Flavor meson localization in 5d brane-world

Kazuo Ghoroku

†Fukuoka Institute of Technology, Wajiro, Higashi-ku
Fukuoka 811-0295, Japan

Abstract

We propose a brane-world, which contains flavor quarks and mesons, by embedding dimensionally reduced D7-brane in both the supersymmetric and non-supersymmetric 5d background which are obtained as the solutions of type IIB supergravity compactified on AdS$_5 \times S^5$. In the supersymmetric case, the RS brane can be put at any point of the fifth coordinate, but it is pushed to the AdS$_5$ boundary in the non-supersymmetric case. We study the localization of the flavor mesons, the fluctuation-modes of D7-brane, on the Randall-Sundrum brane in these backgrounds.

†gouroku@dontaku.fit.ac.jp
1 Introduction

It would be very powerful to use the gravity/gauge correspondence in getting non-perturbative insights of various gauge theories. This idea has been extensively studied in 5d gauged supergravity [1]-[4] which can be formulated as the superstring theory compactified on $\text{AdS}_5 \times S^5$. This theory is useful since it contains various scalar fields which correspond to the relevant and marginal operators of CFT.

The advantage of the 5d supergravity is that it can be reconciled with the brane-world proposed by Randall and Sundrum (RS) [5]. In this brane-world, it is possible to consider 4d gravity since the graviton is localized on the brane. In order to obtain more realistic braneworld, however, it is desirable to set the braneworld in a background which corresponds, in a holographic sense, to the confining gauge theory with flavor quarks as in QCD. In such a brane-world, we expect to find mesons, the quark-bound states, which are localized on the RS brane as well as the graviton.

Recently, a new idea to introduce flavor quarks has been proposed by Karch and Katz [6] by embedding a D7-brane as a probe which is wrapping a contractible $S^3$ of $S^5$ in the $\text{AdS}_5 \times S^5$ background in order to avoid the RR tadpole problem. At the price of this embedding, a tachyonic mode appears on the D7-brane, but the system is stable since the mass is within the Breitenlohner-Freedman bound [7] of the AdS background. Meanwhile, the brane-world is set in the $\text{AdS}_5$ accompanied with the compact $S^5$ as the inner space. In a similar sense, the $S^3$ is considered as the inner space of the embedded D7-brane, and the action of the D7-brane can be reduced to 5d by integrating over the coordinates of $S^3$. Then we can set a system of supergravity and D7-brane which are dimensionally reduced to five dimensions.

The purpose of the present paper is to study the problem of embedding the reduced D7-brane in the 5d background of an appropriate brane-world according to the procedure given in [8]. And we also study the localization of the flavor mesons, which are given as fluctuation modes of the D7-brane, on the RS brane. The analysis is performed for the simple background studied before in [8].

In Section 2, we set up our brane-world model. In Section 3, in this brane-world, the dimensionally reduced D7 brane is embedded. In Section 4, the localization of the mesons are studied, and summary is given in the final section.

2 Setting of a brane-world

Here we set up our model of a simple brane-world to embed a dimensionally reduced D7-brane. Consider the 10d type IIB supergravity which includes the dilaton ($\Phi$), axion ($\alpha$) and five form field strength to obtain $\mathcal{M}_5 \times S^5$ space-time with the following form of the Einstein frame metric,

\[
d s_{10}^2 = \frac{r^2}{L^2} A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^2}{r^2} \left( dr^2 + r^2 d\Omega_5^2 \right),
\]
where $A(r)$ denotes a deviation of $\mathcal{M}_5$ from the exact AdS$_5$. The $\mathcal{M}_5$ part, $ds_5^2 = \frac{r^2}{L^2} A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + L^2 dr^2$, is rewritten by using the new variable $y$, defined as $r = e^{-y/L}$, as follows
\[ ds^2 = \tilde{A}^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \] (2)
where $\tilde{A}^2 = \frac{r^2}{L^2} A^2(r)$. This metric is also obtained from the following 5d theory,
\[ S_g = \int d^4x dy \sqrt{-g} \left\{ \frac{1}{2\kappa^2_5} R - \frac{1}{2} (\partial \Phi)^2 - \frac{1}{2} e^{2\Phi} (\partial \alpha)^2 - V \right\}, \] (3)
where $V = -6/(L^2\kappa^2_5)$.* This action is obtained from 10d supergravity by integrating out the compact $S^5$ part. And this is also equivalent to the 5d gauged supergravity written by dilaton and axion only. In this case, the potential $V$ in $S_g$ is generally given as
\[ V(\phi) = \frac{v^2}{8} \sum_i \left( \frac{\partial W}{\partial \phi_i} \right)^2 - \frac{v^2}{3} W^2, \] (4)
where $W$ represents the superpotential of the theory and $v$ denotes the gauge coupling $v$. In the present case, $\phi_i = \{ \Phi, \alpha \}$ and $V = -3$.

