Statistical approach to thermal evolution of neutron stars

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Abstract. Studying thermal evolution of neutron stars (NSs) is one of a few ways to investigate the properties of superdense matter in their cores. We study the cooling of isolated NSs (INSs) and deep crustal heating of transiently accreting NSs in X-ray transients (XRTs, binary systems with low-mass companions). Currently, nearly 50 of such NSs are observed, and one can apply statistical methods to analyze the whole dataset. We propose a method for such analysis based on thermal evolution theory for individual stars and on averaging the results over NS mass distributions. We calculate the distributions of INSs and accreting NSs (ANSs) in XRTs over cooling and heating diagrams respectively. Comparing theoretical and observational distributions one can infer information on physical properties of superdense matter and on mass distributions of INSs and ANSs.

1. Introduction

We study neutron stars of two types. First, they are isolated middle-aged (10\textsuperscript{2} – 10\textsuperscript{6} yr) NSs born hot in supernova explosions but cooling down mostly via neutrino emission from their superdense cores. They are mainly isothermal inside; a noticeable temperature gradient is available only in thin outer heat blanketing envelopes (e.g., [1, 2]).

Second, we analyze old (t \gtrsim 10\textsuperscript{8} – 10\textsuperscript{9} yr) transiently accreting quasi-stationary NSs in compact binaries with low-mass companions; such systems are called XRTs. These NSs accrete matter from time to time (in the active states of XRTs) from their companions. The accreted matter is compressed under the weight of newly accreted material which is accompanied by nuclear transformations and deep crustal heating [3, 4] with characteristic energy release of 1–2 MeV per accreted nucleon. The accretion episodes are supposed to be neither long nor intense to destroy the internal thermal equilibrium. Nevertheless, the deep crustal heating should be sufficiently strong to keep the NSs warm and power observable thermal emission of ANSs during quiescent states of XRTs [5]. The mean NS heating rate is determined by the mass accretion rate (M) averaged over characteristic cooling times of NSs (\gtrsim 10\textsuperscript{9} yr).

INSs are usually studied by calculating their cooling curves (time dependence of their effective surface temperature T\textsubscript{s}(t) or (equivalently) thermal surface luminosity L\textsubscript{s}(t)). The curves are calculated under different assumptions on the neutrino emission in the stellar core, and then compared with the data.

ANSs in XRTs are investigated by calculating their heating curves, which give average T\textsubscript{s} or L\textsubscript{s} for ANSs in quiescent states as a function of \langle M \rangle (which is also compared with observations).
It is important, that the cooling and heating curves have much in common (e.g., [6, 7]) and allow one to study fundamental physics of NSs. The most important cooling/heating regulators to be tested are:

(i) A level of neutrino luminosity of NSs;
(ii) NS mass \( M \) and equation of state (EOS) of superdense matter in the stellar core which regulate the neutrino emission level;
(iii) Composition of the NS heat blanketing envelope which determines the relation between the internal and surface temperatures.

Observations of INSs and ANSs are rapidly progressing. Let us utilize the accumulated statistics and develop a statistical theory of NS evolution. It will take into account that cooling and heating curves can strongly depend on \( M \). Then one can introduce the probability to find a source in different places of cooling/heating diagrams by averaging these curves over mass distributions of INSs and ANSs. Naturally, these mass distributions can be different. Comparing theoretical and observational distributions over diagrams one can study not only individual cooling regulators mentioned above but also the mass distributions of NSs. A more extended version of this study is published elsewhere [8]. Note that statistical studies of populations of INSs have been performed earlier in several publications (e.g., [9]) but they have been quite different.

2. Statistical theory. Broadening of direct Urca threshold

Let us present the simplest version of the statistical theory. It is based on ordinary theory of NS cooling and heating, e.g., [10, 11]. For illustration, consider NSs with nucleonic cores and one phenomenological EOS in the core described in [12]. The authors of [12] denoted it as HHJ. The maximum NS mass for this EOS is \( M_{\text{max}} = 2.16 \, M_\odot \) (to be consistent with mass measurements \( M \approx 2M_\odot \) of two massive NSs [13, 14]). In our case the radius of the most massive stable NS is \( R = 10.84 \, \text{km} \) and the central density \( \rho_c = 2.45 \times 10^{15} \, \text{g cm}^{-3} \). The powerful direct Urca process of neutrino emission opens at \( M > M_D = 1.72 \, M_\odot \). At lower \( M \) this process is forbidden, while at higher \( M \) it is allowed in the central kernel of a star (at densities \( \rho > \rho_D = 1.00 \times 10^{15} \, \text{g cm}^{-3} \)); then NSs undergo very rapid neutrino cooling.

