Thermodynamic analysis with energy recovery comparison of transcritical CO$_2$ heat pump system using various expansion devices.
Thermodynamic analysis with energy recovery comparison of transcritical CO₂ heat pump system using various expansion devices.

Mohammed Ridha Jawad Al-Tameemi¹, Khuder N Abed¹, Thamer Khalif Salem² and Zhibin Yu³

¹ College of Engineering, University of Diyala, Baqubah, Iraq.
² Department of Mechanical Engineering, Tikrit University, Tikrit Iraq.
³ School of Engineering, University of Glasgow, Glasgow, United Kingdom.

Email: mohammedridha_eng@uodiyala.edu.iq

Abstract. The high irreversibility caused by the expansion valve in the conventional transcritical CO₂ heat pump cycle has been reported as the major drawback on the overall system performance. To overcome this problem and recover some of the energy lost, different isentropic expansion devices such as turbine expander and two phase ejector have been proposed. This study aims to numerically compare the performance of the transcritical CO₂ heat pump in terms of first and second law of thermodynamics. In addition, the energy recovered by the two phase ejector and the turbine expander cycles have been evaluated. The pressure recovery and entrainment ratio in the ejector device were investigated comprehensively. Two numerical models using MATLAB and ASPEN PLUS software have been developed, and REFPROP database was used to estimate the refrigerant thermophysical properties. The results showed that the heating coefficient of performance (COPₜ) of the ejector cycle is higher than that of the turbine and valve cycles by 10.15 % and 20.84 % respectively. In addition, the ejector cycle has the highest second law efficiency (0.1) and the recovered energy is (0.63 kW) compared to (0.107 kW) gained by the turbine cycle. The ejector device has the least exergy destruction (0.2 kW) and can recover 0.7 Mpa of the pressure losses.

1. Introduction
The environmental friendly behaviour and the high efficiency performance of the working fluid are essential demands by the heating and cooling industries for any modern heating systems. The transcritical CO₂ heat pump system could provide such characteristics [11, 17]. However, the system performance of the conventional cycle which utilise an expansion valve is compromised by the high irreversibility (throttling losses) in the expansion process. During this process, the CO₂ velocity rises as a result of the gained kinetic energy. This will lead to high friction losses which would reduce the cooling capacity of the evaporator. As a consequence, the system performance will decrease [8, 14]. Moreover, in the transcritical CO₂ heat pump cycle, the heat rejection process take place in the supercritical region which requires higher pressure ratio. This will lead to higher throttling losses compared with the subcritical cycle [5, 17]. Recent studies have claimed that some of the energy lost can be recovered by using isentropic expansion process (constant entropy) instead of isenthalpic expansion process (constant enthalpy) [9, 14]. This can be achieved by replacing the conversional expansion valve by two phase ejector or turbine expander [8, 16]. The two phase ejector isentropically convert the potential energy of the high pressure working fluid into kinetic energy without consuming mechanical work [4, 6]. The
main parts of the ejector are primary (motive) nozzle, suction nozzle, mixing section, constant area section and diffuser nozzle [10], as shown in Figure 1. The high pressure refrigerant flow enters the ejector through the primary nozzle which is a coverage diverge nozzle. As a consequence, the CO₂ isentropically expands leading to a velocity rise from subsonic to supersonic speed. Then in the mixing zone, the high energy two phase refrigerant entrains the low pressure vapour refrigerant from the suction nozzle. Thus, the two flow are mixed and the momentum are exchanged. Finally, at the exit of the diffuser nozzle, further increase in the mixture pressure take place. This pressure value is greater than the suction pressure in the valve cycle [3, 6, 8].

