Numerical simulations of the angular dependence of magnetization AC losses: coated conductors, Roebel cables and double pancake coils

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Abstract
The AC losses in ReBCO coated conductors are large in situations when the conductors are subjected to a considerable magnetic field, as in rotating machines, transformers and high-field magnets. Roebel cables can reduce the AC losses in these cases. However, computer simulations are needed to interpret the experiments, understand the loss mechanisms, reduce the AC losses by optimizing the Roebel cable and design the cryogenic system. In this paper, we simulate and discuss the AC losses due to an applied magnetic field with an arbitrary angle with respect to the cable and taking into account a realistic anisotropic field dependence of the critical current density. We study the AC losses in the superconductor parts for the limits of very high coupling currents and completely uncoupled strands. The simulations for the uncoupled case also describe a double pancake coil with no transport current. For the simulations, we use two different numerical methods with complementary strengths. This serves as a mutual check of the correctness of the simulation results, which agree with each other. As opposed to what was expected, we found that the AC losses do not only depend on the perpendicular component of the applied magnetic field. We also found that the AC losses for applied fields with an orientation below 7° with the strand surface are reduced by more than one order of magnitude as compared with an untransposed cable. Therefore, we recommend to use Roebel cables for windings with important parallel components, such as transformers and high-field magnets.

(Some figures may appear in colour only in the online journal)

1. Introduction
ReBCO coated conductors present several advantages compared to other superconductors, such as their higher operation temperature (up to liquid nitrogen temperature) and their good performance in large magnetic fields. Nowadays, coated conductors are produced in long lengths with uniform properties, so they are ready to be used in market applications. In particular, power applications are very promising, such as fault-current limiters, cables, rotating machines (including wind generators) and transformers. However, the AC losses in rotating machines and transformers are too high for two reasons. First, the component of the AC magnetic field perpendicular to the tape is large. Second, these applications usually require high-current conductors made of several coated conductors. This results in thick cables with a substantial AC loss contribution from the parallel AC magnetic field. A similar situation arises in high-field [1] or large-scale DC magnets (such as for particle accelerators [2] or fusion). In this case, it is still desirable to reduce the AC losses because they limit the ramp rate and generate significant hysteresis losses, especially for liquid helium cooled magnets. Roebel cables [3–7] reduce the AC losses caused by both components of an external magnetic field [4, 5, 8]. In spite of the progress in Roebel cables, many aspects of their AC losses remain unknown [2].
A similar situation exists for pancake coils, widely used in windings.

Computer simulations are needed to interpret the experiments, understand the loss mechanisms, reduce the AC losses by optimizing the Roebel cable or pancake coil and design the cryogenic system. However, to the best of our knowledge, there are no published simulations for the angular dependence of the magnetization AC losses in Roebel cables and pancake coils.

The published simulations for the angular dependence of the magnetization AC losses are only for a single tape or stacks of tapes. For a single tape, the published results are for a constant critical current density \( J_c \), or an anisotropic magnetic field dependent \( J_c \) [10–14]. However, the anisotropies and field\(^3\) dependences utilized in these articles are not realistic, since they take into account only a generic dependence [11], a dependence extracted from data at high applied fields (\( \geq 0.5 \) T) [10, 15], or without the correction for the self-field [12, 13]. Moreover, [9, 11] calculate neither the complete AC cycle nor the AC losses. The work in [14] was for only two angles (90° and 15°). For stacks, there is only one work for the coupled case (allowing the magnetization currents to close in different tapes) and relatively high applied field amplitudes (50 mT). In addition, all these simulations were done assuming either a simplified anisotropy\(^4\) of \( J_c \) by an elliptical dependence or assuming an isotropic superconductor. In contrast, actual coated conductors present complex anisotropies [16–19].

Simulations on Roebel cables are only for perpendicular applied fields [20, 21] or transport currents [20–22]. For double (or single) pancake coils (or stacks of tapes) the magnetization AC losses of stacks of tapes were simulated only for perpendicular applied fields in [23, 24, 20, 25, 21, 26].

The only measurements on the angular dependence of the AC losses are in [27]. However, there exists extensive work on coated conductors [28, 29] and stacks of them [30–32]. All these measurements for single tapes and stacks were for coated conductors at an early stage of development, with poor artificial pinning and field anisotropies of the critical current approximately elliptical. As a consequence, they may not be representative for the present nano-engineered material [16–18]. For Roebel cables there are also measurements for perpendicular applied fields [20, 33, 7, 27], parallel fields [34] and transport currents [20, 35].

