A Multi-Image Cryptosystem Using Quantum Walks and Chebyshev Map

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1. Introduction

Today, data security and privacy are integral to our daily lives where more than 300 million images are unloaded on the internet daily [1]. Mylio, software company, estimates that 9.3 trillion images will be stored online by 2022 [2]. Considering the varying levels of confidentiality and precious memories in these images, their security and privacy is a major concern to many of us.

Presently, various technologies and cryptographic mechanisms are employed to encrypt, watermark, and secure these images including the use of steganographic data hiding schemes. While data encryption pervades access to unauthorised users by making such images unintelligible, chaotic systems exhibit sensitive dependence on original conditions, which implies that, for any small alteration, the dynamics of the system persistently magnifies the original conditions. This according to [3] implies that “two trajectories with initial conditions that are arbitrarily close will diverge at an exponential rate.” There are many single-image cryptosystems that have been proposed, including those based on colour codes [4], dynamic filtering [5], imitating jigsaw method [6], bit-level permutation [7], DNA sequence operation [8], and particle swarm optimization [9], besides chaos-based image cryptosystems.

Meanwhile, there are two major classifications of chaotic maps: one- and multi-dimensional chaotic systems. One-dimensional (1D) chaotic maps offer low computational complexity, simple architecture, and structure as well as accelerated processing [10]. Despite the enumerated properties, 1D chaotic systems are exposed to attacks due to their
small key space and the chaotic discontinuous ranges of their primary values [11].

Due to the developments in communication networks and prevalence of bulk data processing, multiple-image cryptosystems potentially have important roles in data security and privacy in the emerging era of big data whereas single-image cryptosystems can be utilised to protect multiple images in a per process approach, but these processes pile up to reduce the effectiveness of encryption scheme [12]. To enhance this, various scholars have invested in utilising multi-image cryptosystems to protect multiple images. In doing so, the highlighted properties of chaotic systems are exploited to design multiple-image encryption schemes with many seemingly tamper-proof cryptosystems realised [13–19]. For example, in [13], Shao et al. presented a new multiple-image encryption mechanism using logistic map and quaternion gyrator transform, while in [14], Zarebnia et al. suggested a multi-image encryption approach using chaotic maps and cyclic shift operation. Furthermore, in [15], Li et al. proposed a multi-image encryption approach in wavelet transformation domain based on a chaotic map. Similarly, in [18], Zhang and Wang presented a multi-image encryption mechanism using piecewise linear chaotic map and mixed image elements. While the highlighted schemes offer platforms to secure the images, our push towards the triple-S (speed, security, and size) limits of today’s technologies compels the need for more advanced computing infrastructure.

Quantum computers offer the promise of information processing beyond the capabilities of even the best of today’s supercomputers. The inevitable realisation of such hardware portends huge implications for available cryptosystems, making them vulnerable to many types of violations [20, 21]. Consequently, it is important to consider integrating this apparent “quantumness” into existing schemes to safeguard the confidentiality and integrity of images.

Quantum walks (QWs), which are quantum computing equivalents of the classical (i.e., nonquantum, or digital) random walks, provide high sensitivity to initial parameters and astounding nonperiodicity [22]. Furthermore, they exhibit significant stability and theoretically infinite key space allowance. These properties arise through reversible unitary evolution, nonrandom, quantum superposition of states, and collapse of the wave function due to state measurements [22]. The use of quantum walks in building efficient cryptosystems has been explored in many studies [20, 21, 23–25]. Moreover, QWs have been used to construct exponential speed up algorithms, quantum simulations, universal quantum gates, etc. [22].

For example, in [23], Vlachou et al. proposed a public cryptographic system that utilises QW to generate a public key, which is similar to its use by Yang et al. to propose an image encryption approach based on QW in [25]. In their contribution [20], Abd-El-Atty et al. considered an optical image encryption based on QW. Specifically, they used the alternate QW in a dual encryption strategy that, first, uses double random phase encoding that is executed using permutation followed by substitution, and second, generation of random masks.

Similarly, in [8], Yan and Li utilised the controlled alternate QWs and DNA sequence operations for colour image encryption. They used controlled alternate QWs to generate pseudorandom number generator and two rounds of DNA operation rules to encrypt plaintext images. Furthermore, in [26], Shi et al. proposed a quantum blind signature approach with cluster states using a QW-based cryptosystem. They claim that in the initial signing and verification phases, a message encrypted using QW is sent to the receiver, who requests a blind signature. Subsequently, in the verification phase, the authenticity and integrity of the transmitted message is verified. Finally, in [27], Abd El-Latif et al. considered the use of cascaded QWs with induced chaotic dynamics for cryptographic purposes. Their scheme utilised QWs to construct substitution boxes and provide the necessary chaos inducement and random number generation; the properties they claimed were combined in their reported cryptosystem. Furthermore, the schemes in [20, 27] report high encryption speeds relative to those in competing studies.

