Numerical simulation research on crack propagation of plane steel

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ABSTRACT. There are still two deficiencies in the study of crack propagation in plane steel. One is that there are few studies on the theory of crack cracking and the cracking criterion from the strain field at the crack tip; the other is that there are few studies using the block discrete element method to explore the crack propagation. In view of the above two points, this paper proposes a strain strength criterion to explain the extension of tensile cracks and shear cracks in steel. This strain strength criterion assumes: (1) Tensile cracks grow along the direction of the maximum principal strain. When the principal strain \( \varepsilon \) reaches a critical value, the tensile cracks begin to grow; (2) Shear cracks grow in the direction of the most dangerous stress state. When the Mohr circle exceeds the Mohr Coulomb failure line, the shear crack starts to expand. In addition, this paper applies the strain strength criterion to the block discrete element method to simulate the macro-mechanical response characteristics of steel under load and the propagation and evolution process of cracks. It is shown (1) the stress-strain curve appears elastic, stagnant, fluctuating, and falling back, etc. The basic law, and (2) the characteristics of stable and unstable growth of cracks during the whole loading process.

1. Introduction

Plane steel is an important component in the field of vehicle design and manufacturing. The existence of internal cracks will change the overall mechanical response of the steel. The specific manifestations are: the strength of the crack area is reduced, the stress and strain concentration is obvious; the macro-mechanical properties of the steel appear Non-linear and anisotropic. The propagation of cracks is a fracture problem, which was first systematically studied at the beginning of the 20th century. In 1913, Inglis [1] studied the problem of infinite plates with oblate elliptical holes and discovered the singularity of the stress at the crack tip. In 1921, Griffith [2] conducted theoretical analysis and experimental research on the tensile failure characteristics of pre-cracked glass, and proposed the famous fracture energy balance theory, that is, the growth of cracks depends on the increase of surface energy of the material and the release of strain energy. In 1924, Griffith [3] studied the propagation characteristics of cracks in compressed materials, and proposed the Griffith strength criterion considering tensile crack propagation, and fracture mechanics was established. In 1946, Sneddon [4] theoretically proved that the stress at the crack tip has \( r^{-1/2} \) order singularity. Irwin [5] introduced the concept of stress intensity factor on the basis of considering the stress singularity of the crack tip in 1957, and divided the cracks into three types according to different stress characteristics, namely type I crack (tensile crack) and type II Cracks (shear cracks), type III cracks (tension-shear mixed cracks).
Because the main body of this paper is flat steel, the mixed tensile and shear cracks are not within the scope of the study.

Steel is not a completely brittle material, and its plasticity cannot be ignored, and elastoplastic fracture mechanics is developed from this. In 1960 Dugdale [6] studied the relationship between the shape of the plastic zone at the crack tip and the external load, and proposed the D-M model. In 1961 Wells [7] proposed the COD criterion for crack opening displacement (Crack Opening Displacement) for crack elastoplastic analysis. In 1968, Rice [8] applied the full-quantity theory of plastic mechanics and proposed the use of J integral as a criterion for fracture propagation. Erdogan and Xue Changming (Sih) [9] proposed the maximum circumferential stress criterion to judge the crack initiation position and angle. According to this criterion, the predicted crack initiation direction is in good agreement with the experimental observations.

On the one hand, the numerical simulation [10] method can verify the accuracy and applicability of the fracture theory, on the other hand, it can simulate the propagation of steel cracks and the way of crack evolution in real working conditions, which can provide guidance and basis for theoretical and practical engineering. With the rapid improvement of computing power, the discontinuous numerical method based on the assumption of discontinuity and large displacement [11] has proved to have excellent simulation capabilities. The more widely used discontinuous numerical methods include manifold element method (NMM), discontinuous deformation method (DDA) and discrete element method (DEM). Among them, the discrete element method is the focus of this thesis. The discrete element method treats the material as a collection of individual blocks or particles. This method allows the discrete bodies to produce limited displacement and rotation, or even complete separation; and can automatically identify new contacts generated by the calculation process. Discrete element method uses explicit time domain to solve the motion mode of discrete body, including block discrete element program UDEC [12] and particle discrete element program PFC [13]. Since the discrete element allows large deformation and can reflect nonlinear behavior, it can well simulate the crack propagation behavior of steel under load. This paper focuses on the study of the block discrete element method. Its principle is to discretize the calculation area into blocks through a finite number of intersecting discontinuous surfaces. Each block is then divided into many grids by finite difference or finite element methods. In this way, the stress, strain and displacement of the block are calculated. The deformation solving method of the model is based on the explicit fast Lagrangian algorithm. The mechanical contact behavior between blocks is expressed as normal stiffness and tensile strength criterion in the normal direction, and tangential stiffness and shear strength criterion in the tangential direction. By introducing viscous damping, dynamic relaxation techniques are used to solve static problems.

