SIGNS OF PLANETARY MICROLENSING SIGNALS

CHEONGHO HAN
Department of Physics, Institute for Basic Science Research, Chungbuk National University, Chongju 361-763, Korea; cheongho@astroph.chungbuk.ac.kr

AND

KYONGAE CHANG
Department of Physics, Chonju University, Chonju 360-764, Korea; kchang@chonju.ac.kr

Received 2002 November 10; accepted 2003 July 21

ABSTRACT

An extrasolar planet can be detected via microlensing from the perturbation it makes in the smooth lensing light curve of the primary. In addition to the conventional photometric microlensing, astrometric observation of the center-of-light motion of the source star image provides a new channel of detecting and characterizing extrasolar planets. It was known that the planet-induced astrometric signals tend to be positive while the photometric signals can be either positive or negative. In this paper, we analytically show the reason for these tendencies of microlensing planetary signals.

Subject headings: gravitational lensing — planets and satellites: general

1. INTRODUCTION

A microlensing event occurs when a lensing object approaches very close to the line of sight toward the background source star. Because of lensing, the lensed star appears to be split into two images. The locations and magnifications of the individual images are

\[ \theta_\pm = \frac{1}{2} \left( \zeta \pm \sqrt{\zeta^2 + 4 \frac{\zeta}{\z_\text{E}}} \right) \theta_\text{E}, \]  

and

\[ A_\pm = \frac{\z_\text{E}^2 + 2}{2\z\sqrt{\z^2 + 4}} \pm \frac{1}{2}, \]  

where \( \zeta \) is the projected lens-source separation vector normalized by the Einstein ring radius \( \theta_\text{E} \). The Einstein ring represents the effective lensing region around the lens within which the source star flux is magnified greater than \( 3/\sqrt{5} \). For a typical Galactic microlensing event, the Einstein ring radius is

\[ \theta_\text{E} \sim 0.72 \text{ mas} \left( \frac{m}{0.5 \text{ M}_\odot} \right)^{1/2} \left( \frac{D_{\odot}}{8 \text{ kpc}} \right)^{-1/2} \left( \frac{D_\text{s}}{D_{\odot}} - 1 \right)^{1/2}, \]  

where \( m \) is the lens mass and \( D_{\odot} \) and \( D_\text{s} \) are the distances to the lens and source star, respectively. For a rectilinear lens-source relative motion, the separation vector is related to the lensing parameters by

\[ \zeta = \left( \frac{t - t_0}{t_\text{E}} \right) \hat{\xi} + \beta \hat{\eta}, \]  

where \( t_\text{E} \) is the time required for the source to transit \( \theta_\text{E} \) (Einstein timescale), \( \beta \) is the closest lens-source separation normalized by \( \theta_\text{E} \) (impact parameter), \( t_0 \) is the time at that moment, and the unit vectors \( \hat{\xi} \) and \( \hat{\eta} \) are parallel with and perpendicular to the direction of the relative lens-source motion, respectively. The image with the higher magnification \( A_\pm \) (major image) is located outside of the Einstein ring, while the other image with the lower magnification \( A_- \) (minor image) is located inside of the Einstein ring. The separation between the two images, \( |\theta_+ - \theta_-| = (\zeta^2 + 4)^{1/2} \theta_\text{E}, \) for a typical Galactic event is very small and, thus, they cannot be resolved. However, a lensing event can be identified from its characteristic smooth and symmetric light curve (Paczyński 1986), which is represented by

\[ A = A_+ + A_- = \frac{\z_\text{E}^2 + 2}{\z\sqrt{\z^2 + 4}}. \]  

For a more detailed description about microlensing, see Paczyński (1996).

If a lensing event is caused by a star having a planet and the planet happens to locate close to the path of one of the two images produced by the primary star, the planet perturbs the nearby image and the event can exhibit noticeable deviations from the light curve of a single lens event (Mao & Paczyński 1991). It is empirically known that if the planet perturbs the major image, the resulting deviation in the lensing light curve becomes positive,\(^1\) while the deviation becomes negative if the planet perturbs the minor image (Gaudi & Gould 1997; Wambsganss 1997; Bozza 1999).

