Quantum inflaton, primordial metric perturbations and CMB fluctuations

F J Cao\textsuperscript{1,2}

\textsuperscript{1} Departamento de Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid, Avenida Complutense s/n, 28040 Madrid, Spain
\textsuperscript{2} LERMA, Observatoire de Paris, Laboratoire Associé au CNRS UMR 8112, 61, Avenue de l’Observatoire, 75014 Paris, France
E-mail: franco@fis.ucm.es

Abstract. We compute the primordial scalar, vector and tensor metric perturbations arising from quantum field inflation. Quantum field inflation takes into account the nonperturbative quantum dynamics of the inflaton consistently coupled to the dynamics of the (classical) cosmological metric. For chaotic inflation, the quantum treatment avoids the unnatural requirements of an initial state with all the energy in the zero mode. For new inflation it allows a consistent treatment of the explosive particle production due to spinodal instabilities. Quantum field inflation (under conditions that are the quantum analog of slow roll) leads, upon evolution, to the formation of a condensate starting a regime of effective classical inflation. We compute the primordial perturbations taking the dominant quantum effects into account. The results for the scalar, vector and tensor primordial perturbations are expressed in terms of the classical inflation results. For a N-component field in a $O(N)$ symmetric model, adiabatic fluctuations dominate while isocurvature or entropy fluctuations are negligible. The results agree with the current WMAP observations and predict corrections to the power spectrum in classical inflation. Such corrections are estimated to be of the order of $m^2/[NH^2]$ where $m$ is the inflaton mass and $H$ the Hubble constant at horizon crossing. This turns to be about 4\% for the cosmologically relevant scales. This quantum field treatment of inflation provides the foundations to the classical inflation and permits to compute quantum corrections to it.

1. Introduction

Inflation is a stage of accelerated expansion in the very early Universe \cite{1,2}. The present observations make inflationary cosmology the leading theoretical framework to explain the homogeneity, isotropy and flatness of the Universe, as well as the observed features of the cosmic microwave background \cite{3}. In most inflation models the inflaton background dynamics for these models is usually studied in a classical framework and in order to have a long inflationary period it is necessary that the field rolls down very slowly: for these models various conditions have been obtained which are different realizations of what we will call here the \textit{classical slow roll condition}, $\dot{\varphi}^2 \ll |m^2| \dot{\varphi}^2$. This condition guarantees that there is inflation ($\ddot{a} > 0$) and that it lasts long enough. ($\varphi$ is the classical inflaton field, $m$ its mass, and the dot denotes cosmic time derivative. We use the tilde, $\tilde{\ldots}$, to denote the quantities in classical inflation.)

However, since the energy scale of inflation is so high (the GUT scale), it is necessary a full \textit{quantum} field theory description for the matter. Only such a quantum treatment permits a
consistent description of particle production and particle decays. We address here this problem and show how the slow roll conditions can be generalized.

2. Quantum field inflation
The action for quantum field inflaton is \( S_q = \tilde{S}_q + S_m + \delta \tilde{S}_q + \delta S_m \), where \( \tilde{S}_q + S_m \) describes the dynamics of the background, and \( \delta \tilde{S}_q + \delta S_m \) that of the perturbations. The important difference with classical inflation is that the dynamics of the inflaton background \( (S_m) \) is computed here in quantum field theory. The gravitational terms have the same expressions as in the classical inflaton dynamics. The gravitational action and its perturbation are

\[
\tilde{S}_{gr} + \delta \tilde{S}_{gr} = -\frac{1}{16\pi G} \int \sqrt{-g} \, d^4x \, R
\]

where \( G \) is the universal gravitational constant, and \( R \) is the Ricci scalar for the complete metric \( g_{\mu\nu} \). In our treatment we consider semiclassical gravity: the geometry is classical and the metric obeys the semiclassical Einstein equations where the r. h. s. is the expectation value of the quantum energy momentum tensor. (Quantum gravity corrections are at most of order \( \sim m/M_{\text{Planck}} \sim M_{\text{GUT}}/M_{\text{Planck}} \sim 10^{-6} \) and can be neglected.) In order to implement a nonperturbative treatment, we consider a \( N \)-component inflaton field \( \vec{\chi} \). The quantum matter action is

