Impact of Differential Dust Settling on the SED and Polarization: Application to the Inner Region of the HL Tau Disk

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Abstract

The polarimetric observations of the protoplanetary disk around HL Tau have shown the scattering-induced polarization at ALMA Band 7, which indicates that the maximum dust size is ∼100 μm, while the spectral energy distribution (SED) has suggested that the maximum dust size is approximately a millimeter. To solve the contradiction, we investigate the impact of differential settling of dust grains on the SED and polarization. If the disk is optically thick, a longer observing wavelength traces more interior layers, which would be dominated by larger grains. We find that the SED of the center part of the HL Tau disk can be explained with millimeter-sized grains for a broad range of turbulence strength, while 160 μm-sized grains cannot be explained unless the turbulence strength parameter αt is lower than 10−5. We also find that the observed polarization fraction can be potentially explained with a maximum dust size of 1 mm if αt ≤ 10−5, although models with 160 μm-sized grains are also acceptable. However, if the maximum dust size is ∼3 mm, the simulated polarization fraction is too low to explain the observations even if the turbulence strength is extremely small, indicating a maximum dust size of ≤1 mm. The degeneracy between 100 μm- and millimeter-sized grains can be solved by improving the ALMA calibration accuracy or polarimetric observations at (sub)centimeter wavelengths.

Unified Astronomy Thesaurus concepts: Protoplanetary disks (1300); Planet formation (1241)

1. Introduction

Measurement of dust sizes in protoplanetary disks with different ages is key to understanding how and when dust grains grow into larger bodies. The spectral energy distribution (SED) of disks is one of the best ways to constrain the dust size. If the disk is optically thin at observing wavelengths, the spectral slope of the intensity traces the slope of the dust absorption opacity and hence allows us to estimate the dust size (e.g., Calvet et al. 2002; Draine 2006). In the optically thick regime, the emission is lower than that of the blackbody because self-scattering reduces the apparent disk brightness (Miyake & Nakagawa 1993; Birnstiel et al. 2018; Carrasco-González et al. 2019; Liu 2019; Zhu et al. 2019; Sierra & Lizano 2020). Since the scattering behavior is sensitive to the dust size, we can constrain the dust size from the observed SED even in the optically thick regime (Ueda et al. 2020).

Polarimetric observations at millimeter wavelengths are also useful for constraining dust sizes in protoplanetary disks. Recent Atacama Large Millimeter/submillimeter Array (ALMA) polarimetric observations have shown that many disks show a scattering-induced polarization pattern at the observing wavelength of ∼1 mm (e.g., Bacciotti et al. 2018; Hull et al. 2018; Dent et al. 2019; Sadavoy et al. 2019). Because self-scattering induces the polarization effectively when the observing wavelength is comparable to 2πa_{max}, where a_{max} is the maximum dust radius, these observations indicate the prevalence of 100 μm-sized grains in disks, even though they should already contain millimeter-sized grains.

The disk around the HL Tau young (class I) star is one of the most intensively studied protoplanetary disks. The recent multiwavelength analysis on the HL Tau disk showed that the inner part of the disk contains ∼millimeter-sized dust grains (Carrasco-González et al. 2019). On the other hand, the ALMA polarimetric observations have shown the transition of the polarization pattern: scattering-induced polarization at ALMA Band 7, alignment-induced polarization at ALMA Band 3, and a mixture of them at ALMA Band 6 (Stephens et al. 2017; Kataoka et al. 2017). This clear trend indicates that the maximum dust size in the HL Tau disk is ∼100 μm.

One solution to the contradiction is the differential settling of dust grains caused by disk turbulence (Sierra & Lizano 2020; Brungräber & Wolf 2020; Ohashi et al. 2020; see also Liu 2020 for the polarization of a class 0 object). Since the vertical mixing is less efficient for larger grains, they settle more to the midplane than smaller grains (Dubrulle et al. 1995; Youdin & Lithwick 2007). Therefore, if the disk is optically thick, the shorter observing wavelength traces the higher layer, where smaller grains dominate. The difference in the observed dust sizes due to the differential settling might make the interpretation of the observations more complicated.

In this paper, we investigate the impact of differential settling on both the SED and polarization of the inner part of the HL Tau disk. We describe the observational data in Section 2. The setup of the numerical simulations is described in Section 3. Section 4 gives the comparison between the observations and simulations. The discussion and summary are in Sections 5 and 6.

