CP Violation Beyond the Standard Model and Tau Pair Production in $e^+e^-$ Collisions

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Abstract:
We show that the CP-violating dipole form factors of the tau lepton can be of the order of $\alpha/\pi$ in units of the length scale set by the inverse $Z$ boson mass. We propose a few observables which are sensitive to these form factors at LEP2 and higher $e^+e^-$ collision energies.

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1 Introduction

In order to clarify whether or not the Kobayashi-Maskawa phase in the quark mixing matrix is the sole cause of CP violation in nature, as many CP tests as possible – which are in particular sensitive to other conceivable CP-violating interactions – should be performed, also outside of the kaon system. One possibility is the search for leptonic CP violation. As to the tau lepton a number of proposals have been made in this connection. The OPAL and ALEPH detector groups have demonstrated by detailed investigations of $Z \to \tau^+\tau^-$ at LEP that sensitive CP symmetry tests at the few per mill level can be performed in high energetic $e^+e^-$ collisions. Specifically they have obtained upper bounds on the CP-violating weak dipole form factor of the tau lepton that have recently reached a level well below $10^{-17}$ cm.

In this letter we investigate for a number of CP-violating interactions the possible size of the CP-violating dipole form factors of the tau lepton. Moreover we propose a few observables that are sensitive to these form factors at LEP2 energies and at energies of a presently discussed high luminosity linear $e^+e^-$ collider.

2 Models

Detectable CP violation in tau production and decay requires new CP-violating interactions involving leptons. Here we discuss only possible effects in tau pair production. (For a discussion of tau decay, see[6, 7, 12, 13].) These interactions would induce in the $e^+e^- \to \tau^+\tau^-$ scattering amplitude electric (EDM) and weak (WDM) dipole form factors of the tau lepton through radiative corrections. For a number of models the CP-violating contribution to the one-loop $T$ matrix element is, in the limit of vanishing electron mass, of the form

$$T_{CP} = -e \left[ J^\mu_\gamma \frac{d^Z_\gamma(s)}{s} + \frac{1}{s_W c_W} J^\mu_Z \frac{d^Z_\tau(s)}{s - m_Z^2} \right] \times \bar{u}_\tau(k_\tau) \sigma_{\mu\nu} \gamma_5 k^\nu v_\tau(k_\bar{\tau}),$$

where $J^\mu_\gamma = -\bar{v}_e \gamma^\mu u_e$, $J^\mu_Z = -\bar{v}_e \gamma^\mu (1 - \gamma_5) u_e / 4 - s_W^2 J^\mu_Z$, $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, $k = k_\tau + k_\bar{\tau}$, $s = k^2$. (In the vicinity of the $Z$ resonance the $Z$ width must of course be taken into account in the $Z$ propagator.) The form factors are ultraviolet finite if the interactions are renormalizable. Depending on the model and on the c.m. energy the $d^Z_\tau(s)$ can have also absorptive parts.

Since it is known experimentally that quarks and leptons are pointlike particles up to a scale of order $10^{-16}$ cm one may consider the length scale set by the inverse $Z$ boson mass to be the natural scale for quark and lepton EDMs and WDMs. Therefore we write

$$d_\tau = e \delta \frac{\delta}{m_Z}.$$
In the models discussed below \(d_\gamma^\tau Z\) are generated by radiative corrections at one loop. In general they may be expected to be of the order of a typical electroweak correction, that is, of order \(\alpha/\pi\). Moreover \(d_\gamma^\tau Z\) are chirality-flipping form factors which are, on general grounds, proportional to some fermion mass \(m_F\). Thus we have schematically \(\delta \sim (\alpha/\pi) \times (m_F/m_Z)\). The fermion mass need not be the tau mass but can be the mass of a fermion \(F \neq \tau\) in the loop, which may be much larger than \(m_{\tau}\). (In the case of the one-loop contributions to the EDM \(m_F\) is always the mass of the fermion in the loop.) Hence there is no a priori argument that \(\delta\) must be suppressed by powers of \(m_{\tau}\) contrary to the claim of [13].

These chirality flipping form factors lead to an incoherent contribution to \(d\sigma/d\cos\theta_{\tau}\), which is proportional to \(\sin^2\theta_{\tau}\) and is bilinear in \(d_\gamma^\tau Z\). For \(|\delta| \ll 1\) this distribution is therefore not very sensitive. Moreover, it does not constitute a CP test: a magnetic moment form factor induces a term \(\sim \sin^2\theta_{\tau}\), too. Obviously searches for a term [11] in the scattering amplitude should be done with CP-odd observables whose expectation values are (to good approximation) linear in \(d_\gamma^\tau Z\).

