Abstract: We observed multiple-collision free-electron laser (FEL)-Compton backscattering in which a multi-bunch electron beam makes head-on collisions with multi-pulse FELs in an optical cavity, using an infrared FEL system in the storage ring NIJI-IV. It was demonstrated that the measured spectrum of the multiple-collision FEL-Compton backscattering gamma rays was the summation of the spectra of the gamma rays generated at each collision point. Moreover, it was demonstrated that the spatial distribution of the multiple-collision FEL-Compton backscattering gamma rays was the summation of those of the gamma rays generated at each collision point. Our experimental results proved quantitatively that the multiple collisions in the FEL-Compton backscattering process are effective in increasing the yield of the gamma rays. By applying the multiple-collision FEL-Compton backscattering to high-repetition FEL devices such as energy recovery linac FELs, an unprecedented high-yield gamma-ray source with quasi-monochromaticity and wavelength tunability will be realized.

Keywords: free-electron laser; Compton backscattering; gamma ray; multiple collision

1. Introduction

Backward Compton scattering, which is inelastic scattering between a photon and a relativistic electron, can generate a high-energy photon [1]. In astronomy, backward Compton scattering is recognized as a factor that causes a spectral shift of the cosmic microwave background [2]. In actuality, it has been experimentally demonstrated using coherent synchrotron radiation and transition radiation that backward Compton scattering causes wavelength conversion to a broadband light beam [3,4]. However, currently, a practical application of backward Compton scattering is to obtain quasi-monochromatic X-ray and gamma-ray beams using an intense laser as the incident light. This laser Compton scattering (LCS) gives the best quasi-monochromatic X-ray and gamma-ray sources which have sufficient intensity and controllable polarization in the energy region over keV [5,6]. LCS gamma rays using electron beams of storage rings have long been applied to studies in the field of the nuclear physics [7–10]. Recently, laser performance has been dramatically improved, and studies using electron beams with high-intensity and short-pulse lasers in linear accelerators have been actively promoted [6,11–13].

Free-electron laser (FEL) Compton backscattering, in which a resonator-type FEL generated by a relativistic electron beam is used as the incident photon, has been studied at several accelerator facilities [14–19]. Because the resonator-type FEL oscillates in an optical cavity comprising high-reflectivity mirrors [20], there is no loss caused by transporting a laser to a relativistic electron beam, and the intracavity power of the resonator-type FEL is high enough to generate intense gamma rays via backward Compton scattering. The FEL-Compton backscattering gamma-ray beam has been
used as a high-yield, quasi-monochromatic, and wavelength-variable gamma-ray source [21]. However, in order to counter the LCS gamma-ray source using an external high-power laser, it is necessary to further increase the yield of the gamma rays. Multiple-collision FEL-Compton backscattering [22], in which a multi-bunch electron beam makes head-on collisions with multi-pulse FELs in an optical cavity, is one of the most effective solutions. By realizing multiple-collision FEL-Compton backscattering with a high-repetition electron beam of an energy recovery linac (ERL), it is possible to obtain a high-yield gamma-ray source far superior to existing quasi-monochromatic gamma-ray sources.

Although it has been considered a priori that multiple-collision FEL-Compton backscattering gamma rays can be described as the superposition of the individual FEL-Compton backscattering gamma rays generated at each collision point, experimental results have not been reported in detail. It is important for the progress of science to show the evidence. We thus investigated the properties of multiple-collision FEL-Compton backscattering gamma rays using an infrared FEL system in the storage ring NIJI-IV [23]. In this paper, we describe the expected spatial distribution and spectrum of the multiple-collision FEL-Compton backscattering gamma rays and explain the experimental setup using the infrared FEL system in the NIJI-IV. Then, the measured spatial distributions and spectra of the multiple-collision FEL-Compton backscattering gamma rays are reported, and it is proved that the multiple-collision FEL-Compton backscattering gamma rays can be described as the superposition of the backward Compton scattering gamma rays generated at each collision point.

