Time-dependent backgrounds of two dimensional string theory from the $c = 1$ matrix model

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Abstract

The aim of this paper is to use correspondence between solutions in the $c = 1$ matrix model collective field theory and coupled dilaton-gravity to a massless scalar field. First, we obtain the incoming and outgoing fluctuations for the time-dependent backgrounds with the lightlike and spacelike boundaries. In the case of spacelike boundaries, we have done here for the first time. Then by using the leg-pole transformations we find corresponding tachyon field in two dimensional string theory for lightlikes and spacelikes boundary.

Keywords: Two Dimensional String Theory; Matrix Model; Collective Field Theory; Dilaton-Gravity; Tachyon.

1 Introduction

String theory in low space-time dimensions is an exactly solvable theory. Indeed a theory with $1 \leq D \leq 2$ is a toy model for solving many problems which haven’t exact solutions in higher dimensions. However, the string theory in two dimension [1, 2, 3] has more physics than other models. On the other hand, matrix model [4, 5, 6] is a powerful mathematical tools to solve many problems of two dimensional string theory. For example, by using the matrix model one can obtain the simple linear equation of motion, while the string theory yields to the non-linear equation. We would like to consider the $2D$ string theory from the $c = 1$ matrix model [7, 8, 9].

As we know there are $D - 2$ transverse degrees of freedom for any string. Therefore in two
dimensional string theory there are no transverse degrees of freedom, and only degree of freedom for the on-shell field in this theory is the massless bosonic field \( T(t, \phi) \), which called the massless tachyon. We must note the graviton and dilaton in two dimensional target space haven’t the on-shell degrees of freedom and they are not physical particles. Since the string coupling is an exponential function of space coordinate \( \phi \), then strings are free at the \( \phi = -\infty \) and there are no interactions between them, but they strongly coupled to each other at \( \phi = \infty \). On the other hand, existence of a cosmological constant term, proportional to \( e^\phi \), prevents the creation of large positive terms of the \( \phi \) in path integral. In the other word there are a wall which strings (massless bosons or tachyon fields) can’t pass from it and scattered after the striking to this wall. This wall often called Liouville or tachyonic wall. One can consider such a system via the matrix quantum mechanics. Indeed, the matrix model is a dual description of two dimensional string theory. The matrix with large size is used in the matrix model. For the \( N \times N \) matrix there are \( N \) pair of the \((x_i, y_i)\) eigenvalues which distribute continuously on the two dimensional surface at the \( N \to \infty \) limit. This surface give some information about particles density. Behavior of this system is similar to the fermionic system, therefore one can interpret the matrix model as a free fermionic system. Since the \( \frac{1}{N} \) factor is proportional to Plank’s constant, \( \hbar \), then the \( N \to \infty \) limit is corresponding to the classical limit. In that case according to Liouville theorem, particles in phase space move similar to an incompressible fluid which known as the Fermi sea. This fermionic representation give the microscopic description of two dimensional string theory, while the macroscopic description is Das-Jevicki collective field theory [10]. Das-Jevicki collective field theory includes all of the string interactions in two dimensions. The matrix model description of two dimensional string theory is an example of gauge/gravity correspondence, so the gauge theory lives on the boundary of space-time in the gravity theory. Here we have the matrix quantum mechanics as a gauge theory which at the \( N \to \infty \) limit lives in one dimension (time). On the other hand we have Liouville string theory as a gravity theory which lives in two dimensional target space-time. In Ref. [11] such a correspondence considered, where exact solutions of the \( c = 1 \) matrix model collective field theory and coupled dilaton-gravity to a massless scalar field (tachyon field) have written and the relation between scalar field at the lightlike boundary of space-time in both theory have shown. Also they considered an simple example of the time-dependent background with the lightlike boundary. In mentioned Ref. Karczmarek and Wang have pointed the future work with the spacelike boundaries. So, we take the motivation and progress from their paper. Our goal in this paper is to consider other time-dependent backgrounds, in particular the backgrounds with the spacelike boundaries, and obtain the incoming and outgoing fluctuations. Then by using above correspondence and the leg-pole transformations we will find tachyon fields. For this reason we will consider several time-dependent backgrounds with the lightlike and spacelike boundaries [12, 13, 14, 15, 16], and by using the same method of Ref. [11] will obtain ratio of the outgoing to the incoming tachyon fields. In the section 2 we review the correspondence between the \( c = 1 \) matrix model and coupled dilaton-gravity to the massless scalar field and also consider the leg-pole transformations which is relation between the tachyon fields in the string theory and fluctuations in the collective field theory [11]. Then in the section 3 we consider solutions with the lightlike
boundaries, so after calculation of fluctuations we can obtain the tachyon fields. In the section 4 we consider the solutions with the spacelike boundaries and similar to the section 3 try to specify the tachyon fields. Finally in the section 5 some conclusions and discussions are given.

