Testing QCDF factorization with phase determinations in $B \to K\pi, K\rho$, and $K^*\pi$ decays

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The success of QCD factorization (QCDF)\textsuperscript{1,2} in predicting branching ratios for charmless $B$ decays to light pseudoscalar and vector mesons and the small CP asymmetries measured at BABAR, Belle and LHCb show that the final-state interaction terms in these decays, as predicted by QCDF, is not large. For penguin-dominated charmless $B$ decays to light pseudoscalar and vector mesons, the phase appearing in the decay amplitude is the relative phase between the isospin $I = 3/2$ tree and $I = 1/2$ penguin amplitudes, as in $B \to K\pi, K\rho$, and $K^*\pi$ decays. Since all four modes for $B \to K\pi, K\rho,$ and $K^*\pi$, respectively, have similar branching ratios, the interference terms are quite small, making a determination of these phases more difficult than for the Cabibbo-favored decays $D \to K\pi, \bar{K}\rho,$ and $K^*\pi$, for which a large $\delta_{3/2} - \delta_{1/2} = (86 \pm 8^\circ)$ has been obtained.\textsuperscript{3} Since the tree amplitude is known at the leading order in $\Lambda_{QCD}/m_b$\textsuperscript{2} and is consistent with the measured tree-dominated decay rates, knowledge of the tree amplitude then allows a simple determination of the phase in the decay rate in the tree amplitude using the measured decay rates. This is similar to the extraction of the final-state interaction phases in the interference term between $pp \to J/\psi \to e^+e^-$ and $pp \to e^+e^-$\textsuperscript{4} and in the process $J/\psi \to 0^-0^-$\textsuperscript{4} via three-gluon and one-photon exchange interference terms\textsuperscript{5}. By expressing the $B \to PP, PV$ decay amplitudes in terms of the $I = 1/2$ and $I = 3/2$ isospin amplitudes\textsuperscript{6,7}, the relative phase of the two isospin amplitudes can be obtained from the magnitudes of the isospin amplitudes and the decay rates, as knowledge of the three sides of the triangle formed with the decay amplitude and the other two sides, the two isospin amplitudes, allows a determination of the three angles of the triangle and the corresponding relative phases of the amplitudes. This is possible for the penguin-dominated $\Delta S = 1$, $B \to PP, PV$ decays for which all the decay rates have been measured, and since QCDF predictions for the $I = 2$ $B^+ \to \pi^+\pi^0, \rho^0\pi^+, \rho^+\pi^0$ tree-dominated decays agree rather well with experiments as shown in the table below and in \textsuperscript{8}, the $I = 2$ amplitudes in these decays could be taken as the $I = 3/2$ tree amplitudes in penguin-dominated $B \to PP, PV$ decays with $SU(3)$ breaking effects in the $B \to K, K^*$ form factors and decay constants involving $K, K^*$ meson taken into account.\textsuperscript{8}

With the $I = 3/2$ tree amplitude known, the three sides of the triangle formed with the decay rate, the $I = 1/2$ and $I = 3/2$ isospin amplitude allows a determination of the three angles and the relative phase between the sides. In this paper we will present a determination of the relative phase between the $I = 3/2$ and $I = 1/2$ amplitudes using the QCDF $I = 3/2$ amplitude and the measured decay rates. It is remarkable that the phase extracted from experiments differs only slightly from the QCDF values. This shows that there is no large final-state interaction strong phase in $B \to K\pi, K\rho$, and $K^*\pi$ decays.

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are given in Sec. III.

\[ \Delta S = 1 B \to PP, PV \text{ DECAY IN QCD FACTORIZATION} \]

The \( B \to M_1M_2 \) decay amplitude in QCDF for \( B = B^- \), \( B^0 \) is given by \([11],[12]\):

\[
A(B \to M_1M_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{ps}^* \times \left( -10 \sum a_i^p (M_1M_2|O_i|B)_H + \sum_{i} f_B f_M f_{M_1} b_i \right), \tag{1}
\]

where the QCD coefficients \( a_i^p \) contain the vertex corrections, penguin corrections, and hard spectator scattering contributions, the hadronic matrix elements \( \langle M_1M_2|O_i|B \rangle_H \) of the tree and penguin operators \( O_i \) are given by the factorization model \([12],[10]\), and \( b_i \) are the annihilation terms. The values for \( a_i^p \) are computed from the expressions in \([11],[12]\) at the renormalization scale \( \mu = m_b \), with \( m_b = 4.2 \) GeV, are:

\[
\begin{align*}
&\ a_4^c = -0.031 - 0.010 i + 0.0009 \rho_H \exp(i\phi_H), \\
&\ a_4^u = -0.027 - 0.017 i + 0.0009 \rho_H \exp(i\phi_H), \\
&\ a_6^c = -0.045 - 0.003 i, \quad a_6^u = -0.042 - 0.013 i, \\
&\ a_8^c = -0.0004 - 0.0001 i, \quad a_8^u = -0.0004 - 0.0001 i, \\
&\ a_{10}^c = -0.0011 - 0.0001 i - 0.0006 \rho_H \exp(i\phi_H), \\
&\ a_{10}^u = -0.0011 + 0.0006 i - 0.0006 \rho_H \exp(i\phi_H). \tag{2}
\end{align*}
\]

for \( i = 4, 6, 8, 10 \). For other coefficients, \( a_i = a_i^c = a_i^u \):

\[
\begin{align*}
&\ a_1 = 1.02 + 0.015 i - 0.012 \rho_H \exp(i\phi_H), \\
&\ a_2 = 0.156 - 0.089 i + 0.074 \rho_H \exp(i\phi_H), \\
&\ a_3 = 0.0025 + 0.0030 i - 0.0024 \rho_H \exp(i\phi_H), \\
&\ a_5 = -0.0016 - 0.0034 i + 0.0029 \rho_H \exp(i\phi_H), \\
&\ a_7 = -0.00003 - 0.00004 i + 0.00003 \rho_H \exp(i\phi_H), \\
&\ a_9 = -0.009 - 0.0001 i + 0.0001 \rho_H \exp(i\phi_H). \tag{3}
\end{align*}
\]

where the complex parameter \( \rho_H \exp(i\phi_H) \) represents the end-point singularity in the hard-scattering corrections \( X_H = (1 + \rho_H \exp(i\phi_H)) \ln \left( \frac{m_b}{m} \right) \) \([11],[12]\). For the annihilation terms, for \( B \to PP \) decays, we have:

\[
\begin{align*}
&\ b_2 = -0.0041 - 0.0071 \rho_A \exp(i\phi_A) - 0.0019(\rho_A \exp(i\phi_A))^2, \\
&\ b_3 = -0.0071 - 0.016 \rho_A \exp(i\phi_A) - 0.0093(\rho_A \exp(i\phi_A))^2, \\
&\ b_4^c = -0.0012 - 0.00016 \rho_A \exp(i\phi_A) + 0.000003 \rho_A \exp(i\phi_A)^2. \tag{4}
\end{align*}
\]

where \( b_i \) are evaluated with the factor \( f_B f_M f_{M_1} f_{M_2} \) included and normalized relative to the factor \( f_K F_0^{B \pi}(m_B^2 - m^2) \) in the factorizable terms, and \( \rho_A \), like \( \rho_H \), appears in the divergent annihilation term \( X_A = (1 + \rho_A \exp(i\phi)) \ln \left( \frac{m_b}{m} \right) \).

The \( B \to K \pi \) decay amplitude with the factorizable part \([9]\) and the annihilation term \([11],[13]\) is:

\[
\begin{align*}
A(B^+ \to K^+ \pi^0) &= -i \frac{G_F}{\sqrt{2}} f_K F_0^{B \pi}(m_K^2)(m_B^2 - m^2) \\
&\ (V_{ub} V_{us}^* a_1 + (V_{ub} V_{us}^* + V_{cb} V_{cs}^*)[a_4 + a_{10} + (a_6 + a_8) r_H]) \\
&\ -i \frac{G_F}{\sqrt{2}} f_{F_B} F_0^{B \pi}(m_B^2)(m_B^2 - m^2) \\
&\times (V_{ub} V_{us}^* a_2 + (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) \times \frac{3}{2}(a_9 - a_7)) \\
&\ -i \frac{G_F}{\sqrt{2}} f_{F_K} f_{F_{KB}}(m_K^2)(m_B^2 - m^2) \\
&\times (V_{ub} V_{us}^* b_2 + (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) \times (b_3 + b_3^w)) \tag{5}
\end{align*}
\]

and for \( B^0 \) :

\[
\begin{align*}
A(B^0 \to K^0 \pi^-) &= -i \frac{G_F}{\sqrt{2}} f_K F_0^{B \pi}(m_K^2)(m_B^2 - m^2) \\
&\ (V_{ub} V_{us}^* a_1 + (V_{ub} V_{us}^* + V_{cb} V_{cs}^*)[a_4 + a_{10} + (a_6 + a_8) r_H]) \\
&\ -i \frac{G_F}{\sqrt{2}} f_{F_B} f_{F_{KB}} \left( [V_{ub} V_{us}^* + V_{cb} V_{cs}^*] \times (b_3 - \frac{b_3^w}{2}) \right) \tag{6}
\end{align*}
\]

