On accelerated flow of MHD powell–eyring fluid via homotopy analysis method

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Abstract. The aim of this article is to obtain the approximate analytical solution for incompressible magnetohydrodynamic (MHD) flow for Powell–Eyring fluid induced by an accelerated plate. Both constant and variable accelerated cases are investigated. Approximate analytical solution in each case is obtained by using the Homotopy Analysis Method (HAM). The resulting nonlinear analysis is carried out to generate the series solution. Finally, Graphical outcomes of different values of the material constants parameters on the velocity flow field are discussed and analyzed.

1. Introduction

Many fluids including household items namely, cosmetics, butter and toiletries, paints, lubricants, certain oils, jellies, jams, soaps, soups, marmalades etc have different rheological characteristics and are referred to the non-Newtonian fluids. The rheological properties of all these fluids cannot be explained by using a one constitutive relationship between shear stress and shear rates which is quite different than the viscous fluids [1], [2]. The understanding about the non-Newtonian fluids compelled many to suggest more models of non-Newtonian fluids. Generally, the classification of non-Newtonian fluid models is given under three categories which are called the rate, differential and integral types [3]. Now the interest in the present analysis is to investigate the MHD flow of Eyring Powell fluid induced by an accelerated plate. Here the considered fluid model is complex and it has preference over the power-law fluid in the two folds. Firstly, it is deduced from kinetic theory of liquid rather than empirical relation as in the case of power–law mode. Secondly, it correctly reduces to Newtonian fluid behaviour at low and high shear rates. Because of this we choose the Powell–Eyring model in this work [4], [5]. On the other hand, the constitutive equations of non-Newtonian fluids offer exciting challenges to the researchers to find out their solutions. The equations become very complex when non-Newtonian fluid is discussed in the presence of magnetohydrodynamic. The effects of MHD flows have gained an increasing interest because of the wide range of its applications in geophysics or in engineering such as the optimization of the solidification process of metal alloys and metals, the control underground spreading of chemical wastes, and pollutants. MHD is the study of the interaction of conducting fluids with electromagnetic
phenomena. The flow of an electrically conducting fluid in the presence of magnetic field is of importance in various areas of technology and engineering such as MHD power generation and MHD pumps. Despite this fact various researchers are still making their interesting contributions in the field (e.g. refer to recent studies by many researchers namely Tan and Masuoka [6 -14].)

Now a new study on the accelerated MHD flow of Powell–Eyring fluid is considered. The MHD flow is induced by an accelerated plate. Two explicit examples of constant and variable accelerations subject to a rigid plate are taken into account. Constitutive equations of Powell–Eyring fluid are used. The solution to the resulting non linear problem is computed by utilizing the HAM technique [15 - 17]. The graphs results are plotted to illustrate the variations of embedded flow parameters.

2. Formulation of the problem

Let us choose a Cartesian coordinate system \((x, y, z)\). Consider the Powell–Eyring fluid bounded by an infinite accelerated plate at \(y = 0\) (\(y\)–axis is taken normal to the plate). The fluid is an incompressible and electrically conducting by exerting an applied magnetic field \(B\), parallel to the \(y\)–axis. The electric field is not taken into the consideration and magnetic Reynolds number is small. The applied and induced magnetic fields are chosen zero. The Lorentz force \(\mathbf{J} \times \mathbf{B}\) under these conditions is equal to \(-\sigma \nabla V\). Here \(\mathbf{J}\) is the current density, \(V\) is the velocity field, \(\sigma\) is electrical conductivity of fluid.

The extra stress tensor \(\mathbf{S}\) for Powell-Eyring fluid satisfies the constitutive equations as given in [5], and is in the following form

\[
\mathbf{S} = \mu \nabla V + \frac{1}{\beta} \sinh \left(\frac{1}{c} \nabla V\right), \sinh \left(\frac{1}{c} \nabla V\right) \approx\frac{1}{1} \nabla V - \frac{1}{6} \left(\frac{1}{c} \nabla V\right)^3, \left\{\frac{1}{c} \nabla V\right\} \ll 1
\]  

where \(\mu\) is the dynamic viscosity of fluid and \(\beta\) and \(c\) are the material constants of the Powell-Eyring fluid model. In the absence of body force the equation becomes

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \left[\mu + \frac{1}{6 \beta c^2} \frac{\partial^2 u}{\partial y^2}\right] \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \mathbf{v}
\]

2.1. Flow due to constant accelerated plate

Consider an incompressible Powell-Eyring fluid over an infinite plate at \(y = 0\). Initially the fluid as well as the plate is at rest. At \(t = 0^+\) the plate starts to move with constant acceleration \(A\) in the \(x\)-direction. In the absence of a pressure gradient in the flow direction, the governing equation and the appropriate boundary and initial conditions are

\[
\frac{\partial u}{\partial t} = \left[\nu + \frac{1}{6 \beta c^3} \frac{\partial^2 u}{\partial y^2}\right] - \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \mathbf{v}
\]

The initial and boundary conditions for a constant accelerated plate are

\[
u(0,t) = At \quad \text{for} \quad t > 0, \quad u(y,t) \to 0 \quad \text{as} \quad y \to \infty
\]

\[
u(y,t) = 0, \quad \frac{\partial u(y,t)}{\partial t} = 0 \quad \text{when} \quad t = 0, \quad y > 0
\]
where \( A \) has dimension of \( \frac{L}{T^2} \) and \( \nu = \frac{\mu}{\rho} \) is the kinematic viscosity of the fluid.

