Modeling non-local electrodynamics in superconducting films: the case of a right angle corner

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Abstract

We consider magnetic flux penetration in a superconducting film with a concave corner. Unlike convex corners, where the current flow pattern is easily constructed from Bean’s critical state model, the current flow pattern at a concave corner is highly nontrivial. To address the problem, we do a numerical flux creep simulation, where particular attention is paid to efficient handling of the non-local electrodynamics, characteristic of superconducting films in the transverse geometry. We find that the current stream lines at the concave corner are close to circular, but the small deviation from exact circles ensures that the electric field is continuous inside the sample. Yet, the electric field is, as expected, very high at the concave corner. At low fields, the critical state penetration is deeper from the concave corner than from the straight edges, which is a consequence of the electrodynamic non-locality. A magneto-optical experiment on YBa₂Cu₃Oₓ displays an almost perfect match with the magnetic flux distribution from the simulation, hence verifying the necessity of including electrodynamic non-locality in the modeling of superconducting thin films.

(Some figures may appear in colour only in the online journal)

1. Introduction

The macroscopic magnetic properties of hard type-II superconductors can to a large extent be described by the critical state model, first formulated by Bean [1, 2]. It is based on the assumption that the magnitude of the current cannot exceed the critical current density j_c, thus limiting the ability of the superconductor to carry transport current or to shield applied magnetic fields. The original critical state model was formulated for bulk samples, but it has later been extended to thin films, where additional shielding currents j < j_c flow in the region beyond the flux penetration front. Analytical results have been found for an infinite strip [3, 4] and circular disk [5, 6], while for less symmetric shapes or disconnected geometries the critical state has only been determined numerically [7].

Dynamics is disregarded in Bean’s critical state model, in the sense that the response to an applied magnetic field is instantaneous. In particular, the model does not take into account flux creep, a process which is particularly pronounced in high-T_c superconductors. To model flux creep, time must be taken explicitly into consideration. Conventionally, this is done by assuming a highly nonlinear E–J relation before solving Maxwell’s equations. For thin films, the main obstacle for an efficient implementation of a numerical scheme is the handling of the boundary conditions, given the non-locality of the governing equations. Brandt has derived solutions for selected geometries, such as rectangles [8], disks and rings [9], or arbitrary shapes [10], based on a matrix inversion method. Although being accurate, these solutions scale poorly with system size and need O(N²) operations for each time step, where N is the number of discrete points in the
The conjugate gradient method, which scales like $O(N^{1.4})$ [11, 12] or hybrid real-space—Fourier-space methods scaling as $O(N \log(N))$ [13–15].

One remarkable property of the Bean model is that the current stream line pattern can be drawn simply by adding lines with constant spacing, starting from the contour of the sample edge. When there is a constriction, the stream lines must adapt by bending, which leads to the formation of so called d-lines, i.e., lines in the flow pattern where the stream lines change direction discontinuously. These d-lines are recognized also in the corresponding magnetic flux distribution, due to an almost total local suppression of the magnetic field in their vicinity. By examination of measured flux distributions, e.g., obtained by magneto-optical imaging (MOI), it has been found that the Bean model to a large extent explains the flux penetration patterns of hard superconductors. For example, the d-line created by a circular nonconducting hole (antidot) is parabolic [16–18], a row of antidots give asymptotically straight d-lines at an angle given by the antidot fraction [15], while a convex right-angled corner, e.g., as in a rectangular sample, gives straight d-lines making 45° with the meeting edges [19].

Despite its vast success, the Bean model procedure for drawing critical current stream line patterns is ambiguous at a concave corner. This is apparent in figure 1, demonstrating two possible ways to join stream lines as they pass a 90° concave corner. In (a) the equidistant stream lines are straight and meet at a 45° d-line, as in the case of a convex corner. In (b) the current stream lines consist partly of circular segments, and there are no traces of d-lines. The constructions (a) and (b) are just two out of infinitely many possible solutions that conserve current and give constant current density [20]. However, only one construction can be correct, and to identify it a more comprehensive analysis must be made.

