Optical conductivity of a 2DEG with anisotropic Rashba interaction at the interface of LaAlO$_3$/SrTiO$_3$

Alestin Mawrie and Tarun Kanti Ghosh

Department of Physics, Indian Institute of Technology-Kanpur, Kanpur-208 016, India

E-mail: amawrie@iitk.ac.in

Received 18 January 2016, revised 22 April 2016
Accepted for publication 10 May 2016
Published 24 August 2016

Abstract

We study optical conductivity of a two-dimensional electron gas with anisotropic $k$-cubic Rashba spin–orbit interaction formed at the LaAlO$_3$/SrTiO$_3$ interface. The anisotropic spin splitting energy gives rise to different features of the optical conductivity in comparison to the isotropic $k$-cubic Rashba spin–orbit interaction. For large carrier density and strong spin–orbit couplings, the density dependence of Drude weight deviates from the linear behavior. The charge and optical conductivities remain isotropic despite anisotropic nature of the Fermi contours. An infinitesimally small photon energy would suffice to initiate interband optical transitions due to degeneracy along certain directions in momentum space. The optical conductivity shows a single peak at a given photon energy depending on the system parameters and then falls off to zero at higher photon energy. These features are lacking for systems with isotropic $k$-cubic Rashba spin–orbit coupling. These striking features can be used to extract the information about nature of the spin–orbit interaction experimentally and illuminate some light on the orbital origin of the two-dimensional electron gas.

Keywords: optical properties of low-dimensional, mesoscopic and nanoscale materials and structures, conductivity phenomena in semiconductors and insulators, spin–orbit coupling

(Some figures may appear in colour only in the online journal)
to the lifting of the spin-degeneracy of the six $t_2g$ orbitals in STO [17]. Moreover, the $d_{xy}$ orbitals are confined in the $x$–$y$ plane and are localized at the interface due to impurities and electron–phonon coupling [18], whereas the electrons associated with the $d_{xz}$ and $d_{yz}$ orbitals [18] are itinerant and contribute to transport. One of the major concerns is to understand the nature of the SOI of the charge carriers at the oxide interface. In [17, 19], a $k$-linear Rashba SOI for $d_{xy}$ orbital and an isotropic $k$-cubic for $d_{xz}$ and $d_{yz}$ orbitals were proposed. The magneto-transport measurement of 2DEG at the oxide interface has indicated the existence of $k$-cubic RSOI and is modeled using the isotropic $k$-cubic SOI, $H^\text{iso}_\text{SOI}$ [10, 20].

On the other hand, the first-principle calculations suggested anisotropic non-parabolic spin-split branches for the $d_{xz}$ and $d_{yz}$ orbitals [21]. Two recent polarization-dependent ARPES revealed non-isotropic Fermi contours of the 2DEG at the oxide interface [22]. Very recent theoretical study [23] predicted that these orbitals are characterized by $k$-cubic but anisotropic Rashba spin–orbit interaction whose form is given by $H^\text{ani}_\text{SOI} = \alpha(k^2 - k_0^2)\mathbf{k} \times \mathbf{\sigma} \cdot \mathbf{\xi}$. The spin splitting energy and Fermi contours become highly anisotropic as a result of this anisotropic SOI. In this paper we will refer this as anisotropic RSOI. This form of the anisotropic RSOI [23] enables to explain the experimental observations of the anisotropic spin susceptibility [24, 25] successfully. It is also shown [23] that the anisotropic RSOI leads to different behavior of the spin Hall conductivity, in comparison to the isotropic $k$-cubic RSOI.

The spectroscopic measurement of the absorptive part of the optical conductivity can probe the spin-split energy levels. Theoretical studies of the optical conductivity of various charged systems with an isotropic $k$-linear Rashba SOI have been carried out [26–29, 31]. It is shown that the optical transition takes place for a certain range of photon energy depending on the carrier density and spin–orbit coupling constant. At zero temperature, it takes a box-like function and its value is $\sigma_X^\text{iso} = 3e^2/16\hbar$, independent of carrier density and spin–orbit coupling strength.

