Method to extract the primary cosmic ray spectrum from very high energy $\gamma$-ray data and its application to SNR RX J1713.7-3946

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Abstract

Supernova remnants are likely to be the accelerators of the galactic cosmic rays. Assuming the correctness of this hypothesis, we develop a method to extract the parent cosmic ray spectrum from the VHE gamma ray flux emitted by supernova remnants (and other gamma transparent sources). Namely, we calculate semi-analytically the (inverse) operator which relates an arbitrary gamma ray flux to the parent cosmic ray spectrum, without relying on any theoretical assumption about the shape of the cosmic ray and/or photon spectrum. We illustrate the use of this technique by applying it to the young SNR RX J1713.7-3946 which has been observed by H.E.S.S. experiment during the last three years. Specific implementations of the method permit to use as an input either the parameterized VHE gamma ray flux or directly the raw data. The possibility to detect features in the cosmic rays spectrum and the error in the determination of the parent cosmic ray spectrum are also discussed.

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1 Introduction

There is no doubt that the main part of cosmic rays (CR) till the knee is produced in the Milky Way [1], and it seems fair to say that the favored site for CR production are the young supernova remnants (SNR). In fact, the turbulent gas of SNR is a large reservoir of kinetic energy [2] and this environment can support diffusive shock waves acceleration [3]. In recent times, great progresses have been made both in the observation and in the understanding of SNR. In particular, the new generation imaging Cherenkov telescopes, in particular H.E.S.S. [4], allowed to observe the very high energy (VHE) gamma rays emitted by SNR, which are possibly generated by the decay of $\pi^0$ and $\eta$ produced by collision between the accelerated hadrons and the surrounding gas. It is not yet possible, however, to exclude that (part of) the observed radiation is produced by electromagnetic processes. In order to reach a definitive proof of the hadronic origin of the VHE gamma radiation, more detailed studies are needed. In this respect, new data at high (100 TeV or larger) and low ($E_{\gamma} \sim m_{\pi}/2$) energies, improved theoretical modeling and possibly observations of VHE neutrinos (see e.g., [5]) will be extremely important.

The hypothesis that VHE gamma radiation from young SNR originates from hadronic processes deserves the most serious attention and consideration. New and crucial observations are being collected and the hadronic origin seems to be favored for certain SNR, such as Vela Jr [6] and RX J1713.7-3946 [7]. In this paper we take the hadronic origin has a working hypothesis and we address the question of what we learn on SNR cosmic ray spectra from VHE $\gamma$-ray data. This question has a precise quantitative character and we answer it in the most direct way. Namely, we calculate semi-analytically the (inverse) operator which relates an arbitrary gamma ray flux to the parent cosmic ray spectrum, without relying on any theoretical assumption about the shape of the cosmic ray and/or photon spectrum. We then illustrate the possible applications of our method by considering the H.E.S.S. data of RX J1713.7-3947 that reached an impressive accuracy in the energy range from 300 GeV to 300 TeV [9]. We remark that in this case (and, more in general, whenever the source shows non trivial spectral features) the approximation of power law distribution and the many techniques of calculations tailored to this assumption are not adequate.

The plan of the paper is as follows. In Sect. 2 we formulate the problem and we obtain a general, analytical solution. In Sect. 3 we consider possible applications of our results. First, we derive the parent cosmic ray flux of RX J1713.7-3946 by using suitable parameterizations of the gamma ray data. Then, we extract the information directly from the observational data. This

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1 See also [8] for a recent analysis leading to different conclusions.
second technique requires fewer assumptions and allows to propagate the observational errors easily. However, when applied to noisy data it requires a sort of image processing (Gaussian smearing) to produce a reasonable result. In Sect. 4 we summarize our results, putting emphasis on the possible applications of our method.

2 How to invert the relation between the photon and the CR spectrum

2.1 Formulation of the problem

We assume that the VHE photon flux in SNR has a hadronic origin, i.e., gamma-ray are produced by a flux of high energy cosmic ray protons interacting with an hydrogen ambient cloud having density \( n \). Inelastic proton-proton interactions result in the production of \( \pi^0 \) and \( \eta \)-mesons which subsequently decay producing gamma-rays. It is important to note that SNR are ‘transparent targets’ for cosmic rays, as can be understood by very simple estimates. The column density of the system is indeed much smaller than the TeV-proton and photon interaction lengths (\( \lambda_p \equiv m_p/\sigma \sim 40 \text{ gr/cm}^2 \) and \( X_0 \sim 60 \text{ gr/cm}^2 \), being:

\[
dz = n \text{ dl } m_p = 1.5 \times 10^{-3} \text{ gr/cm}^2
\quad \text{for } n = 100 \text{ prot./cm}^3, \text{ dl} = 3 \text{ pc}.
\]

The possibly overestimated value for \( n \) corresponds to proton number density in a typical molecular cloud that could be associated to the SNR, and the distance \( dl \) is the one covered in 1,000 yr at a speed of 3,000 km/s. In other words, the proton and photon interaction probabilities (equal to \( dz/\lambda_p \) and \( dz/X_0 \) respectively) are \( 10^{-5} \) or smaller, so that proton multiple interactions and/or re-absorption of the produced photons are absent for the typical conditions of a young SNR.

The gamma-ray flux \( \Phi_{\gamma}[E_{\gamma}] \) produced on a detector placed at a distance \( R \) by cosmic ray protons interacting with a ‘transparent’ medium can be written as:

\[
\Phi_{\gamma}[E_{\gamma}] = \frac{c}{4\pi R^2} \int d^3 r \ n[r] \int_{E_{\gamma}}^{\infty} dE_p \frac{dn_p[r,E_p]}{dE_p} \frac{d\sigma_{\gamma}[E_p,E_{\gamma}]}{dE_{\gamma}}
\]

where \( d\sigma_{\gamma}/dE_{\gamma} \) is the inclusive cross-section for \( \gamma \) production. Here \( n \) and \( dn_p/dE_p \) are the target hydrogen number density and the cosmic ray proton number density (per unit energy)

\[
\frac{1}{4\pi} \frac{dn_p[r,E_p]}{dE_p} \rightarrow \frac{dn_p[r,E_p,n]}{d\Omega_p dE_p}
\]

where \( dn_p/d\Omega_p dE_p \) is the CR proton number density per unit energy and unit solid angle, \( n \) is the unit vector in the direction connecting the SNR to the detector and we have taken into account that the produced photons are almost collinear with CR protons.
respectively. Both of them depend on the position inside the SNR, indicated by the coordinate vector $r$.

