Influence of Local Void Fraction Distribution on Turbulent Structure of Upward Bubbly Flow in Vertical Channel

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Abstract. Turbulent structure of upward dilute bubbly flow with 1 mm bubbles in a vertical channel is investigated experimentally. Small amount of surfactant is added to water to avoid bubble coalescence and to control local void fraction distribution. Liquid phase velocity is measured using two-dimensional laser Doppler Velocimetry. In 1-Pentanol solution of 20 ppm, bubbles have half-slip surface and migrate strongly toward the channel wall due to the shear-induced lift force which leads to wall-peaked distribution of local void fraction. On the other hand, in Triton X-100 solution of 2 ppm, bubbles become fully-contaminated and do not migrate toward the wall or the channel centre due to near-zero lift force, causing uniform distribution of local void fraction in the wall-normal direction. Once bubbles accumulate near the wall, transport of turbulent energy produced by the wall shear towards the channel centre is blocked. Then turbulence induced by the bubble motion becomes dominant in a wide core region (so-called pseudo turbulence). By contrast, in the case of the uniform distribution of bubbles, a mechanism of a turbulent energy transport which is the same as that of a single-phase turbulence still exists and furthermore the bubble-induced turbulence is added on it.

1. INTRODUCTION
Bubbly flows are often encountered in nature. They are also observed in many industrial processes such as the flows in chemical reactors, aeration tanks and heat exchangers. Because of the wide application of these flows, a lot of investigations on their turbulent structure have been carried out over the past three decades.

The early studies on the measurement of bubbly flows were conducted by Serizawa et al. (1975) and Theofanous and Sullivan (1982), in which the local measurements of turbulent flow structure in pipe were made using a Hot-Film Anemometer and a Laser Doppler Anemometer (LDA or LDV), respectively. Following these studies, Wang et al. (1987), Lance and Bataille (1991), Serizawa et al. (1991), Liu (1993), Liu and Bankoff (1993a, 1993b), Moursali et al. (1995) and Marié et al. (1997) provided experimental information on: the local void fraction profile and its dependence on bubble size; the effect of initial bubble size and the bubble-induced liquid turbulent.

On the information of the phase distribution of bubbly flow in upward pipe flow at relatively lower void fraction, it is well known that the bubbles migrate toward the wall and the local void fraction has a wall-peak shape or a saddle shape (Serizawa et al., 1975; Wang et al., 1987). There are lots of likely causes of this transverse migration of bubbles; however, the mechanism has not been clarified. Most of existing experimental studies have been conducted with such factors as large deformable bubbles, no dilute bubbly flow and turbulence which complicate the bubbly flow structure. Kashinsky et al. (1993) investigated a laminar bubbly flow in a vertical pipe and reported the wall-peak distribution of the
local void fraction. Single bubble behaviour in vertical channel flow and turbulent boundary layer with vertical plate have been investigated and the forces acting on the bubble was discussed (Žun, 1980; Felton and Loth, 2001).

So et al. (2002) investigated upward dilute bubbly flow in a vertical channel with mono-dispersed 1 mm spherical bubbles by the addition of surfactant into the tap water to avoid bubble coalescences. In 3-Pentanol solution of 20 ppm, these 1 mm bubbles strongly migrate toward the wall. It was observed that highly accumulated bubble near the wall formed crescent like shape horizontal bubble cluster. From their LDV measurement of bubbly flow, plug like flow structure and the bubble-induced turbulence in a core region were reported. Local void fraction distribution has a great influence on the bubbly flow turbulent structure. Therefore, clarification of the mechanism of the transverse bubble migration is important to analyse turbulent modification due to the presence of bubbles.

