THE STUDY OF CHIRAL RESTORATION USING THE QUARK-GLUON MIXED CONDENSATE \( g\langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle \)
IN LATTICE QCD AT FINITE TEMPERATURE

T. DOI\(^1\), N. ISHII\(^2\), M. OKA\(^1\) AND H. SUGANUMA\(^1\)

\(^1\) Department of Physics, Tokyo Institute of Technology, Ohokayama 2-12-1, Meguro, Tokyo 152-8551, Japan
E-mail: doi@th.phys.titech.ac.jp

\(^2\) Radiation Laboratory, The Institute of Physical and Chemical Research, (RIKEN), Hirosawa 2-1, Wako, Saitama, 351-0198, Japan

The quark-gluon mixed condensate \( g\langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle \) is studied using SU(3)\(_c\) lattice QCD with the Kogut-Susskind fermion at the quenched level. Using the lattices as \( \beta = 6.0 \) with \( 16^3 \times N_t(N_t = 16, 12, 10, 8, 6) \), \( \beta = 6.1 \) with \( 20^3 \times N_t(N_t = 20, 12, 10, 8, 6) \) and \( \beta = 6.2 \) with \( 24^3 \times N_t(N_t = 24, 16, 12, 10, 8) \) in high statistics of 100-1000 gauge configurations, we perform accurate measurement of the thermal effects on \( g\langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle \) as well as \( \langle \bar{q}q \rangle \) in the chiral limit. We find that the thermal effects on both the condensates are very weak except for the vicinity of \( T_c \), while both the condensates suddenly vanish around \( T_c \approx 280\text{MeV} \), which indicates strong chiral restoration near \( T_c \). We also find that the ratio \( m_0^2 \equiv g\langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle / \langle \bar{q}q \rangle \) is almost independent of the temperature even in the very vicinity of \( T_c \), which means that these two different condensates obey the same critical behavior. This nontrivial similarity between them would impose constraints on the chiral structure of the QCD vacuum near \( T_c \).

1. Introduction

Fruitful phenomena in hadron physics, such as spontaneous chiral-symmetry breaking and color confinement, originate from the non-perturbative nature of QCD, or its nontrivial vacuum structure. The study of the vacuum structure of QCD is especially interesting at finite temperature/density, because it is expected that QCD has various phase structures in the phase diagram, and chiral restoration and color deconfinement can be observed. Considering that the on-going RHIC experiments are actu-

---

*The lattice QCD Monte Carlo simulations have been performed on NEC SX-5 at Osaka University and IBM POWER4 at Tokyo Institute of Technology.
ally tackling this subject, we here present the theoretical study of finite temperature QCD. In this point, condensates are useful physical quantities because they characterize the nontrivial QCD vacuum. At finite temperature, we can probe the change of the QCD vacuum, by examining the thermal effects on the condensates.

Among various condensates, we study the quark-gluon mixed condensate \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) for the following reasons. First, we note that the mixed condensate is a chiral order parameter independent of \( \langle \bar{q}q \rangle \), since it flips the chirality of the quark as

\[
g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle = g\langle \bar{q}R(\sigma_{\mu\nu}G_{\mu\nu})qL \rangle + g\langle \bar{q}L(\sigma_{\mu\nu}G_{\mu\nu})qR \rangle.
\]

Therefore, \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \), as well as \( \langle \bar{q}q \rangle \), are important indicators on the chiral structure of the QCD vacuum. In particular, the thermal effects on both the condensates and the comparison of them will reveal mechanism of the chiral restoration of the QCD vacuum at finite temperature. Second, in contrast to \( \langle \bar{q}q \rangle \), \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) represents a direct correlation between quarks and gluons in the QCD vacuum, and thus characterizes different aspect of the QCD vacuum. Third, the mixed condensate is relevant in various QCD sum rules, especially in baryons\(^1\), light-heavy mesons\(^2\) and exotic mesons\(^3\). In this point, the quantitative estimate of the thermal effects on \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) has an impact on hadron phenomenology at finite temperature through the framework of the QCD sum rule. Therefore, it is desirable to estimate \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) at zero/finite temperature by a direct calculation from QCD, such as lattice QCD simulations.

