Josephson effect in SIFS tunnel junctions with domain walls in weak link region

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We study theoretically the properties of SIFS type Josephson junctions composed of two superconducting (S) electrodes separated by an insulating layer (I) and a ferromagnetic (F) film consisting of periodic magnetic domains structure with antiparallel magnetization direct ions in neighboring domains. The two-dimensional problem in the weak link area is solved analytically in the framework of the linearized quasiclassical Usadel equations. Based on this solution, the spatial distributions of the critical current density, $J_C$, in the domains and critical current, $I_C$, of SIFS structures are calculated as a function of domain wall parameters, as well as the thickness, $d_F$, and the width, $W$, of the domains. We demonstrate that $I_C(d_F, W)$ dependencies exhibit damped oscillations with the ratio of the decay length, $\xi_1$, and oscillation period, $\xi_2$, being a function of the parameters of the domains, and this ratio may take any value from zero to unity. Thus, we propose a new physical mechanism that may explain the essential difference between $\xi_1$ and $\xi_2$ observed experimentally in various types of SFS Josephson junctions.

It is well known that properties of Josephson structures with ferromagnetic (F) material in a weak link region depends on relation between the complex decay length, $\xi$, ($\xi^{-1} = \xi_1^{-1} + i\xi_2^{-1}$) and geometrical parameters of these junctions [1]-[3]. If F metal is in the dirty limit and exchange energy, $H$, sufficiently exceeds the critical temperature of superconducting (S) electrodes, $\pi T_C$, then from Usadel equations it follows that $\xi_1 \approx \xi_2$. However, it was demonstrated experimentally [4]-[12] that there could be a noticeable difference between $\xi_1$ and $\xi_2$, previously the difference has been attributed either to the presence of strong paramagnetic scattering in the F layer [7], or to violation of the dirty limit conditions in ferromagnetic material [12], [13]. However, application of the first of the mechanisms for the experimental data interpretation requires the existence of unreasonably strong paramagnetic scattering in the weak link material [7]. The relation between an electron mean free, $\ell$, and $\xi_1$, $\xi_2$ in typical experimental situation is also closer to the dirty limit conditions, $\ell \lesssim \xi_1$, $\xi_2$ rather than to the clean one.

In this article we prove that the existence of a ferromagnetic domain walls in F layer can also lead to appearance of substantial differences between $\xi_1$ and $\xi_2$ even in the absence of strong scattering by paramagnetic impurities, and under the fulfilment of the dirty limit conditions in the F material.

Fig.1. Geometry of the considered SIFS Josephson junction and its enlarged part, which includes two halves of domains and domain wall separating them. The insulating barrier I has a small transparency (shown by a blue line).

Model. Consider multilayered SIFS structure presented in Fig.1. It consists of superconductor electrode (S), insulator (I) and FS bilayer as an upper electrode. We assume that the F film has a thickness, $d_F$, and that it subdivides into domain structure with antiparallel direction of magnetization vector in the neighboring domains. The width of the domains is $W$ and they separated by atomically sharp domain walls oriented per-
pendicular to SF interfaces. Due to periodicity of the structures we, without any loss of generality, can perform our analysis within its half of the period, that is from $-W/2$ to $W/2$. This element is enlarged in Fig.1. It consists of two halves of domains and domain wall separating them.

We will suppose that the condition of dirty limit is fulfilled for all metals and that effective electron-phonon coupling constant is zero in F material. We will assume further that either temperature $T$ is close to the critical temperature of superconducting electrodes $T_C$ or the suppression parameters $\gamma_{BS} = R_{BS}A_{BN}/\rho_F \xi_F$ at SF interface is large enough to permit the use of the linearized Usadel equations in F film of the structure. We will characterize the FF interface (domain wall) by the suppression parameter $\gamma = 1$, and the suppression parameter $\gamma_{BF} = R_{BF}A_{BF}/\rho_F \xi_F$, which can take any value. Here $R_{BS}, R_{BF}$ and $A_{BN}, A_{BF}$ are the resistances and areas of the SF and FF interfaces, $\xi_S$, and $\xi_F$ are the decay lengths of S, F materials, while $\rho_S$ and $\rho_F$ are their resistivities, $D_F$ is diffusion coefficient in the F metal.

