Charge and color breaking minima and constraints on the MSSM parameters

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Abstract

The MSSM potential can have unphysical minima deeper than the physically acceptable one. We point out that their presence is quite generic in SO(10) unification with supergravity mediated soft terms. However, at least for moderate values of $\tan \beta$, the physically acceptable vacuum has a life-time longer than the universe age. Furthermore, by discussing the evolution of the universe in an inflationary scenario, we show that the correct vacuum is the natural expectation. This is not the tendency in different cases of unphysical minima. Even in the SO(10) case, the weak assumptions necessary for this may prevent to use the $\tilde{\nu}h^u$ direction for baryogenesis "à la" Affleck-Dine.


1 Introduction

In the supersymmetric limit the MSSM potential possesses some flat direction and many almost flat directions, lifted only by Yukawa interactions with small couplings. In some portions of the MSSM parameter space the soft terms give negative corrections to the potential along such directions, so that new other unwanted minima appear beyond the SM-like one [1, 2, 3]. Typically the vacuum expectation values in the other minima break electric charge and/or colour and are of order \( M_Z / \lambda_f \), where \( \lambda_f \) are the Yukawa couplings of the matter fermions \( f \).

What kind of restrictions on the theory implies the possible presence of other vacuum states beyond the SM-like one?

One extremal point of view consists in barring all the regions of parameter space where other deeper minima are present. In this case one can derive strong constraints on the soft terms [3] and, more interestingly, some scenarios would be excluded on these grounds: this happens, for example, in string theory when supersymmetry breaking is dominated by the dilaton [3]. Similarly, as shown in section 3, the presence of deeper minima also afflicts a SO(10) unified theory with supergravity mediated soft terms.

At the other extremum, the weakest possible requirement on the theory is obtained accepting the possibility that the ‘true’ (physical) vacuum in which we live be a ‘false’ [4] (unstable) one [4] that some mechanism has selected among the other minima. In this case one must at least require that the lifetime of the unstable SM-like vacuum be bigger than the universe age. The quantum tunneling rate is proportional to an exponentially small factor \( \exp\left(-1/\lambda_f^2\right) \) which makes the SM-like minimum stable enough in all cases except in decays towards possible minima with vacuum expectation values of order \( M_Z / \lambda_i \) (or eventually even of order \( M_Z / \lambda_\beta \) and \( M_Z / \lambda_\tau \), if \( \tan \beta \) is large). In such a case the restrictions on the theory are extremely weak. In practice the only constraint derived so far is a weak upper bound on the top quark trilinear soft term, \( A_t \) [5, 6]. We will show that in the large \( \tan \beta \) case another well defined region of the parameter space must be excluded.

The big difference between the constraints obtained following the two extremal attitudes shows that a better understanding of the problem is needed. Before excluding something due to the presence of a bad minimum it is necessary to ensure that it is bad enough. In other words, we must investigate what other unacceptable consequences can have the possible presence of other minima beyond the SM-like one. While their presence does not constitute a problem for particle physics at the Fermi scale, there can be bad cosmological consequences. It may happen that the universe evolution (as now we understand it) can not naturally end up in the desired SM-like vacuum, preferring instead some other unphysical minimum.

For example if the universe has passed through a hot phase where the temperature was bigger than \( M_Z \) and possibly of the order of the vacuum expectation values of some unphysical minimum, it is also necessary to impose that the SM-like vacuum be stable under thermal fluctuations. This requirement turns out to be very weak [6, 7].

It seems however that a cosmological scenario compatible with particle physics must contain a stage of inflation. Such a scenario is most naturally realized assuming a random initial distribution of the order of the gravitational scale for the various vacuum expectation values [8]. If the potential during inflation would be the usual low energy MSSM one, then the natural end point of universe evolution would be the ‘biggest’ eventual unphysical minimum with the largest vacuum expectation values. In this case it would be natural to avoid all the regions of the parameter space in which another ‘bigger’ (and not necessarily deeper) minimum is present. The bounds would be even (slightly) stronger than the strong ones.

However the same positive vacuum energy which gives rise to inflation also produces effective soft terms of order \( H [9] \), where \( H \) is the Hubble constant during inflation. In general, such new contributions to the soft terms are not directly linked to the low energy ones. Even when unphysical

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1To avoid confusion we will call ‘unphysical’ all unacceptable vacua different from the SM-like one.
minima are generically present in the low energy potential, the inflationary potential can be safe. We will show that this naturally happens in the SO(10) case discussed in section 3. As discussed in section 4, in such cases the inflationary cosmological evolution chooses the SM-like minimum avoiding the other ones.

This paper is organized as follows. In section 2 we briefly review the unphysical minima which turn out to be more significant and discuss their main characteristics. In section 3 we will show that in an SO(10) theory with supergravity mediated soft terms the presence of a phenomenologically unacceptable deeper minimum is quite generic. In the large tan$\beta$ region the resulting lifetime of the SM-like vacuum can be smaller than the universe age. Finally, in section 4 we will discuss if we can tolerate the presence of other minima or if they constitute a problem for cosmology.

