Neutron-loaded outflows in gamma-ray bursts

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ABSTRACT
Relativistic neutron-loaded outflows in gamma-ray bursts are studied at their early stages, before deceleration by a surrounding medium. The outflow has four components: radiation, electrons, protons and neutrons. The components interact with each other and exchange energy as the outflow expands. The presence of neutrons significantly changes the outflow evolution. Before neutrons decouple from protons, friction between the two components increases their temperatures by many orders of magnitude. After the decoupling, the gradual neutron decay inside the outflow has a drag effect on the protons and reduces their final Lorentz factor.

Interesting effects also take place at small r < 10^{12} cm. Neutrons are initially accelerated together with protons because they are collisionally coupled, and the last n-p collisions before decoupling cause a significant heating.

We assume in this paper a simple hydrodynamic picture of expansion driven by thermal pressure and study the basic dynamics of a uniform neutron-loaded outflow. We do not consider internal shocks or possible dynamical effects of magnetic fields. The paper is organized as follows. In section 2, we briefly review neutron-free outflows, which have been studied previously in detail (see Piran 2004 for a review). In section 3, we derive equations describing neutron-loaded outflows and calculate example numerical models. Results are discussed in section 4.

INTRODUCTION

A neutron component in γ-ray bursts (GRBs) was proposed by Derishev, Kocharovsky & Kocharovsky (1999a,b), and detailed calculations of nuclear composition show that free neutrons are inevitably present among ejected baryons (Beloborodov 2003b; hereafter B03b). Any plausible central engine of GRBs is dense and at least mildly degenerate, which leads to its neutronization. During the explosion, the expanding neutron-rich material may undergo nucleosynthesis: neutrons tend to recombine with protons to form deuterons. In the present paper, we study the dynamics of neutron-loaded outflows at early stages of their expansion, since they are decelerated by an external medium.

The presence of neutrons changes the theoretical picture of GRB explosion. Firstly, they may develop a somewhat smaller Lorentz factor than protons. When such a decoupling takes place, the last n-p collisions lead to emission of observable multi-GeV neutrinos (Derishev et al. 1999a, hereafter DKK99a; Bahcall & Mészaros 2000; Mészaros & Rees 2000a). Secondly, neutrons decay with time. The decay impacts the external blast wave at radii \( r \approx 10^{16} \rightarrow 10^{17} \) cm because even an exponentially small number of survived neutrons carry an energy much larger than the rest-energy of external medium (Beloborodov 2003a).

In the present paper, we study the dynamics of neutron-loaded outflows at early stages of their expansion, \( r < 10^{16} \) cm, before they are decelerated by an external medium. The neutrons decay gradually at all radii \( r \lesssim 10^{17} \) cm, and at small \( r \) the decay occurs inside the GRB outflow. The decay turns out important at \( r \) as small as \( 10^{12} \) cm.

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2 NEUTRON-FREE OUTFLOW

We model the GRB outflow as a steady wind with duration \( t_{GRB} \), luminosity \( L \), and baryon mass outflow rate \( \dot{M} \). We assume spherical symmetry. This is a good approximation also for jets with constant opening angle greater than \( 1/\Gamma \) – such a jet behaves as a part of a spherically symmetric outflow. The main parameter of the problem is

\[
\eta = \frac{L}{\dot{M} c^2} \sim 10^2 - 10^3. \tag{1}
\]

We consider three components in this section: radiation, electrons and protons. At small radii \( r \approx 30 R_0 \), the outflow is dominated by \( e^\pm \) pairs, all components maintain a common temperature and cool adiabatically. An interesting evolution begins at \( r > 50 R_0 \) where \( e^\pm \) can be neglected.
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2.1 Opaque stage

2.1.1 Outflow acceleration

As long as the outflow is optically thick, electrons, protons and radiation behave as a single relativistic fluid. The fluid has four-velocity \( U^\alpha = (\Gamma_p c, \Gamma_p \beta_p c, 0, 0) \) in spherical coordinates \((t, r, \theta, \phi)\). The trapped radiation is isotropic in the fluid frame and described by the blackbody law with a temperature \( T_r \).

The total stress energy tensor of the fluid is then given by (e.g. Misner, Thorne & Wheeler 1973)

\[
T^{\alpha \beta} = \left( \frac{4}{3} a T^4 + \rho_p c^2 \right) \frac{U^\alpha U^\beta}{c^2} + \frac{1}{3} a T^4 g^{\alpha \beta},
\]

where \( a = 7.56 \times 10^{-15} \) erg cm\(^{-3}\) K\(^{-4}\) is the radiation constant and \( g^{\alpha \beta} = \text{diag}(-1, 1, r^2, r^2 \sin^2 \theta) \) is Minkowski metric in coordinates \((t, r, \theta, \phi)\). The plasma contribution to the energy density is taken equal to \( \rho_p c^2 \) where \( \rho_p \) is the proper mass density of protons; small contributions from \( e \) and \( p \) thermal motions and the electron rest mass are neglected. The four-vector of baryon mass flux is given by,

\[
F^\alpha = \rho_p U^\alpha.
\]

The outflow dynamics is governed by the conservation laws,

\[
\frac{\partial T^{tt}}{\partial t} = \frac{1}{r^2} \frac{\partial (r^2 T^{r r})}{\partial r},
\]

\[
\frac{\partial T^{r r}}{\partial t} = \frac{1}{r^2} \frac{\partial (r^2 F^r)}{\partial r}.
\]

As long as \( r < t_{\text{GRB}} c \Gamma_p^2 \) (which we assume to be satisfied thereafter) the outflow may be described as a steady-state wind, so that \( \partial T^{tt} / \partial t = 0 \). Then the conservation laws give

\[
L = 4 \pi r^2 c \beta_p^2 \rho_p \left( \frac{4}{3} a T^4 + \rho_p c^2 \right) = \text{const},
\]

\[
\dot{M} = 4 \pi r^2 c \beta_p \Gamma_p \rho_p = \text{const}.
\]

The ratio of these two constants is the parameter \( \eta \) (eq. D).

The description of fluid dynamics will be complete once we specify the evolution of radiation temperature \( T_r \) with radius. The photon-to-baryon ratio in GRB outflows is \( 10^5 \) (see e.g. eq. 68 in B03b) and their internal energy is strongly dominated by radiation. Energy exchange between radiation and plasma practically does not affect \( T_r \) and it follows the adiabatic law,

\[
T_r = T_0 \left( \frac{n}{n_0} \right)^{1/3},
\]

\[
n = n_0 \left( \frac{r}{R_0} \right)^{-2} \frac{1}{\Gamma_p \beta_p},
\]

where \( n = \rho_p / m_p \) is the proton number density; \( T_0 \) and \( n_0 \) are constants defined at the base of the outflow \( r = R_0 \approx 10^6 - 10^7 \) cm,

\[
T_0 \equiv \left( \frac{3 L}{16 \pi \Gamma_0 \beta_0 R_0^2 c a} \right)^{1/4} \approx 10^{10} \frac{L_{52}}{R_{07}^{3/2}} \text{K},
\]

1 At radii \( r < t_{\text{GRB}} c \Gamma_p^2 \) the leading and trailing parts of the outflow are causally disconnected. Therefore the outflow behaves as part of a steady wind despite the fact that geometrically it is a thin shell already at \( r > t_{\text{GRB}} c \).

\[
n_0 \equiv \frac{L}{4 \pi \Gamma_0 \beta_0 R_0^2 m_p c^4 \eta} \approx 6 \times 10^{26} \frac{L_{52}}{R_{07}^{3/2}} \left( \frac{\eta}{\text{RIO}} \right)^{-1} \text{cm}^{-3},
\]

where \( \Gamma_0 = \Gamma_p (R_0) \) and we assume

\[
\Gamma_0 \beta_0 = 1.
\]

Equations (3, 4) and (5) give a closed description of the outflow dynamics at the opaque stage of its expansion. Combining the equations we find

\[
\Gamma_p (1 + x) = \eta,
\]

where

\[
x \equiv \frac{4}{3} \frac{a T^4}{n m_p c^2} = x_0 \left( \frac{n}{n_0} \right)^{1/3},
\]

\[
x_0 = \frac{\eta}{\Gamma_0} - 1,
\]

and obtain the algebraic equation for \( \Gamma_p (r) \),

\[
\left( \frac{\eta}{\Gamma_0} - 1 \right) \left( \Gamma_p^2 - 1 \right)^{1/6} = x_0 \left( \frac{r}{R_0} \right)^{-2/3},
\]

(see also DKK99a).

