A methodology for the performance evaluation of low-cost accelerometer and magnetometer sensors in geomatics applications

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ABSTRACT
This paper presents a methodology and its software implementation for the performance evaluation of low-cost accelerometer and magnetometer sensors for use in geomatics applications. A known mathematical calibration model has been adopted. The method was completed with statistical methodologies for adjusting observations and has been extended to calculate accuracies for the attitude, heading, and tilt angles estimation that are of interest to geomatics applications. The evaluation method consists of two stages. First, the evaluation method reviews the total magnitude of acceleration or the strength of the magnetic field. Second, the evaluation is more detailed and concerns the determination of mathematical parameters that describe both accelerometer and magnetometer working model. A software tool that implements the evaluation model has been developed and is applied both in accelerometer and magnetometer measurement data-sets acquired from a low-cost sensor system.

1. Introduction
The micro-electro-mechanical systems (MEMS) sensor technology is currently adopted massively in all kinds of devices and is available at an affordable cost. Examples include mobile devices such as smartphones and tablets, which are equipped with the necessary sensors that have the potential to be used in geomatics applications.

Geomatics applications can use inertial and magnetometer sensor systems in order to estimate the attitude and the heading of a body frame or of an image sensor (Patonis 2012). The basic sensors that are required to determine the system’s orientation in space are accelerometers and magnetometers, while gyroscope systems are used as supplementary, mainly in kinematic applications, to normalize and stabilize the orientation results.

Mobile devices that contain low-cost inertial, magnetometer, and image sensors, among other applications, can be used in augmented reality (AR) applications that project digital georeferenced elements on the screen of a mobile device along with the real-time camera feed. An example of a geomatics AR application is a close range photogrammetry survey tool (Patonis 2016) that includes a utility for the direct exterior orientation estimation of digital photos taken by the mobile device’s camera and a stake out utility in order to point out the position of georeferenced elements in the real world.

A typical commercial low-cost sensor system can achieve accuracies up to 0.5 degrees in the attitude angles and 1 degree in the heading angle as to the North. This order of accuracy is possible to be exploited by close range geomatics applications that do not require high accuracy or can be used for providing approximate values to process more complex methodologies (Patias et al. 2016).

The question is whether the low-cost technology, which is available through mobile devices, can be used in applications that have specific accuracy requirements. The sensor models and the specifications of the low-cost sensors or else called “automotive grade” sensors that are included in smartphones and tablets are usually unknown (Aicardi et al. 2014; Dabove, Ghinamo, and Lingua 2015). These sensors show good repeatability, but have errors that prevent their direct usage in applications that require steady performance and accuracy. The study of the literature shows that these mechanisms can work with constant accuracy and reliability, provided that they will undergo calibration procedures involving high precision instruments (VectorNav 2016). In later reports, researches are directed to calibration methodologies that do not rely on laboratory equipment (Fong, Ong, and Nee 2008; Frosio, Pedersini, and Borghese 2009; Skog and Händel 2006). The only way to ensure the sensors system efficiency, in achieving the required accuracy, is to perform an evaluation precision control.
Taking into account the aforementioned issues, a methodology and a software tool have been developed in order to evaluate the reliability of these sensors. The aim of the methodology is to evaluate accelerometer and magnetometer sensor systems in order to determine their operational condition, while concerning the stability and the accuracy of the results related to geomatics applications.

Regarding the structure of the paper: first, the background introductory of the methodology developed toward the evaluation of low-cost sensors; then, the description of the software development; followed by an example related to sensor’s evaluation and the accuracy estimates on the use of low-cost sensors in geomatics application.

2. Methodology for evaluation of low-cost sensors

One of the objective posed for the design of the evaluation methodology is that it could be applied without using any external laboratory hardware equipment and the procedure ought to be available anywhere, even in field conditions. Another reason for this restriction is that the integrated sensors in mobile devices are not feasible to be physical aligned to a precision dividing head (VectorNav 2016) used in laboratory calibration.

