Resurrection of the Sigma Meson

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Abstract

It is shown from a very general model and an analysis of data on the lightest 0^+ meson nonet that the f_0(980) and f_0(1200) resonance poles are two manifestations of the same s\bar{s} state. Similarly the a_0(980) and the a_0(1450) are likely to be two manifestations of the same q\bar{q} state. On the other hand, the u\bar{u}+d\bar{d} state, when unitarized and strongly distorted by hadronic mass shifts, becomes an extremely broad Breit-Wigner-like background, m_{BW} = 860 MeV, \Gamma_{BW} = 880 MeV, with its pole at s = (0.158-i0.235) GeV^2. This we identify with the \sigma meson required by models for spontaneous breaking of chiral symmetry.

The understanding of the \pi\pi S-wave has been controversial for a long time. Before 1974 (see [1]) one believed in the existence of a broad and light isoscalar resonance (then called \sigma or \epsilon or \eta_0\mp). After the two heavier resonances f_0(980) and f_0(1200 \mp 1300) were established one generally discarded the \sigma assuming it could be replaced by the two heavier ones, which
could complete a light $q\bar{q}$ nonet.

The lightest scalar-isoscalar meson coupling strongly to $\pi\pi$ is of importance in most models for spontaneous breaking of chiral symmetry, like the linear $\sigma$ model or the Nambu–Jona-Lasinio model \[2\], which require a scalar meson of twice the constituent quark mass or $\approx700$ MeV, and a very large $\pi\pi$ width of $\approx850$ MeV. This meson is crucial for our understanding of all hadron masses. Thus most of the nucleon mass is believed to be generated by its coupling to the $\sigma$, which acts like an effective Higgs-like boson for the hadron spectrum. However, the lightest well established mesons with the quantum numbers of the $\sigma$, the $f_0(980)$ and $f_0(1200)$ do not have the right properties. They are both too narrow, $f_0(980)$ couples mainly to $K\bar{K}$, and $f_0(1200)$ is too heavy.

Recently one of us \[3\] showed that one can understand the data on the lightest scalars in a model which includes most well established theoretical constraints: Adler zeroes as required by chiral symmetry, all light two-pseudoscalar (PP) thresholds with flavor symmetric couplings, physically acceptable analyticity, and unitarity. A unique feature of this model is that it simultaneously describes the whole scalar nonet and one obtains a good representation of a large set of relevant data. Only six parameters, which all have a clear physical interpretation, were needed: an overall coupling constant ($\gamma = 1.14$), the bare mass of the $u\bar{u}$ or $d\bar{d}$ state ($m_0 = 1.42$ GeV), the extra mass for a strange quark ($m_s = 100$ MeV), a cutoff parameter ($k_0 = 0.56$ GeV/c), an Adler zero parameter for $K\pi$ ($s_{A_{K\pi}} = -0.42$ GeV), and a phenomenological parameter\[4\] enhancing the $\eta\eta'$ couplings ($\beta = 1.6$).

As expected, the coupling to PP turns out \[3\] to be very large causing the inverse propagators to have very large imaginary parts. Normally, this is expected to result in large widths, but it was shown \[3\] that because of the many nonlinear effects the $a_0(980)$ and the $f_0(980)$ naturally come out narrow. Furthermore, the large flavor symmetry breaking in

\[1\]One could discard the $\beta$ parameter if one also included the next group of important thresholds or pseudoscalar $(0^{-+})$-axial $(1^{+-})$ thresholds, since then the $K\bar{K}_{1B} + c.c.$ thresholds give a very similar contribution to the mass matrix as $\eta\eta'$.\[4\]
the positions of the PP thresholds induce large flavor symmetry breaking in the mass shifts. This makes the physical spectrum quite distorted compared to the simple bare spectrum, which obeys the equal spacing rule and the OZI rule with flavor symmetric couplings.

