Uniform Twistor–Like Formulation of Massive and Massless Superparticles with Tensorial Central Charges

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We construct the manifestly Lorentz-invariant twistorial formulation of the \(N = 1 \) \(D = 4\) superparticle with tensorial central charges which describes massive and massless cases in a uniform manner. The tensorial central charges are realized in terms of even spinor variables and central charge coordinates. The full analysis of the number of conserved supersymmetries has been carried out. In the massive case the superparticle preserves \(1/4\) or \(1/2\) of target-space supersymmetries whereas the massless superparticle preserves two or three supersymmetries.

1. Introduction

In a recent paper \cite{1} we proposed a new relativistic formulation of massive superparticle with tensorial central charges \cite{2}-\cite{9}. The model contains a commuting Weyl spinor as a collection of coordinates of the configuration space and describes a superparticle whose presence breaks two or three of \(N = 1, D = 4\) target-space supersymmetries. It is interesting that in the background of central charges the massive superparticle is equivalent to massive spinning particle \cite{10}, \cite{11} if a quarter of target-space supersymmetry is preserved. In a certain sense the commuting spinor variables of the model play the role of index spinor variables \cite{3}-\cite{4}. This model does not contain any special coordinates for the tensorial central charges. Analogous model of massive superparticle preserving \(1/4\) of target-space supersymmetries has been formulated in \cite{12} without explicit Lorentz covariance.

It should be mentioned that D. V. Volkov and his collaborators have proposed one of the first twistor-like models for the massless superparticle \cite{16} and established the equivalence between the spinning particle and the usual superparticle without central charges at least on the classical level. The idea of identifying the \(\kappa\)-symmetry of the superparticle with the local worldline supersymmetry of the spinning particle has been a basic one for the superfield formulation of massless superparticle theory \cite{16} and its generalization to the superembedding description of superbranes \cite{17}.

In this paper we present a twistorial formulation of the superparticle with tensorial central charges in which massive and massless cases are described in uniform manner. The model uses both the central charge coordinates and the auxiliary bosonic spinor variables simultaneously. Due to the use of spinors the analysis is simplified by reducing the tensorial quantities to scalar ones. For zero mass our model reduces to the twistorial formulation of the massless superparticle with tensorial central charges \cite{18} in which one or two of target-space supersymmetries are broken. In the massive case we have a bitwistorial formulation of the massive superparticle with tensorial central charges preserving \(1/4\) or \(1/2\) of target-space supersymmetries.
2. The formulation of the model

The configuration space of the model is parametrized along with the usual superspace coordinates \( x^\mu, \theta^a, \bar{\theta}^a \) and the tensorial central charge coordinates \( y^{\alpha \beta} = y^{(\alpha \beta)}, \bar{y}^{\dot{\alpha} \dot{\beta}} = \bar{y}^{(\dot{\alpha} \dot{\beta})} \) also by two bosonic spinors \( v^a_\alpha, \bar{v}^a_{\dot{\alpha}} \), \( \bar{v}^a_{\dot{\alpha}} = (v^a_\alpha), a = 1, 2, \alpha = 1, 2, \dot{\alpha} = 1, 2 \). Weyl spinor indices are raised or lowered with the help of the unit invariant skew-symmetric matrices \( \epsilon^{\alpha \beta} = \epsilon^{\dot{\alpha} \dot{\beta}} \), \( A_\alpha = \epsilon^{\alpha \beta} A_\beta, A_\alpha = \epsilon_{\alpha \beta} A^\beta \). The metric tensor \( \eta_{\mu \nu} \) signature is mostly plus and \( \sigma^{(\mu \nu)} = -\delta_\gamma^{\mu \nu} \), \( \sigma^{\mu \nu} \) and \( \sigma \)–matrices conventions of \( \{13\} \). The metric tensors \( v^a_\alpha, \bar{v}^a_{\dot{\alpha}} \) and their positions are interchanged under the complex conjugation.

