Research Article

5-DOF Dynamic Modeling of Rolling Bearing with Local Defect considering Comprehensive Stiffness under Isothermal Elastohydrodynamic Lubrication

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The sliding of the rolling element in the load zone would cause the bearing’s wear and failure at high speed under elastohydrodynamic lubrication (EHL) condition. Aiming at this phenomenon, considering lubrication oil film, time-varying displacement, radial clearance, and comprehensive stiffness, a five degree-of-freedom (DOF) dynamic model of rolling bearing with local defect is proposed based on isothermal EHL and which is validated by experimental data. The variation of oil film stiffness, comprehensive stiffness, and vibration characteristics of rolling bearing is studied under different speeds and loads. The results show that the lubricating oils with different viscosities have a certain influence on the bearing oil film thickness and comprehensive stiffness. As the load increases, the oil film stiffness and comprehensive stiffness would increase, and the oil film thickness would decrease. And as the tangential speed increases, the oil film stiffness would increase, and the oil film thickness and comprehensive stiffness would decrease. The vibration amplitude of the rolling bearing is enhanced with the increase of the rotation speed and the radial load. This model is helpful for the optimization, the correct use of lubricants, and life prediction of rolling bearing.

1. Introduction

Rolling bearing is widely used to support rotating machinery due to its lower frictional resistance and higher mechanical efficiency. The presence of local defects on the surface significantly reduces the life of the rolling bearing and even affects the reliability of the device [1–3]. Therefore, the establishment of a dynamic model of rolling bearing with local defect is helpful for the design, fault diagnosis, and life prediction of rolling bearing. Many scholars have made many efforts on dynamic modeling methods [4–7]. Singh et al. [4], Shah and Patel [5], and Liu and Shao [6] comprehensively reviewed the dynamic modeling and analysis methods used to simulate the vibration characteristics of rolling bearings.

By combining FEM and the kurtosis analysis method, Kiral and Karagülle [8] investigated the dynamic response of rolling bearings under an unbalanced force. Considering the changes of time-varying displacement impulse and contact stiffness, a new defect model was presented by Liu et al. [9], and the effects of different defect sizes on the vibrations of the bearing were studied. Then, a new dynamic model of ball bearing with local defect was proposed by Liu and Shao [10] in terms of edge topography, which was used to identify and estimate the size of defects in the bearing system by different signals. A nonlinear dynamic model of roller bearings with wide-ranging defects was established by Moazen Ahmadi et al. [11] with taking the finite size of roller and damping force into consideration. A model of ball bearing considering Hertz theory was presented by Kong et al. [12], to simulate
the contact force and analyze the mechanism of vibration response in the defect region. Cui et al. [13] established a nonlinear vibration model for fault severity assessment of rolling bearing. A novel model was also proposed for quantitative and positioning estimation for bearing fault in the outer ring by Cui et al. [14], which aimed to investigate the defects of different angular positions and sizes. Considering the shaft, cage, raceway, and ball mass, a 2-DOF dynamic model was established by Patel et al. [15], who studied the vibration responses of single fault and double faults on the raceway surface of rolling bearings. The gears and defective bearings in the gearbox were modeled, which considered rolling elements’ slip, Hertz contact, and time-varying stiffness [16, 17]. Niu et al. [18] presented a three-dimensional dynamic model of an angular contact ball bearing with a defective rolling element. An improved three-dimensional dynamic model with geometric relationship is proposed by Jiang et al. [19]. In this model, the changes of contact force and vibration characteristics of the bearing system were investigated.

The study of the effects about EHL on dynamic of rolling bearings was summarized, and Wijnant et al. [20] developed a computational model for investigating EHL and structural dynamics. Since the drag effect of EHL has not been considered in the quasi-static analysis model, a new bearing dynamic model which considered the effects of EHL and inertial force was proposed by Harris and Crecelius [21]. Wagner et al. [22] presented a full dynamic model to obtain accurate stiffness and deflection of rotor-bearing system. Liu and Shao [23] established a dynamic model to analyze the vibration response of the bearing, in which time-varying deflection excitation, time-varying contact stiffness excitation, and lubricating oil film were considered. Yan et al. [24, 25] investigated the response mechanism of the defective bearing system under EHL condition by establishing a 2-DOF dynamic model. Based on the Reynolds equations, a new computational fluid dynamics method for modeling elastohydrodynamic contacts was proposed, in which fluid-solid interaction and elastic deformation of the solid as well as thermal effects were taken into account [26]. Three different types of models were used to simulate and test the different prefabricated faulty bearings [27]. Polyakov [28] approached the calculation and analysis method of fluid-film and stiffness of bearings. A nonlinear dynamic model of a rotor-bearing system with lubrication was established, and the stiffness and damping coefficients of lubricated contacts were studied [29]. The gyroscope of rolling-rotor system, the composite stiffness, and nonlinear contact force under elastohydrodynamic lubrication were studied, and minimum oil film thickness was calculated [30]. Bizarre et al. [31] calculated the force on an angular contact ball bearing with 5-DOF model, accounted for the effect of the EHL of the rolling element, and provided a comprehensive nonlinear model for evaluating the equivalent parameters of stiffness and damping under different loading conditions.