Here, contaminants in water such as surfactants greatly affect bubble behaviour. It is known that a bubble in contaminated water (surfactant solution) rises much slower than that in super-purified water (e.g. Duineveld, 1995). This phenomenon is explained by Marangoni effect (Frumkin and Levich, 1947). Non-uniform distribution of adsorbed surfactants on a surface of a rising bubble generate surface tension gradient and this causes the tangential stress. Thus, surfactants in water change gradually the boundary condition of the bubble surface from free-slip to no-slip. We (Takagi et al., 2008) reported that stronger Maragoni effect decreases a tendency of a lateral bubble migration of bubbles in an upward channel flow and the local void fraction distribution changes drastically from wall-peaking to uniform with more influential surfactants. This phenomenon is closely related to a shear-induced lift force on a bubble and its behaviour in surfactant solution. Fukuta et al. (2008) investigated numerically the influence of surfactants on the lift force acting on a spherical bubble in simple shear flow and indicated that the lift force decreases from that of a clean bubble, which has nearly the same lift coefficient as Auton’s 0.5 (Auton, 1987), to that of a rigid sphere which is near-zero value by an immobilization of bubble surface due to Marangoni effect at bubble Reynolds number based on the bubble diameter and the relative velocity of the bubble of 100. This result qualitatively explains the lateral migration of bubbles in surfactant solution observed by the bubbly flow experiment (Takagi et al., 2008).

The local void fraction distribution is important information characterizing the bubbly flow structure. Despite strong dependence of surfactant on the local void fraction distribution, extent of contamination in water has not been evaluated in previous bubbly flow experiments in which tap water including some contaminants was used. Our objective is to investigate the effect of the local void fraction distribution on the bubbly flow turbulent structure. By controlling the surfactants in water, two different distributions of bubbles (wall peaking and uniform) are obtained. Liquid-phase velocity is measured using two-dimensional laser Doppler Velocimetry in both cases.

2. EXPERIMENTAL SET-UP

2.1 Experimental Apparatus

The experimental apparatus is shown in figure 1 and consists of a loop water tunnel. The tunnel volume is 270 litters, with a bubble separator at the top of the tunnel to eliminate bubble recirculation. The vertical channel has a thickness of $2H = 40$ mm and a spanwise width of 400 mm with an aspect ratio of 10. The channel height is 2500 mm. The distance from the contraction nozzle to the test section of 2000 mm is long enough for single-phase turbulent and laminar flow to be fully developed. The coordinate $x$ denotes the streamwise direction, $y$ denotes the wall-normal direction and $z$ denotes the spanwise direction. Tap water in the tunnel is filtered through filter paper with a 10 micron pore size and an activated carbon filter, and is pumped vertically upwards through the channel. The output of the pump is controlled by a frequency converter.

In the bubbly flow experiment, air bubbles were injected from the bubble generator which is constructed with 474 stainless steel pipe of 0.1 mm inner diameter. The bubble generator is installed above the contraction nozzle of the tunnel, which is located at $x/H = 100$ downward from the test
2.2 Experimental Conditions

The bulk Reynolds numbers \( Re (=2U_b H/\nu) \), based on the characteristic length of the channel width \( 2H (= 40 \text{ mm}) \) and the bulk mean velocity of liquid phase \( U_b \), is 10100 which corresponds to the friction Reynolds number \( Re_t (=u_\tau H/\nu) \), based on the characteristic length of the half of the channel width \( H \) and the friction velocity of single-phase turbulent channel flow \( u_\tau \), of 300 and to liquid flow rate of 210 litters min\(^{-1}\). The average void fraction is set to be 0.3\% and 0.6\% that are less than 1\%. These amounts of small gas fraction are enough to modify the turbulent structure, larger gas fraction interferes with the optical measurement. Two kinds of surfactants, 1-Pentanol and Triton X-100, are used. The volumetric concentration of 1-Pentanol is 20 ppm for the wall-peaking local void fraction distribution, and of Triton X-100 is 2 ppm for the uniform one. These surfactant concentrations are quite less than the critical micelle concentration and the change of the water viscosity can be negligible.

2.3 Laser Doppler Velocimetry

To measure the liquid flow structure we applied a two-colour fibre LDV system previously described by So et al. (2002). The optical arrangement of the LDV system consisted of a commercial instruments (FLV 8835, Kanomax). Two components of the liquid phase velocity in \( x-y \) plane are measured. In order to measure the same velocity range by the different colour lasers, the optical probe is rotated approximately 45 degrees. In the present experiment, the Kolmogorov length and time scale of single-phase turbulence are estimated to be the order of 100 \( \mu \text{m} \) and 10 msec. Polystyrene particles of 8 \( \mu \text{m} \) in average diameter are added into the liquid phase as tracer particles.

