**Insights in Economical Complexity in Spain: the hidden boost of migrants in international tradings.**

Elena Agliari, Adriano Barra, Andrea Galluzzi, Francisco Requena-Silvente, and Daniele Tantari

1 Dipartimento di Fisica, Sapienza Università di Roma, Italy
2 Dipartimento di Matematica, Sapienza Università di Roma, Italy
3 Department of Economics, Sheffield University, United Kingdom

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We consider extensive data on Spanish international trades and population composition and, through statistical-mechanics and graph-theory driven analysis, we unveil that the social network made of native and foreign-born individuals plays a role in the evolution and in the diversification of trades. Indeed, migrants naturally provide key information on policies and needs in their native countries, hence allowing firm’s holders to leverage transactional costs of exports and duties. As a consequence, international trading is affordable for a larger basin of firms and thus results in an increased number of transactions, which, in turn, implies a larger diversification of international traded products. These results corroborate the novel scenario depicted by “Economical Complexity”, where the pattern of production and trade of more developed countries is highly diversified. We also address a central question in Economics, concerning the existence of a critical threshold for migrants (within a given territorial district) over which they effectively contribute to boost international trades: in our physically-driven picture, this phenomenon corresponds to the emergence of a phase transition and, tackling the problem from this perspective, results in a novel successful quantitative route. Finally, we can infer that the pattern of interaction between native and foreign-born population exhibits small-world features as small diameter, large clustering, and weak ties working as optimal cut-edge, in complete agreement with findings in “Social Complexity”.

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**I. INTRODUCTION**

In this work we aim to merge recent findings in Social Complexity [1,2] with those achieved in Economical Complexity [3,4], in order to deepen our understanding of socio-economical behaviors observed in developed societies. In particular, we examine the role of migratory fluxes on the economical diversification and international trading of the hosting countries.

The emergence and the fitness of economical diversification is nowadays still questioned: classical economic theories prescribe specialization of industrial production for more performing countries [5,6], while recent studies [3,4,7,8] show that diversification of products plays a key role in modern economies. Quoting Hidalgo, Klinger, Barabasi and Hausmann “inspection of the country databases of exported products shows that successful countries are extremely diversified, in analogy with biosystems evolving in a competitive dynamical environment” [3]. Oversimplifying, the key idea to explain such a diversification is that, if the factors (e.g., technology, capital, institutions, skills) necessary for a country to produce a good are (partially) shared with another good, it will be likely that both goods will be produced [3]. Here, we address a closely related problem: we investigate the diversification of the production of a country by looking at its exports and connecting diversification in trades with social complexity beyond economical complexity. In particular, we quantitatively show that stocks of foreign migrants play a crucial role in the establishment of international trades of diversified products, thus contributing to explain the genesis of the Hidalgo, Klinger, Barabasi and Hausmann picture. In a nutshell, our results (in agreement with recent literature [9,13,14]), suggest that social interactions between native and foreign-born populations allow transferring to local firms a crucial knowledge about policies, needs and duties existing in the foreign countries. Remarkably, this information, coupled with firms’ holder capabilities, permits to decrease the overall potential costs of trading thus allowing a larger number of firms to appear in the global market, which, in turn, implies broader and diversified trades.

Thus, our claim is that the interaction network between migrants and natives spreads the social capital (i.e. the collective resources of the community, including information, expertise and skills) and this enhances the extensive margin of trades, which, in turn, acts as a boost in the diversification of the exported products.

In order to prove these statements, we introduce a statistical-mechanics scaffold (where data can be rationally framed) and, step by step, we check for the empirical confirmation of our assumptions and our theoretical results, by analyzing the test case of Spain. In fact, this country has experienced a (well-documented) influx of migrants since 1998 with a very rapid increase during the period 2000 – 2008 [1][2][15][17] and this constitutes an ideal context to investigate the role of immigrants in creating new trade relationships.

More precisely, our work is structured as follows.

In the first part, devoted to the statistical mechanical analysis, we introduce the simplest possible model (i.e.,
a minimal Hamiltonian) that relates two parties: foreign-born and native people living in a given district of the country. As a result of the interaction between the two parties, natives will -stochastically- decide whether to trade with the country of origin of immigrants. Remarkably, we prove that this model belongs to the class of copying-model \cite{18, 19}, or single-party ferromagnets in the jargon of statistical physics, where native decision-makers alone come to play and they spontaneously behave in an imitative way. Through this approach we are able to quantify the role of immigration in the volume of trades and to include this phenomenon in the framework of the phase transitions. Within this setting, we can also test empirically whether a critical mass of migrants in needed in order to ensure that a positive pro-trade effect of migration exists \cite{17, 20}, in agreement with the pioneering suggestions by Gould \cite{21} and, more recently, with the non-linear theories driven by Chaney’s distorted gravity scheme \cite{22}. Our theoretical findings predict a non-linear dependence, encoded by an hyperbolic tangent, for exports to a given foreign country versus the percentage of immigrants hailing from that country, and are successfully checked by comparison with the Spanish dataset. We conclude the first part of the paper by proving the existence of a net and robust correlation between the degree of product-destination diversification of exports (measured in terms of the Herfindhal index) and the number of migrants as a fraction of the total population.

Finally, our theory also allows us to infer the topological structure of the host society, and this is addressed in the second part of the paper. Interestingly, we find that the society displays small-world features and recovers the Granovetter theory of weak ties \cite{23, 25}. Incidentally, we notice that this is also compatible with recent researches investigating the role of immigrant integration in labor markets \cite{20}.

II. RESULTS

Before introducing our model, a few points must be clarified (and empirically proven to hold):

- Our theory, developed within a classical statistical mechanical perspective, is set at a microscopic level and it accounts for an ensemble of native “decision makers”, whose behavior (i.e., the propensity to undertake an international trade) can be affected by the interaction with migrants. However, the theoretical outcomes of such a model are compared with available data on international trades performed by firms: in principle, it is not obvious that we can switch from the microscopic level (i.e. decision makers), where the whole theory lies, to the mesoscopic level (i.e. firms), where the data analysis is performed. This is allowed if and only if there exists a linear proportionality between the total population and the total amount of firms. Luckily, this is the case in Spain for the considered time window (1998–2012), as corroborated by empirical findings shown in Fig. 1. Thus, as far as scalings are concerned, we can exploit the theoretical predictions for the average behavior of decision makers (stemming from the statistical-mechanics model) to describe the expected attitude of firms (that we infer from empirical data).

