An improved CI EKF data fusion algorithm for multi-sensor time-delay system

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Abstract. In order to solve the problem of time-delay in non-linear multi-sensor information fusion, a simulation model of moving target tracking in sensor networks with the time-delay state and observation system is established. Using the augmented matrix to transform the time-delay system into the non-time-delay system, an improved Covariance Intersection (CI) Extended Kalman filter (EKF) data fusion algorithm for multi-sensor systems with time-delay is presented. This method avoids calculating any two local filter error cross-covariance matrices and greatly reduces the computational complexity and time. Analysing the precision of this method, comparing the precision of improved CI EKF data fusion algorithm and the locally optimal fusion EKF algorithm. The results show that the precision of improved CI EKF data fusion algorithm is higher than that of the local EKF and close to the precision of the optimal EKF fusion.

1. Introduction

The problem of multi-sensor data fusion is how to combine local observations, or how to make local state estimators get a fusion state estimator accurately. In the end, the precision of the fusion is much higher than that of each local state estimator. In the case of multi-sensor information transmission, time-delay cannot be avoided, which is the main cause of the poor performance and instability of the system. Therefore, when multi-sensor information is fusion, eliminating state lag and observation lag in the system is also a hot topic of research [1-4]. In [5], linear minimum variance estimation (EKF) is proposed for discrete-time systems, it is far less than the computational burden of the system augmentation method, only the optimal estimators can be obtained from two Riccati equations with the same dimension as the system. Professor Shuli Sun presented a steady-state EKF method for systems with time-delay [6]. The low computational burden of this algorithm is beneficial to its application at any time. However, the deficiency of this algorithm is that it does not take into account the correlation of local estimation errors [7-9].

Considering the current research situation of distributed estimation, this paper studies the fusion estimation of multi-sensor time-delay systems under the framework of distributed fusion. For the wireless sensor network induced delay and uncertain model, a distributed improved Covariance Intersection (CI) Extended EKF (EKF) data fusion filter estimation method is proposed in this paper.
2. Problem elaboration
Consider discrete multiple time-delay stochastic systems with $L$-sensors:

$$x(t + 1) = \sum_{k=0}^{d} f_i(t, x(t - k)) + \gamma(t) \omega(t)$$

(1)

$$y^{(i)}(t) = \sum_{k=0}^{d} h_i^{(i)}(t, x(t - k)) + v^{(i)}(t) \quad i = 1, 2, ..., L$$

(2)

Where, $x(t) \in \mathbb{R}^n$ is the state of the system. $y^{(i)}(t) \in \mathbb{R}^m$ is the observation of the $i$-th sensor, white noise $v^{(i)}(t) \in \mathbb{R}^m$ and $\omega(t) \in \mathbb{R}^r$, $i = 1, 2, ..., L$ is the observation noise and system noise of the $i$-th sensor, $\gamma(t)$ is the time-varying matrix with proper dimension. $f_i(\cdot)$ and $h_i^{(i)}(\cdot)$ are the non-linear functions of the state system and the observation system. $d \geq 0$ is maximum state lag, $d \geq 0$ is maximum observation lag of the first sensor$(i = 1, 2, ..., L)[10]$.

**Assumptions 1**

- $\omega(t) \in \mathbb{R}^r$ and $v^{(i)}(t) \in \mathbb{R}^m \quad i = 1, 2, ..., L$ is zero mean correlated white noise.

$$E \left[ \begin{bmatrix} \omega(t) \\ v^{(i)}(t) \end{bmatrix} \right] \left[ \begin{bmatrix} \omega(t)^T \\ v^{(i)(t)}(t)^T(k) \end{bmatrix} \right] = \begin{bmatrix} Q_i(t) \\ S^{(i)}(t) \\ Q_{S_i}(t) \end{bmatrix} \delta_{lk}$$

(3)

Where, $Q_i(t) = Q_i^{(i)}(t)$. $E$ is the estimated value. $\delta_{lk}$ is the Kronecker delta function.

