Equilibrium analysis of $N$-seller and $N$-buyer Bargaining Games

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A group of players which contain $n$ sellers and $n$ buyers bargain over the partitions of $n$ pies. A seller (buyer) has to reach an agreement with a buyer (seller) on the division of a pie. The players bargain in a system like the stock market: each seller (buyer) can either offer a selling (buying) price to all buyers (sellers) or accept a price offered by another buyer (seller). The offered prices are known to all. Once a player accepts a price offered by another one, the division of a pie between them is determined. Each player has a constant discounting factor and the discounting factors of all players are common knowledge. In this article, we prove that the equilibrium of this bargaining problem is a unanimous division rate for all players, which is equivalent to Nash bargaining equilibrium of a two-player bargaining game in which the discounting factors of two players are the average of $n$ buyers and the average of $n$ sellers respectively. This result shows the relevance between bargaining equilibrium and general equilibrium of markets.
Introduction

Game theorists have developed the axiomatic approach and sequential approach for two-player bargaining games. With a series of axiomatic assumptions, Nash has proved the unique equilibrium of two-player bargaining games (Nash 1950). Rubinstein (1982) has developed the sequential strategic approach in which two players take turns making alternating offers. In the case where each player has a constant discounting factor ($\delta_1$ and $\delta_2$), the solution is proved to be $(1 - \delta_2)/(1 - \delta_1\delta_2)$. Binmore, Rubinstein and Wolinsky et al. (1986) discussed the relationship between these two approaches.

For the bargaining problems with more than two players, the uniqueness of the perfect equilibrium outcome does not hold even under the condition of common discounting factor (Sutton 1986; Herrero 1985; Haller 1986). This bargaining game is described as multiple players trying to reach an agreement on how to share a pie between them. Chae and Yang (1988), Yang (1992), Krishna and Serrano (1996) have proven that the uniqueness of perfect equilibrium can be achieved by introducing the exit opportunity. Recent related work on $n \geq 3$-person bargaining game also includes Asheim (1992), Winter (1994), Merlo and Wilson (1995), Dasgupta and Chiu (1998), Vannetelbosch (1999), Calvo-Armengol (1999), Chatterjee and Sabourian (2000), Vidal-Puga (2004), De Fontenay and Gans (2004), Kultti and Vartiainen (2008), Santamaria (2009), Torstensson (2009), and Yan (2009).

In this paper we study a bargaining game with $n$ sellers and $n$ buyers. In real world instances, bargaining mostly occurs between sellers and buyers in exchange. When the market is not monopolized, each buyer can freely choose among sellers to make an exchange. However, a buyer(seller) cannot bargain with another buyer(seller). Therefore, this bargaining game is described as below. $n$ sellers and $n$ buyers are about to bargain on how to share $n$ pies. The bargaining process is like the bidding system of stock market. Each seller can either offer a
selling price to all buyers or accept a buying price offered by any buyer. Similarly, each buyer can either offer a buying price or accept a selling price offered by any seller. Here the price is equivalent to the partition rate of a pie. A pair of seller and buyer will share a pie if they can reach an agreement. Each player has a constant discounting factor, which means that the value of pie decreases if a player cannot make an agreement with others within a period of time $t$. The discounting factors and offered prices are known to all players.

We prove that this bargaining game has a bargaining equilibrium with which the bargainers accept a unanimous price

$$p = (n - \sum_{i=1}^{n} \delta_{bi})/(n - \sum_{i=1}^{n} \delta_{si}).$$

where $p$ is the exchange rate or price, $\delta_{si}$ and $\delta_{bi}$ ($i = 1, ..., n$) are the discounting factors of the sellers and buyers respectively.

The remainder of this paper is arranged as below. In Section 2, the preliminary knowledge of sequential approach is introduced. The advantage of moving first can be eliminated by introducing a bidding stage in which two players bid for the right to make the first offer. We also deduce the consistency between the axiomatic approach and the sequential strategic approach. In Section 3, we prove the bargaining equilibrium of a two-seller and two-buyer bargaining game and extend the solution to emphpn-seller and emphpn-buyer case. Section 4 concludes the paper.

