Object Tracking by Least Spatiotemporal Searches

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ABSTRACT

Tracking a car or a person in a city is crucial for urban safety management. How can we complete the task with minimal number of spatiotemporal searches from massive camera records? This paper proposes a strategy named IHMs (Intermediate Searching at Heuristic Moments): each step we figure out which moment is the best to search according to a heuristic indicator, then at that moment search locations one by one in descending order of predicted appearing probabilities, until a search hits; iterate this step until we get the object’s current location. Five searching strategies are compared in experiments, and IHMs is validated to be most efficient, which can save up to 1/3 total costs. This result provides an evidence that “searching at intermediate moments can save cost”.

KEYWORDS
Location-dependent and sensitive, Traffic analysis, Human-centered computing, Pervasive computing, Ubiquitous computing

1 INTRODUCTION

Tracking a car or a person in a city is crucial for urban safety management [20]. For instance, a suspect car is witnessed at a location at a past time, but out of sight shortly. Afterwards, the police have captured this clue, and intend to monitor the car’s whereabouts in order to determine whether need to control it immediately. However, the police cannot know the car’s current location conveniently. The reasons why the police cannot locate the car by GPS or a direct phone call include: the car does not have a GPS receiver; the car’s GPS signal does not upload to the police system; the police do not know who are in or around the car and which phone number can reach them; the police decide not to arouse its suspicions, and etc. As a result, the police choose to find the car through cameras which have covered all over the city.

Note that cameras are multifarious, not only include those for traffic and public surveillance, but also those of citizens’ smart mobile phones, thus it is reasonable to assume that these cameras are distributed densely enough and with long-term recording capability (e.g., videos or images stored with time and location stamp). If that’s not a reality, it will be in the near future, because the concept of collaborative sensing [16-19] is emerging, which can greatly enhance the urban sensing ability by the combination of both stationary infrastructures and mobile phones. Since the quantity of cameras can be up to several millions, and their records are last from the witness moment to current moment, the challenge is “how can we find the car with minimal number of spatiotemporal searches?” Here a spatiotemporal search is defined as the effort that the police make to check whether the car appears at a specified location at a specified moment, given that the city is discretized into non-overlapping locations and the time is discretized into non-overlapping moments, and all camera records have been indexed according to their locations and moments (but not indexed by contents, e.g., license number, because content recognition is only performed on records of a specified location at a specified moment when “searching” within it). The unit cost of a spatiotemporal search can be considered as a constant for simplicity, with regard to either human or artificial intelligence. The unit cost depends on the granularity of the spatiotemporal unit, the volume and quality of camera records, and the technique of content recognition.
Because we need to locate the target object in very short time (e.g., within current 1 minute), the unit cost is mainly measured with other resources instead of time length, such as the number of agents that work in parallel. Particularly, for records from traffic surveillance cameras, automatic license plate detection can be utilized, while for records from mobile phones, since the conditions and qualities are not uniform and compliant, automatic license plate detection may meet difficulties. In that case, citizens or policemen themselves determine whether the target object is in a video or an image. The significance of solving this problem lies in that we can save the total costs (= the amount of searches × the unit cost) to track an object (i.e., find where is it now).

A consensus is that we should firstly search where the object is most likely to be, which is the ability of mobility prediction. Among others, Markov model is a good choice for mobility prediction [22]. It can be trained from historical trajectories and then provide the probabilities of an object appearing at each location in future. Besides the straightforward strategy that all searches are executed at the last time (i.e., the current moment), another strategy worthwhile to explore is that we spend some searches at intermediate moments to gather the object’s additional locations passed through, then we can predict better where the object will be, thus at the last time, we only need fewer searches to find the object finally. Continuing the above instance, the police originally need 100 searches at the last time to find the suspect car, but may only need 20 more searches at the last time if they have already spent 50 searches at intermediate moments, as illustrated in Fig. 1.

![Fig. 1. The framework of spatiotemporal searches.](image)

In this paper, we propose and compare various strategies for object tracking by least spatiotemporal searches, and analyze whether the costs spent in earlier searching can be compensated by the costs saved in later searching. The contributions of this paper can be summarized as:

1) Define a novel problem that minimizing the number of spatiotemporal searches to track an object after a specific time since its last appearance. See Section 2.
2) Propose a strategy to solve the problem with intermediate searching, when and where according to a heuristic indicator which can be estimated from historical trajectories. The heuristic indicator can resolve the conflict of spending the least searches to get the object’s latest location. Design other baseline strategies. See Section 3.
3) Evaluate with real world cars’ trajectories, and validate that the idea of saving costs through intermediate searching is feasible, and the proposed strategy is better than others. See Section 4.

Then Section 5 presents some related work, and Section 6 provides the conclusions.
2 PROBLEM DEFINITION

In order to formally describe the problem of “tracking an object by least spatiotemporal searches”, we first need some notations, see Table 1. Let $C = \{c_1, c_2, ..., c_C\}$ be objects (e.g., cars or persons), $L = \{l_1, l_2, ..., l_{|L|}\}$ be locations of a real world area (e.g., a city), $D = \{d_1, d_2, ..., d_{|D|}\}$ be days, in which $d_j = \{t_{j1}, t_{j2}, ..., t_{jd}\}$, where $t_k$ for each moment (e.g., a minute), $TR = \{tr(c_i, d_j)\}$ be trajectories, in which $tr(c_i, d_j) = \{(l_{j1}, t_{j1}), (l_{j2, t_{j2}, ...}) \mid c_i, d_j\}$. This means in this paper, a trajectory is defined as the sequence of spatiotemporal units representing time-specific locations of a day of an object. Note that an object is only at one location at a moment. If an object is extremely fast and passes through several locations in a moment, we will only preserve the first location of that moment.