2 The flat directions and the unphysical minima

The first example of an unphysical minima has been given in [1] where it has been shown that the (approximately) necessary and sufficient conditions such that the RGE improved MSSM potential do not develop a deeper minimum along the directions

$$
\langle|\langle h_u^0|\rangle| = \langle|\langle \tilde{Q}_u^0|\rangle| = \langle|\langle \tilde{u}_R^0|\rangle| \sim O\left(\frac{M_Z}{\lambda_u}\right),
$$

$$
\langle|\langle h_d^0|\rangle| = \langle|\langle \tilde{Q}_d^0|\rangle| = \langle|\langle \tilde{d}_R^0|\rangle| \sim O\left(\frac{M_Z}{\lambda_d}\right),
$$

$$
\langle|\langle h_e^0|\rangle| = \langle|\langle \tilde{L}_e^0|\rangle| = \langle|\langle \tilde{e}_R^0|\rangle| \sim O\left(\frac{M_Z}{\lambda_e}\right)
$$

(1a) (1b) (1c)

are respectively

$$
A_u^2 < 3(\mu_u^2 + m_{\tilde{Q}}^2 + m_{\tilde{u}_R}^2)
$$

$$
A_d^2 < 3(\mu_d^2 + m_{\tilde{Q}}^2 + m_{\tilde{d}_R}^2)
$$

$$
A_e^2 < 3(\mu_d^2 + m_{\tilde{L}}^2 + m_{\tilde{e}_R}^2)
$$

(2a) (2b) (2c)

where $h^u$ ($h^d$) are the Higgs doublet coupled to up quarks (down quarks and leptons), $m_R$ is the soft mass for the field $R$, $\mu_{u,d}^2 = m_{h_{u,d}}^2 + |\mu|^2$, and the standard notations for the other quantities have been followed. The generation number, which can be 1, 2 or 3, has been omitted. Optimized versions of these directions have then been considered in [2, 3], obtaining slightly more stringent constraints in which $\mu_{u,d}$ is replaced by $m_{h_{u,d}}^2$.

Another potentially dangerous direction have been successively considered in [4]

$$
\langle h^u \rangle = -a^2 \frac{\mu}{\lambda_d}, \quad \langle \tilde{Q}_d \rangle = a \frac{\mu}{\lambda_d}, \quad \text{and} \quad \langle \tilde{L}_\nu \rangle = \frac{\mu}{\lambda_d} \cdot a \sqrt{1 + a^2},
$$

(3)

where $\tilde{L}$ and the down squarks can be of any generation and $a$ is a numerical constant of order one which depends on the shape of the potential. In this case the resulting (approximate) condition to avoid an unphysical minimum is

$$
m_{\tilde{L}}^2 + m_{h_{u,d}}^2 > 0
$$

(4)

where $m_{h_{u,d}}^2$ is expected to be driven negative at some scale greater than $M_Z$ by the top quark Yukawa coupling so that to induce the ‘radiative’ electroweak symmetry breaking. Optimized versions of these directions have been recently obtained in [4]; in particular the down squark may be replaced by another slepton $\tilde{e}$ of a generation different than the one which already appears in (3). While the approximate form of the constraint remains the same as in eq. (4), these similar minima have vacuum expectation values of order $\mu/\lambda_e$.

The typical vacuum expectation value $v$ in all these minima is of the order of the electroweak scale $M_Z$ divided by some Yukawa coupling which may be small. For this reason the RGE improved
Figure 1: The part of all the SO(10) parameter space which does not contain an unphysical minimum (gray area) for different values of $\lambda_t$ at the unification scale. The wino mass parameter, $M_2$, the right-handed selectron mass, $m_{\tilde{e}_R}$, and the selectron $A$-term, $A_e$, are renormalized at the weak scale.

Figure 2: Typical values of leptonic (here $d_e$) and hadronic (here $d_N$) signals of SO(10) unification for different possible supersymmetric particle spectra with (◦) or without (●) a deeper unphysical minimum.

tree level potential is not a good approximation: choosing the renormalization scale at $O(M_Z)$ the neglected full one loop potential contains terms proportional to $\ln v/M_Z$, which may be crucial \cite{10}. A more correct bound is obtained evaluating the running parameters at scales $Q \sim v$. While no significant variation is expected in the case of the first constraint (2), the other constraint (4) becomes ineffective when the scale $Q_0$ at which $m_{\tilde{L}}^2 + m_{\tilde{H}}^2$ becomes negative is smaller than the typical vacuum expectation value $\mu/\lambda_f$. Quantum corrections are very important in this second case, since such scale is mainly determined by the top Yukawa coupling, which radiatively induces the electroweak breaking. This point will be crucial in the following. Nowadays we know that the top quark is much heavier than what was typically assumed ten years ago and that $\lambda_t$ at the GUT scale is larger than about $1/2$. This implies that the scale $Q_0$ is typically larger than $M_Z/\lambda_f$ and that the correct evaluation of the bound (4) is still important but no more essential.

