Light scalars with lepton number to solve the \((g - 2)_e\) anomaly

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Abstract

Scalars that carry lepton number can help mediate would-be lepton-number-violating processes, such as neutrinoless double \(\beta\) decay or lepton-scattering-mediated nucleon-antinucleon conversion. Here we show that such new scalars can also solve the anomaly in precision determinations of the fine-structure constant \(\alpha\) from atom interferometry and from the electron’s anomalous magnetic moment, \(a_e \equiv (g - 2)_e/2\), by reducing \(|a_e|\). Study of the phenomenological constraints on these solutions favor a doubly-charged scalar with mass below the GeV scale. Significant constraints arise from the measurement of the parity-violating asymmetry in Möller scattering, and we consider the implications of the next-generation MOLLER experiment at Jefferson Laboratory and of an improved \(a_e\) measurement.
I. INTRODUCTION

Through tour-de-force efforts in both theory and experiment, the anomalous magnetic moments of both the electron and muon have emerged as exquisitely sensitive probes of physics beyond the Standard Model (SM) [1–8]. For many years, the measured value of the electron’s anomalous magnetic moment $a_e \equiv (g - 2)/2$ was used to determine the most precise value of the fine-structure constant $\alpha \equiv e^2/4\pi\epsilon_0\hbar c$ [8], with the measurement of $a_\mu$ providing sensitivity to new physics at the weak scale, once the hadronic and electroweak contributions were taken into account [6, 10]. In recent years, with the emergence of precise assessments of $a_e$ in QED perturbation theory through fifth-order in $\alpha/\pi$ [1, 11–14] and precise determinations of $\alpha$ [3, 4] from atom interferometry [15, 16], $a_e$ itself, due to its quantum nature, has also emerged as a probe of physics beyond the SM. Indeed the comparison of $a_e$ from its direct experimental measurement with its expected value in the SM, using atom interferometry to fix $\alpha$, yields the most precise test of the SM in all of physics [17].

The SM value of $a_e$ is dominated by the contribution from QED — though contributions from the SM weak gauge bosons $W^\pm, Z^0$ and hadronic effects also exist, these are known to be extraordinarily small, contributing only 0.026 ppb [18–20] and 1.47 ppb [21, 22], respectively, of the total contribution to $a_e^{\text{SM}}$ [1]. The analysis of atom interferometry measurements for $\alpha$ also require the use of QED theory and other observables [8], though the uncertainty in the determined value of $\alpha$ is dominated by that in its measured observable, $h/M_X$, where $h$ is Planck’s constant and $M_X$ is the mass of atomic species $X$. With the most precise experimental result for $a_e$ [2, 23] and $h/M_X$ measurements for Rb [3] or Cs atoms [4] to determine $\alpha$ and thus $a_e^{\text{SM}}$ [1] we report [1]

$$a_e^{\text{EXP}} - a_e^{\text{SM}}[\text{Rb}] = (-131 \pm 77) \times 10^{-14},$$

$$a_e^{\text{EXP}} - a_e^{\text{SM}}[\text{Cs}] = (-88 \pm 36) \times 10^{-14},$$

where here and elsewhere the uncertainties are added in quadrature. In what follows we use the most precise determination of $\alpha$ to define the $a_e$ anomaly, $\Delta a_e \equiv (-88 \pm 36) \times 10^{-14}$ [1, 2, 4], which is a discrepancy of $\sim 2.4\sigma$. For reference we report the anomaly in $(g - 2)_\mu$ as well [5, 24]

$$\Delta a_\mu \equiv a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (2.74 \pm 0.73) \times 10^{-9},$$

for a discrepancy of $\sim 3.7\sigma$, as also determined by Ref. [25], with a sign opposite to that of $\Delta a_e$. Both the relative sign and size of the anomalies suggest distinct mechanisms for their explanation. For example, if weak-scale new physics were to explain $\Delta a_\mu$, scaling as $m_\mu^2$, then its contribution to $\Delta a_e$ would be roughly 10 times too small, $\Delta a_e \simeq 0.7 \times 10^{-13}$ [26, 27]. Thus explaining both anomalies is seemingly not possible in models that differentiate electrons and muons only by their mass — rather, possible solutions should break lepton flavor universality [27–30]. The relatively large size of $\Delta a_e$ also suggests the appearance of new physics below the $\sim 1$ GeV scale.

Several models of light new physics have been proposed that could explain both of the $a_\ell$ anomalies [27–30]. However, the suggestion that new physics at scales below $\sim 1$ GeV, arising from so-called dark, hidden, or secluded sectors, could explain the $a_\mu$ anomaly has existed much longer [31, 32]. Keen interest in such scenarios has been generated not only by

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1 In this paper we define the magnetic moment of a charged lepton $\ell$ as $\mu_\ell = g_\ell e\mathbf{S}/2m_\ell$, with $g_\ell > 0$ and $e = -|e|$ [9].
anomalies in high-energy astrophysics that could arise from dark-matter annihilation [33], but also by an appreciation of the great reaches of untested parameter space possible for their realization [34, 35], which has spurred new experimental initiatives [36, 37]. Although the possibility that a U(1) gauge boson that mixes with the photon [38], a “dark photon,” could explain \( \Delta a_\mu \) has been ruled out [39], solutions involving a new light scalar or pseudoscalar are still possible [40, 41]. Since the dark photon gives a positive contribution to \( a_\ell \) [31, 32], it also cannot address the \( a_e \) anomaly [4].

Models that address both \( a_\ell \) anomalies treat the electrons and muons in different ways. In Ref. [28], a single real scalar is introduced, and, in the electron case, the scalar coupling to a heavy charged fermion, such as the \( \tau \), can be chosen to mediate a two-loop Barr-Zee [42] contribution to \( a_e \) that yields the needed opposite sign. In Ref. [29], models with an abelian flavor symmetry \( L_\mu - L_\tau \) are used to realize different contributions to \( a_e \) and \( a_\mu \), with the suggested consequence that the permanent electric-dipole moment (EDM) of the \( \mu \) could be much larger than supposed from electron EDM limits. In Ref. [27], a complex scalar is introduced with CP-odd couplings to the electron and CP-even couplings to the muon, generating contributions to \( a_{e,\mu} \) of opposite sign. The somewhat disjoint nature of the various simultaneous solutions, and the severity of the constraint from nonobservation of \( \mu \to e\gamma \) [29], suggests that we can address one anomaly without precluding the other. In this paper we show that we can solve the \( a_e \) anomaly by introducing a scalar with lepton number that couples to first-generation fermions only, respecting SM symmetries, supposing that one of the solutions for \( \Delta a_\mu \) proposed in Refs. [27, 40, 41, 43–46], e.g., could also act. The solutions we have found also serve as necessary ingredients in minimal scalar models [47–51] that can also mediate lepton-number violating processes, such as neutrinoless double \( \beta \) decay and scattering-mediated nucleon-antinucleon conversion [51].

Giudice, Paradisi, and Passera have shown that many possible new physics models could generate a shift of \( a_e \) from its SM value [26], considering both models that connect to a change in \( a_\mu \) by \((m_\mu/m_e)^2\) and those that do not. In the latter class they consider models that connect to violations of charged lepton flavor number or lepton flavor universality, as well as models with heavy vector-like fermions [26, 52]. In the last example, Giudice et al. introduced a SU(2) vector-like doublet and singlet, with interactions that can explicitly break lepton number. In what follows we consider a new physics model for \( \Delta a_e \) of a completely different kind — here the scalars carry lepton number, with scalar-fermion interactions that conserve lepton number, and indeed are SM-gauge invariant; and these features are essential to the results we find. Other models pertinent to \( a_e \) [53, 54] that also address the \( \Delta a_e \) anomaly [54] have been proposed. Interestingly, models with a new axial-vector boson also generate contributions that decrease \( |a_e| \) [31, 55], though other empirical constraints exist on these solutions as well [4, 55].