**Definition 1** (spatiotemporal search): From camera records of a day $d_j$, a spatiotemporal search takes an object $c_x$, a location $l_i$, and a moment $t_k$ as inputs, and outputs a Boolean value, i.e., 1 if the object locates $l_i$ at $t_k$, 0 otherwise. Formally,

$$s(c_x, l_i, t_k) = \begin{cases} 1, & (l_i, t_k) \in tr(c_x, d_j) \\ 0, & \text{else} \end{cases}$$ (1)

**Problem 1** (tracking an object by least spatiotemporal searches): Give an object $c_x$, a day $d_x$, a past moment $t_p$ of that day, $c_x$’s location $l_{tp}$ at that moment, a current moment $t_x$, camera records that make the spatiotemporal search $s(c_x, l_i, t_k)$ available in $d_x$ before/after $t_x$, as well as history trajectories $TR'$, in which arbitrary $d_j < d_x$. When and where to execute a spatiotemporal search can base on $TR'$ and the outputs of previous searches. The problem is how to utilize minimal amount of searches to find the object at current moment $t_x$. Formally,

$$\text{give } c_x, d_x, t_x, (l_{tp}, t_p), TR' \text{ before } d_x, s(c_x, l_i, t_k) \text{ in } d_x$$

$$\text{return } l_x, \text{ s.t. } \min_{s(c_x, l_i, t_k) = 1} \{ s(), s(), ..., s(c_x, l_i, t_k) \}$$ (2)

3 STRATEGIES OF SPATIOTEMPORAL SEARCHES

The interaction between the problem and the solution (i.e., the strategy of spatiotemporal searches) can be represented by a Markov decision process, as shown in Fig. 2. At each step, the process is in a state that denotes how much we know about the object’s trajectory, and the strategy chooses an action that denotes when and where to search, based on current state. The process responds at the next step by moving into a new state, and incurring the corresponding cost. From this analysis, we can draw inspiration that the total costs will be determined by the searching difficulty $\alpha$ (influenced by the predictability of the object’s trajectory $\gamma$ and the effectiveness of the mobility prediction algorithm $\delta$) and the efficiency of the searching strategy $\beta$. 

| Symbol | Description | Symbol | Description |
|--------|-------------|--------|-------------|
| $d_j$ | a day, $d_j \in$ the set of days $D$, $d_x$ is the testing day | $\vec{p}(c_x, t_k)$ | the result of the mobility prediction algorithm, e.g., a row of a TPM, which moves from $l_i$ |
| $t_k$ | a moment in a day, $d_j = \{t_{j1}, t_{j2}, ..., t_{jd}\}$; particularly $t_p$ is the start moment, $t_x$ is the end moment | $p(l_j)$ | the probability of the object $c_x$ will locate at $l_j$ at the moment $t_k$, an element of $\vec{p}(c_x, t_k)$ |
| $c_x$ | an object, $c_x \in$ the set of objects $C$ | $\vec{p}'(c_x, t_k)$ | the result of sorting the elements of $\vec{p}(c_x, t_k)$ in descending order |
| $l_i$ | a location, $l_i \in$ the set of locations $L$; $l_{tk}$ means the time-specific location at $t_k$ | $p(l'_j)$ | an element of $\vec{p}'(c_x, t_k)$, $p(l'_j) \geq p(l''_j) \geq ... \geq p(l''_{|L|})$ |
| $(l_k, t_k)$ | a spatiotemporal unit | $m, \hat{n}, \hat{n}()$ | the estimated amount of searches needed |
| $tr(c_x, d_j)$ | a trajectory, $\{(l_{j1}, t_{j1}), (l_{j2}, t_{j2}), ... \mid c_x, d_j\}$ | $m, n$ | the actual amount of searches |
| TR | the set of trajectories, $TR = \{tr(c_x, d_j)\}$ | ALT | All Searching at the Last Time |
| TR' | the set of history trajectories, i.e., before the testing day | IPM | Intermediate Searching at a Parametric Moment |
| $s(c_x, l_k, t_k)$ | the result of a spatiotemporal search, 1 for success, 0 for failure | IEM | Intermediate Searching at an Estimated Moment |
| $p(l_i, l_j)\Delta t$ | the probability of an object that moves from one location to another per a period of time $\Delta t$: $t_p \rightarrow t_x$ | IHMs | Intermediate Searching at Heuristic Moments |
| $TPM_{|L|\times|L|}$ | a transition probability matrix, TPM for short; an element is $p(l_i, l_j)\Delta t, p(l_i, l_j)$ for short | IHUs | Intermediate Searching at Heuristic Spatiotemporal Units |

Table 1: Notations
First of all, we introduce why and how first-order Markov model is applied to predict an object’s future time-specific location. Actually, the mobility prediction algorithm is not the focus and contribution of this paper. Other algorithms (e.g., deep learning based algorithms [23][24]) are not adopted in this work because: first, they cannot be applied to time-specific mobility prediction directly (see Section 5 Related Work); second, spatiotemporal search involves multiple rounds of “predict and search”, the training of deep learning model may cost too much; third, the known locations are very sparse, especially at the beginning, that makes deep learning unable to fully realize its power, whereas Markov model can reach good performance in this situation [22].

Let a transition probability matrix (TPM for short) consists of the probabilities of an object that moves from one location to another after a period of time $\Delta t$, i.e.,

$$TPM_{\Delta t}^{[L_x \times L_y]} = \{p(l_i, l_j) | \Delta t\}$$

$$p(l_i, l_j) | \Delta t = \frac{\# tr() \text{ with } (l_i, t_p) \& (l_j, t_p + \Delta t)}{\# tr() \text{ with } (l_i, t_p)}$$

Then $TPM_{\Delta t}^{[L_x \times L_y]}$ can be used to predict any object’s location $\Delta t = t_x - t_p$ time later. For instance, a car was present at a $l_{45}$ at 8:00 am today, now is 8:30 am, its current location will most probably be $l_{46}$, knowing that $p(l_{45}, l_{46}) | 30\text{min} = 0.51$, and second probably be $l_{47}$, knowing that $p(l_{45}, l_{47}) | 30\text{min} = 0.32$, and so on until all locations are enumerated exhaustively.

To show how to implement the calculation of Equation 4 and those others, toy examples with 5 trajectories, 4 locations and 3 moments are presented, whose dataset is shown in Table 2. Equations of toy examples are not numbered for distinguishing.

