations, i.e. to count the density around ‘branches’. Its interpretation is

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22 One could have a further separation into the immediate environment \( \omega \) and the larger one which is not in contact with the system. However, as per equation (37), this is irrelevant when calculating relative amplitudes.

23 Under normal circumstances, the determination of which axis of the spin the experimenter will project under is also contained in the record configuration, which further restricts the manifold containing the records of the initial configuration.
precisely what we require: it measures the relative configuration space volume between an initial and a final configuration, as transported by a classical field history (for more on this, see [17]). In other words, the classical field history maps an infinitesimal configuration space volume $\delta\phi_i$ to an infinitesimal configuration space volume $\delta\phi_f$, and the Van Vleck determinant gives a ratio of these volumes, measuring the 'spread', or density of paths in the coarse-graining around the given classical one.

Therefore, for two branched (as per definition 6) transition amplitudes $K_\alpha(\phi_i, \phi_1)$ and $K_{\alpha'}(\phi_i, \phi_2)$, since the initial volume $\delta\phi_i$ is the same, the relative 'density', or relative probability associated to the final states is:

$$\frac{\rho K_\alpha}{\rho K_{\alpha'}} = \frac{\Delta_\alpha}{\Delta_{\alpha'}}$$

(41)

Is this a good notion of relative probabilities? If there is no interference we indeed obtain from (4) that $|K_\alpha|^2 = \Delta_\alpha$. Thus if there is no interference we obtain exactly:

$$\frac{|K_\alpha|^2}{|K_{\alpha'}|^2} = \frac{\Delta_\alpha}{\Delta_{\alpha'}}$$

(42)

which reproduces Born’s rule in this particular setting, but was here defined as a ‘density of observers’ since it gives the likelihood of finding oneself in configuration $\phi_\alpha$ relative to configuration $\phi_{\alpha'}$ (both with the same record of course). Thus we trivially find the relative probability of the two outcomes in (40), $\Delta_{\alpha'} = 1$. However I have not yet shown here that the appropriate timeless Born’s rule emerges in full generality, which I leave for future work, relying on the Epistemic Separability Principle proof in [26].

### 5.3 Entanglement, chaos and decoherence

With the formulation presented here, a connection between classical chaotic behavior – as a statement about the divergence of paths in configuration space – and decoherence, is suggested. Suppose that a given Hamiltonian $H_0$ yields an integrable (analytically solvable) dynamical system. It is known that for a system with more than 2 degrees of freedom, an arbitrarily small perturbation $H' = H_0 + \epsilon H_1$ destroys the integrability of the system. Such non-integrable systems share a common feature: exponential sensitivity to initial conditions. For an integrable system, bundles of trajectories spread linearly in phase space. For non-integrable systems, the spreading is exponential in configuration space, which means that our inability to specify initial conditions of the system with the arbitrary precision makes its long term behavior largely unpredictable. Once we include environmental effects, which are generally perturbations of Hamiltonians involving more than 2 degrees of freedom, as expanded on in the previous section, this spreading of dynamical orbits gives rise to decoherence.

At least in this setting, a strict relationship should exist between the Lyapunov exponents and the Van Vleck determinant. In other words, under classical dynamics an initial region in configuration space stretches and shrinks along different directions. These rates are described by the Lyapunov exponent $\Lambda$. Roughly translated to the classical particle context, this implies that the Van Vleck determinant shrinks like $\exp(-\Lambda t)$, which can be seen as a chaos induced shrinking of the Van Vleck. Apart from this, for the moment this connection is completely heuristic, requiring a more precise mathematical formulation.

It should be noted however, that there is a historical disconnect between the concepts of decoherence and chaos. In Zurek and Paz [27], this disconnect is put forward succinctly:

The relation between classical and quantum chaos has been always somewhat unclear and, at times, even strained. The cause of the difficulties can be traced to the fact that the defining characteristic of classical chaos sensitive dependence on initial conditions has no quantum counterpart: it is defined through the behavior of neighboring trajectories, a concept which is essentially alien to quantum mechanics. Moreover, when the natural language of quantum mechanics of closed systems is adopted, an analogue of the exponential divergence cannot be found.

Exactly by defining decoherence from the properties of neighboring trajectories in configuration space of closed systems, which I, unlike Zurek and Paz believe is not a concept entirely alien to quantum mechanics, this formulation by and large bridges this gap.

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24 We assume that there are the same number of focusing points between the records and the final configurations for both coarse-grainings. In the branched case there are usually no focusing points whatsoever.

25 This projection onto configuration space is what makes the assertion compatible with Liouville theorem.
6 Conclusions

As has been said before, the main problem with many worlds is that none of us wants to think that “I am a mere copy of myself”, whatever that means. As is often said “there are many emotions against many-worlds, but very few arguments”. In [28], Wallace writes “According to our best current physics, branches are real”, and this is roughly the position I take in this paper as well. As I explained, my interpretation is not exactly of branches, but there is a sense in which the ontological existence of configuration space is my version of the many worlds scenario. To be more precise, nothing is splitting, all the individual copies of the system already exist in timeless $\mathcal{M}$. If anything, emotions against this view should be more intense, as it does not even posit a notion of continuous “copies of myself”.

Observers in this realm have partial knowledge of their entire configuration, which is encoded in their own instantaneous configuration. This lack of absolute knowledge implies such observers are equally represented in many points of $\mathcal{M}$. It is this uncertainty and large number of copies which makes the frequentist approach valid, and the correct reduced density matrix emerge.

This underlying timeless configuration space, argued convincingly for in [1], doesn’t excise, but changes the role of dynamics. A Baesrsonian interpretation of events is forced upon us: in doing experiments we don’t ‘keep track of an evolving system’, we compare configurations possessing the same records. Since the amplitude distribution in configuration space is dictated by the path integral, it is only natural that we ascribe the high relative frequency of certain relations between record and record-holding configurations to the “passage of time” along given classical paths in configuration space.

All that exists in configuration space is an infinite array of all possible “presents”. Some of these presents have records of other presents, which gives them a distribution of relational quantities within them. The notion of a continuous self is a useful fiction that can be abstracted from these distributions of correlations. From such distributions we can also infer the occurrence of interfering coarse-grainings describing a given transition, which is what we here understand by “superpositions”.

Classicality and decoherence

In the present formulation the problem faced by decoherence is no longer to account for the observable classical reality from the basis of a quantum theory: each configuration is, tautologically, classical, and all we can do is find probabilities for going from an initial configuration to a final one. Decoherence comes in only at the level of suppressing interference effects between alternative histories of the Universe, much like what it does in the consistent histories formalism. I connected the central issue of decoherence, the suppression of interference terms, to the concept of “fading” in the path integrals in configuration space setting.

Now, does decoherence completely resolve the measurement problem? In many researchers opinion this has definitely been the case for some time now. However, even leading figures in the decoherence camp have expressed feelings otherwise. Joos writes in [29]:

Does decoherence solve the measurement problem? Clearly not. What decoherence tells us, is that certain objects appear classical when they are observed. But what is an observation?

At some stage, we still have to apply the usual probability rules of quantum theory.

In his review, Laloe states

Indeed, the emergence of a single result in a single experiment, in other words the disappearance of macroscopic superpositions, is a major issue; the fact that such superpositions cannot be resolved at any stage within the linear Schrödinger equation may be seen as the major difficulty of quantum mechanics. As Pearle nicely expresses it, the problem is to explain why events occur!

A response to these quotes requires two threads. The first, which I have attempted to address in this paper, is the statement that “objects appear classical when observed”. By positing classical configuration space as the ontological (and not just epistemological) arena of physics, objects will always appear classical. “Appearing” is a statement about brain configurations, which are just part of the overall configuration point and thus completely classical. In this way, superposition can only appear explicitly through interference effects.

The present work, goes very much against the modern canon that using classical concepts in order to describe quantum mechanics is ill-conceived. By positing first the existence of a classical configuration space, it straightforwardly answers Pearle and Laloe: i) there are no macroscopic superpositions at any
point of configuration space, superpositions are interference effects between different histories of the system, and ii) events, or “instants” of the classical Universe are all that exists, there is no need to explain them. In the present context, the answer to the question of definite outcomes is unambiguous: it is the ontological existence of configuration space, with each point including the instantaneous brain state of any possible observer that resolves it.

The second thread of a response to the quotes regards how exactly the probability for different configurations arise. This brings us to a further property of the present setting, namely that it implements the Epistemic Separability Principle (ESP) [26] in configuration space in a well-defined way: the outcome of experiments performed by an observer on a specific system do not depend on the physical state of other parts of the system not in contact with it, and previous times for which one does not have a record of. The definition of records is constructed so as to prohibit “re-coherence” of coarse-grainings, and it also guarantees that transitions between configurations which are not a “record - record holding” pair are superfluous for describing relative frequencies of any record-holding configuration, and thus are of very limited interest in doing physics: With this view, there is no need to ever “update the wavefunction” of the Universe at each “measurement”: the initial configurations are effectively embedded in the final configurations themselves.

