Theory of high-$T_c$ superconductivity based on the fermion-condensation quantum phase transition

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A theory of high temperature superconductivity based on the combination of the fermion-condensation quantum phase transition and the conventional theory of superconductivity is presented. This theory describes maximum values of the superconducting gap which can be as big as $\Delta_1 \sim 0.1 \varepsilon_F$, with $\varepsilon_F$ being the Fermi level. We show that the critical temperature $2T_c \simeq \Delta_1$. If there exists the pseudogap above $T_c$ then $2T^* \simeq \Delta_1$, and $T^*$ is the temperature at which the pseudogap vanishes. A discontinuity in the specific heat at $T_c$ is calculated. The transition from conventional superconductors to high-$T_c$ ones as a function of the doping level is investigated. The single-particle excitations and their lineshape are also considered.

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The explanation of the large values of the critical temperature $T_c$, of the maximum value of the superconducting gap $\Delta_1$ and of the relation between $\Delta_1$ and the temperature $T^*$ at which the pseudogap vanishes are, as years before, among the main problems in the physics of high-temperature superconductivity. To solve them, one needs to know the single-particle spectra of corresponding metals. Recent studies of photoemission spectra discovered an energy scale in the spectrum of low-energy electrons in copper oxides, which manifests itself as a kink in the single-particle spectra [1-3]. As a result, the spectra in the energy range (-200-0) meV can be described by two straight lines intersecting at the binding energy $E_0 \sim (50 - 70)$ meV [3,3]. The existence of the energy scale $E_0$ could be attributed to the interaction between electrons and the collective excitations, for instance, phonons [1]. On the other hand, the analysis of the experimental data on the single-particle electron spectra demonstrates that the perturbation of the spectra by phonons or other collective states is in fact very small, therefore, the corresponding state of electrons has to be described as a strongly collectivized quantum state and was named “quantum protectorate” [3]. Thus, the interpretation of the above mentioned kink as a consequence of electron-phonon interaction can very likely be in contradiction with the quantum protectorate concept. To describe the single-particle spectra and the kink, the assumption can be used that the electron system of high-$T_c$ superconductor has undergone the fermion-condensation quantum phase transition (FCQPT). This transition serves as a boundary separating the normal Fermi liquid from the strongly correlated liquid of a new type [3,3] and fulfills the quantum protectorate requirements [3]. The FCQPT appears in many-electron systems at relatively low density, when the effective interaction constant becomes sufficiently large. In ordinary electron liquid, this constant is directly proportional to the dimensionless parameter $r_s \sim 1/p_F a_B$, where $a_B$ is the Bohr radius and $p_F$ is the Fermi momentum. The FCQPT appears at a certain value $r_s = r_{FC}$, and precedes formation of charge-density waves or stripes [17], which are observed in underdoped samples of copper oxides [1]. This is why the formation of the FCQPT in copper oxides can be considered as a quite determinate process stemming from general properties of a low-density electron liquid [3].

In this letter we address the mentioned above problems in the physics of high-temperature superconductivity and demonstrate that these problems can be resolved in a theory based on the combination of the FCQPT and the conventional theory of superconductivity. We show that the FCQPT manifests itself in large values of $\Delta_1$, $T_c$ and $T^*$.

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We trace also the transition from conventional superconductors to high-$T_c$ ones as a function of the parameter $r_s$, or as a function of the doping level.

At $T = 0$, the ground state energy $E_{gs}[\kappa(p), n(p)]$ of two-dimensional electron liquid is a functional of the order parameter of the superconducting state $\kappa(p)$ and of the occupation numbers $n(p)$ and is determined by the known equation of the weak-coupling theory of superconductivity

$$E_{gs} = E[n(p)] + \int \lambda_0 V(p_1, p_2) \kappa(p_1) \kappa^*(p_2) \frac{dp_1 dp_2}{(2\pi)^4}. \quad (1)$$