Notwithstanding the performances of the highlighted schemes, a common problem with the use of chaotic systems in the reported schemes is the low stability due to periodicity of the chaotic mapping. Meanwhile, Chebyshev polynomials satisfy the two important properties of semigroup membership and chaos. For this, our proposed cryptosystem infuses the symmetric and correlational properties of Chebyshev maps [28] to permutate the composite image. Additionally, as a prelude to realising the cipher composite image, Chebyshev mapping is used to substitute the composite image. To further elucidate, the key contributions of this study include the following:

1. Integrating quantum walks with existing chaotic systems to design a multiple-image cryptosystem for safeguarding the confidentiality and integrity of images.
2. Using QW to substitute the composite image and update encryption key parameters.

The performance of our systems is validated via extensive simulation-based experiments to establish its effectiveness and reliability in cryptographic applications involving multiple images. The remainder of the paper is organised as follows: background on both quantum walks, and Chebyshev maps are briefly highlighted in Section 2. Intrigues regarding the use of QWs and Chebyshev mapping to design our proposed cryptosystem are presented in Section 3, while evaluation of the performance of the proposed system is reported and discussed in Section 4. Section 5 concludes this study.

2. Preliminary Background on Quantum Walks and Chebyshev Map

This section presents brief overview of the main building blocks of our proposed cryptosystem (i.e., quantum walks and Chebyshev map).
2.1. Quantum Walks. Exploiting the essential properties of quantum mechanics, notably superposition, unitarity of state evolution, and wave function collapse [29], quantum walks were proposed as equivalents of the classical random walks.

There are two core elements of QWs: coin particle \( H_c = \cos \mu |0\rangle + \sin \mu |1\rangle \) and walker space \( H_p \), both of which exist in a Hilbert space \( H = H_c \otimes H_p \) [22]. To transform the full quantum state, the evolution operator \( \hat{U} \), defined in the following equation, is applied on the full quantum state:

\[
\hat{U} = \hat{S}(\hat{I} \otimes \hat{C}),
\]

where \( \hat{S} \) implies the shift operator and it is depending on the coin system of the walker. Further, the shift operators \( \hat{S} \) of one-walker QW on a cycle of \( V \) nodes can be expressed as

\[
\hat{S} = |(x-1)\mod V, 1\rangle \langle x, 1| + |(x+1)\mod V, 0\rangle \langle x, 0|. 
\]

Additionally, a \( 2 \times 2 \) coin operator, \( \hat{C} \), can be expressed as

\[
\hat{C} = \begin{pmatrix} \cos \omega & \sin \omega \\ \sin \omega & -\cos \omega \end{pmatrix}. 
\]

Hence, the final state \( |\psi_t\rangle \) after \( t \) steps can be stated as follows:

\[
|\psi_t\rangle = (\hat{U})^t |\psi_0\rangle, 
\]

where \( |\psi_0\rangle \) indicates the primary state of the quantum system. Consequently, the probability of locating the particle at position \( x \) after \( t \) steps can be computed by making use of

\[
P(x,t) = |\langle x, 0| (\hat{U})^t |\psi_0\rangle|^2 + |\langle x, 1| (\hat{U})^t |\psi_0\rangle|^2. 
\]

2.2. Chebyshev Map. Chebyshev polynomials are two sequences of polynomials built on the sine and cosine functions, notated as \( T_n(x) \) and \( U_n(x) \). These polynomials exhibit two important properties of semigroup and chaos. Of particular interest are mappings of Chebyshev polynomials for \( a \rightarrow T_n(a) \) since any pair of such mappings commute and \( T_n \otimes T_m = T_{nm} \).

The structure of the described collection of mappings, known as Chebyshev map [28, 30], is among the widely used one-dimensional chaotic systems, which is expressed as

\[
x_{t+1} = \cos(B \times \arccos(x_t)),
\]

where \( x_t \epsilon [-1, 1] \) is the original value of the chaotic map and \( B \epsilon N \) is the control parameter and \( B \geq 2 \).

