EQUILIBRIUM UNDER UNCERTAINTY WITH SUGENO PAYOFF

TARAS RADUL

Institute of Mathematics, Casimir the Great University, Bydgoszcz, Poland;
Department of Mechanics and Mathematics, Lviv National University, Universytetu-
skia st.,1, 79000 Lviv, Ukraine.
e-mail: tarasradul@yahoo.co.uk

Key words and phrases: equilibrium under uncertainty, capacity, Sugeno
integral

Abstract. This paper studies n-player games where players beliefs about
their opponents behaviour are capacities. The concept of an equilibrium under
uncertainty was introduced J.Dow and S.Werlang (J Econ. Theory 64 (1994)
205–224)) for two players and was extended to n-player games by J.Eichberger
and D.Kelsey (Games Econ. Behav. 30 (2000) 183–215). Expected utility
was expressed by Choquet integral. We consider the concept of an equilibrium
under uncertainty in this paper but with expected utility expressed by Sugeno
integral. Existence of such an equilibrium is demonstrated using some abstract
non-linear convexity on the space of capacities.

1. Introduction

The classical Nash equilibrium theory is based on fixed point theory and was
developed in frames of linear convexity. The mixed strategies of a player are prob-
ability (additive) measures on a set of pure strategies. But an interest to Nash
equilibria in more general frames is rapidly growing in last decades. There are
also results about Nash equilibrium for non-linear convexities. For instance, Briec
and Horvath proved in [1] existence of Nash equilibrium point for B-convexity and
MaxPlus convexity. Let us remark that MaxPlus convexity is related to idempotent
(Maslov) measures in the same sense as linear convexity is related to probability
measures.

We can use additive measures only when we know precisely probabilities of all
events considered in a game. However it is not a case in many modern economic
models. The decision theory under uncertainty considers a model when proba-
bilities of states are either not known or imprecisely specified. Gilboa [7] and
Schmeidler [14] axiomatized expectations expressed by Choquet integrals attached
to non-additive measures called capacities, as a formal approach to decision-making
under uncertainty. Dow and Werlang [3] used this approach for two players game
where belief of each player about a choice of the strategy by the other player is
a capacity. They introdced some equilibrium notion for such games and proved
its existence. This result was extended onto games with arbitrary finite number of
players [6].

Kozhan and Zaricznyi introduced in [8] a formal mathematical concept of Nash
equilibrium of a game where players are allowed to form non-additive beliefs about
opponent’s decision but also to play their mixed non-additive strategies. Such
game is called by authors game in capacities. The expected payoff function was
there defined using a Choquet integral. Kozhan and Zaricznyi proved existence
theorem using a linear convexity on the space of capacities which is preserved by Choquet integral.

An alternative to so-called Choquet expected utility model is the qualitative decision theory. The corresponding expected utility is expressed by Sugeno integral. See for example papers [4], [5], [2], [13] and others. Sugeno integral chooses a median value of utilities which is qualitative counterpart of the averaging operation by Choquet integral. It was introduced in [11] the general mathematical concept of Nash equilibrium of a game in capacities with expected payoff function defined by Sugeno integral. To prove existence theorem for this case, it was considered some non-linear convexity on the space of capacities generated by capacity monad structure.

It was noticed in [8] that "there is no direct interpretation of the game in non-additive mixed strategies". So, formal mathematical concept of Nash equilibrium for capacities considered in [8] and [11] has rather theoretical character. We consider in this paper the equilibrium notion from [3] and [6] for a game with expected payoff for capacities considered in [8] and [11].

2. GAMES WITH NON-ADDITIVE BELIEFS

By Comp we denote the category of compact Hausdorff spaces (compacta) and continuous maps. For each compactum $X$ we denote by $C(X)$ the Banach space of all continuous functions on $X$ with the usual sup-norm. In what follows, all spaces and maps are assumed to be in $\text{Comp}$ except for $\mathbb{R}$ and maps in sets $C(X)$ with $X$ compact Hausdorff.