3. RESULTS AND DISCUSSION

3.1 Surfactant Effect on Local Void Fraction Distribution

Figure 2 shows the snapshots of bubbly flow in different surfactant conditions: (a) Tap water, (b) 1-Pentanol solution of 20 ppm and (c) Triton X-100 solutions of 2 ppm. Left ones are view from the front of the channel (\( x-z \) plane) and right ones are view from the side of the channel (\( x-y \) plane). The bulk Reynolds number is 10100 and superficial gas velocity is 2.0 mm s\(^{-1}\) in every cases. The average void fraction is approximately 0.6\%; however, it increases with the stronger Marangoni effect due to
the presence of surfactant. Figure 2 (a) shows the snapshot of bubbly flow without any addition of surfactant into the tap water. The range of the bubble size becomes from 0.6 mm to 4 mm because of the coalescences up to the test section. The large deformed bubbles move upwards with zigzag or spiral motion. Figure 2 (b) and (c) are in the cases of 1-Pentanol solution of 20 ppm and Triton X-100 solution of 2 ppm, respectively. Mono-dispersed 1 mm bubbles are obtained by the prevention of the bubble coalescences. However, the spatial distribution of the bubbles is totally different. In the case of 20 ppm 1-Pentanol solution, the bubbles strongly accumulated near the wall and move upwards as they are sliding on the wall. At the same time, bubble clustering phenomenon can be observed. On the
other hand, in the case of 2 ppm Triton X-100 solution, the bubbles distribute uniformly and do not migrate toward the wall.

The surfactant influence on a bubble can be explained by a Marangoni effect (Levich 1962). Nonuniform surfactant adsorption produces the shear stress on the bubble surface. Regarding as this phenomenon from a continuum scale view point, a surface velocity gets lower and a drag on the bubble increases. That is to say, the Marangoni effect changes the boundary condition on the bubble surface from free-slip to no-slip. In 20 ppm 1-Pentanol solution, drag coefficient of an almost spherical bubble which is approximately 1 mm in diameter in stationary fluid takes the intermediate value between that of a clean spherical bubble and of a rigid sphere. On the other hand, in 2 ppm Triton X-100 solution, it becomes almost as same as that of a rigid sphere. The bubble surface in 20 ppm 1-Pentanol solution can still slip; the bubble surface in 2 ppm Triton X-100 solution almost takes no-slip condition.

Around \( Re_b = 100 \), which is bubble Reynolds number and is defined using the bubble diameter and the relative velocity of a bubble, as is the case with our experiment, the lift coefficient of the rigid sphere is considerably less than that of the clean bubble (Legendre and Magnaudet 1998; Bagchi and Barachander 2002; Kurose and Komori 1999). In the case of 20 ppm 1-pentanol solution, it is assumed that the lift coefficient has an intermediate value and it decreases as the surface condition approaches no-slip condition due to the increase of the concentration. In the case of 2 ppm Triton X-100 solution, the lift coefficient might become as same as that of rigid sphere. Fukuta et al. (2008) reported numerical work on a shear-induced lift force on a spherical bubble in surfactant solution. With decrease of free-slip region on a bubble surface, basically \( C_D \) increases and \( C_L \) decreases. This decrease of the lift coefficient is a main reason why the tendency of the lateral migration of the bubbles toward the wall weakens with the effect of the stronger surfactant effect.

In this study, these two conditions, 1-Pentanol 20 ppm and Triton X-100 2 ppm, are set to be targets for LDV measurement. The bubble size is almost the same and the vocal void fraction distribution is quite different. In 1-Pentanol 20 ppm solution and Triton X-100 2 ppm solution, the local void fraction becomes wall-peaking and uniform distribution, respectively.