This fact about records, together with the self-locating uncertainty implied by the observer being represented by a manifold (and thus possessing an infinite number of copies), ensures, through the mechanism discussed in [26] that the Born rule is recovered. It can be seen as the inner product between a record and the record-holding configuration (in section 4.2), but it is in fact recovered through the counting of the density of the branches, in section 5.2.1.

Given a theory, or the form of an action $S[\gamma]$, we can test it by comparing the relative densities which it predicts through repeated experiments. Some configurations with very small relative density might not seem to its intrinsic observers to obey Born’s rule for the given theory. Although with repetition we can make the likelihood that the fault is with the theory, and not with Born’s rule, very large, we can of course never be sure. This is the Bayesian approach we can follow in this setting: using Occam’s razor to give higher priors to simpler laws and multiplying these priors by the likelihoods of our observations given the postulated laws, to choose the laws with the highest posterior probability.

**Inner product, initial state, and physical states.**

I did not attempt to define an overall Hilbert space for the ‘wave-function of the Universe’. Instead, to approximate the present context as much as possible to the standard quantum mechanics setting, each configuration was associated to a unique state. The state $|\phi\rangle$ over the configuration $\phi$ was given by propagating the vacuum state $|p\rangle$ (over the completely degenerate configuration $p$) to that configuration through the amplitude kernel, yielding the state $|\phi\rangle := \hat{K}(\phi,p)|p\rangle$. Without using a global Hilbert space, I constructed a well-defined inner product which requires it to transport the states back to $|p\rangle$. This inner product identifies $\|\phi\|^2$ with its Born probability, which is also shown to be equal to $\langle \phi_1 | \phi \rangle$, where $\phi$ is record-holding for the record $\phi_1$.

Given two configurations $\phi_1$ and $\phi_2$ (belonging to some record-holding submanifold), the ‘superposition’ of their states, say $|\phi_{12}\rangle := |\phi_1\rangle + |\phi_2\rangle$, might be only formal, since sums of such states don’t necessarily correspond to the state of a third configuration. That is, $|\phi_{12}\rangle$ may not really correspond to a configuration $\phi_{12}$, and can thus be of merely formal utility (perhaps one could say it is not “ontic” [23]). If $|\phi_{12}\rangle$ does not correspond to a configuration, it does not have a Bayesian interpretation (as e.g. the relative frequency of observers). If it does correspond to such a configuration, it represents degeneracy of ways a process can take place, not a superposition of configurations in any physical sense.

**The basis selection problem and contextuality**

The usual criterion for selecting pointer basis for a system is based on the interaction with the environment, and embodies the idea of robustness of correlations. The selection is thus a property of the interaction Hamiltonian, which acts by determining what states lead to stable, perceivable records when the interaction of the system with the environment is taken into account. The fact that the pointer basis is in most cases the position basis is seen as an effect of the manner by which we write local interactions.

Here, I start with the assumption of an underlying real existence of configuration space. Then the fact that the preferred basis is usually given in the position basis is seen by me as very convenient for the formulation in configuration space attempted in this paper. Furthermore, the only possible issues with taking the position basis as the pointer basis of decoherence occur for relativistic theories, as Wallace points out [30] (my emphasis):
In the case of non-relativistic quantum theory [the selection of the preferred basis] is unproblematic. The decoherence-preferred basis is basically a coarse-graining of the position basis, so a collapse rule that collapses the wave-function onto wavepackets fairly concentrated around a particular center of mass position, or a choice of position as the hidden-variable, will do nicely. [...] It is crucial to note what makes this possible. Position has a dual role in non-relativistic quantum theory: it is at one and the same time (a) one of the fundamental microphysical variables in terms of which the theory is defined, and (b) such that a coarse-grained version of it is preferred by the high-level, dynamical, emergent process of decoherence. As such, it is possible to formulate modifications or supplements to non-relativistic quantum theory that are both precisely defined in terms of the microphysical variables used to formulate quantum mechanics, and appropriately aligned with the macrophysical variables picked out by decoherence. Unhappily for modificatory strategies, there does not appear to be a variable in extant relativistic quantum theory in QED, say, or in the Standard Model that manages to play this dual role.

Here no issue can thus arise with choosing an ontological status for the position basis. I interpret reasons (a) and (b) given above differently from Wallace however. I interpret them as being emergent from the primary existence of configuration space, or Platonia, a word invented by Barbour [1].

Even if these considerations still allow degeneracy of choices for coarse-grainings of paths in $\mathcal{M}$, I proposed a method to select preferred ones. Roughly, if there are a finite number of classical paths connecting configurations $\phi_i$ and $\phi_f$, one can select coarse-grainings around those. For configurations $\phi_i$ and $\phi_f$ which have extremal paths between them (ignoring caustics), we can use coarse-grainings for the amplitude $K(\phi_i, \phi_f)$ centered around such paths.\footnote{One should note that Fujiwara’s et al [19] formalization of the path integral through piece-wise classical paths is one of the more robust formal mathematical treatments of the path integral.}

The WKB semi-classical approximation can approximate the amplitude kernel as a weighed sum of phases related to these coarse-grainings. Cecille deWitt has shown how to extend this approximation to arbitrary orders of $\hbar$ in what is termed ‘the semi-classical expansion’, involving higher functional derivatives around the on-shell action. Her work could give a framework for using such “as classical as possible” coarse-grainings to describe effects beyond the usual WKB approximation.

Due to several different reasons however, explained in section 3.1.2 and in appendix C, it is not likely that any two configurations have smooth extremal curves connecting them. This can happen even for finite-dimensional spaces with distributional (non-smooth) potentials for example. The PECs are an extension of the semi-classical expansion to segmented extremal paths, and a minimal PEC is basically the PEC with the least amount of segments that can still represent the same process (up to some degree of approximation). Uniqueness of minimal PECs for a given transition amplitude would thus impose a way to select preferred sets of coarse-grained histories representing that process.

**Why the present formalism is not suitable for fundamentally covariant theories of gravity:**

Refoliation symmetry has undesirable properties for my intents. For starters, refoliation symmetries act on a way that depend not only on the configuration $g_{ab}$, but also on the momenta $\pi^{ab}$ (or extrinsic curvature $K_{ab}$) conjugate to $g$. Furthermore, two paths $\gamma(t),\gamma'(t)$ that cross in $\mathcal{M}$, let’s say at a given point $g_o$, might not cross at all if a refoliation is applied to either path. The same is true even if we consider phase space as opposed to configuration space. Thus definitions like that of records – essential for this work – would be meaningless for such a theory. These unwanted properties emerge from the fact that refoliations don’t act as a group in configuration space, and there is thus no well-defined geometric quotient for such symmetries. Refoliations are not a ‘law of the instant’. This is not the case for the spatial diffeomorphisms – a symmetry that I indeed assume for $S[g]$ – which acts through pull-back on both the metric and on any other physical fields (including the metric momenta). This also does not happen for the group of spatial conformal transformations, which are the symmetries of shape dynamics and do have a group action on the whole configuration space. Similarly, here, one can apply all the definitions for initial and final data, e.g. a conformal spatial geometry for shape dynamics, whereas it is very difficult to define space-time diffeomorphism invariant boundary states, a problem related to the infamous ‘problem of time’ in GR.

Secondly, we don’t require relativity to introduce concepts such as a variable particle number. Choosing a theory that has a fundamental formulation in terms of interacting fields, secures the benefit of not having a definite particle number, and of being able to justify, at a fundamental level, the unity of system and environment. At the same time, not being relativistic, it more clearly indicates when it is possible...
to reduce a description of the entire configuration space $\mathcal{M}$ to fewer parameters. This can be achieved by the effective decoupling of the evolution of fields over different domains of the spatial manifold $M$, which provides an effective (as opposed to fundamental) separation of system and environment, and is in this sense also related to the robustness of this decoupling.

**An application to preferred simultaneity theories**

One cannot use tree-level, or classical effects to distinguish GR and most alternative preferred foliation theories. For instance, in shape dynamics the two theories are classically dynamically similar, except if one is close to a black hole horizon \[31\]. This ensures that, apart from black hole horizons, the classical light-cone and causality relations obey the same form as in relativity. For example, as shown in [32], locally one can always use the symmetries of shape dynamics to enforce the equivalence principle (again, not at the horizon), and thus, due to various uniqueness theorems of QFT’s for Lorentz-symmetric theories [24], it would be difficult to differentiate shape dynamics from general relativity at usual local laboratory experimental conditions, where local gravitational fields are merely background.\(^{27}\) This reflects a more general demand that preferred foliation theories do not enter in obvious contradiction with experiments at well explored regimes.

