Possibility of Solid-Fluid Transition in Moving Periodic Systems

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The steady sliding state of periodic structures such as charge density waves and flux line lattices is numerically studied based on the three dimensional driven random-field XY model. We focus on the dynamical phase transition between plastic flow and moving solid phases controlled by the magnitude of the driving force. By analyzing the connectivity of comoving clusters, we find that they percolate the system under driving forces larger than a certain critical force within a finite observation time. The critical force, however, logarithmically diverges with the observation time, i.e., the moving solid phase exists only within a certain finite time, which exponentially grows with driving force.

KEYWORDS: CDW, flux line lattice, plastic flow, Bragg glass, random field, XY model

The collective transport phenomena of condensed matter with random pinning attract much attention from the viewpoints of solid state physics, nonlinear dynamics and statistical mechanics. There are numerous systems which belong to this class of dynamics, e.g., charge density waves (CDWs), flux line lattices (FLLs), colloidal lattices and Wigner crystals. These systems have spatial periodicity in the absence of random pinning, which modifies the periodic order and pins the system. Under a driving force larger than a certain threshold value, the system starts moving and shows highly nonlinear conduction. This is the depinning transition, which has been investigated extensively. Recently another attractive topic of these systems has been the dynamical phase transition between two nonequilibrium steady states above the depinning threshold field. The "ordered phase" the local DC velocity order. It is closely related to the plastic deformation, which is related to the phase coherence and experimentally observable transport quantities, such as electric current for CDWs and voltage drop for FLLs, are proportional to the phase velocity. It is sufficient in the case that periodicity is one dimensional as CDWs in NbSe$_3$. We consider, however, that the essential feature of the dynamics of higher dimensional periodic systems such as FLLs would be captured. The periodicity of the structure is related to the phase coherence and experimentally observable transport quantities, such as electric current for CDWs and voltage drop for FLLs, are proportional to the phase velocity $	heta(r, t)$.

The elastodynamic model treats internal interaction using the elastic energy $\int dr (\nabla \theta)^2$, which becomes a harmonic coupling, $\sum_{i,j} (\theta_i - \theta_j)^2/2$, in a lattice model. In order to treat plastic deformation, we replace this harmonic coupling with a sinusoidal one, $1 - \cos(\theta_i - \theta_j)$. They are equivalent in the limit where phase differences become zero. Here, the indices $i$'s denote macroscopic domains fixed in the space in which phase coherence is always held. This sinusoidal coupling in-
duces maximum restoring force, i.e., yield stress and allows plastic deformation, so-called phase slip. Phase slip is a process in which the phase difference between neighboring domains increases or decreases by $2\pi$. It results in no change in coupling energy.

The overdamped equations of motion for the phases of domains, $\theta_i$'s, are as follows.

$$\dot{\theta}_i = -\frac{J}{z} \sum_j \sin(\theta_i - \theta_j) - \sin(\theta_i - \beta_i) + f$$  \hspace{1cm} (1)

We choose the units that both of a pinning strength and a dissipation coefficient equal unity. The first term on the right hand side refers to the interaction with neighboring sites. When "site" instead of "domain" hereafter. A periodic boundary condition is imposed. There are two independent parameters, coupling constant $J$ and driving force $f$. In this article, we analyze the $f$ dependence of the dynamics of this model systematically paying special attention to the dependences on system size and observation time.

We numerically solve eq. (1) by the fourth-order Runge-Kutta method. The discretized time step is set at $2\pi/8(J + f)$. The domains are put regularly on the simple cubic lattice in three dimensions. We call this unit “site” instead of “domain” hereafter. A periodic boundary condition is imposed. There are two independent parameters, coupling constant $J$ and driving force $f$. In this article we show mainly the results for $J = 1.0$. All phases are set at the unique value in the initial state. Physical quantities are calculated after some precursor running (typical time is 12900) for the relaxation to the steady state. An initial state is sometimes substituted by the final state for the simulation with a slightly larger $f$. Thermal fluctuation is not taken into account. Simulations are performed with some samples that have different set of $\beta_i$'s. The numbers of samples whose linear sizes are 16, 32, 64 and 128 are 32, 16, 8 and 4, respectively.

