Rotating Nuclei at Extreme Conditions: 
Cranked Relativistic Mean Field Description 

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The cranked relativistic mean field (CRMF) theory is applied for the description of superdeformed (SD) rotational bands observed in $^{153}$Ho. The question of the structure of the so-called SD band in $^{154}$Er is also addressed and a brief overview of applications of CRMF theory to the description of rotating nuclei at extreme conditions is presented.

1. INTRODUCTION

CRMF theory [1,2] represents the extension of relativistic mean field (RMF) theory to the rotating frame and thus provides a natural framework for the description of rotating nuclei at high spin. Available experimental data on rotating nuclei at extreme conditions of large deformation (superdeformation) and fast rotation in different mass regions allow to test the theoretical models (in our case the CRMF theory) in physical situations where pairing correlations are expected to play no or only a minor role. This is an especially important point considering the fact that in the framework of CRMF theory a consistent theoretical description of pairing correlations including fluctuations by number projection is still in a stage of development.

Thus a systematic study of SD bands within CRMF theory has been undertaken. Detailed investigations have been performed in the $A \sim 140 - 150$ [2,3] and in the $A \sim 60$ [4] mass regions. Experimental observables as dynamic moments of inertia $J^{(2)}$, kinematic moments of inertia $J^{(1)}$ in the $A \sim 60$ mass region, absolute ($Q_0$) and relative ($\Delta Q_0$) charge quadrupole moments, effective alignments $i_{\text{eff}}$ and the single-particle ordering in the SD minimum (derived from the analysis of effective alignments) have been confronted with results of CRMF calculations without pairing. It was shown that this theory provides in general good agreement with available experimental data.

All these results give us strong confidence that CRMF theory can be a powerful tool both for the interpretation of experimental data and for the microscopic understanding of the behaviour of rotating nuclei at extreme conditions. Considerable disagreement with experiment has so far only been found in the case of the ‘SD’ band in $^{154}$Er [4]. In the present article, we report on investigations on the structure of SD bands observed recently in $^{153}$Ho [4] with the aim to understand better the origin of the discrepancies found in the $^{154}$Er case.

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Figure 1. (a) Dynamic $J^{(2)}$ moments of inertia of observed bands (linked symbols) versus the ones of calculated configurations. Note that $J^{(2)}$(153Ho(2)) is similar to $J^{(2)}$(152Dy(1)). (b) Experimental (symbols) and calculated (lines) effective alignments (in units of $\hbar$). The effective alignment between bands A and B is defined in Ref. [11] as $i_{A,B}^{\text{eff}}(\Omega_x) = I_B(\Omega_x) - I_A(\Omega_x)$. The band A in the lighter nucleus is taken as a reference, so the effective alignment measures the effect of the additional particle(s). The experimental effective alignment between bands A and B is indicated as “A/B”. The compared configurations differ in the occupation of the orbitals indicated in the figure. The dotted line shows the calculated effective alignment of the $\pi[770]1/2(r=+i)$ orbital below the crossing. The lowest transitions in the observed bands with the transition energies of 602.4 keV (152Dy(1)), 651.3 keV (153Ho(1)), 713.0 keV (153Ho(2)) and 657.0 keV (153Ho(3)) correspond to a spin change $26^+ \rightarrow 24^+$, $29^5^- \rightarrow 27^5^-$, $30^5^- \rightarrow 28^5^-$ and 28.5 $\rightarrow$ 26.5, respectively. Note that in the present formalism only the relative spins are “determined” so shifts of all bands in steps of $\pm 2\hbar$ could not be excluded, see Ref. [5] for details.

2. The nuclei 153Ho and 154Er.

The nucleus 153Ho. Three SD bands have been observed in 153Ho [1]. Their structure, as it follows from CRMF calculations with the NL1 force [12], is discussed below. Considering the large size of the SD shell gaps at $Z = 66$ and $N = 86$, which follows from the doubly magic nature of the 152Dy SD core (conf. $\pi^6v^7\nu^2$) [13,14], the occupation of neutron orbitals in the considered configurations is kept as in the 152Dy SD core. Then the configurations based on different occupations of the proton orbitals by the 67th proton have been calculated. As a result, they are labelled by the proton orbital occupied above the $Z = 66$ SD shell gap.

Band 1. This band undergoes a band crossing at a frequency $\Omega_x \sim 0.6$ MeV, where a large increase in $J^{(2)}$ is observed (Fig. 1a). Such a crossing appears also in the lowest
bands with small signature splitting should be observed if these orbitals are occupied. Thus signature partner argument against the interpretation of the observed bands as based on these orbitals comes due to both the deficiencies of the cranking model and the fact that the calculations have been carried out as a function of rotational frequency but not as a function of spin. The calculated gain in alignment at crossing is very close to the measured one and it would be in perfect agreement with experiment if the crossing would have been calculated at the experimental crossing frequency. The same interpretation of this band has been obtained also in cranked Woods-Saxon calculations at fixed deformation \([11]\).

