Research horizontally loaded pyramidal piles with compacted bottom and their calculation

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Abstract. The results of experimental and theoretical research in work features of a rammed pyramidal pile with compacted bottom for horizontal load in clayey soils are presented. The experimental cast-in situ pyramidal piles with compacted bottom were made at 2 experimental sites, composed of clayey soils. Static horizontal load tests were carried out. At each stage of loading, horizontal displacements were measured at the load application level and at a height of one meter from the top edge of the pile. The “horizontal load – pile head displacement” dependencies were obtained. It was found that the rotation of the pile in the soil occurs without bending, i.e. according to a rigid scheme, and also that compacted bottom prevents horizontal displacement of the pile bottom. Based on the analysis of the results of static tests of piles, a design scheme has been created, in which the pile is considered as a rigid rod buried in a multilayer linearly deformable base with a point of zero displacements at the lower end of the pile. In accordance with the design scheme, an analytical method for calculating a pyramidal pile with compacted bottom for horizontal load and bending moment has been developed. The calculation method allows to determine the horizontal displacement of the pile head in any section along the depth, the angle of rotation of the pile, bending moment and shear force along the length of the pile.

1. Introduction
Cast-in-situ pyramidal piles have been widely used in the USSR since the early 60s of the last century, and several geotechnical schools have carried out a comprehensive study including the development of their structures, immersion technology and methods for calculating their load-bearing capacity \cite{1, 2}. The specific load-bearing capacity of pyramidal piles was higher than that of prismatic piles due to the normal component of soil resistance on the inclined sides of the pile lateral surface. Such piles had a limited length (up to 3-4 m) and a cross-section (60×60 cm–80×80 cm). They were sunk into ground with a help of a rammer, similar to cast prismatic piles. They were used mainly in the form of strip foundations for self-supporting walls of brick and large-panel residential buildings and perceived only vertical load.

Since the beginning of the 80s, pyramid-shaped piles have been used under the columns of frame buildings and structures as single-pile foundations \cite{3}. The efficiency of a single-pile foundation in...
comparison with a pile group foundation lies in the absence of a grillage because it is not a soil bearing element and its volume is of up to 50% of the entire foundation volume.

Piles were made directly at a construction site by concrete casting in pyramid-shaped wells which were punched out by sinking a pyramidal stamp be means of a rammer. Then a reinforcement frame was installed in the well and a concrete mixture was compacted by a vibrator [4, 5].  

This technology made it possible to produce piles with cross-section dimensions of up to 120×120 cm and a length of up to 6-8 m under the columns with cross-section dimensions of up to 60×60 cm. In frame buildings, in addition to the vertical load, horizontal and moment loads always act on foundations. Therefore, an additional advantage of such piles, in the case of their use under the columns of frame buildings, is their increased resistance to horizontal load due to a large size of the cross-section in its upper part. At the same time, in some cases, if it is necessary to increase the load-bearing capacity of the pile due to the vertical load, crushed stone is rammed before concreting the pile into the bottom of the well. As a result, the area of compacting bottom with increased strength is formed in the pile base.

In addition, the technology for constructing foundations under the columns of frame buildings and structures was developed. It was called “foundations in rammed pits” [6, 7]. According to that technology, a well in the ground was formed by repeated dropping and lifting of a pyramidal rammer hung on the pile driver mast. Ramming of crushed stone into the bottom of the well was obligatory. The depth of such wells did not exceed 3 m.

In order to ensure the practical application of pyramidal piles as single-pile foundations, investigation was carried out and a number of methods were proposed for designing the piles under horizontal load. Such piles were considered as absolutely rigid [8, 9], in homogeneous by depth Winkler base, and as flexible in an elastic half-space [10]. However, the compacted bottom of pile base was not taken into account in those design methods.

In this regard, an urgent issue is to study the features of such piles work under horizontal loading and develop a method for the horizontal load.

2. Experimental technique

In order to assess the stress-strain state features of “horizontally loaded pyramidal pile– base” system, the construction of a design scheme and the development of a method for pile calculation under the horizontal load, field experiments were performed at 2 experimental sites, whose soil characteristics are presented in table 1.

