Varying Couplings From Orbifold GUTs

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Abstract

We discuss the variation of gauge couplings in time in the framework of orbifold constructions, due to a change of the extra compact dimension’s size. Models with gauge coupling unification allow to estimate the variation of the strong coupling constant $\alpha_3$ and to relate it to a variation of $\alpha_{em}$. The extra-dimensional construction turns out to be crucial for the model to be compatible with data. Within the presented 5D scenarios, the tower of KK states significantly affects gauge coupling running, leads to low scale unification, and provides a suppression of the $\alpha_3(M_Z)$ variation.
1 Introduction

The recent measurements [1] of quasar absorption lines hint to a variation of the fine structure constant (at redshift range $0.2 < z < 3.7$) with

$$\frac{\delta \alpha}{\alpha} = \frac{\alpha_0 - \alpha}{\alpha} = (-0.57 \pm 0.10) \cdot 10^{-5},$$

where $\alpha_0$ is the value of the fine structure constant at late cosmological times ($z = 3.5$), while $\alpha$ corresponds to the value at present time. These data renewed an interest in the old issue of the possible variation of the nature 'constants' [2], [3], triggered the construction of models [4]-[9], which suggest theoretical frameworks for this phenomenon and have reactivated discussions [10]. The various implications of such models are interesting and deserve detailed studies.

The issue of the variation of the fine structure constant got a new insight within GUTs [5]-[9]. Since above the unification scale the gauge sector is described by one unified gauge coupling, the variation of $\alpha$ also directly determines the time dependence of the QCD coupling $\alpha_3$. This would affect standard big bang nucleosynthesis and high-redshift quasar absorption lines much more than only a variation of the fine structure constant. In the realistic versions of GUT the $\alpha_3$ variation contradicts [9] the available data and only some specific extension may save the situation. This issue can be considered as a new selection rule in building realistic GUTs, even without having a dynamical mechanism for the variation of couplings. Of course, it is also desirable to have a theoretical understanding of this variation. Within superstring and/or higher dimensional constructions the four dimensional couplings often depend on VEVs of some scalar fields coming from higher dimensions. It is then natural to have time dependent couplings, because on the 4D level they are related to some dynamical fields [4].

In this paper we consider 5D SUSY scenarios compactified on an orbifold. Orbifold constructions have recently gained much attention as they present a framework where typical problems of 4D GUTs, such as the Doublet-Triplet splitting problem and strong baryon number violation, are easily avoided [11]-[13]. The GUT symmetry breaking also can be realized in a rather economical way. Interestingly enough, in this setting the source of coupling variation can be the change of the size $R$ of the compact fifth dimension. This framework in addition to a variation of $\alpha$ can also give a variation of $\alpha_3$ (and of other couplings which are expressed by 5D parameters and $R$) without gauge coupling unification. Having a unification condition at a certain scale (interpreted as the GUT scale $M_G$), allows one to relate $\frac{\delta \alpha_3}{\alpha_3}$ to $\frac{\delta \alpha}{\alpha}$ at low energies. The value of $\frac{\delta \alpha_3(M_Z)}{\alpha_3(M_Z)}$ depends on the intermediate thresholds contributing between the scales $M_Z$ and $M_G$. As we will see, with a compactification scale $\mu_0 = \frac{1}{R} \simeq M_G$, the picture does not differ from the one of usual 4D GUTs, having similar implications. However, with $\mu_0 \ll M_G$, the KK states will
participate in renormalization. The latter turn out to play an essential role for \( \frac{\delta \alpha_3(M_Z)}{\alpha_3(M_Z)} \) suppression compared to 4D GUT scenarios (a different possibility for the suppression of \( \frac{\delta \alpha_3}{\alpha_3} \) was discussed in the 1st ref. of [7]). The suppression becomes stronger when many KK states appear below \( M_G \), making the scenario compatible with observations without affecting significantly the predictions of nucleosynthesis. Let us note that a large number of KK states, as was pointed out in [14], can be crucial also in lowering the unification scale down to a few TeV. This was confirmed with explicit models in [15], [16], [13]. A low unification scale makes a model testable in the next generation of high energy experiments [17].