The analysis [3] yielded resonance parameters and pole positions for the four states $a_0(980)$, $f_0(980)$, $f_0(1200)$ and $K^*_0(1430)$, close to their conventional values [4]. In addition, however, one also expects ”image” poles (sometimes called ”shadow” or ”companion” poles), which normally lie far away and do not play a significant rôle. Recently Morgan and Pennington [5] showed that for each $q\bar{q}$ state one expects at least one such image pole, which in principle can be used to distinguish a $q\bar{q}$ state from a meson-meson bound state.

In this letter we report a more detailed search for all relevant poles in the amplitudes of the model of [3]. This reveals interesting and surprising new features, which simultaneously resolve two longstanding puzzles in meson spectroscopy:

- What is the nature of the $f_0(980)$ and $f_0(1200)$?

- Where is the long sought for $\sigma$ meson?

These new, important features can appear when the coupling to S-wave thresholds becomes very strong. In particular, two true resonance poles can emerge near the physical region, although only one $q\bar{q}$ state is present. If the coupling is reduced one of these poles disappears as a distant image pole far from the physical region.

In order to explain this phenomenon in the simplest possible terms we use two theoretical demonstrations. The first is based on the actual model amplitude in [3], which fits the $K\pi$ S-wave data and the $K^*_0(1430)$. We chose the channel with strangeness, because there one has only one $s\bar{d}$ quark model state, whereas in the flavorless case one has the more complicated situation of at least two nearby resonances, the $u\bar{u} + d\bar{d}$ and $s\bar{s}$, which mix in an energy dependent and complex way. By increasing the overall coupling, $\gamma$, we show how a second pole appears. In the second demonstration we chose a simple model for the threshold behaviour, which still has the desired analytic properties, and from which the
poles can be found analytically. This can also be used to demonstrate the phenomenon for the $K\bar{K}$ channel.

In Fig. 1a we show the running mass, $m_0^2 + \text{Re}\Pi(s)$ and the width-like function $-\text{Im}\Pi(s)$ for the $K\pi \to K\pi$ S-wave as was found in [3]. The $K\pi$ partial wave amplitude is obtained from these functions through

$$A(s) = -\text{Im}\Pi_{K\pi}(s)/[m_0^2 + \text{Re}\Pi(s) - s + i\text{Im}\Pi(s)].$$ (1)

This fits the $K\pi$ data well and one finds the resonance parameters listed in the first row of Table I. As one increases $\gamma$ there appears, in addition to the resonance, first a virtual bound state and then a true bound state just below the $K\pi$ threshold (See Figs. 1b,c and Table I). Both poles, the original $K^*_0(1430)$ and the new bound state are then manifestations of the same $s\bar{u}$ state, whose bare mass is kept at 1520 MeV.

For our second way to demonstrate this phenomenon we chose the form factor such that $\Pi(s)$ takes a simple analytic form,

$$\Pi(s) = \bar{\gamma}^2 s_{th}[((s_{th} - s)s_{th})^{1/2} - s_{th}]/s.$$ (2)

See Fig. 2. This still has the desired analytic form and satisfies the dispersion relation (See discussion in Sec 2.7 of [3]). The condition for the poles, $m_0^2 + \Pi(s) - s = 0$, now gives an equation of third degree. If $m_0^2 = s_{th}(1 + \bar{\gamma}^2)$ one has one bound state at threshold $s = s_{th}$ and (for $\bar{\gamma} > 1/2$) a complex conjugate pair at $s = s_{th}[\bar{\gamma}^2 + 1/2 \pm i(\bar{\gamma}^2 - 1/4)^{1/2}]$.

