The Hierarchy Problem and the Cosmological Constant Problem Revisited

— A new view on the SM of particle physics —

Fred Jegerlehner

Deutsches Elektronen-Synchrotron (DESY), Platanenallee 6, D–15738 Zeuthen, Germany
Humboldt-Universität zu Berlin, Institut für Physik, Newtonstrasse 15, D–12489 Berlin, Germany

Abstract
We argue that the Standard Model (SM) in the Higgs phase does not suffer from a “hierarchy problem” and that similarly the “cosmological constant problem” resolves itself if we understand the SM as a low energy effective theory emerging from a cutoff-medium at the Planck scale. We actually take serious Veltman’s “The Infrared - Ultraviolet Connection” addressing the issue of quadratic divergences and the related huge radiative correction predicted by the SM in the relationship between the bare and the renormalized theory, usually called “the hierarchy problem” and claimed that this has to be cured. We discuss these issues under the condition of a stable Higgs vacuum, which allows to extend the SM up to the Planck cutoff. The bare Higgs boson mass then changes sign below the Planck scale, such that the SM in the early universe is in the symmetric phase. The cutoff enhanced Higgs mass term as well as the quartically enhanced cosmological constant term provide a large positive dark energy which triggers the inflation of the early universe. Reheating follows via the decays of the four unstable heavy Higgs particles, predominantly into top-antitop pairs, which at this stage are still massless. Preheating is suppressed in SM inflation since in the symmetric phase bosonic decay channels are absent at tree level. The coefficients of the shift between bare and renormalized Higgs mass as well as of the shift between bare and renormalized vacuum energy density exhibit close-by zeros at about $7.7 \times 10^{14}$ GeV and $3.1 \times 10^{15}$ GeV, respectively. The zero of of the Higgs mass counter term triggers the electroweak phase transition from the low energy Higgs phase and to the symmetric phase above the transition point. Since inflation tunes the total energy density to take the critical value of a flat universe and all contributing components are positive, it is obvious that the cosmological constant today is naturally a substantial fraction of the total critical density. Thus taking cutoff enhanced corrections seriously the Higgs system provides besides he masses of particles in the Higgs phase also dark energy, inflation and reheating in the early universe. The main unsolved problem in our context remains the origin of dark matter. Higgs inflation is possible and likely even unavoidable provided new physics does not disturb the known relevant SM properties substantially. The scenario highly favors to understand the SM and its main properties as a natural structure emerging at long distance. This in particular concerns the SM symmetry patterns and their consequences.

Keywords Higgs vacuum stability · hierarchy problem · cosmological constant problem · inflation

PACS 14.80.Bn · 11.10.Gh · 12.15.Lk · 98.80.Cq

* Invited talk at the Workshop “Naturalness, Hierarchy and Fine Tuning”
RWTH Aachen, 28 February 2018 to 2 March 2018, Aachen,Germany
The Hierarchy Problem and the Cosmological Constant Problem Revisited

Higgs inflation and a new view on the SM of particle physics

Fred Jegerlehner
Humboldt-Universität zu Berlin, Institut für Physik,
Newtonstrasse 15, D-12489 Berlin, Germany
Deutsches Elektronen-Synchrotron (DESY),
Platanenallee 6, D-15738 Zeuthen, Germany

Abstract

We argue that the Standard Model (SM) in the Higgs phase does not suffer from a “hierarchy problem” and that similarly the “cosmological constant problem” resolves itself if we understand the SM as a low energy effective theory emerging from a cutoff-medium at the Planck scale. We actually take serious Veltman’s “The Infrared - Ultraviolet Connection” addressing the issue of quadratic divergences and the related huge radiative correction predicted by the SM in the relationship between the bare and the renormalized theory, usually called “the hierarchy problem” and claimed that this has to be cured. We discuss these issues under the condition of a stable Higgs vacuum, which allows to extend the SM up to the Planck cutoff. The bare Higgs boson mass then changes sign below the Planck scale, such that the SM in the early universe is in the symmetric phase. The cutoff enhanced Higgs mass term as well as the quartically enhanced cosmological constant term provide a large positive dark energy which triggers the inflation of the early universe. Reheating follows via the decays of the four unstable heavy Higgs particles, predominantly into top-antitop pairs, which at this stage are still massless. Preheating is suppressed in SM inflation since in the symmetric phase bosonic decay channels are absent at tree level. The coefficients of the shift between bare and renormalized Higgs mass as well as of the shift between bare and renormalized vacuum energy density exhibit close-by zeros at about $7.7 \times 10^{14}\text{ GeV}$ and $3.1 \times 10^{15}\text{ GeV}$, respectively. The zero of of the Higgs mass counter term triggers the electroweak phase transition from the low energy Higgs phase and to the symmetric phase above the transition point. Since inflation tunes the total energy density to take the critical value of a flat universe and all contributing components are positive, it is obvious that the cosmological constant today is naturally a substantial fraction of the total critical density. Thus taking cutoff enhanced corrections seriously the Higgs system provides besides the masses of particles in the Higgs phase also dark energy, inflation and reheating in the early universe. The main unsolved problem in our context remains the origin of dark matter. Higgs inflation is possible and likely even unavoidable provided new physics does not disturb the known relevant SM properties substantially. The scenario highly favors to understand the SM and its

* Invited talk at the Workshop “Naturalness, Hierarchy and Fine Tuning”
RWTH Aachen, February 28, to March 2, 2018, Aachen, Germany
E-mail: fjeger@physik.hu-berlin.de
main properties as a natural structure emerging at long distance. This in particular concerns the SM symmetry patterns and their consequences.

1 Prelude: Higgs inflation in a nutshell

In order to give a quick overview on what will be the essential conclusion of the analysis I start with this prelude (see [1, 2]). The Standard Model (SM) hierarchy problem [3] is well known and addressed very frequently to motivate Beyond the Standard Model (BSM) scenarios in general and a supersymmetric extension of the SM in particular. The renormalized Higgs boson mass is small, at the Electro-Weak (EW) scale, the bare one is huge due to radiative corrections growing quadratically with the ultraviolet (UV) cutoff, which is assumed to be given by the Planck scale $\Lambda_{\text{Pl}} \sim 10^{19}$ GeV$^1$. The cutoff dependence is illustrated in Fig. 1 assuming the cutoff as a renormalization scale and the SM Renormalization Group (RG) ruling the scale dependence of the SM couplings (see below). It is the RG improved version of Veltman’s “The Infrared - Ultraviolet Connection” [4], where the SM renormalization of the Higgs boson mass ($m_0$ the bare, $m$ the renormalized mass)

$$\begin{align*}
m_0^2 &= m^2 + \delta m^2, \\
\delta m^2 &= \frac{\Lambda_{\text{Pl}}^2}{16\pi^2} C(\mu),
\end{align*}$$

1 The Planck medium, which we may call ether, somehow gets shaped by gravity and quasi-particle interactions emergent in the SM at low energies. It is characterized by the well known fundamental Planck cutoff $\Lambda_{\text{Pl}}$ or equivalently the Planck mass $M_{\text{Pl}}$, which derive from the basic fundamental constants, the speed of light $c$ characterizing special relativity, the Planck constant $\hbar$ intrinsic to quantum physics and Newton’s constant $G_N$ the dimensionful key parameter of gravity. Unified they provide an intrinsic length $\ell_{\text{Pl}}$, the Planck length, which also translates into the Planck time $t_{\text{Pl}}$ and the Planck temperature $T_{\text{Pl}}$:

Planck length: $\ell_{\text{Pl}} = \sqrt{\frac{\hbar G_N}{c^3}} = 1.616252(81) \times 10^{-33}$ cm,

Planck time: $t_{\text{Pl}} = \ell_{\text{Pl}}/c = 5.4 \times 10^{-44}$ sec,

Planck (energy) scale: $M_{\text{Pl}} = \sqrt{\frac{\hbar c}{G_N}} = 1.22 \times 10^{19}$ GeV,

Planck temperature: $T_{\text{Pl}} = \sqrt{\frac{\hbar c}{k_B G_N}} = 1.416786(71) \times 10^{32}$ K.

In our context they define a shortest distance $\ell_{\text{Pl}}$ and beginning of time $t_{\text{Pl}}$, i.e. $t > t_{\text{Pl}}$. The Planck era energy scale equivalently is set by $E_{\text{Pl}} = \Lambda_{\text{Pl}} \equiv M_{\text{Pl}}$ or temperature $T_{\text{Pl}}$, as for most time in the evolution of the early universe, when elementary particle physics is at work and before the epoch of formation of hadrons, particle processes are in thermal equilibrium, with well known exceptions during inflation and the electroweak phase transition.
has been addressed (see also [5–8]). The coefficient function $C(\mu)$ depends on the dimensionless SM couplings, which depend on the renormalization scale logarithmically only. The Higgs mass counter term is huge when we adopt the Planck scale as the cutoff to regulate UV singularities. Is this a problem? Is this unnatural? In the first instance it is a prediction of the SM! At low energy we see what we see (what is to be seen): the renormalizable, renormalized SM as it describes close to all we know up to LHC energies. But what does the SM look like if we go to very high energies even to the Planck scale? Not too far below the Planck scale we start to see the bare theory i.e. a SM with its bare short distance effective parameters, so in particular a very heavy Higgs boson, which likely is moving at most very slowly. The potential energy

$$V(\phi) = \frac{m^2}{2} \phi^2 + \frac{\Lambda}{24} \phi^4$$

(2)

is then large while the kinetic energy $\frac{1}{2} \dot{\phi}^2$ is small, as a dedicated calculation shows. Here we have in mind the cosmological solutions of Einstein’s General Relativity Theory (GRT) for an isotropic universe of constant spatial curvature, parametrized by the Robertson-Walker metric. The field $\phi$ is then a function of the cosmic time $t$ only and $\phi$ is the corresponding time derivative. The Higgs boson contributes to the energy momentum tensor by providing the pressure $p = \frac{1}{2} \dot{\phi}^2 - V(\phi)$ and the energy density $\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi)$. As we approach the Planck scale (bare theory) the slow–roll condition $\frac{1}{2} \dot{\phi}^2 \ll V(\phi)$ is satisfied during some window in the time evolution and then $p \approx -V(\phi)$; $\rho \approx +V(\phi)$ implies $p \approx -\rho$, which closely approximates the equation of state $p_A = -\rho_A$ of Dark Energy (DE) $\rho = \rho_A$, the equivalent of a Cosmological Constant (CC) $\Lambda$. DE follows a very special equation of state, only observed indirectly through Cosmic Microwave Background (CMB) [9–13] pattern and through Supernovae (SN) counts [14, 15]. No lab system observation so far has been reported to my knowledge, although statistical mechanics system like the Ising model obey such a ground state equation (see e.g. [16]). Thus, as a consequence of the hierarchy boost, the SM Higgs boson in the early universe provides a huge dark energy, which mimics strong anti-gravity at work which is inflating the universe! The Friedmann equations together with energy conservation read

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8}{3} \pi G N \rho; \quad \frac{\dot{a}}{a} = -\frac{4}{3} \pi G N (3p + \rho); \quad \dot{\rho} = -3 \frac{\dot{a}}{a} (\rho + p)$$

(3)

and indeed if DE dominates we have $(3p + \rho) \approx -2\rho$ such that we have an accelerated expansion $\ddot{a}/a > 0$ and $\frac{dH}{dt} = H(t) \frac{dt}{dr}$ which implies an exponential growth $a(t) = \exp Ht$ of the radius $a(t)$ of the universe. $H(t) = \dot{a}(t)/a(t)$ is the Hubble constant where $H \propto \sqrt{\Omega_0}$ in a

2 Einstein’s field equation for the metric tensor $g_{\mu\nu}$, which incorporates the gravitational field, is given by $G_{\mu\nu} = 8\pi G N \rho$ where $G = \frac{k}{\hbar^2}$ is the effective interaction constant, $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein curvature tensor (geometry) and $T_{\mu\nu}$ is the energy-momentum tensor (matter and radiation). Cosmology is shaped by Einstein gravity, which together with Weyl’s postulate, that radiation and matter (galaxies etc.) on the cosmological scale behave like an ideal fluid, and the cosmological principle, assuming isotropy of space (implying homogeneity), fixes the form of the metric and of the energy-momentum tensor: 1) the metric (3-spaces of constant curvature $k = \pm 1, 0$) takes the form $ds^2 = (c dt)^2 - a^2(t)(dr^2 / (1 - kr^2) + r^2 d\Omega^2)$, where in the comoving frame $ds = c dt$ with $t$ the cosmic time; 2) the energy-momentum tensor takes the form $T^{\mu\nu} = (\rho(t) + p(t)(t))u^\mu u^\nu - p(t) g^{\mu\nu}$; $u^\mu \equiv \frac{dx^\mu}{dt}$, where $\rho(t)$ is the density and $p(t)$ the pressure of the fluid. As a differential equation for the geometry factor $a(t)$ one obtains Friedmann’s equations (3). One needs $p(t)$ and $\rho(t)$ (which are related by an equation of state characterizing the medium) in order to get the radius of the universe $a(t)$ and its evolution in time. The Higgs potential contributes $T_{\mu\nu} = \Theta_{\mu\nu} = V(\phi) g_{\mu\nu} +$ derivative terms, where $\Theta_{\mu\nu}$ is the symmetric energy momentum tensor of the SM (or extensions of it). Only a scalar potential can contribute a term proportional to $g_{\mu\nu}$, which mimics a cosmological constant.
DE dominated era. Inflation stops quite quickly as the field decays exponentially. The field equation
\[ \ddot{\phi} + 3H \dot{\phi} = -V'(\phi), \]
for a potential dominated by the mass term \( V(\phi) \approx \frac{1}{2} \phi^2 \) represents a harmonic oscillator with friction and leads to Gaussian inflation as established by an analysis of the CMB pattern by the Planck mission [17]. One of the reasons why the inflation phenomenon must have happened in the early universe is that the universe looks flat today while a flat universe in the absence of DE and then formally satisfying the strong energy condition \( \rho + 3p \geq 0 \) which implies \( a(t)/a \leq 0 \), is exponentially unstable in its time evolution. So, different types of solutions of the Friedmann equations at \( \mu_A \neq 0 \) deviate dramatically during the 13.8 billion years life of the universe after the Big Bang. The “flattenization” by inflation is evident as the curvature term \( k/a^2(t) \sim k \exp(-2Ht) \to 0 \) drops exponentially independent of the curvature type. The latter, characterized by the normalized curvature \( k = 0, \pm 1 \), distinguishes flat infinite, spherical closed or hyperbolic open geometries. It is very important to note that the CC given by the Higgs potential \( V(\phi) \) in the symmetric phase is positive in any case, very different from the (much smaller) contribution from the Higgs VEV in the broken phase, which is negative [18]. This already shows that dynamical e'

Inflation tunes the total energy density to be that of a flat space (as if \( k = 0 \)), which according to (3) for \( k = 0 \) requires a specific “critical” energy density
\[ \rho_{0, \text{crit}} = 3H_0/(8\pi G_N) = \mu_{\text{crit}}^4 \]
where \( \mu_{\text{crit}} \approx 0.00247 \text{ eV} \). (5)

With \( H_0 \) the present Hubble constant \( \rho_{0, \text{crit}} \) is the present total energy density. For an arbitrary mixture of dark energy, matter and radiation the first of the equations (3) reads
\[ \rho = \rho_{0, \text{crit}} \left[ \Omega_A + \Omega_m \left( \frac{a_0}{a} \right)^3 + \Omega_\gamma \left( \frac{a_0}{a} \right)^4 \right] \]
where \( \Omega_i = \rho_i/\rho_{0, \text{crit}} \) are the present fractional densities and \( a_0 = a(t_0) \) the spacial metric scale factor at present time \( t_0 \). Including the curvature term \( \Omega_k = -k/(a_0^2 H_0^2) \) we have
\[ \Omega_A + \Omega_m + \Omega_\gamma + \Omega_k = 1 \]
(7)
as an exact equation, and when the curvature term is exponentially suppressed we very accurately have
\[ \Omega_A + \Omega_m + \Omega_\gamma \approx 1 \]
(8)
what is supported strongly by observation (CMB). Whatever constitutes the universe the curvature constant is \( k = +1 \), \( k = 0 \) or \( k = -1 \) according to whether the present density \( \rho_0 \) is greater than, equal to, or less than \( \rho_{0, \text{crit}} \). A higher density \( \rho_0 > \rho_{0, \text{crit}} \) implies a recontraction

---

\( ^3 \) Matter here includes dark matter (DM) and normal baryonic-matter (BM), the non-relativistic stuff; radiation includes all relativistic degrees of freedom: photons, neutrinos and at high energies other SM particles besides the Higgs boson which gets boosted to be heavy by its missing naturalness. Note that normal baryonic matter only emerges after the QCD hadronization phase transition, after protons and neutrons have been formed. In contrast cold dark matter looks must have existed much earlier not to long after Planck time.
of the initially (at the Big Bang) expanding universe, a lower density $\rho_0 < \rho_{0,\text{crit}}$ would not be able to stop the expansion forever.

