$B_{d,s}^0 \rightarrow \mu^- \mu^+$ DECAY IN THE MSSM

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Abstract

We present the results of the complete one-loop computation of the $B_{d,s}^0 \rightarrow l^+ l^-$ decay rate in the MSSM. Both sources of the FCNC, the CKM matrix and off-diagonal entries of the sfermion mass matrices are considered. Strong enhancement of the branching ratio (compared to the SM prediction) can be obtained in the large $\tan \beta \sim m_t/m_b$ regime in which the neutral Higgs boson “penguin” diagrams dominate. We make explicit the strong dependence of this enhancement on the top squarks mixing angle in the case of the chargino contribution and on the $\mu$ parameter in the case of the gluino contribution. We show that, in some regions of the MSSM parameter space, the branching ratio for this process can be as large as $10^{-5\,\ldots\,4}$ respecting all existing constraints, including the CLEO measurement of $BR(B \rightarrow X_s \gamma)$. We also estimate, that for chargino and stop masses $\sim \mathcal{O}(100 \text{ GeV})$ $BR(B_s^0 \rightarrow l^+ l^-)$ with $ll' = e\tau$ or $\mu\tau$ can be of the order of $10^{-11}$ for the still allowed values of the off-diagonal entries in the slepton mass matrix.

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1. Introduction

Extensions of the Standard Model (SM) usually predict new contributions to the flavour changing neutral current (FCNC) processes. For example, adding in the most general way a second doublet of the Higgs fields to the standard theory of electroweak interactions typically leads to large amplitudes of FCNC processes mediated at the tree level by neutral Higgs particles. Restricting appropriately the possible form of couplings of the two Higgs doublets to up- and down-type fermions eliminates such tree level contributions to FCNC processes but, of course, new contributions induced by loops involving the physical charged scalar still remain. Charged Higgs boson contributions to FCNC processes depend however on the same elements of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix as does the standard $W^\pm$ boson contribution and, thus, amplify the effects of the FCNC source that is present in the SM, rather than being an independent new source of such processes. Nevertheless, requiring the effects of the charged Higgs boson not to spoil succesful predictions of the standard theory leads to interesting bounds on the $(M_{H^\pm}, \tan \beta)$ plane where $\tan \beta \equiv v_2/v_1$ is the ratio of the vacuum expectation values of the two Higgs doublets. In particular, in the popular two Higgs doublet model of type II (2HDMII), in which the first doublet couples only to leptons and down-type quarks and the second one couples to up-type quarks only, processes like $B \rightarrow X_s \gamma$ and $K^0\bar{K}^0$ mixing together with $Z^0 \rightarrow b\bar{b}$ constrain the plane $(M_{H^\pm}, \tan \beta)$. In particular, for $\tan \beta \gtrsim 3$, $B \rightarrow X_s \gamma$ requires $H^\pm$ to be heavier than $\sim 165 - 200 \text{ GeV}$ [1].

Supersymmetric models like the MSSM, which are the most popular and best motivated extensions of the SM, apart from containing a charged Higgs boson $H^\pm$, induce yet additional contributions to the FCNC processes. Firstly, in such models the effects of the CKM mixing can be further amplified through the loops involving charginos and squarks. Secondly, as there is no reason why the squark mass matrices should be diagonal in the same (so-called super-CKM) basis as quarks, the sfermion sector of such models is in general a new, independent of the CKM matrix, source of the FCNC processes.

Current experimental data on FCNC processes provide important constraints on these sources of flavour nonconservation in supersymmetric models. (Extensive reviews are the refs. [2] and [3].) Taking the CKM matrix as the only source of the FCNC processes, the current experimental data on $B \rightarrow X_s \gamma$ and $K^0\bar{K}^0$, $B^0\bar{B}^0$ mixings impose some constraints on the MSSM parameter space. These constraints, which correlate masses and composition of charginos and top squarks with the mass of the charged Higgs boson, depend in part on the element $V_{td}$ of the CKM matrix which is not directly measured [3, 4] and become weaker with growing sparticle masses. Effects of the nonzero off-diagonal entries of the sfermion mass matrices are usually analyzed separately [2, 3]. Stringent constraints apply to the entries causing transitions between the first two generations. Bounds on the entries connected to the third generation, which follow from the $B^0\bar{B}^0$ mixing and $B \rightarrow X_s \gamma$ decay are much weaker. Thus, large deviations from the rates predicted in the SM can still be discovered in the forthcoming (or already running) experiments like BaBar (SLAC), BELLE (KEK), CLEO (Cornell), HERA-B (DESY) and LHC (CERN). In this context, particularly interesting process to look at are the decays $B^0_{s(d)} \rightarrow l^+l^-$ because they are clean theoretically being almost free of hadronic uncertainties.
Several papers analysed this process in the MSSM \[5, 6, 7\] under various assumptions and with different approximations. In this paper we perform a complete calculation \[1\] of the process \(B_{s,d}^0 \rightarrow l^- l^+\) in the MSSM with emphasis on qualitative understanding of the dominant effects. We reconfirm that for values of \(\tan \beta < 20\) the rate of this process is not significantly enhanced compared to the prediction of the SM (apart from the case of \(\tan \beta \sim 0.5\) and light \(H^\pm \)) which is not favoured theoretically within the supersymmetric framework. In agreement with earlier papers \[6\] we find that large enhancement of the branching ratio is obtained in the case of large \(\tan \beta\) values due the neutral Higgs boson penguin graphs. This has been previously made explicit in ref. \[7\] (possible role of such contributions to \(K^0-\bar{K}^0\) and \(B^0-\bar{B}^0\) mixings has been emphasized in ref. \[9\]) in which the contributions of charginos as a source of the flavour changing has been considered. We demonstrate strong dependence of the decay rate on the value of the stop mixing angle and explain it using our analytic formulae. Moreover, we extend previous calculations by analysing also the case of the flavour mixing induced by squark mass matrices. In the latter case we find very strong dependence on the \(\mu\) parameter. Finally we correlate the predictions for \(B_{s,d}^0 \rightarrow l^- l^+\) with constraints imposed on the parameter space by other processes, in particular by the measurement by CLEO \[10\] of the \(B \rightarrow X_s \gamma\) branching ratio. We find that even respecting all those constraints, \(BR(B_s^0 \rightarrow \mu^- \mu^+)\) can be enhanced up to \(10^{-(4-5)}\) for \(\tan \beta \sim m_t/m_b \approx 50\). Moreover, for such values of \(\tan \beta\), and the off-diagonal 13 entries of the down-type squark mass matrix saturating the existing bound \[2, 3\], also \(BR(B_d^0 \rightarrow \mu^- \mu^+)\) can be of the same order of magnitude. This means that the unsuccessful search done at CLEO \[11\] already provides a constraint on the 13 off-diagonal entries of the down-type squarks which, for some values of the other MSSM parameters, is stronger than the one given in \[2, 3\]. Finally, we also estimate that for chargino and stop masses \(\sim \mathcal{O}(100\ GeV)\) \(B_s^0 \rightarrow l^+ l'^-\) with \(ll' = e\tau\) or \(\mu \tau\) can be of order \(10^{-11}\) for the still allowed values of the off-diagonal entries in the slepton mass matrix.