In this paper we study the Drude weight and optical conductivities of the 2DEG with anisotropic $k$-cubic RSOI formed at the oxide interface and compare our results with that of the isotropic $k$-cubic RSOI. Firstly, we present the characteristics of the zero-frequency Drude weight as a function of the charge density and strength of the anisotropic RSOI. We find that the Drude weight is strongly modified due to the presence of the anisotropic $k$-cubic SOI. It deviates from the linear density dependence for large carrier density and for strong spin–orbit coupling. The Drude weight decreases with the increase of the strength of RSOI. Secondly, we find that an infinitesimally small photon energy would initiate the inter-band optical transition. This is due to the vanishing spin-splitting energy along certain directions in the momentum space. There is a single peak in the optical conductivity and its value depends on the electron density and strength of the anisotropic RSOI. Moreover, the charge and optical conductivities are isotropic despite the fact that the RSOI is anisotropic. In conventional 2DEG the van Hove singularities largely affects the various physical properties like transport [32], character of plasmons [33], N-type kink in photoluminescence [34], dilute-magnetic semiconductor properties [35] etc. Here as well, the van Hove singularities drastically affects the optical conductivity, thereby responsible for the single peak observed in it. The van Hove singularities are of the same $M_1$ type. These features can be used to find out the nature of the RSOI experimentally.

This paper is organized as follows. In section 2, we describe basic properties of the 2DEG with anisotropic $k$-cubic spin–orbit interaction. In section 3, we present the analytical and numerical results of the Drude weight and the optical conductivity. The summary and conclusions of this paper are presented in section 4.

2. Description of the physical system

The effective Hamiltonian of the electron in $d_{xz}$ and $d_{yz}$ orbitals at the interface of LAO/STO is given by [23]

$$H = \frac{\hbar^2 k^2}{2m^*} + \alpha(k^2 - k_0^2)\mathbf{k} \times \mathbf{\sigma} \cdot \mathbf{\xi},$$

where $m^*$ is the effective mass of the electron, $\alpha$ is the strength of the anisotropic RSOI and $\mathbf{\sigma} = \sigma_x\mathbf{i} + \sigma_y\mathbf{j}$. The above Hamiltonian is valid within the narrow region around the $\Gamma$ point. The anisotropic dispersion relations and the corresponding eigenfunctions are given by

$$E_\mathbf{k}(\mathbf{k}) = \frac{\hbar^2 k^2}{2m^*} + \lambda\alpha k^3|\cos \theta|$$

and

$$\psi_\mathbf{k}(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}} \phi_\mathbf{k}(\mathbf{r})/\sqrt{\Omega}$$

with the spinor

$$\phi_\mathbf{k}(\mathbf{r}) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \lambda \eta_\mathbf{k} e^{i\theta} \\ \sqrt{3} \lambda \eta_\mathbf{k} e^{i\varphi} \end{array} \right).$$

Here $\Omega$ is the surface area of the two-dimensional system, $\lambda = \pm$ denotes the spin-split branches and $\eta_\mathbf{k} = \cos \theta/|\cos \theta|$ with $\theta = \tan^{-1}(k_x/k_y)$ measures the anisotropy of the spectrum. The magnitude of the anisotropic spin-splitting energy is $E_s(\mathbf{k}) = |E_s(\mathbf{k}) - E(\mathbf{k})| = 2\alpha k^3|\cos \theta|$. The spin splitting energy vanishes at $\theta = (2p + 1)\pi/4$ with $p = 0, 1, 2, 3$. On the other hand, the maximum spin-splitting ($E_s^\text{max} = 2\alpha k^3$) occurs at $\theta = p\pi/2$. To allow only the bound states, the wave-vector $\mathbf{k}$ should have an upper cut-off given by $k_0(\pi/4) = \hbar^2/(3m^*\alpha)$ which corresponds to the cut-off energy $E_c = \lambda k_0^3/2$.

