Dynamic Modeling of Cascading Failure in Power Systems

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Abstract—The modeling of cascading failure in power systems is difficult because of the many different mechanisms involved; no single model captures all of these mechanisms. Understanding the relative importance of these different mechanisms is an important step in choosing which mechanisms need to be modeled for particular types of cascading failure analysis. This work presents a dynamic simulation model of both power networks and protection systems, which can simulate a wider variety of cascading outage mechanisms, relative to existing quasi-steady state (QSS) models. The model allows one to test the impact of different load models and protections on cascading outage sizes. This paper describes each module of the developed dynamic model and demonstrates how different mechanisms interact. In order to test the model we simulated a batch of randomly selected N−2 contingencies for several different static load configurations, and found that the distribution of blackout sizes and event lengths from the proposed dynamic simulator correlates well with historical trends. The results also show that load models have significant impacts on the cascading risks. This dynamic model was also compared against a QSS model based on the dc power flow approximations; we find that the two models largely agree, but produce substantially different results for later stages of cascading.

Index Terms—Cascading outages, cascading failures, power system dynamic simulation, differential algebraic equation, power system modeling, power system protection.

I. INTRODUCTION

THE vital significance of studying cascading outages has been recognized in both the power industry and academia [1]–[3]. However, since electrical power networks are very large and complex systems [4], understanding the many mechanisms by which cascading outages propagate is challenging. This paper presents the design of and results from a new non-linear dynamic model of cascading failure in power systems (the Cascading Outage Simulator with Multiprocess Integration Capabilities or COSMIC), which can be used to study a wide variety of different mechanisms of cascading outages.

A variety of cascading failure modeling approaches have been reported in the research literature, many of which are reviewed in [1]–[3]. Several have used quasi-steady state (QSS) dc power flow models [5]–[7], which are numerically robust and can describe cascading overloads. However, they do not capture non-linear mechanisms like voltage collapse or dynamic instability. QSS ac power flow models have been used to model cascading failures in [8]–[11], but these models still require difficult assumptions to model machine dynamics and to deal with non-convergent power flows. Some have proposed models that combine the dc approximations and dynamic models [12], allowing for more accurate modeling of under-voltage and under-frequency load shedding. These methods increase the modeling fidelity over pure dc models but still neglect voltage collapse. Others developed statistical models that use data from simulations [13], [14] or historical cascades [15] to represent the general features of cascading. Statistical models are useful, but cannot replace detailed simulations to understand particular cascading mechanisms in detail. There are also topological models [16]–[19] which have been applied to the identification of vulnerable/critical elements, however, without detailed power grid information, the results that they yield differ greatly and could result in misleading conclusions about the grid vulnerability [20].

Some dynamic models [21], [22] and numerical techniques [23], [24] study the mid-long-term dynamics of power system behavior, and show that mid/long term stability is an important part of cascading outage mechanisms. However, concurrent modeling of power system dynamics and discrete protection events – such as line tripping by over-current, distance and temperature relays, under-voltage and under-frequency load shedding – is challenging and not considered in most existing models. In [25] the authors describe an initial approach using a system of differential-algebraic equations with an additional set of discrete equations to dynamically model cascading failures. The paper describes the details of and results from a new non-linear dynamic model of cascading failure in power systems, which we call “Cascading Outage Simulator with Multiprocess Integration Capabilities” (COSMIC). In COSMIC, dynamic components, such as rotating machines, exciters, and governors, are modeled using differential equations. The associated power flows are represented using non-linear power flow equations. Load voltage responses are explicitly represented, and discrete changes (e.g., component failures, load shedding) are described by a set of equations that indicate the proximity to thresholds that trigger discrete changes. Given dynamic data for a power system and a set of exogenous disturbances that may trigger a cascade, COSMIC uses a
recursive process to compute the impact of the triggering event by solving the differential-algebraic equations (DAEs) while monitoring for discrete events, including events that subdivide the network into islands.

The remainder of this paper proceeds as follows: Section II introduces the components of the model mathematically and describes how different modules interact. In Section III, we present results from several experimental validation studies. Finally, Section IV presents our conclusions from this study.

