Jupiter’s Dynamical Love Number

Dong Lai

Cornell Center for Astrophysics and Planetary Science, Department of Astronomy, Cornell University, Ithaca, NY 14853, USA

Received 2021 March 11; revised 2021 May 8; accepted 2021 May 12; published 2021 July 1

Abstract

Recent observations by the Juno spacecraft have revealed that the tidal Love number \( k_2 \) of Jupiter is 4% lower than the hydrostatic value. We present a simple calculation of the dynamical Love number of Jupiter that explains the observed “anomaly.” The Love number is usually dominated by the response of the (rotation-modified) f-modes of the planet. Our method also allows for efficient computation of high-order dynamical Love numbers. While the inertial-mode contributions to the Love numbers are negligible, a sufficiently strong stratification in a large region of the planet’s interior would induce significant g-mode responses and influence the measured Love numbers.

Unified Astronomy Thesaurus concepts: Solar system gas giant planets (1191)

1. Introduction

The Juno spacecraft recently found an “anomaly” in Jupiter’s tidal Love number: the measured \( k_2 \approx 0.565 \pm 0.006 \) (Durante et al. 2020) appears to be smaller than the theoretical hydrostatic value \( k_2^{(\text{th})} = 0.590 \) (Wahl et al. 2020) by 4%. This discrepancy may be explained in terms of dynamical tides, i.e., Jupiter’s response to the finite-frequency tidal forcings from the Galilean moons (Idini & Stevenson 2021). Here we present a simple calculation that explains this Love number “anomaly” quantitatively. Naive expectation would suggest a \( 1/(\omega^2 - \omega^2) \) enhancement (where \( \omega_{\text{f}} \) is the f-mode frequency of the planet) of the tidal response due to the finite tidal frequency \( \omega \) as compared with the hydrostatic \( \omega = 0 \) response. This expectation is incorrect because it does not include a proper treatment of the rotational (Coriolis) effects on the modes and their tidal responses in a rotating planet. We present a simple, rigorous treatment in this paper (see Idini & Stevenson 2021 for an alternative, lengthy treatment). Our general method also allows for efficient computation of high-order dynamical Love numbers \( k_{lm} \) as well as the inclusion of the contributions to \( k_2 \) from the inertial modes (due to planetary rotation) and g-modes (due to stable stratification in the planetary interior).

2. Dynamical Love Number and Normal Modes

Consider a planet (mass, \( M \); radius, \( R \); and spin angular frequency, \( \Omega_\oplus \)) orbited by a satellite (mass \( M' \)) in a circular orbit with semimajor axis \( a \) and orbital frequency \( \Omega_{\text{orb}} \). We assume the spin axis is aligned with the orbital axis. In the frame corotating with the planet, the \((lm)\)-component of the tidal potential produced by \( M' \) on the planet is

\[
U(r, t) = -A_{lm} r^l Y_{lm}(\theta, \phi) e^{-i\omega t},
\]

where \( A_{lm} = (GM'/d^l+1)W_{lm} \) (with \( W_{lm} \) a dimensionless constant; \( W_{lm} = 0 \) when \( l + m = \text{even} \)), \( r = (r, \theta, \phi) \) specifies the position vector (in spherical coordinates) measured from the center of the planet, and

\[
\omega = m(\Omega_{\text{orb}} - \Omega_\oplus)
\]

is the tidal forcing frequency. It suffices to consider only \( m > 0 \). The relevant nonzero tidal components are \((lm) = (2, 2), (3, 1), (3, 3), (4, 2), (4, 4)\) etc.

