As SNe II contributed in all our considerations as SNe Ia turn on at later times.

Integration of equation (1) gives

\[
\frac{d(\text{Fe/H})}{dt} = \frac{P_{\text{Fe}}}{(\text{H})}. \quad (1)
\]

We only treat Fe contributions from Type II supernovae (SNe II) in all our considerations as SNe Ia turn on at later times. Integration of equation (1) gives

\[
(\text{Fe/H}) = (\text{Fe/H})_p + \frac{P_{\text{Fe}}}{(\text{H})} (t - t^*) \quad (2)
\]

As SNe II contributed \((\text{Fe/H})_0/3\) over the period of \((t - t^*) \sim 10\) Gyr prior to solar system formation, \(P_{\text{Fe}}/(\text{H})\) is estimated to be \(\sim (\text{Fe/H})_0/(30\text{ Gyr})\).

In this model, the baseline Fe enrichment is explained by the prompt inventory and the dispersion in \([\text{Fe/H}]\) at a fixed \(z\) by the range of \(t^*\). It was found that most systems at any given \(z\) have \(t^*\) close to the age \(t(z)\) of the universe at this \(z\). This implies that the “turn-on” of DLA systems occurs rather long after the big bang. Further, essentially all the data lie below the upper bound for \([\text{Fe/H}]\) corresponding to \(t^* = 0\). The above model appears to provide a reasonable description of the data on \([\text{Fe/H}]\) for DLA systems. This is rather remarkable considering that the model ignores infall. According to hierarchical structure formation, infall is essential to formation of DLA systems. Here we examine the chemical evolution of these systems by including infall. We wish to gain some insights into what causes most DLA systems at a given \(z\) to have \(t^*\) close to \(t(z)\) in the closed-system model. We also address whether the prompt inventory is needed to explain the baseline Fe enrichment for DLA systems.

2. CHEMICAL EVOLUTION WITH INFALL

Consider a system of gas and stars with infall of primordial metal-free gas. The equations for evolution of the numbers of Fe and H atoms \([\text{Fe}]\) and \([\text{H}]\) in the gas are

\[
\frac{d[\text{Fe}]}{dt} = P_{\text{Fe}} + \frac{d[\text{Fe}]}{dt}, \quad (3)
\]

where \(P_{\text{Fe}}\) is the Fe production rate of SNe II, \(d[\text{H}]/dt < 0\) is the astration rate, and \(d[\text{H}]/dt > 0\) is the infall rate. When \(d[\text{H}]/dt\) is small compared to \(d[\text{H}]/dt\), \([\text{H}]\) is governed by infall. Equations (3) and (4) give (Qian & Wassenburg 2003)

\[
\frac{d(\text{Fe/H})}{dt} = \frac{P_{\text{Fe}}}{(\text{H})} - \frac{1}{(\text{H})} \frac{d[\text{H}]}{dt} \frac{d(\text{Fe/H})}{dt}, \quad (5)
\]

Formal integration of equation (5) gives

\[
(\text{Fe/H}) = \frac{P_{\text{Fe}}}{(\text{H})} t - \int_0^t \frac{1}{(\text{H})} \frac{d[\text{H}]}{dt} \frac{d(\text{Fe/H})}{dt} dt. \quad (6)
\]

With the integral term interpreted as \(P_{\text{Fe}} t/(\text{H})\), this resembles equation (2) for \((\text{Fe/H})_p = 0\). So the effect of a sustained large infall rate is similar to late “turn-on.”

In general, equations (4) and (5) must be solved together.

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For simplicity, we first consider the case where \( d(H)_{\text{ini}}/dt \gg |d(H)_{\text{ini}}/dt| \) so \((H) \approx (H)_0\). We then have

\[
\frac{dZ_{\text{Fe}}}{dt} = \lambda_{\text{Fe}} - \lambda_{\text{ini}} Z_{\text{Fe}}^*, \tag{7}
\]

where \( Z_{\text{Fe}} \equiv (\text{Fe}/H)/(\text{Fe}/H)_0 \), \( \lambda_{\text{Fe}} \equiv p_{\text{Fe}}/(\text{Fe}/H)_0(H) \), and \( \lambda_{\text{ini}} \equiv d\ln(H)/dt \). If \( dZ_{\text{Fe}}/dt \) is negligible, a quasi–steady state is achieved and \( Z_{\text{Fe}}(t) \) approximately assumes the value \( Z_{\text{Fe}}^\text{QSS} (t) \equiv \lambda_{\text{Fe}}(t)/\lambda_{\text{ini}}(t) \). With the onset of Fe production at \( t_0 \), we have

\[
Z_{\text{Fe}}(t) \approx Z_{\text{Fe}}^\text{QSS} (t) - Z_{\text{Fe}}^\text{QSS} (t_0) \exp \left[ - \int_{t_0}^t \lambda_{\text{ini}}(t') dt' \right] \tag{8}
\]

for \( \lambda_{\text{ini}}(t) \gg |d\ln Z_{\text{Fe}}^\text{QSS}/dt| \). At \( t > t_0 + [\lambda_{\text{ini}}(t_0)]^{-1} \), the exponential term is negligible and \( Z_{\text{Fe}}(t) \approx Z_{\text{Fe}}^\text{QSS} (t) \).

