M Theory from World-Sheet Defects in Liouville String

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Abstract

We have argued previously that black holes may be represented in a $D$-brane approach by monopole and vortex defects in a sine-Gordon field theory model of Liouville dynamics on the world sheet. Supersymmetrizing this sine-Gordon system, we find critical behaviour in 11 dimensions, due to defect condensation that is the world-sheet analogue of $D$-brane condensation around an extra space-time dimension in $M$ theory. This supersymmetric description of Liouville dynamics has a natural embedding within a 12-dimensional framework suggestive of $F$ theory.

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1 Introduction

If one is to understand the relationships between the plethora of apparently consistent classical string vacua and understand non-perturbative effects in string theory, one needs an approach that goes beyond critical strings and conformal field theory. Important ingredients in such an understanding are provided by string soliton configurations such as $D$ branes\cite{1}. These are powerful tools that have facilitated the deeper understanding of string theory that is now provided by duality, which links Type I, IIA, IIB and heterotic strings within a unified framework that takes its name from the 11-dimensional limit called $M$ theory\cite{2}. Some string dualities may find a clearer interpretation within a possible 12-dimensional extension called $F$ theory\cite{3}.

Another approach introduces a Liouville field on the world sheet, which enables one to construct critical string theories in unusual numbers of dimensions\cite{4, 5, 6}. It was also shown that this approach could be used to introduce classical string vacua with interpretations as non-trivial space-time backgrounds such as Friedmann-Robertson-Walker cosmologies\cite{3}. This approach was subsequently extended to the quantum level\cite{7}, where it was argued that higher-genus and non-perturbative effects would deform the theory away from classical criticality, inducing non-trivial dynamics for the Liouville field.

This development was motivated by, and used to accommodate, processes involving string black holes in $1 + 1$ dimensions\cite{4}. In particular, the creation and disappearance of such quantum black holes, as well as transitions between them that correspond to back-reaction of particles on the black-hole background metric, could be encoded in the dynamics of the Liouville field. A representation for this dynamics was found\cite{7} in terms of defects on the world sheet: vortices and monopoles of the Liouville field that could be described in terms of a deformed sine-Gordon model\cite{8}.

More recently, it has been shown that $D$ branes emerge naturally from Liouville string theory\cite{9, 10}. Quantum effects at higher genus introduce fluctuations in the $\sigma$-model couplings of the conformal field theory that describes a classical string vacuum. These are associated with singular configurations in the space of higher-genus moduli, such as long, thin handles that resemble wormholes on the world sheet. The absorption of the divergences associated with these singularities requires quantization of the world-sheet couplings and the introduction of $\sigma$-model counterterms that have a space-time interpretation as $D$ branes\cite{9, 10}. Moreover, $D$ branes in arbitrary space-time dimensions can be described within this approach in terms of vortex and monopole defects in the Liouville theory on the world sheet\cite{11}, just as was the case for $(1 + 1)$-dimensional black holes. When these defects are irrelevant operators in a renormalization-group sense, the world sheet is in a dipole phase, and their correlators with the vertex operators of target-space matter excitations have cuts, in general. Thus the world sheet becomes effectively
open, and $D$ branes arise when suitable boundary conditions are implemented along the cuts $11$. This provides a $D$-brane description of quantum fluctuations in the space-time background, i.e., space-time foam, with microscopic black holes represented by $D$ branes whose appearance and disappearance is described by defect fluctuations on the world sheet.

The next step in this programme, described in this paper, is to supersymmetrize the world-sheet sine-Gordon theory that describes the non-critical Liouville dynamics $8, 12$. We enumerate the phases of sine-Gordon models with $n = 1, 2$ world-sheet supersymmetry, identifying the ranges of space dimensions where their world-sheet vortices and monopoles can be classified as relevant or irrelevant deformations, focusing on the vortices and monopoles of lowest charge, which are constrained by quantization conditions. In the case of $n = 1$ world-sheet supersymmetry, we show that the lowest-charge $|q| = 1$ vortices are irrelevant below $d = 11$ dimensions, as seen in Fig. 1. Likewise, the lowest-charge $|e| = 1$ monopoles are irrelevant above $d = 17$ dimensions. Normal space-time with massive black holes is described by the phase in which vortices are irrelevant. There are Kosterlitz-Thouless $13$ (KT) phase transitions in $d = 11, 17$ dimensions, with the new phenomenon of defect condensation and an unstable plasma phase for intermediate dimensions. The appearance of massless defect configurations in $d = 11, 17$ marks recovery of conformal symmetry. Away from these critical condensation points, one has effectively a $D = (d+1)$-dimensional target theory, but the extra dimension decouples in the conformal theory when $d = 11$, just like the Liouville field in conventional critical string theory.

