p-brane solutions and Beltrami-Laplace operator

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Abstract

Generalization of the harmonic superposition rule for the case of dependent choice of harmonic functions is given. Dependence of harmonic functions from all (relative and overall) transverse coordinates is considered using the Beltrami-Laplace operator. Supersymmetry of IIB 10D supergravity solutions with only non-vanished 5-form field and 11D supergravity solutions is discussed.

1 Introduction

The p-brane solutions are the subject of numerous researches [1]–[40].

In the recent time the considerable progress in the understanding of harmonic superposition rule was achieved (for review see [1, 2] and references therein). Two different approaches to the problem was elaborated. The first approach is initiated by the investigations of the supersymmetry properties of the solutions (see [3]–[20]). The second approach is initiated by the investigations of the Einstein field equations (see [21]–[31]).

The present paper continue the research, which was made in the papers [28, 29]. In these works the case of independent choice of harmonic functions was mainly considered. The condition

\[(\partial_i H)(\partial_i H) \neq 0 \implies g_{i_1 i_1} = g_{i_2 i_2} \]

was necessary condition of dependence of harmonic function from coordinates \(x_i\).

The main subject of the present paper is consideration of the case of dependent choice of harmonic functions and elimination of restriction (1).

The following metric represent the example of p-brane solution without restriction (1) in the case of 11D supergravity

\[
ds^2 = (H_1 H_2)^{\frac{1}{2}} \left( -(H_1 H_2)^{-1} dx_0^2 + H_1^{-1}(dx_1^2 + dx_2^2) + H_2^{-1}(dx_3^2 + dx_4^2) + \sum_{i=5}^{10} dx_i^2 \right) \]

\[\Box H_1(x_3, \ldots, x_{10}) = 0, \quad \Box H_2(x_5, \ldots, x_{10}) = 0,\]

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where the symbol “□” designate the Beltrami-Laplace operator. The Beltrami-Laplace
operator is a covariant object, the using of it let us possibility of more profound under-
standing of p-brane solutions properties. The system of equations (4), (3) is nonlinear
system. The simplest way to build the example of the function \( H_1 \) is to set

\[
H_1 = H_{11}(x_3, x_4)H_{12}(x_5, \ldots, x_{10}),
\]

\[
\eta_{KL}\partial_K\partial_LH_{Ru} = 0, \quad R = 1, 2 \quad u = 1, 2.
\]

## 2 General results

Let us consider the case of arbitrary number of antisymmetric and dilatonic fields in the
space-time of arbitrary dimensions and signature. The Kaluza-Klein theory with extra
time-like dimensions has been considered in [41]. The appropriate action has the following
form

\[
I = \frac{1}{2\kappa^2} \int d^D X \sqrt{|g|} \left( R - \frac{1}{2} (\nabla \bar{\phi})^2 - \sum_{i=1}^k s_i e^{-\bar{\phi}(t)}/2(d_f + 1)! F^{(f)2} \right),
\]

where \( s_i = \pm 1, \bar{\phi} \) is a set of the dilaton fields, \( F^{(f)} \) is a field of \( d_f + 1 \)-form.

We can write extremal and non-extremal solutions in terms of occurrence matrices by
the following way. Let \( \Delta^{(f)} \) and \( \Lambda^{(f)} \) are the electric and magnetic incidence matrices
respectively, the elements of these matrices are zeros or unities. The matrices describe
the antisymmetric fields:

\[
F^{(f)} = \sum_{a=1}^{E_f} dA^{(f)}_a + \sum_{b=1}^{M_f} F^{(f)}_b,
\]

\[
A^{(f)}_a = \epsilon_{aM_1\cdots M_d} \eta_{aM_1} \prod_{i=1}^{d_f} \Delta^{(f)}_{ai},
\]

\[
F^{(f)}_b = \epsilon_{bM_1\cdots M_d} \eta_{bM_1} \prod_{i=0}^{d_f} \Lambda^{(f)}_{bi},
\]

where \( \epsilon^{01\cdots} = \epsilon_{01\cdots} = 1 \) are totally antisymmetric symbols.

