Neutron-Mirror-Neutron Oscillations in a Trap

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Abstract

We calculate the rate of neutron-mirror-neutron oscillations for ultracold neutrons trapped in a storage vessel. Recent experimental bounds on the oscillation time are discussed.

1 Introduction

During the last couple of years we are witnessing the revival of the interest to the "mirror particles", "mirror matter" and "mirror world". The idea of the existence of the hypothetical hidden sector to compensate mirror asymmetry was first explicitly formulated in [1]. The subject has a rich history – see the review paper [2]. The present wave of interest to mirror particles has been to a great extent initiated by the quest for neutron-mirror-neutron oscillations \((n-n')\). It was conjectured that \(n-n'\) oscillations may play an important role in the propagation of ultra high energy cosmic rays and that the oscillation time \(\tau_{osc}\) may be as small as \(\tau_{osc} \sim 1\) s [3]. Implications of mirror particles for cosmology and astrophysics were discussed in a number of papers e.g., [4]. Last year the first experimental data on \(n-n'\) transitions were published with the results \(\tau_{osc} \geq 103\) s [5] and \(\tau_{osc} \geq 414\) s [6]. Possible laboratory experiments to search for \(n-n'\) oscillations were discussed in [7].

Experimental results [5, 6] were obtained using the ultracold neutrons (UCN), i.e., neutrons with the energy \(E < 10^{-7}\) eV storaged in a trap.

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Previously similar experimental setup was used in the search for neutron-antinucleon oscillations (see [8] and references therein). The crucial difference between $n - n'$ and $n - \bar{n}$ oscillations is that $n'$ freely escapes from the trap while $\bar{n}$ either annihilates on the trap walls or gets reflected. Therefore formalism developed for the $n - \bar{n}$ oscillations cannot be adjusted to treat $n - n'$ transitions. Still the two processes have a common point. This is a problem of a correct quantum mechanical description of the UCN wave function (w.f.). Most often it is assumed that the w.f. of the bottled UCN corresponds to a stationary state of a particle inside a potential well [9, 10]. Alternatively, other authors [11] describe oscillations of the trapped neutrons in the basis of the free plane waves. Both pictures do not correspond to the physics of real experiments. The process proceeds in time in three stages. At the first stage the filling of the trap takes place, then the neutrons are kept inside the trap during the storage time (hundreds of seconds), and finally neutrons leave the trap to the detectors. Therefore the w.f. undergoes a complicated evolution which hardly can be described without resorting to approximations. We shall first evaluate the neutron-mirror-neutron oscillations using a stationary w.f. as the initial state w.f. Then we shall do the same using wave packet instead of a stationary w.f.

The paper is organized as follows. We start in Section 2 with the analysis of the oscillations in the stationary w.f. approach. Transitions take place from one of the trap eigenstates. In Section 3 transitions are considered in presence of a superimposed magnetic field. A general equation for the transition rate is derived and the limits of weak and strong field are considered. Section 4 is devoted to the wave packet formalism. The evolution of the UCN wave packet (w.p.) is encoded using the trap Green’s function. Neutron-mirror-neutron transition rate is calculated. In Section 5 the main conclusions are presented and open problems are formulated. Appendix contains comparison between the infinite and finite well models.

## 2 Stationary Wave Function Approach

The problem of neutron-mirror-neutron oscillations in free space can be solved by diagonalization of the time-dependent two-channel Schrödinger equation with the result [12]

$$|\psi_n(t)|^2 = \frac{4\varepsilon^2}{\omega^2 + 4\varepsilon^2} \exp(-\Gamma t) \sin^2 \left(\frac{1}{2} \sqrt{\omega^2 + 4\varepsilon^2} t\right),$$

(1)
where \( \omega = E_n - E_{n'} = |\mu_n|B \) is the energy difference between neutron and mirror neutron due to superimposed magnetic field (mirror neutron does not feel “our” magnetic field), \( \varepsilon = \tau_{\text{osc}}^{-1} \) is the mixing parameter, \( \Gamma_\beta \) is the neutron \( \beta \)-decay width. In arriving at (11) the spatial part of the w.f. was factored out making use of the fact that in free space the w.f.-s of \( n \) and \( n' \) are of the same form. In the trap, however, the situation is different: the neutron is confined while for the mirror neutron the trap walls do not exist. As already mentioned in the Introduction, the description of the trapped UCN is a nontrivial problem. The naive guess would be that inside the trap the neutron w.f. corresponds to a discrete eigenstate. Here we assume that the neutron w.f. is that of a particle in a potential well with the boundary conditions corresponding (in the first approximation) to a complete reflection.

