Research on Spatial Spectrum Constraint of Distributed Passive Radar Imaging

Yanhou Zhang, Shuo Wang*, Hao Zhai, Yusheng Han
Anhui Province Key laboratory of Polarized Imaging Detection Technology
Department of Information Engineering, Army Academy of Artillery and Air Defence Forces Hefei, 230031, China

*Corresponding author e-mail: linuxe_xy@163.com

Abstract. Spatial spectrum is an important tool for radar imaging performance analysis. In order to analyze the effect of spatial spectrum filling on the imaging quality of distributed passive radar, the constraints of spatial spectrum filling are studied. The imaging model of distributed passive radar is established, the imaging resolution characteristics are analyzed, and the influencing factors of spatial spectrum filling are obtained. The filling rule of spatial spectrum is analyzed, and the constraint conditions for the critical azimuth and arc of filling are given. The validity of the constraint conditions is verified by simulation, which has guiding significance for optimization of radar station layout and imaging quality improvement.

1. Introduction
Distributed passive radar is one of the research hotspots of modern new imaging radar [1] [2]. Distributed passive radar does not rely on its own transmitting signals, but uses space radiation sources (such as TV broadcast, satellite and wireless network signals), which is characterized by multiple radiation sources and multiple receivers fully expanding in space relative to the target [3], forming a multi-channel one-time sampling to enhance the target imaging performance. Due to the complexity of imaging configuration of distributed passive radar, the effective characterization of imaging resolution plays a key role in the performance analysis of the whole radar imaging system.

In view of the mapping relationship between the target reflection echo and the target function based on the spatial spectrum [3], the spatial spectrum theory provides a better tool for the imaging performance analysis of distributed passive radar. In reference [4] [5], the factors affecting the distribution range and density of MIMO radar support area are studied, including the number of transceivers, the sampling density and the distribution location of transceivers. Reference [6] [7] further studies the influencing factors of the spatial spectrum distribution of passive radar, analyzes the imaging resolution by using the filling property of spatial spectrum, which can directly reflect the radar imaging configuration and accurately describe the actual resolution; reference [8] [9] establishes the distributed passive radar imaging model based on the echo signal analysis. Compared with the traditional point spread function, the research on distributed passive radar imaging from the perspective of spatial spectrum analysis has the advantages of clear physical expression and clear physical meaning [10]. However, there is no reference on the research on the filling range of echo data.
in the spatial spectrum domain, the applicable range of filling effect and the quantitative relationship between the filling range and imaging resolution.

In order to solve the above problems, this paper establishes a distributed passive radar imaging model from the perspective of target echo signal in polar coordinate system, analyzes the spatial spectrum filling characteristics and resolution characteristics of the distributed passive radar, and summarizes the constraints needed to obtain a relatively dense and uniform spatial spectrum filling form. The relationship between spatial spectrum sampling range and imaging resolution, and the influence of spatial spectrum filling results on the imaging performance of the target are obtained.

2. Distributed passive radar imaging model

The geometry of the distributed passive radar imaging system in spherical coordinates is shown in Figure 1. The imaging system is composed of target, external radiation source and receiver. Suppose the coordinate of one point P of the target is \((r, \theta, \phi)\); the elevation and azimuth of M radiation sources are expressed as \(\theta_i\) and \(\phi_i\) respectively, and the elevation and azimuth of N receivers are expressed as \(\theta_j\) and \(\phi_j\) respectively, then the coordinate of the \(i\)th radiation source is \((r, \theta_i, \phi_i)\), and the coordinate of the \(j\)th receiver is \((r, \theta_j, \phi_j)\).

