Cluster number counts in quintessence models

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Abstract

Even though the abundance and evolution of clusters have been used to study the cosmological parameters including the properties of dark energy owing to their pure dependence on the geometry of the Universe and the power spectrum, it is necessary to pay particular attention to the effects of dark energy on the analysis. We obtain the explicit dark energy dependent rms linear mass fluctuation $\sigma_8$ which is consistent with the CMB normalization with less than 2\% errors for general constant dark energy equation of state $\omega_Q$. Thus, we do not have any degeneracy between $\sigma_8$ and the matter energy density contrast $\Omega_0^m$. When we use the correct value of the critical density threshold $\delta_c = 1.58$ obtained recently [30, 31] into the cluster number density $n$ calculation in the Press-Schechter (PS) formalism, $n$ increases as compared to the one obtained by using $\delta_c = 1.69$ by about 60, 80, and 110\% at $z = 0, 0.5$, and 1, respectively. Thus, PS formalism predicts the cluster number consistent with both simulation and observed data at the high mass region. We also introduce the improved coefficients of Sheth-Tormen (ST) formalism, which is consistent with the recently suggested mass function [36]. We found that changing $\omega_Q$ by $\Delta\omega_Q = -0.1$ from $\omega_Q = -1.0$ causes the changing of the comoving numbers of high mass clusters of $M = 10^{16}h^{-1}M_\odot$ by about 20 and 40\% at $z = 0$ and 1, respectively.
1 Introduction

The formation of the large scale dark matter (DM) potential wells of clusters is solely determined by gravitational physics irrelevant to the gas dynamical processes, star formation, and feedback. Also clusters are the largest virialized objects in the Universe with their abundance and evolution simply related to the linear matter power spectrum. Thus, the abundance of clusters and their distribution in redshift should be determined purely by the geometry of the Universe and the power spectrum of initial density fluctuations. As a result, the clusters of galaxies provide a useful probe of the fundamental cosmological parameters including the investigation into the dark energy equation of state $\omega_Q$, because the linear growth factor $D_g$, the cosmological volume element, as well as the primordial scalar amplitude at horizon crossing $\delta_H$ depend on $\omega_Q$ [1, 2].

Abundance of rich clusters has been commonly used to constrain the matter power spectrum because it is sensitive to the normalization of the power spectrum on cluster scales [3, 4]. The amplitude of matter perturbations is sensitive to the presence of dark energy, through the normalization of the matter power spectrum to the raw large scales probed by the cosmic microwave background (CMB) anisotropies [5]. The normalizations of the matter power spectrum obtained from both methods can be represented by the rms mass fluctuation on $8h^{-1}\text{Mpc}$ scales. However, there exists discrepancy in $\sigma_8$ values resulted from two different normalization methods [6]. While $\sigma_8$ is almost constant in the former method, it drops rapidly for larger $\omega_Q$ because of the increasingly strong integrated Sachs-Wolfe (ISW) effect in the latter.

There have been numerous papers investigating cluster abundances in the quintessential universe [7, 8, 9, 10, 11, 12, 13]. Most of them focus on the influences of dark energy on the background evolution and the growth factor. However, we investigate the dark energy effect on the normalization of the primordial density fluctuation and obtain proper $\sigma_8$ for the cluster abundance. We also use the correct critical threshold density $\delta_c = 1.58$ in our analysis.

In this paper, we briefly review the effects of quintessence field on the matter power spectrum given in Ref. [1]. It has been commonly assumed that there is no significant effects of quintessence field on the normalization $\sigma_8$ because it clusters gravitationally on large length scales but remains smooth like the cosmological constant on small length scales due to the relativistic dispersion of its fluctuations. However, we directly show the effects of the quintessence field on the $\sigma_8(M)$ due to the change in $\delta_H$ in the next section. We also show the effects of quintessence field on both the background evolution and the growth factor. In Sec. 3, we investigate the resulting changes in the cluster abundance. We conclude in Sec. 4. We also consider $\omega_Q < -1$ cases, because the obtained formalisms can be extended to the general constant $\omega_Q$ models.
2 Cosmological Consequences of Quintessence Models

We briefly review and find the influences of the quintessence field on the cosmological quantities in this section. We consider the cold dark matter with the quintessence field (QCDM) in a spatially flat universe. We limit our analysis on the constant equation of state of the dark energy $\omega_Q$ which is proper for the late time behaviors of quintessence models [14, 15]. We are able to apply these solutions to the time-varying $\omega_Q$ by interpolating between models with constant $\omega_Q$ [16, 17].

