Analyses of two and three pion Bose-Einstein Correlations using Coulomb wave functions

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Abstract

Using effective formulas we analyze the Bose-Einstein correlations (BEC) data corrected for Coulomb interactions provided by STAR Collaboration and the quasi-corrected data (raw data with acceptance correction etc) on 2π and 3π BEC by using Coulomb wave function with coherence parameter included. The corresponding magnitudes of the interaction regions turn out to be almost the same: \( R_{\text{Coul}}(2\pi) \simeq \frac{3}{2} R_{\text{Coul}}(3\pi) \). \( R_{\text{Coul}} \) means the size of interaction region obtained in terms of Coulomb wave function. This approximate relation is also confirmed by the core-halo model. Moreover, the genuine 3rd order term of BEC has also been investigated in this framework and its magnitude has been estimated both in the fully corrected data and in the quasi-corrected data.

1 Introduction

One of interesting subjects in heavy ion physics at high energies is the higher order Bose-Einstein correlation (BEC) as well as the normal (2nd order) BEC [1–3]. The investigation of the genuine 3rd order term of BEC is a current important topic. We are also interested in this subject to examine usefulness of a theoretical formulas described by Coulomb wave functions. Moreover, it is well known that information on the magnitude of the interaction region is necessary to estimate the energy density of produced hadrons [4]. Actually several Collaborations have reported interesting results in Refs. [5], [6] and [7].

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Very recently the STAR Collaboration [8] has reported new data on BEC for three negatively charged pions $(3\pi^-)$, with Coulomb corrections imposed, where $R = 5$ fm is assumed as input [5,8–10]. Their data on $2\pi$ BEC have been already published in Ref. [10], whereas the quasi-corrected data (raw data with acceptance correction etc) on $3\pi^-$ BEC have been shown in Ref. [9]. In Table 1 we provide our classification of different kinds of data available [11–14]. We shall be interested here in different kinds of data provided by the same experiment, to know description of them. We shall be able to examine whether our formulation of BEC in terms of Coulomb wave function as given in Ref. [14] is useful when applied to concrete data.

| Category of data | Description |
|------------------|-------------|
| 1) Raw data      | raw data    |
| 2) Quasi-corrected data (Q-CD) | (raw data) $\times K_{\text{acceptance}}$ |
| 3) Gamow corrected data | (raw data) $\times K_{\text{acceptance}} \times K_{\text{Gamow}}$ |
| 4) Coulomb corrected data | (raw data) $\times K_{\text{acceptance}} \times K_{\text{Coulomb}}$ |

$K_{\text{Coulomb}}$ is calculated as $K_{\text{Coulomb}} = \frac{\int \prod d^3x_i \rho(x_i) \cdot |\text{plane w. f.}|^2}{\int \prod d^3x_i \rho(x_i) \cdot |\text{Coulomb w. f.}|^2}$, where $\rho(x_i) = (\beta^2/\pi)^{3/2} \exp[-\beta^2 x_i^2]$ [11–13]. Notice that $R = \text{input}$ ($\beta = 1/\sqrt{2R}$) is necessary for $K_{\text{Coulomb}}$, where $R = 5$ fm is used in Refs. [5,8–10]. For the systematic error bars we assume 5% for $3\pi$ BEC and 0.5% for $2\pi$ BEC.

For various kinds of data in Table 1, we prepare following two methods:

i) A conventional method: Effective formulas derived from the plane wave with the source function, $\rho(x_i)$ are applied to 4) Coulomb corrected data categorized in Table 1.

ii) A method proposed in Ref. [14]: A formula described by the integration of the Coulomb wave function with $\rho(x_i)$ is applied to 2) quasi-corrected data (Q-CD) in Table 1. This will be explained in the next paragraph.

Let us start with STAR data including Coulomb corrections as given by [8,10]. We analyze these data using the following effective formulas, introducing parameters, $R_{\text{eff}}$, $\lambda_2$ and $\lambda_3$,

$$C_2 = 1 + \lambda_2 e^{-(R_{\text{eff}}Q_{12})^2}, \quad (1)$$

The author of Ref. [11] pointed out that the Coulomb correction is more accurate than the Gamow correction.