2 Relation Between The Matrix Model and 2D String Theory

In the first part of this section, we have brief review to the $c = 1$ matrix model and Liouville string theory of coupled dilaton-gravity to the massless scalar field [11], then in the second part, we describe relation between two theories.

As we know the $c = 1$ matrix quantum mechanics describes a system of free fermion with the harmonic oscillator potential. Eigenvalues of the matrix form discrete surfaces in two dimensional phase space which called the Fermi sea. Boundary of the Fermi sea specified by $p_{\pm}(x, t)$ which satisfy the condition $p_+(x_l, t) = p_-(x_l, t)$, where $x_l$ is the most left point of the Fermi sea. Also $p_{\pm}$ are called the left and right chiral components. Local density of fermions is given by difference between $p_+$ and $p_-$. Static Fermi surface with the constant energy $E = \mu$ have an equation as $(p_- x_0)(p_+ x_0) = 2\mu$, where $\mu$ must be negative to have left and right branches without interaction. We note here that any fluctuations along the static background is given by $\eta$ field. So fluctuations come from infinity to finite $x$ and again come back to the infinity. In that case under assumption $\mu > 0$ we have [11],

$$p(x, t) = \sqrt{x^2 - 2\mu} + 2\sqrt{\pi} \partial_x \eta. \tag{1}$$

So for large negative $x$ and fluctuation field $\eta$ one can write, $x = \sqrt{2\mu} \cosh \sigma \approx \sqrt{\frac{\mu}{2}} e^{-\sigma}$ and $\partial_\sigma \eta \approx |x| \partial_x \eta$, respectively. Therefore, for given any time-dependent solution one can write the corresponding solution in form of equation (1) to obtain $\eta$ as an incoming fluctuation $\eta_{in}$. Then outgoing fluctuation given by the following relation [11],

$$\eta_{out} = \eta_{in} - \frac{\sqrt{\pi}}{\mu} (\eta_{in}')^2 + \frac{2\pi}{3\mu^2} (\partial - 1)(\eta_{in}')^3 + \mathcal{O}((\eta_{in})^4). \tag{2}$$

We are going to relate collective field theory $\eta$ to the tachyon field in string theory. Indeed, by using results of Ref. [11], we would like to obtain the tachyon fields for several lightlike and spacelike backgrounds. In the coupled dilaton-gravity to a massless scalar field there are three fields as dilaton, graviton and tachyon. In this theory, we must note that, tachyon is massless and therefore isn’t real tachyon.

At the zero-order there is linear dilaton background $\phi_0$, so by definition $x^\pm = t \pm x$, we have $\phi_0 = x^+ - x^- = 2x$. Usually, such a background absorbs to the tachyon field $T$ by definition of the new field $S = e^{-\phi_0} T$. One can expand $S$ in higher order as $S = S^{(1)} + S^{(2)} + S^{(3)} + \ldots$. Relations between the tachyon field $S$ in two dimensional string theory and the fluctuation field $\eta$ in the collective field theory are given by the leg-pole transformation [11],

$$S_{in}(x^-) = - \int dv K(v - x^-) \eta_{in}(v),$$
\[ \eta_{\text{in}}(\sigma^-) = - \int dv K(\sigma^- - v) S_{\text{in}}(v), \]
\[ S_{\text{out}}(x^+) = \int dv K(x^+ - v + \ln \frac{\mu}{2}) \eta_{\text{out}}(v), \]
\[ \eta_{\text{out}}(\sigma^-) = \int dv K(v - \sigma^- + \ln \frac{\mu}{2}) S_{\text{out}}(v), \]  

(3)

where \( x^\pm = t \pm x \) and \( \sigma^\pm = t \pm \sigma \) are the lightcone coordinates in the string theory and collective field theory respectively. \( K \) is a propagator which their integrals are in terms of delta function [11]. The term of \( \ln \frac{\mu}{2} \) in relation (3) specifies position of Liouville wall. At the \( \mu \to 0 \) limit, Liouville wall is in depth of strong coupling region, in that case one can neglect many scattering from the tachyon background.