For description of the superparticle with tensorial central charges we take the action in twisted-like form

\[
S = \int d\tau L, \\
L = P_\mu \Pi^\mu + Z_{\alpha \beta} \Pi^{\alpha \beta} + \bar{Z}_{\dot{\alpha} \dot{\beta}} \bar{\Pi}^{\dot{\alpha} \dot{\beta}} - \lambda(v^{\alpha a} \dot{v}_a - 2m) - \bar{\lambda}(\bar{v}^{\dot{\alpha} a} \dot{\bar{v}}_a - 2m).
\]

(1)

Here the one-forms

\[
\Pi^\mu = d\tau P^\mu + i d\theta \sigma^\mu \delta \bar{\theta} + i d\bar{\theta} \sigma^\mu \epsilon \theta, \\
\Pi^{\alpha \beta} = d\tau \Pi^{\alpha \beta} = d\bar{y}^{\alpha \beta} + i \theta \sigma^\alpha \epsilon \bar{y}^{\beta}, \\
\bar{\Pi}^{\dot{\alpha} \dot{\beta}} = d\tau \bar{\Pi}^{\dot{\alpha} \dot{\beta}} = d\bar{y}^{\dot{\alpha} \dot{\beta}} + i \bar{\theta} \sigma^{\dot{\alpha}} \epsilon \bar{y}^{\dot{\beta}}
\]

are invariant under global supersymmetry transformations

\[
\delta \theta^\alpha = e^\alpha, \quad \delta \bar{\theta}^\dot{\alpha} = \epsilon^\dot{\alpha}, \\
\delta x^\mu = i \theta \sigma^\mu \delta \bar{\theta} - i \bar{\theta} \sigma^\mu \epsilon \theta, \\
\delta y^{\alpha \beta} = i \theta \sigma^\alpha \delta \bar{y}^{\beta} - i \bar{\theta} \sigma^\alpha \delta \bar{y}^{\beta}, \\
\delta \bar{v}^a_\alpha = 0, \quad \delta \bar{v}^a_{\dot{\alpha}} = 0
\]

acting in the extended superspace parametrized by the usual superspace coordinates \( x^\mu, \theta^a, \bar{\theta}^a \) and by the tensorial central charge coordinates \( y^{\alpha \beta}, \bar{y}^{\dot{\alpha} \dot{\beta}} \).

The quantities \( P_\mu, Z_{\alpha \beta}, \bar{Z}_{\dot{\alpha} \dot{\beta}}, \bar{Z}_{\dot{\alpha} \dot{\beta}} \), which play the role of the momenta for \( x^\mu, y^{\alpha \beta}, \bar{y}^{\dot{\alpha} \dot{\beta}} \), are taken as the sums of products of two bosonic spinors \( v^a_\alpha, \bar{v}^a_{\dot{\alpha}} \), \( \bar{v}^a_{\dot{\alpha}} = (v^a_\alpha), a = 1, 2, \alpha = 1, 2, \dot{\alpha} = 1, 2 \).

\[
P_{\alpha \beta} = P_\mu \sigma^{\mu \alpha \beta}, \quad v^{\alpha a}, \bar{v}^a_{\dot{\alpha}}, \quad Z_{\alpha \beta} = v^a_\alpha v^{\beta b} C_{ab}, \quad \bar{Z}_{\dot{\alpha} \dot{\beta}} = \bar{v}^a_{\dot{\alpha}} \bar{v}^{\dot{\beta} b} \bar{C}_{ab}
\]

(4)

(5)

(6)

where \( C_{ab}, \bar{C}_{ab} = (C_{ab}) \) are symmetric constant matrices. These expressions are completely general with respect to the four–momentum \( P_\alpha \beta \) but imply some constraints on the central charges \( Z_{\alpha \beta}, \bar{Z}_{\dot{\alpha} \dot{\beta}} \). Here we do not give the explicit formulation of these constraints.

Due to the kinematic constraints

\[
v^{\alpha a} \dot{v}_a = 2m, \quad \bar{v}^a_{\dot{\alpha}} \dot{\bar{v}}_a = 2m,
\]

(7)

which are equivalent to

\[
v^{\alpha a} \dot{v}_a = m \delta^a_\alpha, \quad \bar{v}^a_{\dot{\alpha}} \dot{\bar{v}}_a = m \delta^a_{\dot{\alpha}},
\]

(8)

and enter the action \( \{1\} \) with Lagrange multipliers we have \( \det(v^{\alpha a}) = m \) and

\[
P^2 = P_\mu P^\mu = -m^2.
\]

(9)

Thus the constant \( |m| \) plays the role of the mass. It should be noted that the change of the sign of \( m \) is equivalent to antipodal transformations \( v^1_a \leftrightarrow v^2_a \) of bosonic spinors in “internal space” which leaves invariant the quadratic expressions \( \{4\} \) for the energy–momentum vector and central charges of the model.

