Spontaneous baryogenesis in warm inflation

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We discuss spontaneous baryogenesis in the warm inflation scenario. In contrast with standard inflation models, radiation always exists in the warm inflation scenario, and the inflaton must be directly coupled to it. Also, the transition to the post-inflationary radiation dominated phase is smooth and the entropy is not significantly increased at the end of the period of inflation. In addition, after the period of warm inflation ends, the inflaton does not oscillate coherently but slowly rolls. We show that as a consequence of these features of warm inflation, the scenario can well accommodate the spontaneous baryogenesis mechanism, provided that the decoupling temperature $T_D$ of the baryon or the $B - L$ violating interactions is higher than the temperature of radiation during the late stages of inflation.

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I. INTRODUCTION

Inflation gives the most natural solution to some of the problems of standard big bang cosmology such as the horizon problem and the flatness problem, and provides a causal mechanism for the origin of the primordial density perturbations whose present state is being mapped to high precision by observational cosmologists [1]. There are two types of inflation models. The first (to which most of the proposed inflation models belong) is isentropic: any preexisting radiation before the onset of inflation is completely diluted away during inflation, and the radiation must then be regenerated at the end of the phase of inflation during inflationary reheating. The other is nonisentropic and called warm inflation [2] (see also [3]): here radiation is continuously produced by the decay of the inflaton, the scalar field which generates inflation, and this decay in turn supports the slow-roll behavior of the inflaton. In this scenario, the temperature of radiation remains large during the period of inflation, and no nonadiabatic radiation generation mechanism needs to be postulated at the end of inflation.

There are significant differences between the warm inflation scenario and standard isentropic inflation. Most importantly for the purpose explored in this paper, the inflaton should be coupled to ordinary matter, whereas in standard inflation it is usually assumed to be a gauge singlet. Also, since radiation always exists, the transition from the inflationary period to the radiation dominated period is straightforward. In particular, after the end of the inflationary phase the inflaton field will in general still be rolling slowly, rather than oscillating about the minimum of its potential as happens in the standard inflationary models. In addition, in warm inflation the primordial density fluctuations originate from thermal fluctuations rather than quantum fluctuations of the inflaton [4].

A requirement for warm inflation is that the constant rate which describes the decay of the inflaton into particles dominates over the Hubble damping coefficient in the inflaton equation of motion. It is a nontrivial problem to obtain such a large decay width and realize warm inflation. The dynamics of warm inflation has been investigated in the context of quantum field theory [5, 6], but the dissipative dynamics is not yet understood fully. It has been suggested that, in order to realize warm inflation, the inflaton must couple to a very large number of particle species [6]. Some models which achieve this from first principles were proposed [7].

We will show that as a consequence of the above-mentioned differences between warm inflation and standard inflation, it is easier to obtain spontaneous baryogenesis in the context of warm inflation than in the context of standard inflation. Most importantly, the inflaton must be coupled to ordinary matter in a warm inflation scenario, and thus it is natural to assume that it is not a gauge singlet, whereas in standard inflation it is usually assumed to be a gauge singlet. In addition, in the standard isentropic inflation scenario, the inflaton oscillates coherently after inflation. Due to the friction term caused by the current violating operator, this oscillation becomes asymmetric so that a baryon charge or a $B - L$ charge is generated. This mechanism also applies to the oscillation of a Nambu-Goldstone boson like an axion and has been discussed in detail in Ref. [8]. As shown in that reference, the oscillations lead to a suppression of the strength of net baryogenesis over what would be obtained using the naive classical analysis. However, in warm inflation, the inflaton does not oscillate coherently but continues to slowly roll even after inflation.
ends. Thus, the analysis of spontaneous lepto/baryogenesis in the warm inflation scenario will be different. As we will show, the classical analysis is justified in this case and hence baryogenesis will be more efficient.

We shall assume that among the many particles the inflaton $\phi$ couples to, it will also couple - albeit derivatively - to the baryon current or to the $B - L$ current. Following the arguments by Cohen and Kaplan [9], we will show that baryo/leptogenesis may be possible if the inflaton has a derivative coupling to such a current given by

$$\mathcal{L}_{\text{eff}} = \frac{1}{M} \partial_\mu \phi J^\mu, \quad (1)$$

where $M$ is the cutoff scale which describes the physics of baryon number violation. In the above, $J^\mu$ is either the baryon current or the $B - L$ current. Integrating this coupling by parts, we have an interaction term given by

$$\mathcal{L}_{\text{eff}} = -\frac{1}{M} \phi \partial_\mu J^\mu. \quad (2)$$

If baryon number conservation or $B - L$ number conservation is violated, the divergence does not disappear and is replaced by a current violating operator, which can cause baryo/leptogenesis like in Affleck-Dine baryogenesis [10].

