BAYESIAN DECISION MAKING IN DETERMINING OPTIMAL LEASED TERM AND PREVENTIVE MAINTENANCE SCHEME FOR LEASED FACILITIES

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Abstract. Under a business competitive environment, quite a few enterprises choose capital leasing to reduce tax payment and investment risk instead of buying facilities. Since the durability and service life of leased facilities will be longer, the breakdowns and deterioration of leased facilities are inevitable during lease period. Accordingly, in order to reduce the related costs and keep the facility’s health during lease period, preventive maintenances are required to perform to reduce the cost of free-repair warranty and maintain customers’ satisfaction. However, performing preventive maintenance is not easy to scheme due to the scarcity of historical failure data. Accordingly, the study integrates lease and maintenance decisions into a synthetic strategy, and it can be applied under the situation of only expert’s evaluation and/or scare historical failure data by employing Bayesian analyses. In this study, the mathematical models and corresponding algorithms are developed to determine the best preventive maintenance scheme and the optimal term of contract for leased facilities to maximize the expected profit. Moreover, the computerized architecture is also proposed, and it can help the lessor to solve the issue in practice. Finally, numerical examples and the sensitive analyses are provided to illustrate the managerial strategies under different leased period and the preventive maintenance policies.

1. Introduction. In global competitive environment, there are many reasons why companies should lease equipment instead of buying them to increase financial competition. Since 1990s several businesses prefer to lease equipment rather to own them. Equipment leasing provides flexibility and protection against technological obsolescence. Accordingly, leasing strategy allows a company to obtain the benefits in terms of tax, cash outflow and revenue management through leasing instead of buying. In other words, capital leasing can reduce tax payment and avoid investment risk, and therefore many companies would lease their equipment rather than purchasing them. Numerous studied the issue “lease or buy?” in different fixed assets. For example, Nisbet and Ward (2001)[32] studied a lease issue of radiotherapy equipment for medical centers and hospitals, and they gave an indication between purchase and lease in consideration of financial. Aras et al. (2011)[1] proposed optimal pricing policies for remanufacturable leased products. Bourjade et al. (2017)[3]
measured the impact of aircraft leasing choice in term of financial performance and stability.

However, equipment lease will raise new issues for both the lessor and the lessee. The lessor may need to provide maintenance service and free-repair warranty to its lessee during the whole lease period. The expenditures can be taken into consideration when the lessor made the lease contract for the lessee. In general, most of contracts will specify the penalties regarding failing frequently, unsatisfied maintenance service and repair being not performed within a reasonable time limit, etc. (Desai and Purohit, 1998[7]; Kleiman, 2001[27]; Yeh et al, 2011[50]; Schutz and Rezg, 2013[38]; Ben Mabrouk et al., 2016[2]). In other words, lease equipment is often bundled with maintenance and free repair service that offered by the lessor as a package to the lessee under a contract. Zhang et al. (2019)[51] presented their perspective about the issue of lease equipment. They considered that most firms may lease them not own them to avoid to afford the burden of purchasing equipment. However, lessors still should satisfy lessees’ needs with equipment maintenance service to increase their lease intention for expanding market share. Therefore, it is often seen that most lease contracts take maintenance service to attract their lessors.

For repairable equipment, the maintenance actions can be classified into two major types, such as corrective maintenance (CM) and preventive maintenance (PM). CM actions are used to rectify failed equipment back to its healthy state, and PM actions are performed to improve the operational state of the equipment to avoid failures. For CM action, minimal repair is used to maintain operation of equipment. Although the equipment can back to normal operation after minimal repair, the failure frequency is still increasing with time. Moreover, since PM actions during the lease period will impact on both the lessor’s disbursement and the facility’s lifetime, lessor should offer a PM program during the whole lease period. With regard to the service of PM, it can be classified into perfect and imperfect PMs. Perfect PM can restore the equipment’s status to its original condition. However, imperfect PM only restore the equipment’s status between its original and current condition. Nowadays, several PM policies have been studied, and the periodic PM policy is the most popular ones. In this policy, a product is maintained at fixed time intervals, and therefore it will be easy to manage the whole PM schedule. Chun (1992)[5] proposed a periodic PM policy with perfect maintenance under a finite planning horizon. Garg and Sharma (2012) [15] proposed a two-phase approach of PM for an automobile industry, and they utilized an evolution algorithm to evaluate the values of critical parameters for increasing the availability of equipment. Grag et al. (2013)[14] proposed a PM scheduling model for pulping units of paper mills. The effect of PM actions to reliability is formulated based on a maintenance-benefit analysis. Saito et al.(2014)[37] studied periodic replacement issue of PM with minimal repair, a parametric bootstrap method was proposed to deal with such uncertainty analysis. Liu et al.(2017)[28] proposed an optimal PM scheduling model with linear deteriorating jobs. Ebrahimi et al.(2020)[10] also proposed PM scheduling for increasing the reliability of equipment. Niwas and Garg (2018)[33] investigated the performance of industrial facilities with consideration of PM and warranty policy, and the proposed model of the system was developed based on Markov process. Taheri-Tolgari et al. (2019)[42] proposed an inspection policy for production lines with PM actions. Since the defective products originated from improper PM under uncertainty environment, the decision makers need to improve PM scheduling for optimizing production lines. Jin et al. (2020)[24] proposed
an optimal PM strategy for multi-state systems by using dynamic programming. Unlike traditional PM strategies, the PM strategy is formulated using an integral equation and avoid the difficulty of complex semi-Markov processes.

Due to the fact that perfect maintenance might be unrealistic for some facilities in practice, Dagpunar and Jack (1994)[6] proposed an imperfect maintenance model with periodic policy to deal with the cases of unrecoverable systems. Wang and Pham (1999)[44] furtherly extended the existing imperfect maintenance models in production systems. Yeh and Lo (2001)[48] considered that the optimal time interval between two successive PM actions is influenced by the degree of PM, and they indicated that the best way to reduce the related costs is to offer an equal-degree of PM. Kim (2004) et al.[26] tried to explain that it is worthwhile for PM actions during warranty period by using the method of discrete time intervals. Yeh and Chang (2007)[47] considered that the optimal threshold of failure-rate for lease products can be obtained by their proposed method to minimize PM and repair costs. Hu and Zong (2008)[17] refined Pongpech and Murthy’s model (2006)[36] to determine the optimal lease term and PM scheme for minimizing the average cost by means of relaxing some conditions. Das and Sarmah (2010)[8] surveyed the existing PM models in heavy process industries. Their survey included two major categories: models for single-unit systems and for multi-unit systems. Yeh et al. (2009, 2011)[49][50] investigated the effects of various preventive maintenance cost functions on the periodic PM policy for leased products with a Weibull life-time distribution. Their model can decide the optimal PM action and cycle time by providing closed-form solutions. Besides, some new analytical methods regarding PM had been proposed to refine such traditional models. Kim and Ozturkoglu (2013)[25] transformed a traditional PM issue into an integer programming model. However, the solution space could be really large so that a heuristic algorithm had been proposed for solving the model. Jamshidi and Seyyed Esfahani (2015)[22] proposed a bi-objective mathematical model (minimize cost and maximize reliability) of preventive maintenance for a complex system. They utilized a genetic algorithm to find the optimal solution for such NP-complete problem. Garg (2014, 2016)[12][13] utilized a particle swan optimization and fuzzy methodology for analyzing a multi-objective problem regarding system’s reliability, availability and maintainability. Hadjaissa et al. (2016) [16] proposed another bi-objective model of maintenance scheduling for power systems, and they focused on the trade-off effect between makespan and training time for balancing different objectives. Ben Mabrouk et al. (2016)[2] proposed an optimal preventive maintenance model for leased equipment by utilizing an iterative numerical algorithm. They concluded that the lessor may incur the penalty cost if the expected equipment downtime exceeds a pre-specified threshold during the lease period. Shin et al. (2016)[40] utilized a queueing network to deal with PM problems of semiconductor facilities. Their main contribution was to connect PM with scheduling issues to optimize system’s cycle time and the related costs. Sheikhalishahi et al. (2017)[39] utilized simulation techniques to analyze the performances of age-based, failure-based and condition-based PM policies. Ding and Kamaruddin (2015)[9] reviewed the existing studies regarding maintenance policy optimization, and they provided some different views on classifying these models and give useful directions for industries. Iskandar and Husnia (2017)[21] studied the contract of two dimensional lease equipment with involving imperfect preventive maintenance. Their model can give the lessors very useful advices to determine the price of a lease contract. Steeneck and Sarin (2018)[41] applied the issue of lease equipment...
to original equipment manufacturers (OEMs). The management of the lease product’s end-of-life is the focus of the study, and they consider that equipment lease is more advantageous to OEMs in cost control and timing management. Wang et al. (2019)[45] proposed a usage-based lease model with imperfect PM for leased industrial equipment, and OEMs can use pre-leasing upgrade and post-leasing PM strategies to minimize lease service costs.