The spin texture on the $k_x - k_y$ plane can be obtained from the average values of spin vector (in units of $3\hbar/2$)

$$P(\mathbf{k}) = \langle \mathbf{\sigma} \rangle = \lambda \eta_\mathbf{k} \hat{\mathbf{\ell}}$$

where $\hat{\mathbf{\ell}} = -\mathbf{\hat{x}} \sin \theta + \mathbf{\hat{y}} \cos \theta$ is the unit polar vector. The electron spin lies in the $\mathbf{k}$ plane and always locked at right angles to its momentum. The Berry connection [30] is defined as $A_\mathbf{k} = i(\phi_\mathbf{k}^\dagger \nabla_\mathbf{k} \phi_\mathbf{k})$, where $\phi_\mathbf{k}^\dagger$ is the spinor part of the wave function $\psi_\mathbf{k}(\mathbf{r})$. The Berry connection for this system yields $A_\mathbf{k} = -\hat{\mathbf{\ell}}/(2\hbar k)$. Using the expression of the Berry phase [30] $\gamma = \oint A_\mathbf{k} \cdot d\mathbf{k}$, we get

$$\gamma_{\text{ani}} = -\pi$$

for anisotropic case, whereas $\gamma_{\text{iso}} = 3\pi$ for isotropic cubic RSOI [31].
energy increases with the increase of the carrier density. On the other hand, the Fermi energy decreases with the increase of the spin–orbit coupling strength. The Fermi wave vectors \( k^L_\theta(\theta) \) can be obtained numerically from the solutions of the equation \( \hbar^2 k^L_\theta(\theta)^2/2m^* + \lambda \alpha k^3(\theta) \cos 2\theta - E_c = 0 \). The Fermi contours are depicted in figure 5 (color: black).

3. Drude weight and optical conductivity

The complex charge conductivity for a two-level system of charge carriers in presence of a sinusoidal electric field \( \mathbf{E}(\omega) \sim \mathbf{E}_0 e^{i\omega t} \) can be written as \( \Sigma_{\alpha}(\omega) = \sigma_\phi(\omega) + i\sigma_\sigma(\omega) \), where \( \sigma_\phi(\omega) \) is the intra-band induced dynamic Drude conductivity and \( \sigma_\sigma(\omega) \) is the inter-band induced complex optical conductivity.

The absorptive part of the conductivity can be obtained by taking the real part of \( \Sigma_{\alpha}(\omega) \) and is given by

\[
\text{Re}[\Sigma_{\alpha}(\omega)] = D_\omega \delta(\omega) + \text{Re}[\sigma_\alpha(\omega)].
\]

Here, \( D_\omega \) is known as the Drude weight measuring the Drude conductivity \( (\sigma_\alpha = eD_\omega/\hbar) \) for a DC electric field and \( \text{Re}[\sigma_\alpha(\omega)] \) is the optical conductivity as a function of the frequency of the AC electric field with vanishing momentum \( q \rightarrow 0 \). The vanishing momentum of the electric field forces the charge carriers to make a transition from \( \lambda = -1 \) branch to \( \lambda = +1 \) branch such that the momentum is conserved.

3.1. Drude weight

The semi-classical expression for the Drude weight at low temperature is given by [36]

\[
D_\omega = \pi e^2 \sum_\lambda \int \frac{d^2k}{(2\pi)^2} \delta(E - E_\lambda(k)) v^L_\eta(k) d\theta E(k) - E_\lambda(k) + 2k_\eta \sigma_\tau \alpha \lambda \sigma_\alpha(\theta)(\theta) \cos 2\theta \right| - E = 0.
\]

where \( D_\omega = \pi e^2 \sum_\lambda \int \frac{d^2k}{(2\pi)^2} \delta(E - E_\lambda(k)) v^L_\eta(k) d\theta E(k) - E_\lambda(k) + 2k_\eta \sigma_\tau \alpha \lambda \sigma_\alpha(\theta)(\theta) \cos 2\theta \right| - E = 0. \]

