How to Directly Measure Kondo Cloud’s Length

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We propose a method to directly measure, by electrical means, the Kondo screening cloud formed by an Anderson impurity coupled to semi-infinite quantum wires, on which an electrostatic gate voltage is applied at distance \( L \) from the impurity. We show that the Kondo cloud, and hence the Kondo temperature and the electron conductance through the impurity, are affected by the gate voltage, as \( L \) decreases below the Kondo cloud length. Based on this behavior, the cloud length can be experimentally identified by changing \( L \) with a keyboard type of gate voltages or tuning the coupling strength between the impurity and the wires.

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Introduction.— The Kondo effect is a central many-body problem of condensed matter physics.\(^1\)\(^2\). It involves a spin singlet, formed by the spin-spin interaction between a magnetic impurity and surrounding conduction electrons. Deeper understanding of the effect has been achieved by using a quantum dot, that hosts a magnetic impurity spin, under systematic control.\(^3\)\(^–\)\(^6\).

Although the Kondo effect is well known, its spatial features still remain to be addressed. The Kondo spin singlet is formed below the energy scale of Kondo temperature \( T_K \). This implies that the singlet is spatially formed over a conduction-electron region of length scale \( \xi_K = \hbar v_F/(k_B T_K) \); when \( T_K \approx 1 \) K and the Fermi velocity \( v_F \approx 10^5-10^6 \) m/s, \( \xi_K \approx 1 \) \( \mu \)m. The region is called the Kondo screening cloud. There have been several proposals\(^7\)\(^–\)\(^10\) for ways to detect the cloud.

Despite the proposals, there has been no conclusive measurement supporting the existence of the Kondo cloud.\(^2\)\(^2\)\(^2\)\(^2\)\(^2\). The difficulty to detect the cloud arises because it is a spin cloud showing quantum fluctuations with zero averaged spin. The cloud manifests itself in the spin-spin correlation between the impurity and the conduction electrons. However it requires measurements of spin dynamics of time scale \( \hbar/(k_B T_K) \). STM studies probing local density of states may be useful for detecting the cloud.\(^12\)\(^–\)\(^14\)\(^2\)\(^4\)\(^2\)\(^2\). Recent STM measurements\(^2\)\(^5\)\(^2\)\(^8\)\(^2\)\(^8\)\(^2\) show the Kondo effect in the region away from a magnetic impurity, whose spatial extension is however much shorter than \( \xi_K \). Another direction is to study a magnetic impurity in a finite-size system.\(^15\)\(^–\)\(^2\)\(^1\). Because the cloud cannot extend beyond the finite size, the Kondo effect is strongly affected, and suppressed when the system is shorter than \( \xi_K \). There has been no conclusive experimental detection of \( \xi_K \) in this direction.\(^2\)\(^1\).

In this work, we propose a new way of detecting the Kondo cloud, based on the intuition that a change of conduction electrons inside the cloud will affect the Kondo effect. We consider a Kondo impurity formed in a quantum dot coupled to two semi-infinite ballistic quantum wires with electron tunneling amplitude \( t_{WD} \) (see Fig. 1). Electrostatic gate voltages \( V_g \) are applied to the wires (or to only one wire) at distance \( L \) from the dot, modifying indirectly the local density of states \( \rho(\epsilon) \) of conduction electrons nearby the dot (Fig. 2). We find that \( V_g \) does not affect the cloud, when \( L \gg \xi_K \). However, when \( L \ll \xi_K \), the cloud, hence Kondo temperature \( T_K \) and electron conductance \( G \) through the dot, are sensitive to \( V_g \). The crossover between the two regimes occurs at \( L \approx \xi_K \). By measuring \( G \) or \( T_K \) with varying \( L \) or \( t_{WD} \) (Figs. 3 and 4), one can detect the crossover and \( \xi_K \). We use the poor man scaling,\(^2\)\(^9\) numerical renormalization group study (NRG)\(^3\)\(^0\)\(^–\)\(^3\)\(^2\)\(^,\)\(^3\)\(^4\)\(^,\)\(^3\)\(^5\)\(^,\)\(^3\)\(^6\)\(^,\)\(^3\)\(^7\)\(^\)\(^,\)\(^3\)\(^8\)\(^,\)\(^3\)\(^9\)\(^,\)\(^3\)\(^1\)\(^,\)\(^3\)\(^2\)\(^,\) Fermi liquid theory\(^3\)\(^3\), and Fermi liquid theory (Fig. 1).

