Effect of Size of the Computational Domain on Spherical Nonlinear Force-Free Modeling of Coronal Magnetic Field Using SDO/HMI Data

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Abstract The solar coronal magnetic field produces solar activity, including extremely energetic solar flares and coronal mass ejections (CMEs). Knowledge of the structure and evolution of the magnetic field of the solar corona is important for investigating and understanding the origins of space weather. Although the coronal field remains difficult to measure directly, there is considerable interest in accurate modeling of magnetic fields in and around sunspot regions on the Sun using photospheric vector magnetograms as boundary data. In this work, we investigate effects of the size of the domain chosen for coronal magnetic field modeling on resulting model solution. We apply spherical Optimization procedure to vector magnetogram data of Helioseismic and Magnetic Imager (HMI) onboard Solar Dynamics Observatory (SDO with four Active Region observed on 09 March 2012 at 20:55UT. The results imply that quantities like magnetic flux density, electric current density and free magnetic energy density of ARs of interest are significantly different from the corresponding quantities obtained in the same region within the wider field of view. The difference is even more pronounced in the regions where there are connections to outside the domain.

Keywords: Active regions, magnetic fields; Active regions, models; Magnetic fields, corona; Magnetic fields, models; Magnetic fields, photosphere

1. Introduction

The three-dimensional magnetic fields in the higher solar atmosphere provide crucial information toward understanding various solar activities, such as fila-
ment eruptions, flares, and coronal mass ejections (CMEs). To understand the physical mechanisms of these activities in the solar atmosphere, an important step is to find out the underlying structure of the magnetic field above the related active regions. Currently, due to the extremely low density and high temperature, the direct measurement of the magnetic field in the solar chromosphere and corona is not as sophisticated as observation in the photosphere (Lin, Penn, and Tomczyk, 2000; Liu and Lin, 2008). As an alternative to direct measurement of the three dimensional magnetic field in solar atmosphere, numerical modeling are used to infer the field strength in the higher layers of the solar corona from the measured photospheric magnetic field. Thus model assumption is called the force-free field assumption, as the Lorentz force vanishes. This can be obtained by either vanishing electric currents or the currents are co-aligned with the magnetic field lines. It is generally assumed that the magnetic pressure in the corona is much higher than the plasma pressure (small plasma $\beta$) and that therefore the magnetic field is nearly force-free (for a critical view of this assumption see Gary (2001).

Force-free coronal magnetic fields are defined entirely by requiring that the field has no Lorentz force and is divergence free (the solenoidal condition):

$$\nabla \times \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

subject to the boundary condition

$$\mathbf{B} = \mathbf{B}_{\text{obs}} \text{ on photosphere}$$

where $\mathbf{B}$ is the magnetic field and $\mathbf{B}_{\text{obs}}$ is measured vector field on the photosphere. Equation (1) states that the currents are co-aligned with magnetic fields and Equation (2) describes the absence of magnetic monopoles. Using two equations as constraints equations, one can calculate the magnetic field density in a corona volume for photospheric measurements.

In absence of coronal magnetic field measurements, nonlinear force-free (NLFF) reconstruction techniques based on photospheric magnetic field measurements (within their limitations; see DeRosa et al., 2009) are to date one of the few means of approximating the coronal field structure with a near real-time temporal cadence given the spatial resolution provided by the measured field vector (e.g., Wiegelmann and Sakurai, 2012). For more details on the success and future improvements of the reconstruction techniques, we direct the reader to Régnier, 2013.

In the present study, we use vector magnetic field data of HMI with two different field-of-views as an input to our spherical optimization code and compare the results. There has been relatively little consideration of the effect of the size of computational domain on NLFFF modeling to date. Therefore, we investigate whether the size of computational domain chosen for coronal magnetic field modeling significantly influences the resulting solutions to the model. In our experiment, we provide a wider field of view that can accommodate magnetic
connections between the region of interest and both nearby plage and neighboring ARs. We have selected a large region on the Sun observed on 09 March 2012 at 20:55 UT with four ARs of which two of them are in northern hemisphere and the other two in the south. In the companion case, the region of interest with two ARs (AR11429 and AR11430) in the northern hemisphere is cropped more tightly, as if observed with an instrument with a limited field of view. We compare quantities like magnetic flux density, electric current density and free magnetic energy density of ARs of interest obtained from the two different field-of-views. In the study, the same spatial resolution is used for both cases.

