Eavesdropping on the improved three-party quantum secret sharing protocol

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Lin et al. [S. Lin, F. Gao, Q.-y. Wen, F.-c. Zhu, Opt. Commun. 281 (2008) 4553] pointed that the multiparty quantum secret sharing protocol [Z.-j. Zhang, G. Gao, X. Wang, L.-f. Han, S.-h. Shi, Opt. Commun. 269 (2007) 418] is insecure and proposed an improved three-party quantum secret sharing protocol. In this paper, we study the security of the improved three-party quantum secret sharing protocol and find that it is still insecure. Finally, a further improved three-party quantum secret sharing protocol is proposed.

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I. INTRODUCTION

In the last decade, as the principles of quantum mechanics are introduced, a lot of interesting applications are realized in the field of cryptography. Quantum secret sharing (QSS) [1-25] is one of these applications, which allows that a secret message is spitted into several pieces by a boss, and each agent owns a piece, and no subset of agents can be sufficient to extract the boss’s secret message, but the whole set can. The first QSS protocol in which a three-particle entangled Greenberger-Horne-Zeilinger (GHZ) state was used is proposed by Hillery et al. [1]. Although this protocol has elegantly shown the essence of QSS, it is hard to realize experimentally because of the inefficiency as regards the generation of a three-particle entangled state. Recently, in order to increase the practical feasibility, Zhang et al. [15] utilized Einstein-Podolsky-Rosen (EPR) pairs and the five local operations to propose a novel QSS protocol. However, it is a slight pity that this protocol has a drawback of security, which was pointed out by Lin et al. [16]. Lin et al. showed that the last agent may solely obtain half of Alice’s secret messages without the other agents’ helps, moreover, they gave an improvement of Zhang et al. QSS protocol. Later on, Wang et al. [17] claimed that the three-party case in Lin et al. improved protocol [16] is secure, and pointed out that the n-party (n ≥ 4) case is insecure, and they show that the first agent and the last agent in the improved QSS protocol [16] may collaborate to eavesdrop Alice’s secret messages without introducing any error. Obviously, both Lin et al. attack and Wang et al. attack are from the inside dishonest agents in QSS, and the attack power of the

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dishonest agent is very strong because he (she) has a chance to tell a lie during the checking security. Only if the lie is successfully constructed, he (she) can eavesdrop the secret messages without being detected by the other participants. Therefore, we should mainly focus on the dishonest agent’s attack while analyzing the security of the QSS protocol.

In this paper, we study the security of the three-party case of Lin et al. improved QSS protocol [16] and find that it is also insecure. Before describing our attack strategy, first, let us review Lin et al. improved three-party protocol [16] as follows: (1)Bob prepares photons $h$ and $t$ in one of four Bell states: 

$$
\psi_{ht}^\pm = \left( |0\rangle|1\rangle \pm |1\rangle|0\rangle \right)/\sqrt{2},
\phi_{ht}^\pm = \left( |0\rangle|0\rangle \pm |1\rangle|1\rangle \right)/\sqrt{2}.
$$

Then he sends photon $t$ to Charlie and retains photon $h$ in his site. (2)After receiving photon $t$, firstly, Charlie ascertains whether photon $t$ is a single photon [18]. If not, the communication will be terminated. Otherwise, he performs one of the five local operations: $I$, $\sigma_x$, $\sigma_y$, $\sigma_z$, $H$ on photon $t$. The probabilities that the five operations are selected by her are 1/8, 1/8, 1/8, 1/8 and 1/2, respectively. Here, 

$$
I = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|, \quad \sigma_y = |0\rangle\langle 1| - |1\rangle\langle 0|,
\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|, \quad H = \left( |0\rangle\langle 0| - |1\rangle\langle 1| + |0\rangle\langle 1| + |1\rangle\langle 0| \right)/\sqrt{2}.
$$