We interpret the transition of world-sheet vortices from irrelevancy to relevancy in 11 dimensions as the world-sheet transcription of the masslessness of $D$ branes in the strong-coupling 11-dimensional limit of $M$ theory. We also discuss how the theories with $n = 1$ world-sheet supersymmetry that we discuss may be described as marginal deformations of theories with $n = 2$ world-sheet supersymmetry $14$, thereby making more direct contact with string models with $N = 1$ space-time supersymmetry. Finally, we conjecture that the extra dimension provided by the Liouville field with non-trivial dynamics may be interpreted as a world-sheet description of $F$ theory $3$.

2 Sine-Gordon Description of World-Sheet Vortices, Monopoles and $D$ Branes

The world-sheet sine-Gordon action $15, 8$ of such defects in a purely bosonic theory was used in $7$ to describe $(1 + 1)$-dimensional black holes, and in $11$ to describe target-space $D$ branes. For completeness and to set the scene for the supersymmetric extension we use later, we briefly review here the bosonic sine-Gordon formalism of $8$, whose $\sigma$-model
The partition function is:

\[ Z = \int D\tilde{X} \exp(-\beta S_{\text{eff}}(\tilde{X})) \]

\[ \beta S_{\text{eff}} = \int d^2z [2\partial\tilde{X}\partial\tilde{X} + \frac{1}{4\pi}[\gamma_0\omega^{2-2}(2\sqrt{|g(z)|})^{1-2} : \cos(2\pi\sqrt{\beta g}[\tilde{X}(z) + \tilde{X}(\bar{z})]) : + \frac{1}{4\pi}[\gamma_0\omega^{2-2}(2\sqrt{|g(z)|})^{1-2} : \cos(\frac{e}{\sqrt{\beta}}[\tilde{X}(z) - \tilde{X}(\bar{z})]) :] ] \] (1)

where \( \alpha \equiv 2\pi\beta q^2 \), \( \alpha' \equiv e^2/2\pi\beta \), with \( q, e \) the charges of the vortex and monopole solutions:

\[ \partial_z\partial_{\bar{z}}\tilde{X}_v = i\pi \frac{q}{2} [\delta(z - z_1) - \delta(z - z_2)], \quad \partial_z\partial_{\bar{z}}\tilde{X}_m = -i\pi \frac{e}{2} [\delta(z - z_1) - \delta(z - z_2)] \] (2)

Moreover, \( \omega \) is an ultraviolet angular cutoff in (1), and \( \gamma_{v,m} \) are the fugacities for vortices and monopoles. We note that \( \beta^{-1} \) plays the role of an effective temperature in (1), and that the vortex and monopole operators have anomalous dimensions:

\[ \Delta_v = \frac{\alpha}{4} = \frac{\pi}{2} q^2, \quad \Delta_m = \frac{\alpha'}{4} = \frac{e^2}{8\pi\beta} \] (3)

The system (1) is invariant under the \( T \)-duality transformation (8): \( \pi\beta \leftrightarrow \frac{1}{4\pi\beta} \); \( q \leftrightarrow e \).

We identify \( \tilde{X} \) in (1) with the rescaled Liouville field \( \sqrt{\frac{C-25}{3g_s^2}} \phi \) - where \( g_s \) denotes the string coupling, and \( \chi \) is the Euler characteristic of the world-sheet manifold - after subtraction of classical solutions to the equations of motion and spin-wave fluctuations (8). As the analysis of [11] indicates, one has \( e \propto \frac{1}{\sqrt{g_s}} \). This is used in our analysis below, when we discuss the spectrum of stable configurations of the theory. In our approach, we relate \( \beta \) to the Zamolodchikov \( C \) function of the accompanying matter (13): \( \beta \rightarrow \frac{3g_s^2}{\pi(C-25)} \).

In ref. [11] we were interested in the case \( C > 25 \), and we considered only irrelevant deformations, that do not drive the theory to a new fixed point. The world-sheet system was then found to be in a dipole phase (8), and had the general features discussed in the context of the formulation of quantum non-critical Liouville string in (11), namely non-conformal deformations, non-trivial Liouville dynamics, and dependence on an ultraviolet cutoff. We interpreted this system in (11) as a model for space-time foam in the context of \((1 + 1)\)-dimensional string black holes.