2. Observational Data

We analyze images of the HL Tau disk observed at several wavelengths obtained in previous studies. In the following analysis, we focus only on the intensity at the center of the
For the SED analysis, we use images at the observing wavelength of 0.87 (ALMA Band 7), 1.3 (ALMA Band 6), 2.1 (ALMA Band 4), and 7.9 mm (VLA Ka + Q), given by Carrasco-González et al. (2019). The synthesized beam size is set to be $0\arcsec 05 \times 0\arcsec 05$, corresponding to a spatial resolution of 7.35 au with a distance of HL Tau (147 pc; Galli et al. 2018). The Very Large Array (VLA) data are corrected by free-free contamination. For details of the data set, we refer readers to Carrasco-González et al. (2019).

Figure 1 shows the intensity at the center of the observed images. We set the uncertainty in the absolute flux as 10% for ALMA Bands 6 and 7 and 5% for ALMA Band 4, which are quoted from the ALMA official observing guide. The uncertainty in the absolute flux in the VLA observation is assumed to be 5%. These uncertainties might be potentially larger than the nominal values due to, e.g., poor weather (Francis et al. 2020; Ueda et al. 2020). At the center of the images, the signal-to-noise ratio is high enough ($809, 723, 322$, and $60.0$) for ALMA Bands 7, 6, and 4 and VLA observations, respectively, implying that the uncertainty in their intensity is dominated by the flux calibration uncertainty.

The intensity slope between ALMA Bands 6 and 7 follows a spectral index of 2. However, the intensity at ALMA Band 4 is below the intensity extrapolated from the intensity at shorter wavelengths with a spectral slope of 2. This indicates that scattering reduces the intensity at ALMA Band 4 more effectively than at ALMA Bands 6 and 7. The deviation from the spectral slope of 2 is significant at the VLA wavelength, and the spectral index between $\lambda = 2.1$ and 7.9 mm is $\sim 2.5$.

For the polarization analysis, we use images at the observing wavelength of 0.87 (ALMA Band 7), 1.3 (ALMA Band 6), and 3.1 mm (ALMA Band 3), which are given by Kataoka et al. (2017) and Stephens et al. (2017). The angular resolution is $0\arcsec 44 \times 0\arcsec 35, 0\arcsec 37 \times 0\arcsec 24$, and $0\arcsec 51 \times 0\arcsec 41$ for the observation at observing wavelengths of 0.87, 1.3, and 3.1 mm, respectively. For details of the data set, we refer readers to Kataoka et al. (2017) and Stephens et al. (2017).

Figure 2 shows the polarization fraction at the center of the observed images. The observed polarization fraction at $\lambda = 0.87$ and 1.3 mm is $\sim 0.5\%$, while it is less than 0.1% at $\lambda = 3$ mm. At $\lambda = 3$ mm, we have only an upper limit on the polarization fraction, since the polarization has not been detected at the center part of the disk. We set potential errors in the polarization fraction as 0.1%, which corresponds to the ALMA instrumental error. As shown in Kataoka et al. (2017) and Stephens et al. (2017), the measured rms in the intensity at the center part of the disk is well below 0.1%.

The observed polarization might not originate only from self-scattering. If dust grains are aligned with the external field, such as a magnetic or radiation field, the emission from the aligned grains is polarized and cancels out the scattering-induced polarization (e.g., Stephens et al. 2017). Mori & Kataoka (2021) showed that the observed low polarization degree at $\lambda = 3$ mm can be explained by the combination of 0.4% polarization due to self-scattering and 0.55% polarization due to dust alignment, which has a polarization vector perpendicular to that of self-scattering. Therefore, for reference, we also plot the self-scattering component suggested by Mori & Kataoka (2021) in Figure 2 with a gray cross.

3. Radiative Transfer Simulations

In order to investigate the properties of the inner part of the HL Tau disk, radiative transfer simulations are performed with the Monte Carlo radiative transfer code RADMC-3D (Dullemond et al. 2012). In this section, we describe how we model the disk in the simulations.
Figure 3. Midplane temperature profile for different $\epsilon$ values.

In our simulations, the dust surface density is assumed to follow a power-law profile with a Gaussian-like gap,

$$\Sigma_d = \Sigma_0 \left( \frac{r}{1 \text{ au}} \right)^{-0.5} \left[ 1 - d_g \exp \left\{ - \left( \frac{r - r_g}{w_g} \right)^2 \right\} \right],$$  \hspace{1cm} (1)

where $r$ is the midplane distance from the central star and $\Sigma_0$ is the dust surface density at 1 au. We choose a power-law index of 0.5 for the dust surface density because the intensity profile of the inner region of the disk needs a relatively flat density profile. We also run some simulations with a power-law index of 1.0 and 1.5 and find that an index of 0.5 is more preferable. This flat profile is only valid for the inner region we focus on ($\lesssim 20$ au). Although we focus only on the intensity at the center of the disk and not on the radial profile, we consider the first gap, which is located at $r_g = 12$ au with a depth of $d_g$ and width of $w_g$.

