Role of higher order deformations in fission fragment mass distribution of Astatine isotopes within collective clusterization approach

Amandeep Kaur and Manoj K. Sharma
School of Physics and Materials Science, Thapar Institute of Engineering and Technology, Patiala-147004, Punjab, India
E-mail: amanganday@gmail.com

Abstract. The fission fragment mass distribution is used as an investigation tool which assists to disentangle between different modes of fission (symmetric fission and asymmetric fission). In the present work, the fission dynamics of various At isotopes with mass number \( A = 191 \) to 220 formed in \(^{19}F\)-induced reactions at common centre-of-mass energy \( E_{c.m.} = 78.5 \) MeV is explored. The calculations are made within the dynamical cluster-decay model (DCM) using three type of fragmentation such as spherical, \((\beta_2)\)-deformed and \((\beta_2 - \beta_4)\)-deformed with hot compact optimized orientations. The structural effects come into picture when deformation effects are included in the fragmentation potential. The mass distributions of ‘At’ isotopes gets significantly modified after inclusion of deformation effects of decaying fragments. The preformation yield shows a drift from symmetric to asymmetric fission with increase in mass of compound nuclei from \( A_{CN} = 191 \) to 220. The clear signature of fine-structure effects is evident in view of single humped to double humped preformation structure in the fissioning region. However, the mass division of At isotopes is symmetric for spherical choice of fragments. Finally, the analysis of hot (compact) and cold (elongated) configurations of \( \beta_2\)-deformed and \( \theta_{opt} \) oriented fragments is also carried out.

1. Introduction
The fusion-fission mechanism is a complex process which deals with collective motion of nucleons at compound nucleus stage and its subsequent segregation in the decay channel. A comprehensive knowledge of nuclear fission and its pros and cons, is important due to its widespread use in everyday life. Moreover, it offers an extensive assortment of logical research on nuclear characteristics and general physics. The advancement in the nuclear fission research, depends primarily on the improvement of the experimental techniques, the availability of projectile beams and fissioning material etc. Significant progress on experimental and theoretical front has helped immensely to understand this intricate process. The division of excited compound nucleus (CN) in two smaller fragments depends on various factors such as mass number, incident energy, angular momentum, deformations, orientations etc. The explanation of mass distributions on the basis of a systematic dynamical treatment remains one of the premier aspect of fission physics.

In our recent work [1-2], the energy, angular momentum and mass dependence of fission fragment mass distribution has been analyzed by taking the decay fragments of quadrupole...
(\beta_2) deformation with hot compact optimized orientations. This analysis was carried out for wide range of pre-actinide and actinide nuclei formed in $^{19}$F-induced reactions at near and above barrier incident energies. The outcome of these studies suggest that the deformations of decay fragments play important role in deciding the mass division of fissioning nuclei. The present study is focused on the investigation of mass distributions by including higher multipole deformations up to hexadecapole (\beta_2 - \beta_4) with hot compact orientations [3]. For this purpose, the Astatine (At) isotopes with mass number $A_{CN}=191-220$ are chosen which are formed in $^{19}$F-induced reactions at centre-of-mass energy $E_{c.m.}=78.5$ MeV near the Coulomb barrier. Moreover, a relative study of ‘hot compact’ and ‘cold elongated’ optimized configurations of $\beta_2$-deformed fragments is also carried out. This analysis is carried out within the collective clusterization approach of dynamical cluster-decay model (DCM) [4-6]. The description of methodology is given in next section.

2. Dynamical cluster-decay model

The dynamical cluster-decay model (DCM) [4-6] is formulated using quantum mechanical fragmentation theory (QMFT) [7-8] for the charge and mass flow/transfer in nuclear decay processes by utilizing the idea of the mass (and charge) $\eta = (A_1 - A_2)/(A_1 + A_2)$ (and $\eta_Z = (Z_1 - Z_2)/(Z_1 + Z_2)$) asymmetry coordinates. The mass division of excited composite system has been explained by a dynamical treatment of collective asymmetry coordinates. Apart from asymmetry coordinate, another coordinates used to define the nuclear shape, and thus the parameters of our potential are relative separation $R$ between the two nuclei, multipole deformations ($\beta_{\lambda i}=2,3,4...$) and orientations ($\theta_i$) of two nuclei. Here $i=1, 2$ stand for the heavy and light fragments. The combine effect of attractive and repulsive energies on a decay process is examined in terms of collective fragmentation potential $V_R(\eta,T)$, and given as

