Sound speed of scalar field dark energy: weak effects and large uncertainties

Olga Sergijenko
Astronomical Observatory of Ivan Franko National University of Lviv, Kyryla i Methodia str., 8, Lviv, 79005, Ukraine
Main Astronomical Observatory of the National Academy of Sciences of Ukraine, Zabolotnogo str., 27, Kyiv, 03680, Ukraine
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Bohdan Novosyadlyj
Astronomical Observatory of Ivan Franko National University of Lviv, Kyryla i Methodia str., 8, Lviv, 79005, Ukraine

The possibility of reconstruction of Lagrangian for the scalar field dark energy with constant effective sound speed $c_s$ is analyzed. The value of $c_s$ is estimated together with other dark energy parameters ($\Omega_{de}$, $w_0$, $c_s^2$) and rest of cosmological parameters on the basis of data including Planck-2013 results on CMB anisotropy, BAO distance ratios from recent galaxy surveys, magnitude-redshift relations for distant SNe Ia from SNLS3 and Union2.1 compilations and the HST determination of the Hubble constant. For main dark energy and cosmological parameters the following best-fit values and $2\sigma$ confidence limits are obtained: $\Omega_{de} = 0.723^{+0.018}_{-0.025}$, $w_0 = -1.176^{+0.141}_{-0.128}$, $c_s^2 = -1.500^{+0.370}_{-0.102}$, $\Omega_{cdm}h^2 = 0.0221 \pm 0.0005$, $\Omega_{b}h^2 = 0.119^{+0.005}_{-0.003}$, $h = 0.715^{+0.026}_{-0.028}$, $n_s = 0.962^{+0.010}_{-0.014}$, $A_s = (2.209^{+0.102}_{-0.112}) \times 10^{-9}$, $\tau_{rei} = 0.093^{+0.022}_{-0.020}$. It is shown that no value of $c_s$ from the range $[0,1]$ is preferred by the used data because of very weak influence of dark energy perturbations on the large scale structure formation and CMB temperature fluctuations.

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I. INTRODUCTION

Extragalactic and cosmological observational data collected up to now and interpreted in the framework of current physical theories certify that about 70% of the energy-mass content of our World is the dark energy which almost uniformly fills the Universe and accelerates its expansion. Its physical nature is still unknown because of its “darkness” and exclusively cosmological scale “fingerprints”. The explanation of the nature of this mysterious component becomes extremely important for elaboration of physics of galaxies and clusters, cosmology and particle physics beyond the Standard Model.

Among several discussed in the literature hypotheses about the nature of dark energy the hypothesis that it is a scalar field with violated weak or null energy condition seems the most promising in the terms of possibility to be tested by comparison of theoretical predictions with observational data. The scalar field can be quintessential dark energy, phantom one or changing the type from one to another (quintom) at different moments of time, or be the vacuum-like (or $\Lambda$-type) dark energy. The current observational data on supernovae type Ia (SNe Ia) luminosity distances, baryon acoustic oscillations (BAO) in the galaxies space distribution together with the Planck measurements of CMB temperature anisotropy and prefer phantom dark energy at 2$\sigma$ or a bit higher confidential level. The accuracy of obtained in recent years observational data has increased so much that reliable determination of the equation of state parameter $w_0$ (with accuracy of 6%) of dark energy and its density parameter $\Omega_{de}$ (with accuracy 2%) for current epoch became possible. Moreover, the nowadays observational data are so good that they even give the possibility to establish also the time variation of $w_0$ (more precisely the squared adiabatic sound speed $c_a^2 = \rho_{de}/\rho_{de}$ with accuracy 18%). So, its reliable determination may be a matter of expected data in the nearest future. These parameters, however, are not enough to establish definitely the nature of dark energy. Other measurements of dark energy must be done. The determination of effective sound speed of dark energy $c_s$, which is the speed of propagation of dark energy perturbations, is among them.

