Final State Interaction Effects in the $e^+e^- \rightarrow N\bar{N}$ Process near Threshold

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Abstract

We use the Paris nucleon-antinucleon optical potential for explanation of experimental data in the process $e^+e^- \rightarrow p\bar{p}$ near threshold. It is shown that the cross section and the electromagnetic form factors are very sensitive to the parameters of the potential. It turns out that final-state interaction due to slightly modified absorptive part of the potential allows us to reproduce available experimental data. We also demonstrated that the cross section in $n\bar{n}$ channel is larger than that in $p\bar{p}$ one, and their ratio is almost energy independent up to 2.2 GeV.

Key words: Electromagnetic form factors of proton and neutron.
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1 Introduction

At present, QCD can not describe quantitatively the low-energy nucleon-antinucleon interaction, and various phenomenological approaches have been suggested in order to explain numerous experimental data, see, e.g., Refs. [123456] and recent reviews [78]. However, parameters of the models still can not be extracted with a good accuracy from the experimental data [9].

Very recently, renewed interest in low-energy nucleon-antinucleon physics has been stimulated by the experimental observation of a strong enhancement of decay probability at low invariant mass of $p\bar{p}$ in the processes $J/\Psi \rightarrow \gamma p\bar{p}$ [10], $B^+ \rightarrow K^+ p\bar{p}$ and $B^0 \rightarrow D^0 p\bar{p}$ [111213], $B^+ \rightarrow \pi^+ p\bar{p}$ and $B^+ \rightarrow K^0 p\bar{p}$ [14], $\Upsilon \rightarrow \gamma p\bar{p}$ [15]. One of the most natural explanation of this enhancement is final state interaction of the proton and antiproton [161718192021].

A similar phenomenon was observed in the investigation of the proton (antiproton) electric, $G_E(Q^2)$, and magnetic, $G_M(Q^2)$, form factors in the process $e^+e^- \rightarrow p\bar{p}$ [222324]. Namely, it was found that the ratio $|G_E(Q^2)/G_M(Q^2)|$
strongly depends on $Q^2 = 4E^2$ (in the center-of-mass frame) in the narrow region of the energy $E$ near the threshold of $p\bar{p}$ production. Such strong dependence at small $(E-M)/M$ is related to the interaction of proton and antiproton at large distances $r >> 1/M$. Therefore, it is possible to take one of the nucleon-antinucleon interactions determined in [1,2,3,4,5,6] for describing the final state interaction in the process $e^+e^- \rightarrow p\bar{p}$ in order to explain the experimental data. In the present paper, we use the Paris nucleon-antinucleon optical potential $V_{NN}$ which has the form [4,5]:

$$V_{NN} = U_{NN} - i W_{NN},$$

(1)

where the real part $U_{NN}$ is the $G$-parity transform of the well established Paris $NN$ potential for the long- and medium-ranged distances ($r \gtrsim 1$fm), and some phenomenological part for the short distances. The absorptive part $W_{NN}$ of the optical potential takes into account the inelastic channels of $N\bar{N}$ interaction, i.e. annihilation into mesons. It is essential at short distances and depends on the kinetic energy of the particles. Our knowledge of $W_{NN}$ is essentially more restricted than that of $U_{NN}$. Therefore, we hope that experimental data for the cross section of the process $e^+e^- \rightarrow p\bar{p}$ can significantly diminish the uncertainty in $W_{NN}$.