Under the above conditions the proximity problem in the SF part of SIFS junction ($0 \leq x \leq d_F$) reduces to solution of the set of linearized Usadel equations [1]-[3], [14]

$$
\begin{align*}
\frac{\partial^2}{\partial x^2} F_F + \frac{\partial^2}{\partial y^2} F_F - \tilde{\Omega}_+ F_F &= 0, \\ \frac{\partial^2}{\partial x^2} F_F + \frac{\partial^2}{\partial y^2} F_F - \tilde{\Omega}_- F_F &= 0,
\end{align*}
$$

(1), (2)

where $\Omega = \omega/\pi T_C, \tilde{\Omega}_+ = |\Omega| \pm i \hbar \operatorname{sgn}(\omega)$, $h = H/\pi T_C$, $H$, is exchange energy of ferromagnetic material, $\omega = \pi T(2n + 1)$ are Matsubara frequencies. The spatial coordinates in (1), (2) are normalized on decay length $\xi_F$. To write these equations we have chosen the $x$ and $y$ axis in the directions perpendicular and parallel to the SF plane and put the origin in the middle of SF interface to the point, which belongs to the domain wall (see Fig.1).

Equations (1), (2) must be supplemented by the boundary conditions [15]. They have the form

$$
\begin{align*}
\gamma_{BS} \frac{\partial}{\partial x} F_F &= -G_0 \frac{\Delta}{\omega}, \\ \frac{\partial}{\partial x} F_F &= 0,
\end{align*}
$$

(3)

At FF interface ($y = 0, 0 \leq x \leq d_F$) and in the middle of the domains ($y = \pm W/2$, $0 \leq x \leq d_F$) we also have

$$
\gamma_{BF} \frac{\partial}{\partial y} F_F(x,+0) = F_F(x,+0) - F_F(x,-0),
$$

(4)

Here $W$ is the width of the domains, $G_0 = \omega/\sqrt{\omega^2 + \Delta^2}$, $\Delta$ is the modulus of the order parameter of superconducting electrodes. The critical current density, $J_C$, of SIFS Josephson junction is determined by s-wave superconducting correlations at IF interface, which is even function of the Matsubara frequencies

$$
\frac{eJ_C R_N}{2\pi T_C} = \frac{T}{W T_C} \sum_{\omega > 0} \frac{G_0 \Delta}{\omega} \Phi(y),
$$

(6)

where $\Phi(y) = (F_{F,+}(d_F,y) + F_{F,-}(d_F,y))/2$, while the full critical current, $I_C$, is the result of integration of $J_C(y)$ over width of the junction.

$$
\frac{eI_C R_N}{2\pi T_C} = \frac{T}{W T_C} \sum_{\omega > 0} \frac{G_0 \Delta}{\omega} \int_{-W/2}^{W/2} \Phi(y) dy.
$$

(7)

Here, $R_N$, is the normal junction resistance.

**Solution of Usadel equations in FS electrode.** Solution of two-dimensional boundary value problem [11]-[15] in the F layer $(0 \leq x \leq d_F)$ is convenient to find in the form of the Fourier series expansion

$$
F_F(x,y) = \sum_{n=-\infty}^{\infty} A_n(y) \cos \frac{\pi nx}{d_F},
$$

(8)

$$
F_F = \sum_{n=-\infty}^{\infty} B_n(y) \cos \frac{\pi nx}{d_F},
$$

(9)

where

$$
A_n(y) = \frac{Z}{q^+} + a_n \cosh(q_+ \left(y - \frac{W}{2}\right)),
$$

(10)

$$
B_n(y) = \frac{Z}{q^-} + b_n \cosh(q_- \left(y + \frac{W}{2}\right)),
$$

(11)

and coefficients $a_n$ and $b_n$

$$
a_n = -\left[\frac{1}{q^+} - \frac{1}{q^-}\right] \frac{Z q_- S_-}{\delta},
$$