\[
S_m + \delta S_m = \int \sqrt{-g} \, d^4x \left[ \frac{1}{2} \partial_\mu \chi^\nu \partial^\mu \chi^\nu - V(\chi) \right] = \int d^4x \, a^3(t) \left[ \frac{1}{2} (\dot{\chi})^2 - \frac{1}{2} \frac{(\nabla \chi)^2}{a^2(t)} - V(\chi) \right]
\]

where \( \chi = (\chi_1, \ldots, \chi_N) \) and

\[
V(\chi) = \frac{1}{2} m^2 \chi^2 + \frac{\lambda}{8N} (\chi^2)^2 + \frac{Nm^4}{2\lambda} \frac{1 - \alpha}{2},
\]

with \( \alpha \equiv \text{sign}(m^2) = \pm 1 \). For positive \( m^2 \) the \( O(N) \) symmetry is unbroken while it is spontaneously broken for \( m^2 < 0 \). The first case describes chaotic inflation and the second one corresponds to new inflation. The initial state for chaotic inflation is a highly excited field state, i.e., a state with large \( |\varphi| \), while for new inflation is a state with small \( |\varphi| \).

The quantum field \( \vec{\chi} \) can be expanded as its expectation value \( \langle \chi(x) \rangle \) plus quantum contributions which are in general large and cannot be linearized (except for \( k/a \) much larger than the effective mass). We therefore split the quantum contribution \( \chi - \langle \chi(x) \rangle \) into large quantum contributions \( \vec{\varphi}(x) \) (or background), plus small quantum contributions \( \delta \vec{\varphi}(x) \). Thus, we express the \( N \)-component quantum scalar field \( \vec{\chi} \) as

\[
\chi(x) = \langle \chi(x) \rangle + \vec{\varphi}(x) + \delta \vec{\varphi}(x).
\]

(1)

After expanding the quantum matter action using Eq. (1), \( S_m \) stands for the terms without \( \delta \vec{\varphi} \) and describes the inflaton background dynamics, while \( \delta S_m \) stands for the remaining terms which describe the inflaton perturbation dynamics. The dynamics of \( \delta \vec{\varphi}(x) \) can be then linearized, and includes the cosmologically relevant fluctuations, that is those which had exited the horizon during the last \( N_e \approx 60 \) efolds of inflation. In momentum space, let us call \( \Lambda \) the \( k \)-scale that separates the perturbation from the background. Without loss of generality, we can write \( \langle \chi(x) \rangle = (\sqrt{N} \varphi(t), 0) \), and \( \chi(x) = \left( \sqrt{N} \varphi(t) + \varphi_\parallel(x), \varphi_\perp(x) \right) + (\delta \varphi_\parallel(x), \delta \varphi_\perp(x)) \). The mode expansions for the fluctuations are

\[
\varphi_\perp(x, t) = \frac{1}{\sqrt{2}} \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \left[ \tilde{a}_k f_k(t) e^{i\vec{k} \cdot \vec{x}} + \tilde{a}_k^* f_k^*(t) e^{-i\vec{k} \cdot \vec{x}} \right]
\]
and

\[ \delta \vec{\varphi}_\perp (\vec{x}, t) = \frac{1}{\sqrt{2}} \int_A^{\infty} \frac{d^3k}{(2\pi)^3} \left[ \tilde{a}_k f_k(t) e^{i\vec{k} \cdot \vec{x}} + \tilde{a}^\dagger_k f_k^*(t) e^{-i\vec{k} \cdot \vec{x}} \right] \]

, with \( \tilde{a}_k \) and \( \tilde{a}^\dagger_k \) being annihilation and creation operators, respectively, satisfying the canonical commutation relations. \( [\varphi] \) and \( \delta \varphi \) can also be expanded analogously with different operators \( b_k \) and \( \tilde{b}_k \), and modes \( g_k(t) \). The scale \( \Lambda \) is well above the \( k \)-modes that dominate the bulk of the energy, and well below the cosmologically relevant modes. The results are independent of the precise value of \( \Lambda \). This is due to the fact that modes with \( k \gg m \) cannot be significantly excited since the energy density of the universe during inflation must be of the order \( \gtrsim 10^{m^2 M_{\text{Planck}}^2} \).

