Enhancement of spontaneous emission in metal-dielectric multilayer structures accounting losses

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Abstract. We study the emission rate enhancement of the dipole emitter centred in the stratified metal-dielectric metamaterial, characterized by the hyperbolic isofrequency surface. We find out a limited enhancement of the Purcell factor in the layered metamaterial. We demonstrate that the radiative decay rate is strongly depends on a ratio of the thickness of layers and is affected by the level of losses in metal.

1. Introduction

Metamaterials constructed from periodic metal-dielectric multilayer system are generally known as a simple realization of the so-called hyperbolic media [1]. Such hyperbolic metamaterial (HMM) systems have attracted great scientific interest over the past decade due to their special optical properties emanating primarily from hyperbolic dispersion of isofrequency surface. When one of the principal components of the effective permittivity tensor $\varepsilon_{\text{eff}}$ of the diagonal form is opposite in sign to the other two principal components [2]:

$$\varepsilon_{\text{eff}} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix},$$

where components perpendicular and parallel to the anisotropy axis are given by:

$$\varepsilon_\perp = \frac{d_m \varepsilon_m + d_d \varepsilon_d}{d_d + d_m}, \quad \varepsilon_\parallel = \frac{(d_m + d_d) \varepsilon_d \varepsilon_m}{d_m \varepsilon_d + d_d \varepsilon_m}.$$

The hyperbolic dispersion relation for eigenmodes of such structures is given by:

$$\frac{k_x^2 + k_y^2}{\varepsilon_\parallel} + \frac{k_z^2}{\varepsilon_\perp} = \left(\frac{\omega}{c}\right)^2,$$

and depends strongly on frequency, and can be obtained only for a certain ratio of the layer thicknesses.

Hyperbolic metamaterials have attracted great scientific attention as an application for engineering the spontaneous emission. The concept of the radiative decay enhancement in a cavity first proposed by Purcell in 1946 [3] within the framework of nuclear magnetic resonance and is known as the Purcell effect. This effect is described by the Purcell factor, which in the broad sense is defined by the ratio

$$F_p = \frac{\tau_{\text{bulk}}}{\tau_{\text{cav}}}.$$
between lifetimes of spontaneous emission of a point light source in the infinite homogeneous medium to that inserted into a resonant cavity. According to the Fermi golden rule, the radiative decay rate $1/\tau$ in a homogeneous lossless medium is defined by:

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \sum_{k,\sigma} |\langle f | H_{\text{int}}(k, \sigma) | i \rangle|^2 \delta(\hbar \omega_{k,\sigma} - \hbar \omega)$$

where $\langle f | H_{\text{int}}(k, \sigma) | i \rangle$ is the matrix of the perturbation between the final state $|f\rangle$ and the initial state $|i\rangle$, $\hbar \omega$ is the transition energy with the emission of the photon $(k, \sigma)$, with the wave vector $k$ and polarization $\sigma$.

The density of photonic states in HMM diverges [4], affording an enhancement of the spontaneous emission [5]. The Purcell effect provides a means to tune the emission properties of the light source, potentially useful for a variety of applications. Recently, it was predicted theoretically, that for a point dipole-emitter embedded in a metamaterial structure constructed with thinner dielectric layers the Purcell factor rapidly increases [6]. Therefore, it is interesting to study how the enhancement of the Purcell factor depends on the thickness of layers. In addition, structures with metallic layers are also characterized by losses, which should be accurately taken into account. Therefore, we need to clarify how losses affect the electromagnetic properties of the multilayered HMM and the Purcell factor.

2. Results and discussion

We study a silver-dielectric stratified structure shown schematically in figure 1, the dielectric constant of silver is described by the Drude model: $\varepsilon_m = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega+i\gamma)}$, which was fitted to a particular frequency range of the Johnson and Christy bulk dielectric data [7] with the following parameters: $\varepsilon_\infty \approx 4.96$, the plasma frequency $\omega_p \approx 8.98$ eV, and $\gamma = 0.018$ eV is the damping rate. The dielectric constant $\varepsilon_d = 1$, the structure period $D = d_m + d_d = 60$ nm, and the ratio of the thickness of silver and dielectric layers is varied $\eta = \frac{d_m}{d_d}$.

