Contributions from $\Phi_{B_2}$ to the $B \to PP$ decays within the QCD factorization

Qin Chang,¹ Lili Chen,¹ Yunyun Zhang,¹ Junfeng Sun,¹ and Yueling Yang¹

¹Institute of Particle and Nuclear Physics,
Henan Normal University, Xinxiang 453007, China

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Abstract

With the potential for the improvements of measurement precision, the refinement of theoretical calculation on hadronic $B$ weak decays is necessary. In this paper, we study the contributions of $B$ mesonic distribution amplitude $\Phi_{B_2}$ within the QCD factorization approach, and find that $\Phi_{B_2}$ contributes to only the nonfactorizable annihilation amplitudes for the $B \to PP$ decays ($P$ denotes the ground SU(3) pseudoscalar mesons). Although small, the $\Phi_{B_2}$ contributions might be helpful for improving the performance of the QCD factorization approach, especially for the pure annihilation $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ decays.
Because of successive impetus from both experiments and theoretical improvements, the study of nonleptonic $B$ meson weak decays has been one of the hot topics of particle physics. Most of the two-body hadronic $B$ decays with branching ratio larger than $10^{-6}$ have been investigated thoroughly and carefully at the BABAR and Belle experiments [1, 2] in the past years. A huge amount of $B$ meson experimental data will be accumulated at the high luminosity colliders in the near future, about $50\ ab^{-1}$ by the Belle-II detector at the $e^+e^-$ SuperKEKB collider [3] and about $300\ fb^{-1}$ by the LHCb Upgrade II detector at the hadron HL-LHC collider [4, 5]. With the advent of a new age of $B$ physics at the intensity frontier, besides some new phenomena, the unprecedented precision will offer a much more rigorous test on the standard model of elementary particles. The prospective experimental sensitivities for $B$ mesons require more and more accuracy of theoretical calculation.

As is well known, the participation of the strong interactions make it very complicated to calculate the $B$ meson weak decays, especially for the nonleptonic cases. Based on power-counting rules in the heavy quark limits and perturbative QCD theory, some phenomenological models, such as QCD factorization (QCDF) [6–11], perturbative QCD (pQCD) approach [12–15] and so on, have been developed and employed to compute the hadronic matrix elements (HMEs) describing the transformations between the initial $B$ meson and final hadrons through local quark interactions. However, the nonperturbative contributions to HMEs bring theoretical results on branching ratios with many and large uncertainties, particularly for the internal $W$-boson emission and the neutral current processes. To reduce theoretical uncertainties and satisfy the precision requirements of experimental analysis, a careful and comprehensive examination of all possible nonperturbative factors within a phenomenological model is necessary. In this paper, the contributions from the $B$ meson wave functions will be reassessed in detail within the theoretical framework of QCDF.

Wave functions (WFs) or distribution amplitudes (DAs) of the $B$ meson are the essential ingredients of the master formulas in QCDF [7] and pQCD [13] approaches to evaluate the nonfactorizable contributions to HMEs, such as the spectator scattering amplitudes. However, the knowledge of the $B$ mesonic WF and DAs is still limited so far. It is intuitive that the component quarks of a hadron should move with the same velocity to form a color singlet, and thus the valence quarks would share momentum fractions according to their masses. It is expected that the $B$ mesonic DAs should be very asymmetric with $\xi$ at the scales of order $m_b$ or smaller, if the light spectator quark carries a longitudinal momentum
fraction $\xi \sim \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$, where $\Lambda_{\text{QCD}}$ and $m_b$ are respectively the characteristic QCD scale and the mass of $b$ quark. Generally, the $B$ meson is described by two scalar functions up to the leading power in $1/m_b$ [16–19], which is written as [7]

$$
\langle 0|\bar{q}_\alpha(z)[...]|B(p)\rangle = -\frac{i f_B}{4} \left\{ (\not{p}+m_b) \gamma_5 \right\}_{\beta\gamma} \int d\xi e^{-i\xi p^+ z^-} \left[ \Phi_{B1}(\xi) + \frac{\not{n}_-}{2} \Phi_{B2}(\xi) \right]_{\gamma\alpha},
$$

