Particle velocity in noncommutative space-time

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Abstract

We investigate a particle velocity in the $\kappa$-Minkowski space-time, which is one of the realization of a noncommutative space-time. We emphasize that arrival time analyses by high-energy $\gamma$-rays or neutrinos, which have been considered as powerful tools to restrict the violation of Lorentz invariance, are not effective to detect space-time noncommutativity. In contrast with these examples, we point out a possibility that low-energy massive particles play an important role to detect it.

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I. INTRODUCTION

It is believed that general relativity describes the large scale structure of space-time, and it has revealed the history and present states of our universe. The direct evidence for the validity of general relativity in strong gravitational regime will be obtained by the observations of gravitational waves from inspiraling binaries in near future. While little is known about the small scale structure of space-time because gravity should be also quantized consistently, which is not completed yet, in such a regime. The physics of small scale structure is important because there are some phenomena in our universe which such physics may be needed to describe, e.g., the birth of the universe, space-time singularity and ultra-high energy cosmic rays. The last one is one of the main topics of this paper.

Although we do not have the coherent theory of small scale structure, several attempts have been made to extract its information and effects. Among them, the simplest way is modifying the dispersion relation that leads to the violation of Lorentz invariance. We call the theories obtained by this method modified dispersion relation (MDR) models.

The violation of Lorentz invariance appears in the context of string/M theories, where the space-time structure is modified to include the space-time noncommutativity [1]. The space-time noncommutativity also arises as a result of deformation quantization [2]. By using the MDR models, the robustness of the spectrum of a black hole evaporation and of the fluctuation generated in an inflationary cosmology were discussed [3,4]. Remarkably enough, it was discussed that the anomalous detection of the ultra-high energy cosmic rays beyond the Greisen-Zatsepin-Kuzmin cutoff may be observational evidence for the violation of Lorentz invariance [5,6].

It has been discussed that there is a severe constraint on the energy scale where the Lorentz invariance might be violated, i.e., the scale of quantum gravity as $E_{QG} \gtrsim 7.2 \times 10^{16}$ GeV by the detection of $\gamma$-rays from Markarian (Mk) 421 as pointed out in Refs. [7–10]. They examined the energy dependence of the arrival time of photons and compared it with the observations of Mk 421 to obtain this constraint.
In general, however, it is plausible that not only a dispersion relation but also other relations such as energy-momentum conservation laws might be altered in a Planck scale physics. A particle velocity in such models may have qualitative difference from that in the MDR models. To investigate these features, we employ a model called κ-Minkowski space-time, where noncommutativity is introduced as $[x^i, t] = i\lambda x^i$ [11–13], and compare a group velocity in the κ-Minkowski space-time evaluated in our previous paper [6] with that in the MDR models. The properties of this group velocity were also investigated in Ref. [14].

This paper is organized as follows. In Sec. II, we review the previous discussion in the MDR models. After introducing the κ-Minkowski space-time in Sec. III, we discuss the particle velocity in this model in Sec. IV. In Sec. V, we consider the observational possibilities of time delay by comparing a particle velocity in the usual Minkowski space-time with that in the κ-Minkowski space-time and MDR models. We show that the space-time noncommutativity does not affect the velocity of massless particles, which implies that the arrival time analysis by γ-rays is not useful to detect the space-time noncommutativity. We also discuss a possibility that the space-time noncommutativity might be detected by using low-energy massive particles. In Sec. VI, we summarize our results and mention future work. We use the signature $(-, +, +, +)$ and units in which $c = \hbar = 1$ below.