2. Materials and Methods

2.1. Backward Compton Scattering

First, we consider the phenomenon in which one electron bunch collides head-on with a multi-pulse FEL beam in an optical cavity. In this paper, it is assumed that linearly polarized FEL pulses are incident light in the LCS process, and nonlinear effects due to the laser field of the FEL are negligible [24]. The optical cavity is assumed to be a Fabry–Pérot resonator which is constructed using two high-reflectivity mirrors. As shown in Figure 1, the FEL-Compton backscattering gamma rays are observed in a screen which is separated from the collision point by a distance \( l \) and is perpendicular to an axis of the electron-beam orbit (\( z \) axis). The distance \( l \) is selected to be sufficiently larger than the beam sizes of the FEL pulse and the electron bunch at the collision point [12]. Representing the azimuthal and zenith angles in spherical coordinates as \( \theta \) and \( \varphi \), respectively, the differential cross section of the scattered gamma ray on the screen \( O \) is given by

\[
\frac{d\sigma}{d\Omega}(\theta, \varphi) = r_e^2 \left| 1 - \frac{\sin^2 \theta}{\gamma^2 (1 - \beta \cos \theta)^2} \cos^2 \varphi \right| \frac{1}{\gamma^2 (1 - \beta \cos \theta)^2}
\]

(1)

where \( r_e \) is the classical electron radius, \( \beta \) is the electron speed normalized by the speed of light \( c \), and \( \gamma \) is the electron energy in units of its rest mass \( m_e c^2 \), respectively [25]. When \( \gamma \gg 1 \) and \( \theta \ll 1 \) hold, Equation (1) can be approximated as

\[
\frac{d\sigma(\xi, \varphi)}{d\Omega} = 4\gamma^2 r_e^2 \left| 1 - \frac{4\xi^2}{(1 + \xi^2)^2} \cos^2 \varphi \right| \frac{1}{(1 + \xi^2)^2}
\]

(2)

where \( \xi \) equals \( \gamma \theta \) [26]. The yield of gamma rays emitted by an electron bunch \( \#j \) colliding with \( n \) FEL pulses in a circle with radius \( r \) on the center of the screen \( O \) is denoted as \( N_j \). The radius \( r \) is assumed to be sufficiently larger than the beam sizes of the FEL and the electron and to be sufficiently smaller than \( l \). Because the differential cross section of backward Compton scattering is very small, the probability that one electron in the bunch contributes to multiple processes of the backward Compton scattering is extremely low. It can be ignored that electrons lost energy in the backward Compton scattering collide with the FEL pulses again. Because the beam sizes of the electron bunch and the FEL pulses are sufficiently larger than the wavelengths of the scattered gamma rays and the electron bunch length is
much longer than the wavelengths of the scattered gamma rays, spatial and time interference effects among the scattered gamma rays at the collision points can be ignored. Therefore, $N_j$ is expressed via the following equations:

$$N_j = \sum_{i}^{n} \int_{0}^{2\pi} d\phi \int_{0}^{\theta_{ij}} d\sigma_{ij} L_{ij} \sin \theta d\theta$$  \hspace{1cm} (3)$$

$$\theta_{ij} = \arctan \left( \frac{r}{l_{ij}} \right)$$ \hspace{1cm} (4)

where $L_{ij}$ is the luminosity between electron bunch #j and FEL pulse #i at the collision point [23]. Using the number of electrons in #j, $N_{e,j}$, and the number of photons in #i, $N_{p,i}$, the luminosity is given by

$$L_{ij} = \frac{N_{e,j} N_{p,i}}{A_{ij}}$$ \hspace{1cm} (5)$$

$$A_{ij} = 2\pi \sqrt{\sigma_{x,j}^2 + \sigma_{y,j}^2 + \sigma_{p,i}^2}$$ \hspace{1cm} (6)$$

where $A_{ij}$ denotes the effective overlap area between electron bunch #j and FEL pulse #i [16, 27]. The parameters $\sigma_x$, $\sigma_y$, and $\sigma_p$ are the horizontal beam size of the electron bunch, the vertical size of the electron bunch, and the beam size of the FEL pulse at the collision points, respectively. When the electron bunch has slight transverse motion around the axis of the electron-beam orbit, phase factors are added to the angles $\theta$ and $\phi$ in Equation (3). Here it is assumed that the phase factors are negligible. As shown in Equation (3), it is important that the total yield is expressed as the summation of the scattered gamma rays from each collision point.