where the dots denote the path-ordered exponential gauge factor; the light spectator quark moves along the light-like $z_-$ line; $n_- = (1,0,0,-1)$ is a null vector; and the normalization conditions of DAs are [7]

$$
\int_0^1 d\xi \Phi_{B1}(\xi) = 1,
$$

$$
\int_0^1 d\xi \Phi_{B2}(\xi) = 0.
$$

According to the conventions of Refs.[16, 17], $\Phi_{B1} = \phi_B^+$ and $\Phi_{B2} = (\phi_B^+ - \phi_B^-)/2$. Generally, the two functions $\phi_B^\pm$ are not identical, $\phi_B^+ \neq \phi_B^-$, and satisfy the relation $\phi_B^+(\xi) + \xi \phi_B^-(\xi) = 0$ [17]. So, $\Phi_{B2} \neq 0$. The contributions of $\Phi_{B2}$ part are suppressed by the power factor of $\Lambda_{\text{QCD}}/m_b$, compared with those of $\Phi_{B1}$. In the actual calculations for the $B \to PP$ decays with the QCDF approach ($P$ denotes the light $SU(3)$ ground pseudoscalar meson), for example in Ref.[8], only the contributions from $\Phi_{B1}$ part are considered appropriately, while those from $\Phi_{B2}$ part are not included explicitly. It should be pointed out that the value of $\Lambda_{\text{QCD}}/m_b$ is not a negligible number, because the mass of the $b$ quark is finite rather than infinite. It has been shown in Refs.[18–22] that there is a large contribution of $\Phi_{B2}$ to the hadronic $B \to \pi$ transition formfactors within the pQCD approach, and its share could reach up to $\sim 30\%$ with some specific inputs [21, 22]. This means that the contributions of $\Phi_{B2}$ to branching ratios for the $W$ emission processes can reach up to $\sim 70\%$ for some cases. The $\Phi_{B2}$ contribution that were neglected in most cases should be given due attention with the QCDF approach, which is the focus of this paper.

Here, it should be pointed out that a possibly large contribution of $\Phi_{B2}$ to formfactors is present only with the pQCD approach rather than the QCDF approach, due to different understandings on the nature of the hadronic transition formfactors. With the pQCD approach [12–15], it is assumed that the light quark with a soft momentum of $\mathcal{O}(\Lambda_{\text{QCD}})$ in the initial $B$ meson should interact with a hard gluon, so it could receive a large boost in order to form a colorless final state with a light energetic quark originating from the $b$ quark decaying.
FIG. 1: The spectator scattering interactions.

interaction point. It is therefore arguable that the hadronic transition formfactors are computable perturbatively with the help of the Sudakov factor regulation on soft contributions. The hadronic transition formfactors are written as the convolution of wave functions of both the $B$ meson and final hadron. Contrarily, it is argued [7, 17] with the QCDF approach that the hard and soft contributions to the heavy-to-light formfactors have the same scaling behavior, and the hard contributions are suppressed by one power of $\alpha_s$ compared with the soft contributions. Because of the dominance of soft contributions, the formfactors for the transition between $B$ meson and light hadron are not fully calculable with the perturbative QCD theory. So, the formfactors are regarded as nonperturbative inputs with the QCDF approach, and therefore have nothing to do with the $B$ mesonic wave functions.