The theoretical aspects of effective sound speed of dark energy, the impact of its value on the evolution of dark energy and dark matter perturbations, the possibility of its determination as well as the recent attempts of its

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1. The terms “adiabatic sound speed” and “effective sound speed” of dark energy are used in the literature for designation of dark energy values which formally correspond to the thermodynamical ones.
constraining are analyzed in [12–25] (see also books [26–29] and citing therein). They can be summarized as follows: a) evolution of dark energy density perturbations depends on the value of effective sound speed: their amplitudes increase when scales of perturbations are larger than acoustic horizon scale \((k^{-1} > c_s t)\) and decay when they become smaller \((k^{-1} < c_s t)\), b) practically for any \(0 < c_s^2 < 1\) the amplitudes of density perturbations of dark energy are essentially lower than the amplitudes for dark matter and baryon components at current epoch, c) the value of EoS parameter as well as the character of its time variation changes the evolution of density too: the lower initial value of \(w_{de}\), the lower initial amplitude of scalar field density perturbations.

In this paper we discuss the possibility of determination of effective sound speed of quintessential/phantom model of dark energy, for which the phantom is preferable by current observational data. For this type of dark energy the problem of determination of the effective sound speed is complicated by the too weak influence of dark energy perturbations on the matter ones. We discuss also the problem of reconstruction of scalar field dark energy with constant effective sound speed. If the phantom type of dark energy is confirmed then for its reliable reconstruction not only the increasing requirements for accuracy of data, but also radically new ideas for its study would be necessary.

In the analysis we use the simplest minimal coupled scalar field model of dynamical dark energy that can be either quintessential or phantom scalar field with barotropic EoS [30–33]. The existence of analytical solutions for the evolution of such a scalar field, their regularity and applicability for any epoch in the past as well as in the future could make it a useful model to establish the type of dark energy.

The paper is organized as follows: in Sect. II we analyze the principal possibility of reconstruction of scalar field dark energy; in Sect. III we analyze the gravitational instability of scalar field with generalized linear barotropic equation of state and its effects on structure formation; in Sect. IV we present the method, observational data and results of the estimation of 4 dark energy parameters. The conclusions are presented in Sect. V.

II. RECONSTRUCTION OF SCALAR FIELD DARK ENERGY WITH CONSTANT EFFECTIVE SOUND SPEED

Scalar field dark energy can be described in several ways. Among them the most popular one is the phenomenological approach in which the dark energy is assumed to be the perfect fluid described by a small number of parameters, e. g. 3 for a constant equation of state parameter \((\Omega_{de}, w_{de} and c_s^2)\). This is convenient for practical calculations and putting the observational constraints on model parameters, however gives very little information about the physical nature of dark energy. On the other hand, the scalar field approach is well suited for study of physics of dark energy, but is not as usable in practice as the former one. So it is often useful to combine both methods of modeling of dark energy and use in numerical calculations the phenomenological perfect fluid, while study the physical features of scalar fields reconstructed to mimic the behavior of this perfect fluid.

It is well-known that the potential of a scalar field with given Lagrangian can be uniquely reconstructed using only the dark energy density and its equation of state: from their temporal dependences it is easy to determine the temporal dependences of the field potential and its kinetic term. From the temporal dependence of a kinetic term it is possible to obtain the temporal dependence of a field variable which allows the determination of explicit dependence of a potential on field variable. Note that the precision of reconstruction of the potential from observational data is limited by the precision of cosmological parameters estimation (e. g. for the discussion of how the uncertainties in determination of different parameters affect the reconstructed potential in the case of \(w = const\) see [34]). However, the infinite degeneracy of the forms of Lagrangians exists: they lead to the same observable characteristics of dynamics of expansion of the Universe. In 2008 Unnikrishnan [35] showed that the degeneracy is not broken even if we take into account the linear perturbations of dark energy, since different Lagrangians can lead to the same \(w_{de} = const\) and \(c_s^2 = const\).

