On an Intrinsically Local Gauge Symmetric SU(3) Field Theory for Quantum Chromodynamics

Brian Jonathan Wolk*

Abstract. The SU(3)$_c$ gauge theory for eight massless vector gauge fields is assimilated into the formalism which generated the intrinsic local gauge invariant $U(1)$ and $SU(2)_L$ theories [30,31], thus establishing the theoretical structure for intrinsically local gauge invariant quantum chromodynamics. Use is made of both the standard and split algebra of the octonions, the last of the Hurwitz algebras, in devising the technology. Numerous interesting results are obtained, including an explanation of the inherently non-chiral nature of the strong and electromagnetic interactions in contradistinction to the weak interaction. The formalism’s novelty and universality compels contemplation of its potential ability to assimilate the gravitational interaction as well.

Keywords. SU(3)gauge theory, Local gauge invariance, Quantum field theory, Quantum chromodynamics, Strong force, Octonions, Hurwitz algebras, Normed division algebras, Particle physics, Yang–Mills theory, Standard model.

1. Introduction

This is the third$^1$ in a series of papers establishing a new formalism for deriving, and a new conceptual framework for conceiving, the fundamental interactions of nature. Just as the $U(1)$ theory of quantum electrodynamics (QED) and $SU(2)_L$ theory were developed [30,31], herein is developed an intrinsically local $SU(3)$ gauge invariant theory. Since $SU(3)_c$ theory in its unbroken form gives the theory of quantum chromodynamics (QCD) [4,11,12,19], the new formalism can be said to generate QCD in an intrinsically local gauge symmetric environment.

In addition to providing a unified and consistent mathematical rendition of the strong interaction, the simplicity and integrity of the formalism’s

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$^1$ See Ref. [30,31].

*Corresponding author.
approach makes it amenable for conceptually introducing the essential nature of the strong interaction to a broad level of studies within all potential fields of scientific and mathematical expertise. From a didactic standpoint this provides the formalism with a distinct advantage over other frameworks which concern the fundamental forces and which are often deemed inaccessible to all but a mathematical elite.

As an example, college-level mathematics and physics students are made well aware of the existence of the unique normed division (Hurwitz) algebras and their basic attributes. It would be fairly straightforward to introduce a section in the textbooks introducing the structures (operators and fields) of QCD through use of the current formalism, showing how the theory can be quickly built from the ground up within the confines of the Hurwitz algebras. Pure mathematics professors would assuredly find it satisfying—as it is in no doubt intriguing—to teach their students that the forms of the fundamental laws of physics are entirely dependent on the structure of the solely mathematically derived Hurwitz algebras. Further and importantly, such a didactic approach would also provide an excellent avenue for introducing the basics of the Clifford algebra—a mathematical arena becoming ever-important in fundamental physics—to all levels of students, as well as to scientists and mathematicians who do not typically practice in the area of theoretical particle physics.

So to begin, notice is first made that one of the only two algebras in the closed system of the four Hurwitz algebras plays a ubiquitous and necessary role in intrinsically gauging the $\mathbb{U}(1)$ and $\mathbb{SU}(2)$ theories [30,31]. The formalism for $\mathbb{U}(1)$ and $\mathbb{SU}(2)$ depends on and is mandated by the structure of the Hurwitz algebra $\mathbb{Q}$ along with its operator technology, including the operator coupling equation [30]

$$\left(v_0, v\right)\left(w_0, w\right) = (v_0 w_0 - v \cdot w, v_0 w + v w_0 + v \times w).$$

This coupling equation exists for $\mathbb{O}$ as well [1,7,9,25]. Again, $\mathbb{O}$ is the only Hurwitz algebra other than $\mathbb{Q}$ with a non-trivial cross product. Further, the QCD $\mathbb{SU}(3)$ group is a subgroup of $\mathbb{G}_2$ [1,18,20,22,25], which is the automorphism group of $\mathbb{O}$ [1,6,10,20]: $\mathbb{SU}(3)_c \subset \mathbb{G}_2$. Lastly, QCD has been formulated in terms of $\mathbb{O}$ [4,13,23].

