The tetraquark $K_1(1400)$ in the decays

$\tau \rightarrow [\omega(782), \phi(1020)]K^-\nu_\tau$

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Abstract

In the extended Nambu–Jona-Lasinio model, the decay widths of the processes $\tau \rightarrow [\omega(782), \phi(1020)]K^-\nu_\tau$ were calculated. The intermediate strange quark-antiquark mesons in both the ground and the first radially excited states were taken into account. The meson $K_1(1400)$, being a possible candidate for the role of the tetraquark meson, was also taken into account as an intermediate state. The result for the process $\tau \rightarrow \omega(782)K^-\nu_\tau$ is in satisfactory agreement with the experimental data. Simultaneously, the result for the decay $\tau \rightarrow \phi(1020)K^-\nu_\tau$ is in agreement with the experimental data with precision $1.3\sigma$.

1 Introduction

The processes $\tau \rightarrow [\omega(782), \phi(1020)]K^-\nu_\tau$ were intensively investigated from both the experimental [1, 2, 3] and theoretical [4, 5] points of view. In the theoretical works, the resonance chiral theory and G-parity were applied.

In the present paper, the pointed processes are considered in the framework of the extended Nambu–Jona-Lasinio model (NJL) [6, 7, 8, 9, 10, 11]. This model allows one to describe the scalar, pseudoscalar, vector and axial vector meson nonets in both the ground and the first radially excited states in the framework of the chiral symmetry. It has turned out to be very useful for calculation of numerous modes of the $\tau$-lepton decays because in the intermediate states of these decays the ground and the first radially excited mesons give the main contribution [11]. Indeed, in the framework of this model the decays $\tau \rightarrow \pi\pi\nu_\tau$ [12], $\tau \rightarrow \pi[\eta, \eta'(958)]\nu_\tau$ [13], $\tau \rightarrow \pi K\nu_\tau$ [14, 15], $\tau \rightarrow [\eta, \eta'(958)]K\nu_\tau$ [16], $\tau \rightarrow KK\nu_\tau$ [17] were described without any additional arbitrary parameters. These processes have two channels. One of them is the contact channel where the $W$-boson produces the final mesons directly. The other channel includes the intermediate vector mesons in the ground and the first radially excited states. The decay $\tau \rightarrow f_1(1285)\pi\nu_\tau$ with only axial vector meson in the intermediate state was calculated [18]. The series of decays with vector and pseudoscalar mesons in the final states were also described, for example, the decay $\tau \rightarrow \pi\omega(782)\nu_\tau$ [19] including only contact and vector intermediate channel and the decays $\tau \rightarrow \pi[\rho(770), \rho(1450)]\nu_\tau$ [20] including contact, axial vector and pseudoscalar channel. However, similar processes with strange particles are of particular interest.

Typical examples of these decays are the processes $\tau \rightarrow [\omega(782), \phi(1020)]K^-\nu_\tau$. In the present paper, their decay widths are calculated. They include the contact channel and three intermediate channels: axial vector, vector and pseudoscalar ones. The axial vector channel is dominant. In this channel, we take into account the mesons $K_1(1270)$, $K_1(1650)$ and $K_1(1400)$. The last one is considered as a tetraquark state. Since this meson is not included into the nonet and consequently is not described with the NJL model directly, the description of appropriate
vertices may be performed phenomenologicaly by using arbitrary parameters which can be defined
from the independent processes $\tau \rightarrow K_1(1400)\nu_\tau$ and $K_1(1400) \rightarrow \omega(782)K$.

The paper has the following structure. Section 2 contains the quark-meson interaction La-
grangian of the extended NJL model. Section 3 is devoted to the amplitudes of the considered
processes obtained in the framework of the extended NJL model and the numerical results for the
decay widths. In section 4, the corrections from the intermediate tetraquark meson for these pro-
cesses are given. The conclusion contains the discussion of the obtained results and the comparison
with other theoretical works.

2 The interaction Lagrangian of the extended NJL model

When building the extended NJL model, the two main conditions were complied. Firstly,
the insertion of the radially excited meson states should not change the quark condensate. This is
achieved by introduction of the polynomial form factor $F(k_\perp^2) = cf(k_\perp^2)$. The constant $c$
influences only the masses of the radially excited mesons. The rest part

$$f(k_\perp^2) = (1 + dk_\perp^2) \theta(\Lambda^2 - k_\perp^2)$$

influences the mesons interactions. The slope parameter $d$ is definitely fixed from the requirement
of invariability of the quark condensate after including the radially excited meson states; $k_\perp$
is the
relative momentum of quarks in a meson. It is transverse to the meson momenta; $\Lambda$ is the cutoff
parameter.

The second condition is the requirement of the diagonal of the free Lagrangian included the
ground and the first radially excited mesons. This condition is fulfilled with the mixing angles.
Thereby, all model parameters are fixed when building the free Lagrangian. Therefore, it is not
necessary to include additional arbitrary parameters for description of the mesons interactions in
the extended NJL model.

As a result, the fragment of the quark-meson interaction Lagrangian for the mesons included
in our processes takes the form [7, 11]:

$$\Delta L_{\text{int}} = \bar{q} \left[ \frac{1}{2} \gamma^\mu \gamma^5 \sum_{j=\pm} \lambda_j^K \left( A_{K\mu} K_{1\mu}^j + B_{K\mu} K_{1\mu}^j \right) + \frac{1}{2} \gamma^\mu \sum_{j=\pm} \lambda_j^K \left( A_{K\mu} K_{1\mu}^j + B_{K\mu} K_{1\mu}^j \right) \right] q,$$

where $q$ and $\bar{q}$ are the u, d and s quark fields with the constituent masses $m_u = m_d = 280$ MeV,
$m_s = 460$ MeV, the excited meson states are marked with prime,

$$A_M = \frac{1}{\sin(2\theta_M)} \left[ g_M \sin(\theta_M + \theta_M^0) + g_M f_M(k_\perp^2) \sin(\theta_M - \theta_M^0) \right],$$

$$B_M = \frac{-1}{\sin(2\theta_M)} \left[ g_M \cos(\theta_M + \theta_M^0) + g_M f_M(k_\perp^2) \cos(\theta_M - \theta_M^0) \right].$$

Here the index $M$ denotes an appropriate meson.

The form factor $f(k_\perp^2)$ describes the first radially excited mesons and the slope parameter
d depends only on the quark composition of the meson:

$$d_{uu} = -1.784 \times 10^{-6}\text{MeV}^{-2}, \quad d_{ss} = -1.725 \times 10^{-6}\text{MeV}^{-2};$$

$$d_{us} = -1.756 \times 10^{-6}\text{MeV}^{-2}.$$
The transverse relative momentum of the inner quark-antiquark system may be represented as
\[ k_\perp = k - \frac{(kp)p}{p^2}, \]  
where \( p \) is the meson momentum. In the rest system of a meson (see details in [18])
\[ k_\perp = (0, k). \]
Therefore, this momentum may be used as a three-dimensional one.

The parameter \( \theta_M \) is the mixing angle for the ground and the first radially excited mesons [7, 11]:
\[ \theta_{K_1} = 85.97^\circ, \quad \theta_{K^*} = 84.74^\circ, \]
\[ \theta_K = 58.11^\circ, \quad \theta_\omega = 81.8^\circ, \]
\[ \theta_\phi = 68.4^\circ. \]  
Auxiliary values \( \theta_{0M}^0 \) are included for convenience:
\[ \sin (\theta_{0M}^0) = \sqrt{\frac{1 + R_M}{2}}, \]
\[ R_{K_1} = R_{K^*} = \frac{I_{11}}{I_{11}^n}, \quad R_K = \frac{I_{11}^n}{\sqrt{Z_K I_{11}^n}}, \]
\[ R_\omega = \frac{I_{20}}{\sqrt{I_{20}^n}}, \quad R_\phi = \frac{I_{20}^n}{\sqrt{I_{20}^n}}, \]  
where
\[ Z_K = \left(1 - \frac{3}{2} \frac{(m_u + m_s)^2}{M_{K_1}^2}\right)^{-1} \]  
is an additional constant of renormalization appearing in \( K - K_1 \) transitions, \( M_{K_1} = 1272 \) MeV is the mass of the axial vector strange meson [21]. The integrals appearing in the quark loops as a result of renormalization of the Lagrangian:
\[ I_{il}^n = -i \frac{N_c}{(2\pi)^4} \int \frac{f^n(k^2)}{(m_u^2 - k^2)^{i_1}(m_s^2 - k^2)^{i_2}} \theta(\Lambda^2 - k^2) d^4k, \]
\[ \Lambda = 1.03 \text{ GeV} \]  
is the cutoff parameter. Then
\[ \theta_{0K_1}^0 = \theta_{0K^*}^0 = 59.56^\circ, \quad \theta_{0K}^0 = 55.52^\circ, \]
\[ \theta_{0\omega}^0 = 61.5^\circ, \quad \theta_{0\phi}^0 = 57.13^\circ. \]  
The matrices \( \lambda \) are the linear combinations of the Gell-Mann matrices:
\[ \lambda^K_+ = \frac{\lambda_4 + i\lambda_5}{\sqrt{2}}, \quad \lambda^K_- = \frac{\lambda_4 - i\lambda_5}{\sqrt{2}}, \]
\[ \lambda^\omega = \frac{\sqrt{2}\lambda_0 + \lambda_8}{\sqrt{3}}, \quad \lambda^\phi = \frac{-\lambda_0 + \sqrt{2}\lambda_8}{\sqrt{3}}, \]  
\[ (11) \]
where $\lambda_i, i = 1, \ldots, 9$ are the Gell-Mann matrices,
\[
\lambda_0 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\] (13)

The coupling constants are:
\[
g_{K_1} = g_{K^*} = \left(\frac{2}{3} I_{11}\right)^{-1/2}, \quad g_{\omega} = \left(\frac{2}{3} I_{20}\right)^{-1/2},
g_{K} = \left(\frac{4}{Z_K} I_{11}\right)^{-1/2}, \quad g_{\phi} = \left(\frac{2}{3} I_{02}\right)^{-1/2},
g'_{K_1} = g'_{K^*} = \left(\frac{2}{3} I_{11}^2\right)^{-1/2}, \quad g'_{K} = (4I_{11}^2)^{-1/2},
g'_{\omega} = \left(\frac{2}{3} I_{20}^2\right)^{-1/2}, \quad g'_{\phi} = \left(\frac{2}{3} I_{02}^2\right)^{-1/2}.
\] (14)

3 **The processes $\tau \rightarrow [\omega(782), \phi(1020)]K^{-}\nu_\tau$ in the extended NJL model**

The diagrams for the processes $\tau \rightarrow [\omega(782), \phi(1020)]K^{-}\nu_\tau$ are shown in Fig. 1, 2.

![Diagram](image)

**Figure 1:** The contact diagram.

The amplitude of the process $\tau \rightarrow \omega(782)K\nu_\tau$ in the extended NJL model takes the form
\[
\mathcal{M} = -iG_F V_{us} L_\mu \left\{ \mathcal{M}_c + \mathcal{M}_{AV} + \mathcal{M}_V + \mathcal{M}_{PS} + \mathcal{M}_{AV'} + \mathcal{M}_{V'} + \mathcal{M}_{PS'} \right\}^{\mu\nu} e^*_\nu(p_\omega),
\] (15)

where $G_F$ is the Fermi constant, $V_{us}$ is the Cabbibo-Kobayashi-Maskawa matrix element, $L_\mu$ is the lepton current, $e^*_\nu(p_\omega)$ is the polarization vector of the meson $\omega(782)$. The terms in the braces describe the contributions from the contact diagram and from the diagrams with the intermediate axial vector, vector and pseudoscalar mesons in the ground and the first radially excited states:
\[
\mathcal{M}_c = (m_s + m_u) I_{11}^{K\omega} g^{\mu\nu} + i2m_u \left[I_{21}^{K\omega} + (m_s - m_u)m_u I_{31}^{K\omega}\right] e^{\mu\nu\lambda\delta} p_{K\chi} p_{\omega\delta},
\]
Intermediate meson in the extended NJL model: Two terms in the contact contribution describe the axial vector and the vector parts of the contact.

The values $R$ in the amplitude:

$$M_{AV} = \frac{C_{K_1}}{g_{K_1}}(m_s + m_u)I_{K_0K_1}g^{\mu\nu}\left[\frac{q^2 - \frac{3}{2}(m_s + m_u)^2}{M_{K_1}^2 - q^2 - i\sqrt{q^2}G_{K_1}}\right],$$

$$M_{V} = i2m_u C_{K^*} \left[\frac{I_{K_0K^*} + (m_s - m_u)I_{K_0K^*}}{M_{K_0}^2 - q^2 - i\sqrt{q^2}G_{K_0}}\right]\frac{q^2 - \frac{3}{2}(m_s - m_u)^2}{M_{K_0}^2 - q^2 - i\sqrt{q^2}G_{K_0}}e^{\mu\nu}l\psi p_{K_0}\psi,$$

$$M_{PS} = 2(m_s + m_u)Z_K C_K I_{K_0K_1}\left[1 - 6\frac{C_{K_1}g_{K_1}I_{K_0K_1}}{C_J g_{K_1}Z_K}\frac{(m_s + m_u)^2}{M_{K_1}^2}\right]\times\left[1 - I_{K_0K_1}I_{K_0K_1}^*\frac{(m_s + m_u)^2}{M_{K_1}^2}\frac{q^2}{M_{K_1}^2 - q^2 - i\sqrt{q^2}G_{K_1}}\right]\frac{q^2}{M_{K_1}^2 - q^2 - i\sqrt{q^2}G_{K_1}},$$

$$M_{AV'} = e^{i\pi} \frac{C'_{K_1}}{g_{K_1}}(m_s + m_u)I_{K_0K_1}g^{\mu\nu}\left[\frac{q^2 - \frac{3}{2}(m_s + m_u)^2}{M_{K_1}^2 - q^2 - i\sqrt{q^2}G_{K_1}}\right]\frac{1 - \frac{3}{2}(m_s + m_u)^2}{M_{K_1}^2 - q^2 - i\sqrt{q^2}G_{K_1}},$$

$$M_{V'} = e^{i\pi} i2m_u \frac{C_{K^*}}{g_{K^*}}\left[\frac{I_{K_0K^*} + (m_s - m_u)I_{K_0K^*}}{M_{K_0}^2 - q^2 - i\sqrt{q^2}G_{K_0}}\right]\frac{q^2 - \frac{3}{2}(m_s - m_u)^2}{M_{K_0}^2 - q^2 - i\sqrt{q^2}G_{K_0}} e^{\mu\nu}l\psi p_{K_0}\psi,$$

$$M_{PS'} = e^{i\pi} 2(m_s + m_u)Z_K C'_K I_{K_0K_1}\frac{q^2}{M_{K_1}^2 - q^2 - i\sqrt{q^2}G_{K_1}}.$$

Two terms in the contact contribution describe the axial vector and the vector parts of the contact diagram.

The constants $C_M$ and $C'_M$ appear in the quark loops of the $W$-boson transition into the intermediate meson in the extended NJL model:

$$C_M = \frac{1}{\sin(2\theta_{M}^{0})} \left[\sin(\theta_{M} + \theta_{M}^{0}) + R_{M} \sin(\theta_{M} - \theta_{M}^{0})\right],$$

$$C'_M = \frac{-1}{\sin(2\theta_{M}^{0})} \left[\cos(\theta_{M} + \theta_{M}^{0}) + R_{M} \cos(\theta_{M} - \theta_{M}^{0})\right].$$

The values $R$ are defined in (8).

The integrals with the vertices from the Lagrangian in the numerator which were also used in the amplitude:

$$I_{n_1n_2...M...}^{M...} = -i \frac{N_c}{(2\pi)^4} \int \frac{A_{M...}B_{M...}}{(m_u^2-k^2)^{n_1}(m_s^2-k^2)^{n_2}} \theta(\Lambda^2-k^2)\,d^4k,$$

Figure 2: The diagram with the intermediate mesons.
where $A_M, B_M$ are defined in [3].

The branching fractions of the separate contributions of the process $\tau \to \omega(782)K\nu_\tau$ are displayed in Tab. [1].

Table 1: The branching fractions of the process $\tau \to \omega(782)K\nu_\tau$.

|         | Br ($\times 10^{-4}$) |
|---------|------------------------|
|         | Ground | Excited | Sum  |
| $W_{AV}$ | 0.32   | -       | -    |
| $AV$    | 3.09(1.89) | 3.86 $\times 10^{-3}$ | 1.93 |
| $W_V$   | 0.36   | -       | -    |
| $V$     | 1.05(0.19) | 0.19     | 0.54 |
| $PS$    | 0.042  | 1.83 $\times 10^{-3}$ | 0.047 |
| tot     | 2.09   | 0.19    | 2.49 |
| PDG     | 4.1 ± 0.9 |           |      |

Here in the columns the values of the contributions of the ground mesons, their first radially excited states and the summary contributions for the separate channels taking into account the interference are displayed. In the lines $W_{AV}$ and $W_V$, the contributions of the axial vector and vector parts of the contact diagram are shown. In the lines $AV$, $V$, and $PS$, the contributions of the axial vector, vector and pseudoscalar channels are given. In the parentheses, there are summary contributions with the appropriate contact diagram. The total branching fraction calculated in the NJL model is 2.49 $\times 10^{-4}$. In the last line, there is the experimental value [21]. One can see that the deviation of the theoretical result from the experimental data does not exceed 1.8$\sigma$.

The amplitude of the process $\tau \to \phi(1020)K\nu_\tau$ in the extended NJL model

$$
\mathcal{M} = i\sqrt{2}G_FV_{us}L_\mu \{ M_c + M_{AV} + M_V + M_{PS} + M_{AV'} + M_{V'} + M_{PS'} \} \epsilon_{\mu}^{\nu} (p_\phi),
$$

where

$$
M_c = (m_s + m_u)I_{11}^{K\phi} g_{\mu\nu} - i2m_s \left[ I_{12}^{K\phi} - (m_s - m_u)m_s I_{13}^{K\phi} \right] e_{\mu\nu\lambda\delta} p_{K\lambda} p_{\delta},
$$

$$
M_{AV} = \frac{C_{K_1}}{g_{K_1}} (m_s + m_u)I_{11}^{K\phi K_1} \frac{g_{\mu\nu} \left[ q^2 - \frac{3}{2}(m_s + m_u)^2 \right] - q^\mu q^\nu}{M_{K_1}^2 - q^2 - i\sqrt{q^2} \Gamma_{K_1}},
$$

$$
M_V = -i2m_s \frac{Z_{K_1}}{g_{K_1}} \left[ I_{12}^{K\phi \nu} - (m_s - m_u)m_s I_{13}^{K\phi \nu} \right] \frac{\Gamma_{K_1}^\nu}{M_{K_1}^2 - q^2 - i\sqrt{q^2} \Gamma_{K_1}},
$$

$$
M_{PS} = 2(m_s + m_u) \frac{Z_K}{g_K} \frac{C_{K_1} I_{11}^{K\phi K}}{M_{K_1}^2} \left[ 1 - \frac{I_{11}^{K\phi}}{I_{11}^{K\phi K}} \frac{(m_s + m_u)^2}{M_{K_1}^2} \right] \frac{q^\mu q^\nu}{M_{K_1}^2 - q^2 - i\sqrt{q^2} \Gamma_{K_1}},
$$

$$
M_{AV'} = \frac{C_{K_1}}{g_{K_1}} (m_s + m_u)I_{11}^{K\phi K_1} \frac{g_{\mu\nu} \left[ q^2 - \frac{3}{2}(m_s + m_u)^2 \right] - q^\mu q^\nu \left[ 1 - \frac{3}{2} \frac{(m_s + m_u)^2}{M_{K_1}^2} \right]}{M_{K_1}^2 - q^2 - i\sqrt{q^2} \Gamma_{K_1}},
$$

$$
M_{V'} = -i2m_s \frac{C_{K_1}}{g_{K_1}} \left[ I_{12}^{K\phi \nu'} - (m_s - m_u)m_s I_{13}^{K\phi \nu'} \right] \frac{\Gamma_{K_1}^\nu}{M_{K_1}^2 - q^2 - i\sqrt{q^2} \Gamma_{K_1}},
$$

$$
M_{PS'} = 2(m_s + m_u) \frac{Z_K}{g_K} \frac{C_{K'} I_{11}^{K\phi K'}}{M_{K'}^2} \frac{q^\mu q^\nu}{M_{K'}^2 - q^2 - i\sqrt{q^2} \Gamma_{K'}}.
$$

(20)
The branching fractions of the separate contributions of the process \( \tau \to \phi(1020)K\nu_\tau \) are displayed in Tab. 2.

|          | Ground  | Excited | Sum   |
|----------|---------|---------|-------|
| \( W_{AV} \) | 1.36    | -       | -     |
| \( AV \)  | 3.49(0.58) | 0.54    | 1.31  |
| \( W_V \)  | 0.45    | -       | -     |
| \( V \)   | 0.9(0.08) | 3.72 x 10^{-3} | 0.12  |
| \( PS \)  | 0.06    | 0.22 x 10^{-3} | 0.07  |
| tot      | 0.68    | 0.54    | 1.44  |
| PDG      | 4.4 ± 1.6 |         |       |

One can see that the deviation of the theoretical result from the experimental data does not exceed 1.9\( \sigma \).

In all these calculations, the interferences between different contributions were taken into account. The interference between the vector contribution and the other ones are equal to zero due to the antisymmetric tensor in the vector channel.

The obtained results do not exceed 2\( \sigma \) deviation from the experimental data. The meson \( K_1(1400) \) considered as a tetraquark state by us, can also be included in this process. It is involved into the decays \( \tau \to K_1(1400)\nu_\tau \) and \( K_1(1400) \to \omega(782)K \). Therefore, one can expect that the vertex \( K_1(1400)K\phi(1020) \) exists. Accordingly, the influence of this meson on the processes \( \tau \to [\omega(782), \phi(1020)]K^-\nu_\tau \) should be taken into account.

4 The estimation of the contributions from the tetraquark meson

In the work [22], the nonstrange axial vector tetraquark meson was considered. For the vertex of interaction of the strange axial vector tetraquark meson with quarks a similar structure but with nonzero strangeness is used:

\[
K_1^{\text{tqra}} \left( c_1 \bar{u} \gamma^\mu u \bar{u} \gamma^5 s + c_2 \bar{u} \gamma^\mu d \bar{d} \gamma^5 s + c_3 \bar{u} \gamma^\mu s \bar{s} \gamma^5 s + c_4 \bar{u} \gamma^\mu s \bar{u} \gamma^5 u + c_5 \bar{d} \gamma^\mu s \bar{u} \gamma^5 d + c_6 \bar{s} \gamma^\mu s \bar{u} \gamma^5 s \right),
\]

where \( c_i, i = 1, \ldots, 6 \) are constants.

The use of this structure leads to the diagram for the process \( \tau \to K_1(1400)\nu_\tau \) displayed in Fig. 3.

As a result, in the loop of \( W \)-boson transition into the tetraquark meson, the same structure as in the case of the quark-antiquark meson appears. The sum of possible tadpoles gives us the constant \( \text{Const}_1 \). The amplitude takes the form:

\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{us} \text{Const}_1 L_\mu \left\{ g^\mu\nu \left[ q^2 - \frac{3}{2} (m_s + m_u)^2 \right] - q^\mu q^\nu \right\} e^\nu(p_{K_1})
\]

Using the experimental value for this process,

\[
\text{Br}(\tau \to K_1(1400)\nu_\tau)_{\text{EXP}} = (1.7 \pm 2.6) \times 10^{-3},
\]

one can estimate this constant approximately:

\[
\text{Const}_1 = 0.2.
\]
We assume that the structure of the amplitude for the process $K_{1}(1400) \to \omega(782)K$ is similar to the structure of the amplitude in the case of the quark-antiquark state.

Then this amplitude takes the form:

$$M = \text{Const}\, \omega^2 e_{\mu}^{*} (p_{K_{1}}) g^{\mu\nu} e_{\nu}^{*} (p_{\omega}).$$  \hspace{1cm} (24)

Using the experimental value for this process,

$$Br(K_{1}(1400) \to \omega(782)K)_{\text{EXP}} = (1 \pm 1) \times 10^{-2},$$  \hspace{1cm} (25)

one can estimate the constant $\text{Const}\omega^2$ approximately:

$$\text{Const}\omega^2 = 539\text{MeV}.$$  \hspace{1cm} (26)

Then the amplitude for the tetraquark channel of the process $\tau \to \omega(782)K\nu_{\tau}$ takes the form:

$$M = \frac{G_{F}}{\sqrt{2}} V_{us} \text{Const}_{1} \text{Const}_{2} \omega L_{\mu} \frac{g^{\mu\nu} \left[ q^2 - \frac{3}{2} (m_{s} + m_{u})^2 \right] - q^{\mu} q^{\nu} \left[ 1 - \frac{3}{2} (m_{s} + m_{u})^2 \right]}{M_{K_{\text{tetra}}}^2 - q^2 - i\sqrt{q^2 \Gamma_{K_{\text{tetra}}}} e_{\nu}^{*} (p_{\omega})}.$$  \hspace{1cm}

The contribution from the tetraquark channel to the branching fraction of this process is

$$Br(\tau \to \omega(782)K\nu_{\tau})_{\text{tetra}} = 0.11 \times 10^{-4}.$$  \hspace{1cm} (27)

The total branching fraction of this process with the tetraquark channel is

$$Br(\tau \to \omega(782)K\nu_{\tau})_{\text{tot}} = 3.2 \times 10^{-4}.$$  \hspace{1cm} (28)

For the process $\tau \to \phi(1020)K\nu_{\tau}$, the constant $\text{Const}_{1}$ is the same as in the previous case. We estimate the constant $\text{Const}_{2}^\phi$ by multiplication of $\text{Const}_{2}^\omega$ by the factor $\sqrt{\frac{2g_{\omega}}{2g_{\phi}}}$:

$$\text{Const}_{2}^\phi = 693\text{MeV}.$$  \hspace{1cm} (29)

The contribution from the tetraquark channel to the branching fraction of this process is

$$Br(\tau \to \phi(1020)K\nu_{\tau})_{\text{tetra}} = 0.21 \times 10^{-5}.$$  \hspace{1cm} (30)

The total branching fraction of this process with the tetraquark channel

$$Br(\tau \to \phi(1020)K\nu_{\tau})_{\text{tot}} = 2.3 \times 10^{-5}.$$  \hspace{1cm} (31)
5 Conclusion

Our calculations allow us to highlight two main results. Firstly, we have shown that the axial vector channel is the dominant one. Secondly, the important role of the intermediate tetraquark meson $K_1(1400)$ was emphasized. Indeed, the final result of the branching fraction of the decay $\tau \to \omega(782)K\nu_\tau$ is in satisfactory agreement with the experimental data. On the other hand, the value of the branching fraction of the decay $\tau \to \phi(1020)K\nu_\tau$ is beyond the experimental error; however, the deviation does not exceed $1.3\sigma$. It is necessary to emphasize that the experimental data for this process have big errors.

It is interesting to compare our results with those of other theoretical papers. In the work [4], for the decay $\tau \to \omega(782)K\nu_\tau$ the following results were obtained: $3 \times 10^{-4}$, $3.5 \times 10^{-4}$ and $4 \times 10^{-4}$ and for the process $\tau \to \phi(1020)K\nu_\tau$ the results $1.6 \times 10^{-5}$, $1.7 \times 10^{-5}$, $1.8 \times 10^{-5}$ were obtained. In the above mentioned work, the intermediate meson $K_1(1400)$ was taken into account. However, it was considered as a quark-antiquark state mixing with the state $K_1(1270)$.

In the recent paper [5], the following results were obtained: $3.24 \times 10^{-4}$ and $3.1 \times 10^{-4}$ for the process $\tau \to \omega(782)K\nu_\tau$ and $6.82 \times 10^{-5}$ and $6.54 \times 10^{-5}$ for the process $\tau \to \phi(1020)K\nu_\tau$.

We hope to obtain more precise experimental results concerning the processes considered in the present paper.

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