![Fig. 1 3-D configuration of distributed passive radar imaging](image)

In order to analyze simple and retain the complete configuration information in three-dimensional space configuration, this paper builds the model based on the two-dimensional projection imaging plane in three-dimensional configuration of distributed passive radar. The signal form of the \(i\)th external radiation source is supposed as follow.

\[
S_i(t) = u_i(t)e^{j(2\pi f_0 t + \phi)}
\]  

(1)

If the scattering coefficient of the target in the space is \(\sigma(r, \theta, \phi)\) and the corresponding rectangular coordinate of the two-dimensional plane is \(\sigma(x, y)\), then the echo of the \(i\)th external radiation source signal received by the \(j\)th receiver in the two-dimensional plane reflected by the target is (2).

\[
G_j(t) = \iint_{s} \alpha \cdot \sigma(x, y) \cdot u_i(t - \Delta t_j) \cdot e^{j(2\pi f_0 t - \Delta t_j + \phi)} \, dxdy + n_j(t)
\]

(2)

Above, \(s\) is the plane imaging area illuminated by the radiation source, \(\alpha\) is the echo propagation attenuation coefficient, \(\Delta t_j\) is the propagation delay, \(n_j(t)\) and is the received noise. In the actual
scene, the propagation path of the signal can be approximated in the far field, and the echo can be de-carrier and de-noising [9]. The total scattered echo can be expressed as

$$G(f, \theta_i, \varphi_i, \theta_f, \varphi_f) = \int \sigma(x, y) e^{i2\pi f / [(\cos \theta_i \sin \varphi_i + \cos \theta_f \sin \varphi_f) x + y] / c } \, dx \, dy$$

(3)

Using the dual relationship between time and space, spectrum and wavenumber spectrum, the echo information obtained by multi-channel is converted to spatial spectrum for processing.

$$\begin{cases} k_x = f_i (\cos \theta_i \sin \varphi_i + \cos \theta_f \sin \varphi_f) / c \\ k_y = f_i (\cos \theta_i \cos \varphi_i + \cos \theta_f \cos \varphi_f) / c \end{cases}$$

(4)

In the above formula, $i = 1, \ldots, M; j = 1, \ldots, N$, $(k_x, k_y)$ constitutes the spatial spectral domain of the echo, and $k_x, k_y$ constitute the filling points of the spatial spectral domain. Therefore

$$G(k_x, k_y) = \int \sigma(x, y) e^{i2\pi (k_x x + k_y y) / \lambda_f} \, dx \, dy$$

(5)

$G(k_x, k_y)$ is the observation sample in the spatial spectral domain. From formula (5), we can get the Fourier mapping relationship between the target echo $G(k_x, k_y)$ and the target scattering coefficient $\sigma(x, y)$. Based on this formula, we can realize the inversion imaging of the observation target.

3. Analysis of spatial spectrum characteristics

3.1. Analysis of resolution characteristic

It can be seen from the imaging model that each external radiation source and each receiver constitute a sampling channel, which corresponds to a filling point in the spatial spectrum domain. The distributed multi-channel sampling channel maps to the whole spatial spectrum filling range. If the spatial spectrum filling is relatively uniform, the target image can be inverted by formula (5) using IFFT. In the actual configuration, the spatial spectrum filling has the following relations. Let $\lambda_f = c / f_i, 1 / \lambda_f = \gamma_i$.

$$\begin{cases} (k_x - \gamma_i \sin \varphi_i)^2 + (k_y - \gamma_i \cos \varphi_i)^2 = (\gamma_i \sin \varphi_i)^2 \\ (k_x - \gamma_i \sin \varphi_i)^2 + (k_y - \gamma_i \cos \varphi_i)^2 = (\gamma_i \cos \varphi_i)^2 \end{cases}$$

(6)

If the radiation source, target and receiver are assumed to be in the same plane, then the pitch angle of the transceiver is 0 degree. From equation (4), it can be deduced that the filling point of spatial spectrum in two-dimensional configuration satisfies the following relationship.

$$k_x^2 + k_y^2 = 2(f_i / \gamma_i)^2 \left(1 + \cos(\varphi_i - \varphi_f) \right)$$

(7)