A general analysis of the various possibly dangerous flat directions was performed in \cite{3}. In the standard scenario of supergravity mediated soft terms it turns out that the two examples (2) and (4) here reported almost include all the others. The first constraint gives an upper bound on the $A$-terms, while the second one, eq. (4), gives a $\lambda_t$-dependent upper bound on the ratio between gaugino masses $M_{1/2}$ and scalar masses $m_0$ at the GUT scale, that, in the case of universal soft masses at the GUT scale, is approximately $M_{1/2}/m_0 \lesssim (0.5 \div 0.8)$. This restriction can be problematic for some scenarios where such ratio is predicted, for example in supergravity with dilaton dominated supersymmetry breaking \cite{3}.

3 Constraints on the SO(10) parameter space

We will now show that in a very large portion of the parameter space of an SO(10) unified theory with soft breaking terms mediated by minimal supergravity, unphysical deeper minima are present along the directions (3) due to the RGE effects generated by the unified top Yukawa coupling. These same
radiative corrections to the unified soft terms produce significant rates for processes which violate the lepton flavour numbers (such as $\mu \to e\gamma$) and CP (such as $d_\tau$ and $d_N$) \cite{ref1}. For this reason the regions of parameter space in which the unphysical minimum is not present are the same in which the most interesting leptonic signals of SO(10) unification are more difficult to detect (see fig. \ref{fig}.

To give a precise meaning to our computation we will adopt the ‘minimal’ SO(10) model presented in ref. \cite{ref2} in which the only relevant Yukawa coupling above the unification scale is the unified top quark one and the light Higgs doublets $h_u$ and $h_d$ are not unified in a single representation of SO(10), allowing for moderate values of $\tan \beta$. The opposite case of large $\tan \beta \sim m_t/m_b$ will be considered at the end of this section.

Let us explain why in an SO(10) GUT the constraint \cite{ref3} is stronger than in the MSSM case. This is due to two different reasons:

i. New RGE corrections are present from the Planck scale to the GUT scale, with the larger RGE coefficients typical of a unified theory. This means that, for a given $\lambda_t$ value, the scale $Q_0$ at which $m^2_L + m^2_{h_u}$ becomes negative is much larger than in the MSSM. The approximate strong form \cite{ref3} of the condition necessary to avoid minima in the directions \cite{ref3} becomes now more exact.

ii. In an SO(10) theory also the third generation slepton doublet feels the unified Yukawa coupling of the top and becomes lighter than the corresponding sleptons of first and second generation. For this reason the bound \cite{ref4} with a slepton of third generation, $\tilde{\nu}_\tau$, becomes much stronger.

This explains why the bound \cite{ref3} is violated in a very great part of the SO(10) parameter space, approximately for

$$M^2_2 \gtrsim [0.13 - 0.064 \lambda^2_t(M_G)] \cdot m^2_{\tilde{e}_R}$$

where $M_2$, the wino mass parameter, and $m_{\tilde{e}_R}$, the right-handed selectron mass, are renormalized at the weak scale.

In the case of universal soft terms, the precise form of this restriction is shown in figure \ref{fig} that covers all the parameter space for any moderate $\tan \beta$ value. Only in the shaded area inside the thin lines no other minima are present for $\lambda_t(M_G) = 0.5$ (dotted line), 0.75 (dashed line), 1 (dot-dashed line) and 1.25 (continue line). Outside of the corresponding thick lines $m^2_\tau < 0$ and the SM-like minimum is not present. Similar results are obtained including a possible $U(1)_X$ $D$-term contribution at the SO(10) breaking scale or with non universal soft terms at the Planck scale. For example, in such a case, unphysical minima are present in all the parameter space for $\lambda_t(M_G) \gtrsim 1$ and $m^2_{16} \gtrsim 2m^2_{10}$, where $m_{16}$ and $m_{10}$ are the soft masses at the Planck scale for the matter (16) and Higgs (10) SO(10) representations.

Let us made an aside remark. Possible non renormalizable operators, suppressed by inverse powers of the unification mass or of the Planck scale, can lift all the (almost) flat directions of the MSSM potential along which the unphysical minima may be present. Anyhow, the effects of such operators are totally negligible at vacuum expectation values of order $M_Z/\lambda_f$, where $f$ — in the case under examination — can be $d, s, b, e$ or $\mu$. However, a light right handed neutrino mass $M_{\nu_R}$, around $10^{11 \pm 13}$ GeV (which in SO(10) models can be naturally obtained as $\sim M^2_{Z}/M_{Pl}$), is necessary if the usual seesaw mechanism should produce neutrino masses in the range suggested by the MSW solution of the solar neutrino deficit. The superpotential then contains the non renormalizable operators