The density of states is obtained numerically and their characteristics for the two branches are shown in figure 1. The DOS of the anisotropic spin-split levels varies asymmetrically with respect to \( D_0 \). For fixed electron density \( n_e \) and \( \alpha \), the Fermi energy \( (E_F) \) is obtained from the conservation of electron number \( n_e = \int_0^{E_F} \sum D(E)dE \). The variations of the Fermi energy with \( n_e \) and \( \alpha \) are shown in figure 2. The Fermi

\[
D_\omega = \pi e^2 \sum_\lambda \int \frac{d^2k}{(2\pi)^2} \delta(E - E_\lambda(k)) v^L_\eta(k) d\theta E(k) - E_\lambda(k) + 2k_\eta \sigma_\tau \alpha \lambda \sigma_\alpha(\theta)(\theta) \cos 2\theta \right| - E = 0.
\]

Figure 1. Plots showing the density of states in the units of \( D_0 \) for two different values of \( \alpha \). Here \( \alpha = 0.004 \text{ eV nm}^2 \) (solid black) and \( \alpha = 0.006 \text{ eV nm}^2 \) (dotted red).

Figure 2. Plots of the Fermi energy versus \( n_e \) and \( \alpha \). Left panel: plots of the Fermi energy versus density for different values of \( \alpha \). Right panel: plots of the Fermi energy versus \( \alpha \) for different values of carrier density \( n_e \).

In order to obtain two anisotropic Fermi contours \( k^L_\theta(\theta) \), we need to calculate density of states (DOS) and Fermi energy \( E_F \). The density of states of the spin-split energy branches are given by

\[
D_\lambda(E) = \frac{D_0}{2\pi} \int_0^{2\pi} \frac{k^L_\theta(\theta) d\theta}{|k^L_\theta(\theta) + \lambda \alpha D_0(k^L_\theta(\theta)^2) \cos 2\theta|},
\]

where \( D_0 = 2\pi m^*/\hbar^2 \) and \( k^L_\theta(\theta) \) being the solution of the equation \( \hbar^2 k^L_\theta(\theta)^2/2m^* + \alpha \lambda k^3(\theta) \cos 2\theta - E = 0 \). The density of states is obtained numerically and their characteristics for the two branches are shown in figure 1. The DOS of the anisotropic spin-split levels varies asymmetrically with respect to \( D_0 \). For fixed electron density \( n_e \) and \( \alpha \), the Fermi energy \( (E_F) \) is obtained from the conservation of electron number \( n_e = \int_0^{E_F} \sum D(E)dE \). The variations of the Fermi energy with \( n_e \) and \( \alpha \) are shown in figure 2.
For carrying out the numerical calculation, we adopt the following parameters used in [19, 23]: \( n_0 = 3.5 \times 10^{16} \text{ m}^{-2} \) and \( m' m_0 = 1 \), where \( m_0 \) is the bare mass of the electron. In figure 3, the variations of the Drude weight with the carrier density and with the strength of the Rashba spin–orbit interactions are shown. The plots of the Drude weight versus carrier density for three different values of \( \alpha \) are shown in the left panels of figure 3. The analytical expression of \( D_{\text{iso}} \) obtained in [31] clearly shows the deviation from the linear density dependence. Because of the small value of \( \alpha \) considered here, the deviation is not visible in this figure. On the other hand, the Drude weight versus \( \alpha \) for three different values of carrier density are plotted in the right panels of figure 3. The Drude weight decreases with the increase of \( \alpha \) but the decreasing nature of \( D_{\text{iso}} \) for the two different cases is quite different. This important feature would help to know the nature of the RSOI.

### 3.2. Optical conductivity

The generalized Kubo formula of the optical conductivity in terms of the Matsubara Green’s function is given by [36]

\[
\sigma_{\alpha\nu}(\omega) = -\frac{e^2 T}{4\pi^2} \int \frac{d^2 k}{(2\pi)^2} \sum_I \text{Tr} \left[ \hat{G}(\mathbf{k}, \omega_I) \hat{G}(\mathbf{k}, \omega_I + \omega) \right].
\]

(9)

Here, \( \mu, \nu = x, y \), \( T \) being the temperature, \( \omega_I = (2s + 1)\pi T \) and \( \omega_I = 2\pi n T \) are the fermionic and bosonic Matsubara frequencies with \( s \) and \( l \) are integers, respectively.