### 2.2. Solution for Constant Accelerated Flow

Introducing the following dimensionless quantities

\[
\tau = \left( \frac{A^2}{\nu} \right)^{\frac{1}{3}}, \quad \alpha = \left( 1 + \frac{1}{\nu \beta c \rho} \right), \quad M = \frac{\sigma B^2}{\rho} \left( \frac{\nu}{A^2} \right)^{\frac{1}{3}}, \quad B = \frac{1}{2 \beta c \rho} \left( \frac{A^4}{\nu^3} \right), \quad \zeta = \sqrt{\left( \frac{A^3}{\nu^2} \right)} , \quad G = \frac{u}{(\nu A)^{\frac{3}{2}}} \tag{7}
\]

The problem statement reduces to

\[
\frac{\partial G(\zeta, \tau)}{\partial \tau} + MG(\zeta, \tau) = \frac{\partial^2 G(\zeta, \tau)}{\partial \zeta^2} \left( \alpha - B \left( \frac{\partial G(\zeta, \tau)}{\partial \zeta} \right)^2 \right), \quad \zeta, \quad \tau > 0, \tag{8}
\]

\( G(0, \tau) = \tau, \quad \tau > 0, \quad \zeta \to \infty; \tau > 0, \quad G(\zeta, 0) = 0, \quad \frac{\partial G(\zeta, \tau)}{\partial \tau} = 0, \quad \zeta > 0 \)

For the HAM solution we choose the initial guess function in the form

\[
G_0(\zeta, \tau) = \tau e^{-\zeta} \tag{9}
\]

and \( \mathcal{L} = G^* \) as the auxiliary linear operator satisfying

\[
\mathcal{L}(C_1 + C_2 \zeta) = 0, \tag{10}
\]

where \( C_1 \) and \( C_2 \) are arbitrary constants. We consider the auxiliary function \( \mathcal{H}(r, \tau) = \tau \).

It is very important that we have great freedom to take the auxiliary objects in HAM in accordance to the rule of its solution expression. Moreover, any value of the initial operator may be selected which yields rapid convergence in the given domain. In our problem, the flow domain is semi-infinite and therefore having the boundary condition at infinity, an initial operator of the form \( \mathcal{L} = G^* \) which satisfies Eq. (10) is chosen. If \( q \in [0,1] \) is the embedding parameter and \( h \) is the auxiliary nonzero parameter. It is found that the series solution form is given by

\[
G(\zeta, \tau) = \tau e^{-\zeta} + \frac{1}{9} e^{-3 \zeta} (9 e^{3+2 \zeta} h - 9 e^{3+3 \zeta} h + 9 e^{3+2 \zeta} \tau - 9 e^{3+3 \zeta} \tau \\
+9 e^{3+2 \zeta} h \tau \zeta - 9 e^{3+3 \zeta} h \tau \zeta + e^{3} h \tau \zeta^3 - 9 e^{3+3 \zeta} h \tau \zeta - 9 e^{3+3 \zeta} h \tau \zeta \\
+9 e^{3+3 \zeta} h \tau \zeta - 9 e^{3+3 \zeta} h \tau \zeta + 9 e^{3+3 \zeta} h \tau \zeta - 9 e^{3+3 \zeta} h \tau \zeta \\
+9 e^{3+3 \zeta} h \tau \zeta - 9 e^{3+3 \zeta} h \tau \zeta + 9 e^{3+3 \zeta} h \tau \zeta - 9 e^{3+3 \zeta} h \tau \zeta \\
+9 e^{3+3 \zeta} h \tau \zeta = -9 e^{3+3 \zeta} h \tau \zeta + 9 e^{3+3 \zeta} h \tau \zeta - 9 e^{3+3 \zeta} h \tau \zeta + 9 e^{3+3 \zeta} h \tau \zeta \\
+9 e^{3+3 \zeta} h \tau \zeta) + \ldots. \tag{11}
\]

The approximate analytical solution given by equation (11) contains the auxiliary parameter \( h \), which influences the convergence region and rate of approximation for the HAM solution. In Fig.1 clearly show that the range for the admissible values of \( h \) is \(-0.4 \leq h \leq 0.4\). The calculations show that the series solution as given in Eq. (11) converges in the whole region of \( \zeta \) when \( h = 0.13 \).
Figure 1. $h$-curve for constant accelerated case.