The performance of the MEMS sensors can be influenced mainly by temperature. In laboratory conditions, the calibration involves a thermal calibration stage, where all the calibration procedures are repeated at multiple temperatures inside an environmental test chamber (VectorNav 2016). In the current work, the aim is the evaluation of low-cost sensors concluded in devices which usually have no temperature sensors. For this reason, the temperature parameter was not taken into consideration in the evaluation mathematical model. When the evaluation procedure is used to correct the measurements from the sensors, then the procedure can be performed in the field just before the operation of the geomatics application. In that way, the best performance will be achieved bypassing the absence of the temperature sensor.

Both magnetometer and accelerometer sensors in a three orthogonal layout measure constant magnitudes, so the same general evaluation model can be used in both cases. An accelerometer system, in static condition, can measure the vector components of the gravity acceleration by measuring the force that the Earth’s gravitational field pull in to the reference mass of the accelerometer’s mechanism. The gravity acceleration vector for the same place can be considered to have a fixed value, regardless of the system orientation. On the other hand, the magnetometer is a type of sensor that measures the strength of the local magnetic field. The measured local magnetic field is the combination of both Earth’s magnetic field and the magnetic fields generated by nearby objects. A magnetometer system in a three orthogonal layout, with the contribution of the attitude angles, is capable to calculate the heading angle of one axis of the system toward the magnetic North, by measuring the intensity vector components of the magnetic field.

The evaluation methodology presented in this paper concerns two stages. The first stage checks the approximate performance of the system sensors by considering the diagram or the variation of the total magnitude vector measured in multiple positions. In the second and more detailed evaluation stage, the methodology was extended by utilizing statistical data analysis methods for adjusting observations and the law of covariance propagation to estimate the best possible accuracies that the evaluated sensor systems can achieve. The second stage includes the estimation of specific model parameters (such as scale coefficients, biases, axis orthogonality, and the estimations of the standard deviations of the tilts (Tuck 2007) and navigation angles (Patonis 2012)) that are calculated from the sensors measurements and are of great interest to geomatics applications. The detailed analysis of the particular sub stage of the accuracy estimations is given with an evaluation example in Section 5.

Mathematical models for correcting the measurements of an accelerometer system have been formed in works of Skog and Händel (2006), Fong, Ong, and Nee (2008), Frosio, Pedersini, and Borghese (2009). In our present work, a mathematical calibration model (successfully tested in accelerometer sensors by Frosio, Pedersini, and Borghese 2009) has been adopted. And for the first time, it was applied to data-sets acquired from a magnetometer sensor system. In a form of matrices, the mathematical model is:

\[ A = S(V - O) \] (1)

where \( A \) is a matrix with dimensions 3 × 1 containing the corrected values; \( V \) is a matrix with dimensions 3 × 1 containing the original measurements; \( S \) is a square symmetric matrix of coefficients with dimensions 3 × 3. In addition to the scale correction on each axle, described by the diagonal elements, it takes into account the interaction between axes of the sensors; \( O \) is a matrix with dimensions 3 × 1 containing the bias vector.

The analytical equations of the evaluation model are:

\[
\begin{align*}
A_{x1} &= S_{x1}(V_{x1} - O_x) + S_{y1}(V_{y1} - O_y) + S_{z1}(V_{z1} - O_z) \\
A_{y1} &= S_{x1}(V_{x1} - O_x) + S_{y1}(V_{y1} - O_y) + S_{z1}(V_{z1} - O_z) \\
A_{z1} &= S_{x1}(V_{x1} - O_x) + S_{y1}(V_{y1} - O_y) + S_{z1}(V_{z1} - O_z)
\end{align*}
\] (2)

where \( i = 1,...,N \) and \( N \) is the number of orientation positions of the axes of the system; \( x, y, \) and \( z \) are the axes of the system.

Totally, in the matrices \( S \) and \( O \), there are nine unknown parameters. Therefore, measurements, from at least nine triads of axes components in random orientations, are needed. More measurements are required in order to achieve reliable results by the statistical adjustment of Equation (2).
The measurements of MEMS sensors are influenced mainly by thermal and electronic noise, which is usually modeled as additive white Gaussian noise that follows the Gauss distribution (Grewal, Weill, and Andrews 2007). Because of the properties of the noise, the method of mixed equations (Dermanis and Fotiou 1992) can be chosen for estimating the unknown parameters and for the statistical evaluation of the results. This method minimizes the squares of observation errors. According to the methodology, \( N \) mixed equations are formed:

\[
u_i = A_i^x + A_i^y + A_i^z - c^2 = 0 \quad (3)
\]

where \( u \) is the mixed equation; \( c \) is a constant value that depends of the sensor type (for accelerometer, it is the gravity acceleration vector (1 g or 9.81 m/s²); for magnetometer, it is the total strength of the local magnetic field).