(12)

$$
b_n = \left[\frac{1}{q^+} - \frac{1}{q^-}\right] \frac{Z q_+ S_+}{\delta},
$$

(13)

are determined from boundary conditions [4]. Here the coefficients $\delta, C_\pm$ and $S_\pm$ are defined by expressions

$$
\delta = q_- q_+ \gamma_{BF} S_+ S_- + q_- C_+ S_- + q_+ C_+ C_-,
$$

(14)

$$
C_\pm = \cosh\left(q_\pm \frac{W}{2}\right),
$$

(15)
Taking into account the symmetry relation \( q_-(-\omega) = q_+(\omega) \) for \( s \)-wave superconducting component in the F layer at \( x = d_p \) it is easy to get

\[
\Phi(y \geq 0) = \frac{Z}{2} \sum_{n=-\infty}^{\infty} (1-\eta)^n \left[ \frac{1}{q_+^2 n} + \frac{1}{q_-^2 n} - \frac{1}{q_+^2 n} \frac{\delta_q}{\delta} \right], \tag{16}
\]
\[
\Phi(y \leq 0) = \frac{Z}{2} \sum_{n=-\infty}^{\infty} (1-\eta)^n \left[ \frac{1}{q_+^2 n} + \frac{1}{q_-^2 n} - \frac{1}{q_+^2 n} \frac{\delta_q}{\delta} \right], \tag{17}
\]
\[
\delta_q = q_- S - \cosh(q_+ 2y \mp W) - q_+ S + \cosh(q_+ 2y \pm W). \tag{18}
\]

Finally for the critical current from (17), (16) and (17) we have

\[
el_{C1}R_N = \frac{T}{T_C} \sum_{\omega > 0} G_0 \Delta^2 \omega \Re \frac{1}{\sqrt{\Omega} \sinh (d_F \sqrt{\Omega})}, \tag{19}
\]

while the second

\[
el_{C2}R_N = \frac{4h^2 T}{W_d T_C} \sum_{\omega > 0} G_0 \Delta^2 \omega^2 \sum_{n=-\infty}^{\infty} (-1)^n S_+ S_- q_+^2 q_6 \tag{20}
\]
gives the contribution from the domain wall. Here \( \Re(a) \) denotes the real part of \( a \).

Expression (19) reproduces the well-known result previously obtained for single-domain SIFS structures [13]-[15] thereby demonstrating the independence of the critical current on the orientation of the domains magnetization vectors, if they are collinear oriented and the FF interface is fully opaque for electrons.

**Limit of large \( \gamma_{BF} \).** For large values of suppression parameter \( \gamma_{BF} \gg \max \{1,(W q_\pm)^{-1}\} \) expression (20) transforms to

\[
el_{C2}R_N = \frac{4h^2 T}{W_d T_C} \sum_{\omega > 0} G_0 \Delta^2 \omega^2 \sum_{n=-\infty}^{\infty} (-1)^n S_+ S_- q_+^2 q_6 \tag{21}
\]

The sum over \( n \) in Eq. (21) can be calculated analytically using the theory of residues

\[
el_{C2}R_N = \frac{2h T}{W T_C} \sum_{\omega > 0} G_0 \Delta^2 \gamma_{BF} \gamma_{BS} \omega^2 S_1, \tag{22}
\]

\[
S_1 = \Re \left[ \frac{1}{\sqrt{\Omega} \cosh (d_F \sqrt{\Omega} \sinh (d_F \sqrt{\Omega}))} \right] \tag{23}
\]

It is seen that \( I_{C2} \) is vanished as \( (\gamma_{BF} W)^{-1} \) with increase of \( \gamma_{BF} W \) product and scales on the same characteristic lengths \( \xi_1, \xi_2 \) as the critical current for single-domain SIFS structures [15].

**Limit of small \( \gamma_{BF} \).** In the opposite limit, \( \gamma_{BF} \ll \max \{1,(W q_\pm)^{-1}\} \) we have

\[
el_{C2}R_N = \frac{8h^2 T}{W d_T T_C} \sum_{\omega > 0} G_0 \Delta^2 \omega^2 \sum_{n=-\infty}^{\infty} (-1)^n q_+^2 (q_- + q_+)^n, \tag{24}
\]

contribution to the critical current from domain wall region falls as \( W^{-1} \) and decays in the scale of \( \xi_1 \).

