Investigation of the Current and Voltage Waveforms for a TiO$_2$-based Memristor with MATLAB

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Abstract

The first TiO$_2$-based memristor was implemented in 2008 by HP company in Nano-scale to present the predictable behavior of the memristor and voltage-current (V-I) hysteresis curve. Many aspects of this device are unknown yet and having an accurate model can help to control the voltage and current of this device. For this purpose, some of electronic-based soft wares can help like PSPICE or Workbench. This paper is modeling the resistive behavior of this component with non-linear ionic deviance of the TiO$_2$ by simple codes in MATLAB to be controlled by voltage or current. Some of the simulation results are presented with model adjustments for the specifications of this component.

Keywords: Memristor, TiO$_2$, Matlab simulation

1. Introduction

For about 150 years it was believed that the resistance, the inductor, and the capacitor were the only three main passive elements of electronic circuits. In 1960, Professor Lean Chua, using nonlinear circuit theory, proposed an independent differential relationship that coupled the electric charge and magnetic flux in the circuit, and predicted the existence of a fourth element called the memistor. He proved that memristive behavior is achieved by combining only inductor, resistance, and nonlinear capacitors [1]. Although Chua has shown that such a lost component of electronics is very interesting and has many circuit features, at that time and until recently no one had ever presented a real physical model or a sample of a piece made as a memristor. It took almost 40 years to build a real system that had memorization properties. Although Chua has shown that such a lost component of electronics is very interesting and has many circuit features, at that time and until recently no one had ever presented a real physical model or a sample of an implemented memristor. It took almost 40 years to build a real system that had memristive properties. A team from Hewlett Packard (HP) company was researching for the construction of a physical element with a memorial behavior. In 2006, the team concluded that the time-variable derivative of the state in the Chua
dynamic state equation is comparable to the rate of deviation of oxygen vacancies in a nanometer-sized titanium dioxide resistance switch [2-3]. As the investigation continued, in 2008 the team finally managed to build a two-legged component with the resistive features. In addition, they showed that using this element in a Crossbar array could be a good alternative to transistors in future computers [4-5].

In this study, we're going to take a closer look at the theory and interesting features, especially the "soft and hard switching" effects, by expanding previous work on the TiO$_2$-based memristor model by using simple MATLAB codes according to existing mathematical equations at the new types recently suggested in various models of nonlinear ion deviation. In the second part, after this introduction, we discuss the general features of the memristor, the structure, and formulas of the titanium dioxide HP memristor. In the third section, we will review the various models available for this component, review the advantages and disadvantages of these models, and the characteristics of the proposed model. In the fourth part, the HP Memory MATLAB Model is introduced with a nonlinear ion deviation for the voltage or current control type, and some simulations are performed by adjusting the parameters of the model in accordance with the specifications of the memristor. Finally, the results of this article will be reviewed and summarized in the fifth section.

2. The Structure, Properties and Formulas of the Memristor

2.1. General Features of the Memristor

In 1971, Leon Cheva, a professor of electrical engineering at the University of California, Berkeley, arranged the linear relationships between all four basic variables describing the orbital relationships as shown in Figure 1. These four variables are related by six methods. With a sort of mathematical arrangement, he was able to predict the existence of a fourth passive element, a theory that linked the relationship between electric charge and magnetic flux.

![Figure 1: Four basic two-wire elements](image1)

2.2. Structure of the Memristor

Williams and his group presented the physical model of a two-ended electrical device that behaved like an ideal memristor within the limits set in 2008. This model has been in the literature for about 50 years; bipolar nanometer scale, especially in thin-film devices; switching provided a simplified explanation for current-voltage anomalies that occur as hysteretic conductivity, multi-state conductivity, and negative differential resistance. The electrical switching in thin-film devices has recently begun to attract more attention, as such technology can allow scaling in logic and memory circuits beyond the current limits of CMOS technology. Williams and his group proposed a model to explain electrical switching. According to this model, doped TiO$_2$ and undoped TiO$_2$-x regions were formed between two Pt thin films in Pt / TiO$_2$ / Pt structure. The total thickness of the doped and undoped regions is in the order of several nm. The application of the V(t) voltage to the device from outside and the drift of oxygen ions, shifts the boundaries of the two regions as has been shown in figure 2, and switching occurs with the change of the equivalent resistance of the structure. In figure 3, the model for memristor, which is based on the observations of HP researchers, was later called the Linear Model.
There are four fundamental circuit variables, the current $i$, the voltage $v$, the charge $q$ and the flux $\phi$. Chua [6] realized that only five of the six possible combinations are used, so he defines the missing combination to be the memristive one,

$$R(w) = \frac{dq}{d\phi} = \frac{v(t)}{i(t)}$$

(1)

where $R$ is the instantaneous resistance and $w$ is the state variable of the Memristor. The HP Memristor [7-8] was built on drifting the dopant between doped and undoped portion of the material, which models the memristive property. As shown in Fig.(1) $w$ is modeled by the length of the doped region, which is bounded between 0 and device length $D$. The instantaneous resistance of the Memristor is written as [9],

$$R = x.R_{on} + (1 - x)R_{off}$$

(2)

where $x = w/D$, $R_{on}$ is the resistance of the completely doped Memristor, and $R_{off}$ is for the completely undoped. The Memristor resistance as a function of time is driven in [10-11]. We can generalize the resistance for linear dopant drift as,

$$R^2 = R_0^2 - 2kR_0\phi(t), R \in (R_{on}, R_{off})$$

(3)