By using the relation (2) and putting \( \eta_{\text{out}} \) order by order in the leg-pole transformation (3) and find \( S^{(1)}, S^{(2)} \) and \( \ldots \) separately, one can obtain the tachyonic field \( S \).

Before end of this section we would like to write General time-dependent Fermi surface which have a following equation [14, 16],

\[ x^2 - p^2 + \lambda_- e^{-\sigma} (x + p)^r + \lambda_+ e^{\sigma} (x - p)^r + \lambda_- \lambda_+ (x^2 - p^2)^{r-1} = 2 \mu, \]  

(4)

where \( r \) is a non-negative integer, so \( r = 1, 2 \) is corresponding to the classical solution of collective field theory and \( \lambda_\pm \) are finite constant parameters. We will consider some special case of equation (4). Already the \( r = 1 \) solutions in Ref.s [11, 12, 17, 18, 19, 20] and \( r = 2 \) solutions in Ref [14] are discussed.

### 3 Solutions With The Lightlike Boundaries

In this section we consider three time-dependent solutions of the Fermi surface and try to obtain corresponding tachyon field in two dimensional string theory. We follow similar to Ref. [11] to obtain the incoming and outgoing fluctuations \( \eta \) and tachyon field \( S \). The simplest case of time-dependent background is given by [11],

\[ (x + p + \lambda e^t)(x - p) = 2 \mu, \]  

(5)

which is corresponding to the the equation (4) with \( r = 1, \lambda_- = 0 \) and \( \lambda_+ = \lambda \). Here, \( \lambda \) and \( \mu \) are the positive constant. Equation (5) represents a moving hyperbola which its center is at \((x, p) = (\lambda e^t, \lambda e^t)\). By choosing a parameter as \(-\infty < \sigma < \infty\), we can write \( x = \sqrt{2\mu} \cosh \sigma - \frac{\lambda}{2} e^t \) and \( p = \sqrt{2\mu} \sinh \sigma - \frac{\lambda}{2} e^t \) as solutions of the equation (5). For the large negative \( \sigma \) we have \( x - p \gg 1 \), so one can write \( x - p \approx 2x \). Then for the large negative \( x \) one can rewrite the equation (5) as a following,

\[ p \approx \sqrt{x^2 - 2\mu - \lambda e^t}. \]  

(6)

In the equation (6), we separate static and dynamics parts of the equation (5), so for the large negative \( x \) at \( t = 0 \) we have \( x = \sqrt{\frac{\mu}{2}} e^{-\sigma} \) as expected. In order to obtain the incoming
fluctuation field we compare the equation (6) with the equation (1) and find the following equation,

$$
\eta_{\text{in}} \approx \frac{\lambda}{2} \sqrt{\frac{\mu}{2\pi}} e^{t-\sigma}.
$$

(7)

Then by using the equation (2) one can obtain outgoing fluctuation field as,

$$
\eta_{\text{out}} \approx \frac{1}{2\sqrt{\pi}} \left[ \lambda \sqrt{\frac{\mu}{2}} e^{t-\sigma} - \frac{\lambda^2}{4} e^2(t - \sigma) + \frac{\lambda^3}{3\mu} e^3(t - \sigma) \right].
$$

(8)

From the relation (7) it is clear that for the incoming fluctuations we have \( \eta_{\text{in}} \sim e^{t-\sigma} \), then by using the leg-pole transformation (3) we can obtain the incoming tachyon field as

$$
S_{\text{in}} \sim e^{t-x}.
$$

With the similar way one can obtain the outgoing tachyon field for several order as \( S_{\text{out}}^{(n)} \sim \left( \frac{\lambda}{2} \right)^n e^{n(t+x)} \), with \( n = 1, 2, 3, \ldots \). Therefore in the first order, we can find the ratio of the outgoing to the incoming tachyon fields as a following,

$$
\frac{S_{\text{out}}}{S_{\text{in}}} \sim \mu e^{2x} = \mu e^{\phi_0}.
$$

(9)

It tell us that the ratio of the outgoing to the incoming tachyon fields is proportional to inverse of the string coupling.