2. General structure of the amplitude and the SM prediction

The effective Lagrangian describing the \(d_I \bar{d}_J \rightarrow \bar{l}_A l_B\) transition has the general form

\[
\mathcal{L}_{\text{eff}} = \sum_x C_x \mathcal{O}_x
\]

in which \(\mathcal{O}_x\) are the local four-fermion operators and \(C_x\) are their Wilson coefficients (we suppress quark and lepton flavour indices on \(\mathcal{O}_x\) and \(C_x\) as well as on various formfactors which will appear in the following). Four vector-vector operators \(\mathcal{O}_{XY}^V \equiv (\bar{d}_J \gamma_\mu P_X d_I)(\bar{l}_B \gamma^\mu P_Y l_A)\) and four scalar operators \(\mathcal{O}_{XY}^S \equiv (\bar{d}_J P_X d_I)(\bar{l}_B P_Y l_A)\) (where \(X, Y = LL, RR, LR\ and RL\)) contribute to this process. In addition, two tensor operators exist but they do not contribute to this process (their matrix elements vanish when taken between one meson and vacuum states). In the following we will specify the formulae to the case of \(B_{d(s)}^0 \rightarrow \bar{b}d(s)\) decay, hence we will take \(J = 3\) and \(I = 1 = d\) for \(B_d^0\) or \(I = 2 = s\) for \(B_s^0\). Furthermore, due to the pseudoscalar

\[\text{In what follows we display only formulae for the dominant contributions. The plots are, however, based on the programme including contributions from all relevant one-loop diagrams.}\]
nature of the $B_I^0$ mesons we need only two matrix elements\footnote{The second follows from the first one by using the QCD equation of motion for the quark field operators; this fixes their relative sign.}

\[
\langle 0 | \bar{b} \gamma^\mu \gamma^5 d_I | B_I^0(q) \rangle = -i f_{B_I} q^\mu \\
\langle 0 | \bar{b} \gamma^5 d_I | B_I^0(q) \rangle = +i f_{B_I} \frac{M_B^2}{m_{d_I} + m_b}
\]

Using (2) one finds the total $B^0$ width

\[
\Gamma = \frac{M_B}{16\pi} f(x_A^2, x_B^2) \left\{ |a|^2 \left[ 1 - (x_A - x_B)^2 \right] + |b|^2 \left[ 1 - (x_A + x_B)^2 \right] \right\}
\]

where $f(x, y) \equiv \sqrt{1 - 2(x + y) + (x - y)^2}$, $x_A \equiv m_{l_A}/M_B$ and the coefficients $a$ and $b$ are given in terms of the Wilson coefficients as

\[
a = \frac{f_{B_I}}{4} \left\{ (m_{l_B}^2 + m_{l_A}) \left[ C^{V}_{LL} - C^{V}_{LR} + C^{V}_{RR} - C^{V}_{RL} \right] - \frac{M_B^2}{m_b} \left[ C^{S}_{LL} - C^{S}_{LR} + C^{S}_{RR} - C^{S}_{RL} \right] \right\}
\]

\[
b = -\frac{f_{B_I}}{4} \left\{ (m_{l_B}^2 - m_{l_A}) \left[ C^{V}_{LL} + C^{V}_{LR} - C^{V}_{RR} - C^{V}_{RL} \right] - \frac{M_B^2}{m_b} \left[ C^{S}_{LL} + C^{S}_{LR} - C^{S}_{RR} - C^{S}_{RL} \right] \right\}
\]

where we have neglected $m_{d_I}$ compared to $m_b$.

Three groups of diagrams contribute to the Wilson coefficients: Box diagrams, $Z^0$ penguin diagrams and neutral Higgs boson penguin diagrams\footnote{At the one-loop level the photon penguin diagram does not contribute for the $\bar{\ell} \ell$ final state due to the vector current conservation. As long as neutrinos are massless, the $\bar{\ell} \ell'$ final state can appear neither in the SM nor in the 2HDM; in the MSSM the $\bar{\ell} \ell'$ final state can only be due to box contribution provided the slepton mass matrices remain non-diagonal in the lepton mass eigenstate basis.}. Denoting the self energy diagrams on the external quark lines as $-i\Sigma(p)$, with

\[
\Sigma(p) = \Sigma^V_L \not{p} P_L + \Sigma^V_R \not{p} P_R + \Sigma^S_L P_L + \Sigma^S_R P_R,
\]

and vertex corrections to the couplings $\bar{d} J d I Z^0$, $\bar{d} J d I S^0$ and $\bar{d} J d I P^0$, where $S^0 (P^0)$ is a neutral scalar (pseudoscalar), respectively as

\[
+ i \gamma^\mu \left( F^V_L P_L + F^V_R P_R \right) - i \left( F^S_L P_L + F^S_R P_R \right) - \left( F^P_L P_L + F^P_R P_R \right),
\]

one finds (in the approximation $\Sigma(p^2) \equiv \Sigma(0)$, $F(q^2) = F(0)$) the following expressions for the Wilson coefficients generated by various penguin diagrams:

\[
C^{V}_{XY} = -\frac{e}{2s_Wc_W M_Z^2} \hat{F}^{V}_{X} \epsilon_{Y}, \quad X, Y = L, R
\]
from the Z\textsuperscript{0} penguin diagram, with \( c^e_L = 1 - 2s^2_W \), \( c^e_R = -2s^2_W \) and \( s_W \) (\( c_W \)) is the sine (cosine) of the Weinberg angle;

\[
C^S_{LL} = C^S_{LR} = \sum_{k=1,2} \frac{1}{M^2_{H^0_k}} \frac{Z^1_k}{v_1} \hat{F}^S_{L,R} m_l \quad C^S_{RR} = C^S_{RL} = \sum_{k=1,2} \frac{1}{M^2_{H^0_k}} \frac{Z^1_k}{v_1} \hat{F}^S_{R,L} m_l
\]  

from neutral scalar penguin diagrams; and

\[
C^S_{LL} = -C^S_{LR} = \sum_{k=1,2} \frac{1}{M^2_{H^0_{k+2}}} \frac{Z^1_k}{v_1} \hat{F}^P_{L,R} m_l \quad C^S_{RR} = -C^S_{RL} = \sum_{k=1,2} \frac{1}{M^2_{H^0_{k+2}}} \frac{Z^1_k}{v_1} \hat{F}^P_{R,L} m_l
\]  

from neutral pseudoscalar penguin diagrams. We use here (and throughout) the notation of ref. [12] in which \( H^0_k \equiv (h^0, H^0), H^0_{2+k} \equiv (A^0, G^0), H^\pm_k \equiv (H^\pm, G^\pm) \) and \( Z^1_k (Z^1_k) \) denotes the projection of the \( k \)-th physical neutral CP-even (-odd) Higgs boson onto the real (imaginary) part of the neutral component of the Higgs doublet that couples to the down-type quarks. In addition, since at one loop penguin graphs cannot generate transitions \( B^0 \to \bar{\ell} \ell' \), we have set \( m_{l_A} = m_{l_B} = m_l \). In these formulae

\[
\hat{F}^V_{L,R} = F^V_{L,R} + \frac{e^d}{2s_W c_W} \epsilon^d_{L,R} \Sigma^V_{L,R}
\]  

where \( \epsilon^d_L = 1 - 2s^2_W/3 \), \( \epsilon^d_R = -2s^2_W/3 \),

\[
\hat{F}^S_{L,R} = F^S_{L,R} - \frac{Z^1_k}{v_1} C^S_{L,R}
\]  

and

\[
\hat{F}^P_{L} = F^P_{L} + \frac{Z^1_k}{v_1} \Sigma^P_{L}, \quad \hat{F}^P_{R} = F^P_{R} - \frac{Z^1_k}{v_1} \Sigma^S_{R}
\]  

are the full effective vertices including the effects of flavour changing self energy diagrams on the external quark lines. Box diagram contributions to the Wilson coefficients can also be easily found. From eqs. (4,5,8-10) one sees that scalar penguin diagrams contribute only to \( b \) in eq (3) whereas the coefficient \( a \) receives contributions from both \( Z^0 \) and the pseudoscalar penguin diagrams. The relative sign of the \( Z^0 \) and the neutral Goldstone boson contributions to \( a \) should be such that the total contribution is independent of the gauge chosen for the \( Z^0 \) propagator. This is the case if

\[
-m_J \hat{F}^V_{L,R} + m_J F^V_{L,R} = -M_Z \hat{F}^P_{L,R}
\]  

for \( P \) referring to the Goldstone boson. Since the formfactor \( \hat{F}^P_{L,R} \) for the physical pseudoscalar \( A^0 \) is related to the one for \( G^0 \) by the \( SU_L(2) \) symmetry, this relation tests also the relative sign of the \( Z^0 \) and pseudoscalar penguin diagrams.