The supersymmetric solutions of $S_g$ are given by solving the following first order equations [12],
\[ \phi_i' = \frac{v}{2} \frac{\partial W}{\partial \phi_i}, \quad \frac{\tilde{A}'}{\tilde{A}} = -\frac{v}{3} W, \] (5)
where $' = d/dy$. However, for some non-supersymmetric solutions, there exists $W$ which satisfies the above relations (4) and (5). This is the case known as the fake supersymmetry [13].

The solutions are obtained under the ansatzs, $\phi_i = \phi_i(y)$. Here we consider both the fake and the true supersymmetric cases. The AdS$_5$ solution is considered as a simple example for the supersymmetric case,
\[ A = 1, \quad \Phi = \Phi_0, \quad \alpha = 0, \] (6)
where $\Phi_0$ is a constant and $W = -3/2$. As for the fake-supersymmetric case, we adopt the following solution [13, 8],
\[ A = \left( 1 - \left( \frac{r_0}{r} \right)^8 \right)^{1/4}, \quad e^\Phi = \left( 1 + \frac{(r_0/r)^4}{1 - (r_0/r)^4} \right)^{\sqrt{3}/2}, \quad \alpha = 0, \] (7)
where $r_0$ is a constant and
\[ W = -\frac{3}{2} \cosh\left( \sqrt{\frac{8}{3}} \Phi \right). \] (8)

The solutions of this 5d theory can be easily lifted up to 10d. However, when the other scalars are added and they take some non-trivial configurations, it is non-trivial

---

*In the following, we set as $V = -3$ for $\kappa^2_5 = 2$ and $L = 1$.
†The gauge coupling parameter $v$ is fixed from the AdS$_5$ vacuum [8] by fixing the radius of AdS as a unit length. Here we set as $v = -2$. 

2
to lift up the solutions to 10d, and $S^5$ has been deformed to a complicated geometry [9]. In this case, it is not simple to embed the D7 brane in this 10d background.

The brane-world solution is obtained by solving the following system,

$$S = S_g + S_b,$$

where $S_b$ is the action of the RS-brane. Here, we set $S_b$ as follows,

$$S_b = -v \int d^4x dy \sqrt{-g}W(\phi_i)\delta(y - y_b).$$

where $y_b$ denotes the brane position. In solving the equations of motion of $S_g + S_b$, the role of the brane action is to give the boundary conditions for the bulk solution of $S_g$. Meanwhile, these conditions for $\Phi$ and $A$ are equivalent to the above equations (9) evaluated just on the point $y = y_b$. Then, at any point of $y_b$, all the solutions of (9) satisfy the boundary conditions given by $S_b$ of the form of (10). So it is enough to solve equations (9) to obtain the brane-world solution which satisfies the boundary conditions at $y_b$. In other words, it is possible to put the brane at any point on the $y-$axis. This situation is not changed even if the “superpotential” $W$ is given by (8) with the fake-supersymmetric solution given above.

3 Reduced D7-brane and its embedding

In [8], we have given an explicit embedding of D7-brane as a probe in the 10d background (11). Rewrite the metric (11) as

$$ds_{10}^2 = \frac{r^2}{L^2} A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^2}{r^2} \left( dr^2 + r^2 d\Omega_3^2 \right)$$

$$= \frac{r^2}{L^2} A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^2}{r^2} \left( \sum_{m=4}^7 (dX^m)^2 + \sum_{i=8}^9 (dX^i)^2 \right),$$

and the four dimensional coordinates $\{X^m\}$ are further rewritten in the polar coordinate as

$$\sum_{m=4}^7 (dX^m)^2 = d\rho^2 + \rho^2 d\Omega_3^2,$$

where $\rho^2 = \sum_{m=4}^7 (X^m)^2$. Then, the world-volume of the D7-brane is taken as $\{x^\mu, \rho, \Omega_3\}$. As mentioned above, the D7-brane is wrapping on the contractible $S^3$, and we notice the relation $r^2 = \rho^2 + (X^8)^2 + (X^9)^2$ in this case.