Next, we adopt the usual definition of the adimensional distribution function $F_\gamma [x, E_p]$, according to which:

$$
\frac{d\sigma_\gamma}{dE_\gamma} = \frac{\sigma[E_p]}{E_p} F_\gamma \left[ \frac{E_\gamma}{E_p}, E_p \right]
$$

where $\sigma$ is the total inelastic p-p cross-section given by (10):

$$
\sigma[E_p] = 34.3 + 1.88 \ln(E_p/1\text{TeV}) + 0.25 \ln[E_p/1\text{TeV}]^2 \text{mb}
$$

Hadronic interactions are affected by quite large uncertainties and independent calculations of $F_\gamma [x, E_p]$ may differ at the 20% level (12). In this work, we use a simple analytic formula presented in (10) (see appendix A) which describes the results of public available SIBYLL code (13) with a few per cent accuracy over a large region of the parameter space ($x \geq 10^{-3}, E_p > 100\text{GeV}$). By using rel. (2), we can rewrite eq. (1) as:

$$
\Phi_\gamma[E_\gamma] = \int_{E_\gamma}^{\infty} \frac{dE_p}{E_p} \Phi_p[E_p] F_\gamma \left[ \frac{E_\gamma}{E_p}, E_p \right]
$$

where we introduce the important quantity:

$$
\Phi_p[E_p] = \frac{c \sigma[E_p]}{4\pi R^2} \int d^3r \ n[r] \frac{dn_p[r, E_p]}{dE_p}
$$

The function $\Phi_p[E_p]$ is the quantity which is constrained by and most directly related to the VHE gamma ray observations. It has the dimensions of a differential flux, and below we will use $\text{cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}$. In the following, we will refer to $\Phi_p[E_p]$ as the effective cosmic ray flux from the SNR and we will show how this quantity can be calculated starting from the photon flux $\Phi_\gamma[E_\gamma]$.

When comparing with theoretical predictions, one should note that the effective CR flux encodes not only the energy distribution of cosmic ray protons but also the (weak) energy dependence of the cosmic ray interaction probability in the SNR. We note, in fact, that eq. (5) can be rewritten as:

$$
\Phi_p[E_p] = \frac{c N \sigma[E_p]}{4\pi R^2} J_p[E_p]
$$

where $N = \int d^3r \ n[r]$ is the total amount of target hydrogen in the observed system and $J_p[E_p]$ given by:

$$
J_p[E_p] = \frac{1}{N} \int d^3r \ n[r] \frac{dn_p[r, E_p]}{dE_p}
$$

is the weighted average the CR energy distribution in the SNR with a weight function proportional to the target hydrogen distribution. A part from the constant term $c N/(4\pi R^2)$ (which
can be deduced if independent information on the SNR distance and on the amount of target hydrogen are given), the two functions $\Phi_p[E_p]$ and $J_p[E_p]$ differs by the energy-dependent factor $1/\sigma[E_p]$. It should be noted that the cross section is slowly varying with energy, so that the main spectral features of $\Phi_p[E_p]$ (such as the position and the sharpness of a cutoff/transition) always reflects the spectral features of $J_p[E_p]$. In the region where the spectrum is approximated by a power law, the energy dependence of $\sigma[E_p]$ accounts for a small difference between the spectral indices of $J_p[E_p]$ and $\Phi_p[E_p]$ which can be easily quantified being of the order of $d\ln \sigma/d\ln E_p \sim 0.07$ in the energy range of interest. It is clear that the above formalism can be applied to any gamma transparent source (not only to SNR) where the VHE gamma are of hadronic origin, such as a SNR-molecular cloud association [11].

The question of what can we learn on the effective cosmic ray flux $\Phi_p$ from $\Phi_\gamma$ boils down to the task of inverting an integral equation (eq. (4)). In the rest of Sect. 2, we argue that, assuming a quasi-scaling behavior of hadronic cross sections (accurate at the few percent level), this problem can be solved in good approximation by applying the differential operator:

$$D = \sum_{n=0}^{5} a_n \left( \frac{E dE}{dE} \right)^n$$

where $a_n$ are appropriate numerical coefficients given in sect. 2.3. Stated more clearly, we claim that the approximate inverse of eq. (1), symbolically written as $\Phi_\gamma = F[\Phi_p]$, is simply given by $\Phi_p \approx D[\Phi_\gamma]$. This result does not rely on any theoretical assumption about the shape of the cosmic ray and/or photon spectrum. We thus provide a simple method to extract and study possible spectral features of the parent cosmic ray flux in the SNR directly from the observed VHE gamma radiation.

Finally, to help the readers who are more interested in applications than in the formal derivation of our results, we anticipate that, to tackle the mathematical problem, it was necessary to introduce a number of definitions. In this paper, we indicate the natural logarithm of proton and photon energies with the symbols $\varepsilon_p$ and $\varepsilon_\gamma$ (see eq. (10) in the next section). Moreover, it is convenient to multiply the cosmic ray and the photon fluxes by a power laws in energy according to $\varphi_p = \Phi_p \cdot (E_p/1\text{TeV})^\alpha$ and $\varphi_\gamma = \Phi_\gamma \cdot (E_\gamma/1\text{TeV})^\alpha$, where $\alpha$ is an appropriate coefficient. We remark that the “fluxes” $\varphi_p$ and $\varphi_\gamma$ are related by a differential operator of the kind (8) for any value of $\alpha$. The values of the numerical coefficients $a_n$ can be easily calculated for any adopted value for $\alpha$ (see eq. (20) and related discussion).

