Lessons from cosmic history: the case for a linear star formation – \( \text{H}_2 \) relation

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ABSTRACT

Observations show that star formation in galaxies is closely correlated with the abundance of molecular hydrogen. Modelling this empirical relation from first principles proves challenging, however, and many questions regarding its properties remain open. For instance, the exact functional form of the relation is still debated and it is also unknown whether it applies at \( z > 4 \), where carbon monoxide observations are sparse. Here, we analyse how the shape of the star formation–gas relation affects the cosmic star formation history and global galaxy properties using an analytic model that follows the average evolution of galaxies in dark matter haloes across cosmic time. We show that a linear relation with an \( \text{H}_2 \) depletion time of \( \sim 2.5 \) Gyr, as found in studies of nearby galaxies, results in good agreement with current observations of galaxies at both low and high redshift. These observations include the evolution of the cosmic star formation rate density, the \( z \sim 4–9 \) UV luminosity function, the evolution of the mass–metallicity relation, the relation between stellar and halo mass, and the gas-to-stellar mass ratios of galaxies. In contrast, the short depletion times that result from adopting a highly super-linear star formation–gas relation lead to large star formation rates, substantial metal enrichment (\( \sim 0.1 \) \( Z_\odot \)) and low gas-to-stellar mass ratios already at \( z \gtrsim 10 \), in disagreement with observations. These results can be understood in terms of an equilibrium picture of galaxy evolution in which gas inflows, outflows and star formation drive the metallicities and gas fractions towards equilibrium values that are determined by the ratio of the accretion time to the gas-depletion time. In this picture, the cosmic modulation of the accretion rate is the primary process that drives the evolution of stellar masses, gas masses and metallicities of galaxies from high redshift until today.

Key words: galaxies: evolution – galaxies: high-redshift – galaxies: ISM – galaxies: star formation.

1 INTRODUCTION

The relation between the star formation rate (SFR) per unit area, \( \Sigma_{\text{SFR}} \), and the gas surface density (of either molecular or neutral gas) has been a focal point of many galaxy evolution studies (Schmidt 1959, 1963; Kennicutt 1989, 1998; Wong & Blitz 2002; Gao & Solomon 2004; Heyer et al. 2004; Krumholz & McKee 2005; Krumholz & Thompson 2007; Bigiel et al. 2008; Robertson & Kravtsov 2008; Gnedin, Tassis & Kravtsov 2009; Krumholz, McKee & Tumlinson 2009b; Daddi et al. 2010a,b; Genzel et al. 2010; Gnedin & Kravtsov 2010, 2011; Ostriker, McKee & Leroy 2010; Schruba et al. 2010; Bigiel et al. 2011b; Feldmann, Gnedin & Kravtsov 2011, 2012; Narayanan et al. 2011; Rahman et al. 2011, 2012; Lada et al. 2012). This relation holds over several orders of magnitude in \( \Sigma_{\text{SFR}} \) with a relatively small scatter if measured on ~kpc-sized patches of galaxies or for whole galaxies. Yet, despite these efforts, we still do not quite understand how this correlation comes about and, embarrassingly, what the precise functional form of this relation is.

The reason for the latter is that neither SFRs nor molecular hydrogen (\( \text{H}_2 \)) masses are directly accessible in extragalactic observations. SFRs are inferred from a combination of ultraviolet (UV) and infrared (IR) observations, in order to capture the energy output of both obscured and unobscured star formation. Cold molecular gas is measured indirectly from emission lines of tracer molecules, such as carbon monoxide (CO). While observational studies of nearby galaxies and galaxies out to \( z \sim 3 \) seem to show that the relation between \( \Sigma_{\text{SFR}} \) and \( \Sigma_{\text{HI}} \) is close to linear (Bigiel et al. 2008; Daddi et al. 2010a,b; Genzel et al. 2010), there is no clear theoretical understanding why this is so and, furthermore, there is the concern that systematics in the observables bias these results. For...
instance, it has been suggested that the star formation–gas relation is super-linear with a power-law exponent of $\sim 1.4$ on kpc scales, but that IR emission uncorrelated to recent star formation reduces the inferred slope, see Liu et al. (2011), but cf. Rahman et al. (2011). Alternatively, systematic variation of the CO/H$_2$ conversion factor with CO intensity, SFRs or gas surface density could potentially disguise a highly super-linear relation and make it appear linear (Narayanan et al. 2011, 2012a), cf. Feldmann et al. (2012). Finally, the fitting itself has been targeted as a potential source of bias. A Bayesian re-analysis of the data presented by Bigiel et al. (2008) results in tentative evidence that the star formation–gas relation is mildly sub-linear and shows substantial variations from one galaxy to the next (Shetty, Kelly & Bigiel 2013).

Theoretical explanations of the star formation–gas connection predict a variety of non-linear relationships, see e.g. Krumholz & McKee (2005), Krumholz et al. (2009b), Ostriker et al. (2010), Ostriker & Shetty (2011) and Krumholz, Dekel & McKee (2012). For instance, Krumholz et al. (2009b) estimate that the power-law exponent is mildly non-linear ($\sim 1.33$) at high gas surface densities ($\gtrsim 100$ M$_\odot$ pc$^{-2}$), while Ostriker & Shetty (2011) predict a much steeper, quadratic relation at such gas surface densities. This gas surface density regime is clearly highly relevant because it corresponds to typical gas surface densities of star-forming galaxies at $z \gtrsim 1$ and higher (e.g. Genzel et al. 2010, see also Section 3.2). A significantly super-linear star formation–gas relation has also been suggested by recent work based on (non-cosmological) galaxy evolution simulations that include $\sim$pc-scale modelling of various stellar feedback channels (Hopkins, Quataert & Murray 2011; Hopkins et al. 2013).

In this paper, we take a very different approach to put constraints on the functional form of the star formation–gas relation. We study how global galaxy properties vary depending on the chosen star formation–gas relation and compare our model predictions with observations. In particular, we show that the evolution of the cosmic SFR density (SFRD; Lilly et al. 1996; Madau et al. 1996; Steidel et al. 1999) at $z \sim 4$–10 (Bouwens et al. 2007, 2012a; Behroozi, Wechsler & Conroy 2012) is highly sensitive to the adopted form of the star formation–gas relation, and, hence, that it can be used to test its functional form.

The paper is organized as follows. In Section 2, we describe the analytic model which underlies our predictions and present the star formation–gas relations that enter our analysis. In Section 3, we first demonstrate that the model reproduces the observed stellar mass–metality relation, the stellar mass–halo mass relation and the gas-to-stellar mass ratios of galaxies reasonably well if a linear star formation–gas relation is chosen. We show that predictions based on a strongly super-linear star formation–gas relation are in conflict with observations. We then present how the cosmic star formation history is affected by the functional form of the star formation–gas relation. Potential caveats of our approach are discussed in Section 3.3. In Section 4, we put our findings in a broader context, showing that galaxy evolution can be thought of as an evolution along a line of equilibrium points driven by the cosmic modulation of the accretion rate. We summarize our findings in Section 5 and close the paper with a comparison to previous work.

2 THE MODEL

The analytic model that is used in the remainder of the paper follows closely the approach presented by Bouché et al. (2010) and Krumholz & Dekel (2012). At its most basic level, the model tracks the average evolution of the dark matter, gas and stellar content of haloes using theoretically or observationally motivated accretion and transformation rates for the different mass components. For the convenience of the reader, we begin by summarizing the main equations that enter the model and refer the reader to Krumholz & Dekel (2012) for more details. We then discuss the various star formation models that we explore in this work and conclude the section with a short description of the actual set-up of the model.

In the following, we adopt cosmological parameters consistent with Wilkinson Microwave Anisotropy Probe 7-year data (Komatsu et al. 2011): $\Omega_m = 0.27$, $\Omega_b = 0.046$, $\Omega_\Lambda = 0.73$, $h = 0.7$ and $\sigma_8 = 0.81$.

2.1 Basic equations

Accretion rates. The total (baryon plus dark matter) accretion rate at time $t$ on to a halo of a given mass $M_{\text{halo}}$ is (Neistein, van den Bosch & Dekel 2006; Neistein & Dekel 2008; Krumholz & Dekel 2012)

$$M_{\text{halo}} = 1.06 \times 10^{12} M_\odot \left( \frac{M_{\text{halo}}}{10^{12} M_\odot} \right)^{1.14} \frac{D(t)}{D(t_0)}^2,$$  

(1)

Here, $D(t)$ is the linear growth factor normalized to unity at the present epoch. The (average) evolution of the mass of a halo can be computed from its initial mass and the time- and mass-dependent accretion rate (equation 1). The corresponding accretion rates of the gas and stellar components are

$$M_{\text{f,in}} = \epsilon_{\text{in}} f_{\text{g,in}} f_0 M_{\text{halo}},$$  

(2)

$$M_{\text{b,in}} = (1 - f_{\text{g,in}}) f_0 M_{\text{halo}}.$$  

(3)

Here, $\epsilon_{\text{in}}$ specifies which fraction of the accreted gas reaches the galaxy at the centre of the halo, $f_0 = \Omega_b/\Omega_m$ is the cosmic baryon fraction and $f_{\text{g,in}}$ is the gas fraction (in units of $f_0$) of the accreted matter. We assume that accretion of gas on to the disc is completely quenched ($\epsilon_{\text{in}} = 0$) when the halo mass is above the virial shock mass $M_{\text{shock}}(z)/10^{12} M_\odot = \max \left(2, \left(10^{12} M_\odot / (3 M_* p_z)\right) \right)$ (Dekel et al. 2009; Krumholz & Dekel 2012). Here, $M_{*, p_z}(z)$ is the Press–Schechter mass at the given redshift, see Mo & White (2002). Accretion is also quenched when the halo mass is so low that photoinization limits the baryon fraction in the haloes to 50 per cent of $f_0$ or less (Hoeft et al. 2006; Okamoto, Gao & Theuns 2008). Specifically, we assume $\epsilon_{\text{in}} = 0$ if the halo mass is below the mass $M_{\text{cut}}(z)$ given in fig. 3 of Okamoto et al. (2008). We adopt $M_{\text{cut}} = 0$ for $z \geq 9$. We assume that accretion is not quenched ($\epsilon_{\text{in}} = 1$) if the halo mass lies between $M_{\text{cut}}$ and $M_{\text{shock}}$. We compute $f_{\text{g,in}}$ as described by Krumholz & Dekel (2012). In brief, we first compute which fraction of the accretion rate is due to accreted haloes of a given mass. The model provides us directly with the gas fraction of a halo of a given mass and we then calculate $f_{\text{g,in}}$ as the accretion-weighted mean gas fraction (in units of $f_0$) of all accreted haloes plus an additional non-halo contribution of 20 per cent.

Evolution of the gas and stellar mass. Besides accretion, the baryonic components are subject to additional physical processes, such as star formation, stellar mass loss and galactic outflows which we consider next. Specifically, we assume that the gas and stellar masses change according to

$\epsilon_{\text{in}}$ Since the model does not use merger trees, it only follows the mean evolution of haloes. For the quantities studied in this paper (the cosmic star formation history and the gas fraction of galaxies), such an average description is adequate.
Here, \( M_{\text{s,form}} \) is the SFR of the galaxy (see below), \( R \) is the fraction of the stellar mass of a single stellar population that is recycled to the interstellar medium (ISM) via, e.g., stellar winds. For the initial mass function (IMF) assumed in this work (Chabrier 2005), \( R = 0.46 \) is appropriate. The model makes the simplifying assumption of instant recycling which is, however, adequate for our purposes (Krumholz & Dekel 2012). The parameter \( \epsilon_{\text{out}} \) is included to model large-scale galactic outflows. We follow Krumholz & Dekel (2012) and assume \( \epsilon_{\text{out}} = 1 \), i.e. a gas outflow rate equal to the SFR, based on the estimates by Erb (2008). We discuss the impact of varying this and other model parameters in Section 3.3.

