Holographic model of hybrid and coexisting s-wave and p-wave Josephson junction

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Abstract: In this paper the holographic model for hybrid and coexisting s-wave and p-wave Josephson junction is constructed by a triplet charged scalar field coupled with a non-Abelian $SU(2)$ gauge fields in (3+1)-dimensional AdS spacetime. Depending on the value of chemical potential $\mu$, one can show that there are four types of junctions ($s+p-N-s+p$, $s+p-N-s$, $s+p-N-p$ and $s-N-p$). We show that DC current of all the hybrid and coexisting s-wave and p-wave junctions is proportional to the sine of the phase difference across the junction. In addition, the maximum current and the total condensation decays with the width of junction exponentially, respectively. For $s+p-N-s$ and $s-N-p$ junction, the maximum current decreases with growing temperature. Moreover, we find that the maximum current increases with growing temperature for $s+p-N-s+p$ and $s+p-N-p$ junction, which is in the different manner as the behaviour of $s+p-N-s$ and $s-N-p$ junction.
1 Introduction

The study of superconductivity has been at the forefront of condensed matter physics. In particular, what is the origin of high temperature superconductivity is still one of the major unsolved problems of condensed matter theory. Over the past decade, one of the most important result in string theory is the AdS/CFT correspondence, which was first proposed by Juan Maldacena in [1] and states that the strong coupled field living on the AdS boundary can be described with a weakly gravity theory in one higher dimensional AdS spacetime. By applying the AdS/CFT correspondence, ones have firstly achieve success in the study of holographic QCD and heavy ions collisions. In recent, The AdS/CFT correspondence also provides insights into the condensed matter theory. In [2–4], the authors investigated the action of a complex scalar field coupled to a $U(1)$ gauge field in 3+1 dimensional Schwarzschild-AdS black hole and found that below some critical value of the temperature due to the $U(1)$ symmetry breaking, the scalar field which condenses near the horizon could be interpreted as cooper pair-like superconductor condensation. Moreover, Analyzing the optical conductivity of superconducting state, the rate of the width of the gap to the critical temperature is closed to the value of high temperature superconductor. Thus, it is hoped that the holographic model can match properties of the high temperature superconducting behavior. Soon, The p-wave and d-wave holographic
superconductors proposed in [5] and [6, 7], respectively. For reviews on holographic superconductors, see [8–10].

It is known that the Josephson junction is a device made up of two superconductors materials coupled by weak link barrier in [11]. The weak link can be a thin normal conductor which is named as a superconductor-normal-superconductor junction (SNS), or a thin insulating barrier which is named as a superconductor-insulator-superconductor junction (SIS). In recent, by studying the spatial-dependent solution of the action of a Maxwell field coupled with a complex scalar field in an 3+1 dimensional Schwarzschild-AdS black hole background, Horowitz et al. [12] had constructed a 2+1 dimensional holographic model of Josephson junction and found that the sine relation between tuning current and the phase difference of the condensation across the junction. The extension to a four dimensional Josephson junction has been discussed in [13, 14]. The holographic model of Josephson junction array has been constructed with based on a designer multigravity in [15]. With the $SU(2)$ gauge field coupled with gravity, the holographic p-wave Josephson junction has also been discussed in [16]. In [17], a holographic model of 1+1 dimensional superconductor-insulator-superconductor (S-I-S) Josephson junction has been investigated. In [18, 19], with the action of the Einstein-Maxwell-complex scalar field, the authors studied a holographic model of superconducting quantum interference device (SQUID). In [20] a holographic model of Josephson junction in non-relativistic case with a Lifshitz geometry was constructed.

Recently, the holographic approach has been applied to the coexistence and competition order phenomena, in which the the phase diagram have rich structure, such as the competition of two scalar order parameters in the probe limit [21] and with the backreaction of the scalars [22], and the competition of s-wave and d-wave order [23]. Especially, there is another interesting paper about competition and coexisted of s-wave and p-wave order in [24], the authors studied a charged triplet scalar coupled with an $SU(2)$ gauge field in the 3+1 dimensional spacetime and found that the existence of s+p coexisting phases is confirmed. Furthermore, the phase transitions of the holographic s+p wave superconductor with backreaction is investigated in [25].

Since the holographic model of hybrid and s+p coexisting superconductors has been constructed, it is natural to set up a holographic model for hybrid and coexisting s-wave and p-wave Josephson junction. We will study a non-Abelian $SU(2)$ Yang-Mills field and a scalar triplet charged under an $SU(2)$ gauge field in (3+1)-dimensional AdS spacetime and construct the holographic model of hybrid and coexisting s-wave and p-wave Josephson junction, which be related to s-N-s junction in [12] and p-N-p junction in [16]. To construct the holographic model of junction, ones need to tune value of the chemical potential.