We consider a baryonic system that is formed through infall into the potential well of a dark matter halo. The mass \( M \) of the halo grows according to hierarchical structure formation. We assume that baryonic and dark matter are fed into the halo at a fixed mass ratio. The infall rate is then

\[
\lambda_{\text{ini}}(t) = \frac{d\ln M}{dt} = \frac{d\ln M}{dz} \frac{dz}{dt} \tag{9}
\]

With \( t \approx 17(1 + z)^{-3/2} \) Gyr at \( z > 0.5 \), equation (9) gives

\[
\lambda_{\text{ini}}(t) \approx \frac{4.4 \text{ Gyr}}{(\text{Gyr}/t)^{5/3}} \left| \frac{d\ln M}{dz} \right| . \tag{10}
\]

From Figure 6 in Barkana & Loeb (2001), we find that \( |d\ln M/dz| \sim 2.3 \) (within a factor of 2) for \( 0.5 < z < 5 \). As an example, we take \( \lambda_{\text{ini}}(t) = (0.1 \text{ Gyr}^{-1})/\text{Gyr}^{5/3} \) and \( \lambda_{\text{Fe}} = (30 \text{ Gyr})^{-1} \) for \( t > 0 \) (i.e., \( t_0 = 0 \)). We numerically integrate equation (7) and show the evolution of \( \log(\text{Fe}/H) = \log Z_{\text{Fe}} \) in Figure 1. The quasi–steady state value \( Z_{\text{Fe}}^\text{QSS} (t) = \lambda_{\text{Fe}}/\lambda_{\text{ini}}(t) = (\text{Gyr})^{5/3}/300 \) is a good approximation to the exact solution. For the case of evolution from a metal-free initial state without infall, \( Z_{\text{Fe}}(t) = \lambda_{\text{Fe}} t \) (the case of \( t^* = 0 \) in §1). In comparison with this, Fe enrichment in the case of infall is suppressed by a factor of \( \approx \lambda_{\text{ini}}(t)/t \approx 10(\text{Gyr})^{5/3} \), which ranges from 4.5 for \( t = 3.3 \) Gyr \((z = 2)\) to 8.9 for \( t = 1.2 \) Gyr \((z = 5)\). This would explain the late “turn-on” required \( t^* \) to be close to \( t(z) \) in the closed-system model. The data on \( \log(\text{Fe}/H) \) for 96 DLA systems (Prochaska et al. 2003) are shown in Figure 1. It can be seen that the solid curve for the case of infall passes through the median of the body of the data. This curve has a slope of \( d(\text{Fe}/H)/dz \approx -1.1/\left(1 + z \right) \). For \( z \approx 2 \) where most of the data lie, this slope is in good agreement with the estimate of \( d(\text{Fe}/H)/dz \approx -0.26 \) given by Prochaska et al. (2003) for the growth of the mean \( \log(\text{Fe}/H) \) as a function of \( z \). However, the above result does not explain the wide range in \( \log(\text{Fe}/H) \) at a given \( z \).

3. THE DISPERSION IN \( \log(\text{Fe}/H) \)

We now turn to the wide range in \( \log(\text{Fe}/H) \) for DLA systems at a given \( z \). We assume a constant Fe production rate \( \lambda_{\text{Fe}} \) per \( H \) atom in the gas for \( t > t_c \). The infall rate \( \lambda_{\text{ini}} \) is estimated on the basis of hierarchical structure formation, and we now allow the possibility that infall may cease at a time \( t_c \). We assume that the infall rate greatly exceeds the astration rate at \( t < t_c \). Under these assumptions, \( t_0 \) and \( t_c \) determine \( \log(\text{Fe}/H) \) in a baryonic system at time \( t \) (see eqs. [8] and [11]).

