Insight on confinement using scalar field interactions

R. Markazi$^{1,2*}$ and N. El Biaze$^{1,2†}$

$^1$ Materials and renewable energies laboratory, Faculty of science, Ibn Zohr University-Agadir, Morocco.
$^2$ High School of Technology-Guelmim, Ibn Zohr University-Agadir, Morocco.

Abstract

The scalar field plays a fundamental role in the investigation of confinement property characterising many particle physics models. This is achieved by coupling this particle directly with gauge fields at the lagrangian level. We have adopted the same approach to determine a potential [10] as a perturbative series in terms of interquark distance. In order to introduce the gravitational effects and inspired from bag models, we implement a scalar field which interacts both with the vacuum and the electron field. In this context and with presence of the vacuum condensates, it is possible to derive a more accurate expression of the electron energy.

* r.markazi@uiz.ac.ma
† n.elbiaze@uiz.ac.ma
1 Introduction

Bag model is an interesting approach which provides an explanation of confinement property characterising quarks interactions. They are also used to describe some physical phenomena related to the stability of some particles like electron. According to this approach, one supposes the existence of a cavity, which is a space zone filled with scalar condensate. The scalar field appears in many theoretical physics models with different meanings. Sometimes it appears as a massless field and in others as a massive one. In QCD models, it is seen as a pseudo-Goldstone boson[5]. Whereas in cosmology, it is an hypothetical particle considered as a dark matter candidate that constitutes the missing mass of universe [6]. The scalar field can couples in different ways. For example in string theory, it couples to super-Yang Mills gauge fields in curved space whereas in the Brans-Dicke model, it couples to the Ricci curvature[8]. For a long time the electron stability was studied using different approaches. These latter faced several difficulties related to the electron finite size, the origin of its mass and the problem with the relativistic transformation properties of the energy and momentum of the electromagnetic field of the electron. To overcome these difficulties, one introduces a model of an electron based on a charged conducting surface of a cavity in the electromagnetic field. In this way, we get phenomenological solution to the electron stability problem. In fact, the cavity exhibits a surface tension due to the difference of the condensate density between its inside and its outside.

The surface tension depends on the parameters defining the Higgs potential in the electroweak gauge-model theory [9-14]. It allows to establish the expression of the total energy of the electron and its surrounding cavity. The total energy depends on the vacuum expectation value of the Higgs field and the Ginzburg-Landau coherence length [15].

2 Confining potential from scalar field-gluon coupling

The confinement property means that the quarks and gluons cannot exist as separate objects. To investigate it, various QCD models were proposed to derive an interquark potential which exhibits confinement behavior. Such potential can be obtained by considering the coupling of a dilaton $\phi$ to the $4dSU(Nc)$ gauge field like in string theory models[8]. To this end, one suggests the following effective lagrangian:

$$L = -\frac{1}{4F(\phi)}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{2}m^2\phi^2 + J_{a}A_{a}^{\mu},$$

(2.1)
where the coupling $\frac{1}{4 F(\phi)}$ is a function of the dilaton field $\phi$ and $m$ is the dilatonic mass. The form of this coupling can have several expressions according to the theoretical frameworks. The current density $J^a_\nu = g \delta(r) C_a \eta^{\mu}$ appearing in the lagrangian is considered as a point like static source. $C_a$ is the expectation value of the $SU(N_c)$ generators for a normalized spinor in the color space, satisfying the algebra identity $\Sigma_a C_a^2 = \frac{N_c^2 - 1}{2N_c}$.

Using the lagrangian (2.1), one derives the following equations of motion corresponding to $\phi$ and $A_\mu$ fields:

$$\partial^2 \phi - m^2 \phi = \frac{1}{4} \frac{d(1/F(\phi))}{d\phi} G_{\mu\nu} G^{\mu\nu}$$  \hspace{2cm} (2.2)$$

$$\partial_\mu \left( \frac{1}{F(\phi)} G^{\mu\nu}_a \right) + g \frac{1}{F(\phi)} A^b_{\mu} f^{c}_{ab} G^{\mu\nu}_c = -J^a_\nu,$$  \hspace{2cm} (2.3)$$

which can be rewritten as:

$$\frac{d^2}{dr^2} (r \phi) - m^2 (r \phi) = \frac{\mu^2}{2r^3} \frac{d(F(\phi))}{d\phi}$$  \hspace{2cm} (2.4)$$

where $\mu = \frac{g}{4\pi} \sqrt{\frac{N_c^2 - 1}{2N_c}}$.