In this paper, we explore in detail the spontaneous baryo/leptogenesis mechanism in warm inflation. In the next section, we briefly review the warm inflation scenario. In Sec III, we discuss the possibility of spontaneous baryo/leptogenesis in warm inflation. In the final section, we summarize our results.

II. DYNAMICS OF WARM INFLATION

In this section, we briefly review the dynamics of warm inflation. First of all, we assume that the inflaton couples to a large number of particles, generating a large and time-independent dissipative constant $\Gamma$ (related to the decay width). Then, the equation of motion of the inflaton and the time development of the energy density of radiation $\rho_r$ are given by

$$\ddot{\phi} + (3H + \Gamma) \dot{\phi} + V'(\phi) = 0, \quad (3)$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma \dot{\phi}^2 \quad (4)$$

with the potential given by $V(\phi) = \frac{1}{2}m^2\phi^2$. Here, the derivative of the potential is taken with respect to $\phi$, and $H$ is the Hubble parameter given by

$$H^2 = \frac{1}{3M_G^2}(\rho_\phi + \rho_r), \quad (5)$$

with $\rho_\phi$ being the energy density of the inflaton and $M_G \simeq 2.4 \times 10^{18}$ GeV denoting the reduced Planck scale.

For successful warm inflation, we require the following five conditions:

(i) $\rho_r \ll V$,

(ii) $\frac{1}{2}\ddot{\phi}^2 \ll V$,

(iii) $H \ll \Gamma$,

(iv) $|\dot{\rho}_r| \ll 4H\rho_r, \Gamma \dot{\phi}^2$,

(v) $|\dot{\phi}| \ll (3H + \Gamma)\dot{\phi}, V'(\phi)$.

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1 Strictly speaking, as shown in [5, 6, 7], the dissipative coefficient $\Gamma$ has a more complicated form and is in fact nonlocal in time. However, as shown in [4-6], by using the Markovian adiabatic approximation, the dissipative term becomes local in time and has a rather simple form in the high temperature limit. Generally speaking, the dissipative coefficient still depends on the field value. But, if we consider a Yukawa interaction and the self-energy contribution, for example, the dissipative coefficient becomes proportional to the temperature, which is a constant in the context of this paper. Thus, as a first approximation, we assume that the dissipative term is local in time, proportional to the time derivative of the field, and in particular that its coefficient is a constant.
The first two conditions are required in order to have inflation, the third is the criterion for warm inflation as opposed to standard inflation, without the fourth requirement it would be unreasonable to assume that $\Gamma$ is constant, and the final criterion is the slow rolling condition for the inflaton dynamics. If these conditions are satisfied, Eqs. (6) and (4) reduce to

$$\Gamma \dot{\phi} + V' \simeq 0 \iff \dot{\phi} \simeq -\frac{V'}{\Gamma} \simeq -\frac{m^2 \phi}{\Gamma},$$

(6)

$$4H\rho_r \simeq \Gamma \dot{\phi}^2 \iff \rho_r \simeq \frac{\Gamma \dot{\phi}^2}{4H} \simeq \frac{V' \dot{\phi}^2}{4H\Gamma} \simeq \sqrt{\frac{3}{8}} \frac{m^3 M_G \phi}{\Gamma},$$

(7)

$$\iff T_r = \left( \frac{30 \rho_r}{g_\star \pi^2} \right)^{\frac{1}{4}} \simeq \left( \frac{675}{4g_\star^2 \pi^4} \right)^{\frac{1}{4}} \left( \frac{m^3 M_G \phi}{\Gamma} \right)^{\frac{1}{4}},$$

(8)

where $g_\star$ is the number of the relativistic degrees of freedom.