2.1.2 Thermal balance

The evolution of electron temperature \( T_e \) and proton temperature \( T_p \) in general depends on their energy exchange with each other and radiation. It turns out that all components of the neutron-free outflow maintain the common temperature \( T_e \approx T_p \approx T_r \) during the opaque stage. We shall verify this with an accurate thermodynamic calculation.

The electron and proton components obey the first law of thermodynamics,

\[
dU_i = dQ_i + (U_i + P_i) \frac{dn}{n}, \quad i = e, p,
\]

where \( U_i \) is internal energy density of component \( i \), \( dQ_i \) is heat received by component \( i \) per unit volume and \( P_i \) is its pressure; all the quantities are measured in the fluid frame. \( U_i \) and \( P_i \) are related to temperature \( T_i \),

\[
U_i = \frac{nk T_i}{\gamma_i - 1}, \quad P_i = nk T_i, \quad i = e, p,
\]

where \( \gamma_i \) is the adiabatic index of component \( i \); it equals 5/3 for nonrelativistic electrons and protons. Then from equation (16) we obtain the equation for \( T_i \),

\[
\frac{3}{2} \frac{dT_i}{dr} = \frac{T_i}{n} \frac{dn}{dr} + \frac{1}{kn} \frac{dQ_i}{dt'} \frac{1}{c \gamma_i \Gamma_p \beta_p}, \quad i = e, p,
\]

where \( dt' = dt / \Gamma_p \) is time measured in the fluid frame. Electrons exchange energy with photons via Compton scattering and with protons via Coulomb collisions,

\[
\frac{dQ_e}{dt'} = \frac{3}{2} \frac{nk}{m_e} \left( \frac{T_C - T_r}{\tau_{\text{ep}}} + \frac{T_r - T_e}{\tau_{\text{ep}}} \right),
\]

where

\[
\tau_{\text{ep}} = \frac{\sqrt{\pi} m_p}{8 U_{\text{rad}} \sigma_T} c \ln \Lambda \frac{1}{m_e} \left( \frac{k T_p}{m_e c^2} + \frac{k T_r}{m_p c^2} \right)^{3/2},
\]

is the Coulomb timescale (with Coulomb logarithm \( \ln \Lambda \approx 15 \)); see Stepney 1983,

\[
\tau_C = \frac{3 m_e c}{8 U_{\text{rad}} \sigma_T}.
\]
is the Compton timescale, 
\[ U_{\text{rad}} = a \Gamma_r^4, \]

is the radiation energy density, and \( T_C \) is the Compton temperature of radiation. For blackbody radiation \( T_C = T_r \).

The thermal balance for protons reads 
\[ \frac{dQ_p}{dt} = -\frac{3}{2} n_k (T_p - T_r) \tau_p, \]  
(22)

We have neglected here the energy exchange between protons and radiation; it is \( \sim (m_e/m_p)^4 \) times the electron-radiation energy exchange.

### 2.2 Transparent stage

Electron scattering dominates opacity of the outflow, and its optical depth is 
\[ \tau_e = \frac{n_e \sigma_T r}{\Gamma_p} = \frac{L \sigma_T}{4\pi r m_p c^2 \Gamma_p^2 \eta}, \]  
(23)

where \( n_e = n \) is the electron number density. At radii \( r > R_e \) where \( \tau_p < 1 \), the outflow is transparent and equation (8) is no longer valid. The photon luminosity at this point is (see eq. 6),
\[ L_\gamma = \frac{16\pi}{3} R_p^2 c \beta_p \Gamma_p^2 T_r^4, \]  
(24)

where all quantities are taken at \( r = R_e \). Approximately this luminosity escapes to a distant observer as a blackbody radiation with observed temperature \( \Gamma_p T_r(R_e) \).

#### 2.2.1 Outflow acceleration

After the transparency radius, the outflow may still be accelerated by radiation.

A freely propagating photon at an angle \( \theta \) with respect to radius satisfies the relation \( r \sin \theta = \text{const} \) and becomes more beamed at larger \( r \). A typical photon is emitted at angle \( \pi/2 \) in the plasma frame and has initial beaming angle \( \theta_{\text{rad}} \approx 1/\Gamma_p \) at \( r = R_e \). At larger radii the beaming angle decreases as
\[ \theta_{\text{rad}} \approx \frac{R_e}{r} \frac{1}{\Gamma_p(R_e)}. \]  
(25)

One can define a frame where the freely streaming radiation remains approximately isotropic. The velocity of this frame is \( \beta_{\text{rad}} = \cos\theta_{\text{rad}} \) and its Lorentz factor is
\[ \Gamma_{\text{rad}} = \frac{1}{\sin\theta_{\text{rad}}} \approx \frac{r}{R_e} \Gamma_p(R_e). \]  
(26)

The streaming radiation will tend to accelerate the plasma outflow since \( \Gamma_{\text{rad}} > \Gamma_p \). The radiation flux \( F_\gamma = L_\gamma/4\pi r^2 \) exerts a force on an electron moving with velocity \( \beta_p \) (see Beloborodov 2002, eq. A6),
\[ \frac{dp}{dt} = \frac{\sigma_T F_\gamma}{c} \left( 1 - \beta_p \right) \left( 1 + \frac{\Gamma_p}{\Gamma_{\text{rad}}} \right) \]  
(27)

and the plasma Lorentz factor grows,
\[ \frac{d\Gamma_p}{dr} = \frac{\sigma_T L_\gamma}{16\pi^2 r^2 m_p c^3 \Gamma_p^2} \left( 1 - \frac{\Gamma_p^4}{\Gamma_{\text{rad}}^4} \right). \]  
(28)

An easy estimate of the acceleration effect may be obtained neglecting the factor \( \Gamma_{\text{rad}}^4/\Gamma_{\text{rad}}^4 \) in parenthesis. Then equation (28) gives
\[ \Gamma_p^4(r) = \Gamma_p^4(R_e) + \frac{3\sigma_T L_\gamma}{16\pi^2 m_p c^5} \left( \frac{1}{R_e^4} - \frac{1}{r^4} \right). \]  
(29)

The acceleration at the transparent stage is significant if at \( r \gg R_e \) the second term on right-hand side exceeds the first term. This condition is equivalent to \( aT_\gamma^p > nm_e c^2 \) at \( r = R_e \), which requires most of the outflow energy be in radiation at the moment of transparency. The same condition may be expressed in terms of \( \eta \) (e.g. Mészáros & Rees 2000b),
\[ \eta > \eta_{\text{rad}} \approx \frac{\eta}{\Gamma_p^4} \approx 10^3 L_\gamma^{1/4} R_0^{-1/4}. \]  
(30)

#### 2.2.2 Thermal balance

The thermal balance of the outflow at the transparent stage is still given by equations (13), (19) and (22). The Coulomb timescale is given by the same equation (28) and the Compton timescale is given by equation (21) with
\[ U_{\text{rad}}(r) \approx \frac{L_\gamma}{4\pi^2 c \Gamma_p^4}, \]  
(31)

being the radiation density measured in the plasma frame.

