Stress state of two-layer composite elements of curved shape

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Abstract. Two-layer composite structures consisting of a metal base and a polymer coating are widely used in mechanical engineering. To assess the bearing capacity and durability of such structures, methods for calculating their stress-strain state are necessary. The paper theoretically investigates the stress state of a two-layer structural element of a curved profile for two variants - under pure bending and under temperature change. The radial and meridional stresses arising in the polymer coating and the metal base of the element are taken as the studied factors. The geometric, strength, and physical characteristics of the structural element are accepted as the independent factors. Mathematical models are obtained using the plane section hypothesis. The authors investigated how the radial and meridional stresses acting in the polymer coating are related to the ratio of the coating and the metal thicknesses, as well as the metal thickness to the curvature radius. It has been established that the curvature of the metal element and the ratio of the layers thickness have the most significant effect on the magnitude of the radial stresses, and the ratio of the thicknesses on the meridional stresses. The stress state of the polymer coating is mainly determined by the ratio of the metal thickness to the radius of curvature of the boundary layer.

1. Introduction
Nowadays composite materials based on polymer coatings are widely used in the manufacture of various structural elements in mechanical engineering, aircraft manufacturing and other industries. Composite materials are efficient due to their lightness, variability of composition and structure, and, consequently, a wide range of technical properties [1–6].

A characteristic property of a significant number of structural elements, especially in the aircraft industry, is that they are formed by curved surfaces. Examples of the composite materials used by aircraft manufacturers in the USA and Great Britain include: brake flaps for slats, stabilizers (F-5 and Skyhawk A-4 aircraft); a wing of BQM-34F target drone; retraction system doors (CF-14 reconnaissance aircraft), transmission shafts (Wasp/Scout helicopter); external paneling, spars, attachment points and landing gear; monocoque tail booms and other parts for other aircraft and helicopters. According to preliminary estimates, the widespread use of composite materials can reduce the mass of: helicopters by 35%; military transport aircraft by 22%; vertical take-off and landing aircraft by 21% [7–10].

This, in turn, has a significant effect on the stress-strain state of polymer coatings applied to a curved metal base. It should also be borne in mind that during operation, metal structures can be subjected to intense shock, vibration, and temperature loads.

The literature has limited information on stresses in a polymer coating on a curved metal surface. Existing calculation methods are applicable when the metal-coating composition is deformed without
bending during the formation [11-20]. But in real structural elements, areas with a large curvature are, as a rule, in conditions of moment stress state.

This makes theoretical and experimental research relevant in order to assess the stress state of the reinforced polymer coating on the curved surface of metallic mechanical engineering structural elements.

2. Methodology for assessing the stress state of two-layer curved composite elements

2.1. Calculation scheme and mathematical models

Consider the areas located far enough from the coating edge, which allows neglecting the effect of stress concentration at the edge. The selected curved composite element of unit width is bounded by two meridional sections and two sections normal to the meridian, the angle between which is $d\theta$. The composite element is a two-layer curved beam (Figure 1), which curvature can be considered constant in the area corresponding to the angle $d\theta$.

Let us consider two cases of stresses arising in a two-layer beam: under pure bending and under temperature change. These cases correspond to the main types of stresses in the coating – operational and technological (residual). In solving this problem, we accept the following hypothesis: a flat section remains flat even after deformation of a two-layer beam, but it can stretch or contract in the radial direction. This assumption makes it possible to take into account radial stresses $\sigma_r$.

The accepted hypothesis establishes dependence

$$\frac{d}{dr}(\varepsilon_m r) - \varepsilon_r = \frac{\Delta d\theta}{d\theta}, \quad (1)$$

where $r$ – the radius of curvature of the beam’s curved axis; $\varepsilon_r$ – the relative strain acting in the radial direction (radial relative strain); $\varepsilon_m$ – the relative strain acting parallel to the metal-coating interface (meridional relative strain); $\Delta d\theta$ – the change in angle as a result of the beam strain.

Using Hooke’s law [21, 22] for the biaxial stress state and the equilibrium equation, we write

$$\sigma_m = \frac{d}{dr}(\varepsilon_r r) \quad (2)$$

and taking into account the expression (1), we obtain:

$$r^2 \frac{d^2 \sigma_r}{dr^2} + 3r \frac{d\sigma_r}{dr} = E \frac{\Delta d\theta}{d\theta}. \quad (3)$$
where $\sigma_m$ – the meridional stress, $E$ – the modulus of elasticity of the two-layer curved beam material.