### 2 Amplitude of the process

Using the standard definition of the Dirac form factors $F_1(Q^2)$ and $F_2(Q^2)$ [25] of the proton we write, in the nonrelativistic approximation, the amplitude of $N\bar{N}$ pair production near threshold as follows (in units $e^2/Q^2$):

$$T_{\lambda\mu} = \epsilon_\lambda \left\{ \frac{1}{3} \left[ (2E + M)F_1(Q^2) + \frac{E^2 + 2ME}{M}F_2(Q^2) \right] \epsilon_\mu + \frac{EF_2(Q^2) - MF_1(Q^2)}{3M(E + M)} \left[ 3(p' \cdot \epsilon_\mu)p' - p'^2 \epsilon_\mu \right] \right\},$$

(2)

where $\epsilon_\mu$ is a virtual photon polarization vector, and $\epsilon_\lambda$ is the spin-1 function of $N\bar{N}$ pair. Two tensor structures in Eq.(2) correspond to the s-wave and d-wave production amplitudes. The total angular momentum of the $N\bar{N}$ pair is fixed by a production mechanism. The functions $F_1(Q^2)$ and $F_2(Q^2)$ are related in a standard way with the electric and the magnetic form factors of the proton $G_E = F_1 + \frac{Q^2}{4M^2}F_2$ and $G_M = F_1 + F_2$. Near threshold, it is more convenient to work with the Dirac form factors, because at threshold $G_E = G_M$. The amplitude (2) already includes effects of final state interaction. Therefore, the form factors in Eq.(2) should have a pronounced $Q^2$ behavior near threshold.
Our aim is to single out these effects. In order to do that, let us introduce the amplitude $\tilde{T}_{\lambda\mu}$ which has the form of Eq. (2) but with the replacement $F_1 \to \tilde{F}_1$ and $F_2 \to \tilde{F}_2$, where the form factors $F_{1,2}$ do not account for the effect of final state interaction. So, the difference between $T_{\lambda\mu}$ and $\tilde{T}_{\lambda\mu}$ is in their $Q^2$ dependence due to the form factors. In $\tilde{T}_{\lambda\mu}$, the only scale is the nucleon mass $M$ (which is still too small to use perturbative QCD). Therefore, near threshold, where the final state interaction is important, the form factors $\tilde{F}_{1,2}$ are smooth functions of $Q^2$ and can be treated as phenomenological constants.

Now we can construct the amplitude of $NN$ pair production in a certain isospin channel $I = 0, 1$ using the wave function $\Phi_I^{(-\dagger)}(p')$ of the $NN$ pair in momentum space:

$$T^I_{\lambda\mu} = \int \frac{d^3p'}{(2\pi)^3} \Phi_{p\lambda}^{I(-\dagger)}(p') \left\{ \frac{1}{3} \left[ (2E + M)\tilde{F}^I_1 + \frac{E^2 + 2ME}{M^3}\tilde{F}^I_2 \right] e^\mu + \frac{E\tilde{F}^I_2 - M\tilde{F}^I_1}{3M(E + M)} [3(p' \cdot e^\mu) p' - p'^2 e^\mu] \right\} .$$

The Fourier transform of the function $\Phi_{p\lambda}^{I(-\dagger)}(p')$, which is the wave function $\Psi_{p\lambda}^{I(-\dagger)}(r)$ of the $NN$ pair in coordinate space, is the solution of the Schrödinger equation

$$\Psi_{p\lambda}^{I(-\dagger)}(r) \hat{H} = (E - M)\Psi_{p\lambda}^{I(-\dagger)}(r), \quad \hat{H} = \frac{\hat{p}^2}{M} + V_{NN},$$

where $V_{NN}$ is given by Eq. (1). Note that $\Psi_{p\lambda}^{I(-\dagger)}(r)$ is the left eigenfunction of the bi-orthogonal set of eigenfunctions of the non-Hermitian operator $\hat{H}$. Its asymptotic behavior at large distances is

$$\Psi_{p\lambda}^{I(-\dagger)}(r) \approx e^{-ip \cdot r} + \int \frac{E^{pp} e^{pp} \nu_I(r)}{r} .$$

For $p\bar{p}$ production, we have $T^{\bar{p}}_{\lambda\mu} = T^{p\bar{p}}_{\lambda\mu} + T^0_{\lambda\mu}$, while for $n\bar{n}$ $T^0_{\lambda\mu} = T^{p\bar{p}}_{\lambda\mu} - T^0_{\lambda\mu}$.

