Nonlinear straining of shallow and non-shallow spherical shells under thermal and force loadings

M S Ganeeva, V E Moiseeva and Z V Skvortsova

Institute of Mechanics and Engineering, FRC Kazan Scientific Center of RAS, 2/31 Lobachevsky str., Kazan 420111, Russia

E-mail: ganeeva@kfti.knc.ru

Abstract. Numerical results of the study of nonlinear bending and the stability loss of spherical segments with a rigidly embedded edge are obtained under the influence of external normal pressure and temperature changing in the range from cryogenic to increased. The dependence of the material characteristics on the level of the operating temperature is taken into account. A significant dependence of the stress-strain state and the stability of the segment on the nature of the interaction of external normal pressure and temperature is shown. With axisymmetric temperature and force loading, it is established that the critical loads for a shallow segment are greatest in the vicinity of the initial temperature at low temperature stresses, while for the deep spherical shell, the greatest critical loads are observed at cryogenic temperatures with a significant decrease in their values at elevated temperatures.

Introduction

The basic relations and statements of non-linear problems in the theory of thermo elasticity are given in [1-4]. The bending and stability of spherical shells under pressure at elevated temperatures have been studied, in particular, in [5, 6]. It is of interest to expand research on the temperature range from cryogenic to high and take into account material nonlinearity. The dependence of critical loads on the geometry of the shells has not been sufficiently studied. In this paper, we investigate nonlinear straining of spherical segments with a rigidly embedded edge under the action of external pressure and temperature changing in the range from cryogenic to high, taking into account geometric nonlinearity and elastic-plastic deformations, depending on the deep of the segment.

1. Problem statement and solution method

We consider a nonlinear axisymmetric bending and stability of a thin spherical segment subjected to the external normal pressure and temperature changing in the interval from the initial \( T_0 \) to the current \( T \). The parameters of the segment (Figure 1): \( R \) is the radius of the sphere, \( h \) is the thickness, \( H \) is the height of the pole above the base, and \( a \) is the radius of the base. For coordinate lines we take the meridians \( s \), \( 0 \leq s \leq s_e \), and the external normal \( z \), to the middle surface \(-0.5h \leq z \leq 0.5h\).

\( r = R \sin(\theta) \) is the radius of the parallel; \( 0 \leq \theta \leq \theta_e \) is the angle between the rotation axis of the shell \( x \) and the normal to the middle surface.
The relations of the thermo-force problem from the shell theory are applied, which describe the general, geometrically and material nonlinear stress-and-strain state (SSS) at moderate turning [3] under the influence of pressure on the shell P and the temperature difference. It becomes necessary to take into account the dependence of the material properties on temperature \( T \) in the case where there is a significant difference in the working temperature of the shell from the initial temperature \( T_0 \). In this case, the average coefficient of linear temperature expansion in the interval \( [T_0, T] \) should be used in the relations of the thermo force problem [1-3]. The stress-to-deformation relation is described by the equations of the theory of small elastic plastic deformations [7] for the compressed material with a diagram of linear hardening with hardening factor \( \lambda \), elasticity modulus \( E \), Poisson’s coefficient \( \nu \), stress yield \( \sigma_S \), linear temperature expansion coefficient \( \alpha \) and an average coefficient of linear temperature expansion \( \bar{\alpha} \). In the problem’s relations, the dependence of the material characteristics on temperatures \( T_0, T \) is taken into account:

\[
E = E(T), \quad \sigma_s = \sigma_s(T), \quad \bar{\alpha} = \bar{\alpha}(T_0, T). \tag{1}
\]

For the vector of resolving functions \([3]\) \( Y = (T_{11}', Q_1', M_{11}, P, u, w, \partial_1, V)' \), where \( T_{11}', Q_1' \) are the meridian and the shear strain, respectively; \( M_{11} \) is the moment of deflection; \( u \) is the tangent displacement; \( w \) is shell deflection, \( \partial_1 \) is the rotation of the normal, \( V \) is integral deflection:

\[
V(s) = 2\pi \int_0^s w ds, V(0) = 0, V(s_N) = C, \tag{2}
\]

the non-linear resolving system of equations is obtained:

\[
dY/ds = A(s)Y + F(s, Y) + \Delta T(s), \quad 0 \leq s \leq s_N. \tag{3}
\]