For $\bar{\gamma}^2 > 1$ one has a running mass (cf. Fig. 2) quite similar to the one in Fig 1c for the $K\pi$ threshold and to the actual one fitting the data at the $K\bar{K}$ threshold (Fig. 2b and 9a of ref. [3]). If $\bar{\gamma} > 1$ the phase shift passes through $90^\circ$ at $s = \bar{\gamma}^2 s_{th} = m_{BW}^2$. This simplified model is also similar to the actual situation in [3] for the $s\bar{s}$ channel and the $K\bar{K}$ threshold. The $\bar{\gamma}$ of the simplified model is comparable in magnitude to the $\gamma$ used in [3], such that with $\gamma = \bar{\gamma}$ both models give similar $\text{Im}\Pi_{K\bar{K}}(s)$ near the $K\bar{K}$ threshold for $s\bar{s}$. Thus the fact that $\gamma = 1.14 > 1$ actually shows that the real world is not too far from our simplified model. Using $\bar{\gamma} = 1.14$ one would predict $(\text{Re}s_{pole})^{1/2} = 2m_K(\bar{\gamma}^2 + 1/2)^{1/2} = 1329$ MeV.
and $m_{BW} = 2m_K\bar{\gamma} = 1129$ MeV, which is not far from what was actually obtained for the $f_0(1200) : 1202$, and 1186 respectively. In reality, of course, other thresholds and mixing with $u\bar{u} + d\bar{d}$ complicate the picture. For example the $f_0(980)$ and the $a_0(980)$ are probably unstable virtual states (i.e., lying on the second sheet, and not on the third sheet although two thresholds are open at the pole positions).

The situation is also similar for the $a_0(980)$ and $a_0(1450)$ in the $I=1$ channel, although now the Clebsch-Gordan coefficient reduces the effective $\bar{\gamma}$ by $1/\sqrt{2}$. However, the fact that the $\pi\eta$ channel is already open at the $K\bar{K}$ threshold helps in creating a similar situation of a running mass rising fast enough after threshold. Therefore, one can expect a repetition of the phenomenon, such that there could exist a second manifestation of the $I=1$ state, somewhere in the 1.5 GeV region, in addition to $a_0(980)$. This could be the $a_0(1450)$ seen by the Crystal Barrel [3]. And as we shall see below, the model of [3] actually has image poles near this mass, one of which in an improved model and fit could emerge as the $a_0(1450)$. On the other hand, in the strange channel, there is only one important channel open, the $K\pi$ with a Clebsch-Gordan coefficient reducing the coupling compared to $s\bar{s} - K\bar{K}$ by $(3/4)^{1/2}$. This, together with the fact that the $K\pi$ threshold involves two unequal mass mesons, implies that the resonance doubling phenomenon does not appear in the strange sector.

We now look for the actual pole positions in the model of [3], and list the significant ones in Table II. Four of these, which are near the physical region, were already given in [3]: the $f_0(980)$ , $f_0(1200)$ , $a_0(980)$ and $K_0^*(1430)$. However, we now find three new poles. (There are of course more image poles, which we do not list, since these are very far from the physical region.) Note that all the poles in Table II are manifestations of the same nonet. In [3] the Breit-Wigner parameters of the $\sigma$ meson were also given, but it was not specified which pole should be associated with it.

We now unambiguously find that the $f_0(980)$ and the $f_0(1200)$ are two manifestations of the same $s\bar{s}$ state. The dominant pole in $u\bar{u} + d\bar{d}$ is the first pole in Table II at $s = (0.158 - i0.235)\text{GeV}^2$, which gives rise to a very broad (880 MeV) Breit-Wigner-like background with $m_{BW} = 860$ MeV. One can convince oneself that this is the right conclusion by decoupling
the two channels $s\bar{s}$ and $u\bar{u} + d\bar{d}$. This can be done within the model, maintaining unitarity etc., by sending $m_0$ or $m_0 + 2m_s$ (gradually) to infinity. The $\sigma$ pole remains almost at the same position as in Table II even when $m_s \to \infty$, while there is no trace of $f_0(980)$ nor $f_0(1200)$ in $u\bar{u} + d\bar{d}$. The two latter poles remain, however, in the $s\bar{s}$ channel even when this is completely decoupled from $u\bar{u} + d\bar{d}$. The $f_0(980)$ remains near the $K\bar{K}$ threshold whereas $f_0(1200)$ is shifted to somewhat higher values.