The setup.— We describe the wires by the tight-binding Hamiltonian with sites \( j \)’s in wire \( i = l, r \),

\[
H_W = \sum_{i=l,r} \sum_{j=1}^{\infty} \sum_{\sigma = \uparrow, \downarrow} \epsilon_0 n_{ij\sigma} + \langle -t_{ij} c_{ij\sigma}^\dagger c_{i(j+1)\sigma} \rangle + \text{H.c.} \\
- eV_g \sum_{i=l,r} \sum_{j=0}^{\infty} \sum_{\sigma = \uparrow, \downarrow} n_{ij\sigma},
\]

where \( c_{ij\sigma}^\dagger \) creates an electron with spin \( \sigma \) and energy \( \epsilon_0 \) at site \( j \) in wire \( i \), \( n_{ij\sigma} = c_{ij\sigma}^\dagger c_{ij\sigma} \), and \( t \) is the hopping energy. The last term describes the gate voltage \( V_g \) applied in \( |x| > L = Na \), where \( a \) is the lat-

![FIG. 1: (Color Online) Setup for detecting the Kondo cloud. A quantum dot located at \( x = 0 \) hosts a magnetic impurity spin. It couples to quantum wires along \( \hat{x} \) axis, with electron tunneling amplitude \( t_{WD} \). Gate voltage \( V_g \) is applied at distance \( L \) from the dot (in \( |x| > L \)). The Kondo effect becomes sensitive to \( V_g \), as \( L \) decreases below the cloud length.](https://example.com/fig1)
tice spacing. $V_g(x)$ changes at $x = L$ abruptly over the length shorter than the Fermi wave length. The dot Hamiltonian $H_D$ is modeled by the Anderson impurity $[34]: H_D = \sum_{\sigma=\uparrow,\downarrow} \epsilon_d d_\sigma^\dagger d_\sigma + U n_d^\dagger n_d^\phantom{\dagger}$, where $d_\sigma^\dagger$ creates an electron with energy $\epsilon_d$ and spin $\sigma$ in the dot, $n_d = d_\sigma^\dagger d_\sigma$, and $U$ is the electron repulsive interaction. $H_T = -t_{WD} \sum_{\sigma=\uparrow,\downarrow} (c_1^\dagger \sigma d_\sigma^\dagger \text{H.c.) describes} \quad \text{electron tunneling between the wire and the dot.}$

The dot is occupied by a single electron in the Coulomb blockade regime of $\epsilon_d < \epsilon_F$ and $\Gamma(\epsilon_F) \ll -\epsilon_d + \epsilon_F$, $U + \epsilon_d - \epsilon_F$, where $\Gamma(\epsilon) = 2\pi|t_{WD}|^2\delta(\epsilon)$ is the hybridization function between the dot and the wires, the total Hamiltonian of the setup becomes $[2,6,35]

\begin{align*}
H = H_D + H_W + H_T \approx J \vec{s} \cdot \vec{S} + V \sum_{i,\sigma} n_{i,\sigma} + H_W. (2)
\end{align*}

Here, the Kondo impurity spin, $\vec{s} = \sum_{\sigma,\sigma'} d_\sigma^\dagger \vec{\sigma}_{\sigma\sigma'} d_{\sigma'}$, couples with the spin of the neighboring conduction electrons, $\vec{S} = \sum_{\sigma} (c_{1,\sigma}^\dagger + c_{1,\sigma}) \vec{\sigma}_{\sigma\sigma} (c_{1,\sigma} + c_{1,\sigma})/2$, with strength $J = 2|t_{WD}|[-1/(\epsilon_d - \epsilon_F) + 1/(U + \epsilon_d - \epsilon_F)]$. The second term describes the potential scattering with strength $V = t_{WD}^2[-1/(\epsilon_d - \epsilon_F) - 1/(U + \epsilon_d - \epsilon_F)]/2$.

Local density of states.— We show how the gate voltage $V_g$ changes the local density of states $\rho(\epsilon)$ at sites $j = 1$ of the wires. We calculate $\rho(\epsilon)$ by, matching the single-particle wavefunctions of $H_W$ between $j = N$ and $N + 1$, $\rho(\epsilon) = \frac{\sin(q)a^2}{\pi a^2 \sin^2(ka) + \frac{2V_g}{\epsilon} \sin[k(N+1)a] \sin(kNa)]. (3)$

$k$ and $q$ are the wavevectors in $|x| < L$ and $|x| > L$, respectively, satisfying $\epsilon = \epsilon_0 - 2t \cos(ka) = \epsilon_0 - eV_g - 2t \cos(qa)$. $\Gamma(\epsilon) = 2\pi t_{WD}\rho(\epsilon)$ shows resonances with level spacing $\Delta \equiv \pi \hbar v_F/L$; see Fig. [2]. The oscillation amplitude of $\Gamma(\epsilon)$ is proportional to $V_g$. The value of $\Gamma(\epsilon_F)$ of the $L \rightarrow \infty$ limit is denoted as $\Gamma_{\infty}$, which equals the average of $\Gamma(\epsilon)$ around $\epsilon_F$ for finite $L$.