Looking at the research on steel crack propagation at home and abroad, there are still two major deficiencies. First, there are few studies to explain the theory of crack cracking and the cracking criterion from the strain field at the crack tip. Second, there are few studies on crack propagation using the block discrete element method, and the research results are not in-depth. This dissertation will focus on these two major deficiencies in order to propose a new steel crack growth criterion and embed it into the block discrete element program for simulation and numerical simulation.

2. Strain strength criterion

In order to fully describe the mechanism of tensile cracks and shear cracks in steel, based on the crack tip theory and the study of the principal strain field, this paper proposes a strain strength criterion, which includes a criterion for determining tensile cracks based on principal strain and a Mohr Coulomb-based criterion. Judgment criteria for shear cracks. The strain strength criterion assumes:

(1) The tensile crack grows along the direction of the maximum principal strain. When the principal strain $\varepsilon_{\text{max}}$ reaches the critical value, the tensile crack begins to grow;

(2) Shear cracks expand in the direction of the most dangerous stress state. When the Mohr circle of the stress state exceeds the Mohr Coulomb failure line, the shear cracks begin to expand. The critical state expression is:
\[
\begin{align*}
\varepsilon_3 &= -\varepsilon_1 \\
\tau &= c + \sigma \tan \phi
\end{align*}
\] (1)

The stress state is positive for compression and negative for tension. Assuming that the principal stress of a point is \( \sigma_1, \sigma_3 \), for linear elastic materials, \( E\varepsilon_3 = \sigma_3 - \nu\sigma_1 \), when the point is in the limit equilibrium state, the formula (1) can be transformed into:

\[
\begin{align*}
\sigma_1 &= (\sigma_3 + E\varepsilon_i) / \nu \\
\sigma_1 &= 2c \frac{1 + \sin \varphi}{1 - \sin \varphi} + \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3
\end{align*}
\] (2)

Formula (2) The following formula can be obtained according to the geometric relationship of the stress circle in the limit equilibrium state in Figure 1. This is an expression of strength in the form of a piecewise function, which can explain the different fracture characteristics of the material for different stress states. This criterion can judge both tensile crack and shear crack propagation. Moreover, whether it is to judge tensile cracks or shear cracks, the two-way stress can be considered, which is the main difference from the maximum tensile stress criterion. In formula (1), the upper and lower two types are combined, and the cutoff point is:

\[
\sigma_{3d} = \frac{E\varepsilon_i - 2c}{\nu} \frac{1 + \sin \varphi}{1 - \sin \varphi} + \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3
\] (3)

So the critical state of this criterion can be expressed as:

\[
\begin{align*}
\sigma_3 &\leq \sigma_{3d}, \quad \sigma_1 = (\sigma_3 + E\varepsilon_i) / \nu \\
\sigma_3 > \sigma_{3d}, \quad \sigma_1 &= 2c \frac{1 + \sin \varphi}{1 - \sin \varphi} + \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3
\end{align*}
\] (4), (5)

Figure 1. Diagram of Mohr Coulomb’s strength criterion.