### Table 2: Dataset of Toy Examples

| $tr(c_1, d_1)$ | $l_1$ | $l_2$ | $l_3$ |
| $tr(c_1, d_2)$ | $l_2$ | $l_2$ | $l_3$ |
| $tr(c_1, d_3)$ | $l_2$ | $l_1$ | $l_1$ |
| $tr(c_2, d_1)$ | $l_3$ | $l_4$ | $l_4$ |
| $tr(c_2, d_2)$ | $l_2$ | $l_4$ | $l_3$ |

Let $t_p = t_1$, $\Delta t$ is from $t_1$ to $t_3$. Actually we need to calculate $4^4 = 16$ probabilities in TPM, here we only demonstrate how to calculate $p(l_2, l_3)$ according to Equation 4,

$$p(l_2, l_3) | \Delta t = \frac{\# tr() \text{ with } (l_2, t_1) \& (l_3, t_3)}{\# tr() \text{ with } (l_2, t_1)} = \frac{|\{tr(c_1, d_2), tr(c_2, d_2)\}|}{|\{tr(c_1, d_2), tr(c_1, d_3), tr(c_2, d_2)\}|} = \frac{2}{3}$$

The meaning of “$\#$” is “the number of”, thus this example means to get the probability that objects transit from location $l_2$ to location $l_3$ after 2 moments, we statistically count the frequency of historical trajectories that happened to meet these two conditions ($l_2$ at $t_1$ AND $l_3$ at $t_1$ as the numerator, $l_2$ at $t_1$ as the denominator) respectively, and divide them. And so on, until every $p(l_i, l_j)$ are calculated, we get a TPM like this:
3.1 All Searching at the Last Time (ALT)

A straightforward strategy is to execute all searches at the last time. Knowing \((l_t, t_p)\) of object \(c_x\) day \(d_x\)’s trajectory \(tr(c_x, d_x)\), the mobility prediction algorithm (e.g., first-order Markov model) yields descending sorted probabilities \(\text{\(\arrowvert\)}(c_x, t_k)\) in descending order, saved as \(\arrowvert(c_x, t_k) = (p(l'_1), p(l'_2), ..., p(l_{|L|}))\) (in which \(p(l'_1) \geq p(l'_2) \geq \cdots \geq p(l_{|L|})\)), then compute Equation 7. That’s because the location with the largest probability only needs one search and hit, but the location with the second largest probability will be searched and hit after one failed search, and so on. According to the Law of Large Numbers (under the condition that training and testing data are abundant and identically distributed), this is also the estimated amount of searches needed to find \(c_x\) at \(t_k\) in \(d_x\) (note that the ground truth of \(d_x\)’s trajectory is only for performance evaluation, and remains unknown in the process of real world spatiotemporal searching).

\[
\hat{n}(\text{\(\arrowvert\)}(c_x, t_k)) = \sum_{j=1}^{L} j \times \text{\(\arrowvert\)}(c_x, t_k)[j] = \sum_{j=1}^{L} j \times p(l'_j)\]  

(7)

Continuing the example, now we begin to search an object \(c_x\) at \(t_1\) that knowing its location \(l_2\) at \(t_1\). Since the second row of \(\text{\(\arrowvert\)}(c_x, t_2) = (p(l_1) = 1/3, p(l_2) = 0, p(l_3) = 2/3, p(l_4) = 0)\), it is sorted to \(\text{\(\arrowvert\)}(c_x, t_2) = (p(l_3) = 2/3, p(l_1) = 1/3, p(l_2) = 0, p(l_4) = 0). According to Equation 7,

\[
\hat{n}(\text{\(\arrowvert\)}(c_x, t_2)) = 1 \times 2/3 + 2 \times 1/3 + 3 \times 0 + 4 \times 0 = 4/3
\]

That is to say, it is expected that we can find this object with 4/3 searches, because we search \(l_3\) first, which have 2/3 probability to find it and cost 1 searches, if not, then search \(l_1\), which have 1/3 probability to succeed and cost 2 cumulative searches, then \(l_2\) and \(l_4\).

The strategy of spatiotemporal searching aims to determine the location and the moment of each search in \((s(), s(), ..., s(c_x, l_1, t_2))\). According to Definition 1 (spatiotemporal search), a search \(s(c_x, l, t_k)\) has three parameters: object, location, and moment, and we note that traces of object \(c_y\) may help to track object \(c_x\), if \(c_y\) and \(c_x\) are correlated, thus strictly, which object to search is also a parameter needs to be determined, but in this paper we reasonably assume that the target object \(c_x\)’s information can help most and other objects’ can be ignored. We introduce various strategies, including several baselines and a proposed one, detailed as follows.

3.1 All Searching at the Last Time (ALT)

If the denominator = 0, we reset the probability to \(1/4\) with the simple assumption that it will equally distribute at all 4 locations, as shown in the last row of the above TPM.

Generally speaking, knowing more locations the object has passed through will bring higher prediction accuracy, however, that depends on sufficient data. If there are not enough matched history trajectories for training, the high-order Markov model will be inaccurate because of low signal-to-noise ratio or over-fitting. As a result, we only take second-order Markov model into consideration. Its TPM (transition from \(l_i\) to \(l_j\) through \(l_h\)) is calculated as:

\[
\text{TPM}_{t1\rightarrow t3} = \begin{pmatrix}
0 & 0 & 1 & 0 \\
1/3 & 0 & 2/3 & 0 \\
0 & 0 & 0 & 1 \\
1/4 & 1/4 & 1/4 & 1/4
\end{pmatrix}
\]

Similarly to the toy example of Equation 4, according to Equation 6, we can calculate, if knowing the object at location \(l_4\) at \(t_2\), what the transition probability from \(l_2\) to \(l_3\):

\[
p(l_2, l_3) = \frac{\# tr() with (l_2, t_2) & (l_3, t_3)}{\# tr() with (l_2, t_2) & (l_3, t_3)}
\]

(6)

After every \(p(l_i, l_j)\) are calculated, we get a TPM like this:

\[
\text{TPM}_{t1\rightarrow t3, (t4,t2)} = \begin{pmatrix}
1/4 & 1/4 & 1/4 & 1/4 \\
0 & 0 & 0 & 1 \\
1/4 & 1/4 & 1/4 & 1/4
\end{pmatrix}
\]

