Unleashing the aether

Alberto Diez-Tejedor
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, México D.F.
04510, México
E-mail: alberto.diez@nucleares.unam.mx

Abstract. We follow an effective field theory approach to identify the general class of theories that describes a real vector field coupled to gravity at low energies, large distances and long timescales. Here we restrict ourselves to the case in which Lorentz invariance is spontaneously broken at low energies (i.e. at energies below the characteristic scale of the effective theory). We find the constraints on the parameter space of the theory coming from the absence of classical and quantum instabilities. We apply our results to the flat case before closing.

The symmetries of a spacetime severely constrain the transformation properties of its matter content. In flat spacetime for instance Lorentz invariance only allows scalar fields to have a non-vanishing expectation value. However, we do not live in a Lorentz invariant universe: at large scales it is homogeneous and isotropic, but non-static. In an expanding universe no symmetry principle prevents the appearance of non-vanishing tensor fields, as long as they were constant in space and invariant under spatial rotations. Since only bosonic fields can acquire a large occupation number, and if we restrict ourselves to fields of spin less than two, this leaves vector fields as essentially the only additional type of matter field that may be classically relevant in a cosmological context. (By classical we mean that at the quantum level the state of the system is described by a coherent excitation.)

Massless vector-tensor theories were considered long time ago to explore different modifications of gravity [1, 2, 3]. More recently, theories with vectors have experienced a modest revival in connection with inflation [4], the current accelerated expansion of the universe [5] and a possible breakdown of Lorentz invariance [6]. Contrary to other proposals we do not assume here any specific model and consider the most general vector-tensor theory expected to describe our universe at large distances and long timescales. We thus follow a low energy effective action approach and retain all possible terms compatible with the symmetries of the theory, which we assume to consist just of locality, general covariance and a $\mathbb{Z}_2$ symmetry for the vector field (i.e. $A^\mu \rightarrow -A^\mu$). If we are interested in low energies however, we expect only those terms with the least number of field derivatives to appear into the action. In principle, such an action would also contain arbitrary powers of the vector field, but since a vector has dimensions of mass, terms with too many powers of $A^\mu$ must be suppressed by corresponding powers of an energy scale $\Lambda$. If the background value of the field is smaller than $\Lambda$, the dominant terms are hence given by those with no suppression, that is, by the operators of dimension four. Therefore, we just have to consider the most general action with two derivatives acting on the metric and the vector field, but with no vector field operators of dimension higher than four:

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{\beta_1}{2} F_{\mu\nu} F^{\mu\nu} - \beta (\nabla_\mu A^\mu)^2 + \beta_{13} R_{\mu\nu\rho\sigma} A^\mu A^\nu + \beta_4 R A_\mu A^\mu - V \right].$$ (1)
Here $\beta_i$ are some dimensionless coefficients, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor and $V$ a potential for the vector field which, under the previously mentioned assumptions and up to a constant term, is given by

$$V(A_\mu A^\mu) = \lambda (A_\mu A^\mu + M^2)^2.$$  \hfill (2)

Other low energy terms not appearing into the expression (1) can be reabsorbed by the appearing ones after integrating by parts. Notice that, since we deal with massive vectors, a consistent quantum description at low energies does not require gauge invariance, so many different kinetic terms for the vector field are possible. It is also interesting to point out here that for generic values of the potential parameters different from zero Lorentz invariance will be spontaneously broken by the vacuum expectation value of the theory. The action (1) is to some extent a generalization of previously considered theories. Setting $V \equiv 0$ one recovers the massless vector-tensor theories studied in [3], while replacing the potential (2) by a fixed-norm constraint leads to the Einstein-aether models considered in [6]. Although a fixed normed constraint seems to be difficult to justify from a particle physics perspective, it can be naively recovered from a potential of the form (2), at least classically, by taking the limit $\lambda \to \infty$. This is the reason for which we call the theories considered here \textit{aether unleashed}. (Similar kind of actions have been considered for instance in [7] under the name of “bumblebee models”.)

However, in order to consider the aether unleashed as a sensible model to describe the observable universe, we should be sure that at least it is self-consistent in a cosmological setting. With this in mind, let us analyze the conditions that classical and quantum-mechanical stability place on the parameters of the theory.\footnote{For more details about the possible viability of the unleashed aether models please see the reference [8].} The stability of the system will be determined by the dynamics of the small perturbations $b_\mu$ around the background value $\bar{A}_\mu$ of the vector field,

$$A_\mu = \bar{A}_\mu + b_\mu.$$ \hfill (3)

We neglect here metric perturbations, which relies on the assumption that $\Lambda \ll M_G$. In conformal time coordinates, the metric and background field are hence given by

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^2), \quad \bar{A}_\mu = a^{-1} \cdot (\bar{A}(\eta), 0, 0, 0).$$ \hfill (4)

