Free convection from a corrugated heated cylinder with nanofluids in a porous enclosure

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Abstract
Natural convection between a cold square porous enclosure and a hot corrugated cylinder is studied numerically in the current article. The enclosure is filled with a water-base nanofluids suspending metal nanoparticles and the porous layer is modelled applying the Brinkman-Forchheimer law. The finite element method has been utilised to solve the governing equations. Analysis in this studies are: the amplitude of corrugated surface, the number of corrugated surface and the concentration are considered. It is found that the heat transfer of the corrugated cylinder might be slightly better than the heat transfer of the smooth cylinder under specific circumstances, but in general, the heat transfer is reduced by applying the corrugated surface. The heat transfer enhances up to 10% by increasing nanoparticle concentration. The heat transfer rate does not increase linearly by increasing the concentration, but it is proportional to the square root of the concentration.

Keywords
Brinkman-Forchheimer, natural convection, corrugated surface, nanofluids, FEM

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Introduction
Nanoscience and nanotechnology work with extreme tiny things which make human able to figure and design the bonds between atoms inside materials. Choi1 introduced the engineered nanoparticles inside the host fluid to boost thermal performance of the system such as electronic devices and heat exchangers. The latest technological method is applying a porous medium with the nanofluids. This innovative technique was applied by Sun and Pop2 for a porous triangle enclosure where the flush-mounted heater attached to the wall. The conjugate convection were studied by Chamkha and Ismael,3 Sheremet et al.,4 Ismael et al.5 and Mehryan et al.6 They studied the impact of the finite wall and nanofluids conductivity on the overall thermal performance. Localised heated enclosure was considered by Chamkha and Ismael,7 Ghalambaz et al.8 and Sheremet et al.9 applied the modified Tiwari and Das model for the nanofluid properties. Mahdi et al.10 gave a critical examination of utilizing nanofluid with a porous medium. Ghasemi and Siafashi11 studied the influence of various linear temperature distribution of sidewalls on the thermal performance. Mehryan et al.12 considered hybrid nanofluids or suspending more different nanoparticles into the base fluid. They concluded that the reduction of the heat transfer rate is much greater for hybrid fluid compared to the ordinary nanofluids. Emami et al.13 simulated nanofluid free convection in an inclined porous enclosure. The porous

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medium treated using the Brinkman–Forchheimer model and they did not recommend utilizing of nanofluids with porous media for strong convection.

Phenomena of natural convection due to a hot circular cylinder inside enclosures are analysed extensively. One of the practical applications occurs in pipe bringing hot water passes through an enclosure formed by geological building components. The enclosure is often partly filled with the saturated porous insulating material to reduce the thermal performance from the pipe. The convective flows between a cylinder and its surrounding fluid medium lead to a formation of convective cells. In every cell, the fluid circulates in the specific direction of rotation. Oosthuizen and Naylor\textsuperscript{14} studied the below and sides walls in adiabatic condition while the top wall is cold and the cylinder surface maintains in hot temperature. They found that slightly modification in the heat transfer rate with adjusting the cylinder radius. Misirlioglu\textsuperscript{15} investigated the rotating cylinder put in the midle of the differentially heated enclosure. Saleh and Hashim\textsuperscript{16} utilised Darcy model and filled Ag, Cu, Al\textsubscript{2}O\textsubscript{3}, or TiO\textsubscript{2} nanoparticles into the water. Chamkha et al.\textsuperscript{17} found that the cylinder size has a profound effect on the convective flow in the upper half of the enclosure at high Darcy number. Nazari et al.\textsuperscript{18} studied heat transfer convection from a hot elliptical cylinder attached with a porous medium. Sheikholeslami and Shehzad\textsuperscript{19} investigated the annulus enclosure with Darcy model for the porous layer. The annulus enclosure with multilayers of nanofluid and porous medium having sinusoidal cylinders was investigated by Abdulkadhim et al.\textsuperscript{20} They suggested that the sinusoidal cylinder need to move upward to convective flow enhancements. Dogonchi et al.\textsuperscript{21} filled nanofluid in a porous chamber between the warm outer rectangular cylinder and cold inner circular cylinder. They found that the heat transfer enhances with the convection intensity, porosity, concentration and nanoparticles shape aspect. Dogonchi et al.\textsuperscript{22} simulated nanofluids free convection within a porous annulus with diverse configurations of heater. Recently, Alhashash\textsuperscript{23} applied Brinkman-Forchheimer model for the porous layer and found that the thermal performance is enhanced for the half thermally active surface for suspending Al\textsubscript{2}O\textsubscript{3} nanoparticles.

