Super Wilson Loops in Planar QCD

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Abstract

In the planar limit of QCD meson correlation functions can be written as a path-integral for a spin-half particle with each path being weighted by the expectation value of the corresponding Super Wilson Loop. An important quantity in this context is the expectation value of the Super Wilson Loop averaged over loops of fixed length. I obtain the leading and the sub-leading length dependence for this quantity. The leading term, which was also known from the work of Banks and Casher, reflects the fact that chiral symmetry is spontaneously broken in planer QCD, while the sub-leading term implies that at least a finite fraction of paths contributing to the average of the Super Wilson Loop are effectively two-dimensional, thus suggesting a dual string description of planar QCD.

1 Introduction

The planar limit of QCD, namely the limit in which the number of colors $N$ goes to infinity, describes a theory of free and stable mesons and glueballs and could be an appropriate starting point for obtaining hadronic physics from QCD using a $1/N$ expansion [1, 2]. This of course has been the main motivation behind the various efforts towards solving planar QCD. Although

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none of these efforts have been successful, there are features of planar QCD that continue to inspire hope that it can indeed be solved. One of them is the fact that in the planar limit the expectation value of the gauge-invariant observables are given not by the full functional integral over all possible gauge configurations, as one would expect in any quantum field theory, but by its value on a single field, the so called “master-field” [3]. Then there is an attractive possibility which is suggested by strong coupling expansion in lattice gauge theory and by the recent developments in string theory, that this master-field is given by a free string theory [4, 5].

The gauge-invariant observable that best illuminates the possibility of a dual string description of planar QCD are the various Wilson loops [6]. Thus in the planar limit we believe that the expectation value of a Wilson loop should exhibit an area law indicating that the planar QCD is a confining theory. Similarly, the manner in which chiral symmetry is realized is reflected in the behavior of the Wilson loop for a spin-half particle, often referred to as the Super Wilson Loop (SWL). An important quantity for us will be the expectation value of SWL averaged over loops of length $T$, which we will denote by $<\overline{W}>_T$. The fact that chiral symmetry is spontaneously broken in the planar limit [7] implies that $<\overline{W}>_T$ must exhibit a leading behavior of the form $1/\sqrt{T}$ [8]. Once we recall that in $D$ dimensions the probability for a particle to return to the origin after traveling a closed path of length $T$ is proportional to $1/T^{D/2}$, then this leading behavior implies that at least a finite fraction of the paths contributing to the average of a SWL are effectively one-dimensional.

In fact the importance of the SWL in planar QCD is quiet general, arising from the fact that the amplitude for a spin-half particle to follow a given closed path in the presence of an external gauge field is given, apart from the kinematic factors, precisely by the corresponding SWL. This allows us to write meson propagators in planar QCD as a sum over closed paths of a quark with each path being weighted by the expectation value of the associated SWL. An efficient way of writing such path-integrals is to use the worldline techniques [9].

In this work we would like to further explore the behavior of the SWL in the planar limit and relate it to the spectrum of the theory. Specifically we will find that the leading $1/\sqrt{T}$ behavior of the $<\overline{W}>_T$ will ensure the existence of massless pions, and the assumption that the only massless spin-zero mesons are the Nambu-Goldstone bosons will imply that the sub-leading behavior of a SWL must go as $1/T^n$, where $n$ is greater than or equal to one. We
will also find that the difference between the scalar meson correlation function and the pseudo-scalar meson correlation function crucially depends on the sub-leading behaviour of $<\mathbf{W}>_T$, and that only sub-leading behaviour of the form $1/T$ is consistent with the known short-distance behaviour of these correlation functions. This sub-leading behaviour implies that a fraction of paths that contribute to $<\mathbf{W}>_T$ are effectively two-dimensional, and therefore is suggestive of a string description of planar QCD.

The outline of this paper is as follows: In the next section I will write down the connected two-point functions for spin-zero mesons as a path integral for a spin-half particle using the so called vertex operators. These operators play the role of interpolating fields in the worldline representation. In section 3 I will use these operators to relate the behavior of the SWL to the spontaneous breaking of chiral symmetry and in this manner re-derive the leading behaviour for $<\mathbf{W}>_T$. The main results are presented in section 4 where I obtain a relationship between scalar and pseudo-scalar two point functions and use it to obtain the sub-leading dependence of $<\mathbf{W}>_T$ on $T$. In the final section I discuss these results in the context of a possible string description of the planar QCD.