In the massless case \( (m = 0) \) the spinors \( v^1_a \) and \( v^2_a \) are proportional to each other \( v^1_a \sim v^2_a \) as the consequence of the kinematic constraints \( \{5\} \). As a result one obtains a formulation of massless superparticle with one bosonic spinor from which both the massless four–momentum and the tensorial central charge are constructed.

Such a model has been analyzed in \( \{18\} \). The number of preserved SUSY is equal two or three in this model. In the proposed model \( \{4\} \) we use a minimal number of bosonic spinors, which is two, for
constructing the energy-momentum vector with arbitrary mass \(20\). Therefore we regard our formulation as the twistor–like one and concentrate on the massive case in the following.

Coefficients in the expansion of the symmetric central charge matrix \(C_{ab}\) in terms of the Pauli matrices \((\sigma_i)_{a} b\) form a complex dimensionless “internal” three–vector \(C = i(E + iH)\), real and imaginary parts of which we denote by analogy with electrodynamics. Thus
\[
C_{ab} = C_i(\sigma_i)_{ab}, \quad \bar{C}^{ab} = -\bar{C}_i(\sigma_i)^{ab} \tag{10}
\]
and
\[
C_{ab}C^{bc} = -CC\bar{C}_c = (E^2 - H^2 + 2iEH)\delta_{ab}. \tag{11}
\]
\[
C_{ab}\bar{C}^{ab} = 2C\bar{C} = 2(E^2 + H^2). \tag{12}
\]

One can simplify the matrix \(C\) of the central charges using redefinitions of the bosonic spinors with unitary unimodular transformation acting on the indices \(a, b, \ldots\) and leaving intact the four–momentum matrix and kinematic constraints. In fact with some loss of generality we could take the matrix \(C\) to be diagonal from the beginning.

3. \(\kappa\)–symmetry transformations

The variations of bosonic coordinates under the local \(\kappa\)–symmetry transformations \([2], [22]\) has the same form in terms of the variations of odd spinor coordinates as SUSY variations but are opposite in sign
\[
\delta x^a = -i\theta^c \delta \bar{\theta} + i\delta \bar{\theta} \theta^c, \\
\delta y^{\alpha\beta} = -i\theta^{(\alpha} \delta \bar{\theta}^{\beta)}, \quad \delta y^{\bar{\alpha}\bar{\beta}} = -i\bar{\theta}^{(\bar{\alpha}} \delta \bar{\theta}^{\bar{\beta})}, \tag{13}
\]
\[
\delta v_\alpha = 0, \quad \delta \bar{v}_\alpha = 0.
\]
Further, for the one–forms \([2]\) in the action we have
\[
\delta \Pi^a_\tau = -2i\dot{\theta} \theta^a \delta \bar{\theta} + 2i\delta \theta \dot{\bar{\theta}}^a, \tag{14}
\]
\[
\delta \Pi^{a\beta}_\tau = -2i\dot{\theta}^{(\alpha} \delta \bar{\theta}^{\beta)}, \quad \delta \Pi^{\bar{a}\bar{\beta}}_\tau = -2i\bar{\theta}^{(\bar{\alpha}} \delta \bar{\theta}^{\bar{\beta})}. \tag{15}
\]
The corresponding variation of the Lagrangian is
\[
\delta L = 2i(\bar{v}_a \delta \bar{\theta} + C_{ab} \delta \theta^b) \dot{\theta} v^a + 2i(\delta \bar{\theta} v^a + \bar{C}^{ab} \bar{v}_b \delta \bar{\theta}) \dot{\bar{v}}_a. \tag{16}
\]

The most general variations of the Grassmann spinors under \(\kappa\)–symmetry are
\[
\delta \theta^a = \kappa_a \theta^a, \quad \delta \bar{\theta}^a = \bar{\kappa}_a \bar{\theta}^a \tag{17}
\]
with two complex local Grassmann parameters \(\kappa_a(\tau), \bar{\kappa}_a(\tau) = (\bar{\kappa}_a)\). Taking into account the normalization conditions for the bosonic spinors \([8]\) we arrive at
\[
\delta L = 2i(\kappa_a + C^{ab} \kappa_b) \dot{\theta} v^a - 2i(\bar{\kappa}_a + \bar{C}_{ab} \bar{\kappa}^b) \bar{\theta} \dot{v}^a. \tag{18}
\]