Corrugated surfaces are used in enhancing heat transport performance. The literature survey concerning the corrugated surface with porous media has shown that this topic is limited because of the following reason. The first, it is challenging to build the grid generation of the corrugated surface. The second, non-uniformity of the wall orientation brings complexity to apply the numerical technique. In addition, several parameters are needed to be considered in the formulation such as the amplitude, wave number, wave ratio and inter-wall spacing. Murthy et al.\textsuperscript{24} studied impact of the corrugated surface at the bottom wall and concluded that the heat transfer rate is reduced by using the corrugated surface. Kumar\textsuperscript{25} concluded that the high corrugation frequency enhances the free convection. Kumar and Shalin\textsuperscript{26} investigated impact of corrugations surface and a thermally stratified at the vertical wall. Later, Kumar and Shalin\textsuperscript{27} demonstrated that the heat flux has a periodical structure of frequency in the non-Darcian model equal to the vertical corrugated surface. Misirlioglu et al.\textsuperscript{28} applied the cosine corrugation profile and compared their computations with the literature results for an enclosure with smooth walls. They considered some values of Rayleigh number, enclosure aspect ratio and corrugation frequency parameters. Later, Misirlioglu et al.\textsuperscript{29} concluded that the flow circulation and temperature distribution were observed to be sensitive on surface corrugations for tilt angles below 45\textdegree{} at high Rayleigh numbers. The surface waviness influenced heat transfer behaviours were showed by Khanafar et al.\textsuperscript{30} Mansour et al.\textsuperscript{31} added the effect of thermal radiation under non-equilibrium model and found that the thermal performance decreases by enhancing the material conductivity. Sheremet et al.\textsuperscript{32} filled the porous enclosure with nanofluids and the impact of thermal dispersion has been considered using the Forchheimer-Buongiorno model. They reported the heat transfer enhancement with Rayleigh number, corrugation number and dispersion parameter. Cheong et al.\textsuperscript{33} studied the effect of sinusoidal external heating and combustion to the wavy geometry. Hoghoughi et al.\textsuperscript{34} investigated the effect of waviness on heat transfer of nanofluid over a cylindrical heater using local thermal non-equilibrium formulation. They found that at lower corrugation amplitudes, the heat transfer rate is reduced when the cylindrical heater is elevated. Recently, Alhashash and Saleh \textsuperscript{35} treated the porous media using Darcy model and found that the thermal performance were sensitive to the varying of corrugation property, Rayleigh number and nanoparticle volume fraction. Parveen and Mahapatra\textsuperscript{36} studied double diffusive free convection in a wavy enclosure filled with nanofluid under partial heating. Free convection in a heat generating porous medium-filled wavy enclosures using Buongiorno’s nanofluid model was investigated by Ahmed and Rashed\textsuperscript{37} Selimefendigil and Oztap\textsuperscript{38} concluded that the dimension of the porous layer has a small effect on heat transfer and curvature of the upper wall has large effect on heat transfer. Recently, Selimefendigil and Oztap\textsuperscript{39} reported that the amount of enhancement by adding nanoparticle is 112\% and the effect is moved backward for the highest value of Darcy number and the waviness was found to be utilised as a useful tool for convective flow features.

The main objective of the current work is to perform a numerical simulation of free convection between a cold square enclosure and a hot corrugated cylinder
filled with metal-water nanofluids. The numerical investigation is considered for different non-dimensional governing parameters such as the fractional volume fraction, the amplitude of corrugated surface, the number of corrugated surface, nanoparticles concentration and the Darcy number. The waviness and porous material is predicted to contribute a better energy harvesting for example in a solar collector with a composite absorber made of porous material where the absorber is made of a corrugated iron sheet. To the best of our knowledge, investigation of the effect of corrugation parameters on convective flows in a porous enclosure having a hot inner sinusoidal cylinder has not received due attention. Similar research conducted by Nabavizadeh et al. and Safari43 for clear fluid found that the vortices, isotherms, and the number, magnitude and formation of the cells inside the non-porous enclosure strongly depend on the corrugation parameters. Hatami and Safari43 studied a hot cylinder on the free convection heat transfer of nanofluids inside a wavy enclosure.

Later, sequences investigation by Selimefendigil and Oztop44–48 concluded that corrugated surface and nanoparticle concentration parameters can be utilised to control the heat transfer rate.