The extremely tiny upper bound on the electric dipole moment of the electron [20] may at first sight discourage searches for CP violation in tau production. However, these searches make sense because CP-violating interactions of non-universal strength are conceivable that induce a tau EDM and WDM being much larger than those of the electron. A prototype of such an interaction is CP violation by an extended Higgs sector, where the symmetry breaking interactions are unrelated to the mixing of fermion generations [21]. For two-Higgs doublet extensions of the Standard Model with natural flavour conservation the real and imaginary parts of the EDM and WDM were computed for the top quark in [22]. The formulae given there can be readily transcribed to the tau lepton. We get that in this type of models \(\delta\) may become as large as \(10^{-4}\).

In supersymmetric extensions of the Standard Model the \(\tau - \bar{\tau}\) neutralino couplings may contain CP phases and thereby generate a non-zero tau EDM and WDM at one loop. (\(\bar{\tau}\) denotes a scalar tau.) The chirality flip is provided by the neutralino mass. Applying the formulae of [23] we obtain that not too far away from the \(\bar{\tau}\) threshold \(\delta\) can be as large as a few \(\times 10^{-4}\) in the case of the EDM.

Larger effects may be induced by leptoquarks. Leptoquark bosons, which mediate quark-lepton transitions, appear naturally in unified and composite models (see e.g. [24]). Here we are interested only in spin zero leptoquarks with \(SU(3)_C \times SU(2)_L \times U(1)_Y\) invariant couplings to quarks and leptons, which are, moreover, baryon and lepton number conserving. We consider two different types of spin zero leptoquarks: a weak isodoublet \(\chi = (\chi_1, \chi_2)\) with quantum numbers \(\chi(3, 2, \frac{\tau}{3})\) (model I) and a weak isosinglet \(\chi_0\) with quantum numbers \(\chi_0(3, 1, -\frac{1}{3})\) (model II). The corresponding interaction Lagrangians involving the fermions of the third generation are [25]:

\[
\mathcal{L}_I = \lambda_1 (\bar{q}_L \cdot \chi) \tau_R + \bar{\lambda}_1 (\chi^T i \sigma_2 \ell_L) \bar{\tau}_R + h.c., \quad (3)
\]

\[
\mathcal{L}_{II} = \lambda_2 (\bar{t}_R \tau_R) \chi_0^\dagger + \bar{\lambda}_2 (\bar{q}_L \cdot \chi) \bar{\ell}_L + h.c. \quad (4)
\]
Here \( q_L = (t, b)_L \) and \( \ell_L = (\nu_\tau, \tau)_L \), the label \( c \) denotes charge conjugation, and \( \sigma_2 \) is the Pauli matrix acting in the weak isospace.

While leptoquarks that couple to the first and second generation of quarks and leptons are strongly constrained in their masses and couplings (see, e.g. [26–27]), the bounds on third generation leptoquarks are less restrictive [24–29]. An analysis of radiative corrections to observables for \( Z \) boson physics leads to the conclusion that the masses of the doublet \( \chi \) (which we assume to be degenerate in mass) and of \( \chi_0 \) cannot be smaller than about 200 GeV if the couplings of these bosons to \( t \) quarks and \( \tau \) leptons are of weak interaction strength [28, 29].

If \( \text{Im}(\bar{\lambda}_i^i\lambda_i) \neq 0 \) then the \( \tau t\chi_1 \) and \( \tau t\chi_0 \) couplings in (3) and (4), respectively, are CP-violating. In this case the following EDM and WDM form factors of the \( \tau \) lepton are induced to one-loop approximation:

\[
d^*_\tau = e m_t N_C \frac{\text{Im}(\bar{\lambda}_1^i\lambda_i)}{8\pi^2} \frac{1}{s/\beta^2_\tau} [Q_t H(s) - Q_\chi K(s)],
\]

\[
d^\tau = e m_t N_C \frac{\text{Im}(\bar{\lambda}_1^i\lambda_i)}{8\pi^2} \frac{1}{s/\beta^2_\tau} \left[(g_t^V H(s) - g_\chi K(s)),
\right.
\]

with

\[
H(s) = B_0(s, m_t^2, m_t^2) - B_0(m_\tau^2, m_t^2, m_\chi^2) - (m_t^2 - m_\tau^2 - m_\chi^2)C_0(s, m_t^2, m_\chi^2, m_\chi^2),
\]

\[
K(s) = B_0(s, m_\chi^2, m_\chi^2) - B_0(m_\tau^2, m_\tau^2, m_\chi^2) - (m_\tau^2 + m_\chi^2 - m_\chi^2 - s/2)C_0(s, m_\chi^2, m_\tau^2, m_\chi^2).
\]