In order to make calculations tractable and transparent we shall consider the following simple model of a trap. Let it be a one-dimensional square well of width \( L = 1 m \) with walls at \( x = 0 \) and \( x = L \), i.e., the potential of the form

\[
U(x) = \begin{cases} 
V, & x < 0 \\
0, & 0 < x < L \\
V, & x > L 
\end{cases}
\]  

(2)

The height of the potential well depends on the material with the typical value \( V = 2 \cdot 10^{-7} \) eV which will be used in what follows. For such a well the limit for stored UCN velocity is 6.2 m/s. The number of discrete levels in such a trap may be estimated as

\[
M \simeq \frac{L \sqrt{2mV}}{\pi} \approx \frac{10^8}{\pi}.
\]  

(3)

We choose the UCN energy to be \( E = 0.8 \cdot 10^{-7} \) eV. This energy corresponds to a level with quantum number \( j \simeq 2 \cdot 10^7 \). Positions and eigenfunctions of such highly excited states in a finite-depth potential are very close to the same quantities in the infinite well (except for the levels close to the upper edge of the well; we not consider such levels). The finite-depth corrections are considered in the Appendix. The eigenvalues and eigenfunctions for the infinite well are

\[
E_j = \frac{\pi^2 j^2}{2mL^2}, \quad k_j = \frac{\pi j}{L}, \quad j = 1, 2, 3,.. 
\]  

(4)

\[
\varphi_j(x) = \sqrt{\frac{2}{L}} \sin k_j x.
\]  

(5)
Another important quantity characterizing highly excited states is the classical frequency $\omega_{cl}[11]\]

$$\omega_{cl} = \frac{\pi^2}{mL^2} j = \frac{2\pi}{\tau_{cl}} = \delta E_j, \quad (6)$$

where $\tau_{cl}$ is the time of the classical period and

$$\delta E_j = E_{j+1} - E_j \simeq 0.8 \cdot 10^{-14} \text{ eV}$$

is the level spacing. Levels with $j \gg 1$ are almost equidistant. In the semiclassical limit we may also define the trap crossing time $\tau$

$$\tau = \frac{\tau_{cl}}{2} = \frac{mL}{k_j} \simeq 0.26 \text{ s} \quad (7)$$

for $j = 2 \cdot 10^7$. Next we calculate the rate of $(n - n')$ oscillations for the neutron at the $j$th discrete level. The neutron and mirror neutron w.f.-s in a two-component basis are

$$\tilde{\varphi}_j(x) = \sqrt{\frac{2}{L}} \sin k_j x \begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv \varphi_j(x) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (8)$$

$$\tilde{f}_p(x) = \frac{1}{\sqrt{2\pi}} e^{ipx} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv f_p(x) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (9)$$

where $-\infty < p < +\infty$. The $(n - n')$ system is described by the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{W} = \begin{pmatrix} \frac{k^2}{2m} + U & 0 \\ 0 & \frac{p^2}{2m} \end{pmatrix} + \begin{pmatrix} 0 & \varepsilon \\ \varepsilon & 0 \end{pmatrix}. \quad (10)$$

The states (8) and (9) are the eigenstates of $\hat{H}_0$, therefore it is convenient to use the interaction representation. The probability to find at time $t$ a mirror neutron instead of a neutron reads

$$P_{nn'} = \int dp |\langle \tilde{f}_p | \exp \left\{ -i \int_0^t \! dt' \hat{W}_{int}(t') \right\} |\tilde{\varphi}_j>|^2, \quad (11)$$

where $\hat{W}_{int}(t) = e^{i\hat{H}_0 t} \hat{W} e^{-i\hat{H}_0 t}$. In the first order of perturbation theory we get

$$P_{nn'} = \int dp |\langle \tilde{f}_p | \int_0^t \! dt' \hat{W}_{int}(t') |\tilde{\varphi}_j>|^2 =$$
\[ \varepsilon^2 \int dp \int_0^t dt' e^{-i(E_j - E_p)t'} | \langle f_p | \varphi_j \rangle |^2, \quad (12) \]

where \( E_j = \frac{k_j^2}{2m} \), \( E_p = \frac{p^2}{2m} \). The time-dependent integral is a standard one

\[ w(E_p) = \mid \int_0^t dt' e^{-i(E_j - E_p)t'} \mid^2 = \frac{4 \sin^2 \left( \frac{(E_p - E_j)t}{2} \right)}{(E_p - E_j)^2}, \quad (13) \]

The overlap of the w.f.-s reads

\[ g_j(p) = \mid \langle f_p | \varphi_j \rangle \mid^2 = \frac{4k_j^2}{\pi L (p^2 - k_j^2)^2} \sin^2 \left( \frac{pL + \pi j}{2} \right), \quad j = 1, 2, \ldots \quad (14) \]

From (12), (13) and (14) we obtain

\[ P_{nn'} = \varepsilon^2 \int_{-\infty}^{+\infty} dp g_j(p) w(E_p). \quad (15) \]

It is convenient to change integration from \( dp \) to \( dE_p \) taking into account that \( g(p) = g(-p) \).

Then

\[ P_{nn'} = 2m \varepsilon^2 \int dE_p \frac{g(E_p) w(E_p)}{p}, \quad (16) \]

where the factor 2 comes from the fact that two plane waves \( e^{\pm ipx} \) correspond to the same energy \( E_p \). Both functions \( g(E_p) \) and \( w(E_p) \) are peaked at \( E_p = E_j \). According to (14) and (13) the widths \( \Delta E_p^g \) and \( \Delta E_p^w \) of the corresponding maxima are

\[ \Delta E_p^g \simeq \pi/\tau, \quad \Delta E_p^w \simeq 4\pi/t, \quad (17) \]

with \( \tau \) being the trap crossing time. At times \( t \gg \tau \) we may substitute \( g(E_p)/p \) by its value at \( p = k_j \) and take it out of the integral (16). From (13) one gets \( g(E_j) = L/4\pi \). The remaining integration in (16) can be extended to \((-\infty < E_p < +\infty)\) yielding \( 2\pi t \). Collecting all pieces together we obtain

\[ P_{nn'} = \varepsilon^2 \tau t. \quad (18) \]

At very short times \( t \ll \tau \) the function \( w(E_p) \) becomes smoother than \( g(E_p) \). Hence \( w(E_p) \) can be taken out of the integral (16). The remaining integral is time-independent while \( w(E_p) \sim t^2 \). As a result \( P_{nn'} \sim \varepsilon^2 t^2 \) and we can not define the transition probability per unit time (12). On the other hand, Eq. (18) is valid only for times shorter than the neutron \( \beta \)-decay time \( t_\beta \) since we have define the eigenstate (8) neglecting the \( \beta \)-decay. The condition \( \tau \ll t \ll t_\beta \) was with a fair accuracy satisfied in experiments [5, 6].
3 Stationary approach with magnetic field included

The search for \( n - n' \) oscillations is experiments with bottled UCN is based on the comparison of UCN storage with and without superimposed magnetic field \([5, 6]\). It is assumed that there is no mirror magnetic field in the laboratory and therefore the interaction of the neutron with magnetic field lifts the degeneracy and thus suppresses the oscillations.

In magnetic field \( B \) the energy of the trapped neutron becomes equal to

\[
E_j = \frac{k_j^2}{2m} + \mu B, \tag{19}
\]

where \( \mu = -\mu_n = 1.91\mu_N (\mu_N = e/2m_p) \).

Inclusion of the magnetic field does not alter the functions \( w(E_p) \) and \( g(p) \) given by Eq.\-s \([13] \) and \([14] \). There is, however, an important difference between our present considerations and the previous section. As we see from \([19] \) \( w(E_p) \) now peaks at \( p = \pm \sqrt{k_j^2 + 2m\mu B} \) while the maximum of \( g(p) \) is as before at \( p = \pm k_j \). As a result instead of \([16] \) we obtain

\[
P_{nn'} \approx 4\varepsilon^2 t \left( \frac{1}{\mu B} \right) \frac{\sin^2 \left( \frac{k_j L}{2} \right)}{(\mu B)^2} \sqrt{1 + \frac{2m\mu B}{k_j^2}}, \quad j = 2, 4, ...
\]

This equation can be simplified taking into account that the quantities \( (\mu B) \) and \( k_j^2/2m \) differ by many orders of magnitudes. In experiments \([5, 6] \) the value of the magnetic field varied in the interval \((1-2)nT \leq B \leq \text{(few)}\mu T \) which corresponds to \(10^{-16} \text{eV} \lesssim \mu B \lesssim 10^{-13} \text{eV} \), while \( k_j^2/2m \approx 10^{-7} \text{eV} \) (note that the unshielded Earth magnetic field corresponds to \( \mu B \approx 3 \cdot 10^{-12} \text{eV} \)).

Therefore Eq. \([20] \) easily reduces to

\[
P_{nn'} \approx 4\varepsilon^2 t \sin \left( \frac{1}{\mu B} \right) \frac{\sin^2 \left( \frac{k_j L}{2} \right)}{(\mu B)^2}, \tag{21}
\]

where \( \tau \) is the trap crossing time. For our model of the trap described in section 2 we have \( \tau/2 \approx 2 \cdot 10^{14} \text{eV}^{-1} \). Therefore in the limit of weak magnetic field \( B \approx nT \) Eq. \([21] \) yields

\[
P_{nn'} \approx \varepsilon^2 \tau t, \tag{22}
\]
as expected (see (18)). In the opposite limit of strong magnetic field $B \simeq (\text{few}) \mu T$ we have to take into account that the quantities $\tau$ and $B$ in (21) experience fluctuations leading to rapid oscillations of the function $\sin^2(\frac{1}{2} \mu B \tau)$. In particular, the crossing time $\tau$ may vary either due to changes of $L$ at each cossision, or due to variations of the neutron velocity. Substituting the rapidly oscillating quantity in (21) by its mean value equal to 1/2 we obtain the equation describing the neutron-mirror-neutron transitions in strong magnetic field

$$P_{nn'} = \varepsilon^2 \frac{2t}{(\mu B)^2 \tau}. \tag{23}$$

4 The wave packet approach

We now turn to the question formulated in the Introduction, namely to the problem of the UCN w.f. evolution and to the calculation of the oscillations in the wave packet approach. In order to get physically transparent results and to avoid numerical calculations suited to a concrete experiment we assume that UCN coming to the trap from the source are described by the Gaussian wave packets (w.p.) [13].

The w.p. moving from the left and for $t = 0$ centered at $x = 0$ is given by the following expression

$$\Psi_k(x, t = 0) = (\pi a^2)^{-1/4} \exp \left\{ -\frac{(x - x_0)^2}{2a^2} + ikx \right\}, \tag{24}$$

where $a$ is the width of the w.p. and $k$ is its central momentum. The normalization of the w.p. (23) corresponds to one particle in the entire one-dimensional space,

$$\int_{-\infty}^{+\infty} dx \left| \Psi_k(x, t = 0) \right|^2 = 1. \tag{25}$$

Let the UCN energy be equal to the value chosen in Section 2, $E = 0.8 \cdot 10^{-7}$ eV, and let the beam resolution be equal to $\Delta E / E = 10^{-3}$. Thus the set of parameters to be used is

$$E = 0.8 \cdot 10^{-7} \text{ eV}, \quad \lambda = \frac{2\pi}{k} \simeq 10^{-5} \text{ cm}, \quad a \simeq 3.2 \cdot 10^{-3} \text{ cm}. \tag{26}$$

The condition $a \gg \lambda$ ensures the localization of the w.p. We remind that the above value of $E$ corresponds to the level $E_j$ with a very high quantum

\footnote{The problem of the choice of the w.p. parameters will be addressed in the next Section.}
number \( j \simeq 2 \cdot 10^7 \). Next we estimate the number of levels within \( \Delta E \). One has

\[
\Delta j = \frac{\Delta E}{\omega_{cl}} = \frac{v(\Delta k)}{\omega_{cl}} = \frac{L}{\pi a} \simeq 10^4. \tag{27}
\]

The large number of levels forming the w.p. is a necessary condition for the trapped w.p. to be localized (in free space this condition reads \( a \gg \lambda \), see above). The time evolution of the initial w.p. \((24)\) proceeds according to the following law

\[
\Psi_k(x, t) = \int dx' G(x, t; x', 0)\Psi_k(x', 0), \tag{28}
\]

where \( G(x, t; x', t') \) is the trap Green’s function. In the infinite well approximation we may use the spectral decomposition of the Green’s function over the set of eigenfunctions \((5)\) and write

\[
\Psi_k(x, t) = \sum_{j=1}^{\infty} e^{-iE_j t} \phi_j(x) \int_0^L dx' \phi_j^*(x') \Psi_k(x', 0). \tag{29}
\]

The width of the w.p. \((29)\) increases with time according to

\[
a' = a \left[ 1 + \left( \frac{t}{ma^2} \right)^2 \right]^{1/2} \simeq a \left( \frac{t}{ma^2} \right),
\]

where for our model the spreading time is \( ma^2 \simeq 1.7 \cdot 10^{-2} s \) and \( t/ma^2 \simeq 60t(s) \). A so-called collapse time \( t_c \) \([14]\) corresponds to \( a' = L \) and is equal to \( t_c \simeq 500s \). At \( t = t_c \) the w.p. spreads uniformly over the entire well and the stationary regime considered in Section 2 sets in \([15]\). We note in passing that there is another time scale in the problem, the so-called revival time \( t_{rev} = 4mL^2/\pi \simeq 2.10^7 s \) when the w.p. regains its initial shape –see \([14]\) and references therein.

The initial w.p \( \Psi_k(x, 0) \) contains only right running wave – see \((24)\). The trapped w.p. \((29)\) contains both right and left running waves, i.e., it correctly describes reflections from the trap walls. We assume that the point \( x_0 \) (see \((24)\) is not in the immediate vicinity of the trap walls, i.e., \( x_0 \) is at least few times of \( a \) away from the walls. Then the integration in \((29)\) may be extended to the entire one-dimensional space. This yields

\[
F(k, k_j; L, a, x_0) \equiv \int_{-\infty}^{+\infty} dx' \phi_j^*(x') \Psi_k(x', 0) = i \left( \frac{a\sqrt{\pi}}{L} \right)^{1/2} \times
\]
\[ \Psi_k(x, t) = \sum_{j=1}^{\infty} e^{-iE_j t} \varphi_j(x) F_j(k), \]  
\[ \langle \tau \rangle = \sum_j |F_j(k)|^2 \tau(k_j), \]
\[
P_{nn'}^{(2)} = 4\pi m\varepsilon^2 \left\{ \sum_j \frac{F_j(k)}{k_j} \sum_l F^*_l(k) e^{\frac{i}{2}(E_l-E_j)\tau} \left[ \frac{2\sin\left(\frac{(E_l-E_j)\tau}{2}\right)}{(E_j-E_l)} \right] \right. \\
\left. \times \langle f_j|\varphi_j\rangle\langle\varphi_l|f_j\rangle + (j \leftrightarrow l) \right\}.
\]

Replacing summation over \(l\) by integration we obtain
\[
P_{nn'}^{(2)} \simeq 8\varepsilon^2 \sum_j |F_j|^2 \frac{m^2L^2}{k_j^2} = 8\varepsilon^2 \langle \tau^2 \rangle
\]
where
\[
\langle \tau^2 \rangle = \sum_j |F_j|^2 \tau_j^2.
\]

Collecting the two contributions (34) and (37) together we get the final result
\[
P_{nn'} = \varepsilon^2 \langle \tau \rangle t \left( 1 + 8 \frac{\langle \tau^2 \rangle}{\langle \tau \rangle} t \right).
\]

5 Conclusions

We have calculated the rate of neutron-mirror-neutron oscillations for trapped UCN. Two types of the UCN w.f.-s were used: the stationary solution for a particle inside a potential well and the Gaussian w.p. Calculations were performed in the first-order perturbation theory. This approximation is legitimate provided \(P_{nn'} \ll 1\). From (22) and (23) it follows that this condition holds for \(\tau_{nn'} \gg 7s\) and \(\tau_{nn'} \gg 0.1s\) in the weak and strong magnetic fields correspondingly. Obviously the first order perturbation theory describes the transition of the neutron into mirror neutron. The inverse process appears only in the second order in \(\varepsilon\). For the analysis of the experiments [5, 6] first order perturbation theory is a fair approximation.

The above analysis has been performed for a simple one-dimensional trap. We think that such a model correctly describes the principal features of the process. Generalization to the three-dimensional rectangular trap is trivial. The simplest way to generalize our result to the trap with arbitrary geometry is to substitute the crossing time \(\tau\) by the effective crossing time corresponding to a given trap geometry.
Experimental data \[5, 6\] were analyzed using the free space equation \(\Pi\) with the time \(t\) being limited by the crossing time \(\tau\).

Equation \(\Pi\) contains only time dependence since the spatial parts of \(n\) and \(n'\) w.f.-s were factored out using the fact that in free space the coordinate w.f.-s of \(n\) and \(n'\) have the same form. For bottled UCN the situation is different. Neutron is confined inside the trap while mirror neutron freely crosses the trap walls. Therefore the use of the \(\Pi\) to describe oscillations of trapped UCN seems questionable\[2\]. However our accurate approach justifies the analysis of the experimental data based on Eq.(\(\Pi\)) \[5, 6\]. This can be explained by the semiclassical character of the UCN motion inside the trap of macroscopic size. In the stationary approach the typical UCN energy corresponds to the states with \(j \gg 1\), i.e., to the semiclassical part of the spectrum. In the w.p. formalism the classical limit corresponds to \((\Delta j) \sim (j)^{1/2} \rightarrow \infty\) \[16\]. The w.p. \(24), 26\) is close to this limit. For UCN with extremely low energy, \(E \lesssim 10^{-16}\) eV, the oscillation pattern changes. We shall consider this question elsewhere. The fraction of UCN with such energies in experiments is negligible. Another point which deserves a dedicated study is decoherence of the UCN w.f. and subsequent randomization of the oscillation process. This might occur due to dephasing of the w.f. caused by collisions with the trap walls and with the residual gas.

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\[2\]To describe the free space experiments by Eq.\(\Pi\) one still has to supplement it by a boundary condition at \(x = 0\) where the reactor is placed. Otherwise at \(t = \pi\tau_{\text{eff}}/2\) the reactor becomes a source of mirror neutrons. We are grateful to L.B.Okun and M.I.Vysotsky for drawing our attention to this.
6 Appendix

Calculations presented above were performed for the infinite well model of a trap. Here we consider the finite potential and show that there is only a minor difference between the two models. Consider the potential well defined by Eq. (2). Matching the logarithmic derivatives of the w.f.-s at $x = 0$ and $x = L$ we obtain the eigenvalue equation

$$k_j' L = \pi j - 2 \arcsin \frac{k_j'}{\sqrt{2mV}}, \quad (A.1)$$

(the notation $k_j$ is kept for $k_j = \pi j / L$). The small parameter in the problem is

$$\delta = \left( \frac{2}{mVL^2} \right)^{1/2} \simeq 2 \cdot 10^{-8}. \quad (A.2)$$

Expanding (A.1) with respect to $\delta$ we obtain

$$k_j' \simeq \frac{\pi j}{L} (1 - \delta), \quad E_j' \simeq \frac{\pi^2 j^2}{2mL^2} (1 - 2\delta). \quad (A.3)$$

Therefore the levels in the finite well are shifted relative to the infinite well levels by

$$E_j - E_j' \simeq 4 \cdot 10^{-15} \text{ eV}. \quad (A.4)$$

From (A.3) it follows that the spectrum in the finite well (2) is the same as in somewhat wider infinite well

$$L' = L(1 + \delta). \quad (A.5)$$

In the finite well the w.f. penetrates into classically forbidden regions inside the trap walls. However neutron-mirror-neutron transitions inside the walls may be neglected since both the penetration depth $d$ and the collision time $\tau_{\text{coll}}$ are small: $d \sim 10^{-6} \text{ cm}$, $\tau_{\text{coll}} \sim 10^{-8}$s.

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