Testing Scenario Library Generation for Connected and Automated Vehicles, Part I: Methodology

Shuo Feng, Yiheng Feng, Chunhui Yu, Yi Zhang, Member, IEEE, and Henry X. Liu, Member, IEEE

Abstract—Testing and evaluation is a critical step in the development and deployment of connected and automated vehicles (CAVs), and yet there is no systematic framework to generate testing scenario library. This paper aims to provide a general framework to solve the testing scenario library generation (TSLG) problem for different scenario types, CAV models, and performance metrics. In part I of the paper, four research questions are identified: (1) scenario description, (2) metric design, (3) library generation, and (4) CAV evaluation. To answer these questions, a unified framework is proposed. First, the operational design domain of CAVs is considered for scenario description and decision variable formulation. Second, a set of incremental performance metrics are designed including safety, functionality, mobility, and rider’s comfort. Third, a new definition of criticality is proposed as a combination of maneuver challenge and exposure frequency, and a critical scenario searching method is designed based on multi-start optimization and seed-fill method. Finally, with the generated library, CAVs can be evaluated through three steps: scenario sampling, field testing, and index value estimation. The proposed framework is theoretically proved to obtain accurate evaluation results with much fewer number of tests, compared with public road test method. In part II of the paper, three case studies are investigated to demonstrate the proposed methodologies. Reinforcement learning based technique is applied to enhance the method under high-dimensional scenarios.

Index Terms—Testing Scenario Library, Connected and Automated Vehicles, Testing and Evaluation, Safety, Functionality

I. INTRODUCTION

TESTING and evaluation is a critical step in the development and deployment of connected and automated vehicles (CAVs). Testing procedures for human-driven vehicles, such as Federal Motor Vehicle Safety Standards (FMVSS), have been established for a long time. However, current standards only regulate automobile safety-related components, systems, and design features, because all driving tasks are performed by human drivers. For CAVs, it is essential to evaluate the “intelligence” of the vehicle [1], similar to a driver’s license test, which indicates whether a CAV can operate safely and efficiently without human intervention.

Currently, CAV testing and evaluation is mainly conducted via the following steps: simulation test, closed facility test, and public road test. Simulation test is a cost-effective method, but it is difficult to model exact vehicle dynamics and road environment. Public road test is the most realistic method, but has the following problems: First, at the current stage of CAV technology, safety is still a significant issue. At least four fatal crashes have been reported in the past two years involving automatic driving functions [2]. Second, testing on public roads is extremely inefficient. A CAV would have to drive hundreds of millions of miles, sometimes hundreds of billions of miles to validate both safety and reliability at the level of human driven vehicles [3]. The underlying reason is that most scenarios on public roads are not challenging enough to evaluate the performances of a CAV. Only a small portion of the scenarios are critical, which are rare events on public roads. For instance, if we want to evaluate the safety performance (e.g., accident rate) of a CAV by analyzing its reaction to red light running vehicles at signalized intersections, it may require the CAV to pass thousands or even millions of intersections to accumulate enough accident events, which becomes intractable.

Closed facility test, which can test real CAVs in a controlled environment, has its unique advantages over the other two methods. First, testing real CAVs resolves the problem of modeling exact vehicle dynamics in simulation. Second, the closed facility test provides a more controlled and therefore safer environment for CAV testing than the public road test. Third, the closed facility test has potential to greatly improve the testing efficiency, i.e., obtain the evaluation results with the same accuracy by fewer number of tests.

A. Motivation

The key to exploiting the advantages of closed facility test is to generate testing scenario libraries. A testing scenario library is defined as a set of critical scenarios that can comprehensively evaluate certain pre-defined performance metrics. Each scenario in the library has its testing value, which quantitatively measures the criticality of the scenario. After the library is generated, CAVs can be tested in closed facilities by sampling scenarios from the library. Scenarios with smaller testing values are sampled with smaller probabilities. Since the library includes more critical scenarios, the CAV evaluation...
can be performed much more efficiently than that of public road test.

To efficiently and effectively evaluate different CAVs in closed facilities, the testing scenario library generation (TSLG) problem needs to be solved. It can be described as: how to generate a testing scenario library for one scenario type (e.g., car-following), which can be used to accurately and efficiently evaluate different CAVs with a pre-defined performance metric (e.g., safety). The ultimate goal of this paper is to systematically solve the TSLG problem for different scenario types (e.g., both low- and high-dimensional scenarios) and performance metrics.

B. Problem Description and Related Work

Generally speaking, the TSLG problem can be disassembled into four research questions:

1. How to describe a testing scenario and formulate the decision variables? (Scenario Description)
2. What are the performance metrics for CAV evaluation? (Metric Design)
3. How to generate a testing scenario library for a specific performance metric? (Library Generation)
4. How to use the generated library to evaluate CAVs? (CAV Evaluation)

The first question focuses on the description of testing scenarios and decision variable formulation. A scenario describes the temporal development between a sequence of scenes, which include snapshots of the environment (e.g., background vehicles, road information, and environment conditions) [4]. Decision variables denote what requires to be changed in testing scenarios. Most existing studies construct the decision variables by listing all possible influencing factors, which is infeasible when the testing scenarios are complex. To reduce the complexity, Li et al. [5] described testing scenarios as a temporal-spatial combination of assigned tasks, so the decision variables are formulated as the temporal-spatial locations of assigned tasks. Zhou et al. [6] described testing scenarios by several basic scenarios and a set of transition rules. The PEGASUS project [7] proposed a three-level framework to describe testing scenarios, i.e., functional level, logical level, and concrete level. If parameters of the top two levels are pre-determined, then the decision variables include only the parameters of the concrete level. However, all these methods do not consider the operational design domain (ODD) [8] of testing CAVs. Yet testing scenarios outside the ODD are meaningless for CAV evaluation.

