Application of diameter distribution model for volume estimation in *Tectona grandis* L.f. stands in the Oluwa forest reserve, Nigeria

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Abstract: The ability to predict the distribution of diameters in a stand is essential for forest managers to make informed management decisions such as prescription of silvicultural treatments and harvesting regimes. Such information is preferably derived from suitable distribution model. The problem evaluated the performance of four distribution models in describing the structure of the teak stands in Oluwa Forest Reserve, Nigeria. Data were collected from 12 temporary sample plots of 20 × 20 m size in the teak stand. Maximum likelihood estimator was used to fit the distribution models: beta, gamma, Johnson SB, and Weibull to the diameter data from the teak stand. Relative rank-sum derived from four indices was used to conclude on the most suitable distribution for the stand. The results showed that the Weibull distribution was the most suitable function for the teak stand with a relative rank-sum of 4.0. Application of Weibull distribution together with suitable height-diameter and volume models estimated yield of 136.281 m³ ha⁻¹ within timber size class (diameter ≥30 cm). And a total of 309.640 m³ ha⁻¹ was estimated for the stand. Other product specifications were also provided. This would help in the routine management of the stand.

Keywords: Johnson SB - Stand structure - Teak - Weibull distribution - Yield.

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INTRODUCTION

Tree diameter distribution is used to determine the structure of a forest stand (Rouvinen & Kuuluvainen 2005). The ability to predict the distribution of diameters in a stand help forest manager to make informed decisions such as prescription of silvicultural treatments and determination of distribution (Carretero & Álvarez 2013). Knowledge of diameter distributions forms the basis for deciding when a stand can be economically harvested for a specific product (Ekpa et al. 2014). Diameter structure is an important stand characteristic on which we can evaluate the stability, growth, volume production, structure of assortment, and maturity (Gorgoso-Varela & Rojo-Alboreca 2014).

Stand yields have also been predicted based on the assumption that diameter distribution of a stand can be characterized by a probability density function (Poudel & Cao 2013). In forestry, various distribution functions such as normal (Nanang 1998), gamma (Nelson 1964, Gorgoso-Varela et al. 2020), lognormal (Mirzaei et al. 2016), Johnson’s SB (Özçelik et al. 2016, Ogana & Ekpa 2020), beta (Mayrinck et al. 2018, Gorgoso-Varela et al. 2020), and Weibull (Sun et al. 2019, Schmidt et al. 2020, Gorgoso-Varela et al. 2020), logit-logistic (Wang & Rennolls 2005, Ogana et al. 2018) have been successfully used to describe tree diameter distributions of forest stands. The application of distribution to quantify stand volume in production forests has been relatively few in Nigeria (e.g. Ajayi 2013, Ogana & Ekpa 2020). Quantifying yield by size classes is harbinger for prescription of effective silvicultural treatment and harvesting regime for any forest stand.

The *Tectona grandis* L.f. is a fast-growing tropical hardwood tree species belonging to the family of Verbenaceae. Teak is one of the most widely cultivated exotic species in Nigeria because of its good anatomical and physical properties (Miranda et al. 2011). It is also multipurpose tree species and as such, there is continuous demand for its products (Miranda et al. 2011). The teak plantation in Oluwa Forest Reserve is a production forest established for pole and timber. There is dearth of information on product yield for the teak
planted. This information is necessary for the management of the plantation. Therefore, the main purpose of the study was to apply a suitable diameter distribution function to quantify the stand volume by size classes for effective management of the stand.

**METHODOLOGY**

**Study area and data**

The data used in this study were from the teak plantation (about 3 ha size) in the Oluwa Forest Reserve (FR) of Southwestern Nigeria. The Oluwa Forest Reserve is situated between 6º 55′ and 7º 20′ N and longitude 3º 45′ and 4º 32′ E, and occupies an area of 87,816 ha (Ogana & Ekpa 2020). Oluwa has an annual rainfall in the range 1700 to 2200 mm, an average annual temperature of 26°C, and a mean elevation of 123 m above sea level (Onyekwelu et al. 2006). Establishment of large-scale plantations in the reserve started in early 1960s. *Gmelina arborea* Roxb. and *Tectona grandis* L.f. are the dominant plantation species in Oluwa FR.