**Limit of small domain width.** In the opposite case, \( W \ll \Re(q_\pm) \), presentation of the critical current as a sum of \( I_{C1} \) and \( I_{C2} \) is not physically reasonable and for \( I_{C2} \) from (13) we get

\[
el_{C2}R_N = \frac{2h T}{T_C} \sum_{\omega > 0} G_0 \Delta^2 \gamma_{BF} d_T \omega^2 S_2, \tag{25}
\]

\[
S_2 = \sum_{n=-\infty}^{\infty} (-1)^n q_+^2 q_6 \left( q_+ S_+ + q_+ S_- + q_+ S_- C_- \right), \tag{26}
\]

where \( \gamma_{BW} = \gamma_{BF} W/2 \). It is seen that for \( \gamma_{BW} \gg 1 \) expression (25) transforms to (19) and \( I_{C} = I_{C1} \), while in the limit \( \gamma_{BW} \rightarrow 0 \) from (25) it follows that the critical current

\[
el_{C2}R_N = \frac{2h T}{2 T_C} \sum_{\omega > 0} G_0 \Delta^2 \gamma_{BF} \gamma_{BS} d_T \omega^2 S_3, \tag{27}
\]

\[
S_3 = \sum_{n=-\infty}^{\infty} (-1)^n \left( q_+^2 + q_-^2 \right) \gamma_{BW} + 4 \left( q_+^2 q_6 \right), \tag{28}
\]

where \( \gamma_{BW} = \gamma_{BF} W/2 \). It is seen that for \( \gamma_{BW} \gg 1 \) expression (25) transforms to (19) and \( I_{C} = I_{C1} \), while in the limit \( \gamma_{BW} \rightarrow 0 \) from (25) it follows that the critical current

\[
el_{C2}R_N = \frac{2h T}{T_C} \sum_{\omega > 0} G_0 \Delta^2 \gamma_{BF} \gamma_{BS} \omega^2 \Omega \sinh (d_F \sqrt{\Omega}), \tag{29}
\]

is independent on exchange energy and falls with increase of \( d_F \) in the same scale as it is for SINS devices. Previously it was found that such transformation of decay length takes place in a vicinity of domain wall [20]-[33]. In particular, it was shown that if a sharp domain wall is parallel [22], [24] or perpendicular to SF interface [33] and the thickness of ferromagnetic layers, \( d_f \lesssim \xi_F \), then for antiparallel direction of magnetization the exchange field effectively averages out,
and the decay length of superconducting correlations becomes close to that of a single nonmagnetic N metal \( \xi_F = \sqrt{D_F}/2\pi T_C \). The same effect may also take place in S-FNF-S variable thickness bridges \(^{34, 35}\).

For arbitrary values of \( \gamma_{BW} \) the sum over \( n \) in \( (25) \) can be also calculated analytically. The denominator in \( (25) \) has the poles at

\[
n = \pm \frac{i d_F}{\pi} \sqrt{\Omega + 1} \pm \frac{1}{\gamma_{BW} h^2}.
\]

Application of the residue theorem to the summation of the series in \( n \) in the expression \( (25) \) leads to

\[
\frac{e I_C R_N}{2\pi T_C} = \frac{T}{2 T_C} \sum_{p > 0} \frac{G_0 \Delta^2}{\gamma_{BSW}^2} \sqrt{1 - \gamma_{BM}^2} \frac{S_1}{p}, \tag{27}
\]

\[
S_1 = \frac{q}{\sqrt{1 + q \sinh (d_F \sqrt{1 + p})}} - \frac{p}{\sqrt{1 + q \sinh (d_F \sqrt{1 + q})}},
\]

\[
p = 1 - \frac{\sqrt{1 - \gamma_{BW}^2 h^2}}{\gamma_{BW}}, \quad q = 1 + \frac{\sqrt{1 - \gamma_{BW}^2 h^2}}{\gamma_{BW}}. \tag{28}
\]