The linearized equations of the model in a form of matrices are:

\[
AX - BV + W = 0 \quad (4)
\]

where \( X \) is the matrix of unknowns parameters; \( V \) is the matrix of errors; \( A \) is a matrix resulting from the partial derivatives of the mixed equations with respect to the unknown variables; \( B \) is a matrix resulting from the partial derivatives of the mixed equations with respect to the observations; \( W \) is the error closing matrix of the mathematical model equations.

As a convergence criterion for the calibration adjustment, the stabilization (in subsequent iterations) of the variability of the variance estimation (given by the following relation) can be used:

\[
\sigma^2 = \frac{\hat{V}^TP\hat{V}}{f} \quad (5)
\]

where \( P \) is a weight matrix that is calculated as \( P = Q^{-1} \); the matrix \( Q \) has, as diagonal elements, the sampling variations of measurements per axis; \( f \) is the degrees of freedom.

3. Software development for sensor evaluation

For the implementation of the evaluation methodology, an algorithm and a software application have been developed. The programming language for the implementation is Visual Basic. The structure diagram of the application is presented in Figure 1.

The application receives the raw data samples from both accelerometer and magnetometer sensor systems as inputs, and executes independently, for each sensor type, the evaluation adjustment using the mixed equations method. The procedure is performed with a controlled repetitive way, where the final best estimates of the unknown parameters become initial values for the next iteration. The user monitors the progress of solving with the help of the application interface (Figure 2) in real-time and can evaluate the adjustment convergence, as well as the change in values of the unknown parameters and their standard deviations, in order to decide whether to resume the adjustment or whether the current solution is satisfactory. Specifically, when the change of the reference variability \( \sigma^2 \) or the individual change of the parameters estimation is limited to the last decimals, then the solution is stabilized and can be considered to be satisfactory. In a different case, more iterations must be performed.

When the solution is stabilized both for accelerometer and magnetometer systems, then the covariance low is applied to the appropriate mathematical equations in order to calculate the best estimations of pitch, roll, and heading angles per orientation position, and their corresponding deviations.

All the evaluation results mentioned above in addition with some intermediate results, such as the best estimations of the accelerations and the magnetic intensities along with their deviations, are printed in an external file, which is a detailed evaluation report for the examined sensor systems.

4. Sensor evaluation example

The evaluation methodology has been applied to the measurement data acquired from the sensor system of the low-cost inertial measurement unit, SparkFun 9DOF (Figure 3), which includes three types of sensors: three gyroscopes, three accelerometers, and three magnetometers in a three rectangular layout.

The specific low-cost inertial measurement unit works at the frequency of 100 Hz, except the group of the magnetometer sensors that operate at 20 Hz. The transmission of measurements from the sensors is carried out via a serial connection and a USB adaptor.

4.1. Low-cost accelerometer sensor evaluation

A first approximate evaluation of the accelerometer sensors system performance is the comparison of the total acceleration per position. In static condition, a calibrated accelerometer sensor system should provide a constant value independent of the position of the accelerometer sensors. In the case that a significant variation occurs and the target application requires steady accuracy the system must undergo calibration. In case that there is a small but still significant variation, a most sophisticated evaluation must take place.

In the specific example, for the accelerometer evaluation, 15 samples of acceleration components from different position orientations have been used. In Figure 4, it seems to be a noticeable variation in the measurement of the total acceleration in the different positions. Specifically, the standard deviation is 22.3 device units.
For further evaluation of the sensors, the evaluation method is applied to the data acquired from the sensors. The primary statistics of the sample measurement data are tabulated in Table 1.

In Table 1, it can be observed that the mean error value is exactly the same for axes $x$ and $y$ (0.79 device units), but higher for axis $z$ (1.13 device units). This value is 8.8% of the mean total acceleration, having values that range from 216.7 to 286.4 device units. For example, in the extreme case that the deviation percentage comes exclusively from the axis $y$, the error in the roll angle estimation is more than 20 degrees. These values indicate that the accelerometer sensor system is not calibrated.