First consider the case where \( t_0 = 0 \) and infall ceases at \( t_c \). The evolution of \( Z_{\text{Fe}} \) is the same as for the case of infall discussed in §2 until \( t = t_c \). At \( t > t_c \), equation (7) reduces to \( dZ_{\text{Fe}}/dt = \lambda_{\text{Fe}} \), which gives

\[
Z_{\text{Fe}}(t) = Z_{\text{Fe}}(t_c) + \lambda_{\text{Fe}}(t - t_c), \tag{11}
\]

Solutions for \( t_c = 0, 1.5 \) Gyr \((z_c = 4)\) and \( 4.3 \) Gyr \((z_c = 1.5)\) are shown in Figure 2. In all cases of \( t_c > 0 \), the evolution of \( \log(\text{Fe}/H) \) for \( t < t_c \) is along the solid curve labeled “continuous infall.” As illustrated by points A, B, and C at \( z = 3 \) in Figure 2, a wide range of \( \log(\text{Fe}/H) \) bounded by the cases of no infall \((t_c = 0)\) and continuous infall can be produced at a given \( z \) for different \( t_c \) values. Next consider the case where infall is continuous but \( t_c \) may vary. Subsequent to onset of Fe production at \( t_0 \), \( Z_{\text{Fe}} \) grows rapidly and approaches \( Z_{\text{Fe}}^\text{QSS} \) on a timescale of \( \sim \lambda_{\text{ini}}(t_c)^{-1} \) (see eq. [8]). This is shown in Figure 1 for \( t_0 = 1.5 \) Gyr \((z_0 = 4)\), \( 2.1 \) Gyr \((z_0 = 3)\), and \( 3.3 \) Gyr \((z_0 = 2)\). For example, \( \log(\text{Fe}/H) \) in baryonic systems formed at \( t_0 = 2.1 \) Gyr first evolves rapidly, crossing \( \log(\text{Fe}/H) = -3 \), and then grows more slowly toward the quasi–steady state solution. Thus, values of \( \log(\text{Fe}/H) \) below those for the case of \( t_0 = 0 \) may be populated with baryonic systems that have different \( t_c \) values. Then the lower range in \( \log(\text{Fe}/H) \) at a given \( z \) can also be explained by the infall model.

The observed lower bound of \( \log(\text{Fe}/H) \approx -3 \) for DLA systems requires attention. With infall, the growth of \( Z_{\text{Fe}} \) is determined by competition between \( \lambda_{\text{Fe}} \) and \( \lambda_{\text{ini}} Z_{\text{Fe}} \). As \( Z_{\text{Fe}}(t_c) = 0, \lambda_{\text{Fe}} \) governs the initial growth of \( Z_{\text{Fe}} \) so long as \( \lambda_{\text{Fe}}/\lambda_{\text{ini}} \gg Z_{\text{Fe}} \). We
These redshifts correspond to the $t_{\text{infall}}$ in Figure 2 and seem reasonable. For $n\sigma$ halos with $n > 4$, $M_{\text{ic}} = 10^{11} M_{\odot}$ is reached at $z_{\text{inst}} = 4.3$ Gyr. The evolution of $[\text{Fe/H}]$ at $z < 5$ ($t > 1.2$ Gyr) for baryonic systems inside these high-$\sigma$ halos is close to the case of $t_{\text{infall}} = 0$. Similar results are found for $M_{\text{ic}} \sim 10^{11} - 10^{12} M_{\odot}$.

Now consider $t_{\text{ic}}$, which may be taken as the onset of star formation. Following the discussion in Barkana & Loeb (2001), the first stars appear to have formed in $3 - 4 \sigma$ halos of mass $M \sim 10^8 M_{\odot}$ at $z \sim 20 - 30$. For these halos, $t_{\text{ic}}$ is only $0.1$ Gyr. Formation of stars at $z < 20$ appears to require a minimum halo mass of $10^9 - 10^{10} M_{\odot}$. We assume that star formation starts at $t_{\text{ic}}$ when a halo reaches a mass of $M_{\odot} = 5 \times 10^5 M_{\odot}$ [\sigma(M_{\odot}) = 6]. This occurs at redshifts $z_{\text{ic}} = 4$, 3, and 2 for $1\sigma$, $0.89\sigma$, and $0.67\sigma$ halos, respectively. These redshifts correspond to the $t_{\text{ic}}$ in Figure 1 and again seem reasonable.

It follows that at a given $z$, baryonic systems inside different halos are in different stages of evolution. Consider an example using $M_{\odot} = 5 \times 10^5 M_{\odot}$ and $M_{\odot} = 10^{10} M_{\odot}$. At $z = 2.6$, star formation just starts in a $0.8\sigma$ halo ([Fe/H] $\sim$ $-3$). Fe production competes with infall in a $1.1\sigma$ halo ([Fe/H] $\sim$ $-1.9$; see Fig. 1) and uniform growth of [Fe/H] has been going on since infall cessation at $z_{\text{ic}} = 4$ in a $1.9\sigma$ halo ([Fe/H] $\sim$ $-1.4$; see Fig. 2). For a sample of DLA systems at $z = 2.6$, systems with $-3 < [\text{Fe/H}] \leq -1.9$, $-1.9 < [\text{Fe/H}] \leq -1.4$, and $[\text{Fe/H}] > -1.4$ are then associated with $n\sigma$ halos with $0 < n \leq 1.1$, $1.1 < n \leq 1.9$, and $n > 1.9$, respectively. Statistically speaking, the fraction of halos more evolved than an $n\sigma$ halo $f(n) = \frac{M_{\odot} + \int M_{\odot} \exp(-x^2/2)dx}{M_{\odot} + 2\int M_{\odot} \exp(-x^2/2)dx}$, but we expect the occurrence of $3 < [\text{Fe/H}] \leq -1.9$, $-1.9 < [\text{Fe/H}] \leq -1.4$, and $[\text{Fe/H}] > -1.4$ at $z = 2.6$ to be in the ratios of $[F(0.8) - F(1.1)] : [F(1.1) - F(1.9)] : F(1.9) = 0.7 : 1 : 0.3$. Of 19 DLA systems at $2.4 < z < 2.7$, the numbers of systems in the above [Fe/H] intervals are 5, 10, and 4, consistent with expectation from our model.