2 Vertex Operators for Spin-Zero Mesons

To motivate the use of worldline representation let us first recall one of the simplifying features of planar QCD. Consider the Euclidean partition function of a $SU(N)$ gauge theory with one flavor of quark of mass $m$ in the presence of the sources for the scalar and the pseudoscalar mesons,

$$Z[J] = \int_A \exp(-S[A]) \exp(-\Gamma[A,J]), \quad (1)$$

$$-\Gamma[A,J] = \ln \det(-i\partial - A - im - iJ_\sigma - \gamma_5 J_\pi), \quad (2)$$

where $S[A]$ is the Yang-Mill action for the $SU(N)$ gauge field $A$. This can be written as

$$Z[J] = Z[0] \langle \exp(-\Gamma[A,J]) \rangle_{YM}, \quad (3)$$

where the functional average is with respect to the Yang-Mill action $S[A]$. In the planar limit, as we have noted in the introduction, only one gauge

\footnote{In the planar limit the flavor degrees of freedom will not play any dynamical role and there is a genuine Nambu-Goldstone boson corresponding to the spontaneous breaking of $U(1)$ flavour symmetry.}
field configuration, the so called master field $\bar{A}$, contributes to the functional integral $^2$. Therefore, the partition function can be written as

$$Z[J] = Z[0] \exp (-\Gamma[\bar{A}, J]). \quad (4)$$

Once we know $Z[J]$ we can of course obtain various connected two-point functions by differentiating $\ln Z[J]$ with respect to suitable sources. Thus in the planar limit the scalar and pseudo-scalar two-point functions,

$$\Delta_s(X - Y) = \langle \Omega | \bar{\Psi}(X) \Psi(Y) | \Omega \rangle, \quad (5)$$

$$\Delta_p(X - Y) = \langle \Omega | \bar{\Psi} i\gamma_5 \Psi(X) \bar{\Psi} i\gamma_5 \Psi(Y) | \Omega \rangle, \quad (6)$$

can be written as

$$\Delta_p(X - Y) = \left( \frac{\delta}{\delta J^\sigma (X)} \frac{\delta}{\delta J^\sigma (Y)} \Gamma[\bar{A}, J] \right) \bigg|_{J=0}, \quad (7)$$

$$\Delta_s(X - Y) = \left( \frac{\delta}{\delta J^\sigma (X)} \frac{\delta}{\delta J^\sigma (Y)} \Gamma[\bar{A}, J] \right) \bigg|_{J=0}. \quad (8)$$

Now the real part of the fermionic effective action, $\Gamma[\bar{A}, J]$, which is what is required for the above two-point functions, can be written as a path-integral for a spin-half particle $^{10, 11}$.

$$\Gamma[\bar{A}, J] = \int_0^\infty \frac{dT}{T} \exp \left\{ -m^2 \frac{T}{2} \right\}$$

$$\times \int_{x, \psi} \exp \left\{ -(S_0 + S[J]) \right\} \mathcal{W}, \quad (9)$$

where the worldline of a particle is specified by four bosonic coordinates $x_\mu(\tau)$ which satisfy periodic boundary conditions and by six fermionic, or the anti-commuting, coordinates $\psi_\alpha(\tau)$ that satisfy anti-periodic boundary conditions (see Ref. $^{11}$ for details.) The worldline actions $S_0$ and $S[J]$ are given by

$$S_0 = \int_0^T d\tau \left\{ \frac{\dot{x}_{\mu}^2}{2} + \frac{1}{2} \psi_\mu \dot{\psi}_\mu + \frac{1}{2} \dot{\psi}_5 \dot{\psi}_5 + \frac{1}{2} \dot{\psi}_6 \dot{\psi}_6 \right\} \quad (10)$$

$$S[J] = \int_0^T d\tau \left\{ \frac{1}{2} J_{\pi}^2 + i \psi_\mu \psi_5 \partial_\mu J_\pi + mJ_\sigma + \frac{1}{2} J_\sigma^2 + i \psi_\mu \psi_6 \partial_\mu J_\sigma \right\} \quad (11)$$
while the expectation value of the SWL, $\mathcal{W}$, is defined as