For further evaluation of the sensors, the evaluation method is applied to the data acquired from the sensors. The primary statistics of the sample measurement data are tabulated in Table 1.

In Table 1, it can be observed that the mean error value is exactly the same for axes $x$ and $y$ (0.79 device units), but is higher for axis $z$ (1.13 device units). This
indicates that the $z$ sensor has a different behavior, and a lower performance should be expected when it uses measurements acquired from this axis.

Data in Table 1 are used as input to the software tool and the evaluation method. The results from the evaluation application, for the unknown parameters and their standard deviations (in parentheses) are presented in Tables 2 and 3.

According to the evaluation results in Table 2, it seems to be a large bias ($-36.01$ device units) that concerns the axis $z$ and confirms that there is a different behavior of the particular accelerometer system axis. If the values of the evaluation mathematical model parameters are applied as corrections to the original sensor measurements, the mean value $1,000 \pm 0.72$ mg is accomplished for the total acceleration and for all samples.

### 4.2. Low-cost magnetometer sensor evaluation

In order to check the evaluation method on the magnetometer sensors, 17 samples of measurements from different orientations acquired from the SparkFun 9DOF unit were used. The standard deviation of the mean value is 19.87 device units which is 4% of the mean total magnetic intensity, with the values that range from 473.1 to 542.4 device units. The total magnetic intensity calculated per position is shown in Figure 5 and the primary statistics of the sample measurement data are tabulated in Table 4. In case, the deviation percentage comes exclusively from the $y$-axis of the magnetometers, the error in the heading angle estimation is more than 10 degrees. These values indicate that the magnetometer sensor system can be characterized as non-calibrated also.

In Table 4, it can be observed that the mean error value are quite close for axes $x$ and $z$ (3.69 and 3.56 device units), and is smaller in axis $y$ (3.20 device units). Additionally, axis $z$ has smaller standard deviation for the $x$ and $z$ sensors, and a lower performance should be expected when it uses measurements acquired from these axes.

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**Table 1. Sample measurements from accelerometers and their primary statistics.**

| Sample | Measurements per sample | $V_x$ | $V_y$ | $V_z$ | $s_{V_x}$ | $s_{V_y}$ | $s_{V_z}$ |
|--------|-------------------------|-------|-------|-------|------------|------------|------------|
| 1      | 681                     | 7.17  | -17.92| 215.93| 0.77       | 0.79       | 1.10       |
| 2      | 834                     | 249.56| -17.01| 54.04 | 0.80       | 0.79       | 1.06       |
| 3      | 670                     | 195.12| -60.25| -199.03| 0.88       | 0.78       | 1.22       |
| 4      | 717                     | -212.58| -53.85| 94.28 | 0.79       | 0.82       | 1.11       |
| 5      | 554                     | 15.29 | -270.47| -30.27| 0.82       | 0.86       | 1.17       |
| 6      | 560                     | -228.49| -128.54| -37.84| 0.86       | 0.66       | 1.11       |
| 7      | 701                     | -47.81| 10.18 | 211.35| 0.81       | 0.80       | 1.11       |
| 8      | 696                     | -135.41| 209.01| 29.82 | 0.80       | 0.85       | 1.19       |
| 9      | 607                     | -254.25| 24.28 | -69.04| 0.64       | 0.82       | 1.02       |
| 10     | 344                     | 33.26 | 26.37 | 215.26| 0.74       | 0.75       | 1.14       |
| 11     | 724                     | 240.13| 42.66 | 66.91 | 0.86       | 0.84       | 1.11       |
| 12     | 567                     | 242.01| 56.05 | 56.78 | 0.83       | 0.81       | 1.06       |
| 13     | 362                     | 247.68| -35.51| 53.62 | 0.80       | 0.77       | 1.11       |
| 14     | 303                     | 146.88| -7.20 | -245.82| 0.87       | 0.89       | 1.14       |
| 15     | 719                     | -253.53| 1.65 | -73.61| 0.63       | 0.67       | 1.23       |

Mean error value

- $V_x$: 0.79
- $V_y$: 0.79
- $V_z$: 1.13

Standard deviation of each error

- $s_{V_x}$: 0.07
- $s_{V_y}$: 0.06
- $s_{V_z}$: 0.06

**Note:** Device units $V_x$, $V_y$, and $V_z$ are the average values of the samples; $s_{V_x}$, $s_{V_y}$, and $s_{V_z}$ are their sampling standard deviations.
the mean error value than the other axis (0.19 against 0.25 and 0.27 device units).