The gas disc. For the modelling of the star formation (see below) we need to know how the gas is distributed in the galaxy. We make the ansatz that the gas forms an exponential disc, see e.g. Bigiel & Blitz (2012),

\[
\Sigma_g(r) = \Sigma_g(0) \exp^{-r/(R_d)}.
\]

We assume that the scale radius \( R_d \) is set by the initial, pre-collapse angular momentum of the gas (Mo, Mao & White 1998). Specifically,

\[
R_d = 0.5 R_{\text{vir}},
\]

where \( R_{\text{vir}}(M_{\text{halo}}, \mathcal{z}) \) and \( \lambda = 0.07 \) (Krumholz & Dekel 2012) are the virial radius and (median) spin parameter of the halo, respectively. The pre-factor takes into account that the half-mass radius of an exponential disc is \( \sim 1.7 R_d \). The predictions of equation (7) are in reasonable agreement with observations. Bigiel & Blitz (2012) find a mean scale radius of 6.1 ± 0.7 kpc (with significant galaxy-to-galaxy variations) for a sample of roughly Milky Way-(MW)-like galaxies in the local Universe, while equation (7) predicts \( R_d \sim 9 \) kpc for a \( 10^{12} M_{\odot} \) halo at \( \mathcal{z} = 0 \). Reducing \( \lambda \) to a more traditional value \( \lambda \sim 0.04-0.05 \) (Barnes & Efstathiou 1987; Bullock et al. 2001; Dutton et al. 2011) would lead to even better agreement, see also Section 3.3.

Molecular hydrogen. The abundance of molecular gas plays an important role in many of the star formation models that we consider in the paper (see below). We compute the \( \text{H}_2 \) surface density \( \Sigma_{\text{H}_2} = f_{\text{H}_2} \Sigma_g \) following Krumholz, McKee & Tumlinson (2008, 2009a):

\[
f_{\text{H}_2}(\Sigma_g, Z) = \begin{cases} 1 - \frac{0.75}{1 + 0.357} \rho / \Sigma_g , & \text{if } \rho < Z / \Sigma_g , \\ 0, & \text{otherwise} \end{cases}
\]

\[
\rho = \frac{\ln(1 + 0.6 \chi + 0.01 \chi^2) - 0.6 \tau_c}{\chi}
\]

\[
\chi = \frac{3.1}{4.1} (1 + 3.1(Z/Z_{\odot})^{0.365})
\]

\[
\tau_c = 320 c \frac{\Sigma_z}{g \text{ cm}^{-2} \text{ cm}^{-3}} Z / Z_{\odot},
\]

Here, \( Z = M_{\text{Z}/M_\odot} \) is the average metallicity of the gas and we effectively assume that there are no strong metallicity gradients. We note that gas discs in nearby galaxies often show a moderate radial metallicity gradient of the order of \( \sim 0.05-0.1 \) dex per kpc (Zaritsky, Kennicutt & Huchra 1994), corresponding to a factor of \( \sim 1.5-4 \) decrease in oxygen abundance from the centre to the optical edge of the disc (Moustakas et al. 2010). We discuss how metallicity gradients affect our results in Section 3.3.

Metallicity evolution. The dependence of the \( \text{H}_2 \) abundance on the metallicity\(^2\) of the ISM requires that we keep track of the build-up of metals in each galaxy. In the instantaneous recycling approximation and ignoring inflows and outflows, the total metal production rate is proportional to both the yield \( y \) of the stellar population and the SFR, i.e. \( \dot{M}_{\text{Z,form}} = y(1 - R)M_{\text{s,form}} \). Metals that are locked in stellar remnants do not contribute to the metallicity of the ISM, which therefore evolves according to \( M_Z = y(1 - R)M_{\text{s,form}} - Z(1 - R)M_{\text{s,form}} \). As suggested by Krumholz & Dekel (2012), we add three processes to the basic picture. First, a fraction \( \zeta (M_{\text{halo}}) = 0.9 \exp (-M_{\text{halo}}/3 \times 10^{12} M_{\odot}) \) of the newly produced metals (predominantly in a hot, shocked phase) is removed from small haloes by supernovae driven outflows. Secondly, cold and warm gas of the ISM and the associated metals are lost by galactic winds at a rate \( \epsilon_{\text{out}}(M_Z/M_\odot)M_{\text{s,form}} \). Thirdly, inflows from the intergalactic medium (IGM) add metals at the rate \( Z_{\text{IGM}}M_{\text{g,inf}} \). Hence, the total metal mass of a galaxy changes according to

\[
M_Z = \left[ y(1 - R)(1 - \zeta) - (1 - R + \epsilon_{\text{out}})M_Z/M_\odot \right] M_{\text{s,form}}
\]

\[
+ Z_{\text{IGM}}M_{\text{g,inf}}
\]

A single pair instability supernova can enrich haloes at high redshifts to metallicities \( \sim 2 \times 10^{-5} \) (Wise et al. 2012). We therefore adopt the value \( Z_{\text{IGM}} = 2 \times 10^{-5} \) as the metallicity of both the initial gas and the gas accreted on to haloes.

Differences to Krumholz & Dekel (2012). Our implementation of the model follows closely the approach presented by Krumholz & Dekel (2012). The only noteworthy differences are the following.

(i) We added a mass threshold below which gas accretion is quenched due to photoionization.

(ii) The metallicity evolution of the ISM (equation 12) accounts for the locking-up of metals in stellar remnants. Mathematically this amounts to replacing \( \epsilon_{\text{out}} \) in equation 23 of Krumholz & Dekel (2012) with \( \epsilon_{\text{out}} + (1 - R) \) and, hence, is equivalent to a change of the parameter \( \epsilon_{\text{out}} \). We show in Section 3.3 that varying \( \epsilon_{\text{out}} \) by a factor of a few does not strongly affect the SFRDs at \( \mathcal{z} > 2 \).

(iii) We use a variety of different star formation–gas relations (see the next section).

2 More precisely, it is the dust-to-gas ratio that regulates the abundance of \( \text{H}_2 \). We assume that the dust-to-gas ratio scales proportional with metallicity as observed for galaxies enriched to \( Z \gtrsim 0.1 Z_{\odot} \) (Leroy et al. 2011).
cases do not add anything qualitatively new, but rather result in intermediate predictions for galaxy properties. Hence, much can be learned already from a study of the extreme cases. Finally, both a linear and a quadratic star formation–gas relation are supported by various lines of evidence. It is thus clearly of high interest to test their specific implications for the evolution of galaxy properties.

An approximately linear relation is empirically motivated by observational studies of the $\Sigma_{\text{SFR}}-\Sigma_{\text{H}_2}$ relation in the local universe (Bigiel et al. 2008, 2011) and out to, at least, $z \approx 2$ (Genzel et al. 2010). The depletion time in nearby galaxies is $\sim 2-3$ Gyr (Bigiel et al. 2011), but whether this is also true at high redshift is not so clear, see e.g. Tacconi et al. (2013). High-redshift samples typically include only galaxies with large SFRs and gas masses, and, hence, in the case of a slightly super-linear star formation–gas relation it is difficult to distinguish a change in the depletion time with redshift from a change in depletion time due to the increase in gas surface density. Uncertainties related to the CO/H$_2$ conversion factor and the limited spatial resolution of CO observations further complicate an accurate measurement of the depletion time at high redshift.

Our default star formation–gas relation is thus a linear relation with a constant molecular depletion time of 2.5 Gyr (Bigiel et al. 2011):

$$\Sigma_{\text{SFR}} = \frac{\Sigma_{\text{H}_2}}{2.5 \text{ Gyr}}.$$  \hspace{1cm} (13)

The star formation–gas relation is assumed to hold at all redshifts and for all galaxies. Some galaxies may not follow this relation, e.g. massively starbursting galaxies that lie above the so-called main sequence of star formation, but their contribution to the cosmic star formation budget is estimated to be limited ($\sim 10$ per cent or less, see e.g. Rodighiero et al. 2011).

A quadratic star formation–gas has been proposed based on analytic and numerical models of feedback-regulated star formation, e.g. Ostriker & Shetty (2011). Typically the quadratic relationship is expected to hold only at sufficiently high gas surface densities ($\gtrsim 50 \text{M}_\odot \text{pc}^{-2}$), because at lower surface densities the star formation–gas relation reduces to effectively counting the number of molecular clouds in a beam and, hence, should still follow a linear relation. Some theoretical studies that aim at constraining the conversion factor between CO emission and H$_2$ surface density also support the idea of an intrinsically quadratic star formation–gas relation (Narayanan et al. 2012a, but cf. Feldmann et al. 2012).

We therefore also explore the implications of the following star formation–gas relation that is quadratic at gas surface densities $\gtrsim 50 \text{M}_\odot \text{pc}^{-2}$, but linear at smaller surface densities (Ostriker & Shetty 2011):

$$\Sigma_{\text{SFR}} = \begin{cases} 
0.1 \text{M}_\odot \text{yr}^{-1} \text{kpc}^{-2} \left( \frac{\Sigma_{\text{H}_2}}{50 \text{M}_\odot \text{pc}^{-2}} \right)^2, & \text{if } \Sigma_g \geq 50 \text{M}_\odot \text{pc}^{-2} \\
\frac{\Sigma_{\text{H}_2}}{2.5 \text{ Gyr}}, & \text{if } \Sigma_g < 50 \text{M}_\odot \text{pc}^{-2}.
\end{cases} \hspace{1cm} (14)$$

The $z \gtrsim 3$ predictions of this star formation–gas relation are effectively indistinguishable from the predictions of a relation that extends the quadratic relationship to $\Sigma_g < 50 \text{M}_\odot \text{pc}^{-2}$, because surface densities of star-forming galaxies at high redshift typically exceed $50 \text{M}_\odot \text{pc}^{-2}$, see Section 3.2. In the remainder of this paper, we will refer to the relation given by (equation 14) as the strongly super-linear star formation–gas relation.

In Section 3.2, we also test a few additional star formation–gas relations suggested in the literature and explore how they affect the cosmic star formation history. These additional star formation–gas relations are as follows.