### 3.2 Surfactant Effect on Single-Phase Turbulence

To investigate the effect of added surfactant on the fully developed turbulent channel flow at the test section, the streamwise mean velocity and turbulent fluctuations in \( x \) and \( y \) directions are measured by the LDV system. Figure 3 (a) shows the mean velocity profiles in single-phase flow at \( Re = 10100 \), which corresponds to \( Re_b = 303 \) in the cases of tap water, 320 ppm 1-Pentanol solution and 2 ppm

![Figure 3](image.png)

(a) mean velocity profile  
(b) RMS value of fluctuations

Figure 3. LDV results for single-phase flow at \( Re_b = 303 \) (\( Re = 10100 \)) - dependence of surfactant addition on single-phase turbulence. DNS result by Kasagi et al. (1992).
Triton X-100 solution. Figure 3 (a) also shows the comparison of the results with the law of the wall and that of Direct Numerical Simulation at $Re_\tau=300$ (Kasagi et al. 1992). Regardless of the addition of surfactant, the present results show an excellent agreement with the DNS result. The turbulent fluctuations normalized by the wall friction velocity are shown in Figure 3 (b). These profiles do not change with surfactant condition and are also in good agreement with DNS result in both $x$ and $y$ directions. Thus, through the comparison of the results tap water and surfactant solutions, the surfactant effect on single phase turbulence was negligible in present study.

3.3 LDV Results of Bubbly Flow

To compare turbulent structure of two types of local void fraction distribution, wall-peaking and uniform distribution, bubbly flows with 20 ppm 1-Pentanol and 2 ppm Triton X-100 are measured using 2-D LDV system.

Figure 4 shows the LDV results of the bubbly flows at Reynolds number of 10100. $y/H = 0$ and 1 means the channel wall and the centre, respectively. The circle and the triangle markers indicate the case of 1-Pentanol 20 ppm and Triton X-100 2 ppm with the average void fraction of 0.3 %, respectively. The plots with filled markers show the cases with a higher average void fraction of 0.6 % and the cross markers show the case of single-phase turbulence at $Re = 10100$ ($Re_\tau = 300$).

Figure 4 (a) shows mean velocity profiles. In the case of 1-Pentanol 20 ppm, the mean velocity gradient near the wall becomes steep, and the mean velocity increases at the wall-near range around $0 < y/H < 0.4$. Then, at the channel centre around $0.4 < y/H < 1$, the velocity decreases and the plot flattens. Increasing the void fraction, this tendency becomes pronounced. In the case of $f_g = 0.6 \%$, the velocity gradient becomes steeper, and the plot flattens over the wide region in channel core at $0.2 < y/H < 1$. In Triton X-100 2 ppm case, the mean velocity profile does not change from that of single-phase case at any void fraction. Once the local void fraction distribution takes the wall-peaking shape, the buoyancy force due to the accumulated bubbles lifts up the liquid-phase near the wall, and the velocity increases near the wall and decreases in the channel core region at the same time. On the other hand, in uniform distribution case, the acceleration of the liquid-phase does not occur in a particular region, because there is less nonuniformity of density as a mixture in the wall-normal direction.

Figure 4 (b) shows the plots of root-mean-square value of stream-wise velocity fluctuation. In the cases of 1-Pentanol 20 ppm, the stream-wise fluctuation decreases all over the channel. Especially, with higher void fraction condition ($f_g = 0.6 \%$), it becomes more suppressed, and its plots flatten over the wide region in channel core at $0.2 < y/H < 1$. In contrast, in the cases of Triton X-100 2 ppm, it increases in whole channel region compared with that of single-phase turbulence. Here, it is interesting to say that its dependence on the average void fraction is opposite in these two kinds of local void fraction distribution. That is, increasing the average void fraction, the stream-wise fluctuation decreases in the case with wall-peaking distribution of the local void fraction or increases in the case with uniform distribution. This result is important information about the dominant factor which causes a turbulent fluctuation of wall bounded turbulent bubbly flows. Highly accumulate bubbles near the wall change the mechanism of turbulence transport and prevent it toward the channel centre. Stronger the bubbles accumulated near the wall, more difficult the turbulent transport toward the centre becomes. If the bubbles distribute uniformly, the turbulent transport mechanism, which is the same as that of single-phase turbulence, still exists at least in the condition of the present experiment. In this case, bubble induced turbulence is added furthermore, and this is the reason why the fluctuation increases the average void fraction.