Data in Table 4 are used as input to the evaluation method, therefore, the unknown parameters and their standard deviations (in parentheses) are presented in Tables 5 and 6.

If the values of the evaluation mathematical model parameters are applied, as corrections, to the original measurements, the mean value $1,000 \pm 3.44$ (unitless quantity) is accomplished for the total magnetic intensity and for all samples.

5. Accuracy estimation in geomatics applications

The final results that interest geomatics applications is the representation of the inclination of the inertial measurement unit system as to the horizontal plane and the orientation toward North, elsewhere known as attitude and heading. In any cases, it is essential to estimate the standard deviations of the calculated magnitudes. In this way, it is possible to determine the capabilities of the particular sensors and define in what applications this system can be used.

The most costless method to achieve the accuracy estimation for the required magnitudes is to apply statistical data analysis methods to the equations from which these magnitudes are calculated. For this purpose, the evaluation method was extended to estimate the best accuracies that can be achieved using the evaluation model parameters for correcting the sensor readings.

### Table 2. Accelerometer Bias (device units).

| Axis | Accelerometer Bias (standard deviation) |
|------|----------------------------------------|
| $x$  | 3.75 (0.10)                            |
| $y$  | -5.93 (0.40)                           |
| $z$  | -36.01 (0.22)                          |

### Table 3. Rectangular scale factors, non-rectangularity of the axes, and interaction between sensors.

| Axes | $x$ | $y$ | $z$ |
|------|-----|-----|-----|
|      | ($x$) | ($y$) | ($z$) |
| $x$  | 3.82133 | -0.00415 | -0.03069 |
|      | (0.002104) | (0.003601) | (0.002946) |
| $y$  | -0.00415 | 3.77211 | -0.06561 |
|      | (0.003601) | (0.005704) | (0.012013) |
| $z$  | -0.03069 | -0.06561 | 3.95239 |
|      | (0.002946) | (0.012013) | (0.004548) |

### Table 4. Measurement samples from the magnetometers and their primary statistics.

| A/A | Measurements per sample | $V_{x}$ | $V_{y}$ | $V_{z}$ | $sV_{x}$ | $sV_{y}$ | $sV_{z}$ |
|-----|------------------------|--------|--------|--------|--------|--------|--------|
| 1   | 690                    | -330.61| -162.94| -336.37| 3.90   | 3.00   | 3.73   |
| 2   | 655                    | 334.86 | 49.16  | -341.24| 3.62   | 3.45   | 3.43   |
| 3   | 549                    | -297.83| -93.45 | -381.05| 3.56   | 2.87   | 3.73   |
| 4   | 733                    | -291.65| 43.10  | -391.51| 3.54   | 2.90   | 3.58   |
| 5   | 559                    | -365.36| -129.05| -317.98| 3.77   | 3.03   | 3.71   |
| 6   | 1,265                  | -521.07| 149.19 | -20.31 | 3.95   | 3.37   | 3.55   |
| 7   | 1,071                  | -536.87| 36.82  | -59.95 | 4.12   | 3.43   | 3.70   |
| 8   | 758                    | -479.13| 149.70 | 103.91 | 3.66   | 3.32   | 3.06   |
| 9   | 218                    | -148.60| -144.75| 423.18 | 3.50   | 2.93   | 3.65   |
| 10  | 721                    | -431.03| -253.72| 153.29 | 3.42   | 3.08   | 3.34   |
| 11  | 664                    | 289.45 | -27.11 | 383.47 | 3.47   | 3.11   | 3.69   |
| 12  | 575                    | 333.54 | 262.43 | 273.85 | 3.81   | 3.12   | 3.42   |
| 13  | 736                    | -374.05| -184.05| 281.97 | 4.17   | 3.65   | 3.45   |
| 14  | 944                    | -268.87| -376.47| 219.94 | 3.10   | 3.18   | 3.68   |
| 15  | 594                    | 336.99 | -80.15 | 347.10 | 3.52   | 2.95   | 3.85   |
| 16  | 604                    | -241.68| 71.06  | -416.98| 3.86   | 3.41   | 3.58   |
| 17  | 551                    | -423.46| 51.49  | 287.14 | 3.70   | 3.64   | 3.43   |