In this paper, we simulate and discuss the AC losses due to an applied magnetic field at an arbitrary angle with the cable and we take into account a realistic anisotropic field dependence of the critical current density. We study the AC losses in the superconductor parts for the limits of very high coupling currents (coupled case) and completely uncoupled strands (uncoupled case). The 2D simulations for the uncoupled case also describe a double pancake coil with no transport current. Therefore, the results and discussions for the uncoupled case are also valid for double pancake coils. Additionally, we also discuss the details of the AC losses in a single tape. For the simulations, we use two different numerical methods with complementary strengths: the minimum magnetic energy variation (MMEV) and a finite element method (FEM) with the \( H \)-formulation and edge elements (see section 2.1). This serves as a mutual check of the correctness of the simulation results. Moreover, it is also a check of the applicability of the sharp \( E(J) \) relation from the critical-state model (\( E \) and \( J \) are the electric field and the current density, respectively), because MMEV assumes this sharp \( E(J) \) relation and FEM a smooth one. In this paper, we use a power law for the FEM simulations: \( E(J) = E_c (J/J_c)^n \), where \( E_c \) is the voltage-per-length criterion for the critical current density \( J_c \) (usually set equal to \( 10^{-4} \) V m\(^{-1} \)) and \( n \) is the flux-creep exponent.

This paper is structured as follows. In section 2 we outline the simulation models and the anisotropic field dependence of \( J_c \) for the calculations. In section 3 we present and discuss the results for a single tape and a Roebel cable in the coupled and uncoupled cases. For the Roebel cable, we do not only present the AC losses but also the field and current distribution for some cases. Finally, in 4 we present our conclusions.

2. Models

In this section, we first outline both simulation methods, their complementary strengths and important technical details (section 2.1). Afterward, we detail the anisotropic field dependences of \( J_c \) in the simulations: two anisotropic (a realistic one for YBCO and an elliptical one) and one isotropic field dependences (section 2.2).

2.1. Simulation methods

In this paper, we use two different numerical methods to obtain the current distribution, the magnetic field and the AC losses, as we did in a previous work [21]. These models are the minimum magnetic energy variation (MMEV) method and the finite element method (FEM) with \( H \)-formulation and edge elements. The comparison of their results serves as a mutual check of the correctness of the methods, as well as of the assumption of the critical-state model for modeling high-temperature superconductors.

These methods present different strengths. The MMEV method is generally faster than FEM [21]. This is an advantage for performing simulations with many tapes, such as windings [36, 37]. Since MMEV is a user-programmed softwares (the program for this paper is written in FORTRAN language), it is possible to control the processes, make further improvements and include it in larger programs. In addition, it is possible to exploit the vast collection of existing free-source numerical routines in FORTRAN or C++. The FEM model is more versatile because it uses commercial software [38]. In particular, it can simulate non-linear magnetic materials interacting with the superconductor [39] and multi-physics problems, for example the coupling of electromagnetic and thermal effects [40]. Moreover, in contrast to MMEV, the FEM model can simulate over-critical currents because it takes into account a smooth \( E(J) \) relation.

\(^3\) In order to avoid unnecessary repetition, we refer to the magnetic field as simply ‘field’ except when the electric and magnetic fields could be confused with each other.

\(^4\) In this paper, the term ‘anisotropy’ refers to the field anisotropy of \( J_c \).
Both methods are 2D models, reducing the problem to solving the cross-section of the cable. They assume that the transposition length of the cable is much larger than its thickness and width, so the cable is well approximated by a set of infinitely long tapes parallel to each other, as we detailed in [21].

2.1.1. The minimum magnetic energy variation (MMEV) method. The MMEV method assumes the sharp $E(J)$ relation of the critical-state model, where $E$ is the electrical field. It is based on a variational principle proposed by Prigozhin [41], which finds the current distribution by minimizing the magnetic energy variation, and a fast non-standard minimization routine. This routine has been developed incrementally in several articles. First, Sanchez and Navau solved a cylinder in an applied magnetic field under certain restrictions [42]. Later on, Pardo et al. developed the general method for tapes under any combination of applied magnetic field and transport current [43]. The latest stage of the method is published in [44, 45, 37], where [44, 37] and [45] take into account the field dependence of $J_c$ and the interaction with linear magnetic materials, respectively. Recently, we applied the method to Roebel cables [20, 21], but using a constant $J_c$.

2.1.2. The finite element method (FEM) model. The FEM model assumes a smooth $E(J)$ relation, usually a power law $E(J) = E_c (J/J_c)^n$. In this work, we assume that $n$ is constant, neglecting the magnetic field dependence of this parameter. The state variables of the FEM model are the magnetic field components; as a consequence, the implementation of the $J_c(B)$ dependence is straightforward since the magnetic flux components are immediately available from the state variables by means of the $B = \mu_0 H$ relation. More details about the model implementation can be found in [46].