The equation (2.4) may be solved for a given dilaton-gluon coupling $F(\phi)$ for which the interquark potential $V(r)$ is given by the following formula [10]:

$$V(r) = -\frac{g}{4\pi} C \int \frac{F(\phi(r))}{r^2} dr.$$  \hspace{2cm} (2.5)$$

This expression of the potential is very interesting as it generalizes the Coulombian potential formula $V_c(r) \sim \frac{1}{r}$, recovered in the particular case: $F(\phi) = 1$. The main reason for introducing the coupling between the dilaton field and the Yang Mills field strength is to have an alternative way to derive more general potential which takes into account the confinement behavior of interquark interaction. Following this approach, one can get many forms of the interquark potential just by modifying the dilaton-gluon coupling $F(\phi)$. We have shown in [8,12] that the confining terms are directly related to the QCD vacuum condensates describing the nonperturbative effects.

### 3 Influence of scalar field condensate on electron energy

In order to investigate the electron stability, a model [10] based on a scalar field $\phi$ and a $U(1)$ gauge field $A_\mu$ was suggested. The electron charge is supposed to be uniformly distributed on
of a spherical cavity immerged in a vacuum filled with the Higgs condensate. In this approach, the Lagrangian can be written as:

\[ \mathcal{L} = \mathcal{L}^\phi + \mathcal{L}^E \]  

(3.1)

Where the scalar part of the lagrangian is:

\[ \mathcal{L}^\phi = \frac{1}{2} g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \]  

(3.2)

the conventional Minkowski metric tensor \( g_{\mu\nu} = (1, -1, -1, -1) \). To ensure the spontaneous symmetry breaking, \( V(\phi) \) is chosen as \([11]\): 

\[ V(\phi) = -2\mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4. \]

The electric charge of the electron gives rise to the Yang-Mills term:

\[ \mathcal{L}^E = -\frac{1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} \]  

(3.3)

where \( F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \) is the field-strength tensor. In this framework, the stress energy tensor given by the relation:

\[ T_{ij} = \frac{2}{\sqrt{|g|} \partial g^{ij}} (\sqrt{|g|} \mathcal{L}) \]  

(3.4)

can be splitted into two main parts:

\[ T_{ij} = T^H_{ij} + T^E_{ij} \]  

(3.5)

where \( T^H_{ij} \) and \( T^E_{ij} \) are respectively the scalar and the gauge field contributions:

\[ T^H_{ij} = \partial_i \phi \partial_j \phi - g_{ij} \mathcal{L}^H \]  

(3.6)

and

\[ T^E_{ij} = \frac{1}{4} g_{\beta j} F^i_{\lambda \beta} F^{\lambda \beta} + \frac{1}{4} g_{ij} F_{\mu \lambda} F^{\mu \lambda} \]  

(3.7)

Let’s consider the electron charge as a static source confined in a spherical cavity of radius \( R \) and that the scalar field vanishes inside it whereas, it is supposed to have a fixed value at infinity. A coherent length denoted by \( \delta \) was defined in order to determine the transitional space regions which separates between the two different vacuum expectation values of the scalar field. This transitional zone can be seen as a domain wall separating the false vacuum and the true vacuum regions. With these assumptions, the total energy can be written as follows\([10]\).

\[ E_{\text{tot}}(R) = \frac{e^2}{2R} + 4\pi \sigma R^2 \]  

(3.8)
where $\sigma$ is the surface tension due to the scalar field expectation value outside the cavity $\eta$ given by: $\sigma = \frac{2\eta^2}{3\delta}$.

The first term appearing in (3.8) represents the coulomb energy of the cavity surface whereas the second term arises from both the presence of the condensation energy and the coherent length.

4 Gravitational effects on electron energy

The two approaches mentioned above provide two different ways to derive an interaction potential which is able to take into account both the colombian interaction and the confinement behavior.

In the first approach, a direct coupling between the scalar field and the field strength is necessary to obtain such interaction potential whatever is the form of the scalar field space time distribution.