We further suggested in [11] that the field theory (11) also represents \( D \)-brane foam. This is because the correlation functions of these non-conformal deformations with other (conformal) deformations, representing vertex operators for the excitation of matter fields in target space, appear to have cuts, thereby leading to effectively open world sheets. Consider, for example, the scattering of closed string states \( V_T(X) = \exp(ik_MX^M - iEX^0) : M = 1, \ldots D_{cr} - 1, \) where \( D_{cr} \) is the critical space-time dimension, in the presence
of a monopole defect. An essential aspect of this problem is the singular behaviour of the operator product expansion of $V_T$ and a vortex or monopole operator $V_{v,m}$. Treating the latter as a sine-Gordon deformation of (1), computing at the tree level using the free world-sheet action, and suppressing for brevity anti-holomorphic parts, we find

$$\lim_{z \to w} < V_T(X^0, X^i)(z)V_{v,m}(X^0)(w) \ldots > \sim \int d^{Dcr-1}k \int dE \delta^{Dcr-1}(k) \ldots [\delta(\Sigma E + \kappa/\sqrt{\beta})(z-w)^{-\Delta_T-\Delta_{v,m}} + \delta(\Sigma E - \kappa/\sqrt{\beta})(z-w)^{-\Delta_T-\Delta_{v,m}}]$$

(4)

where $\kappa = 2\pi \beta q$ for vortices, $\kappa = e$ for monopoles, $\Delta_T = \frac{E_0^2}{2}$, the energy-conservation $\delta$ functions result from integration over the Liouville field $\tilde{X} \equiv X^0$, and $(\ldots)$ indicates factors related to the spatial momentum components of $V_T$, other vertex operators in the correlation function, etc.. We see that (4) has cuts for generic values of $\Delta_T + \Delta_{v,m}$, causing the theory to become effectively that of an open string.

To relate this observation more closely to $D$ branes, we recall that there are two possible types of conformal boundary conditions, Dirichlet and Neumann, related to each other by appropriate $T$-duality transformations, which in the above picture appear as canonical transformations in the path integral [17, 11]. Thus one may place Dirichlet boundary conditions on the boundaries of the effective world sheet, i.e., along the cuts, thereby obtaining solitonic $D$-brane [1] configurations. Some non-trivial consistency checks of this identification of world-sheet defects with target-space $D$ branes, including derivations of the correct energy and momentum conservation relations during the scattering of light closed-string states off these solitons, have been obtained in [11], using the identification of the zero mode of the Liouville field with the target time. We interpret the resulting world-sheet vortex/space-time $D$-brane system as a representation of space-time foam and microscopic black holes in higher dimensions, as a natural extension of the previous (1+1)-dimensional approach.

### 3 Critical Dimensions in Supersymmetric Sine-Gordon
World-Sheet Theory

In this section we follow the discussion of [8] on the supersymmetrization of the monopole and vortex configurations on the world sheet of the string, with a view to relating their physical interpretation to $M$ theory. We start with a sine-Gordon theory with local $n = 1$ world-sheet supersymmetry, whose (non-chiral) monopole deformation is [8]:

$$V_m = \bar{\psi}\psi : \cos\left[\frac{e}{\beta^{1/2}}(\phi(z) - \phi(\bar{z}))\right] :$$

(5)
where the $\psi, \bar{\psi}$ are world-sheet fermions, which transform as fields of conformal dimensions $(0, \frac{1}{2}), (\frac{1}{2}, 0)$, respectively, $\phi$ is the Liouville field, which corresponds as in [8] and above to the fluctuating quantum part of the conformal scale factor of the original world-sheet metric with the ‘spin-wave’ and classical parts appropriately subtracted, and $\mathbf{...}$ denotes normal ordering. The equivalence of the above $\sigma$ model with Liouville theory implies that the effective ‘temperature’ $1/\beta_{n=1}$ is related to the matter central charge $d$ [8] by:

$$\beta_{n=1} = \frac{2}{\pi(d-9)}$$

where we assume the case $d > 9$. The ‘matter’ central charge $d$ may be considered as the number of dimensions of an effective target space-time in which the theory lives. The deformation (5) is understood to be added to the free-fermion and Liouville actions, and the combined system has $n = 1$ world-sheet supersymmetry. The corresponding deformation for vortex solutions is given by:

$$V_v = \bar{\psi}\psi : \cos[2\pi q^2 \beta_{n=1}^{1/2}(\phi(z) + \phi(\bar{z}))] :$$

where $q$ is the vortex charge. For simplicity, we consider the case where the world-sheet cosmological constant is zero.

Including the conformal dimensions $(\frac{1}{2}, 0), (0, \frac{1}{2})$ of the fermion fields, the conformal dimension $\Delta_v, \Delta_m$ of the vortex and monopole deformations (7),(5) in the holomorphic sector are, respectively:

$$\Delta_v = \frac{1}{2} + \frac{1}{\pi \beta_{n=1}} q^2 = 1/2 + q^2/(d-9)$$

$$\Delta_m = \frac{1}{2} + \frac{1}{8\pi \beta_{n=1}} e^2 = 1/2 + e^2(d-9)/16$$

As usual, relevant deformations have $\Delta_{m,v} < 1$ and irrelevant deformations $\Delta_{m,v} > 1$, whilst marginal deformations have $\Delta_{m,v} = 1$. In the relevant case, the vacuum is unstable with respect to condensation of the corresponding topological world-sheet defects, and the system dissociates into a plasma of the corresponding free charges, whilst in the irrelevant case the vacuum is stable with respect to condensation of the corresponding defects. Marginal deformations correspond to a Kosterlitz-Thouless phase transition [8, 13], and respect the conformal invariance of the system.