The paper is organized as following: In section. 2, we consider non-Abelian $SU(2)$ gauge field coupled with a scalar triplet charged field in (3+1)-dimensional
AdS spacetime and construct a gravity dual model for a (2+1)-dimensional s+p coexisting Josephson junction. We show the numerical results of the equations of motion and study the characteristics of the (2+1)-dimensional s+p coexisting Josephson junction in section 3. The conclusion and discussion are in the last section.

2 Holographic model of hybrid and coexisting s-wave and p-wave Josephson junction

2.1 The model setup

In [24], the author considered the action of a charged scalar field coupled to an SU(2) gauge field, in which the charged scalar field transform as a triplet under the gauge group SU(2). This model can realize a competition and coexisted of s-wave and p-wave orders. We will also adopt the same form of action in the (3+1)-dimensional AdS spacetime:

\[
S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}(R - 2\Lambda) + \frac{1}{g_c^2} \int d^4x \sqrt{-g}(-|D_\mu \Psi^a|^2 - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - m^2 |\Psi^a|^2),
\]

where \( \Lambda = -3/L^2 \) is the negative cosmology constant, \( L \) is the radius of asymptotic AdS spacetime. \( \Psi^a \) is a complex scalar triplet charged under the SU(2) Yang-Mills field and

\[
D_\mu \Psi^a = \partial_\mu \Psi^a + \varepsilon^{abc} A^b_\mu \Psi^c.
\]

The field strength \( F^a_{\mu\nu} \) of the SU(2) gauge theory is given by

\[
F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \varepsilon^{abc} A^b_\mu A^c_\nu,
\]

where the one form \( A = A_\mu dx^\mu = A^a_\mu \tau^a dx^\mu \) and the superscripts \( a \) is the index of generator \( \tau^a \) of SU(2) gauge field with \( a = 1, 2, 3 \). \( g_c \) is the coupling constant of non-Abelian SU(2) Yang-Mills field. In this paper we will consider the probe limit by taking \( g_c \to \infty \) to ignore the backreaction of the matter.

In the probe limit, we still choose a 3 + 1 dimensional planar Schwarzschild-AdS black hole solution as the background geometry with metric

\[
ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2),
\]

where \( x \) and \( y \) are the coordinates of a 2 dimensional Euclidean space. The function \( f(r) \) is

\[
f(r) = \frac{r^2}{L^2}(1 - \frac{r_h^3}{r^3}),
\]

where \( r_h \) is the black hole event horizon radius. The Hawking temperature of the black hole is given by

\[
T = \frac{1}{4\pi} \frac{df}{dr}\bigg|_{r=r_h} = \frac{3r_h}{4\pi L^2}.
\]
The temperature $T$ relates to the black hole event horizon radius $r_h$ and the AdS radius $L$. In the rest of paper, we will work in units which $L = 1$. $T$ is correspondent to the temperature of dual field theory on the AdS boundary.

Variation of the action (2.1) with respect to the scalar field $\Psi^a$ and $A^a_\mu$ lead to the equations of motion respectively.

\begin{align}
\nabla_\mu (D^\mu \Psi^a) + \varepsilon^{abc} A^b_\mu D^\mu \Psi^c - m^2 \Psi^a &= 0 , \\
\nabla^\nu F^a_{\nu \mu} + \varepsilon^{abc} A^b_{\nu \mu} F^c_{\nu \mu} - 2 \varepsilon^{abc} \Psi^b \left( \partial_\mu \Psi^c + \varepsilon^{cde} A^d_\mu \Psi^e \right) &= 0 .
\end{align}

For the hybrid and coexisting s-wave and p-wave Josephson junction, there are several different kinds of matter fields ansatzs. Now, let us consider the one of them as

\begin{align}
\tilde{\Psi}^3 = \tilde{\Psi}_3(r, y), \quad A^1_r = \phi(r, y), \quad A^3_x = \Psi_x(r, y), \quad A^1_y = A_x(r, y), \quad A^1_y = A_y(r, y),
\end{align}

where field function $\tilde{\Psi}^3$, $\theta^a$, $\phi$, $\Psi_x$, $A_r$ and $A_y$ are dependent of the coordinates $r$ and $y$. Thus the holographic model of the Josephson junction would be along the $y$ direction. Without loss of generality, we will consider the $SU(2)$ gauge-invariant of the vector field $A^a_\mu$ and the scalar field $\Psi^a$

\begin{align}
M^a_\mu &= A^a_\mu - D_\mu \theta^a, \quad \Psi^a = \tilde{\Psi}^a + \varepsilon^{abc} \theta^b \tilde{\Psi}^c.
\end{align}

