Tests of the Standard Model in Neutron Beta Decay with Polarized Neutron and Electron and Unpolarized Proton

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We analyse the electron–energy and angular distribution of the neutron $\beta^-$–decay with polarized neutron and electron and unpolarized proton, calculated in Phys. Rev. C 95, 055502 (2017) within the Standard Model (SM), by taking into account the contributions of interactions beyond the SM. After the absorption of vector and axial vector contributions by the axial coupling constant and Cabibbo–Kobayashi–Maskawa (CKM) matrix element (Bhattacharya et al., Phys. Rev. D 85, 054512 (2012) and so on) these are the contributions of scalar and tensor interactions only. The neutron lifetime, correlation coefficients and their averaged values, and asymmetries of the neutron $\beta^-$–decay with polarized neutron and electron are adapted to the analysis of experimental data on searches of contributions of interactions beyond the SM. We use the estimate of the Fierz interference term $b_F = -0.0028 \pm 0.0026$ by Hardy and Towner (Phys. Rev. C 91, 025501 (2015)), the neutron lifetime $\tau_n = 880.2(1.0) \text{s}$ (Particle Data Group, Chin. Phys. C 40, 10001 (2016)) and the experimental data $N_{\exp} = 0.067 \pm 0.011_{\text{stat.}} \pm 0.004_{\text{syst.}}$ for the averaged value of the correlation coefficient of the neutron–electron spin–spin correlations, measured by Kozela et al. (Phys. Rev. C 85, 045501 (2012)). The contributions of $G$–odd correlations are calculated and found at the level of $10^{-5}$ in agreement with the results obtained by Gardner and Plaster (Phys. Rev. C 87, 065504 (2013)).

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I. INTRODUCTION

Recently [1] we have calculated in the Standard Model (SM) the electron–energy and angular distribution of the neutron $\beta^-$–decay with polarized neutron and electron and unpolarized proton by taking into account the contributions of the weak magnetism and proton recoil of order $O(E_n/M)$, where $M$ is an averaged nucleon mass and $E_n$ is the electron energy, and the radiative corrections of order $O(\alpha/\pi)$, where $\alpha$ is the fine–structure constant [2]. These contributions define a complete set of corrections of order $10^{-3}$ to the correlation coefficients of the neutron $\beta^-$–decay with polarized neutron and electron and unpolarized proton. The obtained results together with Wilkinson’s corrections of order $10^{-5}$ [3], which we have also adapted to the correlation coefficients of the neutron $\beta^-$–decay under consideration [1], may provide a robust SM theoretical background for the analysis of experimental data on the searches of contributions of interactions beyond the SM at the level of $10^{-4}$ [4] or even better [5] (see also [1, 6]) if they are supplemented by a complete set of corrections of order $10^{-5}$. This set of corrections is caused by the weak magnetism and proton recoil of order $O(E_n^2/M^2)$, calculated to next–to–next–to–leading order in the large nucleon mass expansion, the radiative corrections of order $O(\alpha E_n/M)$, calculated to next–to–leading order in the large nucleon mass expansion, and the radiative corrections of order $O(\alpha^2/\pi^2)$, calculated to leading order in the large nucleon mass expansion [7, 8]. The first steps towards the experimental searches of contributions of interactions beyond the SM in the neutron $\beta^-$–decay with polarized neutron and electron and unpolarized proton have been done by Kozela et al. [8, 10].
The paper is organized as follows. In section II we give the electron–energy spectrum and angular distribution of the neutron $\beta^-$ decay with polarized neutron and electron and unpolarized proton, which has been calculated within the SM in [1]. The correlation coefficients $A_W(E_e)$, $G(E_e)$, $N(E_e)$, $Q_e(E_e)$ and $R(E_e)$ (see Eq. (1)) are calculated at the level of $10^{-3}$ by taking into account the contributions of the weak magnetism and proton recoil to next–to–leading order in the large proton mass expansion and radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large proton mass expansion [10].

In section III we calculate the contributions of interactions beyond the SM to the correlation coefficients $A_W(E_e)$, $G(E_e)$, $N(E_e)$, $Q_e(E_e)$ and $R(E_e)$, calculated to leading order in the large nucleon mass expansion, and arrive at the correlation coefficients $A_{W, \text{eff}}(E_e)$, $G_{\text{eff}}(E_e)$, $N_{\text{eff}}(E_e)$, $Q_{e, \text{eff}}(E_e)$ and $R_{\text{eff}}(E_e)$ [12–24] (see also [6]). In the linear approximation for vector and axial–vector interactions beyond the SM the obtained contributions are defined by scalar and tensor nucleon–lepton four–fermion couplings beyond the SM only in agreement with [13–24] (see also [6]). A possible dominant role of scalar and tensor interactions beyond the SM has also been discussed by Jackson et al. [13]. In section IV we give the neutron lifetime, correlation coefficients $A_{W, \text{eff}}(E_e)$, $G_{\text{eff}}(E_e)$, $N_{\text{eff}}(E_e)$, $Q_{e, \text{eff}}(E_e)$ and $R_{\text{eff}}(E_e)$ and asymmetries of the neutron $\beta^-$ decay with polarized neutron and electron in the form suitable for the analysis of experimental data of experiments on searches of interactions beyond the SM, including the complete set of the SM corrections of order $10^{-3}$. Wilkinson’s corrections of order $10^{-5}$ [1] and contributions of interactions beyond the SM. The contributions of interactions beyond the SM agree well with the results obtained by Jackson et al. [13, 14] and Severijns et al. [15] up to redefinition of the metric and normalization. In section VII we calculate the $G$–odd corrections to the neutron lifetime and correlation coefficients of the neutron $\beta^-$ decay with polarized neutron and electron and unpolarized proton. We estimate these corrections at the level of $10^{-5}$ and even smaller in agreement with the results obtained by Gardner and Plaster [24]. In section VII we discuss the obtained results and propose some estimates of the values of scalar and tensor coupling constants of interactions beyond the SM. We follow Severijns et al. [15] and use for simplicity a real coupling constant approximation and nucleon–lepton four–fermion couplings with left–handed neutrinos only. The obtained results are adduced in Table I and Table II. For the analysis of the experimental data of experiments on the searches of contributions of interactions beyond the SM at the level of $10^{-4}$ and even better [3] we argue an important role of the theoretical background with the SM corrections of order $10^{-5}$ including Wilkinson’s corrections [1] and corrections, caused by the weak magnetism and proton recoil of order $O(E_e^2/M^2)$, the radiative corrections of order $O(\alpha E_e/M)$, and the radiative corrections of order $O(\alpha^2/\pi^2)$ [6, 8].

II. ELECTRON–ENERGY AND ANGULAR DISTRIBUTION IN THE SM

The electron–energy and angular distribution of the neutron $\beta^-$ decay with polarized neutron and electron, introduced for the first time by Jackson et al. [13, 14] but taken in notations [1], is given by

$$
\frac{d^3\lambda_n(E_e, \vec{k}_e, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e} = \left(1 + 3\lambda^2\right) \frac{G_F^2 |V_{ud}|^2}{8\pi^4} (E_0 - E_e)^2 \sqrt{E_e^2 - m^2_e} E_e F(E_e, Z = 1) \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} + G(E_e) \frac{\vec{\xi}_e \cdot \vec{k}_e}{E_e} + N(E_e) \frac{\vec{\xi}_n \cdot \vec{\xi}_e + Q_e(E_e) \frac{(\vec{\xi}_n \cdot \vec{k}_e)(\vec{\xi}_e \cdot \vec{k}_e)}{E_e + m_e} + R(E_e) \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{\xi}_e)}{E_e},
$$

where $G_F = 1.1664 \times 10^{-11}$ MeV$^{-2}$ is the Fermi weak constant, $V_{ud} = 0.97417(21)$ is the Cabibbo–Kobayashi–Maskawa (CKM) matrix element [2], extracted from the $0^+ \rightarrow 0^+$ transitions, $\lambda = -1.2750(9)$ is the axial coupling constant, which is real [3], $E_0 = (m_n^2 + m_e^2 + m_\pi^2)/2m_n = 1.2927$ MeV is the end–point energy of the electron–energy spectrum, calculated for $m_n = 939.5654$ MeV, and $m_\pi = 938.2721$ MeV and $m_e = 0.5110$ MeV [2]. $\vec{\xi}_n$ and $\vec{\xi}_e$ are unit polarization vectors of the neutron and electron, respectively, $F(E_e, Z = 1)$ is the relativistic Fermi function [14, 25, 24]