3. Proposed Cryptosystem

This section outlines the use of the quantum walks and Chebyshev map highlighted in the previous section to design our proposed multiple-image cryptosystem. In summary, the proposed scheme involves the use of QW to substitute the plain composite image and the use of the substituted composite image to update key parameters of the Chebyshev map and subsequent permutation of the substituted composite image using QW and Chebyshev map. Finally, this permuted image is substituted using the Chebyshev map to realise the cipher composite image.

3.1. Encryption Algorithm. Figure 1 presents a graphical outline of the proposed encryption approach which is executed in the steps enumerated as follows:

1. Merge all plain images (\( Ig_0, Ig_2, \ldots, Ig_0N \)) into one composite image (\( Pim \)), and then convert \( Pim \) into a vector \( IVec \).

\[
IVec = \text{reshape} (Pim, m \times n \times c, 1),
\]

where \( m, n, \) and \( c \) are the dimension of the composite image.

2. Select the primary key parameters \( (V, t, \mu, \omega) \) for operating QWs \( t \) times on a cycle of odd \( V \) nodes to produce a probability distribution \( P \) of size \( V \). Here, the primary state of the coin walker is \( H_c = \cos \mu |0\rangle + \sin \mu |1\rangle \), and \( \mu, \omega \) is used to construct the coin operator \( \hat{C} \) (equation (3)), where \( \mu, \omega \epsilon [0, \pi/2]\).

3. Resize \( P \) to the dimension of the composite image \( IVec \) \( (m \times n \times c) \). This action accommodates composite images of different dimensions. In this study, the bicubic interpolation method [31] is used for the resizing process.

\[
\text{Re} P = \text{resize} (P, [m \times n \times c, 1]).
\]

4. Obtain the key sequence (Key1) via conversion of \( \text{Re}P \) into integer values.

\[
\text{Key1} = \text{floor} (\text{Re} P \times 10^{12}) \mod 256,
\]

where floor is the floor operation that rounds each value of \( \text{Re} P \times 10^{12} \) towards zero (e.g., floor \( 4.736 \) = 4).

5. Execute the Bitwise XOR operation process on the composite image \( IVec \) and key1 to obtain the substituted composite image \( SimVec \) as

\[
SimVec = IVec \oplus \text{Key1},
\]

6. Select the primary key parameters \( (x_0, B) \) for the Chebyshev map, and then update the initial key parameter \( x_0 \) as specified in the following equations:

\[
\delta = \frac{\sum_{i=0}^{\text{max}} \text{SimVec}(i)}{m \times n \times c} \mod 1,
\]

\[
x_u = x_0 + \frac{\delta}{2}.
\]

7. Apply Chebyshev mapping using \( x_u \) and \( B \) for \( m \times n \times c \) times to produce a sequence \( X \).

8. Add the elements of sequence \( X \) to the elements of sequence \( \text{Re}P \) to obtain sequence \( R \) as follows:
\[ R_k = X_k + \text{Re} P_k, \quad \text{for } k = 1, 2, \ldots, m \times n \times c. \] (13)

(9) Arrange the elements of \( R \) in ascending order as \( RA \) and retrieve the index of each element of \( RA \) in \( R \) as \( \text{PerVec} \).

(10) Permutate the substituted composite image \( \text{SimVec} \) using the generated \( \text{PerVec} \) such that
\[ \text{PerIm}(k) = \text{SimVec} (\text{PerVec}(k)), \quad \text{for } k = 1, 2, \ldots, m \times n \times c. \] (14)

(11) Obtain the key sequence (Key2) by transforming the sequence \( X \) into integer values.
\[ \text{Key2} = \text{fix}(X \times 10^{12}) \mod 256. \] (15)

(12) Execute Bitwise XOR process on the permutated composite image \( \text{PerIm} \) and key2 to obtain the cipher composite image \( \text{Cim} \) as
\[ \text{CimVec} = \text{PerIm} \oplus \text{Key2}, \]
\[ \text{Cim} = \text{reshape} (\text{CimVec}, m, n, c). \] (16)

3.2. Decryption Algorithm. The decryption method (Figure 2) of the proposed cryptosystem is the reverse process of the encryption process, and it is executed in the steps enumerated as follows:

(1) Convert the cipher composite image \( \text{Cim} \) into a vector \( \text{CimVec} \).

(2) Use the primary key parameters \((x_0, B)\) for the Chebyshev map to update the initial key parameter \( x_0 \) using \( \delta \).
\[ x_u = \frac{x_0 + \delta}{2}. \] (17)

(3) Apply Chebyshev mapping using \( x_u \) and \( B \) for \( m \times n \times c \) times to produce a sequence \( X \).

