Determination Plastic Properties of a Material by Spherical Indentation Base on the Representative Stress Approach

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Abstract. Under an indentation, the material undergoes a complex deformation. One of the most effective ways to analyse indentation has been the representative method. The concept coupled with finite element (FE) modelling has been used successfully in analysing sharp indenters. It is of great importance to extend this method to spherical indentation and associated hardness system. One particular case is the Rockwell B test, where the hardness is determined by two points on the P-h curve of a spherical indenter. In this case, an established link between materials parameters and P-h curves can naturally lead to direct hardness estimation from the materials parameters (e.g. yield stress (σy) and work hardening coefficients (n)). This could provide a useful tool for both research and industrial applications. Two method to predict p-h curve in spherical indentation has been established. One is use method using C1-C2 polynomial equation approach and another one by depth approach. Both approach has been successfully. An effective method in representing the P-h curves using a normalized representative stress concept was established. The concept and methodology developed is used to predict hardness (HRB) values of materials through direct analysis and validated with experimental data on selected samples of steel.

1. Introduction
One significant advantage of indentation is that it only requires a small amount of materials; this makes it very attractive for the characterization of materials with gradient property where standard specimen is not readily available such as in situ or in vivo tests [1]. However, despite its wide use, the materials behaviour (represented by the hardness or P-h curves) are not explicitly linked with the constitutive material properties. Further work is required to be able to predict indentation resistance (P-h curves and/or hardness) from constitutive materials parameters. On the other side, it is also of great significance to both research and practical use to explore the potential to use indentation data to predict the constitutive materials properties. It may potentially provide a quicker way for material parameter identification and applicable in situation where standard specimen is not available. Developments of both areas require a detailed understanding/program linking constitutive materials properties, P-h curve and hardness from spherical indenters.

Earlier works showed that hardness can be related to the stress of the indented material, σr, corresponding to a representative strain, εr, which represents the mean plastic strain after yielding. The concept coupled with finite element (FE) modelling has been used successfully in analyzing sharp indenters where the representative strain and stress is well defined with a fixed indenter angle [2], [3]. It is of great importance to extend this to spherical indentation and associated hardness system. One
particular case is the Rockwell B test, where the hardens is determined by two points on the P-h curve of a spherical indenter. In this case, an established link between materials parameters and P-h curves can naturally lead to direct hardness estimation from the materials parameters (such as yield stress and work hardening coefficients). This could provide an useful tool for both research and practical applications.

2. Experimental
The material used were steel. The chemical compositions of materials are listed in table 1.

| Material               | Condition | Element Composition (%) |
|------------------------|-----------|-------------------------|
| Carbon Steel 0.10% C   | Norm. at 900°C | C  0.1  Mn  0.5  P  <0.04  S  <0.05  Si  0.100  Ni  0.01 |
| Mild Steel             | N/A       | C  0.3  Mn  0.3  P  0.05  S  0.05  Si  0.122  Ni  490 ppm |
| Carbon Steel 0.54% C   | Norm. at 840°C | C  0.54  Mn  0.9  P  0.055  S  0.014  Si  0.19  Ni  0.014 |

Sample steel used is solid rod-shaped elliptical of 5 mm in diameter and 90 mm long. The tensile tests were performed with extensometer interfaced with a microcomputer, graphical print out of the test can be obtained and test data saved. The machine has a maximum loading capacity of 30 kN, with the readings being accurate to 0.5% of the force. The two main materials were used in this experiment include a carbon steel (0.1% C Steel) and Mild steel specimen. Typical The stress–strain curves of the two main materials tested are shown in figure 1a. Yield stress (σy) identified for the 0.1% C carbon steel is 308.03 MPa, work hardening coefficients (n) is 0.07; the yield stress (σy) for the mild steel is 601.66 MPa, the work hardening coefficient (n) is 0.025. The stress–strain curves and the material properties data are to be used as input to the FE model and to assess the accuracy of the P-h curves based hardness evaluation and inverse material properties. Specimens were sectioned, in order to hardness testing performed using hardness test equipment. The indentation using spherical indenter B scale, with R= 0.79 mm (Diameter steel ball =1/16 in), the indenter is forced into the sample under a preliminary minor load (Fm) = 10 Kgf and followed by a mayor loading (FM) =100. The hardness values is determined by the depth of penetration spherical indenter loaded on test samples following equation HR=130-(h1000-h100)/0.002. The machine is calibrated using a testing block, the variation of the hardness is within 5%. The average HRB values for two sample materials are shown in figure 1b. The Rockwell hardness B (HRB) identified for the 0.1% C is 62.0, and HRB for Mild Steel is 88.0 These HRB hardness data be used to validate the representative stress based hardness evaluation and property prediction program to be developed.