The geological structure of site no. 1 is presented by deluvial-alluvial clays with carbonate inclusions of semi-solid consistency, natural humidity equal to \( W=0.20-0.025 \), density at natural humidity of \( \rho =1.91-1.95 \) t/m\(^3\), porosity coefficient \( e=0.7-0.8 \), index of liquidity \( l_i=0.30-0.25 \), internal friction angle \( \varphi=19 \) deg., cohesion \( c=0.021-0.022 \) MPa, total deformation modulus \( E=18.5-21 \) MPa. Site no. 2 is composed of quaternary deluvial deposits in the form of clays and loams with \( W=0.26-0.32, \rho=1.84-1.87 \) t/m\(^3\), \( e=0.88-0.923, l_i=0.12-0.35, \varphi=19-20 \) deg., \( c=0.030-0.052 \) MPa, \( E=10-18 \) MPa (see table 1).

| No. of sites | Density at natural humidity \( \rho \), t/m\(^3\) | Porosity coefficient \( e \), d.q | Adhesion \( c \), kPa | Angle of internal friction \( \varphi \), deg. | Deformation modulus \( E \), MPa | Index of liquidity \( l_i \), d.q. |
|--------------|-----------------------------|-----------------|----------------|------------------|----------------|-----------------|
| 1            | 1.91-1.95                  | 0.7-0.8         | 21-22         | 19               | 18.5-21.0      | 0.25-0.30       |
| 2            | 1.84-1.87                  | 0.88-0.923      | 30-52         | 19-20            | 10-18          | 0.12-0.35       |

As a working body for the experimental pile installation, hexagonal rammers (site no. 1) and rectangular rammers (site no. 2) with the dimensions given in table 2 were used.
In the process of static testing of the pile at each stage of loading, the horizontal pile displacements at the ground surface level and at a height of 1 m from the ground surface level were measured. The displacements at a height of 1 m were measured using displacement measuring device mounted on a reinforcing bar embedded in concrete into the pile body.

The horizontal load was applied in steps equal to 1/10 of the expected load-bearing capacity of the pile. Each subsequent stage was applied after stabilizing the horizontal displacement at the ground surface level from the previous load stage. Change in the horizontal displacement of no more than 0.1 mm in the last 15 minutes was taken as the stabilization criterion.

3. The experimental research results
The characteristics of the experimental piles and the main test results are given below (table 2 and figure 1). In this case, the load at which the pile displacement at the ground surface level was equal to 10 mm is taken as the maximum pile resistance to horizontal loading.

The “horizontal load – displacement” dependencies have a pronounced nonlinear character. The linear part is observed only under displacements at the ground surface level up to 4 mm. In this connection, the stabilization of displacements takes place in a sufficiently wide range of displacements (more than 20 mm), i.e. significantly larger than is allowed by regulations and norms. At the same time, due to large transverse dimensions of the piles and the availability of reinforcement, its bending is impossible, i.e. the pile works as rigid when rotating in soil without bending. It follows from this, that non-linearity of the “load-displacement” graph occurs due to the non-linear work of the soil base.

| No. of sites | No. of piles | Pile dimensions | Volume of rammed crushed stone, m³ | Pile resistance to horizontal load, kN at displacement of 10 mm | Angle of pile head turning at displacement of 10 mm, rad ∙ 10⁻³ | Depth of zero displacement point location at displacement of 10 mm, m |
|--------------|--------------|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1            | 1            | 112x130        | 2.8                            | 146                             | 19.2                             | 0.52                             |
|              | 2            | 112x130        | 2.8                            | 146                             | 19.2                             | 0.52                             |
|              | 1            | 112x130        | 2.8                            | 146                             | 19.2                             | 0.52                             |
| 2            | 3            | 110x110        | 3.0                            | 107                             | 5.5                              | 1.82                             |
|              |              | 60x60          |                                 |                                 |                                  |                                  |

According to the obtained horizontal displacements at the ground surface level and the angle of rotation of the experimental piles, the depth of the so-called zero displacement point (ZDP) was determined (see table 2). For piles without compacted bottom, ZDP was located at a depth of 0.52 m, while for piles with compacted bottom, it was located at a depth of up to 1.9 m, i.e. close to the pile bottom. Obviously, compacted bottom created a zone of increased strength in the area of the pile bottom, which prevented its horizontal displacement when the pile rotates under horizontal loading. This fact should be taken into account in the design scheme.
4. Calculation method

In order to construct the design scheme of the “horizontally loaded pile – base” system, it is necessary to adopt a soil base model that best corresponds to the physical laws of the pile work in soil. Almost all previously developed methods for calculating piles for horizontal loading were based on the theory of calculating a beam on an elastic base that implemented Winkler contact model, in which the bedding value was the design characteristic of the soil. At the same time, various laws of bedding value variations in depth were considered. Thus, L. Fijiy [11] and M. Khiteniy [12] and some others adopted a scheme of a constant bedding value with depth. Then, in the works of M.T. Devisson [13], K.S. Zavriev [14], a design scheme was developed in which the bedding value changed linearly with depth. For long piles, methods for calculating piles in a multi-layer base were developed [15], including those for pyramidal piles [16].

