Size Agnostic Change Point Detection Framework for Evolving Networks

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Abstract Changes in the structure of observed social and complex networks’ structure can indicate a significant underlying change in an organization, or reflect the response of the network to an external event. Automatic detection of change points in evolving networks is rudimentary to the research and the understanding of the effect of such events on networks. Here we present an easy-to-implement and fast framework for change point detection in temporal evolving networks. Unlike previous approaches, our method is size agnostic, and does not require either prior knowledge about the network’s size and structure, nor does it require obtaining historical information or nodal identities over time. We use both synthetic data derived from dynamic models and two real datasets: Enron email exchange and Ask-Ubuntu forum. Our framework succeeds with both precision and recall and outperforms previous solutions.

1 Introduction

Complex systems of interacting elements, from human (social and organizational) to physical and biological ones, can be modeled as interaction networks, with nodes representing the elements and edges representing their interactions. When the interactions are dynamic, i.e., human and social interactions, a complete model that captures the longitudinal evolution of the system is comprised of a sequence of networks, each portraying a snapshot of the system at a single point in time.

Of specific interest recently is the analysis of changes in dynamic social and complex networks in response to events, and the automatic detection of these points of change, termed Change Point Detection (CPD). Recent works identified changes in the community partitioning of the Enron email exchange immediately after the Californian blackouts [28], and a turling up of conversation networks between traders in response to significant stock price changes [31]. Understanding the network’s reaction to unusual events provides improved abilities to analyze, understand and possibly take
actions in a given system, infer its reaction to external shocks, and aid in predicting organizational and behavioral changes.

Past research for identifying change points used stochastic models, of either scalar values representing the longitudinal data [25], or probabilistic and model-based representations of the network [22, 28, 35], and did not examine the complex network’s structure as manifested through distributions.

The structural properties that are in the focus of our work here are the network’s native statistical distribution, i.e., its degree distribution measure. Distribution functions are a measure of the division of resources within the network, and their relative positions, and are considered a fundamental tool in the understanding of complex systems. Stumpf and Porter [33] have discussed this notable role, claiming that degree distributions serve as an aiding tool for understanding, interpreting and even predicting behaviors in a given system. Bhamidi et al. [4] further showed that degree-distribution measures reflect changes in the underlying structure better than the hyper-parameters of the corresponding network models.

An additional valuable advantage of a degree distribution-based event detection is that it eliminates the need to know in advance the number of nodes in the network at each point in time, and can work with as little information as the sequence of interactions for the periods under inspection. Thus, unlike all previous CPD schemes, the proposed solution here assumes no prior-knowledge of the network, does not require pre-processing, and can be used in an online manner, where new network snapshots are generated on-the-fly.

Here, we devise an online fast change point detection mechanism, utilizing the degree distribution of snapshots of networks in time. The detection mechanism is planned in a manner that does not require to determine exact theoretical fits to the distributions. We conduct a hypothesis testing to assess the significance of the change and differentiate a change signal from local fluctuations.

The contributions of the work are the following:

1. Taking a sliding-window approach for the network interactions, this method can address both the anomaly detection problem, in which there is a significant variation from a norm, and the change point detection problem, which considers a significant change to the norm itself, by computing the significance measure of the change (calculated p-value) over different window sizes.

2. The approach is the first that does not require to know in advance the number of interacting nodes in each stage of the network’s life, and hence can be used online.

3. We investigate the performance of the scheme over both synthetic data and real world data. For the synthetic data we conduct a thorough investigation of several generative models, i.e., random networks and small world networks, with varying rate of events and over different network sizes. This enables us to quantify the reaction of different network models to events. We further show that over two real datasets, the scheme performs better than existing detection schemes, while being faster.