**Mathematical formulation**

A schematic illustration of a porous enclosure with a corrugated hot cylinder is given in Figure 1(a). Bottom, left, right and top walls have constant low temperature. The cylinder having radius \( r \) and moves along the vertical centerline. The cosine corrugation function of the cylinder surface is:

\[
r(\eta) = r_n + A \cos(N\eta) \tag{1}
\]

where \( r_n \) is radius from the base (smooth) circle, \( A \) and \( N \) are amplitude and number of corrugations respectively and \( \eta \) is the angular coordinate.

All of the walls are considered to be impermeable, the fluids within the enclosure is a water-based nanofluids containing \( \text{Ag, Al}_2\text{O}_3, \text{Cu or TiO}_2 \) nanoparticles. Thermophysical properties of these particles were given in.16 Hot cylinder surface brings a heat transfer problem that is, free convection. Brinkman–Forchheimer model is assumed to satisfied and the Boussinesq approximation is considered valid. Based on these assumptions, the continuity, momentum and energy equations can be written as follows:

\[
\nabla \cdot \mathbf{v} = 0, \tag{2}
\]

\[
\frac{(\rho C_p)_{nf}}{\varepsilon} \mathbf{v} \cdot \nabla T = - \nabla \cdot (k_{nf} \nabla T) \tag{4}
\]

where \( \mu = \nu = 0 \) on the bottom, left, right, top walls and cylinder surface. The temperature boundary conditions are:

\[
T = T_c \quad \text{on left, right, and top walls}
\]

\[
T = T_h \quad \text{on cylinder surface}
\]

where \( g \) is the gravity and \( \varphi \) is the nanoparticles concentration. The specific heat capacitance of the nanofluids \((\rho C_p)_{nf}\) is defined as follows:

\[
(\rho C_p)_{nf} = (\rho C_p)_{bf}(1 - \varphi) + \varphi(\rho C_p)_{np} \tag{6}
\]

The nanofluids diffusivity \( \alpha_{nf} \) is defined as

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \tag{7}
\]

The nanofluids density \( \rho_{nf} \) can be determined

\[
\rho_{nf} = (1 - \varphi)\rho_{nf} + \varphi\rho_{np} \tag{8}
\]

The thermal expansion coefficient of the nanofluids \( \beta_{nf} \) is formulated as:

\[
(\rho \beta)_{nf} = (\rho \beta)_{bf}(1 - \varphi) + \varphi(\rho \beta)_{np} \tag{9}
\]

The viscosity ratio for 33nm particle-size in the room temperature was given by Corcione49 as follows.

\[
\frac{\mu_{nf}}{\mu_{bf}} = 1/\left(1 - 34.87(d_{np}/d_{bf})^{-0.3}\varphi^{1.03}\right) \tag{10}
\]

The thermal conductivity ratio is given by Corcione49 as follows:

\[
k_{nf}/k_{bf} = 1 + 4.4R_{e}^{0.4}P_{r}^{0.66}\left(T/T_{p}\right)^{10}(k_{np}/k_{bf})^{0.03}\varphi^{0.66}. \tag{11}
\]
Where $Re_B$ is defined as

$$Re_B = \frac{\rho_{bf} u_B d_{bf}}{\mu_{bf}}.$$  \hspace{1cm} (12)

$$u_B = \frac{2k_{bf} T}{\pi \mu_{bf} d_{bf}^2}. \hspace{1cm} (13)$$

Where $u_B$ is mean Brownian velocity of nanoparticle and $k_{bf} = 1.380648 \times 10^{-23} (J/K)$ is the Boltzmann constant. $d_{bf} = 0.17$nm is the average path of basefluids particles. $d_{bf}$ is the molecular diameter of water given as Corcione:

$$d_{bf} = \frac{6M}{N_A \pi \rho_{bf}}. \hspace{1cm} (14)$$

Now the following non-dimensional variables were introduced:

$$U = \frac{u_B}{\nu_{bf}}, \quad V = \frac{v_B}{\nu_{bf}}, \quad \Theta = \frac{T - T_c}{\Delta T}, \quad R = \frac{r}{\ell}, \quad X = \frac{x}{\ell}, \quad Y = \frac{y}{\ell}, \quad (\text{where } \Delta T = T_h - T_c > 0)$$

This then yields the dimensionless governing equations are: Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \hspace{1cm} (17)$$

