On Physical Interpretation of the
Poynting-Robertson Effect

Letter to the Editor

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Abstract. Comments to the statements of Srikanth (1999) are presented. As for the
standard definition of the Poynting-Robertson drag, the results of Srikanth (1999) are
incorrect. Srikanth’s statements about the isothermality condition and “red-shift” are
also discussed. Srikanth’s results are generalized for the most general case of the equation
of motion when momentum loss per unit time is proportional to $-v/c$ (higher orders are
neglected), where $v$ is (heliocentric) velocity of the particle, $c$ is the speed of light.

1. Introduction

Srikanth (1999) offers three physical viewpoints on the corresponding statements pre-
sented in astronomical literature which is the most referenced on the Poynting-Robertson
effect (P-R effect). It is surprising that astronomers are still not aware of serious phys-
ical errors in paper which they use as the most referenced literature for more than two
decades.

The aim of this contribution is to discuss statements of Srikanth (1999) in a more
detail. Main attention is devoted to the Poynting-Robertson drag (P-R drag). However,
other two physical viewpoints of Srikanth (1999) are also discussed. The discussion is
presented also for the generalization of the P-R effect, not only for perfect absorption and symmetric reemission in particle’s proper frame of reference as it is presented in Srikanth (1999).

2. Poynting-Robertson Drag

The most referenced paper on the P-R effect during the last twenty years is an invited review paper by Burns et al (1979). The definition of the P-R drag, presented in Burns et al (1979) on page 6 states: “The momentum loss per unit time represents ... the Poynting-Robertson drag.”.

Let us define

\[ \frac{DP^\mu}{D\tau} \equiv \frac{D(m \ u^\mu)}{D\tau} = \epsilon \ l^\mu - \frac{\xi}{c^2} w^\mu, \]  

(1)

where the notation of Srikanth (1999) is used (\( l^\mu = (1, \hat{S}) \equiv (1, \hat{r}) \) is not four-vector).

If we take into account the above presented definition of the P-R drag, then the P-R drag term is given as

\[ \left( \frac{DP^\mu}{D\tau} \right)_{P-R \ drag} = - \frac{\xi}{c^2} w^\mu. \]  

(2)

Thus, the statement that “dust reemission is a necessary condition for P-R drag as seen in the heliocentric frame” is correct.

However, Srikanth (1999) defines P-R drag in his own way. Srikanth rewrote Eq. (1) to the form

\[ m \ \frac{Du^\mu}{D\tau} = \epsilon \ (l^\mu - \frac{l_\nu}{c^2} w^\nu) \]  

(3)

and he defines “the second term on the right-hand side is the drag term”. (One must be careful since Eq. (1) yields \( \xi = \epsilon l_\nu \ u^\nu \) – this is the reason of misunderstanding of the situation by Srikanth and other people.) Using this Srikanth’s definition, Srikanth comes to partially correct statement that “the reemission possesses an assymmetry in the heliocentric frame, but this produces no drag” – there is a problem with mass \( m \), since \( m \) increases due to the incident radiation. Since the P-R drag (if we want to use such a term) should be a part of the P-R effect which yields Eq. (3) with constant \( m \), we see that the incident radiation is not able to completely explain the P-R drag. Thus, the standard definition presented above is more convenient and more physical.

2.1. Generalized P-R Effect

We want to generalize the P-R effect (Robertson 1937) in the way that the final equation of motion contains only two terms: the first proportional to unit radius vector \( \hat{S} \equiv \hat{r} \) and
the second to $-v/c$ (higher orders are neglected), where $v$ is (heliocentric) velocity of the particle and $c$ is the speed of light. The most general P-R effect corresponds to the case when the total momentum per unit time of the “outgoing” radiation $p'_o$ is proportional to the “incident” momentum per unit time $p'_i$: 

$$p'_o = (1 - Q'_{PR}) p'_i,$$  

(4)

where the primes denote quantities measured in the proper frame of reference of the particle (see Eq. (122) in Klačka (1992a)). On the basis of Eq. (4) one comes to the following equations (special relativity is used)

$$\frac{dE_p}{d\tau} = Q'_{PR} A' (1 - \gamma w) \gamma U c,$$

$$\frac{dp}{d\tau} = Q'_{PR} A' \left( \mathbf{S} - \gamma w \frac{v}{c} \right) \gamma U,$$  

(5)

where the notation of Klačka (1992a) is used – see Eqs. (133)-(134) in Klačka (1992a); as for the coincidence with notation used in Srikanth (1999), we have $l_\mu u_\nu = w c$, $A' = \sigma$ – cross section of the dust), $U$ – energy density, $u_\mu = (\gamma c, \gamma v)$, $\epsilon = A' U w$ (the right-hand side of Srikanth’s Eq. (2.7) must be divided by $c$). Eqs. (5) can be rewritten into a more compact form:

$$\frac{Dp^\mu}{D\tau} = Q'_{PR} A' \gamma U \left( l^\mu - w \frac{u^\mu}{c} \right).$$  

(6)

The standard definition of the P-R drag (“the momentum loss per unit time”) yields

$$\left( \frac{Dp^\mu}{D\tau} \right)_{P-R \ drag} = - Q'_{PR} A' \gamma U \frac{w u^\mu}{c} \equiv - \xi c^2 u^\mu,$$  

(7)

if the definition $\xi/c^2 \equiv Q'_{PR} A' \gamma U / c$ is used.