We calculate thermal evolution of INSs and ANSs using our generally relativistic cooling code [15] on a dense grid of masses \( M \), from \( 1.1 \, M_\odot \) to \( 2.1 \, M_\odot \). The cooling curves of INSs are obtained by direct running the code. The heating curves of ANSs are calculated as stationary solutions of the heat balance equation (e.g., [16]),

\[
L_h^\infty = L_\nu^\infty + L_\gamma^\infty, \tag{1}
\]

where \( L_h^\infty \) is the averaged deep crustal heating power (redshifted for a distant observer and determined by the time-averaged mass accretion rate \( \langle \dot{M} \rangle \)). The interior of the star is assumed to be isothermal while the internal temperature is related to \( T_s \) by the heat blanketing solution (e.g., [2]). Calculated cooling curves will be plotted on the \( T^\infty - t \) diagram, while heating curves will be plotted on the \( L_\gamma^\infty - \langle \dot{M} \rangle \) diagram. These will be ordinary cooling and heating curves which have been extensively studied by the theory. As a rule, the highest cooling or heating curve refers to the low-mass NSs (with slow neutrino emission) while the lowest curve belongs to the maximum-mass star with highest neutrino cooling rate. The space between the highest and lowest cooling curves is filled by the curves for NSs of different \( M \) but this filling can be very non-uniform (e.g., [17]).

Now we put forward the statistical theory of NS evolution. All NSs will be assumed to have the same internal structure (EOS, neutrino emission properties) but, naturally, they have different parameters such as mass, the amount of light elements in heat-blanketing envelopes, magnetic
fields, rotation, etc. Instead of deterministic cooling/heating curves in appropriate diagrams we introduce the probability to find a star in different places of a diagram. These probabilities are obtained by averaging the cooling/heating curves over statistical distributions of variable parameters such as distributions of masses $M$ and masses $\Delta M_{\text{le}}$ of light elements in the heat-blanketing envelopes. Then the cooling/heating, that initially followed specific trajectories, is replaced by statistical probabilities to find NSs at different stages of their evolution.

In the first approximation it is reasonable to neglect the effects of magnetic fields and rotation on thermal states of NSs. To study INSs and ANSs we introduce the mass distribution functions for these sources, $f_i(M)$ and $f_a(M)$. These functions are naturally different; ANSs should be overall more massive than INSs.

Such distribution functions are taken in the form

$$f_i(M) = \frac{1}{N_i \sqrt{2\pi} \sigma_i} \exp \left( -\frac{(M - \mu_i)^2}{2\sigma_i^2} \right),$$

$$f_a(M) = \frac{1}{N_a \sqrt{2\pi} \sigma_a} \exp \left( -\frac{(\ln[M/M_\odot] - \mu_a)^2}{2\sigma_a^2} \right),$$

where $\sigma_{i,a}$ and $\mu_{i,a}$ are the parameters of the distributions; $N_{i,a}$ are normalization factors. These distributions are used in the mass range from $1.1 \ M_\odot$ to $2.1 \ M_\odot$ for $M < 1.1 \ M_\odot$ and $M > 2.1 \ M_\odot$ they are artificially set to zero. Note that for the normal distribution function $f_i(M)$, $\mu_i$ is the most probable mass. However, for the log-normal distribution $f_a(M)$ the most probable mass is $M_\odot \exp(\mu_a - \sigma_a^2)$. We expect that ANSs should be overall heavier than INSs as a result of accretion.

The heat transparency of the blanketing envelope is determined by the mass $\Delta M_{\text{le}}$ of light (accreted) elements (mainly, hydrogen and helium) in these envelopes. The higher $\Delta M_{\text{le}}$, the larger thermal conductivity in the envelope, and the higher $T_\text{le}$ for a given internal temperature of the star (e.g., [2]). However, $\Delta M_{\text{le}}$ cannot exceed $\Delta M_{\text{le,max}} \approx 10^{-7} M$ because at formally larger $\Delta M_{\text{le}}$ the light elements at the bottom of the heat blanketing envelope transform into heavier ones. We will consider $\Delta M_{\text{le}} \leq \Delta M_{\text{le,max}}$ as a random quantity characterized by a distribution $f_{\text{acc}}(\Delta M_{\text{le}})$

$$f_{\text{acc}}(\Delta M_{\text{le}}) = \text{const at } \Delta M_{\text{le}} \leq 10^{-7} M.$$  

In fact, $f_{\text{acc}}(\Delta M_{\text{le}})$ is uncertain (especially for ANSs); we take (3) as a first approximation.