(i) A relation with a moderately sub-linear exponent 0.84 as suggested by Shetty et al. (2013):

$$\Sigma_{\text{SFR}} = \left( \frac{\Sigma_{\text{H}_2}}{10 \text{M}_\odot \text{pc}^{-2}} \right)^{0.84} 10 \text{M}_\odot \text{pc}^{-2}.$$

(ii) A relation suggested by Krumholz et al. (2009b):

$$\Sigma_{\text{SFR}} = \frac{\Sigma_{\text{H}_2}}{2.6 \text{ Gyr}} \left( \frac{\Sigma_g}{85 \text{M}_\odot \text{pc}^{-2}} \right)^{0.33}, \text{ if } \Sigma_g < 85 \text{M}_\odot \text{pc}^{-2}$$

$$\Sigma_{\text{SFR}} = \frac{\Sigma_{\text{H}_2}}{2.6 \text{ Gyr}} \left( \frac{\Sigma_g}{85 \text{M}_\odot \text{pc}^{-2}} \right)^{0.33}, \text{ if } \Sigma_g \geq 85 \text{M}_\odot \text{pc}^{-2}.$$  \hspace{1cm} (16)

(iii) The 'universal' star formation relation suggested by Krumholz et al. (2012):

$$\Sigma_{\text{SFR}} = 0.01 \frac{\Sigma_{\text{H}_2}}{\text{ff}}, \text{ where } \text{ff} = \min(\text{ff}_\text{GMC}, \text{ff}_\text{T}),$$

and

$$\text{ff}_\text{GMC} = \frac{\pi^{1/4}}{\sqrt{8}} G(\Sigma_{\text{GMC}} \Sigma_g)^{1/4}, \Sigma_{\text{GMC}} = 85 \text{M}_\odot \text{pc}^{-2},$$

$$\text{ff}_\text{T} = \frac{\pi}{8} \frac{1}{\Omega}. \hspace{1cm} (17)$$

In equation (18), $\sigma$ is the gas velocity dispersion on scales comparable to the disc scaleheight (few 100 pc). We set it to 8 km s$^{-1}$, adequate for disc galaxies in the local universe (Walter et al. 2008). High-redshift galaxies may have larger velocity dispersions (Tacconi et al. 2010), but since typically $\text{ff}_\text{GMC} > \text{ff}_\text{T}$ for these galaxies (Krumholz et al. 2012), the choice of $\sigma$ does not matter in this case. Equation (19) includes the angular velocity of galactic rotation $\Omega$. We chose $\Omega = V_{\text{rot}}/(1.7R_\odot)$ as our fiducial value. Here, 1.7$R_\odot$ is the half-mass radius of the gas disc and $V_{\text{rot}}$ is the rotation velocity of the disc at 1.7$R_\odot$. The latter we equate with $V_{\text{max,halo}}$, the maximum circular velocity of a dark matter halo with NFW profile (Navarro, Frenk & White 1997) that hosts the given galaxy. We determine $V_{\text{max,halo}}$ for a halo of given mass and at a given redshift using the fits (converted to our definition of halo mass) by Prada et al. (2012) based on high-resolution N-body simulations (Klypin, Trujillo-Gomez & Primack 2011). The ansatz $V_{\text{rot}} \sim V_{\text{max,halo}}$ has a firm observational justification, at least for disc galaxies in the local universe (Reyes et al. 2012). In compact star formation galaxies at high redshift, however, it is conceivable that $V_{\text{rot}}$ exceeds $V_{\text{max,halo}}$. If the latter is true, we potentially overestimate $\text{ff}_\text{T}$ and, hence, potentially underestimate the SFR at high redshift. As we will show in Section 3.2, this only increases the conflict between this particular star formation–gas relation and observations.

2.3 Running the model

We have implemented the model described in the previous sections as a GNU Octave$^3$ program. We adopt the fiducial choice of Krumholz & Dekel (2012) (see their table 1) for the model parameters, and note that most values are justified based on either theoretical grounds (e.g. the recycling fraction $R$ is a consequence of the adopted IMF) or on observations (e.g. $f_0$, $y$). We discuss the

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$^3$ http://www.gnu.org/software/octave
3 THE FUNCTIONAL FORM OF THE STAR FORMATION–GAS RELATION

In this section, we analyse the importance of the functional form of the star formation–gas relation for various galaxy properties and for the cosmic star formation history in the context of the model of Section 2.1. We will show that a strongly non-linear star formation–gas relation results in predictions that are in conflict with observations, while predictions based on a linear star formation–gas relation match the various observational data better and in most cases remarkably well.

3.1 Metallicities, stellar and gas masses

In Fig. 1, we show the evolution of the stellar mass–metallicity relation as predicted by the model for a linear and a strongly super-linear $\Sigma_{\text{SFH}} - \Sigma_{\text{H}_2}$ relation, see equations (13) and (14). We compare these predictions with observations of galaxies at $z \sim 0$ (Tronconi et al. 2004; Kewley & Ellison 2008), at $z \sim 2$ (Erb et al. 2006; Maiolino et al. 2008) and at $z \sim 3.5$ (Maiolino et al. 2008). Predictions based on a linear relation are in excellent agreement with the observed mass–metallicity relation at $z \gtrsim 2$. At low redshifts and low stellar masses, the model somewhat underpredicts the metallicities. We suspect that this is a result of the particular modelling of the metal enriched blow-outs (parametrized by the $\xi$ parameter), which we plan to address in future work. A strongly super-linear relation, however, severely underpredicts the evolution of the mass–metallicity relation at high redshift. The evolution at $z < 2$ is similar to the one for the linear star formation–gas relation, because at those times typical gas surface densities in galaxies falls below ~50 $M_\odot$ pc$^{-2}$ and the star formation–gas relation is effectively linear, see equation (14). In fact, if we had used a purely quadratic relation down to low gas surface densities, the mass–metallicity relation would not evolve between $z = 9$ and $z = 0$ – in obvious disagreement with observations.

In Fig. 2, we compare the predicted gas-to-stellar mass ratios with observations spanning the redshift range $z \sim 0$–3. A strongly super-linear star formation–gas relation underpredicts both the total gas-to-stellar mass ratio (see dotted lines) and the molecular gas-to-stellar mass ratio (see solid lines). At $z \sim 0.5$, the model predictions are off by a factor of ~3. The disagreement becomes even more severe at higher redshift. At $z \sim 2$, the observed gas-to-stellar mass ratios exceed the predicted ones by an order of magnitude. In contrast, we find good agreement between model predictions and observations for both the normalization and the redshift evolution of the gas-to-stellar mass ratio if the star formation–gas relation is linear. For instance, the predicted fraction $M_{\text{H}_2}/(M_{\text{H}_2} + M_\ast)$ is 39 per cent at $z = 1.5$ and 54 per cent at $z = 2.5$, compared with 34 per cent at $z \sim 1.2$ and 44 per cent at $z \sim 2.3$ as found by Tacconi...
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Figure 2. Gas-to-stellar mass ratio in galaxies as function of their stellar mass. Left: model predictions based on a linear $\Sigma_{\text{SFR}} - \Sigma_{H_2}$ relation, equation (13). Right: model predictions based on the strongly super-linear relation given by equation (14). The solid lines show the predicted molecular gas-to-stellar mass ratio, while dotted lines show the total gas-to-stellar mass ratio. Different colours correspond to different redshifts as indicated in the legend. The circles (arrows) show $>3\sigma$ detections (upper limits) of the $H_2$ fraction of galaxies in the local universe (Saintonge et al. 2011). The circles ($z \sim 0.5$), squares ($z \sim 1.5$) and stars ($z \sim 2.5$) represent measurements of cold gas masses for a sample of galaxies at higher redshift (Magdis et al. 2012b). These latter observations are based on dust IR emission and are thus not affected by uncertainties related to the CO–$H_2$ conversion. A linear $\Sigma_{\text{SFR}} - \Sigma_{H_2}$ relation predicts a large increase in the gas fractions up to $\sim 2$ in good agreement with the observed evolution, followed by only a modest change towards higher redshift, cf. Magdis et al. (2012a).

et al. (2010) based on CO observations of a sample of massive star-forming galaxies. It has been argued that the observed gas masses are potentially overestimated, because of systematic variations of the CO/$H_2$ conversion factor with CO surface brightness that is unaccounted for (Narayanan, Bothwell & Davé 2012b). We therefore also compare our predictions with observations that estimate gas masses based on the thermal emission of dust (Magdis et al. 2012b) to circumvent this potential issue. While this observational method also has its challenges, it is encouraging that we find again good agreement if the star formation–gas relation is linear, but stark disagreement if the relation is strongly super-linear.

The relation between stellar mass and halo mass of a galaxy is also sensitive to the functional form of the star formation–gas relation, see Fig. 3. Again, a linear star formation–gas relation leads to much better agreement with abundance matching results than a strongly super-linear star formation–gas relation. The agreement is

Figure 3. Relation between halo mass and stellar mass of galaxies. Left: model predictions based on a linear $\Sigma_{\text{SFR}} - \Sigma_{H_2}$ relation, equation (13). Right: model predictions based on the strongly super-linear relation given by equation (14). The solid lines represent the model predictions for $z = 0, z = 2, z = 5$ and $z = 8$ (see the legend). The symbols (colour coded according to redshift, see the legend) represent empirical estimates based on the abundance matching technique (Behroozi, Wechsler & Conroy 2013). Because the evolution of the $M_{\text{halo}} - M_*$ relation is weak, we combine, for a given redshift $z$, the abundance matching results from the $z - 1$ (downward pointing triangles), $z$ (squares) and $z + 1$ (upward pointing triangles) redshift bins.
particularly good at high redshifts \( z > 4 \), where, as we show in the next section, the cosmic star formation history is sensitive to changes in the star formation–gas relation.

### 3.2 The cosmic star formation history

In Fig. 4, we present the cosmic star formation history as predicted by the model for a linear star formation–H\(_2\) relation (equation 13). Here, the black solid curve shows the contribution of all galaxies sufficiently bright (\( M_{UV,AB} < -17.7 \)) to be accessible to current observations at \( z \geq 4 \). We estimate the luminosity of a galaxy based on its SFR and dust attenuation. For the latter, we use table 6 of Bouwens et al. (2012b) and we extrapolate the multiplicative dust extinction to 1 (no dust extinction) at \( z = 10 \). For \( z < 3 \), we assume that the dust extinction remains constant at its value at \( z = 3 \). The red solid curve shows the contribution of all galaxies down to the photoionization limit (Okamoto et al. 2008) to the SFRD. The corresponding dashed curves show what happens if we assume that all gas in the disc is molecular. The figure also includes compilations of observational data by Oesch et al. (2012), Bouwens et al. (2012b) and Behroozi et al. (2012).

An important result of Fig. 4 is that a linear star formation–gas relation predicts a cosmic star formation history that is in good agreement with observational data. Interestingly, given the current observational limits the cosmic star formation history at \( z > 2 \) changes only by a moderate amount depending on whether star formation is based on molecular hydrogen or not. We can rephrase this result, namely that quenching of star formation in low-metallicity and hence H\(_2\)-poor galaxies at high redshift is not very important for galaxies accessible to observations (\( M_{UV,AB} < -17.7 \)). It does, however, play a crucial role for fainter galaxies. In fact, considering all galaxies the cosmic SFRD at \( z \sim 10 \) is suppressed by 1.5 orders of magnitude if star formation is allowed to occur only in the molecular gas instead of all the gas in the disc. Obviously, this has implications for the re-ionization contribution by faint galaxies at such redshifts, see e.g. Kuhlen & Faucher-Giguère (2012).

In Fig. 5, we show the predicted UV luminosity function and its evolution with redshift for a linear \( \Sigma_{SFR} - \Sigma_{H_2} \) relation and compare it with observations (Bouwens et al. 2012a). The agreement is excellent considering the simplicity of the model and the various observational uncertainties. At \( z \sim 8 \) the observed luminosity function appears to have a somewhat larger normalization compared to cosmic variance.
with our model predictions, while no such offset is seen at the other redshifts, which could indicate that the $z \sim 8$ observations are affected by cosmic variance. At low luminosities (not shown), the UV luminosity function turns over as a result of the suppression of star formation in low-mass haloes. Unfortunately, the magnitude at which the turn-over takes place depends sensitively on model parameters, e.g. on the disc sizes (parametrized by the spin parameter $\lambda$), and is therefore not a robust prediction of the model. For instance, at $z \sim 4$ the UV luminosity function is predicted to break at $M_{UV} \sim -15.5$ if we use a linear $\Sigma_{SFR} - \Sigma_{H_2}$ relation with our default choice of model parameters. If, however, we decrease $\lambda$ from 0.07 to 0.05 the bright end of the luminosity function remains unaffected, while the turn-over shifts to $M_{UV} \sim -12.5$.

