M/M/1 Queueing Model with Working Vacation and Two Type of Server Breakdown

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Abstract. In this paper the steady state probability distribution of the number of customers in single server Markovian queue is obtained by using matrix geometric approach with working vacation where server may breakdown in working vacation state as well as in busy state. The arrival of the customer is depends upon the server’s state and arrival follows FCFS discipline. The server is made available for rendering alternate service to customers. The inter-arrival time of the customers, service time, vacation duration, and lifetime and repair time of the server follows an exponential distribution. Numerical illustrations are made to examine the validity of analytical results. Sensitivity analysis is also made of in order to discover the outcome of various parameters on the system performance physical appearance.

Key words: Markovian queue, Matrix geometric method, Queue size, State dependent rate, Unreliable server, Working vacation

1. Introduction
The merit of queueing theory is continuously increasing due to breakdown and working vacation where server interrupts the servicing of a customer. There are many real-world problems where server takes vacations whenever there are no more customers waiting in the queue for service and do secondary task. A typical example is often encountered in the post office in the situation when a clerk completes service and finds no customer waiting, he finds himself face to work on a secondary task, say sorting letters, putting the material in a bag etc. Numerous papers have been published on queueing models with server breakdown and vacation. In past, Servi and Finn [11], Jain and Jain [5], Selvaraju and Goswami [10], Guha et al. [2], Yu and Alfa [14] and many others analyzed the Markovian queueing models with server breakdown and working vacation. Shoukry et al. [12] used matrix geometric method in order to find out the stationary state probabilities and then different performance characteristics. Rajadurai et al. [9] discussed M/G/1 queue under the consideration that the server may breakdown in working as well as vacation state. Kempa and Kobielnik [7] analyzed a finite queue with working vacation using embedded markov chain. Recently, Kalyanaraman and Sundaramoorthy [6] discussed a single server queueing system with busy state, repair state and working vacation state.

With the demand of multimedia services and increasing complexity of the modern computer and telecommunication networks, the matrix analytic approach is frequently used to investigate the queues whose behavior can be modeled in terms of multi-dimensional Markov chains. The advantage of matrix geometric method (MGM) is that we can deal such models whose activities are not necessarily exponentially distributed. The MGM method was first introduced by Neuts [8] and is used to find the stationary state probabilities which are worthwhile to find the various performance characteristics. The contribution of several authors including Houdt [3], Ekren [1], Wang et al. [13] etc. are worth noting in this regard. Recently, Zeifmana et al. [15] analyzed a batch arrival markovian queueing model with infinite capacity using matrix geometric method. Huang et al. [4] obtained stability conditions for a single server infinite capacity markovian queue using matrix geometric method.

In the investigation, the work has been extended by including the two types of server breakdowns, which occur during working vacation as well as in busy state of the server. The arriving jobs may be influenced by the status of the server which is subject to breakdown and may be in working vacation, busy and breakdown states. The organization of the paper is as follows. In section 2 we describe the assumptions related to the model. In section 3 the matrix geometric method is presented in order to compute steady-state probabilities. Some performance measures are given in section 4. Numerical illustrations are also made in section 5 to test the tractability of the matrix geometric method outlined. The sensitivity analysis is also supported to explore the effect of various parameters on
performance indices. Finally, in section 6 concluding comments and further views of the study done are given.

2. Model Description
In the queueing model, Chapman-Kolmogorov equations governing the model are constructed by using matrix geometric method. Assuming that the customers follow a Poisson distribution with rate depend on the server status as given below. The arriving rate \( \lambda_i \) of customers depends upon the server's status which is given by

\[
\begin{align*}
0, & \quad \text{working vacation state} \\
1, & \quad \text{server is in busy in rendering service of primary customers} \\
2, & \quad \text{server is in broken down state due to failure when it in working vacation} \\
3, & \quad \text{server is in broken down state due to failure when it was busy in rendering service of primary customers}
\end{align*}
\]

For formulation of mathematical model following assumptions are consider as follows:
1. The server takes a working vacation that is exponential distributed with mean \( 1/\mu \).
2. The server renders services to secondary customers during working vacation with rate \( \mu \), the primary customers are served with rate \( \mu_5 \).
3. The server may breakdown in working vacation (busy) state with rate \( \omega_v \) (\( \omega_b \)) and is repair with rate \( \omega_v \) (\( \omega_b \)).
4. The switch over times from busy (repaired) to working state (busy state) are supposed to be negligible.