Here $E[n(p)]$ is the ground-state energy of normal Fermi liquid, $n(p) = v^2(p)$ and $\kappa(p) = v(p)\sqrt{1 - v^2(p)}$. It is assumed that the pairing interaction $\lambda_0 V(p_1, p_2)$ is weak. Minimizing $E_{gs}$ with respect to $\kappa(p)$ we obtain the equation connecting the single-particle energy $\varepsilon(p)$ to $\Delta(p)$,

$$\varepsilon(p) - \mu = \Delta(p)\frac{1 - 2v^2(p)}{2\kappa(p)}. \quad (2)$$

The single-particle energy $\varepsilon(p)$ is determined by the Landau equation, $\varepsilon(p) = \delta E[n(p)]/\delta n(p)$ \[12\], and $\mu$ is chemical potential. The equation for superconducting gap $\Delta(p)$ takes form

$$\Delta(p) = -\int \lambda_0 V(p, p_1) \kappa(p_1) \frac{dp_1}{4\pi^2} = -\frac{1}{2} \int \lambda_0 V(p, p_1) \Delta(p_1) \frac{dp_1}{\sqrt{\varepsilon(p_1) - \mu}^2 + \Delta^2(p_1)} \frac{dp_1}{4\pi^2}. \quad (3)$$

If $\lambda_0 \to 0$, then, the gap $\Delta(p) \to 0$, and Eq. (2) reduces to the equation proposed in [7]

$$\varepsilon(p) - \mu = 0, \text{ if } 0 < n(p) < 1; \ p_i \leq p \leq p_f. \quad (4)$$

At $T = 0$, Eq. (4) defines a particular state of Fermi liquid with the fermion condensate (FC) for which the modulus of the order parameter $|\kappa(p)|$ has finite values in the $L_{FC}$ range of momenta $p_i \leq p \leq p_f$, and $\Delta_1 \to 0$ in the $L_{FC}$. Such a state can be considered as superconducting, with infinitely small value of $\Delta_1$ so that the entropy of this state is equal to zero. It is obvious, that this state, being driven by the quantum phase transition, disappears at $T > 0$. When $p_i \to p_F \to p_f$, Eq. (4) determines the point $r_{FC}$ at which the FCQPT takes place. It follows from Eq. (4) that the system brakes into two quasiparticle subsystems: the first subsystem in the $L_{FC}$ range is occupied by the quasiparticles with the effective mass $M_{FC}^* \to \infty$, while the second one is occupied by quasiparticles with finite mass $M_{FC}^*$ and momenta $p < p_i$. If $\lambda_0 \neq 0$, $\Delta_1$ becomes finite, leading to finite value of the effective mass $M_{FC}^*$ in $L_{FC}$, which can be obtained from Eq. (2) \[8\]

$$M_{FC}^* \simeq p_F \frac{p_f - p_i}{2\Delta_1}. \quad (5)$$

As to the energy scale, it is determined by the parameter $E_0$:

$$E_0 = \varepsilon(p_f) - \varepsilon(p_i) \simeq 2\frac{(p_f - p_F)p_F}{M_{FC}^*} \simeq 2\Delta_1. \quad (6)$$

Thus, a system with the FC has the single-particle spectrum of a universal form and possesses quantum protectorate features at $T \ll T_f$, with $T_f$ being a temperature, at which the effect of the FCQPT disappears.

We assume that the range $L_{FC}$ is small, $(p_f - p_F)/p_F \ll 1$, and $2\Delta_1 \ll T_f$ so that the order parameter $\kappa(p)$ is governed mainly by the FC \[8\]. To solve Eq. (2) analytically, we take the Bardeen-Cooper-Schrieffer (BCS) approximation for the interaction \[13\]: $\lambda_0 V(p, p_1) = -\lambda_0$ if $|\varepsilon(p) - \mu| \leq \omega_D$, the interaction is zero outside this region, with $\omega_D$ being the characteristic phonon energy. As a result, the gap becomes dependent only on the temperature, $\Delta(p) = \Delta_1(T)$, being independent of the momentum, and Eq. (2) takes the form
Here we set $\Delta_1$ from Eq. (10) that the isotope effect is presented. A more detailed analysis will be published elsewhere.

