The HDMR-hybrid Network Method of Model Approximation for the Stochastic Analysis of Semi-rigid Joint

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Abstract: The huge economic costs with the experimental analysis and computationally expensive with the simulation are the difficulties for the stochastic analysis of engineering structures. The structural data of stochastic analysis is based on the probability of statistical results. Therefore, the focus of structural stochastic analysis is how to simulate the finite element model or the physical model by the method of approximate model with the requirements of the expected accuracy range. For the higher precision sensitivity coefficients, a large number of finite element simulations would be conducted and this process leads to intensive computation. This paper puts forward a methodology that combines the high dimensional model representation (HDMR) method and the hybrid neural network for the approximate model. The advantage of this method is the determination of coupling characteristics of the input parameters, and the complex multidimensional model could be constructed by the limited sample points. The feasibility of the method was applied to semi-rigid connection with multidimensional parameter, and the efficiency and precision were obviously superior to the traditional approximate method.

1. Introduction

In the deterministic analysis of the structure, the mechanical properties of the materials, the load and the error distributions of the randomness of the cross section size would not be taken into account[1]. It is necessary to consider the influence of the randomness of the uncertain parameters on the design, optimization and maintenance in the structures. Large-scale simulation models are indispensable in the stochastic analysis of engineering. With the difficulty of the engineering problems increasing (in most cases, the relationship between the design variables and the response parameters of the object model is complex explicit or implicit form, which is unknown before the analysis) and the growth of the design variable dimensions, the high performance computer would unable to meet the computational costs for a large number of the simulation calculations. In recent years, the approximate model is an effective method to solve the computationally expensive, and the main purpose is the computational efficiency. There are four key steps in the analysis of the approximate model: sampling methods, approximate model analysis, construction methods, evaluation criterion. In the actual modeling process, however, the above four factors are not independent but interaction and mutual dependence.
The focus of the approximate research is getting the mathematical model by the fewer experimental simulation for the stochastic analysis of structures, however, the difficulty of the approximate model would be increasing for the high-dimensional design variables. The object model to be approximated includes both the objective function with explicit form and the engineering real model. In spite of many methods to construct the approximate models, the basic methods could be summarized as the parametric model and the non-parametric model.

A variety of non-parametric model techniques had been studied, such as: finite element[2], response surface method[3,4], artificial neural network, radial basis functions[5,6], Kriging model, support vector regression model[7], additive regression model, projection pursuit regression model[8,9], recursive partitioning regression model and high dimensional model representation[10]. In this paper, a new approximate model method was built by a combination of the high dimensional model representation (HDMR) method with hybrid neural network method. A new approximate model method, HDMR-hybrid method was to be explored, and for representation of the engineering structure, a welding joint is implemented. With the engineering examples, the proposed method is developed for the approximate model with accuracy and efficiently. The simulation model in this paper adopts finite element modeling, and the approximation model adopts HDMR-hybrid network model.

2. The algorithm of HDMR

HDMR model is a multi parameter decoupling model, the basic principle is that a multi parameter function is to be divided into several low dimensional parameter functions according to the coupling among the variables. The total dimension of the random variables is \( N \), \( i, j \) and \( l \) are the dimensional random variable respectively.

\[
f(x) = f_0 + \sum_{i=1}^{N} f_i(x_i) + \sum_{1 \leq i < j \leq N} f_{ij}(x_i, x_j) + \sum_{1 \leq i < j < l \leq N} f_{ijl}(x_i, x_j, x_l) + \cdots + f_{12...N}(x_{12...N})
\]

Where \( f_0 \) is the structural response of a point, \( f_i(x_i) \) is the first order response of the structure, it represents the influence of the random input variables in one dimension \( x_i \) changed on the overall response of the structure alone. \( f_{ij}(x_i, x_j) \) is the second order response of the structure, it represents the influence of a two-dimensional random input variables which changed collectively on the overall response of the structure with other dimensions common variable unchanged. \( f_{12...N}(x_{12...N}) \) represents the whole influence of the \( n \)-dimensional input variables changed on the overall response of the structure and so on.