So far, the mixed condensate \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) at zero temperature has been analyzed phenomenologically in the QCD sum rules\(^4\). In the lattice QCD, there has been no result except for a rather preliminary but pioneering work\(^5\) done long time ago. Recently, new lattice calculations have been performed by us\(^6,7\) using the Kogut-Susskind (KS) fermion, and by other group\(^8\) using the Domain-Wall fermion. At finite temperature, however, there has been no result on \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) except for our early results\(^7\). Therefore, we here present the extensive results of the thermal effects on \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) as well as \( \langle \bar{q}q \rangle \), including the analysis near the critical temperature\(^9\).

2. The Lattice Formalism

We calculate the condensates \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) as well as \( \langle \bar{q}q \rangle \) using SU(3)\(_c\) lattice QCD with the KS-fermion at the quenched level. We note that the KS-fermion preserves the explicit chiral symmetry for the quark mass...
\( m = 0, \) which is a desirable feature to study both the condensates of chiral order parameters.

The Monte Carlo simulations are performed with the standard Wilson action for \( \beta = 6.0, 6.1 \) and \( 6.2. \) In order to calculate at various temperatures, we use the following lattices as

(1) \( \beta = 6.0, \ 16^3 \times N_t \ (N_t = 16, 12, 10, 8, 6), \)
(2) \( \beta = 6.1, \ 20^3 \times N_t \ (N_t = 20, 12, 10, 8), \)
(3) \( \beta = 6.2, \ 24^3 \times N_t \ (N_t = 24, 16, 12, 10, 8). \)

We generate 100 gauge configurations for each lattice. However, in the vicinity of the chiral phase transition point, namely, \( 20^3 \times 8 \) at \( \beta = 6.1 \) and \( 24^3 \times 10 \) at \( \beta = 6.2, \) the fluctuations of the condensates get larger. We hence generate 1000 gauge configurations for the above two lattices to improve the estimate. The lattice units are obtained as \( a \simeq 0.10, 0.09, 0.07 \)fm for \( \beta = 6.0, 6.1, 6.2, \) respectively, which reproduce the string tension \( \sigma = 0.89 \)GeV/fm. We calculate the flavor-averaged condensates as

\[
a^3 \langle \bar{q}q \rangle = -\frac{1}{4} \sum_f \text{Tr} \left[ \langle q^f(x)\bar{q}^f(x) \rangle \right],
\]

\[
a^5 g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle = -\frac{1}{4} \sum_{f, \mu, \nu} \text{Tr} \left[ \langle q^f(x)\bar{q}^f(x) \rangle \sigma_{\mu\nu} G_{\mu\nu} \right],
\]

where SU(4)\(_f\) quark-spinor fields, \( q \) and \( \bar{q} \), are converted into spinless Grassmann KS-fields \( \chi \) and \( \bar{\chi} \) and the gauge-link variable. We adopt the clover-type definition for the gluon field strength \( G_{\mu\nu} \) on the lattice in order to eliminate \( \mathcal{O}(a) \) discretization error. The more detailed formula for the condensates and gluon field strength are given in Ref. 6.

In the calculation of the condensates, we use the current-quark mass \( m = 21, 36, 52 \)MeV. For the fields \( \chi, \bar{\chi}, \) the anti-periodic condition is imposed at the boundary in the all directions. We measure the condensates on 16 different physical space-time points of \( x (\beta = 6.0) \) or 2 points of \( x (\beta = 6.1, 6.2) \) in each configuration. Therefore, at each \( m \) and temperature, we achieve high statistics in total as 1600 data (\( \beta = 6.0 \)) or 200 data (\( \beta = 6.1, 6.2 \)). Note that in the vicinity of the critical temperature, \( 20^3 \times 8 \) (\( \beta = 6.1 \)) and \( 24^3 \times 10 \) (\( \beta = 6.2 \)), we take up to 2000 data so as to achieve high statistics and to guarantee reliability of the results.