\[
\begin{align*}
A(B^0 \to K^0 \pi^0) &= -i \frac{G_F}{\sqrt{2}} f_K F_0^{B \pi}(m_K^2)(m_B^2 - m^2) \\
&\times (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) \left[ a_4 - \frac{1}{2} a_{10} + (a_6 - \frac{1}{2} a_8) r_H \right] \\
&\ -i \frac{G_F}{\sqrt{2}} f_{F_B} f_{F_{KB}} \left( [V_{ub} V_{us}^* + V_{cb} V_{cs}^*] \times (b_3 - \frac{b_3^w}{2}) \right) \tag{7}
\end{align*}
\]

\[
\begin{align*}
A(B^0 \to K^0 \pi^0) &= -i \frac{G_F}{\sqrt{2}} f_K F_0^{B \pi}(m_K^2)(m_B^2 - m^2) \\
&\times (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) \left[ a_4 - \frac{1}{2} a_{10} + (a_6 - \frac{1}{2} a_8) r_H \right] \\
&\ -i \frac{G_F}{\sqrt{2}} f_{F_B} f_{F_{KB}} \left( [V_{ub} V_{us}^* + V_{cb} V_{cs}^*] \times (b_3 - \frac{b_3^w}{2}) \right) \tag{8}
\end{align*}
\]

where \( r_H = \frac{2m_K^2}{(m_b-m_a)(m_b+m_s)} \) is the chirally enhanced terms in the penguin \( O_6 \) matrix element. We also need the \( B^+ \to \pi^+ \pi^0 \) amplitude:

\[
\begin{align*}
A(B^+ \to \pi^+ \pi^0) &= -i \frac{G_F}{2} f_{F_B} F_0^{B \pi}(m_B^2)(m_B^2 - m^2) \\
&\ (V_{ub} V_{us}^*(a_1 + a_2) + (V_{ub} V_{us}^* + V_{cb} V_{cs}^*) \times \frac{3}{2}(a_9 - a_7)) \\
&\times \frac{3}{2}(a_9 - a_7 + a_{10} + a_8 r_H) \tag{9}
\end{align*}
\]

We see that the \( B \to K \pi \) decay amplitudes consist of a QCD penguin(p) \( a_1 + a_6 r_H \), a color-allowed tree(T) \( a_1 \), a color-suppressed tree(C) \( a_2 \), a color-allowed electroweak
penguin (EW) \( a_9 - a_7 \), a color-suppressed electroweak penguin (EWP) \( a_{10} + a_9 r_x \) term.

Similar expressions for the QCD coefficients for \( B \to PV \) decays with hard-scattering corrections and annihilation terms used in the calculations are not shown here, but can be found in \cite{11, 14}. For the CKM matrix elements, since the inclusive and exclusive data on \( |V_{ub}| \) differ by a large amount and the higher inclusive data exceeds the unitarity limit for \( R_{b} = |V_{ub}V_{ub}^*|/|V_{cd}V_{cd}^*| \) with the current value \( \sin(2\beta) = 0.682 \pm 0.019 \) \cite{15}, we shall determine \( |V_{ub}| \) from the more precise \( |V_{cb}| \) data \cite{17}. As mentioned in \cite{18}, we have:

\[
|V_{ub}| = |V_{cb}V_{ub}^*| |V_{ub}^*| \sin \beta \sqrt{1 + \frac{\cos^2 \alpha}{\sin^2 \alpha}}. \tag{10}
\]

With \( \alpha = (93.7 \pm 10.6)° \) \cite{19} and \( |V_{cb}| = (41.78 \pm 0.30 \pm 0.08) \times 10^{-3} \) \cite{21}, we find, neglecting the errors,

\[
|V_{ub}| = 3.56 \times 10^{-3}. \tag{11}
\]

in good agreement with the exclusive data in the range \(|V_{ub}| = 3.33-3.51 \) \cite{22}. A recent UT fit also gives \(|V_{ub}| = (3.61 \pm 0.12) \times 10^{-3} \) and \(|V_{cb}| = (41.53 \pm 0.30 \pm 0.06) \times 10^{-3} \) close to the above values \cite{21}. The measurements of the \( B_s - B_s \) mixing also allow the extraction of \( |V_{ud}| \) from \( |V_{ud}| \) and \( |V_{us}| \). The current determination \cite{22} gives \(|V_{ud}/V_{ts}| = (0.208 \pm 0.008) \) which in turn can be used to determine the angle \( \beta \) from the unitarity relation \cite{23}:

\[
|V_{ud}| = |V_{cb}V_{ub}^*| |V_{ub}^*| \sin \gamma \sqrt{1 + \frac{\cos^2 \alpha}{\sin^2 \alpha}}. \tag{12}
\]

with \(|V_{ub}| = 1\), we find \( \gamma = 67.6° \) which implies an angle \( \alpha = 90.7° \), in good agreement with the new Belle value \( \alpha = (93.7 \pm 10.6)° \) \cite{19} mentioned above. The value \( \gamma = 67.6° \) is also consistent with the current UT fit value \( \gamma = (70.3 \pm 3.7)° \) \cite{21}. In the following in our calculations, we shall use the unitarity triangle values for \( |V_{ub}| \) and \( \gamma \). For other hadronic parameters we use the values in Table 1 of \cite{13} and take \( m_s(2 \text{ GeV}) = 80 \text{ MeV} \). For the \( B \to \pi \) and \( B \to K \) transition form factor, we use the current light-cone sum rules central value \cite{24}:

\[
F_0^{B\pi}(0) = 0.258, \quad F_0^{BK}(0) = 0.33 \tag{13}
\]

The computed branching ratios with \( \rho_A = 1, \rho_H = 1, \phi_H = 0 \) and \( \phi_A = -55° \) as in scenario S4 of \cite{12} are shown in Table 1. As can be seen, QCDF with power corrections from penguin annihilation as in S4 \cite{12, 20} could bring the branching ratios closer to experiments. With a different choice of the annihilation parameters, as given in \cite{27}, one could increase further the predicted decay rates to values consistent with experiments. For the CKM-allowed tree-dominated decays, as shown in Table 1 and in \cite{8}, the predicted \( B^+ \to \pi^+ \pi^0, \rho^0 \pi^+, \rho^+ \pi^0 \) decay rates agree well with experiments. Therefore we can use the QCDF tree amplitude for \( \Delta S = 1 \) \( B \to PP, PV \) in the determination of the phases of the decay amplitudes. For this purpose, one needs to express the \( \Delta S = 1 \) \( B \to PP, PV \) decay amplitudes in terms of isospin amplitudes. Following \cite{6, 7}, we have, for \( B \to K \pi \), in the notation of \cite{7}:

\[
A_{K^{+} \pi^{0}} = \frac{2}{3} B_3 + \sqrt{\frac{2}{3}} (A_1 + B_1),
\]

\[
A_{K^{0} \pi^{+}} = -\frac{\sqrt{2}}{3} B_3 + \sqrt{\frac{2}{3}} (A_1 + B_1),
\]

\[
A_{K^{+} \pi^{-}} = \frac{\sqrt{2}}{3} B_3 + \sqrt{\frac{2}{3}} (A_1 - B_1),
\]

\[
A_{K^{0} \pi^{0}} = \frac{2}{3} B_3 - \sqrt{\frac{1}{3}} (A_1 - B_1), \tag{14}
\]

with \( B_1, B_3 \) the \( I = 1/2 \) and \( I = 3/2 \) isospin amplitudes in terms of the decay amplitudes:

\[
A_1 = \frac{\sqrt{6}}{4} (A_{K^{0} \pi^{+}} + A_{K^{+} \pi^{-}}),
\]

\[
B_1 = \frac{1}{\sqrt{3}} A_{K^{+} \pi^{0}} + \frac{\sqrt{6}}{12} A_{K^{0} \pi^{+}} - \frac{\sqrt{6}}{4} A_{K^{+} \pi^{-}}
\]

\[
B_3 = A_{K^{+} \pi^{0}} - \frac{1}{\sqrt{2}} A_{K^{0} \pi^{+}} \tag{15}
\]

with the expressions in QCDF given by:

\[
A_1 = -i \frac{G_F}{2} f_K F_0^{B\pi} (m_K^2) \frac{\sqrt{3}}{2} (m_B^2 - m_{K^0}^2) \left( V_{ub} V_{us}^* a_1 + (V_{ub} V_{us}^* + V_{ub} V_{us}^*) \frac{1}{2} (a_{10} + (a_{9} + a_{8} r_{a_{10}})) \right)
\]

\[
-\frac{i G_F}{2} f_B f_K f_{\pi} \times \left( V_{ub} V_{us}^* b_2 + (V_{ub} V_{us}^* + V_{ub} V_{us}^*) \frac{1}{3} b_3 \right) \tag{16}
\]

