Cluster states in $^{11}$B, $^{11}$C and $^8$He and their similarity to $^{12}$C

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Abstract. We discuss cluster structures of $^{11}$B($^{11}$C) and $^8$He. The $^{11}$B$(3/2^-_3)$ is found to have a well developed $2\alpha + t$-cluster structure whose features are similar to those of the $^{12}$C$(0^+_2)$. The ground and excited states of $^8$He were also investigated. Dineutron correlations at the surface of $^8$He are analyzed. The mechanism of dineutron correlations at a nuclear surface is also discussed.

1. Introduction

In light stable nuclei, cluster structures are known for a long time as discussed in $^{12}$C and $^{20}$Ne. Recently, various new types of cluster structures were suggested in stable nuclei and also in unstable nuclei[1].

In C isotopes, three-center cluster structures have been attracting great interests as in $^{12}$C, in which developed $3\alpha$-cluster structures are found in excited states[2, 3, 4, 5, 6]. In 2001, Tohsaki et al. proposed a new interpretation of of the $^{12}$C$(0^+_2)$. According to their idea, this state is regarded as a cluster gas state, where three $\alpha$ particles are weakly interacting[7, 8]. Interestingly, similar dilute gas-like states of clusters were suggested in excited states of $^{11}$B($^{11}$C) and $^{16}$O[7, 9, 10, 11].

The $\alpha$ gas states in $^{12}$C and $^{16}$O are often discussed in relation with the alpha condensation in dilute nuclear matter[12] because of bosonic behavior of $\alpha$ particles in the dilute density. From the similar point of view, dineutron correlations are one of the key issues in physics of unstable nuclei. The nuclear force in the $^1S_0$ channel shows a rather strong attraction though it is not strong enough to form a two-neutron bound state. Therefore, spin-zero neutron-neutron correlations may be found in nuclear many-body systems. We here call the spin-zero ($S = 0$) nn pair with strong spatial correlations “dineutron”. In theoretical investigations, it has been suggested that the dineutron correlations may enhance in dilute neutron matter and also at the surface of neutron-rich nuclei[13, 14, 15, 16]. These facts may provide us a possible interpretation of dineutron correlations as virtual dineutrons formed at the nuclear surface.

In this paper, we report the cluster structure of the $3/2^-_3$ state of $^{11}$B($^{11}$C) and discuss its similarity to the $0^+_2$ of $^{12}$C based on theoretical calculations with antisymmetrized molecular dynamics(AMD)[17, 18]. The ground and excited states of $^8$He were also investigated with the AMD method[15]. To analyze dineutron correlations, we investigate the dineutron cluster components in the He isotopes. The mechanism of dineutron formation at nuclear surfaces is also discussed by introducing an ideal dineutron wave function for one dineutron moving around an $\alpha$ particle.
2. Formulation

We here briefly explain the formulation of the present calculations by means of the AMD method. The details of the AMD method for nuclear structure study are described in Refs. [17, 18].

An AMD wave function is given by a Slater determinant of Gaussian wave packets;

$$\Phi_{\text{AMD}}(Z) = \frac{1}{\sqrt{A!}} A\{\varphi_1, \varphi_2, \ldots, \varphi_A\},$$

where the $i$th single-particle wave function is written by a product of spatial($\phi$), intrinsic spin($\chi$) and isospin($\tau$) wave functions as,

$$\varphi_i = \phi_{X_i, \chi_i \tau_i},$$

$$\phi_{X_i}(r_j) \propto \exp\{-\nu(r_j - \frac{X_i}{\sqrt{D}})^2\},$$

$$\chi_i = \left( \frac{1}{2} + \xi_i \right) \chi^+_\uparrow + \left( \frac{1}{2} - \xi_i \right) \chi^+_\downarrow.$$

$\phi_{X_i}$ and $\chi_i$ are spatial and spin functions, and $\tau_i$ is isospin function which is fixed to be a proton or a neutron. Accordingly, an AMD wave function is expressed by a set of variational parameters, $Z = \{X_1, X_2, \ldots, X_A, \xi_1, \xi_2, \ldots, \xi_A\}$.