We need the definition of capacity on a compactum $X$. We follow a terminology of [9]. A function $\nu$ which assign each closed subset $A$ of $X$ a real number $\nu(A) \in [0,1]$ is called an upper-semicontinuous capacity on $X$ if the three following properties hold for each closed subsets $F$ and $G$ of $X$:

1. $\nu(X) = 1$, $\nu(\emptyset) = 0$,
2. if $F \subset G$, then $\nu(F) \leq \nu(G)$,
3. if $\nu(F) < a$, then there exists an open set $O \supseteq F$ such that $\nu(B) < a$ for each compactum $B \subset O$.

We extend a capacity $\nu$ to all open subsets $U \subset X$ by the formula $\nu(U) = \sup \{ \nu(K) \mid K \text{ is a closed subset of } X \text{ such that } K \subset U \}$. It was proved in [9] that the space $MX$ of all upper-semicontinuous capacities on a compactum $X$ is a compactum as well, if a topology on $MX$ is defined by a subbase that consists of all sets of the form $O_{\nu}(F,a) = \{ c \in MX \mid c(F) < a \}$, where $F$ is a closed subset of $X$, $a \in [0,1]$, and $O_{\nu}(U,a) = \{ c \in MX \mid c(U) > a \}$, where $U$ is an open subset of $X$, $a \in [0,1]$. Since all capacities we consider here are upper-semicontinuous, in the following we call elements of $MX$ simply capacities.

There is considered in [8] a tensor product for capacities, which is a continuous map $\otimes : MX_1 \times MX_2 \to MX_1 \times X_2$ such that for each $i \in \{1,2\}$ we have $M(p_i) \circ \otimes = p_i$, where $p_i : X_1 \times X_2 \to X_i$ and $pr_i : MX_1 \times MX_2 \to MX_i$ are natural projections. This definition is based on the capacity monad structure. We give there a direct formula for evaluating tensor product of capacities. For $\mu_1 \in MX_1$, $\mu_2 \in MX_2$ and $B \subset X_1 \times X_2$ we put $\mu_1 \otimes \mu_2(B) = \sup \{ t \in [0,1] \mid \mu_1(\{ x \in X_1 \mid \mu_2(\{ y \in X_2 \mid \{ x \times y \} \cap B \}) = t \} \geq t \}$. Note that, despite the space of capacities contains the space of probability measures, the tensor product of capacities does not extend tensor product of probability measures. It was noticed in [8] that we can extend the definition of tensor product to any finite number of factors by induction.
Lemma 1. Let $A_i$ be a closed subset of a compactum $X_i$ and $\mu_i \in MX_i$ such that $\mu_i(X_i \setminus A_i) = 0$ for each $i \in \{1, \ldots, n\}$. Then $\otimes_{i=1}^n \mu_i(\prod_{i=1}^n X_i \setminus \prod_{i=1}^n A_i) = 0$.

Proof. Consider the case $n = 2$. Let $B$ any compact subset of $(X_1 \times X_2) \setminus (A_1 \times A_2)$. For any $t > 0$ consider the set $K_t = \{x \in X_1 \mid \mu_2((\{x\} \times X_2) \cap B) \geq t\}$. If $x \in A_1$, then $p_2((\{x\} \times X_2) \cap B) \subset X_2 \setminus A_2$, hence $x \notin K_t$. Thus $K_t \subset X \setminus A_1$ and we obtain $\mu_1 \otimes \mu_2(B) = 0$.

The general case could be obtained by induction. \qed

Let us describe the Sugeno integral with respect to a capacity $\mu \in MX$. Fix any increasing homeomorphism $\psi : (0,1) \to \mathbb{R}$. We put additionally $\psi(0) = -\infty$, $\psi(1) = +\infty$ and assume $-\infty < t < +\infty$ for each $t \in \mathbb{R}$. We consider for each function $f \in C(X)$ an integral defined by the formulae

$$\int_X^\text{Sug} f d\mu = \max\{t \in \mathbb{R} \mid \mu(f^{-1}([t, +\infty))) \geq \psi^{-1}(t)\}$$

Let us remark that we use some modification from [12] of Sugeno integral. The original Sugeno integral [15] "ignores" function values outside the interval $[0, 1]$ and we introduce a "correction" homeomorphism $\psi$ to avoid this problem.

Now, we are going to introduce notion of equilibrium under uncertainty for games where belief of each player about opponents behaviour are represented by non-additive measures (or capacities) on $X_i$. We follow definitions and denotation from [3] with the only difference that we use the Sugeno integral for expected payoff instead the Choquet integral.