- The total amount of trades $Y$ is usually defined in terms of two contributions: the amount of firms that perform international trading (i.e. extensive margin $Y_{ext}$) and the amount of money each firm moves in any transaction (i.e. intensive margin $Y_{int}$), namely $Y = Y_{ext} \cdot Y_{int}$, or, in a logarithmic scale, $\log Y = \log Y_{ext} + \log Y_{int}$. Chaney has shown that a reduction in fixed trade costs has a positive impact on $Y_{ext}$ \cite{22}; Peri and Requena have shown that migrants have a positive effect on the extensive margin of trade in Spain, hence deriving that migrants facilitate trade mainly by reducing the fixed costs of exporting \cite{15, 17}. On the other hand, the intensive margin of trades seems to be poorly affected by migration stocks. Thus, our theory is actually devoted to capture the evolution of $Y_{ext}$.

- The database available reports about the total volume of transaction, that is $\log Y$. As a consequence, we first need to prove that the expected linear proportionality between $\log Y$ and $\log Y_{ext}$ is fulfilled by our data, such that, later, we will be authorized to analyze the evolution of $\log Y$ as a function of migrant density inside the host country in order to extrapolate an analogous scaling for $\log Y_{ext}$ too. This proportionality is robustly checked as shown in Fig. 2.

![FIG. 1: Each data point (blue bullet) represents the number of firms versus the population of a given Spanish province (out of 50) for a given year (in the interval 1998 – 2012). The linear proportionality of these quantities is highlighted by binned data (green squares), whose best fit is given by a linear law (red solid line) with slope $\approx 1.02 \pm 0.03$ and goodness $R^2 \sim 0.99$.](image-url)
and individuals can be divided into two groups: we denote with \( N \) the provincial level. Then, for any arbitrary province, we fix the degree of resolution at in-province communities: note that for small \( \gamma \) and natives, being \( N_1 + N_2 = N \). We also define
\[
\gamma \equiv \frac{N_2}{N}, \quad 1 - \gamma \equiv \frac{N_1}{N},
\]
measuring the relative size of the two groups and we introduce \( \Gamma \equiv (1 - \gamma) \), too, the latter representing the normalized number of cross links between the two communities: note that for small \( \gamma \) (and this is the case for Spain), \( \Gamma \sim \gamma \).

Moreover, we introduce variables (i.e. spins), referred to as \( \{\sigma_i\}_{i=1}^{N_1} \) and \( \{\xi_{ij}\}_{\mu=1}^{N_2} \), respectively, such that \( \sigma_i \in \{-1,+1\} \) represents the propensities of the native agent \( i \) to establish \( (\sigma_i = +1) \) or not establish \( (\sigma_i = -1) \) a trade, while the variables \( \xi_{ij} \) represent the quantity of information, either positive \( (\xi_{ij} > 0) \) or negative \( (\xi_{ij} < 0) \), that the \( \mu \)-th immigrant can provide (regarding trading toward his/her country of origin). Otherwise stated, the ensemble \( \{\xi_{ij}\}_{\mu=1}^{N_2} \) represents the social capital of the immigrant community and, in the absence of any additional information, in a mean-field approach, it can be thought of as a collection of Gaussian variables identically and independently distributed.

The diffusion of the social capital and the decisional mechanism can be now described by an Hamiltonian (i.e. a cost function in economical vocabulary) \( \mathcal{H}(\sigma,z;J,\xi) \), dependent on the couplings \( J \) and \( \xi \), encoding for native-native interactions and for native-migrant interactions, respectively (see Fig. 3 left panel).

Now, let us inspect in more details the interaction patterns and the resulting Hamiltonian.

The interaction between a native, say \( i \), and a foreign-born, say \( \mu \), is encoded by the variable \( \xi_{ij}^\mu \in \{0,1\} \) describing the presence \( (\xi_{ij}^\mu = 1) \) or the absence \( (\xi_{ij}^\mu = 0) \) of a connection (e.g. friend, work-mate, acquaintance, familiar) between \( i \) and \( \mu \). The set of variables \( \xi \) generates the topology of the social network between immigrants and natives. Since there exist neither detailed information about individual connections, neither a broadly accepted protocol for their measure, and checking that migratory fluxes are uncorrelated (i.e. the time-scales considered are long enough and migrants comes from a wide range of countries), the most basic assumption one can then pose is simply to consider the completely general set of \( \xi_{ij}^\mu \) as i.i.d. aleatory variables, extracted with probability
\[
\mathbb{P}(\xi_{ij}^\mu = 1) = 1 - \mathbb{P}(\xi_{ij}^\mu = 0) = \xi \}
\]
where \( \theta \in (0,1) \), and \( \xi \in \mathbb{R}^+ \) are parameters province-dependent: in this way, properly tuning \( \theta \) and \( \xi \), the network recovers all the standard regimes (e.g., extreme dilution, finite connectivity, etc.) and, by fitting these parameters over the available data, we can infer the topological features of the actual Spanish network for the analyzed years [29].

Analogously, \( J \) describes the connections among natives and, at this stage, it can be assumed to be arbitrary but endowed with a well defined average value \( J \) (see also Part II for more details), which, in principle, depends on the province \( p \).

Therefore, at the provincial level of resolution, the system can be described by the Hamiltonian
\[
\mathcal{H}(\sigma,z;J,\xi) = -\frac{1}{N_1} \sum_{(i,j)} J_{ij} \sigma_i \sigma_j - \frac{1}{N_1^{1-\theta}} \sum_{i=1}^{N_1} \sum_{\mu=1}^{N_2} \xi_{ij}^\mu \sigma_i \xi_{ij}^\mu.
\]
Note that in the second term in the r.h.s. of the above equation, the normalization factor \( 1/N_1^{1-\theta} \) ensures the linear extensivity of the Hamiltonian or, analogously, that the field \( h_i \) acting on any spin \( \sigma_i \) is \( \mathcal{O}(1) \). In fact, \( h_i = \frac{1}{N_1^{1-\theta}} \sum_{\mu} \xi_{ij}^\mu \sigma_i \xi_{ij}^\mu = \mathcal{O}(1) \), as the expected number of non-null entries in the vector \( \xi_i \), namely the expected number of non-null terms in the sum, is \( \mathcal{O}(N_1^{1-\theta}) \).