1 Two-player Bargaining Game

According to Rubinstein (1982), a two-player bargaining game is described as below:

Two players, 1 and 2, are bargaining on the partition of a pie. The pie will be partitioned only after the players reach an agreement. Each player, in turn offers a partition and his opponent may agree to the offer or reject it. Acceptance of the offer ends the bargaining. After rejection, the rejecting player then has to make a counter offer and so on. If no agreement is
achieved, both players keep their status quo (no gain no loss).

Let \( X \) be the set of possible agreements, \( D \) the status quo (no agreement), and \( x_1 \) and \( x_2 \) the partitions of the pie that 1 and 2 receives respectively. The players’ preference relations are defined on the set of ordered pairs of the type \((x, t)\), where \( t \) is a nonnegative integer and denotes the time when the agreement is reached, \( 0 \leq x \leq 1, x_1 = x \) and \( x_2 = 1 - x \). Let \( >_i \) denote player \( i \)'s preference ordering over \( X \cup \{D\} \). There are the following assumptions.

**A-1.** Disagreement is the worst outcome: for every \((x, t) \in X \times T\) we have \((x, t) >_i D\).

**A-2.** 'Pie’ is desirable: \((x, t) >_i (y, t)\) iff \(x_i \geq y_i\).

**A-3.** 'Time’ is valuable: for every \(x, y \in X, \Delta > 0\), if \((x, t_1) >_i (y, t_1 + \Delta)\) then \((x, t_2) >_i (y, t_2 + \Delta)\).

**A-4.** Stationarity: for every \(x, y \in X, \Delta > 0\), if \((x, t_1) >_i (y, t_1 + \Delta)\) then \((x, t_2) >_i (y, t_2 + \Delta)\).

**A-5.** Continuity: if \((x, t_1) >_i (y, t_2)\), there always exists \(\epsilon \rightarrow 0\) such that \((x + \epsilon, t_1) >_i (y, t_2)\).

**A-6.** Increasing loss to delay: for any \(c_1, c_2 > 0\), if \((x + c_1, t) \sim_i (x, 0)\), \((y + c_2, t) \sim_i (y, 0)\) and \(x_i > y_i\) then \(c_1 \geq c_2\).

The players are with constant discounting factors: each player has a number \(0 \leq \delta_i \leq 1\) such that \((x, t_1) >_i (y, t_2)\) iff \(x_i \delta_i^{t_1} \geq y_i \delta_i^{t_2}\). Under these assumptions, Rubinstein (1982) has proven the following proposition 1.

**Proposition 1:** (a) There exists a unique perfect equilibrium of this bargaining game (b) If at least one of the \(\delta_i\) less than 1 and at least one of them is positive, the bargaining solution is \((x, 0)\), where \(x = (1 - \delta_2)/(1 - \delta_1 \delta_2)\).

Notice that player 1 is supposed to make the first offer. If player 2 makes the first move, the solution would be \(x = (1 - \delta_1)/(1 - \delta_1 \delta_2)\). The player who makes the first offer has an advantage in bargaining and receives a larger partition of the pie than what would be received if another
player had made the first offer.

Binmore, Rubinstein and Wolinsky (1986) gave a procedure to eliminate the advantage of moving first as follows: let the time delay between successive periods be $\Delta$, and represent the discounting factor as $\delta^\Delta$. Then in the limit $\Delta \to 0$, it is indifferent whoever makes the opening demand.

$$
\lim_{\Delta \to 0} x^*(\Delta) = \lim_{\Delta \to 0} y^*(\Delta) = x_{TP}^{N} (>_1,>_2)
$$

where $x^*(\Delta)$ and $y^*(\Delta)$ denote the pair of agreements and $x_{TP}^{N} (>_1,>_2)$ is the time preference Nash bargaining solution.

We introduce a new procedure to eliminate the advantage of moving first as follows: two players bid for the right to make the first offer before bargaining for the partition of the pie. Player 1 offers a bid $w$ ($0 \leq w \leq 1$) to player 2 to exchange the right of moving first. If player 2 accepts the bid, she receives $w$ partition of the pie and player 1 begins the bargaining to divide the rest $1-w$; If player 2 refuses the bid, she wins the right to make the first offer to divide $1-w$ and player 1 receives $w$. We have the following proposition 2.

**Proposition 2**: In the case where two players bid for the right of moving first, the unique bargaining solution is $(x, 0)$, where $x = (1 - \delta_2)/(2 - \delta_1 - \delta_2)$.