For a planet with a mass ratio to the primary star of \( q \), the planetary signal endures for a short period of time of \( \sim \sqrt{q} t_\text{E} \), corresponding to several days for a Jupiter-mass planet \( [q \sim O(10^{-3})] \) and a few hours for an Earth-mass planet \( [q \sim O(10^{-5})] \). However, the strength of the signal depends weakly on \( q \). Then, planets can be detected if lensing events are monitored with a high enough frequency. This frequency can be achieved from the

\(^1\) For typical planet-induced perturbations, the perturbed part of the light curve is composed of one major peak and surrounding deviations with signs opposite to that of the peak deviation. Compared to the peak deviation, the deviations at the wings of the peak deviation are very small. Throughout the paper, therefore, the sign of the planetary deviation implies that of the peak.
MISSION ON SPACE-BASED PLATFORMS, E.G., THE 
EVALUATIONS OF LENSING EVENTS BY USING NEXT GENERATION HIGH-
EXTRASOLAR PLANETS VIA MICROLENSING, SAFIZADEH, DALAL, &
2002, PRIVATE COMMUNICATION).

PLANET (ALBROW ET AL. 1998), AND MICROFUN (D. DEPOY 2002, PRIVATE COMMUNICATION).

AS AN ADDITIONAL CHANNEL TO DETECT AND CHARACTERIZE 
EXTRASOLAR Planets VIA MICROLENSING, SAFIZADEH, DALAL, & 
GIEST (1999) PROPOSED ASTROMETRIC FOLLOW-UP OBSERVATIONS 
OF LENSING EVENTS BY USING NEXT GENERATION HIGH-
PRECISION INTERFEROMETERS, SUCH AS THOSE TO BE MOUNTED ON 
SPACE-BASED PLATFORMS, E.G., THE SPACE INTERFEROMETRY 
MISSION (SIM), AND THOSE TO BE MOUNTED ON VERY LARGE 
GROUND-BASED TELESCOPES, E.G., KECK AND VLT. WHEN AN 
EVENT IS OBSERVED BY USING THESE INSTRUMENTS, ALTHOUGH RESOLVING THE INDIVIDUAL IMAGES IS STILL DIFFICULT, IT IS POSSIBLE TO MEASURE THE CENTER-OF-LIGHT MOTION OF THE LENSED SOURCE STAR IMAGE CAUSED BY THE CHANGE OF THE SEPARATION BETWEEN THE LENS AND SOURCE AND THE RESULTING VARIATION OF THE BRIGHTNESS RATIO BETWEEN THE TWO IMAGES. FOR A SINGLE LENS EVENT, THE SHIFT OF THE IMAGE CENTROID WITH RESPECT TO THE UNLENSED POSITION OF THE SOURCE STAR (CENTROID SHIFT) IS REPRESENTED BY

\[ \delta = \frac{A_t}{A} \xi + A_t \frac{\theta}{A - \xi \theta} - \frac{\xi \theta E}{\xi^2 + \theta E}. \]

The trajectory of the centroid motion (astrometric curve) traces out an ellipse during the event (Walker 1995; Jeong, Han, & Park 1999; Dominik & Sahu 2000). Safizadeh, Dalal, & Giest (1999) showed that planets can be identified from the perturbations in astrometric curves, which are analogous to photometric perturbations in lensing light curves. They pointed out that because of the strong correlation between the photometric and astrometric planetary signals, adding astrometric information to the photometric lensing light curve will greatly help in determining the mass ratio and the projected separation of the planet. In addition, since astrometric lensing observations enable one to determine the absolute mass of the lens system by measuring both the lens proper motion and parallax (Miyamoto & Yoshii 1995; Høg, Novikov, & Polnarev 1995; Walker 1995; Paczyński 1998; Boden, Shao, & van Buren 1998; Gould & Salim 1999), one can determine the absolute mass of the planet. Han & Lee (2002) further investigated the patterns of astrometric deviations caused by planets with various separations and mass ratios. From this investigation, they found an interesting tendency of astrometric planetary signals, where while photometric deviations can become either positive or negative depending on which of the two images produced by the primary is perturbed by the planet, astrometric deviations are positive in all tested events regardless of which image is perturbed. The usefulness of this tendency was soon noticed by Han (2002), who pointed out that the problematic photometric degeneracy between binary source and planetary perturbations (Gaudi 1998) can be unambiguously resolved with the additional astrometric information because the astro-

metric perturbations induced by a faint binary source companion are always negative, which is opposite to the sign of the planet-induced perturbations. However, none of the previous works explained the reason for the known tendencies of photometric and astrometric planetary signals. In this paper, we analytically show why planetary-induced astrometric signals are always positive while photometric signals can be either positive or negative.