Data were collected from twelve sample plots of 20 m × 20 m in size in the teak stand. Diameter measurements of all trees (over bark) at breast height (1.3 m above ground, DBH) were measured with diameter tape to an accuracy of 0.1 cm. Their corresponding height (H) measurements were also taken with hypsometer. The measured variables were used to calculate the basal area (BA, m$^2$) and tree volume (V, m$^3$). The descriptive statistics of the data set is presented in table 1.

| Statistics | DBH (cm) | H (m) | BA (m$^2$) | V (m$^3$) |
|------------|----------|-------|------------|-----------|
| Mean       | 24.1     | 16.1  | 0.051      | 0.375     |
| Standard Deviation | 8.4     | 3.8   | 0.037      | 0.261     |
| Minimum    | 10.0     | 8.4   | 0.008      | 0.038     |
| Maximum    | 54.0     | 29.5  | 0.229      | 1.410     |
| Kurtosis   | 0.55     | 0.26  | 4.384      | 1.091     |
| Skewness   | 0.74     | 0.58  | 1.747      | 1.156     |

N = 379 trees

**Distribution models**

The plot of skewness against kurtosis was first used to narrow the search for a probability density functions (pdf) that described the diameter distribution of the teak data (Fig. 1). The graph showed that the observed point was within the beta and close to Weibull and gamma distributions. Thus, these functions were used to fit the diameter data. Johnson SB function was also evaluated because studies (e.g. Wang & Rennolls 2005) has shown that it occupies the same region in the skewness and kurtosis plane as the beta distribution.

![Figure 1. Skewness and kurtosis plane showing the plots of selected distributions.](http://www.tropicalplantresearch.com/574)
**Beta distribution function:**

\[
f(x) = \frac{1}{(b-a)\beta(a, \beta)} \left( \frac{x-a}{b-a} \right)^{\alpha-1} \left( 1 - \frac{x-a}{b-a} \right)^{\beta-1}
\]  

where, \( f(x) \) is the pdf; \( \alpha, \beta \) represent the shape parameters; \( a, b \) are the location and scale parameters, respectively; \( \Gamma(\cdot) \) is the gamma function. \( a \leq x \leq b, \alpha > 0, \beta > 0. \)

**Gamma distribution function:**

\[
f(x) = \frac{(x-\gamma)\alpha^{-1}}{\beta^\alpha \Gamma(\alpha)} e^{-\frac{(x-\gamma)}{\beta}}
\]  

where, \( \alpha \) is the shape parameter, \( \beta \) represents an inverse scale parameter, \( \gamma \) is the location parameter \((\gamma = 0\) for the two-parameter gamma distribution) and \( \Gamma(\cdot) \) represents the gamma function, with \( x > \gamma, \alpha > 0 \) and \( \beta > 0. \)

**Johnson S_δ distribution** (Johnson 1949):

\[
f(x) = \frac{\xi}{\sqrt{\lambda}} \frac{\lambda}{(x+\lambda)(x-\xi)} \cdot e^{-\frac{1}{2}[\gamma+\delta \ln(x+\lambda)]^2}
\]  

where, \( \xi \) and \( \lambda \) represent the location and scale parameters, respectively; \( \gamma \) and \( \delta \) are the shape parameters; \( \xi < x < \lambda, -\infty < \xi < +\infty, -\infty < \gamma < +\infty, \lambda > 0, \) and \( \delta > 0. \)

**Weibull distribution** (Weibull 1951):

\[
f(x) = \frac{a}{\beta} \left( \frac{x-\gamma}{\beta} \right)^{\alpha-1} e^{-\left( \frac{x-\gamma}{\beta} \right)^{\alpha}}
\]  

where, \( \alpha \) is the shape parameter \((\alpha > 0)\); \( \beta \) represents the scale parameter \((\beta > 0)\); \( \gamma \) is the location parameter.