Momentum equation in the $X$-direction:

$$\left( \frac{\rho_{nf}}{\rho_{bf}} \right) \frac{U}{\ell^2} \frac{\partial U}{\partial X} + \left( \frac{\rho_{nf}}{\rho_{bf}} \right) \frac{V}{\ell^2} \frac{\partial U}{\partial Y} = \left( \frac{\mu_{nf}}{\mu_{bf}} \right) \frac{\partial P}{\partial X}$$

$$+ \left( \frac{\mu_{nf}}{\mu_{bf} \ell} \right) \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)$$

$$- \left( \frac{\rho_{nf} \mu_{nf}}{\rho_{bf} \mu_{bf}} \right) C_F \sqrt{U^2 + V^2} \frac{U}{Pr \sqrt{Da}}$$

$$+ \left( \frac{\rho_{nf} \mu_{nf}}{\rho_{bf} \mu_{bf}} \right) \frac{U}{Da} \hspace{1cm} (18)$$

Momentum equation in the $Y$-direction:

$$\left( \frac{\rho_{nf}}{\rho_{bf}} \right) \frac{U}{\ell^2} \frac{\partial V}{\partial X} + \left( \frac{\rho_{nf}}{\rho_{bf}} \right) \frac{V}{\ell^2} \frac{\partial V}{\partial Y} = - \left( \frac{\rho_{nf}}{\rho_{bf}} \right) \frac{\partial P}{\partial Y}$$

$$+ \left( \frac{\mu_{nf}}{\mu_{bf} \ell} \right) \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right)$$

$$+ \left[ \frac{(\rho \Theta)_{nf}}{(\rho \beta)_{bf}} \right] \frac{1}{Pr} Ra \Theta \hspace{1cm} (19)$$

Energy equation:

$$\frac{U}{\ell} \frac{\partial \Theta}{\partial X} + \frac{V}{\ell} \frac{\partial \Theta}{\partial Y} = \left( \frac{\rho C_p}{\mu_{bf}} \right) \frac{k_{bf}}{k_{nf}} \frac{\partial^2 \Theta}{\partial X^2}$$

$$\times \frac{1}{Pr} \left( \frac{\partial \Theta}{\partial X} \right)^2 + \frac{1}{Da} \left( \frac{\partial \Theta}{\partial Y} \right)^2 \hspace{1cm} (20)$$

The flow pattern is represented by the streamfunctions which is derived from the fluid velocity that can be written as

$$\frac{\partial V}{\partial Y} - \frac{\partial U}{\partial X} = \nabla^2 \Psi.$$  \hspace{1cm} (21)

$Ra = g \rho_{bf} \beta_{bf} \Delta T \ell^3 / (\mu_{bf} \alpha_{bf})$ is the Rayleigh number, $Da = \frac{k_{bf}}{\ell}$ is the Darcy number and $Pr = \nu_{bf} / \alpha_{bf}$ is the Prandtl number, $C_F = 1.75 / \sqrt{150}$ is the Forchheimer constant, $K = \nu^2 \ell^3 / (150[1 - e^2])$ is the permeability, $D_p$ is spherical beads diameter and $k_{eff} = e + (1 - e) k_s / k_{bf}$ is the effective porous medium conductivity. $U = 0, V = 0$ and $\Psi = 0$ on the walls and cylinder surface. The temperature boundary conditions are:

$$\Theta = 0 \quad \text{on left, right, top and bottom walls} \hspace{1cm} (22)$$

$$\Theta = 1 \quad \text{on wavy cylinder}$$

The physical quantity of interest in this investigation is the Nusselt number or heat transfer rate. The dimensionless heat transfer is:

$$Nu(\eta) = \frac{k_{bf} \partial \Theta}{k_{nf} \partial \eta} \hspace{1cm} (23)$$

with $\eta$ is the angular coordinate. The mean Nusselt number $\overline{Nu}$ on the hot corrugated cylinder is calculated by

$$\overline{Nu} = \frac{1}{2 \pi R} \int_0^{2 \pi} Nu(\eta) d\eta \hspace{1cm} (24)$$
Table 1. Grid sensitivity checks for the average Nusselt number and the maximum value of stream function at $Ra = 10^7$, $A = 0.1$, $N = 4$ and $\varphi = 0.03$.

| Predefined mesh size | Domain elements | Boundary elements | $Nu$     | $|\Psi|_{\text{max}}$ | CPU time (s) |
|----------------------|-----------------|-------------------|----------|----------------------|--------------|
| Coarser             | 3593            | 283               | 13.844   | 7.895                | 17           |
| Coarse              | 4874            | 342               | 13.838   | 7.487                | 21           |
| Normal              | 5995            | 369               | 13.872   | 7.341                | 21           |
| Fine                | 8280            | 420               | 13.859   | 7.316                | 25           |
| Finer               | 13466           | 608               | 13.855   | 7.192                | 36           |
| Extra fine          | 29372           | 1042              | 13.855   | 7.123                | 78           |
| Extremly fine       | 36480           | 1042              | 13.854   | 7.114                | 100          |