The SM contribution to the \( B^0 \to \ell \ell \) decay is well known [13] (see also [14]). The Higgs boson couplings to fermions are not enhanced so the scalar and pseudoscalar penguins are
negligibly small. The only important box diagram is the one with two $W^\pm$ which contributes only to $C_{LL}^V$

$$C_{LL}^V = -\frac{1}{16\pi^2} \left( \frac{e}{s_W} \right)^4 \frac{\lambda_{II} x_t}{M_W^2} \frac{1}{1 - x_t} \left[ \frac{1}{1 - x_t} + \log x_t \right]$$

where $x_t \equiv (m_t/M_W)^2$ and $\lambda_{II} \equiv V^*_{tI} V_{tI}$. The effective $\bar{d}_f d_I Z^0$ vertex receives contributions from loops involving both $W^\pm$ and the charged Goldstone bosons. One finds

$$\hat{F}_L^V = \frac{1}{16\pi^2} \frac{e^3}{4 s_W^2 c_W} \lambda_{II} x_t \left[ \frac{x_t - 6}{1 - x_t} - \frac{3x_t + 2}{(1 - x_t)^2} \log x_t \right]$$

and $\hat{F}_R^V = 0$ in the limit of $m_{d_I} = 0$. Adding all one gets

$$BR(B^0_I \to \bar{b}l) = \tau(B^0_I) \left[ \frac{G_F \alpha}{4\pi s_W^2} \right]^{1/2} \frac{f_{B_I} m_{B_I}^2 |\lambda_{II}|^2}{\pi} \left[ 1 - 4 \frac{m_t^2}{M_{B_I}^2} Y_0(x_t) \right]$$

where $\tau(B^0_I)$ is the lifetime of the $B^0_I$ meson and [13, 14]

$$Y_0(x_t) = -\frac{x_t}{8} \left[ \frac{x_t - 4}{x_t - 1} + \frac{3x_t}{(x_t - 1)^2} \log x_t \right]$$

In general, taking QCD corrections into account consists of computing corrections to the Wilson coefficients at the scale $\sim M_Z$, and subsequently evolving the latter from the electroweak scale down to the hadronic scale $\mu_h \sim m_b$. The first step of this procedure amounts to replacing $Y_0(x_t)$ by $Y(x_t) = Y_0(x_t) + (\alpha_s/4\pi) Y_1(x_t)$ [13] where now $x_t = (\bar{m}_t(m_t^2)/M_W^2)$. $Y(x_t)$ can be conveniently parametrized as [16]

$$Y(x_t) = 0.997 \left[ \frac{\bar{m}_t(m_t)}{166\text{GeV}} \right]^{1.55}$$

As far as the evolution is concerned, it has been noted in [16] that the vector operators contributing to $B^0 \to \bar{b}l$ have zero anomalous dimensions. Hence, their Wilson coefficients do not evolve at all, whereas the evolution of the Wilson coefficients of the scalar operators result in multiplying them by $m_b(\mu_h)/m_b(M_Z)$. Consequently, if $C_{XY}^S$ are proportional to $m_b(M_Z)$, their evolution is taken into account if this factor is replaced by $m_b(\mu_h)$ which in turn cancels out with the factor $1/m_b(\mu_h)$ present in eqs. (14). As it will be apparent, whenever the coefficients $C_{XY}^S$ are large, they are indeed proportional to $m_b(M_Z)$. In the SM, including QCD corrections one finds [16]

$$BR(B^0_s \to \bar{b}l) = 4.1 \times 10^{-9} \left[ \frac{\tau(B_s)}{1.54 \text{ ps}} \right] \left[ \frac{f_{Bs}}{245\text{ MeV}} \right] \left[ \frac{|V_{ts}|}{0.040} \right] \left[ \frac{\bar{m}_t(m_t)}{166\text{GeV}} \right]^{3.12}$$

3. Contribution of the extended Higgs sector

As remarked in the introduction, the presence of the physical charged Higgs boson in the extended Higgs sector of the MSSM (or 2HDM) in general enhances the FCNC transition
rates generated by the CKM mixing matrix. This enhancement can appear through the $H^\pm$ contribution to box diagrams, $Z^0$ penguin diagrams and neutral Higgs boson penguin diagrams. The latter type of diagrams can only be important in the large $\tan \beta \gtrsim 30$ regime in which the neutral Higgs boson couplings to the down-type quarks and charged leptons are enhanced by $\tan \beta$ factors.

For low values of $\tan \beta \lesssim 20$ the neutral Higgs boson penguin diagrams are small. It is also easy to check, that for such $\tan \beta$ values no box diagram can give significant contribution. Thus, the only large contribution can be due to the $H^\pm$ contribution to the $Z^0$ penguin diagrams. Computing the relevant self energy diagrams (vector parts thereof) and vertex corrections one arrives at

$$\Delta \hat{F}_L^V = \frac{1}{16\pi^2} \left( \frac{e^3}{s_W c_W} \right)^3 \lambda_{tt} \cot^2 \beta \frac{m_t^2}{M_Z^2} \frac{1}{4} \frac{y_t}{1-y_t} \left[ 1 + \frac{1}{1-y_t} \log y_t \right]$$

$$\Delta \hat{F}_R^V = -\frac{1}{16\pi^2} \left( \frac{e^3}{s_W c_W} \right)^3 \lambda_{tt} \tan^2 \beta \frac{m_t m_{d_4}}{M_Z^2} \frac{1}{4} \frac{y_t}{1-y_t} \left[ 1 + \frac{1}{1-y_t} \log y_t \right]$$

where $y_t \equiv (m_t/M_{H^+})^2$. Taking into account only $\Delta \hat{F}_L^V$ which is enhanced for $\tan \beta < 1$ amounts to replacing $Y(x_t)$ in eq. (17) by

$$Y(x_t) \rightarrow Y(x_t) - \cot^2 \beta \frac{x_t y_t}{8} \left[ 1 + \frac{1}{1-y_t} \log y_t \right] \quad (20)$$

The new contribution has the same sign as $Y(x_t)$ and, therefore, enhances the SM contribution. For example, for $\tan \beta = 0.5$ and $M_{H^+} = M_W$, $BR(B^0 \rightarrow \bar{t}l)$ is enhanced by a factor of $(1 + \frac{1506}{0.997})^2 \approx 6.6$ compared to the SM prediction.

For large $\tan \beta \sim m_t/m_b$ in the case of $B^0_s$ decay, $\hat{F}_R^V$, despite being suppressed by one power of $m_s/M_W$, is about two orders of magnitude larger than $\hat{F}_L^V$ and, for $M_{H^+} \sim 100$ GeV, is of the order of the SM contribution. However, in this regime, there are other contributions which are more important [17, 19].

Firstly, the mixed, $W^\pm H^\pm$, box diagram in which $H^\pm$ couples to the $b$-quark is also $O(\tan^2 \beta)$ and is not suppressed by $m_{d_4}/M_W$. After summation over different types of virtual quarks it gives [16]

$$C_{LR}^S = \frac{1}{16\pi^2} \left( \frac{e}{s_W} \right)^4 \frac{m_t m_{d_4}}{M_W^2} \lambda_H \tan^2 \beta \frac{1}{4} \frac{y_t}{x_H - x_t} \left[ 1 + \frac{1}{x_H - 1} \log x_H - \frac{1}{x_t - 1} \log x_t \right] \quad (21)$$

where $x_H \equiv (M_{H^\pm}/M_W)^2$. The other $W^\pm H^\pm$ box is proportional to $m_{d_4}/M_W$ and hence it is less important. The box diagrams containing two charged scalars (either physical or Goldstone) are suppressed always by $(m_t/M_W)^2$. Therefore, although the $H^\pm H^\pm$ box grows as $\tan^4 \beta$, it is not important even for $\tan \beta \sim 50$.