The matrix Green’s function associated with the Hamiltonian given by equation (1) is

\[
G(k, \omega_n) = \frac{1}{2} \sum_\lambda [I + \mathbf{P}_I(k) \cdot \sigma] G_0(k, \omega_n).
\]

(10)

Here \( I \) is a 2 \times 2 unit matrix and \( G_0(k, \omega_n) = \mathbf{I}(i\hbar \omega_n + \mu_q - E_0(k)) \) with \( \mu_q \) being the chemical potential. It indicates that the optical spectral weight is directly related to the local spin texture \( \mathbf{P}_I(k) \).

Substituting equations (5) and (10) into equation (9), the \( xx \)-component of the longitudinal conductivity reduces to

\[
\sigma_{xx}(\omega) = -\frac{e^2}{i(2\pi \hbar)^2} \int_{0}^{\infty} \int_{0}^{2\pi} \alpha^2 k^5 \cos^2 \theta \sin^2 \theta d\theta d\phi \\
\times \left[ f(E_-) - f(E_+) \right].
\]

(11)

where \( f(E) = [e^{E-n_0} + 1]^{-1} \) is the Fermi–Dirac distribution function with \( \beta = 1/(k_B T) \).

We have carried out the same calculation for other components of the conductivity tensor \( \sigma_{\alpha\beta}(\omega) \). We find that \( \sigma_{xx}(\omega) = \sigma_{yy}(\omega) \) and \( \sigma_{xy}(\omega) = \sigma_{yx}(\omega) = 0 \). Hence the optical conductivity remains isotropic despite the fact that the Fermi contours are anisotropic.

Using the fact that \( \omega > 0 \) and after performing the \( k \) integral, the expression for the absorptive part of the optical conductivity at \( T = 0 \) is given by

\[
\text{Re}[\sigma_{xx}(\omega)] = \frac{3 e^2}{16 \hbar} \iiint d\theta d\phi \left[ \Theta(\mu_-) - \Theta(\mu_+) \right],
\]

(12)

where \( \Theta(x) \) is the unit step function and \( \mu_\pm = \pm k_\perp(k_z) - \mu_0 \) with

\[
k_\perp = k_\perp(\theta) = (\hbar \omega/2\alpha \cos 2\theta)^{1/3}.
\]

This integral cannot be solved analytically due to \( \theta \) dependence of \( k_\perp \).

On the other hand, in isotropic cubic Rashba SOI the closed form expression of the absorptive part of the optical conductivity at \( T = 0 \) K is given by [31]

\[
\text{Re}[\sigma_{xx}^{\text{iso}}(\omega)] = \frac{3 e^2}{16 \hbar} \Theta(\mu_-) - \Theta(\mu_+),
\]

(13)

where \( \mu_\pm = \pm k_\perp(\hat{k}_z) - \mu_0 \) with \( \hat{k}_z = (\hbar \omega/2\alpha \cos 2\theta)^{1/3} \) which is independent of the carrier density and \( \alpha \). Note that simultaneous presence of isotropic Rashba and Dresselhaus SOI leads to anisotropic Fermi contours, in turns produces interesting optical features. Whereas anisotropic RSOI alone gives rise to anisotropic Fermi contours and provides distinct optical features.

