Higgs and SUSY Higgs bosons at future linear collider

Yasuhiro Okada

Institute of Particle and Nuclear Studies, KEK
Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan

Abstract. Theoretical overview on phenomenology of Higgs boson and SUSY Higgs boson at a future linear collider experiment is given as a Higgs and SUSY Higgs working group summary report for LCWS 2000.

INTRODUCTION

Exploring the Higgs sector is one of main motivations for constructing an $e^+e^-$ linear collider (LC). At the future LC with the center of mass energy of 300 - 500 GeV, we can expect to produce $10^4 - 10^5$ Higgs bosons with an integrated luminosity of $100 - 1000$ fb$^{-1}$ if the Higgs boson mass is $100 - 200$ GeV. For such a relatively light Higgs boson, LC can play a role of a Higgs factory and provide qualitatively different information from hadron machines, even when the Higgs particle is discovered at Tevatron or LHC experiment before the start of the LC experiment.

The purpose of the Higgs factory is to study various properties of the Higgs particle. From the production cross section in the $e^+e^- \rightarrow Z^* \rightarrow ZH$ process the Higgs coupling to two gauge bosons can be deduced. This coupling constant is closely related to the mass-generation mechanism of the gauge boson, because both the Higgs-two-gauge-boson coupling and the gauge-boson mass term originate from the same term in the standard model (SM) Lagrangian. By measuring this coupling constant, we can test whether or not the discovered scalar particle really plays a role of the Higgs boson associated with the electroweak symmetry breaking. Measurement of the Higgs boson coupling to fermions is important to understand the fermion mass generation mechanism. Eventually, we would like to reconstruct the Higgs potential itself by measuring three-point as well as four-point Higgs boson

1) Talk given at Linear Collider Workshop 2000 - LCWS 2000, October 24 -28, 2000, Fermilab.
2) Email: yasuhiro.okada@kek.jp
self-coupling constants. These are fundamental questions of particle physics, and the $e^+e^-$ LC is an ideal place to answer these questions.

In the Higgs and SUSY Higgs session of this workshop, most of contributions are related to the above questions. Firstly, how well can various coupling constants be determined in the future $e^+e^-$ and $\gamma\gamma$ colliders? With expected accuracy on those coupling measurements, is it possible to distinguish various models such as the minimal SM, the two Higgs doublet model (2HDM) and the minimal supersymmetric standard model (MSSM)? Within a specific model, for example in the MSSM, how well would various supersymmetric (SUSY) parameters be determined? Since the first question was mainly addressed by the experimental summary talk of this session [1], more theoretical aspects are discussed here. In the following, a short introduction to theoretical considerations on the Higgs boson mass is given first, and then the MSSM Higgs sector and the measurement of anomalous $ZZH$ and $Z\gamma H$ coupling constants are discussed. Finally, the Higgs physics at a $\gamma\gamma$ collider is briefly mentioned.

HIGGS BOSON MASS

The most fundamental parameter of the Higgs sector is the mass of the Higgs boson. Because the role of the Higgs field is to give masses to elementary particles through electroweak symmetry breaking, the mass of the Higgs boson itself reflects the self-interaction of the Higgs field. In the minimal SM with one Higgs doublet field, the Higgs boson mass ($m_H$) is given by $m_H = \sqrt{2\lambda v}$, where $\lambda$ is the self-coupling constant of the Higgs potential, $V = -\mu^2|H|^2 + \lambda|H|^4$, and $v \approx 246$ GeV. This formula implies that the mass of the Higgs boson is closely related to the strength of interaction responsible for the electroweak symmetry breaking and its determination provides us an important clue on the possible mechanism of the electroweak symmetry breaking.

More precisely, we can derive theoretical upper and lower bounds of the Higgs boson mass, if we take a particular model. In the minimal SM, the allowed range is $135 \text{ GeV} \lesssim m_h \lesssim 180 \text{ GeV}$, if we require that the theory is valid up to the Planck scale, $M_{pl} \sim 10^{19}$ GeV. The self-coupling constant $\lambda$ blows up below the Planck scale for a larger Higgs boson mass than the upper-bound, whereas $\lambda$ turns to a negative value and the vacuum stability is not guaranteed if the Higgs boson mass is less than the lower-bound. Similar upper and lower bounds can be obtained for the 2HDM [2] and the Zee model of neutrino mass generation [3] as a function of the cutoff scale. In these cases the allowed range of the mass for the lightest CP-even Higgs boson is $100 \text{ GeV} \lesssim m_h \lesssim 180 \text{ GeV}$ for $\Lambda = 10^{19}$ GeV in the decoupling case where only one SM-like Higgs boson is light compared to other physical state of the Higgs particles. The mass bounds of the lightest CP-even Higgs boson in the 2HDM are shown in Figure 1.