**Electron beam**

**Figure 1.** Schematic diagram of the free-electron laser (FEL)-Compton backscattering in head-on collision.

2.2. Description of the Multiple-Collision FEL-Compton Backscattering Gamma Rays

We consider that $m$ electron bunches per unit time collide head-on with $n$ FEL pulses at collision points in the optical cavity. Using Equations (3)–(6), the total yield of the scattered gamma rays is given by

$$N_{CS} = \sum_{j}^{m} N_j = \sum_{j=1}^{m} \sum_{i=1}^{n} \int_{0}^{2\pi} d\phi \int_{0}^{\theta_{ij}} d\sigma_{ij} L_{ij} \sin \theta d\theta$$ \hspace{1cm} (7)$$

For a storage ring FEL system in which the round-trip time of the FEL pulse in the optical cavity equals the revolution period of the electron bunch, this equation can be rewritten as
\[ N_{CS} = f_r \sum_{j}^m N_j = f_r \sum_{j=1}^m \sum_{i=1}^n \int_{0}^{2\pi} d\phi \int_{0}^{\theta_{ij}} d\theta_{ij} (N_{ij} N_{ij} A_{ij} \sin \theta) \sin \theta d\theta \]  

where \( f_r \) is the revolution frequency of the electron bunch in the case of the storage ring and \( m' \) is the number of electron bunches with which one FEL pulse collides in the optical cavity. When the collision points are close as in linac FELs, Equation (8) shows that the total yield of the scattered gamma rays is approximately proportional to the product of the number of electron bunches per unit time and the number of FEL pulses which collide with the electron bunches in the optical cavity. Then, a high-yield gamma-ray source is expected by using backward Compton scattering between multiple electron bunches and multiple FEL pulses in an optical cavity.

The spectrum of the scattered gamma rays in multiple-collision FEL Compton backscattering will be considered. When the incident light collides head-on with the electron beam in the backward Compton scattering process, the energy of the scattered gamma rays \( (E_p) \) is given by the following approximate equation using the energy of the incident light \( (E_L) \) [25,28]:

\[ E_p = \frac{4\gamma^2 E_L}{1 + \varepsilon^2 + \frac{4E_L}{m_e c^2}}. \]  

As shown in this equation, the energy of the scattered gamma rays for a certain \( E_L \) is determined by the zenith angle regardless of the polarization of the FEL. Using the scattered gamma-ray energy normalized by its maximum energy \( E_{p,\text{Max}} \), which is the scattered gamma-ray energy at \( \theta = 0 \), the differential cross section of backward Compton scattering can be expressed by [16]

\[ \frac{d\sigma(\varepsilon)}{d\varepsilon} = 4\gamma^2 \varepsilon^2 \left( 2\varepsilon - 2\varepsilon^2 + 1 \right), \]  

\[ \varepsilon = \frac{E_p}{E_{p,\text{Max}}}. \]  

When the influence of the electron-beam emittance and energy spread on the backward Compton scattering is negligible, the lower limit of the scattered gamma-ray spectrum is determined by limiting the zenith angle by the collimator. Defining the energy spectrum of the scattered gamma rays which are generated by backward Compton scattering between electron bunch \( #j \) and FEL pulse \( #i \) and which pass through the collimator as \( S_{ij}(\varepsilon) \), the spectrum of the total gamma rays observed behind the collimator is given by the summation of the spectrum \( S_{ij}(\varepsilon) \) from each collision point as given in the following equation:

\[ S(\varepsilon) = \sum_{j}^m S_j(\varepsilon) = \sum_{j}^m \sum_{i}^n S_{ij}(\varepsilon). \]  

Considering the electron bunch length and temporal resolution of the general measurement system for the energy spectrum in the gamma-rays region, it is difficult to identify the spectrum \( S_{ij}(\varepsilon) \) of the FEL-Compton backscattering gamma rays which electron bunch \( #j \) generates at each collision point. The spectrum \( S_j(\varepsilon) \) of whole scattered gamma rays which the electron bunch \( #j \) generates with multiple FEL pulses is observed in actual experiments.