However, it is quite possible that in order to quantum mechanically detect a preferred simultaneity surface at low energies non-local experiments might yield stronger signals since a simultaneity surface is itself non-local by definition. In the presence of multiple classical paths interpolating between an initial and final configuration, one might be able to use semi-classical interference effects to make qualitative predictions for preferred foliation theories of gravity. This still does not require the use of quantum corrections to propagators, in the sense that one needs off-shell processes, or the higher terms in $\hbar$ in (7). Instead, it requires a sum over different “extremal loops”: multiple extremal trajectories that connect the same initial and final configurations.

**Unresolved issues**

There are two unresolved issues that I would like to comment on. Both were mentioned in the text but have not received a satisfactory treatment here.

The first is the matter of the choice of configuration space. One should remember that the theory can only be equivalent to relativistic theories in a given preferred foliation. In that preferred foliation, configuration variables should not invoke time derivatives. Even then, it is not clear (at least to me) that the theory is completely invariant under different choices of configuration variables. For electromagnetic fields, the choice seems to be more ambiguous, a choice of the vector potential $A^\mu$ corresponds basically to a choice of the magnetic field $B^{\alpha \beta}$ as the configuration variable, whereas a choice of $E^\alpha$ would correspond to $A^\alpha$. Choosing either implies that the other is only obtained with the use of a time derivative. Since $A^\mu$ is the standard gauge variable, $A^\mu_i$ seems like a preferred choice for Yang-Mills. Of course, fundamentally one would make a choice of configuration fields for the entire standard model Lagrangian in 3+1 form. Spinor fields are more complicated, see however [33] for a Lagrangian formulation in a non-relativistic setting or as a classical formulation of spin as circulating flow of energy. Ideally, one would like to show that requiring equivalence with relativistic theories demands that any such choice of configuration variables intrinsic to the given preferred foliation should be equivalent, but I have not attacked this problem here.

The second issue which I have not properly dealt with here is the standing of a more familiar notion of unitarity (other than (29)). The obstacle here is the absence of time at a fundamental level. I have toyed with the idea of clock subsystems in appendix E. There I defined a clock subsystem as one part of an entangled bipartite system. The special property of this subsystem is that it has no alternative branches (e.g. in any of the minimal PECs connecting to a given record-holding submanifold $\mathcal{M}_{(r)}$). Thus it can serve as an “anchor” with which we can compare different amplitudes. In this case we recover the usual time-dependent path integral connecting $\phi_r$ to $\mathcal{M}_{(r)}$, as in equation (58), but one should still be able to define under which circumstances this notion can be emergent (other than the one where we completely recover the standard time-dependent setting). This also remains an investigation for future work.

\(^{27}\)For full disclosure, no one has yet performed quantum field theory calculations in shape dynamics. But at the fixed metric background level, it is likely that these conclusions, regarding local QFT experiments, can be extended to shape dynamics.
ACKNOWLEDGEMENTS

I would like to thank Lee Smolin, Daniel Sudarsky and Lucien Hardy for comments, Natalie Schober for help with the English, and Jess Riedel for explanations in decoherence theory. This research was supported by Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research and Innovation.

APPENDIX

A Gravity

A.1 Configuration space and observers

Let me now flush this out in more detail with the example of gravity. For gravity, configuration space is the space $\text{Riem}(M)$, of positive-definite sections of the symmetric covariant tensor bundle $C^\infty_+ (T^* M \otimes_S T^* M)$ over $M$, which forms a cone in the Banach vector space $B := C^\infty(T^* M \otimes_S T^* M)$. This cone has a natural Riemannian structure induced pointwise by the metric $g_{ab}$. Matter fields are further sections over different tensor and spin bundles over $M$. A topological manifold structure is easily given to $\text{Riem}(M)$ since it is just a subspace of a linear Banach space (charts are just the trivial embedding maps and homeomorphic maps between them are well defined). A similar construction works if $M$ has boundaries, with appropriate (Dirichlet) boundary conditions.

Keeping the more general notation and calling the metric fields also by the generic $\phi$, suppose we are given the field $\phi_o$, on $O$, a closed subset of $M$. The manifold $N$, diffeomorphic to $M - O$, will represent in our example the spatial support of the part of the field we don’t have physical access to. We can form the configuration space $\text{Riem}(N)$, with metrics over the manifold $N$ respecting some boundary condition on $\partial N \simeq \partial O$ which is compatible with the boundary conditions of $\phi_o$ (suppose for simplicity that the configuration $\phi_o$ has no isometries). Now we require an embedding map, $\text{Riem}(N) \hookrightarrow \text{Riem}(M)$ to define it as a submanifold of $\text{Riem}(M)$. Calling an element of $\text{Riem}(N)$ by $\phi$, we need an embedding map:

$$\text{Riem}(N) \hookrightarrow \text{Riem}(M)$$

$$\tilde{\phi} \rightarrow \phi$$

where roughly we want $\phi$ to be the result of glueing of the laboratory metric $\phi_o$ to the outside world $\tilde{\phi}$, i.e. $\phi = \phi_o \cup \tilde{\phi}$. Each laboratory field $\phi_o$ then has an infinite number of copies, parametrized by $\phi$, and we need to find a manner by which given $\phi \in \text{Riem}(N)$, for fixed $\phi_o$ we recover a $\phi \in \text{Riem}(M)$. We have a manifold $N$, which we define as diffeomorphic to $M - O$, but we can’t have any unrestricted embedding of $N$ into $M$, for we need to respect the boundary conditions set by $\phi_o$ (remember $\partial \phi_o$ also has no isometries). Thus we define a smooth embedding with fixed boundary matching onto $\partial O$, $f : N \hookrightarrow M$. By construction (the interior of) $\text{Im}(f)$ is an open submanifold of $M$, and we can define the push forward $f_*(\tilde{\phi})$ which then glues onto $\phi_o$ to yield a field for the entire manifold, $\phi \in \text{Riem}(M)$, i.e.

$$\phi := f_*(\tilde{\phi}) \cup \phi_o$$

Note that we are still left with an ambiguity in the choice of embedding map $f$ respecting the given boundary conditions. In fact we have a further ambiguity that I have so far omitted. That is, if we really have only access to $O$, we do not know it as a subset of $M$, and therefore we need to also choose an embedding for it. The joint space of the two embeddings, with matching boundary conditions, is isomorphic to the diffeomorphism group of $M$, and the ambiguity vanishes once we go to the reduced (or relational) configuration space (taking the quotient wrt the diffeomorphisms).\(^{28}\)

This is our first brush with an important issue in dealing with background independent theories. Namely, in the absence of a background for spatial fields, we are obligated to define our physical entities in a relational manner, i.e. relative to each other. Here I will take this relational view, implying that from now on the space of paths $\Gamma[[\phi_i], [\phi_f]]$ is taken to lie in reduced configuration space $[\mathcal{M}]$ (e.g. $\text{Riem}/\text{Diff}$), and interpolate between two points therein (e.g. the equivalence classes $[\phi_i]$ and $[\phi_f]$).

Each element of this space can be written as $\gamma : [0, 1] \rightarrow [\mathcal{M}]$ with $\gamma(0) = [\phi_1]$ and $\gamma(1) = [\phi_2]$, forming a

\(^{28}\) Note that further problems arise for points $\phi$ which have non-trivial isometries. This is exactly the case discussed in Carroll and Sebens in [26], where this ambiguity adds to the degree of what they term ‘self-locating uncertainty’.
groupoid under the usual composition law for curves. One can visualize the path integral (1) much like the usual non-relativistic path integral, the main difference is that the ambient space for the paths is here (infinite-dimensional, reduced) configuration space. Since I will not be using reduced configuration space explicitly, in order to unclutter notation, I will not keep the equivalence class notation \([ \cdot ]\), but it should be understood that this is the setting of this work. Since I will not use the form of the gravitational action explicitly, I leave a brief introduction and a few references to gravitational actions which do not explicitly incorporate refoliation invariance, and yet have a global reparametrization invariance, to the next subsection.