2.1.3. Tape and Roebel cable parameters. The anisotropic field dependences of $J_c$ for the simulations are detailed in section 2.2. These dependences are based on measurements [19] of YBCO coated conductors from SuperPower, Inc. [47]. The functions for $J_c$ describe the experimental anisotropic field dependence with several degrees of approximation: realistic, elliptic and isotropic. For all cases, we do not take into account the metal parts of the tapes in the calculations, ignoring the effects of eddy currents. For the single tape, the dimensions for the simulations are 4.16 mm for the width and 1.4 μm for the thickness. For the Roebel cable, we chose the geometry of a cable composed of 14 strands, which was manufactured at the Karlsruhe Institute of Technology [20]. The strands are 1.98 mm wide, their lateral separation is 200 μm and their vertical separation (i.e. the distance between the superconducting films) is 140 μm. The total critical current of the cable at 77 K (determined with the 1 μV cm$^{-1}$ criterion over a distance of 30 cm) is 465 A. In the simulations, the strands (from now on we call them ‘tapes’) have the following dimensions: width $w = 1.98$ mm and thickness $d = 1.4$ μm. For the power-law resistivity used in the FEM simulations, $E_c = 10^{-4}$ V m$^{-1}$ and $n = 35$ (unless stated otherwise). The frequency of the applied field is 100 Hz.

2.1.4. Coupled and uncoupled cases. For the cable simulations, we distinguish between the coupled and uncoupled cases. The coupled case assumes that the resistance per transposition length between the tapes is very small, so the magnetization current loops can close freely between any tape of the cable. The uncoupled case assumes a very large resistance per transposition length between the tapes, so the current loops must close within each tape. From the computation point of view, this means that for the coupled case there is only one current constraint (zero net current in the whole cable), while for the uncoupled one there are as many current constraints as tapes (zero net current in each tape).

A cable made of untransposed tapes corresponds to the coupled case because the tapes are interconnected at the current leads. Indeed, Polak et al. [48] found experimentally that, for tape lengths of around 10 cm or above, interconnected tapes in parallel are already fully coupled.

A double pancake coil exactly corresponds to the uncoupled case. This is because, for the uncoupled case, the net current in each tape is the same as it is in a coil. Actually, the simulations in this paper are for arrays of infinitely long tapes. This is the limit for coils with a large radius [49]. Then, all the results and discussions for the Roebel cable in the uncoupled case are also valid for double pancake coils.

2.1.5. Discretization of the superconducting domain. Due to their high aspect ratio, ReBCO superconductors are often approximated as 1D objects, where the variation of the electromagnetic quantities along the thickness is neglected. However, in certain cases, this approximation is not correct. In particular, if we consider the cases shown in this paper, this happens for two reasons. First, for the uncoupled case (or a single tape) and low angles and low fields, the AC losses are dominated by the current penetration across the thickness of the tape (figures 8 and 10). Second, for a field dependent $J_c$ and the uncoupled case (or a single tape), taking only one element in the thickness (or a 1D approximation) neglects the influence of the local parallel field on $J_c$. This is because the parallel component of the field is anti-symmetric with respect to the mid-plane of the tape, so both its value at the mid-plane and its average in the thickness vanish. Taking only one element in the thickness induces significant errors for the uncoupled case and single tapes at applied fields below the self-field. The exception is when $J_c$ only depends on the perpendicular component of the magnetic field, which is a good approximation for Bi2223 tapes but not for ReBCO coated conductors.

The simulations use the following number of elements. For the coupled case, there are one element in the thickness and between 100 and 500 elements in the width per tape, with larger values for lower applied magnetic fields. For the uncoupled case and single tapes, there are between 1 and 20 elements in the thickness and 100 and 500 elements in the width. Higher numbers of elements in the thickness and the width are for lower applied magnetic fields. For the particular case of 0° (parallel applied field) and the uncoupled situation,
we use 50 elements in the thickness and 10 in the width. This is sufficient because the aspect ratio of the tapes is very high, so they roughly behave as slabs.

2.2. Angular and field dependence.

In this paper, we study the effect of three different field and angular dependences of the critical current density, $J_c(B, \theta)$: isotropic superconductor, elliptical anisotropy and a realistic field and angle dependence. This latter dependence presents a peak in both the ab and c orientations and contains three different contributions with elliptical anisotropy (figure 2(a)). In the following sections, we study the error in the computations as a consequence of choosing a simplistic angular dependence, i.e. isotropic superconductor or elliptical anisotropy. In order to do so, we assume that the superconductor is perfectly described by the realistic description. Then, we simulate a measurement of the in-field critical current for an applied field in the ab and c directions (figure 2(b)). We do this with a self-consistency method between the magnetic field and $J_c(B, \theta)$, using the same procedure as in [19]. These are the data available in many experiments. With these data, we extract the parameters assuming an elliptical anisotropy. This simplification describes well the critical current in the ab and c directions but not for intermediate angles at large applied magnetic fields. If we simplify further, to an isotropic superconductor, we could only expect to fit the critical current in one orientation. Moreover, it is not possible to fit the field dependence at low applied fields and high applied fields at the same time because the self-field changes the orientation of the local magnetic field. In this paper we choose to take the parameters that fit the critical current at high applied fields in the ab direction. We choose the ab direction because it overestimates the AC losses at high applied fields. This is because the AC loss at high applied fields is proportional to $J_c$ and it is the largest for the ab direction. This choice is in contrast to critical current analyses, where the c direction is usually chosen because it underestimates the critical current.