2.1 The Power Spectrum

The linear perturbation equation for the quintessence field $Q = Q_0 + \delta Q$ is given by

$$\delta \ddot{Q} + 3H \dot{\delta} Q + (k^2 + V_{QQ}) \delta Q = -\frac{1}{2} h \dot{Q}_0,$$

where dots mean the derivatives with respect to the cosmic time, $V_{QQ} = d^2 V/dQ^2|_{Q=Q_0}$, and $h$ is the trace of the spatial metric perturbation [18, 19]. Thus, the Compton wavelength of the quintessence field above which it clusters gravitationally but remains smooth on smaller scales is determined from the wavenumber $k_Q = 2\sqrt{V_{QQ}}$, where $V_{QQ}$ is given by

$$V_{QQ} = \frac{9}{4} \frac{H^2}{c^2} (1 - \omega_Q) \left[ 2(1 + \omega_Q) - \omega_Q \Omega_m(a) \right] + \frac{H^2}{c^2} \frac{1}{1 + \omega_Q} \times$$

$$\left[ \frac{1}{2} \frac{d^2 \omega_Q}{d \ln a^2} + \frac{1}{4(1 + \omega_Q)} \left( \frac{d \omega_Q}{d \ln a} \right)^2 + \left( \frac{9}{4} + 3\omega_Q + \frac{3\omega_Q}{4} \Omega_{de}(a) \right) \frac{d \omega_Q}{d \ln a} \right],$$

where $\omega_Q$ is the equation of state of $Q$ field, $\Omega_m(a)$ and $\Omega_{de}(a)$ are the energy density contrasts of matter and the quintessence, respectively. The linear perturbation of the quintessence field $\delta Q$ grows only on large scales ($k \ll k_Q$), so the quintessence field clusters and affects the evolution of the matter density perturbation $\delta_m$. The Compton wavenumber is determined by two terms in Eq. (2.2). The second term is due to the time variation of $\omega_Q$ and disappears when $\omega_Q$ is a constant. We investigate the contribution of the second term to $k_Q$ when $\omega_Q$ varies slowly. For this purpose, we adopt Chevallier-Polarski-Linder (CPL) parametrization $\omega_Q = \omega_0 + \omega_a (1 - a)$ [20, 21]. In Fig. 1, the dot-dashed, solid, and dashed lines correspond to $(\omega_0, \omega_a) = (-0.9, 0.1), (-0.98, 0.01),$ and $(-0.8, 0.3)$, respectively. It is less than 30% for the slowly varying $\omega_Q$ cases. Thus, we can safely consider only the first term of the wavenumber for those cases. The effects of the quintessence field on the matter power spectrum and the time evolution of gravitational clustering is parameterized by the shape parameter $\Gamma_Q = k_Q/h$ in Ref. [1] and we adopt this shape parameter.
The linear power spectrum for $\delta_m$ in QCDM models is given by

$$P(k, a) = A_Q k^{n_s} T_Q^2(k) \left( \frac{D(a)}{D(a_0)} \right)^2,$$

where $A_Q$ is a normalization, $n_s$ is the spectral index of the primordial adiabatic density perturbations, $T_Q(k)$ is the transfer function, $D$ is the linear growth factor, and the present scale factor normalized as $a_0 = 1$. Even though the transfer function $T_Q(k)$ does depend on $\omega_Q$, its change happens only on large scale $k \leq 0.01 \, h\text{Mpc}^{-1}$. Thus, this correction in the transfer function does not affect the value of the $\text{rms}$ mass fluctuation. The main effect of the quintessence field is on the normalization $A_Q$ which can be written as $A_Q = 2\pi^2 \delta_H^2 (c/H_0)^{n_s+3}$, where [1]

$$\delta_H = 2.05 \times 10^{-5} \alpha_0^{-1} (\Omega_m)^{c_1 + c_2 \ln \Omega_m} \exp[c_3(n_s - 1) + c_4(n_s - 1)^2],$$

with

$$c_1 = -0.789[\omega_Q^{0.0754 - 0.211 \ln |\omega_Q|}, c_2 = -0.118 - 0.0727 \omega_Q, c_3 = -1.037, c_4 = -0.138,$$

$$\alpha = (-\omega_Q)^s \text{ with } s = (0.012 - 0.036 \omega_Q - 0.017 \omega_Q^{-1})(1 - \Omega_m(a))$$

