Parameter-free Stark Broadening of Hydrogen Lines in DA White Dwarfs

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Abstract. We present new calculations for the Stark broadening of the hydrogen line profiles in the dense atmospheres of white dwarf stars. Our improved model is based on the unified theory of Stark broadening from Vidal, Cooper & Smith, but it also includes non-ideal gas effects from the Hummer & Mihalas occupation probability formalism directly inside the line profile calculations. This approach improves upon previous calculations that relied on the use of an ad-hoc free parameter to describe the dissolution of the line wing opacity in the presence of high electric microfields in the plasma. We present here the first grid of model spectra for hot ($T_{\text{eff}} \geq 12,000$ K) DA white dwarfs that has no free parameters. The atmospheric parameters obtained from optical and UV spectroscopic observations using these improved models are shown to differ substantially from those published in previous studies.

1. Introduction

The spectroscopic technique has been the most successful method for determining the atmospheric parameters — $T_{\text{eff}}$ and $\log g$ — of hydrogen-line DA white dwarfs\textsuperscript{1}. The technique consists in comparing the observed and predicted Balmer line profiles, and it has recently been expanded to the Lyman line series in the ultraviolet\textsuperscript{2}. The advantage of this method is that the theoretical profiles are very sensitive to variations of the atmospheric parameters. Since about 80% of the white dwarf population is of the DA type, the spectroscopic technique coupled with high signal-to-noise spectroscopic observations of Lyman or Balmer lines for large samples of DA stars can reveal important details about the mass distribution, the luminosity function, and the formation rate of white dwarfs. Even though this technique yields very accurate results with uncertainties less than 2% in terms of the relative parameters between stars, the absolute values of these parameters may still depend on the physics included in the models. One important aspect of the model atmosphere calculations is the opacity of the hydrogen lines, which are the dominant features observed in optical and UV spectra.

2. Models with a free parameter

In their preliminary analysis of DA white dwarfs,\textsuperscript{1} found a lack of internal consistency between the spectroscopic solutions obtained when an increasing number of Balmer lines was included in the fitting procedure. This is illustrated in the top panel of Figure 1 for a typical DA star where we can see the solution gradually drifting in the $T_{\text{eff}} - \log g$ diagram as more lines are
included in the fit. [3] traced back the problem to the neglect of non-ideal gas effects inside the line profile calculations. Indeed, all white dwarf models rely on the theory of Stark broadening from Vidal, Cooper, & Smith [4] (hereafter VCS), which assumes that the absorber is in an ideal gas. However, [5] has argued that non-ideal effects resulting from the perturbations of neighboring particles should be included directly in the line profile calculations. As a simpler alternative, [3] proposed instead to parameterize the value of the critical field ($\beta_{\text{crit}}$) used in the non-ideal equation of state of Hummer & Mihalas [6, 7] (hereafter HM88) to mimic the non-ideal effects in the line opacity, and in particular in the regions where the line wings overlap. Model spectra calculated with twice the value of the critical field ($\beta_{\text{crit}} \times 2$) were shown to improve the internal consistency significantly, as can be observed in the middle panel of Figure 1. It should be stressed, however, that the use of this free parameter does not imply that HM88 have underestimated the value of the critical field. As explained in detail by [3], this is just a way to simulate the non-ideal effects discussed in [5] by reducing the line wing opacity near the Balmer limit. The effect of this additional free parameter on the line wing opacity is illustrated in Figure 2 where two model spectra are compared. The problem with this approach, of course, is that it has no physical basis, and there is also no reason to believe that this free parameter is the same for various hydrogen states or for different atmospheric parameters. It also has the disadvantage of modifying artificially the various populations in the equation of state. We note that all currently available model spectra of DA white dwarfs [8, 9, 10, 11] make use of this free parameter. Finally, even though the same parameterization is also used for transitions in the UV, it has never been tested empirically in an analysis similar to that displayed in Figure 1.

**Figure 1.** Solutions in a $T_{\text{eff}}$ – $\log g$ diagram for a typical DA star using 1 line (H$\beta$), 2, 3, 4 and 5 lines (up to H$8$) in the fitting procedure (represented by thicker 1σ uncertainty ellipses). The top panel shows the results with the original VCS profiles, while the middle panel also includes the free parameter proposed by [3]. The bottom panel is with our new line profiles (see § 3).

**Figure 2.** Comparison of synthetic spectra at $T_{\text{eff}} = 20,000$ K and $\log g = 8$ without (solid line) and with (dashed line) the parameterization of the critical field ($\beta_{\text{crit}} \times 2$) proposed by [3].
3. Improved Stark profiles
In this work, we provide a framework with no free parameter by improving the physics in the line profile calculations. We briefly describe here our procedure; further details will be provided elsewhere. Our work is based on the unified theory of Stark broadening from VCS. An extension of the VCS calculations in the white dwarf regime has been published by [12], and these tables have been the most commonly used to model the Balmer and Lyman line profiles in DA stars. The unified VCS theory relies on the quasi-static approximation to describe the broadening by protons and a unified classical path theory for the broadening by electrons, which reduces to the impact approximation in the line cores and to the quasi-static approximation in the wings. One major problem for the modeling of DA atmospheres, however, is that the theory considers the absorber to be in an ideal gas, which means that line dissolution due to plasma perturbations is not allowed. Such perturbations are described at length in the HM88 non-ideal equation of state for astrophysical applications. They include, in particular, the high amplitude electric microfields produced by protons and the frequent electronic collisions.