To obtain the Lagrangian for the field fluctuations $b^\mu$ we insert the expansion (3) and the background (4) into the action (1). Expanding to quadratic order (free level) in the small perturbations we arrive at

$$\mathcal{L}_b = - \beta_1 \partial_\mu b_\nu \partial^\nu b^\mu - (\beta - \beta_1)(\partial_\mu b^\mu) - 4 \beta (\mathcal{H} b_0 + \mathcal{H}_0 b_0 \partial_\mu b^\mu) + \beta_1 R_{\mu\nu} b^\mu b_\nu + a^2 (\beta_1 R_{\mu\nu} b^\mu b_\nu + a^2 \mathcal{V} b_\mu b^\mu - 2 a^4 V'' b_\mu b^\mu - 2 a^4 V'' b_\mu b^\mu),$$ \hfill (5)

where indexes are raised with the Minkowski metric $\eta^\mu{}^\nu$, $\mathcal{H} = H/a \equiv d \log a / d\eta$, and a prime denotes a derivative with respect to $A_\mu A^\mu$. For arbitrary parameters $\beta_1$ and $\beta$ the determinant of the Hessian matrix is non-zero and the four components of the vector field are dynamical. For $\beta = 0$ or $\beta_1 = 0$ the determinant of the Hessian vanishes, signalling that some of the vector field components are constrained. This is what happens for instance for the gauge invariant Maxwell theory ($\beta = 0$). These two degenerate cases should be analyzed separately. In aether models the fixed norm constrain requires $b^0 = 0$, eliminating one of the four dynamical fields. This will be no longer true for the case in which the aether is unleashed.

To proceed any further it is convenient to decompose the field $b^\mu$ in irreducible representations of the unbroken symmetry group: the Euclidean group in three dimensions $E(3)$. Expanding as usual

$$b_0 \equiv - u \quad \text{and} \quad b_i \equiv \partial_i r + v_i, \quad \text{with} \quad \partial_i v^i = 0,$$ \hfill (6)

1 For more details about the possible viability of the unleashed aether models please see the reference [8].
Table 1. Stability conditions in the vector sector. The checkmark means that the condition is automatically satisfied.

| Vector  | Low $p$ | High $p$ |
|---------|---------|---------|
| Classical | $\beta_1 m_v^2 \geq 0$ ✓ | |
| Quantum  | $\beta_1 \geq 0$ | $\beta_1 \geq 0$ |

develops the two scalar modes $u$ and $r$ from the two vector modes in $\vec{v}$. (Here we are talking about scalar and vector modes in a 3-dimensional sense). The Lagrangian of the vector sector is then given by

$$L_v = -\beta_1 \partial_\mu \vec{v} \cdot \partial^\mu \vec{v} - a^2 m_v^2 \vec{v} \cdot \vec{v},$$

where $m_v^2 = V'' - 3\beta_1 H^2 - (\beta_1 + 6\beta_4) \dot{H}$. (7)

Notice that the quantity $a^2 m_v^2 / \beta_1$ one would naively identify as the mass of the vector excitations is time-dependent. We can nevertheless quantize the theory under the approximation that the frequency of each mode is constant if the relative change in $\omega^2 = \beta_1 \beta_2 + m_v^2 / \beta_1$ during the characteristic time $\Delta \eta \approx \omega^{-1}$ is small [9]. Under this assumption the vector sector is ghost free for $\beta_1 \geq 0$, and free of classical instabilities for $m_v^2 / \beta_1 \geq 0$. These conditions are summarized in Table 1. Note that if $\beta_1 = 0$, the vector sector is not dynamical.

Regarding the scalar sector, it is more convenient to switch to Fourier space. Assuming again that all explicitly time-dependent quantities are constant we arrive at the action

$$S_s = -\int \frac{d^4 p}{(2\pi)^4} \left( \begin{array}{c} u \\ r \end{array} \right)^\dagger D \left( \begin{array}{c} u \\ r \end{array} \right),$$

where the matrix $D$ is given by

$$D = \left( \begin{array}{cc} \beta \omega^2 - \beta_1 p^2 + m_u^2 & -i(\beta - \beta_1) \omega p - 2\beta H p \\ i(\beta - \beta_1) \omega p - 2\beta H p & -\beta_1 \omega^2 + \beta p^2 + m_v^2 \end{array} \right),$$

and we have also defined

$$m_u^2 = 2V'' - V' + (2\beta + 3\beta_1 + 12\beta_4) H^2 - (2\beta - 3\beta_1 - 6\beta_4) \dot{H}.$$ (10)

Here $p \equiv |\vec{p}|$. The inverse of the matrix $D$ is just the field propagator. In order to find the propagating modes we just have to find the values of $\omega^2$ at which its eigenvalues have poles, or, equivalently, the values of $\omega^2$ at which the eigenvalues of $D$ have zeros. Requiring that $\det D$ vanish we thus arrive at the frequencies of the two propagating scalar modes. Furthermore, demanding them to be real guarantees the classical stability of the system. On the other hand, its quantum character will be determined by the residuals at the poles of the eigenvalues of $D^{-1}$. They should be positive in order to avoid ghosts. The conditions obtained for the stability of the scalar sector are summarized in Table 2. Notice that ghosts are inevitable at high energies. However, we are following an effective field theory approach so it should not represent a serious problem for the self-consistency of the model: for appropriate choices of the free parameters it is still possible to avoid the existence of low-momentum instabilities.