We will concentrate on the $B \to PP$ decays for the moment. Up to power corrections of $1/m_b$, the general QCDF formula of HMEs for an effective operator $\hat{O}_i$ is written as [7],

$$\langle P_1 P_2 | \hat{O}_i | \bar{B} \rangle = F_{0}^{B \to P_1} \int_{0}^{1} dx T_{I}^{I}(y) \phi_{P_2}(x) + F_{0}^{B \to P_2} \int_{0}^{1} dy H_{I}^{I}(x) \phi_{P_1}(y) + \int_{0}^{1} d\xi dx dy T_{II}^{II}(\xi, x, y) \phi_{B}(\xi) \phi_{P_1}(y) \phi_{P_2}(x), \quad (4)$$

where $F_{0}^{B \to P_1}$ denotes the formfactor; $T_{I}^{I}$, $H_{I}^{I}$ and $T_{II}^{II}$ are hard scattering kernels; the mesonic DAs, $\phi_{P_2}(x)$ and $\phi_{P_1}(y)$, are the functions of longitudinal momentum fractions $x$ and $y$ of light quarks.

For the two terms in the first line of Eq.(4), soft contributions are assumed to be embodied in the formfactors $F_{0}^{B \to P_1}$ and DAs. Contributions of $T_{I}^{I}$ and $H_{I}^{I}$ are dominated by hard gluon exchange. So these contributions, which are irrelevant to $B$ mesonic wave functions, are considered as perturbative corrections to the naive factorization formula, which involve only decay constants and formfactors, but no DAs.

The term in the second line of Eq.(4) corresponds to nonfactorizable contributions. The spectator scattering interactions (see Fig.1) entangle the initial $B$ meson with the final
FIG. 2: The weak annihilation interactions, where (a) and (b) are factorizable diagrams, (c) and (d) are nonfactorizable diagrams.

hadrons, which make separating one hadron from others impossible. Therefore, the spectator scattering amplitudes are usually written as the convolution integral of the hard kernels $T^{II}$ and all participating DAs. The hard spectator scattering amplitudes contain the contributions from both $\Phi_{B1}$ and $\Phi_{B2}$, and can be written as

$$H_k(P_1, P_2) = H_k^{B1}(P_1, P_2) + H_k^{B2}(P_1, P_2),$$

(5)

where $P_1$ is the emitted meson; $P_2$ is the recoiled meson that incorporates the spectator quark from $B$ meson into itself; $H_k^{B1}$ ($H_k^{B2}$) is the contribution from $\Phi_{B1}$ ($\Phi_{B2}$); the subscript $k$ on $H_k$ refers to the possible Dirac current structure $\Gamma \otimes \Gamma$ of an operator $\hat{O}$, namely, $k = 1$, 2 and 3 correspond to $\Gamma \otimes \Gamma = (V-A) \otimes (V-A)$, $(V-A) \otimes (V+A)$ and $-2(S-P) \otimes (S+P)$ respectively. After the straightforward calculation, we find that considering the $SU(3)$ flavor symmetry, the expressions of $H_k^{B1}$ and $H_k^{B1}$ are entirely consistent with Eq.(47) and Eq.(48) of Ref.[23], and $H_3^{B1} = 0$. Our calculations also show that $H_k^{B2}$ corresponding to Fig.1(a) and Fig.1(b) are nonzero. Moreover, the terms of both $\int_0^1 \frac{\Phi_{P1}(y)}{y^2} dy$ and $\int_0^1 \frac{\Phi_{P1}(y)}{y^2} dy$ appear in $H_k^{B2}$, where $\Phi_{P1}(y)$ and $\Phi_{P1}(y)$ are the leading twist (twist-2) and twist-3 DAs of the emitted meson $P_1$ and $\bar{y} = 1 - y$. It is clearly seen that with the asymptotic forms of $\Phi_{P1}(y) = 6y\bar{y}$ and $\Phi_{P1}(y) = 1$, the integrals of $\int_0^1 \frac{\Phi_{P1}(y)}{y^2} dy$ and $\int_0^1 \frac{\Phi_{P1}(y)}{y^2} dy$ exhibit logarithmic and linear infrared divergences. Fortunately, because of the opposite sign between the emitted quark and antiquark propagators plus the condition of Eq.(3), the contributions of $H_k^{B2}$ exactly cancel each other out. The total contributions from $\Phi_{B2}$ to spectator scattering amplitudes are zero.