Based on the above derivation and analysis, the following results of the spatial spectrum of any configuration can be obtained by simulating.
Assuming the coverage of the spatial spectrum support domain of echo signal in $k_x$ and $k_y$ dimensions are $\Delta k_{x_{\text{max}}}$ and $\Delta k_{y_{\text{max}}}$ respectively. It is easy to obtain the ideal resolution $d_x$ and $d_y$ of the target inversion image under this configuration.

$$d_x \geq \frac{1}{\Delta k_{x_{\text{max}}}}, \quad d_y \geq \frac{1}{\Delta k_{y_{\text{max}}}}$$

(8)

The theoretical limit resolution of target image in $x$ and $y$ dimensions is defined as

$$\begin{align*}
\rho_x &= \frac{1}{\Delta k_{x_{\text{max}}}} = \frac{1}{(\max\{k_x\} - \min\{k_x\})} \quad \text{(a)} \\
\rho_y &= \frac{1}{\Delta k_{y_{\text{max}}}} = \frac{1}{(\max\{k_y\} - \min\{k_y\})} \quad \text{(b)}
\end{align*}$$

(9)

The results of the above formula can be further approximated. $k_x$ and $k_y$ of formula (4) can be substituted into formula (9) to obtain the denominator.

$$\begin{align*}
\max\{k_x\} - \min\{k_x\} &= \max_{i,j,l} \left\{ f_i (\sin\phi_j + \sin\phi_{j'}) / c \right\} - \min_{i,j,l} \left\{ f_i (\sin\phi_j + \sin\phi_{j'}) / c \right\} \\
&\leq \left( \frac{f_{\text{max}}}{c} \right) \max_{i,j,l} \left\{ \sin\phi_j + \sin\phi_{j'} \right\} - \left( \frac{f_{\text{min}}}{c} \right) \min_{i,j,l} \left\{ \sin\phi_j + \sin\phi_{j'} \right\} \\
&\leq \left( \frac{f_{\text{max}}}{c} \right) (\Delta\phi_j^{\text{max}} + \Delta\phi_{j'}^{\text{max}}) - \left( \frac{f_{\text{min}}}{c} \right) (\Delta\phi_j^{\text{min}} + \Delta\phi_{j'}^{\text{min}}) \\
&= \left( \frac{f_{\text{max}}}{c} \right) (\Delta\phi_j^{\text{max}} + \Delta\phi_{j'}^{\text{max}}) + (\Delta B_j / c) (\Delta\phi_j^{\text{max}} + \Delta\phi_{j'}^{\text{max}})
\end{align*}$$

(10)

Among them, $f_{\text{max}}$ is the highest frequency of the external radiation source $f_i$, $f_{\text{min}}$ is the lowest frequency of the external radiation source $f_i$, $\Delta\phi_j^{\text{max}} = \phi_j^{\text{max}} - \phi_j^{\text{min}}$ and $\Delta\phi_j^{\text{max}} = \phi_j^{\text{max}} - \phi_j^{\text{min}}$ represent the maximum azimuth difference of the external radiation source and receiver respectively, and $\Delta B_j = f_{\text{max}} - f_{\text{min}}$ represents the total bandwidth of the external radiation source. Therefore, the ultimate resolution of the $x$-axis is as follows.

$$\rho_x = \frac{1}{(f_{\text{max}} (\Delta\phi_j^{\text{max}} + \Delta\phi_{j'}^{\text{max}}) / c + B_j (\Delta\phi_j^{\text{max}} + \Delta\phi_{j'}^{\text{max}}) / c)}$$

(11)

Similarly, the ultimate resolution of $y$-axis can be obtained.

$$\rho_y = \frac{1}{(f_{\text{min}} (\Delta\phi_j^{\text{max}} + \Delta\phi_{j'}^{\text{max}}) / 2c + 2B_j / c)}$$

(12)
Although equation (11) and equation (12) cannot reflect the actual resolution of imaging, it is further proved that the resolution of target imaging is affected by the frequency of external radiation source and the distribution of transceiver array (the spread characteristics of transceiver space), the bandwidth of distributed passive radar radiation source and the virtual aperture composed of multiple imaging channels form a complementary relationship, while changing the number of transceiver. It is necessary to consider the distribution of the azimuth of the transceiver, which seriously affects the filling effect of the spatial spectrum, and then affects the horizontal and vertical imaging resolution of the target.