The number of preserved supersymmetries is defined by the number of independent functions \(\kappa_a, \bar{\kappa}_a\) for which \(\delta L = 0\). Hence the equations
\[
\kappa_a + \bar{C}_{ab} \bar{\kappa}^b = 0, \quad \kappa_a + C^{ab} \kappa_b = 0 \tag{19}
\]
should have nontrivial solutions when there is \(\kappa\)–symmetry. These equations can be written in the matrix form
\[
\Delta \, \kappa = 0 \tag{20}
\]
where
\[
\Delta = \begin{pmatrix} \delta_{a}^b & \bar{C}_{ab} \\ \delta_{b}^a & \delta_{b}^a \end{pmatrix} \quad \text{and} \quad \kappa = \begin{pmatrix} \kappa_a \\ \bar{\kappa}_a \end{pmatrix}. \tag{21}
\]
The matrix \(\Delta\) is Hermitian, \(\Delta = \Delta^\dagger\), therefore it is unitary diagonalizable. The number of the independent \(\kappa\)–symmetries (solutions of eqs. \([19]\)) coincides with the number of the zero eigenvalues of the matrix \(\Delta\).

One can easily obtain that
\[
\det \Delta = 1 - C^{ab} \bar{C}_{ab} + \frac{1}{4} C^{ab} C_{ab} C^{cd} C_{cd}. \tag{22}
\]
So the necessary condition for the presence of \(\kappa\)–symmetries (one or more) consists in equality
\[
\det \Delta = 0. \tag{23}
\]
Some algebra gives
\[
\det(\Delta - \lambda 1_4) = \Lambda^4 - \Lambda^2 C^{ab} C_{ab} + \frac{1}{4} C^{ab} C_{ab} C^{cd} C_{cd} = (\Lambda^2 - E^2 - H^2)^2 - 4E \times H || \tag{24}
\]
with \(\Lambda \equiv 1 - \lambda\).
The characteristic equation reads
\[ \lambda^4 - 4\lambda^3 - k_2\lambda^2 + 2k_1\lambda + k_0 = 0 \]  
(25)
where the coefficients are
\[ k_2 = C^{ab}\bar{C}_{ab} - 6 = 2(E^2 + H^2 - 3), \]
\[ k_1 = C^{ab}\bar{C}_{ab} - 2 = 2(E^2 + H^2 - 1), \]
\[ k_0 = \det\Delta = (E^2 + H^2 - 1)^2 - 4|E \times H|^2. \]

Let us now consider all possible eigenvalues of the matrix \( \Delta \).

### 3.1. 3/4 unbroken SUSY

The presence of three zero eigenvalues means that the characteristic equation (25) must be of the form
\[ \lambda^3(\lambda - \lambda_1) = 0. \]
This gives us the conditions \( k_2 = k_1 = k_0 = 0 \) on the coefficients of eq. (25). However as one can see from the explicit expressions for the coefficients in our model the inequality \( k_1 \neq k_2 \) is always fulfilled. Therefore the presence of three zero eigenvalues is not possible in the massive case of the model under consideration. So one can not get three first class fermionic constraints and 3/4 unbroken SUSY in this case.

### 3.2. 1/2 unbroken SUSY

For two zero eigenvalues or 1/2 unbroken SUSY the equation on the eigenvalues \( \lambda^2(\lambda - \lambda_1)(\lambda - \lambda_2) = 0 \) means that \( k_1 = k_0 = 0 \) in eq. (27). This gives us two conditions on parameters of the central charges
\[ C^{ab}\bar{C}_{ab} = 2, \quad C^{ab}\bar{C}_{ab}\bar{C}_{cd} = 4 \]
or equivalently in the 3–vector form
\[ E^2 + H^2 = 1, \]
\[ E \times H = 0. \]
Thus in this case the vectors \( E \) and \( H \) are parallel, and they are not equal to zero simultaneously. If two eigenvalues are zero then two nonzero eigenvalues are both equal to 2.