We consider a $n$-players game $f : X = \prod_{i=1}^n X_i \to \mathbb{R}^n$ with compact Hausdorff spaces of strategies $X_i$. The coordinate function $p_i : X \to \mathbb{R}$ we call payoff function of $i$-th player. For $i \in \{1, \ldots, n\}$ we denote by $X_{-i} = \prod_{j \neq i} X_j$ the set of strategy combinations which players other than $i$ could choose. For $x \in X$ the corresponding point in $X_{-i}$ we denote by $x_{-i}$. In contrast to standard game theory, beliefs of $i$-th player about opponents behaviour are represented by non-additive measures (or capacities) on $X_{-i}$.

For $i \in \{1, \ldots, n\}$ we consider the expected payoff function $P_i : X_i \times MX_{-i} \to \mathbb{R}$ defined as follows $P_i(x_i, \nu) = \int_X^\text{Sug} f d\nu$ where the function $p_i^x : X_{-i} \to \mathbb{R}$ is defined by the formulae $p_i^x(x_{-i}) = p_i(x, x_{-i})$, $x_i \in X_i$ and $\nu \in MX_{-i}$.

We are going to prove continuity of $P_i$. We will need some notations and a technical lemma. Let $f : X \times Y \to \mathbb{R}$ be a function. Consider any $x \in X$ and $t \in \mathbb{R}$. Denote $A_{<t}^x = \{y \in Y \mid f(x, y) \leq t\}$. We also will use analogous notations $A_{\leq t}^x$, $A_{<t}^x$, and $A_{\leq t}^x$.

Lemma 2. Let $f : X \times Y \to \mathbb{R}$ be a continuous function on the product $X \times Y$ of compacta $X$ and $Y$. Then for each $x \in X$, $t \in \mathbb{R}$ and $\delta > 0$ there exists an open neighborhood $O$ of $x$ such that $A_{<t}^x \subset A_{<t+\delta}^x$ ($A_{\leq t}^x \subset A_{\leq t+\delta}^x$) for each $z \in O$.

Proof. Let us prove the first statement of the lemma. We have two compact sets $\{x\} \times A_{<t}^x$ and $\{x\} \times A_{\leq t+\delta}^x$. Consider its open neighborhoods $V = \{(z, y) \in X \times Y \mid f(z, y) < t + \frac{\delta}{2}\}$ and $U = \{(z, y) \in X \times Y \mid f(z, y) > t + \frac{\delta}{2}\}$. Since $\{x\} \times A_{<t}^x$ and $\{x\} \times A_{\leq t+\delta}^x$ are compact, we can choose an open set $O \subset X$ and two open sets $W_1, W_2 \subset Y$ such that $\{x\} \times A_{<t}^x \subset O \times W_1 \subset V$ and $\{x\} \times A_{\leq t+\delta}^x \subset O \times W_2 \subset U$. Consider any $z \in O$ and $y \in Y$ such that $f(z, y) \leq t$. Then $y \notin W_2 \supset A_{\leq t+\delta}$, hence $y \in A_{<t+\delta}^x$.

The proof of the second statement is the same. \qed

Lemma 3. The map $P_i$ is continuous.

Proof. Consider any $x \in X_i$ and $\nu_0 \in MX_{-i}$ and put $P_i(x, \nu_0) = t \in \mathbb{R}$. Consider any $\varepsilon > 0$. By Lemma 2 we can choose a neighborhood $O_1$ of $x$ such that for each
z ∈ O₁ we have $A_{\varepsilon t + \frac{\varepsilon}{2}}^\nu \subset A_{\varepsilon t - \frac{\varepsilon}{2}}^\nu$. Put $V₁ = \{ \nu \in M(X_{-1}) | \psi(A_{\varepsilon t + \frac{\varepsilon}{2}}^\nu) < \psi^{-1}(t + \frac{\varepsilon}{2}) \}$, then $V₁$ is a neighborhood of $\nu₀$.