### 2.2 Notation and quasi scaling approximation

It is useful to perform some changes of variables and rewrite the integral (4) as:

$$\varphi_\gamma[\varepsilon_\gamma] = \int_{-\infty}^{\infty} d\varepsilon_p \varphi_p[\varepsilon_p] f[\varepsilon_\gamma - \varepsilon_p, \varepsilon_p]$$

(9)
where proton and photon energies are expressed in terms of the variables:

\[ \varepsilon_i = \ln \left[ \frac{E_i}{1 \text{ TeV}} \right], \quad i = p, \gamma \]  

The fluxes are rewritten in terms of:

\[
\begin{align*}
\varphi_p[\varepsilon_p] &= \Phi_p[e^{\varepsilon_p}] e^{\alpha \varepsilon_p}, \\
\varphi_\gamma[\varepsilon_\gamma] &= \Phi_\gamma[e^{\varepsilon_\gamma}] e^{\alpha \varepsilon_\gamma},
\end{align*}
\]

and the integral kernel \( f[y, \varepsilon_p] \) is defined by:

\[ f[y, \varepsilon_p] = \theta[-y] \cdot e^{\alpha y} \cdot F_\gamma[e^y, e^{\varepsilon_p}] \]

The Heaviside function is introduced in order to integrate over the entire real axis. The inclusion of the exponential factors in definitions (11) and (12) is, instead, motivated by the fact that the photon and CR proton spectra are expected to decrease approximately as power laws in energy. For a proper choice of the parameter \( \alpha \), the functions \( \varphi_i = \Phi_i \cdot (E_i/1 \text{ TeV})^\alpha \) are, thus, expected to be nearly constant in the relevant energy range, highlighting the deviations from the pure power-law behavior. In the following, we find it convenient to set the value:

\[ \alpha = 2.5 \]  

which is particularly appropriate for the analysis of the H.E.S.S. gamma ray spectrum of the RX J1713.7-3946 supernova remnant. We remark that the “fluxes” \( \varphi_i[\varepsilon_i] \) and the integral kernel \( f[y, \varepsilon_p] \) defined above are easily tractable, since they are square-integrable in \( \varepsilon_i \) and \( y \) respectively.

In Fig. 1, we show the behavior of the integral kernel \( f[y, \varepsilon_p] \) as a function of \( y \) for selected values of \( \varepsilon_p \). We see that the function \( f[y, \varepsilon_p] \) is peaked at \( y = -1.8 \) (which corresponds to \( E_p/E_\gamma = \exp[-y] \approx 6 \)) and that it is marginally dependent on the assumed proton energy. In the following, we assume a quasi-scaling behavior for hadronic cross sections, i.e., we neglect the dependence of \( f[y, \varepsilon_p] \) on \( \varepsilon_p \) and replace:

\[ f[y, \varepsilon_p] \rightarrow f[y] \equiv f[y, \varepsilon_p^0] \]

where \( \varepsilon_p^0 \) is a fixed reference value for the proton energy. We have chosen the value \( \varepsilon_p^0 = 6.9 \) (i.e., \( E_p^0 = 1000 \text{ TeV} \)) which is appropriate to calculate the gamma ray flux in the energy region \( E_\gamma \approx 1 - 1000 \text{ TeV} \) probed by the H.E.S.S. experiment. Our calculations and Fig. 1 show that the quasi-scaling approximation is adequate at the few percent level.

### 2.3 Formal solution of the inverse problem

In the quasi-scaling approximation, we can invert the relation between the effective CR proton flux and the photon flux (a Volterra integral equation of the first type) by a simple semi-analytical method which gives very precise results. We obtain, in fact, a convolution integral:

\[ \varphi_\gamma[\varepsilon_\gamma] = \int_{-\infty}^{\infty} d\varepsilon_p \varphi_p[\varepsilon_p] f[\varepsilon_\gamma - \varepsilon_p] \]
which can be treated by using standard techniques, such as Fourier analysis, finding
\[ \varphi_p[k] = \frac{1}{f[k]} \varphi_\gamma[k] \]  \hspace{1cm} (16)
where \( f[k] \), \( \varphi_\gamma[k] \) and \( \varphi_p[k] \) are the Fourier transforms of the functions \( f[y] \), \( \varphi_\gamma[\varepsilon_\gamma] \) and \( \varphi_p[\varepsilon_p] \) respectively. We note that the inclusion of the exponential factor in the definition of the \( \varphi_i[\varepsilon_i] \) ensures that the Fourier transforms \( \varphi_\gamma[k] \) and \( \varphi_p[k] \) exist.

The function:
\[ h[k] \equiv \frac{1}{f[k]} \]  \hspace{1cm} (17)
deﬁnes, in Fourier space, the operator which inverts eq. (15). We see from Fig. 2 that \( \text{Abs}[h[k]] \) is fast increasing with \( |k| \). This can be understood in simple terms by noting that the integral kernel \( f[y] \) has a half-width-half-maximum equal approximately to \( \delta y \simeq 1.0 \) (see Fig. 1). Correspondingly, the Fourier transform \( f[k] \) has a characteristic width \( \delta k \simeq 1/(2\pi\delta y) \sim 0.16 \) and its inverse function \( h[k] \) has a sharp increase for \( |k| \geq \delta k \). In physical terms, this has the important consequence that any feature in the photon spectrum on scales smaller than \( \delta \varepsilon_\gamma \leq \delta y \) will be greatly amplified in the parent CR proton spectrum (more discussion in the next section).

The behavior of \( h[k] \) at large \( k \) depends on the regularity of \( f[y] \) and its derivatives. In this case, we can expand \( h[k] \) to ﬁfth order in a Taylor series:
\[ h[k] \simeq \sum_{j=0}^{5} h_j k^j \]  \hspace{1cm} (18)

\(^3\) We calculate Fourier transforms according to the standard definition: \( \varphi[\varepsilon] = \int dk \varphi[k] \exp[2\pi ik\varepsilon] \).

\(^4\) The functions \( \varphi_i[\varepsilon_i] \) decrease exponentially for \( |\varepsilon_i| \to \infty \) provided that the differential energy spectra \( \Phi_i[E_i] \) decrease slower than \( E_i^{-\alpha} \) at low energy and faster than \( E_i^{-\alpha} \) at high energy.
where \( h_j = (1/j!) \frac{d^j h}{dk^j}|_{k=0} \), with a few per cent accuracy in the relevant range \(|k| < 2.5\). We remark that the photon flux \( \varphi_\gamma[\varepsilon_\gamma] \) is sampled by H.E.S.S. experiment in bins \( \delta \varepsilon_\gamma = \delta E_\gamma/E_\gamma \simeq 0.2 \) or larger and, thus, only the region \(|k| < 1/(2\delta \varepsilon_\gamma) = 2.5 \) carries physical information.