II. HYBRID SYSTEM MODELING IN COSMIC

A. Hybrid differential-algebraic formulation

Dynamic power networks are typically modeled as sets of DAEs [26]. If one also considers the dynamics resulting from discrete changes such as those caused by protective relays, an additional set of discrete equations is added, which results in a hybrid DAE system [27]. Let us assume that the state of the power system at time \( t \) can be defined by three vectors: \( x(t) \), \( y(t) \), and \( z(t) \), where:

- \( x \) is a vector of continuous state variables that change with time according to a set of differential equations

\[
\frac{dx}{dt} = f(t, x(t), y(t), z(t)) \tag{1}
\]

- \( y \) is a vector of continuous state variables that have pure algebraic relationships to other variables in the system:

\[
g(t, x(t), y(t), z(t)) = 0 \tag{2}
\]

- \( z \) is a vector of state variables that can only take integer states (\( z_i \in \{0, 1\} \))

\[
h(t, x(t), y(t), z(t)) < 0 \tag{3}
\]

When constraint \( h_i(...) < 0 \) fails, an associated counter function \( d_i \) (see II-B) activates. Each \( z_i \) changes state if \( d_i \) reaches its limit.

The set of differential equations (1) represent the machine dynamics (and/or load dynamics if dynamic load models are included). In COSMIC the differential equations include a third order machine model and somewhat simplified governor and exciter models in order to improve computational efficiency without compromising the fundamental functions of those components. In particular, the governor is rate and rail limited to model the practical constraints of generator power control systems. The governor model incorporates both droop control and integral control, which is important to mid/long term dynamic stability modeling, especially in isolated systems [26].

The algebraic constraints (2) encapsulate the standard ac power flow equations. In this study we implemented both polar and rectangular power flow formulations. Load models are an important part of the algebraic equations, which are particularly critical components of cascading failure simulation because i) they need to represent the aggregated dynamics of many complicated devices and ii) they can dramatically change system dynamics. The baseline load model in COSMIC is a static model, which can be configured as constant power (P), constant current (I), constant impedance (Z), exponential (E), or any combination thereof (ZIPE) [28].

As Fig. 1 illustrates, load models can have a dramatic impact on algebraic convergence. Constant power loads are particularly difficult to model for the off-nominal condition. Numerical failures are much less common with constant I or Z loads, but are not accurate representations of many loads. This motivated us to include the exponential component in COSMIC.

During cascading failures, power systems undergo many discrete changes that are caused by exogenous events (e.g., manual operations, weather) and endogenous events (e.g., automatic protective relay actions). The discrete event(s) will consequently change algebraic equations and the systems dynamic response, which may result in cascading failures, system islanding, and large blackouts. In COSMIC, the endogenous responses of a power network to stresses are represented by (3). These discrete responses are described in detail in II-B and II-C.

B. Relay modeling

Major disturbances cause system oscillations as the system seeks a new equilibrium. These oscillations maybe naturally die out due to the interactions of system inertia, damping, and exciter and governor controls. In order to ensure that relays do not trip due to brief transient state changes, time-delays are added to each protective relay in COSMIC.

We implement in this model two types of time-delayed triggering algorithms: fixed-time delay and time-inverse delay. These two delay algorithms are modeled by a counter function, \( d \), which is triggered by (3). The fixed-time delay triggering activates its counter/timer as soon as the monitored signal exceeds its threshold. If the signal remains beyond the threshold, this timer will continue to count down from a preset value until it runs out then the associated relay takes actions. Similarly, the timer will recover if the signal is within the threshold and will max out at the preset value. For the time-inverse delay algorithm, instead of counting for the increment of time beyond (or within) the threshold, we evaluate the area over (or under) the threshold by integration (based on Euler’s rule).
Five types of protective relays are modeled in COSMIC: over-current (OC) relays, distance (DIST) relays, and temperature (TEMP) relays for transmission line protection; as well as under-voltage load shedding (UVLS) and under-frequency load shedding (UFLS) relays for stress mitigation. OC relays monitor the instantaneous current flow along each branch. DIST relays represent a Zone 1 relay that monitors the apparent admittance of the transmission line. TEMP relays monitor the line temperature, which is obtained from a first order differential equation

\[ T_i = r_i F_i^2 - k_i T_i \]  

where \( T_i \) is the temperature difference relative to the ambient temperature (20 °C) for line \( i \), \( F_i \) is the current flow of line \( i \), \( r_i \) and \( k_i \) are the are heating and time constants for line \( i \) [12]. \( r_i \) and \( k_i \) are chosen so that line \( i \)'s temperature reaches 75 °C (ACSR conductors) if current flow hits the rate-A limit, and its TEMP relay triggers in 60 seconds when current flow jumps from rate-A to rate-C. The threshold for rate-A limit, and its TEMP relay triggers in 60 seconds when

\[ P \]

are lower than the specified thresholds, the UVLS relay or UFLS relay will shed a 25% load. While it would have been possible to integrate the temperature relays into the trapezoidal integration used for the other differential equations, these variables are much slower than the other differential variables in \( x \). Instead, we computed \( T_i \) outside of the primary integration system using Euler’s rule.