3. The Analysis
The matrix geometric method developed by Neuts [8], for obtaining the stationary state probabilities is employed. For this purpose, generator matrix \( Q \) of a continuous time Markov process with the structure shown below:

\[
Q = \begin{bmatrix}
H_0 & J_0 & 0 & 0 & 0 \\
I_0 & H_1 & J_1 & 0 & 0 \\
0 & I_1 & H_1 & J_1 & 0 \\
0 & 0 & I_1 & H_1 & J_1 \\
. & . & . & . & . \\
. & . & . & . & .
\end{bmatrix}
\]

In the initial portion, \( H_0, I_0 \) and \( J_0 \) contain the transition rate with in level 0, from level 1 to 0 and level 0 to 1, respectively. The repetitive portion, \( H_1, I_1 \) and \( J_1 \) contain the transition rate from level 1 to 1, level 1 to i-1 and level i
to level \(i+1\) \((i=1, 2, 3, \ldots)\), respectively. The submetrics are given by

\[
H_i = \begin{bmatrix}
-(\Lambda_0 + \nu_i + \varepsilon + \omega_i) & \varepsilon & \omega_i & 0 \\
0 & -(\Lambda_1 + \nu_{b} + \omega_{b}) & 0 & \omega_{b} \\
\rho_i & 0 & -(\Lambda_2 + \rho_i) & 0 \\
0 & \rho_{b} & 0 & -(\Lambda_3 + \rho_{b})
\end{bmatrix},
I_i = \begin{bmatrix}
\nu_i & 0 & 0 & 0 \\
0 & \nu_{b} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}_{4 \times 4}
\]

The balance equations are

\[
-A_0 X_0^T + X_0^T I_0 = 0^T
\]

(2)

\[
J_0 X_0^T + X_1^T (H_1 - I_0 - J_1) + X_2^T I_1 = 0^T
\]

(3)

\[
J_0 X_0^T + X_1^T (H_1 - I_1 - J_1) + X_2^T I_1 = 0^T,
\]

(4)

where \(0\) is a column vector of zeros.

Now let \(X\) be the vector of steady state probabilities associated with \(Q\), such that

\[
X Q = 0
\]

(5)

and the normalizing condition

\[
X e = 1
\]

(6)

where \(e\) is a vector of 1’s.

We partition \(X\) as \(X = [\pi_0, \pi_1, \pi_2, \pi_3, \ldots\]

with sub vectors \(\pi_0 = [\pi_{0,0}, 0, 0, 0, 0]\) and \(\pi_1 = [\pi_{1,0,0}, \pi_{1,0,1}, \pi_{1,0,2}, \pi_{1,0,3}, \ldots],\ i \geq 1.\)

The vector of stationary probabilities \(\pi_i, i \geq 1\) is a function only of the transition rates between states with \(i-1\) queued customers and states with \(i\) queued customers and can be expressed in terms of \(\pi_1\) as follows:

\[
\pi_i = \pi_1 r^{i-1},
\]

(7)

\[
\pi_i = \pi_{i-1} r
\]

(8)

Here \(r\) is called the rate matrix and is the minimal nonnegative solution of the matrix polynomial

\[
J_1 + r H_1 + r^2 I_1 = 0
\]

(9)

Sub vectors \(\pi_0, \pi_1\) can be obtained from

\[
(p_0, p_1) \begin{bmatrix}
H_0 & J_0 \\
I_0 & (H_1 + r I_1)
\end{bmatrix} = 0
\]

(10)

and \(p_1 = p_0 r.\)

The probabilities \(p_0, p_1\) are obtained using the normalizing condition,
\[
\begin{align*}
\pi_0 e + \pi_1 (I-r)^{-1}e &= 1 \\
\end{align*}
\]

where \(e\) is a column vector of 1’s. The above equation together with equation (10) yields a unique solution. The above method can also be applied (approximately) in the case of finite models, as outlined below. In many circumstances, this method provides a good approximation for the steady-state probabilities of a finite model.