Different methods for data of $2\pi$ BEC in Refs. [15,16] will be explained in the third paragraph.
\[ C_3 = 1 + \lambda_3 \sum_{i>j} e^{-(R_{eff} Q_{ij})^2} + 2\lambda_3^{3/2} e^{-0.5(R_{eff} Q_3)^2}, \]

(2)

where \( Q_{ij}^2 = (k_i - k_j)^2 \) and \( Q_3^2 = (k_1 - k_2)^2 + (k_2 - k_3)^2 + (k_3 - k_1)^2 \). Our estimated parameters for \( 2\pi^- \) and \( 3\pi^- \) BEC are shown in Table 2. To reproduce \( R \approx 6.3 \text{ fm} \) reported by STAR Collaboration, we have to skip first two data points, see Fig. 1(b).

![Graphs](image)

Fig. 1. (a) Analysis of data [10] by Eq. (1). (b) Data with (⊙) are skipped in analysis. (c) Analysis of data [8] by Eq. (2).

Table 2
Data (Coulomb correction with \( R = 5 \text{ fm} \) (input)) on \( 2\pi \) and \( 3\pi \) by STAR Collaboration [8,10] and our analyses of data by Eqs. (1) and (2).

| Data  | Theor. | \( C \)   | \( R_{eff} \) [fm] | \( \lambda_2 \) or \( \lambda_3 \) | \( \chi^2/N_{dof} \) |
|-------|--------|---------|------------------|-------------------------------|-----------------|
| \( 2\pi \) [10] | Eq. (1) | 0.999±0.003 | 7.84±0.20 | 0.62±0.02 | 30.8/24 |
| \( 2\pi \) [10] | Eq. (1) | 0.996±0.003 | 6.52±0.31 | 0.43±0.03 | 8.32/22 |
| \( 3\pi \) [8,9] | Eq. (2) | 0.999±0.012 | 7.83±0.26 | 0.57±0.02 | 5.32/35 |

On the other hand, in Ref. [14] we have proposed another theoretical formulation of the Coulomb wave function approach to BEC containing the degree of coherence parameter \( \lambda \). We shall explain it in the next paragraph in detail. Here it will be applied to the raw data (with the acceptance corrections etc included) and its result will be then compared with the previous one in...
paragraph 3. Therein we shall analyze also the raw data on $2\pi^-\text{BEC}$. In 4th paragraph, we are going to analyze the data by the core-halo model [13,14]. Our concluding remarks will be in the final paragraph. It will also contain discussion of the problem of phases among the three 2nd order BEC correlation functions $C_2$, which will be done by introducing a parameter $\lambda$ (which can attain complex values). Finally we compare our results with those obtained by means of formula given in Ref. [17].

2 A theoretical formula for charged $3\pi^-\text{BEC}$.

We shall provide now brief explanations of our formulas (see Ref. [14] for more details). The charged $3\pi^-\text{BEC}$ should be described by the following products of three Coulomb wave functions, see Fig. 2.

\[
\frac{N^{(3\pi^-)}}{N_{BG}} \approx \frac{1}{6} \int d^3x_i \rho(x_i) \left[\prod_{i=1}^{3} \psi_{k_1,k_2}^{\text{C}}(x_1, x_2) \psi_{k_2,k_3}^{\text{C}}(x_2, x_3) \psi_{k_3,k_1}^{\text{C}}(x_3, x_1) + \psi_{k_1,k_2}^{\text{C}}(x_1, x_3) \psi_{k_2,k_3}^{\text{C}}(x_3, x_2) \psi_{k_3,k_1}^{\text{C}}(x_2, x_1) + \psi_{k_1,k_2}^{\text{C}}(x_2, x_1) \psi_{k_2,k_3}^{\text{C}}(x_1, x_3) \psi_{k_3,k_1}^{\text{C}}(x_3, x_2) + \psi_{k_1,k_2}^{\text{C}}(x_2, x_3) \psi_{k_2,k_3}^{\text{C}}(x_3, x_1) \psi_{k_3,k_1}^{\text{C}}(x_1, x_2) \right]
\]

