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Motion Parameter Identification and Motion Compensation for Shipborne HFSWR by Using the Reference RF Signal Generated at the Shore

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Abstract: The shipborne high-frequency surface wave radar (HFSWR) platform produces six degrees of freedom (DOF) motion at sea, which affects the performance of radar target detection and remote sensing of ocean surface dynamics parameters. Motion compensation can mitigate the effect of six-DOF motion, but motion parameters (including amplitude and angular frequency) need to be known. Motion parameters obtained by using high precision sensors are affected by the precision error and time delay, thus affecting the effect of motion compensation. To obtain the motion parameters accurately and in real time, a method of identifying the motion parameters by using an artificially transmitted reference radio frequency (RF) signal generated at the shore is proposed. Based on the results of the parameter identification, the reference RF signal and the first-order radar cross-sections (RCSs) modulated by six-DOF motion of the shipborne HFSWR platform can be compensated. The identification of angular frequency is divided into two steps: (1) Preliminary identification results are obtained by using the reference RF signal; (2) the pattern search method is used to further improve the identification accuracy of angular frequency. The amplitude of translation (including surge and sway) can be identified accurately through the reference RF signal. Due to the small amplitude of rotation (including roll, pitch, and yaw), it needs to be identified by the reference RF signal and pattern search method. After identifying the motion parameters, division in the time domain is used for motion compensation. Through the simulation results, both translation and rotation have good motion compensation effects. In addition, the method of using high precision sensors to obtain motion parameters and compensation is compared with the method in this paper, the simulation results of motion compensation show that the latter is better.

Keywords: motion parameter identification; motion compensation; six-DOF; reference RF signal; the first-order RCS; pattern search

1. Introduction

High-frequency surface wave radar (HFSWR) can provide real-time, all-weather, low-cost surveillance beyond the horizon. It has been successfully applied in ocean surface moving target detection and remote sensing of ocean surface dynamics parameters [1–7]. Shipborne HFSWR has the following advantages over shore-based HFSWR: The platform is mobile and flexible, it is not
restricted by the coastline, so it can achieve real-time monitoring of the open sea. However, the interaction between the ocean waves and the platform produces six degrees of freedom (DOF) platform motion, which modulates the echo signal of shipborne HFSWR. Some experimental and theoretical analyses have been carried out to explain the effects of platform motion on radar Doppler spectra. For example, some researchers have shown that platform motion could be viewed as phase modulation of radar Doppler spectra [8–11]. On this basis, models of HFSWR scattering cross-sections with different platform motions in the frequency-domain have been developed. Walsh et al. derived first- and second-order ocean surface radar cross-section (RCS) models for floating platforms that include surge [12,13], indicating that surge can generate additional peaks in the radar Doppler spectra. Based on the Walsh model, some researchers have added forward motion or motion with some DOF to the RCS model, and have analyzed the effects of different kinds of motion on radar Doppler spectra [14–17]. Subsequently, Ma et al. extended the bistatic RCS model of floating platforms to include both surge and sway [18]. They pointed out that there would be more symmetrical peaks caused by the combined motion in the radar Doppler spectra. Walsh et al. derived the first- and second-order bistatic RCS models with multi-frequency six-DOF oscillation motion and described platform motion with a more realistic motion model [19]. Chang et al. derived the first-order shipborne HFSWR time-domain RCS model, including six-DOF platform motion, which was based on the modulation of motion for the antenna pattern and array steering vector [20]. The model was also compared to Walsh’s model to verify the validity. These studies show that platform motion affects radar target detection and remote sensing of ocean surface dynamics parameters. Therefore, in order to improve the performance of moving target detection and the extraction accuracy of ocean surface dynamics parameters, motion compensation methods should be used to recover the RCSs modulated by six-DOF motion.

At present, motion compensation methods are mainly divided into two categories: (1) Recover the Doppler spectra contaminated by motion of the platform to extract ocean surface information from the spectra [21]; and (2) recover the antenna pattern and array steering vector of the shipborne HFSWR to reduce the effects of platform motion on radar receiving antennas [22,23]. However, in [21], the parameters of the platform’s six-DOF motion are assumed to be known. Motion parameters (including amplitude and angular frequency) can be obtained using high precision sensors, including the Inertial Navigation System (INS) and the Global Navigation Satellite System (GNSS) Receiver, etc. However, the precision error and time delay will affect the effects of motion compensation. In addition, in order to suppress the spreading clutter and to improve target signal detection performance, the common method is the space–time adaptive processing (STAP) [11,24]. However, it may adversely affect remote sensing of ocean surface dynamics parameters. In order to give consideration to the performance of radar target detection and remote sensing of ocean surface dynamics parameters, we pay more attention to the research of motion compensation methods. Thus, it is necessary to put forward a method to accurately identify motion parameters which does not depend on sensors.

For shore-based HFSWR, radio frequency (RF) signals have been studied for many years. The single-frequency RF signal is shown as a line spectrum in the Doppler spectra. If the RF signal appears in the low Doppler frequency region, it will affect the detection of targets and the remote sensing of ocean surface dynamics parameters. In this situation, the RF signal is usually regarded as an interference signal. Meanwhile, for shipborne HFSWR, there are few studies on the characteristics of RF signals, and there are almost no reports on using RF signals to identify ship motion parameters. Zhou et al. proved that the position of the single-frequency RF signal in the Doppler spectra is related to its transmitting frequency and the radar sweep period [25]. Therefore, we may select an appropriate transmitting frequency for the RF signal so that it appears outside the region of the Doppler spectra that contains useful information. This avoids interference with the detection and extraction of useful information. This signal is called the reference RF signal. When the shipborne HFSWR receiving antennas are modulated by six-DOF motion, the reference RF signal is also modulated by six-DOF motion, and the motion characteristics are reflected in the Doppler spectra. Therefore, the six-DOF motion parameters can be accurately identified by the reference RF signal.
This paper presents a method to identify the six-DOF motion parameters of the shipborne HFSWR platform by using the reference RF signal. The main innovation points of this paper are as follows: (1) The frequency-domain expression of the reference RF signal modulated by six-DOF motion of the shipborne HFSWR platform is derived and the expression is used to identify the parameters of six-DOF motion of the platform; and (2) since the motion compensation method mentioned in this paper requires a more accurate angular frequency, the pattern search method is used to further improve the identification accuracy of angular frequency. The amplitude of rotation (including roll, pitch, and yaw) also needs to use a pattern search method to improve the identification accuracy. The results of the parameter identification are applied to the method of motion compensation. The results show that this method has a positive compensation effect for both the reference RF signal and the first-order RCS.

This paper is organized as follows. In Section 2, a radar echo model containing the reference RF signal and a six-DOF motion model of the shipborne HFSWR platform are presented. Then, the feasibility of motion compensation and of using the reference RF signal to identify motion parameters are analyzed. The method of motion parameter identification is also given. In Section 3, the identification results of the six-DOF motion parameters of the platform are given, and the simulation results of the motion compensation of the reference RF signal and the first-order RCS are presented. In addition, the method of using high precision sensors to obtain motion parameters and compensation is compared with the method in this paper. In Section 4, the influence of motion parameter identification on compensation is discussed, and suggestions for improving motion compensation are given. Section 5 gives the conclusions of the research and puts forward suggestions for future studies.

2. Methods

2.1. Physical Model

Shipborne HFSWR operates on a frequency modulated interrupted continuous wave (FMICW). The relative positions of the shipborne HFSWR platform, the emission source of the reference RF signal, ocean surface, and shore are shown in Figure 1. The reference RF signal source is located onshore.

