Nonlinear Helmholtz equations with sign-changing diffusion coefficient

Abstract
In this talk, we study nonlinear Helmholtz equations with sign-changing diffusion coefficients on bounded domains of the form $-\text{div}(\sigma(x) \nabla u) - \lambda c(x) u = g(x, u)$. Using weak $T$-coercivity theory, we can establish the existence of an orthonormal basis of eigenfunctions of the linear part $-c(x)^{-1} \text{div}(\sigma(x) \nabla u)$. Then, all eigenvalues are proved to be bifurcation points and we investigate the bifurcating branches both theoretically and numerically. As a fundamental example, we look at some one-dimensional model, we obtain the existence of infinitely many bifurcating branches that are mutually disjoint, unbounded, and consist of solutions with a fixed nodal pattern.