Figure 2. $h$-curve for variable accelerated case.

Figure 3. Velocity profiles for different values of $M$.

Figure 4. Velocity profiles for different values of $B$.

Figure 5. Velocity profiles for different values of $\alpha$.

Figure 6. Velocity profiles for different values of $\tau$. 
2.3. Solution for Variable Accelerated Flow

Following similar methodology of solution as described previously, one can obtain for variable case

\[
S(\xi, \delta) = \delta^2 e^{-\xi} + \frac{1}{9} e^{-3-3\xi} (18e^{3+2\delta} h\delta - 18e^{3+3\delta} h\delta^2 + 9e^{3+2\delta} \delta^2 - 9e^{3+3\delta} \delta^2
+ 9e^{3+2\delta} hH\delta^2 - 9e^{3+3\delta} hH\delta^2 + Ke^{3+3\delta} h\delta^6 - Ke^{3+3\delta} h\delta^6 - 18e^{2+3\delta} h\delta^6
+ 18e^{3+3\delta} h\delta^6 - 9e^{2+3\delta} \delta^2 \xi + 9e^{3+3\delta} \delta^2 \xi - 9e^{2+3\delta} hH\delta^2 \xi + 9e^{3+3\delta} hH\delta^2 \xi - e^{1+\xi} hH \delta^6 \xi
+ e^{3+3\delta} hH \delta^6 \xi - 9e^{3+2\delta} h\delta^2 \lambda + 9e^{3+3\delta} h\delta^2 \lambda + 9e^{2+3\delta} h\delta^2 \lambda \xi - 9e^{3+3\delta} h\delta^2 \lambda \xi) + \ldots
\]  

(12)

Figure 7. Velocity profiles for different values of \(H\).

Figure 8. Velocity profiles for different values of \(K\).

Figure 9. Velocity profiles for different values of \(\lambda\).

Figure 10. Velocity profiles for different values of \(\delta\).

3. Results and discussion

In this section we concerns with the variations of embedded flow parameters in the solution expressions. Hence Figures 3 - 6 and 7 - 10 have been displayed in order to illustrate such variations. These graphs have been obtained for the MHD flow of Powell–Eyring induced by accelerated plate case. We note that Figs. 2 – 5 have been sketched for the constant accelerated flow case whereas the Figs. 7 – 10 are shown for the case of variable accelerated flow. Fig. 3 is prepared to show the effects of applied magnetic field...
(Hartman number) $M$ on the velocity profile. Keeping $\alpha, B, \tau$ fixed and varying $M$, it is noted that the velocity profile decreases by increasing the magnetic field parameter $M$. Clearly, we see that with increasing the values of $M$, the velocity profile of $G(\zeta, \tau)$ decreases, this is because of the effects of the transverse magnetic field on the electrically conducting fluid which gives rise to a resistive type Lorentz force which tends to slow down the motion of the fluid. Fig. 4 shows that the effects of the material constant parameter $B$ on the velocity profile when $M, \alpha, \tau$ are fixed. It is very interesting to see that by increasing the parameter $\delta$ would lead to a decrease in the velocity profile (this is much related to increase in the boundary layer thickness). This is due to the fact that increasing the values of $B$ would lead to an increasing in the friction forces, and, thus, slow down the motion of the fluid. Fig. 5 indicate that the effects of the parameter $\alpha$ on the velocity profile when $M, B, \tau$ are fixed. It is very interest to notice that by increasing the parameter $B$, this would lead to an increase in the velocity profile (this is much related to decrease in the boundary layer thickness). This is in fact true because by increasing the values of $\alpha$ would then reduce the friction forces, and, thus, assists the flow of the fluid considerably; and hence the fluid moves with greater velocity. The variation of $\tau$ on the magnitude of velocity components is sketched in Fig. 6. It is found that the magnitude of velocity components increases with dimensionless time. In addition, the variations of material constant parameters in variable accelerated flow (i.e. Figures 7 to 10) are seen to be qualitatively similar to the effects plotted for constant accelerated flow (i.e. Figures 3 to 6).

4. Conclusions

In this work, the problems on constant and variable accelerated MHD flows of Powell–Eyring induced by accelerated plate are obtained and solved analytically. The approximate analytical solutions are well established by using HAM and then the following important observations have been noted thoroughly via the graphical results:

i. The magnitudes of the velocity components in Newtonian fluid are greater than Powell–Eyring fluid in both cases of constant and variable accelerated flows.

ii. The magnitudes of velocity components for constant and variable accelerated flows and MHD Powell–Eyring fluid are less than that of the hydrodynamic fluid.

iii. The behaviours of parameter $M$ on the magnitude of velocity components is quite opposite to that of parameter $B$.

iv. The features of embedded parameters in constant and variable accelerated cases are found to be similar in a qualitative sense.

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