Axisymmetric plasma mode in disk with two-dimensional electron gas

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Abstract. A theory of resonant absorption in disk with two-dimensional electron gas has been developed when the wavelength of external radiation is comparable with the radius of the disk. We have analyzed analytically and numerically the position and the linewidth of the resonant peak, which corresponds to excitation of the axisymmetric (quadrupole) plasma mode in the disk. We have found dependence of the plasma resonance characteristics on a dimensionless retardation parameter that is equal to the product of the plasma frequency and the radius of the disk divided by the speed of light. We have shown that the linewidth is not merely the sum of collisional and radiation decay, but also contains 'interference' terms.

1. Introduction

Plasma excitations in two-dimensional (2D) electron systems are of interest because they have a gapless dispersion law and their frequency can be continuously varied by changing the carrier density \( n \) by means of a gate voltage or magnetic field. One of the simplest and restricted in any directions geometry of 2D systems is a disk. Plasma waves in this geometry have been studied for over 30 years [1, 2] and still are interesting because they can couple directly to the electromagnetic radiation in contrast to the plasmons in the infinite system. Recently this geometry has intensively investigated in graphene structures [3–5]. However, most studies were done in quasielectrostatic or non-retarded regime, when the size of the sample is much smaller than the wavelength of the exciting electromagnetic radiation. In this limit plasma wavelength is much smaller than the wavelength of excitation radiation, the electromagnetic retardation is absent (the speed of light can be assumed infinite), the induced magnetic fields are negligible, and the plasma oscillations are described by the Poisson equation. The importance of electromagnetic retardation was demonstrated in [6–8]. In addition, it turned out that in such system the 'dark' (axisymmetric) plasma mode is rather easily excited [7–9].

The integer number of plasma wavelength should fit on the diameter and the perimeter of the disk to quantize the plasma wave in it. Therefore the eigenmodes of the system are characterized by the radial number \( n_r = 1, 2, 3, ... \) and the orbital (angular) momentum one \( l = 0, \pm 1, \pm 2, ... \) [2, 10]. The mode with \( l = 0 \) corresponds to axisymmetric mode in which charges and current move only along the radius of the disk. This mode is also called breathing mode or dark mode, since it has zero dipole moment and therefore rather weakly interacts with a plane electromagnetic wave [7, 11]. If the reader is well acquainted with the quantum mechanics, then he probably expects that fundamental mode (with the lowest frequency) corresponds to the axisymmetric mode. However, the correct plasma modes order in the quasielectrostatic (non-retarded) approximation in the 2D disk is \( l = \pm 1, \pm 2, \pm 3, 0 \ldots \) [10].
In this paper we analyze the frequency and damping of axisymmetric plasma mode in 2D disk by calculating the absorption of an electromagnetic wave taking electromagnetic retardation into account. Numerical calculations are widespread for this case [12, 13]. Here, by analogy with [10], we use the method of decomposition of the unknown components of the total current density into a basis set and determine the dependences of the absorption peak position and linewidth on the parameters of the system. Using one properly chosen basis function one can obtain analytical results with high accuracy.

2. Problem statement

Let us consider two-dimensional electron gas in disk with radius \( R \) assuming that the conductivity is described by the dynamical Drude model \( \sigma = ne^2\tau/m(1 - i\omega\tau) \), where \( n \) is the carrier concentration, \( \tau \) is the relaxation time, \( e \) and \( m \) are the charge and the effective mass of carriers. The total electric field as well as the total current is the sum of the external and induced fields, \( E^{\text{tot}} = E^{\text{ext}} + E^{\text{ind}} \). Assuming that every function oscillates with frequency \( \omega \), we write down the Maxwell’s equations in terms of the electrical scalar and vector potentials:

\[
\left( \Delta + \frac{\omega^2}{c^2} \right) A(r, z) = -\frac{4\pi}{c} j(r) \delta(z)
\]  

We apply the Hankel transform \( F(p) = \int_0^\infty r dr J_l(pr) f(r) \) for Eq. (1) in the cylinder coordinates \((r, \phi)\), and express the vector potential through the total density current in plane of the disk. After that we make the inverse Hankel transform and find a selfconsistent equation, that connects the total current density with the external electric field for each angular momentum \( l \). We focus only on the axisymmetric mode with \( l = 0 \) and choose the corresponding harmonic of external electric field as \( E^{\text{ext}} = (E_r, E_\phi) = E_0(1, 0) \), without considering how this field is created.