In the MSSM, the upper-bound on the the lightest CP-even Higgs boson mass ($m_h$) is determined without reference to the cut-off scale of the theory. This is
because the self-coupling of the Higgs field is completely determined by the gauge coupling constants at the tree level. Taking into account the top and stop one-loop corrections, $m_h$ is given by

$$m_h^2 \leq m_Z^2 \cos^22\beta + \frac{3}{2\pi^2} \frac{m_t^4}{v^2} \ln \frac{m_{stop}^2}{m_t^2},$$

(1)

where $\tan \beta \equiv \langle H_2^0 \rangle / \langle H_1^0 \rangle$ is the ratio of the vacuum expectation values of two Higgs fields [4]. In the above expression, we have assumed that the left- and right-handed stop squarks have the same mass and there is no mixing among two states. In the literature more precise calculation is available, and it is concluded that $m_h$ is bounded by about 130 GeV, even if we take the stop mass to be a few TeV. Because the expected mass range is somewhat lower than that derived under assumption that the minimal SM is valid up to the Planck scale, the discovery of the Higgs boson around 120 GeV would be a strong indication of the MSSM.

In extended versions of SUSY model, the upper-bound of the lightest CP-even Higgs boson can be determined only if we require that any of dimensionless coupling constants of the model does not blow up below some cut-off scale. For the SUSY model with an extra gauge singlet Higgs field, the bound is about 150 GeV, which is a slightly larger than the upper-bound for the MSSM case. Because there is a new tree level contribution to the Higgs mass formula, the maximum value corresponds to a lower value of $\tan \beta$, which is quite different from the MSSM case where the

**FIGURE 1.** The upper and lower bounds of the lightest CP-even Higgs boson mass in the type I and II 2HDM for the cutoff scale $\Lambda = 10^{19}$ GeV. $M$ is the soft-breaking mass of the discrete symmetry and the heavy Higgs boson masses become approximately $M$ for the large $M$ limit.(In the MSSM, $M = m_A$.) For comparison, the SM Higgs mass bound obtained from one-loop renormalization group equations and the lightest CP-even Higgs boson mass in the MSSM for $m_{stop} = 1$ TeV are shown.
Higgs mass becomes larger for large $\tan \beta$. In Ref. [5] the upper-bound of the lightest CP-even Higgs boson was calculated for SUSY models with gauge-singlet or gauge-triplet Higgs field and the maximal possible value was studied in those extensions of the MSSM. It was concluded that the mass bound can be as large as 210 GeV for a specific type model with a triplet-Higgs field for a stop mass of 1 TeV. The mass bound was also studied for the SUSY model with extra matter fields. In this model the upper-bound becomes larger due to loop corrections of extra matter multiplets. If the extra fields have $\bar{5} + 10 + 5 + \bar{10}$ representations in SU(5) GUT symmetry, the maximum value of the lightest CP-even Higgs boson mass becomes 180 GeV for the case that the squark mass is 1 TeV [6].

**Detectability of at least one Higgs boson at LC**

Because the above upper-bound of the lightest CP-even Higgs boson is at most about 200 GeV, at least one of the Higgs boson is kinematically accessible in any type of SUSY models for the $e^+e^- \rightarrow \gamma \text{Higgs}$ process with $\sqrt{s} = 500$ GeV. It is, however, important to know whether such a Higgs boson can be discovered for reasonable integrated luminosity, because the production cross section and decay modes depend on details of models. In the MSSM, it was shown that at least one of two CP-even Higgs bosons can be detectable through $e^+e^- \rightarrow Zh$ process with relatively low integrated luminosity ($\sim 10\text{fb}^{-1}$). In the model with a singlet Higgs field there are three CP-even Higgs bosons, and if the lightest one becomes singlet-dominated, its mass bound alone does not guarantee the discovery of the Higgs boson because its coupling to gauge bosons is very suppressed. Even in such a case we can show that at least one Higgs boson with a sizable SM Higgs component exists below or around the upper-bound of the lightest CP-even Higgs boson. In fact, we can calculate a minimal production cross section which means that at least one of three CP-even Higgs bosons has a larger production cross section than that value in the $e^+e^- \rightarrow Zh$ process. The minimal cross section turns out to be one third of the SM Higgs production cross section evaluated at the upper-bound value. This production cross section is typically 20 - 50 fb for $\sqrt{s} = 300 - 500$ GeV and large enough to guarantee the discovery of at least one Higgs boson independently of its decay modes [7]. Furthermore, in more general SUSY models with arbitrary number of the Higgs fields, it was shown that the Higgs signal is observable at $e^+e^- \rightarrow Zh$ with $\sqrt{s} = 500$ GeV and integrated luminosity of 500 fb$^{-1}$ [8]. This is true even if the Higgs bosons decay to invisible modes because we can measure a recoil mass distribution at the LC experiment.