$$(\lambda^\nu \frac{1}{M_{\nu_R}}\lambda^\nu)(Lh^u)^2,$$

which lift the vacuum degeneracy precisely along the very dangerous direction under examination $\langle \tilde{\nu}_r \rangle \sim \langle h^u_0 \rangle$. The corresponding dangerous minima are erased if $m_{\nu_r} \gtrsim \lambda^2_f M_Z$. The mass of a stable $\tau$ neutrino must be smaller than $m_{\nu_\tau} \lesssim 100$ eV in order not to overclose the universe (an unstable $\tau$ neutrino can be heavier) and a mass around 10 eV is preferred for the (necessary?) hot dark matter
of the universe. In this case the unphysical minima with \( f = e, d \) are no more present and those for \( f = \mu, s \) only marginally affected. In the case \( f = b \) non renormalizable operators are totally irrelevant.

We can then conclude that at least the unphysical minimum in (3) along the \( h^u \nu_\tau, \ b_L, \ b_R \) direction is present in the low energy potential of an SO(10) GUT for quite generic initial tree level values of the soft terms. The small part of the parameter space in which unphysical minima are not present can be characterized as follows:

- A light chargino is present in the spectrum and the squarks and the gluinos cannot be significantly heavier than sleptons (apart from a light stau), especially if the unified top quark Yukawa coupling is near at its IR-fixed point \( [1], \lambda_t(M_G) \gtrsim 1 \).
- With this particular spectrum, the gluino mass RGE contribution to squark masses can not efficiently restore their flavour universality, so that flavour and CP violating signals of SO(10) unification in the leptonic sector (\( \mu \rightarrow e\gamma, \mu \rightarrow e \) conversion, \( d_e \)) are not more important than the corresponding signals in the hadronic sector (\( d_N, \epsilon_K, \epsilon'_K, \Delta m_B, b \rightarrow s \gamma, \ldots \)). We illustrate this point in figure 2. We remember that lepton signals are correlated among them, and that, at a less extent, the same happens for the hadronic signals also \( [1] \). For this reason we have chosen the electric dipoles of the electron, \( d_e \), and of the neutron, \( d_N \), as representatives of leptonic and hadronic signals and we have plotted in fig. 2 their predicted values in the plane \( (d_N, d_e) \) for randomly chosen samples of reasonable supersymmetric spectra and \( \lambda_t \) values. We clearly see that an observable \( d_e \) one order of magnitude larger than \( d_N \), which is a possible distinguishing feature of SO(10) unification \( [12] \), is accompanied by an unphysical minimum.

However in the next section we will argue that unphysical minima of this kind do not necessarily constitute a problem and that there is no reason to restrict the parameter space to its small region where they are not present.

It is now interesting to study what happens in the large \( \tan \beta \) case. The now significant \( \lambda_\tau \) coupling below the unification scale (together with \( \lambda_{e_\tau} \) if \( M_{\nu_R} < M_G \)) will make the third generation sleptons even lighter and the bound \( [1] \) even stronger. The most important difference is however that the vacuum expectation values in the eventual unphysical minima can now be of order \( M_Z/\lambda_b \sim M_Z \). The quantum tunneling rate of the SM-like vacuum into the unphysical vacuum would no longer contain a safe exponentially small factor \( \exp(\Delta/\lambda_\beta^2) \). It may happen that the resulting lifetime be shorter than the universe age so that the corresponding regions must be excluded.

Is such an unphysical minimum again an almost generic feature of an SO(10) GUT with large \( \tan \beta \)? Of course, an appropriate numerical analysis is necessary to delimitate the excluded regions. It is however easy to see that the ‘best’ part of the parameter space is safe. Let us remember that it is not possible to satisfy the conditions for a correct electroweak symmetry breaking with large \( \tan \beta \sim m_t/m_b \),

\[
\frac{\mu B}{\mu_u^2 + \mu_d^2} \approx \frac{1}{\tan \beta} \quad \text{and} \quad \mu_u^2 \approx -\frac{M_Z^2}{2},
\]

without fine tuning. A minimum fine tuning of order \( \tan \beta \) is necessary even in the ‘best’ region \( [13] \) where

\[
M_t, \mu, A_f, B \sim M_Z, \quad \text{and} \quad m_0^2 \sim \frac{M_Z^2}{\tan \beta} \sim (1 \text{ TeV})^2.
\]

Accepting to pay the price of some fine-tuning we can exploit the full predictability of SO(10) gauge unification considering models where the higgs doublets and all the third generation Yukawa couplings are unified. In this case the resulting predictions are phenomenologically correct and a non zero \( U(1)_X \) \( D \)-term contribution \( m_X^2 \sim m_0^2 \) is necessary to split the \( h^u \) soft mass from the \( h^d \) one and satisfy the minimum conditions (6). In this way a possible but narrow region is obtained. The resulting particular low energy spectrum has been explored in \( [13, 14] \) and the associated rates for lepton flavour violating
processes have been computed in [14], showing that large $m_0 \sim 1$ TeV sfermion masses are also preferred for compatibility with the experimental upper bounds.

