Neutrino Anomalies and 
Quasi-Dirac Neutrinos

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We discuss possibility of describing solar, atmospheric and LSND results 
with four neutrinos forming two quasi-degenerate pairs. The simplest versions 
of this 2 + 2 scheme with either $\nu_e$ or $\nu_\mu$ mixing exclusively with sterile neutrino is disfavored by the SNO and atmospheric neutrino results respectively. 
A generalized scheme with sterile state participating in both the solar and atmospheric oscillations is still allowed. We show that the complex pattern of mixing needed for this purpose follows from a simple $L_e + L_\mu - L_\tau - L_s$ symmetry. Specific form of $L_e + L_\mu - L_\tau - L_s$ symmetric mass matrix is determined from experimental results. Two theoretical schemes which lead to this form and a proper breaking of $L_e + L_\mu - L_\tau - L_s$ symmetry are discussed.

1. INTRODUCTION

Both positive and negative results on neutrino oscillation searches have provided very important clues on possible patterns of neutrino masses and mixing \[.\] These results include experiments detecting solar and atmospheric neutrinos as well as laboratory experiments such as LSND, CHOOZ etc. The observed solar and atmospheric neutrino deficits provide concrete ground to believe in neutrino oscillations. Along with the LSND results, they give important and by now well-known \[\] information on neutrino masses.

The latest input in the analysis of neutrino spectrum is results of solar neutrino deficit
seen at SNO [2]. This experiment finds lower neutrino flux in their charged current events compared to the flux inferred from the elastic scattering at SuperKamioka [3] which receives contributions from the charged as well as the neutral current processes. The difference in these two fluxes is consistent with \( \nu_e \) predominantly converting to active flavours only. The global analysis of the solar neutrino data [4] shows that complete conversion of the solar \( \nu_e \) to sterile neutrino is a disfavored possibility allowed at 3\( \sigma \) level that too mainly in case of the vacuum oscillations only.

The implications of the above results for the neutrino masses are as follows.

- The oscillation interpretation of data requires [1] three different mass scales (\( \Delta_{\text{LSND}} \sim eV^2 \), \( \Delta_A \sim (5 - 8) \cdot 10^{-3} eV^2 \) and \( \Delta_S \leq 10^{-4} eV^2 \)) and four light neutrinos to account for these (mass)\(^2\) differences.

- The schemes with hierarchical masses is highly disfavored from the point of view of explaining all neutrino anomalies [5,6]. In this scheme, the allowed LSND probability is found to be smaller than the observed [7] one when negative results of neutrino oscillation searches at Bugey and CDHS are taken into consideration.

- All the experimental results before SNO could be understood [5] in terms of a simple picture in which four neutrinos group themselves into two pairs with a gap of order \( \sqrt{\Delta_{\text{LSND}}} \). There are two versions:
  
  (a) in which one of the pairs consists of \( \nu_e, \nu_s \) and accounts for the solar neutrino anomaly. The other is responsible for \( \nu_\mu - \nu_\tau \) oscillations of atmospheric neutrinos.
  (b) corresponds to converse possibility with \( \nu_\mu - \nu_s \) accounting for the atmospheric neutrino deficit, solar neutrinos converting themselves completely to active components. Both these possibilities have been termed as 2 + 2 schemes [5,8] of neutrino masses.

The above possibilities can be incorporated naturally into theoretical schemes for neutrino masses and there have been number of models [9–13] realizing these possibilities. The possibility (b) was already disfavored by the observed absence [14] of the matter effects in atmospheric neutrino data. Now the SNO results [4] strongly disfavors (a). Thus it becomes a challenging task to understand experimental results within the four neutrino scenario and build necessary framework to account for this. We discuss simple four neutrino schemes in this paper which can simultaneously explain the solar, atmospheric and LSND results.

While the possibilities (a) and (b) above are the simplest realization of the 2+2 schemes they are not exhaustive. In general, a sterile state can simultaneously but partially influence
both the solar and atmospheric neutrino oscillations. The most general possibility in this context was worked out in [8]. It was shown that the LSND results can be accounted for in 2+2 scheme without conflicting with the negative searches of neutrino oscillations if \( \nu_e \) and \( \nu_\mu \) reside mainly in different mass pairs. The unitarity of mixing matrix then dictates the following mixing pattern among four neutrinos [8]:

\[
\nu_e = \cos \theta_S \nu_1 + \sin \theta_S \nu_2 + \mathcal{O}(\epsilon) , \\
\nu_\mu = \cos \theta_A \nu_3 + \sin \theta_A \nu_4 + \mathcal{O}(\epsilon) , \\
\nu_\tau = \sin \alpha (-\sin \theta_A \nu_3 + \cos \theta_A \nu_4) + \cos \alpha (-\sin \theta_S \nu_1 + \cos \theta_S \nu_2) + \mathcal{O}(\epsilon) , \\
\nu_s = \cos \alpha (-\sin \theta_A \nu_3 + \cos \theta_A \nu_4) - \sin \alpha (-\sin \theta_S \nu_1 + \cos \theta_S \nu_2) + \mathcal{O}(\epsilon) .
\] (1)