However, for short piles, it is not necessary to make calculation in a multi-layer base, since within a small depth (up to 3-4 m), it is unlikely that there are layers of soil with very different characteristics.

To construct the design scheme of a horizontally loaded pyramidal pile with compacted bottom, the following prerequisites are accepted.

1. The base of the pile in depth is assumed to be single layered with the bedding value that varies in a linear relationship with depth $K_z$

$$K_z = K_l \frac{z}{l}$$ (1)

where $K_l$ is the bedding value at the depth $l$; $z$ is the current depth coordinate; $l$ is the length of the pile in soil.

2. The size of the cross-section of pile $d_z$ varies with depth in linear relationship

$$d_z = d_0 (1 - \xi z); \quad \xi = \frac{(d_0 - d_n)}{d_0 d}$$ (2)

where $d_0$ and $d_n$ are the cross-section size of the top and bottom of the pile, respectively.
3. The pile bending rigidity is assumed to be infinitely large, and therefore the horizontal pile displacement in the depth $U_z$ is determined from expression

$$U_z = U_0 - \varphi_0 z$$

where $U_0$ and $\varphi_0$ are the horizontal displacement and the angle of pile rotation at the ground surface level.

4. The compacted bottom prevents the horizontal displacement of the lower end of the pile, so in the design scheme, the displacement of the pile bottom is assumed to be equal to 0.

5. According to the local deformations model, the soil resistance on the frontal surface of the pile $q_z$ is proportional to its displacement $U_z$ and the bedding value $K_z$

$$q_z = U_z K_z d_z,$$

The bedding value $K_l$ is found from the condition of settlement equality determined by the theory of local deformations and elastic half-space

$$K_l = \omega \frac{E_0 A_r}{(1-\nu^2) d_{av}},$$

where $\omega$ is the dimensionless coefficient that depends on the $l/d$ ratio; $E_0$ is the modulus of deformation; $A_r$ is an empirical coefficient determined by the results of processing experimental data when testing the experimental piles; $\nu$ is the Poisson’s ratio; $d_{av}$ is the average size of the side of the pile cross-section.

It is also possible to determine the bedding value in accordance with CR 24.13330-2011 by Table C. 1 of Appendix C.

The design scheme is shown in figure 2.

![Figure 2. Design scheme of a horizontally loaded pyramidal pile with compacted bottom in the base.](image)

In accordance with the experimental results, we accept the following boundary conditions

$$Z = l; U_l = 0; M_l = 0; Q_l = R,$$

where $R$ is the reactive force that occurs at the pile lower end level.

From the condition of the acting and reactive forces equilibrium and according to the design scheme, we write down the bending moment $M_z$, the transverse force $Q_z$ in an arbitrary cross-section
of a pyramidal pile at the depth \( z \), arising from the action of the horizontal load \( H_0 \) and the bending moment \( M_0 \) to the pile

\[
\begin{align*}
M_z &= M_0 + H_0z - M^*_z \\
Q_z &= H_0 - Q^*_z
\end{align*}
\]

(7)

where \( M^*_z \) and \( Q^*_z \) are the bending moment and the transverse force in the arbitrary cross-section \( z \) resulted from the soil resistance \( q_z \) at the depth \( z \), which can be determined as the follows

\[
\begin{align*}
M^*_z &= d_0 U_0 K_l \int_0^1 (1 - \xi \bar{z})(1 - \frac{z}{l_0}) \frac{z}{l}(z - \bar{z}) d\bar{z} \\
Q^*_z &= d_0 U_0 K_l \int_0^1 (1 - \xi \bar{z})(1 - \frac{z}{l_0}) \frac{z}{l}(z - \bar{z}) d\bar{z}
\end{align*}
\]

(8)

After integrating expressions (8) and substituting them in (7) and taking into account boundary conditions (6), we obtain the system of two equations. After making some appropriate transformations regard to \( U_0 \) and \( \varphi_0 \), we obtain expressions for determining these values.

\[
\begin{align*}
U_0 &= M_0 + H_0 \delta_2 - R \delta_5 \\
\varphi_0 &= M_0 \delta_3 + H_0 \delta_4 - R \delta_6
\end{align*}
\]