For \( B_1 \), we have:

\[
B_1 = -i \frac{G_F}{2} f_K F_0^{B\pi} (m_K^2) \frac{2}{3} (m_B^2 - m_{K^0}^2) \left( V_{ub} V_{us}^* a_1 + (V_{ub} V_{us}^* + V_{ub} V_{us}^*) \frac{1}{2} (a_{10} + a_{8} r_{a_{10}}) \right)
\]

\[
-\frac{i G_F}{2} f_K F_0^{B\pi} (m_K^2) \frac{2}{3} (m_B^2 - m_{K^0}^2) \left( V_{ub} V_{us}^* a_2 + (V_{ub} V_{us}^* + V_{ub} V_{us}^*) 2 (a_{9} - a_{7}) \right)
\]

\[
-\frac{i G_F}{2} f_B f_K f_{\pi} \left( V_{ub} V_{us}^* b_2 + (V_{ub} V_{us}^* + V_{ub} V_{us}^*) b_3 \right) \tag{17}
\]
and for $B_3$

$$B_3 = -i \frac{G_F}{\sqrt{2}} f_K f_0 B_{\pi^0}(m_K^2) (m_B^2 - m_K^2) \left( V_{ub} V_{us}^* a_1 + (V_{ub} V_{us} + V_{cb} V_{cs}^*) - \frac{3}{2} (a_{10} + a_8 r_\chi) \right)$$

$$- i \frac{G_F}{\sqrt{2}} f_K f_0 B_{\pi^0}(m_K^2) (m_B^2 - m_K^2) \left( V_{ub} V_{us}^* a_2 + (V_{ub} V_{us} + V_{cb} V_{cs}^*) \times \frac{3}{2} (a_9 - a_7) \right)$$

(18)

We see that $B_3$ does not contain the strong penguin $a_4$ and $a_6$ terms. In the $SU(3)$ limit, apart from the small electroweak penguin terms, the main contribution to $B_3$ comes from the large color-favored $(a_1 + a_2)$ term, as in $B^+ \to \pi^+\pi^0$ decay, for which QCDF without the strong penguin contributions, is quite reliable, as can be seen from the good agreement with experiments for $B^+ \to \pi^+\pi^0$, $\rho^0\pi^+$, $\rho^+\pi^0$ decays shown in Table I. The relation between $B_3$ and the $B^+ \to \pi^+\pi^0$ decay amplitude can also be obtained in a general proof based on a model-independent approach to charmless $B \to PP$ decays, given recently in [23]. In terms of the $SU(3)/U(3)$ invariant amplitudes, one has, putting aside the CKM factor:

$$T_{\pi^+\pi^0}^{B_u} = \frac{8}{\sqrt{2}} C_{15}^T$$

$$T_{\rho^0\pi^+}^{B_u} = \frac{1}{\sqrt{2}} (C_3^T - C_6^T + 3 A_{15}^T + 7 A_{15}^T)$$

$$T_{\rho^0\pi^0}^{B_u} = (C_3^T - C_6^T + 3 A_{15}^T - C_{15}^T)$$

(19)

From Eq. (19), we get:

$$B_3 = \sqrt{2} T_{\rho^0\pi^+}^{B_u} - T_{\rho^0\pi^0}^{B_u} = \frac{8}{\sqrt{2}} C_{15}^T = T_{\pi^+\pi^0}^{B_u}$$

(20)

in agreement with QCDF in the $SU(3)$ limit. This relation can also be derived in a simple manner by using the topological amplitudes. We have [8]:

$$A_{K^+\pi^0} = -\frac{1}{\sqrt{2}} (p^* + t^* + c')$$

$$A_{\pi^+\pi^0} = -\frac{1}{\sqrt{2}} (t + c)$$

$$B_3 = -(t^* + c')$$

(21)

showing $B_3 = \sqrt{2} A_{\pi^+\pi^0}$ in the $SU(3)$ limit.