Let us take a closer look at the possibility of reconstruction of the functional form of Lagrangian in case of fields with \(c_s^2 = const\) and arbitrary (variable in time) \(w_{de}\). The equation of state parameter of dark energy \(w_{de} \equiv p_{de}/\rho_{de}\) and its effective (rest frame) sound speed \(c_s^2 \equiv \delta p_{de}/\delta \rho_{de}\) are defined by the field Lagrangian as follows:

\[
\rho_{de} = \frac{L}{2X \frac{\partial L}{\partial \phi} - L}, \tag{1}
\]

\[
c_s^2 = \frac{2X \frac{\partial^2 L}{\partial \phi^2} - \frac{\partial L}{\partial \phi}}{2X \left(\frac{\partial L}{\partial \phi}\right)^2 + \frac{\partial^2 L}{\partial X^2}}. \tag{2}
\]

Assuming that \(c_s^2 = const\) we obtain from (2) the general form of the Lagrangian:

\[
L = V X^{\frac{1 + c_s^2}{2c_s^2}} - U, \tag{3}
\]

where \(U = U(\phi)\) and \(V = V(\phi)\) are the potentials and \(X = \phi_j \dot{\phi}^{ij}/2\) is the kinetic term for the field \(\phi\). The temporal dependence of the potential \(U\) is unambiguously determined from (1) as:

\[
U = \frac{\rho_{de}(c_s^2 - w_{de})}{1 + c_s^2}. \tag{4}
\]

We see that the product \(V X^{\frac{1 + c_s^2}{2c_s^2}}\) is also determined unambiguously, but \(V\) and \(X\) separately are not. Really,
the explicit expressions for their temporal dependences can be obtained in the form:

\[ V = V_0(w_{de} - c_s^2)\rho_{de}, \]
\[ X = \left( \frac{1}{V_0} \frac{c_s^2}{1 + w_{de}} \right)^{\frac{2}{1 + c_s^2}}. \]

Here \( V_0 \) is an arbitrary integration constant, for determination of which we have no condition. So, as expected, in the case of constant \( c_s^2 \) the infinite degeneracy of reconstructed Lagrangians cannot be broken. On the other hand for any fixed \( c_s^2 \neq 0 \) Lagrangian does not contain undefined constant. For special case \( c_s^2 = 1 \) the Lagrangian \( 8b \) yields the canonical form \( L = X - U \).

The dependence of dark energy density on time or scale factor \( a \) is obtained by integration of the continuity equation \( \dot{\rho}_{de} = 0 \) and has the general form for any dependence of EoS parameter \( w_{de} \) on scale factor:

\[ \rho_{de} = \rho_{de}^{(0)} a^{-3(1 + \tilde{w}_{de})}, \]
\[ \tilde{w}_{de} = \frac{1}{\ln a} \int_1^a w_{de}(a) d\ln a, \]

where the dark energy density at current epoch \( \rho_{de}^{(0)} \) is determined by the dimensionless parameter \( \Omega_{de} \equiv \frac{8\pi G \rho_{de}^{(0)}}{3H_0^2} \). For the constant EoS parameter \( \tilde{w}_{de} = w_{de} \). In this paper we consider the scalar field model with generalized linear barotropic EoS \( p_{de} = c_s^2 \rho_{de} + C \)

\[ \rho_{de}^{(0)}(1 + w_0 a^{-3(1+c_s^2)}) + c_s^2 w_0 \]