We know that $\rho_{0,\Lambda} = c_4 \Lambda_0$ today is about $\mu_4 \Lambda_0 = 0.00171$ eV which in a flat universe must be a fraction of the critical density, and actually has been determined to amount to $69.2 \pm 1.2$%. Since the non-DE components drop with a power of the radius $a(t)$ as time goes on, the asymptotic behavior is determined by $\rho_A$ solely. Friedmann solutions in GRT with non-vanishing cosmological constant have been discussed in [19, 20].

We see that the large positive cosmological constant provided by the SM Higgs boson gets tamed by inflation to be part of the critical flat space density. So also the cosmological constant problem may turn out to get its natural explanation (see below).

As inflation is strongly supported to have happened by observation, we must assume the existence of an appropriate scalar field, and the Higgs field is precisely such a field we need and within the SM it has the properties which promote it to be the inflaton. In contrast to other inflaton models the Higgs inflaton is special because almost all properties are known or predictable! Below, I will argue that the SM in the Higgs phase does not suffer from a “hierarchy problem” and that similarly the “cosmological constant problem” resolves itself if we understand the SM as a low energy effective theory emerging from a cutoff medium at the Planck scale.

Adopting a bottom-up approach, I discuss these issues under the condition of a stable Higgs vacuum, by predicting the behavior of the SM when approaching the Planck era at high energies. SM Higgs inflation as exposed in this prelude may look pretty simple but in fact is rather subtle, because of the high sensitivity to the SM parameters uncertainties and a high sensitivity to higher order SM effects. In any case my preconditions are: (i) a stable Higgs vacuum and a sufficiently large Higgs field at $M_{\text{Pl}}$, (ii) physics beyond the SM should not spoil main features of SM i.e. no Supersymmetry (SUSY), no Grand Unified Theories (GUT) etc. pretending to solve the hierarchy problem, and/or affecting the SM RG flow substantially! Here we have to assume that a kind of desert in the heavy particle spectrum is extending effectively up to the Planck scale. This is not so far beyond the grand desert usually assumed in the context of GUTs. This does not exclude new physics which we know to exist, like dark matter, Majorana neutrinos or axions, for example.

Slow-roll inflation in general has been investigated in [21–26] in the 80’s mostly as a top-down approach. For an alternative non-minimal gravity Higgs inflation approach I refer to [27–33]. For yet another Higgs inflation scenario see [34].

In this prelude I have outlined what a correct interpretation of the “hierarchy problem” likely looks like, i.e. the predicted SM hierarchy patterns is not a problem, rather is the solution for what we need to trigger inflation in the early universe. In the following I will discuss the hierarchy issue in a broader context and discuss in some detail the intricacies of the cosmological constant problem and Higgs boson inflation. I will try to convince the reader that the Higgs boson inevitably delivers dark energy and the consequent inflation is strongly supported by a self-consistent perturbative SM calculation [1, 2]. The approach is highly predictive and limited mainly by the uncertainties of the knowledge of the SM parameters and the accuracy of the perturbative calculations of the matching conditions between measured and MS parameters and the MS renormalization group coefficients.

2 The Higgs boson discovery – the SM completion

With the discovery of the Higgs boson by ATLAS [35] and CMS [36] in 2012 a last major but often questioned building block of the electroweak SM has been experimentally
verified. The existence of an elementary scalar has been found to be required to render the electroweak massive gauge theory renormalizable in 1964 by Englert and Brout and by Higgs [37, 38]. The key mechanism turned out to be a Spontaneous Symmetry Breaking (SSB) mechanism of the non-Abelian $SU(2)_L$ gauge sector responsible for the weak interactions. The corresponding Higgs mechanism generates masses for all massive particles while not affecting the renormalizable UV behavior of the massless unbroken theory. Now, remarkably, the SM Higgs boson mass has been found in very special mass range \[ 125.18 \pm 0.16 \text{ GeV}, \] which seems matches the possibility to extrapolate the SM up to the Planck scale. Knowing the Higgs mass \( M_H \) and using the mass coupling relation valid in the Higgs phase, we also know the Higgs self-coupling \( \lambda \) and hence the Higgs potential \[ V = \frac{m^2}{2} H^2 + \frac{\lambda}{4} H^4, \] which is the object in our focus. Perturbativeness and vacuum stability of the Higgs potential are the key issues in this context (for early considerations see [39–44]).

Fig. 2 adopted from an analysis by Hambye and Riesselmann in 1996 [45] illustrates the possible impact to have a Higgs mass in a window extending up to the Planck scale. Knowing the Higgs mass \( M_H \) and using the mass coupling relation valid in the Higgs phase, we also know the Higgs self-coupling \( \lambda \) and hence the Higgs potential \[ V = \frac{m^2}{2} H^2 + \frac{\lambda}{4} H^4, \] which is the object in our focus. Perturbativeness and vacuum stability of the Higgs potential are the key issues in this context (for early considerations see [39–44]).

Later estimates have been improved after more precise SM parameters like the QCD coupling \( \alpha_s \) and top quark mass \( M_t \) became available, see e.g. [46, 54–57]. Given the Higgs self-coupling all relevant SM parameters are known. While the RG evolution equations in the symmetric phase of the SM have been known for a long time to two loops, recently also the three loop result have been calculated in [58–60] in the \( \overline{\text{MS}} \) scheme. The \( \overline{\text{MS}} \) input parameters, which are most suitable to parametrize the high energy tail, have to be gotten via the matching conditions from the experimentally measured ones (see [46, 56, 57, 61–66] and references therein).

### 2.1 The SM running parameters

In Fig. 3 we plot the evolution of the SM couplings as a function of the log of the energy scale. As we learn from Fig. 3 the amazing thing is that the perturbation expansion turns out to work up to the Planck scale! In our analysis, for the input parameters specified below, we have no Landau pole or other singularities and the Higgs potential remains stable. This likely opens a new gate to precision cosmology of the early universe [1, 46, 56]. The remarkable
interrelations between SM couplings may be summarized as follows: the $U(1)_Y$ coupling $g_1$ is screening (IR free), the $SU(2)_L$ coupling $g_2$ and the $SU(3)_c$ coupling $g_3$ are antiscreening (UV free) as expected. In contrast the top quark Yukawa coupling $y_t$ and the Higgs self-coupling $\lambda$, which are screening if standalone (IR free, like QED), as part of the SM change their behavior from IR free to UV free, such that perturbation theory works the better the higher the energy in these couplings as well. What happens is that QCD effects dominate the behavior of the top Yukawa coupling RG provided $g_3 > \frac{3}{4} y_t$ in the gaugeless ($g_1, g_2 = 0$) approximation, which is satisfied. Similarly, the top quark Yukawa effect dominates the Higgs coupling RG provided $\lambda < \frac{3}{4} (\sqrt{5} - 1) y_t^2$, which also holds in the gaugeless ($g_1, g_2 = 0$) limit. These conditions are satisfied in the SM with the given parameters and extend to higher orders as far as these are known. We note that the Abelian hypercharge coupling $g_1$, although increasing with energy, stays small up to $\Lambda_{\text{Pl}}$ such that it does not affect perturbativeness. Note that in spite of its increasing behavior $g_1 < g_2 < g_3$ at Planck scale. Interestingly there we have an inverted $g_3 < g_2$ hierarchy of the non-Abelian gauge couplings. In the focus is the Higgs self-coupling because it may not stay positive $\lambda > 0$ up to $\Lambda_{\text{Pl}}$. In fact a 3 $\sigma$ significance for meta-stability is claimed e.g. in [46,56] (see right panel of Fig. 2). Calculating previously missing 2-loop contributions to the matching conditions the significance for missing stability could be reduced to a 1 $\sigma$ gap in [66]. The existence of a zero in $\lambda(\mu)$ crucially depends on the precise size of the top Yukawa coupling $y_t$, which actually seems to decide about the stability of our world. Note that $\lambda = 0$ would be an essential singularity! Uncertainties here have to be reduced by more precise input parameters and better established EW matching conditions. For our input parameters the Higgs coupling decreases up to the zero of the beta–function $\beta_\lambda$ at $\mu = 3.5 \times 10^{17}$ GeV, where $\lambda$ is small but still positive and above $\mu$ increases with energy up to $\mu = M_{\text{Pl}}$.

I think our discussion shows that ATLAS and CMS results may have “revolutionized” particle physics in an unexpected way, namely showing that the SM has higher self-consistency (conspiracy) than expected and previous arguments for the existence of new physics may turn out not to be compelling. Also the absence so far of any established new physics signal at the LHC may indicate that commonly accepted expectations may not be satisfied. On the one hand it seems to look completely implausible to assume the SM to be essentially valid up to Planck energies, on the other hand the high tide of speculations about physics beyond the SM have been of no avail. Within the context of GUTs, a large gap in the par-
particle spectrum, the “grand desert” up to the GUT scale at about \(10^{16}\) GeV, still is a widely accepted hypothesis. So why the “grand desert” could not extend a little further namely to \(10^{19}\) GeV? The central issue for the future is the very delicate conspiracy between SM couplings, which makes precision determination of SM parameters more important than ever. This mainly challenges accelerator physics, the LHC experiments and the future ILC/FCCee projects (top quark and Higgs boson factories), which could improve the precision values for \(\lambda, y_t\) and \(\alpha_s\). Still important are lower energy hadron facilities, which should provide more precise hadronic cross sections to reduce hadronic uncertainties in \(\alpha(M_Z)\) and \(\alpha_s(M_Z)\). This could open a new gate to precision cosmology of the early universe, in case the Higgs boson inflation scenario outlined in Sect. 1 could be hardened.

3 Thoughts on guiding principles and paradigms in particle physics

In last decades “solving the hierarchy problem” has been a strong motivation to find possible extensions of the SM. Guiding principles often have played an important role for progress in science although they afterwards turned out to miss the point they suggested natural laws should follow. Most prominent are symmetry principles. Related group theory is beautiful mathematics but is not always mapping the real or supposed physical problem it was proposed to describe. Kepler already dreamed of the Platonic bodies (regular polyhedra) to rule celestial mechanics of planets. After his attempt to prove this by analyzing celestial data, finally Kepler’s laws resulted from his investigation. Kepler’s model is completely false, the interplanetary distances it predicts are not sufficiently accurate, and Kepler was scientist enough to accept this eventually. But it is an excellent example of how truth and beauty do not always fit together. The widespread string theory paradigm assumes that a simple stringy structure at and beyond the Planck scale could explain what we observe down on earth, actually a rather complex real world. That something simple looks complicated when seen from far away is certainly not a very natural expectation. In any case concepts like the ones we are addressing here: Naturalness, Hierarchy and Fine-Tuning make no sense by themselves, but require to specify the context.

While “solving the hierarchy problem” seems to fail as a route to new physics, in contrast, the concept of the minimal renormalizable extension of Fermi’s weak interaction theory turned out to be impressively successful in constructing piece by piece the electroweak SM. It lead to the introduction of the massive intermediate spin 1 bosons \(W^\pm\) in charged current processes, the prediction of neural currents and the need for a \(Z\) boson. The resulting \(U(1)_Y \otimes SU(2)_L\) gauge theory, renormalizable in the massless case, required a scalar spin 0 boson, the Higgs boson, as a trick to generate masses of the weak gauge bosons and the fermions, without spoiling renormalizability. Spontaneous symmetry breaking, a mechanism known from condensed matter physics, turned out to be the key mechanism for a renormalizable massive gauge theory. Another issue concerning minimality versus non-minimality are GUTs, where fermions are necessarily populating higher representations while the fundamental ones are not occupied. Therefore the typical leptoquarks necessarily suffering up in GUTs are unnatural as an emergent phenomenon.

---

Footnote: The unification paradigm celebrated its triumphal success in Maxwell’s electromagnetism, which unified electrical and magnetic laws and predicted electromagnetic waves. In contrast as we know the electroweak theory is not a true unification, it rather regulates the mixing of electromagnetic and weak interaction phenomena. At the heart is \(\gamma - Z\) mixing and \(Z\) resonance (heavy-light) physics, which manifests itself most convincingly in electron positron annihilation into \(Z\) bosons. All further unification attempts so far are missing confirmation.
A convincing solution of the SM’s hierarchy problem is known to be provided by a supersymmetric extension of the SM. SUSY cannot be an exact symmetry because it would predict a degenerate mass spectrum while phenomenologically the states of the SUSY mirror world all must be heavier than the SM particles. This leads to a very complicated world as a broken SUSY scenario not only is doubling the spectrum at once but also leaving too much freedom, with about 100 unknown symmetry breaking parameters. This makes such extensions not really predictive without additional assumptions. At the end phenomenological constraints require a SUSY version which would not be solving the hierarchy problem really, rather it would only be shifting the amount of fine-tuning required.

In addition, the hierarchy fine-tuning problem if being solved by a super-symmetrization of the SM creates new problems as we know. First, a second Higgs doublet field needs to be introduced, which as such is an interesting option. However, in order not to be in conflict with the absence of tree level Flavor Changing Neutral Currents (FCNC)\(^5\) one has to impose \(R\)-parity, which is not less a fine tuning, although FCNCs can be forbidden by a simple discrete symmetry. \(R\)-parity is not required by renormalizability, it is not naturally emergent in a low energy effective theory and thus looks to be ad hoc. A generic SUSY extension as such would be in contradiction with observation right away. This also illustrates that naturalness is a doubtful concept: \(R\)-parity is a symmetry which forbids FCNCs but what is natural about it?

In my opinion the dogma surrounding the so called hierarchy or fine tuning problem turned out to be a complete failure. Similarly, the GUT paradigm has not lead to any experimentally confirmed predictions which would support this concept. I think the minimal renormalizable QFT paradigm is back. Thereby “minimal” is crucial, many higher renormalizable structures like a GUT extension of the SM are not natural in that sense. Interestingly, a missing third fermion family or an additional fourth family would spoil important properties of the SM, such as the RG flow, and the Higgs boson then could not be a candidate for the inflaton. Another very important special feature of the SM is its tree level accidental custodial symmetry, which is violated by many of the proposed extensions of the SM and then create a different fine tuning problem [67], in all cases which violate the tree level SM relation \(\cos^2 \theta_W \frac{M_Z^2}{M_W^2} = 1\), which is strongly supported by experimental data as the latter precisely confirm SM predictions of the radiative corrections to this relation.

One also has to keep in mind that precision tests of the SM already revealed a test in depth of its quantum structure, besides large corrections from the running fine structure constant \(\alpha(s)\), the running of the strong coupling \(\alpha_s(s)\) and the large top Yukawa \(\lambda_t(s)\) effect as contributing to the \(\rho = G_{NC}/G_{\mu}(0)\) parameter, for example, subleading corrections amount to a 10\(\sigma\) deviation from the SM prediction when taking into account leading order corrections only. Thus the SM is on very solid grounds better than everything else we ever had.

On the other hand the view that the SM is a low energy effective theory of some cutoff system at the Planck energy scale \(M_{Pl}\) appears to be consolidated. This also puts QFT on a firm mathematical basis. A crucial point is that \(M_{Pl}\), providing the scale for the low energy expansion in powers \(E/M_{Pl}\), is exceedingly large, very far from what we can see! A dimension 6 operator at LHC energies is suppressed by \((E_{LHC}/M_{Pl})^2 \approx 10^{-30}\). This seems to motivate a change in paradigm from the view that the world looks simpler the higher the energy to a more natural scenario which understands the “cutoff SM” as the “true world” seen from far away, with symmetries emerging from not resolving the details of the short distance structure. In the low energy expansion one is “throwing away” an infinite tower of shorter distance information carried by the suppressed so called \textit{irrelevant} operators.