Schuster et al [21] investigated the flux and current distributions in a cross-shaped superconductor, a geometry that contains both convex and concave corners. Their MOI experiment showed no d-lines at the concave corners, hence excluding the sharp turn illustrated in figure 1(a). An accompanying flux creep simulation gave stream lines close to circular, as in figure 1(b). At the same time, a theoretical estimate showed that exactly circular stream lines diverge as $E \propto 1/r$ close to the corner and the $E$ field will make a jump where the circular stream lines connect with the straight lines running parallel to the edges.

As pointed out by Gurevich and Friesen [22, 20], the jumps in the Bean results might be a artifact arising because the Bean model does not take into account the electric field and the Faraday law, $\mathbf{B} = -\nabla \times \mathbf{E}$. A more reliable approach is to obtain the steady state current patterns using the powerful and general hodograph method. Such calculations showed that, in the limit of negligible flux creep, neither figure 1(a) nor (b) are correctly representing the stream line pattern near a concave corner. However, the solution of [20, 22] did not include the history of the magnetization process leading to the current distribution. Neither did it include the specific non-locality effects crucially important in the transverse geometry. Thus, it still remains an open question what is the correct current stream line pattern in a thin superconductor with a concave corner, and subjected to a perpendicular magnetic field.

This work considers in detail the flux penetration and current flow in a thin superconductor having the simplest shape that includes a concave corner, namely that shown in figure 1. A general simulation method for flux dynamics in thin samples of almost any shape is described in section 2. The approach takes special care to model the non-locality of the equations in an efficient way. Section 3 reports and discusses the simulations results, which include distributions of magnetic field, shielding current and electrical field. A direct comparison with MOI experiments on a YBa$_2$Cu$_3$O$_y$ film in an increasing applied magnetic field is reported in section 4. Finally, section 5 gives the conclusions.

2. Simulation model

The numerical scheme is quite general and can be applied to thin planar superconductors of arbitrary shape, provided the thickness $d$ is much smaller than any lateral dimension, and the whole sample can be embedded inside a rectangular area [14, 15]. We also require that the external field $H_x$ is applied in the $z$-direction, transverse to the sample plane. Then, the flux dynamics is found by solving Maxwell’s equations in the quasi-static (eddy current) approximation, where the superconducting properties enters the equations through a highly nonlinear $E$–$J$ relation that characterizes the sharp vortex depinning transition happening when the sheet current $J$ approaches the value of the critical sheet current $J_c = d/\rho_c$. A realistic approximation for many materials is a power law [8–10, 22, 20]

$$E = \rho J/d, \quad \rho = \rho_0 (J/J_c)^{n-1},$$  \hspace{1cm} (1)

where $E$ is the electric field, $\rho$ is the resistivity, and $\rho_0$ is a resistivity constant. The creep exponent $n$ is usually large, with the Bean model corresponding to the limit $n \to \infty$. Note that the model also works for an Ohmic conductor, where $n = 1$. 

![Figure 1. Two different stream line patterns near a concave corner with flow of a constant-magnitude current density, $j_c$, as in the Bean model.](image-url)
Since current is conserved, \( \nabla \cdot \mathbf{J} = 0 \), we can introduce the local magnetization \( g = g(\mathbf{r}, t) \) as
\[
\frac{\partial g}{\partial y} = J_y, \quad \frac{\partial g}{\partial x} = -J_y,
\]
where \( \mathbf{r} = (x, y) \). Outside the sample, \( g = 0 \). The integral of \( g \) gives the magnetic moment, \( m = \int d^2r \, g(\mathbf{r}) \).

For quasi-static situation, the \( B-J \) relation is given by the non-local Biot–Savart law, which can be rewritten to a \( B_z - g \) relation as
\[
B_z/\mu_0 = H_a + \hat{Q}g,
\]
with the operator \( \hat{Q} \) given by
\[
\hat{Q}g(\mathbf{r}) = \mathcal{F}^{-1} \left[ \frac{k}{2} \mathcal{F}[g(\mathbf{r})] \right],
\]
where \( \mathcal{F} \) is the 2D spatial Fourier transform, \( k = |\mathbf{k}| \), and \( \mathbf{k} \) is the wavevector. The inverse relation is
\[
\hat{Q}^{-1} \varphi(\mathbf{r}) = \mathcal{F}^{-1} \left[ \frac{2}{k} \mathcal{F}[\varphi(\mathbf{r})] \right],
\]
where \( \varphi \) is an auxiliary function.