$$\mathcal{W} = \left\langle \text{Tr}_c \hat{P} \exp \left\{ i \int_0^T d\tau \{ \dot{x}_\mu A_\mu - \frac{1}{2} \psi_\mu F_{\mu\nu} \psi_\nu \} \right\} \right\rangle_{YM}$$  \hspace{1cm} (12)

$$= \text{Tr}_c \hat{P} \exp \left\{ i \int_0^T d\tau \{ \dot{x}_\mu \bar{A}_\mu - \frac{1}{2} \psi_\mu \bar{F}_{\mu\nu} \psi_\nu \} \right\}$$  \hspace{1cm} (13)

where $F_{\mu\nu}$ is the Yang-Mills field strength tensor while the trace is over the color degrees of freedom.

Using this path-integral representation of the one loop fermionic effective action in (7) and (8) we can readily obtain the worldline representation for the scalar and the pseudoscalar two-point functions. For this purpose it is convenient to define a worldline average of any functional $F[x, \psi]$ as

$$< F >_{wl} = \int_0^\infty \frac{dT}{T} \exp \left\{ -\frac{m^2}{2} T \right\} \int_{x,\psi} \exp \{ -S_0 \} \mathcal{W} F$$  \hspace{1cm} (14)

then the two-point functions can be written as

$$\Delta_p (X - Y) = < V_\pi (X) V_\pi (Y) >_{wl},$$  \hspace{1cm} (15)

$$\Delta_s (X - Y) = < V_\sigma (X) V_\sigma (Y) >_{wl},$$  \hspace{1cm} (16)

where the pseudo-scalar vertex operator $V_\pi (X)$ is given by

$$V_\pi (X) = i \int_0^T d\tau \psi_\mu \gamma_5 \partial_\mu \delta (x(\tau) - X),$$  \hspace{1cm} (17)

while the scalar vertex operator $V_\sigma (X)$ is

$$V_\sigma (X) = \int_0^T d\tau \left\{ m \delta (x(\tau) - X) \right\}$$  \hspace{1cm} (18)

$$+ i \psi_\mu (\tau) \psi_\nu (\tau) \partial_\mu \delta (x(\tau) - X) \right\}.$$  \hspace{1cm} (18)

In Eqs (15) and (16) a contact term which will not play any role in the present investigation has been neglected\(^2\). It will be convenient to write the scalar vertex operator as

$$V_\sigma (X) = m V_0 (X) + V_s (X),$$  \hspace{1cm} (19)

\(^2\)The contact term is of the following form $\int_0^T d\tau \delta (X - x(\tau)) \delta (Y - x(\tau))$. 

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where

\[ V_0(X) = \int_0^T d\tau \delta(x(\tau) - X), \]

\[ V_s(X) = i \int_0^T d\tau \bar{\psi}_\mu(\tau) \psi_6(\tau) \partial^\mu \delta(x(\tau) - X). \]

It is useful to observe that formally the above vertex operators can be defined as

\[ V_\pi(X) = \left( \frac{\delta}{\delta J_\pi(X)} S[J] \right)_{J=0}, \]

\[ V_\sigma(X) = \left( \frac{\delta}{\delta J_\sigma(X)} S[J] \right)_{J=0}, \]

where \( S[J] \) is given by \( S \).

Finally, we note that the left hand side of (15) and (16) represents a formal sum of all the planar diagrams which have quark line only at the boundary of the diagram. Since in the planar limit these are the only diagrams that contribute to the meson correlation function we could have written these equations without any reference to a master-field.