**Fitting method and evaluation statistics**

The method of maximum likelihood estimation was used to fit the functions to the teak data through the `optim` function in R (R Core Team 2017). Evaluation was based on Kolmogorov-Smirnov (Dn), Anderson-Darling (AD), Cramer-von Mises (W^2) and Bayesian information criterion (BIC). The smaller the indices, the better the distribution fit the data set. Furthermore, relative rank sum proposed by Poudel & Cao (2013) was applied to decide on the best distribution for the teak stand. It is expressed:

\[
R_i = 1 + \frac{(m-1)(S_i-S_{min})}{S_{max}-S_{min}}
\]  

where, \( R_i \) represents the relative rank of distribution \( i \) \((i = 1, 2, \ldots, m)\); \( m \) is the number of distribution assessed (4 distributions), \( S_i \) is the fit index value of distribution \( i; S_{max} \) and \( S_{min} \) represent respectively the maximum and minimum values of \( S_i \). Relative rank is a real number between 1 (best) and 4 (worst). For each distribution, the relative ranks were summed for the four evaluation statistics. Distribution with the smallest relative rank sum was selected as the candidate distribution for the teak stand.

**Application of the distribution**

| Model name | Form | Reference | Eq. |
|------------|------|-----------|-----|
| **H-D models** | | | |
| Chapman-Richards | \( h = 1.3 + b_0(1 - e^{-b_1 d})^{b_2} \) | Richards (1959) | [7] |
| Korf | \( h = 1.3 + b_0 e^{-b_1 d+b_2} \) | Lundqvist (1957) | [8] |
| Logistic | \( h = 1.3 + \frac{b_0}{1 + b_1 e^{-b_2 d}} \) | Pearl & Reed (1920) | [9] |
| Naslund | \( h = 1.3 + \frac{d^2}{(b_0 + b_1 d)^2} \) | Nåslund (1936) | [10] |
| Weibull | \( h = 1.3 + b_0 \left( 1 - e^{b_1 d^2 b_2} \right) \) | Yang et al. (1978) | [11] |
| **Volume Equations** | | | |
| Combined variable | \( v = b_3 + b_4 d^2 h \) | Laar & Akçar (2007) | [12] |
| Constant form factor | \( v = b_1 d^2 h \) | Laar & Akçar (2007) | [13] |
| Logarithmic | \( v = b_1 d^2 b_2 h b_3 \) | Laar & Akçar (2007) | [14] |
| Generalized logarithmic | \( v = b_0 + b_1 d^2 b_2 h b_3 \) | Laar & Akçar (2007) | [15] |

**Note:** \( b_0, b_1, b_2 \) = model parameters; \( h = \) total tree height (m); \( d = \) diameter at breast height (cm); \( v = \) volume (m^3); \( e = \) base of the natural logarithm.
The best distribution based on the evaluation statistics was applied to estimate the volume per hectare for specified diameter classes. To do this, suitable height-diameter (H-D) and volume models for the teak stand were first identified; so that the mean height for a given diameter class can be estimated. The diameter class midpoint and estimated mean height were inserted in the volume equation to derive the class mean volume. The product of the mean volume and the density (trees ha\(^{-1}\)) produced the volume per ha of that class. Since there are no suitable H-D and volume models for the teak stand in Oluwa FR, the candidate models in table 2 were evaluated.

The models were assessed based on mean absolute bias (MAB), root mean square error (RMSE) and BIC. The smaller the values of the indices, the better the model. Relative rank was also used to decide on the best H-D and volume models for the teak stand.

