Search for the Optimal Shape of Metal Spatial (space) Structures

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Abstract. The main disadvantages spatial structures are: increased requirements for the manufacturing elements accuracy; high consumption of material due to the having of "extra" elements; high material consumption due to the specified unification of elements. It is possible to reduce material consumption by searching optimal shape of the spatial structures and to refuse the idea of maximum unification of elements. A reduction in material consumption, if other things being equal, should provide a cheaper construction. The reasonable choice of the best project from the set of acceptable options can be made only with the use of optimization methods. The following are the author's suggestions on parametric optimization task formulation for spatial bar metal structures. To solve this problem, it is suggested to use a method of calculation similar to the method suggested by prof. Ya.I. Olkov. Proposed for use direct search method on all spatial construction nodes coordinates variation. In suggested method node coordinate change cause change in geometrical shape of the whole construction. Geometrical shape changes cause original construction to turn to a new one. For all new constructions only original topology is similar. This method allows to analyze effectiveness of constructions with the same topology but different: height; node amount; belt grids of various geometric shapes. It is proposed to organize the solution of the problem in the form of a dynamic iterative process in which several levels of cycles of enumeration of parameters are realized. To implement the solution of the described problem, we proposed a computer implementation algorithm.

1. Introduction

Spatial structures belong to a wide class of spatial hinged-rod structures [1]. The most common of them are flat double-layer grids and double-layer braced barrel vaults. Structures have improved rigidity. Spatial structures installation is faster and more economical than traditional coating designs. The simplicity of some joint types of rods with nodal elements and high precision manufacturing allow to achieve this effect.

There are lots of advantages, but this constructions also have disadvantages. The main disadvantages are: increased requirements for the manufacturing elements accuracy; high consumption of material due to the having of "extra" elements; high material consumption due to the specified unification of elements.

It is possible to reduce material consumption by searching optimal shape of the spatial structures and to refuse the idea of maximum unification of elements. The practice of designing hinged rod metal
structures shows that structural changes and changes in shape can give a better economic effect than optimization of parameters in a given form.

The main criterion for applied design effectiveness has always been its low cost. There are many scientific studies comparing the cost-effectiveness of spatial structures with traditional structures [2-6]. These studies prove the effectiveness of use of this type of structures and their advantages over traditional ones. The main factor determining the economic efficiency of steel structures is their material consumption. A reduction in material consumption, if other things being equal, should provide a cheaper construction. The reasonable choice of the best project from the set of acceptable options can be made only with the use of optimization methods.

The optimal construction design is a targeted choice of its parameters, providing best result according to a given criterion. The criteria can be economic - material consumption, cost, etc.; physical - the principle of equally stressed structures, minimum mass, maximum perceived load, etc. When setting and solving the optimization problem, it is important to take into account the maximum number of significantly influencing features of the future design. It is important to initially determine which design parameters are constant and which ones will be found.

Optimal design problems may have differences in the accepted quality criterion, in the number of unknown parameters, in the limitations imposed on the objective function. To date, the most studied is the problem of determining the optimal distribution of material within a statically indefinable system. The complexity of solving such a problem is that, even with an unchanged external load, forces arising in the elements depend on the ratio of elements stiffness in statically indefinable system.

There are three types of optimal design problems:
1. Tasks related to constructions with a given topology and geometry. This is a reverse task of structures theory, or a direct task of design [7].
2. A much more hard task is search of construction geometry. This task is called generalized problem of the theory of structures [7].
3. Even more complex tasks are synthesis tasks. In these tasks topology, geometry of constructions and elements can vary. In tasks called "Maxwell- Michell" [8] only loads and only support nodes are given initially. There is a lot of literature about solution of this task, but results hadn't got wide practical use, because resulting forms of designs often are unacceptable by stability and impossible to manufacture.

2. Materials and methods
The following are the author's suggestions on parametric optimization task formulation for spatial bar metal structures. To solve this problem, it is suggested to use a method of calculation similar to the method suggested by prof. Ya.I. Olkov [9]. He suggested a method for optimizing hinged rod metal structures called the "dynamic search method". Two ideas served as the basis for this method: the Hooke-Jeeves direct search algorithm; the suggestion of the volume function surface uniqueness with variation of one structure node coordinates with the remaining nodes fixed. The disadvantage of this method is volume function surface uniqueness idea is not finally proven.

Proposed for use direct search method on all spatial construction nodes coordinates variation. In suggested method node coordinate change cause change in geometrical shape of the whole construction. Geometrical shape changes cause original construction to turn to a new one (Fig. 1). For all new constructions only original topology is similar (Fig. 2). This method allows to analyze effectiveness of constructions with the same topology but different: height (Fig. 1); node amount; belt grids of various geometric shapes (Fig. 1).

We suggest to vary any construction parameters - construction height on support \( h \); length of spans \( Lx', Ly', Lx^b, Ly^b \); amount of nodes \( nx', ny', nx^b, ny^b \); belt grids curvature values \( f_{x'}, f_{y'}, f_{x^b}, f_{y^b} \); supports scheme \( t_{ip_{sup}} \); rods geometry \( F_i \), elements material \( E \) and so on. Then task becomes multiparametric. Complex multiparametric task solution can be found by solving oneparametric tasks step-by-step solution. For this, it is necessary to determine the importance of each parameter and its
dependence on others, that is, to establish a hierarchical dependence of the parameters. Then the solution to the low-level problem can be taken as initial data for solving the higher-level problem. According to the degree of influence on each other, the listed parameters can be arranged in the following row (Fig. 3): I - characteristics of the elements \((F_i,E,\gamma)\); II - geometry of waist nets \((fx^i, fy^i, fx^b, fy^b)\); III - diagram of the support structure \((tip_{sup})\); IV - general construction geometry \((Lx^i, Ly^i, Lx^b, Ly^b, h, n_x^i, n_y^i, n_x^b, n_y^b)\).

