Control and manipulation of electromagentically induced transparency in a nonlinear optomechanical system with two movable mirrors

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We consider an optomechanical cavity made by two moving mirrors which contains a Kerr-down conversion nonlinear crystal. We show that the coherent oscillations of the two mechanical oscillators can lead to splitting in the electromagnetically induced transparency (EIT) resonance, and appearance of an absorption peak within the transparency window. In this configuration the coherent induced splitting of EIT is similar to driving a hyperfine transition in an atomic Lambda-type three-level system by a radio-frequency or microwave field. Also, we show that the presence of nonlinearity provides an additional flexibility for adjusting the width of the transparency windows. The combination of an additional mechanical mode and the nonlinear crystal suggests new possibilities for adjusting the resonance frequency, the width and the spectral positions of the EIT windows as well as the enhancement of the absorption peak within the transparency window.

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I. INTRODUCTION

The coherent interaction of laser radiation with multi-level atoms can induce interesting phenomena such as electromagnetically induced transparency (EIT) and electromagnetically induced absorption (EIA). EIT is a technique for turning an opaque medium into a transparent one and EIA is a technique for enhancement of absorption of light around resonance. These techniques have been used widely to manipulate the group velocity of light for storage of quantum information, and for enhancement of nonlinear processes.

Theoretical studies and technological advances in nanofabrication, laser cooling and trapping have made it possible to reach a considerable control over the light-matter interaction in an optomechanical system. Optomechanically induced transparency (OMIT) and absorption (OMIA) are notable examples of light beam control in optomechanical systems. IN OMIT which has been predicted theoretically and demonstrated experimentally, the anti-Stokes scattering of an intense red-detuned optical "control" field brings about a modification in the optical response of the optomechanical cavity making it transparent in a narrow bandwidth around the cavity resonance for a probe beam. In analogy to the atomic EIT, the happening of OMIT is accompanied by a sharp negative derivative of the dispersion profile of the cavity near the resonance and subluminal group velocity for the probe field. In the atomic EIT the possibility of modification of the probe laser absorption, splitting and reshaping of the EIT peak and reduction of the EIT linewidth have been studied widely.

In this work we are interested in the engineering and control of the probe response, specially OMIT resonance, in the presence of an additional mechanical mode and the Kerr-down conversion nonlinearity. To this end, we consider a cavity with two moving mirrors, driven by a strong coupling and a weak probe field which contains a nonlinear crystal consisting of a Kerr medium and a degenerate optical parametric amplifier (OPA). This exploration is motivated by the following reasons. First, in a recent theoretical work it has been shown that the coherent coupling between the two cavity modes and the mechanical mode of a moving mirror in a double cavity configuration of an optomechanical system leads to the appearance of an absorption peak within the transparency window. In this configuration by changing the power of the electromagnetic field the switching between EIT and EIA is possible. This model is quite general and a variety of systems, which can be modeled by three coupled oscillators, can make the same response. A driven Fabry-Perot cavity with two vibrating mirrors can be effectively described by three coupled oscillators whenever a slight difference in the mechanical frequencies leads to the center-of-mass-relative-motion coupling. In this configuration the two mechanical oscillators are coupled to a single cavity mode.

Second, an OPA inside a cavity can considerably improve the optomechanical coupling, the normal mode splitting (NMS), and the cooling of the mechanical mirror. This kind of cooling process which is accompanied by the enhancement of the effective damping rate of the mirror can be used to increase the width of the transparency window and reduce the group velocity of a propagating probe pulse.

Third, it has been predicted that by tuning the Kerr nonlinearity in an optomechanical cavity one can use the cavity energy shift to reduce the photon number fluctuation and provide a coherently-controlled dynamics for the mirror.