In calculations of $^{11}\text{B}(^{11}\text{C})$ and $^{12}\text{C}$, we performed energy variation after spin parity projection (VAP) within the AMD model space in the same way as done in Ref. [10]. For $^8\text{He}$ we applied the AMD+GCM, in which we performed superposition of a number of AMD wave functions obtained by energy variation with constraints.

For analysis of dineutron correlations, we introduce dineutron condensate wave functions as follows,

$$\Psi_{\text{cond}}(B, b_n) \equiv n_0 \int \prod_{i=1}^{k} d^3S_i \exp\left\{ -\frac{(S_i - S_{C})^2}{B^2} \right\} \Phi_{\text{Brink}}(S_C, S_1, \cdots S_k),$$

where $n_0$ is the normalization factor and $\Phi_{\text{Brink}}(S_C, S_1, S_2, \cdots S_k)$ is the Brink wave function for the $C + k(2n)$-cluster system consisting the core($C$) and $k$ dineutrons($2n$) as,

$$\Phi_{\text{Brink}}(S_C, S_1, \cdots S_k) \equiv A\{\phi_{C}(S_C)\phi_{b_n}^2(S_1) \cdots \phi_{b_n}^2(S_k)\}.$$  (6)

Here, the wave function of the $i$th $2n$, $\phi_{b_n}^2(S_i)$, is given by the $(0s)^2$ state with the size $b_n$ localized around $S_i$.

3. The $3/2^-$ state of $^{11}\text{B}(^{11}\text{C})$

Details of the AMD calculations for $^{11}\text{B}(^{11}\text{C})$ are described in Ref. [10]. We adopt the effective nuclear interactions consisting of the central force, the spin-orbit force and Coulomb force.

The calculated energy levels of negative-parity states in $^{11}\text{B}$ are in reasonable agreement with the experimental data. The $3/2^-$ state at 8.5 MeV is a candidate of the cluster gas-like state. This state has characteristic features different from ordinary states, namely, it has the weak M1 and GT transitions and the strong monopole transitions to the lower states. These features of the $3/2^-$ state are reproduced well the present calculations.

By analyzing the wave functions of the excited states, we find that $2\alpha + t$ is a three-center cluster state with loosely bound $2\alpha + t$. In $^{11}\text{B}$, the root-mean-square radius of the $3/2^-$ state is 3.1 fm which is remarkably large compared with that of the ground state (2.5 fm), though it is not as large as that (3.3 fm) of the $^{12}\text{C}(0^+_2)$. Moreover, the wave function of the $3/2^-$ state can not
be expressed by a dominant single Slater determinant but it is described by a linear combination of various spatial configurations of three-cluster positions. It means that the 3/2\(^{-}\) state is the well-developed 2\(\alpha + t\) cluster state, where three clusters have no geometric configuration but they are rather freely moving. The cluster features of the 3/2\(^{-}\) state are quite similar to those of \(^{12}\)C(0\(^{+}\), 7.65 MeV), which is known to be a dilute gas-like 3\(\alpha\) state. Therefore, the 3/2\(^{-}\) state is considered to be a weakly interacting cluster state in a dilute density, and it is likely to be a cluster gas. Recently, Yamada et al. suggested that two \(\alpha\) clusters in the 3/2\(^{-}\) state do not condensate in the \(S\) orbit[19]. Further analysis of the present wave functions are required to get a more definite conclusion. We will report more details in a future paper.

4. Dineutron correlation in \(^{8}\)He

Details of the AMD calculations for \(^{8}\)He as well as \(^{6}\)He are explained in Ref. [15].

In the calculated results, the 2\(^{+}\) state is the lowest excited state, and the 0\(^{+}\) state appears as well as the 1\(^{-}\) and 3\(^{-}\) states. An interesting result is that the 0\(^{+}\) state has a remarkably large neutron radius (\(r_{\text{r.m.s.}}\approx3.1\) fm) compared with the ground state (\(r_{\text{r.m.s.}}\approx2.6\) fm). The reason is a well developed \(^{4}\)He + 2\(n + 2n\) structure. In the wave function of the 0\(^{+}\) state large amplitudes are found for AMD basis states with various spatial configurations of \(^{4}\)He + 2\(n + 2n\). This may indicate a gas-like cluster feature where dineutrons are rather freely moving around the \(^{4}\)He core.