With the black hole background (2.4) and the above ansatz (2.9), the equations of matter fields (2.7) and (2.8) can be written as:

\begin{align}
\partial_r^2 \Psi_3 + \frac{1}{r^2 f} \partial_y^2 \Psi_3 + \left( \frac{f'}{f} + \frac{2}{r} \right) \partial_r \Psi_3 + \left( - \frac{m^2}{f} + \frac{\phi^2}{f^2} - M^2_r - \frac{M^2_y}{r^2 f} \right) \Psi_3 &= 0 , \\
\partial_r^2 \phi + \frac{1}{r^2 f} \partial_y^2 \phi + \frac{2}{r} \partial_r \phi - \frac{\phi \Psi_3}{r^2 f} - \frac{2 \phi \Psi_3}{f} &= 0 , \\
\partial_r^2 \Psi_x + \frac{1}{r^2 f} \partial_y^2 \Psi_x + \frac{f'}{f} \partial_r \Psi_x + \left( \frac{\partial^2}{f^2} - M^2_r - \frac{M^2_y}{r^2 f} \right) \Psi_x &= 0 , \\
\partial_y^2 M_r - \partial_r \partial_y M_r - M_r \Psi_3^2 - 2 \Psi_3 M_r \Psi_x^2 &= 0 , \\
\partial_r^2 M_y - \partial_r \partial_y M_r + \frac{f'}{f} (\partial_r M_y - \partial_y M_r) - \frac{M_y \Psi_x^2}{r^2 f} - \frac{2 \Psi_3 M_y^2}{f} &= 0 , \\
\partial_r M_r + \frac{1}{r^2 f} \partial_y M_y + \frac{2}{\Psi_3} (M_r \partial_r \Psi_3 + \frac{M_y}{r^2 f} \partial_y \Psi_3) + \left( \frac{f'}{f} + \frac{2}{r} \right) M_r &= 0 , \\
\partial_r M_y + \frac{1}{r^2 f} \partial_y M_y + \frac{2}{\Psi_x} (M_r \partial_r \Psi_x + \frac{M_y}{r^2 f} \partial_y \Psi_x) + \frac{f'}{f} M_r &= 0 ,
\end{align}

where a prime denotes derivative with respect to $r$. In our paper, we will work with the case $m^2 \geq -9/4$ in order to satisfy the BF bound [26]. Let us have observation of these Eqs. (2.11a)-(2.11g). If $\Psi_x$, $A_r$ and $A_y$ are turned off, $\Psi_3$ and $\phi$ are only
dependent on $r$, the remaining two equations will be the same as the equations of the s-wave holographic superconductivity. Similarly, if we turn off $\Psi_3, A_r, A_y, \Psi_x$ and $\phi$ are only dependent on $r$, the two remaining equations will be the equations which describe the p-wave condensation. Next, we will consider the Josephson junction. If $\Psi_x$ is only turned off, the remaining equations are the same equations of the pure s-wave Josephson junction. In a similar way, if we only turn off $\Psi_3$, we will get the pure p-wave Josephson junction. So we can get the so-called s+p coexisting phase Josephson junction under this ansatz (2.9). It is obvious that Eqs. (2.11a)-(2.11g) are coupled and nonlinear, so we need to solve them numerically instead of solving them analytically.

Near the AdS boundary ($r \to \infty$), the matter fields take the asymptotic forms

\begin{align*}
\Psi_3 &= \frac{\Psi_3^-(y)}{r^{\Delta_-}} + \frac{\Psi_3^+(y)}{r^{\Delta_+}} + \ldots, \\
\phi &= \mu(y) - \frac{\rho(y)}{r} + O\left(\frac{1}{r^2}\right), \\
\Psi_x &= \Psi_x^-(y) + \frac{\Psi_x^+(y)}{r} + O\left(\frac{1}{r^3}\right), \\
M_r &= O\left(\frac{1}{r^3}\right), \\
M_y &= \nu(y) + \frac{J}{r} + O\left(\frac{1}{r^2}\right).
\end{align*}

Here, the dimensions of the operations $\Psi_3^{(\pm)}$ are

$$\Delta_\pm = \frac{3 \pm \sqrt{9 + 4m^2}}{2}.\quad (2.17)$$

According to the AdS/CFT dictionary, $\Psi_3^{(\pm)}$ is considered as the source of the scalar operation of s-wave condensation, and $\Psi_3^{(\mp)}$ is the corresponding expectation value of the operator. Meanwhile, $\Psi_x^{(\pm)}$ is the source of the vector operation of p-wave condensation, and $\Psi_x^{(\mp)}$ is the corresponding expectation value of the operator. $\mu(y), \rho(y), \nu(y)$ and $J$ are the chemical potential, charge density, velocity of the superfluid and the constant current in dual field, respectively [27–33].