3. Results

3.1. Experimental Setup Using the Infrared FEL System in the NIJI-IV

The infrared FEL system in the storage ring NIJI-IV was used to observe the spatial distribution and the spectrum of FEL-Compton backscattering gamma rays at multiple collision points. Figure 2 shows the scheme of the infrared FEL system and the measurement configuration of the FEL-Compton
backscattering experiments. The NIJI-IV can store up to 16 electron bunches in its circumference of 29.6 m [29]. The energy of the electron is usually operated at 310 MeV in infrared FEL experiments. The fundamental FEL oscillations in the wavelength region of 0.84–2.68 μm have been demonstrated with this system [30,31]. An ETLOK-III planar optical klystron was incorporated into the infrared FEL system as an insertion device [32]. It has two identical undulator sections of length 1.4 m and one dispersive section of length 0.72 m. The undulator period in the undulator section is 0.2 m, and the number of the periods is 7. Although the value of the deflection parameter $K$ is tunable from 1.28 to 10.4, it was fixed at 3.03 in the demonstration experiments of the multiple-collision FEL-Compton backscattering. As shown in Figure 3, the wavelength of the FEL oscillation used in these experiments was 1.53 μm. The finesse of the optical cavity was $2.6 \times 10^3$ at the wavelength of 1.53 μm. The energy spread of the electron beam increased up to approximately $10^{-3}$ due to bunch heating caused by the FEL oscillations. However, the electron beam sizes did not increase at the collision points of the FEL-Compton backscattering on oscillations because the dispersion function was almost zero in the long straight section where the ETLOK-III was installed.

![Figure 2](image-url)  
**Figure 2.** Schematic layout of the FEL-Compton backscattering experiments in the storage ring NIJI-IV. The cross symbols are the collision points in the optical cavity and the numerals under the names of collision points are the numbers of FEL-Compton backscattering processes per revolution period at each collision point. The collision points without magnetic field are shown in red.

![Figure 3](image-url)  
**Figure 3.** Spectra of the infrared FELs at the FEL-Compton backscattering experiments in (a) the two-bunch operation and (b) the three-bunch operation.

Because the optical cavity length of the infrared FEL system is 14.8 m, it can store up to 16 FEL pulses as well as the electron bunches. However, the maximum number of electron bunches that could oscillate in the FEL was four due to the coupled-bunch instability of the electron beam in the
NIJI-IV [31,33]. The monochromaticity of the FEL-Compton backscattering gamma rays is degraded by the magnetic field at the collision points [34]. By controlling the arrangement of electron bunches, multiple collision points can be set outside the optical klystron ETLOK-III or quadrupole magnets. As shown in Figure 2, there are three collision points (C3, C–2, C–3) without magnetic field in the optical cavity. At least two electron bunches are required for generating gamma rays via backward Compton scattering with the storage ring FELs. Because an electron bunch collides with only one FEL pulse in the optical cavity in the two-bunch operation of the NIJI-IV, the number of collisions per revolution period is two, which is the same as the number of electron bunches.

