Scalar field dark energy perturbations and the Integrated Sachs Wolfe effect

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Dark energy perturbations affect the growth of matter perturbations even in scenarios with non-interacting dark energy. We investigate the Integrated Sachs Wolfe (ISW) effect in various canonical scalar field models with perturbed dark energy. We do this analysis for models belonging to the thawing and freezing classes. We show that between these classes there is no clear difference for the ISW effect. We show that on taking perturbations into account, the contribution due to different models is closer to each other and to the cosmological constant model as compared to the case of a smooth dark energy. Therefore considering dark energy to be homogeneous gives an overestimate in distinction between different models. However there are significant difference between contribution to the angular power spectrum due to different models.

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The accelerated expansion of the universe has been confirmed by various observations, including Supernova type Ia observations observations of Cosmic Microwave Background and large scale structure [1]. The simplest dark energy model, the cosmological constant $\Lambda$ [2-4] has the fine tuning problem which lead to proposals for various dark energy models to explain the current accelerated expansion of the universe. In these models, the equation of state of dark energy is a function of time and typically the dark energy component is assumed to be a homogeneous component of the universe. Dark energy is described by an ideal fluid or by a scalar field and the parameters of dark energy models are well constrained by observations. However, if the background evolution for different models is the same, we cannot distinguish between them purely on the basis of distance measurements.

Since dark energy affects the background evolution of the universe, it affects the growth of perturbations [5,10,23,25]. In addition dark energy contributes via its inherent perturbations. If dark energy is a cosmological constant, then it does not cluster. The gravitational field $\Phi$ begins to decay when the cosmological constant starts to dominate [23,25]. For the case $\omega \neq -1$, in the matter dominated epoch, the potential $\Phi$ remains at a constant value and decays when dark energy contribution becomes important and this rate of decay depends on the value of $\omega$. For a canonical scalar field, dark energy perturbations are correlated with the matter perturbations leading to an enhancement in matter perturbations [23], whereas they are suppressed if dark energy is a barotropic fluid compared to the corresponding case of a homogeneous dark energy model [24]. The difference is due to the fact that the pressure gradients evolve differently in these model and the homogeneous limit is arrived at differently for the two classes of models. In particular, the evolution of perturbations in different dark energy models is expected to break the degeneracy between those which cannot otherwise be distinguished by distance measurements [5-8].

Dark energy perturbations affect the low l quadrupole in the CMB angular power spectrum through the ISW effect [7,8]. The ISW effect can distinguish a cosmological constant from other models of dark energy [12,13]. The difference in models is expected to make a significant contribution to integrated effects such as the ISW effect [13]. At scales smaller then the Hubble radius, dark energy can be assumed to be homogeneous and for these scales, a fluid model is good approximation to a scalar field model [21]. For large scales the behavior of perturbation growth depends on specific models and the growth factors deviate from each other (for scalar tensor gravity models, it has been shown that the dark energy perturbations are significant even at small scales [21]). Since dark energy perturbations are significant at large scales, i.e., scales larger then Hubble radius, the effect of dark energy perturbations is expected to show prominently in the Integrated Sachs Wolfe (ISW) effect.

In this Report, we study the ISW effect in various (canonical) scalar field models, if the dark energy is considered to be a smooth component or if it contributes to structure formation by way of its perturbations. As in [23,25], we choose the Newtonian gauge

$$ ds^2 = (1 + 2\Phi)dt^2 - a^2(t) \left[ (1 - 2\Phi) dx^2 + dx^2 \right] $$

where $\Phi$ is the gauge invariant potential [10]. The linearized Einstein equations obtained from this metric are

$$ \frac{k^2}{a^2} \Phi + 3 \frac{a}{a} \left( \Phi + \frac{a}{a} \Phi \right) = -4\pi G [\rho_{NR} \delta_{NR} + \rho_{DE} \delta_{DE}] $$

$$ \frac{\ddot{\Phi}}{a} + 2 \frac{\dot{\Phi}}{a} \Phi + \frac{\dot{\Phi}}{a} = -4\pi G [\rho_{NR} v_{NR} + \rho_{DE} v_{DE}] $$

