Modeling and Analysis of SCMA Enhanced D2D and Cellular Hybrid Network

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Abstract

Sparse code multiple access (SCMA) has been recently proposed for the future wireless networks, which allows non-orthogonal spectrum resource sharing and enables system overloading. In this paper, we apply SCMA into device-to-device (D2D) communication and cellular hybrid network, targeting at using the overload feature of SCMA to support massive device connectivity and expand network capacity. Particularly, we develop a stochastic geometry based framework to model and analyze SCMA, considering underlaid and overlaid mode. Based on the results, we analytically compare SCMA with orthogonal frequency-division multiple access (OFDMA) using area spectral efficiency (ASE) and quantify closed-form ASE gain of SCMA over OFDMA. Notably, it is shown that system ASE can be significantly improved using SCMA and the ASE gain scales linearly with the SCMA codeword dimension. Besides, we endow D2D users with an activated probability to balance cross-tier interference in the underlaid mode and derive the optimal activated probability. Meanwhile, we study resource allocation in the overlaid mode and obtain the optimal codebook allocation rule. It is interestingly found that the optimal SCMA codebook allocation rule is independent of cellular network parameters when cellular users are densely deployed. The results are helpful in the implementation of SCMA in the hybrid system.

I. INTRODUCTION

The fifth generation (5G) wireless networks are expected to be a mixture of network architectures with a number of ambitious goals, including exploding mobile data volume (1000×
improvement over 2010 by 2020 [1] and massive connected devices (around 50 billion by 2020 [2]), etc. Device-to-device (D2D) communication and cellular hybrid network is a promising architecture to achieve these goals [3]. In particular, due to the proximity feature of D2D communication devices, spatial resources can be effectively exploited, thereby improving spectrum utilization and network capacity. Moreover, since direct local transmission is enabled to bypass the cellular base station (BS), considerable traffic can be offloaded from BSs. Hence, huge number of devices can be simultaneously supported.

The coexistence of cellular network and D2D network can be generally categorized into underlaid mode and overlaid mode [4]–[7]. In the underlaid mode, cellular users and D2D users simultaneously transmit data over the same resources (e.g., spectrum resources). In consequence, cross-tier interference exists between cellular users and D2D users. In order to combat the overwhelming interference, efficient resource sharing and interference management mechanisms have been proposed in [8], [9] such that the benefits of proximity transmissions can be fully exploited. In terms of the overlaid mode, orthogonal resources are partially allocated to cellular users and D2D users, respectively [10], [11]. Since only co-tier interference exists, the performance degradation caused by interference can be more readily handled. Nevertheless, it is crucial to wisely implement resource allocation in order to better utilize the available resources. In [11], authors show that, even in the overlaid case, D2D communication can significantly improve the per-user average rate through carefully tuning frequency allocation for D2D pairs.

In both coexisting modes, efficient multiple access methods should be applied in order to support more cellular users and D2D users. One of the most prevalent methods is orthogonal frequency-division multiple access (OFDMA) [12]. Using OFDMA, orthogonal subcarriers or OFDMA tones are allocated to individual users so as to mitigate the interference over one tone. In order to improve spectrum reuse, D2D users can be underlaid with cellular network, i.e., OFDMA tones are reused by cellular users and D2D users. Accordingly, cross-tier interference is introduced between cellular users and D2D users, especially when users are densely distributed.

1In the following, we use OFDMA tones to replace OFDMA orthogonal resources.
To avoid the mutual interference and improve the quality of service (QoS) of individual users, it is preferable to use the overlaid mode, where OFDMA tones are separately allocated to cellular users and D2D users. However, due to the scarcity of spectrum resources, limited OFDMA tones are available such that the number of users that can be simultaneously served is critically restricted. Therefore, it is challenging to admit more users over the limited spectrum resources. Recently, sparse code multiple access (SCMA) has been proposed for 5G wireless networks, which has the potential to enable massive connectivity by allowing non-orthogonal spectrum resource sharing. Since users occupying the same OFDMA resources can be distinguished using different SCMA constellations, more orthogonal resources, i.e., codebooks, can be provided by SCMA compared to OFDMA. As a result, overloading gain can be yielded by SCMA. Besides, depending on the resource allocation rule, the number of users occupying the same codebooks will be decreased as more orthogonal resources are to be shared. Hence, the interference over one codebook could be mitigated, thereby boosting the performance of individual users. For these reasons, it is intuitively more suitable for SCMA to be applied in the D2D and cellular hybrid network, especially when communication devices are densely deployed. However, since SCMA codewords are spread to multiple OFDMA tones, great difficulty has been encountered in analytically characterizing the interference statistics of cellular users and D2D users. Thus, the benefits of SCMA in enhancing the hybrid network performance remain to be explored. Worse still, as overloading gain can only be achieved when codebooks are uniquely occupied by individual users, it is doubtful whether the overloading gain can also be achieved when codebook reuse is enabled.

Motivated by the above discussion, we consider a D2D and cellular hybrid network, where SCMA is employed by both uplink cellular transmissions and D2D transmissions. Using stochastic geometry, we present a tractable model to evaluate the performance of cellular network and D2D network by characterizing the signal-to-interference ratio (SIR) statistics of a typical cellular network.

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2 Overloading can be achieved when the number of available SCMA layers is greater than the number of OFDMA tones. For instance, using typical SCMA setting, 6 SCMA layers can be multiplexed over 4 OFDMA tones. Therefore, system can be overloaded by admitting more users. As will be discussed later, each SCMA layer is assigned with one SCMA codebook.
uplink and a typical D2D link. Moreover, the potential of SCMA in supporting more active users and enhancing spectrum utilization has been investigated using area spectral efficiency (ASE). The main contributions and outcomes of this paper are summarized as follows:

- **Analytical Framework:** To our best knowledge, no previous work has investigated the performance of SCMA in large scale networks. To fill in the gap, we have developed an analytical framework for the design and analysis of SCMA enhanced D2D and cellular hybrid network. The framework could capture the impact of important parameters on the performance of the hybrid network, including user spatial distributions, wireless channel propagation model, D2D mode selection rule and SCMA parameters, etc.

- **SCMA VS OFDMA:** Based on the analytical framework, closed-form expressions of approximate system ASE are derived in the D2D underlaid/overlaid cellular network. Furthermore, we compare the performance of SCMA with OFDMA, in the underlaid mode. In particular, we quantify the gain of SCMA over OFDMA, termed ASE gain. Through the comparison of ASE gain with overloading gain, it is numerically shown that the overloading gain can be almost achieved even when codebook reuse is allowed.

- **System Design Guidance:** In the underlaid mode, we enable D2D transmitters to use an activated probability to control the generated cross-tier interference to cellular users. The optimal activated probability is obtained such that proportional fairness utility function is maximized. In the overlaid mode, we study SCMA codebook allocation for cellular network and D2D network. Specifically, we find out the optimal codebook allocation rule when cellular users are densely deployed, in order to maximize the proportional fairness utility function. The results can serve as a guideline for the efficient design of SCMA mechanisms and resource allocation in the D2D and cellular hybrid network.

The remainder of this paper is organized as follows. We first present the system model and performance metrics in Section II. Performance analysis of D2D underlaid cellular network is then presented in Section III, where comparison between OFDMA and SCMA is made. In Section IV we evaluate the performance of SCMA in the D2D overlaid cellular network. Numerical results are given in Section V and conclusions are provided in Section VI.
II. SYSTEM MODEL

A. Network Model

We consider a D2D and cellular hybrid network (see Fig. 1), consisting of cellular uplinks and unidirectional D2D links. Stochastic geometry model is used to characterize the locations of BSs and users in the hybrid network [8], [15], [16]. In particular, cellular BSs are assumed to be spatially distributed over the infinite two-dimensional plane $\mathbb{R}^2$, according to a homogeneous Poisson Point Process (HPPP) $\Pi_{BS} = \{BS_i\} (i \in \mathbb{N})$ with intensities $\lambda_{BS}$. The locations of uplink cellular users and D2D transmitters are also modeled using HPPPs $\Pi_U = \{U_j\}$ and $\Pi_D = \{DT_k\} (j, k \in \mathbb{N})$ with intensity $\lambda_U$ and $\lambda_D$, respectively, and constant transmit power $P_U$ and $P_D$, respectively. The cellular user $U_j$ is associated with the geographically closest BS, the distance between which is denoted by $r_{U,j}$. The D2D transmitter $DT_k$ connects to an intended D2D receiver $DR_k$ with isotropic direction at distance $r_{D,k}$ away. Similarly with [6], [17], we assume that $r_{D,k}$ follows the Rayleigh distribution with probability density function (PDF) given by

$$f_{r_{D,k}} (x) = 2\pi \xi x \exp \left( -\xi \pi x^2 \right), \quad x \geq 0. \quad (1)$$
According to (1), $r_{D,k}$ can be very large, which, however, makes D2D transmission lose its proximity merit. Therefore, for practical concerns, we introduce a distance based mode selection rule for D2D users [6], [18]. Specifically, $DT_k$ is allowed to use D2D mode only when $r_{D,k}$ is smaller than a distance threshold $\tau_{\text{dis}}$. Otherwise, $DT_k$ uses cellular mode with transmit power $P_U$ by connecting to the nearest BS. Thus, $\tau_{\text{dis}}$ serves as a tunable parameter to control the traffic load from D2D network to cellular network. According to this rule, a new Poisson Point Process (PPP) $\Pi_{UT} = \{U_j\}$ is formed by uplink cellular users and cellular mode D2D transmitters with intensity $\lambda_{UT} = \lambda_U + \lambda_{D} \mathbb{I}(r_{D,k} > \tau_{\text{dis}})$. Meanwhile, transmitters selecting D2D mode form a PPP $\Pi_{DT} = \{DT_k\}$ with intensity $\lambda_{DT} = \lambda_{D} \mathbb{I}(r_{D,k} \leq \tau_{\text{dis}})$. Besides, we consider that uplink cellular users and D2D transmitters always have data to transmit.