2.2.1. Realistic field and angle dependence. Many ReBCO coated conductor tapes present an angular dependence with a peak in the ab and c directions. Reference [19] found an expression based on three contributions that describes a coated conductor manufactured by SuperPower, Inc. [47]. Here, we use a simplified version of that field and angle dependence of $J_c$, which reproduces the main features of the experimental critical current in [19].

$$J_c(B, \theta) = \max[J_{c,ab}(B f_{ab}(\theta)), J_{c,c}(B f_{c}(\theta)), J_{c,i}(B f_i(\theta))]$$

(1)

with

$$J_{c,ab}(B) = \frac{J_{0p}}{1 + B/B_{0ab}}$$

(2)

$$J_{c,c}(B) = \frac{J_{0p}}{1 + B/B_{0c}}$$

(3)

$$J_{c,i}(B) = \frac{J_{0i}}{1 + B/B_{0i}}$$

(4)

and

$$f_{ab}(\theta) = \sqrt{\cos^2 \theta + u_{ab}^2 \sin^2 \theta},$$

(5)

$$f_{c}(\theta) = \sqrt{u_c^2 \cos^2 \theta + \sin^2 \theta},$$

(6)

$$f_i(\theta) = \sqrt{\cos^2 \theta + u_i^2 \sin^2 \theta}.$$  

(7)

The angle $\theta$ is defined as shown in figure 1. We take the following values for the parameters: $J_{0p} = 4.9 \times 10^{10}$ A m$^{-2}$, $J_{0i} = 3.2 \times 10^{10}$ A m$^{-2}$, $B_{0ab} = 4.6$ mT, $B_{0c} = 2.0$ mT, $B_{0i} = 32$ mT, $\beta = 0.48$, $\alpha = 0.9$, $u_{ab} = 8.3$, $u_c = 1.8$ and $u_i = 1.7$.

2.2.2. Elliptical anisotropy. Now, we consider the Blatter’s [50] scaling law for the field and angular dependence, also known as elliptical anisotropy:

$$J_c(B, \theta) = J_{c,e}[B f_e(\theta)]$$

(8)

with

$$J_{c,e}(B) = \frac{J_{0e}}{1 + B/B_{0e}^B},$$

(9)

$$f_e(\theta) = \sqrt{\cos^2 \theta + u_e^2 \sin^2 \theta}.$$  

(10)

For this field and angle dependence we set the parameters in the following way. We take the parameters for which the critical current $I_c$ as a function of the applied magnetic field $B_a$ for the elliptical dependence, equations (8)–(10), fits the best to $I_c$ for the realistic dependence, equations (1)–(7), if the applied field is in the ab or c directions (see figure 2(b)). For low applied magnetic fields, the critical current is smaller.
Figure 2. Calculated critical current using equations (1)–(10) if the self-field is ignored (continuous lines) or if it is taken into account by means of the numerical method in [19] (dashed lines). (a) The applied magnetic field is, from top to bottom, 20, 60, 100 and 200 mT. (b) The lines are for 0° and 90°, from top to bottom (see figure 1 for the definition of the angle θ).

than the integration of $J_c(B_a, \theta)$ over the volume because the self-field increases the local magnetic field. The parameters for the elliptical anisotropy are $J_{oe} = 4.602 \times 10^{10}$ A m$^{-2}$, $B_{0e} = 4.6$ mT, $u_e = 2.015$ and $\beta$ takes the same value as for equations (2) and (3), $\beta = 0.48$.

2.2.3. Isotropic superconductor. Finally, the simplest case is that of an isotropic superconductor, with no angular dependence of $J_c$.

$$J_{c,iso}(B, \theta) = \frac{J_0}{(1 + B/B_0)^\beta},$$

Figure 2. Calculated critical current using equations (1)–(10) if the self-field is ignored (continuous lines) or if it is taken into account by means of the numerical method in [19] (dashed lines). (a) The applied magnetic field is, from top to bottom, 20, 60, 100 and 200 mT. (b) The lines are for 0° and 90°, from top to bottom (see figure 1 for the definition of the angle θ).

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For this field dependence, we take the same parameters as the field dependence for the elliptic case in the $ab$ direction, equation (3): $B_0 = 4.6$ mT, $J_0 = 4.9 \times 10^{10}$ A m$^{-2}$ and $\beta = 0.48$ (if we wanted to take the dependence in the $c$ direction, the isotropic field dependence would be with $B_0 = B_{0e}/u_e = 2.283$ mT).

