Mixed platoon control of automated and human-driven vehicles at a signalized intersection: dynamical analysis and optimal control

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Abstract

The emergence of Connected and Automated Vehicles (CAVs) promises better traffic mobility for future transportation systems. Existing research mostly focused on fully-autonomous scenarios, while the potential of CAV control at a mixed traffic intersection where human-driven vehicles (HDVs) also exist has been less explored. This paper proposes a notion of ”1 + n” mixed platoon, consisting of one leading CAV and n following HDVs, and formulates a platoon-based optimal control framework for CAV control at a signalized intersection. Based on the linearized dynamics model of the ”1 + n” mixed platoon, fundamental properties including stability and controllability are under rigorous theoretical analysis. Then, a constrained optimal control framework is established, aiming at improving the global traffic efficiency and fuel consumption at the intersection via direct control of the CAV. A hierarchical event-triggered algorithm is also designed for practical implementation of the optimal control method between adjacent mixed platoons when approaching the intersection. Extensive numerical simulations at multiple traffic volumes and market penetration rates validate the greater benefits of the mixed platoon based method, compared with traditional trajectory optimization methods for one single CAV.

Keywords: Connected and Automated Vehicle, Signalized intersection, Mixed traffic system, Optimal control

1. Introduction

As planned points of conflict in urban traffic networks, intersections play a critical role in traffic mobility optimization. Existing research has shown that the frequent stop-and-go and idling behavior of individual vehicles when approaching the intersection is the main cause of traffic congestion and casualties \cite{1,2}. Accordingly, trajectory optimization for individual vehicles at the intersection has attracted significant attention. In particular, the emergence of Connected and Automated Vehicles (CAVs) has provided new opportunities for improving the traffic performance at intersections. Compared to traditional human-driven vehicles (HDVs), CAVs can acquire accurate information of surrounding traffic participants and traffic signal phase and timing (SPAT) through vehicle-to-vehicle and vehicle-to-infrastructure communication \cite{3}, and their velocity trajectories in approaching intersections can be directly optimized in the pursuit of higher traffic efficiency and lower fuel consumption. Multiple methods have been applied to CAV control at signalized intersections, including model predictive control \cite{4,5}, fuzzy logic \cite{6,7} and optimal control \cite{8,9}. Moreover, the potential of cooperative control of traffic signals and CAVs has also been recently discussed \cite{10}.

The aforementioned research mainly considered a fully-autonomous scenario—the market penetration rate (MPR) of CAVs is 100%. In practice, however, it might take decades for all the HDVs in current transportation systems to be transformed into CAVs. Instead, a more practical scenario in the near future is a mixed traffic system where CAVs and HDVs coexist \cite{11}. Existing research on mixed traffic intersections mostly focused on estimation of traffic states and optimization of traffic signals \cite{12,13}. For example, Priemer et al. utilized dynamic programming to estimate the queuing length in mixed traffic environment and then optimized the traffic phase time based on the
Despite these existing work, the topic of CAV control, i.e., trajectory optimization of CAVs, in mixed traffic intersections has not been fully discussed. To tackle this problem, several works regarded HDVs as disturbances in the control of CAVs \([20, 22]\), or focused on the task of collision avoidance based on the prediction of HDVs’ behaviors \([9, 23]\). It is worth noting that most of these methods were limited to improving the performance of CAVs themselves in their optimization frameworks, instead of optimizing the global traffic flow consisting of both HDVs and CAVs at the intersection. Two notable exceptions are in \([24, 25]\), which attempted to improve the performance at signalized intersection from the perspective of the so-called \textit{mixed platoon}. They enumerated several possible formations consisting of HDVs and CAVs, and investigated its effectiveness through small-scale simulation experiments.

However, a general and explicit definition of mixed platoon have not been clarified, and fundamental properties of mixed platoon at intersections have been less explored. Moreover, a specific optimal control framework for mixed platoon at intersections with global consideration of improving the entire traffic performance is still lacking.

In fact, considering the interaction between adjacent vehicles on the same lane, it is easy to understand that velocity trajectories of CAVs could have certain influence on those of surrounding vehicles, especially the vehicles following behind them. Accordingly, the driving strategies of CAVs could have a direct impact on the performance of entire mixed traffic intersection; such impact might even be negative when inappropriate CAV strategies are adopted \([22]\). By contrast, when taking the performance of entire mixed traffic into explicit consideration, the optimization of CAVs’ trajectories could bring further benefits to traffic mobility. Such idea of improving the global traffic performance via controlling CAVs has been recently proposed as \textit{Lagrangian Control} of traffic flow \([26]\), which has been discussed in various traffic scenarios, including closed ring road \([11]\), open straight road \([27]\), traffic bottleneck \([28]\), and non-signalized intersection \([29]\). Regarding signalized intersections, the potential of this notion has not been well understood.

In this paper, we focus on the scenario of a signalized intersection where HDVs and CAVs coexist, and aim at improving the performance of the entire mixed traffic intersection through direct control of CAVs. To address this problem, we propose a novel framework which separates the traffic flow into “1 + n” microstructures consisting of one leading CAV and \(n\) following HDVs. Such microstructure is particularly common in the near future when the MPR is relatively low, which is named as “1 + n” mixed platoon. We discuss the possibility of letting the first CAV lead the motion of following \(n\) HDVs in approaching intersections, and show how to enable the CAV to benefit the global traffic mobility in the proposed structure of mixed platoon. First, we present the dynamics model of “1 + n” mixed platoon systems based on linearized car-following models, and investigate its fundamental properties, including open-loop stability and controllability. Then, we establish the optimal control framework for the mixed platoon and design a hierarchical algorithm to improve the global traffic mobility performance at a signalized intersection. Specifically, our contributions are as follows:

1. The notion of the “1 + n” mixed platoon is proposed for trajectory optimization of CAVs at signalized intersections in mixed traffic environment. Rather than enumerating the mixed platoon formations \([24, 25]\), we provide an explicit definition of mixed platoon. Based on the linearized dynamics model, we perform rigorous theoretical analysis of its fundamental properties, including open-loop stability and controllability. Our theoretical results reveal that the “1 + n” mixed platoon is always stable and controllable under a very mild condition, regardless of the platoon size \(n\).

2. An optimal control framework is established for the “1 + n” mixed platoon at a constant traffic SPAT intersection scenario. Instead of being limited to optimizing the performance of the CAVs only \([4, 9, 20, 23]\), our optimal control formulation aims at improving the global performance of the entire mixed traffic intersection. Precisely, the velocity deviations and fuel consumption of all the vehicles in the mixed platoon are under explicit consideration. Moreover, unlike \([24, 25]\), we optimize the terminal velocity setting to maximize the traffic throughput.

3. Finally, a hierarchical algorithm is proposed to accomplish the optimal control framework of “1 + n” mixed platoons, which can be applied in any MPRs of mixed traffic environment. An event-triggered section is designed.
Figure 1: Illustration for the mixed traffic intersection. Red vehicles represent CAVs, which can transmit and receive vehicle information and the vehicle is fully autonomous driving. Black vehicles represent HDVs, which can only transmit ego vehicle information to other vehicles and are controlled by car following model.

to avoid potential collisions of adjacent mixed platoons. Large-scale traffic simulations are conducted at multiple traffic volumes and MPRs, and it is observed that the proposed mixed platoon based control method surpasses the traditional intersection control method for single CAV [4, 5] in both traffic efficiency and fuel consumption.

The remaining of this paper is organized as follows. Section 2 introduces the problem of the system modeling for the proposed “1 + n” mixed platoon. Section 3 presents the system dynamics analysis, optimal control framework and algorithm design. The simulation results are shown in Section 4 and Section 5 concludes this paper.

2. Problem Statement

In this section, we firstly introduce the scenario setup, and then present the dynamical modeling of individual vehicles and the mixed platoon systems.

2.1. Scenario Setup

In this paper, we consider a typical signalized intersection scenario in the mixed traffic environment, where HDVs and CAVs coexist; see Fig. 1 for illustration. A traffic light is deployed in the center to guide individual vehicles to drive through the intersection. Note that CAVs follow the instructions from a central cloud coordinator, which collects information from all the involved vehicles around the intersection and calculates the optimal velocity trajectories for each CAV. The design of the control strategies for the central cloud coordinator is presented in Section 3.

Motivated by previous works on signalized intersections [15, 30, 31], we separate the intersection into three zones, as illustrated in Fig. 1 The red square area in the center is named as the Merging Zone (MZ), which is the area of
potential lateral collision of the involved vehicles. The ring area between two dashed lines is called the Observation Zone (OZ), where the CAVs and HDVs are allowed to perform lane-changing behaviors. The area between OZ and MZ is the Control Zone (CZ), where the CAVs are under direct control from the central cloud coordinator. The specific range of each zone is discussed in Section 4.

Similar to existing research [24, 25], the following assumptions are needed to facilitate the control design for signalized intersection controls, as well as the system modeling and dynamics analysis.

1. All the vehicles are connected vehicles, which means that both the CAVs and the HDVs are able to transmit their velocity and position to the central cloud coordinator through wireless communication, e.g., V2I communication [32]. An ideal communication condition without communication delay or packet loss is under consideration.

2. All the CAVs are capable of fully autonomous driving, which follow the velocity trajectories assigned from the central cloud coordinator after they enter CZ. Regarding the HDVs, they are controlled by human drivers, for which we assume a general car-following model to describe its driving behavior (see Section 2.2 for details).

3. Lane changing is not permitted in CZ. As shown in [22, 33], unexpected lane changing behaviors might worsen the traffic efficiency, especially near the intersection. Therefore, lane changing is only allowed in OZ, while in CZ, we only need to focus on the longitudinal behavior of each vehicle.

2.2. Dynamical Modeling of Mixed Platoon Systems

In this paper, we propose the notion of “1 + n” mixed platoon as shown in Fig. 2. It consists of one leading CAV and n following HDVs. Specifically, each CAV is designed as the leading vehicle of the mixed platoon, which leads the motion of the following n HDVs with the aim of improving the performance of the entire mixed platoon when passing the intersection.

Consider the n following HDVs in the mixed platoon illustrated in Fig. 2. We denote the position and velocity of vehicle i at time t as \(x_i(t)\) and \(v_i(t)\), respectively. The headway distance of vehicle i from vehicle \(i - 1\) is defined as \(d_i(t) := x_{i-1}(t) - x_i(t)\). Then, \(\dot{x}_i(t)\) and \(\dot{v}_i(t) = v_{i-1}(t) - v_i(t)\) represents the acceleration of vehicle i, and the relative velocity of vehicle i from vehicle \(i - 1\), respectively.

A great many efforts have been made in previous works to describe the HDVs’ car-following dynamics, with several significant models developed, e.g., Gipps [17], OVM [18] and IDM [19]. As shown in the literature [11, 34], most of them can be expressed as the following general form (\(i = 1, \ldots, n\))

\[
\dot{v}_i(t) = F(d_i(t), \dot{d}_i(t), v_i(t)),
\]

(1)

which means that the acceleration of vehicle i is determined by its headway distance, relative velocity and its own velocity.

In this paper, we require that the mixed platoon passes the intersection at a pre-specified equilibrium velocity \(v^*\). In equilibrium traffic state, we have \(\dot{d}_i(t) = 0\) for \(i = 1, \ldots, n\), and thus each vehicle has a corresponding equilibrium headway distance \(d^*\), where it holds that

\[
F(d^*, 0, v^*) = 0.
\]

(2)
Then we employ the deviation of the current state \((d_i(t), v_i(t))\) of vehicle \(i\) from the equilibrium state \((d^*, v^*)\) as its state variable, given by
\[
\begin{align*}
\tilde{d}_i(t) &= d_i(t) - d^*, \\
\tilde{v}_i(t) &= v_i(t) - v^*.
\end{align*}
\]  
(3)

Applying the first-order Taylor expansion to \((1)\) leads to the linearized dynamics of HDVs around the equilibrium state \((d^*, v^*)\), given as follows \((i = 1, \ldots, n)\)
\[
\begin{align*}
\tilde{\dot{d}}_i(t) &= \tilde{v}_{i-1}(t) - v_i(t), \\
\tilde{\dot{v}}_i(t) &= \alpha_1 \tilde{d}_i(t) - \alpha_2 \tilde{v}_i(t) + \alpha_3 \tilde{v}_{i-1}(t),
\end{align*}
\]  
(4)

with \(\alpha_1 = \frac{\partial F}{\partial d}, \alpha_2 = \frac{\partial F}{\partial v}, \alpha_3 = \frac{\partial F}{\partial \alpha}\), evaluated at the equilibrium state \((d^*, v^*)\).

Regarding the leading CAV, indexed as vehicle 0, its acceleration signal is utilized as the control input \(u(t)\). Then, the longitudinal dynamics of the leading vehicle can be expressed as the following second-order form
\[
\begin{align*}
\tilde{\dot{x}}_0(t) &= \tilde{v}_0(t), \\
\tilde{\dot{v}}_0(t) &= u(t).
\end{align*}
\]  
(5)

It is worth noting the acceleration signal of the leading CAV is also the only external control input of the entire system of the mixed platoon illustrated in Fig. 2. Lumping the state of both the leading CAV and the following HDVs yields the state vector of the entire mixed platoon system, given as follows
\[
X(t) = \begin{bmatrix} x_0(t) & v_0(t) & \tilde{d}_1(t) & \tilde{v}_1(t) & \cdots & \tilde{d}_n(t) & \tilde{v}_n(t) \end{bmatrix}^T.
\]  
(6)

Based on \((4)\) and \((5)\), the state-space model of the mixed platoon system is obtained
\[
\dot{X}(t) = AX(t) + Bu(t),
\]  
(7)

where \((A \in \mathbb{R}^{(2n+2) \times (2n+2)}, B \in \mathbb{R}^{(2n+2) \times 1})\)

\[
A = \begin{bmatrix}
C_1 & 0 & \cdots & \cdots & 0 & 0 \\
A_2 & A_1 & 0 & \cdots & \cdots & 0 \\
0 & A_2 & A_1 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & A_2 & A_1 & 0 \\
0 & \cdots & \cdots & 0 & A_2 & A_1
\end{bmatrix}, \quad
B = \begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_2 \\
B_2
\end{bmatrix},
\]

with \(A_1 = \begin{bmatrix} 0 & -1 \\ \alpha_1 & -\alpha_2 \end{bmatrix}\), \(A_2 = \begin{bmatrix} 0 & 1 \\ 0 & \alpha_3 \end{bmatrix}\), \(B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\), \(B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\), \(C_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\).

**Remark 1.** Previous work on control of individuals vehicles at signalized intersections mostly focused on the dynamics \((5)\) of CAVs only; see, e.g., \([5, 8, 9]\). Considering the interaction between neighboring vehicles, in this paper we focus on the dynamics \((7)\) of the entire mixed platoon system consisting of both the CAV and its following HDVs, and seek to improve the overall performance via controlling the CAV only. A similar idea has been recently proposed by Stern et al. \([26]\), known as Lagrangian Control of traffic flow, where CAVs are utilized as mobile actuators to control the entire mixed traffic system. The effectiveness of this notion has been validated in various scenarios, including closed ring road \([11]\), open straight road \([27]\), traffic bottleneck \([28]\), and nonsignalized intersection \([29]\). To the best of our knowledge, the feasibility of this notion has not been explicitly discussed in the scenario of signalized intersections.
3. Methodology

Firstly we analyze the open-loop stability and controllability of the proposed “1 + n” mixed platoon systems. Based on the derived stable and controllable conditions, an optimal control framework is proposed to optimize the CAVs’ driving strategy in a single mixed platoon. Finally, an event-triggered algorithm is established to solve the collision problem of different mixed platoons.

3.1. Open-Loop Stability Analysis

Based on the model (6) of the mixed platoon system, we first consider its open-loop stability as shown in Definition:\[\text{Def}1\] when the leading CAV has no external control input, i.e., \(u(t) = 0\).

**Definition 1 (Asymptotic Stability \([35]\)).** An LTI system \(\dot{x} = Ax\) is said to be asymptotically stable if all the eigenvalues of \(A\) have negative real parts.

Existing research have revealed the stability criterion of the linearized car-following model (4) of one single HDV, shown in Lemma:\[\text{Lem}1\].

**Lemma 1 ([34]).** The linearized car-following model (4) is stable if and only if \(\alpha_1 > 0, \alpha_2 > 0\).

Regarding the “1 + n” mixed platoon system, where there exist \(n\) following HDVs, we have the following result.

**Theorem 1.** The “1 + n” mixed platoon system (7) is open-loop stable if and only if \(\alpha_1 > 0, \alpha_2 > 0\), which is irrelevant to the platoon size \(n\).

**Proof.** The method of mathematical induction is employed. Firstly, we consider the initial case of \(n = 1\), i.e., there is only one HDV following the leading CAV. To obtain the eigenvalue \(\lambda\) of \(A_{n=1}\), we need to solve \(|\lambda I - A_{n=1}| = 0\), leading to \(\lambda^4 + \alpha_2 \lambda^3 + \alpha_1 \lambda^2 = 0\), whose solutions are

\[
\lambda_1 = 0, \lambda_2 = 0, \quad \lambda_3 = -\frac{\alpha_2}{2} - \frac{\sqrt{\alpha_2^2 - 4\alpha_1}}{2}, \quad \lambda_4 = -\frac{\alpha_2}{2} + \frac{\sqrt{\alpha_2^2 - 4\alpha_1}}{2}.
\]

The two zero eigenvalues indicate that the system is not asymptotically stable, but can be made critically stable. According to Definition\[\text{Def}1\] we have the stability criterion as \(\alpha_1 > 0, \alpha_2 > 0\).

Then, we assume the system is stable when \(n = k\), i.e., all the eigenvalues of \(A_{n=k}\) have negative real parts, and focus on the case where \(n = k + 1\). The system matrix can be expressed as

\[
A_{n=k+1} = \begin{bmatrix}
A_{n=k} & 0 & 0 \\
1 & 0 & -1 \\
\alpha_3 & \alpha_1 & -\alpha_2
\end{bmatrix}.
\]

Given a block matrix

\[
P = \begin{bmatrix} A & B \\ C & D \end{bmatrix},
\]

if \(D\) is invertible, it holds that\[\text{Det}\]

\[
|P| = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A - BD^{-1}C| |D|.
\]

According to the initial state assumption, we have that

\[
\begin{bmatrix} \lambda & 1 \\ -\alpha_1 & \lambda + \alpha_2 \end{bmatrix}
\]
is invertible, and thus it is obtained that
\[
|\lambda I - A_{n-k+1}| = |\lambda I - A_{n-k}| = |\lambda I - A_{n-k}| (\lambda^2 + \alpha_2 \lambda + \alpha_1). \tag{10}
\]
According to the assumption, we only need to consider the solutions of \(\lambda^2 + \alpha_2 \lambda + \alpha_1 = 0\), which are stable if and only if \(\alpha_1 > 0, \alpha_2 > 0\). Hence, we have that \(A_{n-k+1}\) is stable if and only if \(\alpha_1 > 0, \alpha_2 > 0\), which completes the proof of Theorem 1 according to the method of mathematical induction.

Note that two zero eigenvalues shown in (8) always exist, and thus the mixed platoon system (7) is critically stable, but not asymptotically stable in the open-loop case. In fact, it can be easily seen that the zero eigenvalues are brought by the leading CAV. When \(\alpha_1 > 0, \alpha_2 > 0\), the subsystem consisting of the following HDVs is strictly asymptotically stable.

3.2. Controllability Analysis

One goal of the control of the mixed platoon is to pass the intersection with a pre-specified equilibrium velocity, and controllability is a fundamental property to depict the feasibility of this goal. Particularly, if controllability holds for the mixed platoon, then the mixed platoon system can be moved to any desired state under the control input of the CAV. The formal definition and one useful criterion of controllability is shown as follows.

Definition 2 (Controllability [35]). The dynamical system \(\dot{x}(t) = Ax(t) + Bu(t)\), or the pair \((A, B)\), is controllable if and only if, for any initial state \(x(0) = x_0\), any time \(t_f > 0\) and any final state \(x_f\), there exists an input \(u(t)\) such that \(x(t_f) = x_f\).

Lemma 2 (Popov-Belevitch-Hautus criterion [35]). In a continuous-time LTI system \((A, B)\) of size \(n\), the system is controllable if and only if for every eigenvalue \(\lambda\), \(\text{rank}(\lambda I - A, B) = n\). System \((A, B)\) is uncontrollable if and only if there exists \(\rho \neq 0\), such that \(\rho^T A = \lambda \rho^T\), \(\rho^T B = 0\).

Our result regarding the controllability of the \(1 + n\) mixed platoon is as follows.

Theorem 2. The “1 + n” mixed platoon system (7) is controllable when the following condition holds, which is irrelevant to the platoon size \(n\).
\[
\alpha_1 - \alpha_2 \alpha_3 + \alpha_3^2 \neq 0. \tag{11}
\]

PROOF. Assume that the mixed platoon system (7) is uncontrollable. According to Lemma 2, there exists a scalar \(\lambda\) and a non-zero vector \(\rho = [\rho_{01}, \rho_{02}, \rho_{11}, \rho_{12}, \ldots, \rho_{n1}, \rho_{n2}]^T\), where \(\rho_{ij} \in \mathbb{R}\), which satisfy
\[
\rho^T (A - \lambda I) = 0, \quad \rho^T B = 0. \tag{12}
\]
From \(\rho^T B = 0\), we have \(\rho_{02} = 0\). From \(\rho^T (A - \lambda I) = 0\), it is obtained that
\[
\begin{cases}
-\lambda \rho_{01} = 0, \\
\rho_{01} - \lambda \rho_{02} + \rho_{11} + \alpha_3 \rho_{12} = 0,
\end{cases} \tag{13}
\]
and
\[
-\rho_{n1} - (\alpha_2 + \lambda) \rho_{n2} = 0, \tag{14}
\]
and for \(i = 1, \ldots, n\),
\[
\begin{cases}
-\lambda \rho_{i1} + \alpha_1 \rho_{i2} = 0, \\
\rho_{i1} + (\alpha_2 - \lambda) \rho_{i2} + \rho_{(i+1)1} + \alpha_3 \rho_{(i+1)2} = 0.
\end{cases} \tag{15}
\]
According to (15), we have
\[
(\lambda^2 + \alpha_2 \lambda + \alpha_1) \rho_{i1} = (\alpha_3 \lambda + \alpha_1) \rho_{(i+1)1}, \quad i = 1, \ldots, n, \tag{16}
\]
and when \( i = 1 \), it holds that \( \lambda \rho_{11} = \alpha_1 \rho_{12} \). Substituting it into (13) yields

\[
(\alpha_3 \lambda + \alpha_1) \rho_{11} = 0.
\]  

(17)

From (14), we have

\[
(\lambda^2 + \alpha_2 \lambda + \alpha_1) \rho_{n1} = 0.
\]  

(18)

It can be easily examined that when \( \alpha_1 - \alpha_2 \alpha_3 + \alpha_2^2 \neq 0 \), the two equations \( \lambda^2 + \alpha_2 \lambda + \alpha_1 = 0 \) and \( \alpha_3 \lambda + \alpha_1 = 0 \) cannot be satisfied simultaneously. Therefore, we consider the following three cases: (1) \( \alpha_1 - \alpha_2 \alpha_3 + \alpha_2^2 = 0 \), \( \alpha_3 \lambda + \alpha_1 = 0 \), and \( \alpha_3 \lambda + \alpha_1 \neq 0 \); (2) \( \alpha_1 - \alpha_2 \alpha_3 + \alpha_2^2 \neq 0 \), \( \alpha_3 \lambda + \alpha_1 = 0 \), and \( \alpha_3 \lambda + \alpha_1 \neq 0 \). In each case, it can be obtained that \( \rho_{11} = \rho_{i2} = 0 \), \( i = 0, 1, \ldots, n \) by combing \( \rho_{02} = 0 \) and equations (16), (17), (18). This contradicts the requirement that \( \rho \neq 0 \), which indicates that the assumption does not hold. Therefore, the system \((A, B)\) is controllable when condition (11) holds.

**Remark 2.** Theorem 2 indicates that the “1 + n” mixed platoon is controllable with regard to the control input of the leading CAV when condition (11) holds. This result indicates that through controlling the leading CAV directly, one has complete control of the motion of the following HDVs without changing their natural driving behaviors. This property allows for the feasibility of designing the control input of the single CAV with the aim of improving the performance of the entire “1 + n” mixed platoon. Note that condition (11) is consistent with previous controllability analysis in [11, 37], which focused on mixed platoons in a closed ring-road traffic system. Interested readers are referred to [38] to make further investigations on the controllability property of the “1 + n” mixed platoon when the following n HDVs have heterogeneous dynamics in (1).

### 3.3. Optimal Control Framework

After dynamical analysis of the fundamental properties of the proposed “1 + n” mixed platoon, in the following section we proceed to establish an optimal control framework for the mixed platoon at the signalized intersection.

#### 3.3.1. Cost Function

In our optimal control framework of the “1 + n” mixed platoon, the main control objective is to let the CAV reach the stopping line of the intersection when the traffic signal turns green, and meanwhile the following HDVs are stabilized at an desired equilibrium velocity \( v^* \), as discussed in (2). Moreover, we also aim at minimizing the fuel consumption of the entire mixed platoon during its process of approaching the intersection. Accordingly, we define the following cost function in the Bolza form

\[
J = \varphi(X(t_f)) + \int_{t_0}^{t_f} L(X(t), u(t)) \, dt,
\]  

(19)

where \( t_0 \) and \( t_f \) denote the time when the CAV enters CZ, and the time when the CAV enters MZ, respectively.

As the terminal cost function in (19), \( \varphi(X(t_f)) \) measures the deviation of the system final state from the desired state, which is defined as

\[
\varphi(X(t_f)) = \omega_1 (x_0(t_f) - x_{tar})^2 + \omega_2 \sum_{i=0}^{n} (v_i(t_f) - v^*)^2,
\]  

(20)

where \( \omega_1 \) and \( \omega_2 \) denote the weight coefficients for penalty of the position deviation of the leading CAV and the velocity deviation of all the vehicles in the mixed platoon, respectively. \( x_0(t_f) \) denotes the position of the leading CAV at \( t = t_f \). \( x_{tar} \) is the target final position of the leading CAV, which refers to the position of the stopping line at the intersection. The specific choice of the desired equilibrium velocity \( v^* \) and the target position \( x_{tar} \) of the CAV will be discussed in Sections 3.3.2 and 3.3.3 respectively.

In (19), \( L(X(t), u(t)) \) denotes the transient fuel consumption of the mixed platoon at time \( t \), which is defined as

\[
L(X(t), u(t)) = G_0(t) + \sum_{i=1}^{n} G_i(t),
\]  

(21)
where $G_0(t)$ and $G_i(t)$, $i = 1, \ldots, n$ represent the transient fuel consumption of the leading CAV and the following HDVs, respectively. Similar to recent work [24, 9], we utilize the Akcelik’s fuel consumption model for the specific model to calculate transient fuel consumption [39]

$$G(t) = \alpha + \beta_1 P_T(t) + \left( \beta_2 m a^2 v_i(t) \right)_{a>0},$$

(22)

where $m$ is the vehicle mass, and the term $\left( \beta_2 m a^2 v_i(t) \right)_{a>0}$ represents the extra power used for vehicle acceleration. $P_T$ denotes the total power to drive the vehicle, which contains the engine dragging power, moment of inertia, air friction and other energy loss; it can be computed by

$$P_T(t) = \max \{ 0, d_1 v_i(t) + d_2 v_i(t)^2 + d_3 v_i(t)^3 + m a v_i(t) \}.$$  

(23)

A typical setup for parameter values in the Akcelik’s fuel consumption model (22) and (23) is shown in Table 1.

**Remark 3.** Note that the minimization of fuel consumption is one typical control objective for control of individual vehicles at the intersection, which is known as the eco-approaching behavior [5, 8, 9]. However, existing research mostly focused on the behaviors of the CAVs themselves; such consideration might limit the potential of CAVs in improving traffic performance, especially in mixed traffic flow where HDVs also exist. One of the major distinctions in our optimal control framework from previous results [5, 8, 9] lies in the explicit consideration of the fuel consumption of both CAVs and HDVs. This framework allows one to improve the fuel economy of the entire mixed traffic intersection via direct control of only CAVs.

### 3.3.2. Terminal Velocity

We proceed to discuss how to design the desired equilibrium velocity $v^*$, which also represents the terminal velocity in the terminal cost function [20]. Existing research mostly focused on the control of the CAV alone, and thus they typically set the terminal velocity of the CAV as the highest limited velocity in order to improve traffic efficiency at the intersection; see, e.g., [4, 9]. Considering that there might exist other HDVs at the intersection, we reveal that this setup in previous works might not be the optimal choice for the entire mixed intersection.

When designing the terminal velocity, we aim at maximizing the number of vehicles that can pass the intersection in an equilibrium state during a constant green phase time $T_{\text{Green}}$. Take one “1 + n” mixed platoon for example, i.e., there is one leading CAV and $n$ following HDVs in the platoon. From the equilibrium equation (2) in the HDVs’ car-following model, it can be inferred that the equilibrium headway distance $d^*$ relies on the equilibrium velocity $v^*$. Thus, the number $n$ of the following HDVs can be described as in (24). For constant green light phase time, our optimization goal is to maximize the number $n$ of the following HDVs in $T_{\text{Green}}$. Accordingly, we have the following result.

**Theorem 3.** Consider the $1 + n$ mixed platoon system consisting of one CAV and $n$ HDVs given by the general car-following model $F(\cdot)$ in (1). The optimal equilibrium velocity $v^*$, i.e., the optimal target velocity, during a constant
green phase time $T_{\text{Green}}$ can be obtained by solving the following optimization problem.

$$\arg \max_{v^*} \quad n = \frac{v^* T_{\text{Green}}}{d^*},$$
$$\text{subject to : } \quad F(d^*, 0, v^*) = 0. \quad (24)$$

For trade-off between model fidelity and computational tractability, we consider the OVM model \cite{18} as the specific car-following model in our optimal control formulation. In OVM, the general expression (1) of HDVs’ car-following dynamics becomes

$$\dot{v}_i = \kappa [V_{\text{des}}(d_i) - v_i], \quad (25)$$

where $V_{\text{des}}$ is the driver’s desired velocity at headway distance $d_i$, given by

$$V_{\text{des}}(d_i) = V_1 + V_2 \tanh[C_1 (d_i - L_{\text{veh}}) - C_2], \quad (26)$$

with $L_{\text{veh}}$ denoting the vehicle length. One typical setup of OVM parameter values is shown in Table 2. Note that both open-loop stability and controllability hold in this parameter setup, as discussed in Sections 3.1 and 3.2.

If the mixed platoon is in equilibrium state, the leading CAV and following HDVs run at the same velocity, i.e., $F(\cdot) = \dot{v}_i = 0$ holds for the following HDVs. Thus, we obtain the relationship between velocity and headway distance in equilibrium state based on the OVM model as follows. Substituting $F(\cdot) = \dot{v}_i = 0$ into equation (26) yields

$$d_i = \left( \arctan\left(\frac{v_i - V_1}{V_2}\right) + C_2 \right) \ast \frac{1}{C_1} + L_{\text{veh}}. \quad (27)$$

In Fig. 3, it can be clearly seen that HDVs’ headway distance is a nonlinear function to HDV velocity in equilibrium state. And there exists a maximum number of HDVs corresponding to the optimal equilibrium velocity $v^*$ and equilibrium headway distance $d^*$. The optimal terminal velocity $v^*$ can be obtained by solving (24).

Remark 4. Our result is consistent with the typical observations from the perspective of macroscopic traffic theory. Denote $\rho(x, t)$, $v(x, t)$, $q(x, t)$ as the traffic density, traffic flow velocity and traffic flow volume at position $x$ and time $t$, respectively. The fundamental Lighthill-Whitham-Richard model is commonly employed to depict the relationship among them, which is given by \cite{40}

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial q(x, t)}{\partial x} = 0. \quad (28)$$

Based on (28), the relationship between traffic flow volume and density can be described as

$$q(x, t) = Q(\rho(x, t)), \quad (29)$$

where $Q$ is usually a convex and non-monotonic function—typical results reveal that the traffic flow volume $q$ usually grows up first and then drops down as the increase of the traffic density $\rho$. Similarly, our result suggest that the optimal target velocity for the mixed platoon is not “the higher, the better”.

Figure 3: Relationship between terminal platoon speed and HDV number. With the increase of mixed platoon velocity, the HDVs’ headway distance increase non-linearly as shown in Fig. 3(a). Thus for constant traffic SPAT, the number of HDVs is a convex function to mixed platoon velocity as shown in Fig. 3(b). OVM is taken as the car-following model, whose parameters are shown in Table 2.
3.3.3. Constraints

For practical implementation of the obtained controller of the CAV, there also exist several constraints that need to be taken into consideration, including process constraints and terminal constraints. Regarding process constraints, first is the safety constraint, which means that each vehicle in the mixed platoon should keep a safe distance $d_{\text{safe}}$ from the preceding vehicle.

\[ x_i(t) - x_{i-1}(t) - L_{\text{veh}} \geq d_{\text{safe}}, \text{ for } t_0 \leq t \leq t_f, i = 1, 2, \ldots, n. \] (30)

Second is the practical constraint of the value of the velocity and acceleration of each vehicle in the mixed platoon. Denote $v_{\text{max}}$ as the maximum velocity, $a_{\text{min}}$ and $a_{\text{max}}$ as the minimum and maximum acceleration, respectively. Then, it should be satisfied that

\[ 0 \leq v_i(t) \leq v_{\text{max}}, \text{ for } t_0 \leq t \leq t_f, i = 0, 1, 2, \ldots, n; \] (31)

\[ a_{\text{min}} \leq a_i(t) \leq a_{\text{max}}, \text{ for } t_0 \leq t \leq t_f, i = 0, 1, 2, \ldots, n. \] (32)

For the terminal constraint, we mainly focus on the terminal position of the CAV. Note that the deviation of the terminal position of the CAV from the target position $x_{\text{tar}}$ has been penalized in the terminal cost function (20). Here, we further add a solid constraint to require the CAV to neither pass the stopping line nor keep a large spacing away from it, given as follows

\[ 0 \leq x_0(t_f) \leq x_{0,\text{max}}(t_f), \] (33)

where $x_{0,\text{max}}$ denotes the maximum tolerance spacing of the CAV from the stopping line.

Remark 5. Note that the process constraint (30) only focuses on the longitudinal position inside a “1 + n” mixed platoon. The collisions between two adjacent “1 + n” mixed platoons are not considered in the constraints in the optimal control formulation. To address the safety constraint between different mixed platoons, we introduce an event-triggered algorithm, which is presented in Section 3.4.

3.3.4. Optimal Control Formulation

Lumping the aforementioned design of the cost function and the constraints, the overall optimal control problem can be formulated as follows

\[ \arg \max_u J = \varphi(X(t_f)) + \int_{t_0}^{t_f} L(X(t), u(t))dt, \] (34)

subject to: (7), (19), (30), (31), (32), (33), given : $X(t_0)$.

Before solving Problem (34), the optimal terminal velocity $v^*$ needs to be calculated first by solving Problem (24). In addition, the terminal time $t_f$ needs to be pre-determined, which is discussed in the proposed algorithm in Section 3.4. To solve Problem (34) numerically, which is a high-order nonlinear optimal control problem, we transform it into a nonlinear programming (NLP) problem by employing the pseudo-spectral method [41]. Several practical packages can be directly utilized to address this problem; see, e.g., the GPOPS (General-Purpose Optimal Control Software) toolbox [42] with parameter setups shown in Table 3.

| Parameter                          | Value                  |
|------------------------------------|------------------------|
| setup.nlp.solver                   | SNOPT                  |
| setup.nlp.snoptoptions.tolerance   | $2 \times 10^{-3}$     |
| setup.scales.method                | automatic-bounds       |
| setup.derivatives.derivativelevel  | second                 |
| setup.mesh.tolerance               | $10^{-2}$              |
| setup.mesh.iteration               | 8                      |
| setup.mesh.method                  | hp1                    |
| setup.method                       | RPMintegration         |

Table 3: GPOPS Parameters
3.4. Algorithm Design

In this section, we firstly introduce the benchmark algorithm for control of CAVs at signalized intersections. Note that only the constraints in the mixed platoon are considered in Section 3.3.3. To avoid the potential collision between mixed platoons, we also present our event-triggered algorithm design for practical implementation of the “1 + n” mixed platoon based method.

3.4.1. Benchmark Algorithm

The basic benchmark algorithm is the predictive cruise control (PCC) algorithm proposed by Asadi et al. [4]. After obtaining traffic SPAT by V2I technology in advance, a practical method was proposed to choose the target green phase window, shown as follows

\[
\left[ \frac{D_i}{r_i}, \frac{D_i}{g_i} \right] \cap \left[ v_{\text{min}}, v_{\text{max}} \right],
\]

(35)
where $D_i$ is the distance from the CAV $i$ to the stopping line; $r_i$ is the start time of the next red phase; $g_i$ is that of the next green phase; $v_{\text{min}}$ and $v_{\text{min}}$ are the CAV velocity limitations. Then, the CAV obtains the corresponding target velocity and schedules an optimum velocity trajectory through model predictive control (MPC).

However, PCC cannot consider preceding HDVs (especially the queuing ones) into optimization. Instead of PCC, we consider an improvement inspired by Yang’s queuing length adjustment method [5]. The distance from the CAV $i$ to the stopping line $D_i$ is optimized by

$$D_i^* = \frac{v_i}{v_i + v_{\text{AC}}} [D_i + v_{\text{AC}} (r_i - t)],$$

(36)

where $v_i$ is the current velocity of CAV $i$, and $v_{\text{AC}}$ is the velocity of the traffic upstream flow shock wave. In the rest of this paper, this modified PCC algorithm is named as PCC+.

It can be inferred that even if the queuing length is considered in the optimization by (36), the inner core of the PCC+ algorithm is still designed for one CAV alone, which makes it a qualified benchmark algorithm to make comparisons with the mixed platoon based algorithm. Note that the similar target time window chosen method (35) is employed in our algorithm.

3.4.2. Algorithm Design for Mixed Platoon

In this paper, we consider the traffic light as constant SPAT. We design a hierarchical mixed platoon based algorithm as shown in Fig. 4. In Section 3.3, the velocity trajectory planning process has been explained in detail. However, there still remains one problem. In our expectation, the decision-making and trajectory planning process are made when the CAV arrives at the border between OZ and CZ. But one-time planning cannot forecast the future trajectories of the preceding vehicles, which means there are still collision risks when the CAV is undertaking the planned velocity trajectory in CZ. To solve this problem, there are basically two solutions. One is called shooting heuristic [23], which estimates the maximum possible trajectory boundaries of the preceding vehicle. The drawback is that the models’ deviation makes it difficult to guarantee prediction accuracy. The other method is receding horizon [15], which allows the CAV to make planning in constant time period. However, this method brings huge computation burden for real-time implementation.

In fact, the trajectory interference seldom happens when the traffic flow density is not saturated. So distributing extra computation brings very limited benefit. Thus, we design an event-triggered method to guarantee safety with minimum calculation. The CAV state is separated into four states, uncontrolled, computation, controlled and re-computation.

(1) uncontrolled

All the CAVs which arrive in OZ are in uncontrolled state, as shown in the white boxes in Fig. 4. It means that both CAVs and HDVs are not automatically controlled. And only in uncontrolled state are the vehicles allowed to change lanes.

(2) computed

When the CAV arrives at CZ, its state becomes computed, as shown in the orange boxes in Fig. 4. At this time point, the CAV firstly receives the traffic SPAT and chooses its target green phase window. Based on the time window, the optimal velocity trajectory is calculated by optimal control framework as shown in Section 3.3.

As mentioned before, the target window is also chosen by the method in (35). Differently, only one CAV is considered in [4], and thus it’s straightforward to set the target window as the closest one. In our research, however, there is an optimal terminal platoon velocity as designed in Section 3.3.2. If the target velocity is set as the maximum velocity $v_{\text{max}}$, the optimization on the velocity trajectory becomes meaningless. So the limitation on the $v_{\text{max}}$ (35) is sacked as to

$$\left[ \frac{d_1}{r_{1i}}, \frac{d_1}{g_{1i}} \right] \cap \left[ v_{\text{min}}, v_{\text{max}} + v^* \right],$$

(37)

in which $v^*$ means the optimal velocity calculated from (24).

As for the platoon size, the determined platoon size must not be larger than the corresponding maximum number of HDVs in [24]. Otherwise, the excess vehicles will be excluded in the optimization. Note that if the following
vehicle in the platoon is CAV, its control will be set as car-following model so that the following CAV can be regarded the same as a HDV.

(3) controlled

When the CAV is in CZ, it’s in controlled state, as the green boxes showed in Fig. [4]. In this state, CAV executes the velocity trajectory planned in computed state. If there is a preceding vehicle, the CAV simply check the safety distance constraint (30) with the preceding vehicle in each step. If not, the CAV’s state is changed into re-computed state.

(4) re-computed

In re-computed state, the CAV is too close to the preceding vehicle, as the blue boxes showed in Fig. [4]. At first, we should decide whether the remaining distance is long enough for the second optimization for CAV. We judge if the distance from CAV to stopping line is larger than $k_c L_{control}$, where $L_{control}$ is the length of CZ, and $k_c \in [0, 1]$. If yes, the CAV can be re-computed state when the preceding vehicle drives away. If not, the CAV is switched to car-following model control.

4. Simulation Results and Discussion

In this section, we evaluate the effectiveness of the proposed optimal control framework in Section 3.3 and the corresponding algorithm in Section 3.4 based on large-scale traffic simulations. The nonlinear OVM model (25) is utilized to model the dynamics of HDVs. Two types of simulation experiments are conducted, considering multiple traffic volumes and MPRs.

4.1. Simulation Environment and Evaluation Index

The traffic simulation is conducted in SUMO, which is widely used in traffic researches [43]. The simulation is run on Intel Core i7-7700 processor @3.6GHz. Some simulation parameters are shown in Table 4.

Denote $t_i^{in}$ as the time step when vehicle $i$ enters CZ, and $t_i^{out}$ as the time step when it exits CZ. In this way, it means that the vehicle spends $t_i^{out} - t_i^{in}$ time to travel through CZ, while it spends $L_{control} / v_{max}$ time to travel through CZ in free drive condition. Accordingly, Average Travel Time Delay (ATTD) is chosen to measure the average traffic efficiency for the mixed platoon, as shown in (38). Meanwhile, fuel consumption per 100KM is chosen as the evaluation index to measure the fuel economy.

\[
\text{ATTD} = \frac{1}{n} \sum_{i=0}^{n} \left( t_i^{out} - t_i^{in} - \frac{L_{control}}{v_{max}} \right) .
\]  

Table 4: Simulation Parameters

| Parameters                          | Symbol | Value |
|------------------------------------|--------|-------|
| Simulation step (s)                | $T_s$  | 0.1   |
| Maximum acceleration (m/s$^2$)     | $a_{max}$ | 3    |
| Minimum acceleration (m/s$^2$)     | $a_{min}$ | -6   |
| Terminal position weight           | $\omega_1$ | $10^5$ |
| Terminal velocity weight           | $\omega_2$ | $10^4$ |
| Maximum velocity (m/s)             | $v_{max}$ | 15   |
| Minimum velocity (m/s)             | $v_{min}$ | 0    |
| Control zone length (m)            | $L_{control}$ | 300  |

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4.2. Simulation Results at Different Traffic Volumes

We first conduct simulation in 50% MPR of 100 vehicles, which means there are randomly distributed 50 CAVs and 50 HDVs driving into the intersection. Then further comparisons are made between the proposed Mixed Platoon (MP) algorithm and the benchmark algorithm PCC+ shown in Section 3.4 at different traffic volumes.
In Fig. 6, there are three groups of vehicle trajectories in 700 veh/(h·l), which represent no control input (colored in black), PCC+ control (colored in blue) and proposed mixed platoon control (colored in magenta), respectively. It can be observed that in Fig. 6(a) the queuing at the signalized intersection is gradually accumulating, which means the intersection is over-saturated in 700 veh/(h·l). On the contrary, with PCC+ and MP applied, the queuing accumulation is constrained in CZ range. It means queuing effect is limited in one intersection rather than spreading to the upstream intersection, which is important to urban traffic control. The reason is that the vehicles’ stop-and-go behavior before the stopping line lowers the green phase utilization, which worsens the traffic efficiency. With CA V control applied to CA Vs, the CA Vs’ stop-and-go behavior is avoided as much as possible. Moreover, compared with PCC+ control in Fig. 6(b), MC control in Fig. 6(c) creates much bigger gaps between trajectory blocks, which creates more optimization space for CA V. Generally speaking, the existence of these gaps mean that the CAV algorithm brings lower traffic saturation and higher traffic mobility at the intersections [16].

4.3. Simulation Results in Different MPRs

In the last simulation, we test the performance of the MP algorithm in all MPRs and traffic volumes. ATTD and fuel consumption of MP are shown in Fig. 7(a) and 7(b). As can be observed, ATTD and fuel consumption get worse as traffic volume increase while the penetration of CA V helps improve the traffic conditions. The improvements of the MP algorithm are shown in Table 5.

To present the result in a more straightforward way, the 0% MPR is chosen as the reference value to evaluate the MP algorithm improvement, i.e., how much improvement MP algorithm brings compared to the case where all the vehicles are HDVs. As shown in Fig. 7(c) and 7(d), the red lines in the figure denote the reference plane of 0% MPR. The surface beneath the reference plane means the algorithm reduces the ATTD or fuel consumption, which benefits
Figure 7: Algorithm performance comparison in different traffic volumes and different MPRs. Fig. 7(a) and 7(b) are the ATTD and fuel consumption respectively. 0% MPR is chosen as the reference value to evaluate the MP algorithm traffic benefits, which is shown as the red lines in Fig. 7(c)−7(f).

When the MPR is lower than 20%, the MP algorithm has negative influence on the evaluation index. The reason is that the CAV usually takes a “slow down in advance” strategy to avoid idling before the stopping line, which inevitably brings the velocity fluctuation of the following HDVs. When the MPR is quite low, the accumulative dynamics of a series of HDVs magnifies the fluctuation, and thus the disadvantages brought by the fluctuation exceed the advantages.
brought by CAV control. Several research has pointed out that as MPR increases, the CAV’s benefits on traffic may not increase in positive correlation. For instance, Ala et al. [22] proposed Eco-CACC algorithm which includes queuing length into CAV control, where low MPR results show negative influence on fuel consumption because of HDV’s unexpected behaviour.

It can be observed that when the MPR is higher than 40%, MP algorithm has positive influence on both traffic efficiency and fuel consumption in all traffic volumes. In the critical-saturated volume of around 700 veh/(hour · lane), MP improves traffic efficiency by 60% and fuel consumption by 25%. Even in the over-saturated volume of 900−1000 veh/(hour · lane), it still has about 10% improvement on the traffic efficiency and 5% on the fuel consumption. In medium or high MPR, velocity fluctuation is limited in the overall optimal control of the mixed platoon, achieving the improvement of traffic efficiency and fuel consumption. All of these results confirm the great benefits of the MP algorithm on traffic performance in the mixed traffic intersection.

5. Conclusions

In this paper, the notion of “1+ n” mixed platoon for CAV control in mixed traffic intersections has been proposed. Assigned as the leading vehicle of n HDVs, the potential of the CAV in improving the global traffic mobility at the intersection has been revealed. Based on rigorous theoretical analysis, we have pointed out that the proposed mixed platoon formation is open-loop stable and controllable in a very mild condition, which is irrelevant to the mixed platoon size n. An optimal control framework has been established with consideration of velocity deviation and fuel consumption of the whole mixed platoon, where the terminal velocity has also been optimized to improve traffic throughput. Furthermore, a hierarchical event-triggered algorithm has been designed to solve the collision problem between adjacent mixed platoons, which can be applied in any MPR of mixed traffic environment. Traffic simulations have verified the effectiveness of the proposed optimal control method.

One future research direction is to address the problems of possible heterogeneous dynamics and model uncertainties in HDVs’ behaviors in the optimal control framework for the “1 + n” mixed platoon. Considering that multiple “1 + n” mixed platoons could pass the intersection in the same green phasing time, especially at a relatively high MPR, another interesting topic is to apply cooperative control algorithms to multiple adjacent mixed platoons to further improve the overall performance. Finally, field experiments are also needed for further validation of the proposed algorithm.

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