Conditions (i) and (ii) imply the dominance of the vacuum energy. The first is satisfied for $\phi \gg \phi_{\text{end}} \equiv \sqrt{\frac{3}{2}} \frac{m}{\Gamma} M_G$,

(9)

and the second is then obeyed for all values of $\phi$ provided that

$$m \ll \Gamma.$$  

(10)

Thus, warm inflation ends at $\phi = \phi_{\text{end}}$. At that time, the temperature of radiation becomes

$$T_{\text{end}} \equiv T_r(\phi = \phi_{\text{end}}) \sim m \sqrt{\frac{M_G}{\Gamma}}.$$  

(11)

The condition (iii) implies that the dominant friction term in the inflaton equation of motion is given by the coupling to other particles rather than by the Hubble expansion and is satisfied for

$$\phi \ll \sqrt{\frac{6}{m}} \frac{\Gamma}{M_G}.$$  

(12)

The condition (iv) implies the constancy of the energy density of radiation and is valid as long as $\phi > \phi_{\text{end}}$. The last requirement is the so-called slow-roll condition and is also satisfied if $m \ll \Gamma$. Combining these results, we conclude that warm inflation takes place while $\phi$ is in the range given by

$$\phi_{\text{end}} \leq \phi \leq (\Gamma/m) M_G$$

(13)

provided that $m \ll \Gamma$.

The number $N(\phi)$ of e-foldings of inflation between when the inflaton field has the value $\phi$ in the range given by Eq. (13) and when $\phi = \phi_{\text{end}}$ can easily be estimated and yields the following relation between $\phi_N$ (the initial value of $\phi$ which gives $N$ e-foldings) and $N$:

$$N = \int H dt \simeq \frac{\Gamma}{\sqrt{6} m M_G} (\phi_N - \phi_{\text{end}}),$$

(14)

with the result $\phi_N \simeq \sqrt{6} N m M_G / \Gamma$. Taking the scale of the cosmic microwave background (CMB) anisotropies measured by the Cosmic Background Explorer (COBE) satellite to correspond to $N = 60$, then

$$\phi_{\text{COBE}} = \phi_{60} \sim 150 m M_G / \Gamma \sim 150 \phi_{\text{end}}.$$  

(15)

For $T_r > H$, which corresponds to $\phi < (M_G/m \Gamma)^{1/3} M_G$, thermal fluctuations dominate over quantum fluctuations. As shown in the last reference of [7] (see also [11]) the root mean square of fluctuations of the inflaton is given by

$$\langle (\delta \phi)^2 \rangle \simeq \frac{1}{2\pi^2} \sqrt{\Gamma T_r H},$$

(16)
Based on these initial conditions for fluctuations generated during inflation, the final amplitude of the curvature perturbation $\Phi_A$ (the relativistic gravitational potential in longitudinal gauge - see [12]) on a comoving scale whose physical wavelength equals the Hubble radius during the period of warm inflation at $\phi = \phi_N$ is given by [11, 13, 14]

$$\Phi_A \sim f H \sqrt{\left\langle (\delta \phi)^2 \right\rangle} \sim 0.02 \left( \frac{\Gamma^9 \phi_N^3}{M_G^9 m^3} \right)^{1/8},$$

(17)

where $f = 3/5$ ($2/3$) in the matter (radiation) domination (this result was derived using the full general relativistic theory [12] of linear cosmological fluctuations in [14]). The COBE normalization of CMB anisotropies requires $\Phi_A \simeq 3 \times 10^{-5}$ at $N \simeq 60$ [15]. This leads to the requirement

$$\Phi_A(N = 60) \sim 0.1 \left( \frac{\Gamma}{M_G} \right)^{1/8} \sim 10^{-5}.$$  

(18)

which yields $\Gamma \sim 10^{13}$ GeV.

In addition, the spectral index $n_s$ can be estimated to be [11, 14]

$$n_s - 1 = \frac{\dot{\phi}}{H} \frac{d}{d\phi} \ln \Phi_A \sim \frac{3\sqrt{6} m M_G}{8 \Gamma \phi_N},$$

(19)

$$\sim 0.006 \quad \text{for} \quad \phi = \phi_{COBE} \sim 150 \left( \frac{m M_G}{\Gamma} \right).$$

(20)

Even after warm inflation ends, the friction term in the equation of motion of the inflaton $\phi$ is still large so that the inflaton continues to slow-roll instead of oscillating coherently. Hence, the discussion of spontaneous baryogenesis in the reheating stage done in Ref. [9] does not directly apply to the case of warm inflation. After warm inflation ends, the dynamics of the inflaton is given by

$$\left\{ \begin{array}{l}
\phi = \phi_{\text{end}} \exp \left\{ -\frac{m^2}{\Gamma} (t - t_{\text{end}}) \right\}, \\
\dot{\phi} = -\sqrt{\frac{3}{2} m^2 M_G} \exp \left\{ -\frac{m^2}{\Gamma} (t - t_{\text{end}}) \right\},
\end{array} \right. \quad (21)$$

with $t_{\text{end}} \simeq \Gamma/m^2$.

