Robust Control Design for Rotary Inverted Pendulum Balance

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Abstract

Background/Objectives: Rotary inverted pendulum is a system that is often used in the domain of control theory to perform experiments. It is a complex yet interesting system that helps better understand concepts of control mechanism. It is an example of a non-linear oscillator. The pendulum is under actuated and extremely non-linear due to the gravitational forces and the coupling arising from centripetal forces. The experiment aims at swinging up the pendulum and balancing it in the upright position.

Methods/Statistical Analysis: This paper describes the existing system of Furuta Inverted Pendulum manufactured by Quanser. The system consists of a motor-run rotary arm which in turn controls a pendulum. PID controller is a very commonly used controller to set the process parameters at the desired values. It can be easily tuned and also helps make use of trial and error mechanism. Cascade controller makes use of two PID controllers. Findings: The main aim of this work is to compare the performance of two controllers, namely PID controller and Cascade controller, in controlling the pendulum in the upright position and to detect their performance. The criteria of comparison would be factors including low settling time and a stable output which would make the control action more efficient and smooth. The simulation results obtained in Matlab Simulink for Conventional PID and Cascade PID controllers are included. From the simulation results the Rise time and settling time has improved by using cascade PID controller but comparatively PID has given less noise.

Applications/Improvements: This work can be made use of in a number of scenarios like flight control, control of rocket propeller, unicycle, Segway transporters (which are self-balancing battery powered vehicles), two-wheel robots etc.

Keywords: Cascade PID Controller, Balance Control, PID Controller, Robust Control, Rotary Inverted Pendulum

1. Introduction

The Furuta Inverted Pendulum is a Quanser product which is frequently used in experiments based on control theory. A number of controllers have been used in the control mechanism of the inverted pendulum to date.

The main task that is associated with the system is to swing up and balance the pendulum in the upright position. This concept can be made use of in a number of scenarios like flight control, control of rocket propeller, unicycle, Segway transporters (which are self-balancing battery powered vehicles), two-wheel robots etc.

2. System Description

The Rotary Inverted Pendulum (RIP) system consists of an SRV02 servo motor which runs a gear to rotate a rotary arm which in turn affects the motion of inverted pendulum. There are two encoders in the system. One is fitted at the junction to check the angular position of the arm (label 2). The arm in turn is connected to a pendulum at the other end which can freely rotate about the axis of the arm. Another Encoder (label 1) is used to measure the angle of deflection of the pendulum. The values sensed by the encoders are fed back into the system in order to control the position of the pendulum. The main controller that is being used here is PID controller. Another
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The approach also includes the use of cascade controller which makes use of 2 PID controllers in cascade4,7.

The Rotary inverted pendulum is a complex; non-linear yet effective system which helps us learn and tackle numerous problems in the field of control systems. It is highly reliable and is very effective in providing good and accurate results without much distortion. The system is also very user friendly and opens up a wide range of possibilities for further research.

Figure 1 shows the system. The different parts include: Encoder of pendulum – used to measure the angle of deflection of the pendulum, Motor and Encoder of rotary arm – used on the rotary arm to check the deflection of rotary arm and to rotate the arm, Pendulum – The freely rotating part, Rotary arm – Controlled by motor with one end supporting to the pendulum, Output gear – Controls the rotary arm movement.

As the voltage is supplied to motor it rotates and changes the gear which in turn imparts motion to the rotary arm. As the arm moves, the pendulum also starts to oscillate and the swing up control swings it up till it reaches the inverted position.

Now the inverted pendulum is held in upright position by PID Controller. Basically the deflection angle of pendulum is input and is tried to be controlled at 0. When the pendulum is in upright position the angle between the pendulum and vertical axis is 0 which is to be maintained to hold it in upright position. The angle is measured by encoder at 1. The controlling of this is done by movement of rotary arm which is controlled by the motor.

The direction of rotation of motor forms the basis of this system control. Let’s say when arm is moving in clockwise direction positive voltage is applied to motor and when it is moving in anticlockwise direction negative voltage is applied. When the pendulum reaches the desired position, balance control takes over. Now as α deviates from zero to +α rotary arm moves in clockwise direction (i.e. positive voltage is supplied to motor) to bring α back to zero. Similarly as α deviates to −α the rotary arm moves in anticlockwise direction (i.e. negative voltage is supplied to motor) to bring α back to zero.

Figure 2 shows that Rotary inverted pendulum conventions, where

\[ \alpha : \text{angle of deflection of pendulum} \]
\[ \theta : \text{angle of deflection of rotary arm} \]
\[ r : \text{length of rotary arm} \]
\[ m : \text{centre of mass of pendulum} \]
\[ l_p : \text{length of pendulum} \]

The two main degrees of freedom involved in the system are the angles α and θ.

The Table 1 describes the specifications of the system including details of potentiometer, tachometer, motor and the plant in general1. These specifications give us an idea about the measurements made by the system during the experiment. Note that the potentiometer and the tachometer both have a measuring range of +5 to -5 V.

