Proximity effects in superconductor-ferromagnet heterostructures

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The very special characteristic of the proximity effect in superconductor-ferromagnet systems is the damped oscillatory behavior of the Cooper pair wave function in a ferromagnet. In some sense, this is analogous to the inhomogeneous superconductivity, predicted long time ago by Larkin and Ovchinnikov (1964), and Fulde and Ferrell (1964), and constantly searched since that. After the qualitative analysis of the peculiarities of the proximity effect in the presence of the exchange field, the author provides a unified description of the properties of the superconductor-ferromagnet heterostructures. Special attention is paid to the striking non-monotonous dependance of the critical temperature of the multilayers and bilayers on the ferromagnetic layer thickness and conditions of the realization of the ”π”-Josephson junctions. The recent progress in the preparation of the high quality hybrid systems permitted to observe on experiments many interesting effects, which are also discussed in the article. Finally, the author analyzes the phenomenon of the domain-wall superconductivity and the influence of superconductivity on the magnetic structure in superconductor-ferromagnet bilayers.

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I. INTRODUCTION

Due to their antagonistic characters, singlet superconductivity and ferromagnetic order cannot coexist in bulk samples with realistic physical parameters. Ginzburg (1956) was the first to set up theoretically the problem of magnetism and superconductivity coexistence taking into account the orbital mechanism of superconductivity destruction (interaction of the superconducting order parameter with a vector-potential \( \mathbf{A} \) of the magnetic field). After the creation of BCS theory, it became clear that superconductivity (in the singlet state) can be also destroyed by the exchange mechanism. The exchange field, in the magnetically ordered state, tends to align spins of Cooper pairs in the same direction, thus preventing a pairing effect. This is the so-called paramagnetic effect (Saint-James et al., 1969). Anderson and Suhl (1959) demonstrated that ferromagnetic ordering is unlikely to appear in the superconducting phase. The main reason for that is the suppression of the zero wave-vector component of the electronic paramagnetic susceptibility in the presence of superconductivity. In such situation the gain of energy for the ferromagnetic ordering decreases and instead of the ferromagnetic order the non-uniform magnetic ordering should appear. Anderson and Suhl (1959) called this state cryptoferromagnetic.

The 1977 discovery of ternary rare earth (RE) compounds (RE)Rh\(_4\)B\(_4\) and (RE)Mo\(_6\)X\(_8\) (X=S, Se) (as a review see, for example, Maple and Fisher, 1982) provided the first experimental evidence of magnetism and superconductivity coexistence in stoichiometrical compounds. It turned out that in many of these systems, superconductivity (with the critical temperature \( T_c \)) coexists rather easily with antiferromagnetic order (with the Néel temperature \( T_N \)), and usually the situation with \( T_N < T_c \) is realized.

The more recent discovery of superconductivity in the quaternary intermetallic compounds (RE)Ni\(_2\)B\(_2\)C (as a review see, for example, Müller and Narozhnyi, 2001) gives another example of antiferromagnetism and superconductivity coexistence.

Indeed, superconductivity and antiferromagnetism can coexist quite peacefully because, on average, at distances of the order of the Cooper pair size (superconducting coherence length) the exchange and orbital fields are zero. Much more interesting a re-entrant behavior of the superconductivity was observed in ErRh\(_4\)B\(_4\) and HoMo\(_6\)S\(_8\) (Maple and Fisher, 1982). For example, ErRh\(_4\)B\(_4\) becomes superconductor below \( T_c = 8.7 \) K. When it is cooled to the Curie temperature \( \Theta \approx 0.8 \) K an inhomogeneous magnetic order appears in the superconducting state. With further cooling the superconductivity is destroyed by the onset of a first-order ferromagnetic transition at the second critical temperature \( T_{c2} \approx 0.7 \) K. HoMo\(_6\)S\(_8\) gives another example of the re-entrant superconductivity with \( T_c = 1.8 \) K, \( \Theta \approx 0.74 \) K, and \( T_{c2} \approx 0.7 \) K.

In these compounds at Curie temperature, following the prediction of Anderson and Suhl (1959) a non-uniform magnetic order appears. Its presence was confirmed by neutron scattering experiments. The period of this magnetic structure is smaller than the superconducting coherence length, but larger than the interatomic distance. In some sense this structure is a realization of the compromise between superconductivity and ferromagnetism: for the superconductivity it is seen as an antiferromagnetism, but for the magnetism it looks like a ferromagnetism. Theoretical analysis, taking into account both orbital and exchange mechanisms and magnetic anisotropy (as a review see Bul'evskii et al., 1985), revealed that the coexistence phase is a domain-like structure with very small period. The region of magnetism and superconductivity coexistence in ErRh\(_4\)B\(_4\) and HoMo\(_6\)S\(_8\) is narrow, but in HoMo\(_6\)Se\(_8\) the domain coexistence phase survives till \( T = 0 \) K.

The first truly ferromagnetic superconductors UGe\(_2\) (Saxena et al., 2000) and URhGe (Aoki et al., 2001) have been discovered only recently, and apparently the coexistence of superconductivity with ferromagnetism is possible due to the triplet character of the superconducting pairing. Indeed, the superconductivity in URhGe (Aoki et al., 2001) appears below 0.3 K in the ferromagnetic phase which has the Curie temperature \( \Theta = 9.5 \) K; this makes the singlet scenario of superconductivity rather improbable.

Though the coexistence of singlet superconductivity with ferromagnetism is very unlikely in bulk compounds, it may be easily achieved in artificially fabricated layered ferromagnet/superconductors (F/S) systems. Due to the proximity effect, the Cooper pairs can penetrate into the F layer and induce superconductivity there. In such case we have the unique possibility to study the properties of superconducting electrons under the influence of a huge exchange field acting on the electron spins. In addition, it is possible to study the interplay between superconductivity and magnetism in a controlled manner, since varying the layer thicknesses we change the relative strength of two competing orderings. The behavior of the superconducting condensate under these conditions is quite peculiar.

Long time ago Larkin and Ovchinnikov (1964), and Fulde and Ferrell (1964) demonstrated that in a pure ferromagnetic superconductor at low temperature the superconductivity may be non-uniform. Due to the incompatibility of
of ferromagnetism and superconductivity it is not easy to verify this prediction on experiment. It occurs that in S/F systems there exists some analogy with the non-uniform superconducting state. The Cooper pair wave function has damped oscillatory behavior in a ferromagnet in contact with a superconductor. It results in many new effects that we discuss in this article: the spatial oscillations of the electron’s density of states, the non-monotonous dependance of the critical temperature of S/F multilayers and bilayers on the ferromagnet layer thickness, the realization of the Josephson "π"- junctions in S/F/S systems. The spin-wave effect in the complex S/F structures gives another example of the interesting interplay between magnetism and superconductivity, promising for the potential applications. We discuss also the issues of the localized domain-wall superconductivity in S/F bilayers and the inverse influence of superconductivity on ferromagnetism, which favors the non-uniform magnetic structures. An interesting example of atomic thickness S/F multilayers is provided by the layered superconductors like Sm$_{1.85}$Ce$_{0.15}$CuO$_4$ and RuSr$_2$GdCu$_2$O$_8$. For such systems the exchange field in F layer also favors the "π"-phase behavior, with an alternating order parameter in adjacent superconducting layers.

Note that practically all interesting effects related with the interplay between the superconductivity and the magnetism in S/F structures occurs at the nanoscopic range of layers thicknesses. The observation of these effects became possible only recently due to the great progress in the preparation of high-quality hybrid F/S systems. The experimental progress and the possibility of potential applications in its turn stimulated a revival of the interest to the superconductivity and ferromagnetism interplay in heterostructures. It seems to be timely to review the present state of the research in this domain and outline the perspectives.

II. PARAMAGNETIC LIMIT AND QUALITATIVE EXPLANATION OF THE NON-UNIFORM PHASE FORMATION

A. The (H, T) phase diagram

For a pure paramagnetic effect, the critical field of a superconductor $H_p$ at $T = 0$ may be found from the comparison of the energy gain $\Delta E_n$ due to the electron spin polarization in the normal state and the superconducting condensation energy $\Delta E_s$. Really, in the normal state, the polarization of the electron gas changes its energy in the magnetic field by

$$\Delta E_n = -\chi_n H^2 / 2, \quad (1)$$

where $\chi_n = 2\mu_B^2 N(0)$ is the spin susceptibility of the normal metal, $\mu_B$ is the Bohr magneton, $2N(0)$ is the density of electron states at Fermi level (per two spin projections), and the electron $g$ factor is supposed to be equal to 2.

On the other hand, in a superconductor the polarization is absent, but the BCS pairing decreases its energy by

$$\Delta E_s = -N(0) \Delta_0^2 / 2, \quad (2)$$

where $\Delta_0 = 1.76 T_c$ is the superconducting gap at $T = 0$. From the condition $\Delta E_n = \Delta E_s$, we find the Chandrasekhar (1962) - Clogston (1962) limit (the paramagnetic limit at $T = 0$)

$$H_p(0) = \frac{\Delta_0}{\sqrt{2}\mu_B}. \quad (3)$$

Note that it is the field of the first-order phase transition from a normal to a superconducting state. The complete analysis (Saint-James et al., 1969) demonstrates that at $T = 0$ this critical field is higher than the field of the second order phase transition $H_{pI}^{(2)}(0) = 4\Delta_0/2\mu_B$, and the transition from a normal to a uniform superconducting state is of the second-order at $T^* < T < T_c$ only, where $T^* = 0.56 T_c$, $H^* = H(T^*) = 0.61 \Delta_0 / \mu_B = 1.05 T_c / \mu_B$. However, Larkin and Ovchinnikov (1964), and Fulde and Ferrell (1964) predicted in the framework of the model of pure paramagnetic effect the appearance of the non-uniform superconducting state with a sinusoidal modulation of the superconducting order parameter at the scale of the superconducting coherence length $\xi_s$ (the FFLO state). In this FFLO state, the Cooper pairs have a finite momentum, compared with zero momentum in conventional superconductors. Recently Casalbuoni and Nardulli (2004) reviewed the theory of the inhomogeneous superconductivity applied to the condensed matter and quantum chromodynamics at high density and low temperature.

The critical field of the second-order transition into FFLO state goes somewhere above the first-order transition line into a uniform superconducting state (Saint-James et al., 1969). At $T = 0$, it is $H^{FFLO}(0) = 0.755 \Delta_0 / \mu_B$ (whereas $H_p = 0.7 \Delta_0 / \mu_B$). This FFLO state only appears in the temperature interval $0 < T < T^*$, and is sensitive to impurities
the ferromagnetic transition the inhomogeneous magnetic order appears (Maple and Fisher, 1982; Bulaevskii ferromagnet a magnetic field acting on electron spins. To demonstrate this, we consider the simplest case of the 1D superconductor. Of modulation of the superconducting order parameter is related to the Zeeman’s splitting of the electron’s level under the exchange Hamiltonian

$$H_{int} = \int d^3r \Psi^+(r) \left\{ \sum_i J(r - r_i) S_i \sigma \right\} \Psi(r),$$

where $$\Psi(r)$$ is the electron’s spinor operator, $$\sigma = \{\sigma_x, \sigma_y, \sigma_z\}$$ are the Pauli matrices, and $$J(r)$$ is the exchange integral. Below the Curie temperature $$\Theta$$, the average value of the localized spins $$\langle S_i \rangle$$ is non-zero, and the exchange interaction may be considered as some effective Zeeman field $$H^{eff} = \frac{\mu_B}{\mu_n} \int J(r) d^3r$$, where $$n$$ is the concentration of localized moments, and the spin quantization z-axis is chosen along the ferromagnetic moment. It is convenient to introduce the exchange field $$h$$ as

$$h = \mu_B H^{eff} = \langle S_i^z \rangle n \int J(r) d^3r = s(T)h_0,$$

where $$s(T) = \langle S_i^z \rangle / \langle S_i^z \rangle_{T = 0}$$ is the dimensionless magnetization and $$h_0$$ is the maximum value of an exchange field at $$T = 0$$. The exchange field $$h$$ describes the spin-dependent part of the electron’s energy and the exchange Hamiltonian Eq. (4) is then simply written as

$$H_{int} = \int d^3r \Psi^+(r) h \sigma_z \Psi(r).$$

If we also want to take into account the proper Zeeman field of magnetization $$M$$, then we may simply replace $$h$$ in Eq. (6) by $$h + 4\pi M \mu_B$$. The reader is warned that in principle, if the exchange integral is negative, the exchange field may have the direction opposite to the magnetic moments and the interesting compensation Jaccarino-Peter (1962) effect is possible. However, in the ferromagnetic metals, the contribution of the magnetic induction to the spin splitting is several order of magnitude smaller than that of the exchange interaction and may be neglected. In the case of the Ruderman-Kittel-Kasuya-Yosida (RKKY) mechanism of the ferromagnetic ordering, the Curie temperature $$\Theta \sim h_0^2 / E_F$$ and in all real systems the exchange field $$h_0 >> \Theta, T_c$$. This explains that the conditions of singlet superconductivity and ferromagnetism coexistence are very stringent. Indeed, if $$\Theta > T_c$$ the exchange field in a ferromagnet $$h >> T_c$$, which strongly exceeds the paramagnetic limit. On the other hand, if $$\Theta < T_c$$ then, instead of the ferromagnetic transition the inhomogeneous magnetic ordering appears (Maple and Fisher, 1982; Bulaevskii et al., 1985). The very high value of the exchange field in ferromagnet permits us to concentrate on the paramagnetic effect and neglect the orbital one (note that well below the Curie temperature the magnetic induction $$4\pi M$$ in ferromagnets is of the order of several $$\mu_0$$ only).

C. Why does the Fulde-Ferrell-Larkin-Ovchinnikov state appear?

What is the physical origin of the superconducting order parameter modulation in the FFLO state? The appearance of modulation of the superconducting order parameter is related to the Zeeman’s splitting of the electron’s level under a magnetic field acting on electron spins. To demonstrate this, we consider the simplest case of the 1D superconductor.

In the absence of the field, a Cooper pair is formed by two electrons with opposite momenta $$+k_F$$ and $$-k_F$$ and opposite spins ($$\uparrow$$) and ($$\downarrow$$) respectively. The resulting momentum of the Cooper pair $$k_F + (-k_F) = 0$$. Under a magnetic field, because of the Zeeman’s splitting, the Fermi momentum of the electron with spin ($$\uparrow$$) will shift from $$k_F$$
to \( k_1 = k_F + \delta k_F \), where \( \delta k_F = \mu_B H/v_F \) and \( v_F \) is the Fermi velocity. Similarly, the Fermi momentum of an electron with spin \( (\downarrow) \) will shift from \(-k_F\) to \( k_2 = -k_F + \delta k_F \) (see Fig. 2). Then, the resulting momentum of the Cooper pair will be \( k_1 + k_2 = 2\delta k_F \neq 0 \), which just implies the space modulation of the superconducting order parameter with a resulting wave-vector \( 2\delta k_F \). Such type of reasoning explains the origin of the non-uniform superconducting state formation in the presence of the field acting on electron spins, and, at the same time, demonstrates the absence of a paramagnetic limit (at \( T \to 0 \)) for the 1D superconductor (Buzdin and Polonskii, 1987). For 3D (Larkin and Ovchinnikov, 1964 and Fulde and Ferrell, 1964) or 2D (Bulaevskii, 1973) superconductors, it is not possible to choose the single wave vector \( \delta k_F \) which compensates the Zeeman splitting for all electrons on the Fermi surface (as \( \delta k_F \) depends on direction of \( v_F \)), and the paramagnetic limit is preserved. However, the critical field for a non-uniform state at \( T = 0 \) is always higher than for a uniform one. However, the critical field for a non-uniform state at \( T = 0 \) is always higher than for a uniform one. At finite temperature (when \( T \gtrsim \mu_B H \)), the smearing of the electrons distribution function near the Fermi energy decreases the difference of energies between the non-uniform and uniform states. As it follows from the microscopical calculations, at \( T > T^* = 0.56 T_c \) the uniform superconducting phase is always more favorable (Saint-James et al., 1969).

D. Generalized Ginzburg-Landau functional

Qualitatively, the phenomenon of the FFLO phase formation and the particularities of the proximity effect in S/F systems may be described in the framework of the generalized Ginzburg-Landau expansion. Let us first recall the form of the standard Ginzburg-Landau functional (see, for example, De Gennes, 1966)

\[
F = a |\psi|^2 + \gamma |\nabla \psi|^2 + \frac{b}{2} |\psi|^4, \tag{7}
\]

where \( \psi \) is the superconducting order parameter, and the coefficient \( a \) vanishes at the transition temperature \( T_c \). At \( T < T_c \), the coefficient \( a \) is negative and the minimum of \( F \) in Eq. (7) is achieved for a uniform superconducting state with \( |\psi|^2 = -\frac{a}{\gamma} \). If we consider also the paramagnetic effect of the magnetic field, all the coefficients in Eq. (7) will depend on the energy of the Zeeman splitting \( \mu_B H \), i.e., an exchange field \( h \) in the ferromagnet. Note that we neglect the orbital effect, so there is no vector-potential \( A \) in Eq. (7). To take into account the orbital effect in the Ginzburg-Landau functional, we may substitute the gradient by its gauge-invariant form \( \nabla \to \nabla - \frac{2a}{c} A \). Usually, the orbital effect is much more important for the superconductivity destruction than the paramagnetic one. It explains why in the standard Ginzburg-Landau theory there is no need to take into account the field and temperature dependence of the coefficients \( \gamma \) and \( b \). However, when the paramagnetic effect becomes predominant, this approximation fails. What are the consequences? If it was simply some renormalization of the coefficients in Ginzburg-Landau functional, the general superconducting properties of the system would basically be the same. However, the qualitatively new physics emerges due to the fact that the coefficient \( \gamma \) changes its sign at the point \((H^*, T^*)\) of the phase diagram, see Fig. 1. The negative sign of \( \gamma \) means that the minimum of the functional does not correspond to an uniform state anymore, and a spatial variation of the order parameter decreases the energy of the system. To describe such a situation it is necessary to add a higher order derivative term in the expansion (7), and the generalized Ginzburg-Landau expansion will be:

\[
F_G = a(H,T) |\psi|^2 + \gamma(H,T) |\nabla \psi|^2 + \frac{\eta(H,T)}{2} |\nabla^2 \psi|^2 + \frac{b(H,T)}{2} |\psi|^4, \tag{8}
\]

The critical temperature of the second order phase transition into a superconducting state may be found from the solution of the linear equation for the superconducting order parameter

\[
a \psi - \gamma \Delta \psi + \frac{\eta}{2} \Delta^2 \psi = 0. \tag{9}
\]

If we seek for a non-uniform solution \( \psi = \psi_0 \exp(iq r) \), the corresponding critical temperature depends on the wave-vector \( q \) and is given by the expression

\[
a = -\gamma q^2 - \frac{\eta}{2} q^4. \tag{10}
\]
Note that the coefficient $a$ may be written as $a = \alpha(T - T_{cu}(H))$, where $T_{cu}(H)$ is the critical temperature of the transition into the uniform superconducting state. In a standard situation, the gradient term in the Ginzburg-Landau functional is positive, $\gamma > 0$, and the highest transition temperature coincides with $T_{cu}(H)$; it is realized for the uniform state with $q = 0$. However, in the case $\gamma < 0$, the maximum critical temperature corresponds to the finite value of the modulation vector $q_0^2 = -\gamma/\eta$ and the corresponding transition temperature into the non-uniform FFLO state $T_{cu}(H)$ is given by

$$a = \alpha(T_{cu} - T_{cu}) = \frac{\gamma^2}{2\eta}.$$  

(11)

It is higher than the critical temperature $T_{cu}$ of the uniform state. Therefore, we see that the FFLO state appearance may simply be interpreted as a change of the sign of the gradient term in the Ginzburg-Landau functional. A more detailed analysis of the FFLO state in the framework of the generalized Ginzburg-Landau functional shows that it is not an exponential but a one dimensional sinusoidal modulation of the order parameter which gives the minimum energy (Buzdin and Kachkachi, 1997; Houzet et al., 1999). In fact, the generalized Ginzburg-Landau functional describes new type of superconductors with very different properties, and the whole theory of superconductivity must be redone on the basis of this functional. The orbital effect in the framework of the generalized Ginzburg-Landau functional may be introduced by the usual gauge-invariant procedure $\nabla \rightarrow \nabla - \frac{2e}{\hbar c} A$. The resulting expression for the superconducting current is quite a special one and the critical field may correspond to the higher Landau level solutions as well as new types of vortex lattices may exist (Houzet and Buzdin, 2000; Houzet and Buzdin, 2001).

III. PROXIMITY EFFECT IN FERROMAGNETS

A. Some generalities about superconducting proximity effect

The contact of materials with different long-range ordering modifies their properties near the interface. In the case of a superconductor-normal metal interface, the Cooper pairs can penetrate the normal metal at some distance. If the electrons motion is diffusive, this distance is of the order of the thermal diffusion length scale $L_T \sim \sqrt{D/T}$, where $D$ is the diffusion constant. In the case of pure normal metal the corresponding characteristic distance is $\xi_T \sim v_F/T$. Therefore the superconducting-like properties may be induced in the normal metal, and usually this phenomenon is called the proximity effect. At the same time the leakage of the Cooper pairs weakens the superconductivity near the interface with a normal metal. Sometime this effect is called the "inverse proximity effect", and it results in the decrease of the superconducting transition temperature in thin superconducting layer in contact with a normal metal. If the thickness of a superconducting layer is smaller than some critical one, the proximity effect totally suppresses the superconducting transition. All these phenomena and the earlier experimental and theoretical works on the proximity effect were reviewed by Deutscher and de Gennes (1969).

Note that the proximity effect is a rather general phenomenon not limited by the superconducting phase transition. For example, in the case of the surface magnetism (White and Geballe, 1979) the critical temperature at the surface can be higher than the bulk one. In the result the magnetic transition at the surface induces the magnetisation nearby. On the other hand, the volume strongly affects the surface transition characteristics.

However, the unique and very important characteristic of the superconducting proximity effect is the Andreev reflection revealed at the microscopic level. Andreev (1964) demonstrated how the single electron states of the normal metal are converted into Cooper pairs and explained the mechanism of the transformation at the interface of the dissipative electrical current into the dissipationless supercurrent. An electron with an energy below the superconducting gap is reflected at the interface as a hole. The corresponding charge $2e$ is transferred to the Cooper pair which appears on the superconducting side of the interface. The manifestation of this double charge transfer is that for a perfect contact the sub-gap conductance occurs to be twice the normal state conductance. The classical work by Blonder, Tinkham and Klapwijk (1982) gives the detailed theory of this phenomenon.