We also can choose a neighborhood $O₂$ of $x$ such that for each $z \in O₂$ we have $A_{\varepsilon t - \frac{\varepsilon}{2}}^\nu \subset A_{\varepsilon t - \frac{\varepsilon}{2}}^\nu$. Put $V₂ = \{ \nu \in M(X_{-1}) | \psi(A_{\varepsilon t - \frac{\varepsilon}{2}}^\nu) > \psi^{-1}(t - \frac{\varepsilon}{2}) \}$, then $V₂$ is a neighborhood of $\nu₀$. Put $O = O₁ \cap O₂$ and $V = V₁ \cap V₂$. Consider any $(z, \nu) \in O \times V$. Since $(z, \nu) \in O₁ \times V₁$, we have $\psi(A_{\varepsilon t + \frac{\varepsilon}{2}}) \leq \nu(A_{\varepsilon t + \frac{\varepsilon}{2}}^\nu) < \psi^{-1}(t + \frac{\varepsilon}{2})$. Hence $P₁(z, \nu) < t + \varepsilon$.

On the other hand, since $(z, \nu) \in O₂ \times V₂$, we have $\nu(A_{\varepsilon t - \frac{\varepsilon}{2}}) \geq \nu(A_{\varepsilon t - \frac{\varepsilon}{2}}^\nu) = \nu(X_{-1} \setminus A_{\varepsilon t - \frac{\varepsilon}{2}}^\nu) \geq \nu(X_{-1} \setminus A_{\varepsilon t - \frac{\varepsilon}{2}}) = \nu(A_{\varepsilon t - \frac{\varepsilon}{2}}^\nu) > \psi^{-1}(t - \frac{\varepsilon}{2})$. Hence $P₁(z, \nu) > t - \varepsilon$ and the map $P₁$ is continuous. □

For $\nuᵢ ∈ M(X_{-i})$ denote by $Rᵢ = \{ x ∈ Xᵢ | Pᵢ(x, \nuᵢ) = \max\{Pᵢ(x, \nuᵢ) | z ∈ Xᵢ \} \}$ the best response correspondence of player $i$ given belief $\nuᵢ$. The set $Rᵢ$ is well defined and compact by Lemma 3.

A belief system $(\nu₁, \ldots, \nuₙ)$, where $\nuᵢ ∈ M(X_{-i})$, is called an equilibrium under uncertainty with Sugeno payoff if for all $i$ we have $\nuᵢ(X_{-i} \setminus \bigcap_{j \neq i} Rⱼ) = 0$.

The main goal of this paper is to prove the existence of such equilibrium. Since Sugeno integral does not preserve linear convexity on $M(X)$, we can not use methods from [3] and [6]. We will use some another natural convexity structure on the space of capacities which has the biniarity property (has Helly number 2).

3. Binary convexity on the space of capacities

Consider a compactum $X$. There exists a natural lattice structure on $M(X)$ defined as follows $\nu ∨ μ(A) = \max\{\nu(A), μ(A)\}$ and $\nu ∧ μ(A) = \min\{\nu(A), μ(A)\}$ for each closed subset $A ⊂ X$ and $\nu, μ ∈ M(X)$. The lattice $M(X)$ is a compact complete sublattice of the lattice $[0, 1]^X$ with natural coordinate-wise operations. The lattice $M(X)$ has a greatest element and a least element defined as $μ₁X(A) = 1$ for each $A \neq \emptyset$, $μ₁X(∅) = 0$ and $μ₀X(A) = 0$ for each $A \neq X$, $μ₀X(X) = 1$.

By convexity on $M(X)$ we mean any family $C$ of closed subsets which is stable for intersection and contains $M(X)$ and the empty set. Elements of $C$ are called $C$-convex (or simply convex). See [10] for more information about abstract convexities.

A convexity $C$ on $M(X)$ is called $T₂$ if for each distinct $x₁, x₂ ∈ M(X)$ there exist $S₁, S₂ ∈ C$ such that $S₁ ∪ S₂ = X, x₁ ∉ S₂$ and $x₂ ∉ S₁$. Let $L$ be a family of subsets of a compactum $X$. We say that $L$ is linked if the intersection of every two elements is non-empty. A convexity $C$ is called binary if the intersection of every linked subsystem of $C$ is non-empty.