Putting $\Delta_1 \to 0$, Eq. (7) reduces to the BCS equation. On the other hand, assuming that $E_0 \leq 2\omega_D$ and omitting the second integral in the right hand side of Eq. (7), we obtain

$$\Delta_1(0) = \frac{\lambda_0 p_F (p_f - p_F)}{2\pi} \ln(1 + \sqrt{2}) = 2\beta \varepsilon_F \frac{p_f - p_F}{p_F} \ln(1 + \sqrt{2}),$$

where the Fermi energy $\varepsilon_F = p_F^2 / 2M_F^*$, and dimensionless coupling constant $\beta = \lambda_0 M_F^*/2\pi$. Taking the usual values of the dimensionless coupling constant $\beta \approx 0.3$, and $(p_f - p_F)/p_F \approx 0.2$, we get from Eq. (7) the large value of $\Delta_1(0) \approx 0.1 \varepsilon_F$, while for normal metals one has $\Delta_1(0) \approx 10^{-3} \varepsilon_F$. Taking into account the omitted integral, we obtain

$$\Delta_1(0) \approx 2\beta \varepsilon_F \frac{p_f - p_F}{p_F} \ln(1 + \sqrt{2}) \left(1 + \beta \ln \frac{2\omega_D}{E_0}\right).$$

It is seen from Eq. (9) that the correction due to the second integral is small, provided $E_0 \approx 2\omega_D$. Below we show that $2T_c \approx \Delta_1(0)$, which leads to the conclusion that there is no isotope effect since $\Delta_1$ is independent of $\omega_D$. But this effect is restored as $E_0 \to 0$. Assuming $E_0 \approx \omega_D$ and $E_0 > \omega_D$, we see that Eq. (7) has no standard solutions $\Delta(p) = \Delta_1(0)$ because $\omega_D < \varepsilon(p \approx p_f) - \mu$ and the interaction vanishes at these momenta. The only way to obtain solutions is to restore the condition $E_0 < \omega_D$. For instance, we can define the momentum $p_D < p_f$ such that

$$\Delta_1(0) = 2\beta \varepsilon_F \frac{p_f - p_D}{p_F} \ln(1 + \sqrt{2}) = \omega_D,$$

while the other part in the $L_{FC}$ range can be occupied by a gap $\Delta_2$ of the different sign, $\Delta_1(0)/\Delta_2 < 0$. It follows from Eq. (10) that the isotope effect is presented. A more detailed analysis will be published elsewhere.

At $T \to T_c$, Eqs. (5) and (6) are replaced by the equation, which is valid also at $T_c \leq T \ll T_f$:

$$M_{FC}^* \approx p_F \frac{p_f - p_i}{4T_c}, \quad E_0 \approx 4T_c; \quad \text{if } T_c \leq T: \quad \Delta_1(0) = p_F \frac{p_f - p_i}{4T}, \quad E_0 \approx 4T.$$ 

Equation (7) is replaced by its conventional finite temperature generalization

$$1 = N_{FC} \lambda_0 \int_{0}^{E_0/2} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1(T)^2}} \tanh \frac{\sqrt{\xi^2 + \Delta_1(T)^2}}{2T} + N_{L} \lambda_0 \int_{E_0/2}^{\omega_D} \frac{d\xi}{\sqrt{\xi^2 + \Delta_1(T)^2}} \tanh \frac{\sqrt{\xi^2 + \Delta_1(T)^2}}{2T}.$$ 