With the isotropic dependence we do not find the parameters that fit the critical current for the realistic dependence. This is because an isotropic critical current density cannot describe the in-field critical current of a tape with an anisotropic superconductor, not even for a particular orientation.

3. Results and discussion

3.1. AC losses for a single tape

In this section we present and discuss the AC losses in a single tape for an applied magnetic field with several orientations and the three angular dependences of section 2.2 (results in figures 3 and 4). The orientation angle $\theta$ is defined as shown in figure 1. We present the loss factor relative to the applied magnetic field amplitude, $Q/B_m^2$ ($Q$ is the loss per cycle and length and $B_m$ is the applied field amplitude) or to the component of the applied magnetic field perpendicular to the tape surface, $Q/B_{per}^2$.

The results from the two simulation methods agree (figures 3 and 4). However, there is a small discrepancy at large amplitudes because of the finite flux-creep exponent for the FEM simulations (exponent 35). Indeed, when we increase the flux-creep exponent to 101, this discrepancy vanishes (figure 3).

As expected, the AC losses decrease with decreasing orientation angle [51, 10, 12, 28, 29] (see figure 3). This is because the AC losses in a thin tape are dominated by the
The shoulder at low applied magnetic fields for low angles (curve for $3^\circ$ in figures 3 and 4) is because of the current penetration across the thickness [14]. When the perpendicular component of the applied field is low, the AC losses due to the parallel component become important.

The loss factor relative to the perpendicular applied magnetic field, $Q/B_{\text{per}}^2$, is generally not independent of the angle (figure 4), in contrast to the conclusions of earlier work [10, 12, 28–32]. However, there actually is this angle independence for large and moderate angles ($\geq 30^\circ$). Apart from the shoulder at low amplitudes, for low angles ($\leq 15^\circ$) the peak of $Q/B_{\text{per}}^2$ increases, shifts to lower fields and becomes slightly narrower. If we compare $3^\circ$ with $90^\circ$, the peak for $3^\circ$ is at around three times lower $B_{\text{per}}$, resulting in around three times larger AC losses for $B_{\text{per}}$ close to the peak for $3^\circ$. The difference in AC losses is similar for large $B_{\text{per}}$. This angle dependence of $Q/B_{\text{per}}^2$ is caused by the anisotropy and field dependence of the critical current density.

The results for the isotropic field dependence and the elliptical one reveal the different effects of the anisotropy and the field dependence, as follows.

The increase in the peak in $Q/B_{\text{per}}^2$, its shift to lower fields and its narrowing when decreasing the angle are due to the field dependence because these effects are already present for the isotropic $J_c$ (figure 4(c)). In particular, the shift of the peak of $Q/B_{\text{per}}^2$ to lower $B_{\text{per}}$ with decreasing angle is caused by the following factor. For the same $B_{\text{per}}$, lower angles correspond to higher applied field magnitudes and, consequently, a lower average $J_c$ in the AC cycle. Then, the tape saturates for lower $B_{\text{per}}$ and the loss factor shifts accordingly. The increase in the peak of $Q/B_{\text{per}}^2$ and its narrowing is due to the stronger field dependence of $J_c$ as a function of $B_{\text{per}}$ for lower $B_{\text{per}}$ [52]. Indeed, the field dependence of (11) as a function of $B_{\text{per}}$ is $J_c = J_0/(1 + B_{\text{per}}/B_0|\sin \theta|)^2$, resulting in an effective field constant $B_{\text{eff}} = B_0|\sin \theta|$, which decreases with the angle $\theta$ and, therefore, strengthens the field dependence with decreasing angle.

An elliptical anisotropy weakens the angular dependence of $Q/B_{\text{per}}^2$, with respect to the realistic or isotropic dependence (figure 4(b)). This is because the angular dependence in $J_c$ of (10) becomes $f_\theta(\theta) \approx u_\epsilon |\sin \theta|$ for large enough angles $\theta$, resulting in a $B_{\text{per}}$ dependence independent of the angle, $J_c = J_0/(1 + B_{\text{per}}u_\epsilon/B_0)^2$. The angle range where $J_c$ becomes independent of $B_{\text{per}}$ becomes wider for larger anisotropies, that is larger $u_\epsilon$. This explains the published angle independence of $Q/B_{\text{per}}^2$, as a function of $B_{\text{per}}$ for Bi2223, with a large field anisotropy [51].