The remaining part of the embedding procedure is to determine the shape of the D7-brane in its outer two dimensional space, $(X^7, X^8)$. This is performed by solving the equations of motion of two scalar fields in the D7-brane action, which is written as

$$S_{D7} = -\tau_7 \int d^8\xi \left( e^{-\Phi} \sqrt{G} + \frac{1}{8!} e^{i_1...i_8} A_{i_1...i_8} \right),$$

3
Table 1: Coordinate conventions

|     | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|---|---|---|---|---|---|---|---|---|---|
| RS-brane |   |   |   |   |   |   |   |   |   |   |
| $x^\mu$ | $r$, $\Omega_5$ |   |   |   |   |   |   |   |   |   |
| D7-brane | $x^\mu$ | $\rho$, $\Omega_3$ | $X^i$ |   |   |   |   |   |   |   |

where $\mathcal{G} = -\det(G_{i,j})$, $i, j = 0 \sim 7$. $G_{i,j}$ denotes the induced metric and $\tau_7$ is the tension of D7 brane. The eight form potential $A_{i_1 \cdots i_8}$, which is Hodge dual to the axion, couples to the D7 brane minimally. It is obtained as $F_{(9)} = dA_{(8)}$ in terms of the Hodge dual field strength $F_{(9)}$ \[10\]. We take the gauge, $\xi_i = \{x^\mu$, $\rho$, $\theta^j\}$, where $\theta^j$ ($j = 1 \sim 3$) denote the angle variables of $S^3$, and we make an ansatz for the two scalar fields $X^8(\xi)$ and $X^9(\xi)$ as $X^9 \equiv w(\rho)$ and $X^8 = 0$, without loss of generality. Then, the induced metric and the explicit form of the action \[13\] are obtained as \[8\]

$$ds_8^2 = e^{\Phi/2} \left( \frac{r^2}{L^2} A(r)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1 + (w')^2}{r^2} L^2 d\rho^2 + \frac{\rho^2}{r^2} L^2 d\Omega_3^2 \right).$$ (14)

$$S_{D7} = -\tau_7 \int d^8\xi \sqrt{\epsilon_3} \left( L_{D7}^{\text{cl}} + L_{D7}^{\text{fluc}} \right),$$ (15)

$$L_{D7}^{\text{cl}} = \rho^3 \left( e^\Phi A(r)^4 \sqrt{1 + (w')^2 + C_8(r)} \right)$$ (16)

where $C_8(r)$ denotes the contribution of the eight form potential, and the fluctuations of D7 around its classical configuration are included in $L_{D7}^{\text{fluc}}$.

Here $w(\rho)$ is solved independently of the angle variables $\theta^i$ on $S^3$, so the form of the solution $w$ is preserved even if the action was reduced to 5d by integrating over $\theta^i$. As a result, we can reduce the above action as

$$S_{D7}^{(5d)} = -2\pi^2 \tau_7 \int d^4x^\mu d\rho \left( L_{D7}^{\text{cl}} + L_{D7}^{\text{fluc}} \right).$$ (17)

Thus we obtain the D7-brane action which is dimensionally reduced to the 5d. Here we notice the relation, $r^2 = \rho^2 + w^2(\rho)$, for $r$ and $\rho$. We preserve the interpretation of $r$ or $\rho$ as the energy scale of the field theory on the brane, so we select the solution of $w(\rho)$ such that it could give the one to one correspondence between $r$ and $\rho$ \[8\].