2. Instrumentation and Data Set

The Helioseismic and Magnetic Imager (HMI) is part of the Solar Dynamics Observatory (SDO), which provides the first uninterrupted time series of space-based, full-disk, vector magnetic field observations of the Sun with a 12-minute cadence [Schou et al., 2012]. HMI consists of a refracting telescope, a polarization selector, an image stabilization system, a narrow band tunable filter and two 4096 pixel CCD cameras with mechanical shutters and control electronics. Photospheric line-of-sight LOS and vector magnetograms are retrieved from filtergrams with a plate scale of 0.5 arc-second. From filtergrams averaged over about ten minutes, Stokes parameters are derived and inverted using the Milne-Eddington (ME) inversion algorithm of [Borrero et al., 2011].

The transverse components of vector magnetic fields suffer from the so-called 180° ambiguity. The inversion applied to each pixel cannot resolve the inherent 180° azimuth ambiguity in the transverse field direction. Therefore, the 180° ambiguity has been resolved by minimum energy method [Metcalf, 1994]. As described in [Leka et al., 2009], in weak-field areas, the minimization may not return a good solution due to large noise. The noise level is ≈10G and ≈100G for the longitudinal and transverse magnetic field, respectively. Therefore, in order to get a spatially smooth solution in weak-field areas, we divide the magnetic field into two regions, i.e., strong-field region and weak-field region, which is defined to be where the field strength is below 200 G at the disk center, and 400 G on the limb. The values vary linearly with distance from the center to the limb. For more detailed descriptions of HMI data processing and production techniques, we direct the reader to [Hoeksema et al., 2014].

3. Method

We solve the force-free Equations (1)-(3) by using an optimization principle as proposed by Wheatland, Sturrock, and Roumeliotis (2000) and extended by Wiegelmann (2004) and Wiegelmann and Inhester (2010) in the form

\[ L = L_f + L_d + \nu L_{\text{photo}} \]  

\[ L_f = \int_V \omega_f(r, \theta, \phi) B^{-2} |(\nabla \times B) \times B| r^2 \sin \theta dr d\theta d\phi \]
Figure 1. (a) Full-disk SDO/HMI magnetogram on 09 March 2012 at at 20:55UT and (b) Zoomed region from the full-disk in (a). (c) The corresponding full-disk AIA 171 Å image and (d) A composite AIA (171, 193, and 211 Å) image of the region. The red rectangle in all figures outlines region of interest and the yellow one indicates the bottom boundary of larger computational box.

\[ L_d = \int_{V} \omega_d(r, \theta, \phi) |\nabla \cdot \mathbf{B}|^2 r^2 \sin \theta dr d\theta d\phi \]

\[ L_{\text{photo}} = \int_{S} (\mathbf{B} - \mathbf{B}_{\text{obs}}) \cdot \mathbf{W}(\theta, \phi) \cdot (\mathbf{B} - \mathbf{B}_{\text{obs}}) r^2 \sin \theta d\theta d\phi \]
where $L_f$ and $L_d$ measure how well the force-free equation [Equation (1)] and divergence-free condition [Equation (2)] are fulfilled, respectively. $\omega_f$ and $\omega_d$ are weighting functions, which are one in the region of interest and drop to zero in a 32 pixel boundary layer toward the lateral and top boundaries of the computational domain. In this work, we implement the surface integral term, $L_{\text{photo}}$, in Equation (4) to work with boundary data of different noise levels and qualities (Wiegelmann and Inhester, 2010; Tadesse et al., 2011). This allows deviations between the model field $B$ and the input field, i.e. the observed $B_{\text{obs}}$ surface field, so that the model field can be iterated closer to a force-free solution. $W(\theta, \phi)$ is a space-dependent diagonal matrix whose elements $(w_{\text{los}}, w_{\text{trans}}, w_{\text{trans}})$ are inversely proportional to the estimated squared measurement error of the respective field components. Because the line-of-sight photospheric magnetic field is measured with much higher accuracy than the transverse field, we typically set the component $w_{\text{los}}$ to unity, while the transverse components of $w_{\text{trans}}$ are typically small but positive. In regions where transverse field has not been measured or where the signal-to-noise ratio is very poor, we set $w_{\text{trans}} = 0$. In order to control the speed with which the lower boundary condition is injected during the NLFFF extrapolation, we have used the Lagrangian multiplier of $\nu = 0.001$ as suggested by Tadesse et al. (2013a).