An important constraint on the charges of the monopole and vortex configurations is imposed by the single-valuedness of the partition function on a spherical world sheet [8]. We recall that the monopole and vortex configurations are related by a $T$-duality transformation which corresponds to a temperature inversion [8]:

$$e \leftrightarrow q \quad \pi \beta \leftrightarrow \frac{1}{4\pi \beta}$$

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This implies the co-existence of both deformations if either one is present. Single-valuedness of the partition function then requires the following relation between the monopole and vortex charges:

$$2\pi \beta_n = 4q/(d - 9) = e, \quad q, e \in \mathbb{Z}$$

(10)

It is easy to verify that this condition imposes equality between the conformal dimensions (8) of the vortex and monopole defects: $\Delta_v = \Delta_m$.

![Conformal Dimensions for Various (q, e)](chart.png)

Figure 1: Chart of the conformal dimensions of $n = 1$ world-sheet supersymmetric sine-Gordon defects in integer dimensions $9 < d < 22$: the operators are marginal for $\Delta_v = \Delta_m = 1$. Vortices with $|q| = 1$ are linked by solid lines, vortices with $|q| = 2$ by long-dashed lines, monopoles with $|e| = 1$ by short-dashed lines, and monopoles with $|e| = 2$ by dotted lines. In each case, the charges of the T-dual defects given by the quantization condition (10) are also indicated. Note that the world-sheet vacuum is stable for $d < 11$ and $d > 17$, and dissolves into an unstable plasma phase of free $|q| = 1$ and/or $|e| = 1$ defects at intermediate dimensions.

We consider first the case with the lowest possible vortex charge $|q| = 1$. We see from (8) that defect deformations are irrelevant, and the vacuum is stable, if

$$0 < d - 9 < 2q^2 = 2 \rightarrow 9 < d < 11$$

(11)

The quantization condition (10) then implies that the allowed monopole charges in this case are:

$$e = 4q/(d - 9) = \pm 4/(d - 9)$$

(12)
The vacuum for this monopole is also stable in the range (11), since this monopole deformation (3) is also irrelevant, with the same conformal dimension as the lowest-lying vortex. The conformal dimensions of vortices with $|q| = 1$ (and of the corresponding monopole charges $e$) are shown in Fig. 1 for allowed integer values of $d > 9$, linked by solid lines to guide the eye.

Some comments are in order about defects with larger charges. If one restricts oneself to the region $9 < d < 11$\footnote{We recall that $d$ is the central charge of the matter system, which need not be integer.}, higher-charge monopole configurations with

$$e^2 > \frac{8}{d-9}, \quad e \in \mathbb{Z}$$

are irrelevant deformations. In the particular case of standard critical strings with $d = 10$, the stable monopole configurations are those with charges $e > 2\sqrt{2}$, i.e., the integers $|e| = 3, 4, 5, \ldots$, whilst for $d = 9$ all finite $|e|$ configurations are unstable. As already mentioned, we see from the analysis of (11) that one can identify $1/e^2 \propto g_s$, the string coupling in the case of a $D$-brane $\sigma$-model. Thus the above infinite tower of stable discrete monopole configurations corresponds to an infinite tower of massive $D$-brane states, with masses $\propto e^2$ in appropriate units. The conformal dimensions of $|q| = 2$ vortices (and the corresponding monopoles) are shown in Fig. 1, linked by long-dashed lines to guide the eye, and we see explicitly that they are irrelevant for all $d$.

Consider now the case of the lowest monopole charge $e = \pm 1$, when the quantization condition (11) implies that the allowed vortex charges are:

$$q = \pm(d - 9)/4$$

The corresponding defect operators are both irrelevant, and the vacuum stable, for

$$d > 17$$

The conformal dimensions of $|e| = 1$ monopoles are shown in Fig. 1, linked by short-dashed lines to guide the eye. Also shown in Fig. 1 are the conformal dimensions of $|e| = 2$ monopoles, which are never relevant, linked by dotted lines.

The monopole and vortex operators are both marginal, and the theory conformal, for:

$$\frac{(d-9)e^2}{16} = \frac{q^2}{(d-9)} = \frac{1}{2}$$

which, on account of the quantization condition (11), singles out the two limiting cases:

$$d = 11, |q| = 1, |e| = 2; \quad d = 17, |e| = 1, |q| = 2$$
which are clearly visible in Fig. 1. At these points, the monopole and vortex configurations are both at their Kosterlitz-Thouless (KT) transition points \[13\]. For \(11 < d < 17\), the world sheet is in an unstable plasma phase with free defects.