For Finite Model

For this case, we have

\[
Q = \begin{bmatrix}
H_0 & J_0 & 0 & 0 & 0 & \ldots & \ldots \\
I_0 & H_1 & J_1 & 0 & 0 & \ldots & \ldots \\
0 & I_1 & H_1 & J_1 & 0 & \ldots & \ldots \\
0 & 0 & I_1 & H_1 & J_1 & \ldots & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\
\end{bmatrix}
\]

where

\[
H'_1 = \begin{bmatrix}
-(\Lambda_B + \nu_v + \varepsilon + \omega_v) & \varepsilon & \omega_v & 0 \\
0 & -(\Lambda_1 + \nu_B + \omega_B) & \omega_B & 0 \\
\rho_v & 0 & \rho_v & 0 \\
0 & \rho_B & 0 & \rho_B & 1 + 4
\end{bmatrix}
\]

The global balance equations (4) holds only for \(1 \leq n \leq M-1\). The additional equation is

\[
C_1 \pi_{M-1} + H'_1 \pi_M = 0^T, \quad n = M
\]

4. Performance Measures

Let us assume that generator matrix is irreducible. It is only possible when the matrices \(H_0\) and \(H_1\) are non-singular i.e. we can compute the inverse of these matrices. The rate matrix \(r\) can be calculated using initial iteration, \(r(0) = 0\) and \(r\) is obtained recursively by rewriting as

\[
r(n+1) = -J_1 (H_1^{-1}) - r^2(n) I_1 (H_1^{-1}) \text{ for } n > 0.
\]

Having calculated the rate matrix \(r\), the stationary probability vectors are calculated. Using these probabilities, the various performance measures have been established as follows:

- The probable number of customers in the system is in working vacation state or on vacation

\[
E[V] = \sum_{i=0}^{\infty} i \pi_{i,0}
\]

(13)

- The probable number of customers in busy state

\[
E[B] = \sum_{i=1}^{\infty} i \pi_{i,1}
\]

(14)
The probable number of customers in the system in broken down state from working vacation state, is

$$E[D_1] = \sum_{i=1}^{\infty} i \pi_{i,2}$$

(15)

The probable number of customers in broken down state from busy state, is given by

$$E[D_2] = \sum_{i=1}^{\infty} i \pi_{i,3}$$

(16)

Overall number of customers in the system

$$E[N] = E[V] + E[B] + E[D_1] + E[D_2]$$

(17)

Throughput is obtained using

$$TP = \mu_v \sum_{i=1}^{\infty} \pi_{i,0} + \mu_B \sum_{i=1}^{\infty} \pi_{i,1}$$

(18)

The average delay is given by

$$D = \frac{E[N]}{TP}$$

(19)

5. Numerical illustration and Sensitivity Analysis

We fix the different system parameters for evaluating probability vectors and various performance characteristics by using software MATLAB.