Secondly, there are neutral Higgs boson penguin diagrams. It turns out [16], that the dominant i.e. $\sim \tan \beta$ part of the genuine $d_J d_I S^0(P^0)$ vertex correction cancels out [3] and the

\footnote{This cancellation is even simpler in the case of the MSSM than in the case of the 2HDM(II) considered in [16].}
only contribution arises from the scalar parts of the self energies of external quarks (for $B^0$ decay it is $\Sigma^S_R$ which is dominant):

$$\Sigma^S_L = \frac{1}{16\pi^2} \left( \frac{e}{s_W} \right)^2 m_t \lambda t x t \left[ \frac{x_H - 1}{x_H - x_t} \log x_H - \frac{x_t}{x_H - 1} \log x_t \right]$$

where $x_H = (M_{H^\pm}/M_W)^2$.

Using the formulae (11,13) and the fact that in the MSSM for neutral CP-even scalars for large values of $\tan \beta$ the following relations hold

$$\sin^2 \alpha \approx 1, \quad M^2_h \approx M^2_A \quad \text{for} \quad M_A < M_Z$$
$$\cos^2 \alpha \approx 1, \quad M^2_{H^0} \approx M^2_A \quad \text{for} \quad M_A > M_Z$$

$$M^2_{H^+} = M^2_A + M^2_W$$

we can now summarize the dominant contribution of the extended Higgs sector to the the coefficients $a$ and $b$ given by eqs. (4,5) [16]:

$$a = \frac{1}{16\pi^2} \frac{f_{B_s}}{2} \left( \frac{e}{s_W} \right)^4 \frac{m_t}{M^2_W} \lambda t X H - \frac{M^2_{B_s}}{8M^2_W} \tan^2 \beta \log \frac{r}{r - 1}$$

$$b = -\frac{1}{16\pi^2} \frac{f_{B_s}}{2} \left( \frac{e}{s_W} \right)^4 \frac{m_t}{M^2_W} \lambda t Z (x_t) - \frac{M^2_{B_s}}{8M^2_W} \tan^2 \beta \log \frac{r}{r - 1}$$

where $r \equiv 1/y_t = (M_{H^\pm}/m_t)^2$. Since $\frac{\log r}{r^2} > 1$, the CP-odd neutral Higgs exchange interferes destructively with the SM contribution. Figures [6] show the contribution of the extended Higgs sector of the MSSM (assuming that sparticles contribute negligibly) or of the 2HDM(II) [6] to $BR(B^0_{s,d} \to \mu^- \mu^+)$. These results which should be compared with the SM results $4 \times 10^{-9}$ and $1 \times 10^{-10}$, respectively, agree for $\tan \beta \sim 30$ with the ones given in [16] and, for smaller values of $\tan \beta$, update the computations done earlier in refs. [8, 17].

4. Chargino contribution

Another source of amplification of the flavour changing transitions induced by the CKM matrix is the chargino sector of the MSSM. Assuming that the squark mass matrices are diagonal in the super-CKM basis, the first result is that in the whole relevant parameter space the box diagram contribution to any of the Wilson coefficients remains small compared to the SM contribution. Furthermore, the $Z^0$ penguin can change the predicted $BR(B^0_{s,d} \to \mu^- \mu^+)$ by no more than $\sim 5$-10% for $\tan \beta \sim 2$ and $\sim 20$% for $\tan \beta \sim 0.5$. The magnitude and sign of this

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5 The well-known large radiative corrections to $h^0$ and $H^0$ masses do not spoil these relations. Moreover, these corrections do not affect the neutral Higgs boson penguin contributions because they always modify significantly only the mass of that Higgs boson which almost does not couple to the down-type quarks and charged leptons.

6 In the case of the 2HDM(II) the subleading in $\tan \beta$ contributions of the genuine vertex corrections in the CP-even Higgs boson penguin may be different than in the MSSM because the dimensionfull couplings $H^+ H^0 H^0(h^0)$ differ in both models [17]. Still, unless these couplings in the 2HDM(II) are very large (and numerically very different from their MSSM counterparts) so as to enhance the otherwise subleading contribution, figs [6] should be fairly representative also for the 2HDM(II) results.
contribution depends, apart from the masses of the sparticles involved, also on the chargino composition and on the mixing angle of the top squarks. For natural stop composition i.e. when the lighter stop is predominantly right-handed and the mixing angle is not too large \[18\], the chargino loop contribution to the Wilson coefficients has opposite sign to that of the top quark loop and, hence, decreases the rate of the \(B^0_{s,d} \rightarrow \mu^- \mu^+\) decay. This is very similar to the opposite, as compared to the SM, sign of the chargino-stop loop contribution to \(R_b \equiv \frac{\Gamma(Z^0 \rightarrow b\bar{b})}{\Gamma(Z^0 \rightarrow hadr)}\) \[19\] since, in view of the smallness of the box contribution, the two calculations are very similar. We conclude that in the whole range of the MSSM parameter space the box and \(Z^0\) penguin diagrams arising from chargino exchanges do not change the order of magnitude of the \(B^0_{s,d} \rightarrow \mu^- \mu^+\) decay rate.

Huge contribution to this rate can be however induced for large \(\tan \beta \gtrsim 30\) by neutral Higgs boson penguins. This has been first made explicit in ref. \[7\] in the approach based on the effective Lagrangian method. Our numerical results are obtained by full computation of all relevant Feynman diagrams. Here we present only the derivation of the approximate formulae summarizing the dominant effects. To this end we consider the limit in which all soft SUSY breaking parameters, except for the ones which determine the Higgs potential, are much larger than the electroweak scale. In this limit, which allows us to work in the symmetric phase of the theory (i.e. with \(v_i = 0\)) in which sfermions still have definite chirality, we can construct the effective theory by integrating out sparticles (but not the Higgs fields). In this construction, threshold corrections shown in fig. \[2\] give rise to the effective Yukawa interactions.

Figure 1: Contribution of the Higgs sector of the MSSM or 2HDM(II) to \(BR(B_{s,d}^0 \rightarrow \mu^- \mu^+)\) as a function of the charged Higgs boson mass for \(\tan \beta = 0.5\) (solid lines), 2 (dashed), 25 (dotted) and 50 (dot-dashed).
Figure 2: Diagrams giving rise to $\Delta_u Y_d$ and $\Delta_d Y_d$ respectively, in the construction of the effective theory.

of the down-type quarks summarized by

$$\mathcal{L}_{\text{eff}} = -\epsilon_{ij} (Y_d + \Delta_d Y_d)^{BA} H_i^d q_j^A d^c B - (\Delta_u Y_d)^{BA} H_i^u q_i^A d^c B + h.c.$$  \hspace{1cm} (25)$$

where $A, B$ are the generation indices and we work in the language of two-component Weyl spinors. In order to diagonalize the quark mass matrix arising after the electroweak symmetry breaking we perform first the standard CKM rotations (diagonalizing the original matrix $Y_d^{BA}$) followed by the infinitesimal rotations

$$d^A \to \left(1 + \Delta V^{D\dagger}_{L,R}\right)^{AB} d^B, \quad d^A \to d^B \left(1 + \Delta V^{D\dagger}_{R,L}\right)^{AB}$$

with $\Delta V^{D\dagger}_{L,R}, \Delta V^{D\dagger}_{R,L}$ satisfying $\Delta V^{D\dagger}_{L,R} = -\Delta V^{D\dagger}_{R,L}$. Diagonal mass matrix for down-type quarks is obtained with

$$- (\Delta'_d Y_d)^{AB} + \frac{v_2}{v_1} (\Delta'_u Y_d)^{AB} = (\Delta V^{D\dagger}_L)^{AB} Y_d^B + Y_d^A (\Delta V^{L\dagger}_R)^{AB}$$

where $Y_d^A$ are already diagonal and $\Delta'_u(d) Y_d$ are related to the original $\Delta_u(d) Y_d$ by the rotation diagonalizing the original $Y_d^{RA}$. This leads to the effective Yukawa couplings of the neutral Higgs bosons of the form

$$\mathcal{L} = -\frac{1}{\sqrt{2}} d^c \left(-Y_d Z_{1k}^R + \Delta'_d Y_d Z_{2k}^R - \tan \beta \Delta'_u Y_d Z_{1k}^R\right) d H_k^0 + h.c. + \frac{i}{\sqrt{2}} d^c \left(Y_d Z_{1k}^H + \Delta'_d Y_d Z_{2k}^H + \tan \beta \Delta'_u Y_d Z_{1k}^H\right) d H_{k+2}^0 + h.c.$$\hspace{1cm} (28)$$

