Core Phase Transitions for Embedded Topological Defects

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Abstract

Vortices in superfluid \( ^3\text{He}-\text{B} \) have been observed to undergo a core transition. We discuss the analog phenomenon in relativistic field theories which admit \textit{embedded} global domain walls, vortices and monopoles with a core phase structure. They are present in scalar field theories with approximate global symmetries which are broken both spontaneously and \textit{in parts} explicitly. For a particular range of parameters their symmetric core exhibits an instability and decays into the nonsymmetric phase.

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1 Introduction and Conclusions

In the superfluid phases of liquid $^3$He occur the most complicated known vacuum states of condensed matter, in which many symmetries are simultaneously broken. These symmetries manifest in the physical properties of the quantized vortex lines in the two superfluid phases of $^3$He under rotation.

More specifically in the B-phase vortices of two types appear: those with an axisymmetric core in the high pressure regime and the ones with a non-axisymmetric core for low pressure which possess distinct topological characteristics. As such in passing from the high to the low pressure phase there is a distinct core transition characterized by both a change in the rotational symmetry as well as of the topology of the core structure \[1, 2\]. Direct experimental observation of such a transition indicated the spontaneous breaking of axial symmetry \[3\]. The one quantum vortex (high pressure one) undergoes a dimerization into a pair of half-quantum vortices (low pressure vortices) producing a novel topological feature - the transformation of point vortices or boojums into one another after circling either one half quantum vortex.

A first order phase transition usually follows from such a change in the topology at the "bifurcation process" of the vortex core. In equivalent terms this is thought of as a topological transition between two inequivalent ways in which vorticity can flare out into the momentum space \[1\].

In this talk we present the analog phenomenon in quantum field theory. We review recent work of examples of global defects which admit a core phase structure \[8, 9\]. It is a result of deformations of the vacuum manifolds as a result of partial explicit breaking of the global symmetries. More specifically global embedded defects such as domain walls, vortices and monopoles arise in scalar field theories that exhibit a partial explicit breaking of an spontaneously broken global symmetry $U(1)$ to $Z_2$ for domain walls $SU(2)$ to $U(1)$ for vortices and $SO(4)$ to $SO(3)$ for monopoles. For particular values of parameters the defect core exhibits a transition, in analogy with their superfluid $^3$He – $B$ vortices, from a symmetric phase to a non-symmetric one. We will present our results in detail for the case of global domain walls vortices. In both cases we will identify the parameter ranges for stability of the configurations with either a symmetric or a non-symmetric core. For the case of a domain wall wall we will discuss results of a simulation for an expanding bubble of a domain wall.

Interesting implications of such core phase transition for cosmic defects...
relate to nontrivial dynamics between defects with nonsymmetric cores as well as to novel realization of topological inflation. Defects have been thought to seed an inflating phase in the very early universe. This is because their core is an effective trap of vacuum energy. In our case the ones which undergo a core phase transition offer a novel kind of an inflating seed with the core relaxing its vacuum energy and thus exiting from an otherwise eternally inflating phase. Core reheating and the emergence of latent heat from a defect-core are few of the interesting new phenomena that core phase transitions provides us.

2 Domain Walls with NonSymmetric Core

We consider a model with a $U(1)$ symmetry explicitly broken to a $Z_2$. This breaking can be realized by the Lagrangian density

$$L = \frac{1}{2} \partial_\mu \Phi^* \partial^\mu \Phi + \frac{CC^2}{2} |\Phi|^2 + \frac{m^2}{2} Re(\Phi^2) - \frac{h}{4} |\Phi|^4$$

where $\Phi = \Phi_1 + i\Phi_2$ is a complex scalar field. After a rescaling

$$\Phi \to \frac{m}{\sqrt{h}} \Phi$$
$$x \to \frac{1}{m} x$$
$$M \to \frac{1}{\alpha m}$$

The corresponding equation of motion for the field $\Phi$ is

$$\ddot{\Phi} - \nabla^2 \Phi - (\alpha^2 \Phi + \Phi^*) + |\Phi|^2 \Phi = 0$$

The potential takes the form

$$V(\Phi) = -\frac{m^4}{2h} (\alpha^2 |\Phi|^2 + Re(\Phi^2) - \frac{1}{2} |\Phi|^4)$$

For $\alpha < 1$ it has the shape of a "saddle hat" potential i.e. at $\Phi = 0$ there is a local minimum in the $\Phi_2$ direction but a local maximum in the $\Phi_1$ (Fig 1).
Figure 1: (a) The domain wall potential has a local maximum at $\Phi = 0$ in the $\Phi_1$ direction. (b) For $\alpha > 1$ ($\alpha < 1$) this point is a local maximum (minimum) in the $\Phi_2$ direction.