The FEL-Compton backscattering gamma rays were observed behind the downstream mirror chamber. A lead collimator with a bore diameter of 10 mm and thickness of 100 mm was positioned at a distance of 0.4 m from the downstream cavity mirror in the air. A Ta2O5/SiO2 multilayer mirror, which had a quartz substrate, was used as the cavity mirror [35,36]. The gamma-ray beam passing through the collimator was measured using a LaBr3(Ce) scintillation detector with diameter and length of 25 mm and 51 mm [34], respectively. The output signals from this detector were divided and fed into a constant fraction discriminator and a shaping amplifier. The timing output from the constant fraction discriminator was sent to a time-to-amplitude converter as a start signal, and a 1/16 subharmonic signal was used as the stop signal for the time-to-amplitude converter. The output signals from the time-to-amplitude converter and shaping amplifier were recorded using a dual multi-channel analyzer for temporal and energy analysis. Because the time resolution was 1.3 ns, this system could distinguish each gamma-ray pulse even in the full-bunch operation, where the pulse interval was 6.2 ns. On the other hand, the energy resolution of this system was 3.1% for a 1.2 MeV energy gamma ray. Because it was much larger than the energy spread of the electron beam caused by the FEL oscillations, this system could not observe the influence of changes in the energy spread on the FEL-Compton backscattering gamma rays. For measurements of the spatial distribution of the gamma rays, imaging plates (IP) with a spatial resolution of 50 × 50 µm² were utilized [37]. The IP were inserted just before the lead collimator. Because it was necessary to expose the gamma rays to the IP for more than one minute, it was not possible to identify the spatial distribution of the gamma rays at each collision point.

3.2. Measured Spectra of the FEL-Compton Backscattering Gamma Rays Generated at Each Collision Point

In order to clarify the characteristics of the multiple-collision FEL-Compton backscattering gamma rays, the spectra of the gamma rays generated at each collision point were individually measured during the two-bunch operation. Because the electron bunch collided head-on with one FEL pulse in the two-bunch operation, it was possible to identify the spectrum of the gamma rays generated at each collision point. By arranging the electron bunches asymmetrically in the storage ring, we could select any collision point of the FEL-Compton backscattering [34]. When the bunch-fill pattern was #0 and #5, the collision points were C3 and C–3, respectively. In the case of the bunch-fill pattern of #0 and #6, the collision points were C2 and C–2, respectively. Figure 4 shows the observed spectra of the FEL-Compton backscattering gamma rays generated at C3, C2, C–2, and C–3. A full-energy peak with a sharp edge in the high-energy side was observed near 1.2 MeV. The broad humps of lower energy were caused by the escape of gamma rays through Compton scattering inside the small detector without full-energy deposition. The electron-beam currents were 8.7 mA for the bunch-fill pattern of (#0, #5) and 11.0 mA for the bunch-fill pattern of (#0, #6). The FEL powers extracted from a cavity mirror, the transmittance of which was approximately 1 × 10⁻⁴ at the target wavelength of 1530 nm, were 0.50 mW for the bunch-fill pattern of (#0, #5) and 0.88 mW for the bunch-fill pattern of (#0, #6) in the air, respectively. The yields of the FEL-Compton backscattering gamma rays passing through the lead collimator were 4.4 × 10⁴ photons/s for the bunch-fill pattern of (#0, #5) and 6.9 × 10⁴ photons/s for the bunch-fill pattern of (#0, #6), respectively, in consideration of the efficiency of the measurement system. The collision point C2 was in the optical klystron, and the yield of the gamma rays generated at C2 was less than one-tenth of that generated at the other points. The FEL-Compton backscattering gamma-ray spectra simulated via EGS5 code, which can simulate photon
transport in any material \[34, 38\], at the position of the detector are also described in Figure 4. We note that the measured spectra of the gamma rays were in good agreement with the calculated spectra. If the multiple-collision FEL-Compton backscattering gamma rays are described as the superposition of the individual FEL-Compton backscattering gamma rays generated at each collision point, the measured spectrum of the multiple-collision FEL-Compton backscattering gamma rays should be reproduced using the calculated spectra in Figure 4.

![Figure 4](image-url)

**Figure 4.** Measured energy spectra of the FEL-Compton backscattering gamma rays for the bunch-fill patterns of (a) \(\#0, \#5\) and (b) \(\#0, \#6\). The calculated spectra are indicated by the red broken lines. The opening half angles of the collimator were 0.47, 0.52, 0.84, and 1.0 mrad for the collision points of \(C_3, C_2, C_{-2}\), and \(C_{-3}\), respectively.