In the left panel of Fig. 1, a spatial configuration of the local DC velocity, $\omega_{\text{DC}}^i = (\dot{\theta}_i(t))_T$, is shown. Here, $(\cdots)_T$ denotes time averaging for the observation time $T$ and the resolution of DC velocity is given by $2\pi/T$. Note that we show the results of two dimensional systems, which show qualitatively similar behaviors to three dimensional systems but show the domain structure of $\omega_{\text{DC}}^i$ more clearly. In the right panel we show the difference in $\omega_{\text{DC}}^i$:

$$|\Delta \omega_{\text{DC}}^{i,j}| = |\omega_{\text{DC}}^i - \omega_{\text{DC}}^j|,$$

for each bond between neighboring sites. When $|\Delta \omega_{\text{DC}}^{i,j}| > 2\pi/T$, phase slip occurs on the bond at least once during the observation time $T$. We focus on this phase slip process to discuss the spatial correlation of motion instead of the direct spatial correlation of DC local velocity.\textsuperscript{13} We define such a bond as a “disconnected” bond. Otherwise, if $|\Delta \omega_{\text{DC}}| < 2\pi/T$, the bond is “connected” and two sites belong to the same cluster. When both of the pair sites are pinned, i.e., $\omega_{\text{DC}}^i, \omega_{\text{DC}}^j < 2\pi/T$, $|\Delta \omega_{\text{DC}}|$ is less than $2\pi/T$ but we regard such a bond as disconnected, which forms nonmoving solid clusters. We analyze a bond percolation transition by controlling driving force. The moving solid phase is characterized by an infinitely large cluster, which is made from connected sites without phase slip. The percolating phase is thus the moving solid phase. The driving force in Fig. 1 is slightly below the critical point and a fractal domain structure appears.

In order to perform finite size scaling, we divide the system into subsystems, whose linear dimension $L$ is smaller than that of a real sample $L_{\text{max}}$. Then we determine that percolation occurs if a certain cluster reaches the two opposite sides of each subsystem. The statistics of subsystems and samples yields percolation probability $P(f, L, T)$, which monotonically grows with $f$ and becomes smaller as $T$ increases.

The reason we perform finite size scaling in terms of $L$ and not $L_{\text{max}}$ as usual is the following. We have found that the whole system with a finite $L_{\text{max}}$ eventually falls into a periodic motion above some threshold driving force. Therefore, for a given driving force, we choose to work on a system with sufficiently large $L_{\text{max}}$ to eliminate this artificial periodic motion and analyze the connectivity of clusters at various length scales $L(< L_{\text{max}})$.

In Fig. 2, $P(f, L, T)$’s for various $L$’s and $T$’s are plotted as functions of $f$. We consider the large $L$ limit at fixed $T$ first and the $T$ dependence next. Percolation probability grows with $f$ and its shape comes closer to that of a step function of $f$ as $L$ increases. Finite size scaling can be performed in the same way as the stochastic percolation. The curves for different $L$’s converge on the universal one as the driving force is scaled as $f = [(f - f_c)/f_c]^{1/\nu}$ with the suitable critical force $f_c$ and the critical exponent $\nu$. The correlation length diverges as $[(f - f_c)/f_c]^{-\nu}$. A good conversion is obtained for each $T$ as shown in Fig. 2 then $f_c(T)$’s and $\nu(T)$’s are obtained. The fraction of connected bonds at the critical
point is 0.10, which is smaller than that for the stochastic percolation, 0.2488, due to the attractive correlation of connected bonds.

The magnitude of the critical driving force for the percolation transition obtained from the above analysis depends on $T$. The $T$ dependence of $f_c$ for several $L_{\text{max}}$'s is shown in Fig. 3. It is expressed as

$$f_c(T) = f_0 \ln(T/t_0),$$

for a long $T$. Here, $f_0$ and $t_0$ are constants. We expect that eq. (2), which is consistent with the idea of shaking temperature as mentioned later, holds for a sufficiently long $T$ and large $L_{\text{max}}$ and the reason for the deviation between the present data and eq. (2) for a smaller $T$ or a smaller $L_{\text{max}}$ is considered as follows. For $T < 2000$, the $f_c$'s obtained from the simulations are a little larger than that expected from eq. (2). This discrepancy can be due to the existence of pinned or very low velocity sites, which cannot be candidates of the percolating cluster. They are defects of the percolation transition, of which effects are not taken into account in eq. (2). The percolation transition is not so sensitive to the existence of such defects when they are rare and isolated. They are not a minority, however, below $f = 0.80$, e.g., the fractions of such sites with $\omega_{DC} < 0.050$ are 0.51, 0.090 and 0.00054 for $f = 0.75, 0.80$ and 0.85, respectively. Here, the local depinning threshold force equals 0.10.) The critical fraction of the connected bonds becomes larger in the lattice with such defects and $f_c$ becomes higher. These defects decrease rapidly with $f$ and have less effect on a higher $f_c$ for a longer $T$.

Another disagreement occurs at a longer $T$, which is caused by the finiteness of the real sample size $L_{\text{max}}$. For a finite $L_{\text{max}}, f_c$ exhibits saturating behavior to a finite value above a certain $T$, which increases with $L_{\text{max}}$.

As mentioned before, this is caused by the falling of the system into limit cycle motion when the phase coherence length becomes comparable to $L_{\text{max}}$. For $T \leq 51500 \ (f_c < 0.870)$, the results for $L = 64$ and 128 hardly differ and they are considered to show $f_c(T)$ for $L_{\text{max}} = \infty$.

