Is anisotropic flow really acoustic?

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The flow harmonics for charged hadrons \((v_n)\) and their ratios \((v_n/v_2)\), are studied for a broad range of transverse momenta \((p_T)\) and centrality \((\text{cent})\) in Pb+Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV. They indicate characteristic scaling patterns for viscous damping consistent with the dispersion relation for sound propagation in the plasma produced in the collisions. These scaling properties are not only a unique signature for anisotropic expansion modulated by the specific shear viscosity \((\eta/s)\), they provide essential constraints for the relaxation time, a distinction between two of the leading models for initial eccentricity, as well as an extracted \((\eta/s)\) value which is insensitive to the initial geometry model. These constraints could be important for a more precise determination of \((\eta/s)\).

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Azimuthal anisotropy measurements are a key ingredient in ongoing efforts to pin down the precise value of the transport coefficients of the plasma produced in heavy ion collisions at both the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). The Fourier coefficients \(v_n\) are routinely used to quantify such measurements as a function of collision centrality \((\text{cent})\) and particle transverse momentum \(p_T\):

\[
dN/d\phi \propto \left(1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos n(\phi - \psi_n)\right),
\]

where \(\phi\) is the azimuthal angle of an emitted particle, and \(\psi_n\) are the azimuths of the estimated participant event planes \(1, 2\);

\[
v_n(p_T) = \langle \cos n(\phi - \psi_n) \rangle,
\]

where the brackets denote averaging over particles and events. The distribution of the azimuthal angle difference \((\Delta \phi = \phi_n - \phi_p)\) between particle pairs with transverse momenta \(p_T^a\) and \(p_T^b\) (respectively) is also commonly used to quantify the anisotropy \(2, 3\):

\[
\frac{dN_{\text{pairs}}}{d\Delta \phi} \propto \left(1 + \sum_{n=1}^{\infty} 2v_{n,n}(p_T^a, p_T^b) \cos n(\Delta \phi)\right),
\]

where the latter factorization has been demonstrated to hold well for \(p_T \lesssim 3\) GeV/c for particle pairs with a sizable pseudorapidity gap \(\Delta \eta \gtrsim 0.8\).

The coefficients \(v_n(p_T, \text{cent})\) (for \(p_T \lesssim 3 - 4\) GeV/c) have been attributed to an eccentricity-driven hydrodynamic expansion of the plasma produced in the collision zone \(0.1 \lesssim t \lesssim 2\). That is, a finite eccentricity moment \(\varepsilon_n\) drives uneven pressure gradients in- and out of the event plane \(\varepsilon_n\), and the resulting expansion leads to the anisotropic flow of particles about this plane. In this model framework, the values of \(v_n(p_T, \text{cent})\) are sensitive to the magnitude of both \(\varepsilon_n\) and the transport coefficient \(\eta/s\) (i.e. the specific shear viscosity or ratio of shear viscosity \(\eta\) to entropy density \(s\)) of the expanding hot matter \(3, 11, 14, 18\). Thus, \(v_n(p_T, \text{cent})\) measurements provide a crucial bridge to the extraction of \(\eta/s\) from data.

Recent estimates of \(\eta/s\) from \(v_n\) measurements \(11, 12, 16, 17, 19-24\) have all indicated a small value \((\eta/s\sim 1-4)\) times the lower conjectured bound of \(1/4\pi\) \(29\). Recent 3+1D hydrodynamic calculations, which have been quite successful at reproducing \(v_n(p_T, \text{cent})\) measurements \(26-28\), have also indicated a similarly small value of \(\eta/s\lesssim 2/4\pi\). However, the precision of all of these extractions has been hampered by significant theoretical uncertainty, especially those arising from poor constraints for the initial eccentricity and the relaxation time. One approach to the resolution of this issue is to target these uncertainties for systematic study, with the aim of establishing reliable upper and lower bounds for \(\eta/s\). An alternative approach, adopted in this work, is to ask whether better constraints for these theoretical bottlenecks can be developed to aid precision extractions of \(\eta/s\)?

The acoustic nature of anisotropic flow (i.e. it is driven by pressure gradients), a transparent way to evaluate the strength of the dissipative effects which reduce the magnitude of \(v_n(p_T, \text{cent})\), is to consider the attenuation of sound waves in the plasma. In the presence of viscosity, sound intensity is exponentially damped \(e^{-t/\Gamma_s}\) relative to the sound attenuation length \(\Gamma_s\). This can be expressed as a perturbation to the energy-momentum tensor \(T_{\mu\nu}\);

\[
\delta T_{\mu\nu}(n, t) = \exp(-\beta n^2)\delta T_{\mu\nu}(0), \quad \beta = \frac{2\eta}{3s R^2 T},
\]

which incorporates the dispersion relation for sound propagation, as well as the spectrum of initial \((t = 0)\) perturbations associated with the eccentricity moments.

The latter reflects the collision geometry and its associated density driven fluctuations. Here, the viscous coeffi-
cent \( \beta \propto \eta/s \), \( t \propto \bar{R} \) is the expansion time, \( T \) is the temperature, \( k = n/R \) is the wave number (i.e. \( 2\pi R = n\lambda \) for \( n \geq 1 \)) and \( \bar{R} \) is the transverse size of the collision zone.

The viscous corrections to \( v_n \) implied in Eq. 3 do not indicate an explicit \( p_T \)-dependence. However, a finite viscosity in the plasma results in an asymmetry in the energy-momentum tensor which manifests as a correction to the local particle distribution \( f \) at freeze-out \[22\]:

\[
f = f_0 + \delta f(\bar{p}_T), \quad \bar{p}_T = \frac{p_T}{T},
\]

where \( f_0 \) is the equilibrium distribution and \( \delta f(\bar{p}_T) \) is its first order correction. The latter leads to the \( p_T \)-dependent viscous coefficient \( \beta(\bar{p}_T) \propto \beta/p_T^n \), where the magnitude of \( \alpha \) is related to the relaxation time \( \tau_R(p_T) \).

Equations 3 and 4 suggest that for a given centrality, the viscous corrections to the flow harmonics \( v_n(p_T) \), grow exponentially as \( n^2 \):

\[
\frac{v_n(p_T)}{\varepsilon_n} \propto \exp\left(-\beta'n^2\right),
\]

and the ratios \( (v_n(p_T)/v_2(p_T))_{n\geq3} \) can be expressed as:

\[
\frac{v_n(p_T)}{v_2(p_T)} = \frac{\varepsilon_n}{\varepsilon_2} \exp\left(-\beta'(n^2 - 4)\right),
\]

indicating that they only depend on the eccentricity ratios and the relative viscous correction factors. Note as well that Eq. 5 shows that the higher order harmonics \( v_{n,n\geq3} \), can all be expressed in terms of the lower order

\[
\text{harmonic } v_2, \text{ as has been observed recently}[6, 32]. \text{ For a given harmonic, Eq. 5 can be linearized to give}
\]

\[
\ln\left(\frac{v_n(p_T)}{\varepsilon_n}\right) \propto -\frac{\beta''}{R},
\]

which indicates a characteristic system size dependence \( (1/R) \) of the viscous corrections.

If validated, the acoustic dissipative patterns summarized in Eqs. 3 and 7 indicate that estimates for \( \alpha, \beta \) and \( \varepsilon_n/\varepsilon_2 \) can be extracted directly from the data. Here, we perform validation tests for these dissipative patterns with an eye toward more stringent constraints for \( \tau_R, \eta/s \) and the distinction between different eccentricity models.

The data employed in our analysis are taken from measurements by the ATLAS collaboration for Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} [2, 30] \). These measurements exploit the event plane analysis method (c.f. Eq. 1), as well as the two-particle \( \Delta \phi \) correlation technique (c.f. Eq. 2) to obtain robust values of \( v_n(p_T, \text{cent}) \) for a sizable \( \Delta p_T \) gap between particles and the event plane, or particle pairs. We divide these values by \( \varepsilon_n(\text{cent}) \) and plot them as a function of \( n \), to make an initial test for viscous damping compatible with sound propagation in the plasma produced in these collisions. Monte Carlo Glauber (MC-Glauber) simulations were used to compute the number of participants \( N_{\text{part}}(\text{cent}) \) and \( \varepsilon_n(\text{cent}) \) from the two-dimensional profile of the density of sources in the transverse plane \( \rho_s(r_\perp) \). The weight \( w(r_\perp) = r_\perp^n \)

\[23\] was used to compute \( \varepsilon_n(\text{cent}) \).

The open circles in Figs. 1 (a)-(d) show representative examples of \( v_n/\varepsilon_n \) vs. \( n \) for several \( p_T \) cuts, for the

\[
\text{FIG. 1. (a)-(d) } v_n/\varepsilon_n \text{ vs. } n \text{ for charged hadrons for several } p_T \text{ selections in 20-30\% central Pb+Pb collisions at } \sqrt{s_{NN}} = 2.76 \text{ TeV; (e) } \beta \text{ vs. } p_T \text{ for the same centrality selection; (f) } \beta \text{ vs. } p_T \text{ from the analysis of the results from viscous hydrodynamical calculations } [22] \text{ for } \delta f \propto p_T^n \text{ and } \delta f \propto p_T^3 \text{. The } v_n \text{ data are taken from Refs. 3, 30; the dashed and dotted curves represent fits (see text).}
\]
20-30% centrality selection. The dashed curves which 
indicate fits to the data with Eq. 5 confirm the ex-
pected exponential growth of the viscous corrections to 
v_n, as n^2. The p_T-dependent viscous coefficients β(p_T) 
obtained from these fits, are summarized in Fig. 1 (e); 
they show the expected 1/p_T^4 dependence attributable to 
delta f(p_T). Note that a similar dependence is obtained for 
fits to the results of viscous hydrodynamical calculations, 
as illustrated in panel (f). The latter indicates that the 
p_T dependence of β allows a distinction between the two 
sets of calculations which use different input assumptions 
for δf(p_T). The dotted curve in panel (e) is a fit which 
gives the values α ∼ 0.58 and β ∼ 0.12. Similar results 
were obtained for a broad range of centrality selections.

Additional constraints can be obtained from the ratios 
of the flow harmonics (v_n(p_T)/v_2(p_T))_{n≥3} (cf. Eq. 6), as 
well as the dependence of v_n(p_T)/ε_n on the transverse 
size of the collision zone (cf. Eq. 4). The open symbols 
in Fig. 2 show the values of (v_n(p_T)/v_2(p_T)) for n = 3, 4 and 5, for each of the centrality selections indicated. 
A simultaneous fit to these ratios was performed with 
Eq. 4 to extract β and ε_n/ε_2 at each centrality. Small 
variations about the previously extracted value of α ∼ 
0.58 were used to aid the convergence of these fits. The 
filled symbols in Fig. 2 show the excellent fits achieved; 
they confirm the characteristic dependence of the relative 
viscous correction factors expressed in Eq. 4. They also 
confirm that the relationship between v_2 and the higher 
order harmonics stems solely from “acoustic scaling” of 
the viscous corrections to anisotropic flow. The extracted 
values for ε_n/ε_2, α and β are summarized and discussed 
below.

Figures (a) and (b) gives a more transparent view of 
the influence of system size on the viscous corrections. 
Fig. (a) shows that v_2,3 increases for 140 ≤ N_part ≤ 340 
as would be expected from an increase in ε_2,3 over the 
same N_part range. For N_part ≤ 140 however, the de-
creasing trend of v_2,3 contrasts with the increasing trends 
for ε_2,3, suggesting that the viscous effects due to much 
smaller system sizes, serve to suppress v_2,3. This is con-
firmed by the dashed curves in Fig. (b) which validate 
the expected linear dependence of ln(v_n/ε_n) on 1/R (cf. 
Eq. 7) for the data shown in Fig. (a). A similar depen-
dence was observed for other p_T selections. The slopes of 
these curves serve as an important additional constraint 
for β.

Figures (c) - (e) show a comparison between the ε_n/ε_2 
ratios extracted from the fits shown in Fig. (open sym-
bols), and those obtained from model calculations (filled 
symbols). For the 5-50% centrality range, the compar-
ison shows good agreement between the extracted ratios 
and those obtained from MC-Glauber calculations with 
weight ω(r_1) = r_1^n [32]. A similarly good agreement 
with the ratios obtained from a Monte Carlo implemen-
tation [34] of the factorized Kharzeev-Levin-Nardi (KLN) 
model [35] is not observed. For the 0-5% most cen-
tral collisions, the extracted values of ε_n/ε_2 are larger 
than the values obtained from either eccentricity model. 
This difference could result from an overestimate of ε_2 in 
the 0-5% centrality selection, for the initial eccentricity 
models considered.

The fits shown in Fig. 2 also give values for α and 
β, which are summarized in Figs. (f) and (g); they are 
essentially independent of centrality. This suggests that,
FIG. 3. (a) $v_{2,3}$ vs. $N_{\text{part}}$ for $p_T = 1 - 2$ GeV/c: (b) $\ln(v_n/\varepsilon_n)$ vs. $1/R$ for the data shown in (a): (c - e) centrality dependence of the $\varepsilon_n/\varepsilon_2$ ratios extracted from fits to $(v_n(p_T)/v_2(p_T))_{n \geq 3}$ with Eq. 6 $\varepsilon_n/\varepsilon_2$ ratios for the MC-Glauber [33, 37] and MC-KLN [34] models are also shown: (f) extracted values of $\beta$ vs. centrality: (g) extracted values of $\alpha$ vs. centrality (see text).

FIG. 4. (a) $\ln(v_n/\varepsilon_n)$ vs. $n^2$ from viscous hydrodynamical calculations for three values of specific shear viscosity as indicated. (b) $\ln(v_n/\varepsilon_n)$ vs. $n^2$ for Pb+Pb data. The $p_T$-integrated $v_n$ results in (a) and (b) are for 0.1% central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [38]; the curves are linear fits. (c) $\beta$ vs. $4\pi\eta/s$ extracted from the curves shown in (a) and (b).

Within errors, the full data set for $v_n(p_T, \text{cent})$ can be understood in terms of the eccentricity moments coupled to a single (average) value for $\alpha$ and $\beta$ (respectively). This observation is compatible with recent viscous hydrodynamical calculations which have been successful in reproducing $v_n(p_T, \text{cent})$ measurements with a single $\delta f(\hat{p}_T)$ ansatz and an average value of $\eta/s$ [26, 27]. Therefore, these values of $\alpha$ and $\beta$ should provide an important set of constraints for detailed model calculations.

To demonstrate their utility, we have used the results from recent viscous hydrodynamical calculations [38] to calibrate $\beta$ and make an estimate of $\eta/s$. This is illustrated in Fig. 4. The $p_T$-integrated $v_n$ results from viscous hydrodynamical calculations for three separate $\eta/s$ values, for 0.1% central Pb+Pb collisions are shown in Fig. 4(a). They indicate the expected linear dependence
of $\ln(v_n/\varepsilon_n)$ on $n^2$, as well as the magnitude of $\eta/s$. The calibration curve or $\beta$ vs. $4\pi n/\ell$, obtained from linear fits to the curves in Fig. 4(a), is shown in Fig. 4(c). The $p_T$-integrated $v_n$ data shown in Fig. 4(b), also validates the expected linear dependence of $\ln(v_n/\varepsilon_n)$ on $n^2$ for the same $\varepsilon_n$ values employed in Fig. 4(a). We use the slope of this curve in concert with the calibration in Fig. 4(c) to obtain the estimate $(4\pi n/\ell) \approx 2.2 \pm 0.2$, which is in reasonable agreement with recent $(\eta/s)$ estimates. Here, it is noteworthy that our calibration procedure leads to a $(\eta/s)$ value which is insensitive to the initial geometry model employed. Further calculations are undoubtedly required to reduce model driven calibration uncertainties. However, our analysis clearly demonstrates the value of the relative magnitudes of $v_n$ as an important constraint.

In summary, we have presented a detailed phenomenological study of viscous damping of the flow harmonics $v_n$ and their ratios $(v_n/v_2)_{n \geq 2}$, for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Within a parametrized viscous hydrodynamical framework, this damping can be understood to be a consequence of the acoustic nature of anisotropic flow. That is, the observed viscous damping reflects the detailed scaling properties inferred from the dispersion relation for sound propagation in the plasma produced in these collisions. These patterns give a unique signature for anisotropic expansion modulated by viscosity, and provide straightforward constraints for the relaxation time, a distinction between two of the leading models for initial eccentricity, as well as an extracted $(\eta/s)$ value which is essentially independent of the initial eccentricity. Such constraints could be crucial for a more precise determination of the specific shear viscosity $\eta/s$.

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