Possible Effects of $^3$He Impurities and Shearing on the Formation of Locally Amorphous Supersolid $^4$He driven by a Pressure Gradient

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Abstract. Possible anomalous states of $^4$He crystal relevant to the possible onset of supersolidity in $^4$He crystal were reported recently. Here, by treating the $^4$He crystal locally as an amorphous matter and using the transition-state model together with the specific activation volume as well as activation energy, we observe a series of sudden change of the shearing stresses (which directly relates to the local transport resistance) at corresponding onset temperatures of $^4$He crystal for different activation volumes considering the role of $^3$He concentration. We found that once the pressure forcing increases for fixed concentration of $^3$He the transition temperature decreases which qualitatively agrees with previous results. We also investigate the possible effects of different shear strain rates as well as the pressure gradient upon the nearly frictionless transport of locally amorphous solid $^4$He within a confined cylindrical domain for a fixed $^3$He concentration. The tuning of different shear strain rates was found to play a crucial role in the formation of possible supersolidity in $^4$He crystal.

1. Introduction
The majority of recent experiments [1-2] search for evidence of supersolid $^4$He [3-4] (with a tiny $^3$He concentration built-in) from different microscopic points of view (say, the spin-lattice relaxation time and the spin-spin relaxation time at the temperatures corresponding to the onset of superfluidity of solid $^4$He considering the role of $^3$He impurities [1] and the microscopic excitations produced by a direct shear strain considering the rotational responses of solid $^4$He attributed to possible supersolidity [2]). As reported in [5]: High quality crystals show a transition from a normal stiff state below 20 mK to an anomalous soft state at higher temperature (around 100 mK) that is very sensitive to traces of $^3$He impurities and to very small stresses. It is yet open for the relation between the softening and the possible transition to a supersolid state of solid $^4$He [5].
Proposal about Bose-condensed supersolid state might exist in solid $^4$He at sufficiently low temperatures (as originally proposed in [6-8]) has been theoretically confirmed by describing it as a Gross-Pitaevskii fluid of delocalized quantum vacancies [9]. As remarked in [9], the superfluid is an intrinsic property of the pure crystal, which is locally enhanced by imperfections, seems to account for the low and reasonably invariant genuine superfluid transition (cf. [10]) and the large variations in the quantity of superflow (cf. [11]). Meanwhile Andreev explained the experimental results (cf. [3,4]) using atomic-scale tunneling two-level systems (presuming...
solid $^4$He contains a population of inertially active crystal excitations). However, Anderson still argued: *Why small concentrations of $^3$He produce large effects remains an open question* [9] although there are continuous studies of $^3$He’s role in the formation of supersolidity [13-15]. In this short paper we shall investigate possible effects of $^3$He impurities and shearing on the transport of locally amorphous solid $^4$He (driven by a pressure gradient) in a confined domain (presumed to be cylindrical). Our focus will be the onset temperature of the possible superfluidity (frictionless states) of solid $^4$He.

2. Formulations

With the Eyring’s transition-state model [16-18] (of stress-biased thermal activation), structural rearrangement is associated with a single energy barrier (height) $E$ that is lowered or raised linearly by a (shear) yield stress $\tau$. If the transition rate is proportional to the shear strain rate (with a constant ratio: $K_0 \approx 2V_a/V_m$), we can calculate the shear stress

$$\tau = E/V_a + (k_BT/V_a) \ln(\dot{\xi}/K_0\nu_0),$$

(1)

where $V_a$ is the activation volume, $\dot{\xi}$ is the shear strain rate, $\nu_0$ is an attempt frequency [16,18], e.g., for temperatures $(T)$ being $O(1)$ K: $\nu_0 \approx k_B/h \sim O(10^{11})$ (1/sec) with $k_B$ being the Boltzmann constant and $h$ the Planck constant. Normally, the value of $V_a$ is associated with a typical volume required for a molecular shear rearrangement. Here $V_m = A_2A_3A_1$, $A_2A_3$ is the cross-section of the transport unit on which the shear stress acts and $A_1$ is the perpendicular distance between two neighboring layers of (composite) particles sliding past each other [16]. We have (using the boundary perturbation series [17,18]), after using the forcing parameter $\Psi = -(r_2/2\tau_0)(dp/dz)$ ($r_2$ is the mean outer radius of the cylindrical domain, $\tau_0 = 2k_BT/V_a$, and $dp/dz$ is the pressure gradient along the axis of the cylindrical domain or the transport direction) $\xi = \dot{\xi}_0 \sinh(\Psi) + \text{HOT}$ with the small wavy-roughness effect being the first order perturbation which is rather small and thus neglected (HOT means the higher order contributions). The (referenced) shear rate is

$$\dot{\xi}_0 = \frac{2V_a k_BT}{h} \exp\left(-\frac{E}{k_BT}\right),$$

(2)

which is a function of temperature, the activation energy $(E)$, the activation volume, and the length scale. $K_0\nu_0$ in Eq. (1) is temperature dependent and the value could be traced in [18]. The remaining task is to fix the value of $\Psi$ by prescribing $r_2$ and $dp/dz$ with different temperatures. Once the detailed or corresponding geometric scales in experimental setup were unknown (closely relevant to our formulations), we can select $|dp/dz| = 1$ (or $r_2 = 1$) for convenience. After all these, the remaining in the equation (1) is the unique relationship between $V_a$ and $T$ for a fixed $\tau$. Note that most of the mathematical derivations could be found in [17,18].