We note that the outflow is subject to significant Compton cooling even at \( r > R_e \) because the scattering rate per electron is still very high at \( R_e \) (it is \( n_e/n \approx 10^5 \) higher than the scattering rate per photon). The Compton temperature \( T_C \) appearing in equation (19) represents the effective temperature of radiation observed from the plasma frame and is proportional to the average photon energy in this frame. \( T_C \) may be evaluated as follows.

The typical photon at \( r > R_e \) has angle \( \theta(r) = \theta_{\text{rad}}(r) \) and constant energy \( \epsilon(r) = \epsilon(R_e) \Gamma_p(R_e) \) in the lab frame. Its energy in the plasma frame is
\[ \epsilon'(r) = c \Gamma_p(1 - \beta_p \cos\theta_{\text{rad}}) \Gamma_p(R_e) \approx \epsilon(R_e) \Gamma_p(R_e) \frac{1}{2} \left( \frac{1}{\Gamma_p} + \frac{\Gamma_p}{\Gamma_{\text{rad}}} \right). \]

The Compton temperature changes with radius as
\[ \frac{T_C(r)}{T_C(R_e)} = \frac{\epsilon'(r)}{\epsilon'(R_e)} \approx \frac{1}{2} \left( 1 + \frac{\Gamma_p^2}{\Gamma_{\text{rad}}^2} \right), \]  
(32)

where \( T_C(R_e) = T_r(R_e) \). Equation (32) completes the description of Compton energy exchange at the transparent stage.

### 2.3 Numerical models

Numerical models of outflows with \( \eta = 200 \) and 800 are shown in Figures 1 and 2. Both models have \( \eta < \eta_{\text{rad}} \) and the outflow Lorentz factor saturates before transparency.

Upper panels in Figures 1 and 2 show the evolution of the Lorentz factor. As long as enthalpy exceeds rest-mass energy \( (x > 1) \), the outflow accelerates with \( \Gamma_p \propto r \). When most of the internal energy is converted to kinetic energy \( (x < 1) \), \( \Gamma_p \) tends to a constant asymptotic value \( \Gamma_{\text{pf}} = \eta \).

The characteristic saturation radius \( R_s \) is where \( x = 1; \Gamma_p = \eta/2 \) at this radius and
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Figure 1. Lorentz factor (upper panel) and temperature (lower panel) as functions of radius for a neutron-free outflow with $L = 10^{52}$ erg/s, $\eta = 200$ and $R_0 = 10^7$ cm. Electrons never decouple thermally from protons and their common temperature is shown by the solid curve. Dashed-dotted curve shows the radiation temperature.

Figure 2. Same as Fig. 1 but for $\eta = 800$. $T_p$ and $T_e$ are shown by solid and dashed curves, respectively. The thermal decoupling $T_e > T_p$ takes place at radii $r \sim 10^{12} - 10^{16}$ cm.

Figure 3. Same as Fig. 2 but for $\eta = 8 \times 10^3$. The outflow becomes transparent before the acceleration stage ends.

\[ R_0 = 2^{-1/4} \eta R_0. \]  

Solutions for temperatures $T_r$, $T_e$, and $T_p$ are shown in the lower panels of Figures 1 and 2. At very small radii where $kT \lesssim m_e c^2 = 511$ keV, the number density of $e^\pm$ pairs is comparable to that of photons, the energy density is $(11/4) a T^4$ and the temperature is reduced by the factor of $(4/11)^{1/4} \approx 0.78$; this correction is neglected in the figures. As long as the outflow is optically thick, all three components maintain a common temperature $T$ via Coulomb collisions and Compton scattering, and $T$ decreases adiabatically with index $\hat{\gamma} = \frac{4}{3}$. The outflow becomes transparent at radius

\[ R_t = \frac{L \sigma_T}{4 \pi m_e c^4 T_p^3} \eta < \eta_{rad}. \]  

At the transparent stage the plasma is still tracking the temperature of (freely streaming) photons: $T_e \approx T_p \approx T_r \approx \text{const}$ until the electrons decouple either from radiation ($e$-$\gamma$ decoupling) or from the protons ($e$-$p$ decoupling).

The $e$-$\gamma$ decoupling occurs at a radius $R_{e\gamma}$ where the Compton time-scale (eq. 21) exceeds the expansion time-scale $\tau_{exp} = R/T_p c$,

\[ R_{e\gamma} = \frac{2 L \sigma_T}{3 \pi m_e c^3 T_p^3} \approx \frac{2 \sigma_T L}{3 \pi m_e c^3 \eta^{2/3}} \left( \frac{R_0}{R_t} \right)^{2/3}. \]

The $e$-$p$ decoupling may happen or not depending mostly on the value of $\eta$. The two possible cases are illustrated in Figures 1 and 2:

(i) Electrons and protons are still coupled at $R_{e\gamma}$ ($\tau_{exp} < \tau_{e\gamma}$) and begin a common adiabatic cooling, $T_p = T_e \propto n_e^{1/3}$. They will not decouple later because $\tau_{exp} = \text{const}$ while $\tau_{e\gamma}$ keeps increasing. This regime takes place at $\eta \lesssim 650$ and is illustrated in Figure 1.

(ii) Electrons decouple from protons before $R_{e\gamma}$. The radius of $e$-$p$ decoupling is found from condition $\tau_{exp} = \tau_{e\gamma}$. Using
The mean life-time of neutrons in their rest frame is 

\[ T_e \approx T_r(R_e) = T_e(R_s)(R_e/R_s)^{-3/2} \] in equation (39) for \( \tau_{\text{ep}} \) (and neglecting the small term \( kT_p/m_p c^2 \)), we get 

\[ R_{\text{ep}} \simeq \left( 68 \pi m_p c^4 \right)^{-1} \frac{L}{\eta^3 T_9^{3/2}} \left( \frac{R_e}{R_s} \right), \]

where \( T_s = 2T_0/\eta \) is the temperature at the saturation radius. At \( R_{\text{ep}} \ll r < R_e \gamma, \) \( T_e \approx T_s \gg T_p \). At \( r > R_e \gamma \), the electrons cool down adiabatically and \( \tau_{\text{ep}} \approx \text{const} \) while \( \tau_{\text{e-p}} \) keeps increasing. Therefore, electrons and protons eventually regain the thermal coupling. This regime takes place at high \( \eta \gtrsim 650 \) and is illustrated in Figure 2.

Figure 3 shows a model with \( \eta = 8 \times 10^3 > \eta_{\text{ad}} \). In this case, the outflow acceleration continues after the transparency radius

\[ R_e \simeq R_0 \eta_{\text{ad}} \left( \frac{\eta_{\text{ad}}}{\eta} \right)^{1/3}, \quad \eta > \eta_{\text{ad}} \]

and \( \Gamma_p \) saturates at

\[ \Gamma_{p,f} \approx \eta_{\text{ad}} \left( \frac{\eta}{\eta_{\text{ad}}} \right)^{1/9} \eta, \]

after a few \( R_e \). Since density is lower than in the previous examples, the \( e-p \) decoupling happens much earlier than the \( e-\gamma \) decoupling (\( R_{\text{ep}} \ll R_e \)).