where \( w_0 \) is the EoS parameter at the current epoch, \( a = 1 \). They essentially simplify the analysis without reducing generality. For such scalar field its phenomenological density \( \rho_{de} \) and pressure \( p_{de} \) are analytical functions of \( a \) for any values of the constants \( c_s^2 \) and \( w_0 \) defining the type and the dynamics of scalar field. Both have the clear physical meaning: \( w_0 \) is the EoS parameter \( w_{de} \) at current epoch, \( c_s^2 \) is asymptotic value of the EoS parameter \( w_{de} \) at early times (\( a \to 0 \)) for \( c_s^2 > -1 \) and in far future (\( a \to \infty \)) for \( c_s^2 < -1 \). The asymptotic value of \( w_{de} \) in the opposite time direction is \(-1 \) in both cases. So, the Lagrangian of such scalar field
model of dark energy can be reconstructed accurately up to a constant \( V_0 \) if parameters \( \Omega_{de}, w_0, c_s^2 \) and \( c_a^2 \) are given or determined using observational data. In Fig. 1 we present the dependences \( U(a), V(a) \) and \( X(a) \) for different values of \( c_a^2 \).

One can see that the potential \( U(a) \) changes slightly with the value of \( c_s^2 \) from the range \([0,1]\). Moreover, for quintessential scalar field the differences occur in the past, while for phantom scalar field in the future. The potential \( V(a) \) seems to be more sensitive to \( c_a^2 \), but indefiniteness of constant \( V_0 \) cancels this advantage: one can renormalize the potentials so that these lines superimpose. The situation is better for kinetic terms \( X(a) \) (bottom panels): each curve for each value of effective sound speed is distinguishable in the past for both fields. So, we would hope to find some observational data which give the possibility to constrain the value of \( c_s^2 \) for quintessential or phantom scalar field.

III. EFFECTS OF THE SOUND SPEED OF SCALAR FIELD ON THE CMB AND LARGE SCALE STRUCTURE

The value of effective sound speed of dynamical dark energy defines the evolution of density and velocity perturbations when the rest of parameters are fixed [10–20]. For illustration of this effect we integrate the evolution equations for perturbations of densities and velocities of each component (dark energy, cold dark matter, baryons, relativistic components) and metric in synchronous gauge. The evolution of density perturbations for scalar field with [19] and any \( c_s^2 \) can be described either by the set of 2 first-order differential equations presented in [32] (equations (14)-(15)) or by the single second-order differential equation:

\[
\ddot{\delta}_{de} + A\dot{\delta}_{de} + B\delta_{de} + S = 0, \quad (11)
\]

where

\[
A = \frac{\ddot{a}}{a} \left[ 1 - 6w_{de} + 3c_s^2 - \frac{18\left(\frac{\dot{a}}{a} - \left(\frac{\ddot{a}}{a}\right)^2\right)(c_s^2 - c_a^2)}{k^2 + 9\left(\frac{\dot{a}}{a}\right)^2(c_s^2 - c_a^2)} \right],
\]

\[
B = 3\frac{\ddot{a}}{a} \left( c_s^2 - w_{de} \right) - 9\left(\frac{\dot{a}}{a}\right)^2 \left[ w_{de} - c_s^2 + c_a^2 (c_s^2 - w_{de}) \right]
+ \frac{6\left(\frac{\dot{a}}{a} - \left(\frac{\ddot{a}}{a}\right)^2\right)(c_s^2 - c_a^2)(c_s^2 - w_{de})}{k^2 + 9\left(\frac{\dot{a}}{a}\right)^2(c_s^2 - c_a^2)}
+ c_s^2k^2 + 9\left(\frac{\dot{a}}{a}\right)^2 c_s^2 (c_s^2 - c_a^2)
\]

\[
S = \frac{1}{2}(1 + w_{de}) \left[ \ddot{c}_s^2 + 6\frac{\dot{a}}{a}\frac{\ddot{a}}{a} - 3\left(\frac{\dot{a}}{a}\right)^2 \right]
- \frac{18\left(\frac{\dot{a}}{a} - \left(\frac{\ddot{a}}{a}\right)^2\right)(c_s^2 - c_a^2)}{k^2 + 9\left(\frac{\dot{a}}{a}\right)^2(c_s^2 - c_a^2)}.
\]