In some situations, the historical reliability data of newly developed equipment may be insufficient, and therefore the equipment’s deterioration is not easily to estimate. In such situations, Bayesian analysis should be a reasonable approach by means of utilizing experts’ opinions and few relevant data to estimate the systems’ deterioration. Accordingly, the issue motivates the study to optimize lessor’s maintenance and capital lease decisions under the situation of scare data. Moore and Chen (1984)[30] utilized a Bayesian approach to analyze the problem of leasing or purchasing facilities under uncertainty. Tax and finance were major issues in their study. Papazoglou (1999)[34] applied Bayesian decision analysis to deal with reliability certification works. The study utilized the existing prior assessment of uncertainty and the further information from testing components to estimate the reliability. Percy (2002)[35] proposed several prior distributions for deterioration models, and the concept regarding predictive elicitation can specify his model’s hyper-parameters. In order to determine the best time to overhaul the whole system, Huang (2004)[18] developed a Bayesian decision support system for deteriorating repairable facilities. Fang and Huang (2010)[11] also utilized a Bayesian analysis in dealing with free-repair warranty problems, and the deterioration behavior of the product was assumed to a non-homogeneous Poisson process (NHPP) with a power-law intensity function. Yeh and Fang (2014)[46] proposed a Bayesian model to deal with pro-rata warranty problem under insufficient reliability data. Walter and Flapper (2017)[43] proposed a condition-based PM policy by using Bayesian analysis, and the failure of the system is modeled by a Weibull distribution to measure the system’s status. Cao and Xie (2019)[4] also proposed a condition-based PM policy for dealing with a stockless production system. In their model, the cost of PM depends on its duration, and the duration of PM impacts on the lead time.

As mentioned above, the objective of the study is to develop an analytical model and efficient solution algorithm for solving the leased equipment problems. In this model, different PM schemes are taken into consideration for efficiently reducing the related costs during the lease period. NHPP is employed to describe the successive failure times of the deteriorating leased equipment. Moreover, the lease revenue, residual value and penalty cost are also included in the proposed model. Due to the fact that the manufacturers (lessor) did not have sufficient historical data to estimate the deterioration of newly developed equipment, this study will employ Bayesian analysis to solve the problem in such situation. In order to solve the problem efficiently and carry out practical applications, the heuristic solution algorithms and the computerized implementation architecture are proposed in the study. It will be helpful to some industries who want to push their high-price equipment to the leasing market such as production facilities or large scale machines.

2. The decision of optimal preventive maintenance and lease term.

2.1. Preventive maintenance policy. In order to maintain customers’ satisfaction, the reliability of leased equipment should be managed to an acceptable level. Accordingly, providing PM service is necessary to avoid potential disasters caused
by severe failures. In this study, a scheduled periodic PM strategy is considered since it is easier to manage in practice for the lessor. Moreover, the failure times of the leased equipment are assumed to be drawn from a non-homogeneous Poisson process (NHPP) with a specific intensity function, and it would undergo \( N-1 \) PM actions during the length of lease period \( T_L \), where the periodic inter-PM times are equally designated to be \( x \). It is assumed that any breakdown which occurs within two PM actions would lead to a minimal repair, and the repair time is negligible, and thus the breakdown process of the system can be modeled by an NHPP with exponential or power law intensity function. The whole system will be replaced at the \( N \)th PM action. Figure 1 shows the timeline of the PM model.

![Timeline of the PM Model](image)

**Figure 1. Timeline of the PM Model**

The following notations are used in the analysis throughout this study:

- \( T_L \): the length of lease (contract) period.
- \( M_P \): the preventive maintenance scheme, and different scheme can impact on the different degree of the system’s recovery.
- \( V_{\text{residual}}(T_L, \rho) \): the time segment for time-discounting factor. (A year is usually used as the time segment for time-discounting)
- \( V \): the present value of an equipment (production or investment cost of an equipment)
- \( \rho \): the depreciation rate of equipment after every maintenance
- \( V_{\text{residual}}(T_L, \rho) \): the residual value of equipment at time \( T_L \), and its form is given by Eq. (15)
- \( R(T_L, \varepsilon) \): the present value of the total rental of the leased equipment with consideration of the time discount factor \( \varepsilon \), and its form is given by Eq. (14)
- \( x \): the time interval between two scheduled PM actions.
- \( \lambda(t) \): the intensity function of exponential or power law form.
- \( \alpha \): the scale factor of exponential or power law intensity function.
- \( \beta \): the shape factor of exponential or power law intensity function.
- \( f(\alpha, \beta) \): the prior probability distribution of intensity function.
- \( f'(\alpha, \beta) \): the posterior probability distribution of intensity function.
- \( \delta^{M_P} \): the age reduction factor in effective age for maintenance scheme due to the \( k \)th PM activity, where
- \( t_k^- \): the effective age of the deteriorating system before the \( k \)th PM action.
- \( t_k^+ \): the effective age of the deteriorating system after the \( k \)th PM action.
- \( C_{\text{mr}} \): the expected cost of a minimal repair.
where $\Phi(T_L, x, \delta, \alpha, \beta)$ is the expected failure number within the lease period $T_L$. Figure 2 illustrates the maintenance scheme which is described in terms of the failure times.

![Figure 2. Maintenance Scheme under Imperfect Recovery](image-url)
2.2. **Evaluation of repair and maintenance expense.** The expected disbursement during lease period includes both the repair expense and the maintenance expense. The repair cost includes two parts: (1) the expected cost of performing a minimal repair ($C_{mr}$); (2) the penalty cost ($C_{penalty}$) as long as the actual repair time exceed the time limit. During the lease period, any breakdown of the lease facility would be rectified by minimal repairs. Moreover, the repair time $t_r$ can be regarded as random variable that follows a Gamma probability distribution. The expected probability of the time of a minimal repair exceeding the tolerable waiting time $\varphi$ will be as follow:

$$G(t_r) = \int_\varphi^\infty \frac{\eta^\omega t_r^{\omega-1}}{\Gamma(\omega)} e^{-\eta t_r} dt_r \tag{4}$$

The parameters $\omega$ and $\eta$ can be estimated by $\omega = E(t_r)^2 / \sigma(t_r)^2$ and $\eta = E(t_r) / \sigma(t_r)^2$ from historical data or engineers. Once the repair time exceeds a pre-specified time limit $\varphi$, the penalty cost will be produced to the lessor.