Before proceeding, we need to introduce a parameter \( \beta \) to tune the degree of stochasticity in the system, in such A. PART ONE: Insights from Statistical Mechanics

First, we need to set a proper length-scale: as the migration-trade relation is known to be an in-province phenomenon [24] [16], we fix the degree of resolution at the provincial level. Then, for any arbitrary province, we denote with \( N \) its population and notice that the \( N \) individuals can be divided into two groups: \( N_1 \) natives and \( N_2 \) foreign-born, being \( N_1 + N_2 = N \). We also define
\[
\gamma \equiv \frac{N_2}{N}, \quad 1 - \gamma \equiv \frac{N_1}{N},
\]
for a given Spanish province (out of 50) for a given year (in the interval \( 1998 - 2012 \)). Binned data (green squares) are best-fitted by a straight line (red solid line) \( y = ax \), being \( a \approx 0.006 \), and \( R^2 \approx 0.89 \).
a way that for \( \beta \to 0 \) the system behaves completely randomly, while as \( \beta \to \infty \) the system deterministically relaxes to the configuration corresponding to the minimum of the cost function. Thus, the partition function \( Z \) of the model defined by the Hamiltonian \( (3) \) reads as

\[
Z = \sum_{\sigma} e^{\beta \sum_{i,j} \sigma_i \sigma_j} \int d\mu(z) e^{\beta \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \xi_i \sigma_i \xi_j},
\]

where we called \( d\mu(z) \) the standard Gaussian measure. Crucially, by a direct comparison of the arguments in the exponents of Eqs. (4) and (5), respectively, we see that the bipartite interactions between natives and immigrants (i.e. those \( \propto \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \xi_i \sigma_i \xi_j \) in the first line) are stored in an effective coupling \( \tilde{J}_{ij} \) between couples of local decision makers alone (i.e. those \( \propto \sum_{i,j} \sum_{\mu=1}^{N_2} \xi_i \sigma_i \xi_j \) in the second line). Such a coupling is Hebbian-like \( (29) \) as

\[
\tilde{J}_{ij} = \frac{\sum_{\mu=1}^{N_2} \xi_i \xi_j}{N^2 (1 - \bar{\gamma})},
\]

The “order parameter” for this model is given by \( M(\sigma) = \frac{1}{N} \sum_{i=1}^{N_1} \sigma_i \bar{\sigma}_{i,1} \), namely the fraction of individuals inclined to an international trade (i.e., the amount of spins positively aligned). This order parameter is equivalent (upon translation) to \( m(\sigma) = \frac{1}{N} \sum_{i=1}^{N_1} \sigma_i \), namely the standard magnetization of the system (in its ferromagnetic interpretation \( (27, 28) \)). Now, it is worth recalling that the linear proportionality between decision makers and firms in the Spanish provinces (see Fig. 1) allows inferring only scalings and proportionality relations (but not exact values) for the amount of trading firms. Therefore, there is no loss of information in using the (mathematically more convenient) \( m \) instead of \( M \), and hereafter we will retain the former observable to quantify the extensive margin of trades \( Y_{ext} \). Moreover, as explained in the previous section, the evolution in \( Y_{ext} \) can be related to the evolution of trades \( Y \) as a whole.

By applying the standard statistical-mechanical machinery (see Appendix A for a detailed derivation), we attain the following self-consistent equation for \( m \):

\[
m = \tanh(\beta \bar{J} m + \beta^2 \xi^2 \Gamma m). \tag{7}
\]

This is the main formula in this first part as, following the scaling \( m \propto Y \) argued above, it relates the growth of trades with the percentage of migrants (we recall \( \Gamma = \gamma(1 - \gamma) \approx \gamma \)). The agreement between Eq. (7) and the Spanish test case is reported in Fig. 4 and deepened in the Data Analysis Section.

Remarkably, Eq. (7) also contains information regarding the critical percentage of migrants that must be reached before they start to influence new trade relationships. To extract such information, we exploit the statistical physics know-how of phase transitions: when the argument of the hyperbolic tangent is smaller than one the only solution for Eq. (7) is \( m = 0 \). However, as the argument gets larger than one, non-zero solutions appear and we can expand the hyperbolic tangent as

\[
m \sim \beta(\bar{J} + \beta \xi^2 \Gamma)m - \frac{\beta^2}{3}(\bar{J} + \beta \xi^2 \Gamma)^3m^3 + O(m^4), \tag{8}
\]

and, excluding the paramagnetic solution \( (m = 0) \), we get

\[
m \sim \sqrt{\frac{3}{\beta^2(\bar{J} + \beta \xi^2 \Gamma)^2}}[\beta(\bar{J} + \beta \xi^2 \Gamma) - 1] = a \sqrt{\Gamma - \Gamma_c}, \tag{9}
\]

where \( a = 3\xi^2 / [\beta(\bar{J} + \beta \xi^2 \Gamma)]^3 \) and \( \Gamma_c = (1 - \bar{J}) / (\beta \xi)^2 \). From the previous equation we see that as far as \( \Gamma < \Gamma_c \) no real solution to this equation exists. Thus, when the percentage of migrants within a given province is smaller than \( \Gamma_c \), trades can of course take place, but the related international market is not influenced by the presence of migrants within the province itself.

Three important aspects of the relation between migration and trading are thus coded in equation Eq. (7):

- The relation between migrant density and growth of trades is non-linear, as these observables are related via an hyperbolic tangent.
- There exists a critical value for the fraction of migrants, that reads as
  \[
  \Gamma_c = \frac{1 - \beta \bar{J}}{\beta^2 \xi^2}, \tag{10}
  \]
  beyond which they start to have a net effect on international trading for the host province. Notice that \( \Gamma_c \) is stochastic (via \( \bar{\gamma} \)) and, in principle, province dependent through \( \bar{J} \) and \( \xi \). In fact, we stress that the previous derivation holds for any arbitrary province and, in general, the parameter set \( (N, J, \xi) \) is province dependent, in such a way that the outline for \( m \) versus \( \Gamma \) as well as the critical value \( \Gamma_c \) vary with the province. However, note that, in principle, \( \Gamma_c \) can be vanishing.
- There is a saturation effect for large enough \( \Gamma \) as the hyperbolic tangent is a bounded function that eventually reaches a plateau. Exhaustion levels in bilateral exports have already been linked with migrant saturation effects as, for instance, in the experimental works discussed in [9].
1. Data Analysis

We check our findings versus empirical data for the test-case of Spain. The overall dataset is obtained by merging two sources: trade data come from ADUANAS-AEAT dataset provided by Ministerio de Economia y Hacienda, and demographic data come from the Spanish Statistical Office (INE).

