‘Desert’ in Energy or Transverse Space?

C. Bachas

Laboratoire de Physique Théorique de l’Ecole Normale Supérieure
24 rue Lhomond, 75231 Paris Cedex 05, France

Abstract

I review the issue of string and compactification scales in the weak-coupling regimes of string theory. I explain how in the Brane World scenario a (effectively) two-dimensional transverse space that is hierarchically larger than the string length may replace the conventional ‘energy desert’ described by renormalizable supersymmetric QFT. I comment on the puzzle of unification in this context.

1. The SQFT Hypothesis

String/M-theory is a higher-dimensional theory with a single dimensionful parameter, which can be taken to be the fundamental string tension or the eleven-dimensional Planck scale. The theory has on the other hand a large number of ‘dynamical parameters’ characterizing its many distinct semiclassical vacua, such as compactification radii or sizes of defects localized in the compact space. Understanding how the Standard Model and Einstein gravity arise at low energies in one of those vacuum states is a central outstanding problem of String/M-theory.

1Based on talks given at the conferences ‘22nd Johns Hopkins workshop’ (Göteborg, August 1998), ‘Fundamental interactions: from symmetries to black holes’ in honor of François Englert (Brussels, March 1999) and ‘From Planck Scale to Electroweak Scale’ (Bad Honnef, April 1999).
The usual hypothesis is that the string, compactification and Planck scales lie all close to one another, and that the physics at lower energies is well described by some effective four-dimensional renormalizable supersymmetric quantum field theory (SQFT), which must include the Minimal Supersymmetric Standard Model (MSSM) and some hidden sectors. I will refer to this picture of the world as the ‘SQFT hypothesis’. In this picture the breaking of the residual supersymmetry and the generation of the electroweak scale are believed to be triggered by non-perturbative gaugino condensation – a story that is however incomplete because of the problems of vacuum stability and of the cosmological constant.

The minimal version of the SQFT hypothesis is obtained when there are no light fields charged under $SU(3)_c \times SU(2)_{ew} \times U(1)_Y$, besides those of the MSSM. This is the ‘energy desert’ scenario – a slight misnommer since the ‘desert’ may be populated by all sorts of stuff coupling with gravitational strength to ordinary matter. The minimal unification scenario is supported, as is well known [2, 3, 4], by two pieces of strong, though indirect evidence: (a) the measured low-energy gauge couplings do meet when extrapolated to higher energies with the MSSM $\beta$-functions, and (b) the energy $M_U \sim 2 \times 10^{16} GeV$ at which they meet is in the same ballpark as the string scale. A detailed analysis within the weakly-coupled heterotic string [5] leads, in fact, to a discrepancy of roughly one order of magnitude between the theoretical point of string unification, and the one that fits the low-energy data. This is a small discrepancy on a logarithmic scale, and it could be fixed by small modifications of the minimal scenario [3].

Besides being simple and rather natural, the minimal SQFT hypothesis makes thus two quantitative predictions which fit the low-energy data to better than one part in ten.
2. The Weakly-Coupled Heterotic Theory

The SQFT hypothesis is particularly compelling in the context of the weakly-coupled heterotic string [7]. Both the graviton and the gauge bosons live in this case in the ten-dimensional bulk, and their leading interactions are given by the same order in string perturbation theory (i.e. the sphere diagram). This leads to the universal relation between the four-dimensional Planck mass ($M_P$) and the tree-level Yang-Mills couplings [8],

$$M_P^2 \sim M_H^2/g_{YM}^2,$$  \hspace{1cm} (2.1)

independently of the details of compactification. If we assume that $g_{YM} \sim o(1)$, then the heterotic string scale ($M_H$) is necessarily tied to the Planck scale. Furthermore, the standard Kaluza-Klein formula for the four-dimensional gauge couplings is

$$1/g_{YM}^2 \sim (RM_H)^6/g_H^2,$$  \hspace{1cm} (2.2)

with $R$ the typical radius of the six-dimensional compact space and $g_H$ the dimensionless string coupling. Pushing the Kaluza-Klein scale ($M_{KK} \sim R^{-1}$) much below $M_H$ requires therefore a hierarchically-strong string coupling, and invalidates the semiclassical treatment of the vacuum. Of course all radii need not be equal but, at least in orbifold compactifications, T-duality allows us to take them all larger or equal to the string length, and then the above argument forbids any single radius from becoming too large.

