Development of a Fuzzy Model to Estimate the Head of Gaseous Petroleum Fluids Driven by Electrical Submersible Pumps

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\textbf{ABSTRACT}

This paper proposes a fuzzy model to estimate the head of gaseous petroleum fluids (GPFs) driven by electrical submersible pumps (ESPs). The proposed fuzzy model is an alternative to widely used empirical models. Numerical and analytical models have been also proposed to estimate heads of GPFs in ESPs, which have failed to reliably serve the function. The developed fuzzy model evidently outperforms comparable empirical models in terms of accuracy and presents a mean absolute estimation error of 52.4\% less than the most accurate existing empirical model.

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1. Introduction

Electrical submersible pumps (ESPs) are effective and economical devices to lift large volume of fluid from downhole under different well conditions [1, 2]. Selection of ESP size is a crucial matter, as over- or under-sizing leads to premature equipment failure or low petroleum fluid recovery. When liquid is pumped, the size of ESPs is selected based on the manufacturer curves. These curves present the output fluid head versus liquid volumetric flow rate for each ESP size. However, in some reservoirs, ESPs should pump two-phase fluid with high gas content. In this case, manufacturer curves are invalid. The alternative is development of models to estimate the head of gaseous petroleum fluids (GPFs) produced by ESPs.

These models have been investigated since 1980s [3]. Apart from head-estimating models, which are the focus of this work, some other models have also been developed to estimate surging or stability border [4], gas bubble size [5] or \textit{in situ} gas volume fraction [6]. These models are outside the scope of this paper.

Analytical, numerical and empirical methods have been employed to develop head-estimating models for GPFs in ESPs. Analytical models have been derived based on mass and momentum balances [7, 8]. However, their derivation process includes unrealistic assumptions and/or oversimplification of complex physics of two-phase fluids. Numerical models have been formulated based on one-dimensional two-fluid conservations of...
mass and momentum along streamlines and require the prediction of surging initiation in ESPs which is not an easy task [9, 10]. Therefore, analytical and numerical models are yet to be practically used to model GPFs in ESPs; while, empirical models are widely trusted alternatively [11, 12].

2. Models in use

In this section, the homogenous model (an old and simple analytical model) and a number of empirical head-estimating models of ESP which are used for GPFs are briefly introduced. The parameters of the empirical models have been identified using the data collected from experiments on diesel fuel/carbon dioxide mixtures. These mixtures are similar to petroleum fluids. Aforementioned experimental data have been presented in [13]. Empirical models identified based on the data of experiments on air/water mixtures, e.g. the ones detailed in [14–16], have been excluded from this paper.

2.1. Model 1

The first and the oldest model of GPFs is the homogenous model. The basis of this analytical model is oversimplification of two-phase physics of GPFs. In this model, first, the head of a liquid flow, with the same flow rate as the GPF's, is determined from the manufacturer’s curve. Then, this head ($H_l$) is modified with the assumption that the fluid motion is homogenous i.e. liquid and gas have equal speeds:

$$\hat{H}_m = \left( (1 - \alpha)\rho_l + \alpha \rho_g \right) H_l,$$

where $\rho$, $H$ and indices $l$, $g$ and $m$ stand for density, head, gas, liquid and mixture respectively. $\alpha$ is the gas void fraction. $\hat{\cdot}$ shows that the head is estimated rather than being experimentally measured.

2.2. Model 2

The second model was developed by Turpin et al. in 1986 [17]:

$$\hat{H}_m = H_l \exp \left( 346,430 \left( \frac{q_g}{p_{in}q_l} \right)^2 - 410 \left( \frac{q_g}{p_{in}q_l} \right) \right),$$

where $q_l$ and $q_g$ are liquid and gas volumetric flow rates in gallons per minutes (gpm), $p_{in}$ is the intake pressure in psi.

2.3. Model 3

This model was proposed by Sachdeva et al. in 1992 [18]:

$$\hat{H}_m = \frac{K_2}{\rho_m g} p_{in} E_1 \alpha^{E_2} q_l^{E_3}.$$

The values of $E_1$, $E_2$, $E_3$ and $K_2$ are listed in [11] for multiple stages of electrical submersible pumps. As an example, for eight stages of I-42B radial ESP, $K_2 = 1.1545620$, $E_1 = 0.943308$, $E_2 = -1.175596$ and $E_3 = -1.300093$. Similar to Model 1, Equation (3) is convertible to a linear equation through taking algorithm.
2.4. Model 4

This model was presented by Zhou and Sachdeva in 2010 [11]:

\[
\hat{H}_m = H_{\text{max}} K_3 (C p_{in})^{\alpha E_4} (1 - \alpha)^{E_5} \left(1 - q_m/q_{\text{max}}\right)^{E_6}, (4)
\]

where \(C\) is the pressure unit coefficient, e.g., 1, 1000 or 0.145 for psi, ksi or kPa. \(H_{\text{max}}\) and \(q_{\text{max}}\) are nominal maximum head and flow rate which can be handled by the ESP; \(q_m\) is mixture or GPF flow rate where \(q_m = q_l + q_g = q_g/\alpha\). Model 4 seems to be a modified version of Model 3. In this model, when gas void fraction and flow rate equal zero, estimated head is \(H_{\text{max}}\).