\(^5\) FCNCs are automatically absent in the SM, as it is also highly established by experiment.
The hierarchy problem requires to take the relationship between the bare UV and the renormalized IR regime as testable physics. Here Kenneth Wilson’s 1971 solution [68] of a problem which has been persisting for about 75 years, surrounding the critical indices of phase transitions in condense matter systems, has shed new light on the role cutoffs may play in physical laws. Wilson’s renormalization semi-group, based on integrating out irrelevant details of the short distance structure opened the quantitative approach of constructing low energy effective quantum field theories from systems which’s short distance structure has an intrinsic cutoff, like an atomic lattice or an atomic gas or fluid (see e.g. [69]). The key low energy emergent structure notably turned out to reveal renormalizable Euclidean quantum field theory. The latter exhibits analyticity in a way which makes it equivalent to a Minkowski quantum field theory. The latter hence is incorporating quantum mechanics as an emergent structure. As I will argue in the following, cutoffs in particle physics are unavoidable in understanding the relationship between a bare and a renormalized theory (see e.g. [70]). In such a context renormalizability is an emergent property like all structures required in order renormalizability to be manifest. In our context cutoffs are indispensable for understanding early cosmology in a bottom-up way [1]. This opens the possibility of an alternative understanding of inflation, reheating, baryogenesis and all that [21, 22, 24–26]. As in condensed matter physics the connection between macroscopic long distance physics (at laboratory scales) and the microscopic underlying cutoff system (high energy events as they were natural in the early universe) turn out to have a physical meaning.

I remind that the SM’s naturalness problems and fine-tuning problems have been made conscious by G. ’t Hooft [3] long time ago as a possible problem in the relationship between macroscopic phenomena which follow from microscopic laws (a condensed matter system inspired scenario). Soon later the “hierarchy problem” had been dogmatized as a kind of fundamental principle. In fact the hierarchy problem of the SM seems to be the key motivation for all kind of extensions of the SM. It is therefore important to reconsider the “problem” in more detail.

One of my key points concerns the different meaning a possible hierarchy problem has in the symmetric and in the broken phase of the SM. In order to understand the point we have to remember why we need the Higgs particle in the SM. The Higgs boson is necessary to get a renormalizable low energy effective electroweak theory [71]. Interestingly, one scalar particle is sufficient to solve the renormalizability problems arising from each of many different massive fields in the SM, of which each causes the problem independently of the others. The point is that this one particle has to exhibit as many new forces as there are individual massive states [72]. All required new interactions are in accordance with the SM symmetry structure in the symmetric phase as we know. The taming of the high energy behavior of course requires the Higgs boson to have a mass in the ballpark of the other given heavier SM states, if it would be much heavier it would not serve its purpose in the low energy regime. It would lead to the so called “delayed unitarity” phenomenon [73]. Note that the Higgs boson has to cure the unphysical mass effects for the given gauge boson masses $M_W$, $M_Z$ and fermion masses $M_f$, via adequate Higgs exchange forces, where the coupling strength is proportional to the mass of the massive field coupled. A very heavy Higgs boson eventually would decouple and thus miss to restore renormalizability of the massive vector-boson gauge theory. Interestingly, in the symmetric phase the SM gauge-boson plus chiral fermions sector is renormalizable without the Higgs-boson and Yukawa sectors and scalars are not required at all to cure the high energy behavior, because it is renormalizable on its own structure. Therefore, in the symmetric phase the mass-degenerate Higgs fields in the

---

6 We denote on-shell masses by capital, MS masses by lower case letters as in [1]
complex Higgs doublet can be as heavy as we like. Since unprotected by any symmetry, naturally we would expect the Higgses indeed to be very heavy. Indeed the “origin” of the Higgs mass is very different in the broken phase, where all the masses, including the Higgs mass itself, are generated by the Higgs mechanism [37, 38]. This we learn from the relation $m_H^2 = \frac{1}{4} \lambda v^2$, holding in the broken phase. In the symmetric phase the effective Higgs mass is dynamically generated by the Planck medium, as we will argue below. Therefore, the usual claim that the SM requires to be extended in such a way that quadratic divergences are absent has no foundation. Purely formal arguments based on perturbative counterterm adjustments do not lead any further.

The hierarchy problem in particular addresses the presence of quadratic UV divergences related with the SM Higgs mass term. Infinities in physical theories are the result of idealizations and show up as singularities in a formalism or in models. UV singularities in general plague the precise definition as well as concrete calculations in quantum field theories (QFT)\(^7\). A closer look usually reveals infinities to parametrize our ignorance or mark the limitations of our understanding or knowledge. One particular consequence of UV divergences in local QFTs is that a vacuum energy is ill-defined as it is associated with quartically divergent quantum fluctuations.

This is another indication which tells us that local continuum QFT has its limitation and that the need for regularization is actually the need to look at the true system behind it. In fact the cutoff system is more physical and does not share the problems with infinities which result from the idealization realized in the large cutoff limit or lattice continuum limit. In any case the framework of a renormalizable QFT, which has been extremely successful in particle physics up to highest accessible energies, is not able to give answers to the questions related to vacuum energy and hence to all questions related to dark energy, accelerated expansion and inflation of the universe.

Since the SM exhibits non-Abelian couplings like the $U(1)_Y$ coupling $g_1$ or the Higgs self-coupling $\lambda$ at scales beyond the zero of the $\beta_1$ function, also lattice calculations [76–78] strongly suggest that in fact the theory requires a finite cutoff, because the continuum limit at infinite cutoff would be trivial.

It is thus natural to consider the SM to be what we observe as the Low Energy Effective SM (LEESM), the renormalizable tail of the real cutoff system sitting at the Planck scale. As a consequence all properties required by renormalizability, gauge symmetries, chiral symmetry, anomaly cancellation and the related fermion family structure, as well as the existence of an elementary scalar, the Higgs boson, naturally emerge as a consequence of the low energy expansion\(^8\). Remind that the emergence of SM structures in a low energy expansion is a well investigated subject. It is often advocated as tree-unitary requirement but is easily reinterpretable as a low energy expansion where non-renormalizable effects are suppressed by inverse powers in the cutoff. These mechanisms are calculable within perturbation theory [79–85]. The infinite tower of higher order operators is suppressed to be invisible. Only a few operators are non-relevant and effectively observable, and this is

---

\(^7\) Taming the infinities we encounter in the theory of elementary particles i.e. of quantum field theories, specifically of the SM, by completing it with a cutoff, often called the UV–completion of a QFT, is as old as QFT itself. Actually, it took 20 years from Dirac 1928 (Dirac hole theory of relativistic electron–photon interaction [pre-QED]) to Feynman, Schwinger and Tomonaga in 1948 who found how to deal with the large cutoff limit and making QED a predictive theory. For non-Abelian gauge theories proposed by Yang and Mills in 1954 [74] it took another 17 years until a renormalizable formulation was found by ’t Hooft in 1971 [75] (actually by circumventing a cutoff regularization).

\(^8\) It is interesting to note that statistical mechanical systems with short-range exchange and long-range multipole interactions exhibit vector bosons and graviton modes which follow from a multipole expansion of a static potential [79]. In this sense the existence of gauge bosons is naturally expected (emergent).
what makes the world look much simpler, than a possibly chaotic Planck medium. In reality
infinities related to the relevant operators are replaced by eventually very large but finite
numbers, and I will show that sometimes such huge effects are needed to understand the
real world. I will argue that cutoff enhanced effects are responsible for triggering the Higgs
mechanism not very far below the Planck scale and the inflation of the early universe, as
outlined already in Sect. 1.

The history of our universe we can trace back 13.8 billion years close to the Big Bang,
when the expansion of the universe was ignited in a “fireball”, an extremely hot and dense
state when all structures and at the end all atoms, nuclei and nucleons were disintegrated
to a world of elementary particles only. So the SM provides the key information for what
has happened in the early universe, and high energy accelerator experiments are testing
processes which only took place in nature then. If the Higgs boson is the source of a dark
energy which triggered inflation its discovery could mark a milestone in our understanding
of the dynamics of the very early universe. The origin of the cold dark matter remains a
mystery, which can have many different explanations.

I think that questions concerning the early universe can be addressed only in the LEESM
“extension” of the SM as such, given by a local QFT supplied by cutoff effects in a mini-
mal way. As we know, in a renormalizable QFT all renormalized quantities as a function of
the renormalized parameters and fields in the limit of a large cutoff are finite and devoid of
any cutoff relicts! Here we should remember the Bogoliubov-Parasiuk renormalization the-
orem which states that order by order in perturbation theory the renormalized Green’s func-
tions and matrix elements of the scattering matrix (S-matrix) are free of ultraviolet diver-
gences. The theorem specifies a concrete procedure (the Bogoliubov-Parasiuk R-operation)
for subtraction of divergences, establishes correctness of this procedure, and guarantees the
uniqueness of the obtained results, modulo reparametrizations, which are controlled by the
renormalization group. In other words, in the low energy world cutoff effects are not ac-
cessible to experiments. Consequently, the hierarchy problem cannot be addressed within
the renormalizable, renormalized (like all observables) SM. In this framework all indepen-
dent parameters are free and have to be supplied by experiments. In this sense within the
renormalized QFT the hierarchy problem is a pseudo problem.

To my knowledge the only non-perturbative definition of a renormalizable local quan-
tum field theory is the possibility to put in on a lattice by discretization of space-time. This
again may be taken as an indication that the need for a cutoff actually is an indication that the
cutoff exists in the real(er) world. In this sense the lattice QFT is the true(er) system than its
continuum tail. Of course, there are many ways to introduce a cutoff and actually we cannot
know what the cutoff system looks like truly. This is not a real problem if we are interested in
the long range patterns mainly, the only thing we have to care is that the underlying system
is in the universality class of the SM. This in particular concerns the observable degrees of
freedom and the emergent symmetries at work, which require the particles to be grouped pre-
dominantly in the simplest (lowest dimensional) representation of the corresponding sym-
metry groups. The simplest symmetry groups with singlets, doublets and triplets are the
most natural ones to emerge, as realized within the SM’s $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$, gauge
symmetry pattern [80–85]. More on how the SM may emerge the reader may find in the
Appendix.
4 The Hierarchy Problem revisited

In [86] already I outlined the flaws I see in common argumentation concerning the hierarchy issue. As argued above we are addressing the hierarchy problem in the LEESM “extension” of the SM. Specifically, we have in mind an implementation of the SM on a Planck lattice (see e.g. [87]). The only important point is that we can perform a low energy expansion in the corresponding cutoff. It is an accepted fact that the SM predicts a huge gap between the renormalized and the bare Higgs boson mass. In the LEESM point of view this prediction is what promotes the Higgs boson to be a promising candidate for the inflaton. The hierarchy gap showing up is not something we have to avoid. Now would-be infinities are replaced by eventually very large but finite numbers, and I will show that sometimes such huge effects provide what we need to understand established phenomena like inflation.

One thing we should remind here: the bare suitably regularized theory has always been the true one. Renormalization always has just been a reparametrization. The bare theory, assumed to exhibit a cutoff of some sort, shows a cutoff dependent large-cutoff tail, sometimes called “preasymptote” [1, 70], which is equivalent to a renormalized local QFT in the universality class of the cutoff-system. Thereby it is not important that the bare cutoff system exhibits all symmetries the long range tail will have, as the symmetries of the LEET are emergent. In fact by a reparametrization of parameters and fields of the preasymptotic theory (renormalizable tail) the residual cutoff dependence is completely removable (see [70] and references therein). Because the renormalized tail has lost all information about the cutoff, it is nonsensical to say that in the LEET we would naturally expect the Higgs mass to be of the order of the cutoff.

However, in the LEESM “extension” of the SM bare parameter turn into physical parameters of the underlying cutoff system as the “true world” at short distances. Then the hierarchy problem is the problem of “tuning to criticality”, which concerns the $\text{dim} < 4$ relevant operators, in particular the mass terms. In the symmetric phase of the SM there is only one mass to be renormalized, the others being forbidden by the known chiral and gauge symmetries. For the Higgs field mass which appears in the Higgs potential the fine tuning to criticality has the familiar form

$$m_0^2(\mu_1 = M_{Pl}) = m^2(\mu_2 = M_H) + \delta m^2(\mu_1, \mu_2);$$

$$\delta m^2 = \frac{\Lambda_{Pl}^2}{16\pi^2} C(\mu) \quad (9)$$

with a coefficient typically $C = O(1)$. To keep the renormalized mass at some small value, which can be seen at low energy, formally $m_0^2$ has to be adjusted to compensate the huge number $\delta m^2$ revealed by the perturbative SM calculation such that about 35 digits must be adjusted in order to get the observed value below the electroweak scale. Is this a problem?

One thing is obvious: our fine-tuning relation exhibits quantities (in the LEESM all observable in principle) at very different scales, the renormalized ones at low energy and the bare ones when approaching the Planck scale. As long as we have no direct access to the Planck physics there is no proven conflict.

Actually, a closer look reveals that in the Higgs phase there is no hierarchy problem in the SM! Why? It is true that in the relation (9) both $m_0^2$ and $\delta m^2$ formally may be expected many many orders of magnitude larger than $m^2$. However, in the broken phase $m^2 \propto \nu^2(\mu_0)$ is $O(\nu^2)$ not $O(M_{Pl}^2)$. Since $\nu$ is the result of spontaneous symmetry breaking (non symmetric ground state) it is per se a low energy parameter related to the emergence of long range order. Thus in the broken phase the Higgs boson is expected to be naturally light. That the
Fig. 4 Dimensionful SM running MS parameters \( m \) and \( v = \sqrt{\frac{6}{\lambda}} m \). Error bands include SM parameter uncertainties and a Higgs boson mass range 125.5 ± 1.5 GeV which essentially determines the widths of the bands.

Higgs mass likely is \( O(M_{Pl}) \) in the symmetric phase is what realistic inflation scenarios are demanding. In the broken phase, characterized by the non-vanishing Higgs field vacuum expectation value (VEV) \( v(\mu) \neq 0 \), all the masses are determined by the well known mass-coupling relations

\[
\begin{align*}
m^2_H(\mu) &= \frac{1}{4} g_2^2(\mu) v^2(\mu); \\
m^2_W(\mu) &= \frac{1}{4} (g_2^2(\mu) + g_1^2(\mu)) v^2(\mu); \\
m^2_Z(\mu) &= \frac{1}{4} g_2^2(\mu) v^2(\mu) + \frac{1}{4} g_1^2(\mu) v^2(\mu); \\
m^2_f(\mu) &= \frac{1}{2} y_f^2(\mu) v^2(\mu); \\
m^2_H(\mu) &= \frac{1}{3} \lambda(\mu) v^2(\mu).
\end{align*}
\]

Here we consider the parameters in the \( \overline{\text{MS}} \) renormalization scheme, \( \mu \) is the \( \overline{\text{MS}} \) renormalization scale, which we have to identify with the energy scale of the physical processes or equivalently with the corresponding temperature in the evolution of the universe. The RG equation for \( v^2(\mu^2) \) follows from the RG equations for masses and massless coupling constants using one of these relations. The evolution of the \( \overline{\text{MS}} \) versions of \( m \) and \( v \) are shown in Fig. 4. As a key relation we use \[63\]

\[
\mu^2 \frac{d}{d\mu^2} v^2(\mu^2) = 3 \mu^2 \frac{d}{d\mu^2} \left[ \frac{m^2_H(\mu^2)}{\lambda(\mu^2)} \right] \equiv v^2(\mu^2) \left[ \frac{\gamma_m}{\lambda} - \frac{\beta_\lambda}{\lambda} \right],
\]

where \( \gamma_m = \mu^2 \frac{d}{d\mu^2} \ln m^2 \) and \( \beta_\lambda = \mu^2 \frac{d}{d\mu^2} \lambda \). We write the Higgs potential as \( V = \frac{\lambda}{4} H^4 + \frac{1}{2} H^2 \), which fixes our normalization of the Higgs self-coupling. When the \( m^2 \)-term changes sign and \( \lambda \) stays positive, we know we have a first order phase transition (see below). Funny enough, the Higgs particle gets its mass from its interaction with its own condensate! and thus gets a mass in the same way and in the same ballpark as the heavier SM species, which couple strongest to the Higgs field. As mentioned before the Higgs mass cannot by much heavier than the other heavier particles if renormalizability is to be effective at low and moderate energies. The interrelations \[10\] also show that for fixed \( v \), as determined by the Fermi constant \( G_\mu = 1 / (\sqrt{2} v^2) \), the Higgs cannot get too heavy if perturbation theory should remain applicable.