In order to solve Eq. (2.11a) - (2.11g) numerically, we need to impose the boundary conditions on them. First, we impose the Dirichlet boundary condition on $\Psi_3$ and $\Psi_x$ on the AdS boundary. In this paper, we set

$$\Psi_3^-(y) = 0.\quad (2.18)$$

So $\Psi_3^{(+)}$ is the expectation value of s-wave scalar operator, $\Psi_3^{(+)} = \langle O_s \rangle$. Similarly, we impose the Dirichlet boundary condition on $\Psi_x$ on the AdS boundary.

$$\Psi_x^-(y) = 0.\quad (2.19)$$
\( \Psi^{(+)}_x \) is the expectation value of p-wave vector operator, \( \Psi^{(+)}_x = \langle \mathcal{O}_p \rangle \). Second, we impose the Dirichlet boundary condition on \( \phi \) at the horizon.

\[
\phi(r_h) = 0, \tag{2.20}
\]

\( \phi \) is the \( t \) component of \( A^1_\mu \), and \( \phi(r_h) = 0 \) is to avoid the divergence of \( g^{\mu\nu}A^1_\mu A^1_\nu \). In addition, the matter field functions are independent of \( y \) at the spatial coordinate \( y \to \pm \infty \). So, the boundary conditions of the Eq. (2.11a)-(2.11g) are determined by the value of \( \mu \) and \( J \). The phase difference \( \gamma \) across junction is

\[
\gamma = \Delta \theta^1 - \int A_y dy. \tag{2.21}
\]

Now, we would like to introduce the critical temperature \( T_c \) of the Josephson junction, which is proportional to the chemical potential \( \mu(\infty) \) or \( \mu(-\infty) \), and we set

\[
T_c = \frac{3}{4\pi} \frac{\mu(-\infty)}{\mu_c} \tag{2.22}
\]

where \( \mu_c \approx 3.65 \). In order to describe a hybrid and coexisting s-wave and p-wave Josephson junction, we should construct a chemical potential which can make phase transition occur along the direction \( y \) of the Josephson junction, thus the chemical potential \( \mu(y) \) is dependent on the spatial coordinate \( y \)

\[
\mu(y) = \mu \left\{ 1 - \frac{1}{2 \tanh \left( \frac{y}{2\sigma} \right)} \left[ \tanh \left( \frac{y + \frac{\ell}{2}}{\sigma} \right) - \beta \tanh \left( \frac{y - \frac{\ell}{2}}{\sigma} \right) \right] \right\}, \tag{2.23}
\]

where the chemical potential \( \mu(y) \) is proportional to \( \mu \) and \( \ell \) is the width of Josephson junction. The parameter \( \epsilon, \sigma \) and \( \beta \) controls the steepness, depth of the junction, respectively.

### 2.2 The scaling symmetry

Analyzing the EoM, we found that there is a scaling symmetry in Eq. (2.11a)-(2.11g). These equations are invariant under the following scale transformation:

\[
\begin{align*}
& r \to br, \quad r_h \to br_h, \quad y \to y/b, \quad \phi \to b\phi, \\
& \Psi_x \to b\Psi_x, \quad \Psi_3 \to \Psi_3, \quad f \to b^2 f, \quad f' \to bf', \quad A_y \to bA_y.
\end{align*}
\]

Under this scaling symmetry, we can get behaviors of the following physical quantities

\[
\begin{align*}
& \mu \to b\mu, \quad \Psi_x^{(-)} \to b\Psi_x^{(-)}, \quad \Psi_3^{(+)} \to b\Psi_3^{(+)}, \quad \Psi_3^{(-)} \to b\Psi_3^{(-)}, \quad \Psi_x^{(+)} \to b\Psi_x^{(+)}, \\
& \rho \to b^2 \rho, \quad \Psi_x^{(-)} \to b\Psi_x^{(-)}, \quad \Psi_3^{(+)} \to b\Psi_3^{(+)}, \quad J \to b^2 J.
\end{align*}
\]

Because of this scaling symmetry, we can set the black hole event horizon radius \( r_h = 1 \). In addition, we have set \( L = 1 \), so the temperature \( T \) and the background
geometry is fixed. From Eq. (2.6), we can see that the temperature $T$ changes to $bT$ under scaling transformation. So the following quantities are invariant

$$
\Psi_3^{(+)} / T^{\Delta_+}, \quad \Psi_x^{(+)} / T^2, \quad J / T^2.
$$

These invariant quantities can change with $T / T_c$.