Compared with the leading contributions, the weak annihilation (WA) contributions are thought to be suppressed by one power of $\Lambda_{QCD}/m_b$ [7]. However, the WA contributions are significant and cannot be ignored in practical application of the QCDF approach to the hadronic $B$ decays [8, 23–26]. Therefore, the QCDF master formula of Eq.(4) is general-
ized to estimate the WA contributions. The WA interactions have two types of topologies within the QCDF approach. The nonfactorizable and factorizable topologies respectively correspond to gluon emission from the initial $B$ meson and final quarks, see Fig.2. The factorizable WA amplitudes can be written as the product of the time-like $0 \rightarrow P_1 P_2$ formfactors and the integral of $B$ mesonic WF, see Fig.2(a) and (b). With the normalization condition of Eq.(3), it is clearly seen that $\Phi_{B2}$ contributes nothing to the factorizable WA amplitudes $A_k^f$, where the superscript $f$ means factorizable, $i.e.$, gluon emission from the final quarks; the subscript $k$ has the same meaning as that of $H_k$ in Eq.(5). The nonfactorizable WA amplitudes, corresponding to Fig.2(c) and (d), can be written as the convolution integral of all participating hadronic DAs, and contain the contributions from both $\Phi_{B1}$ and $\Phi_{B2}$.

$$A_k^i = A_k^i B_1 + A_k^i B_2,$$

(6)

where the superscript $i$ means gluon emission from the initial $B$ meson; $A_k^i B_1 (A_k^i B_2)$ is the contribution from $\Phi_{B1}$ ($\Phi_{B2}$). The expressions of $A_k^i B_1$ have been explicitly given by Eq.(62) of Ref.[8] and Eq.(54) of Ref.[23]. Here, we will give the new components $A_k^i B_2$.

$$A^{B_2} = \pi \alpha_s \int_0^1 dx \int_0^1 dy \frac{\Phi_{P2}(x) \Phi_{P1}(y)}{1 - x \bar{y}} - r_x^P \int_0^1 dy \frac{\Phi_{P2}(x) \Phi_{P1}(y)}{1 - x \bar{y}},$$

(7)

$$A_1^{i B_2} = -A^{B_2} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \left\{ 2 \frac{\Phi_{P2}(x) \Phi_{P1}(y)}{1 - x \bar{y}} - r_x^P \int_0^1 dy \frac{\Phi_{P2}(x) \Phi_{P1}(y)}{1 - x \bar{y}} \right\},$$

(8)

$$A_2^{i B_2} = A^{B_2} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \left\{ 2 \frac{\Phi_{P2}(x) \Phi_{P1}(y)}{1 - x \bar{y}} - r_x^P \int_0^1 dy \frac{\Phi_{P2}(x) \Phi_{P1}(y)}{1 - x \bar{y}} \right\},$$

(9)

$$A_3^{i B_2} = -A^{B_2} \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \left\{ 2 r_x^P \frac{x \Phi_{P2}(x) \Phi_{P1}(y)}{1 - x \bar{y}} + r_x^P \frac{\Phi_{P2}(x) \Phi_{P1}(y)}{1 - x \bar{y}} \right\},$$

(10)

where the factor $r_x^P = \frac{2 m_\pi^2}{m_k (m_{q1} + m_{q2})}$.

It is easy to find that contributions from $\Phi_{B2}$ to the WA amplitudes are nonzero, because the moment parameter $\langle \xi \rangle$$_{B2}$ is nonzero. Hence, $\Phi_{B2}$ may present nontrivial effects on the observables of hadronic $B$ decays, especially for the WA dominant ones.