In (7) \( B_0 \) and \( C_0 \) denote the standard scalar 2- and 3-point functions [32]. Further, \( \beta_\tau = (1 - 4m_\tau^2/s)^{1/2} \), \( N_C = 3 \), \( Q_t = 2/3 \), \( g_t^V = 1/4 - 2s_W^2/3 \), and \( g_\chi = T_3^\chi - Q_\chi s_W^2 \). The results for models I and II are obtained by inserting into (5), (6) the quantum numbers \((Q_\chi, T_3^\chi) = (\frac{5}{3}, \frac{1}{2}) \) and \((-\frac{1}{3}, 0)\), of \( \chi_1 \) and \( \chi_0 \), respectively.

The chirality flip is provided by the mass of the top quark. The form factors (5), (6) cross, as functions of the c.m. energy, the \( \chi \bar{\chi} \) and \( t\bar{t} \) thresholds. Above the lower of the two the EDM and WDM develop imaginary parts. In order to illustrate the possible size of the form factors the real and imaginary parts of the EDM and WDM are plotted in Fig.1 for the leptoquark doublet model with \( m_t = 180 \) GeV and choosing \( m_{\chi_1} = 200 \) GeV. Using the results of [28, 29] and taking the CP phase to be maximal, we get \( |\text{Im}(\bar{\lambda}_1^i\lambda_i)| \leq 0.44 \).

Larger couplings are tolerable if \( \chi_1 \) is heavier; but this would not increase the EDM and WDM as compared to the case exhibited in Fig. 1. From a numerical analysis we conclude that in the leptoquark doublet model the real

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\[3\]For discussions of other CP-violating effects due to scalar leptoquarks, see for instance [30, 31].
part of the EDM may be as large as $0.3 \times 10^{-18} e$ cm above the $Z$ resonance. The real part of the WDM is smaller than $\text{Red}_\gamma$ by a factor of about 4. The numerical value of $\text{Red}_\gamma$ in model II is smaller by a factor of 2.8 than in model I.

One may expect that the couplings of the scalar leptoquarks in (3) and (4) are of the Higgs boson type. Then the couplings of $\chi$ and $\chi_0$ would be proportional to the mass of the right-handed fermion involved. That is, $\lambda \sim m_\tau/M$ and $\tilde{\lambda} \sim m_t/M$, where $M$ is some mass scale. Analogous relations hold for the couplings to the first and to the second generation of quarks and leptons. Furthermore one may expect that inter-generation couplings are suppressed by small mixing angles and hence cannot become more important than generation-diagonal couplings. If this is the case, one gets the following scaling relation for the electron, muon, and tau dipole moments:

$$d_e : d_\mu : d_\tau = m_\mu^2 m_e : m_\tau^2 m_\mu : m_\tau^2 m_\tau$$

(8)

This relation indicates that the tau dipole moments can be of the order of a small electroweak radiative correction – i.e. $|\delta| \simeq 0.001$ as obtained above – whereas the electron EDM is severely suppressed by small fermion masses and hence well below the experimental upper bound of $4 \times 10^{-27} e$ cm. The above type of leptoquark couplings have another amusing feature. They generate also EDMs and WDMs of $u$, $c$, and $t$ quarks which are smaller in magnitude than the corresponding moments of the charged leptons within the same generation.

As a final example we mention tau dipole moments due to heavy Majorana neutrinos. These particles appear naturally, for instance, in grand unified theories. These models are in addition endowed with an extended Higgs sector. If there are charged Higgs boson ($H^+$) couplings to a heavy neutrino $N_\tau$ and the tau lepton,

$$\mathcal{L}_N = (2\sqrt{2}G_F)^{1/2}(\beta_1 m_\tau \bar{N}_\tau \tau_R + \beta_2 m_{N_\tau} \bar{N}_\tau \tau_L)H^+ + h.c.,$$

(9)

with $\text{Im}(\beta_1 \beta_2^*) \neq 0$, then non-zero tau EDM and WDM are generated. The chirality flip comes from the mass of the $N_\tau$, which may be of the order of a few hundred GeV. The computation of the moments is straightforward. Their maximal size is of similar order of magnitude as in the leptoquark model above.