### 3.3. Demonstration of the Multiple-Collision FEL-Compton Backscattering

In order to generate the multiple-collision FEL-Compton backscattering gamma rays, the infrared FELs in the storage ring NIJI-IV were oscillated in the three-bunch operation. Coupled-bunch instability appeared in the electron beam of the storage ring NIJI-IV when there were three or more electron bunches in the storage ring. However, the FEL oscillation stabilized in the three-bunch operation when the electron-beam current exceeded 13 mA. This is because the FEL oscillation caused bunch lengthening due to the bunch heating effect, and the electron density in the bunch decreased considerably \[39\].

In the above-mentioned current region, each electron bunch in the three-bunch operation emitted the same saturated FEL power as that in the single-bunch operation. Using an radio-frequency (RF) knock-out method, we filled RF-buckets \#0, \#5, and \#10 with bunch intervals of 31 ns, 31 ns, and 37 ns, respectively \[26\]. The electron bunches \#0, \#5, and \#10 collided with the FEL pulses at the points \((C_3, C_{-2}), (C_3, C_{-3}),\) and \((C_2, C_{-3})\), respectively. There was no external magnetic field except at the collision point \(C_2\), and the yield of the gamma-ray beam passing through the lead collimator was high. Figure 5 shows the spectra of the FEL-Compton backscattering gamma rays generated by each electron bunch at the electron-beam current of 4.2 mA for bunch \#0, 4.9 mA for bunch \#5, and 5.4 mA for bunch \#10. The FEL power extracted from the cavity mirror was 0.89 mW in the air. The yields of the gamma rays passing through the lead collimator were \(1.7 \times 10^5\) photons/s in consideration of the efficiency of the measurement system. Figure 5 also shows the calculated spectra of the FEL-Compton backscattering gamma rays for electron bunches \#0, \#5, and \#10.
Figure 5. Measured energy spectra of the FEL-Compton backscattering gamma rays generated by electron bunches (a) #0, (b) #5, and (c) #10. The spectra evaluated using the calculated data are indicated by the red broken lines.

The spatial distribution of the FEL-Compton backscattering gamma rays was also measured in the three-bunch operation. The bunch-fill pattern of the electron beam was (#0, #5, #10) as in the measurement of the gamma-ray spectra. Figure 6 shows the two-dimensional distribution of the gamma-ray beam measured by the IP with exposure time of one minute. The electron-beam current was 19.2 mA and the currents of each bunch were almost equal. The FEL power extracted from the cavity mirror was 1.34 mW in the air, and the total yield of the FEL-Compton backscattering gamma rays was evaluated to be approximately $2 \times 10^6$ photons/s. The diameter of the cavity mirror was 2 inches, and the effective area where the gamma-ray beam was not disturbed by the metallic mirror holder was in a circle with a diameter of approximately 50 mm.
4. Discussion

4.1. Spectra of Multiple-Collision FEL-Compton Backscattering

Compared with the experimental data from the two-bunch operation with bunch-fill pattern (#0, #5), the yield of the gamma-ray beam in the three-bunch operation was approximately 3.9 times greater, though the electron-beam current per bunch was approximately 1.1 times greater. This is because the number of collisions per revolution period increased to six in the three-bunch operations. In these calculations using the data in Figure 4, the powers of the FEL pulses with which the electron bunches collided were evaluated in consideration of the electron-beam current of the bunches. The energy spreads were 15% and 13% at the full width at half-maximum for electron bunches #0 and #5, respectively. In order to evaluate the difference between the measurement and the calculated curve, the coefficient of determination $R^2$, which is a statistical measure in regression models that determines the proportion of variance in the dependent variable that can be explained by the independent variable, was calculated. The coefficients of determination calculated for the energy region of 400–1200 keV in Figure 5 were 0.85, 0.91, and 0.89 for electron bunches #0, #5, and #10, respectively. Therefore, the spectra calculated using the FEL-Compton backscattering gamma rays from each collision point were in agreement with the spectra of the measured gamma-ray beams. These experimental results demonstrate that the gamma-ray spectrum generated when one electron bunch collides with multiple FEL pulses can be described as the superposition of the gamma-ray spectra generated at each collision point.