In summary, the thermal evolution of a neutron-free outflow is dominated by adiabatic cooling. The opaque outflow has a common temperature \( T_p = T_e = T_r \) which decreases as \( r^{-1} \) during the acceleration stage and as \( r^{-2/3} \) during the cooling stage. Then, after an intermediate stage where the details of coupling between \( e, p \) and radiation are important, the outflow is again described by a simple adiabatic law \( T_p \approx T_s \propto r^{-4/3} \). The presence of neutrons will change this picture.

3 NEUTRON-LOADED OUTFLOW

We now consider an outflow with a neutron component and denote its initial neutron richness (neutron-to-proton ratio) by \( \xi_0 \). All other assumptions are the same as in section 2. In particular, we consider spherical expansion driven by radiation pressure. Neutrons and protons are injected at \( r = R_0 \) with an initial density ratio \( \xi = \xi_0 \), and then \( \xi \) evolves with radius because neutrons continuously \( \beta \)-decay into protons,

\[ n \rightarrow p + e + \nu. \]

The mean life-time of neutrons in their rest frame is \( \tau_\beta \approx 900 \) s, and the corresponding mean radial of \( \beta \)-decay is

\[ R_\beta = \int_0^{\tau_\beta} c \beta \Gamma_n \, d\tau \simeq 8 \times 10^{15} \left( \frac{\Gamma_n}{300} \right) \text{ cm}, \]

where \( \Gamma_n \) is the neutron Lorentz factor and \( \beta_n \approx 1; \) \( \Gamma_n/\beta \) is the final value of \( \Gamma_n \) achieved at \( r \sim (10^2 - 10^3)R_0 \ll R_\beta \). The neutron population is gradually depleted as \( \exp(-\beta/rR_\beta) \) and the \( n/p \) ratio (measured in the fixed lab frame) evolves with radius as

\[ \xi \equiv \frac{n_\text{n}}{n_\text{p}} = \frac{\xi_0 \exp(-\beta/rR_\beta)}{1 + \xi_0 \left( 1 - \exp(-\beta/rR_\beta) \right)}, \]

where \( n_\text{n} \) and \( n_\text{p} \) are proper densities of the neutron and proton components.

3.1 Outflow acceleration and \( n-p \) coupling

The total luminosity of the outflow (cf. eq. 40) now includes the contribution from neutrons,

\[ L = 4\pi r^2 c \left[ \beta_p \Gamma_p^2 \left( \frac{4}{3} a T_p^3 + p_e c^2 \right) + \beta_n \Gamma_n^2 \rho_n c^2 \right] = \text{const}, \]

where \( \rho_p = n_pm_p \) and \( \rho_n = n_n m_n; \) the thermal energy of the plasma and neutrons has been neglected compared to their rest-mass energy. We allow here the neutron component to have a different Lorentz factor \( \Gamma_n \), which will be close to \( \Gamma_p \) as long as the \( n-p \) coupling is efficient.

The baryon outflow rate is given by

\[ \dot{M} = 4\pi r^2 c (\beta_p \Gamma_p \rho_p + \beta_n \Gamma_n \rho_n) = \text{const}. \]

The outflow expansion is accompanied by adiabatic cooling which determines \( T_e(r) \). Expansion of volume can be described by the decrease of number density of original protons injected at the base of the outflow. We denote this density by \( n \) and distinguish it from the total proton density \( n_p \) that includes the decayed neutrons. They are related by

\[ n_p = 1 + \xi_0 \frac{1 + \xi}{1 + \xi_0}. \]

The radiation temperature at the optically thick stage obeys equation (38). We neglect destruction of neutrons at this stage (see section 3.3) and assume in this section \( \xi = \xi_0 \) and \( n = n_p \). The difference between \( n_p \) and \( n \) will become significant at larger radii comparable to \( R_\beta \).

Acceleration of the optically thick outflow is described by equations (36), (37), and (38), from which we derive

\[ \frac{d\Gamma_p}{dr} = \frac{\Gamma_p}{r} \frac{2x}{2x + 3} - \frac{d\Gamma_n}{dr} \frac{3\xi_0}{2x + 3}, \]

where

\[ x = \frac{4}{3} \frac{a T_p^3}{n_pm_pc^2} = x_0 \left( \frac{n}{n_0} \right)^{1/3}, \quad x_0 = \frac{\eta(1 + \xi_0)}{\Gamma_0} - 1, \quad (40) \]

\[ x(r) = x_0 \left( \frac{r}{R_0} \right)^{-2/3} (\Gamma_p \beta_p)^{-1/3}. \]

The first term on the right-hand side of equation (36) describes the acceleration by radiation pressure; the second term describes the deceleration caused by transfer of momentum to neutrons.

As long as the collisional \( n-p \) coupling is strong, \( \Gamma_n \approx \Gamma_p \), equation (32) yields

\[ \frac{d\Gamma_p}{dr} = \frac{\Gamma_p}{r} \frac{2x}{2x + 3(1 + \xi_0)}. \]

Acceleration of coupled \( n \) and \( p \) begins to saturate when \( x \approx 1 + \xi_0 \) at a radius

\[ R_{sb} \approx \eta R_0, \quad \Gamma_p(R_{sb}) \approx \frac{\eta}{2}. \]

At this point \( \rho_e c^2 \approx a T_p^3 \) where \( \rho_e \approx \rho_p + \rho_n \). In the case of \( \xi_0 \approx 1 \), acceleration saturates when \( x \) is still large because the protons are coupled to a large number of neutrons and have a lot of effective inertia. Decoupling from neutrons at \( r \lesssim R_{sb} \) would allow the protons to accelerate further and reach Lorentz factors \( \Gamma_p \sim \eta(1 + \xi_0) \) if the outflow remains optically thick (Fuller at al. 2000).

Description of \( n-p \) decoupling will require one more equation that specifies momentum exchange between neutron and proton components. Neutrons accelerate because
they collide with the accelerating protons. In its rest frame, a neutron experiences $\Gamma_{\text{rel}} n_p \sigma_{np} c$ collisions per second$^2$ where $\sigma_{np} \approx 3 \times 10^{-28} \text{cm}^2$ and

$$\Gamma_{\text{rel}} = \Gamma_p \Gamma_n (1 - \beta_p \beta_n) \approx \frac{1}{2} \left( \frac{\Gamma_p}{\Gamma_n} + \frac{\Gamma_n}{\Gamma_p} \right)$$

(49)

is the Lorentz factor of the neutron component relative to the plasma component. Assuming isotropic scattering in the center-of-momentum frame, the mean momentum gained by the neutron per collision equals $\tilde{p}$ where $\mu = m_p m_n / (m_n + m_p) \approx m_p/2$ is the reduced mass. The momentum gained by a neutron during time $dt$ is

$$d\tilde{p}_n = \frac{1}{2} n_p \Gamma_{\text{rel}}^2 \sigma_{np} \beta_{\text{rel}} m_p c^2 \, dt.$$ 

(50)

The corresponding change of the neutron Lorentz factor in the lab frame is found from the Lorentz transformation of 4-momentum $d\tilde{p}_n = (0, d\tilde{p}_n),$

$$d\Gamma_n = \Gamma_n \beta_n \frac{d\tilde{p}_n}{m_n c}.$$ 

(51)

and one gets

$$\frac{d\Gamma_n}{dr} = \frac{1}{2} n_p \Gamma_{\text{rel}}^2 \beta_{\text{rel}} \sigma_{np}.$$ 

(52)

Equation (52) closes the set of dynamic equations describing the outflow acceleration at the optically thick stage.