## II. MODIFIED DISPERSION RELATION MODELS

Although there are various ways to modify the dispersion relation, we consider here the form in Ref. [8] as $p^2 + m^2 = E^2[1 + f(E/E_{QG})]$, where $f$ is a model-dependent function and $E_{QG}$ is the effective energy scale of quantum gravity. For simplicity, we assume that $f$ is an analytic function. Although in general, $f$ and $E_{QG}$ may depend on the species and properties of the particles [5], we do not consider this possibility, which implies that the effects of quantum gravity originate from the space-time structure. In the low-energy limit, $E \ll E_{QG}$, the above dispersion relation becomes
\[ p^2 + m^2 = E^2 + \frac{\xi E^n}{E_{QG}^{n-2}}, \quad (2.1) \]

up to the lowest correction. We have chosen \( \xi = \pm 1 \) and \( n \geq 3 \) is the integer, which is determined by the form of the function \( f \). Note that \( E < m \) for \( \xi = 1 \) in the low-momentum limit. This type of dispersion relation also appears in the Liouville string approach to quantum gravity [15].

The velocity \( v_{MDR} \) in this model is obtained by differentiating the dispersion relation (2.1) with respect to \( p \),

\[ v_{MDR} := \frac{dE}{dp} = \frac{2\sqrt{E^2 - m^2 + \xi E^n/E_{QG}^{n-2}}}{2E + n\xi E^{n-1}/E_{QG}^{n-2}}. \quad (2.2) \]

It should be noted that \( v_{MDR} \) depends on the energy even for massless particles because of the correction term. We can make use of the energy dependence to restrict \( E_{QG} \).

Let us consider a \( \gamma \)-ray from the distant source. We approximate the velocity of the \( \gamma \)-ray by expanding Eq. (2.2) by \( E/E_{QG} \) to

\[ v_{MDR} \approx 1 - \frac{\xi(n-1)}{2} \left( \frac{E}{E_{QG}} \right)^{n-2}. \quad (2.3) \]

Although the correction term may be very small, the difference of arrival time depending on the energy of the photons may become large enough to measure if the \( \gamma \)-rays travel a very long distance [7–10]. The time delay is evaluated as

\[ \delta t = \frac{L}{v_{MDR}(E_1)} - \frac{L}{v_{MDR}(E_2)} \approx \frac{(n-1)\xi L}{2E_{QG}^{n-2}}(E_1^{n-2} - E_2^{n-2}), \quad (2.4) \]

where \( L, E_1 \) and \( E_2 \) are the distance from the source to the Earth, amounts of the energy of particles 1 and 2, respectively.

One of the examples of this kind of analyses is the arrival time analysis by \( \gamma \)-rays from Mk 421 (\( \sim 150 \) Mpc from the Earth). It was reported that \( \gamma \)-rays in the energy range between 1 and 2 TeV arrived at the Earth within the time difference \( \sim 200 \) seconds [7]. Then, \( E_{QG} \) is constrained to \( E_{QG} \gtrsim [3.6 \times (n-1)(n-2) \times 10^{13}]^{1/(n-2)} \times 10^3 \) GeV. Since
the value of $n$ has been assumed to be 3 in most of the previous works, it has been concluded that $E_{QG} \gtrsim 7.2 \times 10^{16}$ GeV. We should note, however, that $n$ may be 4 or larger. In this case, the constraint becomes $E_{QG} \gtrsim 1.5 \times 10^{10}$ GeV for $n = 4$ and $E_{QG} \gtrsim 7.6 \times 10^{7}$ GeV for $n = 5$. Hence the constraint may become quite loose compared with the previous reports.

III. $\kappa$-POINCARÈ ALGEBRA AND $\kappa$-MINKOWSKI SPACE-TIME

Here, we review the $\kappa$-Poincarè algebra [16], which has the structure of a Hopf algebra (quantum group) [17]. The generators of the $\kappa$-Poincarè algebra $P_\kappa$ satisfy the following commutation relations:

\[
[M_{\mu\nu}, M_{\rho\sigma}] = i \left( \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\nu\rho} M_{\mu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} \right),
\]

\[
[M_i, p_0] = 0,
\]

\[
[M_i, p_j] = i \epsilon_{ijk} p_k,
\]

\[
[N_i, p_0] = ip_i,
\]

