A Droplet State in an Interacting Two-Dimensional Electron System

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It is well known that the dielectric constant of two-dimensional (2D) electron system goes negative at low electron densities. A consequence of the negative dielectric constant could be the formation of the droplet state. The droplet state is a two-phase coexistence region of high density liquid and low density "gas". In this paper, we carry out energetic calculations to study the stability of the droplet ground state. The possible relevance of the droplet state to recently observed 2D metal-insulator transition is also discussed.

The recent discovery of two-dimensional (2D) metal-insulator transition (MIT) by Kravchenko et al. has challenged the scaling theory of localization in which a 2D MIT is forbidden. A noticeable character of the electron system in these experiments is that \( r_s \), the parameter measures the strength of the Coulomb interaction, is fairly large. We suspect that the electron system may be unstable against phase separation at these large values of \( r_s \). We demonstrated in our previous paper that this assumption alone is sufficient to provide a theoretical description that is consistent with all the known experiments. For a two-dimensional (2D) electron system, there believed to be two phases: a high density Fermi gas phase and a low density insulating Wigner crystal phase. The dielectric constant of the liquid phase becomes negative when \( r_s \simeq 2 \), which indicates that the liquid phase is unstable. At lower densities, the Wigner crystal phase appears around \( r_s \simeq 37 \) in the absence of disorder. This critical value of \( r_s \) appears to be reduced to around \( r_s \simeq 10 \) with disorders. In the intermediate values of \( r_s \), we believe that there is a liquid phase which we think is responsible for the observed MIT.

In this paper, we propose that a droplet state of the electron system resulted from the phase separation of the electrons into this new liquid phase and a low density "gas" phase. Here we call the low density phase "gas" purely for the reason that its density is low. In fact, in the presence of impurities, the "gas" phase is disordered Wigner crystal. To investigate our proposal, we have studied the energetics of such a droplet state. We find that both electron-electron interaction and potential fluctuations are crucial for the formation of the droplet state.

An obvious condition for the droplet state is that the electron gas is unstable. To investigate what possibilities of the instability leads to, we study a simple but physically motivated model. Let us consider electrons in the disc of radius \( b \) with positive background. Imagine that the electron system is shrunk to a new radius \( a < b \) while the positive background remains intact. Clearly the charging energy due to the separation of the electrons from the positive background increases the energy of the system. However there can also be energy gained (decreasing total energy): since for a uniform electron gas the ground state energy \( E_g \) is at its minimum when \( r_s \simeq 2 \), for \( r_s > 2 \) the system gains energy by shrinking the area occupied by electrons. Furthermore, in the presence of disorder, electrons tend to occupy the valleys of the disorder landscape. Thus, a slowly varying disorder potential is in favor for the formation of the droplet state. We calculate the energy changes when electron disc is shrunk from \( b \) to \( a \) to determine whether a spontaneous shrinking can take place.

For electron-electron or electron and positive background interaction, we use screened Coulomb potential. For Si MOSFETs, the image charge in the metal substrate induces the screening and the interaction in the momentum space can be written as

\[
V(k) = \frac{1}{\varepsilon} \frac{2\pi e^2}{k} \frac{1 - e^{-2kD}}{1 - K e^{-2kD}},
\]

where \( D \) is the thickness of the \( Si_2O \) insulating layer and \( \varepsilon = \frac{\varepsilon_1 + \varepsilon_2}{2}, K = (\varepsilon_1 - \varepsilon_2)/(\varepsilon_1 + \varepsilon_2) \), with \( \varepsilon_1 \) and \( \varepsilon_2 \) being the dielectric constants of \( Si \) and \( Si_2O \), respectively. For other systems, such as \( GaAs/AlGaAs \), the screened interaction can be well represented by the following form:

\[
V(r) = \varepsilon^2 e^{-\lambda r},
\]

and the corresponding moment space representation is,

\[
V(k) = \frac{2\pi e^2}{\varepsilon} \frac{1}{\sqrt{k^2 + \lambda^2}}.
\]

Both forms of the interactions define an interaction range \( \xi \). For Si MOSFETs, \( \xi = \frac{\varepsilon_2 D}{\varepsilon_1} \), and for the screened Coulomb potential, \( \xi = \frac{1}{\lambda} \). Outside the range the interaction is strongly screened.
the ground state energy

\[ \alpha \]

\[ \text{to within 10\% comparing with the best available quantum Monte-Carlo results}\] for densities down to 2D electron system [9]. The accuracy of the ground state energy is found to within the ground state energy of 2D electron system [9]. The accuracy of the ground state energy is found to within the ground state energy of 2D electron system [9].