$$
F(E_e, Z = 1) = \left(1 + \frac{1}{2}\right) \frac{4(2\pi m_e)\beta^2 \gamma}{\sqrt{2(3 + 2\gamma)} (1 - \beta^2)^{3/2}} \left| 1 + \gamma + \frac{\alpha}{\beta} \right|^2,
$$

where $\beta = k_e/E_e = \sqrt{E_e^2 - m^2_e}/E_e$ is the electron velocity, $\gamma = \sqrt{1 - \alpha^2} - 1$, $r_p$ is the electric radius of the proton. In the numerical calculations we will use $r_p = 0.841$ fm [28] used in [3], which is smaller than $r_p = 0.875$ fm reported in [29] and used in [1]. The Fermi function Eq. (2) describes the contribution of the electron–proton final–state Coulomb interaction. The analysis of different approximations of the Fermi function Eq. (2) has been carried out by Wilkinson [8] (see also [1]). In the SM the correlation coefficients of the electron–energy and angular distribution Eq. (1) we calculate with the Hamiltonian of $V - A$ weak interactions and the weak magnetism [8]

$$
\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud} \left\{ |\bar{\psi}_p(x)\gamma_\mu(1 + \lambda\gamma^5)\psi_n(x)\rangle + \frac{\kappa}{2M} \epsilon^{\mu
u\rho\sigma} [\bar{\psi}_p(x)\sigma_{\mu\nu}\psi_n(x)] \right\} \frac{|\gamma^\rho(\gamma^\mu - 1) - \gamma^5\gamma^\mu| \psi_e(x)\rangle},
$$

where $\lambda = 0, \pm 1$ and $\kappa = 0, \pm 1$. The calculations were performed in the Jacobi–Bessel approximation with $\kappa = 0$ and $\lambda = 0$.
where \( \psi_p(x), \psi_n(x), \psi_e(x) \) and \( \psi_{\nu_e}(x) \) are the field operators of the proton, neutron, electron and anti-neutrino, respectively, \( \gamma^\mu, \sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) \) and \( \gamma^5 \) are the Dirac matrices; \( \kappa = \kappa_p - \kappa_n = 3.7058 \) is the isovector anomalous magnetic moment of the nucleon, defined by the anomalous magnetic moments of the proton \( \kappa_p = 1.7928 \) and neutron \( \kappa_n = -1.9130 \) and measured in nuclear magneton \([2]\). We would like to remind that for the first time the calculation of the corrections to order \( \alpha/\pi \) was carried out by Bilen’kii and Czarnecki [3]. We would like to remind that for the first time the calculation of the corrections to order \( \alpha/\pi \) to the correlation coefficients of the neutron anomalous magnetic moment of the nucleon, defined by the anomalous magnetic moments of the proton [3], and \( \alpha/\pi \) function \( G(E_e) \) has been calculated by Wilkinson [3]. The other correlation coefficients in the electron–energy and angular distribution Eq. (5) have been calculated in [1]. They are equal to

\[
G(E_e) = -\left(1 + \frac{\alpha}{\pi} f_n(E_e)\right) \left(1 + \frac{1}{M} \frac{1}{1+3\lambda^2} \left(2\lambda^2 - 2(\kappa + 1)\lambda\right) \frac{m_e^2}{E_e}\right),
\]

\[
N(E_e) = +\left(1 + \frac{\alpha}{\pi} f_n(E_e)\right) \frac{m_e}{E_e} \left[-A_0 + \frac{1}{M} \frac{1}{1+3\lambda^2} \left(16 \lambda^2 - \frac{4}{3} \kappa - \frac{2}{3}(\kappa + 1)\right) E_e \right.
\]

\[
- \frac{4}{3} \lambda^2 - \frac{4}{3} \kappa - \frac{2}{3}(\kappa + 1) \lambda E_0 \left.\right] - \frac{1}{M} \frac{A_0}{1+3\lambda^2} \left[-(10\lambda^2 - 4(\kappa + 1)\lambda + 2) E_e \right.
\]

\[
\left.\right] + \left(2\lambda^2 - 2(\kappa + 1)\lambda\right) \left(E_0 + \frac{m_e^2}{E_e}\right)\right\},
\]

\[
Q_{\nu_e}(E_e) = \left(1 + \frac{\alpha}{\pi} h_n^{(2)}(E_e)\right) \left[-A_0 + \frac{1}{M} \frac{1}{1+3\lambda^2} \left(\frac{22}{3} \lambda^2 - \frac{10}{3} \kappa - \frac{10}{3}\lambda - \frac{2}{3}(\kappa + 1)\right) E_e \right.
\]

\[
- \frac{4}{3} \lambda^2 - \frac{4}{3} \kappa - \frac{2}{3}(\kappa + 1) \lambda \left(E_0 + \left(2\lambda^2 - 2(\kappa + 1)\lambda\right) \frac{m_e}{E_e}\right) \right.\]

\[
\left.\right] - \frac{1}{M} \frac{A_0}{1+3\lambda^2} \left[-(10\lambda^2 - 4(\kappa + 1)\lambda + 2) E_e \right.
\]

\[
\left.\right] + \frac{2}{3} \lambda^2 - 2(\kappa + 1)\lambda \left(E_0 + \frac{m_e^2}{E_e}\right)\right\},
\]

\[
R(E_e) = -\frac{m_e}{E_e} A_0 , \quad A_0 = -2 \frac{\lambda(1+\lambda)}{1+3\lambda^2},
\]

where the terms of order \((\alpha/\pi)(E_e/M) < 3 \times 10^{-6}\) are neglected. The correlation coefficients Eq. (5) are defined at the level of \(10^{-3}\) of a complete set of contributions, caused by the weak magnetism and proton recoil of order \(\alpha/\pi\) and radiative corrections of order \(O(\alpha/\pi)\). The functions \(h_n^{(1)}(E_e)\) and \(h_n^{(2)}(E_e)\), defining the radiative corrections to the correlation coefficients of \(N(E_e)\) and \(Q_{\nu_e}(E_e)\), are calculated for the first time in [3].

### III. ELECTRON–ENERGY AND ANGULAR DISTRIBUTION BEYOND THE SM

For the calculation of contributions of interactions beyond the SM we use the effective low–energy Hamiltonian of weak nucleon–lepton four–fermion local interactions, taking into account all phenomenological couplings beyond the SM. In notations [3] such a Hamiltonian takes the form

\[
\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud} \left[ [\bar{\psi}_p(x)\gamma_\mu\psi_n(x)] [\bar{\psi}_e(x)\gamma^\mu(C_V + C_\gamma\gamma^5)\psi_{\nu_e}(x)] + [\bar{\psi}_p(x)\gamma_\mu\gamma^5\psi_n(x)] [\bar{\psi}_e(x)\gamma^\mu(C_A + C_A\gamma^5)\psi_{\nu_e}(x)] \right.
\]

\[
+ [\bar{\psi}_p(x)\psi_n(x)] [\bar{\psi}_e(x)(C_S + C_S\gamma^5)\psi_{\nu_e}(x)] + [\bar{\psi}_p(x)\gamma_\mu\gamma^5\psi_n(x)][\bar{\psi}_e(x)(C_P + C_P\gamma^5)\psi_{\nu_e}(x)]
\]
This is the most general form of the effective low–energy weak interactions, where the phenomenological coupling constants \( C_i \) and \( \tilde{C}_i \) for \( i = V, A, S, P \) and \( T \) can be induced by the left–handed and right–handed hadronic and leptonic currents \[13\]–\[17\] and \[18\]. They are related to the phenomenological coupling constants, analogous to those

\[
M(n \to p e^- \bar{\nu}_e) = -2m_n \frac{G_F}{\sqrt{2}} V_{ud} \left\{ [\overline{\psi}_e \gamma^0 (C_V + \bar{C}_V \gamma^5) \psi_n] - [\overline{\psi}_e \bar{\sigma} \psi_n] \cdot [\bar{u} e^\gamma (\bar{C}_A + C_A \gamma^5) \bar{c}_n] \right\}.
\]