\[
[N_i, p_j] = -i \delta_{ij} \left[ \frac{1}{2\lambda} \left( 1 - e^{2p_0\lambda} \right) + \frac{\lambda}{2} p^2 \right] + i\lambda p_i p_j,
\]

\[
[p_\mu, p_\nu] = 0,
\]

where $M_i \equiv \frac{1}{2} \epsilon_{ijk} M_{jk}$, $N_i \equiv M_{0i}$ and $p_\mu$ are generators of rotation, boost and translation, respectively. The Greek and Roman indicies take the values from 0 to 3 and from 1 to 3, respectively. We abbreviate $\sum_i p_i^2$ as $p^2$. We can recover the ordinary commutation relations of the Poincarè algebra in the limit $\lambda \rightarrow 0$. The dispersion relation is determined by the eigenvalue of the Casimir operator that commutes with all elements in $P_\kappa$:

\[
\frac{2 \cosh(\lambda p_0)}{\lambda^2} - p^2 e^{-\lambda p_0} = \frac{2 \cosh(\lambda m)}{\lambda^2},
\]

where the rest mass $m$ is defined as the energy with $p_i = 0$. The coproducts $\Delta : P_\kappa \rightarrow P_\kappa \otimes P_\kappa$ of the basic generators are

\[
\Delta(M_i) = M_i \otimes 1 + 1 \otimes M_i,
\]
\[
\Delta(N_i) = N_i \otimes 1 + e^\mu_0 \otimes N_i - \lambda \epsilon_{ijk} p_j \otimes M_k, \quad (3.9)
\]
\[
\Delta(p_0) = p_0 \otimes 1 + 1 \otimes p_0, \quad (3.10)
\]
\[
\Delta(p_i) = p_i \otimes 1 + e^\mu_0 \otimes p_i. \quad (3.11)
\]

The above coproducts of \(p_\mu\), (3.10) and (3.11), are interpreted as the non-Abelian addition law of energy-momenta for particles 1 and 2 as

\[
(E_1, p_1) \hat{+} (E_2, p_2) := \left( E_1 + E_2, p_1 + e^\lambda E_1 p_2 \right), \quad (3.12)
\]

where we identify \(p_0\) with energy \(E\). Note that the associativity of the addition law is given by the coassociativity \((\Delta \otimes id) \circ \Delta = (id \otimes \Delta) \circ \Delta\). The coproducts of other elements in \(\mathcal{P}_\kappa\) are extended as \(\Delta(1) = 1 \otimes 1\) and \(\Delta(MM') = \Delta(M)\Delta(M'), \forall M, M' \in \mathcal{P}_\kappa\). We can check the consistency between this extension of the coproducts as an algebra homomorphism and the commutation relation, i.e., \(\Delta [M, M'] = [\Delta M, \Delta M']\). This consistency guarantees that the \(\kappa\)-Poincarè algebra is form-invariant for multi-particle systems.

The asymmetry of the coproducts for the permutation of particles is called noncocommutativity. The noncocommutativity of the coproducts for the translation sector \(T \subset \mathcal{P}_\kappa\) has two important meanings. One is that the noncommutativity of the \(\kappa\)-Minkowski space-time is a direct consequence of the noncocommutativity. Elements in the \(\kappa\)-Minkowski space-time are defined as linear functionals on the translation sector, \(T^* : T \rightarrow \mathbb{C}\). The products in \(T^*\) is defined in terms of coproducts in \(T\), i.e., \(\forall x, y \in T^*\) and \(\forall p \in T\),

\[
\langle xy, p \rangle := \langle x \otimes y, \Delta p \rangle \quad (3.13)
\]
\[
= \sum_a \langle x, p_a(1) \rangle \langle y, p_a(2) \rangle, \quad (3.14)
\]

where we write the coproducts as \(\Delta(p) = \sum_a p_a(1) \otimes p_a(2)\). With the duality relations \(\langle x^\mu, p_\nu \rangle = -i \delta_\mu^\nu\), this leads to the following commutation relations [11]:

\[
[x^i, x^0] = i \lambda x^i, \quad (3.15)
\]
\[
[x^0, x^0] = 0, \quad (3.16)
\]
\[
[x^i, x^j] = 0. \quad (3.17)
\]
The other is that the noncocommutativity leads to a deformed group velocity formula [6], which is different from the usual velocity formula \( \frac{dE}{dp} \) as will be shown in the next section.