In terms of the coexistence of cellular network and D2D network, underlaid mode and overlaid mode are taken into account. In the underlaid mode, all available resources are universally reused by cellular users and D2D users. Consequently, cross-tier interference is introduced between cellular users and D2D users. In contrast, resources are partially allocated to cellular users and D2D users, respectively, such that only co-tier interference exists.

We evaluate the performance of a typical cellular uplink and a typical D2D link. $U_0 (DT_0)$ and $BS_0 (DR_0)$ are the associated transmitter and receiver of the typical cellular (D2D) link, respectively. According to Slivnyak’s Theorem [19], the performance of typical links can be used to evaluate the performance of the other links.

B. Channel Model

Consider that channel gain consists of a path loss component with the path loss exponent $\alpha$ ($\alpha > 2$) and a distance-independent small-scale fading component. Besides, we use independently and identically distributed (i.i.d.) Rayleigh fading, i.e., $h \sim \mathcal{CN}(0, 1)$, to model the small-scale fading.

C. Multiple Access Schemes

We consider OFDMA and SCMA as two multiple access schemes for cellular users and D2D users in the hybrid network. OFDMA is a typical and most widely used multiple access scheme in
wireless networks, especially for the uplink transmission in cellular networks. Moreover, SCMA is designed based on OFDMA. Therefore, it is straightforward and convincing to take OFDMA as a benchmark to demonstrate the advantage of SCMA. In order to facilitate the comparison, we only consider the underlaid case when considering OFDMA. In the following, we describe how to implement the two schemes.

1) OFDMA. For cellular transmission, each cellular user in one cell is randomly and independently assigned with one of \( K \) OFDMA tones, which are orthogonal in time and frequency. Meanwhile, OFDMA tones can be reused by BSs of different cells. In consequence, only inter-cell interference exists, while no intra-cell interference exists. According to this allocation rule, when the number of cellular users within a cell is larger than that of OFDMA tones, randomly selected \( K \) cellular users are activated and the remaining users are kept inactive due to the unavailability of OFDMA resources. For D2D transmission, each D2D pair is also randomly and independently allocated with one OFDMA tone. Since the same OFDMA tones are reused by cellular users and D2D users, mutual interference is introduced.

We denote \( x_{U_0}^O \) and \( x_{DT_j}^O \) as the data symbols sent from \( U_i \) and \( DT_j \), respectively. Accordingly, the received signal at BS\(_0\) can be expressed as

\[
y_{BS_0} = \sqrt{P_U} h_{BS_0,U_0} x_{U_0}^O + \sum_{U_i \in \Pi_{UT}^O} \sqrt{P_U} h_{BS_0,U_i} x_{U_i}^O + \sum_{DT_j \in \Pi_{DT}^O} \sqrt{P_D} h_{BS_0,DT_j} x_{DT_j}^O + n_0, \tag{2}
\]

where \( \Pi_{DT}^O \) denotes the set of active D2D transmitters using the same OFDMA tone with \( U_0 \). \( \tilde{\Pi}_{UT}^O = \Pi_{UT}^O \setminus \{U_0\} \), where \( \Pi_{UT}^O \) is the set of active cellular users using the same OFDMA tone with \( U_0 \). In the following, \( h_{Y,X} \) denotes the channel from \( X \) to \( Y \). \( n_0 \) is the additive Gaussian noise.

Similarly, the received signal at DR\(_0\) using OFDMA can be expressed as

\[
y_{DR_0} = \sqrt{P_D} h_{DR_0,DT_0} x_{DT_0}^O + \sum_{DT_j \in \tilde{\Pi}_{DT}^O} \sqrt{P_D} h_{DR_0,DT_j} x_{DT_j}^O + \sum_{U_i \in \Pi_{UT}^O} \sqrt{P_U} h_{DR_0,U_i} x_{U_i}^O + n_0, \tag{3}
\]

where \( \tilde{\Pi}_{DT}^O = \Pi_{DT}^O \setminus \{DT_0\} \).

2) SCMA. An SCMA encoder is defined as a function, which maps a binary stream of
log_2 (M) bits to a K-dimensional complex codebook of size M [13]. The K-dimensional complex codewords of each codebook are sparse vectors with N_C (2 ≤ N_C < K) non-zero entries. Using SCMA, each SCMA layer is assigned with a codebook and users’ data is transmitted using the codeword from the codebook over K OFDMA tones. An exemplary mapping relationship between SCMA layers and OFDMA tones is illustrated in Fig. 2. Therefore, multiple access is achieved by sharing the same time-frequency resources among the SCMA layers. According to [13], the maximum number of codebooks J is a function of the codeword length K and the number of non-zero elements in the codeword N_C. The generation of codebooks is equivalent to selecting N_C positions out of K elements. Hence, J = C_{N_C}^K. As a result, the received signal after layer multiplexing can be obtained as

\[ y = \sum_{j=1}^{J} \sqrt{P_U} \text{diag} (h_j) x_j + n_0, \]

where x_j is the K × 1 SCMA codeword of the layer j, h_j is the K × 1 channel vector of layer j and n_0 is the vector of additive Gaussian noise. diag (·) is defined as the operation, which turns a vector into a matrix by putting the vector on the main diagonal of the matrix.

Due to the sparsity of SCMA codewords, multi-user detection based on message passing algorithm (MPA) can be implemented at the SCMA receiver with low complexity [20]. Using ideal MPA receiver, codewords from different layers can be decoded without interfering with each other. Therefore, the codebooks allocated to different layers can be considered as orthogonal resources. Consequently, interference occurs only when the same layer or equivalently the same codebook is reused by more than one user. Since J layers can be multiplexed over K resources, we further define the SCMA overloading factor as

\[ \eta_{\text{overload}} = \frac{J}{K}, \]  

(4)
For cellular transmission, each BS randomly and independently selects one of the available codebooks to serve one connected cellular user. Similarly, if the number of the connected cellular users at a BS is larger than that of the available codebooks, randomly selected \( J \) cellular users are kept active. For D2D transmission, each D2D pair is randomly and independently allocated with one codebook to transmit with.

We consider two coexisting modes, i.e., underlaid mode and overlaid mode. Let \( x_{U_i,m}^S \) and \( x_{DT_j,m}^S \) denote the SCMA codewords sent by \( U_i \) and \( DT_j \), respectively, over the \( m \)th OFDMA tone. Then, the received signal at BS\(_0\) is given by

\[
g_{BS_0}^S = \sqrt{P_U^\dagger g_{BS_0,U_0}x_{U_0,m}^S} + \sum_{U_i \in \tilde{\Pi}_{UT}^S} \sqrt{P_U^\dagger g_{BS_0,U_i}x_{U_i,m}^S} + 1_U \sum_{DT_j \in \Pi_{DT}^S} \sqrt{P_D^\dagger g_{BS_0,DT_j}x_{DT_j,m}^S} + n_0, \tag{5}
\]

where \( g_{BS_0,U_i} = \sum_{m=1}^{N_C} h_{BS_0,U_i,m} \), \( g_{BS_0,DT_j} = \sum_{m=1}^{N_C} h_{BS_0,DT_j,m} \) and \( h_{Y,X,m} \) denotes the channel from X to Y over the \( m \)th OFDMA tone. \( 1_U \) is the indicator function, which equals 1 in the underlaid mode and 0 in the overlaid mode. \( P_U^\dagger = \frac{P_U}{N_C} \) and \( P_D^\dagger = \frac{P_D}{N_C} \), since transmit power is assumed to be uniformly allocated over \( N_C \) OFDMA tones. \( \Pi_{DT}^S \) denotes the set of active D2D transmitters, which use the same codebook with BS\(_0\). \( \tilde{\Pi}_{UT}^S = \Pi_{UT}^S \setminus \{U_0\} \), where \( \Pi_{UT}^S \) denotes the set of active cellular users using the same codebook with U\(_0\).

Similarly, the received signal at DR\(_0\) can be expressed as

\[
g_{DR_0}^S = \sqrt{P_D^\dagger g_{DR_0,DT_0}x_{DT_0,m}^S} + \sum_{DT_j \in \tilde{\Pi}_{DT}^S} \sqrt{P_D^\dagger g_{DR_0,DT_j}x_{DT_j,m}^S} + 1_U \sum_{U_i \in \Pi_{UT}^S} \sqrt{P_U^\dagger g_{DR_0,U_i}x_{U_i,m}^S} + n_0, \tag{6}
\]

where \( g_{DR_0,DT_j} = \sum_{m=1}^{N_C} h_{DR_0,DT_j,m} \), \( g_{DR_0,U_i} = \sum_{m=1}^{N_C} h_{DR_0,U_i,m} \) and \( \tilde{\Pi}_{DT}^S = \Pi_{DT}^S \setminus \{DT_0\} \).

### D. Performance Metrics

We use ASE [bits/ (s \cdot Hz \cdot m^2)] to evaluate the performance of the D2D and cellular hybrid network [21]. Specifically, ASE of the cellular network is defined as

\[
\mathcal{A}_C = q_U \lambda_{UT} \mathbb{P} \{\text{SIR}_{BS_0} > \tau_{BS} \} \log (1 + \tau_{BS}), \tag{7}
\]
where $q_U$ is the access probability of cellular users, $\text{SIR}_{BS_0}$ is the SIR at $BS_0$ and $\tau_{BS}$ is the corresponding SIR threshold. Likewise, ASE of the D2D network is defined as

$$A_D = q_D \lambda_{DT} P \{ \text{SIR}_{DR_0} > \tau_{DR} \} \log (1 + \tau_{DR}) , \quad (8)$$

where $q_D$ is the activated probability of D2D transmitters, $\text{SIR}_{DR_0}$ is the SIR at $DR_0$ and $\tau_{DR}$ is the corresponding SIR threshold. Note that the effect of noise is ignored in the interference-limited network.