The second question aims to design performance metrics for CAV evaluation. Most current studies only focus on safety, which is usually assessed by indices, e.g., the disengagement rate or the accident rate on public roads [9][10]. Although safety is the foundation of all CAV applications, a safe but over-conservative CAV may fail in simple driving tasks. Therefore, functionality, which represents the vehicle’s ability to complete driving tasks, should also be included in the evaluation process. Furthermore, mobility and rider’s comfort can be considered as higher level requirements. Although critical scenarios for different performance metrics may differ, the framework of solving the TSLG problem should remain the same.

The third and the key question is how to generate a testing scenario library for a specific performance metric. The most straightforward method is to design a “test matrix” based on expert knowledge, which is similar to the validation of human-driven vehicles [11][12][13]. However, this method relies heavily on the external input, and the accident typology of CAVs may not be reflected in the predefined test matrix. Improvements were made to generate testing scenarios based on particular CAV models. The worst-case scenario evaluation method (WCSE) was proposed to generate testing scenarios with model-based optimization methods [14]. The critical step of WCSE is to model the exact CAV dynamics and driving behaviors, which is not realistic for implementation. To resolve this problem, some black-box model-based methods were proposed. An adaptive searching method was proposed to generate testing scenarios based on a specific black-box CAV model [15]. However, the “black-box” model method requires to conduct real vehicle testing for each step of scenario searching, which is time-consuming and expensive. Moreover, the generated scenarios can only be applied to a specific CAV, which are not suitable for other CAVs. All these methods can only provide some representative scenarios, which cannot comprehensively evaluate CAVs without a testing scenario library. The PEGASUS project [7] proposed an exhaustive method to construct a testing scenario library, which suffers from computational complexity for high-dimensional scenarios.

The fourth question focuses on CAV evaluation with the generated library. For safety evaluation, most existing methods estimate the accident rate of a CAV using a scenario library from Naturalistic Driving Data (NDD), such as naturalistic field operational tests [16] and crude Monte Carlo method [17][18]. However, this method is proved inefficient and intractable for even low-dimensional scenarios [3]. The evaluation efficiency of low-dimensional scenarios was significantly improved by the accelerated evaluation (AE) method proposed by Zhao et al. [10]. The importance sampling technologies were first applied into the CAV evaluation problem. The major idea is to construct an importance function, which attaches more importance to critical scenarios. However, each step of searching the importance function is based on one test run of a real CAV. Thus it is time-consuming and expensive to construct the importance function for high-dimensional scenarios. As a result, under high-dimensional car-following scenarios, the AE method degrades to a white-box method with the assumption of knowing exact CAV models [19], which is usually impossible for real applications. Moreover, the generated scenarios can only be applied to a specific CAV, which is not generic.

Notwithstanding the related studies, all existing methods have limitations in either scenario types that can be handled (e.g., low-dimensional scenarios only), CAV models (e.g., a specific CAV only), or performance metrics (e.g., safety evaluation only). To the best of our knowledge, there is no existing study that integrates all parts of the TSLG problem together and generates libraries for different scenario types,
CAV types, and performance metrics.

C. Contributions

In this paper, a unified framework is designed to solve the entire TSLG problem, where a novel method is proposed for the library generation question.

The four abovementioned research questions are integrated and solved together in the proposed unified framework: (1) The terms scene and scenario defined in [4] are adopted, and the operational design domain (ODD) is considered to formulate the decision variables of testing scenarios (Section II.A). (2) Incremental performance metrics are designed, including safety, functionality, mobility, and rider’s comfort (Section II.B). (3) A method is proposed to generate the testing scenario library, including the criticality definition and critical scenario searching (Section III). (4) How to use the generated library for the CAV test is discussed including scenario sampling, CAV testing, and index estimation (Section IV).

The library generation is the key step in the entire framework (Section III). The basic idea is to define the criticality of scenarios and search the critical scenarios to construct the library. To this end, a new definition of criticality is proposed as a combination of maneuver challenge and exposure frequency. The new definition is fundamentally different from most existing studies, which usually overvalue the infrequent scenarios, e.g., worst-case scenario evaluation [14] and accelerated evaluation [19]. To efficiently search for critical scenarios, multi-start optimization and seed-fill based method is applied, where an auxiliary objective function is designed to provide searching directions for critical scenarios.

Theoretical analysis in Section V provides justifications of the proposed method regarding for both evaluation accuracy and efficiency. Specifically, the proposed method obtains unbiased index estimation of performance metrics (i.e., accuracy), and the estimation variance is zero under certain conditions (i.e., efficiency). Although the conditions are too strict for practical applications, they still indicate the efficiency of the proposed method and provide theoretical foundation for further improvement. Hyper-parameters are also pre-determined and theoretically analyzed, e.g., the threshold of critical scenarios and parameters of sampling policy.

D. Structure of the Papers

This paper is divided into two parts. Overall framework, methodologies, and theoretical analysis are presented in Part I. Implementation problems are answered by case studies in Part II. The proposed method is enhanced by reinforcement learning techniques under high-dimensional scenarios, e.g., the car-following case. Compared with existing methods, the enhanced method shows powerful ability in handling high-dimensional scenarios.

The rest of Part I is organized as follows. Section II formulates the testing scenario library generation problem. Section III proposes the novel method for library generation. The CAV evaluation method with the library is introduced in Section IV. In Section V, theoretical analysis is provided regarding the accuracy and efficiency. Finally, we conclude the paper in Section VI. Notations of all variables are listed in Table I.

II. PROBLEM FORMULATION

A. Decision Variables

The terms scene and scenario defined in [4] are adopted. A scene describes a snapshot of the environment including the scenery and dynamic elements. A scenario describes the temporal development between several scenes in a sequence of scenes. The scenery includes all geo-statically stationary elements, which entails metric, semantic, and topological information about roads and all their components like lanes, lane markings, road surfaces, or the roads’ domain types. The dynamic elements are moving or have the ability to move, e.g., pedestrians and vehicles. Slightly different with the definitions in [4], a scene denotes ground truth of the environment (objective) in this paper, instead of observations (subjective). Therefore, the scene representation is considered to be static.