In this model, it can be written as shown below for linear ionic drift and resistive electronic conductivity in a uniform field with average mobility of oxygen ions representing the moving charge:

$$v(t) = (R_{on} \frac{w(t)}{D} + R_{off}(1 - \frac{w(t)}{D}))i(t)$$

(4)

$$\frac{\partial w(t)}{\partial t} = \mu v \frac{R_{on}}{D} i(t)$$

(5)

From equation (5), one can calculate foe $W(t)$:

$$W(t) = \mu v \frac{R_{on}}{D} q(t)$$

(6)

By replacing (6) in (4):

$$M(q) = R_{off}(1 - \frac{\mu v R_{on}}{D^2} q(t))$$

(7)
The coefficient D corresponds to the total thickness of the doped and undoped regions. M (q) increases 1,000,000 times due to the dependence of the constant M (q) on 1 / D^2 when descending from the micron dimension to the nm dimension. This, in fact, has attracted the attention of scientists with the studies in nanotechnology nowadays of a detail that has been overlooked in a macro dimension. Thus, as any device shrinks towards critical dimensions with a nanometer scale, memristance becomes very important to understand its electronic characteristics.

3. MATLAB Codes for Different Inputs and Simulation Results

MATLAB can be considered as a key program for behavior analysis in Memristor devices. This program can be used for other areas including the low power and high power applications in electrical and electronics engineering [12-15]. In this section for the mentioned doped (TiO_2) and undoped (Ti) blocks, different types of the input voltages under Memristor Parameters: Ron = 100, Ro = 4k condition are given are results are reported.

MATLAB simulation for Ti and TiO_2 blocks was carried out by means of the following codes, and the applied voltage, current, and current-voltage curves were obtained. The important detail is that, for applying the voltage to the structure, two thin layers of the platinum is considered as figure 4 shows.

3.1 Sinusoidal Voltage:

For this state, the Ron and Roff are considered as Ron=54 and Roff=1MΩ. Also equations (2), (3) and (4) are considered for obtaining the current value of the structure. Figure 5 presents the voltage and current waveforms of the structure and the I-V hysteresis curve for this state.

```matlab
clc;
clear all;
close all;
p=2;
v_d(1)=0;
f_p=[];
L=[];
f=5;
f_0=5;
t = 0:0.01./f:10./f;
v=ones(size(t));
d = 9*10^-9;
j=2;
```

```matlab
u_v=30*10^-15;
r_on=54;
r_off=1000000;
r_i=11000;

w(1)=0.9989*d;
w(1)=((r_off-r_i)/(r_off-r_on))*d;
x(1)=w(1)/d;
m(1)=r_on*(w/d)+r_off*(1-w/d);
f_p(1)=1-(2*x-1)^(2*p); %Joglekar window
f_p(1)=j*(1 - ((x-0.5)^2+0.75)^p); %Prodromakis window
for index=2:length(t)
    i(index)=v(index)/m(index-1);
end
```

Figure 4. The proposed structure for applying the different voltages
v_d(index)=(u_v*r_on*i(index)*f_p(index-1))/d;
w(index)=v_d(index)*(t(index)-t(index-1))+w(index-1);
x(index)=w(index)/d;

% f_p(index)=1 - (2*x(index)-1)^(2*p);  %Joglekar window
f_p(index)=j*(1 - ((x(index)-0.5)^2+0.75)^p);  %Prodromakis window
m(index)=r_on*(w(index)/d)+r_off*(1-w(index)/d);
if m(index)<r_on
    m(index)=r_on;
end
if m(index)>r_off
    m(index)=r_off;
end
x(index)=w(index)/d;
end
title('Memristor non linear boundary drift model')
subplot(3,1,1)
plot(v)
xlabel('time(t)')
ylabel('Voltage(V)')
subplot(3,1,2)
plot(i)
xlabel('time(t)')
ylabel('current(i)')
subplot(3,1,3)
plot(v,i)
xlabel('V oltage(V)')
ylabel('Current(A)')

Figure 5. voltage, current and I-V hysteresis curves for sinusoidal input wave.