![Figure 1](image-url)
3. Framework and analysis

3.1. Material model

The power law description, almost used to approximate the plastic behaviour of metal material [4]. A comprehensive framework using power law was developed earlier [5] in material model with reduce young modulus $E^*$, within a specified range of material parameters has been identified a representative plastic strain and used to normalize the loading curvature of material hardening exponent ($n$). This work is concerned with elastic-plastic materials with a particular focus of steel, the stress-strain curves can be represented by Hooke’s law and Von Misses yield criterion with isotropic power law hardening. Under those general conditions, the dependence of the true stress $\sigma$ on the true strain $\varepsilon$ is commonly expressed:

$$
\sigma = \begin{cases} 
E \varepsilon & \text{for } \sigma \leq \sigma_y \\
\sigma_y \left( \frac{\varepsilon}{\sigma_y} \right)^n \varepsilon^n & \text{for } \sigma > \sigma_y 
\end{cases}
$$

(1)

Where $E$ is the Young’s modulus, $n$ the strain hardening exponent and $\sigma_y$ the initial yield stress at zero offset strain. If $\varepsilon_r$ is a particular plastic strain point, the stress at the point representative stress, $\sigma_r$. At this plastic strain point, the stress can also be expressed as:

$$
\sigma_r = \sigma_y \left( 1 + \frac{\varepsilon}{\sigma_y} \varepsilon_r \right)^n
$$

(2)

3.2. Numerical model

Axial symmetric 2-D space FE models were constructed to simulate the indentation response of elastic plastic solids, are shown in figure 2. Model was used due to the symmetry of the spherical indenter. The indenter was assumed to be rigid body as it is much harder than the indented material. The movement of the indenter was simulated by displacing a rigid arc (rigid body) along the Z axis. In the model, the sample size can be changed to ensure that the sample is much larger than the indenter radius/contact area during the indentation to avoid potential sample size and boundary effects [6].

![Figure 2](image)

**Figure 2.** FE Model of the spherical indentation test (R=0.79 mm). Close up view showing the mesh underneath the indenter.

The bottom line of the model was fixed in all degree of freedoms (DOF) and the central line was symmetrically constrained. A gradient meshing scheme has been developed for different regions. While the spherical indenter models used R= 0.79 mm, and specimen model used young’s modulus = 200 GPa, Poisson ratio = 0.2 and material plastic input data are to be used as input to the FE model include a carbon steel (0.1% C Steel) and mild steel. The mesh size is 10µm in the region underneath and around the indenter, while the mesh of other regions used single bias with bias element number 33 and ratio 5 to obtain gradient mesh tightly into underneath and around the indenter to improve the accuracy of the model. The free surface was made at the top and outside surface of specimen. In FE modelling, the accuracy of results is influenced by many factor such as the mesh shape and density,
element type, friction condition and validation of the boundary conditions [6]. In the Spherical FE model the indenter considered as a rigid body to improve the modelling efficiency. A predefined displacement was applied on indenter and the reaction force is recorded on the reference point, representing the overall load on the indenter. The results of simulations FE model Spherical establish will produce p-h curve. Figure 6 shows typical P-h curve during loading and unloading phase of a typical elastic-plastic materials with different indenter sizes (R=0.50 mm; 0.79 mm; and 1.25 mm). The loading curve represents the resistance of material to indenter penetration, while difference between the loading and unloading curve represents the energy loss [7]. The Spherical FE Model developed were validation with analytical solution of elastic material base of relationship using a known analytical solution [6] for indentation of linear elastic materials.

\[
F_z = \left(\frac{16R}{9}\right)^{\frac{1}{2}} \cdot \frac{E}{1-v^2} \cdot \delta^{\frac{3}{2}}
\]  

(3)

