Semileptonic decays of the scalar tetraquark $Z_{bc,\bar{ud}}^0$

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We study semileptonic decays of the scalar tetraquark $Z_{bc,\bar{ud}}^0$ to final states $T_{bc,\bar{ud}}^-\mu^+\nu_\mu$ and $T_{bc,\bar{ud}}^-\mu^+\nu_\mu$, which run through the weak transitions $c \rightarrow se^+\nu_e$ and $c \rightarrow ms^+\nu_s$, respectively. To this end, we calculate the mass and coupling of the final-state scalar tetraquark $T_{bc,\bar{ud}}^-$ by means of the QCD two-point sum rule method: these spectroscopic parameters are used in our following investigations. In calculations we take into account the vacuum expectation values of the quark, gluon, and mixed operators up to dimension ten. We use also three-point sum rules to evaluate the weak form factors $G_i(q^2)$ ($i = 1, 2$) that describe these decays. The sum rule predictions for $G_i(q^2)$ are employed to construct fit functions $F_i(q^2)$, which allow us to extrapolate the form factors to the whole region of kinematically accessible $q^2$. These functions are required to get partial width of the semileptonic decays $\Gamma(Z_{bc}^0 \rightarrow T_{bc}^-\nu)$ and $\Gamma(Z_{bc}^0 \rightarrow T_{bc}^-\nu)$ by integrating corresponding differential rates. The obtained results for the total width $\Gamma_{tot} = (2.37 \pm 0.36) \times 10^{-11}$ MeV and mean lifetime $27.8^{+4.9}_{-3.7}$ ps of the tetraquark $Z_{bc,\bar{ud}}^0$ can be used in experimental investigations of this exotic state.

I. INTRODUCTION

Investigations of double-heavy tetraquarks composed of a heavy $QQ$ diquark ($Q$ is the heavy $c$ or $b$ quark) and a light antidiquark are among interesting topics in physics of exotic hadrons. The interest to such kind of quark configurations is connected with a possible stability of some of them against the strong and electromagnetic decays. The relevant problems were addressed already in the pioneering papers [1,2], in which a stability of the exotic four-quark mesons $QQ\bar{q}q$ and $Q\bar{Q}q\bar{q}$ was examined. It was found that the heavy $Q$ and light $q$ quarks with a large mass ratio $m_Q/m_q$ may form the stable tetraquarks $QQ\bar{q}q$. The similar conclusions were drawn in Ref. [4] as well, in accordance of which the isoscalar $J^P = 1^+$ tetraquark $T_{bc,\bar{ud}}$ lies below the two B-meson threshold and can decay only weakly.

All available theoretical tools of high energy physics were exploited to study properties of double-heavy exotic mesons; the chiral and dynamical quark models, the relativistic quark model and sum rules method were mobilized to calculate their parameters [5,13]. Interest to these mesons renewed after experimental observation by the LHCb Collaboration of the $\Xi^{++} = ccu$ baryon [14]. Its mass was used as an input information in a phenomenological model to estimate the mass of the axial-vector tetraquark $T_{bc,\bar{ud}}^- [15]$. The obtained prediction $m = (10389 \pm 12)$ MeV is 215 MeV below the $B^-\bar{B}^0$ threshold and 170 MeV below the threshold for decay $B^-\bar{B}^0\gamma$, which means that $T_{bc,\bar{ud}}^-$ is stable against the strong and electromagnetic decays and dissociates only weakly. The conclusion about the strong-interaction stability of the tetraquarks $T_{bc,\bar{ud}}^-$ and $T_{bc,\bar{ud}}^0$ was made in Ref. [16] on the basis of the relations derived from heavy-quark symmetry. The mass $m = 10482$ MeV of the axial-vector tetraquark $T_{bc,\bar{ud}}^0$ found there is 121 MeV below the open-bottom threshold.

In Ref. [17] we calculated the spectroscopic parameters of the axial-vector tetraquark $T_{bc,\bar{ud}}^- [15]$ and analyzed also its semileptonic decay to the scalar state $Z_{bc,\bar{ud}}^0$. Our result for its mass $m = (10035 \pm 260)$ MeV confirms once more that it is stable against the strong and electromagnetic decays. We evaluated the total width and mean lifetime of $T_{bc,\bar{ud}}$ using the semileptonic decay channels $T_{bc,\bar{ud}}^- \rightarrow Z_{bc,\bar{ud}}^0 l^\pm \nu_l$, where $l = e, \mu$ and $\tau$. The predictions $\Gamma = (7.17 \pm 1.23) \times 10^{-8}$ MeV and $\tau = 9.18^{+1.90}_{-1.34}$ fs provide information useful for experimental investigation of the double-heavy exotic mesons. Details of performed analysis and references to earlier and recent articles devoted to different aspects of the doubly and fully heavy tetraquarks can be found in Ref. [17].