We shall use also the general incidence matrices: the field matrix \( \Delta_{RK} \) and the brane
matrix \( \Upsilon_{RK} \):

\[
\Delta = \begin{pmatrix}
\Delta^{(1)} \\
\vdots \\
\Delta^{(k)} \\
\Lambda^{(1)} \\
\vdots \\
\Lambda^{(k)}
\end{pmatrix}, \quad \Upsilon = \begin{pmatrix}
\Delta^{(1)} \\
\vdots \\
\Delta^{(k)} \\
1 - \Lambda^{(1)} \\
\vdots \\
1 - \Lambda^{(k)}
\end{pmatrix}.
\]

Index \( R \) numerate branes, and index \( K \) numerate coordinates.

Let

\[
ds^2 = \sum_{K,L=0}^{D-1} \left( \prod_R \eta_{R}^{2H^2} \right) \eta_{KL} dX^K dX^L,
\]

\[
\bar{\phi} = 1/2 \sum_R \bar{\alpha}_R H^2 \ln H_R.
\]
In all above equations $h_R$ are constants, and $H_R$ are functions from coordinates. $\varsigma_R = \pm 1$: $\varsigma_R = +1$ for magnetic branes, $\varsigma_R = -1$ for electric branes, $\eta_{KL} = \text{diag}(\pm 1, \ldots, \pm 1)$.

The equations (1), (8), (10), (11) describe the solution of the equations of motion under the following restrictions (16)–(22) for functions the $H_R$, incidence matrices and other parameters of solution.

Let us introduce the following designations:

$$\Box = g^{KL} \partial_K \partial_L,$$

(13)

under the used gauge conditions (Fock-De Donder gauge) $\Box$ is Beltrami-Laplace operator.

$$(\partial f, \partial g) = g^{KL} \partial_K f \partial_L g,$$

(14)

$I(R,R') = \sum_{D=1}^{D=1} \Delta_{RK} \Delta_{R'K}$, $\varsigma_R = -1$ or $\varsigma_{R'} = -1$

$$\sum_{D=1}^{D=1} \Delta_{RK} \Delta_{R'K} - 2$, $\varsigma_R = +1$ and $\varsigma_{R'} = +1$

(15)

for $I(R,R')$ we shall use the term intersection index.

Using these designations we can write the restrictions for the solution parameters in the following form

$$\Box H_R = 0,$$

(16)

$$\Upsilon_{RK} \partial_K H_R = 0,$$

(17)

$$\Upsilon_{R'N} \partial_N H_R \Upsilon_{RM} \partial_M H_{R'} = 0,$$

(18)

$s_{I|R}$ describes $F^{(l)} \prod_K (\eta_{KK})^{\Delta_{RK}} = \varsigma_R,$

(19)

$$\ln H_R = \frac{\varsigma_R}{2} \sum_{R'} \varsigma_{R'} h_{R'}^2 \left\{ I(R,R') - \frac{d_R d_{R'}}{D - 2} + \tilde{\alpha}_R \tilde{\alpha}_{R'} \right\} \ln H_{R'},$$

(20)

$$\ln H_{R'} = \frac{\varsigma_R}{2} \sum_{R'} \varsigma_{R'} h_{R'}^2 \left\{ I(R,R') - \frac{d_R d_{R'}}{D - 2} + \tilde{\alpha}_R \tilde{\alpha}_{R'} \right\} \ln H_R,$$

(21)

if $I(R,R') = d_R - 1 = d_{R'} - 1,$

(22)

where the indices $R$ and $R'$ describe the same field $F^{(l)}$,

then

$$\begin{cases}
\varsigma_R = \varsigma_{R'} \\
\sum_{M} \Upsilon_{RM} = \Upsilon_{R'M} = 0 \partial_M H_R \partial_M H_{R'} = 0 \\
(\partial_M H_R, \partial_N H_{R'}) |_{\Upsilon_{RM} = \Upsilon_{R'M} = 0, \Upsilon_{RN} = \Upsilon_{R'M} = 1} = 0
\end{cases},$$

or

$$\begin{cases}
\varsigma_R \neq \varsigma_{R'} \\
(\partial_M H_R \partial_{M2} H_{R'} - \partial_M H_R \partial_{M2} H_{R'}) |_{\Upsilon_{RM1} = \Upsilon_{RM2} = \Upsilon_{R'M1} = \Upsilon_{R'M2} = 0} = 0
\end{cases}.$$

The equation (19) for the standard ($-, +, \ldots, +$) signature gives restrictions $\Delta_{a0}^{(l)} = 1$, $\Lambda_{a0}^{(l)} = 0$.