In Fig. 6, we show plots analogous to Fig. 4 for four non-linear star formation–gas relations: a moderately sub-linear relation (equation 15), the moderately super-linear relation postulated by Krumholz et al. (2009b) (equation 16), the ‘universal relation’ proposed by Krumholz et al. (2012) (equation 17) and the strongly super-linear relation (equation 14). The figure clearly demonstrates that a super-linear star formation–gas relation results in SFRDs higher than observed at $z \sim 4-10$. The discrepancy is particularly large for strongly non-linear star formation–gas relations, such as those given by equations (14) and (17). These relations result in SFRDs that are inconsistent with observations. For instance, at $z = 8 (z = 10)$, they are one (two) orders of magnitude above observations. Moderately non-linear relations such as those given by equation (15) and (16)
are in tension with observations, but are probably not excluded given the uncertainties of the model.

Similar to Fig. 4, the observable SFRD changes little whether star formation is tied to the abundance of molecular gas or to all the gas in the disc. Specifically, a sub-linear star formation–gas relation leads to a difference no larger than ~0.5 dex, while there is almost no discernible change if the relation is super-linear. Hence, we find that quenching of star formation in low-metallicity and, thus, H$_2$-poor galaxies does not have a significant impact on the observable cosmic star formation history. In contrast, the predicted total star formation history that includes all galaxies below the observational limits can change noticeably depending on whether the star formation–gas relation is based on H$_2$ or all the gas. For instance, the difference is ~1 dex for the moderately super-linear star formation–gas relation suggested by Krumholz et al. (2009b). The difference is even larger for a linear or sub-linear star formation–gas relation.

3.3 Caveats

Our findings come with a number of potential caveats. First, our approach is based on the assumption that a single, unchanging star formation–gas relation, potentially parametrized by local galaxy properties such as the gas metallicity, or the free-fall time, etc., holds from early times until the present. Secondly, the model obviously idealizes many aspects of star formation in high-redshift galaxies. For instance, it assumes that the gas is always arranged in an exponential disc (likely not true during major mergers) and that accreted haloes donate all their cold gas to the central galaxy. Furthermore, since the model only makes predictions for the mean evolution of haloes of a given mass, it ignores variations in the accretion histories and possible environmental dependences. Such, admittedly interesting, complexities are best studied in numerical simulations. However, demanding requirements in terms of simulation volume and resolution, see e.g. Schaye et al. (2010), limit the systematic exploration of the parameter space of feedback and star formation models using this approach. Nonetheless, we plan to compare the predictions of our model with dedicated high-resolution cosmological simulations in future work.

Additional caveats are as follows.

(i) Model parameters. We use the same set of parameters as in Krumholz & Dekel (2012). Most of these parameters have a theoretical or empirical justification (we refer to reader to Krumholz & Dekel 2012). When we vary the model parameters within justifiable ranges, we find that the mass ejection rate into the IGM per SFR, $\epsilon_{\text{out}}$, the metallicity of the IGM, $Z_{\text{IGM}}$, and the spin of the halo, $\lambda$, have the largest effect on the predicted cosmic star formation history. The impact of these changes is largest if a linear star formation–gas relation is used, and smaller if the relation is highly non-linear and, hence, star formation is accretion limited early on. However, even for a linear relation we can vary each of these parameters by a factor of ~3 without any qualitative changes in the model predictions. For instance, increasing $\epsilon_{\text{out}}$ by such a factor decreases the SFRD at $z \sim 1 - 2$ by ~0.5 dex, but leads to little change at higher redshifts (e.g. at $z = 4$ the decrease in the SFRD is only 0.2 dex). Reducing $Z_{\text{IGM}}$ by a factor of 3 delays star formation in low-mass haloes and results in SFRDs that are reduced by a similar factor. Decreasing $\lambda$ from 0.07 to 0.05 increases the SFRD at $z \sim 4 - 10$ by a factor of ~2–3. The predicted observable cosmic star formation history is less affected (~0.2 dex typically).

We stress that none of the tested combination of model parameters would bring the predictions for the strongly super-linear star formation–gas relation (equation 14) in agreement with the observations of the cosmic star formation history, gas-to-stellar fraction and mass–metallicity relation. This is not surprising because the model predictions for a strongly super-linear star formation–gas relation are primarily driven by the short gas-depletion time. The latter implies that star formation in galaxies can keep up with the accretion of gas on to galaxies and results in relatively low gas fractions, high SFRs and high metallicities at early times. In contrast, a linear star formation–gas relation with a long gas-depletion time ensures that the SFRs at high-redshift fall short of the gas-accretion rates and, hence, that galaxies have relatively large gas fractions, low metallicities and low stellar masses.

The equilibrium approach presented in Section 4.3 allows a further assessment of the importance of the various model parameters. According to equations (21)–(23), galaxies have metallicities, gas fractions and stellar fractions that depend on the model parameters and on the ratio $r$ between the SFR and gas-accretion rate. For instance, the chosen value for $Z_{\text{IGM}}$ will affect the metallicity of a galaxy if $r$ is small compared with $Z_{\text{IGM}}$. However, since $r$ is in fact much larger than $Z_{\text{IGM}}$ for galaxies at $z \sim 0-10$, see Section 4.2, the choice of $Z_{\text{IGM}}$ does not strongly alter the predicted metallicities of galaxies in this redshift range. Similarly, we can understand why changing $\epsilon_{\text{out}}$ within a factor of a few does not strongly affect galaxy properties at high redshifts. According to equations (26)–(28), only the combination $1 + r'(1 - R + \epsilon_{\text{out}})$ enters the equilibrium predictions. Here, $r'$ is the ratio between the matter accretion rate and the gas-depletion time. At early times $r' \ll 1$ and, hence, the precise value of $\epsilon_{\text{out}}$ does not matter as long as $r'(1 - R + \epsilon_{\text{out}}) \ll 1$.

(ii) Missing physics. Given the simplicity of the model it is possible that we are missing an important physical process that would affect the model predictions and potentially change our conclusions. For instance, a mechanism that increases the mass ejection rate of the winds with redshift would help to mitigate some of the problems of a highly non-linear star formation–gas relation. We demonstrate this in Fig. 7 where we show the model predictions for a hypothetical scenario in which $\epsilon_{\text{out}} = 1$ for $z < 2$ and $\epsilon_{\text{out}} = z - 1$ for $z \geq 2$. By choosing this particular redshift dependence of $\epsilon_{\text{out}}$, even a strongly super-linear star formation–gas relation can be brought into agreement with observations of the cosmic star formation history. However, this does not necessarily mean that other observational constraints are met as well. The figure shows, for instance, that this particular model does not reproduce the $M_{\text{halo}} - M_*$ relation at any redshift and the gas-to-stellar mass ratios at $z \gtrsim 1$. A simple, linear star formation–gas relation with an H$_2$ depletion time of 2.5 Gyr (as observed in the local universe) does not require the assumption of additional ad hoc (and fine-tuned) physics to reproduce global galaxy properties from high redshifts until today, see Section 3.1.

(iii) Metal mixing. We assume that metals in the discs of galaxies are efficiently mixed, i.e. that the gas has a spatially uniform metallicity. In reality, we expect some metallicity gradients, since accretion of metal-poor gas occurs preferentially at the outer edge of the disc. Numerical simulation by Kuhlen et al. (2012) indicate that the metallicity difference is only modest for the majority of the gas in the discs of high-redshift galaxies. In order to test the importance of metal mixing, we modified the model to allow only local enrichment (i.e. metals produced in a given radius bin are added only to that radial bin). This somewhat extreme case of no-mixing lowers the predicted SFRDs by ~3 if we use a linear star formation–gas relation, but makes almost no difference if the relation is strongly non-linear.

(iv) Dust. The model assumes that the dust-to-gas ratio (which determines the H$_2$ abundance) scales proportional to the metallicity.
At low metallicities \((Z < 0.1Z_\odot)\), it is unclear whether this assumption is justified. A potential concern is that at high \(z\) the dust production may lag behind the metal production, limiting the formation of \(H_2\) and thus star formation. Empirically, there is evidence for a substantial presence of dust in high-redshift galaxies from absorption line studies of damped Ly\(\alpha\) systems at \(z \sim 1.5–3.5\) (e.g. Pettini et al. 1994; Prochaska & Wolfe 2002) and from the thermal emission of dust in quasar hosts at \(z \sim 4–6\) (e.g. Omont et al. 2001; Bertoldi & Cox 2002). Both AGB stars (e.g. Valiante et al. 2009) and supernovae (e.g. Todini & Ferrara 2001) have been proposed as formation channels for dust at such high redshifts. If supernovae dominate the dust production, we expect a dust production timescale of \(\sim 10^7\) yr, which is shorter than the gas-accretion time \((\sim 1–2 \times 10^8\) at \(z = 10\)) and, hence, not a limiting factor.

\(v\) Molecular hydrogen. The \(H_2\) abundances are estimated based on the equilibrium model by Krumholz et al. (2009a). For MW-like, turbulent ISM conditions the \(H_2\) formation times \(t_{H_2,\text{form}}\) are short \((\sim 10^6\) yr; Glover & Mac Low 2007), but they could be higher in high-redshift galaxies. With \(t_{H_2,\text{form}} \propto (Z \rho)^{-1}\) (Hollenbach, Werner & Salpeter 1971) and gas densities in the high-\(z\) galaxies that are likely an order of magnitude higher than in the MW, we may expect formation times of \(\lesssim 10^4\) yr for gas with metallicities \(Z \gtrsim 2 \times 10^{-5}\), which is sufficiently short to not limit star formation. In addition, numerical simulations of star formation in the ISM show that decreasing the metallicity from solar to a hundredth solar postpones the onset of star formation only by a few Myr (Glover & Clark 2012). Hence, we do not expect that star formation is limited by the \(H_2\) formation time even in the case of a highly non-linear star formation–gas relation.

### 4 THE DRIVERS OF GALAXY EVOLUTION

So far we have demonstrated that the functional form of the star formation–gas relation has a large impact on a variety of global galaxy properties, such as the overall metallicities, SFRs and the gas-to-stellar fractions. In this section, we analyse this result using the framework of the model introduced in Section 2 in order to learn more about the actual drivers of galaxy properties across cosmic history. To simplify the discussion, we will focus on only two of the introduced star formation–gas relations: the linear star formation–\(H_2\) relation (equation 13) and the strongly super-linear star formation–\(H_2\) relation (equation 14). After discussing the role of metal enrichment (Section 4.1), we show how the competition between star formation and gas accretion drives the evolution of basic galaxy properties (Section 4.2). Finally, in Section 4.3, we present a novel equilibrium picture of galaxy evolution that captures the interplay between star formation, gas accretion and outflows, and in which average galaxy properties are determined by the functional form of the star formation–gas relation. Galaxies remain in equilibrium since birth and may drop out of equilibrium only at very late times when the gas accretion on to their haloes is quenched. In this picture, the evolution of galaxy properties across cosmic time is a simple consequence of the modulation of the external accretion rate.

#### 4.1 Surface densities of galaxies and the role of metal enrichment

In this section, we discuss why the functional form of the star formation–gas relation has a large impact on the SFRD at high redshift. In Fig. 8, we show the evolution of the typical gas surface density (here \(\Sigma_5(r/2)\), the surface density at the gas halfmass radius) of galaxies residing in haloes of different mass. At \(z \sim 20–30\), this surface density is typically in the range \(300 – 1000 M_\odot\) pc\(^{-2}\). As the universe expands, gas surface densities in galaxies decrease and reach values in agreement with observations of nearby galaxies (\(\sim 10 M_\odot\) pc\(^{-2}\) for galaxies in haloes with masses \(\sim 10^{11} – 10^{12} M_\odot\); e.g. Wong & Blitz 2002). Galaxies in haloes above the virial shock quenching mass (\(\gtrsim 2 \times 10^{12} M_\odot\)) experience a fast decline in their gas surface density at low redshift caused by the shut-down of gas accretion.