Theoretical and experimental studies have investigated the performance of the two phase ejector cycle extensively. These studies revealed that using the ejector as an expansion device can reduce the work of the compressor and consequently increase the COP of the ejector cycle as well as reduce the evaporator size [6, 8, 12-15, 17]. A theoretical investigation conducted by Li and Groll [9] showed around 16% improvement of the COP for the ejector transcritical CO₂ heat pump system. Whereas, two other studies reported an improvement in the theoretical COP by 22% and 21% respectively [3, 5]. Similarly, an experimental work by Elbel and Hrnjak [6] concluded that the COP for the ejector cycle has been improved by 18%, and 14.5% of the energy lost has been recovered. Whereas, a more recent study pointed out that the COP of the ejector cycle can be improved by up to 30% as well as the exergy lost could be reduce by 25% [7]. Furthermore, the ejector device has more advantages including simple structure, inexpensive and no maintenance required [3, 12].

On the other hand, the two phase turbine expander can recover some of the energy lost throughout the expansion process by converting the high kinetic energy into mechanical work. This recovered energy could be used to drive a compressor or an electric motor [16]. A theoretical and experimental study was conducted by Yang et al. [16] to study the transcritical CO₂ water-to-water heat pump system with turbine expander. The throttling valve was replaced by turbine expander to recover the energy lost in the expansion process. An experimental data was obtained from a test rig by varying the compressor output pressure between (7.5 - 9.5 MPa). In addition, a steady-state mathematical model was developed and the accuracy of the model was verified by comparing the simulation results with the experimental data. The results showed that the cooling COP of the system could significantly vary with changing the mass flow rate and temperature of the water entering the evaporator. It will also slightly increase the optimal high pressure.

The aim of the current study is to investigate and compare the overall efficiency performance of the transcritical CO₂ heat pump system in term of first and second law of thermodynamics using a valve, two phase ejector and a turbine expander as expansion devices. In addition, the energy recovered by the ejector and turbine cycles is evaluated and analyzed. Moreover, the effect of pressure recovery and entertainment ratio on the ejector design parameters were studies.
2. Modelling of the cycles

The schematic diagram of the two phase ejector heat pump system is shown in Figure 2.

![Schematic diagram of the two phase ejector heat pump system](image)

Figure 2. Schematic diagram of heat pump system with two phase ejectors.

The pressure enthalpy (P-H) diagram for the three cycles are represented in Figures 3. For simulation purposes, thermodynamic equilibrium model is assumed across the entire cycles components, i.e. the velocity, pressure and density of the refrigerant phases are equal. The compression process for all cycles is assumed to be an adiabatic process. As shown in Figure 3, for the ejector cycle, the CO\textsubscript{2} enters the compressor at a recovery pressure (P\textsubscript{c}) state (1\textsubscript{ejc}) and exit at a discharged pressure (P\textsubscript{d}) state (2\textsubscript{ejc}).

![P-H diagram for the three CO\textsubscript{2} heat pump cycles](image)

Figure 3. P-H diagram for the three CO\textsubscript{2} heat pump cycles

For the other two cycles, the compression process is slightly different. The refrigerant enters the compressor at an evaporator pressure (P\textsubscript{e}) state (10) and compressed to reach a gas cooler pressure (P\textsubscript{d}) state.
state (2). The isentropic efficiency of the compressor $\eta_{\text{comp}}$ is adopted from Ahammed et al. [3]. After that, the heat rejection process take place in the gas cooler under the assumption of constant pressure, which is represented by state (2ejc-3) for the ejector cycle and state (2-3) for the valve and turbine cycles. The working fluid is assumed to leave the gas cooler at discharged pressure (Pd) state (3) with a temperature (T3) of 35 °C. Next, the CO$_2$ is adiabatically expanded in the expansion devices into two phase flows. In the ejector, the refrigerant is assumed to enter the primary nozzle at a stagnation condition, where it expands and exits at state (4ejc). A constant pressure model is adopted from Li and Groll [9]. This model state that the pressure will drop to a value less than the evaporator pressure by 0.3 bar which represent the mixing pressure (Ps) in the mixing section. While for the turbine and the valve cycles, the Carbon dioxide expands to evaporation pressure (Pe) at state (4t) and state (4) respectively. The isentropic efficiency correlations for the turbine ($\eta_{\text{turb}}$) is adopted from a measured experimental data by Yang et al. [16]. While the isentropic efficiency for the ejector nozzles ($\eta_{\text{pm}}, \eta_{\text{sn}}$) are assumed at 85% based on Ahammed et al. model [3].

