Fractional vortices in a nano-sized superconducting composite structure (d-dot) with a twin boundary

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Abstract. In order to investigate effects of twin boundary (TB) on structure of spontaneous half-quantized vortices (SHQVs) in a nano-sized composite structure that consists of d- and s-wave superconductors (d-dot), we investigate magnetic field distribution with a d-dot model which has a single TB using anisotropic two-component Ginzburg-Landau equations and finite element method. We find that fractional vortices appear at the edges of TB and that the fractional vortices merge with SHQVs when effective interaction between d- and s-wave SCs is small.

1. Introduction
Corner junctions made with d- and s-wave superconductors (SCs) are called π-junctions, because the phase between d- and s-wave SCs is different by due to symmetry of order parameter in d-wave SC [1]. With quantization of fluxoids including this phase differences, half-quantized vortices spontaneously appear around π-junctions with zero magnetic field [2].

A d-dot, which is a nano-sized composite structure that consists of a d-wave SC embedded in an s-wave matrix [3,4] has four -junctions and then four spontaneous half-quantized vortices (SHQVs). If the size of the d-wave SC in d-dot is sufficiently small than penetration depth of the s-wave SC, SHQVs form anti-ferromagnetic configuration. So d-dot has two arrangements of SHQVs, which are doubly degenerated stable states controlling by an external field or current [5,6] we can use it as a classical bit with. It is also predicted that we can use superposition of these stable states as quantum bit by reducing the size of d-dot’s to increase transition probability between both states [7].

Fujii et al. made d-dot’s that consist of YBa2Cu3O7-δ (YBCO) and Pb, but SHQVs were not observed [8]. One of reasons of this result seems to be defects in YBCO, especially twin boundaries (TBs). Smilde et al. reported TBs suppress the SHQVs [9].

In this paper, to clarify effects of TB on structures of SHQVs in a d-dot, we investigate magnetic field distribution with a d-dot model that has a single TB using anisotropic two-component Ginzburg-Landau (GL) equations and finite element method (FEM). We show that fractional vortices appear at the edges of TB and that the fractional vortices merge with SHQVs when effective interaction between d- and s-wave SCs is small.
2. Methods

In order to incorporate effects of TBs into the GL equations, we introduce an anisotropic effective mass into the Gor’kov equations [10] as follows.

\[
\begin{aligned}
\{ \begin{array}{c}
\frac{\mathrm{i} \hbar \omega_n}{2m_s} \left( -\mathrm{i} \hbar \frac{\partial}{\partial x} + \frac{\mu}{c} A_x \right)^2 - \frac{1}{2m_s} \left( -\mathrm{i} \hbar \frac{\partial}{\partial y} + \frac{\mu}{c} A_y \right)^2 + \mu \\ -\frac{1}{2m_s} \left( -\mathrm{i} \hbar \frac{\partial}{\partial x} + \frac{\mu}{c} A_x \right)^2 - \frac{1}{2m_s} \left( -\mathrm{i} \hbar \frac{\partial}{\partial y} + \frac{\mu}{c} A_y \right)^2 + \mu 
\end{array} \} \tilde{G}(x,x';\omega_n) + \int \! dx'' \Delta(x,x'') F^\dagger(x'',x';\omega_n) = \delta(x-x') \\
F(x,x';\omega_n) - \int \! dx'' \Delta(x,x'') \tilde{G}(x'',x';\omega_n) = 0
\end{aligned}
\]

(1)

(2)

where \( \tilde{G} \) and \( F \) are the single particle normal and anomalous Green’s functions, respectively. \( m_s \) and \( m_e \) are the effective mass along \( x \)- and \( y \)-axis, respectively. \( \omega_n = (2n+1)\pi T \) is the Matsubara frequency. The order parameter in real space is given

\[
\Delta^\ast(x,x') = V(x-x')^T \sum_{\alpha} F^\dagger(x,x';\omega_n)
\]

(3)

\[
V(k,k') = V_x + V_y (\tilde{k}_x^2 - \tilde{k}_y^2) (\tilde{k}_x^2 - \tilde{k}_y^2)
\]

(4)

where \( V(k,k') \) is an effective pairing interaction between two electrons in momentum space, and \( V_x \) and \( V_y \) are interaction constants for \( s \)- and \( d \)-wave pairings [11], respectively.