These lines of thought lead to considering the octonion algebra as a structure for generalizing the formalism used for $\mathbb{U}(1)$ and $\mathbb{SU}(2)$ in order to generate an intrinsically local $\mathbb{SU}(3)$ gauge invariant Lagrangian for QCD.

2. The Octonions with Gell–Mann Basis

The goal of the current paper is not to frame QCD in terms of the octonions. This has already been achieved [4,13,23], the results of which will be used herein.

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2 The two algebras being the quaternions $\mathbb{Q}$ and octonions $\mathbb{O}$ [1,18,20,25].
3 The four being \{R, C, Q, O\} [20].
4 The dim(3) cross product for $\mathbb{Q}$ and the dim(7) cross product for $\mathbb{O}$ [1,3,7,22].
The goal of the paper instead is to generate an intrinsically local $SU(3)$ gauge invariant formalism. To do so requires QCD to be framed within the structure of the two octonion algebras which up to isomorphism form the only distinct octonion algebras over the reals $[1,9,22,25]$, namely for this paper: (1) the standard octonion algebra (with basis isomorphic to the Gell–Mann elements $\{\lambda_a\}$), designated as $G$; and (2) the split-octonion algebra (in the isomorphic Zorn-algebra representation) $[3,17,22,26]$, designated as $Z$.

The relation between the standard octonion basis elements $\{e_a\}$ and the eight Gell–Mann basis elements $\{\lambda_a\}$ is straightforward. The $\{\lambda_a\}$ satisfy the well-known commutation relations $[5,11,12]$

$$[\lambda_a, \lambda_b] = 2i f_{abc} \lambda_c,$$  

(2)

where $\{a, b, c = 0 − 7\}$ and the $f_{abc}$ are the non-zero structure constants $[5,11,12,19,21]$. Set $\lambda_0 \equiv \lambda_8$ used in the standard rendition of the Gell-Mann elements. A bijective mapping between $\{e_a\}$ and $\{\lambda_a\}$ relates $G$ to QCD and allows definition of new basis elements $\{\ell_a\}$ $[4,13,23]$

\[
\begin{align*}
e_0 &\mapsto \frac{\sqrt{3}}{2} \lambda_0 \equiv \ell_0 \\
e_1 &\mapsto \lambda_1 \equiv \ell_1 \\
e_2 &\mapsto \lambda_2 \equiv \ell_2 \\
e_3 &\mapsto \lambda_3 \equiv \ell_3 \\
e_4 &\mapsto \frac{1}{2} \lambda_4 \equiv \ell_4 \\
e_5 &\mapsto \frac{1}{2} \lambda_5 \equiv \ell_5 \\
e_6 &\mapsto -\frac{1}{2} \lambda_6 \equiv \ell_6 \\
e_7 &\mapsto -\frac{1}{2} \lambda_7 \equiv \ell_7.
\end{align*}
\]

(3)

with associated commutation relations and structure constants $[4]$

$$[e_a, e_b] \rightarrow i [\lambda_a, \lambda_b] \forall abc = 123;$$

$$[e_a, e_b] \rightarrow \frac{i}{2} [\lambda_a, \lambda_b] \forall abc = 147, 246, 257, 345, 165, 376;$$

$$[e_a, e_b] \rightarrow \frac{\sqrt{3}}{2} i [\lambda_a, \lambda_b] \forall abc = 450, 670.$$ 

\[
\begin{align*}
f_{abc} &\mapsto f_{abc} \forall abc = 123; \\
f_{abc} &\mapsto \frac{1}{2} f_{abc} \forall abc = 147, 246, 257, 345, 165, 376; \\
f_{abc} &\mapsto \frac{\sqrt{3}}{2} f_{abc} \forall abc = 450, 670.
\end{align*}
\]

(4)

(5)

Using $\{\ell_a\}$ within the formalism’s machinery will permit deduction of part of the $SU(3)_c$ gauge field tensor’s interaction forms necessary for generating an intrinsically local gauge invariant Lagrangian.