4. DISCUSSION AND CONCLUSIONS

We have treated the chemical evolution of DLA systems considering astration and infall. It is assumed that the H in the gas is initially controlled by the infall of primordial metal-free baryonic matter and the Fe production rate per H atom in the gas is constant subsequent to the onset of star formation in a system. With the infall rate estimated from the standard scenario of hierarchical structure formation, this model yields an explanation for the data on [Fe/H] in DLA systems. It is shown that the slow growth of [Fe/H] with decreasing $z$ is a direct consequence of competition between enrichment by Fe production and dilution by infall. It is argued that the upper range of [Fe/H] at a given $z$ results from the different times at which individual halos associated with protogalaxies reach a mass of $10^{11} M_{\odot}$ and infall ceases or diminishes. It is also argued that the lower range of [Fe/H] results from the different times at which individual halos reach a minimum mass of $10^8 - 10^{10} M_{\odot}$ required to initiate astration. Because the initial growth of [Fe/H] up to $-3$ is very rapid subsequent to the onset of astration, the probability of observing DLA systems with [Fe/H] $< -3$ is very small. The approach presented here appears to be a reasonably quantitative description of the chemical evolution of DLA systems that is compatible with the paradigm of hierarchical structure formation. Of the available data, only three points lie outside the upper bound of the model, and these are still within a factor of 2 of this bound.
We assumed that the astration rate is small compared to the infall rate during the infall phase. If we assume a constant astration rate $\alpha$ per H atom in the gas consistent with the Fe production rate, then equation (4) reduces to
$$\frac{d(H)}{dt} = -\alpha_0 + d(H)_{\text{inf}}/dt,$$
which can be solved explicitly. As long as $d(H)_{\text{inf}}/dt > \alpha_0$, $H$ increases. When infall ceases, $H$ reaches the maximum value $H_{\text{max}}$ and the star formation rate also reaches the maximum value $\alpha(H)_{\text{max}}$. Subsequently, the gas will be depleted by star formation on a timescale of $\sim 10^8$ Myr. When a protogalaxy would reach its maximum star formation rate depends on when its associated halo reaches a mass of $\sim 10^{11} M_\odot$ and infall ceases. This occurs at $z \approx 1.6$ and 4.3 for 1 $\sigma$ and 2 $\sigma$ halos, respectively. The extent to which the approach outlined here can be used to quantitatively explain the cosmic star formation history remains to be explored.

In earlier works (Wasserburg & Qian 2000a; Qian & Wasserburg 2002), we proposed that very massive ($\geq 100 M_\odot$) stars (VMSs) produced the elements from C to the Fe group to explain the observed jump in the abundances of heavy $r$-process elements (Ba and above) at $[\text{Fe/H}] \approx -3$. The VMSs produced no heavy $r$-elements but dominated chemical evolution at $[\text{Fe/H}] < -3$. This evolution resulted in a prompt inventory of “metals” corresponding to $[\text{Fe/H}] \approx -3$ in the IGM. Cessation of VMS activities and the rapid occurrence of a hypothesized “metals” corresponding to in the IGM. Cessation $[\text{Fe/H}]$ explains the observed jump in the abundances of heavy elements (Ba and above) at $[\text{Fe/H}] \approx -3$. The VMSs produced $[\text{Fe/H}]$ to $\approx -3$ for $1 \sigma$ and $2 \sigma$ halos, respectively. The extent to which the approach outlined here can be used to quantitatively explain the cosmic star formation history remains to be explored.

We wish to dedicate this Letter to Allan Sandage for galaxies of reasons. We thank Jason Prochaska for making available the new data that stimulated us to further consider DLA systems. Intense questioning by Michael Norman on the requirement of VMSs and a prompt inventory in our earlier models also provided a great stimulus. This work was supported in part by DOE grants DE-FG02-87ER40328, DE-FG02-00ER41149 (Y. Z., Q.), and DE-FG03-88ER13851 (G. J. W.), Caltech Division Contribution 8906(1108).

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