Using the dust surface density, the dust volume density $\rho_d$ is calculated as

$$\rho_d = \frac{\Sigma_d}{\sqrt{2\pi h_d}} \exp \left( -\frac{z^2}{2h_d^2} \right),$$  \hspace{1cm} (2)

where $z$ is the vertical height from the midplane and $h_d$ is the scale height of the dust disk.

The radial temperature profile is given by the brightness temperature in the Rayleigh–Jeans limit at Band 7, $T_{b,B7}(r)$:

$$T(r) = \epsilon T_{b,B7}(r) = \epsilon \frac{c^2}{2k_b\nu_{B7}^2} I_{\nu_7}(r),$$  \hspace{1cm} (3)

where $\nu_{B7}$ and $I_{\nu_7}$ are the observing frequency and intensity at ALMA Band 7. The true temperature would be different from $T_{b,B7}$ even if the disk is optically thick because scattering reduces the observed intensity. Therefore, we introduce a parameter $\epsilon$ to fit the observed intensity at Band 7. The temperature profiles for different $\epsilon$ values are shown in Figure 3. Although $\epsilon$ would also be a function of the radius, since the dust population would change with the radius, we set $\epsilon$ to be constant with the radius for simplicity. The brightness temperature should be lower than the true disk temperature because of the scattering-induced intensity reduction, as well as the optical depth effect.

The dust scale height is assumed to be a mixing–settling equilibrium (Dubrulle et al. 1995; Youdin & Lithwick 2007),

$$h_d = h_g \left( 1 + \frac{St}{1 + St} \right)^{-1/2},$$  \hspace{1cm} (4)

where $h_d$ is the gas scale height given as $h_g = c_s/\Omega_K$ and $\alpha_t$ is the stress-to-pressure ratio (Shakura & Sunyaev 1973). The Stokes number, $St$, is calculated as

$$St = \frac{\pi \rho_{int} d}{2 \Sigma_g},$$  \hspace{1cm} (5)

where $\rho_{int}$ is the material density of dust and $d$ is the dust radius. The gas surface density $\Sigma_g$ is assumed to be $1000(r/\text{au})^{-0.5}$ g cm$^{-2}$, where the power-law index is the same as the dust surface density.

At each radial grid, the vertically integrated dust size distribution is assumed to be a power-law distribution ranging from 0.1 $\mu$m to $d_{\text{max}}$ with a power-law index of 3.5. The dust size distribution ranging from 10 $\mu$m to $d_{\text{max}}$ is logarithmically divided into 15 size bins per decade, and grains smaller than 10 $\mu$m are represented with a single size bin. For each size bin, Equation (4) is applied. Opacities are calculated using DSHARP dust optical constants published in Birnstiel et al. (2018); see also Henning & Stognienko 1996; Draine 2003; Warren & Brandt 2008 for the optical constants of each dust component. The dust grains are assumed to be spherical compact grains with a material density of 1.675 g cm$^{-3}$. To treat full anisotropic scattering, the Mie matrices are calculated using the Mie theory, specifically the Bohren–Huffman program (Bohren & Huffman 1983). Figure 4 shows the absorption opacity $\kappa_{\text{abs}}$ and effective scattering albedo $\omega_{\text{eff}}$ obtained from our dust model. The effective scattering albedo is defined as

$$\omega_{\text{eff}} = \frac{\kappa_{\text{sc}}^{\text{eff}}}{\kappa_{\text{abs}} + \kappa_{\text{sc}}^{\text{eff}}},$$  \hspace{1cm} (6)

where $\kappa_{\text{sc}}^{\text{eff}}$ is the effective scattering opacity considering the effect of forward scattering (Heney & Greenstein 1941); the explicit form is given by, e.g., Birnstiel et al. (2018). The sum of the absorption and effective scattering opacity is the extinction opacity $\kappa_{\text{ext}} = \kappa_{\text{abs}} + \kappa_{\text{sc}}^{\text{eff}}$. One of the most important features is that the effective scattering albedo has a peak at $\lambda \sim 2\pi a$, which means that the scattering-induced intensity reduction is the most effective at $\lambda \sim 2\pi a$ (see Ueda et al. 2020).

In the simulation, we use the spherical coordinate and the theta coordinate ranges from $\pi/3$ to $2\pi/3$, where $\pi/2$ corresponds to the midplane. To accurately solve the vertical settling, the calculation domain is divided into 800 grids for the theta direction, and $3 \times 10^6$ photon packages are used for each simulation.

4. Results

4.1. Spectral Energy Distribution

In this section, we compare the observed and simulated SED of the center part of the HL Tau disk to see the impact of the differential settling on the SED.

