We address the dynamical map of entanglement and coherence in a four qubit maximally entangled GHZ states coupled with classical environments driven by fractional Gaussian noise. The system-environment coupling is assumed in four different schemes: common, bipartite, tripartite, and independent system-environment configuration. We show that entanglement preservation can be modeled in multipartite GHZ-like states using parameter optimization in the current local environments except for the independent configuration. The decay is characterized by monotonous functions in time and the exact fluctuating behaviour of the local fields, as well as entanglement sudden death and birth revivals, are completely suppressed. Not only noise but also the nature of qubit-environment coupling and the number of independent environments coupled, have dephasing effects on the entanglement and coherence preservation. Furthermore, as the Hurst parameter of fractional Gaussian noise is increased, the decay becomes delayed initially. Finally, the four-qubit GHZ state is found to be a good resource for quantum information processing that can withstand noise dissipation, particularly under a common noise source.

Keywords: Entanglement, decoherence, classical fluctuating fields, fractional Gaussian noise

I. Introduction

Quantum computers have emerged as leading-edge advanced devices that are far superior in functionality and most certain in practical applications. The quantum mechanical phenomena which control the tasks to be accomplished are the major preoccupations and working concepts of quantum computers. Therefore, physical phenomena and the corresponding functional concepts are just as important to consider as quantum computers [1, 2]. Entanglement and coherence, among many other phenomena, are undeniably the most significant for efficient quantum mechanical operations [3, 4]. Without a doubt, entanglement and coherence are at the root of nearly all quantum computing applications. For this reason,
the dynamics of entangled and coherent quantum systems remain a valuable pathway. As a result, these phenomena are discovered to be extremely resourceful and to play a vital role in the successful practical deployment and better functioning of quantum mechanical protocols.

Because of the non-local nature that classical interpretations cannot account for, entanglement is often used to distinguish quantum and classical correlations [5, 6]. Thus, entanglement has received a lot of attention and is being studied for new applications, such as quantum communication [7], quantum cryptography [8], dense coding [9], teleportation [10], quantum secure direct communication and quantum key distribution [11, 12]. For effective quantum information operations, we need entanglement preservation in quantum processing to execute various quantum information processing protocols [13]. Besides, coherence also is at the forefront of quantum computation, where the "0" and "1" states of the qubits superpose, which leads to an acceleration in many traditional algorithms. But when a state is decoherent, all its quantumness is lost and its usefulness drops.

The quantum information in progress is frequently connected to a surrounded medium and can never be fully isolated [14–16]. One of the key drawbacks of the effective use of quantum information processing protocols is the physical interaction between quantum systems and their environments that results in entanglement and coherence degradation and are termed as disentanglement and decoherence [17–19]. The environmental defects that are responsible for these dephasing effects are known as environmental noise, which limits the system’s initial state entanglement and coherence. The environmental noise has two different interpretable forms, which are classical and quantum interaction pictures. We will use the classical noisy interpretation instead of the quantum one because it allows us to explore the time evolution of the quantum systems with a larger number of degrees of freedom [20–26]. For practical quantum information processing, it is critical to optimize and describe the evolution of quantum systems in the presence of such fatal interfering classical environmental noises in particular. This can provide solutions to avoid interruption and losses in the quantum mechanical protocols.

Several quantum systems have been extensively researched to date for the dynamics and protection of quantum correlations and coherence under different classical environmental noisy effects. This includes static, dynamic, and coloured noises, all of which have yielded significant outcomes. Quantum correlations were increasingly fragile in the presence of these noises, and they were deteriorated or entirely disappeared.[17–26]. In addition, recent quantum research advances explore important measures, methods, and dynamical characterizations of different entangled quantum systems, such as bipartite, tripartite, and multiparty qubit systems. Here, multi-qubit systems outperform single-qubit and two-qubit systems in terms of channel capacities, cryptographic behaviour, and information transmission [27–30]. We will study the dynamics of a multi-partite system because of the advantages it has over other quantum systems.

This paper investigates dynamics of the entanglement and coherence for a system of four non-interacting qubits initially prepared as maximally entangled Greenberger-Horne-Zeilinger (GHZ) state when coupled to the classical noisy environments described by fractional Gaussian (FG) noise [31–33]. This kind of Gaussian noise arises due to the random motion of the particles in the diffusion process. In the current study, the dynamics of the quantum systems can also undergo such disorder and thus should be carefully analyzed. The FG noise has been used to investigate weather information, traffic control analysis, and electrical measurements because of its higher time scale correlations [34]. This noise has also been
used to obtain Tsallis permutation entropy [35], long-range and short-range dependence [36] and multi-scale pattern images [37].

Here, we suggest coupling the four non-interacting qubits to the classical fluctuating environments in four different configurations: common (CSE), bipartite (BSE), tripartite (TSE), and independent system-environment (ISE) configurations. We coupled all four qubits to a single common environment in the first case. In the second case, two pairs of qubits are independently coupled to two classical environments. While in the third situation, the state is coupled with three environments and, each qubit in ISE configuration is connected with four individual classical environments. The focus of the current configurations is to study the variation in the FG noisy effects over the initially encoded non-local correlation and coherence in the system of four qubits. In the presence of the current noisy situation, various measures such as quantum negativity, purity, and decoherence will evaluate the dynamics and protection of the multi-partite entanglement and coherence.

This paper is assembled as: In Sec.II, the multipartite entanglement, purity, and environmental decoherence estimators are illustrated. Sec.III, describes the physical model, FG noise application, and various types of system-environment coupling configurations. Sec.IV, deals with the findings and the subsequent discussions. Sec.V, represents the conclusive remarks based on the investigation undertaken.

II. Entanglement and coherence measures

This section describes the quantifiers used to evaluate multipartite entanglement and coherence.

A. N-partite negativity

Entanglement in an arbitrary N-qubit entangled system in a mixed state can be quantified by taking the average of the bipartite entanglement measures over all the possible bipartitions of the N-qubit system [38]. This is among the most useful and practical strategy proposed for the quantification of the global amount of genuine entanglement in multipartite systems and can be mathematically expressed as:

$$\mathcal{N}^{(N)}(\rho) = \frac{2}{N} \sum_{k=1}^{N/2} \left( \frac{1}{n_{\text{bipart}}^{(k)}} \sum_{P=1}^{n_{\text{bipart}}^{(k)}} \mathcal{N}^{P[k|N-k]}(\rho) \right),$$  \hspace{1cm} (1)

where $k|N - k$ represent the bi-partitions of the N-qubit system with $k$ qubits in one block and the remaining $N - k$ ones in another block. $P[k|N - k]$ is used to specify a precise combination of $k$ and $N-k$ qubits in constituting the bipartition $k|N-k$. Thus, $n_{\text{bipart}}^{(k)}$ stands for the total possible non-equivalent concrete bi-partitions $P[k|N - k]$. $\mathcal{N}^{P[k|N-k]}(\rho)$ denotes the bipartite entanglement (in terms of negativity) associated to the concrete bipartition $P[k|N - k]$. As mentioned above, the entanglement associated with any given bipartition of the N-qubit system is evaluated by means of negativity, defined for an arbitrary N-qubit system in a mixed state $\rho$ as:

$$\mathcal{N}^{P[k|N-k]}(\rho) = \sum_{j} |\lambda_j(\rho^{T_j})| - 1.$$  \hspace{1cm} (2)
Where $\lambda_j(\rho^{T_I})$ are the eigenvalues of the partial transpose $\rho^{T_I}$ of the total density matrix with respect to the subsystem $I$ which is constituted by the $k$ qubits of the given bipartition $P[k|N-k]$.