For this range of values of $\alpha$ the equation of motion admits the well known static kink solution

$$\Phi_1 = \Phi_R \equiv \pm (\alpha^2 + 1)^{1/2} \tanh\left(\frac{\alpha^2 + 1}{2} x\right)$$

(7)

$$\Phi_2 = 0$$

(8)

It corresponds to a symmetric domain wall since in the core of the soliton the full symmetry of the Lagrangian is manifest ($\Phi(0) = 0$) and the topological charge arises as a consequence of the behavior of the field at infinity ($Q = \frac{1}{2}(\Phi(-\infty) - \Phi(\infty))/(\alpha^2 + 1)^{1/2}$).

For $\alpha > 1$ the local minimum in the $\Phi_2$ direction becomes a local maximum but the vacuum manifold remains disconnected, and the $Z_2$ symmetry remains. This type of potential may be called a "Napoleon hat" potential in analogy to the Mexican hat potential that is obtained in the limit $\alpha \to \infty$ and corresponds to the restoration of the $S^1$ vacuum manifold.

The form of the potential however implies that the symmetric wall solution may not be stable for $\alpha > 1$ since in that case the potential energy favors a solution with $\Phi_2 \neq 0$. However, the answer is not obvious because for $\alpha > 1$, $\Phi_2 \neq 0$ would save the wall some potential energy but would cost additional gradient energy as $\Phi_2$ varies from a constant value at $x = 0$ to $0$ at infinity. Indeed a stability analysis was performed by introducing a small perturbation about the kink solution reveals the presence of negative modes for $\alpha > \alpha_{\text{crit}} = \sqrt{3} \approx 1.73$ For the range of values $1 < \alpha < 1.73$ the potential takes the shape of a "High Napoleon hat". We study the full non-linear
static field equations obtained from (6) for a typical value of $\alpha = 1.65$ with boundary conditions

\begin{align}
\Phi_1(0) &= 0 & \lim_{x \to \infty} \Phi_1(x) &= (\alpha^2 + 1)^{1/2} \\
\Phi_2'(0) &= 0 & \lim_{x \to \infty} \Phi_2(x) &= 0
\end{align}

Using a relaxation method based on collocation at gaussian points \cite{7} to solve the system (6) of second order non-linear equations we find that for $1 < \alpha < \sqrt{3}$ the solution relaxes to the expected form of (7) for $\Phi_1$ while $\Phi_2 = 0$ (Fig. 2). For $\alpha > \sqrt{3}$ we find $\Phi_1 \neq 0$ and $\Phi_2 \neq 0$ (Fig. 3) obeying the boundary conditions (13), (14) and giving the explicit solution for the non-symmetric domain wall. In both cases we also plot the analytic solution on (7) stable only for $\alpha < \sqrt{3}$ for comparison (bold dashed line). As expected the numerical and analytic solutions are identical for $\alpha < \sqrt{3}$ (Fig. 2).

We now proceed to present results of our study on the evolution of bubbles of a domain wall. We constructed a two dimensional simulation of the field evolution of domain wall bubbles with both symmetric and non-symmetric core. In particular we solved the non-static field equation (6) using a leapfrog algorithm \cite{7} with reflective boundary conditions. We used an $80 \times 80$ lattice and in all runs we retained $\frac{dt}{dx} \simeq \frac{1}{3}$ thus satisfying the Cauchy stability criterion for the timestep $dt$ and the lattice spacing $dx$. The initial conditions were those corresponding to a spherically symmetric bubble with initial field
ansatz
\[ \Phi(t_i) = (\alpha^2 + 1)^{1/2} \tanh\left(\frac{\alpha^2 + 1}{2}(\rho - \rho_0)\right) + i \ 0.1 \ e^{-|x| - \rho_0} \frac{x}{|x|} \]  

where \( \rho = x^2 + y^2 \) and \( \rho_0 \) is the initial radius of the bubble. Energy was conserved to within 2% in all runs. For \( \alpha \) in the region of symmetric core stability the imaginary initial fluctuation of the field \( \Phi(t_i) \) decreased and the bubble collapsed due to tension in a spherically symmetric way as expected.

For \( \alpha \) in the region of values corresponding to having a non-symmetric stable core the evolution of the bubble was quite different. The initial imaginary perturbation increased but even though dynamics favored the increase of the perturbation, topology forced the \( \text{Im}\Phi(t) \) to stay at zero along a line on the bubble: the intersections of the bubble wall with the y axis (Figs. 4, 5). Thus in the region of these points, surface energy (tension) of the bubble wall remained larger than the energy on other points of the bubble. The result was a non-spherical collapse with the x-direction of the bubble collapsing first (Fig. 5).

