Beyond the Quasi-Particle picture in Nuclear Matter calculations using Green's function techniques

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Widths of low-lying states in nuclei are of the order of 30 MeV. These large widths are a consequence of the strong interactions leading to a strongly correlated many body system at the typical densities of nuclear matter. Nevertheless "traditional" Brueckner calculations treat these states as quasiparticles i.e. with spectral functions of zero widths. The width is related to the imaginary part of the selfenergy and is included selfconsistently in an extension of the Brueckner theory using $T$-matrix and Green's function (GF) techniques. A more general formulation applicable also to non-equilibrium systems is contained in the Kadanoff-Baym (KB) equations while still maintaining the basic many-body techniques of Brueckner theory. In the present work the two-time KB-equations are timestepped along the imaginary time-axis to calculate the binding energy of nuclear matter as a function of density, including the spectral widths self-consistently. These zero temperature calculations are compared with quasi-particle calculations. The inclusion of the selfconsistent widths are found to add several MeV to the binding. The spectral widths are due to the long-ranged correlations. Short ranged correlations decrease rather than increase the binding. The method is easily extended to non-zero temperatures where the importance of the widths are expected to increase.

I. INTRODUCTION

Nuclei are strongly correlated many-body systems. Traditional many-body theories such as Brueckner’s are based on a quasi-particle picture ab initio neglecting the widths associated with the strong interactions when calculating the binding energy. The spectral functions are approximated by $\delta$–functions. The Green’s function approach with in-medium $T$-matrix interactions extend the traditional approach by including the spectral-functions selfconsistently. The calculations work with complex rather than only real self-energies. [1–3] In transport theories this is somewhat analogous to replacing the Boltzmann equation with the Kadanoff-Baym (KB) equation. [4]

The nuclear matter correlations are well illustrated by regarding the occupation-numbers in Fig 1. [5].

![Fig. 1. Occupation numbers in the correlated medium at normal and twice the normal nuclear matter density as indicated by the thick curves. The thinner curve shows the occupation numbers for the uncorrelated (quasi-particle) system.](image)

In the quasiparticle approximation the occupation-numbers are that of the fermi-distribution at zero temperature. The correlated medium shows a depletion of about 20% at normal nuclear matter density and a number conserving tail and there exists some experimental evidence of this “redistribution”.

The depletion is mainly due to the short ranged part of the interaction while as shown in ref [6], the long-ranged part, defined by the Moszkowski-Scott (MS) separation method [7], is mainly responsible for the widths i.e. the imaginary part of the selfenergy. This statement is in agreement with the extended quasiparticle approximation (EQP) for the spectral-function given by [8–10]
\[ S_{EQP}(\mathbf{p}, \omega) = 2\pi \hbar \delta(\omega - \omega_0)Z'(\mathbf{p}) - \frac{2\hbar m \Sigma^+(\mathbf{p}, \omega)}{(\omega - \omega_0)^2} \]

where

\[ Z'(\mathbf{p}) = 1 + \left( \frac{\partial \text{Re} \Sigma^+(\mathbf{p}, \omega)}{\partial \omega} \right)_{\omega=\omega_0} \]

In the first term the factor \( Z' \) that shows the depletion of the strength depends only on (the \( \omega \)-dependence) the real part of the selfenergy while the width i.e. the imaginary part of the selfenergy is contained in the second term.

It is of interest to find the effect of these widths in a many-body calculation of nuclear properties, e.g. the binding energy and saturation of nuclear matter. This can conveniently be done using Green’s function techniques allowing for complex self-energies including the widths selfconsistently, going beyond the quasi-particle picture. [1–3]

The effect of correlations on the problems of transport going beyond the Boltzmann equation by using the KB-equation has been investigated [11,12] and is of current interest. [13] Here the equilibrium solution of these same KB-equations is used to find the effect of the widths in nuclear matter binding energy.

This method based on non-equilibrium GF methods differs then from the equilibrium methods mainly in that rather than working with energy- (\( \omega \)-) dependent GF’s these depend on two times and in the equilibrium on the difference between the two times with \( \omega \) being the conjugate of this time-difference.

Some earlier results using a not very realistic local gaussian interaction and a second order selfenergy showed the effect of the widths to increase the binding of nuclear matter by 0.5 MeV at \( k_F = 1.4 \) and by 1.5 MeV at \( k_F = 1.8 \text{fm}^{-1} \). [14] A more realistic interaction will be used here.

Group-renormalisation techniques have been used to derive a \( V_{low-k} \) interaction that is independent of the various NN-potential models used as input [15]. It is somewhat analogous to the long-ranged part of the MS separation method [16] that considers the radial dependence of the interaction while the \( V_{low-k} \) is displayed in momentum space. A cutoff in momentum-space is chosen to be \( \Lambda = 2 \text{fm}^{-1} \).