RESULTS

Fits of the distribution models

The estimated parameters of the beta, gamma, Johnson SB and Weibull distributions and their corresponding evaluation statistics are presented in table 3. The results showed that the Weibull distribution had the smallest Kolmogorov-Smirnov (Dn), Anderson-Darling (AD), Cramer-von Mises (W\(^2\)) and Bayesian information criterion (BIC) of 0.0309, 0.2649, 0.0378, and 2773, respectively with a least relative rank sum of 4.00. This was followed by beta and Johnson SB distributions with relative rank sum of 7.50 and 11.42, respectively. The gamma distribution had the largest relative rank sum (15.67), as such, ranked last. Thus, based on the evaluation statistics the Weibull was selected as the most suitable distribution for the teak stands.

The graph of the observed and predicted diameter distributions from beta, gamma, Johnson SB and Weibull of the teak stand is presented in figure 2. The predicted distributions were reasonable and compared well with the observed. The diameter distribution of the teak stand was slightly skewed to the right (i.e., positive skewness).

Table 3. Estimated parameters and the evaluation statistics of the four distributions.

| Distributions | Parameters | Evaluation statistics |
|---------------|------------|-----------------------|
|               | location   | scale | shape1 | shape2 | Dn | AD | W\(^2\) | BIC | \(\Sigma R\) |
| Beta          | 9.1639     | 75.323 | 2.2061 | 7.5754 | 0.0319 | 0.3168 | 0.0448 | 2780 | 7.50 |
|               | (1.35)     | (1.42) | (1.40)  | (3.33)  |
| Gamma         | 0.3556     | 8.5675 |         |         | 0.0394 | 0.6354 | 0.0908 | 2781 | 15.67 |
|               | (4.00)     | (4.00) | (4.00)  | (3.67)  |
| Johnson SB    | 7.1264     | 62.3499 | 1.488  | 1.3566 | 0.0359 | 0.4206 | 0.0624 | 2782 | 11.42 |
|               | (2.76)     | (2.26) | (2.39)  | (4.00)  |
| Weibull       | 9.4408     | 16.4736 | 1.8083 |         | 0.0309 | 0.2649 | 0.0378 | 2773 | 4.00* |
|               | (1.00)     | (1.00) | (1.00)  | (1.00)  |

Note: Values in parenthesis are the relative ranks; \(\Sigma R\): relative rank sum; * means selected distributions.

Figure 2. Observed and fitted beta, gamma, Johnson SB and Weibull distributions of the teak stand.

Height-diameter and volume models and application

The results of the fitted H-D models showed that Logistic model was the best function for estimating tree height in the teak stand (Table 4). It had the smallest MAB, RMSE and BIC of 2.175, 2.892, 1990 with a
relative rank sum of 3.24. The Weibull and Chapman-Richards H-D functions also performed well with relative rank sum of 6.60 and 7.27, respectively. In fact, the Weibull model, had the smallest mean bias. The Korf model had the largest relative rank sum of 13.59. The five H-D models produced tree height trajectories that are consistent with biological realism: monotonic increment, inflection point and asymptote (Fig. 3).

In the case of the fitted volume functions, the generalised Logarithmic had the smallest MAB and RMSE of 0.046 and 0.068, respectively with the least relative rank sum of 3.01. The indices of Logarithmic volume model were approximately like those of the generalised Logarithmic model. It had relative rank sum of 3.23. Constant form factor model had the largest MAB, RMSE and BIC; as such ranked last. Though, the generalised Logarithmic had the smallest relative rank sum, it was not selected as the candidate volume model for the teak stand because one of its parameters was not significant (p > 0.05). The residual plots of the selected height-diameter (Logistic H-D) and volume (Logarithmic) models are presented in figure 4a and b. Both residual plots showed a relative horizontal band within the range of ±4 m and ±0.4 m³ for the Logistic H-D and Logarithmic volume models, respectively.