According to [11], for eight stages of I-42B radial ESP, \(K_3 = 1.971988, E_4 = 1.987838, E_5 = 9.659664\) and \(E_6 = 0.905908\).

2.5. Summary and limits of empirical models

All presented models have three input variables amongst \(p_{in}, p_l, p_g, q_l, q_g, q_m\) or \(\alpha\). Two other potential input variables, pump rotational speed and temperature have not been considered in empirical models yet. All presented models have been developed based on the data collected at a fixed rotational speed of 3500 rpm. The estimated head can be adapted for other rotational speeds using ‘affinity laws’ [2, 11], which are outside the scope of this paper.

3. Fuzzy model

In this research, a mixture of carbon dioxide and diesel fuel pumped by eight stages of an I42B radial ESP was modelled using a linear Sugeno type fuzzy inference system [19–22]. This fuzzy model is comparable with empirical models presented in Section 2.

The data used to develop, validate and test the fuzzy model are the same as the data used to identify the parameters of empirical models 1–4. These experimental data, reported in [13], present maximum heads up to 55 ft and cover a wide range of gas void fractions (0–0.5) and intake pressures (50–400 psi).

Inspired by existing empirical models, a single output of \(H_m\) and three inputs of \(p_{in}, q_m\) and \(\alpha\) were opted for the fuzzy model. Also, similar to existing empirical models, temperature and rotational speed were not considered in modelling.

The proposed fuzzy model has \(n\) rules. Each rule receives all inputs and has a membership function per input. The output of each membership function is a membership grade. In this research, for \(j\)th rule and \(i\)th input \((u_i)\), the Gaussian membership function of (5) was employed to produce a membership grade \(\mu_{ij}\).

\[
\mu_{ij} = \exp \left( -\frac{(u_i - c_{ij})^2}{2\sigma_{ij}^2} \right), (5)
\]

where \(c_{ij}\) and \(\sigma_{ij}\) are the centre and width of the membership function, respectively. The product of membership grades of a rule was considered as the weight of the rule, as shown in the denominator of (6). Weight of a rule is a number between zero and one. Moreover, any rule has an output which is a linear combination of its inputs, as shown in the numerator.
The output of the whole model is the weighted sum of rules outputs:

$$
\hat{H}_m = \frac{\sum_{j=1}^{n} \left( \frac{\text{jth rule output}}{\sum_{j=1}^{n} \prod_{i=1}^{3} \mu_{ij}} \right) \left( \sum_{i=1}^{3} a_{ij} u_i + a_j \right) \prod_{i=1}^{3} \mu_{ij}}{\sum_{j=1}^{n} \prod_{i=1}^{3} \mu_{ij}}.
$$

(6)

In order to develop the fuzzy model for GPFs in ESPs, two steps were taken: (i) Model generation: finding the number of rules, $n$, and initial estimation of model parameters, $a_{ij}$, $a_j$, $c_{ij}$ and $\sigma_{ij}$. (ii) Model identification: determining model parameters accurately. Both of these steps as well as test were carried out using 101 sets of experimental data; where each set includes the head of fluid, $H_m$, as the output and three inputs $p_{ij}$, $q_m$ and $\alpha$.

Subtractive clustering technique, detailed in [23], was used for the model generation with these coefficients: Range of Influence $= 0.5$, Squash Factor $= 1.25$, Accept Ratio $= 0.1$ and Reject Ratio $= 0.05$. The result is a model with $n = 3$ rules. Each rule (e.g. jth rule) has four output parameters ($a_{1j}$, $a_{2j}$, $a_{3j}$ and $a_j$ in (6)) and three membership functions; each membership function has two parameters as presented in (5). As a result, each rule is of 10 parameters, and the fuzzy model has 30 parameters in total.

For model identification, first, the ‘model error’, $E$, was defined to represent the discrepancy of real and estimated (with $^\wedge$) value of the head:

$$
E = \frac{\sum_{\text{for a series of data}} \left( H_m - \hat{H}_m \right)^2}{\text{number of data sets}}.
$$

(7)

In this research, 69 data sets were used as the ‘training data’. The model error calculated for the training data is called the ‘training error’. The parameters of the model were adjusted (or trained) using an iterative algorithm [23] so as to minimise the training error. The training algorithm, at each iteration, includes the least square of error [24] to adjust the parameters of the rules’ outputs ($a_{ij}$, $a_j$) and error backpropagation with gradient (or steepest) decent method [25] to adjust the parameters of membership functions ($c_{ij}$ and $\sigma_{ij}$). At each iteration, the model error for another series of 25 data sets, namely the ‘validation data’, is also calculated: the ‘validation error’. At a point, the validation error starts to increase, while the training error continues to decrease. This situation is called overfitting and is a sign to stop the iterative algorithm of identification [21].