Testing scenarios should be consistent with the operational design domain (ODD) of testing CAVs. The ODD describes the specific conditions under which a given CAV is intended to function [8]. To define the capability boundaries, the following information is required at a minimum in the ODD: roadway types, geographic area, speed range, and environmental conditions. Therefore, most of the scenery and part of dynamic elements have been specified in the ODD. The determination of remaining parts of scenarios is the critical step to generate testing scenarios. If the remaining parts are denoted as a vector of decision variables $x$, e.g., acceleration profiles of background vehicles, a testing scenario is generated with each realization of $x$.

To formulate the decision variables $x$ of testing scenarios, three levels of scenarios were defined in the PEGASUS project [7], i.e., functional scenario, logical scenario, and concrete scenario. The functional scenario is the high-level specification, which has textual description of a class of scenarios, e.g., cut-in scenarios. The logical scenario denotes one line of evolution, e.g., a vehicle cuts in front of the ego vehicle.
from the right lane. The concrete scenario contains the fully
defined sequence of scenes, e.g., a vehicle cuts in front of the
ego vehicle from the right lane with relative distance $R = 5m$
and relative speed $\dot{R} = 1m/s$. In this example, if the ODD
has specified the functional and logical scenario, the decision
variables can be denoted as $x = (R, \dot{R})$.

This method provides a simplified way to formulate the
vector of decision variables if the ODD is defined following
the same structure. For more generic ODD, however, the
method has limitations. For examples, if the logical scenario
has not been specified in the ODD, the vector needs to include
more variables to define parameters in the logical scenario,
e.g., $x = (R, \dot{R}, l)$, where $l$ denotes whether a vehicle cuts
from the left or right lane. Moreover, if the functional scenario
has not been determined, e.g., the ODD just specifies a three-
lane highway, then all the car-following, cut-in, and over-
taking scenarios could coexist. In this case, it is intractable
to directly include all functional scenarios. Instead, the vector
of decision variables can be formulated as the number of BVs
and corresponding trajectories of each BV. Furthermore, if the
scenery is also not completely defined, e.g., the number of lane
changes with the highway segment, the vector should include
the spatial variables of scenery.

In summary, the vector of decision variables needs to be
formulated according to the ODD. If the ODD is defined
following some specific structures, e.g., the three-level struc-
ture in the PEGASUS project [7], then the vector can be
formulated in the simplified way. For less specified ODD, the
vector should include temporal variables of dynamic elements
and spatial variables of scenery, e.g., trajectories of all traffic
participants and spatial development of road parameters. If $\theta$
denotes the pre-determined parameters in the ODD, then a
testing scenario can be specified by one realization of $x$ and $\theta$,
and a library is constructed by a critical set of decision
variables $x \in \Phi$.

### B. Performance Metrics

Performance metrics define what aspects a CAV needs
to be evaluated. Most existing studies focus only on safety
evaluation, which is essential but insufficient for a commerci-
alized CAV. In this paper, we define the performance metrics
to reflect people’s incremental expectations towards CAVs,
including safety, functionality, mobility, and rider’s comfort,
as shown in Fig. 1.

Safety is the foundation of all CAV applications, which is
usually assessed by the accident rate during the test without
human intervention or the disengagement rate [9][10]. Taking
the commonly used scenario, i.e., cut-in scenario, for an
example, a background vehicle (BV) changes its lane in front
of a CAV in the adjacent lane with pre-determined parameters,
i.e., cut-in distance and speed difference. Whether an accident
(e.g., conflict or crash) may happen or not depends on the
CAV’s response to the BV’s maneuvers. After a certain number
of tests with varying parameters, the accident rate of the
CAV could be estimated, which is used to indicate the safety
performance in the lane-change scenario.

The second level of the performance metric is functionality,
which is defined by whether a CAV can complete a given task
in a specific scenario. Considering a scenario that a CAV needs
to make a lane change to the right and exit the highway within
a certain distance, several BVs are driving on the right lane
following pre-determined parameters (e.g., initial distance to
the CAV, acceleration profiles). If the CAV is very conservative
and keeps a long safety distance with surrounding vehicles, it
may fail to complete the lane-change task before the freeway
exit. In the case, the vehicle may pass the safety evaluation but
fail in the functionality evaluation. Similar to safety evaluation,
the functionality of a CAV can be evaluated by the failure rates
of the CAV in completing certain driving tasks with different
environment settings and BVs’ trajectories.

We believe both safety and functionality are critical for CAV
evaluation at the current technology maturity level. Unless a
CAV can safely complete all driving tasks without human
interventions, it may not be accepted by the general public.
In Part II of this paper, three case studies are designed to
evaluate both safety and functionality.

For higher level requirements, mobility and rider’s comfort
should also be considered into the evaluation scope. Mobility
is utilized to measure the travel efficiency in completing a
series of driving tasks, while rider’s comfort measures the
physical and psychological feeling of passengers. Case studies
of these two metrics will be investigated in future work.

### C. Index Estimation

Specific indices are designed to measure the performance
metrics, e.g., the accident rate for safety performance and the
failure rate for functionality performance. If we denote the
event of interest (e.g., accident) as $A$, the accident rate in the
ODD is denoted as $P(A|\theta)$, where $\theta$ denotes the predefined
parameters by the ODD.