3.2. Analysis of constraint condition

From the analysis of the previous section and the further analysis of equation (7), it can be found that the filling points of the spatial spectrum in the two-dimensional configuration are distributed on different circles with the center as the origin and the radius as $2(f_i/c)^2[1 + \cos(\phi) - \phi]$. Under the limit conditions, if there are enough transceivers and the external radiation source is a fixed frequency signal, the distribution of the spatial spectrum will be a circle with the maximum radius as $2f_i/c$, and the simulation is shown in Fig. 3. The ultimate resolution obtained by filling spatial spectrum is $c/4f_i$.

![Fig. 3 Full filling of 2D spatial spectrum](image)

Equation (6) can be simplified as follows.

$$
\begin{align*}
\left( k_x^2 - \sin \phi \right)^2 + \left( k_y^2 - (f_i/c) \cos \phi \right)^2 &= (f_i/c)^2 \\
\left( k_x^2 - (f_i/c) \sin \phi \right)^2 + \left( k_y^2 - (f_i/c) \cos \phi \right)^2 &= (f_i/c)^2 
\end{align*}
$$

(13)

The filling points of spatial spectrum must meet the above equations and each channel filling points are obtained by the intersection of two arcs whose centers are $((f_i/c) \sin \phi, (f_i/c) \cos \phi)$ and $((f_i/c) \sin \phi, (f_i/c) \cos \phi)$ respectively and whose radius is $f_i/c$. As shown in Fig. 4, the red dotted line is the receiver, the blue dotted line is the external radiation source, and the black box is the spatial spectrum filling point, that is, any filling of the spatial spectrum domain is above the intersection of the two arcs. The position of the outer radiation source filling the center of the circle is determined by the azimuth of the outer radiation source and the carrier wavelength of the outer radiation source, while the position of the receiver filling the center of the circle is determined by the azimuth of the receiver and the carrier wavelength of the outer radiation source. The center locus of these two arcs meets the circle with the original point as the center and $f_i/c$ as the radius, as shown in the black solid line circle in the figure. When the azimuth of the radiation source and the receiver are the same, i.e.
similar to the common configuration of the transceiver, the filling circle of the radiation source and the filling circle of the receiver are overlapped. The intersection point is the intersection point of the extension line connecting the origin and the filling circle center and the filling circle. At this time, the distance between the intersection point and the filling circle center is $2f_i/c$. It is further verified that the maximum filling radius of the spatial spectrum field is $2f_i/c$ under the limit condition.

![Fig. 4 Schematic diagram of spatial spectrum filling](image)

![Fig. 5 Schematic diagram of the intersection of receiving and sending filling arcs](image)

It can be seen from Fig. 4 that the filling point of spatial spectrum domain is located on multiple arcs under two-dimensional condition, and it will show non-uniform characteristics after mapping to two-dimensional grid. When the relationship between the azimuth angle of external radiation source $\varphi_i$ and the azimuth angle of receiver $\varphi_j$ satisfies $|\varphi_i - \varphi_j| = \pi / 2$, the tangent of the filling arc of external radiation source and the filling arc of receiver at the intersection point is vertical, as shown in Fig. 5, the spatial spectrum is relatively uniform. Based on this, a constraint condition is proposed to form a relatively regular uniform spatial spectrum.

In order to obtain a relatively uniform spatial spectrum filling form, the four corners of the filling domain can be obtained from Fig. 2, which are determined by the difference $\Delta \varphi = |\varphi_i - \varphi_j|$ between the azimuth of external radiation source and receiver. Therefore, only the critical conditions
\[ |\phi_{\max} - \phi_{\min}| = \pi / 2 \text{ and } |\phi_{\min} - \phi_{\max}| = \pi / 2 \] of azimuth distribution need to be satisfied, which is the basic spatial spectrum constraint conditions. If the basic conditions are satisfied, in order to obtain a more compact and uniform spatial spectrum filling pattern, the number of external radiation sources and receivers should be equal, and the distribution of station azimuth should be equal.