2 Some Bounds from Experimental Data

Several authors have studied the possibility that the variation in the fine structure constant \( \alpha \) is the consequence of the (more fundamental) variation of the unified coupling \( \alpha_G \) either at a fixed GUT scale \( M_G \) [5] or at a varying one [8, 9]. In view of this, it is clear that the variation in \( \alpha \) should be accompanied by variations in the other gauge couplings, but possibly also by variations in Yukawa couplings and mass scales such as the SUSY and electroweak scales. As we will see, it turns out that the knowledge of the mechanism which is behind the variation of the fundamental constants is at least as important as the knowledge of the model of unification.

The variations of various observables are constrained by observational data and if we trust these data one can check various scenarios. It is beyond the scope of this paper to consider precise mechanisms of (N=1) SUSY breaking, EW symmetry breaking nor do we precise the way \( M_{Pl} \) varies. We will therefore be forced to introduce several dimensionless quantities (\( \rho, \rho_a, \bar{\rho}_e, \ldots \)) to parameterize our ignorance. In fact, in the kind of models we consider here, these make only sub-leading (but non-negligible) contributions to the variations of the observables we are interested in.

A variation in the strong coupling implies a variation in the QCD scale \( \Lambda_c \). The latter can be roughly expressed through \( M_Z \) as

\[
\Lambda_c = M_Z \exp \left( -\frac{2\pi}{9\alpha_3(M_Z)} \right),
\]

where thresholds are neglected. Therefore,

\[
\frac{\delta \Lambda_c}{\Lambda_c} = \left( \frac{2\pi}{9} \alpha_3^{-1} + \bar{\rho}_e \right) \frac{\delta \alpha_3}{\alpha_3},
\]

The variation of a mass scale \( m \) is naturally to be understood as measured in units of some non-varying mass scale \( M \), i.e. \( \delta m = \delta s \cdot M \) where \( s = m/M \). In the next section we will choose \( M = M_G \).
where $\rho_c$ includes possible threshold contributions in the range $M_Z - \Lambda_c$ and is expected to have a sub-leading effect. The resulting variation in the masses of the hadrons is constrained by some cosmological observables such as the abundance of elements [18] and the high redshift quasar absorption lines [19, 20].

The $^4$He abundance $Y_4$ is expressed as $Y_4 = \frac{2n_n/n_p}{1+n_n/n_p}$, where $n_n/n_p \simeq 0.8 \cdot e^{-Q/T_D} (\simeq 1/7)$ is the ratio of neutron to proton density at the time of nucleosynthesis, with a decoupling temperature $T_D \simeq 0.8$ MeV and mass difference $Q = m_n - m_p = B + C\alpha\Lambda_c \simeq 1.29$ MeV ($B = 2.05$ MeV, $C \sim -1$). $T_D$ is determined by the expansion rate, $T_D \sim (M_{Pl}G_F^2)^{-1/3}$. Having a variation of couplings/scales, $M_{Pl}$ and $G_F$ can also vary (as we already pointed out, one mass scale, not necessarily $M_{Pl}$, can be fixed as a reference scale since only the variation of dimensionless quantities has a physical meaning). Without detailed knowledge of the variation law one can parameterize our ignorance with $\frac{\delta(M_{Pl}G_F^2)}{M_{Pl}G_F^2} \equiv \rho$ and therefore $\frac{\delta T_D}{T_D} = -\rho \frac{\delta \alpha}{\alpha}$. Taking into account all this, one obtains

$$\frac{\delta Y_4}{Y_4} = \frac{B - Q}{T_D} \frac{1}{1 + n_n/n_p} \left( \frac{\delta \alpha}{\alpha} + \frac{\delta \Lambda_c}{\Lambda_c} + \frac{Q}{B - Q} \frac{\delta T_D}{T_D} \right) \simeq 0.8(1 + R - 0.6\rho) \frac{\delta \alpha}{\alpha},$$

(4)

where we defined

$$\frac{\delta \Lambda_c}{\Lambda_c} = R \frac{\delta \alpha}{\alpha}. \quad (5)$$