These researches mentioned above help to reveal the vibration characteristics of rotor-bearing systems caused by local defects. To our best knowledge, however, the phenomenon of local defect during a rolling element passing over the raceway surface is rarely studied under lubrication conditions. Thus, the purpose of this article is to reveal the comprehensive stiffness of lubrication oil film under different radial load or different velocity. The changes of stiffness under different parameters of lubrication oil film are researched, the vibration characteristics when the rolling element passes through the defect of the raceway surface during isothermal EHL are explored, and the influence of the amplitude of vibration in the faulty bearing is analyzed. It can be concluded that the changes of vibration characteristic of the rolling bearing are affected by different speeds and loads.

2. Establishment of Dynamic Model

Based on the EHL theory, considering time-varying displacement excitation, radial clearance, and lubrication oil film, a 5-DOF dynamic model of shaft-bearing-housing system is proposed. This model is based on the following assumptions:

(i) The outer ring is fixed, and the inner ring rotates with shaft.
(ii) The contact between mating surfaces (i.e., the ball and the inner ring or the outer race) is considered as a point Hertz contact, which ignores the force of volume, such as gravity.
(iii) All translational motions are limited to the two-dimensional plane (x-y plane) and the rolling element rotates around z-axis.
(iv) The lubrication oil is a Newtonian fluid which is incompressible; the viscosity value is constant along the thickness of the lubrication oil film.
(v) Thermal effects are ignored based on isothermal consideration.
(vi) The variation pressure of lubrication oil film in the vertical direction is ignored.

A technical flowchart for the dynamic modeling of rolling bearing is shown in Figure 1. It contains the internal and external factors of dynamic model, and the calculation of damping and stiffness of bearing. Finally, the dynamic model is established by Newton’s second law.

2.1. Calculation of Contact Stiffness between Rolling Element and Raceways. The contact stiffness of the rolling elements with inner and outer raceways is caused by the load of rolling bearing. The calculation of the contact stiffness is defined as follows [32]:

\[
\begin{align*}
    k_{in-re} & = \frac{2 \sqrt{2} \left( E/1 - \mu^2 \right)}{3 \sum \rho_m} \left( \frac{1}{\delta_{in}} \right)^{3/2}, \\
    k_{ou-re} & = \frac{2 \sqrt{2} \left( E/1 - \mu^2 \right)}{3 \sum \rho_{ou}} \left( \frac{1}{\delta_{ou}} \right)^{3/2},
\end{align*}
\]

where \( \mu \) is the Poisson ratio; \( k_{in-re} \) and \( k_{ou-re} \) are the contact stiffness between the rolling element and inner raceway and
outer raceway, respectively; \( \sum \rho _{in} \) and \( \delta _{in}^* \) are the curvature and contact displacement of the inner raceway, respectively; and \( \sum \rho _{ou} \) and \( \delta _{ou}^* \) are the curvature and contact displacement of outer raceway, respectively.

2.2. Equations of Isothermal Point Contact EHL. The basic equations of isothermal point contact EHL are as follows [33].

The Reynolds equation is as follows:

\[
\frac{\partial}{\partial x} \left( \frac{p h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{p h^3}{\eta} \frac{\partial p}{\partial y} \right) = 12u_r \frac{\partial (ph)}{\partial x},
\]

where \( u_r \) is the entrainment speed; \( x \) and \( y \) are the coordinate variables, respectively; \( p \) and \( \eta \) are density and viscosity of lubricating oil, respectively; and \( h \) and \( p \) are the thickness and pressure of the lubrication film, respectively.

The film thickness equation is as follows:

\[
h(x, y) = h_0 + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} + \nu(x, y),
\]

where \( R_x \) and \( R_y \) are the combined curvature radii in the \( x \) and \( y \) directions, respectively.