Note that the above conditions, which define the case with 1/2 unbroken SUSY, are equivalent to
\[ C^{ab}\bar{C}_{ab} = 2, \quad C^{ab}\bar{C}_{ab}\bar{C}_{cd} = 2 \]
which are obtained by the Fierz transformation
\[ C^{ab}\bar{C}_{ab}\bar{C}_{cd} = 2(C^{ab}\bar{C}_{ab})^2 - 2C^{ab}\bar{C}_{bc}\bar{C}_{cd}. \]

Due to the first condition \( C^{ab}\bar{C}_{ab} = \delta^a_b + A^a_b \), the matrix \( A \) is traceless, \( A^a_b = 0 \) and Hermitian, \( A^+ = A \). But due to the second condition we obtain the equation \( A^a_bA^b_a = 0 \) which gives us \( A^a_b = 0 \). Thus in the case of two \( \kappa \)–symmetries (1/2 SUSY preserved) the coefficient matrix of the central charges is unitary
\[ C^{ac}\bar{C}_{cd} = \delta^a_c. \]

The solutions of eqs. (13), provided that condition (23) is fulfilled, can be obtained after the diagonalization of the matrix \( \Delta \)
\[ \Delta_{\mathrm{diag}} \equiv \left( \begin{array}{cc} 0 & 1 \\ 0 & 2 \end{array} \right) = V\Delta V^{-1}, \]
with
\[ V = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} \delta^a_b & -\bar{C}_{ab} \\ C^{ab} & \delta^a_b \end{array} \right). \]

To verify the unitarity of the matrix \( V \) and the equality (27) we have used the condition (26). Thus eq. (13) takes a simple form
\[ \Delta_{\mathrm{diag}} \kappa' = 0 \]
(28)
where \( \kappa' = V \kappa \). Obviously the solution of eq. (28) is
\[ \kappa' = (\bar{\nu}^a_0). \]

However the condition of mutual conjugacy \( \bar{\kappa}^a = \bar{\kappa}^a_0 \) of the upper and lower part of the column \( \kappa \) should be taken into account. To this end let us represent the symmetric unitary matrix \( C \) as a square of a symmetric unitary matrix \( \sqrt{C} \), whose explicit form is not required. Then for an arbitrary real odd two-component quantity \( \rho \) the quantity \( \nu = \sqrt{C}\rho \) satisfies the required conjugation condition. Thus we have demonstrated that the parameter space of the \( \kappa \)–transformations is actually a two–dimensional real space.

Eigenvectors corresponding to the eigenvalue 2 can be obtained in the similar way. But now \( \kappa' = (\bar{\nu}'_0) \) where \( \bar{\nu}' = \sqrt{C}\rho' \) and this space is parameterized by two arbitrary real odd quantities collected in the two-component “vector” \( \rho' \).
3.3. 1/4 unbroken SUSY

For a single zero eigenvalue or for 1/4 unbroken SUSY we have the single condition

\[ k_0 = \det \Delta = 1 - C^{ab} \bar{C}_{ab} + \frac{1}{4} C^{ab} C_{ab} \bar{C}^{cd} \bar{C}_{cd} = 0 \]

which in term of the vectors \( \mathbf{E} \) and \( \mathbf{H} \) has the form

\[ |\mathbf{E}^2 + \mathbf{H}^2 - 1| = 2|\mathbf{E} \times \mathbf{H}|. \]

In this case the characteristic equation is

\[ \lambda(\lambda - 2)(\lambda - 1 - \sqrt{C^{ab} C_{ab} - 1}) \times (\lambda - 1 + \sqrt{C^{ab} C_{ab} - 1}) = 0 \]

and the three nonzero eigenvalues are \( \lambda = 2, \lambda = 1 \pm \sqrt{C^{ab} C_{ab} - 1} \).

As it has been noted above the arbitrary symmetric matrix \( C \) can be reduced to the diagonal form

\[ C' = \begin{pmatrix} \rho_1 e^{i\varphi_1} & 0 \\ 0 & \rho_2 e^{i\varphi_2} \end{pmatrix} = VC V^{-1} \]

by means of the “internal” \( SU(2) \)–transformation \( V \). Here \( \rho_1, \rho_2 \) and \( \varphi_1, \varphi_2 \) are real. One can easily obtain that \( \rho_1^2 = |\mathbf{E}^2 + \mathbf{H}^2| \pm \sqrt{\mathbf{E}^2 \times \mathbf{H}^2} \). The case when \( \rho_1 = \rho_2 = 1 \) and the matrix \( C' \) is unitary has been considered in the previous subsection. Now we have

\[ C^{ab} \bar{C}_{ab} = \rho_1^2 + \rho_2^2 \]

and

\[ \det \Delta = (\rho_1^2 - 1)(\rho_2^2 - 1). \]

The eigenvalues of the matrix \( \Delta \) are 1, \( \rho_1 \) and 1, \( \rho_2 \).