3 Numerical results

In this section we will solve the coupled and nonlinear Eqs. (2.11a)-(2.11g) numerically. Before we solve these equations, we will have coordinate transformations,

$$
z = 1/r, \quad \tilde{y} = \tanh(\frac{y}{\sigma}).
$$

It is more convenient to impose boundary conditions at $z = 1, z = 0$ and $\tilde{y} = \pm 1$ rather than at $r = 1, r = +\infty$ and $y = \pm \infty$. There are four kinds of Josephson junction.

(i) The s+p-N-s+p Josephson junction consists of the s+p coexisting phase in the both leads, and the normal phase between them.

(ii) The s+p-N-s Josephson junction consists of the s+p coexisting phase in the left lead, the conventional s-wave phase in the right lead, and the normal phase between them.

(iii) The s+p-N-s Josephson junction consists of the s+p coexisting phase in the left lead, the conventional p-wave phase in the right lead, and the normal phase between them.

(iv) The s-N-p Josephson junction consists of the conventional s-wave phase in the left lead, the conventional s-wave phase in the right lead, and the normal phase between them.

We can tune the chemical potential $\mu(y)$ to realize the four cases. In [24], when the value of $\Delta_+ \text{ or } m^2$ is in a special region, as the temperature decreases, the p-wave condensation will appear first and increase, the s-wave condensation will not appear. When temperature continues to decrease and reaches the critical temperature $T_{c1}^{sp}$, the s-wave condensation will appear and increase, the p-wave condensation will decrease. When temperature reaches another critical temperature $T_{c2}^{sp}$, the p-wave condensation will disappear. So when the temperature is in the region $T_{c2}^{sp} \sim T_{c1}^{sp}$, the s-wave phase and p-wave will coexist. When the temperature is higher than $T_{c1}^{sp}$, there is only p-wave phase. When the temperature is lower than $T_{c2}^{sp}$, there is only s-wave phase. In order to construct the junction of s+p coexisting phase, we can write the region of chemical potential as $\mu_{c1} \sim \mu_{c2}$ in [24].

3.1 s+p-N-s+p Josephson junction

In this subsection, in order to obtain the model of s+p-N-s+p Josephson junction, we need tune the value of chemical potential $\mu(y)$ at $y = \pm \infty$ such that it is in the
Figure 1. We represent the components $\mu(y)$ and $A_y$ of Yang-Mills fields. (Left) The profile of $\mu(y)$. (Right) The figure of $A_y$. The parameters are $m^2 = -33/16$, $\mu = 7.6$, $\epsilon = 0.25$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 1$. We can see that these two figures are symmetry.

**s+p coexisting region $\mu_{c1} \sim \mu_{c2}$.** Because the superconductor phases in the both leads are symmetrical, the phase difference $\gamma$ can be obtained by (2.21)

\[
\gamma = - \int_{-\infty}^{+\infty} dy[\nu(y) - \nu(\pm\infty)].
\]

The profiles of $A_{y1}$ and $A_{y2}$ are showed in Fig. 1.

We show the relationship between the DC current $J$ and the phase difference $\gamma$ across the junction on the left panel of Fig. 2. From the figure, we can see $J$ is proportional to the sine of $\gamma$. 

\[
J/T_c^2 \approx \begin{cases} 
0.464 \sin \gamma & \text{for } m^2 = -33/16, \\
0.539 \sin \gamma & \text{for } m^2 = -1031/500.
\end{cases}
\]

Note we only obtain the phase difference in the interval $(-\pi/2, \pi/2)$, and we can see the points which represent our numerical data fit sine line very well. We can see that when the value of $m^2$ increases, the maximum current $J_{\max}$ will grow. The dependence of $J_{\max}$ on the temperature $T$ is shown on the right panel of Fig. 2. The graph shows that $J_{\max}/T_c^2$ increases with the growing $T/T_c$, however, in the s-N-s or p-N-p Josephson junction [12, 16], $J_{\max}/T_c^2$ decays with the increasing $T/T_c$. The reason is that in the s-wave and p-wave coexisting region [24], the condensation of s-wave decreases and the condensation of p-wave increases with growing temperature, respectively. But the total condensation would decrease when the temperature drops. So, the $J_{\max}$ decreases with the rising $T$.