We determined the mass and coupling of the scalar four-quark meson $Z_{bc,\bar{ud}}^0$ (hereafter $Z_{bc}^0$) as well [17], because these parameters were necessary to evaluate the width of the semileptonic decay $T_{bc,\bar{ud}}^- \rightarrow Z_{bc}^0 l\bar{\nu}_l$. For these purposes we employed the QCD sum rule approach and found $m_Z = (6660 \pm 150)$ MeV. This prediction is considerably below the threshold 7145 MeV for strong decays of $Z_{bc}^0$ to heavy mesons $B^-D^+$ and $B^-D_s^0$. The state $Z_{bc}^0$ cannot decay to a pair of heavy and light mesons as well; this fact differs it qualitatively from the open charm-bottom scalar tetraquarks $c\bar{d}\bar{b}\bar{s}$ and $c\bar{b}s\bar{d}$, which decay to $B_c \pi$ and $B_c \eta$ mesons [18], respectively. The thresholds for the electromagnetic decays $Z_{bc}^0 \rightarrow B^-D^+\gamma$ and $B^-D_s^0\gamma$ exceed 7600 MeV and are higher than the mass of $Z_{bc}^0$. In
other words, the tetraquark \( Z_{bc}^0 \) as the state \( T_{bc, \bar{u}\bar{d}}^- \) is the strong- and electromagnetic-interaction stable particle.

The scalar and axial-vector states \( bc \bar{c} \bar{d} \) were subjects of interesting theoretical investigations with, sometimes, controversial predictions. In fact, the analysis performed in Ref. \[13\] showed that \( Z_{bc}^0 \) resides 11 MeV below the threshold 7145 MeV for \( S \)-wave resonances for the masses of the scalar and axial-vector mesons. Computations of the ground-state \( QQ\bar{c}\bar{d} \) tetraquarks’ masses carried out in the context of the Bethe-Salpeter method led to similar conclusions \[10\]. The mass of \( Z_{bc}^0 \) found there (for some set of used parameters) equals to 6.93 GeV and is lower than the relevant strong threshold. On the contrary, for the masses of the scalar and axial-vector \( bc \bar{c} \bar{d} \) states the heavy-quark symmetry predicts 7229 MeV and 7272 MeV \[10\], which means that they can decay to ordinary mesons \( B^-D^+ / \overline{B^0}D^0 \) and \( B^+D^- \), respectively. The charged exotic scalar mesons \( Z_{bc, \bar{c} \bar{d}}^0 \) and \( Z_{bc, \bar{c} \bar{d}}^+ \) were explored by means of the QCD sum rule method as well \[20\]: the mass of these particles \( m = (7.14 \pm 0.10) \) GeV is higher than our prediction for \( m_Z \).

The recent lattice simulations prove the strong-interaction stability of the \( I(J^P) = 0(1^+) \) four-quark meson \( Z_{ud, \bar{c} \bar{d}}^0 \) with the mass in the range 15 to 61 MeV below \( \overline{B}B^* \) threshold \[21\]. But, because of theoretical uncertainties the authors could not determine whether this tetraquark would decay electromagnetically to \( \overline{B}B^* \) or can transform only weakly. Another confirmation of the \( bc \bar{c} \bar{d} \) tetraquarks stability came from Ref. \[22\]: there it was demonstrated that both the \( J^P = 0^+ \) and \( 1^+ \) isoscalar tetraquarks \( bc \bar{c} \bar{d} \) are stable against the strong decays. The isoscalar \( J^P = 0^+ \) state is also electromagnetic-interaction stable, whereas \( J^P = 1^+ \) may undergo the electromagnetic decay to \( \overline{B}D^\gamma \).

In light all of these theoretical predictions, it becomes evident that decays of the tetraquark \( Z_{bc}^0 \) are sources of a valuable information about this exotic meson. In the present work we explore the semileptonic decays of the tetraquark \( Z_{bc}^0 \), which are important for some reasons. First of all, \( Z_{bc}^0 \) may be produced copiously at the LHC \[23\], hence it is necessary to fix processes, where it has to be searched for. The second reason is exploration of the tetraquark \( T_{bc, \bar{u}\bar{d}}^- \) itself, and decay channels appropriate for its discovery. As usual, all states classified till now as candidates to tetraquarks were seen through their decays to conventional mesons. If a tetraquark is stable against strong and electromagnetic decays, then it should be observed due to products of its semileptonic decays. In the case under discussion at the first stage \( T_{bc, \bar{u}\bar{d}}^- \) decays to \( Z_{bc}^0 \) and \( \bar{l}\nu_l \). But, because the scalar tetraquark \( Z_{bc}^0 \) does not transform directly to conventional mesons, one needs to consider its semileptonic decays.

The weak decays of \( Z_{bc}^0 \) can proceed through different channels. Its dominant decay modes are \( Z_{bc}^0 \rightarrow T_{bc, \bar{u}\bar{d}}^- e^+\nu_e \) and \( Z_{bc}^0 \rightarrow T_{bc, \bar{u}\bar{d}}^- \mu^+\nu_\mu \), which run due to transitions \( e \rightarrow W^+s \) and \( W^+ \rightarrow \bar{l}\nu_l \). The channels triggered by the processes \( c \rightarrow W^+d \) and \( W^+ \rightarrow \bar{l}\nu_l \) lead to creation of the tetraquark \( T_{bc, \bar{u}\bar{d}}^- \) and are suppressed relative to the first modes by a factor \( |V_{cd}|^2 / |V_{cb}|^2 \approx 0.05 \). The similar arguments can be applied to other semileptonic decays of \( Z_{bc}^0 \) generated by a chain of transformations \( b \rightarrow W^- c \rightarrow c(l\nu) \) and \( b \rightarrow W^- u \rightarrow u(l\nu) \), respectively. In fact, the Cabibbo-Kobayashi-Maskawa (CKM) matrix element \( |V_{bc}| \), which is small numerically, and the ratio \( |V_{ub}/V_{cb}|^2 \approx 0.01 \) demonstrates a subdominant nature of the decays \( b \rightarrow c(l\nu) \) and \( b \rightarrow u(l\nu) \). Therefore, we saturate the total width of \( Z_{bc}^0 \) by two dominant decay modes.