**Assumptions 2**

Initial status $x(-k), k = 0, 1, ..., \bar{d} \quad \bar{d} = \max \{d, d_i, i = 1, 2, ..., L\} \quad \omega(t)$ and $v^{(i)}(t), i = 1, 2, ..., L$ are independent of each other, and

$$E[x(-k)] = \mu_x, E[(x(-k) - \mu_x)(x(l) - \mu_x)^T] = P_{0x}(k,l) \quad k, l = 0, 1, ..., \bar{d}$$

(4)

Introduction of augmented state $X(t) = [x(t)^T \ldots x(t-\bar{d})]^T$, the observation equation of each sensor is given as an augmented observation equation, System (1) and (2) can be converted into the following equivalent model:

$$X(t + 1) = f(t, X(t)) + \Gamma(t) \omega(t)$$

(5)

$$Y^{(i)}(t) = h^{(i)}(t, X(t)) + V^{(i)}(t) \quad i = 1, 2, ..., L$$

(6)

Where, $\Gamma(t) = \begin{bmatrix} \gamma(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

The fusion filter of multi-sensor systems with time-delay is based on the improved CI EKF data fusion algorithm under assumption 1 and 2.

For system (1) and system (2), basing on state $(x(t), x(t-1), \ldots)$ and observation $(y(i)(t), y(i)(t-1), \ldots)$ solving local steady-state EKF $\hat{x}(t|t)$ equivalent to the following:

$$\hat{x}(t|t) = [I \quad 0] \hat{X}(t|t)$$

(7)

From equation (8) to solve $\hat{X}(t|t)$, then the local steady-state EKF can be obtained. From the local steady-state EKF $\hat{X}(t|t)$, the CI fusion steady-state EKF $\hat{X}_{CI}(t|t)$ can be obtained.

It's important to note that, for the system (2), when $d = 0$, the observed delay vanishes and only the state delay is left. Therefore, discrete state stochastic systems with $L$ sensors can be obtained.

$$x(t + 1) = \sum_{k=0}^{d} f_i(t, x(t - k)) + \Gamma(t) \omega(t)$$

(9)

$$y^{(i)}(t) = h^{(i)}(t, X(t)) + v^{(i)}(t) \quad i = 1, 2, ..., L$$

(10)

For system (1), when $d = 0$, the state time delay vanishes and only the observation time delay is left. Therefore, discrete observation time-delay stochastic systems with $L$ sensors can be obtained.

$$x(t + 1) = f(t, x(t)) + \Gamma(t) \omega(t)$$

(11)
\[ y^{(i)}(t) = \sum_{k=0}^{d_i} h^{(i)}(t, x(t-k)) + v^{(i)}(t) \quad i = 1, 2, \ldots, L \]  

(12)

3. Locally optimal steady-state EKF

**Lemma 1** In that case of assumptions 1 and 2, Augmented system with optimal EKF [11]:

\[ \hat{X}^{(i)}(t+1 \mid t+1) = \hat{X}^{(i)}(t+1 \mid t) + K^{(i)}(t+1)e^{(i)}(t+1) \]  

(13)

\[ \hat{X}^{(i)}(t+1 \mid t+1) = f^{(i)}(X^{(i)}(t \mid t-1)) + K^{(i)}_p(t)e^{(i)}(t) \]  

(14)

\[ e^{(i)}(t+1) = Y^{(i)}(t+1) - h(X^{(i)}(t+1 \mid t)) \]  

(15)

\[ K^{(i)}(t+1) = P^{(i)}(t+1 \mid t)H^{(i)T}(t+1)Q^{-1}_e(t+1) \]  

(16)

\[ Q^{(i)}(t+1) = H^{(i)}(t+1)P^{(i)}(t+1 \mid t)H^{(i)T}(t+1) + Q_e(t+1) \]  

(17)

\[ K_p^{(i)}(t) = [\Phi^{(i)}(t)P(t \mid t-1)H^{(i)T}(t) + \Gamma^{(i)}(t)S^{(i)}(t)]Q^{-1}_e(t) \]  

(18)

\[ P^{(i)}(t+1 \mid t+1) = [I_n - K^{(i)}(t+1)H^{(i)}(t+1)]P^{(i)}(t+1 \mid t) \]  

(19)

**Lemma 2** For system (5) and (6), The local steady-state EKF one-step predictor is:

\[ \hat{X}^{(i)}(t+1 \mid t) = \Psi_p^{(i)}X_n(t \mid t-1) + K_p^{(i)}Y(t) \]  

(20)

\[ \Psi_p^{(i)} = \Phi[I_n - KH], K_p^{(i)} = \Phi K_i \]  

(21)

\[ K_i = \Sigma_i H^T[I_n \Sigma_i H^T + R_i]^{-1} \]  

