Fine-tuning in GGM and the 126 GeV Higgs particle

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SUSY breaking mediation

- Supergravity
  - No control over mixing between families $\rightarrow$ large FCNC

- Gauge mediation
  - SUSY is spontaneously broken $\rightarrow$ singlet $\langle X \rangle = X + \theta^2 F$
  - breaking is transmitted through messengers $W = \lambda \bar{\Phi} X \Phi$
  - messengers $\bar{\Phi}, \Phi$ interact with MSSM fields only via gauge interactions

Diagram:
- Hidden sector
- Visible sector
- $\bar{\Phi}, \Phi$
- $X$
- MSSM
Gauge mediated soft terms

\[ \lambda \xrightarrow{\phi} \lambda \quad \xrightarrow{\phi} \quad M_i = \frac{\alpha_i}{4\pi} \frac{F}{X} \]

\[ \tilde{f} \quad + \quad \tilde{f} \quad + \ldots \]

\[ \Rightarrow m_f^2 = 2 \sum_i C_i(f) \left( \frac{\alpha_i}{4\pi} \right)^2 \left| \frac{F}{X} \right|^2 \]
GGM soft terms

Meade, Shih and Seiberg 0801.3278
Gauge mediated soft terms can be expressed by just six parameters

- Three gaugino masses
  \[ M_1 = \frac{\alpha_1}{4\pi} m_Y, \quad M_2 = \frac{\alpha_2}{4\pi} m_w, \quad M_3 = \frac{\alpha_3}{4\pi} m_c, \]

- Three parameters determining scalar masses \( \Lambda_c^2, \Lambda_w^2, \Lambda_Y^2 \)
  which give

  \[ m_f^2 = 2 \left[ C_3(f) \left( \frac{\alpha_3}{4\pi} \right)^2 \Lambda_c^2 + C_2(f) \left( \frac{\alpha_2}{4\pi} \right)^2 \Lambda_w^2 + C_1(f) \left( \frac{\alpha_1}{4\pi} \right)^2 \Lambda_Y^2 \right], \]

- Only negligible A-terms are generated.
Two specific models Carpenter et al. 0805.2944

- GGM1
  \[ W_{GGM1} = X_i (y^i \bar{Q} Q + r^i \bar{U} U + s^i \bar{E} E), \]
  with three independent parameters \( \Lambda_Q, \Lambda_U, \Lambda_E \)

- GGM2
  \[ W_{GGM2} = X_i (y^i \bar{Q} Q + r^i \bar{U} U + s^i \bar{E} E + \lambda^i_q \bar{q} q + \lambda^i_l \bar{l} l), \]
  with five independent parameters \( \Lambda_Q, \Lambda_U, \Lambda_E, \Lambda_q, \Lambda_l \)
fine-tuning definition

- fine-tuning from parameter $a$

$$
\Delta_a = \left| \frac{\partial \ln m^2_Z}{\partial \ln a} \right|.
$$

- fine-tuning coming from a whole set of parameters $a_i$

$$
\Delta = \max_{a_i} \Delta_{a_i}.
$$
FT in mSUGRA

\[ \Delta \]

\[ \Delta m_h \]

\[ \Delta \]

\[ m_h \text{ [GeV]} \]

\[ \Delta \]

\[ \Delta \]
FT in GGM

\[ \Delta = \Delta_{fullGGM} \]

\[ \Delta = \Delta_{GGM2} \]

\[ \Delta = \Delta_{GGM1} \]

\[ m_h [\text{GeV}] \]
reducing fine-tuning

Assuming that parameters are not independent of each other, but instead are functions of some fundamental parameters. For example, if gaugino masses $M_i$ are given functions of parameter $M_{\frac{1}{2}}$ we obtain

$$M_i = f_i(M_{\frac{1}{2}}),$$

$$\Delta M_{\frac{1}{2}} = \left| \frac{\partial \ln M^2_Z}{\partial \ln M_{\frac{1}{2}}} \right| = \left| M_{\frac{1}{2}} \frac{f'_i(M_{\frac{1}{2}}) \partial \ln M^2_Z}{f_i(M_{\frac{1}{2}}) \partial \ln M_i} \right|. $$

If $f_i$ are simply proportional to $M_{\frac{1}{2}}$ one finds

$$\Delta M_{\frac{1}{2}} = \left| \sum_{i=1}^{3} \frac{\partial \ln M^2_Z}{\partial \ln M_i} \right|. $$

If these functions were logarithms

$$M_i(M_{\frac{1}{2}}) = \tilde{m} \ln \frac{M^2_{\frac{1}{2}}}{Q}, \quad \Delta M_{\frac{1}{2}} = \left| \sum_{i=1}^{3} \frac{\tilde{m} \partial \ln M^2_Z}{M_i \partial \ln M_i} \right|. $$
$\Lambda_i \propto \Lambda_j \propto \mu$

$\Delta_{fullGGM}$  
$\Delta_{GGM2}$  
$\Delta_{GGM1}$
fine-tuning from only gauge mediated soft terms

Brummer and Buchmuller 1201.4338
discrepancy between measurement and SM prediction:

\[ \delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (2.8 \pm 0.8) \times 10^{-9}. \]

The simplest approximation of SUSY contribution

\[ \delta a_\mu^{\text{SUSY}} \approx \left( \frac{g_1^2 - g_2^2}{192\pi^2} + \frac{g_2^2}{32\pi^2} \right) \frac{m_\mu^2}{M_{\text{SUSY}}^2} \tan \beta, \]

Problem: We need heavy superpartners \((M_{\text{SUSY}})\)
$g_\mu - 2$ and FT

**SUSY contribution to muon $g-2$**

- $\delta a^\text{fullGGM}_{\mu}$
- $\delta a^\text{GGM2}_{\mu}$
- $\delta a^\text{GGM1}_{\mu}$

**Fine-tuning**

- $\Delta^\text{fullGGM}$
- $\Delta^\text{GGM2}$
- $\Delta^\text{GGM1}$
Conclusions

1. GGM predicts smaller fine-tuning than mSUGRA
2. for $m_h = 126$ GeV fine-tuning always larger than 100 unless one includes only gauge mediated soft terms
3. including $g_{\mu} - 2$ raises fine-tuning about four times, but it's still possible to obtain $g_{\mu} - 2$ within $1\sigma$ bound
4. decrease of the Higgs mass down to 123 GeV reduces the fine-tuning by a factor of 2.