In the following, $F(\cdot)$ and $f(\cdot)$ denote the cumulative distribution function (CDF) and PDF, respectively. Meanwhile, the notations used throughout this paper are summarized in Table I.

III. D2D UNDERLAIĐ CELLULAR NETWORK

In this section, we first compare the performance of OFDMA and SCMA in the underlaid mode through ASE. Afterward, we endow D2D users with an activated probability to balance the cross-tier interference and search for the optimal activated probability of D2D users to maximize the utility function defined based on proportional fairness.

A. OFDMA VS SCMA

In this part, we investigate the performance of OFDMA and SCMA. To this end, we first determine the access probability of cellular users, which is defined as the probability that a BS uses available orthogonal resources (OFDMA tones or SCMA codebooks) to serve its associated cellular users, according to the following lemma.

**Lemma 1.** The probability that a BS allocates one orthogonal resource to one of the associated cellular users is given by

$$q_U = 1 - \sum_{m=N_R+1}^{\infty} \frac{m - 1}{m} P \{ N_U = m \} , \quad (9)$$

where $N_R$ is the number of available resources and $N_U$ is the number of cellular users served by the BS.
### Table I
SUMMARY OF NOTATIONS

| Symbol | Meaning | Symbol | Meaning |
|--------|---------|--------|---------|
| BS<sub>i</sub>, U<sub>j</sub> | i<sup>th</sup> BS, j<sup>th</sup> uplink user | y<sub>DR<sub>0</sub></sub>, y<sub>S<sub>0</sub></sub> | received signals at DR<sub>0</sub> using OFDMA and SCMA |
| DT<sub>m</sub> | m<sup>th</sup> D2D transmitter | | |
| DR<sub>n</sub> | n<sup>th</sup> D2D receiver | P<sub>U</sub><sup>T</sup>, P<sub>D</sub><sup>T</sup> | P<sub>U</sub><sup>T</sup><sup>T</sup> = \( P<sub>U</sub><sub>N</sub> \), P<sub>D</sub><sup>T</sup><sup>T</sup> = \( P<sub>D</sub><sub>N</sub> \) |
| Π<sub>BS</sub> | Π<sub>BS</sub> = \{BS<sub>i</sub>\} | Π<sub>U</sub>, Π<sub>UT</sub> | sets of uplink cellular users and D2D transmitters |
| Π<sub>U</sub> | uplink cellular users set | η<sub>UT</sub>, η<sub>SDR</sub> | η<sub>P</sub><sup>U</sup><sup>T</sup> = \( \frac{\eta<sub>P</sub><sub>U</sub>}{\eta<sub>U</sub><sub>NC</sub>} \), η<sub>P</sub><sup>D</sup><sup>T</sup> = \( \frac{\eta<sub>P</sub><sub>D</sub>}{\eta<sub>D</sub><sub>NC</sub>} \) |
| Π<sub>UT</sub> | set of Π<sub>U</sub> and D2D users in cellular mode | Π<sub>UT</sub><sup>S</sup><sup>U</sup>, Π<sub>UT</sub><sup>S</sup><sup>D</sup> | set of active cellular users and D2D transmitters using one SCMA codebook |
| Π<sub>D</sub> | D2D transmitters set | Π<sub>DT</sub><sup>S</sup><sup>U</sup>, Π<sub>DT</sub><sup>S</sup><sup>D</sup> | |
| Π<sub>DT</sub> | set of D2D users in D2D mode | Π<sub>DT</sub><sup>S</sup><sup>U</sup><sup>T</sup><sup>U</sup>, Π<sub>DT</sub><sup>S</sup><sup>U</sup><sup>T</sup><sup>D</sup> | sets of active cellular users and D2D transmitters using one SCMA codebook |
| λ<sub>BS</sub> | intensity of Π<sub>BS</sub> | λ<sub>UT</sub><sup>S</sup><sup>U</sup>, λ<sub>UT</sub><sup>S</sup><sup>D</sup> | |
| λ<sub>U</sub>, λ<sub>UT</sub> | intensities of Π<sub>U</sub> and Π<sub>UT</sub> | Π<sub>UT</sub><sup>S</sup><sup>U</sup>, Π<sub>UT</sub><sup>S</sup><sup>D</sup> | |
| λ<sub>D</sub>, λ<sub>DT</sub> | intensities of Π<sub>D</sub> and Π<sub>DT</sub> | | |
| P<sub>U</sub><sup>T</sup>, P<sub>D</sub><sup>T</sup> | transmit power of U<sub>i</sub> and DT<sub>m</sub> | Π<sub>OT</sub><sup>S</sup><sup>U</sup><sup>T</sup><sup>U</sup>, Π<sub>OT</sub><sup>S</sup><sup>U</sup><sup>T</sup><sup>D</sup> | sets of uplink cellular users and D2D transmitters using one SCMA codebook |
| r<sub>U,j</sub> | distance from U<sub>j</sub> to BS<sub>j</sub> | Π<sub>OT</sub><sup>S</sup><sup>U</sup><sup>T</sup><sup>U</sup>, Π<sub>OT</sub><sup>S</sup><sup>U</sup><sup>T</sup><sup>D</sup> | sets of active cellular users and D2D transmitters using one SCMA codebook |
| r<sub>D,m</sub> | distance from DT<sub>m</sub> to DR<sub>n</sub> | Π<sub>OT</sub><sup>S</sup><sup>U</sup><sup>T</sup><sup>U</sup>, Π<sub>OT</sub><sup>S</sup><sup>U</sup><sup>T</sup><sup>D</sup> | set of active cellular users and D2D transmitters using one SCMA codebook |
| τ<sub>dis</sub> | mode selection threshold | \( τ<sub>BS</sub><sup>T</sup><sup>U</sup><sup>T</sup><sup>U</sup>, τ<sub>BS</sub><sup>T</sup><sup>U</sup><sup>T</sup><sup>D</sup> | SIR thresholds at BSs and D2D receivers |
| ξ | D2D link length parameter | \( τ<sub>BS</sub><sup>T</sup><sup>U</sup><sup>T</sup><sup>U</sup>, τ<sub>BS</sub><sup>T</sup><sup>U</sup><sup>T</sup><sup>D</sup> | SIR thresholds at BSs and D2D receivers |
| α, δ | path loss exponent, \( δ = 2 \) | \( τ<sub>BS</sub><sup>T</sup><sup>U</sup><sup>T</sup><sup>U</sup>, τ<sub>BS</sub><sup>T</sup><sup>U</sup><sup>T</sup><sup>D</sup> | SIR thresholds at BSs and D2D receivers |
| h<sub>Y,X</sub> | Rayleigh fading from X to Y | η<sub>overload</sub> | overloading factor |
| h<sub>Y,X,m</sub> | \( h<sub>Y,X</sub> \) at the \( m \)th OFDMA tone | η<sub>ASE</sub> | ASE gain |
| K | number of OFDMA tones | η<sub>ASE</sub><sup>U</sup>, η<sub>ASE</sub><sup>D</sup> | ASE gain when \( τ<sub>BS</sub> = τ<sub>DR</sub> \) |
| M | codebook size | q<sub>U</sub>, q<sub>D</sub> | cellular access probability |
| N<sub>C</sub> | number of non-zero elements in SCMA codewords | q<sub>U</sub><sup>0</sup>, q<sub>S</sub><sup>0</sup> | access probabilities of cellular users using OFDMA and SCMA |
| y<sub>BS<sub>0</sub></sub>, y<sub>S<sub>0</sub></sub> | received signals at BS<sub>0</sub> using OFDMA and SCMA | | |

**Proof:** See Appendix A-I in [22].

According to [23], the probability mass function (PMF) of \( N_U \) can be obtained as

\[
P\{N_U = m\} = \frac{\lambda<sub>UT</sub>^m}{\Gamma(m+b)} \left( \frac{E[N_U]}{b} \right)^m \Gamma(b+bE[N_U])^{-m},
\]

where \( b = 3.575 \), \( \Gamma(\cdot) \) is the standard gamma function and \( E[N_U] = \frac{\lambda<sub>UT</sub>}{\lambda<sub>BS</sub>} \) is the average number of cellular users connected to one BS.

We substitute \( N_R \) with \( K \) and \( J \), respectively, in Lemma \([\Pi]\) to obtain the access probabilities \( q<sub>U</sub><sup>0</sup> \) and \( q<sub>S</sub><sup>0</sup> \) when OFDMA and SCMA are used. Then, the densities of active cellular users using the same OFDMA tone and the same SCMA codebook are \( q<sub>U</sub><sup>0</sup>\lambda<sub>UT</sub><sub>N</sub>\) and \( q<sub>S</sub><sup>0</sup>\lambda<sub>UT</sub><sub>NC</sub>\), respectively.
Next, we analyze the ASE of the underlaid scenario when OFDMA is applied. According to the definitions of ASE in Section II-D, the coverage probabilities $\text{CP}_{\text{OU}}^{\text{BS}} = \mathbb{P}\{\text{SIR}_{\text{BS}0}^{\text{O}} > \tau_{\text{BS}}\}$ and $\text{CP}_{\text{OU}}^{\text{DR}} = \mathbb{P}\{\text{SIR}_{\text{DR}0}^{\text{O}} > \tau_{\text{DR}}\}$ should be calculated in order to derive ASE. Based on (2), the SIR at BS$_0$ using OFDMA in the underlaid mode can be expressed as

$$\text{SIR}_{\text{BS}0}^{\text{O}} = \frac{P_U r_{U,0}^{-\alpha} \| h_{\text{BS}0, U0} \|^2}{I_{\text{C,C}}^O + I_{\text{C,D}}^O},$$

where $I_{\text{C,C}}^O = \sum_{U_i \in \Pi_{\text{UT}}^O} P_U \| h_{\text{BS}0, U_i} \|^2 \| U_i - \text{BS}_0 \|^{-\alpha}$ is the inter-cell interference from active cellular users using the same OFDMA tone with $U_0$ outside the coverage of BS$_0$ and $I_{\text{C,D}}^O = \sum_{\text{DT}_j \in \Pi_{\text{DT}}^O} P_D \| h_{\text{BS}0, \text{DT}_j} \|^2 \| \text{DT}_j - \text{BS}_0 \|^{-\alpha}$ is the interference from active D2D transmitters using the same OFDMA tone with $U_0$.