In the top panels of Fig. 2 we present the results of integration of such equations by CAMB [38, 39] in cosmological model with the quintessential scalar field (left) and the phantom one (right) for different values of effective sound speed: \( c_s^2 = 1, 0.5, 0.1, 0.01, 0.0 \). All perturbations are computed for the cold dark matter rest frame. The density perturbations of cold dark matter (\( \delta_{cdm} \)) and baryons (\( \delta_b \)) are presented only for model with dark energy with \( c_s^2 = 1 \) (top thick solid lines). For the scalar field the absolute values of density perturbations (\( \abs{\delta_{de}} \)) are presented.

Taking the adiabatic initial conditions for matter perturbations and treating the dark energy as subdominant (probe) in the gravitational potential of matter (see equations (18) in [32]) means that \( \delta_{m}^{\text{(init)}} \sim -h^{\text{(init)}}, \delta_{de}^{\text{(init)}} \sim -(1 + w_{de})h^{\text{(init)}} \). So, in the early Universe, when scale of perturbation is superhorizon, the matter density perturbations and the quintessential scalar field (1 + \( w_{de} > 0 \)) ones have the same sign: positive matter density perturbation – positive scalar field one, negative matter density perturbation – negative scalar field one. The amplitude of dark matter density perturbations increase monotonically changing the rate at the transition from radiation-dominated epoch to matter-dominated one. The baryon-photon density perturbation starts to oscillate after entering into its sound horizon and continues up to recombination epoch. The temporal behavior of scalar field density perturbations is more complicated since it depends on the value of effective sound speed of scalar field and the wave number of perturbation. As it is shown for \( k = 0.05 \) Mpc\(^{-1}\), the amplitude of \( \delta_{de} \) changes sign from “+” to “-” after entering its own acoustic horizon and then freezes at some value for \( c_s^2 = 1 \), changes sign from “+” to “-” after entering its own acoustic horizon and then freezes at some value after few oscillations for \( c_s^2 \sim 0.5 \), does not change the sign but freezes at some value for \( 0 < c_s^2 < 0.1 \), and increases monotonically for \( c_s^2 = 0 \).

For phantom scalar field (1 + \( w_{de} < 0 \)) the initial amplitude of \( \delta_{de} \) is lower and has the sign opposite to the sign of \( \delta_{m}(a_{\text{init}}) \), but dependence of \( \abs{\delta_{de}(a)} \) is similar to the corresponding dependences for quintessence scalar field.

Note that for the dark energy perturbations their signs and magnitudes are strongly gauge-dependent at superhorizon scales [14].

To analyze the effect of dark energy perturbations on matter ones we have calculated the matter power spectra for models with the same main parameters but different \( c_s^2 \). In the bottom panels of Fig. 2 we present the relative differences \( 1 - P_m(k; c_s^2 \neq 1)/P_m(k; c_s^2 = 1) \) for \( c_s^2 = 0.5, 0.1, 0.01, 0.0 \), which illustrate the influence of reducing the value of effective sound speed on the matter power spectrum: suppression of power at large scales (\( k < 0.001 \) Mpc\(^{-1}\)) for the quintessential scalar field and enhancement for the phantom one, enhancement of power at intermediate scales (0.001 \( < k < 0.1 \) Mpc\(^{-1}\)) for the quintessential scalar field and small sup-
FIG. 2: Top panels: Evolution of density perturbations of cold dark matter (top thick solid line), baryons (blue solid line) and scalar field dark energy with different values of effective sound speed ($c_s^2=1, 0.5, 0.1, 0.0$). Bottom panels: Effect of the sound speed of scalar field dark energy on the matter power spectrum. Left column – quintessential scalar field with $w_0 = -0.9$, $c_s^2 = -0.5$; right column – phantom scalar field with $w_0 = -1.1$, $c_s^2 = -1.5$. The rest of parameters correspond to the model $p_3$ from [8].

expression for the phantom one. At $k > 0.1$ Mpc$^{-1}$ the effect is completely absent for $c_s \neq 0$. Therefore, the form of the matter power spectrum at large and intermediate scales is sensitive to the value of $c_s$, more sensitive for the quintessential scalar field and less sensitive for the phantom one. At intermediate scales, for which we have the observational data, the differences for models with $0 \leq c_s \leq 1$ are within 2% for the quintessential scalar field and within 0.5% for the phantom one, while observational uncertainties are not less than 10% [40, 41] (see also Fig. 5 in [33]).