We denote the polymer layer of the curved beam by index 1, and the metal layer by index 2 (Fig. 1). Then the solution of equation (3) for radial stresses can be written as:

$$\sigma_{r_i} = 0.5E_i \frac{\Delta d\theta}{d\theta} \ln r + a_i r^{-2} + b_i,$$

where $i = 1, 2$.

From the equilibrium equation we obtain the equation for determining the meridional stresses:

$$\sigma_{r_i} = 0.5E_i \frac{\Delta d\theta}{d\theta} (1 + \ln r) + a_i r^{-2} + b_i.$$  

(5)

To find $a_i$, $b_i$ and $\Delta d\theta/d\theta$ constants, we use the boundary conditions and the compatibility condition for the strain of the polymer and metal layers on the their interface. The boundary conditions lies in the radial stresses $\sigma_{r_i}=0$ on the curved surfaces of the two-layer beam and the stresses are reduced to the moment on the end surfaces.

2.2. Assessment of the stress state under pure bending

We obtain expressions for determining radial and meridional stresses in the two-layer curved structural element under pure bending.

We introduce additional notation: $k_1=1+(h_1/R)$, $k_2=1-(h_2/R)$, $k_3=1+(h_2/R)$, $k_4=r/R$, $k_5=1-\mu_1$, $k_6=1-\mu_2$, $k_7=1-(h_1/R)$, $k_8=1+\mu_2$, $k_9=1+\mu_1$, $e=E_1/E_2$,

where $h_1$ and $h_2$ – the coating and metal thickness; $R$ – the radius of curvature of the boundary layer; $\mu_1$ and $\mu_2$ – the Poisson`s ratios for the coating and metal, respectively; $E_1$ and $E_2$ – the tensile moduli of the coating and metal material.

After determining the constants and solving equations (4) and (5), these expressions are written in the form:

$$\sigma_{r_1} = -\frac{M e^{(h_1/R)}}{Z_i h_1^2} \left[ D_1 \left( \frac{1}{k_4^2} - \frac{1}{k_2^2} \right) + \ln \frac{k_4}{k_1} \right];$$

(6)

$$\sigma_{m_1} = -\frac{M e^{(h_1/R)}}{Z_i h_1^2} \left[ D_1 \left( \frac{1}{k_4^2} + \frac{1}{k_2^2} \right) + \ln \frac{k_4}{k_1} + 1 \right];$$

(7)

$$\sigma_{r_2} = -\frac{M e^{(h_2/R)}}{Z_i h_2^2} \left[ D_2 \left( \frac{1}{k_4^2} - \frac{1}{k_2^2} \right) + \frac{1}{e} \ln \frac{k_4}{k_2} \right];$$

(8)

$$\sigma_{m_2} = -\frac{M e^{(h_2/R)}}{Z_i h_2^2} \left[ D_2 \left( \frac{1}{k_4^2} + \frac{1}{k_2^2} \right) + \frac{1}{e} \left( 1 + \ln \frac{k_4}{k_2} \right) \right].$$

(9)

Here: $Z_i = e[D_1 \ln k_1 + D_2 \ln k_2 + 0.25(k_1^{-2} - k_2^{-2})]$;

$$D_1 = \frac{(e \ln k_1 - \ln k_2)(1 + \mu_2 + k_2^{-2} k_6) + (k_5 \ln k_1 - k_6 \ln k_2)(1 - k_2^{-2})}{e(1 - k_1^{-2})(1 + \mu_2 + k_2^{-2} k_6) - (1 - k_2^{-2})(1 + \mu_2 + k_2^{-2} k_6)};$$

$$D_2 = \frac{(1 - k_2^{-2})(k_5 \ln k_1 - k_6 \ln k_2) + (\ln k_1 - e^{-1} \ln k_2)(1 + \mu_2 + k_3^{-2} k_6)}{e(1 - k_1^{-2})(k_5 + k_2^{-2} k_6) - (1 - k_2^{-2})(k_5 + k_7^{-2} k_6)}.$$
The direction of moment \( M \) indicated in Figure 1 is taken as positive value. Indices 1 and 2 in expressions (6)–(9) relate to the coating and metal layers, respectively.

2.3. Assessment of stress state under temperature change
When the temperature of the two-layer curved beam changes by variable \( \Delta t \) due to differences in the mechanical properties of the layer material, technological stresses arise.