3. The Formalism

As with the quaternionic operator $\eta$ used for the $U(1)$ and $SU(2)$ technologies $[30,31]$, the fundamental $SU(3)$ octonionic operator for $G$ is defined as $\eta_{SU(3)_G} = \ell_0 \gamma_0 + \ell_i \gamma_i; i = 1 − 7$. The Clifford elements are simply extended to
have the properties akin to the Clifford–Dirac set,\(^5\) with \(\gamma_a \gamma^a = 8; a = 0 - 7.\) We define \(\gamma_{SU(3)} = \ell_i \gamma_i\) for later use.

Crucially for the \(SU(3)\) technology, Eq. (1)’s mathematical machinery used to intrinsically gauge \(U(1)\) and \(SU(2)\) also exists for \(\Omega [1,7,9,18,22]\), where \(v = (v_0, v)\) and \(w = (w_0, w)\) are now octonionic operators, there is an extended dot product and \(v \times w\) is the \(\text{dim}(7)\) cross product.

\(SU(3)_c\) requires a triplet Dirac field \(\Psi [11,12,19,21,24]\), and the Lie algebra \(su(3)\) requires eight fields for the gauge field \(G_\mu(x) = \{G^i_\mu(x) : i = 0 - 7\} [11,12].\) The \(SU(3)\) tensor is similar in form to that for \(SU(2) [12]\)

\[
F_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - 2g(G_\mu \times G_\nu),
\]

with pivotal distinction being the structure of the non-linear interaction term \(G_\mu \times G_\nu,\) for which we now begin construction.

**i. The non-linear component \(G_\mu \times G_\nu\)**

For \(SU(3)_c\) the cross product of the gauge fields is different than for \(SU(2).\) We have for the definition of the \(SU(3)_c\) cross product of QCD \([5,12]\)

\[
(B \times C)_i = \sum_{i=0}^7 f_{ijk} B_j C_k.
\]

Using Eq. (1) the coupled operator \(\eta_{SU(3)}\)\(^6\) is given by

\[
\eta_{SU(3)} = \gamma_0 \gamma_{SU(3)} + \gamma_{SU(3)} \gamma_0 + \left(\gamma_{SU(3)} \times \gamma_{SU(3)}\right)
= \left\{\gamma_0, \gamma_{SU(3)}\right\} + \frac{1}{2} \left[\gamma_{SU(3)}, \gamma_{SU(3)}\right],
\]

where the relation \(a \times b = \frac{1}{2} [a, b]\) has been used, and \(\{,\}\) is the anti-commutator.

At this point for \(SU(2)\) the two components are separately analyzed \([31]\), with \(\gamma \times \gamma\) being sufficient to provide the \(W_\mu\) gauge field connections and the remaining operator \(\gamma_0 \gamma\) generating chiral asymmetry \([31]\). However, two issues immediately arise for the \(SU(3)\) formalism.

Firstly, with \(SU(3)\) there is no chiral asymmetry as with the weak interaction, and so \(\gamma_0 \gamma_{SU(3)}\) appears misplaced.

Secondly, Eq. (8)’s \(\text{dim}(7)\) cross product \(\gamma_{SU(3)} \times \gamma_{SU(3)}\) provides a summation and bijective mapping between Clifford fields and the gauge fields on only seven terms \((\sum_i^7).\) To form a bijection with Eq. (7)’s rendition of the cross product for \(G_\mu \times G_\nu\) a summation on eight terms is required, corresponding to the eight gauge fields. The expression \(\gamma_{SU(3)} \times \gamma_{SU(3)}\) is missing the zeroth operator component \(\left(\gamma_{SU(3)} \times \gamma_{SU(3)}\right)_0.\)

\(^5\) See Ref. [30], Eq. (1).