Further research and analysis, based on the filling shown in Fig. 2, the four sides of the spatial spectrum filling domain outline basically determine the size and shape of the spatial spectrum filling region. The distribution range of spatial spectrum is fixed when the frequency of radiation source and the azimuth distribution range of transceiver are fixed, that is, the maximum values of \( k_x \) and \( k_y \) are fixed. When the number of transceivers is changed, only the density of spatial spectrum in this fixed area is changed. The radial of each side is determined by the azimuth of the radiation source and the receiver. The curvature formula (14) of the arc can be defined to calculate the distortion of the spatial spectrum filling area. The smaller the curvature \( K \) is, the closer the difference of azimuth \( \Delta \phi \) is to \( \pi / 2 \). The closer the filling area is to rectangle, the more regular and uniform the filling of the whole spatial spectrum.

\[ K = \frac{(k_x' - k_x)(k_y' - k_y)}{((k_x'^2 + k_y'^2)^{3/2})} \tag{14} \]

4. Simulation experiment and analysis

In order to verify the effectiveness of the spatial spectrum constraint conditions proposed in this paper, the air target is selected as the research object, and its two-dimensional strong scattering point model is given. Nine coordinate points (- 4, - 2), (4,0), (2, - 2), (2,2), (4,0), (6,0) are selected to form the target, as shown in Fig. 6.

The base station signal of terrestrial television is selected as the external radiation source, and the frequency of the external radiation source is in the range of 480 MHz ~ 632 MHz, and the distribution range and number of different azimuth angles of the transceiver are respectively set for simulation experiment. Set the number of radiation sources as \( M = 12 \), and the number of receivers as \( n = 12 \). Set the distribution range of transmitter azimuth as \( \phi_i \in [\pi / 4, \pi / 2] \), \( \phi_i \in [\pi / 2, 3\pi / 4] \), \( \phi_i \in [3\pi / 4, \pi] \), \( \phi_i \in [\pi, 5\pi / 4] \) in turn, and require equal interval distribution. The distribution range of receiver azimuth is \( \phi_i \in [0, \pi / 4] \), where \( i = 1... M, j = 1... N \). Then the relationship between the radiation source and the receiver azimuth meets \( |\phi_i - \phi_j| = \pi / 4 \), \( |\phi_i - \phi_j| = \pi / 2 \), \( |\phi_i - \phi_j| = 3\pi / 4 \), \( |\phi_i - \phi_j| = \pi \), and the simulation results are shown in Fig. 7(a)(b)(c)(d) in turn.
It can be seen from Fig. 7 that, in order to obtain a relatively dense and uniform spatial spectrum filling pattern, on the premise of equal spacing between the transmitter and the receiver's station layout azimuth angle, the tangent of the transmitter and the receiver's filling arc at the intersection point should also meet the orthogonal relationship as far as possible, which is the basic constraints in this paper.

Then set the azimuth distribution range of 20 radiation sources as $\phi_i \in [-3\pi/5, -\pi/2]$, the azimuth distribution range of 10 receivers as $\phi_j \in [-\pi/10, 0]$, change the number of receivers, equal interval distribution, and the simulation results are shown in Fig. 8.
Based on the simulation results of Fig. 7 and Fig. 8, it can be found that the relatively uniform spatial spectrum distribution can be obtained only by ensuring that the azimuth difference between the radiation source and the receiver meets the critical conditions. Changing the number of transceivers does not affect the filling shape of the entire spatial spectrum, but the density of the spatial spectrum distribution changes. In addition, from the distribution range of the receiver, it can be found that $K_r$ is limited by the distribution range of the radiation source and receiver azimuth, and the smaller the curvature is, the smaller the radian is. Therefore, when the distribution range of the radiation source and receiver azimuth is small, the radian of the filling arc will be smaller, which shows that the whole arc is relatively straight, that is to say, the azimuth of the transceiver configuration can ensure a more regular space under the condition of small corner. Interspectral filling region.

