Chaotic diffusion in multi-scale turbulence

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ABSTRACT

Turbulence plays a very important role in determining the transport of energy and particles in tokamaks. This work is devoted to studying the chaotic diffusion in multi-scale turbulence in the context of the nonlinear wave-particle interaction. Turbulent waves with different scales of characteristic wavelengths can interact with the same group of charged particles when their phase velocity is close to the velocities of the charged particles. A multi-wavenumber standard mapping is developed to model the chaotic diffusion in multi-scale turbulence. The diffusion coefficient is obtained by calculating the correlation functions analytically. It is found that the contribution of the largest scale turbulence dominates the deviation from the quasi-linear diffusion coefficient. Increasing the overlap parameters of the smaller scale turbulence by just increasing the wavenumber cannot make the diffusion coefficient to be the quasi-linear diffusion coefficient for a finite wave amplitude. Especially, in two-scale turbulence, the diffusion coefficient is mostly over the quasi-linear diffusion coefficient in the large wavenumber (of the smaller scale turbulence) limit. As more scales of components are added in the turbulence, the diffusion coefficient approaches the quasi-linear diffusion coefficient. The results can also be applied to other resonance-induced multi-scale turbulence in Hamiltonian systems with 1.5 or 2 degrees of freedom.

Keywords: chaotic diffusion, multi-scale, turbulence, wave-particle interaction

1. Introduction

Turbulence plays a very important role in determining the transport of energy and particles in tokamaks and it shows multi-scale behaviours[1]. Quasi-linear(QL) theory is widely applied for the charged particles interacting with turbulent waves. The weak warm-beam instability is firstly applied to use the QL theory to describe the saturation of the Langmuir turbulence and the evolution of the electron velocity distribution function[2, 3]. As the radio-frequency (RF) waves (such as, lower hybrid waves, electron cyclotron waves, ion cyclotron waves) serve as sources for plasma heating and current drive, the QL theory is applied to the codes in the modeling for the langmuir waves[4, 5]. The micro-instabilities of different characteristic wave lengths in tokamaks, such as trapped electron modes (TEM) with the characteristic wave length of the order of the ion Lamor radius and electron temperature gradient modes (ETG) with the characteristic wave length of the order of the electron Lamor radius, can produce multi-scale turbulence as their resonances are overlaped. The QL theory is also used to model the turbulence transport produced by these micro-instabilities[6]. The diffusions of different scales of modes are treated to be decoupled in the modeling, e.g., the QL diffusion coefficient for the langmuir waves is calculated as

\[ D = \frac{e^2}{m^2} 2\pi \sum_k E_k^2 \delta(\omega_k - kv)[2] \]

in which there is no coupling term between each wave, where, \( E_k \) is the electric field of each wave with the wavenumber \( k \).

However, QL diffusion is only valid for a time less than the discretization time \( t_d=2\pi/k_{typ}\delta v_\parallel \) (\( k_{typ} \) is the typical wavenumber and \( \delta v_\parallel \) is the interwave spacing of the wave phase velocity)[7] or in the large resonance overlap parameter limit in a long time[8, 9]. Test particle simulations show that the electron heat transport in the multi-scale turbulence is not always the sum of the independent contributions of each component when the nonlinear effects are considered[10]. The transport processes of different scales can be nonlinearly coupled, such as nonlinear coupling between small scale high-frequency turbulence and larger scale low frequency fluctuations in the edge[11], multi-scale interactions between small-scale turbulence and large scale magnetic islands[12]. Zaslavsky et al considered wave-particle interaction at double resonance[13], they found that the mismatch between the waves’ resonant velocities play an important role on the quasi-periodical exchange of energy between the waves, but their study is limited to just two waves. Actually, it is very time-consuming and costs too much computing power to numerically calculate a mass of particle orbits in multi-scale turbulence, e.g., only two-scale
turbulence is considered in the simulations of Ref.[10], and few works give general characteristics of the chaotic transport in multi-scale turbulence.