The hermitian conjugate amplitude is

\[
M^\dagger(n \to p e^- \bar{\nu}_e) = -2m_n \frac{G_F}{\sqrt{2}} V_{ud}^* \left\{ [\overline{\psi}_e \gamma^0 (C_{\bar{V}} + C_{\bar{V}} \gamma^5) \psi_n] - [\overline{\psi}_e \bar{\sigma} \psi_n] \cdot [\bar{u} e^\gamma (C_{\bar{A}} + \bar{C}_A \gamma^5) \bar{c}_n] \right\}.
\]

The contributions of interactions with the strength, defined by the phenomenological coupling constants \( C_P \) and \( \bar{C}_P \), may appear only of order \( O(C_P E_e/M) \) and \( O(C_P E_e/M) \) and can be neglected to leading order in the large nucleon mass expansion. We have also neglected the contributions of the neutron–proton mass difference. The squared absolute value of the amplitude Eq. \(6\), summed over polarizations of massive fermions, is equal to

\[
\sum_{\text{pol.}} |M(n \to p e^- \bar{\nu}_e)|^2 = 8m_n^2 G_F^2 V_{ud}^2 E_e \left\{ \frac{1}{3} \left[ |C_V|^2 + |\bar{C}_V|^2 + 3|C_A|^2 + 3|\bar{C}_A|^2 + |C_S|^2 + 3|\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2 \right] + \frac{m_e}{E_e} \text{Re} \left( C_V C_S + \bar{C}_V \bar{C}_S - 3C_A C_T - 3C_A \bar{C}_T + \bar{C}_A C_S + \bar{C}_A \bar{C}_T \right) - \bar{C}_A \bar{C}_T + C_A \bar{C}_S + C_A \bar{C}_S \right) \right. 
\]

\[
+ \frac{\bar{C}_A \bar{C}_T + C_A \bar{C}_S + C_A \bar{C}_S - 2C_A C_T - 2\bar{C}_A \bar{C}_T - 2C_A \bar{C}_S + 2\bar{C}_A C_S - 2C_A C_T - 2\bar{C}_A \bar{C}_T}{E_e} \left( \bar{C}_A \bar{C}_T + C_A \bar{C}_S + C_A \bar{C}_S - 2C_A C_T - 2\bar{C}_A \bar{C}_T - 2C_A C_T - 2\bar{C}_A \bar{C}_T \right) \right) \}
\]
In Eq. (10) the structure of the contributions of interactions beyond the SM agrees well with the structure of the corresponding expressions obtained by Jackson et al. [13, 14]. The first term on the second line of Eq. (10) is the Fierz interference term. It appears as a result of the calculation of the traces over the Dirac matrices on the same footing as it appeared in the paper by Lee and Yang [11] (see the Appendix of Ref. [11] and [48]). In the linear approximation corresponding expressions obtained by Jackson et al. in Eq. (10) the structure of the contributions of interactions beyond the SM agrees well with the structure of the electron–energy and angular distribution Eq. (1), taking into account the contributions of interactions beyond the SM, can be transcribed into the form

\[ \sum_{\text{pol.}} |M(n \to p e^- \bar{\nu}_e)|^2 = 8m_n^2 G_F^2 |V_{ud}|^2 E_c E_e (1 + 3\lambda^2) \left\{ \left[ 1 + \frac{1}{2} \frac{1}{1 + 3\lambda^2} (|C_S|^2 + |C_T|^2)^2 \right] + \frac{m_e}{E_e} \frac{1}{1 + 3\lambda^2} \text{Re}\left( (C_S - C_T) + 3\lambda (C_T - C_T) \right) + \frac{\xi_n \cdot \bar{\xi}_e}{E_e} \left( -A_0 - \frac{1}{1 + 3\lambda^2} \text{Re}\left( C_S C_T + C_S C_T^* + 2C_T C_T^* \right) \right) \right\} + \frac{m_e}{E_e} \frac{1}{1 + 3\lambda^2} \text{Re}\left( C_S C_T + C_S C_T^* + 2C_T C_T^* \right) \]

where we have replaced \( C_j \) with \( j = V, A \) by \( C_V = 1 + \delta C_V, C_A = -1 + \delta C_A \) and \( C_A = \lambda + \delta C_A \) and neglected also the contributions of the products \( \delta C_V C_A, \delta C_A C_A \) and so on for \( j = V, A \) and \( k = S, T \). Following [15] we have absorbed the contributions the vector and axial vector interactions beyond the SM by the axial coupling constant \( \lambda \) and the CKM matrix element \( V_{ud} \).

Thus, the electron–energy and angular distribution Eq. (11), taking into account the contributions of interactions beyond the SM, can be transcribed into the form

\[ \frac{d^3 \lambda_\alpha(E_e, \bar{k}_e, \xi_n, \bar{\xi}_e)}{dE_e d\Omega_e} = \frac{(1 + 3\lambda^2) G_F^2 |V_{ud}|^2}{8\pi^4} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) \zeta^{(SM)}(E_e) \]

\[ \times \left\{ 1 + \zeta^{(BSM)}(E_e) + b_F \frac{m_e}{E_e} \left\{ 1 + A_{\text{eff}}(E_e) \frac{\xi_n \cdot \bar{\xi}_e}{E_e} + G_{\text{eff}}(E_e) \frac{\xi_n \cdot \bar{\xi}_e}{E_e} + N_{\text{eff}}(E_e) \xi_n \cdot \bar{\xi}_e \right\} + Q_{\text{e,eff}}(E_e) \frac{\xi_n \cdot (\bar{k}_e \times \bar{\xi}_e)}{E_e (E_e + m_e)} + R_{\text{eff}}(E_e) \frac{\xi_n \cdot (\bar{k}_e \times \bar{\xi}_e)}{E_e (E_e + m_e)} \right\} \]

where the indices “SM” and “BSM” mean “Standard Model” and “Beyond Standard Model”, respectively. The correlation coefficient \( \zeta^{(SM)}(E_e) \) is given by Eq. (11), whereas the analytical expressions for the correlation coefficient \( \zeta^{(BSM)}(E_e) \) and the Fierz interference terms \( b_F \) are aduced in Eq. (12). Other correlation coefficients are defined by

\[ A_{\text{eff}}(E_e) = \frac{A^{(SM)}(E_e) + A^{(BSM)}(E_e)}{1 + \zeta^{(BSM)}(E_e) + b_F \frac{m_e}{E_e}}, \quad G_{\text{eff}}(E_e) = \frac{G^{(SM)}(E_e) + G^{(BSM)}(E_e)}{1 + \zeta^{(BSM)}(E_e) + b_F \frac{m_e}{E_e}}, \]

\[ N_{\text{eff}}(E_e) = \frac{N^{(SM)}(E_e) + N^{(BSM)}(E_e)}{1 + \zeta^{(BSM)}(E_e) + b_F \frac{m_e}{E_e}}, \quad Q_{\text{e,eff}}(E_e) = \frac{Q_{\text{e}}^{(SM)}(E_e) + Q_{\text{e}}^{(BSM)}(E_e)}{1 + \zeta^{(BSM)}(E_e) + b_F \frac{m_e}{E_e}}, \]

\[ R_{\text{eff}}(E_e) = \frac{R^{(SM)}(E_e) + R^{(BSM)}(E_e)}{1 + \zeta^{(BSM)}(E_e) + b_F \frac{m_e}{E_e}}. \]