In this way, the relationship between the response value of the structure and the random variables can be expressed by the hierarchical design. This representation method can both reflect the functional relationship between the random variables and the structural response and the coupling relationship among the random variables.

HDMR based on the center point method has been used to adopted for the determination of \( f_0 \) belong to function (1), that is CUT-HDMR. Firstly, it determined \( d=(d_1, d_2, \ldots, d_N) \) as a reference point in the feasible domain model of the random variables, then \( f_0 \) was the structural response value of the reference point, the first order response value of the structure, the two order response value and the nth order response value would be determined by the center point.

\[
f_0 = f(d)
\]

\[
f_i(x_i) = f_i(x_i, d^i) - f(d)
\]

\[
f_{ij}(x_i, x_j) = f_{ij}(x_i, x_j, d^{ij}) - f_i(x_i) - f_j(x_j) + f_0
\]

Where \( f_i(x_i, d^i) \) is the structural response which the \( i \)-th dimension variable is taken as \( x_i \) and the other dimension parameters are all taken as the center point \( d \). \( f_{ij}(x_i, x_j, d^{ij}) \) is the structural response which the \( i \)-th and \( j \)-th dimension variables are taken as \( x_i \) and \( x_j \), the other dimension parameters are
all taken as the center point \( d \), \( d \) is any value in the feasible region. \( f_i(x_i) \), \( f_{ij}(x_i, x_j) \), \( f_{12,…N}(x_{12,…N}) \) are fitted by Lagrange polynomial interpolation usually. The function between the structure response value and the multidimensional variable would be expressed hierarchically by the above algorithm. In order to improve the accuracy of the function model, increasing the number of functions is a traditional way, however, the advantage of HDMR method is that it can combine with other approximate model construction methods, exert the advantages of different methods, transform the physical model (object model) to the mathematical model (approximate model) gradually.

3. The method of HDMR-hybrid network

In this section, the neural networks is combined with the high-dimensional model representation method. The weight value and threshold decide the accuracy of the neural network. In the training process, regardless of which algorithm is used, the initial weights and thresholds are all random. Therefore, even if the same information in the training for the same neural network, there will comes to a different accuracy of the network outputs. Hybrid neural network with high accuracy refers to a new network structure which extracting the weights and thresholds from the established network. So that the training process of the neural network is no longer blind random but the best results of the last training for the initial weights and thresholds. Hybrid neural network is based on the multiple neural network training, each training uses the last final weight value and threshold. Hybrid neural network contains 5-8 BP networks could achieve higher accuracy generally.

Parameter description:

\( y(x_i) \) is the structural response of the object model, including finite element simulation or functions; \( f(x_i) \) is the structural response of the approximate model (HDMR-hybrid network);

(1) Assume that the total dimension of the random variables is \( n \) and the precision is \( \theta \), the structural response calculated by the object model is denoted by \( y(x_i) \). The central point of the \( n \)-dimensional variables in the domain is \( d=[d_1, d_2…d_n] \), the response of the structure at the point \( d \) is calculated by the object model and represented by \( y_0, f_0=y_0 \).

(2) Construct the uncoupled term \( f_i(x_i) \) of the approximation model in the interval of each dimension domain of the \( n \)-dimensional variables. Take the \( i \)th dimension variable as an example, two sample points are constructed at each end of the domain, the response values of the structure at the sample points are calculated by object model and denoted by \( y_{ij}(x_{il}, d^i) = y([d_1, d_2,⋯, x_{ils}, ⋯, d_n]) \) and \( y_{ij}(x_{ir}, d^i) = y([d_1, d_2,⋯, x_{irs}, ⋯, d_n]) \), then \( f_{ij}(x_i) = y_{ij}(x_{il}, d^i) - f_0 \), \( f_{ir}(x_i) = y_{ij}(x_{ir}, d^i) - f_0 \). The center point and the two end points are as input variables, the structural responses is calculated by object model are as output variables, neural network simulate \( f_i(x_i) \), the first simulation is expressed as \( f_{ij}(x_i) \).