For $\nu, μ ∈ M(X)$ we denote $[\nu, μ] = \{ α ∈ M(X) | ν ∧ μ ≤ α ≤ ν ∨ μ \}$. It is easy to see that $[\nu, μ]$ is a closed subset of $M(X)$. We consider on $M(X)$ a convexity $C_X = \{ [\nu, μ] | ν, μ ∈ M(X) \}$.

Lemma 4. The convexity $C_X$ is binary.

Proof. Let $B$ is a linked subfamily of $C$. It is enough to prove that intersection of every three elements of $B$ is not empty by Proposition 2.1 from [10]. Consider any $[μ₁, ν₁], [μ₂, ν₂], [μ₃, ν₃] ∈ B$. We can suppose that $μ_i ≤ ν_i$ for each $i ∈ \{1, 2, 3\}$. We denote by $ν_A = \bigwedge\{ νᵢ | i ∈ A \}$ and $μ_A = \bigvee\{ μᵢ | i ∈ A \}$ for each $A ⊂ \{1, 2, 3\}$. It is enough to prove that $μ₁₂₃(B) ≤ ν₁₂₃(B)$.

Suppose the contrary then there exists a closed set $B ⊂ X$ such that $\nu₁₂₃(B) < μ₁₂₃(B)$. We can choose $i ∈ \{1, 2, 3\}$ such that $νᵢ₂₃(B) < μᵢ(B)$. Without loss of generality we can suppose that $i = 1$. Since the family $\{μ₁, ν₁, [μ₂, ν₂], [μ₃, ν₃]\}$ is linked, we have $μ₁(B) ≤ μ₁₂(B) ≤ ν₁₂(B)$. Hence $ν₁(B) < μ₁(B)$. But then
\( \nu_3(B) \leq \nu_3(B) < \mu_1(B) \leq \mu_1(B) \) and we obtain a contradiction with the fact that \([\mu_3, \nu_3] \cap [\mu_3, \nu_3] = \emptyset \). 

**Lemma 5.** The convexity \( C \) is \( T_2 \).

**Proof.** Consider any \( \mu, \nu \in MX \) such that \( \mu \neq \nu \). Then there exists a closed subset \( A \subset X \) such that \( \mu(A) < \nu(A) \). We can suppose that \( \mu(A) < \nu(A) \). Put \( a = \frac{\mu(A) + \nu(A)}{2} \) and consider sets \( O_1 = \{ \alpha \in MX \mid \alpha(A) \geq a \} \) and \( O_2 = \{ \alpha \in MX \mid \alpha(A) \leq a \} \). Consider the capacity \( \nu_1 \in MX \) defined as follows \( \nu_1(C) = 0 \) if \( A \setminus C \neq \emptyset \), \( \nu_1(C) = a \) if \( A \subset C \) and \( C \neq X \) and \( \nu_1(X) = 1 \) for a closed subset \( C \subset X \). Then we have that \( O_1 = [\nu_1, \mu_1X] \subset CX \).

Analogously, we can consider the capacity \( \nu_2 \in MX \) defined as follows \( \nu_2(\emptyset) = 0 \), \( \nu_2(C) = a \) if \( \emptyset \neq C \subset A \) and \( \nu_2(C) = 1 \) if \( C \setminus A \neq \emptyset \) for a closed subset \( C \subset X \). Then we have that \( O_1 = [\mu_0X, \nu_2] \subset CX \). Obviously, \( O_1 \cup O_2 = MX \) and \( \mu \notin O_1 \) and \( \nu \notin O_2 \).

**4. THE MAIN RESULT**

We will prove existence of equilibrium introduced in Section 2, moreover we will show that each \( \nu_i \) could be represented as tensor product of capacities on factors.

By a multimap (set-valued map) of a set \( X \) into a set \( Y \) we mean a map \( F : X \to 2^Y \). We use the notation \( F : X \to Y \). If \( X \) and \( Y \) are topological spaces, then a multimap \( F : X \to Y \) is called upper semi-continuous (USC) provided for each open set \( O \subset Y \) the set \( \{ x \in X \mid F(x) \subset O \} \) is open in \( X \). It is well-known that a multimap is USC iff its graph is closed in \( X \times Y \).

Let \( F : X \to X \) be a multimap. We say that a point \( x \in X \) is a fixed point of \( F \) if \( x \in F(x) \). The following counterpart of Kakutani theorem for binary convexity was obtained in [11].