The road test method is essentially an index estimation.
Taking the cut-in case as an example, if a testing CAV drives
on public roads, experiences $n$ cut-in scenarios defined by the
ODD, and has $m$ accident events, the accident rate can be
estimated by

$$P(A|\theta) \approx \frac{m}{n}. \quad (1)$$

The theoretical justification is provided as follow. Denote the
decision variable vector of cut-in scenarios as $x \in \mathcal{X}$, where
the feasible region $\mathcal{X}$ is defined by the ODD. The experienced
cut-in scenarios in the public road test follow the distribution
of $P(x|\theta)$, i.e., $x_i \sim P(x|\theta), \ i = 1, \cdots, n$. Then we can
estimate the index as

$$P(A|\theta) = \sum_{x \in X} P(A|x, \theta)P(x|\theta),$$

$$\approx \frac{1}{n} \sum_{i=1}^{n} P(A|x_i, \theta), x_i \sim P(x|\theta), \quad \text{(2)}$$

where the last two equivalences are derived by Monte Carlo theory [20]. As proved in [3], however, because the accident is a rare event, the required number of tests $n$ is intolerably large for reasonable estimation accuracy.

The importance sampling technique was introduced in this field by [10] to improve the efficiency. If an importance function $q(x)$ is properly constructed as

$$q(x) \in [0, 1]; \sum_{x \in X} q(x) = 1,$$

and testing scenarios are sampled via the importance function, the index could be estimated by

$$P(A|\theta) = \sum_{x \in X} P(A|x, \theta)P(x|\theta),$$

$$= \sum_{x \in X} \frac{P(A|x, \theta)P(x|\theta)}{q(x)} q(x),$$

$$\approx \frac{1}{n} \sum_{i=1}^{n} \frac{P(x_i|\theta)}{q(x_i)} P(A|x_i, \theta), x_i \sim q(x).$$

Intuitively, if the importance function $q(x)$ increases the probability of sampling critical scenarios, more critical scenarios will be tested, so the required number of tests can be reduced, i.e., the evaluation method becomes more efficient. As shown in [10], because most scenarios in the ODD are uncritical, an importance function constructed by heuristic principles would significantly improve the safety evaluation efficiency in cut-in scenarios. For complex scenarios and other metrics, however, the construction of a proper importance function still remains a problem.

D. Assumptions of Testing CAVs

The following assumptions are applied to testing CAVs in this paper:

**Assumption 1.** Testing CAVs are well-developed so that the event of interest $A$ is a rare event on public roads.

**Assumption 2.** Testing CAVs share some “common features” of behaviors.

**Remark 1.** Assumption 1 is the basic requirement of CAVs before deployment. For example, if a CAV has accidents frequently in the road test, it is far from deployment, and testing of the CAV is not critical.

**Remark 2.** Assumption 2 represents a mild condition for many CAVs. Different types of CAVs may have common features as well as unique features brought by their own manufacturers. The common features capture fundamental functions of a well-developed vehicle behavior, e.g., keep safe distances and interact with surrounding vehicles. It is similar to human driving vehicles, where different drivers have different driving habits, but common features exist among all drivers. A well-generated library should be designed towards the common features of CAVs and includes more critical scenarios for most CAVs.

E. Objective of Testing Scenario Library

The essence of a testing scenario library is the generation of the importance function $q(x)$. A small set of critical scenarios with $q(x) > \gamma$ constructs the library, where $\gamma$ is a threshold, and the value of $q(x)$ determines the sampling probability of each scenario. If the importance function can attach more importance to critical scenarios, then the library leads to more efficient CAV evaluation. Therefore, the objective of testing scenario library is to construct the importance function which attaches more importance to critical scenarios. In the following sections, we will elaborate how to define the criticality of scenarios (Section III.A), how to search critical scenarios (Section III.B), and how to construct the importance function based on the scenario criticality (Section IV).

III. TESTING SCENARIO LIBRARY GENERATION

To generate the testing scenario library, the criticality of scenarios is defined, and the searching method is designed for efficiently searching critical scenarios. An illustration of the entire framework is shown in Fig. 2. The proposed definition provides theoretical foundation to construct the optimal importance function and indicates that both maneuver challenge and exposure frequency are critical for CAV evaluations, which is fundamentally different from most existing studies.

A. Definition of Criticality

The criticality of a scenario measures the importance in evaluating a performance metric. In ISO 26262 [21], the risk assessment of a scenario was defined as a combination of severity of injuries, exposure classification, and controllability classification. The exposure classification denotes the relative expected exposure frequency of the scenario where the injury can possibly happen. The controllability classification denotes the relative likelihood that the driver can act to prevent the injury.

Inheriting the concepts of the risk assessment, we define the criticality of scenarios as

$$V(x|\theta) \overset{\text{def}}{=} P(S|x, \theta)P(x|\theta),$$

where $\theta$ denotes the specified parameters in the ODD, $x$ denotes the vector of decision variables, and $S$ denotes the event of interest (e.g., accident) with a surrogate model (SM) of CAVs. The SM is designed to encode the common features of CAVs (see Assumption 2). As discussed in Remark 2, a well-generated library should include more critical scenarios for most CAVs, and the introduction of the SM contributes to achieving this goal. An ideal SM should be calibrated from actual CAV driving data similar to human driving model calibration [22]. At the current stage, however, there is very little open CAV data available for public research. Therefore, we propose to calibrate the SM based on the human driving
data, i.e., NDD. It is a reasonable starting point because of the following reasons. First, the common features of human drivers are the natural baselines for CAV evaluation. Critical scenarios for human drivers are the most straightforward testing scenarios for CAVs. Second, CAV is essentially an application of “artificial intelligence”, the purpose of which is to mimic and outperform “human intelligence”. Many CAV algorithms are obtained by imitating human driving behaviours, e.g., end-to-end learning method [23][24]. Third, a “human-like” CAV can improve safety in a mixed traffic condition, where CAVs and human-driven vehicles coexist on the roadway. A similar concept of “roadmanship” was recently proposed for CAV evaluation [25]. Therefore, it is reasonable to represent the common features of CAVs based on human naturalistic driving data.