Scalars that carry lepton number also appear in neutrino mass models. Although the smallness of the neutrino masses can be elegantly ascribed to a seesaw mechanism with a new-physics scale of some \( M_N \sim 10^{10–15} \text{GeV} \) [56–59], there are many alternate possibilities [60]. In type II seesaw models [61–65], e.g., the see-saw scale can be below the electroweak scale. The neutrino masses can also be generated radiatively [64, 66–73]. The scalars of interest to us appear in different contexts. For example, weak-isospin singlet scalars appear in radiative mass models [67, 68, 72], whereas weak-isospin triplet scalars appear in light type II seesaw models and other mass models [74]. If the scalar also couples to SM weak gauge bosons, as in the latter case, scalar masses of less than \( \sim 100 \text{ GeV} \) are severely constrained by existing experimental limits on neutrinoless double \( \beta \) decay [74]. Here we suppose, as in Ref. [51],
that scalars with lepton number need not explain the numerical size of the neutrino mass, so that we take no stance on the precise origin of the neutrino masses and mixings. Moreover, we consider minimal scalar models with weak-isospin triplet and singlet scalars that couple to first-generation fermions only — such a scenario is much less constrained, evading severe constraints, e.g., from the $\mu$ lifetime and $\mu \rightarrow e\gamma$ decay [72].

We conclude this section by outlining the content of our paper — we begin, in Sec. II, by describing the scalar models we employ in more detail. Thereafter, in Sec. III, we discuss the contributions to $a_e$ in these models, providing our detailed computations in the appendix for clarity. We describe the sets of possible couplings and masses that solve the $a_e$ anomaly before turning to the constraints on these models from parity-violating electron scattering in Sec. IV and considering other possible constraints in Sec. V. In our analysis we focus on scalars of less than $O(10\text{GeV})$ in mass, making our analysis complementary to that of Ref. [74], who analyzed constraints on doubly charged scalars with masses in excess of that. We conclude with a discussion of the experimental prospects in Sec. VI.

II. SCALARS WITH LEPTON NUMBER

Minimal scalar models are extensions of the SM that respect its gauge symmetries and do not impact its predictive power, because the new interactions possess mass dimension 4 or less. Such models have been primarily employed in the study of baryon-number-violating and/or lepton-number-violating processes [47–51, 75]. In what follows we introduce new scalars with definite representations under the SU(3)$_c \times$SU(2)$_L \times$U(1)$_Y$ gauge symmetry of the SM that also carry nonzero lepton number $L$ and construct their minimal interactions by requiring Lorentz and SM gauge invariance. Generally, there are three possible scalars $X_i$ that couple to SM leptons only, all carrying $L = -2$. We have two weak isospin singlets: $X_1$ with hypercharge $Y = 2$ that couples to right-handed fermions, where we employ the convention that the electric charge $Q_{em} = T_3 + Y$ in units of $|e|$ and $T_3$ is the third component of weak isospin, and $X_2$ with hypercharge $Y = 2$ that couples to left-handed fermions. There is also one weak isospin triplet $X_3$ with $Y = 1$ that couples to left-handed fermions. Denoting a right-handed lepton of generation $a$ as $e^a$ and the associated left-handed lepton doublet as $L^a$, the possible scalar-fermion interactions mediated by each $X_i$ are of form

$$-g^{ab}_1 X_1 (e^a e^b), \quad -g^{ab}_2 X_2 (L^a \varepsilon L^b), \quad -g^{ab}_3 X_3 (L^a \xi^A L^b),$$  \hspace{1cm} (4)$$

where $\varepsilon = i\tau^2$ is a totally antisymmetric tensor, $\xi^A \equiv ((1 + \tau^3)/2, \tau^1/\sqrt{2}, (1 - \tau^3)/2)$, and $\tau^A$ are Pauli matrices with $A \in 1, 2, 3$ [51]. The symmetries of the scalar representations under weak isospin SU(2) fix the symmetry of the associated coupling constant under $a, b$ interchange, with $g^{ab}_1$ and $g^{ab}_3$ symmetric and $g^{ab}_2$ antisymmetric. Thus only $X_1$ and $X_3$ can couple to first-generation leptons exclusively. In Eq. (4) we adopt 2-spinors such that the fermion products in parentheses are Lorentz invariant, and we map to 4-spinors via $(e_{L,Ra} \mu_{L,Rb}) \rightarrow (e_{a \tau}^T C P_{L,R} \mu_{\beta})$ where $C$ and $P_{L,R} = (1 \mp \gamma_5)/2$ are in Weyl representation [76]. We have chosen the arbitrary phases [77] that appear such that $C = i\gamma^0\gamma^2$ and the charge-conjugate field $\psi^c$ is $\psi^c \equiv C(\bar{\psi})^\dagger$. Thus the scalar-fermion interactions for each of these
scalars are of form
\[ \mathcal{L}_{X_1} \supset -g_1^{ab} \bar{X}_1 e^a_R e^b_R + \text{H.c.}, \]
\[ \mathcal{L}_{X_2} \supset -g_2^{ab} \bar{X}_2 (\bar{\nu}_L^a \nu^b_L - \bar{\nu}_L^b \nu^a_L) + \text{H.c.}, \]
\[ \mathcal{L}_{X_3} \supset -g_3^{ab} \left( \bar{X}_3^1 \nu^a_L \nu^b_L + \bar{X}_3^2 \frac{1}{\sqrt{2}} \left( \bar{\nu}_L^a \nu^b_L + \bar{\nu}_L^b \nu^a_L \right) + \bar{X}_3^3 \bar{\nu}_L^a \nu^b_L \right) + \text{H.c.} \] (5)

In what follows we assume that \( X_1 \) and \( X_3 \) couple to first-generation fermions only, whereas for \( X_2 \) we assume only \( 1 \leftrightarrow 3 \) couplings exist, since the existing constraints on intergenerational mixing are less severe in that case \([72]\). We analyze the pertinent constraints there in Sec. V.

### III. NEW SCALAR CONTRIBUTIONS TO \( a_e \)

In minimal scalar models the new scalars can carry electric charge, so that two types of Feynman diagrams can contribute to \( a_e \) at leading order: one in which the photon attaches to the internal charged fermion line and a second in which the photon attaches to the charged scalar line. We find that \( X_1 \) and \( X_3 \) can contribute to \( a_e \) through both diagrams, whereas in the case of \( X_2 \) only the second diagram appears. The contributions to \( a_e \) from \( X_1 \) and \( X_2 \) have been previously studied \([72]\). Although we agree with Ref. \([72]\) for the computation of \( \Delta a_e \) from \( X_2 \), our computation of \( \Delta a_e \) from \( X_1 \) does not — indeed, our result differs from theirs by a factor of \(-4\). Consequently we find that the contribution to \( \Delta a_e \) from each scalar is negative definite. Since this result is key to our paper, and subtleties exist in the computation of \( \Delta a_e \), we present our computation in detail in the appendix. In this section we compile our results and evaluate their consequences. We evaluate the contribution of each possible new scalar to \( a_e \) independently, terming this \((\delta a_e)_{X_1}\).