Second case we consider in this section already introduced in \([12, 17]\). We use from the following equation of the Fermi surface and will obtain the tachyon field. One can write the time-dependent Fermi surface with the lightlike boundary which represents a moving hyperbola as,

$$
(x + p + 2\lambda_- e^{-t} \frac{x + p}{x - p} + 2\lambda_+ e^{t})(x - p) = 2\mu,
$$

(10)

where \( \lambda_\pm \) is arbitrary non-negative constant. The center of hyperbola (10) placed at \((x, p) = (-\lambda_+ e^t - \lambda_- e^{-t}, -\lambda_+ e^t - \lambda_- e^{-t})\). Equation (10) is corresponding to the general equation (4) with \( r = 1 \) and infinitesimal \( \lambda_\pm \), so one can neglect the second order of \( \lambda_\pm \). Here, there are some special cases, for example the solutions with \( \lambda_- = 0 \) and \( \lambda_+ = \frac{\lambda}{2} \), are the same as solutions of equation (5).

We would like to consider other special case with \( \lambda_+ = \lambda_- = \lambda \). In that case at the \( t = 0 \) filled part of Fermi sea is centered at \((x, p) = (-2\lambda, 0)\). After the time revolution, fermions (tachyon fields) coming from \( x = -\infty \) to the finite point \( x = -2\lambda - \sqrt{2\mu} \), then coming back to \( x = -\infty \). Similar to previous case and under assumption of very small \( \lambda \), one can rewrite the equation (10) as,

$$
p \approx \sqrt{x^2 - 2\mu} - 2\lambda e^t - \frac{\mu \lambda}{x^2} e^{-t}.
$$

(11)

Then by comparing the equations (1) and (11), the incoming fluctuation field will obtained as a following,

$$
\eta_{\text{in}} \approx \lambda \sqrt{\frac{\mu}{2\pi}} (e^{t-\sigma} + e^{-(t+\sigma)}),
$$

(12)

and by using the equation (2) one can obtain the outgoing fluctuation field as,

$$
\eta_{\text{out}} \approx \frac{1}{\sqrt{\pi}} \left[ \lambda \sqrt{\frac{\mu}{2}} (e^{t-\sigma} + e^{-(t+\sigma)}) - \frac{\lambda^2}{2} (e^{t-\sigma} + e^{-(t+\sigma)})^2 - \frac{\lambda^3}{3\mu} (e^{t-\sigma} + e^{-(t+\sigma)})^3 \right].
$$

(13)
Now to calculate tachyon field we use from the leg-pole transformations (3). For incoming and outgoing tachyon field we find that $S_{\text{in}} \sim e^{-x}(e^{t} + e^{-t}) = 2e^{-x} \cosh t$, and $S_{\text{out}}^{(n)} \sim \mu e^{nx}(e^{t} + e^{-t})^{n} = 2^{n} \mu e^{nx} \cosh^{n} t$, respectively. Therefore in the first order we find that the ratio of the outgoing to the incoming tachyon fields is proportional to inverse of the string coupling. We must note that our result for the tachyon field is agree with results of Ref. [12]. We must note that in the higher order of the outgoing tachyon field the ratio of the outgoing to the incoming tachyon fields is depend to the space-time coordinates.

Finally we consider the other interesting solution which satisfy the equation of motion. It will be written as,

$$(x + p + 2\lambda_{-}(x + p)^{2} + 2\lambda_{+}e^{t})(x - p) = 2\mu.$$  \hspace{1cm} (14)

Just like to previous case we assume that $\lambda_{+} = \lambda_{-} = \lambda$ and then for large $x - p$ one can obtain,

$$p \approx \sqrt{x^{2} - 2\mu - 2\lambda e^{t}}.$$  \hspace{1cm} (15)

So the incoming fluctuation obtained as the following relation,

$$\eta_{\text{in}} \approx \lambda \sqrt{\frac{\mu}{2\pi}} (e^{t-\sigma}).$$  \hspace{1cm} (16)

Again similar to previous case one can write the outgoing fluctuation and then by using the leg-pole transformation (3) obtain the corresponding tachyon fields. After all one can obtain the ratio of outgoing to incoming tachyon fields proportional to inverse of the string coupling.

$$\frac{S_{\text{out}}}{S_{\text{in}}} \sim \mu e^{\phi_{0}},$$  \hspace{1cm} (17)

which is similar to the equation (9), therefore solution (14) behave as first case of this section given by the equation (5).