Given QCDF for the CKM-favored tree-dominated decay amplitudes, the $SU(3)$ breaking effects can be automatically taken into account in the QCDF expressions for penguin-dominated decays. The point we made in this paper is that QCDF works well for processes with large color-favored tree contribution, but without the strong penguin terms. The agreement with experiments for $B^+ \to \pi^+\pi^0$, $\rho^0\pi^+$, $\rho^+\pi^0$ measured branching ratios and the rather well-known short-distance Wilson coefficients for the tree operator shows that the central values for the form factors and decay constants involved are consistent with experiments and can be used in QCDF calculations with penguin-dominated decays. Thus the uncertainties for the QCDF branching ratios depend only on the accuracy of the measured $B^+ \to \pi^+\pi^0$, $\rho^0\pi^+$, $\rho^+\pi^0$ branching ratios, which are 10% while the theoretical errors and uncertainties in the current QCDF calculations are quite large [8] [11] [12]. This shows the advantage of using the measured $B^+ \to \pi^+\pi^0$, $\rho^0\pi^+$, $\rho^+\pi^0$ branching ratios to obtain the correct form factor values for QCDF calculations of the $B \to K\pi, K\rho, K^+\pi$ decay rates and in particular for the $I = 3/2$ isospin amplitude $B_3$, though the $SU(3)$ relation between $B_3$ and the $B^+ \to \pi^+\pi^0$ branching ratio in Eq. (20) or Eq. (21) is useful for a qualitative argument that $B_3$ is exactly the $B^+ \to \pi^+\pi^0 I = 2$ amplitude in the $SU(3)$ limit. For the penguin-dominated decays, we do not expect QCDF to produce a correct penguin amplitude in the $B \to K\pi, K\rho, K^+\pi$ decays which

| Decay         | $A \times 10^6\text{GeV}$(QCDF) | $BR \times 10^6$(QCDF) | $BR \times 10^6$(exp.) | 15, 16 |
|---------------|--------------------------------|------------------------|------------------------|--------|
| $B^+ \to \pi^+\pi^0$ | 2.162 - 1.112 i | 5.535 | 5.5 ± 0.4 |        |
| $B^+ \to \rho^0\pi^+$ | 0.925 - 2.752 i | 7.732 | 8.3 ± 1.2 |        |
| $B^+ \to \rho^+\pi^0$ | 1.863 - 3.055 i | 11.744 | 10.9 ± 1.4 |        |
| $B^+ \to K^+\pi^0$ | 0.725 + 3.244 i | 10.266 | 12.94 ± 0.52 |        |
| $B^+ \to K^0\rho^+$ | 0.162 + 4.399 i | 18.002 | 23.79 ± 0.75 |        |
| $B^0 \to K^+\pi^0$ | 0.887 + 4.180 i | 15.782 | 19.57 ± 0.53 |        |
| $B^0 \to K^0\rho^0$ | -0.016 - 2.817 i | 6.863 | 9.9 ± 0.5 |        |
| $B^+ \to K^+\rho^0$ | 1.422 + 0.4.483 i | 2.052 | 3.7 ± 0.5 |        |
| $B^+ \to K^0\rho^0$ | 2.463 - 0.363 i | 5.637 | 8.0 ± 1.5 |        |
| $B^0 \to K^+\rho^0$ | 2.608 + 0.466 i | 5.943 | 7.0 ± 0.9 |        |
| $B^0 \to K^0\rho^0$ | -2.164 + 0.411 i | 4.107 | 4.7 ± 0.6 |        |
| $B^+ \to K^+\pi^0$ | -1.495 + 0.786 i | 2.589 | 8.2 ± 1.8 |        |
| $B^+ \to K^0\pi^0$ | -1.876 - 0.022 i | 3.206 | 10.1 ± 0.9 |        |
| $B^0 \to K^+\pi^0$ | -1.657 + 0.946 i | 3.084 | 8.4 ± 0.8 |        |
| $B^0 \to K^0\pi^0$ | 1.003 + 0.128 i | 0.867 | 3.3 ± 0.6 |        |

**TABLE I:** The measured and computed QCDF branching ratios shown with the QCDF amplitudes for $B \to PV$ decays.
could have power correction terms like the penguin annihilation mentioned in the literature [12-23,27], especially for the predicted $K^+\pi$ branching ratios which are below the measured values by more than 30%.

### III. Determination of Phases of the \(\Delta S = 1 \rightarrow PP, PV\) Decay Amplitudes

With the $I = 3/2$ amplitude given by QCDF, we now proceed to the determination of the relative phase between the tree and penguin amplitudes.