The collisional coupling between the proton and neutron components is friction, which inevitably dissipates energy and heats both components. This heating will be included in the plasma thermal balance below (section 3.5).

### 3.2 n-p decoupling and transparency

As long as the n-p coupling is efficient, $\Gamma_p \approx \Gamma_n$, the relative velocity $\beta_{\text{rel}} \ll 1$ may be evaluated using equations (49) and (52),

$$\beta_{\text{rel}} \approx \frac{2\Gamma_p}{n_p \sigma_{np}} \frac{2x + 3(1 + \xi_0)}{r + (9/8)\rho_0 c^2/a T_p^3},$$

(53)

The decoupling of $\Gamma_n$ from $\Gamma_p$ may happen at $r < R_{sb}$ where $a T_p^3 > \rho_0 c^2$, and according to equation (53)

$$\beta_{\text{rel}} \approx \frac{2\Gamma_p}{n_p \sigma_{np}} r, \quad r < R_{sb}.$$ 

(54)

The decoupling condition is expressed by setting $\beta_{\text{rel}} = 1$ in equation (53). The $\beta_{\text{rel}}$ approaches unity at $r \approx R_{sb}$ if the outflow has a sufficiently high $\eta$,

$$\eta > \eta_c \approx \left[ \frac{L \sigma_{np}}{4 \pi R_0 m_p c^3 (1 + \xi_0)} \right]^{1/4} = \frac{4.8 \times 10^2}{(1 + \xi_0)^{1/4}} \left( \frac{L_{\text{52}}}{R_0 \gamma} \right)^{1/4}.$$ 

(55)

Then the decoupling radius is given by

$$R_{np} \approx \left[ \frac{L \sigma_{np}}{8 \pi R_0 n_p m_p c^3 (1 + \xi_0)} \right]^{1/3} R_0,$$ 

(56)

and the neutron Lorentz factor at decoupling is

$$\Gamma_{nf} \approx \frac{R_{np}}{R_0} \approx \frac{\eta_{nf}^{1/3}}{\eta_c^{1/3}}, \quad \eta > \eta_c.$$ 

(57)

No significant decoupling $\Gamma_p > \Gamma_n$ happens for $\eta < \eta_c$. This is so despite the fact that $\eta < \eta_c$ does not exclude $\rho_0 c^2 < \pi R_0^2 < \rho_c c^2$ at $r > R_{sb}$, so there may be enough energy in radiation to accelerate protons to $\Gamma_p > \Gamma_{nf}$. A detailed analysis and numerical models show that if $\beta_{\text{rel}}$ is small at $r \approx R_{sb}$, it remains small at $r > R_{sb}$. Therefore, to a good approximation, $\eta > \eta_c$ may be taken as a true condition for decoupling of $\Gamma_p$ from $\Gamma_n$. Hereafter we focus on $\eta > \eta_c$.

The decoupled neutrons do not affect the remaining e-p-γ outflow until a much larger radius where $\beta$-decay becomes important (see below). Therefore saturation of proton acceleration and transition to transparency may be described as if there were no neutrons. This ‘neutron-free’ outflow has luminosity

$$\dot{L} = \dot{L} - \Gamma_{nf} M c^2 \frac{\xi_0}{1 + \xi_0},$$

(58)

and mass outflow rate

$$\dot{M} = \frac{\dot{M}}{1 + \xi_0}.$$ 

(59)

Equations of section 2 then apply if one replaces $L \rightarrow \dot{L}$, $\dot{M} \rightarrow \dot{M}$ and $\eta \rightarrow \eta = \dot{L}/\dot{M} c^2$.

The outflow is still opaque to radiation at the n-p decoupling radius. This follows from the small ratio of cross sections $\sigma_{np}/\sigma_T \sim 1/20$. If $e^\pm$ cascade initiated by $n^\gamma$ is neglected (see DKK09a and section 3.7 below), transparency comes soon after $R_{np}$. In this case, one can show that an outflow with $\Gamma_p > \Gamma_n (\eta > \eta_c)$ becomes transparent before saturation of $\Gamma_p$.

This may be seen from the following relation,

$$\eta \approx \eta_{rad} = \left[ 1 + \xi_0 \left( 1 - \frac{\Gamma_{nf}}{\Gamma_n} \right) \right]^{3/4} \left( \frac{\sigma_T}{\sigma_{np}} \right)^{1/4} \eta_c,$$

(60)

where (cf. eq. 30)

$$\eta_{rad} \approx \left( \frac{L \sigma_T}{4 \pi R_0 m_p c^2} \right)^{1/4}.$$ 

(61)

$\eta > \eta_c$ implies $\eta > \eta_{rad}$ and hence $\Gamma_p$ does not reach the maximum possible value $\Gamma_{p,\text{max}} = \eta$. The outflow becomes transparent and $\Gamma_p$ saturates at (cf. eq. 58)

$$\Gamma_{p} \approx \eta_{rad} \left( \frac{\eta}{\eta_{rad}} \right)^{1/9}.$$ 

(62)

It is not much larger than $\Gamma_{nf}$. The $e^\pm$ cascade initiated by $n^\gamma$ decay may prolong the opaque stage and increase $\Gamma_{nf}$.

### 3.3 Deceleration by $\beta$-decay

Next, we consider larger radii $r > R_c$ when the outflow is composed of decoupled radiation with luminosity $\dot{L}_r$, decoupled neutrons and plasma. The neutrons are no longer a fluid; they retain a mildly-relativistic velocity dispersion acquired at their last collisions at $r \sim R_{np}$, which implies that $\Gamma_n$ varies by a factor $< 2$. We will neglect this dispersion...
and assume that all neutrons have equal Lorentz factors $\Gamma_n$ after decoupling.

The neutrons continuously $\beta$-decay in the outflow and create a source of protons and electrons moving with respect to the plasma frame with a Lorentz factor $\Gamma_{rel}$. This beam shares momentum with the plasma, heats it and reduces the plasma Lorentz factor $\Gamma_p$.

The outflow luminosity at the transparent stage may be approximated as

$$L = 4\pi r^2 c \left[ \beta_p \Gamma_p^2 (\rho_p c^2 + h_p) + \beta_n \Gamma_n^2 (\rho_n c^2) \right] + L_\gamma = \text{const.} \quad (63)$$

We include here the enthalpy of proton component $h_p = U_p + P_p$ for completeness. As we shall see the plasma is heated to a high temperature at large radii $r \sim R_\beta$, however, $h_p$ remains smaller than $\rho_p c^2$. The enthalpy is related to proton temperature by

$$h_p = \frac{\hat{\gamma}}{\hat{\gamma} - 1} n_p k T_p, \quad (64)$$

where $\hat{\gamma}$ is the adiabatic index of the proton component, which is between 4/3 and 5/3. From equation 12 and using the first law of thermodynamics (eqs. 16 and 17) we derive,

$$\frac{d \Gamma_p}{dr} = \frac{1}{1 + x_p(2 - \hat{\gamma})} \left[ \frac{\xi (\Gamma_n - \Gamma_p)}{R_\beta} + \frac{2(\hat{\gamma} - 1)}{r} \right] \Gamma_p x_p \rho_p c^2 \right] - \frac{\Gamma_p \hat{\gamma} \sigma_p L \gamma}{\rho_p c^2} \frac{d \hat{\gamma}}{dr} + \frac{\sigma_p L \gamma}{16\pi^2 \rho_p c^2 \Gamma_p^2} \left( 1 - \frac{1}{\Gamma_p^2 \Gamma_{rel}^2} \right), \quad (65)$$

where $x_p = h_p / \rho_p c^2$. The last term describes the radiative acceleration (see section 2.2.1).