Here we shall present how the anisotropic RSOI alone gives rise to some unique features of the optical conductivity. We first evaluate \( \text{Re}[\sigma_{xx}(\omega)] \) numerically using the parameters \( \alpha = 0.004 \text{ eV nm}^{-3} \), \( n_0 = 3.5 \times 10^{16} \text{ m}^{-2} \) and \( m' m_0 = 1 \) as used in [19, 23] and shown in the lower panel of figure 4. For comparison with the isotropic case, we plot \( \text{Re}[\sigma_{xx}^{\text{iso}}(\omega)] \) which appears as the rectangular box on the right side of the lower panel of figure 4. We depict \( \epsilon_0(\theta) = 2\alpha [k_z^2(\theta)^2 \cos 2\theta] \) in the top panel of figure 4. The contribution to optical conductivity arises from the shaded angular region. The optical
transitions from $\lambda = -1$ to $\lambda = +1$ occur when the photon energy satisfies the inequality $0 < h\omega < \epsilon_\ast(\theta)$. One can see that an infinitesimally small photon energy can initiate the optical transition, in complete contrast to the isotropic SOI case. This is due to the presence of the degenerate lines $\theta = (2p + 1)\pi/4$. There is a single peak of the $\text{Re} \left\{ \sigma_\omega(\omega) \right\}$ at $h\omega = \epsilon_\ast(\pi/2) = 2\alpha[k^2(\pi/2)]^3$ and the optical conductivity becomes zero when $h\omega > \epsilon_\ast(\pi/2) = 2\alpha[k^2(\pi/2)]^3$. For better understanding of these features, we plot the constant energy-difference curves $E_\delta(k) = \epsilon_\omega$ for $\epsilon_\omega = \epsilon_\ast(\pi/2) = \epsilon_1$ ($C_1$: dashed) and $\epsilon_\omega = \epsilon_\ast(\pi/2) = \epsilon_2$ ($C_2$: solid) in figure 5. The area intercept by the curves $C_i$ with $i = 1, 2$ and the Fermi contours $(k^2)_{\alpha}$ are responsible for the $k$-selective optical transitions as shown in figure 5. It should be noted that the anisotropic k-cubic band is well separated from the other two bands with k-linear SOI for the parameters used in [19, 23]. As a result, the contribution to the optical conductivity from other two bands having k-linear SOI is ruled out since they occur at $h\omega$ much larger than $\epsilon_\ast(\pi/2) \approx 0.9$ meV.

The overall behavior of the optical spectra can be understood from the joint density of states which is given as

$$D(\omega) = \int \frac{d^3k}{(2\pi)^3} \left[ \delta(E_\delta(k)) - \delta(E_\ast(k)) \right] d\omega.$$  

Figure 4. Top panel: plots of $\epsilon_\omega(\theta)$ versus $\theta$. Middle panel: plots of the joint density of states versus $h\omega$ with $k_0 = \sqrt{2\pi}n_e$. Bottom panel: the real part of the optical conductivity as a function of photon energy $h\omega$.

Figure 5. Plots of the Fermi contours $k_i^0(\theta)$, $k_f^0(\theta)$, the constant-energy difference curves $C_1: E_\delta(k) = \epsilon_1$ and $C_2: E_\delta(k) = \epsilon_2$.

It can be reformulated as

$$D(\omega) = \frac{1}{(2\pi)^3} \int \frac{dC[f(E_\delta(k))] - f(E_\ast(k))]}{|\partial E_\delta(k)|} \omega \omega \omega.$$  

Here $C$ is the line element along the contour. The joint density of states versus $h\omega$ is plotted in the middle panel of figure 4. The location of the single peak and the region of zero optical conductivity are nicely described by the joint density of states. It can be seen from equation (14) that any peak may arise whenever $|\partial E_\delta(k)|$ attains a minimum value. For the present problem, the singular points are at $k_s = (k, \pi/4)$. The single peak appears at $\epsilon_\omega = \epsilon_\ast(\pi/2)$ in the joint density of states corresponds to the well known van Hove singularity. The asymmetric spin-splitting at the Fermi contours along the $k_s = k_s = 0$ lines is the reason for the appearance of the peak at $\epsilon_\ast(\pi/2)$.

There are three different types of the singularity [37] depending on the nature of change of the energy gap around the singular points $k_s$. Using the Taylor series expansion of $E_\delta(k)$ around $k_s$ as $E_\delta(k) = E_\delta(k_s) + \sum a_i(p)(k_i - k_{si})^2$ with the expansion coefficients $2a_i(p) = \frac{\partial^2 E_\delta(k)}{\partial e_i^2}$, the coefficients $a_i$ are as follows $a_i(p) = \alpha k[5 + 7(-1)^{i_i}]$ and $a_i(p) = \alpha k[5 - 7(-1)^{i_i}]$. The sign of the coefficients will determine the type of classification of the various singular points. One can easily find that the signs of $a_i$ and $a_i$ at different singular points are $(-1)^i$ and $(-1)^{i+1}$, respectively. Therefore, every singularities are all of the same class $i.e.$ $M_1$ type.