**Precision electroweak measurements v.s. the Higgs boson mass**

Since present electroweak measurements are precise enough to be sensitive to virtual loop effects, a useful constraint on the Higgs boson mass is obtained in the minimal SM. Taking account of the direct measurement of the $W$ boson and top...
quark mass in addition to various electroweak data at LEP and SLD experiments, the 95% upper bound on the SM Higgs boson mass is about 210 GeV \cite{9}. This value means that the Higgs boson is detectable in the future LC with $\sqrt{s} = 500$ GeV. Because this bound is obtained within the minimal SM, it is very interesting whether or not this kind of bound can be derived in other models.

In general 2HDM, unlike the MSSM case, we cannot derive useful mass bound if we do not consider the condition on the validity of the theory up to some cut-off scale. The question whether we can conclude that at least one of Higgs boson will be discovered at LC based on present knowledge of the electroweak measurements was considered in Ref. \cite{10} and it was shown that the following four conditions could be satisfied simultaneously: (1) The light Higgs boson (CP-even ($h$) or CP-odd ($A$)) has no $ZZ/WW$ couplings. (2) The heavy CP-even Higgs boson ($H$) is too heavy to be produced at LC with $\sqrt{s} = 500 - 800$ GeV. (3) The production cross section for $e^+e^- \rightarrow t\bar{t}h$ or $b\bar{b}h$ ($t\bar{t}A$ or $b\bar{b}A$) are not large enough for discovery in these modes even with extremely large integrated luminosity (2500 fb$^{-1}$). (4) These parameters are still reasonably consistent with the present electroweak precision test. Although no direct signal on the Higgs sector is expected in this particular parameter space, we can distinguish this model if we improve the electroweak measurement including the $W$ boson mass determination at the Giga-Z option of the LC experiment \cite{10,11}.

**MSSM HIGGS SECTOR**

In the MSSM, physical Higgs states are CP-even Higgs bosons ($h, H$), a CP-odd Higgs boson ($A$) and a pair of charged Higgs bosons ($H^\pm$). At the tree level, the Higgs potential is parametrized by three mass parameters in addition to gauge

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{higgs_masses.png}
\caption{Higgs boson masses as a function of the CP-odd Higgs boson mass in MSSM.}
\end{figure}
coupling constants. It is known that the radiative corrections, mainly due to top and stop loop diagrams, can give important modification to the Higgs potential, so that the lightest CP-even boson mass receives a large radiative correction. Three dimensionful parameters can be taken as two vacuum expectation values, in other words $v$ and $\tan \beta$, and one of Higgs boson masses, which is usually taken as the CP-odd Higgs boson mass ($m_A$). In addition, the Higgs sector depends on the top and the stop masses through the radiative correction. More precisely, the parameters appearing in the formula of the radiative correction are not just one stop mass, but two stop masses, trilinear coupling constant for stop sector ($A_t$), the higgsino mass parameter $\mu$, and sbottom masses, etc. (If we use more precise formula, we need to specify more input parameters.) Once these parameters are specified, we can calculate the mass and the mixing of the Higgs sector. The Higgs mixing parameter $\alpha$ is defined by $ReH_1^0 = (v \cos \beta - h \sin \alpha + H \cos \alpha)/\sqrt{2}$, $ReH_2^0 = (v \sin \beta + h \cos \alpha + H \sin \alpha)/\sqrt{2}$, where $H_1^0$ and $H_2^0$ are neutral component of two Higgs doublet fields. $\alpha$ is a function of independent parameters ($m_A, \tan \beta, m_t, m_{stop}$, etc).

The masses of the CP-even and the CP-odd Higgs bosons as well as the charged Higgs bosons are shown as a function of $m_A$ in Fig. 2. It is important to distinguish two regions in this figure. Namely, when $m_A$ is much larger than 150 GeV, $H, A$ and $H^\pm$ states become approximately degenerate and the mass of $h$ approaches to its upper-bound value for each $\tan \beta$. This limit is called the decoupling limit. In this limit, $h$ has properties similar to the SM Higgs boson, and the coupling of the heavy Higgs bosons to two gauge-boson states is suppressed. On the other hand, if $m_A$ is less than 150 GeV, the lightest CP-even Higgs boson has a sizable components of the SM Higgs field and the other doublet field. Since LEP experiments have already excluded a significant portion of the parameter space, it is quite possible that the Higgs boson will be found in the decoupling region. If we discover only one Higgs boson at either Tevatron or LHC experiments in the mass region consistent with the MSSM, precise measurements of the Higgs particle’s properties are important to distinguish this from the SM Higgs boson, especially in the decoupling and near-decoupling regions. The future LC is the most suitable place to perform such precise measurements. In the following subsections, we discuss various ways to distinguish the MSSM from other models through measurements with respect to the Higgs sector.