Similarly, according to (3), the SIR at DR$_0$ using OFDMA in the underlaid mode is given by

$$\text{SIR}_{\text{DR}0}^{\text{O}} = \frac{P_D r_{D,0}^{-\alpha} \| h_{\text{DR}0, \text{DT}_0} \|^2}{I_{\text{D,D}}^O + I_{\text{D,C}}^O},$$

where $I_{\text{D,D}}^O = \sum_{\text{DT}_j \in \Pi_{\text{DT}}^O} P_D \| h_{\text{DR}0, \text{DT}_j} \|^2 \| \text{DT}_j - \text{DR}_0 \|^{-\alpha}$ is the interference from other D2D transmitters over the same OFDMA tone with DR$_0$ and $I_{\text{D,C}}^O = \sum_{U_i \in \Pi_{\text{UT}}^O} P_U \| h_{\text{DR}0, U_i} \|^2 \| U_i - \text{DR}_0 \|^{-\alpha}$ is the interference from active cellular users over the same OFDMA tone with DR$_0$.

Based on (10) and (11), we derive ASE using the following proposition.

**Proposition 1.** Considering that OFDMA is used in D2D underlaid cellular networks, the ASE of cellular network is given by $A_C^{\text{OU}} = q_U^O \lambda_{\text{UT}} \text{CP}_B^{\text{OU}} \log (1 + \tau_{\text{BS}})$, where

$$\text{CP}_B^{\text{OU}} = \lambda_{\text{BS}} \frac{\lambda_{\text{BS}}}{\lambda_{\text{BS}} + \frac{2\lambda_{\text{UT}} \tau_{\text{BS}}}{K(\alpha - 2)} H y F_1 + \frac{2\pi \lambda_{\text{DT}} (\tau_{\text{BS}} \eta_{\text{B}})^\alpha}{\alpha \sin (\frac{\alpha \pi}{2})}},$$

In (12), $H y F_1 = \binom{2}{1} F_1 (1, 1 - \delta, 2 - \delta, -\tau_{\text{BS}})$, where $\binom{2}{1} F_1 (\cdot, \cdot, \cdot)$ denotes the hypergeometric function, $\eta_{\text{B}} = \frac{P_{\text{DU}}}{P_{\text{U}}}$ and $\delta = \frac{2}{\alpha}$. The D2D network ASE is given by $A_D^{\text{OU}} = \lambda_{\text{DT}} \text{CP}_D^{\text{OU}} \log (1 + \tau_{\text{DR}})$, where

$$\text{CP}_D^{\text{OU}} = \pi \xi \rho_{\text{O}}^{-1} \left(1 - e^{-\rho_0 \tau_{\text{dis}}} \right).$$

(13)
In (13), \( \rho_O = \pi \xi + \frac{2\pi^2 \rho_{D_R} (\lambda_{D_T} + q_{\Omega} \lambda_{U_T} \eta_{\rho^2})}{K \alpha \sin \left(\frac{\pi}{m}\right)} \).

**Proof:** See Appendix A □

Note that it is easy to obtain \( \lambda_{U_T} = \lambda_U + \lambda_{D} e^{-\xi \pi \delta_{\text{dis}}} \) and \( \lambda_{D_T} = \lambda_D \left(1 - e^{-\xi \pi \delta_{\text{dis}}} \right) \) according to the mode selection rule introduced in Section II. We notice from Proposition I that the ASE achieved by cellular networks increases with the intensity of active cellular users, i.e., \( q_{\Omega} \lambda_{U_T} \).

However, given the number of OFDMA tones and intensity of BSs, \( q_{\Omega} \lambda_{U_T} \) cannot be further improved when \( \lambda_{U_T} \) is sufficiently large. Consequently, cellular network ASE is limited. In order to increase cellular network ASE, it is intuitive to increase the intensity of active cellular users, which can be accomplished using SCMA.

According to (5), when SCMA is applied, the SIR at BS0 in the underlaid scenario can be expressed as

\[
\text{SIR}_{BS0}^U = \frac{\sum_{m=1}^{N_C} P_{U,U,0}^I \|h_{BS0,U,0,m}\|^2}{I_{C,C}^{S} + I_{C,D}^{S}}, \tag{14}
\]

where \( I_{C,C}^{S} = \sum_{U_i \in \Pi_{U}^{S}} \sum_{m=1}^{N_C} P_{U}^I \|h_{BS0,U_i,m}\|^2 \) and \( I_{C,D}^{S} = \sum_{D_T_j \in \Pi_{D_T}^{S}} \sum_{m=1}^{N_C} P_{D}^I \|h_{BS0,D_T_j,m}\|^2 \).

Likewise, based on (6), the SIR at DR0 is given by

\[
\text{SIR}_{DR0}^U = \frac{\sum_{m=1}^{N_C} P_{D_{DR0},D_T^0}^I \|h_{DR0,D_T^0,m}\|^2}{I_{D,D}^{S} + I_{D,C}^{S}}, \tag{15}
\]

where \( I_{D,D}^{S} = \sum_{D_T_j \in \Pi_{D_T}^{S}} \sum_{m=1}^{N_C} P_{D}^I \|h_{DR0,D_T^0,m}\|^2 \) and \( I_{D,C}^{S} = \sum_{U_i \in \Pi_{U}^{S}} \sum_{m=1}^{N_C} P_{U}^I \|h_{DR0,U_i,m}\|^2 \).

In order to derive ASE, we should first calculate the coverage probabilities at BS0 and DR0.

Note from (14) and (15) that the SCMA codeword is spread over more than one OFDMA tone, which brings about difficulty to obtain the exact results of coverage probabilities in explicit form. Accordingly, the methods utilized in [22] cannot be applied. Targeting at providing design insights through the results, we use approximations to derive the closed-form expressions of \( \text{CP}_{BS}^U = \mathbb{P}\{\text{SIR}_{BS0}^U > \tau_{BS}\} \) and \( \text{CP}_{BS}^U = \mathbb{P}\{\text{SIR}_{DR0}^U > \tau_{DR}\} \) according to the following proposition.

**Proposition 2.** Considering that SCMA is used in D2D underlaid cellular networks, the ASE
achieved by cellular network is given by \( \mathcal{A}^{SU}_{C} = q^S_U \lambda_{UT} \text{CP}^{SU}_{BS} \log(1 + \tau_{BS}) \), where \( \text{CP}_{BS} \) is approximated as

\[
\text{CP}^{SU}_{BS} \approx \frac{\lambda_{BS}}{\lambda_{BS} + \frac{q^S_U \lambda_{UT}}{2} H y F_2 + \frac{2 \pi \lambda_{DT}(\tau_{BS})^{\delta}}{J_\alpha(\frac{4\pi}{\alpha})} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1)\alpha} + 1 \right)}.
\]

In (16), \( H y F_2 = 2 F_1(N_C, -\delta, 1 - \delta, -\tau_{BS}) - 1 \) and \( \tau_{BS} = \frac{\tau_{BS}}{N_C} \). The ASE achieved by D2D network is given by \( \mathcal{A}^{SU}_{D} = \lambda_{DT} \text{CP}^{SU}_{DR} \log(1 + \tau_{DR}) \), where \( \text{CP}^{SU}_{DR} \) is approximated as

\[
\text{CP}^{SU}_{DR} \approx \pi \xi \rho_S^{-1} \left( 1 - e^{-\rho_S \tau_{dis}^2} \right).
\]

In (17), \( \rho_S = \pi \xi + \frac{2 \pi^2 \bar{\tau}_{DR}(\lambda_{DT} + q^S_U \lambda_{UT})^{\delta}}{J_\alpha(\frac{4\pi}{\alpha})} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1)\alpha} + 1 \right) \) and \( \bar{\tau}_{DR} = \frac{\tau_{DR}}{N_C} \).

**Proof:** See Appendix B.

In Appendix B, we have used \( W_{BS_0, U_0} \sim \exp(N_C^{-1}) \) to approximate \( G_{BS_0, U_0} \sim \text{Gamma}(N_C, 1) \) for analytical tractability. \( W_{BS_0, U_0} \) and \( G_{BS_0, U_0} \) have the same first moment. Meanwhile, it can be readily shown that higher moments of \( W_{BS_0, U_0} \) and \( G_{BS_0, U_0} \) are close especially when \( N_C \) is small. Note that the typical value of \( N_C \) equals 2. Besides, the power loss caused by the small-scale fading is much smaller than that caused by pathloss. Therefore, using such approximation will not exert much influence on the accuracy of the coverage probability analysis provided in Proposition 2, which will be shown in Section V.

Comparing the results given by Proposition 1 and Proposition 2, we derive the ASE gain of SCMA over OFDMA as

\[
\eta_{ASE} = \frac{\mathcal{A}^{SU}_{C} + \mathcal{A}^{SU}_{D}}{\mathcal{A}^{OU}_{C} + \mathcal{A}^{OU}_{D}}.
\]

By definition, if the same SIR thresholds are applied by cellular users and D2D users, i.e., \( \tau_{BS} = \tau_{DR} \), the ASE gain is equivalent to

\[
\hat{\eta}_{ASE} = \frac{q^S_U \lambda_{UT} \text{CP}^{SU}_{BS} + \lambda_{DT} \text{CP}^{SU}_{DR}}{\lambda_{DT} \text{CP}^{OU}_{BS} + \lambda_{DT} \text{CP}^{OU}_{DR}}.
\]

\(^3\text{Note that the subscripts of } W \text{ and } G \text{ have the same meaning as that for } h.\)
where the numerator denotes the intensity of users that are successfully admitted using SCMA and the denominator denotes the intensity of users that are successfully admitted using OFDMA. Therefore, we could compare $\hat{\eta}_{\text{ASE}}$ with $\eta_{\text{overload}}$ defined in (4) to examine whether overloading gain can be achieved when codebook reuse is enabled, which will be shown in Section V.