$$+(0.098 + 0.029 \omega_Q - 0.085 \omega_Q^{-1}) \ln \Omega_m(a).$$

Note that the notation $\alpha_0 = \alpha(a_0)$ and $\delta_H$ is the amplitude at horizon crossing. We use $\delta_H = 2.05 \times 10^{-5}$ to be consistent with WMAP 7 year data, and the cosmological parameters $\Omega_m^0 = 0.272$, $h = 0.7$, $n_s = 0.963$, $\Omega_b^0 = 0.0456$, and $\sigma_8 = 0.809$ from WMAP + BAO + $H_0$ measurements given in Ref. [22]. The $\text{rms}$ linear mass fluctuation at top-hat smoothing scale $R$ is given by

$$\sigma_R^2(a) = \left\langle \left( \frac{\delta M}{M(R, a)} \right)^2 \right\rangle = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k, a) |W(kR)|^2 \, dk,$$

where the filtering radius $R$ is the Lagrangian radius of a halo of mass $M$ at the present time, $R = (\frac{3M}{4\pi \rho_m(a_0)})^{1/3}$, and $W(kR) = \frac{3}{(kR)^2} \left( \sin[kR] - (kR) \cos[kR] \right)$ is the top-hat window function.
Traditionally, the cluster abundance is used to put a constraint on the dispersion of the density contrast at the scale $8 \, h^{-1} \text{Mpc}$, denoted $\sigma_8$. We also denote the mass inside a sphere of radius $R_8 = 8 \, h^{-1} \text{Mpc}$ as $M_8 = 5.95 \times 10^{14} \Omega_{m}^0 \, h^{-1} \, M_{\odot}$ where $M_{\odot}$ means the solar mass and we use the present critical energy density $\rho_{\text{crit}}^0 = 2.775 \times 10^{11} \, M_{\odot} \, h^2 \, \text{Mpc}^{-3}$. Now we show the present matter power spectra and $\sigma_8$ values for different values of $\omega_Q$ in Fig. 2. In the left panel of Fig. 2, the dot-dashed, solid, and dashed lines correspond to the matter power spectrum for $\omega_Q = -1.1$, $-1.0$, and $-0.8$, respectively. The differences in the power spectra of $\omega_Q = -1.1$ and $-0.8$ from $-1.0$ are about 7 and 17 %, respectively. The amplitude of the power spectra are comparable with the one in Ref. [23]. In the right panel of Fig. 2, we also compare the COBE normalized $\sigma_8$ (solid line) with the one from Ref. [4] (dashed one). There exists the discrepancy in the dependence of $\sigma_8$ on $\omega_Q$ between the two different normalizations other than the magnitudes. It is easy to understand this as shown in Ref. [6]. For the fixed value of $\Omega_m^0$, the dark energy dominates the cosmic expansion earlier as $\omega_Q$ increases and thus enhancing the dynamics of the gravitational potential which results in an increasing integrated Sachs-Wolfe (ISW) effect on large scales. This effect is properly shown in the COBE normalized $\sigma_8$ only. Also $\sigma_8$ obtained from the X-ray cluster population as in Ref. [4] suffers from the uncertainties due to the uncertainties in modelling. Thus, we use the COBE normalized $\sigma_8$ in our cluster abundance calculation.

Even though the COBE normalized $\sigma_8$ is the proper and the accurate one, we need to express it as a function of $M$ in the cluster abundance calculation. When $\omega_Q = -1$, $\sigma_8$ is well known [24] and we can parameterize $\omega_Q$ dependence of $\sigma_8$ by $\sigma_8(\omega_Q) = (-\omega_Q)^{0.72+0.36\omega_Q} \sigma_8(\omega_Q = -1)$ with less than 2 % errors for $-1.1 \leq \omega_Q \leq -0.6$. For the cosmological parameters adopted from the
Figure 3: $\sigma(M, z = 0)$ for different values of $\omega_Q = -1.1, -1.0, \text{and } -0.8$ (from top to bottom).

With WMAP, we can express

$$\sigma(M, z) \simeq (-\omega_Q)^{0.72+0.36\omega_Q} \left( 3.90 - 0.215 \log \left( \frac{M}{h^{-1}M_\odot} \right) \right) \left( \frac{D_g(z)}{D_g(z_0)} \right).$$

(2.7)

We show $\sigma(M, z = 0)$ for different values of $\omega_Q$ in Fig. 3. The dot-dashed, solid, and dashed lines correspond to $\omega_Q = -1.1, -1.0, \text{and } -0.8$, respectively.