Putting $\Delta_1(T \to T_c) \to 0$, we obtain from Eq. (12)

$$2T_c \approx \Delta_1(0),$$

with $\Delta_1(0)$ being given by Eq. (9). By comparing Eqs. (5), (11) and (13), we see that $M_{FC}^*$ and $E_0$ are almost temperature independent at $T \leq T_c$. Now a few remarks are in order. One can define $T_c$ as the temperature when $\Delta_1(T_c) \equiv 0$. At $T \geq T_c$, Eq. (12) has only the trivial solution $\Delta_1 \equiv 0$. On the other hand, $T_c$ can be defined as a temperature at which the superconductivity vanishes. Thus, we deal with two different definitions, which can lead to different temperatures. It was shown that in the case of the d-wave superconductivity, taking place in the presence of the FC, there exist a nontrivial solutions of Eq. (12) at $T_c \leq T \leq T^*$ corresponding to the pseudogap state.
It happens when the gap occupies only such a part of the Fermi surface, which shrinks as the temperature increases. Here $T^*$ defines the temperature at which $\Delta_1(T^*) \equiv 0$ and the pseudogap state vanishes. The superconductivity is destroyed at $T_c$, and the ratio $2\Delta_1/T_c$ can vary in a wide range and strongly depends upon the material’s properties, as it follows from consideration given in \cite{14,15}. Therefore, provided there exists the pseudogap above $T_c$, then $T_c$ is to be replaced by $T^*$, and Eq. (13) takes the form

$$2T^* \simeq \Delta_1(0).$$

(14)

The ratio $2\Delta_1/T_c$ can reach very high values. For instance, in the case of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, where the superconductivity and the pseudogap are considered to be of the common origin, $2\Delta_1/T_c$ is about 28, while the ratio $2\Delta_1/T^* \simeq 4$, which is also valid for various cuprates \cite{16}. Thus, Eq. (14) gives good description of the experimental data. We remark that Eq. (7) gives also good description of the maximum gap $\Delta_1$ in the case of the d-wave superconductivity \cite{14,15}, because the different regions with the maximum absolute value of $\Delta_1$ and the maximal density of states can be considered as disconnected \cite{17}. Therefore, the gap in this region is formed by attractive phonon interaction which is approximately independent of the momenta. According to the model proposed in \cite{9}, the doping level $x$ presents a straight line crossing the abscissa at the point ($x_{FC} - x$) $\sim (r_s - r_{FC}) \sim (p_f - p_i)/p_F$. The value $x_{FC}$ matches $r_{FC}$ when defining the point at which the FCQPT takes place. We assume that the dopant concentration $x_{FC}$ corresponds to the highly overdoped regime at which slight deviations from the normal Fermi liquid are observed \cite{13}. Then, from Eqs. (8) and (9) it follows that $\Delta_1$ is directly proportional to $(x_{FC} - x)$. From Eq. (14) one finds that the function $T^*(x)$ presents a straight line crossing the abscissa at the point $(x_{FC} \simeq x)$, while in the vicinity of this point $T^*$ merges with $T_c$ and both of them tends to zero.

Now we turn to the calculations of the gap and the specific heat at the temperatures $T \rightarrow T_c$. It is worth noting that this consideration is valid provided $T^* = T_c$, otherwise the considered below discontinuity is smoothed out over the temperature range $T^* - T_c$. For the sake of simplicity, we calculate the main contribution to the gap and the specific heat coming from the FC. The function $\Delta_1(T \rightarrow T_c)$ is found from Eq. (12) upon expanding the right hand side of the first integral in powers of $\Delta_1$ and omitting the contribution from the second integral on the right hand side of Eq. (12). This procedure leads to the following equation

$$\Delta_1(T) \simeq 3.4T_c\sqrt{1 - \frac{T}{T_c}}. \quad (15)$$

Thus, the gap in the spectrum of the single-particle excitations has quite usual behavior. To calculate the specific heat, the conventional expression for the entropy $S$ \cite{13} can be used

$$S = 2 \int [f(p) \ln f(p) + (1 - f(p)) \ln(1 - f(p))] \frac{dp}{(2\pi)^2}, \quad (16)$$

where

$$f(p) = \frac{1}{1 + \exp[E(p)/T]}; \quad E(p) = \sqrt{(\varepsilon(p) - \mu)^2 + \Delta^2_1(T)}. \quad (17)$$