Often an extreme point of view is taken: all particles naturally should have masses \( O(M_{Pl}) \) i.e. \( v = O(M_{Pl}) \). This would mean that the symmetry is not recovered at the high (=bare) scale and the notion of spontaneous symmetry breaking would be obsolete! Of course this makes no sense. In a perturbative calculation within a cutoff regulated theory formally one finds \( v = O(M_{Pl}) \) but in the broken phase \( \delta m^2_{H} \) is huge negative, which requires
The Hierarchy Problem, Dark Energy and Higgs Inflation

Fig. 5 Spontaneous magnetization $M = M(T)$ as a function of temperature $T$. $T_c$ is the critical temperature above which $M(T) \equiv 0$ for all $T > T_c$. Furthermore, $M(T) \to 0$ as $T \to T_c$ may be as small as we like depending on the distance $T - T_c$ from criticality. Note that $M(0)$ is not given by what would correspond to the cutoff of the ferromagnetic system, even if it would be measured in units of the cutoff.

The Hierarchy Problem, Dark Energy and Higgs Inflation

Spontaneous magnetization $M = M(T)$ as a function of temperature $T$. $T_c$ is the critical temperature above which $M(T) \equiv 0$ for all $T > T_c$. Furthermore, $M(T) \to 0$ as $T \to T_c$ may be as small as we like depending on the distance $T - T_c$ from criticality. Note that $M(0)$ is not given by what would correspond to the cutoff of the ferromagnetic system, even if it would be measured in units of the cutoff.

A well known prototype for long range order is the magnetization in a ferromagnetic spin system illustrated in Fig. 5.10.

The analogy shows us that $v/M_{Pl} \ll 1$ is not unnatural since $v \neq 0$ emerges only below a critical temperature, which is not in a simple way related to $M_{Pl}$. We note that the EW scale is set by $v(\mu)$. At low energy $v(0) = 1/(\sqrt{2}G_\mu)^{1/2} \approx 246$ GeV and $v(\mu)$ is monotonically decreasing with increasing $\mu$ and vanishing at $\mu_0 \sim 10^{16}$ GeV: $v(\mu) \to 0$ when $\mu \leq \mu_0$, as we will see later. The phase transition (PT) point is a point of non-analyticity i.e. exhibits singular behavior and physics in the ordered phase and the disordered phase are very different.

Considering a ferromagnet one has to tune the temperature $T$ to criticality in order to find the PT point. What is tuning the temperature to criticality in the SM? The answer is the expansion of the universe, which provides a scan in temperature (see also [89]). The

In the unitary gauge we can avoid problems related to Elitzur’s theorem [88], which claims that an order parameter cannot be associated with SSB of a non-Abelian gauge theory. In a physical gauge, on physical Hilbert space, Higgs ghost fields are absent and a Mexican hat potential is a phantom as it only exist if ghost space is taken into the display. A physical Mexican hat potential would imply the existence of three Nambu-Goldstone bosons.

As an example we may consider an Ising ferromagnet in $D = 2$ dimensions, $J$ is the nearest neighbor (n.n.) spin coupling between the spins on a lattice

$$H(\sigma) = -J \sum_{\langle \sigma \rangle} \sigma_i \sigma_j ; \quad P_\beta(\sigma) = \frac{e^{-\beta H(\sigma)}}{Z_\beta} ; \quad Z_\beta = \sum_\sigma e^{-\beta H(\sigma)}.$$  

Here $\beta = \frac{1}{k_B T}$, where $k_B$ is the Boltzmann constant. The Onsager solution for the critical temperature reads

$$\sinh^2 \left( \frac{J}{k_B T_c} \right) = 1 ; \quad T_c = \frac{J}{k_B \ln(1 + \sqrt{2})}$$  

and the magnetization is given by $M = \left( 1 - \left[ \sinh(2J) \right]^{-2} \right)^{1/4}$, depending on temperature $T$ and n.n. spin interaction strength $J$. For more details see e.g. [69].
maximum value of $v(\mu)$ is achieved at $\mu = 0$, why should the magnitude of $v(0)$ be set by the Planck scale, while when we increase the energy, after reaching the symmetric/disordered phase transition point, the VEV is actually vanishing identically?

This shows that the Higgs boson mass renormalization equation is not a static equation but is subject to a sophisticated dynamics driven by the expansion of the universe.

In the symmetric phase at very high energy we see the bare system. There the Higgs field is a collective field exhibiting an effective mass generated by radiative effects within the Planck system such that $m_0^2 = \delta m^2$ at $M_{Pl}$. In particle physics a radiatively induced mass is known from the Coleman-Weinberg mechanism [90], now in the symmetric phase and applied to the Planck medium. Such mechanism, which is natural in this context, eliminates a possible fine-tuning problem at all scales. There are many examples in condensed matter systems, like the effective mass of the photon in the superconducting phase (Meissner effect) or the effective mass of the effective field which encodes the spin-singlet electron (Cooper pairs) in the Ginzburg-Landau (GL) model [91] of superconductivity. The latter directly corresponds to the Abelian Higgs model. Emerging as an effective field from the hot Planck system, which exhibits all types of excitations, it is also pretty obvious that the Higgs field couples to all these modes which we see as Yukawa and Higgs to gauge boson couplings in the SM. That these couplings exhibit the symmetries of the SM is again due to the fact that only the renormalizable tail can be seen at low energies. All Planck system modes which do not conspire, as SM degrees of freedom and their couplings do within the SM, are not visible at long distances.

On the one hand we know that astronomy and astrophysics are unthinkable without the input from laboratory physics in general and particle physics in particular. On the other hand, it is not new that particle physics is learning from cosmology. What is required to explain inflation, baryogenesis, nucleosynthesis, CMB patterns, dark matter, etc.? If the SM has an extrapolation up to the Planck scale evidently one is able to confront SM predictions with physics established to have happened in the early universe. In contrast to the old paradigm of an empty vacuum: we know that the ground state of the world is filled with dark energy, with a Higgs condensate and quark and gluon condensates. All these effects have been showing up at certain times and play a key role in the evolution of the universe. Obviously, there are plenty of questions to be answered in order to get a better understanding of how the universe has been shaped after the Big Bang.

5 Running SM parameters trigger the Higgs mechanism

We remind that all dimensionless couplings satisfy the same RG equations in the broken and in the unbroken phase and are not affected by any power cutoff dependencies. This is as it has to be because the Higgs mechanism (SSB) does not alter the UV behavior. The evolution of the SM couplings in the $\overline{MS}$ scheme up to the Planck scale has been investigated in [45, 46, 55–60, 65, 95, 96] recently, and has been extended to include the Higgs VEV and the masses in [1, 64]. Except for $g_1$, which increases very moderately, all other

11 Originally the Ginzburg-Landau theory of superconductivity has been proposed as a macroscopic phenomenological effective theory describing type-I superconductors without reference to microscopic properties. Later Bardeen-Cooper-Schrieffer could explained superconductivity from its microscopic structure in their BCS-theory [92]. Afterwards Gor'kov derived the GL-theory [93] showing that in some limit all GL parameters have a microscopic interpretation. In addition, Abrikosov showed that GL-theory also models type-II superconductors [94]. The effective GL-theory thus efficiently describes a rich variety if superconducting systems, without the need of a detailed microscopic understanding.
Table 1 Comparison of $\overline{\text{MS}}$ parameters at various scales: Running couplings for $M_H = 126$ GeV and $\mu_0 = 1.4 \times 10^{10}$ GeV. Note that $\lambda = 0$ is an essential singularity and the theory cannot be extended beyond a possible zero of $\lambda$. Remind that $\tau = \sqrt{6m_t^2/\lambda}$ i.e. $v(\lambda) \to \infty$ as $\lambda \to 0$. Besides the Higgs boson mass $m_H = \sqrt{2}m$ all masses $m_t = g_1 v \to \infty$ would yield a different cosmology.

| Coupling \ Scale | $M_0$ | $M_1$ | $\mu_0$ | $M_{Pl}$ | My Findings [Deg] | Degrassi et al. 2013 [Deg] |
|------------------|-------|-------|---------|---------|-------------------|--------------------------|
| $g_1$            | 1.2200 | 1.1644 | 0.5271 | 0.4886 | 1.1644 | 0.4873 |
| $g_2$            | 0.6530 | 0.6496 | 0.5249 | 0.5068 | 0.6483 | 0.5057 |
| $g_1$            | 0.3497 | 0.3509 | 0.4333 | 0.4589 | 0.3587 | 0.4777 |
| $g_t$            | 0.9347 | 0.9002 | 0.3872 | 0.3510 | 0.9399 | 0.3823 |
| $\sqrt{\lambda}$| 0.8983 | 0.8586 | 0.3732 | 0.3749 | 0.8733 | 0.1113 |
| $\lambda$        | 0.8070 | 0.7373 | 0.1393 | 0.1405 | 0.7626 | -0.0128 |

Table 1 shows that the relevant running $\overline{\text{MS}}$ parameters at Planck scale are of comparable size in the range 0.51 for $g_2$ being the largest here and 0.35 for $g_t$ being the smallest, with $\sqrt{\lambda}$ at 0.375 slightly larger in our normalization. It tells us that approximations like the gaugeless limit ($g_1 = g_2 = 0$) or assuming $\lambda = 0$ relative to other couplings are not viable approximations near $M_{Pl}$.

For what follows we take up what Shaposhnikov et al. [33] say about vacuum stability in their conclusion: “Although the present experimental data are perfectly consistent with the absolute stability of the Standard Model within the experimental and theoretical uncertainties, one should not exclude the possibility that other experiments will be able to establish the meta-stability of the electroweak vacuum in the future.” But, based on a slightly modified evaluation of $\overline{\text{MS}}$ parameters [64] (which revealed vacuum stability), we adopt the view:

12 Most groups are adopting essentially the same input parameters from [46, 56, 65] and an effective potential and hence find the vacuum to lose stability at about surprisingly low $\mu \sim 10^{10}$ GeV [input not independent]. Keep in mind: the Higgs boson mass miraculously turns out to have a value as it was expected form vacuum stability. It looks like a tricky conspiracy with other couplings to reach this “purpose”. If it misses to stabilize the vacuum, why does it just miss it almost not?
“Although other evaluations of the matching conditions seem to favor the meta-stability of the electroweak vacuum within the experimental and theoretical uncertainties, one should not exclude the possibility that other experiments and improved matching conditions will be able to establish the absolute stability of the Standard Model in the future.”

Running couplings can affect dramatically the quadratic divergences and the interpretation of the hierarchy problem. Quadratic divergences have been investigated at one loop in [4] (see also [5, 98, 99]), at two loops in [6–8]. At \( n \) loops the quadratic cutoff dependence is of the form

\[
\delta m_{ff}^2 = \frac{\Lambda^2}{16\pi^2} C_n(\mu) \tag{12}
\]

where the \( n \)-loop coefficient only depends on the gauge couplings \( g_1, g_2, g_3 \), the Yukawa couplings \( y_f \) and the Higgs self-coupling \( \lambda \). Neglecting the numerically insignificant light fermion contributions, the one-loop coefficient function \( C_1 \) may be written as

\[
C_1 = 2 \lambda + \frac{3}{2} \frac{g_1^2}{2} + \frac{9}{2} \frac{g_2^2}{2} - 12 y_f^2 \tag{13}
\]

and is uniquely determined by dimensionless couplings. The latter are not affected by quadratic divergences such that standard RG equations apply. Surprisingly, as first pointed out in [7], taking into account the running of the SM couplings, the coefficient of the quadratic divergences of the bare Higgs mass correction can vanish at some scale. In contrast to our evaluation Hamada et al. actually find the zero to lie above the Planck scale, having adopted input MS parameters from [46]. In our analysis, relying on matching conditions for the top quark mass analyzed in [64], we get a scenario where \( \lambda(\mu^2) \) stays positive up to the Planck scale and looking at the relation between the bare and the renormalized Higgs mass we find \( C_1 \) and hence the Higgs mass counterterm to vanish at about \( \mu_0 \sim 1.4 \times 10^{16} \text{ GeV} \), not very far below the Planck scale. The next-order correction, first calculated in [6,8] and confirmed in [7] reads

\[
C_2 = C_1 + \frac{\ln(\frac{\mu^2}{\Lambda^2})}{16\pi^2} \left[ 18 g_1^4 + g_2^4 (-\frac{7}{6} g_1^2 + \frac{9}{2} g_2^2 - 32 g_3^2) \right] - \frac{107}{8} g_4^4 - \frac{63}{8} g_2^4 - \frac{15}{4} g_2^2 g_4^2 + \lambda (-\frac{27}{4} g_1^2 + g_2^2 + 3 g_3^2) - \frac{2}{3} \lambda^2 \right], \tag{14}
\]

and numerically does not change the one-loop result significantly. The same results apply for the Higgs potential parameter \( m^2 \), which corresponds to \( m^2 \pm \frac{1}{2} m_{ff}^2 \) in the broken phase. For scales \( \mu < \mu_0 \) we have \( \delta m^2 \) large negative, which is triggering spontaneous symmetry breaking by a negative bare mass \( m_0^2 = m^2 + \delta m^2 \), where \( m \) again denotes the renormalized mass. The phase transition is illustrated in Fig. 6. The jump in the vacuum energy

\[
\Delta V(\phi_0) = -\frac{m_{eff}^2 \nu^2}{8} = -\frac{\lambda \nu^4}{24} \sim -9.6 \times 10^8 \text{ GeV}^4 \approx -(176.0 \text{ GeV})^4 \tag{15}
\]

of negative sign and 50 orders of magnitude off \( \Lambda_{\text{CMB}} \) is taking place here. However, it is small relative to the \( O(M_{Pl}^4) \) size \( V(0) = \langle V(\phi) \rangle \), which will be discussed in Sect. 6. At \( \mu = \mu_0 \) we have \( \delta m^2 = 0 \) and the sign of \( \delta m^2 \) flips, implying a phase transition to the symmetric phase. Finite temperature effects [100–103], which must be included in a realistic scenario, turn out not to change the gross features of our scenario, unless \( \mu_0 \) would turn out to lie much closer to \( \Lambda_{\text{Pl}} \) [1]. It turns out that the change in the effective mass from the Wick ordering of the Lagrangian due to a non-vanishing \( \langle \Phi^* \Phi \rangle \), discussed in the following section, produces...
The Hierarchy Problem, Dark Energy and Higgs Inflation

Fig. 6 The Higgs mechanism transition in the SM. Left: the zero in $C_1$ and $C_2$ for $M_H = 125.9 \pm 0.4$ GeV. Right: shown is $X = \text{sign}(m^2_{\text{bare}}) \times \log_{10}(|m^2_{\text{bare}}|)$, which represents $m^2_{\text{bare}} = \text{sign}(m^2_{\text{bare}}) \times 10^X$.