In this equation \( A(s) \) is the \( 8 \times 8 \) dimension factor matrix; \( F(s, Y) \) is the vector of geometrically and material nonlinear terms \( 8 \times 1 \); \( \Delta T(s) \) is the vector of temperature terms \( 8 \times 1 \) with expressions \( \bar{\alpha} \cdot (T - T_0) \). At shell base \( s = s_N \) the conditions for anchorage are considered:

\[
u = 0, \quad w = 0, \quad \partial_1 = 0, \quad V = C. \tag{4}
\]

Nonlinear boundary problem (3), (4) is solved using the method of successive approximations. The process of successive approximations for the given temperature range is organized for a number of leading parameter values. The best value of the leading parameter ensures the process convergence, providing a clear full picture of the distinctive curve of the load parameters and deflection as well as finding the limit points. The basis of the used technique is the algorithm for the continuation of the numerical solution with respect to the integral parameter which is determined according to the half-wave deflection diagram [8]. Let the meridian of the shell be broken into three parts as follows:

\[
s_i \leq s \leq s_j, \quad V_1 = a_1 \int_{s_i}^{s_j} w ds, \quad V_1(s_i) = 0, \quad V_1(s_j) = C_1; \tag{5}
\]

\[
s_i \leq s \leq s_j, \quad V_2 = a_2 \int_{s_i}^{s_j} w ds + C_1, \quad V_2(s_i) = C_1, \quad V_2(s_j) = C_2; \tag{6}
\]
\[ s_j \leq s \leq s_N, \quad V_3 = a_i \int_{s_j}^{s_N} wds + C_2, \quad V_j(s_j) = C_2, \quad V_j(s_N) = C_3. \]  

(7)

Depending on the values of the coefficients \( a_i, i = 1,3 \), the part on the meridian of the shell is allocated on which a monotonic change in the value of the integral deflection parameter in the boundary conditions (4) will be observed (Table 1).

| \( a_1 \) | \( a_2 \) | \( a_3 \) | Leading parameter \( C \) |
|---------|---------|---------|-----------------|
| 1       | 1       | 1       | \( C_1 + C_2 + C_3 \) |
| 1       | 0       | 0       | \( C_1 = C_2 = C_3 \) |
| 0       | 1       | 0       | \( C_1 = C_2 \) |
| 0       | 0       | 1       | \( C_1 \) |

Table 1. Variants of application of the algorithm

Below are the results of the calculations in which \( T_0 = 20^\circ C \). The characteristics of the material of the segment are shown in Table 2.

| \( T \), \( ^\circ C \) | \( E \cdot 10^5 \), MPa | \( \sigma_N \), MPa | \( \sigma_B \), MPa | \( \alpha(T) \cdot 10^5 \cdot 1/^\circ C \) | \( \tilde{\sigma}(T/T_0) \cdot 10^5 \cdot 1/^\circ C \) | Source |
|-------------------|--------------------|-----------------|-----------------|-----------------|-----------------|-----|
| -253              | 2.23               | 520             | 1420            | 0.08            | 1.12            | [9, 10] |
| -193              | 2.12               | 448             | 1000            | 0.66            | 1.32            | [10, 11, 2] |
| -103              | 2.09               | 380             | 1420            | 1.42            | 1.49            |     |
| -70               | 2.03               | 250             | 1000            | 1.60            | 1.58            |     |
| 20                | 1.74               | 660             | 1000            | 1.80            | 1.72            |     |
| 300               | 1.53               | 392             | 1420            | 2.07            | 1.79            |     |
| 500               | 1.48               | 343             | 1420            | 2.17            | 1.82            |     |
| 600               |                    |                  |                 |                 |                 |     |

The following designations of the SSS stages are used: I – non-axisymmetric wave formation along the parallel, II – axisymmetric loss of stability. Load waveforms in the direction of the parallel with the number of waves \( k \) are obtained from the solution of the problem of nonaxisymmetric loss of stability under axisymmetric loading [12].

2. SSS of the spherical segment, depending on its geometric dimensions and material characteristics

2.1 SSS of the non-shallow shell

The geometric parameters of the segment and the material characteristics are as follows: \( a = 200 \) mm, \( h = 1 \) mm, \( H = 100 \) mm, \( a / h = 200, \quad H / a = 0.5, \quad J / x = 0.9273, \quad v = 0.3, \quad \lambda = 0.9 \). In Table 3 the dimensionless parameters of the load \( P / E_0 \), the maximum values of deflection \( w / h \) and the stress intensity \( \sigma_i / E_0 = (\sigma_{11}^i + \sigma_{22}^i - \sigma_{11} \sigma_{22})^{0.5} / E_0 \) for different temperatures \( T \) are shown. The observed complex picture of the segment SSS is determined by the simultaneous action of two main factors – external pressure \( P \) and temperature stresses. There is a significant sensitivity of the SSS of the spherical segment to the temperature change \( T \).

