The Ability of Students’ Mathematical Proof in Determining the Validity of Argument Reviewed from Gender Differences

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Abstract. This research aims to describe the ability of students’ mathematical proof in determining the validity of argument reviewed from gender differences. The subjects of this research were one male and one female student of the fifth semester of Mathematic Education study program. The subjects were selected based on the highest mathematics ability which was assessed from their previous assignments and tests. In addition, the communication ability of the subjects was also considered in order to facilitate the researcher in conducting interviews. Based on the result of the test with direct and indirect proof, it could be concluded that the subjects were able to: 1) mention all facts/premises and write about what should be shown (conclusion) in direct proof and write additional premise in indirect proof; 2) connect facts/premises to concepts which must be mastered; 3) use equivalent concept to manipulate and organize the proof; 4) use the concept of syllogism and tollens mode to obtain the desired conclusion; 5) construct mathematical evidence systematically, and logically; 6) complement the reason for each step appropriately. The difference was that the male subject wrote the final conclusion, while the female subject did not write the final conclusion on the proof.

1. Introduction

Mathematics has many abstract objects, though not every abstract object is mathematics. Gagne [1] classifies mathematical objects into direct and indirect objects. The direct objects of mathematics includes facts, concepts, principles, and skills. While the mathematics indirect objects consist of the ability of investigate, the ability of problem solving, transfer of learning, self-discipline, and appreciation of mathematical structure. For this case, the ability to investigate must be completed by providing the evidence. In mathematics, evidence has an important role as a method used to test the knowledge and very different from the way the inductive method works in the empirical activities [2].

According to the Educational Development Center (in [3]), evidence in a mathematical proof is a series of logical arguments that explain the truth of a statement. These arguments can be derived from the premise of the statement itself, definitions and other theorems. Logical means every step in the argument is justified by the preceding steps.

Evidence in mathematics, also called mathematical proof, is one aspect that must be considered in learning mathematics. This is as recommended by NCTM that proof is part of mathematics curriculum at all levels of education. The National Council of Teachers of Mathematics [4] states that students should be able to: know reasoning and proof as fundamental aspects of mathematics; create a conjecture and check the truth of the conjecture; develop and evaluate arguments and mathematical proofs; select and use various types of reasoning and verification methods. Therefore, the ability to do mathematical
proving must be possessed by all mathematics students. On the contrary, the fact showed that the students lack of this ability. Some studies ([5], [6], [7], [8]) suggest that many students have difficulty in mathematical proofing. Fadillah and Jamilah (2014) revealed that students have difficulty in proving the very abstract concept of algebra structure. The students are less familiar with the examples related to the concept and not yet accustomed to deductive proof. This is in line with the results of Hidayanto (2010) which states that students have difficulty in performing formal proof from a statement in Calculus I. Taufik’s research result also shows that (2014) students have difficulty in proving propositions using mathematical induction. Furthermore, Prihandoko ([8]) reveals that students have difficulty to identify the elements and reasoning patterns in many theorems and definitions. In addition, Prihandoko added that many students are still accustomed to using inductive reasoning patterns and have not been familiar with valid proving types. Students also do not adhere to the basic principles of mathematical logic and set theory, so they can not prove the theorems correctly.

Proof in mathematics has a role of verification and justification, but in education, proof is used for explanation [9]. The verification and explanation in this research is used to determine the validity of the arguments in making a conclusion. Skill in constructing a conclusion requires an understanding of concepts and capabilities connecting the known arguments to learned concepts (definitions, axioms, postulates, theorems and etc.) through systematic and deductive algorithms. Since the mathematical proof is one of the mathematical abilities that must be mastered by students, then it is necessary also to give more attention. One of the important factor is gender differences. Gender is a visible difference between male and female seen from society’s value.

Gender differences certainly lead to physiological differences and affect the psychological differences in learning. According to Susento ([10]) gender differences not only result in different abilities in mathematics, but also how to acquire mathematical knowledge. This statement is reinforced by Keitel ([11]) that gender, social and cultural influence mathematics learning. Similarly, the results of the study by Zhu ([12]) suggested that gender differences affect students’ problem-solving abilities, in which the problem-solving abilities of male are better than female for middle and upper classes.

There is no research has examined directly the relation of mathematical proof to gender differences. Some researches only relate mathematical abilities in general to gender differences. Therefore, it is important to examine the ability of mathematical proof in determining the validity of the argument viewed from gender differences.