The variations of the peak height ($\sigma_{\text{peak}}$) with $n_e$ and $\alpha$ are shown in figure 6. It strongly depends on the Fermi energy. We also define a width $\Delta = \epsilon_\ast(\pi/2) - \epsilon_\ast(\pi/2)$, the difference between peak position and the position beyond which $\sigma_\omega(\omega)$ vanishes. Its variation with $\alpha$ as well as $n_e$ are shown in figure 7. It shows that $\Delta$ increases with the increase of $n_e$ as well as $\alpha$. 

5
The spin–orbit interaction experimentally and would help in understanding the orbital origin of the two-dimensional electron gas at the oxide interface.

References

[1] Winkler R 2003 Spin–Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems (Berlin: Springer)
[2] Zutic I, Fabian J and Das Sarma S 2004 Rev. Mod. Phys. 76 323
[3] Rashba E I 1960 Sov. Phys. Solid State 2 1109
[4] Bychkov Y A and Rashba E I 1984 J. Phys. C: Solid State Phys. 17 6039
[5] Dresselhaus G 1955 Phys. Rev. 100 580
[6] Das B, Miller D C, Datta S, Reifenberger R, Hong W P, Bhattacharyya P K, Sing J and Jaffe M 1989 Phys. Rev. B 39 1411
[7] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 Phys. Rev. Lett. 78 1335
[8] Hasan M Z and Kane C L 2010 Rev. Mod. Phys. 82 3045
[9] Qi X L and Zhang S C 2011 Rev. Mod. Phys. 83 1057
[10] Xu Y, Miotkowski I, Liu C, Tian J, Nam H, Aldisoust N, Hu J, Shih C-K, Hasan M Z and Chen Y P 2014 Nat. Phys. 10 956
[11] Rashba E I and Sherman E Ya 1988 Phys. Lett. A 129 175
[12] Winkler R 2000 Phys. Rev. B 62 4245
[13] Minkov G M, Sherstobitov A A, Germanenko A V, Rut O E, Lariovna V A and Zvonkov B N 2005 Phys. Rev. B 71 165312
[14] Nakamura H, Koga T and Kimura T 2012 Phys. Rev. Lett. 108 206601
[15] Moriya R et al 2014 Phys. Rev. Lett. 113 086601
[16] Ohtomo A and Hwang H Y 2004 Nature 427 423
[17] Caviglia A D, Gariglio S, Cancellieri C, Sace B, Fete A, Gabay M, Thiel S, Hammerl G, Mannhart J and Triscone J M 2008 Nature 456 624
[18] Bell C, Harashima S, Kozuka Y, Kim M, Kim B G, Hikita Y and Hwang H Y 2009 Phys. Rev. Lett. 103 226802
[19] Caviglia A D, Gariglio S, Cancellieri C, Sace B, Fete A, Reyren N, Gabay M, Morpurgo A F and Triscone J M 2010 Phys. Rev. Lett. 105 236802
[20] Umansky V, de-Picciotto R and Heiblum M 1997 Appl. Phys. Lett. 71 683
[21] Lee G K B and MacDonald A H 2013 Phys. Rev. B 88 041302
[22] Popovic Z S, Satpathy S and Martin R M 2008 Phys. Rev. Lett. 101 256801
[23] Zhong Z, Toth A and Held K 2013 Phys. Rev. B 87 161102
[24] Kim Y, Lutchyn R M and Nayak C 2013 Phys. Rev. B 87 245121
[25] van Heeringen L W, de Wijjs G A, McCollam A, Maan J C and Fasolino A 2013 Phys. Rev. B 88 205140
[26] Cancellieri C, Reinele-Schmitt M L, Kobayashi M, Strocov V N, Willmott P R, Fontaine D, Ghosez Ph, Filippetti A, Delugas P and Fiorrentini V 2014 Phys. Rev. B 89 124142
[27] King P D C, Walker S M, Tamai A, de la Torre A, Eknaphakul T, Buaphet P, Mo S-K, Meevasana W, Bahramy M S and Baumberger F 2014 Nat. Commun. 5 3414
[28] Zhou J, Shan W and Xiao D 2015 Phys. Rev. B 91 241302
[29] Li L, Richter C, Mannhart J and Aschoori R C 2011 Nat. Phys. 7 762
[30] Bert J A, Kalisky B, Bell C, Kim M, Hikita Y, Hwang H Y and Moler K A 2011 Nat. Phys. 7 767
[31] Lu W and Lin L B 2008 J. Phys.: Condens. Matter 16 1777
[32] Yang C H, Xu W, Zeng Z, Lu F and Zhang C 2006 Phys. Rev. B 74 075321