Figure 5 shows the simulated SED at the center of the disks with a maximum dust size of 160 $\mu$m for different $\alpha_t$ values.
1.98 in each model, respectively. The simulated images are convolved with the same beam size as the observations. (\textit{a}) The simulated SED of the center part of the HL Tau disk with a maximum dust size of 160 \( \mu m \). Here \( \Sigma_0 \) and \( \epsilon \) are tuned to fit the observed intensity at \( \lambda = 0.87 \) and 7.9 mm in the no-settling limit. All adopted parameters in our calculations are summarized in Table 1.

The intensity steeply decreases with \( \lambda \geq 3 \) mm because the disk becomes optically thin. In this regime, the intensity does not depend on the turbulence strength, since all grains can be seen and hence the vertical distribution does not matter. In contrast, in the optically thick regime (\( \lambda < 3 \) mm), the model with weaker turbulence yields higher intensity. This is because weaker turbulence makes the grains more likely to settle down to the midplane, which makes the effective grain size at the photosphere smaller. Since the scattering albedo of 160 \( \mu m \) grains has a peak at \( \lambda \sim 1 \) mm (Figure 4), the SED has a dip at \( \lambda \sim 1 \) mm due to a scattering-induced intensity reduction in the no-settling model. The depth of the dip at \( \lambda \sim 1 \) mm is smaller for models with weaker turbulence, because smaller grains have a smaller scattering albedo at \( \lambda \sim 1 \) mm.

To more quantitatively see the impact of differential settling, we plot a breakdown of the extinction optical depth coming from each dust size at a radial position of 3 au in Figure 6. Since the spatial resolution of the observing beam is 7.35 au, the intensity at the center of the images mainly traces the radial location at \( \sim 3 \) au. The optical depth is calculated only with grains above the \( \tau = 1 \) surface. In Figure 6, we also plot the model with extremely weak turbulence (\( \alpha_t = 10^{-7} \)) for reference. At ALMA Band 7 (\( \lambda = 870 \mu m \)), more than 70\% of the observed emission comes from grains with radii of 100 \( \mu m < a \leq 160 \) \( \mu m \) in the no-settling model, while it is less than 5\% when \( \alpha_t = 10^{-6} \). In the model with \( \alpha_t = 10^{-6} \), almost the half of the observed emission comes from grains smaller than 10 \( \mu m \). In contrast, at ALMA Band 3 (\( \lambda = 3 \) mm), the breakdown is almost the same in all models because the disk is almost optically thin.

Figure 7 shows the simulated SED for the different maximum dust sizes. In Figure 7, we plot the results of tuned models where the temperature and surface density profile (i.e., \( \Sigma_0 \) and \( \epsilon \)) are tuned to fit the observed intensity at \( \lambda = 0.87 \) and 8 mm in each simulation. Basically, the temperature is constrained from the intensity at the shorter wavelength (ALMA Band 7), since the disk would be optically thick. For a given temperature, the dust surface density is constrained from the intensity at the longer wavelength (VLA Ka + Q).

Since the different maximum dust sizes have different absorption/scattering opacities (Figure 4), the combination of the temperature and optical depth for the observed SED to be reproduced is also different for different dust models. For each dust model, we fixed the gap structure for simplicity. Even though the obtained intensity profile is slightly different from the observed one at the gap region for some \( \alpha_t \) models, we confirmed that the slight difference in gap structure does not affect the SED and polarization significantly.

We clearly see that the models with a maximum grain size of 160 \( \mu m \) cannot reproduce the observed intensity profile if \( \alpha_t > 10^{-5} \). This is because 160 \( \mu m \)-sized grains reduce the intensity efficiently at ALMA Band 7 and hence cannot reproduce the observed low intensity at ALMA Band 4. If \( \alpha_t \leq 10^{-5} \), the effective dust size is very small, owing to the settling of large grains; hence, the observed intensity profile

![Figure 4. Absorption opacity (top) and effective scattering albedo (bottom) for different dust sizes as a function of observing wavelength.](image)

![Figure 5. Simulated SED of the center part of the HL Tau disk with a maximum dust size of 160 \( \mu m \). Here \( \Sigma_0 \) and \( \epsilon \) are fixed to be 36.7 g cm\(^{-2}\) and 1.98 in each model, respectively.](image)
Fixed 1000 μm yield the polarization fraction consistent with the observations at ALMA Bands 6 and 7 when \( \alpha_t \lesssim 10^{-5} \) while predicting a very low polarization fraction when \( \alpha_t \gtrsim 10^{-4} \). In contrast to these models, the models with \( a_{\text{max}} = 2900 \mu m \) predict a very low polarization fraction compared to the observations even if \( \alpha_t \lesssim 10^{-5} \). Since the other polarization mechanisms might take place and cancel out the scattering-induced polarization, a polarization fraction higher than the observations would be acceptable. Therefore, we cannot exclude the possibility that the maximum dust size is 160 μm and the turbulence strength is higher than 10^{-5} from the polarization analysis. All models that potentially account for the observed polarization fraction at ALMA Bands 6 and 7 predict a polarization fraction higher than the observation at ALMA Band 3, indicating that the other polarization mechanisms are necessary to explain the observed polarization fraction at ALMA Band 3 (Mori & Kataoka 2021).