### 2.1 Coupled channels basis

In the presence of tensor forces, the states with angular momentum $L$ and $L + 2$ are coupled, while the total angular momentum $J$ is conserved. As a result, the final-state wave function $\Psi_{p\lambda}^{I(-\dagger)}(r)$ can be represented in the form

$$\Psi_{p\lambda}^{I(-\dagger)}(r) = \sum_{JMa} D^{\alpha IJM*}_\lambda \Psi^{\alpha IJM*}(r) + \sum_{JM} F^I_{\lambda} \nu_J^I(r) Y_J^M(n) ,$$

where $D^{\alpha IJM*}_\lambda$ and $F^I_{\lambda}$ are the Clebsch-Gordan coefficients and the form factors, respectively.
where \( n = r/r \),

\[
\Psi^{\alpha J M}(r) = u_J^{\alpha I}(r)Y_{JM}^{l-1}(n) + w_J^{\alpha I}(r)Y_{JM}^{l+1}(n) \quad (\alpha = 1, 2),
\]

and

\[
v_J^{\alpha I}(r)Y_{JM}(n)
\]

are three independent solutions of the Schrödinger equation having total angular momentum \( J \). In Eqs. (7) and (8), \( Y_{JM}^{l}(\theta, \phi) \) is the vector spherical harmonic, which is an eigenfunction of the operators \( L^2, J^2 \), and \( J_z \), where \( J = L + S \) and \( S = 1 \). In order to find the coefficients \( D^{\alpha J M}_\alpha \) and \( F^{J M}_\alpha \) in Eq. (5), we substitute the asymptotics of the functions \( u_J^{\alpha I}(r), w_J^{\alpha I}(r) \) and \( v_J^{\alpha I}(r) \) at large \( r \) in Eq. (6) and use Eq. (5). Then we obtain

\[
D^{\alpha J M}_\alpha = 4\pi \sum_L \sum_m \langle J M | L m \lambda \rangle Y^{*}_{LM}(\hat{p}) A^{\alpha L}_1, \quad F^{J M}_\alpha = 4\pi \sum_L \sum_m \langle J M | J m \lambda \rangle Y^{*}_{JM}(\hat{p}) A^{\alpha J}_1,
\]

where the coefficients \( A^{\alpha L}_1 \) and \( A^{\alpha J}_1 \) are related to incoming flux in the asymptotics of the radial functions \( u_J^{\alpha I}(r), w_J^{\alpha I}(r) \) and \( v_J^{\alpha I}(r) \). Making the Fourier transform of the expansion Eq. (6), substituting the result in Eq. (3), and performing the integration over the angles of the vector \( p' \), we obtain

\[
T^{J \mu}_\lambda = \int \frac{p'^2 dp'}{(2\pi)^3} \left\{ \sqrt{\frac{4\pi}{3}} \left[ \frac{1}{2}(2E + M) \tilde{F}_1^{\alpha} + \frac{E^2 + 2ME}{3M} \tilde{F}_2^{\alpha} \right] \right. \\
\times \sum_\alpha D^{\alpha 1 \mu \ast}_\lambda f_1^{\alpha}(p') + \sqrt{\frac{2\pi}{3M(E + M)}} p'^2 \sum_\alpha D^{\alpha 1 \mu \ast}_\lambda g_1^{\alpha}(p') \right\},
\]

where \( f_1^{\alpha}(p') \) and \( g_1^{\alpha}(p') \) are the corresponding radial parts of the wave functions in momentum space. Integrals over \( p' \) are related to the values of radial wave functions at \( r = 0 \).