Our optimization method uses vector field values $B_{\text{obs}}$ over the entire lower boundary as boundary conditions at the photosphere. The inconsistency of the boundary data with the force-free assumption can lead to poor model solutions. Therefore, we use a spherical preprocessing procedure to remove most of the net force and torque from HMI boundary data to be more consistent with NLFFF modeling (Wiegelmann, Inhester, and Sakurai, 2006; Tadesse, Wiegelmann, and Inhester, 2009).

There are no vector magnetic field measurements for the side and top boundaries of a localized domain. Therefore, we have to make assumptions about these fields before performing a NLFFF extrapolation. We assumed the lateral and upper boundaries of the computational domain as current-free. In order to initialize our NLFFF code, we calculated potential field from SDO/HMI data set using preprocessed radial field components ($B_r$) by spherical harmonic expansion method. Therefore, the potential magnetic field is used with values of radial field matching the initial preprocessed lower boundary values, and this field provides also the initial field at all points in the volume except the lower boundary. The computational box is a wedge-shaped volume $V$ with six boundary surfaces (four lateral side boundaries, top and photospheric boundaries). This box enables us to study the connectivities between ARs and their surroundings for the large field-of-views.

4. Results

The main purpose of this work is to investigate whether the size of computational domain chosen for coronal magnetic field modeling significantly influences the resulting solutions to the model. In our investigations, we provide two SDO/HMI vector magnetic field boundaries data from two different field of views to our
Figure 2. Surface vector magnetic field solutions at radial height of $h = 30$ Mm obtained from (a) the boundary of region of interest (FOV1) with smaller field of view and (b) the boundary of corresponding region of interest (FOV2*) cropped from the region with larger field of view (FOV2). (c) Magnetic field difference between a and b. The color coding and the yellow arrows show $B_r$ and transverse components of the magnetic field, respectively.
Field lines of the NLFF model obtained from (a) the boundary of region of interest with smaller field of view and overlaid on the composite AIA (171, 193, and 211 Å) image and (b) the same field lines in a) rotated to the limb (c) the boundary of corresponding region of interest (FOV2*) cropped from the region with larger field of view (FOV2). (d) the same field lines in c) rotated to the limb.

Figure 3. Field lines of the NLFF model obtained from (a) the boundary of region of interest with smaller field of view and overlaid on the composite AIA (171, 193, and 211 Å) image and (b) the same field lines in a) rotated to the limb (c) the boundary of corresponding region of interest (FOV2*) cropped from the region with larger field of view (FOV2). (d) the same field lines in c) rotated to the limb.

spherical code. We extracted the two boundary data from full-disk HMI data observed on 09 March 2012 at at 20:55UT. The first data set accommodates the region of interest, which has two ARs (AR11429 and AR11430) located in the northern hemisphere (see, Figure 1). The photospheric surface boundary of the computational box over the region of interest is shown by the red rectangles in Figure 1 (referred as FOV1, thereafter). This region is cropped more tightly, as if observed with an instrument with a limited field of view. The other boundary data has a wider field of view that can accommodate magnetic connections between the region of interest and neighboring ARs (referred as FOV2, thereafter).
Figure 4. Surface vector electric current densities at radial height of $h = 30$ Mm obtained from (a) the boundary of region of interest with smaller field of view and (b) the boundary of corresponding region of interest (FOV2*) cropped from the region with larger field of view (FOV2). The color coding and the yellow arrows show $J_r$ and transverse components of electric current densities, respectively.

It has four ARs of which two of them are in northern hemisphere and the other two in the south. The photospheric surface boundary of the larger computational box is shown by the yellow rectangles in Figure 1.