Based on these reasons, we first investigate the effect
of the additional mechanical oscillator on the OMIT resonance. We will show that if the coupling field oscillates close to the mechanical resonance frequencies there are two occasions of two-photon resonance for the probe and coupling lasers. Consequently, the coherent oscillations of the two mechanical oscillators give rise to splitting of the OMIT resonance, and appearance of an absorption peak within the transparency window. The coherent induced splitting of OMIT resonance in this configuration is similar to driving a hyperfine transition in an atomic Λ-type three-level system by a radio-frequency or microwave field.

Then we explore how EIT and EIA resonances respond to the presence of a Kerr-down conversion nonlinearity in the cavity. We will show that in the presence of Kerr-down conversion nonlinearity one can effectively control the width of the transparency window. Also, we demonstrate that to achieve a desirable control over the OMIT resonance the presence of both nonlinearities is needed.

In addition, for the three-mode nonlinear optomechanical system we show that the coherent oscillation of the center-of-mass mode which is responsible for the absorption peak and splitting in the transparency window, increases. This results in the increment of the central peak absorption.

Briefly, the combination of an additional mechanical mode and the nonlinear crystal suggests new possibilities for "engineering" the OMIT resonance.

II. THE PHYSICAL MODEL

The model we consider is an optomechanical cavity with two vibrating mirrors which contains a Kerr-down conversion nonlinear crystal (Fig. 1). The vibrating mirrors are treated as two independent quantum mechanical harmonic oscillators with resonance frequency $\Omega_k$, effective mass $m_k$, and energy decay rate $\gamma_k$ ($k = 1, 2$), coupled to a common cavity mode having the resonance frequency $\omega_0$. The nonlinear crystal is composed of a degenerate OPA and a nonlinear Kerr medium. The cavity mode is coherently driven by a strong input coupling laser field with frequency $\omega_c$ and amplitude $\varepsilon_c$ as well as a weak probe field with frequency $\omega_p$ and amplitude $\varepsilon_p$ through the left mirror. Furthermore, the system is pumped by a coupling beam to produce parametric oscillation and induce Kerr nonlinearity in the cavity. When the detection bandwidth is chosen such that it includes only a single, isolated, mechanical resonance and mode-mode coupling is negligible we can restrict to a single mechanical mode for each mirror so that the mechanical Hamiltonian of the mirrors is given by

$$ H_m = \sum_{k=1}^{2} \left( \frac{p_k^2}{2m_k} + \frac{1}{2}m_k\Omega_k^2 q_k^2 \right). $$

Furthermore, in the adiabatic limit, in which the mirror frequencies are much smaller than the cavity free spectral range $c/2L$ ($c$ is the speed of light in vacuum and $L$ is the cavity length in the absence of the intracavity field) the photon scattering into the other modes can be neglected and we can restrict the model to the case of single-cavity mode $|1\rangle$. We also assume that the induced resonance frequency shift of the cavity and the nonlinear parameter of the Kerr medium are much smaller than the longitudinal-mode spacing in the cavity. It should be noted that in the adiabatic limit, the number of photons generated by the Casimir, retardation, and Doppler effects is negligible. Under this condition, the total Hamiltonian of the system can be written as

$$ H = H_0 + H_1, $$

where

$$ H_0 = \hbar\omega_0 a^\dagger a + \hbar m_c a^\dagger a (q_1 - q_2) + i\hbar (s_{in}(t) a^\dagger - s_{in}^\dagger(t) a), $$

$$ H_1 = i\hbar G (e^{i\theta} a^{\dagger 2} - e^{-i\theta} a^2) + \hbar \eta a^{\dagger 2} a^2. $$

