Abstract  The largest gap in our understanding of nature at the fundamental level is perhaps a unified description of gravity and quantum theory. Although there are currently a variety of theoretical approaches to this question, experimental research in this field is inhibited by the expected Planck-scale suppression of quantum-gravity effects. However, the breakdown of spacetime symmetries has recently been identified as a promising signal in this context: a number of models for underlying physics can accommodate minuscule Lorentz and CPT violation, and such effects are amenable to ultrahigh-precision tests. This presentation will give an overview of the subject. Topics such as motivations, the SME test framework, mechanisms for relativity breakdown, and experimental tests will be reviewed. Emphasis is given to observations involving antimatter.

Keywords  Lorentz breaking · CPT violation · quantum gravity

1 Introduction

Present-day physics rests on two distinct theories: the Standard Model (SM) describing the microscopic quantum world of elementary particles and General Relativity (GR) governing the macroscopic world dominated by gravity. These two theories are generally considered to be two low-energy aspects of a single, more fundamental framework believed to operate at the Planck scale. Such a fundamental framework must consistently unify quantum mechanics and gravity. Although there are numerous theoretical approaches to this subject, experimental progress appears to be inhibited by the expected Planck-suppression of deviations from established physics. Experimental quantum-gravity investigations therefore rely largely on ultrahigh-precision searches for Planck-suppressed effects at attainable energies.

Promising candidate effects within this context are violations of Lorentz and CPT invariance [1]. These symmetries form cornerstones of both the SM and GR, so that any measured deviations necessarily imply new physics. Moreover, small Lorentz and

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CPT breakdown can be motivated by various approaches to physics beyond the SM and GR [2]. An additional motivation for Lorentz and CPT tests is provided by the fundamental character of these symmetries: they should be supported as firmly as possible by experimental evidence.

At presently attainable energy regimes, such Lorentz- and CPT-breaking effects are described in great generality by the Standard-Model Extension (SME) [3]. The SME is a field-theory framework that incorporates both the usual SM and GR as limiting cases. The additional Lagrangian terms of the SME are taken as small Lorentz- and/or CPT-violating corrections; these corrections are constructed to involve all operators for Lorentz violation that are scalars under coordinate changes. This broad scope ensures that essentially any current or near-future experiment can be analyzed with regards to Lorentz and CPT breakdown. A number of studies have been performed within the SME [4], which confirm its solid theoretical foundation. To date, the SME has provided the basis for the identification and analysis of numerous experimental investigations [1, 5]. For example, the SME leads to modifications in one-particle dispersion relations [6], which in turn could cause Cherenkov radiation in the vacuum [7]. The absence of this effect at colliders leads to stringent limits on Lorentz violation in QED [8]. For other limits in electrodynamics, see, e.g., Ref. [9].

This talk is focussed on another class of tests, namely those involving antimatter. CPT symmetry implies that a particle and its corresponding antiparticle have certain identical properties, such as the magnitudes of mass, charge, gyromagnetic ratio, etc. This suggests that matter–antimatter comparisons can be excellent tools in the search for CPT violation. In Sec. 2, we review the idea behind the construction of the SME, and we comment on the relation between Lorentz and CPT symmetry. Section 3 is dedicated to phenomenology. In particular, a number of matter–antimatter comparisons are discussed within the context of the SME.

2 The SME test framework

For the identification and analysis of suitable experiments, a test model is needed. The derivation of such a test model through a limiting process faces various obstacles. One of these is the multitude of candidate underlying models that can accommodate Lorentz and CPT violation: there is presently no single realistic underlying theory, whose low-energy limit can serve as the test framework. Moreover, for some candidate models, the low-energy limit is not unique or unknown. For these reasons, the test model will be constructed by hand with the goal of greatest possible generality. This ensures the widest applicability and relative independence from the underlying physics.

On the one hand, the test model should describe general breakdown of Lorentz and CPT symmetry. On the other hand, this breakdown should be carefully controlled in the sense that other, physically desirable properties are left unaffected. One of these properties is coordinate independence. Coordinates are a mathematical tool for the description of physical processes; they are purely a product of human thought, and therefore they should not acquire physical significance. Violating Lorentz invariance while keeping coordinate independence intact can be achieved by maintaining the usual Minkowski structure of spacetime but including preferred directions modeled by background vectors and tensors.

