Investigation of stresses and deformations of ball mill support axles taking into account the temperature field by numerical method

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Abstract. In the article, the stresses and deformations of the ball mill support axes are studied numerically, taking into account the temperature field. Expressions are given that determine the values of the stress-strain state of the external surface of the ball mill trunnion, depending on the temperature of the internal and external surfaces. The analysis of the behavior of stresses and deformations when changing the temperature parameters of the trunnion is based on the numerical method. Constant values and parameters of the trunnion are set, and the temperature ranges of the external and internal surfaces of the loading and unloading trunnions are set. A numerical simulation of deformations and stresses is presented, described in the form of dependency graphs with varying parameters—the temperature of the internal and external surface of the trunnion within specified limits. The dependences of stresses and deformations of the external surface on temperature changes are analyzed. It is shown that as the temperature difference increases, the stresses in both the circumferential and axial directions increase, and the deformations in the circumferential and axial directions monotonously increase. The unloading axle is subject to a more complex stress-strain state under the influence of a temperature field, and the external surface of the axle has the highest values of stresses and deformations.

1. Introduction
When operating a ball mill, the dangerous cross-section of the bottoms is the transition point of the cylindrical part of the trunnion to the conical one [1, 2]. Under the influence of constant loads of the housing with grinding bodies and material, the simultaneous action of gravity and rotation due to the moment of external forces, a bend occurs [3].

The trunnion has the form of a hollow cylinder, which is pinched at the end and rotates around the longitudinal axis [4]. The stress-strain state of the ball mill pin is estimated on the basis of a mathematical model that includes a complete system of equilibrium equations that determine the
relations of elastic-plastic deformation, taking into account the effects of cyclic loading of the material, with appropriate initial and boundary conditions.

According to [5, 6, 7], in the first approximation, the solution of this problem can be obtained using engineering approaches based on the simplest relations of the theory of elasticity, resistance of materials and mechanics of materials [8, 9, 10]. The bending of a rotating axle is caused by the simultaneous action of gravity and rotation due to the moment of external forces, rotation when bending a cylindrical part leads to the formation of a complex stress-strain state [8]. Modeling such a process requires the use of the Euler-langrangian approach, tracing the history of deformation of material particles, including the loading process and elastic-plastic loading processes with the appearance of secondary plastic deformations and changes in the yield strength of the material, taking into account the features of alternating deformation.

The stresses of the external surface in the axial direction are calculated based on [9] with the summation of stresses arising under the influence of gravity and rotation:

$$\sigma_z = \frac{\alpha E(T_{R_z} - T_{R_l})}{1-\nu} \left[ \frac{R_1^2 - R_z^2}{R_z^2 - R_l^2} \frac{1}{2 \ln \left( \frac{R_z}{R_l} \right)} \right] + \sigma_{z0}, \quad (1)$$

According to [9], the stresses on the outer surface of the ball mill shaft in the circumferential direction are determined:

$$\sigma_{\varphi} = \frac{\alpha E(T_{R_z} - T_{R_l})}{1-\nu} \left[ \frac{R_1^2 - R_z^2}{R_z^2 - R_l^2} \frac{1}{2 \ln \left( \frac{R_z}{R_l} \right)} \right] \quad (2)$$

Deformations of the external surface of the axle in the axial, circumferential and radial directions are found according to [9]:

$$\varepsilon_z = \frac{1}{2(1-\nu)} \left[ \frac{(1-3\nu)(R_z^2T_{R_z} - R_l^2T_{R_l}) - (1+\nu)(R_z^2T_{R_z} - R_z^2T_{R_l}) + (1-\nu)(R_z^2T_{R_z} - R_l^2T_{R_l})}{R_z^2 - R_l^2} \right] + \varepsilon_{z0}, \quad (3)$$

$$\varepsilon_{\varphi} = \frac{1}{2(1-\nu)} \left[ \frac{(1-3\nu)(R_z^2T_{R_z} - R_l^2T_{R_l}) - (1+\nu)(R_z^2T_{R_z} - R_z^2T_{R_l}) + (1-\nu)(R_z^2T_{R_z} - R_l^2T_{R_l})}{R_z^2 - R_l^2} \right], \quad (4)$$

$$\varepsilon_r = \frac{1}{2(1-\nu)} \left[ \frac{(1-3\nu)(R_z^2T_{R_z} - R_l^2T_{R_l}) + (1+\nu)(R_z^2T_{R_z} - R_z^2T_{R_l}) + 2\nu(T_{R_z} - T_{R_l})}{R_z^2 - R_l^2} \right], \quad (5)$$

These expressions (1) – (5) determine the values of the stress-strain state of the external surface of the ball mill pin, depending on the temperature of the internal and external surfaces.