Suppose the result of the mobility prediction algorithm is in the form of a vector (e.g., a row of TPM), \(\text{\(\arrowvert\)}(c_x, t_k) = (p(l_1), p(l_2), ..., p(l_{|L|}))\), in which each element denotes the probability of the object \(c_x\) will locate at \(l_j\) at the moment \(t_k\). We can calculate the average amount of searches needed to find \(c_x\) at \(t_k\) in TR’ like this: sort the elements of \(\text{\(\arrowvert\)}(c_x, t_k)\) in descending order, saved as \(\text{\(\arrowvert\)}(c_x, t_k) = (p(l'_1), p(l'_2), ..., p(l_{|L|}))\) (in which \(p(l'_1) \geq p(l'_2) \geq \cdots \geq p(l_{|L|})\)), then compute Equation 7. That’s because the location with the largest probability only needs one search and hit, but the location with the second largest probability will be searched and hit after one failed search, and so on. According to the Law of Large Numbers (under the condition that training and testing data are abundant and identically distributed), this is also the estimated amount of searches needed to find \(c_x\) at \(t_k\) in \(d_x\) (note that the ground truth of \(d_x\)’s trajectory is only for performance evaluation, and remains unknown in the process of real world spatiotemporal searching).

\[
\hat{n}(\text{\(\arrowvert\)}(c_x, t_k)) = \sum_{j=1}^{L} j \times \text{\(\arrowvert\)}(c_x, t_k)[j] = \sum_{j=1}^{L} j \times p(l'_j)
\]

(7)
3.2 Intermediate Searching at a Parametric Moment (IPM)

The steps of Intermediate Searching at a Parametric Moment (IPM) strategy are:

1. a parametric moment is specified, denoted as \( t_k \in (t_p, t_x] \);
2. knowing \((l_{tp}, t_p)\) of \( tr(c_x, d_x)\), the mobility prediction algorithm (e.g., first-order Markov model) yields descending sorted probabilities \( \vec{p}'(c_x, t_k) \);
3. then execute searches at \( t_k \) from locations with higher probability to lower, until the object is found, i.e., \( s(c_x, l'_m, t_k) = 1 \); (actually Step 2 and 3 adheres ALT strategy)
4. knowing \((l_{tp}, t_p)\) and \((l'_m, t_k)\) of \( tr(c_x, d_x)\), the mobility prediction algorithm (e.g., second-order Markov model, or first-order Markov model with updated location) yields descending sorted probabilities \( \vec{p}'(c_x, t_k) \);
5. finally execute searches at \( t_x \), until the object is found, i.e., \( s(c_x, l'_m, t_x) = 1 \), and \( m + n \) is the actual amount of searches that found \( c_x \) at \( t_x \) in \( d_x \).

In addition, the model can be trained offline. The time complexity of offline training is \( O(n) \), and the time complexity of online testing is also \( O(n) \).

3.3 Intermediate Searching at an Estimated Moment (IEM)

Intermediate Searching at an Estimated Moment (IEM) strategy is the same as IPM strategy except Step 1. It estimates the optimal moment that can minimize the total amount of searches. The optimal moment \( t_{opt} \) is found by traversing all moments from \( t_p \) to \( t_x \). For each moment \( t_k \in (t_p, t_x] \), simulated IPM strategy is rehearsed. Since the actual amount of searches in simulated IPM strategy remains unknown, we take estimated amount of searches instead, i.e., \( \hat{n} \) and \( \hat{t} \) are computed with Equation 7, therefore can avoid the costs of searching from massive camera records in this step. Note the object’s location at \( t_k \) remains unknown, but we know \( \vec{p}(c_x, t_k) = (p(l_1), p(l_2), ..., p(l_{|I|})) \), thus \( \hat{n} \) is calculated as Equation 10:

\[
\begin{align*}
  t_{opt} &= \arg \min_{t_k \in (t_p, t_x]} (\hat{n} + \hat{t} | \text{IPM}(t_k)) \\
  \hat{n} &= \sum_{j=1}^{|I|} p(l_j) \times \hat{t} \left( \frac{\vec{p}(c_x, t_j)}{\text{IPM}(t_k)} | \text{tr}(c_x, d_x) \text{ with } (l_{tp}, t_p) \right) \\
  \hat{t} &= \sum_{j=1}^{|I|} p(l_j) \times \hat{t} \left( \frac{\vec{p}(c_x, t_j)}{\text{IPM}(t_k)} | \text{tr}(c_x, d_x) \text{ with } (l_{tp}, t_p) \right) \\%
\end{align*}
\]

Then IPM strategy is truly performed using \( t_{opt} \) as the parametric moment.

Continuing the toy examples, when we want to estimate the amount of searches assuming that we knowing an extra location between the start moment and the end moment, but the extra location is unknown during estimating, we need to adapt Equation 7 to Equation 10, in which \( p(l_j) \) is the probability of the extra location, and get the weighted sum. For example, we begin to search an object \( c_x \) at \( t_5 \) that knowing its location \( l_2 \) at \( t_1 \). To calculate Equation 10, we first prepare \( \text{TPM}_{t1→t2}, \text{TPM}_{t1→t3,(t1,t2)}, \text{TPM}_{t1→t3,(t2,t3)}, \text{TPM}_{t1→t3,(t3,t2)}, \text{TPM}_{t1→t3,(t3,t1)}, \text{TPM}_{t1→t3,(t2,t3)} \), while actually we only need the second row of these TPMs, respectively as:

\[
\begin{align*}
  \vec{p}(c_x, t_2) &= (p(l_1) = 1/3, p(l_2) = 1/3, p(l_3) = 0, p(l_4) = 1/3) \\
  \vec{p}(c_x, t_3) &= (p(l_1) = 1, p(l_2) = 0, p(l_3) = 0, p(l_4) = 0) \\
  \hat{n} &= 1 \text{ by Equation 7} \\
  \hat{t} &= 1 \text{ by Equation 7} \\
  \vec{p}(c_x, t_1) &= (p(l_1) = 0, p(l_2) = 0, p(l_3) = 1, p(l_4) = 0) \\
  \hat{n} &= 1 \text{ by Equation 7} \\
  \hat{t} &= 2 \text{ by Equation 7} \\
  \vec{p}(c_x, t_2) &= (p(l_1) = 1/4, p(l_2) = 1/4, p(l_3) = 1/4, p(l_4) = 1/4) \\
  \hat{n} &= 5/2 \text{ by Equation 7} \\
  \hat{t} &= 1 \text{ by Equation 7} \\
\end{align*}
\]

According to Equation 10, we get \( \hat{n} = 1/3 \times 1 + 1/3 \times 1 + 0 \times 5/2 + 1/3 \times 1 = 1 \), which means if we know the object’s location at \( t_2 \), we only need one more search to find it at \( t_3 \), probably. In addition, the model can be trained offline. The time complexity of offline training is \( O(n^2) \), and the time complexity of online testing is \( O(n) \).