Interestingly, in the models of millimeter-sized grains, the simulated polarization fraction at ALMA Bands 6 and 7 reaches the maximum when \( \alpha_t \sim 10^{-5} \) and cannot be higher even if the turbulence strength gets lower than 10^{-5}. This is because, in the weak turbulence regime (\( \alpha_t \lesssim 10^{-5} \)), almost all grains settle to the midplane in the same manner. Figure 9 shows a breakdown of extinction optical depth coming from each dust size at a radial position of 20 au of fixed models of \( a_{\text{max}} = 1000 \) and 2900 μm. We clearly see that the breakdown of the optical depth coming from each dust bin does not change with the turbulence strength for a very weak turbulence regime. If \( \alpha_t \ll \text{St} \ll 1 \), the dust scale height is

\[
h_d \sim \frac{\alpha_t}{\sqrt{\text{St}}} h_g.
\]

In this regime, the scale heights of the different dust populations change with \( \alpha_t \) in the same way. In other words, large grains cannot settle to the midplane with leaving small grains in the upper layer. Therefore, the location of the \( \tau = 1 \) can barely be reproduced. The models with \( \alpha_t \lesssim 10^{-5} \) yield the intensity corresponding to the lower and upper limits of the intensity at ALMA Bands 7 and 4, respectively.

In contrast, if the maximum dust size is 1 or 2.9 mm (Figure 7, middle and right), the observed intensity profile can be reproduced for a broad range of the turbulence strength because millimeter-sized grains have a high scattering albedo at ALMA Band 4. This is consistent with the result of Carrasco-González et al. (2019). Although the SED depends on the turbulence strength even in models with millimeter-sized grains, it is difficult to constrain the turbulence strength from the SED with the given intensity accuracy.

### 4.2. Polarization Fraction

As previous studies have shown, polarization should be the most effective when \( \lambda \sim 2\pi a_{\text{max}} \) in the no-settling limit. However, if differential settling takes place, the polarization behavior would depend on how many large grains exist above the surface where the vertical optical depth is unity. In this section, we investigate whether the observed polarization can be explained with millimeter-sized grains or needs 100 μm-sized grains, as previously expected.

Figure 8 shows the polarization fraction caused by self-scattering at the center of the HL Tau disk. The simulated images are convolved with the synthesized beam size of 0.3". It is worth noting that the beam size for the polarization analysis is six times larger than that for the SED analysis, meaning that the beam in the polarimetric observation averages a broader region.

If the maximum dust size is 160 μm, the simulated polarization fraction is \(~1\%\) at \( \lambda = 1\text{–}3 \) mm for \( \alpha_t \gtrsim 10^{-4} \), which is higher than the observed values. If \( \alpha_t \lesssim 10^{-5} \), the simulated polarization fraction at ALMA Band 7 is consistent with the observation but higher at ALMA Band 6. If \( \alpha_t = 10^{-6} \), the simulated polarization fraction at ALMA Band 7 is too small to explain the observation. The models with

| Model    | \( a_{\text{max}} \) (μm) | \( \alpha_t \) | \( \Sigma_0 \) (g cm\(^{-2}\)) | \( \tau_{0,8\text{mm}} \) | \( \epsilon \) | \( r_g \) (au) | \( d_g \) | \( w_g \) (au) |
|----------|----------------|-------------|-------------------------------|----------------|------|--------|------|------|
| Fixed    | 160 μm         | All         | 36.7                          | 0.59           | 1.98 | 12     | 0.9  | 4.5  |
|          | 1000 μm        | All         | 24.3                          | 8.9            | 2.28 | 12     | 0.97 | 5    |
|          | 2900 μm        | All         | 30.0                          | 86             | 2.05 | 12     | 0.98 | 9    |

| No settling | 160 μm | 10^{-2} | 36.7 | 0.59 | 1.98 | 12 | 0.9 | 4.5 |
|-------------|--------|---------|------|------|------|----|----|----|
| 10^{-3}     | 38.3   | 0.62    | 1.92 | 12   | 0.9 | 4.5 |
| 10^{-4}     | 46     | 0.74    | 1.71 | 12   | 0.9 | 4.5 |
| 10^{-5}     | 58.3   | 0.94    | 1.35 | 12   | 0.9 | 4.5 |
| 10^{-6}     | 60.0   | 0.96    | 1.32 | 12   | 0.9 | 4.5 |