When voltage magnitude or frequency signals at load Bus \( i \) are lower than the specified thresholds, the UVLS relay or UFLS relay will shed a 25% (default setting) of the initial \( P_{d,i} \) to avoid the onset of voltage instability and reduce system stress. In order to monitor frequency at each load bus, Dijkstra’s algorithm [29] and electrical distances [30] were used to find the generator (and thus frequency from \( x \)) that is most proximate to each load bus. Both the UVLS and UFLS relays used a fixed-time delay of 0.5 seconds.

C. Solving the hybrid DAE

Because of its numerical stability advantages, COSMIC uses the trapezoidal rule [31] to simultaneously integrate and solve the differential and algebraic equations.

Whereas many of the common tools in the literature [32], [33] use a fixed time step-size, COSMIC implements a variable time step-size in order to trade-off between the diverse timescales of the dynamics that we implement. We select small step sizes during transition periods that have high deviation or oscillations in order to keep the numerical within tolerance; the step sizes increase as the oscillations dampen toward steady-state values.

When a discrete event occurs at \( t_d \), the complete representation of the system is provided in the following equations:

\[ 0 = x + \frac{t_d - t}{2} (f(t) + f(t_d, x_h, y_h, z)) \]  

\[ 0 = g(t_d, x_+, y_+, z) \]  

\[ 0 > h(t_d, x_+, y_+) \]  

\[ 0 = d(t_d, x_+, y_+) \]  

where, \( t \) is the previous time point, and \( d \) is the counter function mentioned previously. Because of the adaptive step-size, COSMIC retains \( t_d \) from \( t = t + \Delta t_d \), in which \( \Delta t_d \) is found by linear interpolation.

Every time a discrete event happens (\( h < 0 \) and \( d = 0 \)), COSMIC stops solving the DAE for the previous network configuration, processes the discrete event(s), then resumes the DAE solver using the updated initial condition. COSMIC deals with the separation of a power network into sub-networks (unintentional islanding) using a recursive process, which is illustrated in Fig. 2. If islanding results from a discrete event, the present hybrid DAE separates into two sets of DAEs, which can be represented as \( f_1, g_1, h_1, d_1 \), and \( f_2, g_2, h_2, d_2 \). COSMIC treats the two sub-networks the same way as the original one, integrates and solves these two DAE systems in parallel, and synchronizes two result sets in the end.

D. Validation

To validate COSMIC, we compared the dynamic response in COSMIC against commercial software – PowerWorld [33] – using the classic 9-bus test case [34]. From a random contingency simulation, the Mean Absolute Error (MAE) between the results produced by COSMIC and PowerWorld was within 0.011%. Since COSMIC adopted simplified exciter and governor models that are not included in many commercial packages, several of the time constants were set to zero or very close to zero in order to obtain agreement between the two models.

III. EXPERIMENTS AND RESULTS

In this section, we present experimental results from validation tests on three test systems: the 9-bus system [34], the 39-bus system, and the 2383-bus system [35], which is an equivalenced system based on the year 2000 winter snapshot for the Polish network.
TABLE I
THE NUMBER OF LINEAR SOLVES FOR THE 39-BUS CASE.

|      | 0-1 MW | 1-200 MW | 200-500 MW | 500-1000 MW | 1000-3000 MW | 3000-6000 MW |
|------|--------|----------|------------|-------------|--------------|--------------|
| Polar | 3084   | 6839     | 10523      | 5994        | 41067        | 10278        |
| Rec.  | 3035   | 6724     | 10324      | 5846        | 4052         | 9501         |
| Tests | 176    | 38       | 14         | 32          | 4            | 2            |
| % Dec. | 1.59%  | 1.60%    | 1.89%      | 2.46%       | 1.33%        | 7.6%         |

1: The non-zero rates of the Jacobian matrices for polar and rectangular forms are 0.0376% and 0.0383% respectively; 2: the number of tests; 3: The percentage decrease in the number of linear solves, rectangular vs. polar.