The \( \nu_{1,2} \) and \( \nu_{3,4} \) represent two quasi-Dirac pairs of neutrinos with definite masses \( m_\nu_i \) \((i = 1, 4)\). \( \theta_S, \theta_A \) respectively denote mixing angles relevant for the solar and atmospheric neutrino oscillations. The angle \( \alpha \) determines the amount of sterile component in these two fluxes. The splittings \( \Delta_{12}, \Delta_{34} \) \((\Delta_{ij} \equiv m_{\nu_j}^2 - m_{\nu_i}^2)\) respectively account for the solar and atmospheric oscillations. As is seen, the \( \nu_e \) and \( \nu_\mu \) reside in different mass pairs apart from small \( \mathcal{O}(\epsilon) \) corrections. The parameter \( \epsilon \) thus decides the amplitude for the LSND oscillations.

Two examples of pure active sterile mixing discussed above correspond to special cases of eq.(1) with \( \alpha = \pi/2 \) (case (a)) and \( \alpha = 0 \) (case (b)). While these extreme cases are ruled out, intermediate possibility with non-zero \( \alpha \) is still allowed. The phenomenology of this case was studied in [8,15] and was updated after SNO results in [16]. Basically, the same parameter \( \alpha \) determines amount of sterile component in the solar as well as atmospheric neutrino flux and thus can be constrained by both experiments. It was found in [16] that the solar as well as atmospheric data can be fitted with a non-zero \( \alpha \) in two possible ways. Either \( \alpha \) is large in which case the sterile state mainly appears in the solar neutrino flux. The best fit in this case is obtained for

\[
\sin^2 \alpha \approx 0.8 .
\] (2)

The possibility (a) above is allowed in this case only at 99%CL. The other case corresponds to small \( \alpha \) and sterile component mainly in the atmospheric neutrino flux. The best fit value of \( \alpha \) is given in this case by

\[
\sin^2 \alpha \approx 0.1 .
\] (3)

\(^1\)\( \alpha \) defined here differs from the one in [8] by \( \pi/2 \).
The possibility (b) above can be allowed around 95%CL in this case. It follows that complete conversion of \( \nu_e \) to \( \nu_s \) in the solar flux or \( \nu_\mu \) to \( \nu_s \) in the atmospheric flux is a disfavored possibility although best fit value of \( \alpha \) is not very far from these two limiting cases. General mixing pattern in eq.\([1]\) is a more favored possibility. A priori, this mixing pattern looks complex but as we discuss below, it follows if 4X4 neutrino mass matrix respects a \( L_e + L_\mu - L_\tau - L_s \) symmetry. We write down a specific ansatz based on this symmetry in the next section where we also show that \( L_e + L_\mu - L_\tau - L_s \) symmetric mass matrix can be determined from the experimental results. Next two sections are devoted to realization of the mass matrix based on \( L_e + L_\mu - L_\tau - L_s \) symmetry. Last section summarizes the results obtained.

**II. \( L_e + L_\mu - L_\tau - L_s \) SYMMETRY AND NEUTRINO MIXING**

Let us parameterize the most general 4X4 matrix in the basis \( (\nu_e, \nu_\mu, \nu_\tau, \nu_s) \) as follows:

\[
\mathcal{M}_\nu = \begin{pmatrix} \Delta & m \\ m^T & \Delta' \end{pmatrix}
\]

(4)

Here each sub-block is a 2X2 matrix. The existence of pseudo-Dirac pairs implies some partially broken \( U(1) \) symmetry. Such symmetry follows if 2X2 matrices \( \Delta, \Delta' \) are sub-dominant compared to \( m \) in eq.\([4]\). Let us choose \( \Delta = \Delta' = 0 \) as a first approximation. Then for arbitrary \( m \), \( \mathcal{M}_\nu \) is invariant under \( L_e + L_\mu - L_\tau - L_s \) symmetry. It is possible to exactly diagonalize eq.\([4]\) in this case. This is done in two steps. First, \( m \) is diagonalized by the following 2X2 rotations \( R \):

\[
R \beta \ m \ R_\alpha^T = \text{Diag.}(m_1, m_2)
\]