B. Purity

Purity is the quantifier of pureness and coherence in a quantum system. This measure mathematically can be written as [43]:

$$P(t) = \text{Tr}(\rho_{abcd}(t))^2.$$  \hspace{1cm} (3)

Where $\rho_{abcd}(t)$ is the time evolved density matrix of the system and for $m$-dimensional system, the multipartite state will be completely disordered and decoherent at $P(t) = \frac{1}{m}$. For a completely pure and coherent state, $P(t) = 1$. Any other values between this upper and lower bound will show the corresponding amount of purity and coherence.

C. Decoherence

The loss of coherence and quantum information caused by the unavoidable interaction of the environment with the qubits, which causes them to change their quantum states uncontrollably, is referred to as decoherence. Von Neumann entropy approach here will estimate the decoherence measurement as [43]:

$$V(t) = -\text{Tr}[\rho_{abcd}(t)\ln\rho_{abcd}(t)],$$  \hspace{1cm} (4)

where $\rho_{abcd}(t)$ is the time evolved density matrix of the system. With the coherent quantum condition, $V(t) = 0$, while any other output value of this measure will display the corresponding decoherence amount.

III. The model and dynamics

Our physical model comprises four identical non-interacting qubits with equivalent energy splitting $e_n$ coupled to classical fluctuating fields in four different configuration models. In the first case, all the four qubits are subjected to a single classical environment and is known as a common system-environment (CSE) configuration. In the second and third situations, we present bipartite (BSE) and tripartite system-environment (TSE) configurations where the system is coupled with two and three independent classical environments, respectively. In the final case, each qubit is exposed to an independent classical environment and is named as an independent system-environment (ISE) configuration. Note that, each environment contains a single FG noise source. The stochastic Hamiltonian governs the current physical model, which reads as [43]:

$$H_{abcd}(t) = H_a(t) \otimes I_b \otimes I_c \otimes I_d + I_a \otimes H_b(t) \otimes I_c \otimes I_d + I_a \otimes I_b \otimes H_c(t) \otimes I_d + I_a \otimes I_b \otimes I_c \otimes H_d(t),$$  \hspace{1cm} (5)

where $H_n(t)$ is the Hamiltonian for a single qubit and is written as $H_n(t) = e_n I + \omega \Delta_n(t) \sigma^x$ with $n \in \{a, b, c, d\}$. Here, $I$ and $\sigma^x$ are the identity and Pauli matrices operating on the qubit sub-spaces. $\Delta(t)$ is the classical stochastic parameter flipping between $\pm 1$ while $\omega$
FIG. 1. Upper panel: Schematic diagram shows the present configurations used in this paper for four non-interacting maximally entangled qubits, $Q_1$, $Q_2$, $Q_3$ and $Q_4$ in common (a) and bipartite (b), tripartite (c) and independent system-environment configuration (d) described by fractional Gaussian noise shown by the black-brownish shaded regions. The black-reddish wavy line shows the action of noise over the dynamics of qubits and the black narrow wavy lines with reducing amplitude indicates the dephasing effects. The lines joining the qubits show the non-local correlation with entanglement monogamy between every two qubits. The joining lines ensure that the four qubits are prepared in a single state with equal energy splitting represented by the same sizes and shapes.

is the coupling constant for the system-environment interaction. Next, the time evolved density matrix for the system is obtained by [44]:

$$\rho_{abcd}(t) = U_{abcd}(t)\rho_o U_{abcd}(t)\dagger,$$

(6)

where, $U_{abcd}(t) = \exp[-i \int_{t_0}^{t} H_{abcd}(s) ds]$ and is called time-unitary operation with $\hbar = 1$ and $\rho_o$, the initial density matrix for the system of four qubits and is given by [45]:

$$\rho_o = \frac{I_{(16 \times 16)}}{16}(1 - p) + p|GHZ\rangle\langle GHZ|,$$

(7)

with $0 \leq p \leq 1$ and $|GHZ\rangle$ is the four qubit maximally entangled Greenberger-Horne-Zeilinger state and is defined as $|GHZ\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$. 
A. Fractional Gaussian noise

We will cover the complete FG noise application in this section. To include the stochastic process, the β-function, which adds the noisy effects to the dynamics of the system, must be described, which read as [46, 47]:

$$\beta(t) = \int_0^t \int_0^t dx dx' k(x - x').$$  \hfill (8)

The auto-correlation function of the noise, that introduces the noise to the system phase in the local fields, is written as:

$$K_{FG}(t - t') = \frac{1}{2}(|t|^{2H} + |t'|^{2H} - |t - t'|^{2H}).$$  \hfill (9)

Now, one can get the corresponding β-function by substituting the auto-correlation function from Eq.(9) into Eq.(8) as:

$$\beta_{FG}(\tau) = \frac{\tau^{2H+2}}{2H + 2},$$  \hfill (10)

where, H is known as the Hurst parameter, which ranges between 0 and 1. [48]. By averaging the elements of the time evolved density matrix over the noise determines the resulting dephasing effects and can be computed as \langle \exp[\pm i\phi_n(t)] \rangle = \langle \exp[-\frac{1}{2}\beta_{FG}(\tau)(n)^2] \rangle with \phi_n(t) = n\lambda\Delta_n(t). When FG noise is introduced to CSE configuration, the corresponding final density matrix can be obtained by [49]:

$$\rho_{CSE}(\tau) = \langle U_{abcd}(t)\rho_o U_{abcd}(t)\rangle_{\phi_a},$$  \hfill (11)

where, \phi_a = \phi_b = \phi_c = \phi_d. In the case of BSE configuration, the final density matrix can be obtained by:

$$\rho_{BSE}(\tau) = \langle \langle U_{abcd}(t)\rho_o U_{abcd}(t)\rangle_{\phi_a} \rangle_{\phi_b},$$  \hfill (12)

where, we assume \phi_c = \phi_a and \phi_d = \phi_b. Similarly, for the TSE configuration, the final density matrix can be computed by:

$$\rho_{TSE}(\tau) = \langle \langle \langle U_{abcd}(t)\rho_o U_{abcd}(t)\rangle_{\phi_a} \rangle_{\phi_b} \rangle_{\phi_c},$$  \hfill (13)

we assume \phi_d = \phi_c. Finally, for the ISE configuration, the final density matrix is obtained by:

$$\rho_{ISE}(\tau) = \langle \langle \langle U_{abcd}(t)\rho_o U_{abcd}(t)\rangle_{\phi_a} \rangle_{\phi_b} \rangle_{\phi_c} \rangle_{\phi_d}. $$  \hfill (14)

In the above equations, \langle \Psi \rangle indicates the average over all the possible values of the FG noise phase applied on the state Ψ.