The layout of the paper is as follows. In § 2, we describe the basics of planetary microlensing. In § 3, we derive the relation between photometric and astrometric planetary signals and explain the reasons for the empirically known properties of the signs of planetary signals. We summarize and conclude in § 4.

2. BASICS OF PLANETARY MICROLENSING

The planetary lensing behavior is described by the formalism of binary lensing with a very low mass-ratio companion. If a source star located at \( \zeta = \xi + iy \) in complex plane is lensed by two point-mass lenses with the individual locations of \( \zeta_{L,i} \) and \( \zeta_{L,j} \) and the mass fractions of \( m_i \) and \( m_j \), respectively, the locations of the resulting images \( \zeta = x + iy \) are obtained by solving the lens equation, which is represented by

\[ \zeta = x + \sum_i \frac{m_i}{\zeta_{L,i} - z}, \]

where \( z \) denotes the complex conjugate of \( z \), and all lengths are normalized by the combined Einstein ring radius. Since the lens equation describes a mapping from the lens plane to the source plane, finding image locations \( (x, y) \) for a given source position \( (\xi, \eta) \) requires inverting the lens equation. Although the lens equation for a binary lens system cannot be algebraically inverted because of its nonlinearity, it can be expressed as a fifth-order polynomial in \( z \), and thus the image positions can be obtained by numerically solving the polynomial equation (Witt 1990). Since the lensing process conserves the source star surface brightness, the magnification of each image is equal to the area ratio between the image and the un lensed source, and mathematically it is given by the Jacobian of the mapping equation evaluated at the image position,

\[ A_t = \left| 1 - \frac{\partial^2 \xi}{\partial z \partial \xi} \right|^{-1}. \]

Then, the total magnification is the sum of the magnifications of the individual images, i.e., \( A = \sum A_t \). Since the position of the image centroid is equal to the magnification-weighted mean position of the individual images, the centroid shift is given by

\[ \delta = \frac{\sum A_t z_i}{A} - \zeta, \]

where \( z_i \) and \( \zeta \) are the vector notations of each image position and the location of the un lensed source position, respectively.

Because of the very small mass ratio of the planet to the primary, the planetary lensing behavior is well described by that of a single lens for most of the event duration. However, noticeable deviations can occur when the source passes the region close to caustics. The caustics are the main new
features of binary lensing and refer to the set of source positions at which the magnification of a point source becomes infinity. For a planetary case, the caustics are located along or very close to the primary-planet axis (x-axis), and its location on the x-axis is approximated by

\[ x_c \sim x_p - \frac{1}{x_p} \],

where \( x_p \) is the position of the planet (Griest & Safizadeh 1998). Caustics are located within the Einstein ring when the planetary separation is in the range of \( 0.6 \leq x_p \leq 1.6 \). Since the size of the caustic, which is directly proportional to the planet detection efficiency, is maximized when the planet is in this range, this range is referred to as the “lensing zone” (Gould & Loeb 1992). The location of the caustic in the source plane corresponds to the region near one of the two images created by the primary in the lens (or image) plane. From the lens plane point of view, therefore, noticeable deviation occurs when the planet is located close to one of the two images produced by the primary.

Major image perturbation refers to the case in which the deviation is caused by a planet located near the major image. Since the major image is located outside of the Einstein ring, major image perturbations are caused by planets with separations greater than \( \theta_E \), i.e., \( |x_p| > 1 \) (wide planet). In this case, one finds from equation (10) that \( \text{sign}(x_c) = \text{sign}(x_p) \) and \( |x_c| < |x_p| \), implying that the caustic is located on the same side of the planet with respect to the center of mass between the primary and planet.