![Figure 1. The diagram of the relative position of the shipborne high-frequency surface wave radar (HFSWR) platform, the emission source of the reference radio frequency (RF) signal, ocean surface, and shore.](image-url)
The ocean surface is divided into small patches for calculation convenience, and these patches are considered as ideal point targets. In this research, based on the time-domain echo signal model established by Chang et al. [20], the single-frequency RF signal model proposed by Zhou et al. was added [25]. The echo model is studied at a certain distance, so the value of the propagation attenuation is determined by this distance in the model. The research method is similar to reference [20]. The transmitting power and the maximum propagation distance of the reference RF signal source are described in detail in the following text. This indicates that this research is feasible in practice. In this case, the whole echo signal model can be expressed as:

$$x(R,n) = \frac{P_r G_r G_c \lambda^2}{(4\pi)^2 R_s^4} \sum_{r,s} \sigma(R_r, \theta_s, n) a(n) g(\theta_s, n)p(\theta_s, n)$$

$$+ I(R_r, \theta_s, n) a(n) g(\theta_s, n)p(\theta_s, n) + e(n)$$

where $P_r$ is the transmitting power of the radar, $G_r$ and $G_c$ are the gain of transmitting and receiving antennas, $\lambda$ is the wavelength, $R_s$ is the distance of each patch from the platform, $\sigma(R_r, \theta_s, n)$ is the first-order RCS of the ocean surface, $\theta_s$ is the included angle between the line that connects each patch and the platform and the Y-axis. The reference position of $\theta_s$ is the Y-axis and the clockwise direction is the positive direction. $n$ denotes the n-th sweep period. Both $r$ and $s$ are positive integers. The intervals of $r$ and $s$ are $[R/R + c/(4B/\Delta R), R/R + c/(4B/\Delta R)]$ and $[1, 2\pi/\Delta \theta]$, and $\Delta \theta$ and $\Delta R$ describe the dimension of each patch. $c$ is the speed of light. $B_r$ is the bandwidth of radar. $I(R_r, \theta_s, n)$ is the single-frequency reference RF signal after the first fast Fourier transform (FFT). $R_r$ is the distance between the RF source and the platform. $\theta_s$ is the included angle between the line that connects the RF source and the platform and the opposite extension of the Y-axis. To increase the detection range of radar, $R_r$ is usually more than 30 km away from the platform. In the coherent integration time (CIT) of radar, the values $R_r$ and $\theta_s$ are approximately unchanged. $g(\theta_s, n)$ is the receiving antenna pattern, which only changes with the roll and pitch of the platform; $a(n)$ and $p(\theta_s, n) \in C^{M \times 1}$ are the amplitude and the phase of the array steering vector. $M$ is the number of receiving antennas ($M$ is an even number). $x(R,n) \in C^{M \times 1}$ is the echo signal data vector. $e(n) \in C^{M \times 1}$ is the background noise [26, 27].

According to the theory of free-space path loss (FSPL) [28], under the condition of uniform and lossless media in all directions, the loss of electromagnetic wave can be expressed as:

$$L_{os} = 32.44 + 20 \log R_r + 20 \log f_c$$

where $L_{os}$ is the propagation loss, $f_c$ is the radar carrier frequency. Since the loss of electromagnetic waves in the real environment are higher than that in the ideal situation, the actual transmitting power of the reference RF signal should be higher than the theoretical transmitting power.

The motion of the shipborne HFSWR platform can be viewed as the superposition of six-DOF motion [29, 30]. The six-DOF motion includes surge, sway, heave, yaw, roll, and pitch. Figure 2 shows the physical model of the shipborne HFSWR platform and the six-DOF motion of the platform. The origin of coordinate $O$ is the center of gravity of the platform. The antenna array is equally spaced on the side of the hull with the center of gravity of the platform. The antennas are numbered sequentially from prow to stern. $w$, $h$, $d$, and $l$ are represented as the distance between the centerline of the deck and the receiving antennas, the distance between the platform’s center of gravity and the deck, the distance between adjacent receiving antennas, and the height of the receiving antennas, respectively. $x$, $y$, $z$, $\psi$, $\phi$, and $\varphi$ represent the displacement of the platform’s surge, sway, and heave, and the angles of roll, pitch, and yaw, respectively.
The six-DOF motion of the platform changes the amplitude \( a(n) \) and phase \( p(\theta, n) \) of the array steering vector, and the roll and pitch also change the receiving antenna pattern \( g(\theta, n) \). Therefore, it is necessary to quantitatively express the six-DOF motion of the platform to accurately represent the changes of the array steering vector and the receiving antenna pattern. According to the work of Walsh et al., the vertical motion components do not result in additional Doppler effects [12]. Therefore, neither heave nor the vertical components of roll and pitch in vertical direction were considered. The horizontal motion vectors caused by surge, sway, yaw, roll, and pitch can be respectively expressed as [12,16,17,19]:

\[
\begin{align*}
\delta\hat{\rho}_{01}(t) &= a_1 \cos(a_1 t) \delta\hat{\rho}_{01} \\
\delta\hat{\rho}_{02}(t) &= a_2 \cos(a_2 t) \delta\hat{\rho}_{02} \\
\delta\hat{\rho}_{03}(t) &= \theta_1(t) \delta\hat{\rho}_{03} \\
\delta\hat{\rho}_{04}(t) &= \theta_1(t) \delta\hat{\rho}_{04} \\
\delta\hat{\rho}_{05}(t) &= \theta_1(t) \delta\hat{\rho}_{05}
\end{align*}
\]

where \( a_1 \) and \( a_2 \) and \( \omega_1 \) and \( \omega_2 \) are the amplitudes and angular frequencies of surge and sway, respectively. \( \theta_1(t) \), \( \theta_2(t) \), and \( \theta_3(t) \) are the rotation angles of yaw, roll, and pitch, respectively, which are the functions of time. \( \delta\hat{\rho}_{01} \), \( \delta\hat{\rho}_{02} \), \( \delta\hat{\rho}_{03} \), \( \delta\hat{\rho}_{04} \), and \( \delta\hat{\rho}_{05} \) are the motion directions, respectively. \( \theta_1(t) \), \( \theta_2(t) \), and \( \theta_3(t) \) can be respectively expressed as:

\[
\begin{align*}
\theta_1(t) &= a_1 \cos(\omega_1 t) \\
\theta_2(t) &= a_2 \cos(\omega_2 t) \\
\theta_3(t) &= a_3 \cos(\omega_3 t)
\end{align*}
\]

where \( a_1 \), \( a_2 \), and \( a_3 \) and \( \omega_1 \), \( \omega_2 \), and \( \omega_3 \) are the amplitudes and angular frequencies of yaw, roll, and pitch, respectively. The changing rules of \( x \), \( y \), \( \varphi \), \( \psi \), and \( \phi \) with time satisfy Equations (3)–(7).
2.2. Motion Compensation with Known Motion Parameters

The method of motion compensation in this research was to use division in the time domain after the first FFT to recover the antenna pattern and array steering vector to a level without motion. The idea of motion compensation is using identified motion parameters to get the estimate of the antenna pattern, as well as the array steering vector, and dividing the true value by the estimate. The higher the precision of motion parameters is, the closer the estimate is to the real value, and the better effects of the motion compensation will be.

This section includes compensation for the reference RF signal and the first-order RCS. The validity of the compensation of the reference RF signal is analyzed. Based on the same principle, the influence of six-DOF motion on the first-order RCS can also be corrected.

Pitch is taken as an example in the following sections and a compensation analysis of the six-DOF motion is attached to each section.

2.2.1. Expression of the Reference RF Signal

According to Zhou et al. [25] and features of our radar system. Assume the radar transmitting signal which is not to be interrupted is the same as the radar local oscillator signal. The reference RF signal $I(t, n)$ is multiplied by the complex conjugate of radar local oscillator signal, $S^*(t)$, and passed through a low-pass filter and an A/D converter. After that, the first FFT is performed for range separation. Finally, a number of sweeps are coherently processed, using the second FFT, to get the Doppler spectra.

The reference RF signal is a stationary sinusoidal incident on the radar receiver, in the $n$th sweep, the signal is expressed as:

$$I(t, n) = a_i e^{j2\pi f_i t + \phi_{n0}}$$  \hspace{1cm} (11)

where $a_i$ is the amplitude of the reference RF signal, $f_i$ is the frequency of the reference RF signal. The value of $f_i$ needs to be within $[f_r - B_r/2, f_r + B_r/2]$. $B_r$ is the radar bandwidth. This ensures that the reference RF signal can be received by the radar. $\phi_{n0}$ is the initial phase of the reference RF signal.

After the first FFT, the reference RF signal is expressed as:

$$I(R, \theta, n) = F(I(t, n) S^*(t))(f) H(f)$$

$$= I_0 e^{j\pi f t} e^{j(2\pi f_0 t + \phi_{n0})} H(f)$$  \hspace{1cm} (12)

where $F[\cdot](f)$ is the Fourier transformation operator. $f_{r0} = f_r - f_0$, $f_0$ is the start frequency of radar local oscillator signal. $T$ is the sweep period. $k$ is the sweep slope, $k = B_r/T$. $f = k \cdot t_d = (B_r/T)(2R/c)$, and $t_d$ is the echo time delay. Since $R$ is a constant during a CIT, $f$ is also a constant, $I_0 = a_i(1-j)(2k)$. $H(f)$ is the frequency response of the low-pass filter whose cutoff frequency is $f_c$. $H(f) = \begin{cases} 1, & f \leq f_c \\ 0, & f > f_c \end{cases}$.