\( \theta_v = 4, \ \theta_b = 2, \ \omega_v = 0.5, \ \omega_b = 0.1, \ \theta_a = 5, \ \theta_d = 2, \ \theta = 3. \) For homogenous arrival rate we set \( \theta_a = \theta_b = \theta_d = 0 \) whereas for heterogeneous arrival rate we choose \( \theta_v = 0, \ \theta_b = 1.1, \ \theta_a = 0.8, \ \theta = 1.2. \) We find the sub-matrices \( H_0, I_0, J_0, H_1, I_1 \) and \( J_1 \) as.

\[
H_0 = \begin{bmatrix} -0.50 \\ 6.00 & 5.00 & 0 \\ -1.00 & 2.60 & 0 & 2.00 \\ 0.50 & 0 & -5.50 & 0 \\ 0 & 0.10 & 0 & -2.50 \end{bmatrix}
I_0 = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
J_0 = \begin{bmatrix} 0.50 & 0 & 0 & 0 \\ 0 & 0.50 & 0 & 0 \\ 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0.50 \end{bmatrix}
\]

The minimal non-negative square rate matrix \( r \) is given as
The probability vectors are obtained using (10) and (11) as follows:

\[
\begin{align*}
\pi_0 &= [0.0529] \\
\pi_1 &= [0.0529, 0.0584, 0.05, 0.374] \\
\pi_2 &= [0.0228, 0.0327, 0.0257, 0.1465] \\
\pi_3 &= [0.0112, 0.0167, 0.0128, 0.06]
\end{align*}
\]

By using above probability vectors, we calculate various performance measures given in equations (20)-(26) as

\[
E[V] = 1.2227, \quad E[B] = 0.1589, \quad E[D_1] = 0.1346, \quad E[D_2] = 0.8591 \\
E[N] = 2.3754, \quad TF = 5.6120, \quad D = 0.4233
\]

In order to perform the sensitivity analysis of various parameters on system performance measures such as probable number of customers in the system \( E[N] \), throughput (TP) and average delay (D) the numerical results are displayed in tables 1-4 and figures 1-6.

In table 1 we examine the effect of various parameters on the expected number of customers in different states i.e. in (i) idle \( E[I] \) (ii) busy \( E[B] \) and (iii) breakdown during working vacation \( E[D_1] \) and (iv) breakdown during busy state \( E[D_2] \) by fixing \( \omega = 0.5, \quad \omega_b = 0.1, \quad \omega_v = 2, \quad \omega = 3, \quad \varphi = \varphi_v = \varphi_b = \varphi = 0.2100 \). From table 1 it is observed that expected number of customers in different states increases as \( \omega_b \) increases with exception for \( \omega \) and \( \omega_v \). It is also noted that \( E[I] \), \( E[B] \), \( E[D_1] \) and \( E[D_2] \) decrease as \( \omega, \omega_b, \omega_v, \omega \) and \( \omega_b \) increases except with respect to \( \omega_b \) for which decreasing trend is found.

\[
\begin{bmatrix}
0.0977 & 0 & 0.0888 & 0 \\
0.0614 & 0.2500 & 0.0558 & 0.2000 \\
0.0096 & 0 & 0.0996 & 0 \\
0.0036 & 0.0125 & 0.0033 & 0.2100
\end{bmatrix}
\]

Table 1. The expected number of customers in different states
Table 2. The effect of \((v, b)\) on throughput TP, expected number of customer \(E[N]\) and average delay \(D\)

| \(v\) | \(E[I] \) | \(E[B] \) | \(E[D_{1}] \) | \(E[D_{2}] \) | \(\omega_v \) | \(E[I] \) | \(E[B] \) | \(E[D_{1}] \) | \(E[D_{2}] \) |
|---|---|---|---|---|---|---|---|---|---|
| 0.6 | 1.29 | 0.18 | 0.15 | 0.92 | 0.1 | 1.24 | 0.16 | 0.13 | 0.85 |
| 0.8 | 1.46 | 0.23 | 0.19 | 1.01 | 0.3 | 1.23 | 0.16 | 0.13 | 0.86 |
| 1 | 1.81 | 0.32 | 0.28 | 1.09 | 0.5 | 1.22 | 0.16 | 0.13 | 0.86 |
| \(b\) | \(E[I] \) | \(E[B] \) | \(E[D_{1}] \) | \(E[D_{2}] \) | \(\omega_b \) | \(E[I] \) | \(E[B] \) | \(E[D_{1}] \) | \(E[D_{2}] \) |
| 1.0 | 1.28 | 0.16 | 0.17 | 0.82 | 0.1 | 1.22 | 0.16 | 0.13 | 0.86 |
| 3.0 | 1.24 | 0.16 | 0.14 | 0.85 | 0.3 | 2.16 | 0.26 | 0.22 | 0.65 |
| 5.0 | 1.21 | 0.16 | 0.13 | 0.87 | 0.5 | 2.48 | 0.31 | 0.25 | 0.59 |