Using definitions $\epsilon \equiv A' U w$, $\xi/c^2 \equiv Q'_{PR} A' \gamma U / c$, we can rewrite Eq. (6) into the form

$$\frac{Dp^\mu}{D\tau} \equiv \frac{D(m u^\mu)}{D\tau} = \epsilon l^\mu - \epsilon (1 - Q'_{PR}) l^\mu - (\xi/c^2) u^\mu.$$  

(8)

On the basis of $u_\mu u^\mu = c^2$, $u_\mu D u^\mu/D\tau = 0$, we can rewrite Eq. (8) as

$$c^2 \frac{dm}{d\tau} = \epsilon l^\mu u_\mu - \epsilon (1 - Q'_{PR}) l^\mu u_\mu - \xi,$$

$$m \frac{D(l^\mu - l^\nu u_\nu / c^2 u^\mu)}{D\tau} = \epsilon \left( l^\mu - l^\nu u_\nu / c^2 u^\mu \right) - \epsilon (1 - Q'_{PR}) \left( l^\mu - l^\nu u_\nu / c^2 u^\mu \right).$$  

(9)

Eq. (8) is generalization of Eqs. (2.3) and (2.4) in Srikanth (1999). We see that the interaction of the electromagnetic radiation with the particle is important in understanding the P-R drag: constant mass $m$ and $Q'_{PR}$ coefficient.

If we consider Eqs. (127)-(128) in Klačka (1992a), we can easily obtain (special relativistic form)
\[ m \frac{dv}{dt} = -\gamma^{-1} w A' (1 - Q'_{PR}) U \left( S - \frac{v}{c} \right) \]  

(10)

for the effect of the “outgoing” radiation. We see that

\[
\left( \frac{dv}{dt} \right)_{out} = 0 \iff Q'_{PR} = 1 .
\]

(11)

The case represented by Eq. (11) is the case which Srikanth wanted to stress. However, the change of momentum is nonzero (for \( v \neq 0 \)), since the change of mass is nonzero (the value is independent on \( Q'_{PR} \)):

\[
\left( \frac{dp}{dt} \right)_{out} \neq 0 , \text{ since } \left( \frac{dm}{dt} \right)_{out} = -\gamma^{-1} w^2 U A' / c .
\]

(12)

The P-R effect as a whole is important. Which part we call a P-R drag is not important – however, a better physical access seems to be in terms of loss of momentum per unit time.

3. Isothermality

Eqs. (22), (25) and (26), Eqs. (60) and (77), and, Eqs. (133) and (141) in Klačka (1992a) show that isothermality condition \( dm / d \tau = 0 \) implies conservation of energy only in the dust’s rest frame. Lorentz transformations (in special relativity) immediately show that energy changes in heliocentric reference frame. One can immediately see this also from the well-known relation for energy \( E = \gamma m c^2 \), which yields \( dE/dt = \gamma^3 m v \cdot dv/dt \) for \( dm/dt = 0 \) – only in the rest frame of the particle is \( dE/dt = 0 (v = 0) \).

However, the situation is similar to the law of reflection, which states: “The angle of incidence equals the angle of reflection, and the incident and reflected rays are in the same plane.”. It is supposed that physicists know that this formulation is correct in the rest frame of the reflecting surface and the angles do not equal for observer who is moving with respect to the reflecting surface.

Thus, the statement that isothermality condition implies that the dust emits as much as it absorbs (for \( Q'_{PR} = 1 \)) is not incorrect – one must only bear in mind that the formulation holds only in the dust’s rest frame. Physically educated man should know this.

4. Red-shift

Srikanth (1999) states that the factor \( l^\mu u_\mu / c \) in Eq. (2.7) does not represent red-shift. This statement is correct repetition of the detailed discussion in Klačka (1992a) – part. 2.4, Eqs. (78)-(92), mainly Eqs. (78)-(79) and Eqs. (84)-(86).
5. Discussion

As we see, astronomers still do not understand physics of the P-R effect. For the purpose of help in understanding the P-R effect author has published papers in which the most general case of the P-R effect is derived in several ways: Klačka: (1992a, 1992c, 1993a, 1993b, 1993c, 1994a). These papers discuss physics in detail and present physical errors in published papers, also. Application to orbital motion is presented in Klačka: (1992b – correct statement below Eq. (22) is: “Equations (8)-(9) and (11) still hold. Transformation $\mu \rightarrow \mu (1 - \beta)$ must be done in Eq. (10), now.”, 1994b, 1999), Klačka and Kaufmannová (1992). Since even the most general case of the P-R effect still represents a very special form of interaction between electromagnetic radiation and dust particle, other forms of equation of motion must exist. As for other forms of equation of motion for dust particle due to interaction with electromagnetic radiation we refer to Klačka (1993d, 1993e, 1994c, 1994d, 2000), Klačka and Kocifaj (1994), Kocifaj and Klačka (1999).