3.2 Triangular Voltage:
While between the two pelatin layers for Ti and TiO2 layers, MATLAB simulation was carried out by means of the following codes, and the current and current-voltage curves passing through the structure were obtained according to the applied voltage. Here are the codes and curves:

clear all;
p=40;
v_d(1)=0;
f_p=[];
L=[];
f=5;
t = 0:0.01/10:f;
v=sawtooth(t/(0.01.*pi),0.5);
d = 9*10^(-9);
j=2;
u_v=30*10^(-15);
*r_on=10.58;
*r_off=1000000;
r_i=7500;
%
w(1)=0.9989*d;
w(1)=((r_off-r_i)/(r_off-2.*r_on))*d;
x(1)=w(1)/d;
m(1)=2.*r_on*(w/d)+r_off*(1-w/d);
f_p(1)=1 - (2*x-1)^2*(2*p);

% Joglekar window
f_p(1)=j*(1 - ((x-0.5)^2+0.75)^p);
% Prodromakis window

for index=2:length(t)
    i(index)=v(index)/m(index-1);
w(index)=v_d(index)*t(index-1));
    x(index)=w(index)/d;
    f_p(index)=1 - (2*x(index)-1)^2*(2*p);
    % Joglekar window
    f_p(index)=j*(1 - ((x(index)-0.5)^2+0.75)^p);
    % Prodromakis window
    m(index)=2.*r_on*(w(index)/d)+r_off*(1-w(index)/d);
    if m(index)<r_on
        m(index)=r_on;
        end
    if m(index)>r_off
        m(index)=r_off;
        end
    x(index)=w(index)/d;
end
time('Memristor non linear boundary drift model')
subplot(2,1,1)
plot(v)
xlabel('time(t)')
ylabel('Voltage(V)')
subplot(2,1,2)
plot(v,i)
xlabel('Voltage(V)')
ylabel('Current(A)')

V hysteresis curve for the triangular voltage signal with amplitude equal to -Figure 6 presents the input voltage, output current and I 1V
3.3 Random Voltage with Different Magnitudes:

While the MATLAB simulation for Ti and TiO2 layers is between the two platinum layers, the following codes were obtained and the current and current-voltage curves passing through the structure were obtained according to the applied voltage. The results were obtained by changing the waveform of the voltage. Here are the codes and curves:

```matlab
clc;
clear all;
close all;
p=20;
v_d(1)=0;
f_p=[];
L=[];
f=5;
t = 0:0.01./f:10./f;
v=(t).*(sawtooth(t./(0.01.*pi),0.5)-1)./2;
d = 9*10^-9;
j=2;
u_v=10*10^-15;
or=10.58;
off=1000000;
io=7500;

% Joglekar window
w(1)=0.9989*d;
w(1)=((r_off-r_i)/(r_off-r_on))*d;
x(1)=w(1)/d;
m(1)=2.*(r_on*(w/d)+r_off*(1-w/d));

% f_p(1)=1 - (2*x-1)^(2*p);  % Joglekar window
f_p(1)=j*(1 - ((x-0.5)^2+0.75)^p);  % Prodromakis window

for index=2:length(t)
    i(index)=v(index)/m(index-1);
    v_d(index)=(u_v*r_on*i(index)*f_p(index-1))/d;
    w(index)=v_d(index)*t(index-1)-i(index-1)+w(index-1);
    x(index)=w(index)/d;
    % f_p(index)=1 - (2*x(index)-1)^(2*p);  % Joglekar window
end
```

Figure 6. Voltage, current and I-V hysteresis curves for triangular input wave.
\[ f_p(index)=j*(1 - ((x(index)-0.5)^2+0.75)^p); \% \text{Prodromakis window} \]
\[ m(index)=2.*r_{on}*w(index)/d+r_{off}*(1-w(index)/d); \]
\[
\text{if } m(index)<r_{on} \\
\quad m(index)=r_{on}; \\
\text{end} \\
\text{if } m(index)>r_{off} \\
\quad m(index)=r_{off}; \\
\text{end} \\
\text{end} \\
\]
\[ x(index)=w(index)/d; \]
\[ \text{end} \]

\[ \text{title('Memristor non linear boundary drift model')} \]
\[ \text{plot(abs(v),abs(i))} \]
\[ \text{xlabel('Voltage(V)')} \]
\[ \text{ylabel('Current(A)')} \]

with maximum amplitude \( V \) hysteresis curve for the random voltage signal—figure 7 presents the input voltage, output current and I-2V-equal to

![Figure 7. Voltage, current and I-V hysteresis curves for sinusoidal input wave.](image_url)

**4. Conclusion**

Based on the results, it should be considered that a specified memristor block, presents completely different behaviors for different types of the input voltages. It means that for any memristor type, a specified value of the voltage should be applied. One of the main important parameters in behavior of a memristor device is the frequency of the input voltage signal that causes receiving to different hysteresis curves. Also, the changes of the ratio for \( R_{ON} \) and \( R_{OFF} \) can directly change the slope of the I-V curve. All these reactions are modeled in this study. One should take into account that memristors are categorized in the passive components type and could not give the energy to the circuit, so they need to be compacted with circuits including the active components like transistors which can amplify or switch the electronic signals. Circuits including memristors and transistors have some advantages like: higher efficiency, less components numbers and need to less power to be activated.
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