IV. Results and discussions

The major results for the dynamics of negativity, purity, and coherence for a system of four qubits initially prepared as mixed entangled GHZ state are given in this section for each configuration.
A. Common system-environment configuration

The dynamics of quantum correlation and coherence under FG noise for the four qubit GHZ state are examined in this section. The focus is made on the coupling of all four qubits to a single classical fluctuating environment, whose final density matrix can be obtained by using Eq.(11). The analytical relations for N-partite negativity, purity, and decoherence measures given in Eqs.(1), (3), and (4) can be put into the following forms:

\[ N_{CSE}^{(N)}(t) = \frac{1}{32} \left( \sum_{i=1}^{7} A_i \right), \]
\[ P_{CSE}(t) = \frac{1}{32} \left( 19 + e^{-64 \beta} + 12e^{-16 \beta} \right), \]
\[ V_{CSE}(t) = -A_8 - \frac{1}{8}(A_9) \log \left[ \frac{1}{16} A_9 \right]. \]

where
\[ A_1 = -24 + 2\text{Abs} \left[ 1 - e^{-32 \beta} \right] + 4\text{Abs} \left[ 1 - e^{-8 \beta} \right] + 4\text{Abs} \left[ 1 + e^{-8 \beta} \right], \]
\[ A_2 = 4e^{-8\Re[\beta]} \text{Abs} \left[ -1 + e^{8 \beta} \right] + 4e^{-8\Re[\beta]} \text{Abs} \left[ 1 + e^{8 \beta} \right] + 4\text{Abs} \left[ 2 - e^{-16 \beta} \sqrt{e^{16 \beta} \left( 3 + e^{16 \beta} \right)} \right], \]
\[ A_3 = 4\text{Abs} \left[ 2 + e^{-16 \beta} \sqrt{e^{16 \beta} \left( 3 + e^{16 \beta} \right)} + e^{-32\Re[\beta]} \text{Abs} \left[ 1 + 3e^{32 \beta} + \sqrt{1 - 6e^{32 \beta} + 12e^{48 \beta} + 9e^{64 \beta}} \right] \right], \]
\[ A_4 = \text{Abs} \left[ 3 - e^{-32 \beta} \left( 1 + \sqrt{1 - 2e^{32 \beta} + 16e^{48 \beta} + e^{64 \beta}} \right) \right], \]
\[ A_5 = \text{Abs} \left[ 3 + e^{-32 \beta} \left( 1 + \sqrt{1 - 2e^{32 \beta} + 16e^{48 \beta} + e^{64 \beta}} \right) \right]. \]
\[ A_6 = 2 \text{Abs} \left[ 3 - e^{-32\beta} \left( -1 + \sqrt{1 - 6e^{32\beta} + 12e^{48\beta} + 9e^{64\beta}} \right) \right], \]
\[ A_7 = \text{Abs} \left[ 3 + e^{-32\beta} \left( 1 + \sqrt{1 - 6e^{32\beta} + 12e^{48\beta} + 9e^{64\beta}} \right) \right], \]
\[ A_8 = \frac{1}{8} \left( 1 - e^{-32\beta} \right) \log \left[ \frac{1}{8} \left( 1 - e^{-32\beta} \right) \right], \]
\[ A_9 = 7 + e^{-32\beta} - e^{-32\beta} \sqrt{1 - 10e^{32\beta} + 48e^{48\beta} + 25e^{64\beta}}. \]

Fig. 2 explores the time evolution of entanglement and decoherence for the four qubit GHZ state, when coupled to a single source of FG noise. We found that negativity, and purity are decreasing functions of entanglement and coherence. In contrast, decoherence measure remained an increasing function of coherence decay. This demonstrates that FG noise significantly degrades multipartite entanglement and coherence. According to the negativity, entanglement does not completely disappear, making the CSE configuration an excellent resource for extended memory properties. In comparison, the rate of disentanglement differs from the rate of decoherence for the four qubit states. As can be seen that the saturation time of the curves are not same in the cases of negativity, purity and decoherence measure. For a quantum system, entanglement can be protected by designing the system-environment coupling with a single noise source, contributing to improving the efficiency of quantum mechanical protocols. The saturation of entanglement and coherence at different time intervals shows that the current FG noise has different impact on the two phenomenon. The negativity, purity and decoherence measures showed a monotonic decline of the entanglement and coherence rather than any rebirths after the first death, which is comparable to the results obtained under Gaussian noises in [46, 47, 49, 50]. This confirms that in the CSE configuration under FG noise, the system loses information that is not recoverable. As shown in [20, 21, 31], the overall dephasing effects of the FG noise differ completely from non-Gaussian noises, such as random telegraph and static noises. Under the FG noise, the previously studied bipartite Bells state has achieved full separability [46], however, entanglement is not totally lost for the current four qubit states suggesting it a useful resource for quantum dynamics and information preservation. At the upper bound of \( H \), entanglement and coherence remained more robust. Thus, increasing the \( H \) exponent improves entanglement and coherence survival time; nonetheless, the total FG noisy dephasing effects are unavoidable.

B. Bipartite system-environment configuration

This section examines the dynamics of entanglement and coherence when the four qubits are coupled with two separate environments, each with an equal number of qubits. The final density matrix of the current configuration is given by Eq.(12). The analytical results of negativity, purity, and decoherence are obtained using Eqs.(1), (3), and (4) and takes the forms:

\[ \mathcal{N}_{\text{BSE}}^{(N)}(t) = \frac{1}{192} e^{-16\text{Re}[\beta]} \left( \sum_{i=1}^{12} B_i \right), \] (18)
\[ P_{\text{BSE}}(t) = \frac{1}{16} e^{-32\beta} \left( 1 + e^{16\beta} + 8e^{24\beta} + e^{28\beta} + 5e^{32\beta} \right), \] (19)
\[ V_{BSE}(t) = -B_1 - B_4 - \frac{1}{16} (B_{15}) \log \left( \frac{1}{16} B_{15} \right). \] (20)

where

\[ B_1 = 2e^{12\text{Re}[\beta]} - 176e^{16\text{Re}[\beta]} + 16e^{16\text{Re}[\beta]} \text{Abs} \left[ 1 - e^{-16\beta} \right] + 20e^{16\text{Re}[\beta]} \text{Abs} \left[ 1 - e^{-2\beta} \right], \]

\[ B_2 = 4e^{16\text{Re}[\beta]} \text{Abs} \left[ 1 - e^{-16\beta} + e^{-8\beta} - e^{-2\beta} \right] + 4e^{16\text{Re}[\beta]} \text{Abs} \left[ 1 - e^{-16\beta} - e^{-8\beta} + e^{-2\beta} \right] + 8e^{14\text{Re}[\beta]} \text{Abs} \left[ -1 + e^{2\beta} \right] + 12\text{Abs} \left[ -1 + e^{8\beta} \right], \]

\[ B_3 = 8e^{8\text{Re}[\beta]} \text{Abs} \left[ -1 + e^{8\beta} \right] + 12\text{Abs} \left[ -1 + e^{14\beta} \right] + 4\text{Abs} \left[ 1 - e^{8\beta} + e^{14\beta} + e^{16\beta} \right], \]

\[ B_4 = 6e^{16\text{Re}[\beta]} \text{Abs} \left[ 3 + e^{-2\beta} - e^{-8\beta} \sqrt{e^{8\beta} \left( 16 + e^{4\beta} - 2e^{6\beta} + e^{8\beta} \right)} \right] + 6e^{16\text{Re}[\beta]} \text{Abs} \left[ 3 + e^{-2\beta} + e^{-8\beta} \sqrt{e^{8\beta} \left( 16 + e^{4\beta} - 2e^{6\beta} + e^{8\beta} \right)} \right], \]

\[ B_5 = 4e^{6\text{Re}[\beta]} \text{Abs} \left[ e^{2\beta} + e^8 + 2e^{10\beta} + \sqrt{e^{4\beta} \left( 1 - 2e^{6\beta} + 16e^{8\beta} + e^{12\beta} \right)} \right] + 8e^{16\text{Re}[\beta]} \text{Abs} \left[ 2 + e^{-8\beta} + e^{-2\beta} + e^{-10\beta} \sqrt{e^{4\beta} \left( 1 - 2e^{6\beta} + 16e^{8\beta} + e^{12\beta} \right)} \right], \]

\[ B_6 = 4e^{16\text{Re}[\beta]} \text{Abs} \left[ 2 + e^{-8\beta} + e^{-2\beta} + e^{-10\beta} \sqrt{e^{4\beta} \left( 1 - 2e^{6\beta} + 16e^{8\beta} + e^{12\beta} \right)} \right] + 3\text{Abs} \left[ 1 + e^{8\beta} + 2e^{16\beta} - \sqrt{1 + 2e^{8\beta} - 3e^{16\beta} + 12e^{24\beta} + 4e^{32\beta}} \right], \]