The deformation formula is expressed as

\[
\nu(x, y) = \frac{2}{\pi E} \int_0^1 \frac{p(s,t)}{\sqrt{(x-s)^2 + (y-t)^2}} \, ds \, dt,
\]

where \( p(s,t) \) represents the oil film pressure at point \((s,t)\) and \( E \) is the modulus of elasticity.

The isothermal viscous-pressure equation is as follows:

\[
\eta = \eta_0 \exp \left[ (1 + 5.1 \times 10^{-9} p) \right],
\]

where \( \kappa \) is the pressure viscosity index, approximately 0.1~0.5, and \( \eta_0 \) is the lubricant viscosity under normal pressure.

The dimensionless film thickness parameter \( h/R_x \) is expressed as a function of three dimensionless parameters. These three parameters are as follows [34]:

(i) Speed parameter:

\[
U = \frac{\eta_0 \mu}{E' R_x}, \quad R_x = \frac{d_b}{2} \left( 1 + \gamma \right), \quad \gamma = \frac{d_b \cos \alpha}{D_p},
\]

(ii) Load parameter:

\[
W = \frac{w}{E' R_x^2}, \quad E' = \frac{E}{1 - \mu^2},
\]

(iii) Material parameter:

\[
G = \alpha' E',
\]

where \( d_b \) is the diameter of rolling element, \( \alpha \) is the contact angle, \( D_p \) is the pitch diameter of bearing, \( \alpha' \) is the viscosity coefficient of lubricant, \( E \) is the elastic modulus, and \( E' \) is the equivalent elastic modulus.

The dimensionless minimum film thickness is as follows:

\[
H_{min} = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68k}).
\]

3. Mechanism of Rolling Element’s Slide

Under the lubrication condition, the elastohydrodynamic lubrication oil film with a certain thickness can be maintained between the rolling element and the raceway in bearing. The presence of oil film changes the contact of rolling element with raceways and the kinetic characteristics of rolling element with raceways can be analyzed. During the actual operation of many rotating machineries, the surfaces of bearing parts are prone to wear.

Due to the characteristic of radial clearance of the bearing, the radial force applied to the rolling elements by the inner ring in the unload area is 0, it changes when the rolling element is biting from load zone to unload zone, and this process is a
gradual acceleration process. Thus, the relative sliding between the rolling element and the raceway occurs.

The rolling bearing operates under radial load, while the rolling element also maintains a cyclic rotational movement, as shown in Figure 2. In Figure 2, $F$ is the radial load, $w_i$ is the rotation speed of bearing, and $C_{do}$ and $\theta_{do}$ are the depth and angle of defect, respectively. The running position of the rolling element is a circular process, “unload zone (Ball1), boundary (Ball2), load zone (Ball3).”

The kinetic characteristic of the rolling element is as presented in Figure 3. In Figure 3, $w_b$ is the rotation speed of rolling element, $w_c$ is the rotation speed of cage, and $V_M$, $V_E$, and $V_F$ are tangential velocities.

When the rolling element passes over the defect, $V_M$ is the rolling element’s linear velocity at the point M which rotates around the center of the inner ring. Therefore, $V_M$ can be expressed as

$$V_M = w_i \cdot \frac{D_i}{2},$$  \hspace{1cm} (10)

where $D_i$ is the diameter of inner raceway.

As shown in Figure 3, the speed $V_E$ located at point E is the sum of the rolling element’s line speed $V_M$ and its rotation speed $V_B$. $V_E$ can be expressed as

$$V_E = w_i \cdot \left( \frac{D_i}{2} + \delta_{in} + \delta_{ou} + H_r \right) + w_b \cdot \left( \frac{d_b}{2} - \delta_{in} \right),$$  \hspace{1cm} (11)

where $\delta_{in}$ and $\delta_{ou}$ are the deflection of the inner and outer raceways, respectively, and $H_r$ is the maximum radial displacement.

The speed $V_F$ located at point F can be expressed as

$$V_F = w_c \cdot \left( \frac{D_b}{2} + H_r \right) - w_b \cdot \left( \frac{d_b}{2} - \delta_{ou} \right),$$  \hspace{1cm} (12)

where $D_b$ is the diameter of outer raceway.