By taking the time derivative of equation (3) and inverting it, we get
\[
\dot{g} = \hat{Q}^{-1} [B_z/\mu_0 - H_a].
\]
This equation is solved by discrete integration forward in time.

Regarding the discretization of space, the key point is that both \( \hat{Q} \) and \( \hat{Q}^{-1} \) are direct products in Fourier space (FFTs). However, the derivation leading to the simple form for \( \hat{Q} \) and \( \hat{Q}^{-1} \) has neglected the sample boundary, which means that also the vacuum surrounding the sample must be explicitly included in the calculations. The total area of calculations is thus a rectangle of dimension \( L_x \times L_y \) including both sample and vacuum. The solution will be periodic on this larger rectangular area, but will, apart from that, lack symmetries, so that the general FFT must be used, rather than the sine and cosine versions.

Thus, in order to integrate equation (6) forward in time, \( \dot{B}_z \) must be known everywhere in the embedding \( L_x \times L_y \) rectangle at time \( t \). Our strategy is to find \( \dot{B}_z \) inside the sample from the material law, while in the vacuum \( \dot{B}_z \) is found implicitly from the condition \( \dot{g} = 0 \), as described below.

Starting with the superconductor itself, it obeys the material law, equation (1), which, when combined with the Faraday law, \( \dot{B}_z = -\nabla \times \mathbf{E} \), gives
\[
\dot{B}_z = \nabla \cdot (\rho \nabla g)/d.
\]

For all iteration steps, \( i = 1 \ldots s \), \( \dot{B}_z^{(i)} \) is fixed inside the superconductor by equation (7). At \( i = 1 \), an initial guess is made for \( \dot{B}_z^{(0)} \) outside the sample, and \( \dot{g}^{(0)} \) is calculated from equation (6). In general, this \( \dot{g}^{(i)} \) does not vanish outside the superconductor, but an improvement can be obtained by
\[
\dot{B}_z^{(i+1)} = \dot{B}_z^{(i)} - \mu_0 \hat{Q} \dot{g}^{(i)} + C^{(i)}.
\]
The projection operator \( \hat{Q} \) is unity outside the superconductor and zero inside. Also, the output of the operation should be shifted to satisfy \( \int d^2r \, \hat{Q} \dot{g}^{(i)} = 0 \). The constant \( C^{(i)} \) is determined by requiring flux conservation,
\[
\int d^2r [\dot{B}_z^{(i+1)} - \mu_0 \hat{H}_a] = 0.
\]

Thus, at each iteration \( i \), \( \dot{B}_z^{(i+1)} \) is calculated for the outside area. The procedure is repeated until after \( i = s \) iterations \( \dot{g}^{(i)} \) becomes sufficiently uniform outside the sample. Then, \( \dot{g}^{(s)} \) is inserted in equation (6), which brings us to the next time step, where the whole iterative procedure starts anew.

The non-dimensional form of the equations are particularly simple when the applied field is ramped with constant rate \( \dot{H}_a \neq 0 \) and \( J_a \) and \( n \) are both constants. We define the sheet current constant
\[
J_0 \equiv J_a \left( \frac{\rho_0 \mu_0 H_a}{\rho_0 J_c} \right)^{1/n},
\]
where \( w \) is some lateral length and the rest of the parameters have been defined previously. The time constant is defined as
\[
t_0 \equiv J_0/\dot{H}_a.
\]
The dimensionless form of the material law, equation (1), becomes
\[
\tilde{E} = \tilde{\rho} \tilde{J}, \quad \tilde{\rho} = J_n^{-1},
\]
where \( \tilde{J} = J/J_0 \) and \( \tilde{E} = E d/(\rho_0 J_0) \). The time evolution, equation (6), becomes
\[
\frac{\partial \tilde{g}}{\partial t} = \tilde{Q}^{-1} \left[ \frac{\partial \tilde{B}_z}{\partial t} - 1 \right].
\]
where \( \tilde{g} = g/(w J_0) \), \( \tilde{g}^{-1} = \tilde{Q}^{-1}/w \), \( \tilde{t} = t/t_0 \), and \( \tilde{B}_z = B_z/(\mu_0 J_0) \). Finally, the Faraday law, equation (7), becomes
\[
\frac{\partial \tilde{B}_z}{\partial \tilde{t}} = \tilde{\nabla} \cdot (\tilde{\rho} \tilde{\nabla} \tilde{g}),
\]
where \( \tilde{\nabla} \equiv w \nabla \). This means that the only free parameter in the model is \( n \). A practical consequence of this is that simulations only need to be run once for each geometry and each value of \( n \), and the result for any combination of dimensional parameters can be found simply by rescaling the solution.