(4) Obtain the key sequence (DKey1) by converting the sequence \( X \) into integer values.
\[ \text{DKey1} = \text{fix}(X \times 10^{12}) \mod 256. \] (18)

(5) Execute Bitwise XOR operation on \( \text{CimVec} \) and \( \text{DKey1} \) to obtain the permutated composite image \( \text{PerIm} \).

(6) Use the primary key parameters \((V, t, \mu, \omega)\) for operating QWs \( t \) times on a cycle of odd \( V \) nodes to produce a probability distribution \( P \) of size \( V \).

(7) Resize \( P \) to the dimension of the composite image \( \text{CimVec} \) \((m \times n \times c)\).
\[ \text{Re} P = \text{resize} (P, [m \times n \times c \ 1]). \] (19)

(8) Add the elements of sequence \( X \) to the elements of sequence \( \text{Re} P \) to obtain sequence \( R \).

(9) Arrange the elements of \( R \) in ascending order (as \( RA \)) and retrieve the index of each element of \( RA \) in \( R \) as DPerVec.
(10) Rearrange (or depermutate) the permutated composite image \( \text{PerIm} \) using the generated \( \text{DPerVec} \) such that
\[
\text{DPerIm}(\text{DPerVec}(k)) = \text{PerIm}(k), \quad \text{for } k = 1, 2, \ldots, m \times n \times c.
\]

(11) Obtain the key sequence (DKey2) via conversion of \( \text{ReP} \) into integer values.
\[
\text{DKey2} = \text{fix}(\text{ReP} \times 10^{12}) \mod 256. \tag{21}
\]

(12) Execute the Bitwise XOR operation process on the depermutated composite image \( \text{DPerIm} \) and DKey2 to obtain the DimVec and decrypted composite image \( \text{DecIm} \) as
\[
\text{DimVec} = \text{DPerIm} \oplus \text{DKey2},
\]
\[
\text{DecIm} = \text{reshape}(\text{DimVec}, m, n, c). \tag{22}
\]

(13) Decompose the decrypted composite image \( \text{DecIm} \) to get the decrypted images (\( \text{Dig01}, \text{Dig02}, \ldots, \text{Dig0N} \)).

4. Experimental Validation

To appraise the performance of the proposed multi-image cryptosystem, experiments executed using a laptop with Intel Core™ i5-2450M, 6 GB RAM and implemented using MATLAB R2016b are reported in this section. The dataset used are sourced from the Signal and Image Processing Institute (SIPI) database in [32]. Four images each of size \( 512 \times 512 \) and labelled as Ig01 through Ig04 (see Figure 3) were selected and used for the experiments.

Whereas the initial values for key parameters (i.e., \( V, t, \mu, \omega, x_0 \), and \( B \)) can be selected according to their ranges, in this study, the used initial parameter settings for running the QWs are set as \( V = 301, t = 325, \mu = 0, \) and \( \omega = \pi/6 \). Similarly, initial parameter settings for iterating the Chebyshev map are set as \( x_0 = 0.6743 \) and \( B = 55 \). Figure 4 presents the plain composite image consisting of input images Ig01 (Baboon), Ig02 (Airplane), Ig03 (Boat), and Ig04 (Peppers) as well as the resulting cipher composite image obtained using our multi-image cryptosystem and the decrypted composite image obtained using our multi-image cryptosystem for the provided initial key parameters. To guarantee the effectiveness of our cryptosystem, in addition to the composite images, we analysed the properties of the input images (Ig01, Ig02, Ig03, and Ig04) and their respective decomposed cipher composite image versions (Cipher-Ig01, Cipher-Ig02, Cipher-Ig03, and Cipher-Ig04) as presented in Figure 5.

The remainder of this section presents an extensive analysis of our proposed scheme using the dataset introduced earlier (in Figure 3). However, throughout the ensuing discussion and performance evaluation, we shall refer to the pristine or undulterated input images (i.e., Ig01, Ig02, Ig03, and Ig04 and their composite image) as...
the pristine group whilst their corresponding ciphered versions are collectively referred to as the cipher group.

4.1. Correlation Analysis. To assess the concordance between neighbouring pixels $X$ and $Y$ in an image, we use correlation coefficient, $\text{Corr}$, where it is established that pristine (i.e., unadulterated) images exhibit correlation values near 1 in each direction and that of a cipher image from a well-designed cryptosystem should be close to 0 [27]. To calculate the correlation coefficients for our multiple-image cryptosystem, we picked $10^4$ pairs of neighbouring pixels per direction at random and $\text{Corr}$ is computed using

\[
\text{Corr} = \frac{\sum_{k=1}^{N} (a_k - \bar{a})(b_k - \bar{b})}{\sqrt{\sum_{k=1}^{N} (a_k - \bar{a})^2 \sum_{k=1}^{N} (b_k - \bar{b})^2}},
\]

(23)

where $a_k$ and $b_k$ are the values of the two adjacent pixel values, and $N$ denotes the total number of adjacent pixel pairs per direction.