Due to the fact that the expected failure number of the facility within the lease period is important in evaluating the repair cost, the estimation of the facility’s deterioration is critical to the lessor. Given that the failure process of the facility is assumed to be an NHPP with a specific intensity function $\lambda(t)$, the expected number of failure for the lease period $[0, T_L]$ under the time interval of PM $x$ and the age reduction factor in effective age $\delta$ is $\Phi(T_L, x, \delta, \alpha, \beta)$. According the above mentioned, the total repair cost during the lease period is given by

$$\left( C_{mr} + C_{penalty} \int_\varphi^\infty G(t_r) dt_r \right) \Phi(T_L, x, \delta, \alpha, \beta) \tag{5}$$

With regard to preventive maintenance during the lease period, it is worthwhile for the lessor to carry out this maintenance only if the reduction of the repair expense exceeds the maintenance cost. Due to the mechanical aging of the deteriorating system, the maintenance cost will get higher and higher for sequential PM activities over the lease period. Jayabalan and Chaudhuri (1992) [23] suggested an approach to estimate the maintenance cost per unit time during the lease period, which is given by

$$C_{pm_k} = C_F (1 + \tau(k - 1)x) \tag{6}$$

where $C_F$ denotes the base cost of performing a PM activity, and $\tau$ denotes the periodically increasing rate of the PM cost. Therefore, the total PM cost should be given by

$$C_{pm} (C_F, \tau, x, T_L) = \sum_{k=1}^{T_L/x-1} C_{pm_k} = \sum_{k=1}^{T_L/x-1} C_F (1 + \tau(k - 1)x) \tag{7}$$

Moreover, different preventive maintenance scheme will influence on the degree of the system’s recovery but it also bring different maintenance expense to the lessor. Suppose that the several maintenance schemes $M_P = \{M^{k_1}_p, M^{k_2}_p, ..., M^{k_p}_p \}$ can be chosen, and the corresponding preventive maintenance expense and the expected failure times can be rewritten as follows:

$$C_{pm}^{M_p} (C_F^{M_p}, \tau^{M_p}, x, T_L) = \sum_{k=1}^{T_L/x-1} C_{pm_k}^{M_p} = \sum_{k=1}^{T_L/x-1} C_F^{M_p} (1 + \tau^{M_p}(k - 1)x) \tag{8}$$
and
\[ \Phi(T_L, x, \delta^{M_q^p}, \alpha, \beta) = \frac{T_L}{x} - \sum_{k=0}^{x-1} \int_{kx}^{(k+1)x} \lambda(t - \delta^{M_q^p}kx) \, dt \] 

(9)

2.3. Failure process. In this study, the failure process of the leased equipment is assumed as a time-dependent NHPP. The time-dependent tendency of NHPP can be modeled by the intensity function of the form \( \lambda(t) = \alpha f(t, \beta) \), where \( \alpha \) is the scale factor, \( \beta \) is the rate of deterioration, \( t \) is the elapsed time, and \( f(\cdot) \) is the deteriorating function which characterizes the deteriorating behavior of the process. The two commonly used models are exponential and power law, and their forms are given as follows:

\[ \lambda(t) = \alpha e^{\beta t} \]  

(10) 

and

\[ \lambda(t) = \alpha \beta^t (t - 1) \]  

(11) 

To exponential model, if the deterioration rate \( \beta > 0 \) with an initial failure rate \( \alpha \) the failure rate of the system will increase exponentially over time. In most cases, power law model is more flexible and manageable to describe the deteriorating behavior of aging systems than other models. However, when \( \beta \) is equal to one, the NHPP degenerates to an HPP with a constant failure intensity \( \alpha \). For \( \beta < 1 \) the failure intensity is decreasing show the reliability of the system is strengthening, and for \( \beta > 1 \) the failure intensity is increasing show the reliability of the system is weakening. Moreover, for \( 1 < \beta < 2 \) the failure intensity curve is concave downward, and for \( \beta > 2 \) the failure intensity curve is concave upward.

2.4. Optimal maintenance schedule without consideration of lease revenue. In general, the manufacturer (lessor) should arrange the optimal maintenance schedule by using the most economical method for free-repair warranty policy. The cost of free-repair warranty includes repair, penalty and preventive maintenance costs. Besides, since the initial investment of an equipment is the most important expenditure of the lessor, the lessor can take it as an amortized expenditure for calculating the average cost per unit time. The residual value of an equipment can be regarded as a deduction of the investment. According to the above mentioned, if the lessor has not made the plan for the revenue of rent, the expected average cost per unit time for performing the periodic PM action \( N \) times within the system life \( T_L \) can be given by

\[ MC(N) = \frac{T_L}{x} \sum_{k=1}^{N-1} C_{pmk}^p + \left( C_{mr} + C_{penalty} \int_{t_r}^{\infty} G(t_r) \, dt_r \right) \Phi(T_L, x, \delta^{M_q^p}, \alpha, \beta) + V - V_{residual} \]  

(12)
Proposition 1. If the intensity function \( \lambda(t) = \alpha e^{\beta t} \) or \( \lambda(t) = \alpha t^{\beta - 1} \) is strictly increasing, and the two inequalities \( MC(N + 1) \geq MC(N) \) and \( MC(N) < MC(N - 1) \) are both hold, the convexity of the cost function \( MC(N) \) with respect to \( N \) can be assured.

Proof. For \( MC(N + 1) \geq MC(N) \), we have 

\[
\sum_{k=1}^{N} C_{pmk}^{M_{k}^{p}} + \left( C_{mr} + C_{penalty} \int_{\varphi}^{\infty} G(t_{r}) dt_{r} \right) \Phi \left( (N + 1)x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) + V - V_{residual} \]

\[
\Rightarrow \frac{(N + 1)x}{N} \sum_{k=1}^{N-1} \frac{C_{pmk}^{M_{k}^{p}}}{(N + 1)x} + \left( C_{mr} + C_{penalty} \int_{\varphi}^{\infty} G(t_{r}) dt_{r} \right) \Phi \left( N x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) \geq 0
\]

\[
\Rightarrow \left( C_{mr} + C_{penalty} \int_{\varphi}^{\infty} G(t_{r}) dt_{r} \right) \left\{ \Phi \left( (N + 1)x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) - \frac{1}{N} \Phi \left( N x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) \right\} \geq \left( \frac{1}{N} \right) (V - V_{residual}) - C_{pmN} + \frac{1}{N} \sum_{k=1}^{N-1} C_{pmk}^{M_{k}^{p}}
\]

\[
\Rightarrow \left( C_{mr} + C_{penalty} \int_{\varphi}^{\infty} G(t_{r}) dt_{r} \right) \left\{ N \Phi \left( (N + 1)x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) - (N + 1) \Phi \left( N x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) \right\} \geq (V - V_{residual}) - NC_{pmN} + \sum_{k=1}^{N-1} C_{pmk}^{M_{k}^{p}}
\]

\[
\Rightarrow \Phi \left( (N + 1)x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) - (N + 1) \Phi \left( N x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) \geq \frac{V - V_{residual} - NC_{pmN} + \sum_{k=1}^{N-1} C_{pmk}^{M_{k}^{p}}}{C_{mr} + C_{penalty} \int_{\varphi}^{\infty} G(t_{r}) dt_{r}}
\]