A.2 A few global reparametrization invariant actions

The more naive form of such actions is the so-called Bahrlein-Sharpe-Wheeler action [34]. This is obtained, after the 3+1 split of the Einstein-Hilbert action, upon rewriting the extrinsic curvature

\[
K_{ij} = \frac{1}{2N} k_{ij},
\]

where

\[
k_{ij} = \dot{g}_{ij} - L^{\xi}g_{ab},
\]

where \(N\) is the lapse and \(\xi^i\) is the shift vector field in the 3+1 decomposition. Variation by the lapse yields the equation

\[
N = \sqrt{T/R}
\]

where \(T = G^{ijkl}k_{ij}k_{kl}\) with \(G^{ijkl}\) the inverse DeWitt supermetric, and \(R\) the intrinsic spatial Ricci scalar curvature. Reinstating this solution into the action one obtains a “Jacobi-type” action [35]

\[
S = \int dt \int d^3x \sqrt{g} \sqrt{T/R}
\]

which is globally reparametrization invariant. Variation of the action by the shift gives similarly an equation for itself, in terms of \(g_{ij}\) and \(\dot{g}_{ij}\), called ‘the thin-sandwich equation’. This is not always solvable, but when it is, one can reinstate both the lapse and shift into the extrinsic curvature, and thus find the appropriate embedding of the hypersurface in space-time, in terms of the initial data \((g_{ij}, \dot{g}_{ij})\).

Upon a Legendre transformation, one finds that there is primary constraint (or secondary, depending on how one counts constraints) on the momenta, which is the present incarnation of the infamous (super) Hamiltonian scalar constraint – which is the responsible for on-shell refoliation symmetry in the Hamiltonian formalism of general relativity.

One could more thoroughly deal with this in the conformal-thin-sandwich approach, which has better solvability properties than the thin-sandwich problem. There, one defines as initial data merely an initial conformal class of geometries, and transverse-traceless component of the velocity of the conformal class. One then solves the constraints in terms of a conformally weighted vector field and lapse [36]. There are more straightforward ways of building gravitational theories of transverse-traceless fields with 2 degrees of freedom and no refoliation constraint. See [37] for an example which is spatially conformal diffeomorphism invariant, but not generally space-time covariant and clearly differs from general relativity even locally, and [7] for an introduction to shape dynamics, which is also a theory incorporating spatial conformal diffeomorphism invariance but which largely matches general relativity in the absence of black holes.

A different action with similar properties could be given by [37]: We thus have the action:

\[
S = \int dt \int_M \sqrt{gd^3x} \sqrt{C^{ab}C_{ab}(T^{TT} \dot{g}^{TT})}
\]

(44)

where The cotton tensor used here is defined as

\[
C^{ab} := \epsilon^{acd} \nabla_c \left( R^b_d - \frac{1}{4} \delta^b_d R \right)
\]

(45)

The Cotton tensor is symmetric, transverse and traceless:

\[
C^{ab} = C^{(ab)}, \quad C^{ab} = 0, \quad C^a_a = 0
\]

(46)

It also homogeneously scales conformally with weight \(-5/2\). Thus under \(g_{ab} \rightarrow e^{4\phi}g_{ab}\) we get \(C^{ab} \rightarrow e^{-10\phi}C^{ab}\).

\[
\dot{g}^{TT} = (\dot{g}_{ab} + (\omega_P[g])_{(ab)} + \omega_P[g]g_{ab})
\]

(47)

are the traverse traceless metric velocities, corrected by infinite-dimensional gauge connections on Riem, \(\omega_P[g]\) and \(\omega_P[g]\), which project the velocities into the space orthogonal to the conformal diffeomorphism
fibers. To stress, this is a fully $C$-invariant action with the same number of degrees of freedom as GR, but which does not have a Hamiltonian constraint. Equation (44) is furthermore a purely geodesic-type action in Riem, with just one global lapse and thus one global notion of time, as such it also possesses inherent value in a relationalist setting.

### B More on the DeWitt expansion

Proposition 8 in [17] states that for a single extremal path $\gamma$ between $\phi_i$ and $\phi_f$, the propagation kernel gives

$$K(\phi_i, \phi_f) = e^{i\mathcal{S}[\gamma]/\hbar} \int_{\mathcal{X}} e^{i\Sigma[\gamma, x]/\hbar} dw(x)$$

where we have a lot of nomenclature to explain. First $x \in \mathcal{X}$ are vector fields along $\gamma$ which vanish at $\phi_i$ and $\phi_f$.

$$\Sigma[\gamma, x] = \sum_{n=3}^{\infty} (1/n!) S^{(n)}_{\gamma}(x, \ldots, x)$$

where $S^{(n)}_{\gamma}$ denotes the $n$-th variation of the action around $\gamma$, and it acts on the n-tuple of vector fields $x, \ldots, x$. The pseudo-measure $w$ on $\mathcal{X}$ is given by $w(\mathcal{X}) = \Delta^{1/2}_{\gamma}$.

Jacobi vector fields correspond to geodesic variations. They are the also called Synge vector fields, or $u$-vector fields. To make a long story short, one finds that although the integrand yields a power expansion in $\hbar$, the integral gives a power expansion in $\hbar$. The terms can be regrouped into a power expansion in $\hbar$.

$$S'(\gamma) \cdot x_p x_p = u' W^{-1}_{ij} u^j$$

where $W^{-1}_{ij}$ is the propagator corresponding to the Jacobi equation (or the Feynman Green function corresponding to the action, which also corresponds to the inverse of the van-Vleck matrix), and $u'$ are Jacobi vector fields.

$$\lim_{p=\infty} K_p(\phi_i, \phi_f) = K(\phi_i, \phi_f)$$

where $K_p$ is given by:

$$K_p(\phi_i, \phi_f) = e^{i\mathcal{S}[\gamma]} \Delta^{1/2} \int_{\mathcal{X}} D_p(u) \exp (i\Sigma_p(u))$$

where $\mathcal{B}$ is the linear Banach tangent space, and one uses the Jacobi equation to propagate the measure to the endpoints of the interval.

To make a long story short, one finds that although the integrand yields a power expansion in $\hbar^{-1}$, the integral gives a power expansion in $\hbar$. The terms can be regrouped into a power expansion in $\hbar$, yielding

$$K(\phi_i, \phi_f) = \Delta^{1/2} (2\pi)^{-\nu/2} e^{i\mathcal{S}[\gamma]/\hbar} \sum_{n=0}^{\infty} (i\hbar)^n A_n$$

where $A_0 = 1$ and the next order contribution is

$$i\hbar A_1 = \frac{1}{\int_{\mathcal{X}} dw(x)} \int_{\mathcal{X}} \left( \frac{1}{\hbar} S^{(4)}(x)^4 - \frac{1}{2\hbar^2} \left( \frac{1}{3} S^{(3)}(x)^3 \right)^2 \right) dw(x)$$

And the others can be calculated order by order. Although this work was accomplished for finite-dimensional configuration spaces, it was extended to the infinite-dimensional Banach one required here in [15]. The formal treatment of the time-slicing approach used here in the context of piece-wise extremal curves was developed in [19].

31
C Geodesically and dynamically connected manifolds

The Jacobi version of the Maupertuis principle establishes some instances when dynamics can be viewed as geodesic motion in an associated Riemannian manifold. That is, if the action can be written as

\[ S = \int (T - V) dt \]

defined in \((M, h)\), where \(h\) is the Jacobi metric, conformally related to \(g\) by \(h = 2(E - V)g\). This I will call the Jacobi procedure, whereby one builds a metric in configuration space whose geodesics are extremals of the action. For any finite-dimensional metrically complete Riemannian manifold, the Hopf-Rinow theorem guarantees that two points of the manifold will be connected by a geodesic.\(^{29}\)

However, even for finite-dimensional spaces, there is already here a very robust obstruction to this procedure: namely, non-smooth potentials \(U\) (such as a Dirac \(\delta\)-function potential, which are ubiquitous in physics). In that case, it can be shown that there might be no extremal paths connecting two given points \(a\) and \(b\). For questions on the generic behavior of “critical connectedness” in Hamiltonian systems, see [18].

Furthermore, reduced configuration space is a stratified manifold once one includes gravity, and I am not aware of an analogous Hopf-Rinow theorem in this setting (geodesics that reach a highly symmetric configuration might stop there, making the reduced configuration space metrically incomplete for example). But the even deeper issue here is that unlike the usual Jacobi procedure, the actions we use for the totality of configuration space, e.g. (43), don’t reproduce even Finslerian metrics [38]. Therefore it is very unlikely that any two points will be critically connected.

Tubular bundles In the definition of PECs, Definition 5, we used the notion of the width around each of the extremal segments, but this was not made rigorous at the time. To make the width of each “tube” determinate in this context we require the concept of a tubular neighborhoods. A tubular neighborhood of a given path \(\gamma\) in \(\mathcal{M}\) is roughly a small tube around \(\gamma\). More precisely, a tubular neighborhood of a submanifold \(L\) embedded in a Riemannian manifold \(N\) is a homeomorphism between the normal bundle of \(L\) and an open set of \(N\), for which the zero section reduces to the identity on \(L\). Using the canonical supermetric in Riem, one can define a tubular neighborhood orthogonal to a given curve (that the exponential map is still well defined in this infinite-dimensional setting was proven in [39]).