We consider the time series for exports \{Y_{y,p}\} and for the fraction of immigrants \{\gamma_{y,p}\}, along the range of years \(y = 1998, ..., 2012\) and for the 50 provinces \(p = 1, ..., 50\) making up the country (EUROSTAT NUTS III definition). Thus, our time range is made of \(N_y = 15\) years and our geographic set is made of \(N_p = 50\) provinces.

Preliminarily, as we start from historical series, we check that at least one of the observables \(Y\) and \(\gamma\) is monotonically increasing with respect to the years \(y\), and \(\gamma(y)\) satisfies this request. Thus, we are allowed to invert \(\gamma(y) \rightarrow y(\gamma)\) and look at the evolution of \(Y\) as a function of \(\gamma\), so to obtain \(Y(\gamma)\) that must then be suitably binned and averaged (see Fig. 11 for details on this procedure).

The whole set of provinces constitutes our pool, namely we consider different provinces as independent realizations (or, otherwise stated, extractions) of the same system. This means that the trades of a given province are taken to depend only on the fraction of immigrants within the province itself. While there is general consensus on this, the consistency of such an hypothesis is shown in [32], where the authors prove that the proximity (meant as geographical closeness) is fundamental for the diffusion of the social capital and therefore for the growth of trades.

For each province \(p\), we can measure the percentage of immigrants \(\gamma_p\) and plot \(Y_p\) versus \(\Gamma_p \sim \gamma_p\), as shown in Fig. 11 for some exemplary cases. Note that theoretical predictions (see Eq. 7) are in remarkable agreement with the empirical behaviour. We performed extensive fits over all the provinces available according to Eq. 11, which we report hereafter as

\[
m = \tanh \left( \frac{(1 - b)\Gamma}{\Gamma_c} + b \right) m,
\]

where we highlighted the critical density \(\Gamma_c = (1 - b)/(\beta \xi)^2\) and we posed \(b = \beta J\). While fitting, an extra, province-dependent, parameter referred to as \(a\), has to be introduced in order to account for the fact that, due to the scaling between \(Y\) and \(m\), the former is in principle not bounded. The best-fit coefficients are collected in Fig. 3. Notably, we checked that these results are in full consistency with the analogous parameters that one would obtain when fitting with the more explicit square root function \((\xi)\), at least as far as small values of \(\Gamma\) are considered. In particular, we notice that \(\log(a)\) is roughly uniformly distributed along the range \((12, 19)\), suggesting that the extent of exports varies over several orders of magnitude, according to the province considered. On the other hand, \(\Gamma_c\) looks Poissonian-like distributed and is peaked around 0.003, suggesting that

![FIG. 4: Exports \(Y\) versus \(\Gamma\) for three different provinces as explained by the legend; we choose the three largest provinces for the sake of readability and for consistency with the analysis of the following sections, however, we checked that analogous plots hold also for the other provinces. In this plot each data point corresponds to a different year. The solid lines represent the best fit according to Eq. 11 and the goodness of the fit is \(R^2 = 0.94\) (Madrid), \(R^2 = 0.97\) (Barcelona), and \(R^2 = 0.95\) (Valencia).](image1)

![FIG. 5: Histograms for the best-fit coefficients \(\Gamma_c\) (upper panel), \(a\) (middle panel), and \(b\) (lower panel) obtained by fitting \(Y_p\) versus \(\Gamma_p\) according to Eq. 11 for each province \(p\). Notice that, due to the broad range along which \(Y_p\) (and, accordingly, \(a\)) spans, we represent the histogram of \(\log(a)\).](image2)
2. Bilateral trades

In order to get a finer picture, and to deepen the possible existence of a country-dependent critical threshold $\Gamma_c$, we fragment the migrant party into several subsets, each corresponding to a different country of origin and then we analyze the trades $\Gamma_{p,f}$ performed between any province $p$ and any foreign country $f$ as a function of the related fraction of immigrants $\Gamma_{p,f}$. Of course, results are expected to be much more noisy, as we are dealing with considerable smaller datasets and the intrinsic fluctuations are only partially smoothened by the central limit theorem. Nonetheless, it is worth checking whether the previous results are still valid at this less coarse-grained level, and inferring the country-dependent critical masses. We focused on the three major Spanish cities, namely Madrid (Fig. 6), Barcelona (Fig. 7) and Valencia (Fig. 8) and on the foreign countries for which the size of immigrant communities are larger and span along a wide interval in the time window considered, in order to get more accurate and reliable fits.

By fitting data according to Eq. 11 we derive estimates for $\Gamma_c$ which, in general, depend on both $p$ and $f$, as shown in Fig. 9. In particular, $\Gamma_c$ follows a distribution peaked around $\Gamma_c \approx 10^{-5}$, that is consistent with the previous value $\sim 3 \cdot 10^{-3}$ as migrants come from $O(10^2)$ different countries.

We finally notice that $\Gamma_c$ seems to slightly vary with the size of the hosting population, consistently with expected finite size effects.

Lastly, we checked that there is a clear correlation between the critical value $\Gamma_c$, obtained for trades between $p$ and $f$, and the size $N_2$ of the community of migrants hailing from $f$ and resident in $p$. This linear correlation is confirmed for the four largest cities we analyzed in detail (i.e. Madrid, Barcelona, Valencia, Sevilla) as shown in Fig. 10.

3. The relation between migrants and products diversification

Having proved that the amount of trades is positively influenced by migration, we still have to check that also the diversification of exports is enhanced, namely, that migration plays a significant role in the modern theory of Economical Complexity.

In order to keep this analysis as simple as possible, we do not deal with recent complexity measures \[8,33\], but we follow the simplest possible route (leaving for future works possible improvements).

The export portfolio of a province is composed of products and destinations. That is, a province can export several products to a single destination or export the same product to several destinations. Thus, the basic unit in the export portfolio is a product-destination pair. We define $K$ as the total number of product-destination pairs in the export portfolio of a province. Products are defined using the HS1996 product classification \[45\]. Destinations are defined as countries with more than 1 million population in 2010. There are 4507 products and 154 countries, so the total number of product-destination $K$ pairs is 694078.

To account for the distribution of export sales across product-destination pairs, we use the export share of each product-destination pair in total export value so to cap-
FIG. 8: Trades performed by the province of Valencia with different foreign countries as a function of the related immigrant density; different countries are depicted in different colours as specified by the legend. Data (bullets) are fitted via Eq. [11] (solid line). The foreign countries considered are those where $\Gamma$ spans over the largest interval in such a way that fits can be more accurate.