| Tuned      | 1000 μm | 10^{-2} | 24.3 | 8.9  | 2.28 | 12 | 0.97 | 5   |
|-------------|---------|---------|------|------|------|----|-----|-----|
| 10^{-3}     | 23.7   | 8.7     | 2.30 | 12   | 0.97 | 5  |
| 10^{-4}     | 26     | 9.5     | 2.15 | 12   | 0.97 | 5  |
| 10^{-5}     | 32.7   | 12.0    | 1.75 | 12   | 0.97 | 5  |
| 10^{-6}     | 36     | 13.2    | 1.6  | 12   | 0.97 | 5  |

|                | No settling | 2900 μm | 10^{-2} | 29.3 | 84.0 | 2.09 | 12 | 0.98 | 9   |
|----------------|-------------|---------|---------|------|------|------|----|-----|-----|
| 10^{-3}        | 28.7       | 82.3    | 2.12    | 12   | 0.98 | 9  |
| 10^{-4}        | 19.3       | 55.3    | 2.23    | 12   | 0.98 | 9  |
| 10^{-5}        | 19.3       | 55.3    | 2.1     | 12   | 0.98 | 9  |
| 10^{-6}        | 19.3       | 55.3    | 2.02    | 12   | 0.98 | 9  |

\( \alpha_t = 10^{-5} \) and \( \alpha_t = 10^{-6} \) are all from Table 1. All 36.7 0.59 1.98 12 0.9 4.5

\( \alpha_t = 10^{-5} \) and \( \alpha_t = 10^{-6} \) are all from Table 1. All 30.0 86 2.05 12 0.98 9
surface relative to the dust scale height of each dust population does not change with \( \alpha_t \) (Figure 10). The critical dust radius above which the dust grain settles to the midplane \( (\alpha_t \sim \Sigma) \) can be estimated from Equation (5):

\[
a_{\text{crit}} \sim 11.4 \left( \frac{1.675 \, \text{g cm}^{-3}}{\rho_{\text{int}}} \right)^{-1} \times \left( \frac{\Sigma_g}{300 \, \text{g cm}^{-2}} \right)^{-1} \left( \frac{10^{-5}}{\alpha_t} \right)^{1/2} \mu\text{m}. \tag{8}
\]

From Equation (8), small grains can be left in the upper layer only if \( \alpha_t \lesssim 10^{-5} \).

From the polarization analysis, we can exclude the possibility that the maximum dust radius is \( >3 \) mm at the center part of the HL Tau disk. Even though the model with \( a_{\text{max}} = 160 \, \mu\text{m} \) predicts a higher polarization fraction than the observations, we cannot exclude the possibility that the maximum dust radius is \( 160 \, \mu\text{m} \) because the other polarization mechanisms might take place and reduce the polarization fraction due to self-scattering in the observing wavelength. However, by considering both the SED and polarization, we conclude that the maximum dust size is \( \lesssim 1 \) mm and the turbulence strength parameter is \( \lesssim 10^{-5} \).

5. Discussion

5.1. Is Turbulence Indeed Very Weak?

Our results show that the turbulence strength needs to be very low \( (\alpha_t \lesssim 10^{-5}) \). The efficient dust settling in the HL Tau disk is consistent with the indication from the geometrical thickness of the rings at the outer region (Pinte et al. 2016) and the previous SED analysis (Kwon et al. 2011, 2015). Although we put a upper limit on the turbulence strength as \( \alpha_t \sim 10^{-5} \), the lower limit would be potentially set from the infrared observations. Kwon et al. (2011) has shown that the HL Tau disk needs small grains that are vertically well mixed with gas to explain its high mid- and far-infrared emission. From Equation (8), dust grains smaller than \( 10 \, \mu\text{m} \) also settle to the midplane if \( \alpha_t \lesssim 10^{-7} \), indicating that \( \alpha_t \) needs to be \( \sim 10^{-5} \), not well below \( 10^{-7} \). The near-infrared scattered-light observations would be helpful to constrain how many small grains are mixed up. For example, the infrared observation toward the HD 163296 disk has shown that the outer region of the disk needs very weak turbulence corresponding to \( \alpha_t \sim 10^{-5} \) to explain the nondetection of the scattered light (Muro-Arena et al. 2018). However, the characterization from the scattered light is difficult for the HL Tau disk because it is still embedded in a massive envelope that prevents us from investigating the disk surface structure (Beckwith et al. 1986, 1990).

The very weak turbulence is consistent with the recent nonideal MHD models where the magnetorotational instability (MRI) is suppressed at the disk midplane (e.g., Gammie 1996; Bai 2017). However, even if MRI is suppressed, pure hydrodynamic instabilities would induce turbulence and lift up dust grains (e.g., Flock et al. 2017, 2020).