2. Methods and materials
The analysis of the behavior of stresses and deformations when changing the temperature parameters of the trunnion on the basis of (1) – (5) is carried out by the numerical method [10]. When conducting the study, the values of the constant and parameters of the trunnion are set numerically.

A mill with a standard size with diameter of 3.2×15 m with the following parameters is considered:

- the size of the axle:
  $$d = 1.4 \text{ m}, \quad l = 0.92 \text{ m}. \quad (6)$$

- the radii of the inner and outer surfaces are equal:
  $$R_1 = 0.525 \text{ m}, \quad R_2 = 0.7 \text{ m}. \quad (7)$$

Constant values for the common axle material – 35L steel have the following values, according to [11]:
Poisson's ratio: \[ \nu = 0.25; \] (8)

elastic modulus: \[ E = 201 \, 000 \, \text{MPa}. \] (9)

Coefficient of linear expansion of steel 35L for temperatures from 20°C to 200°C – \[ \alpha = 12 \, \text{deg}^{-1}. \]

To determine the stresses and strains, you need to set the temperatures of the internal and external surfaces of the trunnion. The temperature of the inner surface depends on the temperature of the feed material:

- for the loading pin – the clinker temperature varies within:
  \[ T_{R_1} = 70\ldots100^\circ\text{C}; \]

- for discharge trunnion is the temperature of the cement varies between:
  \[ T_{R_1} = 100\ldots150^\circ\text{C}. \]

To measure the temperature of the external surface of the ball mill trunnion, thermocouples are used.

The temperature of the external surface of the loading and unloading trunnion varies within:

- loading axle: \[ T_{R_2} = 35\ldots45^\circ\text{C}; \]
- unloading of the axle: \[ T_{R_2} = 40\ldots65^\circ\text{C}. \]

In the framework of the mathematical package Maple, equations (1) – (5) are solved when the parameters – the temperature of the internal and external surface of the trunnion are varied within the specified limits:

\[ T_{R_1} = 70\ldots150^\circ\text{C}, \] (10)

\[ T_{R_2} = 30\ldots70^\circ\text{C}. \] (11)

The solution of numerical modeling of strains and stresses, according to the equations (1) to (5) described in the form of graphs according to the temperature of the inner and outer surfaces in figures 1 – 2.

The graphs of stresses in the outer surface of the axle in the axial and circumferential direction is shown in figure 1. Voltage \( \sigma_z \) slightly exceed the voltage \( \sigma_\varphi \), this is because in the axial direction is added to the load of gravity \( \sigma_{z_0} \) (figure 2), accounting for 1% of \( \sigma_z \).

Analysis of the dependence of external surface stresses on temperature changes indicates that as the temperature difference between the internal and external surfaces of the trunnion increases, the stresses in both the circumferential and axial directions increase. The minimum voltage values are taken when the temperature of the inner surface approaches the temperature of the outer surface – when the temperature is equal to \( T_{R_1} = T_{R_2} \), the voltage in both directions is zero – \( \sigma_\varphi = \sigma_z = 0 \). The maximum value of the external surface stress under the influence of the temperature field in the circumferential and axial directions at a given temperature range is \( \sigma_\varphi = \sigma_z = 172 \, \text{MPa} \), which is achieved at a temperature difference of \( \Delta T_R = 120 \, ^\circ\text{C} \).

The change in the deformations of the external surface of the trunnion in the axial, circumferential and radial directions when the temperatures of the external and internal surfaces change is shown in figure 2.
The dependence of deformations in the radial direction under the influence of the temperature field is significantly inhomogeneous: a decrease in the temperature of the inner surface leads to a decrease in deformations, while an increase in the temperature of the outer surface leads to an increase in deformations. Thus, the deformations of the external surface in the radial direction increase from the internal cylindrical surface of the trunnion to the external: the maximum values in the specified temperature range are $\varepsilon_r = 4.102$ mm at the temperature of the internal surface $T_{R_1} = 67^\circ C$ and the external surface $T_{R_2} = 68^\circ C$.