Where ‘\( F_z \)’ is the reaction force, ‘\( R \)’ is the indenter radius; ‘\( E \)’ and ‘\( v \)’ is the Young’s modulus and Poisson’s ratio of material, respectively. ‘\( \delta \)’ is the indentation depth. As shown in the figure 3a, visible the trend analysis in accordance with the numerical force–displacement data simulation FE Model and resulting using the following analytical. This indicates a statistically curve fitting the data equally well and the FE model is congruous with the analytical model. The correlation coefficients between these two curves using a least square regression method is within 99.9%. The FE spherical model was further validated by comparing the P-h curve with an elastic-plastic material model and published result data [8]. In the FE model, the material properties used were depicted directly from the published work. As shown in figure 3b the numerical results showed good agreement with the experimental data, which suggests that the FE model is valid and the results are accurate. One development in this work that has directly contribute to this process is the development of FE model. In this process firstly developed. Then the key parameter, such as dimensional data (e.g. sample size, indenter size) or material properties are identified and the parameters to be studied can be defined as variables. For example, in the initial model development process, the mesh size can be set as a variable to determine the best mesh density; the fictional coefficients at the indenter-sample interface can be set as a variable to determine the best values. In this stage, the main purpose is to find the P-h curve and material properties relationship, so the yield stress and the work hardening coefficients were set as variables. In this work encompassing a domain of Yielding strength \( \sigma_y \) from 100 to 900 MPa and strain hardening exponent \( n \) varying from 0.0 to 0.3 and Poisson’s ratio \( v \) was fixed at 0.2.

Figure 3a. Comparison between the FE numerical force–displacement (p-h curve) data and analytical solution with elastic material model. (\( R=0.5, 0.79, 1.25 \) mm.) and Figure 3b. Comparison of numerical results with published experimental data [8] of indentation with a spherical indenter (\( R=1.25 \) mm) showing the validation of FE model with elastic-plastic materials.
4. Effect of material properties and new analysing approaches of P-h curves

4.1. Full curve fitting approach and results

A Comprehensive parametric study using procedure developed was conducted representing the range of parameters of mechanical behavior found in common engineering metals. Poisson’s ratio was fixed at 0.3, Young modulus $E=200$ GPa, the yield strength ($\sigma_y$) 100 to 900 MPa, and strain hardening from 0.0 to 0.3. The results of simulations with FE model Spherical establish will produce P-h curve. After evaluation of several approaches, two approaches have been found to be effective in representing the curves with adequate/acceptable accuracy. Figure 4-5 shows the two approached proposed. The first method is to use second order polynomial fitting in the form of:

$$ P = C_1 h^2 + C_2 h $$

(4)

The second fitting approach to be explored to represent the curve is using the force at different indentation depths. Most continuous machine can be depth controlled, and the representative stress is known to be depth dependent (angle dependent), so this potentially can be used as a more robust method. If the correlation between the force at different depth and the constitutive material properties and/or the representative stress is established, then the full P-h curve can be determined. This potentially can provide an effective way and physically meaningful way by using the power of computer simulation as large set of data has to be processed. For spherical indenter, the angle changes with the increasing depth, no fixed representative strain is readily available. However, in general, based on the deformation mechanism of an indentation process, the material deformation is controlled by the elastic deformation and the yielding, so we propose to use an effective representative stress which potentially could be linked to the C1 and C2 parameters, thus representing the full P-h curve. In the equation, C2 is linear term in the equation, so the fitting was conducted directly associating C2 to E/$\sigma_r$, this term represents the balance of elastic and plastic properties. Figure 4 plots the C2 vs. E/$\sigma_r$ with different representative strain for C2. It clearly shows that there is a reasonable correlation between these C2 and E/$\sigma_r$, and the fitting is influenced by the representative strain used. At a representative strain of 0.01 (Fig. 4), the fitting is reasonable with the best correlation coefficient.

$$ C_2 = 3566.9 \left( \frac{E}{\sigma_r} \right)^{0.855} $$

(5)

Theoretically, C1 is a second order coefficient, so it could potentially be linked to the representative stress follow the C1/$\sigma_r$ vs E/$\sigma_r$ according to Eq. 4 and Eq. 5. But the results is not very good with a effective representative strain of 0.07. Other strain level has been explored the results were equally not satisfactory. So a physical based hypothesis has been evaluated. From energy point of view, the resistance to indentation consists of elastic resistance and plastic resistance. The nonlinearity and linearity of the curve should reflect a balance between elastic deformation and plastic deformation, which can be represented by E/$\sigma_r$. The relationship directly between C1 and E/$\sigma_r$, has been explored with different representative strains. Some typical results are shown in figure 5. The fitting are much better than that for fitting between C1/ $\sigma_r$ vs E/ $\sigma_r$. Comparing the correlation with different representative strains, the most effective reference strain is $\varepsilon r = 0.07$ (figure 5.(d)), which give an equation

$$ C_1 = 3606.8 \left( \frac{E}{\sigma_r} \right) (-1.252) $$

(6)