The relationship of between $J_{\max}/T_c^2$ and $\ell$ is shown on the left panel of Fig. 3. The $J_{\max}/T_c^2$ will decay with $\ell$ exponentially and change larger when $m^2$ becomes
Figure 2. (Left) The current $J/T_c^2$ as sine function of the difference phase $\gamma$. (Right) The $J_{\text{max}}/T_c^2$ increases with growing $T/T_c$. The black lines are sine curves and the points with $m^2 = -33/16$ (red), $-1031/500$ (blue) are numerical results. The parameters are $\mu = 7.6$, $\epsilon = 0.25$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 1$.

larger. The total condensation in $y = 0$ when the current $J = 0$ is the sum of s-wave and p-wave condensation. Here, we define

$$\langle O(0) \rangle_{J=0} = \langle O_s(0) \rangle_{J=0}/T_c^\Delta + \langle O_p(0) \rangle_{J=0}/T_c^2. \quad (3.2)$$

We plot the $\langle O(0) \rangle_{J=0}$ on the right panel in Fig. 3. The $\langle O(0) \rangle_{J=0}$ also decays with the growing $\ell$ exponentially and the $\langle O(0) \rangle_{J=0}$ becomes larger with increasing $m^2$.

$$\begin{cases} J_{\text{max}}/T_c^2 \approx 25.12 e^{-\ell/0.7505} \\ \langle O(0) \rangle_{J=0} \approx 22.03 e^{-\ell/(2 \times 0.7534)} \end{cases} \quad \text{for} \quad m^2 = -33/16.$$  

We can obtain the coherence length (0.7505, 0.7534) from the above two equations, respectively. The error of two values is about 0.4%.

$$\begin{cases} J_{\text{max}}/T_c^2 \approx 24.08 e^{-\ell/0.7856} \\ \langle O(0) \rangle_{J=0} \approx 20.66 e^{-\ell/(2 \times 0.8295)} \end{cases} \quad \text{for} \quad m^2 = -1031/500.$$  

The coherence length (0.7856, 0.8295) is obtained from the above two equations, respectively. The error of two values is about 5.6%.

3.2 s+p-N-s Josephson junction

In this subsection, the model of the s+p-N-s Josephson junction would be constructed. We tune the chemical potential $\mu(y)$ such that $\mu(-\infty)$ is in the s+p
The maximum current $J_{\text{max}}$ as exponential function of $\ell$. (Right) The total condensate $\langle \mathcal{O}(0) \rangle_{J=0}$ as exponential function of $\ell$. Our numerical results are the points with $m^2 = -33/16$ (red), $-1031/500$ (blue). In all the plots, we use $\mu = 7.6$, $\epsilon = 0.25$, $\sigma = 0.5$ and $\beta = 1$. The numerical data fits exponential curves well.

To calculate the phase difference $\gamma$, we use

$$\gamma = -\int_{-\infty}^{0} dy [\nu(y) - \nu(-\infty)] - \int_{0}^{+\infty} dy [\nu(y) - \nu(+\infty)].$$

(3.3)

First, we show the profiles of $A^1_t$ and $A^1_y$ in Fig. 4, with the parameters $m^2 = -33/16$, $\mu = 8.3$, $\epsilon = 0.3$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 1.286$.

The dependence of the current $J$ on the phase difference $\gamma$ across junction is shown on the left panel of Fig. 5. The figure shows that $J$ is proportional to the sine of $\gamma$.

$$J/T_c^2 \approx 0.4095 \sin \gamma, \quad \text{for} \quad m^2 = -33/16.$$  

(3.4)

From the above result, we can see our numerical data which is drawn with the red points fits sine line very well. The dependence of $J_{\text{max}}$ on the temperature $T$ is shown on the right panel of Fig. 5. The graph shows that $J_{\text{max}}/T_c^2$ decreases with the growing $T/T_c$.

The dependence of $J_{\text{max}}$ on the width $\ell$ of the gap is shown on the left panel of the Fig. 6. The figure shows that $J_{\text{max}}/T_c^2$ decays with growing $\ell$ exponentially. The dependence of $\langle \mathcal{O}(0) \rangle_{J=0}$ on the width $\ell$ of the gap is shown on the right panel of
Figure 4. We represent the components $\phi$ and $A_r$ of Yang-Mills fields. (Left) The profile of $\phi$. (Right) The figure of $A_r$. The parameters are $m^2 = -33/16$, $\mu = 8.3$, $\epsilon = 0.3$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 1.286$. We can see that these two figure are not symmetrical.

Figure 5. (Left) The current $J/T_c^2$ as sine function of phase difference $\gamma$. (Right) The $J_{\text{max}}/T_c^2$ decays with the growing $T/T_c$. The parameters are $m^2 = -33/16$, $\mu = 8.3$, $\epsilon = 0.3$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 1.286$. The numerical data fits exponential curves well.

The Fig. 6. The figure predicts that $\langle O(0) \rangle_{J=0}$ decays with growing $\ell$ exponentially.

$$ \begin{align*}
J_{\text{max}}/T_c^2 &\approx 27.07 e^{-\ell/0.7138} \\
\langle O(0) \rangle_{J=0} &\approx 26.91 e^{-\ell/(2\times0.6567)}
\end{align*} $$

for $m^2 = -33/16$.