Minor image perturbation, on the other hand, refers to the case in which the planet perturbs the minor image. The minor image is inside the Einstein ring, and thus minor image perturbations are caused by planets with separations less than \( \theta_E \), i.e., \( |x_p| < 1 \) (close planet). Unlike the single caustic formed by the wide planet, the close planet causes formation of two caustics, which are located symmetrically with respect to the primary-planet axis, i.e., the x-axis. For planets in the lensing zone, however, the caustics are located very close to the x-axis, and thus their positions can also be approximated by eq. (10). Since \( \text{sign}(x_c) \neq \text{sign}(x_p) \), the caustics are located on the opposite side of the planet with respect to the center of mass.

3. SIGNS OF PLANETARYPERTURBATIONS

In this section, we analytically derive the relation between photometric and astrometric microlensing signals of planets and explain the reason for the empirically known properties of the signs of the signals.

We begin with the case in which the major image is perturbed by a planet. Let us define \( \epsilon \) as the fractional photometric deviation, i.e.,

\[ \epsilon = \frac{A_p - A}{A} \],

where \( A_p \) and \( A = A_+ + A_- \) represent the magnifications with and without the perturbation, respectively. Since the minor image is not perturbed by the planet (see Appendix), the perturbed magnification is \( A_p = A_{p+} + A_- \), where \( A_{p+} \) represents the magnification of the perturbed major image. Then, the magnification excess can be written as

\[ \epsilon = \frac{A_{p+} - A_+}{A} = \frac{A_{p+} - (A - A_-)}{A} \].

By inverting equation (12), the perturbed major image magnification is expressed in terms of \( \epsilon \) by

\[ A_p = (1 + \epsilon)A - A_- \].

The location of the perturbed image centroid is represented by

\[ \varphi_p = \frac{A_{p+} \theta_+ + A_- \theta_-}{A_{p+} + A_-} \]

where \( \theta_+ \) and \( \theta_- \) are the positions of the unperturbed major and minor images, respectively. Here we use an approximation that the change in the position of the major image due to the planetary perturbation is small. By plugging equation (13) into equation (14), the centroid position is expressed in terms of \( \epsilon \) by

\[ \varphi_p = \frac{(1 + \epsilon)A \theta_+ + A_- (\theta_+ - \theta_-)}{1 + \epsilon A} \].

By definition, the astrometric planetary signal is the difference between the centroid positions with and without the planetary perturbation, i.e.,

\[ \Delta \varphi = \varphi_p - \varphi \],

where \( \varphi = (A_+ \theta_+ + A_- \theta_-)/A \). From equation (15) and (16), one finds that the relation between the astrometric and photometric perturbations in the case of the major image perturbation is

\[ \Delta \varphi = \frac{\epsilon A_+}{1 + \epsilon A} (\theta_+ - \theta_-) \]

One finds a similar relation in the case of the minor image perturbation:

\[ \Delta \varphi = -\frac{\epsilon A_+}{1 + \epsilon A} (\theta_+ - \theta_-) \]

If we define the signs of astrometric shifts such that the direction toward the major image from the lens position is positive, the sign of the term \( \theta_+ - \theta_- \) is also positive. Magnification is always positive by definition, and thus the signs of both terms \( A_+ / A \) and \( A_- / A \) are also positive. Then, one finds the relation between the signs of photometric and astrometric perturbations in the case of the major image

\[ \Delta \varphi = \frac{2q}{x - x_p} \frac{1}{\sqrt{x^2 - 1/x^2}} \times (x - x_p)^{-2} \]

(see Appendix). During the time of maximum perturbation when \( x_p \to x \), therefore, the photometric perturbation dominates over the astrometric perturbation, i.e., \( \Delta \varphi / \Delta x \propto \Delta x / x \). This implies that the astrometric perturbation to the centroid shift derives largely from the photometric perturbation rather than the astrometric shifts in the positions of the individual images.
perturbation:
\[
\begin{align*}
\Delta \varphi_p > 0 & \quad \text{if } \epsilon > 0, \\
\Delta \varphi_p < 0 & \quad \text{if } -1 < \epsilon < 0.
\end{align*}
\]

In the case of the minor image perturbation, the relation is
\[
\begin{align*}
\Delta \varphi_p < 0 & \quad \text{if } \epsilon > 0, \\
\Delta \varphi_p > 0 & \quad \text{if } -1 < \epsilon < 0.
\end{align*}
\]

As shown in the Appendix, the flux of the major image is magnified ($\epsilon > 0$) when it is perturbed by the planet, while the flux of the minor image is demagnified ($-1 < \epsilon < 0$). Then, the sign of the astrometric perturbation is positive in both cases of major and minor image perturbations. Therefore, while the sign of the photometric planetary signal is either positive or negative, depending on whether the major or minor is perturbed, astrometric signals are always positive, regardless of the type of image perturbations.