The outer ring speed $V_X$ is 0. Therefore, the calculation formulas for the relative sliding velocity are as follows:

$$\Delta V_j = w_c \cdot \left( \frac{D_i}{2} + \delta_{in} + \delta_{ou} + H_r \right) + w_b \cdot \left( \frac{d_b}{2} - \delta_{in} \right) - w_i \cdot \frac{D_i}{2},$$

$$\Delta V_o = w_c \cdot \left( \frac{D_b}{2} + H_r \right) - w_b \cdot \left( \frac{d_b}{2} - \delta_{ou} \right).$$  \hspace{1cm} (13)
4. Dynamic Modeling of Bearing-Housing System

The 5-DOF simplified model of bearing system is shown in Figure 4, which takes the shaft, bearing, and housing into consideration as a whole. In Figure 4, $k_p$ and $c_p$ are the stiffness and damping between pedestal and outer ring, respectively, $k_s$ and $c_s$ are the stiffness and damping between inner ring and shaft, respectively, $k_r$, $c_r$, and $m_r$ are the stiffness, damping, and mass of unit resonator, and $X_s$, $Y_s$ and $X_p$, $Y_p$ are the coordinate axes of inner and outer ring, respectively. The subscript $s$ indicates shaft and inner ring, and $p$ indicates the bearing housing and outer ring.

In order to investigate the vibration characteristics of the bearing-housing system, the unit resonator is added to the 4-DOF dynamic model to simulate the resonance response of
the system. A 5-DOF simplified model is proposed, with the inner and outer races having each 2-DOF, and another DOF is the unit resonator.

The unit resonator is an extra spring-quality system which is included in the model with higher damping to enlarge the vibration response frequency of bearing [13, 16, 17] and can adjust its stiffness and damping coefficient.

4.1. Contact Model of Rolling Bearing. The Hertzian contact forces in the x and y directions can be written as follows [14]:

\[
\begin{bmatrix}
 f_x \\
 f_y
\end{bmatrix} = \sum_{i=1}^{N_b} K_{in-ou} \cdot \varsigma_0 \cdot \delta_i^{3/2} \cdot \begin{bmatrix}
 \sin \theta_i \\
 \cos \theta_i
\end{bmatrix},
\]

where \( \delta_i \) and \( \theta_i \) are the radial deformation and angle of \( i \)-th rolling element, respectively; \( N_b \) is the number of balls; \( \varsigma_0 \) is the Dirac function; and \( K_{in-ou} \) is comprehensive stiffness.

The Dirac function \( \varsigma_0 \), as Equation (15) is introduced to determine whether the rolling element enters into load zone.

\[
\varsigma_0 = \begin{cases} 
1, & \delta_i > 0, \\
0, & \delta_i \leq 0.
\end{cases}
\]

4.2. Time-Varying Displacement

4.2.1. Time-Varying Displacement Caused by Defect. The radial displacement would change when the rolling element passes over the defect. When the mass center of the rolling element is just on the center line, the maximum displacement change is \( H_r \). As shown in Figure 5, \( H_d \) is the time-varying displacement, and the geometric position relationship is defined as follows:

\[
H_r = \frac{d_c}{2} - \sqrt{\left(\frac{d_c}{2}\right)^2 - \left(\frac{w_d}{2}\right)^2},
\]

where \( w_d \) is the length of defect.

In the process of the rolling element in entering into and exiting out of the defect, the radial displacement changes from 0 to \( H_r \) and from \( H_r \) to 0, respectively. The process can be approximated as a sine wave [6]; therefore the radial displacement \( H_d \) caused by the defect is defined as

\[
H_d = \begin{cases} 
H_r \cdot \sin \left[ \frac{\pi}{\theta_{do}} \cdot (\mod(\theta_i, 2\pi) - \theta_0) \right], \\
0, & 0 \leq \mod(\theta_i, 2\pi) - \theta_0 \leq \theta_{do}, \\
0, & \text{others},
\end{cases}\]

where \( \theta_{do} \) is the angle of defect and \( \theta_0 \) is the initial reference position (\( \theta_0 \) is the first ball’s angle relative to the x-axis); the formula can be expressed as

\[
\theta_i = \frac{2\pi (i - 1)}{N_b} + w_c t + \theta_0, \quad \text{(18)}
\]

The connection between cage speed \( w_c \) and shaft rotation speed \( w_i \) can be obtained as follows:

\[
w_c = \frac{w_i}{2} \left( 1 - \frac{d_c}{D_p} \right). \quad \text{(19)}
\]

4.2.2. Time-Varying Displacement with Oil Film. The radial deformation when the bearing outer ring is defective is defined as follows:

\[
\Delta = \left( x_s - x_p \right) \sin \theta_i + \left( y_s - y_p \right) \cos \theta_i - c, \quad \text{(20)}
\]

where \( c \) is the radial clearance, the subscript \( s \) indicates shaft and inner ring, and \( p \) indicates the bearing housing and outer ring.

