Could one find petroleum using neutrino oscillations in matter?

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Abstract. – In neutrino physics, it is now widely believed that neutrino oscillations are influenced by the presence of matter, modifying the energy spectrum produced by a neutrino beam traversing the Earth. Here, we will discuss the reverse problem, i.e. what could be learned about the Earth’s interior from a single neutrino baseline energy spectrum, especially about the Earth’s mantle. We will use a statistical analysis with a low-energy neutrino beam under very optimistic assumptions. At the end, we will note that it is hard to find petroleum with such a method, though it is not too far away from technical feasibility.

Recently, neutrino physics and especially neutrino oscillations have drawn a lot of attention in the field of physics. This is mainly due to the successes of the Super-Kamiokande and SNO experiments [1–3], which have strongly indicated that neutrinos are massive particles and that they are oscillating among different flavours. As far as we know today, the neutrinos come in three flavours [4], i.e. the electron, muon, and tau neutrino. Neutrino oscillations are a purely quantum mechanical phenomenon due to interference among the flavours. In order to reveal further basic properties of neutrinos and to pursue the mounting evidence for neutrino oscillations, so-called neutrino factories have been proposed [5,6]. Exploiting some of the properties of neutrinos, various approaches to neutrino absorption tomography, a method in some sense similar to X-ray tomography, have been suggested to obtain information on the interior of the Earth [7–13]. However, these techniques face several difficulties involving extremely high-energetic neutrino sources, large detectors, and the prerequisite of many baselines. As a completely different approach, the question has been raised if one could use the fact that neutrino oscillations are influenced by the presence of matter [16–18] to perform neutrino oscillation tomography [19–21], which would, in principle, be possible with only one single baseline. However, movable detectors, such as a grid of photomultipliers hanging from a movable floating pontoon, have been proposed to be used together with an upgraded CERN beam [22]. In comparison to geophysics, one could access the matter density profile directly

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Fig. 1 – The PREM (Preliminary Earth Reference Matter) density profile [26, 27] (dashed curve) as well as one possible reconstructed profile from ref. [21] close to the 1σ contour (solid curve) for a baseline length of 11,736 km and the parameters used in ref. [21]. In this paper, the matter density profile was cut into layers of constant matter densities to be measured by a low-energy neutrino beam. Since the matter density of every layer was treated as a free parameter and the contours of a high-dimensional parameter space cannot be directly displayed, we only show one representative of a possible matter density profile close to the 1σ contour to demonstrate the inability to resolve short-scale fluctuations, i.e., one cannot exceed the precision illustrated by this reconstructed matter density profile.

with the neutrino oscillation tomography method as opposed to measuring the seismic wave velocity profile (see, e.g., refs. [23, 24]). We will use a rather simple approach to see what could be learned about cavities in the Earth’s mantle from neutrino oscillations in matter under very optimistic assumptions.

Thus, let us now assume a future scenario at the mid 21st century, when neutrino factories have been operating already for some decades, and the neutrino mixing parameters, such as mixing angles, phases, and mass squared differences, have been measured more accurately. At this time, we will probably know much more about neutrino oscillation technology to build larger neutrino sources producing higher neutrino event rates than proposed today. It was shown in ref. [21] that one can, in such a scenario, reconstruct the symmetric Earth’s matter density profile from a single neutrino baseline energy spectrum up to a certain precision. Since the operators in the Hamiltonian describing neutrino propagation through different layers of matter are, in general, non-commuting (see, e.g., ref. [23]), a single baseline supplies more information on the matter density profile than a single baseline in neutrino absorption tomography. In other words, interference effects among the quantum mechanical transition amplitudes of different density layers contain this information. However, one cannot resolve density fluctuations of small amplitudes around the average value of the matter density profile, as can be seen in fig. [1]. But what about cavities in the Earth’s mantle? How large do they have to be in order to be identified in a single neutrino baseline energy spectrum? All these questions points towards neutrino oscillation tomography, i.e. using neutrino oscillations in matter to learn something about the structure and composition of the matter inside the Earth.

In order to have a neutrino beam very sensitive to matter density fluctuations in the Earth’s mantle, we use a low-energy neutrino beam with about 500 MeV for our investigation, as it is right now often proposed for upgraded conventional beams (so-called superbeams) [28, 30].
Fig. 2 – A baseline configuration from a source $S$ to a detector $D$ with a baseline length of $L = 1,000 \text{ km}$, which reaches a maximum depth of about 20 km under the Earth’s surface, but has an average depth of about 13 km. The neutrinos are propagating in matter of constant density of about $2.9 \text{ g/cm}^3$ and crossing a cavity of length $l$ centred at a distance $d$ from the source. The matter density in the cavity is $\rho$.

In addition, we make very optimistic assumptions for cross sections, beam characteristics, energy uncertainties, and detector properties, since a possible experimental setup at this time can only be estimated. Let us take 20 energy bins between 300 MeV and 500 MeV at a cross section proportional to $E^{1.66}$ in this energy range, where $E$ is the energy, such that we could see as many as an accumulated 10,000 events per energy bin at 500 MeV to be folded with the neutrino oscillation transition probabilities. This should be a reasonable guess without taking into account too many yet unknown problem-specific details. For the neutrino oscillations we choose the channel $\nu_\mu \to \nu_e$ in a three-flavour neutrino oscillation analysis with the parameter values $\theta_{13} = 5^\circ$, $\theta_{12} = \theta_{23} = 45^\circ$ (bimaximal mixing), $\Delta m_{21}^2 = 3.65 \cdot 10^{-5} \text{ eV}^2$, $\Delta m_{32}^2 \approx \Delta m_{31}^2 = 2.5 \cdot 10^{-3} \text{ eV}^2$ and the CP phase $\delta = 0$. (Of course, the number of events in this channel depends on the value of the mixing angle $\theta_{13}$.)