Fig. 7 $X$ as displayed in the right panel of Fig. 6 including leading finite temperature correction to the potential $V(\phi, T) = \frac{1}{2}(g_T^2 T^2 + m^2_0) \phi^2 + \frac{\lambda}{4!} \phi^4 + \cdots$ with $g_T = \frac{1}{2} \left[ 3y_t^2 + g_1^2 + 4y^2_t + \frac{3}{2} \lambda \right]$ from [104] affecting the phase transition point. Left: for the bare case $[m^2, C_1]$. Right: with adjusted effective mass from vacuum rearrangement $[m^2, C_1] = C_1 + \lambda$. In the case $\mu_0$ sufficiently below $M_{\text{Pl}}$, the case displayed here, finite temperature effects affect the position of the phase transition little, while the change of the effective mass by the vacuum rearrangement is more efficient. The finite temperature effect with our parameters is barely visible a larger shift of the transition point as one may learn from Fig. 7, where the finite temperature effects are displayed. What do we learn from this analysis? The Higgs mechanism is dynamically triggered as temperature in the universe drops below $\mu_0$. In the low energy phase the Higgs boson mass $M_H$ substitutes $\sqrt{2m}$ and in fact has to be calculated from the vacuum rearrangement (see Fig. 11). Now $m_H$ turns into an emergent mass determined by the mass-coupling relation (10) like for all other massive particles in the Higgs phase. At the transition point we have $\delta m^2 = 0$ and no hierarchy problem. While above $\mu_0$ the shift $\delta m^2$ is physical and emergent from the interaction in the Planck medium, below $\mu_0$ the shift $\delta m^2$ loses its physical meaning because at this point the enhanced cutoff effects are nullified and the access to cutoff effects is lost (renormalizable low energy phase). Below $\mu_0$ we still use $\delta m^2_H$ in perturbative mass renormalization, where it is now large negative if we still insist in using the now physics-wise inaccessible Planck scale as a UV cutoff. I would say that argumentation based on (9) now turns into formal nonsense. Not only the magnitude of the cancellation is arbitrary, it also has the wrong sign, for what could be related to a physical mass. The physical mass is determined by the curvature at the minimum of the potential. Key outcome of our calculation is the observation that the SM at high enough sub-Planckian energies undergoes a transition into the symmetric phase [1], presuming a stable vacuum. Fig. 1 displays the SM prediction for the effective Higgs mass as a function of the energy.
5.1 Vacuum stability and effective potential

The classical Higgs potential (2) if bounded from below for \( \lambda > 0 \) has a trivial minimum for \( m^2 > 0 \) at \( \phi_0 = 0 \), and non-trivial one at \( \phi_0^2 = -\frac{\lambda_0}{2} \) for \( m^2 < 0 \). When the classic potential turns unstable, because \( \lambda \) is running to be negative, the analysis of the vacuum stability has to be based on the effective potential, which is obtained by including the quantum corrections \([90, 105]\). The effective potential is not an observable and therefore is gauge- and scale-dependent. In the Landau gauge and \( \overline{\text{MS}} \) scheme it can be written as \([106–109]\) (also see \([47]\))

\[
V_{\text{eff}}(\phi(t)) = \frac{1}{2} m^2(t) \phi^2(t) + \frac{1}{24} \lambda(t) \phi^4(t) + V_1 + V_2 + V_3 + V_{\text{rem}},
\]

with

\[
V_1 = \kappa \left\{ \frac{3}{2} m_H^2(t) \left[ \ln \frac{m_W^2(t)}{\mu^2(t)} - \frac{5}{6} \right] + \frac{3}{4} m_Z^2(t) \left[ \ln \frac{m_Z^2(t)}{\mu^2(t)} - \frac{5}{6} \right] - 3 m_t^2(t) \left[ \ln \frac{m_t^2(t)}{\mu^2(t)} - \frac{3}{2} \right] \right. \\
\left. + \frac{1}{4} m_H^4(t) \left[ \ln \frac{m_H^2(t)}{\mu^2(t)} - \frac{3}{2} \right] + \frac{3}{4} m_Z^4(t) \left[ \ln \frac{m_Z^2(t)}{\mu^2(t)} - \frac{3}{2} \right] \right\},
\]

where \( \kappa = 1/(4\pi^2) \) and \( m_i \) are the masses of different particles in the background of the classical Higgs source field \( \phi \) of the generating functional for the irreducible Higgs vertex functions, which upon renormalization is given by \( \phi(t) = Z_\phi(t) \phi \). Thus we have

\[
\begin{align*}
\bar{m}_W^2(t) &= \frac{1}{4} g_2^2(t) \phi^2(t), \\
\bar{m}_Z^2(t) &= \frac{1}{4} (g_2^2(t) + g_1^2(t)) \phi^2(t), \\
\bar{m}_t^2(t) &= \frac{1}{2} y_t^2(t) \phi^2(t), \\
\bar{m}_H^2(t) &= m^2 + \frac{1}{2} \lambda \phi^2(t), \\
\bar{m}_Z^4(t) &= m^2 + \frac{1}{6} \lambda \phi^2(t).
\end{align*}
\]

---

13 As a part of the SM Lagrangian the Higgs potential term considered so far gets reparametrized by a change of the effective parameters and the effective Higgs field and by appropriate counterterms only, as long as perturbation theory does not break down. All perturbative physics is obtained as usual by means of the renormalizable Lagrangian, written in terms of the quantized fields, and the corresponding Feynman rules. Also note that the Higgs contribution to the energy-momentum tensor of Einstein gravity is represented by the symmetric energy-momentum tensor

\[
\Theta_{\mu\nu} = \frac{\partial L}{\partial (\partial_{\mu} \phi)} \partial_{\nu} \phi - \partial_{\nu} L, \quad \text{where} \quad L(\phi) = \frac{1}{2} \phi_{\mu} \partial^2 \phi - V(\phi),
\]

in terms of the Higgs part of the bare SM Lagrangian.

14 As shown in \([90]\), the potential satisfies the RG equation

\[
\left( \mu^2 \frac{\partial}{\partial \mu^2} + \sum \gamma_i \frac{\partial}{\partial \gamma_i} + \gamma \phi \frac{\partial}{\partial \phi} \right) V = 0
\]

where \( \lambda_i = m^2, \lambda_i g' = g_1, g = g_2, g_3 = g_1, g_6 \) with corresponding beta-functions \( \beta_i \) and \( \gamma \) the anomalous dimension of the Higgs field. The RG as usual is solved along characteristic curves where \( t \) parametrizes the position on the curve. The solution reads

\[
V(\mu, \lambda_i, \phi) = Z_{\lambda}^2(t) V(\mu(t), \lambda_i(t), \phi),
\]

with \( Z_{\alpha}(0) = 1, \lambda_i = \lambda_i(0) \) and \( \phi = \phi(0) \).
The effective potential as derived in the symmetric phase include the would-be Higgs ghosts $G$ contribution as physical degrees of freedom, in the broken phase Higgs ghosts are massless in the Landau gauge (would-be Nambu-Goldstone bosons). In the symmetric phase they contribute as three additional Higgs particles. As we know the Higgs boson mass in the broken phase ($m^2 < 0$) is $M_H^2 = -2m^2 = \frac{1}{2}v^2$, where $v$ refers to the EW vacuum. Two-loop corrections $V_2$ have been calculated in [107, 108] and may be found in more condensed form in [46]. $V_3$ includes the leading three-loop corrections computed in [110]. The remainder $V_{\text{rem}}$ represents the higher-order contributions, which include also the higher dimension operators starting at four loops [108, 111, 112]:

$$V_{\text{rem}} \sim \lambda \phi^4 \sum_{L=4} \left( \frac{\lambda^2}{M_{\phi}^2} \right)^{L-4},$$

(19)

where $L$ is number of loops.

The wavefunction renormalization of the Higgs field takes the form

$$\phi(t) = Z_\phi(t) \phi_c = \exp \left\{ \int_0^t \gamma(\tau) d\tau \right\} \phi(0), \quad \phi(0) = \phi_c,$$

(20)

where $\gamma(t) = d\ln Z_\phi/dt$ is the anomalous dimension of the Higgs field:

$$\gamma = \kappa \left[ \frac{9}{4} g_2^4 + \frac{3}{4} g_1^2 + 3 \Gamma \right] + \kappa^2 \left[ g_2^6 \left( \frac{27}{4} g_1^2 - 20 g_3^2 - \frac{45}{8} g_2^2 - \frac{85}{24} g_1^2 \right) + \frac{271}{32} g_2^2 - \frac{9}{16} g_2^2 g_1^2 - \frac{431}{96} g_1^4 - \frac{1}{6} t^4 \right] + \ldots$$

(21)

Finally, the scale $\mu(t)$ is related to the running parameter $t$ by

$$\mu(t) = \mu \gamma(t), \quad \text{i.e.} \quad t = \ln \mu(t)/\mu,$$

(22)

where $\mu$ is a fixed scale, that we will take equal to the physical top quark mass, $M_t$ as a reference point. Observable physical predictions up to perturbative truncation errors do not depend on the choice of the renormalization scale. This can be used in order to keep radiative corrections moderate by choosing $\mu(t) = \phi(t)$ which avoids large logarithms at any given $t$, since then $\ln m_t^2(t)/\mu^2(t) = \ln g^2(t)/4$ etc. (see [108]). One also may choose $\mu(t) = \phi_c$ in which case $\ln m_t^2(t)/\mu^2(t) = \ln g^2(t)/4 + 2\Gamma$ etc. The correction $\Gamma = \int_0^t \gamma(\mu) d\ln(\mu)$ stems from the field renormalization factor $Z_\phi$.

As elaborated in [47] for high Higgs fields the effective potential may be cast into the simple form where it is dominated by the quartic term

$$V_{\text{eff}} \approx \frac{A_{\text{eff}}(\phi)}{24} \phi^4$$

(23)

and $A_{\text{eff}}(\phi)$ depends on $\phi$ the same as the running coupling $A(\mu)$ depends on the running scale $\mu = \phi_c$ with modified coupling [65]

$$A_{\text{eff}} \approx A + \kappa \left[ \frac{9}{4} g_2^4 \left( \ln \frac{g_2^2}{4} - \frac{5}{6} + 2\Gamma \right) + \frac{9}{8} (g_2^2 + g_1^2)^2 \left( \ln \frac{g_2^2 + g_1^2}{4} - \frac{5}{6} + 2\Gamma \right) - 18 g_1^4 \left( \ln \frac{2}{2} - \frac{3}{2} + 2\Gamma \right) + \frac{3}{2} x^2 \left( \ln \frac{|\lambda|}{2} - \frac{3}{2} + 2\Gamma \right) + \frac{1}{2} x^2 \left( \ln \frac{|\lambda|}{2} - \frac{3}{2} + 2\Gamma \right) \right] + \ldots$$

(24)
Fig. 8 The bare versus the effective Higgs coupling and the effective potential for the parameter set [Jeg] of Table 1. Left: the effective Higgs self-coupling $\lambda_{\text{eff}}$ governing the SM effective potential $V_{\text{eff}} = \frac{1}{2} \phi^4$ for large fields. “CEQ” is one-loop improved from [47], “Deg” is two-loop improved from [46]. The correction $\Delta \lambda_{\text{eff}}$ represents the corrections included in “Deg” relative to $\lambda_{\text{bare}}$. Right: the bare potential compared with different approximations of the effective potential one-loop improved “Deg-1”, two-loop improved “Deg-2” and three-loop improved “Mar-3” [110].

up to less relevant corrections. The crucial point is that for parameters as [Deg] in Table 1 the correction term is positive all up to the Planck scale. At the EW scale the leading positive $\lambda$-term dominates $\lambda_{\text{eff}}$ up to scales where $\lambda$ approaches a zero and there changes sign, if such a zero exists, which depend on the precise input values at the EW scale. In the vicinity above the zero of $\lambda$ actually $\lambda_{\text{eff}}$ remains positive and such stabilizes the Higgs vacuum to somewhat higher scales but also turns negative to a metastable state before reaching the Planck scale (see Fig. 3 in [46]). In contrast for the parameter set [Jeg] the correction $\Delta \lambda_{\text{eff}}$ also starts positive but at higher scales takes negative values but these are small and are not affecting the positivity of $\lambda_{\text{eff}}$ itself as seen in the left panel of Fig. 8. In the stable vacuum scenario the effective potential radiative corrections are moderate and do not affect the main pattern discussed in the previous as well as in the following sections. The quantum

Fig. 9 The form of the effective potential for the Higgs field $\phi$ which corresponds to the stable, critical and metastable electroweak vacuum. The pattern displayed admitting for two minima at non-zero field values requires the effective potential to exhibit even powers of $\phi$ up to $\phi^8$. $v$ is the location of the EW minimum and $\phi_{\text{min}} \gg v$ is the value of a new minimum.
corrections modify the shape of the potential such that a second minimum at some higher (Plank) scale may be induced (see Fig. 9 and Fig. 7 in [46]). As first discussed in [108], a second minimum is also obtained when a transient instability emerges above our Higgs transition point \( \mu_0 \) when the bare mass term gets positive and actually gets huge because of the quadratic cutoff enhancement. For specially fine tuned parameters this may happen also if radiative corrections are not yet included. In any case the existence of a second minimum depends significantly on the higher order corrections. Depending on the values of the Higgs boson and top quark masses the lifetime of the EW vacuum can be larger or smaller than the age of the Universe. The first case corresponds to metastable scenario.

For stationary points \( \phi_0 \) much larger than the electroweak scale \( \lambda_{\text{eff}} \ll 1 \) and the curvature of the potential is given by [48]

\[
\frac{\partial^2 V_{\text{eff}}}{\partial \phi^2}(0) \bigg|_{\phi=\phi_0} = \frac{1}{2} \left( \beta_4 - 4 \gamma \lambda_3 \right) \phi_0^2 \approx \frac{1}{2} \beta_4 \phi_0^2
\]

and in order to have a second minimum of the potential \( \beta_4 \), which we know is negative at EW scales, must have passed a zero which actually happens at about \( \mu_0 \approx 10^{17} \) GeV. In fact what happens for the two parameter sets [Jeg] and [Deg] is shown in Fig. 10, which also illustrates the significance of the radiative corrections. In the stability case the effective potential does not alter the main picture, while in the metastable case a second minimum is also missing and the potential turns unbounded from below in the Planck regime. Since the tunneling rate to the Planck regime is exceedingly low, the EW vacuum still looks to be stable. As follows from the SM RG, the Higgs self-coupling \( \lambda \) is the only SM dimensionless coupling that can change sign with the scale variation since its beta-function \( \beta_4 \) contains parts which are not proportional to \( \lambda \).

In our case, where \( \lambda(\mu) > 0 \) up to \( M_{\text{Pl}} \), in the early phase of the expanding universe, the effective potential is approximated by

\[
V(\phi) \approx V(0) + \frac{\lambda_{\text{eff}}}{24} \phi^4,
\]

and the correction turns out not to be significant for what concerns the scenario as such. Actually, the upshot of the two-loop analysis in [108] has been that “the requirement that the electroweak vacuum remains stable turns out to be essentially identical to the requirement that \( \lambda \) remains positive”.

Fig. 10 The effective potential including 1-,2-, and leading 3-loop [46, 110] corrections, with \( \mu = \phi \) as a scale. Left: for parameter set [Jeg] (stable vacuum). Right: for parameter set [Deg] (metastable case); the EW vacuum is tunneling into the bottomless potential. The tunneling time by far exceeds the age of the universe and hence looks very stable for us.
One has to keep in mind that a metastable EW ground-state in a globally unstable potential, as found in commonly accepted analyses [33, 46, 65], very likely does not model what truly happens at the Planck scale. It could signal the need of an extension of the SM including new physics or that the analysis underestimates uncertainties.