Combining the results of the appendix, Eqs. (A.26) and (A.34), as appropriate, we find that the contribution to \( \Delta a_e \) from \( X_1 \) is
\[ (\delta a_e)_{X_1} = -\frac{m_e^2 |g_1|^{11}|^2}{4\pi^2} \left( \int_0^1 dz \frac{z(1-z)^2}{(1-z)^2 m_e^2 + zm_1^2} + 2 \int_0^1 dz \frac{z(1-z)^2}{z^2 m_e^2 + (1-z)m_1^2} \right), \] (6)
so that \((\delta a_e)_{X_1} \leq 0\) and finite for all \(m_{X_1} > 0\). Moreover, the contribution to \( \Delta a_e \) from \( X_2 \) from Eq. (A.35) is
\[ (\delta a_e)_{X_2} = -\frac{4m_e^2 |g_2|^{13}|^2}{16\pi^2} \left( \int_0^1 dz \frac{z(1-z)}{m_{X_2}^2 - zm_e^2} \right), \] (7)
where we have set the mass of the neutrino \( \nu_j \) to zero here and elsewhere, as it is known to be very small \([7]\). The 4 in the numerator appears because \( g_2^{13} = -g_3^{31} \), as in Ref. \([72]\).

Here \( M_{X_2} < m_e \) leads to a singularity in the parameter integral arising from on-mass-shell intermediate states; we avoid this possibility if \( M_{X_2} > m_e \). For \( M_{X_2} < m_e \) we would replace the integral in Eq. (7) with its principal value, though in that region \((\delta a_e)_{X_2} > 0\). Finally, the contribution to \( \Delta a_e \) from \( X_3 \) is
\[ (\delta a_e)_{X_3} = -\frac{m_e^2 |g_3|^{11}|^2}{4\pi^2} \left( \int_0^1 dz \frac{z(1-z)^2}{(1-z)^2 m_e^2 + zm_{X_3}^2} + 2 \int_0^1 dz \frac{z(1-z)^2}{z^2 m_e^2 + (1-z)m_{X_3}^2} \right. \]
\[ \left. + \frac{1}{2} \int_0^1 dz \frac{z(1-z)}{m_{X_3}^2 - zm_e^2} \right). \] (8)
Here, too, by choosing $M_{X_3} > m_e$ we would avoid the inconvenience of a singularity in the parameter integral; in the $M_{X_3} < m_e$ region the integral would be replaced by its principal value, noting that in this case $(\delta \alpha_e)_{X_3} < 0$ for $M_{X_3} > 0$. Thus we observe that each of the three lepton-number-carrying scalars possible in minimal scalar models could solve the $\alpha_e$ anomaly — we need only choose a scalar mass and scalar-fermion coupling consistent with the empirical value of $\Delta \alpha_e$, and a broad range of choices are possible. Thus we see that the $\Delta \alpha_e$ anomaly could also potentially be solved by very light mass scales, beyond the reach of existing accelerator experiments. Nevertheless, in what follows we consider scalars with masses $M_{X_i} > m_e$, as that mass region loosely avoids astrophysical constraints, such as those from stellar cooling [78]. We note, however, that new particles with masses $M_{X_i} < m_e$ may be possible if their interactions do not permit them to escape an astrophysical environment [79] — and our lepton-number-carrying scalars may well be of that class. We also consider $M_{X_i} < 8$ GeV on $X_1$ and $X_3$ to avoid collider constraints [80, 81] on same-sign dileptons. We note that both $X_2$ and $X_3$ can induce a contribution to the magnetic moment of a massive Dirac neutrino; we consider this further in Sec. V.

We now summarize our solutions for the $\Delta \alpha_e$ anomaly. Working in the $M_{X_i} \gg m_e$ limit and considering $\Delta \alpha_e$ at 95% CL we find that the masses and scalar-fermion couplings of each $X_i$ must satisfy

$$
3.2 \times 10^{-6} \leq \frac{m_e}{M_{X_1}} |g_{11}^i| \leq 9.7 \times 10^{-6},
$$

$$
6.5 \times 10^{-6} \leq \frac{m_e}{M_{X_2}} |g_{1j}^j| \leq 2.0 \times 10^{-5},
$$

$$
3.4 \times 10^{-6} \leq \frac{m_e}{M_{X_3}} |g_{1j}^3| \leq 1.0 \times 10^{-5},
$$

where $j \neq 1$. We show the exact numerical solutions for $|g_{ij}^i|$ and $M_{X_i}$ for $i = 1, 3$ in Fig. 1, along with other pertinent constraints and their future prospects — the mass range we show is selected to evade both stellar cooling and collider bounds. In this mass range, $X_2$, even with the assumption of $1 \leftrightarrow 3$ couplings only, is significantly constrained by branching ratio measurements of semileptonic $\tau$ decay — we update the analysis of Ref. [72] in Sec. V. We develop the established and expected constraints from parity-violating Møller scattering, which act on $X_1$ and $X_3$, in the next section. Here we wish to emphasize, in addition to providing the solutions we have shown, that the measured value of $\Delta \alpha_e$ also constrains new physics; that is, the upper value of Eqs. (9,10,11) serves as the boundary of a 95% CL exclusion. That is, we can exclude

$$
\frac{m_e}{M_{X_1}} |g_{11}^i| > 9.7 \times 10^{-6},
$$

$$
\frac{m_e}{M_{X_2}} |g_{1j}^j| > 2.0 \times 10^{-5},
$$

$$
\frac{m_e}{M_{X_3}} |g_{1j}^3| > 1.0 \times 10^{-5},
$$

as these regions of parameter space yield values of $|\Delta \alpha_e|$ that are too large — these regions, for $X_1$ and $X_3$, appear above the shaded black bands in Fig. 1. In contrast, the regions below the black band in Fig. 1 give values of $|\Delta \alpha_e|$ that are too small — although the latter region does not explain the anomaly, these regions of parameter space are not excluded by the $\Delta \alpha_e$ result, because the scalars we have introduced need not solve the $\Delta \alpha_e$ anomaly.
FIG. 1: Our solution for the $a_e$ anomaly in scalar mass $M_{X_i}$ versus the magnitude of the $X_i ee$ coupling, $|g_{1i}^1|$, for scalars (a) $X_1$ and (b) $X_3$, compared with existing and anticipated experimental constraints. The black dashed line shows our solution for $\Delta a_e$ in $|g_{X_i}^1|$ with $M_{X_i}$ for its experimental central value, with the black band enclosing the solutions bounded by that for $\Delta a_e$ taken at 95% CL. Note that values of $|g_{X_i}^1|$ above the black band produce a $|\Delta a_e|$ that is too large and are thus excluded by the measurement; we refer to the text for further discussion. We also show the excluded region at 90% CL from the measurement of parity-violating Møller scattering from the E158 [82] experiment (solid boundary), as well as the excluded region anticipated from the expected sensitivity of the planned MOLLER experiment (dashed boundary) at Jefferson Laboratory [83, 84], if no departure from the SM is observed. Constraints on the lightest $M_{X_i}$ masses can come from the measured running of $\alpha(s)$; please see Sec. V.