The proposed definition is a conceptual generalization of the risk assessment in ISO 26262 [21]. The left term \( P(S|x, \theta) \) measures the probability that CAVs encounter the event of interest in the scenario. The severity is encoded by determining the interested event, and the controllability classification is encoded by the probability. The right term \( P(x|\theta) \) denotes the probability of the scenario occurring on public roads, which encodes the exposure classification. Different from the classification methods in ISO 262262, we generalize the concepts from safety to generic metrics, introduce the concept of SM, and define the criticality in a quantitative way. The justifications of this definition are theoretically proved regarding the evaluation accuracy and efficiency in Section V.

To calculate the criticality, \( P(x|\theta) \) can be obtained from NDD, and \( P(S|x, \theta) \) is obtained by simulations of the SM.

The definition also indicates that both maneuver challenge \( P(S|x_i, \theta) \) and exposure frequency \( P(x_i|\theta) \) are critical for CAV evaluations. This is fundamentally different from most existing studies, which usually overvalue the infrequent scenarios. For instance, the worst-case scenario evaluation [14] focuses on the worst-case (i.e., most dangerous) scenarios for safety evaluation. The accelerated evaluation method for the car-following scenarios [19] maximizes the likelihood of the occurrence of accidents (e.g., crash or conflict), which generates the most infrequent scenarios. All these methods essentially focus on the most infrequent scenarios, which happen to be the most challenging scenarios for safety evaluation. However, for functionality evaluation, as an example, there is no explicit relation between the maneuver challenge (i.e., difficulty) and exposure frequency. All existing methods overvalue the challenging part but ignore the exposure frequency of scenarios. Taking an extreme example for conceptual explanation, the scenario that a meteor hitting a car is extremely dangerous...
but we cannot evaluate the performances of CAVs based on testing results from these extremely low frequent scenarios. The common and challenging scenarios are more critical for CAV evaluation.

B. Critical Scenario Searching

The next problem is how to search critical scenarios in the whole scenario space. The basic idea is to find local critical scenarios by optimization methods and then search their neighbor scenarios. However, directly using the criticality function as the objective function is problematic. As discussed in Assumption 1, most scenarios are uncritical with zero criticality and zero gradient of criticality, i.e., local minimal. If a scenario is uncritical, its criticality function provides little information of searching direction for critical scenarios. Therefore, the optimization process degrades to a random sampling process, which is inefficient for complex scenarios.

To resolve this issue, an auxiliary objective function is designed to guide searching directions, and the seed-fill method is applied to search neighbor scenarios. To overcome issues brought by high dimensions, the searching method is enhanced by reinforcement learning (RL) techniques for specific cases, e.g., car-following case.

First, an auxiliary objective function is designed as the combination of maneuver challenge and exposure frequency, similar to criticality definition. An example of the auxiliary objective function of the cut-in case for safety evaluation is shown as

\[
\min_{x} J(x) = \min_{x} (\text{mnpETTC}(x) + w \times d(x, \Omega)),
\]

where \(x\) denotes the vector of decision variables (i.e., the cut-in distance \(R\) and speed difference \(\dot{R}\)) of the cut-in scenario. The first term is the minimal normalized positive enhanced time-to-collision (mnpETTC) during testing, which measures the danger level (i.e., maneuver challenge) of scenario \(x\). The value of ETTC is calculated based on a surrogate car-following model [26]

\[
\text{ETTC}(t) = \frac{-\dot{R}(t) - \sqrt{\dot{R}^2(t) - 2u_r(t)R(t)}}{u_r(t)},
\]

where \(u_r\) is the relative acceleration. The minimal positive ETTC measures the most dangerous point during a testing scenario. To make the metric comparable with exposure frequency, a normalization factor is applied. The second term is a normalized distance between the scenario and a high exposure frequency zone (i.e., \(\Omega\)) in NDD (e.g., 95% percentile), which measures the exposure frequency of the scenario. \(w\) is a weight parameter to balance the two terms. Note the aim of the auxiliary objective function is to provide searching directions for critical scenarios. Therefore, the roughness of the auxiliary objective function (e.g., caused by the value of \(w\)) is acceptable.

Second, a commonly used multi-start optimization method is applied to obtain a number of local critical scenarios. Specifically, multiple initial points are generated by space-filling methods (e.g., random sampling). After solving the optimization problem from each initial point, local critical scenarios are obtained. The parameters from the ODD are considered as constraints, e.g., speed limit, acceleration limit, perception range, etc. The number of initial points increases with the dimensions of the decision variables. Fortunately, the dimension of the decision variables can be greatly reduced by exploiting their specific structures, e.g., independence properties, and the method can be enhanced by RL techniques (see Part II for examples).

Third, using the local critical solutions as starting points, other critical scenarios are expanded by the seed-fill method. Seed-fill, also called flood-fill, is a basic method in computer graphics [27] that determines the area connected to a given node in multi-dimensional arrays. The key idea is to exhaustively explore the critical points of unexplored space rather than all of the space from the starting point outwards [28]. The criticality function instead of the auxiliary objective function is calculated in this step. The threshold of critical scenarios is theoretically analyzed in Section V.

To illustrate the searching method, two typical non-convex objective functions, i.e., Peaks function and Ackley function [29], are studied, as shown in Fig. 3 (a, c). The fifty and one-hundred initial searching points are sampled for the two functions respectively. In this illustration, the criticality function is calculated by the normalized objective function, and the threshold is manually selected. As shown in Fig. 3 (b, d), critical scenarios of the both functions are effectively obtained by the proposed searching method as red areas.