Our cooling code [15] allows us to study the effects of superfluidity on evolution of NSs. To reduce the number of variable parameters, we employ a semi-phenomenological approach. In particular, we will broad out artificially a step-like density dependence of the neutrino emissivity $Q_D$ provided by the direct Urca process [18]. In the absence of superfluidity the direct Urca process switches on sharply with increasing density, from $Q_D = 0$ at $\rho < \rho_D$ to finite $Q_D$ at $\rho \geq \rho_D$. Moreover, in our model HHJ EOS, NS cores consist of neutrons with admixture of protons, electrons and muons, and we have two direct Urca processes, electronic and muonic ones (e.g., [19]). Accordingly, we have two density thresholds for the onset of the these processes (and the emissivities of both processes – if open – are the same). The density threshold for the muonic process is always higher than for the electronic one. Accordingly, when we increase $M$ (or $\rho_\text{e}$) the electronic direct Urca always switches on first. It sharply (by $6-7$ orders of magnitude) increases the neutrino luminosity of the NS, and becomes the leading one. The switch on of the muonic process with further increase of $M$ is relatively unimportant (although included in the calculations). A sharp step-like onset of the direct Urca process is known to be incompatible with observations. One needs to broaden its threshold. We will describe this broadening on a phenomenological level by multiplying the direct Urca neutrino emissivities by a broadening factor $b$. For instance, for the electronic direct Urca we take

$$Q_D = Q_D0 \ b(x), \quad b(x) = 0.5 \ [1 + \text{erf}(x)],$$

(4)
where $Q_{D0}$ is the threshold emissivity, $x = (\rho - \rho_D)/(\alpha \rho_D)$, erf$(x)$ is the error function, so that $b(x) \to 0$ at $x \to -\infty$ and $b(x) \to 1$ at $x \to \infty$, and $\alpha$ is a parameter assumed to be small, $\alpha \ll 1$. This parameter determines a narrow range of densities $|\rho - \rho_D| \sim \alpha \rho_D$ in which the direct Urca process gains its strength. Similar broadening is introduced for the muonic direct Urca process but it is almost unimportant.

For example, the broadening of the direct Urca threshold can be provided by proton superfluidity (e.g., [19]). This superfluidity is characterized by the proton critical temperature $T_{cp}(\rho)$ (e.g., [20]). The critical temperatures are very model dependent. A scatter of theoretical $T_{cp}(\rho)$ is large, so that we do not rely on specific theoretical models but consider $T_{cp}(\rho)$ on phenomenological level. Proton superfluidity is expected to be strong in the outer core of the NS (with $T_{cp}(\rho) \gtrsim 3 \times 10^9$ K) but becomes weaker or fully disappears in the inner core, at a few nuclear matter densities. As long as it is strong, it greatly suppresses the direct Urca process (even if it is formally allowed) by the presence of a large gap in the energy spectrum of protons. When proton superfluidity weakens with growing $\rho$, the superfluid suppression is removed and the direct Urca becomes very powerful. It switches on after exceeding some threshold density, but not very sharply, as if the threshold is broadened.

3. Results and conclusions

Some results are presented in Figs. 1 and 2 (more details can be found in [8]). The probabilities are calculated by averaging over neutron star masses in accordance with (2) and over the amount of light elements in the heat blanketing envelopes (3). The probability distributions are plotted by grayscaling (in relative units). White regions refer to zero or very low probability. The direct Urca threshold is broadened in accordance with (4) at $\alpha = 0.1$. We see that the theoretical distributions describe the data reasonably well.

![Figure 1](image1.png)

Figure 1: Probability to find an INS in different places of the $T_{\infty} - t$ plane compared with observations. Dotted lines show 11 “reference” cooling curves for stars with iron envelopes and masses $M = 1.1, 1.2, \ldots, 2.1 M_\odot$ (see text for details).

![Figure 2](image2.png)

Figure 2: Same as in Fig. 1 but for ANSs in the $L_{\infty} - \langle \dot{M} \rangle$ plane.