(In the lagrangian (28) $Y_d$ is diagonal and the rest of the notation is explained below eq. (11)) which in general generates the FCNC transitions. Note that the correction $\Delta_d Y_d$ dissapeared altogether as it should, since it cannot cotribute to the FCNC transition.

\footnote{Strictly speaking eq. (27) must hold only for off diagonal elements; for $A = B$ the relation $m_{dA} = -Y_d^A v_1 / \sqrt{2}$ is corrected but the net result is that in eq. (28) $Y_d^A \equiv -\sqrt{2} m_{dA} / v_1$ again.}
The replacement of corrections contain one power of tan β given by eqs. (32) to the formfactors arises only from the self energy diagrams (the 1PI vertex phase in which the electroweak symmetry is broken) reveals that the dominant contributions approach (in which one computes both, the self energy corrections the 1PI vertex diagrams, in the subleading).

Replacing \( A \) (right formfactors are given by the Hermitean conjugation; they involve \( Y \) and the initial part of the lagrangian \[12\]) in eqs. (9,10) yields the full vertex formfactors

\[
\frac{Y_u}{\lambda} = \frac{\lambda_l}{\lambda} Y_u Y_l^2 A_l m_{C_j} C_0(m_j^2, M^2_{L}, M^2_{R})
\]

where we have replaced \( \mu \) with \( m_{C_j} \) and the sign \( \pm \) keeps track of the sign of \( \mu \). Using eq. (28) in eqs. (31) yields the full vertex formfactors

\[
\tilde{F}_L^S = \frac{1}{\sqrt{2}} \Delta_u Y_d^{\frac{2}{3}} [Z^2_R - Z^1_R \tan \beta] \approx -\frac{1}{\sqrt{2}} \Delta_u Y_d Z^1_R \tan \beta \\
\tilde{F}_L^P = \frac{1}{\sqrt{2}} \Delta_u Y_d [Z^2_H + Z^1_H \tan \beta] \approx \frac{1}{\sqrt{2}} \Delta_u Y_d Z^1_H \tan \beta
\]

(right formfactors are given by the Hermitean conjugation; they involve \( Y_d^I \) and are, therefore, subleading).

Detailed comparison of the above simplified calculation with the standard diagramatic approach (in which one computes both, the self energy corrections the 1PI vertex diagrams, in the phase in which the electroweak symmetry is broken) reveals that the dominant contributions given by eqs. (32) to the formfactors arise only from the self energy diagrams (the 1PI vertex corrections contain one power of tan β less). Moreover, the comparison shows that one should replace \( A_t \) by \( \tilde{A}_t \equiv A_t + \mu \cot \beta, M_{tL}, M_{tR} \) with the true mass eigenstates \( M_{t1}, M_{t2} \) and justifies the replacement of \( \pm \mu \) by the mass of the lighter chargino.

Using eqs. (32) we get

\[
a = \frac{f_B}{4} \frac{1}{16 \pi^2} \frac{\lambda_l}{M_{w}^2} \left( \frac{e}{s_w} \right)^4 \left[ Y(x_t) - \frac{M^2_B}{8 M_{w}^2} \tan^2 \beta \frac{\log r}{r - 1} \pm \frac{M^2_B}{8 M_{w}^2} m^2_{l} \tan^3 \beta A_l m_{C_j} C_0 \right]
\]

\[
b = \frac{f_B}{4} \frac{1}{16 \pi^2} \frac{\lambda_l}{M_{w}^2} \left( \frac{e}{s_w} \right)^4 \left[ -\frac{M^2_B}{8 M_{w}^2} \tan^2 \beta \frac{\log r}{r - 1} \pm \frac{M^2_B}{8 M_{w}^2} m^2_{l} \tan^3 \beta A_l m_{C_j} C_0 \right]
\]
Figure 3: $\text{BR}(B_{s(d)}^0 \rightarrow \mu^- \mu^+)$ as a function of the stop mixing angle $\theta_t$ for $\tan \beta = 50$, the lighter chargino mass 100 GeV and different values of $M_A$. Solid, dashed and dotted lines correspond to $(M_{\tilde{t}_2}, M_{\tilde{t}_1})$ equal (240,500), (400, 700) and (300, 850) GeV, respectively. In the left (right) pannels $M_2/\mu = 10(-1)$, where $M_2$ is the usual $SU(2)$ gaugino mass parameter.
Figure 4: $BR(B_{s(d)}^0 \rightarrow \mu^-\mu^+)$ as a function of the lighter chargino mass for $\tan\beta = 50$, $M_A = 200$ and 1000 GeV. Solid (dashed) lines correspond to $(M_{\tilde{t}_2}, M_{\tilde{t}_1})$ equal to $(m_{C_1}, 3m_{C_1})$ and the stop mixing angle $\theta_t = 10^\circ$ ($30^\circ$) whereas dotted (dash-dotted) lines to $(3m_{C_1}, 5m_{C_1})$ and $\theta_t = 10^\circ$ ($30^\circ$), respectively. In the left (right) pannels $M_2/\mu = 10(-1)$. 
Knowing that \( Y(x_t) \approx 1 \) these formulae allow for a quick estimate of the effects. It is important to note that the contribution of charginos to the rate grows as \( \tan^6 \beta \) and therefore can be much larger than the contribution of the Higgs sector.

Figure 3 shows the dependence of the full branching ratio \( BR(B_s^0 \to \mu^- \mu^+) \), including the SM, Higgs boson and chargino contributions, on the mixing angle of the top squarks for some values of the other MSSM parameters. The minimum around \( \theta_t \approx 0 \) corresponding to \( \tilde{A}_t \approx 0 \) is clearly seen. Incidentally this plot also supports the replacement of \( \mu \) by \( \pm m_{C_1} \) in eq. (31) because very similarly (up to a reflection \( \theta_t \to -\theta_t \) which follows from different signs of \( \mu \)) looking curves in the left and right panels have the same \( m_{C_1} \) but distinctly different \( \mu \). Another important feature of the chargino contribution is that it does not vanish if all sparticle mass parameters are scaled uniformly: \( m_{\tilde{t}_1} \to \lambda M_{\tilde{t}_1}; m_{\tilde{C}_1} \to \lambda m_{\tilde{C}_1}, \mu \to \lambda \mu, A_t \to \lambda A_t \). This is clear from the fact that in such a case \( C_0 \to \lambda^{-2} C_0 \). This is illustrated in figure 3 which shows \( BR(B_s^0 \to \mu^- \mu^+) \) as a function of the lighter chargino mass for \((M_{\tilde{t}_2}, M_{\tilde{t}_1})\) equal to \((m_C, 3m_C)\) and \( \theta_t = 10^\circ \) (solid lines), \((m_C, 3m_C)\) and \( \theta_t = 30^\circ \) (dotted lines) and \((3m_C, 5m_C)\) and \( \theta_t = 30^\circ \) (dashed-dotted lines). In fact, keeping the stop mixing angle fixed requires that \( \tilde{A}_t \) scales as \( \lambda^2 \) rather than as \( \lambda \), which explains the growth of the rates with \( m_{C_1} \) in fig. 4.