![Figure 1. The dipole emitter centred in the stratified metal-dielectric metamaterial.](image)

We study the Purcell factor for a point dipole emitter embedded in the central dielectric layer of the metal-dielectric stratified media. We assume that the dipole-source is enclosed between two identical mirrors formed by semi-infinite metal-dielectric periodical structures. Therefore the system can be treated as a cavity, and to calculate the Purcell factor we apply an approach based on the quantum theory of the spontaneous emission (SpE) from an active microcavity [8]. Reflectivity and transmittance through layered media were calculated by applying the transfer matrix approach [9].

Figure 2 illustrates the dependence of the Purcell factor over the in-plane component of the wavevector $k_{||}$. We have calculated separately the contribution of two orientations of the dipole: perpendicular (figure 2 (a,b,c)) and parallel (figure 2 (d,e,f)) to the interface of layers.
Figure 2. The dependence of the Purcell factor over the in-plane component of the wave vector. With the dipole orientation respectively perpendicular (a,b,c) and parallel (d,e,f) to the interface of semi-infinite periodic mirrors. The period of the structure is fixed D=60 nm, while the ratio of the thickness of layers $\eta = \frac{d_m}{d_d}$ is varied: $\eta = 0.5$ (a, d), $\eta = 1$ (b, e), and $\eta = 2$ (c, f).

When the dipole is perpendicular to the interface of layers, only the case corresponding to TM polarization significantly contribute to the Purcell factor. The surface waves exist at interfaces between materials with opposite signs of the real part of their dielectric permittivity, $\text{Re}[\varepsilon_m] = -\varepsilon_d$, defining the resonant limit of a single surface plasmon polariton (SPP). We observe a sharp maximum of the Purcell factor at the frequency of the SPP. Additionally, there are two main branches (indicated by red vertical lines in figure 2) contributing to the Purcell factor: first branch related to the odd modes $\omega_+$, which have frequencies higher than the respective frequency for a single interface SPP; second branch corresponds to the even modes $\omega_-$, which have lower frequencies [10]. Decreasing metal layers thickness we can see an interesting property of odd modes: the confinement of the coupled SPP to the metal layers decreases, as a result the mode evolves into a plane wave propagating in the homogeneous dielectric layer. In contrast, for the even modes the confinement to the metal increases while decreasing thickness of metal layers, as a consequence the propagation length is reduced.

In case when the dipole is parallel to the interface of layers, both the TE and TM polarizations contribute to the Purcell factor. We present the evolution of the Purcell factor as a function of $k_\parallel$ in figure 3 (a,d), calculated independently for TE and TM polarization respectively. For TE polarization, we observe an enhancement of spontaneous emission at the edge of the stop-band, when the Bloch wave vector becomes imaginary. The dependence of the Bloch wave vector $K_B$ is defined by:

$$\cos(K_B D) = \cos(k_{1z}d_1) \cos(k_{2z}d_2) - \frac{1}{2} \left( \frac{Z_1^{s,p}}{Z_2^{s,p}} + \frac{Z_2^{s,p}}{Z_1^{s,p}} \right) \sin(k_{1z}d_1) \sin(k_{2z}d_2),$$

where $k_{(1,2)z} = \left( \frac{\varepsilon_{(1,2)}}{c} \right)^2 - k_\parallel^2_1$, $Z_i^{s} = k_{iz}$ for TE and $Z_i^{p} = \frac{\varepsilon_i}{k_{iz}}$ for TM polarization. The dependence of the $K_B$ over the in-plane component of the wave vector $k_\parallel$ is illustrated in figure 3, independently real and imaginary parts of $K_B$ for TE and TM polarizations respectively. For TM polarization we observe the enhancement of the spontaneous emission at the frequencies of the SPP, and both branches related to odd and even coupled SPP modes (indicated by black vertical lines). We note that the contribution of the SPP exhibit only for TM polarization.

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Figure 3. The dependence of the Purcell factor over the in-plane component of the wave vector when the dipole is parallel to the layers. We present TE-(a) and TM-(d) polarizations independently. The dependence of the real (b,e) and imaginary (c,f) parts of the Bloch wave vector over the in-plane component of the wave vector, for TE-(b,c) and TM-(e,f) polarization correspondingly. The thickness of metal and dielectric layers: \( d_m = d_d = 30 \) nm.

3. Conclusions
In summary, we have calculated the spontaneous emission rate of the dipole emitter embedded in the stratified metal-dielectric structure. Our modeling has demonstrated the increase of the molecular decay rates at the frequencies related to the SPP, and both odd and even coupled SPP modes. We have shown results introducing realistic level of losses into the problem. We have observed that the Purcell factor significantly depend on the ratio of the layers thicknesses.

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