Variation of Time Domain Failure Probabilities of Jack-up with Wave Return Periods

Ahmad Idris, Indra S H Harahap and Montassir Osman Ahmed Ali
Department of Civil and Environmental Engineering, Universiti Teknologi Petronas
Bandar Universiti, Seri Iskandar, 32610, Malaysia.

Corresponding Author: aidris.civ@buk.edu.ng

Abstract: This study evaluated failure probabilities of jack up units on the framework of time dependent reliability analysis using uncertainty from different sea states representing different return period of the design wave. Surface elevation for each sea state was represented by Karhunen-Loeve expansion method using the eigenfunctions of prolate spheroidal wave functions in order to obtain the wave load. The stochastic wave load was propagated on a simplified jack up model developed in commercial software to obtain the structural response due to the wave loading. Analysis of the stochastic response to determine the failure probability in excessive deck displacement in the framework of time dependent reliability analysis was performed by developing Matlab codes in a personal computer. Results from the study indicated that the failure probability increases with increase in the severity of the sea state representing a longer return period. Although the results obtained are in agreement with the results of a study of similar jack up model using time independent method at higher values of maximum allowable deck displacement, it is in contrast at lower values of the criteria where the study reported that failure probability decreases with increase in the severity of the sea state.

Keywords: Jack up; Wave load; Sea surface elevation; Return period

1. Introduction
Jackup is a movable offshore structure used in oil and gas industry for exploration and production in relatively shallower waters of depths of up to 150 meters. The units which are towed on demand to an offshore site are installed in position and operated for a period of up to 12 months before it is removed and towed to another offshore location. During the operation, Jackup unit is expected to withstand the most severe environmental conditions expected to occur at least once in a given return period. These environmental conditions for a specific site may not possibly be captured at the design stage of the unit due to their mobility and the variations of environmental conditions between offshore locations. Consequently, before a jack up unit is offered for use in a given offshore location, the suitability of the unit for safe operation in the site is checked in accordance with the guidelines for Site Specific Assessment (SSA) of Jackup. Published by the society for naval architects and marine engineers (SNAME) [1] as well as the International standards organization (ISO) [2], the guideline for SSA of Jackup units required the evaluation of such suitability in different frameworks of safety assessment with increasing complexity. In each stage, when the unit satisfied the safety criteria for the stage, it is qualified for use in the location. When it fails, a more realistic assumption in the structure and environmental conditions is made and the unit is further assessed using the same assessment framework.
However, when the unit fails the safety criteria of the framework, it is not disqualified; rather, it is further assessed in a more complex assessment framework until a decision can be made.

One of the safety frameworks recommended for SSA of Jackup is the use of reliability theories. They are those theories that are concerned with the estimation of the probability of failure of structural element of the structure as a whole. Reliability analysis is divided into time independent and time dependent reliability approach. In time independent approach, the load is assumed to be deterministic and structural material degradation is non-existent. Statistics of peak values of loads and resistance are used to evaluate the probability that such extreme design values are not exceeded using time independent reliability solution method. In time dependent approach, it is assumed that for dynamically responding structures subjected to stochastic loading, extreme loads may not necessarily result in extreme structural response. Hence, time histories of the structural response are required to establish the probability that the structure can safely perform its function at any time during the excitation [3]. In both of the two reliability approaches, the aim is to determine the probability that a given response value does not exceed a prescribed maximum value.

Reliability analysis of Jackup units as reported in the literature such as in [4]–[6] uses the expected most severe wave in a given return period to evaluate the Jackup structural response. Peak responses due to several random waves are then used to develop the statistical distribution of extreme waves. The distribution function is then used to evaluate the probability of failure of the units. It is shown in the study of [7] that when the statistical distribution functions are used, the failure probability increases with increase in the return period at higher values of maximum allowable displacement and decreases at lower values.

This study aims to investigate the effect of increasing return period on Jackup time dependent failure probability. Different sea state parameters representing different return periods will be selected and used in time dependent reliability analysis. The wave surface elevation will be modelled using Karhunen-Loeve expansion method and the wave load will be propagated on Jackup model in time domain analysis. Structural response due to wave load will be analyzed using the VanMarckes time dependent reliability models to evaluate the failure probabilities.