4. Results and discussion

The accuracy of the model was tested with 17 data sets used for neither training nor validation, namely the ‘test data’. The ‘test error’, as an accuracy criterion, was calculated for the model using the test data as follows:

$$
\text{Test Error} = \frac{\sum_{\text{for test data}} \left| H_m - \hat{H}_m \right|}{17}.
$$

(8)
Table 1. Test error for different models in ft.

| Model | Error |
|-------|-------|
| M1    | 8.8572|
| M2    | 7.4046|
| M3    | 13.488|
| M4    | 4.6695|
| Fuzzy | 2.2224|

Table 2. Mean of absolute head estimation error in ft for different models at various operating areas.

| Pin  | a   | M1   | M2   | M3   | M4   | Fuzzy |
|------|-----|------|------|------|------|-------|
| 50   | 0.10| 4.66 | 14.7 | 24.1 | 5.00 | 0.84  |
| 50   | 0.15| 12.1 | 11.9 | 11.4 | 7.32 | 0.79  |
| 50   | 0.20| 15.8 | 8.63 | 6.85 | 8.63 | 0.48  |
| 50   | 0.30| 16.4 | 6.63 | 5.06 | 4.34 | 0.67  |
| 50   | 0.40| 17.7 | 2.33 | 1.13 | 3.07 | 0.44  |
| 100  | 0.10| 5.66 | 3.21 | 22.5 | 5.00 | 0.84  |
| 100  | 0.15| 6.28 | 4.61 | 13.3 | 5.94 | 1.53  |
| 100  | 0.20| 8.25 | 4.96 | 10.4 | 6.68 | 0.77  |
| 100  | 0.30| 10.1 | 10.3 | 7.76 | 4.65 | 3.20  |
| 100  | 0.40| 11.7 | 6.40 | 3.81 | 2.89 | 0.61  |
| 400  | 0.30| 5.47 | 3.73 | 9.92 | 5.84 | 5.06  |
| 400  | 0.40| 4.45 | 2.95 | 8.27 | 4.30 | 1.00  |
| 400  | 0.50| 5.69 | 9.04 | 7.51 | 5.79 | 0.78  |

Figure 1. Real and estimated head (by five models) for a mixture of carbon dioxide and diesel fuel pumped by eight stages of an I-42B radial ESP; intake pressure is 50 psi and gas void fraction is 0.1.

The developed fuzzy model presents a test error of 2.2 ft or 4% of the maximum head, far smaller than currently used empirical models as shown in Table 1. Such a small test error means that the fuzzy model is cross-validated [26, 27].

Table 2 and Figures 1–3 present the estimation accuracy of different models at different operating areas. In this paper, an operating area is a collection of work conditions with same intake pressure and gas void ratio, e.g. $P_{in} = 100$ psi and $\alpha = 0.2$. The results presented in this table have been calculated for the whole available experimental data in each operating area, not only the test data.

According to Table 2, the developed fuzzy model evidently outperforms all other comparable empirical models in 12 operating areas out of 13. Only in one operating area, the
Figure 2. Real and estimated head (by five models) for a mixture of carbon dioxide and diesel fuel pumped by eight stages of an I-42B radial ESP; intake pressure is 100 psi and gas void fraction is 0.15.

Figure 3. Real and estimated head (by five models) for a mixture of carbon dioxide and diesel fuel pumped by eight stages of an I-42B radial ESP; intake pressure is 400 psi and gas void fraction is 0.4.

fuzzy model stands second in terms of accuracy, at a pressure of 400 psi and gas void ratio of 0.3. The performance of the fuzzy model at this particular operating area was inspected as detailed in the following.

In the aforementioned area of operating (11th row of Table 2), 12 data samples are available, where 6 were used for training (or modelling). That is, the ratio of the training data to the entire data is 50% for this operating area; this ratio is 62.16% in total; however, this slight discrepancy cannot be a convincing reason for model inaccuracy; while other operating areas with similar ratios witness an excellent performance of the fuzzy model. The real issue is that the training data do not cover most of the range of flow rates in this operating area. The range of flow rate in this operating area is [47 69] gpm; while, five samples of the training data in this operating area (out of 6) have a flow rate of 60 gpm or above. The only other sample has a flow rate of 47 gpm, nothing between 47 and 60. This overlooked range
is exactly where high errors appear. As a conclusion, in practice, randomly distributed training data is better to be double-checked prior to modelling to ensure that these data cover all operating areas appropriately.

5. Conclusion

This paper first presented existing models which are used to estimate the head of GPFs in ESPs. Empirical models are widely trusted and applied for this estimation purpose; while, analytical and numerical models are yet to be relied for practice. Afterwards, a fuzzy model was generated, trained and cross-validated as an alternative to existing head-estimating models. Finally, the developed fuzzy model was shown to outperform all the presented empirical models in terms of accuracy.

Disclosure statement

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