The coherence length $(0.7138, 0.6567)$ can be obtained from these equations. The error of these two values is about 8%.
3.3 s+p-N-p Josephson junction

In this subsection, let us to continue to study the s+p-N-p Josephson junction. We tune the chemical potential $\mu(y)$ such that $\mu(-\infty)$ is in the s+p coexisting region $\mu_{c1} \sim \mu_{c2}$ and $\mu(+\infty) < \mu_{c1}$ is in the pure p-wave phase region. We still calculate the phase difference $\gamma$ from Eq. (3.3). The profiles of $A^1_t$ and $A^3_y$ are shown in Fig. 7, with the parameters $m^2 = -33/16$, $\mu = 6.3$, $\epsilon = 0.0$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 0.6$.

The dots which be determined by the DC current $J$ and the phase difference $\gamma$ across the junction is shown on the left panel of Fig. 8. With data fitting, we can see $J$ is proportional to the sine of $\gamma$.

$$J/T_c^2 \approx \begin{cases} 0.4999 \sin \gamma & \text{for } m^2 = -33/16, \\ 0.5318 \sin \gamma & \text{for } m^2 = -1031/500. \end{cases}$$

From the figure, it is shown that when the value of $m^2$ increases the maximum current $J_{\text{max}}$ will grow. The dependence of $J_{\text{max}}$ on the temperature $T$ is shown on the right panel of Fig. 8. The graph shows that $J_{\text{max}}/T_c^2$ increases with the growing $T/T_c$, which is for the same reason as the case in s+p-N-s+p junction.

In the left panel of Fig. 3, the dependence of $J_{\text{max}}/T_c^2$ on $\ell$ is shown. We can see that the $J_{\text{max}}/T_c^2$ decays with $\ell$ exponentially and change larger when $m^2$ becomes larger. Furthermore, we show the the dependence of $\langle \mathcal{O}(0) \rangle_{J=0}$ on $\ell$ in the right panel in Fig. 9. The $\langle \mathcal{O}(0) \rangle_{J=0}$ also decays with the growing $\ell$ exponentially and the
We represent the components $\phi$ and $A_y$ of Yang-Mills fields. (Left) The profile of $\phi$. (Right) The figure of $A_y$. The parameters are $m^2 = -33/16$, $\mu = 6.3$, $\epsilon = 0.0$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 0.6$. We can see that these two figures are not symmetrical.

$\langle O(0) \rangle_{J=0}$ becomes larger with increasing $m^2$.

\[
\begin{align*}
J_{\text{max}}/T_c^2 &\approx 7.843e^{-\ell/0.7376} \\
\langle O(0) \rangle_{J=0} &\approx 15.61e^{-\ell/(2\times 0.6021)}
\end{align*}
\]

for $m^2 = -33/16$.

From the above result, we can obtain the coherence length $(0.7376, 0.6021)$ from the above two equations, respectively. The error of two values is about 18.4%.

\[
\begin{align*}
J_{\text{max}}/T_c^2 &\approx 8.129e^{-\ell/0.7558} \\
\langle O(0) \rangle_{J=0} &\approx 14.68e^{-\ell/(2\times 0.6337)}
\end{align*}
\]

for $m^2 = -1031/500$.

Similarly, the coherence length $(0.7558, 0.6337)$ is obtained from the above two equations, respectively. The error of two values is about 16.2%.

### 3.4 s-N-p Josephson junction

In this subsection, we will construct the hybrid model of s-wave and p-wave Josephson junction, namely, the s-N-p Josephson junction. We tune the value of chemical potential $\mu(y)$ such that $\mu(-\infty) > \mu_c2$ is in the pure s-wave region and $\mu(+\infty) < \mu_c1$ is in the pure p-wave phase region. The phase difference $\gamma$ can be obtained in Eq. (3.3). The profiles of $A^1_x$ and $A^1_y$ are shown in Fig. 10, with the parameter $m^2 = -33/16$, $\mu = 7$, $\epsilon = 0.0$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 0.6$.

We show that the dependence of the current $J$ on the phase difference $\gamma$ across junction on the left side of Fig. 11, in which it is shown that $J$ is proportional to the sine of $\gamma$. 

\[
\langle O(0) \rangle_{J=0}
\]
\[ J/T_c^2 \approx 0.1166 \sin\gamma, \quad \text{for} \quad m^2 = -33/16. \quad (3.5) \]

The dependence of \( J_{\max} \) on the temperature \( T \) is shown on the right panel of Fig. 11, which shows that \( J_{\max}/T_c^2 \) decreases with the growing \( T/T_c \).