The region of interest, FOV1, was well isolated from the surrounding with the quiet Sun region. However, it was magnetically connected to the ARs in the southern hemisphere by trans-equatorial loops crossing the lateral boundary of FOV1 in the south (see, Figures 1c and d). During modeling the 3-D magnetic field of FOV1, the photospheric magnetic field outside this region is ignored. This
Figure 5. Free magnetic energy densities at radial height of $h = 30$ Mm obtained from (a) the boundary of region of interest with smaller field of view and (b) the boundary of corresponding region of interest (FOV2*) cropped from the region with larger field of view (FOV2).
approach is the better justified the more the region of interest is isolated. Often, however, active regions are not always completely isolated but magnetically connected with other active regions (Wiegelmann, 2004). In our previous work (Tadesse et al., 2013b), we have studied the connectivity between two active regions one in the northern hemisphere and the other in the south. The study revealed that there were substantial number of fluxes shared between the ARs and as the result there were trans-equatorial loops carrying electric current. Therefore, those fluxes and electric currents crossing the lateral boundary of the region of interest might have effect on magnetic field solutions, electric current density and free magnetic energy.

4.1. Magnetic Field Solutions

To calculate three-dimensional NLFF magnetic field solutions, we minimize the functional $L$ given by Equation (4) using preprocessed photospheric boundary data for the two cases. In the first case, the boundary data of FOV1 (region in the red rectangles of Figure 1) has been provided to our spherical optimization code as an input. We compute the solution on a number of different horizontal grid scales, using 3-levels multigrid (e.g., $75 \times 108 \times 216$, $150 \times 216 \times 432$, and $300 \times 432 \times 864$), with the results of the coarser resolution used to initialize the next, finer resolution. The solution from the coarser grids are interpolated onto finer grids as the initial state for the magnetic field in the computational domain of the next larger box. This gives a better starting equilibrium in the full resolution box, an improvement on an initial potential field. The region of interest (FOV1) corresponds to a wedge shape box with the field of view of $[r_{\min} = 1R_\odot : r_{\max} = 2.5R_\odot] \times [\theta_{\min} = 5.5^\circ : \theta_{\max} = 40^\circ] \times [\phi_{\min} = 160^\circ : \phi_{\max} = 230^\circ]$. In the second case, the boundary data of FOV2 (region in the yellow rectangles of Figure 1) has been provided to the code as an input. We compute the solution on a number of different horizontal grid scales, using the same 3-levels multigrid (e.g., $75 \times 216 \times 216$, $150 \times 432 \times 432$, and $300 \times 864 \times 864$). This region (FOV2) corresponds to a wedge shape box with the large field of view of $[r_{\min} = 1R_\odot : r_{\max} = 2.5R_\odot] \times [\theta_{\min} = -30^\circ : \theta_{\max} = 40^\circ] \times [\phi_{\min} = 160^\circ : \phi_{\max} = 230^\circ]$, twice the size of FOV1.

Once the 3D magnetic configuration is reconstructed for the two cases, we plot the surface vector magnetic field solutions of the region of interest from the two individual cases at the coronal height of $h = 30$ Mm (see, Figure 2). For the comparison of the two magnetic field solutions in the region of interest that are obtained from the two field-of-views (FOV1 and FOV2), we cropped the volume of region of interest that corresponds to FOV1 from FOV2 (referred as FOV2*, thereafter and where FOV2* = FOV2 $\cap$ FOV1, is an intersecting region of FOV2 and region of interest, FOV1 ). Figure 2a and b show the vector fields solution that are obtained from the boundary data of the region of interest from FOV1 and FOV2*, respectively. We also plot the difference in the magnetic field solutions of FOV1 and FOV2* over the region of interest in Figure 2c. The difference in magnetic field vector is higher in the area where the field lines are connected to outside the domain.
In order to evaluate the three-dimensional NLFFF of the region of interest from the two different solutions, we have calculated the following quantities like

\[ L_1 = \langle \sum_i \frac{|J_i \times B_i|^2}{|J_i|^2} \rangle, \quad L_2 = \langle \sum_i |\nabla \cdot B_i|^2 \rangle \]