In order to better investigate the $\Phi_{B2}$ contributions and eliminate other pollution, the pure WA decays $B_d \rightarrow K^+ K^-$ and $B_s \rightarrow \pi^+ \pi^-$ will be restudied in this paper. Although their branching ratios are tiny, they have been measured accurately by now [27].

$$\mathcal{B}(B_s \rightarrow \pi^+ \pi^-) = (6.7 \pm 0.8) \times 10^{-7},$$

(11)
\[ \mathcal{B}(B_d \rightarrow K^+K^-) = \left(8.0 \pm 1.5\right) \times 10^{-8}. \] (12)

With the asymptotic twist-2 and -3 DAs, \( \Phi_P(u) = 6 u \bar{u} \) and \( \Phi_P^0(u) = 1 \), the integrals in Eq.(8-10) exhibit logarithmic and linear infrared divergences. For an estimation of the WA contributions from \( \Phi_{B2} \), these divergent endpoint integrals will be parameterized by the commonly used notations within the QCDF approach [8, 23, 26].

\[ \int_0^1 \! \! \frac{du}{u} \rightarrow X_A, \quad (13) \]
\[ \int_0^1 \! \! \frac{du}{u^2} \rightarrow X_L, \quad (14) \]
\[ \int_0^1 \! \! \frac{\ln u}{u} \rightarrow -\frac{1}{2} X_A^2, \quad (15) \]
\[ \int_0^1 \! \! \frac{\ln u}{u^2} \rightarrow X_L - X_A - X_L X_A. \quad (16) \]

The phenomenological parameters \( X_A \) and \( X_L \), are usually treated as universal for hadronic \( B \) decays in previous literatures [8, 23–26].

With the above parameterization scheme, the WA amplitudes can be rewritten as

\[ A_{1,B2}^i = -A_{B2}^B \left\{ 12 \left( \pi^2 - 6 \right) r_P r_P \left[ \frac{\pi^2}{6} + \frac{1}{2} X_A^2 + X_L X_A + X_A - X_L \right] \right\}, \quad (17) \]
\[ A_{2,B2}^i = A_{B2}^B \left\{ 72 \left( X_A - 1 \right)^2 r_P r_P \left[ \frac{\pi^2}{6} + \frac{1}{2} X_A^2 + X_L X_A + X_L^2 \right] \right\}, \quad (18) \]
\[ A_{3,B2}^i = -A_{B2}^B \left\{ \left[ \frac{\pi^2}{6} + \frac{1}{2} X_A^2 - X_A \right] \left( 2 r_P^2 - r_P^2 \right) + r_P^2 \left( X_A X_L - X_L + 1 \right) \right\}. \quad (19) \]

The parameters of \( X_A \) and \( X_L \) including part of strong phases are complex, and are usually parameterized as [8, 23–26]

\[ X_A = (1 + \rho_A e^{i\phi_A}) \frac{m_B}{\Lambda_h}, \quad (20) \]
\[ X_L = (1 + \rho_A e^{i\phi_A}) \frac{m_B}{\Lambda_h}, \quad (21) \]

where \( \Lambda_h = 0.5 \text{ GeV} \) [8, 23], and \( \phi_A \) is an undetermined strong phase. In addition, according to the relations given by Refs.[16, 17], the moment parameter in Eq.(7) is

\[ \langle \xi \rangle_{B2} = \frac{1}{2} \left( \langle \xi \rangle_+ - \langle \xi \rangle_- \right) = \frac{\Lambda}{3 m_b}, \quad (22) \]
with \( \langle \xi \rangle_+ = 2 \langle \xi \rangle_- = \frac{4}{3} \frac{A}{m_b} \) and \( \bar{A} = m_B - m_b \approx 0.55 \text{ GeV} \) [16]. Using the exponential type model for \( B \) meson DAs

\[
\phi^{+}_{B_q}(\xi) = N^+ \xi \exp(-\frac{\xi m_{B_q}}{\omega_{B_q}}), \tag{23}
\]

\[
\phi^{-}_{B_q}(\xi) = N^- \exp(-\frac{\xi m_{B_q}}{\omega_{B_q}}), \tag{24}
\]

where \( N^\pm \) is the normalization constant determined via \( \int_0^1 \phi^\pm_{B_q}(\xi)d\xi = 1 \), one can obtain \( \langle \xi \rangle_{B_2} = 0.042 \pm 0.01 \) with the shape parameter \( \omega_{B_q} = 0.45 \pm 0.10 \text{ GeV} \) for \( B_s \) meson [28], and \( \langle \xi \rangle_{B_2} = 0.039 \pm 0.01 \) with \( \omega_{B_d} = 0.42 \pm 0.10 \text{ GeV} \) for \( B_d \) meson [22], which are basically in agreement with the estimation of Eq.\( \langle 22 \rangle \).