The expansion ([18]) allows to express the parent proton spectrum as a function of the photon flux and its derivatives. By exploiting the properties of Fourier transforms one easily obtains:

\[
\varphi_p[\varepsilon_p] = \sum_{j=0}^{5} a_j \frac{d^j \varphi_\gamma}{d \varepsilon_\gamma^j}[\varepsilon_\gamma = \varepsilon_p]
\]  

(19)

where \( a_j = h_j / (2\pi i)^j \). The coefficients \( a_j \) depend on the value of the parameter \( \alpha \) adopted in eqs. ([11],[12]). In our case (\( \alpha = 2.5 \)) the relevant coefficients are given by \( a_0 = 20.85, a_1/a_0 = -2.336, a_2/a_0 = 2.113, a_3/a_0 = -0.9034, a_4/a_0 = 0.1718 \) and \( a_5/a_0 = -9.79 \cdot 10^{-3} \). For a different choice \( \alpha \to \alpha - \beta \), the coefficients \( a_i \) have to be replaced by:

\[
a_i \to \sum_{j=i}^{5} \frac{j! \beta^{j-i}}{i! (j-i)!} a_j
\]

(20)

For instance, if we set \( \beta = \alpha \) which corresponds to the particular situation considered in eq. ([18]) (i.e., \( \varphi_p = \Phi_p \) and \( \varphi_\gamma = \Phi_\gamma \)), we immediately obtain \( a_0 = .1148, a_1 = 2.390, a_2 = 5.205, a_3 = 4.225, a_4 = 1.031 \) and \( a_5 = -0.2041 \).

The above equations are the main results of this paper and, in the next section, we will discuss the possible applications to real data. Here, we note that rel. ([19]) is remarkably simple if the photon spectrum can be approximated by a power law, i.e., \( \Phi_\gamma \propto E_\gamma^{-\Gamma} \) or, equivalently, \( \varphi_\gamma \propto \exp[(\alpha-\Gamma)\varepsilon_\gamma] \). We obtain, in fact: \( \varphi_p[\varepsilon_p] = \mathcal{Y}[\Gamma] \varphi_\gamma[\varepsilon_p] \) with \( \mathcal{Y}[\Gamma] \equiv \sum_{j=0}^{5} a_j (\alpha-\Gamma)^j \) which shows that the ratio between the effective cosmic ray flux and the photon flux at a fixed energy is given by the function \( \mathcal{Y}[\Gamma] \) which only depends on the photon spectral index \( \Gamma \). The function \( \mathcal{Y}[\Gamma] \) can be compared with the spectrum weighted moments \( Z[\Gamma] \) displayed in Fig. 5.5 of [14]. We obtain a good agreement by noting that, in the assumption of [14], one has \( Z[\Gamma] \simeq \Gamma/(2 \mathcal{Y}[\Gamma]).\)

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5 The physically significant range \(|k| < 2.5\) has been estimated by applying the sampling theorem.
Finally, we remind that the differential operator in the r.h.s. of eq. (19) is the inverse of the integral operator in eq. (15), which is obtained in the quasi-scaling assumption for hadronic cross-section. One can go beyond this approximation by using eq. (19) as zero-order solution and calculating perturbatively the effects of deviations from the quasi-scaling assumption. We used this approach to check that the corrections in the parent cosmic ray spectrum obtained from (19) are small in comparison to the hadronic cross-section uncertainties. We do not discuss the numerical implementation here to avoid unnecessary complications, but we provide the relevant details in the appendix B.

3 Applications

3.1 The young SNR RX J1713.7-3946

The RX J1713.7-3946 Supernova Remnant has been observed by H.E.S.S. during three years from 2003 to 2005 [9]. The $\gamma$-ray spectrum obtained by combining the observation of the three years is shown in Fig. 3. The data extend over three decades, exploring the energy interval $E_\gamma = 0.3 - 300$ TeV. The energy resolution of the experiment is equal to about 20% and the photon spectrum is sampled in 25 bins $\delta\varepsilon_\gamma = \delta E_\gamma / E_\gamma \simeq 0.2$ plus three larger bins at high energy.

The described data allow to obtain important conclusions, as discussed in [9]. First, they show that there is a significant emission at energy larger than 30 TeV, implying the existence of primary particles of at least that energy. Moreover, the data show a non trivial dependence on energy. In particular, there is a significant deviation from the simple power law behavior, as can be understood from Fig. 3. The solid line is the best fit power law spectrum with spectral index $\Gamma = 2.32$ that does not provides an acceptable fit of the data since $\chi^2$/d.o.f. $\sim 145.6/25$ (i.e., can be rejected at 9σ).

From a theoretical point of view, one expects that the photon spectrum can be described by power law at low energy with a “cutoff” above a certain energy $E_c$ related to the properties of the primary particles acceleration mechanism. This kind of behavior is usually parameterized in the form of a broken power law (BPL):

$$\Phi_\gamma = I \left( \frac{E_\gamma}{1 \text{TeV}} \right)^{-\Gamma_1} \left( 1 + \left( \frac{E_\gamma}{E_c} \right)^{1/S} \right)^{-S(\Gamma_2-\Gamma_1)}$$ \hspace{1cm} [BPL case] \hspace{1cm} (21)

where $\Gamma_1$ and $\Gamma_2$ are the low and high energy spectral indices and $S$ quantifies the sharpness of the transition from $\Gamma_1$ and $\Gamma_2$, or by an exponential cutoff (EC) with exponent $\beta$:

$$\Phi_\gamma = I \left( \frac{E_\gamma}{1 \text{TeV}} \right)^{-\Gamma} \exp \left[ - \left( \frac{E_\gamma}{E_c} \right)^{\beta} \right]$$ \hspace{1cm} [EC case] \hspace{1cm} (22)

To help readability, in Figs. 3-6 we use the logarithm to basis 10, denoted by log.
Figure 3: The $\gamma$-ray spectrum of the RX J1713.7-3946 Supernova Remnant obtained by H.E.S.S. experiment. The black line represents the best fit to the data in the assumption of a power law behavior of the photon spectrum. The blue lines are obtained by assuming that the photon spectrum follows the BPL given by eq. (21) in the assumption that the transition parameter is $S = 0.6$ (blue solid line) or $S = 0.45, 0.75$ (blue dotted lines). The red lines refers to the EC case, see eq. (22), in the assumption that $\beta = 0.5$ (red solid line) or $\beta = 1.0$ (red dotted line).

The value $\beta \sim 0.5$, which describes a relatively smooth cutoff, has been considered in [10, 15].