Non-minimality of the Higgs sector

In the production of the light Higgs boson, the number of the Higgs signals provide important information on the model behind the Higgs sector. Because the main decay mode of the light Higgs boson is $h \rightarrow b\bar{b}$, $\sigma(e^+e^- \rightarrow Z h)B(h \rightarrow b\bar{b})$ is a quantity expected to be measured precisely. In Fig. 3, the MSSM parameter region where $\sigma(e^+e^- \rightarrow Z h)B(h \rightarrow b\bar{b})$ differs from the SM prediction by 6% is shown for $m_t = 175$ GeV and $m_{stop} = 1$ TeV. Because the statistical accuracy of this quantity at the first stage of the future LC experiment is expected to reach a few % level,
the deviation will be clear within this parameter region. The contour extends to the line corresponding to \( m_A = 500 \) GeV. In this figure two other lines are drawn. One corresponds to the case in which the total width of the Higgs boson is larger than 200 MeV, so that the recoil mass distribution in the \( e^+e^- \rightarrow Zh \rightarrow l^+l^-h \) provides the direct measurement of this quantity. The other line is the limit where the charged Higgs boson pair production is observable at the LC with \( \sqrt{s} = 500 \) GeV.

**Higgs boson branching ratios v.s. the heavy Higgs boson mass**

It was pointed out that the following ratio of the branching ratios

\[
R_{cc+gg/bb} = \frac{B(h \rightarrow c\bar{c}) + B(h \rightarrow gg)}{B(h \rightarrow bb)}
\]

is a sensitive probe to \( m_A \) [12]. In the MSSM, the Higgs sector is the same as type II 2HDM at the tree level, so that the up(down)-type Yukawa coupling constant is associated with \( H_2 \) (\( H_1 \)), and \( B(h \rightarrow c\bar{c}) \) (\( B(h \rightarrow bb) \)) is proportional to \( \cos^2 \alpha/\sin^2 \beta \) (\( \sin^2 \alpha/\cos^2 \beta \)). Because \( h \rightarrow gg \) is induced predominantly by the internal top quark loop, the dependence on the angles is the same as the \( h \rightarrow c\bar{c} \) mode. As a result, \( R_{cc+gg/bb} \) is proportional to \( 1/(\tan \beta \tan \alpha)^2 \). Taking into account

![Non-minimality Contour](image)

**FIGURE 3.** Parameter space in the MSSM where deviation from the SM is expected to be observable at \( e^+e^- \) LC. Three lines corresponds to: (1) The total width of the lightest CP even Higgs boson mass is larger than 200 MeV, (2) \( \sigma(e^+e^- \rightarrow Zh)Br(h \rightarrow bb) \) deviates from the SM value more than 6% (3) \( \sigma(e^+e^- \rightarrow H^+H^-) > 10 \text{ fb} \) for \( \sqrt{s} = 500 \) GeV. This figure is provided by A. Miyamoto.
the top and the stop loop corrections to the Higgs potential, we can relate these angles to other input parameters and derive the following approximate formula.

\[ R_{cc+gg/bb} \simeq \left( \frac{m_A^2 - m_h^2}{m_A^2 + m_Z^2} \right)^2 R_{cc+gg/bb}(SM), \]  

(3) where \( R_{cc+gg/bb}(SM) \) is the same ratio evaluated in the SM. In Fig.4, \( R_{cc+gg/bb}(SM) \) is shown for \( m_h = 120 \text{ GeV} \). (\( \tan \beta \) is solved for each \( m_A \) to give this Higgs boson mass.) This figure shows that the above formula is actually a very good approximation and the ratio is almost independent of other parameters like \( m_{\text{stop}} \) and \( A_t \). The expected statistical error for 100 fb\(^{-1} \) is also shown [13]. We can see that this quantity is useful to constrain the mass scale of the heavy Higgs bosons.