6 The cosmological constant – dark energy provided by the Higgs scalar

It is crucial that in the early universe both terms in the Higgs potential are positive in order to condition slow-roll inflation during long enough time. In fact the quadratically and quartically cutoff enhanced terms in the Higgs potential enforce the condition \( \frac{1}{2} \dot{\phi}^2 \ll V(\phi) \) and given the Higgs boson pressure \( p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) \) and the Higgs energy density \( \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \), we arrive at the equation of state \( w = p/\rho \approx -1 \) characteristic for dark energy and the equivalent cosmological constant (CC) (see e.g. [16,113–115] and references therein). A first remarkably precise measurement of the dark energy equation of state \( w = -1.01 \pm 0.04 \), has been obtained by the Planck mission [12, 13] recently. A more detailed study [2] shows that the enhanced Higgs boson effective mass alone actually does not provide a sufficient amount of inflation which is required to inflate the causal CMB cone to include the full CMB sky.\(^{15}\)

One important quantity we have not taken into account so far is the vacuum energy \( V(0) = \langle V(\phi) \rangle \). A key point is that in the LEESM scenario the vacuum energy is a calculable quantity. In the symmetric phase \( SU(2) \) symmetry implies that while \( \langle \Phi(x) \rangle \equiv 0 \) the composite field \( \Phi^+ \Phi(x) \) is a singlet such that the invariant vacuum energy is given just by simple Higgs field loops

\[
\langle H^2 \rangle =: \left( \begin{array}{c}
\hline
\hline
\end{array} \right) ; \quad \langle H^4 \rangle = 3 \left( \langle H^2 \rangle \right)^2 =: \left( \begin{array}{c}
\hline
\hline
\end{array} \right)
\]

where

\[
\langle 0 | \Phi^+ \Phi | 0 \rangle = \frac{1}{2} \langle 0 | \hat{H}^2 | 0 \rangle \neq \frac{1}{2} \xi ; \quad \xi = \frac{\Lambda^2_{\text{Pl}}}{16\pi^2}.
\]

This provides a CC given by

\[
V(0) = \langle V(\phi) \rangle = \frac{m^2}{2} \xi + \frac{\Lambda^2}{8} \xi^2.
\]

A Wick ordering type of rearrangement of the Lagrangian also leads to a shift of the effective mass

\[
m^2 = m^2 + \frac{\Lambda}{2} \xi.
\]

For our values of the \( \overline{\text{MS}} \) input parameters the zero in the Higgs mass counter term and hence the phase transition point gets shifted downwards as follows

\[
\mu_0 \approx 1.4 \times 10^{16} \text{ GeV} \rightarrow \mu'_0 \approx 7.7 \times 10^{14} \text{ GeV}.
\]

\(^{15}\) This is the Horizon problem: the finite age \( t \) of the universe together with the finite speed of light \( c \) allows us to see to distances \( D_{\text{hor}} = ct \) at most. The CMB sky is much larger \( [4 \xi_{\text{CMB}} = 4 \cdot 10^7 \text{ ly}] \) than the causally connected patch \( [D_{\text{CMB}} = 4 \cdot 10^7 \text{ ly}] \) at the time of last scattering \( t_{\text{CMB}} \) when the CMB decoupled from matter, but no such spot shadow is seen, as we know.
The shift is shown in the right panel of Fig. 7. We notice that the SM predicts a huge CC at $M_{Pl}$:

$$\rho_0 \approx V(\phi) \sim 2.77 M_{Pl}^4 \sim 6.13 \times 10^{76} \text{GeV}^4$$

(31)

exhibiting a very weakly scale dependence (running couplings) and we are confronted with the question how to get rid of this huge quasi-constant? Remember that $\rho_0$ has no direct dependence on $a(t)$. An intriguing structure again solves the puzzle. The effective CC counterterm has a zero, which again is a point where renormalized and bare quantities are in agreement:

$$\rho_{\Lambda 0} = \rho_\Lambda + \frac{M_{Pl}^4}{(16\pi^2)^2} X(\mu)$$

(32)

with $X(\mu) = \frac{1}{2} (2C(\mu) + \lambda(\mu))$ which has a zero close to the zero of $C(\mu)$ when $2C(\mu) = -\lambda(\mu)$.

Note that $C(\mu)$ = $-\lambda(\mu)$ is the shifted Higgs transition point.

Again we find a matching point between low energy and high energy world: $\rho_{\Lambda 0} = \rho_\Lambda$ where the memory of the quartic Planck scale enhancement gets lost, as it should be since we know that the low energy phase does not provide access to cutoff effects.

Crucial point is that

$$X(\mu) = 2C + \lambda = 5\lambda + 3g_1^2 + 9g_2^2 - 24y_t^2$$

(33)

acquires positive bosonic contribution and negative fermionic ones, with different scale dependence. $X$ can change a lot (pass a zero), while individual couplings are weakly scale dependent with $g_2(M_Z)/g_2(M_{Pl}) \sim 2.7$ the biggest and $g_1(M_Z)/g_1(M_{Pl}) \sim 0.76$ the smallest change. Obviously, the energy dependence of any of the individual couplings would by far not be able to sufficiently diminish the originally huge cosmological constant. Only the existence of a zero is able to provide the dramatic compensation of the huge cutoff sensitive prefactor.

At the Higgs transition point, as soon as $m'^2 < 0$ for $\mu < \mu'_0$, the vacuum rearrangement of the Higgs potential takes place. As a result at the minimum $\phi_h$ of the potential we should get $V(0) + V(\phi_h) \sim 0.00171 \text{eV}^4$ about the observed value of today's CC (see Fig. 11). How can this be? Indeed, at the zero of $X(\mu)$ we have $\rho_{\Lambda 0} = \rho_\Lambda$ and one may expect that like the Higgs boson mass another free SM parameter is to be fixed by experiment here\(^\text{16}\).

---

\(^\text{16}\) The appearance of an non-vanishing $v$ provides a large negative contribution which however by far does not compensate the large positive offset $\langle V(\phi) \rangle$ we have from the symmetric phase. A more accurate analysis would have to take into account subleading effects from the chiral phase transition of QCD as well.
expect $\rho_{\Lambda}$ to be naturally small, since the $A_{p_0}^4$ term is nullified at the matching point. Note that the huge cutoff prefactors act as amplifiers of small changes in the effective SM couplings. But how small we should expect the low energy effective CC to be? In fact, in the LEESM scenario neither the Higgs mass nor the CC are really free parameters in the low energy world. The Higgs mass, more precisely the Higgs self coupling, has to be constrained to a window where the Higgs potential remains stable up to the Planck scale, and the CC which triggers inflation gets tuned down by inflation to lie in the ballpark of the critical density of a flat universe.

6.1 A self-organized cosmological constant?

Implications of inflation we already outlined in Sect. 1 after Eq. (5). Our analysis of the LEESM showed that the CC is very much time dependent especially through the running of the SM parameters and phase transitions taking place in the evolution of the universe. The typical problem is that in general one gets a CC which is way too big and this looks to create a tremendous fine-tuning problem. For the SM this concerns the contribution to the vacuum density via the Higgs VEV in the broken phase, as well as the contributions from spontaneous breakdown of chiral symmetry, which are much to big and even of wrong sign. Interestingly, our Higgs inflation scenario predicts a large positive DE which actually implies that $\rho_{\text{Higgs}} \gg \rho_{\text{crit}}$, which implied that at Planck time $k = +1$ and $\Omega_k$ in Eq. (7) evaluated at Planck time is large negative if $a(t_{\text{Pl}})$ is of Planck size. It is important to keep in mind that in Big Bang cosmology $\rho_{\text{rad}}$ at the beginning is always dominated by the radiation density since $\rho_{\text{rad}} \propto a(t)^{-4}$ grows fastest when $a(t)$ gets smaller when going back in time. Because they also scale with inverse powers in $a(t)$, also the matter density as well as the curvature term first overshoot the CC supplied by Higgs boson effects, since $\rho_{\text{A}} \sim (1.29 M_{\text{Pl}})^4$ is of comparatively moderate size, although extremely big relative to the critical density. However, if inflation is at work, the final vacuum density is fixed, whatever the initial density has been. Given that $\Omega_{\text{tot}} = \Omega_{\Lambda} + \Omega_{\text{DM}} + \Omega_{\text{BM}} + \Omega_{\text{rad}} = 1$ with $1 > \Omega_{\text{DM}} > \Omega_{\text{BM}} > \Omega_{\text{rad}} > 0$ we know that $\Omega_{\Lambda}$ being positive must be of order $\Omega_{\text{tot}}$, actually a fraction of it. As a non-vanishing $\rho_{\Lambda 0}$ at Planck time is needed it is not unlikely that the other components contributing to the total energy density do not saturate the bound. Actually we know that normal matter including the tiny radiation density represent about 5% of the critical density only. This means that the fine-tuning is dynamically enforced by inflation and the value of today’s dark energy density

$$\rho_{\Lambda 0} = \mu_{\Lambda 0}^4; \quad \mu_{\Lambda 0} = 0.00171 \text{ eV}$$

looks all but exotic. While $\Omega_{\text{rad}}$ and very likely $\Omega_{\text{BM}}$ are essentially SM predictions if we include the $B + L$ violating dimension 6 four-fermion operators, $\Omega_{\text{DM}}$ is the only missing piece which remains an open problem and definitely requires additional beyond the SM physics. This also concerns contributions from quark and possible gluon condensates, which we do not explicitly consider here.

Provided SM parameters indeed support a stable Higgs potential up to $M_{\text{Pl}}$, inflation and the CC itself are SM ingredients leading to a highly self-consistent conspiracy which shapes the universe. Fig. 12 shows the development of the quadratically and the quartically enhanced terms in the symmetric phase of the SM, and its matching to the low energy world.
ever, the interaction term is actually dominating for a short time after the initial Planck
are huge and start to dominate quickly. Because of the large initial field strength
well satisfied, by the fact that in the symmetric phase the mass term as well as
V of the parameters and perturbative approximations. In LEESM cosmology the form of the
mology is characterized by the fact that almost everything is known, within uncertainties
In contrast to standard scenarios of modeling the evolution of the early universe, SM cos-

7 Inflation and reheating

In contrast to standard scenarios of modeling the evolution of the early universe, SM cos-
mology is characterized by the fact that almost everything is known, within uncertainties
of the parameters and perturbative approximations. In LEESM cosmology the form of the
potential is given by the bare Higgs potential $V(\phi) = \frac{\lambda}{2} \phi^2 + \frac{\lambda_{int}}{4!} \phi^4$ as part of the SM La-
grangian, the parameters are known, calculable in terms of the low energy parameters, the
only unknown is the magnitude of the Higgs field. The latter must be large – trans-Planckian
– in order to get the required number of $e$-folds $N$ given by

$$N = \ln \left( \frac{a(t_f)}{a(t_i)} \right) = \int_{t_i}^{t_f} H(t) dt \approx -\frac{8\pi}{M_{Pl}^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi. \quad (35)$$

The second form is obtained using the field equation (4). Note that $N$ is determined entirely
by the scalar potential. Needed is $N \geq 60$ in order to cover the CMB causal cone. By definition, $\exp N$ is the expansion factor $a(t_f)/a(t_i) = \exp H(t_f - t_i)$, where $a(t)$ is the Friedmann-
Robertson-Walker radius of the universe at cosmic time $t$. $t_i$ denotes the begin of inflation
and $t_f$ the end of inflation and $H$ the Hubble constant. For our set of $\overline{MS}$ input parameters we
require $\phi_0 = \phi(\mu =\ M_{Pl}) \approx 4.5 \ M_{Pl}$. Shortly after start the slow-roll condition $V(\phi) \gg \frac{1}{2} \dot{\phi}^2$ is
well satisfied, by the fact that in the symmetric phase the mass term as well as $V(\phi) \equiv \langle V(\phi) \rangle$
are huge and start to dominate quickly. Because of the large initial field strength $\phi_0$, however,
the interaction term is actually dominating for a short time after the initial Planck
time $t_{Pl}$. The field equation $\dot{\phi} + 3H\phi = -V'(\phi)$ then predicts a dramatic decay of the field,
$\phi(t) = \phi_0 e^{-E_0/(3H)}$ with $E_0 = \sqrt{\lambda/3} = 4.3 \times 10^{17}$ GeV. $V_{int} \gg V_{mass}$ and shortly after
$\dot{E_0} = m^2/(3\sqrt{V(\phi)}) \approx 6.6 \times 10^{17}$ GeV, $V_{mass} \gg V_{int}$ $[\ell^2 = 8\pi G_N/3]$, such that in almost no
time, still under slow-roll conditions, the mass term dominates and for what follows the field
equation predicts an exponential decay followed by harmonic oscillation setting in. The universe
thus undergoes an epoch of Gaussian inflation as confirmed by observation [17]. The
time evolution is displayed in Fig. 13 and it is very interesting to see which term dominates
during which time slice. Obviously inflation gets stopped by the fast decay of the Higgs field
(see Fig. 14), in spite of the fact that a CC $V(\phi)$ persists to be substantial at first.
A highly non-trivial challenge is the calculation of the spectral indices

\[ \varepsilon \equiv \frac{M_{\text{Pl}}^2}{8\pi} \frac{1}{2} \left( \frac{V'}{V} \right)^2; \quad \eta \equiv \frac{M_{\text{Pl}}^2}{8\pi} \frac{V''}{V}, \]

which sensitively depend on the form of the potential and which have been extracted from the observed CMB radiation fluctuation spectrum. From the theory side they are constrained by the slow-roll criteria \( \varepsilon \ll 1 \) ensuring \( p \approx -\rho \), and \( \varepsilon, \eta \ll 1 \) which ensures that the slow-roll condition hold for long enough time, while maintaining \( \dot{\phi} \ll 3H\dot{\phi} \) before oscillations start. These conditions also should be satisfied in order to ensure the required amount of inflation. In our case we are confronted with a SM prediction modulo the unknown Higgs field \( \phi(\mu = M_{\text{Pl}}) \). The calculation presented in [2] shows that a prediction of \( \eta \) is delicate, but present uncertainties in predicting the bare Higgs potential at post-inflation times certainly do not allow us the draw definite conclusions. In view that there are many predictions which look to work surprisingly well, I would be surprised if the Higgs boson inflation would not work in predicting at the end also the spectral exponents in agreement with observation.

7.1 Reheating by Higgs boson decays

In the symmetric phase all four Higgs fields are physical and very heavy and rather unstable. The Yukawa couplings at inflation times are pretty well known and the Higgs bosons decay predominantly (largest Yukawa couplings) into as yet massless top-antitop pairs and lighter fermion-antifermion pairs \( H, \phi \rightarrow t\bar{t}, b\bar{b}, \cdots, H^+ \rightarrow t\bar{b}, \cdots, H^- \rightarrow \bar{t}b \cdots \) and are thereby reheating the young universe which just had been cooled down dramatically by inflation. Pre-heating is suppressed in SM inflation as in the symmetric phase bosonic decay channels like \( H \rightarrow WW \) and \( H \rightarrow ZZ \) are absent at tree level. The CP violating decays \( H^+ \rightarrow t\bar{d} \) [rate \( \propto y_{t,d}V_{td} \)], \( H^- \rightarrow b\bar{u} \) [rate \( \propto y_{b,u}V_{ub} \)] likely are important for baryogenesis. After the electroweak phase transition which closely follows the Higgs transition, where \( m^2 \) in the Higgs potential changes sign, the now heavy top quarks decay into normal matter as driven by CKM [116] couplings and phase space. At these scales the \( B + L \) violating dimension 6 operators [117–119] can still play a key role for baryogenesis and together with decays like \( t \rightarrow d^{*}\nu \) provide CP violating reactions during a phase out of thermal equilibrium. For details see [1,2,86].
A very different model of Higgs inflation, which has barely something in common with our LEESM scenario, is the Minkowski-Zee-Shaposhnikov et al. [27–33] so called non-minimal SM inflation scenario. It is based on the following points: i) Einstein gravity has to be extended by adding $G_{\mu\nu} \rightarrow G_{\mu\nu} + \xi (\Phi^* \Phi) R_{\mu\nu}$ to Einstein’s equation. On the source side the model is assuming the renormalized low energy $T_{\mu\nu}$ supplied by the renormalized SM (no relevant operator enhancement). The new term is a direct coupling of the gauge invariant Higgs field singlet operator $\Phi^* \Phi$ to the scalar Ricci curvature $R$. This extra term violates the equivalence principle, yet so far without observable consequences. ii) Choose $\xi$ large enough in order to get a sufficient amount of inflation, which requires a rather large value $\xi \sim 10^4$. The entire inflation pattern then essentially depends on $\xi$ only (inflation “by hand”). In case $\xi = O(1)$ the added non-minimal coupling term is tiny and does not affect our LEESM or standard inflation scenarios. iii) Assume quadratic and quartic SM divergences are absent (argued by dimensional regularization (DR) and \(\overline{\text{MS}}\) renormalization). iv) Assume the SM to be in broken phase at Planck scale, which looks unnatural since SSB is a low energy phenomenon, which assumes the symmetry to be restored at the short distance scale!

Note: 1) It is well possible maybe even likely that such non-minimal gravity couplings of the Higgs field exist and could play a role when curvature is very high. However, the coupling $\xi$ would rather be $O(1)$ than fine-tuned to be about $\xi \sim 10^4$. 2) DR and \(\overline{\text{MS}}\) renormalization are possible in perturbation theory only. There is no corresponding non-perturbative formulation (simulation on a lattice) or measuring prescription (experimental procedure). It is based on a finite part prescription (singularities nullified by hand), which can only be used to calculate quantities which do not exhibit any singularities at the end. As elaborated earlier in Sect. 4, the hierarchy problem cannot be addressed within the dimensionally regularized SM or adopting the \(\overline{\text{MS}}\) scheme.