Let us now look at a beam configuration as it is shown in fig. 3. In this configuration, the neutrino beam, propagating in approximately constant matter density in the Earth’s mantle, crosses a cavity with matter density $\rho$ and length $l$ centred at a distance $d$ from the source $S$. We will speak about $l$ and $d$ as the size and position of the cavity, respectively. Assuming a neutrino energy spectrum measured at the detector $D$ produced with the parameter values above, what can we learn about the parameters $d$ and $l$? Since the phase shift in neutrino oscillations will depend on the matter density contrast between the surrounding matter and the cavity, we assume a rather small matter density within the cavity, i.e. $\rho \approx 1 \text{ g/cm}^3$, corresponding to a cavity filled with water. The matter density contrast would be much larger for air-filled cavities and much smaller for a porous rock, which may act as a petroleum trap.

Figure 3 shows the results of a two-parameter statistical analysis of a cavity centred at $d_0 = 300 \text{ km}$ for different values of $l_0 \in \{50, 100, 200\} \text{ km}$. In these plots, the true position $d_0$ and size $l_0$ of the cavity assumed or measured is marked with a cross. The contour lines tell us for the fixed reference values that we will measure a value within the $1\sigma$ contour with 68.3%, within the $2\sigma$ contour with 95.5% and within the $3\sigma$ contour with 99.7% probability.\footnote{Note that for energies lower than about 1 GeV there are quite large uncertainties, which will need to be reduced by future experiments. However, it turns out that our application is rather insensitive to the slope of the cross section, i.e. the coefficient of the energy dependence.}
Fig. 3 – The 1σ (solid curve), 2σ (dotted curve) and 3σ (dashed curve) contours in the statistical analysis of a true cavity centred at $d_0 = 300$ km. In each plot, the value $l_0$ for the true cavity length is given in the title as well as the true position is marked with a cross.

Hence, these contours help us to estimate the measurability of the parameters $d$ and $l$. Closed contours mean that we can really detect this cavity within the configuration assumed on a statistical basis on the respective significance level. Contours open to the left only constrain the values within these, i.e. for a certain position $d$, cavities larger than a certain size $l$ can be excluded from the measured neutrino energy spectrum. Small enough cavities, however, could be located anywhere.

In the plots in fig. 3, large cavities with $l \approx 200$ km can be clearly detected on the 3σ level, though with some degeneracy in position and uncertainty in size. The degeneracy in the position $d$ basically comes from the periodicity in neutrino oscillations and this may be resolved by using additional knowledge coming from geophysics. Note that two-flavour neutrino oscillations cannot distinguish time-inverted matter density profiles [32], which means that the symmetry in the plots with respect to the “$d = 500$ km line” is only destroyed by three-flavour effects. Furthermore, the areas fulfilling $d - l/2 < 0$ and $d + l/2 > 1,000$ km are excluded by definition. Smaller cavities, such as for $l \approx 100$ km, can only be seen on the 1σ level, or even not at all, such as for $l \approx 50$ km. The reason for this is the short cavity length compared with the characteristic length scale of neutrino oscillations, which is of the order of 1,000 km, as well as the quite small matter density contrast. Generally speaking, in any quantum mechanical problem, the influence of a perturbing potential depends on its integral, i.e. the length scale of the perturbation times its amplitude.

The result of the analysis is, however, not only dependent on the cavity size $l_0$, but also on the cavity position $d_0$. Figure 3 shows the result of a calculation with $d_0 = 500$ km and $l_0 = 200$ km, i.e. the cavity is situated on the very centre of the baseline. In this case, the degeneracy in the position on the 1σ level is three compared with two above. In addition, the cavity can clearly be located on the 2σ level. However, on the 3σ level, it cannot be proven to exist at all. Thus, the position of the cavity slightly modifies the statistics and has to be taken into account.

Coming back to our original idea, the search for petroleum, what can we conclude from the above analysis? Because we used very optimistic estimates for the source and detector, as well as we neglected energy uncertainties, backgrounds, and cross section and mixing parameter uncertainties, it may be very hard to exploit this idea. Moreover, the cavity, i.e. the porous rock acting as petroleum trap, has to be quite large and it has to have a rather large density.
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Fig. 4 – The $1\sigma$ (solid curve), $2\sigma$ (dotted curve) and $3\sigma$ (dashed curve) contours in the statistical analysis of a true cavity with $d_0 = 500\text{ km}$ and $l_0 = 200\text{ km}$.

contrast to the surrounding matter in order to be detected. One can easily imagine that reducing the density contrast or taking into account backgrounds and other uncertainties would make the closed contours vanish, i.e. the corresponding cavity could not be detected anymore. However, with this sort of analysis one should be able to see larger scale structures, such as cavity systems, though conventional geophysical methods could be more successful and less expensive. Thus, neutrino oscillation tomography may not be the very best way to look for cavities in the Earth’s mantle; however, it could, in the far future, have different applications such as for the Earth’s core or other planets. Finally, as a curiosity, taking the recent development and progress in neutrino physics into account, it is interesting to observe that neutrino oscillation tomography is not as far away from technical feasibility as one may expect.

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