The first term in $H_0$ is the free Hamiltonian of the cavity field with the annihilation (creation) operator $a (a^\dagger)$, frequency $\omega_0$ and decay rate $\kappa$. $H_m$ is the free Hamiltonian of the mirrors given by Eq. (1), the third term describes the optomechanical coupling between the cavity field and the mechanical oscillators due to the radiation pressure force, and the last term in $H_0$ describes the driving of the intracavity mode with the input laser amplitude $s_{in}(t)$. Also, the two terms in $H_1$ describe, respectively, the coupling of the intracavity field with the OPA and the Kerr medium; $G$ is the nonlinear gain of the OPA which is proportional to the pump power driving amplitude, $\theta$ is the phase of the field driving the OPA, and $\eta$ is the anharmonicity parameter proportional to the third order nonlinear susceptibility $\chi^{(3)}$ of the Kerr medium. We will solve this problem for the total driving field $s_{in}(t) = (\varepsilon_c + \varepsilon_p e^{-i(\omega_0 - \omega_s)t}) e^{-i\omega_s t}$, where $\varepsilon_c = \sqrt{2\kappa P_c/\hbar \omega_c}$, $\varepsilon_p = \sqrt{2\kappa P_p/\hbar \omega_p}$ and $P_c$ ($P_p$) are, respectively, the amplitude and power of the input coupling.
the parameters such that only one solution exists and the system is coupled only to the relative motion of the two mirrors and \( \Delta = \omega_0 - \omega_c + g_m(q_1^2 - q_2^2) + 2\eta|a_s|^2 = \Delta_0 + 2\eta|a_s|^2 \) is the effective detuning of the cavity which includes both the radiation pressure and the Kerr medium effects. It is obvious that the optical path and hence the cavity detuning are modified in an intensity-dependent way. Since the effective detuning \( \Delta \) satisfies a fifth-order equation, it can have five real solutions and hence the system may exhibit multistability for a certain range of parameters. In our work we choose the parameters such that only one solution exists and the system has no bistability. Now we consider the perturbation made by the probe field. The quantum Langevin equations for the fluctuations are given by

\[
\delta a = -(i\Delta_1 + \kappa)|\delta a| - ig_ma_s|\delta q_1 - \delta q_2| + (2Ge^{i\theta} - 2im\Omega_2^2|\delta a|^1 + s_{in}(t)),
\]

\[
\delta q_1 = \delta p_k, \quad (k = 1, 2),
\]

\[
\delta p_k = -m_k\Omega_k^2|\delta q_k| + (-1)^k h_{gm}|\delta a^1 + \delta a| - \gamma_k|\delta p_k|, \quad (k = 1, 2),
\]

where \( \delta q_k \) denotes the new equilibrium position of the movable mirrors and \( \Delta = \omega_0 - \omega_c + g_m(q_1^2 - q_2^2) + 2\eta|a_s|^2 = \Delta_0 + 2\eta|a_s|^2 \) is the effective detuning of the cavity which includes both the radiation pressure and the Kerr medium effects. It is obvious that the optical path and hence the cavity detuning are modified in an intensity-dependent way. Since the effective detuning \( \Delta \) satisfies a fifth-order equation, it can have five real solutions and hence the system may exhibit multistability for a certain range of parameters. In our work we choose the parameters such that only one solution exists and the system has no bistability. Now we consider the perturbation made by the probe field. The quantum Langevin equations for the fluctuations are given by

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\]

where \( \Delta_1 = \Delta_0 + 4\eta|a_s|^2 \). It is evident that the cavity mode is coupled only to the relative motion of the two mirrors, and it is therefore convenient to rewrite the above equations in terms of the fluctuations of the relative and center-of-mass coordinates:

\[
\delta Q = \frac{m_1}{M}\delta q_1 + \frac{m_2}{M}\delta q_2, \quad \delta P = \delta p_1 + \delta p_2,
\]

\[
\delta q = \delta q_2 - \delta q_1, \quad \delta \mu = \frac{\delta p_2}{m_2} - \frac{\delta p_1}{m_1},
\]

where \( M = m_1 + m_2 \) and \( \mu = m_1m_2/M \) are the effective masses of the relative and center-of-mass modes, respectively. The linearized quantum Langevin equations for the fluctuation operators of these coordinates take the forms

\[
\langle \delta a \rangle = -(i\Delta_1 + \kappa)|\delta a| + ig_m a_s|\delta q_1 - \delta q_2| + (2Ge^{i\theta} - 2im\Omega_2^2|\delta a|^1 + s_{in}(t)),
\]