The above low-energy description of Lorentz and CPT breaking with background vectors and tensors has several advantages. First, coordinate changes are still imple-
mented by the usual Lorentz transformations. However, we remind the reader that selecting a different coordinate system must be clearly distinguished from rotations and boosts of the experimental set-up. It is these rotation and boost transformations (i.e., the particle Lorentz transformations), under which the symmetry is lost. The second advantage of this description is that it can be motivated by candidate fundamental theories: Most approaches to underlying physics are based on Minkowskian manifolds in four or more spacetime dimensions. Once this structure is contained in the theory, it can typically not be removed by considering a particular low-energy solution, such as the vacuum. Indeed, one can think of the background vectors and tensors as vacuum expectation values of Planck-mass fields. A third advantage is that a fully dynamical and microscopic description at presently attainable energies is relatively straightforward, as is reviewed next.

The starting point for the construction of the SME is essentially the entire body of established physics in the form of the SM Lagrangian $L_{\text{SM}}$ and the Einstein–Hilbert Lagrangian $L_{\text{EH}}$. This ensures that Lorentz and CPT breaking in all known physical systems can be accommodated. The next step involves adding small corrections $\delta L_{\text{LIV}}$ constructed by contracting the background vectors and tensors with ordinary SM and gravitational fields to form coordinate-independent scalars:

$$L_{\text{SME}} = L_{\text{SM}} + L_{\text{EH}} + \delta L_{\text{LIV}}.$$  \hfill (1)

Sample terms contained in the flat-spacetime limit of $\delta L_{\text{LIV}}$ are

$$\delta L_{\text{LIV}} \supset b_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi, \ (r_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi)^2, \ (k_F)^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}, \ldots.$$  \hfill (2)

Here, $\psi$ and $F$ are a conventional spinor field and a conventional gauge field strength, respectively. The nondynamical $b_\mu$, $r_\mu$, and $(k_F)^{\alpha\beta\gamma\delta}$ are small Lorentz-violating background vectors and tensors assumed to be generated by underlying physics. Experimental tests seek to constrain or measure these vectors and tensors. We finally mention that the minimal SME (mSME) is restricted by further physical requirements, such as translational invariance, the usual gauge symmetries, and power-counting renormalizability. For example, the mSME does not contain the $r_\mu$ term present in the above expression (2).

In what follows, we focus on the mSME. Within the mSME, a subset of all Lorentz-breaking corrections also violates CPT symmetry. For instance, in the expression (2) the $b_\mu$ term is CPT violating, while the $(k_F)^{\alpha\beta\gamma\delta}$ correction preserves CPT. The questions arises, as to whether we have missed terms that violate CPT invariance but preserve Lorentz symmetry. One answer to this question is given by Greenberg’s rigorous “anti-CPT theorem” [10]: the theorem roughly states that in any local, unitary, relativistic point-particle field theory CPT violation implies Lorentz breakdown. It follows that under these mild assumptions, CPT tests also probe Lorentz symmetry. This result offers the possibility for a further class of CPT-violation searches in addition to instantaneous matter–antimatter comparisons: probing for sidereal effects in matter–antimatter and other systems. We finally remark that relaxing the condition of effective unitarity, and thus observable probability conservation, can generate CPT breakdown without Lorentz violation [11].
3 Experimental tests of Lorentz and CPT symmetry

Most theoretical mSME prediction for atomic systems follow similar lines of reasoning. The first step is the assumption of vanishing Lorentz and CPT violation in electrodynamics. This step is justified for most mSME coefficients in the photon sector because astrophysical observations constrain their size to such a degree that they can be ignored for present-day atomic physics tests. The remaining photon coefficients can be absorbed into other sectors of the mSME. The next step is the extraction of the modified Dirac equation for the electron as well as the one for the proton (and the neutron, if needed). From these equations, the relativistic-quantum-mechanics Hamiltonian is determined. To do so, the unconventional time derivatives must be removed by a field redefinition [12]. One then proceeds with a generalized Foldy–Wouthuysen transformations that decouples the large and small spinor components. The emerging form of the Hamiltonian can then be used to extract the pieces for the particle and the antiparticle. From these, the modified nonrelativistic Pauli equation for the particle and the one for the antiparticle can be obtained [13]. As a result of CPT violation, these two Pauli equations are inequivalent. Lorentz- and CPT-violating corrections to atomic spectra can then be calculated employing conventional perturbation-theory methods. What follows is a brief description of various results for a number of physical systems.

The unmixed 1S–2S transition in (anti)hydrogen. The experimental resolution of the transition involving the unmixed spin states is expected to be roughly one part in $10^{-18}$. This sensitivity seems promising considering the likely Planck suppression for quantum-gravity effects. However, the leading-order mSME analysis shows that there are the same shifts for free H and $\bar{H}$ in both the initial and final states with respect to the ordinary levels. As a consequence, this particular transition is less useful for detecting leading-order mSME effects. Non-vanishing corrections to this transition in the context of the mSME are generated via relativistic effects that contain two further powers of the fine-structure constant $\alpha \simeq \frac{1}{137}$ [14]. A Planck-suppressed energy shift would therefore exhibit a further suppression by a factor in excess of $10^{4}$.