Andreev reflection plays a primary role for the understanding of quantum transport properties of superconductor/normal metal systems. The interplay between Andreev reflection and proximity effect was reviewed by Panetier and Courtois (2000). The reader can find a detailed description of the Andreev reflection in the normal metal-superconductor junctions in the framework of the scattering theory formalism in the review by Beenakker (1997). Recent review by Deutscher (2005) is devoted to the Andreev reflection spectroscopy of the superconductors.
The physics of the oscillating Cooper pair wave function in a ferromagnet is similar to the physics of the superconducting order parameter modulation in the FFLO state - see section II.C. Qualitative picture of this effect has been well presented by Demler, Arnold, and Beasley (1997). When a superconductor is in a contact with a normal metal the Cooper pairs penetrate across the interface at some distance inside the metal. A Cooper pair in a superconductor comprises two electrons with opposite spins and momenta. In a ferromagnet the up spin electron (with the spin orientation along the exchange field) decreases its energy by $h$, while the down spin electron increases its energy by the same value. To compensate this energy variation, the up spin electron increases its kinetic energy, while the down spin electron decreases its. In the result the Cooper pair acquires a center of mass momentum $2\delta k_F = 2h/v_F$, which implies the modulation of the order parameter with the period $\pi v_F / h$. The direction of the modulation wave vector must be perpendicular to the interface, because only this orientation is compatible with the uniform order parameter in the superconductor.

To get some idea about the peculiarity of the proximity effect in S/F structures, we may start also from the description based on the generalized Ginzburg-Landau functional Eq. (8). Such approach is adequate for a small wave-vector modulation case, i. e. in the vicinity of the $(H^*, T^*)$ point of the $(H,T)$ phase diagram, otherwise the microscopical theory must be used. This situation corresponds to a very weak ferromagnet with an extremely small exchange field $h \approx \mu_B H^* = 1.05 T_c$, which is non realistic as usually $h >> T_c$. However, we will discuss this case to get a preliminary understanding of the phenomenon. We address the question of the proximity effect for a weak ferromagnet described by the generalized Ginzburg-Landau functional Eq. (8). More precisely, we consider the decay of the order parameter in the normal phase, i. e. at $T > T_c$ assuming that our system is in contact with another superconductor with a higher critical temperature, and the $x$ axis is choosen perpendicular to the interface (see Fig. 3).

The induced superconductivity is weak and to deal with it, we may use the linearized equation for the order parameter (9), which is written for our geometry as

$$a \psi - \gamma \frac{\partial^2 \psi}{\partial x^2} + \frac{\eta}{2} \frac{\partial^4 \psi}{\partial x^4} = 0. \quad (12)$$

The solutions of this equation in the normal phase are of the type $\psi = \psi_0 \exp(kx)$, with a complex wave-vector $k = k_1 + ik_2$, and

$$k_1^2 = \frac{|\gamma|}{2\eta} \left( \sqrt{1 + \frac{T - T_{ci}}{T_{ci} - T_{cu}}} - 1 \right), \quad (13)$$
$$k_2^2 = \frac{|\gamma|}{2\eta} \left( 1 + \sqrt{1 + \frac{T - T_{ci}}{T_{ci} - T_{cu}}} \right). \quad (14)$$

If we choose the gauge with the real order parameter in the superconductor, then the solution for the decaying order parameter in the ferromagnet is also real

$$\psi(x) = \psi_0 \exp(-k_1 x) \cos(k_2 x), \quad (15)$$

where the choice of the root for $k$ is the condition $k_1 > 0$. So the decay of the order parameter is accompanied by its oscillation (Fig. 3b), which is the characteristic feature of the proximity effect in the considered system. When we approach the critical temperature $T_{ci}$ the decaying wave-vector vanishes, $k_1 \to 0$, while the oscillating wave-vector $k_2$ goes to the FFLO wave-vector, $k_2 \to \sqrt{\frac{\xi T_c}{\eta}}$, so a FFLO phase emerges. Let us compare this behavior with the standard proximity effect (Deutscher and De Gennes, 1969) described by the linearized Ginzburg-Landau equation for the order parameter

$$a \psi - \frac{\partial^2 \psi}{\partial x^2} = 0, \quad (16)$$

with $\gamma > 0$. In such case $T_c$ simply coincides with $T_{cu}$, and the decaying solution is $\psi = \psi_0 \exp(-x/\xi(T))$, where the coherence length $\xi(T) = \sqrt{\gamma/\alpha}$ (Fig. 3a). This simple analysis brings in evidence the appearance of the oscillations of the order parameter in the presence of an exchange field. This is a fundamental difference between the proximity effect in S/F and S/N systems, and it is at the origin of many peculiar characteristics of S/F heterostructures.
In real ferromagnets, the exchange field is very large compared with superconducting temperature and energy scales, so the gradients of the superconducting order parameter variations are large too, and can not be treated in the framework of the generalized Ginzburg-Landau functional. To describe the relevant experimental situation we need to use a microscopical approach. The most convenient scheme to do this (see Appendix A and B) is the use of the Boboliubov-de Gennes equations or the Green’s functions in the framework of the quasiclassical Eilenberger (Eilenberger, 1968) or Usadel (Usadel, 1970) equations.

If the electron scattering mean free path $l$ is small (which is usually the case in S/F systems), the most natural approach is to use the Usadel equations for the Green’s functions averaged over the Fermi surface (Appendix). Linearized over the pair potential $\Delta(x)$, the Usadel equation for the anomalous function $F(x, \omega)$ depending only on one coordinate $x$ is

$$
\left( |\omega| + i h \cdot \text{sgn}(\omega) - \frac{D}{2} \frac{\partial^2}{\partial x^2} \right) F(x, \omega) = \Delta(x),
$$

where $\omega = (2n + 1) \pi T$ are the Matsubara frequencies, and $D = \frac{1}{3} v_F l$ is the diffusion coefficient. In the F region, we may neglect the Matsubara frequencies compared to the large exchange field ($h >> T_c$), and the pairing potential $\Delta$ is absent (we assume that the BCS coupling constant $\lambda$ is zero there). This results in a very simple form of the Usadel equation for the anomalous function $F_f$ in the ferromagnet

$$
\frac{ih \text{sgn}(\omega)}{2} F_f - D_f \frac{\partial^2 F_f}{\partial x^2} = 0,
$$

where $D_f$ is the diffusion coefficient in the ferromagnet. For the geometry in Fig. 3 and $\omega > 0$, the decaying solution for $F_f$ is

$$
F_f(x, \omega > 0) = A \exp\left( -\frac{i + 1}{\xi_f} x \right),
$$

where $\xi_f = \sqrt{\frac{D_f}{h}}$ is the characteristic length of the superconducting correlations decay (with oscillations) in F-layer (see Table I). Due to the condition $h >> T_c$, this length is much smaller than the superconducting coherence length $\xi_s = \sqrt{\frac{D_f}{2 \pi T_c}}$, i.e. $\xi_f << \xi_s$. The constant $A$ is determined by the boundary conditions at the S/F interface. For example, in the case of a low resistivity of a ferromagnet, at first approximation the anomalous function in a superconductor $F_s$ is independent on coordinate and practically the same as in the absence of the ferromagnet, i.e. $F_s = \Delta / \sqrt{\Delta^2 + \omega^2}$. If, in addition, the interface is transparent then the continuity of the function $F$ at the F/S boundary gives $A = \Delta / \sqrt{\Delta^2 + \omega^2}$. For $\omega < 0$, we simply have $F_f(x, \omega < 0) = F_f^\ast(x, \omega > 0)$. In a ferromagnet, the role of the Cooper pair wave function is played by $\Psi$ than decays as

$$
\Psi \sim \sum_\omega F(x, \omega) \sim \Delta \exp\left( -\frac{x}{\xi_f} \cos\left( \frac{x}{\xi_f} \right) \right).
$$

We retrieve the damping oscillatory behavior of the order parameter Eq. (15), Fig. 3b. The important conclusion we obtain from the microscopic approach is that in the dirty limit the scale for the oscillation and decay of the Cooper pair wave function in a ferromagnet is the same.

In the case of a clean ferromagnet the damped oscillatory behavior of the Cooper pair wave function remains, though at zero temperature the damping is non-exponential and much weaker ($\sim \frac{1}{3}$). Indeed, the decaying solution of the Eilenberger equation in the clean limit (see Appendix B) is

$$
\tilde{f}(x, \theta, \omega) \sim \exp\left( -\frac{2(\omega + i h)x}{v_{Ff} \cos \theta} \right),
$$

where $\theta$ is the angle between $x$-axis and Fermi velocity in a ferromagnet, and $v_{Ff}$ is its modulus. After averaging over the angle $\theta$ and summation over the Matsubara frequencies $\omega$ we obtain

$$
\Psi \sim \sum_\omega \int_0^\pi \tilde{f}(x, \theta, \omega) \sin \theta d\theta \sim \frac{1}{x} \exp\left( -\frac{x}{\xi_f} \right) \sin\left( \frac{x}{\xi_{2f}} \right).
$$
Here the decaying length $\xi_f = \frac{\mu_f}{2\pi}$, and the oscillating length $\xi_{2f} = \frac{\mu_f}{2\pi}$ (see Table I). At low temperature $\xi_f \to 0$ and the Cooper pair wave function decays very slowly $\sim \frac{1}{x} \sin\left(\frac{\pi x}{\xi_f}\right)$. An important difference with the proximity effect for the normal metal is the presence of the short-ranged oscillations of the order parameter with the temperature independent period $2\pi \xi_{2f}$. In contrast with the dirty limit in a clean ferromagnet the characteristic lengths of the superconducting correlations’ decay and oscillations are not the same. Halterman and Valls (2001) performed the studies of the ferromagnet-superconductor interfaces on the basis of the self-consistent numerical solution of the microscopical Bogoliubov-de Gennes equations. They clearly observed the damped oscillatory behavior of the Cooper pair wave function of the type $\Psi \sim \frac{1}{x} \sin\left(\frac{\pi x}{\xi_f}\right)$.

We may conclude that at low temperatures the proximity effect in clean ferromagnet metals is long-ranged. On the other hand, in the dirty limit the use of the Usadel equations gives the exponential decay of $\Psi$. This is due to the fact that the Usadel equations are obtained by averaging over the impurities configurations. Zyuzin et al. (2003) pointed out that at distances $x >> \xi_f$ the anomalous Green’s function $F$ (as well as the Cooper pair wave function) has a random sample-specific sign, while the modulus does not decay exponentially. This circumstance leads to the survival of the proximity effect in the dirty ferromagnet at distances $x >> \xi_f$. The use of the Usadel equations at such distances may be misleading. However, from the practical point of view the range of interest is $x < 5\xi_f$, because at larger distances it is difficult to observe the oscillating phenomena on experiment. In this range the use of the Usadel equation is adequate.

The characteristic length of the induced superconductivity variation in a ferromagnet is small compared with a superconducting length, and it implies the use of the microscopic theory of the superconductivity to describe the proximity effect in S/F structures. In this context, the calculations of the free energy of S/F structures in the framework of the standard Ginzburg-Landau functional (Ryazanov et al., 2001a; Ryazanov et al., 2001b) can not be justified. Indeed, the possibility to neglect the higher gradient terms in the Ginzburg-Landau functional implies that the length scale of the variation of the order parameter must be larger than the correlation length. In the ferromagnet the correlation length is $\xi_f = \sqrt{\frac{D_f}{\hbar}}$ in the dirty limit and $\xi^0_f = \sqrt{\frac{v_F}{\hbar}}$ in the clean limit. We see that they coincide with the characteristic lengths of the order parameter variation in a ferromagnet. Therefore the higher gradient terms in the Ginzburg-Landau functional will be of the same order of magnitude as the term with the first derivative.

C. Density of states oscillations

Superconductivity creates a gap in the electronic density of states (DOS) near the Fermi energy $E_F$, i. e. the DOS is zero for an energy $E$ in the interval $E_F - \Delta < E < E_F + \Delta$. So, it is natural, that the induced superconductivity in S/N structures decreases DOS at $E_F$ near the interface. Detailed experimental studies of this phenomenon have been performed by Moussy et al. (2001). Damped oscillatory dependence of the Cooper pair wave function in ferromagnet hints that a similar damped oscillatory behavior may be expected for the variation of the DOS due to the proximity effect. Indeed, the DOS $N(\varepsilon)$, where $\varepsilon = E - E_F$ is the energy calculated from the Fermi energy, is directly related to the normal Green function in the ferromagnet $G_F(x, \omega)$ (Abrikosov et al., 1975)

$$N_f(\varepsilon) = N(0) \text{Re} G_f(x, \omega \to i\varepsilon), \quad (23)$$

where $N(0)$ is the DOS of the ferromagnetic metal. In a dirty limit taking into account the relation between the normal and anomalous Green functions $G^2_f + F^2_f = 1$ (Usadel, 1970), and using for $F_f = \frac{\Delta}{\sqrt{\Delta^2 + \omega^2}} \exp\left(-\frac{i\omega}{\xi_f}\right)$, we directly obtain the DOS at the Fermi energy ($\varepsilon = 0$) in a ferromagnet (Buzdin, 2000) at the distance $x >> \xi_f$

$$N_f(\varepsilon = 0) \approx N(0) \left(1 - \frac{1}{2} \exp\left(-\frac{2x}{\xi_f}\right) \cos\left(\frac{2x}{\xi_f}\right)\right). \quad (24)$$

This simple calculation implies $\Delta < < T_c$. An interesting conclusion is that at certain distances the DOS at the Fermi energy may be higher than in the absence of superconductor. This contrasts with the proximity effect in the S/N systems. Such behavior has been observed experimentally by Kontos et al. (2001) in the measurements of the DOS by planar-tunneling spectroscopy in Al/Al₂O₃/PdNi/Nb junctions, see Fig. 4.

For the PdNi layer thickness 50 Å we are at the distance when the term $\cos\left(\frac{2x}{\xi_f}\right)$ in Eq. (24) is positive and we have the normal decrease of the DOS inside the gap due to the proximity effect. However, for PdNi layer thickness 75 Å the $\cos\left(\frac{2x}{\xi_f}\right)$ term changes its sign and the DOS becomes a little bit larger than its value in the normal effect. Such inversion of the DOS permits us to roughly estimate $\xi_f$ for the PdNi alloy used by Kontos et al. (2001) as 60 Å.
At the moment, there exist only one experimental work on the DOS in S/F systems, while several theoretical papers treat this subject more in details. In a series of papers Halterman and Valls (2001, 2002, 2003) performed extensive theoretical studies of the local DOS behavior in S/F systems in a clean limit in the framework of the self-consistent Bogoliubov-De Gennes approach. They calculated the DOS spectra on both S and F sides and took into account the Fermi wave vectors mismatch, interfacial barrier and sample size.

Fazio and Lucheroni (1999) performed numerical self-consistent calculations of the local DOS in S/F system in the framework of the Usadel equation. The influence of the impurity scattering on the DOS oscillations has been studied by Baladić and Buzdin, (2001) and Bergeret et al. (2002). An interesting conclusion is that the oscillations disappear in the clean limit. In this context it is quite understandable, that the calculations of the DOS oscillations made in the ballistic regime for the ferromagnetic film on the top of the superconductor (Zareyan et al., 2001, Zareyan et al., 2002) depend essentially on the boundary conditions at the ferromagnet-vacuum interface. Sun et al. (2002) used the quasiclassical version of the Bogoliubov-De Gennes equations for the numerical calculations of the DOS in the S/F system with semi-infinite ferromagnet. They obtained in the clean limit the oscillations of the DOS and presented a quantitative fit of the experimental data of Kontos et al. (2001). Astonishingly, in the another quasiclassical approach on the basis of Eilenberger equations the oscillations of DOS are absent in the case of an infinite electron mean free path (Baladie and Buzdin, 2001 and Bergeret et al., 2002).

DOS oscillations in ferromagnets hint on the similar oscillatory behavior of the local magnetic moment of the electrons. The corresponding magnetic moment induced by the proximity effect may be written as

$$\delta M = i\mu_B N(0) \pi T \sum_{\omega} (G_f(x,\omega,h) - G_f(x,\omega,-h)).$$

Assuming the low resistivity of a ferromagnet in the dirty limit at temperature near $T_c$, the magnetic moment is

$$\delta M = -\mu_B N(0) \pi \frac{\Delta^2}{2T_c} \exp\left(-\frac{2x}{\xi_f}\right) \sin\left(\frac{2x}{\xi_f}\right).$$

Note that the total electron’s magnetic moment in a ferromagnet being

$$M = \delta M + \mu_B N(0)h.$$ 

Similarly to the DOS the local magnetic moment oscillates, and curiously in some regions it may be higher than in the absence of superconductivity. Proximity effect also induces the local magnetic moment in a superconductor near the S/F interface at the distance of the order of superconducting coherence length $\xi_s$.

The proximity induced magnetism was studied on the basis of the Usadel equations by Bergeret al. (2004a, 2004b) and Krivoruchko and Koshina (2002). Numerical calculations of Krivoruchko and Koshina (2002) revealed the damped oscillatory behavior of the local magnetic moment in a superconductor at the scale of $\xi_s$ with positive magnetization at the interface. On the other hand Bergeret al. (2004a) argued that the induced magnetic moment in a superconductor must be negative. This is related to the Cooper pairs located in space in such a way that one electron of the pair is in superconductor, while the other is in the ferromagnet. The direction along the magnetic moment in the ferromagnet is preferable for the electron of the pair located there and this makes the spin of the other electron of the pair (located in superconductor) to be antiparallel.

The microscopic calculations of the local magnetic moment in the pure limit in the framework of Bogoliubov-de Gennes equations (Halterman and Valls, 2004) also revealed the damped oscillatory behavior of the local magnetic moment but at the atomic length scale. Probably in the quasiclassical approach the oscillations of the local magnetic moments disappear in the clean limit, similarly to the case of DOS oscillations. The magnitude of the proximity induced magnetic moment is very small, and at present time there are no manifestations of this phenomena on experiment.

D. Andreev reflection at the S/F interface

The spin effects play an important role in the Andreev reflection at the S/F interface. Indeed, an incident spin up electron in ferromagnet is reflected by the interface as a spin down hole, and in the result a Cooper pair of electrons
with opposite spins appears in a superconductor. Therefore the both spin up and spin down bands of electrons in ferromagnet are involved in this process. De Jong and Beenakker (1995) were the first to demonstrate the major influence of spin polarization in ferromagnet on the subgap conductance of the S/F interface. Indeed, in the fully spin-polarized metal all carriers have the same spin and Andreev reflection is totally suppressed. In general, with the increase of the spin polarization the subgap conductance drops from the double of the normal state conductance to a small value for the highly polarized metals. Following de Jong and Beenakker (1995) let us consider a simple intuitive picture of the conductance through a ballistic S/F point contact. Using the language of the scattering channels (subbands which cross the Fermi level), the conductance at $T = 0$ of a ferromagnet-normal metal contact is given by the Landauer formula

$$G_{FN} = \frac{e^2}{h} N.$$  

(28)

The total number of scattering channels $N$ is the sum of the spin up $N_\uparrow$ and spin down $N_\downarrow$ channels $N = N_\uparrow + N_\downarrow$, and the spin polarization implies that $N_\uparrow > N_\downarrow$. In the case of the contact of the superconductor with the non-polarized metal all electrons are reflected as the holes, which doubles the number of scattering channels and the conductance itself. For the spin-polarized metal where $N_\uparrow > N_\downarrow$, all the spin down electrons will be reflected as the spin up holes. However, only the part $N_\downarrow / N_\uparrow < 1$ of the spin up electrons can be Andreev reflected. The subgap conductance of the S/F contact is then

$$G_{FS} = \frac{e^2}{h} \left( 2N_\downarrow + 2N_\uparrow \frac{N_\downarrow}{N_\uparrow} \right) = 4 \frac{e^2}{h} N_\downarrow.$$  

(29)

Comparing this expression with Eq. (28) we see that $G_{FS}/G_{FN} = 4N_\uparrow/(N_\downarrow + N_\uparrow) < 2$ and $G_{FS} = 0$ for the full-polarized ferromagnet with $N_\downarrow = 0$. If the spin polarization is defined as $P = (N_\uparrow - N_\downarrow)/(N_\downarrow + N_\uparrow)$, then the suppression of the normalized zero-bias conductance gives the direct access to the value of $P$:

$$\frac{G_{FS}}{G_{FN}} = 2(1 - P).$$  

(30)

The subsequent experimental measurements of the spin polarization with Andreev reflection (Upadhyay et al., 1998; and Soulen et al., 1998) fully confirmed the efficiency of this method to probe the ferromagnets. The Andreev point contact spectroscopy permits to measure the spin polarization in a much wider range of materials (Zutic, Fabian and Das Sarma, 2004) comparing with the spin-polarized electron tunneling (Meservey and Tedrow, 1994).

However, the interpretation of the Andreev reflection data on the conductance of the S/F interfaces and the comparison of the spin polarization with the one obtained from the tunneling data, may be complicated by the band structure effects (Mazin, 1999). Zutic and Valls (1999, 2000), Zutic andDas Sarma (1999) generalized the results of the theoretical analysis of Blonder, Tinkham and Klapwijk (1982) to the case of the S/F interface. An interesting striking result is that in the absence of the potential barrier at the S/F interface, the spin polarization could increase the subgap conductance. The condition of perfect transparency of the interface is $v_{F\uparrow}v_{F\downarrow} = v_s^2$, where $v_{F\uparrow}$ and $v_{F\downarrow}$ are the Fermi velocities for two spin polarizations in ferromagnet, and $v_s$ is the Fermi velocity in superconductor. Vodopyanov and Tagirov (2003a) proposed a quasiclassical theory of Andreev reflection in F/S nanocontacts and analyzed the spin polarization calculated from the conductance and tunneling measurements.

Note that a rather high spin polarization has been measured in CrO$_2$ films $P = 90\%$ and in La$_{0.7}$Sr$_{0.3}$MnO$_3$ films $P = 78\%$ (Soulen et al., 1998). The spin-polarized tunneling data for these systems is lacking.

Another interesting effect related with the crossed Andreev reflection has been predicted by Deutscher and Feinberg (2000) (see also Deutscher, 2004 and Yamashita, Takahashi and Maekawa, 2003). The electric current between two ferromagnetic leads attached to the superconductor strongly depends on the relative orientation of the magnetization in these leads. If we assume that the leads are fully polarized, then the electron coming from one lead cannot experience the Andreev reflection in the same lead. However, this reflection is possible in the second lead, provided its polarization is opposite, and the distance between the leads is smaller than the superconducting coherence length. The resistance between the leads will be high for the parallel orientation of the magnetizations and low for the antiparallel orientation.
IV. OSCILLATORY SUPERCONDUCTING TRANSITION TEMPERATURE IN S/F MULTILAYERS AND BILAYERS

A. First experimental evidences of the anomalous proximity effect in S/F systems

The damped oscillatory behavior of the superconducting order parameter in ferromagnets may produce the commensurability effects between the period of the order parameter oscillation (which is of the order of $\xi_f$) and the thickness of a F layer. This results in the striking non-monotonous superconducting transition temperature dependence on the F layer thickness in S/F multilayers and bilayers. Indeed, for a F layer thickness smaller than $\xi_f$, the pair wave function in the F layer changes a little and the superconducting order parameter in the adjacent S layers must be the same. The phase difference between the superconducting order parameters in the S layers is absent and we call this state the "0"-phase. On the other hand, if the F layer thickness becomes of the order of $\xi_f$, the pair wave function may go through zero at the center of F layer providing the state with the opposite sign (or $\pi$ shift of the phase) of the superconducting order parameter in the adjacent S layers, which we call the "$\pi$"-phase. The increase of the thickness of the F layers may provoke the subsequent transitions from "0"- to "$\pi$"-phases, what superpose on the commensurability effect and result in a very special dependence of the critical temperature on the F layer thickness. For the S/F bilayers, the transitions between "0" and "$\pi$"-phases are impossible; the commensurability effect between $\xi_f$ and F layer thickness nevertheless leads to the non-monotonous dependence of $T_c$ on the F layer thickness.