Similarly, the approximation of the Fresnel integral is under the condition $|f| \leq f_c$ and $f_{r0} \square f_c$.

According to Equation (12), the signal forms a sinusoidal sequence during each sweep period. After the second FFT, the position of the reference RF signal in the Doppler spectra is a constant value, which can be expressed as:

$$f_d = \begin{cases} \text{mod}(f_r, f_c), & \text{mod}(f_r, f_c) \leq f_c/2 \\ \text{mod}(f_r, f_c) - f_c, & \text{others} \end{cases}$$  \hspace{1cm} (13)

where $f_c = 1/T$. 


2.2.2. Antenna Pattern and Array Steering Vector Modulated by Pitch

- **Antenna pattern**

  Pitch of the platform changes the antenna pattern and array steering vector [20]. In this case, the antenna pattern is expressed as:

  \[
  g_{05}(\theta_j, n) = \frac{\cos(k_c l_u \sin(\arctan(\tan(\Delta \phi) \sin(\theta_j)))) - \cos(k_c l_u)}{\cos(\arctan(\Delta \phi) \sin(\theta_j))}
  \]  

  (14)

  where \( k_c \) is the wave number and \( k_c = 2\pi/\lambda \). \( \Delta \phi \) refers to the angle changes of pitch.

  When \( \Delta \phi \) is within \( 5^\circ \), \( \sin(\Delta \phi) \) is approximately equal to \( \Delta \phi \), \( \cos(\Delta \phi) = 1 \), \( \tan(\Delta \phi) = \arctan(\Delta \phi) = \Delta \phi \). In this case, Equation (14) is equivalent to:

  \[
  g_{05}(\theta_j, n) = \frac{\cos(k_c l_u \sin(\arctan(\Delta \phi \sin(\theta_j)))) - \cos(k_c l_u)}{\cos(\Delta \phi \sin(\theta_j))}
  \]  

  (15)

  to simplify Equation (15), it can be expressed as:

  \[
  g_{05}(\theta_j, n) = 1 - \cos(k_c l_u)
  \]  

  (16)

  It can be seen that the antenna pattern is a fixed value when the pitch angle changes a little, and that it has nothing to do with \( \theta_j \) and \( n \). Therefore, \( g_{05}(\theta_j, n) \) is represented by the constant \( g_{05} \).

  Similarly, the antenna pattern generated by roll \( g_{04}(\theta_j, n) \) can also be equivalent to \( 1 - \cos(k_c l_u) \).

  Since only roll and pitch change the antenna pattern, the antenna pattern modulated by six-DOF motion is expressed as:

  \[
  g_{05}(\theta_j, n) = g_{04}(\theta_j, n) \cdot g_{05}(\theta_j, n) = (1 - \cos(k_c l_u))^2
  \]  

  (17)

- **Array steering vector**

  In the absence of motion, the array steering vector is expressed as:

  \[
  \mathbf{p}(\theta_j) = \begin{bmatrix}
  e^{\frac{2\pi i \sin(\theta_j) M - 1}{\lambda}} \\
  \vdots \\
  e^{\frac{2\pi i \sin(\theta_j) 1}{\lambda}} \\
  e^{\frac{2\pi i \sin(\theta_j) - 1}{\lambda}} \\
  \vdots \\
  e^{\frac{2\pi i \sin(\theta_j) M - 1}{\lambda}} \\
  \end{bmatrix}
  \]  

  (18)

  where \( M \) rows of the matrix represent \( M \) antennas. Antennas are located, in turn, from prow to stern.

  The array steering vector modulated by pitch is expressed as:

  \[
  a_{05}(n) \cdot \mathbf{p}_{05}(\theta_j, n) = \cos(\Delta \phi) \cdot e^{-\frac{2\pi i \sin(\theta_j) M - 1}{2}}
  \]  

  (19)
where $a_{0_5}(n)$ and $p_{0_5}(\theta, n)$ are the amplitude and the phase variance of the array steering vector modulated by pitch, $a_{0_5}(n) = \cos(\Delta \phi)$. $\odot$ is the Hadamard product. Applying the same analytical method, let $\sin \phi = \phi$, $\cos \phi = 1$; then Equation (19) is simplified to:

$$a_{0_5}(n) \cdot p_{0_5}(\theta, n) = e^{-\frac{2\pi \sin \theta \cdot \phi}{\lambda} \cdot n} \cdot e^{\frac{2\pi \sin \theta \cdot \phi}{\lambda} \cdot n} = e^{i\phi \cdot \cos(\alpha \tau)} \cdot p(\theta) \quad (20)$$

where $a'_5 = -\frac{2\pi}{\lambda} \cdot \sin \theta \cdot a_5$, the change rule of $\phi$ satisfies Equation (7).

Similarly, the array steering vector modulated by six-DOF motion is expressed as:

$$a_{0_6}(n) \cdot p_{0_6}(\theta, n) = \cos(\Delta \phi) \cdot \cos(\Delta \psi) \cdot e^{-\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n} \cdot e^{\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n}$$

$$\begin{bmatrix}
\frac{e}{\lambda} & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n
\end{bmatrix}$$

$$= e^{-\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n} \cdot e^{\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n}$$

Equation (21) can be simplified to:

$$a_{0_6}(n) \cdot p_{0_6}(\theta, n) = e^{\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n} \cdot e^{-\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n}$$

$$\begin{bmatrix}
\frac{e}{\lambda} & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n
\end{bmatrix}$$

$$= e^{-\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n} \cdot e^{\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n}$$

where $a_{0_6}(n)$ and $p_{0_6}(\theta, n)$ are the amplitude and the phase variance of the array steering vector modulated by six-DOF motion, $a_{0_6}(n) = \cos(\Delta \phi) \cdot \cos(\Delta \psi)$. Equation (21) can be simplified to:

$$a_{0_6}(n) \cdot p_{0_6}(\theta, n) = e^{\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n} \cdot e^{-\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n}$$

$$\begin{bmatrix}
\frac{e}{\lambda} & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n & \frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n
\end{bmatrix}$$

$$= e^{-\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n} \cdot e^{\frac{2\pi \sin \theta \cdot \phi \cdot \psi}{\lambda} \cdot n}$$

where $a'_1 = -\frac{2\pi}{\lambda} \cdot \sin \theta \cdot a_1$, $a'_2 = -\frac{2\pi}{\lambda} \cdot \cos \theta \cdot a_2$, $a'_3 = \frac{2\pi}{\lambda} \cdot \sin \theta \cdot a_3$, $a'_4 = \frac{2\pi}{\lambda} \cdot \cos \theta \cdot a_4$, $a'_5 = -\frac{2\pi}{\lambda} \cdot \sin \theta \cdot a_5$. The change rule of $x$, $y$, $\phi$, $\psi$, $\theta$ satisfies Equations (3)–(7).

2.2.3. Motion Compensation for the Reference RF Signal
For easy analysis and presentation, take the antenna closest to the prow, that is, the first row of the array steering vector, and is modulated by pitch as an example. While the reference RF signal of the antenna modulated by pitch would be expressed as:

\[
I_{05,1}(R, \theta, n) = I(R, \theta, n) a_{05}(n) g_{05}(\theta, n) p_{05,1}(\theta, n) e^{j[a_{05}\cos(a_{05}a_{05}n)]}
\]

where \( p_{05,1}(\theta, n) \) is the steering vector phase of the antenna closest to the prow modulated by pitch. \( p(\theta) \) is the steering vector phase of the antenna without motion. Therefore, if the parameters of pitch \( a_0 \) and \( \omega_p \) are known, the reference RF signal modulated by pitch and received by the antenna may be compensated as follows:

\[
I(R, \theta, n)p(\theta) = \frac{1}{g_{05}(\theta, n)} I_{05,1}(R, \theta, n) e^{j[a_{05}\cos(a_{05}a_{05}n)]}
\]

similarly, if the parameters of six-DOF, \( a_i \), and \( \omega_i \) \( (i = 1, 2, 3, 4, 5) \) are known, the reference RF signal modulated by six-DOF motion and received by the antenna may be compensated as follows:

\[
I(R, \theta, n)p(\theta) = \frac{1}{g_{05}(\theta, n)} I_{05,1}(R, \theta, n) e^{j[\sum_{i=1}^{5}\omega_i\cos(a_{05}a_{05}n)]}
\]