$$V_R(\eta,T) = \sum_{i=1}^{2}[V_{LDM}(A_i,Z_i,T)] + \sum_{i=1}^{2}[\delta U_i] \exp(-T^2/T_0^2) + V_C(R,Z_i,\beta_{\lambda i},\theta_i,T)$$
$$+ V_P(R,A_i,\beta_{\lambda i},\theta_i,T) + V_t(R,A_i,\beta_{\lambda i},\theta_i,T).$$

(1)

Here, $V_{LDM}$ is $T$-dependent liquid drop energy of Davison et al. [9] and $\delta U$, the “empirical” shell correction, also made $T$-dependent to vanish exponentially with $T_0=1.5$ MeV. $V_C$, $V_P$ and $V_t$ are respectively, the $T$-dependent Coulomb, the nuclear proximity and the centrifugal potentials for deformed and oriented nuclei. For more details see refs. [4-5].

In order to determine the fragment mass distributions and energetically favoured nascent fragments, the preformation probability ($P_0$) is evaluated using mass-asymmetry $\eta$-dependent Schrödinger wave equation at fixed $R = R_0$,

$$\left\{ -\frac{\hbar^2}{2\sqrt{B_{\eta \eta}} \partial \eta} \frac{1}{\sqrt{B_{\eta \eta}}} \frac{\partial}{\partial \eta} + V_R(\eta,T) \right\} \psi^\nu(\eta) = E^{\nu} \psi^\nu(\eta),$$

(2)

with $\nu = 0,1,2,3...$ referring to ground-state ($\nu=0$) and excited-states solutions, and the ground state $P_0$ given by the solution of Eq. (2) as

$$P_0 = |\psi(\eta(A_i))|^2 \sqrt{B_{\eta \eta}} \frac{2}{A_{CN}},$$

(3)

Here, $B_{\eta \eta}$ is the smooth classical hydrodynamical mass [10]. Apart from this, the penetrability ($P$) of decaying fragments is calculated using the Wentzel-Kramers-Brillouin (WKB) approximation,

$$P = \exp(-2 \int_{R_a}^{R_b} \frac{2}{h} \{2\mu[V(R) - Q_{eff}]\}^{1/2} dR)$$

(4)
with $V(R_a, T) = V(R_b, T) = TKE(T) = Q_{eff}(T)$ for the two turning points. $V(R_a, T)$ acts like an effective $Q$ value of decay, $Q_{eff}(T)$, and $TKE(T)$ as the total kinetic energy of decaying fragments. The first turning point of penetration path, $R_a$ is defined as

$$R_a(T) = R_1(\alpha_1, T) + R_2(\alpha_2, T) + \Delta R(T) = R_3(\alpha, T) + \Delta R(T),$$

(5)

with radius vectors $R_i$ ($i = 1, 2$)

$$R_i(\alpha_i, T) = R_{0i}(T) \left[ 1 + \sum_{\lambda} \beta_{\lambda i} Y_{\lambda 0}^{0}(\alpha_i) \right]$$

(6)

and $T$-dependent nuclear radii $R_{0i}$ of the equivalent spherical nuclei, $R_{0i}(T) = [1.28A_{i}^{1/3} - 0.76 + 0.8A_{i}^{1/3}](1 + 0.0007T^2)\text{fm}$. $\Delta R$ is the only parameter of the model known as neck-length parameter, which assimilates the neck formation effects. For $\ell$-partial waves, the decay or fragment’s production cross-section is given by

$$\sigma(A_1, A_2) = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1)P_0P; \quad k = \sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}},$$