(9)

where \( \delta_1 \ldots \delta_6 \) are unit displacements determined by the following formulas

\[
\begin{align*}
\delta_1 &= \frac{\phi_2}{\eta}; \delta_2 = \frac{\phi_2 - \phi_3}{\eta}; \delta_3 = \frac{\phi_4}{\eta}; \delta_4 = \frac{\phi_4 - \phi_3}{\eta}; \delta_5 = \frac{\phi_3}{\eta}; \delta_6 = \frac{\phi_3}{\eta}
\end{align*}
\]

(10)

Values \( \Phi_1 \ldots \Phi_4 \) are determined by formulas

\[
\begin{align*}
\Phi_1 &= \frac{d_0 K_l}{12l} (6l^2 - 4l^3); \Phi_2 = \frac{d_0 K_l}{12l} (4l^2 - 3l^3); \\
\Phi_3 &= \frac{d_0 K_l}{12l} (2l^2 - l^3); \Phi_4 = \frac{d_0 K_l}{12l} (l^6 - 0.6l^5)
\end{align*}
\]

(11)

Value \( \eta \) is determined by formula

\[
\eta = \Phi_2 \Phi_3 - \Phi_1 \Phi_4
\]

(12)

Value \( R \) is determined from condition

\[
U_1 = U_0 - \varphi_0 l
\]

(13)

Substituting formulas (9) into (13) and solving it with regard to \( R \), we obtain the following formula

\[
R = \frac{M_0 (\delta_1 - l \delta_3) + H_0 (\delta_2 - l \delta_4)}{\delta_5 - l \delta_6}
\]

(14)

Values of the bending moment \( M_z \) and the transverse force \( Q_z \) in the cross-section \( z \) are determined by formulas

\[
\begin{align*}
M_z &= M_0 + H_0z + U_0 \Phi_3 - \varphi_0 \Phi_4 \\
Q_z &= H_0 + U_0 \Phi_4 - \varphi_0 \Phi_2
\end{align*}
\]

(15)

Here, when determining the values \( \Phi_1 \ldots \Phi_4 \) in formulas (11), we should take \( l = z \).

According to this method, calculations of the experimental piles were performed in two variants – with and without compacted bottom. The bedding value was determined by formula (5) with the coefficient of \( A_r \) =3.5. The horizontal load in the calculations was taken from the experimental graphs for each pile in figure 1, equal to the load when displacing the pile head by 10 mm.

Table 3 shows the results of the calculations in comparison with the experimental data.

As can be seen, according to the developed method, the results of calculations give a discrepancy of 8-20 % with the experimental data. The resulting convergence takes place only for the displacement in the range of 10 cm. However, as can be seen from the graphs of the “load – displacement” dependence obtained experimentally, these dependencies are nonlinear. Therefore, in order to obtain
data on the deformation of the pile over the entire range of displacements, additional studies should be carried out to take this factor into account.

Table 3. Comparison of calculation results with experimental data.

| No. of piles | Experimental results | Calculation results |
|--------------|----------------------|---------------------|
|              | Displacement of pile head $U_0$ at ground surface level, mm | Angle of pile head rotation $\phi_0$ at ground surface level, rad$\cdot$10$^{-3}$ | Displacement of pile head $U_0$ at ground surface level, mm | Angle of pile head rotation $\phi_0$ at ground surface level, rad$\cdot$10$^{-3}$ |
| 1            | 10.0                 | 19.2               | -                     | 11.9                   | -                     | 6.0                   |
| 2            | 10.0                 | 5.3                | 12.2                  | 18.2                   | 4.3                   | 9.1                   |
| 3            | 10.0                 | 5.5                | 10.8                  | 16.4                   | 3.6                   | 7.6                   |

5. Conclusion
1. Static tests of cast-in-situ pyramidal piles with compaction bottom formed by ramming crushed stone into the stamped well were carried out. According to the test results, the diagram of “horizontal load – pile displacement” at the ground surface level and the diagram of “horizontal load – angle of pile rotation” were obtained. Based on these data, the points of zero displacements of the experimental piles were determined and the impact of compacted bottom on the horizontally loaded pile deformation was identified.

2. Based on the analysis of the test results of the experimental piles, the design scheme of the horizontally loaded pyramidal pile was constructed and the method for calculating such piles for horizontal load in a linearly deformed (Winkler) base with the bedding value that varied linearly in depth was developed. The calculation method allows determining the horizontal displacement and the angle of pile rotation, as well as the bending moment and the transverse force in any pile cross-section in depth.

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