B. Optimizing Activated Probability of D2D Users

In this part, we enable D2D users to use an activated probability to contend for available resources. In particular, at the beginning of each time slot, each D2D transmitter randomly and independently tosses a coin to determine whether to keep active during this time slot. Then, the activated D2D transmitters use SCMA for data transmission. Given that the activated probability of each D2D transmitters is $q_D$, the set of active D2D transmitters is a thinned PPP with intensity $q_D \lambda_{\text{DT}}$. Hence, $q_D$ is a tunable parameter to control cross-tier interference from D2D network to cellular network. Specifically, a large $q_D$ would enhance D2D network performance by activating more D2D transmissions, but result in overwhelming cross-tier interference to cellular transmissions, and vice versa. Meanwhile, D2D transmissions are fully under control of BSs. Specifically, BSs decide how many D2D users are active through tuning the activated probability. After the activated probability is set, this scheme can be independently performed by D2D users without channel estimation and channel information interaction. Therefore, this scheme serves as a simple but effective semi-centralized interference management method. Although suboptimal compared to conventional centralized scheme, it can provide a performance lower bound to these sophisticated schemes.

In the following, we intend to balance the tradeoff by optimizing $q_D$ based on a proportional fairness utility function. To this end, we first quantify the effect of $q_D$ on the hybrid system performance according to the following corollary.

**Corollary 1.** Considering that D2D transmitters are activated with probability $q_D$ in the underlaid scenario, the ASE of cellular network using SCMA is given by $\hat{A}_{\text{SU}}^{\text{C}} = q_U^S \lambda_{\text{UT}} C_{\text{BS}}^{\text{SU}} \log (1 + \tau_{\text{BS}})$.
where $\hat{\CP}^{SU}_{BS}$ is approximated as

$$\hat{\CP}^{SU}_{BS} \approx \frac{\lambda_{BS}}{\lambda_{BS} + \frac{q^S_{U,T}}{H y F_2} + \frac{2\pi q^D_{U,T}(\eta_{BS}p)^\delta N_C}{J_n\sin\left(\frac{\theta}{\delta}\right)} \prod_{n=2}^{N_C} \left(\frac{2}{(n-1)\alpha} + 1\right)}.$$  \(\text{(20)}\)

The ASE of D2D network is given by

$$\hat{\CP}^{SU}_{DR} \approx \frac{\pi \xi}{\rho^S} \left(1 - e^{-\rho^S_{\text{dis}}^2}\right).$$  \(\text{(21)}\)

In (27), $\rho^S = \pi \xi + \frac{2\pi^2 \rho^S_{\text{dis}} (q^D_{U,T} + q^S_{U,T}p)^\delta N_C}{J_n\sin\left(\frac{\theta}{\delta}\right)} \prod_{n=2}^{N_C} \left(\frac{2}{(n-1)\alpha} + 1\right).$

**Proof:** The proof is similar to that in Appendix B and thus is omitted due to space limitation.

We then search for the optimal activated probability $q^*_{D}$ according to the following objective function

$$q^*_{D} = \arg \max_{q^D} u_U \left(\hat{\CP}^{SU}_{C}, \hat{\CP}^{SU}_{D}\right),$$  \(\text{(22)}\)

where $q^D \in [0, 1]$ and $u_U \left(\hat{\CP}^{SU}_{C}, \hat{\CP}^{SU}_{D}\right)$ is a utility function that is in different forms according to different design targets. In our work, we define the utility function based on the most commonly used proportional fairness, i.e.,

$$u_U \left(\hat{\CP}^{SU}_{C}, \hat{\CP}^{SU}_{D}\right) = \log \hat{\CP}^{SU}_{C} + \log \hat{\CP}^{SU}_{D},$$  \(\text{(23)}\)

where $\hat{\CP}^{SU}_{C}$ and $\hat{\CP}^{SU}_{D}$ are given by Corollary 1. According to Corollary 1, it is observed that $q^D$ has a complicated influence on $\hat{\CP}^{SU}_{C}$ and $\hat{\CP}^{SU}_{D}$. Although $q^*_D$ can be numerically obtained, a closed-form solution is unobtainable. In the following, we consider two special cases such that the approximate $q^*_D$ can be derived in closed-form.

We first consider $\rho^S_{\text{dis}}^2$ in Corollary 1 satisfies $\rho^S_{\text{dis}}^2 \rightarrow 0$. This corresponds to the case, where
The intensities of cellular users and D2D transmitters are small and mode selection threshold is small. Under this condition, we give the approximate $q^*_D$ in the following theorem.

**Theorem 1.** In SCMA enhanced D2D underlaid cellular network, when $\rho^S_T^2 \tau^2_{\text{dis}} \rightarrow 0$, the optimal activated probability that maximizes the proportional fairness utility function is $q^*_D = 1$.

**Proof:** According to Corollary 1, when $\rho^S_T^2 \tau^2_{\text{dis}} \rightarrow 0$, the coverage probability at the typical D2D receiver using SCMA is approximated as $\hat{C}_{\text{DR}}^{SU} = \pi \xi \tau^2_{\text{dis}}$, which is due to the first order Taylor expansion of $e^{-\rho^S_T^2 \tau^2_{\text{dis}}}$. Therefore, the objective function defined in (22) turns into

$$q^*_D = \arg \max_{q_D} \log \left( \frac{q_D^S \lambda_U \lambda_{\text{BS}} \lambda_{\text{DT}} \pi \xi \tau^2_{\text{dis}} q_D}{Q_1 + Q_2 q_D} \right) \log (1 + \tau_{\text{BS}}) \log (1 + \tau_{\text{DR}}),$$

where we denote $Q_1 = \lambda_{\text{BS}} + \frac{q_D^S \lambda_U}{J} H y F_2$ and $Q_2 = \frac{2\pi \lambda_{\text{DT}} (\hat{\tau}_{\text{BS}})^2}{J \alpha \sin (\frac{\pi}{\alpha})} \prod_{n=2}^{NC} \left( \frac{2}{(n-1)\alpha} + 1 \right)$ for notation simplicity. As $\log (\cdot)$ is a monotonically increasing function, $q^*_D$ can be obtained by searching for the optimal $q_D$ to maximize $\bar{u}_U(q_D) = \frac{q_D}{Q_1 + Q_2 q_D}$. It is easy to prove that $\nabla \bar{u}_U(q_D) > 0$ and $\nabla^2 \bar{u}_U(q_D) < 0$ when $q_D \in [0, 1]$, where $\nabla \bar{u}_U(q_D)$ and $\nabla^2 \bar{u}_U(q_D)$ denote the first and second derivatives of $\bar{u}_U$, respectively. Therefore, $\bar{u}_U(q_D)$ is a concave function and an increasing function of $q_D$. As $q_D \in [0, 1]$, the optimal $q_D$ to maximize $\bar{u}_U(q_D)$ equals 1.

It can be seen from Theorem 1 that when $\rho^S_T^2 \tau^2_{\text{dis}} \rightarrow 0$, all the D2D transmitters should contend for available resources such that the proportional fairness utility function can be maximized. This coincides with the intuition. In particular, when cellular users and D2D users are sparsely distributed, spectrum resources cannot be fully exploited. Therefore, enabling full access of D2D users can boost the spectrum utilization of D2D network without significantly degenerating the performance of cellular network.

Next, we consider $\tau_{\text{dis}} \rightarrow \infty$. In this case, all the D2D users select D2D mode for data transmission. The optimal $q_D$ to maximize (22) is given by the following theorem.

**Theorem 2.** In SCMA enhanced D2D underlaid cellular network, when $\tau_{\text{dis}} \rightarrow \infty$, the optimal
activated probability to maximize (22) is given by

\[ q_D^* = \begin{cases} \sqrt{\frac{Q_1 Q_4}{Q_2 Q_3}}, & \text{if } Q_1 Q_4 < Q_2 Q_3, \\ 1, & \text{if } Q_1 Q_4 \geq Q_2 Q_3, \end{cases} \]

where \( Q_3 = \frac{2 \pi \tilde{\tau}_D \tilde{\tau}_D^*}{J_D \sin \left( \frac{2 \pi}{\alpha} \right)} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1)\alpha} + 1 \right) \) and \( Q_4 = \xi + \frac{2 \pi \tilde{\tau}_D \tilde{\tau}_D^*}{J_D \sin \left( \frac{2 \pi}{\alpha} \right)} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1)\alpha} + 1 \right) \).

Proof: When \( \tau_{\text{dis}} \to \infty \), all the D2D users select D2D mode, i.e., \( \mathbb{P}(\tau_{D,k} \leq \tau_{\text{dis}}) = 1 \).

According to Corollary 1, D2D network ASE degenerates into \( \hat{A}_D^{\text{SU}} = \frac{\lambda_D \xi}{Q_3 + Q_4 q_D} \log (1 + \tau_{\text{DR}}) \) and cellular network ASE is given by \( \hat{A}_C^{\text{SU}} = \frac{q_D^* \lambda_D \lambda_{\text{UT}}}{Q_1 + Q_2 q_D} \log (1 + \tau_{\text{BS}}) \). Since \( \log(\cdot) \) is a monotonically increasing function, the original optimization problem (22) can be converted into

\[ q_D^* = \arg \min_{q_D} \tilde{u}_U(q_D), \text{ where } \tilde{u}_U(q_D) = (Q_1 + Q_2 q_D) (Q_3 + Q_4 q_D^{-1}). \]

We obtain the second derivative of \( \tilde{u}_U(q_D) \) as \( \nabla^2 \tilde{u}_U(q_D) = \frac{2 Q_1 Q_4}{q_D^2} \). Since \( Q_1 > 0 \) and \( Q_4 > 0 \), it is obvious that \( \nabla^2 \tilde{u}_U(q_D) > 0 \) when \( q_D \in [0,1] \). According to (24), \( \tilde{u}_U(q_D) \) is a convex function of \( q_D \).