The data for INSs include (1) PSR J1119–6127, (2) RX J0822–4300, (3) PSR J1357–6429, (4) Vela, (5) PSR B1706–44, (6) PSR J0538+2817, (7) B2334+61, (8) PSR B0656+14, (9) Geminga, (10) PSR B1055–52, (11) RX J1856.4–3754, (12) PSR J2043+2740, (13) RX
J0720.4–3125, (14) PSR J1741–2054, (15) XMMU J1731–347, (16) Cas A NS, (17) PSR J0357+3205 (18) Crab, (19) PSR J0205+6449.

XRTs in Fig. 2 are (1) Aql X-1, (2) 4U 1608–522, (3) MXB 1659–29, (4) NGC 6440 X-1, (5) RX J1709–2639, (6) IGR 00291+5934, (7) Cen X-4, (8) KS 1731–260, (9) 1M 1716–315, (10) 4U 1730–22, (11) 4U 2129+47, (12) Terzan 5, (13) SAX J1808.4–3658, (14) XTE J1751–305, (15) XTE J1814–338, (16) EXO 1747–214, (17) Terzan 1, (18) XTE J1807–294, (24) NGC 6440 X-2. The references to observational data can be found in [8].

Our results are preliminary and illustrative. We have taken one EOS of nucleon matter in the NS core. Moreover, we have introduced the broadening of the direct Urca threshold and distribution functions over NS masses phenomenologically. Nevertheless, we have varied the broadening of the direct Urca threshold \( \alpha \) in (4), and typical mass ranges of INSs and ANSs. Using such ‘trial and error’ procedure we have obtained a reasonable agreement with observations of NSs of both types for \( \alpha = 0.1, \mu_i = 1.4, \sigma_i = 0.15, \mu_a = 0.47, \) and \( \sigma_a = 0.17. \)

Lower \( \alpha \) would result in two distinctly different populations of NSs of each type (INSs and ANSs), warmer \((M \leq M_D)\) and colder \((M > M_D)\) ones, in disagreement with the data. At \( \alpha \approx 0.1 \) these populations merge in one population (for NSs of each type) which qualitatively describes the data. Larger \( \alpha \) result in overabundance of colder stars, in contradiction with the data [8].

The selected mass distribution functions (2) and the obtained values of their parameters do not contradict observations and theoretical expectations [21] but they are not unique. The most probable mass for ANSs is 1.55 M_⊙, whereas for INSs it is 1.4 M_⊙. Thus, ANSs are overall heavier than INSs as it was expected.

The advantage of our approach is that it explains all the data. The explanation essentially requires (i) the presence of the direct Urca process in the inner cores of massive NSs (to interpret the coldest ANSs); (ii) a moderate broadening of the direct Urca threshold \( \alpha \approx 0.1 \) to merge the populations of warm and colder NSs of each type into one (observable) population; and (iii) higher typical masses of ANSs [to explain the very cold accreting source 13 (SAX 1808.4–3658) and the absence of very cold middle-aged INSs]. In this case the averaging over the masses of light elements in (3) plays relatively minor role but is helpful to explain the existence of warmer isolated and accreting NSs. Nevertheless, these stars can be explained [8] by assuming the presence of strong proton superfluidity in NSs with \( M < M_D \). This superfluidity suppresses the modified Urca process, which is the major process of neutrino emission in low-mass NSs. Such stars will become slower neutrino coolers, and hence warmer sources (e.g., [11], and references therein). The required broadening of the direct Urca threshold can be produced by weakening of proton superfluidity in the massive stars. Therefore, the obtained explanation, in physical terms, can be reached by assuming the presence of proton superfluidity in neutron star cores. This superfluidity should be strong in low-mass stars but weakens in high-mass ones whose neutrino emission is greatly enhanced by the direct Urca process (as discussed above).

The present scenario is different from the minimal cooling model [22, 23]. The latter model assumes that the enhanced neutrino cooling is produced by the neutrino emission due to the triplet-state pairing of neutrons. This enhancement is much weaker than that due to the direct Urca process; it cannot explain the observations of the coldest XRTs (such as SAX 1808.4–3658).

The disadvantage of our analysis is that it should be elaborated. For instance, one can try other EOSs of superdense matter. We expect that the results will be similar but rescaled with respect to the values of \( M_D \) for new EOSs. One can also try different mass distributions of INSs and ANSs. In addition, the distribution of light elements in the heat blanketing envelopes is not entirely arbitrary but is regulated by diffusion of ions. Another issue would be to include the effects of rotation and magnetic fields, and also the effects of nucleon superfluidity.
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