For the above objects filled with different spatial spectrum (From left to right are the filling results of Figure 7 (a) (b) (d) and figure 8 (b), respectively.), point by point matching filtering method [9] is used for inversion imaging, and the imaging results are as follows.

![Fig. 9 Target imaging results](image)

The simulation results show that the spatial spectrum (more regular and dense) constructed under the constraints can obtain better imaging quality and higher target inversion resolution.

5. Conclusion
Through the above analysis and experiments, it can be concluded that in order to obtain a relatively compact and uniform spatial spectrum filling form, one is to have a sufficient number of radiation sources and receivers, with equal spacing distribution of azimuth, the other is to meet the critical orthogonal relationship of the difference between the azimuth of radiation sources and the azimuth of
receivers, and the third is to meet the distribution of small azimuth angle or in a limited opening angle.
If we want to get a relatively regular filling form of spatial spectrum and a large enough support area,
we need to make a reasonable compromise on the distribution of each other's position angle. Finally,
the validity of the spatial spectrum filling constraint is verified by the simulation results, and the
imaging quality of the distributed passive radar can be improved by the reasonable arrangement of the
transceiver.

Acknowledgments
The work is supported by the Natural Science Foundation of Anhui Province under Grant No.
1808085MF163.
The corresponding author is Shuo Wang*.

References
[1] S. Brisken, M. Moscadelli, V. Seidel and C. Schwark, "Passive radar imaging using DVB-S2,"
2017 IEEE Radar Conference (RadarConf), Seattle, WA, 2017, pp. 0552-0556.
[2] D. Cristallini, I. Pisciottano and H. Kuschel, "Multi-Band Passive Radar Imaging Using Satellite
Illumination," 2018 International Conference on Radar (RADAR), Brisbane, QLD, 2018, pp.
1-6.
[3] Z. Yin, H. Zhao, T Wang and W. Chen, "Passive imaging using multiple navigation satellites as
illuminator of opportunity," Signal Processing (ICSP) 2014 12th International Conference on,
2014, pp. 2026-2030.
[4] Y. Wu, D.C. Munson, “Multistatic synthetic aperture imaging of aircraft using reflected
television signals,” Proceedings of SPIE - The International Society for Optical Engineering,
2001, pp.1-13.
[5] C. Yang, J. Shen, Z. Jiang, C. She and X. Zou, "A Multi-Channel Radar Forward-Looking
Imaging Algorithm Based on Super-Resolution Technique," 2018 China International SAR
Symposium (CISS), Shanghai, 2018, pp. 1-4.
[6] H. Xu, C. Liu, X. He, D. Wang and W. Chen, "Analysis of Distributed Passive Radar Imaging
and Space-spectrum Property," Modern Radar, vol.33(9), pp. 42-47, 2011.
[7] S. Wang, J. Wang, "Distributive passive radar imaging algorithm based on multiple external
illuminators, " Journal of XiDian University(natural science), vol.33(6), pp. 907-910,2006.
[8] S. Wang, W. Chen, "Reconstruction characteristic and station layout optimization of distributed
radar sparse imaging," Journal of University of Science and Technology of China, vol.44(4),
pp. 303-309, 2014.
[9] T. Wang, B. Liu, X. Ling, Y. Liu and W. Chen, "Sparse Passive Radar Imaging Under
Transceiver Position Errors," Radar Science and Technology, vol.16(3), pp. 29-34,2018.
[10] M. Conti, F. Berizzi, M. Martorella, E. D. Mese, D. Petri and A. Capria, "High range resolution
multichannel DVB-T passive radar," in IEEE Aerospace and Electronic Systems Magazine,
vol. 27, no. 10, pp. 37-42, Oct. 2012