The predicted oscillatory type dependence of the critical temperature (Buzdin and Kuprianov, 1990; Radovic et al., 1991) was subsequently observed experimentally in Nb/Gd (Jiang et al., 1995), Nb/CuMn (Mercaldo et al., 1996) and Nb/Co and V/Co (Obi et al., 1999) multilayers, as well as in bilayers Nb/Ni (Sidorenko et al., 2003), trilayers Fe/V/Fe (Garifullin et al., 2002), Fe/Nb/Fe (Mühlge et al., 1996), Nb/[Fe/Cu] layers (Vélez et al., 1999) and Fe/Pb/Fe (Lazar et al., 2000).

The strong pair-breaking influence of the ferromagnet and the nanoscopic range of the oscillation period complicate the observation of this effect. Advances in thin film processing techniques were crucial for the study of this subtle phenomenon. The first indications on the non-monotonous variation of $T_c$ versus the thickness of the F layer was obtained by Wong et al. (1986) for V/Fe superlattices. However, in the subsequent experiments of Koorevaar et al. (1994), no oscillatory behavior of $T_c$ was found, while the recent studies by Garifullin et al. (2002) of the superconducting properties of Fe/V/Fe trilayers even revealed the re-entrant $T_c$ behavior as a function of the F layer thickness. Bourgeois and Dynes (2002) studied amorphous Pb/Ni bilayer quench-condensed films and observed only monotonic depairing effect with the increase of the Ni layer thickness. In the work of Sidorenko et al. (2003), the comparative analysis of different techniques of the sample preparation was made and the conclusion is, that the molecular beam epitaxy (MBE) grown samples do not reveal $T_c$ oscillations, whereas magnetron sputtered samples do. This difference is attributed to the appearance of magnetically "dead" interdiffused layer at the S/F interface which plays an important role for the MBE grown samples. The transition metal ferromagnets, such as Fe, have a strongly itinerant character of the magnetic moment which is very sensitive to the local coordination. In thin Fe layers, the magnetism may be strongly decreased and even vanished. Probably the best choice is to use the rare-earth ferromagnetic metal with localized magnetic moments. This has been done by Jiang et al. (1995) who prepared the magnetron sputtered Nb/Gd multilayers, which clearly revealed the $T_c$ oscillations, Fig. 5.

The curves show a pronounced non-monotonous dependence of $T_c$ on the Gd layer thickness. The increase of $T_c$ implies the transition from the "0"-phase to the "$\pi$"-phase. Note that the previous experiments on the MBE grown Nb/Gd samples (Strunk et al., 1994) only revealed the step-like decrease of $T_c$ with increasing Gd layer thickness. The comprehensive analysis of different problems related to the samples quality was made by Chien and Reich (1999). Aarts et al. (1997), studied in detail the proximity effect in the system consisting of the superconducting V and ferromagnetic $V_{1-x}Fe_x$ alloys and demonstrated the important role of the interface transparency for the understanding of the pair-breaking mechanism.

B. Theoretical description of the S/F multilayers

To provide the theoretical description of the non-monotonous dependence of $T_c$, we consider the S/F multilayered system with a thickness of the F layer $2d_f$ and the S layer $2d_s$, see Fig. 6.

The $x$-axis is chosen perpendicular to the layers with $x = 0$ at the center of the S layer. The "0"-phase case corresponds to the same superconducting order parameter sign in all S layers (Fig. 6a) while in the "$\pi$"-phase the sign of the superconducting order parameter in adjacent S layers is opposite (Fig. 6b). In the case of a S/F bilayer, the anomalous Green function $F(x)$ has zero derivative at the boundary with vacuum, see Eq. (32) below. It is just the case for the function $F(x)$ in the "0"-phase at the centers of the S and F layers. So the superconducting...
characteristics of a S/F bilayer with thicknesses $d_s$ and $d_f$ of the S and F layers respectively are equivalent to that of the S/F multilayer with double layer thicknesses $(2d_s$ and $2d_f$).

The approach based on the quasiclassical Eilenberger (1968) or Usadel (1970) equations is very convenient to deal with S/F systems (see Appendix B). In fact, it is much simpler than the complete microscopical theory, it does not need the detailed knowledge of all the characteristics of the S and F metals, and is applicable for scales larger than the atomic one. Then, it must work for thicknesses of the layers in the range $20 - 200 \ \text{Å}$, which is of primary interest for S/F systems.

In the dirty limit, if the electron elastic scattering time $\tau = l/v_f$ is small, more precisely $T_c \tau \ll 1$ and $h\tau \ll 1$, the use of the Usadel equations is justified. The second condition, however, is much more restrictive due to a large value of the exchange field ($h \gg T_c$). The Usadel equations deal only with the Green’s functions $G(x, \omega)$ and $F(x, \omega)$ averaged over the Fermi surface. Moreover, to calculate the critical temperature of the second-order superconducting transition in S/F systems, it is enough to deal with the limit of the small superconducting order parameter ($\Delta \to 0$) in the Usadel equations. This linearization permits to put $G = \text{sgn}(\omega)$ and in the form linearized over $\Delta$, the Usadel equation for the anomalous function $F_s$ in the S region is written as

$$
\left( |\omega| - \frac{D_s}{2} \frac{\partial^2}{\partial x^2} \right) F_s = \Delta(x),
$$

where $D_s$ is the diffusion coefficient in the S layer. In the F region, the exchange field is present while the pairing potential $\Delta$ is absent, and the corresponding Usadel equation for the anomalous function $F_f$ is just the Eq. (18).

The equations for $F_s$ and $F_f$ must be supplemented by the boundary conditions. At the superconductor-vacuum interface, the boundary condition is simply a zero derivative of the anomalous Green function, which implies the absence of the superconducting current through the interface. The general boundary conditions for the Usadel equations at the superconductor-normal metal interface have been derived by Kupriyanov and Lukichev (1988) and near the critical temperature they read

$$
\left. \frac{\partial F_s}{\partial x} \right|_{x=0} = \frac{\sigma_f}{\sigma_s} \left. \frac{\partial F_f}{\partial x} \right|_{x=0},
$$

$$
F_s(0) = F_f(0) - \xi_n \gamma_B \left. \frac{\partial F_f}{\partial x} \right|_{x=0},
$$

where $\sigma_f$ ($\sigma_s$) is the conductivity of the F-layer (S-layer above $T_c$). The parameter $\gamma_B$ characterizes the interface transparency $T = \frac{1}{R} = \frac{1}{R_A^s}$ and is related to the S/F boundary resistance per unit area $R_b$ via the following simple relationship $\gamma_B = \frac{R_b \sigma_f}{\sigma_s}$ (Kupriyanov and Lukichev, 1988). By analogy with the superconducting coherence length $\xi_s = \sqrt{\frac{D_s}{2\pi T_c}}$, we introduce the normal metal coherence length $\xi_n = \sqrt{\frac{D_f}{2\pi T_c}}$. The presented form of the boundary conditions corresponds to the S/F interface $x = 0$ and the positive direction of the $x$ axis chosen along the outer normal to the S surface (i.e. the $x$ axis is directed from the S to the F metal). It is worth notify that the boundary conditions for the Usadel equations (Kupriyanov and Lukichev, 1988) have been obtained for superconductor/normal metal interfaces, and their applicability for S/F interfaces is justified, provided that the exchange field in the ferromagnet is much smaller than the Fermi energy, i.e. $h << E_F$. For a ferromagnet with localized moments, such as Gd, this condition is always fulfilled, while it becomes more stringent for transition metals and violated for half-metals. Recently Vodopyanov and Tagirov (2003b) obtained the boundary conditions for Eilenberger equations in the case of a strong ferromagnet. They used them to study the critical temperature of a S/F bilayer when ferromagnet is in the clean limit. Nevertheless the important question about the boundary conditions for Usadel equations at the interface superconductor/strong ferromagnet is still open.

Provided the solutions of Usadel equations in the F and S layers are known, the critical temperature $T_c^*$ may be found from the self-consistency equation for the pair potential $\Delta(x)$ in a superconducting layer

$$
\Delta(x) = \pi T_c^* \lambda \sum_{\omega} F_s(x, \omega),
$$

where $\lambda$ is BCS coupling constant in S layer (while in F layer it is supposed to be equal to zero). This equation is more convenient to write in the following form

$$
\Delta(x) \ln \frac{T_c^*}{T_c} + \pi T_c^* \lambda \sum_{\omega} \left( \frac{\Delta(x)}{|\omega|} - F_s(x, \omega) \right) = 0,
$$

where $\lambda$ is BCS coupling constant in S layer.
where $T_c$ is the bare transition temperature of the superconducting layer in the absence of the proximity effect.

The Usadel equations provide a good basis for the complete numerical solution of the problem of the transition temperature of S/F superlattices. Firstly such a solution has been obtained for a S/F system with no interface barrier by Radovic et al. (1988, 1991), using the Fourier transform method, and this case was treated analytically by Buzdin and Kuprianov (1990) and Buzdin et al. (1992). The role of the S/F interface transparency has been elucidated by Proshin and Khusainov (1997), (for more references see also the review by Izyumov et al. 2002) and Tagirov (1998). Recently Fominov et al. (2002), performed a detailed analysis of the non-monotonous critical temperature dependence of S/F bilayers for arbitrary interface transparency and compared the results of different approximations with exact numerical calculations.

Below we illustrate the appearance of the non-monotonous superconducting transition temperature dependence for the case of a thin S-layer, which has a simple analytical solution. More precisely, we consider the case $d_s < \xi_s$, which implies that the variations of the superconducting order parameter and anomalous Green’s function in the S layer are small. We may write the following expansion up to the $x^2$ order term for $F_s$ in the S layer centered at $x = 0$:

$$F_s(x, \omega) = F_0 \left(1 - \frac{\beta \omega}{2} x^2\right), \quad (35)$$

where $F_0$ is the value of the anomalous Green’s functions at the center of the S-layer, and the linear over $x$ term is absent due to the symmetry of the problem in both $0^\circ$- and $\pi^\circ$-phases (see Fig. 4). Putting this form of $F_s$ into the Usadel equation (31), we readily find

$$F_0 = \frac{\Delta}{\omega + \tau_s}, \quad (36)$$

where we have introduced the complex pair-breaking parameter $\tau_s^{-1} = \frac{D_s}{2} \beta \omega$, and in the first approximation over $d_s/\xi_s \ll 1$, the pair potential $\Delta$ may be considered as spatially independent. The pair-breaking parameter $\tau_s^{-1}$, is directly related to the logarithmic derivative of $F_s$ at $x = d_s$

$$\frac{F'_s(d_s)}{F_s(d_s)} \simeq -d_s \beta \omega = -\frac{2d_s \tau_s^{-1}}{D_s}. \quad (37)$$

The boundary conditions Eq. (32) permit us to calculate the parameter $\tau_s^{-1}$, provided the anomalous Green function in the F layer is known:

$$\tau_s^{-1} = \frac{D_s}{2d_s \sigma_s} \frac{\sigma_f}{1 - \xi_n \gamma_B F_f^*(d_s)/F_f(d_s)}, \quad (38)$$

C. $0^\circ$- and $\pi^\circ$-phases

The solution of the Usadel equation (18) in the F layer is straightforward but different for $0^\circ$- and $\pi^\circ$-phases. Let us start first with a $0^\circ$-phase. In such a case (see Fig. 6a), we must take as a solution for $F_f(x)$ at $\omega > 0$ in the interval $d_s < x < d_s + 2d_f$ the function symmetrical relative to the plane $x = d_s + d_f$, i. e.

$$F_f(x, \omega > 0) = A \cosh \left[\frac{i + 1 \xi_f}{\xi_f} (x - d_s - d_f)\right]. \quad (39)$$

Therefore the pair-breaking parameter $\tau_{s,0}^{-1}$ for $0^\circ$-phase at $\omega > 0$ is

$$\tau_{s,0}^{-1}(\omega > 0) = \frac{D_s}{2d_s \sigma_s} \frac{\sigma_f}{\xi_f} \frac{\tanh \left[\frac{i + 1 \xi_f}{\xi_f} d_f\right]}{1 + \frac{i + 1 \xi_f}{\xi_f} \xi_n \gamma_B \tanh \left[\frac{i + 1 \xi_f}{\xi_f} d_f\right]}, \quad (40)$$

and does not depend on the Matsubara frequencies $\omega$. For a negative $\omega$ we simply have $\tau_{s,0}^{-1}(\omega < 0) = (\tau_{s,0}^{-1}(\omega > 0))^*$. 

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Now, let us address the case of the "π"-phase. The only difference is that in such case we must choose the asymmetrical solution for \( F_f(x) \)

\[
F_f(x, \omega > 0) = B \sinh \left( \frac{i + 1}{\xi_f} (x - d_s - d_f) \right),
\]

(41)

and the corresponding pair-breaking parameter \( \tau_{s,\pi}^{-1} \) is given by the expression

\[
\tau_{s,\pi}^{-1}(\omega > 0) = (\tau_{s,\pi}(\omega < 0))^* = \frac{D_s \sigma_f i}{2d_s \sigma_s} \frac{\coth \left( \frac{i + 1}{\xi_f} d_f \right)}{1 + \frac{i + 1}{\xi_f} \gamma_B \coth \left( \frac{i + 1}{\xi_f} d_f \right)}.
\]

(42)

We see that in all cases the pair-breaking parameter \( \tau_{s}^{-1} \) is complex and depends on the sign of the Matsubara frequency only but not on its value. As a result, with the help of the self-consistency equation (34), we obtain the following expression for the critical temperature \( T_c^* \) of the S/F multilayer

\[
\ln \frac{T_c^*}{T_c} = \Psi \left( \frac{1}{2} \right) - \text{Re} \Psi \left( \frac{1}{2} + \frac{1}{2\pi T_c^* \tau_s} \right),
\]

(43)

where \( \Psi \) is the Digamma function, and the pair-breaking parameter \( \tau_{s}^{-1} \) is given by Eqs. (40) and (42) for the "0"- and "π"-phases respectively. This type of expression for \( T_c^* \) reminds the corresponding formula for the critical temperature of a superconductor with magnetic impurities (Abrikosov and Gor’kov, 1960), though the "magnetic scattering time" \( \tau_s \) is complex in our system. If the critical temperature variation is small \((\frac{T_c - T_c^*}{T_c} << 1)\), the formula for the critical temperature shift Eq. (43) may be simplified

\[
\frac{T_c - T_c^*}{T_c} = \frac{\pi}{4T_c} \text{Re} (\tau_{s}^{-1}).
\]

(44)

D. Oscillating critical temperature

To illustrate the oscillatory behavior of the critical temperature, we consider the case of a transparent S/F interface \( \gamma_B = 0 \). The critical temperatures \( T_{c,0} \) and \( T_{c,\pi}^* \) for the "0"- and "π"-phases respectively, are

\[
\frac{T_c - T_{c,0}}{T_c} = \frac{\pi}{4T_c \tau_0} \left( \frac{\sinh(2y) - \sin(2y)}{\cosh(2y) + \cos(2y)} \right),
\]

(45)

\[
\frac{T_c - T_{c,\pi}^*}{T_c} = \frac{\pi}{4T_c \tau_0} \left( \frac{\sin(2y) + \sinh(2y)}{\cos(2y) - \cosh(2y)} \right),
\]

(46)

where \( \tau_0^{-1} = \frac{D_s \sigma_f i}{2d_s \xi_f} \) and \( 2y = 2d_f/\xi_f \) is the dimensionless thickness of the F layer. The critical temperature variation versus the F layer thickness is presented in Fig. 7.

We see that for the small F layer thicknesses, the "0"-phase has a higher transition temperature. The first crossing of the curves \( T_{c,0}(y) \) and \( T_{c,\pi}^*(y) \) occurs at \( 2y_c \approx 2.36 \) and in the interval of thickness \( 2.36 \xi_f < 2d_f < 5.5 \xi_f \), the "π"-phase has a higher critical temperature. The oscillations of the critical temperature rapidly decay with the increase of \( y \), and it is not realistic to observe on experiment more than two periods of oscillations.

In the general case, the F-layer thickness dependence of the critical temperature Eq. (43) may be written for "0"-phase in the following form convenient for numerical calculations

\[
\ln \frac{T_{c,0}^*}{T_c} = \Psi \left( \frac{1}{2} \right) - \text{Re} \left\{ \frac{1}{2} + \frac{2T_c}{T_{c,0}^* \gamma_B \tau_0 + 1} \coth \left[ (1 + i)y \right] \right\},
\]

(47)

where the dimensionless parameter \( \gamma_0^{-1} = 1/(4\pi T_c \tau_0) \) and \( \gamma = \gamma_B (\xi_n/\xi_f) \). The corresponding formula for the critical temperature for the "π"-phase is simply obtained from Eq. (47) by the substitution \( \coth \to \tanh \).
In Fig. 8, we present the examples of calculations of the thickness dependence of the critical temperature for S/F multilayers for different interface transparencies.

The oscillations of the critical temperature are most pronounced for transparent interface $\tilde{\gamma} = 0$, and they rapidly decrease with the increase of the boundary barrier (at $\tilde{\gamma} \gtrsim 2$ the oscillations are hardly observable). Note that, for certain values of the parameters $\tilde{\tau}_0$ and $\tilde{\gamma}$, the $T_{c0} (d_f)$ dependence may show the infinite derivative, which indicates the change of the order of the superconducting transition from second-order to the first-order one. This question was studied in detail by Tollis (2004). The increase of the boundary barrier not only decreases the amplitude of the critical temperature oscillations, but also it decreases the critical thickness of F layer $y_c$, corresponding to the "0"-"$\pi$"-phase transition. The limit $\tilde{\gamma} = \gamma_B (\xi, s_f/\xi_f) \gg 1$ is rather special one. In such case the S/F interface barrier becomes a tunnel barrier, and the critical thickness $y_c$ may be much smaller than 1. Indeed, if the critical temperature variation is small, (more precisely if $\tilde{\gamma} \tau_0 >> 1$), the condition $\text{Re}(\tau_{s,\tilde{0}}^{-1}) = \text{Re}(\tau_{s,\tilde{1}}^{-1})$ is realized at

$$d_f^* = \frac{\xi_f}{2} \left(\frac{3}{\tilde{\gamma}}\right)^{1/3},$$

and the mechanism of the "0"-"$\pi$"-phase transition is now related to the peculiarity of tunneling through the F layer. This is very different from the case of low interface transparency, when the transition occurs due to the spatial oscillations of the anomalous Green’s function. It must be very difficult to observe the low transparency regime of the "0"-"$\pi$"-transition with the help of the critical temperature measurements due to the fact that at $\tilde{\gamma} >> 1$ the oscillations of $T_{c0} (d_f)$ become very small. On the other hand, the measurements of the critical current in S/F/S Josephson junctions may be the adequate technique to reveal the "0"-"$\pi$"-transition in this regime (see next Section).

It is interesting to note that for small thicknesses of F layer ($d_f < \xi_f$) the critical temperature decreases with the increase of the interface barrier (provided the condition $\tilde{\gamma} (d_f/\xi_f) < 1$ is fulfilled) - see Fig. 8. Such a counterintuitive behavior may be explained in the following way. The low penetration of the barrier prevents the quick return of the Cooper pair from thin F layer. Therefore, the Cooper pair stays for a relatively long time in the F layer before going back to the S layer. In the results, the pair-breaking role of the exchange field in the F layer occurs to be strongly enhanced.

The cases of S/F bilayers or F/S/F threelayers with parallel magnetization are equivalent to the "0"-phase case for the multilayers (with double F layers thickness) and the corresponding $T_{c0} (d_f)$ dependence reveals a rather weak non-monotonous behavior in the case of finite transparency of the S/F interface (see Fig. 8). The comparison of the experimental data of Ryazanov et al. (2003) for the critical temperature of the bilayer Nb/Cu$_{0.43}$Ni$_{0.57}$ vs the thickness of the ferromagnetic layer with the theoretical fit (Fominov et al., 2002) is presented in Fig. 9.

Now let us address a question, if it is possible to have a transition into a state with the phase difference another than 0 and $\pi$? For example the state with the intermediate phase difference $0 < \varphi_0 < \pi$ may be expected at F layer thicknesses near $d_f^*$'. The numerical calculations of Radovic et al. (1991) indeed revealed the presence of the intermediate phase. However, the relative width of the region of its existence near $d_f^*$ was very small - around several percents only. On the other hand, the analytical calculations show that for the thin S layer case the states without current (corresponding to the highest $T_{c0}^*$) are possible only for the phase difference 0 or $\pi$. Also, in the S/F/S junctions the transitions between "0"- and " $\pi$"-states are discontinuous - see discussion in the next section. Probably the narrow region of the "$\varphi_0$"-phase existence obtained by the numerical calculations (Radovic et al., 1991) is simply related with its accuracy $\sim 1\%$, and the width of this region may decrease with the increase of the accuracy. Nevertheless there is another mechanism of the realization of the "$\varphi_0$"-phase due to the fluctuations of the thickness of F layer. In such case near the critical F layer thickness $d_f^*$ the regions of "0"- and " $\pi$"-phases would coexist. If the characteristic dimensions of these regions are smaller than the Josephson length in S/F structure, then the average phase difference would be different from 0 and $\pi$ (Buzdin and Koshelev, 2003).

The quasiclassical Eilenberger and Usadel equations are not adequate for treating the strong ferromagnets with $h \sim E_F$ because the period of Green’s function oscillations becomes comparable with the interatomic distance. On the other hand, the approach based on the Bogoliubov-de Gennes equations in clean limit is universal. Halterman and Valls (2003, 2004a) applied it to study the properties of clean S/F multilayers, at low temperature. They obtained the excitation spectrum through numerical solution of the self-consistent Bogoliubov-de Gennes equations and discussed the influence of the interface barrier and Fermi energy mismatch on the local density of states. Comparing the energy of the "0"- and " $\pi$" phases Halterman and Valls confirmed the existence of the transitions between them with the increase of F layer thickness. It is of interest that the local density of states is quite different in the "0"- and " $\pi$" phases, and its measurements could permit to trace the "0"-" $\pi$" transition. In the more recent work Halterman and Valls (2004b) showed that a lot of different order parameter configurations may correspond to the local energy minima in S/F heterostructures.
The calculations of the energy spectrum in the S/F/S system in "0" and "$\pi$" phases on the basis of Eilenberger equations were performed by Dobrosavljevic-Grujic, Zikic and Radovic (2000) for $s$-wave superconductivity and $d$-wave superconductivity (Zikic et al., 1999). The large peaks in the density of states were attributed to the spin-split bound states appearing due to the special case of the Andreev reflection at the ferromagnetic barrier.