To check the correlation of the prediction for \( BR(B_{s(d)}^0 \to \mu^- \mu^+) \) with the results for \( BR(B \to X_s \gamma) \) we have performed scans over the relevant parameter space of the MSSM. We took the following ranges: \( 100 < m_{C_1} < 1000 \) GeV, \( 0.1 < |M_2/\mu| < 10, 1 < M_{\tilde{t}_2}/m_{\tilde{C}_1} < 10, 1 < M_{\tilde{t}_1}/M_{\tilde{t}_2} < 5 \) and \(-60^\circ < \theta_t < 60^\circ \) and rejected points for which \( \Delta \rho_{\text{squarks}} > 6 \times 10^{-4} \) and \( M_h < 107 \) GeV. For calculating \( BR(B \to X_s \gamma) \) we have used the routine based on refs. [20, 21] including the NLO matching conditions at the scale \( M_Z \) for the top and charged Higgs contribution as in [22, 23] and only the LO ones for the supersymmetric contribution [24, 3].

We have not used the available NLO matching conditions for the supersymmetric particles since they are computed under the specific assumptions about the sparticle spectrum, not necessarily satisfied in the scan and, moreover, they are not valid for large values of \( \tan \beta \). The theoretical uncertainty is taken into account by computing the rate for \( \mu_t = 2.4 \) and 9.6 GeV and then by shifting its larger (smaller) value upward (downward) by the added in quadratures errors related to the uncertainties in \( \alpha_s, m_t, m_c/m_b, |V_{tb}V_{ts}^*/V_{cb}|^2 \), and higher order electroweak corrections; we do not take into account the variation of the scale \( \mu_W \). For a given set of parameters of the MSSM the \( BR(B \to X_s \gamma) \) value shown in fig. 5 corresponds to the lowest (highest) edge of the resulting band of theoretical predictions, if the whole band is above (below) the range allowed by CLEO [10], and to the central point of the overlap of the theoretical and CLEO bands in the case such an overlap exists.

The results of the scans, shown in fig. 5 demonstrate that the the CLEO result for \( BR(B \to X_s \gamma) \) does not eliminate the points corresponding to the largest values of \( BR(B_{s(d)}^0 \to \mu^- \mu^+) \) and even does not exhibit any definite correlation between the two rates, especially for those points for which \( BR(B_s^0 \to \mu^- \mu^+) \) is very large. This is mainly due to the fact that the (LO) chargino contribution to \( BR(B \to X_s \gamma) \) decreases with growing sparticle masses whereas its contribution to \( BR(B_{s(d)}^0 \to \mu^- \mu^+) \) does not.\footnote{For the same reason we do not expect strong constraints from the \( K^0 \bar{K}^0 \) or \( B^0 \bar{B}^0 \) mass differences, which...
Figure 5: $\text{BR}(B \to X_s \gamma)$ versus $\text{BR}(B_{s(d)}^0 \to \mu^- \mu^+)$ for $\tan \beta = 50$, $M_A = 200$ GeV in panels a) and b), $\tan \beta = 50$, $M_A = 600$ GeV in panel c) and $\tan \beta = 30$, $M_A = 200$ in panel d).

Limits from CLEO on $\text{BR}(B \to X_s \gamma)$ and on $\text{BR}(B_{d}^0 \to \mu^- \mu^+)$ are also shown by solid lines.

Computation of $\text{BR}(B \to X_s \gamma)$ will not change this picture qualitatively. Moreover, fig. 5 shows that the present CLEO bound $\text{BR}(B_{d}^0 \to \mu^- \mu^+) < 6.2 \times 10^{-7}$ (shown in the upper-right plot by the vertical solid line) already puts some weak constraints on the MSSM parameter space in the case of large $\tan \beta \sim m_t/m_b$ and $M_A \lesssim 300$ GeV.

5. Flavour changing induced by sfermion mass matrices

Up to now we have assumed that the fermion and sfermion mass matrices are flavour diagonal in the same basis (the so-called super-CKM basis). In this section we consider the effects of nondiagonal entries in the sfermion mass matrices. It is customary to parametrize such nondiagonal entries by the so-called dimensionless mass insertions [2, 3]:

$$ (\delta^{K}_{XY})^{IJ} \equiv \frac{(\Delta M^{2}_{K})_{IJ}^{IY}}{\sqrt{(M^{2}_{K})_{XX}^{II} (M^{2}_{K})_{YY}^{II}}} \quad (35) $$

A priori also depend on the chargino and stop parameters.
where \( X, Y = L, R, K = u, d, l, (M_K^2)_{XX} \) are the diagonal elements of the \( XX \) blocks of the full mass squared matrices, and \( (\Delta M_K^2)_{XY} \) are the off diagonal entries of the \( XY \) blocks. Most of these insertions are bounded by the experimental data (for review, see refs. \([2, 3]\)). In the case of the \( B^0 \rightarrow \bar{l}l'1^+ \) decay the relevant insertions are \( (\delta_{XY})_{LL}^{I3} \) and \( (\delta_{XY})_{L}^{I3} \), \( I = 1, 2 \).

The first interesting point is to check the effects of the slepton mass insertions which are the only source of the decays \( B^0 \rightarrow \bar{l}l' \) (through the box diagrams with charginos in the loop). Very strong bounds from nonobservation of the transition \( \mu \rightarrow e\gamma \) exist only on the \( (\delta_{LL})_{12}^{I1} \) insertions \([2]\) whereas in the case of the \( B^0 \rightarrow \bar{l}l' \) decay most important are the insertions \( \delta_{LL}^I \) on which the bounds are weaker \([4]\). Taking \( m_{C_1} = 100 \text{ GeV} \), light stop \( M_t \approx 100 \text{ GeV} \) and adjusting the slepton sector mass parameters so to keep \( M_t \gtrsim 90 \text{ GeV} \), \( M_b \gtrsim 50 \) for \( (\delta_{LL})_{13(23)}^{I(3)} \approx 0.9 \) we get

\[
\begin{align*}
BR(B^0 \rightarrow \bar{l}l') &\lesssim 1.6 \times 10^{-11} \\
BR(B^0_{d} \rightarrow \bar{l}l') &\lesssim 3.8 \times 10^{-13}
\end{align*}
\]

where \( ll' = e\tau \) or \( \mu\tau \). (The largest rates are obtained for \( |M_2/\mu| \lesssim 1 \) and small stop mixing angle \( \theta_t \); the result scale approximately as \( |\delta_{LL}|^2 \).) For other parameters (heavier stops and charginos) branching fractions for these processes are, of course, smaller.

We now discuss the effects of the flavour non-diagonal mass insertions in the down-type squark mass matrix and return to the \( ll \) final states. The approximate formulae accounting for the effects of the insertions \( (\delta_{XY})_{IJ} \) are easily derived in the so-called mass insertion method \([2, 3]\) in which flavour off diagonal elements of the sfermion mass squared matrices are treated as additional interactions. Usually, the linear approximation in \( (\delta_{XY})_{IJ} \) is sufficient to account for the results obtained with the full diagramatic calculation. In the case of a nonzero \( (\delta_{XY})_{I3} \) insertion the dominant contribution is expected to come from the diagrams involving gluinos, due to their strong coupling, \( g_s = \sqrt{4\pi\alpha_s} \), to quarks and squarks. \([9]\) (This expectation is confirmed by the numerical computation in which all one-loop contributions are taken into account.) Since at one-loop there are no box diagrams with gluinos we are left only with the \( Z^0 \) and neutral Higgs boson penguin diagrams. As previously, the latter type of penguin diagrams is important only for large values of \( \tan\beta \). Another important remark is that because the change of flavour in the gluino diagrams does not originate from the CKM mass matrix, the rates of the \( B^0_{d} \) decays need not be suppressed compared to the rates of the \( B^0_s \) decays.