It is seen that for \( \gamma_{BW} h \leq 1 \) s-wave superconducting correlations decay exponentially into the F metal without any oscillations with two characteristic scales, \( \xi_{11} = \xi_F (\Omega + p)^{-1/2} \), and, \( \xi_{12} = \xi_F (\Omega + q)^{-1/2} \). If \( \gamma_{BW} \) tends to zero then one of the damping characteristic scale \( \xi_{11} \) goes to that \( \xi_F \Omega^{-1/2} \) of SINF junctions (see \( (26) \)), while the other \( \xi_{12} \) goes to zero. With \( \gamma_{BW} \) increase \( \xi_{11} \) reduces, whereas \( \xi_{12} \) increases, so that at \( \gamma_{BW} h = 1 \) they become equal to each other \( \xi_{11} = \xi_{12} = \xi_F (\Omega + h)^{-1/2} \). Further increase of \( \gamma_{BW} h \) leads to appearance of the damped oscillations in \( I_C (d_F) \) dependence with the ratio

\[
\frac{\xi_1}{\xi_2} = \frac{\sqrt{\gamma_{BW}^2 h^2 - 1}}{\sqrt{\gamma_{BW} \Omega + 1)^2 + \gamma_{BW}^2 h^2 - 1} + \Omega \gamma_{BW} + 1}, \tag{29}
\]

which monotonically increase from zero at \( \gamma_{BW} h = 1 \) up to that of single domain SINF junctions

\[
\frac{\xi_1}{\xi_2} = \frac{h}{\sqrt{\Omega^2 + h^2 + \Omega}}, \tag{30}
\]

in the limit \( \gamma_{BW} \to \infty \).

From \( (29, 30) \) we can conclude that the existence of domain structure in the F layer of SIFS devices can significantly modify the relation between \( \xi_1 \) and \( \xi_2 \) extracted from experimental studies of \( I_C (d_F) \) dependence in SIFS tunnel junctions.

This conclusion is valid not only in the limit of small domain width.

**Arbitrary values of the domain width.** For arbitrary values of the width of the magnetic domains to calculate the dependence of \( I_C (d_F) \) is necessary to use the general expression \( (18) \). Figure 2 gives the \( I_C (d_F) \) curves calculated for \( H = 10 \pi T_C, \gamma_{BF} = 0 \) and for a set of widths \( W/\xi_F \). It is seen that in full accordance with the analytical analysis given above for \( W \) smaller than \( 0.78 \xi_F, I_C \) falls monotonically with \( W \) increase. At \( W \gtrsim 0.78 \) there is a transformation from a monotonic dependence of \( I_C (d_F) \) to a damped oscillatory one. It is interesting to note that in the vicinity of the transition the critical current decays even faster than for large \( W \). To illustrate this result, we make a fit of the calculated curves by the simple expression

\[
I_C (d_F) = A \exp(-d_F/\xi_1) \cos(d_F/\xi_2 + \varphi),
\]

which is ordinary used for estimation of the decay lengths \( \xi_1 \) and \( \xi_2 \) from an experimental data \( 36, 37 \).

At the first step we define \( \xi_2 \)

\[
\xi_2 = (d_{F2} - d_{F1})/\pi
\]

from the positions of the first, \( d_{F1} \), and the second, \( d_{F2} \), 0-\( \pi \) transitions in \( I_C (d_F) \) dependence and put

\[
\varphi = \pi/2 - d_{F1}/\xi_2
\]

in order to get \( I_C (d_{F1}) = 0 \). The decay length \( \xi_1 \) is determined from the ratio of magnitudes of critical current taken in two points having equal phase of oscillation:

\[
\xi_1 = \pi \xi_2 \ln \left[ \frac{I_C (d_{F1} + \xi_2 \pi/2)}{I_C (d_{F2} + \xi_2 \pi/2)} \right]
\]
and normalization constant $A$

$$A = \frac{I_C(d_F) + \xi_2 \pi/2}{\exp(-d_F/\xi_1) \cos(d_F/\xi_2 + \varphi)}$$

has been determined by direct calculation of magnitude in the certain point between 0-π transitions. If the position of the second 0-π transition exceeds 10 $\xi_F$, we suppose that $\xi_2$ is infinite and $I_C(d_F)$ dependence can be fitted by function

$$J_C(d_F) = A \exp(-d_F/\xi_1).$$

The results of the fitting procedure are presented in Fig.3-Fig.5, which give the decay lengths $\xi_1$ and $\xi_2$ as well as their ratio $\xi_1/\xi_2$ calculated at $T = 0.5 T_C$, $H = 10 \pi T_C$ for a set of suppression parameter $\gamma_{BF} = 0; 0.3; 1$. Thin vertical lines in Fig.3, Fig.4 give values on the x-axis, at which there is a transition from a monotonous exponential decay of $I_C(d_F)$ to the damped oscillation lows. Thin horizontal lines in Fig.3 - Fig.5 provide the asymptotic values of $\xi_1$, $\xi_2$ and $\xi_1/\xi_2$ in the limit $W \gg \xi_F$, which are coincide with the magnitudes calculated for single domain SIFS junction for given temperature $T = 0.5 T_C$ and exchange energy $H = 10 \pi T_C$.