3.4 Intermediate Searching at Heuristic Moments (IHMs)

Intermediate Searching at Heuristic Moments (IHMs) strategy is shown as the pseudocode Algorithm 1. The basic idea is in a greedy way: each step we spend the least searches to get the object’s latest location; iterate the step until we get the object’s current location. That’s because the latest appearance is the most helpful for finding it at the current moment, while the amount of searches needed to find it again after its previous appearance will also increase as the timespan becomes larger. Therefore, an indicator (shown as Equation 11) is defined as the ratio of the needed amount (estimated by Equation 7) to the timespan, and each step we search locations in descending order at a heuristics moment which can minimize this indicator, as Line 7 and
8 of the pseudocode shows.

\[
\text{indicator}(t_k) = \frac{\hat{h}(\bar{p}(c_x, t_k))}{t_k - t_{\text{cur}}} \tag{11}
\]

Algorithm 1. IHMs of Spatiotemporal Searches

Input: \(c_x, d_x, t_x, l_{tp}, t_p, \text{TR}'\) before \(d_x\), \(s(c_x, l_t, t_k)\) in \(d_x\)

Output: \(l_{\text{cur}}, \text{amount}\)

1. initiate \(l_{\text{cur}} = l_{tp}, t_{\text{cur}} = t_p, \text{amount} = 0\);
2. partially known \(t(0, t_{\text{cur}}) = \emptyset\);
3. \text{while} \(t_{\text{cur}} < t_x\)
4. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
5. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
6. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
7. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
8. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
9. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
10. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
11. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
12. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
13. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
14. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);

In addition, the model can be trained offline. The time complexity of offline training is \(O(n^3)\), and the time complexity of online testing is \(O(n^2)\).

3.5 Intermediate Searching at Heuristic Spatiotemporal Units (IHUs)

Intermediate Searching at Heuristic Spatiotemporal Units (IHUs) strategy is shown as the pseudocode Algorithm 2. The basic idea is also in a greedy way similar to IHMs, but with two different aspects: 1) each step we search at a heuristic spatiotemporal unit (instead of searching locations in descending order at a heuristic moment) which can minimize an indicator, and update the mobility prediction result no matter the object is found or not in this unit; 2) the indicator uses \(1/\bar{p}(c_x, t_k)\) (instead of \(h(\bar{p}(c_x, t_k))\)) to estimate the amount of searches needed to find \(c_x\) at \(l_j, t_k\) in \(d_x\), e.g., if the probability of finding an object is 1/3, then we can expect that we will find the object by executing 3 searches.

\[
\text{indicator}(l_j, t_k) = \frac{1}{\bar{p}(c_x, t_k)(l_j)} \tag{12}
\]

Algorithm 2. IHUs of Spatiotemporal Searches

Input: \(c_x, d_x, t_x, l_{tp}, t_p, \text{TR}'\) before \(d_x\), \(s(c_x, l_t, t_k)\) in \(d_x\)

Output: \(l_{\text{cur}}, \text{amount}\)

1. initiate \(l_{\text{cur}} = l_{tp}, t_{\text{cur}} = t_p, \text{amount} = 0\);
2. partially known \(t(0, t_{\text{cur}}) = \emptyset\);
3. \text{while} \(t_{\text{cur}} < t_x\)
4. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
5. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
6. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
7. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
8. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
9. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
10. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
11. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
12. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
13. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);
14. \(t(0, t_{\text{cur}}) = t(p, t_{\text{cur}}) = \emptyset\);

In addition, the model cannot be trained offline. We need to rematch the training data after each search is completed, so the time complexity of its online training and testing is \(O(n^3)\).
4 EVALUATION

4.1 Dataset Description and Preprocessing

The dataset is derived from the trajectories of 19 thousands taxis in Chengdu, China, August 2014. We choose a rectangular urban area about 30 km x 30 km and time from 7:00 to 21:59 of 17 days, which makes the data scoped and denser. A raw trajectory of a day of a car is composed with thousands of GPS waypoints, each of which indicates the car’s identification, the latitude and longitude, the date and time, as shown in Fig. 3. As mentioned before, the urban area is discretized into locations with each one covers about 1 km x 1 km (900 locations in total), and the time is discretized into moments with each one lasts 1 minute (900 moments in total). Then these raw trajectories are discretized accordingly, as shown in Fig. 4. If a car passes through several locations in a moment, we only preserve the first location of that moment.

We assume objects in the same dataset are with the same spatiotemporal characteristics so that normal history trajectories can help the mobility prediction of a target object. Objects with abnormal trajectories may be hard to search (i.e., with larger amount of searches) but still can be searched. To this end, it is justified that using taxi traces for experiments. The granularity of spatiotemporal units is a priori setting, which depends on factors like the granularity of data sensing and application tasks, and affects the cost of a spatiotemporal search. We will study this issue in the future.