2.2 Volume Element and Growth Factor

Friedmann’s equation for the QCDM is written as

$$H^2(z) = H_0^2 \left[ \Omega_m^0 (1 + z)^3 + \Omega_{de}^0 (1 + z)^3(1 + \omega_Q) \right].$$

(2.1)

Thus, the cosmic volume per unit redshift is also given by

$$V(z) = \int_0^z 4\pi d_A^2(z') \left| \frac{cdt}{dz} \right| (z') dz'.$$

(2.2)

where $d_A = \frac{c}{H(0)} \int_0^z \frac{dz'}{H(z')}$ is the angular diameter distance between redshifts 0 and $z$. $V(z)$ is the proper volume of a sphere of radius $z$ around the observer. We show the dependence of $V(z)$ on $\omega_Q$ in the left panel of Fig. 4. As $\omega_Q$ increases, $V(z)$ decreases.

The sub-horizon-scale linear perturbation equation with respect to the scale factor $a$ is well known [25], given by

$$\frac{d^2 \delta}{da^2} + \left( \frac{d \ln H}{da} + \frac{3}{a} \right) \frac{d \delta}{da} - \frac{4\pi G \rho_m}{(aH)^2} \delta = 0.$$

(2.3)
Figure 4: a) $V(z)$ for different values of $\omega_Q = -1.1, -1.0, \text{ and } -0.8$ (from top to bottom). b) $D_g(z)$ for the same values of $\omega_Q$ as in the left panel.

The exact analytic growing mode solution $D_g$ of $\delta$ for any value of the constant $\omega_Q$ is well known [16, 26, 27]:

$$D_g(Y) = c_1 Y^{3\omega_Q - 1} \frac{1}{6\omega_Q} \Gamma\left[\frac{1}{2} - \frac{1}{2\omega_Q}, \frac{1}{2} + \frac{1}{3\omega_Q}, \frac{3}{2} - \frac{1}{6\omega_Q}, -Y\right]$$

$$+ c_2 F\left[-\frac{1}{3\omega_Q}, \frac{1}{2\omega_Q}, \frac{1}{2} + \frac{1}{6\omega_Q}, -Y\right],$$

where $\Omega = \Omega_0^{\Omega_m}, Y = \Omega a^{3\omega_Q}, F$ is the hypergeometric function, and $c_1$ and $c_2$ are related to each other

$$\frac{c_1}{c_2}(a_i, \Omega, \omega_Q) = 2a_i^{1 - 3\omega_Q} \Omega^{\frac{1}{2} - \frac{1}{3\omega_Q}} (9\omega_Q - 1) \left( -\left(1 + 3\omega_Q\right) \times \right)$$

$$\Gamma\left[\frac{3}{2} - \frac{1}{2\omega_Q}, \frac{3}{2} + \frac{1}{3\omega_Q}, \frac{5}{2} - \frac{1}{6\omega_Q}, -Y_i\right]$$

$$3(3\omega_Q + 1)(\omega_Q - 1) \left( Y_i(3\omega_Q + 2) F\left[\frac{3}{2} - \frac{1}{2\omega_Q}, \frac{3}{2} + \frac{1}{3\omega_Q}, \frac{5}{2} - \frac{1}{6\omega_Q}, -Y_i\right] \right)$$

$$+ (1 - 9\omega_Q) F\left[-\frac{1}{2\omega_Q} + \frac{1}{2} + \frac{1}{3\omega_Q}, \frac{3}{2} - \frac{1}{6\omega_Q}, -Y_i\right]$$

where $Y_i = a_i^{3\omega_Q} \Omega$. In the right panel of Fig. 4, we show the behaviors of the growth factor $D_g$ for different dark energy models (i.e. for different values of $\omega_Q$) when $\Omega_0^{\Omega_m} = 0.272$. The dot-dashed, solid, and dashed lines correspond to $\omega_Q = -1.1, -1.0, \text{ and } -0.8$, respectively. As $\omega_Q$ decreases, $D_g$ maintains the linear growth factor proportional to $a$ for a longer time.
Figure 5: a) The comoving number density of clusters $n$ of mass greater than $M$ for different values of $z = 0, 0.5, 1.0, \text{and} 2.0$ (from top to bottom) when $\omega_Q = -1$ and $\delta_c = 1.58$. The circular ($z \simeq 0$) and triangular ($0.18 \leq z \leq 0.85$) dots represent the data from Ref. [33]. b) Errors of $n$ when we use the correct threshold density contrast $\delta_c = 1.58$ instead of 1.69 for different values of $z = 0, 0.5, \text{and} 1.0$ (from bottom to top).