In the previous analysis the spin-orbit and magnetic scattering were ignored. Demler, Arnold, and Beasley (1997) theoretically studied the influence of the spin-orbit scattering on the properties of S/F systems and demonstrated that it is quite harmful for the observation of the oscillatory effects. A similar effect is produced by the magnetic scattering which at some extend is always present in S/F systems due to the non-stoichiometry of the F layers (and it may be rather large when the magnetic alloy is used as F layer). The calculations of the critical temperature of the S/F multilayers in the presence of the magnetic scattering were firstly performed by Tagirov (1998). In the framework of the formalism presented in this section it is very easy to take into account the magnetic diffusion with the spin-flip scattering time $\tau_m$ - it is enough to substitute the exchange field $h$ in the linearized Usadel equation (17) by $h - \text{sgn}(\omega)\tau_m^{-1}$. This renormalization leads to the decrease of the damping length and the increase of the oscillation period, which makes the $T_c(d_f)$ oscillations less pronounced (Tagirov, 1998).

V. SUPERCONDUCTOR-FERROMAGNET-SUPERCONDUCTOR "$\pi$"-JUNCTION

A. General characteristics of the "$\pi$"-junction

A Josephson junction at equilibrium has usually a zero phase difference $\varphi$ between two superconductors. The energy $E$ of Josephson junction may be written as (see for example De Gennes, 1966a)

$$ E = \frac{\Phi_0 I_c}{2\pi c} (1 - \cos \varphi), \quad (49) $$

where $I_c$ is the Josephson critical current, and the current-phase relation is $I_c(\varphi) = \frac{2e}{h} \frac{\partial E}{\partial \varphi} = I_c \sin \varphi$. At the standard situation, the constant $I_c > 0$, and the minimum energy of a Josephson junction is achieved at $\varphi = 0$. However, in the previous section it has been demonstrated that in the S/F multilayers the transition into the "$\pi$"-phase may occur. This means that for the Josephson S/F/S junction (with the same thickness of F layer which corresponds to the "$\pi$"-phase in the multilayered system) the equilibrium phase difference would be equal to $\pi$, and it is natural to call such a junction the "$\pi$"-junction. For the "$\pi$"-junction, the constant $I_c$ in the equation (49) is negative, and the transition from "0"- to "$\pi$"-state may be considered as a change of the sign of the critical current, though the experimentally measured critical current is always positive and equals to $|I_c|$. The S/F/S junctions would reveal the striking non-monotone behavior of the critical current as a function of F layer thickness. The vanishing of the critical current signals the transition from "0"- to "$\pi$"-state.

The possibility of the negative Josephson coupling was firstly noted by Kulik (1966), who discussed the spin-flip tunneling through an insulator with magnetic impurities. Bulaevskii et al. (1977) put forward the arguments that under certain conditions such a spin-flip tunneling could dominate the direct tunneling and lead to the "$\pi$"-junction appearance. Up to now there are no experimental evidences of the "$\pi$"-coupling in the Josephson junctions with magnetic impurities. On the other hand, Buzdin et al. (1982) showed that in the ballistic S/F/S weak link $I_c$ displays damped oscillations as a function of the thickness of the F layer and its exchange field. Later, Buzdin and Kuprianov (1991) demonstrated that these oscillations remain in the diffusive regime and so, the "$\pi$"-coupling is the inherent property of the S/F/S junctions. The characteristic thickness of F layer corresponding to the transition from the "0"- to "$\pi$"-phase is $\xi_f = \sqrt{\frac{D_f}{h}}$, and it is rather small ($10 - 50$ Å) in the typical ferromagnets because of the large value of the exchange field ($h \gtrsim 1000$ K). So, the experimental verification of the "$\pi$"-coupling in S/F/S junction was not easy, due to the needed very careful control of the F layer thickness. Finally the first experimental evidence for a "$\pi$"-junction was obtained by Ryazanov et al. (2001a) for the Josephson junction with a weakly ferromagnetic interlayer of a Cu$_{10}$Ni$_{1-x}$ alloy. Such choice of F layer permitted to have a ferromagnet with a relatively weak exchange field ($h \sim 100 - 500$ K) and, therefore the relatively large $\xi_f$ length.

B. Theory of "$\pi$"-junction

The complete qualitative analysis of the S/F/S junctions is rather complicated, because the ferromagnetic layer may strongly modify superconductivity near the S/F interface. In addition, the boundary transparency and electron mean free path, as well as magnetic and spin-orbit scattering, are important parameters affecting the critical current.
To introduce the physics of "$\pi$"-coupling, we prefer to concentrate on the rather simple approach based on the Usadel equation and consider the S/F/S junction with a F-layer of thickness $2d_f$, (see Fig. 10) and identical S/F interfaces. In the case of small conductivity of F layer or small interface transparency $\sigma_f \xi_f / \sigma_S \xi_f << \text{max}(1, \gamma_B)$ we may use the "rigid boundary" conditions (Golubov et al., 2004) with $F_s(-d_f) = \Delta e^{-i\gamma/2}/\sqrt{\omega^2 + \Delta^2}$ and $F_s(d_f) = \Delta e^{i\gamma/2}/\sqrt{\omega^2 + \Delta^2}$.

The solution of Eq. (18) in a ferromagnet satisfying the corresponding boundary conditions is written as

$$F(x) = \frac{\Delta}{\sqrt{\omega^2 + \Delta^2}} \left\{ \frac{\cos (\varphi/2) \cosh (kx)}{(\cosh (kd_f) + k\gamma_B \xi_n \sinh (kd_f))} + \frac{i \sin (\varphi/2) \sinh (kx)}{(\sinh (kd_f) + k\gamma_B \xi_n \cosh (kd_f))} \right\},$$

where the complex wave-vector $k = \sqrt{2(\omega + is\text{sign} \omega) h}/D_f$. This solution describes the $F(x)$ behavior near the critical temperature. Note, that in principle, at arbitrary temperature, the boundary conditions are different from the Eq. (32), see for example (Golubov et al., 2004). However, in the limit of low S/F interface transparency ($\gamma_B >> 1$), when the amplitude of the $F$ function in F-layer is small, we may use the linearized Usadel equation (18) at all temperatures. The only modification in the boundary conditions Eq. (32) is that $F_s$ must be substituted by $F_s/|G_s|$ and $\gamma_B$ by $\gamma_B/|G_s|$, where the normal Green function in superconducting electrode $G_s = \omega/\sqrt{\omega^2 + \Delta^2}$. Taking this renormalization into account in the explicit form Eq. (50), we may use it in the formula for the supercurrent

$$I_s(\varphi) = \frac{ieN(0)}{\sqrt{2}} \pi T S \sum_{-\infty}^{\infty} \left( \frac{d}{dx} \tilde{F} - \frac{d}{dx} \tilde{F} \right),$$

where $\tilde{F}(x, h) = F^*(x, -h)$, $S$ is the area of the cross section of the junction and $N(0)$ is the electron density of state for a one spin projection. This expression gives the usual sinusoidal current-phase dependence $I_s(\varphi) = I_c \sin(\varphi)$ with the critical current

$$I_c = \frac{eSN(0)D_f \pi T}{\sum_{-\infty}^{\infty} \frac{\Delta^2}{\omega^2 \tanh (2kd_f)(1 + \Gamma^2_\omega k^2) + 2k\Gamma_\omega}},$$

where $\Gamma_\omega = \gamma_B \xi_n / |G_s|$. This expression may be easily generalized to take into account the different interface transparencies $\gamma_{B1}, \gamma_{B2} >> 1$, it is enough to substitute in Eq. (52) $\Gamma^2_\omega \rightarrow \gamma_{B1} \gamma_{B2} (\xi_n / |G_s|)^2$ and $2\Gamma_\omega \rightarrow (\gamma_{B1} + \gamma_{B2}) \xi_n / |G_s|$. Near $T_c$ and in the case of transparent interface $\gamma_B \rightarrow 0$ (Buzdin and Kuprianov, 1991)

$$I_c = \frac{eSN(0)D_f \pi \Delta^2}{2T_c} \frac{\text{Re} \left[ \frac{k}{\sinh (2kd_f)} \right]}{\Gamma_\omega},$$

where $2y = 2d_f/\xi_f$ is the dimensionless thickness of the F layer, $R_n = 2d_f / (\sigma_f S)$ is the resistance of the junction ($\sigma_f = 2e^2N(0)D_f$ is the conductivity of the F layer), and $V_0 = \frac{\Delta^2}{\omega^2}$. The dependence $I_c, R_n/V_0$ vs. $2y$ is presented in Fig. 11. The first vanishing of the critical current signals the transition from "0"- to "\pi"-state. It occurs at $2y_c \approx 2.36$ which is exactly the critical value of F layer thickness in S/F multilayer system corresponding to the "0"- "\pi"-state transition, i. e. to the condition $T_c \rightarrow T_c^{\pi}$ in the Eqs.(45). The theoretical description of the S/F/S junctions with arbitrary interface transparencies near the critical temperature was proposed by Buzdin and Baladie (2003).

At low temperature or low S/F barrier the amplitude of the anomalous Green’s function $F_f(x)$ is not small and we need to use the complete (non-linearized) Usadel equation. In the limit of large thickness of F layer $d_f >> \xi_f$ and $\gamma_B = 0$, the analytical solution was obtained by Buzdin and Kuprianov (1991), and the critical current is

$$I_cR_n = 64\sqrt{2} \frac{\Delta}{e} \mathcal{F} \left( \frac{\Delta}{T} \right) 2y \exp(-2y) \sin \left( 2y + \frac{\pi}{4} \right),$$

where $\mathcal{F}$ is the dimensionless thickness of the F layer, $R_n = 2d_f / (\sigma_f S)$ is the resistance of the junction, and $V_0 = \frac{\Delta^2}{\omega^2}$. The dependence $I_c, R_n/V_0$ vs. $2y$ is presented in Fig. 11. The first vanishing of the critical current signals the transition from "0"- to "\pi"-state. It occurs at $2y_c \approx 2.36$ which is exactly the critical value of F layer thickness in S/F multilayer system corresponding to the "0"- "\pi"-state transition, i. e. to the condition $T_c \rightarrow T_c^{\pi}$ in the Eqs.(45). The theoretical description of the S/F/S junctions with arbitrary interface transparencies near the critical temperature was proposed by Buzdin and Baladie (2003).
with the function
\[ F\left(\frac{|\Delta|}{T}\right) = \pi T \sum_{\omega>0} \frac{|\Delta|}{\sqrt{2\Omega} + \sqrt{\Omega + \omega}} , \]

where \( \Omega = \sqrt{\omega^2 + |\Delta|^2} \), and \( F\left(\frac{|\Delta|}{T}\right) \approx \frac{|\Delta|}{2T_0} \) at \( T \approx T_c \) while at low temperature \( T << T_c \), the function \( F\left(\frac{|\Delta|}{T}\right) \approx 0.071 \).

Note that in the clean limit \((\tau h >> 1)\) the thickness dependence of the critical current is very different (Buzdin et al., 1982) and near \( T_c \) it is

\[ I_c R_n = \frac{\pi \Delta^2}{4e} \frac{\sin\left(\frac{4hd_f}{v_f}\right)}{\left(\frac{4hd_f}{v_f}\right)} , \]

i.e. the critical current decreases \( \propto 1/d_f \) and not exponentially like in the dirty limit case. In general, in the clean limit the S/F proximity effect is not exponential, but a power low one.

The expression (56) was obtained on the basis of Eilenberger equations. In the case of a strong ferromagnet \( h \lesssim E_F \), the period of the oscillations of the Green’s functions becomes of the order of the interatomic distance, and this approach does not work anymore. Using the technique of the Bogoliubov-de Gennes equations, Cayssol and Montambaux (2004) demonstrated that the quasiclassical result (56), where the only relevant parameter for the critical current oscillations being \( h d_f/v_F \), is not applicable for the strong ferromagnets. This is related to the progressive suppression of the Andreev reflection channels with the increase of the exchange energy.

In the framework of the Bogoliubov-de Gennes equations Radovic et al. (2003) studied the general case of the ballistic S/F/S junction for a strong exchange field, arbitrary interfacial transparency and Fermi wave vectors mismatch. The characteristic feature of such ballistic junction is the short-period geometrical oscillations of the supercurrent as the function of \( d_f \) due to the quasiparticle transmission resonances. In the case of strong ferromagnet, the period of "0" – "\( \pi \)" oscillations becomes comparable with the period of geometrical oscillations, and their interplay provides very special \( I_c(d_f) \) dependences. Also Radovic et al. (2003) demonstrated that the current-phase relationship may strongly deviate from the simple sinusoidal one, and studied how it depends on the junction parameters. While the temperature variation of \( I_c \) is usually a monotonic decay with increasing temperature, near the critical thickness \( d_f \) corresponding to the "0" – "\( \pi \)" transition, a nonmonotonic dependence \( I_c \) on temperature was obtained. Radovic et al. (2001) showed that at low temperature the characteristic multimode anharmonicity of the current-phase relation in clean S/F/S junctions implies the coexistence of stable and metastable "0 – "0" – "\( \pi \) – " states. As a consequence, the coexistence of integer and half-integer fluxoid configuration of SQUID was predicted. Note that for strong ferromagnets the details of the electrons energy bands become important for the description of the properties of S/F/S junction.

The weak link between \( d \)–wave superconductors may also produce the \( \pi \) shift effect (as a review, see for example Van Harlingen, 1995). The situation of the Josephson coupling in a ferromagnetic weak link between \( d \)–wave superconductors was studied in the clean limit theoretically by Radovic et al. (1999).

It is interesting that in the limit \( kd << 1 \) (i.e. \( d_f << \xi_f \) ) the oscillations of the anomalous function in the F layer are absent, but as it has been noted previously, for the case of the low transparency of the barrier \( \gamma_n >> 1 \), the critical current can nevertheless change its sign. Indeed, in this limit, the expression for the critical current Eq. (52) reads

\[ I_c = e N(0) D_0 \pi T S \sum_{\omega} \frac{2 |\Delta|^2}{\omega^2 + |\Delta|^2} \frac{1}{\gamma_n^2 \xi_f^2 2d_f} \left( \frac{1}{k^2} - \frac{2d_f^2}{3} - \frac{1}{\gamma_n \xi_f d_f k^4} \right) \frac{|\omega|}{\sqrt{\omega^2 + |\Delta|^2}} . \]

Usually at experiment, the Curie temperature \( \Theta \) of ferromagnet is higher than the superconducting critical temperature \( T_c \). For RKKY mechanism of ferromagnetic transition \( \Theta \sim h^2/E_F \) and so the exchange field \( h \) occurs to be much larger than the superconducting critical temperature \( T_c \). In the case of the itinerant ferromagnetism, the exchange field is usually several times higher than the Curie temperature and also the limit \( h >> T_c \) holds. Taking this into account and performing the summation over Matsubara frequencies of the first two terms in the brackets of the Eq. (57), we finally obtain
\[ I_c = \frac{eN(0)SD_f\Delta\xi_f^2}{4\xi_f^2 d_f \xi_n^2} \]

\[
\left\{ \frac{\Delta}{h} \left[ \Psi \left( \frac{1}{2} + \frac{h}{2\pi T} \right) - \Psi \left( \frac{1}{2} + \frac{\Delta}{2\pi T} \right) + \text{c.c.} \right] + \frac{2\pi T \Delta \xi_f^2}{\gamma_B \xi_n d_f} \sum_{\omega > 0} \frac{\omega}{(\omega^2 + \Delta^2)^{3/2}} - \frac{4\pi}{3} \left( \frac{d_f^2}{\xi_f^2} \right) \tanh \left( \frac{\Delta}{2T} \right) \right\}. \tag{58} \]

We start with the analysis of \( I_c \) over \( d_f \) dependence in the limit of very large \( \gamma_B \) (more precisely when \( \gamma_B \gg \frac{\Delta}{2\pi} \)). In such case we may neglect the term proportional to \( 1/\gamma_B \) in the brackets of Eq. (58), and then we obtain that at \( T \to 0 \) the transition into the \( \pi \)-phase occurs (\( I_c \) changes its sign) at

\[ d_f^c \approx \xi_f \sqrt{\frac{3\Delta(0)}{h} \ln \left( \frac{h}{\Delta(0)} \right)}. \tag{59} \]

Indeed the condition \( d_f^c \ll \xi_f \) is satisfied. In the case of very low boundary transparencies, the relevant formula obtained in (Buzdin and Baladie, 2003) near the critical temperature in the limit \( (T_c/h) \to 0 \) also reveals the crossover between "0"- and "\( \pi \)"-phase. On the other hand, no transition into \( \pi \)-phase was obtained in the analysis of S/F/S system by Golubov et al. (2002b), which is apparently related to the use of the gradient expansion of the anomalous function in ferromagnet when only the first term has been retained.

It is interesting to note that the critical F-layer thickness \( d_f^c \), when the transition from "0"- to "\( \pi \)"-phase occurs, depends on the temperature. The corresponding temperature dependences are presented in Fig. 12 for different value of \( (T_c/h) \) ratios. We see that \( d_f^c(T) \) decreases when the temperature decreases. This is a very general feature and it is true also for the subsequent "0"- "\( \pi \)" transitions occurring at higher F-layer thickness. So for some range of F-layer thicknesses the transition from "0"- to "\( \pi \)"-phase is possible when the temperature lowers.

For the case of moderately large \( \gamma_B \), i.e. when \( 1 \ll \gamma_B \ll \frac{\Delta}{2\pi} \), the terms with \( \Psi \) functions in Eq. (58) can be neglected, and at \( T = T_c \) the critical thickness \( d_f^c \) is

\[ d_f^c (T = T_c) = \frac{\xi_f}{2} \left( \frac{3\xi_f}{\gamma_B \xi_n} \right)^{1/3}, \tag{60} \]

while at \( T \to 0 \) the critical thickness is somewhat smaller \( d_f^c (T = 0) = \frac{\xi_f}{2} \left( \frac{6\xi_f}{\pi \gamma_B \xi_n} \right)^{1/3} \). The critical F layer thickness, given by Eq. (60), naturally coincides with the corresponding expression Eq. (48) obtained for S/F multilayers in the limit \( h \gg T_c \). The examples of different non-monotonous \( I_c(T) \) dependences for low barrier transparency limit \( \gamma_B \gg \frac{\Delta}{2\pi} \) are presented in Fig. 13. In fact, in the limit of low barrier transparency and thin F layer, we deal with the superconducting electrons tunneling through ferromagnetically ordered atoms. The situation is in some sense reminiscent the tunneling through magnetic impurities, considered by Kulik (1966) and Bulaevskii et al. (1977). What may be more relevant is the analogy with the mechanism of the "\( \pi \)"-phase realization due to the tunneling through a ferromagnetic layer in the atomic S/F multilayer structure, which we consider in the section 7.

Fogelström (2000) considered the ferromagnetic layer as a partially transparent barrier with different transmission for two spin projections. In some sense this work may be considered as a further development of Bulaevskii et al. (1977) approach. The Andreev bound states appearing near the spin-active interface within the superconducting gap are tunable with the magnetic properties of the interface. This can result to the switch of the junction from "0"- to "\( \pi \)"-state with changing the transmission characteristics of the interface. This approach was also applied by Andersson, Cuevas and Fogelström (2002) to study the coupling of two superconductors through a ferromagnetic dot. They demonstrated that the realization of the "\( \pi \)"-junction is possible in this case as well. In the framework of the Bogoliubov-de Gennes approach Tanaka and Kashiwaya (1997) analyzed the system consisting of two superconductors separated by \( \delta \)-functional barrier with the spin-orientation dependent height.

Similarly to the case of S/F multilayers we may discuss the question of the existence of the S/F/S junction with arbitrary equilibrium phase difference \( \varphi_0 \). Naturally, the form Eq. (49) for the energy of the junction may give the minima at \( \varphi = 0 \) and \( \varphi = \pi \) only. A more general expression for the Josephson junction energy takes into account the higher order terms over the critical current which leads to the appearance of the higher harmonics over \( \varphi \) in the current-phase relationship. Up to the second harmonic, the energy is

\[ E = \frac{\Phi_0 I_c}{2\pi c} (1 - \cos \varphi) - \frac{\Phi_0}{2\pi c} \frac{I_2}{2} \cos 2\varphi, \tag{61} \]
and the current is

\[ j(\varphi) = I_c \sin \varphi + I_2 \sin 2\varphi. \] (62)

If the sign of the second harmonic term is negative \( I_2 < 0 \), then the transition from "0"- to "\( \pi \)-phase will be continuous, and the realization of the "\( \varphi_0 \)-"-junction becomes possible. In general, the "\( \varphi_0 \)-"-junction may exist if \( j(\varphi_0) = 0 \) and \( (\partial j/\partial \varphi)_{\varphi_0} > 0 \). The calculations of the current-phase relationships for different types of S/F/S junctions (Golubov et al., 2004, Radovic et al., 2003 and Cayssol and Montambaux, 2004) show that \( (\partial j/\partial \varphi) < 0 \), and therefore the transition between "0"- and "\( \pi \)-states occurs to be discontinuous.

The presence of the higher harmonics in the \( j(\varphi) \) relationship prevents the vanishing of the critical current at the transition from "0"- to "\( \pi \)-state. This is always the case when the transition occurs at low temperature. Theoretical studies of the properties of clean S/F/S junctions at \( T < T_c \) (Buzdin et al., 1982, Chthelkatchev et al., 2001, and Radovic et al., 2003) confirm this conclusion.

Zyuzin and Spivak (2000) argued that the mesoscopic fluctuations of the critical current may produce the "\( \pi/2 \)-"-junction. Such situation is possible when the thickness of F layer is close to \( 2d_f \). The spatial variations of the thickness of F layer lead to the appearance of the second harmonic term in Eq. (62) with \( I_2 < 0 \) (Buzdin and Koshelev, 2003), and thus the realization of the "\( \varphi_0 \)-"-junction becomes possible at \( 2d_f \approx 2d_f^c \).

C. Experiments with "\( \pi \)-"-junctions

The temperature dependence of the critical thickness \( d_f^c \) is at the origin of the observed by Ryazanov et al. (2001a) very specific temperature dependence of the critical current \( I_c(T) \) (see Fig. 14). With decreasing temperature for specific thicknesses of the F layer (around 27 nm), a maximum of \( I_c \) is followed by a strong decrease down to zero, after which \( I_c \) rises again.

This was the first unambiguous experimental confirmation of the "0-\( \pi \)" transition via the critical current measurements. Ryazanov et al. (2001a) explained their results by a model with a small exchange field \( h \sim T_c \). The Cu\(_2\)Ni\(_{1-x}\) alloy used in their experiments has the Curie temperature \( \Theta \sim 20-30K \) and this implies that the exchange field must be higher 100K. In consequence, it seems more probable that the thickness of the F layer was in the range \( d_f^c(0) < d_f < d_f^c(T_c) \), which provides the strong non-monotinous temperature dependence of \( I_c \). Also, the experimental estimate of \( \xi_f \sim 10 \text{ nm} \) is too large for expected value of the exchange field.