The H.E.S.S. gamma ray spectrum of the RX J1713.7-3946 supernova remnant has been fitted with a BPL with parameters $\Gamma_1 = 2.00 \pm 0.05$, $\Gamma_2 = 3.1 \pm 0.2$ and $E_c = 6.6 \pm 2.2$ obtaining a $\chi^2$/d.o.f. $\sim 29.8/23$ (see Fig. 3 blue solid line). It should be noted that the sharpness parameter $S$ was kept fixed in the fit, with an adopted value equal to $S = 0.6$. Different choices for $S$, however, are possible. As an example, equally good fits of the data are provided by the blue dashed line which corresponds to $S = 0.75$, $\Gamma_1 = 1.97$, $\Gamma_2 = 3.22$ and $E_c = 7.97$ and by the blue dotted line which corresponds to $S = 0.45$, $\Gamma_1 = 2.03$, $\Gamma_2 = 2.96$ and $E_c = 5.64$.

Alternatively, the H.E.S.S. data can be fitted with an EC with parameters $\Gamma = 1.79 \pm 0.06$, $E_c = 3.7 \pm 1.0$ and $\beta = 0.5$ as it is shown by the red solid line in Fig. 3 obtaining a $\chi^2$/d.o.f. $\sim 34.3/24$. For comparison, we also show with a red dotted line the best fit which obtained by assuming “pure” EC (i.e., $\beta = 1.0$). In this case, one obtains $\Gamma = 2.04 \pm 0.04$, $E_c = 17.9 \pm 3.3$ and a slightly worse fit to the data $\chi^2$/d.o.f. $\sim 39.5/24$ (i.e., a goodness of fit of 2.4%).

As discussed in [7, 9], the observed spectral shape seems to favor the hadronic origin. In the following, we assume the hadronic origin as a working hypothesis and we discuss what the data can tell us about the primary proton spectrum in RX J1713.7-3946.
Figure 4: **Left Panel:** The effective CR flux \( \Phi_p[\gamma_p] \) from the SNR RX J1713.7-3946 obtained from BPL and EC parameterizations of the gamma-ray flux measured by the H.E.S.S. experiment. The blue lines are obtained from the best-fit BPL parameterizations with sharpness parameter \( S = 0.6 \) (blue solid line), \( S = 0.45 \) (blue dotted line) and \( S = 0.75 \) (blue dashed line). The red lines correspond to best-fit EC parameterization with \( \beta = 0.5 \) (red solid line) and \( \beta = 1.0 \) (red dotted line). **Right Panel:** The CR energy distribution \( J[p] \) calculated from eq. (6) by assuming \( R = 1 \) kpc and \( N = 3.6 \times 10^{59} \).

### 3.2 Using parameterized fluxes

If we accept the BPL and EC parameterizations as reliable descriptions of the photon spectrum, we can calculate the effective CR flux from the SNR by simply applying eq. (19) to the functional forms (21) and (22). The results of this procedure are shown in Fig. 4 (left panel) where the blue lines are obtained from the BPL parameterizations of the photon flux, while the red lines refer to the EC case. We remind that the effective cosmic ray flux encodes not only the energy distribution of cosmic ray protons but also the (weak) energy dependence of the cosmic ray interaction probability in the SNR. For this reason we also show (right panel) the CR energy distribution in the SNR calculated according to eq. (6). We assume \( R = 1 \) kpc and \( N = 3.6 \times 10^{59} \) which corresponds to 300 solar masses of target hydrogen. This value is motivated if gamma ray emission is due to a molecular cloud-SNR association of the type proposed in [17] which seems to be consistent with the observations of NANTEN [18]; see [19] for a theoretical model. In this work, we do not aim to discuss the precise value of \( N \), which would only affect the normalization of the CR energy distribution. We focus instead on the CR spectral properties which are directly determined by the observed photon spectrum. We remark a few important points.

**Accuracy of the inversion.** The obtained CR fluxes can be used in rel. (15) in order to check the accuracy of the inversion method. In all cases, the re-calculated photon fluxes agree with the input photon flux \( (i.e., \text{adopted in rel. (19)}) \) at the level of few parts per thousand in the energy range \( E_\gamma = 1 - 1000 \) TeV. This show that the differential operator on the r.h.s. of eq. (19) is the
inverse of the integral operator in eq. (15) with very good accuracy, especially when compared with the uncertainties in the hadronic cross-section (at the ∼ 20% level) or with the accuracy of the quasi scaling approximation (at the level of few percent or better).

**Cutoff/transition in the CR spectrum.** The calculated CR spectra indicate, in all cases, that there is a significant number of protons at high energy. Protons should be efficiently accelerated up to an energy equal to about ∼ 100 TeV, in order to explain the observed data. We see that the cutoff/transition region in the CR spectrum is in the energy region $E_p = 30 - 150$ TeV. It is interesting to note that a photon flux with a smooth EC ($\beta = 0.5$) corresponds to an effective CR flux $\Phi_p[E_p]$ which is well described by the simple functional form $E_p^{-\Gamma} \exp[-E/E_c]$ with $\Gamma \simeq 1.79$ and cutoff energy $E_c \simeq 113$ TeV, in reasonable agreement with the conclusion of [15]. The corresponding CR energy distribution $J_p[E_p]$ is well fitted by the same functional form, with the same cutoff energy and with $\Gamma \simeq 1.86$. Compare with the discussion after eq. (7).

**Other features in CR and photon spectrum.** We note that the differences between the CR spectra are much larger than the differences between the input photon fluxes. In the BPL case, the calculated CR spectra have a complex behavior in the energy range $E_p = 3 - 30$ TeV. Similarly, in the EC case the obtained curves differ substantially in the energy region $E_p = 30 - 150$ TeV. This is not an artifact of the inversion method which is accurate at the level of few parts per thousand or better. It simply reflects the fact that any sharp feature in the photon flux is amplified in the parent CR spectrum. In particular, the sharper is the transition/cutoff in the photon spectrum (i.e., the smaller is the $S$ in eq. (21) or the larger is the $\beta$ in eq. (22)), the more complex is the behavior of the calculated CR flux.

**Dilution of spectral features.** The previous point can be understood in terms of the properties of hadronic cross sections. It is basically related to the fact that the photon spectrum is not supposed to have any sharp feature if it is originated by hadronic processes. The integral kernel $f[y]$ has, in fact, a characteristic width $\delta y \sim 1.0$. Consequently, features in the CR spectrum on scales $\delta \varepsilon_p \leq \delta y$ are washed-out by convolution (15). Conversely, if we observe features in the photon spectrum on scales $\delta \varepsilon_\gamma \leq \delta y$ we are forced by rel. (19) to postulate a complicated behavior of the parent CR proton flux which may be difficult or impossible to justify. As an example, BPL fits of the observational data with $S \leq 0.4$ correspond to parent CR spectra which become negative in the region $E_p = 3 - 30$ TeV and are, thus, not acceptable.