We sketch how the change of $\rho(\epsilon)$ by $V_g$ affects the Kondo effect in different regimes of $L$. For $L \gg \xi_K$ (i.e., $T_{K\infty} \gg \Delta$), the Kondo temperature is determined by $\Gamma_{\infty}$ as $T_{K\infty} \sim \sqrt{\Gamma_{\infty}U/2\pi} \exp[\pi(\epsilon_d - \epsilon_F)(U + \epsilon_d - \epsilon_F)/(2V_{\infty}U)]$, and the cloud size is $\xi_{K\infty} = \hbar v_F/(k_B T_{K\infty})$. In this regime, the average $\Gamma_{\infty}$ of many resonances of $\Gamma(\epsilon)$ around $\epsilon_F$ determines the Kondo effect, sensitively to $V_g$. On the other hand, for $L \ll \xi_K$ (i.e., $T_{K\infty} \ll \Delta$), the resonance of $\Gamma(\epsilon)$ located at $\epsilon_F$, namely $\Gamma_0 \equiv \Gamma(\epsilon_F)$, determines the Kondo effect, resulting in $T_K = T_{K0} \sim \sqrt{\Gamma_0 U/2\pi} \exp[\pi(\epsilon_d - \epsilon_F)(U + \epsilon_d - \epsilon_F)/(2V_{0}U)]$ and $\xi_K = \hbar v_F/(k_B T_K)$. In this case, $V_g$ affects conduction electrons within the cloud, and modifies $T_K$.

We will discuss how $T_K$ changes between $T_{K0}$ and $T_{K\infty}$ as a function of $L/\xi_K$ in the two possible situations, case A where one changes $L$ with keeping $t_{WD}$ constant, and case B where $t_{WD}$ changes and $L$ remains constant.

**Kondo temperature.—** We compute $T_K$, using the poor man scaling and the NRG $[32]$. In the poor man scaling, the renormalization of $J \rightarrow J + J^2(\int_{D_0} - \int_{D_{\infty}})\delta(\epsilon_F)/|\epsilon|$ is performed with reducing the energy bandwidth of the wire from $D_0$ to $D$, and stopped at the bandwidth where $J^2 \int \delta(\epsilon_F)/|\epsilon|$ is comparable with $J$. The final bandwidth provides $T_K$,

\begin{align*}
\ln\frac{T_K}{T_{K\infty}} &\approx \frac{eV_g \cos(k_F(2L + a))}{2t \sin^2(k_F a)} \pi L \\ &\equiv \frac{2L}{\xi_K} \cos(k_F(2L + a)) (4)
\end{align*}

where $\xi_K \equiv \int_{-\infty}^\infty \frac{dy}{\cos(y)/y'}$, $k_{F,n} = \frac{2\pi n}{2L + a}$, and $n$ is an integer. Equation (4) is obtained by putting $k \rightarrow k_F \equiv k(\epsilon_F)$, $q \rightarrow q(\epsilon_F)$, $L \gg a$, $V_F \equiv \frac{1}{k_B T_K} |k_F| = (2at/\hbar) \sin(k_F a)$, $|eV_g| \ll 2t \sin^2(k_F a)$, and the linearization of $\epsilon \equiv \epsilon_F + \hbar v_F(k_F - k_{F,0})$ into Eq. (3); $k \rightarrow k_F$ and $q \rightarrow q(\epsilon_F)$ are valid within the small energy scale of $T_K$. We remark that $\xi_K$ depends on $L$ and $V_g$ in Eq. (4).

In Eq. (4), the term $2L/\xi_K$ gives the information on the cloud, while another $L$ dependence of the $2k_F$ oscillation appears because resonance centers in $\Gamma(\epsilon)$ shift across $\epsilon_F$ as $L$ changes. One can focus on the former. In case A, where one changes $L$, one can reduce the effect of the $2k_F$ oscillation, by considering the situation that $\epsilon_F$ is located near the bottom of an energy band where the $2k_F$ term slowly oscillates, or by considering the resonance condition of $k_F = k_{F,n}$ where the $2k_F$ term provides the maximum value; see Eq. (4). The resonance condition can be achieved at each value of $L$, by tuning an additional gate voltage applied to the entire region of the wires (not shown in Fig. [1]) with monitoring the conductance through the dot. On the other hand, in case B, where one changes $t_{WD}$ with keeping $L$ constant, the term $\cos(k_F(2L + a))$ is constant, hence can be ignored.