Then he sends photon $t$ to Alice. (3)After receiving photon $t$, Alice randomly switches between the control mode and the message mode. In the control mode, Alice randomly selects one action from the two choices: One is that she lets Bob use \{\{0\}, \{1\}\} or \{\{h\} = \left( |0\rangle + |1\rangle \right)/\sqrt{2}, \{v\} = \left( |0\rangle - |1\rangle \right)/\sqrt{2}\} to measure photon $h$, and tell her his measurement outcome and initial Bell state. Then Alice requires Charlie to announce their operations. The other is that Alice first lets Charlie announce their operations, and then asks Bob to perform a measurement on photon $h$ and tell her his measurement outcome and initial Bell state. Next, Alice uses correct measuring basis to measure the $t$ photon. By comparing her measurement outcome with the deduced outcome, Alice can judge whether quantum channel is secure. If the quantum channel is attacked, the communication is aborted. Otherwise, the transmission goes on to Step (1). In the message mode, Alice performs a unitary operation ($I$, $\sigma_x$, $\sigma_y$, $\sigma_z$) on photon $t$ to encode her secret messages. After her encoding, Alice sends all the $t$-photons of message mode as a sequence ($t$-sequence) to Bob in one communication. Before sending $t$-sequence, Alice prepares a certain number of single photons (checking photons) randomly in one of four states: $|0\rangle$, $|1\rangle$, $|h\rangle$, $|v\rangle$, and inserts these checking photons into $t$-sequence. And then, she sends $t$-sequence to Bob. (4)After Bob receives the sequence, Alice tells Bob the positions of checking photons in the $t$-sequence and the initial states of all checking photons. Bob picks out the checking photons and uses the suitable basis to measure them. And then, by comparing his measurement outcomes with the initial states, Bob can judges whether quantum channel between Alice and him is secure. After confirming that no eavesdropping exists, they can extract Alice’s secret messages if Bob and Charlie collaborate. (5)At last, Alice announces a small part of the secret messages so that Bob and Charlie can execute the message authentication process.
II. SECURITY LEAK OF LIN ET AL. IMPROVED THREE-PARTY QSS PROTOCOL

We can see that, in Lin et al. improved three-party protocol [16], Alice inserts the checking photons into the t-sequence in order to prevent Charlie from eavesdropping. By making a single-photon measurement on each checking photon, Alice may judge whether the quantum channel between Alice and Bob is safe. Though the process of the security-check is added, their improved three-party quantum secret sharing protocol is still insecure. In what follows, we will detailedly analyse why it isn’t secure.

In advance, Bob prepares two EPR photon pairs. Suppose that one pair is in $\psi^{-}_{ab}$ and the other $\phi^+_{ht}$. According to Step (1), Bob sends photon $t$ to Charlie. After receiving photon $t$, Charlie performs one of the five local operations: $I$, $\sigma_x$, $\sigma_y$, $\sigma_z$, $H$ on it, and then sends photon $t$ to Alice. When photon $t$ is traveling between Charlie and Alice, Bob intercepts it, and stores it well. At the same time, Bob sends photon $b$ (that is from $\psi^{-}_{ab}$), instead of photon $t$, to Alice. Alice randomly switches the control mode and the message mode. In the control mode, when Alice requires Bob to make single-photon measurement on photon $h$, firstly, Bob immediately makes Bell state measurement on photons $a$ and $h$. Obviously, this is the entangle swapping process of Bell states. Suppose that the operation performed by Charlie is $H$, the whole system state can be written as follows:

$$
\psi^{-}_{ab} \otimes \phi^+_{ht} \rightarrow \psi^{-}_{ab}H\phi^+_{ht} = \frac{1}{2}[\phi^+_{ah}(\psi^{-}_{bt} - \phi^{-}_{bt}) + \phi^{-}_{ah}(\phi^+_{bt} - \psi^{-}_{bt}) - \psi^+_{ah}(\phi^+_{bt} + \psi^{-}_{bt}) + \psi^+_{ah}(\phi^+_{bt} + \psi^{-}_{bt})]
$$

(1)

Suppose that Bob’s Bell state measurement outcome on photons $a$ and $h$ is $\psi^+_{ah}$. According to Equation (1), photons $b$ and $t$ must be in $\phi^+_{bt} + \psi^{-}_{bt})$. Next, Bob makes a comparison for $\psi^+_{ah}$ and $\psi^{-}_{ab}$, and gets the unitary operation $\sigma_z$. This kind of Bell state comparison method and its comparison steps can be consulted in the paper [26]. By the way, here the four operations $I$, $\sigma_x$, $\sigma_y$, $\sigma_z$ are similar to the four operators $U_1$, $U_2$, $U_3$, $U_4$ in the paper [26]. After making Bell state measurement, Bob uses $\{|0\rangle, |1\rangle\}$ or $\{|h\rangle, |v\rangle\}$ to measure photon $t$ and tells Alice his single-qubit measurement outcome. According to Step (3), Bob still needs to tell Alice his initial Bell state. In order that his replacing trick is not detected by Alice, Bob may not directly say that the initial Bell state is $\phi^+_{ht}$, but should tell the lie that it is $\sigma_z\phi^+_{ht} = \phi^{-}_{ht}$ ($\sigma_z$ is just the gotten operator that Bob makes the Bell state comparison). So his replacing action will not be detected by Alice. Here, we can’t help asking why Bob’s replacing action isn’t detected? Continuing to analyse, the key of the question will be obtained. We know, that Alice deduces which state her and Bob’s hand photons are in depends on the information published by Bob and Charlie. Only if it is satisfied that Alice’s deducing state and the $\phi^+_{bt} + \psi^{-}_{bt})$ are the same states, Bob’s replacing trick can not be detected by Alice. From the above content, we can see that the initial state published by Bob is $\phi^{-}_{ht}$ and the operation published by
Charlie is $H$, so Alice can only deduce as follows:

$$\phi_{ht}^- \rightarrow H\phi_{ht}^- = \phi_{ht}^- + \psi_{ht}^-$$

(2)

Indeed, the $(\phi_{ht}^+ + \psi_{ht}^-)$ and the $(\phi_{ht}^+ + \psi_{ht}^-)$ are the same states. Hence, Bob’s replacing trick does not introduce any error. As for the other Bell states that Bob’s measurement outcomes on photons $a$ and $h$ are, the law exists as of old. Here, for saving space of a whole page, we don’t list out the others’ deducing processes again. So Bob may evade Alice’s security-check successfully in the case that Charlie’s operation is $H$. If Charlie’s operation is one of four unitary operations: $I$, $\sigma_x$, $\sigma_y$, $\sigma_z$, is Bob’s replacing trick also feasible? The answer is ”yes”. Suppose that the two Bell states prepared by Bob are still $\psi_{ab}^-$ and $\phi_{ht}^+$, and the operation performed by Charlie is $I$. So the whole system state may be written as follows:

$$\psi_{ab}^- \otimes \phi_{ht}^+ \rightarrow \psi_{ab}^- \otimes I\phi_{ht}^+ = \frac{1}{2}(-\phi_{ah}^+\psi_{bt}^- + \phi_{ah}^+\psi_{bt}^- - \psi_{ah}^-\phi_{bt}^- + \psi_{ah}^-\phi_{bt}^-)$$

(3)

Assume that Bob’s measurement outcome on photons $a$ and $h$ is $\phi_{ah}^-$. According to Equation (3), photons $b$ and $t$ must be in $\psi_{bt}^+$. Similarly, Bob gets the unitary operation $\sigma_x$ by comparing $\phi_{ah}^-$ with $\psi_{ah}^-$ [26]. So, he may tell the lie that it is $\sigma_x \phi_{ht}^+ = \psi_{ht}^+$ when Alice requires him to publish the initial Bell state. According to Bob’s $\psi_{ht}^+$ and Charlie’s $I$, Alice deduces that $I\psi_{ht}^+ = \psi_{ht}^+$. It is evident that the $\psi_{ht}^-$ and the $\psi_{bt}^+$ are the same states. So Bob is also able to do his replacing trick in the case that Charlie’s operation is one of four unitary operations. In a word, no matter what Charlie’s operation is, Bob’s replacing trick is not detected by Alice in the control mode. When the message mode is switched into, since Alice doesn’t know that Bob has done the replacing trick and regards photon $b$ as photon $t$ as of old, she encodes her secret messages by performing a unitary operation on photon $b$. Then she sends photon $b$ back to Bob. Bob very easily gets Alice’s secret messages by making Bell state measurement on photons $a$ and $b$ without Charlie’s helps.

So far, we have successfully proposed a attack for Lin et al. improved three-party secret sharing protocol [16]. Meanwhile, we have also proved Wang et al. statement that the three-party case of Lin et al. improved protocol is secure is not correct. Obviously, that our attack and Wang et al. attack [17] are put together shows that Lin et al. improved QSS protocol [16] is completely insecure. Next, let us discuss how to further modify Lin et al. improved three-party QSS protocol so that it can resist this kind of attack. For the integrity, we describe the modified Lin et al. improved three-party protocol as follows in brief.

(1') Bob prepares a batch of EPR photon pairs, which each pair is randomly in one of four Bell states. He takes out photon $t$ from each EPR pair to form one sequence, called $t$ sequence. The partner photon $h$ in one EPR pair forms another sequence, called $h$ sequence. Then Bob sends the $t$ sequence to Charlie.