number of customer \(E[N]\) and average delay \(D\)

| \((\omega_v, \omega_b)\) | TP | \(E[N]\) | D |
|---|---|---|---|
| \((4, 2)\) | | | |
| 1 | 5.75 | 5.76 | 1.99 | 2.12 | 0.35 | 0.37 |
| 2 | 5.66 | 5.67 | 2.24 | 2.39 | 0.40 | 0.42 |
| 3 | 5.61 | 5.63 | 2.38 | 2.52 | 0.42 | 0.45 |
| 4 | 5.58 | 5.60 | 2.46 | 2.60 | 0.44 | 0.46 |
| 5 | 5.56 | 5.58 | 2.51 | 2.66 | 0.45 | 0.48 |
| \((2, 4)\) | | | |
| 1 | 5.82 | 5.83 | 1.66 | 1.75 | 0.28 | 0.30 |
| 2 | 5.74 | 5.75 | 1.83 | 1.94 | 0.32 | 0.34 |
| 3 | 5.69 | 5.70 | 1.93 | 2.05 | 0.34 | 0.36 |
| 4 | 5.66 | 5.67 | 2.00 | 2.13 | 0.35 | 0.37 |
| 5 | 5.63 | 5.65 | 2.06 | 2.18 | 0.36 | 0.39 |
| \((4, 4)\) | | | |
| 1 | 7.78 | 7.79 | 1.62 | 1.71 | 0.21 | 0.22 |
| 2 | 7.67 | 7.68 | 1.80 | 1.91 | 0.23 | 0.25 |
| 3 | 7.60 | 7.61 | 1.91 | 2.03 | 0.25 | 0.27 |
| 4 | 7.55 | 7.57 | 1.99 | 2.11 | 0.26 | 0.28 |
| 5 | 7.51 | 7.54 | 2.04 | 2.17 | 0.27 | 0.29 |

Table 3. The effect of \((\omega_v, \omega_b)\) on throughput TP, expected number of customer \(E[N]\) and average delay \(D\)
Table 4. The effect of \((\omega_v, \omega_b)\) on throughput \(TP\), expected number of customer \(E[N]\) and average delay \(D\)

| \((\omega_v, \omega_b)\) | \(\mathfrak{U}\) | \(TP\) | \(E[N]\) | \(D\) |
|--------------------------|----------------|--------|----------|------|
|                          | \(\text{Arrival Rate}\) | Homo. | Hetero. | Homo. | Hetero. | Homo. | Hetero. |
| (0.5, 0.1)               | 1               | 5.75  | 5.76    | 1.99  | 2.12    | 0.35  | 0.37    |
|                          | 2               | 5.66  | 5.67    | 2.24  | 2.39    | 0.40  | 0.42    |
|                          | 3               | 5.61  | 5.63    | 2.38  | 2.52    | 0.42  | 0.45    |
|                          | 4               | 5.58  | 5.60    | 2.46  | 2.60    | 0.44  | 0.46    |
|                          | 5               | 5.56  | 5.58    | 2.51  | 2.66    | 0.45  | 0.48    |
| (0.1, 0.5)               | 1               | 5.48  | 5.51    | 3.20  | 3.47    | 0.58  | 0.63    |
|                          | 2               | 5.39  | 5.42    | 3.52  | 3.81    | 0.65  | 0.70    |
|                          | 3               | 5.35  | 5.39    | 3.65  | 3.94    | 0.68  | 0.73    |
|                          | 4               | 5.33  | 5.37    | 3.72  | 4.01    | 0.70  | 0.75    |
|                          | 5               | 5.32  | 5.36    | 3.76  | 4.05    | 0.71  | 0.76    |
| (0.5, 0.5)               | 1               | 5.49  | 5.52    | 3.18  | 3.45    | 0.58  | 0.62    |
|                          | 2               | 5.40  | 5.43    | 3.50  | 3.79    | 0.65  | 0.70    |
|                          | 3               | 5.36  | 5.39    | 3.63  | 3.93    | 0.68  | 0.73    |
|                          | 4               | 5.34  | 5.37    | 3.70  | 4.00    | 0.69  | 0.74    |
|                          | 5               | 5.33  | 5.36    | 3.75  | 4.04    | 0.70  | 0.75    |