Although the lightest CP-even Higgs boson is likely to be discovered in the LHC experiment, whether or not other Higgs states \( (H, A, H^\pm) \) can be found depends on the SUSY parameters. There is a large portion of the parameter space with \( m_A \gtrsim 200 \text{ GeV} \) and \( 2 \lesssim \tan \beta \lesssim 10 \) where only one lightest CP-even Higgs boson will be discovered. If this is the case, it would be very important that possible range of the heavy Higgs boson mass could be inferred from branching ratio measurements at the first stage LC experiment with \( \sqrt{s} = 300 - 500 \text{ GeV} \), so that we can set the target energy of the second stage LC experiment.

\[ R_{cc+gg/bb} \equiv \frac{\text{Br}(h \to c\bar{c}) + \text{Br}(h \to gg)}{\text{Br}(h \to bb)}, \]

**FIGURE 4.** \( R_{cc+gg/bb} \) is shown as a function of \( m_A \) for different choices of SUSY parameters. Expected statistical error for 100 fb\(^{-1} \) is also shown.
**SUSY loop correction to bottom Yukawa coupling at large tan β**

When \( \tan \beta \) is as large as 50, SUSY loop corrections generate important contribution to the bottom-Higgs Yukawa coupling [14]. Including one-loop induced coupling the top and bottom Yukawa couplings with the neutral Higgs fields are given by

\[
L_{\text{Yukawa}} = y_t \bar{t}_L t_R H_2^0 + y_b \bar{b}_L b_R H_1^0 + \epsilon_b \bar{b}_L b_R H_2^{0*} + h.c. \tag{4}
\]

The \( \epsilon_b \) term is induced by loop diagrams with internal sbottom-gluino and stop-chargino. Thus the Higgs sector becomes effectively general-type 2HDM, not restricted to the type-II model. The bottom mass is expressed by

\[
m_b = y_b (1 + \epsilon_b \tan \beta) \frac{v}{\sqrt{2}} \cos \beta. \tag{5}\]

Although \( \epsilon_b \) is typically \( O(10^{-2}) \), the correction to \( m_b \) enters with a combination of \( \epsilon_b \tan \beta \) which can be close to \( O(1) \) for a large value of \( \tan \beta \). Precise calculation of various branching ratios was presented taking account of these corrections [15]. For example, the ratio of \( B(h \to \tau^+ \tau^-) \) and \( B(h \to b \bar{b}) \) is modified to

\[
R_{\tau\tau/bb} \equiv \frac{B(h \to \tau^+ \tau^-)}{B(h \to b \bar{b})} = \left( \frac{1 + \epsilon_b \tan \beta}{1 - \epsilon_b / \tan \alpha} \right)^2 R_{\tau\tau/bb}(SM), \tag{6}\]

where \( R_{\tau\tau/bb}(SM) \) is the same ratio evaluated in the SM. \( R_{\tau\tau/bb} \) is the same as \( R_{\tau\tau/bb}(SM) \) if the SUSY loop effect to the \( b \bar{b} H_2^{0*} \) vertex is negligible. We also notice that in the decoupling limit with \( m_A \to \infty \), \( \tan \alpha \) is approaching to \( -1 / \tan \beta \), so that the ratio reduces to the SM prediction. This is because the properties of the lightest CP-even Higgs boson become similar to those of the SM Higgs boson, independent of how the two Higgs doublet fields couple to the fermions. In actual evaluation, however, the approach to the asymptotic form is slow for large \( \tan \beta \), so that sizable deviation from the SM prediction may be observed.

The SUSY vertex correction also modifies the previous discussion on constraining \( m_A \) from \( R_{cc+gg/bb} \). This problem was investigated and it was shown that the effect on \( R_{cc+gg/bb} \) is within a few \% for \( \tan \beta = 10 \), although the uncertainty is too large for \( \tan \beta = 50 \) [16]. This means that for the parameter space where the LHC experiment would not find the heavy Higgs boson, \( R_{cc+gg/bb} \) can still give important clue on the heavy Higgs mass scale. The theoretical uncertainty to the branching ratio from input parameters such as \( \alpha_s, m_b \) and \( m_c \) was also studied, and the error is about 8\% for \( R_{cc+gg/bb}(SM) \) and \( R_{\tau\tau/bb}(SM) \) if we know \( \alpha_s \) within 2\%, the \( \overline{\text{MS}} \) running quark masses, \( m_b(m_b) \) within 3\% and \( m_c(m_c) \) within 5\%. The mass determination at this level from lattice gauge theory is very important for Higgs physics.

Once we measure \( B(h \to b \bar{b}) \), \( B(h \to c \bar{c}) \) and \( B(h \to \tau^+ \tau^-) \) with enough precision, we can determine the correction factor \( \Delta_b \equiv \epsilon_b \tan \beta \) [17]. If \( x(y) \) is defined
as $B(h \rightarrow b\bar{b})/B(h \rightarrow \tau^+\tau^-)$ $B(h \rightarrow c\bar{c})/B(h \rightarrow \tau^+\tau^-)$ normalized by the corresponding SM values, we get $\Delta_b = (1 - \sqrt{x})/\left((\sqrt{x} - \sqrt{y})\right)$. Determination of $\Delta_b$ is very interesting because non-zero value provides a hint for the SUSY loop effect on the vertex correction.