3 Mass Function and Cluster Number

The mass function $f(M)$ from the Press-Schechter (PS) formalism is related to the comoving number density $dn$ of objects in the range $dM$ as $dn = \left( \rho_0^0 / M \right) |d\ln \sigma / dM| f(M) dM$ [28, 29]. PS theory which relates the comoving number density of virialized objects to their mass is given by

$$dn(M, z) = \sqrt{2 \rho_m^0 \rho_m} \left| \frac{d\ln \sigma}{d\ln M} \right| \delta_c \left( \exp \left[ -\frac{\delta_c^2}{2\sigma^2} \right] \right) dM,$$

(3.1)

where the critical density threshold $\delta_c = \rho_{\text{linear}} / \rho_m$ is predicted for a spherical overdensity of radius $R = (3M/4\pi \rho_m)^{1/3}$ and mass $M$ according to the linear theory. However, the PS mass function is known to predict too many low mass clusters and too few high mass clusters, as well as too few clusters at high $z$ [9]. One remark is that the correct value of $\delta_c = 1.58$ was recently obtained independent of the value of $\omega_Q$ instead of the well-known value of 1.69 [30, 31, 32]. If we use this correct value of $\delta_c$, then the PS formalism shows the improved predictions for the number densities of both low and high mass clusters. We show this in Fig. 5. In the left panel of Fig. 5, we show the comoving number density of clusters $n$ of mass greater than $M$ for $\omega_Q = -1$. The solid, dashed, dot-dashed, and long dashed lines correspond to $z = 0, 0.5, 1.0, \text{and} 2.0$, respectively. The circular ($z \simeq 0$) and triangular ($0.18 \leq z \leq 0.85$) dots represent the data from Ref. [33]. We also show the changes of $n$ at high and low masses at different redshifts in the right panel of Fig. 5. At the low mass $M = 10^9 [h^{-1} M_\odot]$, there is about 2% decrease in $n$ at present. At the high mass $M = 10^{16} [h^{-1} M_\odot]$, $n$ increases about 58% today. Also at high $z$, $n$ increases about 75 and 114% at $z = 0.5$ and 1.0, respectively.
Using the correct value of $\delta_c$, we can predict more massive clusters and more clusters at high $z$. However, we may still have too many low mass clusters even by using correct value of $\delta_c$ from the PS formalism. This is shown in Fig. 6. If we strictly limit the mass of clusters as bigger than $10^{14}M_\odot$, then PS formalism might be good enough for the cluster abundance calculation. Also there might be other mechanisms than only gravity for the low mass cluster formations. In any case, PS formalism shows the deviation from the simulation at low mass region. Thus, we need to consider another popular numerical fit for the differential mass function given by Sheth and Tormen (ST) [34, 35],

$$f_{\text{ST}}(\sigma) = A\sqrt{\frac{2b}{\pi}} \exp \left[ -\frac{b\delta_c^2}{2\sigma^2} \right] \left[ 1 + \left( \frac{\sigma^2}{b\delta_c^2} \right)^p \right] \frac{\delta_c}{\sigma},$$

(3.2)

where $A = 0.3222$, $b = 0.75$, and $p = 0.3$ are three parameters tuned to fit with numerical simulations. $A$ is fixed by the normalization that all dark matter particles reside in halos. However, ST mass function deviates from the simulation results by as much as 40% at the high mass end [36]. Thus, a new fitting function for $f(\sigma)$ is given in Ref. [36] by adding one extra parameter into ST,

$$f_{\text{mod}}(\sigma, z) = \tilde{A}\sqrt{\frac{2\tilde{b}}{\pi}} \exp \left[ -\frac{\tilde{b}\delta_c^2}{2\sigma^2} \right] \left[ 1 + \left( \frac{\sigma^2}{\tilde{b}\delta_c^2} \right)^{\tilde{p}} \right] \left( \frac{\delta_c \sqrt{\tilde{b}}}{\sigma} \right)^{\tilde{q}},$$

(3.3)

where $\tilde{A} = 0.333a^{0.11}$, $\tilde{b} = 0.788a^{0.01}$, $\tilde{p} = 0.807$, and $\tilde{q} = 1.795$. Recently, there is another mass function $f_{\text{Manera}}$ which is similar to the original ST formalism [37].