A posteriori, this result is natural also in the light of the mixing angles, $\delta_S$, found for the physical states (see Table II). At the $\sigma$ pole, as well as for energies $\lesssim 900$ MeV $\delta_S$ is small along the real $s$ axis. Thus the $\pi\pi$ amplitudes below this energy are dominated by the $\sigma$ and only slightly perturbed by the $s\bar{s}$ and $K\bar{K}$ channels. The $f_0(980)$ and the near octet $f_0(1200)$ owe their existence to the $s\bar{s} \to K\bar{K}$ channel dynamics and have a comparatively small mixing with the $\sigma$, also evident from the rather small mixing angle $\delta_S$ of these states.

In conclusion, the $\sigma$ meson exists in the $u\bar{u} + d\bar{d}$ channel and we tentatively name it $f_0(600)$. This is the scalar meson required by models for dynamical breaking of chiral symmetry. It has the right mass and width and large $\pi\pi$ coupling, thus dominating $\pi\pi$ scattering below 900 MeV. The very large width of the $\sigma$ explains why it has been difficult to find in the data, without a sufficiently sophisticated model.

The $f_0(980)$ and $f_0(1200)$ are both manifestations of the same $s\bar{s}$ quark model state. Similarly we believe that the $a_0(980)$ and the $a_0(1450)$ are two manifestations of the $I=1$ $qq$ quark model state. In the $s\bar{u}$ and $u\bar{u} + d\bar{d}$ systems the image poles of the $K_0^*(1430)$ and $\sigma$, respectively, are sufficiently far from the physical region and therefore do not give rise to additional resonances. We emphasize again that all the states discussed in this paper are manifestations of the same quark model nonet, which naturally can be assumed to be the $^3P_0$ nonet. When unitarized, the $^3P_0$ naive quark model spectrum is strongly distorted, and

\footnote{The near octet nature of this state is also supported by the small branching ratio of 0.02 to $\eta\eta$ found by GAMS2000 \cite{gams2000}, since the $8-\eta\eta$ coupling nearly vanishes for the conventional pseudoscalar mixing angle.}
results not in 4, but in 6 different physical resonances of different isospin.

One could argue that the two states $f_0(980)$ and $a_0(980)$ are a kind of $K\bar{K}$ molecules, since these have a large component of $K\bar{K}$ in their wave function. However, the dynamics of these states is quite different from that of normal two-hadron bound states. In particular, it is very different from the hyperfine interaction suggested by Weinstein and Isgur [8]. If one wants to consider them as molecules, it is the $K\bar{K} \rightarrow s\bar{s} \rightarrow K\bar{K}$ interaction which creates their binding energy. Although they may spend most of their time as $K\bar{K}$ they owe their existence to the $s\bar{s}$ state. And in general, it is not obvious which of the two states, $f_0(980)$ or $f_0(1200)$, would be the extra state and which is $q\bar{q}$, since one can well imagine situations (with different $m_0$ and $\gamma$) where either one of these is removed to being a distant image pole. Therefore, one should rather consider both as two different manifestations of the same $q\bar{q}$ state.
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FIG. 1. (a) The running mass and -Im\Pi(s) which fits the $K\pi$ S wave data. In (b) and (c) the overall coupling, $\gamma$, is increased whereby first a virtual bound state (b) and then a bound state (c) appears below the $K\pi$ threshold in addition to the $K_0^*$ resonance whose 90° mass gets shifted to slightly lower energies. See text and Table I.
FIG. 1. (b) As in fig 1a the running mass and -Im\Pi(s) but with the overall coupling, \( \gamma \), is increased to 1.34 whereby first a virtual bound state appears below the \( K\pi \) threshold in addition to the \( K_0^* \) resonance whose 90\(^\circ\) mass gets shifted to slightly lower energies. See text and Table I.
FIG. 1. (c) The running mass and -ImΠ(s) but with the overall coupling, $\gamma$, is increased to 1.53 whereby a bound state appears below the $K\pi$ threshold in addition to the $K_0^*$ resonance whose $90^\circ$ mass gets shifted to slightly lower energies. See text and Table I.
FIG. 2. The running mass and $-Im\Pi(s)$ for the simple model function (2) and the $K\bar{K}$ threshold using $\tilde{\gamma} = 1.14$. Choosing $m_0$ such that there is a bound state at the threshold, there is another resonance as seen from the second crossing of $s$ with the running mass. This crossing point gives the 90° or Breit-Wigner mass of the second state.
TABLES