We also include in the figure the evolution of the critical surface density \(\Sigma_{\text{cr}}(f_{H_2} = 0.5)\) required to turn half the gas molecular. We note that there is a one-to-one correspondence between this critical surface density and the metallicity of the gas, see equation (8). Initially, the critical surface density is large \(\sim 3000 M_\odot\) pc\(^{-2}\) (corresponding to \(Z_{\text{ISM}} = 2 \times 10^{-3}\)), but it decreases with time as metal enrichment by star formation boosts the molecular content of the gas. Clearly, the choice of the star formation–gas relation has a large impact on the evolution of this critical surface density or, equivalently, the metallicity. If the star formation–gas relation is linear, haloes with \(z = 0\) masses below \(\sim 3 \times 10^{10} M_\odot\) are unable to...
Figure 8. Evolution of the gas surface density at the disc half-mass radius ($\Sigma_g(r_{1/2})$) and evolution of the surface density at which the molecular fraction is 50 per cent ($\Sigma_g(f_{\text{H}_2}=0.5)$) for galaxies in haloes of different masses. Left: for a linear star formation–gas relation (equation 13). Right: for a strongly super-linear star formation–gas relation (equation 14). The solid and dot-dashed lines show how $\Sigma_g(r_{1/2})$ and $\Sigma_g(f_{\text{H}_2}=0.5)$, respectively, change across cosmic time for galaxies that, by $z=0$, reside in the centre of a halo of the specified mass (see colour-coding in the legend). The expansion of the universe, the consumption of gas and the quenching of gas accretion result in a reduction of $\Sigma_g(r_{1/2})$ with time. Star formation increases the metallicity and thus decreases $\Sigma_g(f_{\text{H}_2}=0.5)$ (equation 8). A strongly super-linear star formation–gas relation leads to early enrichment (to $\sim 0.1 Z_\odot$ already at $z \gtrsim 10$). Enrichment proceeds much more gradually in the case of a linear $\Sigma_{\text{SFR}} - \Sigma_{\text{H}_2}$ relation.

significantly increase the metallicity of the gas. In contrast, efficient enrichment can take place in such haloes at early times ($z > 10$) if the relation is strongly super-linear.

In Fig. 9, we compare the ratio between the typical gas surface density, $\Sigma_g(r_{1/2})$, and the critical surface density, $\Sigma_g(f_{\text{H}_2}=0.5)$. $\text{H}_2$-based star formation can only take place throughout the disc if this ratio is $\gtrsim 1$. If the star formation–gas relation is linear, gas in galaxies with $z = 0$ halo masses below $\sim 3 \times 10^{10} M_\odot$ remains effectively un-enriched and thus atomic. Hence, star formation is almost completely quenched in haloes of such mass. However, galaxies in more massive haloes are able to turn a substantial fraction of their gas reservoir into $\text{H}_2$. In fact, the condition $\Sigma_g(r_{1/2}) \sim \Sigma_g(f_{\text{H}_2}=0.5)$ is satisfied increasingly earlier for haloes of higher mass, e.g. at $z \sim 10$ ($z \sim 4.5$) for haloes with $z = 0$ masses of $10^{11} M_\odot$ to $10^{12} M_\odot$. If the star formation–$\text{H}_2$ relation is strongly super-linear, galaxies generally reach metallicities of $Z \sim 2 \times 10^{-3}$ (a tenth solar) quickly after a single Pop III star initially enriches the gas to $Z \sim 2 \times 10^{-5}$. In this case, most galaxies, even those in low-mass haloes, have fully molecular gas disks and are able to form stars efficiently at early times. The metallicity-dependent quenching of star formation has thus little impact on the cosmic star formation history in this case.

Returning to the discussion of the linear star formation–gas relation, it is important to point out that the lack of early enrichment is not the reason that $\lesssim 3 \times 10^{10} M_\odot$ haloes are inefficient star formers at observationally accessible redshifts $z \lesssim 10$. To show this, we re-run the model for the linear relation with a 100 times higher initial gas metallicity of the haloes, i.e. with $Z = 2 \times 10^{-3}$ instead of $2 \times 10^{-5}$, see Fig. 9. Clearly, the ratio between typical and critical gas surface density is not affected in a significant way at $z \lesssim 10$. In particular, galaxies with $z = 0$ halo masses below $3 \times 10^{10} M_\odot$ still do not contribute to star formation at these redshifts. Hence, while star formation is regulated by the presence of $\text{H}_2$ and, thus, by the metal enrichment of the gas, the level of metal enrichment is determined primarily by the functional form of the star formation–gas relation. A non-linear relation leads to fast enrichment and, hence, to galaxy discs that are fully molecular and have large SFRs even in low-mass haloes at high redshifts. In contrast, a linear relation delays the enrichment of the gas and therefore limits star formation to sufficiently massive haloes.

This leaves us with one open question, namely how can the star formation–gas relation have such a strong impact on the metal enrichment in haloes? The metallicity evolution is determined by the SFR (increases metal mass and decreases gas mass; equation 12) and the gas-accretion rate (increases gas mass and metal mass; equation 4). Rearranging these equations it is easy to show (see Appendix A) that the metallicity $Z$ is constant, i.e. $\dot{Z} = 0$, if

$$ r = \frac{\dot{M}_{\text{star}}}{\dot{M}_{\text{gas}}} = \frac{Z - Z_{\text{ISM}}}{1 - (1 - \zeta)} $$

(20)

Given an SFR to gas-accretion rate ratio $r$, the equilibrium metallicity is therefore

$$ Z_{eq}(r) = ry(1 - \zeta) + Z_{\text{ISM}} $$

(21)

We demonstrate in Appendix A that this equilibrium solution is (linearly) stable under most circumstances. Intuitively, the stability of the equilibrium is a simple consequence of the interplay between gas accretion and metal enrichment via star formation. Gas inflows from the IGM dilute the metallicity of the ISM at a rate that is proportional to both the gas-accretion rate and to the difference between the current ISM metallicity and the metallicity of the accreted gas. This implies that the dilution rate of the ISM metallicity by gas inflows will be smaller or larger depending on the current metallicity of the ISM. In contrast, the metal production rate by stellar evolution processes is typically less sensitive to the ISM metallicity (see Appendix A for a more elaborate discussion). In equilibrium, the dilution rate of metals via gas inflows matches the enrichment rate via star formation. Hence, whenever the current ISM metallicity
is below its equilibrium value, the dilution rate is reduced relative to its value for \( Z_{eq} \), while the enrichment rate remains the same. Consequently, the ISM metallicity increases. Vice versa if the ISM metallicity is above its equilibrium value.

The answer to our question above is therefore that a change of the functional form of the star formation–gas relation changes the ratio \( r \) (assuming that the gas-accretion rate and the gas mass of the galaxy remain unaffected) and, hence, the equilibrium metallicity of a galaxy. This argument will be made rigorous in Section 4.3.

Equation (21) has a few interesting properties. First, it demonstrates that, for a given ratio \( r \), galactic outflows of warm/cold ISM material (parametrized by \( \varepsilon_{ou} \)) play no role for the ISM metallicity. While outflows carry metals away they also remove a corresponding amount of gas leaving the equilibrium metallicity unchanged. But since the true metallicity of the ISM is likely close to \( Z_{eq}(r) \), it also remains unaffected. Secondly, \( Z_{eq}(r) \) depends on \( \xi \) (and thus on halo mass) and on \( Z_{IGM} \). Hence, an analysis of the mass or redshift dependence of the metallicities of high-redshift galaxies (combined with measured SFRs and mass accretion rates, estimated e.g. from their halo masses) could be used to learn more about supernova driven blow-outs (parametrized by \( \zeta \)) or the IGM enrichment (\( Z_{IGM} \)).

In the next section, we study the connection between the star formation–gas relation and the driver of the metallicity evolution in galaxies, the ratio of SFR to gas-accretion rate.

### 4.2 Gas accretion and star formation

Previously we have demonstrated that the functional form of the star formation–gas relation has a profound impact on the SFR of galaxies at high redshifts and we have attributed this effect to the altered metal enrichment. In this section, we will show that the metal enrichment is intrinsically linked to the star formation–gas relation via the balance between star formation and gas accretion in haloes.

In Fig. 10, we plot the evolution of the \( r \), the ratio of the SFR and the gas-accretion rate, or equivalently the ratio of the accretion time (gas mass divided by the gas-accretion rate) to the gas-depletion time (gas mass divided by SFR), for haloes of a range of masses. At \( z \gtrsim 3 \), a linear star formation–gas relation (equations 13) results in a gas-depletion time that is significantly longer (it is 2.5 Gyr if all gas is molecular and even longer otherwise) than the gas-accretion time \((\sim 2-3 \times 10^7 \text{ yr at } z = 30, \sim 1-2 \times 10^8 \text{ yr at } z = 10, \sim 7 \times 10^8 \text{ yr at } z = 5)\). Hence, star formation is ‘depletion limited’ at those redshifts. Changing the functional form of the star formation–gas relation alters the gas-depletion times, but has only a small effect on the gas-accretion rates, which are determined primarily by the mass accretion rates on to dark matter haloes. A strongly super-linear star formation–gas relation, for instance, results in gas-depletion times as low as \( \sim 2 \times 10^8 \text{ yr at } z > 10 \). As a consequence, star formation is ‘accretion limited’ (the SFR is of the same order as the gas-accretion rate) over most of cosmic history in this case.

Equation (20) allows us to understand the consequences of a depletion limited versus an accretion limited star formation. Consider a gas disc with a given metallicity \( Z \). Such a gas disc will increase or decrease its metallicity until it matches the metallicity \( Z_{eq} \) specified by equation (21) for the given SFR to gas-accretion rate ratio \( r \). For a stellar mass loss fraction of \( R = 0.46 \), a stellar metal yield of \( \eta = 0.069 \), and \( \xi = 0.9 \) (a reasonable choice for all but the most massive haloes at \( z \sim 4-10 \)), we obtain \( Z_{eq} = 0.1 Z_{\odot} \) for \( r = 0.53 \), but \( Z = 0.01 Z_{\odot} \) for \( r = 0.048 \). Therefore, if star formation is accretion limited, the gas will get enriched to \( \sim 0.1 Z_{\odot} \) or more on the gas-accretion time-scale. Accretion-limited star formation, and the rapid enrichment to \( Z \sim 0.1 Z_{\odot} \), has indeed been seen in many hydrodynamical simulations with a super-linear star formation model, see e.g. Davé, Finlator & Oppenheimer (2006) and Finlator, Oppenheimer & Davé 2011. The equilibrium metallicity is small if \( r \) is small, i.e. if the depletion time is much longer than the gas-accretion time. Hence, metal enrichment is delayed until \( z \sim 2-3 \) in the case of a linear star formation–gas relation with a sufficiently large (>Gyr) depletion time.

To summarize, the functional form of the star formation–gas relation determines whether star formation in a given halo at a given time is depletion limited or accretion limited. This is crucial, because the evolution of the metallicity in a halo is driven by the competition between gas accretion and gas depletion (equation 21). A linear star formation–gas relation delays metal enrichment until late times and, hence, suppresses star formation at high redshifts.
contrast, a strongly super-linear star formation–gas relation boosts metal enrichment and star formation at high redshifts.

A quantity similar to the ratio between SFR and gas-accretion rate has been studied empirically by Behroozi et al. (2013) using abundance matching techniques. In their work, the baryon equivalent accretion rate (BEAR; \( f_{b} M_{\text{halo}} \)) is used as a proxy for the unknown gas-accretion rate. They find (see fig. 1 in Behroozi et al. 2013) that the SFR to BEAR ratio is roughly constant out to \( z \approx 3 \) and speculate that it might be time independent for much of cosmic history.