The value of $\frac{\delta Y_4}{Y_4}$, at times corresponding to $z \sim 10^{10}$, is constrained [18]

$$\frac{\delta Y_4}{Y_4} = (-5.6 \pm 7.2) \cdot 10^{-3}. \quad (6)$$

If a time independent $\frac{\dot{Y}_4}{Y_4}$ is assumed, then using in (4) the value (1) (which corresponds to $z = 0.5 - 3.5$) we can verify that (6) is indeed satisfied (unless $|R - 0.6\rho| \gtrsim 10^3$, which is never realized as we will see below). The situation can be drastically changed for a time dependent $\frac{\dot{Y}_4}{Y_4}$, since integration in a range of large redshifts can cause a strong enhancement in $\frac{\delta Y_4}{Y_4}$. However, since the present data doesn’t have any information about the law by which $\frac{\dot{Y}_4}{Y_4}$ may vary, we are unable to judge this situation.

Other observables that one can use to constrain any model are obtained from high precision observations of quasar absorption lines [19]:

$$X \equiv \alpha^2 g_p \frac{m_e}{m_p}, \quad \text{with} \quad \frac{\delta X}{X} = (0.7 \pm 1.1) \times 10^{-5} \quad (z = 1.8),$$

(7)

As it turns out, for the considered scenarios $\rho$ will not play a significant role. This allows one to carry out the analysis without knowledge of $\rho$. 

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and measuring the wavelengths of molecular hydrogen transitions in the early universe [20]

\[ Y \equiv \frac{m_p}{m_e} \quad \text{with} \quad \frac{\delta Y}{Y} = (3.0 \pm 2.4) \times 10^{-5} \quad (z = 3). \]  

(8)

Using

\[ \frac{\delta X}{X} = 2 \frac{\delta \alpha}{\alpha} - \frac{\delta Y}{Y}, \]  

(9)

(where we neglect the variation of \( g_p^{6} \)) one sees that at 1σ level there is an inconsistency between the data (7), (8), and the measured variation (1) of the fine structure constant. However, taking data at 2σ level (in the spirit of ref.[9]), from (9) it follows that the measurements are consistent with each other. Because of this, to derive further bounds on \( \mathcal{R} \), the corresponding data at 2σ level is used. Taking into account \( m_p \sim \Lambda_c \) and eq.(5), we have \( \frac{\delta X}{X} = (2 - \mathcal{R} + \rho_e) \frac{\delta \alpha}{\alpha} \) and \( \frac{\delta Y}{Y} = (\mathcal{R} - \rho_e) \frac{\delta \alpha}{\alpha} \), where \( \rho_e \) is introduced to parameterize the variation of \( m_e \), \( \frac{\delta m_e}{m_e} = \rho_e \frac{\delta \alpha}{\alpha} \). Using eqs.(1), (7) and (8) - all at 2σ level - we finally obtain

\[ -2.1 + \rho_e \lesssim \mathcal{R} \lesssim 4.9 + \rho_e. \]  

(10)

To check whether this limit is satisfied for a given model or not, we need to know \( \mathcal{R} \) (and \( \rho_e \)).

3 The Framework

Consider 5D SUSY models compactified on a circle of radius \( R \) which is allowed to vary. This is the source of time dependent couplings. We will discuss scenarios with gauge coupling unification at \( M_G \), not much below the 5D fundamental mass scale \( \Lambda_5 \). It is natural to assume that the dimensionfull 5D gauge coupling \( \alpha_5^{-1} \) is close to the fundamental scale. With \( R \gg M_5^{-1} \) (which is indeed satisfied in the models considered below) the 4D unified coupling \( \alpha_G \) at the \( M_G \) scale (and its vicinity) has the following \( R \)-dependence:

\[ \alpha_G^{-1}(M_G) \approx 2\pi R \alpha_5^{-1}. \]  

(11)

Apart from the change in \( \alpha_G^{-1} \) we have a variation in the masses of the Kaluza-Klein states which, as we will see below, can contribute significantly (at least for \( RM_G \gg 1 \)) to the variation of gauge couplings at low energies. To estimate this effect we should renormalize couplings from high energies down to low scales.

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6See however discussions in [6].