The correlation coefficients is over 93%. Further increasing or decreasing of the representative strains shows no improvement in correlation of the fitting. So this is the value in predicting the P-h curves to evaluate its accurate With the relationship between C1 and C2 with $\sigma_r$, the P-h curve can determined with a known set of material properties (yield stress and work hardening coefficients). Some typical examples are shown in figure-5. In each figure (a-c), the predicted P-h curves using the representative stress full curve fitting approach (solid line) and Finite element data (symbols) was plotted together. In
each case, the comparison is in a reasonable agreement. The full curve fitting approach showed a reasonable accuracy in predicting the P-h curves with known material properties.

![Figure 4. Plot the C vs. E/σr with different representative strains for the full curve approach.](image)

![Figure 5. Plot the C1 vs. E/σr with different representative strains for the full curve approach.](image)

4.2 Hardness prediction base on P-h curves an inverse material parameter estimation

The concept and methodology developed is to be use to predict hardness value (HRB) of material through direct analysis and validated with experimental data on selected sample of steel. Experimental test have been performed on 0.1% Carbon steel and Mild steel including tensile test and Rockwell hardness test (HRB). The Rockwell hardness is predicted directly base on P-h curve predicting using the representative stress equation established. **Figure 6a.** shows the P-h curves of the two steel samples from direct FE modelling (using the stress strain curves as input material properties) and the P-h curves predicted using equations (4-6). As shown in the figure, the curves are in a good agreement with each other. From the P-h curves, the indentation depth at a load of 100N and the 1000N is depicted and used to calculate the HRB following the equation:

$$HRB = 130 - (h_{1000N} - h_{100N})/0.002$$

(7)

**Figure 6b.** plots the HRB values calculated based on the data in comparison with the experimental results. In all cases, the prediction based on the representative approach (equations 4-6) showed a good agreement with the test data and the FE data. Works has been conducted on other to materials, and the prediction results showed a similar degree of agreement. This suggests that the P-h curve based approach for predicting HRB values is valid and accurate.
Base on the concept prediction of hardness base on P-h curves an inverse material parameter estimation developed, it is implied that the hardness values could be potentially being directly linked to the representative stresses over the material ranges studied. This is investigated by firstly determine the HRB values of materials over a wide range of properties, then a direct hardness-$\sigma_r$ relation is explored using data fitting process. Figure 6 shows the correlation between the HRB vs $\sigma_r$.

![Figure 6a. Comparison of the P-h curs of the two steel specimen from FE model and representative stress equations. Figure 6b. Comparison between experiment and prediction Rockwell hardness value (HRB) with FE modelling for Carbon Steel 0.10 % C and. Mild Steel.](image)

$$HRB/\sigma_r = 0.0748 \ln \left( \frac{E}{\sigma_r} \right) - 0.2945$$ (8)

These relationships (eqs.8) established allow direct hardness prediction from material properties. This is assessed using the two steel materials as example, the predicted HRB showed a similar level of agreement with the experimental data. In the case of the 0.1 C Steel, the HRB values is within 107% of the measured value; In the case of Mild steel, the HRB is within 102 % of the measured value. Similar agreement has been found in other materials (within 5% error range) as shown in figure 6(b). This suggests that these can be used to predict the hardness values with sufficient accuracy with the measurement error ranges.

5. Conclusions
In this work, the relationships between constitutive materials parameters ($\sigma_y$ and $n$) of elasto-plastic materials, indentation P-h curves and hardness with spherical indenters has been systematically investigated by combining representative stress analysis and FE modelling using steel as a typical model material group. The main outcomes of work has formed a frame work of models to predict indentation P-h curves from constitutive. An approach to predict the P-h curves from constitutive material properties has been developed and evaluated based on the relationship between the curvature and material properties and representative stress. Two new approaches to characterise the P-h curves of spherical indentation have been developed and evaluated. One is the full curve fitting approach while the other is depth based approach. In the full curve fitting approach, the relationship between an effective representative stress with the first and second order coefficients of a polynomial fitting line of the P-h was established. In the depth approach the relationship between force and representative stress with varying representative strain has been established. Both approaches were proven to be adequate/effective in predicting indentation P-h curves.
The approaches (i.e. predict hardness from P-h curves) established was successfully used to produce hardness values of a wide range of material properties, which is then used to establish the relationship between the hardness values (HRB) with representative stress.

6. References
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