The harmonicity conditions (16) will be a subject of special discussion in the section 3.

The equation (20) in the case of “independent” choice of functions the $H_R$ gives equations for $h_R$ and the characteristic equations, we shall refer this equation as the precharacteristic equation.

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1 The important restriction (18) was absent in the initial version of this paper due to the mistake in calculations. This restriction was initially introduced by H. Liu and C. N. Pope in the paper [10].

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3
The energy-momentum tensor can be written in the following form

\[ T_{KL} = \sum_{RR'} T_{KL}^{RR'}, \quad (23) \]

where the terms \( T_{KL}^{RR'} \) corresponds to pair of branes. The restriction (22) guarantee the vanishing of terms \( T_{KL}^{RR'} \) with \( R \neq R' \).

### 3 Harmonicity conditions

Equation (16) now is not simple condition of harmonicity, using equation (13) one can rewrite it in the following form

\[ g^{KL} \partial_K \partial_L H_R = 0, \quad (24) \]

and using (11)

\[ \sum_{K,L=0}^{D-1} \left( \prod_{R \neq R'} H_R^{h_R} \eta_{RR'}^{Y_{RR}} \right) \eta^{KL} \partial_K \partial_L H_R = 0. \quad (25) \]

These equations are nonlinear. The simplest way to satisfy this equation is to set restriction (1) for coordinate dependence of the functions \( H \). In this case

\[ \eta^{KL} \partial_K \partial_L H_R = 0, \quad (26) \]

e.i. we have several linear differential equations.

The other simple way to construct \( H_R \) functions is to set

\[ H_R = \prod_u H_{Ru}(x_{a_1}^u, \ldots, x_{a_n}^u), \quad g_{a_i}^{a_j} = g_{a_i}^{a_j}, \quad \eta_{MN} \partial_M \partial_N H_{Ru} = 0, \quad (27) \]

taking into account the restriction (18). \( H_{Ru} \) satisfy restriction (1) and depend upon different coordinates. One can find the examples of such solution for 11D supergravity in (2)–(5) and (50)–(55).

### 4 Precharacteristic equations

#### 4.1 Case of independent choice of \( H_R \)

If we want to select the \( H_R \) functions by an independent way, then, using the precharacteristic equations (20), we have

\[ \frac{h_R^2}{2} \left\{ d_R - \frac{d_R^2}{D-2} + \tilde{\alpha}_R^2 \right\} = 1, \quad (28) \]

\[ I(R, R') - \frac{d_R d_{R'}}{D-2} + \frac{\tilde{\alpha}_R \tilde{\alpha}_{R'}}{2} = 0, \quad R \neq R'. \quad (29) \]

The equation (28) gives the values of \( h_R \) in terms of the action parameters. The equation (29) is the characteristic equation in the case of independent choice of \( H_R \). It gives the intersection index.
4.2 Case of dependent choice of $H_R$

4.2.1 General statements

In this section we assume restriction (1) in general case one has linear system (20) of precharacteristic equations for $\ln H_R$ and conditions $\Box H_R = 0$. One can construct maximal set of linearly independent $-\ln H_R$ functions. Let us numerate elements of this set by indices $\varpi, \varrho, \nu$, as $C_\varpi, C_\varrho, C_\nu$, etc.

For arbitrary $C_R = -\ln H_R$ one has

$$C_R = \sum_{\varrho = 1}^{P} a_{R\varrho} C_\varrho. \quad (30)$$

In these terms the equation $\Box H_R = 0$ has the following form

$$\sum_{\varrho, \varpi = 1}^{P} a_{R\varrho} a_{R\varpi} (\partial C_\varrho, \partial C_\varpi) = \sum_{\varrho = 1}^{P} a_{R\varrho} \Box C_\varrho. \quad (31)$$

Taking into account equations $\Box e^{-C_\varrho} = 0$ one can rewrite this equation

$$\sum_{\varrho \neq \varpi} a_{R\varrho} a_{R\varpi} (\partial C_\varrho, \partial C_\varpi) = \sum_{\varrho = 1}^{P} a_{R\varrho} (1 - a_{R\varrho}) \Box C_\varrho. \quad (32)$$

Obviously if $P = 1$ the only way to satisfy this condition is to set $a_{R\varrho} \in \{0, 1\}$.