### III. BARYO/LEPTOGENESIS IN WARM INFLATION

In this subsection, we show that spontaneous baryo/leptogenesis can easily be realized in warm inflation. As is mentioned briefly in the Introduction, we assume that the inflaton couples derivatively to the $B - L$ current via an interaction Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{M} \partial_\mu \phi J_{B - L}^\mu,$$

(22)

where $M$ is the cutoff scale. Assuming that $\phi$ is homogeneous, the above coupling becomes

$$\mathcal{L}_{\text{eff}} = \frac{\dot{\phi}}{M} n_{B - L} = \frac{\dot{\phi}}{M} (n_{B - L} - n_{\overline{B} - \overline{L}}) = \mu(t) n_{B - L},$$

(23)

with $\mu(t)$ defined as

$$\mu(t) \equiv \frac{\dot{\phi}}{M}.$$  

(24)

In contrast to the standard inflation models, radiation always exists in close to thermal equilibrium. Then, if the time derivative $\dot{\phi}$ is effectively nonzero, $\mu(t)$ becomes the effective time-dependent chemical potential, which induces

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2 Final means when the scale reenters the Hubble radius at late times.
the $B-L$ asymmetry even in thermal equilibrium. Such a thermal equilibrium baryo/leptogenesis scenario is discussed in the context of quintessence \[16, 17, 18\]. However, for this mechanism to work in the context of warm inflation, the decoupling temperature of the $B-L$ violating operator would have to be fine-tuned to be equal to the temperature of the radiation at the end of warm inflation. Obviously, it needs to be lower or equal - else there would be no thermal equilibrium for baryo- or leptogenesis to occur towards the end of the period of inflation. But the decoupling temperature cannot be lower either. Since $\phi$ decays exponentially after warm inflation ends, then if the decoupling temperature were lower than $T_{\text{end}}$, there would be a time interval after the end of warm inflation during which $B-L$ violating processes would be in thermal equilibrium but the chemical potential for $B-L$ number would be effectively zero, and during which therefore the $B-L$ number density would be driven to zero. Because of this fine-tuning, we do not consider this possibility any further.

In this paper, we consider another possibility, namely one in which the $B-L$ asymmetry is generated dynamically like in the Affleck-Dine baryogenesis scenario. In this case, the upper bound on the decoupling temperature for $B-L$ violating processes no longer is present. Taking the coupling (22) into account, the equation of motion of the inflaton is changed to

$$
\dot{\phi} + (3H + \Gamma)\phi - \frac{1}{M}(\dot{n}_{B-L} + 3Hn_{B-L}) + V'(\phi) = 0.
$$

(25)

When the $B-L$ current is not conserved, the divergence of the current does not disappear and is replaced by a current violating operator. We simply assume that the $B-L$ current is not conserved and such an operator just gives rise to an additional decay channel for the inflaton. In fact, if such a derivative coupling is, for example, derived from the Yukawa coupling or something like that, the inflaton interacts with the particles with the $B-L$ charges, which causes the violation of the $B-L$ current. As stated before and shown in \[3, 4, 5\], the dissipative term of the inflaton has a complicated form and is not necessarily local in time. However, by using the adiabatic-Markovian approximation \[3, 4, 5\], the dissipative term in the equation of motion of the inflaton can be approximated as a local term which is proportional to the time derivative of the inflaton. The coefficient of proportionality $\Gamma_{B-L}$ is still complicated and depends on the form of the interactions, but, for simplicity, we assume it is a constant. It is straightforward to extend the analysis to the case of a coefficient which depends on the field value. Thus, the equation of motion of the inflaton is changed to

$$
\dot{\phi} + (3H + \Gamma + \Gamma_{B-L})\phi + V'(\phi) = 0.
$$

(26)

If $\Gamma_{B-L}$ is much smaller than $\Gamma$, the dynamics of the inflaton is not changed much. Comparing the two Eqs. (25) and (26)\(^4\), it follows that the time evolution of the $B-L$ number density is given by

$$
\dot{n}_{B-L} + 3Hn_{B-L} = -M\Gamma_{B-L}\dot{\phi}.
$$

(27)