TABLE II
COMPARISON OF LINEAR SOLVES FOR THE 2383-BUS CASE.

|      | 0-1 MW | 1-2000 MW |
|------|--------|-----------|
| Polar | 867.77 | 692.81    |
| Rec.  | 867.84 | 693.10    |
| Tests | 2494   | 556       |
| % Dec. | -0.009% | -0.04%   |

4: The non-zero rates of the Jacobian matrices for polar and rectangular formulations are 0.000856% and 0.000872% respectively.

A. Polar formulation vs. rectangular formulation in computational efficiency

COSMIC includes both polar and rectangular power flow formulations. In order to compare the computational efficiency of the two formulations, we conducted a number of \( N - 1 \) and \( N - 2 \) experiments using two different cases, the 39-bus and 2383-bus systems. The amount of time that a simulation requires and the number of linear solves \( (Ax = b) \) are two measures commonly used to evaluate the computational efficiency of a model. Compared to the first metric, the number of linear solves describes computational speed independently of specific computing hardware, and it is adopted in this study.

Tables I and II compare the two formulations with respect to the number of linear solves. For the 39-bus case, 45 \( N - 1 \) experiments and 222 randomly selected \( N - 2 \) experiments were conducted, and each of them finished at 50 seconds and lost the same amount of power demand. As shown in Table II, the performances of the two methods were similar in terms of number of linear solves; however, the rectangular formulation required fewer linear solves and shows various improvements (e.g., positive decrease rectangular vs. polar) for different demand losses.

For the 2383-bus test case, we simulated 2494 \( N - 1 \) and 556 \( N - 2 \) contingencies; Table II shows the results. There was no significant improvement for the rectangular formulation over the polar formulation, and the number of linear solves that resulted from both forms were almost identical.

One can also notice that solving the 2383-bus case required fewer linear solves than for the 39-bus case. This suggests that some branch outages have a higher impact on a smaller network and cause more dynamic oscillations than on a larger network such as the 2383-bus case.

B. Relay event illustration

To depict the functionality of how protective relays integrate with COSMIC’s time delay features, we implemented the following example using the 9-bus system. The initial event was a single-line outage from Bus 6 to Bus 9 at \( t = 10 \) seconds. The count-down timer of the DIST relay for branch 5-7 was activated with a \( t_{\text{preset-delay}} = 0.5 \) seconds. As shown in Fig. 3, the system underwent a transient swing following the one-line outage. Right after 0.5 seconds, \( t_{\text{delay}} \) ran out \( (P_1) \) and branch 5-7 was tripped by its DIST relay, which resulted in two isolated islands. Meanwhile, the thresholds for UVLS relays were set to 0.92 pu. Note that the magenta voltage trace violated this voltage limit at about \( t = 10.2 \) seconds \( (P_2) \). Because this trace continued under the limit after that, its UVLS timer counted down from \( t_{\text{preset-delay}} = 0.5 \) seconds till \( t = 10.7 \) seconds \( (P_3) \), where its UVLS relay took action and shed 25% of the initial load at this bus. The adjacent yellow trace illustrates that the UVLS relay timer was activated as well but with a small lag. This UVLS relay never got triggered because before its \( t_{\text{delay}} \) emptied out the load shedding at \( P_2 \) put this yellow trace back upon threshold, and its \( t_{\text{delay}} \) was restored to 0.5 seconds.

C. Cascading outage examples using the 39-bus and the 2383-bus power systems

The following experiment demonstrates a cascading outage example using the IEEE 39-bus case (see Table III for a summary of the sequential events). The system suffered a strong dynamic oscillation after the initial two exogenous events (branches 2-25 and 5-6). After approximately 55 seconds the first OC relay at branch 4-5 triggered. Because the monitored current kept up-crossing and down-crossing its limit, it delayed the relay triggering based on the time delay algorithms for the protection devices. Load shedding at two buses (Bus 7 and Bus 8) occurred around \( t = 55.06 \) seconds, then another two branches (10-13 and 13-14) shut down after OC relay trips at \( t = 55.28 \) seconds. These events separated the system into two islands. At \( t = 55.78 \) seconds, two branches (3-4 and 17-18) were taken off the grid and this resulted in another island. The system eventually ended up with three isolated networks. However, one of them was not algebraically solvable due to a dramatic power imbalance, and it was declared as a blackout area.
Fig. 4. The sequence of events for an illustrative cascading failure in the 2383-bus network. Numbers show the locations and sequence of line outages. Number 0 with black highlights denotes the two initial events. Other sequential numbers indicate the rest of the branch outages. In this example, 24 branches are off-line and causes a small island (the blue colored network) in the end. The dots with additional red square indicate buses where load shedding happens.