**Proof**: If player 2 accepts $w$ and player 1 makes the first move, according to Proposition 1, two players will receive $x_1$ and $x_2$ respectively, where

$$
x_1 = \frac{1 - \delta_2}{1 - \delta_1 \delta_2} (1 - w)
$$

and $x_2 = 1 - x_1$.

On the other hand, if player 2 declines player 1’s bid, two players will receive $x_1^*$ and $x_2^*$, where

$$
x_1^* = w + \frac{\delta_1 (1 - \delta_2)}{1 - \delta_1 \delta_2} (1 - w)
$$
\[ x_2^* = 1 - x_1^* . \]

It is obvious that there should be \( x_1 = x_1^* . \) Then we have

\[
\begin{align*}
  w &= \frac{(1 - \delta_1)(1 - \delta_2)}{2 - \delta_1 - \delta_2} \\
  x_1 &= \frac{1 - \delta_2}{2 - \delta_1 - \delta_2} \\
  x_2 &= \frac{1 - \delta_1}{2 - \delta_1 - \delta_2}
\end{align*}
\]

With the bidding procedure, the bargaining equilibrium remains fixed no matter which player makes the first offer and it is indifferent for each player to make an offer or to accept the opponent’s offer. If we regard player 1 as the seller and player 2 the buyer, \( p = x_1 / x_2 = (1 - \delta_2) / (1 - \delta_1) \) can be treated as the price of exchange. Specifically, two players share the pie equally when \( \delta_1 = \delta_2 \) or \( p = 1 \). Player 2 receives the whole pie when \( p = 0 \). Player 1 receives the whole pie when \( p = \infty \). Proposition 2 also shows the consistency between the strategic bargaining approach and the Nash bargaining solution. Let \( u_1 \) and \( u_2 \) be the von Neumann-Morgenstern utilities of two players. If we define

\[
p_N = \frac{1 - \delta_2}{1 - \delta_1} = \arg \max (u_1(p)u_2(p))
\]

the outcome of Proposition 2 is exactly Nash bargaining solution.

2 **A \( N \)-seller and \( N \)-buyer Bargaining Game**

Consider a market where there are only two goods, \( A \) and \( B \). Participants in this market negotiate to exchange \( A \) for \( B \) or to exchange \( B \) for \( A \). Without loss of generality, let’s assume that those who want to exchange \( A \) for \( B \) are the sellers and those who want to exchange \( B \) for \( A \) are the buyers. The price \( p \) is then the exchange ratio of the amount of \( B \) to \( A \) and it is the only factor
that every player cares for in the bargaining. Each seller prefers a higher price and each buyer prefers a lower price. The assumptions here are A-1 to A-3 and that each player can freely choose their bargaining partner. The bargaining game is described as below.

A group of players that contains $n$ sellers and $n$ buyers bargain over the partitions of $n$ pies (each pie is of size of 1). A seller (buyer) has to reach an agreement with a buyer (seller) on the division of a pie. Each player has a constant discounting factor and their discounting factors are known to all players ($\delta_{si}$ and $\delta_{bi}$ for the sellers and buyers respectively, $i = 1, ..., n$, $0 < \delta_{si} < 1$, $0 < \delta_{bi} < 1$). The players bargain in a system like the stock market: each seller (buyer) can either offer a selling (buying) price to all buyers (sellers) or accept a price offered by another buyer (seller). The offered prices are known to all. Once a player accepts a price offered by another player, the division of a pie between them is determined and both players quit the bargaining. The players who do not make any agreement with others remain to the next round and this process will continue until no further agreements are possible or the value of pies decrease to zero.

When an agreement is made, which side offers the price (and which side accepts it) is not important since both sides only care for the price of the agreement. In order to simplify the analysis, we assume that the bargaining process in each round is as follows. The sellers first offer their selling prices. The buyers choose whether or not to accept them. If a buyer does not accept any selling offer, he/she has to offer a buying price. And then the sellers who do not have an agreement choose whether or not to accept a buying price. A round of bargaining ends. The players who do not have an agreement with others enter the next round of bargaining.