For \( \tan\beta \) values not too large, only the \( Z^0 \) penguin contribution can be important. Direct computation shows however, that in the formula \([11]\) terms linear in the mass insertions \( (\delta_{LL(RR)})_{I3} \) cancel out completely between the self energy \( \Sigma_X^{l} \) and the proper vertex corrections.

\[9\text{Ref. } [3] \text{ gives } (\delta_{LL})_{12}^{I2} < 0.2 \left( \frac{M_{\tilde{\chi}_1^0}}{\mu_{\tilde{\chi}_1^0}} \right)^2, (\delta_{LL})_{13}^{I3} < 700 \left( \frac{M_{\tilde{\chi}_1^0}}{\mu_{\tilde{\chi}_1^0}} \right)^2 \text{ and } (\delta_{LL})_{23}^{I3} < 100 \left( \frac{M_{\tilde{\chi}_1^0}}{\mu_{\tilde{\chi}_1^0}} \right)^2, \text{ which in most cases are superseded by } \delta_{LL}^I \lesssim 1 \text{ - the limit in which one of the sneutrinos becomes tachionic.}
\]

\[10\text{For nonzero } (\delta_{XY})_{I3} \text{ insertion also neutralinos contribute; moreover a nonzero } (\delta_{LL})_{I3} \text{ insertion induces, via the CKM matrix (see e.g. } [3, 12]\), nonzero } (\delta_{LL})_{I3} \text{ insertions which affect in principle the chargino contribution. Both these effects are small but are taken into account in our numerical code.} \]
tion $F_X^V$ ($X = L, R$). Because of that, the effects of the nonzero $(\delta_{\text{LL(RR)}}^d)^{13}$ mass insertions, even taking into account their quadratic and higher contributions in gluino exchanges as well as neutralino diagrams, are small for $\tan \beta$ values for which the neutral Higgs boson penguin graphs are negligible. Larger effects could come only from nonzero $(\delta_{\text{LR}}^d)^{13}$ mass insertions which, however, are strongly constrained [2, 3]: $| (\delta_{\text{LR}}^d)^{13} | < 0.07 (m_{\text{max}}/1 \text{TeV})$, $| (\delta_{\text{LR}}^d)^{23} | < 0.03 (m_{\text{max}}/1 \text{TeV})$ (where $m_{\text{max}} \equiv (\max(M_{\text{sq}}, m_{\tilde{g}})$). Respecting these constraints, $BR(B_s^0 \rightarrow \mu^- \mu^+) (BR(B_d^0 \rightarrow \mu^- \mu^+)$ remains of order $4 \times 10^{-9}$ ($10^{-10}$).

In the case of large $\tan \beta$ we have to compute in the linear approximation in the mass insertions both the scalar parts of the self energies and the 1PI vertex corrections to the couplings $\tilde{d}_j d_l S^0 (P^0)$. For the self energies we get

$$\Sigma_L^S = \frac{1}{16 \pi^2} \frac{8}{3} g_s^2 m_{\tilde{g}} \left\{ \left( \Delta M_D^2 \right)_{LR}^{IJ} C_0 (m_{\tilde{g}}, M_D^2, M_D^2) + \left( \Delta M_D^2 \right)_{LL}^{IJ} m_b (A_b + \mu \tan \beta) D_0 (m_{\tilde{g}}, M_D^2, M_D^2, M_D^2) \right\}$$

where $D_0$ is the standard four-point function

$$D_0 (a, b, c, d) = \frac{1}{a-b} [C_0 (a, c, d) - C_0 (b, c, d)] \tag{38}$$

$m_{\tilde{g}}$ is the gluino mass and $M_D$ is the average mass of the two bottom squarks. Similar formula is obtained for $\Sigma_R^S$ with the replacement $(\Delta M_D^2)_{LL} \rightarrow - (\Delta M_D^2)_{RR}$.

In the same approximation, for the vertex correction $\tilde{d}_j d_l P^0$ we get

$$F_L^P = \frac{1}{16 \pi^2} \frac{8}{3} g_s^2 m_{\tilde{g}} \left\{ \frac{1}{v_1} Z_H^{1k} \left( \Delta M_D^2 \right)_{LR}^{IJ} C_0 (m_{\tilde{g}}, M_D^2, M_D^2) + \frac{e}{2 s_W M_W} \left( m_{d_l} \left( \Delta M_D^2 \right)_{RR}^{IJ} + \left( \Delta M_D^2 \right)_{LL}^{IJ} m_d \right) Z_H^{2k} \tan \beta D_0 (m_{\tilde{g}}, M_D^2, M_D^2, M_D^2) \right\} \tag{39}$$

where for the three-linear soft term we have used $A_D^H \equiv Y_d^H A_b$. $F_R^P$ is similar, with $(\Delta M_D^2)_{LL}^{IJ} \leftrightarrow - (\Delta M_D^2)_{RR}^{IJ}$. Combining (39) and (37) according to (13), we see that $(\Delta M_D^2)_{RL}^{IJ}$ cancels out. Moreover, since the CP-odd scalar $A^0$ whose coupling to leptons is enhanced, corresponds to $k = 1$ and $Z_H^{21} = \cos \beta \approx 0$, the second line in (39) is suppressed compared to the third one. Therefore, we can write:

$$\tilde{F}_L^P \approx \frac{1}{16 \pi^2} \frac{8}{3} g_s^2 \frac{e}{2 s_W M_W} m_b \tan^2 \beta \mu \left( \Delta M_D^2 \right)_{LL}^{IJ} m_{\tilde{g}} D_0 (m_{\tilde{g}}, M_D^2, M_D^2, M_D^2) \tag{40}$$

where we have retained only $(\Delta M_D^2)_{LL}^{IJ}$ which in $\tilde{F}_L^P$ is multiplied by $m_{d_l} = m_b$ and neglected $(\Delta M_D^2)_{RR}^{IJ}$ which is multiplied by $m_{d_l}$ (in $\tilde{F}_R^P$ it is the other way around). Similar calculation leads to

$$\tilde{F}_L^S \approx \frac{1}{16 \pi^2} \frac{8}{3} g_s^2 \frac{e}{2 s_W M_W} m_b \tan^2 \beta \mu \left( \Delta M_D^2 \right)_{LL}^{IJ} m_{\tilde{g}} D_0 (m_{\tilde{g}}, M_D^2, M_D^2, M_D^2) \tag{41}$$
Figure 6: $BR(B_s^0 \to \mu^-\mu^+)$ for $\tan \beta = 50$, $M_A = 200$ GeV and $(\delta^d_{LL})^{23} = 0.1$ as a function of the $\mu$ parameter. In the left panel $(m_{\tilde{g}}, A_t = A_b)$ equals: (300,0) GeV (solid line), (300,250) GeV (dashed), (800,0) GeV (dotted) and (800,250) GeV (dash-dotted); $(m^2_Q)_{33} = (500 \text{ GeV})^2$, $(m^2_D)_{33} = (300 \text{ GeV})^2$, $(m^2_X)_{KK} = (600 \text{ GeV})^2$ for $K \neq 3$. In the right panel $(m_{\tilde{g}}, A_t = A_b)$ equals: (800,0) GeV (solid line), (800,450) GeV (dashed), (1500,0) GeV (dotted) and (1500,450) GeV (dash-dotted); $(m^2_Q)_{33} = (900 \text{ GeV})^2$, $(m^2_D)_{33} = (m^2_D)_{33} = (700 \text{ GeV})^2$, $(m^2_X)_{KK} = 1000 \text{ GeV}^2$ for $K \neq 3$.

