Wigner Functions in High Energy Physics

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Abstract. Recent developments are (meta)reviewed in the applications of Wigner functions to describe the observed single particle spectra and two-particle Bose-Einstein (or Hanbury Brown – Twiss) correlations in high energy particle and nuclear physics, with examples from hadron-proton and Pb + Pb collisions at CERN SPS.

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1. Introduction

High energy physics attempts to determine what are the most basic constituents of matter and how these most elementary forms of matter interact with each other. The method to achieve this goal is to build accelerators at the highest available energies. High energy particle physics studies the collisions of elementary particles and attempts to determine the interaction between as few particles as possible. High energy heavy ion physics attempts to create a new medium with the highest currently available volumes and energy densities so that the in-medium interactions of the elementary particles could be determined. These studies can be utilized to learn about the form of matter in the first few microseconds of the early Universe as well as about the behaviour of matter in the center of dense (neutron) stars.

Why are Wigner functions applied in high energy physics? Wigner functions are quantum analogies of the classical phase-space distributions. In conventional field theory, e.g. perturbative QCD, the calculation of the momentum distributions of the observable multi-particle final state can be performed in momentum space only. Usually, a description using Wigner functions can be equivalently rewritten into a calculation in momentum space, too. In some cases, the properties the phase space description becomes important in high energy physics. Currently, such important

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topics are the search for new states of matter in high energy heavy ion collisions using correlation techniques [1], and the precise determination of the mass of the W-boson in $e^+e^-$ annihilation at LEP [2]. In the first case, a new state of strongly interacting matter is searched for, where quarks and gluons are deconfined, not bound into the usual hardonic states. Theoretical calculations suggest [3] that a new phase of matter can be reached if the temperature, or, the energy density can be sufficiently high, ($T \geq 170$) MeV, at the currently reached finite net baryon densities. The temperature, the energy and the baryon density are local variables, they depend on the phase space evolution of the matter (and of course on the applicability of statistical and thermodynamical concepts in high energy heavy ion collisions). In case of the W-mass reconstruction in $e^+e^-$ collisions at LEP2, the aim is to decrease the error on the estimate that comes from the invariant mass estimates of 4-jet decays. However, Bose-Einstein symmetrization effects may introduce a non-perturbative “cross-talk” between the jets[2]. Hence pions that come from the decay of the $W^+$ may prefer to appear with momenta close to that of another like-charged pion from the decay of the $W^-$, due to the bosonic nature of the pions. Calculations suggested that this modification of the momentum distribution may result in a systematic error as big as 100 MeV in the reconstructed W mass, which is the biggest systematic error on this observable and should be reduced to reach the expected level [4]. However, the most recent paper by the L3 collaboration finds no evidence for inter-W Bose-Einstein correlation effect in fully hadronic W decays at LEP2 [11].

Quantum-statistical correlations are observed in high energy particle physics in all kind of particle reactions ranging from the “most elementary” $e^+e^-$ reactions, see e.g.[12] through hadron-proton reactions, see e.g. [13], to the “most complex” heavy ion reactions, (see ref. [14] for a recent summary of the data), essentially in all experimentally accessible energy regions. These quantum-statistical correlations are correlations of intensities of particles, and they are attributed to either the Bose-Einstein statistics of meson pairs, or, to the Fermi-Dirac statistics of the baryons. These statistics are dependent, in turn, on the phase-space densities of particles, so it is not a surprise, that Wigner functions turn out to be fundamental when one attempts to describe the experimentally measured two-, three- or higher order intensity correlations. As this note is very brief and attempts only to create a narrow bridge between the current quantum optical studies and the theory and experiment dealing with quantum statistical correlations and Wigner functions in high energy physics, let us quote some of the more detailed recent reviews in the field [14, 4–9].

Thus Wigner functions have a broad application in the theory and experiment of high energy physics, as they provide a natural tool to model these reactions in terms of phase space distributions. In particular, they are very popular in describing quantum statistical correlations (Bose-Einstein or Fermi-Dirac correlations) of the emitted particles. Currently, Gaussian models dominate [7], and the coupling between the coordinate space and the momentum space distributions is frequently modelled with the help of hydrodynamics [5]. State of art results attempt to quantify and to characterize the deviations from Gaussian structure. Shell of fire type
Fig. 1. This figure shows some structures in the Wigner function $S(x, k)$ when integrated over the momentum, and plotted as a function of the $(t, r)$ temporal and longitudinal as well as in the $(r_x, r_y)$ transverse coordinates. Till now, non-Gaussian structure of Wigner functions has been reconstructed in the transverse distribution of hadron-proton reactions at CERN SPS energies only, ref. [5].

The Wigner function formalism has been extended not only to thermal, or, in the quantum statistical sense, chaotic sources, but also to coherent particle emitting sources, for example, the pion laser model [15]. In discussions of in-medium hadron mass modification, two-mode squeezed states of hadrons were studied in terms of Wigner functions, both for bosons [19] and for fermions [20]. In principle, squeezing appears in these papers due to sudden disintegration of a medium.
where particles propagate with modified mass, so the situation is very similar to the one considered in quantum optics in ref. [21]. The only essential difference is that the field theoretical formulation results in two-mode squeezed states, hence quanta with opposite momenta and opposite quantum numbers will be correlated. Wigner functions appear in the formalism because the strength and the width of these back-to-back correlations depends on the phase-space distribution of quanta in the mass-modified medium [19, 20].

From a broader perspective, it seems that Wigner functions will be indispensable tools in high energy physics as long as effects sensitive to the phase-space evolution of these reactions will be investigated.

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b. Dedicated to the memories of Gy. Marx and E. P. Wigner.

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