The occupation probability formalism of HM88 has been introduced in the context of white dwarf spectra since the work of [8]. According to this formalism, the probability that one electronic state is bound to the atom is obtained from the integral of the electric microfield probability distribution up to a given value for the critical field $\beta_{\text{crit}}$. The critical field is roughly defined as the point where one Stark state for a particular level with a principal quantum number $n$ “crosses” another Stark state associated with a higher level $n + 1$. At the critical field, we expect that a bound electron on a level $n$, and a superposition of the accessible Stark states due to the electronic collisions and fluctuations in the direction of the microfield, will undergo a cascade of transitions to higher levels all the way up to the continuum. The occupation probability obtained with this formalism can then be used to compute the hydrogen state populations. The treatment of the amplitude of the line opacity must also be modified to take into account the fact that a transition to an upper “dissolved” atomic level must be treated as a bound-free opacity rather than a bound-bound transition — the so-called pseudo-continuum opacity [7].

The late Mike Seaton was the first to introduce the HM88 non-ideal effects directly inside the line profile calculations so that both the spectral shape and the amplitude are affected in a consistent way [5]. However, his calculations made for the Opacity Project rely on an approximate electronic broadening theory that is inappropriate in the context of white dwarf atmospheres. We have therefore performed our own implementation of these non-ideal effects in a version of the VCS code provided to us by M. Lemke. In the case of proton perturbations, the line wing opacity will be reduced with respect to the ideal gas case since the high electric microfields contribute more importantly to the bound-free opacity by dissociating the atoms. We therefore use here a truncation of the proton microfields in the calculations of the quasi-static proton line broadening and include corrections due to electron perturbations as well. This is the first time these non-ideal effects are included directly into the VCS theory, and that DA models are calculated with line profiles that are coherent with the equation of state, without any free parameter. In Figure 3, we compare our improved line profile calculations with the original VCS profiles. Also shown for comparison are the approximate calculations of [5].

4. Results
4.1. Balmer lines
We now discuss the astrophysical implications of our improved line profiles. We calculated two grids of model atmospheres using (1) our new line profile calculations and (2) the original VCS theory using the tables computed by [12], but with twice the value of the critical field ($\beta_{\text{crit}} \times 2$) as proposed by [3]. Both grids range from 13,000 to 40,000 K in $T_{\text{eff}}$ and from 6.5 to 9.5 in log $g$. In this analysis, we chose to restrain our calculations to this range of effective temperature to
avoid the uncertainties related to convective energy transport, neutral broadening, and non-LTE effects.

First of all, we can already see in the bottom panel of Figure 1 that our line profiles have significantly improved the internal consistency between the solutions obtained from various Balmer lines compared to the previous calculations displayed in the two upper panels. We find, however, that the quality of the fits (not shown here) are nearly identical for all grids. In the following, we analyze the PG spectroscopic sample of DA white dwarfs from [13], but we restrict the range of temperature to that of our model grid. This yields a sample of 250 objects out of the 348 DA stars for the complete PG sample. Our spectroscopic data and fitting procedure are identical to those described in [13]. The atmospheric parameters for each star are obtained using both model atmosphere grids, and the log $g$ values are then converted into masses using evolutionary models appropriate for white dwarfs with thick hydrogen layers (see [13] for details). Our results are presented in Figure 4 in a mass versus $T_{\text{eff}}$ diagram. We can see that the distributions with both grids are significantly different. In particular, the masses derived with our improved models are globally larger and the $T_{\text{eff}}$ values are also higher (see also Fig. 1).

As seen from Figure 5, the mean mass of this PG sample is shifted by $\Delta M/M_\odot = +0.034$ when our new models are used. The shape of the mass distribution remains statistically equivalent for both grids, however, and the dispersions are comparable.

As discussed by [1], even though the spectroscopic technique provides very accurate individual determinations of atmospheric parameters, the absolute values of log $g$ may suffer from a zero-point offset and it is therefore important to compare the individual measurements with results obtained from independent techniques. The only reliable independent observational constraints for large samples of white dwarfs are trigonometric parallax measurements. We thus compare the absolute visual magnitudes $M_V$ derived from the measured parallaxes (combined with $V$) with the absolute magnitudes obtained from the spectroscopic values of $T_{\text{eff}}$ and log $g$ following the procedure described at length in [14]. The results presented here are based on the high signal-to-noise spectroscopic sample of [15] for 92 DA white dwarfs with known parallaxes. The comparisons are shown in Figure 6 for both model grids. Again, we have restrained the analysis.
Figure 4. Mass versus $T_{\text{eff}}$ distributions for the PG sample in the range $40,000 \, \text{K} > T_{\text{eff}} > 13,000 \, \text{K}$. Results are shown for both our improved line profiles (top panel) and our old models (bottom panel). Lines of constant masses at 0.55 and 0.70 $M/M_\odot$ are shown as a guide.