In order to deepen the understanding of the strain strength criterion, the following discusses \( \sigma_1 - \sigma_3 \) the manifestation of the criterion under the coordinate axis and its failure critical line, and compares it with Griffith criterion, Mohr Coulomb criterion, etc., so that it can be better applied to practical engineering. go with. Under the coordinate axis \( \sigma_1 - \sigma_3 \), the strain intensity criterion expression is shown in formulas (4) and (5). If the critical value \( E\varepsilon_i = \sigma_1 \) is assumed, draw the two criteria under the same coordinate axis \( \sigma_1 - \sigma_3 \), as shown in Figure 2. First of all, for the strain intensity criterion, it is a segmented broken line \( 1 / \nu \) in the coordinate system. When \( \sigma_3 \leq \sigma_{3d} \) is a line segment with a slope, when the stress state is above the line segment, tensile cracks are generated. This line segment intersects the Mohr Coulomb intensity line at point 4. When \( \sigma_3 > \sigma_{3d} \) is a straight
line with a slope, the intercept on the axis is \( \frac{1 + \sin \varphi}{1 - \sin \varphi} \), when the stress state is above the straight line, shear cracks will occur. For Griffith criterion, it is a segmented curve in the coordinate system, when \( \sigma_1 + 3 \sigma_3 \leq 0 \) is a straight line, no matter what the value is, \( \sigma_3 = -\sigma_1 \) tensile cracks will occur as long as it is. Secondly, when \( \sigma_1 + 3 \sigma_3 > 0 \) is a quadratic curve, the intercept on the axis is, when the stress state is above the curve, \( \sigma_1 \) tensile cracks will occur \( \sigma_1 \). Comparing the two available, Griffith's strength judgment condition is more stringent, and its failure zone in the coordinate system is also smaller than the strain Mohr criterion, that is, the strain strength criterion is safer because it can resist tensile cracks. Shear cracks are well explained and judged.

Further study, the Griffith criterion and Mohr Coulomb criterion are combined to obtain a segmented broken line, which is also a method that can judge both tensile cracks and The failure line of shear cracks has been studied in the literature, and it is called the Mohr Coulomb criterion considering stretching. In order to distinguish this article, it is called the stress intensity criterion. The main difference between the stress intensity criterion and the strain intensity criterion is on the broken line and the line segment in the figure, that is, in the judgment of tensile cracks. Line considers the effect of confining pressure in the process of tensile cracking, while line, as part of the Griffith criterion, only focuses on the effect of maximum tensile stress. The line segment T-4 is more conservative in damage judgment than T-3. Line segments 1-T and 2-T are in a two-way tension state.

![Figure 2. Comparison of Griffith criterion, stress intensity criterion and strain intensity criterion.](image)

In order to facilitate the judgment of tensile cracks and shear cracks and their application to numerical simulations, a crack judgment parameter system related to the strain intensity criterion is established and two crack judgment factors are defined:

\[
\begin{align*}
f_i &= \sigma_3 - \nu \sigma_1 + E \varepsilon_i \\
f_s &= \sigma_1 - \frac{1 + \sin \varphi \sigma_1}{1 - \sin \varphi} \sigma_3 - 2c \frac{1 + \sin \varphi}{1 - \sin \varphi}
\end{align*}
\]

3. **Block discrete element model**

The UDEC program developed by ITASCA is an ideal tool for studying materials with discontinuous characteristics (i.e., internal cracks in steel). As a discrete element program, it follows the basic principles of the discrete element method. In the discrete element method, the steel is cut into a
collection of discrete bodies, and the cracks in the steel are regarded as the contact surface between
the discrete bodies. In the model calculation, by introducing contact and taking contact as the boundary
condition of the block, the continuous mechanics method is used to describe the deformation
characteristics of the block, and the discontinuous mechanics method is used to describe the
mechanical behavior of the contact. The discrete element method is mainly based on the following
assumptions: the contact characteristics between blocks can be divided into angle-side contact and
edge-side contact; the contact force $k_n$ between $k_s$ blocks is mainly provided by the normal stiffness
and tangential stiffness; the stiffness is related Viscous damping can absorb the kinetic energy of the
movement process; the failure forms of contact are tensile and shear failures.