Recent systematic studies of the thickness dependence of the critical current in junctions with Cu\(_2\)Ni\(_{1-x}\) alloy as a F layer (Ryazanov et al., 2004), have revealed very strong variation of \( I_c \) with the F layer thickness. Indeed, the five orders change of the critical current was observed in the thickness interval (12 - 26) nm. The natural explanation of such a strong thickness dependence is the magnetic scattering effect which is inherent to the ferromagnetic alloys.

The presence of rather strong magnetic scattering in Cu\(_2\)Ni\(_{1-x}\) alloy S/F/S junctions was noted also by Sellier et al. (2003). The magnetic scattering strengthens the decrease of the critical current with the increase of the F layer thickness, and at the same time it increases the period of \( I_c(2d_f) \) oscillations. The general expression for the \( I_c(2d_f) \) dependence, taking into account the magnetic scattering is given in Appendix B, Eq. (101). The attempts to describe the experimental data of Ryazanov et al. (2004) on the \( I_c(2d_f) \) dependence with the help of this expression provided hints on the existence of the another minimum \( I_c(2d_f) \) at smaller F layer thickness - around 10 nm. The very recent experiments with the junctions with the F layer thicknesses up to 7 nm have confirmed this prediction (Ryazanov et al., 2005) - see Fig.15. The existence of the first "0-\( \pi \)" transition at \( 2d_f \approx 11 \text{ nm} \) means that previously reported transitions in Cu\(_2\)Ni\(_{1-x}\) junctions were actually the transitions from "\( \pi \)-" to "0-" phase (and not as was assumed, from "0" to "\( \pi \)-" phase). It means also that now it is possible to fabricate the "\( \pi \)-" -junctions with a 10\(^8\) times higher critical current. Note, that the first measurements (Frolov et al., 2004) of the current-phase relation in S/F/S junction with Cu\(_{0.47}\)Ni\(_{0.53}\) F layer provided no evidence of the second harmonic in \( j(\varphi) \) relationship at the "0"-"\( \pi \)-" transition. These measurements were performed using the junction with F layer thickness around 22 nm, i. e. near the second minimum on the \( I_c(2d_f) \) dependence. The much higher critical current near the first minimum (at \( 2d_f \approx 11 \text{ nm} \) may occur to be very helpful for a search of the second harmonic.

The results of Ryazanov et al. (2001a) on the temperature induced crossover between 0- and \( \pi \)-states were recently confirmed in the experiments of Sellier et al. (2003). Kontos et al. (2002) observed the damped oscillations of the critical current as a function of F layer thickness in Nb/Al/Al\(_2\)O\(_3\)/PdNi/Nb junctions. The measured critical current with the theoretical fit (Buzdin and Baladie, 2003) are presented in Fig. 16. Blum et al. (2002) reported the strong oscillations of the critical current with the F layer thickness in Nb/Cu/Ni/Cu/Nb junctions.
Bulaevskii et al. (1977) pointed out that "π"-junction incorporated into a superconducting ring would generate a spontaneous current and a corresponding magnetic flux would be half a flux quantum $\Phi_0$. The appearance of the spontaneous current is related to the fact that the ground state of the "π"-junction corresponds to the phase difference $\pi$ and so, this phase difference will generate a supercurrent in the ring which short circuits the junction. Naturally the spontaneous current is generated if there are any odd number of "π"-junctions in the ring. This circumstance has been exploited in a elegant way by Ryazanov et al. (2001c) to provide unambiguous proof of the "π"-phase transition. The authors (Ryazanov et al., 2001c) observed the half-period shift of the external magnetic field dependence of the transport critical current in triangular S/F/S arrays. The thickness of F layers of the S/F/S junctions was chosen in such a way that at high temperature the junctions were the usual "0"-junctions, and they transformed into the "π"-junctions with the decrease of the temperature (Ryazanov et al., 2001a).

Guichard et al. (2003) performed similar phase sensitive experiments using dc SQUID with "π"-junction. The total current $I$ flowing trough the SQUID is the sum of the currents $I_a$ and $I_b$ flowing through the two junctions, $I = I_a + I_b$. If the junctions have the same critical currents $I_c$ and both are "0"-junctions, then $I_a = I_c \sin \varphi_a$ and $I_b = I_c \sin \varphi_b$, where $\varphi_a$ and $\varphi_b$ are the phase differences across the junctions. Neglecting the inductance of the loop of SQUID, the phase differences satisfy the usual relation (Barone and Paterno, 1982), $\varphi_a - \varphi_b = 2\pi \Phi/\Phi_0$, where $\Phi$ is the flux of the external magnetic field through the loop of the SQUID. The maximum critical current of the SQUID will be $I_{\text{max}} = 2I_c \cos(\pi \Phi/\Phi_0)$. In the case when one of the junctions (let us say b) is the "π"-junction with the same critical current, the current flowing through it $I_b = -I_c \sin \varphi_b = I_c \sin(\varphi_b + \pi)$. Therefore the maximum critical current of the SQUID in this case will be $I_{\text{π max}}^\pi = 2I_c \cos(\pi \Phi/\Phi_0 + \pi/2)$, and the diffraction pattern will be shifted of half a quantum flux. If both junctions are the "π"-junctions the diffraction pattern will be identical to the diffraction pattern of the SQUID with two "0"-junctions. Namely this was observed on experiment by Guichard et al. (2003) with SQUID containing junctions with PdNi ferromagnetic layers, see Fig. 17.

Recently Bauer et al. (2004) measured with the help of micro Hall-sensor the magnetization of a mesoscopic superconducting loop containing a PdNi ferromagnetic "π"-junction. These measurements also provided a direct evidence of the spontaneous current induced by the "π"-junction.

VI. COMPLEX S/F STRUCTURES

A. F/S/F spin-valve sandwiches

The strong proximity effect in superconductor-metallic ferromagnet structures could lead to the phenomenon of spin-orientation-dependent superconductivity in F/S/F spin-valve sandwiches. Such type of behavior was predicted by Buzdin et al. (1999) and Tagirov (1999) and recently has been observed on experiment by Gu et al. (2002). Note that a long time ago De Gennes (1966b) considered theoretically the system consisting of a thin S layer in between two ferromagnetic insulators. He argued that the parallel orientation of the magnetic moments is more harmful for superconductivity because of the presence of the non-zero averaged exchange field acting on the surface of the superconductor. This prediction has been confirmed on experiment by Hauser (1969) on In film sandwiched between two Fe$_2$O$_4$ films and Deutscher and Meunier (1969), on a In film between oxidized FeNi and Ni layers, see Fig. 18. Curiously, the experiments of Deutscher and Meunier (1969) correspond more to the case of the metallic F/S/F sandwiches as the authors report rather low interface resistance.

To consider the spin-orientation effect in metallic F/S/F sandwiches we use the notations analogous to that of section 4. More precisely, to have a direct connection with the corresponding formula of Section 4, we assume that the thickness of the F layers is $d_f$ and the S layer - $2d_s$, see Fig. 19.

Also, to provide a simple theoretical description we consider the case $d_s \ll \xi_s$ with only two orientations of the ferromagnetic moments: parallel and antiparallel. The case of arbitrary orientations of the ferromagnetic moments needs the introduction of triplet components of the anomalous Green’s functions. The first attempt of such analysis was made by Baladié et al. (2001), but on the basis of the incomplete form of the Usadel equation. The full correct calculations for this case has been performed by Volkov et al. (2003), Bergeret et al. (2003), and Fominov et al. (2003a).

In fact, we only need to analyze the case of the antiparallel orientation of the ferromagnetic moments because the case of the parallel orientation is completely equivalent to the "0" – phase in S/F multilayered structure (Section 4) with the F layers two times thinner than in a F/S/F sandwich. In other words, our choice of notations permits for the parallel orientation case to use directly the corresponding expressions for the critical temperature for the "0" – phase from Section 4. To analyze the antiparallel orientation case, we follow the approach used in Section 4, but we need
written as mutual orientation of ferromagnetic moments is hardly observable. Antiparallel alignment case decreases the spin-valve effect, and for the parameter \( \tau_s^{-1} \) determined by the expression

\[
\frac{4d_s \tau_s^{-1}}{D_s} = 2d_s \beta \omega \simeq \frac{F_s'(-d_s)}{F_s(-d_s)} - \frac{F_s'(d_s)}{F_s(d_s)} \frac{d_s}{2} \left[ \frac{F_s'(d_s)}{F_s(d_s)} + \frac{F_s'(-d_s)}{F_s(-d_s)} \right] ^2.
\]

(64)

Let us suppose that the exchange field is positive (+\( h \)) in the right F layer and then for \( d_s + d_f > x > d_s \)

\[
F_f(x, \omega > 0) = A \cosh \left[ \frac{i + 1}{\xi_f} (x - d_s - d_f) \right],
\]

(65)

while for the left F layer, the exchange field is negative and for \( -d_s - d_f < x < -d_s \) we have

\[
F_f(x, \omega > 0) = B \cosh \left[ \frac{1 - i}{\xi_f} (x + d_s + d_f) \right].
\]

(66)

Taking into account the explicit form of the function \( F_f(x) \) and the boundary conditions (32), we may see that for the antiparallel alignment case \( \frac{F'_s(d_f)}{F_s(d_f)} = - \left( \frac{F'_s(-d_f)}{F_s(-d_f)} \right) ^* \) and the pair-breaking parameter for this case \( \tau_s^{-1} = \tau_s^{-1}_{s,AP} \) may be written as

\[
\tau_s^{-1}_{s,AP} \simeq - \frac{D_s}{2d_s} \text{Re} \left( \frac{F'_s(d_s)}{F_s(d_s)} \right) + \frac{D_s}{2} \left[ \text{Im} \left( \frac{F'_s(d_s)}{F_s(d_s)} \right) \right] ^2.
\]

(67)

The second term in the right-hand side of the eq. (67) may be important only in the limit of very small \( d_f \) and we will omit it further. The boundary conditions Eqs. (32) permit us to calculate the parameter \( \tau_s^{-1} \), provided the anomalous Green function in the F layer is known. For the parallel alignment of the ferromagnetic moments it is just \( \tau_s^{-1} = \tau_s^{-1}_{s, P} \), where \( \tau_s^{-1} \) is given by the Eq. (40), while for the antiparallel alignment it is just

\[
\tau_s^{-1}_{s,AP} = \text{Re} \left( \tau_s^{-1}_{s,0} \right) = \text{Re} \left( \tau_s^{-1}_0 \right).
\]

(68)

In result, we obtain the following simple formula for the critical temperature \( T_c^P \) for the parallel orientation and \( T_c^{AP} \) for the antiparallel one

\[
\ln \frac{T_c^P}{T_c} = \Psi \left( \frac{1}{2} \right) - \text{Re} \left\{ \frac{1}{2} + \frac{1}{2\pi T_c^P \tau_s,0} \right\},
\]

(69)

\[
\ln \frac{T_c^{AP}}{T_c} = \Psi \left( \frac{1}{2} \right) - \Psi \left\{ \frac{1}{2} + \text{Re} \left( \frac{1}{2\pi T_c^{AP} \tau_s,0} \right) \right\}.
\]

(70)

The different kinds of \( T_c(d_f) \) curves are presented in Fig. 20.

We see that the interface transparency is the important factor, controlling the spin-valve effect in F/S/F structures. It is interesting that the optimum condition for the observation of this effect in the case of the non-negligeable interface transparency is the choice \( d_f \sim (0.1 - 0.4) \xi_f \).

In the case when the F layer thickness exceeds \( \xi_f \), the critical temperature practically does not depend on \( d_f \). This case for the transparent S/F interface (\( \gamma_B = 0 \)) was considered by Buzdin et al. (1999), and the critical temperatures for the parallel and antiparallel alignments are presented in Fig. 21. The finite interface transparency strongly decreases the spin-valve effect, and for the parameter \( \gamma_B > 5 \) the dependence of the critical temperatures on the mutual orientation of ferromagnetic moments is hardly observable.
The thermodynamic characteristics of F/S/F systems were studied theoretically by Baladié and Buzdin (2003) and Tollis (2004) in the framework of Usadel formalism and it was noted that the superconductivity always remains gapless.

Bagrets et al. (2003) developed a microscopic theory of F/S/F systems based on the direct solution of the Gor’kov equations for the normal and anomalous Green’s functions. The main mechanism of the electron scattering in F layers was supposed to be of the s – d type. The results of this microscopical analysis were in accordance with the quasiclassical approach and provided a reasonable quantitative description of the experimental data of Obi et al. (1999) on $Tc(d_f)$ dependence in Nb/Co multilayers.

Krunavakarn et al. (2004) generalized the approach of Fominov et al. (2002) to perform exact numerical calculations of the nonmonotonic critical temperature in F/S/F sandwiches. They demonstrated also that the Takahashi-Tachiki (1986) theory of the proximity effect is equivalent to the approach based on the Usadel equations.

Bozovic and Radovic (2002) studied theoretically the coherent transport current through F/S/F double-barrier junctions. The exchange field and the interface barrier reduce the Andreev reflection due to the enhancement of the normal reflection. Interestingly, that the conductance is always higher for parallel alignment of the ferromagnetic moments. The similar conclusion was obtained in work of Yamashita et al. (2003). Such behavior is related with the larger transmission for the normal tunneling current in this orientation. The calculations also revealed the periodic vanishing of Andreev reflection at the energies of geometrical resonance above the superconducting gap.

The case of insulating F layers (De Gennes, 1966b) corresponds to the situation when the superconducting electrons feel the exchange field only on the surface of S layer. We may describe this case taking formally the limit $d_f \to 0$ with $\tau_{\alpha,\beta}^{-1} = i\hbar \frac{\alpha}{2\pi}$, where $\alpha$ is the distance of the order of the interatomic one, which describes the region near the S/F interface where the exchange interaction (described by the exchange field $h$) with electron spins takes place. In fact it simply means that, for the parallel orientation case, the superconductor is under the influence of the averaged exchange field $\bar{h} = h \frac{\alpha}{2\pi}$, while for the antiparallel orientation this field is absent. Careful theoretical analysis of the system consisting of the superconducting film sandwiched between two ferromagnetic semiconducting insulators with differently oriented magnetization was performed by Kulic and Endres (2000) for both singlet and triplet superconductivity cases. In the case of a triplet superconductivity, the critical temperature depends not only on the relative orientation of the magnetization but also on its absolute orientation.

B. S-F-I’-F’-S heterostructures and triplet proximity effect

A bunch of theoretical works was devoted to the analysis of more complex S/F systems. Proshin et al. (2001) (see also Izyumov et al. 2002) studied the critical temperature of S/F multilayers with alternating magnetization of adjacent F layers. The same authors (Izyumov et al., 2000 and Izyumov et al. (2002)) also proposed the 3D LOFF state in F/S contacts. However, this conclusion was based on controversial boundary conditions, corresponding to the different in plane 2D wave-vectors on the both sides of the contact - see the comment by Fominov et al. (2003b) and the reply of Khusainov and Proshin (2003).

Koshina and Krivoruchko (2001) (see also Golubov et al. 2002a) studied the Josephson current of two proximity S/F bilayers separated by an insulating (I) barrier and demonstrated that in such S/F-I-F/S contact the $\pi$-phase may appear even at very small F layer thickness (smaller than $\xi_f$). The mechanism of the $\pi$-phase transition in this case is related to the rotation on $\pi/2$ of the phase of the anomalous Green’s function $F$ on the S/F boundary in addition to the jump of its modulus. To demonstrate this we consider the thin F layer of the thickness $d_f << \xi_s$ in contact with a superconductor. If the $x = 0$ corresponds to the S/F interface, and $x = d_f$ is the outer surface of the F layer, then the solution of the linearized Usadel equation in the ferromagnet is

$$ F_f(x,\omega > 0) = A \cosh \left[ \frac{i + 1}{\xi_f} (x - d_f) \right]. \quad (71) $$

Using the boundary condition Eq. (32) we may easily obtain

$$ F_f(x,\omega > 0) \approx F_f(0,\omega > 0) = \frac{F_s(0,\omega > 0)}{1 + 2\gamma B \xi, d_f / \xi_f^2}. \quad (72) $$

In the case of a rather low interface transparency, $\gamma B \xi, d_f / \xi_f^2 >> 1$, the jump of the phase of the $F$ function at the interface is practically equal to $-\pi/2$:
\[ F_{f}(0, \omega > 0) \approx F_{s}(0, \omega > 0) \exp(-i \frac{\pi}{2}) \frac{\xi_{f}^{2}}{\gamma_{\theta} \xi_{s} d_{f}}. \]  

Koshina and Krivoruchko (2001) and Golubov et al. (2002a) argued that at each S/F interface in the S/F-I-F/S contact the phase jump $-\pi/2$ occurs, and the total phase jump in the equilibrium state would be $\pi$.

Kulic and Kulic (2001) calculated the Josephson current between two superconductors with a helicoidal magnetic structure. They found that the critical current depends on the simple manner on the relative orientation $\theta$ of the magnetic moments on the banks of contact:

\[ I_{c} = I_{c0} (1 - R_{\pm} \cos \theta), \]  

where $R_{-}(R_{+})$ corresponds to the same (opposite) helicity of the magnetization in the banks. Depending on the parameters of the helicoidal ordering, the value of $R_{\pm}$ may be either smaller or larger than 1. If $R_{\pm} > 1$, than $I_{c}$ may be negative for some misorientation angles $\theta$, which means the realization of the $\pi$ - phase. Interestingly that tuning the magnetic phase $\theta$, it is possible to provoke a switch between 0 - and $\pi$ - phase. As it may be seen from Eq. (74), the critical current of the Josephson junction is maximal for the antiparallel orientation ($\theta = \pi$) of the magnetizations in the banks.

Bergeret et al. (2001a) studied the Josephson current between two S/F bilayers and pointed out the enhancement of the critical current for an antiparallel alignment of the ferromagnetic moments. They demonstrated that at low temperatures the critical current in a S/F-I-F/S junction may become even larger than in the absence of the exchange field (i.e. if the ferromagnetic layers are replaced by the normal metal layers with $h = 0$). More in details (taking into account different transparency of S/F interfaces and different orientations of the magnetization in the banks) these junctions were studied theoretically by Krivoruchko and Koshina (2001), Golubov et al. (2002a), Chchelkatchev et al. (2002) and Li et al. (2002). Blanter and Hekking (2004) used Eilenberger and Usadel equations to calculate the current-phase relation of Josephson junction with the composite F layer, consisting of two ferromagnets with opposite magnetizations. Bergeret et al. (2001b) and Kadigrobov et al. (2001) analyzed in the framework of Usadel equations the proximity effect in S/F structures with local inhomogeneity of the magnetization. They obtained an interesting conclusion that the varying in space magnetization generates the triplet component of the anomalous Green’s function ($\sim \langle \Psi_{1} \Psi_{1} \rangle$) which may penetrate in the ferromagnet at distances much larger than $\xi_{f}$. It is not however the triplet superconductivity itself because the corresponding triplet order parameter would be equal to zero, unlike the superfluidity in He3, for example. In general, the triplet components of the anomalous Green’s function always appear at the description of the singlet superconductivity in the presence of rotating in space exchange field. For example, they were introduced by Bulaevskii et al. (1980) in the theory of coexistence of superconductivity with helicoidal magnetic order. An important finding of Bergeret et al. (2001b) and Kadigrobov et al. (2001) was the demonstration of the fact that in some sense the triplet component is insensitive to the pair-breaking by the exchange field. Therefore its characteristic decaying length is the same as in the normal metal, i.e. $\xi_{T,d} = \sqrt{\frac{\xi_{f}}{2\pi T}}$. The triplet long-range proximity effect could explain the experiments on S/F mesoscopic structures (Giroud et al., 1998 and Petrovskh et al., 1999), where a considerable increase of the conductance below the superconducting critical temperature was observed at distances much larger than $\xi_{f}$.

In their subsequent works Bergeret et al. (2003) and Volkov et al. (2003) studied the unusual manifestation of this triplet component in S/F multilayered structures. The most striking effect is the peculiar dependence of the critical current in multilayered S/F structures on the relative orientation of the ferromagnetic moments. For the collinear orientation, the triplet component is absent, and provided the thickness of the ferromagnetic layer $d_{f} >> \xi_{f}$, the critical current is exponentially small. On the other hand, if the orientation of the magnetic moments is noncollinear then the triplet component of the superconducting condensate appears. Its decaying length $\xi_{T,d}$ is much larger than $\xi_{f}$, and namely this triplet component realizes the coupling between the adjacent superconducting layers. When the thicknesses of F layers are in the interval of $\xi_{T,d} >> d_{f} >> \xi_{f}$, then this coupling occurs to be strong. In result, the critical current is maximal for the perpendicular orientation of the adjacent ferromagnetic moments, and it may exceed many times the critical current for their parallel orientation. Due to the mesoscopic fluctuations (Zyuzin et al. 2003), the decay of the critical current for collinear orientation of the magnetic moments is not exponential. Nevertheless, for this orientation it would be very small, and this circumstance do not change the main conclusion on the existence of the long range triplet proximity effect. A lot of interesting physics is expected to emerge in the case.
of S/F systems with genuine triplet superconductors. For example, the proximity effect would be strongly dependent on the mutual orientation of the magnetic moments of the Cooper pairs and ferromagnets.

The long range triplet proximity effect was predicted to exist in the dirty limit. An interesting question is how it evolves in the clean limit. In this regime there is no characteristic decaying length for the anomalous Green’s function in a ferromagnet (see Eqs. (21),(22)), and the angular behavior of the critical current in S/F multilayers may be quite different. If, for example, we apply the Eilenberger equations for the description of clean S/F/F'/S structure with antiparallel ferromagnetic layers with equal thicknesses, the exchange field completely drops (Blanter and Hekking, 2004). Therefore, the critical current will be the same as for the non magnetic interlayers. In this case it is difficult to believe that for the perpendicular orientation of the magnetic moments the critical current could be even higher. The microscopical calculations in the framework of the Bogoliubov-de Gennes equations of the properties of S/F multilayers with non-collinear orientation of the magnetic moments would be of substantial interest.

Barash et al. (2002) studied the Josephson current in S-FIF-S junctions in clean limit within the quasiclassical theory of superconductivity, based on the so-called Ricatti parametrization (Schopol and Maki, 1995). They obtained the striking nonmonotonic dependences of the critical current on the misorientation angle of the ferromagnetic moments. However, even for a rather high transparency of I barrier \((D = 0.8)\), the maximum of the critical current occurred for the antiparallel orientation of the magnetic moments.