From these discussions eq. (2) is expected to hold up to the infinite $T$. This means that $f_c(T)$ diverges logarithmically as $T$ approaches infinity and the moving solid phase does not exist in the long time limit based on the present definition. The supplemental simulation indicates that this behavior does not change for a system with a stronger coupling.

From another point of view, eq. (2) is regarded as a type of “phase boundary” between the plastic flow and moving solid phases in the force-time plain (See Fig. 3). Considering the observation with fixed $f$, the crossover time $\tau(f)$ is obtained as

$$\tau(f) = t_0 \exp(f/f_0).$$

The system behaves as if it were a moving solid in an observation time shorter than $\tau(f)$. Beyond this time, cracks of plastic deformation, which are sheets of phase slip bonds, propagate to the macroscopic scale and fluid like property is revealed.

Equation (3) can be regarded as a thermal activation process, $\tau \propto \exp(V/k_B T_{\text{eq}})$, if an effective temperature proportional to $f^{-1}(\approx \omega_{DC}^{-1})$ is supposed. The inverse of $f_0$ is then proportional to the potential barrier $V$. This is consistent with the idea of “shaking temperature” proposed by Koshelev and Vinokur.

Next, we discuss the universality of the percolation transition for different observation times. In Fig.4, the $T$ dependence of the inverse of the critical exponent $\nu$ is
shown. There is a tendency for large $L_{\text{max}}$’s, 64 and 128, where $\nu^{-1}$ has $T$-independent value $\approx 0.8$. The inverse of $\nu$ becomes smaller and approaches zero as $T$ increases. $\nu^{-1} = 0$ means that $P(f, L, T)$ does not depend on $L$, which manifests that the system falls into the limit cycle motion, i.e., a finite size effect. Such effect appears more clearly in the critical exponent than in the critical force, because the former is determined by the behavior in the whole critical regime. So we expect the percolation transition discussed here is universal with respect to the observation time.

This universality is confirmed by the property of the percolating cluster at the critical point. Cluster size is identified with the number of contained sites and we define $s_c(L, T)$ as the size of the maximum cluster in a subsystem at $f = f_c(T)$. In Fig. 5, $s_c(L, T)$’s for several $T$’s are plotted as functions of subsystem size $L$. They are expressed as $s_c(L, T) \approx 0.35L^{2.45}$ with a fractal dimension of 2.45 and show little $T$ (or $f_c(T)$) dependence. The large deviation of the data for $L_{\text{max}}=64$ at the longest $T$ is due to starting the limit cycle motion.

The universality of the transition for different $T$’s means that the fluctuation of DC velocity has a scaleless spatial pattern, which depends on neither $f$ nor the phase coherence length, which grows with $f$, if one chooses proper time scale, i.e., sees the DC velocity in the resolution of $2\pi/\tau(f)$.

In conclusion, we numerically investigate the possibility of the dynamical phase transition between plastic flow and moving solid phases, which are distinguished by the existence of the long range order of local DC velocity. By analyzing the percolation of no-phase slip bonds and its observation time dependence, we found that the moving solid phase becomes unstable in a finite lifetime.

The condition for the connected bond, that no phase slip occurs, is necessary too. For example all bonds necessarily take phase slips if thermal fluctuation exists. It is important, however, to note that the local symmetry limit $T \rightarrow \infty$ $\Delta \omega_{\text{PC}}^{ij} = 0$ is destroyed from the beginning due to the random field. The bond steadily takes a phase slip in the same direction no matter how rarely it occurs.

The crossover time $\tau(f)$ can be defined clearly; the system behaves like a moving solid in a shorter time scale than $\tau(f)$ and macroscopic plastic deformation occurs beyond $\tau(f)$. This crossover time increases exponentially with the driving force. Its characteristic scale of growth $f_0 = 0.025$ is very small compared with other scales such as pinning strength ($=-1$), therefore the crossover time increases very rapidly in the narrow region of $f$ and overcomes the macroscopic time scale. This is a possible reason the moving solid phase is observed in experiments. The situation is similar to the case of structure glasses, whose viscosity grows quite large and they show slow dynamics, then it is hard to distinguish whether an equilibrium phase transition exists.

We focused on the macroscopic plastic deformation and distinguished between plastic flow and moving solid. This stance is different from the conventional interest in the liquid-crystal(Bragg glass) transition. Although it seems natural that these transitions occurs at the same time, the absence of the moving solid phase discussed here is not immediately related to the absence of the long range periodic order. For example the spatial phase order in a long span is possible in the plastic flow phase if the propagation of plastic deformation along the domain boundary is temporally localized and leaves no change before and after it. On the other hand we see that the saturation of phase correlation length to the system size results limit cycle motion and plastic deformation is suppressed. Then if the transition from liquid to crystal or Bragg glass, which is not observed in the range of our simulation, occurs at finite $f$, the present transition would happen at the same time.

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