IV. CONSTRAINTS FROM PARITY-VIOLATING MØLLER SCATTERING

The parity-violating asymmetry $A_{PV}$ in the low-momentum-transfer scattering of longitudinally polarized electrons from unpolarized electrons has been measured to a precision of 17 ppb in the E158 experiment at SLAC, yielding a determination of the value of the effective weak mixing angle $\sin^2 \theta_W^{\text{eff}}$ to $\simeq 0.5\%$ precision [82]. In contrast, in a future experiment planned at the Jefferson Laboratory [83, 84], the MOLLER collaboration expects to measure $A_{PV}$ to an overall precision of 0.7 ppb [84], to determine $\sin^2 \theta_W^{\text{eff}}$ to $\simeq 0.1\%$ precision [84], with a commensurate improvement of $A_{PV}$ as a test of new physics. The determination of the weak mixing angle relies on the theoretical assessment of $A_{PV}$ in the SM [85–90], for which electroweak radiative corrections are important [86–90]. Nevertheless, as per usual practice [74, 83, 84], we use the tree-level formula for $A_{PV}$ of Ref. [85] to determine the sensitivity of the existing and planned $A_{PV}$ measurements to new physics. Only the doubly-charged scalars, $X_1$ and $X_3$, couple to two electrons, so that they contribute in $s$-channel to Møller scattering, i.e., $e^-(p) + e^-(k) \rightarrow X_i \rightarrow e^-(p') + e^-(k')$. Since we are considering constraints on light scalars, the value of $s$ is important, so that we note that both E158 and MOLLER are fixed-target experiments with an electron beam energy of $E = 50$ GeV for E158 [82] and $E = 12$ GeV for the MOLLER experiment [84]. Thus we have $s \simeq 2m_eE$, with $\sqrt{s} \simeq 0.23$ GeV for E158 — we label this “$\sqrt{s}$” in Fig. 1 — and $\sqrt{s} \simeq 0.11$ GeV for MOLLER. If a measured value of $A_{PV}$ agrees with SM expectations, then a model-independent constraint on new four-electron contact interactions follows, such
as those of either left-left or right-right form [91]

\[ \mathcal{H}_{\text{new}} = -\frac{g_{\xi e}^2}{2\Lambda^2} \bar{\psi}_{x} \gamma^\mu \psi_x \gamma^\mu \psi_x \]  

(15)

for \( \xi = L, R \). For the MOLLER experiment, e.g., we would have the lower bound [83, 84]

\[ \frac{\Lambda}{\sqrt{|g_{RR}^2 - g_{LL}^2|}} = \frac{1}{\sqrt{2G_F|\Delta Q_W^e|}} \simeq 7.5 \text{ TeV}, \]  

(16)

at 67% CL, where \( \Lambda \) is the mass scale of new physics and \( G_F \) is the Fermi constant. We note

that the error in the weak charge of the electron \( \Delta Q_W^e \), where \( Q_W^e \equiv 1 - 4\sin^2 \theta_W \) [84] in the SM, is \( \pm 5.1 \times 10^{-3} \) for the E158 experiment [82] and is expected to be \( \pm 1.1 \times 10^{-3} \) for the MOLLER experiment [84]. Interpreting both results at 90% CL yields \( \Lambda/\sqrt{|g_{RR}^2 - g_{LL}^2|} \simeq 2.7 \text{ TeV} \) and \( \Lambda/\sqrt{|g_{RR}^2 - g_{LL}^2|} \simeq 5.7 \text{ TeV} \) for the E158 and MOLLER experiments, respectively.

Returning to the possibility of doubly-charged scalars, we rewrite the interactions of Eq. (5) as

\[ \mathcal{H} \supset g_{i11}X_i \bar{\psi}^cP_{\xi}^i \psi + g_{i11}^*X_i^* \bar{\psi}P_{-\xi}^i \psi^c, \]

(17)

where \( i \) denotes model 1 or 3. Here \( \xi_1, -\xi_1 \) are \( R, L \) and \( \xi_3, -\xi_3 \) are \( L, R \), respectively. Computing the S-matrix for Möller scattering, \( e^-(p) + e^-(k) \to e^-(p') + e^-(k') \):

\[ \langle p'k'|T\left(\frac{1}{2!}(-i)^2 \int d^4x \mathcal{H}(x) \int d^4y \mathcal{H}(y) \right) |pk \rangle, \]

(18)

and noting that \( \bar{\psi}^c(x)P_{\xi}^i \psi(x) \bar{\psi}^c(y)P_{\xi}^i \psi^c(y) \) and \( \bar{\psi}^c(y)P_{\xi}^i \psi(y) \bar{\psi}^c(x)P_{\xi}^i \psi^c(x) \) generate the same contribution to the S-matrix, we have

\[ -|g_{i11}|^2\langle p'k'|T\left(\int d^4x X_i(x) \bar{\psi}^c(x)P_{\xi}^i \psi(x) \int d^4y X_i^*(y) \bar{\psi}^c(y)P_{\xi}^i \psi^c(y) \right) |pk \rangle. \]

(19)

After contracting \( X_i \) and \( X_i^* \), applying a Fierz transformation [92], and working in the \( s \ll M_{X_i}^2 \) limit, we extract the effective Hamiltonian

\[ \mathcal{H}_{\text{eff}} = -\frac{|g_{i11}|^2}{2M_{X_i}^2} \bar{\psi}^c \gamma^\mu P_{\xi}^i \psi \bar{\psi} \gamma^\mu P_{\xi}^i \psi. \]

(20)

Comparing with Eq. (15), we identify \( g_{RR} \equiv |g_{i11}| \) and \( g_{LL} \equiv |g_{i11}^*| \). For definiteness we note that Eq. (15) follows from the use of the \( Z_0 \) interaction in Ref. [85] to compute \( A_{\text{PV}} \), with \( v = g_{RR} + g_{LL}, a = g_{RR} - g_{LL}, g_0 = 1/2 \), which also yields \( |g_{RR}^2 - g_{LL}^2|/\Lambda^2 \leftrightarrow \sqrt{2G_F|\Delta Q_W^e|} \) as used in Eq. (16). Previously the relations \( |g_{RR}|^2 \equiv |g_{i11}|^2/2 \) and \( |g_{LL}|^2 \equiv |g_{i11}^*|^2/2 \) have been used to set the effective mass scale \( \Lambda \) for the doubly-charged scalars [74, 83, 84]; however, as we have shown, those 2’s should not appear. In our current analysis we wish to constrain light scalars, so that \( s \ll M_{X_i}^2 \) need no longer be satisfied. We note that we may still safely use \( A_{\text{PV}} \) as computed in Ref. [85] because \( g_{\xi e}^2 s/(2(s - M_{X_i}^2)\pi \alpha) \ll 1 \) can be satisfied nonetheless. Thus at low scales, we replace \( \Lambda/\sqrt{|g_{RR}^2 - g_{LL}^2|} \) by \( \sqrt{|s - M_{X_i}^2|/|g_{i11}|} \) to find the constraints

\[ \frac{\sqrt{|s - M_{X_i}^2|}}{|g_{i11}|} \geq 2.7 \text{ TeV}, \quad \frac{\sqrt{|s - M_{X_i}^2|}}{|g_{i11}|} \geq 5.7 \text{ TeV}, \]  

(21)
at 90% CL for the E158 [82] and MOLLER experiments [84], respectively. Thus we note that
the constrained region depends on the center-of-mass energy for each experiment and that
if \( M_{X_i} \ll \sqrt{s} \), only the coupling constants \( g_{i1} \) are constrained. In particular, if \( M_{X_i} \ll \sqrt{s'} \),
the E158 constraint becomes \( |g_{i1}| \leq 8.58 \times 10^{-5} \), whereas if \( M_{X_i} \ll \sqrt{s} \), the MOLLER
constraint becomes \( |g_{i1}| \leq 1.9 \times 10^{-5} \). The exclusion limits from Eq. (21) as a function of
\( M_{X_i} \) are shown in Fig. 1. One can find that indeed both the solid (red) and dashed (olive)
curves become straight lines as \( M_{X_i} \) grows much bigger than \( \sqrt{s'} \) and \( \sqrt{s} \). Moreover, as \( M_{X_i} \)
becomes much smaller than \( \sqrt{s'} \) and \( \sqrt{s} \), the solid (blue) and dashed (purple) curves become
flat, so that only a coupling constant constraint emerges. In the regions for which \( M_{X_i} \) is
very close to either \( \sqrt{s} \) or \( \sqrt{s'} \), the constraints of Eq. (21) demand a very small coupling
constant, though the evaluation of \( A_{PV} \) can become non-trivial — it may be necessary to
replace the scalar propagator by a Breit-Wigner form to find a definite result. However, for
the region shown in Fig. 1 this is not needed.