The linear response of the planet to the tidal forcing is specified by the Lagrangian displacement, \( \xi(r, t) \), of a fluid element from its unperturbed position. In the rotating frame of the planet, the equation of motion takes the form

\[
\frac{\partial^2 \xi}{\partial t^2} + 2\Omega_{\text{orb}} \times \frac{\partial \xi}{\partial t} + C \cdot \xi = -\nabla U,
\]

where \( C \) is a self-adjoint operator (a function of the pressure and gravity) acting on \( \xi \) (see, e.g., Lynden-Bell & Ostriker 1967). A free mode of frequency \( \omega_\alpha \) (in the rotating frame) with \( \xi_\alpha(r, t) = \xi_\alpha(r) e^{-i\omega_\alpha t} \propto e^{im\theta-i\phi t} \) satisfies

\[
-\omega_\alpha^2 \xi_\alpha - 2i\omega_\alpha \Omega_{\text{orb}} \times \xi_\alpha + C \cdot \xi_\alpha = 0,
\]

where \( \alpha \) denotes the mode index, which includes the azimuthal number \( m \). We carry out phase-space mode expansion (Schenk et al. 2002)

\[
\left[ \frac{\partial}{\partial \xi / \partial t} \right] \xi_\alpha(r, t) = \sum_{\alpha'} c_{\alpha\alpha'}(r) \left[ \xi_{\alpha'}(r) - i\omega_{\alpha'} \xi_{\alpha}(r) \right],
\]

where \( c_{\alpha\alpha'}(r) \) is the mode amplitude. Using the orthogonality relation \( \langle \xi_\alpha, 2i\Omega_{\text{orb}} \times \xi_{\alpha'} \rangle + (\omega_\alpha + \omega_{\alpha'}) \langle \xi_\alpha, \xi_{\alpha'} \rangle = 0 \) (for \( \alpha \neq \alpha' \)), where \( \langle A, B \rangle = \int d^3 r \rho(A \cdot B) \), we find (Lai & Wu 2006)

\[
\frac{d\xi_{\alpha\alpha}}{dt} + i\omega_{\alpha\alpha} \xi_{\alpha\alpha} = \frac{iQ_{\alpha\alpha,lm}}{2\xi_{\alpha\alpha}} A_{lm} e^{-i\omega_{\alpha\alpha} t},
\]

where

\[
Q_{\alpha\alpha,lm} = \langle \xi_\alpha, \nabla(r^l Y_{lm}) \rangle = \int d^3 r X_{lm}(\delta_\alpha^\alpha)^* \bar{X}_{lm},
\]

\[
\bar{X}_{lm} \equiv \omega_{\alpha\alpha} + \langle \xi_\alpha, \Omega_{\text{orb}} \times \xi_{\alpha'} \rangle,
\]

and we have used the normalization \( \langle \xi_\alpha, \xi_{\alpha'} \rangle = 1 \). In Equation (7), \( \delta_\alpha^\alpha = -\nabla \cdot (\rho_\alpha \bar{X}_{\alpha\alpha}) \) is the Eulerian density perturbation associated with the eigenfunction \( \xi_\alpha \).
Equation (6) has the solution

\[ c_\alpha(t) = \frac{Q_{\alpha, lm}}{2\varepsilon_\alpha(\omega_\alpha - \omega)} A_{lm} e^{-i\omega t}. \]  

(9)

The gravitational perturbation associated with the density perturbation \( \delta \rho(r, t) = \sum\alpha c_\alpha(t) \delta \rho_\alpha(r) \), evaluated at the planet’s surface \( (r = R) \), is

\[ \delta \Phi(r, t)|_{r=R} = -\sum\alpha c_\alpha(t) \frac{4\pi}{2l+1} \frac{GMQ_{\alpha, lm}}{R} Y_{lm}. \]  

(10)

Thus, the tidal Love number is

\[ k_{lm} = \frac{\delta \Phi}{U} \bigg|_{r=R} = \frac{2\pi}{2l+1} \sum \frac{\bar{Q}_{\alpha, lm}^2}{\varepsilon_\alpha(\bar{\omega}_\alpha - \bar{\omega})}. \]  

(11)

In the above equation, the tidal overlap coefficient \( Q_{\alpha, lm} \) and the mode frequencies \( \bar{\omega}_\alpha \) and \( \bar{\varepsilon}_\alpha \) are in units where \( G = M = R = 1 \), i.e., \( \bar{\omega}_\alpha = \omega_\alpha/\sqrt{GM/R^3} \), etc. Note that for a given \( m > 0 \), the sum in Equation (11) includes modes with positive \( \omega_\alpha \) and negative \( \omega_\alpha \), corresponding to prograde (with respect to the planet’s rotation) and retrograde modes.