The above discussion shows that the tau EDM can be of the order of $d_\tau = e|\delta|/m_Z$ with $|\delta| \simeq \text{a few} \times 10^{-3}$. In the leptoquark models the WDM is smaller by a factor of about four.

### 3 CP-odd correlations

The above form factors can be traced with appropriate observables in tau pair production. Here we consider unpolarized $e^+ e^-$ collisions above the $Z$ boson resonance and decays of $\tau^+$ into the following channels:

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \rightarrow A + \bar{B} + X,$$

(10)
where \( A, B = \pi, \rho, \) and \( \ell = e, \mu \). Generic CP symmetry tests for these reactions are as follows. Consider observables \( \mathcal{O} \) which change sign under a CP transformation. One can prove that in the case of unpolarized and transversely polarized \( e^+e^- \) collisions and CP-invariant phase space cuts

\[
< \mathcal{O} >_{A\bar{B}} + < \mathcal{O} >_{B\bar{A}} \neq 0
\]

is an unambiguous signal of CP violation \[33\]. More specifically, non-zero tau EDM and WDM induce a number of CP-odd spin-momentum correlations in the \( \tau^+\tau^- \) system \[4\]. For instance they lead to non-zero expectation values of the following CP- and T-odd observables involving the \( \tau^\pm \) spins (\( \sigma_i^\pm \) are the Pauli matrices with \( \pm \) referring to the respective spin spaces, and \( \hat{e}, \hat{k} \) are the directions of the incoming positron and of the \( \tau^+ \) in the overall c. m. frame, respectively):

\[
\begin{align*}
\mathcal{O}_1 &= (\hat{e} \times \hat{k}) \cdot (\sigma_+ - \sigma_-), \\
\mathcal{O}_2 &= \hat{k} \cdot \sigma_+ (\hat{e} \times \hat{k}) \cdot \sigma_- - \hat{k} \cdot \sigma_- (\hat{e} \times \hat{k}) \cdot \sigma_+ 
\end{align*}
\]

(12)

Non-zero Red\( _{Y,Z}^\tau \) generate for instance a tau polarization normal to the scattering plane which differs in sign for \( \tau^+ \) and \( \tau^- \). This makes \( < \mathcal{O}_1 > \neq 0 \). Absorptive parts from CP-invariant interactions in the scattering amplitude lead to equal \( \tau^\pm \) normal polarizations and thus cancel in \( < \mathcal{O}_1 > \). An analogous statement applies to the longitudinal-normal spin-spin correlation \( < \mathcal{O}_2 > \). A closer inspection reveals that \( \mathcal{O}_1 \) has a higher sensitivity to Red\( _{Y,Z}^\tau \) than to Red\( _{Y}^\gamma \). For \( \mathcal{O}_2 \) the opposite holds. If one takes the sums instead of the differences \( \|12\| \) one projects onto CP-invariant absorptive parts.

The tau spins are analysed by the decay distributions of the charged prongs. Below we consider only the channels \( \pi\pi, \pi\rho, \pi\ell, \rho\rho \), and \( \ell\ell \) that have a good \( \tau \)-spin analyzer quality. Spin-momentum correlations like \( \|12\| \) can be translated into correlations among the momenta of the charged particles \( A, \bar{B} \) and the charge conjugated modes.

In \[4\] a number of correlations involving momenta in the overall c. m. frame were computed for various \( e^+e^- \) collision energies. If the tau momentum directions are known one can construct observables with a substantially higher sensitivity. For the channels with only two neutrinos in the final state the tau direction of flight can be reconstructed up to a two-fold ambiguity. This ambiguity can in principle be resolved by means of the information obtained from a precise vertex detector \[4\]. Resolution of this ambiguity is, however, not absolutely necessary (for details, see \[4, 13\]).

One can read off from the \( \tau^+\tau^- \) production and decay density matrices the following CP- and T-odd observables for tracing non-zero Red\( _{Y,Z}^\gamma \). (Below \( \hat{p}_+, \hat{p}_- \) denote the momentum directions of the charged final state particles taken in the respective \( \tau^+ \) and \( \tau^- \) rest systems).

\[
\begin{align*}
\mathcal{O}_1^{Re} &= T^{Re} + (\hat{k} \cdot \hat{p}_+)(\hat{k} \times \hat{p}_-) \cdot \hat{e} - (\hat{k} \cdot \hat{p}_-)(\hat{k} \times \hat{p}_+) \cdot \hat{e}, \\
\mathcal{O}_2^{Re} &= T^{Re} + 4(\hat{e} \times \hat{k}) \cdot (\hat{p}_+ + \hat{p}_-),
\end{align*}
\]