3.4 Summary of the dynamical model

Our dynamical model of the neutron-loaded outflow may be summarized as follows. It is described by different equations before and after transparency, and the equations match at the transparency radius. Before transparency, $r < R_\tau$, we neglect the decay of neutrons. The evolution of $\Gamma_p$ and $\Gamma_n$ is found from equations (15) and (16).

After transparency, $r > R_\tau$, we neglect the $n$-$p$ collisions and assume $\Gamma_n = \text{const.}$ We take into account the $\beta$-decay which becomes increasingly important at larger radii. The evolution of $\Gamma_p$ is described by equation (66). It includes the heating term $d \Gamma_p / dr$ which may be non-negligible at $r \sim R_\beta$. This term couples the dynamics with the proton thermal balance which is discussed in section 3.5 below.

The two descriptions match at the beginning of the coasting phase $\frac{d \Gamma_p}{dr} = 0$ where neither $n$-$p$ collisions nor $\beta$-decay affects the outflow.

3.5 Thermal balance

Thermal evolution of neutrons and protons obeys the first law of thermodynamics (eq. 18). Using equations 17 and 18 we derive

$$\frac{(1 + \xi) d}{dr} \left[ \frac{T}{(\hat{\gamma} - 1)(1 + \xi)} \right] = T_\gamma \frac{d n}{dr} + \frac{1}{kn_p} \frac{d Q_p}{d \Gamma_p} \left. \frac{1}{\Gamma_p \beta_p} \right), \quad (66)$$

Here $d Q_i / dt'$ ($i = e, p$) are the heating rates of electrons and protons.

The thermal balance of protons is significantly changed compared to the neutron-free case because of two effects: (i) frictional heating due to $n$-$p$ collisions and (ii) heating due to $\beta$-decay.

The rate of $n$-$p$ collisions per unit volume in the plasma frame is

$$n_{ep} = n_n n_p \sigma_n p c,$$

where $n_n = \Gamma_{rel} n_n$ is neutron density measured in the plasma frame. The mean energy dissipated per collision is $(1/2)(\Gamma_{rel} - 1)m_p c^2$ assuming that the relative bulk velocity is isotropized in collisions. This gives the frictional heating rate,

$$\frac{d Q_p}{dt'} = \frac{1}{2} \Gamma_{rel} (\Gamma_{rel} - 1) m_p c^2 \frac{\sigma_n}{\Gamma_{rel} \Gamma_p}, \quad (67)$$

Heating also results from the gradual $\beta$-decay of the neutron component. The decay rate per unit volume is Lorentz invariant and in the plasma frame (where the decay time is $\Gamma_{rel} \tau_\beta$) may be written as

$$\frac{d n_n}{dt'} = n_n \frac{n_n}{\Gamma_{rel} \tau_\beta} = n_n \frac{n_n}{\Gamma_{rel} \tau_\beta} , \quad (68)$$

The decay products form an $e$-$p$ beam with velocity $v_{rel}$ in the plasma frame, which immediately dissipates its relative kinetic energy into heat. We will assume that this heat goes entirely to the proton component. The dissipated energy per decayed neutron is $(\Gamma_{rel} - 1)m_p c^2$, which gives the heating rate,

$$\frac{d Q_\beta}{dt'} = (\Gamma_{rel} - 1) m_p c^2 \frac{2 n_n}{\Gamma_{rel} \tau_\beta}, \quad (69)$$

Protons also exchange energy with electrons via Coulomb collisions with rate $d Q_{ep} / dt'$ (eq. 22). The net thermal balance of the proton component is then given by

$$\frac{d q_p}{dt'} = - \frac{d Q_{ep}}{dt'} + \Gamma_{rel} n_n (\Gamma_{rel} - 1) m_p c^2 \left( \frac{1}{2} n_n \sigma_n p c + \frac{1}{\Gamma_{rel} \tau_\beta} \right) \quad (70)$$

The $\beta$-decay weakly affects the plasma thermal balance as long as $(\Gamma_{rel} \tau_\beta)^{-1} \ll n_n \sigma_n p c$. The decay becomes important at large radii, after the outflow becomes transparent.

The thermal-balance equation for electrons is similar to the neutron-free case (section 2),

$$\frac{d Q_e}{dt'} = \frac{3}{2} n_p k (T_e - T_\gamma) + \frac{d Q_{ep}}{d \Gamma_p} \frac{d \Gamma_p}{dr} \quad (71)$$

3.6 Destruction of neutrons by inelastic collisions

We assume in this paper that neutron richness $\xi$ changes only as a result of $\beta$-decay and neglect other channels of neutron conversion to protons. In fact, near the decoupling radius $R_{np}$ the collisions between neutrons and protons become sufficiently energetic to produce pions (DKK99a). Thus, some collisions are inelastic, which may destroy the neutron component.

Inelastic $n$-$p$ collisions may convert neutron to proton $n + p \rightarrow p + p + \pi^+$ as well as proton to neutron $n + p \rightarrow n + n + \pi^+$. These reactions have equal cross section which
has been measured down to the 140 MeV threshold (e.g. Daum et al. 2002). The reactions have equal rates and do not change neutron richness \( \xi \).

Inelastic \( n-n \) collisions may destroy the neutron component via reactions \( n+n \rightarrow d+\pi^- \) and \( n+n \rightarrow n+p+\pi^- \). These reactions have same cross sections as reactions \( p+p \rightarrow d+\pi^+ \) and \( p+p \rightarrow n+p+\pi^- \), respectively, which have been studied in experiments (Shimizu et al. 1982). The cross section of \( d\pi^- \) channel does not exceed 0.1 of the elastic cross section and is less important. The main reaction of neutron destruction is \( n+n \rightarrow n+n+p+\pi^- \). Its cross section becomes comparable to the elastic cross section when neutrons collide with relative energy \( E_{nn} > 700 \) MeV and quickly decreases at smaller \( E_{nn} \).

The rate of elastic \( n-n \) collisions is \( \xi \) times higher than the rate of \( n-p \) collisions, which equals the expansion rate at decoupling. Therefore, the timescale of \( n-n \) collisions at \( r \sim R_{np} \) is \( \xi^{-1} \) times shorter than the expansion time \( \tau_{exp} \), and at large \( \xi \) neutrons may be treated as Maxwellian gas. This gas is heated by \( n-p \) collisions which dissipate energy \( (\Gamma_{rel} - 1)mpc^2/2 \approx m_p\xi\tau_{rel}/4 \) per collision, and the heating rate of neutrons is equal to that of protons (eq. \( 67 \)).

