Adaptation of Zohdy’s Method for Controlled-Source Audio-frequency Magnetotelluric (CSAMT) Data Interpretation with Layered Model

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Abstract. Surveys with Controlled-Source Audio-frequency Magnetotellurics (CSAMT) are often performed in scalar mode for practical considerations. In such case, CSAMT data are obtained from electric and magnetic fields oriented along and perpendicular to the traverse line, respectively. With closely spaced sounding stations along a profile, layered or 1D model can be considered as sufficient to obtain an overall view of the subsurface resistivity distribution in the study area. Zohdy’s method that was used to infer resistivity variation with depth from Vertical Electrical Sounding (VES) data is extended for CSAMT data interpretation. An initial model is determined from apparent resistivity sounding data using skin depth principle or its alternative, e.g. Bostick transform. With the number of layers equals to the number of data (in frequency or period), the discrepancy of calculated from observed data can be used to modify iteratively the layer parameters (resistivity and thickness). Preliminary tests using synthetic data associated with simple synthetic models showed satisfactory results. On-going study is still underway to assess smooth resistivity variation with depth and to correlate models from stations along a profile to obtain a quasi-2D resistivity model. Application for CSAMT data from a geothermal prospect is also sought.

1. Introduction

Controlled-Source Audio-frequency Magnetotellurics (CSAMT) uses artificial signal source operating in the 0.1 Hz to 10 kHz frequency band to overcome randomness and low signal-to-noise (S/N) ratio of the natural electromagnetic (EM) field used in MT. In most cases, a grounded electric dipole (2-4 km in length oriented along a predefined x-axis) is employed in CSAMT. A multi-channel receiver records the electric field simultaneously at several dipoles along a profile parallel to the x-axis ($E_x$) and the perpendicular magnetic field ($H_y$). Therefore, only scalar impedance $Z_{xy} = E_x/H_y$ is obtained [1,2]. The distance between CSAMT stations along a profile is limited to around 100-200 m (see Figure 1).

With apparent good lateral resolution, CSAMT data can be interpreted adequately using layered or 1D model. In this paper we adopt Zohdy’s method [3] to infer subsurface resistivity variation with depth from CSAMT data. Zohdy’s method was first proposed for modeling of geo-electrical sounding data [3] and was also successfully applied for MT data [4,5]. With slight modifications for initial model (layers’ thicknesses) and its iterative adjustments, promising results were obtained in terms of synthetic model recovery and data misfits.
Figure 1. CSAMT field set-up with the electric dipole and transmitter (Tx) powered by an electrical power generator (PG). The receiver (R) records data from orthogonal components of electric (Ex) and magnetic (Hy) fields [1].

2. Method
Zohdy’s method is based on successive refinements of an initial layered (1D) model to fit the VES (Vertical Electrical Sounding) data. The sounding data in terms of apparent resistivity variations with electrode spacing (AB/2) are converted to resistivity versus layer’s interface depth representing the initial model. Subsequent modifications of the interface depths and resistivities of the layers are performed until a convergence, i.e. fitting between observed and calculated data, is reached [3]. Adaptation of Zohdy’s method for MT data employs the well-known Bostick transform to obtain the initial model [6]. Similar scheme for model (interface depths and resistivities) adjustments are done to fit the MT sounding data [4,5].

In the original scheme of Zohdy’s method for both VES and MT data, initial interface depths are usually over-estimated. For VES data, the investigation depth is approximately equivalent to AB/5 up to AB/4 rather than to AB/2 [7]. For MT data, the investigation depth represented by the skin depth is also over-estimated since it is an approximation for a homogeneous medium [8]. Therefore, in the first stage of the algorithm interface depths are scaled down by a multiplication with a number smaller than 1, for example with 0.9 which means that the thickness of each layer is reduced by 10%. The multiplication is repeated until the misfit cannot be reduced anymore.

In the second stage, the resistivity of j-th layer at k-th iteration is updated to reduce further the misfit according to the following formula [3],

\[ \rho_{j}^{k+1} = \frac{\rho_{a,j}^{obs}}{\rho_{a,j}^{cal}} \times \rho_{j}^{k} \]  

(1)

where \( \rho_{j} \), with \( cal \) and \( obs \) super-scripts represent j-th apparent resistivity for calculated and observed data respectively, with \( j = 1, 2, \ldots, NL \). The algorithm requires that there is one-to-one correspondence between apparent resistivity data and resistivity of layers, i.e. the number of layers (NL) is equal to the number of the sounding data (ND). Equation (1) states that the resistivity of a layer will be increased if the corresponding calculated datum is smaller than the observed datum, and vice-versa.
Following our previous works on VES [9] and MT data [10] the subsurface interval between minimum \((d_\text{min})\) and maximum \((d_\text{max})\) depths can be discretized into \(NL\) layers with a uniform interval in the logarithmic scale. With such depth partition, we have layers’ thicknesses increasing with depth that represent the decrease of the resolution with depth. The depth partition of the subsurface can be expressed by,

\[
d_j = d_{j-1} \times \exp(\ln(d_{\text{max}} - d_{\text{min}})/NL)
\]

where \(d_j\) is the depth interface of the \(j\)-th layer with \(d_1 = d_{\text{min}}\). Then, the layers’ thicknesses can be calculated as the difference between two consecutive depth interfaces. However, other possibility for discretization of the subsurface does exist to make the depth partition of the model represents the equivalent data interval (e.g. period range for MT or CSAMT data). For that purpose, we choose a minimum thickness \(t_{\text{min}} = t\), as a starting point and increase it with depth with a factor \(t\) slightly greater than 1,

\[
t_{j+1} = t_j \times t_{\text{inc}}
\]

where \(t_j\) is the thickness of the \(j\)-th layer. For CSAMT data in our case \(t_{\text{min}}\) and \(t\) were chosen between 30 to 50 m and 1.05 respectively, that means there is 5% thickness increase with depth. With fixed thicknesses, the required model adjustment is only for resistivity of each layer.