Using the commonly used notations in the QCDF approach [8, 23–26], the amplitudes for the pure WA decays \( B_d \to K^+K^- \) and \( B_s \to \pi^+\pi^- \) are written as

\[
\mathcal{A}(B_s \to \pi^+\pi^-) = i \frac{G_F}{\sqrt{2}} f_{B_s} f_{\pi} \left\{ V_{ub} V_{us} \left[ b_1 + 2 b_4 + \frac{1}{2} b_{4,EW} \right] + V_{cb} V_{cs} \left( 2 b_4 + \frac{1}{2} b_{4,EW} \right) \right\}, \tag{25}
\]

\[
\mathcal{A}(B_d \to K^+K^-) = i \frac{G_F}{\sqrt{2}} f_{B_d} f_{K} \left\{ V_{ub} V_{ud} \left[ b_1 + 2 b_4 + \frac{1}{2} b_{4,EW} \right] + V_{cb} V_{cd} \left( 2 b_4 + \frac{1}{2} b_{4,EW} \right) \right\}, \tag{26}
\]

where the Fermi weak coupling constant \( G_F \approx 1.166 \times 10^{-5} \text{ GeV}^{-2} \) [1]; \( f_{B_q} \), \( f_{\pi} \) and \( f_K \) are decay constants; \( V_{ij} \) (\( i = u, c \) and \( j = d, s, b \)) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element. The definition of parameter \( b_i \) is

\[
b_1 = \frac{C_F}{N_c^2} C_1 A_i^1, \tag{27}
\]

\[
b_4 = \frac{C_F}{N_c^2} \left[ C_4 A_i^1 + C_6 A_i^2 \right], \tag{28}
\]

\[
b_{4,EW} = \frac{C_F}{N_c^2} \left[ C_{10} A_i^1 + C_8 A_i^2 \right], \tag{29}
\]

where \( C_F = 4/3 \) is the color factor; \( N_c = 3 \) is the number of colors; \( C_i \) is the Wilson coefficient; \( A_i^k \) is the amplitude building block of Eq.\( \langle 6 \rangle \).

To provide a quantitative estimate of the \( \Phi_{B_2} \) contributions, the inputs listed in Table.I are used in our numerical calculation. Their central values will be regarded as the default inputs unless otherwise specified.

The constraints on annihilation parameters from data are illustrated in Fig.3. It is clearly seen from Fig.3(a) that it is impossible to accommodate simultaneously \( B_d \to K^+K^- \) and \( B_s \to \pi^+\pi^- \) decays within 2\( \sigma \) errors with the same values of \( \rho_A \) and \( \phi_A \) when the \( \Phi_{B_2} \) contributions are overlooked. Other studies of \( B \) decays, such as Refs.[23, 29], have uncovered
TABLE I: The input parameters [1].

| Parameter | Value |
|-----------|-------|
| $m_b$ | 4.78±0.06 GeV |
| $\bar{m}_b$ | 4.18$^{+0.04}_{-0.03}$ (2 GeV) |
| $\bar{m}_s(2 \text{ GeV})$ | 95±5 MeV |
| $m_{B_d}$ | 5279.63±0.15 MeV |
| $f_{B_d}$ | 187.1±4.2 MeV |
| $\tau_{B_d}$ | 1.520±0.004 ps |
| $m_{B_s}$ | 5366.89±0.19 MeV |
| $f_{B_s}$ | 227.2±3.4 MeV |
| $\tau_{B_s}$ | 1.509±0.004 ps |