The vertex correction also modifies the production and the decay of the heavy Higgs bosons [18]. Main effects on the coupling of heavy Higgs boson to the fermions is due to the correction to the relation between the bottom mass and the original bottom Yukawa coupling constant shown in Eq.(5). In SO(10)-like GUT model with Yukawa unification, for example, $\Delta_b$ should be -25% to -18% and the bottom coupling constants to $A$ and $H$ become effectively larger by the same amount. As a result, the production cross sections of $e^+e^- \rightarrow Ab\bar{b}$, $Hb\bar{b}$ below the threshold of the $A H$ associated production is increased, and the $\tau^+\tau^-$ and $c\bar{c}$ branching ratios of the heavy Higgs boson are suppressed.

**Determination of $\tan \beta$ from heavy Higgs boson process**

In SUSY models $\tan \beta$ is a very important parameter. Although it is defined as a ratio of two vacuum expectation values, this parameter appears in various couplings of the MSSM Lagrangian. The consistency check of $\tan \beta$ determined in the Higgs sector, the chargino-neutralino sector as well as the squark-slepton sector will be interesting because these relationships result from SUSY invariance.

In the MSSM, the possible range of $\tan \beta$ is 1 - 70 from the requirement that the Yukawa coupling constants do not blow up below the Planck scale. The range of $\tan \beta \lesssim 2$ is excluded from the LEP II Higgs boson search for reasonable values of stop masses and mixing parameters. Direct determination of $\tan \beta$ is a challenging task for future LC experiment.

One of best ways to determine $\tan \beta$ is to study heavy Higgs boson production and decays, because the branching ratios and production cross sections depend on $\tan \beta$ significantly. For the branching ratios, $A, H \rightarrow t\bar{t}$ and $H^- \rightarrow b\bar{t}$ are dominant for small $\tan \beta$ as long as these modes are kinematically allowed, whereas $A, H \rightarrow b\bar{b}, \tau^+\tau^-$ and $H^- \rightarrow \tau\nu$ becomes important for larger values of $\tan \beta$. For the production processes, $e^+e^- \rightarrow b\bar{b}A$, $b\bar{b}H$ and $e^+e^- \rightarrow tbH^-$ are enhanced for large $\tan \beta$. The question to what extent $\tan \beta$ can be determined at future LC was addressed in Ref. [19]. It was concluded that $\tan \beta$ would be well constrained in the intermediate values of $\tan \beta$ ($3 \lesssim \tan \beta \lesssim 10$). In addition, $e^+e^- \rightarrow tbH^-$ production cross section is enhanced for a large value of $\tan \beta(\gtrsim 50)$.

In this workshop, a new analysis was reported where systematic study was performed using $e^+e^- \rightarrow Hb\bar{b}$, $Ht\bar{t}$, $Ab\bar{b}$, $At\bar{t}$ process at LC with $\sqrt{s} = 500 - 1000$ GeV [20]. Within the MSSM scenario, $\tan \beta$ will be well constrained for $\tan \beta \lesssim 10$. Typical error is $\pm 2$ for $\tan \beta = 3$ and $\pm 4$ for $\tan \beta = 10$. If $\tan \beta$ is determined from the Higgs sector we can use this value as an input parameter for various other SUSY processes such as chargino production and stau decays, so that we can perform quantitative tests of SUSY Lagrangian.
Single charged Higgs boson production

Main production process of the charged Higgs boson at LC is $e^+e^- \rightarrow H^+H^-$, so that the mass reach is kinematically restricted to $\sqrt{s}/2$. A single charged Higgs boson production is potentially important because the search for a heavier charged Higgs boson is possible.

For this purpose, a systematic investigation of single charged Higgs boson production processes at $e^+e^-\text{LC}$ has been done \cite{21}. Production cross sections for 14 processes were calculated in the MSSM. These are $e^+e^- \rightarrow \tau^+\nu H^-$, $t\bar{b}H^-$, $W^+H^-$, $e^+\nu H^-$, $Z^0W^+H^-$, $H^+H^-$, $AW^+H^-$, $e^+e^-W^+H^-$, $\nu\bar{\nu}W^+H^-$, $e^+\nu Z^0H^-$, $e^+\nu hH^-$, $e^+\nu HH^-$, and $e^+\nu AH^-$. Among these processes, $e^+e^- \rightarrow W^+H^-$, $e^+e^- \rightarrow \tau^+\nu H^-$ are one-loop induced processes. Generally, the production cross sections are very suppressed ($<10^{-2}$ fb$^{-1}$) below the $H^+H^-$ pair production threshold, but the two processes are found to be promising. Namely, $e^+e^- \rightarrow \tau^+\nu H^-$ for large tan $\beta$ case and $e^+e^- \rightarrow W^+H^-$ for small tan $\beta$ case have production cross sections of $0.01-1$ fb$^{-1}$ for $\sqrt{s} = 500$ GeV, and therefore may be observable with integrated luminosity of 500 fb$^{-1}$.