4. The hypothesis testing we conduct enables a sensitivity measure for a change. First, it enables to set the level of sensitivity of a change according to need. Then, it opens the possibility to detect changes with decreasing sensitivity during a window of time. While current schemes detect reactions to shocks, this scheme can detect gradual changes that follow a clear trend of increasing probability of a change and can be utilized as a predictive framework.
2 Background and Related Works

It is widely accepted that structural properties of a network play a significant role in determining its actors’ behavior \cite{13, 6, 32, 21, 11}. The last decade’s abundance of temporal information paved the path to a further understanding of the dynamics of networks \cite{23}, and findings corroborate that structural properties have a prominent affect on the longitudinal dynamics of networks and their actors \cite{21, 24, 11, 29, 18}.

In this work we investigate the effect of events on social networks. Romero, Uzzy, and Kleinberg in a recent novel work \cite{31} defined these events as mostly exogenous events that are either unexpected, or are extreme, relative to the average \cite{31, 12}. They found a turtling-up of the network as a reaction to an external shock, and measured changes in the clustering coefficient, tie strength and percentage of border edges. Kondor et al. \cite{20} researched the longitudinal structure of the network of the most active Bitcoin users for a period of two years, and searched for important changes in the graph structure by comparing successive snapshots of the active core of the transaction network using principle component analysis (PCA). They found a clear correspondence with the market price of Bitcoin. McCulloh & Carley \cite{25} included in their analysis of change points also cases of endogenous changes, and showed that their detection system can determine that a change has occurred from a longitudinal analysis of the network itself. Using their method, Tambayong \cite{34} examined Sudan’s political networks and found that foreign-brokered signings of multiple peace agreements served as a political solidification point for political actors of Sudan during the recent violent domestic conflict. According to their analysis, this was a catalyst that caused three leaders to have emerged and lead the more compartmentalized yet faction-cohesive political networks of Sudan. In a recent analysis, Peel and Clauset \cite{28} were able to detect external changes during the Enron crisis through a stochastic analysis of the Enron organizational email exchange \cite{19}.

Considering that distributions in complex systems have practical importance as an aiding tool for data interpretation and event prediction \cite{33, 4}, we investigate here the interplay between points of change and this fundamental structural distribution in social organizations and systems.

2.1 Models for Change Point Detection in Networks

In stochastic models of networks, change points are points in time where a change in the system’s norm is detected in a manner that can be significantly differentiated from plain stochastic noise \cite{28, 2, 17, 14, 3}. McCulloh & Carley \cite{25} convert the series of networks to a time series of scalar values for different network measures, and looked for a stable change in these values (as opposed to temporal change, when looking for anomaly detection) using process mining techniques for change points detection \cite{15, 30, 25}.
Methods for CPD differ mainly by the graph features they compute. A model-based approach fits each snapshot to a generative model. For example, General Hierarchical Random graph (GHRG), Generalized Two Block Erdos-Renyi (GBTER), and Kronecker Product Graph Model (KPGM) [28,5,26]. A model-based approach requires a pre-processing phase, for which enough history is pre-known. It further requires that labeled nodal information is known. When taking the degree distribution, we eliminate the need for this extended information, as degree distribution does not require historical information, nor the node names. Moreover, recent analysis found that structural changes are better detected by the degree distribution than by the hyper-parameters of the generative model, for the PA case [4].

A complementary approach, similar in nature to ours, is to extract a large number of features from each consecutive graph snapshots, and find the distance between them [2,22,35,8]. A change is determined if a predefined threshold for the distance is crossed.

Unlike previous works that consider graph features, in our work, we conduct a hypothesis test to determine a change, to provide a certainty level for a change point detection.

3 Detecting Change Points in Networks

We explain our method following Figure 1. A sequence of networks is presented, where a change in the generative model occurs. The change is not tied to a specific structural characteristic. Our framework computes the cumulative distribution function of the degrees (CDF) for each graph, computes the distance, and performs a hypothesis testing to infer how probable is a change given the measured distance between the two CDF’s. Here, we chose the nonparametric Kolmogorov-Smirnov (KS) two-sample test to measure the distance, though other non-parametric statistical methods for measuring the distance may be applied.