(13)

For \( MC(N) < MC(N - 1) \), we have \( MC(N) - MC(N - 1) < 0 \)

\[
\sum_{k=1}^{N-1} C_{pmk}^{M_{k}^{p}} + \left( C_{mr} + C_{penalty} \int_{\varphi}^{\infty} G(t_{r}) dt_{r} \right) \Phi \left( N x, x, \delta_{k}^{M_{k}^{p}}, \alpha, \beta \right) + V - V_{residual} \]

\[
\Rightarrow \frac{N x}{N}
\]
\[
\sum_{k=1}^{N-2} C_{p_{mn_k}}^{M_P} + \left( C_{mr} + C_{penalty} \int_{0}^{\infty} G(t_r)dt_r \right) \Phi \left( (N-1)x, x, \delta^{M_P}, \alpha, \beta \right) \\
+ V - V_{residual} < 0
\]
2.5. Optimal maintenance schedule with consideration of lease revenue. If most of companies intend to lease equipment instead of buying them by the consideration of some financial issues, the lessor will handle the maintenance schedule for the company (lessee) to earn the better profit. By considering the lease revenue and the cost of maintenance service of the equipment, we suppose that the present value of the total rental of the leased equipment is $R(T_L, \varepsilon)$ and the rental will be influenced by the lease period $T_L$ and the time discount factor $\varepsilon$. It means that the longer lease period will be more benefic to the both of the lessee and the lessor because lessee can enjoy more discount and lessor can earn more revenue of leased equipment. The present value of the total rental of the leased equipment can be
formulated as follow:

\[ R(T_L, \varepsilon) = R_0 + R_0 \times (1 - \varepsilon) + R_0 \times (1 - \varepsilon)^2 + \ldots + R_0 \times (1 - \varepsilon)^{(T_L/T_S) - 1} \]

\[ = R_0 \left( \frac{1 - (1 - \varepsilon)^{T_L/T_S}}{\varepsilon} \right) \]  

(15)

where \( T_S \) is the time segment for the time-discounting factor \( \varepsilon \) and it is usually defined as a year.

Moreover, after the usage of the lease period, the residual value of the equipment can be estimated and evaluated by some experts and the lessor can sell the old equipment or reuse it. In other word, the residual value is also a part of the revenue of lease contract. Here, the declining balance depreciation method in Accounting will be used to estimate the residual value of equipment because this method is one of most popular in estimating the value of used equipment. Therefore, the estimated residual value of equipment can be formulated as follow:

\[ V_{\text{Residual}}(T_L, \rho) = V \times (1 - \rho)^{T_L/T_S} \]  

(16)

where \( \rho \) denotes the depreciation rate of equipment after every maintenance. Based on the above discussion, the estimated average profit per unit time of the lessor can be formulated as follow:

\[
\begin{align*}
\text{Max}_{T_L, \pi} \pi &= \left( \frac{1}{T_L} \right) \left\{ R(T_L, \varepsilon) + V_{\text{Residual}}(T_L, \rho) - \sum_{k=1}^{T_L/x-1} C_{\text{pmk}}^{M_f} \right. \\
&\quad \left. - \left( C_{\text{mv}} + C_{\text{penalty}} \int_0^\infty G(t) \, dt \right) \Phi(T_L, x, \delta^{M_f}, \alpha, \beta) - V \right\} \\
\text{Subjectto } &\quad T_L^{\text{Min}} \leq T_L \leq T_L^{\text{Max}}
\end{align*}
\]  

(17)

where \( T_L^{\text{Max}} \) denotes the maximum lifetime of the equipment, and the length of lease contract can’t exceed the equipment’s lifetime.

The subsequent decision analyses for the lessor will be proceed, and the studies regarding leased equipment issues will be also surveyed continually.

3. The Bayesian analysis for optimal maintenance policy and lease period.

3.1. Analysis by natural conjugate probability distribution. In some cases, the lessor does not have sufficient historical data to estimate the deterioration of newly equipment, and therefore we need a more reliable approach to estimate the deterioration under such situation. Bayesian analysis is a reasonable and reliable approach in practice, and it can both take expert opinions and scare historical data into account for better decision making.

Bayesian analysis can be divided into the two phases in the study. The first phase is the prior analysis, and the domain experts need to provide the values of the related four parameters \( \mu_\alpha, \mu_\beta, \sigma_\alpha, \sigma_\beta \) according to their domain knowledge and judgement in order to estimate the system deterioration. The second phase is the posterior analysis. The decision makers will collect additional failure data from engineering experiments if they did not convince the result of the prior analysis. Once the failure data has been collected, the decision makers can use this and the prior judgement of the four parameters to get the more accurate estimation.
However, Bayesian analysis is not easy to perform because the derivation of prior and posterior distributions is involving the use of numerical integration. Especially for the two critical parameters (i.e., $\alpha$ and $\beta$) in the state space, the analysis would be very complicated to deal with. Huang and Bier (1998, 1999) [19] [20] proposed natural conjugate prior distributions for the exponential model and the power law model for deteriorating systems which are of the forms

$$f(\alpha, \beta) = K\alpha^m e^{\beta(d(m+1)Z - ac\beta^{-1}(e^{\beta Z} - 1)}$$

(18)

and

$$f(\alpha, \beta) = K\alpha^{m-1} \beta^{m-1}(e^{-d \frac{m}{z}})^{\beta-1} e^{-\alpha c\beta}$$

(19)

where $K$ is a normalizing factor to ensure that the integration of each distribution can be equal to 1. The major merit of the natural conjugate prior distributions is to facilitate a straightforward analysis instead of the complicated computation which usually occurs in a Bayesian updating process. According to Huang and Bier (1998, 1999), the four moments in Equations (18) and (19) (i.e. $c$, $d$, $m$, and $z$) can be obtained by solving the derived equations of $\mu_\alpha$, $\mu_\beta$, $\sigma_\alpha$, and $\sigma_\beta$.

Suppose that the further investigation will be needed, and the optimal sample size $n$ has been carefully examined and the failure data is collected as $D^{(n)} = t_1, t_2, t_3, \ldots, t_n$. The posterior distribution of $\alpha$ and $\beta$ can be gained without further computation by the property of natural conjugate family, and which are given by

$$f(\alpha, \beta | D^{(n)}) \propto L(D^{(n)} | \alpha, \beta) f(\alpha, \beta)$$

$$= K' \alpha^m + n e^{\beta(d(m+1)Z + \sum_{i=1}^n t_i) - \alpha \beta^{-1}(e^{\beta n} + c(0.29272 - 1))}$$

(20)

and

$$f(\alpha, \beta | D^{(n)}) \propto L(D^{(n)} | \alpha, \beta) f(\alpha, \beta)$$

$$= K' \alpha^{m+n-1} \beta^{m+n-1} \left( e^{-d \frac{m}{z}} \prod_{i=1}^n t_i \right)^{\beta-1} e^{-\alpha(cz + t_n \beta)}$$

(21)

respectively for the power law and the exponential deteriorating models, where $K'$ is a normalizing factor to ensure each of the distribution sums up to unity, and $L(D^{(n)} | \alpha, \beta)$ is the likelihood function, and which are given by

$$\alpha^n e^{\beta \sum_{i=1}^n t_i - \alpha \beta^{-1} e^{\beta t_n}}$$

(22)

and

$$\alpha^n \beta^n (\prod_{i=1}^n t_i)^{\beta-1} e^{-\alpha t_n \beta}.$$ 

(23)

respectively for exponential and power law models.