We can also define differentiation, integration and Fourier transformation [18]. The plane wave \( \psi(E,p) = e^{i\mathbf{p} \cdot \mathbf{x}} e^{iEt} \) in the \( \kappa \)-Minkowski space-time introduced in [19,20] respects the non-Abelian addition law of energy-momenta in the sense

\[
\psi(E_1,p_1)\psi(E_2,p_2) = e^{i\mathbf{p}_1 \cdot \mathbf{x} e^{iE_1 t}} e^{i\mathbf{p}_2 \cdot \mathbf{x} e^{iE_2 t}} = \psi(E_1+E_2,p_1+e^{\lambda E_1} p_2).
\]

**IV. VELOCITY FORMULA**

From the properties in the \( \kappa \)-Minkowski space-time in Sec III, we can establish group velocity formulae. For this purpose, we consider infinitesimal changes \( \Delta E \) and \( \Delta p \) in \( E \) and \( p \), respectively, as a result of adding \((\Delta E', \Delta p')\) as

\[
(E,p) \rightarrow (\Delta E', \Delta p') = (E + \Delta E, p + \Delta p).
\]

By the addition law (3.12), we have

\[
(\Delta E', \Delta p') = (\Delta E, \frac{\Delta p}{e^{\lambda E}}).
\]

Next, we construct a wave packet by superposing plane waves. Here we only consider two waves for simplicity, whose momenta and amounts of energy are different infinitesimally from each other [21].

\[
I = \psi(E-\Delta E,p-\Delta p) + \psi(E+\Delta E,p+\Delta p) \\
\cong 2e^{i\mathbf{p} \cdot \mathbf{x} e^{iEt}} \cos \left[ \frac{\Delta p}{e^{\lambda E}} \cdot \left( x + \frac{e^{\lambda E} \Delta Et}{\Delta p} \right) \right],
\]

where we neglected the terms that vanish in the limit \( \Delta p \to 0 \). The group velocity \( \mathbf{v}_I \) of this wave packet can be written as

\[
\mathbf{v}_I := e^{\lambda E} \frac{dE}{dp}.
\]
There remains ambiguity in constructing the wave packet because of the noncommutativity of the space-time. Another possibility is

\[(\Delta E', \Delta p') + (E, p) = (E + \Delta E, p + \Delta p).\]  \hspace{1cm} (4.5)

In this case, the corresponding group velocity \(v_r\) is

\[v_r := \left(1 - \lambda \frac{dE}{dp}\right)^{-1} \frac{dE}{dp}.\]  \hspace{1cm} (4.6)

These velocities can be expressed explicitly in terms of the functions of \(E\) and \(m\) by using the dispersion relation. By the definitions of \(v_l\) and \(v_r\), we find

\[v_l = \frac{e^{\lambda E/2} \sqrt{2[\cosh(\lambda E) - \cosh(\lambda m)]}}{|e^{\lambda E} - \cosh(\lambda m)|} e,\]  \hspace{1cm} (4.7)

\[v_r = \frac{e^{-\lambda E/2} \sqrt{2[\cosh(\lambda E') - \cosh(\lambda m)]}}{|e^{-\lambda E} - \cosh(\lambda m)|} e,\]  \hspace{1cm} (4.8)

where \(e := p/|p|\). We find that the velocities have the same direction as that of the momenta. Note also that there is a correspondence between the transformations \(\lambda \to -\lambda\) and \(v_l \to v_r\).