When the value of \( \Delta \) is greater than 0, it means that the rolling element enters into load zone. The contact deformation formula is defined as follows:

\[
\chi_i = \Delta - H_d = \left( x_s - x_p \right) \sin \theta_i + \left( y_s - y_p \right) \cos \theta_i - c - H_d.
\]

The defect of the outer ring is regarded as a small rectangular spall as shown in Figure 2. Considering the thickness of oil film during the rolling element enters into the defect, time-varying displacement \( \delta_i \) is defined in the following equation:
\[
\delta_i = \chi_i - H = (x_s - x_p)\sin \theta_i + (y_s - y_p)\cos \theta_i - c - H_d - h,
\]
(22)

Therefore, the Hertz contact force is calculated as follows:

\[
\begin{bmatrix}
    f_x \\
    f_y
\end{bmatrix} = \sum_{i=1}^{N_b} \left(K_{in-out} \cdot \varsigma_o \cdot \begin{bmatrix}
    (x_s - x_p)\sin \theta_i \\
    (y_s - y_p)\cos \theta_i \\
    -c - H_d - h \\
    (x_s - x_p)\sin \theta_i \\
    (y_s - y_p)\cos \theta_i \\
    -c - H_d - h
\end{bmatrix} \right)^{3/2} \begin{bmatrix}
    \sin \theta_i \\
    \cos \theta_i
\end{bmatrix}.
\]
(23)

4.3. Comprehensive Stiffness Affected by Oil Film

4.3.1. Time-Varying Stiffness with Oil Film. It is a basis for the stiffness of the bearing to analyze the vibration characteristic during bearing operation. In line with the definition of stiffness, the oil film stiffness \( K_{film} \) between rolling elements and raceway can be expressed as

\[
K_{film} = \frac{dF}{dH_0}
\]
(24)

With the case of sufficient lubricating oil, the stiffness of the outer or inner ring considered has two parts which are shown in Figure 6, and those are the Hertzian contact stiffness \( k_{in/out-re} \) and the oil film stiffness \( k_{hi/ho} \), respectively. In Figure 6, \( K_i \) and \( c_i \) are the stiffness and damping of inner ring, respectively, and \( K_o \) and \( c_o \) are the stiffness and damping of outer ring, respectively.

The stiffness of the inner and outer rings of the bearing can be expressed as

\[
\begin{align*}
    K_i &= (k_{in-re}^{-1} + k_{hi}^{-1})^{-1}, \\
    K_o &= (k_{ou-re}^{-1} + k_{ho}^{-1})^{-1},
\end{align*}
\]
(25)

where \( k_{hi} \) and \( k_{ho} \) are the stiffness between inner ring and oil film and between outer ring and oil film, respectively.

Under the condition of elastic fluid lubrication, the comprehensive stiffness of the bearing is formed by a series
connection of the inner and outer ring. The equivalent contact stiffness $K_{\text{in-out}}$ of the bearing can be expressed as

$$K_{\text{in-out}} = \frac{1}{(K_1^{-1} + K_\sigma^{-1})}$$  \hfill (26)
viscosity of lubrication oil is 0.05 Pa·s, the viscosity coefficient is \( \alpha = 2.2 \times 10^{-3} \) m²/N [35], and entrainment speed is \( \omega_0 = 3 \) m/s. Based on Equations (9) and (24), the oil film thickness and the stiffness of the bearing would change with the radial load \( F \), which is shown as Figure 7.

In Figure 7, with the increasing of radial load, the bearing oil film stiffness would increase, and the oil film thickness would decrease.

Based on Equation (26), the comprehensive stiffness of the bearing would change as the radial load changes, as shown in Figure 8. And it is shown that the comprehensive stiffness of the bearing is strengthened with the increase of radial load. With the radial load increasing, the change of comprehensive stiffness from 100 N to 600 N is sharp, the change from 600 N to 1200 N is slow, and there is almost no change from 1200 N to 2000 N.

4.3.3. Changes of Oil Film Thickness and Stiffness with Different Oil Entrainment Velocities. When the dynamic viscosity of lubrication oil is 0.05 Pa·s, the viscosity coefficient is \( \alpha = 2.2 \times 10^{-3} \) m²/N, and the radial load is \( F = 1500 \) N. The oil film thickness and stiffness of the bearing would change with the oil entrainment velocity \( \omega_0 \), as shown in Figure 9.

As can be seen from Figure 10, as the entrainment velocity increases, the bearing oil film stiffness would increase, and the oil film thickness would decrease.