Several physical conclusions can be drawn directly from the non-dimensional equations. First, equation (13) with (14) inserted is a non-local diffusion equation with \( \tilde{\rho} \) as a very nonlinear diffusion constant. The non-locality prevents scaling solutions like in bulk [23], but will still give plateaus
in the current where $J \approx 1$. Second, the level of these plateaus in dimensional units is $J_0$, not $J_c$ as one could naively expect. Thus, $J > 1$ does not imply $J > J_c$, since $J_0 < J_c$ for parameters corresponding to most type-II superconductors. Third, the current constant depends on ramp rate as $J_0 \propto \mu_0 H_a^{1/n}$. Such a ramp rate dependent current gives also a flux penetration depending on ramp rate, and this has indeed been measured by [24] in a strip of high-$T_c$ superconductor.

In this paper the tildes are omitted when reporting the results in dimensionless units.

We chose the lateral length scale $w$ as the half-width of the shortest side of the superconducting corner. The long sides of the sample have length $6$. The sample is embedded in a square with $L_x = L_y = 9$, discretized on a $512 \times 512$ equidistant grid. Note that since the geometry is very non-symmetric, only 25% of the grid points lie within the sample, the rest are in the vacuum, needed to fulfil the boundary conditions. Nevertheless, the method is fast and the simulations of this paper can be run on a personal computer.

3. Simulation results

The initial state is a flux free superconductor, and we use a flux creep exponent of $n = 29$. As the applied magnetic field is increased with constant rate the magnetic flux gradually enters the sample from the edges, as seen in figure 2, showing snapshot shots of the $B_z$-distribution at $H_a = 0.3$, 0.6 and 1.3. In the figure, the image brightness represents the magnitude of $B_z$, which everywhere is directed parallel to the applied field.

At $H_a = 0.3$, the most visible feature is that the edges light up with field values much larger than the applied field, while the sample interior is black in an area representing the Meissner state, where the flux is expelled from the superconductor. Already at this low field, the concave corner shows enhanced field values and deeper flux penetration as compared to the penetration from the straight edges.

At $H_a = 0.6$, the flux front has advanced considerably, and now the concave corner is filled with a fan of enhanced flux density. In the convex corners, the d-lines start developing, but they are not yet distinct lines, but rather wedge-shaped regions of vanishing flux density.

At $H_a = 1.3$, the flux has penetrated essentially the entire sample, and the superconductor is now described by a fully developed critical state. The d-lines are clearly seen in all convex corners, where they make $45^\circ$ angles with the meeting edges, as expected for a superconductor with isotropic $J_c$. In the concave corner, on the other hand, there is no sign of any d-line. Quite the contrary, as the flux density is there higher than in the surrounding regions. In particular, in the corner point itself the magnetic field is greatly amplified. Evidently, the current stream line pattern responsible for this flux distribution must be close to figure 1(b), and definitely not like in figure 1(a).

Figure 3 shows color-coded images of the sheet current magnitude, $J$, together with the stream lines of $\mathbf{J}$, calculated as the contour lines of the local magnetization $g$. At $H_a = 0.3$ the critical state with $J \approx 1$ has only penetrated a short distance from the edge, while most of the sample is in the Meissner state, where $J < 1$, as expected for a thin superconductor in a small magnetic field. The outermost stream lines can be followed around the sample perimeter where they, along each straight edge segment, stack with equal spacing, in accordance with the critical state model. At the concave corner the stream lines turn $90^\circ$ in a gradual way, forming lines resembling circle segments while maintaining an essentially constant separation. Note that the two innermost stream lines, as they leave the critical state in the concave corner, cross over into the Meissner state area. This is a clear manifestation of the non-locality of the governing equations and cannot be found within the bulk case of the Bean model.