Following the articles [8,10] we use dimensionles frequency parameters \( v = \omega/\omega_0 \) and the retardation parameter \( \Gamma = ne^2/mc \), where \( \omega_0^2 = 2\pi ne^2/mR \) is a plasma frequency, which can be considered as quasielectrostatic plasma frequency in infinite system taken for the wave vector \( q = 1/R \). Finally, the equation for \( l = 0 \) has the following form

\[
\left( \gamma - i\frac{A}{v} \right) j_r(x) + i \int_0^1 \int_0^\infty dx' dx' dp' p \sqrt{p^2 - A^2v^2} J_1(xp)J_1(x'p)j_r(x') = \frac{Ac}{2\pi} E_0,
\]  

where \( J_1(z) \) is the Bessel function of the first order, \( \gamma = 1/\tau, \Gamma = ne^2/mc \), and \( x = r/R \). We assume that imaginary part of the square root \( \sqrt{p^2 - A^2v^2} \) is negative since it corresponds to outgoing from disk waves.

Boundary conditions follow from vanishing of the current at the edge of the disk, \( j_r(1) = 0 \), and finiteness of the charge density in the center of the disk, \( j_r(0) = 0 \).

The Eq. (2) can hardly be solve analytically. By analogy with [10, 14] we use the method of decomposition of the components of total current density in a Taylor-like series \( j_r(x) = \sum_{n=1}^N C_n f_n(x), \) where \( f_n(x) = x \cdot (1 - x)^n \). To obtain analytical results we use only first term in the series and compare it with numerical calculation for \( N = 3 \). Multiplying the Eq. (2) by \( x^2(1-x)^m \) for each \( m = 1, \ldots, N \) and then integrating over \( x \) between 0 and 1 we reduce it to matrix equations on the coefficients \( C_n \), which can be easily solved. Such choice of function set allows us to take analytically the integral over \( x \) and \( x' \).

After that we calculate the absorption power \( P(\omega) = \int_0^R (jE) 2\pi r dr \).
3. Results and discussion

The dependence of the absorption power as a function of frequency is shown in Fig. 1. One can see three peaks, which correspond to the excitation of plasma waves in the 2D disk with radial numbers \( n_r = 1, 2, 3 \). Let us consider only the main resonance (\( n_r = 1 \)). The dependence of the resonant frequency and linewidth on retardation parameter is shown in Fig. 2 and Fig. 3.

In order to get analytical results for the position and linewidth of the resonance we use only one function. The integral over \( p \) can be considered as a function of value \( Av \) so we use its representation in

Figure 1. Dependence of the absorption power in the disk as a function of the dimensionless frequency of incident electromagnetic radiation \( \omega/\omega_0 \) for the retardation parameter \( A = \omega_0 R/c = 0.17 \) and \( \gamma = \Gamma \). Three basis functions were taken into account. Peaks correspond to the excitation of plasma waves in the disk with radial number \( n_r = 1, 2, 3 \).

Figure 2. Dependence of the main (lowest) resonant frequency with \( n_r = 1 \) in dimensionless units \( \omega_{\text{max}}/\omega_0 \) on the retardation parameter \( A \) for \( \gamma = \Gamma \) (blue circle), \( \gamma = 0.2\Gamma \) (red square) and \( \gamma = 0.01\Gamma \) (green diamond). Three basis functions were taken into account for numerical points. The solid lines correspond to Eq. (5).
the Taylor series near $A\nu = 0$. Thus, we have the following expression

$$C_1 = \frac{5}{2\pi} \frac{\nu A c E^0}{A\nu/\Gamma - iv^2 + iI(A\nu)}, \tag{3}$$

where

$$I(a) \approx 3.493 - 0.169a^2 - 0.012a^4 - 0.005ia^5 + O(a^6). \tag{4}$$

The position of resonance, $\omega_{\text{max}}$, was found from the equation $P'(\omega_{\text{max}}) = 0$, where prime is the frequency derivative. The linewidth was obtained from the approximation of the resonance peak by the Lorenz fit, i.e. $\Delta \omega = 2\sqrt{2P(\omega_{\text{max}})/P''(\omega_{\text{max}})}$. Finally, we get

$$\omega_{\text{max}} = 1.869\omega_0 \left[1 - 0.084A^2 - 0.0096A^4\right] + ... \tag{5}$$

$$\Delta \omega = \frac{\gamma}{2} \left[1 - 0.169A^2 - 0.053A^4\right] + 0.031A^4\Gamma + ... \tag{6}$$

The linewidth contains two contributions: the collisional ((the first term in Eq. (6))) and the radiative decay (the last term in Eq. (6)). The radiative decay, which is determined by the quadrupole radiation, increases with the increase of the retardation parameter, opposite to the collisional decay. That is a reason for the changing of the qualitative dependence of the absorption linewidth as function of the retardation parameter for different values of the electron relaxation time $\tau = 1/\gamma$ in Fig. 3. The linewidth is not merely the sum of collisional and radiation decay, but also contains interference terms, which are depend on combinations of $\gamma$ and $\Gamma$. By tuning this parameters one can optimize the quality factor of the disk plasmon.

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