Substituting equations (1), (2) and (4) into equation (3), we derive self-consistent equations for \( s \)- and \( d \)-wave order parameters [12], \( \Delta_s \) and \( \Delta_d \),

\[
\begin{aligned}
\Delta_s' &= 2\lambda_s \left[ \frac{V_x}{V_y} \Delta_s \ln \left( \frac{2e^\hbar \omega_0}{\pi k_BT} \right) - 2\lambda_s \left( \frac{V_x}{V_y} \right) \left( \frac{\epsilon_F}{2m_s} \Pi_1 + \frac{\epsilon_F}{2m_s} \Pi_1^\dagger \right) \Delta_s + \left( \frac{\epsilon_F}{4m_s} \Pi_1 + \frac{\epsilon_F}{4m_s} \Pi_1^\dagger \right) \Delta_s^\dagger + \left[ \frac{1}{2} \lambda_D \right] \Delta_s^\dagger \Delta_s \right] \\
\Delta_d' &= 2\lambda_d \Delta_s \ln \left( \frac{2e^\hbar \omega_0}{\pi k_BT} \right) - 2\lambda_d \left( \frac{V_x}{V_y} \right) \left( \frac{\epsilon_F}{4m_s} \Pi_1 + \frac{\epsilon_F}{4m_s} \Pi_1^\dagger \right) \Delta_s + \left( \frac{\epsilon_F}{4m_s} \Pi_1 + \frac{\epsilon_F}{4m_s} \Pi_1^\dagger \right) \Delta_s^\dagger + \left[ \frac{1}{2} \lambda_D \right] \Delta_s^\dagger \Delta_s
\end{aligned}
\]

(5)

(6)

Here \( \gamma \) is the Euler constant, and \( \omega_D \) is a cutoff frequency for interactions. Also

\[
\alpha = 7\zeta(3)/8(\pi k_BT)^2, \quad \epsilon_F = \hbar^2 k_F^2 / 2m_s = \hbar^2 k_F^2 / 2m_y \quad \text{and} \quad \lambda_d = N(0)\epsilon'_F / 2 \quad \text{where} \quad N(0) \quad \text{is a density of states at the Fermi surface. And} \quad \Pi = -\hbar \nabla_R - (2e/c) A_R \quad \text{where} \quad R = (x + x') / 2 .
\]

Note that gradient terms and anisotropic coupling terms between \( \Delta_s \) and \( \Delta_d \) in equations (5) and (6) contain anisotropic mass. In order to analyze how these terms affect d-dot’s, we solve equations (5) and (6) using the FEM [3].

In this calculation, we use material parameters of YBCO and Pb for d- and s-wave SCs, respectively.

Figure 1 shows (a) a schematic diagram and (b) a calculation model of a d-dot with a single TB. We consider following system; an s-wave matrix is \( L \times L \) \( (L = \xi_{s0} = 50\xi_{d0} = 80\text{nm}) \) and an embedded d-wave region is \( L/2 \times L/2 \) \( (L/2 = 40\text{nm}) \). The single TB is shown by a straight line (yellow line) in the d-wave SC region from the middle point of the bottom to the middle point of the left side, which divides the d-wave SC region into a left lower domain (orange area) and a right upper domain (purple area). Effective masses of electrons in the left lower domain are switched in the right upper domain.

Here, \( \xi_{s0} \) and \( \xi_{d0} \) are the coherent length of d- and s-wave SCs at 0 K.

We assume that the s-wave order parameter exists in the s-wave SC region and penetrates into d-wave SC region. On the other hand, the d-wave order parameter exists in the d-wave SC region and
penetrates into the $s$-wave SC region. So, interactions between $s$- and $d$-wave order parameters around interface of $d$- and $s$-wave SCs are important for appearance of SHQVs.

Note that in this calculation, ratio of interaction constants for $s$- and $d$-wave pairings $V_s, V_d$ are different between $d$- and $s$-wave SC regions.

Figure 1. (a) A Schematic diagram and (b) a calculation model of a d-dot with a single TB. The $s$-wave matrix is $L \times L$ ($L = \xi_s/0 = 50\xi_d/0 = 80nm$) and the embedded $d$-wave region is $L/2 \times L/2$ ($L/2 = 40nm$). The TB is shown by a straight yellow line which divides a central $d$-wave square into a left lower domain (orange area) and a right upper domain (purple area). Effective masses of Cooper pairs in the left lower domain are switched in the top right domain. Here, $\xi_x/0$ and $\xi_y/0$ are the coherent length of $d$- and $s$-wave SCs at 0 K.

3. Results and Discussions

Figure 2 shows magnetic field distributions for isotropic system with $m_x/m_y = 1.0$ (a), and for anisotropic system with $m_x/m_y = 2.0$ (b), respectively. In figures 2 (a) and (b), red and blue colors mean $H_x < 0$ and $H_x > 0$, respectively. In this case we set ratios of $V_x, V_y$ in $d$-wave SC ($V_x/V_y)_d = 1.0$ and in $s$-wave SC ($V_x/V_y)_s = 1.0$, respectively.