\[
\langle \delta q_1 \rangle = \delta p_k, \quad (k = 1, 2),
\]

\[
\delta p_k = -m_k\Omega_k^2|\delta q_k| + (-1)^k h_{gm}|\delta a^1 + \delta a| - \gamma_k|\delta p_k|, \quad (k = 1, 2),
\]

where we have defined the relative motion frequency \( \Omega_c^2 = (m_2\Omega_1^2 + m_1\Omega_2^2)/M \), damping rate \( \gamma_r = (m_2\gamma_1 + m_1\gamma_2)/M \) and also the center-of-mass frequency \( \Omega_{cm}^2 = (m_1\Omega_1^2 + m_2\Omega_2^2)/M \) and damping rate \( \gamma_{cm} = (m_1\gamma_1 + m_2\gamma_2)/M \). The above equations show that even though the cavity mode interacts only with the relative motion mode, there is a coupling between the center-of-mass and relative motion modes when \( \Omega_1 \neq \Omega_2 \) or \( \gamma_1 \neq \gamma_2 \). We will show that the presence of this coupling makes the switching from EIT to EIA possible. Now we use a fairly standard procedure for the investigation of the probe response. Defining \( \delta = \omega_p - \omega_c \), we use the following ansatz

\[
\langle \delta a \rangle = A_+ e^{-i\theta t} + A_- e^{i\theta t},
\]

\[
\langle \delta q_1 \rangle = A_+^* e^{-i\theta t} + A_-^* e^{i\theta t},
\]

\[
\langle \delta q \rangle = q e^{-i\theta t} + q^* e^{i\theta t},
\]

\[
\langle \delta Q \rangle = Q e^{-i\theta t} + Q^* e^{i\theta t}.
\]

In the original frame \( A_- \) and \( A_+ \) oscillate at \( \omega_p \) and \( 2\omega_c - \omega_p \), respectively. Using the input-output relation \([39]\), we obtain

\[
\varepsilon_{out} + \varepsilon_p e^{-i\omega_p t} + \varepsilon_{p} e^{-i\omega_p t} = 2\kappa(a_s + \delta a)e^{-i\omega_t}.\]

Substituting Eq.\([10]\) into Eq.\([9]\), we obtain the following equations

\[
(\Theta + i\delta) A_+ + \Gamma(A_+^* + ig_m a_s q + e_p) = 0,
\]

\[
\Gamma^* A_+ + (\Theta^* + i\delta)(A_+^* - ig_m a_s q) = 0,
\]

\[
h_{ga_s}(A_- + (A_+)^*) + \mu\kappa_2 q + \Lambda Q = 0,
\]

\[
M_{cm}(\delta) Q + \Lambda q = 0,
\]

where we have defined

\[
\Theta = -(\kappa + i\Delta_1),
\]

\[
\Gamma = 2Ge^{i\theta} - 2im\Omega_2^2,
\]

\[
\Lambda = \mu(\Omega_2^2 - \Omega_1^2 + i\delta_2 (\gamma_2 - \gamma_1)),
\]

\[
\chi_{cm}(\delta) = \delta^2 - \Omega_{cm}^2 + i\delta_2 \gamma_{cm}.
\]

From the Eq.\([13c]\) we find that when \( \Omega_2 = \Omega_1 \) and \( \gamma_1 = \gamma_2 \), \( \Lambda = 0 \) and thus the center-of-mass motion is fully
decoupled from the cavity mode and the relative motion. While whenever \( \Lambda \neq 0 \) the three modes are all coupled.