3.2. Calculation of expectation of failures for prior and posterior analyses. According to the above, the forms of the probability distributions for the prior and the posterior analyses have been known from equations (18), (19), (20) and (21) for the power law and the exponential deteriorating models, and we can use these to calculate the expectations of the systems’ failures. Due to fact that the expectations with such complex probabilities cannot be obtained in closed-form expressions, we need to utilize numerical integration methods to get the expectations.
of the failure times during the lease period $T_L$ (without consideration of any PM) under exponential or power law model in the prior analysis as follows:

\[
\int_0^{T_L} \int_0^\infty \int_0^\infty \lambda(t) f(\alpha, \beta) d\beta d\alpha dt = \int_0^{T_L} \int_0^\infty \int_0^\infty \alpha e^{\beta t} K \alpha^m e^{\beta d(m+1)z - \alpha c \beta^{-1}} (e^{\beta z} - 1) d\beta d\alpha dt \tag{24}
\]

and

\[
\int_0^{T_L} \int_0^\infty \int_0^\infty \lambda(t) f(\alpha, \beta) d\beta d\alpha dt = \int_0^{T_L} \int_0^\infty \int_0^\infty \alpha \beta^{\beta - 1} K \alpha^{m-1} \beta^{m-1} (e^{-dz^m})^{\beta - 1} e^{-\alpha cz^\beta} d\beta d\alpha dt \tag{25}
\]

Moreover, the calculation of the expectations would become complex if we took the PM program into consideration during the lease period $T_L$. Equations (26) and (27) present the mathematical forms for exponential and power law model in the prior analysis.

\[
E_{\text{Prior}} \left[ \Phi(T_L, x, \delta M_p, \alpha, \beta) \right] = \sum_{k=0}^{T_L/x-1} \int_{kx}^{(k+1)x} \int_0^\infty \int_0^\infty \lambda(t - \delta M_p kx) f(\alpha, \beta) d\beta d\alpha dt
\]

and

\[
E_{\text{Prior}} \left[ \Phi(T_L, x, \delta M_p, \alpha, \beta) \right] = \sum_{k=0}^{T_L/x-1} \int_{kx}^{(k+1)x} \int_0^\infty \int_0^\infty \alpha e^{\beta(t - \delta M_p kx)} K \alpha^m e^{\beta d(m+1)z - \alpha c \beta^{-1}} (e^{\beta z} - 1) d\beta d\alpha dt \tag{26}
\]

and

\[
E_{\text{Prior}} \left[ \Phi(T_L, x, \delta M_p, \alpha, \beta) \right] = \sum_{k=0}^{T_L/x-1} \int_{kx}^{(k+1)x} \int_0^\infty \int_0^\infty \lambda(t - \delta M_p kx) f(\alpha, \beta) d\beta d\alpha dt
\]

\[
E_{\text{Prior}} \left[ \Phi(T_L, x, \delta M_p, \alpha, \beta) \right] = \sum_{k=0}^{T_L/x-1} \int_{kx}^{(k+1)x} \int_0^\infty \int_0^\infty \alpha \beta^{\beta - 1} (t - \delta M_p kx)^{\beta - 1} K \alpha^{m-1} \beta^{m-1} (e^{-dz^m})^{\beta - 1} e^{-\alpha cz^\beta} d\beta d\alpha dt \tag{27}
\]

Similarly, the expectation of failures for the posterior analysis cannot be obtained in a closed form expression either. The numerical integration methods are also employed to calculate the expectations of the failure times for the posterior analysis,
which are given by

\[ E_{\text{Posterior}} \left[ \Phi(T_L, x, \delta^{M_p}, \alpha, \beta) \right] \]

\[ = \sum_{k=0}^{T_L/x-1} \int_{kx}^{(k+1)x} \int_0^\infty \int_0^\infty \lambda \left( t - \delta^{M_p} k x \right) f \left( \alpha, \beta | D(n) \right) d\beta d\alpha dt \]

\[ = \sum_{k=0}^{T_L/x-1} \int_{kx}^{(k+1)x} \int_0^\infty \int_0^\infty \alpha e^{\beta \left( t - \delta^{M_p} k x \right)} \]

\[ \times K^\beta \alpha^{m+n-1} \beta^{m+n-1} \left( e^{-d z m} \prod_{i=1}^{n} t_i \right)^{\beta-1} e^{-\alpha (e z^\beta + t_n^{\beta})} d\beta d\alpha dt \]

and

\[ E_{\text{Posterior}} \left[ \Phi(T_L, x, \delta^{M_p}, \alpha, \beta) \right] \]

\[ = \sum_{k=0}^{T_L/x-1} \int_{kx}^{(k+1)x} \int_0^\infty \int_0^\infty \lambda \left( t - \delta^{M_p} k x \right) f \left( \alpha, \beta | D(n) \right) d\beta d\alpha dt \]

\[ = \sum_{k=0}^{T_L/x-1} \int_{kx}^{(k+1)x} \int_0^\infty \int_0^\infty \alpha e^{\beta \left( t - \delta^{M_p} k x \right)} \]

\[ \times K^\beta \alpha^{m+n-1} \beta^{m+n-1} \left( e^{-d z m} \prod_{i=1}^{n} t_i \right)^{\beta-1} e^{-\alpha (e z^\beta + t_n^{\beta})} d\beta d\alpha dt \]

for exponential and power law models respectively. The posterior analysis can carry out with the consideration of both prior knowledge and sampling information.

To proceed with the Bayesian analysis, the two objective functions respectively represent the optimal expected profit for the prior analysis and the posterior analysis based on equation (17) are constructed as follows:

\[ \begin{align*}
\text{Max} & \quad E_{\text{Prior}} \left[ \pi \right] = \left( \frac{1}{T_L} \right) \left\{ R(T_L, \varepsilon) + V_{\text{Residual}}(T_L, \rho) - V - \sum_{k=1}^{T_L/x-1} C_{pm_k} \right\} \\
\text{Subjectto} & \quad T_L^{\min} \leq T_L \leq T_L^{\max} \\
\text{Max} & \quad E_{\text{Posterior}} \left[ \pi \right] = \left( \frac{1}{T_L} \right) \left\{ R(T_L, \varepsilon) + V_{\text{Residual}}(T_L, \rho) - V - \sum_{k=1}^{T_L/x-1} C_{pm_k} \right\} \\
\text{Subjectto} & \quad T_L^{\min} \leq T_L \leq T_L^{\max}
\end{align*} \]

3.3. Solution algorithm. According to the mathematical analysis and discussion, the deduction process can be used to develop the solution algorithm for the proposed model. The solution algorithm can be divided two parts. One is for the prior analysis and the other is the posterior analysis. In the prior analysis, the domain experts of the lessor need to investigate the form of the prior probability distribution
of the leased equipment and then give the prior judgement on the four critical parameters in advance in order to reasonably estimate the expected repair cost in the future. It should be noticed that the calculation of the expected repair cost will need numerical method like Monte Carlo integration (Müller and Parmigiani, 1995 [31]). Although the numerical integration may consume some computation-resource, it can complete the computation within a tolerable time. The other expenditures and income (the investment of leased equipment, the estimated residual value and the PM cost) will be easily obtained by the equations provided in Section 2. Moreover, the financial manager of the lesser also need to estimate the total lease revenue with time discount in the future from the lessees. According to these financial information, the lessor can easily obtain the optimal decision in the prior analysis.