\[
u_1^{\alpha}(0) = \frac{1}{2\pi^2} \int p^2 f_1^{\alpha}(p) dp, \quad w_1^{\alpha''}(0) = -\frac{1}{15\pi^2} \int p^4 g_1^{\alpha}(p) dp,
\]

where \( w_1^{\alpha''}(0) \) is the second derivative at \( r = 0 \). The amplitude Eq. (10) becomes

\[
T^{J \mu}_\lambda = \frac{1}{\sqrt{4\pi}} \left[ \frac{1}{3}(2E + M) \tilde{F}_1^{\alpha} + \frac{E^2 + 2ME}{3M} \tilde{F}_2^{\alpha} \right] \sum_\alpha D^{\alpha 1 \mu \ast}_\lambda u_1^{\alpha}(0) \\
+ \frac{5}{4} \frac{E \tilde{F}_2^{\alpha} - M \tilde{F}_1^{\alpha}}{M(E + M)\sqrt{2\pi}} \sum_\alpha D^{\alpha 1 \mu \ast}_\lambda w_1^{\alpha''}(0).
\]
Using Eq. (9) for the coefficients $D$, we can present the amplitude in the form

$$T_{\lambda\mu} = G_0^l \delta_{\lambda\mu} - \sqrt{4\pi} G_2^l (1\mu|2m\ 1\lambda) Y_{2m}^* (\hat{p}),$$  \hspace{1cm} (13)

where

$$G_0^l = \left[ \frac{1}{3} (2E + M) \bar{F}_1^l + \frac{E^2 + 2ME}{3M} \bar{F}_2^l \right] \sum_{\alpha} A^{I\alpha}_0 u_{1\alpha}^{I\alpha}(0)$$

$$+ \frac{5\sqrt{2}}{4} \frac{E \bar{F}_2^l - M \bar{F}_1^l}{M(E + M)} \sum_{\alpha} A^{I\alpha}_0 u_{1\alpha''}^{I\alpha}(0),$$ \hspace{1cm} (14)

$$G_2^l = \left[ \frac{1}{3} (2E + M) \bar{F}_1^l + \frac{E^2 + 2ME}{3M} \bar{F}_2^l \right] \sum_{\alpha} A^{I\alpha}_2 u_{1\alpha}^{I\alpha}(0)$$

$$+ \frac{5}{2\sqrt{2}} \frac{E \bar{F}_2^l - M \bar{F}_1^l}{M(E + M)} \sum_{\alpha} A^{I\alpha}_2 u_{1\alpha''}^{I\alpha}(0).$$ \hspace{1cm} (15)

The observable electric and magnetic form factors are expressed in terms of $G_0^l$ and $G_2^l$ in the following way

$$G_E^l = G_0^l + \sqrt{2} G_2^l \frac{Q}{2M}, \quad G_M^l = G_0^l + \frac{1}{\sqrt{2}} G_2^l.$$ \hspace{1cm} (16)

3 \hspace{0.5cm} pp production

The proton form factors are the sum of isoscalar and isovector form factors. The differential cross section for pp production is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4Q^2} \left[ (G_0^p(Q^2))^2 (1 + \cos^2 \theta) + \frac{4M^2}{Q^2} |G_E^p(Q^2)|^2 \sin^2 \theta \right],$$ \hspace{1cm} (17)