2. Methodology

2.1 Wave modelling

Jackup structural design as well as SSA is performed to ensure that a unit can withstand the most severe wave event expected to occur at least once within a certain duration called return period. The SSA guideline in [2] requires that such units be assessed using a wave with a minimum return period of 50 years. A wave with a specified return period can be represented in terms of the significant wave height $H_s$ and zero crossing period $T_z$ corresponding to that return period. In this study, three different and commonly used return periods were selected and the corresponding parameters are given in table 1.

| Return Period (Years) | $H_s$ (m) | $T_z$ (m) |
|------------------------|-----------|-----------|
| 100                    | 12.00     | 10.81     |
| 1000                   | 13.25     | 11.36     |
| 106                    | 16.45     | 12.66     |
Wave surface elevation corresponding to each set of environmental parameters representing a return period was represented using Karhunen-Loeve expansion method. The procedure for sea state representation implemented in [8] is adopted. The wave profile and subsequently, wave kinematics and wave load were evaluated for the wave signal using the eigenfunctions of Prolate Spheroidal Wave Functions.

2.2 Structural Modelling

Simplified Jackup model is used in this study. The model consists of three legs and hull with uniform properties. The legs and hull are considered as beam elements with stiffness of 7.5m4 and 150m4 respectively. Mass of the hull is taken as 16x106Kg and that of the legs as 1.9x106Kg. Elastic modulus of the material is 200GPa and effect of marine growth is neglected. Full dynamic analysis was performed using numerical schemes in ANSYS software as shown in Figure 1 to obtain time dependent response of the structure due to stochastic wave loading. Dynamic analysis was performed for each of the terms in the expansion and the response at each discrete time step was analyzed to evaluate the required parameters as required by the reliability procedure.

![Figure 1. ANSYS image of the Jackup idealised model used in the study](image)

2.3 Reliability Analysis

For a discretized time interval of a stationary response process over a longer period of time, the time dependent probability of failure with outcrossing rates can be approximated as [9]

\[ P_f(t_0) = 1 - e^{-\sum_{\alpha} \lambda_\alpha(t_0)} \]  

(1)

And the probability of safe operation, which is the probability that the limit state will not be violated, can be obtained as;
\[ P_f(t_i) = 1 - P_r(t_i) \]  

(2)

Where the probability of failure at initial time is assumed to be zero and \( \alpha(t_i) \) is the decay rates obtained by modifying the rate of crossings. The Van Marckes approximation of the decay rates is can be calculated from [10];

\[ \alpha(t_i) = 2\nu_b \frac{1 - e^{-\frac{1}{2}(\nu_0)^2/r(t_i)}}{1 - e^{-\frac{1}{2}}} \]  

(3)

The parameter \( q(t_i) \) is the bandwidth parameter that describes the bandwidth of the structural response. \( \nu_b(t_i) \) Are the up crossing rates at time \( t_i \) and \( r(t_i) \) is the instantaneous reliability index at time \( t_i \). They are calculated according to [11] using Equations 4 to 6 as;

\[ q(t_i) = \sqrt{1 - \frac{a_1(t_i)^2}{a_0(t_i)a_2(t_i)}} \]  

(4)

\[ \nu_b(t_i) = \frac{1}{2\pi} \sqrt{\frac{a_2(t_i)}{a_0(t_i)}} e^{-\frac{r(t_i)^2}{2}} \]  

(5)

\[ r(t_i) = \frac{b(t_i)}{\sqrt{a_0(t_i)}} \]  

(6)

Where: \( a_0 \), \( a_1 \) and \( a_2 \) are the mean square standard deviation, covariance between the response and the derivative of its Hilbert’s transform and mean square derivative of the standard deviation at time \( t_i \) respectively. This represents the first three spectral moments of the system response [12].

3. Results and Discussions

The procedure for wave surface elevation modelling using the method of KLE with least number of independent sources of uncertainty outlined in [8] is implemented. The Slepian frequency \( C \) for each of the return periods was optimized to obtain the wave profile as shown in figure 2. The number of terms required to adequately represent the wave is selected from the number of dominant eigenvalues from the solution of the Fredholm integral equation cast with P-M spectral energy density.
Figure 2. Profile of the realization of wave surface elevation for different return periods

The simulated wave elevation was then used to develop wave kinematics and subsequently, the stochastic wave load. The load representing the worst condition for each given return period was propagated through the structure to obtain the stochastic dynamic response. The analysis of response to determine the probability of limit state exceedance was performed using the procedure outlined in section 2.3.