Table 4. Parameters and fit indices of the height prediction models for the teak data

| Models          | Parameters | Evaluation statistics |        |        |        |        |
|-----------------|------------|-----------------------|--------|--------|--------|--------|
|                 | b₀         | b₁                   | b₂     | b₃     | MAB    | RMSE   | BIC    | ∑R     |
| H-D models      |            |                       |        |        |        |        |        |        |
| Chapman-Richards| 20.020     | 0.070                | 1.248  |        | 2.176  | 2.901  | 1993   | 7.27   |
|                 |            | (1.47)               | (2.80) | (3.00) |        |        |        |        |
| Korf            | 25.200     | 9.546ns              | 0.932  |        | 2.185  | 2.912  | 1996   | 13.59  |
|                 |            | (3.59)               | (5.00) | (5.00) |        |        |        |        |
| Logistic        | 19.291     | 3.429                | 0.109  |        | 2.175  | 2.892  | 1990   | 3.24*  |
|                 |            | (1.24)               | (1.00) | (1.00) |        |        |        |        |
| Naslund         | 1.444      | 0.195                |        |        | 2.191  | 2.908  | 1990   | 10.20  |
|                 |            | (5.00)               | (4.20) | (1.00) |        |        |        |        |
| Weibull         | 19.673     | 0.039                | 1.159  |        | 2.174  | 2.900  | 1993   | 6.60   |
|                 |            | (1.00)               | (2.60) | (3.00) |        |        |        |        |
| Tree volume Equation |            |                       |        |        |        |        |        |        |
| Combined variable| 0.085      | 2.53x10⁻⁵            |        |        | 0.058  | 0.079  | -873   | 6.84   |
|                 |            | (2.44)               | (2.18) | (2.22) |        |        |        |        |
| Constant form factor | 2.96x10⁻⁵ |            |        |        | 0.071  | 0.096  | -720   | 12.00  |
|                 |            | (4.00)               | (4.00) | (4.00) |        |        |        |        |
| Logarithmic     | 2.22x10⁻⁴ | 1.434                | 0.996  |        | 0.047  | 0.069  | -978   | 3.23*  |
|                 |            | (1.12)               | (1.11) | (1.00) |        |        |        |        |
| Gen. Logarithmic| -0.035ns   | 4.25x10⁻⁴           | 1.331  | 0.919  | 0.046  | 0.068  | -977   | 3.01   |
|                 |            | (1.00)               | (1.00) | (1.01) |        |        |        |        |

Note: Values in parenthesis are the relative ranks; ∑R: relative rank sum; * means selected models; ns: not significant.

Figure 3. Scatterplot and fitted height trajectories of the five H-D models.

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The Weibull distribution, Logistic H-D and Logarithmic volume models were used to estimate the yield of the teak stand (Table 5). The table showed the density and yield by 5 cm diameter class. The largest volume of 67.782 m$^3$ ha$^{-1}$ was observed between 25–30 cm diameter class with a density of 150 trees ha$^{-1}$. Diameter classes 20–25 cm and 30–35 cm had 59.681 and 57.948 m$^3$ ha$^{-1}$, respectively. Thus, majority of the yield of the teak was within 20–35 cm diameter class. The total volume was 309.640 m$^3$ ha$^{-1}$.