3 Chiral Condensate and the SWL

In the planar limit the expectation value of the chiral condensate can be written as the worldline average of the scalar vertex operator \( V_\sigma \)

\[ < \Omega | \bar{\Psi} \Psi(X) | \Omega > = i < V_\sigma(X) >_{wl}. \]

Denoting the worldline average of \( V_\sigma \) by \( V_\chi \) and using (19) we obtain

\[ V_\chi = m < V_0(X) >_{wl} + < V_s(X) >_{wl} \]

where we have used the fact that \( < V_s(X) >_{wl} \) vanishes as it is odd in the Grassmann variable \( \psi_6 \). Using our definition of the worldline average, (14), we obtain the path-integral representation for the chiral condensate,

\[ V_\chi = m \int_0^\infty \frac{dT}{T} \exp \left\{ - \frac{m^2}{2} T \right\} \int_{x,\psi} \exp \{-S_0\} \bar{W} \int_0^T d\tau \delta(x(\tau) - X). \]
In performing the path-integral over $x(\tau)$ it is convenient to separate the zero mode which arises because of the translation invariance. One way of doing this is to write the path $x(\tau)$ as
\[
x(\tau) = \bar{x} + y(\tau),
\]
(26)
where
\[
\bar{x} = \frac{1}{T} \int_0^T d\tau x(\tau),
\]
\[
\int_0^T d\tau y(\tau) = 0.
\]
The path-integral then can be decomposed into an integral over $\bar{x}$ and a path-integral over $y(\tau)$,
\[
V_\chi = m \int_0^\infty \frac{dT}{T} \exp \left\{ -\frac{m^2}{2} T \right\} \int_{y,\psi} d\bar{x} \exp\{-S_0\} \mathcal{W} \int_0^T d\tau \delta(\bar{x} + y(\tau) - X),
\]
(27)
writting the delta function in the momentum representation and then doing the integral over $\bar{x}$ leads to
\[
V_\chi = m \int_0^\infty \frac{dT}{T} \exp \left\{ -\frac{m^2}{2} T \right\} \int_{y,\psi} \exp\{-S_0\} \mathcal{W}.
\]
(28)
This is essentially the expression for chiral-condensate that Banks and Casher had obtained in [8], except that instead of trace over gamma-matric es we have an integral over the Grassmann variables $\psi$.

The spontaneous breaking of chiral symmetry requires that $V_\chi$ does not vanish in the limit of the current quark mass $m$ going to zero. This is possible if the worldline average of the SWL of length $T$ has the following asymptotic behaviour
\[
< \mathcal{W} >_T \equiv \lim_{T \to \infty} \int_{y,\psi} \exp\{-S_0\} \mathcal{W} = \frac{C_0}{T^{1/2}} + \mathcal{O}\left(\frac{1}{T^n}\right),
\]
(29)
where $C_0$ is some constant and $n > \frac{1}{2}$. It is interesting to note that in $D$ dim
\[
\lim_{T \to \infty} \int_{y,\psi} \exp\{-S_0\} \propto \frac{1}{T^{D/2}},
\]
(30)
this can be thought of as the probability for a free particle to return to its starting point after following a closed path of length $T$. The same probability in the presence of external gauge fields is given by (29), thus the effect of $\mathbf{W}$ is to increase the probability for the existence of a closed path of length $T$ from its value in four dimension to that of one-dimension. Correspondingly, in the Minkowski space-time the effect of $\mathbf{W}$ is to increase the amplitude for process in which a quark-antiquark pair is created and lasts for a (proper) time of the order of $T$ from its value in four-dimension to that of its value in one-dimension. As emphasized in [8], it is the spin-dependent interaction, described by the SWL, which is responsible for the existence of effectively one-dimensional paths that ensures that the leading behavior of $<\mathbf{W}>_T$ is proportional to $1/T^{1/2}$.

4 Chiral Limit of Planar QCD

In the previous section we found the asymptotic behaviour of the SWL which is responsible for the spontaneous breaking of chiral symmetry. Now we would like to see how this asymptotic behaviour influences the spectrum of the spin-zero mesons. Consider first the scalar two-point function (8) which can be written in the worldline representation as

$$\Delta_s(k) = <V_s(k)V_s(-k)>_{wl} + m^2 <V_0(k)V_0(-k)>_{wl},$$

(31)

where the vertex function $V_s$ and $V_0$ are given by (21) and (20) and we have used the fact that the cross term $<V_0V_s>_{wl}$ vanishes being odd in the Grassmann variable $\psi_6$. Next we notice that

$$<V_\pi(k)V_\pi(-k)>_{wl} = <V_s(k)V_s(-k)>_{wl}$$

(32)

as the right hand side can be obtained from the left hand side by simply relabeling $\psi_6$ as $\psi_5$ and $\psi_5$ as $\psi_6$ in (14). Using this in (31) we obtain the following relation between the scalar and the pseudo-scalar two-point function