The dependence of \( J_{\max} \) on the width \( \ell \) of the gap is shown on the left panel of the Fig. 12, which shows that \( J_{\max}/T_c^2 \) decays with growing \( \ell \) exponentially. The
dependence of $\langle O(0) \rangle_{J=0}$ on the width $\ell$ of the gap is shown on the right panel of the Fig. 12, which predicts that $\langle O(0) \rangle_{J=0}$ decays with growing $\ell$ exponentially.

$$
\begin{cases}
J_{\text{max}}/T_c^2 \approx 9.842e^{-\ell/0.6798} \\
\langle O(0) \rangle_{J=0} \approx 12.95e^{-\ell/(2 \times 0.6362)}
\end{cases}
$$

for $m^2 = -33/16$.

The coherence length $(0.6798, 0.6362)$ can be obtained from these equations. The error of these two values is about 6.4%.

4 Conclusion and discussion

In this paper, we set up a holographic model for hybrid s-wave and p-wave DC Josephson junction with a scalar triplet charged under the SU(2) gauge field in the background of 3+1 dimensional AdS black hole. We get a set of partial differential equations of fields that are nonlinear and coupled and solve them numerically. We construct a new chemical potential $\mu(y)$ and tune the parameters in it, so the s+p-N-s+p junction, s+p-N-s junction, s+p-N-p junction and s-N-p junction can be obtained respectively. For the four kinds of junctions, we find the DC is proportional to the sine of the phase difference across the junction and the coherence lengths are different. We also study the relationship between $J_{\text{max}}/T_c^2$ and $\ell$, the total condensation $\langle O(0) \rangle_{J=0}$ and $\ell$, $J_{\text{max}}/T_c^2$ and $T/T_c$, respectively.

The reason we take $m^2 = -33/16$ is that when $m^2 < -33/16$, the region of s+p coexisting is too small, when $m^2 > -33/16$, the value in the region of s+p
Figure 11. (Left) The current $J/T_c^2$ as sine function of $\gamma$. (Right) The $J_{\text{max}}/T_c^2$ decays with growing $T/T_c$. In all the plots, we use $m^2 = -33/16$, $\mu = 7$, $\epsilon = 0.0$, $\sigma = 0.5$, $\ell = 3$ and $\beta = 0.6$. The numerical data fit exponential curves well.

Figure 12. (Left) The maximum current $J_{\text{max}}/T_c^2$ as exponential function of $\ell$. (Right) The total condensate $\langle O(0) \rangle_{J=0}$ as exponential function of $\ell$. In all the plots, we use $m^2 = -33/16$, $\mu = 7$, $\epsilon = 0.0$, $\sigma = 0.5$ and $\beta = 0.6$. The numerical data fits exponential curves well.

coexisting is too large for junction, the numerical results are not good. It is well known that the Josephson period is $2\pi$ in the $p_y$ wave s-N-p junction, so our ansatz just corresponds to $p_y$ wave junction. To our surprise, the periods of current are also $2\pi$ in remaining three kinds of junctions. For s+p-N-s+p junction and s+p-N-N-p junction, we take different values of $m^2 = -33/16, -1031/500$ and find that when the value becomes larger, the $J$, $J_{\text{max}}/T_c^2$ and $\langle O(0) \rangle_{J=0}$ will become larger. The reason we take another value of $m^2 = -1031/500$ is that the region of s+p
coexisting is small, we should take the another value of $m^2$ approaches the $m^2 = -33/16$. The maximum current increases with the growing temperature in s+p-N-s+p and s+p-N-p junction. Except the s+p-N-s+p junction, the phase difference should be get by \( \gamma = -\int_{-\infty}^{\infty} dy [\nu(y) - \nu(-\infty)] - \int_{c}^{+\infty} dy [\nu(y) - \nu(+\infty)] \) without loss of generality. When $c \neq 0$, the relationship between current and phase difference is \( J/T_c^2 = (J_{\text{max}}/T_c^2)\sin(\gamma + \phi) \), where $\phi$ is the origin phase difference and $\phi \neq 0$. So it is more convenient to set $c = 0$ to make $\phi = 0$.

Note that our model also can describe s-wave superconductor, p-wave superconductor, s+p coexisting superconductor, s-wave junction and p-wave junction. The p-wave contains $p_x$ wave and $p_y$ wave, the Josephson period is $\pi$ and $2\pi$, respectively. In the present study, this ansatz just describes $p_y$ wave. So, it should be of great interest to construct an ansatz which can describe $p_x$ wave and $p_y$ wave, respectively.

Finally, we have studied the hybrid and coexisting s-wave and p-wave junction with the probe limit, and we would like to study these kinds of junctions with the gravity backreaction in the future. So far, we have studied the hybrid and coexisting s-wave and p-wave DC Josephson junction in the AdS black hole background. It is meaningful to study these DC junctions by taking an AdS soliton as the geometry background in the further.

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