2.2.4. Motion Compensation Verification for the First-Order RCS

The reference RF signal is compensated in accordance with Section 2.2.3. If the same method can be applied to compensate for the first-order RCS, the modulated Doppler spectra can be restored, as well as the radar’s performance of the detection of targets and remote sensing of ocean surface dynamics parameters. This is of great practical significance. When analyzing the first-order RCS, the same method as Section 2.2.3 is also adopted, the antenna closest to the prow was again taken as an example. The expression of the first-order RCS after six-DOF modulation received by the antenna may be compensated as follows:

\[
\sum_{i=1}^{5}\sigma(R, \theta, n)p_i(\theta) = \frac{1}{g_{05}(\theta, n)} \sum_{i=1}^{5}\sigma_{05}(R, \theta, n) e^{j[\sum_{i=1}^{5}\omega_i\cos(a_{05}a_{05}n)]}
\]

Notice that \( \theta_i \) is not a constant, and it varies in the interval \([-0.5\pi, 0.5\pi]\). Since Equation (17) shows that \( g_{05}(\theta, n) = (1-\cos(kL))^2 \) is independent with \( \theta \). From Equations (25) and (26), both the reference RF signal and the first-order RCS may be compensated in the same way. In theory, if the six-DOF motion parameters \( a_i \) and \( \omega_i \) \( (i = 1, 2, 3, 4, 5) \) are known, the echo signal will be compensated well.

2.3. Identifiability Analysis of Motion Parameters by the Reference RF Signal

Section 2.2 explains that both the reference RF signal and the first-order RCS can be compensated as long as the motion parameters can be identified. Therefore, the method to obtain the motion parameters is particularly important. Since the Doppler spectra can reflect the frequency and energy information of the signal, it is hoped that the motion parameters can be obtained by the Doppler spectra of the RF signal modulated by motion.

Therefore, the second FFT is needed to obtain the frequency-domain expression of the reference RF signal modulated by motion.

Next, the frequency-domain expression of the reference RF signal was derived by taking pitch as an example, and it is shown that the motion parameters \( a_i \) and \( \omega_i \) can be identified through this expression. Then, the frequency-domain expression of the reference RF signal modulated by six-DOF motion was given, indicating that all parameters of six-DOF motion can be identified.
2.3.1. Identifiability Analysis of the Pitch Motion Parameters by the Reference RF Signal

If the antenna closest to the prow was taken as an example. Expand Equation (23) and get:

\[
I_{051}(R, \theta, n) = I(R, \theta, n)g_{05}(\theta, n)p_{051}(\theta, n) \\
= g_{05}I_0e^{j\phi_0}e^{j\frac{\omega}{\lambda}(\theta_0 - \theta_0')} \frac{e^{-j[\omega n\cos(\alpha n T) + \omega_0 n T]}}{e^{\frac{(M-1)\pi}{2}}} \\
(27)
\]

where \( \omega_t = 2\pi f_d \). The second FFT was performed using Equation (28), and the expression of the reference RF signal modulated by pitch in the frequency-domain can be obtained. The process is as follows:

\[
I_{051}(\omega) = \sum_{n=1}^{\infty} I(R, \theta, n)g_{05}(\theta, n)p_{051}(\theta, n)e^{-j\omega n T} \\
= \sum_{n=1}^{\infty} g_{05}I_0e^{j\phi_0}e^{j\frac{\omega}{\lambda}(\theta_0 - \theta_0')} \frac{e^{-j[\omega n\cos(\alpha n T) + \omega_0 n T]}}{e^{\frac{(M-1)\pi}{2}}} \\
= g_{05}I_0e^{j\phi_0}e^{j\frac{\omega}{\lambda}(\theta_0 - \theta_0')} \sum_{n=1}^{\infty} e^{-j\omega n T(\omega - \omega_0)}e^{j\omega n T} \\
(28)
\]

where for \( e^{\omega x} \), it satisfies the following mathematical relationship:

\[
e^{\omega x} = J_0(x) + 2\sum_{m=1}^{\infty} i^m J_m(x)\cos(m\phi) \\
(29)
\]

where \( J(x) \) is the Bessel function, the subscript \( m \) is an integer, representing the order of the Bessel function.

Putting \( e^{j\omega n T} \) into Equation (29) gives:

\[
e^{j\omega n T} = J_0(a_s') + 2\sum_{m=1}^{\infty} i^m J_m(a_s')\cos(m\omega n T) \\
(30)
\]

There is also the following relationship:

\[
2\cos(m\omega n T) = e^{jmnT} + e^{-jmnT} \\
(31)
\]

put Equations (30) and (31) into Equation (28) to obtain:

\[
I_{051}(\omega) = g_{05}I_0e^{j\phi_0}e^{j\frac{\omega}{\lambda}(\theta_0 - \theta_0')} \frac{e^{-j[\omega n\cos(\alpha n T) + \omega_0 n T]}}{e^{\frac{(M-1)\pi}{2}}} \\
\left\{ J_0(a_s') + \sum_{m=1}^{\infty} i^m J_m(a_s')\left(e^{jmnT} + e^{-jmnT}\right) \right\} \\
(32)
\]

according to the properties of the discrete Fourier transform, then:

\[
I_{051}(\omega) = 2\pi g_{05}I_0e^{j\phi_0}e^{j\frac{\omega}{\lambda}(\theta_0 - \theta_0')} \frac{e^{-j[\omega n\cos(\alpha n T) + \omega_0 n T]}}{e^{\frac{(M-1)\pi}{2}}} \\
\left( J_0(a_s')\delta(\omega - \omega_j) + \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} i^m J_m(a_s')\left(\delta(\omega - \omega_j + l\omega_0)\right) \right) \\
(33)
\]

Equation (33) is the expression of the reference RF signal in the frequency-domain modulated by the pitch of the antenna closest to the prow. Due to the combination of the Bessel and Dirac functions of different orders, many peaks appear in the Doppler spectra. According to the reason for the peaks, they can be divided into the reference RF peak and motion-induced peaks. The reference RF peak is caused by the reference RF signal. In Equation (33), this is the term containing \( J_0(a_s') \).

The peak has the largest amplitude (energy) in the Doppler spectra, and its amplitude decreases because of pitch. The amplitude of \( a_s \) can be identified by the amplitude change of \( J_0(a_s') \).
Motion-induced peaks are caused by pitch. In Equation (33), these are the terms containing \( J_m(a'_x) \). In the Doppler spectra, the peaks are symmetrical to the reference RF peak and are distributed in pairs. Their amplitudes decrease with the increasing distance from the reference RF peak. The position of the peaks in the Doppler spectra is determined by angular frequency \( \omega \) and \( m \) of pitch. Thus, the angular frequency \( \omega \) can be identified.

Equation (33) illustrates the relationship between parameters \( a \) and \( \omega \) and the peaks of the reference RF signal modulated by pitch in the Doppler spectra. This indicates that the motion parameters \( a \) and \( \omega \) can be identified by the reference RF signal. The specific identification method is introduced in the section of Motion parameter identification method (Section 2.4.).

2.3.2. Identifiability Analysis of Six-DOF Motion Parameters by the Reference RF Signal

Applying the same derivation method, the expression of the reference RF signal in the frequency-domain modulated by six-DOF motion of the antenna closest to the prow is expressed as:

\[
I_{0,1}(\omega) = \sum_{n=0}^{\infty} I(R, \theta, n) a_0(n) g_0(\theta, n) p_{0,1}(\theta, n) e^{-j\omega T} = g_0 I e^{j\omega T} e^{-j\omega T} \sum_{n=0}^{\infty} e^{-j\omega T(n-a_k)} \prod_{\ell=1}^{5} \left( J_0(a'_x) + \sum_{m=1}^{\infty} J_m(a'_x) \left( e^{j\omega_T(m+1)} + e^{-j\omega_T(m+1)} \right) \right)
\]

because the amplitude of \( J_m(a'_x) \) is small under the condition of \( m \geq 1 \), it has little effect on the actual results when multiplied by two or more terms, and thus is omitted for simplifying the expression. The simplified expression is as follows:

\[
I_{0,1}(\omega) = 2\pi g_0 I e^{j\omega T} e^{-j\omega T} \sum_{\ell=1}^{5} \left( J_0(a'_x) + \sum_{m=1}^{\infty} J_m(a'_x) \left( e^{j\omega_T(m+1)} + e^{-j\omega_T(m+1)} \right) \right)
\]

where \( a_0(n) = \cos(\Delta\phi)\cos(\Delta\psi) = 1 \), \( J_4 = J_4(a'_x)J_4(a'_x)J_4(a'_x)J_4(a'_x) \), \( J_5 = J_5(a'_x)J_5(a'_x) \).