The exotic mesons \( T_{bc, \bar{u}\bar{d}}^- \) can bear different quantum numbers. We treat \( T_{bc, \bar{u}\bar{d}}^- \) as a scalar particle, and in what follows denote it by \( T \). To calculate the width of aforementioned decays, one needs the mass and coupling of the tetraquark \( T \); they enter as parameters to the sum rules for the weak form factors that determine width of the decays. The spectroscopic parameters of this tetraquark can be extracted from the two-point correlation function by means of the sum rule approach, which is one of the powerful nonperturbative tools in QCD \[24, 25\]. It can be applied to compute spectroscopic parameters and decay width not only of the conventional hadrons but also the exotic states [for the recent review, see Ref. \[26\].

In the present work the mass and coupling of \( T \) are calculated by taking into account vacuum expectation values of various quark, gluon, and mixed local operators up to dimension ten. The weak form factors \( G_i(q^2) \), \( (i = 1, 2) \) are extracted from the three-point sum rules, which allow us to find numerical values of \( G_i(q^2) \) at momentum transfer \( q^2 \) accessible for the QCD sum rule computations. Later we fit \( G_i(q^2) \) by functions \( F_i(q^2) \), and extrapolate them to a whole domain of physical \( q^2 \). The fit functions are used to integrate the differential decay rates and obtain the width of the semileptonic decays \( \Gamma \left( Z_{bc}^0 \rightarrow T e^+\nu_e \right) \) and \( \Gamma \left( Z_{bc}^0 \rightarrow T \mu^+\nu_\mu \right) \).

This article is structured in the following form: In Sec. \[11\] we derive the QCD two-point sum rules for the mass and coupling of the tetraquark \( T \), and find their numerical values. In Sec. \[12\] the QCD three-point correlation functions are utilized to get sum rules for the form factors \( G_i(q^2) \). Here we carry out also numerical analysis of derived sum rules and determine the fit functions, and evaluate the width of the semileptonic decays of concern. In Sec. \[14\] we analyze the obtained results and make our conclusions.

## II. SPECTROSCOPIC PARAMETERS OF THE TETRAQUARK \( T_{bc, \bar{u}\bar{d}}^- \)

The spectroscopic parameters of the tetraquark \( T \) are important to calculate the width of the exotic \( Z_{bc}^0 \) meson’s semileptonic decays. The \( T \) state contains four quarks \( b, s, u, \) and \( d \) of different flavors and has the heavy-light structure. In other words, the \( b \)-quark and \( s \)-quark,
which is considerably heavier than \( q = u, d \), groups to form the heavy diquark, whereas the antidiquark is built of light \( u \) and \( d \) quarks. This is the main difference of \( T \) and the famous resonance \( X(5568) \); the latter has the same quark content, but \( b \) and \( s \) quarks are distributed between a diquark and an antidiquark [27]. The scalar tetraquark \( T \) can be composed using diquarks of a different type. The ground-state scalar particle \( T \) should be composed of the scalar diquark \( e^{abc}[b^2_0 C\gamma_5 s_c] \) in the color antitriplet and flavor antisymmetric state and the antidiquark \( e^{ade}[\bar{d}^\gamma_5 C\bar{d}_a] \) in the color triplet state. The reason is that they are most attractive diquark configurations, and exotic mesons composed of them should be lighter and more stable than four-quark mesons made of other diquarks [28]. Therefore, we assume that \( T \) has such favorable structure, and accordingly choose the interpolating current \( J(x) \)

\[
J(x) = \epsilon \bar{c}[b^T_0(x)C\gamma_5 s_c(x)][\bar{d}(x)\gamma_5 C\bar{d}_e(x)],
\]

where \( \epsilon = e^{abc}e^{ade} \). In this expression \( a, b, c, d \) and \( e \) are color indices and \( C \) is the charge-conjugation operator.

The mass and coupling of the tetraquark \( T \) can be obtained from the QCD two-point sum rules. To derive the sum rules for the mass \( m_T \) and coupling \( f_T \) of \( T \), we analyze the correlation function

\[
\Pi(p) = i \int d^4xe^{ipx}\langle 0 | T\{J(x)J^\dagger(0)\}|0 \rangle. \tag{2}
\]

To find the phenomenological side of the sum rule \( \Pi^{\text{phys}}(p) \), we treat \( T \) as a ground-state particle and use the “ground-state + continuum” scheme. Then \( \Pi^{\text{phys}}(p) \) contains a contribution of the ground-state particle and contributions arising from higher resonances and continuum states

\[
\Pi^{\text{phys}}(p) = \frac{\langle 0 | J[T(p)](T(p)) J^\dagger(0) \rangle}{m_T^2 - p^2} + \ldots, \tag{3}
\]

which are denoted in Eq. (3) by dots. This expression for the phenomenological side is obtained by inserting into the correlation function \( \Pi(p) \) a full set of relevant states and carrying out integration in Eq. (2) over \( x \).