The case of a single preserved SUSY is reached if only one of the moduli of the nonzero elements in the diagonal matrix \( C' \) is equal to 1, for definiteness let it to be \( \rho_1, \rho_1 = 1 \). After the diagonalization of the matrix \( C \) the eq. \( \Delta' K' = 0 \)

requires vanishing of all entries in \( K' \) except for \( \text{Im} e^{i\varphi_1/2} K'_1 = \nu \) which is arbitrary. This value plays a role of the parameter of the single unbroken SUSY. Further, for the \( \kappa \)–symmetry parameters \( \kappa \) one has

\[ \kappa = U^{-1} \begin{pmatrix} i e^{-i\varphi_1/2} \\ 0 \end{pmatrix}. \]

where \( U \) is a unitary unimodular matrix diagonalizing the matrix \( C_{ab} \).

Thus we have shown that the model of the massive superparticle described by the twistor–like action \( \mathcal{L} \) possesses one or two independent local \( \kappa \)–transformations which correspond to BPS configurations preserving 1/4 or 1/2 of the target–space supersymmetry. The case with 3/4 unbroken supersymmetry is not realized in the massive case of the presented model.

4. Constraints of the model

Phase space of the model is parametrized by the coordinate variables

\[ q^A = (x^\mu, y^{\alpha \beta}, \bar{y}^{\dot{\alpha} \dot{\beta}}; \theta^a, \bar{\theta}^a; \bar{\theta}_a, \bar{\theta}_a) \]

and by corresponding canonically conjugate momenta

\[ p_A = (p_\mu, \bar{z}_{\alpha \beta}, \bar{\xi}_{\dot{\alpha} \dot{\beta}}; \pi_\alpha, \bar{\pi}_{\dot{\alpha}}; \omega^a, \bar{\omega}_a). \]

We take the standard definition of the Legendre transformation \( p_A = \partial L / \partial q^A \) and of the graded Poisson brackets \( \{ q^A, p_B \} = \delta^A_B \) for all basic phase variables.

The Lagrangian \( \mathcal{L} \) is homogeneous with respect to all velocities, therefore the expressions for all momenta lead to the primary constraints

\[ D_\alpha = -i\pi_\alpha - P_\alpha \bar{\theta}^\dot{\beta} - \theta^A Z_\beta \approx 0, \]

\[ \bar{D}_\dot{\alpha} = (D_\alpha) = -i\bar{\pi}_{\dot{\alpha}} - \theta^A P_\beta \bar{Z}_{\dot{\beta}} \approx 0; \]

\[ T_{\alpha \beta} = P_\alpha - P_\beta \approx 0; \]

\[ R_{\alpha \beta} = Z_{\beta} - Z_\alpha \approx 0; \]

\[ \bar{R}_{\dot{\alpha} \dot{\beta}} = \bar{Z}_{\dot{\beta}} - \bar{Z}_{\dot{\alpha}} \approx 0; \]

\[ \omega^a \approx 0, \quad \bar{\omega}^\dot{a} \approx 0 \]

where \( P_\alpha, Z_\alpha, \bar{Z}_{\dot{\alpha}} \) have the expressions \( \mathcal{L} \)–in terms of the bosonic spinors. In addition, the whole system of constraints includes the kinematic constraints

\[ v^{\alpha a} v_{\alpha a} - 2m \approx 0, \quad \bar{v}_{\dot{\alpha} \dot{a}} \bar{v}^\dot{\alpha} \dot{a} - 2m \approx 0 \]
which are explicitly introduced into the action. The kinematic constraints are secondary ones if Lagrange multipliers are assigned to canonical variables. Any other constraints do not appear in the model.