Fig. 2. Diagram reflecting three-charged particles Bose-Einstein Correlation (BEC) and Coulomb potential ($V_c$). $\times$ means the exchange effect of BEC.
Here \( \psi_{k_i,k_j}^C(x_i, x_j) \) are the Coulomb wave functions of the respective 2-body collision expressed as,

\[
\psi_{k_i,k_j}^C(x_i, x_j) = \Gamma(1 + i\eta_{ij})e^{\pi\eta_{ij}/2}e^{ik_i \cdot r_{ij}} F[-i\eta_{ij}, 1; i(k_{ij}r_{ij} - k_{ij} \cdot r_{ij})],
\]

with \( r_{ij} = (x_i - x_j), k_{ij} = (k_i - k_j)/2, r_{ij} = |r_{ij}|, k_{ij} = |k_{ij}| \) and \( \eta_{ij} = ma/k_{ij} \). \( F[a, b; x] \) and \( \Gamma(x) \) are the confluent hypergeometric function and the Gamma function, respectively.

In order to introduce a parameter of the degree of coherence (\( \lambda \)) in Eq. (3), we decompose the Coulomb wave functions assigning to amplitudes \( A(i) \) the following Coulomb wave functions (taken in our calculation in the plane wave approximation (PWA)):

\[
A(1) = \psi_{k_1,k_2}^C(x_1, x_2)\psi_{k_2,k_3}^C(x_2, x_3)\psi_{k_3,k_1}^C(x_3, x_1)
\]

\[
\text{PWA} \quad e^{ik_1 \cdot r_{12}}e^{ik_2 \cdot r_{23}}e^{ik_3 \cdot r_{31}} = e^{(3/2)i(k_1 \cdot x_1 + k_2 \cdot x_2 + k_3 \cdot x_3)},
\]

\[
A(2) = \psi_{k_1,k_2}^C(x_1, x_3)\psi_{k_2,k_3}^C(x_2, x_3)\psi_{k_3,k_1}^C(x_3, x_1)
\]

\[
\text{PWA} \quad e^{ik_1 \cdot r_{13}}e^{ik_2 \cdot r_{23}}e^{ik_3 \cdot r_{31}} = e^{(3/2)i(k_2 \cdot x_2 + k_3 \cdot x_3 + k_3 \cdot x_2)},
\]

\[
A(3) = \psi_{k_1,k_2}^C(x_2, x_1)\psi_{k_2,k_3}^C(x_1, x_3)\psi_{k_3,k_1}^C(x_3, x_2)
\]

\[
\text{PWA} \quad e^{ik_1 \cdot r_{21}}e^{ik_2 \cdot r_{23}}e^{ik_3 \cdot r_{31}} = e^{(3/2)i(k_3 \cdot x_3 + k_3 \cdot x_3 + k_1 \cdot x_1)},
\]

\[
A(4) = \psi_{k_1,k_2}^C(x_2, x_3)\psi_{k_2,k_3}^C(x_1, x_3)\psi_{k_3,k_1}^C(x_1, x_2)
\]

\[
\text{PWA} \quad e^{ik_1 \cdot r_{23}}e^{ik_2 \cdot r_{21}}e^{ik_3 \cdot r_{31}} = e^{(3/2)i(k_1 \cdot x_1 + k_2 \cdot x_2 + k_3 \cdot x_3)},
\]

\[
A(5) = \psi_{k_1,k_2}^C(x_3, x_1)\psi_{k_2,k_3}^C(x_1, x_2)\psi_{k_3,k_1}^C(x_2, x_3)
\]

\[
\text{PWA} \quad e^{ik_1 \cdot r_{31}}e^{ik_2 \cdot r_{23}}e^{ik_3 \cdot r_{31}} = e^{(3/2)i(k_3 \cdot x_3 + k_3 \cdot x_3 + k_2 \cdot x_2)},
\]

\[
A(6) = \psi_{k_1,k_2}^C(x_3, x_2)\psi_{k_2,k_3}^C(x_2, x_1)\psi_{k_3,k_1}^C(x_1, x_3)
\]

\[
\text{PWA} \quad e^{ik_1 \cdot r_{32}}e^{ik_2 \cdot r_{21}}e^{ik_3 \cdot r_{31}} = e^{(3/2)i(k_1 \cdot x_1 + k_2 \cdot x_2 + k_3 \cdot x_1)}.\]