(3) To determine whether the neural network established by the two endpoints and the center point satisfies the accuracy requirements. Take the point \( x_{iln} \) between the upper end point and the center point, the point \( x_{ln} \) between the lower end point and the center point at \( i \)-dimensional variable respectively, the responses of the structure at points \( x_{iln} \) and \( x_{ln} \) are calculated by object model. The center point, the upper and lower endpoints, \( x_{iln} \) and \( x_{ln} \) as input variables, the structural responses by object model are as output variables, the neural network is constructed to simulate \( f_i(x_i) \) again, the second simulation is \( f_{2i}(x_i) \). Calculate \( f_{2i}(x_{iln})-f_{ij}(x_{iln}) \) and \( f_{2i}(x_{ln})-f_{ij}(x_{ln}) \), if \( f_{2i}(x_{iln})-f_{ij}(x_{iln}) \leq \theta \) and \( f_{2i}(x_{ln})-f_{ij}(x_{ln}) \leq \theta \), then \( f_{ij}(x_i) \) meet the accuracy requirement, \( f_{ij}(x_i) \) could simulate \( f_i(x_i) \); if \( f_{2i}(x_{iln})-f_{ij}(x_{iln}) > \theta \) or \( f_{2i}(x_{ln})-f_{ij}(x_{ln}) > \theta \), the neural network need to be constructed by re-sampling points until \( f_{ij}(x_i) \) to meet the accuracy requirement.

(4) Repeat steps 2 and 3 to construct all uncoupled terms.

(5) Determine whether \( f_{ij}(x_i, x_j) \) exists, take the \( i \)th and \( j \)th dimension variables as the example,
construct a new sample point \(x_{ij} = [d_1, d_2, \ldots, x_i, d_i, \ldots, d_n]\). Comparing the structural response value of \(x_{ij}\) in the object model and the neural network calculated value \(f(x_{ij}) = f_u + f_y(x_i) + f_y(x_j)\), if \(y_{ij} - f(x_{ij}) \leq \theta\), then the coupling terms of the \(i^{th}\) and \(j^{th}\) variables have no effect on the structural response; if \(y_{ij} - f(x_{ij}) > \theta\), then \(f_{ij}(x_i, x_j)\) need to be constructed, the method refers to steps 2 and 3.

(6) The high-dimensional coupling terms are constructed until the approximation model satisfies the precision requirement.

It is not necessary to judge the nonlinear order of each dimension approximation model for the network structure constructed by the neural network is nonlinear. It is only to determine whether the number of sample points meet the precision requirement. The first-order coupled term can satisfy the accuracy of the approximation model, in general the higher-order coupling term is not needed except to the high-order nonlinear function.

4. Semi-rigid connection approximate model

This beam column joint is modeling by finite element analysis, the types of elements including SOLID95, TARGE170, CONTA174 and PRETS179. Each component of the extended end plate connection was modeled by solid element SOLID95, all the interfaces of the contact including the interfaces between the extended end plate and the column flange, the bolt and the end plate. The bolt and column flange were modeled by TARGE170 and CONTA174 elements. The pretension force of bolts was modeled by PRETS179 elements. Based on the available test data, detailed geometry of the specimen are shown in Table 2, the average characteristics of the connection are set out in Table 1, where \(E\) is the elastic modulus, \(E_{st}\) is the strain hardening modulus, \(f_y\) is yield strength, \(f_u\) is the tensile stress, \(\varepsilon_e\) is the elastic strain, \(\varepsilon_{st}\) is the strain of the initial hardening point, \(\varepsilon_{uni}\) is the uniform strain, \(\varepsilon_u\) is the ultimate strain.