There is actually a loophole in the above reasoning. If some compact dimensions are much larger than the heterotic string length, loop corrections to the inverse squared gauge couplings will generi-
cally grow like a power of radius $\sqrt{2}$. It is thus logically conceivable that even though the observed low-energy gauge couplings are of order one, their tree-level values are hierarchically smaller. Since it is the tree-level couplings that enter in the relation (2.1), the heterotic string scale could thus in principle be significantly lower than the four-dimensional Planck mass [11].

The main motivation for contemplating such possibilities in the past was the search for string models with low-energy supersymmetry broken spontaneously at tree level. Existing heterotic vacua of this type employ a string variant [12] of the Scherk-Schwarz mechanism [13], which breaks supersymmetry in a way reminiscent of finite-temperature effects. The scale of (primordial) breaking is proportional to an inverse radius, so that lowering it to the electroweak scale requires the opening of extra dimensions at the TeV – a feature shown [14] to be generic in orbifold models.

Insisting on tree-level breaking is, on the other hand, only a technical requirement – there is no reason why the breaking in nature should not have a non-perturbative origin. Furthermore, Scherk-Schwarz compactification has not so far lead to any new insights on the problems of vacuum selection and stability. Thus, there seems to be little theoretical motivation at this point for abandoning the SQFT hypothesis, and its successful unification predictions, in heterotic string theory.

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2In special models, such as orbifolds without N=2 sectors, these large threshold corrections can be made to vanish at one-loop. The evolution of gauge couplings with energy is thus unaffected by the opening of large extra dimensions [1]. However, since $g_{H}$ must in these models be hierarchically strong, the semiclassical string vacuum cannot be trusted.

3Power corrections to gauge couplings have been also recently invoked as a way to speed up the unification process [10].

4For more general compactifications, the limit of supersymmetry restoration is also known to be a singular limit [3], even though there is no precise relation between the scale of symmetry breaking and some Kaluza Klein threshold.
3. Brane World and Open String Theory

The story is different in the theory of (unoriented) open and closed strings, in which gauge and gravitational interactions have different origins. While the graviton (a closed-string state) lives in the ten-dimensional bulk, open-string vector bosons can be localized on defects [16] – the worldvolumes of D(irichlet)-branes [17]. Furthermore while closed strings interact to leading order via the sphere diagram, open strings must be attached to a boundary and thus interact via the disk diagram which is of higher order in the genus expansion. The four-dimensional Planck mass and Yang-Mills couplings therefore read

\[ \frac{1}{g_{YM}^2} \sim (R_{||} M_I)^{6-n} / g_I , \quad M_P^2 \sim R_{||}^n R_{\perp}^{6-n} M_I^8 / g_I^2 , \]  

(3.3)

where \( R_{\perp} \) is the typical radius of the n compact dimensions transverse to the brane, \( R_{||} \) the typical radius of the remaining (6-n) compact longitudinal dimensions, \( M_I \) the type-I string scale and \( g_I \) the string coupling constant. As a result (a) there is no universal relation between \( M_P, g_{YM} \) and \( M_I \) anymore, and (b) tree-level gauge couplings corresponding to different sets of branes have radius-dependent ratios and need not unify.

A few remarks before going on. First, we are here discussing a theory of unoriented strings, because orientifolds [19] are required in order to cancel the tension and RR charges of the (non-compact) space-filling D-branes. Second, using T-dualities we can ensure that both \( R_{\perp} \) and \( R_{||} \) are greater than or equal to the string scale [16]. This may take us either to Ia or to Ib theory (also called I or I’, respectively) – I will not make a distinction between them in what follows. Finally, it should be stressed that D-branes are the only known defects which can localize non-abelian gauge interactions in a perturbative setting. Orbifold fixed points can at most ‘trap’
matter fields and abelian vector bosons (from twisted RR sectors).\textsuperscript{5}