\(^6\) The scalar portion of the operator \(\eta_{SU(3)}\), namely \(\eta_{SU(3)} \equiv \Omega_{SU(3)}\), is but a scalar which has no effect on the physics and thus may be disregarded.
There thus appears at first blush to be prohibitive obstacles to a tenable formulation of the $SU(3)$ technology. We might consider the operator expression \( \{ \gamma_0, \gamma_{SU(3)} \} \) to determine if it can be used to overcome these obstacles.

If the standard octonion unit basis \( \{ e_a \} \) abiding by the constraints [1, 4, 13]

\[
e_0 e_i = e_i e_0,
\]

were being used in the $G$-technology, then the expression \( \{ \gamma_0, \gamma_{SU(3)} \} \) would pose a problem for further development of the formalism.

Fortunately the basis elements of $G$ do not commute as in Eq. (9), and are not subject to its constraints. Instead $G$ abides by the commutation relations set forth in Eqs. (2–5). As such the basis element \( \ell_0 \) attached to \( \gamma_0 \) must also be considered in analyzing the expression \( \{ \gamma_0, \gamma_{SU(3)} \} \). Applying the commutation relations to the explicit expression gives

\[
\{ \ell_0 \gamma_0, \gamma_{SU(3)} \} = \gamma_0 \ell_0 \gamma_{SU(3)} + \gamma_{SU(3)} \gamma_0 \ell_0
\]

\[
= \gamma_0 \gamma_4 \ell_0 \ell_4 + \gamma_0 \gamma_5 \ell_0 \ell_5 - \gamma_0 \gamma_4 \ell_4 \ell_6 - \gamma_0 \gamma_5 \ell_5 \ell_6
\]

\[
+ \gamma_0 \gamma_6 \ell_6 \ell_6 + \gamma_0 \gamma_7 \ell_6 \ell_7 - \gamma_0 \gamma_6 \ell_7 \ell_6 - \gamma_0 \gamma_7 \ell_7 \ell_6
\]

\[
= \left( 2 \sum_{i=1}^{8} f_0'_{ik} \gamma_0 \gamma_k \right) \ell_i.
\]

where the structure constants\(^8\) ensure these are the only operator terms which survive, and we have also used \( \gamma_i \gamma_0 = -\gamma_0 \gamma_i \) and that the commutation relations for \( \{ \ell_a \} \) bring in an extra negative sign in writing the expression as a summation. Equation (10) gives the four terms

\[
\{ (f_{054}^' \gamma_0 \gamma_5) \ell_4, (f_{054}^' \gamma_0 \gamma_4) \ell_5, (f_{067}^' \gamma_0 \gamma_7) \ell_6, (f_{076}^' \gamma_0 \gamma_6) \ell_7 \}. \quad (11)
\]

Consider now Eq. (7)’s rendition of the zeroth term operator:

\[
(\gamma_{SU(3)} \times \gamma_{SU(3)})_0 = (f_{0jk} \gamma_j \gamma_k) \ell_0. \quad (12)
\]

This reduces to the four operator terms

\[
\{ (f_{054}^' \gamma_4 \gamma_5) \ell_0, (f_{054} \gamma_5 \gamma_4) \ell_0, (f_{067} \gamma_7 \gamma_6) \ell_0, (f_{076} \gamma_6 \gamma_7) \ell_0 \}. \quad (13)
\]

We now make use of a fundamental principle of QCD, that of gluon confinement [11, 12]. Because of gluon confinement which of the eight gauge fields is involved in any given interaction is not determinable, as they are not individually observable [12, 16, 28].\(^9\) The gauge fields are thus taken as indistinguishable for purposes of writing the field tensor $F_{\mu \nu}$.