These velocities were also investigated by Lukierski and Nowicki and the following facts were pointed out in Ref. [14]: (i) \(v_l := |v_l|, v_r := |v_r| \leq 1\) for all energies, (ii) \(dv_l/dE > 0, dv_r/dE > 0\), and (iii) \(v_r\) has a classical velocity addition law, i.e., the addition of parallel velocities \(v_{r1}\) and \(v_{r2}\) becomes

\[v_{r12} = \frac{v_{r1} + v_{r2}}{1 + v_{r1}v_{r2}}.\]  \hspace{1cm} (4.9)

If the boost generator \(N_i\) was an even function for \(\lambda\), this addition law would hold even for \(v_l\) because of the correspondence mentioned above. However, this is not the case. We postpone the interpretation of this asymmetry as future work.

Next, we discuss the application of the above velocity formulae. In the MDR models, since the energy scale of quantum gravity \(E_{QG}\) is introduced perturbatively (see Eq (2.3)), it is reasonable to apply the velocity formulae under the condition on \(E \ll E_{QG}\). While if we apply the velocity formulae in the \(\kappa\)-Minkowski space-time, the energy range is not restricted.
Let us examine the case beyond the quantum gravity scale, i.e., $|\lambda E| \gg 1$. Since we can obtain the information about $v_\ell$ by using the transformation $\lambda \rightarrow -\lambda$ to $v_\ell$, we only examine $v_\ell$ below. We evaluate the velocity $v_\ell$ in the following limits (see Table I.). When $\lambda > 0$ and $E/m \gg 1$, we can find that the velocity of massive particles approaches 1 much faster than that in the Minkowski space-time as the energy of the particle increases. However, for $\lambda < 0$ and $E/m \gg 1$, the difference of the velocity from 1 becomes large as the mass of the particle increases. Note that if $|\lambda(E - m)| \ll 1$, we obtain $|\lambda m| \gg 1$ by using the condition $|\lambda E| \gg 1$. Since $E - m$ is written as $m(1/\sqrt{1 - v_M^2} - 1)$ in the Minkowski space-time, where $v_M$ is the velocity in the Minkowski space-time, we can rewrite the condition $|\lambda(E - m)| \ll 1$ as $|\lambda m(1/\sqrt{1 - v_M^2} - 1)| \ll 1$, which leads to $v_M \ll 1$ because of $|\lambda m| \gg 1$. Then, we find $v_\ell \simeq v_M \sqrt{2\lambda m}$ and $v_\ell \simeq e^{\lambda m}v_M \sqrt{-2\lambda m}$ for $\lambda > 0$ and for $\lambda < 0$, respectively. Thus, we find that $v_\ell$ for the case $|\lambda m| \gg 1$ is quite different from $v_M$, which is a good approximation for describing a velocity of macroscopic bodies in our world under the conditions we are considering. To describe a velocity of macroscopic bodies in the $\kappa$-Minkowski space-time, we must consider carefully what are the energy and the momentum, since these quantities are obtained by a total sum of those of elementary particles according to the addition law (3.12). The above discrepancy may be explained in this reason. Below, we only consider elementary particles and restrict the discussion to the case $|\lambda m| \ll 1$. 

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TABLES

TABLE I. Approximation of the velocity \( v_l \) in the case \(|\lambda E| \gg 1\)

| \( \lambda > 0 \) | \( E/m \gg 1 \) | \( |\lambda(E - m)| \ll 1 \) |
|---|---|---|
| \( \lambda > 0 \) | \( 1 + e^{-2\lambda E} \left[ \frac{1}{2} - \cosh^2(\lambda m) \right] \) | \( 2\sqrt{\lambda(E - m)} \) |
| \( \lambda < 0 \) | \( \frac{1}{\cosh(\lambda m)} \) | \( 2e^{\lambda m} \sqrt{\lambda(m - E)} \) |
V. MEASUREMENTS OF THE EFFECTIVE SCALE OF “QUANTUM GRAVITY” BY MASSIVE PARTICLES

In this section, we compare $v_l$ with $v_{MDR}$ and discuss the possibility of detection of effective scale of quantum gravity by observations and experiments. The behavior of the velocities is quite different depending on the mass and energy of the particle. Hence, we consider two limiting cases: (i) the “relativistic” case ($m \ll E$) and (ii) “non-relativistic” case ($m \approx E$) [22].