In this particular narrow ‘good’ region $m_h^2 \approx \mu^2 \approx -M_Z^2/2$ is much smaller than $m_{\tilde{L}_3}^2$, so that the bound (4) necessary to avoid the unphysical minimum (3) is already practically included in the obvious $m_{\tilde{L}_3}^2 > 0$ condition.

The same conclusion cannot be reached if, for example, $\mu \sim m_0$ or $m_0 \sim M_Z$ which are, however, regions accessible with a worse fine tuning [13] and that for this reason were discarded in the analysis in [14] making possible more defined predictions for lepton flavour violating rates.

These qualitative considerations are confirmed by the numerical calculation. For example in figure 3 we show the region of the $(M_2, |\mu|)$ plane where only the SM-like minimum is present (dark gray area) in the case of a minimal SO(10) theory with large $\tan \beta$ [14]. The figure is valid for small values of the selectron $A$-term, $A_e \sim M_Z$, and universal soft masses at the Planck scale larger than the $Z$ mass as in (7). In the white region delimited by the various continue lines, along which some particle becomes too light, the SM-like minimum is not present [14]. In the remaining light gray area, which extends outside of the preferred region (4), the SM-like minimum is present together with other minima since the bound (4) is violated. It is interesting that if the tunneling rate of the SM-like vacuum were sufficiently large one could exclude such regions on a more solid base than fine tuning considerations.

We recall that, in standard cosmology, the probability that the unstable SM-like vacuum has survived until today is is given by

$$p \approx \exp \{ - (M_Z T)^4 e^{-S[\varphi_i^B]} \}$$

(8)

where $T \sim 10^{10}$ yrs is the universe age, $\varphi_i^B(x)$ is the ‘bounce’ (a particular field configuration which interpolates between the true and the false vacuum) and $S[\varphi_i^B]$ its action [4]. In our case $\varphi_i = \{ h_u, \tilde{u}_e, b_L, \tilde{b}_R \}$ involves more than one field so that finding the bounce is a cumbersome numerical problem. However, we can approximate the problem with a single field one restricting the trajectories in field space to the deepest direction (3). This is a good approximation for large field values, that
give a dominant and large contribution to the bounce action, since the potential goes down only quadratically towards the unphysical minimum. In such approximation the relevant Lagrangian is

\[ \mathcal{L} = 2Z(a) |\partial h^u|^2 + \{m_2^2 h^u|^2 - \frac{|\mu|}{\lambda_b} m_3^2 h^u|} \]  \tag{9}  

where \( m_2^2 \equiv |m_{h,0}^2 + m_{\nu,1}^2|, \) \( m_3^2 \equiv m_{\nu}^2 + m_{b_L}^2 + m_{b_R}^2 \), and

\[ a = \frac{h^u}{|\mu/\lambda_b|} \geq 0, \quad Z(a) = \frac{8a^2 + 10a + 3}{8a(a + 1)}. \]  \tag{10}  

At 'large' field values \( a \gtrsim 1 \) where this Lagrangian is realistic we can further approximate \( Z \approx 1 \). Note that we are now going to compute the tunneling rate between two minima using an approximated potential that does not have any minimum. Since this might seem suspect, it is better to add some word of comment. The unphysical minimum appears at the (high) scale at which the scale-dependent \( m_{h,0}^2 + m_{\nu,1}^2 \) term becomes positive. We can however neglect this scale dependence, since the behavior of the potential beyond the 'escape' point, \( \varphi_1^B(0) \) that in our case if of \( \mathcal{O}(\text{TeV}) \), is irrelevant, and, in fact, the potential could even be unbounded from below along the direction of the unstable minimum at \( a \approx 0 \), we only need to proceed carefully and consider the potential as the limit of a 'conventionally shaped' one, different from our approximated form only for \( a \to 0 \). In this sense we can work with a potential without minima. While being a bit crazy, our approximation constitutes a great simplification, since now the bounce action depends only on one dimensionless ratio \( \mu/m_3^2 \). Moreover, expressing the Lagrangian in terms of a \( h^u \) field normalized in units of \( \mu/\lambda_b \) as in eq. (14), we can see that, due to the particular form of the approximated potential, \( S[\varphi_1^B] \propto (\mu/\lambda_b)^2 \). These considerations fix the bounce action to be

\[ S[\varphi_1^B] \approx c \cdot \frac{2\pi^2 \mu^2 (m_{\nu}^2 + m_{b_L}^2 + m_{b_R}^2)^2}{|m_{\nu}^2 + m_{h,0}^2| \beta}. \]  \tag{11}  