$$\Delta_p(k) = \Delta_s(k) - m^2 <V_0(k)V_0(-k)>_{wl},$$

(33)

which can be written in a more suggestive form as

$$\Delta_p(k) - \Delta_s(k) = -m^2 <V_0(k)V_0(-k)>_{wl}.$$  

(34)
Consider this relation in the chiral limit, if
\[
\lim_{m \to 0} < V_0(k) V_0(-k) >_{wl} = 0
\] (35)
then we would have
\[
\Delta_p(k) = \Delta_s(k),
\] (36)
which would correspond to the situation when chiral symmetry is not spontaneously broken. In the planar limit we know that chiral symmetry is indeed spontaneously broken \[7\] therefore
\[
\lim_{m \to 0} m^2 < V_0(k) V_0(-k) >_{wl} \neq 0.
\] (37)

We also know that planar QCD is a theory of an infinite number of free, stable, and non-interacting, mesons and glueballs \[2\]. This in turn implies that the meson correlation functions are of the form of an infinite sum of simple poles. In particular the scalar meson correlation function \(\Delta_s(k)\) has the following form
\[
\Delta_s(k) = \sum_{n=1}^{\infty} \frac{Z_{s,n}^2}{k^2 + M_{s,n}^2}.
\] (38)
where \(Z_{n,s} = \langle \Omega | \bar{\Psi} \Psi | n \rangle\) is the matrix element for \(\bar{\Psi} \Psi\) to create \(n^{th}\) scalar meson from the vacuum. Similarly the pseudo-scalar correlation function can be written as
\[
\Delta_p(k) = \frac{Z_{\pi}^2}{k^2 + M_{\pi}^2} + \sum_{n=1}^{\infty} \frac{Z_{p,n}^2}{k^2 + M_{p,n}^2},
\] (39)
where we have separated the pion pole and the pion mass vanishes in the chiral limit as
\[
\lim_{m \to 0} M_{\pi}^2 = C_\chi m,
\]
where \(C_\chi\) is related to the chiral condensate. Combining (34) with (38) and (39) leads to
\[
\Delta_P(k) - \Delta_s(k) = -m^2 < V_0(k) V_0(-k) >_{wl}
\]
\[
= \frac{Z_{\pi}^2}{k^2 + M_{\pi}^2} + \sum_{n=1}^{\infty} \frac{Z_{p,n}^2}{k^2 + M_{p,n}^2} - \sum_{n=1}^{\infty} \frac{Z_{s,n}^2}{k^2 + M_{s,n}^2},
\]

Consider the above expression in the \( \lim k \to 0 \) with a small but finite current quark mass

\[
\lim_{k \to 0} -m^2 < V_0(k) V_0(-k) >_{wl} = \frac{1}{m} \frac{Z^2_\pi}{C_x} + \sum_{n=1}^{\infty} \frac{Z^2_{p,n}}{M^2_{p,n}} - \sum_{n=1}^{\infty} \frac{Z^2_{s,n}}{M^2_{s,n}}. \tag{40}
\]

Now in the limit of \( k \to 0 \) we can evaluate the left hand side of the above expression

\[
\lim_{k \to 0} -m^2 < V_0(k) V_0(-k) >_{wl} = -m^2 \int_0^\infty dT \exp \left\{ -\frac{m^2}{2} T \right\} \times \int_y \exp \{ -S_0 \} \nabla. \tag{41}
\]

where I have used

\[
V_0(k) = \int_0^T d\tau \exp\{ik \cdot y(\tau)\}. \tag{42}
\]

As noted in Eq. (29), if chiral symmetry is spontaneously broken then we must have the following asymptotic behaviour

\[
\lim_{T \to \infty} < \nabla >_{T} \equiv \lim_{T \to \infty} \int_{y, \psi} \exp\{ -S_0 \} \nabla \equiv C_0 \sqrt{T} + C_1 T^n + \cdots \tag{43}
\]

with some constants \( C_0 \) and \( C_1 \). The second term on the right hand side with \( n > \frac{1}{2} \) represents the next to the leading term in the asymptotic expansion. Using this in (41) leads to