We are now ready to embed the reduced D7-brane in the 5d brane-world. As done in 10d, it is embedded as a probe such that the given background configuration is not altered. This is performed by solving the equation of motion for $w$, which is obtained from $S_{D7}$ only in the brane-world background. However we impose $Z_2$ symmetry with respect to the coordinate $y$ as,

$$w(y) = w(|y - y_b|).$$ (18)
Then the equation of \( w \) is obtained as

\[
\tau_7 \left( \delta w \frac{L_{D7}^{cl}}{F(\rho)} - \frac{2w' G(\rho)}{(1 + w' \rho)^{3/2}} \delta (\rho - \rho_b) \right) = 0,
\]

(19)

where \( w' = dw/d\rho \), \( \delta w \frac{L_{D7}^{cl}}{F(\rho)} = \partial w \frac{L_{D7}^{cl}}{F(\rho)} - \partial_\rho \left( \partial w' \frac{L_{D7}^{cl}}{\rho} \right) \), \( F(\rho) = \rho^3 e^\Phi A^4 \) and \( G(\rho) = (\rho + w^2)/\rho^2 \). And \( \rho_b \) denotes the position of the brane on the coordinate \( \rho \), defined as \( e^{-2 y_b/L} (= \tau_b^2) = \rho_b^2 + w^2(\rho_b) \). The \( \delta \)-function of Eq. (19) comes from the \( Z_2 \) symmetry at \( y = y_b \), and the factor \( G(\rho) \) appears when the \( \rho \)-derivative is changed to the \( y \)-derivative. For \( \rho < \rho_b \), the field equation for \( w \), \( \delta w \frac{L_{D7}^{cl}}{F(\rho)} = 0 \), is written as

\[
w'' + \frac{3}{\rho} w' + 2 K(1)(\rho w' - w) = 0
\]

(20)

where \( K(1) = \partial_x \log(e^\Phi A^4) \), and the boundary condition,

\[
w'(\rho_b) = 0.
\]

(21)

For the super-symmetric background \( \square \), \( K(1) = 0 \). Then we find the solution \( w'(\rho) = 0 \) \((w = \text{const.})\) which preserves the super-symmetry of the system \( \square \), and the condition (21) is satisfied at any \( \rho \). While, for the non-supersymmetric background \( \square \), \( K(1) \neq 0 \), then \( w'(\rho) \neq 0 \) and the boundary condition is not satisfied except at \( \rho = 0 \) and \( \rho = \infty \) since \( w(\rho) \) decreases monotonically with \( \rho \). For finite \( \rho_b \), then, the condition (21) selects the super-symmetric solution. And the RS brane in the non-supersymmetric background is pushed toward the boundary, \( \rho_b = \infty \).

As another possible boundary condition, we may consider \( G(\rho_b) = 0 \), and we get

\[
w(\rho_b)w'(\rho_b) + \rho_b = 0.
\]

(22)

This is equivalent to \( \partial_\rho r(\rho_b) = 0 \). Meanwhile \( r(\rho) \) must increase monotonically with \( \rho \) since it should be a single valued function of \( \rho \) due to the requirement of the one to one correspondence between \( r^2(= \rho^2 + w(\rho)^2) \) and \( \rho \). Thus, the points which satisfy the condition, \( \partial_\rho r(\rho_b) = 0 \), are again restricted to \( \rho_b = 0 \) and \( \rho_b = \infty \). Then this condition is not useful as the boundary condition. By the same reason, \( F(\rho_b) = 0 \) is also rejected as a useful boundary condition. After all, we find that only the supersymmetric solution satisfies the meaningful boundary condition (21).

Here we remember the asymptotic form of \( w \) at large \( \rho \),

\[
w(\rho) = m_q + c/\rho^2, \quad c = -\langle \bar{\Psi} \Psi \rangle
\]

(23)

and the meaning of the parameters in it. Namely \( m_q \) represents the quark-mass and \( c \) is the vacuum expectation value of bi-linear operators of quark fields \( (\Psi) \) in the dual gauge theory. Then we can see \( \langle \bar{\Psi} \Psi \rangle = 0 \) and \( w = m_q \) in the supersymmetric case. As a result, the chiral symmetry is preserved.

In the above analysis, we have ignored the interaction between D7 and RS brane since D7 should be treated as a probe. However, the situation is not changed even if we
solve the equation of $w$ by taking into account of the brane action as, $S_{D7} + S_b$, since $S_b$ does not depend on $w$. In other words, the shape of the D7-brane is independent of the RS brane. So we need the back-reaction from the bulk in order to change the situation of the embedding. But this is out of the present work.