\[ B_7 = 3\text{Abs} \left[ 1 + e^{8\beta} + 2e^{16\beta} + \sqrt{1 + 2e^{8\beta} - 3e^{16\beta} + 12e^{24\beta} + 4e^{32\beta}} \right] + 3e^{16\text{Re}[\beta]} \text{Abs} \left[ 2 + e^{-16\beta} + e^{-8\beta} + e^{-16\beta} \sqrt{1 + 2e^{8\beta} - 3e^{16\beta} + 12e^{24\beta} + 4e^{32\beta}} \right], \]

\[ B_8 = 3\text{Abs} \left[ 1 + e^{14\beta} + 2e^{16\beta} - \sqrt{1 + 2e^{14\beta} - 4e^{16\beta} + 16e^{24\beta} + e^{28\beta} - 4e^{30\beta} + 4e^{32\beta}} \right] + 3\text{Abs} \left[ 1 + e^{14\beta} + 2e^{16\beta} + \sqrt{1 + 2e^{14\beta} - 4e^{16\beta} + 16e^{24\beta} + e^{28\beta} - 4e^{30\beta} + 4e^{32\beta}} \right], \]

\[ B_9 = 8e^{16\text{Re}[\beta]} \text{Abs} \left[ 3 - e^{-16\beta} \left( -1 + \sqrt{1 - 2e^{16\beta} + 16e^{24\beta} + e^{32\beta}} \right) \right] + 8e^{16\text{Re}[\beta]} \text{Abs} \left[ 3 + e^{-16\beta} \left( 1 + \sqrt{1 - 2e^{16\beta} + 16e^{24\beta} + e^{32\beta}} \right) \right], \]

\[ B_{10} = 6e^{16\text{Re}[\beta]} \text{Abs} \left[ 3 - e^{-8\beta} \left( -1 + \sqrt{1 + 14e^{8\beta} + e^{16\beta}} \right) \right] + 6e^{16\text{Re}[\beta]} \text{Abs} \left[ 3 + e^{-8\beta} \left( 1 + \sqrt{1 + 14e^{8\beta} + e^{16\beta}} \right) \right], \]

\[ B_{11} = 3e^{16\text{Re}[\beta]} \text{Abs} \left[ 2 + e^{-8\beta} - e^{-16\beta} \left( -1 + \sqrt{1 + 2e^{8\beta} - 3e^{16\beta} + 12e^{24\beta} + 4e^{32\beta}} \right) \right] + 3e^{16\text{Re}[\beta]} \text{Abs} \left[ 2 + e^{-8\beta} - e^{-16\beta} \left( -1 + \sqrt{1 + 2e^{8\beta} - 3e^{16\beta} + 12e^{24\beta} + 4e^{32\beta}} \right) \right], \]

\[ B_{12} = 3e^{16\text{Re}[\beta]} \text{Abs} \left[ 2 + e^{-16\beta} \left( 1 + \sqrt{1 + 2e^{14\beta} - 4e^{16\beta} + 16e^{24\beta} + e^{28\beta} - 4e^{30\beta} + 4e^{32\beta}} \right) \right], \]

\[ B_{13} = \frac{1}{4} \left( 1 - e^{-16\beta} \right) \log \left[ \frac{1}{8} \left( 1 - e^{-16\beta} \right) \right], \]
\[ B_{14} = \frac{1}{8} e^{-16\beta} (-1 + e^{8\beta})^2 \log \left[ \frac{1}{8} e^{-16\beta} (-1 + e^{8\beta})^2 \right], \]
\[ B_{15} = 5 + e^{-16\beta} + 2e^{-8\beta} - e^{-16\beta} \sqrt{1 + 4e^{8\beta} - 2e^{16\beta} + 52e^{24\beta} + 9e^{32\beta}}. \]

In Fig.3, we plot the time evolution of negativity, purity, and decoherence to explore the characteristic behaviour of the entanglement and coherence for the system of four qubits under \textit{FG} noise in \textit{BSE} configuration. According to the negativity and entanglement witnesses, when the \textit{BSE} configuration is used, the entanglement is more susceptible to the \textit{FG} noise. Unlike the \textit{CSE} configuration, the state faces greater mixedness and coherence loss, according to purity and decoherence measures. This implies a strong propensity of the \textit{FG} noise to limit information exchange between the system and associated bipartite environments. The qualitative dynamics were consistent across negativity, purity and decoherence measures, while the quantitative features differed slightly. The decoherence rate is different from the relevant disentanglement rate due to the interaction between the system and the bipartite environment. As shown in [46], the current qualitative temporal evolution of entanglement is similar to the results achieved for a system of two qubits employing quantum negativity and quantum discord. Non-Gaussian noises such as random telegraph and static noise, in contrast to \textit{FG} noise, have been discovered to support strong entanglement sudden death and birth revivals [20, 21, 31].

C. Tripartite system-environment configuration

In the current configuration, the four qubits are linked to three separate environments described by \textit{FG} noise. The final density matrix of the current configuration is given by
Eq. (13). To provide details on entanglement and coherence dynamics, we use quantum negativity, purity, and decoherence measures from Eqs. (1), (3), and (4). The corresponding numerical simulations are as under:

\[
N_{TSE}(t) = \frac{1}{192} \left(-192 + \sum_{i=1}^{10} C_i \right), \tag{21}
\]

\[
P_{TSE}(t) = \frac{1}{16} \left(5 + 8e^{-8\beta} + 3e^{-4\beta} \right), \tag{22}
\]

\[
V_{TSE}(t) = -C_{11} - C_{12} - C_{12} - \frac{1}{8}C_{14} \log \left(\frac{1}{16}C_{14} \right). \tag{23}
\]

where

\[
C_1 = 32e^{-4Re[\beta]} + 16\text{Abs} \left[1 - e^{-12\beta}\right] + 74\text{Abs} \left[1 - e^{-4\beta}\right] + 12\text{Abs} \left[1 - e^{-2\beta}\right] + 4\text{Abs} \left[1 - e^{-12\beta} + e^{-4\beta} - e^{-2\beta}\right],
\]

\[
C_2 = 4\text{Abs} \left[1 - e^{-12\beta} - e^{-4\beta} + e^{-2\beta}\right] + 16e^{-4Re[\beta]} \text{Abs} \left[-1 + e^{2\beta}\right] + 14e^{-4Re[\beta]} \text{Abs} \left[-1 + e^{4\beta}\right],
\]

\[
C_3 = 8e^{-4Re[\beta]} \text{Abs} \left[1 + e^{4\beta}\right] + 12e^{-4Re[\beta]} \text{Abs} \left[3 + e^{4\beta}\right] + 12e^{-12Re[\beta]} \text{Abs} \left[-1 + e^{8\beta}\right],
\]

\[
C_4 = 12e^{-12Re[\beta]} \text{Abs} \left[-1 + e^{10\beta}\right] + 4e^{-12Re[\beta]} \text{Abs} \left[1 - e^{8\beta} - e^{10\beta} + e^{12\beta}\right] + 4e^{-12Re[\beta]} \text{Abs} \left[1 + e^{8\beta} + e^{10\beta} + e^{12\beta}\right],
\]

\[
C_5 = 4e^{-8Re[\beta]} \text{Abs} \left[2e^{4\beta} + e^{6\beta} + e^{8\beta} + \sqrt{e^{8\beta}} (16 + e^{4\beta} - 2e^{6\beta} + e^{8\beta})\right] + 6\text{Abs} \left[3 + e^{-2\beta} - e^{-8\beta} \sqrt{e^{8\beta}} (16 + e^{4\beta} - 2e^{6\beta} + e^{8\beta})\right],
\]

FIG. 4. Dynamics of negativity (left), purity (center) and decoherence (right) for the multipartite GHZ state subjected to tripartite system-environment coupling configuration with $H = 0.9$ against the time evolution parameter $\tau = 1$. 