CPD frameworks as the ones discussed in the previous section divide the data to consecutive snapshots according to a natural division derived from the nature of the data, such as daily or weekly snapshots of organizational frameworks, or monthly graphs of votes. In methods measuring the distance between features extracted from two consecutive graph snapshots [2,22,35], a change is detected if the measured distance is bigger than an arbitrarily predefined threshold value. But distance measures work well mainly for large sample sizes. When the sample size is small, a large distance can be measured, crossing the threshold value. This can lead to a false positive result that a change occurred, when there is merely a fluctuation in the network that should be identified as noise, and is considered a false positive inference, as demonstrated in Figure 2. To avoid these types of false inference we suggest the use of a sliding window over several graph snapshots, and computing the CDF across the entire window, as is the case in Figure 4. A complementary situation occurs when windows that are set too large conceal an event within them, thus hiding the point of change. This would correspond with a false negative inference, and is demonstrated in Figure 3. A solution for this problem is the use of a sliding window to find the exact point of change within the window, as is used in [28]. An alternative approach to measuring a distance between windows is to try and fit a theoretical statistical distribution to each network snapshot, and determine whether they are derived from the same model. This is, however, a rather time and computational-intensive approach. To fit data to
a statistical theoretical model requires both to find a fit and to reject other possible theoretical statistical distributions [7]. Hence, we compare distances across windows, as described in Figure 4.

We conduct a hypothesis testing for understanding whether the distance between the degree distributions asserts that they come from the same model, or from two different generative models. We measure the distance between the cumulative degree distributions of consecutive snapshots. For any two consecutive windows, let us define their graphs representations as \( g_i, g_{i+1} \). The null hypothesis is that the cumulative distributions measured for any two consecutive snapshots, \( g_i, g_{i+1} \), are drawn from the same distribution, \( G_{Null} \), in which case no change has occurred between the windows. To test the hypothesis we generate synthetic datasets from the distribution of \( g_i \) and find their distributions. The standard approach for generating samples for hypothesis testing is bootstrapping, which generates samples by randomly re-sampling (with replacement) the data [9]. We use here the nonparametric Kolmogorov-Smirnov (KS) two-sample test. The method is considered robust and is widely used. Yet, when comparing two distributions using too few samples it can fail to reject a false null hypothesis. The probability for that diminishes as the number of nodes interacting in each snapshot increases, and it is best to create snapshots that contain, as a rule of thumb, at least 50 nodes each.

For two consecutive graph snapshots \( g_i, g_{i+1} (i \in \{1, 2, \ldots \}) \) we denote the two generated corresponding cumulative degree distribution functions by \( S_i(x), S_{i+1}(x) \). Given the CDF degree distribution \( S_j(x), j \in i, i+1 \) for graph \( g_j : S_j(x) = P_j(x \leq X) \) we compute the KS statistic \( D \), defined as the maximal difference between the two
Fig. 2: False Positive: Distance measure is large as sample size is too small, although graphs come from same generative model.

Fig. 3: False Negative: Fluctuations conceal each other and decrease the measured distance between two networks.

Fig. 4: The use of a sliding window over several graph snapshots decreases the probability of a false positive estimation of a change.
Size Agnostic Change Point Detection Framework for Evolving Networks

empirical distributions, as described by Equation 1. The KS null hypothesis is that the two samples where drawn from the same distribution.

\[ D_{i,i+1} = \sup_x |S_i(x) - S_{i+1}(x)| \]

The KS null hypothesis is rejected with significant level \( \alpha \) if the computed distance \( (D_{i,i+1}) \) is greater than some critical value.

As explained before, a large KS distance \( D_{i,i+1} \) measured between \( S_i(x) \) and \( S_{i+1}(x) \) doesn’t necessarily indicate a signal in our framework. We would like to test how rare such distance \( D_{i,i+1} \) is. We define \( g_i \) as the base model graph, and conduct a hypothesis testing, with a null hypothesis that the distance \( D_{i,i+1} \) between the base model graph distribution and the consecutive one is not rare for samples taken from the same statistical model.