(22)

Where, \( \Phi(t) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_1}{\partial x_d} & \frac{\partial f_2}{\partial x_d} & \cdots & \frac{\partial f_m}{\partial x_d} \end{bmatrix}, H^{(i)}(t) = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_2}{\partial x_1} & \cdots & \frac{\partial h_m}{\partial x_1} \\ \frac{\partial h_1}{\partial x_2} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_m}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_1}{\partial x_d} & \frac{\partial h_2}{\partial x_d} & \cdots & \frac{\partial h_m}{\partial x_d} \end{bmatrix}. \Sigma_i \text{ satisfied the steady-state Riccati equation:} \]

\[ \Sigma_i = \Phi[\Sigma_i - \Sigma_i H^T(H\Sigma_i H^T + R_i)^{-1}] \Phi^T + \Gamma Q \Gamma^T \]  

(23)

In equation (23), \( \Sigma \) is the error variance matrix of steady-state EKF predictor \( \hat{X}_i(t \mid t-1) \) [11,12].

4. Improved CI EKF data fusion

In two sensor systems, the covariance \( P_1 \) and \( P_2 \) of the two sensors are known, but the mutual covariance \( P_{12} \) is unknown, application of CI method [13-15], can obtain CI steady-state EKF fusion filter:

\[ \hat{X}_{CI}(t \mid t) = P_{CI}[\omega P_1^{-1} \hat{X}_1(t \mid t) + (1-\omega)P_2^{-1} \hat{X}_2(t \mid t)] \]  

(24)

\[ P_{CI} = [\omega P_1^{-1} + (1-\omega)P_2^{-1}]^{-1} \]  

(25)

When \( P_1 \) and \( P_2 \) is known, and the mutual covariance \( P_{12} = 0 \) is known, there have optimal weighted fusion estimate \( \hat{x}_m \) and its fusion error variance matrix \( P_m \):

\[ P_m^{-1} = P_1^{-1} + P_2^{-1} \]  

(26)

\[ P_m^{-1} \hat{x}_m = P_1^{-1} \hat{x}_1 + P_2^{-1} \hat{x}_2 \]  

(27)

Compare equation (24) with (25), if \( P_1^{-1} \) and \( P_2^{-2} \) are magnified \( \omega \) and \( 1-\omega \) times respectively:

\[ \hat{P}_1^{-1} = \omega P_1^{-1}, \hat{P}_2^{-1} = (1-\omega)P_2^{-1} \]  

(28)
that defined by equation (25) can be expressed in the form of an unrelated valuation equation:

\[ P_{CI}^{-1} = \hat{P}^{-1} + \tilde{P}^{-1} \]  

(29)

Therefore, the CI fusion value \( \hat{x}_{CI} \) can be defined in the form of an unrelated valuation fusion equation (28).

\[ P_{CI}^{-1} \hat{x}_{CI} = \hat{P}^{-1} \hat{x}_1 + \tilde{P}^{-1} \hat{x}_2 \]  

(30)

Substitute the equation (28) into the equation (29) and (30), the CI fusion algorithm can be obtained:

\[ P_{CI}^{-1} = \omega \hat{P}^{-1}_1 + (1 - \omega) \tilde{P}^{-1}_2 \]  

(31)

\[ P_{CI}^{-1} \hat{x}_{CI} = \omega \hat{P}^{-1}_1 \hat{x}_1 + (1 - \omega) \tilde{P}^{-1}_2 \hat{x}_2 \]  

(32)

Where, \( \omega \in [0, 1] \), minimization performance index:

\[ \min_{\omega} \text{tr} P_{CI} = \min_{\omega \in [0, 1]} \{ [\omega \hat{P}^{-1}_1 + (1 - \omega) \tilde{P}^{-1}_2] \} \]  

(33)

In the nonlinear optimization problem, the optimal weight coefficient \( \omega_0 \) can be obtained by fast searching with the Fibonacci method [12]. In this paper, \( \omega = 0.1, \ldots, 0.9 \) for search, obtain the optimal weight coefficient \( \omega_0 = 0.39136 \).