FIG. 9: Critical density $\Gamma_c$ estimated by fitting data for trading versus $\Gamma$ according to the theoretical law Eq. [11]. Analysis are performed for the four largest provinces: Madrid, Barcelona, Valencia, Sevilla depicted in different symbols and colors as explained by the legend. We added Sevilla to confirm with a fourth point the trend depicted by the first three cities. For each available foreign country $f$ we fit the data for trades versus $\Gamma$ according to the tanh law of Eq. [11] and we derive an estimate for $\Gamma_c$ with the related $R^2$, which are plotted in the topmost panel. The most reliable fits (i.e. $R^2$ close to 1) suggest that $\Gamma_c$ are scattered around $10^{-5}$, similarly for the four provinces analyzed. Focusing on estimates corresponding to $R^2 > 0.85$, we build the histogram of $\Gamma_c$ (shown in the middle panel) and calculate the arithmetic average to get $\bar{\Gamma}_c$, which is plotted in the bottom panel as a function of the population of the related province.

The relative importance of each pair for exports. The Herfindahl index $N_H$ is a simple calculation of concentration of exports that uses such export shares: the larger the number $N_H$, the more concentrated (less diversified) the export portfolio of the province is. Therefore, if migrants do really contribute to diversification of exports, we should expect a negative correlation between $N_H$ and $\Gamma$. More precisely, the $N_H$ index is calculated as

$$N_H = \sum_{i=1}^{K} \left( \frac{x_i}{X} \right)^2,$$

where $x_i$ is the value of export in product-destination $i$ and $X$ is the total value of exports. One can further normalise $N_H$ to get an index $n_H$ whose values lie between 0 and 1. Results are shown in Fig. [11] where the negative correlation between $n_H$ and the percentage of...
migrants within the province is manifest. Thus, at least for small percentages, that is $\Gamma \sim \gamma$, there is a positive correlation between export portfolio diversification and the density of migrants in a particular province. We can therefore derive that migrants act as facilitators of trade by reducing international transaction costs.

B. PART TWO: Insights from Graph Theory

The interaction between natives and immigrants was described in terms of a bipartite graph (see Fig. 3, left panel). The statistical mechanics analysis shows that if the local agents $i$ and $j$ both interact with some foreign-born individual $\mu$, i.e. $\xi_i^{\mu}, \xi_j^{\mu} \neq 0$, then the agents $i$ and $j$ can be thought of as directly interacting via an effective coupling $\tilde{J}_{ij} \sim \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$ (see Fig. 3, right panel and Eq. [9]). We now focus on such emergent network, referred to as $G$ and, through calibration with available data, we try to infer information for the test case of Spain.

1. A glance at the theory

The topological properties of $G$ have been formerly mathematically investigated in [25, 29, 30] and here we review the main points.

A global characterization of the graph $G$ can be attained in terms of the average link probability $p$: considering a generic couple of nodes, say $i$ and $j$, keeping a mean-field perspective, we can write

$$p = 1 - \prod_{\mu=1}^{N_2} [1 - P(\xi_i^{\mu} = 1)P(\xi_j^{\mu} = 1)]$$

Equation (13)

$$= 1 - \left(1 - \frac{\xi^2}{N^{2\theta}}\right)^{\gamma N},$$

Equation (14)

where in Eq. (13) the term in the square brackets represents the probability that the contribution $\xi_i^{\mu} \xi_j^{\mu}$ in the sum $\tilde{J}_{ij}$ is equal to zero and the product over $\mu$ returns the probability that all entries $\mu = 1, ..., N_2$ are null such that, finally, the complementary of this quantity provides the probability that at least one entry is non-null, that is, that $\tilde{J}_{ij} > 0$: in Eq. (14) we used the homogeneity of pattern entries $\xi$ and the definition of $\gamma$.

The average degree of $G$ therefore reads as $\bar{d} = pN_1$.

Now, as $\theta$ and $\xi$ are tuned, the emerging graph can range from fully-connected to completely disconnected [29, 30]. From a mean-field perspective, we can distinguish the following topological regimes:

- $\theta < 1/2$, $p \to 1$, $\bar{d} \to N$ $\Rightarrow$ Fully connected (weighted) graph.
- $\theta = 1/2$, $p \sim 1 - e^{-\xi^2}$, $\bar{d} = O(N)$ $\Rightarrow$ Linearly extensive degree.

- $1/2 < \theta < 1$, $p \sim \xi^2 N^{1-2\theta}$, $\bar{d} = O(N^{2(1-\theta)})$ $\Rightarrow$ Extreme dilution regime: $\lim_{N \to \infty} \bar{d}^{-1} = \lim_{N \to \infty} d/N = 0$.
- $\theta = 1$, $p \sim \xi^2 \gamma/N$, $\bar{d} = O(N^0)$ $\Rightarrow$ Sparse (weighted) graph; $\xi^2 \gamma = 1$ corresponds to the percolation threshold.

Summarizing, large values of $\theta$ determine a disconnected graph with vanishing average degree. Therefore, $\theta$ coarsely controls the connectivity regime of the network, while $\xi$ and $\gamma$ allow a finer tuning.

As the graph $G$ is meant to describe the mutual interactions among the decision makers inside a society, it is worth investigating whether it also exhibits any of the small-world hallmarks. Indeed, as shown in [25, 29, 30], this is the case: for instance, in the proper parameter range, $G$ is shown to display a small diameter and a high clustering coefficient. In fact, the definition in Eq. (9) (i.e. the Hebbian kernel) implicitly endows couplings with “transitivity”: if $i$ and $j$ are connected as they share acquaintances among immigrants, and the same holds for $i$ and $z$, then $j$ and $z$ are also likely to share any acquaintance. Otherwise stated, interactions based on sharing (i.e., matching non-null entries) intrinsically generate a clustered society.

Up to now we just focused on the bare topology, yet the graph $G$ is weighted and we can wonder whether, even from this perspective, the graph exhibits typical features of social networks.

In particular, according to the strength of weak ties theory by Granovetter [23, 24], the degree of overlap of two individuals’ neighbourhood varies directly with the strength of their tie to one another. If the two individuals are acquaintances (rather than close friends), there is little overlap. Consistently, in the graph $G$ weak ties connect individuals sharing a small number (possibly only one) of connections in the immigrant community.

Finally, as shown in [25, 35, 39], weak ties also turn out to be crucial in order to maintain the network connected: by cutting (a relatively small number of) weak link the network gets fragmented into several components.

2. Inferring the topological properties

The parameters into play are $\gamma, \theta$ and $\xi$; their values determine the topology of the emergent network and are also expected to affect the growth of trades (see e.g. Eq. 4). Let us now try to estimate them starting from empirical data.