7This bound is obtained from the unweighted data (eq.(8)) of the second reference in [20]. With the weighted data we would need to go to 3σ to satisfy (9). In that case the range for \( (\mathcal{R} - \rho_e) \) would be shifted.
The gauge coupling evolution from a high scale \( \mu \) down to the weak scale in one loop approximation reads:

\[
\alpha_a^{-1}(M_Z) = \alpha_a^{-1}(\mu) + \frac{b_a}{2\pi} \ln \frac{\mu}{M_Z} + P_a + \Delta_a, \tag{12}
\]

with \( P_a \) the contribution of the KK states (if \( \mu > \mu_0 = 1/R \)) and \( \Delta_a \) the contribution of other possible (logarithmic) thresholds. Having the unification condition \( \alpha_a(M_G) = \alpha_G(M_G) \) at \( M_G \), from the three equations (12) we get

\[
\alpha_3^{-1} = \frac{12}{7} \alpha_2^{-1} - \frac{5}{7} \alpha_1^{-1} + (P_3 - \frac{12}{7} P_2 + \frac{5}{7} P_1) + (\Delta_3 - \frac{12}{7} \Delta_2 + \frac{5}{7} \Delta_1), \tag{13}
\]

\[
\ln \frac{M_G}{M_Z} = \frac{5\pi}{14} (\alpha_1^{-1} - \alpha_2^{-1}) - \frac{5\pi}{14} (P_1 - P_2) - \frac{5\pi}{14} (\Delta_1 - \Delta_2), \tag{14}
\]

\[
\alpha_2^{-1} = \alpha_1^{-1} - \frac{1}{2\pi} \ln \frac{M_G}{M_Z} - P_2 - \Delta_2, \tag{15}
\]

where we have taken into account that \((b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)\). Expressions (13)-(15) will be useful for estimating the unification picture within specific models.

In the following, we will consider two types of compactification, which give different mass spectra for KK states. With compactification on an \( S^1/Z_2 \) orbifold, all states have definite \( Z_2 \) parity \( P = \pm 1 \) under \( y \to -y \) (fifth space-like dimension) and the masses of KK states are \( \frac{n}{R} \), where \( n \) denotes the quantum number in the KK mode expansion. In this case

\[
P_a = \frac{\hat{b}_a}{2\pi} \sum_{n=1}^{N_0} \ln \frac{M_G R}{n}, \tag{16}
\]

where \( N_0 \) is the truncation number of the KK tower\(^8\) \((N_0 = \lceil \frac{M_G^2}{\mu_0} \rceil)\), and \( \hat{b}_a \) is a model-dependent group theoretical factor.

With compactification on an \( S^1/Z_2 \times Z'_2 \) orbifold the bulk states have parities \((\pm 1, \pm 1)\) and \((\pm 1, \mp 1)\) with masses \( \frac{2n+2}{R} \) and \( \frac{2n+1}{R} \), respectively, for the corresponding KK states. In this case we have

\[
P_a = \frac{\gamma_a}{2\pi} \sum_{n=0}^{N} \ln \frac{M_G R}{2n+2} + \frac{\delta_a}{2\pi} \sum_{n=0}^{N'} \ln \frac{M_G R}{2n+1}, \tag{17}
\]

\((N = \lceil \frac{M_G}{2\mu_0} \rceil - 1), \quad N' = \lceil \frac{M_G}{2\mu_0} - \frac{1}{2} \rceil)\), where \( \gamma_a \) and \( \delta_a \) are group-theoretical factors corresponding to states with parities \((\pm 1, \pm 1)\) and \((\pm 1, \mp 1)\) respectively.