5. Precision analysis

**Theorem 1** The actual error variance matrix of improved CI EKF data fusion \( \tilde{P}_{CI} \) is:

\[ \tilde{P}_{CI} = P_{CI} [\omega^2 \hat{P}^{-1}_1 + \omega(1 - \omega) \hat{P}^{-1}_1 \tilde{P}^{-1}_2 \hat{P}^{-1}_1 + \omega(1 - \omega) \tilde{P}^{-1}_2 \hat{P}^{-1}_1 \tilde{P}^{-1}_2 + (1 - \omega)^2 \tilde{P}^{-1}_2] | P_{CI} \]  

(34)

Where, \( P_{CI} = P_{12} \). \( \tilde{P}_{CI} \) that defined by (32) and (33) is an upper bound of \( \tilde{P}_{CI} \):

\[ \tilde{P}_{CI} \leq P_{CI} \]  

(35)

**Theorem 2** Precision relationship between locally EKF fusion [15]:

\[ \text{tr} P_{CI} \leq \text{tr} \tilde{P}_{CI} \leq \text{tr} \tilde{P}_{CI} \]  

(36)

From the equation (32), we know that the unknown covariance intersection is independent of this upper bound \( P_{CI} \). And this upper bound gives all possible, uniform common bounds for the actual precision of any CI fusion device with any unknown mutual covariance, and this is the bound for the worst possible precision. However, real precision will not be close to this bound [17].

6. Simulation results and analysis

Based on the tracking problem of a moving target in the scene of a sensor network, three sensor nodes are established to sample the moving state of the moving target respectively. The system equation can be expressed approximately:

\[ x(t + 1) = \sum_{k=0}^{d} f_k(t, x(t - k)) + \gamma(t) \omega(t) \]  

(37)

\[ y^{(i)}(t) = \sum_{k=0}^{d} h_k^{(i)}(t, x(t - k)) + v^{(i)}(t) \quad i = 1, 2, 3 \]  

(38)

Where, \( x(k) = (s(k), \dot{s}(k), \ddot{s}(k))^T \), \( s(k), \dot{s}(k), \ddot{s}(k) \) are the position, the velocity, and the acceleration at \( kT_1 \) time respectively. Where, \( T_1 \) is the sampling period, \( T_1 = 0.1 \)s.

Convert the system to a lag-free system:

\[ X(t + 1) = f(t, X(t)) + \Gamma(t) \omega(t) \]  

(39)

\[ Y^{(i)}(t) = h^{(i)}(t, X(t)) + V^{(i)}(t) \quad i = 1, 2, 3 \]  

(40)

Where, \( \Gamma(t) = \begin{bmatrix} \gamma(t) & 0 & 0 & 0 \end{bmatrix}^T \).
The parameter in the status system is
\[
\phi = \begin{bmatrix} 1 & T_k & T_k^2 / 2 \\ 0 & 1 & T_k \\ 0 & 0 & 1 \end{bmatrix}, \quad \gamma = \begin{bmatrix} T_k^2 / 2 \\ T_k \\ 1 \end{bmatrix}.
\]

The observation system is composed of three sensors to monitor the position, the velocity, and the acceleration of a moving target. The parameters of the observation matrix and noise variance in the model are selected as follows:

\[
h_0^{(1)} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad h_1^{(1)} = \begin{bmatrix} 0.3 & 0.8 & 0 \end{bmatrix}, \quad h_2^{(1)} = \begin{bmatrix} 0.1 & 0 & 1 \end{bmatrix}, \quad h_0^{(2)} = \begin{bmatrix} 1 & 0.2 & 0 \end{bmatrix}, \quad h_1^{(2)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \quad h_2^{(2)} = \begin{bmatrix} 0 & 0.1 & 0.96 \end{bmatrix}, \quad h_0^{(3)} = \begin{bmatrix} 1 & 0 & 0.15 \end{bmatrix}, \quad h_1^{(3)} = \begin{bmatrix} 0.1 & 0.94 & 0 \end{bmatrix}, \quad h_2^{(3)} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}
\]

\[
Q = 4I, \quad R_1 = 4I, \quad R_2 = 9I, \quad R_3 = 25I, \quad d_1 = 1, \quad d_2 = 2, \quad t = 1, \ldots, 100.
\]

Simulate 100 times. In order to demonstrate the effectiveness of the proposed distributed improved CI EKF data fusion algorithm, then compared with the distributed weighted fusion method. The results are shown in figure 1 and figure 2.