Mean error value: 3.69 3.20 3.56
Standard deviation of each error: 0.27 0.25 0.19

Note: Device units $V_{x}$, $V_{y}$, and $V_{z}$ are the mean values of the samples; $sV_{x}$, $sV_{y}$, and $sV_{z}$ are their sampling standard deviations.
The variations and co-variabilities of the calculated accelerations can be estimated by applying the law of covariance propagation (Dermanis 1986) to the equations for the calculation of gravity acceleration components (Equation 2), as shown in Equation (6).

$$\hat{C}_{A_x,A_y,A_z} = \begin{bmatrix} A_1 & A_2 \\ \hat{e}_x & 0 \\ 0 & \hat{e}_\nu \\ \hat{e}_\nu & A_3^T \end{bmatrix}$$ (6)

where $\hat{C}_{A_x,A_y,A_z}$ is the covariance matrix of acceleration estimations that include variations and co-variations of the calculated acceleration components per axis and system orientation; $A_i$ is a matrix that is formed by the partial derivatives of the analytical equations for the acceleration calculation per axis with respect to the unknown parameters; $\hat{e}_i$ is the covariance matrix of the unknown parameter estimations; $\hat{e}_\nu$ is the covariance matrix of the error estimations.

The covariances that are calculated, is an indicator for the evaluation of low-cost accelerometer capabilities.

Table 5. Magnetometer bias (device units).

| Axis | Magnetometer Bias (standard deviation) |
|-----|---------------------------------------|
| $x$ | $-11.93$ (1.30)                      |
| $y$ | $9.09$ (3.56)                        |
| $z$ | $1.64$ (0.97)                        |

Table 6. Scale factors, non-rectangularity of the magnetometer axes, interaction between sensors.

| Axes | $x$ | $y$ | $z$ |
|------|-----|-----|-----|
| $x$  | $1.899,71$ | $0.02453$ | $-0.011,56$ |
|      | $(0.005,890)$ | $(0.010,554)$ | $(0.004,157)$ |
| $y$  | $0.024,53$ | $1.85863$ | $-0.003,54$ |
|      | $(0.010,554)$ | $(0.022,643)$ | $(0.011,250)$ |
| $z$  | $-0.011,56$ | $-0.00354$ | $2.157,01$ |
|      | $(0.004,157)$ | $(0.011,250)$ | $(0.006,629)$ |

Table 7. Standard deviations of acceleration components, tilt, and rotation angles.

| Orientation position | $a_x$(mg) | $a_y$(mg) | $a_z$(mg) | $\sigma_{x,\text{deg}}$ | $\sigma_{y,\text{deg}}$ | $\sigma_{z,\text{deg}}$ |
|---------------------|-----------|-----------|-----------|-------------------------|-------------------------|-------------------------|
| 1                   | $1.07$    | $2.22$    | $1.14$    | $0.06$                  | $0.13$                  | $0.13$                  |
| 2                   | $0.82$    | $1.78$    | $1.33$    | $0.08$                  | $0.10$                  | $0.29$                  |
| 3                   | $1.38$    | $3.54$    | $1.72$    | $0.10$                  | $0.20$                  | $0.29$                  |
| 4                   | $1.03$    | $1.41$    | $1.59$    | $0.08$                  | $0.08$                  | $0.14$                  |
| 5                   | $1.33$    | $1.26$    | $3.23$    | $0.08$                  | $0.13$                  | $0.19$                  |
| 6                   | $0.94$    | $1.14$    | $2.13$    | $0.06$                  | $0.06$                  | $0.27$                  |
| 7                   | $1.13$    | $2.12$    | $1.04$    | $0.07$                  | $0.12$                  | $0.12$                  |
| 8                   | $1.31$    | $1.92$    | $3.27$    | $0.09$                  | $0.15$                  | $0.24$                  |
| 9                   | $0.72$    | $1.82$    | $2.08$    | $0.13$                  | $0.10$                  | $0.44$                  |
| 10                  | $1.05$    | $2.08$    | $1.21$    | $0.06$                  | $0.12$                  | $0.12$                  |
| 11                  | $0.86$    | $1.93$    | $1.52$    | $0.09$                  | $0.11$                  | $0.26$                  |
| 12                  | $0.90$    | $2.00$    | $1.60$    | $0.08$                  | $0.11$                  | $0.29$                  |
| 13                  | $0.87$    | $1.72$    | $1.40$    | $0.08$                  | $0.10$                  | $0.26$                  |
| 14                  | $1.40$    | $4.22$    | $1.93$    | $0.10$                  | $0.24$                  | $0.29$                  |
| 15                  | $0.69$    | $1.71$    | $2.13$    | $0.13$                  | $0.10$                  | $0.66$                  |
| Average value       | $1.03$    | $2.06$    | $1.82$    | $0.09$                  | $0.12$                  | $0.26$                  |
| Standard deviation of each error | $0.24$ | $0.81$ | $0.68$ | $0.02$ | $0.05$ | $0.14$ |