On the other hand, modes that are relevant for the large scale structure and the CMB are today in the range from \( 0.1 \text{ Mpc} \) to \( 10^3 \text{ Mpc} \). These scales at the beginning of inflation correspond to physical wavenumbers in the range \( e^{N_T - 60} 10^{16} \text{ GeV} < k < e^{N_T - 60} 10^{20} \text{ GeV} \) where \( N_T \) stands for the total number of e-folds (see for example Ref. [5]). Therefore, there is an intermediate \( k \)-range of modes which are neither relevant for the background nor for the observed perturbation. \( \Lambda \) is inside this \( k \)-range, and the results are independent of its particular value. In usual cases we can safely choose for \( \Lambda, 10^m \lesssim \Lambda \lesssim 10^3 e^{N_T - 60} m \).

2.1. Quantum field inflation dynamics

We now describe the main features of the background dynamics, i.e., the \( a, \varphi \) and \( (\varphi_\parallel, \varphi_\perp) \) dynamics. We treat the inflaton as a full quantum field, and we study its dynamics in a selfconsistent classical space-time metric (consistent with inflation at a scale well below the Planck energy density). The dynamics of the space-time metric is determined by the semiclassical Einstein equations, where the source term is given by the expectation value of the energy momentum tensor of the quantum inflaton field

\[ \mathcal{G}_{\mu\nu} = 8\pi m_P^2 \langle T_{\mu\nu} \rangle \]

Hence we solve self-consistently the coupled evolution equations for the classical metric and the quantum inflaton field.

The amplitude of the quantum fluctuations for a set of modes can be large (in quantum chaotic inflation due to the initial state, and in new inflation due to spinodal instabilities). This implies the need of a non-perturbative treatment of the evolution of the quantum state, and therefore we use the large \( N \) limit method. In the large \( N \) limit, the longitudinal quantum contributions \( \varphi_\parallel \) are subleading by a factor \( 1/N \) [3, 7, 8]. Thus, the evolution equations for the inflaton background are

\[ \ddot{\varphi} + 3H \dot{\varphi} + \mathcal{M}^2 \varphi = 0, \]

and

\[ \ddot{f}_k + 3H \dot{f}_k + \left( \frac{k^2}{a^2} + \mathcal{M}^2 \right) f_k = 0, \]

with

\[ \mathcal{M}^2 = m^2 + \lambda \varphi^2 + \frac{\lambda}{2} \int_R \frac{d^3k}{(2\pi)^3} |f_k|^2, \]

and for the scale factor \( (H \equiv \dot{a}/a) \) we have

\[ H^2 = \frac{8\pi}{3 m_P^2} \rho \]

and

\[ \frac{\rho}{N} = \frac{1}{2} \varphi^2 + \frac{\mathcal{M}^4 - m^4}{2\lambda} + \frac{m^4}{2\lambda} \frac{1 - \alpha}{2} + \frac{1}{4} \int_R \frac{d^3k}{(2\pi)^3} \left( |f_k|^2 + \frac{k^2}{a^2} |f_k|^2 \right) \],
where $\rho = \langle T^{00} \rangle$ is the energy density. The pressure ($p \delta^j_i = \langle T^j_i \rangle$) is given by

$$\frac{p + \rho}{N} = \varphi^2 + \frac{1}{2} \int_R \frac{d^3k}{(2\pi)^3} \left( |\dot{f}_k|^2 + \frac{k^2}{3a^2} |f_k|^2 \right).$$

The index $R$ denotes the renormalized expressions of these integrals [4, 6]. This means that we must subtract the appropriate asymptotic ultraviolet behaviour in order to make convergent the integrals. The evolution equations for the expectation value and for the field modes are analogous to damped oscillator equations, and the inflationary period ($\ddot{a} > 0$) corresponds to the overdamped regime of these damped oscillators.