Computational methodology

The dimensionless governing equations are solved numerically by employing finite element method (FEM). The basic idea of finite element is dividing the computational domain into smaller elements or subdomains (finite elements). The solution element of the mathematical calculation is generated by a triangular mesh. The triangular mesh arrangements calibrate specifically for fluid dynamics problem as shown in Figure 1(b) for an extra fine mesh size selection. The governing equations are transformed into a set of integral equations by utilising Galerkin weighted residual technique. Calculating of residuals is done by changing the approximations into the governing equations. The integration involved in each term of the continuity, momentum and energy equations is done by using Gauss’s quadrature method. An automatic Newton method is applied to solve the system nonlinear algebraic equations in the form of matrix. The iteration approach is adapted through PDEs solver Comsol. The convergence criterion of the Comsol solver is set to the default setting.

To achieve grid-independence, some tests were performed to make certain of the results are free from the mesh calibration. Comsol default mesh size, that is, coarser, coarse, normal, fine, finer, extra fine and extremely fine are considered as tabulated in Table 1. The results in this table are found to be consistent by refining the mesh size. The test suggests that an extra fine mesh size was chosen for all the calculations performed in this work. As a validation, present results (left) of streamlines are compared with that obtained by Kumar* (right) for the sinusoidal corrugation surface at $Ra = 5 \times 10^6$, $Da = 10^{-5}$, $A = 0.05$, $N = 4$, 3 without suspending nanoparticles as shown in Figure 2.

Additional validation in Figure 3 is performed by a comparison with the published work of Alhashash et al.* which is a natural convection inside non-porous annulus at $A = 0.4$, $R_b = 0.2$, $Pr = 0.71$, $Ra = 10^4$, $\varepsilon = 0.9999$, $Da = 10^7$. Comparison of current (bottom) isotherms with* (top) was conducted for different corrugation number, $N = 4$ (left) and $N = 8$ (right). This figure also summarises the $Nu$ values. In each case, the isotherms and Nusselt number are significantly consistent with reported in the literature. These comprehensive validations confirm the reliability of the present computations.

Results and discussion

The analysis in this simulation are performed in the following domain of the related dimensionless groups: the amplitude of corrugated surface, $0 \leq A \leq 0.3$, the number of corrugated surface, $3 \leq N \leq 6$, the radius, $0.1 \leq R_b \leq 0.25$, the cylinder position, $-0.15 \leq \delta \leq +0.15$, the Darcy number, $10^{-5} \leq Da \leq 10^{-2}$. The nanoparticles are argentum (Ag), alumina (Al$_2$O$_3$), copper (Cu) and titania (TiO$_2$) dispersed in the host fluid. The Rayleigh number is taken as $Ra = 10^7$. Figure 4 exhibits the influences of various values of Darcy numbers and corrugated amplitude on streamlines at $R_b = 0.2$, $\delta = 0.0$, $N = 4$ and $\varphi = 0.03$ for Al$_2$O$_3$. Figure 4(a) presents the impact of changing the flow motion with the wavy cylinder set at low amplitude value ($A = 0.05$). Heated the cylinder surface leads the flow occurred in the enclosure. The streamlines generate pair of rotating cells surrounded the cylinder, one next to the left wall and one close to the right wall. When the streamlines circulated as cells in the clockwise rotation (negative signs), the maximum strength of the streamlines is appointed by $\Psi_{\text{max}}$. The strength of the convective flow increases by the increases of Darcy number. The clockwise and anticlockwise cells move toward the top wall. The streamline magnitude is almost stagnant by varying the corrugation amplitude ($A = 0.1$). The active hot surface enlarges by increasing the corrugation amplitude, but the flow movement becomes harder in particular in the gaps between active surface, so these regions are almost no current or stagnant. The clockwise and anticlockwise cells breaks up and the streamlines magnitude were suppressed by increasing the corrugation amplitude ($A = 0.2$). The multiple contra-rotative cell is still persistent at the highest amplitude value ($A = 0.3$). It seems that the
**Figure 2.** Comparison of current streamlines with literature results for the sinusoidal corrugation surface at $Ra = 5 \times 10^6$, $Da = 10^{-5}$, $A = 0.05$, $N = 4$, 3.

**Figure 3.** Comparison of current isotherms and $\bar{Nu}$ with literature result for different corrugation number, $N = 4$ (left) and $N = 8$ (right) at $A = 0.4$, $Pr = 0.71$, $Ra = 10^5$, $\varepsilon = 0.9999$, $Da = 10^2$. 
multiple cells circulation locally suppresses the fluid flow as compared to the Figure 4(a).