A potential model was derived by inverse scattering techniques [17] resulting in a separable NN-potential. The input was experimental scattering phaseshifts and deuteron data. When used to calculate nuclear matter bindings this potential showed excellent agreement with the Bonn-potential except in the \( ^3P_1 \) case. In the spirit of the MS-method and the \( V_{low-k} \), a separable potential \( V(\Lambda; k, k') \) has been calculated by inverse scattering fitting all phaseshifts (exactly) for momenta less than \( \Lambda = 2 \text{fm}^{-1} \). The diagonal elements of the \( ^1S_0 \)-potential shown in Fig 2 agrees with those shown by the authors of \( V_{low-k} \). The off-diagonal elements show a deviation similar to the spread of the different potential-models considered in the \( V_{low-k} \) publications. The dependence of \( V(\Lambda; 0,0) \) on the cutoff \( \Lambda \) also shows perfect agreement especially for the \( ^1S_0 \) state. (Fig 3)

This serves to show that this separable potential also is a good representaion of the \( V_{low-k} \) and can be considered a “realistic” interaction..

II. KADANOFF-BAYM CALCULATIONS

The so defined separable interaction \( V(\Lambda = 2; k, k') \) is used in a Kadanoff-Baym [4] formalism. The earlier computer-program used in ref. [14] allowed only for a local interaction. [18] A revised program allows for the separable interaction to be used. Only S-states are used as these are the major contrbutions to the widths. The KB-equations are timestepped along the imaginary time-axis with selfconsistent (complex) self-energies. With the imaginary time \( \tau \) sufficiently large the resulting correlated GF’s correspond to a temperature \( T = 0 \). [11,12] When subsequently time-stepping along the real axis the system is stationary and the total energy is calculated from

\[ E = -\frac{i}{4} \int \frac{d^3p}{(2\pi)^3} \left( \frac{p^2}{2m} + \frac{1}{\partial t} - \frac{1}{\partial t'} \right) G^>(\mathbf{p}, \tau, \tau')_{\tau=t'} \]

which is equivalent to

\[ E = \frac{1}{2} \int_{-\infty}^{+\infty} f(\omega) d\omega \int_{-\infty}^{+\infty} \frac{d^3p}{(2\pi \hbar)^3} [p^2/2m - \omega] S(\mathbf{p}, \omega) \]

where \( S \) is the spectral function and \( f \) the fermi-distribution.

In accordance with previous results [6] discussed above the selfenergy can be calculated from the long-ranged part of the NN-interaction in a second order Born approximation.
The binding energy is compared with a quasi-particle calculation performed by replacing the correlated GF’s in the KB equations with the free ones which have zero widths.  

In Fig 4 the upper curve shows the result of the quasiparticle calculation in a "standard" Brueckner calculation with the separable interaction and the Bonn-B deuteron data as in ref. [17]. The difference between the quasi particle and selfconsistent spectral function calculations is added to get the lower curve thus showing the effect of going beyond the quasi-particle approximation.

\(^1\) An extra factor of 1/2 is then required in the expression for the total energy [19,20]
III. CONCLUSIONS

Fig 4 shows an increase in binding of about 2 MeV due to the widths of the spectral function relative to the quasi-particle result while the results of full $T$-matrix calculations [1,3] show a decreased rather than an increased binding. These two results are however not directly comparable.

Various methods and approximations were discussed at length in refs [9,10] where one notable result obtained was that the binding calculated with spectral functions gave a binding of 17.4 MeV while the Brueckner (quasi-particle) calculation gave 20.0 MeV. This seems comparable to the findings in refs [1,3].

But the result shown in Fig 4 only includes the effect of the width of the spectral function while in relation to Fig 1 and eq (1) another effect of the strong correlations in nuclei was pointed out: The depletion of occupation-numbers due to short-ranged correlations. An estimate of the corrections to the binding energy related to this depletion can be made as follows. The depletion is directly related to the third order "rearrangement" energy that is positive and when included in the definition of single-particle energies in the calculation of the Brueckner $K$-matrix will lead to an increase of about one MeV in the binding energy. [21,22] An additional correction is for the reduced strength of the spectral function in eq (4). This is given by the factor $Z'$ in eq (1) shown by the reduced occupation-numbers in Fig 1 to be $\approx 0.8$ so that the binding due to this correction is reduced by $\approx 20\%$ i.e. $\approx 3$ MeV. The total effect associated with the depletion would thus be a binding decreased by $\approx 2MeV$. This would thus essentially cancel the effect of the width shown in Fig 4.

The above are estimates that could be off by MeV's and are only meant to illustrate that there are opposing effects that each are not small justifying full $T$-matrix calculations such as referred to above.

These spectral corrections are expected to increase with density and temperature and would be important for a theoretical determination of the Nuclear Equation Of State. Of course these calculations are still limited by our limited knowledge of nuclear forces and many body effects.

The method of imaginary time-stepping used here has the numerical advantage over the conventional $T$-matrix calculations of not explicitly having to integrate over sharply peaked spectral functions as a function of $\omega$. For the study of non-equilibrium phenomena the method can also be used with constraints to define an initial condition. [11] It still has to be improved however to incorporate a full $T$-matrix calculation rather than just a second order Born
calculation. Only $S$-states were included to find the energy correction due to the width. This may not be sufficient.

In summary it can now be conclusively stated that selfconsistent spectral functions should (and perhaps equally important now can) be included in any serious consideration of nuclear properties being it in ground state, excited state or in a state of non-equilibrium.

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