Our conclusions above do not apply to the two degenerate limits $\beta = 0$ and $\beta_1 = 0$. For $\beta = 0$ the conditions for the stability of the vector sector coincide with those summarized in Table 1. However, only one scalar mode propagates now. Its stability conditions can be found in Table 3. Notice that some of them are the same as in the vector sector, and that if the
Table 2. Stability conditions in the scalar sector for $\beta_1 \neq 0$ and $\beta \neq 0$. The cross means that the condition cannot be satisfied.

| Scalar $(\beta_1 \neq 0, \beta \neq 0)$ | Low $p$ | High $p$ |
|--------------------------------------|---------|---------|
| Classical $\beta_1 \cdot m_v^2 > 0, \beta \cdot m_u^2 < 0$ | | $(\beta_1^{-1} - \beta^{-1})(m_u^2 + m_v^2) - 4\beta_1^{-1}H^2 > 0$ |
| Quantum $\beta_1 > 0, \beta < 0$ | | X |

Table 3. Stability conditions in the scalar sector for the cases $\beta = 0$ and $\beta_1 = 0$ respectively.

| Scalar | Low $p$ | High $p$ |
|--------|---------|---------|
| Classical $\beta_1 m_v^2 > 0$ | $m_v^2 m_u^2 < 0$ | |
| Quantum $\beta_1 > 0$ | $m_v^2 - m_u^2 > 0$ | |

| Scalar | Low $p$ | High $p$ |
|--------|---------|---------|
| Classical $\beta m_u^2 < 0$ | $m_v^2 (m_u^2 - 4\beta H^2) < 0$ | |
| Quantum $\beta < 0$ | $m_u^2 - m_v^2 - 4\beta H^2 < 0$ | |

theory is stable both in the low and high momentum limits it will be stable for all momenta. If $m_u^2 = 0$ the theory does not have any propagating scalar degrees of freedom, as it happens in the electromagnetic case. For $\beta_1 = 0$ the vector sector is not dynamical. Once again, only one scalar mode propagates; the corresponding stability conditions are given in Table 3. If $m_v^2 = 0$ there are no propagating modes.

As an application of the results presented here, let us briefly discuss the stability conditions obtained for the simplest possible case in which the vector field sits at the minimum of the potential. This is generically possible only in flat spacetime (i.e. $H = 0$, $m_v^2 = 0$ and $m_u^2 = 4\Lambda M^2$). If $\beta, \beta_1 \neq 0$ the spectrum contains two vector and two scalar degrees of freedom, three of them (the two vectors plus a scalar) massless: the Nambu-Goldstone bosons associated with the spontaneous breaking of the three generators of Lorentz boosts. However, at low momenta, the theory is either classically or quantum-mechanically stable, but not both. The only way to escape this conclusion is to choose a well-behaved set of massless modes ($\beta > 0$ and $\beta_1 > 0$) and push the massive mode out the domain of validity of the effective theory (taking $\beta (\Lambda/M)^4 \lesssim 1$). In this sense, the aether unleashed models with $\beta, \beta_1 \neq 0$ can be seen as a high-momentum completion of the Einstein-aether theories. The instability can be also averted by setting $\beta = 0$ or $\beta_1 = 0$. For $\beta = 0, \beta_1 > 0$ two well-behaved massless vector modes propagate. On the other hand, if $\beta_1 = 0$ there are not propagating modes.

References
[1] Will C M and Nordtvedt K J 1972 Conservation laws and preferred frames in relativistic gravity. I. Preferred-frame theories and an extended PPN formalism Astrophys. J. 177 757
[2] Hellings R W and Nordtvedt K 1973 Vector-metric theory of gravity Phys. Rev. D 7 3593
[3] Will C M 1981 Theory And Experiment In Gravitational Physics (UK: Cambridge University Press)
[4] Ford L H 1989 Inflation driven by a vector field Phys. Rev. D 40 967
[5] Armendariz-Picon C 2004 Could dark energy be vector-like? J. Cosmol. Astropart. Phys. JCAP07(2004)007
[6] Jacobson T and Mattingly D 2001 Gravity with a dynamical preferred frame Phys. Rev. D 64 024028
[7] Kostelecky V A and Potting R 2009 Gravity from spontaneous Lorentz violation Phys. Rev. D 79 065018
[8] Armendariz-Picon C and Diez-Tejedor A 2009 Aether Unleashed J. Cosmol. Astropart. Phys. JCAP12(2009)018
[9] Mukhanov V and Winitzki S 2007 Introduction to quantum effects in gravity (UK: Cambridge University Press)