[Graph showing dynamics of negativity, purity, and decoherence over time for a specific system-environment configuration.]
yielded the same results, indicating that they are accurate and valid, as well as a direct
environments, the results are drastically different [20, 21, 31, 43, 44]. All of the measures
phase. When such entangled states are combined with non-Gaussian noises in classical
GHZ systems subjected to Gaussian noisy effects [46, 47, 50]. The fact that the current results
differ from those obtained for the BSE systems subjected to Gaussian noisy effects
in Fig.4. We noticed that the negativity, and purity of the four qubits GHZ state
improves the system’s memory attributes. The state becomes disentangled and decoheres
for the configurations is different, such as entanglement and coherence preservation time.
The general qualitative dynamics appear to be similar, the quantitative features measured
for the configurations is significant, the decline was monotonic, and no revivals of entanglement were
observed. This implies that the current Gaussian classical environments do not allow for the
entanglement of sudden death and birth phenomena, implying that the system does not have
information backflow. This matches the decay observed for bipartite and tripartite entangled
systems subjected to Gaussian noisy effects [46, 47, 50]. The fact that the current results
differ from those obtained for the BSE and CSE configurations is significant. In the current
case, entanglement and coherence were more fragile and suffered a greater loss. Although
the general qualitative dynamics appear to be similar, the quantitative features measured
for the configurations is different, such as entanglement and coherence preservation time.
With higher $H$ values, there is more robustness in the entanglement and coherence, which
improves the system’s memory attributes. The state becomes disentangled and decoheres
partially in the TSE configuration due to the dephasing effects caused by the FG noise
phase. When such entangled states are combined with non-Gaussian noises in classical
environments, the results are drastically different [20, 21, 31, 43, 44]. All of the measures
yielded the same results, indicating that they are accurate and valid, as well as a direct

\begin{align*}
\mathcal{C}_6 &= \frac{8}{8} \left[ 1 + 2e^{-4\beta} + e^{-2\beta} - e^{-8\beta} \sqrt{e^{8\beta}(16 + e^{4\beta} - 2e^{6\beta} + e^{8\beta})} \right] + \\
6\text{Abs} \left[ 3 + e^{-2\beta} + e^{-8\beta} \sqrt{e^{8\beta}(16 + e^{4\beta} - 2e^{6\beta} + e^{8\beta})} \right], \\
\mathcal{C}_7 &= \frac{4}{4} \left[ 1 + 2e^{-4\beta} + e^{-2\beta} + e^{-8\beta} \sqrt{e^{8\beta}(16 + e^{4\beta} - 2e^{6\beta} + e^{8\beta})} \right] + 
6e^{-16}\text{Re}[\beta] \left[ e^{4\beta} + 3e^{12\beta} - \sqrt{e^{8\beta} - 2e^{16\beta} + 17e^{24\beta}} \right], \\
\mathcal{C}_8 &= \frac{6}{6} \left[ e^{4\beta} + 3e^{12\beta} + \sqrt{e^{8\beta} - 2e^{16\beta} + 17e^{24\beta}} \right] + 
8\text{Abs} \left[ 2 + e^{-12\beta} + e^{-4\beta} - e^{-16\beta} \sqrt{e^{8\beta} - 2e^{16\beta} + 17e^{24\beta}} \right] + 
8\text{Abs} \left[ 2 + e^{-12\beta} + e^{-4\beta} + e^{-16\beta} \sqrt{e^{8\beta} - 2e^{16\beta} + 17e^{24\beta}} \right], \\
\mathcal{C}_9 &= \frac{3}{3} \left[ \sqrt{e^{8\beta} - 2e^{16\beta} + 17e^{24\beta}} - 2e^{20\beta} + 3e^{28\beta} - 2e^{30\beta} + e^{32\beta} \right], \\
\mathcal{C}_{10} &= \frac{3}{3} \left[ \sqrt{e^{8\beta} - 2e^{16\beta} + 17e^{24\beta} - 2e^{26\beta} + 3e^{28\beta} - 2e^{30\beta} + e^{32\beta}} \right], \\
\mathcal{C}_{11} &= \frac{1}{4} \left( 1 - e^{-4\beta} \right) \log \left( \frac{1}{4} \left( 1 - e^{-4\beta} \right) \right) - \frac{1}{8} \left( 1 - e^{-12\beta} + e^{-8\beta} - e^{-4\beta} \right) \log \left( \frac{1}{8} \left( 1 - e^{-12\beta} + e^{-8\beta} - e^{-4\beta} \right) \right), \\
\mathcal{C}_{12} &= \frac{1}{8} \left( -1 + e^{4\beta} \right)^2 \left( 1 + e^{4\beta} \right) \log \left( \frac{1}{8} \left( -1 + e^{4\beta} \right)^2 \left( 1 + e^{4\beta} \right) \right), \\
\mathcal{C}_{13} &= \frac{1}{8} \left( -1 + e^{4\beta} \right) \left( 1 + e^{4\beta} \right)^2 \log \left( \frac{1}{8} \left( -1 + e^{4\beta} \right) \left( 1 + e^{4\beta} \right)^2 \right), \\
\mathcal{C}_{14} &= 3 + e^{-12\beta} + e^{-8\beta} + 3e^{-4\beta} - e^{-16\beta} \sqrt{e^{8\beta} + 2e^{12\beta} - e^{16\beta} - 4e^{20\beta} + 63e^{24\beta} + 2e^{28\beta} + e^{32\beta}}.
\end{align*}

The key findings obtained for the dynamics of the negativity, purity, and coherence for multipartite GHZ state when coupled to TSE configuration with FG noise are reported in Fig.4. We noticed that the negativity, purity of the four qubits GHZ state were decreasing functions of entanglement and coherence in time due to the FG noisy effects in the TSE configuration. Decoherence, on the other hand, has remained a time-dependent function of coherence loss. The decline was monotonic, and no revivals of entanglement were observed. This implies that the current Gaussian classical environments do not allow for the entanglement of sudden death and birth phenomena, implying that the system does not have information backflow. This matches the decay observed for bipartite and tripartite entangled systems subjected to Gaussian noisy effects [46, 47, 50]. The fact that the current results differ from those obtained for the BSE and CSE configurations is significant. In the current case, entanglement and coherence were more fragile and suffered a greater loss. Although the general qualitative dynamics appear to be similar, the quantitative features measured for the configurations is different, such as entanglement and coherence preservation time.
correlation between them.