The influences of Darcy numbers and corrugation amplitude on isotherms at \( R_b = 0.2, \; \delta = 0.0, \; N = 4 \) and \( \varphi = 0.03 \) for \( \text{Al}_2\text{O}_3 \) are showed in Figure 5. At small corrugation amplitude (\( A = 0.05 \)) with the Darcy number value is low (\( Da = 10^{-4} \)), the isotherms pattern seem to appear with circular form because of the generating fluid flow by porosity permeability is weak. By taking the Darcy number (\( Da = 10^{-2} \)), the isotherms start bending. Then, at higher Darcy number, the temperature distribution substitute to the mushroom formation and the flow almost not clashed adjacent to the top wall. This due to the fast flow circulation and the nanoparticles disperse broader with denser thermal boundary layer at the top enclosure. Substituting corrugations amplitude do not give any considerably visual alteration of the temperature distribution for relative small undulations. When \( A = 0.2 \), there are two plumes in the enclosure because of that the upwelling heat that appears in left top and the right top portion above the cylinder. By the fact that with increasing the amplitude, the active surface volume also increases and it increases the temperature in the upper region. A stronger amplitude of undulation value makes the hot surface closer to the cold surface.

Figure 6 shows the effects of Darcy numbers and radius on streamlines at \( R_b = 0.2, \; \delta = 0.0, \; A = 0.2 \) and \( \varphi = 0.03 \) for \( \text{Al}_2\text{O}_3 \). The wavy cylinder starts to transport the heat to the inside fluid due to the higher temperature of the cylinder compared to that of the fluid inside the enclosure. Increasing Darcy number lends to enlarge the porosities and more nanoparticles are diffused within recirculating zones. The streamlines cells approaching the left and right walls tend to expand horizontally. The streamlines move away from the bottom wall upward direction to the cylinder. At higher Darcy number value, the counterclockwise cell and the clockwise cell move closer to the top boundary and change to elongated shape, which caused packing the streamlines with high intensity at the top segment of the enclosure. Figure 6(b) depicts that the strength of the flow movement decreases by the rise of corrugation number and the similar streamlines cells tend to expand vertically due to the changing of the hot area. The pattern of vortices changes from unicellular to bicalcular. The multiple contra-rotative cell is still persistent at \( N = 5 \) with weaker flow circulation than case \( N = 4 \). Increasing the corrugation number to 6 tends to merge the bicalcular counterclockwise and clockwise circulation returned to ‘bean’ shape. Applying a higher \( N \) value, the strength of the fluid circulation increases the \( \Psi_{\text{min}} \) significantly.

Figure 7 displays the effects Darcy numbers and corrugation number on isotherms for \( R_b = 0.2, \; \delta = 0.0, \; A = 0.2 \) and \( \varphi = 0.03 \) for \( \text{Al}_2\text{O}_3 \). At the beginning, the isotherms pattern surrounding the inner boundary tends to take a circular shape, which at low Darcy number value. When the value of Darcy number increases (from left to right), the isotherms pattern moves toward the top boundary, which tends to increase the intensity at the top segment of the enclosure. For a higher value of Darcy number (\( Da = 10^{-2} \)), the currents are strong enough to push heat closer to the left and right walls. The isotherm lines are very much packed and highly dense at the upper wall boundaries indicating the large temperature gradient. The mushroom shapes were also obtained at \( N = 3 \) for \( Da = 10^{-2} \). Increasing the corrugation number up to 4 creates two plumes in the enclosure because of that the upwelling heat that appears in the left top and the right top portion above the cylinder. On the other hand the pattern of isothermals is about the same at \( N = 5 \). The isotherms expand horizontally because the flow strongly hits the top boundary. This flow was accelerated by rising maximum temperature due to increasing the heated surface area from adding corrugation number. The mushroom shape reappears for the \( N = 6 \). The isotherms are symmetric along the geometric centerline. This is due to the fact that the wavy cylinder was kept symmetric at all \( N \).

Figure 8 shows the effects of Darcy numbers and radius on streamlines for \( \delta = 0.0, \; A = 0.2 \) and \( \varphi = 0.03 \) for \( \text{Al}_2\text{O}_3 \). At a small cylinder, the streamlines exhibit a clockwise and anticlockwise cells pressing the cylinder. The clockwise and anticlockwise cells break up at \( R_b = 0.2 \) and the streamlines magnitude were increased by increasing the cylinder size. The multiple contra-rotative cell still persistent at \( R_b = 0.25 \) with stronger flow circulation than case \( R_b = 0.2 \). Double eye vortices with similar size was observed at small Darcy number. Mainly, the wavy cylinder size affects the flow circulation position where the flow circulates near the upper region at a smaller cylinder.