One preprocessing operation worth to mention is how to deal with data missing, i.e., some time-specific locations are unknown in a trajectory, because of GPS signal loss or out of the scope. There are two situations: 1) the unknown locations are at the start or end of the trajectory; 2) the unknown locations are between two known locations. For the first situation, if the time with unknown locations lasts less or equal than 5 moments, the car is deemed at the same location as recent moment with known location; otherwise, the trajectory is discarded. For the second situation, if the time with unknown locations lasts less or equal than 10 moments, and the distance of two known locations are no longer than 15 km threshold, the car’s locations are interpolated according to uniform rectilinear movement; otherwise, the trajectory is discarded. Finally about 70% trajectories are remained, of which 0.3% time-specific locations are missing but complemented.

| Car ID | Longitude  | Latitude  | Date and Time    |
|--------|------------|-----------|------------------|
| 1      | 104.039163 | 30.597572 | 2014/8/4 15:15:52|
| 1      | 104.126565 | 30.599733 | 2014/8/4 07:03:04|
| 1      | 104.042833 | 30.599851 | 2014/8/4 15:13:33|
| 1      | 104.127154 | 30.600009 | 2014/8/4 07:03:34|
| 1      | 104.126921 | 30.600271 | 2014/8/4 07:03:56|
| ...    | ...        | ...       | ...              |
| 1      | 104.043161 | 30.600716 | 2014/8/4 15:15:07|

Fig. 3. Raw trajectories with GPS waypoints.

| Object | Day   | Location | Moment |
|--------|-------|----------|--------|
| c₁     | 2014/8/4 | l₅ | 7:00  |
|        |        | l₆ | 7:01  |
|        |        | ?(l₄₇) | 7:02 |
|        |        | l₈ | 7:03  |
|        |        | ... | ...   |
|        |        | l₉₂ | 21:58 |
|        |        | ?(l₉₂) | 21:59 |

Fig. 4. Discretized and complemented trajectories.

The 99,265 trajectories of former 14 days are used as training data, i.e., TR’. The 21,963 trajectories of latter 3 days are testing data, i.e., to run a test, a trajectory is picked from them as the ground truth of tr(cₓ, dₓ), and in each experiment, we will run 21,963 tests exhaustively and average the results.
4.2 Performance of Mobility Prediction

Usually the Top-N accuracy is adopted as the performance metric of mobility prediction algorithms. However, in this paper, the mission of mobility prediction is to guide the least spatiotemporal search, and to estimate the amount of searches needed. Top-N accuracy is not a proper metric of this mission. For instance, given two mobility prediction results like \((0.5(l_1), 0.3(l_2), 0.2(l_3))\) and \((0.5(l_1), 0.1(l_2), 0.4(l_3))\), as well as the ground truth \(l_3\), their Top-1 accuracy are the same (i.e., 0), but the amounts of searches are different (i.e., 3 and 2 respectively). Therefore, we choose directly the amount of searches that actually executed as the metric of mobility prediction algorithms.

Beside the second-order Markov model that may outperform the first-order Markov model, another plausible improvement is the time-specific first-order Markov model, whose TPM is trained from only trajectories with the same specified start moment, as Equation 13 and 14. Likewise, the time-specific second-order Markov model is also tried.

\[
TPM^{[T] \times [T]}_{\Delta t, t_p} = \{p(l_i, l_j)|\Delta t, t_p) \}
\]

\[
p(l_i, l_j)|\Delta t, t_p = \frac{\# tr() \ with \ (l_i, t_p) \ & \ (l_j, t_p + \Delta t)}{\# tr() \ with \ (l_i, t_p)}, \quad tr() \in TR'
\]

From Fig. 5, we can see that although the other three models are better than the first-order Markov model in terms of Top-5 accuracy, the first-order Markov model outperforms the other three in terms of the amount of searches. That is to say, the correlation between Top-N accuracy and the amount of searches is not strong. Fundamentally, Top-N accuracy is a metric of classification, while the amount of searches is a metric of ranking. If the performance is measured with the amount of searches, its impact to the total cost is straightforward, i.e., better mobility prediction algorithm will result lower total cost.

Fig. 5. Different performance metrics of mobility prediction algorithms. (a) (b) are the Top-5 accuracy (higher is better); (c) (d) are the amount of searches (lower is better).
The number of trajectories passing through given locations reflects the amount of data that can be used to train a mobility prediction model. Fig. 6 shows the statistics of trajectories passing through different locations in the training data. For mobility prediction based on the first-order Markov model, most locations have lots of (about one thousand in average) matched history trajectories, while for the other three models, most locations have very few (dozens in average) trajectories. Therefore, since there are not enough matched history trajectories for training, time-specific models or high-order Markov models will be inaccurate because of low signal-to-noise ratio or over-fitting.

**Fig. 6. The statistics of trajectories passing through different locations.** The first deep blue bar means there are about 100 locations (out of total 900 locations) that are passed through by 0 to 100 trajectories, and etc. For second-order Markov models, the first location is fixed to a very busy location \( l_{495} \) and the moment of the second locations is 5 minutes later. For time-specific models, the moment is fixed to 8:00.

The mobility prediction algorithm is also in charge of estimating the amount of searches needed, as in IEM or IHMs strategy. Here we show how good the estimation is provided by the first-order Markov model in Fig. 7. When the transition time (aka \( \Delta t = \text{end moment} - \text{start moment} \)) is shorter than 35 min, the estimated amount of searches almost coincides with the actual amount, but when the transition time becomes longer, the estimated amount is obviously smaller than the actual amount. That means the effectiveness of the first-order Markov model is attenuated in long-term mobility prediction, and as a consequence, the strategies who rely on it may perform worse. The phenomenon in Fig. 10 also verifies this opinion.

**Fig. 7. The amount of searches estimated by the first-order Markov model.**

The reason why first-order Markov model is attenuated when transition time becomes long lies in the predictability of trajectories. Fig. 8 to show the distribution of end locations after different transition time (the start location is fixed to a very busy location \( l_{495} \)). It is clear that the end location distribution is more dispersed after a longer transition time. This also makes the data sparsity problem even worse.
Fig. 8. The distribution of end locations after different transition time (start location: $l_{495}$).

4.3 Results of Different Searching Strategies

Five searching strategies that described in Section 3, i.e., All Searching at the Last Time (ALT), Intermediate Searching at a Parametric Moment (IPM), Intermediate Searching at an Estimated Moment (IEM), Intermediate Searching at Heuristic Moments (IHMs), and Intermediate Searching at Heuristic Spatiotemporal Units (IHUs), are compared under different experimental settings. The first setting is fixing $\Delta t$ to 30 min, and assigning the start moment (aka past moment) from 7:00 to 21:30 at 1-minute intervals; the second setting is fixing the start moment to 8:00, and assigning the end moment (aka current moment) from 8:05 to 9:20 at 5-minute intervals; the third setting is fixing the start moment to 8:00, and the end moment to 8:30, but the start locations are divided into four groups according to their busyness. Since IPM needs to specified a parameter beforehand, here we simply specify it right in the middle of the time, i.e., $t_p + \Delta t/2$, and discuss variable parameter in Subsection 4.5.