TABLE I. The 90° “Breit-Wigner” mass and width together with the pole positions of the $K_0^*(1430)$ resonance in units of MeV. The first row shows the result from the actual fit to data, while the second and third rows show how the resonance parameters are shifted as one increases $\gamma$. At the same time, first a virtual bound state and then a true bound state appears when $\gamma$ is increased. The bare $s \bar{s}$ mass is 1520 MeV throughout. See also Fig. 1.

| $\gamma$ | $m_{BW}$ | $\Gamma_{BW}$ | $m_{pole} = Re(s_{pole})^{1/2}$ | $-Im(s_{pole})/m_{pole}$ | Comment |
|----------|----------|---------------|-------------------------------|--------------------------|---------|
| 1.14     | 1349     | 498           | 1441                          | 270                      | No nearby image pole |
| 1.34     | 1260     | 780           | 1460                          | 404                      | A virtual bound state appears at 548 MeV |
| 1.53     | 1100     | 1100          | 1496                          | 441                      | A bound state appears at 632 MeV |

TABLE II. Poles in the S-wave $PP \to PP$ amplitudes [3] The first resonance is the $\sigma$ which we name $f_0(600)$. The two following are both manifestations of the same $s \bar{s}$ state. The $f_0(980)$ and $a_0(980)$ have no approximate Breit Wigner-like description, and the $\Gamma_{BW}$ given for $a_0(980)$ is rather the peak width. The two last entries are image poles of the $a_0(980)$, one of which in an improved fit could represent the $a_0(1450)$. The $f_0(1200)$ and $K_0^*(1430)$ poles appear simultaneously on two sheets since the $\eta \eta$ and the $K \eta$ couplings, respectively, nearly vanish. The mixing angle $\delta_S$ for the $f_0(600)$ or $\sigma$ is with respect to $u \bar{u} + d \bar{d}$, while for the two heavier $f_0$’s it is with respect to $s \bar{s}$.

| resonance | $m_{BW}$ | $\Gamma_{BW}$ | $\delta_{S,BW}$ | $m_{pole}$ | $-Im(s_{pole})/m_{pole}$ | $\delta_{S,pole}$ | Sheet | Comment |
|-----------|----------|---------------|-----------------|------------|--------------------------|-------------------|-------|---------|
| $f_0(600)$| 860      | 880           | $(-9 + i8.5)^{\circ}$ | 397        | 590                      | $(-3.4 + i1.5)^{\circ}$ | II    | The $\sigma$ meson. Near $u \bar{u} + d \bar{d}$ state |
| $f_0(980)$| -        | -             | -               | 1006       | 33.7                     | $(0.4 + i39)^{\circ}$ | II    | First near $s \bar{s}$ state |
| $f_0(1200)$| 1186     | 360           | $(-32 + i1)^{\circ}$ | 1202       | 338                      | $(-36 + i2)^{\circ}$ | III,IV | Second near $s \bar{s}$ state |
| $K_0^*(1430)$| 1349   | 498           | -               | 1441       | 320                      | -                 | II,III| The $s \bar{d}$ state |
| $a_0(980)$| 987      | $\approx 100$| -               | 1084       | 270                      | -                 | II    | First I=1 state |
| $a_0(1450)$| -        | -             | -               | 1566       | 578                      | -                 | III   | Image pole to $a_0(980)$ |
| $a_0(1450)$| -        | -             | -               | 1748       | 690                      | -                 | V     | Image pole to $a_0(980)$ |