![Figure 1. The transformation of the original design into a new one.](image)

![Figure 2. The topological scheme of all designs is the same.](image)

It is proposed to organize the solution of the problem in the form of a dynamic iterative process in which several levels of cycles of enumeration of parameters are realized. The number of levels of cycles is chosen by the researcher, therefore he can choose the level of complexity of the task. For example, if you want to determine the distribution of the material in a structure with parameters
known to the general geometry and known fixing, then the first level problem is solved (Fig. 3). This is a direct design task. If you want to determine the optimal geometry of the structure, then the second level problem is solved. If you need to determine the best option for securing the structure, then the third level problem is solved. The fourth level allows you to determine the best solution for buildings with different overall dimensions.

Figure 3. The hierarchical structure of cycles of iterative calculations of a multiparameter problem.

Such a task is dynamic. Changing parameters affects all design characteristics. The algorithm for solving this problem is iterative. For example, conducting a search for the optimal height of a structure with curvilinear belt grids vary \( h \) and \( f_x^l, f_y^l, f_x^b, f_y^b \), while the cycle \( h \) is external and \( f_x^l, f_y^l, f_x^b, f_y^b \) nested. The calculation cycles are organized so that all other parameters are accepted unchanged, and the user sets the change step. From iteration to iteration, the geometry of the upper or lower belt grids of the structure changes. This causes a change not only in the design itself, but also in external factors. A change in the geometry of the structure leads to a change in the main loads. The weight of the structure is changing. The form of snow accumulation on the coating changes and, as a result, the load from the snow. This makes us, at each stage of the iteration, recount the external loads on the structure based on new conditions. At each level, after determining the internal forces in the elements, an optimal selection of elements is performed. When selecting elements, strength and stiffness constraints, stability constraints, structural constraints are taken into account. Then, the calculation of the accepted quality criterion is performed. For the quality criterion, it is proposed to take the mass of the structure. At each level, the process of selecting the geometric characteristics of the elements is performed cyclically until the mass function changes. When the mass of the structure ceases to change, the process of selecting elements stops. The resulting value of the mass of the structure is remembered. Then the parameters change and the process repeats again. After enumerating all possible parameter values, a transition to a higher level is performed, that is, we change the parameter \( h \). With a new value \( h \), we successively carry out each nested cycle of parameter changes \( f_x^l, f_y^l, f_x^b, f_y^b \). As a result, we get an array of structural mass values. We can choose the minimum value of the mass of the structure from all found. For the minimum mass value, we will know the values of all design parameters. The design with these parameters is optimal in weight.

To implement the solution of the described problem, we proposed a computer implementation algorithm (Fig. 4). For this algorithm, a mathematical model of the problem [10-16] is created. In the
model, the mathematical dependencies of the objective function (1) and constraint functions (2) - (4) are recorded through the design parameters.

\[
M = \gamma \times \Psi_a \times \left[ \sum_{j=1}^{n} \left( \frac{N_j^p + \sum_{j=1}^{m} P_j \times z_j}{\sigma_a} \right)^2 \right] \times \left( x_i^b \times x_i^e \right) + \left( y_i^b \times y_i^e \right)^2 + \left( z_i^b \times z_i^e \right)^2 \]  (1)

1. Physical restrictions (strength/stability of rods):
\[
\frac{N_i}{F_i} \leq \sigma_a \]  (2)

2. Geometrical restrictions (deformation compatibility conditions):
\[
\begin{cases}
R_{11} \times z_1 + \cdots + R_{im} \times z_m = -P_{im} \\
\vdots \\
R_{nm} \times z_1 + \cdots + R_{nm} \times z_m = -P_{nm}
\end{cases} \]  (3)

3. Rigidity restrictions (restriction of maximum structure main node bending):
\[
f \leq \hat{f} \]  (4)

4. Constructive restrictions and restrictions on geometrical parameters changes limit of structure.

3. Results
The «Mirage» and «PSMK» programs were used as a tool for implementing our method of optimizing metal structural structures. The «Mirage» program performs static structural analysis. The «PSMK» program performs the optimal selection of structural elements. These programs are combined by a
program written by the author. The author's program directs the algorithm for changing the form of the structural structure and the process of exchanging data between programs. This program consists of several blocks that perform the following work:

1) preparation of initial information for the static structural analysis in the «Mirage» program. Launch of the «Mirage» program with the necessary parameters;
2) transfer of calculation results from the «Mirage» program to the «PSMK» program. Launch of the «PSMK» program;
3) calculation of the mass value of the structure. Saving the mass value and design parameters.
4) changing the design parameters, transferring the results of the «PSMK» program to the «Mirage» program. Launch of the «Mirage» program.

4. Conclusions
On the basis of the research the following conclusions are made:

1. A technique is proposed for the optimal design of double-layer grids structures with an arbitrary geometry of belt grids.
2. A mathematical model has been created for the optimal design of double-layer grids structures with arbitrary geometry of belt grids.
3. A program has been written for the optimal design of spatial structures with arbitrary geometry of belt grids.

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