Analysis of the vertices $B_s B K^*$ and $D_s D K^*$ with QCD Sum Rules

A Cerqueira Jr$^1$, B Osório Rodrigues$^2$, M E Bracco$^3$

$^1$ Centro Universitário de Volta Redonda, UniFOA, Av. Paulo Erlei Alves Abrantes 1325, 27240-560, Volta Redonda, RJ, Brazil
$^2$ Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro, RJ, Brazil
$^3$ Faculdade de Tecnologia, Universidade do Estado do Rio de Janeiro, Rod. Presidente Dutra Km 298, Pólo Industrial, 27537-000, Resende, RJ, Brazil

E-mail: angelo.cerqueira.cunha@gmail.com

Abstract. In this work, we calculated the form factors and coupling constant of the vertex $D_s D K^*$ using the QCD Sum Rules. The calculation of the form factors were performed for the cases $D$, $D_s$ and $K^*$ off shell. Using the similarities between $B$ and $D$ mesons, we also calculated the form factors and coupling constant of the vertex $B_s B K^*$. The coupling constants of the vertices $B_s B K^*$ and $D_s D K^*$ were compared with each other through the Heavy Hadron Chiral Perturbation Theory (HHChPT). We found a difference of 19% between $B_s B K^*$ and $D_s D K^*$ coupling constants using the HHChPT relation.

1. Introduction

In previous works, we calculated the form factors and coupling constants for the beauty vertices $B_s^* B K[1]$ and $B_s B^* K[2]$. The calculation of these vertices was made in comparison with the results of the vertices $D_s^* D K$ and $D_s D^* K[3]$. Through the relations of Heavy Hadron Chiral Perturbation Theory (HHChPT) between coupling constants of the vertices with $D$ mesons and $B$, we verified the consistency of our results for the vertices $B_s^* B K$ and $B_s B^* K$ when compared with the vertices $D_s^* D K$ and $D_s D^* K$, respectively. The coupling constants of these vertices were calculated in order to contribute to the study of hadronic molecules. These hadronic molecules are considered as weakly-bound states of two or more hadrons. In Ref [4], the author uses effective lagrangians to describe the molecular state. This treatment was extensively applied in decays of $D_{s0}$ and $D_{s1}[5]$ mesons as molecular structure, respectively, $D^* K$ [6] and $D K[7]$. In these studies, use was made of a quantum field approach based on effective lagrangians that depend on the coupling constants of vertices like $B_s^* B K$, $B_s B^* K$ and $B_s B K^*$. A precise knowledge about their values is of great importance.

The determination of the coupling constant, $g_{D_s D K^*}$, using QCD Sum Rules is important to compare with the result obtained using the HHChPT relation. The coupling $g_{D_s D K^*}$ was used to estimate the coupling $g_{B_s B K^*}$ using HHChPT in Ref [2]. This estimated value using HHChPT was useful to compare with the coupling $g_{B_s B K^*}$ obtained from QCD Sum Rules. Both results of the coupling $g_{B_s B K^*}$ are in agreement in Ref [2].
2. The QCD Sum Rules
The three-point correlation functions can be evaluated by two different approaches, one known as the OPE side, or QCD side, and the other known as the phenomenological side. In the OPE side the hadrons are described by their interpolating currents. In the phenomenological side the hadrons are described by hadronic states. The starting point of the calculation is the three point correlation function,

\[ \Pi(p,p') = \int d^4xd^4y \langle 0 | T \{ j_1(x) j_2(y) j_3(0) \} | 0 \rangle e^{ip'x}e^{-iq'y}. \] (1)

In the OPE side, a Wilson’s operator product expansion is applied, the OPE side is obtained from Eq. 1. The OPE side is described as a sum of the perturbative and non-perturbative contributions to the correlation function. The non-perturbative term includes the contribution of all the QCD condensates. In this work, the only non-perturbative term we calculate is the contribution from quark condensates, i.e., \( \Pi^{\text{non-pert}} = \Pi^{(qq)} \).

In the phenomenological side, we use the same correlation function of Eq. 1, where we perform a sum over a set of hadronic states. Applying the quark-hadron duality, the quantities of interest (masses, form factors, coupling constants) are obtained matching both sides before a Borel Transform.