To calculate the ground state energy of the electron gas with the screened Coulomb interaction, we use the variational correlated-basis-function (CBF) method [9]. This method has been applied to the bare Coulomb interaction and is proved to provide rather accurate result for the ground state energy of 2D electron system [9]. The accuracy of the ground state energy is found to within 10\% comparing with the best available quantum Monte-Carlo results [9] for densities down to \( r_s = 20 \). In the CBF approach, there is a variational variable \( \alpha \) for which the ground state energy \( E_g \) has to be minimized

\[
E_g(r_s, \alpha) = \frac{A(\alpha)}{r_s^2} - U_c\left(\frac{\sqrt{\alpha}}{r_s}\right) \quad \text{(Ry.)}
\]

\[
A(\alpha) = A_B^0(\alpha) + A_F^0(\alpha) + A_F^1(\alpha) + A_F^2(\alpha) + \cdots
\]

\[
A_B^0(\alpha) = 1
\]

\[
A_F^0(\alpha) = -\frac{16}{\pi} \int_0^1 \left[ 2 \arccos(y) - y(1-y^2)^{1/2} \right] dy
\]

\[
A_F^1(\alpha) = -\frac{2}{\pi} \int_0^1 dy_1 \int_0^1 dy_2 \int_0^1 dy_3 \int_0^\pi d\theta_1 \frac{y_1^2}{2}\frac{y_2^2}{2}\frac{y_3^2}{2}
\]

\[
\times \left\{ 1 - \exp \left[ -y_{12}^2/2\alpha \right] \right\}
\]

\[
\times \exp \left[ -\left( y_1^2 + y_2^2 + 2y_3^2 \right)/2\alpha \right]
\]

\[
\times I_0 \left[ \alpha^{-1} \left( y_1^2 y_3^2 + y_2^2 y_3^2 + 2y_1 y_2 y_3 \cos \theta \right)^{1/2} \right],
\]

where \( 1 \text{Ry} = \frac{\hbar^2}{2m_e} \), \( a_B = \frac{\hbar^2}{m_e^2} \), and \( y_{12} = y_1 - y_2 \).

\[
U_c\left(\frac{\sqrt{\alpha}}{r_s}\right)
\]

is the cohesive energy which depends on the special form of the interaction,

\[
U_c\left(\frac{\sqrt{\alpha}}{r_s}\right) = \frac{1}{2} \int \exp \left[ -\left( \frac{k}{2\sqrt{\alpha}} \right)^2 \right] V(k) \frac{d^2k}{(2\pi)^2}.
\]

Figure 4 shows the calculated results for the ground state energy. The screening effect raises the ground state energy because the electron correlation is suppressed.

The shrinking of the electron disc will cause the redistribution of the charge, which will raise extra electrostatic energy because the positive background is fixed. The charge distribution can be written as

\[
\rho(k) = 2N \left( \frac{J_1(ka)}{ka} - \frac{J_1(kb)}{kb} \right),
\]

where \( N \) is the total number of the electrons in the disk. The charging energy is

\[
E_c = \frac{1}{2N} \int \frac{d^2k}{(2\pi)^2} V(k)|\rho(k)|^2
\]

\[
= \frac{1}{\pi a_B^2} \int_0^{\infty} dx x \left( x \right)^2 \left( \frac{J_1(x\gamma)}{\gamma} - J_1(x) \right)^2,
\]

where \( \gamma = \frac{r_s}{r_s^*} \) is the ratio of the radii after and before shrinking electron disc. The charging energy shows distinct forms for different interaction ranges. When the interaction range \( \xi \) is much larger than the radius of the disk, \( \xi \gg b \), the electron-electron interaction can be roughly considered as the bare Coulomb interaction. In this case, the electrostatic charging energy is

\[
E_c \approx 4b \left( 0.290545 - \frac{1}{\pi} \ln \left| 1 - \gamma \right| \right) (1 - \gamma)^2 + \cdots \text{ (Ry.)}
\]