Using \( A(i) \), \( i = 1, 2, \ldots \), we obtain the following three groups denoted as \( F_i \)’s,

\[ A_{\text{plane}}(1) = e^{i(k_1 \cdot x_1 + k_2 \cdot x_2 + k_3 \cdot x_3)}, \quad A_{\text{plane}}(2) = e^{i(k_1 \cdot x_1 + k_2 \cdot x_2 + k_3 \cdot x_2)}, \ldots \]

See Ref. [2].
\[ F_1 = \frac{1}{6} \sum_{i=1}^{6} A(i) A^*(i) \xrightarrow{\text{PWA}} 1 , \]  
\[ F_2 = \frac{1}{6} [A(1) A^*(2) + A(1) A^*(3) + A(1) A^*(6) + A(2) A^*(4) + A(2) A^*(5) + A(3) A^*(4) + A(3) A^*(5) + A(4) A^*(6) + A(5) A^*(6) + \text{c. c.}] \xrightarrow{\text{PWA}} \text{BEC between two-charged particles} \]  
\[ F_3 = \frac{1}{6} [A(1) A^*(4) + A(1) A^*(5) + A(2) A^*(3) + A(2) A^*(6) + A(3) A^*(6) + A(4) A^*(5) + \text{c. c.}] \xrightarrow{\text{PWA}} \text{BEC among three-charged particles} \]

We have to assign the degree of coherence (real number of \( \sqrt{\lambda} \)) to the cross marks (x) in Fig. 2. We obtain finally that

\[
\frac{N(3\pi^-)}{N_{BG}} = C \int \prod_{i=1}^{3} d^3 x_i \rho(x_i) \cdot [F_1 + \lambda F_2 + \lambda^{3/2} F_3] ,
\]

where \( \rho(x) = (\beta^2/\pi)^{3/2} \exp[-\beta^2 x^2] \) (\( \beta = 1/\sqrt{2}R \)) and \( \lambda \) is the degree of coherence. Moreover, we can also derive simple formula which uses only plane wave approximation (PWA). It can be written as

\[
\frac{N(3\pi^-)}{N_{BG}} = \text{Eq. (7)} \xrightarrow{\eta_{ij} \to 0} C \left( 1 + 3\lambda \sum_{i>j} e^{-\frac{9}{4} R^2 Q_{ij}^2} + 2\lambda^{3/2} e^{-\frac{9}{8} R^2 Q_{ij}^2} \right) ,
\]

Actually, Eq. (8) is also applied for the Gamow corrected data, because it contains no Coulomb effect. \( R \) stands for \( R_{\text{Gamow}} \).

3 Analyses of data on BEC.

a) Analyses of data on 3\( \pi^- \) BEC by Eqs. (7) and (8): In Ref. [9] quasi-corrected data at \( \sqrt{s_{NN}} = 130 \) GeV have been reported. We can confirm them by using method proposed in Refs. [13,14] and results are shown in Table 3 and Fig. 3. Eq. (7) and Eq. (8) with the Gamow correction \( K_{\text{Gamow}} \) are used in the present analyses. It is very interesting that the corresponding values of size parameter \( R \) estimated by Eqs. (7) and (8) are almost the same.

b) Analyses of data on 2\( \pi^- \) BEC: In Ref. [10] quasi-corrected data (Q-CD) on 2\( \pi^- \) BEC are given, which we have analyzed using following theoretical formula:
Fig. 3. (a) Analysis of quasi-corrected data (Q-CD) on $3\pi^−$ BEC [8] by Eq. (7). (b) Analysis of quasi-corrected data (Q-CD) on $2\pi^−$ [10] by Eq. (9).