Mappings, such as the Fermi mapping[14], the standard mapping[15] and the web mapping[16](more mappings can be found in Ref.[17]), are very good tools for studying the regular and chaotic dynamics because they are distilled from complex nonlinear physical problems and the key physical mechanisms are retained in them besides they are power-saving for numerical calculation. Among them, the standard mapping, independently proposed by Taylor and Chirikov[18], describes the equidistantly distributed nonlinear resonances in phase space, which is a universal, generic description of area-preserving mappings with divided phase space when integrable islands are surrounded by chaotic seas. Various dynamical systems and mappings can be locally reduced to the standard mapping, so it plays an important role in the study of classical and quantum chaos [17, 19, 20]. It has been used to model the turbulent transport of the charged particles in the wave-particle interaction[21] and more recently it was used to model the tokamak edge electron diffusion in the lower hybrid antenna electric field[22, 23]. Deviations from the QL diffusion are found when the kick amplitude $K$ satisfy the Chirikov overlap condition[15], or more exactly, when $K$ is over the Greene’s criterion $K_c=0.971635...[24]$ in the standard mapping. For the standard mapping, the wave phases are all zeros and the ratio of the diffusion coefficient to the QL diffusion coefficient oscillates as $K$ increases[15, 21]. As the waves are random-phased, the diffusion coefficient never falls below the QL diffusion coefficient after rising above it[25] and self-consistent simulations of weak warm-beam instability show that the growth rate is enhanced when the diffusion coefficient is over the QL diffusion coefficient[7, 26]. All the considerations of these mappings above are in one scale turbulence.

Inspired by a so-called incommensurate standard map which describes the dynamics of cold atoms in a kicked optical lattice with an incommensurate potential[27], a more general form of the standard mapping which includes multi-scale case is developed and the diffusion coefficient in multi-scale turbulence is analytically derived. Surprisingly, it will be shown that the diffusion coefficient deviates the QL value when a large-scale component is added in a very chaotic/QL behaved small scale turbulence with strongly overlapping resonances. The article is arranged as follows: the model for the chaotic diffusion in multi-scale turbulence is presented in the second section. The analytical and numerical results of the chaotic diffusion coefficient are illustrated in the third section. Finally, the conclusions are in the last section.

2. The model for the chaotic diffusion in multi-scale turbulence

The electrostatic turbulent waves in one-dimensional configuration can be described as

$$E(x,t) = \sum_{m,l} E_{m,l} \exp\left(ik_{m,l}x-\omega_{m,l}t\right) + c.c. \tag{1}$$

It is assumed that $a$ is the length of the 1-dimension configuration space, the characteristic wavenumber of the turbulent waves are $k_{m,l}=2\pi\eta_{m,l}/a$, the frequency is $\omega_{m,l}=\omega_0$, where, $\eta_{m,l}, m$ and $l$ are integers, $\omega$ is the minimum characteristic frequency in the turbulent system.

Given the condition $\eta_{m,l+1}>>\eta_{m,l}$, the characteristic wavelengths $a/\eta_{m,l}$ and $a/\eta_{m,l+1}$ are in different scales. When the electric field $E_{m,l}$'s satisfy the Chirikov overlap condition at a phase velocity $\omega_{m,l}=\eta_{m,l}\omega_0/\eta_{m,l}$, $\omega_{m,l} = \omega_{m,l}(m=0, 1, ... r)$ for different scales of turbulent modes (where, $\omega_{m,l}$ is the resonant frequency of the modes with the maximum characteristic wave length, the total number of scales is $r+1$), we set $E_{m,l} = E_{0,0}$, so that the resonance-overlap condition can be extended to the whole phase space, $E(x,t) = \sum_{m=0}^{r} \sum_{l=-\infty}^{\infty} E_{m,0} \exp\left(ik_{m,l}x-l\omega_0 t\right) + c.c. \tag{2}$

The normalized variables are used by rescaling the distance with $a/2\pi$, and the time with $2\pi/\omega$, the dynamics of charged particles (with the mass $\mu$ and the charge $q$) in the presence of such turbulent modes can be described as

$$\frac{dx}{dt} = p$$

$$\frac{dp}{dt} = -\frac{i}{2} \sum_{m=0}^{r} K_m \sum_{l=-\infty}^{\infty} \exp\left(i\eta_{m,l}x-2\pi\tau t\right) + c.c.. \tag{3}$$

where, $K_m = 2i(2\pi)^4 E_{m,0}q/(\mu a \omega_0^3)$.

Finally, a multi-wavenumber standard mapping is derived as

$$p_{n+1} = p_n + \sum_{m=0}^{r} K_m \sin\left(\eta_{m,n}x_n\right), \tag{4}$$

$$x_{n+1} = x_n + p_{n+1}\text{ which describes multi-scale turbulence as }\eta_{m,n+1}>>\eta_{m,n}.$$
Although $\eta_m$ may be a fractional number mathematically in the mapping (4), all the $\eta_m$’s can be set as integers due to the fact that the fractional number can be transformed to the ratio of two integers and the fractional number case can be easily set in the mapping equivalently by using integers. The calculated diffusion coefficient which will be shown in the third section can be applied to the incommensurate standard map in Ref.[27], in which the diffusion coefficient is only numerically given.