where dot is the derivative with respect to time $t$. The potential for the matter peculiar velocity is given by $v_{NR}$ with $\delta u_{i} = \nabla_{i} v_{NR}$. The perturbed quantities have been
time [12]. The temperature perturbation is given by
\[ \tau = \frac{\Delta T}{T_{\text{CMB}}} = -\frac{2}{c^2} \int_{z^*}^{z_0} dz \frac{d\Phi}{B^2} \]
and the angular auto correlation power spectrum is given by
\[ C_l = \frac{2}{\pi} \int k^2 dk P(k) I_l^2(k) \]
where \( P(k) \propto k^n T^2(k) \) is the present power spectrum with \( T(k) \) being the transfer function. For the transfer function, we adopt the fitting function given by [11]
\[ T(q \equiv k/\Omega_{\text{NR}} h M^{-1}) = \frac{\ln[1 + 2.34q]}{2.34q} \left[ 1 + 3.89q + (16.2q)^2 + (5.47q)^3 + (6.71q)^4 \right]^{-0.25} \]
and the function \( I \) is given by
\[ I_l(k) = -2 \int dz \frac{d\Phi(z)}{dz} j_l[k\chi(z)]dz \]
where \( j_l \) is the spherical Bessel function and \( \chi \) is the coordinate distance.

We use the classification scheme described in [22] for the freezing and thawing models. The scalar field potentials considered in our analysis are the 'thawing' potentials: the exponential potential [13], \( V(\phi) = M^4 \exp(-\sqrt{\alpha \phi}/M_P) \) (henceforth, model T1). Polynomial (concave) potential [14, 15], \( V(\phi) = M^{4-n} \phi^n \) (model T2) and for ‘freezing’ behavior, the Inverse power potential [16], \( V(\phi) = M^{4+n} \phi^{-n} \) (model F1) and \( V(\phi) = M^{4+n} \phi^{-n} \exp(\alpha \phi^2/M_P^2) \) (model F2). In the freezing type models, the scalar field rolls down a steep potential, remains subdominant and at late times dominates and drives the acceleration of the expansion of the universe. The thawing models have a nearly flat potential, the equation of state starts at \( w = -1 \) at early times and deviates from this value (thaws) at late times. Therefore, the dark energy equation of state evolves very differently in these classes of models.

For freezing potential referred to as F1, the scalar field rolls down the steep potential at early times and at late times freezes at the value \( w = -1 \). The epoch of freezing depends on the fine tuning of cosmological parameters. In this model, if the scalar field is frozen before the present time, the duration of the matter dominated phase is too small and there is insufficient structure formation [23]. Therefore, the scalar field needs to freeze in far future. In this model, for \( n = 2 \), the dark energy equation of state reaches a maximum of approximately \(-0.7 \), and the initial \( \phi \) can be fine tuned such that at the present epoch \( \Omega_{\text{NR}} \approx 0.3 \). For the potential F2, the freezing behavior is achieved earlier and the matter dominated phase is sufficiently long. For a given \( n \), the present value of the equation of state deviates
The dotted line denotes the thawing exponential potential \( n = 2\). The dashed line corresponds to potential this analysis. The solid line corresponds to the scalar field (freezing) potential \( V(\phi) = M^4 + \phi^{-n} \exp(\alpha \phi^2/M_P^2) \) (F1) where we have taken \( n = 2\). The dotted line denotes the thawing exponential potential \( V(\phi) = M^4 \exp(-\sqrt{\alpha \phi}/M_P) \) (T1) with \( \alpha = 1\) and the dot-dashed line corresponds to \( V(\phi) = M^4 - \phi^n \) (T2) where we have taken \( n = 2\). The thick solid line corresponds to the ΛCDM model.

The line styles corresponding to different models are chosen to be same as in [25]. The plot on the right shows the ratios of values of angular power spectrum for a freezing and a thawing model. The models are respectively, model F1 with \( \alpha = 0.8\) and \( n = 4\) and model T1 with the value \( \alpha = 1\).

FIG. 2: In this figure the plot on the left shows the angular power spectrum (ISW) for the different models considered in this analysis. The solid line corresponds to the scalar field (freezing) potential \( V(\phi) = M^4 + \phi^{-n} \) (F1) where we have taken \( n = 2\). The dashed line corresponds to potential \( V(\phi) = M^4 + \phi^{-n} \exp(\alpha \phi^2/M_P^2) \) (F2) with parameters \( \alpha = 0.8\) and \( n = 4\). The dotted line denotes the thawing exponential potential \( V(\phi) = M^4 \exp(-\sqrt{\alpha \phi}/M_P) \) (T1) with \( \alpha = 1\) and the dot-dashed line corresponds to \( V(\phi) = M^4 - \phi^n \) (T2) where we have taken \( n = 2\). The thick solid line corresponds to the ΛCDM model.