The density perturbations of scalar fields with different $c_s$ should affect the temperature fluctuations of cosmic microwave background (CMB) because of Sachs-Wolfe effect [42] with different strength. Really, Fig. 3 illustrates the suppression of power at large angular scales (low spherical harmonics) for quintessential scalar field and the enhancement for phantom one. The differences for models with $0 \leq c_s \leq 1$, shown in the bottom panels, are within 15% for $w_0 = -0.9$ and within 6% for $w_0 = -1.1$, while observational uncertainties are within 30% [2, 43] (see also Fig. 6 in [33] and Fig. 5 in [8]). So, the arbitrating power of CMB data for $c_s$ is also weak. On the other hand the uncertainty in determination of $c_s$ increases the uncertainty in determination of the ratio of tensor and scalar modes $r \equiv T/S$. However, the possible tension between upper limit $r < 0.11$ at 95% C.L. given by Planck Collaboration [3] and 1σ-range $0.15 \leq r \leq 0.27$ given by BICEP2 Collaboration [44] might be removed in the model with scalar field as dark energy.\footnote{Both collaborations have presented the results for $\Lambda$CDM model.}

Let us finally discuss the impact of effective sound speed and other dark energy parameters on the scale dependence of dark matter and dark energy perturbations. In Fig. 4 the transfer functions for dark matter and dark energy are presented for the current epoch (for discussion of dark energy transfer functions see also [45]). Here the perturbations for each component are calculated in its own rest frame, so the dark energy transfer func-
FIG. 3: Top panels: The angular power spectrum of CMB temperature fluctuations at large angular scales for model with scalar field dark energy with different values of squared effective sound speed ($c_s^2=1, 0.5, 0.1, 0.01, 0.0$). Bottom panels: The relative differences of $C_l$'s for models with different values of squared effective sound speed. Left column – quintessential scalar field with $w_0=-0.9$, $c_a^2=-0.5$; right column – phantom scalar field with $w_0=-1.1$, $c_a^2=-1.5$. The rest of parameters correspond to model $p_3$ from [8].

In all panels the main cosmological parameters are taken from $p_3$ in [8]. In the upper row the transfer functions for different values of $c_a^2$ are shown (for quintessential field here $w_0=-0.9$, $c_a^2=-0.5$, for phantom $w_0=-1.1$, $c_a^2=-1.5$). We see that the effect of value of the effective sound speed on the unobservable scale dependence of transfer function of dark energy is quite strong, while the effect on observable scale dependence of cold dark matter transfer function is negligible. It is interesting to note that the scale dependence of dark energy transfer function with $c_a^2=0$ coincides with the scale dependence of dark matter transfer functions (superimposed lines in both panels) despite the sufficiently different equation of state and different frames. As it can be seen in the middle panels, the value of $w_0$ has virtually no effect on the scale dependence of both dark matter and dark energy transfer functions (here for quintessential field $c_s^2=-0.5$, $c_a^2=1$, for phantom $c_s^2=-1.5$, $c_a^2=1$). In the bottom row the scale dependences of transfer functions are shown for different values of adiabatic sound speed. For the phantom field (right panel, $w_0=-1.1$, $c_s^2=1$) the value of $c_s^2$ has almost no influence on the scale dependence of transfer functions for both dark components. For the quintessential field (left panel, $w_0=-0.9$, $c_s^2=1$) the value $c_s^2=0$ leads to the visibly larger suppression of cold dark matter transfer function than other considered values, the suppression of dark energy transfer functions is different for all considered values of $c_s^2$.

