Age and covid fatality
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ABSTRACT
It is widely acknowledged that the distribution of Covid cases and that of Covid deaths by age constitute a factor that deserves to be taken into account in assessing and comparing quantitative indicators of Covid-related mortality. The single most widely employed measure of Covid mortality is the so-called Case Fatality Rate (CFR), which is just the ratio of Covid deaths to Covid cases. The CFR is essentially a measure of central tendency. The present note outlines a procedure, drawing on the standard literature on income inequality, for deriving a measure of Covid mortality which supplements information on average mortality with information on its dispersion across a population’s age cohorts.

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1. The problem
The measure most widely resorted to in the measurement of COVID mortality is the so-called case fatality rate, or CFR for short. The CFR is a straightforward measure of central tendency, simply the proportion of all COVID cases that have resulted in mortality. Many researchers have pointed out that concentrating solely on the CFR as a measure of mortality could be misleading, as the statistic ignores the age-distribution of COVID cases and deaths (Green, Peer, and Nitzan 1920; Dudel et al. 2020; Mukhopadhyay 2020; Philip, Ray, and Subramanian 2020). For instance, the case fatality rate for India is comparably small when set beside that of a country like West Germany, which is somewhat surprising considering that West Germany has a vastly more developed health infrastructure than India. What, however, is not reckoned in this comparison is the fact that the age 60 + population accounts for a much smaller proportion of the population in India than in West Germany, and that age-specific mortality rates are distributed differently in the two countries, with both COVID cases and COVID deaths displaying relatively greater concentration among the younger age cohorts in India as compared to West Germany. This is a specific instance of a general proposition made famous by the evolutionary biologist Stephen Jay Gould (1981), when he observed: ‘… our culture encodes a strong bias either to neglect or ignore variation. We tend to focus instead on measures of central tendency, and as a result we make some terrible mistakes, often with considerable practical import.’

So is there a way of incorporating distributional considerations, to supplement the information conveyed by the CFR average, in the measurement of COVID mortality? The present note is an attempt in this direction. The method employed relies on old and widely-used protocols in the quantitative assessment of dispersion. Specifically, in what follows, we borrow, for the purposes of the current exercise and context, elements relating to the ‘Atkinson-Kolm-Sen’ welfare approach to the measurement of income inequality (Kolm 1969; Atkinson 1970; Sen 1973) and Amartya Sen’s (1976) approach to the formulation of a welfare indicator of ‘real national income’.

2. Mortality illfare functions and ‘best’ equivalent mortality outcomes
A = {a_1, \ldots, a_K} is a set of age categories into which the population is partitioned, and arranged in ascending order of magnitude: a_j > a_{j+1}, j = 1, \ldots, K - 1. N = \{n_1, \ldots, n_K\} is a set displaying the numbers of covid cases in each age category. The total number of covid cases is n = \sum_{j=1}^{K} n_j. D = \{d_1, \ldots, d_K\} is a set showing the number of covid deaths in each age category, with d_j \leq n_j for all...
The total number of covid deaths is \( d = \sum_{j=1}^{K} d_j \). A pair \((N, D)\) will be called a covid situation, or just situation, for short. For any situation \((N, D)\), the case fatality rate – call it \( C \) – is the proportion of covid cases resulting in death: \( C(N, D) = d/n \).

For any situation \((N, D)\) and all \( j \in \{1, \ldots, K\} \), let \( w_j \) be a weight attached to the \( j \)th age category. Given any situation, a mortality illfare function \( L \) can be written as an age-weighted sum of age-specific mortalities. For all \((N, D)\), the illfare function normalized for case population size can be expressed as:

\[
L(N, D) = (1/n) \sum_{j=1}^{K} w_j d_j.
\]  

(1)

The quantity \( L \) in Equation (1) is an indicator of the ‘illfare’ arising from mortality, and it is commonly accepted that, other things equal, death acquires greater (adverse) significance the younger the age at which it occurs. This would be reflected in a weighting structure for the mortality illfare function such that, for all ages \( a_j \) and \( a_k \), \( w_j > w_k \) whenever \( a_j < a_k \), which expresses the judgement that, other things equal, it is worse for a person to die younger rather than older. This is not necessarily an ‘age-ist’ value-judgement. With John Donne, we would agree that ‘every man’s death diminishes [us]’, but this is not inconsistent with the view that a younger person’s death is more tragic, and so diminishes us more, than an older person’s death.

As has been pointed out by Sen (1978) and Broome (1989), Atkinson’s approach to measuring inequality was to reckon the efficiency loss of social welfare, expressed in equivalent income units, due to the deviation of an income distribution from the welfare-maximising distribution. In what follows, we resort to a similar procedure for reckoning the ‘efficiency loss’, in equivalent mortality terms, that arises from an age-distribution of covid cases and deaths that is sub-optimal in a specific sense. To this end, we resort to Atkinson’s device of comparing any outcome against the standard of the best outcomes available to that society, given its age distribution and given the total number of deaths under the given outcome.