The correlation coefficients with index “SM” are given by Eqs. (14) and (15). These expressions should be also supplemented by Wilkinson’s corrections of order \( 10^{-5} \) [8], calculated for the neutron \( \beta^- \)-decay under consideration in [8]. The Fierz interference term \( b_F \), the correlation coefficient \( b_E \) and the correlation coefficients with index “BSM” are
given by

\[
\begin{align*}
   b_F &= \frac{1}{1 + 3\lambda^2} \text{Re}((C_S - \bar{C}_S) + 3\lambda (C_T - \bar{C}_T)), \\
   b_E &= \frac{1}{1 + 3\lambda^2} \text{Re}(\lambda (C_S - \bar{C}_S) + (1 + 2\lambda) (C_T - \bar{C}_T)), \\
   \zeta^{(\text{BSM})}(E_e) &= \frac{1}{2} \frac{1}{1 + 3\lambda^2} \left(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2\right), \\
   A_W^{(\text{BSM})}(E_e) &= -\frac{1}{1 + 3\lambda^2} \text{Re}(C_S \bar{C}_T^* + \bar{C}_S C_T^* + 2C_T \bar{C}_T^*), \\
   G^{(\text{BSM})}(E_e) &= \frac{1}{1 + 3\lambda^2} \text{Re}(C_S \bar{C}_S^* + 3C_T \bar{C}_T^*), \\
   N^{(\text{BSM})}(E_e) &= \frac{m_e}{E_e} \frac{1}{1 + 3\lambda^2} \text{Re}(C_S \bar{C}_T^* + \bar{C}_S C_T^* + |C_T|^2 + |\bar{C}_T|^2) + b_E, \\
   Q_e^{(\text{BSM})}(E_e) &= \frac{1}{1 + 3\lambda^2} \text{Re}(C_S \bar{C}_T^* + \bar{C}_S C_T^* + |C_T|^2 + |\bar{C}_T|^2) - b_E, \\
   R^{(\text{BSM})}(E_e) &= \frac{1}{1 + 3\lambda^2} \text{Im}(\lambda (C_S - \bar{C}_S) + (1 + 2\lambda)(C_T - \bar{C}_T)).
\end{align*}
\]

In Eq. (14) the structure of the contributions of interactions beyond the SM agrees well with the structure of corresponding expressions taken in the linear approximation with respect to vector and axial–vector interactions beyond the SM obtained by Jackson et al. [13, 14]. We would like to emphasize that our analysis of the contributions of the Fierz interference term \( b_F \) differs from that by Gardner and Plaster [24]. Indeed, unlike Gardner and Plaster [24] we have included the Fierz interference term to the correlation coefficient \( 1 + \zeta^{(\text{BSM})}(E_e) + b_F m_e/E_e \), related to the contributions of interactions beyond the SM to the neutron lifetime, and taken into account the traces of the Fierz interference term in all correlation coefficients \( X_{\text{eff}}(E_e) \) for \( X = A_W, G, N \) and \( Q_e \). The neutron lifetime, the averaged values of correlation coefficients and the asymmetries of the neutron \( \beta^- \)–decay with polarized neutron and electron, taking into account the contributions of interactions beyond the SM, we calculate in section IV. We represent these expressions in the form suitable for the analysis of experimental data of experiments on the searches of interactions beyond the SM.

**IV. NEUTRON LIFETIME, AVERAGED VALUES OF CORRELATION COEFFICIENTS AND ASYMETRIES OF NEUTRON \( \beta^- \)–DECAY WITH POLARIZED NEUTRON AND ELECTRON**

For the analysis of experimental data of experiments on the searches of interactions beyond the SM in the neutron \( \beta^- \)–decay with polarized neutron and electron we propose to use a complete set of contributions of scalar and tensor interactions beyond the SM including linear, crossing and quadratic terms, which are given in Eq. (14).

**A. Neutron lifetime**

Having integrated the electron–energy and angular distribution Eq. (12) with contributions of interactions beyond the SM we get

\[
\tau_n^{-1}(\vec{\xi}_n, \vec{\xi}_e) = \tau_n^{-1}\left(1 + \frac{1}{2} \frac{1}{1 + 3\lambda^2} \left(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2\right) + \left\langle \frac{m_e}{E_e} b_F + \langle N_{\text{eff}}(E_e) \rangle \vec{\xi}_e \cdot \vec{\xi}_e \right\rangle, \right)
\]

Here we have denoted

\[
\langle N_{\text{eff}}(E_e) \rangle = \left\langle N^{(\text{SM})}(E_e) + \frac{1}{3} \left(1 - \frac{m_e}{E_e}\right) Q_e^{(\text{SM})}(E_e) \right\rangle_{\text{SM}} + \left\langle N^{(\text{BSM})}(E_e) \right\rangle_{\text{BSM}}. \quad (16)
\]

For the calculation of the averaged value \( \langle N_{\text{eff}}(E_e) \rangle \) we use the electron–energy density

\[
\rho_e(E_e) = \rho_e^{(\text{SM})}(E_e) \left(1 + \frac{1}{2} \frac{1}{1 + 3\lambda^2} \left(|C_S|^2 + |\bar{C}_S|^2 + 3|C_T|^2 + 3|\bar{C}_T|^2\right) + b_F \frac{m_e}{E_e}\right), \quad (17)
\]
where the electron–energy density $\rho^{(SM)}_e(E_e)$ is defined by Eq. (D-59) of Ref. [3]. The notation $\langle \ldots \rangle_{SM}$ means that the integration over the electron–energy spectrum is carried out with the electron–energy density $\rho^{(SM)}_e(E_e)$. Then, $\langle N^{(BSM)}(E_e) \rangle_{SM}$ is equal to

$$\langle N^{(BSM)}(E_e) \rangle_{SM} = \left( \frac{2}{3} + \frac{1}{3} \langle m_e/E_e \rangle_{SM} \right) b_E + \left( \frac{1}{3} + \frac{2}{3} \langle m_e/E_e \rangle_{SM} \right) \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + C_S C*T + |C_T|^2 + |C_T|^2),$$

(18)

where $\langle m_e/E_e \rangle_{SM} = 0.6556$ and $\tau_n = 879.6(1.1)\text{s}$. Recent analysis of the experimental data on the neutron lifetime, carried out by Czarnecki et al. [44], has led to the favoured neutron lifetime $\tau_{(favour\text{ed})} = 879.4(6)\text{s}$ and the favoured axial coupling constant $\lambda^{(favour\text{ed})} = -1.2755(11)$, which agree very well with $\tau_n = 879.6(1.1)\text{s}$ and $\lambda = -1.2750(9)$, respectively [6].

B. Averaged values of correlation coefficients

In terms of the Fierz interference term $b_F$ and the correlation coefficient $b_E$ the phenomenological scalar and tensor coupling constants $\text{Re}(C_S - C_S^*)$ and $\text{Re}(C_T - C_T^*)$ are defined by

$$\text{Re}(C_S - C_S^*) = \frac{3\lambda^2 + 1}{3\lambda^2 - 2\lambda - 1}(-1 + 2\lambda)b_F + 3\lambda b_E),$$

$$\text{Re}(C_T - C_T^*) = \frac{3\lambda^2 + 1}{3\lambda^2 - 2\lambda - 1}(\lambda b_F - b_E).$$

(19)

The averaged values of the correlation coefficients $A_{W,\text{eff}}(E_e)$, $G_{\text{eff}}(E_e)$, $N_{\text{eff}}(E_e)$, $Q_{e,\text{eff}}(E_e)$ and $R_{\text{eff}}(E_e)$, taking into account the contributions of the SM and interactions beyond the SM, are given by

$$\langle A_{W,\text{eff}}(E_e) \rangle_{SM} = \langle A^{(SM)}_W(E_e) \rangle_{SM} - \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + C_S C*T + 2C_T C_T^*) =$$

$$= -0.12121 - \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + C_S C*T + 2C_T C_T^*) ,$$

$$\langle G_{\text{eff}}(E_e) \rangle_{SM} = \langle G^{(SM)}(E_e) \rangle_{SM} - \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + 3C_T C_T^*) =$$

$$= -1.00242 - \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + 3C_T C_T^*) ,$$

$$\langle N_{\text{eff}}(E_e) \rangle_{SM} = \langle N^{(SM)}(E_e) \rangle_{SM} + \langle m_e/E_e \rangle_{SM} \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + C_S C*T + |C_T|^2 + |C_T|^2) + b_E =$$

$$= 0.07767 + 0.6556 - \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + C_S C*T + |C_T|^2 + |C_T|^2) + b_E ,$$

$$\langle Q_{e,\text{eff}}(E_e) \rangle_{SM} = \langle Q^{(SM)}_e(E_e) \rangle_{SM} + \langle m_e/E_e \rangle_{SM} \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + C_S C*T + |C_T|^2 + |C_T|^2) - b_E =$$

$$= 0.12279 + 0.6556 - \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C*T + C_S C*T + |C_T|^2 + |C_T|^2) - b_E ,$$

$$\langle R_{\text{eff}}(E_e) \rangle_{SM} = \langle R^{(SM)}(E_e) \rangle_{SM} + \frac{1}{1 + 3\lambda^2} \text{Im}(\lambda(C_S - C_S^*) + (1 + 2\lambda)(C_T - C_T^*)) =$$

$$= 0.0089 + \frac{1}{1 + 3\lambda^2} \text{Im}(\lambda(C_S - C_S^*) + (1 + 2\lambda)(C_T - C_T^*)) .$$

(20)

For the calculation of $\langle X_{\text{eff}}(E_e) \rangle$, where $X = A_{W}, G, N, Q_e$ and $R$, we have used the electron–energy density Eq. (17).