Computation of \( \Pi^{\text{phys}}(p) \) can be continued by introducing the matrix element of the scalar tetraquark

\[
\langle 0 | J[T(p)] \rangle = f_T m_T. \tag{4}
\]

After simple manipulations we get

\[
\Pi^{\text{phys}}(p) = \frac{f_T^2 m_T^2}{m_T^2 - p^2} + \ldots \tag{5}
\]

At the next step one should choose in \( \Pi^{\text{phys}}(p) \) some Lorentz structure and fix the corresponding invariant amplitude. The correlation function \( \Pi^{\text{phys}}(p) \) contains only the trivial structure \( \sim I \), therefore the amplitude \( \Pi^{\text{phys}}(p^2) \) is given by the function from Eq. (5).

We need also to determine \( \Pi(p) \) by employing the perturbative QCD and express it in terms of the quark propagators. For these purposes, we utilize the explicit expression of the interpolating current \( J(x) \) and calculate \( \Pi(p) \) by contracting in Eq. (2) the relevant heavy and light quark fields. As a result, we get

\[
\Pi^{\text{OPE}}(p) = i \int d^4xe^{ipx}\epsilon\epsilon'\epsilon''\epsilon'''\text{Tr}\left\{\gamma_5 S_{bb'}(x)\gamma_5 S_{cc'}(x)\right\}
\]

\[
\times \text{Tr}\left\{\gamma_5 S_{dd'}(-x)\gamma_5 S_{uu'}(-x)\right\}, \tag{6}
\]

where \( S_b(x) \) and \( S_{uu(d,s)}(x) \) are the heavy \( b \)- and light \( u(d,s) \)-quark propagators, respectively. Here we also use the shorthand notation

\[
\tilde{S}(x) = CST(x)C. \tag{7}
\]

The explicit expressions of the heavy and light quark propagators can be found in Ref. [24], for example. They contain the perturbative and nonperturbative components: the latter depends on vacuum expectation values of various quark, gluon, and mixed operators which generate dependence of \( \Pi^{\text{OPE}}(p) \) on the nonperturbative quantities.

The sum rule can be extracted by equating the amplitudes \( \Pi^{\text{phys}}(p^2) \) and \( \Pi^{\text{OPE}}(p^2) \), which is the first stage of the analysis. Afterwards, we apply the Borel transformation to both sides of this equality, which is required to suppress contributions of higher resonances and continuum states. Next, we carry out the continuum subtraction by invoking the assumption on the quark-hadron duality. The obtained equality can be used to derive sum rules for \( m_T \) and \( f_T \), but there is a necessity to find the second expression. As usual, it is obtained from the first equality by applying the operator \( d/d(-1/M^2) \). We also follow this recipe and find

\[
m_T^2 = \frac{\int_{-M^2}^{s_0} ds \rho_{\text{OPE}}(s)e^{-s/M^2}}{\int_{-M^2}^{s_0} ds \rho_{\text{OPE}}(s)e^{-s/M^2}}, \tag{8}
\]

and

\[
f_T^2 = \frac{1}{m_T^2} \int_{-M^2}^{s_0} ds \rho_{\text{OPE}}(s)e^{(m_T^2 - s)/M^2}. \tag{9}
\]

where \( \mathcal{M} = m_b + m_s \). In Eqs. (8) and (9) \( \rho_{\text{OPE}}(s) \) is the two-point spectral density, which is proportional to the imaginary part of the correlation function \( \Pi^{\text{OPE}}(p) \). It is seen also that the obtained sum rules have acquired a dependence on the auxiliary parameters \( M^2 \) and \( s_0 \). The first of them is the Borel parameter introduced during the corresponding transformation. The \( s_0 \) is the continuum threshold parameter that separates the ground-state and continuum contributions to \( \Pi^{\text{OPE}}(p^2) \) from one another.

Apart from \( M^2 \) and \( s_0 \), which are specific for each considering problem, Eqs. (8) and (9) contain vacuum
aries of the region within of which the Borel parameter \( s \) and the continuum threshold \( s_0 \) for which we use the extracted invariant amplitude \( \Pi^{\text{OPE}} \) where \( \Pi(M^2, s) \) is the Borel-transformed and subtracted invariant amplitude \( \Pi^{\text{OPE}}(p^2) \), at \( M^2 = 4.8 \text{ GeV}^2 \) is 0.18, whereas at \( M^2 = 3.4 \text{ GeV}^2 \) it amounts to 0.63. These two values of \( M^2 \) determine the boundaries of the region within of which the Borel parameter can be varied. The lower limit of \( M^2 \) should meet also the very important constraint: the minimum of \( M^2 \) has to ensure the convergence of the operator product expansion (OPE). This restriction is quantified by the ratio

\[
R(M^2) = \frac{\Pi^{\text{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)}.
\]  

Here \( \Pi^{\text{DimN}}(M^2, s_0) \) denotes a contribution to the correlation function of the last term (or a sum of last few terms) in OPE. Numerical analysis shows that for \( \text{DimN} = \text{Dim}(8 + 9 + 10) \) this ratio is \( R(3.4 \text{ GeV}^2) = 0.013 \), which guarantees the convergence of the sum rules. Additionally, at minimal value of the Borel parameter the perturbative term gives 62% of the total result exceeding considerably the nonperturbative contributions.