**Table 5.** Estimated density and yield of the teak stand from Weibull distribution.

| L (cm) | U (cm) | CM (cm) | H (m) | V (m$^3$) | RF | Density (trees ha$^{-1}$) | Yield (m$^3$ ha$^{-1}$) | Utilisation purpose |
|--------|--------|---------|-------|-----------|----|--------------------------|------------------------|---------------------|
| 5      | 10     | 7.5     | 9     | 0.0356    | 0.00220 | 2                         | 0.065                 | Fuelwood (DBH ≤11.0 cm) |
| 10     | 15     | 12.5    | 11.6  | 0.0954    | 0.12865 | 106                      | 10.150                | Pole (DBH: 12–29.5 cm)  |
| 15     | 20     | 17.5    | 14.1  | 0.1877    | 0.22987 | 190                      | 35.682                |                     |
| 20     | 25     | 22.5    | 16.2  | 0.3091    | 0.23347 | 193                      | 59.681                |                     |
| 25     | 30     | 27.5    | 17.8  | 0.4527    | 0.18105 | 150                      | 67.782                |                     |
| 30     | 35     | 32.5    | 18.8  | 0.6074    | 0.11536 | 95                       | 57.948                | Timber (DBH ≥30 cm)    |
| 35     | 40     | 37.5    | 19.5  | 0.7734    | 0.06235 | 52                       | 39.879                |                     |
| 40     | 45     | 42.5    | 20    | 0.949     | 0.02909 | 24                       | 22.831                |                     |
| 45     | 50     | 47.5    | 20.2  | 1.1242    | 0.01185 | 10                       | 11.017                |                     |
| 50     | 55     | 52.5    | 20.4  | 1.3105    | 0.00425 | 4                        | 4.606                 |                     |
| Total  | 825    |         |       |           |           | 825                      | 309.640               |                     |

**Note:** L = Class lower limit; U = Class upper limit; CM = Class midpoint; H = Class mean height; V = Class mean volume; RF = Relative frequency.

**DISCUSSION**

This study has evaluated four distributions for describing the structure and estimating the stand volume of the teak plantation in the Oluwa FR. Of the functions evaluated, the Weibull was the most suitable distribution for the teak stand. The beta, Johnson SB and gamma also provided good fits comparable to the observed diameter distribution.

The suitability of the Weibull distribution in predicting diameter distribution in both even-aged and uneven-aged stands are well documented in forestry literature. For example, Sun et al. (2019) found the Weibull distribution to be adequate in describing the diameter distribution in the uneven-aged mixed stands of oak and pine. Similarly, Gorgoso et al. (2012) also reported good performance with Weibull distribution in characterising the diameter distribution of *Pinus pinaster* Ait, *Pinus radiata* D. Don and *Pinus sylvestris* L. However, some researchers such as Mateus & Tomé (2011), Mayrinck et al. (2018), Ogana & Ekpa (2020), etc., reported better fits with the Johnson SB compared to beta and Weibull distributions. Even though the beta and Johnson SB occupy more region in the skewness-kurtosis plane compare to the Weibull (Wang & Rennolls 2005), the Weibull was the preferred distribution for the teak stand based on the evaluation statistics. In
addition, it is the only distribution that provides good efficiency in calculating the relative frequency of trees by diameter class due to its closed-form cumulative distribution function (cdf) compares to those of beta, gamma and Johnson SB distributions which require numerical integration.

Modelling the diameter distribution of a forest stand is an integral part of forest management and planning (Nord-Larsen & Cao 2006). This study has provided quantifiable information on product specification for the management of the teak stands. If for example, a diameter size of ≥30 cm is specified for timber, a total yield of 136.281 m³ ha⁻¹ (summation of volumes in classes 30–55 cm) would be obtained from the stand. The product classification was based on Shamaki & Akindele (2014). Other products such as fuelwood, pole, rafter, etc., can be estimated from the system. Effective development of a diameter distribution yield system requires plot data comprising of age, site index (or average dominant height), density, height and diameter for specify sample trees (Burkhart & Tomé 2012). Such that future stand and diameter distribution can be projected for the stands. Though age is a limiting factor in the study at hand, the result can be still be used in the routine management of the teak stand.

CONCLUSION

It is advantageous in forest management to use appropriate statistical distribution in predicting the condition of a forest stand so that silvicultural treatments and harvesting regimes could be prescribed. In this study, we found the Weibull distribution function to be suitable for the teak stand in the Oluwa Forest Reserve. Application of the Weibull distribution model with fitted Logistic height-diameter and Logarithmic volume models provided information on per hectare volume by diameter class for the teak stands. Since the wise use of forest resources is paramount to the forest managers and planners, this information is germane for making decisions on product specification and overall management of the teak stand in Oluwa forest reserve.

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