Consequently, the results in (24) can be obtained by solving \( \nabla \tilde{u}_U(q_D) = 0 \). \( \square \)

It is worth noting from Theorem 2 that \( q_D^* \) is an increasing function of the number of available codebooks \( J \). We intuitively explain the reason as follows. As earlier discussed, \( q_D \) is a parameter that can balance the cross-tier interference from D2D transmissions to cellular transmissions. According to Corollary 1, increasing \( q_D \) will improve the D2D network ASE. However, more cross-tier interference will be introduced, resulting in the decrease of cellular network ASE. Therefore, the proportional fairness utility function defined in (23) cannot be maximized by excessively increasing \( q_D \) when the interference from D2D network is overwhelming. Given small \( J \), the number of D2D users that reuse one codebook with cellular users is large. Consequently, \( q_D \) has been kept small in order to mitigate the cross-tier interference, thereby maximizing (23).

In contrast, when \( J \) is large, the interference becomes “sparse” over one codebook such that \( q_D \) can be moderately increased to maximize (23).

IV. D2D OVERLAIDED CELLULAR NETWORK

In this section, we consider the D2D overlaid cellular network, where \( J_C \) and \( J_D \) SCMA codebooks are allocated to cellular users and D2D users, respectively. Different from underlaid
mode, no cross-tier interference exists between cellular users and D2D users in the overlaid mode. Therefore, codebook allocation becomes more dominant in affecting the hybrid system performance. Here, we intend to find the optimal codebook allocation rule based on a given utility function

\[ J^*_C = \arg \max_{J_C} u_O (A^{SO}_C, A^{SO}_D), \quad (25) \]

where \( A^{SO}_C \) and \( A^{SO}_D \) are the ASEs of cellular network and D2D network, respectively, and

\[ u_O (A^{SO}_C, A^{SO}_D) = \log A^{SO}_C + \log A^{SO}_D. \quad (26) \]

Note that optimizing \( J_C \) is equivalent to optimizing \( J_D \) in (25), since \( J = J_C + J_D \). In order to achieve the optimization goal, we first determine \( A^{SO}_C \) and \( A^{SO}_D \) according to the following corollary.

**Corollary 2.** Considering that SCMA is used in D2D overlaid cellular network, the ASE achieved by cellular network is given by

\[ A^{SO}_C = \hat{q}_U \lambda_{UT} C^{SO}_{BS} \log (1 + \tau_{BS}), \]

where \( C^{SO}_{BS} \) is approximated as

\[ C^{SO}_{BS} \approx \frac{\lambda_{BS}}{\lambda_{BS} + \hat{q}_U \lambda_{UT} H y F}. \quad (27) \]

In (27), \( \hat{q}_U \) is calculated by substituting \( N_R \) with \( J_C \) in (2). The ASE achieved by D2D network is given by

\[ A^{SO}_D = \lambda_{DT} C^{SO}_{DR} \log (1 + \tau_{DR}), \]

where \( C^{SO}_{DR} \) is approximated as

\[ C^{SO}_{DR} \approx \frac{\pi \xi}{\rho_O} \left( 1 - e^{-\rho_O d^2} \right). \quad (28) \]

In (28), \( \rho_O = \pi \xi + \frac{2\pi^2 \delta^2}{J_{DR} \sin(\frac{\alpha}{2})} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1)\alpha} + 1 \right) \).

**Proof:** We describe the sketch of the proof. When codebooks are partially allocated to cellular users and D2D users, respectively, no cross-tier interference exists between them. Therefore,
the SIR at BS$_0$ is given by

$$\text{SIR}^\text{SU}_{\text{BS}_0} = \frac{\sum_{m=1}^{N_C} P_U^{\text{f}} U_{0,m}^{-\alpha} \| h_{\text{BS}_0, U_{0,m}} \|^2}{\tilde{I}^\text{S}_{C,C}},$$

where $\tilde{I}^\text{S}_{C,C}$ is the inter-cell interference from active cellular users outside the coverage of BS$_0$. Note that $\tilde{I}^\text{S}_{C,C}$ is different from $I^\text{S}_{C,C}$ in (14), as only $J_C$ codebooks are allocated to cellular users. Using similar approach as (38) in Appendix B, the coverage probability in (27) can be obtained and hence cellular network ASE is derived. Similarly, D2D network ASE can be derived as well.

The detail of the derivation steps is omitted due to space limitation.

According to (9), $\tilde{q}^\text{S}_U$ in (27) is a function of $J_C$. Therefore, the influence of codebook allocation in the overlaid scenario also has complicated impact on the ASE. In consequence, it is hard to obtain the closed-form expression for $J^*_C$. Next, we consider the case, where cellular users are densely deployed such that $\lambda_{UT} \gg J_C \lambda_{BS}$. Note that it is well accepted that the dense scenario is a common scenario in future wireless networks. Under this condition, a proportion of cellular users would be inactivated due to the unavailability of SCMA codebooks even if all the codebooks are allocated to cellular network. In this case, the density of active cellular users is $J_C \lambda_{BS}$. Replacing $\tilde{q}^\text{S}_U \lambda_U$ with $J_C \lambda_{BS}$ in Corollary 2, cellular network ASE degenerates into

$$\tilde{A}^\text{SO}_C = \frac{J_C \lambda_{BS} \log(1 + \tau_{BS})}{H y F_2 + 1}. \quad (29)$$

Besides, if D2D users select cellular mode for data transmission, they will be blocked with a higher probability due to the dense deployment of cellular users. Therefore, we enable all the D2D users to select D2D mode by setting $\tau_{\text{dis}} \rightarrow \infty$. In this case, $J^*_C$ can be derived according to the following theorem.

**Theorem 3.** We consider an SCMA enhanced D2D overlaid cellular network, where cellular users are densely deployed. When $\tau_{\text{dis}} \rightarrow \infty$, the optimal $J_C$ that maximizes the proportional...


fairness utility function defined in (26) is given by

\[ J^* = \text{round} \left( J + Q_6 - \sqrt{Q_6^2 + JQ_6} \right), \]  

(30)

where \( Q_6 = \frac{2\pi \tilde{\tau}_D \lambda_D T}{\xi \alpha \sin \left( \frac{2\pi}{n} \right)} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1)\alpha} + 1 \right) \) and round (·) denotes the round operation.

**Proof:** According to Corollary 2, when \( \tau_{\text{dis}} \to \infty \), \( \text{CP}^\text{SO} \) in (28) degenerates into

\[ \text{CP}^\text{SO} = \frac{\xi}{\xi + \frac{2\pi \tilde{\tau}_D \lambda_D T}{J_D \alpha \sin \left( \frac{2\pi}{n} \right)} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1)\alpha} + 1 \right) \}. \]  

(31)

Based on (29) and (31), the utility function in (26) turns into

\[ u_O (A^\text{SO}_C, A^\text{SO}_D) = \log A^\text{SO}_C + \log A^\text{SO}_D, \]  

(32)

where \( A^\text{SO}_C = J_C Q_5 \) and \( A^\text{SO}_D = \frac{\lambda_D T \log (1 + \tilde{\tau}_{\text{DR}})}{1 + \frac{Q_6}{J_C}} \). For notation simplicity, we denote \( Q_5 = \frac{\lambda_{BS} \log (1 + \tau_{\text{BS}})}{H y F + 1} \) and \( Q_6 = \frac{2\pi \tilde{\tau}_D \lambda_D T}{\xi \alpha \sin \left( \frac{2\pi}{n} \right)} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1)\alpha} + 1 \right) \). Accordingly, we rewrite (32) as

\[ u_O (A^\text{SO}_C, A^\text{SO}_D) = \log \left( \frac{Q_5 \lambda_D T \log (1 + \tilde{\tau}_{\text{DR}}) J_C}{1 + \frac{Q_6}{J_C}} \right). \]  

(33)

In (33), \( Q_5 \lambda_D T \log (1 + \tilde{\tau}_{\text{DR}}) > 0 \) and \( \log (\cdot) \) is a monotonically increasing function. Therefore, maximizing (33) is equivalently to maximizing \( \tilde{u}_O (J_C) = \frac{J_C}{1 + \frac{Q_6}{J_C}} \). We then calculate the second derivative of \( \tilde{u}_O (J_C) \) as \( \nabla^2 \tilde{u}_O (J_C) = -\frac{2Q_6 (J + Q_6)}{(1 + \frac{Q_6}{J_C})^3 (J - J_C)^3} \). It is easy to check that \( \nabla^2 \tilde{u}_O (J_C) < 0 \) when \( J_C \in [0, J] \). Hence, \( \tilde{u}_O (J_C) \) is concave function of \( J_C \). The optimal \( J_C \) to maximize \( \tilde{u}_O (J_C) \) can be derived by solving \( \nabla \tilde{u}_O (J_C) = 0 \).

Note from (30) in Theorem 3 that the optimal \( J_C \) that maximizes the proportional fairness utility function is a function of parameters of D2D network rather than those of cellular network. This is due to the assumption that cellular users are densely deployed. According to (30), cellular network ASE \( \tilde{A}^\text{SO}_C \) increases linearly with \( J_C \). In other words, once one more codebook is provided, the codebook can be fully utilized by the ultra-densely deployed cellular users, thereby linearly improving \( \tilde{A}^\text{SO}_C \). Therefore, cellular network parameters will not affect the optimal \( J_C \).
when we target at maximizing the utility function defined based on proportional fairness in (26).