Finally, for all cycles, the stream is assumed to be fully evaporated at the exit of the evaporator with evaporation temperature (Te) of 2 °C (state 10) to re-enter the compressor again. Furthermore, the refrigerant total mass flow rate ($\dot{m}_{\text{tot}}$) is assumed to be (0.05 kg/s). In the valve and turbine cycles, this mass will circulate through all parts of the system. Whereas for the ejector cycle, the $\dot{m}_{\text{tot}}$ in the mixing chamber is the sum of $\dot{m}_p$ (the mass rate running from the gas cooler through the motive nozzle) and $\dot{m}_e$ (the mass rate passing from the evaporator through the suction nozzle). In the diffuser section (diverge nozzle in Figure 1), the pressure of the refrigerant increases to reach the recovery pressure (Pe). After that, the mixture leaves the ejector at state (7) to enter the separator. In the separator, the wet refrigerant flow is separated into a saturated vapor (state 1ejc) and saturated liquid (state 8). The vapor will be re-compressed again, while the liquid CO$_2$ will expand in the valve of the ejector cycle at evaporator pressure (Pe) (state 9).

For the ejector cycle simulation, a feedback throttle valve adopted from Li and Groll model [9] was assumed. The purpose of this valve is to maintain the mass balance in the system by shifting excess vapor exiting the ejector back to the evaporator. This will keep the vapor fraction (CO$_2$ quality) equal to the mass ratio of the primary over the total refrigerant flow.

In order to evaluate the overall performance of the three cycles in terms of first and second law of thermodynamics, a steady state one dimensional mathematical model written by MATLAB software is developed. This code is linked to REFPROP database in order to obtain the thermophysical properties of the refrigerant in different states across these cycles including pressure, temperature, enthalpy, entropy and quality. In addition, the P-H diagram is obtained from this code. In this simulation, a scaler vector of discharged pressure ranged between (8 to 12 Mpa) is set in order to identify the optimum gas cooler discharged pressure that could produce the maximum value of $COP_h$.

The coefficient of performance ($COP_h$) for the three cycles is calculated from the following equations:

$$COP_{h,\text{Valve}} = \frac{\dot{m}_{\text{tot}}(h_2-h_3)}{\dot{m}_{\text{tot}}(h_{10}-h_2)}$$  \hspace{1cm} (1)

$$COP_{h,\text{Turbine}} = \frac{\dot{m}_{\text{tot}}(h_2-h_3)}{\dot{m}_{\text{tot}}(h_{10}-h_2)}$$  \hspace{1cm} (2)

$$COP_{h,\text{Ejector}} = \frac{\dot{m}_p(h_2-\text{h}_{1ejc})}{\dot{m}_p(h_2-\text{h}_{1ejc})}$$  \hspace{1cm} (3)

The exergy destruction for each thermodynamic process is carried out under the assumption that the environment temperature ($T_o$) is 27 °C and the reference temperature ($T_r$) is 35 °C.

$$I_{\text{expan,Valve}} = \dot{m}_{\text{total}} \times T_0 \times (S_4 - S_3)$$  \hspace{1cm} (4)

$$I_{\text{expan,Turbine}} = \dot{m}_{\text{total}} \times T_0 \times (S_{4t} - S_3)$$  \hspace{1cm} (5)

$$I_{\text{expan, ejc}} = \sum I_{\text{pm}} + I_{\text{sn}} + I_{\text{diff}} + I_{\text{mx}}$$  \hspace{1cm} (6)