As for $\theta$, we can derive it through an indirect measure: we expect that the number of links between locals and immigrants is lower bounded by the number of mixed marriages $M_{\text{mixed}}$. In fact, a mixed marriage yields, in general, several “mixed acquaintances” between the family members and the friends of the two parties. In complete generality, the probability of mixed marriage $p_{\text{mixed}}$
Mixed marriages in Spain have been thoroughly investigated in [1, 2] and from those data we can fit the ratio $M_{\text{mixed}}/N_2 \sim N^{1-\theta}$ inferring an estimate for $\theta$. As shown in Fig. [12] the number of normalized mixed marriages is roughly constant with respect to $N$, that is $\theta \approx 1$. As a consequence, $\theta \leq 1$.

Now, a value of $\theta$ strictly smaller that 1 would imply that the number of connections between the two parties grows indefinitely with $N$ (or, analogously, with $N_1$ or $N_2$), and this is certainly not realistic (it would imply infinite energy in order to sustain such a network and the linear extensivity of its related thermodynamics would breaks down). Thus, the experimental argument for the lower bound coupled with the theoretical argument for the upper bound implies $\theta = 1$ (as intuitive).

Finally, we need to estimate $\xi$. According to Eq. [2] and having fixed $\theta = 1$, $\xi$ represents the average number of local acquaintances displayed by an immigrant. In our analysis we bound $\xi$ in between 1 (we expect that any immigrant has at least one link with the local community) and 20: there are several sociological studies trying to estimate the average number of acquaintances (familiars and/or friends) of a member of societies. In particular, in [40, 41] this analysis is performed in Spain finding that this number is $\xi = O(10)$, similarly to other European countries.

According to these estimates for $\theta, \xi$, and $\gamma$ we expect a sparse graph and we can check whether the emergent graph is indeed clustered. In Fig. [13] (upper panel) we show the ratio between the average clustering coefficient $c(\gamma, \xi)$ measured in a numerical realization of $G$ and the clustering coefficient $c_{\text{ER}}$ of an analogous Erdős-Rényi graph. More precisely, $c(\gamma, \xi)$ is measured as a function of $\xi$ and $\gamma$, varied within the ranges empirically detected as described above; for each choice of parameters we can derive an average degree $\langle s \rangle$ which is used to estimate $c_{\text{ER}}$, namely $c_{\text{ER}} = \langle d \rangle / N_1$. As long as $c/c_{\text{ER}} > 1$, $G$ is highly clustered and this occurs in a wide region of the plane $(\Gamma, \xi)$, especially in the region of high dilution: in the parameter range considered the graph $G$ turns out to be small world.

Lastly, we address the problem of the existence of weak ties within the network generated by the Hebbian rule (eq. 5). To this task, we have numerically built over-percolated networks at various sizes and then, for each sample, we performed two types of dilution: the former is purely random, namely we delete a fraction of links extracted according to a uniform distribution, the latter

![Figure 12: Data on mixed marriages for each province along the years 1998-2012 are drawn from the local offices of Vital Records and Statistics (Registro Civil) and divided by the related size $N_2$ of the immigrant community. Raw data (blue bullets) are properly binned (green squares) to highlight the effective behaviour with respect to the overall size $N$ of the related province. The red line shows the lack of dependence on $N$, for $M_{\text{mixed}}/N_2$, in the large $N$ limit.](image1)

![Figure 13: Upper panel: Comparison between $c_{\text{ER}}$ and $c$, as a function of $\Gamma$ and $\xi$. The empirical values of $\Gamma$ for the test-case provinces are also shown. Lower panel: Average size of the giant component obtained by bond-percolating $G$, finding that number is $\sim 10^4$. This analysis is performed in Spain finding that this number is $\xi = O(10)$, similarly to other European countries.](image2)
is deterministic, namely we delete links selecting those corresponding to the weakest coupling. We can then compare the results (shown in Fig. 13, lower panel). If weak ties effectively play a crucial role in keeping different communities connected together, then the deterministic percolation should break the giant component first (i.e., at higher values of network’s connectivity) as this is the case, hence, at least numerically, we definitely confirm that these Hebbian networks are small worlds.

III. CONCLUSIONS AND OUTLOOKS

The recent results of Hidalgo, Klinger, Barabasi and Hausmann [1 4], as well as those by Pietronero, Caldarelli, Gabrielli and their coworkers [7 8] play as breakthrough in the modern theory of Economical Complexity: while classical economic theories prescribed specialization in the industrial production of most developed countries, their investigations clearly show that nowadays the production of such countries are actually extremely diversified. However, in these papers, how diversification affects international trades is not deepened and this is the goal of the present work.

Our approach is framed within the scaffold of Statistical Mechanics, a well consolidated stochastic tool in Theoretical Physics that aims to detect emergent and collective behaviors sharing attributes over the details, and it is supported by extensive data analysis for the test-case of Spain. The resulting theory plays as a new dowel in this modern mosaic of Economical Complexity, shedding lights on the way diversification of exports is achieved due to a continuous swarming of natives and migrants in interaction. These exchanges of information are fundamental to allow firm’s holders to leverage transactional costs thus tacitely allowing a larger basin of firms to appear on the international market.

From a practical economical perspective, our results suggest the existence of a (eventually very small) critical threshold $\Gamma_c$ in the percentage of migrants present in the host community before a boost in international trading is achieved, as well as a saturation effect, in agreement both with the Chaney distorted gravity scheme as well as with recent non linear models by Egger et al. [9] and the (related) pioneering suggestions of Gould [21].

It is worth highlighting that, through an analogy with phase transitions, we can quantitatively find the probability distribution of the critical threshold, that, when considering migrant’s from all over the world as a whole, is Poissonian-like distributed with peak at 0.3% of the whole population, while, when considering migrants from a specific country, decreases to values $\Gamma_c \sim 10^{-5}$, whose scaling is in agreement with the observation that trading nations are $\sim 10^2$.

Summarizing, under the assumption of not so large migrant’s densities, the effect of immigrant’s networking on exports is always significant, robust and stable across goods. Indeed, we can safety state that migrants play a significant role in the modern theory of Economical Complexity.

Further outlooks may cover more complex measures of product’s complexity in order to better tackle the outlined (indirect) influence of migrants to the global market and other nations should be considered beyond Spain to give more ground to the theory as a whole.