The gas surface density is also an important parameter for the dust settling, although we fixed it as \( 1000 \, (r/\text{au})^{-0.5} \) g cm\(^{-2}\). As shown in Equation (4), the settling behavior is determined by the ratio of \( \Sigma \) to \( \alpha_t \) and hence by \( \alpha_t \Sigma \). Therefore, if the gas surface density is 10 times lower than our model, 10 times higher \( \alpha_t \) is acceptable. If the gas surface density is 10 times lower than our model, the dust-to-gas mass ratio is an order of 0.3 or higher. In this case, the streaming instability might be operating, and dust grains might be converted into planetesimals (e.g., Youdin & Goodman 2005). If the dust-to-mass ratio is high enough, turbulence can no longer lift the dust grains due to the drag force from dust to gas, which also helps the streaming instability take place at the midplane (Lin 2019).

5.2. Impact of Other Polarization Mechanisms

As mentioned above, this study focuses only on the polarization induced by self-scattering. However, in observed disks, the other polarization mechanisms would operate and potentially cancel out the self-scattering polarization. Our models with \( a_{\text{max}} = 1 \, \text{mm} \) and \( \alpha_t \lesssim 10^{-5} \) predict a polarization fraction comparable to the observations at ALMA Bands 6 and...
while it is significantly higher at ALMA Band 3, where the observed value is less than 0.1%. This indicates that the polarization fraction originating from the other mechanisms should be negligible at \( \lambda \sim 1 \) mm and \( \lesssim 0.3\% \) (the polarization vector should be perpendicular to that from self-scattering) at ALMA Band 3. The polarization induced by dust alignment is also sensitive to the dust size (Guillet et al. 2020). Guillet et al. (2020) showed that the alignment-induced polarization can be more effective at ALMA Band 3 than at ALMA Bands 6 and 7 if \( a_{\text{max}} \sim 250 \) \( \mu \)m, while it is less sensitive to the observing wavelength if \( a_{\text{max}} \sim 1 \) mm. This implies that if both self-scattering and alignment are taken into account, the true maximum dust size might be between 100 \( \mu \)m and 1 mm.

The nonuniformity of the polarization at Band 6 indicates that the alignment-induced polarization takes place at this wavelength (Stephens et al. 2017), which seems to be in conflict with our millimeter-sized dust model. However, in our model, we focus only on the optically thick inner region where the differential settling has a strong impact on the scattering-induced polarization. The optical depth would decrease with the radial distance; hence, the differential settling would have a smaller impact at a more outer region. This indicates that a polarization mechanism other than self-scattering can dominate the polarization at the outer region, while self-scattering dominates at the inner region.

In addition to these uncertainties, the dust shape and internal structure also affect the polarization efficiency, as well as the alignment efficiency of dust grains (Tazaki et al. 2019; Kirchschlager et al. 2019; Kirchschlager & Bertrang 2020; Guillet et al. 2020), which makes the interpretation of the disk polarization more complicated. A comprehensive study including these effects would be necessary to understand the complex polarization behavior from the HL Tau disk.

### 5.3. Effect of Vertical Temperature Structure

In our simulations, the temperature structure is assumed to be vertically isothermal. However, the vertical temperature structure in protoplanetary disks is not necessarily isothermal in the vertical direction and might have an impact on the SED.

If the disk is passively heated by the central star, the temperature of the disk layer above the absorption surface for the stellar light would be higher than that of the layer below the surface. The absorption surface of the disk is typically \( \sim 4–5 \) times above the scale height of the disk, which is far enough above the layer observed by ALMA and the VLA. Therefore,
the assumption of the vertically isothermal temperature structure will be valid if the HL Tau disk is passively heated. If the disk is heated by gas accretion, the disk interior has a higher temperature than the upper layer (e.g., Chiang & Goldreich 1997). The internal heating affects the SED, since the longer observing wavelength traces the hot lower layer (Sierra & Lizano 2020). The observed SED shows that the intensity at ALMA Band 4 is lower than the intensity extrapolated from the intensity at shorter wavelengths with a spectral slope of 2. This indicates that the internal heating is not so effective, otherwise the SED needs an extremely strong intensity reduction at ALMA Band 4.

5.4. Polarization at (Sub)centimeter Wavelengths

Our results showed that the observed polarization fraction can be explained with a maximum dust size of 1000 μm, but 160 μm–sized grains might also be acceptable if the other polarization mechanisms operate. Our results also showed that the polarization fraction at λ = 8 mm is 0.1% for $a_{\text{max}} = 160 \mu m$, while it is ~4% for $a_{\text{max}} = 1000 \mu m$. This is because the polarization efficiency has a peak at $\lambda \sim 2\pi a_{\text{max}}$ (Kataoka et al. 2015), and the dust settling has a little impact on the polarization at $\lambda \sim \text{cm}$ owing to the low optical depth. Therefore, the polarimetric observation at (sub)centimeter wavelengths using, e.g., the ngVLA will be a good tool to solve the degeneracy and know the true dust sizes in protoplanetary disks.