3. Numerical Results and Discussion

After intensive calculations and calibration, we firstly select an activation energy: $9.8 \times 10^{-23}$ Joule (considering the binding energy [1,13,14,15] with 16 ppm of $^3$He) associated with $r_2 = 10^{-3}m$ and $|dp/dz| \approx 8.5 \times 10^6$ Pa/m. The data shown in Fig. 1 illustrates a sudden change of the shearing stresses (which directly relates to the local transport resistance) at $T \sim 0.35$ K (with $V_a \approx 1.94 \times 10^{-27}$ m$^3$). The transport below $T \sim 0.3$ K ($\dot{\xi}_0 \approx 0.65$ sec$^{-1}$) is almost frictionless. We noticed that, as remarked in [19]: The strong impurity effect is likely to be caused by $^3$He, whose local concentration is much higher than the averaged concentration.

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Note: The original text refers to Fig. 1, which is not included in the image. The description in the text assumes that the reader is familiar with the content of Fig. 1.
Figure 1. Calculated shear stresses (or resistance) using an activation energy $9.8 \times 10^{-23}$ Joule (cf. the binding energy [1,13,14,15] of $^3$He). There is a sharp decrease of shear stress around $T \sim 0.35$ K ($V_a \sim 1.9 \times 10^{-27}$ m$^3$). The transport below $T \sim 0.3$ K ($\xi_0 \approx 0.65$ sec$^{-1}$) is almost frictionless.

Figure 2. Calculated shear stresses (or resistance) using an activation energy $9.8 \times 10^{-23}$ Joule (cf. the binding energy [1,13,14,15] of $^3$He). We increase $|dp/dz|$ (Pa/m) from around $8.5 \times 10^6$ (Fig. 1) to around $42.5 \times 10^6$ (here). There is a sharp decrease of shear stress around $T \sim 0.33$ K ($V_a \sim 3.6 \times 10^{-28}$ m$^3$). The transport below $T \sim 0.28$ K ($\xi_0 \approx 0.11$ sec$^{-1}$) is almost frictionless. The shift (of possible onset of supersolidity) to lower temperatures for the increasing pressure (gradient) qualitatively resembles those in [1,10] considering the fixed $^3$He concentration.

To observe the increasing pressure (gradient) effects, we fix $r_2$ and $E$ (the same as in Fig. 1) but set $|dp/dz| \approx 4.257 \times 10^7$ Pa/m. As shown in Fig. 2, the possible onset temperature (of supersolidity) moves to lower value : $T \approx 0.33$ K ($V_a \sim 3.6 \times 10^{-28}$ m$^3$). The shift (of possible onset of supersolidity) to lower temperatures for the increasing pressure (gradient) qualitatively resembles those in [1,10] considering the fixed $^3$He concentration. The transport below $T \sim 0.28$ K ($\xi_0 \approx 0.11$ sec$^{-1}$) is almost frictionless.
Figure 3. Calculated shear stresses (or resistance) using different activation energy $5 \times 10^{-24}$, $1.2 \times 10^{-22}$ Joule. We increase $|\Delta p/\Delta z|$ (Pa/m) from around $6.0817 \times 10^7$ (lower $E$) to around $6.0818 \times 10^7$ (higher $E$). There is a sharp decrease of shear stress around $T \sim 0.5$ K ($V_a \sim 7.9 \times 10^{-28}$ m$^3$). The transport below $T \sim 0.47$ K ($\xi_0 \approx 0.38$ sec$^{-1}$ for higher $E$) is almost frictionless ($\xi_0 \approx 9 \times 10^9$ sec$^{-1}$ for lower $E$).

Finally we present the combination effect of $E$ (cf. the lower concentration of $^3$He in [11]), $|\Delta p/\Delta z|$ with fixed $V_a$ ($r_2 = 5 \times 10^{-4}$ m) in Fig. 3. The latter (trend) resembles those in [20,21]. The possible onset temperature ($\sim 0.5$K) of supersolidity of solid $^4$He matches with that in [11]. The sharp onset occurs only for lower $E$ (with higher $\xi_0$) even both $V_a$ are the same.

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