Relations (3.3) tell us that type I string theory is much more flexible (and less predictive) than heterotic theory. The string scale $M_I$ is now a free parameter, even if one insists that both $g_{YM}$ and $g_I$ be kept fixed and of $o(1)$. This added flexibility can be used to remove the order-of-magnitude discrepancy between the unification and string scales \textsuperscript{20}. A much more drastic proposal \textsuperscript{21, 22, 23} is to lower $M_I$ down to the experimentally-allowed limit $\sim o$(TeV). Keeping for instance $g_I$, $g_{YM}$ and $R_{\parallel} M_I$ of order one, leads to the condition

$$R_{\perp}^n \sim M_p^2 / M_I^{2+n}.$$  \hspace{1cm} (3.4)

A TeV string scale would then require from n=2 millimetric to n=6 fermi-size dimensions transverse to our Brane World – the relative weakness of gravity being in this picture attributed to the transverse spreading of gravitational flux.

What has brought this idea \textsuperscript{4} into sharp focus \textsuperscript{21} was (a) the realization that submillimeter dimensions are not at present ruled out by mesoscopic gravity experiments,\textsuperscript{6} and (b) the hope that lowering $M_I$ to the TeV scale may lead to a new understanding of the gauge hierarchy. Needless to say that a host of constraints (astrophysical and cosmological bounds, proton decay, fermion masses etc.) will make realistic model building a very strenuous exercise indeed. Finding type I vacua with three chiral families of quarks

\textsuperscript{5}Non-perturbative symmetry enhancement is of course a possibility, as has been discussed for instance in \textsuperscript{15}. The great success of the perturbative Standard Model makes one, however, reluctant to start with a theory in which $W$ bosons, and all quarks and leptons do not correspond to perturbative quanta.

\textsuperscript{6}For early discussions of a Brane Universe see \textsuperscript{24}.

\textsuperscript{7}That such experiments do not rule out light scalar particles, such as axions, with gravitational-force couplings and Compton wavelengths of a millimeter or less, had been already appreciated in the past \textsuperscript{25}. The Kaluza-Klein excitations of the graviton are basically subject to the same bound.
and leptons is already a non-trivial problem by itself \[26\]. None of these difficulties seems, however, \textit{a priori} fatal to the Brane World idea, even in its most extreme realization \[27\].

4. Renormalization Group or Classical Supergravity?

Although the type I string scale could lie anywhere below the four-dimensional Planck mass\[^8\] I will now focus on the extreme case where it is close to its experimental lower limit, $M_I \sim \mathcal{O}(\text{TeV})$. Besides being a natural starting point for discussing the question of the gauge hierarchy, this has also the pragmatic advantage of bringing string physics within the reach of future accelerator experiments. This extreme choice is at first sight antipodal to the minimal SQFT hypothesis: the MSSM is a stable renormalizable field theory, and yet one proposes to shrink its range of validity to one order of magnitude at most! Nevertheless, as I will now argue, the Brane World and SQFT scenarios share many common features when the number of large transverse dimensions in the former is exactly two \[29,30\].

The key feature of the SQFT hypothesis is that low-energy parameters receive large logarithmic corrections, which are effectively resummed by the equations of the Renormalization Group. This running with energy can account for the observed values of the three gauge couplings, and of the mass matrices of quarks and leptons, in a way that is relatively ‘robust’\[^9\]. Furthermore the logarithmic sensitivity of parameters generates naturally hierarchies of scales, and has been the key ingredient in all efforts to understand the origin of the $M_Z/M_P$ hierarchy in the past \[31\].

\[^8\] Arguments in favour of an intermediate string scale were given in \[28\].

\[^9\] One must of course assume initial conditions for the RG equations, typically imposed by unification and by discrete symmetries, but there is no need to know in greater detail the physics in the ultraviolet regime.
Consider now the Brane World scenario. The parameters of the effective Brane Lagrangian are dynamical open- and closed-string moduli. These latter, denoted collectively by $m_K$, include the dilaton, twisted-sector massless scalars, the metric of the transverse space etc. Their vacuum expectation values are constant along the four non-compact space-time dimensions, but vary generically as a function of the transverse coordinates $\xi$. For weak type-I string coupling and large transverse space these variations can be described by a Lagrangian of the (schematic) form

$$L_{\text{bulk}} + L_{\text{source}} \sim \int d^n\xi \left[ \frac{1}{g_I^2} (\partial_\xi m_K)^2 + \frac{1}{g_I} \sum_s f_s(m_K) \delta(\xi - \xi_s) \right].$$  \hspace{1cm} (4.1)