\(^8\)See ref.[13] for more details.
Now we investigate the effects of a variation $\delta R$ of $R$, the size of the 5th dimension. Since only the variation of the ratio of mass scales has physical meaning, for convenience we will treat $M_G$ as a fixed reference scale. Then other scales will be time dependent. If the number of KK states is few orders of magnitude larger than the number of the other thresholds, then the latter will have sub-leading effect. This statement gets more support if all matter and MSSM higgs doublets live on a brane and have $R$ independent couplings. We discuss this at the end of sect. 4.1. Taking all this into account, from eqs.(11) and (12) we find

$$\alpha^{-1} \frac{\delta \alpha_a}{\alpha_a} = - A_a \frac{\delta R}{R},$$

where $\alpha_a$ are the couplings at $M_Z$ and

$$A_a = A_a^0 + \rho_a,$$

$$A_a^0 \equiv \alpha_G^{-1} + \frac{1}{2\pi} \left\{ \hat{b}_a N_0 \gamma_a (N + 1) + \delta_a (N' + 1) \right\} \text{ for } S^1/Z_2 \text{ orbifold},$$

$$A_a^0 \equiv \alpha_G^{-1} + \frac{1}{2\pi} \left\{ \hat{b}_a N_0 \gamma_a (N + 1) + \delta_a (N' + 1) \right\} \text{ for } S^1/Z_2 \times Z'_2 \text{ orbifold},$$

We have assumed that $\alpha_5$ does not vary in time, i.e. the time dependence in $\alpha_G$ is caused only by the variation of $R$. In (19) $\rho_a$ denote the effects of the variation of the thresholds $\Delta_a$ in the $M_G - M_Z$ range. These include the effects of the variations in SUSY and EW scales. They depend on the origin of SUSY and EW symmetry breaking and the mechanism of generation of fermion masses. Without specifying these, it is still possible to demonstrate the effects caused by the KK states.

In addition to the three equations (18) we have the relation $\alpha^{-1}(M_Z) = (5/3)\alpha_1^{-1}(M_Z) + \alpha_2^{-1}(M_Z)$. This allows us to eliminate $\frac{\delta R}{R}$ and to express $\frac{\delta \alpha_3}{\alpha_3}$ through $\frac{\delta \alpha}{\alpha}$:

$$\frac{\delta \alpha_3}{\alpha_3} = \frac{\alpha}{\alpha} \frac{3A_3}{5A_1 + 3A_2} \frac{\delta \alpha}{\alpha}. \quad (21)$$

On the r.h.s. of this equation $\alpha_3$ is taken at $M_Z$, while $\alpha$ at the electron mass. The reason for the latter is that $\frac{\delta \alpha}{\alpha}$ is measured at low scales and the combination $\alpha^{-1} \frac{\delta \alpha_3}{\alpha_3}$ is scale invariant in the leading order approximation. Using (3) and (19)-(21) we can express $R$ as

$$R \approx \frac{2\pi}{9\alpha} \frac{3A_3^0}{5A_1^0 + 3A_2^0} \left( 1 + \frac{9\alpha_3}{2\pi} \frac{\overline{\rho}_c}{A_3^0} + \frac{\rho_3}{A_3^0} - \frac{5\rho_1 + 3\rho_2}{5A_1^0 + 3A_2^0} \right), \quad (22)$$

where we have assumed that $\overline{\rho}_c$, $\rho_a$ have sub-leading effects. For a given model one can estimate unification by using eqs.(12), calculate all $A_a$ factors, and then determine $R$ from (22). In the case that all KK states lie above $M_G$, they are irrelevant for the running and we have $A_a^0 = \alpha_G^{-1} \approx 24$. Therefore,

$$R_{4D \text{ GUT}} \approx 36 \left( 1 + 0.17 \overline{\rho}_c + 0.04(\rho_3 - \frac{5}{8}\rho_1 - \frac{3}{8}\rho_2) \right), \quad (23)$$
and we recover the result for 4D GUTs [5]-[9] (there was not presented expression including \(\rho_c, \rho_a\) in these papers). The value of \(\mathcal{R}\) in (23) should be compared with the upper bound of (10). In order to satisfy this bound, either \(\rho_c\) must equal to \(-5.39\) with 1% accuracy or \(\rho_3 - \frac{5}{7}\rho_1 - \frac{3}{7}\rho_2 = -22.9\) with the same accuracy (these are nothing but fine tunings with 1% accuracy). It is hard to imagine having such cancellations. Because of this, the 4D GUTs turn out to be inconsistent [9] with data.