(2') After receiving the $t$ sequence, firstly, Charlie ascertains whether the photons in $t$ sequence are a single photon [18]. If the multi-photon signal is detected, the communication will be terminated. Otherwise,
he performs one of the five local operations: $I$, $\sigma_x$, $\sigma_y$, $\sigma_z$ and $H$ on each photon in the $t$ sequence. The probabilities that the five operations are selected by him are 1/8, 1/8, 1/8, 1/8 and 1/2, respectively. Next, Charlie prepares a certain number of decoy photons, which each decoy photon is randomly in one of four states: $|0\rangle$, $|1\rangle$, $|h\rangle$, $|v\rangle$. He inserts these decoy photons into the $t$ sequence, and then sends the $t$ sequence to Alice.

(3') After confirming that Alice receives the $t$ sequence, Charlie tells Alice the position of each decoy photon in the $t$ sequence and the state of each decoy photon. Afterwards, Alice uses the appropriate measuring basis to measure each decoy photon. So she can judge whether the quantum channel between hers and Charlie is secure. Next, what Alice still needs to do is the same as that in Step (3).

Steps (4'), (5') are same to Steps (4), (5) in Lin et al. improved three-party QSS protocol.

Up to now, we have proposed a further improved three-party QSS protocol (Note that we only give the further improved three-party QSS case here). In contrast to Lin et al. improved protocol [16], our further improved protocol only adds one security-check process between Charlie and Alice. By the way, this process is also realized by utilizing some photons randomly in four states: $|0\rangle$, $|1\rangle$, $|h\rangle$, $|v\rangle$. If Bob uses the above replacing trick to attack this further improved protocol, his eavesdropping action will fail because this process exists. The reason is that, firstly, Bob doesn’t know the positions of decoy photons in $t$ sequence and the states of decoy photons, secondly, both the decoy photon and the photon $t$ in $t$ sequence are in maximally mixed state $\rho = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$ for Bob so that he can’t distinguish them, when he replaces the $t$ sequence with another sequence prepared by him, Bob inevitably introduces some errors to the decoy photons. As a result, his replacing trick will be detected. In other words, our further improved three-party QSS protocol can stand against the above proposed attack.

In the end, we discuss the securities of channels in this further improved three-party QSS protocol one by one. From Steps (2') and (3'), we see that the checking photons are used to assure the security of the channel between Alice and Bob, and the decoy photons the security of the channel between Charlie and Alice. Moreover, the decoy photons and the checking photons are same and the purposes using them are also same, that is, to analyse whether the eavesdropping exists. As a matter of fact, the procedures that utilize decoy photons and checking photons to check eavesdropping are equivalent to the security checking in BB84 protocol [27]. Since BB84 protocol has been proved to be unconditionally secure [28,29,30], both of the Alice-Bob channel and the Charlie-Alice channel are also safe in this further improved protocol. Now, there leaves only the Bob-Charlie channel whose security is not discussed. If we regard Bob as one party, and regard two of Charlie and Alice as the other party, the security checking procedure for the Bob-Charlie channel is essentially the same as that in BBM92 protocol [31]. Similarly, since BBM92 protocol has been proved to be secure [32,33], his eavesdropping would be detected if the outside eavesdropper attacks the
Bob-Charlie channel. On the basis of these analysis, this further improved three-party QSS protocol is secure.

III. SUMMARY

We have shown that, by doing the replacing trick, Bob is able to solely eavesdrop Alice’s secret messages without introducing any error in Lin et al. improved three-party QSS protocol. That is, an efficient attack has been proposed for Lin et al. improved three-party QSS protocol by us. Obviously, the trait of this attack is that the Bell state comparison and the entanglement swapping are employed. By the way, the Bell state comparison [26] has been also used in the proposed attack strategy [25]. In addition, after giving this attack, we further modify Lin et al. improved three-party protocol so that it can stand against this attack. Finally, by means of analyzing each channel, we discuss the security of this further improved three-party QSS protocol.

Note added—Recently, we detect that the proposed attack in this paper has a small drawback. The drawback mainly focuses on that the correlation of the two particles in one Bell state are not always identical after the local operation $H$ is performed on one of the two particles. For example,

$$H\psi^{+}_{bd} = (|0\rangle|-\rangle + |1\rangle|+\rangle)_{bd}/\sqrt{2} = (|0\rangle|+\rangle - |1\rangle|-\rangle)_{tb}/\sqrt{2}.$$

This means the proposed attack needs to be further optimized.

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