(5)

where \( \alpha, \beta \) denote the angles of rotations and \( m_{1,2} \) are the eigenvalues of \( m \). Given eq.\([5]\), the following 4X4 matrix diagonalizes \( \mathcal{M}_\nu \) when \( \Delta, \Delta' \) are zero:

\[
R' R \mathcal{M}_\nu R R^T = \text{Diag.}(m_1, -m_1, m_2, -m_2)
\]

(6)

where,

\[
R = \begin{pmatrix} R_\beta & 0 \\ 0 & R_\alpha \end{pmatrix}
\]

(7)
and
\[ R' = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}. \] (8)

Eqs. (7,8) together imply the following mixing among neutrinos:
\[ \nu_\alpha \equiv U_{\alpha i} \nu_i = (R' R')^T_{\alpha i} \nu_i. \] (9)

Explicitly,
\[ \nu_e = \frac{c_\beta}{\sqrt{2}} (\nu_1 + \nu_2) + \frac{s_\beta}{\sqrt{2}} (\nu_3 + \nu_4), \]
\[ \nu_\mu = -\frac{s_\beta}{\sqrt{2}} (\nu_1 + \nu_2) + \frac{c_\beta}{\sqrt{2}} (\nu_3 + \nu_4), \]
\[ \nu_\tau = \frac{c_\alpha}{\sqrt{2}} (\nu_1 - \nu_2) + \frac{s_\alpha}{\sqrt{2}} (\nu_3 - \nu_4), \]
\[ \nu_s = -\frac{s_\alpha}{\sqrt{2}} (\nu_1 - \nu_2) + \frac{c_\alpha}{\sqrt{2}} (\nu_3 - \nu_4). \] (10)

The above pattern coincides with eq.(1) if \( s_\beta \ll 1 \) and \( \theta_A = \theta_S = \pi/4 \). It is easily seen that \( \beta \) governs the amplitude of LSND oscillations. Eq.(10) implies
\[ \sin^2 2\theta_{LSND} \approx 4(U_{e4} U_{\mu4} + U_{e3} U_{\mu3})^2 = \sin^2 2\beta \]
when \( m_{\nu_{1,2}} \ll m_{\nu_{3,4}} \sim \text{eV} \). The observations at LSND then implies that \( \sin^2 2\beta \sim 3 \cdot 10^{-3} \). \( \theta_A = \theta_S = \pi/4 \) is a prediction of the model which arise as a consequence of the \( L_e + L_\mu - L_\tau - L_s \) symmetry. The perturbation \( \Delta, \Delta' \) which would cause the splitting of the neutrino masses would also change this prediction somewhat but one will get two large mixing angles needed on phenomenological grounds.

Eq.(10) leads to the following survival probability in disappearance experiment such as CHOOZ [17]
\[ P_{ee} = 1 - 4U_{e3} U_{e4}^2 \sin^2 \frac{\Delta_{34} t}{4E} - 2(U_{e1}^2 + U_{e2}^2)(U_{e3}^2 + U_{e4}^2) \]
\[ \approx 1 - \frac{s_\beta^2}{2} \frac{\Delta_{34} t}{4E} \sin^2 2\beta . \] (11)

This probability is correlated with the LSND angle. Its oscillatory part is suppressed in view of the LSND results on \( s_\beta \). The average term is smaller than the present limit set by CHOOZ [17] experiment but it can be significant.
It is clear that the $L_e + L_\mu - L_\tau - L_s$ symmetric mass matrix leads to phenomenologically desirable pattern naturally. Its breaking is needed to generate the mass splittings. Details depend upon the form and strength of the perturbation $\Delta, \Delta'$ which is yet unspecified. But the following observation is relevant. One can calculate the (mass)$^2$ differences using perturbation theory assuming that $\Delta, \Delta'$ have general structure with all elements having equal strength. Using the unperturbed eigenfunctions following from $U$ in eq.(9), we get

$$
\Delta_A \equiv (m_{\nu_4}^2 - m_{\nu_3}^2) \approx m_2 \mathcal{O}(\delta)
$$

$$
\Delta_S \equiv (m_{\nu_2}^2 - m_{\nu_1}^2) \approx m_1 \mathcal{O}(\delta) .
$$

(12)

It follows that

$$
\frac{\Delta_S}{\Delta_A} \approx \frac{m_1}{m_2} \mathcal{O}(1) .
$$

(13)