To compare the magnitude of the variation of the strong coupling in KK models with the one in the 4D GUTs, we introduce the quantity

\[
\kappa \equiv \frac{8A_3^0}{5A_1^0 + 3A_2^0},
\]

which measures the suppression of \(\frac{\delta\alpha_3}{\alpha_3}\) (see (21)) compared to the one in 4D GUTs. It is clear that in the absence of KK states we obtain \(\kappa \sim 1\). Once more we emphasize that this is usually the case for realistic 4D GUTs [9]. Such a \(\kappa\) is unsatisfactory as it is too large when compared with the constraint which (taking into account (22)) follows from eq.(10)

\[
-0.06 \lesssim \kappa \left( 1 + 0.17 \, \rho_c + \frac{\rho_3}{A_3^0} - \frac{5\rho_1 + 3\rho_2}{5A_1^0 + 3A_2^0} \right) \lesssim 0.15.
\]

Inspection of eqs.(20) and (24) shows that the presence of a large number of KK modes may significantly change this situation, hopefully by suppressing the variation of the strong coupling. As we will show now, this is indeed the case for low scale unification models. The point is that for large \(N_0\), in order to maintain the successful prediction for \(\alpha_3(M_Z)\), we need

\[
\hat{b}_3 = \frac{12}{7} \hat{b}_2 - \frac{5}{7} \hat{b}_1.
\]

(For the \(S^1/Z_2 \times Z_2'\) orbifold we must replace \(\hat{b}_a \rightarrow \gamma_a + \delta_a\) and \(N_0 \rightarrow N + N' + 2\). We then have \(\alpha_3^{-1} \approx (12/7)\alpha_2^{-1} - (5/7)\alpha_1^{-1} = 1/0.116\). On the other hand, since for \(N_0 = 0\) the GUT scale \(M_G\) is given approximately by \(\ln \left( M_G/M_Z \right) \approx (5\pi/14)(\alpha_1^{-1} - \alpha_2^{-1}) \approx 33\) we have low scale unification if and only if

\[
\hat{b}_2 < \hat{b}_1.
\]

Combining the last two equations we obtain

\[
\hat{b}_3 < \frac{5}{8} \hat{b}_1 + \frac{3}{8} \hat{b}_2,
\]

which, in view of the definition of \(\kappa\), is nothing but the condition that \(\kappa < 1\). Further insight can be gained by using (12) to rewrite \(\kappa\) as

\[
\kappa = \frac{8 \alpha_3^{-1}(M_Z) + \alpha_1}{3 \alpha^{-1}(M_Z) + \alpha_2},
\]

inspecting
where

\[ o_1 \approx \frac{3}{2\pi} \ln \frac{M_G}{M_Z} - \Delta_3 + \frac{\hat{b}_3}{4\pi} \ln (M_G R) \] (30)

\[ o_2 \approx -\frac{6}{\pi} \ln \frac{M_G}{M_Z} - \frac{5}{3} \Delta_1 - \Delta_2 + \frac{5\hat{b}_1 + 3\hat{b}_2}{12\pi} \ln (M_G R) \] (31)

are of order \( O(1) \) in the case of low scale unification. While \( o_2 \) is much smaller than \( \alpha^{-1} \), \( o_1 \) is still \( \sim \log_{10} (M_G/M_Z) \) and therefore

\[ \kappa \approx \frac{8}{3} \frac{\alpha_3^{-1}(M_Z)}{\alpha^{-1}(M_Z)} (1 + o) = 0.16(1 + o), \] (32)

(for \( \alpha_3(M_Z) = 0.12 \)) where the model dependent quantity \( o \) is usually smaller than one as we will see in the next section. From (32) we already see that (25) can be easily satisfied with \( \kappa \) being near to the upper bound. Therefore, a large number of KK states can suppress the relative variation of \( \alpha_3 \).

Finally, we want to emphasize that the introduction of more extra dimensions doesn’t necessarily lead to a stronger suppression of the variation of \( \alpha_3 \). At least in the simple case, when the extra compact dimensions form a torus of trivial shape and equal size radii, the power law function \( P_a/\hat{b}_a \) should be replaced by \( (P_a/\hat{b}_a)^{\delta} \) (\( \delta = \) number of extra dimensions). Limitations from scales require the \( P_a/\hat{b}_a \) to have nearly the same value as \( P_a \) in the case of \( \delta = 1 \). Therefore, the relation between low scale unification and a suppressed \( \delta\alpha_3 \), as well as eq.(32), remain unchanged. However, it is not excluded that with more elaborated compactifications a different behavior emerges.