Applying the same analytical method, according to Equation (35), all parameters of six-DOF motion can be identified.

This shows that it is feasible to identify the motion parameters of six-DOF motion. Next, the identification method of the motion parameters is given.
2.4. Motion Parameter Identification Method

The motion parameters include amplitude $a_i$ and angular frequency $\omega_i$, $i = 1, 2, 3, 4, 5$. The identification methods of these two motion parameters are presented herein, taking pitch as an example.

The process of motion parameter identification is as follows: (1) Identify the preliminary values of amplitude $a_i$ and angular frequency $\omega_i$ using the reference RF signal; (2) use the method of pattern search to identify angular frequency $\omega_i$ with high precision; (3) if it is rotation, the same method (pattern search) is used to improve the identification accuracy of the corresponding amplitude $a_i$. The specific identification process is as follows.

2.4.1. Identification of Motion Amplitude

In the absence of motion, the reference RF signal received by the antenna closest to the prow in the frequency-domain is expressed as:

$$I_r(\omega) = 2\pi I e^{i\theta_0} e^{i\frac{\pi}{2}(f-f_0)} e^{-i\frac{(M-1)\pi}{2}\sin\delta} \cdot \delta(\omega - \omega_p)$$

(36)

At this time, the antenna pattern and the array steering vector do not change, the amplitude of the reference RF peak is not modulated, and there is no motion-induced peak. The motion parameter $a_i$ can be identified by combining Equations (33) and (36). Both the reference RF peak and the motion-induced peaks can be used to identify motion parameter $a_i$. Finally, the reference RF peak was selected, because the amplitude of the reference RF peak is large and the relative error is small. The specific steps for identifying motion parameter $a_i$ are as follows:

1. According to Equation (36), the amplitude $A_i$ of the reference RF peak without motion is calculated.
2. According to Equation (33), the amplitude $A_{i'}$ of the reference RF peak modulated by pitch is calculated.
3. $A$ and $A_{i'}$ satisfy the following relationship:

$$\frac{A_{i'}}{A} = J_0\left(a_{i'}\right)$$

(37)

According to Equation (37), the value of a Bessel function of zero order can be obtained.

4. By the properties of the zero-order Bessel function, we can invert $a_{i'}$.

5. According to the relation between $a_{i'}$ and $a_i$, the identification result of motion parameter $a_i$ can be obtained, denoted as $a_i^*$. The identification method is also applicable to the motion of other DOF motions. Therefore, the motion parameters $a^*$ are obtained.

2.4.2. Identification of Angular Frequency

- Preliminary identification of angular frequency

According to Equation (33), when $m = 1$, the motion-induced peaks are the first-order motion-induced peaks. In the Doppler spectra, the distance between the first-order motion-induced peaks and the reference RF peak is the frequency of pitch $f_s$. Therefore, the preliminary identification result of $\omega_j$ can be obtained directly through the Doppler spectra: $\omega_j = 2\pi f_s$, denoted as $\omega_j^{(0)}$. 


In particular, the sum of a finite number of sweep periods was used as the CIT in the actual situation. The identification accuracy of angular frequency $\omega_i$ is affected by the frequency resolution $F_r$, expressed as:

$$F_r = \frac{1}{T \cdot N}$$  \hspace{1cm} (38)

where $N$ is the number of sweep periods. It can be seen from Equation (38) that increasing the number of sweep periods improves the frequency resolution $F_r$, and the preliminary identification result of the angular frequency $\omega_i^{(0)}$ is closer to the theoretical value $\omega_i$.

However, in actual radar information processing, the accuracy of angular frequency will improve with the increasing number of sweep periods, but this will lead to a great reduction of the real-time performance of target detection. Therefore, this method is not feasible. If the preliminary identification results of angular frequency are directly used for motion compensation, motion of different frequencies will be introduced on the basis of the original motion, which will result in more motion-induced peaks and will make the motion compensation results worse. Therefore, in the following research, a method of searching the motion parameter $\omega$ with high-precision is proposed by combining with motion compensation algorithms.

- High-precision identification of angular frequency

The method of identifying $\omega$ with high-precision is to search the parameters on the basis of the preliminary identification results $\omega^{(0)}$, and it is effective for the motion of all DOF. The specific method is as follows.

Search $\omega$ within the interval $\left[\omega^{(0)} - F_r, \omega^{(0)} + F_r \right]$. The result of the i-th search is denoted as $\omega^{(i)}$, and each search returns the amplitude of a certain motion-induced peak at the angular frequency $\omega^{(i)}$. The amplitude of the peak in the i-th search is expressed as $f(\omega^{(i)})$. In order to facilitate the return of amplitudes and the evaluation of motion compensation effects, a pair of motion-induced peaks with the largest amplitude (i.e., the first-order motion-induced peaks) are selected. Since the motion-induced peaks are symmetric about the reference RF peak, one of them can be chosen arbitrarily. When $\omega^{(i)}$ is the true value $\omega$, $f(\omega^{(i)})$ receives a unique minimum, denoted as $f_{\min}(\omega^{(i)})$. The best motion compensation effect is obtained at this point.

In order to give consideration to the results of motion compensation and the efficiency of searching with high-precision, the termination condition of the search was set. The termination condition can be determined by the amplitude of the motion-induced peaks after motion compensation. The test shows that when the absolute value of the maximum allowable error of the angular frequency is $|e_{\text{max}}| \leq 10^{-4}$ Hz, the maximum of the amplitude of the motion-induced peaks caused by the inaccurate identification of angular frequency $\omega$ are less than 5 dB, and it can be considered that the precision of angular frequency $\omega$ meets the motion compensation requirements. Thus, $e_{\text{max}} = 10^{-4}$ Hz was selected as the termination condition, it is also the search step size of angular frequency $\omega$. When the termination condition is satisfied, the motion parameters $\omega^*$ are obtained.

Since there is no specific expression for $f(\omega^{(i)})$, the derivative and gradient cannot be calculated, and the function characteristics are unknown. Only the global minimum value point is known as the target point, and the amount of calculation is large when calculating $f(\omega^{(i)})$. From what has been discussed above, it is necessary to choose an optimization method that can cover the entire search interval as soon as possible on the premise of the unknown function characteristics and fewer search times.

The pattern search method is not required to provide a function expression, but should have the advantage of fewer search times (compared to genetic algorithms and simulated annealing
algorithms) and should be able to adjust the search center according to every search result in time; thus, the pattern search method was used.

In particular, due to the small amplitude of rotation, it is necessary to expand the amplitude \( a \) when searching for the corresponding \( \omega \) to increase the amplitude of the first-order motion-induced peaks. This improves the accuracy and efficiency of the pattern search algorithm.

In addition, the pattern search algorithm is implemented in parallel computing, which greatly improves the speed of calculation and guarantees real-time performance of the search with high precision.

### 2.4.3. Correction of the Amplitude of Rotation

As the amplitude of rotation is small, large relative errors will yield after the identification of the reference RF signal, which will affect the results of motion compensation to some extent. Therefore, the motion compensation effects can also be improved by using the pattern search method to correct the amplitude of rotation after the angular frequency is accurately identified.

This method is similar to search for high-precision angular frequency. The amplitude identified by the reference RF signal is taken as the initial results. The amplitude of the peak is expressed as \( g(a^0) \). When \( a^0 \) is the true value \( a \), \( g(a^0) \) receives a unique minimum, denoted as \( g_{\text{min}}(a^0) \). In this way, amplitude \( a \) of rotation can obtain a more accurate value. Therefore, the effects of motion compensation are further improved.

Different from the high-precision identification of angular frequency, the correction of amplitude does not need to be of too high precision. When the maximum allowable error of the amplitude \( |a_{\text{max}}| \leq 0.1^\circ \), the motion-induced peaks of rotation are basically lower than 5 dB. This satisfies the requirement of motion compensation.

Similarly, since the amplitude of rotation is small, it is also necessary to expand the amplitude \( a \) during the correction.

### 3. Simulation Results

In this research, the ocean wave spectrum was calculated using the Pierson–Moskowitz model with a cardioid directional distribution of a wind-driven sea \([31,32]\). The simulation was used to analyze the motion parameter identification and compensation results of the reference RF signal and the first-order RCS with different DOF motions.