However, the ST mass function is known to deviate from the simulation at the high mass end [36]. This problem can be cured when we use the correct $\delta_c$. Thus, we introduce a simple but quite
similar to the results in Ref. [36] by using the original ST formalism:

\[ f_{\text{LN}}(\sigma, z) = 0.32 \sqrt{\frac{2(0.67)}{\pi}} \exp \left[ -\frac{0.67\delta_c^2}{2\sigma^2} \right] \left[ 1 + \left( \frac{\sigma^2}{0.67\delta_c^2} \right)^{0.32} \right] \frac{\delta_c}{\sigma}. \] (3.4)

The comparison between different mass functions \( f(\sigma) \) is given in Fig. 7. The dashed, long dashed, solid, and dot-dashed lines correspond to \( f_{\text{ST}}, f_{\text{Manera}}, f_{\text{LN}}, \) and \( f_{\text{mod}} \), respectively. As we can see in this figure, we can produce enough high mass clusters from the simple ST formalism when we use the correct \( \delta_c = 1.58 \).

We show the comoving number density of the clusters for \( \omega_Q = -1.0 \) at different \( z \) in the left panel of Fig. 8. The solid, dashed, dot-dashed, and large dashed lines correspond to \( z = 0, 0.5, 1.0, \) and 2.0, respectively. In the right panel of Fig. 8, we show the relative errors of the comoving number densities for two different models at two different redshifts. For example, the solid line represents the relative errors of \( n \) between \( \omega_Q = -1.1 \) and \( -1.0 \) at \( z = 0, |n(> M)_{\omega_Q=-1.1,z=0} - n(> M)_{\omega_Q=-1.0,z=0}| / n(> M)_{\omega_Q=-1.0,z=0} \times 100 \) (\%). At low mass \( M = 10^9 h^{-1} M_\odot \), the differences of \( n \) between two different models \( (\omega_Q = -1.1 \) and \( -1.0) \) are only 0.5 and 1 % at \( z = 0 \) and 1, respectively. At this mass, the differences of \( n \) between \( \omega_Q = -0.8 \) and \( -1.0 \) are also only 1 and 4 % at \( z = 0 \) and 1, respectively. However, the differences of \( n \) between \( \omega_Q = -1.1 \) and \( -1.0 \) at high mass \( M = 10^{16} h^{-1} M_\odot \) are 28 and 48 % at \( z = 0 \) and 1, respectively. For the two models \( \omega_Q = -0.8 \) and \( -1.0 \), the differences become 58 and 80 % at two different \( z = 0 \) and 1, respectively.
Figure 8: a) The comoving number density for the mass function given in Eq. (3.4) when $\omega_Q = -1$ at $z = 0, 0.5, 1,$ and $2$ (from top to bottom). b) The relative errors of comoving number density for the two different dark energy models between $\omega_Q = -1.1$ and $-1.0$ at two different redshifts $z = 0$ (solid) and $z = 1$ (dashed) and for models between $\omega_Q = -0.8$ and $-1.0$ at $z = 0$ (dot-dashed) and $1$ (long dashed).

4 Conclusions

We obtain the dark energy dependence of the mass fluctuation which is consistent with the CMB normalization. Even though we limit our analysis for the constant $\omega_Q$, we can extend our analysis for the slowly time varying $\omega_Q$ or use the interpolation of the constant $\omega_Q$ to study the general time varying $\omega_Q$.

We use the correct critical density threshold contrast $\delta_c$ to calculate the comoving number density of clusters for both PS and ST formalism. We show that PS formalism with this correct value of $\delta_c$ can predict the consistent cluster number with both the simulation and the observed data at the high mass region. Even though, the PS formalism with this correct value of $\delta_c$ still predicts too many low mass clusters, this might be due to other mechanisms for the cluster formation at the low mass region in addition to gravity. Thus, PS formalism might be good enough to explain the cluster abundance.

We obtain that the dark energy dependence of the comoving number densities for the high mass $M = 10^{16} h^{-1} M_\odot$ is about $25$ and $40 \%$ at two different redshifts $z = 0$ and $1$. Thus, observation will need to be as accurate as this level to probe the property of dark energy from the cluster physics.

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