Computing the relevant Wilson coefficients we finally find for the coefficients $a$ and $b$:

\begin{align}
\frac{a}{m_l} &= \frac{1}{16\pi^2} \frac{f_B}{2} \left( \frac{e}{s_W} \right)^4 \lambda_{tI} \\
&\times \left[ Y(x_t) - \frac{8}{3} g_s^2 \frac{s_W}{e} \frac{M_B^2}{M^2_{A0}} \frac{\delta^d_{LL}}{\lambda_{tI}} \tan^3 \beta m_{\tilde{g}} \mu (m_{\tilde{g}}, m^2_D, m_D^2, M^2_D) \right] \\
\frac{b}{m_l} &= \frac{1}{16\pi^2} \frac{f_B}{2} \left( \frac{e}{s_W} \right)^4 \lambda_{tI} \\
&\times \left[ -\frac{8}{3} g_s^2 \frac{s_W}{e} \frac{M_B^2}{M^2_{A0}} \frac{\delta^d_{LL}}{\lambda_{tI}} \tan^3 \beta m_{\tilde{g}} \mu (m_{\tilde{g}}, m^2_D, m_D^2, M^2_D) \right]
\end{align}

in which we have displayed also the SM contribution to allow for easy estimate of the magnitude of the gluino contribution. It is essential that the dominant effect is due to the $LL$ insertion and
not the LR one which is much more strongly constrained [2, 3]. Similarly as in the case of the chargino contribution through the neutral Higgs boson penguin graphs, also the gluino (and neutralino) contribution is proportional to tan$^6 \beta$ and does not vanish when all SUSY mass parameters are uniformly scaled up (provided the dimensionless mass insertion is kept fixed). Figure 6 shows the result of the full diagramatic computation of the SM and gluino exchange contributions to $BR(B^0_s \to \mu^- \mu^+)$ as a function of the $\mu$ parameter for $(\delta_{LL}^d)^{23} = 0.1$ and tan$\beta = 50$, $M_A = 200$ GeV. The minimum for $\mu = 0$ is clearly seen. The gluino contribution scales approximately as $|\delta_{LL}^d)|^2$.

Figure 7 shows the results of the general scan over the MSSM parameter space in the form of the scatter plot $BR(B \to X_s \gamma)$ versus $BR(B^0_s \to \mu^- \mu^+)$ for $(\delta_{LL}^d)^{23} = 0.1$ and versus $BR(B^0_d \to \mu^- \mu^+)$ for $(\delta_{LL}^d)^{13} = 0.05$. The parameters have been varied in the following ranges: 100 < $m_{C_1}$ < 600 GeV, 0.1 < |$M_2/\mu$| < 5, $m_\tilde{g} = 3M_2$, $-60^\circ < \theta_1 < 60^\circ$, $A_b = A_t$, 0.5 < $m_{\tilde{b}}/m_{C_1}$ < 1.5, 1 < $M_{\tilde{t}_1}/M_{\tilde{t}_2}$ < 5, 0.25 < ($m_\tilde{g}^2$)$_{33}/m_\tilde{g}^2$ < 2.25. For other entries of the squark mass matrices we took ($m_\tilde{t}_K^2$) = ($m_\tilde{D}^2$)$_{33}$. All points for which $M_\tilde{b} < 107$ GeV, $\Delta\rho_{\text{squark}} > 6 \times 10^{-4}$ (as well as points with too light stops) have been rejected. Results for $(\delta_{RR}^d)^{13}$ are similar.

In agreement with the bounds given in refs. [2, 3], the measured by CLEO [10] $BR(B \to X_s \gamma)$ does not constrain the rate of the $BR(B^0_s \to \mu^- \mu^+)$ decay (nor does it exhibit any particular correlation with the latter) and the latter can attain values of the order of $10^{-4}$, respecting all the relevant phenomenological constraints. As expected, whenever the gluino contribution is dominant the rates of the $B^0_s \to \mu^- \mu^+$ and $B^0_d \to \mu^- \mu^+$ decays are comparable which means that $BR(B^0_d \to \mu^- \mu^+)$ can also be as large as $10^{-4}$ for $(\delta_{LL}^d)^{13} = 0.1$ (in the plot, we took $(\delta_{LL}^d)^{13} = 0.05$ in order to satisfy the bound $(\delta_{LL(\text{RR})}^d)^{13} < 0.2(m_{\text{max}}/1\text{TeV})$ [2, 3] for almost all points in the scan; however the biggest effects are for $m_{\text{max}}$ large for which $(\delta_{LL(\text{RR})}^d)^{13}$ can be larger). It follows, that for such values of the MSSM parameters, the current CLEO bound, $BR(B^0_d \to \mu^- \mu^+) < 6.2 \times 10^{-7}$ [11] puts constraints on $(\delta_{LL(\text{RR})}^d)^{13}$ which are much stronger than the ones given in [2, 3].

6. Conclusions

We have performed a complete one loop diagramatic calculation of the decay rate of the $B_{s(d)}^0$ mesons into charged leptons. Both possible sources of the FCNC processes, the CKM mixing matrix and the off diagonal entries of the sfermion mass matrices, have been considered. For values of tan$\beta$, for which the neutral Higgs boson penguin graphs are negligible the rates of these decays in the MSSM remain of the order of the SM prediction.

Large enhancement of the SM prediction can occur for tan$\beta \gg 1$ provided the additional Higgs bosons predicted by the MSSM are not too heavy (all the large contributions behave as $1/M_A^2$, where $M_A$ is the mass of the CP-odd neutral Higgs boson). The contribution of the Higgs sector grows like tan$^4 \beta$ and can give $BR(B^0_s \to \mu^- \mu^+) \approx 2 \times 10^{-8}$ for tan$\beta \approx m_t/m_b$. Dominant effects of the chargino sector grow as tan$^6 \beta$ and depend strongly on the top squark mixing. For tan$\beta \sim m_t/m_b$ and substantial mixing of the top squarks they can give $BR(B_{s(d)}^0 \to \mu^- \mu^+)$ up to $5 \times 10^{-5}(10^{-6})$ respecting other phenomenological constraints including the measurement of $BR(B \to X_s \gamma)$. Large effects, growing as tan$^6 \beta$ and exhibiting strong dependence on
Figure 7: \( BR(B \to X_s \gamma) \) versus \( BR(B^0_s \to \mu^- \mu^+) \) (with \( (\delta_{LL}^d)^{23} = 0.1 \)) and \( BR(B^0_d \to \mu^- \mu^+) \) (with \( (\delta_{LL}^d)^{13} = 0.05 \)) for \( \tan \beta = 50, M_A = 200 \) GeV in panels a) and b), \( \tan \beta = 50, M_A = 600 \) GeV in panel c) and \( \tan \beta = 30, M_A = 200 \) in panel d). Limits from CLEO on \( BR(B \to X_s \gamma) \) and on \( BR(B^0_d \to \mu^- \mu^+) \) are also shown by solid lines.
the $\mu$ parameter, can be also induced by the off diagonal elements of the down-type squark mass matrix. As we have shown, $BR(B_{s(d)}^0 \rightarrow \mu^-\mu^+)$ is sensitive to the 23 (13) off-diagonal entries of the LL and RR blocks of these matrices, which are not so strongly constrained by $BR(B \rightarrow X_s\gamma)$. For $\tan\beta \sim m_t/m_b$ and $M_A \lesssim 200$ GeV these effects can easily give $BR(B^0 \rightarrow \mu^-\mu^+)$ larger than $10^{-4}$. It is also interesting that even for $BR(B^0 \rightarrow \mu^-\mu^+)$ these effects can be so large that they could exceed the present CLEO limit \cite{11} which, therefore, already now puts constraints on the MSSM parameter space.

Finally it is important to stress that, both types of effects growing as $\tan^6\beta$ do not necessarily decrease as sparticles become heavy. However, they are sensitive to the mass scale of the extended Higgs sector. Thus, large deviation from the SM prediction observed in these decays, apart from being a signal of supersymmetry, would have important implications on the Higgs search at the LHC.

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