The solution of the discrete element model is based on the explicit fast Lagrangian algorithm. The
contact force between blocks can be solved by physical equations; the motion of a single block follows
the equation of Newton's second law of motion. The whole calculation process is carried out back and
forth between the physical equations of all contacts and the equations of motion of all blocks. If the
block is a deformable body, then the motion characteristics of the triangular mesh nodes inside the
block can also be solved, and then the stress state of all the meshes can be obtained through the
constitutive equation of the block.

Although the UDEC block can be deformed and moved, the unit block itself cannot be broken.
Therefore, if the numerical model is defined completely based on the test specimen, that is, the steel
is defined as a continuous block, and the prefabricated crack is defined as a discontinuous surface, then
the model will not be able to crack propagation. Moreover, this model definition method is not
allowed in the UDEC program, because the discontinuous surfaces in UDEC must be connected to
each other to cut the block, and any internal or partial penetration cracks will be deleted during the
calculation of the model. Therefore, in the UDEC program, the test specimens must be numerically
equivalent in order to realize the simulation of the propagation of the prefabricated crack. Therefore,
many scholars use the method of introducing virtual joints to approximate the mechanical behavior of
cracked steel.

The final effect of the numerical simulation test model. The vertical displacement of the bottom
end of the sample is fixed, and the upper end is loaded. In order to load uniformly and conveniently
record the load and displacement, a loading plate is placed on the top of the sample. In order to obtain
the stress and strain inside the model block, a deformable body is used for the block. Therefore, after
the hexagonal unit block is generated, the interior of the block is divided into a triangular mesh, and
the maximum side length of the mesh is less than 1mm. This model sets up virtual cracks for all
potential cracking areas to provide a path for crack propagation. The virtual crack has high strength
and can work with the block to simulate the mechanical properties of the steel, such as stress and
deformation. Different from the general discrete element calculation method, the cracking of the high-
strength virtual crack in this model is not caused by the failure of its contact characteristics, but the
equivalent stress state at the position of the virtual crack satisfies the strength criterion, and the virtual
crack is assigned Material parameters of real cracks. With the replacement of material parameters, the
overall strength of the specimen will be weakened, which effectively simulates the nonlinear behavior
of the new cracks during the cracking process. When the discrete element method is used for
simulation, the material parameters and contact parameters of the sample block have a greater impact
on the simulation results. For the deformed block, the Mohr-Coulomb plastic model (cons=3) is used,
and for the crack surface, the contact-Coulomb slip model (jcons=2) is used.

4. Numerical simulation results
In this section, starting from the numerical simulation of the 45° crack inclination angle specimen,
general common problems such as crack initiation and propagation will be discussed. Then, the
numerical results of samples with different crack inclination angles will be compared, and the
characteristics of different samples will be summarized.
The whole stress-strain curve of the numerical simulation test and the tensile-shear crack development curve are shown in the figure, and the special moments are marked in the figure. In order to explore the whole process of loading, the initiation and propagation of tensile cracks and shear cracks, the fracture process of numerical samples is analyzed in four stages below.

(1) Elasticity stage. At this stage, the stress-strain curve of the specimen is linear, and the unit blocks and virtual cracks in the numerical model work together. When loaded to the marking point 1, the crack initiated and the initiation stress was 24.6Mpa.

(2) The stage of stable crack propagation. At this stage, the overall stress and strain relationship is still linear. Although the tip of the pre-crack cracks, it is different from tensile brittle fracture. Compression fracture is a progressive failure process, and the further expansion of the crack requires a continuous increase in external load. This stage can be regarded as the accumulation stage of crack growth. With the continuous enrichment of stress and strain, the crack approaches the critical state of unstable growth.

(3) Unsteady crack propagation stage. Marking point 2 is the demarcation point with the previous stage. At this demarcation point, the stress-strain curve of the sample drops sharply, and the crack suddenly expands. The slope of the tensile crack curve in the figure also increased, approaching 90, and the number of cracks increased by an order of magnitude. The steep drop of the stress-strain curve and the rapid expansion of the cracks indicate the instability and expansion of the cracks at this stage. At this time, the crack growth does not need to rely on external load growth to drive, just maintain the stress level, the crack will automatically expand. On the other hand, the propagation of cracks will increase the weak areas of the steel and weaken the overall strength of the steel, which makes the stress unable to be maintained at this level. The passive reduction of stress and the propagation of cracks release the stress concentration at the crack tip. After that, the stress-strain curve will fluctuate up and down, which is similar to the yield phase in the material strength test.