The dimensionless proportionality constant \( c \) can be easily computed since, under our approximations, it is also possible to find analytically the bounce \( h^{uB} \) in terms of the Bessel function \( J_1 \),

\[ h^{uB}(r) = \frac{|\mu|m_3^2}{2\lambda_b m_2^2} \times \begin{cases} b + (1 - b)J_1(m_2r) & \text{for } 0 \leq r \leq r_* \smallskip \cr 0 & \text{for } r \geq r_* \end{cases} \]

where \( r \) is the Euclidean 4-radius, \( j_1(x) \equiv 2J_1(x)/x \), \( m_2r_* \approx 5.14 \) is the position of its first minimum, and \( b \approx 1/8.56 \) has been chosen in such a way that \( h^{uB}(r_*) = 0 \). The relatively large value of \( r_* \), due to the slow quadratic decrease of the potential at large \( h^u \), give rise to a correspondingly large proportionality constant in eq. (14), \( c \approx 90 \).

Let us now apply these results to the minimal SO(10) case with universal soft terms at the Planck scale and large \( \tan \beta \), so that \( \lambda_b \approx 0.9 \). In this model there are peculiar correlations between the soft parameters. In particular there exists no deep interior area where the bound is sufficiently strongly violated, as shown in figure. For this reason, in all the allowed parameter space, \( S[\varphi_1^B] \approx 10^{4_{-5}} \) is quite large, so that the lifetime of the SM-like minimum is always much larger than the universe age. The fact that the bounce action is always much larger than the limiting value ensures that the quality of our approximation is much better than what had been sufficient. For more general but less interesting supersymmetric models with large \( \tan \beta \) — for example not imposing exact SO(10) relations at the unification scale — the parameter space is more various and the life-time of the SM-like vacuum can be shorter than the universe age.

4 Cosmological evolution and unphysical minima

We now come back to the question raised in the introduction. What are the consequences of the possible presence of phenomenologically unacceptable minima other than the SM-like one? For example,
is it necessary to restrict an SO(10) theory to the narrow part of its parameter space where unphysical minima are not present?

As discussed in the introduction there can be cosmological problems: if the low energy potential possesses more than one minimum it may be impossible, or unnatural, that the SM-like minimum be the actually populated one. Implementing this condition requires considering the universe evolution, so that the answer is not only a matter of particle physics at the Fermi scale. While to obtain precise results would be necessary to choose some particular well defined cosmological model, the general ideas at the basis of our present understanding of universe evolution are, for example, sufficient to clarify the situation in the SO(10) case presented in the previous section. For this reason, we will not adopt any particular cosmological model but rather we will base our discussion only on its most well established features. We can summarize them as follows. A period of inflation seems to be an essential feature of any consistent cosmological model. After inflation the inflaton field decays in a time of order $\Gamma_I^{-1}$ giving rise to the standard ‘hot big-bang’ scenario with a reheating temperature $T_R \sim (\Gamma_I M_{Pl})^{1/2}$. The natural way of obtaining a sufficient amount of inflation consists in assuming random out of equilibrium initial values of the order of the Planck scale for the various fields. In such a case the non zero modes of the fields are rapidly red-shifted away and the various fields begin to move towards a minimum.

When more than one minimum is present, the most natural final vacuum state is the one with the largest vacuum expectation value. For example, in the case of a single field whose potential possesses two minima at different scales in field space $v \sim M_Z$ and $v' \sim v/\lambda_f$ the relative probability is at least $p/p' \sim \lambda_f$, even in the case where the depths are comparable. This shows that the unphysical minima with the smallest $\lambda_f$ are the more dangerous ones, even if not deeper than the SM-like one. Here we are assuming that the unknown mechanism responsible of the small present value of the cosmological constant does not treat the SM-like minimum as special.

We thus reach the conclusion that the bounds can be even (slightly) stronger than in the extremal case of excluding only all deeper unphysical minima. However such conditions can not be directly applied to the low energy potential, since, during inflation, the potential along the flat directions is expected to be significantly different from the low energy one \[1\]. The reason is that the same positive vacuum energy density $V = |F_I|^2 + \cdots$ which give rise to inflation with a Hubble constant $H^2 \sim V/M_{Pl}^2$ also produces supergravity mediated soft terms of order $H$ \[1\]. If the inflaton field $\varphi_I$ were more directly coupled to the SM fields than the usual ‘hidden’ sector responsible of low energy supersymmetry breaking, the resulting effective soft terms would be even bigger. For example $\varphi_I$ may have a Yukawa coupling to some charged field with mass of the order of the GUT scale \[1\]. In the opposite case, for example in no-scale models, it is also possible that the inflationary soft terms are zero at tree level \[1\]. In general the inflationary contributions to the soft terms are not directly linked to the low energy ones neither in the supergravity case. In fact, a vacuum expectation value of the inflaton away from the minimum and of the order of the Planck scale, $\langle \varphi_I \rangle \sim M_{Pl}$, can distort the Kähler potential.