4.2.2 Possible way for solutions construction

Similar to above one can find solutions with $a_{R\varrho} \in \{0, 1\}$ for arbitrary $P$. In this case

$$\sum_{\varrho \neq \varpi} (\partial C_\varrho, \partial C_\varpi) = 0.$$

The simplest way to satisfy this equation is to set $(\partial C_\varrho, \partial C_\varpi) = 0$ for every pair $\varrho, \varpi$ for which there exists $R a_{R\varrho} = a_{R\varpi} = 1$. For building of this construction using some of $C_R$ functions as $C_\varrho$ functions is not necessary, one can use any functions for which $\Box e^{-C_\varrho} = 0$.

If $s = 0$, we can combine only two $C_\varrho$ functions. In this case one can build solutions using us $C_\varrho a\Re f$ and $b\Im f$, where $a, b = constant$ and $f$ is complex analytical function.

4.2.3 Examples of solutions families

Let us build, for example, the family of such solutions. We will discuss the system with one $d + 1$-form ($N = 1, d_I = d$) and one or zero dilaton ($n = 0, 1, \alpha(I) = \alpha$, if $n = 0$ we will set $\alpha = 0$). Let

$$l = \frac{d^2}{D - 2} - \frac{\alpha^2}{2}, \quad (33)$$

$$I(R, R') = I \neq l, \quad R \neq R'. \quad (34)$$

We will consider the case with one electric and $N$ magnetic terms in ansatz for $d$-form (case with one magnetic and $N$ electric terms may be constructed totally similarly).
The subscript $\varepsilon$ designates the electric term, and the subscript $\mu$ numerates the magnetic terms. Let

$$C_\varepsilon = \sum_\mu C_\mu,$$  \hspace{1cm} (35)

$$(\partial C_{\mu_1}, \partial C_{\mu_2}) = 0, \quad \mu_1 \neq \mu_2,$$  \hspace{1cm} (36)

where the functions $C_\mu$ are chosen by an independent way. The equations (20) for the functions $C_\mu$ have the form

$$C_\mu = \frac{h_\mu^2}{2} (d - l) C_\mu + \sum_{\mu' \neq \mu} \frac{h_{\mu'}^2}{2} (I - l) C_{\mu'} - \frac{h_\varepsilon^2}{2} (I - l) C_\varepsilon.$$  \hspace{1cm} (37)

Taking into account (35) and independent choice of the functions $C_\mu$ we can find

$$h_\mu^2 = h_\varepsilon^2 = h^2 = \frac{2}{d - I}.$$  \hspace{1cm} (38)

Now equation (20) for the function $C_\varepsilon$

$$C_\varepsilon = \frac{h_\mu^2}{2} (d - l) C_\varepsilon - \sum_{\mu'} \frac{h_{\mu'}^2}{2} (I - l) C_{\mu'}.$$  \hspace{1cm} (39)

is the identity.

We can find another similar solutions family, if we consider $\mathcal{N} + 1$ electric or magnetic terms. We will designate one of them by the subscript $\varepsilon$, and numerate all others by the subscript $\mu$. We assume $C_\varepsilon = \sum_\mu C_\mu$ (that is similar to above), and

$$I(\varepsilon, \mu) = l - c_\varepsilon,$$  \hspace{1cm} (40)

$$I(\mu_1, \mu_2) = l + c_\mu, \quad \mu_1 \neq \mu_2.$$  \hspace{1cm} (41)

Similarly to above from the equation (20) for the functions $C_\mu$ we can find

$$h_\mu^2 c_\mu = h_\varepsilon^2 c_\varepsilon,$$

$$\frac{h_\mu^2}{2} (d - l - c_\mu) = 1.$$  \hspace{1cm} (42)

From the equation (20) for the function $C_\varepsilon$ we finally find

$$c_\mu = c_\varepsilon = c,$$  \hspace{1cm} (42)

$$h_\varepsilon^2 = h_\mu^2 = h^2 = \frac{2}{d - l - c}.$$  \hspace{1cm} (43)

The examples of solutions from these solutions families in are (56), (57). All other non-trivial configurations of these families in considered case are prohibited by the restriction (22).
5 The case of 11D supergravity

Let us consider in the bosonic sector of the 11D supergravity

\[ I = \frac{1}{2\kappa^2} \int d^{11}x \sqrt{-g} \left( R - \frac{F^2}{48} \right) + \frac{b}{2\kappa^2} \int A \wedge F \wedge F, \]  

(44)

The bosonic sector of \( D = 11 \) supergravity consists of a metric \( g \) and a three-form potential \( A, F = dA \).