Our null hypothesis then assumes that the distance between the snapshots’ distributions is typical for distances between distributions sampled from the base model graph distribution. The null hypothesis is rejected with significance \( p \) if in \( (1-p) \) percent of the times the measured distance between \( S_i(x) \) and the sampled distributions is smaller than \( D_{i,i+1} \), as depicted in Equation 2.

Following the bootstrap procedure \[9\] we generate \( j = 1000 \) new samples from \( S_i(x) \) and measure the distance \( d_{i,j} \) between \( S_i(x) \) and each of its bootstrap samples. We test the hypothesis by computing the fraction of times a KS test will yield a distance \( D_{i,i}, j \in \{1...10000\} \), that is at least as big as \( D_{i,i+1} \).

\[ p = \frac{|D_{i,i+1} > \{d_{i,j}\}|}{|\{d_{i,j}\}|} \]

A confidence level \( \alpha \) may now be chosen to reject the null hypothesis, depending on the acceptable false positive rate. This confidence level corresponds to the sensitivity of the change, and can be tunable.

4 Detecting Changes over Different Network Types

We conduct several experiments to evaluate the performance of our framework, on both large synthetic datasets and real networks. First, we investigate the performance of the framework on synthetic networks generated by several generative models. Each such generative model enables us to investigate the framework’s behavior for different structural characteristics. As our method is based on the degree distribution of the network it is agnostic to any changes in the network size. Hence, we expect our framework to detect changes across network snapshots that may gain or lose nodes during the network’s lifetime. At first we considered to use a preferential-attachment growing network as one of the models. However, this model is specifically designed to explain the emergence of hubs in networks and the long tail distribution of real-world networks degrees, and thus is designed to create a specific degree distribution, which is what we try to find. For generative models we therefore employ the Erdős-Rényi (ER) random networks model and the Caveman model. In each experiment the network model alternates between two configurations that differ in their hyper parameters. The number of changes is set to 100, distributed randomly. Then, the number of consecutive snapshots of the network drawn from the model configuration, \( x \), is chosen from a normal distribution \( x \sim N(\mu = 4, \sigma^2 = 2) \), such that the average number of consec-
Random Graphs:
We start with the Erdős-Rényi (ER) random graph model \[10\]. The model for random graphs \( G(n, p) \) assumes a fixed number of nodes \( n \). Edges connect node pairs independently with probability \( p \). Low values of \( p \) entail that the number of edges is substantially lower than the number of nodes, and the model generates small components in tree forms. As \( p \) increases, and reaches \( p > o\left(\frac{1}{n}\right) \), the network changes to suddenly form a giant component, a phase transition that has a distinct influence on the structure of the network.

We then perform two experiments for this model type, as described here, and detailed in Figures 5a, 5b:
- Experiment 1 - A change in the hyper-parameters of the ER model transitions the network between the two network states of fragmented \((p << o(\frac{1}{n}))\) and connected \((p > o(\frac{1}{n}))\). The networks configurations are the following. Each configuration consists of 200 nodes, and the model’s hyper parameter is either \( p = 0.003 \), i.e., fragmented, or \( p = 0.01 \), i.e., connected.
- Experiment 2 - The ER networks consist of 200 nodes each, and the model’s hyper parameter is either \( p = 0.1 \), \( p = 0.15 \), i.e., both times the network is connected, and there is a slight change in the connectedness. It is safe to assume that the subtleness of the change in the generative model of the random network will make it harder to identify the change.

Caveman Model:
The ER model generates graph with small clustering coefficients, which lack the capability to represent communities. Social networks are often characterized as having highly connected communities that form rare interactions in between, and form a Small World. For example, in an organization you may expect intensive interactions between actors within departments and sparse interactions between actors belonging to different departments. A generative model for a small world network is the Caveman \[36\].