ANOMALOUS COUPLING MEASUREMENT

At $e^+e^-\text{LC}$ experiment, a great number of light Higgs bosons are expected to be produced through the $e^+e^- \rightarrow ZH \rightarrow f\bar{f}H$ process. We can put constraints on anomalous coupling constants for the $ZZH$ and $Z\gamma H$ vertexes, because these coupling constants generate additional contribution to the $ZH$ production process through interference with the SM process \cite{22}. Up to dimension-five operators these anomalous couplings are given by

$$L_{\text{eff}} = (1 + a_z) \frac{g_Z m_Z}{2} HZ_{\mu}Z^\mu + \frac{g_Z}{m_Z} \sum_{V=\gamma,Z} [b_V HZ_{\mu\nu}Z^{\mu\nu} + c_V (\partial_{\mu}Z_{\nu} - \partial_{\nu}Z_{\mu})V^{\mu\nu} + \tilde{b}_V HZ_{\mu\nu}\tilde{V}^{\mu\nu}]$$

where $V_{\mu} = A_{\mu}$ or $Z_{\mu}$ and $V_{\mu\nu} = \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu}$, $\tilde{V}^{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}V^{\alpha\beta}$. For $\sqrt{s} = 500$ GeV and integrated luminosity of 300 fb$^{-1}$, expected constraints on these coupling constants were studied for $m_H = 115$ GeV. In particular, effects of helicity measurements of $\tau$ lepton ($\epsilon_\tau = 50\%$), charge identification of the bottom quark ($\epsilon_b = 60\%$) as well as beam polarization ($|P_{e^-}| = 80\%$, $|P_{e^+}| = 45\%$) were investigated. These options turns out to be useful to improve the measurements of anomalous coupling constants. With these options the above parameters can be constrained to the level of $10^{-4}$. In particular, the improvement is significant for the $Z\gamma H$ coupling measurement. This is because the interference term between the SM amplitude and the amplitude with the $Z\gamma H$ couplings is suppressed without these options due to the almost axial-vector nature of the $e^+e^-Z$ coupling in the SM.
PHOTON-PHOTON COLLIDER

In the photon-photon option of the LC experiment, we can directly measure $\Gamma(H \to \gamma\gamma)B(H \to bb)$ by the s-channel Higgs boson production process. Assuming that $B(H \to bb)$ will be already precisely known from the $e^+e^-$ LC experiment, we can obtain the two photon partial width of the Higgs boson. Combined with the information on $B(H \to \gamma\gamma)$, the total decay width will be derived. Because $\Gamma(H \to \gamma\gamma)$ is induced by heavy particle loops, accurate determination of $\Gamma(H \to \gamma\gamma)$ is very interesting as a possible window to new physics.

In order to estimate the accuracy, we have to evaluate rates of the signal and background processes in the SM. In this workshop, recent developments on the radiative corrections for these processes were reported [23,24]. In the SM background process, $e^+e^- \to bb$, the Born cross section for $J_z = 0$ channel is suppressed by $m_b^2/s$ compared to that for $J_z = \pm 2$ channel, so that we can use polarization of initial photons to suppress the background. On the other hand, there is a large QCD correction to this process and careful treatment of higher order corrections is necessary. The radiative corrections to the SM signal and background processes are under control and it was reported that the measurement of two photon decay width with a statistical accuracy of 1.4% would be possible for a Higgs boson mass of 115 GeV using the TESLA parameters [24].

One example of possible role in identifying a new physics from the two photon partial width of the Higgs boson was discussed [25]. In 2HDM, it may be possible that only a light Higgs boson is found at LHC and $e^+e^-$ LC experiments and its couplings to gauge boson and fermions are found to be consistent with the SM Higgs boson within the expected accuracy of these experiments. Even in such a situation, the deviation of $\Gamma(H \to \gamma\gamma)$ from the SM value can be as large as 10%, so that the two-photon decay width measurement may be able to provide the first hint for non-minimality of the Higgs sector.