### 3 Vortices with Nonsymmetric Core

We have generalized our analysis for domain walls to the case of a scalar field theory that admits global vortices. We consider a model with an SU(2)
Figure 4: Initial field configuration for a non-symmetric spherical bubble wall with $\alpha = 3.5$. 

Initial Non-Symmetric Wall Bubble, $\alpha = 3.5$
Figure 5: Evolved field configuration \((t = 14.25, 90\) timesteps) for a non-symmetric initially spherical bubble wall with \(\alpha = 3.5\).
symmetry explicitly broken to $U(1)$. Such a theory is described by the Lagrangian density:

$$L = \frac{1}{2} \partial_\mu \Phi^\dagger \partial^\mu \Phi + \frac{M^2}{2} \Phi^\dagger \Phi + \frac{m^2}{2} \Phi^\dagger \tau_3 \Phi - \frac{h}{4} (\Phi^\dagger \Phi)^2$$

(12)

where $\Phi = (\Phi_1, \Phi_2)$ is a complex scalar doublet and $\tau_3$ is the $2 \times 2$ Pauli matrix. After rescaling as in equations (2)-(4) we obtain the equations of motion for $\Phi_{1,2}$:

$$\partial_\mu \partial^\mu \Phi_{1,2} - (\alpha^2 \pm 1) \Phi_{1,2} + (\Phi^\dagger \Phi) \Phi_{1,2} = 0$$

(13)

where the $+$ (-) corresponds to the field $\Phi_1$ ($\Phi_2$).

Consider now the ansatz

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} f(\rho)e^{i\theta} \\ g(\rho) \end{pmatrix}$$

(14)
Figure 7: Field configuration for a non-symmetric-core global string with $\alpha = 2.8$.

with boundary conditions

$$
\lim_{\rho \to 0} f(\rho) = 0, \quad \lim_{\rho \to 0} g'(\rho) = 0 \quad (15)
$$

$$
\lim_{\rho \to \infty} f(\rho) = (\alpha^2 + 1)^{1/2}, \quad \lim_{\rho \to \infty} g(\rho) = 0 \quad (16)
$$

This ansatz corresponds to a global vortex configuration with a core that can be either in the symmetric or in the non-symmetric phase of the theory. Whether the core will be symmetric or non-symmetric is determined by the dynamics of the field equations. As in the wall case the numerical solution of the system (21) of non-linear complex field equations with the ansatz (22) for various values of the parameter $\alpha$ reveals the existence of an $\alpha_{cr} \approx 2.7$

For $\alpha < \alpha_{cr} \approx 2.7$ the solution relaxed to a lowest energy configuration with $g(\rho) = 0$ everywhere corresponding to a vortex with symmetric core (Fig. 6).

For $\alpha > \alpha_{cr} \approx 2.7$ the solution relaxed to a configuration with $g(0) \neq 0$ indicating a vortex with non-symmetric core (Fig. 7). Both configurations are dynamically and topologically stable and consist additional paradigms of the defect classification discussed in the introduction.
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References

[1] G.E.Volovik and V.P.Mineev, JETP\textbf{56}, 579(1982).
[2] M.M.Salomaa and G.E.Volovik, \textit{Rev.Mod.Phys.}\textbf{59}, 550, 1987.
[3] Y.Kondo, J.S.Korhonen, M.Krusius, V.V.Dmitriev, Y.M.Mukharsky, E.B.Sonin and G.E.Volovik, \textit{Phys.Rev.Lett.}\textbf{67}, 81, 1991.
[4] A.Vilenkin, E.P.S.Shellard, in "Cosmic Strings and other Topological Defects" Cambridge U. Press, 1994.
[5] A.Vilenkin, \textit{Phys.Rep.}\textbf{121}, 263, 1985; J.Preskill, \textit{Ann.Rev.Nucl.Part. Sci.}\textbf{34}, 461, 1984.
[6] R. Rajaraman, 'Solitons and Instantons’ North Holland Pub., 1987.
[7] W. Press et. al., \textit{Numerical Recipes}, Cambridge U. Press, 2nd ed., 1993.
[8] M. Axenides and L. Perivolaropoulos, \textit{Phys.Rev.D.}\textbf{56},1973, 1997.
[9] M.Axenides,L.Perivolaropoulos and M.Trodden, in \textit{Phase Transitions in the Core of Global Embedded Defects}, e-
[10] A. Linde \textit{Phys. Lett.} \textbf{B327}, 208 (1994).
[11] A. Vilenkin, \textit{Phys. Rev. Lett.} \textbf{72}, 3137 (1994).