Minijet production in ultra-relativistic heavy-ion collisions is discussed by taking semi-hard parton rescatterings explicitly into account. At both RHIC and LHC energies we find sizable effects on global characteristics of the nuclear collision like the initial multiplicity and transverse energy of the minijets. The dependence of these quantities on the cutoff that separates soft and hard interactions becomes much smoother after the inclusion of the rescatterings. This allows to define an energy and centrality dependent saturation cutoff and to push perturbative computations to rather low values of the cutoff. As an application we compute the charged multiplicity at mid rapidity and compare it to the recent RHIC data.

In nuclear collisions at ultra-relativistic energies the initial production mechanism is the liberation of a great number of partons from the nuclear wave-functions in a very short time from the beginning of the collision. Then, this dense system of partons, also called minijet plasma, will then evolve, possibly thermalizing and giving rise to the quark gluon plasma. The signals of its creation are very sensitive to the minijet plasma initial conditions, that in turn can be related to final state bulk observables like the charged particle multiplicity and their transverse energy. On the other hand, at RHIC and LHC an increasingly large fraction of the event is expected to be due to hard and semi-hard interactions, so that we can try to gain control on the initial conditions by using perturbative QCD.

The usual way to do this is to take into account the Eikonalized minijet cross-section \( \sigma_{mj} = \int d^2\beta (1 - \exp[-\sigma_J T_{AB}(\beta)]) \), where \( \beta \) is the nuclear impact parameter, \( T_{AB}(\beta) = \int d^2b \tau_A(b - \beta) \tau_B(b) \), \( \sigma_J = \int dx dx' G(x) \sigma_H(x x') G(x') \), \( G \) is the parton distribution function and \( \tau_{A(B)} \) are the nuclear thickness functions; \( \sigma_H(x x') \) is the perturbative parton-parton cross-section defined with an infrared cutoff \( p_{cut} \), that separates soft and semi-hard scatterings; here and in the following the flavour indices are suppressed for simplicity. In this way we are describing different parton pairs interacting in different points in the plane transverse to the beam; we will call these events

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disconnected collisions, see Fig. a. In each collision two back-to-back jets are produced, so that the minijet multiplicity is easily seen to be

\[ N_{\text{jet}}^{\text{eik}}(\beta) = 2N_{\text{col}}(\beta) = 2\sigma_J T_{AB}(\beta) = 2 \int dx dx' d^2b \Gamma_A(x, b - \beta) \sigma_H(xx') \Gamma_B(x', b), \]

where \( \Gamma_A(x, b) = G(x)x_A(b) \).

There are two problems with this expression. First, it diverges when \( p_{\text{cut}} \to 0 \) due to the inverse power divergence of the perturbative cross-section \( \sigma_H \). Second, at very high energy and large atomic numbers a typical projectile parton is traversing a dense target nucleus, so that the chance of scattering more than once may become non-negligible. To have a quantitative feeling on this, one can take the average number of parton-parton collisions in (1), drop the integrations over \( x \) and \( b \), divide by the number of incoming partons, \( \Gamma_A \), and obtain the average number of collisions per incoming parton, \( \langle N_{\text{scat}}(x, b) \rangle = \int dx' \sigma_H(xx') \Gamma_B(x', b) \). For example, at LHC with a cutoff \( p_{\text{cut}} = 2 \text{GeV} \) a parton in the central rapidity region suffers on average two to three collisions over most of the target transverse area. In conclusion, parton rescatterings must be taken into account in the description of the collision dynamics.

The simplest rescattering event is a three-parton elastic scattering. It can be shown that in the limit \( t/s \to 0 \) and for infinite number of colours this cross-section factorizes in terms of two two-body elastic cross-sections. Therefore the process can be pictured as two successive semi-hard scatterings against two different target partons (see Fig. b). We can try to extend this picture to an arbitrary number of scatterings in the following way. We consider the multi-parton distributions \( D_A^n(\vec{x}, \vec{b}) \), i.e. the probability densities of having \( n \) partons in the A nucleus with fractional momenta \( x_i \) and transverse coordinates \( b_i \). If we neglect correlations inside the nuclei the multi-parton distributions are Poissonian: \( D_A^n(\vec{x}, \vec{b}) = \prod_n \Gamma_A(x_i, b_i) \ldots \Gamma_A(x_n, b_n) \exp[-\int dx dx' d^2b \Gamma_A(x, b)] \). Then, we consider the probability that at least one semi-hard interaction occurs between \( n \) partons from A and \( m \) partons from B: \( P_{nm} = 1 - \prod_{i=1}^n \Pi_{j=1}^m (1 - \hat{\sigma}_{ij}) \), where \( \hat{\sigma}_{ij} = \sigma_H(x_i, x_j) \delta^{(2)}(b_i - b_j) \) is the probability of a semi-hard scattering between the \( i \)-th parton from A and the \( j \)-th from B. One can then obtain the minijet cross-section by convoluting the multi-parton distributions and the semi-hard probability: \( \sigma_{mj} = \sum_{n,m=1}^\infty D_A^n P_{nm} D_B^m \). In this way we are describing both the disconnected collisions and the rescatterings (see Fig. b). The minijet multiplicity is no longer related in a simple way to the number of collisions, however if we choose a fixed scale \( Q = p_{\text{cut}} \) for the distribution functions and the running coupling constant, we can obtain a simple expression for it:

\[ N_{\text{jet}}(\beta) = \int dx dx' d^2b \Gamma_A(x, b - \beta) \left( 1 - e^{-\int dx' \sigma_H(xx') \Gamma_B(x', b)} \right) \{ A \leftrightarrow B \} p_{\text{cut}} \to \infty N_{\text{jet}}^{\text{eik}}(\beta) \]