Table 1 presents the outcomes of the correlation coefficient test, where we see that the correlation coefficient values of the cipher images are close to 0. Similarly, the correlation distributions for the red, blue, and green channels of the composite image are presented in Figures 6–8, respectively. It is apparent from these outcomes that no intelligible information about the composite image can be obtained from the figures.

4.2. Histogram Analysis. Histogram analysis is a widely used image evaluation that reflects the frequency distribution of pixels in an image [27]. A well-designed cryptosystem should produce ciphers with uniformly distributed histogram. Figure 9 presents the histograms of the plain composite image and its corresponding cipher composite image. From these plots, we can deduce that the distributions of the plain composite image are markedly different for each channel whilst those for the ciphered composite image have flat histograms, which indicates the absence of useful information. Furthermore, we employ the chi-square measure ($\chi^2$) to quantify the pixel distribution in the histograms as defined in

\[
\chi^2 = \sum_{k=0}^{255} \frac{(f_k - N)^2}{N},
\]

(24)

where $f_k$ is the pixel value with frequency $k$, and $N$ is the dimension of the image.

It is instructive that by assuming a significant level $\alpha = 0.05$, then for the maximum greyscale value $\chi^2(255) = 293.3$. Therefore, an image with $\chi^2$ less than $\chi^2(255)$ has a uniform pixel distribution. Otherwise, the histogram is considered nonuniformly distributed. Based on this, Table 2 presents results of the $\chi^2$ test for our pristine and cipher groups, respectively. The outcomes show that all images in the cipher group have $\chi^2$ values less than the threshold value of 293.3, which conforms with the flat-out plots of their histograms presented earlier in
Figure 5: Pristine (or input) images (a) Ig01, (b) Ig02, (c) Ig03, and (d) Ig04 and their respective ciphered versions (e) Cipher-Ig01, (f) Cipher-Ig02, (g) Cipher-Ig03, and (h) Cipher-Ig04.

Table 1: Correlation coefficient test for the pristine and cipher groups for input and composite images.

| Group     | Image                | Colour component | Horizontal | Direction |
|-----------|----------------------|------------------|------------|-----------|
|           |                      |                  | R          | G         | B         |
| Pristine  | Plain composite      | R                | 0.9280     | 0.9527    | 0.9489    |
|           |                      | G                | 0.9501     | 0.9617    | 0.9312    |
|           |                      | B                | 0.9614     | 0.9695    | 0.9476    |
| Cipher    | Cipher composite     | R                | −0.0005    | 0.0002    | −0.0009   |
|           |                      | G                | −0.0001    | −0.0003   | 0.0009    |
|           |                      | B                | −0.0002    | −0.0005   | −0.0001   |
| Pristine  | Ig01                 | R                | 0.8678     | 0.9196    | 0.8574    |
|           |                      | G                | 0.7709     | 0.8649    | 0.7464    |
|           |                      | B                | 0.8843     | 0.9081    | 0.8389    |
| Cipher    | Cipher-Ig01          | R                | 0.0006     | 0.0002    | 0.0006    |
|           |                      | G                | −0.0006    | −0.0007   | 0.0002    |
|           |                      | B                | −0.0005    | −0.0001   | 0.0005    |
| Pristine  | Ig02                 | R                | 0.9651     | 0.9726    | 0.9412    |
|           |                      | G                | 0.9679     | 0.9630    | 0.9382    |
|           |                      | B                | 0.9534     | 0.9613    | 0.9336    |
| Cipher    | Cipher-Ig02          | R                | 0.0001     | 0.0008    | −0.0009   |
|           |                      | G                | −0.0009    | −0.0007   | 0.0009    |
|           |                      | B                | −0.0003    | 0.0005    | 0.0003    |
| Pristine  | Ig03                 | R                | 0.9542     | 0.9542    | 0.9427    |
|           |                      | G                | 0.9657     | 0.9725    | 0.9533    |
|           |                      | B                | 0.9729     | 0.9713    | 0.9562    |
| Cipher    | Cipher-Ig03          | R                | 0.0008     | −0.0008   | −0.0001   |
|           |                      | G                | 0.0007     | 0.0009    | 0.0007    |
|           |                      | B                | −0.0007    | −0.0007   | −0.0004   |
| Pristine  | Ig04                 | R                | 0.9657     | 0.9647    | 0.9590    |
|           |                      | G                | 0.9856     | 0.9837    | 0.9750    |
|           |                      | B                | 0.9696     | 0.9660    | 0.9505    |
| Cipher    | Cipher-Ig04          | R                | −0.0009    | 0.0006    | −0.0006   |
|           |                      | G                | 0.0009     | −0.0005   | −0.0007   |
|           |                      | B                | −0.0002    | −0.0001   | 0.0001    |
| Average values for cipher group | | R | −0.00012 | | | | | |
|           |                      | G | −0.00008 | | | | | |
|           |                      | B | 0.00002 | | | | | |
4.3. Entropy Analysis. Information entropy $E(X)$ is a test used to compute the concentration of pixel values per bit-level in an image $E(X)$. It is considered a measure of efficiency of a cryptosystem [27], and it can be computed using