We first analyze the bargaining problem of two sellers and two buyers and then extend it to the case of $n$ sellers and $n$ buyers.
2.1 A two-seller and two-buyer bargaining game

Consider the bargaining problem of two sellers $S_1$ and $S_2$ and two buyers $B_1$ and $B_2$ whose discounting factors are $\delta_{s1}$, $\delta_{s2}$, $\delta_{b1}$ and $\delta_{b2}$ respectively. Let $F$ be the set of all strategies of the players who offer the partitions, and $G$ the set of all strategies of the players who have to respond to an offer. The outcome of this bargaining can be expressed by the quad $(x_1, t_1, x_2, t_2)$ where $x_1, x_2 \in X$ denote the partitions of two pies that two pairs of players agree with respectively, $t_1, t_2$ denote the time when the agreements are made. Notice that $t \to \infty$ denotes ‘disagreement’ between a seller and a buyer. We have the following proposition.

**Proposition 3**: A two-seller and two-buyer bargaining game has a bargaining equilibrium, a unanimous partition of both pies.

**Proof**: We first prove that, if there exists a bargaining equilibrium, it must be a unanimous partition of two pies (unanimous partition is a necessary condition).

Without loss of generality, suppose that the seller $S_1$ and the buyer $B_1$ reach the agreement of partition $(x_1, t_1)$ and $S_2$ and $B_2$ reach the agreement of partition $(x_2, t_2)$. For the outcome $(x_1, t_1, x_2, t_2)$ to be a perfect equilibrium partition, there must be $x_1 = x_2$ and $t_1 = t_2$.

If $x_1 \neq x_2$, without loss of generality, let’s assume $x_1 > x_2$.

(1) If $t_1 > t_2$, $B_1$ could offer a buying partition $x' (x_2 < x' < x_1)$ at time $t_2$ and $S_2$ would accept the offer instead of $x_1$ so that both players improved their payoffs. Thus, $(x_1, t_1, x_2, t_2)$ is not an equilibrium.

(2) If $t_1 < t_2$, $S_2$ could offer a selling partition $x' (x_2 < x' < x_1)$ at time $t_1$ and $B_1$ would accept it so that both players improved their payoffs. So $(x_1, t_1, x_2, t_2)$ is not an equilibrium.

(3) If $t_1 = t_2$, either $B_1$ or $S_2$ would offer a partition $x' (x_2 < x' < x_1)$ to improve their payoffs.
Whenever there are $x_1 \neq x_2$, we can always find a pair of seller and buyer who can be better off by making a deal with new partition $x' (x_2 < x' < x_1)$. Thus the equilibrium must be a unanimous partition of two pies.

Second, we prove that no player has incentive to deviate from a unanimous partition unilaterally (unanimous partition is a sufficient condition). Given that four players have reached a unanimous partition, in which the partition is $x$ between each pair of seller and buyer. A seller would deviate only if there was a buyer who would accept a higher price than $x$. A buyer would deviate only if there was a seller who would accept a lower price than $x$. Obviously, two conditions are conflict with each other. Thus no player could deviate from a unanimous partition unilaterally.

Every player will receive the minimum payoff if they cannot reach a unanimous partition. Realising that they have to reach a unanimous partition, the sellers and buyers actually bargain on a price with which no single player can be better off by deviating from it. Both sellers prefer higher prices to lower prices while both buyers prefer lower prices to higher prices. The bargaining game turns out to be a two-player game, in which one side is the group of sellers and the other side is the group of buyers. According to Proposition 1, there is a unique equilibrium of the two-player bargaining game. Let $x^*$ denote the partition in this equilibrium. In order to determine $x^*$ by adopting a sequential approach, we assume that the sellers first offer a partition to the buyers. If the buyers accept it, two agreements are made and the game ends. Otherwise, the buyers make a counter offer on the discounted pie. The game continues until either two agreements or disagreement is made.

Following Rubinstein (1982) and Osborne and Rubinstein (1990), we define a group of
functions $v_i (i = s_1, s_2, b_1, b_2)$ as follows.

$$v_i(x, t) = \begin{cases} y, & \exists y \in X \text{ such that } (y, 0) \sim_i (x, t) \\ 0, & \forall y \in X \text{ there is } (y, 0) >_i (x, t) \end{cases}$$

This means that for any $(x, t)$, either there is a unique $y \in X$ such that player $i$ is indifferent between $(x, t)$ and $(y, 0)$, or every $(y, 0)$ is preferred by $i$ to $(x, t)$. In order for two sellers and two buyers to reach a unanimous partition, there should be $v_{s_1} = v_{s_2}$ and $v_{b_1} = v_{b_2}$. This means that two sellers (buyers) are equal in the bargaining no matter what values their discounting factors are.