The desired maximum distance between the reference RF signal source and the shipborne HFSWR platform is about 80 km. The sensitivity of the radar receiver is \(-40 \text{ dBm}\). It can be worked out that the theoretical transmitting power of the reference RF signal source is 25 W. However, in practice, a larger transmitting power is adopted to deal with the adverse effects brought by many factors. Therefore, the power used in the simulation is 35 W, and the amplitude of the reference RF signal \( a_0 \) is determined according to the transmitting power.

The values of simulation parameters were set according to the real ship and the radar parameters in the field experiment were conducted in July 2019. The radar system was installed on a large shipborne platform, and the receiving antenna array with eight antennas \((M = 8)\) was a uniform linear array. The parameters were as follows. For the shipborne HFSWR platform: \( w = 8 \text{ m}, h = 5 \text{ m}, d = 14 \text{ m}, l_a = 2 \text{ m} \). For the reference RF signal: \( f_r = 4720128 \text{ Hz}, R_s = 50 \text{ km}, \theta_s = 46^\circ, \phi_{\theta_0} = 45^\circ \). For the radar system: \( f_c = 4.7 \text{ MHz}, f_0 = 4.67 \text{ MHz}, T = 0.128 \text{ s}, N = 2048, B_c = 60 \text{ kHz}, \Delta R = 2.5 \text{ km}, \text{CIT} = 262.144 \text{ s}, F_r = 3.81 \times 10^{-3} \text{ Hz} \). The signal-to-noise ratio (SNR) = 70 dB. Hamming windows were used in the simulation to reduce spectrum leakage. The pattern search algorithm in the optimization toolbox of MATLAB was selected for searching for angular frequency \( \omega \) and correcting amplitude \( a \).
The flow chart of the complete algorithm is shown in Figure 3.

**Figure 3.** Flow chart of the complete algorithm in MATLAB. HFSWR, shipborne high-frequency surface wave radar; RF, radio frequency; FFT, fast Fourier transform; RCS, radar cross-section; DOF, degrees of freedom.

Through the observation of the sea state in that field experiment in July 2019, the sea state at that time was about level 2–3. Table 1 shows the motion parameters of the six-DOF of the shipborne HFSWR platform. The motion parameters of rotation were obtained by INS IMU720-G. Since the INS cannot obtain the motion parameters of translation (including surge and sway), the motion parameters of translation were selected according to [19], the real ship and the sea state in the experiment were also fully considered.
Table 1. The motion parameters of the six-degrees of freedom (DOF) of the shipborne HFSWR platform.

|               | Surge  | Sway   | Yaw    | Roll   | Pitch   |
|---------------|--------|--------|--------|--------|---------|
| Amplitude     | 1.335 m| 1.093 m| 1.373° | 2.225° | 1.263°  |
| Angular Frequency (rad/s) | 0.7351 | 0.8105 | 0.7603 | 0.8859 | 0.7037 |

3.1. Preliminary Results of Motion Parameter Identification and Compensation Incorporating the Reference RF Signal and the First-Order RCS

Table 2 shows the preliminary identification results of the motion parameter $a$ and $\omega$, which were directly identified by the reference RF signal. For surge, sway, and roll, the identification errors of amplitude $a$ are within 0.3%. Meanwhile, for yaw and pitch, the identification errors of amplitude $a$ are 3.58% and 20.43%, respectively. The amplitude of translation is large and the identification errors are small, while the amplitude of rotation is small and the identification errors are large. The identification error of roll is small and it is accidental. When the parameter of roll changes, the error also changes. The errors of the angular frequency $\omega$ are within the frequency resolution $F_r$.

Table 2. The results and analysis of the six-DOF motion parameter identification.

|               | Surge  | Sway   | Yaw    | Roll   | Pitch   |
|---------------|--------|--------|--------|--------|---------|
| Amplitude     | Theoretical Value | 1.335 m | 1.093 m | 1.373° | 2.225° | 1.263° |
|               | Preliminary Value | 1.339 m | 1.092 m | 1.424° | 2.229° | 1.005° |
|               | Error   | 0.30%  | 0.09%  | 3.58%  | 0.18%  | 20.43% |
|               | Final Value | 1.339 m | 1.092 m | 1.389° | 2.229° | 1.254° |
|               | Error   | 0.30%  | 0.09%  | 1.17%  | 0.18%  | 0.71%  |
| Angular Frequency (rad/s) | Theoretical Value | 0.7351 | 0.8105 | 0.7603 | 0.8859 | 0.7037 |
|               | Preliminary Value | 0.7194 | 0.7911 | 0.7433 | 0.8872 | 0.6955 |
|               | Error   | 0.0157 | 0.0194 | 0.017  | 0.0013 | 0.0082 |
|               | Final Value | 0.7351 | 0.8105 | 0.7603 | 0.8859 | 0.7037 |

Figure 4 shows the results of motion compensation errors using the preliminary identification results of the motion parameters by taking pitch as an example. Figure 4a,b shows the results of the reference RF signal and the RCS, respectively. To analyze the effects of motion compensation, each graph contains two curves, including Modulation deviation and Motion compensation errors, respectively. The curve of Modulation deviation is the difference between the curve modulated by pitch and the curve without motion. The curve of Motion compensation errors is the difference between the curve after compensation and the curve without motion. These two curves are almost identical, which indicates that the motion of pitch is not compensated. This shows that if the preliminary identification results of the amplitude $a$ and the angular frequency $\omega$ are used for compensation, the motion compensation method is ineffective. Therefore, the pattern search method is necessary to improve the identification accuracy of angular frequency $\omega$ and amplitude $a$. 
Figure 4. Motion compensation error analysis of reference RF signal and first-order RCS with the preliminary results by taking pitch as an example. (a) The reference RF signal case; (b) The RCS case.

3.2. Final Results of Motion Parameter Identification and Compensation Incorporating the Reference RF Signal and the First-Order RCS

Table 2 also shows the final identification results of the motion parameter \( a \) and \( \omega \), by using pattern search to improve identification accuracy. It can be seen from Table 2 that the accuracy of angular frequency satisfies \( |e_{\omega_{max}}| \leq 10^{-4} \text{Hz} \). The errors of yaw and pitch are calibrated to 1.17% and 0.71%, respectively, and the accuracy is greatly improved than before. Figure 5a shows the high-precision searching process of the angular frequency of pitch \( \omega_5 \) as an example. As the amplitude of pitch \( a_5 \) is small, the real amplitude and the identified amplitude are enlarged by 50 times in the search of \( \omega_5 \) to improve the search efficiency of \( \omega_5 \). \( e_{\omega_{max}} = 10^{-4} \text{Hz} \) is selected as the maximum allowable error of \( \omega_5 \) in the high-precision search, which is also the termination condition of iteration. As shown in Figure 5a, after 31 searches, the termination condition is satisfied, and function \( f(\omega_5^{(31)}) \) reaches the minimum value. In this case, \( f_5 = 0.1120 \text{Hz} \) and \( \omega_5 = 2\pi f_5 \).

Figure 5b shows the correction process of the amplitude of pitch \( a_5 \) as an example. Similarly, the real amplitude and the identified amplitude are enlarged by 180 times during the correction. \( e_{a_{max}} = 0.1^\circ \) is the termination condition of iteration. As shown in Figure 5b, after 12 searches, the termination condition is satisfied, and function \( g(a_5^{(12)}) \) reaches the minimum value. Convert the amplitude to angle, \( a_5 = 1.254^\circ \).

Figure 5. The searching process using pattern research. (a) The case of angular frequency of pitch \( \omega_5 \). (b) The case of amplitude of pitch \( a_5 \).
Figures 6–11 show the motion compensation results of surge, sway, yaw, roll, pitch, and motion with six-DOF, respectively, using the final identification results of the parameters. In order to fully show the effects of motion compensation, each graph contains three curves, including Fixed, Before compensation, and After compensation, respectively. It can be seen from these figures that the amplitude and position of the motion-induced peaks generated by different DOF motions are different, which were determined by the amplitude $a_i$ and angular frequency $\omega_i$ of the motion models. Due to the usage of the final identification results of the motion parameters, the reference RF signal and the RCS achieved good motion compensation results.

**Figure 6.** The simulation of reference RF signal and first-order RCS for surge with the final results. (a) The reference RF signal case. (b) The RCS case.