The QL diffusion coefficient of each scale of component $D_{m,QL} = \frac{K_m^2}{4}$. The corresponding Chirikov overlap parameter
\[
s_m = \frac{2\sqrt{\eta_m K_m}}{\pi}.
\]
Note that the overlap parameter $s_m$ with larger $\eta_m$ increases more rapidly as $K_m$ increases.

The Greene’s criterion for each scale is $K_m = \eta_m K_s = 0.9716\eta_m$ without considering other scales. When considered the effects of other scales of components, the criterion should be smaller as the Kolmogorov-Arnold-Moser surfaces can be easier broken when more spatial perturbations of other scales are presented.

The chaotic diffusion coefficient can be measured by initiating an ensemble of 40,000 particles which are randomly distributed in configuration space with the same initial momentum $p_0$. The slope $\langle (p_{1N} - p_0)^2 \rangle / (2N)$ of this ensemble gives the diffusion coefficient when $N$ is large. In the numerical simulation, $N = 400$.

3. Results

In this section, the calculated chaotic diffusion coefficient in multi-scale turbulence is shown. Numerical results will be used to validate the analytical calculations.

For our interests, the wave amplitudes of each scale of modes are of the same order, otherwise, the terms of the lower orders can be neglected in the equation of motion. Without loss of generality, all the $K_m$’s are assumed to be the same, $K_m = K^*(m=0,1,\ldots)$ and $\eta_0 = 1$ in the cases to be shown.

The method to calculate such an area-preserving map is given in Ref.[28-30]. The calculated chaotic diffusion coefficient is
\[
D \approx \sum_{m=0} D_{m,QL} + \Delta_2 + \Delta_3,
\]
where, the correction term $\Delta_j$ is the phase-space-averaged impulse correlation function $\Delta_j = \langle (p_1, p_0) (p_j, p_0) \rangle$
\[
\Delta_j = \sum_{m=0} D_{m,j} (j = 2, 3),
\]
the contributions to the second and third correction term of each scale is
\[
D_{m,2} = -\frac{1}{4} K_m \left[ \chi_2 (\eta_m, 0, \eta_m) + \chi_2 (-\eta_m, 0, -\eta_m) \right]
\]
and
\[
D_{m,3} = -\frac{1}{4} \sum_{j=0}^{m} K_j \left[ \chi_3 (\eta_m, 0, 0, \eta_j) + \chi_3 (-\eta_m, 0, 0, -\eta_j) - \chi_3 (\eta_m, 0, 0, -\eta_j) - \chi_3 (-\eta_m, 0, 0, -\eta_j) \right],
\]
respectively, the characteristic function $\chi_2$ and $\chi_3$ are
\[
\chi_2 (m_0, m_1, m_2) = \delta_{m_0, m_2} \sum_{m_0} \sum_{m_1} \sum_{\eta_1} \sum_{\eta_2} \sum_{n_1} \sum_{n_2} \sum_{n_0} \prod_{j=0}^{r} J_n (m_j K_n)
\]
and
\[
\chi_3 (m_0, m_1, m_2) = \sum_{m_0} \sum_{m_1} \sum_{m_2} \sum_{\eta_1} \sum_{\eta_2} \sum_{n_1} \sum_{n_2} \sum_{n_0} \prod_{j=0}^{r} J_n (m_j K_n)
\]
respectively, and $J_n (x)$ is Bessel function of the first kind.

From Eq.(5-8), it is seen that the chaotic diffusions driven by each group of turbulent modes in different scales are coupled and they are decoupled as the correction terms are zero. To demonstrate this more clearly, we set $r=1$ which correspond to the turbulence with two scales of components. The contributions to the second and third correction terms of each scale are derived as,
\[ D_{a,2} = \frac{K_m^2}{4} \left[ \sum_{n=0}^{\infty} J_z (\eta_n K_m \eta_n K_1) + \sum_{n=0}^{\infty} J_z (-\eta_n K_m \eta_n K_1) \right] \]  

(9)

and

\[ D_{a,3} = \frac{K_m^2}{4} \sum_{n=0}^{\infty} K_n \left[ \sum_{n=0}^{\infty} J_z (\eta_n K_m \eta_n K_1) + \sum_{n=0}^{\infty} J_z (-\eta_n K_m \eta_n K_1) \right] \]  

(10)

respectively, where \( (n)J_q(x,y) = \sum_{\eta=0}^{\infty} J_{\eta q} (x) J_q (y) \).

