Stochastic gravitational-wave background from primordial black hole scenario after GW150914 and GW151226

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Advanced LIGO’s discovery of gravitational-wave events GW150914 and GW151226 has stimulated extensive studies on the origin of binary black holes. Supposing the gravitational-wave events could be explained by binary primordial black hole (PBH) mergers, we investigate the corresponding stochastic gravitational-wave background (SGWB) and point out the possibility to detect this SGWB spectrum, in particular from the subsolar mass PBHs, by the Advanced LIGO in the near future. We also use the non-detection of SGWB to give a new independent constraint on the abundance of PBHs in dark matter.

Introduction.— Recently the gravitational wave (GW) events GW150914 and GW151226 have been discovered by the Advanced LIGO detectors [1, 2]. Both GW signals are well consistent with the mergers of binary black holes (BBHs). The GW150914 event originates from two relatively heavy coalescing black holes with masses of $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$ [1], while the GW151226 event originated from two coalescing black holes with masses of $14^{+8}_{-4} M_{\odot}$ and $7^{+2}_{-2} M_{\odot}$ [2]. The local merger rate of the BBH mergers has been inferred to be $3.4^{+8.6}_{-2.8} \, \text{Gpc}^{-3} \, \text{yr}^{-1}$ for GW150914, and $37^{+92}_{-31} \, \text{Gpc}^{-3} \, \text{yr}^{-1}$ for GW151226 [3]. Here the uncertainties are given at 90% confidence level (CL). The Advanced LIGO’s discovery robustly demonstrate that the BBHs indeed exist and can inspiral and merge within the age of the Universe.

The origin of these black holes and the formation mechanism of black hole binaries are still under debates. The possibility that these black holes are of primordial origin and constitute a fraction of dark matter is considered in [4–6]. Some other recent related works are given in [7–11]. The primordial black holes (PBHs) could be produced by direct gravitational collapse of a primordial overdensity in the early Universe [12, 13]. At the formation redshift $z_f$, the PBH mass is roughly equal to the horizon mass, namely $M_{\text{BH}} \approx \frac{4}{\pi} \rho_f (H_f^{-1})^3 \sim 30 M_{\odot} [4 \times 10^{11}/(1+z_f)]^2$. Thus, the PBHs formed deeply in the radiation dominated era could possibly claim responsibility for the GW150914 and GW151226 events. The local merger rate of binary PBH mergers has also been showed consistent with Advanced LIGO’s estimation [4, 5].

However, one could have different mechanisms to form PBH binary systems. Two PBHs might pass by each other accidentally and then form a binary due to energy loss by gravitational radiation [4]. To account for the estimated GW event rate, the PBHs need to contribute most of dark matter in this model. On the other hand, in fact, two nearby PBHs can form a binary due to tidal force from the third neighboring PBH [5, 14]. The fraction of PBHs in dark matter is required to be relatively small in this model to be compatible with the estimated local merger rate.

Stochastic gravitational-wave background (SGWB) from BBHs is produced from the incoherent superposition of all the merging binaries in the Universe [15–22]. This background is potentially measurable at Advanced LIGO/Virgo detectors’ projected final sensitivity [22]. Recently, the SGWB following the mechanism of PBH binaries formation in [4] has been studied in [23]. While in this Letter, we aim to investigate the SGWB energy density spectrum from binary PBH mergers in the scenario of [5], under the assumption that the observed GW events originate from binary PBH mergers. The SGWB spectrum from different mass PBH binaries, particularly from subsolar mass PBH binaries, is studied. Currently, the nature of dark matter is still lack of direct experimental evidence. PBHs could be a reasonable candidate of dark matter. We utilize the non-detection of SGWB by Advanced LIGO detectors to give a new independent upper limit on the abundance of PBHs in dark matter.