where $\beta = v/c$ and $C$ is the Coulomb distortion factor. We omitted here effects of Coulomb-nuclear interference. Using the form factors $\bar{F}_i^l$ as free parameters, we fit the observed energy behavior of the cross section up to 100 MeV of proton kinetic energy. We found that using the original Paris $NN$ potential [26] it is impossible to reproduce the energy behavior of the cross section. The cross section falls down too steeply becoming more than one order of magnitude smaller than the observed one at 10 MeV of proton c.m. kinetic energy. We verified that possible smooth dependence of the form factors $\bar{F}_i^l$ on $Q$ in a narrow energy region near threshold (for instance, $\bar{F}_i^l \propto 1 + b(Q^2 - 4M^2)/8M^2$ with $b \sim 1$), which is not related to final state interaction, does not change this result. The result is also insensitive to variation of most parameters of the potential. Variation of the energy independent parameters
is absorbed by the fitting parameters $\tilde{F}_i$. However, if we modify the only parameter, the energy dependence of absorptive part of the triplet potential $W_{NN}$, decreasing it by a factor of $8 \div 10$, we obtain a good fit of the cross section (see Fig.1). Modification of the energy dependence of the real part of the potential is not so important. Note that the values of the parameters, which are responsible for the energy dependence of the potential, are not well known. In the two versions of the Paris $NN$ potential, [5] and [26], these parameters differ from each other up to factor 2. In order to clarify the importance of the energy dependence of the absorptive potential, we calculated S-matrix elements and corresponding phase shifts for $J = 1$, $S = 1$ at different energies. It turns out that the phase shifts obtained with our strong modification of the absorptive potential differ only slightly as compared to those obtained in [5]. The only noticeable modification appeared in the energy dependence of the parameter $\eta$ directly related to absorption. In contrast, $|\Psi_{1^1M}(0)|^2$, see Eq. (7), is much more sensitive to this modification. Therefore, the uncertainty in determination of the parameters of energy dependence in the absorptive part of the triplet potential from the scattering data is apparently larger than factor two, and it is necessary to use also another data. In the process $e^+e^- \to p\bar{p}$ we have a unique situation where the quantum numbers of $p\bar{p}$ pair are fixed, $J = S = 1$. However, the absorptive part of $NN$ effective potential is not universal and may be different in different scattering channels, as it takes place in Nijmegen potential [27]. Thus, the modification of the energy dependence in the absorptive part of the potential for $J = S = 1$ channel does not allow us to make any conclusions on other channels and a new fit of absorptive potential in these channels should be performed.

Using the parameters obtained from the fit of the cross section, we simultaneously reproduce the energy behavior of the form factors ratio $|G_E/G_M|$ (see Fig. 2). Emphasize that the form factor $G_E$ differs from the form factor $G_M$ due to the contribution of D-wave only, see Eq.(16). Therefore, the strong energy dependence of the ratio $|G_E/G_M|$ clearly indicates the importance of D-wave even in the vicinity of threshold. Having obtained the amplitudes for the two isospins, we have calculated the cross section and the ratio $|G_E/G_M|$ for the process $e^+e^- \to n\bar{n}$. The corresponding results are shown in Figs 1,2 by the dashed lines. It is seen that the final state interaction leads to strong enhancement of both quantities in $n\bar{n}$ channel as well. It is interesting that the cross section in $n\bar{n}$ is larger than that in $p\bar{p}$ channel, and their ratio is almost energy independent up to 2.2 GeV.

Very recently, the final state interaction in $e^+e^- \to p\bar{p}$ has been discussed in Ref. [28]. The authors of the paper have presented many arguments in favor of importance of final state interaction in $p\bar{p}$ production near threshold. Using Jülich model for $NN$ interaction they have calculated the contribution of $^3S_1$ partial wave to the $e^+e^- \to p\bar{p}$ cross section near threshold. However, since they have neglected the $^3D_1$ partial wave contribution, they have not
been able to reproduce the ratio $|G_E/G_M|$ which is equal to unity in their approximation.

Fig. 1. Fit of the cross section of $pp$ production (solid line). Data are from Ref.[24]. Dashed line: the cross section for $n\bar{n}$ production

Fig. 2. Ratio of $|G_E/G_M|$ (solid line). Data are from Ref.[24]. Dashed line: the ratio for a neutron.

In summary, we calculated the effects of final state interaction in the reactions $e^+e^- \rightarrow p\bar{p}$ and $e^+e^- \rightarrow n\bar{n}$ near threshold. We found that the last version of the Paris $NN$ potential [26] does not reproduce the energy dependence of the observed cross section. A smooth dependence of the form factors $\tilde{F}_i$ on $Q$ in a narrow energy region near threshold, which is not related to final state interaction, does not change this result. Variation of the parameters responsible for the energy dependence of the imaginary part of the potential in the channel with $J = S = 1$, and $L = 0, 2$ allows us to reproduce both the energy dependence of the cross section and the ratio $|G_E/G_M|$ for $p\bar{p}$ production. We obtained that the cross section in $n\bar{n}$ channel is larger than that in $p\bar{p}$ one, and their ratio is almost energy independent up to 2.2 GeV.

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