The non–linear equations of motion of the SRV02 system as follows:

\[ Q_1 = \tau - B \dot{\theta} \]
\[ Q_2 = -B \alpha \]

Here \( Q_1 = \tau - B \dot{\theta} \) and \( Q_2 = -B \alpha \)are the external non-conservative forces acting on the system with respect to the generalizied coordinates α and θ. They are derived through Euler-Lagrange equation.
adjust the control mechanism in order to minimize the error and get the desired output. In this case the values have been set to $K_p = 1.4$, $K_i = 6.8$ and $K_d = 0.02$. The LQR controller used in the existing model is replaced by a PID controller in Matlab Simulink.

According to Figure 3, the system is connected to a PID controller where the proportional, integral and derivative parameters are set by the user. A signal generator provides a signal to the controller which in turn provides a control signal to the RIP. Note that the control signal also depends on the feedback from the system. Here only one degree of freedom is assumed to perform the control action i.e. $\alpha$. The parameter fed back is angle of deflection $\alpha$ of the pendulum. This is because the other parameter which is the angle of deflection $\theta$ of the rotary arm is considered to be automatically controlled by the voltage input.

In cascaded control, Figure 4, two PID controllers are used in a cascaded form, the $k_p$, $k_d$ and $k_i$ values for both PID controllers are very different from each other. The response of PID controller is expected to be much faster and better.

### Table 1. System specifications

| SPECIFICATION                        | VALUE    | UNITS |
|--------------------------------------|----------|-------|
| Plant Dimension [L x W x H]          | 15 x 15 x 18 | Cm     |
| Plant Weight                         | 1.2      | Kg    |
| Nominal Voltage                      | 6        | V     |
| Motor Maximum Continuous Current [recommended] | 1   | A     |
| Motor Maximum Speed [recommended]    | 6000     | RPM   |
| Potentiometer Bias Power             | +12      | V     |
| Potentiometer Measurement Range      | +5       | V     |
| Tachometer Bias Power                | +12      | V     |
| Tachometer Measurement Range         | +5       | V     |
| Tachometer Sensitivity               | 0.0015   | V/RPM |
| Encoder Resolution                   | 4096     | Counts/rev. |
| Gear Ratio- high gear configuration  | 70       | n/a   |

### Table 2. State space matrix

| DESCRIPTION                        | VALUE |
|------------------------------------|-------|
| State Matrix                       | $0 \ 0 \ 0 \ 1$
|                                    | $0 \ 0 \ 0 \ 1$
|                                    | $0 \ 80.3 \ -45.8 \ -0.95$
|                                    | $0 \ 122 \ -44.1 \ -1.40$ |
| State-Space input Matrix           | $B$    |
|                                    | $0$    |
|                                    | $0$    |
|                                    | $83.4$ |
|                                    | $80.3$ |
| State-Space output Matrix          | $C$    |
|                                    | $1 \ 0 \ 0 \ 0$ |
|                                    | $0 \ 1 \ 0 \ 0$ |
| State-Space Matrix                 | $D$    |
|                                    | $0$    |
|                                    | $0$    |
| Open-Loop Poles                    | $OL$   |
|                                    | $[-48.42, \ 7.06, \ -5.86 \ and \ 0]$ |

### Figure 3. Simulink block diagram of PID controller.
One is used to control angle $\alpha$ while the other controls angle $\theta$. Each angle is fed back to the corresponding controllers through encoders in order to carry out the required control mechanism.

5. Result and Discussion

Figures 5, 6 and 7, show the simulation response of the system by using PID and cascade controllers. The output has a slight overshoot after which it gives a stable output. It can also be noted that the settling time of the output is very low.

Figure 8, Figure 9 and Figure 10 show the hardware response of the system by using PID and Cascade controllers. The control signal gives a slight overshoot after which it comes down to a stable state. The settling time of the response is low as can be seen. After the settling of the overshoot the stabilized output seems to slightly elevate which shows that the speed of the rotary arm steadily increases after a certain angle. The stability can be maintained by providing a slight external force to keep the angle within the range.
Table 3. Software Time response

| PARAMETERS         | PID  | CASCADE |
|--------------------|------|---------|
| Rise time for α    | 0.05 | 0.04    |
| Settling time for α| 0.5  | 0.15    |
| Noise              | Less | More    |

Table 4. Hardware Time response

| PARAMETERS         | PID  | CASCADE |
|--------------------|------|---------|
| Rise time for α    | 0.3  | 1       |
| Rise time for θ    | 0.2  | 1.1     |
| Settling time for α| 9    | 14      |
| Noise              | Less | More    |

6. Conclusion

In this work we have designed a controller for rotary inverted pendulum and successfully controlled it. The controllers are used to swing up the pendulum and balance it in upright position. Tables 3 and 4 shows that the Simulation results of the controllers in Mat lab Simulink and the response of each controller in hardware respectively. The performance of PID controller can be improved by tuning the parameters. From the experimental results both the controllers are capable in maintaining the rotary inverted pendulum stable. But the cascade controller has given robust performance in rise time and settling time comparatively to the single PID controller in balancing the pendulum in inverted position.

7. References

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