At $H_a = 0.6$ the domains of constant current density have grown in size mainly by penetrating deeper, but also by filling larger parts of all corners. All the features commented for $H_a = 0.3$ are still present here. In addition, the current now bends quite sharply at the convex corners, where the d-lines become increasingly more well defined.

At $H_a = 1.3$, the full penetration state has been reached and the current density is everywhere $J \approx 1$. Note that in the central part of the sample, there are now several stream lines making small closed loops. At the same time, the d-line extending from the large convex corner is longer than the
Figure 3. Distribution of sheet current magnitude $J$ and current stream lines at applied perpendicular fields $H_a = 0.3$, 0.6 and 1.3. The images are color coded so that the brighter red the larger is $J$, see the color bar.

Figure 4. Distribution of electric field $E$ at applied magnetic fields $H_a = 0.3$, 0.6 and 1.3 during ramping of the magnetic field. The images are color coded to show the variation in the magnitude of $E$, see the color bar. Note that $E$ is not calculated in the vacuum.

Figure 5. The distribution of electric field $E$ close to the convex corner at $H_a = 1.3$.

Figure 6. The distribution of electric field $E$ close to the concave corner at $H_a = 1.3$.

thin superconductors with concave corners are far more susceptible to such dramatic events occurring than samples with only convex corners or without corners at all.

As mentioned, the current stream lines near the concave corner appear to be nearly circular. Yet, the simulation results shown in figures 4 and 5 deviate from the $E$ field generated

others. This overall pattern is fully compatible with the critical state model rule for constructing equidistant stream lines. In particular, one finds that the length of the long d-line should be a factor $2(2 - \sqrt{2})$, or 17\% longer than the others, in very good agreement with the simulation result. Moreover, the construction implies that the curved part of the central d-line consists of two parabolic curves meeting at the end of the long d-line.

Figure 4 shows the distribution of the electric field magnitude $E$. At the straight edges, the results are as expected from the critical state model in an infinite strip, where the $E$ field grows almost linearly from $E = 0$ at the flux front towards the maximum at the edges [8]. Also in agreement with the critical state model is the strong suppression of $E$ close to the convex corners, where at the d-lines $E = 0$, signifying absence of flux traffic. However, the most striking feature in the $E$ maps is the spot of very high field at the concave corner, which is strongly present in all three panels.

Figure 5 focuses at this spot at $H_a = 1.3$. The result appears similar to the numerical result of [21]. The maximum value is $E \sim 4.5$, which is much larger than $E \sim 1$ at the straight edges. This high electrical field is a sign of intensive flux traffic through the concave corner.

Since high electrical fields are known to trigger thermomagnetic avalanches [25–27], it is thus clear that...
by perfectly circular stream lines, as calculated in [21]. In particular, the maximum $E$ is not found exactly at the corner, but in two spots located on each side of the corner. The $E$ field is also smooth everywhere inside the sample, and in particular there is no discontinuity in $E$ at angles $0^\circ$ and $90^\circ$. Hence, the stream lines obtained from the simulations are not simply circular segments connected with straight lines. This motivates a closer inspection of the flow pattern.

Shown in figure 6 is the detailed current stream line pattern near the concave corner at the applied field $H_0 = 1.3$. For direct comparison the figure also shows stream lines shaped as concentric circular segments connecting to straight lines running parallel to the edges, and having a spacing corresponding to $J = 1$. The two patterns clearly deviate, as the calculated stream lines are compressed as they pass the corner, implying that there is an enhanced current density, $J > 1$, in the region. The enhanced current density is a consequence of the finite creep exponent, and values for current density are consistent with the electrical field map in figure 5, where the highest electrical field is $E \sim 4.5$, giving a sheet current of $4.5^{1/20} \approx 1.05$, i.e., a 5% increase in the current density.

The stream lines of figure 6 begin to curve some distance before they meet the corner sector, thus the region of curves lines cover a sector wider than $90^\circ$. In this way, the stream lines change direction more gradually and the unphysical discontinuity in $E$, mentioned earlier, is avoided. The current pattern of figure 6 seems to be consistent with the analytical calculation of [22, 20].

One open question that remains is if the electric field is finite at the tip of the corner. A simulation, like the present work, cannot address this question due to the limited spatial resolution. Also analytically, a definite answer to this question is not easily found, when properly including non-local and nonlinear electrodynamics, under non-stationary conditions. The analytical calculations of [22] and [20] for steady state and local electrodynamics suggest that the electric field is divergent, yet weaker than the $1/r$ divergence found by naive Bean model considerations.