(4) The crack grows steadily until the specimen is broken. After marking point 5, due to the end confinement effect, the crack entered a stable expansion stage when it expanded to the vicinity of the end. At this stage, the strength of the specimen has increased, but the stress-strain curve is relatively flat, the shape of the crack and the development of the high-strain area are relatively slow. According to the crack development curve, the new cracks before this stage are basically tensile cracks, and there is no obvious initiation of shear cracks. But when the peak intensity is approaching, the shear crack curve rises rapidly from the 0 axis.

Figure 3. The evolution cloud diagram of the principal stress field during the numerical simulation process.
Figures 3 and 4 respectively show the evolution of the principal stress field and tensile principal stress distribution during the numerical simulation test, and the marking points (0-6) correspond to Fig. 6 one by one. The blue point in the tensile principal stress distribution map indicates that the maximum principal stress at this location is tensile. The greater the distribution density of the stress points, the higher the stress concentration in the region. Combining these two figures, we can clearly observe the change trend of tensile principal stress in the whole process of crack propagation. At the beginning of the test, as the load increased, stress concentration appeared at the tip of the crack, and it was a tensile action, and there was basically no tensile stress in other positions of the sample. The shape of the tensile stress concentration area at the crack tip is composed of two vertical symmetrical sectors. From Mark 1 to Mark 2, the tensile stress has been greatly accumulated in both range and size. In a very short time thereafter, the cracks destabilized and expanded and the stress was released. Moreover, the stress concentration at the location where the macro cracks are generated is also reduced. For signs 3 and 4, the phenomenon of tensile stress concentration and nucleation appears at the tip of the macro crack. In general, the macroscopic cracks crack, the tensile stress nucleation area gradually develops to both ends, and the tensile stress coverage is gradually increasing. It can be found that the tensile stress is released after the wing crack macroscopically cracks when the specimen marked 6 is broken.

Figure 5 shows the stress-strain curve and the tensile-shear crack development curve of different samples (15°, 30°, 60°, 75° inclination angle samples) throughout the loading process. Marked point 1 in the figure indicates the crack initiation state, and 4 indicates the crack initiation state. Peak intensity state. Through observation, we can first find that the extended discrete element algorithm that adopts the strain strength criterion can basically simulate the crack growth process in steel. The stress-strain curve in the numerical test process can basically reflect the nonlinearity and strength weakening in the failure process of the cracked material. And the alternating process of crack propagation and instability.
Figure 5. Numerical simulation of stress-strain and tensile-shear crack development curve of specimens with different inclination angles.

Before the new cracks cracked, the specimens all showed linear elastic characteristics, that is, the stress-strain curve was basically a straight line. After the crack initiates, the overall resistance of the sample is weakened due to the creation of a new fracture surface and the weakening of material parameters. As the loading progresses, although the cracks grow slowly at certain stages, the stress
concentration range and density at the crack tip continue to increase, resulting in stable growth. When a wide range of stresses converge and nucleate, the crack growth will accelerate, forming an unstable growth, and the stress will drop at this time. Similar to the crack propagation process of the 45° sample, the unsteady growth phenomenon can also be observed in each curve, but the unsteady growth time is very short in some locations, which shows that the curve has a small steep drop and the crack grows rapidly. However, in the process of crack propagation, the sequence of the stages of stable and unsteady growth is not the same. In most cases, the cracks are stable, propagated and unsteady propagated alternately. Through stress reduction and crack growth, the concentrated stress is released, and the crack re-enters a stable growth, which makes the stress-strain curve fluctuate and the crack grows at a non-uniform speed.