We introduce notation \( \eta_1 = (\alpha_1 - \alpha_2)\Delta E_1 \) and \( \eta_2 = (\alpha_1 - \alpha_2)\Delta E_2 \),

where \( \alpha_1 \) and \( \alpha_2 \) – the linear expansion coefficients for the coating and metal, respectively.

Then, dependencies (4) and (5) for determining radial and meridional stresses take the form:

\[
\sigma_{r1} = \frac{\eta_1}{Z_2} \left[ \frac{D_3 \left( \frac{1}{k_4^2} - \frac{1}{k_1^2} \right)}{e} + \ln \frac{k_4}{k_1} \right] ;
\]

\[
\sigma_{m1} = \frac{\eta_1}{Z_2} \left[ -\frac{D_3 \left( \frac{1}{k_4^2} + \frac{1}{k_1^2} \right)}{e} + \ln \frac{k_4}{k_1} + 1 \right] ;
\]

\[
\sigma_{r2} = \frac{\eta_2}{Z_2} \left[ D_4 \left( \frac{1}{k_4^2} - \frac{1}{k_2^2} \right) + \ln \frac{k_4}{k_2} \right] ;
\]

\[
\sigma_{m2} = \frac{\eta_2}{Z_2} \left[ -D_4 \left( \frac{1}{k_4^2} + \frac{1}{k_2^2} \right) + \ln \frac{k_4}{k_2} + 1 \right] .
\]

3. Results of calculation and discussion
Figures 2–7 show the results of calculations according to dependences (6)–(13), which describe radial stresses under pure bending and temperature changes for the coating applied to a convex surface of a metal element provided: \( e = E_1 = 0.1E_2; \mu_1 = 0.15; \mu_2 = 0.35. \)

The nature of the change in radial and medial stresses in a reinforced polymer coating depending on the ratio of the layer thickness of the polymer coating \( h_1 \) and the metal \( h_2 \) under temperature change is shown in Figure 2 and 3.

As it can be seen from the results obtained, radial stresses \( \sigma_{r1} \) depend on the curvature of the metal element and the ratio of the thicknesses of the layers, and meridional stresses \( \sigma_{m1} \) mainly depend on the ratio of thicknesses. The increase of \( \sigma_{r1} \) and decrease of \( \sigma_{m1} \) are, apparently, one of the reasons for the fact known in practice: the strength of the coating on the curved surface of the structural element decreases with increasing thickness.

The nature of the change in radial and medial stresses in a reinforced polymer coating depending on the ratio of the metal thickness \( h_2 \) to the radius of curvature of the boundary layer \( R \) under pure bending and temperature change is shown in Figure 6–7.
Figure 2. Dependence of radial stresses on the ratio of the layer thicknesses of the coating and the metal under temperature change.

Figure 3. Dependence of meridional stresses on the ratio of the layer thicknesses of the coating and the metal under temperature change.

Figure 4. Dependence of radial stresses on the ratio of the thickness of the metal to the radius of curvature under pure bending.

Figure 5. Dependence of radial stresses on the ratio of the thickness of the metal to the radius of curvature under temperature change.

Figure 6. Dependence of meridional stresses on the ratio of the thickness of the metal to the radius of curvature under pure bending.

Figure 7. Dependence of meridional stresses on the ratio of the thickness of the metal to the radius of curvature under temperature change.

The analysis of the curves shows that tensile stresses $\sigma_{rl}$ normal to the adhesive surface increase with increasing curvature. This poses a risk of normal cleavage of the coating from the metal surface.
At the same time, the meridional stresses $\sigma_{m1}$ on the free surface of the coating are reduced, which weakens the resistance of the coating to stretching strain. This pattern of stress variation is observed both under pure bending and temperature change.

4. Conclusion
Assuming that the plane section hypothesis is fulfilled, a mathematical model is obtained – dependences (6)–(13), which describe the stress state of the polymer coating.

When determining radial and meridional stresses in a two-layer curved mechanical engineering component, we took into account pure bending and technological stresses arising in the materials of the layers of a two-layer curved beam under temperature change.

According to the obtained mathematical models, the ratio of the metal’s thickness to the radius of curvature of the boundary layer $h_2/R$ is the most important parameter on which the stress distribution in the polymer coating depends.

Tensile stresses normal to the adhesive surface increase with increasing curvature, which can lead to cleavage of the coating from the metal surface. Stresses on the free surface of the coating are reduced, which weakens the resistance of the coating to stretching strain.

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