For antisymmetric field and metric we have the following

\[ F = \sum_{a=1}^{E} dA_a + \sum_{b=1}^{M} F_b, \]  

(45)

\[ A_{aM_1M_2M_3} = \epsilon_{M_1M_2M_3} h_a \Lambda_{M_3}^{-1} \prod_{i=1}^{3} \Delta_{aM_i}, \]  

(46)

\[ F^b_{M_0M_1M_2M_3} = \epsilon^{M_0M_1M_2M_3} h_b |g|^{-1/2} \partial_K \Lambda_{bM_3}^{-1} \prod_{i=0}^{3} \Lambda_{kM_i}^{(j)} \]  

(47)

\[ ds^2 = \sum_{K,L=0}^{D-1} \left( \prod_{R} H_{R}^{\kappa h_k} \{ \Delta_{RK}^{-\frac{3}{2}} \} \right) \eta_{KL} dX^K dX^L, \]  

(48)

Precharacteristic equation has the following form

\[ \ln H_R = \frac{c_R}{2} \sum_{R'} \zeta_{R'} h_{R'} \left\{ I(R, R') - \frac{1}{3} \right\} \ln H_{R'}. \]  

(49)

If we set \( h_R = 1 \) and \( I(R, R') = 1, R \neq R' \), then the precharacteristic equation is an identity.

For example, let us consider the following incidence matrix

\[ \begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\end{array} \]

In this incidence table each column represent value of world index and each line represent value of R index. There is a sign in every box, which correspond to \( \Delta_{RL} = 1 \). For electric term the sign is “\( \circ \)”, and for magnetic terms the sign is “\( \bullet \)”. A blank box corresponds to \( \Delta_{RL} = 0 \).

The correspondent metric has the form

\[ ds^2 = (H_1H_2H_3)^\frac{1}{2} \left\{ -(H_1H_2H_3)^{-1} dx_0^2 \right\} + H_1^{-1}(dx_1^2 + dx_2^2) + H_2^{-1}(dx_3^2 + dx_4^2) + H_3^{-1}(dx_5^2 + dx_6^2) + (dx_7^2 + dx_8^2 + dx_9^2 + dx_{10}^2). \]  

(50)

According to (27) we can set

\[ H_1 = H_{12}(x_3, x_4)H_{13}(x_5, x_6)H_{10}(x_7, x_8, x_9, x_{10}), \]  

(51)

\[ H_2 = H_{21}(x_1, x_2)H_{23}(x_5, x_6)H_{20}(x_7, x_8, x_9, x_{10}), \]  

(52)

\[ H_3 = H_{31}(x_1, x_2)H_{32}(x_3, x_4)H_{30}(x_7, x_8, x_9, x_{10}), \]  

(53)

\[ \forall i \neq j \in \{1, 2, 3\} \ : \ H_{ij} = 1 \quad \text{or} \quad H_{ji} = 1, \]  

(54)

\[ \partial_K \partial_K H_{Ru} = 0, \quad R = 1, 2, 3, \quad u = 0, 1, 2, 3. \]  

(55)
Let us demonstrate two new solutions from the solutions families described in previous section.

These are solutions with $b = 0$:

- the case with one electric and two magnetic terms, $I = 0$, $h^2 = 2/3$.

\[
 ds^2 = H_1^{2/9} (H_2 H_3)^{4/9} \left( (H_1 H_2 H_3)^{-1/3} (-d^2 x_0^2 + dx_1^2 + dx_2^2) \right.
\]
\[
\left. + H_3^{-2/3} (dx_3^2 + dx_4^2 + dx_5^2) + H_2^{-2/3} (dx_6^2 + dx_7^2 + dx_8^2) \right)
\]
\[
+ (dx_9^2 + dx_{10}^2) \right) \quad (56)
\]

- the case with three magnetic terms, $c = 1$, $h^2 = 2$.

\[
 ds^2 = (H_1 H_2 H_3)^{4/3} \left( (H_1 H_2 H_3)^{-2} (-d^2 x_0^2 + dx_1^2) \right.
\]
\[
\left. + (H_2 H_3)^{-1} (dx_2^2 + dx_3^2 + dx_4^2) \right)
\]
\[
+ (H_1 H_3)^{-2} dx_5^2 + H_1^{-1} (dx_6^2 + dx_7^2) + (H_1 H_2)^{-2} dx_8^2
\]
\[
+ (dx_9^2 + dx_{10}^2) \right) \quad (57)
\]

The last solution exists also for the signatures $(+, \ldots, +)$ and $(-, - , +, \ldots , +)$.