![Figure 6](image1.png)

Figure 6. Left panel: plots of $\sigma_{\text{peak}}$ versus $\alpha$ for fixed values of $n_e = 4.0 \times 10^{16}$ m$^{-2}$ (dashed) and $n_e = 3.0 \times 10^{16}$ m$^{-2}$ (solid). Right panel: plots of $\sigma_{\text{peak}}$ versus $n_e$ for different values of $\alpha = 0.004$ eV nm$^3$ (solid) and $\alpha = 0.006$ eV nm$^3$ (dashed).

![Figure 7](image2.png)

Figure 7. Left panel: plots of $\Delta$ versus $\alpha$ for fixed values of $n_e = 3.5 \times 10^{16}$ m$^{-2}$ (dashed) and $n_e = 3.0 \times 10^{16}$ m$^{-2}$ (solid). Right panel: plots of $\Delta$ versus $n_e$ for different values of $\alpha = 0.004$ eV nm$^3$ (solid) and $\alpha = 0.006$ eV nm$^3$ (dashed).

4. Summary and conclusion

We have studied the Drude weight and optical conductivity for 2DEG with $k$-cubic anisotropic RSOI at the oxide interface. We have presented the variation of the zero-frequency Drude weight with the carrier density as well as the strength of the anisotropic spin–orbit coupling. For anisotropic RSOI, the Drude weight deviates from the linear density dependence. It is indicated that the spectral weight is directly related to the local spin texture in momentum space. We found that the charge and optical conductivities remain isotropic although the Fermi contours are anisotropic. It is found that an infinitesimally small photon energy can trigger inter-band optical conductivity. This is due to the fact that the spin-splitting energy vanishes along the certain directions in $k$ space. We found a single peak in the optical conductivity whose value depends on the Fermi energy. We have shown that the van Hove singularities responsible for the single peak in the optical conductivity are of the same $M_1$ type. The different features of the conductivity can determine the information of the nature of the spin–orbit interaction experimentally and would help in understanding the orbital origin of the two-dimensional electron gas at the oxide interface.
[28] Wong A and Mireles F 2010 Phys. Rev. B 81 085304
[29] Li Z, Marsiglio F and Carbotte J P 2013 Sci. Rep. 3 2828
[30] Berry M V 1984 Proc. R. Soc. A 392 45
[31] Mawrie A and Ghosh T K 2016 J. Appl. Phys. 119 044303
[32] Simmons J A, Lyo S K, Harff N E and Klem J F 1994 Phys. Rev. Lett. 73 2256
[33] Cheng S and Gerhardts R 2002 Phys. Rev. B 65 085307
[34] Orlita M, Grill R, Hlidek P, Zvara M, Dohler G H, Malzer S and Byszewski M 2005 Phys. Rev. B 72 165314
[35] Simserides C 2007 Phys. Rev. B 75 195344
[36] Mahan G D 1981 Many Particle Physics (New York: Springer)
[37] Hamaguchi C 2010 Basic Semiconductor Physics (New York: Springer)