Using the canonical supermetric in Riem \(G_{abcd} := \frac{1}{2}(g^{ac}g^{bd} + g^{bc}g^{ad})\) (i.e. the one with zero DeWitt parameter), one obtains a local distance function \(d(\phi_1, \phi_2)\) that could determine the radius of each tubular neighborhood. But this radius offers very little insight into the actual dynamical behavior of a system. Let us now discuss a distance function in configuration space for establishing this width which is more closely related to decoherence properties. From the properties of the amplitude kernel, one can check that the following functional:

\[ d(\phi_1, \phi_2) := |\log(|K(\phi_1, \phi_2)|)| \] (53)

satisfies the requirements for a distance functional: i) it is non-negative, ii) zero only when \(\phi_1 = \phi_2\), iii) symmetric, and iv) satisfies the triangle inequality (since \(K(\phi_1, \phi_3) \leq K(\phi_1, \phi_2)K(\phi_2, \phi_3)\)).\(^{30}\)

However, one should note that the construction of the normal bundle to the path is still determined by the canonical supermetric in Riem – which defined what it means to be orthogonal to the path. One could also make this notion of orthogonality dynamical if one can form a pointwise metric in configuration space using the following definition:

\[ \langle \delta \phi_1, \delta \phi_2 \rangle(\phi) := \frac{d}{dt} \frac{d}{ds} K(\phi + t\delta \phi_1, \phi + s\delta \phi_2)|_{t=s=0} \] (54)

\(^{29}\) For Finsler metrics and infinite-dimensions there are caveats (the geodesics come arbitrarily close to any given endpoint in infinite-dimensions for example), but nothing that would bother us here.

\(^{30}\) We are using \(\log(|K(\phi_1, \phi_2)|)\) because for the inner product introduced later, in section 4.2, \(\log(|(\phi_1|\phi_2)|)\) would not satisfy property ii) above of the distance functional (53).
This always gives the pointwise Jacobi metric if one can be defined, however this is not always the case. For example, for the BSW Lagrangian (43), \( L = G_{abc \delta} g_{ab} g_{\delta \epsilon} h^c h^\delta \) which does not form even a Finslerian metric \([38]\), as mentioned above. In either case, the radius \( r \) of the tubular neighborhood would be given by the distance (53) such that a congruence of paths in such a tubular neighborhood would still allow for an exhaustive coarse-graining.

D Semi-classical decoupling

To make the manner in which the semi-classical decoupling of section 5.1 more precise, let’s start by choosing any one-dimensional parameter \( t \in [0,1] \) parametrizing a classical curve \( \phi(t) \) from \( \phi^r := \phi(0) \) to \( \phi^f := \phi(1) \). Suppose there exists a 1-parameter family \( \{ \phi^r \} \) of diffeomorphisms\(^{31}\) such that \( (h^*_t \phi^r_t(O), h^*_t \phi^r_t(M-O)) = (\phi^O, \phi^M-O) \), \( t \in [0,1] \) where \( \phi^r \) (resp. \( \phi^r_{M-O} \)) is a classical field history on the space of field configurations over \( O \) (resp. \( M-O \)), going from a local record configuration \( \phi^O := h^*_0 \phi^r_0 \) (resp. \( \phi^M-O := h^*_0 \phi^r_{M-O} \)) to \( \phi^O := h^*_1 \phi^f_1 \) (resp. \( \phi^M-O := h^*_1 \phi^f_{M-O} \)). If the totality of independent classical field histories \( \phi^O \) and \( \phi^M-O \) account for all complete classical trajectories \( \phi^r \) between \( \phi^f \) and \( \phi^f \) through (55) (for some choice of \( h_1 \) for each path), I will say that \( O \) is semi-classically decoupled from \( M-O \) in the propagator \( K[\phi^r, \phi^f] \). This means that for each classical partial field history \( \phi^O \), and, independently, each partial classical field history \( \phi^M-O \), there exists a complete classical field history \( \phi^r \), and every complete history can be decomposed into two partial classical histories. A further subtlety arises, because this decoupling is always approximate, for we can never truly disconnect a laboratory from the rest of the Universe. In the case of Riem(\( \mathcal{M} \)), one can make the degree with which this decoupling is achieved more precise, using the simple canonical positive definite (super)metric \( G_{abc \delta} \) in Riem(\( \mathcal{M} \)) induced by the pointwise metric \( g_{ab} \). One can then say the system is approximately decoupled in this norm:

\[
\inf_{h_t \in \text{Diff}(\mathcal{M})} \| (h^*_t \phi^r_t(O) \oplus h^*_t \phi^r_t(M-O)) - (\phi^O \cup \phi^M-O) \|_{\phi^r} \leq \epsilon, \quad t \in [0,1]
\]

where the norm is taken at the metric \( g_{ab}^t = \phi^t \), and I have heuristically used the direct sum sign \( \oplus \) to signify a sum of tensor fields with complementary supports on \( \mathcal{M} \). I will define the decoupling (up to \( \epsilon \) precision) if it satisfies equation (56) for all classical field histories \( \phi^r \) (resp. \( \phi^O \) and \( \phi^M-O \)) between \( \phi^r \) and \( \phi^f \).

E Pointer basis and clock subsystems: an example

Now I can attempt to define a semi-classical pointer basis for a given subsystem \( \phi^O \) and coarse-graining \( \{ O_\alpha \}_{\alpha \in \mathcal{I}} \) as another subsystem, or subspace of partial field configurations \( \phi^O \), entangled with \( \phi^O \), such that \( \text{whenver} \ \phi^O \ \text{contains a partial record} \ \phi^r \ \text{of} \ \phi^O \ \text{the given exclusive} \ \epsilon \text{-exhaustive coarse-grained set} \ \text{of partial histories between} \ \phi^O \ \text{and} \ \phi^O \ \text{contains a single contribution} \). This roughly states that alternative histories for this subsystem – which is entangled with another subsystem – have negligible interference.

Using minimal PECS one can make this definition more independent of the choice of coarse-graining by demanding that the definition be valid up to some \( N \) in the truncation of equation (8). That is, in a region \( \mathcal{U} \subset \mathcal{M} \), one can define the pointer basis subsystem \( \mathcal{U}_O \), entangled with \( \mathcal{U}_O \) by requiring that if a minimal PEC exists between \( \phi^r \) and \( \phi^f \) (where \( \phi^r, \phi^f \in \mathcal{U} \)), it has at most one partial piece-wise extremal path (up to \( N \) segments) interpolating between the partial \( \phi^r_\gamma \) and \( \phi^O \), and each extremal segment is entangled (in the sense of (39) for \( J = 1 \)). As usual in this paper, I will concern myself solely with the semi-classical case (i.e. \( N = 1 \)).\(^{32}\) In this case we can partially parametrize the record holding configuration by the branch of the pointer basis subsystem it is on. That, is, for \( \gamma O \) the unique extremal path between \( \phi^r_\gamma \) and \( \phi^O \), we would have:

\[
K_{\mathcal{U}_O \mathcal{U}_O}^O (\phi^r_\gamma \cup \phi^r_\gamma, \phi^O \cup \phi^O) = A e^{i S[\gamma]} \sum_{k \in \mathcal{I}} (\Delta_{\gamma})^{1/2} e^{i S[\gamma]}[\gamma^k]
\]

\(^{31}\)And conformal transformations if one assumes that the model is also scale relational. This diffeomorphism accounts for different manners in which we can embed the partial fields into \( \mathcal{M} \), to make up the complete field.

\(^{32}\)Note that the pointer basis subsystem need not be defined on the entirety of \( \mathcal{M} \). I.e. a pointer basis may “dissipate” or cease to be semi-classically decoupled form the rest of the field in some regions of \( \mathcal{M} \).
where I have denoted one of the possible end points of the $O'$ system along the $j$-th branch of the $O$ system by $\phi_{O'}^{(j)}$ (note that the $O'$ system may still have interference effects).