The analysis of supersymmetry conditions is totally similar to the section “Supersymmetry in 11D Supergravity” in the paper [28], if the brane configuration satisfy the following condition

\[
 I(R, R') = 1, \quad R \neq R', \quad (58)
\]

then one can choose the signs of constants $h_R$ by a supersymmetric way.

6 The case of IIB 10D supergravity

Let us consider in the bosonic sector of IIB supergravity only one field: 5-form $F_5$ (i.e $d = 4$), then one can write the following action

\[
 I = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left( R - \frac{1}{2} 5! F_5^2 \right), \quad (59)
\]

with the additional restriction

\[
 * F_5 = F_5. \quad (60)
\]
In the p-brane case the equation (60) may be rewritten as statement, that there is numeration of branes, for which
\[ H_a = H_{a\bar{a}}, \] (61)
\[ h_a = h_{a\bar{a}}, \] (62)
\[ \Delta_a \Lambda + \Lambda \bar{\Lambda}_{a\bar{a}} = 1, \] (63)
where we introduce the pairs of indices \( a, \bar{a} \), which correspond to the self-dual pairs of the electric and magnetic branes.

For IIB supergravity theory restrictions (61)–(63) are not compatible with the characteristic equations (29) for the independent harmonic functions. We can write, using the restrictions (61)–(63), the precharacteristic equations (20) in the following form
\[ \ln H_a = \sum_{a'} \frac{h_{a'}^2}{2} \left\{ 2I(a, a') - d \right\} \ln H_{a'}. \] (64)

If the functions \( H_a \) are chosen by an independent way, then we can find
\[ h_a = h_{a\bar{a}} = \pm 1/\sqrt{2}, \] (65)
\[ I(a, a') = 2. \] (66)

Let us consider supersymmetry of such solutions. The supersymmetry takes place if there exist the Killing spinor \( \tilde{D}_M \epsilon = 0, \) (67)
\[ \tilde{D}_M = \partial_M + \frac{1}{4} \omega_{MAB} \Gamma_{AB} + \frac{i}{8\sqrt{2} \cdot 120} \Gamma^{K_1...K_5} \Gamma_M F_{K_1...K_5}. \] (68)
The symbols \( \Gamma_A \) are 10D Dirac matrices
\[ \{ \Gamma_A, \Gamma_B \} = 2 \eta_{AB}, \] (69)
\[ \Gamma_{AB...C} = \Gamma_{[AB...C]}. \] (70)

In terms of the considered solutions one can write
\[ \tilde{D}_M = \partial_M - \sum_a \frac{h_a}{4\sqrt{2}} \partial_M \ln H_a i(\Gamma(a) + \Gamma(\bar{a})) \] (71)
\[ - \sum_a \frac{h_a}{2\sqrt{2}} \{ 2\Delta_a - 1 \} \Gamma_M^N \partial_N \ln H_a \frac{h_a \sqrt{2} + \frac{i}{2} (\Gamma(a) + \Gamma(\bar{a}))}{2}, \]
where
\[ \Gamma(a) = \frac{1}{4!} \prod_{\{A | \Delta_a A = 1 \}} \Gamma_A, \] (72)
\[ \Gamma(\bar{a}) = \frac{1}{6!} \prod_{\{A | \Lambda_{a\bar{a}} = 1 \}} \Gamma_A. \] (73)

Using sentence (63) we can write
\[ \frac{h_a \sqrt{2} + \frac{i}{2} (\Gamma(a) + \Gamma(\bar{a}))}{2} = \pm 1 \pm \frac{i}{2} (\Gamma(a) + \Gamma(\bar{a})), \] (74)
i.e. this term is proportional to the sum of two projectors. Let us consider the spinor subspace for which

\[ \Gamma_0 \cdots \Gamma_9 \cdot \epsilon = -\epsilon. \] (75)

The term (74) is a projector for this subspace, so similar to 11D case (see [28]) if the brane configuration satisfy the condition \( I(a, a') = 2 \), then one can choose the signs of constants \( h_a \) by a supersymmetric way.

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