Time Dilation contra Hamiltonian Mechanics

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Abstract. It is observed that, the introduction of delayed electromagnetic interaction to account for the finite speed of “light” (actually all electromagnetic interaction) leads to different time variables where non relativistic mechanics has only one. By carefully identifying and taking these various times into account, solutions are found for certain conundrums, including in particular the twin of clock paradox. An important consequence of this analysis is that it leads to a direct generalization of mechanics including special relativistic effects.

1. The Issue
Most often time dilation in Special Relativity (SR) is so discussed as to impart the impression that it is indeed mysterious, but in the end, an inevitable physical effect with a rigorously logical explanation. Nevertheless, despite this mostly socially imposed mandate, new physics students and sceptically inclined veterans continue to raise objections. This included, surprisingly for many, an originator and the main proponent of time dilation, namely: Albert Einstein.[1] Einstein first proposed in 1905 that time dilation and what became known as Lorentz-Fitzgerald space contraction are kinematic consequences of the structure of Lorentz Transforms. At that time this assertion was found by many to be amazing and mystical, but just another “Natural Wonder.” Nevertheless, 13 years later, Einstein was moved to publish an “explanation” for the effect based, not on kinematics at all as in his first paper, but based on the dynamics involved in accelerations at the start, reversal and termination of the traveling twin.[2] From the structure of the argument in this latter paper, it is clear that Einstein felt obliged to respond to critics with serious professional status, because he had lost confidence in the persuasiveness of his original arguments. Still, he advanced an explanation based on dynamics years after von Laue[3] responding to Langevins published arguments to the effect that the magnitude of a time dilation is proportional to the duration of uniform flight, i.e., to kinematics, and not that of the of any acceleration involved.[4]

Moreover, a consequence of Special Relativistic time dilations, as generally understood nowadays, is that the motion of interacting systems, for example a collection of charged particles, cannot be described by some sort of generalized Hamiltonian Mechanics, because the various interacting particles do not have a common proper-time variable to serve as a system indexing parameter in the manner that time plays in this role in non-relativistic mechanics. 1

Insofar as the essential feature introduced into mechanics by Special Relativity is the delay in the interaction (or force between) between particles due to the finite velocity of light (actually

1 The literature is replete with conundrums associated with direct-interaction formulations of relativistic mechanics. Many problems result from various arbitrary choices, for example, the inclusion of advanced interaction without empirical support.
the speed of any electromagnetic interaction) as encoded by Coulomb’s Law: \( F = \frac{Fe_1e_2}{r_{12}^2} \) where \( r_{12} \) is to be evaluated on the past light light cone of each absorbing particle. Now, if there exists no possible relativistic formulation of Hamiltonian Mechanics, this follows only when by introducing “time of flight delay” as due to a finite “speed of light,” somehow the fundamental assumption upon which Hamiltonian Mechanics was derived is rendered invalid. The relevant fundamental assumption was one way or another based on the Principle of Conservation of Energy, which, in other words, comprises the assumption that all work done by or on any other particles in the system must be sourced within the system under consideration; i.e., there is no influence by or on any of the system’s particles from or to the exterior of the system.[5] Now, it is arguably unreasonable to suppose that introducing interaction delay must, or could even, break this conservation principle. In turn, this consideration is tantamount to asserting that there must exist a relativistic formulation of Hamilton Mechanics satisfying this principle. Rejecting it, as is nowadays customary, on the other hand, must be a consequence of a serious misunderstanding pertaining to the current interpretation or employ of Special Relativity. The task remains to identify it.

These arguments are supported by the fact that no experiment to date has verified the physical sister-effect to time dilation, namely: Lorentz-Fitzgerald contraction. Furthermore, experiments widely held to verify time dilation by demonstration, namely by employ of muon decay in the atmosphere or in the storage ring at CERN, do not logically “prove” the reality of this effect. To do so would require that there exists no alternate explanation for the observations, which, being a negative statement is in principle unprovable. The fact is, in the final analysis, these experiments count photo-detections of, in addition to primary, also secondary particles, the generation of which is very difficult to control or even measure. For these experiments, however, it is necessary that all possible alternate explanations be excluded. Arguably, this has not been accomplished, especially in view of the conflicting issues outlined above.

