A new class of de Sitter vacua in String Theory Compactifications

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We revisit the stability of the complex structure moduli in the large volume regime of type-IIB flux compactifications. We argue that when the volume is not exponentially large, such as in Kähler uplifted dS vacua, the quantum corrections to the tree-level mass spectrum can induce tachyonic instabilities in this sector. We discuss a Random Matrix Theory model for the classical spectrum of the complex structure fields, and derive a new stability bound involving the compactification volume and the (very large) number of moduli. We also present a new class of vacua for this sector where the mass spectrum presents a finite gap, without invoking large supersymmetric masses. At these vacua the complex structure sector is protected from tachyonic instabilities even at non-exponential volumes. A distinguishing feature is that all fermions in this sector are lighter than the gravitino.

It has been known for decades that String Theory has low energy solutions describing a four-dimensional universe with negative or zero cosmological constant, with the extra six dimensions “compactified” (for a review see [1-4]). From the four-dimensional point of view the compactified space is described by a set of fields called the moduli which describe, roughly, the size and shape of the extra dimensions. A much harder question is whether string theory can describe a four-dimensional universe with broken supersymmetry and a positive cosmological constant, a so-called de Sitter vacuum (dS), with a metastable compactification. In type IIB String Theory this question has been answered positively in a few scenarios, the best studied being the KKLT [5] constructions, Large Volume Scenarios (LVS) [6, 7] and the so-called Kähler uplifted vacua [8-14]. The effective low energy theories describing these models typically involve hundreds of moduli fields, which can be divided into two classes: Kähler moduli and complex structure moduli. In addition we also have the dilaton, whose expectation value determines the string coupling constant. The interactions among all these fields are given by a complicated scalar potential, what makes a detailed perturbative stability analysis of these vacua unfeasible except in very simplified scenarios. In type-IIB flux compactifications, at the classical level, the scalar potential is induced by the presence of background fluxes (higher dimensional generalisations of electromagntic fields) on the compact space [15]. Due to a Dirac condition these fluxes need to be quantised, and are therefore characterised by a set of integers. This leading contribution of the scalar potential depends only on the dilaton and the complex structure moduli (for short, the complex structure sector), and therefore it is necessary to take into account quantum effects in order to fix the remaining Kähler moduli.

To make the problem more tractable, it is often assumed that the background fluxes provide an effective stabilisation mechanism for the complex structure sector, and it is not considered any further. The consistency of this approach has been checked for KKLT vacua [16-21] and Large Volume Scenarios [22, 23]. Here we discuss this matter for Kähler uplifted dS vacua.

In the large volume regime of type IIB flux compactifications, both for LVS and Kähler uplifted dS vacua, the stabilisation of the Kähler moduli is a result of the competition between the leading non-perturbative and $\alpha'$ (radiative) quantum corrections [1]. For these corrections to be under control it is necessary that the volume of the compactification, which belongs to the Kähler sector, has a large expectation value compared to the string length. A large compactification volume is also essential for the consistency of the 4-dimensional supergravity description of these models, and in particular for the Kaluza-Klein scale to be large compared to the supersymmetry breaking scale [7, 23]. In LVS the vacuum obtained in this way have a negative cosmological constant (AdS), and thus additional interactions are needed to make the cosmological constant positive. Kähler uplifted vacua are particularly interesting because the dS vacuum is achieved without the need for extra ingredients (matter or branes), just with an appropriate tuning of the parameters. The downside of the latter models is that the volume is fixed only at moderately large values, narrowing the regime of validity of the effective field theory.

**Stability of the complex structure sector**— An underlying assumption of many constructions based on the scenarios above is that, with the right choice of fluxes, the complex structure sector can be stabilised at a supersymmetric configuration where the masses of fermions and scalars are much larger than the relevant cosmolog-

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1 See [24-26] for discussions on the effect of string loop ($g_s$) corrections in these models.
cal energy scales. In that case, this sector can be safely integrated out, and then the attention is focused on the stabilisation of the lighter Kähler moduli, which is much trickier. While this is a reasonable starting point, we will argue that, at least in Kähler uplifted scenarios, this assumption becomes untenable as the number of complex structure moduli increases. We will show that this observation leads to further constraints on the parameter space of the model which are more restrictive than those derived from the consistency of the effective field theory. Moreover, for very large number of moduli – typical numbers are in excess of $O(100)$ – a new class of stable vacua emerges, in which the fermions of the complex structure sector are all lighter than the gravitino.