4 Meson localization

In the next, we study the fluctuation-modes of the embedded D7 brane. Some of them are trapped on the RS brane, and they might be observed as mesons in our 4d world. Then, the trapped modes are defined as the normalizable one for the integration over $\rho$.

4.1 For finite $\rho_b$

In this case, only the supersymmetric embeddings are allowed. So the background is given by (6), and the quadratic parts of the fluctuations, $\phi_8 = X^8$, $\phi_9 = X^9 - w$ and vector, are written as

$$L_{D7}^{(2)} = \rho^3 \left\{ \frac{1}{2} \rho^2 \sum_{i=8,9} (\partial \phi_i)^2 + F_{ab} F^{ab} + \cdots \right\}$$

where $w$ is a constant, $w(\rho) = w(\infty) \equiv m (= m_q/2\pi)$. The dots denote the higher order terms which can be neglected here.

For the scalar, we obtain the same field equation for $\phi^8$ and $\phi^9$, then they are denoted by $\phi$ for the simplicity. The field-equation is written as

$$\partial_\rho^2 \phi + \frac{3}{\rho} \partial_\rho \phi = -\frac{M^2 R^4}{(m^2 + \rho^2)^2} \phi,$$

where $M^2$ is defined as $\eta^{\mu\nu} \partial_\mu \partial_\nu \phi = M^2 \phi$. In the present case, $\phi$ is $Z_2$ symmetric at the brane position $y_b$, so we must solve the equation (25) by imposing the following boundary condition,

$$\phi'(\rho_b) = 0.$$  

Before studying the trapping in the brane-world by using the boundary condition (26), we consider the normalizable modes which could be observed at the boundary $\rho_b = \infty$. This analysis is useful for finding the localized modes on the brane since the mass eigen-values of the localized modes on the branes at different $\rho_b$ should be related to each other by the smooth functions of $\rho_b$ in all the region $0 < \rho_b < \infty$. In this sense, the spectra, which should be observed on the boundary, are considered as the one at the starting point.

The Eq.(26) is solved as

$$\phi = (\rho^2 + m^2)^{-\alpha} \left( c_1 F(-\alpha, -\alpha + 1, 2; -\rho^2/m^2) + c_2 \rho^{-2} F(-\alpha - 1, -\alpha, 0; -\rho^2/m^2) \right).$$

(27)
where \( c_1 \) and \( c_2 \) are arbitrary constants, and \( \alpha = (1 + \sqrt{1 + M^2 L^4/m^2})/2 \). In order to find the trapped modes, consider the normalizability condition for \( \phi \),
\[
\int_0^{\rho_b} d\rho \, \rho^3 \left( \frac{L^2}{\rho^2 + m^2} \right)^2 \phi^2(\rho) < \infty.
\] (28)

We estimate this condition for (i) \( m \neq 0 \) and (ii) \( m = 0 \) separately. For the case of (i), the above integral near \( \rho \sim 0 \) is approximately estimated as
\[
\int_0 d\rho \, \rho^3 \phi^2(\rho) < \infty.
\] (29)

Meanwhile, the solution (27) is expanded near \( \rho \sim 0 \) as,
\[
\phi = c_1 (1 + c_1^1 \rho^2 + \cdots) + c_2 \rho^{-2}(1 + c_2^1 \rho^2 + \cdots),
\] (30)
where \( c_1^1, c_2^1 \) are the calculable coefficients. Then we find that the solution of \( c_2 = 0 \) satisfies (29). However, for the case of (ii), the condition (29) is replaced by
\[
\int_0 d\rho \, \rho \phi^2(\rho) < \infty.
\] (31)

Then we must take as \( c_1 = c_2 = 0 \), namely \( \phi = 0 \), to satisfy (31). In other words, there is no localized state in the case of \( m = 0 \) for the supersymmetric case. So the interesting case is restricted to the case of massive quark.