D. Independent system-environment configuration

When each of the four qubits is connected to an independent classical environment, the dynamics of entanglement and coherence under FG noise are described in this section. Eq. (14) represents the current configuration’s final density matrix. We use quantum negativity, purity, and decoherence measures from Eqs. (1), (3), and (4) to provide details on the entanglement and coherence dynamics, which are as follows:

\[
N_{ISE}^{(N)}(t) = \frac{1}{8} e^{-8\text{Re}[\beta]}(D_1 + D_2),
\]

\[
P_{ISE}(t) = \frac{1}{8} \left(1 + e^{-16\beta} + 6e^{-8\beta}\right),
\]

\[
V_{ISE}(t) = -D_3 - D_4 - D_5.
\]

where,

\[D_1 = 4e^{4\text{Re}[\beta]} - 8e^{8\text{Re}[\beta]} + 2e^{8\text{Re}[\beta]}\text{Abs}\left[1 - e^{-8\beta}\right] + 3e^{8\text{Re}[\beta]}\text{Abs}\left[1 - e^{-4\beta}\right] + 3\text{Abs}\left[-1 + e^{4\beta}\right],\]

\[D_2 = \text{Abs}\left[-1 + e^{4\beta}\right]^2 + \text{Abs}\left[1 + e^{4\beta}\right]^2 + e^{4\text{Re}[\beta]}\text{Abs}\left[3 + e^{4\beta}\right] + \text{Abs}\left[1 + 3e^{4\beta}\right],\]

\[D_3 = \frac{1}{2} \left(1 - e^{-8\beta}\right) \log \left[\frac{1}{8} \left(1 - e^{-8\beta}\right)\right],\]

\[D_4 = \frac{3}{8} e^{-8\beta} \left(-1 + e^{4\beta}\right)^2 \log \left[\frac{1}{8} e^{-8\beta} \left(-1 + e^{4\beta}\right)^2\right],\]

\[D_5 = \frac{1}{8} e^{-8\beta} \left(1 + 6e^{4\beta} + e^{8\beta}\right) \log \left[\frac{1}{8} e^{-8\beta} \left(1 + 6e^{4\beta} + e^{8\beta}\right)\right].\]

Fig. 5 investigate the dynamics of negativity, purity, and decoherence for four qubit GHZ state when coupled to ISE configuration driven by FG noise. In line with previous findings, negativity and purity remained lowering functions of entanglement and coherence in time. Decoherence remained an increasing function of coherence decay. In this case, the detrimental degrading effects are completely different quantitatively, with the resulting loss being significantly higher. The entangled sudden death and birth phenomenon is completely halted, and the decay is completely monotone. Because there is no flow of lost information from the environment back to the system, this translates to irreversible decay. When connected to independent environments, the current entanglement and coherence decay for the four-qubit system is substantially greater than that seen for the tripartite maximally entangled state described in [47]. We found that the independent noisy sources and decay have an intrinsic relationship in which both are directly related. Aside from that, the slopes show a reduced loss as H increases, signifying that the system’s memory characteristics have improved. This contradicts the findings of previous studies for a variety of noisy parameters, as given in [20, 21, 31, 43, 44, 47, 50–52]. The disentanglement and decoherence rates remained different at a comparable time for the four qubits. As a result of the decoherence generated by the coupling of the system with the environment, the system becomes entangled. This arrangement could be useful for quantum mechanical protocols that require a great deal of
FIG. 5. Dynamics of negativity (left), purity (center) and decoherence (right) for the multipartite GHZ state subjected to independent system-environment coupling configuration with $H = 0.9$ against the time evolution parameter $\tau = 1$.

channel capacity. The current qualitative dephasing effects are consistent with those obtained in independent coupling with $FG$ and $OU$ noise for bipartite and tripartite states [46, 47, 50]. The decay generated by non-Gaussian noises, on the other hand, is discovered to have a completely different nature [20, 21, 31, 43, 44]. All of the metrics produced the same results, suggesting that the results are consistent and valid and that they have a strong relationship.

E. Detailed time evolution analysis for four qubit GHZ state

In the presence of $FG$ noise, the dynamics of negativity, purity, and decoherence metrics for a system of four maximally entangled qubits coupled to various classical environmental configurations are investigated in this section. The major aim is to see how different $H$ values affect the $FG$ noisy effects concerning the time evolution parameter. The distinction between noisy effects induced by diverse environmental configurations will be investigated in greater detail and a range of settings in this research.

1. Time evolution of the entanglement and coherence within CSE configuration

In Fig.6, the time evolution of negativity, purity, and decoherence for the four qubit GHZ state when connected to the CSE configuration with $FG$ noise is fully studied. The current major findings are entirely consistent with previous ones for the same configuration and display the least degradation. As a result, the CSE configuration is ideal for successfully implementing quantum mechanical protocols. Furthermore, the non-local correlation and coherence become more retained as $H$ grows. This property of the $H$ exponent can engi-
FIG. 6. Dynamics of negativity (left), purity (center) and decoherence (right) for the multipartite GHZ state subjected to common system-environment coupling configuration with $H = 10^{-2}$ (green), 0.5 (blue) and 0.9 (red) against the time evolution parameter $\tau = 3$.

neer the memory characteristics that an environment requires. For different $H$ values, the slopes are nearly spaced, illustrating the discrete nature of the noise. The slopes eventually achieve a single saturation level, suggesting the same amount of decay and death point, even for varied values of $H$. In addition, as $H$ increases, the slopes change from green to red. This includes the entanglement’s initial robustness and coherence for increasing $H$ choices. It is important to emphasise, however, that the state does not become fully unentangled and incoherent. The decay observed is monotonous, and no rebirths have been witnessed, showing that the entangled sudden death and birth event has been off-limited by the FG noise. In the bipartite and tripartite states, the same decay pattern has been observed [46, 47, 50]. Because the rates of coherence and entanglement do not coincide, we conclude that system-environment coupling-induced decoherence is a major cause of disentanglement. The maxima and minima of the entanglement and coherence measures are quite close to each other, indicating that they are completely in agreement. In contrast to the current situation, static and dynamic noise has resulted in strong entanglement revivals and unprecedented decay [20, 21, 31]. When compared to the preservation intervals of entanglement and coherence in GHZ-class states, the current preservation intervals are quite short [53, 54]. This illustrates that, unlike the supportive character of the parameter $H$, the noisy phase of FG noise is extremely devastating to the memory properties of the environments, resulting in rapid entanglement and coherence loss.

2. Time evolution of the entanglement and coherence within BSE configuration

The dynamics of entanglement and coherence in the BSE configuration under the influence of FG noise are explored in detail in Fig.7. In this coupling strategy, we couple an equal number of qubits to two separate classical environments. The time evolution of the GHZ state differs significantly from that depicted in Fig.6 for the CSE design. Because of the FG noise in BSE, the negativity, entanglement witness, purity, and decoherence suffer from increased entanglement and coherence degradation. We noticed a shift in the slopes from greater to lower decay rates as $H$ increased. The statement is implied from the shift of the green slopes towards the red ones for increasing $H$. The robustness in the initial encoded entanglement and coherence in the four qubits are interpreted in this respect, but the entire preservation time is unaffected. Despite this robustness, the negative impacts of the FG noisy phase, which result in the system’s entire separability, cannot be disregarded.
FIG. 7. Dynamics of negativity (left), purity (center) and decoherence (right) for the multipartite
GHZ state subjected to bipartite system-environment coupling configuration with $H = 10^{-2}$
green), 0.5 (blue) and 0.9 (red) against the time evolution parameter $\tau = 3$.

There is no entanglement of sudden death and birth revivals and only a monotonous dete-
rioration has been observed. This suggests no backflow of lost information from the $BSE$
to the system, confirming the system’s permanent entanglement and coherence degradation.
This is in line with the findings for bipartite and tripartite states under Gaussian noises
examined in [46, 47, 50, 53, 54]. The measured decoherence speed appears to diverge from
the rate of entanglement degradation, which is consistent with the observations in Fig.6.
Finally, because the maxima and minima in the measurement agree, all the measures agree.
The described dynamical features for quantum systems driven by non-Gaussian disturbances
depart greatly from the existing dynamics of the four qubits, as shown in [50–52, 55].