However, if the result of the prior analysis would not be convinced or accepted by the lessor, it would be needed to collect the failure data of the lease equipment to construct the posterior probability distribution. Similarly, the calculation of the expected repair cost in the posterior analysis will also need numerical integration method. After collecting the related information regarding all expenditures and incomes, the lessor can perform the posterior analysis to get the optimal solution. The illustration of the solution algorithm in detail can be seen in Figure 4.

3.4. Computerized implementation architecture. In order to deal with such complicated mathematical problem, a computerized application system is necessary to implement in obtaining the optimal decision. In enhancing the manageability, the whole system is divided into two subsystems. The model management system is implemented for engineers and domain experts to maintain the database and the model base. Furthermore, the decision support system is developed for decision makers to provide the informative knowledge. In operating the model management system, the engineers should inspect the parameters of costs, failure intensity functions, probability of repair time, depreciation, discount rate, lease payment, residual value... etc. in advance. After obtaining these data, the engineers can store them into the database by using the model management system. Moreover, the deterioration of the new facility is hard to evaluate because the reliability data is insufficient. Therefore, the domain experts of reliability engineering need to evaluate the statistical characteristics and related parameters according to their experience. Besides, if a posterior analysis is desirable for the decision maker, the engineers need to collect the facilities’ failure data from some engineering experiments. With regard to storing or accessing the database and model base, a data formalizing mechanism is applied to transform the inconsistent data, and this mechanism can help us to store or access the data more efficiently. Besides, a computation engine might be needed to deal with the complexity in obtaining the optimal solution. By utilizing an application programming interface (API), the system developers can use computation engine to proceed with all the mathematical analyses for the decision support system. Figure 5 illustrates the computerized implementation architecture of our system.

4. Application and sensitivity analysis.

4.1. Application of prior and posterior analyses. Suppose that an industrial equipment manufacturer will propose a capital lease project to attract customers in the industrial facilities market since the most of companies prefer to lease it than to purchase it for financial and tax issues. In this lease project, the manufacturer
Figure 4. Flowchart for the Bayesian Solution Algorithm
(lessor) will provide a maintenance program and free-repair warranty service to customers (lessee) during the lease period. Since the new release equipment has not been proceeded by complete reliability tests, the equipment’s deterioration will be difficult to estimate. In order to deal with the problem under insufficient historical reliability-test data, a Bayesian analysis will be applied to estimate the equipment’s deterioration. The Bayesian analysis can be divided into the two phases: the first phase is the prior analysis which is evaluated by reliability domain experts; the second phase is the posterior analysis which needs for collecting failure data from engineering experiments. In the first phase, the experts’ opinions are appealed to estimate the parameters for the facility’s deterioration. After careful evaluation and prediction, the facility may deteriorate in accordance with an NHPP with a power law intensity function, and the joint distribution of $\alpha$ and $\beta$ can be modeled by the natural conjugate prior distribution with the four prior parameters $\mu_\alpha = 1.60$, $\mu_\beta = 2.10$, $\sigma_\alpha = 1.10$, $\sigma_\beta = 0.80$. Besides, the repair time is assumed to Gamma probability distribution, and the average and the standard deviation of the repair time are estimated to be 9 and 5 hours, and however the tolerable waiting time is only 4.5 hours according to the customers’ satisfaction survey. Therefore, the parameters $\omega$ and $\eta$ can be deduced as 3.24 and 0.36 by the simultaneous equations.
\[ \omega = E(t_r^2) / \sigma^2 \] and \[ \eta = E(t_r) / \sigma(t_r) \] and the expected penalty cost for each repair will be $139.765. \( C_{\text{penalty}} \int_\varphi \psi^\phi_{\text{penalty}} dt_r = 170 \int_4.5^\infty 0.36^3.24_{\text{penalty}} - 1 \psi^\phi_{\text{penalty}} dt_r \)

Moreover, the engineering department proposed the three different maintenance plans, and each plan will lead to different repair and maintenance costs during the lease period because of different systems’ recovery. Although the degree of plan1’s age reduction factor is lower than plans 2 and 3’s, its PM cost and the increasing rate are also lower the others. Accordingly, it is hard to judge which maintenance plan is better for the lessor. Due to the fact that the contract of the lease term will impact on the revenue, the residual value and the related costs, the lessor should carefully make the decision regarding the lease term to maximize its average profit. Based on the above mentioned, the detailed information of three maintenance plans with the experts’ evaluation regarding the lease facility’s deterioration is presented as table 1.

### Table 1. The detailed information of three maintenance plans

| Parameters for the deterioration judged by experts | Maintenance Plan 1 | Maintenance Plan 2 | Maintenance Plan 3 |
|--------------------------------------------------|--------------------|--------------------|--------------------|
| Parameters for the deterioration judged by experts | \( u_\alpha = 1.60, u_\beta = 2.10, \sigma_\alpha = 1.10, \sigma_\beta = 0.80 \) | \( \delta^{M^1} = 0.7 \) | \( \delta^{M^2} = 0.8 \) | \( \delta^{M^3} = 0.9 \) |
| Age reduction factors | \( \delta^{M^1} = 0.7 \) | \( \delta^{M^2} = 0.8 \) | \( \delta^{M^3} = 0.9 \) |
| Base cost for a PM action | \( C^{M^1} = 600 \) | \( C^{M^2} = 750 \) | \( C^{M^3} = 900 \) |
| Periodically increasing rates of PM cost | \( \tau^{M^1} = 0.2 \) | \( \tau^{M^2} = 0.25 \) | \( \tau^{M^3} = 0.25 \) |
| Depreciation rate | \( \rho = 0.15 \) | \( z = 0.5 \) years; \( T_{L} = 0.5 \) year | \( T_{L}^{\text{Min}} = 2 \) years; \( T_{L}^{\text{Max}} = 12 \) years |
| Interval of PM; Time segment | \( T_{L}^{\text{Min}} = 2 \) years; \( T_{L}^{\text{Max}} = 12 \) years | \( T_{L}^{\text{Min}} = 2 \) years; \( T_{L}^{\text{Max}} = 12 \) years | \( T_{L}^{\text{Min}} = 2 \) years; \( T_{L}^{\text{Max}} = 12 \) years |
| The minimal and maximal planned lease terms | \( R_0 = 9800 \) | \( \epsilon = 0.02 \) | \( V = 9800 \) |
| Rental of per half-year | \( \rho = 0.15 \) | \( z = 0.5 \) years; \( T_{L} = 0.5 \) year | \( T_{L}^{\text{Min}} = 2 \) years; \( T_{L}^{\text{Max}} = 12 \) years |
| Penalty cost for repair time over the time limit | \( C_{\text{penalty}} = 170 \) | \( E(t_r) = 9 \) hours | \( \sigma(t_r) = 5 \) hours |
| Expectation of performing a minimal repair | \( \epsilon = 0.02 \) | \( V = 9800 \) | \( \varphi = 4.5 \) hours |
| Standard deviation of performing a minimal repair | \( \sigma(t_r) = 5 \) hours | \( \varphi = 4.5 \) hours | \( C_{\text{penalty}} = 170 \) |
| Tolerable waiting time limit for performing a minimal repair | \( \varphi = 4.5 \) hours | \( C_{\text{penalty}} = 170 \) | \( C_{\text{penalty}} = 170 \) |
| Expected cost of performing a minimal repair | \( C_{\text{penalty}} = 170 \) | \( C_{\text{penalty}} = 170 \) | \( C_{\text{penalty}} = 170 \) |