In Figure 10, there is a clear trend of decreasing in the comprehensive stiffness of the bearing with the entrainment velocity increasing.

4.4. Calculation of Damping. The internal damping coefficient \( C_{da} \) of the bearing can be expressed as [36]

\[
C_{da} = \frac{\eta_b \cdot K_b}{\omega_{ext}},
\]

(27)

where \( \omega_{ext} \) is the excitation frequency and \( \eta_b \) and \( K_b \) are the loss parameter and stiffness of the bearing.

4.5. Analysis of Sliding Friction. The sliding frictional force generated by the rolling element entering into the slip area of load zone can be given as

\[
F_f = \mu' \cdot f = \mu K_{in-ou} \cdot \frac{\omega}{\omega_{BPFO}},
\]

(28)

where \( \mu \) is the coefficient of friction.

Equation (23) is substituted into Equation (28) to obtain the calculating friction force equation (29) in different directions; \( \mu \) is referred to in reference [37]:

\[
\left[ \begin{array}{c}
F_{f_x} \\
F_{f_y}
\end{array} \right] = \sum_{i=1}^{N_k} \mu_k K_{in-ou} \cdot c_o \cdot \left[ \begin{array}{c}
(x_s - x_p) \sin \theta_i \\
(y_s - y_p) \cos \theta_i \\
-c - H_d - h \\
(x_s - x_p) \sin \theta_i \\
(y_s - y_p) \cos \theta_i \\
-c - H_d - h
\end{array} \right] \left[ \begin{array}{c}
\sin \theta_i \\
\cos \theta_i
\end{array} \right].
\]

(29)

5. Establishment of Dynamic Equations of Bearing-Housing System and Experimental Validation

5.1. Dynamic Equations of Bearing-Housing System. The outer ring fixed on the pedestal is modeled as 2-DOF system with translational motion in \( x \) and \( y \) directions. According to Newton’s law of motion, Equation (30) of motion can be written as

\[
\begin{align*}
\begin{cases}
 m_p x''_p + c_p x'_p + k_p x_p &= f_x + F_{f_x}, \\
m_p y''_p + c_p y'_p + k_p y_p &= f_y + F_{f_y},
\end{cases}
\end{align*}
\]

(30)

where \( m_p \) is the mass of the pedestal with outer race and \( f_x \) and \( f_y \) are the contact forces defined in Equation (23).

The inner ring is fixed rigidly to the shaft and it has 2-DOF system with a constant rotation speed. According to Newton’s law of motion, Equation (31) is as follows:

\[
\begin{align*}
\begin{cases}
m_i x'_i + c_i x''_i + k_i x_i &= f_x - F_{f_x}, \\
m_i y'_i + c_i y''_i + k_i y_i &= F - F_{f_y} - f_y,
\end{cases}
\end{align*}
\]

(31)

where \( m_i \) is the mass of the shaft with inner race and \( x_i, y_i \) are \( x \) and \( y \) displacement of inner race center of mass, respectively.

The unit resonator is connected in \( y \) direction. According to Newton’s law of motion, Equation (32) can be expressed as follows:

\[
m_i y'_i + c_i \left( y_i - y_p \right) + k_i \left( y_i - y_p \right) = 0,
\]

(32)

where \( y_i \) is the displacement of unit resonator.
From Equations (30)–(32), a 5-DOF equation of motion is obtained as

\[
\begin{align*}
    m_x \ddot{x} + c_x \dot{x} + k_x x + f_x + F_{fx} &= 0, \\
    m_y \ddot{y} + c_y \dot{y} + k_y y + f_y + F_{fy} &= F, \\
    m_p \ddot{x}_p + c_p \dot{x}_p + k_p x_p - f_x - F_{fx} &= 0, \\
    m_p \ddot{y}_p + (c_p + c_r) \dot{y}_p - k_r y_r - c_r \dot{y}_r, \\
    (k_p + k_r) y_p - f_y - F_{fy} &= 0, \\
    m_r \ddot{y}_r + c_r (\dot{y}_r - \dot{y}_p) + k_r (y_r - y_p) &= 0,
\end{align*}
\]

Equation (33) can be calculated by using the fourth-order Runge–Kutta method. The solution procedure of dynamic model is shown as Figure 11. Set the initial parameters in the x-axis and y-axis directions, the radial load is \(F = 500\, \text{N}\), \(F_p = 0\), and the step size is set as \(1 \times 10^{-4}\).