The AC losses for the realistic case present the main features of the isotropic case regarding the position, height and width of the peak. This is because the anisotropy of $J_c$ is not very strong. The main difference from the isotropic case is that the angular dependence of $Q/B_{\text{per}}^2$ is smaller, especially for $B_{\text{per}}$ below the peak. This is because for low fields, the realistic $J_c$ approaches an elliptical dependence (figure 2(a)), therefore a weaker angular dependence of $Q/B_{\text{per}}^2$.
3.2. Magnetic field and current distribution in a Roebel cable

The magnetic field distributions for the coupled and uncoupled cases are very different (figure 5). For the coupled case, the superconductor shields the applied magnetic field as much as possible in all the cable volume. For the uncoupled case, the superconductor only shields the volume of each tape individually. This is in accordance with the published results for perpendicular applied magnetic fields [23, 21]. In addition, for the uncoupled case, the superconductor shields the perpendicular component of the applied magnetic field within each stack of tapes but not in the gap between the stacks, where it concentrates. As a result, the angle of the magnetic field in this gap increases. Another consequence is that the magnetic field within each stack is parallel to the tape surfaces. Moreover, the magnetic field in between the tapes of the stack is uniform. Actually, this is always the case when the magnetic field in the horizontal separation between tapes is parallel to their surface [21].

In some cases it is not enough to take only one element in the thickness of the tape (1D approximation), see section 2.1.5. This is the case for the uncoupled situation and low applied magnetic fields. An example is for an applied magnetic field at 7° and 20 mT of amplitude, figure 8, where the AC losses are dominated by the current penetration across the thickness of the tape (figure 10). In contrast, for an applied field of, for example, 15° and 50 mT of amplitude (figure 6), one element in the thickness is sufficient for AC loss and magnetic field calculations. This is because the main AC loss contribution is from the penetration across the width of the tape and the average parallel magnetic field in the tapes thickness is nonzero.

As expected, the current distribution between the coupled and uncoupled cases is very different (figures 6 and 8). For the coupled case, the current penetrates roughly as in a monoblock. With an oblique applied magnetic field, the current penetrates faster from the top left and bottom right corners, figures 6(a) and 8(a). This is consistent with the calculations for one single tape [9] and the field penetration in figure 5(a). For the uncoupled case, the net current in each tape is zero with qualitatively similar current penetration in each tape, except in the tapes at the boundaries of the cable (figures 6(b) and 8(b)).
The average current density $J$ in the thickness of the tapes for both simulation methods agree for both the coupled (a) and uncoupled (b) cases (solid and dashed lines are for the MMEV and FEM results, respectively). The different lines are for tapes 1–4 (in the direction of the arrow) counting from the left in figures 5 and 6. The situation is the same as in those figures.

The only similarity between the coupled and uncoupled cases is that the current distribution is anti-symmetric with respect to the central point of the tape. This is because of the geometry of the cable and the applied magnetic field.

The current distribution for the 1D approximation (figure 7) allows an accurate quantitative comparison between the simulation methods (MMEV and FEM), rather than a qualitative comparison between current distributions from colour maps like figures 6 or the comparison of the AC losses in a logarithmic scale, such as in figures 3, 4, 9 and 10. The simulation methods agree very well, better than for a constant $J_c$ [21]. Moreover, the effect of the smooth $E(J)$ relation for the FEM simulations is not very important for low applied field amplitudes [21]. With increasing amplitude, the local electro-motive force due to the applied magnetic field increases, and so does the local electric field and $J$ for a smooth $E(J)$ relation. This is indirectly seen in the higher AC losses at high amplitudes for the FEM model (figures 9 and 10).

The sheet current density, $K$, at the peak of the AC cycle presents the following features (figure 7). Thanks to the inversion point-symmetry, it is enough to study one half of the cable, for example the leftmost half in figure 5. For both cases, the sharp peaks in figure 7 correspond to the boundary between the regions with sheet current density equal and below its critical value ($K_c = J_c d$). This is because at this boundary $|K| = K_c$ and the magnetic field is minimum (it vanishes for the coupled case, while for the uncoupled case only its perpendicular component vanishes). At the region in between the sharp peaks and the edge of the tape, $|K|$ becomes $K_c$. There, $|K|$ decreases toward the edge because the magnetic field increases. For the FEM simulations, there are small distortions close to the interface between the critical and under-critical regions (corresponding to the position of the sharp peaks). These distortions are not necessarily due to numerical errors. Indeed, a superconductor with a smooth $E(J)$ relation presents a time-retarded response compared to the critical-state model [53]. Then, at the peak of the AC field, the maximum current penetration is still not fully developed.

Figure 7. The average current density $J$ in the thickness of the tapes for both simulation methods agree for both the coupled (a) and uncoupled (b) cases (solid and dashed lines are for the MMEV and FEM results, respectively). The different lines are for tapes 1–4 (in the direction of the arrow) counting from the left in figures 5 and 6. The situation is the same as in those figures.