In Fig. 11, we show our predictions for the SFR to BEAR ratio using a similar colour scheme and axis ranges as in fig. 1 of Behroozi et al. (2013). We compute BEAR based on the halo mass evolution formula provided in the appendix of Behroozi et al. (2013). We find that a linear, \( H_{2} \)-based star formation–gas relation can reproduce their fig. 1 reasonably well. Most importantly, the ratio between SFR and BEAR peaks around halo masses of \( \sim 10^{13} \, M_{\odot} \) at \( z \lesssim 2 \). Some features seen in Fig. 11, such as the complete lack of star formation in sufficiently low-mass haloes, may be related to the fact that the model follows the average evolution of galaxy properties and, at least in its current form, ignores the possibility of scatter caused by, e.g. the variability of the accretion rates or scatter in the disc sizes of galaxies.

### 4.3 An equilibrium view on galaxy evolution

Not only does the ratio between SFR and gas-accretion rate regulate the metallicity in galaxies (see equation 21), it also determines the gas fraction \( f_{g} = M_{g}/M_{\text{halo}} \), stellar fraction \( f_{s} = M_{s}/M_{\text{halo}} \) and the metal fraction \( f_{\text{metal}} = M_{\text{metal}}/M_{\text{halo}} \) of galaxies.

Specifically, we can show using equations (2)–(5) that for a given ratio \( r \equiv \frac{M_{\text{out}}}{M_{\text{in}}} \),

\[
\dot{f}_{g} = 0 \text{ if } f_{g} = f_{g, eq}(r) \text{ with } \\
\dot{f}_{g, eq}(r) = \left[ 1 - r(1 - R + \epsilon_{\text{out}}) \right] \epsilon_{\text{in}} f_{g, in} f_{b},
\]

\[
\dot{f}_{s} = 0 \text{ if } f_{s} = f_{s, eq}(r) \text{ with } \\
f_{s, eq}(r) = \left( 1 - f_{s, in} \right) f_{s} + \epsilon_{\text{in}} f_{g, in} f_{b} (1 - R) r,
\]

\[
\dot{f}_{\text{Z}} = 0 \text{ if } f_{\text{Z}} = f_{\text{Z}, eq}(r) \text{ with } \\
f_{\text{Z}, eq}(r) = Z_{\text{eq}}(r) f_{g, eq}(r).
\]

The equilibrium values of other properties, such as the gas-to-stellar mass ratio \( f_{g s} = f_{g}/f_{s} \) or the baryon fraction \( f_{b} + f_{s} \), can be obtained by combining these equations. For instance, \( f_{g s} = 0 \text{ if } f_{g s} = f_{g s, eq}(r) \) with

\[
\frac{f_{g s, eq}(r)}{f_{s, eq}(r)} = \frac{1 - (1 - R + \epsilon_{\text{out}}) r}{1 - f_{s, in} + (1 - R) r}.
\]
changes in metal abundance. There can be no equilibrium state for $r > 1/(1 - R + \epsilon_{\text{out}})$, because it would lead to negative equilibrium gas fractions, see equation (22). However, $r > 1/(1 - R + \epsilon_{\text{out}})$ is a physical possibility and applies, e.g., to massive galaxies at low redshifts that have their gas accretion quenched, but are still star forming. These galaxies are obviously not in an equilibrium state, but rely on their diminishing gas reservoir for star formation.

The stability of the equilibrium state implies, for instance, that if $f_g < f_{g,\text{eq}}(r)$ (and it is sufficiently close to the equilibrium value), the gas fraction increases and continues to do so until it reaches $f_{g,\text{eq}}(r)$. Similarly, if $f_g > f_{g,\text{eq}}(r)$, the gas fraction decreases until it matches the equilibrium value. We can therefore draw important conclusions for the gas fractions and gas-to-stellar mass ratios from Fig. 10. A small SFR to gas-accretion rate ratio of, e.g., $r = 0.048$ corresponds to an equilibrium metallicity of $Z = 0.01 Z_{\odot}$, to an equilibrium gas fraction $f_g/\delta_f \sim 0.83$ (0.46) and to an equilibrium gas-to-stellar mass ratio $f_{g,\text{eq}} \sim 6.8$ (0.9). Here, we assume $f_{g,\text{eq}} = 0.9$ ($f_{g,\text{eq}} = 0.5$) and the default parameters of the model: $\epsilon_{\text{in}} = 1$, $R = 0.46$, $\epsilon_{\text{out}} = 1$. Hence, depletion-limited star formation implies low metallicities, large gas fractions and large gas-to-stellar mass ratios. In contrast, equilibrium metallicities are higher and equilibrium gas fractions and gas-to-stellar mass ratios are much lower if star formation is accretion limited. For instances, an SFR to gas-accretion rate ratio $r = 0.53$ corresponds to $Z_{\text{eq}} = 0.1 Z_{\odot}$, $f_{g,\text{eq}}/f_s \sim 0.17$ (0.09) and $f_{g,\text{eq}} \sim 0.46$ (0.14) for $f_{g,\text{in}} = 0.9$ ($f_{g,\text{in}} = 0.5$).

In the context of our model the baryonic properties of galaxies (of given halo mass and at a given redshift) are specified by $f_z$, $f_{g,z}$ and $f_s$. In more abstract terms, the vector $(f_z, f_{g,z}, f_s)$ represents the baryonic state of a galaxy. Equations (22)–(24) denote a curve\(^4\) of fixed points parametrized by $r$ in this baryonic state space. We demonstrate in Appendix A that if $(f_z, f_{g,z}, f_s)$ is reasonably close to one of the fixed points it will approach it. As the cosmic accretion rate on to a halo changes, $r$ changes as well and the galaxy transits smoothly from one equilibrium state to the next. Strong perturbations, such as major mergers, will also allow a galaxy to leave its current fixed point, but we would expect that such events are relatively rare. Hence, galaxy evolution is largely an evolution along this curve of fixed points driven by the external modulation of the gas-accretion rate.

Instead of using $r$ it is also possible to phrase the equilibrium metallicities, gas fractions and stellar fractions in terms of the ratio $r'$ between the matter accretion time ($M_{\text{halo}}/M_{\text{halo}}$) and the gas-depletion time ($M_f/M_{\text{gas}}$). Using $r'$ allows for a conceptually clean separation between gravitational processes (determining matter accretion times) and baryonic physics (determining gas-depletion times). The conversion from one formulation to the other is straightforward. Using the definitions of $r$ and $r'$ and equation (2), it is easy to show that $f_g = \frac{r}{1 + r' (1 - R + \epsilon_{\text{out}})} f_{g,\text{in}} f_s$. Hence, in equilibrium, see equation (22),

$$r' = \frac{r}{1 - r (1 - R + \epsilon_{\text{out}})}, \quad \text{and} \quad r'' = \frac{r'}{1 + r' (1 - R + \epsilon_{\text{out}})}.$$  

Plugging the latter equation into equations (21)–(23) results in the following alternative formulation for the equilibrium baryonic state of a galaxy:

$$Z_{\text{eq}}(r') = \frac{r' y (1 - R) (1 - \xi)}{1 + r' (1 - R + \epsilon_{\text{out}})} + Z_{\text{RM}},$$  

$$f_{g,\text{eq}}(r') = \frac{\epsilon_{\text{in}} f_{g,\text{in}} f_s}{1 + r' (1 - R + \epsilon_{\text{out}})},$$  

$$f_s,\text{eq}(r') = (1 - f_{g,\text{in}}) f_s + \frac{r' \epsilon_{\text{in}} f_{g,\text{in}} f_s (1 - R)}{1 + r' (1 - R + \epsilon_{\text{out}})}.$$  

The idealized equilibrium picture that we outline here assumes that galaxies can adapt on short time-scales to changes in the SFR and the gas accretion rate. However, the matter accretion time $M_{\text{halo}}/M_{\text{halo}}$ sets a natural time-scale for gas fractions, stellar fractions and metallicities to reach their equilibrium values. Since the accretion time is a significant fraction of the Hubble time at any given redshift, we expect that the stellar fractions, gas fractions and metallicities lag somewhat behind their equilibrium predictions. Hence, we expect that gas fractions are somewhat underestimated by the equilibrium formalism, while stellar fractions and metallicities are overestimated, as seen in Fig. 12. In addition, the derivation of the equilibrium equations assumes that model parameters and halo masses remain fixed, which is not always a good assumption.

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\(^4\) The parameter $f_{g,\text{in}}$ can affect the predicted equilibrium gas fractions and gas-to-stellar mass ratios noticeably. The values chosen here are for illustration purposes, but we stress that we compute $f_{g,\text{in}}$ self-consistently in the model (see Section 2).

\(^5\) This is only approximately true, because some of the terms in equations (22)–(24) are dependent on more than just $r$, e.g., on the halo mass. This also implies that different galaxies will in general follow somewhat different curves of fixed points during their evolution, see Fig. 12.
Figure 12. Comparison of the gas fractions and metallicities at $z \geq 1.5$ as predicted by the dynamical model (see Section 2) and by the equilibrium scenario (see Section 4.3) for a linear $\Sigma_{\text{SFR}}-\Sigma_{\text{H}}$ relation (equation 13; left-hand panel) and a strongly super-linear relation (equation 14; right-hand panel). The solid lines show the predictions of the dynamical model for a galaxy residing in the progenitor of a $10^{12} \, M_\odot$ (at $z = 0$) halo. The shaded areas show the corresponding results for haloes with $z = 0$ masses of $10^{11} \, M_\odot$ (lower boundary) and $10^{13} \, M_\odot$ (upper boundary), respectively. Symbols highlight predictions at the specific redshifts 10, 8, 6, 4, 3 and 2 as indicated in the figure. The dot–dashed lines show the equilibrium predictions based on a straightforward application of equations (26) and (27) with $r' = t_{\text{acc}}/t_{\text{dep}}$. The value of $r'$ is indicated in the figure. Rescaling $r'$ can substantially improve the accuracy of the equilibrium formalism, see the text. The dashed line shows the result of rescaling $r'$ by a factor of 0.45 (left-hand panel) and 0.9 (right-hand panel), respectively. There is excellent agreement between the predictions of the dynamical model (solid line) and those of the rescaled equilibrium scenario (dashed line).

In order to test whether the equilibrium picture can be used to make quantitative predictions, we show the $f_\text{g}-Z$ projection of the baryonic state space in Fig. 12. We plot both the metallicities and gas fraction as computed by the dynamical model, and also the corresponding equilibrium quantities based on equations (26) and (27).

The equilibrium formalism describes the overall trend between $f_\text{g}$ and $Z$ well, but for a given $r'$ it can underpredict the gas fractions and overpredict the metallicities. The disagreement can be reduced if $r'$ (or $r$) is scaled by a factor $\sim 0.45/\log_{10} Z$ before applying equations (26) and (27) (or equations 21 and 22). Here, $n \in [1, 2]$ is the power-law exponent $n$ of the star formation–gas relation.

Our equilibrium picture of galaxy formation differs from the ‘equilibrium condition’ of a steady-state model (Fialator & Davé 2008; Bouché et al. 2010; Dutton, van den Bosch & Dekel 2010; Davé, Fialator & Oppenheimer 2012) in which the SFR in a given halo is balanced by the net gas-accretion rate. Most importantly, we demonstrate that galaxies can be in a stable equilibrium even if the ‘equilibrium condition’ is not even remotely satisfied. Specifically, galaxies can have little star formation and can massively build up their gas reservoirs and, yet, be in a proper equilibrium state.