To test our framework against networks with varying sizes we generated a sequence of unlabeled networks, \( g_i \in G \), while using the Caveman model. The number of nodes for each snapshot was randomly selected from a uniform distribution \(||g_i|| \sim U(200, 1000)\). To prevent a sample size bias while calculating the KS distance we randomly sampled 200 nodes from each network and calculated the distance between the two samples degree distribution.
- Experiment 3 - The Caveman-based networks are drawn from the a model containing 200 nodes, as explained above, and \( C = 5 \) communities each. The change in the hyper-paramter between the two configurations is in the rewire probability \( p \). In the 1\textsuperscript{st} configuration, visualized in Figure 5c \( p = 0.4 \). In the 2\textsuperscript{nd} \( p = 0.7 \), leading to a more inter-connected network, as is visualized in Figure 5d.

Detection Performance
Table 1 describes the performance of our detection framework for the three described experiments. Note, that for the ER networks (exp1, exp2) we get a perfect recall. The

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\[1\] Recently Zhang et al. \[37\] suggested a generalization for dynamic random networks, in which the dynamic process is governed by a continuous Markov-process. As we need to study the change point detection problem, requiring the change in the generative model hyper parameters, we could not employ their model.
degree distribution of an ER random graph with edge probability $p = \lambda n$ follows a Poisson distribution with probability mass function: $e^{-\lambda} \frac{\lambda^k}{k!}$ with mean $\lambda$ and skewness $\lambda^{-0.5}$. A change in $\lambda$ differentiates two ER generative models and will be projected to the networks’ CDF, thus detectable by our model. This may explain the perfect detection (Recall=1) of all events in our synthetic data tests. However, the variance of a Poison distribution is $\lambda$ as well. As the variance $\lambda$ increases, the chances of mistakenly find two samples drawn from the same model as not sharing the same distribution increase. This explains our relative low precision.

As true positive events (change points) were detected with significance that exceeds 99%. We repeated the experiments while increasing the CPD threshold from 90% to 99%. This test resulted with Recall = 1.0 and Precision = 0.89. This corresponds to changing the sensitivity of framework, as discussed before.

| Experiment | Model & Main Structural Property | Precision Mean, Std | Recall Mean, Std |
|------------|----------------------------------|---------------------|-----------------|
| exp1       | ER: Phase transition $p = \{0.003, 0.010\}$ | 0.767, 0.03         | 1.0, 0.0        |
| exp2       | ER: Connected $p = \{0.1, 0.15\}$          | 0.671, 0.02         | 1.0, 0.0        |
| exp3       | Caveman: Communities $p = \{0.4, 0.7\}$     | 1.0, 0              | 0.961, 0.01     |
4.1 Detection of Events Changing Real Networks

We tested our framework against two real world datasets. The first, the Enron email exchange between 151 employees, mostly managers [19]. We generated weekly networks from the emails interactions similar to [28, 35]. Figure 6 describes our framework’s performance, compared to both the real events, and to the GHRG-based detection framework by Peel and Clauset [28]. Our framework detected 13 out of 14 change points, resulting in during the period of the second half of 2001 where many events affected Enron.

The second dataset is the interactions on the stack exchange web site Ask Ubuntu [27], and generated monthly networks. We assume that a new Ubuntu release might affect the community, and extracted the ground truth from the Ubuntu site’s detailing release.

![Fig. 6: Enron emails exchange during the second half of 2001, where many events took place. Real events denoted by blue rhombuses, True positive detections by a green star followed by the window length. In grey at the top is the results of the baseline GHRG model [28]. Our framework outperforms with recall = 0.9 and Precision = 0.9.](image)

![Fig. 7: Ask Ubuntu forum exchange. Release events denoted by blue rhombuses, True positive detections by a green star followed by the window length. Our framework outperforms with recall = 0.8 and Precision = 0.57.](image)

The results for the Caveman model (exp 3) yield excellent results of perfect precision (100%) and near-perfect recall (96%), showing that a community structure of networks lends itself naturally to our detection framework.
Figure 7 shows our results. Our framework detected almost all the events with high confidence: Recall = 0.8, and Precision = 0.57. Clearly, there might be external events that are not version releases that we are not aware of.