IV. CAV Evaluation with the Library

After the generation of library, the remaining problem is how to use the generated library to evaluate CAVs. As shown in Fig. 2 three steps are designed, i.e., scenario sampling, CAV testing, and index estimation. The importance function is constructed based on the generated library.
A. First Step: Scenario Sampling

The first step is to sample testing scenarios according to the generated library. The major challenge is how to balance exploitation and exploration. Critical scenarios are obtained based on the surrogate model (SM), which usually has dissimilarity compared with the testing CAV. Therefore, the generated library may miss some critical scenarios when testing a specific CAV. To solve this issue, besides sampling scenarios from the library according to their criticality values (i.e., exploitation), the scenarios outside the library is also sampled with a small probability (i.e., exploration).

To better understand the trade-off between the exploitation and exploration, we compare the greedy sampling policy and \( \epsilon \)-greedy sampling policy. The greedy sampling policy greedily exploits the scenarios in the library. This policy, all testing scenarios are sampled based on the normalized criticality values. The \( \epsilon \)-greedy sampling behaves greedily most of the time, but with small probability \( \epsilon \), it selects scenarios randomly outside the library with equal probability (i.e., exploration). This simple yet efficient method is commonly used for balancing exploitation and exploration [30].

The selection of \( \epsilon > 0 \) is theoretically analyzed in Section V.

From the perspective of Monte Carlo estimation, the testing probability distributions in Eq. (8)9 essentially construct the importance function. By involving the domain knowledge of CAVs and NDD, this construction method outperforms the general methods of importance sampling techniques (e.g., Cross Entropy method [31][32]). It can be applied for both low- and high-dimensional scenarios (see Part II) and provides the theoretical foundation for constructing the optimal importance function (see Section V).

B. Second Step: CAV Testing

The second step is to test the CAV with sampled scenarios. To provide a controllable, safe, and cost-effective testing environment, the augmented reality (AR) testing environment [33] is applied. Fig. 4 is an illustration of the AR platform designed for Mcity, a newly established closed CAV testing facility at the University of Michigan. The platform combines the real-world testing facility and a simulation platform together. Movements of testing CAV in the real world are transmitted to the simulation platform by roadside units (RSUs), and the information of simulated BVs is fed back to testing CAV. The traffic control in the real world is synchronized with simulation. In this way, BVs in the simulation and testing CAV in the real-world can interact with each other.

The initial conditions and maneuvers of BVs are determined by the sampled testing scenarios and imported in the AR platform as virtual vehicles. The testing CAV is running in the real testing facility, which responds to the maneuvers of virtual BVs. The testing can be repeated easily by sampling different scenarios from the library, which results in different BV movements. The total number of testing is determined by the required evaluation precision and confidence level [10][34][35]. For example, at a confidence level 100(1 - \( \alpha \))\%, to ensure the relative half-width of the estimation error is smaller than a predefined constant \( \beta \), the number of tests needs to be larger than

\[
\frac{z_{\alpha}^2}{\beta^2 \mu^2 \sigma^2},
\]

where \( z_{\alpha} \) is a constant, and \( \sigma, \mu \) can be estimated by estimation variance and expectation.

C. Third Step: Index Estimation

After the testing results are collected in the second step, the third step is to estimate the index value of the performance metric. As shown in Eq. (4), the index value can be estimated as

\[
\hat{P}(A|\theta) = \frac{1}{n} \sum_{i=1}^{n} P(x_i|\theta) P(A|x_i, \theta),
\]

where \( n \) denotes the total number of the sampled testing scenarios, \( P(x_i|\theta) \) denotes the importance function, i.e., either \( P_1(x_i|\theta) \) or \( P_2(x_i|\theta) \) depending on the choice of sampling policy, and \( P(A|x_i, \theta) \) is estimated by the testing results.

V. THEORETICAL ANALYSIS

In this section, the accuracy and efficiency of the proposed methods are validated by theoretical analysis, and choices of hyper-parameters, i.e., the threshold of critical scenarios and \( \epsilon \), are discussed.

To simplify the notations, we omit the pre-determined parameters in the ODD and define the following notations as

\[
\begin{align*}
    f_A(x) &= P(A|x, \theta), \\
    f_S(x) &= P(S|x, \theta), \\
    p(x) &= P(x|\theta), \\
    q_1(x) &= P_1(x|\theta), \\
    q_2(x) &= P_2(x|\theta), \\
    \mu &= P(A|\theta), \\
    \mu_S &= P(S|\theta), \\
    \hat{\mu} &= \hat{P}(A|\theta), \\
    W &= \sum_{x_i \in \Phi} P(S|x_i, \epsilon) P(x_i|\epsilon).
\end{align*}
\]
A. Accuracy Analysis

In this subsection, we prove that the proposed method can obtain unbiased (i.e., accurate) index estimation with $\epsilon$-greedy sampling policy. For greedy sampling policy, an additional condition is required for the unbiasedness, which indicates the superiority of the $\epsilon$-greedy sampling policy.

**Theorem 1.** The proposed evaluation method can obtain the unbiased index estimation for CAVs, namely

$$E(\hat{\mu}) = \mu, \quad (13)$$

under one of the following conditions:

1. with greedy sampling policy and $f_A(x) = 0, \forall x_i \notin \Phi$;
2. with $\epsilon$-greedy sampling policy.

**Proof.** We first prove the theorem under the condition (2). By the law of total probability, we obtain the right term of Eq. (13) as

$$\mu = P(A|\theta) = \sum_{x_i \in X} P(A|x_i, \theta)P(x_i|\theta).$$

Introducing the sampling probability $P_2(x_i|\theta)$ as Eq. (6), we obtain

$$P(A|\theta) = \sum_{x_i \in X} \frac{P(A|x_i, \theta)P(x_i|\theta)}{P_2(x_i|\theta)} \bar{P}_2(x_i|\theta).$$

By Monte Carlo principle [20], if we sample $x_i \sim P_2(x_i|\theta)$ for $n$ times, we have the estimation as

$$\hat{\mu} = \hat{P}(A|\theta) = \frac{1}{n} \sum_{i=1}^{n} \frac{P(A|x_i, \theta)P(x_i|\theta)}{P_2(x_i|\theta)},$$

as shown in Eq. (12). As $P_2(x_i|\theta) > 0$ for all scenarios and the Central Limit Theorem [36], when $n$ is large, $\hat{P}(A|\theta)$ follows approximately the normal distribution with the mean

$$E(\hat{\mu}) = \mu,$$

which concludes the theorem under condition (2).