As shown in [1], by taking the sum of the $B^+$ and $B^0$ absolute square of the amplitudes $|A|^2$ or the decay rates, from Eqs. (14), we have:

\[
|A_1 + B_1|^2 = |A_{K^+\pi^0}|^2 + |A_{K^0\pi^0}|^2 - \frac{2}{3}|B_3|^2 \tag{22}
\]

\[
|A_1 - B_1|^2 = |A_{K^+\pi^-}|^2 + |A_{K^0\pi^0}|^2 - \frac{2}{3}|B_3|^2. \tag{23}
\]

With the lengths of the sides $A_1 + B_1$ and $A_1 - B_1$ given by the decay rates of the four $B \rightarrow K\pi$ decay modes in Eqs. (22,23), the angles of the triangle formed with the decay amplitude, $B_3$, and with $A_1 + B_1$ and $A_1 - B_1$, respectively. This gives us the relative phase between the $I = 3/2$ tree and the $I = 1/2$ penguin amplitudes for a precise test of the QCDF. Clearly, isospin amplitudes are needed to obtain the phases in $B \rightarrow K\pi,\bar{K}\rho$, and $K^\star\pi$ decays which are in the interference term between $B_3$ and $A_1 + B_1$ and between $B_3$ and $A_1 - B_1$, and each of the length $|A_1 + B_1|$ and $|A_1 - B_1|$ depends on the branching ratios of two decay modes. Let $\delta_{1,2}$ be the relative phase between $B_3$ and $A_1 + B_1$, and between $B_3$ and $A_1 - B_1$, respectively, from Eqs. (14) and using Eqs. (22,23), we have:

\[
\cos(\delta_1) = \frac{\sqrt{3}(|A_{K^+\pi^+}|^2 - |A_{K^0\pi^0}|^2 - |B_3|^2/3)}{4|B_3||A_1 + B_1|} \tag{24}
\]

\[
\cos(\delta_2) = \frac{\sqrt{3}(|A_{K^+\pi^-}|^2 - 2|A_{K^0\pi^0}|^2 + |B_3|^2/3)}{4|B_3||A_1 - B_1|}. \tag{25}
\]

Since all the four penguin-dominated decay modes have similar decay rates, the differences $|A_{K^0\pi^0}|^2 - 2|A_{K^+\pi^+}|^2$ and $|A_{K^0\pi^0}|^2 - 2|A_{K^0\pi^0}|^2$ become small, errors and uncertainties in the measured decay rates would make it difficult to obtain a correct value for $\cos(\delta_1)$ and $\cos(\delta_2)$. Another problem which could affect the phase determination is the consistency of the four measured decay rates imposed on by an isospin relation between the decay rates which is given as [22-23], with QCDF values for $|B_3|^2$ and $\text{Re}(B_3^*B_1)$:

\[
|A_{K^+\pi^-}|^2 - 2|A_{K^0\pi^0}|^2 = -\left| |A_{K^0\pi^0}|^2 - 2|A_{K^+\pi^+}|^2 \right| - \left\{ \frac{4}{3}|B_3|^2 + \frac{8}{\sqrt{3}}\text{Re}(B_3^*B_1) \right\}_{K\pi} \tag{26}
\]

This relation gives a branching ratio $8.98 \times 10^{-6}$ for $B^0 \rightarrow K^0\pi^0$ to be compared with the measured value of $(9.93 \pm 0.49) \times 10^{-6}$ which produces a cancellation in the quantity $|A_{K^+\pi^+}|^2 - 2|A_{K^0\pi^0}|^2$ in Eq. (25) and a phase $\delta_2$ near 90°, which deviates largely from the phase between $B_3$ and $A_1 + B_1$, in contradiction with isospin analysis, since $|B_1|$ is small compared with $|A_1 + B_1|$ and $|A_1 - B_1|$, the difference $\delta_2 - \delta_1$ should be small. Using the above estimated branching ratio for $B^0 \rightarrow K^0\pi^0$, one would obtain $\delta_2 = 75.199°$, close to the value $77.296°$ for $\delta_1$, consistent with isospin analysis. Thus a correct value for $\delta_2$ consistent with $\delta_1$ requires a lower value for $B^0 \rightarrow K^0\pi^0$ branching ratio. This lower value for $B^0 \rightarrow K^0\pi^0$ could turn out to be the correct value, as over the years, the $B^0 \rightarrow K^0\pi^0$ branching ratio has decreased to the present value.

The phases for $B \rightarrow K\rho$ and $B \rightarrow K^*\pi$ decays can be obtained from the above expressions by making a straightforward substitution with the $K\rho$ and $K^*\pi$ decay rates. In Table III we give the relative isospin phases $\delta_{1,2}$ for $B \rightarrow K\pi, K\rho$, and $K^*\pi$ obtained from QCDF and from the measured decay rates.