3. F-mode Contribution

Although Equation (11) is exact, computing the mode parameters for a rotating planet model generally requires numerical calculations. In most situations, the sum in Equation (11) is dominated by f-modes as they have the largest tidal overlap \( Q_{\alpha, lm} \). For planetary rotation rate \( \Omega_\alpha \) much less than the breakup rate \( \sqrt{GM/R^3} \), (e.g., \( \bar{\Omega}_\alpha = 0.288 \) for Jupiter), the effect of rotation on the modes can be treated perturbatively (e.g., Unno et al. 1989). Let \( \omega_\alpha (> 0) \) be the mode frequency of a nonrotating planet, then for a given \( m > 0 \), the sum in Equation (11) includes

\[ \varepsilon_\alpha \simeq \pm \omega_0, \]

\[ \omega_\alpha = \pm \omega_0 - mC\Omega_\alpha, \]

(12)

(13)

with

\[ mC = \int_0^\infty 3\rho \xi_0 (\xi_0 \times \xi_0) \]  

(14)

\[ = \int_0^R dr \rho r^2(2\xi_0 \xi_\perp + \xi_\perp^2). \]

where \( \xi_0(\mathbf{r}) = \{ \xi_0(r) \mathbf{r} + \xi_\perp(r) \nabla \} Y_{lm} \) is the mode eigenvector of a nonrotating planet. To a good approximation, we can also set \( Q_{\alpha, lm} \) (for the \( \{ \alpha \} = \{ lm \} \) mode) to be the nonrotating value, i.e.,

\[ Q_{\alpha, lm} \simeq Q_l. \]  

(15)

Thus, Equation (11) reduces to

\[ k_{lm} \simeq \left( \frac{2\pi}{2l+1} \right) \frac{Q_l^2}{\omega_0^2 - (mC\Omega_\alpha + \bar{\omega})^2}. \]  

(16)

For an incompressible planet model \( (n = 0 \) polytrope), the \( l = 2 \) mode (Kelvin mode) has

\[ \bar{\omega}_0 = \frac{2}{\sqrt{5}}, \quad Q_2 = \frac{3}{2\pi} \left( \frac{1}{2} \right)^{1/2}, \quad C = \frac{1}{2}. \]  

(17)

### Table 1

| Oscillation Modes of a Nonrotating Polytropic \( (n = 1) \) Planet Model |
|-----------------------------|--------|--------|
| \( \omega_l \) | \( Q_l \) | \( C \) |
| \( \Gamma_1 = 2 \) | \( l = 2 \) | \( f \) | 0.1227E+01 | 0.5579E+00 | 0.4991E+00 |
| \( p1 \) | 0.3462E+01 | 0.269E-01 | 0.1119E+00 |
| \( p2 \) | 0.397E-01 | 0.405E-01 | 0.8627E-01 |
| \( l = 4 \) | \( f \) | 0.2037E+01 | 0.5979E+00 | 0.2489E+00 |
| \( p1 \) | 0.4409E+01 | 0.4623E-01 | 0.6984E-01 |
| \( \Gamma_1 = 2.4 \) | \( l = 2 \) | \( f \) | 0.1230E+01 | 0.5580E+00 | 0.4985E+00 |
| \( g1 \) | 0.468E+00 | -0.131E-01 | 0.1057E+00 |
| \( g2 \) | 0.327E+00 | 0.307E-02 | 0.1342E+00 |
| \( g3 \) | 0.252E+00 | -0.8961E-03 | 0.1464E+00 |
| \( l = 3 \) | \( f \) | 0.170E+01 | 0.585E+00 | 0.3317E+00 |
| \( g1 \) | 0.568E+00 | -0.1216E-01 | 0.3321E-01 |
| \( g2 \) | 0.411E-00 | 0.325E-02 | 0.5622E-01 |
| \( g3 \) | 0.325E+00 | -0.1031E-02 | 0.6603E-01 |

