Nuclear symmetry energy and the role of the tensor force

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Using the Hellmann–Feynman theorem we analyze the contribution of the different terms of the nucleon-nucleon interaction to the nuclear symmetry energy \(E_{\text{sym}}\) and the slope parameter \(L\). The analysis is performed within the microscopic Brueckner–Hartree–Fock approach using the Argonne V18 potential plus the Urbana IX three-body force. We find that the main contribution to \(E_{\text{sym}}\) and \(L\) is due to the tensor component of the nuclear force.

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The nuclear symmetry energy, defined as the difference between the energies of neutron and symmetric matter, and in particular its density dependence, is a crucial ingredient to understand many important properties of isospin-rich nuclei and neutron stars [1–3]. Experimental information on the density dependence of the symmetry energy \(E_{\text{sym}}(\rho)\) below, close to and above saturation density \(\rho_0\) can be obtained from the analysis of data of isospin diffusion measurements [4], giant [5] and pygmy resonances [6], isobaric analog states [7], isoscaling [8] or meson production in heavy ion collisions [9, 10]. Accurate measurements of the neutron skin thickness \(\delta R = \sqrt{(R^p_n)} - \sqrt{(R^p_p)}\) in heavy nuclei, via parity-violating electron scattering experiments [11, 12] or by means of antiprotonic atom data [13, 14], can also help to constraint \(E_{\text{sym}}(\rho)\), since its derivative is strongly correlated with \(\delta R\) [15]. Additional information on \(E_{\text{sym}}(\rho)\) can be extracted from the astrophysical observations of compact objects which open a window into both the bulk and microscopic properties of nuclear matter at extreme isospin asymmetries [3]. In particular, the characterization of the core-crust transition in neutron stars [10, 20], or the analysis of power-law correlations, such as the relation between the radius of a neutron star and the equation of state [21] can put stringent constraints on \(E_{\text{sym}}(\rho)\). Theoretically \(E_{\text{sym}}(\rho)\) has been determined using both phenomenological and microscopic many-body approaches. Phenomenological approaches, either relativistic or non-relativistic, are based on effective interactions that are frequently built to reproduce the properties of nuclei [22]. Since many of such interactions are built to describe systems close to the symmetric case, predictions at high asymmetries should be taken with care. Skyrme–Hartree–Fock [23] and relativistic mean field [24] calculations are the most popular ones among them. Microscopic approaches start from realistic nucleon-nucleon (NN) interactions that reproduce the scattering and bound state properties of the free two-nucleon system and include naturally the isospin dependence [25]. The in-medium correlations are then built using many-body techniques that microscopically account for isospin asymmetric effects such as, for instance, the difference in the Pauli blocking factors of neutrons and protons in asymmetric matter. Among this type of approaches the most popular ones are the Brueckner–Bethe–Goldstone (BBG) [26] and the Dirac–Brueckner–Hartree–Fock (DBHF) [27] theories, the variational method [28], the correlated basis function (CBF) formalism [29], the self-consistent Green’s function technique (SCGF) [30] or, recently, the V_{lowq} approach [31]. Nevertheless, in spite of the experimental [32] and theoretical [33] efforts carried out to study the properties of isospin-asymmetric nuclear systems, \(E_{\text{sym}}(\rho)\) is still uncertain. Its value \(E_{\text{sym}}\) at saturation is more or less well established (~ 30 MeV), and its behavior below saturation is now much better known [34]. However, for densities above \(\rho_0\), \(E_{\text{sym}}(\rho)\) is not well constrained yet, and the predictions from different approaches strongly diverge. Why \(E_{\text{sym}}(\rho)\) is so uncertain is still an open question whose answer is related to our limited knowledge of the nuclear force, and in particular of its spin and isospin dependence [35, 36].

In this letter we analyze the contribution of the different terms of the NN interaction to \(E_{\text{sym}}\) and the slope parameter \(L = 3 \rho_0 \partial E_{\text{sym}}(\rho)/\partial \rho\). The analysis is carried out with the help of the Hellmann–Feynman theorem [42] within the framework of the microscopic Brueckner–Hartree–Fock (BHF) approach [20]. We employ the Argonne V18 (Av18) potential [43] supplemented with the Urbana IX three-body force [44] which for the use in the BHF approach is reduced to an effective two-body density-dependent force by averaging over the third nucleon [46]. We find that the tensor term of the nuclear force gives the largest contribution to both \(E_{\text{sym}}\) and \(L\).

The BHF approach is the lowest order of the BBG many-body theory [24]. In this theory, the ground state energy of nuclear matter is evaluated in terms of the so-called hole-line expansion, where the perturbative diagrams are grouped according to the number of independent hole-lines. The expansion is derived by means of the in-medium two-body scattering G-matrix. The G-matrix, that takes into account the effect of the Pauli principle on the scattered particles and the in-medium
potentials to the energy of neutron matter $E_{NM}$, the kinetic contribution to the symmetry energy becoming then negative. We also note that the kinetic contribution to $L$ is smaller than the corresponding one of the FFG ($L_{FFG} \sim 29.2$ MeV). The major contribution to both $E_{sym}$ and $L$ is due to the potential part. Note that, in fact, this contribution is practically equal to the total value of $E_{sym}$ and it represents $\sim 78\%$ of $L$.