Note that the codebook allocation rule, which is studied in this section, is a non-real-time method, since we have used the tools of stochastic geometry to evaluate the average system performance. Therefore, designing a real-time method is out of the scope of this paper.

V. NUMERICAL RESULTS

Numerical results and Monte Carlo simulation results have been provided in this section to demonstrate the performance of the SCMA enhanced D2D and cellular hybrid network. Default system parameters are set as $K = 20$, $N_C = 2$, $\alpha = 4$, $\tau_{BS} = \tau_{DR} = 10$ dB, $P_U = 20$ dBmW and $P_D = 20$ dBmW. Note that although we consider that $K = 20$ OFDMA tones are available, it is practically infeasible to design $C_{20}^2$ codebooks due to the limitation of number of available constellations. Nonetheless, it is practical to design 6 codebooks out of 4 OFDMA tones according to the mapping shown in Fig. 2. Therefore, $J = 30$ codebooks are available in simulations. Note that numerical results and simulation results are drawn by lines and markers, respectively, in the following figures.

Fig. 3 shows the cellular network ASE, D2D network ASE and system ASE when code division multiple access (CDMA), OFDMA and SCMA are applied, respectively, as a function
of the intensity of cellular users and intensity of D2D users. Note that the ASE analysis of the CDMA system is based on the results in [25]. It is obvious from Fig. 3a (Fig. 3b) that cellular (D2D) network ASE first increases and then keeps stable as the intensity of cellular (D2D) users increases. We take the case in Fig. 3a for example. Given small $\lambda_u$, more cellular users can be served when $\lambda_u$ increases. Therefore, spectrum resources can be better exploited by admitting more cellular users into the system. When $\lambda_u$ is sufficiently large, limited number of cellular users can be served due to the association rule such that cellular network ASE hardly increases. In this case, SCMA significantly outperforms OFDMA due to the overloading gain harvested by SCMA, i.e., more orthogonal resources (codebooks) can be provided by SCMA compared to OFDMA. Therefore, more cellular users can be kept active. Besides, we see that the performance of the CDMA network is worse than that of the OFDMA network. The reason can be explained as follows. Using CDMA, interference is averaged out, as CDMA signals are spread over all the available tones. Nonetheless, the interference becomes overwhelming as the user density increases. Hence, when cellular users are densely deployed, it is preferable to use OFDMA, where orthogonal resources are separately allocated to cellular users such that no inter-user interference exists within one cell. In addition, it is observed that little gaps exist between numerical results and simulation results when SCMA is applied. As discussed in Section III-A, we use an exponentially distributed random variable to approximate the small scale fading, which is Gamma distributed. The approximation can provide high accuracy, as we select $N_C = 2$, which is the typical SCMA setting.

It is shown from Fig. 3a and Fig. 3b that the asymptotic system ASE gains of SCMA over OFDMA can reach $\frac{A_{SU} + A_{SU}^D}{A_{SU}^U + A_{SU}^D}$, $\lambda_u \to \infty$ 138.74% and $\frac{A_{SU} + A_{SU}^D}{A_{SU}^U + A_{SU}^D}$, $\lambda_D \to \infty$ 140.12%, respectively, under the given system parameters. From the above results, we conclude that SCMA can serve as an efficient multiple access scheme in the D2D hybrid cellular network to enable massive connectivity.

Fig. 4 shows the ASE as a function of the mode selection threshold $\tau_{dis}$ in the underlaid mode and overlaid mode. When $\tau_{dis} = 0$ m, all the D2D users select cellular mode. In consequence, better ASE performance can be achieved by the underlaid mode. This is obvious since more
codebooks are available in the underlaid mode. When $\tau_{\text{dis}}$ increases, more D2D users select D2D mode for data transmission. Therefore, it is clear from Fig. 4 that D2D network ASE increases with $\tau_{\text{dis}}$. In contrast, cellular network ASE decreases with $\tau_{\text{dis}}$ in both coexisting modes, since the available resources cannot be fully exploited when the number of cellular users decreases. Besides, as the number of active D2D transmitters increases with $\tau_{\text{dis}}$, the cross-tier interference from D2D network to cellular network becomes overwhelming with the increasing $\tau_{\text{dis}}$ in the underlaid mode. Therefore, cellular network ASE is degraded faster in the underlaid mode. As a result, the overlaid mode outperforms the underlaid mode in terms of system ASE performance when $\tau_{\text{dis}}$ is large under this system setting.

Fig. 5 compares the ASE gain $\hat{\eta}_{\text{ASE}}$ defined in (19) and overloading factor $\eta_{\text{overload}}$ defined in (4) as a function of the number of available OFDMA tones in the system. In order to make a direct and comprehensive comparison, we simulate the heavily loaded cellular network, where only cellular users exist with $\lambda_U \gg J\lambda_{\text{BS}}$ at each $K$ and no D2D users are activated. It should be noted that we have assumed $C^{N_C}_K$ codebooks can be designed for the comparison. It can be seen that, although ASE gain is always smaller than the overloading factor, the overloading gain can be almost achieved even when codebook reuse is enabled. The reduction of overloading
Figure 5. ASE gain v.s. overloading factor. System parameters are set as $\lambda_{BS} = 5 \times 10^{-5}$ BSs/m$^2$ and $\lambda_{D} = 0$ users/m$^2$.

Figure 6. The impact of D2D activated probability in D2D underlaid cellular network. System parameters are set as $\lambda_{BS} = 5 \times 10^{-5}$ BSs/m$^2$, $\lambda_{U} = 1 \times 10^{-3}$ users/m$^2$ and $\lambda_{D} = 2.5 \times 10^{-3}$ users/m$^2$.

Gain primarily stems from the inter-cluster interference, which is caused by codebook reuse in different cells. In addition, it is shown that the ASE gain increases linearly with $K$, which is optimistic. Nevertheless, the result is based on the assumption that the number of codebooks increases with the number of OFDMA tones as $J = \binom{NC}{K}$. This is achieved by searching for $K-1$ constellations at each $K$ according to $[13, 14]$. Unfortunately, the number of constellations is limited due to practical concerns. Consequently, the performance improvement of SCMA over OFDMA is thus limited.
We next explore the impact of D2D activated probability $q_D$ on the performance of underlaid system in Fig. 6. Fig. 6a shows the cellular network ASE, D2D network ASE and system ASE as a function of $q_D$. The tradeoff between cellular network ASE and D2D network ASE caused by $q_D$ is clearly presented in Fig. 6a, namely, cellular network ASE and D2D network ASE, respectively, decreases and increases with the increasing $q_D$. Fig. 6b shows the utility function defined in (23) as a function of $q_D$. Particularly, when $\tau_{\text{dis}} \to \infty$, the utility function is an increasing function of $q_D$ such that the optimal $q_D$ equals 1, as indicated by Theorem 1. Otherwise, when $\tau_{\text{dis}}$ is small, it is shown that $q_D^*$ increases with $\xi$. This is because a larger $\xi$ will lead to a smaller average D2D link length. Therefore, increasing $\xi$ would improve $\hat{A}^*_{SU_D}$, thereby increasing the proportional fairness utility function.

We finally investigate the performance of the D2D overlaid cellular network when different codebook allocation schemes are applied in Fig. 7. Fig. 7a shows the cellular network ASE, D2D network ASE and system ASE as a function of the number of codebooks allocated to cellular network. It is shown that allocating more codebooks to cellular (D2D) network would enhance the ASE of the corresponding network. This is due to the resource allocation rule introduced in Section II-C. In particular, each cellular (D2D) user is randomly assigned with one of the available codebooks. If more codebooks are allocated, one codebook will be reused by less users,
thereby resulting in less interference over one codebook. Meanwhile, Fig. 7b shows the utility function defined in (26) as a function of the number of codebooks allocated to cellular users $J_C^*$. It is shown from the comparison between numerical results and simulation results that $J_C^*$ obtained in Theorem 3 is of high accuracy.

VI. Conclusion

In this paper, we have presented a stochastic geometry based framework to investigate the performance of SCMA in D2D and cellular hybrid network, considering underlaid mode and overlaid mode. In the underlaid mode, we analytically compared SCMA with OFDMA from the perspective of ASE and quantified the ASE gain of SCMA over OFDMA in closed form. Though approximations are used, it has been shown that high accuracy can be provided. More importantly, we concluded via the comparison results that SCMA is capable of supporting massive D2D connections, as well as significantly enhancing system ASE especially in the heavily loaded network. Therefore, SCMA can be considered as a candidate of effective multiple access schemes in future 5G wireless networks. In addition, spectrum sharing in the two coexisting modes has been studied. Specifically, the optimal D2D activated probability has been derived in the underlaid mode and the optimal codebook allocation rule has been obtained in the overlaid mode. In both cases, the optimization targets are to maximize the proportional fairness utility function. The results can serve as a guide to help tune system design parameters in SCMA enhanced D2D and cellular hybrid network.