Thus, hierarchy in the solar and atmospheric scales is linked in this case to intra-splitting between two pairs and one generically needs $m_1 \ll m_2$. We shall present an example where eq.(13) is realized naturally with $\mathcal{O}(1)$ parameter being exactly 1. $\mathcal{M}_\nu$ is characterized by four parameters $\alpha, \beta, m_1$ and $m_2$. All these parameters are approximately determined phenomenologically: $m_2$ and $\beta$ from LSND results, $\alpha$ from general fit [16] to solar and atmospheric data and $m_1$ from eq.(13). Thus $\mathcal{M}_\nu$ gets phenomenologically determined in the symmetric limit.

Let us parameterize $\mathbf{m}$ as follows:

$$
\mathbf{m} = \begin{pmatrix}
a_1 & a_2 \\
A_1 & A_2
\end{pmatrix} .
$$

(14)

The elements $a_i$ and $A_i$ ($i = 1, 2$) are determined in terms of mixing and masses as follows:

$$
A_2 = m_2 c_\beta c_\alpha + m_1 s_\beta s_\alpha \approx m_2 c_\alpha ,
$$

$$
A_1 = m_2 s_\alpha c_\beta - m_1 s_\beta c_\alpha \approx m_2 s_\alpha ,
$$

$$
a_2 = m_2 c_\alpha s_\beta - m_1 s_\alpha c_\beta ,
$$

$$
a_1 = m_2 s_\beta s_\alpha + m_1 c_\beta c_\alpha .
$$

(15)

The parameters on the RHS are directly determined from experiments. Since both $s_\beta$ and $m_1$ are required to be small, the above equations imply that

$$
\frac{a_1}{A_1} \sim \frac{a_2}{A_2} \ll 1
$$

(16)
and also
\[
\frac{a_1}{A_1} - \frac{a_2}{A_2} \approx \mathcal{O}(1) \frac{\Delta_S}{s_\alpha c_\alpha \Delta_A},
\] (17)

which itself is small. Successful model should realize the above hierarchy and we present
two specific examples.

III. RADIATIVE SCHEME

We showed that \( L_e + L_\mu - L_\tau - L_s \) symmetry leads to phenomenologically consistent
2+2 model and identified a specific structure for the four neutrino mass matrix in this limit.
We now discuss a radiative scheme which realizes this structure and also provides necessary
breaking of the \( L_e + L_\mu - L_\tau - L_s \) symmetry. Our starting point is the observation that
a part of \( L_e + L_\mu - L_\tau - L_s \) symmetry namely, \( L_e + L_\mu - L_\tau \) already arise as an accidental and
approximate symmetry in the Zee model \cite{18} of neutrino masses. Thus it is natural to start
with this model. We add a singlet neutrino \( \nu_s \) to it and impose an associated singlet lepton
number symmetry \( U(1)_S \) which is carried by \( \nu_s \), rest of the fermions remaining unchanged
under it. Due to inherent \( L_e + L_\mu - L_\tau \) symmetry of the Zee model, the radiatively generated
\( 4 \times 4 \) matrix automatically displays approximate \( L_e + L_\mu - L_\tau - L_s \) symmetry although we
start with only a \( U(1)_S \) symmetry. The Higgs fields of the model are two doublets \( \phi_{1,2} \) and
a charged singlet \( h_1^+ \) as in Zee model \cite{18} and an additional charged singlet \( h_2^+ \) carrying the
same \( U(1)_S \) charge as \( \nu_s \). The leptonic Yukawa couplings in the model are then given by
\[
- \mathcal{L}_Y = f_{ij} \bar{\ell}_i^L \ell_j L h_1^+ + \beta_i \bar{\nu}_s e_i R h_2^+ + g_{ij} \bar{\ell}_i L e_j R \tilde{\phi}_1 + \text{H.c.}. \] (18)

Note that the \( U(1)_S \) symmetry forbids the Dirac coupling between \( \ell_L \) and \( \nu_s \) as well as
the Majorana mass for the \( \nu_s \). Above Yukawa couplings are automatically invariant under
Lepton number as in Zee model. Lepton number as well as \( U(1)_S \) symmetry is broken softly
in Higgs sector through the following terms:
\[
\mu^2 h_1^+ h_2^- + \gamma_i \phi_1 \phi_2 h_i^+. \] (19)