(7)

where $P_0$ refers to $\eta$-motion and $P$ to $R$-motion. $\mu = m_{A_1}A_2/(A_1 + A_2)$ is the reduced mass, and $\ell_{\text{max}}$ is maximum angular momentum fixed where the cross-sections of light particles become negligibly small.
Table 1. DCM-calculated fission cross-sections $\sigma_{\text{fission}}$ of $^{211}$At$^*$ compound nucleus formed in $^{19}$F+$^{192}$Os reaction at $E_{\text{c.m.}} \sim 78.5$ MeV for spherical, $\beta_2$-deformed and $\beta_2-\beta_4$-deformed fragmentation choices, which show nice agreement with the experimental [11] fission cross-section. The optimized values of neck-length parameter $\Delta R$ the most probable fission fragments are also shown.

| $\Delta R$ (fm) | $\ell_{\text{max}}$ (h) | Most probable fission fragments ($A_2$) | $\sigma_{\text{fission}}^{\text{DCM}}$ (mb) | $\sigma_{\text{fission}}^{\text{Expt.}}$ (mb) |
|----------------|-----------------|---------------------------------|------------------------|------------------------|
| Spherical fragmentation | 0.740 | 138 | 73-105 | 6.14 | 6.36 |
| ($\beta_2$)-deformed fragmentation | 0.868 | 140 | 70-91 | 6.32 | 6.36 |
| ($\beta_2-\beta_4$)-deformed fragmentation | 0.740 | 145 | 68-92 | 6.56 | 6.36 |

3. Calculations and results

A comparative analysis of the fragmentation potential of $^{211}$At$^*$ CN is carried out by using the spherical, the $\beta_2$ deformed, and the $\beta_2-\beta_4$ deformed cases, at $E_{\text{c.m.}}=78.5$ MeV. The calculations are made using optimum hot (compact) orientations of decaying fragments. The variation of fragmentation potential plotted in Fig. 1 clearly suggest that the potential surfaces are nearly smooth for spherical choice of decay fragments. However, fragmentation potential show significant structural variation after the inclusion of deformation effects. In other words, the symmetric fission path for spherical choice of fragmentation gets converted to the asymmetric one when deformation effects are included in the potential. Note that the minima in the fragmentation potential is the maxima of the preformation probability $P_0$ distribution as evident from Figs. 2 (c,g,k), respectively, for spherical, $\beta_2$ and $\beta_2-\beta_4$ deformed approaches for $^{211}$At$^*$ CN. Preformation distribution affirms that the symmetric path is followed by $^{211}$At$^*$ nucleus for the choice of spherical fragmentation. However, asymmetric mass fragments show higher preformation probability $P_0$ for the case of deformed fragmentation. It is observed that the fragments on the peaks of preformation profile are spherical or near-spherical (with very small magnitude of quadrupole and hexadecapole deformations), whereas the fission fragments at the valleys are highly deformed. This indicates that the deformations of decaying fragments play important role in the fission mass distribution. Further, the fission cross-sections are calculated at $E_{\text{c.m.}}=78.5$ MeV, using the $\ell$-partial wave method for these three choices of fragmentation, and the results are presented in Table 1. The calculated cross-sections for spherical as well as deformed fragmentation find nice agreement with experimental data [11].

Next, the isotopic analysis of mass distribution is carried out in Figs. 2(a-l) which represent the preformation probability $P_0$ of At$^*$ isotopes (with mass number $A_{\text{CN}}=191-220$) for spherical, ($\beta_2$)- and ($\beta_2-\beta_4$)- deformed choice of decay fragments. The calculated $P_0$ is plotted at common centre-of-mass energy $E_{\text{c.m.}}=78.5$ MeV. The optimized values of neck-length parameter $\Delta R$ for all the three choices of fragmentation as tabulated in table 1 is used to calculate the isotopic mass distributions. Figs. 2(a-d) show that At$^*$ isotopes mainly decay in symmetric fragments for the case of spherical fragmentation. Whereas, a transition from symmetric fission (one peak) to asymmetric fission (double peak) is observed as one goes from lighter to heavier isotope for the case of ($\beta_2$)- and ($\beta_2-\beta_4$)-deformed choice of fragments. As discussed earlier that the fragments at the peak of preformation profile are spherical (or near spherical), whereas the fragments at the dips havehigher deformation values. Thus, one can conclude that the deformations of decay fragments play important role to decide the mass division of fissioning nuclei. Apart from this, the most probable decay fragments near Z=35 for lighter and Z=50 for complementary
Figure 2. DCM-calculated preformation probability $P_0$ for the At$^*$ isotopes with mass number $A_{CN}=191-220$, formed in $^{19}$F-induced reactions at common $E_{c.m.}\sim 78.5$ MeV for (a-d) spherical, (e-h) ($\beta^-\lambda$)-deformed and (i-l) ($\beta^-\beta_4$)-deformed fragmentation choices. Fission fragments appear at the peak of preformation yield for the case of ($\beta^-\lambda$) as well as ($\beta^-\beta_4$) deformed choices of fragmentation.