Given the mortality illfare function described by Equation (1), it is clear that the ‘least inefficient’, or illfare-minimising, pattern of deaths is the one for which all the deaths are re-allocated among the oldest of the old. This is the ‘least-bad-case’ scenario of ‘mortality-illfare’ for any given number of deaths. One can now ask:

What proportion of the total case population constituted by cohorts in the oldest-of-the-old age categories must die, so that the resulting aggregate illfare is the same as the actual level of illfare that obtains under the existing distribution of deaths?

What is involved in this reckoning can be conveyed with the help of a simple numerical example.

Suppose we have four age categories \( a_1, a_2, a_3, a_4 \), that \( N = \{n_1, n_2, n_3, n_4\} = \{10, 7, 4, 3\} \), \( D = \{d_1, d_2, d_3, d_4\} = \{2, 2, 2, 2\} \), and that \( [w_1, w_2, w_3, w_4] = [10, 7, 4, 1] \). Note that \( d = 8 \), \( n = 24 \), and the case fatality rate is 8/24, or around 0.33. Using the data above, it can be verified that the aggregate illfare associated with the situation \((N, D)\) is given by \( L(N, D) = \sum_{j=1}^{K} w_j d_j = 10 \times 2 + 7 \times 2 + 4 \times 2 + 1 \times 2 = 44 \). We now ask: if all the cases in the oldest age category die, will the resulting aggregate illfare be the same as what obtains under the existing distribution of deaths, namely a welfare level of 44? No, because \( w_4 n_4 = 1 \times 3 = 3 \), which is less than 44. So we now consider the two oldest age categories, and ask if the illfare arising from loading all the deaths in these categories matches the existing level of illfare? Again no, because \( w_3 n_3 + w_4 n_4 = 7 \times 4 + 1 \times 3 = 30 < 44 \). We therefore move further down the age-ladder, and find that \( w_2 n_2 + w_3 n_3 + w_4 n_4 = 7 \times 7 + 4 \times 4 + 1 \times 3 = 68 < 44 \). We have now generated more illfare than what obtains under the existing dispensation, namely a welfare level of 44. Let \( n_2^* \) be a number such that \( w_2 n_2^* + w_3 n_3 + w_4 n_4 = (L(N, D) - w_1 n_1) / w_1 \). It can be easily verified from the data for our example that \( n_2^* = 25/7 \approx 3.57 \). It follows then that the fraction of the oldest population that must die so that the resulting illfare is exactly the same as what obtains under the actual distribution is given by the quantity \((1/n)(n_2^* + n_3 + n_4) = (1/24)(3.57 + 4 + 3) = 10.57/24 \approx 0.44 \). Recall that the actual case fatality rate is 0.33. We can obtain a welfare-equivalent situation, in which the case fatality rate is 0.44, by loading all the deaths on the oldest of the old population. This higher case fatality rate of 0.44 is what might be called the ‘least-inefficient-equivalent’ CFR. The derivation of this equivalent CFR can be generalized along the following lines.

First, given any situation \((N, D)\) and the associated illfare \( L(N, D) = (1/n) \sum_{j=1}^{K} w_j d_j \), let the \( q \)th age category \( a_q \) be the largest age for which the following weak inequality holds:

\[
(1/n) \sum_{j=q}^{K} w_j n_j \geq L(N, D)[ = (1/n) \sum_{j=1}^{K} w_j d_j].
\]  

(2)

Define \( n_q^* = \left[ L(N, D) - \sum_{j=q+1}^{K} w_j n_j \right] / w_q \). \( n_q^* \) will be the same as \( n_q \) if (2) is a strict equality, and less than \( n_q \) if (2) is a strict inequality.) Then, it is clear that if the existing distribution of deaths is reallocated in such a way that death is assigned to all \( n_k \) cases at age \( a_k \), all \( n_{k-1} \) cases at age \( a_{k-1} \), \ldots, down the line to all \( n_{q+1} \) cases at age \( a_{q+1} \), and \( n_q^* \) cases at age \( a_q \), then this...
situation would be welfare-equivalent to the status quo situation \((N, D)\). It follows then that the ‘least-inefficient-equivalent’ case fatality rate – call it \(C^e\)–is given by:

\[
C^e = (1/n) \left[ p_0^* + \sum_{j=q+1}^{K} n_j \right].
\]

It should be obvious that \(C^e \geq C\).

