Estimation of propagation characteristics at remote locations based on estimation of arriving waves by compressed sensing

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Abstract: Propagation characteristics have locality in a multipath fading environment. The locality is utilized for various applications using radio wave. Recently, on the other hand, new techniques to break the locality have been studied. In the techniques, received signals at arbitrary locations are estimated from remote receiving points. We present in this letter a new estimation technique of received signals utilizing Compressed Sensing (CS). In the technique, we estimate directions of arrival and each complex amplitude of arriving waves by CS, and calculate a received signal at a remote location based on the estimated results. We present the technique and evaluate the estimation performance through computer simulations.

Keywords: compressed sensing, estimation of received signal, multipath environment

Classification: Antennas and Propagation

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1 Introduction

Propagation characteristics of radio waves are local in a multipath fading environment. Various techniques based on the characteristics have been developed. An example is found in secret key agreement that is part of wireless physical layer security [1, 2]. In this technique, secret keys are made at legitimate two users individually using the propagation characteristics of wireless channels between the users in order to transmit messages securely. A common key is shared between the legitimate users based on the reciprocity of the propagation characteristics. When we are remote from the users, we cannot eavesdrop on the messages because we do not know the secret keys.

On the other hand, new techniques to break the locality have been studied in recent years. If we can estimate the received signals at the legitimate destinations, the possibility of eavesdropping in the physical layer security increases. Thus the estimation techniques of received signals at remote locations can be applied to various radio techniques.

In this letter, we regard complex amplitude at a remote location as the received signal. In the previous study [3], we estimated the complex amplitude at the remote location based on the directions of arrival (DOAs) and each complex amplitude of the arriving waves. We estimated the DOAs by the Multiple Signal Classification (MUSIC) method, and the complex amplitude by the least squares method. In this letter, we estimate DOAs and complex amplitude simultaneously by Compressed Sensing (CS). CS is a technique to obtain a solution from an underdetermined linear system [4, 5]. Based on the DOAs and each complex amplitude of arriving waves, we can calculate a received signal at a remote location. We have evaluated the basic performance of the estimation technique using CS [6]. In addition to that, we evaluate the influence of number of arriving waves in this letter.

2 Estimation of received signal at an arbitrary remote location by compressed sensing

CS is a technique to provide an optimum solution of an underdetermined system taking advantage of the prior knowledge that the true solution is sparse. We consider the linear system having $M$ equations and $N$ unknowns as

$$y = Ax + n$$

(1)

where $A \in \mathbb{C}^{M \times N}$ is a sensing matrix, $n \in \mathbb{C}^M$ is a noise vector, $x \in \mathbb{C}^N$ is an unknown vector, and $y \in \mathbb{C}^M$ is a measurement vector. In the case of $M < N$, we cannot obtain the true solution generally. However, in the case that the true solution
is a sparse vector, in other words, the number of the non-zero elements of $x$ is less than that of the equations, we can calculate the optimum solution. This is the basic idea to obtain the optimum solution by CS. CS is utilized for various fields including communications [7, 8].

![Diagram of Assumed Environment and Estimation Model](image)

Fig. 1. Assumed environment and estimation model.

Fig. 1 shows the assumed environment and the estimation system used in the estimation of the received signals. Assuming a two-dimensional space, an $M$-elements circular array is placed, and the radius of the element arrangement is $R$. In the polar coordinate, the origin of the coordinate is set at the center of the circle. The position of the target point where the received signal is estimated is denoted $(r, \varphi)$. The whole angular range of the estimation, $-180^\circ \sim 180^\circ$ in this case, is divided into $N$ small angular bins. $\theta_n$ is the angle of the $n$-th angular bin. $x_n$ is the $n$-th element of the vector $x$. We consider it as the complex amplitude of the arriving wave contained within the $n$-th angular bin. The phase of the complex amplitude is defined at the origin. $y_m$ is the $m$-th element of the vector $y$ and is the received signal detected by the $m$-th antenna element. We can calculate the optimum solution $\hat{x}$ by CS, setting the estimation system as the number of the antenna elements $M$ is larger than the number of arriving waves $L$. We also assumed the vector $x$ is sparse, in other words, the number of angular division $N$ is much larger than $L$.