In LVS and Kähler uplifted vacua, the potential that stabilises the moduli is a small deformation of the tree-level potential, with quantum corrections suppressed by the volume $V$ of the compact Calabi-Yau space \[ V = V_{\text{tree-level}} + m_{3/2}^2 \times O(\hat{\xi}/V), \] (1)

Here the parameter $\hat{\xi}/V$ characterises the magnitude of the leading quantum corrections, and we have written explicitly its dependence on the volume for clarity (see 10, 25, 26). The tree-level potential is positive semi-definite and is of the “no-scale” type: it is flat for the Kähler moduli leaving undetermined the expectation values of these fields, and in particular the overall volume $V$ and gravitino mass $m_{3/2}$. The dilaton and complex structure moduli are stabilised at a supersymmetric configuration (3) that is determined by the fluxes and by the geometric and topological properties of the compactified space [13, 15]. This configuration defines a Minkowski vacuum where, in general, supersymmetry is broken by the Kähler moduli.

An important point is that, if we ignore quantum corrections, there is a relation between the masses of the fermions $m_\lambda$ (with $\lambda$ running through the complex structure moduli and dilaton) and the squared masses of the scalars in the complex structure sector \[ \mu_{\pm \lambda}^2 = (m_{3/2} + m_\lambda)^2. \] (2)

At tree level, there are no instabilities in the supersymmetric sector, since the potential is non-negative and the no-scale vacuum is Minkowski. But note that, for every fermion in the supersymmetric sector with the same mass as the gravitino, there is a massless scalar in the tree-level spectrum. The sign of the $m_{3/2}^2 \times O(\hat{\xi}/V)$ quantum corrections is unknown so these massless scalars are not protected and can become perturbatively unstable (tachyonic, $\mu^2 < 0$). The same is true for sufficiently light scalars, to which we turn next, but first we need to characterise the spectrum of fermion masses.

The model — The tree-level fermion mass spectrum of the complex structure sector is determined by the geometry of the internal space and by the configuration of background fluxes. However, the high complexity of these theories make a detailed calculation impractical in generic compactifications, so instead, we will follow a statistical approach. Intuitively, it is clear that, as the number of complex structure moduli increases, so does the probability that there are fermions with tree-level masses close to the gravitino mass, and with it the expected percentage of very light scalars that are susceptible of becoming tachyonic by the effect of $\hat{\xi}/V$ corrections. This intuition can be made quantitative in the framework of random matrix theory (RMT) [31–37], and was confirmed in great detail in 24.

The idea is to promote to random variables the entries of the fermion mass matrix and then to characterise the spectrum of these matrices using standard techniques from RMT [33–35]. The universality theorems in RMT ensure that the result depends only mildly on the (unknown) distribution of the couplings for sufficiently large matrices [33, 35], and therefore is insensitive to the details of the compactification [14]. Assuming that all complex structure moduli can be treated on equal footing, i.e. statistical isotropy in field space, the appropriate ensemble to represent the fermion mass matrix is the Altland-Zirnbauer CI matrix ensemble [33, 35]. Proceeding in this way, and using the relation (2), the authors of [24] constructed a random matrix model to characterise the tree-level scalar mass spectrum of the complex structure sector in type IIB flux compactifications. In the limit where the number of (complex) fields is large, $N \to \infty$, the spectral density $\rho(\mu^2)$ for the tree-level scalar masses converges with order one probability to a particularly simple form

\[ \rho(\mu^2) = \frac{2 N m_{3/2}^2}{\pi m_h^2 \mu} \left[ \Theta \left( m_h^2 - (m_{3/2} + \mu)^2 \right) \sqrt{m_h^2 - (m_{3/2} + \mu)^2} + \Theta \left( m_h^2 - (m_{3/2} - \mu)^2 \right) \sqrt{m_h^2 - (m_{3/2} - \mu)^2} \right], \]

where $\Theta(x)$ is the Heaviside theta function. It is important to stress that the spectral density \[ is just the most likely scalar mass spectrum predicted by the random matrix theory model. Thus, it is possible to find vacua with a different mass spectrum, but they occur with an exponentially suppressed probability [10, 12].