Note. \( \omega_0 \) and \( Q_l \) are the mode frequency and tidal overlap coefficient (Equation (7)), both in units such that \( G = M = R = 1 \), and \( C \) is defined in Equation (14). The planet’s density profile is that of \( n = 1 \) polytrope (with the equation of state \( P \propto \rho^2 \)). The first model has \( \Gamma_1 \) (the adiabatic index) equal to \( \Gamma = 1 + 1/n \), and we list the properties for the f-mode and the first radial-order p-mode. The second model has \( \Gamma_1 = 2.4 \) throughout the planet, and the third model has \( \Gamma_1 = 2.4 \) only in two regions \( \mathcal{R}/\mathcal{R} \) (0.5, 0.7), \( \mathcal{R}/\mathcal{R} \) and \( \Gamma_1 = 2.4 \) otherwise (the transition width is 0.025R; see Equation (27)), and we list the properties for the f-mode and the first three radial-order g-modes. Note that when \( |\Omega_\perp| \ll 1 \), the quoted \( Q_l \) values are only accurate in 2–3 significant figures.

Thus,

\[ k_2 \equiv k_{22} \simeq \frac{3}{2} \left[ 1 - \frac{5}{4}(\Omega_\perp + \bar{\omega})^2 \right]^{-1} \]

(18)

with \( \bar{\omega} = 2(\bar{\Omega}_\perp - \bar{\Omega}_0) \).

Giant planets are approximately described by a \( n = 1 \) polytrope (corresponding to \( P \propto \rho^2 \)). Table 1 lists the numerical values of \( \omega_0, Q_l \) and \( C \) for several nonrotating polytropic models (with different levels of stratification; see Section 5). These are computed using the standard method of asteroseismology (e.g., Unno et al. 1989). For \( l = m = 2 \) tidal resonance (and \( n = 1 \)), \( 2C \approx 1 \), we have

\[ k_2 \simeq 0.520 \left[ 1 - 0.664(\bar{\Omega}_\perp + \bar{\omega})^2 \right]^{-1}. \]

(19)

Applying to the Jupiter–Io system: Jupiter has \( \bar{\Omega}_0 = 0.288 \) (with spin period 9.925 hr and \( 2\pi(R^3/GM)^{1/2} = 2.863 \) hr), Io has \( \bar{\Omega}_\perp = 0.0674 \) (orbital period 1.769 days), so \( \bar{\omega} = -0.441 \). Thus the hydrostatic and dynamical \( k_2 \) values are

\[ k_2^{(hs)} = 0.550, \quad k_2 = 0.528 = 0.960 k_2^{(hs)}. \]

(20)

This explains the 4% discrepancy between \( k_2 \) and \( k_2^{(hs)} \). Note that our static \( k_2^{(hs)} \) does not agree with the value (0.590) from
Wahl et al. (2020). This could arise for two reasons: (i) the simple \( n = 1 \) polytropic model does not precisely represent Jupiter’s internal structure and (ii) in deriving Equation (16), we have neglected order \( \tilde{\Omega}_s^2 \) corrections to the mode frequency and the tidal overlap, including those from the oblate planet shape.\(^1\)

Using the data from Table 1, we can easily check that the contributions from p-modes to \( k_2 \) (and \( k_{lm} \)) are negligible.

Figure 1 shows the dynamical corrections \( \delta_{lm} \equiv \Delta k_{lm}/k_{lm} = (k_{lm} - k_{lm}^{(0)})/k_{lm}^{(0)} \) to Jupiter’s tidal Love number as a function of the orbital frequency \( \Omega_{orb} \) of the perturbing satellite (in units of the spin frequency \( \Omega_s \)). All results (solid curves) are computed using the \( n = 1 \) (isentropic) polytropic model, except that the dotted-dashed curve is for \( k_3 = k_{32} \) computed using the \( n = 0.9 \) polytropic model. The vertical dashed lines specify the orbital frequencies of Io, Europa, Ganymede, and Callisto (from right to left). The \( \sim 4\% \) ”anomaly” of \( k_2 \) observed by Juno can be explained by the planetary model with \( n \approx 1 \).