\[
\lim_{k \to 0} -m^2 < V_0(k) V_0(-k) >_{wl} = \tilde{C}_0 m + \tilde{C}_1 m^{2n-2}, \tag{45}
\]

with constants \( \tilde{C}_0 \) and \( \tilde{C}_1 \) related to \( C_0 \) and \( C_1 \) by known factors. Comparing the above result with (40) we find that the first term on the right hand side of the above expression arises from the pion pole with \( M^2_\pi \) proportional to the current quark mass \( m \), while the second term implies that \( n \geq 1 \), assuming that in the chiral limit only massless spin-zero particle are the Nambu-Goldstone bosons.
From (45) we also see that the difference between the pseudo-scalar and the scalar two-point function crucially depends on whether $n = 1$ or $n > 1$. Consider first the possibility that $n > 1$. If that is the case, in the chiral limit (45) is consistent only with

$$\lim_{m \to 0} \Delta_\pi(k) - \Delta_\sigma(k) = -m^2 < V_0(k)V_0(-k) >_{wl}$$

$$= \frac{Z^2_\pi}{k^2 + M^2_\pi},$$

(46)

with $M^2_\pi$ vanishing linearly with the current quark mass $m$, but we know the momentum dependence of the right hand side of the above equation for large momentum from [12] and it goes as $1/k^4$ in the chiral limit. Therefore, the possibility of $n > 1$ is excluded and the long distance behaviour of the expectation value of SWL averaged over paths of length $T$ is given by

$$\lim_{T \to \infty} < W >_T = C_0 \frac{T^{1/2}}{T} + C_1 T + ...$$

(47)

This result has an interesting geometrical interpretation, the meaning of the leading term was already discussed in section (3). Let us consider now the sub-leading term, recalling again that in $D$ dimensions the probability of a particle to return to the origin after transversing a path of length $T$ goes as $T^{-D/2}$, therefore the sub-leading behaviour of the form of $T^{-1}$ implies that a finite fraction of paths that contribute to $< W >_T$ are effectively two-dimensional.

Let us consider (47) in the context of a possible dual string description of the planar QCD. In such a description mesons would appear as open strings with quark or an anti-quark as the end point of the string. The leading term in $< W >_T$ then suggests that there must be a spin-string interaction between quarks and the string connecting them. This spin-string interaction leads to an effective short range spin-spin interaction which is responsible for the $1/T^{1/2}$ behaviour of the leading term. Now consider the sub-leading term. This term is quite natural in a string description of mesons, as it implies that at least a finite fraction of paths that contribute to the average of the expectation value of the SWL are effectively two-dimensional.
5 Conclusions

There is a persistent hope that planar QCD can be solved by reformulating it as a string theory. In the planar limit the observables that seems most amenable to a string representation are the various types of Wilson loops. For planar QCD with quarks the appropriate Wilson loop in terms of which one can write the various meson Greens functions is the Wilson loop associated with a spin-half particle or the SWL. As we have seen in previous sections, an important quantity that probes the nature of the meson spectrum in the planar limit is the expectation value of the SWL averaged over loops of fixed length $T$, which we have denoted by $\langle W \rangle_T$.

In the present work we have been able to determine the leading and the subleading $T$ dependence for this quantity. They in turn suggest a string model of planar QCD in which there is a spin-string interaction between the spin of the quarks and the string connecting them. The spin-string interaction leads to an effective spin-spin interaction between the spin of the quark and the spin of the anti-quark which is responsible for the leading behavior of $\langle W \rangle_T$ and is responsible for the spontaneous breaking of chiral symmetry. The sub-leading term is more directly connected with the string behaviour as it suggests that at least a finite fraction of the paths contributing to $\langle W \rangle_T$ are effectively two-dimensional.

The present analysis therefore implies that in seeking a dual string representation of planar QCD one must include spin-string interaction in order to correctly reflect the manner in which chiral symmetry is realised. One approach, which has been investigated in [13], is to seek a string representation for the expectation value of the SWL. For SWL in a compact $U(1)$ gauge theory, which is known to be confining [14], one indeed finds that the strings describing the SWL have spin-string interaction which leads to an effective spin-spin interaction between the spin of the quark and the anti-quark. Of course the more important and interesting challenge is to find a string representation for the SWL in planar QCD which correctly incorporates the spin-string interaction.

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