3. The vertex \( D_s DK^* \)
The coupling constant of the vertex \( D_s DK^* \) was calculated using the three mesons off shell. In the case where the \( D \) meson is off shell in Fig. 1(a), there is non-perturbative contribution from the strange quark condensate (Fig. 1(d)). The \( D_s \) off shell diagram (Fig. ??(b)) has contribution from the light quark condensate (Fig. 1(e)), the \( K^* \) off shell (Fig. 1(c)) is the only diagram that has no contribution from quark condensate.

3.1. \( D_s \) off shell
Using in Eq. 1 the interpolating currents written in terms of the quark fields: \( j_3 = j_5^{D_s} = ic\gamma_5s \), \( j_2 = j_2^{D} = \bar{q}\gamma_c \) and \( j_1 = j_\mu^{K^*} = \bar{q}\gamma_\mu s \), we obtain the OPE side, given by:

\[ B_{M^2}B_{M'^2}[\Pi^{\text{OPE}}_{\mu}(p,p')] = -\frac{1}{4\pi^2} \int_{s_{\text{min}}}^{\infty} \int_{t_{\text{min}}}^{\infty} dsdu \frac{3}{2\pi \Lambda^2} \{ p_{\mu} [A(2p \cdot k - p \cdot pt + mcms - m_s^2) - k \cdot pt + m_s^2] + p_{\mu} [B(2p \cdot k - p \cdot pt + mcms - m_s^2) + k \cdot p - m_s^2 + mcms] \} e^{-s/M^2}e^{-u/M'^2}. \] (2)

The Borel transform gives the exponentials in Eq. 2. The Borel Transform is useful to eliminate the contribution of higher moments of excited states. The contribution of the strange quark condensate, is given by:

\[ B_{M^2}B_{M'^2}[\Pi^{[ss]}] = -mc_{ss}e^{-m^2_s/M^2}p_{\mu}'. \] (3)

The phenomenological side yields the following equation,

\[ B_{M^2}B_{M'^2}[\Pi^{\text{phen}}_{\mu}(p,p')] = f_{K'}f_{D}f_{D_s}m_{K'}m_{D_s}^2m_{D}^2y_{D_s,DK^*}(Q^2) \left( \frac{m_{mc} + m^2_s}{(Q^2 + m^2_{D_s})} \right) \left( \frac{m^2_D - m^2_{D_s}}{m_{K'}^2} + 1 \right) e^{-m^2_{D_s}/M^2}e^{-m^2_{K'}/M'^2}, \] (4)
Figure 1: The diagram (a) corresponds to the perturbative OPE term of $D$ meson off shell, The diagrams (b) and (c) correspond to the perturbative OPE term of $D_s$ and $K^*$ mesons off shell, respectively; and the diagrams (d) and (e) correspond to the strange quark and light quark condensates contributions, respectively.

where $f_D$, $f_{D_s}$ and $f_{K^*}$ are the decay constants of the mesons, the term $Q^2$ is the transferred momentum and $g_{D_sDK^*}^{(Q^2)}$ is the form factor.

Figure 2(a) shows the pole and continuum contribution and figure 2(b) the Borel mass stability of the $D$ off shell case. The tensor structure chosen is $p'$ and it has a good sum rule and contribution of strange quarks condensates. The Borel mass is stable for values greater than $4.0 GeV^2$ and the contribution of the pole is greater than the continuum contribution. The values used for the continuum thresholds are $\Delta s = \Delta u = 0.5 GeV$. The stability of the Borel mass allowed us to choose its value equal to $M^2 = 6.0 GeV^2$. The plots shown in Fig.2 were made fixing $Q^2 = 1.0 GeV^2$. The data of the form factors were fitted by an exponential function, given by:

$$g_{D_sDK^*}^{(D)} = 1.32 e^{-Q^2/18.16}.$$  (5)
So, the value of the coupling constant is given by:

\[ g_{D_sDK^*}^{(D)} = 1.67. \]  \hfill (6)

This value is obtained extrapolating the form factor to the negative value of the squared mass of the meson off shell. The value of the function in this point, where \( Q^2 = -m^2 \), gives the value of the coupling constant.