Figure 1. Dynamical correction \( \Delta k_{lm}/k_{lm} = (k_{lm} - k_{lm}^{(0)})/k_{lm}^{(0)} \) to Jupiter’s tidal Love number as a function of the orbital frequency \( \Omega_{orb} \) of the perturbing satellite (in units of the spin frequency \( \Omega_s \)). All results (solid curves) are computed using the \( n = 1 \) (isentropic) polytropic model, except that the dotted-dashed curve is for \( k_3 = k_{32} \) computed using the \( n = 0.9 \) polytropic model. The vertical dashed lines specify the orbital frequencies of Io, Europa, Ganymede, and Callisto (from right to left). The \( \sim 4\% \) ”anomaly” of \( k_2 \) observed by Juno can be explained by the planetary model with \( n \approx 1 \).

For \( n = 1 \) polytrope, this gives the hydrostatic values

\[
k_{lm}^{(0)} \approx 0.550, 0.213, 0.219, 0.121, 0.123
\]

for \((lm) = (22), (31), (33), (42), (44)\), \(2\) though these hydrostatic values may not correspond to the “true” values for Jupiter because of the simplicity of the polytropic model and the \( \tilde{\Omega}_s^2 \) corrections (see above), the dynamical corrections \( \delta_{lm} \) shown in Figure 1 are likely robust.

Our results depicted in Figure 1 can be compared with those of Idini & Stevenson (2021) obtained using much more complicated calculations (see their Table 2). Our \( \delta_{22}, \delta_{33} \) and \( \delta_{42} \) values (evaluated for the orbital frequencies of Io, Europa, Ganymede, and Callisto) agree reasonably well with theirs, but our \( \delta_{31}, \delta_{42} \) values are a factor of a few smaller.

As the results of this section are based on a perturbative treatment of the rotational effects on f-modes, we may ask to what extent the results are modified when a more accurate treatment of f-modes is used. As an example, in computing \( k_{42} \), we have only included the f-mode with \( l = 4, m = 2 \). But because of rotation, the \((42)\)-potential also couples to the \( l = 2, m = 2 \) f-mode, with the coupling coefficient \( Q_{22,42} \sim \tilde{\Omega}_s^2 \) (\( \sim 0.08 \) for Jupiter). Thus we expect that our \( k_{42} \) value to be accurate at the 10% level. The errors in other \( k_{lm} \) are expected to be smaller. These estimates are borne out in quantitative calculations (J. Dewberry & D. Lai 2021, in preparation).

4. Inertial-mode Contribution

In addition to f-modes and p-modes, a rotating planet possesses a spectrum of inertial modes supported by Coriolis force.

For \( n = 1 \) polytrope, the \( m = 2 \) inertial modes have been computed by Xu & Lai (2017) using a spectral code. The mode properties are

\[
\omega_+ = 0.556\Omega_s, \quad \varepsilon_+ = 0.28\Omega_s, \quad \tilde{\Phi}_+ = 0.015\tilde{\Omega}_s^2
\]

for the prograde mode, and

\[
\omega_- = -1.10\Omega_s, \quad \varepsilon_- = -0.55\Omega_s, \quad \tilde{\Phi}_- = 0.010\tilde{\Omega}_s^2
\]

for the retrograde mode, where \( \tilde{\Phi}_\pm \) is the tidal coupling coefficient \( Q_{n,22} \). As \( \omega < 0 \), we can write the inertial-mode contribution of \( k_2 \) as

\[
k_{2,lm} = \frac{2\pi}{5} \left[ \frac{\tilde{\Phi}_+^2}{\varepsilon_+^2 (\omega_+^2 + |\omega|)} + \frac{\tilde{\Phi}_-^2}{\varepsilon_-^2 (|\omega_-|^2 - |\omega|)} \right].
\]