\[ \langle \theta_J \rangle = \arcsin \sum_i \frac{|J_i| \sigma_i}{|J_i||B_i|}, \quad C_{\text{vec}} = \frac{\sum_i B_i^* \cdot B_i}{\sqrt{\left( \sum_i |B_i^*|^2 \right) \left( \sum_i |B_i|^2 \right)}} \]

where \( \sigma_i = \frac{|J_i \times B_i|}{|J_i||B_i|} = |\sin \theta_i| \). \( L_1 \), \( L_2 \), \( \theta_J \), and \( C_{\text{vec}} \) are Lorentz-force, divergence of magnetic field, the average angle between magnetic field and electric current density, and vector correlation between the magnetic field from FOV1 respectively. We normalize the magnetic field with the average photospheric field and length scale of solar radius. Table 1 shows that the force-freeness \( (L_1) \) and divergence \( (L_2) \) conditions are best fulfilled for the magnetic field solutions of FOV2\(^*\) extracted from larger field-of-view. Similarly, the magnetic field solution of FOV1 has larger average angle between magnetic field and electric current density compared to that of FOV2\(^*\), indicating that the magnetic field solution of FOV1 are less force-free that that of FOV2\(^*\). The vector correlations values between NLFF magnetic fields and the corresponding potential fields from the two solutions indicate that magnetic field solution of FOV1 is somewhat close to potential field than the solution of FOV2\(^*\).

In our previous works (Tadesse et al., 2012a; Tadesse et al., 2012b; Tadesse et al., 2013b), we studied the connectivity between many neighboring ARs which were found to share a significant amount of magnetic flux compared to their internal flux connecting one polarity to the other. In this study, we have used the same method to calculate the total ("shared") flux, \( |\Phi| \), for all field lines starting from where \( B_r > 100 \text{G} \) in the region of interest, FOV1, and leaving the volume through the lateral boundary of FOV1 towards the rest of the region of FOV2 outside of FOV2\(^*\). Table 1 shows that there are more magnetic field lines crossing the lateral boundary of FOV1 for the magnetic field solution of FOV2\(^*\) than those of FOV1. This indicates that there is large flux shared between the region of interest and its neighboring region, if we include all the connecting regions in the computational domain.

In order to compare our 3-D field reconstructions with observation, we plot the selected field lines of the NLFFF solutions for the two data sets and we overlay the field lines with corresponding AIA composite (171, 193, 211 Å) image (see Figure 3a and c). From those figures, one can see that the field lines of NLFFF model solutions obtained from the two field-of-views have significant difference. In Figures 3b and d, we rotate the same selected field lines to the limb. The field lines reconstructed from the boundary data of FOV1 deviate from observation than the one obtained from the boundary data of FOV2. Especially for those field lines crossing the southern lateral boundary of FOV1, the deviation is more pronounced. There are spatial correspondence between the overall shape of the magnetic field lines of FOV2\(^*\) and the EUV loops. Those qualitative comparisons between NLFFF model magnetic field lines and the observed EUV loops of AIA images indicate that the NLFFF model provides a more consistent field for large field-of-views.
Table 1. Evaluation of the reconstruction quality for the 3-D magnetic field solutions of FOV1 and FOV2*.

|         | $|\Phi|$ | $L_1$ | $L_2$ | $\langle \theta_J \rangle$ | $C_{vec}$ | $E_{nlff}/E_{pot}$ |
|---------|---------|-------|-------|-----------------------------|-----------|-------------------|
| FOV1    | 2.3     | 1.9   | 1.37  | 5.1                         | 0.94      | 1.17              |
| FOV2*   | 2.9     | 0.8   | 0.23  | 3.4                         | 0.78      | 1.23              |

4.2. Electric Current Densities of the Two Field Solutions

Once we have calculated the 3-D magnetic field solutions for the two field-of-views, we computed the vector current density $J$ as the curl of the field:

$$J = \frac{1}{4\pi} \nabla \times B,$$

where $B$ is nonlinear force-free 3D magnetic field solution. We use finite difference method to solve for the electric current density from curl of $B$. We plot electric current density vector $J$ of the region of interest. Figures 4(a) and (b) show the electric current densities on the layer at the corona height of $h = 30\text{Mm}$ from the two magnetic field solutions. There is substantial difference in electric current densities of the two solutions.