C. Asymmetries of the neutron $\beta^-$–decay with polarized neutron and electron

Asymmetry of neutron–electron spin–momentum correlations: electron asymmetry

For the electron asymmetry $A_{\text{exp}}(E_e)$ [10] we obtain the following expression (see [6])

$$A_{\text{exp}}(E_e) = \frac{N^+_A(E_e) - N^-_A(E_e)}{N^+_A(E_e) + N^-_A(E_e)} = \frac{1}{2} \beta A_{W,\text{eff}}(E_e) P_\alpha(\cos\theta_1 + \cos\theta_2),$$

(21)
where $P_n = |\vec{\Omega}_n| \leq 1$ is the neutron spin polarization, $\beta$ is the electron velocity and $N^+_A(E_e)$ are the numbers of events of the emission of the electron forward (+) and backward (−) with respect to the neutron spin into the solid angle $\Delta \Omega_{12} = 2\pi(\cos\theta_1 - \cos\theta_2)$ with $0 \leq \varphi \leq 2\pi$ and $0 \leq \theta_1 \leq \theta_s \leq \theta_2$. They are determined by $\S$ (see also $\S$)

$$N^+_A(E_e) = 2\pi N(E_e) \int_{\theta_1}^{\theta_2} \left( 1 + A_{W,\text{eff}}(E_e) P_n e \cos \theta_e \right) \sin \theta_e d\theta_e =$$

$$= 2\pi N(E_e) \left( 1 + \frac{1}{2} A_{W,\text{eff}}(E_e) P_n e (\cos \theta_1 + \cos \theta_2) \right) (\cos \theta_1 - \cos \theta_2),$$

$$N^-_A(E_e) = 2\pi N(E_e) \int_{\pi-\theta_1}^{\pi-\theta_2} \left( 1 + A_{W,\text{eff}}(E_e) P_n e \cos \theta_e \right) \sin \theta_e d\theta_e =$$

$$= 2\pi N(E_e) \left( 1 - \frac{1}{2} A_{W,\text{eff}}(E_e) P_n e (\cos \theta_1 + \cos \theta_2) \right) (\cos \theta_1 - \cos \theta_2),$$

(22)

where $N(E_e)$ is the normalization factor equal to

$$N(E_e) = (1 + 3\lambda^2) \frac{G^2_F |V_{ud}|^2}{8\pi^4} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) \zeta^{(\text{SM})}(E_e) \times \left( 1 + \frac{1}{2} \frac{1}{1 + 3\lambda^2} ([C_S]^2 + [\bar{C}_S]^2 + 3|C_T|^2 + 3|\bar{C}_T|^2) + b_F \frac{m_e}{E_e} \right).$$

(23)

The correlation coefficient $A_{W,\text{eff}}(E_e)$ in Eq. (21) is given by

$$A_{W,\text{eff}}(E_e) = \frac{A^{(\text{SM})}_W(E_e) - \frac{1}{1 + 3\lambda^2} \text{Re}(C_S \bar{C}_T^* + \bar{C}_S C_T^* + 2C_T \bar{C}_T^*))}{1 + \frac{1}{2} \frac{1}{1 + 3\lambda^2} ([C_S]^2 + [\bar{C}_S]^2 + 3|C_T|^2 + 3|\bar{C}_T|^2) + b_F \frac{m_e}{E_e}}$$

(24)

with the correlation coefficient $A^{(\text{SM})}_W(E_e)$ equal to $\S$.

$$A^{(\text{SM})}_W(E_e) = \left( 1 + \frac{\alpha}{\pi} f_n(E_e) \right) \Lambda_0 \left( 1 - \frac{1}{M} \frac{1}{2(1 + \lambda)(1 + 3\lambda^2)} \left( A^{(W)}_1 E_0 + A^{(W)}_2 E_e + A^{(W)}_3 \frac{m_e}{E_e} \right) \right),$$

$$A^{(W)}_1 = \frac{2}{3} \left( -3\lambda^3 + (3\kappa + 5) \lambda^2 - (2\kappa + 1) \lambda - (\kappa + 1) \right) = -2 \left( \lambda - (\kappa + 1) \right) \left( \lambda^2 - \frac{2}{3} \lambda - \frac{1}{3} \right),$$

$$A^{(W)}_2 = \frac{2}{3} \left( -3\lambda^4 + (3\kappa + 12) \lambda^3 - (9\kappa + 14) \lambda^2 + (5\kappa + 4) \lambda + (\kappa + 1) \right) = -2 \left( \lambda - (\kappa + 1) \right) \left( \lambda^3 - 3\lambda^2 + \frac{5}{3} \lambda + \frac{1}{3} \right),$$

$$A^{(W)}_3 = -4 \lambda^2 (\lambda + 1) \left( \lambda - (\kappa + 1) \right).$$

(25)

The electron asymmetry Eq. (24) can be used for the extraction of contributions of interactions beyond the SM from the new experimental data, which can be obtained in new runs of experiments on the searches of interactions beyond the SM with cold and ultracold neutrons $\S$.

### Asymmetry of electron spin–momentum correlations

For the polarized electron and unpolarized neutron and proton the correlation coefficient $G_{\text{eff}}(E_e)$ defines the following electron–energy and angular distribution

$$\frac{d^3 \lambda_n(E_n, \vec{k}_e, \vec{E}_e)}{dE_e d\Omega_e} = (1 + 3\lambda^2) \frac{G^2_F |V_{ud}|^2}{8\pi^4} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) \zeta^{(\text{SM})}(E_e) \times \left( 1 + \frac{1}{2} \frac{1}{1 + 3\lambda^2} ([C_S]^2 + [\bar{C}_S]^2 + 3|C_T|^2 + 3|\bar{C}_T|^2) + b_F \frac{m_e}{E_e} \right) \left( 1 + G_{\text{eff}}(E_e) \frac{\vec{E}_e}{E_e} \cdot \vec{k}_e \right).$$

(26)

Following Kozela et al. $\S$ (see also $\S$) the corresponding asymmetry can be defined as follows

$$G_{\text{exp}}(E_e) = \frac{d^3 \lambda_n(E_n, \vec{E}_e, \vec{k}_e)}{dE_e d\Omega_e} \div \frac{-d^3 \lambda_n(E_n, \vec{E}_e, \vec{k}_e)}{dE_e d\Omega_e} = \beta G_{\text{eff}}(E_e) P_{\parallel},$$

(27)
where \( P_{||} \) is the longitudinal polarization of the electron. The signs \( (\pm) \) mean parallel and anti-parallel polarizations of the electron with respect to its momentum. For the comparison with Ref. [10], we have to set \( P_{||} = \sigma_L \). The correlation coefficient \( G_{\text{eff}}(E_e) \) is equal to

\[
G_{\text{eff}}(E_e) = \frac{G^{(\text{SM})}(E_e) - \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C_T + 3C_T C_T^*)}{1 + \frac{1}{2 + 3\lambda^2} (|C_S|^2 + |C_T|^2 + 3|C_T|^2) + b_F \frac{m_e}{E_e}},
\]

where \( G^{(\text{SM})}(E_e) \) is given in Eq. (5).