**Figure 7.** The simulation of reference RF signal and first-order RCS for sway with the final results. (a) The reference RF signal case. (b) The RCS case.
Figure 8. The simulation of reference RF signal and first-order RCS for yaw with the final results. (a) The reference RF signal case. (b) The RCS case.

Figure 9. The simulation of reference RF signal and first-order RCS for roll with the final results. (a) The reference RF signal case. (b) The RCS case.

Figure 10. The simulation of reference RF signal and first-order RCS for pitch with the final results. (a) The reference RF signal case. (b) The RCS case.
Figure 11. The simulation of reference RF signal and first-order RCS for six-DOF motion with the final results. (a) The reference RF signal case. (b) The RCS case.

The amplitude of surge and sway is large, and the amplitude of the motion-induced peaks in the Doppler spectra is also large. However, due to accurate identification of the motion parameters $a_1$, $a_2$, $\omega_1$, and $\omega_2$, the motion compensation results of the reference RF signal and the RCS are good.

Although the amplitude of yaw is small, the receiving antennas far from the rotation center of the platform still produce a relatively large displacement; thus, motion-induced peaks with large amplitudes are generated in the Doppler spectra. Due to searching of angular frequency $\omega_3$ with high precision and accurately correct amplitude $a_3$ through the same method, the motion compensation also obtained a good effect. All of the motion-induced peaks are basically lower than 5 dB.
Although the amplitude of roll and pitch is small, the antenna pattern and array steering vector are both changed; thus, the amplitude of motion-induced peaks is large in the Doppler spectra. The effects of motion compensation are even better than yaw. The motion-induced peaks are almost completely restored by improving the corresponding accuracy of angular frequency and amplitude.

When considering the platform’s motion of six-DOF, maximum motion-induced peaks of 5 dB are still generated after motion compensation, which is mainly caused by yaw. Increasing the amplitude of yaw can improve the identification accuracy of amplitude \( a_y \), as well as the motion compensation results.

### 3.3. Method Comparison

The method of using high precision sensors to obtain motion parameters and compensation is compared with the method in the paper. In the comparison, the methods of motion compensation are the same (division in the time domain), only the methods to obtain motion parameters are different. Taking the effects of motion compensation as the standard to assess the quality of motion parameters obtaining methods. The index of motion compensation \( C_I \) is defined as follows:

\[
C_I = \sum_{i=1}^{N} |B_1(i) - B_2(i)|
\]  

(39)

where \( B_1(i) \) and \( B_2(i) \) represent each point on the curve After compensation and the curve Fixed (i.e., shore-based), respectively. Take the absolute value of the difference between \( B_1(i) \) and \( B_2(i) \), and sum them all up (a total of \( N \) points). The smaller the sum value is, the less the difference between the curve After compensation and the curve Fixed is, the better the compensation effect will be.

The sensor for measuring translation adopts the StarFire GNSS Receiver SF-5050 produced by NavCom, the sensor for measuring rotation adopts the INS MGC R3 produced by Kongsberg. The precision error and time delay of the sensors are the main factors affecting the motion compensation. Table 3 shows the comparative results of motion compensation.

|            | Precision Error | Time Delay (ms) | \( C_I \) of Sensor Method | \( C_I \) of Reference RF Signal Method |
|------------|-----------------|-----------------|-----------------------------|---------------------------------------|
| Surge      | 0.05 m          | 40              | 0.0287                      | 0.0013                                |
| Sway       | 0.05 m          | 40              | 0.0097                      | 1.7894e-04                           |
| Yaw        | 0.08°           | 5               | 0.0047                      | 0.0038                                |
| Roll       | 0.01°           | 5               | 3.1084e-04                  | 2.2756e-05                           |
| Pitch      | 0.01°           | 5               | 4.2219e-04                  | 2.7745e-04                           |

As shown in Table 3, for surge and sway (i.e., translation), the compensation results using the reference RF signal are significantly better than the results using high precision sensors. For yaw, roll, and pitch (i.e., rotation), although the results of the two methods are of the same order of magnitude, the results using the reference RF signal are still better. Figure 12 shows the compensation results of the first-order RCS for sway by using the two methods separately.
Figure 12. The results of motion compensation comparison of the first-order RCS for sway using the reference RF signal and high precision sensors.

Figure 12 contains two curves, including Errors using the reference RF signal, and Errors using high precision sensors, respectively. The solid line represents the difference between the curve After compensation using the reference RF signal and the curve Fixed. The dotted line represents the difference between the curve After compensation using high precision sensors and the curve Fixed. Through the motion compensation results in Figure 12, the curve Errors using the reference RF signal is more than 30 dB lower than the curve Errors using high precision sensors. Therefore, the simulation results show that the method using the reference RF signal is better.

4. Discussion

According to the presented analysis, it is feasible to identify six-DOF motion parameters using the reference RF signal. Subsequently, all effects caused by platform motion can be successfully eliminated during motion compensation. The identification accuracy of the motion parameters of six-DOF has a great influence on motion compensation. Thus, improving the identification precision of motion parameters is the main method to improve the compensation effects.

Due to the influence of frequency resolution $r_F$, the identification accuracy of angular frequency $\omega$ is not accurate; thus, the identification accuracy of $\omega$ needs to be further improved. The results show that the compensation requires high precision of the angular frequency $\omega$ and that the maximum allowable error of $\omega$ has to satisfy $|\varepsilon_{\omega_{\text{max}}}| \leq 10^{-4} \text{ Hz}$. However, the actual identification results of $\omega$ are far from achieving this accuracy. Therefore, in the identification of angular frequency $\omega$, after the preliminary identification results of the angular frequency are obtained by using the reference RF signal, the pattern search method is needed to further improve the accuracy of the angular frequency.

The small amplitudes of actual motion lead to the identification results of amplitude $a$ being inaccurate, thus affecting the motion compensation results. For translation, the amplitude of motion is large and the relative errors are small when identifying parameter $a$, and thus, the identification accuracy of the parameters is high. For rotation, the amplitude of motion is small, which introduces large relative errors, thus affecting the identification results of parameter $a$ and adversely affecting the motion compensation. Therefore, the identification accuracy will improve if the amplitude of rotation increases. This, in turn, will be more conducive to obtaining accurate motion parameters and better motion compensation results in higher sea states.
Since the amplitude identification of rotation is not accurate and this causes inaccurate motion compensation results, measures should be taken to improve the rotation compensation results. Yaw has a considerable effect on the Doppler spectra. Due to its small motion amplitude and inaccurate identification, the motion-induced peaks are not completely eliminated after motion compensation. In the actual shipborne experiment, the radar receiving antennas should be as close to the center of rotation of the platform as possible without affecting the radar performance to reduce the modulation of the yaw to the receiving antennas. In addition, the pattern search method can be used to correct the amplitude \(a\) in order to improve the identification accuracy of \(a\) and the compensation results. For roll and pitch, there is also the problem of small motion amplitudes \(a\). Therefore, the compensation effects can be further improved by the pattern search method.

In addition, the method of using high precision sensors to obtain motion parameters and compensation is compared with the method in this paper. The motion compensation method is the division in the time domain and the same motion compensation index is used to assess the quality of motion parameters obtaining methods. The simulation results show that the method in this paper is better no matter whether it is translation or rotation.

This research also suggests problems for further study. For example, the platform’s six-DOF motion is simply described as a single-frequency cosine function, which is not accurate for actual sea states. Additionally, virtual targets and wind direction are not included in the simulation model. In the future, more factors must be considered, such as virtual targets and wind direction. This research intended to use the sum of multiple cosine functions to describe the six-DOF motion of the platform, and the motion compensation algorithm can also be improved. The compensation results of this method in virtual targets and different wind direction conditions will be analyzed to make the simulation environment more similar to real sea states in future research. Finally, this research will also try to use field data to complete the motion compensation.

5. Conclusions

In this paper, a method of motion parameter identification based on the reference RF signal generated at the shore with known frequency was proposed, and motion compensation for the six-DOF motion of the shipborne HFSWR platform was completed by using the results of the parameter identification. The reference RF signal was added to the echo of the shipborne HFSWR, and the feasibility of motion compensation was analyzed. The results show that motion compensation for the reference RF signal and the first-order RCS can be completed by identifying the motion parameters. The expression of the reference RF signal modulated by six-DOF motion in the frequency-domain was derived. This expression was used to analyze the feasibility of using the reference RF signal to identify the motion parameters, and the method of the parameter identification was given.