**A plausibility test for the hadronic origin assumption.** The presence (or the absence) of features in the observed photon flux on scales $\delta \varepsilon_\gamma \leq \delta y$ may be used, in principle, as plausibility criterion to reject (or support) the hadronic origin of the observed $\gamma$-ray fluxes. Interestingly, in the

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7 The energy resolution of the H.E.S.S. experiments ($\delta \varepsilon_\gamma \simeq 0.2$) is sufficiently good, in principle, to test whether there is some sharp feature in the photon spectrum.
case of EC parameterization, the experimental data prefers a smooth cutoff ($\beta \sim 0.5$) which corresponds to a simple behavior of the primary photon flux. In the BPL case, our results shows instead that statistical errors are too large to arrive at any relevant conclusion about the sharpness of the transition. Thus, the fine structures of the primary CR spectra in the energy range $E_p = 3 – 30$ TeV are not significantly constrained by the data.

### 3.3 Using the raw data

The differential operator on the r.h.s. of eq. (19) is the inverse of the integral operator in eq. (15) and it allows to obtain the CR spectrum directly from the photon flux, independently of any theoretical assumption. The CR spectrum, however, depends on high order derivatives of the photon flux which are generally known with bad accuracy and, moreover, introduce complicated correlations between the values of the CR flux extracted at two different energies. This makes difficult to infer the parent CR flux directly from noisy data and one could be tempted to conclude that a parameterization of the gamma ray flux is, in fact, necessary. In this section we propose a non-parametric procedure (based on Gaussian smearing) that avoids these difficulties and moreover permits to evaluate the error on the inferred CR spectrum.

**The ‘smoothing’ procedure.** The relevance of the high order terms in rel. (19) depends on the scale of the features that we probe in the CR spectrum. If we are interested in scales $\delta \varepsilon_p \geq \delta$, we can define the *smoothed CR spectrum* as follows:

$$ \tilde{\varphi}_p[\varepsilon, \delta] = \int_{-\infty}^{\infty} d\varepsilon \varphi_p[\varepsilon] r[\varepsilon_p - \varepsilon, \delta] $$

where:

$$ r[\varepsilon, \delta] = \frac{1}{\sqrt{2\pi}\delta} \exp \left[-\frac{\varepsilon^2}{2\delta^2}\right] $$

In Fourier space, this is equivalent to apply a Gaussian filter to $\varphi_p[k]$ with a width equal to $\Delta k = 1/(2\pi\delta)$. We remind, for clarity, that $\varphi_p = \Phi_p (E_p/1\text{TeV})^{2.5}$ where the effective cosmic ray flux $\Phi_p$ is defined in eq. (5). Equivalently, we can define the *smoothed CR energy distribution* by:

$$ \tilde{\jmath}_p[\varepsilon, \delta] = \int_{-\infty}^{\infty} d\varepsilon \ j_p[\varepsilon] r[\varepsilon_p - \varepsilon, \delta] $$

where $j_p = J_p \times (E_p/1\text{TeV})^{2.5}$ and $J_p$ is given in eq. (7). By using eq. (6), it is possible to show that:

$$ \tilde{\jmath}_p[\varepsilon, \delta] \simeq \frac{4\pi R^2}{c N \sigma(\varepsilon_p)} \tilde{\varphi}_p[\varepsilon, \delta] $$

with a few per cent accuracy, in the energy range of interest. We will then focus on the calculation of $\tilde{\varphi}_p$, showing that it can be simply estimated from observational data.
The ‘smoothed’ CR spectrum. Applying the differential operator of eq. (19), we find that the smoothed CR spectrum $\overline{\varphi}_p$ is related to the photon flux by a convolution integral:

$$\overline{\varphi}_p[\varepsilon_p, \delta] = \int_{-\infty}^{\infty} d\varepsilon_{\gamma} \varphi_{\gamma}[\varepsilon_{\gamma}] \rho[\varepsilon_p - \varepsilon_{\gamma}, \delta]$$

(27)

where the convolving function $\rho[\varepsilon, \delta]$ is given by:

$$\rho[\varepsilon, \delta] = r[\varepsilon, \delta] \sum_{i=0}^{5} A_i \varepsilon^i$$

(28)

and the coefficients $A_i$ are equal to:

$$A_0 = a_0 - \frac{a_2}{\delta^2} + 3 \frac{a_4}{\delta^4}, \quad A_1 = -\frac{a_1}{\delta^2} + 3 \frac{a_3}{\delta^4} - 15 \frac{a_5}{\delta^6}, \quad A_2 = \frac{a_2}{\delta^4} - 6 \frac{a_4}{\delta^6},$$

$$A_3 = -\frac{a_3}{\delta^6} + 10 \frac{a_5}{\delta^8}, \quad A_4 = \frac{a_4}{\delta^8}, \quad A_5 = -\frac{a_5}{\delta^{10}}.$$  

(29)

We can apply rel. (27) directly to the raw data, as it is explained in the following. We indicate with $\varphi_i \pm \Delta \varphi_i$ the value of the photon flux measured in the $i$-th bin, centered at the photon energy $\varepsilon_i$ and covering the energy range $(\varepsilon_{i,\text{inf}}, \varepsilon_{i,\text{sup}})$. We approximate the photon flux by:

$$\varphi_{\gamma}[\varepsilon_{\gamma}] = \sum_i \varphi_i W_i[\varepsilon_{\gamma}]$$

(30)

where $W_i[\varepsilon_{\gamma}]$ are rectangular functions describing the various energy bins (i.e., $W_i[\varepsilon_{\gamma}] \equiv 1$ for $\varepsilon_{i,\text{inf}} \leq \varepsilon_{\gamma} \leq \varepsilon_{i,\text{sup}}$ and zero elsewhere). We immediately obtain from eq. (27) the relation:

$$\overline{\varphi}_p[\varepsilon_p, \delta] = \sum_i \varphi_i \overline{w}_i[\varepsilon_p, \delta]$$

(31)

where:

$$\overline{w}_i[\varepsilon_p, \delta] = \int_{\varepsilon_{i,\text{inf}}}^{\varepsilon_{i,\text{sup}}} d\varepsilon_{\gamma} \rho[\varepsilon_p - \varepsilon_{\gamma}, \delta] \simeq \rho[\varepsilon_p - \varepsilon_i, \delta] \Delta \varepsilon_i$$

(32)

and, in the last step, we have assumed that $\delta \gg \Delta \varepsilon_i = \varepsilon_{i,\text{sup}} - \varepsilon_{i,\text{inf}}$. Eq. (31) gives the desired expression of the (smoothed) CR flux directly from the gamma ray data.