Second law efficiency is defined for each cycle as follow:

$$\eta_{\text{exergy,Valve}} = 1 - \frac{\sum I_{\text{Valve,cycle}}}{W_\ell}$$  \hspace{1cm} (7)

$$\eta_{\text{exergy, Turbine}} = 1 - \frac{\sum I_{\text{Turbine,cycle}}}{W_\ell+W_{t, out}}$$  \hspace{1cm} (8)
The energy recovered by the ejector and turbine cycles:

\[
\text{Energy recovered}_{\text{ejector, cycle}} = W_\text{e} - W_{\text{e, ejc}}
\]

\[
\text{Energy recovered}_{\text{turbine, cycle}} = W_\text{e} - W_{\text{t, out}}
\]

The pressure recovery in the ejector cycle is determined by:

\[
\text{Pressure recovery}_{\text{ejector cycle}} = P_c - P_e
\]

3. Results and discussion

The comparison between the steady state results for the three heat pump cycles conducted using MATLAB code and ASPEN software are shown in Table 1. These values were obtained at a calculated optimum discharge pressure of around 8.7 Mpa.

| Table 1. Steady state results of the three heat pump cycles employing three different expansion devices. |
|-----------------|-----------------|-----------------|-----------------|
| Ejector cycle  | Turbine cycle   | Expansion valve |
| MATLAB | ASPEN PULSE | MATLAB | ASPEN PULSE | MATLAB | ASPEN PULSE |
| COP_h | 4.9398 | 4.912 | 4.4848 | 4.425 | 4.0879 | 4 |
| Exergy efficiency | 0.1065 | 0.1 | 0.0929 | 0.091 | 0.0823 | 0.0815 |
| Exergy destruction in the expansion device (watt) | 0.2087 | 0.2 | 0.4871 | 0.41 | 0.6959 | 0.67 |
| Cooling capacity (kW) | 2.2280 | 2.2 | 3.6120 | 3.6 | 3.7809 | 3.6 |
| Heating capacity (kW) | 2.7648 | 2.6 | 4.9060 | 4.9 | 5.0054 | 5 |

Table 1 shows that the ejector cycle has achieved the highest values in terms of first and second law of thermodynamics. For instant, from MATLAB results, the COP_h of the ejector cycle is higher by 10.15% and 20.84% than the turbine and expansion valve respectively. While, the second law efficiency of the ejector cycle is higher by 14.64% and 29.40% compare to the turbine and valve cycles respectively. The irreversibility in the ejector device is lower by 57.15% and 70.01% than turbine and valve cycles respectively. In contrast, the turbine and valve cycles have achieved nearly similar heating and cooling capacities for this case study, which were higher than that of the ejector cycle.

Furthermore, the results plotted in Table 1 shows a good agreement between the results obtained from MATLAB and ASPEN PLUS models. Thus, MATLAB code can be used confidently to conduct further evaluations in this study.

3.1 Comparison of the energy recovered by the ejector and turbine cycles

Figures 4 and 5 show the energy recovered by the ejector and turbine cycles, respectively. For the ejector cycle (Figure 4), the energy recovered rises sharply with increasing the discharge pressure, reaching 0.63 kW at the optimum gas cooler pressure. In contrast, the energy recovered by the turbine cycle significantly decline with initial change in discharge pressure, achieving a value of 0.107 kW at the optimum pressure then gradually increase thereafter. Generally, the energy recovered by the ejector cycle is significantly higher than that for the turbine cycle. This is due to less compression work in the ejector cycle compared to the turbine cycle.
The effect of entrainment ratio on ejector cycle design parameters

The entrainment ratio of the ejector device has significant contribution to the ejector design and performance. In this section, the effects of the entrainment ratio on other design parameters are studied. Figure 6 shows that the COPₘ is significantly affected by the ejector entrainment ratio. As the discharge pressure increased from (8-12 Mpa), the entrainment ratio also increases. In addition, the COPₘ increased sharply to reach a maximum value of 4.9 then declined afterward. At this point, the entrainment ratio recorded a value of 0.55. This indicate that the COPₘ of the ejector cycle can reach its maximum value when the mass of the secondary flow (mₛ) is approximately half the value of the primary mass flow (mₚ).