In order for two sellers (buyers) to form the same bargaining strategy, there should be

$$v_{s_1}(x, t) = v_{s_2}(x, t) = \frac{1}{2}(\delta^f_{s_1} + \delta^f_{s_2})x$$

$$v_{b_1}(x, t) = v_{b_2}(x, t) = \frac{1}{2}(\delta^f_{b_1} + \delta^f_{b_2})x$$

The intersection of $y_{s_1} = v_1(x_{s_1}, 1)$ and $x_{b_1} = v_1(y_{b_1}, 1)$ reflects the unanimous partition $(x^*, y^*)$. This can be expressed as Fig.1.

From Fig.1, we have

$$x^* = \frac{2(2 - \delta_{b_1} - \delta_{b_2})}{4 - (\delta_{b_1} + \delta_{b_2})(\delta_{s_1} + \delta_{s_2})}$$

$$y^* = \frac{(\delta_{b_1} + \delta_{b_2})(2 - \delta_{s_1} - \delta_{s_2})}{4 - (\delta_{b_1} + \delta_{b_2})(\delta_{s_1} + \delta_{s_2})}$$

When the sellers and buyers bid for the right of making the first offer, the advantage of first offer can be eliminated. The process is that the sellers first offers a bid $w$ $(0 \leq w \leq 1)$ to the buyers to exchange the right of first offer. If the buyers accept the bid, each buyer receives $w$ partition of a pie and the sellers begin the bargaining to divide the rest of pies; If the buyers refuse the bid, they win the right to make an offer first and each seller receives $w$.

If the buyers accept the bid, each seller will receive

$$x_1 = \frac{2(2 - \delta_{b_1} - \delta_{b_2})}{4 - (\delta_{s_1} + \delta_{s_2})(\delta_{b_1} + \delta_{b_2})} (1 - w)$$
Figure 1: Perfect equilibrium \((x^*, y^*)\) for the bargaining game of two-pair of sellers and buyers.

If the buyers incline the bid, each seller will receive

\[
x_2 = \frac{(\delta_{s1} + \delta_{s2})(2 - \delta_{b1} - \delta_{b2})}{4 - (\delta_{s1} + \delta_{s2})(\delta_{b1} + \delta_{b2})}(1 - w) + w
\]

Obviously, there should be \(x_1 = x_2\). Then we have

\[
x_1 = \frac{2 - \delta_{b1} - \delta_{b2}}{4 - \delta_{b1} - \delta_{b2} - \delta_{s1} - \delta_{s2}} \quad (6)
\]

Each buyer receives

\[
y_1 = 1 - x_1 = \frac{2 - \delta_{s1} - \delta_{s2}}{4 - \delta_{b1} - \delta_{b2} - \delta_{s1} - \delta_{s2}} \quad (7)
\]

Hence,

\[
p = x_1/y_1 = \frac{2 - \delta_{b1} - \delta_{b2}}{2 - \delta_{s1} - \delta_{s2}} \quad (8)
\]

According to Proposition 3, a patient player receives the same price as an impatient player in the bargaining. This counterintuititve-seeming result can be explained as below. Because every player would like to choose the impatient player to be their bargaining opponent, the impatient
player could increase their share by threatening to change their bargaining opponent. Similarly, the patient player had to lower their share because of their opponent’s threat of changing their bargaining opponent. Consequently, a unanimous price will be reached so that the sellers (buyers) receive equal partition no matter how patient or impatient they are.

![Graph showing the relationship between two-player bargaining and two-pair of players bargaining.](image)

Figure 2: Relationship between two-player bargaining and two-pair of players bargaining.

Fig.2 shows the relationship between the two-player bargaining equilibrium and the two-seller and two-buyer bargaining equilibrium. If the players bargain with each other independently, the equilibrium of two players bargaining will be either $A$, $C$ or $B$, $D$. When they bargain together, the equilibrium will be $(x^*, y^*)$, which is a point inside the quadrilateral $ABCD$.

### 2.2 The $n$-seller and $n$-buyer bargaining game

Let $S_i$ and $B_i$ ($i = 1, \ldots, n$) denote the sellers and buyers respectively, and $\delta_{s_i}$ and $\delta_{b_i}$ the discounting factors of the sellers and buyers respectively. We have Proposition 4 as below.