For case A under the resonance situation of $k_F = k_{F,n}$, the poor man scaling in Eq. (4) is plotted as a function of $L/\xi_K$ (rather than $L/\xi_K$) in Fig. [5a]. As expected, $T_K$...
in our case). On the other hand, for \( L \ll \xi_{K\infty} \), where low-energy states mainly contribute to the Kondo effect, the NRG shows the same behavior of \( \ln (T_K/T_{K\infty}) \sim -\ln (L/\xi_K) \) as the poorman scaling.

Next, we discuss case B where one changes \( t_{WD} \) with keeping \( L \) constant (hence \( \Delta = \pi \hbar v_F/L \) is constant). Figure 4 shows the NRG result of \( G(T) \) for different \( \xi_k \)'s. We obtain \( T_K \) from the high-temperature behavior of \( G(T) \) in the same way as above, by choosing the temperature at which \( G(T) = G(0, L \to \infty)/2 \). The result of \( T_K \) agrees with case A; see the inset of Fig. 4(b). We below suggest another way to see the cloud from the low-temperature behavior of \( G(T) \). Note that as \( T \) changes across \( \Delta \), \( G(T) \) can show a jump due to the resonance structure of \( \rho(\epsilon) \), as shown for \( L/\xi_K = 26 \) in Fig. 4(a).

We describe the regime of \( T \ll T_K, \Delta \), using the fixed-point Hamiltonian of the Fermi liquid theory\cite{25},

\[
H_{\text{low}} \simeq \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \frac{1}{\pi \rho(\epsilon_F)} \sum_{k'\sigma} \left( \frac{\epsilon_k + \epsilon_{k'}}{2T_K} + \delta_F \right) c^\dagger_{k\sigma} c_{k'\sigma} + \frac{1}{\pi T_K \rho^2(\epsilon_F)} \sum_{k_1,k_2,k_3,k_4} c^\dagger_{k_1} c^\dagger_{k_2} c^\dagger_{k_3} c_{k_4} + \delta_F c^\dagger_{k} c_{k+1} c_{k+2} c^\dagger_{k+3} + \delta_F c^\dagger_{k} c_{k+1} c^\dagger_{k+2} c_{k+3}.
\]  

where \( c^\dagger_{\sigma} \) creates an electron with momentum \( k \), spin \( \sigma \), and energy \( \epsilon_k \). \( \delta_F \) is the phase shift by the potential scattering, which occurs as the particle-hole symmetry is broken. Although \( \rho \) depends on \( \epsilon \), we take for simplicity \( \rho(\epsilon_F) \) in Eq. (6) as a crude approximation.

The second term of Eq. (6) describes elastic scattering of electrons by the Kondo singlet, with scattering phase shift \( \delta_F = \pi/2 + (\epsilon - \epsilon_F)/T_K + \delta_F \). The third term shows repulsive interactions that break the Kondo singlet, and contributes to inelastic T-matrix \( t_{in} \) as \( -\pi \rho(\epsilon_F) \Im t_{in}(\epsilon) = (\epsilon - \epsilon_F)^2 + \pi^2 T^2_K/(2T^2_K) \). By com-

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**FIG. 3:** (Color Online) Case A under the resonance condition of \( k_F = k_{F\infty} \). In this case, one changes \( L \) with keeping \( t_{WD} \) (hence \( T_{K\infty} \) and \( \xi_{K\infty} \)) constant. (a) Kondo temperature \( T_K \) as a function of \( L \), obtained by the poorman scaling (blue solid curve) and NRG (green dashed). The two approaches show qualitatively the same overall behavior that \( T_K \) drastically changes for \( L \ll \xi_{K\infty} \), while \( T_K \sim T_{K\infty} \) for \( L \gg \xi_{K\infty} \); their discrepancy in \( L \gg \xi_{K\infty} \) is discussed in the text. (b) NRG result of the temperature \( T \) dependence of conductance \( G \) for various values of \( L/\xi_{K\infty} \). We choose \( T_{K\infty}/2t = 0.28 \), \( \epsilon_0/2t = 0.925 \), \( eV_p/2t = 0.125 \), and \( U/2t = 3.6 \).