In the isotropic case, figure 2 (a), only four SHQVs appear on each corners of $d$-wave domain and form antiferromagnetic configuration [3]. In the anisotropic cases, figure 2 (b), shapes and configuration of magnetic field distributions are almost same as figure 2 (a), but we can also see additional vortices in solid circles (green circles). These additional vortices are fractional vortices, not SHQVs, because peak values of these additional vortices are less than SHQVs’ in figure 2 (a).
Figure 2. Density plots of magnetic field distributions for (a) isotropic system with $m_j/m_x = 1.0$, and for (b) anisotropic system with $m_j/m_x = 2.0$, respectively. We set ratios of $V_d$ and $V_x$ in $d$-wave SC ($V_d/V_x) = 1.0$ and in $s$-wave SC ($V_d/V_x) = 1.0$, respectively. In (a) and (b), red and blue colors mean $H > 0$ and $H < 0$, respectively. In (a), only four SHQVs appear on each corners of $d$-wave domain and form antiferromagnetic order. But in (b), We can see fractional vortices in solid circles (green circles).

One can understand how these fractional vortices appear as below. The distributions of supercurrents around the corners are in an elliptic form, because the penetration depth $\lambda$ is inversely proportional to the square root of effective mass

$$\lambda = \left(\frac{c^2 m^*}{4\pi n_{se}^2}\right)^{1/2}$$

Here $e^*$ and $m^*$ is charge and effective mass of cooper pair i.e. $e^* = 2e$, $m^* = m_x$ or $m_y$. Around the corner junctions, supercurrent $j_s(r)$ is large. Then they decrease continuously from the corner to the middle point of the corners. Because penetration depth changes due to the change of $m^*$ across the TB, there is finite imbalance of $j_s(r)$ at the edges of the TB. This supercurrent imbalance along the boundary of $d$- and $s$-wave SCs, especially in solid circles (green circles), generate fractional vortices.

To analyze dependence of interaction between $d$- and $s$-wave SCs, we obtain magnetic field distribution for the case of ($V_d/V_x) = (V_x/V_d) = 0.1$, respectively. Figure 3 shows density plots of magnetic field distributions for (a) isotropic system with $m_j/m_x = 1.0$, and for (b) anisotropic system with $m_j/m_x = 2.0$, respectively, (($V_d/V_x) = (V_x/V_d) = 0.1$). In figure3 (a), only four SHQVs also appear on each corners of $d$-wave domain and form antiferromagnetic order as same as figure 2 (a). But the distribution of each SHQV concentrate more inside of $d$-wave region as dashed lines indicating. This is because efficient penetration depth in $d$-wave region becomes larger than the case of figure 2. In figure 3 (b), fractional vortices cannot be distinguished from SHQVs because SHQVs more spread inside $d$-wave region and merge with the nearest parallel fractional vortices.
Figure 3. Density plots of magnetic field distributions for (a) isotropic system with \( m_y/m_x = 1.0 \), and for (b) anisotropic system with \( m_y/m_x = 2.0 \), respectively. ((\( V_s/V_d \))\(_d\) = (\( V_d/V_s \))\(_s\) = 0.1). In (a) and (b), red and blue colors mean \( H_z > 0 \) and \( H_z < 0 \), respectively. In (a), only four SHQVs appear on each corners of d-wave domain and form antiferromagnetic order as same as figure 2 (a), but the distribution of each SHQV concentrate more inside d-wave region as dashed lines indicating. In (b), we can’t distinguish fractional vortices from SHQVs because SHQVs more spread inside d-wave region and merge with the nearest parallel fractional vortices.

4. Summary

In conclusion, in order to analyze the effects of TBs on structure of magnetic field distribution in d-dot, phenomenologically, we calculated magnetic field distribution in a d-dot model with a single TB using modified two-component GL equations that include anisotropic mass terms and the FEM. We find that fractional vortices appear at the edges of TB beside SHQVs and that the fractional vortices merge with the nearest parallel SHQVs when effective interaction between d- and s-wave SCs is small.

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References

[1] Tsuei C C and Kirtley 2000 Rev. Mod. Phys. 72 969
[2] Hilgenkamp H, Ariando, Smilde H, Blank D H A, Rijnders G, Rogalla H, Kirtley J R and Tsuei C C 2003 Nature 422 50
[3] Kato M, Ishida T, Koyama T and Machida M, 2012 Superconductors -Materials, Properties and Applications- (InTech, Croatia) Chap.13, p. 319
[4] Ishida T, et al. 2006 Physica C 437 104
[5] Ako M, Machida M, Koyama T, Ishida T and Kato M 2004 Physica C 412-414 544
[6] Ako M, Machida M, Koyama T, Ishida T and Kato M 2005 Physica C 426-431 122
[7] Koyama T, Machida M, Kato M and Ishida T 2005 Physica C 426-431 1561
[8] Fujii M, et al. 2005 Physica C 426 104
[9] Smilde H J H, Ariando, Blank D H A, Gerritsma G J, Hilgenkamp H and Rogalla H 2002 Phys. Rev. Lett. 88 057004
[10] Gor’kov L P 1960 Sov. Phys. JETP 9 1364
[11] Xu J H, Ren Y and Ting C S 1995 Phys. Rev. B 52 7663
[12] Fujita N, Kato M and Ishida T 2015 Physica C 518 44