Tables 2-4 show the effect of different sets of service rate \((b_v, b_b)\), failure rate \((\omega_v, \omega_b)\) and repair rate \((\tilde{b}_v, \tilde{b}_b)\), respectively on \(E[N]\), \(TP\) and \(D\) by varying vacation rate \(\mathfrak{U}\) for homogenous and heterogeneous arrival rates. We see that with the variation in \(\mathfrak{U}\), \(TP\), \(E[N]\) and \(D\) are smaller for homogenous arrival rate in comparison to heterogeneous arrival rate. From tables 2, 3 and 4 it is observed that throughput decreases as \(\mathfrak{U}\) increases. But the expected number of customers and average delay increase with \(\mathfrak{U}\).
Figures 1–3 and figures 4–6 depict the effect of various parameters on expected number of customers $E[N]$ and average delay $D$, respectively for different sets of homogenous arrival rate and heterogeneous arrival rate. From all the figures it is observed that $E[N]$ and $D$ are higher for heterogeneous arrival rate in comparison to homogenous arrival rate as we except. By increasing the service rate ($\mu$, $\nu$), the expected number of customers $E[N]$ and average delay $D$ decrease; the effect of $\nu$ is quite visible for lower value whereas $\mu$ does not contribute significantly as clearly seen in figures 1 and 4. Figure 2 exhibits that $E[N]$ and $D$ do not change as $\omega_v$ increases, however $\omega_b$ makes linear noticeable effect in the direction. In figures 3 and 6 we examine the effect of repair rate ($\rho$, $\lambda$) on $E[N]$ and $D$, respectively and note that $E[N]$ and $D$ first decrease significantly for increasing values of $\rho$ and become almost constant after a threshold value of $(\rho, \lambda) = (2, 3)$.

Concluding, the probable numbers of customers, throughput and average delay are always lower for homogeneous arrival rate in comparison of heterogeneous arrival rate. The breakdown during vacation period does not affect performance indices, this is quite remarkable observation. Also, failure during busy period lowers down queue length which is somewhat different from the pattern with respect to other parameters.

Figure 1. Expected queue length $E[N]$ by varying service rate (a) $\nu$, (b) $\mu$. 
Figure 2. Expected queue length $E[N]$ by varying failure rate (a) $\omega_v$ (b) $\omega_b$.
Figure 3. Expected queue length $E[N]$ by varying service rate (a) $\rho_v$ (b) $\rho_b$.

Figure 4. Delay $D$ by varying service rate (a) $v_v$ (b) $v_b$. 
6. Concluding Remarks

In this paper, we have developed the queueing model with working vacation, unreliable server and state dependent rates. Matrix geometric method is employed to analyze Markov process which has a certain block structure. The incorporation of state dependent rate and working vacation makes our model more applicable than the other existing models. The utilization of server during vacation may be helpful as cost-effective tool in many congestion situations encountered in production systems, transportation systems and telecommunication systems, etc. We have obtained the probability vectors, which have been
further used to determine the various performance measures.
The sensitivity analysis performed shows the effects of
different parameters on system performance which can be used
for optimal control design.

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