8 Remark on trans-Planckian Higgs fields

If the SM Higgs is the inflaton, sufficient inflation requires trans-Planckian magnitude Higgs fields at the Planck scale. At the cutoff scale the low energy expansion obviously gets obsolete and likely we cannot seriously argue with field monomials and the operator hierarchy appearing in the low energy expansion. What is important is that the field is decaying very fast (see Fig. 14)). Formally, given a truncated series of operators in the potential, the highest power is dominating when approaching the trans-Planckian regime. One then expects that for some time the $\phi^4$ term of the potential is dominating, the decay of the field is then exponential, for higher dimensional operators it is faster than exponential, such that the field
very rapidly reaches the Planck- and sub-Planck regime. This means that the mass term is dominating after a very short period and before the kinetic term becomes relevant and slow-roll inflation ends. So fears that in low energy effective scenarios with trans-Planckian fields higher order operators would mess up things are not in any sense justified\footnote{As mentioned earlier, the constructive understanding of LEETs is Wilson’s renormalization semi-group, based on integrating out short distance fluctuations. This produces all kinds, mostly of irrelevant higher order interactions. A typical example is the Ising model, which by itself seen as the basic microscopic system has simple nearest neighbor interactions only and by the low energy expansion develops a tower of higher order operators, which at the short distance scale are simply absent altogether. Such operators don’t do any harm at the intrinsic short distance scale. As a minimal fairly realistic Planck system representative in the universality class of the SM, we may consider the lattice SM, a SM generalization of lattice QCD, which also is expected to be behaved decently at the short distance scale.}. Obviously, without the precise knowledge of the Planck physics, very close to the Planck scale we never will be able to make a precise prediction of what is happening. This however seems not to be a serious obstacle to quantitatively describe inflation and its properties as far as they can be accessed by observation. The LEESM scenario in principle predicts not only the form of the effective potential not far below the Planck scale but also its parameters and the only quantity not fixed by low energy physics is the magnitude of the field at the Planck scale. We also have shown that taking into account the running of the parameters is mandatory for understanding inflation and reheating and all that.

Trans-Planckian fields are not unnatural in a low energy effective scenario as the Planck medium exhibits a high temperature and temperature fluctuations imply amply of higher excitations (a chaos). While the Planck medium will never be accessible to direct experimental tests, a phenomenological approach to constrain its effective properties is obviously possible, especially by CMB data \cite{120}.

In the extremely hot Planckian medium, the Hubble constant in the radiation dominated state with effective number $g_\ast(T) = g_B(T) + \frac{7}{8} g_f(T) = 102.75$ of relativistic degrees of freedom is given by $H = \ell \sqrt{\rho} \approx 1.66 (k_B T)^2 \sqrt{102.75} M_{Pl}^{-1}$, at Planck time $H_i \approx 16.83 M_{Pl}$ such that a Higgs field of size $\phi_i \approx 4.51 M_{Pl}$, is not unexpectedly large and could as well also be larger.

Often it is argued that trans-Planckian field are unnatural in particular in a LEET scenario \cite{121}. I cannot see any argument against strong fields and LEET arguments (ordering operators with respect to a polynomial expansion and their dimension) completely loose their sense when $E/\Lambda_{Pl} > \sim 1$.

Provided the Higgs field decays fast enough, towards the end of inflation we expect the mass term to be dominant such that a Gaussian fluctuation spectrum prevails. The quasi-constant cosmological constant $V(0)$ at these times mainly enters the Hubble constant $H$ and does not affect the fluctuation spectrum.

9 Remarks for the skeptic

How do our results depend on the true UV completion? In other words, how realistic are the numbers I have presented?

In order to answer these questions we have to stress once more the extreme size of the cutoff, $M_{Pl} \gg \cdots$, from what we can see!, which lets look what we can explore to be ruled by fundamental principles like the Wightman axioms (the “Ten Commandments” of QFT) or extensions of them as they are imposed in deriving the renormalizable SM. In the LEESM approach many things are much more clear-cut than in condensed matter systems,
where cutoffs are directly accessible to experiment and newer as far away and also lattice QCD simulations differ a lot, as cutoffs are always close-by, such that lattice artifacts affect results throughout before extrapolation to the continuum.

We also have to stress that taking actual numbers too serious is premature as long as even the realization of vacuum stability is in question. Detailed results evidently depend sensitively on accurate input values and on the perturbative approximations used for the renormalization group coefficients as well as for the matching relations needed to get the $\overline{MS}$ input parameters in terms of the physical (on-shell) ones. After all we are attempting to extrapolate over 16 orders of magnitude in the energy scale. Such attempt may look to be megalomaniac, but it is a bottom-up approach which appears to lead to a reasonable very possible scenario for inflation at work. And it is a very modest claim relative to those who attempt to construct the TOE.

The next question is how close to $M_{Pl}$ can we trust our extrapolation? It is very important to note that above the EW scale $[v \sim 250 \text{ GeV}]$ perturbation theory seems to works the better the closer we are near the Planck cutoff, vacuum stability presupposed. As long as we are talking about the perturbative regime we can expand perturbative results in powers of $E/\Lambda_{Pl}$ up to logarithms. Then we have full control over cutoff dependence to order $O((E/\Lambda_{Pl})^2)$, corresponding to dim $\geq 6$ operator corrections. Effects $O(E/\Lambda_{Pl})$, related to dim 5 operators, only show up in special circumstances e.g. in scenarios related to generating neutrino masses and mixings and the sea-saw mechanism.

The true problem comes about when we approach the Planck scale, where the expansion in $E/\Lambda_{Pl}$ completely breaks down. Especially, it does not make sense to talk about a tower of operators of increasing dimensions. This does not mean that everything gets out of control. If the “ether” would be something which can be modeled by a lattice SM, implemented similar to lattice QCD, one could still make useful predictions, which eventually could be tested in cosmological phenomena. In condensed matter physics it is well known that an effective Heisenberg Hamiltonian allows one to catch essential properties of the system, although the real structure cannot be expected to be reproduced in the details. One also should keep in mind that models like the mentioned Ginzburg-Landau effective theory of superconductivity perfectly models the phase transition between type I, type II superconductivity and normal state, without reference to the true microscopic structure. In any case it is always possible to find out to what extent the description fits to reality.

It is well known that long range physics appears as field theory naturally from underlying classical statistical systems exhibiting short-range exchange interactions (e.g. nearest-neighbor interactions on a lattice system) [68] (for lectures on the topic see e.g. [69]). The Planck system besides such typical short-range interactions certainly exhibits a long-range gravitational potential, which develops multipole excitations showing up as spin 1, spin 2, etc. modes at long distances [79].

In our context what is important is that the quadratic and quartic enhancements are persisting, as well as the running (screening or anti-screening effects) of couplings and their competition and conspiracy, which are manifest in the existence of the zeros of the enhanced terms, provided these zeros are not to close to the cutoff. A look at Fig. 6 shows that such effects can be dramatic fairly well below the cutoff. Again, the perturbativeness, together with the fact that leading corrections to these results are by dim 6 operators, let us expect that result are reliable at the $10^{-4}$ level up to $10^{17}$ GeV, which is in the middle of the symmetric phase already. Once the phase transition has happened, the running is anyway weak and if cutoff effect are starting to play a role they cannot spoil the relevant qualitative features concerning triggering inflation, reheating and related phenomena.
Lattice SM simulations in the appropriate parameter range of vacuum stability, keeping top quark Yukawa and Higgs self-energy couplings to behave asymptotically free, which requires to include simultaneously besides the Higgs system also the top Yukawa sector and QCD, could help to investigate such problems quantitatively. Experience from lattice QCD simulations may not directly be illustrative since usually the cutoff is rather close and a crucial difference is also the true non-perturbative nature of low energy QCD.

In any case, not to include the effects related to the relevant operators (dim < 4) simply must give wrong results. Even substantial uncertainties, which certainly show up closer to the cutoff, in power-like behaved quantities seem to be an acceptable shortcoming in comparison to not taking into account the cutoff enhancements at all (as usually done).

10 Summary and conclusions

A cutoff regularized SM with the Planck mass as a cutoff is considered to exhibit the relevant features of the physical Planck world in the sense that it resides in the same universality class with respect to its long range behavior. The SM we observe at low energy is then the emergent renormalizable effective theory of the Planck medium. All conditions which usually have to be imposed as principles to ensure renormalizability are emergent as a result from the low energy expansion. Long distance physics is related the short distance physics by Wilson’s renormalization semi-group. Taking into account renormalization effects and the “running” of the parameters is mandatory in order to understand what has been happening in the evolution of the universe.

In this scenario the Higgs field/particle has two different functions in our world: 1) it has to render the effective low energy electroweak theory (massive vector-boson and fermion sector) renormalizable. In place of fermion mass terms we have fermion Yukawa couplings to start with, while gauge boson mass terms enter via the kinetic Higgs term through the covariant derivative which exhibit the gauge fields. In the broken low energy phase the Higgs field forms a vacuum condensate which provides masses to all massive fields including the Higgs boson itself. Key point are the many new Higgs field exchange-forces necessary to render the low energy amplitudes renormalizable. 2) in the symmetric phase there exist four very heavy Higgs bosons \((H,\phi^0,\phi^+,\phi^-)\) which generate a huge positive dark energy, which triggers inflation. After inflation has ended and we are out of equilibrium the Higgs bosons are decaying predominantly into the fermions pairs with largest Yukawa couplings (predominantly at this stage still massless top–antitop pairs), which provides the reheating of the inflated universe. The universe cooling further down then pushes the universe into the Higgs phase, where the particles acquire their masses. The predominating heavy quarks decay into the lighter ones which later form the baryons and normal matter. This scenario is possible because of the quadratically enhanced Higgs boson mass and the quartically enhanced dark energy, which show up in the symmetric phase of the SM before the transition into the Higgs phase. The existence of such relevant operator effects in my opinion are supported by observed inflation patterns, meaning that the hierarchy as well as the cosmological constant “problems” reflect important properties of the SM needed to understand the evolution of the early universe (for different opinions see [122–126]). Consolidation of our bottom-up path to physics near the Planck scale will sensibly depend on progress in high precision physics around the EW scale \(v\). Especially, Higgs and a top-pair factories will play a key role in this context.

A final remark concerning the verifiability of our Higgs inflation scenario: any discovery of physics beyond the SM which has its motivation in the presumed hierarchy problem of
the SM, like a supersymmetric extension, extra dimensions etc., would spoil the delicate balance of SM effective couplings, on which our scenario relies. Also Grand Unified Theory extensions or even such straightforward extensions like a fourth fermion family, would likely rule out the Higgs as an inflaton and as the source of the dark energy.

Concerning the presumed fine tuning problem: the scales $M_{Pl}$ and $v$ represent different physics regimes and there is no reason why they should not vastly differ, one is related to gravity (Planck medium) the other to long range order at low energies induced by low energy SM interactions, which trigger spontaneous symmetry breaking meaning the symmetry is broken by a non-symmetric ground state solution. The critical point nevertheless is the actual value of $v$ which in non-vanishing only below a critical temperature. While in a condensed matter system one is adjusting the temperature by hand, the key problem seems to be that in particle physics we cannot adjust the temperature. But this the expanding universe can do for us. In fact, the hot big bang universe provides a scan of the temperature spectrum and automatically triggers the phase transition at some point, as the calculations show. For more details I refer to my original articles [1, 2] and to my Krakow Lectures [127].

The scenario I advocate requires a change of paradigm, to one where the SM with its structure is emergent from a Planck cutoff medium following a minimal self-organized “strategy”, i.e. conspiracies which make structures to be visible at large distances. This looks like a version of an anthropic principle at work. It lets look the SM to be more natural than many of the BSM scenarios we have heard about during the last about 45 years. Although the SM started to turn out to work well and to work better than ever expected, it still is considered to suffer from presumed flaws. Yes, the SM misses dark matter, singlet neutrinos, axions and likely more, but all of these have room in an emergence scenario. This is in contrast to the top-bottom philosophy behind the most popular BSM physics scenarios like string theory, supersymmetry or grand unification, which assume that the short distance world is intrinsically highly symmetric and symmetries are broken spontaneously only, because renormalizability is always assumed. We know that the SM as seen at low energies is a spontaneous broken gauge theory which gets more symmetric as we go to higher energies because mass operators in a high energy expansion turn into irrelevant operators. This may have lead to a wrong generalization concerning what we have to expect on the path to higher energies. This view overlooks the fact that a tower of possible symmetry breaking irrelevant operators of the low energy expansion turn into relevant operators in a high energy expansion which break symmetries seen at low energy. Certainly existing non-renormalizable higher order effects are commonly ignored, assuming renormalizability to be a fundamental principle. The dream that a simple “theory of everything” should exist at the Planck scale looks to me turned out as an illusion not more. The opposite very probably is true, the world looks more complex the closer we look, and symmetries emerge from not resolving the detailed structure behind. And the final truth remains something we only can get closer but remains unreachable.

I think the LEESM scenario has a good chance to find its confirmation along the lines described in this article. Many aspects need be checked and possibly modified. Admittedly, there are many open questions which should be investigated more thoroughly. One conclusion seems to be unavoidable, namely that the SM Higgs provides dark energy which affects both early as well as late cosmology.
11 Appendix: How natural is the minimal SM?

Often it is considered that it would be more natural to have a left-right symmetric world including mirror fermions. The following consideration, which goes back to Veltman [128], is instructive as it helps to understand why parity violation is quite natural and why QED conserves parity. It has a lot to do with the assumption of a minimal Higgs system. I reproduce a version which I had presented in [129] some time ago, but provides much deeper insight within the context of our LEESM scenario.

In order to try to derive the SM let us make the following assumptions:

1) local field theory
2) interactions follow from a local gauge principle
3) renormalizability
4) masses derive from the minimal Higgs system
5) ν_R which we know must exist does not carry hypercharge.