Table 3

| Data Type | Analysis Method | $R$ [fm] | $\lambda$  | $\chi^2/n.d.f.$ |
|-----------|----------------|---------|------------|-----------------|
| a) $3\pi^−$ BEC Data | quasi-corrected data (Q-CD) Eq. (7) | 5.34 ± 0.24 | 0.56 ± 0.02 | 2.80/35 |
|          | $(Q\text{-CD})/K_{\text{Gamow}}$ Eq. (8) | 5.03 ± 0.20 | 0.61 ± 0.03 | 6.41/35 |
| b) $2\pi^−$ BEC Data | quasi-corrected data (Q-CD) Eq. (9) | 8.75 ± 0.31 | 0.58 ± 0.02 | 23.0/25 |
|          | $(Q\text{-CD})/K_{\text{Gamow}}$ Eq. (10) | 7.41 ± 0.20 | 0.61 ± 0.02 | 39.4/25 |

$N^{(2^+ \text{ or } 2^-)}/N^{BG} \simeq C \int \prod_{i=1}^{2} d^3x_i \rho(x_i) \cdot [G_1 + \lambda G_2], \quad (9)$

where

$$G_1 = \frac{1}{2} \left( |\psi_{k_1k_2}^C(x_1, x_2)|^2 + |\psi_{k_1k_2}^C(x_2, x_1)|^2 \right)$$

and

$$G_2 = \text{Re} \left( \psi_{k_1k_2}^{C\ast}(x_1, x_2) \psi_{k_1k_2}^C(x_2, x_1) \right).$$

Applying the plane wave approximation (PWA) to the above equation\(^7\), we get

\(^7\) It is worth while to mention a different method for $2\pi$ BEC utilized in Refs. [15] based on theoretical studies of Ref. [16]. The following formula is applied to $2\pi$ BEC data (Q-CD) including the momentum resolution in Pb + Au collision at 40, 80 and 158 AGeV [15],

$$C_2 = C[1 + \lambda'((1 + G)F^* - 1)].$$

Here $G = \exp[-R_L^2 q_{L0}^2 - R_O^2 q_{O0}^2 - R_S^2 q_{S0}^2 - 2R_{OL}^2 q_{L0} q_{O0}]$, $R_L$, $R_O$, $R_S$ and cross-term $R_{OL}$ are the Gaussian source radii, whose source functions are assumed. $q_{\text{inv}}^2 = (k_1 - k_2)^2 - (E_1 - E_2)^2$, and $F^* = w(k_1) \cdot (F_{\text{Coul}}(q_{\text{inv}}) - 1) + 1$, where $F_{\text{Coul}}(q_{\text{inv}})$ denotes the Gaussian integration of the Coulomb wave function times the source function with $R = 5$ fm. See Ref. [15] for the notations, where the Monte Carlo
obtain the following expression,
\[ C_2/K_{\text{Gamow}} = [1 + \lambda_2 e^{-(R_{\text{eff}} Q_{12})^2}], \tag{10} \]
where \( R_{\text{Gamow}} = R_{\text{eff}} \) (cf., Eq. (1)). The results are also given in Table 3. It is worth to notice that estimated values of the size parameter \( R \) coincide with each other. Actually, in case of plane wave approximation (PWA) they are reduced to the following value:
\[ \frac{3}{2} R_{\text{Coul}}(3\pi) \simeq R_{\text{plane}}(3\pi) \approx R_{\text{plane}}(2\pi) \tag{11a} \]
\[ R = \frac{3}{2} \times 5 \text{ fm} \approx 7.5 \text{ fm} \tag{11b} \]
The interaction region of \( R_{\text{plane}}(2\pi) \) and \( R_{\text{plane}}(3\pi) \) may be explained by the relation:
\[ R(\text{Au}) = 1.2A^{\frac{1}{4}}(\text{Au}) \sim 7 \text{ fm}. \tag{12} \]