These soft symmetry breaking terms radiatively generate neutrino masses. The mass matrix
generated at the 1-loop level has the following structure:
\[ M_\nu = \begin{pmatrix} 0 & \delta & a_1 & a_2 \\ \delta & 0 & A_1 & A_2 \\ a_1 & A_1 & 0 & \delta' \\ a_2 & A_2 & \delta' & 0 \end{pmatrix}, \quad (20) \]

where

\begin{align*}
a_1 & \sim \frac{C}{v_1} f_{e\tau} m_\tau^2 ; \\
A_1 & \sim \frac{C'}{v_1} f_{\mu\tau} (m_\tau^2 - m_\mu^2) , \\
a_2 & \sim C' (f_{e\tau} \beta m_\tau + f_{e\mu} \beta m_\mu) ; \\
A_2 & \sim C' f_{\mu\tau} \beta m_\mu , \\
\delta & \sim \frac{C}{v_1} f_{e\mu} m_\tau^2 ; \\
\delta' & \sim C' f_{\tau\mu} \beta m_\mu , \quad (21) \end{align*}

where we have neglected the electron mass. \( C \) and \( C' \) in the above equations are given by

\[ C \sim \frac{1}{16\pi^2} f(m_h) ; \quad C' \sim \frac{1}{16\pi^2} g(m_h) \quad (22) \]

\( f \) and \( g \) are dimensionless functions of parameters in Higgs potential (denoted collectively by \( m_h \)) including the soft symmetry breaking terms displayed in eq.(19).

Note that the \( \delta, \delta' \) are determined by the muon mass and rest by the tau as well as muon mass. Thus, eq.(21) has the perturbative structure required in our ansatz, eq.(1). One can calculate splitting among neutrino masses generated by \( M_\nu \) in eq.(20). Treating \( \delta, \delta' \) as perturbation and using the unperturbed eigenfunctions given in eq.(10), we get

\begin{align*}
m_{\nu_1} & \sim m_1 - (\delta c_\beta s_\beta + \delta' c_\alpha s_\alpha) , \\
m_{\nu_2} & \sim -m_1 - (\delta c_\beta s_\beta + \delta' c_\alpha s_\alpha) , \\
m_{\nu_3} & \sim m_2 + (\delta c_\beta s_\beta + \delta' c_\alpha s_\alpha) , \\
m_{\nu_4} & \sim -m_2 + (\delta c_\beta s_\beta + \delta' c_\alpha s_\alpha) . \quad (23) \end{align*}

This leads to the following splittings:

\begin{align*}
\Delta_A & \approx -4m_2 (\delta c_\beta s_\beta + \delta' c_\alpha s_\alpha) , \\
\frac{\Delta_S}{\Delta_A} & \approx -\frac{m_1}{m_2} . \quad (24) \end{align*}

The hierarchy in the solar and atmospheric scale is insensitive to strength of perturbation and is solely determined by the ratio of masses of the Dirac pairs. Eq.(21) leads to

\[ \frac{a_1}{A_1} \approx \frac{a_2}{A_2} \approx \frac{f_{e\tau}}{f_{\mu\tau}} . \]
The required hierarchy in eq. (16) can be obtained if \( f_{e\tau} \) and \( f_{\mu\tau} \) are hierarchical. Eq. (17) now assumes the following form

\[
\left( \frac{a_1}{A_1} - \frac{a_2}{A_2} \right) \sim \frac{f_{e\tau}}{f_{\mu\tau}} \frac{m^2_\mu}{m^2_\tau} - \frac{\beta_\mu}{\beta_\tau} \frac{f_{e\mu}}{f_{\mu\tau}} \frac{m_\mu}{m_\tau} .
\]

The difference on the LHS can thus be naturally small. It is possible to determine the basic parameters \( f_{e\mu}, f_{e\tau}, f_{\mu\tau} \) and \( \beta_{\mu,\tau} \) directly using eqs. (15, 24). Little algebra gives,

\[
\beta_\tau \approx \frac{C m_\tau c_\alpha}{m_1} ; \quad \frac{\beta_\mu}{\beta_\tau} \approx -\frac{\Delta_A m_\tau}{m_\mu^2} \frac{1}{m_\mu s_\alpha c_\alpha} ;
\]

\[
f_{\mu\tau} \approx \frac{m_2 v_1 s_\alpha}{C} ; \quad \frac{f_{e\tau}}{f_{\mu\tau}} \approx s_\beta + \frac{\Delta_S c_\alpha}{\Delta_A s_\alpha} ;
\]

\[
f_{e\mu} \approx \frac{\Delta_S m_2^2}{\Delta_A \Delta_A c_\alpha} .
\]