4 The Models

In this section we present detailed models of low scale unification and confirm the conclusions of the previous section.

4.1 Low scale unified 5D SUSY \( G_{321} \) models

Consider the 5D SUSY \( G_{321} \) model on an \( S^{(1)}/Z_2 \) orbifold. The latter is crucial in order to have 4D \( N = 1 \) SUSY at the orbifold fixed point (identified with our 4D world). As it will turn out, suppression of \( \delta\alpha_3/\alpha_3 \) on the \( M_Z \) scale occurs with a large number of KK states below the GUT scale \( M_G \). Because of this, we consider specific extensions [15], [13] which naturally lead to low scale unification.

a) In addition to gauge fields and \( \eta \) matter families in the bulk, we introduce the bulk states \( E_{N=2}^{(i)} = (E, \overline{E})^{(i)} (i = 1, 2) \), where the \( E^{(i)}, \overline{E}^{(i)} \) are \( SU(3)_c, SU(2)_L \) singlets with
$U(1)_Y$ hypercharges 6, −6, resp., in $1/\sqrt{60}$ units. With orbifold parities for fragments $(E^{(1)}, \overline{E}^{(2)}) \sim +, (E^{(2)}, \overline{E}^{(1)}) \sim −$, only $E^{(1)}, \overline{E}^{(2)}$ states will have zero modes with some 4D mass $M_E$. With this setting, using (13)-(15) we get

$$
\alpha_3^{-1} = \frac{12}{\ell} \alpha_2^{-1} - \frac{5}{\ell} \alpha_1^{-1} + \frac{3}{\ell\pi} \ln \frac{M_G}{M_E},
$$

$$
\ln \frac{M_G}{M_Z} = \frac{5\pi}{14} (\alpha_1^{-1} - \alpha_2^{-1}) - \frac{3}{14} \ln \frac{M_G}{M_E} - S,
$$

$$
\alpha_1^{-1} = \frac{\ln \frac{M_G}{M_Z}}{2\pi} + \frac{1 - 2\eta}{\pi} S,
$$

where we have taken into account that $(b_1, b_2, b_3)^E = (\frac{6}{5}, 0, 0)$, $(\hat{b}_1, \hat{b}_2, \hat{b}_3) = (\frac{18}{5}, -2, -6) + 4\eta(1, 1, 1)$. Using (24), (33), by straightforward calculations one can verify that with increase of $N_0$ the value of $\kappa$ decreases. However, the increase of $N_0$ is limited ($\lesssim 30$) from (33) in order to have $M_G \gtrsim 1$ TeV. From (33) we see that we can have unification in the range $10$ TeV-$10^6$ GeV varying $N_0$ between 0 and 30 and eqs. (33) do not give a preferable choice for $N_0$. However, if we want $\kappa$ suppressed, this dictates large $N_0 (= 30)$ and therefore low scale unification. More precisely, for $N_0 = 30$, $M_E \simeq M_G$, $\mu_0 = 316$ GeV, $\eta = 0$, we have $M_G \simeq 10$ TeV and $\kappa \simeq 0.2$. Since the contributions of $\overline{P}_e$, $\rho_a$ can provide some partial cancellations (which have not to occur with a high accuracy), this value of $\kappa$ can easily satisfy the bound (25).