**Theorem 1.** Let \( C \) be a \( T_2 \) binary convexity on a continuum \( X \) and \( F : X \to X \) is a USC multimap with values in \( C \). Then \( F \) has a fixed point.

We use definitions and notations from Section 2.

**Theorem 2.** There exists \( (\mu_1, \ldots, \mu_n) \in M(X_1) \times \cdots \times M(X_n) \) such that \( (\mu_1^*, \ldots, \mu_n^*) \) is an equilibrium under uncertainty with Sugeno payoff, where \( \mu_i^* = \bigotimes_{j \neq i} \mu_j \).

**Proof.** For each \( i \in \{1, \ldots, n\} \) consider a multimap \( \gamma_i : \prod_{j=1}^n M(X_j) \to M(X_i) \) defined as follows \( \gamma_i(\mu_1, \ldots, \mu_n) = \{ \mu \in M(X_i) \mid \mu(X_i \setminus R_i(\mu_i^*)) = 0 \} \). It follows from the definition of topology on \( M(X_i) \) that \( \gamma_i(\mu_1, \ldots, \mu_n) \) is a closed subset of \( M(X_i) \) for each \( \mu_1, \ldots, \mu_n \in \prod_{j=1}^n M(X_j) \). Consider \( \nu \in M(X_i) \) defined as follows \( \nu(A) = 1 \) if \( A \cap R(\mu_i^*) \neq \emptyset \) and \( \nu(A) = 0 \) otherwise. Then we have \( \gamma_i(\mu_1, \ldots, \mu_n) = [\mu \times \nu, \nu] \), hence \( \gamma_i(\mu_1, \ldots, \mu_n) \in CX_i \).

Define a multimap \( \gamma : \prod_{j=1}^n M(X_j) \to \prod_{j=1}^n M(X_j) \) by the formulae \( \gamma(\mu_1, \ldots, \mu_n) = \prod_{j=1}^n \gamma_j(\mu_1, \ldots, \mu_n) \). Let us show that \( \gamma \) is USC. Consider any pair \( (\mu, \nu) \in \prod_{j=1}^n M(X_j) \times \prod_{j=1}^n M(X_j) \) such that \( \nu \notin \gamma(\mu) \). Then there exists \( i \in \{1, \ldots, n\} \) and a compactum \( K \subset X_i \setminus R_i(\mu_i^*) \) such that \( \nu(K) > 0 \). Put \( O_\alpha = \{ \alpha \in \prod_{j=1}^n M(X_j) \mid \alpha(K) > 0 \} \). Then \( O_\alpha \) is an open neighborhood of \( \nu \). It follows from Lemma 1 and continuity of tensor product that there exists an open neighborhood \( O_{\mu,i} \) of \( \nu \) such that for each \( \alpha \in O_{\mu,i} \) we have \( R(\alpha_i^*) \cap K = \emptyset \). Hence for each \( (\alpha, \beta) \in O_{\mu,j} \times O_{\nu,j} \) we have \( \beta \notin \gamma(\alpha) \) and \( \gamma \) is USC.

We consider on \( \prod_{j=1}^n M(X_j) \) the family \( C = \{ \prod_{i=1}^n C_i \mid C_i \in CX_i \} \). It is easy to see that \( C \) forms a \( T_2 \) binary convexity on a continuum \( \prod_{j=1}^n M(X_j) \) (let us remark that each \( M(X_j) \) is connected). Then by Theorem 1 \( \gamma \) has a fixed point \( \mu = (\mu_1, \ldots, \mu_n) \in \prod_{j=1}^n M(X_j) \). Let us show that \( (\mu_1^*, \ldots, \mu_n^*) \) is an equilibrium
under uncertainty. Consider any $i \in \{1, \ldots, n\}$. Then $\mu_i(X_i \setminus R_i(\mu^*_i)) = 0$. We have by Lemma $\prod_i \mu^*_i(\prod_{j \neq i} X_i \setminus \prod_{j \neq i} R_j(\mu^*_j)) = 0$. □

**Remark 1.** Many results of our could be deduced from general results obtained in [11] but we give direct (not difficult) proofs here because otherwise it would require introducing additional categorical notions.

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