3. Time evolution of the entanglement and coherence within TSE configuration

Fig.8 depicts the time evolution of entanglement by negativity while coherence by purity
and decoherence measures. We describe a hybrid environmental configuration in which two
qubits are exposed to two environments independently, while the other two are connected to
a common classical environment. In the current case, entanglement and coherence decay at

FIG. 8. Dynamics of negativity (left), purity (center) and decoherence (right) for the multipartite
GHZ state subjected to tripartite system-environment coupling configuration with $H = 10^{-2}$
green), 0.5 (blue) and 0.9 (red) against the time evolution parameter $\tau = 3$.

a faster rate than in the $BSE$ and $CSE$ configurations, as seen in Figs.2, 3, 6 and 7. This
demonstrates that the absolute preservation amounts and time of non-local correlation and
coherence are influenced not only by noise but also by the number of environments involved. This shows that the absolute preservation amount and time of non-local correlation and coherence are determined not just by noise but also by the number of environments involved. There has been no proof of entanglement, sudden death, or birth. As a result, following the first loss, there is no information exchange between the system and the environment, as specified in [46, 47, 50, 53, 54]. This also signifies a permanent loss of information rather than a temporary one. In contrast, because of non-Gaussian noises and other non-local correlations, information is temporarily lost with clear entangled sudden death and birth events [20, 21, 43, 44, 55]. The observed multipartite coherence is more transient than the entanglement of the same kind. This implies that the system’s entanglement is lost due to the generation of decoherence between the system and the environment. Because their maxima and minima are in perfect concordance, the qualitative results provided by negativity are in strict agreement with those of purity and decoherence.

4. Time evolution of the entanglement and coherence within ISE configuration

Fig. 9 illustrates the dynamics of negativity, purity, and decoherence for the multipartite GHZ state in ISE configuration. The qubits are connected to four distinct classical environments, each of which produces FG noise. The current adverse effects of such system-

![Graph](image)

FIG. 9. Dynamics of negativity (left), purity (center) and decoherence (right) for the multipartite GHZ state subjected to tripartite system-environment coupling configuration with $H = 10^{-2}$ (green), 0.5 (blue) and 0.9 (red) against the time evolution parameter $\tau = 3$.

evironment interaction are considered being the most detrimental. When a tripartite state is subjected to three independent environments driven by Gaussian and non-Gaussian noise, the decay varies from the current case [47, 55]. On the other hand, the deterioration produced by the four independent environments is substantially greater than that caused by the three independent environments. This means that the number of independent classical environments has a significant impact on the rate of degradation. As a result, a larger number of environments should be avoided for better performance of quantum mechanical hardware and protocols where entanglement is a required feature. According to the metrics, the entanglement decay rate does not match the coherence rate at the comparable time. This explains why the two phenomena for a single system are so distinct. In agreement with the previous results reported in Figs. 6, 7 and 8, for the increasing values of $H$, the slopes show lesser decay. This entanglement and coherence robustness only lasts for the first intervals and has no relevance to the overall preservation period. The decline is pure
exponential, and there is no entanglement sudden death and birth. The noxious effects of OU and FG noise on the tripartite and bipartite states described in [46, 47] are consistent with the current findings. However, the preservation intervals observed for tripartite and multipartite GHZ-like states under various Gaussian noises are different than observed here [53, 54]. The dynamics of entanglement, quantum negativity, quantum discord, and coherence, on the other hand, were found to be more resourceful under static, dynamic, and colourful noises, with fewer and momentary losses, as given in [20, 21, 43–45, 50, 52, 55].

5. Comparative dynamics in different environmental configurations

Fig. 10. Dynamics of negativity (left), purity (center) and decoherence (right) for the multipartite GHZ state subjected to common (blue slopes), and bipartite system-environment coupling (green slopes) with $H = 0.1$ (non-dashed) and $0.7$ (dashed) against the time evolution parameter $\tau = 3$.

Fig.10 evaluate dynamics of the entanglement and coherence in four qubit GHZ-like state when coupled to CSE (green), BSE (blue), TSE (red) and ISE configuration (black) under the FG noisy dephasing effects. Here, we introduce the dynamics of the state under two different conditions, namely, short-range dependence (SD) and long-range dependence (LD). For the FG noise phase, the SD is defined by the range $0 \leq H_{SD} \leq 0.5$ and LD by $0.5 \leq H_{LD} \leq 1$. For the LD, $\lim_{n \to \infty} \beta(n) n^{2H-2} > 0$, in the defined range of the $H$ parameter. For LD, if $\Sigma_n \beta(n) = +\infty$, then the relative auto-correlation function will have decay much slower, and its correlations for the process are not summable. We found the LD had greater entanglement and coherence preservation than the SD. For the transition from SD to LD, the slopes shift from non-dashed to dashed ends. The CSE configuration was found to be the most resourceful and to have the least decay. On the contrary, greater entanglement and coherence degradation is encountered at the ISE configuration. The decay has been demonstrated to be more likely as the number of environments increases. This indicates that the amount of decay is determined not only by noise and proper input parameter values but also by the number of environments involved. Finally, the most deterioration has been observed in the ISE configuration. This is due to an increase in the number of noisy sources and environments. To achieve lower decay rates, the increase in classical environments and noisy sources should be reduced. The remaining findings are completely consistent with the previous ones.
V. Conclusion

Quantum correlations and coherence must be thoroughly investigated before they can be put into practice in order for quantum information processing to be successful. When using quantum systems for non-local correlations and coherence, quantum mechanical protocols become inefficient due to decoherence effects. Quantum state spaces must be optimized for noise parameters in order to avoid these negative consequences. This requires precise characterization of various noises and optimal noise parameter settings. Entanglement and coherence are studied in this paper for a system of four non-interacting Greenberger-Horne-Zeilinger (GHZ) qubits constructed as mixed entangled states and then exposed to fractional Gaussian noise. System-environment coupling models such as common (CSE), bipartite (BSE), tripartite (TSE), and independent (ISE) configurations have been investigated in the present work.

We proved that entanglement and coherence decay monotonically under the influence of FG noise. For the bipartite and tripartite systems, under the influence of various types of Gaussian and non-Gaussian noises, similar results have been observed [46, 47, 50, 53, 54]. It is also found useful that the proper fixing of the noise parameters and selection of the system-environment coupling can reduce the detrimental effects. Under the FG noise, the four qubit GHZ-like state successfully carried entanglement and coherence for a finite interaction time, however, with short preservation intervals, which is inconsistent with the results obtained under OU noise in [47]. It’s also worth noting that the state achieves a single saturation level through monotonic decay. In any of the classical environmental configurations used, no apparent entanglement sudden death and birth revivals were observed. The results obtained for bipartite and tripartite states under the influence of static and dynamic noises, according to [20, 21, 31, 43], contradict this finding. As a result, the transformation of free separable GHZ-class states into resource states is halted, resulting in permanent information loss.

The CSE configuration has proven to be the environment most likely to support entanglement and coherence for an indefinite period. The same is reported in [47, 55] for entangled tripartite states when subjected to three independent environments, however, with different preservation intervals and quantum correlations amounts. Meanwhile, the ISE configuration degrades entanglement and coherence more than the other models. Under FG noise, the four qubit GHZ-like states become separable in ISE configuration and remained entangled in all other environments, which strongly contradicts the results in [46, 47, 53, 54]. Moreover, the amount of decay has been noticed to be regulated by the number of environments and increases directly with the increase in environments. Thus, to implement effective quantum-mechanical protocols, it is proposed to design configurations with minimal environments.

The Hurst exponent (H) has been observed to have the unusual property of initially protecting entanglement and coherence. This directly opposes the properties of nearly all the noisy parameters described in [20, 21, 31, 43–45, 47, 50–52, 55]. Besides, we found no exact solution for avoiding the relative noisy effects in any possible situation. Hence, quantum correlations, coherence and information initially encoded in any kind of quantum systems subjected FG noise will ultimately face decaying effects.