Of course, $K^{O'O'}_{c}(\phi^{(j)}_O \cup \phi^{(j)}_{O'}, \phi^{(j)}_O \cup \phi^{(j)}_{O'})$ can be non-fading for different $j$ (i.e. belonging to different semi-classical pointer branches). In general $\phi^{(j)}_O(t) \neq \phi^{(j)}_{O'}(t)$ for $j \neq k$ and any choice of parametrization of the branches. Only when one has $\phi^{(j)}_{O'}(t) = \phi^{(j)}_{O}(t')$ can one use the pointer subsystems as an “anchor” to compare relative amplitudes between the branches of the entangled system, and this, I am proposing, is the role of a clock.\footnote{Perhaps a good model for a global clock in this sense would be a field component which cannot be semi-classically decoupled, e.g. whose classical evolution equations are non-local. Then one could use the configuration of this system on $\mathcal{M}$ to globally parametrize semi-classical branches. This is roughly the role of ‘dust models’ for parametrizing time in refoliation invariant theories.}

A clock subsystem has only one (entangled) branch, for which we can write the non-fading amplitudes in the given region $U$ as:

$$K_{ci}^{O'O'}(0 \cup \phi^{(j)}_{O'}(0), t \cup \phi^{(j)}_{O'}(t)) = A(t) \sum_{k \in I} (\Delta_k)^{1/2} e^{iS[\phi^{(k)}_O]}$$

(58)

In this case we can define the ‘equal time inner product’ (equal clock configuration inner product would be more accurate in the present circumstance) through (30) for these particular type of configurations.

**Tri-orthogonal decomposition** In standard quantum mechanics, when the density matrix of a bi-partite systems has degenerate eigenvalues, decoherence does not necessarily lead to a unique pointer basis, and indeed can give rise to physically distinct ones (represented by non-commuting projectors). However, when the system contains at least three subsystems, the tri-orthogonal decomposition theorem guarantees that this decomposition is indeed unique. That is, for

$$|\psi_i\rangle = \sum_{j} |\psi^{(1)}_i\rangle |\psi^{(2)}_j\rangle |\psi^{(3)}_i\rangle$$

and $\langle \psi^{(k)}_i | \psi^{(k)}_j \rangle = \delta_{ij}$, the decomposition is unique.

In the present case, the ‘branching of the wave-function going forward in time’ is not as important as in consistent histories. Here all configurations exist timelessly, and all one can do is be a Bayesian and calculate relative amplitudes between configurations with the same record. Minimal PECs for example can aid us in finding preferred coarse-grainings for a given transition amplitude $K(\phi_r, \phi)$, and we will have different minimal PECs for different $\phi$. It is also important to note that using the notion of vector sum of (33), the equality $|\psi_1\rangle + |\psi_2\rangle = |\psi'_1\rangle + |\psi'_2\rangle$ can be said to only have concrete meaning (as opposed to a formal one) if there are configurations $\psi'_1$ and $\psi'_2$ such that

$$K(\cdot, \psi_1)K(\psi_1, \phi_r) + K(\cdot, \psi_2)K(\psi_2, \phi_r) = K(\cdot, \psi'_1)K(\psi'_1, \phi_r) + K(\cdot, \psi'_2)K(\psi'_2, \phi_r)$$

which might already dilute the basis ambiguity.

Nonetheless, we still have a notion of branches emanating from a given record (see definition 6). Suppose that we have a tri-partite system entangled to a clock, then with a few extra assumptions we can use a product of inner products (30) for the partial configuration spaces and apply the same tri-orthogonal decomposition theorem (at least for finite-dimensional subsystems).

Let us be more explicit. Suppose we have that given record configuration $\phi_r = \phi_1(t_0) \oplus \phi_2(t_0) \oplus \phi_3(t_0) \oplus \phi_4(t_0)$, where the last partial field is a physical clock, to which all of the other partial field configurations are semi-classically coupled. Suppose further that we have a family of semi-classical branches of order 1 emanating from $\phi_r$ (according to definition 6), i.e. extremal curves departing from $\phi_r$,

$$\phi^{i}(t) = \phi_1^{i}(t) \oplus \phi_2^{i}(t) \oplus \phi_3^{i}(t) \oplus t \, , \, i \in I$$

which maintain the semi-classical decoupling and span the set of record-holding configurations with the first order amplitude. Now, we make the natural assumption for the manner in which we have coupled the systems that

$$K(\phi^{i}(t), \phi^{i}(t)) = \prod_{k} K(\phi_{k}^{i}(t), \phi_{k}^{i}(t))$$

From the “equal clock inner product”, and the tri-orthogonal decomposition theorem this then implies that we have a unique choice in how to decompose the decohered paths contributing to leading order to those configurations that possess a record of $\phi_r$ at clock configuration $t$.\footnote{Perhaps a good model for a global clock in this sense would be a field component which cannot be semi-classically decoupled, e.g. whose classical evolution equations are non-local. Then one could use the configuration of this system on $\mathcal{M}$ to globally parametrize semi-classical branches. This is roughly the role of ‘dust models’ for parametrizing time in refoliation invariant theories.}
F  Hartle’s gravitational consistent histories

As an example of a consistent histories theory that works in space time, and is close to what I have attempted here, let me briefly describe the work of Hartle [40]. Although the construction is largely schematic, some issues can already be abstracted from it. The first point where it diverges from the standard quantum mechanical approach is in the fact that space-time diffeomorphism invariance makes it impossible to define local observables in time. This means that we cannot find alternative “histories” specified at a specific moment in time, and thus do not expect a formulation of “states in space-like surfaces and unitary evolution between these surfaces.” Thus for Hartle unitarity would only be recovered in certain semi-classical limits of the background spacetime.

To further quote [40]:

The fine-grained histories are not ‘single-valued’ in any geometrically defined variable labelling a space-like surface [...]. More precisely, there is no geometrical variable that picks out a unique spacelike surface in all geometries.

In a footnote at the end of this quote, Hartle adds a comment that is of interest to us:

Spaces like surfaces labeled by the trace of the extrinsic curvature $K$ foliate certain classes of classical spacetimes obeying the Einstein equation. However, there is no reason to require that non-classical histories be foliable in this way. It is easy to construct [four–] geometries where surfaces of a given $K$ occur arbitrarily often.

For such a space-time theory, initial and final configurations, let’s say the geometries $g_i$ and $g_f$, cannot be observable, since they are not gauge-invariant. Other concepts which are central to this work, such as that of records, are also difficult to define in the space-time setting. In theories that are not fundamentally descriptive of a four-geometry however, such as shape dynamics, where configuration space only contains spatially constant $K$’s, all of these issues are side-stepped. Although there are many similarities with Hartle’s quantum theory of spacetime geometry approach, my insistence on theories that are intrinsically 3+1 in form, gives many new insights, and avoids criticisms made to the spacetime formulation.

In this work, I have largely ignored the issue of a time-ordering in $\mathcal{M}$, and it is a big question in attempts of formulating quantum gravity in configuration space (for instance through the WdW equation). Apart from Hartle’s quote above, another criticism towards implementing $K$ as the time variable for paths in GR is argued against because it is seen as “just a choice of gauge”. However, for shape dynamics this is not so. Whenever translatable to space-time solutions, shape dynamics requires monotonically increasing, spatially constant extrinsic curvature. This could be promoted to a physical principle: “thou shalt not consider paths whose related gravitational momenta have non-increasing constant trace”. Thus viewed from a shape dynamics viewpoint, the path integral for gravity could in principle be calculated on conformal superspace (Riem/Conf×Diff(M)), with a time-ordering for paths given by $K$. 34

G  “Counter-intuitive” experiments, from this perspective

To get to grips with the formalism, I will present some heuristic examples from the semi-classical path integral in configuration space point of view.

G.1 Double-slit

The double slit experiment’s ubiquity in the literature allows me to skip a detailed explanation of the setup and results of the experiment. Suffice it to say that it is one of the hallmarks of quantum theory, as it most clearly displays the “particle-wave” duality. To emphasize the point of view espoused in this paper, “quantum wave properties” are abstracted from interference effects, which can occur when non-fading histories exist between a record with the same given final configuration.35 As I intend to show, this explanation can go quite far in helping to hone our intuition about such effects.

34 As mentioned before however, this is a rather naive simplification of the state of affairs. In fact the shape dynamics Hamiltonian depends on the momenta as well. In principle one should consider the path integral in the cotangent space $T^*(\text{Riem/Conf×Diff(M)})$. Work on the precise formulation and 1-loop contributions are underway.

35 Note that since I am looking at a configuration space of fields, rather than particles, classical wave effects are allowed, but fundamentally different than classical wave effects.
By setting up the experiment, as I mentioned, we are determining part of the field configuration $\phi_{\text{O}}$ and fixing a submanifold of $\mathcal{M}$ determined by this partial field configuration (i.e. those points that on $O$ match our setting, but might differ otherwise), let’s call this submanifold $\mathcal{N}_i$. Everything not contained in $O$ would be usually ascribed to the “environment”, which we so far don’t care too much about. As explained in the previous section, we move the description to reduced phase space, where each point corresponds to a submanifold of $\mathcal{M}$. We then calculate the transition amplitude of going from $\mathcal{N}_i$ to configurations where a fluorescent patch on the screen around the specific point $x^* $ has appeared, and the clock now reads $t^*$. Note that one sums only over coarse-grained histories that end up with the same final configuration, i.e. dot at $x^*$ and clock on the wall pointing at $t^*$.