Further, it would be of interest to study the comparative role of hot (compact) and cold (elongated) optimized orientations for $\beta^-\lambda$ deformations in context of decay pattern of $^{211}$At$^*$ nucleus at considered $E_{c.m.}\sim 78.5$ MeV. The hot configuration corresponds to the lowest interaction radius and highest potential barrier, however the cold configuration refers to the largest interaction radius and lowest interaction barrier. A significant change in the structure of fragment mass distribution is observed with these two different approaches as shown in Fig. 3. The triple humped mass distribution of $^{211}$At$^*$ nucleus becomes dominantly symmetric, when the orientations of decaying fragments changes from hot to cold configurations. Interestingly, this result is true for all the considered At$^*$ isotopes. This indicates that along with the deformations, orientations of decaying fragments play equally important role in the fission dynamics.

4. Summary

In this work, the fission fragment mass distributions of At$^*$ isotopes is analyzed at common $E_{c.m.}=78.5$ MeV for three choice of fragmentation, such as spherical, ($\beta^-\lambda$)-deformed and ($\beta^-\beta_4$)-deformed. The DCM-calculated results suggest that symmetric fission forms the main decay mode for the choice of spherical fragmentation. However, some structure effects appear in the mass distribution after the inclusion of deformation effects, and the mass distribution changes from symmetric to asymmetric division when neutrons are added in $^{191}$At$^*$ nucleus. It is also
Figure 3. Preformation probability $P_0$ of $^{211}\text{At}^*$ nucleus at $E_{c.m.}\sim 78.5$ MeV for $\beta_2$-deformed choice of fragments with ‘optimum’ orientations for (a) hot orientations and (b) cold orientations.

observed that fragments at the peaks of preformation profile are spherical in nature, whereas fragments at valleys are highly deformed. Moreover, the most probable heavy fission decay fragments are appearing near $Z=35$ for lighter and $Z=50$ for complementary heavy fission fragment. Further, the comparative analysis of ‘hot’ and ‘cold’ optimized orientations of decaying fragments having $\beta_2$-deformations, indicate contrasting behaviour of the fission fragment mass distributions for these two approaches.

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6. References
[1] Kaur A and Sharma M K 2019 Eur. Phys. J. A 55 89
[2] Kaur A and Sharma M K 2019 Indian J. Pure Appl. Phys. 57 576
[3] Gupta R K, Manhas M and Greiner W 2006 Phys. Rev. C 73 054307
[4] Singh B B, Sharma M K and Gupta R K, 2008 Phys. Rev. C 77 054613
[5] Kaur A and Sharma M K 2019 Phys. Rev. C 99 044611
[6] Kaur A, Kaur G, Patra S K and Sharma M K 2019 Nucl. Phys. A 990 94
[7] Fink H J, Maruhn J, Scheid W and Greiner W 1974 Z. Phys. 268 321
[8] Maruhn J and Greiner W 1974 Phys. Rev. Lett. 32 548
[9] Davidson N J, Hsiao S S, Markram J, Miller H G and Tzeng Y 1994 Nucl. Phys. A 570 61c
[10] Kröger H and Scheid W 1980 J. Phys. G: Nucl. Part. Phys. 6 L85
[11] Banerjee T et al. 2017 Phys. Rev. C 96 014618