Once the ‘least-inefficient-equivalent CFR’ \(C^e\) has been identified, it is a simple matter to reckon the welfare loss arising from a sub-optimal age-distribution of deaths, given the age-distribution of cases. Specifically, one can see that \((C^e - C)\) is a measure of the extent to which a situation ‘loses’ from not having all its deaths loaded on the oldest populations. Let us use the symbol \(Q\) to denote this net efficiency loss in proportionate terms, and define the ‘efficiency loss parameter’ as follows. For any situation \((N, D)\):

\[
Q(N, D) = \frac{[C^e(N, D) - C(N, D)]}{C(N, D)}.
\]  

Before proceeding further, let us define \(D'\) (given \(N\)), as the distribution of covid mortalities derived from the actual distribution \(D\) by redistributing the same number of deaths \(d\) that obtains in \(D\) among the oldest of the old, starting from the oldest and working one’s way down till the number of deaths \(d\) is exhausted. Let \(L(N, D')\) be the illfare level corresponding to the situation \((N, D')\). One can now ask: cannot the efficiency loss arising from a suboptimal age-distribution of deaths simply be written as the ratio of the illfare level corresponding to the situation \((N, D)\) to the illfare level corresponding to the situation \((N, D')\), that is, as the quantity \(L(N, D')/L(N, D)\)? The difficulty is that this normalization is not invariant to linear transformations of the illfare function: under such transformation, the quantity \([L(N, D) + R]/[L(N, D') + R]\) would depend on the precise value of the real number \(R\) that is chosen: as a result, and as Atkinson (1970, 250) observes, these ‘particular numerical values’ would have ‘no meaning.’ By contrast, the measure of inefficiency we have called \(Q\) is invariant with respect to linear transformations of the underlying illfare function.\(^1\)

It is a simple matter to note that the ‘least-inefficient equivalent CFR’ or ‘effective mortality rate’ can be written as a function of both the measure of central tendency \(C\) and the efficiency loss parameter \(Q\), a function which is increasing in each of its arguments. Given (2) and (3), we can see that, for any situation \((N, D)\):

\[
C^e(N, D) = C(N, D)[1 + Q(N, D)].
\]

Equation (4) is precisely analogous to Sen’s (1978) ‘distributionally adjusted’ measure of ‘real national income’, given by \(\mu(1 - G)\), where \(\mu\) is mean income, and \(G\) is the Gini coefficient of inequality in the distribution of income.\(^2\) The ‘least-inefficient equivalent CFR’ \(C^e\) is reminiscent of Atkinson’s (1970) ‘equally distributed equivalent income’, which can be written in such a way as to decompose it into its central tendency and dispersion components: this enables us ‘…to separate “shifts” in the distribution from changes in its shape …’ (Atkinson 1970, 245).

Much of what we have done has been to translate – mutatis mutandis – concepts from the distributional analysis of income to the distributional analysis of COVID mortality.

3. Some axioms for a mortality indicator

A mortality indicator is a function \(M\) which, for every situation \((N, D)\), specifies a real number that is intended to capture the extent of mortality associated with that situation. We consider now a small set of properties which may be regarded as desirable for a mortality indicator \(M\) to possess. These four axioms require, respectively, that the mortality indicator should be increasing in the number of deaths; that a young-to-old transfer of deaths should cause a decrease in the value of the indicator; that such a decline should be greater at the lower than at the upper ends of the age spectrum; and that the indicator should be invariant to case and mortality population replications. The relevant axioms are stated below.

**Monotonicity.** For all situations \((N, D), (N', D')\):

\[
M(N, D) > M(N', D') \text{ whenever } N = N' \text{ and } [d_i > d'_j \forall j \in \{1, \ldots, K\} \text{ and } d_j > d'_j \text{ for some } j].
\]

To state the next two axioms, we need to define the notion of one situation being derived from another through a young-to-old transfer of mortality. For any pair of situations \((N, D), (N', D')\), the latter will be said to have been derived from the former through a young-to-old change in mortality involving the age categories \(a_j\) and \(a_k\) whenever \(N = N'\), and \([d_i = d'_j \forall i \neq j, k\) for some \(j, k\) satisfying \(a_j < a_k\) and \(0 < d_j - d'_j = d'_k - d_k = \Delta_{jk} \leq d_j]\).

**Aversion to Young Deaths.** For all situations \((N, D), (N', D')\):

\[
M(N, D) > M(N', D') \text{ whenever } (N, D) \text{ is derived from } (N, D') \text{ through a young-to-old transfer of mortality involving the age categories } a_j \text{ and } a_k; \text{ (ii) } (N', D') \text{ is derived from } (N, D) \text{ through a young-to-old transfer of mortality involving the age categories } a_p \text{ and } a_q; \text{ (iii) } a_j < a_p \text{ and } a_k < a_q; \text{ (iv) } a_q - a_p = a_k - a_j; \text{ (v) } d_j = d_p \text{ and } d_k = d_q, \text{ and (vi) } \Delta_{jk} = \Delta_{p,q}.
\]

\(\Delta_{jk}\) is a real number that is intended to possess. These four axioms require, respectively, that the mortality indicator should be increasing in the number of deaths; that a young-to-old transfer of deaths should cause a decrease in the value of the indicator; that such a decline should be greater at the lower than at the upper ends of the age spectrum; and that the indicator should be invariant to case and mortality population replications. The relevant axioms are stated below.