To summarize we can say that, while the necessary bounds are possibly very strong, they do not depend only on physics at the Fermi scale and their computation requires a knowledge of physics at the Planck scale. The well established features of cosmological evolution alone are not sufficient to determine if a particular unphysical minimum is present even in the inflationary potential so that to decide if it has intolerable consequences which force to exclude its presence. On the contrary, the identification of the inflaton with some particular field of known behavior (such as the dilaton) would allow to precise the bounds on the low energy theory.

It is interesting that we can obtain a definite answer in the SO(10) case where an unphysical minimum is present in the low energy potential for quite generic values of the soft terms due to combined effect of $\lambda_I$-induced renormalization corrections at all scales from $M_{Pl}$ down to $M_Z$. We now show that unphysical minima do not afflic the potential during inflation with soft terms of order $H$. We recall that $H \sim 10^{14}$ GeV is the preferred value for which the quantum de Sitter fluctuations of the inflaton give rise to density perturbation which produce the observed amount of large scale
inhomogeneities. In the case where the dominant contribution to inflationary soft terms is transmitted by some field at or below the GUT scale, the unified top quark Yukawa coupling can no longer affect the potential distorting it to a dangerous form. Even in the simplest case of supergravity mediated soft terms, the quantum corrections at scales greater than \( H \) alone, are not sufficient for creating unphysical minima. We can conclude that, in SO(10) unification (but, of course, also in more general models) quantum corrections do not generate unphysical minima along the direction (3) in the inflationary potential, at least until \( H \) is sufficiently large. Of course unphysical minima can reappear when inflation is over.

In a generic case it is possible that, just as in the SO(10) case, the unphysical minimum of the low energy potential is not present in the inflationary potential. We now show that in such cases the cosmological problems disappear. In fact, if the potential during inflation does not contain other minima, when inflation is over, the various fields \( \varphi \) have already efficiently rolled towards zero \( \varphi_0 \), ending up, ultimately, in the SM-like minimum. It is again crucial that the inflationary soft terms be of the order of the inflationary Hubble constant, so that the ‘forcing’ \( V' \) term in the inflationary field equations

\[
\ddot{\varphi} + 3H \dot{\varphi} + V'(\varphi) = 0, \quad V(\varphi) = \mathcal{O}(H^2)|\varphi|^2 + \cdots + \frac{|\varphi|^{n+4}}{M^n}
\]

is at least of the same order of the ‘damping’ term.

Let us assume that the inflationary soft masses squared \( \mathcal{O}(H^2) \) be positive. This is indeed what happens for a minimal form of the Kähler potential \( \mathcal{K} \). In this case the fields evolve as \( \varphi(t) \sim \varphi_0 e^{-Ht} \cos Ht \), and, even starting from Planck scale values \( \varphi_0 \sim M_{\text{Pl}} \), at the end of inflation \( \varphi < M_Z \) provided that the number of e-foldings be sufficiently big. This may be difficult to obtain for a given initial starting condition, but it is also exactly what is necessary if inflation should dilute unwanted species and produce the necessary homogeneity and isotropy. Moreover this is naturally obtained in the ‘chaotic’ inflationary scenario we have assumed, where the regions in which the most favorable starting conditions are satisfied are the ones which expand experiencing a huge amount of inflation.

We have shown that inflationary cosmology avoids unphysical minima present only in the low energy MSSM potential. If they were instead present also in the inflationary potential the ‘largest’ minimum would be preferred as the true vacuum of the universe.

It is also possible that another minimum is present in the inflationary potential but not in the low energy MSSM potential. This happens if one soft mass squared \( \mathcal{O}(H^2) \) is negative and, for example, the usual quantum corrections at energies \( E \ll H \) stabilize the potential at low field values, \( \varphi \geq M_Z \). In this case the fields roll in the moving minimum \( \langle \varphi \rangle(t) \sim [H(t)M^{n+1}]^{1/(n+2)} \) following it \( \varphi_0 \) and then relaxing towards the SM-like minimum when the Hubble constant \( H \) and the temperature \( T \) became small enough that the other minima disappear and only the SM-like minimum is present. This is not a problematic situation. On the contrary, it has been shown \( \langle \delta \varphi \rangle \sim H^2 \) that if lepton (or baryon) number and CP are broken in the inflationary minimum a baryon asymmetry is produced, which may be easily the source of the observed one. Since the viability of alternative mechanisms for generating it at the electroweak scale is not ensured, another minimum in the inflationary potential is welcome. We only need to point out that if a corresponding unphysical minimum, or one sufficiently coupled to it in the inflationary field equations, is present also in the low energy MSSM potential, then one has to worry that the SM-like minimum might not be the populated one.