Because \( M^2 \) and \( s_0 \) are the auxiliary parameters, the mass \( m_T \) and coupling \( f_T \) should not depend on them. But in real calculations there is a residual dependence of \( m_T \) and \( f_T \) on these parameters. Therefore, the choice of \( M^2 \) and \( s_0 \) should minimize these non-physical effects. The working windows for the parameters \( M^2 \) and \( s_0 \) given by Eq. \( (11) \) satisfy these conditions as well. To visualize effects of \( M^2 \) and \( s_0 \) on the mass \( m_T \) and coupling \( f_T \) we depict them in Figs. 1 and 2 as functions of these parameters. As is seen both \( m_T \) and \( f_T \) depend on \( M^2 \) and \( s_0 \), which is a main source of the theoretical uncertainties inherent to the sum rule computations. For the mass \( m_T \) these uncertainties are small \( \pm 3\% \), because the relevant sum rule is the ratio of the integrals of the functions \( s_0^{\text{OPE}}(s) \) and \( \rho^{\text{OPE}}(s) \) which smooths these effects, but even in the case of the coupling \( f_T \) they do not exceed \( \pm 24\% \) part of the central value.

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Our calculations for the spectroscopic parameters of the tetraquark \( T \) lead to the following results:

\[
\begin{align*}
  m_T &= (5380 \pm 170) \text{ MeV}, \\
  f_T &= (2.1 \pm 0.5) \times 10^{-3} \text{ GeV}^4.
\end{align*}
\]  

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FIG. 1: The mass of the tetraquark \( T \) as a function of the Borel parameter \( M^2 \) at fixed \( s_0 \) (left panel) and as a function of the continuum threshold \( s_0 \) at fixed \( M^2 \) (right panel).
The mass of the tetraquarks $T$ allows us to see whether this four-quark meson is strong-interaction stable or not. As we have emphasized above, $T$ contains the same quark species like the resonance $X(5568)$, but differs from it by an internal organization. The resonance $X(5568)$ with the content $sbd$ was originally studied in our work [27]. It is a scalar particle, but has the heavy diquark-antidiquark structure. The mass of the resonance $X(5568)$ evaluated there

$$m_X = (5584 \pm 137) \text{ MeV} \quad \text{(15)}$$

is higher than the mass of the tetraquark $T$; structures with a heavy diquark and a light antidiquark seem are more compact than ones composed of a pair of heavy diquark and antidiquark. The resonance $X(5568)$ is unstable against the strong interactions and decays to the conventional mesons $B^{0}_{s} \pi^{+}$. It is clear that $T$ cannot decay to such final states, but its quark content and quantum numbers does not forbid $S$-wave decays to $B^{0}_{s}K^{-}/K^{0}B^{-}$-mesons, thresholds of which $5774/5777$ MeV however, are above the mass $m_T$. Thresholds for $P$-wave decays of the scalar tetraquark $bsbd$ are higher than $m_T$ as well. The possible electromagnetic decay $T \rightarrow B^{-}K_{1}\gamma$ may be realized only if $m_T \geq 6552$ MeV, which is not the case. Therefore, transformation of the tetraquark $T$ to ordinary mesons runs only due to its semileptonic decays.

III. SEMILEPTONIC DECAYS $Z_{bc}^{0} \rightarrow T e^{+}\nu_{e}$ AND $Z_{bc}^{0} \rightarrow T \mu^{+}\nu_{\mu}$

In this section we explore the semileptonic decays $Z_{bc}^{0} \rightarrow T e^{+}\nu_{e}$, and $Z_{bc}^{0} \rightarrow T \mu^{+}\nu_{\mu}$ of the scalar four-quark meson $Z_{bc}^{0}$. The spectroscopic parameters of $Z_{bc}^{0}$ from Ref. [17], as well as the mass and coupling of the final-state tetraquark $T$, obtained in the previous section provide necessary information to calculate the differential rate and width of these decays.

The decay $Z_{bc}^{0} \rightarrow T \nu \bar{\nu}$ runs through the sequence of transformations $c \rightarrow W^{+}s$ and $W^{+} \rightarrow \nu \bar{\nu}$, and, due to the mass of the state $Z_{bc}^{0}$, processes with $l = e$ and $\mu$ are kinematically allowed ones. At the tree level the transition $c \rightarrow s$ is described by the effective Hamiltonian

$$\mathcal{H}^{\text{eff}} = \frac{G_{F}}{\sqrt{2}} V_{cs} \bar{s} \gamma_{\mu}(1 - \gamma_{5}) c \bar{l} \gamma_{\mu}(1 - \gamma_{5}) l, \quad \text{(16)}$$

where $G_{F}$ is the Fermi coupling constant and $V_{cs}$ is the CKM matrix element. Sandwiching $\mathcal{H}^{\text{eff}}$ between the initial and final tetraquarks, and factoring out the lepton fields we get the matrix element of the current

$$J_{\mu}^{\text{eff}} = \bar{s} \gamma_{\mu}(1 - \gamma_{5}) c. \quad \text{(17)}$$

In terms of the weak form factors $G_{i}(q^{2})$ this matrix element has the form

$$\langle T(p')|J_{\mu}^{\text{eff}}|Z(p)\rangle = G_{1}(q^{2}) P_{\mu} + G_{2}(q^{2}) q_{\mu}, \quad \text{(18)}$$

where $p$ and $p'$ are the momenta of the tetraquarks $Z_{bc}^{0}$ and $T$, respectively. In Eq. (18) the form factors $G_{1}(q^{2})$ and $G_{2}(q^{2})$ parameterize the long-distance dynamics of the weak transition. Here we also use $p_{\mu} = p'_{\mu} + p_{\mu}$ and $q_{\mu} = p_{\mu} - p'_{\mu}$. The $q_{\mu}$ is the momentum transferred to the leptons, and evidently $q^{2}$ changes within the limits

$$m_{l}^{2} \leq q^{2} \leq (m_{Z} - m_{T})^{2}, \quad \text{where } m_{l} \text{ is the mass of a lepton } l.$$