The world-sheet cuts generated by the non-integer anomalous dimensions in \[1\], which lead to effectively open world sheets in the construction of \[11\], disappear at the conformal points \[17\], and the above world-sheet analysis is no longer valid. Instead, at a KT point, the system of defect dipoles dissociates into a plasma of free charges, and the world-sheet system undergoes a phase transition. At these KT points the infinite tower of discrete massive \(D\)-brane states \[13\] disappears \[2\].

4 Physical Interpretation of Vortex Condensation as a Representation of \(M\) Theory.

We now seek to relate this discussion to the manner in which the eleventh dimension first appeared in \(M\) theory, namely via \(D\)-brane condensation in the strong-coupling limit of conventional 10-dimensional strings with space-time supersymmetry \[1, 2\]. We recall that, like any other solitons, the target-space masses of \(D\) branes \(m_D \sim 1/g_s\), where \(g_s\) is the string coupling. These masses are reminiscent of those of conventional string winding modes if the winding radius \(R \to \infty\) as \(g_s \to \infty\). Thus \(D\)-brane condensation as \(m_D \to 0\) was shown to correspond to the appearance of an eleventh large dimension, leading to a theory whose low-energy field-theoretical limit was 11-dimensional supergravity.

We have shown previously how \(D\) branes appear naturally in the world-sheet Liouville \(\sigma\)-model approach to non-critical string theory \[3\]. Moreover, we have presented arguments that vortex defects in the world-sheet Liouville field theory provide an explicit representation of \(D\) branes \[11\]. Furthermore, we have identified in the previous section two critical dimensions \(d^*\) of the \(n = 1\) supersymmetric sine-Gordon model, where the world-sheet defects interpreted as target-space black holes and \(D\) branes condense, namely \(|q| = 1\) vortices in \(d^* = 11\) and \(|e| = 1\) monopoles in \(d^* = 17\). We claim that this provides an explicit world-sheet representation of \(D\)-brane condensation.

It is natural to seek to identify the 11-dimensional case with the corresponding critical limit of \(M\) theory: quite apart from anything else, the 17-dimensional case has no known field-theoretical limit. However, there are two apparent puzzles to be resolved before this putative interpretation of the \(d^* = 11\) theory can be established.

\footnote{This transition bears some analogy with the finite-temperature deconfinement transition in QCD, where an infinite set of massive bound states disappears.}
One apparent puzzle is that in non-critical string theory we identify the Liouville field with an extra target space-time dimension, which would seem to provide us with $d^* + 1 = 12$ dimensions. This ‘non-critical’ target space-time must have indefinite signature, with at least one time-like dimension provided by the Liouville field construction [3], and the fact that $d > 9$. However, we recall that both the $|q| = 1$ vortex and $|e| = 2$ monopole deformations are exactly marginal at $d^* = 11$, so that the $n = 1$ supersymmetric version of (2) is conformal. In such a case, as in the case of conventional critical string theory, the background target-space time fields $g^i$, that deform the $\sigma$-model when coupled to the appropriate vertex operators, depend only on the $d^*$ coordinates, due to the conformal nature of the pertinent $\sigma$ model. One way to see this is to recall that the Liouville mode acts in general as a covariant local renormalization scale on the world sheet of the non-critical string [7], but that this scale is redundant for a critical theory. Thus the effective target-space field theory at $d^*$ is completely characterized by fields that are independent of the Liouville field. Hence the supersymmetric sine-Gordon theory at the KT critical point is effectively an 11-dimensional theory, and may be identified with $M$ theory.

Of course, the simple sine-Gordon model (1) and its $n = 1$ supersymmetric extension do not manifest all the mathematical structure of string theory, and hence $M$ theory. This should be reflected in consistency conditions on the possible choices of matter and space-time degrees of freedom that must be satisfied by any critical string construction. Our identification of vortex condensation in $n = 1$ sine-Gordon Liouville theory should be understood by analogy with the $Z_3$ Potts model description of the finite-temperature QCD deconfinement phase transition. The reduced system captures the essence of the critical behaviour of the full system.

The second apparent puzzle is that, since we have been working with $n = 1$ supersymmetric sine-Gordon theory on the world sheet, we do not have $N = 1$ supersymmetry in target space, which would require $n = 2$ supersymmetry on the world sheet. There is also the issue of the signature of the $d^*$ dimensions, which can be resolved in a supersymmetric target-space theory. The next section is devoted to a discussion of these two issues.