**Monotonicity.** For all situations \((N, D), (N', D'):\)

\[
M(N, D) > M(N', D') \text{ whenever } N = N' \text{ and } \exists j \text{ satisfying } a_j < a_k \text{ and } 0 < d_j - d'_j = d'_k - d_k = \Delta_{jk} \leq d_j.
\]

To state the next two axioms, we need to define the notion of one situation being derived from another through a young-to-old transfer of mortality. For any pair of situations \((N, D), (N', D')\), the latter will be said to have been derived from the former through a young-to-old change in mortality involving the age categories \(a_j\) and \(a_k\) whenever \(N = N'\), and \([d_i = d'_j \forall i \neq j, k\) for some \(j, k\) satisfying \(a_j < a_k\) and \(0 < d_j - d'_j = d'_k - d_k = \Delta_{jk} \leq d_j]\).

**Aversion to Young Deaths.** For all situations \((N, D), (N', D'):\)

\[
M(N, D) > M(N', D') \text{ whenever } (N, D) \text{ is derived from } (N, D') \text{ through a young-to-old transfer of mortality involving the age categories } a_j \text{ and } a_k; \text{ (ii) } (N', D') \text{ is derived from } (N, D) \text{ through a young-to-old transfer of mortality involving the age categories } a_p \text{ and } a_q; \text{ (iii) } a_j < a_p \text{ and } a_k < a_q; \text{ (iv) } a_q - a_p = a_k - a_j; \text{ (v) } d_j = d_p \text{ and } d_k = d_q, \text{ and (vi) } \Delta_{jk} = \Delta_{p,q}.
\]
Finally, for the next axiom, we define the notion of an \( r \)-fold replication of a situation as follows. If \( N = \{n_1, \ldots, n_j, \ldots, n_K\} \) and \( D = \{d_1, \ldots, d_j, \ldots, d_K\} \), while \( N' = \{n_1, \ldots, n_j, \ldots, n_K\} \) and \( D' = \{d_1, \ldots, d_j, \ldots, d_K\} \), then \( (N, D) \) is an \( r \)-replication of \( (N, D) \), where \( r \) is any positive integer.

Population Neutrality. For all situations \( (N, D), (N', D') : M(N, D) = M(N', D') \) whenever the latter situation is derived from the former by an \( r \)-replication, for any positive integer \( r \).

4. Two Specific systems of weighting, and Two Specific ‘Adjusted’ CFRs

4.1. A First Example

One system of weights which reflects an intrinsic ethical objection to the unfairness of unequal life-spans and upholds the notion that a young life is more valuable than an old life is the following one:

\[ v_{ij} = a_j^{-\alpha}, \quad \forall j \in \{1, \ldots, K\}, \alpha \geq 0. \]  

(5)

On the Right Hand Side of (5), the parameter \( \alpha \) lends itself to interpretation as a measure of one’s aversion to death at a young age, with such aversion seen as increasing with \( \alpha \). With the weighting system as in (5), we have a family of mortality illfare functions parameterized by \( \alpha \), and given, for any situation \((N, D)\) by:

\[ L_\alpha(N, D) = (1/n) \sum_{j=1}^{K} a_j^{-\alpha} d_j. \]  

(6)

Using the illfare function in (6), and proceeding along the lines indicated in section 2, one can derive a specific family of ‘adjusted’ CFRs; and the family of indicators corresponding to the mortality illfare function of Equation (6) is yielded, in terms of a suitable adaptation of the notation employed in Section 2, by:

\[ C^e_\alpha = (1/n) [n_{q(0)}^{*\alpha} + \sum_{j=q(a)+1}^{K} n_j]. \]  

(7)