In Figure 1, we present the geometry of an example planetary lens system in which the major image is perturbed by a planet. The coordinates are centered at the center of mass of the lens system and the positions of the primary and the planet are marked by a cross. The diamond-shaped figure represents the caustic and the big dashed circle is the combined Einstein ring. The straight line with an arrow represents the source trajectory. The small solid circle on the source trajectory represents the source position at the moment of the major image perturbation and the elongated figures at around $\xi = -1.4$ and $0.7$ represent the images corresponding to the source position. The inset shows the blowup of the region around the major image. The solid and dotted curves running in almost parallel with the source trajectory are the trajectories of the image centroid with and without the planet-induced perturbation, respectively. The mass ratio and the separation (normalized by $E$) of the planet are $q = 3 \times 10^{-3}$ and $a = 1.3$, respectively, and the source star has an angular radius of $0.02E$. Lower panels: The astrometric (left panel) and light curves (right panel) with (solid curves) and without (dotted curves) the planetary perturbation, respectively.

Fig. 2.—Variation of the image shape perturbed by a planet depending on the source position with respect to the caustic. Left panels show the locations of the source star (small solid circle) with respect to the caustic (diamond-shaped figure), and right panels show the resulting images (closed figures drawn by solid curves) corresponding to the source positions. The figures drawn by dotted curves in the right panels are the unperturbed images. The lens system is the same as in Fig. 1, and the planet perturbs the major image.
the magnified flux of the major image, the image centroid is additionally shifted farther away from the unlensed source position toward the major image, causing also positive deviations in the astrometric curve (lower left panel). Figure 2 shows several more examples of perturbed and unperturbed images to illustrate the mentioned trend of image perturbation applies to general cases of perturbations.

In Figure 3, we present the geometry of an example planetary lens system in which the minor image is perturbed by a planet. From the comparison of the unperturbed and perturbed minor images, one finds that the perturbed minor image is demagnified, in contrast with the magnification of the perturbed major image. Astrometrically, demagnification of the minor image causes the image centroid to be further shifted toward the major image, resulting in positive deviations whose sign is the same as that of the astrometric deviations caused by the major image perturbation.

4. CONCLUSION

We investigated the properties of planetary signals in microlensing light curves and centroid shift trajectories. We derived analytic relation between the photometric and astrometric planetary signals and explained the reason for the empirically known properties of planetary signals, where the photometric signal can be either positive or negative, depending on which of the two images produced by the primary is perturbed, while the astrometric signal is always positive, regardless of which image is perturbed.

We thank B. S. Gaudi and V. Bozza for making very helpful comments on planetary microlensing properties. C. H. was supported by the Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARCSEC) of Korea Science and Engineering Foundation (KOSEF) through Science Research Program (SRC) program. K. C. was supported by Korea Astronomy Observatory (KAO).

APPENDIX A

SIGNS OF PHOTOMETRIC PERTURBATIONS

By treating the planet-induced deviation as a perturbation, Bozza (1999) derived the expression for microlensing magnification of a lens with a planet by expanding the Jacobian of the lens equation to the first order in q (see eq. [37] of his paper),

\[ A_p = A + \frac{A}{z^2 - 1} \left\{ 2q(x^2 - y^2)\left[(x - x_p)^2 - y^2 \right] + 4(x - x_p)^2 y^2 \frac{4(x \Delta x + y \Delta y)}{z^2} \right\} , \] (A1)

where \( A \) is the unperturbed magnification, \( z = (x, y) \) is the position vector to the image location, the \( x \)-axis is parallel to the primary-planet axis, \( x_p \) is the planet position on the axis, and \( (\Delta x, \Delta y) \) represents the change of the image position induced by the planet. The perturbation becomes maximum when the source crosses the \( x \)-axis. At this moment, the images are also located along or very close to the \( x \)-axis. Then, by using the approximations of \( y \to 0, \Delta y \to 0, \) and \( z \to x \), one can express the equation into a one-dimensional form of

\[ A_p = A \left\{ 1 + \frac{1}{x^2 - 1} x^2 \left[ \frac{2q}{(x - x_p)^2} - \frac{4 \Delta x}{x} \right] \right\} . \] (A2)