![Fig3](image1)

**Fig3** Dependence of decay length $\xi_1$ as a function of domain width $W$ calculated at $T = 0.5 T_C$, $H = 10 \pi T_C$ and $\gamma_{BF} = 0; 0.3; 1$.

![Fig4](image2)

**Fig4** Dependence of decay length $\xi_2$ as a function of domain width $W$ calculated at $T = 0.5 T_C$, $H = 10 \pi T_C$ and $\gamma_{BF} = 0; 0.3; 1$.

![Fig5](image3)

**Fig5** The ratio of decay lengths $\xi_1/\xi_2$ as a function of domain width $W$ calculated at $T = 0.5 T_C$, $H = 10 \pi T_C$ and $\gamma_{BF} = 0; 0.3; 1$.

It is seen that the transition point at which monotonous decay of $I_C(d_F)$ dependence transforms to a damped oscillation behavior the smaller the larger is suppression parameter $\gamma_{BF}$. Interestingly, in the vicinity of this transition decay length $\xi_1$ is even smaller compare to its magnitude in the limit of large $W$.

It is also necessary to note that despite of the fact that the transition takes place at $W < \xi_F$, the difference between $\xi_1$ and $\xi_2$, as it follows from Fig.5, exists even for large domain width: the ratio $\xi_1/\xi_2$ is only around 0.8 at $W = 4 \xi_F$ and very slowly tends to the following from Fig.5 the single domain value 0.95 with $W$ increase. This fact permits us to conclude that the difference between $\xi_1$ and $\xi_2$ experimentally observed in SFS Josephson structures based on dilute magnetic alloys can be also the consequence of existence of magnetic domains in the F layer.