The total output field \( \varepsilon_t \), at the probe frequency is given by

\[
\varepsilon_t = 2\kappa A_\perp /\varepsilon_p = \frac{2\kappa}{d(\delta)} \{ \kappa - i(\Delta_1 + \delta) - i f(\delta) \}, \tag{14}
\]

where

\[
f(\delta) = \hbar g_m^2 a_s^2 / \chi(\delta), \tag{15a}
\]

\[
\chi(\delta) = \mu \chi_x(\delta) - \frac{\Lambda^2}{M \chi_{cm}(\delta)}, \tag{15b}
\]

\[
d(\delta) = (\kappa - i\delta)^2 + \Delta_1^2 - |\Gamma|^2 + 2(\Delta_1 + iM(\Gamma)) f(\delta). \tag{15c}
\]

The real part \( \langle \varepsilon R \rangle \) and imaginary part \( \langle \varepsilon I \rangle \) of the field amplitude \( \varepsilon_t \), respectively, show the absorptive and dispersive behavior of the output field at the probe frequency. These quantities can be measured by homodyne technique \[40\].

The structure of the output field has some main characteristics, arising from the nonlinearity of the system and the freedom in choosing equal or unequal mechanical frequencies and damping rates. To understand the coupling-field-induced modification of the probe response and its structure we present the results and numerical calculations in the next section.

III. RESULTS AND DISCUSSIONS

In this section, we first consider the bare cavity optomechanical system and investigate the condition in which the coherent coupling between the mechanical and optical modes leads to OMIT and OMIA. Then we examine the effects of the Kerr-down conversion nonlinearity on these phenomena.

A. Bare cavity

To simplify our treatment for the bare cavity we can use the reasonable rotating wave approximation (RWA) to neglect the far off-resonance lower sideband \( (A^+ \simeq 0) \) in the resolved sideband regime \( (\kappa \ll \Omega_k, k = 1, 2) \) [13]. In the resolved sideband regime the normal mode splitting occurs [41,13]. In this approximation \( \varepsilon_t \) is simplified to the following form

\[
\varepsilon_t \simeq \frac{2\kappa}{\kappa - i(\Delta_0 - \delta) + i \hbar g_m^2 a_s^2 / \chi(\delta)} . \tag{16}
\]

In what follows we investigate the two cases of equal and different mechanical frequencies separately.

1. Equal mechanical frequencies and damping rates \( (\Lambda = 0) \)

First, we consider the case in which the frequencies and damping rates of the two mechanical oscillators are the same, i.e., \( \Omega_1 = \Omega_2 = \omega_m \) and \( \gamma_1 = \gamma_2 = \gamma_m \). As mentioned before, in this condition the radiation pressure is only coupled to the relative position of the two mirrors and the center-of-mass becomes an isolated quantum oscillator. Therefore \( \chi(\delta) = \mu \chi_x(\delta) \). When \( \omega_p \) is close to the cavity frequency \( (\omega_p \sim \omega_0) \) and the coupling field \( \omega_c \) drives the cavity on its red sideband \( (\Delta_0 \sim \omega_m) \) the structure of the resonance response of the output field \( \varepsilon_t \) is simplified to that of a cavity with one movable mirror and effective mass \( 2\mu \):

\[
\varepsilon_t \simeq \frac{2\kappa}{\kappa - ix + \{ \beta / (\gamma_m / 2 - ix) \}}, \tag{17}
\]

where \( \beta = \hbar g_m^2 a_s^2 / 2\mu \) and \( x = \delta - \omega_m \) is the detuning from the line center. Therefore the denominator of the response function is quadratic in \( x \).

2. Different mechanical frequencies and equal damping rates \( (\Lambda \neq 0) \)

Now we consider the case in which the frequency of the mechanical oscillators is different \( \Omega_1 \neq \Omega_2 \) but their damping rates are equal \( \gamma_1 = \gamma_2 = \gamma_m \). The new aspect of this condition is the coupling between the center-of-mass and the relative motion modes which results in the anomalous EIA in the optomechanical cavity. When \( \omega_p \) is close to the cavity frequency \( (\omega_p \sim \omega_0) \) and the coupling field \( \omega_c \) is red tuned by an amount \( \omega_m = (\Omega_1 + \Omega_2) / 2 \) the response of the system is simplified to the following form