As noted by Bozza (1999), the planet induces two types of perturbations: the first type caused by the slight change of the image position (the term \( \propto \Delta x / x \)) and the second type resulting from the change of the lens equation due to the planet [the term
The fractional astrometric perturbation is $\Delta x / x \propto (x - x_p)^{-1}$ (Bozza 1999). Then, during the time of maximum perturbation, when $x \rightarrow x_p$, the second type of perturbation dominates, and the magnification excess is approximated by

$$
eq \frac{A_p - A}{A} \sim \frac{2q}{(x - x_p)^3} \frac{1}{(x^2 - 1/x^2)} .$$  \tag{A3}

From equation (A3), one finds that the planet’s approach close to one image (located at $x_1$) causes little perturbation on the other image (located at $x_2$) because $(x_1 - x_p)^2 \ll (x_2 - x_p)^2$, implying that only one image is perturbed by the planet. In addition, since both $q$ and the term $(x - x_p)^{-2}$ are positive, the sign of the photometric perturbation is determined by the remaining term $x^2 - 1/x^2$. Because the major image is located outside of the Einstein ring ($x > 1$), and vice versa in the case of the minor image perturbation, one finds that

$$\text{sign} \left( x^2 - \frac{1}{x^2} \right) = \begin{cases} (+) & \text{for major image perturbation}, \\ (-) & \text{for minor image perturbation} . \end{cases} \tag{A4}$$

Therefore, the flux of the major image is magnified when it is perturbed by the planet, while the flux of the minor image is demagnified because of planetary perturbations, i.e.,

$$\text{sign} (\epsilon) = \begin{cases} (+) & \text{for major image perturbation} , \\ (-) & \text{for minor image perturbation} . \end{cases} \tag{A5}$$

REFERENCES

Afonso, C., et al. 2001, A&A, 378, 1014
Albrow, M., et al. 1998, ApJ, 509, 687
Alcock, C., et al. 1996, ApJ, 463, L67
———. 1997, ApJ, 491, 436
Boden, A. F., Shao, M., & van Buren, D. 1998, ApJ, 502, 538
Bond, I., et al. 2001, MNRAS, 327, 868
Bozza, V. 1999, A&A, 348, 311
Dominik, M., & Sahu, K. C. 2000, ApJ, 534, 213
Gaudi, B. S. 1998, ApJ, 506, 533
Gaudi, B. S., & Gould, A. 1997, ApJ, 486, 85
Gould, A., & Loeb, A. 1992, ApJ, 396, 104
Gould, A., & Salim, S. 1999, ApJ, 524, 794
Griest, K., & Safizadeh, N. 1998, ApJ, 500, 37
Han, C. 2002, ApJ, 564, 1015
Han, C., & Lee, C. 2002, MNRAS, 329, 163
Hag, E., Novikov, I. D., & Polnarev, A. G. 1995, A&A, 294, 287
Jeong, Y., Han, C., & Park, S.-H. 1999, ApJ, 511, 569
Mao, S., & Paczyński, B. 1991, ApJ, 374, L37
———. 1992, ApJ, 396, L37
Miyamoto, M., & Yoshii, Y. 1995, AJ, 110, 1427
Paczyński, B. 1986, ApJ, 304, 1
———. 1996, ARA&A, 34, 419
———. 1998, ApJ, 484, L23
Rhie, S. H., et al. 1999, ApJ, 522, 1037
Salizzadeh, N., Dalal, N., & Griest K. 1999, ApJ, 522, 512
Udalski, A., Szymański, M., Kaluzny, J., Kubiak, M., Mateo, M., Krzemiński, W., & Paczyński, B. 1994, Acta Astron., 44, 227
Walker, M. A. 1995, ApJ, 453, 37
Wambsganss, J. 1997, MNRAS, 284, 172
Witt, H. J. 1990, A&A, 236, 311

No. 2, 2003 SIGNS OF PLANETARY MICROLENSING SIGNALS 1075