Figure 7 shows that the quality of CO₂ at the ejector outlet declines as the entrainment ratio increases. The quality decreased from 0.74 to reach approximately 0.65 at the optimum value of entrance ratio. The decline in the CO₂ quality is due to the rise in the CO₂ mass fraction entrained from the evaporator (which is mostly vapour) against the mass fraction of the primary fluid.
Similarly, the exergy efficiency follows the same behaviour of the COP in relation to entrainment ratio (Figure 6). As the entrainment ratio approaches the optimum value, the exergy efficiency peaked to a value of 0.0106 before declining sharply, as shown in Figure 8. In contrast, the increase in the entrainment ratio has a negative impact on the pressure recovered by the ejector device, as shown in Figure 9. With the increase in the discharged pressure (Pd), the velocity of the CO₂ flow declined at the motive nozzle exit which causes decrease in the enthalpy of the flow (\( h = u + PV \)) and drops in the pressure at the ejector exit (Pe). This lead to decrease the difference between the Pc and Pe, (see equation (12)).

### 3.3 The effect of pressure recovery on ejector cycle design parameters

Figure 10 illustrates the pressure recovery of the ejector cycle and the refrigerant velocity (U6) in the diffuser section in correlation with the gas cooler pressure. The initial rise in discharged pressure caused significant reduction in the amount of the recovered pressure. However, after the optimum discharge pressure is reached, the pressure recovery started to improve gradually. This can be attributed to the mathematical relation between Pc, entropy and enthalpy at state 6, which are dependent on the refrigerant velocity at that state.

![Figure 8](image1.png)

**Figure 8.** Relation between second law efficiency and entrainment ratio.

![Figure 9](image2.png)

**Figure 9.** Relation between pressure recovery and entrainment ratio.

![Figure 10](image3.png)

**Figure 10.** Relation between pressure recovery, CO₂ velocity and discharge pressure.

![Figure 11](image4.png)

**Figure 11.** Relation between pressure recovery and COPₜₘ. 
Figure 11 shows the relation between the pressure recovery and the COP of the ejector cycle. It shows that the pressure recovered by the ejector device declines as COP decreases, however, at the highest COP value, the two phase ejector has recovered around 0.7 Mpa which explains the higher coefficient of performance of the ejector cycle compared with the other cycles.

Figure 12 demonstrates the relation between the enthalpy of CO₂ at ejector exit and the pressure recovery of the ejector cycle. It shows that as enthalpy of the CO₂ increases, the pressure recovery slightly declines then increases. This explained as follow, the decline in the velocity of the CO₂ at the diffuser section and the amount of pressure recovered causes decline in the enthalpy at the ejector exit. After that, the enthalpy rise significantly with the improvement in the velocity and pressure recovery (as shown in Figure 10).

4. Conclusion
A simulation model has been developed to study and compare the thermodynamic performance of three transcritical CO₂ heat pump cycles utilizing different throttling devices. The mathematical modelling is conducted using MATLAB software linked to REFPROP database in order to obtain the thermophysical properties of the refrigerant in different thermodynamic states across the cycles. ASPEN plus software was used as a bench mark to validate the results obtained from the MATLAB model. The steady state comparison is based on the first and second law of thermodynamics, and the heating and cooling capacities for the three cycles. In addition, the energy recovered by the ejector and the turbine cycles is analysed. Furthermore, the pressure recovery and the entrainment ratio correlations with other design parameters for the ejector cycle were illustrated. The results show that, in term of first and second law of thermodynamics, the ejector cycle has achieved the height COP and exergy efficiency values of around 4.9 and 0.1, respectively. Also, the exergy destruction by the ejector device (0.2 kW) is the least of all three devices. For the heating and cooling capacities, the valve and the turbine cycles have achieved nearly similar results, which were higher than that of the ejector cycle. The ejector cycle has shown the potential advantage of energy recovery which is around five time higher than the turbine cycle.