As given in Eq. (21), $\phi$ decays exponentially after warm inflation so that, except for the dilution due to the adiabatic expansion of the universe, $n_{B-L}$ changes significantly only during the short period $\Delta t \simeq \Gamma/m^2$ after inflation. Then, the ratio between the $B-L$ number density $n_{B-L}$ and the entropy density $s = \frac{g_{*}}{4\pi^{2}}g_{s}T^{3}$ can be roughly estimated to be

$$
\frac{n_{B-L}}{s} \simeq \frac{M\Gamma_{B-L}m^{3}}{\Gamma_{G}}M_{G}\Delta t \sqrt[\frac{1}{2}]{\frac{2\pi^{2}}{45}g_{s}T^{3}}
$$

$$
\simeq 0.02\frac{M\Gamma_{B-L}}{m^{2}}\left(\frac{\Gamma}{M_{G}}\right)^{\frac{1}{2}}.
$$

(28)

Here we took $g_{*} \sim 100$. If $T_{\text{end}}$ is higher than the temperature of the electroweak phase transition, a part of the $B-L$ asymmetry at that time is converted into the baryon asymmetry through the sphaleron processes \[19\]. Then,

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\(^3\) Strictly speaking, $\Gamma_{B-L}$ should be included in $\Gamma$. However, since we want to pay special attention to the term related to the $B-L$ current, we keep it separate.

\(^4\) As for this comparison, a subtlety was raised in Ref. \[3\]. While Eq. (26) is an operator equation, Eq. (28) is obtained after vacuum averaging. The authors of \[3\] showed that the average value $\langle \dot{n}_{B-L} \rangle$ is complicated and not given by the above simple comparison when the inflaton oscillates coherently. However, in our case, the inflaton does not oscillate coherently but slowly rolls so that this subtlety does not matter. In particular, for the values of the parameters which we use, the classical approximation is justified, and there is no energy problem as discussed in the second reference of \[3\].
the baryon-to-entropy is given by
\[ \frac{n_B}{s} \simeq \frac{8}{23} \frac{n_{B-L}}{s} \simeq 0.01 \frac{M\Gamma_{B-L}}{m^2} \left( \frac{\Gamma}{M_G} \right)^{\frac{1}{2}} \simeq 3 \times 10^{-5} \frac{M\Gamma_{B-L}}{m^2}, \] (29)
where we have used $\Gamma/M_G \sim 10^{-5}$. If we take $M\Gamma_{B-L}/m^2 \sim 3 \times 10^{-6}$, then $n_B/s \sim 10^{-10}$. It is thus quite easily possible to obtain the observed baryon to entropy ratio.

Finally, we comment on the decoupling temperature of the $B - L$ violating interactions. For example, we consider the following Lagrangian for lepton number violation:
\[ \mathcal{L}_E = \frac{2}{v} \bar{l} l H H + \text{H.c.}, \] (30)
where $v$ is the scale characterizing the interaction which can be related to the heavy Majorana mass for the right-handed neutrino (in the context of the see-saw mechanism) in the following way:
\[ m_\nu = \frac{4}{v} \langle H \rangle^2. \] (31)
In the above, $l$ and $H$ represent the left handed lepton doublet and the Higgs doublet, respectively. Then, the lepton number violating rate of this interaction is
\[ \Gamma_E \sim 0.04 \frac{T^3}{v^2}. \] (32)
The decoupling temperature is now calculated as
\[ T_D \sim 3 \times 10^{11} \text{GeV} \left( \frac{v}{10^{14} \text{GeV}} \right)^2, \]
\[ \sim 2 \times 10^{14} \text{GeV} \left( \frac{m_\nu}{0.05 \text{eV}} \right)^{-2}, \] (33)
where we set the number of effective degrees of freedom for relativistic particles to be 100. On the other hand, taking $m \ll \Gamma, \Gamma/M_G \sim 10^{-5}$ into account, we can obtain an upper bound on $T_{\text{end}}$ of the form
\[ T_{\text{end}} \sim m \sqrt{\frac{M_G}{\Gamma}} \ll 3 \times 10^{-8} M_G \sim 7 \times 10^{10} \text{GeV}. \] (34)
Thus, the decoupling temperature $T_D$ is much higher than $T_{\text{end}}$.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we have discussed spontaneous baryo/leptogenesis in warm inflation. Though radiation always exists in warm inflation, spontaneous baryogenesis in thermal equilibrium does not work in general because for such a baryogenesis mechanism to be successful would require a fine-tuning of the decoupling temperature of the $B - L$ violating interaction. Instead, we considered another possibility, in which the $B - L$ asymmetry is generated dynamically. In the standard inflation models, the inflaton oscillates coherently after inflation. On the other hand, in warm inflation, the inflaton still slowly rolls even after inflation ends. We have shown that spontaneous baryogenesis can be implemented rather easily in this situation. We have shown that during the short period just after warm inflation ends, a sufficient $B - L$ asymmetry can be generated to explain the presently observed baryon asymmetry.

