Transfer and storage of vortex states in light and matter waves.

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We theoretically explore the transfer of vortex states between atomic Bose-Einstein condensates and optical pulses using ultra-slow and stopped light techniques. We find shining a coupling laser on a rotating two-component ground state condensate with a vortex lattice generates a probe laser field with optical vortices. We also find that optical vortex states can be robustly stored in the atomic superfluids for long times (> 100 ms).

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One of the most dramatic developments in recent experiments on atomic Bose-Einstein condensates (BECs) is the creation and study of quantized vortices and vortex lattices \cite{1, 2}. In this Letter we show that light modes with orbital angular momentum can be efficiently stored in superfluid vortex configurations and later rewritten back onto light fields. This also provides a feasible mechanism to transfer BEC vortex lattice ground states to light pulses. The strong light-matter coupling can be implemented using the recent technology of ultra-slow and stopped light pulses in atomic vapors \cite{3, 4, 5, 6}, based on the method of electromagnetically-induced transparency (EIT) \cite{7}. In these experiments, an optical field’s amplitude and phase pattern was written onto superpositions of atomic states, stored for several milliseconds, and then re-written back onto the original light field and output.

One of the unique characteristics of slow light is the strong yet coherent action of the matter fields on the light fields \cite{8}. We here study the robust storage and transfer of coherent information in terms of vortex configurations, for experimentally feasible parameters of a two-component $^{87}$Rb BEC. We find that shining a coupling laser pulse can generate a probe pulse which contains optical vortices corresponding to the vortices originally in the atomic fields. If vortices are present in both BECs, the vortex circulation of one BEC component is reversed in the probe field. This light field could potentially be input into a second BEC which is optically connected with the first BEC (even if it is spatially distant), allowing transfer of vortex states between BECs.

The strong coupling between the quantized optical and BEC vortices is an interesting phenomenon in its own right, as a demonstration of nonlinear superfluid-light optics and may be useful in imaging vortex lattices. But it also points to possibilities for information processing and storage. Experiments have seen advances in the generation of Laguerre-Gaussian (LG) modes of light with orbital angular momentum as well as applications of these modes to quantum information \cite{8}. It may be possible to utilize modes with several different angular momentum quantum numbers to build a quantum information architecture based on a larger alphabet than the traditional two-state systems \cite{10}. However, optically based quantum information schemes suffer from the difficulty of trapping and storing optical fields.

Here we perform a comprehensive analysis of how LG beams can be written into BECs, stored in the atomic fields, and later rewritten onto light fields. We find one can choose parameters so the coherent BEC dynamics do not dissipate the information during the storage time, allowing storage fidelities on the order 70\% for several hundred milliseconds with typical parameters. This fidelity

FIG. 1: Writing an atomic vortex lattice onto a light field. The optical density (top row) $D_i = N_\sigma \int dz |\psi_i|^2$ (black=0, white=55) and the phase $\phi_i$ in the $z=0$ plane (bottom row) for a vortex lattice (a) in |1⟩ and (b) in |2⟩. (c) $n_p$ (the number of probe photons output per cross sectional area $\sigma$) upon a switch-on of the coupling field (black=0, white=25) and the phase profile of $\phi_p$, which contains vortices of both circulation. (d) Same quantities as (c) for a non-rotating ground state BEC in |1⟩, resulting in vortices of only one circulation. (e) A scatter plot of the transfer efficiency $T = n_p/n_2$, where $n_2$ is the number of |2⟩ atoms per $\sigma$ before the switch-on, versus $f_{12}D_1$ for the case in (e) (open circles, red online) and for (d) (closed circles, blue online). The solid curve shows the estimate $1 - (1 + f_{12}D_1)^{-1/2}$.
can be further improved by using BECs with larger optical densities. Moreover, the phase information in vortex lines can exhibit very long life times of tens of seconds or can even be energetically stable [1]. While similar LG beams have been proposed as a method to generate vortices in BECs [11] and in experiments vortices have been imprinted on BECs [2]; our scheme here is very different: In the nonlinear light-matter coupling both amplitude and phase are robustly and simultaneously exchanged, allowing the controlled storage of information. Moreover, slow light and EIT are now inspiring important coherent matter wave technology applications, e.g., BEC shock waves [8], probing decoherence in BECs [12], optical black holes [13], storage and production of nonclassical states [14, 15], quantum information processing [16], low light level nonlinear optics [17], and optical processing [18].

We consider two BECs of Rb-87 atoms, in stable internal states |1⟩ ≡ |5S1/2, F = 2, MF = +1⟩ and |2⟩ ≡ |5S1/2, F = 1, MF = −1⟩, with macroscopic wavefunctions ψ1 and ψ2. These are connected, by resonant, + z propagating coupling and probe light fields, Ωc and Ωp (each of wavelength λ = 780 nm), to the excited state |3⟩ ≡ |5P1/2, F = 2, MF = 0⟩, which decays at Γ = (2π) 6 MHz, forming a Δ three-level structure. Here the Rabi frequencies Ωr(r, t) = −δc : Ei(r, t) (p = 1, c = 2) are defined in terms of the atomic dipole matrix elements d3 and the slowly varying envelope (SVE) of the electric fields Ei (with the rapid phase rotation at the optical frequencies and optical wavenumbers factored out). The light propagation then follows from SVE approximation to the Maxwell’s equations [19]:

\[ \left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \Omega_i = -\frac{ik}{\hbar \epsilon_0} d_{3i} \cdot P_{3} \cdot \hat{z}. \]  

(1)

Here the SVE of the polarization is d3 : P3 ≈ f3D′2N|ψ1⟩|ψ3⟩, N is the total number of BEC atoms, f3 = 1/4 and f3 = 1/12 are dimensionless oscillator strengths, D = √4ε0ω0Γ/k is the reduced dipole matrix element and σ = 3X2/2π is the resonant cross-section. The BEC wavefunctions ψ1, ψ2 evolve according to generalized Gross-Pitaevskii equations (GPEs):

\[ i\hbar \frac{\partial}{\partial t} \psi_i = \left( H_0 + \sum_{j=1,2} U_{ij}|\psi_j|^2 \right) \psi_i + \hbar \Omega_i \psi_3, \]

(2)

where H0 = −ℏ2∇2/2m + V, with a perfectly overlapping trapping potential for both states \( V(r) = \frac{m}{2} [\omega_{x}^2 (x^2 + y^2) + \omega_{z}^2 z^2] \). In the interaction coefficients \( U_{ij} = 4\pi \hbar^2 a_{ij}/m \), the aij are the scattering lengths between atoms in states |i⟩ and |j⟩. For 87Rb, \( a_{12} = 5.5 \) nm and \( a_{11} : a_{12} : a_{22} : 0.97 : 1.0 : 1.03 [20] \). Often in practice, and throughout this Letter, we will be working in a regime where, whenever the light fields are present, the atomic internal state dynamics and light field couplings are on a much faster time scale and dominate the external dynamics. In the context of slow light experiments we can adiabatically eliminate \( \psi_3 \cong -i(\Omega_p \psi_1 + \Omega_c \psi_2)/\Gamma \) in Eqs. (1, 2). Then the last term in Eq. (2) results in both coherent exchange between |1⟩, |2⟩ as well as absorption into |3⟩. In our model, atoms which populate |3⟩ and then spontaneously emit are assumed to be lost from the BECs.

Vortex lattices can be produced by imparting angular momentum to the system with a time-dependent rotating potential. We numerically find the ground state of a two-component BEC rotating along the z axis at the speed \( \bar{\omega} = 0.3 \omega_z \), by evolving the GPEs [2] in imaginary time in the absence of the light fields in the rotating frame, obtained by replacing \( H_0 \) by \( H_0 - \bar{\omega} L_z \), with \( L_z = i\hbar (y\partial_x - x\partial_y) \). At every time step, we separately normalized the wavefunctions to fix the atom number in each component. The full 3D integration, without imposing any symmetry on the solution, is performed in a ‘pancake-shaped’ trap \( \omega_z = 0.1 \omega_z = (2\pi) 10 \) Hz, with \( N = 2.76 \times 10^5 \), on a spatial grid of 2562 × 32.

In Fig. 1 we present a calculation of such a two-species lattice. The phases of the wavefunctions \( \phi_i \) indicate a lattice of singly-quantized vortices in each component, with the positions of the vortex cores in the two species offset from each other. The filling of the vortex cores by the other BEC significantly increases the core size, as compared to vortices in a single-component BEC, and makes them observable even without a ballistic expansion. The 3D results are qualitatively similar to the previous 2D calculations [21]. The rectangular vortex lattice pattern is recognizable close to the trap center, but becomes distorted close to the BEC boundaries. The total density in the center is \( 6.9 \times 10^{13} \text{cm}^{-3} \). The vortex core positions \( \phi_1, \phi_2 \) change very little along z.

To model the writing of this vortex lattice to the probe field \( \Omega_p \), we numerically solved Eqs. (1, 2) when a coupling field with a peak input value of \( \Omega_p^{0} (2\pi) 4 \) MHz was switched on suddenly (in a time 0.8 μs). When this happens the coherence \( |\psi_1⟩|\psi_2⟩ \) acts to generate a probe field \( \Omega_p \), in such a way that the contributions to the polarization in Eq. (1) cancel out, and a dark state is formed. The resulting output intensity and phase pattern of \( \Omega_p \) is shown in Fig. 1(c).

To understand the results, we note that solving Eq. (1), ignoring the negligible z variation of \( \psi_1/\psi_2 \), shows the fields eventually reach the asymptotic values \( \Omega_p = -\Omega_c (f_{13}|\psi_1|^2)/(f_{13}|\psi_1|^2 + f_{23}|\psi_2|^2) \) and \( \Omega_c = \Omega_p \psi_1/\psi_2 \). Note that \( \Omega_p \to 0 \) whenever either \( \psi_1 \), \( \psi_2 \to 0 \), giving vanishing intensity at all the phase singularities. The phase \( \phi_p \) is determined by the relative BEC phase \( \phi_p = \phi_2 - \phi_1 + \pi \) (the coupling field acquires no phase shift here). Thus \( \phi_p \) contains singularities with one circulation at the location of vortices in \( \psi_2 \) and of the opposite circulation at vortices in \( \psi_1 \).

In the columns where \( |\psi_2/\psi_1| \ll 1 \) the coupling field coherently generates the probe field via coherent trans-
fers from \(|2\rangle\) and \(|1\rangle\) so that the number of probe photons output per area \(\sigma\), \(n_p\), equals the original number of \(|2\rangle\) atoms per \(\sigma\), \(n_2\). Deviations from this ideal limit are due to absorptions in a ‘preparation’ region (equal to one optical depth) before the dark state is established. When the optical depth of \(|1\rangle\), \(f_{13}D_1 \gg 1\), this loss is relatively small. On the other hand, near the vortex cores of \(|1\rangle\) where \(\psi_1 \to 0\) there is not enough coherence to generate \(n_2\) photons. Instead \(\Omega_c\) is attenuated by absorption and eventually the dark state is established primarily via incoherent spontaneous emission events. We found that the transfer efficiency \(T = n_p/n_2\) for each column can be determined by the optical depth, agreeing very well with the prediction \(1 - 1/\sqrt{f_{13}D_1}\) which approaches unity as \(D_1 \to \infty\); see Fig. 3(c). The overall output efficiency integrated over \(x, y\) is 35%.

Motivated by the fact that the fidelity improves with \(D_1\) we also considered coupling the \(\psi_2\) vortex lattice of Fig. 1(b) to a vortex-free, non-rotating ground state BEC in \(|1\rangle\), with \(N = 3 \times 10^6\) [Fig. 1(d)]. There the peak optical density \(f_{13}D_1\) is 70 and is > 35 throughout the region occupied by \(|2\rangle\). To a good approximation, the probe intensity simply reflects \(\psi_2\) [see Fig. 1(b)] and only contains vortices of one helicity. The fidelity is now higher [Fig. 1(c)] with the overall efficiency 85%.

Having determined that vortex states can indeed be written from atoms to light fields we now consider the possibility of storing information, input in the form of vortices in light fields, in a two-species BEC. For this study, we switch the roles of the two stable \(^{87}\)Rb states so \(|3\rangle\) is coupled to \(|2\rangle\) \(\equiv \{|F = 2, M_F = +1\}\) by \(\Omega_c\) and to \(|1\rangle\) \(\equiv \{|1, -1\}\) by \(\Omega_p\). A BEC is initially in the (non-rotating) ground state of \(|1\rangle\), labelled \(\psi_1^{(G)}(r)\), in an isotropic trap \(\omega_z = \omega_r = (2\pi)21\) Hz, with \(N = 4 \times 10^6\), \(R_T\) = 25 \(\mu\)m, peak density of \(1.5 \times 10^{14}\) cm\(^{-3}\), and a peak optical density of \(f_{13}D_1 = 213\). The initial coupling field is at \(\Omega_{(in)}^k = (2\pi)8\) MHz [see Fig. 2(a)], while the LG probe input reads (in cylindrical coordinates \(z\)):

\[
\Omega_p^{(in)}(r, \theta, t) = \Omega_p^{(o)}(r) \left( \frac{\sqrt{2}}{w} \right)^{|m|} e^{-\frac{r^2}{w^2}} e^{im\theta} e^{-\frac{z^2}{2z^2}},
\]

where \(w\) is the beam waist size. Each probe photon in the LG mode with \(|m| \geq 1\) contains a vortex at \(r = 0\) with a vanishing intensity and \(m\) units of orbital angular momentum.

We consider an \(m = 1\) mode with \(\tau_0 = 0.25\) \(\mu s\), \(w = 6.3\) \(\mu m\) and \(\Omega_p^{(o)} = (2\pi)4\) MHz. This pulse contains \(N_p^{(in)} = 3.83 \times 10^6\) photons and propagates slowly, with little attenuation or distortion, through the BEC, with the group velocity \(v_g \propto \Omega_c^2/|\psi_1|^2\). As it is input, the pulse induces coherent transfer of atoms from \(|1\rangle\) to \(|2\rangle\). Once the pulse is completely input (\(0.6\) \(\mu s\)), \(N_2\) nearly reaches \(N_p^{(in)}\); see Fig. 2(c). Because of the finite bandwidth of the EIT transmission window there is some non-adiabatic loss from the BEC during the propagation, due to absorption, and so the number of spontaneous emission events \(N_{loss}\) reaches about \(0.21N_p^{(in)}\) at this point.

This pulse is sufficiently weak \(|\Omega_p| \ll |\Omega_p^{(in)}|\) that the coupling field \(\Omega_c\) is nearly homogenous across the BEC and \(\psi_1 \simeq \psi_1^{(G)}\) during the propagation. Then \(\psi_2\) is coherently driven into the dark state \(\psi_2 = -\psi_1^{(G)}(\Omega_p/\Omega_c^{(in)})\) and so acquires the amplitude and vortex phase pattern of the input LG mode. At \(t = 0.6\) \(\mu s\), \(\Omega_c\) is switched off in about \(0.2\) \(\mu s\), and \(N_2\) and \(N_{loss}\) are unaffected by this rapid switch-off; see Fig. 2(c). Both \(\Omega_c\) and \(\Omega_p\) smoothly ramp to zero intensity and the LG mode is then stored in \(\psi_2\). Figures 2(d-e) show the relative density profiles.
|ψ^2|^2/ρ, where ρ = |ψ_i|^2 + |ψ_2|^2 (note the density is cylindrically symmetric [23]), and cuts of the density and phase. The slow light propagation introduces virtually no phase gradient along z or r.

For times short compared to the BEC dynamics, determined by the GPEs [2] with Ω = 0 (typically milliseconds), any spatial mode can then be robustly stored. However, much longer storage can be achieved if the pulse parameters are chosen such that ψ_1 and ψ_2 remain nearly stationary. In the weak probe limit |ψ_1| ≈ ψ_1(G), while ψ_2 will evolve according to an effective potential V_{2-eff}(r), determined by the sum of the centrifugal potential m^2/r^2, the trap V(r) and the mean field potential U_{12}|ψ_1|^2; see Fig. 2(b). Our choice of width w and length (which is governed by the pulse length τ_0 and group velocity), are such that ψ_2 nearly matches the ground state of the potential V_{2-eff}, which is trapping and approximately harmonic for a_1 > a_12. This results in the dynamics being greatly suppressed, as seen in the second and third panels of Figs. 2(d-e). The dominant motion is a residual dipole sloshing in z, an unavoidable artifact of the group velocity and pulse length having a small dependence on r. For our parameters 148 ms ≈ the half-period of V_{2-eff}, so ψ_2 at this time is nearly the mirror image of ψ_2 at the switch-off time.

After an arbitrary time, one can then switch the coupling field back on, regenerating a probe pulse according to Ω_p = −ω_c |ψ_2/ψ_1|^2 which is then output. Fig. 2(c) shows N_2 and N_{loss} upon a switch-on at 148 ms. The total accumulated losses after the pulse has been output is 0.33N_{p}^{(in)} and so N_{p}^{(out)} = 0.67N_{p}^{(in)} photons are output, nearly identical to the 63% transmission we would expect if one had propagated the LG mode through the BEC without stopping and storing it. Furthermore, the vortex phase profile has been preserved and little other unwanted phase gradients are introduced.

We performed a series of numerical calculations of output pulses for the case in Fig. 2 but with switch-ons at various times. Since, ultimately, useful information storage requires an alphabet with several different angular momentum states, we also varied m. The dynamics are sufficiently suppressed and the variation of the storage efficiency T = N_{p}^{(out)}/N_{p}^{(in)} with storage time is not significant; see Fig. 2(D). The variations of T can be accurately estimated by the model of [23]. They are due to spatial features generated in the BECs during the dynamics which get written into small temporal features on the revived probe fields, resulting in additional EIT bandwidth loss. We will report more details of these studies elsewhere.

The general utility of this method for storing fields with a non-uniform phase profile depends not only on the number of photons output but also on maintaining the phase pattern during the storage. However, if, e.g., the vortex is pinned down by an external potential [23], the topological stability of the vortices in BECs makes the winding number m very robust, even in the presence of substantial unwanted phase gradients.

We have shown how vortices can be written from atomic to optical fields and vice-versa, providing a unique nonlinear technique to create optical vortices as well as directly image phase singularities in BECs. This method should also allow the transfer of vortex configurations between BECs which are optically connected. Furthermore, we considered the ability of a BEC to robustly store a vortex state input from light fields. While the case studied here allowed 70% efficiency, this number can be further improved by using larger BECs (with the loss ∝ 1/√D_1). We have found that an appropriate choice of parameters can suppress the BEC dynamics. The ultimate storage times should then only be limited by decoherence due to the small inelastic collision rates and thermal and quantum fluctuations (> 100 ms) [2].

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