5 Relation to Models with $N = 1$ Space-Time Supersymmetry

As already mentioned, local $n = 1$ world-sheet supersymmetry does not admit a supersymmetric target space-time interpretation, for which one needs $n = 2$ local world-sheet supersymmetry. In the context of the monopole and vortex deformations discussed in earlier sections, an $n = 2$ locally supersymmetric sine-Gordon model may easily be constructed [8], as a straightforward generalization of the $n = 2$ super-Liouville theory [12].
The latter originates from a world-sheet \( n = 2 \) supergravity (local supersymmetry). The simplest model \([12]\) is the \( O(2) \)-symmetric \( n = 2 \) supergravity, which is a two-dimensional theory admitting the string interpretation \([13]\), in the sense of giving rise to a consistent four-dimensional target space-time. The particle content of the \( n = 2 \) supergravity multiplet is one graviton (zweibein) field \( e_{\mu}^{a} \), two Majorana or one Dirac gravitino \( \chi^{\mu} \), and a vector field \( A_{\mu} \). The supergravity multiplet is coupled to a set of \( D \) complex matter multiplets, consisting of a complex scalar \( X \), the Dirac spinor \( \psi \) and a complex auxiliary field \( F \). We do not give here details of the action, since we only use the theory for illustrative purposes: details may be found in \([19]\).

The Liouville dressing of this theory is facilitated by picking the conformal gauge: \( e_{\mu}^{a} = e^{\phi} e_{\mu}^{a} \), \( \chi_{\mu} = \frac{1}{2} \gamma_{\mu} (\eta_{1} + i \eta_{2}) \), \( A_{\mu} = \frac{1}{2} \epsilon_{\mu \nu} \partial^{\nu} \rho \). One has also gauge-fixing and reparametrization Fadeev-Popov ghosts, which contribute \( c_{gh} = -6 \) to the central charge. The ghost sector of the \( n = (2,2) \) world-sheet local supersymmetric string consists of the \((b,c)\) ghosts for general coordinate invariance, contributing \((-26, -26)\) to the central charge, the two bosonic ghosts of \( n = (2,2) \) supersymmetry: \((\beta^{1}, \gamma^{1}), (\beta^{2}, \gamma^{2})\), each of conformal weight \((3/2, -1/2)\) and contributing \((22,22)\) to the central charge, and the fermionic ghost fields \((f,g)\) of the local \( O(2) \simeq U(1) \) gauge algebra of the \( N = 2 \) superconformal symmetry, which have conformal weight \((1,0)\) and contribute \((2,2)\) to the conformal anomaly. The total contribution of the ghost sector to the central charge is, therefore, \( c_{gh} = -6 \), implying that under conformal transformations the effective ghost-induced gravity action transforms as: \( S_{gh} \rightarrow S_{gh} + \frac{1}{48\pi} c_{gh} S_{sl} \), where \( S_{sl} \) is the super-Liouville \( n = 2 \) \( O(2) \) action \([12]\):

\[
S_{sl} = \frac{1}{2\pi} \int d^{2} z (\bar{\phi} \partial \phi - \bar{\rho} \partial \rho + \sqrt{g} R^{(2)} \phi - \sqrt{g} R^{(2)} \rho - \sum_{i=1,2} [\bar{\eta}_{i} \bar{\eta}_{i} - \eta_{i} \partial \eta_{i}])
\]  

(18)

Notice the wrong sign of the second scalar field \( \rho \), which indicates that this field is a ghost. Going to a fiducial metric, one can cast the above action in a more familiar form where the central charge deficit \( Q = \sqrt{\frac{1-d}{2}} \), with \( d \) the ‘matter’ central charge, appears in front of the curvature terms as usual. The super-Liouville theory can be made physical, by appropriate Wick rotations of the \( \phi, \rho \) fields, for \textit{any} \( d \), even \( d > 1 \). This is due to the fact that the fields \( \rho \) and \( \phi \), since their kinetic terms have opposite signs, simply interchange their rôles as one crosses the value \( d = 1 \) \([12]\).

The presence of monopoles does not alter the above features. In particular, the monopole deformation of the \( n = 2 \) theory, which respects \( n = 2 \) local supersymmetry of the conformally-fixed action, reads \([3]\):

\[
\bar{\eta}_{1} \eta_{1} \bar{\eta}_{2} \eta_{2} : \cos \left[ \frac{e}{\rho_{n=2}^{1/2}} (\phi(z) - \phi(\bar{z})) \right] :
\]

(19)
and there is a corresponding expression for the vortex, with

$$\beta_{n=2} = \frac{4}{\pi(1 - d)} \quad (20)$$

The fields $\eta_i$ in (19) are conformal fields of dimension $(1/2, 0)$, so that the above $n = 2$ monopole deformation has conformal dimension

$$\Delta_{n=2,m} \geq 1 \quad (21)$$

in each sector. Therefore, the monopole and vortex deformations are *irrelevant* for every $d$, in accord with the above statement that the super-Liouville theory is defined for every $d$. The value $d = 2$ corresponds to the critical four-dimensional target space-time string of [18], for which the Liouville fields decouple.