2. The cause

Analysis of the technicalities involved in integrating relativistic equations of motion leads to the following insight: Every instantaneous electromagnetic interaction is specified by three different time instants: the instant of emission (in the past), the instant of absorption/reception (at the present) and a universal variable or system time which serves as a index. For systems of classical mechanics within which it is taken that the interactions are simultaneous, all three of these instances are equal. On the other hand, for relativistic systems, the instants of emission for the various interactions are all in the past, while the instants of reception can be arbitrary so long as they are such that they are located at intersections of the past light cone with the orbit for each other particle in the system under consideration. Naturally, the most convenient choice for these instants is that they are all equal to an indexing parameter’s instantaneous value. The only restraint here, however, is that the past orbits of all particles in the system must be known (i.e.; previously calculated).

Given these distinctions, it is arguable that, the mysterious logical conflicts described above arise from failing to take into consideration the exact identity of a particular time instant under consideration, thereby admitting an invalid substitution or misidentification. An example of such erroneous analysis or misinterpretation of a physical quantity which can be illustrated on a Mankowski-chart, is the perpetual-puzzle, namely, the clock or “twin” paradox.

This chart is composed of two overlaid charts, one for the fixed frame (stay-at-home twin) and another for the traveling twin. Overlaying these two charts is advantageous because it enables a direct comparison of the relative time intervals for both parties. What is to be seen from this combination chart is that if one mistakenly takes the time of flight for the traveler as the difference between the start of the trip and the instant at which the traveler intersects the
MINKOWSKI CHARTS FOR RELATIVE MOTION

Figure 1. Overlaid Minkowski charts for one leg of the twin’s journey showing that in order to display a dilated time for the traveler the time of intersection of the traveler with the turn-around point must (incorrectly) be read from the intersection of the traveler’s orbit with the world line of the turn-around point on the fixed chart.

world line of the the turnaround point as depicted on the fixed chart, the result is the traditional “dilated” interval. However, the traveler’s motion is not to be found on the fixed chart (he’s not fixed!) but on his own, i.e., moving or “traveling chart. On this chart the world line of the turnaround point is displaced away from the starting point in accordance with the displacement of every point on that chart as given by the Lorentz factor and as portrayed on these charts as found on the eigenlength isobars (hyperbolas). In short, this analysis shows that “time dilation” is a (relativistic) perspective effect seen by the fixed twin. It alters the appearance of relatively moving entities but not their ontic substance. Whatever his reports sent back to the stay-at-home twin show, when the traveler actually returns, after deceleration so that he too in present in the fixed frame again, his biological (proper) age will equal that of his twin sibling.

In short, this analysis leads to the insight that, the Lorentz Group structure pertains to the event coordinates of the reception of electromagnetic interaction and not those of the emission events.

One more or less obvious implication of this structure is a (idealized) means of calibrating a system of clocks extending over an arbitrary although practically limited, region of space. The key concept for a coordination method involves distributing identically built and functioning
clocks and instruments to measure the intensity of electromagnetic pulses emitted by an arbitrary standard clock. Then, the standard clock is to broadcast the intensity of its calibration pulses at the source to all other clocks. Thereafter, the diminished force of the pulse as measured at a clock to be coordinated with the standard is then used in Coulomb’s Law, \( F = kq_1q_2/r \), to find the value of \( r \) and time of flight, \( t \) as given by \( t = r/c \). The \( t \)'s so calculated are the transmission delay times, and give the absolute time as defined by the standard clock at which the signals were emitted.

3. A consequence

A consequence of the current understanding and usage of the Lorentz-Group structure is that, this structure does not admit a Hamiltonian formulation of relativistic mechanics. This assertion is based on the notion that, there does not exist an integrable formulation of relativistic mechanics for a system of, for example, point particles involving a set of coupled equations of motion: one equation for each particle. As is well known, there are several versions of action-at-distance mechanics to be found in the literature, but when examined in detail carefully, it can be seen that they include not only delayed interaction, as richly empirically verified, but also advanced interaction; i.e., on the forward light-cone from the future, as is utterly unverified empirically. In addition, it is asserted that, time relativistic time dilation, the amount for each particle of system being different, renders the structure self-inconsistent.

However, it is the contention herein that, by carefully taking the identity of the various symbols for ‘time, in particular its meaning within the formulations as time-of-emission, time-of-absorption and time-of-observation, it is seen that these objections in fact do not pertain. In non-relativistic mechanics, because the interaction speed is considered to be infinite, these three different ‘times are all equal; a fact that renders significant simplifications. The view proposed here is that, the times of absorption and the time experienced by an observer are equal, but the times of emission for interactions are delayed.