**Asymmetry of neutron–electron spin–spin correlations**

For the decay electrons in the polarization states with polarization \( P_{e\perp} \) or \( \sigma_T \), in the notation of Ref. [10], lying in the decay plane spanned by the neutron spin polarization vector \( \vec{\xi}_n \) and electron momentum \( \vec{k}_e \) (see Fig. 1 of Ref. [10]), we may define the asymmetry [34], caused by the neutron–electron spin–spin correlations

\[
N_{\exp}(E_e) = N_{\text{eff}}(E_e) P_p P_{e\perp} \cos \gamma,
\]

where we have denoted \( \vec{\xi}_n \cdot \vec{\xi}_e = P_p P_{e\perp} \cos \gamma \). The correlation coefficient \( N_{\text{eff}}(E_e) \) is given by

\[
N_{\text{eff}}(E_e) = \frac{N^{(\text{SM})}(E_e) + \frac{m_e}{E_e} \frac{1}{1 + 3\lambda^2} \text{Re}(C_S C_T^* + C_T C_T^* + |C_T|^2) + b_F \frac{m_e}{E_e}}{1 + \frac{1}{2 + 3\lambda^2} (|C_S|^2 + |C_T|^2 + 3|C_T|^2) + b_F \frac{m_e}{E_e}},
\]

where \( N^{(\text{SM})}(E_e) \) is defined in Eq. (5). The results, obtained in this section, can be used for the analysis of the experimental data of experiments on the searches of interactions beyond the SM in the neutron \( \beta^- \)-decay with polarized neutron and electron and unpolarized proton.

**V. G–Odds Correlations**

The \( G \)-parity transformation, i.e. \( G = C e^{i\pi I_2} \), where \( C \) and \( I_2 \) are the charge conjugation and isospin operators, was introduced by Lee and Yang [45] as a symmetry of strong interactions. According to the \( G \)-transformation properties of hadronic currents, Weinberg divided hadronic currents into two classes, which are \( G \)-even first class and \( G \)-odd second class currents [46], respectively. Following Weinberg [46] and Gardner and Plaster [24], the \( G \)-odd contribution to the matrix element of the hadronic \( n \to p \) transition in the \( V - A \) theory of weak interactions can be taken in the following form

\[
\langle p(\vec{k}_p, \sigma_p)|J^{(+)\mu}_n(0)|n(\vec{k}_n, \sigma_n)\rangle_{G-\text{odd}} = \bar{u}_p(\vec{k}_p, \sigma_p)\left( \frac{m_e}{M} f_3(0) + i \frac{1}{M} \sigma_{\mu\nu}^p \gamma^5 q^\nu g_2(0) \right) u_n(\vec{k}_n, \sigma_n),
\]

where \( J^{(+)\mu}_n(0) = V_{ud}^{(+)\mu}(0) - A_{ud}^{(+)\mu}(0) \), \( \bar{u}_p(\vec{k}_p, \sigma_p) \) and \( u_n(\vec{k}_n, \sigma_n) \) are the Dirac wave functions of the proton and neutron [47]: \( f_3(0) \) and \( g_2(0) \) are the phenomenological coupling constants defining the strength of the second class currents in the weak decays. The contribution of the second class currents Eq. (31) to the amplitude of the neutron \( \beta^- \)-decay in the non-relativistic baryon approximation is defined by

\[
M(n \to p^- \vec{\nu}_e)_{G-\text{odd}} = -2m_n \frac{G_F}{\sqrt{2}} V_{ud} \left\{ f_3(0) \frac{m_e}{M} [\psi_p^l \bar{\psi}_p][\bar{u}_e(1 - \gamma^5)\nu_b] + g_2(0) \frac{1}{M} [\bar{\psi}_p^l(\vec{\sigma} \cdot \vec{k}_p)\nu_b][\bar{u}_e\gamma^0(1 - \gamma^5)\nu_b] \right\}
\]

\[
+ -g_2(0) \frac{E_0}{M} [\bar{\psi}_p^l(\vec{\sigma} \cdot \vec{k}_p)\nu_b][\bar{u}_e\gamma^0(1 - \gamma^5)\nu_b] \right\},
\]

where we have kept only the leading \( 1/M \) terms in the large baryon mass expansion. The hermitian conjugate contribution is

\[
M^\dagger(n \to p^- \vec{\nu}_e)_{G-\text{odd}} = -2m_n \frac{G_F}{\sqrt{2}} V_{ud} \left\{ f_3(0) \frac{m_e}{M} [\bar{\psi}_p^l \psi_p][\bar{v}_e(1 + \gamma^5)u_c] + g_2(0) \frac{1}{M} [\bar{\psi}_p^l(\vec{\sigma} \cdot \vec{k}_p)\psi_p][\bar{v}_e\gamma^0(1 + \gamma^5)u_c] \right\}
\]

\[
- -g_2(0) \frac{E_0}{M} [\bar{\psi}_p^l(\vec{\sigma} \cdot \vec{k}_p)\psi_p][\bar{v}_e\gamma^0(1 + \gamma^5)u_c] \right\}.\]
The contributions of the $G$–odd correlations to the squared absolute value of the amplitude of the neutron $\beta^-$–decay of polarized neutron and electron and unpolarized proton, summed over polarizations of massive fermions, are equal to

$$\sum_{\text{pol.}} \left( M^1(n \rightarrow pe^- \nu_e) M(n \rightarrow pe^- \nu_e) G_{-\text{odd}} + M^1(n \rightarrow pe^- \nu_e) G_{-\text{odd}} M(n \rightarrow pe^- \nu_e) \right) = 8 m_e^2 G_F^2 |V_{ud}|^2 \frac{1}{M}$$

$$\times \left\{ \frac{2 \text{Re} f_3(0)}{E_e} m_e \left( \xi_n \cdot \bar{\xi}_e - \frac{(\xi_n \cdot \bar{k}_e)(\bar{k}_e \cdot \xi_e)}{E_e(E_e + m_e)} \right) + 2 \lambda \text{Im} f_3(0) m_e \frac{\xi_n \cdot (\bar{k}_e \times \xi_e)}{E_e} \right\} + 2 \lambda \text{Re} f_2(0) \left( - \frac{4}{3} E_0 + \frac{2}{3} E_e \right) \frac{\xi_n \cdot \bar{k}_e}{E_e} + \left( \frac{4}{3} E_0 - \frac{1}{3} E_e \right) \frac{m_e}{E_e} \left( \xi_n \cdot \xi_e \right) + \left( \frac{4}{3} E_0 + \frac{2}{3} E_e \right) m_e \frac{(\xi_n \cdot \bar{k}_e)(\bar{k}_e \cdot \xi_e)}{E_e(E_e + m_e)} \right\}$$

$$+ 2 \lambda \text{Re} f_2(0) \left( - \frac{8}{3} E_0 + \frac{2}{3} E_e \right) \frac{\xi_n \cdot \bar{k}_e}{E_e} + \left( \frac{8}{3} E_0 - \frac{2}{3} E_e \right) m_e \frac{(\xi_n \cdot \xi_e)}{E_e} \right\}. \quad (34)$$

For the relative $G$–odd contributions to the correlation coefficients we obtain the following expressions

$$\frac{\delta \zeta \left( E_e \right) G_{-\text{odd}}}{\zeta^{\text{SM}} \left( E_e \right)} = \frac{2}{1 + 3 \lambda^2} \frac{1}{M} \left\{ \text{Re} f_3(0) \frac{m_e^2}{E_e} + \lambda \text{Re} f_2(0) \left( \frac{4 E_0 - m_e^2}{E_e} \right) \right\}, \quad (35)$$

$$\frac{\delta A_W \left( E_e \right) G_{-\text{odd}}}{A_W^{\text{SM}} \left( E_e \right)} = \frac{2}{1 + 3 \lambda^2} \frac{1}{M} \frac{\text{Re} f_2(0)}{A_0} \left\{ \left( - \frac{8}{3} \lambda - \frac{4}{3} \right) E_0 + \left( \frac{2}{3} \lambda - \frac{2}{3} \right) E_e \right\} - \delta \zeta \left( E_e \right) G_{-\text{odd}}, \quad (35)$$

$$\frac{\delta G \left( E_e \right) G_{-\text{odd}}}{G^{\text{SM}} \left( E_e \right)} = \frac{2 \lambda}{1 + 3 \lambda^2} \frac{4 E_0}{M} \text{Re} f_2(0) - \delta \zeta \left( E_e \right) G_{-\text{odd}}, \quad (35)$$