![Figure 2: The pole and continuum conditions (a) and the Borel mass stability (b).](image)

### 3.2. \( D_s \) off shell

Starting from Eq. 1 with \( D_s \) off shell(fig. 1(b)) and using the same interpolating currents, we obtain the OPE side:

\[
B_{M^2} B_{M'^2} [\Pi_{\mu}^{QCD}(p,p')] = -\frac{1}{4\pi^2} \int_{s_{\min}}^{\infty} \int_{u_{\min}}^{\infty} ds du \frac{3}{2\pi \sqrt{\lambda}} \left\{ p_\mu [A(p \cdot p' - m_c m_s - 2p \cdot k) - k \cdot p'] + p'_\mu [B(p \cdot p' - m_c m_s - 2p \cdot k) + k \cdot p] \right\} e^{-s/M^2} e^{-u/M'^2}. \]  \hfill (7)

The diagram of the \( D_s \) off shell has a contribution of the light quark condensate \( \langle q\bar{q} \rangle \), given by:

\[
B_{M^2} B_{M'^2} [\Pi_{\mu}^{(q\bar{q})}(p,p')] = \langle q\bar{q} \rangle \left[ m_c p'_\mu - m_s p_\mu \right] e^{-m_c^2/M^2} e^{-m_s^2/M'^2}. \]  \hfill (8)

Starting from the correlation function Eq. 1, through the calculation of hadronic states, we obtain the following expression for the phenomenological side:

\[
B_{M^2} B_{M'^2} [\Pi_{\mu}^{\text{phen}}(p,p')] = -i f_{K^*} f_{D_s} m_{K^*} m_{D_s} m_{D_s}^2 g_{D_sDK^*}^{(D)} (Q^2) \times \left[ -2p_\mu + p'_\mu \left( 1 - \frac{m_{D_s}^2 - m_{D_s}^2}{m_{K^*}^2} \right) \right] e^{-m_{D_s}^2/M^2} e^{-m_{K^*}^2/M'^2}. \]  \hfill (9)
In the case of the $D_s$ off shell, there is a contribution of light quark condensate. The tensor structure chosen to calculate the form factors is the $p_\mu$ structure. This structure shows Borel mass stability (Fig. 3(b)) and conditions of pole and continuum (Fig. 3(a)). The values of the continuum thresholds are $\Delta s = \Delta u = 0.5 GeV$ and the value fixed for the Borel mass is $M^2 = 3.5 GeV^2$. The plots shown in Fig. 3 were made fixing $Q^2 = 1.0 GeV^2$. The form factors were fitted by an exponential function,

$$g^{(D_s)}_{D_sDK^*} = 1.15 e^{-Q^2/27.08}.$$  \hspace{1cm} (10)

The value of the constant is given by:

$$g^{(D_s)}_{D_sDK^*} = 1.35.$$  \hspace{1cm} (11)

Figure 3: The pole and continuum conditions (a) and the Borel mass stability (b).

### 3.3. $K^*$ off shell

Starting from the correlation function of the $K^*$ off shell (Fig. 1(c)), we obtain the OPE side:

$$B_{M^2}B_{M^2}[\Pi^QCD_\mu(p,p')] = \frac{1}{4\pi^2} \int_{s_{\text{min}}}^{\infty} \int_{u_{\text{min}}}^{\infty} dsdu \frac{3}{\sqrt{\lambda}} \{p_\mu[A(p \cdot p' + m_c m_s - m^2_s) + m^2_c - k \cdot p' - m_c m_s] + p'_\mu[B(p \cdot p' + m_c m_s - m^2_s) - k \cdot p + m^2_c]e^{-s/M^2}e^{-u/M'^2}\} e^{-m^2_k/M^2}e^{-m^2_{K^*}/M'^2}.$$  \hspace{1cm} (12)

This diagram does not have contribution of quark condensate. The phenomenology gives the following expression:

$$B_{M^2}B_{M^2}[\Pi^{\text{phen}}_\mu(p,p')] = -\frac{f_D f_{K^*} f_{D_s} m_K m_{D_s} m^2_{D_s} m^2_{K^*}}{(m_s m_c + m^2_c)} \frac{g^{(K^*)}_{D_sDK^*}(Q^2)}{(Q^2 + m^2_{K^*})} [p_\mu \left(1 - \frac{m^2_{D_s}}{m^2_{K^*}}\right) + p'_\mu \left(1 - \frac{m^2_{D_s}}{m^2_{K^*}}\right)] e^{-m^2_{D_s}/M^2}e^{-m^2_{D_s}/M'^2}.$$  \hspace{1cm} (13)
Figure 4 shows the pole and continuum condition (a) and the Borel mass stability (b). Our result has only the contribution of the tensor structure $p_{\mu}$. The other tensor structure does not yield the necessary conditions for a good sum rule. For a good sum rule, the Borel mass must show stability and the pole must have a contribution greater than the continuum. The values of the continuum thresholds are $\Delta s = \Delta u = 0.5\, GeV$ and the Borel mass is $M^2 = 2.0\, GeV^2$. The plots shown in Fig. 4 were made fixing $Q^2 = 1.0\, GeV^2$. The data of this diagram were fitted also by an exponential function,

$$g_{D_s DK^*}^{(K^*)} = 1.13e^{-Q^2/2.41}. \quad (14)$$

Then, the value of the coupling constant is given by:

$$g_{D_s DK^*}^{(K^*)} = 1.63. \quad (15)$$

![Figure 4: The pole and continuum conditions (a) and the Borel mass stability (b).](image)

### 3.4. Results

The table 1 shows the values of masses and decay constants used in this work.

| Quantity | c (GeV) | s (GeV) | b (GeV) | $D_s$ | $D$ | $K^*$ | $B$ | $B_s$ |
|----------|---------|---------|---------|-------|-----|-------|-----|-------|
| $m$      | 1.20    | 0.13    | 4.20    | 1.97  | 1.87| 0.89  | 5.20| 5.40  |
| $f$ (MeV) | -       | -       | -       | 280   | 200 | 220   | 191 | 227   |

The value of the $K^*$ decay constant is in [9]. The other decay constants were taken from [3]. The results of the form factors are shown in the Fig. 5. The value of the light quark condensate is, $-0.23\, GeV^3$ and the value of strange quark condensate is, $0.8(0.23\, GeV)^3$. 


The result of the coupling constant of the vertex $D_sDK^*$ is given by:

$$g_{D_sDK^*} = 1.56.$$  (16)

This value is an average value for this constant considering the three values obtained for each meson off shell. We performed the same calculation substituting the $D$ meson by the $B$ meson and the value found for the coupling constant is $g_{B_sBK^*} = 3.30 \pm 0.10$.

4. Conclusions

The coupling constant for the vertex $B_sBK^*$ estimated by the Heavy Hadron Chiral Perturbation Theory (HHChPT)

$$g_{B_sB_sL} = g_{D_sD_sL} \frac{m_B}{m_D},$$

using the values the coupling constant $g_{D_sDK^*}$, is given by

$$g_{B_sBK^*} = 4.10.$$  (17)

The result of the $B_sBK^*$ from QCD Sum Rules is

$$g_{B_sBK^*} = 3.3.$$  (18)

There is a difference about 19%. But, these are preliminaries results. The next step is to perform an error analysis of these results.

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References

1. Cerqueira Jr A, Osório Rodrigues B, Bracco M E 2012 *Nuc. Phys. A* **874** 130–142
2. Cunha Júnior A C. Fatores de forma em processos com mósons *B*. 2013. 133 f. Tese(Doutorado em física)-Instituto de Física Armando Dias Tavares, Universidade do Estado do Rio de Janeiro, Rio de Janeiro, 2013.
3. Bracco M E, Cerqueira Jr A, Chiapparini M, Lozeá A, Nielsen M 2006 *Phys. Lett. B* **641** 286–293
4. Faessler A, Gutsche T, Lyubovitskij V E, Ma Y L 2008 *Phys. Rev. D* **077** 114013
5. Faessler A, Gutsche T, Kovalenko S, Lyubovitskij V E 2007 *Phys. Rev. D* **076** 014003
6. Faessler A, Gutsche T, Lyubovitskij V E, Ma Y L 2007 *Phys. Rev. D* **076** 114008
7. Faessler A, Gutsche T, Lyubovitskij V E, Ma Y L 2007 *Phys. Rev. D* **076** 114005
8. Particle Data Group, Review of Particle Physics 2014 *Chin. Phys. C* **38**
9. Fang Su, Yue-Liang Wu, Ci Zhuang, Yi-Bo Yang 2011 *Eur. Phys. J* **C72** 1914