5 Conclusions

Our framework for size-agnostic detection of changes proved to work across different generative models and real datasets. During the work we have identified an interesting trade-off between precision and recall of detection when considering the size of the network and detectability. We intend to further study this trade-off in future research. We further plan to try and quantify the nature of the change in the distribution in response to different events.

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References

1. URL http://www.agsm.edu.au/bobm/teaching/BE/Enron/timeline.html
2. Akoglu, L., Faloutsos, C.: Event detection in time series of mobile communication graphs. In: Army Science Conference, pp. 77–79 (2010)
3. Akoglu, L., Tong, H., Koutra, D.: Graph based anomaly detection and description: a survey. Data Mining and Knowledge Discovery 29(3), 626–688 (2015)
4. Bhamidi, S., Jin, J., Nobel, A., et al.: Change point detection in network models: Preferential attachment and long range dependence. The Annals of Applied Probability 28(1), 35–78 (2018)
5. Bridges, R.A., Collins, J.P., Ferragut, E.M., Laska, J.A., Sullivan, B.D.: Multi-level anomaly detection on time-varying graph data. In: Advances in Social Networks Analysis and Mining (ASONAM), 2015 IEEE/ACM International Conference on, pp. 579–583. IEEE (2015)
6. Burt, R.S.: The network structure of social capital. Research in organizational behavior 22, 345–423 (2000)
7. Clauset, A., Shalizi, C., Newman, M.E.J.: POWER-LAW DISTRIBUTIONS IN EMPIRICAL DATA AARON. http://arxiv.org/abs/0706.1062v2 (2009). DOI 10.1109/ICPC.2008.18
8. Donnat, C., Holmes, S., et al.: Tracking network dynamics: A survey using graph distances. The Annals of Applied Statistics 12(2), 971–1012 (2018)
9. Efron, B., Tibshirani, R.J.: An introduction to the bootstrap. CRC press (1994)
10. Erdős, P., Rényi, A.: On the evolution of random graphs. Publ Math Inst Hungar Acad Sci 5, 17–61 (1960)
11. Fowler, J.H., Christakis, N.A., et al.: Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the framingham heart study. Bmj 337, a2338 (2008)
12. Gilbert, C.G.: Unbundling the structure of inertia: Resource versus routine rigidity. Academy of Management Journal 48(5), 741–763 (2005)
13. Granovetter, M.: The strength of weak ties: A network theory revisited. Sociological theory 1(1), 201–233 (1983)
14. Gupta, M., Gao, J., Aggarwal, C., Han, J.: Outlier detection for temporal data. Synthesis Lectures on Data Mining and Knowledge Discovery 5(1), 1–129 (2014)
15. Hawkins, D.M., Qiu, P., Kang, C.W.: The changepoint model for statistical process control. Journal of quality technology 35(4), 355–366 (2003)
16. Haynie, D.L.: Delinquent peers revisited: Does network structure matter? 1. American journal of sociology 106(4), 1013–1057 (2001)
17. Hirose, S., Yamashiki, K., Nakata, T., Fujimaki, R.: Network anomaly detection based on eigen equation compression. In: Proceedings of the 15th ACM SIGKDD international conference on Knowledge discovery and data mining, pp. 1185–1194. ACM (2009)
18. Ilany, A., Booms, A.S., Holekamp, K.E.: Topological effects of network structure on long-term social network dynamics in a wild mammal. Ecology letters 18(7), 687–695 (2015)
19. Klimt, B., Yang, Y.: Introducing the Enron Corpus. Machine Learning (2004)