$$\frac{\delta N \left( E_e \right) G_{-\text{odd}}}{N^{\text{SM}} \left( E_e \right)} = \frac{2}{1 + 3 \lambda^2} \frac{1}{M} \left\{ \lambda \text{Re} f_3(0) E_e + \text{Re} f_2(0) \left( \frac{8}{3} \lambda + \frac{4}{3} \right) E_0 - \left( \frac{2}{3} \lambda + \frac{2}{3} \right) E_e \right\} - \delta \zeta \left( E_e \right) G_{-\text{odd}}, \quad (35)$$

$$\frac{\delta Q_e \left( E_e \right) G_{-\text{odd}}}{Q_e^{\text{SM}} \left( E_e \right)} = \frac{2}{1 + 3 \lambda^2} \frac{1}{M} \left\{ - \lambda \text{Re} f_3(0) m_e + \text{Re} f_2(0) \left( \frac{8}{3} \lambda + \frac{2}{3} \right) E_0 - \left( \frac{2}{3} \lambda + \frac{2}{3} \right) E_e + m_e \right\} - \delta \zeta \left( E_e \right) G_{-\text{odd}}, \quad (35)$$

$$\frac{\delta R \left( E_e \right) G_{-\text{odd}}}{R_e^{\text{SM}} \left( E_e \right)} = \frac{2 \lambda}{1 + 3 \lambda^2} \frac{k_e}{M} \frac{1}{\alpha A_0} \text{Im} \left( f_3(0) - g_2(0) \right), \quad (35)$$

For $\lambda = -1.2750$ we get

$$\frac{\delta \zeta \left( E_e \right) G_{-\text{odd}}}{\zeta^{\text{SM}} \left( E_e \right)} = 1.85 \times 10^{-4} \text{Re} f_3(0) \frac{m_e}{E_e} + \left( - 2.39 \times 10^{-3} + 2.36 \times 10^{-4} \frac{m_e}{E_e} \right) \text{Re} f_2(0), \quad (36)$$

$$\frac{\delta A_W \left( E_e \right) G_{-\text{odd}}}{A_W^{\text{SM}} \left( E_e \right)} = -1.85 \times 10^{-4} \text{Re} f_3(0) \frac{m_e}{E_e} + \left( - 5.73 \times 10^{-3} + 5.96 \times 10^{-3} \frac{E_e}{E_0} - 2.36 \times 10^{-4} \frac{m_e}{E_e} \right) \text{Re} f_2(0), \quad (36)$$

$$\frac{\delta G \left( E_e \right) G_{-\text{odd}}}{G^{\text{SM}} \left( E_e \right)} = -1.85 \times 10^{-4} \text{Re} f_3(0) \frac{m_e}{E_e} - 2.36 \times 10^{-4} \text{Re} f_2(0) \frac{m_e}{E_e}, \quad (36)$$

$$\frac{\delta N \left( E_e \right) G_{-\text{odd}}}{N^{\text{SM}} \left( E_e \right)} = \left( - 5.00 \times 10^{-3} \frac{E_e}{E_0} - 1.85 \times 10^{-4} \frac{m_e}{E_e} \right) \text{Re} f_3(0) + \left( - 5.73 \times 10^{-3} - 2.03 \times 10^{-3} \frac{E_e}{E_0} \right.\right.$$
The $G$-odd correction to the neutron lifetime is

$$\frac{1}{\tau_n^{\text{eff}}} = \frac{1}{\tau_n^0} \left[ 1 + \frac{2}{1 + 3\lambda^2} \frac{1}{M} \left( \text{Re} f_3(0) \left( \frac{m_e^2}{E_e} \right)_{\text{SM}} + \lambda \text{Reg}_2(0) \left( 4E_0 - \left( \frac{m_e^2}{E_e} \right)_{\text{SM}} \right) \right) \right],$$

where $(m_e/E_e)_{\text{SM}} = 0.6556$. For $|\text{Re} f_3(0)| < 0.1$ and $|\text{Reg}_2(0)| < 0.01$ the contributions of the $G$-odd correlations to the neutron lifetime and correlation coefficients can appear at the level of $10^{-5}$ or even smaller. This agrees well with results obtained by Gardner and Plaster [24].

VI. CONCLUSION

In this paper we have continued our work on the precision analysis of the neutron lifetime and the correlation coefficients of the electron–energy and angular distribution of the neutron $\beta^-$–decay with polarized neutron and electron and unpolarized proton. The correlation coefficients, calculated within the SM with Wilkinson’s corrections [1], we have supplemented by the contributions of interactions beyond the SM. Since the contributions of vector and axial vector interactions beyond the SM, calculated to linear approximation, can be absorbed by the axial coupling constant $\lambda$ and the CKM matrix element $V_{ud}$ [6], the observable contributions of interactions beyond the SM are defined by scalar and tensor interactions only. We have taken into account a complete set of contributions of scalar and tensor interactions, which have been calculated in the linear approximation for the vector and axial–vector interactions beyond the SM [6] and to leading order in the large nucleon mass expansion. The neutron lifetime and asymmetries of the neutron $\beta^-$–decay with polarized neutron and electron, calculated in section IV can be used for the analysis of experimental data of experiments on the searches of contributions of interactions beyond the SM.

Numerical estimates of contributions of interactions beyond the SM

Using the results, obtained in this paper, we may attempt to make some estimates of the values of scalar and tensor coupling constants and contributions of scalar and tensor interactions to observables. Following [2] and [3] we assume that the contributions of scalar and tensor interactions beyond the SM to neutron lifetime are at the level $10^{-4}$. As we have shown (see Eq. (19)), the real parts of the scalar and tensor coupling constants are defined in terms of the Fierz interference term $b_F$ and the correlation coefficient $b_G$. According to recent analysis by Hardy and Towner [49], the value of the Fierz interference term is equal to $b_F = -0.0028 \pm 0.0026$. This allows to analyse the values of the Fierz interference term from the interval $-0.0054 \leq b_F \leq -0.0002$. For an estimate of the strengths of the scalar and tensor interactions beyond the SM we accept for simplicity the approximation by real scalar and tensor coupling constants and nucleon–lepton four–fermion interactions with the left–handed neutrinos only [18], i.e. $C_S = -C_S$ and $C_T = -C_T$. In order to fit the mean value of the experimental data on the averaged value of the correlation coefficient of the neutron–electron spin–spin correlations $\langle N_{\text{eff}}(E_e) \rangle = 0.6570$ we have to set $b_F = -0.0107$. In Table I we give the values of the scalar and tensor coupling constants and their contributions to the neutron lifetime and measurable correlation coefficients. One may see that for the Fierz interference term equal to $b_F = -0.0054$ the contribution of interactions beyond the SM to the neutron lifetime is at the level of $3\sigma$ with respect to the world averaged value.
\[ \tau_\text{n} = 880.2(1.0) \text{s} \] and the experimental one \( \tau_\text{n} \approx 880.2(1.2) \text{s} \). This provides sufficiently strong deviation of the theoretical value of the neutron lifetime of about 3 s from the experimental value \( \tau_\text{n} = 877.7 \pm 0.7 \text{stat.} \pm 0.3 \text{syst.} \), which has been reported by the UCNA Collaboration. If we want to keep the contribution of interactions beyond the SM to the neutron lifetime at the level of 1 \( \sigma \) or \( 10^{-3} \) we have to restrict the values of the Fierz interference term as follows: \(-0.0017 \leq b_F \leq -0.0002\). However, in order to keep the contribution of interactions beyond the SM to the neutron lifetime at the level of \( 10^{-4} \) or even smaller we have to analyse the contributions of the Fierz interference term with the values taken from the interval \(-0.0004 \leq b_F \leq -0.0002\).

In Fig. 1 we plot the theoretical electron asymmetry, calculated within the SM (blue dotted line) with \( A_{W}^{\text{SM}}(E_e) \) (see Eq. (25)) only and within the SM with the contributions of interactions beyond the SM, given by Eq. (41) and calculated for the Fierz interference term \( b_F = -0.0002 \) (red dotted line) and \( b_F = -0.0004 \) (green dotted line). The vertical lines in Fig. 1 define the experimental electron–energy region of the electron asymmetry observation.