For the theorem under condition (1), we have

$$P(A|x_i, \theta) = 0, \forall x_i \notin \Phi$$

and

$$P_1(x_i|\theta) = 0, \forall x_i \notin \Phi.$$

Therefore, the feasible space of decision variables can be changed from $X$ to $\Phi$, i.e., all scenarios outside $\Phi$ are un-critical. Similar as the proof of the theorem under condition (2), the theorem under condition (1) can be proved. $\square$

**Remark 3.** The condition $f_A(x_i) = P(A|x_i, \theta) = 0, \forall x_i \notin \Phi$ means that if a scenario is not included in the library, then the event of interest has zero probability to happen for the specific CAV in that scenario. It indicates that all critical scenarios for the specific CAV have been included in the library. That is the reason why the greedy policy can be applied. However, considering the diversity of CAVs, this condition is strict for real applications, so $\epsilon$-greedy policy is more robust and suitable.

B. Efficiency Analysis

In this subsection, we prove that the estimation variance is small and even zero under certain conditions. Because the minimal number of tests is determined by the estimation variance (see Eq. (11)), the proposed method is proved to be efficient. The conditions imply that the major estimation variance comes from the “dissimilarity” between the SM and the testing CAV. Although the dissimilarity between cannot be completely eliminated, it provides the theoretical foundation for progressively improving the evaluation efficiency (i.e., construct better importance function) by mitigating the dissimilarity.

**Theorem 2.** The estimation variance is zero, i.e., $Var(\hat{\mu}) = \sigma^2/n = 0$, under the following conditions

1. with the greedy sampling policy;
2. $f_A(x) = 0, \forall x_i \notin \Phi$;
3. There exists a constant $k > 0$ such that $f_A(x) = kf_S(x)$, $\forall x \in X$.

**Proof.** According to the Monte Carlo method with importance
Substituting Eq. (17) into Eq. (16), we obtain

\[
\sigma^2 = \sum_{x_i \in \Phi} \left( \frac{f_A(x_i)p(x_i)}{q_1(x_i)} \right)^2 q_1(x_i) - \mu^2, \\
= \sum_{x_i \in \Phi} \left( \frac{f_A(x_i)p(x_i) - \mu q_1(x_i)}{q_1(x_i)} \right)^2, \\
= \sum_{x_i \in \Phi} \frac{p^2(x_i)}{q_1(x_i)} \left( f_A(x_i) - \mu \frac{q_1(x_i)}{p(x_i)} \right)^2, \tag{14}
\]

where the second equivalence is obtained by

\[
\sum_{x_i \in \Phi} q_1(x_i) = 1.
\]

By condition (2) and Eq. (5), we have

\[
q_1(x_i) = P(S|\theta, x_i)P(x_i|\theta)/W, \\
= f_S(x_i)p(x_i)/W. \tag{15}
\]

Substituting Eq. (15) into Eq. (14), we obtain

\[
\sigma^2 = \sum_{x_i \in \Phi} \frac{p^2(x_i)}{q_1(x_i)} \times \left( f_A(x_i) - \mu \frac{W}{f_S(x_i)} \right)^2.
\]

Moreover, by the conditions (1-3), we have

\[
\frac{\mu}{W} = \frac{P(A|\theta)}{W}, \\
= \frac{\sum_{x_i \in \Phi} P(A|x_i, \theta)P(x_i|\theta)}{\sum_{x_i \in \Phi} P(S|x_i, \theta)P(x_i|\theta)}, \\
= k. \tag{17}
\]

Substituting Eq. (17) into Eq. (16), we obtain

\[
\text{Var}(\hat{\mu}) = \sigma^2/n = 0,
\]

which concludes the theorem.

\[\square\]

**Remark 4.** Theorem 2 shows the ideal results under strict conditions. As shown in Eq. (11), if the estimation variance is zero, the minimal number of tests is one, which is too ideal. Although the conditions are impossible to completely hold for real applications, they imply the sources of the estimation variance and provide theoretical foundation for further improvement of the method. In the proposed method, conditions (1)-2 are replaced by the \(\epsilon\)-greedy sampling policy, which introduces a certain amount of estimation variance (see Theorem 3). Moreover, the dissimilarity between the SM and the testing CAV violates condition (3), which is the major source of estimation variance. If the dissimilarity can be mitigated progressively, the constructed importance function will be better, and the evaluation efficiency will be further improved.

### C. Choices of Hyper-parameters

In this subsection, we provide a method to determine the hyper-parameters, i.e., \(\epsilon\) and the threshold of critical scenarios. As discussed in Remark 4, the \(\epsilon\)-greedy sampling policy introduces new estimation variance. In Theorem 3 the introduced estimation variance is quantitatively analyzed, and \(\epsilon\) is determined by not introducing estimation variance in critical scenarios. The threshold of critical scenarios also introduces the estimation variance, because the criticality differences among uncritical scenarios are neglected. In Theorem 4 the relation between the threshold and the introduced estimation variance is quantitatively analyzed.