\[
\varepsilon_t \simeq \frac{2\kappa}{\kappa - ix + \frac{2\beta}{\delta_1 x + b_1 - \Lambda^2 / \mu M / \delta_2 x + b_2}}, \tag{18}
\]

where \( \delta_1 = \omega_m + \Omega_r, b_1 = \omega_m^2 - \Omega_r^2 + i\omega_m \gamma_m \) and \( \delta_2 = \omega_m + \Omega_{cm}, b_2 = \omega_m^2 - \Omega_{cm}^2 + i\omega_m \gamma_m \). Therefore the denominator of the response function is cubic in \( x \).

To illustrate the numerical results we show the probe field absorption and dispersion profiles for the bare cavity in Fig.2 for the two cases of equal and different mechanical frequencies. We use the following set of experimentally realizable parameters [44]: \( P_c = 6 \text{ mW}, \lambda = 2\pi c / \omega_c = 1064 \text{ nm}, \Omega_1 = 2\pi \times 10^6 \text{ Hz}, m_1 = m_2 = 12 \text{ ng}, \kappa / \Omega_1 = 0.02, \gamma_1 / 2\pi = \gamma_2 / 2\pi = 200 \text{ Hz}, \) and \( L = 6 \text{ mm} \). The figure clearly shows the splitting of the transparency window due to an additional coherency in the system.

Physically, in the two mode optomechanical system \( (\Lambda = 0) \) when the coupling field \( \omega_c \) is red detuned by an amount \( \omega_m (\Delta_0 \sim \omega_m) \) and \( \omega_p \) is close to the cavity frequency the optomechanical system behaves like a three-level Lambda medium for the probe field as shown in Fig.3. The intense coupling laser field "dresses" the
mechanical mode. In this view, the OMIT can be seen as a level splitting like an Autler-Towns doublet [45], as shown in Fig. 3. The coherent cancellation of the two resonances in the middle of the doublet, at the two-photon resonance, provides the system transmissive in a narrowband around the cavity resonance for the probe field.

Similarly, for the three-mode system (Λ ≠ 0) we can describe the happening of anomalous EIA based on a level diagram structure. In Fig. 4 the |1⟩ ↔ |3⟩ transition is the excitation at cavity frequency and the coupling laser is red tuned by an amount ω_m = (Ω_1 + Ω_2)/2 to induce EIT. The splitting is due to the fact that there are two occasions of two-photon resonance for the coupling lasers.

![FIG. 4. Level diagram structure for the OMIA. The |1⟩ ↔ |3⟩ transition is the excitation at cavity frequency. The coupling laser is red tuned by an amount ω_m = (Ω_1 + Ω_2)/2 to induce EIT. The splitting is due to the fact that there are two occasions of two-photon resonance for the coupling lasers.](image)

B. Nonlinear cavity

Now we investigate the effect of the Kerr-down conversion nonlinearity on the total output field amplitude ε_t. Although the nonlinearity does not alter the level diagram structure of the OMIT, it manifests itself in the steady-state response of the system (Eq. 5), in the optomechanical coupling rate g_m a (Eq. 6), and in the parameter Γ_+ which is responsible for a direct coupling between A_− and A_+ (Eq. 12).
we have plotted the param-

are Ω = 2π × 10^7 Hz and Ω = 1.05Ω. Other parameters are the same as those in Fig.2.

In the OMIT condition the optomechanical coupling rate g_m_a_s is equivalent to the Rabi frequency in the atomic EIT [13]. The dependence of a_s on the nonlinearity can be used to control the width of the transparency window which is related to the effective mechanical damping rate γ_eff. This parameter is approximately given by [18] [19] [51]

\[ \gamma_{\text{eff}} = \gamma_m (1 + C), \]