Figure 9 displays the effects Darcy numbers and radius on isotherms for \( N = 4, \; \delta = 0.0, \; A = 0.2 \) and \( \varphi = 0.03 \) for \( \text{Al}_2\text{O}_3 \). The isotherms are symmetric along the geometric centerline with an upwelling plume. The isotherm lines are highly denser close to the top wall as Darcy number takes higher. Multi upwelling plume with are observed at \( R_b = 0.2 \) Multi upwelling plume with unsymmetrical formation are obtained at the sufficiently large cylinder. Multi upwelling plume with unsymmetrical shape are observed at the highest Darcy number and the largest cylinder. Since the flow circulation is very strong enough to penetrate the decreased space below, left, top and right the wavy cylinder due to the effect of the separation of the thermal boundary on the walls were occurred.

Figure 10 shows the effect of the corrugation amplitude on the Nusselt number versus the concentration at \( R_b = 0.2, \; \delta = 0.0, \; N = 4 \) and \( Da = 10^{-2} \) for \( \text{Al}_2\text{O}_3 \). The heat transfer is increasing regardless of the corrugation
Figure 4. Streamlines for different Darcy numbers and adjusting the amplitude corrugation, $A$ at $R_b = 0.2$, $\delta = 0.0$, $N = 4$ and $\varphi = 0.03$ for $Al_2O_3$. 
The average Nusselt number increases almost linearly by increasing the nanoparticle volume fraction. In general, the ratio average Nusselt number per unit area of the active surface reduces as the size of the active surface increases. The effect of the nanoparticles concentration to the Nusselt number enhancement is slightly differed at small and high nanoparticles volume fraction for the considered amplitude. The heat

Figure 5. Isotherms for different Darcy numbers and adjusting the amplitude $A$ at $R_b = 0.2$, $d = 0.0$, $N = 4$ and $\phi = 0.03$ for Al$_2$O$_3$. 
transfer rate does not increase linearly by increasing the nanoparticle concentration for the considered parameter. Adding 1\% nanoparticles, the heat transfer can be improved about 10\% at $\varphi = 0.01$. However, increasing concentration from 4\% to 5\%, only about 3\% enhancement was observed.

Figure 6. Streamlines for different Darcy numbers and adjusting the corrugation number, $N$ at $R_b = 0.2$, $\delta = 0.0$, $A = 0.2$ and $\varphi = 0.03$ for $\text{Al}_2\text{O}_3$. 
Figure 11 illustrates the ratio Nusselt number versus Darcy number for various amplitude corrugation at $R_b = 0.2$, $\delta = 0.0$, $\Lambda = 0.2$ and $\varphi = 0.03$ for $\text{Al}_2\text{O}_3$.

Figure 11 illustrates the ratio Nusselt number versus Darcy number for various amplitude corrugation at $R_b = 0.2$, $\delta = 0.0$, $\Lambda = 0.2$ and $\varphi = 0.03$. The normalised Nusselt number suppression appears considerably with high Darcy number values $Da > 10^{-4}$ and further, it slightly increases about $Da > 10^{-3}$. The normalised heat
transfer reduces by taking higher the amplitude for the considered Darcy number. This figure indicates that the amplitude corrugation has an effect to reduces the heat transfer rate.

The influences of different number of corrugation on the ratio Nusselt number against number of amplitude of corrugation surface at \( R_b = 0.2 \), \( \delta = 0.0 \), \( Da = 10^{-2} \) and \( \phi = 0.03 \) for Al\(_2\)O\(_3\) was shown in

**Figure 8.** Streamlines for different Darcy numbers and adjusting the radius, \( R_b \) at \( N = 4 \), \( A = 0.2 \) and \( \phi = 0.03 \) for Al\(_2\)O\(_3\).
Figure 12. Higher amplitude brings a longer heated surface that increases the maximum cylinder temperature. In the other hand, the fluid flow was retarded by decreasing the free space between the hot and cold surfaces. These lead to heat transfer rate steady decreasing by growing the amplitude of the undulations for $N = 5$ and $N = 6$. The heat transfer interacts differently with amplitude increment, at $N = 3$, the
normalise Nusselt number increases with increasing the value of $A$ from 0.1 to 0.15 and then it starts to drop. While when $N = 4$, the normalised heat transfer is greater than unity at $A = 0.05$. It means the heat transfer of the undulated cylinder above the heat transfer of the smooth cylinder. The reduction of Nusselt number appears later. However, increasing $A$ to 0.2 leads to the rise of the heat transfer. This due to the existence of the crest in the gaps between undulated surfaces as shown in streamlines, Figure 4(c).