Merger rate of PBH binaries.— We follow the formation mechanism of the binary PBH mergers proposed in [14] and revisited by [5] to study the merger rate of PBH binaries. The PBHs form deeply in the radiation-dominated epoch and decouple from the background when the average energy density of the PBHs exceeds the background cosmic energy density. Due to the tidal force from the third PBH, the PBH pairs perform elliptical orbital motion and finally merge due to the energy loss via gravitational radiation. Assuming the abundance of PBHs in dark matter to be $f$, i.e. $\Omega_{\text{PBH}} = f \Omega_{\text{DM}}$, the probability that the coalescence occurs in the cosmic time.

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interval \((t, t + dt)\) is given in [5]
\[
dP_t = \begin{cases} \frac{3}{58} \left( \left( \frac{1}{t} \right)^2 + \left( \frac{1}{t_c} \right)^2 \right) \frac{dt}{t}, & \text{for } t < t_c \\ \frac{3}{58} \left( \left( \frac{1}{t} \right)^2 + \left( \frac{1}{t_c} \right)^2 \right) \left( -1 + \left( \frac{1}{t} \right)^2 \right) \frac{dt}{t}, & \text{for } t \geq t_c \end{cases}
\]
where \(T = \frac{3}{170} \left( \frac{f^2 c^4}{(GM_{\text{PBH}})^2} \right) \) and \(t_c = \frac{3}{170} \left( \frac{f^2 c^4 f^{25/3}}{c^3} \right) \) are constants, \(c\) is the speed of light, \(G\) is the gravitational constant, \(M_{\text{PBH}}\) is the mass of PBHs, and \(\bar{x}\) is the physical mean separation of PBHs at an epoch of 8 Mpc. Throughout this Letter we adopt the Hubble constant \(H_0 = 67.8 \text{ km s}^{-1} \text{Mpc}^{-1}\), the fraction of dark matter \(\Omega_{\text{DM}} = 0.270\) and the fraction of non-relativistic matter \(\Omega_{\text{M}} = 0.307\). We only deal with the equal mass scenario of PBH binary in this Letter to give an order of magnitude estimate. The unequal mass scenario was considered in [24] but is still under investigation, due to the lack of PBH mass distribution information.

The merger rate of PBH binaries is expressed by
\[
R_{\text{PBH}}(z) = \frac{3H_0^2 f\Omega_{\text{DM}} dP_t}{8\pi G M_{\text{PBH}}} \frac{dt}{dt}
\]  
(2)

Here the redshift \(z\) is related to the cosmic time \(t\) by \(t = t_0 - \frac{1}{H_0} \int z^{z} d\bar{z}/(1 + \bar{z})\), where \(t_0\) is the age of the Universe and \(E(z) = H(z)/H_0 = (\Omega_M(1 + z)^3 + \Omega_\Lambda)^{1/2}\). Throughout this Letter we adopt the Hubble constant \(H_0 = 67.8 \text{ km s}^{-1} \text{Mpc}^{-1}\), the fraction of dark matter \(\Omega_{\text{DM}} = 0.270\) and the fraction of non-relativistic matter \(\Omega_{\text{M}} = 0.307\). We only deal with the equal mass scenario of PBH binary in this Letter to give an order of magnitude estimate. The unequal mass scenario was considered in [24] but is still under investigation, due to the lack of PBH mass distribution information.

The merger rate of BBH of astrophysical origin can be expressed as a convolution of the star formation rate (SFR) \(R_*\) and time delay distribution \(P(t_d)\)
\[
R_m(z) = \int_{t_{\text{min}}}^{t_{\text{max}}} R_*[|t_c(z) - t_d]|P(t_d) \frac{1 + z}{1 + z_\text{f}} dt_d ,
\]
(3)
where \(t_d\) is the delay time between the formation and the coalescence of binary, \(t_c(z)\) is the cosmic time at coalescence corresponding to redshift \(z\), \(z_\text{f}\) is the redshift at the formation time \(t_c(z)\). The factor \((1 + z)/(1 + z_\text{f})\) converts the rate from formation time to coalescence time due to the cosmological expansion effect. We employ the “Vangioni” SFR model [25] and the time delay model
\[
P(t_d) \sim 1/t_d ,
\]
with \(t_{\text{min}} = 50 \text{ Myr}\) and \(t_{\text{max}} = t_c\). Here for relatively large mass black holes, like those in GW150914, we only consider the SFR at metallicity \(Z < Z_c/2\), following the fiducial model in [22].