We consider here two typical classes of quantum inflation models:

- (i) **Quantum chaotic inflation**, where inflation is produced by the dynamical quantum evolution of a excited initial pure state with large energy density (more details and the generalization to mixed states can be found in [6]). This state is formed by a distribution of excited modes. It can be shown that the initial conditions for a general pure state are given by fixing the complex values of $f_k(0)$ and $\dot{f}_k(0)$. Among these four real (two complex) numbers for each $k$ mode, one is an arbitrary global phase, and another is fixed by the wronskian. The two remaining degrees of freedom fix the occupation number for each mode and the relative phase between $f_k(0)$ and $\dot{f}_k(0)$. The coherence between different $k$ modes turns out to be determined by such relative phases.

- (ii) **Quantum new inflation**, where inflation is produced by the dynamical quantum evolution of a state with small inflaton expectation value, and small occupation numbers for the quantum modes, evolving with a spontaneously broken symmetry potential. (More details can be found in [9].)

The two classes of quantum inflation models have important differences in their initial state and in their background and perturbation dynamics (e.g., spinodal instabilities are present in new inflation and not in chaotic inflation). However, we stress here the common features which allow a unified treatment of the computation for the primordial perturbations generated in these models. In this quantum field inflation framework we have found the following **generalized slow roll condition**

$$\varphi^2 + \int_R \frac{d^3k}{2(2\pi)^3} |\dot{f}_k|^2 \ll m^2 \left( \varphi^2 + \int_R \frac{d^3k}{2(2\pi)^3} |f_k|^2 \right)$$

(2)

which guarantees inflation ($\ddot{a} > 0$) and that it lasts long (for both scenarios). (This condition includes the classical slow roll condition $\varphi^2 \ll m^2 \varphi^2$ as a particular case.) There is a wide class of quantum initial conditions satisfying Eq. (2) and leading to inflation that lasts long enough [6]. The quantum field dynamics considered here leads to **two inflationary epochs**, separated by a condensate formation:

(i) **The pre-condensate epoch**: During this epoch the term

$$D \equiv \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{a^2} |f_k|^2$$

has an important contribution to the energy density while it fastly decreases due to the exponential redshift of the excitations ($k/a \to 0$). This epoch ends at a time $\tau_A$ when the $D$ contribution to the energy density becomes negligible, i.e., the $k^2/a^2$ contribution in the background evolution equations is negligible at $\tau = \tau_A$.

After outward horizon crossing, the time dependence of the modes factorizes and becomes $k$ independent. The $k^2/a^2$ term in the mode evolution equations becomes negligible, and all the modes satisfy the same damped oscillator equation. For $m^2 > 0$ the modes decrease
(due to the damping), while for $m^2 < 0$ they grow (due to spinodal instabilities). At the end of this epoch ($t = \tau_A$) all the relevant modes for the background dynamics have exited the horizon, and the time dependence factorization allows to consider them as a zero mode condensate.

(ii) The post-condensate quasi-de Sitter epoch. The enormous redshift of the previous epoch assembles the quanta into a zero mode condensate, $\tilde{\varphi}_{\text{eff}}$, given by

$$\tilde{\varphi}_{\text{eff}}(t) = \sqrt{N} \varphi(t),$$

and $\tilde{\varphi}_{\text{eff}}^i(t) = \sqrt{\frac{4\pi}{3m^2_{\text{Pl}}}} |f_k(t)|^2$ for $i = 2, \ldots, N$ with constant direction in the field space [due to the $O(N)$ invariance of the potential], and modulus

$$\tilde{\varphi}_{\text{eff}}(t) = \sqrt{\frac{4\pi}{3m^2_{\text{Pl}}}} \rho$$

that verifies the classical equations of motion,

$$\ddot{\tilde{\varphi}}_{\text{eff}} + 3H \dot{\tilde{\varphi}}_{\text{eff}} + \tilde{m}^2 \tilde{\varphi}_{\text{eff}} + \tilde{\lambda} \tilde{\varphi}_{\text{eff}}^3 = 0$$

and

$$H^2 = \frac{8\pi}{3m^2_{\text{Pl}}} \rho$$

where

$$\rho = \frac{1}{2} \tilde{\varphi}_{\text{eff}}^2 + \frac{1}{2} \tilde{m}^2 \tilde{\varphi}_{\text{eff}}^2 + \frac{\tilde{\lambda}}{4} \tilde{\varphi}_{\text{eff}}^4$$