It is a simple matter to note that \( C^0_\alpha \) is exactly the same as the (raw) CFR \( C \), which satisfies what we have called the Monotonicity Axiom in Section 2.2, while, as \( \alpha \) increases, \( C^e_\alpha \) becomes more and more sensitive to the precise age-distribution of mortalities. Without dwelling on the issue elaborately, one can see at play here a context-adapted version of the well-known transfer and transfer-sensitivity axioms invoked in the inequality and poverty measurement literature. Specifically, for parameter values of \( \alpha > 0 \), \( C^e_\alpha \) satisfies a transfer axiom here interpreted as the requirement that, other things equal, a ‘favourable’ mortality transfer (that is, the transfer of a death from a lower age to a higher age) should reduce aggregate ‘effective’ mortality. This property is what we have called the Aversion to Young Deaths Axiom in Section 3. That \( C^e_\alpha \) satisfies the property for strictly positive values of \( \alpha \) follows directly from the fact that the age-weighting function \( a^{-\alpha} \) is a declining function of age for \( \alpha > 0 \). Further, the weighting function is declining and also strictly convex for \( \alpha > 0 \), implying that \( C^e_\alpha \) is more sensitive to a ‘mortality transfer’ at the lower end of the age spectrum than at its upper end, other things equal, for \( \alpha > 0 \). This is what we have called the Aversion-Sensitivity Axiom in Section 3. To summarize, and with reference to the axioms listed in Section 3, \( C^e_\alpha \) satisfies the Monotonicity and Population-Neutrality axioms for all \( \alpha \geq 0 \); and the Aversion-Sensitivity axiom (and therefore the weaker property of Aversion to Young Deaths) for all \( \alpha > 0 \). A distinguished value of \( \alpha \) is 1, which is the smallest integer value of \( \alpha \) for which the Aversion to Young Deaths and Aversion-Sensitivity Axioms are satisfied.

4.2. A Second Example

A justification for weights which decline with age could be based on some notion of the opportunity cost of a lost life, measured, for instance, in terms of expected remaining life at the age of death (variants of which are discussed, for example, by Emanuel et al. 2020 and Reddy 2020). A relevant consideration in this context would be to base ‘value-of-life’ weighting functions on certain aggregate population health indicators, such as Quality Adjusted Life Years (QALYs), or Disability Adjusted Life Years (DALYs), or Standard Expected Years of Life Lost (SEYLL). These are certainly declining functions of age, but being – so to speak – furnished by ‘Nature’, they are not otherwise constrained by any regularity restriction on their curvature (such as strict convexity) that might be imposed by an ethical sensibility evaluating the extent of aversion to young deaths. One way of addressing the issue is discussed, in a spirit of some tentativeness, in what follows.

For specificity, let us consider information on age-specific expectations of life. Country-level data on expected years of life left for different age-groups are available in the World Health Organization’s compilation of life tables (see https://www.who.int/data/gho/data/indicators/indicator-details/GHO/gho-ghe-life-tables-by-country). Expected years of life remaining at age \( a \), \( E(a) \), can be estimated from the data as a linear equation: \( E(a) = A - ba \), where \( A \) and \( b \) are the estimated (positive) intercept and slope coefficients of the regression equation. If \( a_0 \) is the age of the \( j \)th youngest age category, then the suggestion is to have a family of age weighting functions such that, for the \( j \)th youngest
age, the weight is given by $u_{ij} = (A - ba_j)^\beta$, $\beta \geq 0$. (We are assuming here that the expected remaining years of life is strictly positive for every age category.)

The parameter $\beta$ (like the parameter $\alpha$ in the weighting function employed in Section 4.1) is a measure of one’s aversion to death at a young age, with such aversion seen as increasing with $\beta$. This is analogous to the ‘inequality-aversion’ parameter employed by Foster, Greer, and Thorbecke. (1984) in their formulation of a class of poverty indices, which is mimicked, via $\beta$, in the weighting function just advanced, to yield the foundation for a class of ethically flexible mortality indices.

Starting with an illfare function given by $L_d(N, D) = \sum_{j=1}^{K} (A - ba_j)^\beta d_j$, it can be verified that the corresponding ‘adjusted’ CFR, $\hat{C}^e_M$, is just the (raw) CFR $C$ for $\beta = 0$; and that it satisfies the Monotonicity and Population Neutrality axioms for $\beta \geq 0$, the Aversion to Young Deaths Axiom for $\beta \geq 1$, and the Aversion-Sensitivity Axiom for $\beta \geq 2$.

Some questions remain. Should the weight assigned to any given age be the same for all countries? If this is seen as being a desirable requirement, then we may wish to estimate the linear regression equation $E(a) = A - ba$ for some ‘representative’ country which will stand in for all countries. One could, for example, choose Japan as the representative country, by virtue of its being the country with the best recent record of life expectancy. Alternatively, should the equation $E(a) = A - ba$ be estimated separately for each country, employing data specific to the country in question? These are left here as open questions in what is admittedly a sketchy and tentative proposal.