20. Kondor, D., Csabai, I., Szüle, J., Pósfai, M., Vattay, G.: Inferring the interplay between network structure and market effects in bitcoin. New Journal of Physics 16(12), 125,003 (2014). URL [http://stacks.iop.org/1367-2630/16/i=12/a=125003](http://stacks.iop.org/1367-2630/16/i=12/a=125003)
21. Kossinets, G., Watts, D.J.: Empirical analysis of an evolving social network. Science 311(5757), 88–90 (2006)
22. Koutra, D., Vogelstein, J.T., Faloutsos, C.: Deltacon: A principled massive-graph similarity function. In: Proceedings of the 2013 SIAM International Conference on Data Mining, pp. 162–170. SIAM (2013)
23. Lazer, D., Pentland, A.S., Adamic, L., Aral, S., Barabasi, A.L., Brewer, D., Christakis, N., Contractor, N., Fowler, J., Gutmann, M., et al.: Life in the network: the coming age of computational social science. Science (New York, NY) 323(5915), 721 (2009)
24. Leskovec, J., Kleinberg, J., Faloutsos, C.: Graph evolution: Densification and Shrinking Diameters. ACM Transactions on Knowledge Discovery from Data 1(1), 2–es (2007). DOI 10.1145/1217299.1217301. URL [http://portal.acm.org/citation.cfm?doid=1217299.1217301](http://portal.acm.org/citation.cfm?doid=1217299.1217301)
25. McCullloh, I., Carley, K.: Detecting change in longitudinal social networks. Journal of Social Structure 12, 1–37 (2011)
26. Moreno, S., Neville, J.: Network hypothesis testing using mixed kronecker product graph models. In: Data Mining (ICDM), 2013 IEEE 13th International Conference on, pp. 1163–1168. IEEE (2013)
27. Paranjape, A., Benson, A.R., Leskovec, J.: Motifs in temporal networks. In: Proceedings of the Tenth ACM International Conference on Web Search and Data Mining, pp. 601–610. ACM (2017)
28. Peel, L., Clauset, A.: Detecting change points in the large-scale structure of evolving networks, 29th AAAI Conference on Artificial Intelligence (AAAI) pp. 1–11 (2015). URL [http://arxiv.org/abs/1405.0989](http://arxiv.org/abs/1405.0989)
29. Phelps, C.C.: A longitudinal study of the influence of alliance network structure and composition on firm exploratory innovation. Academy of Management Journal 53(4), 890–913 (2010)
30. Priebe, C.E., Conroy, J.M., Marchette, D.J., Park, Y.: Scan statistics on enron graphs. Computational & Mathematical Organization Theory 11(3), 229–247 (2005)
31. Romero, D.M., Uzzi, B., Kleinberg, J.: Social networks under stress. In: Proceedings of the 25th International Conference on World Wide Web, pp. 9–20. International World Wide Web Conferences Steering Committee (2016)
32. Spencer, J.W.: Global gatekeeping, representation, and network structure: a longitudinal analysis of regional and global knowledge-diffusion networks. Journal of International Business Studies 34(5), 428–442 (2003)
33. Stumpf, M.P.H., Porter, M.A.: Critical Truths About Power Laws. Science 335(6069), 665–666 (2012). DOI 10.1126/science.1216142
34. Tambayong, L.: Change detection in dynamic political networks: the case of sudan. In: Theories and Simulations of Complex Social Systems, pp. 43–59. Springer (2014)
35. Wang, Y., Chakrabarti, A., Sivakoff, D., Parthasarathy, S.: Fast change point detection on dynamic social networks. arXiv preprint [arXiv:1705.07325](https://arxiv.org/abs/1705.07325) (2017)
36. Watts, D.J.: Networks, dynamics, and the small-world phenomenon. American Journal of sociology 105(2), 493–527 (1999)
37. Zhang, X., Moore, C., Newman, M.E.: Random graph models for dynamic networks. The European Physical Journal B 90(10), 200 (2017)