6. Conclusion

We have presented several facts which should make physics of the P-R effect more clear.

We have shown that standard definition of the P-R drag yields result which is not consistent with the statement presented in Srikanth (1999). However, physics yields P-R effect as a whole, and, thus, it is not wise to separate the P-R effect into several parts; in principle it is possible, but different definitions may generate useless complications.

The statement that isothermality condition implies that the dust emits as much as it absorbs (for $Q_{PR}' = 1$) is acceptable. We have shown in an elementary manner that it automatically holds only in the dust’s rest frame. The situation is analogous to the law of reflection – it holds also in the rest frame of the reflecting surface.

The problem of the “red-shift” discussed in Srikanth (1999) is correct repetition of the detail discussion presented in Klačka (1992a).

We have generalized equations presented in Srikanth (1999) – Srikanth’s equations hold for $Q_{PR}' = 1$. Interesting results are given by Eqs. (10)-(12).

We have presented several references on the literature which deals with the problematics from the physical point of view.

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References
Burns, J. A., Ph. H. Lamy, and S. Soter 1979. Radiation forces on small particles in the Solar System. *Icarus* **40**, 1-48.

Klačka J. 1992a. Poynting-Robertson effect. I. Equation of motion. *Earth, Moon, and Planets* **59**, 41-59.

Klačka J. 1992b. Poynting-Robertson effect. II. Perturbation Equations. *Earth, Moon, and Planets* **59**, 211-218.

Klačka J. 1992c. Poynting-Robertson effect. In: 30. Liege International Astrophysical Colloquium: Observations and physical properties of small solar system bodies, 343-348.

Klačka J. 1993a. Poynting-Robertson effect: general case. *Earth, Moon, and Planets* **61**, 119-124.

Klačka J. 1993b. Aberration of light and the Poynting-Robertson effect. *Earth, Moon, and Planets* **62**, 239-244.

Klačka J. 1993c. Misunderstanding of the Poynting-Robertson effect. *Earth, Moon, and Planets* **63**, 255-258.

Klačka J. 1993d. Poynting-Robertson effect and orbital motion. *Earth, Moon, and Planets* **61**, 57-62.

Klačka J. 1993e. Radiation forces and nonspherical dust particles. *Earth, Moon, and Planets* **62**, 145-148.

Klačka J. 1994a. Dust particle in the solar gravitational and electromagnetic fields. *Earth, Moon, and Planets* **64**, 55-58.

Klačka J. 1994b. Radial forces and orbital elements. In: Dynamics and Astrometry of Natural and Artificial Celestial Bodies, K. Kurzyńska, F. Barlier, P. K. Seidelmann and I. Wytrzyszczak (eds.), Astronomical Observatory of A. Mickiewicz University, Poznań, Poland, 181-185.

Klačka J. 1994c. Interplanetary dust particles and solar radiation. *Earth, Moon, and Planets* **64**, 125-132.

Klačka J. 1994d. On the stability of the zodiacal cloud. *Earth, Moon, and Planets* **64**, 95-98.

Klačka J. 1999. On the Poynting-Robertson effect and analytical solutions. *Mon. Not. Roy. Astron. Soc.* (submitted; [http://xxx.lanl.gov/astro-ph/0004181](http://xxx.lanl.gov/astro-ph/0004181))

Klačka J. 2000. Electromagnetic radiation and motion of dust particle – a simple model. *Mon. Not. Roy. Astron. Soc.* (submitted; [http://xxx.lanl.gov/astro-ph/0005100](http://xxx.lanl.gov/astro-ph/0005100))

Klačka J., J. Kaufmannová 1992. Poynting-Robertson effect: ‘circular’ orbit. *Earth, Moon, and Planets* **59**, 97-102.
Klačka J., M. Kocifaj 1994. Electromagnetic radiation and equation of motion for a dust particle. In: Dynamics and Astrometry of Natural and Artificial Celestial Bodies, K. Kurzyńska, F. Barlier, P. K. Seidelmann and I. Wytrzyszczak (eds.), Astronomical Observatory of A. Mickiewicz University, Poznań, Poland, 187-190.

Kocifaj M., J. Klačka 1999. Real dust particles and unimportance of the Poynting-Robertson effect. [http://xxx.lanl.gov/astro-ph/9910042]

Srikanth, R. 1999. Physical interpretation of the Poynting-Robertson effect. *Icarus* **140**, 231-234.

Robertson, H. P. 1937. The dynamical effects of radiation in the Solar System. *Mon. Not. R. Astron. Soc.* **97**, 423-438.

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