I will also assume that the evolution of the rest of the field configuration $\phi_{\mathcal{M} \setminus O}$ (the “environment”) is identical for all coarse-grained histories between $\mathcal{N}_i$ and $\mathcal{N}_f(t^*, x^*)$. This allows us to focus our attention to paths which are finitely parametrized, by $t$ and $x,y$, where $y$ is the horizontal and $x$ is the vertical axis.

One can form many coarse-grained histories in $\mathcal{M}$ between $\mathcal{N}_i$ and $\mathcal{N}_f(t^*, x^*)$, but most will have vanishing amplitude to contribute, i.e. most will fade. Robust interference occurs for those coarse-grained paths between $\mathcal{N}_i$ and $\mathcal{N}_f(t^*, x^*)$ that don’t fade with respect to each other.

Clearly, any path that does not go through the slits, and instead has to surmount the enormous potential barrier of the screen will not be nearly extremal, and by the usual steepest descent arguments they will not contribute to first order. To first order, we can start by calculating the contribution of coarse-grained paths that are disjoint bundles around classical paths going through the slits and just use equation (2), since there are more than one classical paths that connect the emission point and $x^*$, one reproduces the usual geometric approximation to the path integral analysis of the double slit, and thus recovers the observed interference effect. For this analysis we can completely dispense with a precise determination of the coarse-grained histories, but we write here for completeness:

$$ C_1 = \{ P_1^1 = (y = 0, x = 0, t = 0), \quad P_1^2 = (y = y, x = x_{\text{upper}}), \quad P_1^3 = (y = y_{\text{screen}}, x = x^*, t = t^*) \} $$

$$ C_2 = \{ P_2^1 = (y = 0, x = 0, t = 0), \quad P_2^2 = (y = y, x = x_{\text{lower}}), \quad P_2^3 = (y = y_{\text{screen}}, x = x^*, t = t^*) \} $$

In our notation we then have $\phi_i = (y = 0, x = 0, t = 0)$ and $\phi_f = (y = y_{\text{screen}}, x = x^*, t = t^*)$, and we take the intermediary projectors to be given by single configurations: $\phi_1 = (y = y, x = x_{\text{upper}})$ and $\phi_2 = (y = y_{\text{screen}}, x = x_{\text{lower}})$. Interference arises because the two coarse-grained histories yield similar amplitudes:

$$ K_1(\phi_i, \phi_f) \approx K(\phi, \phi_i)K(\phi_1, \phi_f) \approx K(\phi, \phi_2)K(\phi_2, \phi_f) \approx K_2(\phi_i, \phi_f) $$

G.2 Welcher Weg and interaction with the environment

The reason one observes interference in the double-slit experiment, according to the usual explanation, is because there is no way, even in principle to obtain information about which path the electron took. The lack of which-path (welcher-weg, or WW) information is fundamental to the observability of interference fringes. The interpretation I am giving here to the same effects is that, either through strong interactions with the electrons, or anything else (such as obtaining explicitly the WW information) that forces the (classical) paths in configuration space to diverge, by definition will not allow them to reach the same final configuration.

I have already illustrated “environment-induced” decoherence in (40), but it is useful to present an example of alternative coarse-grainings that fade, in the way I have defined it. Let us suppose that the experimenter performs a non-destructive measurement at one of the slits, for instance by using linearly polarized light and installing orthogonal circular polarization lenses in the slits, but any “tagging” or “which path” information will suffice. The final configuration obtained solely from the extremal path bundles is e.g. $(x^*, t^*, \text{left-handed polarization})$ for the upper slit and $(x^*, t^*, \text{right-handed polarization})$ for the lower slit. An extremal (classical) path going through the upper slit arrives at the final screen point $x = x^*$ in a total field configuration which is different than the field configuration obtained from evolution by the extremal path passing through the lower slit and arriving at $x^*$. In other words, the extremal path going through the upper slit ends up at a different region in configuration space than the extremal path going through the lower slit. Even though the two particles indeed hit $x^*$ on the screen,

36 In actual electron double slit experiments this is exactly what is done: a particle detector is slid up and down the positions of a “virtual screen”

37 There are different types of mathematical models for such a double slit screen. For now, I assume that the screen is given by a smooth potential barrier, and that taking this potential into account for the action there are indeed extremizing paths that connect $\phi_i$ and $\phi_f$ (which may differ from the naive ones present with no barrier potential).

36
when the entire field configuration is taken into account, it is clear that the end points in configuration space are distinct. It can still happen that there is a coarse-grained history of the field that goes through the upper slit and reaches \((x^*, t^*)\), right-handed polarization, but it will be highly non-classical, and will fade in comparison with the one coming from the lower slit. That is, a coarse grained history going through:\(^{38}\)

\[ C_1 = \{ P_1 = (y = 0, x = 0, t = 0, z), \; P_2 = (y = y_{\text{slit}}, x = x_{\text{upper}}, \text{left-handed}), \; P_3 = (y = y_{\text{screen}}, x = x^*, t = t^*, \text{right-handed}) \} \]

fades, while one going through

\[ C_2 = \{ P_1 = (y = 0, x = 0, t = 0, z), \; P_2 = (y = y_{\text{slit}}, x = x_{\text{upper}}, \text{left-handed}), \; P_3 = (y = y_{\text{screen}}, x = x^*, t = t^*, \text{left-handed}) \} \]

does not fade, but has no interference with the other non-fading coarse-grained history, given by:

\[ C_3 = \{ P_1 = (y = 0, x = 0, t = 0, z), \; P_2 = (y = y_{\text{slit}}, x = x_{\text{lower}}, \text{right-handed}), \; P_3 = (y = y_{\text{screen}}, x = x^*, t = t^*, \text{right-handed}) \} \]

since they end up at different final configurations. The recorded configuration is \(\phi_i = (y = 0, x = 0, t = 0, z)\), the present one \(\phi_f = (y = y_{\text{screen}}, x = x^*, t = t^*, R)\) and the two alternative projections \(\phi_1 = (y = y_{\text{slit}}, x = x_{\text{upper}}, L)\) and \(\phi_2 = (y = y_{\text{slit}}, x = x_{\text{lower}}, R)\). In bra-ket notation then we have:

\[
\frac{K_1(\phi_i, \phi_f)}{K(\phi_i, \phi_f)} \approx \frac{K(\phi_1, \phi_f)K(\phi_1, \phi_f)}{K(\phi_1, \phi_2)K(\phi_2, \phi_f)} \sim \epsilon
\]

where \(\|\epsilon\| << 1\).

This effect of decoherence is in the present interpretation identical to environmentally induced decoherence. It furthermore does not distinguish between environmental decoherence and “intrinsic decoherence” (see [41] for a review). In both cases, extremal paths in configuration space containing the particle going through the two slits are ending up in different points in configuration space, and those that do end up at the same point don’t have the same order of contribution. Whether this happens by considering the configuration of the environment (which can be different for the two paths if they interact with it), or of the gravitational field, or any other aspect of the field configuration for that matter, is thus immaterial, coarse-grained paths are naturally diverging to first order, which is a classical statement reminiscent of chaotic systems.

G.3 Hardy’s paradox

Hardy’s paradox is a gedanken experiment in which a particle and an anti-particle can interact without annihilating each other. It relies on a technique known as ‘interaction-free measurement’ which can be described to ‘see without looking’. Upon its discovery in the early 90’s by Lucien Hardy [42], it was hailed by some to ‘kill all realistic models of quantum phenomena’. In the original Hardy’s setup, one talks about an electron and its antiparticle, a positron. Both of them may be detected in one of two detectors. However, a particular combination of detectors could only be chosen by the pair if the two particles have previously traveled along trajectories that would guarantee that the particles annihilate. Because they annihilate, they can’t get to the detectors.

It is supposed to erase any remaining doubt as to the “reality of paths” of particles. This it accomplishes, but it is very easily describable in terms of paths in configuration space. Let us quickly review the setup of the experiment.

**Setup** The experiment is based on two Mach-Zender interferometers. After being emitted, the electron (positron) on the right (left, resp.) goes through a beam-splitter, which has an equal chance of transmission on two trajectories, \(v^-\) or \(w^-\) (\(v^+\) or \(w^+\)) in figure G.3. The classical dynamics of each path of the electron is such that they converge again on a second beam splitter. The interferometers are tuned so that if the electron (positron) interferometer is operating by itself, constructive interference at the second beam splitter will always excite the detector \(e^-\) (resp. \(e^+\)). In standard ket notation:

\[
|e^-\rangle \rightarrow \frac{|u^-\rangle + i|w^-\rangle}{\sqrt{2}} \rightarrow i|e^-\rangle
\]

\(^{38}\)Note that the clock coordinate for \(P_2\) is immaterial.
Figure 1: One realization of Hardy’s experimental setup. An electron-positron pair goes through an interferometer, where one leg of each is bent so as to overlap.