A new class of vacua — The spectral density \[ depends on two free parameters $m_h$ and $m_{3/2}$, which represent the mass scale of the fermions in the complex structure sector and the gravitino mass, respectively. To be precise, the parameter $m_h$ is defined as the expectation value of the largest fermion mass $m_h \equiv E[m_{max}]$, and is related to the flux energy scale, $m_h \sim M_p/V$.

\[ 2 \text{ See } 37 \text{ for a recent discussion on the applicability of random matrix theory to study flux compactifications.} \]
where $M_p$ stands for the Planck mass. The gravitino mass is determined by the volume and the expectation value of the flux superpotential $W_0$, $m_{3/2} = M_p |W_0|/\sqrt{V}$. Figure 1 shows the typical tree-level spectrum [3] of the complex structure sector. Notice the accumulation of very light scalars in the case when the heaviest fermion is heavier than the gravitino, ($m_h > m_{3/2}$). By contrast, if the heaviest fermion in the complex structure sector is lighter than the gravitino, ($m_h < m_{3/2}$), the scalar density develops a mass gap. In the latter regime it is also possible to find atypical vacua, i.e. deviations from [3], where the smallest scalar mass is comparable to the quantum corrections, however the fraction of such vacua is exponentially suppressed [29].

$$P(\mu_{\min}^2 < \frac{\xi}{\sqrt{V}}) \sim e^{-\frac{4}{3} N \xi \frac{x}{\mu_{\min}^2}}, \quad x = \left(1 - \sqrt{\frac{\xi}{V}}\right) \frac{m_{3/2}^2}{m_h^2} - 1. \quad (4)$$

The conclusion is that, for very large numbers of moduli, $N \sim O(100)$, requiring the de Sitter vacua to be free from tachyonic instabilities in the supersymmetric sector favours vacua with all fermions lighter than the gravitino. We will denote these stable de Sitter configurations “LSF vacua”, which stands for “Light(er) Supersymmetric Fermions”. Note that lighter than the gravitino does not necessarily mean light; the actual fermion masses can easily be in the Grand Unification scale as long as the gravitino is even heavier.

**Comparison with KKLT and LVS regimes**—The KKLT scenario corresponds to fine-tuning the fluxes so that the complex structure moduli have large masses compared with the supersymmetry breaking scale, which is set by the gravitino mass, that is $m_h \gg m_{3/2}$ (left). In this regime the stability of this sector is guaranteed since the tree-level masses are large compared to the contributions induced by quantum effects. In LVS and in Kähler uplifted vacua the absence of fine-tuning of $W_0$ implies that the fermions typically have masses comparable (but not necessarily close) to the gravitino mass, so that generically we have $\mu_{\alpha}^2 \sim O(m_{3/2}^2)$ [1][22][43]. The corrections to the tree-level spectrum can still be consistently neglected in this setting as long as the volume of the compactification is exponentially large, and thus the corrections are tiny, $\frac{\xi}{\sqrt{V}} \sim 10^{-10}$. For Kähler uplifted vacua this is no longer true. Since the volume is not exponentially large, typically we have $\frac{\xi}{\sqrt{V}} \sim 10^{-2} - 10^{-4}$ [9][10][12], implying that the corrections in [1] could in principle induce tachyonic instabilities if some of the complex structure moduli are sufficiently light at tree-level,

$$\mu_{\alpha}^2 |_{\text{tree-level}} \lesssim m_{3/2}^2 \times O\left(10^{-2} - 10^{-4}\right). \quad (5)$$

Figure 2 shows the percentage of scalar moduli estimated using [3] that are light enough to be destabilized by the quantum corrections, for a range of values of $\xi/\sqrt{V}$. Note that, for moderately large volumes $\xi/\sqrt{V} \sim 0.01$, this fraction can rise up to a 6–7%, with the maximum occurring at $m_h = \sqrt{2} m_{3/2}$.

Requiring that the number of light fields, $N_{\text{light}}$, is less than one irrespective of the details of the stabilisation of the complex structure sector, i.e. regardless of the value of the mass parameter $m_h$, we find a bound for the size of the $\alpha'$ corrections

$$\max\{N_{\text{light}}\} \approx \frac{4N}{\pi} \sqrt{\frac{\xi}{\sqrt{V}}} \ll 1 \quad \Rightarrow \quad \frac{\xi}{\sqrt{V}} \ll \frac{\pi^2}{16N^2}. \quad (6)$$

Note that in a generic compactification with hundreds
fermions in the supersymmetric sector are massless. The Massless Fermion Limit (MFL) is not always realised at a physical vacuum, because the massless condition may require non-integer values of the fluxes that are not actually realised. However, it provides the “lamppost” near which actual stable vacua may be found.