In this letter, we estimate the received signal $\hat{s}$ at the target point considering the phase difference based on the distance from the origin to the target point by the following formula

$$\hat{s} = \sum_{n=1}^{N} \hat{x}_n \exp\left\{-j2\pi \frac{r}{\lambda} \cos(\theta_n - \varphi)\right\}$$

where $\lambda$ is the wavelength of the carrier frequency of the radio waves. In the above estimation formula, all arriving waves are assumed plane waves.

3 Evaluation of estimation performance

We evaluate estimation performance of received signals at an arbitrary remote location by CS via computer simulations. Table I summarizes the assumed values of the multipath environment and the estimation system.
Fig. 2(a) shows the amplitude ratio of the estimated received signal to the actual at the two target points \( (r, \varphi) = (10\lambda, 30^\circ) \) and \( (100\lambda, 30^\circ) \) where the assumed number of the arriving waves is 3, and the number of the antennas is 20. In the figure, 100 independent trials are carried out varying the DOAs and the phases of the arriving waves randomly. In the case of \( r = 10\lambda \), the errors of the estimation are very small. On the other hand when \( r = 100\lambda \), the errors are large. Here we use 90% value of the distribution of the amplitude ratio (estimation error) as a measure of the estimation accuracy. The 90% value is a value of the amplitude ratio where ninety percent of the absolute values of the amplitude ratios of the independent trials are below the value. For example, the 90% value of the estimated result at \( r = 10\lambda \) is 0.14 dB, and that at \( r = 100\lambda \) is 1.9 dB.

Fig. 2(b) shows the 90% value over the variation of the distance to the target points. The value of angle of the target point \( \varphi \) is fixed to 30°. We calculate the received signals with both the estimated DOAs and the actual. We add intentional angular error \( \psi \) to each actual DOA in order to consider the effect of the DOA estimation error. In the figure, \( \pm \psi \) is added as the error where the sign is random. In the case of \( \psi = 0^\circ \), the 90% values are very small. It appears that we can obtain complex amplitude of arriving waves by CS correctly enough to estimate the received signal at a remote location. The results shown in the figure show the errors of the estimated DOAs cause the degradation of the estimation of the received signals. It is shown that the 90% values obtained using the estimated DOAs are close to that with 0.02° intentional error. Since DOA is discretely expressed with 0.01° separation in the estimation, it inherently includes the quantization error up to 0.005°. However the above result shows the DOA estimation error is larger than the quantization error. Therefore, we see that the improvement of the DOA estimation accuracy is a key in order to improve the whole estimation performance.

Fig. 2(c) shows the influence of the variation of the number of the arriving waves \( L \). It corresponds to the variation of the sparsity of the unknown vector. We assume four cases of the number of the elements, and vary the number of the waves from 1 to 20. The target point is fixed at \( (10\lambda, 30^\circ) \) in this case. Assuming 3 dB as a requirement of estimation accuracy, the maximum numbers of the waves satisfying the requirement are about 3 for 10 elements, 5 for 20 elements, and about 8 for 30 elements.

### Table 1. Values of environment and estimation system.

| Multipath environment         |                         |
|------------------------------|--------------------------|
| Number of arriving waves \( L \) | 1~20                     |
| Arriving angle               | Random \((-180^\circ\~180^\circ)\) |
| Complex amplitude of arriving waves | Amplitude: Unity       |
|                              | Phase: Random \((-180^\circ\~180^\circ)\) |
| SNR                          | 40 dB                    |
| Estimation system            |                         |
| Number of antennas \( M \)   | 10~40                    |
| CS                           | Number of snapshots 100  |
| Bin size                     | 0.01°                    |
elements, and 12 for 40 elements. Therefore, we have good estimation where the number of the arriving waves is less than around a quarter of the number of the elements.

4 Conclusion

In this letter, we present a technique to estimate received signals at an arbitrary remote location based on estimation of arriving waves by CS. We show that the errors of the DOA estimation deteriorate the estimation. It is our future task to improve the DOA estimation accuracy to improve the whole estimation performance.