Eqs. (15, 24) reveal that the required magnitudes of \( a_{1,2} \) may be comparable to the perturbation \( \delta, \delta' \). In this case, some of the perturbative results may change. It is therefore appropriate to perform exact diagonalization of matrix in eq. (20). We have done this numerically choosing values of parameters around the ones given in eq. (26). The specific values chosen are

\[
\beta_\mu = -4.3 \cdot 10^{-3} ; \quad \beta_\tau = 2.7 \cdot 10^{-2}
\]

\[
f_{e\mu} = 2.1 \cdot 10^{-5} ; \quad f_{e\tau} = 3.6 \cdot 10^{-7} ; \quad f_{\mu\tau} = 8.6 \cdot 10^{-6} .
\]

We chose \( C \sim C' \sim 0.01 \) for definiteness. The above choice when substituted in eq. (20) leads to the following values for the solar and atmospheric scales:

\[
\Delta_S = 2.3 \cdot 10^{-5} \text{eV}^2 , \quad \Delta_A = 3.1 \cdot 10^{-3} \text{eV}^2 , \quad \Delta_{LSND} = 0.3 \text{eV}^2 .
\]

The mixing matrix is given by

\[
U^T \approx \begin{pmatrix}
0.826 & 0.57 & 0.02 & 0.017 \\
-0.024 & -0.011 & 0.71 & 0.71 \\
0.52 & 0 & -0.75 & 0.24 & -0.28 \\
-0.23 & 0.33 & 0.64 & -0.65 \\
\end{pmatrix} .
\]
This can be seen to correspond to the following values for various parameters:

\[
\begin{align*}
\sin^2 2\theta_A &= 0.99; & \sin^2 2\theta_S &= 0.88 \\
\sin^2 2\theta_{LSND} &= 2.7 \cdot 10^{-3}; & \sin^2 2\theta_{\text{chooz}} &= 1.4 \cdot 10^{-3} \\
\end{align*}
\]

\( s^2_\alpha \approx 0.15 \)  

The solar mixing angle is reduced significantly compared to its maximal value in the absence of perturbation. This is welcome since strictly maximal mixing is not favored at least in the two generation analysis of the solar data \[4\]. Clearly, there would be ranges in the basic parameters of the model which would reproduce the correct mixing and masses.

We note that the structure of the mass matrix in eq.\((20)\) coincides with the one discussed in \[13\] but the underlying model presented here is much simpler. More importantly, phenomenological emphasis here is very different. We have shown that the basic structure in eq.\((20)\) displaying approximate \(L_e + L_\mu - L_\tau - L_s\) symmetry provides a concrete realization of the generalized \(2 + 2\) model which is fully consistent with all the neutrino anomalies even after inclusion of SNO results.

**IV. SEESAW MODEL**

We now discuss how the ansatz of section (2) can be derived in seesaw type scheme. There are two ways of obtaining a light sterile neutrino in seesaw model. One is to assume that the mass matrix of the right handed neutrino is singular \[19\]. This can be done through some symmetry \[20\]. The massless RH neutrino resulting from this singular matrix picks up a mass through its Dirac coupling with the active neutrino. The RH neutrino remains strictly massless and can provide the sterile state if its Dirac coupling is also forbidden by a symmetry. Example of this was recently presented in \[21\]. Alternative possibility is to add a sterile state to the conventional seesaw picture and impose symmetry which keeps it light. Examples of this possibility were discussed in \[10,12\]. Consider an active state \(\nu_L\), its RH partner \(\nu_R\) and a sterile (left-handed) state \(\nu_s\) with the following mass matrix in the basis \((\nu_L, \nu_s, \nu_R)\):

\[
\begin{pmatrix}
0 & 0 & m_D \\
0 & 0 & m_S \\
m_D & m_S & M
\end{pmatrix}
\]  

\(31\)
This mass matrix leads to a massless, a light \((\sim \frac{m_2^2 + m_S^2}{M})\) and a heavy \((\sim M)\) neutrino. The sterile state mixes with the active state and influences the phenomenology. Crucial points to note are the absence of Dirac mass term between \(\nu_s\) and \(\nu_L\) and the absence of the Majorana mass term for \(\nu_s\). This can be achieved by means of some symmetry, e.g. \(R\) symmetry as in \([10]\). This matrix was proposed as a model for solving the solar neutrino anomaly through \(\nu_e - \nu_s\) mixing. Now we generalize the above idea to obtain the ansatz discussed in the last section.