Tables II and III show the partial wave, and the spin (S) and isospin (T) channel decompositions of the potential part of $E_{NM}$, $E_{SM}$, $E_{sym}$ and $L$ at $\rho_0$. Contributions up to $J = 8$ have been considered. We observe that the spin-triplet ($S = 1$) and isospin-singlet ($T = 0$) channel,
TABLE III: Spin (S) and isospin (T) channel decomposition of the potential part of $E_{NM}$, $E_{SM}$, $E_{sym}$ and $L$. Units are given in MeV.

| $(S, T)$ | $E_{NM}$ | $E_{SM}$ | $E_{sym}$ | $L$ |
|----------|----------|----------|----------|-----|
| (0, 0)   | 0        | 5.600    | −5.600   | −21.457 |
| (0, 1)   | −29.889  | −23.064  | −6.825   | −17.950 |
| (1, 0)   | −49.836  | 49.836   | 90.561   |     |
| (1, 1)   | −4.362   | −2.224   | −2.138   | 0.450 |

and in particular the $^3S_1$ wave, gives the largest contribution to both $E_{sym}$ and $L$. This is due, as we explicitly show in the following, to the effect of the tensor component of nuclear force that dominates the potential contribution to the symmetry energy and $L$, mainly through the $^3S_1 - ^1P_1$ channel. Note that this channel, which gives the major contribution to the energy of symmetric matter, does not contribute to neutron matter. Note also that isospin-triplet ($T = 1$) channels give similar contributions to both $E_{NM}$ and $E_{sym}$ which almost cancel out in $E_{sym}$. Similar arguments have been pointed out by other authors [33–42].

Next, we analyze the role played by the different terms of the nuclear force, particularly the one of the tensor, and the slope parameter of the potential part of $E_{NM}$, $E_{SM}$, $E_{sym}$ and $E_{sym}$. Our study has been done within the framework of the BHF approach using the Av18 potential (denoted as $\langle V \rangle$) and the reduced Urbana force (denoted as $\langle U \rangle$). Units are given in MeV.

As we said above, the Urbana IX three-body force is reduced to an effective density-dependent two-body force when used in the BHF approach. For simplicity, in the following we refer to it as reduced Urbana Force. This force is made of 3 components of the type $u_p \langle r_{ij} \rangle \rho^O_{ij}$ where $O^3_{ij} = 1, (\vec{s}_i \cdot \vec{s}_j)(\vec{r}_i \cdot \vec{r}_j), (\vec{S} \cdot \vec{S})$, introducing additional central, spin, and tensor terms (see e.g., Baldo and Ferreira in Ref. [46] for details).

The separate contributions to $E_{NM}$, $E_{SM}$, $E_{sym}$ and $L$ from the various components of the Av18 potential and the reduced Urbana force are given in Table IV. The contribution from the tensor component to $E_{sym}$ and $L$ (contributions $\langle V_{Si} \rangle$ and $\langle V_{Si}(\vec{s}, \vec{r}) \rangle$ from the Av18 potential, and $\langle U_{Si}(\vec{s}, \vec{r}) \rangle$ from the reduced Urbana force) is 36.056 MeV and 69.968 MeV, respectively. These results clearly confirm that the tensor force gives the largest contribution to both $E_{sym}$ and $L$. The contributions from the other components are either negligible, as for instance the contribution from the charge symmetry breaking terms ($\langle V_{T_{ij}} \rangle$, $\langle V_{T_{ij}} \rangle$, $\langle V_{S_{ij}} \rangle$, $\langle V_{S_{ij}(\tau_1 + \tau_2)} \rangle$), or almost cancel out.

In summary, using the Hellmann–Feynman theorem we have evaluated the separate contribution of the different terms of the nuclear force to the nuclear symmetry energy $E_{sym}$ and the slope parameter $L$. Our study has been done within the framework of the BHF approach using the Av18 potential plus an effective density-dependent two-body force deduced from the Urbana IX three-body one. Our results show that the potential part of the nuclear Hamiltonian gives the main contribution to both $E_{sym}$ and $L$. The kinetic contribution to $E_{sym}$ is very small and negative in agreement with the recent results of Xu and Li [41]. We have performed a partial wave, and a spin-isospin channel decomposition of the potential part of $E_{sym}$ and $L$, showing that the major contribution to them is given by the spin-triplet ($S = 1$) and isospin-singlet ($T = 0$) channel. This is due, as we have ex-
explicitly shown, to the dominant effect of the tensor force which gives the largest contribution to both $E_{\text{sym}}$ and $L$. In conclusion, our results confirm the critical role of the tensor force in the determination of the symmetry energy and its density dependence.

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