$$E(X) = \sum_{k=0}^{255} p(x_k) \log_2 \frac{1}{p(x_k)},$$

(25)

where $p(x_k)$ indicates the probability of $x_k$.

Theoretically, grey scale images with $2^b$ values should have an optimal entropy of $8$ bits. In other words, when efficiently ciphered, such an image should have an entropy value close to 8. However, this global entropy does not consider the apparent randomness in such ciphered images, which motivates the use of a modified or local entropy measure is used to atone for the randomness. The local entropy is the average values of global entropy in non-overlapping blocks (i.e., 1936 pixels within the block).

Like in the global entropy, values of local entropy closer to 8 are indications of an efficient cryptosystem. Table 3 presents a summary of local and global entropies for the pristine and cipher groups of our performance analysis. As deduced from the table, all global entropies for the cipher group are within a difference of 0.0002 from the expected value of 8, which validates the efficiency of our proposed scheme in terms of its ability to withstand different entropy-based attacks.

4.4. Differential Test. In addition to performing creditably in the statistical tests, a well-designed cryptosystem should exhibit sensitivity to tiny modifications in the composition of its pristine version. In this study, we utilise two measures: the number of pixels change rate (NPCR) and unified average changing intensity (UACI) defined in equations (26) and (27) for the differential test [33] to assess these modifications.

$$\text{NPCR}(C1, C2) = \frac{\sum_{i,j} Df(i, j)}{T} \times 100\%,$$

(26)

$$Df(i, j) = \begin{cases} 0, & \text{if } C1(i, j) = C2(i, j), \\ 1, & \text{if } C1(i, j) \neq C2(i, j), \end{cases}$$

$$\text{UACI}(C1, C2) = \frac{1}{T} \left( \sum_{i,j} \left| C1(i, j) - C2(i, j) \right| / 255 \right) \times 100\%,$$

(27)

where $T$ is the total pixels within the image, while $C1$ and $C2$ are cipher composite images for a pristine (i.e., original or plane) composite image with just one bit modified.

Computations of the NPCR and UACI (in %) when making tiny modifications in pixel value at position (1, 1) from 164 to 165 of the red channel of the pristine composite image are presented in Table 4. Theoretically, for an image to be considered uniformly distributed, its respective NPCR and UACI values should be as close to 99.6% and 33.33% as possible.

Results reported in Table 4 show that, on average, our proposed scheme is within 0.01% and 0.1% of the expected ranges of the NPCR and UACI, respectively, which validates its capability to withstand differential attacks.

4.5. Contrast Test. Contrast analysis ($T$) is a statistical measure used to assess the local intensity variation in an image. $T$ can be defined as presented in the following equation [34]:

$$T = \sum_{i,j} |i - j|^2 p(i, j),$$

(28)

where $p(i, j)$ indicates the number of grey level co-occurrence matrices.

Given an image, high contrast values signify presence of significantly distinct grey levels while lower values indicate constant grey levels. The outcomes of the contrast test presented in Table 5 show that the cipher images obtained from our scheme exhibit higher contrast values relative to their corresponding pristine versions.

4.6. Key Sensitivity Test. A robustly efficient cryptosystem should show sensitivity to changes in the composition of its secret key. In other words, to withstand attacks, such as brute-force attacks, an efficient cryptosystem must be sensitive to alterations to its secret key parameters.