On the other hand, at large \( \rho \), the normalizability condition (28) is approximated for any \( m \) as,
\[
\int_0^\infty \frac{d\rho}{\rho} \phi^2(\rho) < \infty.
\] (32)

And the solution of \( c_2 = 0 \), the first term of (27), is expanded at large \( \rho \) as,
\[
\frac{\phi}{c_1} = b_0(\alpha)(1 + b_1(\alpha)/\rho^2 + \cdots) + d_0(\alpha)\rho^{-2}(1 + d_1(\alpha)/\rho^2 + \cdots),
\] (33)
where the coefficients \( b_0(\alpha) \) and \( d_0(\alpha) \) are the functions of \( \alpha \). From (32), we demand \( b_0(\alpha) = 0 \). As a result, we get \( \alpha = n + 1 \) with the integer \( n \) [14], and we thus find infinite series of discrete meson mass.

We now return to the case of the brane-world. In this case, \( \rho_b \) is finite and we need the boundary condition (26) at \( \rho_b \) instead of the normalizable condition at large \( \rho \) (32) given above. In order to satisfy (26), \( b_0(\alpha)/d_0(\alpha) \) should be fixed to an appropriate value \( f \), which depends on \( \rho_b \),
\[
b_0(\alpha)/d_0(\alpha) = f(\rho_b),
\] (34)
and \( f \) should satisfy the condition, \( f(\infty) = 0 \). From this equation, we could find the mass eigen-values. Here, we can see this statement by a numerical analysis. For different three values of \( \rho_b \), the value of \( \phi' \) as a function of \( \alpha \) is shown in the Fig.1. We find that the values of \( \alpha \) at each zero point of \( \phi' \) shift to larger \( \alpha \) side from the integer point, which are given by \( \alpha = n + 1 \) for \( \rho_b = \infty \), smoothly when \( \rho_b \) decreases.

We could conclude as follows. Also on the RS brane, in the present supersymmetric braneworld, we can see the meson mass-spectra, which shift to the massive side when the brane moves to the smaller \( \rho_b \).
Fig. 1: $\phi'$ vs $\alpha$ for $m = 1$. The curves a, b and c represent $\{\phi' \times 10^9, \rho_b = 10^3\}$, $\{\phi' \times 10^6, \rho_b = 10^2\}$ and $\{\phi' \times 10^3, \rho_b = 10\}$ respectively.

### 4.2 Non-supersymmetric background

For non-supersymmetric background, we must take $\rho_b = \infty$. In this case, the field equations for $\phi^8$ and $\phi^9$ are complicated, but the equations for both fields have the same asymptotic solution with the one given for the supersymmetric case by the equation (30) at small $\rho$ and (33) at large $\rho$ respectively. In the present case, however, the normalizable condition has a different form from (28) and it is given as

$$\int_0^\infty d\rho \rho^3 e^{\Phi} \left( \frac{L^2}{Ar^2} \right)^2 \phi(\rho)^2 < \infty$$

where $\phi$ denote $\phi^8$ or $\phi^9$.

In the small $\rho$ region, we find the following approximated condition

$$\int_0 d\rho \rho^3 \phi(\rho)^2 < \infty.$$  \hspace{1cm} (36)

It should be noticed that this condition is not changed even if $m = 0$ because, in the limit of $\rho \to 0$, we find

$$r^2 = \rho^2 + w^2(\rho) \to w^2(0) \neq 0$$

for the non-supersymmetric case and that $e^\Phi$ and $A$ approach to the constant [8]. This is an important point to be discriminated from the supersymmetric case. And (36) is satisfied by the first one of the two independent solutions given in (30). Then we expect to find normalizable modes or the meson spectra on the RS brane for any value of $m$.

At large $\rho$, (33) is approximated by (32), and the asymptotic solution for $\phi$ is also the same form with (33). Then, we can choose the second series of (33) as the normalizable solution, and many number of normalizable modes are found for both $\phi^8$ and $\phi^9$. Especially, for the case of $m = 0$, we find one mass-less mode for $\phi^8$ as the Nambu-Goldstone boson due to the spontaneous chiral symmetry breaking. Actually, these points have been shown in [15, 16] when the RS brane is absent. In
the present case, from the $Z_2$ symmetry, we need new conditions for the modes are 
$\partial_\rho \phi^8|_{\rho = \infty} = \partial_\rho \phi^9|_{\rho = \infty} = 0$. And all the normalizable solutions satisfy this. Then it would be straightforward to find the meson-spectra on the RS brane.