In order to quantify the percentage share in the electric current, we first identified those field lines carrying electric currents and emanating from the region of interest, FOV1 (crossing its lateral boundary on the south) and ending into the region in FOV2 outside of FOV1. The ratio of total unsigned electric current density flux due to those electric current carrying field lines connecting those two regions to the total unsigned electric current density flux due to all field lines with current emanating from FOV1 gives us the percentage share in the electric current between the two regions. For the case of electric current density calculated from the 3-D field of larger field-of-view, we found that 21.6% of positive/negative polarity of the ARs in FOV2* (region of interest) in the northern hemisphere is connected to positive/negative polarity of ARs in FOV2 outside FOV1 crossing the southern lateral boundary of FOV1. However, there is no electric current density crossing the southern lateral boundary of FOV1 for the case of magnetic field solution obtained from FOV1 boundary data. This might be due to the fact that we initialized the lateral boundaries of FOV1 with the potential field.

4.3. Free Magnetic Energies of the Two Field Solutions

We estimate the free magnetic energy budget above the potential-field state, the difference between the extrapolated NLFFF and the potential field with the same normal boundary conditions in the photosphere [Régnier and Priest](#).
The free magnetic energy budget is a measure of the magnetic energy that can be stored in a magnetic configuration or can be released during an eruptive or reconnection event. We therefore estimate the upper limit to the free magnetic energy budget in excess to the potential field state, associated with coronal currents, by:

\[ E_{\text{free}}^m = E_{\text{nlff}}^m - E_{\text{pot}}^m, \]  

where the magnetic energy \( E_m \) is computed in the coronal volume \( V \) as follows:

\[ E_m = \frac{1}{8\pi} \int_V B^2 r^2 \sin \theta dr d\theta d\phi. \]  

Our result for the estimation of the total free magnetic energy budget above the potential-field state throughout the computational volumes of the region of interest from FOV1 and FOV2* have 17% and 23% (see, Table 1) more energy than the corresponding potential-fields, respectively. In Figure 5, we plot total surface free magnetic energy density of the two magnetic field solutions relative to the potential one at the corona height of \( h = 30 \) Mm. From those figures show that there is free magnetic energy difference in the area where loops are connected to outside the domain.

5. Discussion

In this study, we have investigated effects of the size of the domain chosen for coronal magnetic field modeling on resulting model solution. We applied spherical Optimization procedure to vector magnetogram data of Helioseismic and Magnetic Imager (HMI) onboard Solar Dynamics Observatory (SDO with four ARs observed on 09 March 2012 at 20:55UT. In this study, we have used spherical multigrid technique where we computed the solution on a number of different horizontal grid scales, with the results of the coarser resolution used to initialize the next, finer resolution. The solution from the coarser grids were interpolated onto finer grids as the initial state for the magnetic field in the computational domain of the next larger box. This gives a better starting equilibrium in the full resolution box, an improvement on an initial potential field.

In order to study the effects of the size of the domain over large area of the Sun, we have selected a well isolated, but magnetically connected region of interest with two ARs and located in the northern hemisphere. We have used two boundary data with two different computational boxes, one exactly accommodating the region of interest and the other accommodating the region of interest and its southern surrounding region with two additional ARs. We computed the 3-D magnetic field solutions from the two boundary data in the two computational boxes with different field-of-views.

For the comparison with the solution from the smaller box over the selected region of interest, we extracted the 3-D magnetic field solution of the region of interest from the field solution of large field-of-view. We have compared the magnetic field lines of the two magnetic field solutions with the observed corresponding EUV loops. The comparison shows that the field lines of the modeling
fields computed within the larger field-of-view generally agree with the coronal loop images than the one computed within the smaller box. In the study, we found that there are substantial differences in magnetic fluxes, electric current densities and free magnetic energies at a selected coronal height from the two solutions. The difference is even more pronounced in the regions where there are connections to outside the domain. This is due to the fact that the fluxes and electric currents crossing the lateral boundary of the region of interest have effect on magnetic field solutions, electric current density and free magnetic energy. Therefore, one has to consider the region of magnetic connectivity while modeling the magnetic field of the region of interest in spherical geometry.

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