with $\tilde{\lambda} = \frac{\lambda}{2N}$, $\tilde{m}^2 = m^2$, $\tilde{\alpha} \equiv \text{sign}(\tilde{m}^2) = \pm 1$. The pressure is given by $p + \rho = \dot{\tilde{\varphi}}_{\text{eff}}^2$. Therefore, the background evolution in this period can be effectively described by a classical scalar field obeying the evolution equation and with initial conditions defined at $t = \tau_A$. Moreover, it is important to stress that the initial conditions for $\tilde{\varphi}_{\text{eff}}$ are fixed by the quantum state:

$$\tilde{\varphi}_{\text{eff}}(\tau_A) = \sqrt{\frac{4\pi}{m^2_{\text{Pl}}}} \rho \tau_A^{\frac{1}{2}}$$

Also the value of $\tau_A$ depends on the full quantum evolution before the formation of the condensate. $\tau_A$ is therefore a function of the coupling, the mass and the quantum initial conditions.

The previous result shows that after the formation of the condensate (both for chaotic and for new inflation), the background dynamics can be described by an effective classical background inflation whose action structure, parameters (mass and coupling) and initial conditions are fixed by those of the underlying quantum field inflation.

In chaotic inflation the total number of efolds $N_T$ decreases if the initial state had excited modes with non-zero wavenumber (for constant initial energy). For example, if the initial energy is concentrated in a shell of wavenumber $k_0$ and for simplicity the quadratic term dominates the potential, we have $N_T \approx 4\pi \frac{m^2_{\text{Pl}}}{m^2} \left(1 + \frac{k_0}{k_0/m^2}\right)$ (where the classical result is recovered at $k_0 = 0$). We have shown that there are enough efolds even for $k_0 \sim 80 m$ for reasonable choices of the initial energy density ($\rho_0 = 10^{-2}m^4_{\text{Pl}}$) and of the parameters (for instance, $N m^2/|\lambda m^2_{\text{Pl}}| = 2 \cdot 10^5$).
2.2. Primordial perturbations in quantum field inflation

The relevant primordial perturbations are those that exited the horizon during the last efolds of inflation \[1\]. As we have seen in the previous subsection the background \((a, \varphi, \varphi_\parallel\text{ and } \varphi_\perp)\) dynamics during the last efolds in the quantum field inflation scenarios (both chaotic and new) are effectively classical. This will allow to compute the relevant primordial perturbations for these scenarios and express them in terms of the known perturbations for the corresponding single-field classical scenarios.

The more general metric perturbation \(\delta g_{\mu\nu}\) can be decomposed as usual in scalar, vector and tensor components \[4, 10, 11\]. In the linear approximation, scalar, vector and tensor perturbations evolve independently and thus can be considered separately \[10\].

Scalar perturbations We now compute the scalar metric perturbations to the background, these are tightly coupled to the inflaton perturbations, and therefore, both perturbations have to be studied together \[10, 12\]. We have shown that the background dynamics for quantum field inflation can be separated in two epochs: before and after the formation of the condensate. As the first one is short, in the more natural scenarios the last \(N_c \simeq 60\) efolds take place after the formation of the condensate. Thus, the cosmologically relevant scales of the perturbations exit the horizon when the condensate was already formed. Therefore, the dynamics of the perturbations after the formation of the condensate is well approximated by that given by the effective classical inflation background. In addition the \(O(N)\) invariance of the potential implies that the isocurvature scalar perturbations are negligible, and that the background inflaton has a straight line trajectory in field space giving for the power spectrum of adiabatic scalar perturbations in the quantum field inflation\[4\] the result

\[
|\delta_k^{(S)}(m^2, \lambda)|^2 = |\tilde{\delta}_k^{(S)}(m^2, \frac{\lambda}{2N})|^2
\]

where \(|\tilde{\delta}_k^{(S)}(m^2, \lambda)|^2\) is the power spectrum of scalar perturbations for the single-field classical background inflation (evolution equations and initial conditions of the post-condensate regime). The corrections due to the next to leading order in large \(N\) are estimated to be of the order of \(\frac{m^2}{N^2 H^2}\), while the correction due to non-vacuum initial conditions caused by the dynamics previous to the condensate formation are estimated to be of the order of \(\frac{m^2}{N^2}\), where \(m\) is the inflaton mass and \(H\) the Hubble constant at the moment of horizon crossing. An upper estimate gives about 4% for each of them in the cosmologically relevant scales.