4 Analysis of data by core-halo model

To confirm the results obtained previous paragraph, we use the core-halo model proposed in Ref. [2,13,14]. Introducing parameters \( f_c = \langle n_{\text{core}} \rangle / \langle n_{\text{tot}} \rangle \) (the fraction of multiplicity from the core part), \( p_c = \langle n_{\text{co}} \rangle / \langle n_{\text{core}} \rangle \) (the fraction of coherently produced multiplicity from the core part), and \( p = \langle n_{\text{chaos}} \rangle / \langle n_{\text{core}} \rangle \) (chaoticity parameter defined in the laser optical approach), we have the following expressions with \( p = 1 - p_c \),

\[ \frac{N^{(3-)}}{N_{BG}} = C \left[ \int \prod_{i=1}^{3} d^3 x_i \rho_c(x_i) \cdot (F_1 + f_c^2 p^2 F_2 + f_c^3 p^3 F_3) \right. \]
\[ + \left. \int \prod_{i=1}^{3} d^3 x_i \delta^3(x_2) \left( 2f_c^2 p(1-p)F_2 + 3f_c^3 p^2(1-p)F_3 \right) \right], \tag{13} \]
\[ \frac{N^{(2-)}}{N_{BG}} = C \left[ \int \prod_{i=1}^{2} d^3 x_i \rho_c(x_i) \cdot (G_1 + f_c^2 p^2 G_2) \right]. \]

method is also introduced. In this approach the effective degree of coherence \( \lambda = \lambda'w(kt) \cdot (F_{\text{Coul}}(q_{\text{inv}}) - 1) \) is expressed by the parameter \( \lambda' \), the weight function \( w(kt) \) and \( F_{\text{Coul}}(q_{\text{inv}}) \), where \( k_t = \frac{1}{2}|k_{1t} + k_{2t}|. \) Finally, physical information on \( R_L, R_O, R_S \) and \( R_{OL} \) are obtained as function of \( k_t \). Various theoretical formulas with \( w(kt) = 1 \) are investigated in Ref. [16].
\[ + \int \prod_{i=1}^{2} d^3 x_i \rho_c(x_i) \cdot \delta^3(x_2) 2 f_c^2 p(1-p) G_2. \] (14)

Here \( \rho_c \) stands for the source function of the core part. The halo part is reflected by the delta function.

Our results are shown in Table 4 and Fig. 4. As seen in them, the interaction ranges of \( R \) (core part) are estimated in the ranges of \( 5.3 \text{ fm} < R_{Coul}(3 \pi) < 7 \text{ fm} \) and \( 8.7 \text{ fm} < R_{Coul}(2 \pi) < 11 \text{ fm} \), respectively. The common region between BEC of \( 3 \pi^- \) and \( 2 \pi^- \) is roughly described by \( 0.6 \lesssim p \lesssim 1.0 \) and \( 0.75 \lesssim f_c \lesssim 0.85 \). Actually we have confirmed Eq. (11), \( 1.5 R_{Coul}(3 \pi) \sim R_{Coul}(2 \pi) \).

Table 4
Typical results from analysis of data by STAR Collaboration in terms of Eqs. (13) and (14). The effective degrees of coherence are defined by \( \lambda_3^* = f_c^2 (p^2 + 2p(1-p)) + f_c^3 (p^3 + 3p^2(1-p)) \) and \( \lambda_2^* = f_c^2 (p^2 + 2p(1-p)) \), respectively. Note that figures at \( p = 1 \) reproduce previous results in Table 3.

| \( p \) | 1.0 | 0.8 | 0.6 |
|-----|-----|-----|-----|
| \( 3 \pi \) BEC |
| \( f_c \) | 0.75±0.02 | 0.78±0.02 | 0.85±0.02 |
| \( \lambda_3^* \) | 0.97 | 1.00 | 1.00 |
| \( R \) (fm) | 5.34±0.24 | 5.99±0.28 | 6.48±0.32 |
| \( \chi^2/N_{dof} \) | 2.80/34 | 1.39/34 | 0.98/34 |
| \( 2 \pi \) BEC |
| \( f_c \) | 0.76±0.01 | 0.78±0.02 | 0.83±0.02 |
| \( \lambda_2^* \) | 0.58 | 0.59 | 0.58 |
| \( R \) (fm) | 8.75±0.31 | 10.16±0.38 | 11.12±0.42 |
| \( \chi^2/N_{dof} \) | 23.0/24 | 16.3/24 | 16.1/24 |