The result of the first setting is shown as Fig. 9. Apparently, IHMs outperforms other strategies at all start moments. The second strategy is IHUs. The bottom three, i.e., ALT, IPM, and IEM, are almost equivalent in terms of the amount of searches. They need respectively about 33, 40, 50 spatiotemporal searches to find a car disappear at half an hour earlier. We can also discover that different start moments influence the searching difficulty, e.g., the easiest start moments are near 7:00, 14:10, or 18:00, while the hardest start moments are near 8:30, 12:00, 16:30, or 21:30. This means in general it is more difficult to track a car during rush hours.

Fig. 9. Results of different searching strategies (fix $\Delta t$).
The result of the second setting is shown as Fig. 10. Again IHMs outperforms other strategies at all end moment. Longer \( \Delta t \) leads more costs (i.e., the amount of searches) to find a car, which is consistent with common sense. The superiority of IHMs reaches the largest when \( \Delta t = 35 \) min. In particular, when \( \Delta t < 35 \) min, the rank of costs is IHMs < IHUs < IPM ≈ IEM < ALT; when \( 35 \) min \( \leq \Delta t < 60 \) min, the rank is IHMs < IEM ≈ ALT < IHUs < IPM; and when \( \Delta t \geq 60 \) min, all strategies perform similarly except IPM. That reveals IPM has no ability to adapt different searching difficulty. ALT as the simplest strategy, the other strategies intend to perform better than it, however, that is not always true. The polylines in Fig. 9 show the difference between the other strategies and ALT (some details in Subsection 4.4). Thus for the question whether the costs spent in earlier searching can be compensated by the costs saved in later searching, the answer is depending on primary the searching difficulty and secondary the intermediate searching strategy. If a car is out of sight for a long time, intermediate searching seems no help; while if a car is out of sight for a short time, an efficient searching strategy can save up to 1/3 costs.

![Fig. 10. Results of different searching strategies (fix \( t_p \)).](image)

The result of the third setting is shown as Fig. 11. Still IHMs outperforms other strategies at all start locations. The busyness of a location is characterized as how many trajectories pass through it. We divide all 900 locations into four groups according to their busyness as shown in Fig. 11 (b). We can see that busier start location leads more costs to track a car.

Since IHMs performs the best under all three settings, we let it to be the proposed one in this paper. The following subsections make detailed comparisons of IHMs and other strategies respectively, in order to explain why IHMs can save the amount of searches most.

### 4.4 Detailed Comparison of IHMs vs. ALT

Remind that IHMs’ indicator is the ratio of the estimated amount of searches to the timespan (according to Equation 11), thus the inclination angle of the green curve in Fig. 7 indicates this ratio, as show in Fig. 12. Note that the actual ratio in Fig. 12 is the average of all testing trajectories, so for a particular testing trajectory, the actual ratio adheres to the same trend but with small fluctuations. Since ALT always executes searches at the last time, the amount of searches is equal to the area of the

| Busyness of a location | Range of \( \# \text{tr}() \) | Ave. of \( \# \text{tr}() \) | # locations |
|------------------------|--------------------------|-----------------|------------|
| low                    | [0, 100)                 | 14              | 665        |
| medium                 | [100, 300)               | 182             | 118        |
| high                   | [300, 800)               | 512             | 100        |
| extreme                | [800, \( \infty \))      | 1007            | 17         |

![Fig. 11. Results of different searching strategies (fix \( t_p \) and \( \Delta t \)).](image)
light red rectangle in Fig. 12. Whereas IHMs executes searches at the moment that has smallest estimated ratio, the amount of searches is equal to the area of the deep red rectangle in Fig. 12. According to the trend of the indicator, when $\Delta t$ is shorter than about 4 min or longer than about 60 min, IHMs will be performed the same as ALT (because the last time has the smallest ratio); otherwise, IHMs will executes searches by multiple steps, of which each step spans about 4 minutes (because a timespan of 4 min has the smallest ratio). The area difference between the deep red rectangle and the light red rectangle reaches the largest when $\Delta t = 35 \text{ min}$ apparently.

![Fig. 12. The trend of IHMs' indicator.](image)

**4.5 Detailed Comparison of IHMs vs. IPM and IEM**

According to the above subsection, if $\Delta t$ is between 4 min and 60 min, we can benefit from searches at intermediate moments. Here we fix $\Delta t = 30 \text{ min}$, set start moment to 8:00, 12:00, 16:00, and 20:00 respectively, and compare the three strategies with searches at intermediate moment/s, i.e., IHMs, IPM, and IEM. Fig. 13 shows the result. Note that the parameter of IPM traverses all moments from $t_0$ to $t_\times$. Different intermediate searching moment leads different performance, where the best is reached at $t_p + 4 \text{ min}$ or $t_\times - 4 \text{ min}$. Apparently that is because 4 min has the lowest cost-timespan ratio. If only one intermediate moment is available, IEM can find it correctly, which is the moment that lets IPM reach its lower bound. IHMs outstands because it can exploit about $30 \text{ min} / 4 \text{ min} \approx 7$ intermediate moments, while the other two only can exploit one intermediate moment.