Figure 1. The comparison of \( D/D_{0,QL} \) as a function of \( K \) between the numerical result (red circles) and theoretical result (black solid line), the larger scale contribution (blue dashed line) and smaller scale contribution (green solid line) for \( r=1 \), \( K_0=K_1=K \), \( \eta_0=1 \) and \( \eta_1=10 \).

The comparison of \( D/D_{0,QL} \) as a function of \( K \) between the numerical result and theoretical result, the larger scale contribution \( (D_{0,QL}+D_{1,2}+D_{1,3})/D_{0,QL} \) and smaller scale contribution \( (D_{1,QL}+D_{1,2}+D_{1,3})/D_{0,QL} \) are shown in Figure 1. The theoretical result agrees well with the numerical result.

It is also seen that the larger scale contribution accounts for the slow-varying and large amplitude oscillation as \( K \) increases. On the contrary, the smaller scale one accounts for the fast varying and small amplitude oscillation. It indicates that the contribution of the largest scale components dominates the correction terms of the diffusion coefficient. The fast-varying oscillation is due to the rapid increase of the overlap parameter of the smaller scale modes as \( K \) increases.

For the case with just single scale (i.e., \( K_i=0 \), \( K_i \gg 1 \)), the diffusion coefficient,

\[ D \approx D_{1,QL} \frac{K_i^2}{2} J^2 (\eta_i K_i) \frac{K_i^2}{2} \left[ J^2 (\eta_i K_i) - J^2 (\eta_i K_i) \right] \]  

(11)

approaches the QL diffusion coefficient as the normalized wavenumber \( \eta_0 \) increases to a large value.

However, for the multi-scale case, as the normalized wavenumber \( \eta_m \) of the smaller scale turbulent modes increases to a large value, the contributions to the correction terms of smaller scales vanished but those of the larger scale turbulent components exist, the diffusion coefficient approaches a function of \( K_m \)'s,

\[ D \to \sum_{n=0}^{c_{\infty}} D_{n,QL} + D_{1,2} + D_{1,3} \]  

(12)

\[ = \sum_{n=0}^{c_{\infty}} \frac{K_m^2}{4} J_z (\eta_n K_m) \frac{K_m^2}{2} \left[ J_z (\eta_n K_m) - J_z (\eta_n K_m) \right] \]  

\[ \times \left[ J_z (\eta_n K_m) - J_z (\eta_n K_m) \right] \]  

which is NOT just the QL diffusion coefficient, indicating that the strongly resonant overlap of the smaller scale turbulence cannot make the diffusion coefficient to be the QL diffusion coefficient by just increasing the wavenumbers of the smaller
scale components for a finite wave amplitude. In other words, the diffusion coefficient deviates the QL value when a large scale component is added in a very chaotic/QL behaved small scale turbulence with strongly overlapping resonances.

Figure 2. The comparison of $D/D_{QL}$ as a function of $K$ between the numerical result (red squares for one scale, black circles for two scales, blue crosses for three scales) and theoretical result (red dashed line for one scale, black dash-dot line for two scales, blue solid line for three scales).

To demonstrate the characteristics of the diffusion in the large $\eta_{m}$ limit, three cases, i.e., turbulence including one scale of components: $r=0, \eta_{0}=1, K_{0}=K$, two scales of components: $r=1, \eta_{0}=1, \eta_{1}=100, K_{0}=K_{1}=K$, three scales of components: $r=2, \eta_{0}=1, \eta_{1}=100, \eta_{2}=10000, K_{0}=K_{1}=K_{2}=K$, will be compared. The theoretical expressions of $D/D_{QL}$ for these cases are

$$\frac{D}{D_{QL}} = 1 - 2 J_{2}(K) J_{0}(K)$$

which is firstly derived in ref.[21],

$$\frac{D}{D_{QL}} = 2 - 2 J_{2}(K) J_{0}(K) - 2 \left[ J_{1}^{2}(K) J_{0}(K) \right] J_{0}(K)$$

and

$$\frac{D}{D_{QL}} = 3 - 2 J_{2}(K) J_{0}(K) - 2 \left[ J_{1}^{2}(K) J_{0}(K) \right] J_{0}(K)$$

respectively.