To derive the sum rules for the form factors $G_{i}(q^{2})$, $i = 1, 2$ we begin from the three-point correlation function

$$\Pi_{\mu}(p, p') = i^{2} \int d^{4}x d^{4}y e^{i(p'y - px)} \times \langle 0|T\{J(y)J_{\mu}^{\text{eff}}(0)J^{\dagger}(x)\}|0\rangle, \quad \text{(19)}$$

where $J(y)$ and $J^{\dagger}(x)$ are the interpolating currents for the states $T$ and $Z_{bc}^{0}$, respectively. The current $J(y)$ has
be defined above by Eq. \( \text{[17]} \): for \( J^Z(x) \) we use the expression \( \text{[18]} \)

\[
J^Z(x) = b^T(x) C \gamma_5 c_b(x) \left[ \pi_0(x) \gamma_5 C \pi_0^T(x) - \pi_b(x) \gamma_5 C \pi_b^T(x) \right].
\]

(20)

The current \( J^Z(x) \) is composed of the \( S \)-wave diquark fields, has the antisymmetric color structure \( [3]_c \otimes [3]_c \otimes [\pi] \otimes [\pi] \) and describes the ground-state tetraquark \( Z_{bc}^0 \).

As usual, we express the correlation function \( \Pi_\mu(p, p') \) in terms of the spectroscopic parameters of the involved particles, and find the physical side of the sum rule \( \Pi_\mu^{\text{phys}}(p, p') \). The function \( \Pi_\mu^{\text{phys}}(p, p') \) can be easily written down as

\[
\Pi_\mu^{\text{phys}}(p, p') = \frac{\langle 0| T(p') (T(p') J_\mu^0 | Z(p) \rangle}{(p^2 - m_Z^2)(p'^2 - m_{Z'}^2)}
\times \langle Z(p) | J^Z | 0 \rangle + \ldots ,
\]

(21)

where we take explicitly into account contribution only of the ground-state particles, and denote by dots effects of the excited and continuum states.

The phenomenological side of the sum rules can be further simplified by rewriting the relevant matrix elements in terms of the tetraquark’s parameters, and employing for \( \langle T(p') | J_\mu^0 | Z(p) \rangle \) its expression through the weak transition form factors \( G_i(q^2) \). To this end, we use Eq. \( \text{[11]} \) and the matrix element of the state \( Z_{bc} \) defined by

\[
\langle Z(p) | J^Z | 0 \rangle = f_Z m_Z.
\]

Then it is not difficult to find that

\[
\Pi_\mu^{\text{phys}}(p, p') = \frac{f_T m_T f_Z m_Z}{(p^2 - m_Z^2)(p'^2 - m_{Z'}^2)}
\times \left[ G_1(q^2) P_\mu + G_2(q^2) q_\mu \right].
\]

(23)

We determine also \( \Pi_\mu(p, p') \) by employing the interpolating currents and quark propagators, which lead to its expression in terms of quark, gluon, and mixed vacuum condensates. In terms of the quark-gluon degrees of freedom \( \Pi_\mu(p, p') \) takes the form

\[
\Pi_\mu^{OPE}(p, p') = i^2 \int d^4x d^4y e^{i(p' y - px)} \bar{c} T \left[ \gamma_\mu (1 - \gamma_5)
\times \slashed{u} \left( -x \gamma_5 \slashed{b} \right) \gamma_5 \slashed{a} (y - x) \gamma_5 \slashed{s} (y) \right] \left\{ \Tr \left[ \gamma_5 \slashed{a'} \gamma_5 \slashed{c} (x - y) \right]
\times \gamma_5 \slashed{u'} \slashed{d} (x - y) - \Tr \left[ \gamma_5 \slashed{d'} \gamma_5 \slashed{c} (x - y) \right] \gamma_5 \slashed{s'} \slashed{a} (x - y) \right\},
\]

(24)

where \( a', b' \) and \( i \) are the color indices of the currents \( J^Z(x) \) and \( J_\mu^0 \), respectively.

We extract the sum rules for the form factors \( G_i(q^2) \) by equating the invariant amplitudes corresponding to the same Lorentz structures in \( \Pi_\mu^{\text{phys}}(p, p') \) and \( \Pi_\mu^{OPE}(p, p') \). After that, we carry out the double Borel transformation over the variables \( p^2 \) and \( p'^2 \) necessary to suppress contributions of the higher excited and continuum states, and finally carry out the continuum subtraction. These manipulations yield the sum rules

\[
G_i(M^2, s_0, q^2) = \frac{1}{f_T m_T f_Z m_Z} \int_{(m_0 + m_c)^2}^{s_0} ds \int_{M_2^2}^{s_0} ds' \rho_i(s, s', q^2) e^{i(m_Z^2 - s)/M_2^2} e^{i(m_{Z'}^2 - s')/M_2^2}.
\]

(25)

Here \( M^2 = (M_0^2, M_2^2) \) and \( s_0 = (s_0, s_0') \) are the Borel and continuum threshold parameters, respectively. It is worth noting that the set \( (M_0^2, s_0) \) describes \( Z_{bc}^0 \), whereas \( (M_2^2, s_0') \) corresponds to the \( T \) tetraquark channel. The spectral densities \( \rho_i(s, s', q^2) \) are calculated as the imaginary parts of the correlation function \( \Pi_\mu^{OPE}(p, p') \) with dimension-five accuracy, and contain both the perturbative and nonperturbative contributions.