To analyse the key sensitivity of our proposed scheme, we considered recovery (i.e., decryption) of the ciphered composite image (in Figure 4(b)) for different modifications in the secret key parameters. The outcomes of this key sensitivity analysis are presented in Figure 10 for changes in $V$, $T$, $\mu$, $\omega$, $x_0$, and $B$, respectively. As manifested in these outcomes, for each modification, the pristine composite image is unrecovered.

Furthermore, by pairing the decrypted cipher image (i.e., in Figure 4(c)) with the decrypted images for slight modifications in secret key parameters (i.e., in Figure 10) and computing their respective pixel difference rate, namely, pairing of the targeted decrypted composite image with those for different alternations in key parameters, we establish the quantitative performance of the key sensitivity of our scheme. Results of our pixel difference rate (in %) are presented in Table 6 and outcomes therein suggest that, despite the slight changes in key parameters, a high pixel difference rate is high (approximately 99.61%) is maintained throughout. Furthermore, the recovered images are composed of unintelligible noise and the recovered images remain incongruous. This is a testimony of our proposed scheme’s sensitivity to alterations to the composition of its secret key parameters.

4.7. Noise and Occlusion Attacks. In the course of its transmission over different communication networks, data can be tampered, lost, or violated. Consequently, an efficient
Figure 6: Correlation distribution for red component of plain composite image in Figure 4(a) (a) and those from the cipher composite image in Figure 4(b) (b).

Figure 7: Correlation distribution for green component of plain composite image in Figure 4(a) (a) and those from the cipher composite image in Figure 4(b) (b).
Figure 8: Correlation distribution for blue component of plain composite image in Figure 4(a) (a) and those from the cipher composite image in Figure 4(b) (b).

Figure 9: Histogram distribution for plain composite image in Figure 4(a) (a) and the cipher composite image in Figure 4(b) (b).
cryptosystem should be designed to curtail such attacks. To assess the ability of our proposed scheme to withstand the enumerated violations, we report outcomes of noise and occlusion attacks.

For the noise attacks, we considered the impact of using Salt and Pepper (S&N) noise to tamper with the ciphered composite image in Figure 4(b). For effective analysis, the density of S&N noise was varied as 0.1, 0.15, and 0.25; outcomes for which are presented in Figures 11(a)–11(c), respectively. Figures 11(d)–11(f) present the recovered versions of the respective images.

Similarly, the occlusion attacks were assessed for different cut-outs occluding the parts of the same ciphered composite in Figure 4(b). The cut-out size was varied to cover 10%, 25%, and 35% of the ciphered image as presented in Figures 12(a)–12(c), while the recovered versions of these images are presented in Figures 12(d)–12(f).

In both analyses, we note the ability to recover clear versions of the input composite image despite the noise and occlusion attacks. This validates the ability of our proposed scheme to overcome such prevalent attacks.

4.8. Randomness Analysis. To validate the randomness of the constructed cipher composite image, the NIST SP 800-22 tests [35] were executed. The main goal of these tests is to detect any nonrandomness property in the constructed cipher composite image. These tests are executed on a sequence of $10^6$ bits from the cipher composite image whose outcomes are presented in Table 7. It is apparent from these outcomes that the constructed cipher composite image obtained using the proposed cryptosystem is completely random.

4.9. Comparative Analysis. To establish the effectiveness of our proposed multi-image cryptosystem, we compare our results alongside those from other chaotic cryptosystems. Results reported in Table 8 show the average values of correlation coefficients UACI, NPCR, and global entropy for our cryptosystem and those indicated in the table. From these outcomes, it is apparent from these outcomes that the proposed scheme is effective and reliable for various cryptographic applications.

### Table 2: Chi-square values for images in the pristine and cipher groups.

| Image            | R          | G          | B          | R          | G          | B          |
|------------------|------------|------------|------------|------------|------------|------------|
| Composite image  | 561115.509 | 466430.302 | 648823.932 | 228.317    | 224.622    | 208.914    |
| Ig01             | 82839.728  | 142800.039 | 79942.617  | 245.619    | 236.990    | 219.396    |
| Ig02             | 67842.492  | 62495.382  | 110785.805 | 238.957    | 265.861    | 272.330    |
| Ig03             | 196697.306 | 130154.716 | 344571.537 | 264.919    | 259.925    | 218.814    |
| Ig04             | 213187.216 | 318382.929 | 491428.177 | 225.269    | 252.263    | 209.951    |
| Average values   | 346432.850 | 348054.274 | 534524.854 | 240.616    | 247.932    | 225.881    |