The analysis of the $\kappa$–symmetry is based on the consideration of the odd constraints. Their Poisson bracket algebra is

$$\{D_a, D_b\} = 2i\bar{P}_{\alpha\beta},$$

$$\{D_a, D_{\beta}\} = 2iZ_{\alpha\beta},$$

$$\{\bar{D}_a, \bar{D}_{\beta}\} = 2i\bar{Z}_{\alpha\beta}.$$  

The analysis of the constraints is simplified when they are projected on the spinors $v^a, \bar{v}_a$. For the fermionic constraints we get

$$D^a \equiv v^a D = -i\bar{v}^a \bar{\theta} + m(v^a \bar{\theta} - C^{ab} \theta b) \approx 0,$$  

$$\bar{D}_a \equiv \bar{D}_a = -i\bar{\theta} - m(\bar{v}_a - \bar{C}_{ab} \theta b) \approx 0.$$  

Due to the kinematic constraints the canonically conjugate momenta for the Grassmann variables $\theta_a \equiv \theta a$ and $\bar{\theta}^a \equiv -\bar{v}^a \bar{\theta} = (\theta_a)$ are $\pi^a = v^a \bar{\theta}, \tilde{\pi}_a = \bar{\pi}_a / m$ and $\{\theta_a, \pi^b\} = \{\bar{\theta}^b, \bar{\pi}_a\} = \delta^b_a$. In terms of these variables the fermionic constraints acquire simple form

$$D^a = -m(i\pi^a + \bar{\theta}^a + C^{ab} \theta b) \approx 0,$$  

$$\bar{D}_a = (\bar{D}_a^a) = -m(i\tilde{\pi}_a + \theta a + \bar{C}_{ab} \bar{\theta}^b) \approx 0.$$  

Their Poisson brackets are

$$\{D^a, D^b\} = 2im^2\delta_{ab},$$  

$$\{D^a, D^b\} = 2im^2C^{ab},$$  

$$\{\bar{D}_a, \bar{D}_b\} = 2im^2\bar{C}_{ab}.$$  

The algebras of the Lorentz–spinor constraints $D_a, \bar{D}_a$ and of the Lorentz–scalar constraints $D^a, \bar{D}_a$ are identical. But in the second case the role of the central charges is played by the Lorentz–scalar quantities $C^{ab}, \bar{C}_{ab}$ instead of $Z_{\alpha\beta}, \bar{Z}_{\alpha\beta}$ and by the static momentum $p^0 = m, \mathbf{p} = 0$ instead of the usual four–momentum. The consideration in terms of quantities with indices $a, b, ...$ is Lorentz covariant due to the use of the bosonic variables $v^a, \bar{v}_a$, which play the role of harmonic variables parametrizing an appropriate homogeneous subspace of the Lorentz group.

The matrix of the Poisson brackets

$$\left(\begin{array}{ccc} \{D_a, D_b\} & \{\bar{D}_a, \bar{D}_b\} & \{D^a, D^b\} \end{array}\right) = 2m^2\Delta.$$  

is in fact the matrix $\Delta$. Its eigenvalues and odd eigenvectors have been found above. Thus, the separation of the first and second class Fermi constraints can be done straightforwardly.

It is convenient to introduce the new constraints

$$\Delta(\lambda) = X(\lambda)\Delta$$

where $D = \left(D^a\right)$ and $X(\lambda) = \left(X^a\right)$ is an even normalized eigenvector of the matrix $\Delta$ with an eigenvalue $\lambda$, i.e. $\Delta X(\lambda) = \lambda X(\lambda)$. The eigenvectors with different eigenvalues are orthogonal, the eigenvectors having the same eigenvalue can be chosen orthogonal. Here we do not need to distinguishing the special notation of different eigenvectors corresponding to the same eigenvalue. The algebra of new constraints takes a very simple form

$$\{\Delta(\lambda), \Delta(\lambda')\} = 2m^2\delta_{\lambda\lambda'}.$$  

Thus the repetition of the analysis which was made in the previous section allows us to obtain the full system of orthonormal eigenvectors $X(\lambda)$ of the matrix $\Delta$ and to construct the first class constraints $D(0)$ which correspond to the zero eigenvalues and generate the $\kappa$–symmetry transformations.

5. Conclusion

In this paper we have constructed the twistor–like model of the superparticle with tensorial central charges. The proposed model uniformly describes cases of massive and massless superparticles. For the description of the degrees of freedom associated with tensorial central charges we have used coordinates of central charges as well as additional bosonic spinors. The latter variables have also been used for the twistor–like representation of the momentum. In the case of zero mass one can obtain the twistor–like formulation of the
superparticle with tensorial central charges preserving $1/2$ and $3/4$ of target–space supersymmetry. In the massive case our model has one or two $\kappa$–symmetries and preserves $1/4$ or $1/2$ of the target–space supersymmetry. The additional bosonic spinors have been used as Lorentz harmonic variables. This allowed us to eliminate auxiliary and gauge degrees of freedom without the violation of the Lorentz invariance.

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