The standard deviations of acceleration components per axis and per orientation result from the diagonal elements of the covariance matrix, and their values are tabulated in Table 7. The results in the calculation of gravity acceleration components are aggregated as follows: $1.03 \pm 0.24$ mg for axis $x$, $2.06 \pm 0.81$ mg for axis $y$, and $1.82 \pm 0.68$ mg for axis $z$.

5.1. Accuracy estimation for the inclination and attitude angles

When there are available measurement data from the accelerometer sensors, only the inclination of the system axes as to the horizontal plane can be estimated by calculating the tilt angles (Tuck 2007) or the attitude angles pitch and roll used in navigation. The calculation of the angles is performed using Equation (7).

$$\text{tilt } x = t_x = \text{pitch} = \tan^{-1}\left(\frac{A_x^2}{\sqrt{A_y^2 + A_z^2}}\right)$$ (7)

$$\text{tilt } y = t_y = \tan^{-1}\left(\frac{A_y^2}{\sqrt{A_x^2 + A_z^2}}\right)$$

$$\text{roll} = \tan^{-1}\left(\frac{A_y}{A_z}\right)$$

By applying the law of covariances to Equation (7), in the form of matrices, it has the covariance matrix of calculated angles per system position:

$$\hat{C}_{\text{pitch,roll}} = A_1 \hat{C}_{A_x,A_y,A_z} A_3^T$$ (8)

where $A_4$ is a matrix that is formed by the partial derivatives of the equations for the calculation of tilt angles or the attitude angles, with respect to the components of acceleration per system position.

The results for the standard deviations of tilt and attitude angles are tabulated in Table 7.

Regarding the estimates of attitude angles, accuracy (calculated from the accelerometer measurements)
values in the subdivision of one degree (0.10–0.26) have resulted. This order of magnitude is less than the precision referred in the specifications of commercial low-cost inertial navigation systems (Xsens 2016). These excessive accuracies can be justified by the fact that they were achieved by an evaluation procedure that adjusts the measurement correction parameters and involves data samples containing a large number of measurements in static condition.

5.2. Accuracy estimation for heading angles

The heading angle of one axis of the system, toward the magnetic North, can be calculated using the horizontal components of the magnetic field intensity, which are parallel to the local horizontal plane. In addition to this, the vertical component is ignored. In order to calculate the horizontal components of the magnetic field intensity, the measurements from the magnetic compass on the axes x, y, and z have been related to the horizontal plane, using the roll and pitch angles (Caruso 2000; Moafipoor and Toth 2007) as shown in Equation (9).

\[
\begin{align*}
X_H &= M_x \cos(\text{pitch}) + M_y \sin(\text{roll}) \sin(\text{pitch}) \\
Y_H &= M_y \cos(\text{roll}) - M_z \sin(\text{roll})
\end{align*}
\]  

(9)

where \(X_H\) and \(Y_H\) are the horizontal components of the magnetic field intensity; \(M_x, M_y,\) and \(M_z\) are the measurements of the magnetometer sensors on axes \(x, y,\) and \(z,\) respectively.