Note that all points besides the last one are emergent structures in a LEESM as we may learn from [79–85] (see also Sect. 3). We admit that the last assumption looks somewhat ad hoc, but nevertheless we make it. From the above assumptions the following picture develops:

- For the gauge interactions the simplest non-trivial possibility is that the fundamental massless matter fields group into doublets and triplets which are the fundamental representations of SU(2) and SU(3), besides possible singlets.
- Since fields are massless all fields can be chosen left-handed. Left-handed particles and left-handed antiparticles at this stage are uncorrelated.
- We must have pairing for particles that are going to be massive, since a mass term (we ignore the possibility to have Majorana fields here) has the form \( \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L \). Notice that for massive particles only, we know which left-handed antiparticle belongs to which left-handed particle to form a Dirac field.
- For SU(3), triplets we must have pairing in order to avoid axial anomalies. SU(3) is the simplest group having complex representations. This allows to put particles in 3 and antiparticles in the inequivalent 3*. As a consequence a rich color singlet structure (≡ hadron spectrum) results. Furthermore, confinement requires SU(3), to be unbroken!
- SU(2)_L is anomaly free and hence there is no anomaly condition associated with this group. To generate mass we have to break SU(2)_L by a Higgs mechanism. The simplest and natural possibility is to choose one Higgs field in the fundamental representation of SU(2)_L. There is no hypercharge for the moment. The Higgs field may be written in the form

\[
\Phi_b = \bar{\phi} \chi_b : \chi_b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

in terms of a 2×2 matrix field \( \tau_i, i = 1,2,3 \) the Pauli matrices

\[
\bar{\phi} = \frac{1}{\sqrt{2}} (H_s + i\tau_i \phi_i).
\]

The covariant derivative being given by

\[
D_\mu \Phi_b = \left( \partial_\mu - i \frac{g}{2} \tau_\mu W_{ab} \right) \Phi_b,
\]

and the SU(2) invariant renormalizable Higgs system

\[
L_{\text{Higgs}} = \left( D_\mu \Phi_b \right)^\dagger \left( D^\mu \Phi_b \right) - \lambda \left( \Phi_b^+ \Phi_b \right)^2 + \mu^2 \left( \Phi_b^+ \Phi_b \right),
\]

(37)
exhibits an extra global $SU(2)_{R}$-symmetry $\chi_{b} \rightarrow V^{+}\chi_{b}$. One easily checks that the transformation

$$\tilde{\Phi} \rightarrow U(x)\tilde{\Phi}V^{+},$$

with $U(x) \in SU(2)_{L,\text{local}}, V \in SU(2)_{R,\text{global}}$ leaves the Higgs Lagrangian invariant. This implies that the fields $(W^{+}, W_{3}, W^{-})$ form an isospin triplet with $M_{Z} = M_{W^{+}}$. Now consider the fermions (still no hypercharge). Since $L_{f}$ and $\Phi_{b}$ are $SU(2)$ doublets there also must exist singlet fermions $R_{f}$, otherwise we would not be able to write down an invariant and renormalizable fermion-Higgs coupling. Therefore $SU(2)$ must be parity violating of V-A-type! The Yukawa term has the general form

$$\mathcal{L}_{\text{Yukawa}} = -\bar{\ell}f_{\tilde{\Phi}}(g_{11}g_{12})R_{f} + \text{h.c.},$$

with 4 complex couplings $g_{ij}$ and $R_{f}$ a “doublet” having two right-handed singlets as entries. Although we have not used hypercharge to restrict these couplings the existence of a global $SU(2)_{R}$-symmetry of the Higgs system allows us to transform the Yukawa couplings

$$\tilde{\Phi}(\cdot)R_{f} \rightarrow \tilde{\Phi}V^{+}(\cdot)WR_{f}$$

to standard form, $V^{+}(\cdot)W = \text{real diagonal}$. Since $V \in SU(2)_{R}$ has 3 parameters and $W$ is an arbitrary unitary matrix with 4 parameters we end up with one free parameter such that the system exhibits a global $U(1)$ invariance. This is not surprising since in the unitary gauge we always can end up only with $\mathcal{L}_{\text{Yukawa}}$ in the simple standard form

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{f} m_{f}\bar{\phi}_{f}\phi_{f}(1 + \frac{H}{v}).$$

(38)

The global $U(1)$ which is a consequence of the minimal Higgs mechanism may be interpreted as a global $U(1)_{Y}$. We are free to assign $Y = 1$ to $\Phi_{b}$, which means nothing else than that we measure $Y$ in units of the $\Phi_{b}$-hypercharge. Then

$$\Phi_{b} = \tilde{\Phi}X_{1}; \chi_{1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

has $Y = -1$, and we may write $\tilde{\Phi} = (\Phi_{b}, \Phi_{t})$. Since we have the global $U(1)_{Y}$ for free, we may assume this symmetry to be local. The covariant derivative for $\tilde{\Phi}$ now reads

$$D_{\mu}\tilde{\Phi} = \partial_{\mu}\tilde{\Phi} + i\frac{g'}{2}B_{\mu}\tilde{\Phi} + i\frac{g}{2}\tau_{a}W_{\mu}^{a}\tilde{\Phi}$$

and we find back the usual Higgs Lagrangian

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2}(\partial_{\mu}H\partial^{\mu}H) + \frac{(H + v)^{2}}{2v^{2}}(M_{Z}^{2}Z_{\mu}^{\mu} + 2M_{W}^{2}W_{\mu}^{\mu}W^{-\mu})$$

$$-\frac{\lambda}{4}H^{4} - \lambda_{t}H^{3} - \frac{1}{2}m_{H}^{2}H^{2}.$$  

(39)

The 3 real fields $\phi_{a}$, $a = 1, 2, 3$ could and have been gauged away and only 3 out of 4 gauge fields can acquire a mass. Hence there must exist one massless field, the photon! Evidently we obtain the relations $g' = g\tan\theta_{W}$ and $\rho = M_{W}^{2}/(M_{Z}^{2}\cos^{2}\theta_{W}) = 1$!
of $M_Z = M_W$, when $g' = 0$.

Now, what can we say about the hypercharge of the fermions?

A left-handed doublet transforms like

$$L \rightarrow e^{\frac{i}{2} Y_L} \nu_L,$$

where $Y_L$ is arbitrary. By inspection of $\mathcal{L}_{\text{Yukawa}}$ we find for the hypercharges of the singlets: $\psi_{1R}$ must have $Y_{1R} = Y_{1L} + 1$ and $\psi_{2R}$ must have $Y_{2R} = Y_{2L} - 1$. One consequence is that $U(1)_Y$ must violate parity. The astonishing thing is that the fermion current which couples to the photon preserves parity. By inspection we find

$$D_\mu L_f = (\partial_\mu - i \frac{g'}{2} Y_L B - \mu - i \frac{g}{2} \tau_3 W_\mu - \cdots) L_f,$$

and the couplings of $L_f$ and $R_f$ to $A_\mu$ read

$$L_f : -i (g \sin \theta_W \frac{\tau_1}{2} + g' \cos \theta_W \frac{Y_L}{2}) A_\mu,$$

$$R_f : -i (g' \cos \theta_W \frac{\tau_3}{2} + g' \cos \theta_W \frac{Y_L}{2}) A_\mu.$$

Because we have $g' \cos \theta_W = g \sin \theta_W = e$ we find the Gell-Mann-Nishijima relation

$$Q = T_3 + \frac{Y}{2},$$

as a consequence of a minimal Higgs structure! What we find is, that, whatever the hypercharge of $L_f$ is $L_f$ and $R_f$ must couple identically to photons. Thus QED must be parity conserving! Furthermore, the charges of the upper (1) and lower (2) components of the doublets satisfy

$$Q_{1L} = Q_{1R}, \quad Q_1 - Q_2 = 1 \quad \text{and} \quad Q_1 + Q_2 = Y_L.$$

So far we have no charge quantization. Here we need a last assumption.

- If $\nu_R$ does not couple to the $U(1)$ gauge field, we have to set $Y_{1R} = 0$ and consequently we must have $Y_{1L} = Y_{2L} = 0$ and $Q_1 = 0, Q_2 = -1$. For the $U(1)_Y$ anomaly cancellation we need lepton-quark duality and the charges of the quarks must have their known values if they appear in three colors. One thus must have the usual charge quantization.

We finally summarize the consequences of the assumptions stated above:

- Breaking $SU(2)_L$ by a minimal Higgs automatically leads to a global $U(1)_Y$, which can be gauged
- parity violation of $SU(2)_L$
- $\rho = M_0^2/(M_Z^2 \cos^2 \theta_W) = 1$
- existence of the photon
- parity conservation of QED
- validity of the Gell-Mann-Nishijima relation
- family structure
- charge quantization
We do not know why right-handed neutrinos are sterile i.e. do not couple to gauge bosons. In the SM of electroweak interactions, neutrinos originally were assumed to be massless i.e. that right-handed neutrinos did not exist. This is definitely ruled out by the observation of neutrino oscillations.

I think this reasoning is able to help understanding how various excitations in the chaotic Planck medium develop a pattern like the SM as a low energy effective structure. Renormalizability as a consequence of the low energy expansion and the very large gap between the EW and the Planck scale plus a certain minimality (not too little but not too much e.g. only up to symmetry triplets) determines the SM structure without much freedom. Three fermion families are required in order CP violation emerges in a natural way, and to make baryogenesis eventually possible within the LEESM scenario as addressed in Sect. 7. We have been emphasizing the high self-consistency of the SM where all essential structures look to be emergent properties in the low energy effective viewport of a cutoff system residing at the Planck scale. “What is not capable of surviving at long distances does not exist there” (Darwin revisited).

Acknowledgments:
I thank the organizers of the Naturalness, Hierarchy and Fine Tuning Workshop, at the RWTH Aachen, for the kind invitation and the support.

References
1. F. Jegerlehner, Acta Phys. Polon. B 45, 1167 (2014).
2. F. Jegerlehner, Acta Phys. Polon. B 45, 1215 (2014).
3. G. ’t Hooft, Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking, NATO Adv. Study Inst. Ser. B Phys. 59, 135 (1980).
4. M. J. G. Veltman, Acta Phys. Polon. B 12, 437 (1981).
5. R. Decker, J. Pecieau, hep-ph/0512126.
6. M. S. Al-sarhi, I. Jack, D. R. T. Jones, Z. Phys. C 55, 283 (1992).
7. Y. Hamada, H. Kawai, K.Y. Oda, Phys. Rev. D 87, 053009 (2013).
8. D. R. T. Jones, Phys. Rev. D 88, 098301 (2013).
9. Mather, J.C. et al., Astrophys. J. (Letter) 354, 37 (1990).
10. Smoot, G. et al., Astrophys. J. (Letters) 396, 1 (1992).
11. C. L. Bennett et al. [WMAP Collaboration], Astrophys. J. Suppl. 208, 20 (2013).
12. P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A1 (2014); ibid. A16 (2014).
13. R. Adam et al. [Planck Collaboration], Astron. Astrophys. 594, A1 (2016).
14. A. G. Riess et al. [Supernova Search Team], Astron. J. 116, 1009 (1998).
15. S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999).
16. S. D. Bass, Acta Phys. Polon. B 45, 1269 (2014).
17. P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A24 (2014).
18. J. Dreitlein, Phys. Rev. Lett. 33, 1243 (1974).
19. J. E. Feltin, R. Isaacman, Rev. Mod. Phys. 58, 689 (1986).
20. V. Sahni, A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000).
21. A. H. Guth, Phys. Rev. D 23, 347 (1981).
22. A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).
23. A. Albrecht, P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).
24. A. D. Linde, Phys. Lett. B 108, 389 (1982); Phys. Lett. B 129, 177 (1983).
25. E. W. Kolb, M. S. Turner, The Early Universe, Front. Phys. 69, 1 (1990).
26. S. Weinberg, Cosmology, Oxford, UK: Oxford Univ. Pr. (2008) 593 p.
27. P. Minkowski, Phys. Lett. 71B, 419 (1977).
28. A. Zee, Phys. Rev. Lett. 42, 417 (1979).
29. F. Bezrukov, M. Shaposhnikov, Phys. Lett. B 659, 703 (2008).
30. J. L. F. Barbon, J. R. Espinosa, Phys. Rev. D 79, 081302 (2009).
31. F. Bezrukov, A. Magnin, M. Shaposhnikov, S. Sibiryakov, JHEP 1101, 016 (2011).
32. F. Bezrukov, M. Shaposhnikov, Phys. Lett. B 734, 249 (2014).
33. F. Bezrukov, J. Rubio, M. Shaposhnikov, arXiv:1412.3811 [hep-ph].
34. Y. Hamada, H. Kawai, K. Y. Oda, Phys. Rev. D 92, 045009 (2015).
35. G. Aad et al. [ATLAS Collab.], Phys. Lett. B 716, 30 (2012); Science 338, 1576 (2012).
36. S. Chatrchyan et al. [CMS Collab.], Phys. Lett. B 716, 30 (2012); Science 338, 1569 (2012).
37. F. Englert, R. Brout, Phys. Rev. Lett. 13, 321 (1964).
38. P. W. Higgs, Phys. Lett. 12, 132 (1964).
39. N. Cabibbo, L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. B 158, 295 (1979).
40. P. Q. Hung, Phys. Rev. Lett. 42, 873 (1979).
41. M. Lindner, Z. Phys. C 31, 295 (1986).
42. B. Grzadkowski, M. Lindner, Phys. Lett. B 178, 81 (1986).
43. M. Lindner, M. Sher, H. W. Zaglauer, Phys. Lett. B 228, 139 (1989).
44. M. Sher, Phys. Rept. 179, 273 (1989).
45. T. Hambye, K. Riesselmann, Phys. Rev. D 55, 7255 (1997).
46. G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Strumia, JHEP 1208, 098 (2012).
47. J. A. Casas, J. R. Espinosa, M. Quiros, Phys. Lett. B 342, 171 (1995); Phys. Lett. B 382, 374 (1996).
48. J. R. Espinosa, M. Quiros, Phys. Lett. B 353, 257 (1995).
49. B. Schrempp, M. Wimmer, Prog. Part. Nucl. Phys. 37, 1 (1996).
50. G. Isidori, G. Ridolfi, A. Strumia, Nucl. Phys. B 609, 387 (2001).
51. J. R. Espinosa, G. F. Giudice, A. Riotto, JCAP 0805, 002 (2008) [arXiv:0710.2484].
52. J. Ellis, J. R. Espinosa, G. F. Giudice, A. H ¨ocker, A. Riotto, Phys. Lett. B 679, 369 (2009).
53. B. Feldstein, L. J. Hall, T. Watari, Phys. Rev. D 74, 095011 (2006).
54. M. Shaposhnikov, C. Wetterich, Phys. Lett. B 683, 196 (2010).
55. M. Holthausen, K. S. Lim, M. Lindner, JHEP 1202, 037 (2012).
56. F. Bezrukov, M.Yu. Kalmykov, B.A. Kniehl, M. Shaposhnikov, JHEP 1210, 140 (2012).
57. S. Alekhin, A. Djouadi, S. Moch, Phys. Lett. B 716, 214 (2012).
58. L. N. Mihaila, J. Salomon, M. Steinhauser, Phys. Rev. Lett. 108, 151602 (2012).
59. A. V. Bednyakov, A. F. Pikelnier, V. N. Velizhanin, JHEP 1301, 017 (2013); Phys. Lett. B 722, 336 (2013); Nucl. Phys. B 875, 552 (2013); Nucl. Phys. B 879, 256 (2014); Phys. Lett. B 737, 129 (2014).
60. K. G. Chetyrkin, M. F. Zoller, JHEP 1206, 033 (2012); JHEP 1304, 091 (2013).
61. J. Fleischer, F. Jegerlehner, Phys. Rev. D 23, 2001 (1981).
62. A. Sirlin, R. Zucchini, Nucl. Phys. B 266, 389 (1986).
63. F. Jegerlehner, M. Yu. Kalmykov, O. Veretin, Nucl. Phys. B 641, 285 (2002); Nucl. Phys. Proc. Suppl. 116, 382 (2003); Nucl. Phys. B 658, 49 (2003).
64. F. Jegerlehner, M. Yu. Kalmykov, B.A. Kniehl, Phys.Lett. B 722, 123 (2013); J. Phys. Conf. Ser. 608, 012074 (2015).
65. D. Buttazzo, G. Degrassi, P.P. Giardino, G. F. Giudice, F. Sala, A. Salvio, A. Strumia, JHEP 1312, 089 (2013).
66. A. V. Bednyakov, B. A. Kniehl, A. F. Pikelnier, O. L. Veretin, Phys. Rev. Lett. 115, 201802 (2015).
67. M. Czakon, J. Gluza, F. Jegerlehner, M. Zralek, Eur. Phys. J. C 13, 275 (2000).
68. K. G. Wilson, Phys. Rev. B 4, 3174 (1971); Phys. Rev. B 4, 3184 (1971).
69. F. Jegerlehner, An Introduction to the Theory of Critical Phenomena and the Renormalization Group, Preprint, ZU PHYSIK University, Bielefeld, May 1976. 158pp. Lausanne Lectures http://www-com.physik.hu-berlin.de/~fjeger/LausanneLectures1.pdf.
70. F. Jegerlehner, Phys. Rev. D 16, 397 (1977).
71. S. L. Glashow, Nucl. Phys. 22, 579 (1961).
72. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
73. J. R. Aharony, M. E. Peskin, B. W. Lynn, S. B. Selipsky, Nucl. Phys. B 309, 221 (1988).
74. C. N. Yang, R. L. Mills, Phys. Rev. 96, 191 (1954).
75. G. ’t Hooft, Nucl. Phys. B 33, 173 (1971); 35, 167 (1971); G. ’t Hooft, M. Veltman, Nucl. Phys. B 50, 318 (1972).
76. M. Lüscher, P. Weisz, Phys. Lett. B 212, 472 (1988); Nucl. Phys. B 290, 25 (1987); Nucl. Phys. B 295, 65 (1988).
77. C. B. Lang, Nucl. Phys. B 265, 630 (1986).
78. D. J. E. Callaway, Phys. Rept. 167, 241 (1988).
79. F. Jegerlehner, Helv. Phys. Acta 51, 783 (1978).
80. M. J. G. Veltman, Nucl. Phys. B 7, 617 (1968).
81. C. H. Llewellyn Smith, Phys. Lett. B 46, 233 (1973).
82. J. S. Bell, Nucl. Phys. B 60, 427 (1973).