In the relativistic case, $m \ll E$, and under the assumptions, $E \ll E_{QG}$ and $E \ll |\lambda^{-1}|$, $v_{MDR}$ and $v_l$ are

$$v_{MDR} \approx 1 - \frac{1}{2} \left(\frac{m}{E}\right)^2 - \frac{\xi(n-1)}{2} \left(\frac{E}{E_{QG}}\right)^{n-2},$$

$$v_l \approx 1 - \frac{1}{2} \left(\frac{m}{E}\right)^2 + \frac{\lambda m^2}{2E},$$

at the lowest order of $m/E$ and $E/E_{QG}$ in the MDR models and $\lambda E$ in the $\kappa$-Minkowski space-time, respectively. When $m = 0$ in the MDR models, $E_{QG}$ can be constrained by the $\gamma$-rays from the Mk 421 as mentioned in Sec. II. However, since $v_l = 1$ for massless particles, (we can confirm this is also true for all order of $\lambda m$ and $\lambda E$), $\lambda$ is not constrained by massless particles. This is an important result since the result notices us that there are a wide variety of candidates for the theory of quantum gravity, some of which the scale of quantum gravity is not constrained by present observations. The situation changes for massive particles since the lowest order correction appears in the coupled form with the mass of the particle in the $\kappa$-Minkowski space-time, while that of the MDR models does not depend on the mass of the particle.

First, we consider neutrinos from supernovae with energy $E_\nu \sim 10^{10}$ eV to detect space-time noncommutativity. We assume that the mass of an electron neutrino and all the parameters necessary to describe neutrino physics are determined by other experiments and observations, and use the delay of the arrival time between the neutrinos and gravitational waves to evaluate the scale of quantum gravity. In this case, the delay of the arrival time is
Since neutrinos are emitted continuously during about 10 s, it is impossible to determine
the time when the neutrino is emitted more accurate than that time scale. For this reason,
δt ≳ 10 s is necessary to detect the effect of quantum gravity. As for λ, since there is no
restriction from the arrival time analysis of γ-rays, λ may take a large value. However,
by considering reaction processes by collider experiments, we can restrict |λ| < 10−12 eV−1
since the threshold of the reaction will change drastically for |λ| > 1/E_{th}, where E_{th} is the
threshold energy in the Minkowski space-time [6]. Then, L becomes far longer than the
horizon scale in the present universe even if |λ| = 10−12 eV−1. Thus, it is difficult to detect
this effect in this phenomena.

Neutrinos from γ-ray bursts in fireball models have a different energy scale. In the
bursts, neutrinos with energy ∼ 10^{14} eV and γ-rays are expected to be radiated away in ∼ 1
s [23]. We show that we cannot detect space-time noncommutativity even if we neglect the
dissipation of the γ-ray. In the E ≫ 1/|λ| case, we can evaluate the delay of the arrival time
of neutrinos compared with the γ-rays from Table I as

\[ \delta t \approx \frac{L m_\nu^2}{2E_\nu} \left( \frac{1}{E_\nu} + \lambda \right), \]  

\[ \text{for } \lambda > 0, \]  

\[ \delta t \approx \frac{L (\lambda m_\nu)^2}{2} \text{ for } \lambda < 0, \]  

where we have used the conditions E/m ≫ 1 and |λ| ≪ 1. If we assume δt ∼ 1 s and
|λ| = 10−12 eV−1, the path of the particle’s travel becomes far longer than the horizon scale
in the present universe. In the E ∼ 1/|λ| case, the arrival time delay cannot be described
in a simple way. There is, however, no qualitative difference from the above case. Hence, it
is difficult to detect space-time noncommutativity by this method.