The ‘equilibrium condition’ of the steady-state model is equivalent to the assumption that the gas mass in a galaxy remains constant. In contrast, our equilibrium condition is that the gas fraction remains constant. The special state with $r = 1/(1 - R + e_{\text{out}})$ (or equivalently $r' \to \infty$) satisfies the ‘equilibrium condition’ of the steady-state model. Indeed, a similar relation for $r$ is often assumed to hold generally in the steady-state approach, see e.g. equation 2 of Davé et al. (2012). However, as equation (22) shows the equilibrium gas fraction close to this equilibrium state is small, and, hence, gas-rich, star-forming galaxies at high redshift will not be in this particular state.

The equilibrium view presented in this section shows why metallicities, gas fractions and stellar fractions of galaxies are strongly correlated. They are all coupled via a single parameter, which can be chosen to be $r$, the ratio between the SFR and the gas-accretion rate, see equations (21)–(23), or $r'$, the ratio between the matter accretion time and the gas-depletion time, see equations (26)–(28). The latter formulation makes the following point particularly transparent. The functional form of the star formation–gas relation determines the gas-depletion time while gravitational processes alone determine the matter accretion time. Hence, for a given matter accretion rate the functional form of the star formation–gas relation determines the equilibrium properties (including the metallicity, gas fraction, stellar fraction) of a given galaxy. Galaxy properties evolve with time primarily due to the modulation of the cosmic accretion rate.

5 SUMMARY AND DISCUSSION

We have studied the implications of the functional form of the star formation–gas relation for the cosmic star formation history for gas accretion in galaxies, and for the evolution of global galaxy properties using a model that follows the average evolution of galaxies in growing dark matter haloes across cosmic time (Krumholz & Dekel 2012). We specifically contrasted the predictions for a linear star formation–gas relation (Bigiel et al. 2011) with those for a strongly super-linear relation (Ostriker & Shetty 2011), but also tested a few other functional forms suggested in the literature (Krumholz et al. 2009b, 2012; Shetty et al. 2013). In the following, we express our statements in terms of a linear versus a super-linear star formation–gas relation. However, we stress that the physically relevant characteristic of the star formation–gas relation is not the actual value of the slope, but the rather the duration of the gas-depletion
time w.r.t. the matter accretion time, see Section 4.3. The adopted linear star formation–gas relation has a constant (and relatively long) depletion time, while the strongly super-linear relation has a galaxy-averaged depletion time that is typically comparable to or even shorter than the matter accretion time. Our main findings are as follows.

(i) We show that a linear $\Sigma_{\text{SFR}} \sim \Sigma_{H_2}$ relation with an $\sim 2.5$ Gyr depletion time results in a mass–metallicity relation, in gas-to-stellar mass ratios and in a stellar-to-halo mass relation that is in reasonable agreement with observations. In contrast, the short gas-depletion times that result from adopting a strongly super-linear star formation–gas relation lead to a stark mismatch between model predictions and observations.

(ii) The functional form of the star formation–gas relation strongly affects the cosmic star formation history at $z \gtrsim 4$. A linear star formation–$H_2$ relation with a sufficiently long gas-depletion time suppresses star formation in haloes with $z = 0$ masses below a few times $10^{10} M_\odot$. If, however, the star formation–gas relation is highly non-linear, star formation can occur in haloes with $z = 0$ masses even below $10^{10} M_\odot$. As a result, SFRDs can change by factors of $\sim 30$ at $z = 10$ ($\sim 5$ at $z = 6$), when switching between a linear and a strongly super-linear star formation–$H_2$ relation, see Figs 4 and 6.

(iii) The functional form of the star formation–gas relation also strongly affects the observable cosmic star formation history, i.e. the SFRD of galaxies with magnitudes accessible to observations (here assumed to be $M_{\text{UVAB}} < -17.7$; Bouwens et al. 2012b). A linear star formation–$H_2$ relation results in an observable cosmic star formation history compatible with observational data, while a strongly super-linear relation severely overpredicts the observable SFRDs at high redshift.

(iv) Whether star formation is based on $H_2$ or on total gas leaves little imprint on the observable cosmic star formation history. However, the total (including galaxies down to the faintest magnitudes) cosmic star formation history can be affected depending on the functional form of the star formation–gas relation. If the relation is linear, $H_2$-based star formation will be suppressed in low-mass (and $H_2$-poor) haloes and the total SFRD will be reduced significantly, e.g., by more than an order of magnitude at $z = 8$. However, if the star formation–gas relation is strongly super-linear, then low-mass haloes will be enriched early on. Hence, low-mass haloes will contain a significant reservoir of $H_2$ and star formation in such haloes will contribute significantly to the SFRD independently of whether star formation is $H_2$-based or not.

(v) For the adopted linear $\Sigma_{\text{SFR}} \sim \Sigma_{H_2}$ relation (equation 13), the gas-depletion time exceeds the gas-accretion time (approximately the free-fall time of the halo) at high $z$. During this time the SFRs are orders of magnitude smaller than the gas-accretion rates. Star formation becomes accretion limited, i.e. SFRs $\sim$ gas-accretion rates, only at relatively recent epochs ($z \lesssim 2$). We show that these predictions are in reasonable agreement with abundance matching predictions by Behroozi et al. (2013). In contrast, star formation is close to accretion limited already at very high redshifts if the star formation–gas relation is strongly super-linear.

(vi) The evolution of metallicities, gas fractions and stellar fractions are driven by the ratio between the SFR and gas-accretion rate or, equivalently, by the ratio between gas-depletion time (set by the star formation–gas relation) and the matter accretion time (set by gravity). For a given star formation–gas relation galaxies evolve towards a (linearly stable) equilibrium state with a gas fraction, stellar fraction and metallicity determined only by the accretion rate. The equilibrium concept presented in this work differs from the steady-state model suggested in the literature (e.g. Davé et al. 2012) and shows that galaxies can be in equilibrium even if their gas masses are growing quickly. We argue that galaxy evolution can be interpreted as an evolution along a set of equilibria driven primarily by the change in the cosmic accretion rate.

As pointed out above, we find that the observed cosmic star formation history is well reproduced if the star formation–gas relation is linear and has an $H_2$ depletion time of $\sim 2.5$ Gyr. Given the simplicity of the model and the uncertainties in the observational data, we do not rule out the possibility that the relation is actually slightly super- or sub-linear (e.g. has an exponent in the range $\sim 0.85$–$1.3$) or has a somewhat different depletion time. However, our findings are difficult to reconcile with the idea of a highly super-linear, especially quadratic, star formation–gas relation.

Many of our findings for a linear star formation–gas relation are almost identical to those found by Krumholz & Dekel (2012) for a non-linear star formation–gas relation, similar to equation (16), with an exponent of $1.33$ at high gas surface densities. However, their star formation–gas relation uses a very large transition surface density ($\Sigma_g = 10^7 M_\odot$ pc$^{-2}$ compared with $\Sigma_g = 10^5 M_\odot$ pc$^{-2}$) used in equation (16) at which the scaling changes from sub-linear to super-linear. This matters because gas surface densities of high-redshift galaxies are of the order of a few $100 M_\odot$ pc$^{-2}$, see Fig. 8. Hence, a transition surface density as large as $\Sigma_g = 850 M_\odot$ pc$^{-2}$ implies that a large fraction of the gas in galaxy discs is subject to a sub-linear star formation–gas relation and that $\Sigma_{\text{SFR}}$ scales super-linearly with $\Sigma_{H_2}$ only in the central regions (where surface densities are larger by a factor of $e^{1.7}$ $\sim 5.5$ than the surface densities at the half-mass radius). Averaging over sub- and super-linear regions results in a close-to-linear relation between $H_2$ masses and SFRs in galaxies, hence, to similar predictions as for the strictly linear star formation–gas relation given by equation (13).

Krumholz & Dekel (2012) argue that star formation based on $H_2$ is an essential ingredient to bring the predicted SFRDs at high redshifts in agreement with observations. We find that metal-dependent quenching of star formation based on $H_2$ has only a small impact (0.3 dex at $z \sim 9$ and less at lower redshift) on the observable SFRDs at high redshift, because it affects primarily faint galaxies below the current detection limits. However, metal-dependent quenching plays an important role for the total (i.e. including galaxies below the detection limits) SFRD if the star formation–gas relation is not highly super-linear. This is in agreement with Krumholz & Dekel (2012) who effectively use an approximately linear star formation–gas relation (see above) to study the total SFRD. In the case of a strongly super-linear relation, however, star formation in galaxies is almost always close to being accretion limited even at $z \sim 10$. Therefore, galaxies get enriched early on and SFRs proceed at a rate that is effectively identical (to within a factor of 2 or so) to the gas-accretion rates, independently on whether a $\Sigma_{\text{SFR}} \sim \Sigma_{H_2}$ or $\Sigma_{\text{SFR}} \sim \Sigma_g$ relation is used.

Schaye et al. (2010) analyse the star formation history of the universe using large-scale cosmological simulations. The star formation in their simulations is modelled via a moderately non-linear ‘Schmidt law’, i.e. a star formation–gas relation based on volumetric densities. Star formation is allowed to take place only in regions with densities above a threshold that is either fixed (the default model) or scales with metallicity. Both recipes result in essentially identical cosmic star formation histories, indicating that metal-dependent quenching of star formation might not very important. However, compared with Krumholz & Dekel (2012), their...
threshold is much lower than the density required for H$_2$ formation (Gnedin & Kravtsov 2011) and the metallicity scaling is much weaker ($\propto$ log $Z^{-1}$; Schaye 2004) than the one expected to regulate the H$_2$ abundance (approximately $\propto$ $Z^{-3}$, see equations 8–11). It is therefore difficult to properly judge the implications of their findings in the context of H$_2$-based star formation.

Kuhlen et al. (2012) study the evolution of the cosmic star formation history in simulations that follow the abundance of H$_2$ directly (using equations 8–11) and compare these to simulations that use a fixed density threshold for star formation. The chosen threshold (50 cm$^{-3}$) corresponds approximately to the transition from the atomic to the molecular phase for gas of solar metallicity. The Schmidt law used in their simulations is moderately non-linear. Hence, we would expect that the two sets of simulations result in very similar predictions for the SFRDs of galaxies with stellar masses larger than $10^{7.75}$ M$_\odot$ change by (only) a factor of $\sim$2. The differences are even smaller when galaxies are selected based on their UV luminosity ($M_{UV,AB} < -17.7$; Kuhlen, private communication).

Another interesting question is the importance of stellar feedback for the regulation of star formation at high $z$. Schaye et al. (2010) found that increasing the exponent of the star formation–gas relation or decreasing the depletion time has little impact on the cosmic star formation history at $z \lesssim 6$ and the authors use this fact to infer that galaxies are able to regulate their star formation via feedback processes.

However, their default star formation–gas relation is non-linear and, hence, increasing its exponent or decreasing its depletion time further has little impact if SFRs are already close to the accretion limit.

Lagos et al. (2011) predict, based on semi-analytical modelling, that the cosmic star formation history is relatively insensitive to the choice of the star formation–gas relation. However, they do not test for a linear star formation–gas relation which star formation depends on a metallicity-dependent H$_2$ fraction. In fact, the tested relations are either independent of the gas metallicity or are moderately super-linear or both. For such relations, we expect that star formation is limited by the gas-accretion rate out to relatively high redshift and thus not very sensitive to the precise functional form of the adopted star formation–gas relation. In addition, Lagos et al. (2011) consider the total cosmic star formation history (cf. red lines in Fig. 6) which is less sensitive to the choice of the star formation–gas relation than the star formation history of galaxies accessible to current observations (cf. black lines in Fig. 6). A similar study by Fu et al. (2012) that includes a linear H$_2$-based star formation–gas relation finds that the cosmic star formation history at high redshift differs depending on the choice of the star formation–gas relation (see their fig. 2), in agreement with our results.