### 4.3. Un-discussed issues

Section 3 has a small list of four axioms which it is suggested it would be desirable for a mortality indicator to satisfy. It appears that an ‘adjusted’ Case Fatality Rate derived from a mortality illfare function based on a normalized age-weighting function that is positive, declining in age, and strictly convex should satisfy the four specified mortality axioms. But there must be many illfare functions of the type just described and therefore many real-valued ‘adjusted’ mortality indicators that satisfy the four stated axioms. How does one choose from among these measures? The set of eligible candidate indicators can presumably be contracted by a combination of additions to, and strengthening of, the list of desirable axioms; and it would be instructive to have a lean and persuasive axiom system with which to uniquely characterize a mortality indicator. Again, one can ask: if $M^*$ is the set of mortality measures that satisfy the four axioms invoked in this paper, can one think of some criterion (analogous, say, to that of Lorenz-dominance in the standard inequality measurement literature) in terms of which it would be possible to pronounce that all members of $M^*$ would agree in their ranking of alternative situations? The suggested quest is for some appropriate ‘unanimity quasi-ordering’. These issues are raised here only in order to point out they are not addressed in this paper, as they would need to be in any more complete and satisfactory account of the problem than this essay furnishes. This makes the present paper incomplete, and deficient in failing to present a more rather than less exhaustive treatment of its subject. But it is hoped that this does not entirely negate the utility of this work, and that other researchers will find it worth their while to supply its shortcomings.

The next section presents a small data-based application of the measure $\hat{C}^e_M$ discussed in Section 4.1.

### 5. An illustrative empirical example: comparing India with seven other low-mortality countries

Before proceeding, please note that what follows is entirely illustrative, based on data for July/August 2020, and does not claim to be reflective of some contemporaneous empirical reality.

Table 1 presents data on the age-distribution of cases and deaths, and aggregate raw CFRs for 8 countries: South Africa, Argentina, South Korea, India, the Philippines, Japan, Colombia and Switzerland. These are countries for which the age-distributions of covid cases and deaths have been collected or estimated in Table 7 of Philip, Ray, and Subramanian (2020), and for which the aggregate CFR, as of August 31, 2020, was relatively ‘low’, that is, below 5%. What happens to the magnitudes and rankings of these countries’ mortality when their fatality rates are adjusted for age-distributions?

This is reflected in the ensuing computations of Tables 2 and 3, for which we have made the following assumptions. First, we assume that the age-distributions of cases and deaths reported in Table 1 are inter-temporally stable and, in particular, valid for August 31, the date for which the country-specific CFRs reported are valid. Second, we assume that all cases and deaths are concentrated at the mid-point age within each age bracket. Third, we take the mid-point age in the last, open-ended age-bracket to be 84.5 years. We do not have any deep justification for these assumptions; but we cannot proceed without some specificity on these issues, and it seems to us that the particular assumptions we have made are not wildly implausible.
In Table 2 we have estimates of both the raw CFR and the measure $C_{e}^{\alpha}$ computed for $\alpha = 1$, which is the smallest (or most moderately distribution-sensitive) integer value of $\alpha$ for which all of the axioms considered in Section 2.3 are satisfied. Tables 1 and 2 present a number of noteworthy features about the mortality rates of the countries under comparison. The raw CFR for India is quite close to that for Korea (1.78% and 1.62% respectively), and is less than one-half of Switzerland’s CFR (4.11%). This pattern is even more strikingly evident in the comparison of the Philippines with South Korea and Switzerland. This is surprising, because one would imagine that the health infrastructure is far more well-developed in both South Korea and Switzerland than in India and the Philippines.

The assessment of differential mortality in the four countries however begins to change once we make corrections for the age-distributions of cases and mortalities, and allow for declining weights on mortality as the age at mortality increases. Specifically, from Table 2 we note that when mortality is measured by the index $C_{e}^{1}$, India’s mortality is more than one-and-a-half times that of South Korea and more than one-half of the way to catching up with Switzerland’s. In fact, in

**Table 2. The CFR, the ‘Adjusted’ Mortality Rate $C_{e}^{1}$, and the ‘Efficiency Loss’ Parameter $Q$ for Eight Countries.**

| Country  | CFR   | $C_{e}^{1}$ | Efficiency Loss Parameter Q | Rank in Descending Order of CFR | Rank in Descending Order of $C_{e}^{1}$ | Rank in Descending Order of Q | Ratio of India’s CFR to Row Country’s CFR | Ratio of India’s $C_{e}^{1}$ to Row Country’s $C_{e}^{1}$ |
|----------|-------|-------------|-----------------------------|----------------------------------|----------------------------------------|-------------------------------|-------------------------------------------|--------------------------------------------------|
| S.Korea  | 0.0162| 0.0183      | 0.1296                      | 1                                | 1                                      | 2                             | 1.10                                      | 1.53                                             |
| Philippines | 0.0162| 0.0270      | 0.6667                      | 1                                | 3                                      | 8                             | 1.10                                      | 1.03                                             |
| India    | 0.0178| 0.0279      | 0.5674                      | 3                                | 5                                      | 7                             | 1.00                                      | 1.00                                             |
| Japan    | 0.0188| 0.0209      | 0.1117                      | 4                                | 2                                      | 1                             | 0.95                                      | 1.35                                             |
| Argentina| 0.0209| 0.0270      | 0.2919                      | 5                                | 3                                      | 3                             | 0.85                                      | 1.03                                             |
| S.Africa | 0.0224| 0.0320      | 0.4286                      | 6                                | 6                                      | 6                             | 0.80                                      | 0.87                                             |
| Colombia | 0.0319| 0.0415      | 0.3009                      | 7                                | 7                                      | 4                             | 0.56                                      | 0.67                                             |
| Switzerland | 0.0411| 0.0535     | 0.3017                      | 8                                | 8                                      | 5                             | 0.43                                      | 0.52                                             |