### 3. The Lattice Results and Discussions

We calculate the condensates \( g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle \) as well as \( \langle \bar{q}q \rangle \) at each quark mass and temperature. We observe that both the condensates show a clear
linear behavior against the quark mass $m$, which can be typically seen in Ref. 6. We therefore fit the data with a linear function and determine the condensates in the chiral limit. Estimated statistical errors are typically less than 5%, while the finite volume artifact is estimated to be about 1%, from the check on the dependence of the condensates on boundary conditions$^6$.

For each condensate, we estimate the thermal effects by taking the ratio between the values at finite and zero temperatures. The renormalization constants are expected to be canceled in this ratio because there is no operator mixing for both the condensates in the chiral limit$^{10}$. In figure 1, we plot the thermal effects on $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$. We find a drastic change of $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ around the critical temperature $T_c \approx 280$MeV. This is a first observation of chiral-symmetry restoration in terms of $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$. We also find that the thermal effects on $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ are very weak below the critical temperature, namely, $T \lesssim 0.9T_c$. In figure 2, the same features can be also seen for $\langle \bar{q} q \rangle$.

![Figure 1](image1.png)  
*Figure 1. Thermal effects on the mixed condensate $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ v.s. the temperature $T$. The vertical dashed line denotes the critical temperature $T_c \approx 280$MeV at the quenched level.*

![Figure 2](image2.png)  
*Figure 2. Thermal effects on the quark condensate $\langle \bar{q} q \rangle$ v.s. $T$.*

We then compare the thermal effects on $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ and $\langle \bar{q} q \rangle$, because these two condensates characterize different aspects of the QCD vacuum. For this purpose, we plot the thermal effects on $m_0^2 \equiv g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle / \langle \bar{q} q \rangle$ in figure 3. From figure 3, we observe that $m_0^2$ is almost independent of the temperature, even in the very vicinity of $T_c$. This very nontrivial result means that both of $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ and $\langle \bar{q} q \rangle$ obey the same critical behavior. This may indicate the existence of the universal behavior of chiral order parameters near $T_c$, and would impose constraints on the chiral structure of the QCD vacuum.
Figure 3. The thermal effects on $m_0^2 \equiv g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle / \langle \bar{q} q \rangle$ v.s. the temperature $T$. This result indicates the same critical behavior between $\langle \bar{q} q \rangle$ and $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$.

In summary, we have studied the thermal effects on $g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle$ as well as $\langle \bar{q} q \rangle$ using SU(3)$_c$ lattice QCD with KS-fermion at the quenched level. While the thermal effects on both the condensates are very weak below $T_c$, we have observed a strong chiral phase transition near $T_c$ as a sudden disappearance of both the condensates. We have also observed that $m_0^2 \equiv g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle / \langle \bar{q} q \rangle$ is almost independent of the temperature even in the very vicinity of $T_c$, which means both the condensates obey the same critical behavior. This nontrivial similarity would impose constraints on the chiral structure of the QCD vacuum near $T_c$. For further studies, a full QCD lattice calculation is in progress in order to analyze the dynamical quark effects on the condensates.

References
1. H.G. Dosch, M. Jamin and S. Narison, Phys. Lett. B220, 251 (1989).
2. H.G. Dosch and S. Narison, Phys. Lett. B417, 173 (1998).
3. J.I. Latorre, P. Pascual and S. Narison, Z. Phys. C34, 347 (1987).
4. V.M. Belyaev and B.L. Ioffe, Sov. Phys. JETP 56, 493 (1982).
5. M. Kremer and G. Schierholz, Phys. Lett. B194, 283 (1987).
6. T. Doi, N. Ishii, M. Oka and H. Suganuma, Phys. Rev. D67, 054504 (2003).
7. T. Doi, N. Ishii, M. Oka and H. Suganuma, Nucl. Phys. A721, 934 (2003); Prog. Theor. Phys. Suppl. 151, 161 (2003); in Proc. “Quark Confinement and the Hadron Spectrum V” (World Sci. 2003) p.381, hep-lat/0212025.
8. T.W. Chiu and T.H. Hsieh, hep-lat/0305016.
9. T. Doi, N. Ishii, M. Oka and H. Suganuma, Nucl. Phys. B (Proc. Suppl.) in press, hep-lat/0309124.
10. S. Narison and R. Tarrach, Phys. Lett. B125, 217 (1983).