The smoothed CR spectrum is a linear combination of the observational values $\varphi_i$ of the photon flux. The functions $\overline{w}_i[\varepsilon_p, \delta]$ describe the contribution that each data point give to the reconstructed spectrum at a fixed energy $\varepsilon_p$. The uncertainty in the CR spectrum can be easily evaluated by propagating linearly the observational errors $\Delta \varphi_i$ obtaining:

$$\frac{\Delta \overline{\varphi}_p[\varepsilon_p, \delta]}{\overline{\varphi}_p[\varepsilon_p, \delta]} = \sqrt{\frac{\sum_i \Delta \varphi_i^2 \overline{w}_i[\varepsilon_p, \delta]^2}{\sum_i \varphi_i \overline{w}_i[\varepsilon_p, \delta]}}$$

(33)

Similarly, the correlation between the values of the CR flux at two different energies can be obtained by calculating:

$$\varrho[\varepsilon_p, \varepsilon'_p, \delta] = \frac{\sum_k \Delta \varphi_k^2 \overline{w}_k[\varepsilon_p, \delta] \overline{w}_k[\varepsilon'_p, \delta]}{\sqrt{\sum_i \Delta \varphi_i^2 \overline{w}_i[\varepsilon_p, \delta]^2} \sqrt{\sum_j \Delta \varphi_j^2 \overline{w}_j[\varepsilon'_p, \delta]^2}}$$

(34)
Figure 5: **Left Panel:** The (smoothed) CR spectrum from the RX J1713.7-3946 SNR deduced from the raw data of the H.E.S.S. experiments. The solid line is obtained by continuing the γ-ray spectrum at low and high energy with the best-fit BPL with sharpness parameter $S = 0.6$, while the dashed line is obtained by using the best-fit EC with $\beta = 0.5$. The shaded area represents the observational uncertainty and is obtained by propagating H.E.S.S. observational errors. **Right Panel:** The (smoothed) CR energy distribution calculated from eq. (26) with $R = 1$ kpc and $N = 3.6 \times 10^{59}$.

**Application to the RX J1713.7-3946 observations.** Before applying the above relations to the H.E.S.S. data we have to choose the smoothing scale $\delta$. The choice of $\delta$ is somewhat arbitrary and depends on the detector, on the quality of the observational data and on the problem under consideration. The H.E.S.S. detector has an energy resolution equal to $\delta E_\gamma = 0.2$ TeV, that suggests to adopt $\delta \gg 0.2$. Moreover, if we accept the hadronic origin assumption, we know that the hadronic interactions themselves introduce a scale, $\delta y \sim 1.0$, below which the photon spectrum is not expected have features large enough to be significant with respect to observational errors. At the same time, we know that “noise” at these small scales is greatly amplified in the calculated CR spectrum. All this suggests to choose $\delta = \delta y = 1.0$ and to focus our attention on the large scale features of the parent CR flux.

Our final results are displayed in Fig. 5. In the left panel we show the smoothed CR spectrum which is obtained from the H.E.S.S. observational data of the RX J1713.7-3946 SNR. In the right panel we show the smoothed CR energy distribution estimated according to eq. (26) with $R = 1$ kpc and $N = 3.6 \times 10^{59}$. We remind that the observational data cover an energy range equal to $E_\gamma = 0.3 - 300$ TeV. In this energy range we have described the photon spectrum according to eq. (30), while at low ($E_\gamma \leq 0.3$ TeV) and high energy ($E_\gamma \geq 300$ TeV) we have continued the photon spectrum by using the best-fit BPL with sharpness parameter $S = 0.6$ (see previous section). The shaded area describes the observational uncertainty in the smoothed CR spectrum which is obtained by propagating the errors in the observational data according to

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8 Strictly speaking, one should know the photon flux at all energies in order use the rel. (31).
correlation index close to 1, while dark colors correspond to values close to -1. The displayed contours correspond to correlation index equal to 0.5 and to -0.5.

Figure 6: Correlation between the values of the (smoothed) proton spectrum obtained at two different energies. Light colors correspond to values of the correlation index close to 1, while dark colors correspond to values close to -1. The displayed contours correspond to correlation index equal to 0.5 and to -0.5.

One sees that the error is less than 10% at low energy and remains smaller than 20% for $E_p \leq 300$ TeV. The correlation between the values of the CR flux at two different energies, evaluated as in eq. (34), is shown in Fig. 6.

In order to estimate the systematic uncertainty introduced by the ignorance of the high and/or low energy behaviour of the photon flux, we also show with a dashed line the smoothed CR spectrum which is obtained by continuing the photon spectrum at high and low energy with the best-fit EC with $\beta = 0.5$. The difference between the solid and the dashed line is smaller than the observational error for $E_p = 3 - 1000$ TeV, showing that the proton spectrum, in this energy range, is directly constrained by observational data. For $E_p \leq 3$ TeV, instead, the two lines behave differently indicating that the systematic uncertainty due to extrapolation is relevant. In this respect, the small bend of the effective CR flux in the energy region $1 - 10$ TeV, at the moment, does not seem to be fully significative. The existence of such a bend would amount to an important physical information on the acceleration mechanism (see e.g., [16]). Thus, it will be interesting to collect new data at energy lower than $E_\gamma = 0.3$ TeV to assess its significance.

In conclusion, the displayed results show that the large scale features of the CR spectrum in the energy range $E_p = 3 - 300$ TeV are well constrained by the observational data. The effective CR flux is roughly described by power-law with spectral index $\Gamma = 1.7 - 2$ at low energy with a cutoff/transition region between $E_p = 30 - 100$ TeV. This conclusion is also consistent with the
one obtained in Sect. 3.2 using parameterized photon fluxes.