The specific heat $C$ is determined by

$$C = T \frac{dS}{dT} \simeq 4N_{FC} \left[ \int_{0}^{E_0} f(E)(1 - f(E)) \left( E^2 + T\Delta_1(T) \frac{d\Delta_1(T)}{dT} \right) d\xi \right] + 4N_L \left[ \int_{E_0}^{\omega_b} f(E)(1 - f(E)) \left( E^2 + T\Delta_1(T) \frac{d\Delta_1(T)}{dT} \right) d\xi \right]. \quad (18)$$
When deriving Eq. (18) we again use the variable $\xi$ and the densities of states $N_{FC}, N_L$, just as before in connection to Eq. (7), and use the notation $E = \sqrt{\xi^2 + \Delta_f^2(T)}$. Equation (18) predicts the conventional discontinuity $\delta C$ in the specific heat $C$ at $T_c$, because of the last term in the square brackets of Eq. (18). Upon using Eq. (15) to calculate this term and omitting the second integral on the right hand side of Eq. (18), we obtain

$$\delta C \simeq \frac{3}{2\pi} (p_f - p_i)p_F. \quad (19)$$

In contrast to the conventional result when the discontinuity is a linear function of $T_c$, $\delta C$ is independent of the critical temperature $T_c$ because the density of state varies inversely with $T_c$ as it follows from Eq. (11). Note, that deriving Eq. (19) we take into account the main contribution coming from the FC. This contribution vanishes as soon as $E_0 \to 0$ and the second integral of Eq. (18) gives the conventional result.

Consider the lineshape $L(q, \omega)$ of the single-particle spectrum which is a function of two variables. Measurements carried out at a fixed binding energy $\omega = \omega_0$, where $\omega_0$ is the energy of a single-particle excitation, determine the lineshape $L(q, \omega) = \omega_0$ as a function of the momentum $q$. We have shown above that $M_{FC}^*$ is finite and constant at $T \leq T_c$. Therefore, at excitation energies $\omega \leq E_0$ the system behaves like an ordinary superconducting Fermi liquid with the effective mass given by Eq. (5) [1]. At $T_c \leq T$ the low energy effective mass $M_{FC}^*$ is finite and is given by Eq. (11). Once again, at the energies $\omega < E_0$, the system behaves as a Fermi liquid, the single-particle spectrum is well defined, while the width of single-particle excitations is of the order of $T$ [1][19]. This behavior was observed in experiments on measuring the lineshape at a fixed energy [3]. It is pertinent to note that recent measurements of the lineshape suggest that quasiparticle excitation even in the $(\pi, 0)$ region of the Brillouin zone of Bi$_2$Sr$_2$CaCu$_2$Q$_{8+\delta}$ (Bi2212) are much better defined than previously believed from earlier Bi2212 data [20]. The lineshape can also be determined as a function $L(q = q_0, \omega)$ at a fixed $q = q_0$. At small $\omega$, the lineshape resembles the one considered above, and $L(q = q_0, \omega)$ has a characteristic maximum and width. At energies $\omega \geq E_0$, quasiparticles with the mass $M_{FC}^*$ come into play, leading to a growth of the function $L(q = q_0, \omega)$. As a result, the function $L(q = q_0, \omega)$ possesses the known peak-dip-hump structure [21] directly defined by the existence of the two effective masses $M_{FC}^*$ and $M_L^*$ [3]. To have more quantitative and analytical insight into the problem we use the Kramers-Krönig transformation to construct the imaginary part $\text{Im}\Sigma(p, \varepsilon)$ of the self-energy $\Sigma(p, \varepsilon)$ starting with the real one $\text{Re}\Sigma(p, \varepsilon)$ which defines the effective mass

$$\frac{1}{M^*} = \left( \frac{1}{M} + \frac{1}{p_F} \frac{\partial \text{Re}\Sigma}{\partial \varepsilon} \right) / \left( 1 - \frac{\partial \text{Re}\Sigma}{\partial \varepsilon} \right). \quad (20)$$