After performing the prior analysis according to the proposed solution algorithm shown in figure 4, the related analyzed results are presented in table 2 and figure 6. According to the information of table 2, the optimal lease terms of maintenance plans 1, 2, 3 should be set to 6, 7, 8 years respectively, and the average profits will be $4033, $3902 and $3,808. Therefore, it can be seen that the highly intensive maintenance plan may be not able to get good effect in obtaining profit. Generally, highly intensive maintenance plans (2 and 3) can reduce expected facilities’ failure times and save repair cost. However, the save of the repair cost cannot cover the increment of PM cost. Although lower intensive maintenance plan (1) will bring serious facilities’ breakdowns at the post-phase, the lessor can adopt the strategy of shorter lease-term to prevent this disadvantage. It can be seen that the average profit of plan 1 will be lower than the other plans if the lease term is over 8.5 years. Before the time threshold, the average profit of maintenance plan 1 is always higher.
than the other plan's. Accordingly, the lessor should adopt maintenance plan 1 and set the contract of lease term within 6 years to pursue the maximal average profit.

Table 2. Expected failures, repair costs, preventive costs, production cost, residual value and average profits per unit and year for maintenance plan 1, 2, 3 estimated by prior analysis

| Time   | Plan 1  | Plan 2  | Plan 3  | Plan 1  | Plan 2  | Plan 3  | Plan 1  | Plan 2  | Plan 3  | Plan 1  | Plan 2  | Plan 3  |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 2      | 3.06    | 2.74    | 2.44    | 2760    | 3495    | 4194    | 1500    | 1343    | 1194    | 98000   | 70805   | 3292    |
| 2.5    | 4.24    | 3.69    | 3.17    | 3600    | 4575    | 5490    | 2077    | 1807    | 1553    | 98000   | 65279   | 3472    |
| 3      | 5.61    | 4.75    | 3.96    | 4500    | 5738    | 6885    | 2746    | 2327    | 1938    | 98000   | 60184   | 3625    |
| 3.5    | 7.17    | 5.93    | 4.79    | 5460    | 6983    | 8179    | 3514    | 2904    | 2349    | 98000   | 55487   | 3752    |
| 4      | 8.85    | 7.23    | 5.69    | 6480    | 8310    | 9972    | 4387    | 3542    | 2786    | 98000   | 51157   | 3854    |
| 4.5    | 10.17   | 8.66    | 6.63    | 7560    | 9720    | 11664   | 5373    | 4243    | 3250    | 98000   | 47164   | 3932    |
| 5      | 13.22   | 10.23   | 7.64    | 8700    | 11213   | 13455   | 6480    | 5031    | 3741    | 98000   | 43483   | 3987    |
| 5.5    | 15.75   | 11.94   | 8.70    | 9900    | 12788   | 15345   | 7716    | 5848    | 4261    | 98000   | 40089   | 4021    |
| *6     | 18.55   | 13.79   | 9.81    | 11160   | 14445   | 17334   | 9090    | 6758    | 4809    | 98000   | 36961   | *4033   |
| 6.5    | 21.65   | 15.80   | 10.99   | 12480   | 16185   | 19422   | 10111   | 7744    | 5598    | 98000   | 34076   | 4025    |
| *7     | 25.08   | 17.98   | 12.23   | 13860   | 18008   | 21609   | 12289   | 8810    | 5993    | 98000   | 31417   | *3998   |
| 7.5    | 28.85   | 20.32   | 13.53   | 15300   | 19913   | 23895   | 14135   | 9959    | 6631    | 98000   | 28965   | 3951    |
| *8     | 32.98   | 22.85   | 14.90   | 16800   | 21900   | 26280   | 16160   | 11194   | 7301    | 98000   | 26704   | 3885    |
| 8.5    | 37.50   | 25.55   | 16.33   | 18360   | 23970   | 28764   | 18375   | 12521   | 8002    | 98000   | 24620   | 3802    |
| 9      | 42.43   | 28.46   | 17.83   | 19980   | 26213   | 31347   | 20795   | 13943   | 8736    | 98000   | 22698   | 3701    |
| 9.5    | 47.81   | 31.56   | 19.40   | 21600   | 28358   | 34029   | 23426   | 15464   | 9504    | 98000   | 20927   | 3583    |
| 10     | 53.65   | 34.88   | 21.03   | 23400   | 30675   | 36810   | 26289   | 17089   | 10306   | 98000   | 19294   | 3448    |
| 10.5   | 59.99   | 38.41   | 22.74   | 25200   | 33075   | 39690   | 29394   | 18822   | 11143   | 98000   | 17788   | 3296    |
| 11     | 66.85   | 42.18   | 24.52   | 27060   | 35588   | 42669   | 32758   | 20668   | 12016   | 98000   | 16400   | 3128    |
| 11.5   | 74.28   | 46.19   | 26.38   | 28980   | 38123   | 45747   | 36396   | 22632   | 12926   | 98000   | 15120   | 2944    |
| 12     | 82.29   | 50.45   | 28.31   | 30960   | 40770   | 48924   | 40324   | 24719   | 13874   | 98000   | 13040   | 2744    |

![Average profit per unit and year](image)

**Figure 6.** Average Profits per Unit and Year for Maintenance Plans 1, 2, 3 Estimated by Prior Analysis

However, if the lessor may not fully convince the result of the prior analysis, it would be needed to collect a part of failure data by proceeding with accelerated deterioration experiments. Due to the issue of cost and time limit, the lessor only can obtain few experimental data to adjust or amend the forecast of the prior analysis. Suppose that the collected data as follows: \( D^{(10)} = 0.49, 1.07, 1.66, 1.94, 2.11, 2.81, 3.19, 3.47, 3.59, 4.12 \). The collected data can be combined with the former experts' opinion of the prior analysis to proceed with the posterior analysis, and the
computation results are summarized in Table 3. According to figure 7, the average profit estimated by the posterior analysis is always lower than the prior analysis. The optimal lease term should be set to 4.5 years, and the average profit need to be adjusted to $3298 also. The difference of estimated profit between the two analyses is over 18%. After considering the collected failure data of the posterior analysis, it can be seen that the results of the prior analysis may be too optimistic because the re-estimating deterioration in the posterior analysis is more serious than in the prior analysis. In other words, the lessor should moderately shorten the term of lease contract to save the increase of repair cost.