V. OTHER CONSTRAINTS

Light scalars that carry lepton number and couple to electrons, in a manner that preserves
SM symmetries, also carry electric charge. As a result, the “beam-dump” experiments that
severely constrain the electron coupling to electrically neutral, light scalars [41, 93] do not
operate, because electrically charged scalars interact with the material of the target or beam
dump and do not escape. Certainly, too, searches for \( s \)-channel resonances in low-energy
Bhabha scattering [94] do not apply to the current case, though an analogous search for a
\( s \)-channel resonance in \( e^-e^- \) scattering should be possible, though the extremely
narrow decay widths associated with the scalar solutions we have found in Fig. 1 may make
a sufficiently sensitive test impracticable. In what follows we consider further constraints
particular to scalars that carry lepton number.

The scalars \( X_1 \) and \( X_2 \) have been previously discussed in the context of a particular
model [67, 68] in which the neutrino masses are generated through radiative corrections [64,
66]. In this paper we do not delve into the origin of neutrino masses. Nevertheless, the
scalars \( X_2 \) and \( X_3^2 \) can potentially mediate additional neutrino mass contributions. We find
it impossible to generate either a Dirac or Majorana neutrino mass at one loop level, so
that our flavor-specific couplings do not in themselves impact the neutrino mass splittings.
Moreover, as \( M_{X_i} \) becomes much smaller than \( \sqrt{s'} \) and \( \sqrt{s} \), the solid (blue) and dashed (purple) curves become
flat, so that only a coupling constant constraint emerges. In the regions for which \( M_{X_i} \) is
very close to either \( \sqrt{s} \) or \( \sqrt{s'} \), the constraints of Eq. (21) demand a very small coupling
constant, though the evaluation of \( A_{PV} \) can become non-trivial — it may be necessary to
replace the scalar propagator by a Breit-Wigner form to find a definite result. However, for
the region shown in Fig. 1 this is not needed.

If neutrinos are massive Dirac particles, then the scalars \( X_2 \) and \( X_3^2 \) can each contribute
to its magnetic moment, though these effects turn out to be extremely small. The largest
contributions in the region of parameter space of interest to us come from \( X_2 \) to \( \mu_{\nu_e} \)
if \( M_{X_2} \approx m_e \) and from \( X_3^2 \) to \( \nu_e \) if \( M_{X_3} \approx m_e \). Employing Eq. (A.35) we find

\[
[\delta \mu_{\nu_e}]_{X_2} \approx -\frac{1}{12} \frac{|g_{21}^3|^2 m_\nu^2}{\pi^2} \frac{m_\nu}{m_e} \mu_B, \quad [\delta \mu_{\nu_e}]_{X_3} \approx -\frac{1}{24} \frac{|g_{31}^1|^2 m_\nu^2}{\pi^2} \frac{m_\nu}{m_e} \mu_B,
\]

(22)

\(^2\) This interaction is the same as model F in our recent work [51].
where for simplicity we have assumed the neutrinos are approximately degenerate, with mass $m_\nu$ and $\mu_B$ the Bohr magneton. From cosmological observations, we have $\sum_j m_j < 0.170$ eV at 95% CL, though the best current limit on $m_{\nu_e}$ from $^3$H $\beta$-decay is $m_{\nu_e} < 2.05$ eV at 95% CL [7]. Thus we see that even with $m_\nu \simeq 2$ eV and $|g_{21}^{13}| = 1$, the largest contribution, $[\delta \mu_{\nu_e}]_{X_2}$, can not be excluded by the current best experimental limit $|\mu|_\nu \sim 2.9 \times 10^{-11} \mu_B$ [95], nor by expected improvements [96, 97].

![Image](image)

**FIG. 2:** Our solution for the $a_e$ anomaly in scalar mass $M_{X_2}$ versus the magnitude of the $X_2 e\nu_\tau$ coupling, $|g_{2}^{13}|$, compared with existing experimental constraints. In this case we have shown our solution over a larger mass range than in Fig. (1), because the collider constraints on same-sign dileptons do not apply [80, 81]. The black dashed line and band are defined as in Fig. 1, but are for $\Delta a_e$ in $|g_{2}^{13}|$ with $M_{X_2}$. We also show the experimentally excluded region at 90% CL from the current error in the measured branching ratio in $\tau \rightarrow e\bar{\nu}_e \nu_\tau$ decay [7]; for $M_{X_2} < m_\tau$ we assume that the $X_2$ width is saturated by $X_2 \rightarrow e^- \bar{\nu}_\tau$ decay and refer to the text for further discussion.

We now turn to constraints from flavor physics, noting the comprehensive analysis of Ref. [72]. Taken altogether, the constraints on flavor-non-diagonal scalar-fermion couplings from the experimental limits on lepton-flavor-violating processes, and from the muon lifetime, are severe. As a result, we have considered first-generation couplings for $X_1$ and $X_3$, and first-third generation couplings for $X_2$ exclusively. Consequently, we need only consider the constraint from the measurement of $\tau \rightarrow e\bar{\nu}_e \nu_\tau$ decay, as the only other constraint, from $e/\mu$ lepton-flavor universality in semileptonic $\tau$ decay, acts similarly.

The scalar $X_2$ can mediate $\tau$ semileptonic decay via $\tau(p) \rightarrow \bar{\nu}_e X_2^* \rightarrow \bar{\nu}_e(k')e^-(p')\nu_\tau(k)$. After a Fierz transformation, we find the decay amplitude can found from the SM result by replacing $G_F^2 \rightarrow |g_{2}^{13}|^4/[2(t-M_{X_2}^2)^2]$, where $t = (p-k')^2$. Working in the $\tau$ rest frame and integrating over the three-body phase space, neglecting all the light lepton masses, yields

$$\Gamma = \frac{m_\tau |g_{2}^{13}|^4}{4\pi^3} \int_0^{m_\tau/2} \frac{d\omega'}{(m_\tau^2 - 2m_\tau\omega' - M_{X_2}^2)^2} \quad (23)$$

where $\omega'$ is the energy of the anti-electron neutrino. For $M_{X_2} > m_\tau$, the integral is well-defined, and for $M_{X_2} \gg m_\tau$ yields the familiar result

$$\Gamma = \frac{m_\tau^5}{192\pi^3} \frac{|g_{2}^{13}|^4}{2M_{X_2}^4} \quad (24)$$
For $M_{X_2} < m_\tau$, a $t$-channel pole appears, which we address by replacing the scalar propagator by a Breit-Wigner form:

$$\frac{1}{(t - M_{X_2})^2} \rightarrow \frac{1}{|t - M_{X_2}^2 + iM_{X_2}\Gamma_{X_2}|^2}. \quad (25)$$

Defining

$$x = \frac{t}{m_\tau^2}, \quad x_X = \frac{M_{X_2}^2}{m_\tau^2}, \quad \bar{\Gamma}_X = \frac{\Gamma_{X_2}}{m_\tau}, \quad (26)$$

we thus have

$$\Gamma = \frac{m_\tau|g^{13}_2|^4}{32\pi^3} \int_0^1 dx \frac{(1-x)^2 x}{(x-x_X)^2 + x_X\bar{\Gamma}_X^2}. \quad (27)$$