4.2. Spatial Distribution of Multiple-Collision FEL-Compton Backscattering

Figure 6 shows that the FEL-Compton backscattering gamma rays had greater dispersion in the vertical direction because the FELs were horizontally linearly polarized waves [40]. Substituting $\varphi = 1/2$ into Equation (2) and converting from the spherical coordinate system to the Cartesian coordinate system, the distribution function of the backward Compton scattering gamma rays on the vertical axis, $N_{CS}(y)$, is given by the following approximate equation:

$$N(y) = 4\gamma^2 e^2 \sum_i \sum_j N_{\gamma j} N_{\gamma i} \left[ 1 + \frac{y}{l_{ij}} \right]^{-2} , \ i \neq j$$  \hspace{1cm} (13)
where the subscripts \( j \) and \( i \) denote the electron bunches (#0, #5, #10) and the FEL pulses (#0, #5, #10) for the storage ring NIJI-IV, respectively. The gamma-ray beam generated at collision point \( C_2 \) was deflected in the horizontal direction and had extra spread due to the electron bunch length. Thus, the contribution of collision point \( C_2 \) to the gamma-ray distribution on the vertical axis (the dotted line in Figure 6) is negligible. As shown in Figure 2, the numbers of collisions per revolution period at the collision points \( C_3 \), \( C_{-2} \), and \( C_3 \) were two, one, and two, respectively. The distributions of the gamma-ray beams generated at each collision point can be evaluated on the vertical axis by taking the sum of \( j \) in Equation (13), as shown in Figure 6b. However, the calculated lines in this figure have a common arbitrary coefficient to match the measurement data. The sum of all contributions from each collision point is also indicated by the red solid line in Figure 6b. The coefficient of determination between the red solid line and the measured distribution of the total gamma-ray beam on the vertical axis is 0.98 in Figure 6b, and the calculation is thus in good agreement with the measured data. The High-Intensity Gamma-ray Source reported the profile of FEL-Compton backscattering gamma rays with nearly 100% linearly polarized FELs [41]. The agreement between the measured and calculated values in the gamma-ray profile indicates that the degree of linear polarization of the NIJI-IV FEL was also close to 100%. Moreover, this experimental result demonstrates that the distribution of the gamma-ray beam generated at the multiple collision points can be described as the superposition of the gamma-ray beams generated at each collision point. To increase the yield of monochromatic and wavelength-variable photon beams in the energy region above keV, multiple-collision FEL-Compton backscattering in an ERL facility will be a key technique [42,43]. We will advance the development of an insertion device which is suitable for backward Compton scattering using ERL-FELs.

5. Conclusions

Backward Compton scattering using a resonator-type FEL is excellent at generating a high-yield, quasi-monochromatic, and wavelength-variable gamma-ray beam. We studied multiple-collision FEL-Compton backscattering, which is an essential technique for increasing gamma-ray yield, using the infrared FEL system in the storage ring NIJI-IV. By controlling the interval between an electron bunch and an FEL pulse in the optical cavity in asymmetric two-bunch operation, the spectra of the FEL-Compton backscattering gamma rays generated at each collision point were measured. Then, when one electron bunch collided head-on with multiple FEL pulses in the optical cavity, it was demonstrated that the measured spectrum of the gamma-ray beam was the summation of the spectra of the gamma rays generated at each collision point. Using an IP, moreover, it was demonstrated that the spatial distribution of the multiple-collision FEL-Compton backscattering gamma rays was the summation of those of the gamma rays generated at each collision point. These experimental results were expected a priori. However, experimental proof is indispensable for the development of science, and the results of our studies are significant in the field of beam physics. Our studies indicate quantitatively that multiple collisions in the optical cavity are effective in increasing photon yields and will contribute to the realization of high-yield, quasi-monochromatic, and wavelength-variable photon beams in the X-ray and gamma-ray regions.

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