This brings us to another important point. The explicit examples of Kähler uplifted vacua constructed to date [9] [12] [13] have been found in models consistent with the supersymmetric truncation of a large sector of the moduli fields [12] [52–58]. This can be achieved by considering special points of the moduli space, for instance fixed points of global symmetries of the moduli space metric, where the majority of the fields can be fixed at a supersymmetric configuration. By this procedure it is possible to obtain a reduced theory involving, in addition to the Kähler moduli, a small fraction of the complex structure fields and the dilaton so that a detailed stability analysis is possible. In particular, in the examples discussed in [9] [12] the complex structure moduli surviving the truncation were fixed at vacua with large supersymmetric masses, i.e. $m_\alpha \gg m_{3/2}$, that is, imposing a large hierarchy between the masses of these fields and the supersymmetry breaking scale. This method ensures the stability of the field configuration in the reduced theory, however it cannot guarantee that the truncated fields are fixed at minima of the potential and, for this reason, neither does it guarantee the consistency of this reduction. It is therefore crucial to understand under what conditions it is possible to ensure the stability of the full set of moduli fields, including the truncated ones.

In paper [51] it is also shown that, when the fraction of complex structure fields surviving the truncation are stabilised at the MFL of a critical point, then all the complex structure fields (including the truncated ones) and the dilaton have a mass equal to $m_{3/2}$ at tree-level, i.e. the full sector is also at the MFL of the vacuum. Given that it is not feasible to check the stability of hundreds of supersymmetric moduli –except perhaps in very special cases–, we would like to suggest a compromise: apply the usual analytic and numerical techniques to check stability of the surviving low-energy sector (typically, the Kähler moduli and the complex moduli that sit at points of enhanced symmetry) and supplement these with the use of random matrix theory techniques to assess the stability of the truncated moduli that do not appear in the low energy description. Here we made use of the random matrix theory model presented in [29] to characterise the mass spectrum of the complex structure sector. Our conclusion –in line with our previous work in [27,29]– is that, in compactifications where the number of complex structure moduli is very large, there is a class of stable flux configurations, not previously considered, in which all fermions of the supersymmetric sector—including truncated ones– are lighter than the gravitino.

of complex structure moduli this bound is much more restrictive than just requiring the $\alpha'$ corrections to be small, $\xi/V \ll 1$. This remark is particularly relevant for dS solutions and inflationary models built with the method of Kähler uplifting that do not satisfy the constraint [9] (see examples in [9] [10] [13] [14] [44] [45]), as this signals the possible presence of tachyonic instabilities. Other models which could be affected by the same issue are those based on LVS vacua where the volume is only moderately large [10] [13]. In all these constructions one could still search for atypical vacua where all fields in the complex structure sector are much heavier than the gravitino, as in KKLT scenarios. However the probability of such vacua is exponentially suppressed as, without fine-tuning $W_0$, the parameters satisfy $m_h \sim m_{3/2}$, and thus [29] [55]

$$P(\mu_{\min}^2 \geq m_{3/2}^2) \sim e^{-\frac{m_{3/2}^2}{m_1^2} N^2} \ll 1.$$  

By contrast, LSF vacua, where all fermions are lighter than the gravitino, occur with probability of order one when $m_h \lesssim m_{3/2}$ ($|W_0| \gtrsim 1$), and thus they are a more natural configuration to stabilise the complex structure sector in this regime. In figure 2 it can be seen that when LSF vacua become dominant, the typical spectrum contains no light fields, a direct consequence of the appearance of the mass gap. Other scenarios which satisfy constraint [9] are [11] [14] [47] [50].

**Discussion**— Having established that LSF vacua are stable, the next question is how to find them. Reference [51] provides a systematic way of looking for LSF vacua by looking in the vicinity of configurations in which all

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**Figure 2.** Percentage of (real) scalars in the complex structure sector with tree-level masses smaller than the size of the leading quantum corrections, $\mu^2 \lesssim m_{3/2}^2$: $\xi/V$. The horizontal axis represents the typical mass scale in this sector, $m_h$. The spectrum of perturbations of the LSF vacua ($m_h < m_{3/2}$), contains no light scalar modes at tree-level. Stability is also ensured if there is a large hierarchy between the masses of the supersymmetric complex structure sector and the supersymmetry breaking scale, $m_h \gg m_{3/2}$ (KKLT regime), or an exponentially large volume, $\xi/V \sim 10^{-10}$ (LVS).
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