Fig. 5. The top panel shows the timeline of all branch outage events listed in Fig. 4, and the lower panel zooms in the associated load-shedding events.

Table III

| No. | Time (sec) | Events |
|-----|------------|--------|
| 1   | 3          | Initial events: branches 2-25 and 5-6 trip |
| 2   | 54.78      | Branch 6-7 is tripped by OC relay |
| 3   | 55.06      | 58.45 MW load shedding at Bus 7 |
| 4   | 55.07      | 130.50 MW load shedding at Bus 8 |
| 5   | 55.28      | Branch 4-14 is tripped by OC relay |
| 6   | 55.28      | Branch 10-13 is tripped by OC relay |
| 7   | 55.28      | Branch 13-14 is tripped by OC relay |
| 8   | 55.28      | 1st islanding event |
| 9   | 55.78      | Branch 3-4 is tripped by OC relay |
| 10  | 55.78      | Branch 17-18 is tripped by OC relay |
| 11  | 55.78      | 2nd islanding event |

Fig. 6 shows the Complementary Cumulative Distribution Function (CCDF) of demand losses for these four groups failed relatively slowly, however, it speeded up as the number of failures increased. Eventually the system condition was substantially compromised, which caused fast collapse and the majority of the branch outages as well as the load shedding events (see lower panel in Fig. 5).

D. N – 2 contingency analysis using the 2383-bus case

Power systems are operated to ensure the N – 1 security criterion so that any single component failure will not cause subsequent contingencies [36]. The modified 2383-bus system that we are studying in this paper satisfies this criterion for transmission line outages. Thus, we assume here that branch outages capture a wide variety of exogenous contingencies that initiate cascades, for example a transformer tripping due to a generator failure.

The experiment implemented here included four groups of 1200 randomly selected N – 2 contingencies for the 2383-bus system. We measured the size of the resulting cascades using the number of relay events and the amount of demand lost. Each group had a different static load configuration. The load configuration for the first group was 100% constant Z load; the second group used 100% E load; the third had 100% constant P load; and the fourth one included 25% of each portion in the ZIPE model. We set TEMP, DIST, UVLS, and UFLS relays active in this experiment and deactivated OC relay due to its overlapping/similar function with TEMP relay.

Fig. 6 shows the Complementary Cumulative Distribution Function (CCDF) of demand losses for these four groups.
of simulations. The CCDF plots of demand losses exhibit a heavy-tailed blackout size distribution, which are typically found in both historical blackout data and cascading failure models [37]. The magenta trace indicates constant Z load, and shows the best performance—in terms of the average power loss and the probability of large blackout—within this set of 1200 random \( N - 2 \) contingencies (listed in Table IV). In contrast, the blue trace (constant E load) reveals the highest risk of large size blackouts (\( > 1000 \text{ MW} \)). The constant P load has a similar trend as the constant E load, due to their similar stiff characteristics; however, the constant E load with this particular exponent, 0.08, demonstrates a negative effect on the loss of load. The one with 25\% Z, 25\% P and 25\% of E performs in the middle of constant P load and constant Z load.

As can be seen in Table IV, the probabilities of large demand losses varies from 2.5\% to 3.5\% for those four load configurations. These results show that load models play an important role in dynamic simulation and may increase the frequency of non-convergence if they are not properly modeled.

### TABLE IV

**The average demand loss, average branch outages and the probabilities of loss of the whole system for different load models.**

| Load Model | Avg. Loss | Avg. BO\(^*\) | Prob. of Largest Loss |
|------------|-----------|---------------|----------------------|
| I\(100\)  | 644.19 MW | 0.2602        | 0.025                |
| P\(100\)  | 889.02 MW | 0.1800        | 0.018                |
| Z\(25\)P\(25\)E\(25\) | 807.11 MW | 0.2042        | 0.032                |

\(I: \text{BO} — \text{branch outages}\).

Fig. 7 shows the CCDF plots of total event length, including all event types, such as branch outages caused by TEMP and DIST relays, and load shedding events by UVLS and UFLS relays. Fig. 8 shows the CCDF of the branch outage lengths only. We can see from these two figures that the distributions of constant P and constant E loads have a comparable pattern, and they are in general less likely to have the same amount of branch outages, relative to the other two configurations.