Figure 4 (c) indicates the plots of root-mean-square value of wall-normal velocity fluctuation. This result shows an overall trend which is the same as that of stream-wise fluctuation. However, in the case of Triton X-100 2 ppm, the increments are less than that of stream-wise fluctuation. In Triton X-100 2 ppm solution, 1 mm bubbles move upright parallel to the wall and seem no to disturb the liquid-phase in the wall-normal direction. The wakes of the bubbles play a great role related to the turbulence.
induced by the bubbly motion, and the momentum of the bubble wake is dominant in the stream-wise
direction not in the wall-normal direction.

Reynolds stress profiles are shown in Fig. 4 (d). In the case of 1-Pentanol 20 ppm, Reynolds stress
decreases all over the channel. At higher void fraction, it becomes almost zero in the wide channel
core region at $0.2 < y/H < 1$. In the case of Triton X-100 2 ppm, the profiles are almost the same as
that in the case of single-phase turbulence. Here, we consider the stress balance of the turbulent
channel flow of the mixture fluid. The total stress $\tau(y)$ is represented by summation of viscous shear
stress and Reynolds stress.

$$\tau(y) = \mu \frac{dU}{dy} - \rho \langle uv \rangle$$

(1)

Local void fraction $\alpha$ as a function of $y$ causes the buoyancy force and equation (2) is derived from the
momentum equation.

$$\tau(y) + \int_0^y \Delta \rho g \left( \alpha(y) - f_s \right) dy = \tau_w \left( 1 - \frac{y}{H} \right)$$

(2)

In the case of uniform distribution which means $\alpha(y) = f_s$, the second term of the left hand side of
equation (2) becomes zero. Therefore, the stress balance is represented as the same as that of single-
phase turbulence, summation of viscous shear stress and Reynolds stress should be balance with the

Figure 4. LDV results of bubbly flow measurement ($Re = 10100$) - dependence of local void fraction
distribution (‘wall-peaking’ for 1-Pentanol 20 ppm and ‘coring’ for Triton X-100 2 ppm) and average
void fraction.
right hand side of eq. (2). This corresponds to the case of Triton X-100 2 ppm. On the other hand, in
the case of 1-Pentanol 20 ppm, \( \alpha(y) \) is not uniform. So, total stress \( \tau(y) \) does not balance with
the right hand side of eq. (2). If the contribution of the second term of the left hand side of equation (2)
due to the accumulation of the bubbles near the wall, total stress decrease in the channel core region.

4. CONCLUSIONS
In 1-Pentanol solution of 20 ppm, bubbles have half-slip surface and migrate strongly toward the
channel wall due to the shear-induced lift force which leads to wall-peaked distribution of local void
fraction. On the other hand, in Triton X-100 solution of 2 ppm, bubbles become fully-contaminated
and do not migrate toward the wall or the channel centre due to near-zero lift force, causing uniform
distribution of local void fraction in the wall-normal direction. Once bubbles accumulate near the wall,
transport of turbulent energy produced by the wall shear towards the channel centre is blocked. Then
turbulence induced by the bubble motion becomes dominant in a wide core region (so-called pseudo
turbulence). By contrast, in the case of the uniform distribution of bubbles, a mechanism of a turbulent
energy transport which is the same as that of a single-phase turbulence still exists and furthermore the
bubble-induced turbulence is added on it. The contamination of water is a key factor to describe not
only the individual bubble motions but also the large-scale turbulent structure of bubbly flow.

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NOMENCLATURE
\[ f_g \] average void fraction \ [-, \%]  
\[ g \] gravity constant \ [m/s^2]  
\[ H \] half length of the channel width \ [m]  
\[ Re \] bulk Reynolds number \ [-]  
\[ Re_t \] friction Reynolds number \ [-]  
\[ U \] mean velocity \ [m/s]  
\[ u \] instantaneous stream-wise velocity \ [m/s]  
\[ U_b \] bulk mean velocity \ [m/s]  
\[ u_t \] friction velocity \ [m/s]  
\[ v \] instantaneous wall-normal velocity \ [m/s]  
\[ x \] coordinate (stream-wise direction) \ [m]  
\[ y \] coordinate (wall-normal direction) \ [m]  
\[ z \] coordinate (span-wise direction) \ [m]  
\[ \prime \] Root-Mean-Square value \ [-]  

\[ \alpha \] local void fraction \ [-]  
\[ \nu \] kinematic viscosity coefficient \ [m^2/s]  

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