Figure 13 shows the effect of different nanoparticles type (a) and different cylinder position (b) on the Nusselt number versus the concentration at $R_b = 0.2$, $\delta = 0.0$, $N = 4$ and $Da = 10^{-2}$ for $Al_2O_3$. argentum, alumina, copper and titania, the heat transfer is similar. Later, when the quantity of nanoparticle is added, the lowest heat transfer performance was observed for the titania. This due to this material is less conductive that brings to low thermal energy and smaller increment in the convection intensity. In general, the heat transfer rate is proportional to the square root of the concentration. Examination of the heat transfer performance of the alumina for various cylinder location was shown in Figure 13(b). A similar effect of nanoparticles to the $Nu$ for the considered position. This suggests a relatively unimportant feature of the cylinder position in terms of heat transfer enhancement.

**Conclusion**

The present study scrutinised the free convection due to different temperature level between a cold enclosure and a hot corrugated cylinder. The partial differential equations of the governing equations are solved numerically utilising the Galerkin finite element method via COMSOL. The computational results for flow and thermal distribution and the heat transfer have been visualised graphically. The corrugation frequencies, nanoparticle properties and Darcy number connected closely to the pattern of streamlines vortices, thermal distribution and heat transfer profiles. The important finding of the present simulation are as follows:

1. The circulation flow structure and thermal pattern of nanofluids depend on the Darcy number, cylinder size, amplitude and number of corrugations. The corrugation parameters capable to control the flow circulation strength, the number of contra-rotative cells and the formation of the thermal plume inside the annulus.

Figure 10. Influences of the corrugation amplitude on the mean Nusselt number versus the concentration at $R_b = 0.2$, $\delta = 0.0$, $N = 4$ and $Da = 10^{-2}$ for $Al_2O_3$.

Figure 11. Influences of Darcy number and the corrugation amplitude on the ratio of Nusselt number for the undulated cylinder and Nusselt number for a smooth cylinder.

Figure 12. Effect of the amplitude and the number of corrugation surface on the ratio of Nusselt number for the undulated cylinder and Nusselt number for a smooth cylinder.
2. The heat transfer of the corrugated cylinder might be better than the heat transfer of the smooth cylinder. However, in general the heat transfer is reduced by applying the corrugated surface.

3. The highest heat transfer performance was observed for the argentum. A relatively unimportant feature of the cylinder position was found for heat transfer enhancement.

4. In general, the heat transfer rate does not increase linearly by increasing the nanoparticle concentration, but it is proportional to the square root of the concentration.

To gain a better physical insight of free convection in the complex annulus filled with nanofluids, experiments should be conducted to see real interaction among waviness, porous material and nanoparticles. All problem considered here could be extended to other type of heat transfer such as in mixed or forced convection where these convection types often happen in real circumstances.

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### Appendix

#### Notation

| Symbol | Description                                      |
|--------|--------------------------------------------------|
| $A$    | corrugation amplitude                            |
| $Da$   | Darcy number                                     |
| $g$    | gravitational acceleration                        |
| $k$    | thermal conductivity                             |
| $\ell$ | width and height of enclosure                    |
| $M$    | molecular weight                                 |
| $N$    | corrugation number                               |
| $Nu$   | Nusselt number                                    |
| $Pr$   | Prandtl number                                    |
| $r$    | base radius                                       |
| $Ra$   | Rayleigh number                                   |
| $T$    | temperature                                       |
| $u, v$ | velocity components in the $x$ and $y$ space coordinates |
| $x, y$ | space coordinates                                |

#### Greek symbols

| Symbol | Description                                      |
|--------|--------------------------------------------------|
| $\alpha_m$ | thermal diffusivity                             |
| $\beta$ | thermal expansion coefficient                    |
| $\delta$ | cylinder position                                |
| $\eta$  | angular coordinate                               |
| $\nu$   | kinematic viscosity                              |
| $\psi$  | stream function                                  |
| $\varphi$ | nanoparticles concentration                      |
| $\eta$  | angular location                                 |
| $\Theta$ | dimensionless temperature                        |
| $\rho$  | density                                          |
| $\mu$   | dynamic viscosity                                |

#### Subscript

| Symbol | Description |
|--------|-------------|
| $c$    | cold        |
| $bf$   | basefluids  |
| $h$    | hot         |
| $nf$   | nanofluids  |
| $np$   | nanoparticles |
| $p$    | porous      |
| $s$    | solid matrix |