This permits establishment of a one-to-one and onto correspondence between the operator terms generated by the expression \( \{ \ell_0 \gamma_0, \gamma_{SU(3)} \} \) of Eq. (11) and those of Eq. (13) generated by \( (\gamma_{SU(3)} \times \gamma_{SU(3)})_0 \).

\(^7\) This differs from the $SU(2)$ technology in which the basis element attached to $\gamma_0$ commutes with all other basis elements and thus need not be expressly written.

\(^8\) Defined for this summation as: $f_{0nm}' \equiv \frac{1}{2} f_{0nm}$. This can be done because the operator summation is not yet bijectively integrated into the summation of Eq. (7).

\(^9\) See, e.g., Ref. [16], p. 91.
As a result and pursuant to the same strategy used for the $SU(2)$ formalism a bijective mapping between operators on eight Clifford fields and those on the eight $SU(3)$ gauge fields can be written:

$$\sum f'_{ijk}\gamma_j \gamma_k G_j G_k \Leftrightarrow \sum f_{ijk} G_j G_k,\quad (14)$$

where the structure constant relations remain as in Eq. (2-5), excepting when $i = 0$ for which $f'_{0nm} \equiv \frac{1}{2}f_{0nm}$ as in Eq. (10). Consequently it has been shown that $\eta \wedge SU(3) G$ generates $G_\mu \times G_\nu$.

Both issues raised above have thus been addressed and resolved through analysis of $\{\gamma_0, \gamma_{SU(3)}\}$. As a result of absorption of $\{\gamma_0, \gamma_{SU(3)}\}$ into Eq. (14) no terms in $\eta \wedge SU(3) G$ remain to permit establishment of an intrinsically chirally asymmetric QCD theory. This is identical to the result for $U(1)$ theory and dissimilar to what was found for the coupled operator of $SU(2)$ theory, in which the component operator $\gamma_0 \gamma$ was not absorbed into the non-linear $SU(2)$ gauge field term.

ii. The Dirac component and linear $\partial_\mu G_\nu - \partial_\nu G_\mu$

In the $U(1)$ technology use of the fundamental quaternionic operators $\partial$ and $\eta$ in Eq. (1) generated the Dirac operator equation and the linear electromagnetic tensor [30]. The same strategy was successfully applied to $SU(2)$ [31]. However, when applied to the $SU(3)$ methodology problems arise.

Using the standard octonionic basis elements $\{e_a\}$ to define the analogous fundamental octonionic operators $\partial = \partial/\partial t + e_i \partial/\partial x_i$ and $\eta = \gamma_0 + e_i \gamma_i$ and applying the coupling equation generates an ‘extended’ Dirac operator $\partial_8 s$ without clear or evident interpretation or application. Other issues arise in the vector operator portion. Thus the identical strategy as was used in the $U(1)$ and $SU(2)$ formalism does not appear to comport with the current methodology’s use of the Hurwitz algebra $\mathbb{O}$ to intrinsically gauge $SU(3)$.

At the same time, neither the formalism nor the Hurwitz algebra structure can be abandoned without forfeiting the architectonic principle of intrinsicaly as previously enunciated [30,31]. Further development of the $SU(3)$ technology must remain within the confines of the Hurwitz algebra $\mathbb{O}$.

As noted above, there are two and only two distinct octonion algebras over the reals. From a theoretical posture it is logical to suppose that these equally fundamental algebras should be exhausted in rendition of the entire formalism. The standard octonion algebra $\mathbb{O}$ has already been utilized in generating the non-linear tensor $G_\mu \times G_\nu$.