Table 3. Expected failures, repair costs, preventive cost, production cost, residual value and average profits per unit and year for prior and posterior analyses

| Time | Prior | Posterior | Repair Cost | PM Cost | $V_{residual}$ | $E[π]$ |
|------|-------|-----------|-------------|---------|----------------|--------|
| 2    | 3.06  | 3.90      | 1500        | 1911    | 1500           | 3292   |
| 2.5  | 4.24  | 5.82      | 2077        | 2856    | 2077           | 3472   |
| 3    | 5.61  | 8.09      | 2746        | 3965    | 2746           | 3625   |
| 3.5  | 7.17  | 10.68     | 3514        | 5233    | 3514           | 3752   |
| 4    | 8.95  | 13.58     | 4387        | 6655    | 4387           | 3854   |
| 4.5  | 10.97 | 16.79     | 5373        | 8226    | 5373           | 3932   |
| 5    | 13.22 | 20.29     | 6480        | 9944    | 6480           | 3987   |
| 5.5  | 15.75 | 24.09     | 7716        | 11805   | 7716           | 4021   |
| 6    | 18.85 | 28.18     | 9090        | 13807   | 9090           | 4033   |
| 6.5  | 21.65 | 32.54     | 10611       | 15946   | 10611          | 4025   |
| 7    | 25.08 | 37.19     | 12289       | 18222   | 12289          | 3998   |
| 7.5  | 28.85 | 42.10     | 14135       | 20631   | 14135          | 3951   |
| 8    | 32.98 | 47.29     | 16160       | 23173   | 16160          | 3885   |
| 8.5  | 37.50 | 52.74     | 18375       | 25845   | 18375          | 3802   |
| 9    | 42.41 | 58.46     | 20793       | 28645   | 20793          | 3701   |
| 9.5  | 47.81 | 64.43     | 23426       | 31573   | 23426          | 3583   |
| 10   | 53.65 | 70.67     | 26289       | 34627   | 26289          | 3448   |
| 10.5 | 59.99 | 77.15     | 29394       | 37805   | 29394          | 3296   |
| 11   | 66.85 | 83.89     | 32758       | 41107   | 32758          | 3128   |
| 11.5 | 74.28 | 90.88     | 36396       | 44532   | 36396          | 2944   |

Figure 7. Average Profits per Unit and Year for Maintenance Plan 1 Estimated by Prior and Posterior Analyses
4.2. Sensitivity analysis. Misjudging on the parameters of $u_\alpha$, $u_\beta$, $\sigma_\alpha$ and $\sigma_\beta$ may impact on the estimations of the average profit and repair cost, and therefore the lessor should pay attention on the possible changes of the estimations. Accordingly, the sensitivity analyses can be performed to estimate the variations in the terms of lease and expected average profit. It is reasonable that if we underestimate $u_\alpha$ and $u_\beta$, then this cause an underestimation of the repair cost, leading to improper decisions such as mistakenly extending the lease term. According to figure 8, the lower estimation of parameters $u_\alpha$ or $u_\beta$ will lead the lessor to extend the term of the leasing contract by utilizing the advantage of the lower growth deterioration. Similarly, misjudging $\sigma_\alpha$ and $\sigma_\beta$ may also result in risky decisions. The lessor can use the strategy of extending-term of lease if the values of standard deviation $\sigma_\alpha$ and $\sigma_\beta$ are larger. Therefore, the decision makers should be very cautious with their judgments. Tables 4 and 5 show the detail information of changing $u_\alpha$, $u_\beta$, $\sigma_\alpha$ or $\sigma_\beta$. It can be seen that the impacts of $u_\alpha$ and $u_\beta$ on expected failure times and repair cost are more than the impacts of $\sigma_\alpha$ and $\sigma_\beta$.

Figure 8. The Impact of $E(\alpha)$, $E(\beta)$, $\sigma(\alpha)$ and $\sigma(\beta)$ on Average Profit

Besides, due to the fact that the related costs can also influence the lease decision, the lessor should notice the possible changes of the related costs. Figure 8 shows the impact of minimal repair cost on the average profit. The lessor should consider shortening the term of lease to react the repair cost increase. However, it still depends on the range of repair cost increase. According to figure 9, it can be seen
that the optimal lease term should be set to 6.5 years when the repair cost is within the range between $200 and $300. Similarly, the base cost of PM and its increasing rate are also impact on the lessor’s average profit and the decision of lease term. According to figure 10, the average profit is estimated to be over $4,500 when the lease term is set to 6.5 years and the base cost of PM is assumed to be $300. With increasing the base cost of PM, the optimal lease term should be shorter to decrease the burden of PM cost. Moreover, the increasing rate of PM cost may go up due to
to the inflation of the related costs. According to figure 11, it can be seen that the impact will be amplified with the term of lease. If the increasing rate is from 0.05 up to 0.35, the decrease of the average profit will be over $1100.

Figure 9. The Impact of Minimal Repair Cost on Average Profit

Figure 10. The Impact of Base Cost for a PM action on Average Profit

Price inflation is an important issue in some developing countries, and it will cause the real revenue decreased with time during the lease period. Therefore, the decision maker must know the influences of price inflation to adjust its strategy of lease term. The discount rate is used to reflect the present value of all the lease revenue in the future. According to figure 12, it can be seen that the optimal lease term should be set to 8 years and the average profit in present value can reach to $5,164. Moreover, with the time discount rate increasing, the lessor should shorten the lease term to decrease the loss of the lease revenue in present value. However, if the discount rate is higher than 0.05, any strategy of extending lease term only
lead to the average profit decreased. Moreover, since the residual value can retrieve a part of the investment of a facility, the change of the depreciation rate should be noticed for estimating the residual value in the future. According to figure 13, different depreciation rates lead to significant difference of average profits in short lease terms. However, the difference will be diminished with extending the lease term. Therefore, the decision maker should adopt the strategy with a longer lease term for ensuring its average profit if the depreciation rate is higher.

5. Conclusion. The objective of this study is to provide the analytical models and the corresponding solution algorithm for solving the leased equipment problem under the situation of scarce historical data. The model is to provide a PM scheme to reduce the related cost within the lease period and also consider the income from the residual value of equipment at the end of the lease period. A nonhomogeneous
Poisson process is employed to describe the successive failure times of the deteriorating leased equipment. According to the results of mathematical and sensitivity analyses, the major findings can be simply arranged as follows: (1) The lease term and PM plans should be carefully evaluated by lessor since it is not easy to estimate the impacts on the expected costs of repair, PM and delay penalty under the uncertainty of the estimated parameters. (2) To proceed with accelerated deterioration experiments will be necessary if the lessor did not fully convince the result of the prior analysis. (3) If the lessor underestimate the parameters of the scale and shape factors, it may cause improperly extending the lease term by the underestimation of repair cost. (4) Misjudging the standard deviations of the scale and shape factors may also lead to risky decisions. If the parameters $\sigma_\alpha$ and $\sigma_\beta$ are larger, it would be advantageous to extend the term of the lease contract. (5) The related costs can also influence the lease decision, the lessor should notice the possible changes of the related costs. (6) It may be advantageous to shorten the lease term to react the repair cost increase but it depends on the range of repair cost increase. (7) The base cost of PM and its increasing rate are also impact on the lessor’s average profit and the decision of lease term. With increasing the base cost of PM, the optimal lease term should be shorter to decrease the burden of PM cost. (8) Price inflation is an important issue in developing countries, and it causes the real revenue decreased with lease period. (9) The lessor should shorten the lease term to decrease the loss of the lease revenue while the time discount rate increasing. (10) The change of the lease equipment’s depreciation rate should be noticed for estimating the residual value in the future. If the depreciation rate is higher, the lessor might adopt the strategy of a longer lease term for ensuring its expected profit. The future work could be performed on refining the proposed model by considering the leased products with two-dimensional deterioration. Due to the fact that the deterioration of equipment depends not only on time but also on usage, under such situation, only considering one of them could distort the estimation of leased equipment’s deterioration. Therefore, two-dimensional failure model would be fit for dealing with such problems. Besides, a bivariate-Weibull probability distribution will be considered in the analytical model in order to construct the two-factor (time, usage) joint
probability distribution. The decision makers can also utilize this analytical model to refine their lease policies.

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