2. Method
This research aimed to describe the ability of mathematical proof in determining the validity of mathematical arguments of both male and female students. The approach of this research was a qualitative approach. The type of this research was descriptive explorative. The subjects of this research were one male and one female student of the fifth semester of mathematics education study program. The reason of choosing the fifth semester students was because the subjects had been taught the introduction to basic mathematics. The way of selecting the research subjects was all of the students given the mathematics ability test and then one male (DZ) and female students (KH) who have the highest mathematics ability and good communication was selected. Furthermore, DZ and KH were given a mathematical direct proof test and continued with the interview. Then, DZ and KH were given a mathematical indirect proof test and interview. Finally, students’ work results were analyzed. This research used triangulation method/technique to obtain the data. The test and interview were conducted to assure the validity of the data.

3. Results
The following data is the description of mathematical test results done by students DZ and KH

3.1 The description data of mathematical direct proving by KH
Based on the test and interview results of KH in mathematical direct proof, KH has (1) good ability to identified statement that will be proved which showed by KH in mentioning all the facts/ premises also writing what should be shown (conclusion) i.e. ¬p; (2) good ability to use the direct proof method that is indicated by using the premises (facts) in making conclusions and relating with the concepts that must be mastered i.e. \( p \rightarrow q \), ¬q \( \lor r \), \( r \rightarrow ¬s \), and \( s \); (3) good ability to organize and manipulate facts known that is shown by the ability of the student to use the concept of equivalent in the proof step i.e. ¬q \( \lor r \equiv q \rightarrow r \); (4) good ability to make connections between the known facts and elements of conclusions is also good that demonstrated by the ability of students to use the concept of tollens mode (if the premises are \( r \rightarrow ¬s \) and \( s \), then conclusion is \( ¬r \)) and syllogism (if the premises \( p \rightarrow q \) and \( q \rightarrow r \), then the conclusion is \( p \rightarrow r \)) in obtaining the desired conclusions; (5) good ability to compile the proof that is demonstrated a systematic and logical flow made in constructing mathematical proof but not completed by final conclusions; And (6) good ability to complete the reasons for each step taken.

### 3.2 The description data of mathematical direct proving by DZ

Based on the test and interview results of DZ in mathematical direct proof, DZ has (1) good ability to identified statement that will be proved which showed by DZ in mentioning all the facts/ premises on the question of proof, and DZ also writes what should be shown (conclusion) i.e. ¬p; (2) good ability to use the direct verification method shown by DZ in using the premises (facts) in providing conclusions and connecting with the concepts that are mastered i.e. \( p \rightarrow q \), ¬q \( \lor r \), \( r \rightarrow ¬s \), and \( s \); (3) good ability to organize and manipulate the facts that is demonstrated by the ability of the student to use the concept of equivalent in the evidentiary step i.e. ¬q \( \lor r \equiv q \rightarrow r \); (4) good ability to make connections between the known facts and elements of the conclusions possessed shown by DZ in using the concept of syllogism (if the premises \( p \rightarrow q \) and \( q \rightarrow r \), then the conclusion is \( p \rightarrow r \)) and tollens mode (if the premises are \( r \rightarrow ¬s \) and \( s \), then conclusion is \( ¬r \)) in obtaining the desired conclusions; (5) good ability to compile the evidence that is demonstrated a systematic and logical flow made in constructing mathematical evidence but not completed by final conclusions; And (6) good ability to complete the reasons for each step taken.

### 3.3 The description data of mathematical indirect proving of KH

Based on the test and interview of subject KH in mathematical indirect proof, KH has (1) good ability to read the evidence by mentioning all the facts/ premises on the matter of proof i.e. ¬p \( \rightarrow r \), \( r \rightarrow ¬s \) and \( s \lor q \), and KH also write the additional premise that ¬q; (2) good ability to use the indirect proof method, she uses the premises and the additional premise in making conclusions and relating to the concepts; (3) good ability to organize and manipulate the known facts, it is demonstrated by the ability of the student to use the concept of equivalent in the proof step; (4) good ability to make connections between the known facts and elements of conclusions, it is demonstrated by the ability of students to use the concept of syllogism (if the premises are ¬p \( \rightarrow r \) and \( r \rightarrow ¬s \), then the conclusion is ¬p \( \rightarrow ¬s \)) and tollens mode in obtaining the desired conclusion i.e. if the premises are ¬q and ¬s \( \rightarrow q \) and then the conclusion is s; (5) good ability to construct evidence is also good, it is demonstrated by the systematic and logical flow made in constructing mathematical proof, but she does’nt write the final conclusion on the proof; And (6) the ability to complete the reasons for each step taken is very good, it is shown in each statement which is accompanied by a logical and precise reason.