### Table 3: Local and global information entropies of the studied dataset.

| Image            | Local entropy | Global entropy |
|------------------|---------------|----------------|
|                 | Pristine group | Cipher group   | Pristine group | Cipher group |
| Composite image  | 6.07368       | 7.90167        | 7.72801        | 7.99995      |
| Ig01             | 6.64443       | 7.90172        | 7.76243        | 7.99978      |
| Ig02             | 5.52864       | 7.90234        | 6.66391        | 7.99975      |
| Ig03             | 6.07741       | 7.90257        | 7.76216        | 7.99979      |
| Ig04             | 6.04964       | 7.90193        | 7.66982        | 7.99978      |
| Average values   | 6.07476       | 7.90205        | 7.51727        | 7.99981      |

### Table 4: Outcomes of NPCR and UACI for slight changes in the pristine composite image.

| Group      | Image          | NPCR (%) | UACI (%) |
|------------|----------------|----------|----------|
| Pristine   | Composite image| 99.60543 | 33.45229 |
|            | Ig01           | 99.60645 | 33.42891 |
|            | Ig02           | 99.60058 | 33.46614 |
|            | Ig03           | 99.60467 | 33.45160 |
|            | Ig04           | 99.61102 | 33.46251 |
| Average values |              | 99.60563 | 33.45229 |
Table 5: Contrast values of the investigated dataset.

| Image          | Pristine group | Cipher group |
|----------------|----------------|--------------|
|                | R          | G          | B          | R          | G          | B          |
| Composite image| 0.349001  | 0.470013  | 0.427496  | 10.540164 | 10.486821 | 10.507776 |
| Ig01           | 0.615911  | 0.762479  | 0.849334  | 10.501574 | 10.491029 | 10.519275 |
| Ig02           | 0.184736  | 0.285026  | 0.133359  | 10.541669 | 10.487719 | 10.497809 |
| Ig03           | 0.294325  | 0.486102  | 0.461583  | 10.536192 | 10.451294 | 10.496693 |
| Ig04           | 0.275149  | 0.302998  | 0.221375  | 10.583426 | 10.517864 | 10.519198 |
| Average values | 0.343824  | 0.461324  | 0.418629  | 10.540605 | 10.486945 | 10.508150 |

Figure 10: Decryption process for the cipher composite image (Figure 4(b)) for different alterations to secret key parameters. (a) Actual key with alteration at $V = 303$. (b) Actual key with alteration at $t = 324$. (c) Actual key with alteration at $\mu = \pi/2$. (d) Actual key with alteration at $\omega = \pi/3$. (e) Actual key with alteration at $x_0 = 0.674300000000001$. (f) Actual key with alteration at $B = 56$.

Table 6: Pixel difference rate for pairings of decrypted composite image (i.e., using the actual key parameters (in Figure 4(c)) and those decrypted using different alterations in secret key parameters (Figures 10(a)–10(f)).

| Image pairing | Difference rate (%) |
|---------------|---------------------|
| Figures 10(a) and 4(c) | 99.61503    |
| Figures 10(b) and 4(c) | 99.60962    |
| Figures 10(c) and 4(c) | 99.60887    |
| Figures 10(d) and 4(c) | 99.60924    |
| Figures 10(e) and 4(c) | 99.61376    |
| Figures 10(f) and 4(c) | 99.61058    |
| Average values   | 99.61118    |
Figure 11: Noise attack. (a–c) Ciphered version of composite image (Figure 4(b)) with S&N noise of varying densities (i.e., 0.1, 0.15, and 0.25) added. (d–f) Recovered versions of the corrupted images.

Figure 12: Continued.
5. Concluding Remarks

Exploiting the stochastic transitions between states, and randomness attributed to quantum walks on the one side and the semigroup membership and chaos of Chebyshev map on the other, our study proposes an efficient multi-image cryptosystem that exhibits nonlinear chaotic dynamic behaviours. Simulation results prove that the proposed cryptosystem is effective and reliable across wide-range cryptographic applications.

Besides the merits ascribed to traditional chaos systems, our cryptosystem is an excellent tool to generate efficient encryption keys, espouse high sensitivity to initial and system parameters, manifest good unpredictability, pseudorandomness, stability, periodicity, and infinite key space needed to efficiently resist brute-force attacks.
In addition to designing reliable multi-image cryptosystems, a major objective of this study is advancing studies to integrate quantum computing models with digital computing models for designing reliable cryptosystems. Currently and in the future, we intend to extend this work towards designing new secret-sharing cryptosystems.

**Data Availability**

The datasets generated and analysed during the current study are available from the corresponding author upon reasonable request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this study and its contents.

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