Vector perturbations The vector metric perturbations do not have any source in their evolution equation, because the energy-momentum tensor for a scalar field does not lead to any vector perturbation. The \((0i)\) components of the Einstein equations, in the absence of vector perturbation sources, gives \(\Delta S_i = 0\), implying that there cannot be any space-dependent vector perturbations [in Fourier space \(k^2 S_i(k) = 0\)]. Therefore, the vector perturbations are negligible (as for classically driven inflation \[11\]). \(|\delta_k^{(V)}(m^2, \lambda)|^2 = 0\).

Tensor perturbations As the energy-momentum tensor of a scalar field do not have tensor perturbations, the tensor metric perturbations do not have any source in their equation. Therefore, the amplitude of tensor perturbations is determined only by the background evolution, which after the condensate formation has an effective single-field classical description. Thus, the tensor perturbations for the quantum inflation scenario are

\[
|\delta_k^{(T)}(m^2, \lambda)|^2 = |\tilde{\delta}_k^{(T)}(m^2, \frac{\lambda}{2N})|^2
\]
where \( |\tilde{\delta}_k^{(T)}(\tilde{m}^2, \tilde{\lambda})|^2 \) is the power spectrum of tensor perturbations for single-field classical inflation.

**Tensor to scalar amplitude ratio** The tensor to scalar ratio \( r \) is defined as \( r \equiv |\delta_k^{(T)}|^2 / |\delta_k^{(S)}|^2 \). From the previous expressions for the spectra of tensor and adiabatic scalar perturbations, it follows that

\[
r(m^2, \lambda) = \tilde{r} \left( m^2, \frac{\lambda}{2N} \right)
\]

where \( \tilde{r}(m^2, \lambda) \) is the tensor to scalar amplitude ratio for single-field classical inflation.

### 3. Conclusions

Quantum field inflation (under conditions that are the quantum analog of slow roll) leads, upon evolution, to the formation of a condensate starting a regime of effective classical single-field inflation. The action structure, parameters (mass and coupling) and initial conditions for the effective classical field description are fixed by those of the underlying quantum field inflation. We show that this effective description allows an easy computation of the primordial perturbations which takes into account the dominant quantum effects (quantum inflaton background and quantum inflaton and metric perturbations), and gives the results in terms of the classical inflation results. In particular, isocurvature scalar perturbations are absent (at first order of slow roll) due to the \( O(N) \) invariance of the potential in agreement with the WMAP data. It is thus the presence of a large symmetry in multi field models that make them compatible with the present observations.

For chaotic inflation, the quantum treatment avoids the unnatural requirements of an initial state with all the energy in the zero mode. For new inflation it allows a consistent treatment of the explosive particle production due to spinodal instabilities. Quantum field inflation provides enough efolds of inflation provided the generalized slow roll condition is fulfilled. In the chaotic quantum field inflation the number of efolds is lower than in classical inflation when modes with non-zero wavenumber are excited initially. Quantum corrections to the scalar perturbations power spectrum due to initial conditions and from next to leading order in large \( N \) are estimated to be at most of the order of 4% each of them.

In summary, the classical inflationary scenario emerges as an effective description of the post-condensate inflationary period both for the background and for the perturbations. Therefore, this generalized inflation provides the quantum field foundations for classical inflation, which is in agreement with CMB anisotropy observations [3].

### Acknowledgments

F. J. C. thanks N. G. Sanchez, H. J. de Vega and D. Boyanovsky for the pleasant collaborations that have lead to these results. F. J. C. acknowledges support from MECD (Spain) through research projects BFM2003-02547/FISI and FIS2006-05895.

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