SUMMARY

We have discussed various examples in which studies of the Higgs sector at future LC experiment provide signals of physics beyond the SM. If in fact a light Higgs boson is found at Tevatron, LHC or LC experiments, the role of LC will be the Higgs factory. By precise determination of production cross section and branching fractions, we can test whether the observed particle corresponds to the SM Higgs boson or the MSSM Higgs boson. Within the MSSM we can put a constraint on the SUSY parameters. If kinematically allowed, heavy charged and neutral Higgs bosons may be produced, and their properties are useful to identify models. The photon-photon option is also useful to search for new physics through loop effects in the two photon decay width of the Higgs boson. These measurements are important to establish or distinguish various candidates of models such as the minimal
SM, MSSM, 2HDM or some other possibilities which may be highlighted in future.

This work was supported by the Grant-in-Aid of the Ministry of Education, Science, Sports and Culture, Government of Japan (No. 09640381), Priority area “Supersymmetry and Unified Theory of Elementary Particles” (No. 707), and “Physics of CP Violation” (No. 09246105).

REFERENCES

1. M. Battaglia, and K. Desch, in these proceedings, hep-ph/0101165.
2. S. Kanemura, T. Kasai and Y. Okada, Phys. Lett. B 471, 182 (1999).
3. S. Kanemura, T. Kasai, G.-L. Lin, Y. Okada, J.-J. Tseng, C.P. Yuan, hep-ph/0011357.
4. Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85, 1 (1991); Phys. Lett. B 262, 54 (1991); J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B 257, 83 (1991). H.E. Haber and R. Hempfling, Phys. Rev. Lett. 66, 1815 (1991).
5. J.R. Espinosa and M. Quirós, Phys. Rev. Lett. 81, 516 (1998).
6. T. Moroi, and Y. Okada, Phys. Lett. B 295, 73 (1992).
7. J. Kamoshita, Y. Okada, and M. Tanaka, Phys. Lett. B 328, 67 (1994); S.F. King and P.L. White Phys. Rev. D 53, 4049 (1996).
8. J.R. Espinosa and J.F. Gunion, Phys. Rev. Lett. 82, 1084 (1999).
9. LEP Electroweak Working Group report, CERN-EP-200-016 (January 21, 2000).
10. J.F. Gunion, in these proceedings, hep-ph/0012199; P. Chankowski, T.Farris, B. Grzadkowski, J.F. Gunion, J. Kalinowski, and M. Krawczyk, Phys. Lett. B 496, 195 (2000).
11. M.E. Peskin and J.D. Wells, hep-ph/0101342.
12. J. Kamoshita, Y. Okada and M. Tanaka, in Proceedings of the Workshop on Physics and Experiments with Linear Colliders, Morioka-Appi, Iwate, Japan 1995 edited by A. Miyamoto et al. (World Scientific, Singapore, 1996); Phys. Lett. B 391, 124 (1997).
13. I. Nakamura and K. Kawagoe, in Proceedings of the Workshop on Physics and Experiments with Linear Colliders, Morioka-Appi, Iwate, Japan 1995 edited by A. Miyamoto et al. (World Scientific, Singapore, 1996); Phys. Rev. D 54, 3634 (1996).
14. K.S. Babu and C. Kolda, Phys. Lett. B 451, 77 (1999); H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, and Y. Yamada, Phys. Rev. D 62, 055006, (2000); M. Carena, D. Garcia, U. Nierste, and C.E.M. Wagner, Nucl. Phys. B 577 88 (2000); H.E. Haber, M. Herrero, H.E. Logan, S. Peñaranda, S. Rigolin, and D.Temes, Phys. Rev. D 63, 055004, (2001).
15. G. Weiglein, in these proceedings.
16. S. Kiyoura and Y. Okada, in these proceedings, hep-ph/0101172.
17. H.E. Logan, in these proceedings, hep-ph/001202; S. Mrenna, in these proceedings.
18. C. Kolda, in these proceedings.
19. J.L. Feng and T. Moroi, Phys. Rev. D 56, 5962 (1997).
20. J.Jiang, in these proceedings; V.Barger, T. Han, and J.Jaing, hep-ph/0006223.
21. S. Kanemura, S. Moretti, K. Odagiri, in these proceedings, hep-ph/0101354: hep-ph/0012030.
22. S. Kamoshita, in these proceedings; K. Hagiwara, S. Ishihara, J. Kamoshita, and B.A. Kniehl, *Eur. Phys. J.* C 14, 457 (2000).
23. O. Yakovlev, in these proceedings, hep-ph/0012233.
24. M. Melles, in these proceedings, hep-ph/0012195.
25. I.F. Ginzburg, M. Krawczyk, and P. Osland, in these proceedings, hep-ph/0101331; hep-ph/0101208; hep-ph/0101229.