References
[1] Al-Tameemi M and Yu Z 2017 Thermodynamic approach for designing the two-phase motive nozzle of the ejector for transcritical CO₂ heat pump system. 9th Int. Conf. on Applied Energy ICAE2017 21-24 August Cardiff UK.
[2] Al-Tameemi M, Yu Z and Younger P 2016 Numerical investigation of the transcritical CO$_2$ Heat pump system employing different expansion devices. 12th IIR Gustav Lorentzen Natural Working Fluids Conf. Edinburgh UK.

[3] Ahammed M E, Bhattacharyya S and RAMGOPAL M 2014. Thermodynamic design and simulation of a CO$_2$ based transcritical vapour compression refrigeration system with an ejector. International Journal of Refrigeration, 45, 177-188.

[4] Chen X, Omer S, Worall M and Riffat S 2013 Recent developments in ejector refrigeration technologies Renewable and Sustainable Energy Reviews 19, 629-651.

[5] Deng J-Q, Jiang P-X, Lu T and Lu W 2007 Particular characteristics of transcritical CO$_2$ refrigeration cycle with an ejector. Applied Thermal Engineering 27, 381-388.

[6] Elbel S and Hrnjak P 2008 Experimental validation of a prototype ejector designed to reduce throttling losses encountered in transcritical R744 system operation. International Journal of Refrigeration 31, 411-422.

[7] Fangtian S and Yitai M 2011 Thermodynamic analysis of transcritical CO$_2$ refrigeration cycle with an ejector Applied Thermal Engineering 31, 1184-1189.

[8] Lawrence N and Elbel S 2013 Theoretical and practical comparison of two-phase ejector refrigeration cycles including First and Second Law analysis. International Journal of Refrigeration 36, 1220-1232.

[9] Li D and Groll E A 2005 Transcritical CO$_2$ refrigeration cycle with ejector-expansion device. International Journal of Refrigeration 28, 766-773.

[10] Lorentzen G 1994. Revival of carbon dioxide as a refrigerant. International Journal of Refrigeration 17, 292-301.

[11] Ma Y, Liu Z and Tian H 2013 A review of transcritical carbon dioxide heat pump and refrigeration cycles Energy 55, 156-172.

[12] Minetto S, Brignoli R, Banasiak K, Hafner A and Zilio C 2013 Performance assessment of an off-the-shelf R744 heat pump equipped with an ejector Applied Thermal Engineering 59, 568-575.

[13] Sarkar J 2012 Ejector enhanced vapor compression refrigeration and heat pump systems-A review Renewable and Sustainable Energy Reviews 16, 6647-6659.

[14] Sumeru K, Nasution H and Ani F N 2012 A review on two-phase ejector as an expansion device in vapor compression refrigeration cycle Renewable and Sustainable Energy Reviews 16, 4927-4937.

[15] Xu X X, Chen G M, Tang L M and Zhu Z J 2012 Experimental investigation on performance of transcritical CO$_2$ heat pump system with ejector under optimum high-side pressure. Energy 44, 870-877.

[16] Yang J L, Ma Y T, Li M X and Hua J 2010 Modeling and simulating the transcritical CO$_2$ heat pump system Energy 35, 4812-4818.

[17] Yari M and Mahmoudi S M S 2011 Thermodynamic analysis and optimization of novel ejector-expansion TRCC (transcritical CO$_2$) cascade refrigeration cycles (Novel transcritical CO$_2$ cycle) Energy 36, 6839-6850.

[18] Zhang Z-Y, Ma Y-T, Wang H-L and Li M-X 2013 Theoretical evaluation on effect of internal heat exchanger in ejector expansion transcritical CO$_2$ refrigeration cycle Applied Thermal Engineering 50, 932-938.

Acknowledgments
The author Mohammed Ridha Jawad Al-Tameemi wish to acknowledge the support from the University of Diyala and the University of Glasgow.