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**FIG. 4:** (Color Online) Case B. In this case, one changes \( t_{WD} \) (hence \( \xi_K \)) with keeping \( L \) constant. (a) The NRG result of \( G(T) \) for different values of \( t_{WD} \); we show the values of \( \xi_K \) instead of \( t_{WD} \). (b) \( T_{eff}(L/\xi_K) \) [defined in Eq. (5)], obtained from the two different approaches of the Fermi liquid theory and the NRG. Inset: The NRG result of \( T_K(L/\xi_{K\infty}) \) exhibits the same behavior as Fig. 3(a). We choose \( L = 100a \), \( \epsilon_0/2t = 0.931 \), \( eV_p/2t = 0.125 \), and \( U/2t = 3.6 \). \( \epsilon_F \) is chosen to be located at a center of a resonance of \( \Gamma(\epsilon) \), for simplicity.
where $\alpha$ and $\beta$ are constants depending on $V_g$ and $k_F$ but independent of $L$. $\delta_p$ is obtained by comparing $G(T = 0)$ with Eq. (7). For nonzero $\delta_p$, $G(T = 0)$ deviates from the unitary-limit value of $2e^2/h$. Note that when $\rho$ is independent of $\epsilon$ and the particle-hole symmetry is preserved, $\delta_p = 0$ and $\beta = 0$, hence, $T_{\text{eff}} \to T_K$. $T_{\text{eff}}$ is obtained by comparing $G(T)$ with Eq. (7) in experiments or in the NRG, while computed from Eq. (8) in the Fermi liquid theory. We plot $T_{\text{eff}}(L/\xi_K)$ in Fig. 4(b), showing good agreement between the NRG and the Fermi liquid theory; their quantitative discrepancy may come from our approximation in the Fermi liquid theory.

The dependence of $T_{\text{eff}}$ on $L/\xi_K$ or $L/\xi_{K\infty}$ is useful for identifying $\xi_K$ in case B, since $\Delta$ is constant so that $T_{\text{eff}}(L/\xi_K)$ directly provides the information of $T_K(L/\xi_K)$; see Eq. (5). For $L \gg \xi_K$, $T_{\text{eff}}$ is almost constant, implying that $T_K$ and $\xi_K$ are independent of $L$. For $L \ll \xi_K$, $T_{\text{eff}}$ (hence $T_K$) depends on $L/\xi_K$. The crossover occurs around $\xi_K \approx L$.

**Discussion.**— Our proposal may be within experimental reach. Case A, where $L$ varies, may be achieved with keyboard-type gate voltages, while one tunes $t_{WD}$ by a gate in case B. A good candidate for our proposal may be a carbon nanotube, where $T_K \sim 1$ K and $\xi_K \sim 1 \mu$m [38].

For both the cases, a (single-mode or multi-mode) wire whose Fermi level $\epsilon_F$ lies near the bottom $E_b$ [van Hove singularity (VHS)] of one of the energy bands is useful to achieve a conclusive evidence of the cloud; our results of Eq. (4) and NRG are applicable to this regime, since they are obtained, taking into account of the energy dependence of $\rho$. In this regime, $\rho(\epsilon)$ is sensitive to $V_g$, hence, it may not be difficult to obtain sizable difference between $T_{K\infty}$ and $T_{K0}$ by $V_g(< \epsilon_F - E_b)$. Our analysis for a single-mode wire is applicable, without modification, to a multi-mode wire, since the band near the VHS governs the Kondo effect dominantly over the other modes. Moreover, the Fermi wave length $k_F^{-1}$ of the band near the VHS can satisfy $k_F^{-1} \gg L$, $\xi_K$, $l_s$, where $l_s$ is the length scale over which $V_g(x)$ spatially changes from 0 to $V_g$ at $x = L$. Then, the $2k_F$ term in Eq. (4) oscillates slowly as a function of $L$ in the range of $L$ where the transition between $T_{K\infty}$ and $T_{K0}$ occurs; in the case of smooth gate potentials with $k_F l_s \gg 1$, the $2k_F$ oscillations will be washed out. Finally, to avoid any effects of $V_g$ on $G$ irrelevant to the Kondo effect, one may consider a quantum dot coupled to three wires, applying $V_g$ to one of the wires and measuring $G$ between the other two.

Note that our system is distinct from the Kondo box (a Kondo impurity in a finite-size system) [17, 21]. In the latter, the cloud is terminated hence strongly modified by the box boundary, hence, it is hard to directly detect $\xi_K$. In contrast, in our system, the cloud is extensible to $|x| > L$ (not suppressed even for $L \ll \xi_K$) and can be only weakly (perturbatively) modified, allowing direct detection of $\xi_K$ and the spatial structure of the cloud.

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