If one of the arms, let’s say \( w^- \) is blocked by some means, then there is an absence of constructive interference and both detectors \( d^- \) and \( c^- \) have an equal chance of being excited. Now, the paths \( w^- \) and \( w^+ \) are bent so that if the electron and positron “take these paths” they will annihilate each other. Thus the detection of a particle on \( d^- \) would indicate the presence of the obstructing positron, but without an annihilation taking place (which is why it is taken as an example of “an interaction-free measurement”). It is the absence of interference that allows the \( d^- \) detector to be excited.

The paradox emerges from the following classical consideration: the detector at \( d^- \) can only be excited if the positron took the path \( w^+ \) and the electron took the path \( v^- \) (otherwise they would have been annihilated). Similarly, a hit on the detector \( d^+ \) would require the electron taking the path \( w^- \) and the positron taking the path \( v^+ \). Thus, the conclusion would be that detectors \( d^+ \) and \( d^- \) can never be excited simultaneously, and yet sometimes (1/16 of the times) they do!

How is this paradox explained in the paths in configuration space approach? By considering paths of the entire configuration, and not just of the individual particles. The formal manipulations are exactly the same, but the picture is much more intuitive. Let’s consider the classical paths between the initial and final configuration point. We have four paths leaving the initial configuration, which I call \( q_1 = (BS_1^l, BS_1^r) \), where both the electron and the positron hit the beam splitter. Following the notation of figure G.3, I will call these four paths \( (v^+, v^-), (w^+, v^-), (v^+, w^-) \) and \( (w^+, w^-) \). Of these, only \( (w^+, w^-) \) diverges from reaching the final point \( q_2 = (BS_2^l, BS_2^r) \) (only a non-classical, fading trajectory would get \( (w^+, w^-) \) back to \( q_2 \)). The other paths can interfere. In particular, the detectors \( d^+ \) and \( d^- \) are jointly excited 1/16 of the times, when paths \( (w^+, v^-), (v^+, w^-) \) constructively interfere.

References

[1] J. Barbour, ‘The End of Time: The Next Revolution in Physics.’ Oxford University Press, 1999.
[2] S. Weinberg, Dreams of a Final Theory. Pantheon press, New York, 1992.
[3] R. Feynman, The Character of Physical Law. Modern books, New York, 1965.
[4] A. Valentini, Elegance and Enigma: The Quantum Interviews. Springer, 2011.
[5] P. Horava, “Quantum Gravity at a Lifshitz Point,” Phys.Rev., vol. D79, p. 084008, 2009.
[6] C. Eling, T. Jacobson, and D. Mattingly, “Einstein-Aether theory,” pp. 163–179, 2004.
[7] H. Gomes, S. Gryb, and T. Koslowski, “Einstein gravity as a 3D conformally invariant theory,” Class. Quant. Grav., vol. 28, p. 045005, 2011.
[8] S. L. Adler, “Incorporating gravity into trace dynamics: the induced gravitational action,” Class.Quant.Grav., vol. 30, p. 195015, 2013.
[9] E. Okon and D. Sudarsky, “On the Consistency of the Consistent Histories Approach to Quantum Mechanics,” *Found.Phys.*, vol. 44, pp. 19–33, 2014.

[10] P. C. Hohenberg, “An Introduction to consistent quantum theory,” *Rev.Mod.Phys.*, vol. 82, pp. 2835–2844, 2010.

[11] J. B. Hartle, “Space-time quantum mechanics and the quantum mechanics of space-time,” 1992.

[12] R. B. Griffiths, *Consistent Quantum Theory*. Cambridge University Press, 2003.

[13] A. R. H. Richard Phillips Feynman, *Quantum Mechanics and Path Integrals*. McGraw-Hill, 1965.

[14] P. Cartier and C. DeWitt-Morette, *Functional Integration Action and Symmetries*. Cambridge Monographs on Mathematical Physics, 2007.

[15] C. J. S. Clarke, “The application of dewitt-morette path integrals to general relativity,” *Comm. Math. Phys.*, vol. Volume 56, pp. 125–146., (1977).

[16] M. Visser, “van Vleck determinants: Geodesic focusing and defocusing in Lorentzian space-times,” *Phys.Rev.*, vol. D47, pp. 2395–2402, 1993.

[17] C. DeWitt-Morette, “The semiclassical expansion,” *Annals of Physics*, vol. 97, no. 2, pp. 367 – 399, 1976.

[18] J. Marsden, “Generalized hamiltonian mechanics a mathematical exposition of non-smooth dynamical systems and classical hamiltonian mechanics,” *Archive for Rational Mechanics and Analysis*, vol. 28, no. 5, pp. 325–361, 1968.

[19] N. Kumano-go, “Feynman path integrals as analysis on path space by time slicing approximation,” *Bulletin des Sciences Mathematiques*, vol. 128, no. 3, pp. 197 – 251, 2004.

[20] A. Albrecht, “Investigating decoherence in a simple system,” *Phys. Rev. D*, vol. 46, pp. 5504–5520, Dec 1992.

[21] A. Ashtekar and J. Lewandowski, “Projective techniques and functional integration for gauge theories,” *J.Math.Phys.*, vol. 36, pp. 2170–2191, 1995.

[22] A. Barvinsky, “Unitarity approach to quantum cosmology,” *Physics Reports*, vol. 230, no. 56, pp. 237 – 367, 1993.

[23] M. F. Pusey, J. Barrett, and T. Rudolph, “On the reality of the quantum state,” *Nature Phys.*, vol. 8, p. 476, 2012.

[24] S. Weinberg, *The Quantum Theory of Fields I*. Cambridge University Press, 2005.

[25] J. Maldacena and L. Susskind, “Cool horizons for entangled black holes,” *Fortsch.Phys.*, vol. 61, pp. 781–811, 2013.

[26] C. T. Sebens and S. M. Carroll, “Self-Locating Uncertainty and the Origin of Probability in Everettian Quantum Mechanics,” 2014.

[27] W. H. Zurek and J. P. Paz, “Decoherence, chaos, and the second law,” *Phys. Rev. Lett.*, vol. 72, pp. 2508–2511, Apr 1994.

[28] D. Wallace, “Decoherence and ontology (or: How I learned to stop worrying and love fapp),”

[29] E. Joos, *Decoherence: Theoretical, Experimental, and Conceptual Problems*. Springer, 2000.

[30] D. Wallace, “Decoherence and its role in the modern measurement problem,” *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, vol. 370, no. 1975, pp. 4576–4593, 2012.

[31] H. Gomes, “A Birkhoff theorem for Shape Dynamics,” *Class.Quant.Grav.*, vol. 31, p. 085008, 2014.

[32] S. Carlip and H. Gomes, “Lorentz Invariance in Shape Dynamics,” *Class.Quant.Grav.*, vol. 32, no. 1, p. 015021, 2015.
[33] M. Rivas, *Kinematical Theory of Spinning Particles: Classical and Quantum Mechanical Formalism of Elementary Particles*. Springer, 2002.

[34] R. F. Baierlein, D. H. Sharp, and J. Wheeler *Phys. Rev.*, vol. 126, p. 1864, 1962.

[35] J. Barbour, “The timelessness of quantum gravity: I the evidence from the classical theory,” *Class. Quant. Grav.*, vol. 11, pp. 2853–2873, 1994.

[36] Y. Choquet-Bruhat, *General Relativity and the Einstein Equations*. Oxford Mathematical Monographs, 2008.

[37] H. Gomes, “Gauge theory in riem: Classical,” *arXiv.org: gr-qc/0807.4405*, 2009 -Submitted to Class. Quant. Grav.

[38] D. Giulini, “What is the geometry of superspace?,” *Phys. Rev.*, vol. D51, pp. 5630–5635, 1995.

[39] D. Ebin, “The manifold of riemmanian metrics,” *Symp. Pure Math.*, AMS, vol. 11,15, 1970.

[40] J. B. Hartle, “Generalizing quantum mechanics for quantum spacetime,” pp. 21–43, 2006.

[41] P. Stamp, “Environmental Decoherence versus Intrinsic Decoherence,” *Phil.Trans.Roy.Soc.Lond.*, vol. A370, p. 4429, 2012.

[42] L. Hardy, “Quantum mechanics, local realistic theories, and lorentz-invariant realistic theories,” *Phys. Rev. Lett.*, vol. 68, pp. 2981–2984, May 1992.