Appendix

A. Proof for Proposition 7

According to (10), we have

$$CP_{BS}^{OU} = \mathbb{P}\left\{ \frac{P_U r_{U,0}^2}{I_{C,C}^O + I_{C,D}^O} > \tau_{BS} \right\} = \mathbb{P}\left\{ \frac{\|h_{BS_0,U_0}\|^2}{P_U} > \frac{\tau_{BS} r_{U,0}^2}{I_{C,C} + I_{C,D}} \right\} = \mathbb{L}_{L_{C,C}^O (s_{BS}^O)} L_{L_{C,D}^O (s_{BS}^O)},$$

(34)
where (a) follows due to \( \| h_{BS_0,U_0} \| ^2 \sim \exp (1) \). \( L_{I_{C,C}^O} (s_{BS}^O) \) and \( L_{I_{C,D}^O} (s_{BS}^O) \), respectively, denote the Laplace transforms of \( I_{C,C}^O \) and \( I_{C,D}^O \) evaluated at \( s_{BS}^O = \frac{r_{BSr_0,U_0}}{P_U} \). We evaluate \( L_{I_{C,C}^O} (s_{BS}^O) \) as

\[
L_{I_{C,C}^O} (s_{BS}^O) = \mathbb{E} \left[ \prod_{U \in \mathcal{U}^T} \frac{1}{1 + s_{BS}^O P_U \| U - BS_0 \|^\alpha} \right]
\]

\[
\begin{align*}
&= \exp \left[ -2 \pi q_U^O \lambda \frac{\lambda UT}{K} \int_{r_{U,0}}^{\infty} l \left( 1 - \frac{1}{1 + s_{BS}^O P_U l^{-\alpha}} \right) dl \right] \\
&= \exp \left[ - \frac{2 \pi q_U^O \lambda UT s_{BS}^O P_U}{K (\alpha - 2)} r_{U,0}^{\alpha - 2} F_1 (1, 1 - \delta, 2 - \delta, -r_{U,0}^{-\alpha} s_{BS}^O P_U) \right], \quad (35)
\end{align*}
\]

where (a) follows due to \( \| h_{BS_0,U_1} \| ^2 \sim \exp (1) \) and (b) follows due to the probability generating functional (PGFL) of PPP and cellular association rule that \( U_0 \) connects to the nearest BS so that interfering cellular users are at least \( r_0 \) away. Therefore, \( L_{I_{C,C}^O} \left( \frac{r_{BSr_0,U_0}}{P_U} \right) \) can be obtained by substituting \( s_{BS}^O = \frac{r_{BSr_0,U_0}}{P_U} \) into (35). Following similar approach, it is easy to derive

\[
L_{I_{C,D}^O} \left( \frac{r_{BSr_0,U_0}}{P_U} \right) = \exp \left[ - \frac{2 \pi^2 \lambda_D T \delta r_{U,0}^2}{K \alpha \sin \left( \frac{\pi \alpha}{2} \right)} \right].
\]

Due to the association rule of cellular users, the CDF of \( r_{U,0} \) can be obtained according to the contact distribution \([19]\), i.e., \( F_{r_{U,0}} (x) = 1 - \exp (-\pi \lambda_{BS} x^2) \) \( x \geq 0 \). Thus, the PDF of \( r_{U,0} \) can be accordingly obtained as \( f_{r_{U,0}} (x) = 2 \pi \lambda_{BS} x \exp (-\pi \lambda_{BS} x^2) \) \( x \geq 0 \). Combining \( L_{I_{C,C}^O} \left( \frac{r_{BSr_0,U_0}}{P_U} \right) \) and \( L_{I_{C,D}^O} \left( \frac{r_{BSr_0,U_0}}{P_U} \right) \) and taking the expectation of \( r_{U,0} \), we derive the coverage probability at the typical cellular BS. Using similar approach, we have

\[
CP_{OU}^{QU} = \exp \left[ - \frac{2 \pi^2 \delta \lambda_D T_D}{K \alpha \sin \left( \frac{\pi \alpha}{2} \right)} \left( \lambda_D T + q_U^O \lambda UT \left( \frac{P_U}{P_D} \right)^\delta \right) \right]. \quad (36)
\]

By taking the expectation of \( r_{D,0} \) in (36) according to the PDF given in (1), we derive the coverage probability at the typical D2D receiver. Note that \( r_{D,0} \in [0, \tau_{dis}] \), since mode selection is applied by D2D transmitters.

B. Proof for Proposition [2]

We denote \( \sum_{m=1}^{N_C} \| h_{BS_0,U_0,m} \| ^2 \) by \( G_{BS_0,U_0} \). Since \( \| h_{BS_0,U_0,m} \| ^2 \sim \exp (1) \), \( G_{BS_0,U_0} \) follows Gamma distribution, i.e., \( G_{BS_0,U_0} \sim \text{Gamma} (N_C, 1) \), which makes it difficult to derive the
explicit-form result of $CP_{BS}^{SU}$. In order to provide analytical tractability, we use a random variable $W_{BS_0,U_0}$ with mean $N_C$, i.e., $W_{BS_0,U_0} \sim \exp\left(\frac{1}{N_C} \right)$ to approximate $G_{BS_0,U_0}$. Note that $G_{BS_0,U_0}$ and $W_{BS_0,U_0}$ have the same first moment. Therefore, we have the approximation

$$CP_{BS}^{SU} \approx \mathbb{P}\left( \frac{P_{U_0}^{\dagger} r_{BS_0,U_0}^{-\alpha} W_{BS_0,U_0}}{I_{C,C}^{\delta} + I_{C,D}^{\delta}} > \tau_{BS} \right) = \mathcal{L}_{I_{C,C}^{\delta}}(s_{BS}^{\delta}) \mathcal{L}_{I_{C,D}^{\delta}}(s_{BS}^{\delta}),$$

(37)

where $\mathcal{L}_{I_{C,C}^{\delta}}(s_{BS}^{\delta})$ and $\mathcal{L}_{I_{C,D}^{\delta}}(s_{BS}^{\delta})$, respectively, denote the Laplace transforms of $I_{C,C}^{\delta}$ and $I_{C,D}^{\delta}$ evaluated at $s_{BS}^{\delta} = \frac{\tau_{BS} r_{U_0}^{\alpha}}{N_C P_{U}^{\delta}}$. We denote $\sum_{m=1}^{N_C} \| h_{BS_0,U_i,m} \|^2$ as $G_{BS_0,U_i}$ and calculate $\mathcal{L}_{I_{C,C}^{\delta}}(s_{BS}^{\delta})$ as

$$\mathcal{L}_{I_{C,C}^{\delta}}(s_{BS}^{\delta}) = \mathbb{E}\left[ \exp\left( - \sum_{U_i \in \Pi_{UT}^{\delta}} s_{BS}^{\delta} P_{U}^{\dagger} G_{BS_0,U_i} \| U_i - BS_0 \|^{-\alpha} \right) \right]$$

$$= \mathbb{E}\left[ \prod_{U_i \in \Pi_{UT}^{\delta}} \frac{1}{1 + s_{BS}^{\delta} P_{U}^{\dagger} \| U_i - BS_0 \|^{-\alpha}} \right]^{N_C}$$

$$= \exp\left[ -2\pi \frac{q_{U_0}^{\delta} \lambda_{UT}}{J} \int_{r_{U_0,0}}^{\infty} l \left( 1 - \frac{1}{1 + s_{BS}^{\delta} P_{U}^{\dagger} l^{-\alpha}} \right)^{N_C} dl \right]$$

$$= \exp\left[ -\frac{\pi q_{U_0}^{\delta} \lambda_{UT}}{r_{U_0,0}^{2} J} \left( 2F_1\left( N_C, -\delta, 1 - \delta, -r_{U_0,0}^{\delta} s_{BS}^{\delta} P_{U}^{\dagger} - 1 \right) - 1 \right) \right],$$

(38)

where (a) is due to the PGFL of PPP. Likewise, we calculate $\mathcal{L}_{I_{C,D}^{\delta}}(s_{BS}^{\delta})$ as

$$\mathcal{L}_{I_{C,D}^{\delta}}(s_{BS}^{\delta}) = \mathbb{E}\left[ \exp\left( - \sum_{DT_j \in \Pi_{DT}^{\delta}} s_{BS}^{\delta} P_{D}^{\dagger} G_{BS_0,DT_j} \| DT_j - BS_0 \|^{-\alpha} \right) \right]$$

$$= \mathbb{E}\left[ \prod_{DT_j \in \Pi_{DT}^{\delta}} \frac{1}{1 + s_{BS}^{\delta} P_{D}^{\dagger} \| DT_j - BS_0 \|^{-\alpha}} \right]^{N_C}$$

$$= \exp\left[ -2\pi \frac{\lambda_{DT}}{J} \int_{0}^{\infty} l \left( 1 - \frac{1}{1 + s_{BS}^{\delta} P_{D}^{\dagger} l^{-\alpha}} \right)^{N_C} dl \right]$$

$$= \exp\left[ -\frac{2\pi^2 \lambda_{DT}}{Jo} \left( s_{BS}^{\delta} P_{D}^{\dagger} \frac{r}{\alpha} \right)^{\delta} \prod_{n=2}^{N_C} \left( \frac{2}{(n-1) \alpha} + 1 \right) \right],$$

(39)
where (a) follows because all D2D transmitters are active.

Substituting $s^S_{BS} = \tau_{BS}^\alpha r_{U,0} \tau_{UT}^2$ into (38) and (39), we have

$$L_{\hat{I}_{C,0}}^L \left( \frac{\tau_{BS}^\alpha r_{U,0} \tau_{UT}^2}{N_C P_U^D} \right) = \exp \left[ -\pi \frac{q_{U}^S \lambda_{UT}}{J} r_{U,0} \left( 2 F_1 \left( N_C, -\delta, 1 - \delta, -\frac{\tau_{BS}}{N_C} \right) - 1 \right) \right]. \quad (40)$$

$$L_{\hat{I}_{D,0}}^L \left( \frac{\tau_{BS}^\alpha r_{U,0}}{N_C P_U^D} \right) = \exp \left[ -\frac{2\pi^2 \lambda_{DT} \left( \frac{\tau_{BS}^\alpha r_{U,0} \tau_{UT}^2}{N_C P_U^D} \right) \delta}{J \alpha \sin \left( \frac{2\pi}{\alpha} \right)} r_{U,0}^N \prod_{n=2}^{\infty} \left( \frac{2}{(n - 1) \alpha} + 1 \right) \right]. \quad (41)$$

The PDF of $r_{U,0}$ can be obtained in Appendix A. Therefore, we derive the approximate coverage probability at the typical BS by combining the results of (40) and (41) into (37) and taking the expectation of $r_{U,0}$.

Using similar approach, we approximate the coverage probability at the typical D2D receiver and then complete the proof. The detail of derivation steps is omitted due to space limitation.

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