This is a supergravity Lagrangian reduced to the $n$ large transverse dimensions, and coupling to D-branes and orientifolds which act as sources localized at transverse positions $\xi_s$. The couplings $f_s(m_K)$ may vary from source to source – they can for instance depend on open-string moduli – and are subject to global consistency conditions. What is important, however, to us is that they are weak in the type-I limit, leading to weak variations,

$$m_K(\xi) = m^0_K + g_I m^1_K(\xi) + \cdots,$$  \hspace{1cm} (4.2)

with $m^0_K$ a constant, $m^1_K$ a sum of Green’s functions etc. For $n = 2$ dimensions the leading variation $m^1_K$ grows logarithmically with the size of the transverse space, $R_\perp$. Since our Standard Model parameters will be a function of the moduli evaluated at the position of our Brane World, they will have logarithmic sensitivity on $M_P$ in this case, very much like the (relevant) parameters of a supersymmetric renormalizable QFT. Similar sensitivity will occur even if

\[\text{In the general case there could be also branes extending only partially into the large transverse bulk. Our discussion can be adapted easily to take those into account.}\]
Let me now discuss the validity of the approximation (4.1). The bulk supergravity Lagrangian receives both $\alpha'$ and higher-genus corrections, but these involve higher derivatives of fields and should be negligible for moduli varying logarithmically over distance scales $\gg \sqrt{\alpha'}$. The source functions, $f_s(m_K)$, are also in general modified by such corrections – our $\delta$-function approximation is indeed only valid to within $\delta \xi \sim o(\sqrt{\alpha'})$. Such source modifications can, however, be absorbed into boundary conditions for the classical field equations at the special marked points $\xi_s$. The situation thus looks (at least superficially) analogous to that prevailing under the SQFT hypothesis: large corrections to low-energy parameters can be in both cases resummed by differential equations with appropriate boundary conditions. There are, to be sure, also important differences: in particular, the Renormalization Group equations are first order differential equations in a single (energy) scale parameter, while the classical supergravity equations are second-order and depend on the two coordinates of the large transverse space.

The analogy between energy and transverse distance is also reminiscent of the holographic idea [32], considered in the context of compactification in [33]. It is, however, important to stress that our discussion here stayed perturbative (and there was no large-N limit involved). I have just tried to argue that large string-loop corrections to the parameters of a brane action can, in appropriate settings, be calculated reliably as the sum of two superficially similar effects: (a) RG running from some low energy scale up to string scale, and (b) bulk-moduli variations over a transverse two-dimensional space of size much greater than string length. The two corresponding regimes – of renormalizable QFT and of reduced
classical supergravity – are a priori different and need not overlap.

5. The Puzzle of Unification

The logarithmic sensitivity of brane parameters on $R_\perp$ can be used to generate scale hierarchies dynamically, exactly as with renormalizable QFT. Gauge dynamics on a given brane, for example, can become strong as the transverse space expands to a hierarchically large size, thereby inducing gaugino condensation and possibly supersymmetry breaking. Rather than discussing such scenarios further, I would now like to return to the main piece of evidence in favour of the SQFT hypothesis: the apparent unification of the Standard Model gauge couplings. Can their observed low-energy values be understood [29, 34] in an equally robust and controlled manner, as coming from logarithmic variations in the (real) space transverse to our Brane World? I don’t yet know the answer to this important question, but let me at least refute the following possible objection: since the three gauge groups of the Standard Model live at the same point in transverse space (or else matter charged under two of them would have been ultraheavy) how can real-space variations split their coupling constants apart? This objection would have been, indeed, fatal if all gauge couplings were determined by the same combination of bulk fields. This is fortunately not the case: scalar moduli from twisted sectors of orbifolds have been, for instance, shown to have non-universal couplings to gauge fields living on the same brane [36, 34]. The logarithmic variations of such fields could split the three Standard Model gauge couplings apart, although it is unclear why this splitting should be in the right proportion.

\[\text{11For another recent idea see } 37.\]
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