5.2. Experimental Validation

5.2.1. Test Rig. The dynamic model is verified on faulty bearing experimental data from Case Western Reserve University; the test rig is shown as Figure 12. A 2-horsepower electric motor, torque sensor, dynamometer, and control electronics make up the test rig. The experimental data are measured by using an accelerometer, and the vibration signals are collected by a 16-channel DAT record.

The bearing used in the experiment is SKF 6205, and the specific parameters of the test bearing are listed in Table 1. The bearing characteristic frequency \(f_{BPFO}\) of the outer ring is as follows:

\[
f_{BPFO} = \frac{N_b}{2} f_s \left(1 - \frac{d_p}{D_p} \cos \alpha\right), \tag{34}
\]

where \(f_s\) is the shaft frequency.

5.2.2. Model Verification. In order to highlight faulty components, the experimental data are processed by using envelope analysis. The Prontrugram method [38] is used to select the appropriate bandwidth and center frequency. The band-pass filter is used to separate the high-frequency resonance signal containing the faulty component from the collected original signal. Finally, the fast Fourier transform (FFT) is performed on the fault signal to obtain the corresponding frequency spectrum for exploring the characteristics of the vibration response.

When the shaft speed is 1772 rpm and the defect size is 0.007 inches, the experimental and simulated vibration signals of the faulty bearing are shown in Figures 13 and 14, respectively. The vibration characteristic frequency of the simulation result is 105.8 Hz, compared with the theoretical
6.1 Vibration Response Frequency under 500 N at Different Rotation Speeds. When the outer ring defect size is 0.014 inches, $F_f = 0$ N, and the bearing radial load is 500 N, the frequency spectrum of bearing vibration response with the

89.84 Hz. It can be clearly known that the difference between the three is within 1%, and the results of the data further verified the correctness of the 5-DOF dynamic model.

When the shaft speed is 1772 rpm, and the defect size is 0.014 inches, the experimental and simulated vibration signals of the faulty bearing are shown in Figures 15 and 16, respectively.

The experimental data are compared with the simulation results in order to prove the accuracy of the model. The physical parameters of the experimental bearing ER-16K are shown in Table 2. When the radial load is 50 N and the rotating speed is 1500 rpm, the experimental and simulated vibration signals of the faulty bearing are shown in Figures 18 and 19, respectively.

The vibration characteristic frequency of the simulation result is 89.11 Hz, compared with the theoretical calculation of 89.24 Hz based on Equation (34), and the experimental result is 105.77 Hz based on Equation (34), and the experimental result is 106.3 Hz. The difference among them is within 1%, so the accuracy of the model can be verified.

The vibration characteristic frequency of experimental signals, but there is a great diversity in the amplitudes of them. The reasons for the difference among them may be related to various factors, which are as follows:

1. The difference between the actual stiffness of bearings caused by installation and load.
2. The actual damping of the bearing is difficult to measure, and there is a certain difference between the simulated damping taken from experience.
3. There are many different factors considered between the simulation and actual working conditions of the bearing. On the other hand, the spectrum component of the experimental signals is more complicated, which may be caused by other noises during testing.

6. Discussion
shaft speed of 1800 rpm, 2000 rpm, 2200 rpm, and 2400 rpm, is shown in Figure 20.

It can be seen from Figures 20(a)–20(d), when the radial load is constant, the characteristic frequency of the faulty outer ring and the harmonic frequencies can be observed very clearly. From these figures, it is obvious that the amplitude of the faulty characteristic frequency and the harmonic frequencies are enhanced with the increasing of rotation speed, and the harmonic frequencies around the characteristic frequency would decrease when there is a local defect on the outer ring surface of the bearing.

As shown in Figure 21, the amplitude of vibration characteristic frequency of the faulty outer ring in rolling bearing increases with the increasing of rotation speed when the defect size and radial load are constant in oil film lubrication condition. From the figure, it can be seen that the change of vibration amplitude would be slow at lower speed and significantly enhanced when the speed is at higher speed.

6.2 Vibration Response Frequency under 1500 N at Different Rotation Speeds. Figure 22 shows the frequency spectrum of
bearing vibration response with the shaft speed of 1800 rpm, 2000 rpm, 2200 rpm, and 2400 rpm when the outer ring defect size is 0.014 inches, $F_f = 0$ N, and the bearing radial load is 1500 N.