It is possible to estimate the variances and covariances of the horizontal components of the magnetic field intensity by applying the law of covariance propagation to Equation (9) as shown in Equation (10):

\[
\begin{bmatrix}
\hat{C}_{X_H1,Y_H1} \\
\hat{C}_{X_H1,Y_H2} \\
\vdots \\
\hat{C}_{X_H1,Y_HN}
\end{bmatrix} = 
\begin{bmatrix}
A_1 & A_2 \\
A_2 & 0 \\
0 & \hat{C}_{\text{pitch}, \text{roll}}
\end{bmatrix} 
\begin{bmatrix}
A_1^T \\
A_2^T
\end{bmatrix}
\]

(10)

where \(\hat{C}_{X_H1,Y_H1}\) is the horizontal components covariance matrix of the magnetic field intensity per system orientation; \(A_i\) is a matrix that is formed by the partial derivatives of the analytical equations for the calculation of the horizontal components of the magnetic field intensity with respect to the intensities of the magnetic field per axis; \(A_i\) is a matrix that is formed by the partial derivatives of the analytical equations, it is for the calculation of the horizontal components of the magnetic field intensity with respect to the attitude angles roll and pitch of the inertial measurement unit axes as to ground reference system, per system position, result from Equation (10), and are tabulated in Table 8.

The heading angle of the inertial measurement unit axis, as to the magnetic north can be calculated with the help of the horizontal components of the magnetic field’s intensity using the Equation (11).

\[
\text{heading (yaw)} = \tan^{-1}\left(\frac{HMY}{HMX}\right)
\]

(11)

The variances and covariances of the heading angles per system orientation are derived by applying the law of covariance propagation, as shown in Equation (12).

\[
\hat{C}_{\text{heading}} = A_s \hat{C}_{H_M1,H_M2} A_s^T
\]

(12)

where \(\hat{C}_{\text{heading}}\) is the covariance matrix of the heading angle estimations per system orientation; \(A_s\) is a matrix that is formed by the partial derivatives of the equations, it is for the calculation of heading angles with respect to the horizontal components of the magnetic field’s intensity per system orientation.

The standard deviations of the calculated heading angles result from Equation (12), and can provide data for calculating the matrix of weights that will accompany the estimates of the heading angles when they will be used as input data to the geomatics applications.

The estimations of precisions (Table 8), as in the case of accelerations, concern the movements involved in the calibration adjustment in static condition and by taking into account large measurement samples for each orientation. It is concluded that the mean accuracy estimation on the heading angle calculation is in the magnitude order of 1 ± 0.5 degrees. However, the magnetometers are not inertial sensors. So in actual field applications, there is always the possibility of additional magnetic fields presence that can affect the measurements and therefore the heading angles result.
6. Conclusions

The complete procedure of our methodology and its software implementation have been developed in order to evaluate low-cost MEMS accelerometer and magnetometer sensor systems. The method is applied without requiring any external equipment, in order to determine whether these systems are efficient for the geomatics applications. The method has the potential to be used as a correction model, in the case that the parameter of temperature is not taken into consideration due to the absence of this sensor. The evaluation methodology is applied on real data acquired from a low-cost sensor accelerometer and magnetometer system.

Two stages of evaluation are presented. First, the evaluation method reviews the total magnitude of acceleration or the strength of the magnetic field depending on the sensor. In the respective example, the accelerometer diagram shows a standard deviation of 8.8% from the mean total acceleration, and the magnetometer diagram shows 4% from the total strength of the magnetic field. These values indicate that both system sensors are non-calibrated and further evaluation is required. Second, the evaluation is more detailed and determines the values of mathematical model parameters that describe the operation of accelerometers and magnetometers. The results show that the methodology works, since the evaluation model adjustment converged in both cases and provided model values that achieve complete verification of the total acceleration or the strength of the magnetic field, respectively.

The sensor problems can be detected within the developed methodology. For example, a significant problem is detected on the axis z (the particular axis showing lower efficiency compared with the other two axes) of the accelerometer system. There are estimations about the accuracies of tilt, pitch, roll, and heading angles that are calculated in relation to measurements from the sensors, while taking into consideration the evaluation model parameters. It turns out that the particular sensor system is considered possible to achieve accuracies of 0.1°–0.4° in pitch, roll, and tilt angles, and 1.5° in the heading angle. The achieved accuracies concern static conditions and large samples of measurements, while adjusting the observation-measurements along with the correction parameters of the calibration model.

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