### 3.4 The description data of mathematical indirect proof of student DZ

Based on the test and interview of subject DZ in mathematical indirect proof, DZ has (1) good ability to read evidence, it’s proven by mentioning all facts / premises on the matter of proof i.e. ¬p \( \rightarrow r \), \( r \rightarrow ¬s \) and s\( \lor q \), and DZ also writes an additional premise that ¬q; (2) good ability to use the direct proof method, using the premises (facts) in showing conclusions and relating with the concepts; (3) good ability to organize and manipulate the known facts, this is demonstrated by the ability of students to use
the concept of equivalent in the evidence step; (4) good ability to make connections between the known facts with elements of conclusions, it is demonstrated by the ability of students to use concepts syllogism (if the premises are \( \neg p \rightarrow r \) and \( r \rightarrow \neg s \), then the conclusion is \( \neg p \rightarrow \neg s \)) and tollens mode in obtaining the desired conclusion ie if the premises are \( \neg q \) and \( \neg s \rightarrow q \) then the conclusion is \( s \); (5) good ability to construct evidence, this is demonstrated by a systematic and logical flow made in constructing mathematical proofs, and also accompanied by final conclusions in proof; And (6) good ability to complete the reasons for each step, each statement is accompanied by a logical and precise reason.

3.5 The comparison of the ability to construct mathematical direct proof.

**Table 1.** The comparison of the Ability to Construct Mathematical Direct Proof of Male and Female Subject

|                      | Student KH (Female) | Student DZ (Male) |
|----------------------|---------------------|-------------------|
| Mentions all facts / premises (4 premises) on the matter of proof, and KH also write what should be shown (conclusion) that is \( \neg p \) | Uses the premises (facts) in drawing conclusions and connecting with the concepts that are mastered | Uses the premises (facts) in drawing conclusions and connecting with the concepts that are mastered |
| Uses the equivalent concept to manipulate and organize the proofs | Uses the concept of syllogism and tollens mode in obtaining the desired conclusions | Uses the concept of syllogism and tollens mode in obtaining the desired conclusions |
| Makes a systematic and logical flow in constructing mathematical proof, but she doesn’t write the final conclusions | Completes the reasons for each step appropriately | Completes the reasons for each step appropriately |
| Completes the reasons for each step appropriately |                      |                    |

Based on the Table 1, there is no difference in the ability to construct mathematical proof. Both of them are able to make excellent mathematical proofs, either in reading evidence, using proof methods, organizing and manipulating facts, making connections between facts, compiling evidence, or complementing the reasons in each step. It is contrary to the research of Triyadi ([13]) stated female students have mathematical connection, mathematical reasoning, mathematical communication and mathematical problem solving better than female students.

3.6 The comparison of the ability to construct mathematical indirect proof.

**Table 2.** The comparison of the Ability to Construct Mathematical Indirect Proof of Male and Female Subject

|                      | Student KH (Female) | Student DZ (Male) |
|----------------------|---------------------|-------------------|
| Mentions all facts / premises (4 premises) on the matter of proof, and write the additional premise that is \( \neg q \) | Uses the premises (premises) and the additional premise in showing conclusions and relates with the concepts that are | Uses the premises (premises) and the additional premise in showing conclusions and relates with the concepts that are |
| Uses the premises (premises) and the additional premise in showing conclusions and relates with the concepts that are |                      |                    |

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Based on the Table 2, it can be concluded that there is little difference in the ability to construct mathematical proofs, it is on the presence or absence of the final conclusions in mathematical proof. It’s contrary to Krusteskii ([14]) that explained the male is better than female in mathematical reasoning, while female is better than male in accuracy, precision and thoroughness think. He also stated that male students have mathematical and mechanical skills better than female students. Likewise the variety sources ([15], [16]) there were gender differences in verbal skills with female outperforming males on many verbal tasks.

4. Conclusions
There are two conclusions that can be drawn based on the results above. The male subject can mention all facts/premises and write what should be shown (conclusion) in direct proof and write additional premise in indirect proof. The subject is able to connect facts/premises to concepts which must be mastered, use equivalent concept to manipulate and organize the proof, use the concept of syllogism and tollens mode to obtain the desired conclusion, construct mathematical evidence systematically, and logically, complements the reason for each step appropriately and he also writes the final conclusion. The female subject also shows the same ability with the male student, except she doesn’t write the final conclusion on the proof.

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