**Theorem 3.** The estimation variance with \(\epsilon\)-greedy sampling can be separated into two parts

\[
\sigma^2 = \sum_{x_i \in \Phi} \frac{p^2(x_i)}{q_2(x_i)} \left( f_A(x_i) - \mu \frac{q_2(x_i)}{p(x_i)} \right)^2, \\
+ \sum_{x_i \in \Phi} \frac{p^2(x_i)}{q_2(x_i)} \left( f_A(x_i) - \mu \frac{q_2(x_i)}{p(x_i)} \right)^2, \tag{18}
\]

and the latter part is zero if \(\epsilon\) is chosen as

\[
\epsilon = 1 - \frac{W}{\mu_S}, \tag{19}
\]

under the condition (3) in Theorem 2.

**Proof.** As the violation of condition (1) and (2) in Theorem 2 the variance in Eq. (14) changes as

\[
\sigma^2 = \sum_{x_i \in \Phi} \frac{p^2(x_i)}{q_2(x_i)} \left( f_A(x_i) - \mu \frac{q_2(x_i)}{p(x_i)} \right)^2, \\
= \sum_{x_i \in \Phi} \frac{p^2(x_i)}{q_2(x_i)} \left( f_A(x_i) - \mu \frac{q_2(x_i)}{p(x_i)} \right)^2, \\
+ \sum_{x_i \in \Phi} \frac{p^2(x_i)}{q_2(x_i)} \left( f_A(x_i) - \mu \frac{q_2(x_i)}{p(x_i)} \right)^2. \tag{20}
\]

Denote the latter part as \(\sigma^2_\Phi\) and we obtain

\[
\sigma^2 = \sum_{x_i \in \Phi} \frac{p^2(x_i)}{q_2(x_i)} \times \left( f_A(x_i) - \mu \frac{(1 - \epsilon)}{W} f_S(x_i) \right)^2. \tag{21}
\]

Similar to Eq. (17), substituting Eq. (18), we yield

\[
\frac{\mu}{W} (1 - \epsilon) = \frac{P(A|\theta)}{P(S|\theta)}, \\
= \frac{\sum_{x_i \in \Phi} P(A|x_i, \theta)P(x_i|\theta)}{\sum_{x_i \in \Phi} P(S|x_i, \theta)P(x_i|\theta)}, \\
= k. \tag{22}
\]

Substituting Eq. (20) into Eq. (19), we obtain

\[
\sigma^2_\Phi = 0,
\]

which concludes the theorem.

\[\square\]

**Theorem 4.** The estimation variance has an upper bound

\[
\sigma^2 < \mu^2 \frac{(m - \epsilon)^2}{\epsilon}, \tag{23}
\]
if under the same conditions in Theorem 3 and the threshold of critical scenarios is determined as

\[ V(x_i|\theta) \geq \frac{m\mu_S}{N(X) - N(\Phi)}, \forall x_i \in \Phi, \]  

(22)

where \( m \geq 1 \) is a constant.

**Proof.** By Eq. (22) and the condition (3) in Theorem 2, we obtain that for \( x_i \notin \Phi \),

\[
P(x_i|A, \theta) = \frac{f_A(x_i)p(x_i)}{\mu} = \frac{k_f(x_i)p(x_i)}{k\mu_S} = \frac{V(x_i|\theta)}{\mu_S m} \geq \frac{N(X) - N(\Phi)}{m(N(X) - N(\Phi))}.
\]

By Theorem 3 and Eq. (9), we obtain

\[
\sigma^2 = \sum_{x_i \notin \Phi} \left( \frac{f_A(x_i) - \mu q_2(x_i)}{q_2(x_i)} \right)^2 \geq \mu^2 \sum_{x_i \notin \Phi} \frac{1}{q^2(x_i)} \left( \frac{f_A(x_i)p(x_i)}{\mu} - q_2(x_i) \right)^2 \geq \mu^2 (N(X) - N(\Phi)) \epsilon \times \sum_{x_i \notin \Phi} \left( \frac{P(x_i|A, \theta) - \epsilon}{N(X) - N(\Phi)} \right)^2 < \mu^2 \frac{(m - \epsilon)^2}{\epsilon},
\]

(24)

where the last inequality considers the fact that \( m \geq 1 \geq \epsilon \), properties of the quadratic function, and the total number of \( x_i \notin \Phi \) as \( N(X) - N(\Phi) \).

**Remark 5.** Eq. (21) shows the introduced estimation variance caused by the threshold of critical scenarios and the \( \epsilon \)-greedy sampling policy. The pre-determined \( m \) has trade-offs, i.e., a large \( m \) decreases the size of the library but increases the upper bound of the introduced estimation variance. \( N(\Phi) \) is related with the choice of the threshold, so the precise value of the threshold needs to satisfy the inequality (22). For practical applications, considering the rareness of critical scenarios, the threshold can be relaxed as \( m\mu_S/N(X) \).

### VI. Conclusions

In this paper, we proposed a unified framework to solve the testing scenario library generation (TSLG) problem for CAV evaluation. The framework can be used to generate testing scenario libraries for different scenario types, performance metrics, and CAV models.

A novel method was proposed to generate testing scenario libraries. The criticality of scenarios was defined as a combination of maneuver challenge and exposure frequency, which is more reasonable than that of most existing studies. A searching method is designed to efficiently obtain the critical scenarios. To evaluate the maneuver challenge of scenarios, the surrogate model (SM) of CAVs was introduced, which contains the common features of CAVs. Although the dissimilarity between the SM and specific CAVs cannot be eliminated, it provides the theoretical foundation for progressively improving the efficiency by mitigating the dissimilarity. We believe that utilizing the domain knowledge (e.g., common features and NDD) has huge potentials for future study in this field. To validate the proposed method, the evaluation accuracy and efficiency were proved by theoretical analysis.

While Part I of the paper provides general framework and methods to the TSLG problem, there are several remaining implementation problems, which will be answered in part II of the paper. For example, we will discuss the construction of surrogate models, auxiliary objective function design for different performance metrics and scenario types, and enhancement of the method using reinforcement learning technique in high-dimensional scenarios.

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