Next, we examine the non-relativistic case, m ∼ E \ll E_{QG} (or |λ−1|). The velocity in
each model are

\[ v_{MDR} \approx \sqrt{1 - \left( \frac{m}{E} \right)^2} \times \]
\[
\left[ 1 + \frac{\xi E^2(1 - n) + nm^2}{E^2 - m^2} \left( \frac{E}{E_{QG}} \right)^{n-2} \right], 
\]

(5.6)

\[
v_l \approx \sqrt{1 - \left( \frac{m}{E} \right)^2 \left( 1 + \frac{\lambda m^2}{2E} \right)}. 
\]

(5.7)

Note that the absolute value of the correction for the velocity in the $\kappa$-Minkowski space-time decreases with the energy, while that in the MDR model increases. Although, in the low-energy limit, the dispersion relation in the $\kappa$-Minkowski space-time has the same form as that in the MDR models, the correction for the velocity is quite different.

Because of the above difference in the correction terms, there is a possibility that the evidence for space-time noncommutativity can be detected in use of the low-energy massive particles. Here, we consider the ultra-cold neutrons with energy $E_n - m_n \sim 10^{-2}$ eV [24]. Since the mass of a neutron $m_n$ is measured with high accuracy, we can estimate the time interval in which the neutron travels the interval $L$ in the Minkowski space-time. If a time lag is obtained in an experiment, it can be interpreted as the effect of space-time noncommutativity. This time lag is calculated in the $\kappa$-Minkowski space-time as

\[
\delta t = \frac{L}{v_l} - \frac{L}{v_M} \approx \frac{L}{v_M} \frac{\lambda m_n^2}{2E_n}. 
\]

(5.8)

By substituting the value of the apparatus [25], $L \sim 100$ m, we have

\[
\delta t \sim 10^{-1} \lambda m_n. 
\]

(5.9)

If the resolution for the measurement of the time lag is $\sim 10^{-10}$ s and $|\lambda| \gtrsim 10^{-18}$ eV$^{-1}$, we can detect space-time noncommutativity.

VI. CONCLUSION

We have investigated what are the qualitative differences of the velocity formula in the $\kappa$-Minkowski space-time from that in the MDR models. Most of the previous papers had adopted the MDR models since the MDR models are among the simplest ones of quantum gravity. However, many of the MDR models do not have physical foundation in how the
correction terms of naturally arise in the dispersion relation. For example, since the usual Lorentz transformation had been used in the previous work, one could not have avoided the existence of a preferred frame as a result. Since we have taken the standpoint that the existence of a preferred frame is not favorable, we have considered the $\kappa$-Minkowski space-time where the deformed Lorentz transformation and the deformed dispersion relation arise as a result of the deformation quantization.

We have found that since massless particles move in a constant speed in the $\kappa$-Minkowski space-time, the arrival time analyses by $\gamma$-rays are not capable to detect the difference from the Minkowski space-time. This example shows that it is difficult to constrain all kinds of Lorentz invariance by a single experiment. Therefore, we need to investigate specific models individually. We have also considered the possibility to detect space-time noncommutativity by low-energy massive particles. In our model, if the resolution for the measurement of the time lag is given by $\sim 10^{-10}$ s, it is possible to constrain $\lambda$ to $|\lambda| \gtrsim 10^{-18} \text{ eV}^{-1}$. Although these features had not been investigated so far, it may be important.

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[21] This is not a Gaussian wave packet. However, it is sufficient to obtain a group velocity. The extension for more general wave packet will be straightforward.

[22] In the MDR models and in the $\kappa$-Minkowski space-time, it is possible that the particle moves very slowly (fast) even if the condition $m \ll E$ ($m \approx E$) is satisfied.

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