In order to see the dineutron correlations in 0\(^{+}\) states in more detail, we assume a 2\(n\) condensate wave function defined Eq.5 for the \(^{4}\)He + 2\(n + 2n\) system and calculated the overlap with the obtained \(^{8}\)He(0\(^{+}\)) wave functions. Here the dineutron size \(b_{n}\) is chosen to be \(b = 1/\sqrt{2\pi}(^{8}\text{He})\). The \(^{8}\text{He}(0^{+})\) has the maximum overlap of about 0.5 with the dineutron condensate wave function \(\Psi_{\text{cond}}(B)\) at \(B = 4 \sim 5\) fm. This means that the dineutron gas-like component is rather large in the calculated \(^{8}\text{He}(0^{+})\).

In contrast to the excited 0\(^{+}\) state, the ground state of \(^{8}\)He shows the dominant \(p_{3/2}\)-shell closed neutron structure with mixing of the dineutron correlations. Here, the dineutron correlations in the \(^{8}\)He(0\(^{+}\)) do not mean the spatially developed dineutron structure but the dineutron correlations at the nuclear surface which are described by four neutrons in the \(p\) shell forming spin-zero \(nn\) pairs.

To clarify the mechanism of dineutron correlations at the nuclear surface, we consider a spin-zero pair of two neutrons in \(S\)-wave channel around a \(^{4}\)He core. Let us calculate energy expectation values of the condensate wave function \(\Psi_{\text{cond}}(B, b_{n})\) defined in Eq. 5 for \(k = 1\) with a \(^{4}\)He core (a toy model for a \(^{6}\)He system) as a function of \(b_{n}\) and \(\beta = \sqrt{B^{2} + b_{n}^{2}}\). \(b_{n}\) indicates the dineutron size and \(\beta\) means the width for the motion of the dineutron mass center. The dineutron is moving in the width-\(\beta\) Gaussian distribution which is equivalent to a size-\(b_{n}\) dineutron in the \(S\) orbit around the \(^{4}\)He core. A wave function \(\Psi_{\text{cond}}\) with a fixed \(\beta\) shows features of two neutrons virtually confined in a finite region around the core. In the energy curve for \(\Psi_{\text{cond}}\) with the fixed value \(\beta = 3\) fm, we find an energy pocket at \(b_{n} \sim 1.5\) fm as seen in Fig. 1. The energy pocket originates in the energy loss of the kinetic term in \(b_{n} \sim \beta\) where two neutrons become the uncorrelated limit occupying the \(S\) orbit around the \(^{4}\)He core and Pauli repulsion against the core neutrons increases. In other words, a \(nn\) pair favors a compact size to avoid the Pauli blocking effect from the core. This result indicates that, when two neutrons are confined in a core potential, they favor to form a compact-size dineutron at the surface of the core.

5. Summary
We discussed cluster structures of \(^{11}\)B\(^{(11}\)C) and \(^{8}\)He. The \(^{11}\)B(3/2\(^{-}\)) was found to have the well developed 2\(\alpha + t\)-cluster structure which shows cluster features similar to those of the \(^{12}\)C(0\(^{+}\)). The ground and excited states of \(^{8}\)He were also investigated. Dineutron correlations at the
Figure 1. Energy expectation values of the condensate wave function $\Psi_{\text{cond}}(B, b_n)$ for one dineutron ($k = 1$) with a $^4$He core. The total energy (left), the kinetic energy (middle), and the potential energy (right) are plotted as a function of $b_n$ with fixed $\beta$ values, $\beta = 3$, 5, and 7 fm. The energies are measured from the $\alpha + n + n$ energy.

surface of $^8$He are analyzed. The mechanism of dineutron correlation at nuclear surfaces is discussed.

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