FIG. 3: The contour plots of branching ratios of $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ decays as functions of the annihilation parameters $\rho_A$ and $\phi_A$ without and with the $\Phi_{B^2}$ contributions in (a) and (b), respectively. The solid curves correspond to the central values of data and the bands correspond to the 2 $\sigma$ constraints.

similar results. It seems not easy to clarify discrepancies between data and the QCDF results with the same set of parameters $\rho_A$ and $\phi_A$. To clam down this situation, the factorizable and nonfactorizable annihilation parameters corresponding to different topologies are introduced in Refs.[30, 31]. However, more annihilation parameters make the method uneconomical and unsatisfactory. Interestingly, by including the $\Phi_{B^2}$ contributions, Fig.3(b) shows overlapping areas of annihilation parameters, which implies that the $\Phi_{B^2}$ contributions are nontrivial for accommodating the tension between data and QCDF predictions for $B(B_d \to K^+K^-)$ and $B(B_s \to \pi^+\pi^-)$. In addition, if theoretical uncertainties from inputs are taken into account, the overlapping bands will be inevitably enlarged. The same annihilation parameters suitable for pure WA hadronic $B$ decays might be obtained with the QCDF approach.
TABLE II: The $CP$-averaged branching ratios in the unit of $10^{-7}$, where the theoretical uncertainties are from the input parameters listed in Table I. Different scenarios are explained in the text.

| decay mode | our results | Ref. [23] |
|------------|-------------|-----------|
| $B_s \to \pi^+\pi^-$ | $A_{k}^{i,B2} = 0$ | 1.49 |
| $B_d \to K^+K^-$ | $A_{k}^{i,B2} = 0$ | 0.79 |

As is shown by Fig.3(b), strict limits on annihilation parameters $\rho_A$ and $\phi_A$ can not be obtained only from experimental data on $B(B_d \to K^+K^-)$ and $B(B_s \to \pi^+\pi^-)$. In principle, considering more $B$ decays, such as a global fit on nonleptonic $B$ decays in Refs.[30, 31], is helpful for extracting the informations of annihilation parameters. However, for many hadronic $B$ decays, other contributions, such as spectator scattering interactions, will complicate the determination of annihilation parameters. How to get annihilation parameter spaces as compact as possible from data is beyond the scope of this paper.

TABLE III: The values of $A_{k}^{i}$ in Eq.(17-19). See text for explanations of different scenarios.

| decay mode | scenario | $A_{k}^{i,B2} = 0$ | $A_{k}^{i,B2} \neq 0$ |
|------------|----------|---------------------|---------------------|
| $B_s \to \pi^+\pi^-$ | S1       | $A_{1}$ 116         | $A_{1}$ 117         |
|            | S2       | 103 $-i$ 64        | 104 $-i$ 67        |
| $B_d \to K^+K^-$ | S1       | $A_{1}$ 115         | $A_{1}$ 116         |
|            | S2       | 102 $-i$ 64        | 103 $-i$ 67        |

It is seen from Fig.3(b) that, in general, the value of $\rho_A$ increase with the increasing value of $|\phi_A|$. A large value of parameter $\rho_A$ will spoil the self-consistency and confidence level of the QCDF approach, and $\rho_A \leq 1$ is proposed in Refs.[8, 23]. The strong phase $\phi_A$ describes the rescattering among hadrons and relates closely to $CP$ violation of nonleptonic $B$ decays. Focusing on the pure WA decays of $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$, to roughly estimate branching ratios, two scenarios based on Fig.3(b) are considered in our numerical
calculation. Scenario S1 is with parameters $\rho_A = 1$ and $\phi_A = 0^\circ$, and scenario S2 is with $\rho_A = 1.2$ and $\phi_A = -40^\circ$. Practically, for the scenario S1, it is intuitive that zero strong phase $\phi_A$ seems a little unnatural. