Symmetric Collaborative Filtering Using the Noisy Sensor Model

Rita Sharma  
Department of Computer Science  
University of British Columbia  
Vancouver, BC V6T 1Z4  
rsharma@cs.ubc.ca

David Poole  
Department of Computer Science  
University of British Columbia  
Vancouver, BC V6T 1Z4  
poole@cs.ubc.ca

Abstract

Collaborative filtering is the process of making recommendations regarding the potential preference of a user, for example shopping on the Internet, based on the preference ratings of the user and a number of other users for various items. This paper considers collaborative filtering based on explicit multi-valued ratings. To evaluate the algorithms, we consider only pure collaborative filtering, using ratings exclusively, and no other information about the people or items.

Our approach is to predict a user's preferences regarding a particular item by using other people who rated that item and other items rated by the user as noisy sensors. The noisy sensor model uses Bayes' theorem to compute the probability distribution for the user's rating of a new item. We give two variant models: in one, we learn a classical normal linear regression model of how users rate items; in another, we assume different users rate items the same, but the accuracy of the sensors needs to be learned. We compare these variant models with state-of-the-art techniques and show how they are significantly better, whether a user has rated only two items or many. We report empirical results using the EachMovie database of movie ratings. We also show that by considering items similarity along with the users similarity, the accuracy of the prediction increases.

1 Introduction

Collaborative filtering is a key technology used to build Recommender Systems on the Internet. It has been used by Recommender Systems such as Amazon.com — a book store on the web, CDNow.com — a CD store on the web, and MovieFinder.com — a movie site on the internet [Schafer, Konstan and Riedl, 1999].

Collaborative filtering (CF) is the process of making predictions whether a user will prefer a particular item, given his or her ratings on other items and given other people's ratings on various items including the one in question. CF relies on the fact that people's preferences are not randomly distributed; there are patterns within the preferences of a person and among similar groups of people, creating correlation. The user for whom we are predicting a rating is called the active user. In collaborative filtering, the main premise is that the active user will prefer items which like-minded people prefer, or even that dissimilar people don't prefer. The problem can be formalized: given a set of ratings for various user-item pairs, predict a rating for a new user-item pair. It is interesting that the abstract problem is symmetric between users and items.

Collaborative filtering has been an active area of research in recent years. Several collaborative filtering algorithms have been suggested, ranging from binary to non-binary rating, implicit and explicit rating. Initial collaborative filtering algorithms were based on statistical methods using correlation between user preferences [Resnick, Iacovou, Suchak, Bergstrom and Riedl, 1994; Shardanand and Maes, 1995]. These correlation based algorithms predict the active user ratings as a similarity-weighted sum of the other users ratings. These algorithms are also referred to as memory based algorithms [Breses, Heckerman and Kadie, 1998]. Collaborative filtering is different to the standard supervised learning task because there are only two attributes, each with a large domain; it is the structure within the domains that are important to the prediction, but this structure is not provided explicitly. Recently, some researchers have used machine learning methods [Breese et al., 1998; Ungar and Fos-
ter, 1998] for collaborative filtering algorithms. These methods essentially discover the hidden attributes for users and items, which explain the similarity between users and items.

Breese et al. [Breese et al., 1998] proposed and evaluated two probabilistic models for model based collaborative filtering algorithms: cluster models and Bayesian networks. In the cluster model, users with similar preferences are clustered together into classes. The model's parameters, the number of clusters, and the conditional probability of ratings given a class are estimated from the training data. In the Bayesian network, nodes correspond to items in the database. The training data is used to learn the network structure and the conditional probabilities.

Pennock et al. [Pennock, Horvitz, Lawrence and Giles, 2000] proposed a collaborative filtering algorithm called personality diagnosis (PD) and showed that PD makes better predictions than other memory and model based algorithms. This algorithm is based on a probabilistic model of how people rate items, which is similar to our noisy sensor model approach.

In this paper we propose and evaluate a probabilistic approach based on a noisy sensor model, which is symmetric between users and items. Our approach is based on the idea that to predict an active user's rating for a particular item, we can use all those people who rated that item and other items rated by the active user as the noisy sensors. We view the noisy sensor model as a belief network. The conditional probability table associated with each sensor reflects the noise in the sensor.

To model how another user (user u) can act as a noisy sensor for the active user a's rating, we need to find a relationship between their preferences. Unfortunately, there is usually very little data, so we need to make a priori some assumptions about the relationship. Here we give two variants of the general idea for learning the noisy sensor model for explicit-multi-valued rating data: one, where we learn a classical normal linear regression model of how users rate items (Noisy1); and another, where we assume that the different users rate items the same and learn the accuracy of the sensor (Noisy2).

In order to avoid a perfect fit with sparse data we add some dummy points before fitting the relationship. We use hierarchical prior to distribute the effect of dummy points over all possible rating pairs.

After learning the noisy sensor model (i.e. the conditional probability table associated with each sensor node), we use Bayes' theorem to compute the probability distribution of the user a's rating of a new item.

We evaluate both Noisy1 and Noisy2 on the EachMovie database of movie ratings and compare them to the state-of-the-art techniques. We also show that symmetric collaborative filtering, which employs both user and item similarity, offers better accuracy than asymmetric collaborative filtering.

2 Filtering Problem and Mathematical Notation

Let N be the number of users and M be the total number of items in the database. S is an N x M matrix of all user's ratings for all items; Su i is the rating given by user u to item i. Let the ratings be on a cardinal scale with m values that we denote v1, v2, ..., vm. Then each rating Sui has a domain of possible values (v1, v2, ..., vm). In collaborative filtering, S, the user-item matrix, is generally very sparse since each user will only have rated a small percentage of the total number of items. Under this formulation, the collaborative filtering problem becomes predicting those Sui which are not defined in S, the user-item matrix.

3 Collaborative Filtering Using the Noisy Sensor Model

We propose a simple probabilistic approach for symmetric collaborative filtering using the noisy sensor model for predicting the rating by user a (active user) of an item j. We use as noisy sensors:

- all users who rated the item j
- all items rated by user a

The sensor model is depicted as a naive Bayesian network in Figure 1. The direction of the arrow shows that the prediction of user a for item j causes the sensor u to take on a particular prediction for item j. The sensor model is the conditional probability table associated with each sensor node. The noise in the sensor is reflected by the probability of incorrect prediction; that is, by the conditional probability table associated with it. To keep the model simple we use the independence assumption that the prediction of any sensor for item j is independent of others, given the prediction of user a for item j.

We need the following probabilities for Figure 1:

- \( Pr(S_{ui}|S_{aj}) \) : the probability of user u's prediction for item j, given the prediction of user a for item j.
- \( Pr(S_{ak}|S_{aj}) \) : the probability of user a's prediction for item k, given the prediction of user a for item j.
sparse data for the Bayes' rule we can have the fraction of rating user a's rating of an unseen item j, using the noisy sensor model. We need the probability table for probabilities:

\[ Pr(S_{aj}) : \text{the prior probability of active user } a\text{'s prediction for item } j. \]

We compute the prior probability distribution \( Pr(S_{aj} = v_t) \) of user a's rating for item j by the fraction of rating \( v_t \) in the training data set, where \( v_t \in \{v_1, v_2, \ldots, v_m\} \).

Given the conditional probabilities table for all sensors, we can compute the probability distribution for user a's rating of an unseen item j, using the noisy sensor model as described in Figure 1. By applying Bayes' rule we can have the following:

\[
Pr(S_{aj} | (S_{1j}, \ldots, S_{Nj}) \land (S_{a1}, \ldots, S_{aM})) = \propto Pr(S_{aj}) \cdot \prod_{u=1}^{N} Pr(S_{uj} | S_{aj}) \cdot \prod_{k=1}^{M} Pr(S_{ak} | S_{aj})
\]

To use the noisy sensor model for collaborative filtering we need the probability table for probabilities: \( Pr(S_{uj} | S_{aj}) \) and \( Pr(S_{ak} | S_{aj}) \).

Consider first the problem of estimating \( Pr(S_{uj} | S_{aj}) \), which is the problem of estimating user u’s rating for item j given user a’s rating of it. There is typically sparse data for the \( m \times m \) probability table and we need to make some prior assumptions about the relationship. We assume that there is a linear relationship with Gaussian error between the preferences of users and, similarly, between the ratings received by the items. Suppose the rating of user a (the independent variable) is denoted by \( x \), and that the rating of user u (the dependent variable) is denoted by \( y \). Suppose that user a and user u co-rated \( n \) items and their ratings over \( n \) co-rated items are denoted by \( n \) pairs of observations \( (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n) \). We want to find a straight line which best fits these points. We assume that the mean of \( y \) can be expressed as a linear function of independent variable \( x \). Since a model based on an independent variable cannot in general predict exactly the observed values of \( y \), it is necessary to introduce the error \( e_i \). For the \( i^{th} \) observation, we have the following:

\[ y_i = \alpha + \beta x_i + e_i \]

We assume that unobserved errors \( (e_i) \) are independent and normally distributed with a mean of zero and the variance \( \sigma^2 \). If \( y_i \) is the linear function of \( e_i \), which is normally distributed, then \( y_i \) is also normally distributed. We assume the same variance for all the observations. The mean and variance of \( y_i \) are given thus:

\[ E(y_i) = \alpha + \beta x_i \]
\[ var(y_i) = \sigma^2 \]

For the \( i^{th} \) observation, the probability distribution function of \( y \) which is normally distributed can be written thus:

\[
P(y = y_i | x = x_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{1}{2\sigma^2} (y_i - \mu_i)^2 \right]
\]

where \( \mu_i = \alpha + \beta x_i \).

The joint probability distribution function (or the likelihood function) denoted by \( LF(\alpha, \beta, \sigma^2) \) is the product of the individual \( P(y_i | x_i) \) over all observations.

\[
LF(\alpha, \beta, \sigma^2) = \prod_{i=1}^{n} P(y_i | x_i)
\]
\[
= \frac{1}{(\sqrt{2\pi\sigma^2})^n} \exp \left[ -\frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - \mu_i)^2 \right]
\]

We apply the maximum likelihood method [Gujarati, 1995] to estimate unknown parameters \( (\alpha, \beta, \sigma^2) \). The likelihood is maximum at the following values of the parameters:

\[
\alpha = 1/n (\sum y_i - \beta \sum x_i)
\]
\[
\beta = n \sum \frac{y_i - \sum y_i x_i}{\sum x_i^2 - \sum x_i^2}
\]
\[
\sigma^2 = \frac{1}{n} \sum (y_i - \alpha - \beta x_i)^2 = \frac{1}{n} \sum e_i^2
\]

After calculating the parameters \( \alpha, \beta \) and \( \sigma^2 \) we can write the expression of the probability distribution of user u’s preference for item j given the user a’s preference for it as follows:

\[
Pr(S_{uj} = x_{uj} | S_{aj} = x_{aj}) = P(y = x_{uj} | x = x_{aj})
\]
The rating is defined as follows:

\[
Pr(S_{aj}|S_{a}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-1}{2\sigma^2} (x_{aj} - (\alpha + \beta x_{aj}))^2\right]
\]

To estimate \(Pr(S_{ak}|S_{aj})\), we use the same model as described above for computing \(Pr(S_{aj}|S_{a})\). In this case the independent variable \(x\) denotes the rating received by item \(j\), while dependent variable \(y\) denotes the rating received by item \(k\). And, the \(n\) pairs of observations \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) are the ratings over item \(j\) and \(k\) by those \(n\) users, who have co-rated both items \(j\) and \(k\).

After computing the probability distribution \(Pr(S_{aj}|S_{a})\) for all users \(a\) who rated item \(j\), and \(Pr(S_{ak}|S_{aj})\) for all items \(k\) rated by user \(a\), we can compute the probability distribution of the user \(a\) rating for item \(j\) using Equation (1). In this model we need to compute \(3 \ast (u + i)\) free parameters \((\alpha, \beta, \text{and } \sigma^2)\), where \(u\) is the number of users who rated the item \(j\) and \(i\) is the number of items rated by user \(a\).

When the linear relationship exceeds the maximum value of the rating scale, we use the maximum value; when it is lower than the minimum value of the rating scale, we use the minimum value.

To predict a rating (for example, to compare it with other algorithms that predict ratings), we predict the expected value of the rating. The expected value of the rating is defined as follows:

\[
E(S_{aj}) = \sum_{i=1}^{m} Pr(S_{aj} = v_i|X) \ast v_i
\]

where \(X = (S_{ij}, \ldots, S_{Nj}) \wedge (S_{a1}, \ldots, S_{aM})\).

### 3.1 K Dummy Observations

While trying to fit lines with sparse data, we often find a perfect linear relationship, even though the sensor isn’t perfect. If there is a perfect fit between users \(a\) and \(u\), then the variance will be zero according to the above calculations. Therefore, the sensor \(u\)’s prediction for item \(j\) will be perfect, or deterministic; that is, the conditional probability table associated with sensor \(u\) will be purely deterministic. We do not want this for our noisy sensor model because a deterministic sensor will discount the effects of other sensors. For example, often one or two co-rated items always have a perfect fit, even though such a user is not a good sensor.

We hypothesize that this problem can be overcome if we add \(K\) dummy observations along with \(n\) observations (co-rated items). We assume that user \(a\) and user \(u\) give ratings over \(K\) dummy items \((K > 0)\) in such a way that their ratings for \(K\) dummy items are distributed over all possible rating pairs. For \(m\) scale rating data there are \(m^2\) possible combination of the rating pairs. We compute the prior distribution of each rating pair by its frequency in the training data. We use the prior distribution of rating pairs for distributing the effect of \(K\) dummy points over all rating pairs like hierarchical prior. This, however, reduces our ability to guarantee the ratings for \(K\) items will be distributed over all possible rating pairs. We have experimented with parameter \(K\), and we found that Noisy1 performs better with \(K = 1\). For subsequent experiments we, therefore, chose \(K = 1\) for Noisy1.

### 3.2 Selecting Noisy Sensors

For determining the reliability of the noisy sensors, we consider the goodness of fit of the fitted regression line to a set of observations. We use the coefficient of determination \(r^2\) [Gujarati, 1995], a measure that tells how well the regression line fits the observations. This coefficient measures the proportion or percentage of the total variation in the dependent variable explained by the regression model. \(r^2\) is calculated as follows [Gujarati, 1995]:

\[
r^2 = 1 - \frac{\sum e_i^2}{\sum (y_i - \bar{y})^2}
\]

where \(\bar{y}\) is the mean of the ratings of user \(u\).

The value of \(r^2\) lies between 0 and 1 \((0 \leq r^2 \leq 1)\). If \(r^2 = 1\), there exists a perfect linear relationship between the preferences of user \(a\) and user \(u\); that is, \(e_i = 0\) for each observation (co-rated item). On the other hand, if \(r^2 = 0\), it means there is no linear relationship between users \(a\) and \(u\) and the best fit line is horizontal line going through the mean \(\bar{y}\). We order the user and item noisy sensors according to \(r^2\). We use the best \(U\) user noisy sensors and best \(I\) item noisy sensors for making the predictions. The parameter settings for \(U\) and \(I\) are explained in the next section.

### 3.3 Variant Noisy2 of Noisy1

The problem with Noisy1 is that we must often fit linear relationships with very little data (co-rated items). It may be better to assume \(a\) priori the linear model and then simply learn the noise. The algorithm Noisy2 is based on the idea that different users rate items the same and, similarly, different items receive the same rating. We assume that the preferences of user \(a\) and user \(u\) are the same; that is, the expected value of user \(u\)’s preference of any item is equal to active user \(a\)’s preference for that item.

\[
E(y_i|x = x_i) = \mu = x_i
\]

We learn the variance of user \(u\)’s prediction. The algorithm Noisy2 can be derived from algorithm Noisy1.
by making the regression coefficients $\alpha = 0$ and $\beta = 1$. In this model we need to compute $(u + i)$ free parameters ($\sigma^2$), where $u$ is the number of users who rated the item $j$ and $i$ is the number of items rated by user $a$. We also add the $K$ dummy observations because the same problem (as discussed in Subsection 3.1) can arise in $Noisy2$. We have experimented with parameter $K$, and we found that $Noisy2$ also performs better with $K = 1$. For subsequent experiments we, therefore, chose $K = 1$ for $Noisy2$ also.

In $Noisy2$ we are not fitting the relationship between user $a$ and user $u$, but we assume an equal relationship. So, it doesn’t make sense to use the coefficient of determination $r^2$ for finding the reliability of the noisy sensors. Rather, to find the reliability of the noisy sensor, we use the variance; the less variance, the more reliable the noisy sensor. We use the best $U$ user noisy sensors and best $I$ item noisy sensors for making the predictions. The parameter settings for $U$ and $I$ are explained in the next section.

4 Evaluation Framework

To evaluate the accuracy of collaborative filtering algorithms we used the training and test set approach. In this approach, the dataset of users (and their ratings) is divided into two: a training set and a test set. The training set is used as the collaborative filtering dataset. The test set is used to evaluate the accuracy of the collaborative filtering algorithm. We treat each user from the test set as the active user. To carry out testing, we divide the ratings by each test user into two sets: $I_a$ and $P_a$. The set $I_a$ contains ratings that we treat as observed ratings. The set $P_a$ contains the ratings that we attempt to predict using a CF algorithm and observed ratings ($I_a$) and training set.

To evaluate the accuracy of the collaborative filtering algorithm we use the average absolute deviation metric, as it is the most commonly used metric [Breese et al., 1998]. The lower the average absolute deviation, the more accurate the collaborative filtering algorithm is. For all users in the test set we calculate the average absolute deviation of the predicted rating against the actual rating of items. Let the number of predicted ratings in the test set for the active user be $n_a$; then the average absolute deviation for a user is given as follows:

$$S_a = \frac{1}{n_a} \sum_{j \in P_a} |p_{a,j} - r_{a,j}|,$$

where $p_{a,j}$ is user $a$'s observed rating for item $j$ and $r_{a,j}$ is user $a$'s predicted rating for item $j$.

These absolute deviations are then averaged over all users in the test set.

4.1 Data and protocols

We compared both versions of our noisy sensor model to PD (Personality Diagnosis) [Pennock et al., 2000] and Correlation (Pearson Correlation) [Resnick et al., 1994]. To compare the performance we used the same subset of the EachMovie database as used by Breese et al. [Breese et al., 1998] and Pennock et al. [Pennock et al., 2000], consisting of 1,623 movies, 381,862 ratings, 5,000 users in the training set, and 4,119 users in the test set. In EachMovie database the ratings are elicited on an integer scale from zero to five. We also tested the algorithms on other subsets to verify that we are not finding peculiarities of the subset.

We ran experiments with different amounts of ratings in set $I_a$ to understand the effect of the amount of the observed ratings on the prediction accuracy of the collaborative filtering algorithm. As in [Breese et al., 1998] for the AllBut1 Protocol, we put a single randomly selected rating for each test user in the test set $P_a$ and the rest of the ratings in the observed set $I_a$. As in [Breese et al., 1998] for each GivenX Protocol, we place $X$ ratings randomly for each test user in the observed set $I_a$, and the rest of the ratings in the test set $P_a$. We did the experiments for $X = 2, 5$ and 10.

4.2 Selecting Noisy Sensors

After learning the noisy sensor model we determine which noisy sensors should be used in making the prediction for the active user. Figure 2 shows the variation of average absolute deviation with best user noisy sensors for different numbers of best item noisy sensors for Noisy1. We used the dataset as described above but the test rating and the observed ratings for each user of the test set were selected randomly.

Figure 2 shows that the average absolute deviation decreases with the increase in number of item sensors. There is no significant improvement in accuracy when the number of item sensors is more than twenty. It also shows that the average absolute deviation first decreases with the increase in number of user sensors and then increases as more user sensors are used for prediction. This is because the large number of user sensors results in too much noise for those user sensors that have good reliability.

From the experiments, we found that both algorithms give better performances with ten-to-twenty item noisy sensors and forty-to-seventy user noisy sensors. For the experiments reported in the following section, we use the best fifty user noisy sensors and.

2We didn't use the test set for finding the number of user and item noisy sensors.
The number of best user noisy sensors for different best twenty item noisy sensors (i.e., $U = 50$ and $I = 20$) for both Noisy1 and Noisy2. The parameters $U$ and $I$ depend on the database. In the case of the Each-Movie database, the number of users are more than the movies, also each user has rated only few movies. Due to this possibility more best user noisy sensors ($U$) are selected than the best item noisy sensors ($I$).

From Figure 2 we also see that the minimum average absolute deviation is .936 when we use both user and item noisy sensors (with sixty user and twenty item noisy sensors). It is .964 when we use only the user noisy sensors, shown by the zero item noisy sensors case, and 1.027 when we use only the item noisy sensors, shown on the y-axis for ten item noisy sensors. This indicates that when we include the item noisy sensors along with the user noisy sensors, the quality of the prediction improves considerably. It also shows that if we use only the item noisy sensors for prediction, then the average absolute deviation becomes greater than when we use only user noisy sensors. Therefore, symmetric collaborative filtering offers better accuracy than asymmetric collaborative filtering.

### 4.3 Comparison with Other Methods

We compared the algorithms Noisy1, Noisy2, Correlation and PD using the same training and test set as Pennock et al. [Pennock et al., 2000]. For each test user in the test set we use the same set of observed ($I_a$) and test ratings ($P_a$) as Pennock et al.

The results of comparing Noisy1, Noisy2, Correlation and PD are shown in Table 1. We re-implemented Personality Diagnosis. Our results for Personality Diagnosis match exactly with those reported in Pennock et al. [Pennock et al., 2000]. We took the results for Correlation directly from Pennock et al. [Pennock et al., 2000]. Noisy1 performed better than PD and Correlation for AllBut1 and Given10 protocols. For Given5 and Given2 protocols Noisy1 performance is better than Correlation but not better than PD. We believe that Noisy1’s poor performance can be explained by the fact the lines that are fitted to very small data sets are often poor fit to the actual relationship. The algorithm Noisy2, based on an equal relationship between users, doesn’t suffer from the same problem, and outperformed all algorithms under all protocols.

Table 1: Average absolute deviation scores on the Each-Movie data for Noisy1, Noisy2, PD and Correlation (note: lower scores are better).

| Algorithm | Protocol |
|-----------|----------|
|           | AllBut1  | Given10 | Given5  | Given2  |
| Noisy2    | .893     | .943    | .974    | 1.012   |
| Noisy1    | .943     | .983    | 1.021   | 1.196   |
| PD        | .964     | .986    | 1.016   | 1.039   |
| Correl    | .999     | 1.069   | 1.145   | 1.296   |

Shardanand and Maes [Shardanand and Maes, 1995] and Pennock et al. [Pennock et al., 2000] proposed that the accuracy of a collaborative filtering algorithm should be evaluated on extreme ratings (very high or very low ratings) for items. The supposition is that, most of the time, people are interested in suggestions about items they might like or dislike, but not about items they are unsure of. Pennock et al. [Pennock et al., 2000] defined the extreme ratings as those which are 0.5 above and 0.5 below the average rating in the subset. To compare the performance of algorithms with extreme ratings we computed the predicted ratings for those test cases from the test set $P_a$ of all protocols, whose observed rating is less than $R - 0.5$ or greater than $R + 0.5$, where $R$ is the overall average rating in the subset.

Table 2: Average absolute deviation scores on Each-Movie data for Noisy1, Noisy2, PD and Correlation for extreme ratings.

| Algorithm | Protocol |
|-----------|----------|
|           | AllBut1  | Given10 | Given5  | Given2  |
| Noisy2    | 1.001    | 1.057   | 1.087   | 1.124   |
| Noisy1    | .997     | 1.063   | 1.125   | 1.249   |
| PD        | 1.029    | 1.087   | 1.128   | 1.163   |
| Correl    | 1.108    | 1.127   | 1.167   | 1.189   |
The results for the extreme ratings are shown in Table 2. The results for extreme ratings show that Noisy1 performs better than Noisy2 for AllBut1 protocol. It also performs better than PD and Correlation over Given10 and Given5 protocols. Noisy2 performs better than the other three algorithms over Given10, Given5 and Given2 protocols.

Table 3: Significance levels of the differences in average absolute deviation between Noisy1 and PD, and between Noisy2 and PD, on EachMovie data (note: low significance levels indicate that the differences in results are unlikely to be coincidental).

| Protocol   | Noisy1 vs. PD | Noisy2 vs. PD |
|------------|---------------|---------------|
| AllBut1    | .0006         | .0001         |
| Given10    | .1377         | .0001         |
| Given5     | .9064         | .0001         |
| Given2     | .9999         | .0001         |

To determine the statistical significance of these results, we computed the significance levels for the differences in average absolute deviation between Noisy1 and PD, and between Noisy2 and PD, on EachMovie data. To do this, we divided the test set for all protocols into 60 samples of equal size and used randomization paired sample test of differences of means [Cohen, 1995]. This method calculates the sampling distribution of the mean difference between two algorithms by repeatedly shuffling and recalculating the mean difference in 10,000 different permutations. The shuffling reverses the sign of the difference score for each sample with a probability of .5.

The statistical significance results of the EachMovie data results are shown in Table 3; it shows the probability of achieving a difference less than or equal to the mean difference. That is, it shows the probability of incorrectly rejecting the null hypothesis that both algorithms' deviation scores arise from the same distribution.

5 Conclusion

In this paper, we have concerned ourselves with symmetric collaborative filtering based on explicit ratings used for making recommendations to a user based on ratings of various items by a number of people, and the user's ratings of various items.

We have described a new probabilistic approach for symmetric collaborative filtering based on a noisy sensor model. We have shown that the noisy sensor model makes better predictions than other state-of-the-art techniques. The results for Noisy2 are highly statistically significant. We have also shown that by including the items similarity along with users similarity, the accuracy of prediction increases. This paper has only considered the accuracy of the noisy sensor model, not on the computational issues involved. It is beyond the scope of this paper to consider the trade-off between off-line and online computation and effective indexing to find the best matches.

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