![Figure 1](image1.png)

(a) Position state output  
(b) Speed state output  
(c) Acceleration state output

Figure 1. Estimation of the position, the velocity, and the acceleration by two methods

![Figure 2](image2.png)

Figure 2. Comparison of MSE Curves of Three Sensor Local Fusion EKF, Weighted Fusion EKF, and CI Fusion Estimator

As can be seen from figure 1, there is a time lag in the system. The distributed CI fusion estimation algorithm proposed in this paper has a good tracking effect. On the other hand, the error covariance of local and fusion estimates is shown in figure 2. The MSE curve of CI fusion is the lowest of each local MSE curve and weighted fusion MSE curve, it can be seen that the actual precision of the CI fusion device trP_{CI} is higher than that of each local filter, and it is close to the precision of the optimal fusion device. And for any mutual covariance, the worst possible precision of the corresponding CI fusion device trP_{CI} is still higher than that of each local filter.
7. Conclusion
In this paper, a three-sensor system with observation delay and state lag is presented, an improved CI steady-state EKF data fusion with consistency is designed and presented. The algorithm uses the augmented matrix to transform the time-delay system into a non-time-delay system. And because the CI algorithm does not need to calculate the mutual covariance of the local estimation, it greatly reduces the amount of calculation and saves the computational time. The calculation can reduce greatly, and saves the calculation time. In this paper, it is proved that the precision of this filter is higher than that of every local EKF. The simulation results show that its precision is very close to the precision of the optimal EKF fusion, therefore, it has good performance and practical value.

References
[1] Julier S J, Uhlmann J K. A non-divergent estimation algorithm in the presence of unknown correlations[C]. Proceedings of the American Control Conference, 1997:264-268.
[2] Julier S, Uhlmann J K. General decentralized data fusion with covariance intersection in: Handbook of multi-sensor data fusion theory and practice[J]. Boca Raton: CRC Press, 2009, 21(2):19-342.
[3] He Y, Wang G H, Lu D J. Multi-sensor Information Fusion and its Application [M]. Beijing: Electronic Industry Press, 2000:167-192.
[4] Han C Z, Zhu H Y, Duan Z S, et al. Multi-source Information Fusion [M]. Beijing: Tsinghua University Press, 2006:158-201.
[5] Sun S L, Deng Z L. Multi-sensor optical information fusion EKF[J]. Automatic. 2004, 40(3): 1017-1023.
[6] Sun S L, Cui P Y. Multi-sensor Scalar Weighted Optimal Information Fusion Steady-state EKF [J]. Control and Decision, 2004, 19(2):208-211.
[7] Deng Z L. Information Fusion Filter Theory and Its Application [M]. Harbin: Harbin University of Technology Press, 2007:108-132.
[8] Zhang P, Qi W J, Deng Z L, et al. Covariance Cross Fusion Robust EKF [J]. Control and Decision, 2012, 27(6):904-908.
[9] Ma J, Sun S L. Distributed Optimal Component Fusion Filter for Stochastic Singular Systems [J]. Journal of Natural Science of Heilongjiang University, 2007, 24(4):508-512.
[10] Zhang P, Qi W J, Deng Z L. Robust Fused EKF Theory and Its Application [M]. Harbin: Harbin University of Technology Press, 2016:35-83.
[11] Deng Z L, Wang X, Gao Y. Modeling and Estimation [M]. Beijing: Science Press, 2007:143-169.
[12] Li Y H, Zhou X J, Liu J L. Robust L1 filter for uncertain time-delay systems based on T-S fuzzy model [J]. Control and Decision, 2016, 31 (5):895–900.
[13] Deng Z L, Gao Y, Mao L, et al. New approach to information fusion steady-state EKF [J]. Automatica, 2005, 41(10):1695-1707.
[14] Jin X B, Du J J, Bao J. Distributed Optimal One-Step Delayed Track Fusion Estimation Based on Pseudo-Measurement [J]. Control Theory & Applications, 2011, 28(10):1451-1454.
[15] Tao, Deng, Z L. Self-tuning information fusion EKF for multi-sensor multi-channel ARMA signals with colored measurement noises and its convergence[J]. Applied Mathematics & Information Sciences, 2012, 6(3):607-618.
[16] Chen B, Hu G Q, Ho D W C, et al. Distributed robust fusion estimation with application to state monitoring systems[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2017, 47 (11):2994-3005.
[17] Geng H, Liang Y, Liu Y, et al. Bias estimation for asynchronous multi-rate multi-sensor fusion with unknown inputs[J]. Information Fusion, 2017, 39:139-153.