The neutron temperature \( kT_n \approx (2/3)E_{nn} \) may be estimated by integrating the heating rate per neutron \( dE_{n}/dr \approx (mpc^2/2\gamma)\Gamma_{rel} \approx (mpc^2/2\gamma)(r/R_{np})^3 \) from \( r = 0 \) to \( r = R_{np} \). This gives a modest value \( kT_n \sim 100 \) MeV. A more accurate calculation, which includes adiabatic cooling of neutrons, may give even lower \( T_n \). We conclude that the mean relative energy of \( n-n \) collisions, \( E_{nn} = 3kT_n \), is well below 700 MeV, so only a tail of the quasi-Maxwellian distribution will contribute to destruction of neutrons.

Therefore most of \( n-n \) collisions at \( r \sim R_{np} \) are expected to be elastic, and a small fraction \( \xi \) of these collisions will convert neutrons to protons. The lifetime of a neutron at \( r \sim R_{np} \) is \( \tau_{n \rightarrow p} \approx \xi^{-1}\tau_{exp} \). An initially high neutron-to-proton ratio \( \xi \) will be reduced by conversion at decoupling if \( \xi^{-1}\tau_{n \rightarrow p} < \tau_{exp} \), i.e., if the initial \( \xi \) exceeds \( \xi^{-1}/2 \).

The exact \( \xi \) and the corresponding upper bound \( \xi_{max} \) may be found with detailed kinetic calculations of collisions. Such calculations are not attempted here, and we consider an optimistic range of \( \xi < 10 \) which might remain intact after decoupling. The case \( \xi_0 = 4 \) is chosen as a main example model.

### 3.7 Pair cascade initiated by \( \pi^0 \)

Collisions between baryons near decoupling can produce neutral pions \( \pi^0 \) via reactions \( p+p \rightarrow n+p+\pi^0 \), \( n+p \rightarrow d+\pi^0 \) and \( n+n \rightarrow n+n+\pi^0 \). Decay of \( \pi^0 \) initiates a pair cascade in the outflow (DKK99a).

The inelastic \( p-p \) collisions are relatively rare at \( \xi > 1 \), especially when protons are cooled by Coulomb collisions with electrons. Reaction \( n+p \rightarrow d+\pi^0 \) has a maximum cross section \( \sim 0.1 \) of the elastic cross section, and yields only \( \sim 0.1 n^0 \) per proton at decoupling.

The cross section of reaction \( n+n \rightarrow n+n+n^0 \) is \( \sim 1/4 \) times smaller than cross section of \( n+n \rightarrow n+n+\pi^- \). These reactions have the same cross sections as the experimentally studied reactions \( p+p \rightarrow p+p+p+\pi^0 \) and \( p+p \rightarrow p+p+n+\pi^+ \), respectively.

Hence, the timescale of \( \pi^0 \) production by a neutron is 4 times longer than \( \tau_{n \rightarrow p} \), which is in turn longer than \( \tau_{exp} \) (section \( 3.2 \)). This implies that up to \( \sim 0.2 \pi^0 \) is produced per neutron at decoupling, which translates to \( \sim 1 \pi^0 \) per electron.

A typical produced \( \pi^0 \) has energy of a few hundred MeV. It immediately decays into two photons, and the high-energy photons are absorbed by the thermal radiation (DKK99a). As a result, a relativistic \( e^\pm \) pair is created which upscatters thermal radiation, and the upscattered photons create a new generation of pairs. A maximum possible pair yield of the initiated cascade, \( Y \approx 0.1 \), would be achieved if the cascade were ‘saturated’, i.e., if the upscattered \( \gamma \)-rays always got absorbed by softer photons (Svensson 1987). Thus maximum 10 per cent of the \( \pi^0 \) energy could possibly be converted into pair rest mass, which corresponds to \( \sim 30 \) \( e^\pm \) per injected \( \pi^0 \). The cascade is not, however, saturated: after a few generations, the upscattered \( \gamma \)-rays are able to escape the thermal radiation (DKK99a). Therefore, a more realistic pair yield is a few per cent. It corresponds to roughly \( 10 e^\pm \) injected per proton in the outflow. Most of these pairs annihilate soon after injection at \( r \sim R_{np} \).

The inelastic \( n-n \) collisions also happen at \( r > R_{np} \) with increasing timescale \( \propto n^{-1} \propto r^2 \), and the pair creation remains significant until \( r \sim 4R_{np} \). The created pairs increase the opacity of the outflow and reduce the timescale of Coulomb interactions between protons and electrons. In numerical models presented below this effect is neglected.

### 3.8 Numerical models

Figures 4 and 5 show two numerical models of neutron-loaded outflows with initial neutron-to-proton ratio \( \xi_0 = 4 \). The first model has \( \eta = 300 \) and the second model — \( \eta = 3 \times 10^4 \). In both cases, \( \eta > \eta_n \), and the decoupling \( \Gamma_p > \Gamma_n \) takes place. It is especially significant in the high-\( \eta \) model.

The evolution of Lorentz factors \( \Gamma_n \) and \( \Gamma_p \) has three stages: (i) Acceleration of both components which ends with different saturated values of \( \Gamma_n \) and \( \Gamma_p \). Note that neutrons decouple earlier in the \( \eta = 3 \times 10^4 \) model and therefore have a smaller \( \Gamma_n \) compared to the \( \eta = 300 \) model. (ii) Coasting stage \( \Gamma_n = const \) and \( \Gamma_p = const \). (iii) Deceleration of the proton component by decaying neutrons. At about the same time the outflow begins to experience deceleration by an external medium. We do not study the external deceleration in the present paper and stop our calculations at \( r = 3 \times 10^{16} \) cm.

The evolution of proton temperature \( T_p \) has an initial decline followed by two pronounced peaks. The first peak happens at the end of neutron acceleration. It is caused by the strong friction between the \( n \) and \( p \) components near the decoupling radius \( R_{np} \). The second peak accompanies the deceleration of protons at \( r \gtrsim R_d \) — it is caused by absorption of decayed neutrons by the proton outflow. The overall thermal evolution is markedly different from the neutron-free case and we describe it below in more detail.

At the beginning of outflow expansion, the thermal evolution is similar to the neutron-free case: all components are thermally coupled at a common temperature. The temperature is controlled by radiation (which strongly dominates heat capacity of the outflow) and decreases according to adiabatic law with \( \gamma = 4/3 \).

The behaviour changes when the Lorentz factor \( \Gamma_p \approx \Gamma_n \) reaches a value about 50. Then the proton temperature
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Figure 4. Evolution of a neutron-loaded outflow with \( \xi_0 = 4 \), \( \eta = 300 \), \( L = 10^{52} \) erg s\(^{-1} \), and \( R_0 = 10^7 \) cm. **Upper panel**: Lorentz factors of proton and neutron components (solid and dotted curves, respectively). **Bottom panel**: Temperatures of protons (solid curve), electrons (dashed curve) and radiation (dash-dotted curve). Vertical dashed lines indicate characteristic radii \( R_{sb} \), \( R_{\tau} \), \( R_{ep} \), \( R_{e\gamma} \) and \( R_{\beta} \) (see the text). The decoupling radius \( R_{np} \) is close to \( R_{sb} \).

Figure 5. Same as Fig. 4 but for \( \eta = 3 \times 10^3 \).