5 Concluding remarks

Because quasi-corrected data at \( \sqrt{s_{NN}} = 130 \) GeV have been reported by STAR Collaboration, we have applied theoretical formula, Eq. (7) to analyze them. Our method and the conventional method for the charged BEC are explained in paragraph 1. From our results, it can be said that both methods are (almost) equivalent to each other (when accounting for the Fourier factor \( (3/2) \) in Eq. (5) in case of charged \( 3 \pi^- \) BEC). From our present study we have known that magnitude of the interaction region of Au + Au collisions at \( \sqrt{s_{NN}} = 130 \) GeV is described by the relation \( R_{Coul}(2 \pi) \sim \frac{3}{2} R_{Coul}(3 \pi) \).
Fig. 4. Sets of $f_c$ and $p$ estimated in analyses of BEC of $2\pi^-$ and $3\pi^-$. Widths of $f_c$’s stand for error bar of $\pm 2\sigma$. Common region are marked by mesh of a net.

whose magnitude is explained by $R(Au) = 1.2 \times A^{1/3}$ fm. This fact is also confirmed at $p = 1$ in Table 4, provided that the core-halo model proposed in Refs. [2,13,14] is used. This kind of study should be applied also to other data at SPS and RHIC energies [18].

Finally we consider problem of the phase factors appearing between three 2nd BEC $C_2(k_1, k_2)$, $C_2(k_2, k_3)$, and $C_2(k_3, k_1)$. According to Ref. [17], we analyze the respective data by the following formula for the 3rd order BEC,

$$C_3 = C\left[1 + \lambda_3 \sum_{i>j} e^{-(R_{\text{eff}} Q_{ij})^2} + 2\lambda_3^{3/2} e^{-0.5(R_{\text{eff}} Q_3)^2} \times W\right],$$

(15)

where $W = \cos(\phi_{12} + \phi_{23} + \phi_{31})$, which, in the simplest form, is parameterized as $W = \cos(g \times Q_3)$.

Similar formula can be obtained by replacing in Fig. 2 and Eq. (8) degree of coherence parameter $\sqrt{\lambda}$ by three complex numbers $(\sqrt{\lambda} e^{i\phi})$:

$$\frac{N(3\pi^-)}{N_{BG}} = C \int \prod_{i=1}^{3} d^3 x_i \rho(x_i) \cdot [F_1 + \lambda F_2 + \lambda^{3/2} F_3 \times W].$$

(16)

Our results are shown in Table 5 and Fig. 5. The genuine 3rd term of BEC in the quasi-corrected data (Q-CD) is explicitly shown in Fig. 5. It should be stressed that the phase factor is playing important role in formula $N(3\pi^-)/N_{BG}$ [19], leading to smaller minimum of $\chi^2$, see Table 5. The smaller $\chi^2$’s in Table 5 than those of Table 4 suggest us the usefulness of Eq. (16). To elucidate this fact, study on $g$ is also necessary in a future [18]8.

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8 There is a possible interpretation as $g = 30 \text{ GeV}^{-1} \simeq 6 \text{ fm}$, provided that $0.2 \text{ GeV \cdot fm} = 1$ is used.
Table 5
Analyses of $3\pi^-$ BEC by means of Eq. (15) and Eq. (16) with $W = \cos(g \times Q_3)$.

| Data           | $R$ [fm]   | $\lambda_3$ or $\lambda$ | $g$ [GeV$^{-1}$] | $\chi^2$/n.d.f. |
|----------------|------------|---------------------------|-------------------|------------------|
| Coulomb CD Eq. (15) | 7.08±0.42  | 0.62±0.03                 | 32.9±6.2          | 1.00/34          |
| (Q-CD) Eq. (16)    | 4.91±0.36  | 0.61±0.04                 | 31.9±8.8          | 0.57/34          |

Fig. 5. (a) Analysis of Q-CD by Eq. (16). (b) The magnitude of the genuine term. (c) Behavior of $\cos(g \times Q_3)$.

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