Here $M$ is the bare mass, while the relevant momenta $p$ and energies $\varepsilon$ are subjected to the conditions: $|p - p_F|/p_F \ll 1$, and $\varepsilon/\varepsilon_F \ll 1$. We take $\text{Re}\Sigma(p, \varepsilon)$ in the simplest form which accounts for the change of the effective mass at the energy scale $E_0$:

$$\text{Re}\Sigma(p, \varepsilon) = -\varepsilon \frac{M_{FC}^2}{M} + \left( \varepsilon - \frac{E_0}{2} \right) \frac{M_{FC}^* - M_L^*}{M} \left[ \theta(\varepsilon - E_0/2) + \theta(-\varepsilon - E_0/2) \right]. \quad (21)$$

Here $\theta(\varepsilon)$ is the step function. Note that in order to ensure a smooth transition from the single-particle spectrum characterized by $M_{FC}^*$ to the spectrum defined by $M_L^*$ the step function is to be substituted by some smooth function. Upon inserting Eq. (21) into Eq. (20) we can check that inside the interval $(-E_0/2, E_0/2)$ the effective mass $M^* \simeq M_{FC}^*$, and outside the interval $M^* \simeq M_L^*$. By applying the Kramers-Krönig transformation to $\text{Re}\Sigma(p, \varepsilon)$, we obtain the imaginary part of the self-energy,

$$\text{Im}\Sigma(p, \varepsilon) \sim \varepsilon^2 \frac{M_{FC}^*}{\varepsilon_F M} + \frac{M_{FC}^* - M_L^*}{M} \left[ \varepsilon \ln \left| \frac{\varepsilon + E_0/2}{\varepsilon - E_0/2} \right| + E_0/2 \ln \left| \frac{\varepsilon^2 - E_0^2/4}{E_0^2/4} \right| \right]. \quad (22)$$
We can see from Eq. (22) that at $\varepsilon/E_0 \ll 1$ the imaginary part is proportional to $\varepsilon^2$; at $2\varepsilon/E_0 \simeq 1$ $\text{Im} \Sigma \sim \varepsilon$; at $E_0/\varepsilon \ll 1$ the main contribution to the imaginary part is approximately constant. This is the behavior that gives rise to the known peak-dip-hump structure. Then, it is seen from Eq. (22) that when $E_0 \to 0$ the second term on the right hand side tends to zero, the single-particle excitations become better defined resembling that of a normal Fermi liquid, and the peak-dip-hump structure eventually vanishes. On the other hand, the quasiparticle amplitude $a(p)$ is given by

$$\frac{1}{a(p)} = 1 - \frac{\partial \text{Re} \Sigma(p, \varepsilon)}{\partial \varepsilon}.$$  \hspace{1cm} (23)

It follows from Eq. (20) that the quasiparticle amplitude $a(p)$ rises as the effective mass $M^*_F C$ decreases. Since $M^*_F C \sim (p_f - p_i) \sim (x_{FC} - x) \[5\]$, we are led to a conclusion that the amplitude $a(p)$ rises as the doping level rises, and the single-particle excitations become better defined in highly overdoped samples. It is worth noting that such a behavior was observed experimentally in so highly overdoped Bi2212 that the gap size is about 10 meV \[18\]. Such a small size of the gap testifies that the region occupied by the FC is small since $E_0/2 \simeq \Delta_1$.

In conclusion, we have shown that the theory of high temperature superconductivity based on the fermion-condensation quantum phase transition and on the conventional theory of superconductivity permits to describe high values of $T_c$, $T^*$ and of the maximum value of the gap, which may be as big as $\Delta_1 \sim 0.1 \varepsilon_F$. We have also traced the transition from conventional superconductors to high-$T_c$ and demonstrated that in the highly overdoped cuprates the single-particle excitations become much better defined, resembling that of a normal Fermi liquid.

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