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Appendix A: Statistical-mechanics analysis

Let us introduce the set of order parameters $\{m^\mu_p(\sigma_p)\}_{\mu=1}^{N_2}$ (where the subscript $p$ refers to a given province $p$) as

$$m^\mu_p(\sigma_p) = \frac{1}{C} \sum_{i=1}^{N_1} \xi^\mu_i \sigma_{(i_p)},$$

(A1)

where $C$ normalizes with respect to the expected number of non null entries, namely

$$C = N_1 \tau(\xi^\mu = 1) = N_1 \xi N^{-\theta} = \xi (1 - \gamma) N^{1-\theta}.$$  

(A2)

Therefore, $m^\mu_p(\sigma)$ is the average will in international trading for Spanish people living in the province $p$ that share the knowledge of the migrant $\mu$.

For the gauge-like symmetry of the model, clearly $\langle m^\mu(\sigma) \rangle = m$ for each $\mu = 1, \cdots, N_2$. Further, as in the dilution regime of empirical interest each decision maker $\sigma_i$ is linked with (at least) one stranger $\mu$, we have that $\langle \sigma(\mu) \rangle = \langle m^\mu(\sigma) \rangle = m$, i.e. $m$ is the averaged predisposition of the whole host community in international trading. This is because

$$\langle m(\sigma) \rangle = \left\langle \frac{1}{N_1} \sum_{i=1}^{N_1} \sigma_i \right\rangle = \langle \sigma_i \rangle$$

$$\langle m^\mu(\sigma) \rangle = \left\langle \frac{1}{C(N)} \sum_{i=1}^{N_1} \xi^\mu_i \sigma_i \right\rangle = \frac{N_1}{C(N)} \langle \xi^\mu_i \sigma_i \rangle$$

$$= \frac{N_1}{C(N)} \tau(\xi^\mu = 1) \langle \sigma_i \rangle = \langle \sigma_i \rangle.$$  

(A3)

In terms of these new order parameters we can write

$$Z = \sum_{\sigma} e^{\frac{m^2}{2} + \gamma m^2 - \frac{\xi^\mu}{2} \sum_{\mu=1}^{N_2} m^\mu_p(\sigma)}$$

(A4)
that can be evaluated straightforwardly with a standard saddle point argument:

\[
Z = \int \prod_{\mu} dm_\mu \int \prod_{\mu} d\hat{m}_\mu e^{\frac{\beta^2 \xi^2 (1 - \gamma)^2}{2} \sum_{\mu=1}^{N_2} m_\mu \hat{m}_\mu \sum_{\sigma} e^{-i \sum_{\mu=1}^{N_2} \hat{m}_\mu m_\mu(\sigma)}}
\]

As well known, only the dominant term contributes to the free energy in the large \(N\) limit, hence

\[
Z = \int \prod_{\mu} dm_\mu \int \prod_{\mu} d\hat{m}_\mu e^{N_1 A(\{m_\mu\}, \{\hat{m}_\mu\})}
\]

where

\[
A(\{m_\mu\}, \{\hat{m}_\mu\}) = \log 2 + \mathbb{E} \log \cosh\left(\frac{-i \sum_{\mu=1}^{N_2} \hat{m}_\mu e^{\mu}}{\xi (1 - \gamma) N^{1 - \theta}}\right)
\]

+ \frac{\beta^2 \xi^2 (1 - \gamma)^2}{2 N_1} \sum_{\mu=1}^{N_2} m_\mu^2 + \frac{i}{N_1} \sum_{\mu=1}^{N_2} \hat{m}_\mu m_\mu.
\]

Taking the sup of \(A\) we get the self-consistent relations of the system

\[
\partial_{m_\mu} A(\{m_\mu\}, \{\hat{m}_\mu\}) = 0 \rightarrow \partial m_\mu = \frac{-\beta^2 \xi^2 (1 - \gamma)^2}{N_1} m_\mu \rightarrow \partial m_\mu = \frac{-i \beta^2 \xi^2 (1 - \gamma)^2}{N_1} \hat{m}_\mu.
\]

\[
\partial_{\hat{m}_\mu} A(\{m_\mu\}, \{\hat{m}_\mu\}) = 0 \rightarrow \partial \hat{m}_\mu = \frac{i \beta^2 \xi^2 (1 - \gamma)^2}{N_1} \hat{m}_\mu m_\mu.
\]

that, once solved together, returns the value of the order parameter as a solution of

\[
m_\mu = \left(\frac{\xi}{N^{\theta}}\right)^{-1} \left(\xi^\mu \tanh(\beta^2 (1 - \gamma) \xi N^{\theta - 1} \sum_{\nu=1}^{N_2} m_\nu e^{\xi^\nu})\right)\left(\xi (1 - \gamma) N^{1 - \theta}\right)^{-1}.
\]

Evaluating explicitly the average over \(\xi^\mu\), we get

\[
m_\mu = \left(\xi^\mu \tanh(\beta^2 (1 - \gamma) \xi N^{\theta - 1} (m_\mu + \sum_{\nu \neq \mu} m_\nu e^{\xi^\nu}))\right)\left(\xi (1 - \gamma) N^{1 - \theta}\right)^{-1}.
\]

Now we can look for the solution \(m_\mu = m\) for each \(\mu\): killing the vanishing term \(N^{\theta - 1} m\) (that goes to zero in the thermodynamic limit) we find that \(m\) obeys

\[
m = \langle \tanh(\beta^2 (1 - \gamma) \eta m) \rangle = \eta m,
\]

where we defined the random variable \(\eta = N^{\theta - 1} \sum_{\nu \neq \mu} \xi^\nu \sim N^{\theta - 1} \sum_{\nu \neq \mu} \xi^\nu\).

Evaluating the momenta of \(\eta\) is straightforward as

\[
\langle \eta \rangle = N^{\theta - 1} \sum_{\nu \neq \mu} \xi^\nu = \gamma \xi;
\]

\[
\text{Var}[\eta] = N^{2(\theta - 1)} \sum_{\nu \neq \mu} \xi^\nu = \mathcal{O}(N^{2(\theta - 1)} N N^{-\theta}) \sim N^0.
\]

This means that in the limit of infinite size, the signal \(\eta\) is deterministic and thus we have

\[
m = \tanh(\beta^2 \xi^2 m).
\]

This formula relates the expected amount of trading firms (and, similarly, the expected volume of international trades) with the fraction \(\gamma\) of foreign-born people in the province considered.

The full Hamiltonian also contains an intra-party interaction term encoded by \(J\), which was not considered in this treatment. Accounting also for this term would simply imply an additional term \(\beta J m\) in the argument of the hyperbolic tangent, namely

\[
m = \tanh(\beta J m + \beta^2 \xi^2 \Gamma m),
\]

where we wrote \(\Gamma = \gamma (1 - \gamma)\) for simplicity.