Since $x_X\bar{\Gamma}_X^2 \ll 1$, we can apply the narrow width approximation [98], i.e.,

$$(x - x_{A'})^2 + x_{A'}\bar{\Gamma}_{A'}^2)^{-1} \rightarrow \frac{\pi}{\sqrt{x_{A'}\bar{\Gamma}_{A'}}} \delta(x - x_{A'}), \quad (28)$$

to find

$$\Gamma = \frac{m_\tau|g^{13}_2|^4}{32\pi^3} \frac{m_\tau M_{X_2}}{\bar{\Gamma}_X} \left(1 - \frac{M_{X_2}^2}{m_\tau^2}\right)^2. \quad (29)$$

Since there is only one decay channel left for $X_2$, $X_2^* \rightarrow e^-\nu_\tau$, we compute

$$\Gamma_{X_2} = \frac{1}{4\pi} M_{X_2}|g^{13}_2|^2 \quad (30)$$

to find

$$\Gamma = \frac{m_\tau|g^{13}_2|^2}{8\pi} \left(1 - \frac{M_{X_2}^2}{m_\tau^2}\right)^2, \quad (31)$$

which, as expected, is identical to our result for $\Gamma(\tau \rightarrow eX_2^*)$. We now turn to the numerical constraints on the scalar-fermion couplings with $M_{X_2}$, given existing measurements of the $\tau \rightarrow e\nu_\tau\nu_\tau$ branching ratio and $\tau$ lifetime. Referring to Ref. [7] for all experimental parameters, we note particularly that $\text{Br}(\tau \rightarrow e\nu_\tau\nu_\tau) = 17.82 \pm 0.04\%$, $\tau_\tau = (290.3 \pm 0.5) \times 10^{-15}\text{s}$, and $m_\tau = 1776.86 \pm 0.12\text{MeV}$. For $M_X \gg m_\tau$, we can constrain, at 90% CL,

$$\frac{|g^{13}_2|^4}{2M_{X_2}^4 G_F^2} \leq \frac{\eta}{\text{Br}(\tau \rightarrow e\nu_\tau\nu_\tau)} \Rightarrow \frac{|g^{13}_2|}{M_{X_2}} \leq 1.0 \times 10^{-3}\text{GeV}^{-1} \quad (32)$$

or

$$\frac{m_\tau^5}{192\pi^3} \frac{|g^{13}_2|^4}{2M_{X_2}^4} \leq \frac{\eta h}{100\tau_\tau} \Rightarrow \frac{|g^{13}_2|}{M_{X_2}} \leq 1.6 \times 10^{-3}\text{GeV}^{-1}, \quad (33)$$

with $\eta = 0.066$. The two estimates differ in that the former implicitly assumes the leading-order formula describes the SM decay rate, though various refinements exist [99]. We note that the numerical limit reported by Ref. [72] in this case is significantly more severe than what we report. In what follows we use our second method to determine the exclusion limit. For $M_{X_2} > m_\tau$ we replace the left-hand side (LHS) of Eq. (33) with Eq. (23). For $M_{X_2} < m_\tau$ we replace the LHS of Eq. (33) with Eq. (31). We report the 90% CL exclusion we have found in Fig. 2, recalling that $(\delta a_e)_{X_2} < 0$ only if $M_{X_2} > m_e$. Thus we see that in this
case the existing empirical data rules out \( X_2 \) as a solution to the \( a_e \) anomaly, at least in a minimal scalar model. More generally, we note that Eq. (31) can be written \([98]\)

\[
\Gamma = \frac{m_{\tau} |g_2|^{13}|^{4}}{8\pi} \left( 1 - \frac{M_{X_2}^2}{m_{\tau}^2} \right)^2 \text{Br}(X_2^* \rightarrow e^-\nu_{\tau})
\]

and that decreasing \( \text{Br}(X_2^* \rightarrow e^-\nu_{\tau}) \) from unity weakens the constraint on \( |g_2^{13}|/M_{X_2} \) in the \( M_{X_2} < m_{\tau} \) region.

Finally, since our scalars can carry electron charge, we evaluate the constraints that follow from direct measurement of the running of \( \alpha(s) \), \( |\alpha(s)/\alpha(0)|^2 \), where \( \alpha \equiv \alpha(0) \). This can be determined from the measured differential cross section for \( e^+e^- \rightarrow \mu^+\mu^-\gamma \), for which the most precise results are in the time-like region below 1 GeV \([100]\) — there the presence of hadronic contributions is established at more than 5\(\sigma\). We evaluate \( \alpha(s) = \alpha/(1 - \Delta\alpha(s)) \) \([101]\), where the leading contribution to the vacuum polarizaton \( \Delta\alpha \) can be readily calculated in scalar QED to yield \([9]\)

\[
\text{Re}\Delta\alpha_{X_2}(s) = -\frac{\alpha}{2\pi} \int_0^1 dx \left( x - 1 \right) \log \left| \frac{M_{X_2}^2}{M_{X_2}^2 - s x} \right|,
\]

\[
= \frac{\alpha}{12\pi} \left[ \log \left( \frac{s}{M_{X_2}^2} \right) - \frac{8}{3} \right] \text{ for } s \gg 4M_{X_2}^2 ; \quad (35)
\]

\[
\text{Im}\Delta\alpha_{X_2}(s) = -i \frac{\alpha}{12} \left( 1 - \frac{4M_{X_2}^2}{s} \right)^{3/2} \Theta(s - 4M_{X_2}^2) . \quad (36)
\]

We note that \( \Delta\alpha_{X_2}(s) \) is 4 times smaller, and \( \text{Re}\Delta\alpha_{X_2}(s) \) runs more slowly, than that for a fermion in QED. The contribution of \( X_2 \) for \( M_{X_2} \leq m_e \) to \( |\alpha(s)/\alpha(0)|^2 \) deviates from unity by less than 0.5% over the \( s \)-range of the experiment, 0.6 < \( \sqrt{s} \) < 0.975 GeV, with an inappreciable \( s \) dependence. Since the individual measurements have a statistical error of \( \leq 1\% \) and an overall systematic error of 1\% \([100]\), the existence of the \( X_2 \) scalar is not constrained. However, the contributions from \( X_1 \) and \( X_3 \) include doubly-charged scalars, and we have \( \Delta\alpha_{X_1}(s) = 4\Delta\alpha_{X_2}(s) \) and \( \Delta\alpha_{X_3}(s) = 5\Delta\alpha_{X_2}(s) \). Although the contributions to \( \alpha(s) \) from \( X_1 \) and \( X_3 \) also have negligibly small slope in the \( s \)-range of interest, they can each generate an appreciable offset from zero. We suppose that the existence of these scalars is limited by the size of the overall systematic error, or offset, in the measurement of \( |\alpha(s)/\alpha(0)|^2 \). Noting the measured data points and their errors in Table 2 of Ref. \([100]\), we require that the overall shift in the theory contribution with a new scalar be less than 0.011 for \( \sqrt{s} < 0.783 \) GeV, the region for which the hadronic contribution is completely captured by the included \( 2\pi \) intermediate state. Thus we estimate \( M_{X_1} > 8.4 \) MeV and \( M_{X_3} > 19 \) MeV, apparently removing \( X_3 \) as a solution of the \( (g - 2)_e \) anomaly. We regard these limits as guidelines rather than exclusions because the new scalars generate contributions that do not impact the measured \( s \) dependence, but, rather, only its overall normalization. Nevertheless, we conclude that \( X_1 \) more likely offers a solution to the \( (g - 2)_e \) anomaly.