5.5. Radial Profile of the Dust Size and Turbulence Strength

In this paper, we focus only on the intensity at the center of the disk images, but it would be worthwhile to refer to the radial distribution. Figure 11 compares the radial intensity profile obtained in the tuned model ($a_{\text{max}} = 1000 \mu m$ and $\alpha_t = 10^{-5}$) with observed one. Our model predicts higher intensity at the gap region, especially at ALMA Band 4. This indicates that the dust size (and hence $\varepsilon$) and/or turbulence strength in the gap would be different from our model. To reduce the intensity of ALMA Band 4, we need a higher albedo.
than our model, suggesting that the turbulence strength is stronger than our model in the gap if the dust size is $d_{\text{max}} = 1000 \mu m$. Although detailed modeling of the radial intensity profile is outside our focus, it will be important to model the radial dependence for understanding how disk properties change across the substructures. Polarimetric observations with higher angular resolution with ALMA would be required to quantify the disk properties in more detail.

We also performed simulations without the gap to check the effect of the gap structure and found that the polarization fraction is slightly higher for the no-gap case. This is because the no-gap model is optically thicker, which makes the effective dust size smaller. We confirmed that the slight difference due to the gap has little impact on our results.

The HL Tau disk also shows the uniform polarization pattern even in the outer region ($\sim$100 au), indicating that the dust size is $\sim$100 $\mu m$ for the entire region. Although we demonstrated that the millimeter-sized grains can explain the observed polarization fraction at the center of the disk, it is not clear that the differential settling can explain the polarization observations in the entire region of the disk. If the disk is optically thin, the differential settling no longer has an impact on the SED and polarization fraction.

Okuzumi & Tazaki (2019) showed that the observed polarization pattern in the HL Tau disk can be explained by the fragmentation of nonsticky icy grains. However, the uniformity of the polarization degree needs a flat gas density profile because the size of the fragments, which is determined through the Stokes number, needs to be uniform. From our results, since the millimeter-sized grains can explain the observed polarization in the optically thick inner region, the gas density profile might not need to be flat. The detailed modeling for the entire disk, including the differential settling, will give us a comprehensive understanding of the dust evolution in the radial direction.

6. Summary

We performed radiative transfer simulations of disks with an analytical model of a settling–mixing equilibrium of dust grains. The simulated SED and polarization fraction were compared with the observations of the protoplanetary disk around HL Tau to constrain the dust size and turbulence strength.

The SED of the center part of the HL Tau disk shows that the intensity slope between ALMA Bands 6 and 7 is consistent with a spectral index of 2 within the error. The observed intensity at ALMA Band 4 is below the value extrapolated from the intensity at ALMA Bands 6 and 7 with a spectral slope of 2. The models with a maximum dust size of 160 $\mu m$ can reproduce the observed SED only if $\alpha_t \lesssim 10^{-5}$, while the models with millimeter-sized grains can reproduce with a broad range of $\alpha_t$.

The polarization analysis allowed us to constrain the turbulence strength more strongly. If the maximum dust size is 160 $\mu m$, the polarization fraction is comparable to or higher than the observed value at ALMA Bands 6 and 7 for $\alpha_t \gtrsim 10^{-5}$. If $\alpha_t \sim 10^{-6}$, the simulated polarization fraction at ALMA Band 7 is lower than the observed value. The models with a maximum dust size of 1 mm can explain the observed polarization fraction at ALMA Bands 6 and 7 if the turbulence strength parameter $\alpha_t$ is $\lesssim 10^{-5}$. Although the observed polarization fraction at ALMA Band 3 is lower than that expected from the models, the other polarization mechanisms might reduce the scattering-induced polarization fraction. If the maximum dust size is 3 mm, the simulated polarization fraction at ALMA wavelengths is significantly lower than the observed values.

To explain both the SED and polarization, a maximum dust size of $\lesssim 1$ mm and turbulence strength parameter of $\lesssim 10^{-5}$ are required. The efficient dust settling in the HL Tau disk is consistent with the previous studies (Kwon et al. 2011, 2015; Pinte et al. 2016). The degeneracy between 100 $\mu m$– and millimeter-sized grains can be solved by the polarimetric observations at (sub)centimeter wavelengths using, e.g., ngVLA. These results showed that the differential settling has a key role in understanding the polarimetric observations on the optically thick inner region of disks.

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Software: RADMC-3D (Dullemond et al. 2012).

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