Figure 6. Evolution of outflows with the same parameters as in Fig. 4 but with different \( \xi_0 = 1 \) (dashed curves) and \( \xi_0 = 10 \) (solid curves). The model with \( \xi_0 = 4 \) from Fig. 4 is shown by the dash-dotted curves. **Upper panel**: Lorentz factors of proton and neutron components. **Lower panel**: proton temperature.

decouples from \( T_e \) and \( T_r \) and begins to grow. This is the result of the frictional heating and the quickly increasing relative velocity between \( n \) and \( p \) components. Only a fraction of the frictional heat is kept by baryons and a quasi-steady energy circulation is maintained in the accelerating outflow: radiation \( \rightarrow \) relative kinetic energy of the \( n \) and \( p \) components \( \rightarrow \) baryonic heat \( \rightarrow \) electrons \( \rightarrow \) radiation. The thermal balance of protons in this circulation controls \( T_p \), and \( T_p \) quickly grows as the outflow approaches the \( n-p \) decoupling radius.

After the decoupling radius, the \( n-p \) collisions become rare and the frictional heating extinguishes. On the other hand, the protons get thermally decoupled from electrons because of a long timescale of Coulomb energy exchange. The subsequent decrease of \( T_p \) is mainly caused by adiabatic cooling.

Adiabatic cooling continues until the heating by \( \beta \)-decay interferes the thermal evolution. The timescale of adiabatic cooling in the proton frame is \( r/c\Gamma_p \) which is proportional to \( r \) during the coasting stage. By contrast, the timescale of \( \beta \)-decay is constant. Therefore the heating of protons by decayed neutrons wins the adiabatic cooling at some point, and \( T_p \) begins to grow as \( T_p \propto r \). This growth continues until the outflow approaches \( R_\beta \) where most of the neutrons decay. The exponential extinction of neutrons implies that heating extinguishes at \( r \sim R_\beta \). Then the outflow again cools adiabatically.

The two main effects of neutrons on the outflow — heating and deceleration of the proton component — are especially strong at high \( \xi_0 \) and high \( \eta \). This is illustrated in Figures 4 and 5 which show models with \( \xi_0 = 1, 4, 10 \) and \( \eta = 300, 3 \times 10^3 \). At high \( \xi_0 \) and/or \( \eta \) the proton density...
of the outflow is low and the n-p decoupling occurs early, leading to a large relative Lorentz factor $\Gamma_{rel}$ between the n and p components. The large kinetic energy of the relative motion is then available for dissipation.

We also note that outflows with very high $\eta$ and $\xi_0$ quickly become transparent and most of their energy is carried away by thermal radiation. This is illustrated by Figure 8 which shows three components of the outflow luminosity: radiation, protons, and neutrons. The radiation luminosity is given by equation (55), whose typical value is several $10^{42}$ erg s$^{-1}$. The sum of the three luminosities $L_{\gamma} + L_{p} + L_{n}$ is constant and equal to $L = 10^{42}$ erg s$^{-1}$. The upper panel corresponds to the model shown in Fig. 4 and the lower panel — to the model shown in Fig. 8.

The effects of neutrons are pronounced in outflows with very high $\eta > \eta^*$, which corresponds to the model shown in Fig. 4, and the lower panel — to the model shown in Fig. 8.

4 DISCUSSION

In this paper we developed the theory of relativistic neutron-loaded outflows. The presence a neutron component significantly affects the early dynamics of GRB explosions. In particular, the plasma temperature is increased by many orders of magnitude, and this heating can compete with other heating mechanisms such as internal shocks or dissipation of magnetic fields in the outflow.

The effects of neutrons are pronounced in outflows with $\eta > \eta_c$. Given by equation (55), whose typical value is several hundred. Then neutrons and protons develop a substantial relative Lorentz factor which leads to strong heating and momentum exchange when neutrons decay. We also note that neutron-rich outflows with very high $\eta \gg \eta_c$ lose most
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Figure 9. Evolution of outflows with the same parameters as in Fig. 4 but with different \( R_0 = 10^8 \) cm (dashed curves) and \( R_0 = 10^9 \) cm (solid curves). The model with \( R_0 = 10^7 \) cm from Fig. 4 is shown by the dash-dotted curves. Upper panel: Lorentz factors of proton and neutron components. Lower panel: proton temperature.

of their energy at the photosphere; their thermal radiation carries most of the energy at the moment of transparency. Therefore, the neutron effects may be especially strong in GRBs with a significant thermal component in the radiation spectrum (a number of such GRBs have been identified recently, see e.g. Ghirlanda 2003; Ryde 2004).

The calculations in this paper focused on the simplest model of a uniform hydrodynamic outflow with weak magnetic fields. We did not consider, for instance, internal shocks (e.g. Rees & Mészáros 1994) and possible pair creation by nonthermal \( \gamma \)-rays generated in the outflow. Pair creation may extend the optically thick stage of expansion, and the trapped radiation may convert its energy more efficiently into bulk kinetic energy of the plasma. A large-scale magnetic field may gradually collimate the outflow so that the conical geometry of expansion does not apply (Vlahakis, Peng & Königl 2003).

The \( \beta \)-decay is likely to affect the development of internal shocks in the outflow. The shocks are caused by a non-uniform profile of the Lorentz factor, and the drag effect of decayed neutrons tends to smoothen this profile. The fastest portions of the outflow are more effectively decelerated and the initial contrast of Lorentz factors may be substantially reduced already at \( r \sim 10^{14} \) cm. This effect constrains the dissipation efficiency of the shocks, which is sensitive to the contrast of Lorentz factors (see Fig. 3 in Beloborodov 2000). In addition, the high temperature of the outflow heated by \( \beta \)-decay may prevent development of the shocks. We defer a detailed study of these effects to a future work.

The impact of neutrons on the prompt burst and its afterglow provides a unique opportunity to link the observed emission with physical conditions in the central engine of the explosion. The very presence of neutrons is a signature of an extremely hot and dense engine. Observable effects of neutrons may shed light on the mechanism of GRB trigger.

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REFERENCES

Bahcall, J. N., & Mészáros, P., 2000, PhRv 85, L1362
Beloborodov, A. M., 2000, ApJ, 539, L25
Beloborodov, A. M., 2002, ApJ, 565, 808
Beloborodov, A. M., 2003a, ApJ, 585, L19
Beloborodov, A. M., 2003b, ApJ, 588, 931 (B03b)
Daum, M., et al. 2002, Eur. Phys. J., C 23, 43
Derishev, V. K., Kocharovsky V. V., Kocharovsky Vi. V., & 1999b, A&A, 345, L51
Fuller, G. M., Pruet, J., & Abazajian, K., 2000, Phys. Rev. Lett., 85, 2673
Ghirlanda G., Celotti, A., & Ghisellini, G., 2003, A&A, 406, 879
Lemoine, M., 2002, A&A Lett., 390, 31
MacFadyen, A. I., & Woosley, S. E. 1999, ApJ, 524, 262
Mészáros, P., & Rees, M. J., 2000a, ApJ, 541, L5
Mészáros, P., & Rees, M. J., 2000b, ApJ, 530, 292
Misner, C. W., Thorne, K. S., & Wheeler, J. A., 1973, Gravitation (San Francisco: Freeman)
Piran T., 2004, Rev. Mod. Phys., 76, 1143
Puet, J., Guiles S., & Fuller, G. M., 2002, ApJ, 580, 368
Rees, M. J., & Mészáros, P., 1994, ApJ, 430, 93
Ryde, F., 2004, ApJ, 614, 827
Shimizu, F., Kubota, Y., Koiso, H., Sai, F., Sakamoto, S., & Yamamoto, S. S. 1982, Nucl. Phys., A386, 571
Stepnøy, S., 1983, MNRAS, 202, 467
Svensson, R. 1987, MNRAS, 227, 403
Vlahakis, N., Peng, F. & Königl, A. 2003, ApJ, 594, L23