Figure shows the relationship between the number of cracks and the strength of the sample at the peak strength of samples with different crack inclination angles. From this observation, it can be found that the total number of tensile shear cracks during failure affects the strength of the specimen. This is because the greater the number of cracks, the more obvious the weakening effect of the discontinuity on the strength of the specimen, and therefore the lower the strength of the specimen. In-depth observation can find that the total number of tensile-shear cracks and the crack strength are not strictly positively correlated. It is manifested in the abnormality of the 45° specimen compared to the 30° specimen. However, this finding just proves that the tensile crack and the shear strength are abnormal. The weakening effect of cut cracks on strength is inconsistent. Because of the comparison between the two, when the 30° specimen is broken, all are tensile cracks, part of the 45° specimen is tensile cracks, and some are shear cracks. Although the total number of cracks in the 45° sample is smaller than that of the 30° sample, the strength of the sample is smaller than the 30° sample due to the occurrence of shear failure cracks. This shows that tensile cracks and shear cracks have different effects on strength weakening, and shear cracks have more obvious effects on strength weakening.

![Figure 6](image_url)

**Figure 6.** The relationship between the number of cracks and the strength of the specimen.

5. Conclusions
This paper proposes a strain strength criterion to explain the propagation of tensile cracks and shear cracks in steel. This strain strength criterion assumes: (1) Tensile cracks grow along the direction of the maximum principal strain. When the principal strain reaches a critical value, the tensile cracks begin to grow; (2) Shear cracks grow in the direction of the most dangerous stress state. When the Mohr circle exceeds the Mohr Coulomb failure line, the shear crack starts to expand.

In addition, this paper applies the strain strength criterion to the block discrete element method to better simulate the macro-mechanical response characteristics of the steel under load and the crack propagation evolution process, which is shown (1) the stress-strain curve appears elastic and stagnant The basic laws of fluctuation and fall, and (2) the characteristics of stable and unstable crack growth during the whole loading process. In summary, after the sample is cracked, the crack will grow
steadily and unsteadily. During the stable growth stage of the tensile crack, the tensile stress continues to accumulate, and to a certain extent, the crack will grow unsteadily.

References
[1] Inglis C E. Stresses in a plate due to the presence of cracks and sharp corners [J]. Inst. Naval Arch. (London) Trans., 1913, 55: 219–230.
[2] Griffith A A. The phenomena of rupture and flow in solids [J]. Philosophical Transactions of the Royal Society of London, 1921, 221(Series A): 163-198.
[3] Griffith A A. The theory of rupture[C]. Proc. 1st Int. Congr. Applied Mech., 1924: 55-63.
[4] Sneddon I N. The distribution of stress in the neighbourhood of a crack in an elastic solid[C]/Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. The Royal Society, 1946, 187 (1009): 229-260.
[5] Irwin G R. Relation of stresses near a crack to the crack extension force [J]. 9th Cong. App. Mech., Brussels, 1957.
[6] Dugdale D S. Yielding of steel sheets containing slits [J]. Journal of the Mechanics and Physics of Solids, 1960, 8(2): 100-104.
[7] Wells A A. Critical tip opening displacement as fracture criterion[C]/Proc. Crack Propagation Symp, 1961, 1: 210-221.
[8] Rice J R. A path independent integral and the approximate analysis of strain concentration by notches and cracks [J]. Journal of Applied Mechanics, 1968, 35(2): 379-386.
[9] Erdogan F, Sih G C. On the crack extension in plates under plane loading and transverse shear [J]. Journal of Fluids Engineering, 1963, 85(4): 519-525.
[10] Wang Shuilin, Ge Xiurun, Zhang Guang. Numerical analysis of crack propagation under compression [J]. Chinese Journal of Rock Mechanics and Engineering, 1999, 18(6): 671-675.
[11] Jiao Yuyong, Zhang Xiuli, Liu Quansheng, et al. Simulating rock crack propagation by discontinuous deformation analysis method [J]. Chinese Journal of Rock Mechanics and Engineering, 2007, 26(4): 682-691.
[12] ITASCA. UDEC 4.0 manual-theory and background. ITASCA Consulting Group, 2004.
[13] ITASCA. PFC manual. ITASCA Consulting Group, 2004.