The same qualitative behaviour has been observed in the case of measures. Consistency and validity can be demonstrated by comparing results that agree. The current four qubit GHZ-class states were described as being a good resource for quantum information pro-
cessing that can withstand noisy effects for both finite and infinite intervals, depending on the type of classical environments involved. The four-qubit state has also been observed to be entangled and coherent compared to the tripartite and bipartite states. Because it has a larger capacity for storing quantum information, the four-qubit state is likely to be more efficient than other quantum systems. As a result, the proper selection of quantum systems, system-environment couplings, and the number of environments, combined with appropriate noise parameter tuning, can improve the performance of quantum mechanical protocols.

VI. Appendix

In this section, we give the details of the final density matrices for the four qubit GHZ state when coupled to different classical environmental configurations driven by FG noise. By using the Eq.(11), the corresponding final density matrix for the CSE configuration takes the form as:

\[ \rho_{CSE}(\tau) = \begin{bmatrix}
X_1 & 0 & 0 & X_2 & 0 & X_2 & X_2 & 0 & 0 & X_2 & 0 & X_2 & 0 & 0 & X_1 \\
0 & X_3 & X_3 & 0 & X_3 & 0 & 0 & X_3 & X_3 & 0 & 0 & X_3 & 0 & X_3 & X_3 & 0 \\
0 & X_3 & X_3 & 0 & X_3 & 0 & 0 & X_3 & X_3 & 0 & 0 & X_3 & 0 & X_3 & X_3 & 0 \\
X_2 & 0 & 0 & X_4 & 0 & X_4 & X_4 & 0 & 0 & X_4 & X_4 & 0 & X_4 & 0 & X_2 \\
0 & X_3 & X_3 & 0 & X_3 & 0 & 0 & X_3 & X_3 & 0 & 0 & X_3 & 0 & X_3 & X_3 & 0 \\
X_2 & 0 & 0 & X_4 & 0 & X_4 & X_4 & 0 & 0 & X_4 & X_4 & 0 & X_4 & 0 & X_2 \\
0 & X_3 & X_3 & 0 & X_3 & 0 & 0 & X_3 & X_3 & 0 & 0 & X_3 & 0 & X_3 & X_3 & 0 \\
X_2 & 0 & 0 & X_4 & 0 & X_4 & X_4 & 0 & 0 & X_4 & X_4 & 0 & X_4 & 0 & X_2 \\
0 & X_3 & X_3 & 0 & X_3 & 0 & 0 & X_3 & X_3 & 0 & 0 & X_3 & 0 & X_3 & X_3 & 0 \\
X_1 & 0 & 0 & X_2 & 0 & X_2 & X_2 & 0 & 0 & X_2 & X_2 & 0 & X_2 & 0 & X_1 
\end{bmatrix}, \tag{27} \]

where

\[ X_1 = \frac{19}{64} + \eta_1 + \eta_2, \quad X_2 = -\frac{1}{64}(1 + \eta_3) + \eta_4, \]
\[ X_3 = \frac{1}{64}(1 - \eta_5), \quad X_4 = \frac{1}{64}(1 + \eta_6) - \eta_7. \]
By utilizing the Eq.(12), final density matrix of the BSE configuration is given as:

$$\rho_{BSE}(\tau) = \begin{bmatrix}
\mathcal{P}_1 & 0 & 0 & \mathcal{P}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{P}_2 & 0 & 0 & \mathcal{P}_1 \\
0 & \mathcal{P}_3 & \mathcal{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{P}_3 & \mathcal{P}_3 & 0 \\
0 & \mathcal{P}_3 & \mathcal{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{P}_3 & \mathcal{P}_3 & 0 \\
\mathcal{P}_2 & 0 & 0 & \mathcal{P}_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{P}_4 & 0 & 0 & \mathcal{P}_2 \\
0 & 0 & 0 & 0 & \mathcal{P}_5 & 0 & 0 & \mathcal{P}_5 & \mathcal{P}_5 & 0 & 0 & \mathcal{P}_5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathcal{P}_6 & \mathcal{P}_6 & 0 & 0 & \mathcal{P}_6 & \mathcal{P}_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathcal{P}_6 & \mathcal{P}_6 & 0 & 0 & \mathcal{P}_6 & \mathcal{P}_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathcal{P}_5 & 0 & 0 & \mathcal{P}_5 & \mathcal{P}_5 & 0 & 0 & \mathcal{P}_5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathcal{P}_6 & \mathcal{P}_6 & 0 & 0 & \mathcal{P}_6 & \mathcal{P}_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathcal{P}_6 & \mathcal{P}_6 & 0 & 0 & \mathcal{P}_6 & \mathcal{P}_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{P}_2 & 0 & 0 & \mathcal{P}_3 & \mathcal{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{P}_3 & \mathcal{P}_3 & 0 \\
0 & \mathcal{P}_3 & \mathcal{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{P}_3 & \mathcal{P}_3 & 0 \\
\mathcal{P}_1 & 0 & 0 & \mathcal{P}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{P}_2 & 0 & 0 & \mathcal{P}_1 
\end{bmatrix} \quad (28)$$

where

$$\mathcal{P}_1 = \frac{1}{32} \kappa_1 (1 + \kappa_2), \quad \mathcal{P}_2 = \frac{1}{32} \kappa_3 (\kappa_4),$$

$$\mathcal{P}_3 = \frac{1}{32} (-1 + \kappa_5 (\kappa_6)), \quad \mathcal{P}_4 = \frac{1}{32} \kappa_7 (1 + \kappa_8),$$

$$\mathcal{P}_5 = \frac{1}{32} \kappa_9 (-1 + \kappa_10 \text{Sinh}[\beta]).$$

For the TSE configuration, we can get the final density matrix by using the Eq.(13) as:

$$\rho_{TSE}(\tau) = \begin{bmatrix}
\mathcal{Q}_1 & 0 & 0 & \mathcal{Q}_2 & 0 & \mathcal{Q}_2 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_2 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_1 \\
0 & \mathcal{Q}_3 & \mathcal{Q}_2 & 0 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_2 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_2 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_3 & 0 \\
0 & \mathcal{Q}_2 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 & 0 \\
\mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 \\
0 & \mathcal{Q}_2 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 & 0 \\
\mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 \\
0 & \mathcal{Q}_2 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 & 0 \\
0 & \mathcal{Q}_2 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 & 0 \\
\mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 \\
\mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 \\
0 & \mathcal{Q}_2 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 & 0 \\
\mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 \\
0 & \mathcal{Q}_2 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 & 0 \\
\mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_4 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_4 & 0 & 0 & \mathcal{Q}_2 \\
\mathcal{Q}_1 & 0 & 0 & \mathcal{Q}_2 & 0 & \mathcal{Q}_2 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_2 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_2 & 0 & 0 & \mathcal{Q}_1 
\end{bmatrix} \quad (29)$$

where

$$\mathcal{Q}_1 = \frac{1}{32} (5 + \lambda_1), \quad \mathcal{Q}_2 = \frac{1}{32} (-1 + \lambda_2),$$

$$\mathcal{Q}_3 = \frac{1}{32} (5 - \lambda_3), \quad \mathcal{Q}_4 = \frac{1}{16} \lambda_4 \text{Sinh}[\beta].$$
Finally, we can obtain the final density matrix for the $TSE$ configuration has the form as:

$$\rho_{TSE}(\tau) = \begin{bmatrix}
R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_1 \\
0 & R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_2 \\
0 & 0 & R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_2 \\
0 & 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 \\
0 & 0 & 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 \\
0 & 0 & 0 & 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 \\
0 & 0 & 0 & 0 & 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 \\
R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_1 \\
\end{bmatrix}$$

where

$$R_1 = \frac{1}{16} \varsigma_1 (1 + \varsigma_2),$$

$$R_2 = \frac{1}{16} (1 - \varsigma_3),$$

$$R_3 = \frac{1}{16} \varsigma_4 (-1 + \varsigma_5)^2.$$
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