In the second approach, more interest was given to the scalar spacetime distribution with the absence of interaction between the scalar field and the electric field.

Till now, we did not take into consideration the contribution of the gravitational effects to the total energy of the electron. Motivated by their importance, we extend the two above approaches by adding a new term to the lagrangian which describes these effects as an interaction between the scalar field and the space-time curvature. In this framework, the scalar field lagrangian becomes:

$$L_\phi = -\frac{1}{2}(\mu^2 + \xi R)\phi^2 - \frac{\lambda}{4}\phi^4$$

(4.1)

Where the coupling between the scalar field $\phi$ and the space-time curvature $R$ is measured by the $\xi$ parameter. We retrieve the previous form of the potential in the particular case $[11]$ $\xi = 0$, where $\mu$ and $\lambda$ are standard parameters.

In our case, the scalar field vacuum expectation value obtained through the minimisation of the potential (4.1), is $\eta^2 = \frac{\mu^2 + \xi R}{\lambda}$. Using this latter, the potential can be rewritten like:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2 - \frac{\lambda}{4}\eta^4$$

(4.2)
Let’s suppose that the cavity radius is an approximation of the space-time curvature. Consequently, the total energy of the electron becomes:

\[ E_{\text{tot}} = \frac{e^2}{2R} + \frac{\mu^2}{\delta \lambda} \pi R^2 + \frac{\xi}{\delta \lambda} R^3 \]  

(4.3)

By analyzing (4.3), we remark that the curvature nature of space time brings a new contribution to the confining part of the total electron energy.

Finally, due to the space-time curvature coming from the gravitational effects, the value of the condensation energy which garantees the confinement behaviour of our model is \( R \) dependent:

\[ \epsilon_{\text{cond}} = -\frac{1}{4\lambda^3} \left( \mu^2 + \xi R \right)^2 \]  

(4.4)

Because particles are not free but instead live in a small space time region, any tentative to get an estimation, for example of the electron radius must take into account both the gravitational and the vacuum effects.

**Acknowledgements**

We would like to thank A. Ihlal and A. Rachidi for their support.

**References**

[1] A. Chodos, R. J. Jaffe, K. Johnson, C. B. Thore and V. F. Weisskopf, Phys. Rev. D9, 3471 (1974).

[2] W. A. Bardeen, M. S. Chanowitz, S. D. Drell, M. Weinstein and T. M. Yan, Phys. Rev. D11, 1094 (1975).

[3] P. A. M. Dirac, Proc. Roy. Soc. 268A, 57 (1962).

[4] N. Seiberg and E. Witten, Nucl. Phys. B426 (1994) 19.

[5] A. Sakharov, JETP Lett. 5 (1967) 24.

[6] R. Dick, Euro. Phys. J. C6 (1999) 701. R. Dick and L.P. Fulcher, Euro. Phys. J. C9 (1999) 271.

[7] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967). [10] A. Salam, in Elementary Particle Theory, ed. W. Svartholm, (Almqquist and Wiskell, Stockholm (1968)).

[8] P. D. B. Collins, A. D. Martin and E. J. Squires, Particle Physics and Cosmology, (John Wiley & Sons, New York (1989)).
[9] A. L. Fetter and J. D. Walecka, Quantum Theory of Many-particle Systems, (McGraw-Hill, New York (1971)).

[10] E. Simanek “Stability of an electron embedded in Higgs condensate” arXiv:1502.00983

[11] E. Bentivegna, V. Branchina, F. Contino, D. ZappalImpact of New Physics on the EW vacuum stability in a curved spacetime background arXiv:1708.01138

[12] M. Chabab, R. Markazi and E. H. Saidi, Euro. Phys. J. C13 (2000) 543;

[13] M. Chabab, N.EL Biaze, R. Markazi and E. H. Saidi, Class.Quant.Grav.18:5085-5096,2001.

[14] C. Barcelo, S. Liberati and M. Visser, Analog gravity from eld theory normal modes, arXiv:gr-qc/0104001.

[15] J. D. Jackson, Classical Electrodynamics, (John Wiley & Sons, Inc., New York (1975)).

[16] T. D. Lee, Particle Physics and Introduction to Field Theory, (Harwood Academic Publishers, Chur, Switzerland (1981)).