Define \( \tilde{\omega} \equiv \omega/\Omega_s \) (and similarly \( \varepsilon_\pm \) and \( \tilde{\Phi}_\pm \)) and \( \tilde{\Phi}_\pm \equiv \tilde{\Phi}_\pm/\tilde{\Omega}_s^2 \), we have

\[
k_{2,lm} = \frac{2\pi}{5} \tilde{\Omega}_s^2 \left[ \frac{\tilde{\Phi}_+^2}{\varepsilon_+^2 (\omega_+^2 + |\omega|)} + \frac{\tilde{\Phi}_-^2}{\varepsilon_-^2 (|\omega_-|^2 - |\omega|)} \right].
\]

For \( n = 1 \) polytrope, this gives

\[
k_{2,lm} = \frac{2\pi}{5} \tilde{\Omega}_s^2 \times 10^{-4} \left[ 8.04 \left( \frac{0.556}{\tilde{\omega}} + \frac{1.82}{1.10 - \tilde{\omega}} \right) \right].
\]

For Jupiter–Io system, \( \tilde{\omega} = 2\Omega_{orb}/\Omega_s = -2 = -1.53 \), it is clear that \( k_{2,lm} \ll 1 \). In general, unless \( |\omega/\Omega_s| \) happens to be very close to 1 (within \( 10^{-3} \)), the contribution of the inertial modes to the Love number is negligible.

5. Stable Stratification and G-mode Contribution

In Sections 3 and 4 we considered fully isentropic models for Jupiter, i.e., the adiabatic index \( \Gamma_1 \equiv (\partial \ln P/\partial \ln \rho)_k \), equals the
include only f-modes and possesses g-modes. The heavy solid curves include the contributions of f-modes and first three radial-order g-modes to $k_{lm}$, the light solid curves include only f-modes (for the $n = 1$ isentropic model, as in Figure 1).

polytropic index $\Gamma \equiv d \ln P/d \ln \rho = 1 + 1/n$. In reality, some regions of the planet may be stably stratified, with $\Gamma_1 \geq \Gamma$. Indeed, the gravity measurement by Juno and structural modeling suggest that Jupiter have a diluted core and a total heavy-element mass of 10–24 Earth masses, with the heavy elements distributed within an extended region covering nearly half of Jupiter’s radius (Wahl et al. 2017; Debras & Chabrier 2019; Stevenson 2020). The composition gradient outside the diluted core would provide stable stratification, and the planet would then possess g-modes. Another stable region may exist between $0.8 - 0.9 R$ and $0.93 R$ (Debras & Chabrier 2019).

To explore of how g-modes influence the tidal love numbers, we consider three simple planetary models, all having a $n = 1$ density profile ($\Gamma = 2$), but with different adiabatic index profiles: (i) $\Gamma_1 = 2.4$ throughout the planet (see Table 1); (ii) $\Gamma_1 = 2.4$ only in the stable region $r = r/R \in [0.5, 0.7]$ (with a transition width of 0.025) and $\Gamma_1 = \Gamma = 1 + 1/n$ otherwise, i.e.,

$$\Gamma_1(r) = 2 + \frac{0.4}{[1 + e^{40(r-0.7)}][1 + e^{-40(r-0.5)}]} \quad (27)$$

(iii) $\Gamma_1 = 2.4$ only in two stable regions $r = r/R \in [0.5, 0.7]$ and $[0.85, 0.93]$ (with a transition width of 0.025) and $\Gamma_1 = \Gamma = 1 + 1/n$ otherwise (see Table 1).

For each model, we compute the f-modes and g-modes of a nonrotating planet (see Table 1), and use Equations (12)–(13) to account for the effect of rotation on the modes. We include only the first three radial-order g-modes in our calculation of $k_{lm}$. The perturbative approach of the rotational effect is approximately valid for these modes as $mC\Omega_\ast$ is less than the mode frequency $|\omega|_m$.