The spin-mixed 1S–2S transition in (anti)hydrogen. For high-precision spectroscopic studies it is often advantageous to confine the atoms under investigation with magnetic fields. For example, a commonly employed set-up involves a Ioffe–Pritchard trap. In the present context of $H$ and $\bar{H}$, both the 1S and 2S level are affected by the usual Zeeman splitting. An mSME calculation shows that the 1S–2S transition involving the spin-mixed states indeed acquires first-order corrections in this set-up [14]. From an experimental viewpoint, a potential disadvantage lies in the fact that this transition is also affected by the trapping magnetic field. It follows that the attainable resolution is constrained by the inhomogeneities in the $B$ field. To obtain resolutions in the vicinity of the natural line width, it seems likely that new experimental techniques must be devised.

Hyperfine Zeeman transitions within the 1S state of (anti)hydrogen. Another possibility in the context of experimental searches for Lorentz and CPT violation in (anti)hydrogen is provided by the measurement of the transition frequency between the Zeeman-split levels within the 1S state itself. Even for vanishing magnetic fields, the mSME predictions contain leading-order signals for such transitions [14]. Similar transitions of this kind (e.g., the usual Hydrogen-maser line) can be resolved with ultrahigh precision in experiments. A measurement of this hyperfine Zeeman line for antihydrogen is expected to be feasible in the near future [15].
Tests involving (anti)protons in Penning traps. Calculations within the mSME reveal that not only energy levels in atoms can acquire corrections, but also the eigenenergies of (anti)protons confined in a Penning trap. In particular, one can demonstrate that only the $b^\mu$-type mSME parameter given in the expression (2) contributes to transition-frequency differences between protons and antiprotons [16]. More precisely, the anomaly transitions acquire opposite corrections for protons and their antiparticles. This fact can be employed to extract clean experimental limits on the $b^\mu$ coefficient for the proton.

Searches for sidereal variations. In addition to instantaneous matter–antimatter comparisons for CPT tests, one can also exploit Greenberg’s “anti CPT theorem” discussed in Sec. 2. CPT breakdown comes with Lorentz violation, which in turn is often associated with rotation-symmetry breaking. It follows that carefully chosen measurements will then be direction dependent. This idea can be exploited as follows. A terrestrial laboratory, and thus the experiment, rotates as a result of the Earth spinning around its axis. This change of orientation will be reflected in a roughly daily modulation of the experiment’s observable. Under certain circumstances, higher harmonics can also occur. This general idea can be applied to a variety of physical systems. For example, modern tests involving Hydrogen masers and using ingenious experimental techniques employ the idea of such sidereal variations [17].

Testing boost symmetry. Lorentz invariance does not only imply isotropy but also symmetry under boosts. Paralleling the above rotation-violation searches, one can again exploit the motion of the Earth, and in particular its orbital motion. However, the Earth reverses its velocity with respect to the Sun about once every half year. This time frame could be impractical for experiments that need to maintain stability throughout such periods. An alternative would be satellite-based Lorentz and CPT tests. Although there are obvious constraints in terms of size and weight as well as financial issues for such tests, there can also be various benefits. For example, the orbit can be selected with an orientation yielding sensitivities to different components of mSME coefficients. Moreover, large velocity changes in short amounts of time can be attained increasing the sensitivity to violations of boost symmetry. Another benefit would be the quiet environment on board a satellite, which clearly offers advantages for ultrahigh-precision experiments. In addition, microgravity conditions can be advantageous for a number of measurements. For example, in fountain-clock-type tests longer interrogation times, and thus better precision, can be attained for freely falling atoms.

Tests involving gravity. In the experimental investigations discussed above, gravitational effects could be neglected and the flat-spacetime limit of the mSME was considered. However, tests involving gravity have recently been one focus of attention [18, 19, 20]. In particular, antimatter, such as antihydrogen, offers the possibility of testing Lorentz and CPT symmetry in the mSME’s gravity sector. For example, the acceleration of antihydrogen in the Earth’s gravitational field could be investigated. We also note that in gravitational contexts, various mSME coefficients that are inaccessible in the flat-spacetime limit now become measurable. Such ideas were discussed in J. Tasson’s talk at this meeting [19], and further details can be found in Ref. 20.

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