Having discussed how inflation can naturally select the SM-like minimum as the physical one even if bigger minima are present in the MSSM potential, before concluding that unphysical minima are not dangerous in these cases, we must first ensure that the universe remains in the SM-like vacuum until the present epoch. This might fail due to the following effects:

- **Quantum fluctuations** of the MSSM fields in de Sitter inflationary universe with \( \langle \delta \varphi^2 \rangle \sim H^2 \) and correlation length of order \( H^{-1} \) \( \mathcal{L} \) are not sufficient to populate the unphysical minima with large vacuum expectation values \( H/\lambda_f \gg H \). At the end of inflation, when \( H(t) \sim M_Z \), such fluctuations could be a problem if an unphysical minima with \( \langle \varphi \rangle \sim M_Z \) is present, as may happen if the trilinear soft term \( A_{\tau} \) of the top quark exceeds the bound \( \mathcal{L} \).

\[
\langle \delta \varphi^2 \rangle \sim H^2, \quad \mathcal{L} \sim H^{-1}
\]
• In the subsequent ‘hot big-bang’ phase after inflation, where the reheating temperature $T_R$ is expected to be bigger than $M_Z$ and possibly of the order of the unphysical minima vacuum expectation values, it is also necessary to ensure that the ‘false’ SM-like vacuum be stable under thermal fluctuations. This requirement turns out to be very weak \cite{6, 7}. It is also possible that high temperature corrections stabilize sufficiently the potential, for example along the direction (3), erasing the corresponding unphysical minima during the high temperature phase after inflation. In this case, excluding a small part of the MSSM parameter space with $m^2_{lr} < 0$ \cite{6}, the symmetric vacuum will do a phase transition towards the SM-like one when the temperature cools down below the electroweak scale.

• Finally, the unstable SM-like vacuum must not undergo quantum tunneling towards a deeper unphysical minimum in the subsequent $10^{10}$ years. The relative probability is $1 - p$, with $p$ given in eq. (8) in terms of the ‘bounce’ action \cite{4} $S[\varphi^B] = O(2\pi^2/\lambda_f^2)$. For moderate values of $\tan\beta$ only minima with vacuum expectation values of order $M_Z/\lambda_t$ can be dangerous, giving rise to a weak upper bound on the trilinear soft term $A_t$ of the top quark \cite{6}.

In the SO(10) case, where unphysical minima with vacuum expectation values of order $M_Z/\lambda_b$ or larger are present in the low energy MSSM potential, the SM-like minimum suffers no instability in the moderate $\tan\beta$ region. On the contrary quantum tunneling effects can be dangerous if $\tan\beta$ is large, excluding some region of the parameter space, as discussed in section \cite{3}.

5 Conclusions

The MSSM potential can have other minima deeper than the SM-like one. Their presence can be quite generic in some scenarios; for example in section \cite{3} we have shown that, in SO(10) unification with supergravity mediated soft terms, unphysical minima with vacuum expectation values of order $M_Z/\lambda_b$ or larger are not present only in small and well defined regions of the parameter space with a peculiar phenomenology. In the large $\tan\beta$ case no unphysical minima are present in the preferred part \cite{3} of the parameter space which is accessible with a minimum fine tuning of order $\tan\beta \sim m_t/m_b$. Due to $\lambda_b \sim 1$, in some region the quantum tunneling rate of the SM-like minimum can be too large and it is clear that such regions must be excluded. On the contrary, if $\tan\beta$ is small, the lifetime of the SM-like vacuum is exponentially larger than the universe age.

In general, it is not clear whether the presence of other deeper minima signals a problem of the theory. For this reason we have studied if unphysical minima have other unacceptable consequences which force to exclude their presence, and consequently the scenarios that predict them. From the point of view of particle physics, the presence of other minima does not affect the physics in the SM-like minimum. It can instead be a problem for cosmology. However, once we have accepted that for some reason the SM-like minimum is the populated one, the conditions necessary to ensure sufficient stability are very weak.

Stronger bounds could be obtained studying why the SM-like vacuum should be the populated one. In particular this question can be attached in an inflationary scenario, where chaotic vacuum expectation values of the order of the Planck scale are suggested as starting conditions in order to obtain the necessary amount of inflation. In this case, we have seen that if more than one vacuum is present in the inflationary potential, the subsequent evolutions naturally prefers the minimum with the largest vacuum expectation value, which in many interesting cases is not the SM-like one. However, even if strong bounds must be imposed on the parameters of the inflationary potential, we presently can not use them to constrain the low energy physics. The reason is that new dominant contributions to the soft terms of the order of the inflationary Hubble constant are present during inflation and we can not compute them without a well defined model of inflation. In particular it turns out that if no other minimum is present during inflation (or at least during a sufficiently long stage of it), then the SM-like minimum, being the basin of the zero point in field space, is naturally selected even when the low energy potential contains other minima. We have shown that this is exactly what
happens in the SO(10) case since the effect which generates the unphysical minima at low energy is not operative during inflation. In this case the SM-like minimum will be selected, provided that the tree level inflationary potential is safe. The opposite assumption is necessary if baryogenesis should be produced by the Affleck-Dine mechanism \[17\], at least along the ‘ideal’ $\bar{\nu}h^u$ direction \[10\].

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