In the other limit \( b \gg \xi \), the interaction is well screened and the electrostatic energy has the form

\[
E_c = 4\xi \left( \frac{1}{r_s^2} - \frac{1}{r_s^\prime 2} \right) \text{ Ry},
\]

where \( r_s \) and \( r_s^\prime \) are the inverse density parameters before and after the shrinking. The total energy difference can be written as

\[
\Delta E_{tot} = E_c(\gamma) + \Delta E_g.
\]

For a small initial radius \( b \ll \xi \), the energy gain \( \Delta E_g \) dominates over the energy loss \( E_c \), thus, there is always finite shrinking.
However, the above conclusion is not true in general as demonstrated in Fig. 3. Figure 3 shows the shrinking distance $\Delta b = b - a$ versus initial radius $b$. $\Delta b$ approaches a constant for large $b$. However, the shrinking shown here can not be considered as a macroscopic shrinking because the typical $\Delta b$ is only about $2a_B$, which is far smaller than the average distance between the electrons, which is $r_s = 15a_B$ for this calculation. Similar behavior has also been observed for other values of $r_s$.

Thus the intrinsic instability is not sufficient to overcome the charging energy cost in order to form the electron droplet state. The system is in a marginally stable situation. However, in real systems, there are always disorders. The low frequency component of a disorder potential forms the potential landscape, and electrons tend to occupy the low potential valleys. We assume that around each local minimum, the disorder potential is isotropic and slowly varying. We expand the disorder potential around the local minimum up to the quadratic term. Therefore, we adopt the following simple model for disorder potential

$$W(r) = V_0 \frac{r^2}{\lambda^2},$$

where $V_0$ is the potential depth from center to edge of the disc. The energy gain by shrinking to a radius $a$ can be evaluated as

$$\Delta E_W = \frac{1}{N} \int W(r) \rho(r) dS = \frac{V_0}{2} \left( \gamma^2 - 1 \right).$$

The total energy difference in the limit $b \gg \xi$ can be written as

$$\Delta E_{tot} = -\left( \frac{V_0}{2} - \frac{4\xi}{r_{sd}} \right) (1 - \gamma^2) + \Delta E_g.$$ 

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FIG. 4. The phase diagram in the $r_s - \xi$ plot. $r_s$ is the initial density parameter of the electron disc. $D$ or $1/\lambda$ represents the interacting range $\xi$ of the electron-electron interaction. Each value of $V_0$ corresponds to a curve in the figure. On the right side of the curve, the electron disc will have a macroscopic shrinking.

Before summarizing, we would like to make several comments. (i) In this paper, we only consider one electron disc, corresponding to one drop in the droplet state. In real systems, electrons tend to occupy valleys of potential fluctuations to give rise many such drops. The size of each drop is determined by local potential depth. (ii) We only consider zero temperature effect in this paper such that the “gas” phase is empty. At a finite temperature, the “gas” phase is occupied by lower density electrons. Thus, a finite temperature enhances the possibility for the droplet state since the density difference between the liquid and the “gas” is smaller and the charging energy is less costly. However, in order to form the droplet state, the temperature has to be below the cohesive energy (the energy cost to remove an electron from the liquid phase to the “gas” phase [4]). (iii) We believe that the recently observed 2D metal-insulator transition might be the percolation transition of the liquid phase in the droplet state [4]. The percolation here is semi-quantum in nature, different from the conventional classic percolation [10]. (iv) In order to have a percolation, the “gas” phase needs to have a much smaller local conductivity than the liquid phase. This requires that a typical length scale of the “gas” region is larger than the localization length of the “gas” phase. The “gas” phase is low in electron density which gives rise to a short localization length. Thus, one may not need a large shrinking of electron drops to realize a percolation transition. We suspect that in experimental samples, a typical drop size is of the order of $\mu$m.

In summary, we have demonstrated that it is possible to have a droplet phase for 2D electron systems at low densities. Both electron-electron interaction and disorder potential fluctuations are important for the formation of the droplet phase.

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