VII. ATOMIC THICKNESS S/F MULTILAYERS

A. Layered ferromagnetic superconductors

In this section, we consider an atomic-scale multilayer F/S system, where the superconducting (S) and the ferromagnetic (F) layers alternate. When the electron transfer integral between the S and F layers is small, superconductivity can coexist with ferromagnetism in the adjacent layers. Andreev et al., (1991) demonstrated that the exchange field in F layers favors the \(\pi\)–phase behavior of superconductivity, when the superconducting order parameter alternates its sign on the adjacent S layers.

Nowadays several type of layered compounds, where superconducting and magnetic layers alternate, are known. For example in Sm\(_{1.85}\)Ce\(_{0.15}\)CuO\(_4\) (Sumarlin et al., 1992), which reveals superconductivity at \(T\_c = 23.5\) K, the superconducting layers are separated by two ferromagnetic layers with opposite orientations of the magnetic moments and the Neel temperature is \(T\_N = 5.9\) K. Several years ago, a new class of magnetic superconductors based on the layered perovskite ruthenocuprate compound RuSr\(_2\)GdCu\(_2\)O\(_8\) comprising CuO\(_2\) bilayers and RuO\(_2\) monolayers has been syntesized (see for example McLaughlin et al., 1999 and references cited there). In RuSr\(_2\)GdCu\(_2\)O\(_8\), the magnetic transition occurs at \(T\_M \sim 130 - 140\) K and superconductivity appears at \(T\_c \sim 30 - 50\) K. Recent measurements of the interlayer current in small-sized RuSr\(_2\)GdCu\(_2\)O\(_8\) single crystals showed the intrinsic Josephson effect (Nachtrab et al., 2004). Apparently, it is a weak ferromagnetic order which is realized in this compound. Though the magnetization measurements give evidence of the small ferromagnetic component, the neutron diffraction data on RuSr\(_2\)GdCu\(_2\)O\(_8\) (Lynn et al., 1992) revealed the dominant antiferromagnetic ordering in all three directions. Later, the presence of ferromagnetic in-plane component of about \((0.1 - 0.3)\mu\_B\) has been confirmed by neutron scattering on isostructural RuSr\(_2\)YCu\(_2\)O\(_8\) (Tokunaga et al., 2001). In addition, in the external magnetic field the ferromagnetic component grows rapidly at the expense of the antiferromagnetic one.

Due to the progress of methods of the multilayer preparation, the fabrication of artificial atomic-scale S/F superlattices becomes possible. An important example is the YBa\(_2\)Cu\(_3\)O\(_7\)/La\(_{2/3}\)Ca\(_{1/3}\)MnO\(_3\) superlattices (Sefrioui et al., 2003 and Holden et al., 2003). The manganese half metallic compound La\(_{2/3}\)Ca\(_{1/3}\)MnO\(_3\) (LCMO) exhibits colossal magnetoresistance and its Curie temperature \(\Theta = 240\) K. The cuprate high-T\(_c\) superconductor YBa\(_2\)Cu\(_3\)O\(_7\) (YBaCuO) with \(T\_c = 92\) K, have the similar lattice constant as LCMO which permits to prepare the high quality YBaCuO/LCMO superlattices with different ratio of F and S layers thicknesses. The proximity effect in these superlattices occurs to be extremely long-ranged. For a fixed thickness of the superconducting layer, the critical temperature is dependent over a thickness of LCMO layer in the 100 nm range (Sefrioui et al., 2003 and Peña et al., 2004). This is very unusual behavior because the YBaCuO and LCMO are strongly anisotropic layered systems with very small coherence length in the direction perpendicular to the layers \((0.1 - 0.3\) nm). Somewhat similar giant proximity effect has been recently reported in the non-magnetic trilayer junctions La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\)/La\(_2\)CuO\(_{4+d}\)/La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) (Bozovic et al., 2004) and in the superconductor-antiferromagnet YBa\(_2\)Cu\(_3\)O\(_7\)/La\(_{0.45}\)Ca\(_{0.55}\)MnO\(_3\) superlattices (Pang et al., 2004). The observed giant proximity effect defies the conventional explanations. Bozovic et al. (2004) suggested that it may be related with resonant tunneling, but at the moment the question about the nature of this effect is open.
B. Exactly solvable model of the "\(\pi\)"-phase

Let us consider the exactly solvable model (Andreev et al., 1991) of alternating superconducting and ferromagnetic atomic metallic layers. For simplicity, we assume that the electron’s motion inside the F and S layers is described by the same energy spectrum \(\xi(p)\). Three basic parameters characterize the system: \(t\) is the transfer energy between the F and S layers, \(\lambda\) is the Cooper pairing constant which is assumed to be non zero in S layers only, and \(h\) is the constant exchange field in the F layers. The Hamiltonian of the system can be written as

\[
H = \sum_{\vec{p}, n, i, \sigma} \xi(p) a_{n i \sigma}^+(p) a_{n i \sigma}(p) + H_{int1} + H_{int2} +
\]

\[
+ t \left[ a_{n i \sigma}^+(p) a_{n+1, i, -\sigma}(p) + a_{n+1, i, -\sigma}^+(p) a_{n i \sigma}(p) + h.c. \right],
\]

\[
H_{int1} = \frac{g}{2} \sum_{\vec{p}, \vec{q}, n, \sigma} a_{n1 \sigma}(p_1) a_{n1, -\sigma}^+(p_1) (-p_1) a_{n1, -\sigma}(-p_2)a_{n1 \sigma}(p_2),
\]

\[
H_{int2} = -h \sum_{\vec{p}, n, \sigma} \sigma a_{n1, -\sigma}^+(p) a_{n1, -\sigma}(p),
\]

where \(a_{n i \sigma}^+\) is the creation operator of an electron with spin \(\sigma\) in the \(n^{th}\) elementary cell and a momentum \(p\) in the layer \(i\), where \(i = 1\) for the S layer, and \(i = -1\) for the F layer, and \(g\) is the pairing constant. The important advantage of this model is that the quasiparticle Green’s functions can be calculated exactly and the complete analysis of the superconducting characteristic is possible. Assuming that the order parameter changes from cell to cell in the manner \(\Delta_n = |\Delta| e^{i k n}\), the self-consistency equation for the order parameter \(|\Delta|\) reads

\[
1 = -\lambda T_c^* \lambda \sum_{\omega} \int_{-\infty}^{\infty} d\xi
\]

\[
\int_0^{2\pi} \frac{d\eta}{2\pi} \frac{\bar{\omega}_+ \bar{\omega}_-}{|\Delta|^2} - \frac{\omega_- - |T_{q+k}|^2}{(\omega_+ \bar{\omega}_+ - |T_q|^2)^2},
\]

where \(\lambda = g N(0)\) and \(\omega_{\pm} = \omega \pm \xi(p), \bar{\omega}_{\pm} = \omega_{\pm} + h\). The quasimomentum \(q\) lies in the direction perpendicular to the layers, and \(T_q = 2t \cos(q/2) \xi(p) q^2/2\). In the limit of a small transfer integral \(t << T_c\), where \(T_c\) is the bare mean-field critical temperature of the S layer in the absence of coupling \((t = 0)\), we arrive at the following equation for the critical temperature \(T_c^*\):

\[
\ln \frac{T_c^*}{T_c} = -\pi T_c^* t^2 \sum_{\omega} \int_0^{2\pi} \frac{d\eta}{2\pi} \frac{4}{|\omega|^2 + h^2} +
\]

\[
+ \pi T_c^* t^4 \cos k \sum_{\omega} \frac{12 \omega^4 - 7 \omega^2 h^2 - h^4}{|\omega|^3 (\omega^2 + h^2)^2}. \tag{77}
\]

The critical temperature \(T_c^*\) is close to the bare critical temperature \(T_c\) and as is seen from Eq. (77), for \(h = 0\), the maximal \(T_c^*\) corresponds to \(k = 0\), i.e. the superconducting order parameter is the same at all layers. It is worth to note that as the exchange field on the F layers grows, tunneling becomes energetically more costly, so the leading term second order in \(t\) falls as \(1/h^2\) for large \(h\) and the critical temperature increases. This is related to the fact that, due to the decrease of the coupling the effective exchange field included on the S layers decreases with the increase of \(h\). For \(h >> T_c\), the coefficient of the \(\cos k\) term has a negative sign and the maximal \(T_c^*\) corresponds to \(k = \pi\), so the transition occurs to the \(\pi\)-phase with an alternating order parameter \(\Delta_n = |\Delta| (-1)^n\). Numerical calculations (Andreev et al., 1991) give for the critical value of the exchange field (at which \(k\) changes from 0 to \(\pi\)) \(h_c = 3.77 T_c\), and the complete \((h, T)\) phase diagram is presented in Fig. 22.

At \(T = 0\) the transition to the "\(\pi\)"-phase occurs at \(h_{\pi} = 0.87 T_c\). The analysis of Prokić et al. (1999) and Houzet et al. (2001) shows that the perpendicular critical current vanishes at the line of the transition from the "\(0\)"- to the
"π"-phase and the Josephson coupled superconducting planes are decoupled. Strictly speaking, the critical current vanishes only in $\sim t^4$ approximation, see Eq. (77). The term $\sim t^8$ gives the contribution $\sim t^8 \cos 2k$, and the critical current at the transition to the "π"-phase will drop to the very small value $\sim I_c (t/T_c)^8$. Note that the sign of the second harmonic in $j(\varphi)$ relation generated by this $\sim t^8$ term is positive, and therefore the transition from "0"- to the "π"-phase is discontinuous.

In result, if the exchange field is in the interval $h_{c0} < h < 3.77 T_c$, the "0-π" transition may be easily observed with the lowering of the temperature due to the nonmonotone behavior of the Josephson plasma frequency and the parallel London penetration (Houzet et al., 2001). However the typical value of the exchange field is rather high and more probable is the situation $h >> T_c$, and so the system will be in the "π"-phase at any temperatures. This is consistent with the recent experiments of Nachtrab et al. (2004) on RuSr$_2$GdCu$_2$O$_8$ presenting no evidence of superconducting planes decoupling with temperature. In RuSr$_2$GdCu$_2$O$_8$, the superconducting pairing is probably of the d-wave type. This case was analyzed theoretically by Prokić and Dobrosavljević-Grujić (1999), and the scenario of the "π"-phase appearance is very close to the case of the s-wave superconductivity. Calculations of electronic density of states by Prokić and Dobrosavljević-Grujić (1999) and Prokić et al. (1999) revealed some changes inherent to the "0-π" transition, but, apparently, the experimental identification of the π-phase in the atomic-scale S/F superlattices is an extremely difficult task. In principle, if the superlattice consists of an even number of superconducting layers, then the phase of the order parameter at the ends will differ by π, and the entire system will function as a Josephson "π"-junction. The spontaneous current in a superconducting loop containing such a "π"-junction could be observed at an experiment analogous to the one made by Bauer et al. (2004).

The model Eq. (75) permits to analyze the transition from the quasi-2D to 3D system with the increase of the transfer integral $t$. At $t \lesssim T_c$, instead of the "π"-phase, the LOFF state with modulation along the superconducting layers appears and the system becomes analogous to the 3D superconductor in an uniform exchange field (Houzet and Buzdin, 2002).

Buzdin and Daumens (2003) considered the spin wave effect in the F/S/F structure consisting of three atomic layers and described by the model Eq. (75). Analogously to the F/S/F spin-wave sandwiches (see Section 6), the critical temperature is maximal for the antiparallel orientation of the ferromagnetic moments. However, at low temperature, the situation is inversed. Namely, the superconducting gap occurs to be larger for the parallel orientation of the ferromagnetic moments. This counter-intuitive result of the inversion of the proximity effect may be understood on the example of the ferromagnetic half-metal. Indeed at $T = 0$, the disappearance of the Cooper pair in a S layer means that two electrons with opposite spin must leave it. If the neighbouring F layers of half-metals are parallel, then, for one spin orientation, they are both insulators and the electron with this spin orientation can not enter it. It results in the impossibility of the pair destruction. On the other hand, for the antiparallel orientation of the F layers, for any electron spin orientation there is an adjacent normal layer and a Cooper pair can leave the S layer. Such behavior contrasts with the diffusive model prediction (Baladie and Buzdin, 2003 and Tollis, 2004) but is in accordance with the $T = 0$ results obtained in the framework of the multiterminal model for S/F hybrid structures (Apinyan and Mélin, 2002). Apparently, it is a special property of the clean limit of the atomic-layer S/F model, and it disappears in the case of several consecutive S layers per unit cell (Mélin and Feinberg, 2004).

VIII. SUPERCONDUCTIVITY NEAR THE DOMAIN WALL

In the previous discussion of the properties of S/F heterostructures, we have implicitly assumed that the ferromagnet has uniform magnetization, i. e. there are no domains. It practice, the domains appear in ferromagnets quite easily and special conditions are usually needed to obtain the monodomain ferromagnet. In standard situation, the size of the domains is much larger than the superconducting coherence length, and $\xi_f << \xi_s$, therefore the Cooper pair will sample the uniform exchange field. However, a special situation with the S/F proximity effect is realized near the domain wall, where the magnetic moments and the exchange field rotate. The Cooper pairs feel the exchange field averaged over the superconducting coherence length. Naturally, such averaged field will be smaller near the domain wall, which leads to the local decrease of the pair-breaking parameter. As the result, we may expect that superconductivity would be more robust near the domain wall. In particular, the critical temperature $T_{cw}$ for the superconductivity localized near the domain wall would be higher than that of the uniform S/F bilayer $T_c$. For bulk ferromagnetic superconductors, the critical temperature of the superconductivity localized near the domain wall was calculated by Buzdin et al., (1984). The experimental manifestations of the domain wall superconductivity in Ni$_{0.80}$Fe$_{0.20}$/Nb bilayers (with Nb thickness around 20 nm) were observed by Rusanov et al. (2004). The Néel-type domain walls in Permalloy (Ni$_{0.80}$Fe$_{0.20}$) are responsible for the local increase of the critical temperature around 10...
nK. The width of the domain walls $w$ in Permalloy films used in (Rusanov et al., 2004) is rather large $w \sim 0.5 \mu m$, i.e. much larger than the superconducting coherence length of niobium. The rotation angle $\alpha$ of the exchange field at the distance $\xi_s$ may be estimated as $\alpha \sim \xi_s/w$, and so the averaged exchange field $h^{av}$ is slightly smaller than the field $h$ far away from the domain wall: $(h - h^{av})/h \sim (\xi_s/w)^2$. Therefore, the relative decrease of the pair-breaking parameter $\tau_{c1}$ in Eq. (40) will be also of the order $\sim (\xi_s/w)^2$. From Eqs. (40, 43) we obtain the following estimate of the local increase of the critical temperature

$$\frac{T_{cw} - T_c^s}{T_c^s} \sim (\xi_s/w)^2,$$

which is of the same order of magnitude as the effect observed on the Ni$_{0.86}$Fe$_{0.20}$/Nb bilayers. Keeping in mind the temperature dependence of the superconducting coherence length $\xi(T) \sim \xi_s \sqrt{T/T_c^s}$, we see that the condition of the domain wall superconductivity appearance is simply $\xi(T_{cw}) \sim w$.

In the case of a very thin domain wall, the variation of the exchange field is a step-like and the local suppression of the pair-breaking parameter occurs at the small distance of the order $\xi_f << \xi_s$ near the domain wall. The situation resembles the enhancement of the superconducting pairing near the twin planes (Khlyustikov and Buzdin, 1987). The variation of the pair-breaking occurring over a distance $\xi_f$ induces a superconducting order parameter over a distance $\xi(T_{cw})$ near the domain wall and the effective relative decrease of the pair-breaking parameter will be of the order of $\xi_f/\xi(T_{cw})$. Therefore, if the shift of the critical temperature of the S/F bilayer is comparable with $T_c$, i.e. $\left((T_c - T_c^s)/T_c^s \sim 1\right)$, the critical temperature $T_{cw}$ of the superconductivity, localized near the domain wall may be estimated from the condition $\frac{T_{cw} - T_c^s}{T_c^s} \sim \xi_f/\xi(T_{cw})$. In result we have

$$\frac{T_{cw} - T_c^s}{T_c^s} \sim (\xi_f/\xi_s)^2,$$

which is around (1-5)% for typical values of $\xi_f$ and $\xi_s$. A small width of the domain walls is expected in experiments of Kinsey, Burnell, and Blamire (2001) on the critical current measurements of Nb/Co bilayers. The domain walls occured to be responsible for the critical current enhancement below $T_c^s = (5.24 \pm 0.05)$ K. In the presence of domains walls the non-zero critical current has been observed at $(5.4 \pm 0.05)$ K, slightly above $T_c^s$.

It is worth to note that the effect of the increase of the critical temperature in the vicinity of a domain wall is weak for very large and very thin domain wall. The optimum thickness, when the effect may be relatively strong is $w \sim \xi_s$.

In the case of a perpendicular easy-axis the branching of the domains may occur near the surface of magnetic film. If the scale of this branching is smaller than the superconducting coherence length, the effective exchange field is averaged, and the pair breaking parameter will be strongly decreased. This mechanism has been proposed in (Buzdin, 1985) to explain the presence of traces of superconductivity at low temperature in re-entrant ferromagnetic superconductors. The similar effect may take place in S/F bilayers and in such case the superconductivity would be extremely sensitive to the domain structure. Rather weak magnetic field would suffice to modify the branching of domains and suppress superconductivity.

Up to now we have concentrated on the interplay between superconductivity and ferromagnetism caused by the proximity effect related to the passing of electrons across the S/F interface. However, if the magnetic field created by the ferromagnet penetrates into a superconductor, it switches on the orbital mechanism of superconductivity and magnetism interaction. The situation when it is the only one mechanism of superconductivity and magnetism interaction is naturally realized in the case, when the ferromagnet is an insulator, or the buffer oxide layer separates the superconductor and the ferromagnet. The hybrid S/F systems have been intensively studied in connection with the problem of the controlled flux pinning. Enhancement of the critical current has been observed experimentally for superconducting films with arrays of submicron magnetic dots and antidots (see, for example Van Bael et al., 2002a and Van Bael et al., 2002b, and references cited therein), and for S/F bilayers with a domain structure in ferromagnetic films (García-Santiago et al., 2000). A theory of vortex structures and pinning in S/F systems at rather low magnetic field has been elaborated by Lyuksyutov and Pokrovsky (1998), Bulaevskii et al. (2000), Erdin et al. (2002) and Milosevic et al. (2002a). This subject is discussed in details in the recent review by Lyuksyutov and Pokrovsky (2004).

The nucleation of the superconductivity in the presence of domain structure has been theoretically studied by Buzdin and Melnikov (2003), and Aladyshkin et al. (2003) in the case of magnetic film with perpendicular anisotropy. The conditions of the superconductivity appearance occur to be more favorable near the domain walls. Recently the manifestation of the domain wall superconductivity was revealed on experiment by Yang et al. (2004). They deposited on the single crystal ferromagnetic BaFe$_{12}$O$_{19}$ substrate a 10 nm Si buffer layer and then a 50 nm Nb film. The
strong magnetic anisotropy of BaFe$_{12}$O$_{19}$ assures that its magnetisation is perpendicular to the Nb film. The very characteristic $R(T)$ dependences and pronounced hysteresis effects have been found in the resistance measurements in the applied field.

A different situation is realized if the magnetization of F layer is lying in the plane (parallel magnetic anisotropy). Then any type of the domain walls will be a source of the magnetic field for the adjacent S layer, and the domain wall locally weakens superconductivity. This idea was proposed by Sonin (1988) to create in a S layer a superconducting weak link (Josephson junction) attached to the domain wall.

Lange et al. (2003) used a nanoengineered lattice of magnetic dots on the top of the superconducting film for the observation of the field-induced superconductivity. The applied external magnetic field provided the compensation of the magnetic field of the dots and increased the critical temperature. The idea of such compensation effect was proposed a long time ago by Ginzburg (1956) for the case of the ferromagnetic superconductors.

The analysis of the superconducting states appearing near the magnetic dots (when the upper critical field depends on the angular momentum of the superconducting nucleus wave function) was done in the works of Cheng and Fertig (1999) and Milosevic et al. (2002b).

IX. MODIFICATION OF FERROMAGNETIC ORDER BY SUPERCONDUCTIVITY

A. Effective exchange field in thin S/F bilayers

The influence of ferromagnetism on superconductivity is strong, and it leads to many experimentally observed consequences. Whether the inverse is true also? In other words, can superconductivity affect or even destroy ferromagnetism? To address this question, we start with comparing the characteristic energy scales for superconducting and magnetic transitions. The energy gain per atom at the magnetic transition is of the order of the Curie temperature $\Theta$. On the other hand the condensation energy per electron at the superconducting transition (Eq. (2)) is much smaller than $T_c$, and it is only about $\sim T_c (T_c/E_F) << T_c$. Usually the Curie temperature is higher than $T_c$ and ferromagnetism occurs to be much more robust compared with superconductivity. Therefore the superconductivity can hardly destroy the ferromagnetism, but it may nevertheless modify it, if such modification do not cost too much energy. The example is the bulk ferromagnetic superconductors ErRh$_4$B$_4$, HoMo$_6$S$_8$ and HoMo$_6$Se$_8$, where, in superconducting phase, ferromagnetism is transformed into a domain phase with the domain size smaller than the superconducting coherence length $\xi_s$ (Maple and Fisher, 1982; Bulaevskii et al., 1985). Similar effect has been predicted by Buzdin and Bulaevskii (1988) for a thin ferromagnetic film on the surface of a superconductor. To illustrate this effect, we consider the S/F bilayer with S layer thickness $d_s$ smaller than the superconducting coherence length $\xi_s$ and the F layer thickness $d_f << \xi_s << d_s$, see Fig. 23.

In the case of a transparent S/F interface, the pair-breaking parameter is given by the Eq. (40), and it is

$$\tau_s^{-1}(\omega > 0) = i \hbar \frac{D_s d_f}{D_f d_s} \frac{\sigma_f}{\sigma_s},$$

which simply means that the effective exchange field in the superconductor $\bar{h} \approx \hbar \frac{d_f}{d_s} \left( \frac{\sigma_f}{\sigma_s} \right)$. The condition of a transparent interface implies that the Fermi momenta are equals in both materials and this permits us to write the effective field as

$$\bar{h} = h \left( \frac{d_f}{d_s} \right) \left( \frac{v_{F_s}}{v_{F_f}} \right),$$

where $v_{F_s}$ and $v_{F_f}$ are the Fermi velocities in S and F layers respectively. Note however that for strong ferromagnets the condition of perfect transparency of the interface is different, $v_{F\uparrow} v_{F\downarrow} = v_f^2$, where $v_{F\uparrow}$ and $v_{F\downarrow}$ are the Fermi velocities for two spin polarizations in ferromagnet (Zutic and Valls, 1999, and Zutic et al., 2004).