As with $B \rightarrow K\pi$ decays, the determination of $\delta_1$ in $B \rightarrow K\rho$ decays is also subject to large uncertainties, with almost a cancellation in the difference $(|A_{K^0\rho^+}|^2 - 2|A_{K^+\rho^-}|^2)$, one would get a value $\delta_1 = 98.791°$, very different from the value 110.638° for $\delta_2$. In fact, using the isospin relation for $K\rho$ given as:

\[
(|A_{K^0\rho^+}|^2 - 2|A_{K^+\rho^-}|^2) = -(|A_{K^+\rho^-}|^2 - 2|A_{K^0\rho^+}|^2) + \left\{ \frac{4}{3}|B_3|^2 - \frac{8}{\sqrt{3}}\text{Re}(B_3^*B_1) \right\}_{K\rho} \tag{27}
\]

we would get a branching ratio $(9.15 \pm 1.2) \times 10^{-6}$ for $B^+ \rightarrow K^0\rho^+$ higher than the measured value of $(8.0^{+1.5}_{-1.4}) \times 10^{-6}$. This predicted branching ratio then gives $\delta_1 = 109.217°$ close to the value 110.638° for $\delta_2$, consistent with the fact that, as $B \rightarrow K\pi$ decays, since $|B_1|$ is small compared with the penguin amplitude $|A_1|$, $\delta_1$ and $\delta_2$ should be close to each other, as seen from the QCDF values given in the Table III.

Similar problem also appears in $B \rightarrow K^*\pi$ decay, as the isospin relation similar to that for $B \rightarrow K\rho$ in Eq. (27) would give a branching ratio $(6.3 \pm 2.2) \times 10^{-6}$ for $B \rightarrow K^*\pi$ decay, lower than the measured value of $(8.2 \pm 1.9) \times 10^{-6}$ for this reason, the phases $\delta_{1,2}$ for $B \rightarrow K^*\pi$ decays are obtained using only the $B^0 \rightarrow K^{+}\pi^- + B^0 \rightarrow K^{0}\pi^0$ decay rates and the isospin relation, as shown in Table III. We note that for $B \rightarrow K\pi$ decays, the errors on the phases $\delta_{1,2}$, are around 15°. This could be due to the large cancellation between the measured branching ratios which, however, have small errors, of the order few percent. For this reason we will not give errors on the phases for the $K\rho$ and $K^*\pi$ decays for which the errors are more than 10 percent. We note also that the errors for $B \rightarrow K\pi$ shown in the Table III, is comparable to the errors found in the determination of
TABLE II: The relative isospin phases given by QCDF and obtained from the measured decay rates for $B \to K\pi, K\rho$, and $K^*\pi$ decays. The numbers marked as “estimated” are the phases obtained with isospin relation as explained in the text. Errors are estimated to be in the range $(10-15)^\circ$.

| Decay | $\delta_1$(deg)(QCDF) | $\delta_1$(deg)(exp.) | $\delta_2$(deg)(QCDF) | $\delta_2$(deg)(exp.) |
|-------|------------------------|------------------------|------------------------|------------------------|
| $B \to K\pi$ | 71.891 | 77.296 ± 15 | 68.968 | 75.199 ± 15 (estimated) |
| $B \to K\rho$ | 113.701 | 109.217 (estimated) | 110.925 | 110.638 |
| $B \to K^*\pi$ | 67.838 | 73.351 (estimated) | 58.194 | 68.078 |

the relative phase between the three-gluon and the one photon annihilation amplitudes of the $\psi(2S)$ decays to pseudoscalar meson pairs, for which a relative phase of $(-82 \pm 29)^\circ$ or $(+121 \pm 27)^\circ$ is found in [29]. What is remarkable with the result we found is that, all the phases for $B \to K\pi, K\rho$, and $K^*\pi$ decays obtained with the central values for the measured branching ratios consistently show only small deviations from the QCDF values. The implication of this result is that one may need power correction terms, probably of perturbative QCD origin, to bring QCDF values close to the measured decay rates, without the need for a strong phase from long-distance rescattering effects.

IV. CONCLUSION

With the tree amplitude known from the QCDF tree-dominated $B \to PP, PV$ decays, we are able to determine the relative phases of the tree-penguin interference term in $B \to K\pi, K\rho$, and $K^*\pi$ decays. We find that the phases in the tree-penguin interference terms differ slightly from the QCDF phases, in particular, with an uncertainty $\pm 15^\circ$ more or less for $B \to K\pi$. Also for $K\rho$ and $K^*\pi$ decays, this uncertainty could be reduced considerably with more precise data with LHCb and the coming super Belle. This would allow a precise test of QCDF.