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1. A. A. Golubov, M. Yu. Kupriyanov, E. Il’ichev, Rev. Mod. Phys. 76, 411 (2004).
2. A. I. Buzdin, Rev. Mod. Phys. 77, 935 (2005).
3. F. S. Bergeret, A. F. Volkov, K. B. Efetov, Rev. Mod. Phys. 77, 1321 (2005).
4. T. Kontos, M. Aprili, J. Lesueur, F. Genet, B. Stephani-dis, and R. Boursier, Phys. Rev. Lett. 89, 137007 (2002).
5. C. Bell, R. Loloe, G. Burnell, and M. G. Blamire, Phys. Rev. B 71, 180501 (R) (2005).
6. V. Shelukhin, A. Tsukernik, M. Karpovski, et al., Phys. Rev. B 73, 174506 (2006).
7. V. A. Oboznov, V. V. Bol’ginov, A. K. Feofanov, V. V. Ryazanov, and A. Buzdin, Phys. Rev. Lett. 96, 197003 (2006).
8. J. W. A. Robinson, S. Piano, G. Burnell, C. Bell, and M. G. Blamire, Phys. Rev. Lett. 97, 177003 (2006).
9. A. A. Bannykh, J. Pfeiffer, V. S. Stolyarov, I. E. Batov, V. V. Ryazanov, and M. Weides, Phys. Rev. B 79, 054501 (2009).
10. F. Born, M. Siegel, E.K. Hollmann, H. Braak, A.A. Golubov, D.Yu Gusakova, and M.Yu Kupriyanov, Phys. Rev. B 74, 140501 (2006).
11. J. W. A. Robinson, F. Chiiodi, M. Egilmez, G. B. Halasz, M. G. Blamire, Sci. Rep. 2, 00699 (2012)
12. Y. Blum, A. Tsukernik, M. Karpovski, and A. Palevski, Phys. Rev. B 70, 214501 (2004).
13. N.G. Pugach, M.Yu Kupriyanov, E. Goldobin, R. Kleiner, and D. Koelle, Phys. Rev. B 84, 144513 (2011).
14. K. D. Usadel, Phys. Rev. Lett. 25, 507 (1970).
15. M. Yu. Kuprianov and V.F. Lukichev, Zh. Eksp. Teor. Fiz. 94, 139 (1988) [Sov. Phys. JETP 67, 1163 (1988)].
16. A. Buzdin and I. Baladie, Phys. Rev. B 67, 184519 (2003).
17. M. Faure, A.I. Buzdin, A. A. Golubov, and M. Yu. Kupriyanov, Phys. Rev. B 73, 064505 (2006).
18. A.S. Vasenko, A.A. Golubov, M.Yu Kupriyanov, and M. Weides, Phys. Rev. B 77, 134507 (2008).
19. A. I. Buzdin, A. S. Mešnikov, and N. G. Pugach, Phys. Rev. B 83, 144515 (2011).
20. N. M. Chchelkatchev and I. S. Burmistrov, Phys. Rev. B 68, 140501(R) (2003).
21. M. Houzet and A. I. Buzdin, Phys. Rev. B 74, 214507 (2006).
22. M. A. Maleki and M. Zareyan, Physical Review B 74, 144512 (2006).
23. I. S. Burnistrov and N. M. Chchelkatchev, Phys. Rev. B 72, 144520 (2005).
24. A.F. Volkov, K.B. Efetov, Phys Rev B 78, 024519 (2008).
25. I.I. Soloviev, N.V. Klenov, S.V. Bakursky, M.Yu Kupriyanov, A.A. Golubov, Pis’ma Zh. Eksp. Teor. Fiz. 101, 258 (2015) [JETP Lett.101, 240 (2015)].
26. B. Crouzy, S. Tollis, D. A. Ivanov, Phys. Rev. B 75, 054503 (2007).
27. J. Linder and K. Halterman, Phys. Rev. B 90, 104502 (2014).
28. T. Baker, A. Richie-Halford, and A. Bill, New J. Phys. 16, 093048 (2014).
29. Ya. M. Blanter and F. W. J. Hekking, Phys. Rev. B 69, 024525 (2004).
30. T. Champel and M. Eschrig, PRB 72, 054523 (2005).
31. Ya. V. Fominov, A. F. Volkov, and K. B. Efetov, Phys. Rev. B 75, 104509 (2007).
32. B. Crouzy, S. Tollis, D. A. Ivanov, Phys. Rev. B 76, 134502 (2007).
33. T. Yu. Karminskaya and M. Yu. Kupriyanov, Pis’ma Zh. Eksp. Teor. Fiz. 85, 343 (2007) [JETP Lett. 86, 61 (2007)].
34. T. Yu. Karminskaya, A. A. Golubov, M. Yu. Kupriyanov, and A. S. Sidorenko, Phys. Rev. B 79, 214509 (2009).
35. A. I. Buzdin, and M. Yu. Kupriyanov, Pis’ma Zh. Eksp. Teor. Fiz. 75, 308 (1991) [JETP Lett. 53, 321 (1991)].
36. A. I. Buzdin, V. V. Ryazanov, Physica C 460, 238 (2007).
Josephson effect in SIFS tunnel junctions with domain walls in weak link region

Fig. 1. Geometry of the considered SIFS Josephson junction and its enlarged part, which includes two halves of domains and domain wall separating them. The insulating barrier I has a small transparency (shown by a blue line).

Fig. 2. Dependence of the critical current of SIFS Josephson junction as a function of thickness of F layer $d_F$ calculated numerically from Eq. (18) for $T = 0.5T_C$, $H = 10\pi T_C$, $\gamma_{BF} = 0$ and for a set of widths $W/\xi_F = 0, 3, 5, 7, 8, 1, 2$.

Fig. 3. Dependence of decay length $\xi_1$ as a function of domain width $W$ calculated at $T = 0.5T_C$, $H = 10\pi T_C$ and $\gamma_{BF} = 0, 3; 1$.

Fig. 4. Dependence of decay length $\xi_2$ as a function of domain width $W$ calculated at $T = 0.5T_C$, $H = 10\pi T_C$ and $\gamma_{BF} = 0; 0.3; 1$.

Fig. 5. The ratio of decay lengths $\xi_1$ and $\xi_2$ as a function of domain width $W$ calculated at $T = 0.5T_C$, $H = 10\pi T_C$ and $\gamma_{BF} = 0; 0.3; 1$. 