VI. SUMMARY

In this paper we have shown that the light scalars with lepton number that appear in minimal scalar models of new physics can generate solutions to the \( \Delta a_e \) anomaly, in that
they act to reduce the size of $|a_e|$. Although our solutions determine only the ratio of the scalar-fermion coupling to mass, we have particularly focussed on new particles with masses in excess of the electron mass and less than 8 GeV, as this mass region should evade both astrophysical cooling constraints and collider bounds. We should note, however, that since the scalars that couple to electrons also carry electric charge, lighter mass candidates could also prove phenomenologically viable, because such particles may be unable to escape an astrophysical environment and contribute to its cooling. We have proposed three solutions to the $\Delta a_e$ anomaly, but we have found that only the two solutions with doubly charged scalars may be possible, because the existing $\tau$ decay data preclude the singly-charged scalar $X_2$ as a solution, at least in a minimal scalar model. As for the doubly-charged scalars, the constraints from parity-violating Møller scattering permit a solution to the $\Delta a_e$ anomaly, with the upcoming MOLLER experiment poised to discover a conflict with the SM or to constrain our proposed solutions yet further.

We have noted, moreover, that the $\Delta a_e$ determination also constrains broad swathes of the scalar-fermion coupling and mass parameter space, as parameters which would give too large a value of $|a_e|$ should be excluded. There are plans to make substantially improved measurements of both the electron and the positron anomalous magnetic moments [17], to better existing measurements by a factor of 10 and 150 [102], respectively. Although this comparison is meant as a CPT test, it can also help affirm our new physics solution to the $\Delta a_e$ anomaly, as the two new measurements could well agree with each other, up to the expected difference in overall sign, but yet disagree with the SM using $\alpha$ determined through atom interferometry. The scalar solutions we have found can also help engender baryon and lepton number violation in low-energy scattering experiments, and we keenly await these studies.

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APPENDIX

Herewith we detail our $a_\ell$ computation for scalars that carry lepton number. The nature of the scalar-fermion interactions in this case, Eq. (5), allows for multiple ways in which the fermion fields can contract, so that it is more efficient to evaluate the time-ordered products of fields directly, rather than to develop Feynman rules for this case.

We have defined $\psi^c$ as $\psi^c \equiv C(\bar{\psi})^T$, noting the charge conjugation matrix $C$ obeys

$$C^\dagger = C = C^{-1} = -C,$$

as well as

$$C(\gamma^\mu)^T = -\gamma^\mu C, \quad C(\sigma^{\mu\nu})^T = -\sigma^{\mu\nu} C.$$  \hfill (A.2)

We first summarize the plane-wave expansions of a Dirac field $\psi(x)$, its charge conjugate
operators obey the anticommutation relations

\[ u^a_p = \sum_s \left( a^a_p \bar{u}(s, p)e^{-ip \cdot x} + b^a_p \bar{u}(s, p)e^{ip \cdot x} \right), \]

\[ (A.3) \]

\[ \bar{u}^a_p = \sum_s \left( a^a_p \bar{u}(s, p)e^{ip \cdot x} + b^a_p \bar{u}(s, p)e^{-ip \cdot x} \right), \]

\[ (A.4) \]

\[ v^c_p = \sum_s \left( a^c_p \bar{v}(s, p)e^{ip \cdot x} + b^c_p \bar{v}(s, p)e^{-ip \cdot x} \right), \]

\[ (A.5) \]

\[ \bar{v}^c_p = \sum_s \left( a^c_p \bar{v}(s, p)e^{-ip \cdot x} + b^c_p \bar{v}(s, p)e^{ip \cdot x} \right). \]

\[ (A.6) \]

We note \( u^c \) and \( v^c \) are defined in the manner of \( \psi^c \), and the creation and annihilation operators obey the anticommutation relations

\[ \{ a^a_p, a^b_q \} = \{ b^a_p, b^b_q \} = (2\pi)^3 \delta^3(p - q)\delta^{rs}. \]

We now summarize all the Wick contractions that can appear. The contractions between \( \psi(x) \), \( \bar{u}(x) \), \( \psi^c(x) \), and \( \bar{v}^c(x) \) and an incoming or outgoing fermion of mass \( m \) are

\[ \bar{u}_a(p, s) = u_a(s, p)e^{-ip \cdot x}\langle 0 | \bar{u}_a(s, p)e^{ip \cdot x}, \]

\[ (A.8) \]

\[ \bar{v}_a^c(p, s) = v_a^c(s, p)e^{-ip \cdot x}\langle 0 | \bar{v}_a^c(s, p)e^{ip \cdot x}, \]

\[ (A.9) \]

where \( |p, s\rangle = \sqrt{2E_p} \delta^3(p - s) \) and \( \langle p, s | \) denote an incoming and an outgoing fermion with momentum \( p \) and spin \( s \), respectively, whereas the contractions to an incoming or outgoing antifermion are

\[ \bar{v}_a^c(k, r) = \bar{v}_a^c(r, k)e^{-ik \cdot x}\langle 0 | \bar{v}_a^c(r, k)e^{ik \cdot x}, \]

\[ (A.10) \]

\[ \bar{u}_a(k, r) = \bar{u}_a(x)e^{-ik \cdot x}\langle 0 | \bar{u}_a(x)e^{ik \cdot x}, \]

\[ (A.11) \]

where, similarly, \( |k, r\rangle \) \( \langle k, r | \) denote an incoming (outgoing) anti-fermion with momentum \( k \) and spin \( r \). The spinor index \( a \) runs from 1 to 4. Different contractions of the internal fermion and antifermion fields can appear. That is,

\[ \bar{u}_a(x)\bar{v}_b(y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{\! p} + m)_{ab}}{p^2 - m^2 + i\epsilon} e^{-ip \cdot (x - y)}, \]

\[ (A.12) \]

\[ \bar{u}_a(x)\bar{v}_b^c(y) = \int \frac{d^4p}{(2\pi)^4} \frac{i[(\not{\! p} + m)C^\dagger]_{ab}}{p^2 - m^2 + i\epsilon} e^{-ip \cdot (x - y)}, \]

\[ (A.13) \]

\[ \bar{v}_a^c(x)\bar{u}_b(y) = \int \frac{d^4p}{(2\pi)^4} \frac{i[C^\dagger(\not{\! p} + m)]_{ab}}{p^2 - m^2 + i\epsilon} e^{-ip \cdot (x - y)}, \]

\[ (A.14) \]

\[ \bar{v}_a^c(x)\bar{v}_b^c(y) = \int \frac{d^4p}{(2\pi)^4} \frac{i[C^\dagger(\not{\! p} + m)C^\dagger]_{ab}}{p^2 - m^2 + i\epsilon} e^{-ip \cdot (x - y)}, \]

\[ (A.15) \]

where \( a \) and \( b \) are spinor indices.

We now can compute the one-loop amplitude associated with the lepton anomalous magnetic dipole moment \( a_\ell \). As shown in Fig. 3, a photon can be attached to either a charged
FIG. 3: Feynman diagrams to illustrate contributions to the anomalous magnetic dipole moment $a_{\ell_a}$ of lepton $a$, where $b$ denotes another lepton, and $X$ denotes a scalar that carries lepton number. Note that if lepton $b$ is electrically neutral, only diagram (b) can contribute to $a_{\ell_a}$.

fermion line or a charged scalar line, and interactions from quantum electrodynamics (QED) and scalar QED are needed:

$$\mathcal{H}_1 \supset -Q e \bar{\psi} \gamma^\mu \psi A_\mu,$$

$$\mathcal{H}_2 \supset -iQ e [\partial^\mu X^* - X (\partial^\mu X^*)] A_\mu,$$

where $Q = -1$ for the electron. Noting Eq. (5), we make the replacements $g_i^{11} \rightarrow g_i$ for $i = 1, 3$ and $e \rightarrow \psi$. Here we consider the contributions from $X_1$ and $X_3$. We address the contribution to $a_\ell$ from $X_2$ later.