Spearman’s $\rho$ between rankings according to CFR and $C_{e}^{1} = 0.7976$.
Spearman’s $\rho$ between rankings according to CFR and $Q = (-).1548$.
Computations based on Data in Table 1.

Table 1. Age-Specific Proportions of Cases and Deaths, and Aggregate CFRs for 8 ‘Low Covid-Mortality’ Countries.

| Age-Group→ | 0- | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80+ | Total |
|------------|----|-------|-------|-------|-------|-------|-------|-------|-----|-------|
| India      | .036 | .081 | .215 | .210 | .168 | .142 | .099 | .038 | .011 | 1     |
| Deaths     | .008 | .009 | .024 | .055 | .135 | .24  | .304 | .157 | .067 | 1     |
| CFR        | .0178|     |      |      |      |      |      |      |     |       |
| South Korea| .17 | .054 | .252 | .127 | .135 | .176 | .3  | .066 | .042 | 1     |
| Deaths     | .0  | .0  | .0  | .007 | .010 | .053 | .136 | .299 | .495 | 1     |
| CFR        | .0162|     |      |      |      |      |      |      |     |       |
| Philippines| .27 | .045 | .252 | .237 | .164 | .134 | .086 | .040 | .015 | 1     |
| Deaths     | .15 | .011 | .025 | .047 | .096 | .195 | .285 | .220 | .109 | 1     |
| CFR        | .0162|     |      |      |      |      |      |      |     |       |
| South Africa| .28 | .042 | .195 | .283 | .211 | .138 | .061 | .028 | .015 | 1     |
| Deaths     | .04 | .002 | .007 | .057 | .106 | .250 | .265 | .196 | .114 | 1     |
| CFR        | .0224|     |      |      |      |      |      |      |     |       |
| Japan      | .18 | .038 | .281 | .171 | .141 | .130 | .082 | .069 | .071 | 1     |
| Deaths     | 0   | 0    | 0    | .001 | .004 | .014 | .033 | .106 | .273 | .569  |
| CFR        | .0188|     |      |      |      |      |      |      |     |       |
| Argentina  | .47 | .069 | .203 | .230 | .183 | .125 | .066 | .037 | .040 | 1     |
| Deaths     | .02 | .002 | .008 | .021 | .080 | .179 | .245 | .396 |     |       |
| CFR        | .0209|     |      |      |      |      |      |      |     |       |
| Colombia   | .37 | .064 | .219 | .237 | .164 | .132 | .079 | .042 | .027 | 1     |
| Deaths     | .02 | .001 | .013 | .032 | .073 | .142 | .231 | .251 | .257 | 1     |
| CFR        | .0319|     |      |      |      |      |      |      |     |       |
| Switzerland| .08 | .036 | .144 | .143 | .155 | .195 | .116 | .086 | .118 | 1     |
| Deaths     | .01 | 0    | 0    | .003 | .004 | .024 | .076 | .203 | .690 | 1     |
| CFR        | .0411|     |      |      |      |      |      |      |     |       |

Source: Data on case and death distributions are from Table 7 of Philip, Ray, and Subramanian (2020); the aggregate CFR for each country as on August 31 is from OurWorldindata; and the age-distributions of cases and deaths are assumed to be valid for August 31.