To understand the relation of $n = 2$ world-sheet supersymmetric models to $n = 1$, we note that this $n = 2$ string has been shown recently to be connected by world-sheet *marginal* deformations to strings with less world-sheet local supersymmetry, such as (1,1) or (1,0) superstrings [14]. This was achieved by adding suitable topological packages to the critical fields of the (2,2) $n = 2$ world-sheet theory, whose action was described by a $BRST$-exact form. In the case of (2,2) string, such packages consist of fermionic quartets ($\lambda_\alpha^i, \rho_\alpha^i$), $\alpha = 1, \ldots, 4$, and their bosonic partner fields ($F_\alpha^i, \overline{F}_i^\alpha$), where $i$ runs over appropriate packages. These fields have conformal weights $(1,0)$, and their world-sheet $BRST$-exact action is

$$I_{top} = i \int d^2 z \sum_{\alpha=1}^{4} Q_{BRST} (F_\alpha^i \partial \rho_\alpha^i) \quad (22)$$

with the $BRST$ transformation $Q_{BRST} \rho_\alpha^i = iF_\alpha^i$, $Q_{BRST} F_\alpha^i = 0$, $Q_{BRST} \overline{F}_i^\alpha = \lambda_\alpha^i$, and $Q_{BRST} \lambda_\alpha^i = 0$. Such packages do not introduce any anomalies, so any number of them can be added to the world-sheet action of the critical fields.

World-sheet supersymmetry breaking was achieved using marginal deformations by twisting the gravitino ghosts ($\beta^2, \gamma^2$) in such a way that their conformal weight (-3/2, -1/2) was twisted to (1/2, 1/2), so that they should be interpreted as ‘matter’ fields in an $n = 1$ locally-supersymmetric world-sheet theory. The twisting is achieved by exactly-marginal deformations of Liouville type, which deform the theory in an appropriate way, so that the new conformal dimensions of the gravitino ghosts, with respect to the stress tensor deformed by the marginal deformations, are the ones appropriate for the $n = 1$ world-sheet supersymmetry [14]. The marginal deformations of the critical $d = 2, n = 2$ theory consist of linear-dilaton terms [3] of the form [3, 14]

$$\sum_{i=1}^{N} \int d^2 z \frac{1}{2\pi} \sqrt{|g|} (\partial \Phi_i \partial \Phi_i - i \frac{1}{2} k^3 R^{(2)} \Phi_i) \quad (23)$$
where $g$ is a world-sheet metric. The coefficients $k_i$ are chosen to be null: $\sum_i k_i^2 = 0$, so that the total central charge of such terms is equivalent to that of the free bosonic system, which decouples from the rest of the theory \cite{14}. In this way the total central charge of the theory is unaffected by the presence of such terms. Compensating twists are necessary in the ‘topological package’ sector, for consistency. For instance, the breaking of $n = 2$ to $n = 1$ world-sheet supersymmetry requires \cite{14} that the conformal dimension $(1,0)$ of the $(\lambda^1_1, \rho^1_1)$ fields be twisted to $(1/2, 1/2)$. Further twists of the remaining ghost fields, and/or the ghost fields of additional topological packages, can lead to further reduction of the world-sheet supersymmetries, thereby connecting the $n = 2$ string to other $\sigma$ models \cite{14}.

These observations are important for our approach, since they indicate the possibility that, even in the non-critical sine-Gordon Liouville case discussed in previous sections, one can find marginal deformations that connect them to theories with $n = 2$ local world-sheet supersymmetry, that admit a supersymmetric target-space interpretation. Supersymmetry can be broken partially by marginal deformations of similar linear-dilaton nature to the ones discussed just above in the critical $(2, 2)$ case. The ghost sector of the theory remains the same: the only difference from the critical model of \cite{18}, for which the Liouville field decouples, is the fact that now the matter theory is more complicated. It contains in general curved $d$-dimensional target-space background coordinates and their world-sheet fermionic partners, which should be $N = 1$ supersymmetric in target space. We do not enter here into the formal construction of such a world-sheet supergravity theory, which does not appear necessary for our present purpose.

Since the ghost sector retains the features of \cite{18}, we observe that the deformed stress tensor of the theory, with respect to which the new conformal dimensions of the various fields are defined, can be arranged such that the gravitino ghost $(\beta^2, \gamma^2)$ is twisted in the same way as previously. This will connect the $n = 2$ world-sheet supersymmetric background to the $n = 1$ theory. As we have already seen, when one admits topologically non-trivial structures on the world sheet, the only conformal points of the $n = 1$ super sine-Gordon model are those corresponding to target dimensions $d = 11$ or 17. Thus we relate strings with $N = 1$ target-space supersymmetry to the $n = 1$ world-sheet supersymmetric representation of the critical limit of $M$ theory discussed in previous sections.