We argue against the interpretation that galaxy properties and the cosmic star formation history are regulated by feedback-driven large-scale galactic outflows. The reason is, as we pointed out in Section 4.3, that for a given star formation–gas relation the matter accretion rate on to the halo is the main regulatory mechanism that determines the gas masses, SFRs, metallicities and stellar masses of galaxies. Of course, the strength of outflows can affect the predicted equilibrium properties, see equations (26)–(28), but only if the mass loading of the outflows is sufficiently large ($e_{out} \gtrsim 1/r$). Of course, star formation could well be feedback-regulated within the ISM. In particular, the existence of a star formation–gas relation that holds on $\sim$ kpc scales may be a consequence of the interplay between turbulence, gravity and feedback in the first place (e.g. Hopkins et al. 2011).

We have demonstrated that the functional form of the star formation–gas relation has a particularly strong impact on galaxy properties at high redshifts. Especially, the gas fractions of galaxies and the mass–metallicity relation differ widely depending on the adopted star formation–gas relation. Ongoing and upcoming observational efforts carried out with, e.g. the Atacama Large Millimeter/sub-millimeter Array, will be crucial to constrain the functional form of the star formation–gas relation and should be able to test many of the specific predictions of our model.

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\[ \frac{\partial}{\partial t} \theta(\mathbf{x}, t) = \mathbf{F}(\mathbf{x}, t) \quad \text{in} \quad \Omega \times (0, T) \]

where \( \mathbf{F} \) represents the force that depends on the state of the system, \( \Omega \) is the domain of interest, and \( (0, T) \) is the time interval.

\section*{APPENDIX A: EQUILIBRIA AND STABILITY ANALYSIS}

The time derivatives of the metallicity \( Z \), metal fraction \( f_x \), gas fraction \( f_g \) and stellar fraction \( f_s \) are

\[ \begin{align*}
\dot{Z} &= \frac{d}{dt}\left[ Z - \frac{Z_{\text{eq}}}{M_{\text{gas}}} \right], \\
\dot{f}_x &= \frac{d}{dt}\left[ f_x - \frac{f_{x,\text{form}}}{M_{\text{gas}}} \right] = g_x(f_z, f_g, f_s).
\end{align*} \]

where \( X \in \{Z, g, s\} \) and we introduced the functions \( g_x \). Combining equation (A1) with equations (4) and (12), we obtain

\[ \begin{align*}
Z &= \frac{1}{M_{\text{gas}}} \left[ \gamma(1 - R)(1 - \xi)M_{\text{form},\text{gas}} - (Z - Z_{\text{IGM}})M_{\text{gas}} \right], \\
&= \frac{M_{\text{gas}}}{M_{\text{gas}}} \left[ \gamma(1 - R)(1 - \xi) - (Z - Z_{\text{IGM}}) \right].
\end{align*} \]

The second equality assumes \( M_{\text{gas}} \neq 0 \) and we introduced \( R = M_{\text{form},\text{gas}}/M_{\text{gas}} \). Clearly, the metallicity is constant if the equilibrium condition (20, 21) is satisfied, i.e. for \( Z_{\text{eq}} = \gamma(1 - R)(1 - \xi) \) or \( Z_{\text{IGM}} \).

Let us assume that the metallicity is perturbed slightly relative to its equilibrium value, i.e. \( Z = Z_{\text{eq}} + \delta \) with a small perturbation \( \delta \). Let us also introduce the shorthands \( A = 1 - R + \epsilon_{\text{out}}, B = \gamma(1 - R)(1 - \xi), C = \epsilon_{\text{out}} f_z, D = (1 - f_g, a h) \), to simplify the notation. The evolution of \( \delta \) can be obtained from the linearized version of equation (A3),

\[ \dot{\delta} = \frac{M_{\text{gas}}}{M_{\text{gas}}} \left[ B \frac{d}{dZ}(Z_{\text{eq}}) - 1 \right]. \]

Hence, we find that \( \delta \to 0 \) if \( B \frac{d}{dZ}(Z_{\text{eq}}) < 1 \) and \( M_{\text{gas}} > 0 \). In particular, if, for a given gas configuration and a non-zero gas inflow, star formation proceeds independently of its metallicity, the equilibrium metallicity is always stable against perturbations. However, if the SFR changes strongly with metallicity the equilibrium can become unstable. In this case, the perturbation initially grows, i.e. gas is enriched or depleted, but eventually \( \frac{d}{dZ} \) becomes small (e.g. when all the gas is molecular) and the system approaches a new equilibrium. We note that this derivation ignores changes to the gas masses (or stellar masses), and hence, it does not prove that \( Z_{\text{eq}} \)
is the metallicity of an equilibrium state that is also stable against these perturbations, which is what we show next.

Consider a galaxy that resides in a halo with mass $M_{\text{halo}}$ and that accretes gas and stars according to equations (4) and (5). The accretion rates are governed by gravitational and not affected by changes in the gas fraction or the metallicity of the galaxy. Also, the scale radius of the gas disc (proportional to the virial radius of the halo) is not affected by such changes (at least in the context of the model of Section 2). We assume that the star formation in the galaxy is described by a power-law relationship

$$M_{\text{star}} = N \left( f_\text{g} f_{\text{H}_2} f_J \right)^n. \quad \text{(A5)}$$

Here, $N$ is a normalization constant (which includes everything that does not change when the gas fraction or metal fraction changes) and $n$ is the power-law exponent of the star formation–gas relation. The assumption that the $H_2$ fraction of the gas depends on $f_g$ and not on $Z$ deserves a comment. Equation (8) shows that the $H_2$ fraction depends primarily on the dust optical depth (equation 11). The latter is proportional to $Z \Sigma_g \propto f_g$.

Let $\delta X = f_\text{g}, \delta Y = f_z, \delta g$ be a small perturbation around the equilibrium state $(f_\text{g}, f_z, f_s, f_X, f_Y, f_g, f_s)$ (see equations 22 and 25). The evolution of the perturbation is governed by the following system of equations

$$\dot{f}_X = \delta X = g_X (f_z, f_g, f_s, f_s, f_s, f_s, f_s),$$

$$\approx \sum_{Y \in \{g, s, l\}} \delta_Y \frac{\partial g_X}{\partial f_Y} (f_{z, eq}, f_{g, eq}, f_{s, eq}).$$

If all eigenvalues of the Jacobian $J_{X, Y} = \frac{\partial g_X}{\partial f_Y}$ have a real value less than zero, then a system in the state $(f_{z, eq}, f_{g, eq}, f_{s, eq})$ is stable against small perturbations.

In the following, we use the additional shorthands $E = \frac{\partial g_X}{\partial f_Y} (f_{z, eq}, f_{g, eq})$ and $T = M_{\text{halo}} / M_{\text{halo}}$. We find that

$$J_{Z, X} = \frac{1}{T} \left( f_{z, eq} - C Z_{\text{IGM}} \left( \frac{n E f_{z, eq}}{f_{z, eq}} - \frac{A}{B f_{g, eq} - A f_{z, eq}} \right) - 1 \right),$$

$$J_{Z, g} = \frac{1}{T} \left( f_{z, eq} - C Z_{\text{IGM}} \left[ \frac{n f_{g, eq} + f_{z, eq}}{f_{g, eq} B f_{g, eq} - A f_{z, eq}} \right] \right),$$

$$J_{Z, s} = 0,$$

$$J_{g, X} = -\frac{1}{T} \left[ \frac{n E}{f_{z, eq}} (C - f_{g, eq}) \right],$$

$$J_{g, g} = -\frac{1}{T} \left[ \frac{n}{f_{g, eq}} (C - f_{g, eq}) + 1 \right],$$

$$J_{g, s} = 0,$$

$$J_{s, X} = \frac{1}{T} \left( \frac{n E f_{s, eq}}{f_{z, eq}} - D \right),$$

$$J_{s, g} = \frac{1}{T} \left( \frac{n E f_{g, eq}}{f_{z, eq}} - D \right),$$

$$J_{s, s} = -\frac{1}{T}.$$

Replacing $(f_{z, eq}, f_{g, eq}, f_{s, eq}, f_X, f_Y, f_g, f_s)$ with the expressions (22)–(24) allows us to write $J_{X, Y}$ as a function of the following parameters: $n, A, B, E, Z_{\text{IGM}}$ and $r$, the ratio between SFRs and gas-accretion rate, for the given equilibrium state. In particular, $C$ and $D$ drop out, which shows that the stability of the equilibrium does not depend on the values of $f_{g, \text{in}}$ or $\epsilon_{\text{in}}$.

$$J_{Z, X} = \frac{1}{T} \left[ \frac{n E Br^2 A + (n E Z_{\text{IGM}} A + B - n E B) r + Z_{\text{IGM}}}{(Br + Z_{\text{IGM}})(Ar - 1)} \right].$$

$$J_{Z, g} = \frac{1}{T} \left[ \frac{(n B A - B A) r - n B + n Z_{\text{IGM}} A - Z_{\text{IGM}} A}{Ar - 1} \right].$$

$$J_{Z, s} = 0,$$

$$J_{g, Z} = \frac{1}{T} \left[ \frac{n E Ar}{(Br + Z_{\text{IGM}})(Ar - 1)} \right],$$

$$J_{g, g} = \frac{1}{T} \left[ \frac{1 + (n - 1) Ar}{Ar - 1} \right],$$

$$J_{g, s} = 0,$$

$$J_{s, Z} = \frac{1}{T} \left[ \frac{27}{50} \frac{rn E}{(Br + Z_{\text{IGM}})(Ar - 1)} \right]$$

$$J_{s, g} = \frac{1}{T} \left[ \frac{27}{50} \frac{n r}{Ar - 1} \right],$$

$$J_{s, s} = -\frac{1}{T}.$$

Clearly, $J$ has a singularity at $Ar - 1$ as expected from equation (22) and it has an eigenvalue $-1$. The convergence to the equilibrium solution occurs on the time-scale $T$ if the other two eigenvalues have real components that are smaller or equal to this eigenvalue.

The general expression for the eigenvalues of $J$ is rather lengthy.

**Figure A1.** Maximum of the real part of the eigenvalues of $JT$ as function of the ratio between SFR and gas-accretion rate. The equilibrium is stable if the maximum of the real part of the eigenvalues is less than zero (horizontal dot-dashed line). Results for a linear ($n = 1$) star formation–$H_2$ relation are shown by the three solid lines. The individual lines correspond to $E = 1$, $E = 0.3$ and $E = 0.1$ from top to bottom. The dashed lines show the corresponding results for a quadratic star formation–$H_2$ relation ($n = 2$). The equilibrium is stable for $r < 1/A$ (vertical dotted line) if $E < 0.5 - 1$ (depending on $n$), i.e. as long as the $H_2$ fraction is not extremely sensitive to changes in the metal fraction.
and we do not reproduce it here. Instead, we compute the real part of the eigenvalues numerically using the default parameters for $A$, $B$ and $Z_{IGM}$. The parameter $B$ depends on $\zeta$ and thus on halo mass. We adopted $\zeta = 0$ which results in the most stringent limit on stability.

We show in Fig. A1 the result of our numerical analysis for $n = 1$ and $n = 2$. The maximum of the real part of the eigenvalues is less than zero (and hence the equilibrium is stable) for any $r < 1/A$ as long as $E$ is sufficiently small ($E \lesssim 0.5 - 1$). Hence, as long as the H$_2$ fraction is not extremely sensitive to changes in the metal fraction, the equilibrium is stable.

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