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Table 3. The CFR, the ‘Adjusted’ Mortality Rate $\tilde{C}_1^e$, and the ‘Efficiency Loss’ Parameter $Q$ for Eight Countries.

| Country | CFR   | $\tilde{C}_1^e$ | Efficiency Loss Parameter $Q$ | Rank in Descending Order of CFR | Rank in Descending Order of $\tilde{C}_1^e$ | Rank in Descending Order of $Q$ | Ratio of India’s CFR to Row Country’s CFR | Ratio of India’s $\tilde{C}_1^e$ to Row Country’s $\tilde{C}_2^e$ |
|---------|-------|-----------------|-------------------------------|--------------------------------|--------------------------------|--------------------------------|---------------------------------|---------------------------------|
| S.Korea | 0.162 | 0.0213          | 0.3148                        | 1                              | 1                              | 2                              | 1.10                            | 3.36                            |
| Philippines | 0.162 | 0.0739          | 3.5617                        | 1                              | 7                              | 8                              | 1.10                            | 0.97                            |
| India   | 0.178 | 0.0716          | 3.0225                        | 3                              | 6                              | 7                              | 1.00                            | 1.00                            |
| Japan   | 0.188 | 0.0238          | 2.6660                        | 4                              | 2                              | 1                              | 0.95                            | 3.01                            |
| Argentina | 0.209 | 0.0485          | 1.3206                        | 5                              | 3                              | 5                              | 0.85                            | 1.48                            |
| S.Africa | 0.224 | 0.0620          | 1.7679                        | 6                              | 4                              | 6                              | 0.80                            | 1.16                            |
| Colombia | 0.0319 | 0.0706          | 2.1322                        | 7                              | 5                              | 4                              | 0.56                            | 1.05                            |
| Switzerland | 0.0411 | 0.0747 | 0.8176 | 8 | 8 | 3 | 0.43 | 0.96 |

Spearman’s $\rho$ between rankings according to CFR and $\tilde{C}_1^e = 0.2738$.
Spearman’s $\rho$ between rankings according to CFR and $Q = (-).2976$.
Computations based on Data in Table 1.

the transition from CFR to $\tilde{C}_1^e$, India’s relative performance vis-à-vis every country other than the Philippines worsens. In the case of the Philippines, its CFR is the same as South Korea’s, but its $\tilde{C}_1^e$ value is nearly one-and-a-half times that of South Korea; and in moving from CFR to $\tilde{C}_1^e$, the Philippines’ mortality rate as a fraction of Switzerland’s, rises from 39% to 50%.

While India’s CFR rank is third in a list of eight countries, its rank according to $\tilde{C}_1^e$ slips down to fifth; the Philippines’ rank slips from 1 to 3, while that of Japan rises from 4 to 2, and of Argentina’s rises from 5 to 3. In pairwise comparisons, we find that both India and the Philippines undergo a rank-reversal, to their disadvantage, vis-à-vis two countries (Japan and Argentina). Overall, country rankings by CFR and country rankings by $\tilde{C}_1^e$ are not the same: Spearman’s rank correlation coefficient between the two sets of rankings is 0.80. What is most striking is the difference in ranking by CFR and ranking by the efficiency loss parameter $Q$: here, Spearman’s rank correlation coefficient is actually negative, at (-)0.16, suggesting that low ‘average’ mortality can be concealed by its considerably sub-optimal age-wise distribution. Everything considered, for the sample of countries just reviewed, the picture of comparative covid mortality is not the same under measurement by $\tilde{C}_1^e$ from what it is under measurement by CFR.

As one might expect, the differential mortality picture becomes more pronouncedly apparent when measurement is undertaken for larger values of $\alpha$. If one’s ethical intuition suggests that the valuation placed on a life should fall away more steeply with age than is allowed for by setting $\alpha$ at 1 in the weighting function of Equation (1), one may opt for $\alpha = 2$. Table 3 presents the results on comparative mortality for the same set of eight countries as in Table 2, but for $\alpha = 2$ instead of for $\alpha = 1$. While avoiding detailed comments on the figures in Table 3, it is worth noting that while countries like the Philippines and India are at the top of the ranking when mortality is measured by CFR, they are close to the bottom when we resort to measurement by $\tilde{C}_2^e$. Also noteworthy is the fact that while the rankings according to the efficiency loss parameter $Q$ under both $\tilde{C}_1^e$ and $\tilde{C}_2^e$ are negatively correlated with the ranking according to CFR, the rankings of $Q$ under $\tilde{C}_1^e$ and $\tilde{C}_2^e$ are quite similar to each other: Spearman’s $\rho$ for the two sets of rankings turns out to be 0.9048.

4. Concluding observations

We do not attempt a comprehensive summary of this paper, but would like to conclude on the note that a more complete and more accurate picture of COVID-mortality would be available if the central-tendency statistic of the case fatality rate were to be supplemented with information on the dispersion of COVID cases and deaths by age. This paper has been concerned with advancing such a more comprehensive measure of mortality, in the expectation that it will furnish a more meaningful picture of cross-country and time-series mortality comparisons than is presently the case.

Notes

1. It is, in fact, invariant with respect to any strictly increasing transformation of the illfare function.
2. It is, of course, well known that Sen’s indicator of real national income is just the equally distributed equivalent income reckoned for a Borda rank-order weighted social welfare function.
3. For suggestive information, see Goldstein, Cassidy, and Walker (2021).

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