5 Summary

Here, we consider a class of background solutions, $M_5 \times S^5$, of IIB superstring theory in order to construct a braneworld in which mesons are trapped on the RS brane. The brane-world is set in the uncompactified 5d part $M_5$ of these solutions. Its effective action can be obtained from the 10d theory by integrating over the inner compact space, $S^5$. They are also obtained from the 5d gauged supergravity with the dilaton and the axion. Any solution of this 5d theory can be easily lifted up to the one of the 10d theory on $M_5 \times S^5$. On the same footing, the action of the D7-brane is reduced to 5d by integrating over $S^3$, which is regarded as the inner compact space of the D7-brane here.

After setting the stable RS brane, which can be put at any position of the fifth coordinate $\rho_b$ we like, for the 5d background given here, the reduced D7 brane is embedded in this background as a probe in the sense that the D7 brane does not change the given 5d configuration. The embedded form is obtained by solving the scalar field equation of the D7 brane action. For finite $\rho_b$, only the supersymmetric embedding is allowed and the quark mass is arbitrary in this case. The chiral symmetry of the corresponding gauge theory is then preserved for the mass-less quark. When the quark mass is finite, the series of meson mass-spectra are observed as trapped states on the RS brane.

In the non-supersymmetric case, on the other hand, the brane is retained on the boundary, $\rho_b = \infty$. In this case, the situation is similar to the case of 10d gauge/gravity correspondence. We can then observe the spontaneous chiral symmetry breaking and infinite series of the trapped mesons on the RS brane. And we assure the existence of the Nambu-Goldstone boson, which is generated as a result of the spontaneous chiral symmetry breaking.

In any case, we could show a model of a braneworld which includes flavor quarks and their bound states on the RS brane. It would be an interesting problem to study the brane-world cosmology including hadrons in terms of the model presented here. We will discuss on this point in the near future.

Acknowledgments: The author would like to thank M. Tachibana and M. Yahiro for useful discussion. This work has been supported in part by the Grants-in-Aid for Scientific Research (13135223) of the Ministry of Education, Science, Sports, and Culture of Japan.
References

[1] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, JHEP 9812 (1998) 022; [hep-th/9810126], JHEP 9905 (1999) 026; [hep-th/9903026], Nucl. Phys. B 596, 451 (2000) [hep-th/9909047].

[2] J. Distler and F. Zamora, Adv. Theor. Math. Phys. 2 (1998) 1405; [hep-th/9810206].

[3] D. Freedman, S. Gubser, K. Pilch, and N. Warner, Adv. Theor. Math. Phys. 3, 363 (1999) [hep-th/9904017].

[4] R. Apreda, D. E. Crooks, N. Evans and M. Petrini, JHEP 0405 (2004) 065 [hep-th/0308006].

[5] L. Randall and R. Sundrum, Phys. Rev. Lett. B83, 4690 (1999), [hep-th/9906064].

[6] A. Karch and E. Katz, JHEP 0206, 043(2003) [hep-th/0205236].

[7] P. Breitenlohner and Z.D. Freedman, Ann. Phys. 144 (1982)249; Phys. Lett. B115(1982)197.

[8] K. Ghoroku and M. Yahiro, Phys. Lett. B604, 235 (2004) [hep-th/0408040].

[9] K. Pilch and N.P. Warner, Phys. Lett. B487 (2000) 22; [hep-th/0002192], Nucl. Phys. B594 (2001) 209; [hep-th/0004063], Adv.Theor.Math.Phys. 4 (2002) 627; [hep-th/0006066].

[10] G. W. Gibbons, M. B. Green and M. J. Perry, Phys.Lett. B370 (1996) 37-44, [hep-th/9511080].

[11] K. Ghoroku, Phys. Rev. D. 69 (2004) 084003, [hep-th/0310060].

[12] K. Skenderis and P.K. Townsend, Phys. Lett. B468 (1999) 46; [hep-th/9909070].

[13] D. Z. Freedman, C. Nunez, M. Schnabl and K. Skenderis, Phys. Rev. D69 (2004) 104027 [hep-th/0312055].

[14] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, JHEP 0307, 049(2003) [hep-th/0304032].

[15] I. Brevik, K. Ghoroku, and A. Nakamura, [hep-th/0505057].

[16] J. Babington, J. Erdmenger, N. Evans Z, Guralnik and I. Kirsch, Phys. Rev. D69 (2004) 066007 [hep-th/0306018].