In fact, in the considered case of thin F and S layers the situation is analogous to the magnetic superconductors with an effective exchange field $\bar{h}$, which may also depend on the coordinates $(y,z)$ in the plane of bilayer. Let us demonstrate this important point. Keeping in mind the domain structure, (see Fig. 23), where the exchange field depends only on the $z$ coordinate, we may write the Usadel equations in F and S layers

$$-\frac{D_f}{2} \left[ G \left( F + \frac{\partial^2}{\partial z^2} F \right) - F \left( \frac{\partial^2}{\partial x^2} G + \frac{\partial^2}{\partial z^2} G \right) \right] + (\omega + i \hbar(z)) F = 0$$

(82)
Now let us perform the averaging procedure by integrating these equations over \( x \). Due to the small thicknesses of \( F \) and \( S \) layers, the Green’s functions \( G \) and \( F \) vary little with \( x \) and may be considered as constants. The integration of the terms with the second derivatives on \( x \) will generate \( \frac{\partial^2}{\partial x^2} \) and \( \frac{\partial^2}{\partial z^2} \) terms taken at the interfaces. At the interfaces with vacuum these derivatives vanish and the boundary conditions Eq.(32) permit us to rely on the derivatives of \( F \) function on both sides of the \( S/F \) interface (the same relation is true for the \( G \) function, due to the normalization condition Eq. (98)). Excluding the derivatives \( (\frac{\partial G}{\partial z})_d \) and \( (\frac{\partial F}{\partial z})_d \), we obtain the standard Usadel equation but for the averaged (over the \( S \) layer thickness) Green’s functions \( \overline{F} \) and \( \overline{G} \):

\[
\left( \omega + i\tilde{h}(z) \right) \overline{F} - \frac{D_s}{2} \left[ G \frac{\partial^2}{\partial z^2} \overline{F} - \overline{F} \frac{\partial^2}{\partial z^2} G \right] = \Delta \overline{G}, \tag{84}
\]

where the effective field \( \tilde{h}(z) = h(z) \frac{d_f}{d_s} \frac{D_s}{D_f} \frac{\sigma_f}{\sigma_s} = \frac{h d_f}{d_s} \frac{\sigma_f}{\sigma_s} v_F \) and the condition \( d_f/d_s \ll 1 \) is used to neglect the small renormalization of \( D_s \) and \( \omega \). The possibility to introduce the effective field \( \tilde{h}(z) \) in the case of a thin bilayer is quite natural and rather general. The same effective field may be introduced in the framework of Eilenberger equations.

**B. Domain structure**

In the case of the uniform ferromagnetic ordering in the \( F \) layer, superconductivity can exist only if \( \tilde{h} \) does not exceed the paramagnetic limit: \( \tilde{h} < 1.24T_c \). This means that the thickness of the \( F \) layer must be extremely small \( d_f < (T_c/h_d) d_s \); even for \( d_s \sim \xi_s \), taking \( T_c \sim 10 \text{ K} \) and \( h \sim 5000 \text{ K} \), the maximum thickness of \( F \) layer only around 1 nm. However, the ferromagnetic superconductors (Maple and Fisher, 1982; Bulaevskii et al., 1985) give us the example of domain coexistence phases with the exchange field larger than the paramagnetic limit.

We may apply the theory of magnetic superconductors (Bulaevskii et al., 1985) to the description of the domain structure with wave vector \( Q \gg \xi_s^{-1} \) in the \( S/F \) bilayer, Fig. 23. The pair-breaking parameter associated with the domain structure is \( \tau_s^{-1} \sim \frac{\Delta^2}{Q} \) (Bulaevskii et al., 1985), where \( v = v_F s \) is the Fermi velocity in \( S \) layer. Let us write the domain wall energy per unit area as \( \sigma/\pi a^2 \), where \( a \) is the interatomic distance. The domain wall energy in the \( F \) film per unit length of the wall will be \( d_f (\sigma/\pi a^2) \). Note that we consider the case of relatively small domain wall thickness \( w \ll Q^{-1} \ll \xi_s \) and the constant \( \sigma \), describing the domain wall energy is of the order of Curie temperature \( T \) for the atomic thickness domain wall but may be smaller for the thick domain wall. The change of the density of the superconducting condensation energy due to the pair-breaking effect of domain structure is of the order of \( N(0)\Delta^2/(\Delta \tau_s) \). Therefore the density (per unit area) of the energy \( E_{DS} \) related to the domain structure reads

\[
E_{DS} \sim N(0)d_s \Delta \tilde{h}^2 / vQ + d_f \frac{\sigma Q}{a^2}. \tag{85}
\]

Its minimum is reached at

\[
Q^2 = \frac{d_s N(0)\Delta a^2 \tilde{h}^2}{\sigma v} \sim \frac{1}{a \xi_0} \frac{d_s \tilde{h}^2}{d_f \sigma v}, \tag{86}
\]

where \( \xi_0 = h v / (\pi \Delta) \). The factor which favors the existence of the domain structure is the superconducting condensation energy \( E_s \sim -N(0)d_s \Delta^2 \) per unit area. The domain structure decreases the total energy of the system if \( E_{DS} + E_s < 0 \), and we obtain the following condition of its existence

\[
T_c \gtrsim \left( \frac{\tilde{h}^2 \sigma d_f}{d_s} \right)^{1/3} = \tilde{h} (\sigma / h)^{1/3}. \tag{87}
\]

Due to the small factor \( (\sigma / h)^{1/3} \ll 1 \) this condition is less restrictive than the paramagnetic limit \( (T_c > 0.66 \tilde{h}) \). Nevertheless the conditions of the formation of the domain structure remain rather stringent. To minimize the \( d_f/d_s \) ratio (and so the effective exchange field) it is better to choose the largest possible \( d_s \) thickness. However, the maximum thickness of the region, where superconductivity will be affected by the presence of \( F \) layer is of the order
of $\xi_s$. Then, even in the case of the bulk superconductor $d_s^{\text{max}} \sim \xi_s$ and the condition of the domain phase formation in such a case reads

$$T_c \gtrsim h \frac{df}{\xi_s} (\sigma/h)^{1/3}. \quad (88)$$

We may conclude that for the domain phase observation it is better to choose a superconductor with a large coherence length $\xi_s$ and the ferromagnet with low Curie temperature and small energy of the domain walls.

The transition into the domain state is a first order one, and as all transitions related with the domain walls, it would be highly hysteretic. This circumstance may strongly complicate its experimental observation. To overcome this difficulty, it may be helpful to fabricate the S/F bilayer with a ferromagnet with a low Curie temperature $\Theta < T_c$. In such case, from the very beginning we may expect the appearance of the non-uniform magnetic structure below $\Theta$. This system in many senses would be analogous to the ferromagnetic superconductors ErRh$_4$B$_4$, HoMo$_6$Se$_8$ and HoMo$_6$Se$_8$.

Bergeret et al. (2000) argued that the appearance of a nonhomogeneous magnetic order in a F film deposited on the bulk superconductor occurs via the second order transition and the period of the structure goes to infinity at the critical point. They considered the helicoidal magnetic structure with a wave vector $Q$ and the magnetic moment lying in the plane of the film. The increase of the magnetic energy due to the rotation of the moments was taken to be proportional to $Q^2$. However, the considered magnetic structure is known to generate the magnetic field at distance $\sim Q^{-1}$ from the film. The contribution coming from this field makes the magnetic energy to be proportional to $Q$ and not to $Q^2$ at a small wave-vector regime. This circumstance qualitatively change the conclusions of Bergeret et al. (2000) and makes the transition into a nonhomogeneous magnetic state a first-order one.

The experiments of Mühge et al., (1998) on the ferromagnetic resonance measurements in the Fe/Nb bilayers revealed some decrease of the effective magnetisation below $T_c$ for the bilayers with $d_f < 1$ nm. This thickness is compatible with the estimate Eq. (88), but the analysis of these experimental data by Garifullin (2002) reveals the possibility of the formation of islands at a small thickness of Fe layer, which may strongly complicate the interpretation of experimental results.

C. Negative domain wall energy

In the previous analysis, the energy of the domain wall was considered to be constant independent of the presence of the superconducting layer. It is a good approximation for a thin domain wall $w \ll \xi_s$. However, the phenomenon of superconductivity localized near the domain walls is the manifestation of the local enhancement of the superconducting condensation energy, which may give a negative contribution to the domain wall energy. We estimate this effect for a thick $w >> \xi_s$ domain wall. The effect is maximum for the S/F bilayer with the relative variation of the critical temperature $(T_c - T_c^*) T_c \sim 1$ at $d_s \sim \xi_s$. We will suppose these conditions to be satisfied. Following the same reasoning as in the case of the domain wall superconductivity, we may estimate the relative local decrease of the pair-breaking parameter as $\delta (\tau_s^{-1})/\tau_s^{-1} \sim (\xi_s/w)^2$. Therefore the local negative contribution to the domain wall energy (per its unit length) coming from the superconductivity reads

$$\delta E_s \sim -N(0)\Delta^2 (\xi_s/w)^2 wd_s. \quad (89)$$

The proper magnetic energy of the domain wall is $E_{DW} \sim df (\sigma/\pi a^2)$, and for a large domain wall $\sigma \sim \Theta (a/w)$. The condition of the vanishing of the total energy of the domain wall $\delta E_s + E_{DW} = 0$ gives

$$\frac{T_c^2}{E_F} \frac{\xi_s^3}{wa} \sim df \sigma \sim \Theta \frac{a}{w} df, \quad (90)$$

where the estimate $d_s \sim \xi_s$ is used. Finally, we may conclude that the energy of the domain wall may be negative for the system with

$$T_c \gtrsim \Theta \frac{a}{l} \frac{df}{\xi_s}. \quad (91)$$

where $l$ is the electron mean free path. We have taken into account that $\xi_s \sim \sqrt{\xi_0 l}$ and $a/\xi_0 \sim T_c/E_F$. If the condition Eq. (91) is fulfilled, the following scenario emerges. The decrease of the temperature below $T^*_c$ will decrease the energy of the domain walls, which are practically always present in a ferromagnet. The concentration of the domain walls
will increase and finally, when the domain wall energy will change its sign, the relatively dense domain structure will appear. The average distance between the domains walls in such a structure would be of the order of the domain wall thickness itself. Note that in the case of the small thickness of the domain wall the superconducting contribution to its energy is negligible and instead of Eq. (91) we obtain the non-realistic condition \( T_c > \Theta (d_f / \xi_f) (\xi_s / l) \). We have taken into account only the exchange mechanism of the interaction between magnetism and superconductivity. The orbital effect gives an opposite contribution to the domain wall energy, related with the out of plane magnetic field near the domain wall, which generates the screening currents in the superconducting layer.

At the present time, there are no clear experimental evidences for the domain structure formation in S/F bilayers. The experiments of Mühge et al., (1998) on the ferromagnetic resonance measurements in the Fe/Nb bilayers revealed some decrease of the effective magnetization below \( T_c^* \) for the bilayers with \( d_f < 1 \) nm. This thickness is compatible with the estimate Eq. (88), but the magnetic moment decreases continuously below \( T_c^* \). In addition the analysis of these experimental data by Garifullin (2002) reveals the possibility of the formation of islands at small thickness of iron layer thus reducing its magnetic stiffness. The condition Eq. (91) is apparently fulfilled in the experiments of Mühge et al., (1998). Therefore the decrease of the domain wall energy may be at the origin of the observed effect.

D. Ferromagnetic film on a superconducting substrate

Bulaevskii and Chudnovsky (2000) and Bulaevskii et al. (2002) demonstrated that the pure orbital effect could decrease the equilibrium domain width in the ferromagnetic film on the superconducting substrate. The ferromagnet with a perpendicular magnetic anisotropy is either an insulator, or it is separated from the superconductor by a thin insulating (e. g. oxide) layer, see Fig.24.

In such case the ferromagnetic film and the superconductor are coupled only by the magnetic field. It is well-known (Landau and Lifshitz, 1982) that the positive energy of the magnetic field favors small domains, so that the stray field does not spread at large distance. On the other hand, the positive domains wall energy favors a large domain size. The balance of these two contributions gives the equilibrium domain width \( l_N \sim \sqrt{\pi d_f} \). In the presence of a superconductor, the screening currents modify the distribution of the magnetic field near the S/F interface and give an additional positive contribution to the energy of the magnetic field. This results in the shrinkage of the domain width. The energy \( E_D \) of the domain structure on the superconducting substrate reads (Bulaevskii and Chudnovsky, 2000 and Bulaevskii et al., 2002)

\[
E_D \sim 3\overline{l} + \frac{2l_N^2}{l} - \frac{16\overline{l}}{(7\zeta(3)) \sum_{k \geq 0} (2k+1)^2 \left( 2k + 1 + \sqrt{(2k+1)^2 + 16\overline{l}^2} \right)}
\]

Here \( \overline{l} = l/(4\pi \lambda) \) and \( \overline{l}_N = l_N/(4\pi \lambda) \) are the reduced widths of domains on a superconducting and normal substrate respectively, and \( \lambda \) is the London penetration depth. The minimization of \( E_D \) over \( \overline{l} \) gives the equilibrium width of domains. In the limit \( \lambda \rightarrow \infty \) the influence of superconductivity vanishes and \( \overline{l} = l_N \). The limit \( \lambda \rightarrow 0 \), when the magnetic field does not penetrate inside the superconductor was considered by Sonin (2002). In this limit the shrinkage of the width of the domains is maximum and \( \overline{l} = \sqrt{2/3} l_N \). Then we may conclude that the influence of superconductivity on the domain structure is not very large and it is even less pronounced in S/F bilayer when the thickness of the S layer becomes smaller than the London penetration depth (Daumens and Ezzahri, 2003).

Helseth et al. (2002) studied the change of the Bloch domain wall structure in a ferromagnetic thin film on the superconducting substrate with the in-plane magnetization of the domains. It occurs that the wall experiences a small shrinkage, which corresponds to the increase of the energy of the domain wall.

Recently, Dubonos et al. (2002) demonstrated experimentally the influence of the superconducting transition on the distribution of the magnetic domains in mesoscopic ferromagnet-superconductor structures. This finding makes quite plausible the observation of the effect predicted by Bulaevskii and Chudnovsky (2000) and Bulaevskii et al. (2002). Rearrangement of the domains normally results in the resistance change in metallic ferromagnets. In this context Dubonos et al. (2002) noted that domain walls’ displacement due to the superconducting transition could be the actual mechanism of the long-range resistive proximity effects previously observed in mesoscopic Ni/Al structures (Petrashev et al., 1999) and Co/Al structures (Giroud et al., 1998). Note also that Aumentado and Chandrasekhar (2001) studied the electron transport in submicron ferromagnet (Ni) in contact with a mesoscopic superconductor (Al) and demonstrated that the interface resistance is very sensitive to the magnetic state of the ferromagnetic particle.
X. CONCLUSIONS

The most striking peculiarity of the proximity effect between superconductor and ferromagnet is the damped oscillatory behavior of the Cooper pair wave function in ferromagnet. It results in the non-monotonous dependence of the critical temperature of S/F bilayers and multilayers on the F layer thickness, as well as in the formation of "\(\pi\)"– junctions in S/F/S systems. The minimum energy of the "\(\pi\)"– junction is realized for the phase difference \(\pm \pi\), and a spontaneous supercurrent may appear in a circuit containing the "\(\pi\)"– junction. Two possible directions of the supercurrent reflect the double-degenerate ground state. In contrast to the usual junction such a state is achieved without external applied field. The Qubit (or quantum bit) is the analog of a bit for quantum computation, describing by state in a two level quantum system (Nielsen and Chuang, 2000). The S/F systems open a way to create an environmentally decoupled (so called "quiet") qubit (Ioffe et al., 1999) on the basis of the S/F/S junction.

The "\(\pi\)"– junctions allow for a realization of the concept of the complimentary logic. In the metal-oxide superconductor logic family the combination of the semiconducting n-p-n junctions with the complimentary p-n-p ones permits to significantly simplify the circuitry. The similar is possible for the Josephson junctions devices and circuits when the "\(\pi\)"– junctions are used (Terzioglu and Beasley, 1998). The logic cells with the "\(\pi\)"– junctions play a role of the complimentary devices to the usual Josephson logic cells.

Recently, Ustinov and Kaplunenko (2003) proposed to use the "\(\pi\)"– junction as a phase shifter in the rapid single-flux quantum circuits. The relatively large geometrical inductance, which is required by the single-flux quantum storage, may be replaced by the much smaller "\(\pi\)"– junction. The advantage of the implementation of the "\(\pi\)"– junctions is the possibility to scale the dimension of superconducting logic circuits down to the submicron size. In addition, the use of the "\(\pi\)"– junction as a phase shifter substantially increases the parameter margins of the circuits.

As it has been discussed in Section III.D the exchange interaction strongly affects the Andreev reflection at the F/S interface presenting a powerful tool to probe ferromagnets and measure their spin polarization.

The structures consisting of "0" and "\(\pi\)"– Josephson junctions can exhibit quite unusual properties. Bulaevskii et al. (1978) demonstrated that the spontaneous Josephson vortex carrying the flux \(\Phi_0/2\) appears at the boundary between "0" and "\(\pi\)"– junctions. A periodic structure consisting of small (comparing with Josephson length) alternating "0" and "\(\pi\)"– Josephson junctions may have any value of an equilibrium averaged phase difference \(\varphi_0\) in the interval \(-\pi < \varphi_0 < \pi\), depending on the ratio of lengths of "0" and "\(\pi\)"– junctions (Mints, 1998; Buzdin and Koshelev, 2003). The S/F heterostructures provide the possibility of the realization of such "\(\varphi\)"– junction with very special two maxima current-phase relation and Josephson vortices carrying partial fluxes \(\Phi_0 (\varphi_0/\pi)\) and \(\Phi_0 (1 - \varphi_0/\pi)\).

The possibility to combine in a controlled manner paramagnetic and orbital mechanisms of the interaction between superconductivity and magnetism makes the physics of S/F heterostructures quite rich and promising for potential applications. Let us mention in this context the recent observation of strong vortex pinning in S/F hybrid structures, the spin valve effect in F/S/F systems and the domain wall superconductivity, which open a large perspective to the creation of new electronics devices. The progress of controllable fabrication of high-quality heterostructures and especially the high-quality interfaces was crucial for the recent breakthrough in this domain. Further development of the microfabrication technology permits to expect another interesting findings in the near future.

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XII. APPENDIX:

A. Bogoliubov-de Gennes equations

As the characteristic length of the induced superconductivity variation in a ferromagnet is small compared with a superconducting length, it implies the use of the microscopic theory of superconductivity to describe the proximity effect in S/F structures. The very convenient microscopical approach to study the superconducting properties in the ballistic regime (the clean limit) in the presence of spatially varying field is the use of the Bogoliubov-de Gennes equations (de Gennes, 1966a). The equations for electron and hole wave functions $u_\uparrow (\mathbf{r})$ and $v_\downarrow (\mathbf{r})$ are

$$
(H_0 - h (\mathbf{r})) u_\uparrow (\mathbf{r}) + \Delta (\mathbf{r}) v_\downarrow (\mathbf{r}) = E_\uparrow u_\uparrow (\mathbf{r})
$$

$$
\Delta^* (\mathbf{r}) u_\uparrow (\mathbf{r}) - (H_0 + h (\mathbf{r})) v_\downarrow (\mathbf{r}) = E_\downarrow v_\downarrow (\mathbf{r}),
$$

where $E_\uparrow$ is the quasiparticle excitation energy, $H_0 = -\hbar^2 \nabla^2 / 2m - E_F$ is the single particle Hamiltonian, $h (\mathbf{r})$ is the exchange field in the ferromagnet, and the spin quantization axis is chosen along its direction. The equations for the wave functions with opposite spin orientation $u_\downarrow (\mathbf{r})$ and $v_\uparrow (\mathbf{r})$ and the excitation energy $E_\downarrow$ are obtained from Eq. (93) by the substitution $h \rightarrow -h$. Note that the solution $(u_\downarrow, v_\uparrow)$ with energy $E_\downarrow$ may be immediately obtained from the solution of Eq. (93), if we choose $u_\downarrow = v_\uparrow$, $v_\downarrow = -u_\downarrow$ and $E_\downarrow = -E_\uparrow$. The pair potential in the superconductor is determined by the self-consistent equation

$$
\Delta (\mathbf{r}) = \lambda \sum_{E_\uparrow > 0} u_\uparrow (\mathbf{r}) v_\downarrow^* (\mathbf{r}) (1 - 2f (E_\uparrow)),
$$

where $f (E)$ is the Fermi distribution function $f (E) = 1 / (1 + \exp (E/T))$, and $\lambda$ is the BCS coupling constant.

Assuming that the Cooper pairing is absent in the ferromagnet, we have $\Delta (\mathbf{r}) = 0$ there. The situations when it is possible to obtain the analytical solutions of the Bogoliubov-de Gennes equations with spatially varying pair potential are very rare. However, these equations provide a good basis for the numerical calculations to treat different aspects of S/N and S/F proximity effects.

B. Eilenberger and Usadel equations for ferromagnets

Another microscopical approach in the theory of superconductivity uses the electronic Green’s functions. The Green’s functions technique for superconductors has been proposed by Gor'kov who introduced in addition to the normal Green’s function $G(\mathbf{r}_1, \mathbf{r}_2)$ the anomalous (Gor’kov) function $F(\mathbf{r}_1, \mathbf{r}_2)$ (see, for example, Abrikosov et al., 1975). This technique is a very powerful tool, but the corresponding Green’s functions in a general case occur to be rather complicated and oscillate as a function of the relative coordinate $\mathbf{r}_1 - \mathbf{r}_2$ on the scale of the interatomic distance. On the other hand, the characteristic length scales for superconductivity in S/F systems are of the order of the layers thicknesses or damping decay length for the induced superconductivity and, then, they are much larger than the atomic length. This smooth variation is described by the center of mass coordinate $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2) / 2$ in the Green’s functions. The very convenient quasiclassical equations for the Green’s functions averaged over the rapid oscillations on the relative coordinate has been proposed by Eilenberger (1968) (and also by Larkin and Ovchinnikov (1968)).

Eilenberger equations are transport-like equations for the energy-integrated Green’s functions $f(\mathbf{r}, \omega, \mathbf{n})$ and $g(\mathbf{r}, \omega, \mathbf{n})$, depending on the center of mass coordinate $\mathbf{r}$, Matsubara frequencies $\omega = \pi T (2n + 1)$ and the direction of the unit vector $\mathbf{n}$ normal to the Fermi surface. For the case of S/F multilayers we may restrict ourselves to the situations when all quantities only depend on one coordinate $x$, chosen perpendicular to the layers. Introducing the angle $\theta$ between the $x$ axis and the direction of the vector $\mathbf{n}$ (direction of the Fermi velocity), we may write the Eilenberger equations in the presence of the exchange field $h(x)$ in the form (see, for example Bulaevskii et al. (1985)
Usadel (1970) equations. In fact, the conditions of the applicability of the Usadel equations are the dependence of the Green’s functions is weak, and the Eilenberger equations can be replaced by the much simpler 

\[ G(x, \omega) = \int \frac{d\Omega}{4\pi} g(x, \theta, \omega), \quad F(x, \omega) = \int \frac{d\Omega}{4\pi} f(x, \theta, \omega), \]

where the function \( f^+(x, n, \omega) \) satisfies the same equation as \( f(x, -n, \omega) \) with \( \Delta \to \Delta^* \) and the presence of impurities is described by the elastic scattering time \( \tau = 1/\nu_f \). The functions \( G(x, \omega) \) and \( F(x, \omega) \) are the Green’s functions averaged over the Fermi surface. The Eilenberger equations are completed by the self-consistency equation for the pair potential \( \Delta(x) \) in a superconducting layer:

\[ \Delta(x) = \pi T \lambda \sum_\omega F(x, \omega). \]  

The BCS coupling constant \( \lambda \) is spatially independent in a superconducting layer, while in a ferromagnetic layer it is equal to zero. In a superconducting layer, the self-consistency equation may also be written in the following convenient form

\[ \Delta(x) \ln \frac{T}{T_c} + \pi T \sum_\omega \left( \frac{\Delta(x)}{\omega} - F(x, \omega) \right) = 0, \]  

where \( T_{c0} \) is the bare transition temperature of the superconducting layer in the absence of proximity effect.