IV. OBSERVATIONAL CONSTRAINTS ON PARAMETERS OF THE SCALAR FIELD

A. Observational data and method

To obtain joint constraints on the main cosmological parameters ($\Omega_b h^2$, $\Omega_{cdm} h^2$, $H_0$, $A_s$, $n_s$, $\tau_{reio}$) along with the independent dark energy ones ($w_0$, $c_a^2$ and $c_s^2$) we use the Monte Carlo Markov chain (MCMC) method implemented in the CosmoMC code [46, 47]. To compute the theory predictions we use the CAMB code assuming the Universe to be spatially flat (this allows the determina-
FIG. 4: The dark matter and dark energy transfer functions for different values of $c_s^2$ (upper panels), $w_0$ (middle) and $c_a^2$ (bottom). Left – quintessence, right – phantom.

The obtained results are presented in Fig. 5 and Table I for 2 combined datasets: Planck+HST+BAO+SNLS3 and Planck+HST+BAO+Union2.1. We see that the value of $c_s^2$ is unconstrained by the used data: for both sets the one-dimensional posteriors for it are virtually flat, the $2\sigma$ ranges cover the full prior range. Unfortunately, this could be expected and agrees with the conclusions of other authors, e. g. [23] who pointed out

B. Results and discussion

The obtained results are presented in Fig. 5 and Table I for 2 combined datasets: Planck+HST+BAO+SNLS3 and Planck+HST+BAO+Union2.1. We see that the value of $c_s^2$ is unconstrained by the used data: for both sets the one-dimensional posteriors for it are virtually flat, the $2\sigma$ ranges cover the full prior range. Unfortunately, this could be expected and agrees with the conclusions of other authors, e. g. [23] who pointed out
that it would not be possible to distinguish surely between $c_s^2 = 0$ and $c_s^2 = 1$ until the availability of cross-correlation of the Planck data on CMB with the expected large scale structure data from LSST.

However, it is important to check whether the free value of effective sound speed affects the possibility to constrain other cosmological parameters and especially dark energy ones. If we compare the presented in Table 1 mean values and 2σ limits for all parameters except for $c_s^2$ with the corresponding mean values and 2σ limits from Table 2 of [8], we see the good coincidence between them. So, we conclude that the problem with determination of value of $c_s^2$ has no effect on the precision and reliability of determination of values of other cosmological parameters. All conclusions about the properties of best-fit models (obtained from the best-fit sample) for both datasets from [8] remain valid in this case.

V. CONCLUSION

We have analyzed the possibility of reconstruction of Lagrangian of scalar field with $c_s^2 = \text{const}$ and found that it is not unambiguous because possible only up to an arbitrary constant. Considering the dark energy in the Universe to be such a scalar field we have found that the influence of the value of effective sound speed on observable quantities is too weak to allow any reliable observational constraints on this parameter. Estimating the value of $c_s^2$ together with other dark energy and cosmological parameters on the basis of datasets Planck+HST+BAO+SNLS3 and Planck+HST+BAO+Union2.1 we have found that the effective sound speed remains unconstrained by these datasets while the constraints on other parameters are in good agreement with those obtained from the same datasets for the classical scalar field ($c_s^2 = 1$) in [8].

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FIG. 5: One-dimensional marginalized posteriors (solid lines) for $\Omega_{de}$, $w_0$ and $c_s^2$; color panels show two-dimensional marginalized posterior distributions in the planes $\Omega_{de} - w_0$, $\Omega_{de} - c_s^2$ and $w_0 - c_s^2$, where solid lines show the 1$\sigma$ and 2$\sigma$ confidence contours. The plots are for Planck+HST+BAO with SNLS3 (left) and Union2.1 (right) SNe Ia compilations.