(13)

where
\[ T^{\text{Re}} = -\left( \hat{\mathbf{k}} \cdot \hat{\mathbf{e}} \right)^2 (\hat{\mathbf{p}}_+ \times \hat{\mathbf{p}}_-) \cdot \hat{\mathbf{k}} + (\hat{\mathbf{k}} \cdot \hat{\mathbf{e}})(\hat{\mathbf{p}}_+ \times \hat{\mathbf{p}}_-) \cdot \hat{\mathbf{e}}. \] (14)

The following CP- and CPT-odd observables are sensitive to \( \text{Im} d_{\gamma,Z}^{\tau} \):

\[ O_{\text{Im}}^1 = T^{\text{Im}} + (\hat{\mathbf{p}}_+ + \hat{\mathbf{p}}_-) \cdot \hat{\mathbf{e}} - (\hat{\mathbf{k}} \cdot \hat{\mathbf{e}})(\hat{\mathbf{p}}_+ + \hat{\mathbf{p}}_-) \cdot \hat{\mathbf{k}}, \]
\[ O_{\text{Im}}^2 = T^{\text{Im}} + 4(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_+)(\hat{\mathbf{e}} \cdot \hat{\mathbf{p}}_-) - 4(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_-)(\hat{\mathbf{e}} \cdot \hat{\mathbf{p}}_+), \] (15)

where

\[ T^{\text{Im}} = -\left( \hat{\mathbf{k}} \cdot \hat{\mathbf{e}} \right)^2 (\hat{\mathbf{p}}_+ + \hat{\mathbf{p}}_-) \cdot \hat{\mathbf{k}} + (\hat{\mathbf{k}} \cdot \hat{\mathbf{e}})(\hat{\mathbf{p}}_+ + \hat{\mathbf{p}}_-) \cdot \hat{\mathbf{e}}. \] (16)

It is worth recalling that non-zero \( \text{Im} d_{\gamma,Z}^{\tau}(s) \) do not necessarily require a new production threshold \( s_{\text{thr}} < s \). Therefore it makes sense to measure the correlations (13), (15) even if no new threshold has been discovered.

CP violation in tau decay would not leave its mark in the correlations (13), (15). For efficient CP tests in tau decay large samples of highly polarized \( \tau^+ \) and \( \tau^- \) leptons are needed.

We have computed the expectation values of (13), (15) in terms of the form factors for the above-mentioned channels at the LEP2 energy \( \sqrt{s} = 175 \) GeV and at \( \sqrt{s} = 500 \) GeV (an energy relevant for a linear \( e^+e^- \) collider). With these calculations one can estimate the 1 s. d. statistical errors with which the form factors can be measured. The results are given for an assumed number of events in Tables 1 - 4. Moreover, these tables contain also the results obtainable with optimal observables \[35, 15, 18\] that have maximized signal-to-noise ratios.

As to LEP2, one obtains practically the same results as those given in Tables 1,2 at a somewhat higher energy, e.g. \( \sqrt{s} = 190 \) GeV. The expected event numbers at LEP2 are roughly those of Tables 1,2. This means that the real part of the tau EDM can be measured with an accuracy of about \( 2 \times 10^{-17} \) e cm. This would be a new result – no direct measurement of comparable sensitivity is available from LEP1. The leptoquark models discussed above indicate that \( |\text{Red}_{\gamma}^{\tau}(s \simeq 175\text{GeV})| \) may be about eight times larger than \( |\text{Red}_{Z}^{\tau}(s = m_Z)| \). This may serve as an incentive to measure this form factor.

The sensitivity to the EDM and WDM is expected to increase with increasing c. m. energy because, schematically, \( <O_\sim \sqrt{s}d_{\gamma}(s)/e \). In view of the a priori unknown functional dependence on \( s \) of the form factors the accuracy estimates of Tables 3,4 for \( \sqrt{s} = 500 \) GeV may be taken as indication what can be achieved at a linear collider. The numbers show that an interesting level of sensitivity can be reached. The event numbers used for these estimates correspond to an integrated luminosity of about \( 20\text{fb}^{-1} \), which is presently discussed \[36\].