Note that the presented form of the Eilenberger equations implies the natural choice of the spin quantization axis along the direction of the exchange field, and the only difference with the standard form of these equations is the substitution \( \Delta \to \Delta^* \). Here also the only difference with the standard form of the Usadel equations is the substitution \( \omega \) by \( \omega + i\hbar(x) \).

Usually, the electron scattering mean free path in S/F/S systems is rather small. In such a dirty limit, the angular dependence of the Green’s functions is weak, and the Eilenberger equations can be replaced by the much simpler Usadel (1970) equations. In fact, the conditions of the applicability of the Usadel equations are \( T_c \tau \ll 1 \) and \( h \tau \ll 1 \). The second condition is much more restrictive due to a large value of the exchange field \( (h \gg T_c) \). The Usadel equations only deal with the Green’s functions \( G(x, \omega) \) and \( F(x, \omega) \) averaged over the Fermi surface:

\[ \frac{-D}{2} \left[ G(x, \omega, h) \frac{\partial^2}{\partial x^2} F(x, \omega, h) - F(x, \omega, h) \frac{\partial^2}{\partial x^2} G(x, \omega, h) \right] + (\omega + i\hbar(x)) F(x, \omega, h) = \Delta(x) G(x, \omega, h), \]

\[ G^2(x, \omega, h) + F(x, \omega, h) F^+(x, \omega, h) = 1, \]

\( D = \frac{1}{4} \nu_F l \) is the diffusion coefficient which is different in S and F regions and the equation for the function \( F^+(x, h, \omega) \) is the same as for \( F(x, \omega, h) \) with the substitution \( \Delta \to \Delta^* \). Here also the only difference with the standard form of the Usadel equations is the substitution \( \omega \) by \( \omega + i\hbar(x) \).

The equations for the Green’s functions in F and S regions must be completed by the corresponding boundary conditions at the interfaces. For the Eilenberger equations they were derived by Zaitsev (1984) and for the Usadel equations by Kupriyanov and Lukichev (1988). These boundary conditions take into account the finite transparency (resistance) of the interfaces - see Eq. (32).

The most important pair-breaking mechanism in the ferromagnet is the exchange field \( h \). However a disorder in the lattice of magnetic atoms creates centers of magnetic scattering. In ferromagnetic alloys, used as the F layer in S/F/S Josephson junctions, the role of the magnetic scattering may be quite important. Note that even in the case of a perfect ordering of the magnetic atoms, the spin-waves will generate magnetic scattering. The natural choice of the spin-quantization axis used implicitly above is along the direction of the exchange field. The magnetic scattering and spin-orbit scattering mix up the up and down spin states. Therefore to describe this situation it is needed to introduce two normal Green’s functions \( G_1 \sim \langle \psi_\uparrow \psi_\uparrow^\dagger \rangle, \ G_2 \sim \langle \psi_\downarrow \psi_\downarrow^\dagger \rangle \) and two anomalous ones \( F_1 \sim \langle \psi_\uparrow \psi_\downarrow \rangle, \ F_2 \sim \langle \psi_\downarrow \psi_\uparrow \rangle \).
The microscopical Green’s function theory of superconductors with magnetic impurities and spin-orbit scattering was proposed by Abrikosov and Gorkov (1960, 1962). The generalization of the Usadel equations (98) to this case gives

\[-\frac{D}{2} \left[ G_1 \frac{\partial^2}{\partial x^2} F_1 - F_1 \frac{\partial^2}{\partial x^2} G_1 \right] + \left( \omega + ih + \left( \frac{1}{\tau_z} + \frac{2}{\tau_x} \right) G_1 \right) F_1 + G_1 (F_2 - F_1) \left( \frac{1}{\tau_x} - \frac{1}{\tau_{so}} \right) + F_1 (G_2 - G_1) \left( \frac{1}{\tau_x} + \frac{1}{\tau_{so}} \right) = \Delta(x) G_1,\]

and the similar equation for \( F_2 \) with the indices substitution \( 1 \leftrightarrow 2 \). Here \( \tau_{so}^{-1} \) is the spin-orbit scattering rate, while the magnetic scattering rates are \( \tau_z^{-1} = \tau_2^{-1} \langle S_x^2 \rangle / S^2 \) and \( \tau_x^{-1} = \tau_2^{-1} \langle S_y^2 \rangle / S^2 \). The rate \( \tau_2^{-1} \) describes the intensity of the magnetic scattering via exchange interaction and we follow the notation of the paper of Fulde and Maki (1966). In the spatially uniform case the equations (59) are equivalent to those of the Abrikosov-Gorkov theory (1960, 1962) (see also Fulde and Maki, 1966). Demler et al. (1997) analyzed the influence of the spin-orbit scattering on the critical temperature of the S/F multilayers. The equations (Demler et al., 1997) corresponds to the limit \( \Delta \to 0, G_{1,2} = 1 \) in (59).

The ferromagnets used as F layers in S/F heterostructures reveal strong uniaxial anisotropy. Then the magnetic scattering in the plane \( (xy) \) perpendicular to the anisotropy axis is negligible. Moreover due to the relatively small atomic numbers of the F layers atoms the spin-orbit scattering is expected to be weak. In such case there is no spin mixing scattering anymore and the Usadel equations retrieve the initial form (98) with the substitution of the Matsubara frequencies by \( \omega \to \omega + G/\tau_\omega \), where \( \tau_\omega^{-1} = \tau_z^{-1} = \tau_x^{-1} \langle S_y^2 \rangle / S^2 \) may be considered as a phenomenological parameter describing the intensity of the magnetic scattering (Buzdin, 1985).

The linearized Usadel equation in the ferromagnet reads

\[ \left( |\omega| + ih \text{sgn} (\omega) + \frac{1}{\tau_s} \right) F_j - \frac{D_j}{2} \frac{\partial^2 F_j}{\partial x^2} = 0. \] (99)

If \( \tau_s T_c \ll 1 \), we may neglect \( |\omega| \) in Eq. (99) and the exponentially decaying solution has the form

\[ F_j (x, \omega > 0) = A \exp (-x(k_1 + ik_2)), \] (100)

with \( k_1 = \frac{1}{\sqrt{\eta}} \sqrt{1 + \alpha^2} + \alpha \) and \( k_2 = \frac{1}{\sqrt{\eta}} \sqrt{1 + \alpha^2} - \alpha \), where \( \alpha = 1/(\tau_s h) \). In the absence of magnetic scattering, the decaying and oscillating wave vectors are the same \( k_1 = k_2 \). The magnetic scattering decreases the characteristic decaying length and increases the period of oscillations. In practice, it means that the decrease of the critical current of S/F/S junction with the increase of \( d_f \) will be more strong. Note that the spin-orbit scattering (in contrast to the magnetic scattering) decreases the pair-breaking effect of the exchange field (Demler et al., 1997) and both scattering mechanisms decrease the amplitude of the oscillations of the Cooper pair wave function. In some sense the spin-orbit scattering is more harmful for these oscillations because they completely disappear at \( \tau_{so}^{-1} > h \). The observation on experiment of the oscillatory behavior of \( T_c \) in S/F multilayers is an indirect proof of the weakness of the spin-orbit scattering.

The expression for \( I_c(2df) \) dependence (54) may be generalized to take into account the magnetic scattering

\[ I_c R_n = 64 \pi T / e \text{Re} \left[ \sum_{\omega > 0} \frac{2q_\omega y \exp(-2q_\omega y) \Phi_\omega}{\sqrt{(1 - q_\omega^2) \Phi_\omega + 1 + 1}} \right], \] (101)

where the functions

\[ \Phi_\omega = \frac{\Delta^2}{(\Omega + \omega)^2}, q_\omega = \sqrt{2i + 2\alpha + 2\omega/h}, \eta_\omega^2 = \frac{\alpha}{\alpha + i + \omega/h}. \] (102)

Near \( T_c \) and in the limit \( h >> T_c \) and \( 2dfk_2 >> 1 \) it possible to obtain the following simple analytical expression for the critical current

37
\[ I_c = \frac{\pi S \sigma_f \Delta^2 k_1}{2 e T_c} \left[ \cos(2d_f k_2) + \frac{k_2}{k_1} \sin(2d_f k_2) \right] \exp(-2d_f k_1). \]  \hfill (103)

We see that due to the magnetic scattering the decaying length of the critical current \( \xi_{f1} = 1/k_1 \) may be substantially smaller than the oscillating length \( \xi_{f2} = 1/k_2 \).

As it has been noted above, the condition of the applicability of the Usadel equations, \( h \tau \ll 1 \), is rather restrictive in ferromagnets due to the large value of the exchange field. Therefore, it is of interest to retain in the Usadel equations the first correction in the parameter \( h \tau \). The first attempts to calculate this correction were made by Tagirov (1998) and Proshin and Khusainov (1998) and resulted in the renormalization of the diffusion constant of the F layer \( D_f \rightarrow D_f(1 - 2i h \tau \text{sign}(\omega)) \). Later on, the similar renormalization has been proposed by Bergeret et al. (2001c) and Baladić and Buzdin (2001). The critical analysis of this renormalization by Fominov et al. (2002) (see also Fominov et al. 2003b and Khusainov and Proshin, 2003) revealed the inaccuracy of this renormalization, but did not provide the answer. The careful derivation of the Usadel equation for an F layer retaining the linear correction over the parameter \( h \tau \) was made by Buzdin and Baladić (2003) and simply resulted in a somewhat different renormalization of the diffusion constant \( D_f \rightarrow D_f(1 - 0.4i h \tau \text{sign}(\omega)) \). The coefficient in the parameter \( h \tau \) occurs to be rather small which provides more confidence in the description of F layers in the framework of the Usadel equations. Note that this renormalization of the diffusion constant increases the decaying characteristic length and decreases the period of oscillations, which is opposite to the influence of the magnetic scattering.

The Usadel equations give the description of Green’s functions only on average. Zyuzin et al. (2003) pointed out that due to the mesoscopic fluctuations, the decay of the anomalous Green’s function \( F_f \) at distances much larger than \( \xi_f \) is not exponential. In result, the Josephson effect in S/F/S systems may be observed even with a thick ferromagnetic layer.

The Eilenberger and Usadel equations adequately describe the weak ferromagnets, where \( h << E_F \) and the spin-up \( v_{F\uparrow} \) and spin-down \( v_{F\downarrow} \) Fermi velocities are the same. When the parameters of the electrons spectra of the spin-up and spin-down bands are very different, the quasiclassical approach fails. However, if the characteristics of the spin bands are similar, the Eilenberger and Usadel equations are still applicable. Performing the derivation of the Eilenberger equation in such case, it may be demonstrated that the Fermi velocity \( v_F \) in Eq. (95) must be substituted by \((v_{F\uparrow} + v_{F\downarrow})/2 \) and the scattering rate \( 1/\tau \) by \((1/\tau_\uparrow + 1/\tau_\downarrow) / 2 \). In consequence, the diffusion coefficient \( D_f \) in the Usadel equation becomes \((1/6)(v_{F\uparrow} + v_{F\downarrow})^2/(1/\tau_\uparrow + 1/\tau_\downarrow) \). Let us stress that such renormalization is justified only for close values of \( v_{F\uparrow} \) and \( v_{F\downarrow} \) (as well as \( \tau_\uparrow \) and \( \tau_\downarrow \)). Otherwise the Bogoliubov-de Gennes equations must be used for the description of the proximity effect in the strong ferromagnets.

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**TABLE I. Characteristic length scales of S/F proximity effect.**

| Thermal diffusion length $L_T$ | $\sqrt{D/T}$ in pure limit |
|---------------------------------|-----------------------------|
| Superconducting coherence length $\xi_s$ | $\frac{\pi T}{T_F}$ in pure limit |
| Superconducting correlations decaying length $\xi_{1f}$ in a ferromagnet | $\frac{\pi T}{T_F}$ in pure limit |
| Superconducting correlations oscillating length $\xi_{2f}$ in a ferromagnet | $\frac{\pi T}{T_F}$ in pure limit |

Figure captions

FIG. 1. The $(T, H)$ phase diagram for 3D superconductor. At temperature below $T^* = 0.56T_c$ the second order transition occurs from the normal to the non-uniform superconducting FFLO phase. The dashed line corresponds to the first order transition into the uniform superconducting state, and the dotted line presents the second order transition into the uniform superconducting state.

FIG. 2. Energy band of 1D superconductor near the Fermi energy. Due to the Zeeman splitting the energy of the electrons with spin orientation along the magnetic field ($\uparrow$) decreases - dotted line, while the energy of the electrons with the opposite spin orientation ($\downarrow$) increases - dotted line. The splitting of the Fermi momenta is $\pm \delta k_F$, where $\delta k_F = \mu_B H/v_F$. The Cooper pair comprises one electron with the spin ($\uparrow$) and momentum $k_F + \delta k_F$, and another electron with the spin ($\downarrow$) and momentum $-k_F + \delta k_F$. The resulting momentum of the Cooper pair is non-zero: $k_F + \delta k_F + (-k_F + \delta k_F) = 2\delta k_F \neq 0$.

FIG. 3. Schematic behavior of the superconducting order parameter near the interface (a) superconductor-normal metal, and (b) superconductor-ferromagnet. The continuity of the order parameter at the interface implies the absence of the potential barrier. In general case at the interface the jump of the superconducting order parameter occurs.

FIG. 4. Measurements of the differential conductance by Kontos et al. (2001) for two Al/Al$_2$O$_3$/PdNi/Nb junctions with two different thicknesses (50 Å and 75 Å) of the ferromagnetic PdNi layer. A 1500-Å-thick aluminium layer was evaporated on SiO and then quickly oxidized to produce a Al$_2$O$_3$ tunnel barrier. Tunnel junction areas were defined by evaporating 500 Å of SiO through masks. A PdNi thin layer was deposited and then backed by a Nb layer.

FIG. 5. Experimental data of Jiang et al. (1995) on the oscillation of the critical temperature of Nb/Gd multilayers vs thickness of Gd layer $d_G$ for two different thicknesses of Nb layers: (a) $d_{Nb} = 600$ Å and (b) $d_{Nb} = 500$ Å. Dashed line in (a) is a fit by the theory of Radovic et al. (1991).

FIG. 6. S/F multilayer. The axe $x$ is chosen perpendicular to the planes of S and F layers with the thicknesses $2d_s$ and $2d_f$ respectively. (a) The curve $\Psi(x)$ represents schematically the behavior of the Cooper pair wave function in "0"- phase. Due to the symmetry reasons the derivative of $\Psi$ (and $F$) is zero at the centers of S and F layers. The
case of the "0"- phase is equivalent to the S/F bilayer with S and F layers thicknesses $d_s$ and $d_f$ respectively. (b) The Cooper pair wave function in "π"- phase vanishes at the centers of F layers and $\Psi(x)$ is antisymmetric toward the center of F layer.

FIG. 7. The dependence of the critical temperature on the thickness of F layer for "0"-phase (solid line) and "π"-phase (dotted line) in the case of the transparent S/F interface. Note that the highest transition temperature $T_c^*$ corresponds to the lowest point. The dimensionless thickness of F layer $2y = 2d_f/\xi_f$ and the first transition from "0"- to "π"- phase occurs at $2d_f = 2.36\xi_f$ The parameter $\tau_0 = \frac{2d_f^2\xi_f^2}{\sigma_d^2 \sigma_f}$. 

FIG. 8. The critical temperature of "0"- phase (solid line) and "π"- phase (dashed line) as a function of the dimensionless thickness of F layer $2y = 2d_f/\xi_f$ for different S/F interface barriers $\gamma = \gamma_B (\xi_n/\xi_f)$.

(a) The dimensionless pair-breaking parameter $\tilde{\gamma}_0 = 4\pi T_c \frac{2d_f^2\xi_f^2}{\sigma_d^2 \sigma_f} = 21$.

(b) The dimensionless pair-breaking parameter $\tilde{\gamma}_0 = 20.05$.

FIG. 9. Variation of the critical temperature of the Nb/Cu$_{0.43}$Ni$_{0.57}$ bilayer with the F layer thickness (Ryazanov et al. 2003). Theoretical fit (Fominov et al., 2002) gives the exchange field value $h \sim 130$ K and the interface transparency parameter $\gamma_B \sim 0.3$.

FIG. 10. Geometry of the S/F/S junction. The thickness of the ferromagnetic layers is $2d_f$ and the both S/F interfaces have the same transparencies, characterized by the coefficient $\gamma_B$.

FIG. 11. Critical current of the S/F/S Josephson junction near $T_c$ as a function of the dimensionless thickness of F layer $2y = 2d_f/\xi_f$. There are no barriers at the S/F interfaces ($\gamma_B = 0$). $R_n$ is the resistance of the junction and $V_0 = \frac{\pi^2 k_B^4}{2e^2}$. 

FIG. 12. Temperature dependences of the critical thickness $2d_f^*$ of F layer, corresponding to the crossover from "0"- to "π"- phase in the limit of very small boundary transparency for different values of the exchange field.

FIG. 13. Non-monotonous temperature dependences of the normalized critical current for low transparency limit: curve 1: $h/T_c = 10$ and $2d_f/\xi_f = 0.84$; curve 2: $h/T_c = 40$ and $2d_f/\xi_f = 0.5$; curve 3: $h/T_c = 100$ and $2d_f/\xi_f = 0.43$.

FIG. 14. Critical current $I_c$ as a function of temperature for Cu$_{0.48}$Ni$_{0.52}$ junctions with different F layers thicknesses $2d_f$. At the thickness of F layer of 27 nm the temperature mediated transition between "0"- and "π"- phases occurs. Adapted from (Ryazanov et al., 2001a).

FIG. 15. Critical current $I_c$ at $T = 4.2$ K of Cu$_{0.47}$Ni$_{0.53}$ junctions as a function of F layer thickness (Ryazanov et al., 2005). Two "0"- "π" transitions are revealed. The theoretical fit corresponds to the Eq. (101) in Appendix B, taking into account the presence of the magnetic scattering with parameters $\alpha = 1/(\tau_0 h) = 1.33$ and $\xi_f = 2.4$ nm. The inset shows the temperature mediated "0"- "π" transition for the F layer thickness 11 nm.

FIG. 16. The experimental points correspond to the measurements of the critical current, done by Kontos et al. (2002) vs the PdNi layer thickness. The theoretical curve is the fit of Buzdin and Baladie (2003). The fitting parameters are $\xi_f \sim 30$ Å and $\frac{\alpha}{\tau_0} \sim 110$ µV.

FIG. 17. Experiments of Guichard et al. (2003) on the diffraction pattern of SQUID with "0"- and "π"-junctions. There is no shift of the pattern between a "0 - 0" and "π - π" SQUIDs. The $\Phi_0/2$ shift is observed between a "0 - π" and "0 - 0" or "π - π" SQUIDs. The "0"- and "π"-junctions were obtained by varying the PdNi layer thickness.

FIG. 18. Earlier observation by Deutscher and Meunier (1969) of the spin-wave effect on In film between oxidized FeNi and Ni layers. The figure presents the resistive measurements of the critical temperature in zero field: dashed line, after application of 1 T field parallel to the ferromagnetic layers; solid line, after application of the -1 T field and subsequently +0.03 T field to return the magnetization of FeNi layer.

FIG. 19. Geometry of the F/S/F sandwich. The thickness of S layer is $2d_s$ and two F layers have identical thicknesses $d_f$.

FIG. 20. Influence of the S/F interface transparency (parameter $\tilde{\gamma}_0 = \gamma_B (\xi_n/\xi_f)$) on the $T_c^*$ vs $d_f$ dependence (Baladie and Buzdin, 2003). The thickness of F layer is normalized to the $\xi_f$. The dimensionless pair-breaking parameter $\tilde{\tau}_0 = 4\pi T_c \frac{2d_f^2\xi_f^2}{\sigma_d^2 \sigma_f}$ is chosen constant and equal to 4. The full line corresponds to the antiparallel case, and the dashed line to the parallel case. One can distinguish four characteristic types behavior: (a) weakly non-monotonous decay to a finite value of $T_c^*$, (b) reentrant behavior for the parallel orientation, and (c) and (d) monotonous decay.
to $T_c^*=0$ with (d) or without (c) switching to a first-order transition in the parallel case. In (d), the dotted line presents schematically the first order transition line.

FIG. 21. The calculate dependence of the superconducting transition temperature vs inverse reduced half-thickness $d^*/d_s$ of the superconducting layer for parallel and antiparallel alignments for the transparent interface ($\gamma_B = 0$) and thick ferromagnetic layer ($d_f >> \xi_f$). The effective length $d^* = (\sigma_f/\sigma_s) (D_s/4\pi T_c) (h/D_f)^{1/2}$.

FIG. 22. The $(T, h)$ - phase diagram of the atomic S/F multilayer in the limit of the small transfer integral $t << T_c$.

FIG. 23. S/F bilayer with domain structure in the ferromagnetic layer. The period $D$ of the domain structure ($D = 2\pi/Q$) is smaller than the superconducting coherence length $\xi_s$.

FIG. 24. The ferromagnetic film with perpendicular anisotropy on a superconducting substrate.
(a) $d_{Nb} = 600\text{Å}$

(b) $d_{Nb} = 500\text{Å}$
$4\tau_0(\Gamma^*_{c-T_c})/\pi$
$T = 4.2 \text{ K}$

$\mathbf{j}_c$, A/cm$^2$

0 state  \hspace{5cm} \pi state

$2d_F$, nm
A graph showing the relationship between $I_cR_n$ ($\mu V$) and $2d_f$ ($\text{Å}$). The data points are marked with error bars, and a curve fits the data points.
The diagram shows the behavior of SQUID loops at different temperatures (T) and fields (B). The loops exhibit periodic behavior with peaks at specific values of current (Ic) and magnetic field (B).

- **SQUID 0-0**
  - T=5.34 K
  - T=5.37 K

- **SQUID 0-π**
  - T=5.35 K
  - T=5.54 K

- **SQUID π-π**
  - T=5.2 K
  - T=5.3 K

The graphs illustrate the variation of current (Ic) and magnetic field (B) with different operational conditions, highlighting the quantum mechanical properties of SQUID devices.