In this case, the load of nonaxisymmetric wave formation along the parallel \( P_H \) and the upper limiting axisymmetric load \( P_B \) are the greatest in the region of cryogenic temperatures \( T \in [-250, -50]^\circ C \). In the region of elevated temperatures \( T \in [200, 500]^\circ C \), a significant decrease in the level of loads, the values of which approach each other, takes place.
Table 3. The dependence of SSS of the non-shallow shell on the temperature

| Stages of SSS | $T^\circ C$ | -250 | -100 | 20  | 50  | 100 | 200 | 300 | 400 | 500 |
|---------------|-----------|------|------|-----|-----|-----|-----|-----|-----|-----|
| I             | -10^6 P_h / E_o | 9.761 | 9.688 | 9.016 | 9.287 | 9.061 | 7.893 | 6.702 | 5.625 | 4.606 |
|               | k         | 15   | 16   | 17   | 17   | 16   | 15   | 14   | 13   | 13   |
|               | w / h     | -1.207 | -0.828 | -0.265 | -0.119 | 0.144 | 0.675 | 1.221 | 1.772 | 2.354 |
|               | 10^3σ / E_o | 4.309 | 3.109 | 1.566 | 1.300 | 1.302 | 1.700 | 1.999 | 2.232 | 2.406 |
| II            | -10^6 P_b / E_o | 11.73 | 10.97 | 9.326 | 9.462 | 9.113 | 7.977 | 6.753 | 5.647 | 4.606 |
|               | w / h     | -1.535 | -1.056 | -0.406 | -0.201 | 0.146 | 0.700 | 1.252 | 1.807 | 2.361 |
|               | 10^3σ / E_o | 4.450 | 3.205 | 1.593 | 1.348 | 1.298 | 1.695 | 1.997 | 2.232 | 2.406 |
|               | (P_b - P_h) / P_h, % | 17 | 12 | 3.3 | 1.8 | 0.6 | 1.1 | 0.8 | 0.4 | 0 |

In Table 4, the comparative data of the segment SSS for a different choice of the dependence of the material characteristics on temperature for the case of full accounting (1) as well as for cases

$$E = E(T), \sigma_s = \sigma_s(T), \alpha = \alpha(T),$$  \hspace{1cm} (8)

$$E = E(T_0), \sigma_s = \sigma_s(T_0), \alpha = \alpha(T_0),$$  \hspace{1cm} (9)

are presented.

The data in the Table 4 demonstrates that the tracking method choice of the material characteristics dependency on the temperature influences on the SSS segment. This influence is particularly significant at the cryogenic temperature levels where the difference $\bar{\alpha} - \alpha$ has a large value (Table 2).

Table 4. Non-shallow segment SSS dependency on material characteristics choice

| Stages of SSS | $T^\circ C$ | -250 | 100 | 400 |
|---------------|-----------|------|-----|-----|
|               | Expression of the material characteristics | (1) | (8) | (9) | (1) | (8) | (9) | (1) | (8) | (9) |
| I             | -10^6 P_h / E_o | 9.761 | 14.50 | 5.375 | 9.061 | 9.067 | 9.796 | 5.625 | 5.485 | 8.275 |
|               | k         | 15   | 17   | 13   | 16   | 16   | 16   | 16   | 13   | 14   |
|               | w / h     | -1.207 | -0.486 | -1.476 | 0.144 | 0.130 | 0.126 | 1.772 | 1.979 | 1.625 |
|               | 10^3σ / E_o | 4.309 | 3.000 | 3.210 | 1.302 | 1.286 | 1.380 | 2.232 | 2.381 | 2.926 |
| II            | -10^6 P_b / E_o | 11.73 | 15.66 | 6.169 | 9.113 | 9.113 | 9.852 | 5.647 | 5.498 | 8.366 |
|               | w / h     | -1.535 | -0.678 | -2.024 | 0.146 | 0.091 | 0.129 | 1.807 | 2.009 | 1.678 |
|               | 10^3σ / E_o | 4.450 | 3.115 | 3.262 | 1.298 | 1.282 | 1.376 | 2.232 | 2.382 | 2.927 |