4 Summary

In this work we assumed the hadronic origin of the gamma radiation emitted by SNR and we addressed the question of what can be learned on the SNR cosmic ray spectrum from VHE $\gamma$-ray data. We summarize here our conclusions:

i) The main result is contained in eq. (19). This equation shows that, in the approximation of quasi-scaling behavior of the hadronic cross sections, the effective CR spectrum defined in eq. (5) can be obtained by applying a simple differential operator to the observed photon flux. This results does not rely on any theoretical assumption about the shape of the proton and/or photon spectrum. It can thus be applied to sources which show non trivial spectral features such SNR RX J1713.7-3946 for which, instead, the commonly adopted approximation of power law distribution and the many techniques of calculations tailored to this case are not adequate.

ii) We have emphasized that the presence (or the absence) of sharp features in the photon spectrum can be used as plausibility criterion to reject (or to support) the assumption that the observed radiation has a hadronic origin. The basic point is that the hadronic processes (convolved with the parent CR flux) lead to a characteristic energy scale below which the produced photon flux is expected to be featureless (see discussion in Sect. 3.2).

iii) Specific implementations of our method permit to calculate the parent CR spectrum either from parameterized VHE fluxes or directly from raw data (see eq. (31)). This second approach requires fewer theoretical assumptions and allows to propagate the observational errors easily (see eqs. (33-34)). However, when applied to noisy data, it requires a sort of image processing (Gaussian smearing, see eq. (23)) to produce reasonable results.

iv) We have applied our method to the young SNR RX J1713.7-3946 which has been observed by the H.E.S.S. experiment during the last three years. We have calculated the CR spectrum both from parameterized photon fluxes and directly from the raw data. The results are summarized in Fig. 4 and Fig. 5. These figures demonstrate that the observational data constrain well the main features of CR flux in the energy range $E_p \simeq 3-300$ TeV; they give, instead, a poor information outside this range, and cannot significantly test the fine structures of the CR spectrum. As a final result, we conclude that the effective CR flux from SNR RX J1713.7-3946 is well described by power-law with spectral index $\Gamma = 1.7 - 2$ at low energy with a cutoff/transition region between $E_p = 30 - 100$ TeV.

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9The “quasi scaling” assumption defined in eq. (14) is accurate at the few per cent level, see Fig. 1 and can be improved as discussed in the appendix B.
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A The functions $F_{\gamma}\left[E_{\gamma}/E_p, E_p\right]$ and $f[y]$

In order to give a self-contained discussion, we give here the function $F_{\gamma}\left[E_{\gamma}/E_p, E_p\right]$ obtained in [10] and used in this paper. We have:

$$F_{\gamma}[x, E_p] = B_{\gamma} \frac{\ln[x]}{x} \left(\frac{1 - x^{\beta_{\gamma}}}{1 + k_{\gamma} x^{\beta_{\gamma}} (1 - x^{\beta_{\gamma}})}\right)^4 \left(\frac{1}{\ln[x]} - \frac{4 \beta_{\gamma} x^{\beta_{\gamma}}}{1 - x^{\beta_{\gamma}}} - \frac{4 k_{\gamma} \beta_{\gamma} x^{\beta_{\gamma}} (1 - 2 x^{\beta_{\gamma}})}{1 + k_{\gamma} x^{\beta_{\gamma}} (1 - x^{\beta_{\gamma}})}\right)$$

where $x = E_{\gamma}/E_p$. The parameters $B_{\gamma}, \beta_{\gamma},$ and $k_{\gamma}$ depend only on the energy of proton and are given by:

$$B_{\gamma} = 1.30 + 0.14 \varepsilon_p + 0.011 \varepsilon_p^2,$$
$$\beta_{\gamma} = \frac{1}{1.79 + 0.11 \varepsilon_p + 0.008 \varepsilon_p^2},$$
$$k_{\gamma} = \frac{1}{0.801 + 0.049 \varepsilon_p + 0.014 \varepsilon_p^2},$$

where $\varepsilon_p = \ln[E_p/1\text{TeV}]$. The function $f[y]$ is obtained by applying eqs. (12) and (14) and is simply given by:

$$f[y] = \theta[-y] \cdot e^{\alpha y} \cdot F_{\gamma}[e^y, 1\text{PeV}]$$

B Improving the quasi scaling approximation

Let us begin by writing the integral equation eq. (5) in abstract terms:

$$\Phi_{\gamma}[E_{\gamma}] = \mathcal{F}[\Phi_p][E_{\gamma}]$$

We showed that a solution of the integral equation $\Phi_{\gamma} = \mathcal{F}_0[\Phi_p]$ in the quasi-scaling approximation $F[x, E_p] \rightarrow F[x, E_p^0]$, accurate at the level of few parts per thousand or better, is given by

$$\Phi_p = \mathcal{D}[\Phi_{\gamma}]$$
In operator terms, we write $\mathcal{D} \Phi_0 = \Phi_0 \mathcal{D} = 1$. For a given gamma-ray flux $\Phi_\gamma$, we can evaluate the goodness of a certain approximation of the proton ‘flux’ $\Phi_p$ from the difference between the assumed gamma-ray flux and the gamma-ray flux re-calculated from the proton ‘flux’:

$$\xi = \frac{\mathcal{F}[\Phi_p] - \Phi_\gamma}{\Phi_\gamma}$$

(37)

In our case, we evaluate the goodness of the quasi-scaling approximation by using eq. (36) for $\Phi_p$. Assuming that $\Phi_\gamma$ is a broken power-law, we find that $\xi$ is smaller than 10% in the range of energies from 100 GeV to 1 PeV, that is already sufficient for our purposes. It is however possible to improve the approximation for the proton flux as follows:

$$\Phi_p = \mathcal{D}[\Phi_\gamma(1 - \xi)]$$

(38)

where $\xi$ is calculated using eq. (36).\textsuperscript{10} Repeating the procedure (namely: assuming again the broken power-law distribution for $\Phi_\gamma$ and plugging eq. (38) into eq. (37)) in order to test the approximation, the newly calculated $\xi$ is smaller than 0.5% in the range from 100 GeV to 1 PeV.

\textsuperscript{10}The formal derivation is simple: From $\Phi = \mathcal{F}[\Psi] = (\Phi_0 + \delta \mathcal{F})[\Psi]$ we get $\Phi_0[\Psi] = \Phi - \delta \mathcal{F}[\Psi]$. Applying $\mathcal{D}$ we find $\Psi = \mathcal{D}[\Phi - \delta \mathcal{F}[\Psi]]$, that can be improved iteratively. If in the r.h.s. of the last equation we use $\Psi = \mathcal{D}[\Phi]$ (i.e., eq. (36)) we get eq. (38) simply applying the definition of eq. (37).
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