**Band 2.** According to the CRMF calculations, we can assign to this band the configuration \([530]1/2(r = +i)\). Assuming this assignment, the experimental values of \(J^{(2)}\) and \(i_{\text{eff}}\) are reasonably well reproduced (see Fig. 1). This assignment corresponds to the one discussed in Ref. \([11]\). Additional confirmation of the interpretation of bands 1 and 2 could be obtained by a precise measurement of the charge quadrupole moments \(Q_0\) relative to the ones of the \(^{152}\text{Dy}(1)\) band. According to the calculations, the occupation of the \([530]1/2(r = +i)\), \([530]1/2(r = -i)\) and \([770]1/2(r = +i)\) orbitals leads to an increase of \(Q_0\) by 0.60 \(\text{eb}\), by 0.65 \(\text{eb}\) (both values are calculated at \(\Omega_x = 0.5\ \text{MeV}\)) and by 1.15 \(\text{eb}\) (calculated at \(\Omega_x = 0.8\ \text{MeV}\)), respectively.

**Band 3.** The features of this band are difficult to explain assuming that the changes of the physical observables with respect to the ones of the \(^{152}\text{Dy}(1)\) band should be governed by an additional proton. At high rotational frequencies, the \(J^{(2)}\) moment of inertia drops considerably below that of the \(^{152}\text{Dy}(1)\) band. This drop is accompanied by the loss in effective alignment \(i_{\text{eff}}\) of \(\approx 0.8\hbar\) in the \(\Omega_x = 0.51 - 0.68\ \text{MeV}\) range (Fig. 1). It was suggested in Ref. \([11]\) that the occupation of the \([523]7/2(r = -i)\) orbital by 67th proton could lead to such features. It seems that this interpretation can be ruled out since the calculated loss of alignment of \(\approx 0.2\hbar\) arises from the interaction between the \((r = -i)\) signatures of the \([523]7/2\) and the \([530]1/2\) orbitals. However, the configuration with the \([530]1/2(r = -i)\) orbital occupied is assigned to band 2 which does not show an increase neither in \(J^{(2)}\) nor in \(i_{\text{eff}}\) expected from such an interaction. A consistent interpretation of this band within a pure single-particle picture is not found in the CRMF calculations either. For example, the effective alignment of the \([532]5/2(r = -i)\) orbital located above the \(Z = 66\ \text{SD shell gap}\) (see Fig. 4 in Ref. \([4]\)) is shown in Fig. 1b. The calculated \(J^{(2)}\) moment of inertia of this configuration is very close to the one of the configuration assigned to band 2 (Fig. 1a). Although the results of calculations are reasonably close to experiment at low frequencies, the loss of \(i_{\text{eff}}\) and the drop in \(J^{(2)}\) at higher frequencies are not reproduced.

The occupation of positive parity \([413]5/2, [404]9/2\) and \([411]3/2\) orbitals located above the \(Z = 66\ \text{SD shell gap}\) (Fig. 4 in Ref. \([3]\)) has also been considered. The strongest argument against the interpretation of the observed bands as based on these orbitals comes from the fact that these orbitals have a small signature splitting. Thus signature partner bands with small signature splitting should be observed if these orbitals are occupied.
The nucleus $^{154}$Er. One band has been observed in $^{154}$Er and it has been discussed as SD \[ \square \]. Two specific features of this band are (i) the $J^{(2)}$ moment of inertia at high frequencies is much lower than the one of the $^{152}$Dy(1) band (Fig. 1a), (ii) the effective alignment in the $^{152}$Dy(1)/$^{154}$Er(1) pair drops by $\approx 2.1\hbar$ in the frequency range $\Omega_x = 0.37 - 0.65$ MeV. These features strongly suggest that this band has a smaller number of high-$N$ orbitals occupied (and thus is less deformed) than the $^{152}$Dy(1) band. Considering available single-particle orbitals above the $Z = 66$ SD shell gap and their impact on physical observables (as deduced from the analysis of $^{153}$Ho), it is clear that this band cannot be described as a 'doubly magic $^{152}$Dy core + 2 additional protons' system. Indeed, the results of calculations for $J^{(2)}$ and $i_{eff}$ of the lowest SD configurations in this nucleus disagree considerably with experiment. The possibility that the observed band belongs to a highly-deformed triaxial minimum predicted in Ref. \[ \square \] has also been checked. Such a minimum with $Q_0 \sim 10\,\text{eb}$ and $\gamma \sim 9^\circ$ exists in CRMF calculations too and it is lower in energy than the SD minimum at $I < 60\hbar$. Fig. 1a shows the $J^{(2)}$ moment of inertia of one of the configurations ($\pi 6^1\nu 6^4(+,-1)$) calculated in this minimum. Although there still is disagreement with experiment, the discrepancy is somewhat smaller than in the case of the SD configurations. However, it is difficult to present a specific configuration assignment for the observed band. The measurements of the transition quadrupole moment of this band will help to resolve the existing problem.

3. Conclusions

CRMF theory has been applied for the study of SD bands observed in $^{153}$Ho. Bands 1 and 2 are reasonably well described, while it was difficult to get a consistent interpretation for band 3 in a pure single-particle picture. Based on these results it was concluded that the band observed in $^{154}$Er and previously discussed as SD is very likely less deformed than the $^{152}$Dy(1) band. A.V.A. acknowledges support from the Alexander von Humboldt Foundation. This work is also supported in part by the Bundesministerium f"ur Bildung und Forschung under the project 06 TM 875.

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