For successful warm inflation, the inflaton must couple to a very large number of particles in order to maintain a large decay width. This may be viewed as a disadvantage, but it also renders it rather likely that interaction terms such as those which we postulate exist. Note that some models for warm inflation motivated by string theory have been proposed. All we assumed in this paper is the existence of a derivative coupling of the inflaton to the $B - L$ current. It would be of interest to explore whether in the string-motivated models for warm inflation such a derivative coupling can be accommodated given the couplings of $\phi$ to a large number of other fields.
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[1] For example, A.D. Linde, Particle Physics and Inflationary Cosmology, (Harwood, Chur, Switzerland, 1990).
[2] A. Berera, Phys. Rev. Lett. 75, 3218 (1995); A. Berera, Phys. Rev. D 54, 2519 (1996) [arXiv:hep-th/9601134];
A. Berera, Phys. Rev. D 55, 3346 (1997) [arXiv:hep-ph/9612239].
[3] I. G. Moss, Phys. Lett. 154B, 120 (1985);
J. Yokoyama and K. Maeda, Phys. Lett. B 207, 31 (1988);
R. Brout and P. Spindel, Nucl. Phys. B348, 405 (1991).
[4] A. Berera and L. Z. Fang, Phys. Rev. Lett. 74, 1912 (1995) [arXiv:astro-ph/9501024];
W. L. Lee and L. Z. Fang, Int. J. Mod. Phys. D 6, 305 (1997) [arXiv:astro-ph/9706101].
[5] A. Berera, M. Gleiser, and R. O. Ramos, Phys. Rev. D 58, 123508 (1998) [arXiv:hep-ph/9803394];
A. Berera and R. O. Ramos, Phys. Rev. D 63, 103509 (2001) [arXiv:hep-ph/0101049];
A. Berera and R. O. Ramos, arXiv:hep-ph/0210301.
[6] J. Yokoyama and A. D. Linde, Phys. Rev. D 60, 083509 (1999) [arXiv:hep-ph/9809409].
[7] A. Berera, M. Gleiser, and R. O. Ramos, Phys. Rev. Lett. 83, 264 (1999) [arXiv:hep-ph/9809583];
A. Berera and T. W. Kephart, Phys. Rev. Lett. 83, 1084 (1999) [arXiv:hep-ph/9904410];
A. Berera and T. W. Kephart, Phys. Lett. B 456, 135 (1999) [arXiv:hep-ph/9811295];
A. Berera, Nucl. Phys. B585, 666 (2000) [arXiv:hep-ph/9904409].
[8] A. Dolgov and K. Freese, Phys. Rev. D 51, 2693 (1995) [arXiv:hep-ph/9410346];
A. Dolgov, K. Freese, R. Rangarajan, and M. Srednicki, Phys. Rev. D 56, 6155 (1997) [arXiv:hep-ph/9610405].
[9] A. G. Cohen and D. B. Kaplan, Phys. Lett. B 199, 251 (1987).
[10] I. Affleck and M. Dine, Nucl. Phys. B249, 361 (1985).
[11] A. N. Taylor and A. Berera, Phys. Rev. D 62, 083517 (2000) [arXiv:astro-ph/0006077].
[12] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. 215, 203 (1992).
[13] W. Lee and L. Z. Fang, Phys. Rev. D 59, 083503 (1999) [arXiv:astro-ph/9901195].
[14] H. P. De Oliveira and S. E. Joras, Phys. Rev. D 64, 063513 (2001) [arXiv:gr-qc/0103089].
[15] C. L. Bennett et al., Astrophys. J. Lett. 464, L1 (1996) [arXiv:astro-ph/9601067].
[16] M. Z. Li, X. L. Wang, B. Feng and X. M. Zhang, Phys. Rev. D 65, 103511 (2002) [arXiv:hep-ph/0112069];
M. Li and X. Zhang, arXiv:hep-ph/0209093.
[17] A. De Felice, S. Nasri, and M. Trodden, Phys. Rev. D 67, 043509 (2003) [arXiv:hep-ph/0207211].
[18] M. Yamaguchi, arXiv:hep-ph/0211163.
[19] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. 155B, 36 (1985);
S. Y. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B308, 885 (1988).
[20] U. Sarkar, arXiv:hep-ph/9809209;
W. Buchmuller, arXiv:hep-ph/0101102.