In the next section we will consider solutions with the spacelike boundaries.

### 4 Solutions With The Spacelike Boundaries

Our main goal of this paper is to consider the Fermi surface with the spacelike boundary [14, 15]. In this section we consider three cases of such solutions and will try to obtain corresponding tachyon field in two dimensional string theory.

The first case is a time-dependent Fermi surface with the spacelike boundary as a following [14],

$$(x + p - e^{2t}(x - p))(x - p) = 2\mu,$$  \hspace{1cm} (18)

which is corresponding to the equation (4) with $r = 2$, $\lambda_{-} = 0$ and $\lambda_{+} = -1$. The equation (18) represents a closed hyperbola. If we set $\lambda = e^{t}(p - x)$ in the equations (5) we will arrive to the equation (18). Also we can see now the equivalence relation with the explicit calculation. For the large negative $x$ one can rewrite the equation (18) as,

$$p \approx \sqrt{x^{2} - 2\mu + 2|x|e^{2t}}.$$  \hspace{1cm} (19)
As before, we compare it with the equation (1), so, for the incoming fluctuation field we find the following expression,

$$\eta_{\text{in}} \approx \frac{\mu}{2\sqrt{\pi}} e^{2(t-\sigma)},$$  \hspace{1cm} (20)

and the outgoing fluctuation field easily obtained as,

$$\eta_{\text{out}} \approx \frac{\mu}{4\sqrt{\pi}} \left[ e^{2(t-\sigma)} - e^{4(t-\sigma)} - 3e^{6(t-\sigma)} \right].$$  \hspace{1cm} (21)

From the equation (20) we see the incoming fluctuation field obtained as

$$\eta_{\text{in}} \sim e^{2(t-\sigma)},$$

therefore by using the relations (3) it is clear that $S_{\text{in}} \sim e^{2(t-x)}$. The outgoing tachyon field for any order is obtained as $S_{\text{out}}^{(n)} \sim \left( \frac{\mu}{2} \right)^2 e^{2n(t+x)}$. In the first order we see that the ratio of the outgoing to the incoming tachyon fields is proportional to inverse of the squared string coupling.

$$\frac{S_{\text{out}}}{S_{\text{in}}} \sim \mu^2 e^{2\phi_0}. \hspace{1cm} (22)$$

Already, we use from $x \sim \cosh \sigma$, but it is valid at the $x \to -\infty$ limit only. Generally, there are Alexandrov coordinates transformations as,

$$x = \sqrt{2\mu} \frac{\cosh \sigma}{\sqrt{1-e^{2\tau}}} \approx \sqrt{\frac{\mu}{2}} \frac{e^{-\sigma}}{\sqrt{1-e^{2\tau}}},$$

$$t = \tau - \frac{1}{2} \ln(1-e^{2\tau}), \hspace{1cm} (23)$$

which at the $\tau \to -\infty$ limit reduce to $x = \sqrt{2\mu} \cosh \sigma$ and $t \to -\infty$, as expected. In terms of coordinates (23), the incoming fluctuation field obtained as,

$$\eta_{\text{in}} \approx \frac{\mu}{4\sqrt{\pi}} \frac{e^{2(\tau-\sigma)}}{(1-e^{2\tau})^2}. \hspace{1cm} (24)$$

The equation (24) reduces to the equation (20) at the $\tau \to -\infty$ limit, where $t = \tau$. In that case to obtain the outgoing fluctuation field $\eta_{\text{out}}$ from the relation (24) we note that all derivative in the equation (2) are in terms of $\sigma$, and $\sigma$-dependent term in (24) is as before [equation (20)]. Therefore after calculation of the incoming and outgoing tachyon fields we will have similar solution with the equation (22).

The second time-dependent fermi surface with the spacelike boundary which represents an open hyperbola is given by [14],

$$(x + p + e^{2t}(x - p))(x - p) = 2\mu, \hspace{1cm} (25)$$

which is corresponding to $r = 2$, $\lambda_- = 0$ and $\lambda_+ = 1$ in general solution (4). Static solution of the equation (25) is obtained at the $t \to -\infty$ limit. Here we will find the same solution as previous case [see equations (20), (21) and (22)]. But in here, Alexandrov coordinates is different with the first case [14]. By using the equation (25) one can obtain the incoming fluctuation field in terms of Alexandrov coordinates as,

$$\eta_{\text{in}} \approx \frac{\mu}{4\sqrt{\pi}} \frac{e^{-2(\tau+\sigma)}}{(1-e^{-2\tau})^2}, \hspace{1cm} (26)$$
which at the $\tau \to -\infty$ limit, where $t = -\tau$, reduces to the expected relation (20).