![Fig. 13. Results of IPM (variable parameter), IEM, and IHMs.](image)
4.6 Detailed Comparison of IHMs vs. IHUs

It is unexpected that IHUs cannot perform as good as IHMs. IHUs tries to figure out which spatiotemporal unit is the best to search each next step, while IHMs tries to figure out which moment is the best to search each next step. However, if we go deep into their searching processes, we can find the reasons: 1) since an element in $\bar{p}(c_x, t_k)$ is only comparable within the specific moment $t_k$, it does not make sense to use $1/ \bar{p}(c_x, t_k)$ to estimate and compare the amount of searches among different moments; 2) staying in the same moment to search locations one by one can fully utilize the information gained from previous failed searches, e.g., at a moment $t_k$ with appearing probabilities $\{0.5(l_1), 0.3(l_2), 0.2(l_3)\}$, the first search at $(l_1, t_k)$ fails will let us know about $t_k$ with $\{0(l_1), 0.6(l_2), 0.4(l_3)\}$, but not so much information about other moments. Fig. 14 shows the order of searched units by IHMs and IHUs in a special case. On average, $1/4$ amount of searches by IHMs hits, while only $1/8$ amount of searches by IHUs hits. Obviously hit searches bring more information than failed ones. That is why IHMs is more efficient than IHUs.

![Fig. 14. The order of searched units by IHMs and IHUs in a special case.](image)

5 RELATED WORK

Tracking an object that is out of sight for a short time is similar to searching a missing object. In the operational research community, Koopman [1] and Stone [2] etc. have developed the theory of optimal search from backgrounds of military, rescue, and law enforcement. For example, Bayesian search theory is being used to locate the remains of Malaysia Airlines Flight 370 [3]. It first formulates reasonable hypotheses about where the object is, constructs a probability density function accordingly, then start to search from the locations of highest probability to of lower probability, and revises the probabilities continuously during the search. ALT (All Searching at the Last Time) can be deemed as an existing approach that applies Bayesian search theory. New challenges arise with the novel problem defined in this paper. First, the traditional theory mostly handles a one-off planning problem that with little or low rate of knowledge update, while in our problem, the output of each search can immediately affect decision making of the next search. Second, the traditional theory merely involves spatial searches at
current moment, while in our problem, we can adopt spatiotemporal searches, which can search at intermediate moments with the help of digital records, e.g., videos, photos, or check-ins [14]. Third, the traditional theory lacks the perspective from real world data (e.g., prefers theoretical analysis and simulation), while in our work, we use real world data to train the mobility prediction model, to estimate the costs, and to evaluate different strategies. In addition, we focus on different searching strategies under the same mobility prediction algorithm (and provide an evidence that searching at intermediate moments can save cost), while existing work focus on different mobility prediction algorithms under the same searching strategy, i.e., ALT.

The idea of gathering more past locations to improve the prediction of future locations seems to be similar with active learning [4, 5, 6]. The difference lies in that: in active learning, we know features of all samples and labels of a few samples, and to decide which unlabeled sample to be labeled next; while in this paper, for an unlabeled sample we lack its features, and to decide which feature to be gathered next. Furthermore, in this paper we have to do much more work other than traditional feature selection, because on the one hand, we assume we cannot query directly an object’s past locations, thus past locations cannot be candidate features to be selected; on the other hand, if we regard “whether an object was at a location at a moment” as a feature, the features will be highly multi-collinear and unbalanced.

Few work dedicatedly focus on time-specific mobility prediction. Nevertheless, both destination prediction [7, 8, 9, 15], next-place prediction [10, 11], and trajectory compression [25] can be extended by combining time prediction to handle time-specific mobility prediction. In our previous work [12], we proposed a Markov based time-specific mobility prediction algorithm, which outperforms the next-place based one. It considers multiple factors including personal habit, weekday similarity, and collective behavior [13]. In this paper, we exploit several variants of this algorithm to take the past moment or past locations of the object into account.

6 CONCLUSIONS

Tracking a car or a person in a city is crucial for urban safety management. How can we complete the task with minimal number of spatiotemporal searches? This paper solves this problem by designing and comparing five searching strategies, and validating that the proposed one named IHMs (Intermediate Searching at Heuristic Moments) is most efficient. The basic idea of IHMs is in a greedy way: each step we figure out which moment is the best to search according to a heuristic indicator, then at that moment search locations one by one in descending order of predicted appearing probabilities, until a search hits; iterate this step until we get the object’s current location. The heuristic indicator is the ratio of the estimated amount of needed searches in order to find the object at the considered moment, to the timespan from the previous moment to the considered moment, called cost-timespan ratio for short. The cost-timespan ratio can resolve the conflict of spending the least searches to get the object’s latest location.

IHMs is one of the strategies with intermediate searching, but not all intermediate searching strategies can beat the simplest ALT (All Searching at the Last Time) strategy all the time. For the question whether the costs spent in earlier searching can be compensated by the costs saved in later searching, the answer is depending on primary the searching difficulty and secondary the intermediate searching strategy. Moreover, the time length of out of sight is the most influential factor to the spatiotemporal searching difficulty, along with other factors include the mobility prediction effectiveness, the start moment and the start location, e.g., it is more difficult to track a car at a busy location during rush hours. In general if a car is out of sight for a long time (≥ 60 minutes), intermediate searching seems no help; while if a car is out of sight for a short time, an efficient searching strategy can save up to 1/3 costs.

In the process of spatiotemporal searches, the mobility prediction algorithm has the mission to guide the least spatiotemporal search, and to estimate the amount of searches needed. We insist that a more proper metric for this mission is the amount of searches, rather than the Top-N accuracy. Under this new metric, the first-order Markov model outperforms its other three variants.

Spatiotemporal searching allows us to search at the past moments, which means we have to record the history of the past first of all. Nowadays video/image data are prevalent for recording in many applications. Thus the strategy proposed in this paper does not lose generality. We agree that tracking a person could be more challenging than tracking a car, however, the basic idea behind remains the same: use history trajectories to help mobility prediction and then guide the intermediate searching at heuristic moments. To apply our strategy in person tracking, we only need replace the car trajectories with person trajectories. The difference only lies in the difficulty of a spatiotemporal search, i.e., the unit cost of it.

In the future, we plan to conduct more theoretical analysis on this novel problem, design more efficient strategies, test on more datasets, and deduce whether they are still applicable if the unit cost of a spatiotemporal search is not a constant but a variable depending on the involved spatiotemporal unit. We will explore more effective mobility prediction algorithm, since it is also a key factor that influences the total costs of spatiotemporal searches, such as use fuzzy matching instead of current exact matching to train prediction model, or directly predict the ranking of locations instead of current probabilities of locations.
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