Figures 2–4 show the results for the dynamical Love numbers based on the three models. It is obvious that significant dynamical correction to the hydrostatic $k_{lm}^{(0)}$ occurs around the resonance, where $\omega_is = \omega$. The “strength” of each resonance is measured by the tidal overlap coefficient, and a large $|Q|$ value implies that the “width” of the resonant feature is larger (see Equation (16)). For the $\Gamma_1 = 2.4$ model, the stratification is strong, the broad/strong resonance with the $g_1$ mode can affect $k_{lm}$ associated with the Galilean moons (Figure 2). For the model with the stable stratified region restricted to $r/R \in [0.5, 0.7]$ (Figure 3), the resonance feature is much weaker/narrower, but still $\Delta k_2/k_2^{(0)}$ becomes $-5\%$ for Io. When the model further includes the stratified region at $r/R \in [0.85, 0.93]$ (Figure 4), the resonance features shift and broaden, and $k_{31}$ becomes affected for the Galilean moons.
Obviously, these results are for illustrative purpose, but they indicate that resonance features due to stable stratification in the planet’s interior may influence the measured dynamical Love numbers. Note that Figures 2–4 do not include contributions from high-order g-modes. These modes (with mode frequencies comparable to $\Omega_2$) become mixed with inertial modes (so-called “inertial-gravity” modes; see Xu & Lai 2017) and cannot be treated using Equations (12)–(13). However, because of their small tidal overlap coefficients, they are unlikely to be important contributors to $k_{lm}$ except for the coincidence of an extremely close resonance.

6. Conclusion

We have derived a general, exact equation (Equation (11)) for computing the dynamical Love number $k_{lm}$ of a rotating giant planet in response to the tidal forcings from its satellites. In most situations, the Love number is dominated by the tidal response of f-modes, and the general expression reduces to Equation (16), which can be easily evaluated using the mode properties of nonrotating planet models (see Table 1). We show that the 4% discrepancy between the measured $k_2$ of Jupiter and the theoretical hydrostatic value can be naturally explained by the dynamical response of Jupiter’s f-modes to the tidal forcing from Io—the key is to include the rotational (Coriolis) effect in the tidal response in a self-consistent way. We also show that the contributions of the inertial modes to the Love number $k_2$ are negligible.

We have also explored the effect of stable stratification in Jupiter’s interior on the Love numbers. If sufficiently strong stratification exists in a large region of the planet’s interior, g-mode resonances may influence the dynamical Love numbers associated with the tidal forcing from the Galilean moons. Thus, precise measurements of various $k_{lm}$ could provide constraints on the planet’s interior stratification. We plan to explore this issue further in a future paper (J. Dewberry & D. Lai 2021, in preparation).

D.L. thanks Ben Idini and Dave Stevenson for useful discussion/communication. This work has been supported in part by NASA grant 80NSSC19K0444.

ORCID iDs
Dong Lai @ https://orcid.org/0000-0002-1934-6250

References
Debras, F., & Chabrier, G. 2019, ApJ, 872, 100
Durante, D., Parisi, M., Serra, D., et al. 2020, GeoRL, 47, e86572
Ho, W. C. G., & Lai, D. 1999, MNRAS, 308, 153
Idini, B., & Stevenson, D. J. 2021, PSJ, 2, 69
Lai, D., & Wu, Y. 2006, PhRvD, 74, 024007
Lynden-Bell, D., & Ostriker, J. P. 1967, MNRAS, 136, 293
Schenk, A. K., Arras, P., Flanagan, E. E., Teukolsky, S. A., & Wasserman, I. 2002, PhRvD, 65, 024001
Stevenson, D. J. 2020, AREPS, 48, 465
Unno, W., Osaki, Y., Ando, H., Saio, H., & Shibahashi, H. 1989, Nonradial Oscillations of Stars (Tokyo: Univ. Tokyo Press)
Wahl, S. M., Parisi, M., Folkner, W. M., Hubbard, W. B., & Militzer, B. 2020, ApJ, 891, 42
Wahl, S. M., Hubbard, W. B., Militzer, B., et al. 2017, GeoRL, 44, 4649
Xu, W., & Lai, D. 2017, PhRvD, 96, 083005