Figure 8. This cross-section shows that the current distributions for the coupled (a) and uncoupled (b) cases are evidently different (the current distribution is calculated with the MMEV model). The situation is for the peak of the AC cycle for an applied field of $\theta = 7^\circ$ and 20 mT of amplitude. This current distribution calculated with 20 elements in the thickness shows the details of the current penetration across the tapes thickness. For better visualization, the horizontal and vertical axis are not to scale and the represented tape thickness is not to scale with the horizontal separation between tapes.
Figure 9. Loss factor, $B_m$ is the applied field amplitude, for a Roebel cable in the coupled case. Solid lines with symbols are calculated by MMEV and dashed lines with symbols are calculated by FEM. The angle of the applied field amplitude is $90^\circ, 60^\circ, 30^\circ, 15^\circ, 7^\circ, 3^\circ$ and $0^\circ$ in the arrow direction.

For the uncoupled case, the sheet current density around the mid-width of the tapes presents a plateau (figure 7(b)), as for perpendicular applied fields, transport currents and pancake coils [21, 36]. The plateau is due to the difference in the local magnetic field at both surfaces of the tape and its height does not depend on $J_c$, as detailed in [21]. This difference in field appears because of the inhomogeneous field created by the other tapes. Another issue is that the sheet current density close to the lowest $y$ edge of each tape is higher than for the highest $y$ edge. The reason is that in these places the sheet critical current density takes the critical value and the magnetic field concentrates at the top left and bottom right corners of each stack of tapes, decreasing $J_c$. For the coupled case, the maximum sheet current density is higher because the superconductor shields better the magnetic field, decreasing the field and increasing $J_c$. The sheet current density in each layer of tapes is qualitatively different. In the first layer (black line in figure 7(a)), the sheet current density roughly decreases with increasing $y$ because the magnetic field increases (figure 5). In the second layer (blue line in figure 7(a)), there is a double peak in the tape at higher $y$ (a double peak for MMEV and single peak for FEM). This is because the magnetic field vanishes in the small region in between the peaks.

When the simulations take into account the penetration into the thickness of the tape, they show that for the uncoupled case and low angles there is a significant penetration across the thickness (figure 8(b)). Actually, for the situation in figure 6(b), the tapes are completely penetrated with current of both signs, with only a small difference corresponding to the height of the plateau in figure 7(b). For the coupled case, the distance of penetration across the thickness is much smaller than across the width, therefore the description with a sheet current density is correct in terms of AC losses (note that the actual aspect ratio of the tape is 4000). Moreover, the parallel magnetic field keeps the same sign in the thickness of the tape and, therefore, its average in the thickness is representative of the average critical current density.

3.3. AC losses for a Roebel cable

Again, the AC losses calculated from both simulation methods agree with each other (figures 9–11). There is a small discrepancy at high amplitudes or low angles and low amplitudes caused by the different physical models of the superconductor (sharp and smooth $E(J)$ relation for MMEV and FEM, respectively).

The results for the coupled case describe the situation of untransposed tapes. Then, the difference between the AC losses for the coupled and uncoupled cases reveal the maximum possible reduction of AC losses by transposition in a Roebel cable. For large applied magnetic fields or low angles, the AC losses for the uncoupled case are lower than for the coupled one (figures 9 and 10). The decrease by uncoupling the tapes is especially important for low angles, with a decrease up to three orders of magnitude for parallel fields and high amplitudes. For angles below $15^\circ$ this reduction is still substantial for all the amplitudes. At high applied fields and angles of $7^\circ$ or lower the losses decrease by more than one order of magnitude compared to the coupled situation.

The AC losses for the coupled case presents the following features (figure 9). It decreases with decreasing angle until it saturates to the value for $0^\circ$. This decrease with decreasing angle is less pronounced than for a single tape (figure 3). The reason is that the coupled case behaves as a single block [23, 24, 21] and the aspect ratio of the Roebel cable
is relatively small, around 5. Then, the difference in the penetration length across the thickness and the width of the cable is less important than for the tape, resulting in a less pronounced angular dependence.

For the uncoupled case (figure 10), the AC losses are qualitatively similar to a single tape (figure 3). However, there are the following differences. First, the peak in the loss factor is at higher amplitudes because of the stacking effect [23, 21, 24]. Second, the peak (or peak at higher amplitudes) is wider than for a single tape because of the concentration of the perpendicular field in the gap between the stacks (figure 5) [23, 21]. Finally, the peak at low applied fields and angles, due to the parallel applied field, becomes more visible. This is for two reasons. First, the peak due to the perpendicular applied field shifts to higher amplitudes, so the two peaks do not overlap. Second, the contribution from the parallel field becomes larger than that of the perpendicular one. The reason is that the stacking effect reduces the AC losses per volume created by the perpendicular field but the losses created by the parallel field remain the same. For 0°, we can compare the simulations with the slab Bean model for a constant \( J_c \) [54, 55] taking \( J_c \) as the self-field critical current of the tape divided by its volume. The simulations are consistent with the Bean model calculations (figure 10). The higher and narrower peak of the loss factor for the simulations is due to the field dependence of \( J_c \) [52]. Then, the Bean model is useful to determine whether the AC losses due to the parallel field are important, assessing the validity of the 1D approximation at low angles without the need of lengthy simulations with several elements in the thickness.