The line styles corresponding to different models are chosen to be same as in [25]. The plot on the right shows the ratios of values of angular power spectrum for a freezing and a thawing model. The models are respectively, model F1 with \( \alpha = 0.8\) and \( n = 4\) and model T1 with the value \( \alpha = 1\).

from \( w = -1\) as \( \alpha\) increases and for a constant \( \alpha\), the value of the equation of state increases with \( n\). As in all other models, at very early time we assume \( w = -1\). If \( \alpha = 0.6\) and \( n = 1\), the value equation of state reaches \( \approx -0.99\) at redshift \( \approx 0.4\) and begins to freeze and if \( \alpha = 0.8\) and \( n = 4\), the maximum value the equation of state reaches is \( w = -0.82\). The values of the equation of state are within the range allowed by distance observations. In canonical scalar field models, dark energy perturbations are correlated with matter perturbations and consequently enhance them [24, 25]. The enhancement in matter perturbations leads to a suppression in power at large angular scales as compared to the homogeneous dark energy case. Compared to the cosmological constant model, the matter perturbations are suppressed and hence ISW effect is stronger. The deviation from the cosmological constant case decreases with an increasing \( l\). We illustrate the above discussion in Fig. 1 where we plot the angular power spectrum for the last model with and without perturbations and compare with those in the ΛCDM model.

For thawing potential T1, larger the value of \( \alpha\), more is the deviation from cosmological constant type behavior and the perturbations contribute more to power at small \( l\) as compared to the ΛCDM model. For this potential [23, 25], if \( \alpha = 1\), the present day equation of state is \( w = -0.86\) and at redshift \( z = 0.81\), the acceleration of the universe starts. For \( \alpha = 0.1\), the present day value of equation of state is \( w = -0.95\). For smaller \( \alpha\) where the present equation of state is closer to \(-1\), there is no significant difference if we assume dark energy to be a smooth component or if we assume dark energy to be a perturbed component. If \( \alpha = 1\) there is a large enhancement of perturbations in matter [23] and this in turn translates to difference in the power at small \( l\) as compared to the smooth dark energy case and that of cosmological constant. For the concave potential (T2) with \( n = 1\) and \( n = 2\), the present day value of equation of state is \( w \approx -0.94\) if we choose \( n = 1\) and reaches \( w \approx -0.3\) (which is disfavored by observations) for \( n = 2\). For a quadratic potential, the gravitational potential initially decays, and in the future the equation of state starts to oscillate between \( w = -1\) and \( w = 1\). At large scales, therefore, there is a large enhancement in matter perturbations as compared to the case when we assume a smooth dark energy [23]. In contrast, if \( n = 1\), the gravitational potential \( \Phi\) continues to decay and does not show any oscillatory behavior. Matter perturbations in the linear potential remain close to those in cosmological constant model and therefore the contribution of the ISW effect to the angular power spectrum remains close. In quadratic scalar field potential case, these per-
turbations are enhanced compared to the ΛCDM model at early times and are suppressed at late times. The contribution to the angular power spectrum due to the ISW effect is therefore the largest including that of freezing potentials. The above results are summarized in Fig. 2.

We have shown the effect of dark energy perturbations on the Integrated Sachs Wolfe effect. We have chosen freezing/thawing type models for this analysis. One expects the freezing type models to have a higher rate of growth of density contrast at early times, since the equation of state of dark energy is further away from that of a cosmological constant. For thawing type models, dark energy perturbations affect the matter perturbations at late times. The matter perturbations remain close to those in cosmological constant model and at small redshifts being to deviate as the equation of state goes away from \( w = -1 \). The density contrast evolution is significantly different for different models and from that of the cosmological constant model. This difference translates into the difference in the Integrated Sachs Wolfe effect. We show that inclusion of perturbations reduces the difference in the evolution of matter density contrast for different models, hence including perturbations increases the degeneracy between different canonical scalar field models and the freezing/thawing classification scheme is no longer valid. The differences between various models, however, are still significant and more observations and cross correlation of ISW effect with large scale structure indicators may provide a way to distinguish between various dark energy models.

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