According to the Results and Discussion section, on one hand, due to the influence of frequency resolution \(F_r\), the identification accuracy of angular frequency \(\omega\) was not high enough, while motion compensation required higher accuracy of the angular frequency \(\omega\). Thus, the pattern search method was used to further improve the accuracy of the angular frequency \(\omega\) on the basis of the preliminary identification results through the reference RF signal. On the other hand, the identification of amplitude \(a\) could be divided into two cases. Due to the large amplitude of translation, the motion parameters could be accurately identified for accurate motion compensation. However, the amplitude of rotation was small, and a large relative error was generated in the identification of amplitude \(a\), which had adverse effects on the motion compensation. Therefore, the method of pattern search for correcting the amplitude \(a\) was put forward to further improve the compensation results of rotation. Through these methods, both translation and rotation had good motion compensation effects. Finally, the method of using high precision sensors to obtain motion parameters and compensation was compared with the method in this paper, the simulation results of motion compensation show that the latter is better.

In future research, the six-DOF motion model of the platform and motion compensation algorithms will be further improved, and more factors such as wind direction and virtual targets will
be considered. In addition, motion compensation will be realized through the combination with field data.

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**Abbreviations**

| Name                  | Definition                                                                 |
|-----------------------|-----------------------------------------------------------------------------|
| $x(R,n)$              | the whole echo signal model                                                 |
| $P_t$                 | the transmitting power of the radar                                         |
| $G_t$                 | the gain of transmitting antennas                                          |
| $G_r$                 | the gain of receiving antennas                                             |
| $\lambda$             | the wave length                                                             |
| $\sigma(R,\theta,n)$ | the first-order RCS of the ocean surface                                    |
| $R_i$                 | the distance of each patch from the platform                               |
| $\theta_i$            | the included angle between the line that connects each patch and the platform and the Y-axis |
| $n$                   | the n-th sweep period                                                       |
| $\Delta \theta$       | the change in angle of each patch                                           |
| $\Delta R$            | the change in range of each patch                                          |
| $c$                   | the speed of light                                                          |
| $B_t$                 | the bandwidth of radar                                                      |
| $I(R,\theta,n)$       | the single-frequency reference RF signal after the first fast Fourier transform (FFT) |
| $R$                   | the distance between the RF source and the platform                         |
| $\theta_i$            | the included angle between the line that connects the RF source and the platform and the opposite extension of the Y-axis |
| $g(\theta,n)$         | the receiving antenna pattern                                               |
| $g_{\alpha}(\theta,n)$ | the antenna pattern modulated by roll                                      |
| $g_{\alpha}(\theta,n)$ | the antenna pattern modulated by pitch                                     |
| $a(n)$                | the amplitude of the array steering vector                                  |
| $M$                   | the number of receiving antennas                                           |
| $C_{M	imes 1}$        | the data vector                                                             |
| $p(\theta,n)$         | the phase of the array steering vector                                      |
| $e(n)$                | the background noise                                                        |
| $O$                   | the center of gravity of the platform                                        |
| $w$                   | the distance between the centerline of the deck and the receiving antennas  |
| $h$                   | the distance between the platform’s center of gravity and the deck          |
| $d$                   | the distance between adjacent receiving antennas                           |
| $l_s$                 | the height of the receiving antennas                                        |
| $x$                   | the displacement of the platform’s surge                                    |
| $y$                   | the displacement of the platform’s sway                                     |
| $z$                   | the displacement of the platform’s heave                                    |
| $\psi$                | the angles of roll                                                          |
| $\phi$                | the angles of pitch                                                         |
$\varphi$ the angles of yaw

$\delta \tilde{\rho}_i(t) (i = 01, 02, 03, 04, 05)$ the horizontal motion vectors caused by surge, sway, yaw, roll, and pitch, respectively

$\delta \phi_i (i = 01, 02, 03, 04, 05)$ the motion directions of surge, sway, yaw, roll, and pitch, respectively

$a_i (i = 1, 2, 3, 4, 5)$ the amplitude of surge, sway, yaw, roll, and pitch, respectively

$\omega_i (i = 1, 2, 3, 4, 5)$ the angular of surge, sway, yaw, roll, and pitch, respectively

$\theta_i (t)$ the rotation angles of yaw (functions)

$\theta_i (t)$ the rotation angles of roll (functions)

$\theta_i (t)$ the rotation angles of pitch (functions)

$f_i$ the frequency of the reference RF signal

$f_c$ the radar carrier frequency

$I(t, n)$ the reference RF signal

$S^* (t)$ the complex conjugate of radar local oscillator signal

$F[f]$ the Fourier transformation operator

$a_i$ the amplitude of the reference RF signal

$I_a = a_i(1- j)/\sqrt{2k}$

$H(f)$ the frequency response of the low-pass filter

$f_c$ the cutoff frequency of the frequency response of the low-pass filter

$f_0$ the start frequency of radar local oscillator signal

$T$ the sweep period

$k$ the sweep slope

$t_s$ the echo time delay

$\varphi_{\text{co}}$ the initial phase of the reference RF signal

$f_d$ the position of the reference RF signal in the Doppler spectra

$f_c = 1/T$

$\omega_d$ $2\pi f_d$

$k$ the wave number

$\Delta \phi$ the angle changes of pitch

$g_{\theta} (\theta, n)$ the antenna pattern modulated by six-DOF motion

$p(\theta)$ the array steering vector without motion (including the amplitude and phase)

$a_{\omega} (n)$ the amplitude of the array steering vector modulated by pitch

$p_{\omega} (\theta, n)$ the phase of the array steering vector modulated by pitch

$a_{\omega} (n)$ the amplitude of the array steering vector modulated by six-DOF motion

$p_{\omega} (\theta, n)$ the phase of the array steering vector modulated by six-DOF motion

$p_{\theta_{01}} (\theta, n)$ the steering vector phase of the antenna closest to the prow without motion

$p_{\theta} (\theta)$ the steering vector phase of the antenna closest to the prow modulated by six-DOF motion

$a_i^* (i = 1, 2, 3, 4, 5)$ the amplitude after parameter identification of surge, sway, yaw, roll, and pitch, respectively

$\omega_i^* (i = 1, 2, 3, 4, 5)$ the angular frequency after parameter identification of surge, sway, yaw, roll, and pitch, respectively

$\sum_{r,s} \sigma (R_{rs}, \theta, n)$ the first-order RCS (sum)
\( I_{0\omega_3}(R, \theta, n) \)  
the reference RF signal of the antenna closest to the prow modulated by pitch (after the first FFT)

\( J_m(x) \)  
the Bessel function

\( m \)  
the order of the Bessel function

\( I_{0\omega_3}(\omega) \)  
the reference RF signal in the frequency-domain modulated by pitch of the antenna closest to the prow (after the second FFT)

\( I_{0\omega}(\omega) \)  
the reference RF signal in the frequency-domain modulated by six-DOF motion of the antenna closest to the prow (after the second FFT)

\( I_{1\omega}(\omega) \)  
the reference RF signal received by the antenna closest to the prow without motion in the frequency-domain (after the second FFT)

\( A \)  
the amplitude of the reference RF peak without motion

\( A_{\omega} \)  
the amplitude of the reference RF peak modulated by pitch

\( f_5 \)  
the frequency of pitch

\( \omega_5^{(i)} \)  
the preliminary identification result of \( \omega_5 \)

\( F_r \)  
the frequency resolution

\( N \)  
the number of sweep periods

\( \omega_i^{(0)} \)  
the preliminary identification results of angular frequency

\( \omega_i^{(i)} \)  
the result of the i-th search of angular frequency

\( f(\omega_i^{(i)}) \)  
the amplitude of the reference RF peak during the i-th search (search angular frequency)

\( f_{\text{min}}(\omega_i^{(i)}) \)  
a unique minimum amplitude of the reference RF peak (search angular frequency)

\( e_{\text{max}1} \)  
the maximum allowable error of the angular frequency

\( g(a_i^{(i)}) \)  
the amplitude of the reference RF peak during the i-th search (search amplitude)

\( a_i^{(i)} \)  
the result of the i-th search of amplitude

\( g_{\text{min}}(a_i^{(i)}) \)  
a unique minimum amplitude of the reference RF peak (search amplitude)

\( e_{\text{max}2} \)  
the maximum allowable error of the amplitude of rotation

\( C_I \)  
the index of motion compensation

\( B_1(i) \)  
each point on the curve after motion compensation

\( B_2(i) \)  
each point on the curve without motion (shore-based)

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