Uniting this line of reasoning with the observations that: (1) Nature seems to prefer the split-octonions to the standard octonions for rendition of the $U(1)$ theory for electromagnetism [17]; and (2) the $U(1)$ theory of electromagnetism concerns only a linear tensor term equivalent in form to $\partial_\mu G_\nu - \partial_\nu G_\mu$ (the component yet to be generated in this formalism), leads to an interrogation of the split-octonion algebra to determine whether when

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10 In which the coupled operator’s vector portion $\eta \partial_8$ integrated in its entirety into the Maxwell field tensor.
couched within the current formalism it might produce a technology from which the coupled linear $\partial_\mu G_\nu - \partial_\nu G_\mu$ and Dirac components can be generated.

The split-octonion algebra is isomorphic to the Zorn algebra $\mathcal{Z}$ [3,22,25,26], permitting representation as said. Elements of $\mathcal{Z}$: $A \in \mathcal{Z}$, are $2 \times 2$ matrices under standard scalar addition and multiplication and abide by the binary composition rule [3,18,22,25,26]

$$AB = \begin{pmatrix} \alpha & a \\ b & \beta \end{pmatrix} \begin{pmatrix} \kappa & k \\ t & \tau \end{pmatrix} = \begin{pmatrix} \alpha \kappa + \phi (a,t) & \alpha k + a \tau + \varphi [b,t] \\ \kappa b + t \beta + \psi [a,k] & \beta \tau + \phi (b,k) \end{pmatrix},$$ (15)

where $\{(\alpha,a), (\beta,b),(\kappa,k),(\tau,t)\}$ are as $\{(v_0,v)\}$ and $\{(w_0,w)\}$ in the coupling Eq. (1) for the $U(1)$ and $SU(2)$ formalism [30,31], and the dim(3) dot product $(x,y)$ and commutator $[x,y]$ are as in Eq. (1) with $(\phi,\varphi,\psi) = (\pm 1, \pm \frac{1}{2}, \pm \frac{1}{2})$.

For the current formalism $(\phi,\varphi,\psi) = (-1, +\frac{1}{2}, +\frac{1}{2})$ is put.\(^{11}\)

We next generalize the fundamental quaternionic operators by defining the aesthetically symmetric $\mathcal{Z}$-operators

$$\eta = \begin{pmatrix} \gamma_0 & \gamma \\ \gamma & \gamma_0 \end{pmatrix}, \quad \vartheta = \begin{pmatrix} \partial_0 & \nabla \\ \nabla & \partial_0 \end{pmatrix},$$ (16)

with the usual quaternionic definitions for the operators $\partial$ and $\eta$ being [30,31]

$$\partial = \partial_0 + \nabla \quad \eta = \gamma_0 + \gamma.$$ (17)

Precisely as in forming the coupled operator $\eta \partial$, using the binary composition rule we form the coupled operator $\eta \partial$:

$$\begin{pmatrix} \gamma_0 & \gamma \\ \gamma & \gamma_0 \end{pmatrix} \begin{pmatrix} \partial_0 & \nabla \\ \nabla & \partial_0 \end{pmatrix} = \begin{pmatrix} \gamma_0 \partial / \partial t - \gamma \cdot \nabla & \gamma_0 \nabla + \gamma \partial / \partial t + \gamma \times \nabla \\ \gamma_0 \nabla + \gamma \partial / \partial t + \gamma \times \nabla & \gamma_0 \partial / \partial t - \gamma \cdot \nabla \end{pmatrix},$$ (18)

which using the $U(1)$ symbolism can be written compactly as [30]

$$\eta \partial \equiv \begin{pmatrix} \partial \nabla & \nabla \partial \end{pmatrix}.$$ (19)

The structure of the resultant coupled operator field is undoubtedly revealing. Considering the component-wise structure for all $A \in \mathcal{Z} = \begin{pmatrix} \alpha & a \\ b & \beta \end{pmatrix}$, we now simply resect one of the coupled operator forms: $(\alpha,a)$ or $(\beta,b)$, from the coupled field operator $\eta \partial$, thereby giving the selfsame coupled equation for the Dirac and linear $\partial_\mu G_\nu - \partial_\nu G_\mu$ operators as was generated for the $U(1)$ and $SU(2)$ formalism, namely: $(\gamma_0 \partial / \partial t - \gamma \cdot \nabla, \gamma_0 \nabla + \gamma \partial / \partial t + \gamma \times \nabla)$. This is the desired result. Construction of the linear terms of $F_{\mu\nu}$ as well as the Dirac operator equation governing the triplet field $\Psi$ follows as with $U(1)$ and $SU(2)$ [30,31].