[1] A. Barra, et al., An analysis of a large dataset on immigration integration in Spain: The statistical mechanics perspective of Social Action, Sc. Rep. 4, 4174, (2014).

[2] E. Agliari, et al., A stochastic approach for quantifying immigrants’ interactions, New J. Phys. 16, 103034, (2014).

[3] C.A. Hidalgo, B. Klinger, A.L. Barabasi, R. Hausmann, The Product Space Conditions the Development of Nations, Science 317, 482–487, (2007).

[4] C.A. Hidalgo, R. Hausmann, The building blocks of Economical Complexity, Proc. Natl. Acad. Sci. USA 106, 10570-10575, (2009).

[5] D. Ricardo, On the Principles of Political Economy and Taxation, John Murray (1817).

[6] I.L. Flam, M. Flanders, Trade Theory, MIT Press, Cambridge (1991).

[7] A. Tachella, et al., A new metrics for Countries Fitness and Products Complexity, Nature Sci. Rep. 2, 723, (2002).

[8] M. Cristelli, et al. Measuring the intangibles: a Metrics for the Economic Complexity of Countries and Products, PLoS One 8, 8, e70726 (2013).

[9] P. Egger, M. Elrich, D. Nelson, Migration and trade, World Economy 35, 216-241, (2012).
C. Ozgen, P. Nijkamp, J. Poot, Immigration and innovation: Evidence from Canada, Canad. J. Econ. 31(1), 47–62 (1998).

J. Hunt, M. Gauthier-Loiselle, How much does immigration boost innovation?, IZA Disc. PP. 3921, Institute for the Study of Labor, Bonn (2009).

C. Ozgen, P. Nijkamp, J. Poot, Immigration and innovation in European regions, IZA Disc. PP. 5676 (2011).

J. Partridge, H. Furtan, Increasing Canada’s international competitiveness: is there a link between skilled immigrants and innovation? Amer. Agricult. Econ. Meet. (Orlando, Florida), (2008).

S. Rashidi, A. Pyka, Migration and innovation, FZID Disc. PP. 77 – 2013, (2013).

G. Peri, F. Requena-Silvente, The Trade Creation Effect of Immigrants: evidence from the remarkable case of Spain, Canadian Journ. of Econom. 43(4), 1433-1459, (2010).

A. Artal-Tur, V.J. Pallardo’-Lopes, F. Requena-Silvente, The trade-enhancing effect of immigration networks: new evidence on the role of geographics proximity, Econom. Lett. 116, 554-557, (2012).

G. Serrano-Domingo, F. Requena-Silvente, Re-examining the migration-trade link using province data: an application of the generalized propensity score, Econom. Modelling 32, 247-261, (2013).

L. Rendell, et al., Why Copy Others? Insights from the Social Learning Strategies Tournament, Science 328, 5975, 208-213 (2010).

A. Bandaura, Social learning theory, General Learning Press, New York, (1977).

M. Aleksynska, G. Peri, Isolating the Network Effect of Immigrants in Trade, The World Economy 37(3), 434-455, (2014).

D. Gould, Immigrant links to the home country: empirical implications for U.S. bilateral trade flows, Rev. Econom. & Statist. 76:(2), 302–316 (1994).

T. Chaney, Distorted gravity: The intensive and extensive margins of International Trade, Amer. Econ. Rev. 98:(4), 1707-1721, (2008).

M.S. Granovetter, The Strength of Weak Ties, Amer. J. of Sociology 78, 1360-80, (1973).

M.S. Granovetter, The Strength of the Weak Tie: Revisited, Sociol. Theory 1, 201-33, (1983).

A. Barra, E. Agliari A statistical mechanics approach to Granovetter theory, Physica A, 391, 3017–3026 (2012).

A. P. Damm, Neighborhood Quality and Labor Market Outcomes: Evidence from Quasi-Random Neighborhood Assignment of Immigrants, University Press of Southern Denmark, Odense (2013)

A. Barra, The mean field Ising model thought interpolating techniques, J. Stat. Phys. 132, 5, 787-809, (2008).

R.S. Ellis, Large deviations and statistical mechanics, Springer, New York (1985).

E. Agliari, A. Barra A Hebbian approach to complex network generation, Europhys. Lett. 4, 10002, (2011).

A. Barra, E. Agliari Equilibrium statistical mechanics on correlated random graphs, J. Stat., P02027 (2011).

G. Bianconi, Mean field solution of the Ising model on a Barabasi-Albert network, Phys. Lett. A. 303, 166, (2002).

M.G. Herander, L. A. Saavedra, Exports and the structure of immigrant-based networks: the role of geographic proximity, Rev. Econ. Stat. 87, 2, 323-335, (2005).

G. Caldarelli, et al., A Network Analysis of Countries Export Flows: Firm Grounds for the Building Blocks of the Economy, PLoS ONE 7(10): e47278 (2012).

T. Beck, et al., New tools in comparative political economy: The Database of Political Institutions,The World Bank Economic Review 15 (1):165-176, (2001).

A. Barrat, M. Weigt, On the properties of small-world network models, Europ. Phys. J. B 13, 547, (2000).

E. Agliari, C. Cioli, E. Guadagnini, Percolation on correlated random networks, Phys. Rev. E 84, 031120 (2011).

D. Callaway, M.E.J. Newman, S.H. Strogats, D.J. Watts, Network robustness and fragility: Percolation on random graphs, Phys. Rev. Lett. 85, 5468 (2000).

R. Albert, A. L. Barabasi Statistical mechanics of complex networks, Rev. Mod. Phys. 74, 47-97 (2002).

A. Barrat, M. Barthelemy, A. Vespignani, Dynamical processes in complex networks, Cambridge University Press (2008).

Capital social: confianza, redes y asociacionismo en 13 paises, Unidad de Estudios de Opinion Publica (2006).

C. McCarty, P. D. Killworth, H. Russell Bernard, E. C. Johnsen, G. A. Shelley, Comparing Two Methods for Estimating Network Size, Human Organization 60, 1 (2001).

S. Girma, Z. Yu, The link between Immigration and Trade: Evidence from United Kingdom, Weltwirtschaftliches Archiv 138, 1 (2002).

This means that exports from a province to a given foreign country do not receive any stimuli by immigrants coming from that country but living in a different province

We excluded "special" product categories (HS98 and HS99) from COMTRADE database