For comparison, we illustrate the merger rate of PBH binaries \(R_{\text{PBH}}(z)\) and the merger rate of astrophysical origin \(R_m(z)\) in Fig. 1. Both merger rates are normalized to two different local merger rate values, consistent with the GW150914 and GW151226 events, respectively. Since the astrophysical black holes formation tracks the SFR, the first stars do not exist at sufficiently high redshift, and thus there are few mergers. While the PBHs have a larger merger rate at high redshift, because the PBHs already exist in the early Universe. Our merger rate of PBH binaries is also larger than that in [23], where the authors assume \(f \sim 1\) to account for the observed local merger rate and adopt a different formation mechanism of PBH binaries.

\[
\Omega_{\text{GW}} = \frac{\nu d\rho_{\text{GW}}}{\rho_c} d\nu
\]
(4)

where \(d\rho_{\text{GW}}/d\nu\) is the gravitational radiation energy density in the frequency interval \((\nu, \nu + d\nu)\), \(\rho_c = 3H_0^2 c^2/8\pi G\) is the critical energy density of the Universe. For the SGWB produced by binary PBH mergers, \(\Omega_{\text{GW}}\) can be expressed as an integral over the redshift, namely
\[
\Omega_{\text{GW}}(\nu) = \frac{\nu}{\rho_c H_0} \int_{0}^{\text{z_{sup}}} R_{\text{PBH}}(z) \frac{R_{\text{PBH}}(z) dE_{\text{GW}}(\nu_s) d\nu_s}{(1 + z) E(z)} (\nu_s) dz .
\]
(5)

The integrand denominator \((1 + z)\) converts the merger rate from source frame to observer frame. Here \(\nu_s\) is the frequency in source frame, corresponding to the observing frequency \(\nu\) through \(\nu_s = (1 + z)\nu\), \(dE_{\text{GW}}/d\nu_s(\nu_s)\) is the inspiral-merger-ringdown energy spectrum of BBH in terms of \(\nu_s\) with non-precessing spin correction [27, 28], and \(z_{\text{sup}} = \min(\zeta_{\text{max}}, \nu_{\text{cut}}/\nu - 1)\), where \(\nu_{\text{cut}}\) is the cut off frequency given the energy spectrum of BBH and \(\zeta_{\text{max}}\) is the maximum redshift predicted by the PBH model.

Fig. 2 shows the SGWB energy density spectra due to binary PBH mergers as a function of observed frequency. For the PBH binaries, we employ the identical chirp mass and local merger rate to the GW events [3], but assume non-spinning and equal component mass binaries. For comparison, we also illustrate the SGWB spectra of astrophysical BBH origin here. The shaded regions denote the 90% statistical uncertainties, due to the propagation of uncertainty from the local merger rate. The black
curves denote the 1σ sensitivity of the LIGO-Virgo network expected for two first observing runs O1 (2015-16, solid) and O2 (2016-17, dashed), and for two years at the design sensitivity in O5 (2020-22, dot-dashed) [22]. If a model-predicting spectrum intersects a black curve, it has expected SNR ≥ 1.

From Fig. 2, the SGWB energy density spectrum from binary PBH mergers looks similar to that of astrophysical origin, and it is difficult to distinguish them. However, we also notice a bump around the peak of the SGWB spectrum from astrophysical BBH mergers due to the peak of SFR. This feature might help us to distinguish PBHs from astrophysical black holes. Note that our results on the SGWB spectrum are different from those given in [23] due to the different behaviors of PBH binaries merger rate. We predict a larger SGWB which is accessible for Advanced LIGO O5 even though the abundance of PBHs is less than that in [23].