The loss factor relative to the perpendicular applied field \( Q / B_{per}^2 \) (figure 11) does not depend only on \( B_{per} \), as is the case for a single tape (figure 3). With decreasing angle, the shift in the peak and its increase (or the peak at high amplitudes for low angles) is less pronounced than for the single tape. However, for low \( B_{per} \), the increase of the loss factor due to the parallel component of the applied field is more important.

In real Roebel cables, the angular dependence of \( Q / B_{per}^2 \) could be larger. This is because there can be misalignment between the tapes, they can be slightly deformed (for example making a circular arch) or the superconductor layer could have a non-negligible roughness—see for example pictures in [27]. All these effects will result in larger contributions of the parallel applied field. Although the formation of a critical state is questionable for ideal thin superconducting films in a parallel applied field [56], the imperfections in the superconducting layer will originate a critical state in a similar way as in thin films in a perpendicular applied field. In addition, coupling currents will also increase the AC losses for a parallel applied field.

The AC losses for low angles will be important for long solenoids, as in certain transformers. Therefore, we recommend to characterize the AC losses in Roebel cables and tapes also at low angles of the applied field, thereby avoiding the error committed extrapolating \( Q / B_{per}^2 \) for low angles assuming an angular independence of \( Q / B_{per}^2 \).

4. Summary and conclusions

In this paper, we have presented the main features of the magnetic field, current distribution and AC losses for an applied magnetic field with arbitrary orientation, based on two independent numerical simulation methods. For the Roebel cable, we have taken into account the coupled and uncoupled cases, corresponding to the limits of very high coupling currents and negligible coupling currents, respectively. The simulations for the uncoupled case are also valid for a double pancake coil with no transport current, therefore the results and conclusions for the Roebel cable in the uncoupled case are also applicable to double pancake coils. The simulations have taken into account a realistic anisotropic field dependence of \( J_c \). This dependence is in accordance with \( J_c \) measurements within around 20% error. Therefore, the results from the simulations are representative for real tapes, in contrast to those from simple dependences, such as the elliptical or isotropic ones.

Concerning the two simulation methods in this paper, we have seen that all the results agree with each other, serving as a mutual check. In addition, the qualitative results for the field and current distributions are well explained by magnetostatic considerations.

For both single tapes and Roebel cables in the uncoupled case, we found that the AC losses do not only depend on the perpendicular component of the applied field, contrary to the published works [10, 12, 28–32]. For low angles between the applied field and the tapes plane and moderate and large amplitudes, the extrapolated AC losses from purely perpendicular applied fields, assuming independence of the parallel component of the applied field, are around three times lower than the actual ones, therefore this difference is not

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**Figure 11.** The normalized loss factor relative to the perpendicular applied field amplitude, \( Q / B_{per}^2 \), for the uncoupled case does not only depend on \( B_{per} \) only. Solid lines with symbols are calculated by MMVE and dashed lines with symbols are calculated by FEM. The angle of the applied field amplitude is 90°, 60°, 30°, 15°, 7° and 3° from top to bottom.
negligible. The discrepancy is even larger for low amplitudes. The AC losses also depend on the parallel component of the applied field for two reasons. First, at low applied fields and low angles, the AC losses due to the penetration across the thickness are important; second, because for YBCO the field anisotropy in \( J_c \) is relatively weak. Then, \( J_c \) reduces significantly with increasing parallel component of the applied field for the same perpendicular component of the applied field.

For coupled Roebel cables, the AC losses due to the parallel and perpendicular components of the applied field are of the same order of magnitude for all the amplitudes. This is because of the low aspect ratio of the cable, around 5.

Finally, we have found that the highest potential of Roebel cables is to reduce the AC losses at applied fields with low angles with the tape surface. For angles of \( 7^\circ \) or lower, the Roebel cable reduces the AC losses by more than one order of magnitude, as compared to a cable made of untransposed tapes with the same dimensions. The reduction can be of up to three orders of magnitude for perfectly parallel applied fields. This is a much better reduction than the factor of 2 for perpendicular applied fields [20, 21].

In conclusion, transposed cables, such as Roebel cables, should be used in windings with an important parallel applied field, such as the low-voltage winding of transformers. For the characterization of the AC losses of these cables, it is necessary to measure also at low angles with the tape surface (\( \lesssim 15^\circ \)). This consideration is also valid for double (or single) pancake coils to be part of a long solenoid, such as the high-voltage winding of a transformer. As future work, we propose comparison with experiments, the prediction of the influence of intermediate coupling currents and the role of striated tapes in the Roebel cable.

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