We interpret it as the density of projectile partons \( \Gamma_A \) times the probability of having at least one semi-hard scattering, and the exponent in \( \hat{\sigma} \) as the opacity of the target. As \( p_{\text{cut}} \) goes to infinity the target opacity becomes smaller and smaller, therefore the projectile is traversing a very dilute target, the chance of scattering twice or more becomes negligible and the minijet multiplicity approaches the Eikonal estimate. On the contrary, as the cutoff decreases the opacity increases, the target becomes almost black and the multiplicity reaches a finite limit: roughly speaking

\[ N_{\text{jets}} \to_{p_{\text{cut}}/\sqrt{s} \to 0} N_{\text{lim}} = \int \Gamma_A. \]
This basically expresses the fact that a collision cannot free more partons than were present in the incoming nuclear wave-function. The effect of the rescatterings is shown in Fig. 2a, where the minijet multiplicity tends to saturate below 2 GeV at LHC and 1 GeV at RHIC. Note that the dependence on the cutoff has become much smoother than in the Eikonal estimate, making the choice of the cutoff less critical. An analogous behaviour is observed also in the initial transverse energy and for the $k$-factor dependence of these quantities. 

It is possible to show that in (3) we are taking into account not only the jets that had $n \geq 1$ semi-hard scatterings, but also those that had at least one semi-hard scattering but also any number of soft ones. Indeed one semi-hard scattering is enough to shadow completely the soft ones, so that in (3) the soft cross-section does not appear. In this way, we are neglecting only the minijets that had just soft scatterings. This, together with (3), allows to define a saturation cutoff $p_{sat}$: we require that the number of jets be near the limit and we call $p_{sat}$ the cutoff at which this happens; for example we can require

$$N_{jet}(p_{cut} = p_{sat}) = 80\% N_{lim}(p_{cut} = p_{sat}).$$

Finally, we compute the initial conditions at this cutoff: $N_{jet}^{sat} = N_{jet}(p_{cut} = p_{sat})$. 

Now, we have to check if $p_{sat}$ is far enough form $\Lambda_{QCD}$, in which case we can trust these perturbative computations and say that the semi-hard minijets saturate the initial production mechanism. At RHIC energies this method is at the border of applicability, and at LHC it should be reliably applicable (Fig. 2b). The percentage chosen in (4) is an arbitrary parameter, however it happens that $p_{sat}$ sits where the plateau in the minijet multiplicity begins (Fig. 2c), so that choosing a different value does not change too much the results.

With these tools we can study the charged particle multiplicity. We assume isentropic expansion of the minijet plasma in the central rapidity region and direct proportionality between minijets and final hadrons, $N^{ch}(\sqrt{s}, \beta) = 0.9 \frac{2}{3} N_{jet}^{sat}(\sqrt{s}, \beta)$, and compute the number of participant nucleons at fixed energy and centrality in a Glauber model: $N_{part} = \int d^2 b \tau_A(b-\beta) (1-\exp[-\sigma_{pp}(\sqrt{s}) T_B(b)])$, where $\sigma_{pp}$ is the inelastic proton-proton cross-section. To study non-central collisions with Wood-Saxon thickness functions, we have to enforce the collision geometry by assigning the nuclei a radius $R$ and by computing $p_{sat}$ inside the overlap area defined by this radius. The reason is that in its periphery a nucleus is very dilute, therefore in that region it makes no sense to impose that it becomes black. The results are shown in Fig. 3a as a function of the c.m. energy and of the number of participants.

For central collisions the dependence on the energy is strikingly similar to the EKRT saturation model, however in our case the saturation is determined dynamically and is related to the blackness of the target, moreover no shadowing corrections have been included in the parton distribution functions. For $R$ equal to the Woods-Saxon radius, $R_{WS}$, the slope of the
Figure 3: Charged particle density at mid-rapidity per participant nucleon pair.  
Left: Center of mass energy dependence.  Right: Centrality dependence.

centrality dependence at $\sqrt{s} = 130$ GeV compares very well with the experimental data\cite{4} down to 180 participants; it underestimates PHENIX data but is not bad compared to BRAHMS and PHOBOS. Remember also that we are neglecting *ab initio* up to 20% of soft partons in the initial conditions computation, and that the saturation is not expected to work well for peripheral collisions. At this energy the dependence on the radius $R$ is rather strong, but it decreases already at $\sqrt{s} = 200$ GeV, and almost disappears at LHC energies. Note also that the slope of the curve increases with the energy signaling an increasing role played by purely semi-hard events. This is in sharp contrast with the EKRT model, where an almost flat behaviour is found.

In summary, the inclusion of the rescatterings in the dynamics of the collision allows to define global observables also at very low values of the cutoff, in a region of interface between perturbative and non-perturbative interactions. The minijet multiplicity tends to saturate and to reach a limit given by the number of partons in the initial nuclear wave functions. This allows to define a dynamical saturation cutoff at which the initial conditions can be estimated. The results compare satisfactorily to RHIC data at $\sqrt{s} = 130$ GeV, where the model is at the edge of its applicability. As the energy increases it should be more and more reliable.

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