**Proposition 4**: The $n$-seller and $n$-buyer bargaining problem has an equilibrium: a unanimous
price \( p_n \),

\[
p_n = (n - \sum_{i=1}^{n} \delta_{b_i})/(n - \sum_{i=1}^{n} \delta_{s_i})
\]  

(9)

**Proof:** According to the proof of Proposition 3, a bargaining equilibrium for this bargaining game must be a unanimous price. It is easy to verify that, if there are different prices, at least one pair of seller and buyer can be better off by changing their decisions.

Consider the two largest coalitions that contains all sellers and buyers respectively. Let \( p^* \) denotes the price determined by these two coalitions. \( p^* \) must be an equilibrium because no player can be better off by leaving their coalition. For example, a seller would leave the coalition only if there was a buyer who would accept \( p > p^* \). However, any buyer would leave their coalition only if there was \( p < p^* \). Let \( \delta_{s_A} \) and \( \delta_{b_A} \) be the average discounting factor of the coalitions of sellers and buyers respectively.

\[
\delta_{s_A} = (\delta_{s_1} + \delta_{s_2} + \cdots + \delta_{s_n})/n
\]  

(10)

\[
\delta_{b_A} = (\delta_{b_1} + \delta_{b_2} + \cdots + \delta_{b_n})/n
\]  

(11)

According to (1), there must be

\[
p^* = \frac{1 - \delta_{b_A}}{1 - \delta_{s_A}} = \frac{n - \sum_{i=1}^{n} \delta_{b_i}}{n - \sum_{i=1}^{n} \delta_{s_i}}.
\]

Then, \( p^* \) is the unanimous price \( p^* = p_n \).

The equilibrium of the \( n \)-seller and \( n \)-buyer bargaining game can be expressed as Fig.3. Two coalitions reach the agreement of \((x^*, y^*)\). If two coalitions bid for the right to make the first offer, the advantage of making the first offer will be removed. Then, each seller receives \( x_1 \),

\[
x_1 = \frac{n - \sum_{i=1}^{n} \delta_{b_i}}{2n - \sum_{i=1}^{n} (\delta_{s_i} + \delta_{b_i})}
\]  

(12)
Each buyer receives $y_1$,

$$y_1 = \frac{n - \sum_{i=1}^{n} \delta_{s_i}}{2n - \sum_{i=1}^{n} (\delta_{s_i} + \delta_{b_i})}$$

(13)

Figure 3: Perfect equilibrium ($x^*$, $y^*$) for the bargaining game of $n$-pair of sellers and buyers.

Imagine that there exists a pair of representative seller and buyer, $S_A$ and $B_A$, whose discounting factors, $\delta_{s_A}$ and $\delta_{b_A}$, are the average of discounting factors of all sellers and all buyers respectively. The equilibrium of a $n$-pair players bargaining game is equivalent to the equilibrium of two representative players bargaining game. Let $u_{s_A}(p)$ and $u_{b_A}(p)$ be the von Neumann-Morgenstern utility functions of the representative seller and buyer respectively. If we define

$$p_N = \frac{n - \sum_{i=1}^{n} \delta_{b_i}}{n - \sum_{i=1}^{n} \delta_{s_i}} = \arg \max (u_{s_A}(p)u_{b_A}(p)),$$

the outcome of Proposition 4 is consistent with Nash bargaining equilibrium.
3 Conclusions

The bargaining games of two-seller and two-buyer, and further \( n \)-seller and \( n \)-buyer are analyzed by using a sequential approach. The equilibria of these bargaining games are unanimous prices determined by average discounting factors of all sellers and all buyers. Every player is a price taker when \( n \) is large because individual player has trivial power to determine the bargaining price. This conclusion has the potential of being extended to the problems of market-clearing prices in perfectly competitive markets.

In this study, the \( 2n \) players are assumed to bargain together so that the bargaining problem can be considered as a game with complete information in which each player knows the discounting factors of all players. This may be unrealistic in real world market, especially when the number of players is relatively large. It sometimes takes time, money, and other resources for the players to retrieve certain information. If the cost of information is taken into consideration, the players will have to restrict their negotiations within a limited group of players. Then the propagation of information will have an influence on the players’ strategies and different prices are possible in this circumstance.

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