It is important to emphasize that the contributions of scalar and tensor interactions beyond the SM to the correlation coefficients \( \bar{N}_{\text{eff}}(E_e) \) and \( N_{\text{eff}}(E_e) \), caused by neutron–electron spin–spin correlations, are of order \( 10^{-2} \) and do not practically depend on the values of the Fierz interference term taken from the interval \(-0.0054 \leq b_F \leq -0.0002\). The contributions of interactions beyond the SM to these correlation coefficients are practically defined by the correlation coefficient \( b_F \), which we have set equal to \(-0.0107\). Of course, our estimate depends strongly on the experimental mean–value \( N_{\exp} = 0.067 \pm 0.011_{\text{stat}} \pm 0.004_{\text{syst}} \) of the correlation coefficient of the neutron–electron spin–spin correlations, which we have accepted as a signal for a trace of contributions of interactions beyond the SM. Of course, such an assumption seems to be sufficiently strong if we take into account that the experimental value \( N_{\exp} = 0.067 \pm 0.011_{\text{stat}} \pm 0.004_{\text{syst}} \) agrees with the theoretical one \( \langle N^{(\text{SM})}(E_e) \rangle_{\text{SM}} = 0.07767 \), calculated in the SM, within one standard deviation.

Of course, the contributions of interactions beyond the SM of order \( 10^{-2} \) to the correlation coefficients \( N_{\text{eff}}(E_e) \), \( Q_{\text{eff}}(E_e) \) and \( \bar{N}_{\text{eff}}(E_e) \) seem to be unreal, and we have to keep them at the level of \( 10^{-4} \). In Table II we give some estimates of the scalar and tensor coupling constants obtained for the Fierz interference term \( b_F = -0.0002 \) and \( |b_E| \sim 10^{-4}\).

### Table I: Scalar and tensor coupling constants of interactions beyond the SM and their contributions to the neutron lifetime and the measurable correlation coefficients of the neutron \( \beta^- \)-decay with polarized neutron and electron for the correlation coefficient \( b_E = -0.0107\).

| \( b_F \) | \( b_E \) | \( C_S \) | \( C_T \) | \( \Delta \tau_n^{(\text{SM})}/\tau_n \) | \( \langle N^{(\text{SM})}(E_e) \rangle_{\text{SM}} \) | \( \langle N^{(\text{SM})}(E_e) \rangle_{\text{SM}} \) | \( \langle A_{\text{W}}^{(\text{SM})}(E_e) \rangle_{\text{SM}} \) | \( \langle A_{\text{W}}^{(\text{SM})}(E_e) \rangle_{\text{SM}} \) |
|---|---|---|---|---|---|---|---|---|
| -0.0002 | -0.0017 | 0.0186 | 0.0050 | 6.0 \times 10^{-3} | -0.00944 | -0.01067 | 4.0 \times 10^{-5} | 4.0 \times 10^{-5} |
| -0.0054 | -0.0107 | 0.0149 | 0.0080 | 3.5 \times 10^{-3} | -0.00942 | -0.01066 | 6.3 \times 10^{-5} | 6.3 \times 10^{-5} |
| -0.0017 | -0.0107 | 0.0175 | 0.0059 | 10^{-3} | -0.00944 | -0.01067 | 4.7 \times 10^{-5} | 4.7 \times 10^{-5} |
| -0.0004 | -0.0107 | 0.0184 | 0.0051 | 1.9 \times 10^{-3} | -0.00944 | -0.01067 | 4.1 \times 10^{-5} | 4.1 \times 10^{-5} |

### Table II: Scalar and tensor coupling constants of interactions beyond the SM and their contributions to the neutron lifetime and the measurable correlation coefficients of the neutron \( \beta^- \)-decay with polarized neutron and electron for the correlation coefficient \( b_E = -0.0107\).

| \( b_F \) | \( b_E \) | \( C_S \) | \( C_T \) | \( \Delta \tau_n^{(\text{SM})}/\tau_n \) | \( \langle N^{(\text{SM})}(E_e) \rangle_{\text{SM}} \) | \( \langle N^{(\text{SM})}(E_e) \rangle_{\text{SM}} \) | \( \langle A_{\text{W}}^{(\text{SM})}(E_e) \rangle_{\text{SM}} \) | \( \langle A_{\text{W}}^{(\text{SM})}(E_e) \rangle_{\text{SM}} \) |
|---|---|---|---|---|---|---|---|---|
| -0.0002 | -0.0002 | +2.1 \times 10^{-4} | +2.1 \times 10^{-4} | 1.3 \times 10^{-4} | -1.77 \times 10^{-4} | -2.00 \times 10^{-4} | +2.95 \times 10^{-8} |
| -0.0002 | +0.0002 | -4.9 \times 10^{-4} | -2.5 \times 10^{-4} | 1.3 \times 10^{-4} | +1.77 \times 10^{-4} | +2.00 \times 10^{-4} | -3.99 \times 10^{-9} |
| -0.0002 | -0.0001 | +3.3 \times 10^{-5} | +1.6 \times 10^{-4} | 1.3 \times 10^{-4} | -8.85 \times 10^{-5} | -1.00 \times 10^{-4} | +1.08 \times 10^{-8} |
| -0.0002 | +0.0001 | -3.2 \times 10^{-5} | +7.1 \times 10^{-4} | 1.3 \times 10^{-4} | +8.85 \times 10^{-5} | +1.00 \times 10^{-4} | -5.93 \times 10^{-9} |

It is interesting that for \( b_F = -0.0002 \) and keeping the value of the correlation coefficient \( b_E \) at the level of \( 10^{-4} \), i.e. \( |b_F| \sim |b_E| \sim 10^{-4} \), we get the results, which are adduced in Table II. One may see that for the Fierz interference term \( b_F \) and the correlation coefficient \( b_E \) kept at the level of \( 10^{-4} \) the values of scalar and tensor coupling constants at the level of \( 10^{-5} \) – \( 10^{-4} \). In this case the contributions of scalar and tensor interactions beyond the SM can be taken in the linear approximation and fully defined by \( b_F \) and \( b_E \). As a result, expected experimental mean values of the correlation coefficient \( \langle N_{\text{eff}}(E_e) \rangle \) of the neutron–electron spin–spin correlations, averaged over the electron–energy spectrum, may appear, for example, from the interval \( 0.07747 \leq \langle N_{\text{eff}}(E_e) \rangle \leq 0.07787 \).
Towards robust SM theoretical background with corrections to order $10^{-5}$ for analysis of experimental data of experiments on searches of interactions beyond the SM at the level of $10^{-4}$

It is obvious that the analysis of experimental data of experiments on the searches of contributions of interactions beyond the SM at the level of $10^{-4}$ or even better [5] demands a robust SM theoretical background with corrections at the level of $10^{-5}$. These are i) Wilkinson’s corrections [1] and ii) corrections of order $O(\alpha E_e/M^2)$ defined by the weak magnetism and proton recoil, calculated to next–to–next–to–leading order in the large nucleon mass expansion, the radiative corrections of order $O(\alpha E_e/M)$, calculated to next–to–leading order in the large nucleon mass expansion, and the radiative corrections of order $O(\alpha^2/\pi^2)$, calculated to leading order in the large nucleon mass expansion [5]. These theoretical corrections should provide for the analysis of experimental data of "discovery" experiments the required $5\sigma$ level of experimental uncertainties of a few parts in $10^{-5}$ [1]. An important role of strong low–energy interactions for a correct gauge invariant calculation of radiative corrections of order $O(\alpha E_e/M)$ and $O(\alpha^2/\pi^2)$ as functions of the electron energy $E_e$ has been pointed out in [8]. This agrees with Weinberg’s assertion about important role of strong low–energy interactions in decay processes [50]. A procedure for the calculation of these radiative corrections to the neutron $\beta^-$–decays with a consistent account for contributions of strong low–energy interactions, leading to gauge invariant observable expressions dependent on the electron energy $E_e$ determined at the confidence level of Sirlin’s radiative corrections [32], has been proposed in [5].

We would like to accentuate that such a SM background should be also very important for the extraction of the contributions of the $G$–odd correlations which can be of order $10^{-5}$ in agreement with the estimates obtained by Gardner and Plaster [24]. Because of specific energy dependence the contributions of the $G$–odd correlations can be experimentally distinguished from the contributions of the scalar and tensor interactions beyond the SM and the SM background of order $10^{-5}$ discussed above.

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