Figure 5. Mass distributions for the PG sample used in Figure 4. The mean masses are reported in the figure.
to stars above $T_{\text{eff}} = 13,000$ K. The agreement is satisfactory within the parallax uncertainties for both model grids. Hence, despite the fact that our new models yield higher values of $T_{\text{eff}}$ and $\log g$ (i.e., smaller radii), these two effects almost cancel each other and the predicted luminosities (or $M_V$) remain unchanged. Another important constraint is provided by the bright white dwarf 40 Eri B for which a very precise trigonometric parallax and visual magnitude have been measured by Hipparcos. These measurements yield $M_V = 11.01 \pm 0.01$, while we predict $M_V = 11.02 \pm 0.07$ and $10.97 \pm 0.07$ based on our new and old models, respectively. Although both determinations agree within the uncertainties with the observed value, our new grid provides an exact match to the measured $M_V$ value.

![Figure 6](image-url)  

**Figure 6.** Comparison of the absolute visual magnitudes obtained from trigonometric parallax measurements and from the spectroscopic analysis of DA stars above $T_{\text{eff}} \sim 13,000$ K using both model grids.

### 4.2. Lyman lines

This section presents a preliminary investigation of our model predictions for the Lyman lines. We analyze the FUSE spectra of two bright white dwarfs, GD 71 and Sirius B, both of which are in the temperature range considered in this work. In Figure 7, we show $\chi^2$ fits of the entire FUSE spectra obtained with our new models to determine the atmospheric parameters given in the figure. The quality of the fits is very good. We performed similar fits with our old grid and we find that the quality of the fit is comparable, although our new atmospheric parameters are shifted by $+60$ K in $T_{\text{eff}}$ and by $+0.06$ in $\log g$ for both stars. Thus, the shifts in $\log g$ are similar to those observed in the optical. It should be noted, however, that an increase of both $T_{\text{eff}}$ and $\log g$ by $\sim 1\%$ results in fits that are equally good from visual inspection, so the values of the best fit parameters reported here should be taken with caution. Recent analyses by [11, 16] suggest that the $\log g$ values derived from the optical and the UV agree in this range of $T_{\text{eff}}$, and this conclusion should not be affected by our new models.

The temperature shifts in the UV, however, are much lower than those in the optical (see, e.g., Fig. 1). Since the UV spectra were calibrated using DA models with the older line profiles, the data reduction might have to be revisited before we confirm these results. For instance, we show in Figure 8 the predicted spectra over the visible and UV regions for two flux standards (GD 71 and G191-B2B) used for the FUSE and HST calibrations among others. The atmospheric parameters for these LTE models are obtained from the best fits to the observed Balmer lines.
of each star using both grids (hotter models have been calculated for this exercise). The results indicate that the absolute predicted fluxes, as well as the slopes of the energy distributions, are very different even up to the high temperature of G191-B2B ($T_{\text{eff}} \sim 60,000$ K).

**Figure 7.** Fits to two FUSE spectra using our improved model spectra; $T_{\text{eff}}$, $\log g$, and the solid angle are considered free parameters. Interstellar features, including all cores of the lines, have been removed from the observed spectra. The atmospheric parameters obtained from the minimization procedure are given in each panel.

We finish this section by noting that the non-ideal effects for the lower Lyman lines are still not well understood. The critical field in this case becomes so large that it must have been created by a single proton close to the absorber in such a way that the approximation of the dipolar proton-absorber interaction fails completely. For instance, exact calculations show that $n = 1, 2$ and 3 have no crossings. Current white dwarf model atmospheres (this code, TLUSTY) neglect non-ideal effects for Lyman $\alpha$, Lyman $\beta$ and in some cases Lyman $\gamma$. In other words, there is no pseudo-continuum opacity, and ideal gas profiles are used for these lines. This significantly changes the predicted UV spectra for $T_{\text{eff}} < 30,000$ K. For H$\alpha$, non-ideal effects are already negligible with the HM88 theory and no modification is necessary. We plan to study further this problem with a larger sample of FUSE spectra.

5. Conclusion
We have combined for the first time the unified theory of Stark broadening from VCS with the non-ideal equation of state of HM88 to build model atmospheres for DA white dwarfs that are consistent to first order in terms of the physics of the equation of state and the line profiles, without a free parameter. We have shown that in the range 40,000 K $> T_{\text{eff}} > 13,000$ K, the revised mean mass for the PG sample is significantly larger than previous determinations. The atmospheric parameters we obtain with our new model spectra are in excellent agreement with
Figure 8. Theoretical spectra for the flux standards GD 71 (left) and G191-B2B (right) with our improved models (dotted lines) and our old grid (solid lines) using the atmospheric parameters obtained from fits to the Balmer lines.

trigonometric parallax measurements. Future work will extend our analysis to the more complex regime of white dwarf stars at higher and lower effective temperatures.

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