![Figure. 1 Geometry of the specimen](image)

| components | \(E\) (MPa) | \(E_{st}\) (MPa) | \(f_y\) (MPa) | \(f_u\) (MPa) | \(\varepsilon_{st}\) | \(\varepsilon_{uni}\) | \(\varepsilon_u\) |
|------------|------------|----------------|--------------|--------------|----------------|----------------|------------|
| Plate and column | 209856 | 2264 | 340.12 | 480.12 | 0.015 | 0.224 | 0.361 |
| Beam web | 208332 | 1856 | 299.12 | 446.25 | 0.016 | 0.235 | 0.464 |
| Beam flange | 209496 | 1933 | 316.24 | 462.28 | 0.016 | 0.235 | 0.299 |
| Bolt | 223166 | - | 857.33 | 913.78 | - | - | 0.272 |
### Table 2: Actual geometry of the connection

| Column (mm) | Beam (mm) |
|-------------|-----------|
| h           | h         |
| b           | b         |
| $t_t$       | $t_f$     |
| $w$         | $w$       |
| $H_{up}$    | $H_{low}$ |
| 376.0       | 300.45    |
| 307.5       | 150.5     |
| 40.21       | 10.76     |
| 21.0        | 7.2       |
| 175.0       | 1200      |
| 219.0       | 1002.5    |

### Table 3: Analysis results of the approximate model for example 4

| Statistics | $R^2$ | $R_{AAE}$ | $R_{MAE}$ | Samples |
|------------|-------|-----------|-----------|---------|
| ordinary network | 0.8275 | 0.3941   | 0.8179    | 50      |
| HDMR-hybrid   | 0.9521 | 0.1021   | 0.5312    | 50      |

In this section, the sub-term of the model with multi-parameter decoupling is represented by the hybrid network, the accuracy of the approximation model is evaluated from the global and local evaluation indexes $R^2$, $R_{AAE}$ and $R_{MAE}$, respectively. The ordinary network and HDMR-hybrid network are used to simulate the joint respectively, the number of the sample points to construct the approximate model is 50. The comparison results are shown in table 3.

$$
R^2 = 1 - \frac{\sum_{i=1}^{m} [y(a_i) - f(a_i)]^2}{\sum_{i=1}^{m} [y(a_i) - \bar{y}(a_i)]^2} \quad (5)
$$

Where $f(a_i)$ is the output response of the HDMR-hybrid network, $y(a_i)$ is the true response of the structure, $\bar{y}(a_i)$ is the average of the $m$ real responses. If the value of $R^2$ is closer to 1, it shows that the approximation model is more accurate for the structure simulation.

$$
R_{AAE} = 1 - \frac{\sum_{i=1}^{m} |y(a_i) - f(a_i)|}{m \times STD} \quad (6)
$$

$STD$ represents the standard deviation of the true response of the structures, $m$ is the number of sample points. The smaller the value of $R_{AAE}$, the more accurate the approximation model is.

$$
R_{MAE} = \frac{\max\{|y(a_i) - f(a_i)|, \ldots, |y(a_i) - f(a_i)|\}}{STD} \quad (7)
$$

$R_{MAE}$ reflects the maximum error between the approximation model and the solid model in a local region. The smaller the $R_{MAE}$ value is, the higher the accuracy of the approximation model is. $R^2$ and $R_{AAE}$ are the global evaluations, $R_{MAE}$ is the local evaluation of the approximation model.

### 5. Conclusion

In this paper, the approximation model method is different from the general high-dimensional model representation method or neural network method, but the sub-items of the high-dimensional model are simulated by the neural network, and then these are integrated into a multi-level and multi-network structure approximation model. From the results of semi-rigid joint analysis, the accuracy of this HDMR-hybrid is much higher than general neural network. This approximation model has the ability of self-learning adaptation by neural network and the advantages of HDMR model to simulate high-dimensional coupling terms. If the finite element simulation is carried out for $10^6$ times and its operation time is as long as several months, it is not realized. Therefore, only 50000 Monte Carlo simulations are carried out as the exact solution. Monte Carlo method of finite element simulation takes about 10 days, the time of HDMR-hybrid including the training network and the simulation is...
about 1.7 hours. The computational efficiency is significantly improved.

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