### E. Comparison with a dc cascading outage simulator

A number of authors have implemented quasi-steady state (QSS) models using the dc power flow equations to investigate cascading outages [5]–[7]. We conducted two experiments to compare COSMIC with the dc QSS model described in [5] with respect to the overall probabilities of demand losses and the extent to which the patterns of cascading from the two models agree.

1) **The probabilities of demand losses:** For the first experiment, we computed the CCDF of demand losses in both COSMIC (with the constant impedance load model) and the dc model using the same 1200 branch outage pairs from III-D. From Fig. 9 one can learn that the probability of demand losses in the dc simulator is lower than that of COSMIC for the same amount of demand losses. In particular, the largest demand loss in dc simulator is much smaller than in COSMIC (2639 MW vs. 24602 MW, with probabilities 0.08\% vs. 2.5\%). This large difference between them is not surprising because the dc model is much more stable and does not run into problems of numerical non-convergence. Also, the protection algorithms differ somewhat between the two models. In addition, some of the contingencies do produce large blackouts in the dc simulator, which causes the fat tail that can be seen in Fig. 9.

Numerical failures in solving the DAE system greatly
The second comparison experiment was to study the patterns of cascading between these two models. This comparison provides additional insight into the impact of dynamics on cascade propagation patterns. In order to do so, we compared the sets of transmission lines that failed using the “Path Agreement Measure” introduced in (9) [12]. The relative agreement of cascade paths, \( R(m_1, m_2) \), is defined as follows. If models \( m_1 \) and \( m_2 \) are both subjected to the same set of exogenous contingencies: \( C = \{c_1, c_2, c_3, \ldots \} \), \( R(m_1, m_2) \) measures the average agreement in the set of dependent events that result from each contingency in each model. If contingency \( c_i \) results in the set \( A_i \) of dependent branch failures in model \( m_1 \) and the set \( B_i \) of dependent branch failures in model \( m_2 \), \( R(m_1, m_2) \) is defined as:

\[
R(m_1, m_2) = \frac{1}{C} \sum_{\forall i \in C} |A_i \cap B_i|/|A_i \cup B_i|
\]

The experiment measured \( R(m_1, m_2) \) between COSMIC (with a \( Z_{25}I_{25}P_{25}E_{25} \) load configuration) and the dc simulator for 336 critical branch outage pairs. These branch outage pairs were selected by using the random chemistry algorithm proposed in [5] because they are more likely to cause cascading failures. Table V shows that the average \( R \) between the two models for the whole set of sequences is 0.1948, which is relatively low. This indicates that there are substantial differences between cascade paths in the two models. Part of the reason is that the dc model tends to produce longer cascades and consequently increase the denominator in (9). In order to control this, we computed the \( R \) only for the first 10 branch outage events. The average \( R \) increases to 0.3487, and some of cascading paths show a perfect match (\( R = 1 \)). This shows how the cascading paths resulting from COSMIC and the dc simulator have a better agreement in the early stages, when non-linear dynamics are less pronounced.

### IV. Conclusions

This paper describes a method for and results from simulating cascading failures in power systems using full non-linear dynamic models. The new model, COSMIC, represents a power system as a set of hybrid discrete/continuous differential algebraic equations, simultaneously simulating protection systems and machine dynamics. Several experiments illustrated the various components of COSMIC and provided important general insights regarding the modeling of cascading failure in power systems. By simulating 1200 randomly chosen \( N - 2 \) contingencies for a 2383-bus test case, we found that COSMIC produces heavy-tailed blackout size distributions, which are typically found in both historical blackout data and cascading failure models [37]. However, the relative frequency of very large events may be exaggerated in dynamic model due to numerical non-convergence (about 3% of cases). More importantly, the blackout size results show that load models can substantially impact cascade sizes — cases that used constant impedance loads showed consistently smaller blackouts, relative to constant current, power or exponential models. In addition, the contingency simulation results from COSMIC were compared to corresponding simulations from a dc power flow based quasi-steady-state cascading failure simulator, using a new metric. The two models largely agreed for the initial periods of cascading (for about 10 events), then diverged for later stages when dynamic phenomena drive the sequence of events.

Together these results illustrated that detailed dynamic models of cascading failure can be useful in understanding the relative importance of various features of these models. The particular model used in this paper, COSMIC, is likely too slow for many large-scale statistical analyses, but comparing detailed models to simpler ones can be helpful in understanding the relative importance of various modeling assumptions that are necessary to understand complicated phenomena such as cascading.
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