where \( C = 2\hbar (g_m a_s)^2/m \omega_m \gamma_m \) denotes the optomechanical cooperativity of the cavity. [14] [13] [52]. In Fig.6 we have plotted the absorption profile for different values of \( G, \theta \) and \( \eta \). It shows that by controlling these parameters the width of the transparency window can be increased or decreased in comparison with that of a bare cavity. It should be noted that in the presence of only one of the two nonlinearities we cannot control the transparency window desirably. This can be explained by the fact that according to Eq.(13), in the absence of optomechanical coupling \( (g_m = 0) \) there would be an absorption peak near the modified resonance condition of the cavity \( \delta = \sqrt{\Delta^2_i - |\Gamma|^2} \). Therefore the nonlinear parameters should be chosen such that \( \sqrt{\Delta^2_i - |\Gamma|^2} \approx \Delta \), otherwise the control and probe fields induce a radiation-pressure force oscillating at the frequency \( \delta \), which is not close enough to the resonance frequency of the moving mirrors to induce coherent oscillations in them. This feature leads to disappearance of OMIT in the output probe field.

Also, according to Eq.(12), in the presence of nonlinearity there is a direct coupling between \( A_- \) and \( A_+ \) because of the factor \( \Gamma \). Therefore it seems that in contrast to the bare optomechanical cavity the Stokes scattering of the light from the strong intracavity coupling field is no longer negligible. In Fig.7 we have plotted the parameter \( 2\epsilon |A_+|/\epsilon_p \) as a function of the normalized frequency \( x/\omega_m \) for a bare cavity and a cavity with Kerr-down conversion nonlinearity. As is seen, in the dip of the transparency window \( 2\epsilon |A_+|/\epsilon_p \) reaches its local minimum for a nonlinear cavity and its local maximum for a bare cavity. Hence even though in the presence of nonlinearity outside the OMIT window the lower sideband can also be tuned by the strong coupling field, but the contribution of the Stokes scattering around the cavity resonance is more negligible for a nonlinear cavity.

Now we consider the probe response in the presence of Kerr-down conversion nonlinearity for the second case \( (\Omega_1 \neq \Omega_2) \). As stated before, the OMIT splitting and appearance of the central absorption peak are due to an additional coherent oscillation in the system which is provided by the fluctuations in the center-of-mass mode, i.e.,
FIG. 8. (Color online) The real parts of (a) the normalized parameter $g_m a_s q/\varepsilon_p$, (b) the normalized parameter $g_m a_s Q/\varepsilon_p$ and (c) the field amplitude $\varepsilon_1$ versus the normalized frequency $x/\omega_m$ for a bare cavity ($G = \eta = 0$) (red line) and a nonlinear cavity with $G = 10^7$ Hz, $\eta = 0.09$ Hz, $\theta = \pi/2$ (blue dashed line). The mechanical frequencies are $\Omega_1 = 2\pi \times 10^7$ Hz and $\Omega_2 = 1.06\Omega_1$. Other parameters are the same as those in Fig. 6.

\langle \delta Q \rangle. Figures (a) and (b) illustrate the effect of the nonlinearity on $q$ and $Q$, respectively. They show a shift in the coherent oscillations of $q$ which leads to the broadening of the width of the transparency windows and an increase in the coherent oscillations of the center-of-mass mode $Q$ which results in the enhancement of central peak absorption (Fig. (c)).

In conclusion, we have studied theoretically the effect of an additional mechanical mode and a Kerr-down conversion nonlinear crystal on the EIT resonance in an optomechanical system with two movable mirrors. We have shown that the coherent oscillations of the two mechanical oscillators can lead to splitting in the EIT resonance, and appearance of an absorption peak within the transparency window. This configuration is similar to driving a hyperfine transition in an atomic A-type three-level system by a radio-frequency or microwave field. Also, we have shown that in the presence of Kerr-down conversion nonlinearity by controlling the nonlinear parameters $G$, $\eta$ and $\theta$ the width of transparency can be adjusted to be greater or smaller than that of a bare cavity. The combination of an additional mechanical mode and nonlinear crystal suggests new possibilities for manipulating and controlling the EIT resonance in the optomechanical systems.

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