\(^{11}\) As in Ref. [26].
4. The Lagrangian

The intrinsically local gauge invariant Lagrangian for \(SU(3)\) also follows as in the \(U(1)\) or \(SU(2)\) formalism [30,31]:

\[
\mathcal{L}[\eta \partial_{SU(3)} \otimes \eta \eta_{SU(3)}] = i e c \gamma^\mu \partial_\mu \Psi - m c^2 \overline{\Psi} \Psi - \frac{1}{16 \pi} F^{\mu \nu} \cdot F_{\mu \nu} - (g \overline{\Psi} \gamma^\mu \lambda \Psi) \cdot G_\mu.
\] (20)

Of particular interest is that since the formalism’s octonion algebra for the eight gluon fields uses the \(\{\lambda_a\}\)-basis the currents \(J_a\) can thereby be written using this basis, thus permitting the Lagrangian term analogous with that for the \(U(1)\) formalism [12]. The Lagrangian \(\mathcal{L}[\eta \partial_{SU(3)} \otimes \eta \eta_{SU(3)}]\) describes three equal-mass Dirac fields in interaction with eight massless gauge fields as well as the self-interaction of the eight gauge fields [11,12,19].

5. Conclusion

The use of the Hurwitz algebras for generating the form and content of the fundamental interactions of nature is appealing from a theoretic as well as an aesthetic standpoint, and is worthy of serious consideration. For such a finding would have architectonic theoretical implications, answering various unanswered questions and providing those character traits one looks for in a viable theory, namely: universality, rigidity, portability, uniqueness and closure.\(^{12}\)

The aesthetic and theoretic appeal of the theory is evident. The formalism reveals its power and simplicity in inevitably generating the correct forms for the operator fields, as well as in establishing chirality constraints for the three interactions. The formalism and its results arise as of necessity from application of the closed and unique Hurwitz algebras, within which the Clifford algebra substructure is kept intact. Given these considerations it is natural to query as to how gravitation might be assimilated into the technology.\(^{13}\)

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\(^{12}\) See Ref. [29], pp. 17, 105–106, 133–135, 147–152. Regarding portability, see also Ref. [2]. Regarding closure the following observation is often made: There are three gauge theories of the fundamental interactions deriving from the three simple symmetry groups \(U(1)\), \(SU(2)\) and \(SU(3)\). “Why is there no fundamental force following from \(SU(4)\)? Nobody knows!” Ref. [24], p. 3. The answer is now readily given within this formalism. The mathematical fact that the Hurwitz algebras are closed from above at \(\mathbb{O}\) mandates that there be no fundamental force following from \(SU(4)\) or higher symmetry group.

\(^{13}\) **A brief footnote on gravitation.** Unfortunately there exists no fifth Hurwitz algebra above \(\mathbb{O}\) to attempt gravitational assimilation via the same methodology as used for \(U(1)\), \(SU(2)\) and \(SU(3)\). And yet the General Relativity Theory and Standard Theory exhibit profound similarities, such as the covariant derivative (minimal coupling) formulation [8,14,15,27]. Indeed, Maiani states that, “[matterless] Yang–Mills theory is non-trivial, actually a simplified version of Einstein’s theory of gravity.” Ref. [16], p. 31.
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Brian Jonathan Wolk
Independent
3551 Blastrone Road, Suite 105
Tallahassee FL32301
USA
e-mail: Brian.Jonathan.Wolk@gmail.com

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