Thus consider the following Lagrangian (in retrospect, its rationalization is obvious):\(^2\)

\[
\mathcal{L} = \sum_{i}^{N} \left( -m_i c (\mathbf{x}_i \cdot \mathbf{x}_j)^{1/2} - e_i \sum_{j \neq i} e_j \int_{-\infty}^{\tau} \delta (\mathbf{x}_i (\tau) - \mathbf{x}_j (\tau))^2 \right) d\tau. \tag{1}
\]

By executing a variation, \( \delta \int \mathcal{L} d\tau = 0 \), the following equations of motion are deduced:

\[
m_i \ddot{x}_i^\mu (\tau) = \frac{e_i}{c} \sum_{j \neq i} (\mathbf{F}_{Rel})^\mu_{\nu} (\dot{x}_i (\tau))_{\nu}, \quad i = 1, \ldots, N. \tag{2}
\]

This set of coupled equations differ from similar non-relativistic equations in that they include interaction on light cones rather than instantaneous interactions. It is a set of differential-delay equations. Such equations have been studied relatively infrequently and pose certain challenges for their solution. To begin, Cauchy initial or boundary conditions are insufficient to determine a particular solution. Instead of point-like values at particular times, or values and derivative values of time derivatives on some two dimensional surface, whole segments of orbits for all particles on orbits from their intersection with the past light cone and the present; i.e., the value of \( \tau \) from which their integration is to commence must be given. Whatever their complexity and difficulty solving analytically, it seems obvious that they can be machine integrated if the required initial data is available.

\(^2\) The upper limit on the integral below is chosen so as to include only retarded interaction. This is done on the authority of empirical evidence. This has the advantage of precluding from the start many mystical features greatly complicating the mathematics.
Note that the solutions cannot be what an observer would see, however, because the delay in interaction pertains to the electromagnetic signals used to observe the various particles constituting the system. What an observer “sees” at any given instant would be the intersections of his past light cone with the orbits of the particles. At most, an observer could be collocated with one particle only, affording for this particle an observation at the absolute present moment while all other observations would be historical information. This circumstance might well afford perceiving some time intervals as dilated and some displacements as contracted as expressed in the observed values, i.e. as space-time perspective effects only.

4. Multi particle relativistic wave equations

At the present, special relativistic treatments of electromagnetically interacting particles are restricted to situations involving a single charged particle bathed in an external electromagnetic field upon which the particle has no influence. This limit is a direct consequence of the lack of a relativistic multi-body formulation of mechanics. The formulation described above seems to remedy this deficit and leads, with arguments parallel to those used to deduce nonrelativistic wave equations, to relativistic equations involving systems of interaction charged particles. For example, the following equation for bosons is a parallel of the most familiar Schrödinger Equation:

\[ \hbar \frac{\partial \Psi}{\partial \tau} = \sum_{i} \left( \frac{1}{2m_i} \right) \left( \hbar \Box_i - \sum_{j \neq i} A_j (\tau) \right)^2 \Psi, \]  

where the interaction potential is given by:

\[ A_j (\tau) = e_j \int_{-\infty}^{\tau} x_j (\tau) \delta \left( (x_i (\tau) - x_j (\tau))^2 \right) d\tau. \]  

An equation for Fermions might be:

\[ \hbar \frac{\partial \Psi}{\partial \tau} = \sum_{i} \left( \frac{1}{2m_i} \right) \left( \sigma \cdot \nabla_i + \Pi \frac{\partial}{\partial \text{ict}} - \sigma \cdot A_i - \Pi A_4 \right)^2 \Psi, \]  

When factored, this equation becomes the parallel to the Dirac Equation. Analytical solution of these equations can be expected to be even more difficult than for the underlying mechanical equations, Eq. (2).

5. Summary

Careful attention to the distinction among time variables as they are employed in relativistic mechanics resolves several long standing conundrums. In particular it reveals the ambiguities involved in the twin or clock paradox. It can be seen that time dilation is a perspective effect that leaves ontic quantities unchanged while their appearance is subject to aberration. Likewise, it is seen that there exists a self consistent definition of absolute time and a means of using it to calibrate a system of distributed clocks. However, the finite transmission speed of electromagnetic interaction, specifically including the light signals and the like used for observation and measurement, results in perspective effects. In addition to the communication and observation signals, which are generally thought of as being passive with no back-action on the observed entity, in fact are just relatively weak electromagnetic interactions subject to

The upper limit of the integral of the definition of a Dirac delta function is chosen to exclude advanced interaction. This is done on the authority of empirical evidence and avoids considerable inconsistent mathematics.
all the same physical effects as are interactions transmitting orbit modifying mechanical forces. This includes, of course, relativistic perspective also.

6. References
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