For numerical computations of \( G_i(M^2, s_0, q^2) \) one needs to employ various parameters, values some of which are collected in Eq. \( \text{[11]} \). The mass and coupling of the tetraquark \( Z_{bc}^0 \) and \( (M_2^2, s_0') \) are borrowed from Ref. \( \text{[17]} \), whereas for \( m_T \) and \( f_T \), and \( (M_2^2, s_0') \) we use results of the previous section.

To obtain the width of the decay \( Z_{bc}^0 \to T \ell \nu_T \) we have to integrate the differential decay rate \( d\Gamma/dq^2 \) within the kinematical limits \( m_1^2 \leq q^2 \leq (m_Z - m_T)^2 \), whereas the QCD sum rules lead to reliable results only for \( m_1^2 \leq q^2 \leq 1.25 \text{ GeV}^2 \). To cover all values of \( q^2 \) we replace the weak form factors by the functions \( F_i(q^2) \), which at accessible for the sum rule computations \( q^2 \) coincide with \( G_i(q^2) \), but can be extrapolated to the whole integration region.

In the present work for the fit functions we utilize the analytic expressions

\[
F_i(q^2) = f_i^0 \exp \left[ c_i^1 \frac{q^2}{m_Z^2} + c_i^2 \left( \frac{q^2}{m_Z^2} \right)^2 \right].
\]

(26)

Here, \( f_i^0 \), \( c_i^1 \) and \( c_i^2 \) are fitting parameters, values of which are presented below

\[
f_1^0 = 0.144, c_1^1 = 7.68, c_1^2 = 1505.10, \]

\[
f_2^0 = 3.282, c_2^1 = 7.69, c_2^2 = 1504.40.
\]

(27)

In Fig. \( 3 \) as an example, we plot the sum rule predictions for the form factor \( G_1(q^2) \) and the fit function \( F_1(q^2) \): It is seen that the fit function coincides well with the sum rule predictions in the region \( m_1^2 \leq q^2 \leq 1.25 \text{ GeV}^2 \). The differential rate \( d\Gamma/dq^2 \) of the semileptonic decay \( Z_{bc}^0 \to T \ell \nu_T \) is given by the formula
\[
\frac{d\Gamma}{dq^2} = \frac{1}{64\pi^3 m_Z^2} G_F^2 |V_{cs}|^2 \lambda (m_Z^2, m_T^2, q^2) \left( \frac{q^2 - m_T^2}{q^2} \right)^2 \left\{ (2q^2 + m_T^2) \left[ |G_1(q^2)|^2 \left( \frac{q^2}{2} - m_Z^2 - m_T^2 \right) - |G_2(q^2)|^2 \frac{q^2}{2} \right]
\right.

\left. + (m_T^2 - m_Z^2) Re \left[ G_1(q^2) G_2^*(q^2) \right] \right) + \frac{q^2 + m_T^2}{q^2} \left[ |G_1(q^2)|^2 (m_T^2 - m_Z^2)^2 + |G_2(q^2)|^2 q^4 + 2(m_Z^2 - m_T^2) q^2 \right]
\times Re \left[ G_1(q^2) G_2^*(q^2) \right],
\]

where

\[
\lambda (m_Z^2, m_T^2, q^2) = \left[ m_Z^4 + m_T^4 + q^4 - 2 \left( m_Z^2 m_T^2 + m_Z^2 q^2 + m_T^2 q^2 \right) \right]^{1/2}.
\]

To fulfill the numerical computations using Eq. (28) one also needs the Fermi coupling constant \( G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \) and CKM matrix element \( |V_{cs}| = 0.997 \pm 0.017 \). Obtained results for the width of semileptonic decays \( Z_{bc}^0 \to T\nu_l (l = e, \mu) \) read

\[
\Gamma (Z_{bc}^0 \to T e^+ \nu_e) = (1.19 \pm 0.26) \times 10^{-11} \text{ MeV},
\]

\[
\Gamma (Z_{bc}^0 \to T \mu^+ \nu_\mu) = (1.18 \pm 0.25) \times 10^{-11} \text{ MeV}.
\]

These results can be used to evaluate the total width and mean lifetime of the tetraquark \( Z_{bc}^0 \), and estimate branching ratios of its weak decay channels.

![Figure 3: The sum rule predictions for the weak form factor \( G_1(q^2) \) and the fit function \( F_1(q^2) \).](image)

**IV. ANALYSIS AND CONCLUSIONS**

We saturate the total width of \( Z_{bc}^0 \) by two dominant semileptonic decay modes. Then, the total width of the tetraquark \( Z_{bc}^0 \) and its mean lifetime are equal to:

\[
\Gamma_{\text{tot.}} = (2.37 \pm 0.36) \times 10^{-11} \text{ MeV},
\]

\[
\tau = 2.78^{+0.49}_{-0.37} \times 10^{-11} \text{ s}.
\]

As is seen, the scalar tetraquark \( Z_{bc}^0 \) is narrower than the master particle \( T_{bb, \bar{u}\bar{d}}^- \) and its mean lifetime \( 27.8^{4.9}_{3.7} \) ps is considerably longer than the same parameter for \( T_{bb, \bar{u}\bar{d}}^- \).