The comparisons of $D/D_{QL}$ as a function of $K$ between the numerical results and theoretical results for these three cases are shown in Figure 2. It is seen that the theoretical results match the numerical results well except the values at $K\approx 1$. For $K \approx 1$, more correction terms in the evaluation of Eq. (5) need to be retained. For one scale case, the theoretical result also fails to match the numerical results at $K=2\pi n (n=1,2,3,\ldots)$ where the accelerator modes exist, whose existence relies on the the spacial set of the wave phases(all zero)[25]. For cases of two and three scales, the accelerator modes are not observed in our simulation because they are destroyed by the strong resonance overlap of the smaller scales. For the case of two scales, $D/D_{QL}$ is mostly over 2. The maximums of oscillation amplitude of $D/D_{QL}$ around the corresponding QL diffusion coefficient are >0.5,~0.5 and ~0.1, respectively. As the total number of the different scales of components increases, the diffusion coefficient approaches the QL diffusion coefficient for $K>1$ which is because the correction terms are multiplied by more and more $J_{0}(K)$ whose absolute value is below unit as $K>1$. 


As the diffusion properties of the standard mapping is shown to be non-universal in the framework of the wave-particle interaction due to the correlated initial phases[31], a number of uncorrelated phases are used in the mapping. The randomly phased multi-wavenumber standard mapping can be written as,

\[
p_{x+n} = p_n + \sum_{a=0}^{L} K_a a_n \sin (i n_x x_n - \phi_{n,a}),
\]

\[
x_{x+n} = x_n + \frac{p_{x+n}}{L}
\]

where,

\[
\phi_{n,a} = \text{random}, L \text{ is the number of the different wave phases in each scale. When the total number of uncorrelated phases } L \text{ of each scale is large enough, the results will converge to the condition with all uncorrelated phases. Cases with the same parameters with the zero-phased cases with used for comparison, i.e., turbulence including one scale of components: } r=0, \quad \eta_0 = 1, \quad K_0 = K, \quad \text{two scales of modes: } r=1, \quad \eta_0 = 1, \quad \eta_1 = 100, \quad K_0 = K_1 = K, \quad \text{three scales of components: } r=2, \quad \eta_0 = 1, \quad \eta_1 = 100, \quad \eta_2 = 10000, \quad K_0 = K_1 = K_2 = K. \text{ The results are shown in Figure 3. It is seen that the diffusion coefficient never falls below the QL diffusion coefficient after rising above it which is consistent with findings in Ref.[25, 32]. For these three cases, the maximums of the deviations from the QL diffusion coefficient of } D/D_{QL} \text{ are about 1.2, 0.2 and 0.1, respectively. Same conclusions with the zero-phased results can be deduced as follows, the strongly resonant overlap of the smaller scale turbulence cannot make the diffusion coefficient to be the QL diffusion coefficient for a finite wave amplitude while increasing the total number of the different scales of components can make the diffusion coefficient approach the QL diffusion coefficient.}

Figure 3 The comparison of } D/D_{QL} \text{ as a function of } K \text{ between the numerical results of one scale (red squares), two scales (black circles), three scales (blue crosses). The lines are guides to the eyes. In the numerical calculations, } L=50 \text{ and } N=50L.

4. Conclusions

In conclusion, the multi-wavenumber standard mapping is used to model the charged particle motion in multi-scale turbulence which is power-saving for numerical calculation and the chaotic diffusion coefficient is derived analytically.

When the resonance overlap condition is satisfied in each scale, the chaotic diffusions driven by each group of components in different scales are coupled. The coefficient is obtained by calculating the correlation functions analytically. It is found that the contribution of the largest scale components dominates the deviation from the QL diffusion coefficient. Increasing the overlap parameters of the smaller scale component by just increasing the wavenumber cannot make the diffusion coefficient to be the QL diffusion coefficient for a finite wave amplitude. In other words, the diffusion coefficient deviates the QL value when a large scale component is added in a very chaotic/QL behaved small scale turbulence with strongly overlapping resonances. Especially, for the turbulence with two different scales of components, the diffusion coefficient is mostly over the QL diffusion coefficient in the large wavenumber (of the smaller scale component) limit. As more scales of turbulent components with the amplitudes of the same order are presented, the diffusion coefficient approaches the QL diffusion coefficient.
coefficient which gives a new evidence for the validation of QL theory in the modelling for the turbulence in fusion plasmas in which three or more scales of components may be coupled.

Due to the generality of the standard mapping, the conclusions can also be applied to other resonance-induced multi-scale turbulence in Hamiltonian systems with 1.5 or 2 degrees of freedom, such as the motion along a chaotic magnetic field with multi-scale magnetic turbulence.

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