Graphs of the dependence of deformations in the circumferential and axial directions with varying temperatures indicate that the deformations monotonously increase with increasing temperatures of the internal and external surfaces of the trunnion. The minimum values of deformations at a given temperature range are $\varepsilon_\varphi = 3.86$ mm, $\varepsilon_x = 4.39$ mm are assumed at the internal surface temperature $T_{R_1} = 73^\circ C$ and external surface $T_{R_2} = 23^\circ C$, maximum values $\varepsilon_\varphi = 4.6$ mm, $\varepsilon_x = 5.8$ mm at $T_{R_1} = 165^\circ C$ and $T_{R_2} = 55^\circ C$.

The temperature of the external surface in production conditions is assumed to be constant on the entire external surface along the axis, as the cooling liquid is poured. Let's consider the change in the stress-strain state of the loading and unloading trunnion at a constant temperature of the external
surface, averaged according to the readings of the sensors of the ball mill No. 13 with the standard size with diameter of 3.2×15 m at JSC "Belgorod cement".

The average external surface temperature values determined from the sensor readings have the following values:

- loading axle:
  \[ T_{R_2} = 42^\circ C; \]  
  \[ (12) \]

- unloading of the axle:
  \[ T_{R_2} = 58^\circ C. \]  
  \[ (13) \]

The temperature of the inner surface of the trunnion axis is changed: when loading increases along the advancing material from the heat to the body, the value of \( T_{R_1} \) varies according to (10), while unloading decreases along the progress of the material from the housing to the discharge pipe, while \( T_{R_1} \) varies according to (11).

3. Results and discussions

According to (1) – (5), graphs of stress and strain dependences of the loading and unloading trunnion of a ball mill with a standard size with diameter of 3.2×15 m at an assumed constant temperature of the external surface (12) – (13) and variation of the internal surface temperature in a given limit (10) are constructed (figures 3 – 4).

![Graphs of the dependence of the external surface stresses of the loading pin on the internal surface temperature at a constant temperature of the external surface \( T_{R_2} = 42^\circ C \): a) in the circumferential direction; b) in the axial direction.](image)

The graphs of the dependence of the stresses on the external surface of the loading pin in the circumferential and axial directions when the temperature of the internal surface varies, shown in figure 3, indicate a linear character: with an increase in the temperature \( T_{R_1} \), the stresses in these directions \( \sigma_\phi \) and \( \sigma_z \) grow. The maximum value of the voltage \( \sigma_\phi = \sigma_z = 87 \text{ MPa} \) is assumed at \( T_{R_1} = 103^\circ C \).
Figure 4. Graphs of the stress dependence of the external surface of the discharge pin on the internal surface temperature at a constant temperature of the external surface $T_{R_2} = 58°C$: a) in the circumferential direction; b) in the axial direction.

The graphs of the dependence of the stresses on the outer surface of the unloading pin in the circumferential and axial directions when the temperature of the inner surface varies, shown in figure 4, also indicate a linear character: as the temperature of the inner surface increases $T_{R_1}$, the stresses in these directions $\sigma_\phi$ and $\sigma_z$ increase. The maximum value of the voltage $\sigma_\phi = \sigma_z = 138 \text{ MPa}$ is assumed at $T_{R_1} = 153°C$.

Graphs of the dependence of deformations of the external surface of the loading and unloading trunnions when the temperature of the internal surface varies in the radial, circumferential and axial directions are shown in figures 5 – 6.

Deformations in the radial direction of the loading and unloading trunnions have an inverse relationship: when the temperature of the inner surface increases, the deformations monotonously decrease. Maximum strain values are reached at the internal surface temperature $T_{R_1} = 73°C$ for the loading axle and $T_{R_1} = 102°C$ for the unloading axle (figure 5).

Figure 5. Graphs of the dependence of deformations of the external surface of the loading pin when the temperature of the internal surface varies: a) in the radial direction; b) in the circumferential direction; c) in the axial direction.