We consider the conventional seesaw picture with three left-handed and three right-handed neutrinos and add to it a sterile state \(\nu_s\) which would remain light. The lightness can be ensured by a structure which is generalization of eq.\((31)\) to three generations. We demand separate conservation of \(L_e + L_\mu - L_\tau\) and the lepton number \(L_s\) corresponding to sterile state \(\nu_s\). This would lead to \(L_e + L_\mu - L_\tau - L_s\) symmetric 4X4 matrix of the ansatz, eq.\((3)\). We however need to break this symmetry softly in order to obtain realistic mass spectrum. Our model thus has two sets of mass terms: \(\mathcal{L}_m\) which respect the symmetry and \(\mathcal{L}_m'\) which break it softly. The symmetric part is given by

\[
-\mathcal{L}_m = m_{ee}\bar{\nu}_e\nu_{eR} + m_{e\mu}\bar{\nu}_e\nu_{\mu R} + m_{\mu\mu}\bar{\nu}_\mu\nu_{\mu R} + m_{\tau\tau}\bar{\nu}_\tau\nu_{\tau R} + \text{H.c.} \\
+ \frac{1}{2}(M_{ee}\bar{\nu}_e\nu_{eR} + M_{e\tau}\bar{\nu}_e\nu_{\tau R} + M_{\mu\tau}\bar{\nu}_\mu\nu_{\tau R} + M_{\mu\mu}\bar{\nu}_\mu\nu_{\mu R}) .
\]  

Due to the combined effects of two \(U(1)\) symmetries, the \(\nu_s\) remains massless and decoupled from rest of the fermions in eq.\((32)\). Its couplings with active neutrinos are induced entirely by the soft symmetry breaking sector \(\mathcal{L}_m'\) chosen as follows:

\[
-\mathcal{L}_m' = p_e\bar{\nu}_s\nu_{eR} + p_{\mu}\bar{\nu}_s\nu_{\mu R} + p_{\tau}\bar{\nu}_s\nu_{\tau R} + \text{H.c.} \\
+ \frac{1}{2}M_{ee}\bar{\nu}_e\nu_{eR} .
\]  

Note that all terms in the above equation connect only the sterile states and are soft in the technical sense- they would not lead to any divergences.\(^2\) Arbitrary choice of soft terms is consistent with this technical requirement. We have added all possible singlet mass terms above except for the direct mass term \(\bar{\nu}_s\nu_s\) and some additional mass terms among the RH neutrinos. Addition of the latter terms do not make any qualitative change. The omission of \(\bar{\nu}_s\nu_s\) can be justified if soft terms are assumed to come through normalizable couplings of

\(^2\) Models using such soft symmetry breaking were proposed in \([22]\).
sterile state with standard model singlets $\eta_1, \eta_2, \eta$ carrying the $(L_e + L_\mu - L_\tau, L_s)$ charges $(-1, 1), (1, 1)$ and $(-2, 0)$ respectively. These fields cannot couple to $\bar{\nu}_s\nu_s$ but would lead to rest of the soft terms displayed in eq.(33). In the following, we assume that Dirac masses $p_{e,\mu,\tau}$ are smaller than the scale in $M_R$. This may be achieved through $R$ symmetry as in [10]. Here we simply make this choice which is stable against radiative corrections.

The effective 4X4 matrix emerging after seesaw mechanism can be written in the form of eq.(4) with

$$m = \frac{1}{M_{\mu\tau}} \begin{pmatrix} m_{e\mu}m_{\tau\tau} & m_{e\mu}(p_\tau + p_\mu z + p_\mu z^2/y) + m_{ee}(p_e y + p_\mu z) \\ m_{\mu\mu}m_{\tau\tau} & m_{\mu\mu}(p_\tau + p_\mu z + p_\mu z^2/y) + m_{\mu e}(p_e y + p_\mu z) \end{pmatrix}$$

$$\Delta = \frac{1}{y M_{\mu\tau}} \begin{pmatrix} (m_{ee}y + m_{e\mu}z)^2 & (m_{ee}y + m_{e\mu}z)(m_{\mu e}y + m_{\mu\mu}z) \\ (m_{ee}y + m_{e\mu}z)(m_{\mu e}y + m_{\mu\mu}z) & (m_{\mu e}y + m_{\mu\mu}z)^2 \end{pmatrix}$$

$$\Delta' = \frac{1}{M_{\mu\tau}} \begin{pmatrix} 0 & m_{\tau\tau}p_\mu \\ m_{\tau\tau}p_\mu & 2p_\tau p_\mu + \left(\frac{p_e y + p_\mu z}{y}\right)^2 \end{pmatrix}$$

Here, $y = \frac{M_{e\tau}}{M_{ee}}$ and $z = \frac{M_{ee}}{M_{e\tau}}$.