The semileptonic decays of \( Z_{bc}^0 \) occur via the following channels:

i) \( Z_{bc}^0 \to T e^+ \nu_e \),

ii) \( Z_{bc}^0 \to T \mu^+ \nu_\mu \).

The channels i) and ii) lead to appearance of the strong- and electromagnetic-interaction stable tetraquark \( T \equiv T_{bb, \bar{u}\bar{d}}^- \) that at next stages of the process dissociates weakly. The branching ratio for production, for example, of the final state \( T e^+ \nu_e \) is given by

\[
\text{BR}(Z_{bc}^0 \to T e^+ \nu_e) = \Gamma (Z_{bc}^0 \to T e^+ \nu_e) / \Gamma_{\text{tot.}}.
\]

It is not difficult to find that

\[
\text{BR}(Z_{bc}^0 \to T e^+ \nu_e) \simeq \text{BR}(Z_{bc}^0 \to T \mu^+ \nu_\mu) \simeq 0.5.
\]

The decays of \( T_{bb, \bar{u}\bar{d}}^- \) can be analyzed by the same way. The decay modes of \( T_{bb, \bar{u}\bar{d}}^- \) at the final state contain the tetraquark \( T_{bb, \bar{u}\bar{d}}^- \) and two opposite sign leptons accompanying by corresponding neutrinos \( e^- e^+ \nu_e \bar{\nu}_e \), \( e^- \mu^+ \bar{\nu}_e \bar{\nu}_\mu \), \( e^- e^- \nu_e \bar{\nu}_e \), \( \mu^+ e^- \nu_\mu \bar{\nu}_e \), \( e^- \nu_e \bar{\nu}_e \), \( \mu^+ \nu_\mu \bar{\nu}_e \), and \( \tau^+ e^- \nu_\tau \bar{\nu}_e \) and \( \tau^- e^+ \nu_\tau \bar{\nu}_e \). The branching ratios of these channels can be found using the fact, that \( \text{BR}(T_{bb, \bar{u}\bar{d}}^- \to Z_{bc}^0 e^- \bar{\nu}_e) \simeq \text{BR}(T_{bb, \bar{u}\bar{d}}^- \to Z_{bc}^0 \mu^+ \bar{\nu}_e) = 0.37 \) and \( \text{BR}(T_{bb, \bar{u}\bar{d}}^- \to Z_{bc}^0 \tau^- \bar{\nu}_e) = 0.26 \) (see, Ref. [17]). For some of decay channels we find:

\[
\text{BR}(T_{bb, \bar{u}\bar{d}}^- \to T e^- e^+ \nu_e \bar{\nu}_e) \simeq 0.185,
\]

\[
\text{BR}(T_{bb, \bar{u}\bar{d}}^- \to T \mu^+ \mu^- \nu_\mu \bar{\nu}_e) \simeq 0.185,
\]

\[
\text{BR}(T_{bb, \bar{u}\bar{d}}^- \to T \tau^- e^+ \nu_\tau \bar{\nu}_e) \simeq 0.130.
\]

We have explored the dominant semileptonic decays of the scalar tetraquark \( Z_{bc}^0 \) to \( T e^+ \nu_e \) and \( T \mu^+ \nu_\mu \) and estimated branching ratios of these final states. Because \( Z_{bc}^0 \) is stable against strong and electromagnetic decays, semileptonic modes are very important for its experimental studies: in accordance with recent analyses the production rate of the tetraquarks with the heavy diquark \( bc \)
at the LHC would be higher by two order of magnitude than four-quark mesons with $bb$ [24].

Another issue studied here is decays of the tetraquark $T^{bc,ud}_-$ We have analyzed its decay chains consisting of sequential weak transformations to final states with $T$ and evaluated their branching ratios. These calculations are important to fix processes, where the axial-vector tetraquark $T^{bc,ud}_-$ should be searched for.

In the present work the four-quark meson $T^{bc,ud}_-$ has been treated as a scalar particle. But $Z^0_{bc}$ can decay also to exotic mesons with another quantum numbers. By including into analysis these options one can open up new decay modes of $Z^0_{bc}$, and improve predictions for the branching ratios presented above.

Because $T^{bc,ud}_-$ itself is presumably a strong- and electromagnetic-interaction stable state, the weak decays explored in the present work implies generation of ordinary mesons at the second (for $Z^0_{bc}$) or third (for $T^{bc,ud}_-$) legs of weak transformations. But a situation, when some of subdominant weak modes after first or second transitions create four-quark mesons that are unstable against the strong interactions cannot be excluded as well. Then, conventional mesons may appear, even with very small branching ratios, at earlier phases of the $T^{bc,ud}_-$ and $Z^0_{bc}$ tetraquarks' decays. In other words, detailed investigating non-leading semileptonic decays of $Z^0_{bc}$, as well as calculating $T^{bc,ud}_-$ weak decays is necessary to reveal a quark content and quantum numbers of mesons that may appear at final stages of these processes. Studies of these problems, left beyond the scope of the present work, are required to gain more precise knowledge about properties of the exotic $T^{bc,ud}_-$ and $Z^0_{bc}$ states.

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