Figure 3. Mean crossing rates as a function of deck displacement

Figure 3 shows the mean crossing rates as a function of deck displacement for each return period. This shows the mean rate at which the limit state function crosses from safe to un-safe region by varying the maximum allowable displacement. It can be seen that the rate of crossing decreases and gradually closes in to zero as the criteria becomes large. It can also be seen that the magnitude of crossing rates becomes higher as the return period is increased due to increases in the overall magnitude of the excitation.
Time dependent failure probability using the VanMarckes approximation of crossing rates as a function of deck displacement is evaluated. This was achieved by varying the maximum allowable displacement and computing the upper bound of the failure probability for the jack up unit in deck displacement. Figure 4 shows the variation of the upper bound of probability that the deck displacement will not exceed the allowable value during the period of the excitation. It can be seen from the figure that the magnitude of the probabilities increases with increase in the return period used. The overall results obtained shows that the probabilities obtained using the time dependent frameworks are higher in magnitude than those obtained using time independent methods such as those presented in [7], [13]. This means that while the unit is considered to have failed when it is analyzed using time in-dependent reliability methods, it can be qualified for use when the time dependent method is used. However, in the time dependent method used in this study, the probability values increase with increase in return period for all the maximum allowable displacements considered. This is in contrast to the result in [7] where the probabilities decrease with increase in the return period at lower values of the allowable displacement.

4. Conclusions
This study have evaluated time dependent reliability of jack up unit using different return periods that are normally used in the analysis, design and safety assessment of the units. For each of the three return periods selected and used, the sea state parameters representing a given return period were used in simulating the wave using KLE approach. Time dependent reliability analysis was performed to evaluate the upper bound of failure probabilities for the jack up unit subjected to stochastic excitation from random wave loading representing an extreme event for a given return period. It has been shown that the magnitude of failure probabilities increases with increase in the return period for all the maximum allowable deck displacements considered.

Reference
[1] SNAME 2008 Guidelines for Site Specific Assessment of Mobile Jack-Up Units (T&R Bullitin 5-54) p 366
[2] International Standardization Organization 2009 Petroleum and natural gas industries - site-specific assessment of mobile offshore units - part 1: 19905-1

[3] Stefanou G 2009 The stochastic finite element method: Past, present and future Computer Methods in Applied Mechanics and Engineering 198 pp 1031–1051

[4] Mirzadeh J, Kimiae M and Cassidy M J 2016 Performance of an example jack-up platform under directional random ocean waves Appl. Ocean Research 54 pp 87–100

[5] Mirzadeh J, Kimiae M and Cassidy M J 2015 A framework to efficiently calculate the probability of failure of dynamically sensitive structures in a random sea Ocean Engineering 110 pp 215–226

[6] Azadi M R E A 2012 The Effect of Possible Spud-Can Punch Through on the Reliability Index of Neka Drilling Type Jack-up Platform Journal of Civil Engineering Science 1 pp 80–89

[7] Cassidy M J, Taylor P H, Eatock Taylor R and Houlsby G T E 2001 Evaluation of long-term extreme response statistics of jack-up platforms Ocean Engineering 29 pp 1603–1631

[8] Idris A, Harahap I and Ali M 2017 Efficiency of Trigonometric and Eigen Function Methods for Simulating Ocean Wave Profile Indian Journal of Science and Technology 10

[9] Ditlevsen O 1986 Duration of visit to critical set by Gaussian process Probabilistic Engineering Mechanics 1 pp 82–93

[10] Vanmarcke E H 1975 On the Distribution of the First-Passage Time for Normal Stationary Random Processes Journal Applied Mechanics 42 p 215

[11] Valdebenito M A, Jensen H A, Labarca, He J, Au S K and Beck J L 2014 Estimation of first excursion probabilities for uncertain stochastic linear systems subject to Gaussian load Probabilistic Engineering Mechanics 24 pp 418–425

[12] He J 2009 Numerical calculation for first excursion probabilities of linear systems Probabilistic Engineering Mechanics 24 pp 418–425

[13] Cassidy M J 1999 Non-Linear Analysis of Jack-Up Structures Subjected to Random Waves p 230