The analysis of dependences of deformations of the external surface in the circumferential and axial directions is monotonously increasing: as the temperature of the internal surface increases, the
deformations increase (figure 6). The maximum values of deformations for a given range of internal surface temperature have the following values:

- in the circumferential direction: \( \varepsilon_\varphi = 4.12 \text{ mm at } T_{R_1} = 104^\circ\text{C} \) – for the loading axle, \( \varepsilon_\varphi = 4.49 \text{ mm at } T_{R_1} = 154^\circ\text{C} \) – for the discharge axle;

- in the axial direction: \( \varepsilon_x = 4.63 \text{ mm at } T_{R_1} = 104^\circ\text{C} \) – for the loading axle, \( \varepsilon_x = 5.25 \text{ mm at } T_{R_1} = 154^\circ\text{C} \) – for the discharge axle.

![Graphs of the dependence of deformations of the external surface of the unloading pin when the temperature of the internal surface varies: a) in the radial direction; b) in the circumferential direction; c) in the axial direction.](image)

The graphs of dependencies of stresses and deformations of the external surface presented in figures 3-6 prove that the discharge axle is more susceptible to a complex stress-strain state under the influence of a temperature field: the largest values of stresses and deformations are on the external surface of the discharge axle.

4. Summary
Using a numerical method for a mill size with diameter of 3.2×15 m, the analysis of the dependence of stresses and deformations occurring on the external surface of the trunnion on the temperature difference of its internal and external surfaces was carried out. It was found that with an increase in the temperature difference in \( \Delta T_R = 120^\circ\text{C} \) (\( T_{R_1} = 157^\circ\text{C}, T_{R_2} = 37^\circ\text{C} \)), the stresses in both the circumferential and axial directions grow and amount to \( \sigma_\varphi = \sigma_x = 172 \text{ MPa} \), the deformations in the circumferential and axial directions monotonously increase and reach \( \varepsilon_\varphi = 4.6 \text{ mm}, \varepsilon_x = 5.8 \text{ mm} \). The maximum strain value in the radial direction is \( \varepsilon_c = 4.102 \text{ mm} \). The unloading axle is subject to a more complex stress-strain state under the influence of a temperature field, and the external surface of the axle has the highest values of stresses and deformations.

5. References
[1] Markova O, Aulov V, Lavrenchuk A and Fedorenko M 2012 Analysis of methods for calculating wear of contact surfaces of friction International scientific and practical conference of students, postgraduates and young scientists "Youth and scientific and technical progress" (Gubkin) vol 1 (Gubkin: Publishing house of BSTU) 65-69
[2] Fedorenko M and Markova O 2014 Oscillatory processes in grinding mills of the cement industry International scientific and practical conference "Technique and technology of modern production" (Penza) 113-116
[3] Bogdanov V, Ilyin A and Semikopenko I 2008 Basic processes in the production of construction materials (Belgorod: Publishing house BSTU named after V. G. Shukhov) 550
[4] Gologorsky E, Dotsenko A and Ilyin A 2006 Operation and repair of equipment of construction industry enterprises (Moscow: Architecture) 503
[5] Boyarshinov M 2013 Estimation of the stress-strain state of a rotating long cylinder Bulletin of PNRPU Mechanics 1 25-38
[6] Birger I, Shorr B and Iosilevich G 1979 Calculation of the strength of machine parts (Moscow: Mashinostroenie) 702
[7] Erdedi A and Erdedi N 2012 Theoretical mechanics. Resistance of materials (Moscow: Akademiya) 320
[8] Svetlitsky V 2009 Construction mechanics of machines. Mechanics of rods (Moscow: Fizmatlit) 408
[9] Bestuzheva O, Bondarenko J and Khanin S 2019 Mathematical description of the stress-strain state of the ball mill axle under the action of gravity and rotation Bulletin of the Belgorod state technological University named after V. G. Shukhov 3 128-133
[10] Bakhvalov N, Zhidkov N and Kobelkov G 2003 Numerical methods (Moscow: Nauka) 630
[11] Argatov I and Dmitriev N 2003 Fundamentals of the theory of elastic discrete contact (Moscow: Politechnika) 240