**Constraining PBH abundance with SGWB**—PBHs are one of the candidates of dark matter. The PBH abundance in dark matter can be constrained from a variety of observations, including the microlensing events caused by massive astrophysical compact halo objects (MACHOs) [29, 30], the gas accretion effect of PBHs on CMB [6, 31] and so on [8]. A new independent constraint can be given utilizing the current bound on SGWB from the non-detection. Our results are showed in Fig. 3 where the black solid curve denotes the current upper limit from O1, and the black dashed and dot-dashed curves illustrate the best constraint we can potentially reach in the future using O2 and O5’s sensitivity, respectively. For comparison, constraints on f from a variety of microlensing [29, 30] and CMB [6, 31] are also plotted. We see that up to now the microlensing and CMB give the tightest constraints on mass range $10^{-7} M_\odot \lesssim M_{\text{PBH}} \lesssim 1 M_\odot$ and $M_{\text{PBH}} \gtrsim 1 M_\odot$, respectively. The current constraint on f around $M_{\text{PBH}} \sim 1 M_\odot$ from the upper limit of SGWB given by Advanced LIGO’s O1 is comparable to those from microlensing and CMB, and the future constraints become much better from O2 and O5. Further improvements of the microlensing and CMB constraints are improbable in the near future. Therefore, Advanced LIGO’s O2 and O5 will provide the best constraints on f around $M_{\text{PBH}} \sim 1 M_\odot$.

Applying the current tightest constraints on f from microlensing and CMB, Fig. 4 shows the SGWB spectra from binary PBH mergers for different chirp masses. In particular, the SGWB generated by subsolar mass PBHs has opportunity to be detected by Advanced LIGO’s O2 and O5. Another aspect is that the subsolar mass PBH mergers can also contribute to higher frequency band SGWB spectrum.

**Discussion.**—In this Letter, we use the mechanism of binary PBH mergers proposed in [5, 14] to calculate the SGWB spectrum. As an application of GW astronomy, the non-detection of SGWB could place an independent constraint on the PBH abundance in dark matter in the mass range $0.1 M_\odot - 100 M_\odot$, and it can provide the best constraints from Advanced LIGO’s O2 and O5 around $M_{\text{PBH}} \sim 1 M_\odot$. And we find that the SGWB generated by subsolar mass PBH mergers is potentially detectable by Advanced LIGO even with the current most stringent constraints on the abundance of PBHs.

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**FIG. 2:** The SGWB energy density spectra as a function of the observed frequency and the expected sensitivities of Advanced LIGO’s O1, O2 and O5. The chirp mass and local merger rate of the PBHs are assumed to be the same as the estimation of the GW150914 and GW151226 events [3]. The different cut-off frequency of SGWB from $M_e = 8.9 M_\odot$ is due to the different choice of symmetric mass ratio and effective spin parameter.

**FIG. 3:** The constraints on PBH fraction versus mass from the non-detection of SGWB with Advanced LIGO/Virgo O1, O2 and O5 sensitivity. The constraints on f from microlensing [29, 30], and CMB with WMAP data [31] are extracted from [8]. The CMB constraint with Planck data is from [6]. The CMB constraints are plotted by dot dashed line because they are highly model dependent.
For each chirp mass, the PBH mass abundance in dark matter is given by the most stringent constraint from microlensing and CMB.

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FIG. 4: The SGWB spectra from binary PBH mergers for different chirp masses $M_c = 0.1M_{\odot}, 1M_{\odot}, 10M_{\odot}$ and $20M_{\odot}$. For each chirp mass, the PBH mass abundance in dark matter is given by the most stringent constraint from microlensing and CMB.