The simulations of this work are based on a power-law $E-J$ relation, which is fairly representative for high-$T_c$ superconductors in the flux creep regime [28, 8–10, 22, 20]. Most of the results should also be applicable for conventional type-II superconductors, even those that do not follow an exact power law, as long as the $E-J$ relation is strongly nonlinear close to $I_c$. In general, one would use higher $n$ values for conventional superconductors, since flux creep is less pronounced there. In any case $n$ is not a material constant, but depends on temperature. The simulation formalism of this work is also valid when the whole or parts of the sample is in a flux flow state, where $E \propto J$. The flow patterns would in this case be much different from the presented results.

It is appropriate now to compare the simulations with results of MOI experiments.

4. Experiments

A film of $\text{YBa}_2\text{Cu}_3\text{O}_x$ was made by laser ablation on a (100) SrTiO$_3$ substrate. Details of the preparation can be found in [29]. The sample has a thickness of $d = 300$ nm with the c-axis oriented perpendicular to the film plane. The critical temperature $T_c$, measured by magnetic susceptibility, was 89.9 K. The critical current density of the film is $j_c = 8.0 \times 10^{10}$ A m$^{-2}$ at 45 K, the temperature where the magneto-optical images were recorded.

The MOI investigation was performed using a bismuth-substituted ferrite garnet film with in-plane magnetization as Faraday rotating sensor, placed directly on the sample surface. The sample was mounted on the cold finger of a continuous He flow cryostat with an optical window. Images of the flux distribution were recorded with a digital camera through a polarized light microscope using crossed polarizers. In this way the image brightness represents the magnitude of the flux density. Note that the experimental images are slightly more blurred than the simulation results in figure 2. This is partly due to a small gap between the garnet film and the superconductor created by a mirror layer on the sensor film, and partly because the Faraday rotation signal is an average across a 4 µm thickness of garnet film. For more details of the method and setup see [30] and [31].

Shown in figure 7 are two images of the flux penetration in the sample after an initial zero-field cooling to 45 K. Then the applied perpendicular magnetic field was slowly increased, and the images in (a) and (b) were recorded at $B_0$ of 16.5 and 44 mT, respectively. The image (a) was slightly contrast enhanced to allow the location of the flux front to be clearly seen.

In the concave corner region it is evident that the flux front has advanced deeper than from the long straight edges of the sample. The swollen region covers a sector slightly exceeding $90^\circ$. The similarity between this experimental image and the simulated result for $H_0 = 0.6$ shown in figure 2 is striking.

Also on the other side of the sample, near the main convex corner, the experimental images show a nontrivial behavior.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{The current stream lines resulting from the simulation at $H_0 = 1.3$ (full red line) and for comparison, stream lines including circular segments (dotted black line).}
\end{figure}
The flux front associated with the lower edge penetrates slightly deeper near the main convex corner. The dashed line included in panel (b) is parallel to the edge, and serves as a guide to the eye. The formation of the extra long d-line in the main corner is also clearly seen in the image (b). The same swelling effect is seen to occur if one follows the similar flux front down along the vertical part of the sample. The dashed lines in panel (a) are also drawn parallel to the edges, and help to illustrate that the effect is present also far from full penetration, although then much less pronounced. The agreement between these experimental results and the numerical simulations is striking down to the very fine details.

5. Conclusions

In superconducting films, the electrodynamic non-locality implies that calculations, in principle, must consider the whole sample and the full magnetic history. This is particularly evident at the concave corner considered in this work, where the current stream line pattern is nontrivial, unlike the convex corner where the stream line pattern can be drawn using a simple Bean model procedure. The flux creep simulation conducted in this work shows that the stream line pattern at the concave corner deviates from exact circles, and this small deviation prevents unphysical jumps in the electrical field. Nevertheless, the electrical field is very high at the concave corner, and samples with such corners should thus be particularly susceptible for nucleation of thermomagnetic avalanches. The magneto-optical experimental work shows striking similarity with the simulation and verifies the necessity of properly including the electrodynamic non-locality when modeling thin films in the transverse geometry.

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