Now we consider the third case of the time-dependent Fermi surface given by the following equation [12, 13],

$$e^{-2t}(x + p - \lambda_+ e^t)^2 + e^{2t}(x - p - \lambda_- e^{-t})^2 = 2\mu. \quad (27)$$

We are going to consider special case with $\lambda_+ = \lambda_- = -1$. In that case equation (27), for infinitesimal $\mu$, represents a closed Fermi surface which at the initial time $t = 0$ is a circle with the radius $\sqrt{2\mu}$ centered in $(x, p) = (-1, 0)$. After the time revolution this circle changes to an moving ellipse. In here there are the condition $\lambda > \sqrt{2\mu}$ [13], which under assumption of $\mu \ll 1$ will satisfied.

To write the equation (27) in the form of the equation (1) we note that the described universe by the equation (27) is in weak coupling at both early and late times. It means that the situation with $t \to -\infty$ and $x - p \gg 1$ is equivalent to the situation with $t \to 0$ and $x + p \gg 1$. Therefore, after rescaling $2(\mu - 1) \to \mu$ and for the large negative $x$ one can rewrite the equation (27) as a following,

$$p \approx \sqrt{x^2 - 2\mu + 4e^t(1 - |x|e^t)}. \quad (28)$$

As a result one can find the incoming fluctuation field as,

$$\eta_{in} \approx -\frac{1}{\sqrt{\pi}} \left[ \sqrt{2\mu e^{t-\sigma}} + \frac{\mu}{\sqrt{2}} e^{2(t-\sigma)} \right], \quad (29)$$

and the outgoing fluctuation field to second order obtained as,

$$\eta_{out} \approx -\frac{1}{\sqrt{\pi}} \left[ \sqrt{2\mu e^{t-\sigma}} + \frac{\mu}{\sqrt{2}} e^{2(t-\sigma)} \right] - \frac{1}{\mu\sqrt{\pi}} \left[ \sqrt{2\mu e^{t-\sigma}} + \frac{\mu}{\sqrt{2}} e^{2(t-\sigma)} \right]^2. \quad (30)$$

by using equations (29), (30) and the leg-pole transformation (3) we will find the tachyon field in terms of both lightlike and spacelike tachyon field which obtained already in the equations (5) and (18). Indeed for the incoming tachyon field one can obtain $S_{in} \sim e^{t-x} + e^{2(t-x)}$ and for outgoing tachyon field at the first order one can obtain $S_{out} \sim \mu e^{t+x} + \mu^2 e^{2(t+x)}$. Here we see that, even at the first order, the ratio of the outgoing to the incoming tachyon fields obtained in terms of space-time coordinates.

## 5 Conclusion

In this paper we consider several time-dependent Fermi surface with the null and spacelike boundaries and obtained the tachyon fields by using correspondence between the $c = 1$ matrix model and two dimensional string theory. Already in Ref. [11] simple time-dependent background with the lightlike boundary considered. Here we calculated ratio of the outgoing to incoming tachyon fields for other time dependent backgrounds with the lightlike and spacelike boundaries. In the case of time-dependent Fermi sea with null boundaries we found that ratio of the outgoing to the incoming tachyon fields is proportional to inverse of the
string coupling. Same calculations for solutions with the spacelike boundaries [equations (18) and (25)] show that the ratio of the outgoing to the incoming tachyon fields is proportional to inverse of the squared string coupling. Finally for a solution in the form of equation (27), which defined closed cosmological universe, we obtained a combining tachyon field proportional to the lightlike and spacelike tachyon fields. In summary we considered three solutions of the Fermi sea with lightlike boundaries in equations (5), (10) and (14), and three solutions of the Fermi sea with spacelike boundaries in equations (18), (25) and (27), then obtained the tachyon fields corresponding to the fluctuation fields.

It may be interesting to consider any time-dependent solutions such as \( x^2 - p^2 = 1 + (x - p)^3 e^{3t} \) [13], which is corresponding to \( r = 3 \) in relation (4), and try to obtain tachyon field.

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