b) A similar effect can be obtained by extending the 5D $G_{321}$ model with bulk states $U_e^{(i)}_{N=2} = (U^c, \overline{U}^c)^{(i)}$, $L^{(i)}_{N=2} = (L, \overline{L})^{(i)}$ ($i = 1, 2$), where $U^c$ and $L$ have $G_{321}$ quantum numbers $(\overline{3}, 1, 4)$ and $(1, 2, -3)$ resp. This extension also allows for low scale unification. With orbifold parity prescriptions $(U^{c(1)}, \overline{U}^{c(2)}, L^{(1)}, \overline{L}^{(2)}) \sim +, (U^{c(2)}, \overline{U}^{c(1)}, L^{(2)}, \overline{L}^{(1)}) \sim −$ we will have now $(b_1, b_2, b_3) = (\frac{26}{5}, 0, -4) + 4\eta(1, 1, 1)$. Small values of $\kappa$ are still realized for large values of $N_0$. Namely, for zero mode masses $\sim M_G$ of additional vector-like states and for $N_0 = 30$, $\eta = 0$, $\mu_0 \simeq 316$ GeV unification holds at $M_G \simeq 10$ TeV. In this case we obtain $\kappa \simeq 0.21$.

Let us note that within scenarios a) and b) it is possible to introduce a pair of MSSM Higgs doublets on the brane (without KK excitations) and make an additional extension by introducing a vector like pair of $N = 2$ SUSY doublet supermultiplets in the bulk with masses for zero modes near $M_G$. In this case, the KK spectra will be precisely the same, RG analysis will not be altered and we still have $\kappa \simeq 0.2$. However, since in this case all matter and MSSM higgses have only brane couplings, their masses and non gauge couplings will not depend on $R$ at tree level and therefore are time independent at the leading order. Because of this, the assumptions made for our estimates become selfconsistent.
4.2 Low scale unified 5D SUSY $SU(4)_c \times SU(2)_L \times SU(2)_R$ model

The other model we will present here has as its GUT symmetry the Pati-Salam gauge group $SU(4)_c \times SU(2)_L \times SU(2)_R$ [21]. By compactifying the 5th dimension on $S^1/Z_2 \times Z'_2$ in a suitable way one obtains a 4D SUSY model (on a fixed point) with $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)'$ gauge symmetry. Obviously, this is still not the symmetry of the standard model and we will therefore introduce a symmetry breaking $SU(2)_R \times U(1)' \rightarrow U(1)_Y$ by the Higgs mechanism at an intermediate scale. The field content consists of the minimal set of gauge and matter supermultiplets one needs to introduce to obtain the MSSM fields at low energies, a set of Higgs supermultiplets to break $SU(2)_R \times U(1)'$ at the intermediate scale, and a set of four 5D supermultiplets $R^r$, doublets of $SU(2)_R$. This model (called III'-susy422 in ref.[13] - see this ref. for more details) allows low scale unification with $M_G \gtrsim 10^{5.8}$ GeV.

Using the expressions given in [13] for the gauge couplings $\alpha_a(M_G)$ (eqs. (7.64)-(7.66) of that reference) it is straightforward to calculate $\kappa$

$$\kappa \simeq \frac{24 - \frac{3}{2\pi} \left( N + 1 \right)}{24 + \frac{9}{4\pi} \left( N + 1 \right)}.$$  \hfill (34)

Therefore, small $\kappa$ favors large $N$ and therefore low scale unification. The phenomenological bounds [22] on the value of the intermediate scale imply $N \leq 29$. For $N = 29$ we obtain $\kappa \simeq 0.2$ - a value which easily satisfies the bounds, as pointed out in the previous subsection.

5 Conclusions

We have shown that the extra-dimensional constructions provide not only the source for a variation of couplings but also offer possibilities of building GUTs with phenomenological implications compatible with data. We have demonstrated this in concrete 5D SUSY scenarios, which give unification in multi TeV scales. A large number of KK states below the GUT scale, plays an important role for the suppression of $\frac{\Delta \alpha_3(M_Z)}{\alpha_3(M_Z)}$ as well as for low scale unification.

The change in the extra dimension’s size is related to the radion dynamical field, which should have a mass close to the Hubble parameter $\sim 10^{-33}$ eV (at $z = 0.5 - 3.5$ redshift) in order to be relevant for a variation of gauge couplings [4]. In this way the radion can appear as a quintessence field. All this crucially depends on the radion dynamics and particularly on a mechanism through which the radion gets stabilized, but still having small oscillations near its minimum. Detailed studies of concrete examples are highly desirable as they allow to discuss other possible cosmological and astrophysical implications [23].
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