Another way in which one can relate this $n = 1$ sine-Gordon model for world-sheet defects to the $n = 2$ sine-Gordon model is via condensation of the supersymmetric fermions of the $n = 2$ super Liouville sector. To see how this may arise, consider the Thirring-type monopole deformation \cite{19} of the $n = 2$ theory as an example. World-sheet fermion interactions may lead to condensation of the $\eta_2$ fermions, $< \bar{\eta}_2 \eta_2 > = \sigma \neq 0$, which makes the vertex operator look like the $n = 1$ monopole deformation \cite{20}, up to a difference $\beta_{n=2} \neq \beta_{n=1}$. Since we are interested only in the conformal points, this difference can be absorbed in a constant rescaling of the Liouville field in the $n = 2$ theory. The relative normalization, then, between the Liouville kinetic term and the monopole deformation
can be cast into the same form as that of the standard $n = 1$ theory, discussed in section 3, by giving an appropriate value to the fermion condensate $\sigma$. This fixed fermion condensate breaks the second supersymmetry, which corresponds to a $U(1)$ gauge symmetry. This can be achieved in a conformal invariant way only if the central matter charge of the $n = 2$ theory is $d = 11$ or $17$. The above procedure, therefore, describes an alternative mechanism for relating $n = 2$ and $n = 1$ sine-Gordon theories without violating the conformal invariance of the underlying $\sigma$-model theory.

6 Conjectures Concerning $F$ Theory

If one is to maintain space-time supersymmetry, the maximum number of space-time dimensions which is allowed for a consistent field-theory interpretation is $d = 11$, if one accepts that there should be no massless particles with spins higher than two [20]. However, there is another stringy reason which restricts the maximum number of dimensions: $d \leq 11$. As we discussed recently in [11] and reviewed briefly in section 1, a world-sheet theory with monopoles leads to $D$ branes in target space. This was seen by noting that non-conformal monopole deformations in the region $9 < d < 11$ induce cuts in the correlation functions (4) of the defect vertex operators with the vertex operators describing the excitation of target-space matter fields, leading to an effectively open world sheet. Dirichlet boundary conditions can then be placed along the cuts by canonical $T$-duality transformations in the appropriate $\sigma$-model path integral [11, 17], to obtain solitonic $D$ branes in target space.

It was apparent from our non-critical string construction that the target space of such a $d$-brane is a priori $11 + 1 = 12$-dimensional. This is the maximum number of dimensions in which supersymmetry can exist, if the 12-dimensional space time has $(10,2)$ signature [3, 21]. This is because such a signature admits a spinor which is both Majorana and Weyl [3, 22]. Due to the Liouville dynamics [3] for $d > 9$, this implies that the conformal point $d=11$ must have $(10, 1)$ signature for supersymmetry to appear in the full non-critical string, viewed as a critical one in a different target-space dimension. We also note that, among Minkowski space times admitting $p$ branes, the 11-dimensional manifold is the maximal one admitting super $p$ branes [4].

Our construction of $D$ branes, via topological defects on the world sheet of the non-critical Liouville string, provides a natural framework for the appearance of such a 12-dimensional theory, for which the 11-dimensional supergravity appears as a fixed conformal point. We therefore conjecture that the above construction may lead to a $\sigma$-model description of $F$ theory.

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In this construction, the critical string theories appear as conformal fixed points - effectively in 'equilibrium' - of the Liouville string - non-critical and 'non-equilibrium' in general [11] - which describes the bulk of theory space. Each fixed point in the space of string theories has its own time-like coordinate $X^0$, which is a reversible coordinate in the Einstein sense, so that a fixed-point theory is Lorentz invariant. In addition, in the bulk of the theory space there is a second time-like coordinate, the Liouville field $\phi$, with respect to which evolution is irreversible in general [7]. This means that the general $d + 1$-dimensional target-space $F$ theory is 'non-equilibrium', and does not in general have a simple field-theoretical interpretation [1]. This type of deviation from conventional Lorentz-invariant field theory cannot be ignored in general, since defect deformations cause the theory to fluctuate away from the conformal points, without necessarily driving the theory to a new fixed point. Our proposal for managing the appearance of these two times is to identify the zero modes of the a priori distinct quantum fields $X^0$ and $\phi$ [7] in the neighbourhood of each fixed point. We have demonstrated that this identification reproduces correctly the metric of the $1 + 1$-dimensional string black hole [7], and implements correctly energy-momentum conservation in closed-string scattering off a $D$-brane target, including quantum recoil effects [10]. We believe that this approach may cast some light on the mysterious structure of $F$ theory.

Acknowledgements

The work of D.V.N. is supported in part by D.O.E. Grant DEFG05-91-GR-40633.

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