The resistivity modification according to equation (1) is usually unstable resulting oscillations of model and misfit with iterations. Therefore, a damping factor to limit the step size in the resistivity refinement should be introduced. Such strategy can reduce model and misfit oscillations and to speed up convergence. We follow the formulation of Cao et al. [11] such that the resistivity update is as follows,

\[
\rho_j^{k+1} = \rho_j^k + c \left( \frac{\rho_{\text{obs}}}{\rho_{\text{cal}}^{k+1}} - 1 \right) \rho_j^k
\]

where \(c\) is a damping factor less than 1, i.e. between 0.1 to 0.5 for most cases. If \(c = 1\) the resistivity modification is identical to the original Zohdy’s method represented by equation (1).

We apply the modified Zohdy’s method to obtain resistivity variation with depth from CSAMT sounding data. For that purpose, we use the 1D CSAMT forward modelling algorithm of Li et al. [12] that is originally for layered model with azimuthal anisotropy. To simplify the problem, the anisotropy is set to 1, i.e. no anisotropy, and only one component of the resulting impedance tensor \((Z)\) is used to represent the scalar impedance.

### 3. Results with Synthetic Data

The modified Zohdy’s method was applied to invert synthetic data representing H- and K-type sounding curves associated with simple 3-layer synthetic models (Model-1 and Model-2). The model parameters, more particularly the thicknesses were chosen to conform with the period range of typical CSAMT data having shallow to moderate depth investigation (Table 1).

| Layer | Model-1: H-type | Model-2: K-type |
|-------|----------------|----------------|
|       | Resistivity (Ohm.m) | Thickness (m) | Resistivity (Ohm.m) | Thickness (m) |
| 1     | 100            | 200            | 100            | 200            |
| 2     | 10             | 500            | 500            | 500            |
| 3     | 500            | $\infty$       | 10             | $\infty$       |
The synthetic model response was calculated for transmitter-receiver distance of 5 km, at a nominal 5 points per decade between the period of 0.001 to 10 sec. or 4 decades. A relatively small number of CSAMT data, i.e. a total of 21 periods or frequencies, was intended to restrict the number of layers involved and to stabilize the inversion or iterations. To simulate real or field data, 5% Gaussian noise was added to the calculated model response (real and imaginary parts) independently. Ideally, different forward modelling code should be used for creating the synthetic data and for inversion to avoid systematic or cyclic errors. However, with limited availability of the 1D CSAMT forward modelling algorithm, we used the same code of Li et al. [12] for both purposes.

The inversion results for synthetic data are presented in Figure 2 for Model-1 and Model-2. Although a damping factor was used to limit the step size of the model parameter (i.e. resistivity) refinement, resistivity oscillations in the inverse models seem unavoidable. However, resistivity variation with depth from the inverse models recover the synthetic models relatively well. We can observe that the high resistivity contrast or transition is more difficult to recover from the inversion process, i.e. from 10 Ohm.m to 500 Ohm.m in Model-1 and from 500 Ohm.m to 10 Ohm.m in Model-2, both for the resistivity transition from the second layer to the last or third layer (see Figure 2).

The fit between the synthetic and calculated data is satisfactory (Figure 3). The minimum misfits in terms of chi-squared error between the synthetic and calculated data are 1.36 at iteration 65 and 1.68 at iteration 42 for H- and K-type sounding curves, respectively. The misfits as function of iterations for both model-1 and model-2 are presented in Figure 4.

![Figure 2](image_url)

**Figure 2.** Resistivity-depth variations from 1D inversion of synthetic CSAMT data, inverse model (red) compared to synthetic model (dashed): (a) model-1, H-type sounding curve with $t_\text{min} = 30$ m and $t_\text{inc} = 1.05$, (b) model-2, K-type sounding curve with $t_\text{min} = 40$ m and $t_\text{inc} = 1.05$.

### 4. Conclusion

The Zohdy’s method [3] and its alternative algorithms [e.g. 4,5] belong to the class of inversion modelling called the model domain inversion or approximate inverse mapping (AIM) [8]. The modified Zohdy’s method applied to synthetic CSAMT data showed satisfactory results in terms of synthetic model recovery and low misfit between “observed” or synthetic data and model response (between 1.0 to 2.0). Further refinement for smoothing resistivity – depth variation from the obtained model is necessary. Such smooth resistivity variation with depth at CSAMT stations along a profile can be further smoothed and correlated to obtain a quasi-2D resistivity model from CSAMT data. The latter exploits
the apparent high lateral resolution from closely spaced stations commonly performed in scalar CSAMT surveys [13]. In the perspective, the method will be applied to invert field CSAMT data from a geothermal prospect with shallow to moderate depth target for cap-rock delineation.

Figure 3. Comparison of CSAMT synthetic data (blue dots) and inverse model response (red) for: (a) model-1, H-type sounding curve with minimum misfit 1.36 and (b) model-2, K-type sounding curve with minimum misfit 1.68.

Figure 4. Misfit as function of iteration for: (a) model-1, H-type sounding curve with minimum misfit 1.36 at 65-th iteration, (b) model-2, K-type sounding curve with minimum misfit 1.68 at 42-nd iteration.
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