2.2 Shallow segment SSS
Segment geometric parameters and material characteristics are as follows: $a=200$ mm, $h=1$ mm, $H=20$ mm, $a / h = 200$, $H/a=0.1$, $\theta_0 = -0.1993$, $\nu=0.3$, $\lambda=0.9$. Calculation results are presented in the Table 5 and Figure 2. In this problem the highest loads $P_h, P_b$ are achieved when heated to $T = 50^\circ C$.

The loads level gets significantly lower for the higher temperatures and for the cryogenic temperatures. The values $P_h, P_b$ get closer to each other in the lower temperatures interval.
**Figure 2.** SSS of shallow segment at temperature $T = 400^\circ C$.

**Figure 3.** Influence of the geometric parameters of the segment on the character of the critical loads distribution in the temperature interval $T \in [-250, 500]$. 
Table 5. Shallow segment SSS dependency on temperature

| Stages of SSS | $T^\circ C$ | -250 | -200 | -100 | -50 | 20 | 50 | 100 | 200 | 400 |
|---------------|-------------|------|------|------|-----|----|----|-----|-----|-----|
| I             | $-10^6 P_h / E_o$ | 0.347 | 0.356 | 0.393 | 0.523 | 0.915 | 1.111 | 0.840 | 0.653 | 0.472 |
|               | $k$         | 5    | 5    | 5    | 6   | 7  | 4  | 4   | 4   | 4   |
|               | $w / h$     | -7.049 | -6.188 | -3.109 | -1.930 | -0.512 | 0.152 | 1.335 | 3.028 | 5.547 |
|               | $10^3 \sigma / E_o$ | 3.580 | 3.210 | 2.459 | 1.906 | 0.938 | 0.708 | 1.419 | 1.916 | 2.391 |
| II            | $-10^6 P_h / E_o$ | 0.349 | 0.370 | 0.646 | 0.804 | 0.993 | 1.280 | 1.001 | 0.725 | 0.482 |
|               | $w / h$     | -7.289 | -6.624 | -4.264 | -2.479 | -0.622 | 0.256 | 1.452 | 3.136 | 5.625 |
|               | $10^3 \sigma / E_o$ | 3.563 | 3.182 | 2.450 | 1.995 | 1.013 | 0.911 | 1.395 | 1.897 | 2.387 |
|               | $(P_B - P_h) / P_h, \%$ | 0.6 | 4 | 39 | 35 | 8 | 13 | 16 | 10 | 2 |

$T \in [-250, -200]$ and in the vicinity of the elevated temperature $T = 400^\circ C$.

In Figure 2 the dependencies $P(w)$ in the pole $\theta = 0$ and the vicinity of the base $\theta = 0.15$, of the deflection curve $w(\theta)$ and the stress intensity $\sigma_i(\theta) = (\sigma_{i1}^2 + \sigma_{i2}^2 - \sigma_{i1}\sigma_{i2})^{0.5}$ along the meridian of the segment for temperature $T = 400^\circ C$ are shown. From Figure 2 it can be seen that at elevated temperatures, dangerous SSS is localized around the rigidly embedded base, and it varies insignificantly with an increase in external normal pressure, remaining close to the SSS caused by heating. Significant straining is observed with increasing loading in the vicinity of the pole $\theta = 0$.

In Figure 3 the dependencies of critical loads $P_h$, $P_h$ on temperature $T$ for the shallow segments examined in [13] with geometric dimensions $a / H = 200$, $H / a = 0.1$; $a / H = 333.3$, $H / a = 0.18$ and for an elevated shell with dimensions $a / H = 200$, $H / a = 0.5$ are shown. When the external normal pressure and temperature are combined in action on the spherical segments, the characteristics of the SSS cannot be estimated in advance, suggesting that the absence of temperature stresses at $T = T_0$ while contributing to increased critical loads $P_h$, $P_h$. In particular, if the loads $P_h$, $P_h$ are greatest for shallow segments in the vicinity of the temperature $T = T_0$ in the absence or smallness of temperature stresses, then for the deep spherical shell, the greatest critical loads are observed in the region of cryogenic temperatures with a significant decrease in its values at elevated temperatures.

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