In conclusion, the expectation values of the above observables are of the order \( <O_\sim \sqrt{s}d_{\gamma}(s)/e = (\sqrt{s}/m_Z)^\delta \). We have shown that \( \delta \) can be of the order of a few\( \times 10^{-3} \). At LEP the OPAL and ALEPH experiments have shown that correlations of this type can be measured with an accuracy of
a few per mill. Therefore it is worthwhile to perform such CP tests also at LEP2 and at a future high luminosity linear $e^+e^-$ collider.

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Figure Caption

Fig. 1 The real and imaginary parts of the tau dipole moments due to leptoquarks (model I) in units of Im(\(\tilde{\lambda}_1\lambda_1\)) \(\times 10^{-18}\) cm for \(m_{\chi_1} = 200\) GeV. EDM (a) and WDM (b).
| Channel | Events | Re $d_\gamma^\tau$ [10$^{-18}$ecm] | Re $d_Z^\tau$ [10$^{-18}$ecm] |
|---------|--------|---------------------------------|---------------------------------|
| $\pi - \pi$ | 100 | 44 35 | 17 15 |
| $\pi - \rho$ | 400 | 48 43 | 11 10 |
| $\rho - \rho$ | 400 | 106 95 | 18 17 |
| $\ell - \ell$ | 800 | 142 61 | 17 11 |
| $\ell - \pi$ | 600 | 55 33 | 19 8 |
| $\ell - \rho$ | 1200 | 85 53 | 73 9.8 |
| combined | | 25 18 | 6.9 4.4 |

Table 1: 1 s. d. accuracy with which the real parts of the $\tau$ dipole form factors can be measured at $\sqrt{s} = 175$ GeV for a given number of events. The event numbers include the charge conjugated modes.

| Channel | Events | Im $d_\gamma^\tau$ [10$^{-18}$ecm] | Im $d_Z^\tau$ [10$^{-18}$ecm] |
|---------|--------|---------------------------------|---------------------------------|
| $\pi - \pi$ | 100 | 24 20 | 29 23 |
| $\pi - \rho$ | 400 | 17 15 | 32 28 |
| $\rho - \rho$ | 400 | 28 26 | 71 60 |
| $\ell - \ell$ | 800 | 27 17 | 87 39 |
| $\ell - \pi$ | 600 | 32 12 | 34 21 |
| $\ell - \rho$ | 1200 | 123 15 | 54 3 |
| combined | | 11 6.6 | 16 12 |

Table 2: 1 s. d. accuracy with which the imaginary parts of the $\tau$ dipole form factors can be measured at $\sqrt{s} = 175$ GeV.
| Channel | Events | $O_1^{\text{Re}}$ | Optimal | $O_2^{\text{Re}}$ | Optimal |
|---------|--------|-----------------|----------|-----------------|----------|
| $\pi - \pi$ | 100 | 17 | 13 | 7.5 | 6.6 |
| $\pi - \rho$ | 400 | 18 | 16 | 4.9 | 4.3 |
| $\rho - \rho$ | 400 | 40 | 35 | 7.7 | 7.3 |
| $\ell - \ell$ | 800 | 54 | 22 | 7.3 | 5.0 |
| $\ell - \pi$ | 600 | 21 | 12 | 8.1 | 3.5 |
| $\ell - \rho$ | 1200 | 32 | 20 | 31 | 4.2 |
| combined | | 9.6 | 6.7 | 3.0 | 1.9 |

Table 3: 1 s. d. accuracy with which the real parts of the $\tau$ dipole form factors can be measured at $\sqrt{s} = 500$ GeV.

| Channel | Events | $O_1^{\text{Im}}$ | Optimal | $O_2^{\text{Im}}$ | Optimal |
|---------|--------|-----------------|----------|-----------------|----------|
| $\pi - \pi$ | 100 | 9.1 | 7.1 | 12 | 9.9 |
| $\pi - \rho$ | 400 | 6.5 | 5.3 | 14 | 12 |
| $\rho - \rho$ | 400 | 10 | 9.2 | 30 | 26 |
| $\ell - \ell$ | 800 | 10 | 6.2 | 37 | 17 |
| $\ell - \pi$ | 600 | 12 | 4.2 | 15 | 9.2 |
| $\ell - \rho$ | 1200 | 47 | 5.2 | 23 | 15 |
| combined | | 4.0 | 2.3 | 7.0 | 5.1 |

Table 4: 1 s. d. accuracy with which the imaginary parts of the $\tau$ dipole form factors can be measured at $\sqrt{s} = 500$ GeV.
Fig. 1a

Im $d^\gamma$

Re $d^\gamma$
Fig. 1b