For the first case, the interaction is

$$\mathcal{H} \supset -eQ \bar{\psi} \gamma^\mu \psi A_\mu + g_i X_i \bar{\psi} P_\xi \psi + g_i^* X_i^* \bar{\psi} P_{\xi'} \psi^c,$$

where $P_\xi = (1 + \xi \gamma^5)/2$ is the chiral projection operator with $\xi = \pm 1$ for R or L. Hermitian conjugation of the second term results in the third term, in which $\xi' = -\xi$. The one-loop contribution comes from the $H^3$ term of the S-matrix:

$$\langle p' | T \left( \frac{1}{3!} (-i)^3 \int d^4 x \mathcal{H}(x) \int d^4 y \mathcal{H}(y) \int d^4 z \mathcal{H}(z) \right) | p q \rangle,$$

where $q$ represent the momenta of incoming photon, and $p$ and $p'$ denote the momenta of the incoming and outgoing leptons, respectively. Since there are 3! ways of arranging relative interactions in $\mathcal{H}$ that generate the same matrix element we have

$$\langle p' | T \left( (-i)^3 g \int d^4 x g_i X_i \bar{\psi} P_\xi \psi \int d^4 y g_i^* X_i^* \bar{\psi} P_{\xi'} \psi^c \int d^4 z (-eQ) \bar{\psi} \gamma^\mu \psi A_\mu \right) | p q \rangle.$$

There are four different ways of contracting the fields in Eq. (A.20):

$$\langle p' | \left[ \left[ \int d^4 x X_i \bar{\psi}^c (P_\xi)_{a\alpha} \psi_a \int d^4 y X_i^* \bar{\psi} (P_{\xi'})_{b\beta} \psi_b \int d^4 z \bar{\psi} \gamma^\mu \psi A_\mu \right] p q \right],$$

$$\langle p' | \left[ \left[ \int d^4 x X_i \bar{\psi}^c (P_\xi)_{a\alpha} \psi_a \int d^4 y X_i^* \bar{\psi} (P_{\xi'})_{b\beta} \psi_b \int d^4 z \bar{\psi} \gamma^\mu \psi A_\mu \right] p q \right].$$
fermion contractions: there is only one way to contract all the scalars, we show it separately from the four different contractions, and they contribute identically to the one-loop amplitude. Since after combining all of the contractions and dropping the factor \((2\pi)^4\delta^4(p + q - p')\), the total matrix element is

\[
i\mathcal{M}^\mu = 4Qe|g_i|^2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p')P_{\ell'}(k' - m_b)\gamma^\mu(k' - m_b)P_\mu u(p)}{(k^2 - m^2 + i\epsilon)(k'^2 - m'^2 + i\epsilon)((k + p')^2 - M^2_{X_i} + i\epsilon)},
\]

where \(k' = k + q\) and \(m_b\) and \(M_{X_i}\) are the masses of the charged lepton and scalar in the loop, respectively — the overall 4 comes from the different contractions we have noted. We find that Eq. (A.25) contributes to \(a_{\ell_a}\) as

\[
\delta a_{\ell_a} = \frac{Qg_i g_i^*}{4\pi^2} \int_0^1 dz \frac{m_a^2 z(1 - z)^2}{(z^2 - z)m_a^2 + zM^2_{X_i} + (1 - z)m_b^2},
\]

where \(m_a\) is the mass of external lepton \(a\). Note that the final result is independent of \(\xi\).

We now move to the second case. The interaction is

\[
\mathcal{H} = -iQ' e[\partial^\mu X_i X_i^* - X_i(\partial^\mu X_i^*)]A_\mu + g_i X_i \bar{\psi} P_\mu \psi + g_i^* X_i^* \bar{\psi} P_\mu \psi^c,
\]

where the charged scalar has \(Q' = 2\), if it couples to two electrons. Here, too, there are four different contractions, and they contribute identically to the one-loop amplitude. Since there is only one way to contract all the scalars, we show it separately from the four different fermion contractions:

\[
\langle p' | \int d^4x \bar{\psi}^c(P_{\ell'})_{a\alpha'} \psi_{a\mu'} \int d^4y \bar{\psi}_b(P_{\ell'})_{b\alpha'} \psi^c_{b\mu'} \int d^4z A_\mu[p\ q],
\]

\[
\langle p' | \int d^4x \bar{\psi}^c(P_{\ell'})_{a\alpha'} \psi_{a\mu'} \int d^4y \bar{\psi}_b(P_{\ell'})_{b\alpha'} \psi^c_{b\mu'} \int d^4z A_\mu[p\ q],
\]

\[
\langle p' | \int d^4x \bar{\psi}^c(P_{\ell'})_{a\alpha'} \psi_{a\mu'} \int d^4y \bar{\psi}_b(P_{\ell'})_{b\alpha'} \psi^c_{b\mu'} \int d^4z A_\mu[p\ q],
\]

\[
\langle p' | \int d^4x \bar{\psi}^c(P_{\ell'})_{a\alpha'} \psi_{a\mu'} \int d^4y \bar{\psi}_b(P_{\ell'})_{b\alpha'} \psi^c_{b\mu'} \int d^4z A_\mu[p\ q],
\]

with

\[
ig_i g_i^* Q' e \left[ X_i(x)X_i^*(y)(i\partial^\mu X_i(z))X_i^*(z) - X_i(x)X_i^*(y)X_i(z)(i\partial^\mu X_i^*(z)) \right].
\]

After combining all of the contractions and dropping the factor \((2\pi)^4\delta^4(p + q - p')\), we find the one-loop matrix element in the second case is

\[
i\mathcal{M}^\mu = -4g_i g_i^* Q' e \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p')(k + p' + m_b)u(p)(k + k')^\mu}{(k^2 - M^2_{X_i})(k'^2 - M^2_{X_i})((k + p')^2 - m_b^2)},
\]
with \( k' = k + q \), which contributes to \( a_{\ell a} \) as

\[
\delta a_{\ell a} = -\frac{Q' g_1^b g_2^b}{4\pi^2} \int_0^1 dz \frac{m_a^2 z (1 - z)^2}{(z^2 - z) m_a^2 + zm_b^2 + (1 - z) M_{X_i}^2},
\]

(A.34)

noting that this result is independent of \( \xi \), too. To compute the final contribution to \( \delta a_{\ell a} \) from either \( X_1 \) or \( X_3^2 \) we add those of Eqs. (A.26, A.34).

The computation of \( \delta a_{\ell e} \) from \( X_2 \), or from \( X_3^2 \), is more straightforward in that only a single set of fermion contractions exists. We find from \( X_2 \), where \( Q' = 1 \) for the scalar that couples to an electron and a neutrino, that

\[
\delta a_{\ell e} = -\frac{Q' 4 |g_2^b|^2}{4\pi^2} \int_0^1 dz \frac{m_a^2 z (1 - z)^2}{(z^2 - z) m_a^2 + zm_b^2 + (1 - z) M_{X_i}^2},
\]

(A.35)

where \( m_b^2 = m_{\nu_b}^2 \). To find the contribution from \( X_3^2 \) we replace \( 2 g_2^b \) with \( \sqrt{2} g_2^{1b} \) and note that \( m_b^2 \) is just \( m_{\nu_b}^2 \).

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