The characteristic frequency of the faulty outer ring and the harmonic frequencies can be observed clearly from Figures 22(a)−22(d). It is easy to show that the increasing of the harmonic frequencies may result in the vibration amplitudes decrease. Comparing Figure 22 with Figure 20, with the increasing of radial load, the characteristic frequency of the faulty outer ring at the same speed is slightly changed, but the amplitude of vibration response is significantly increasing. And the amplitudes of $2 \times f_{BPFO}$ and $3 \times f_{BPFO}$ are also increased. From these figures, it can be known that the radial load is an important factor affecting the amplitude of vibration at the same working conditions.

The trend of amplitude of characteristic frequency is shown in Figure 23 under different loads when the defect size is 0.014 inch and the rotation speed is 1772 rpm. When the defect size and rotation speed are constant in oil film lubrication condition, with the increasing of radial load, the amplitude of the characteristic frequency of the faulty outer ring in rolling bearing would increase. During the radial load from 500 N to 1400 N, the amplitude of characteristic frequency would change rapidly, which can accelerate the failure rate of bearings under operating conditions, but it appears to be relatively gentle and slow after 1400 N. If the bearings would work under these loads, the vibration fluctuation range of the mechanical equipment will be reduced, which can avoid huge economic losses and casualties.

6.3. Vibration Response Frequency at Different Rotation Speeds with Sliding Friction. The vibration response of bearing with sliding friction is shown in Figure 24 under the shaft speed of 1800 rpm, 2000 rpm, 2200 rpm, and 2400 rpm when outer ring defect size is 0.014 inch, $F_f \neq 0$ N, and the bearing radial load is 1500 N.

From Figures 24(a)−24(d), it can be seen that the characteristics and the harmonic frequencies of the faulty outer ring are very clear. And the amplitudes of harmonic frequencies near the characteristic frequency may decrease gradually. Comparing Figure 24 with Figure 22, when the radial load and the defect size are the same, and the sliding friction force is not zero, the characteristic and harmonic frequencies of the faulty outer ring at the same speed are almost unchanged, but the amplitude of vibration response would significantly increase. The amplitudes of the second and third frequencies are also basically strengthened. By comparing the amplitudes of the vibration response, it can be obtained that the occurrence of sliding friction will have a certain adverse effect on the vibration of the defective bearing.

6.4. Comparison of Vibration Characteristics with Friction Force or Frictionless Force. It is well known that the existence of friction force will affect the vibration response to a certain extent. The amplitude trends of vibration response of the fault characteristic frequency with friction and no friction are shown in Figure 25 at this load under different rotation speeds.

As shown in Figure 25, it can be known that the vibration response of the bearing would increase whether there is friction force or not under different rotation speeds. However, the vibration amplitude of the fault characteristic frequency at this load under the friction force is intuitively slightly higher than that under the frictionless force. It is shown that the friction force is an important factor affecting the amplitude of vibration, which can strengthen the vibration characteristics of the bearing and cause the bearing failure rapidly.
It can be seen from Figure 26 that the amplitude of the vibration response of the bearing would be basically increasing regardless of whether considering the friction force or not under different radial loads. During the radial load from 1400 N to 3000 N, the amplitude of vibration response would be changed gently. However, it can be clearly obtained that the amplitude of the fault characteristic frequency at this speed under friction force would increase more quickly than that under no friction when the radial force is beyond 3000 N. It is also shown that the friction force is an important factor affecting the amplitude of vibration when the radial load is heavy.

7. Conclusions

The excitation mechanism of deep groove ball bearing with defect was explored. The following conclusions are drawn:

1. Based on isothermal EHL theory, a 5-DOF dynamic model of rolling bearing with localized defect is proposed and validated by experimental data, which provides a theoretical basis for the design and life prediction of rolling bearing.

2. The simulated results show that radial load and rolling speed will affect the change of the lubricating oil film characteristics and the comprehensive stiffness of the bearing, which will indirectly cause the change of the vibration characteristics. With the increase of load, the oil film stiffness and comprehensive stiffness would increase, and the oil film thickness would decrease. The change of the comprehensive stiffness would not be obvious when the load reaches heavy load. And with the increase of tangential speed, the oil film stiffness would increase, and the oil film thickness and comprehensive stiffness would decrease. These results can be helpful for the correct use of lubricant.

3. The results show that rotation speed, load, and friction force are all important internal and external factors which affect the coupling vibration response caused by local defect excitation. Different loads and rotation speeds can be coupled with friction force to generate vibration response signals, which indirectly affects the stability of bearing system, accelerates bearing failure, and shortens service life. The vibration amplitude of the bearing is enhanced with the increase of rotation speed and the radial load and the vibration amplitude of bearing which considered the friction force is stronger than no friction.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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