Comparing eq.(14) with eq.(34) we find that

$$\frac{a_1}{A_1} \approx \frac{a_2}{A_2} \approx \frac{m_{e\mu}}{m_{\mu\mu}} \ll 1$$

provided the elements of the Dirac neutrino mass matrix obey the hierarchy

$$m_{ee} \ll m_{e\mu} \sim m_{\mu e} \ll m_{\mu\mu}$$

This hierarchy is a natural assumption in many seesaw models. It would follow, for example, if the Dirac mass matrix has Fritzch type structure. This structure also leads to

$$\frac{a_1}{A_1} - \frac{a_2}{A_2} \sim \frac{m_{e\mu}m_{\mu e}}{m_{\mu\mu}^2} \frac{p_e y + p_\mu z}{p_\tau + p_e z + p_\mu z^2/y}$$

This hierarchy is close to the one required on the phenomenological grounds, see eqs.(16,17). Thus basic $L_e + L_\mu - L_\tau - L_s$ symmetric ansatz is reproduced in this model under natural assumptions.

The symmetry breaking parameters $p_{e,\mu,\tau}$ are still arbitrary. While many choices are possible, let us identify a particularly simple one. This corresponds to choosing $z, p_\mu$ much
less than the rest. The matrices $\Delta, \Delta'$ of eqs. (35,36) assume a simple form in the limit $m_{ee}, p_{\mu}, z$ tending to zero:

$$\Delta = \frac{y}{M_{\mu\tau}} \begin{pmatrix} 0 & 0 \\ 0 & m^2_{\mu e} \end{pmatrix} ; \quad \Delta' = \frac{y}{M_{\mu\tau}} \begin{pmatrix} 0 & 0 \\ 0 & p^2_e \end{pmatrix}. \quad (39)$$

Treating non-zero elements in this matrix as perturbation we can evaluate the mass splitting which turns out to be

$$\Delta_S \approx -2m_1 \frac{y}{M_{\mu\tau}} (s^2_\beta p^2_e + s^2_\alpha m^2_{\mu e})$$

$$\Delta_A \approx -2m_2 \frac{y}{M_{\mu\tau}} (c^2_\beta p^2_e + c^2_\alpha m^2_{\mu e})$$

$$\frac{\Delta_S}{\Delta_A} \approx \frac{m_1 s^2_\beta p^2_e + s^2_\alpha m^2_{\mu e}}{m_2 c^2_\beta p^2_e + c^2_\alpha m^2_{\mu e}}. \quad (40)$$

This can be consistent with the phenomenological requirement for proper choice of parameters.

The above exercise though illustrative in nature, shows that it is indeed possible to integrate a singlet neutrino in seesaw picture in a way that leads to generalize 2 + 2 model and particular ansatz discussed in section(2). Gross features of the ansatz, particularly the required $L_e + L_\mu - L_\tau - L_s$ symmetric matrix $m$ follows under the standard seesaw assumptions. The full matrix with a broken $L_e + L_\mu - L_\tau - L_s$ symmetry is complex but proper symmetry breaking can be achieved due to large number of parameters in the model.

**V. SUMMARY**

The three sets of experimental results namely, the observed solar and atmospheric neutrino fluxes as well as probable oscillations seen at LSND calls for a consistent theoretical explanation. 2 + 2 schemes with four neutrinos were considered an attractive mechanism to explain all anomalies. In these schemes, the $\nu_e$ or $\nu_\mu$ was exclusively assumed to convert to a sterile state. This scenario is not supported by the results of the solar neutrino experiment at SNO. Simple generalization which can still explain all the experimental results assumes that sterile neutrino flux is simultaneously but partially present in the solar as well as atmospheric neutrino fluxes. We have shown that this possibility follows naturally when neutrino mass matrix is $L_e + L_\mu - L_\tau - L_s$ symmetric. Assumption of this symmetry in fact allows us to completely reconstruct neutrino mass matrix from the experimental results. We have undertaken this exercise in this paper. We have also shown that resulting structure follows
naturally in two of the conventional schemes based on Zee model and the seesaw model for neutrino masses.

Note: Before posting this paper, we noticed a preprint (hep-ph/0110243) by K.S. Babu and R. N. Mohapatra who also advocate the use of $L_e + L_\mu - L_\tau - L_s$ symmetry as an explanation of the neutrino anomalies. Model presented in this paper is different from the models considered here.

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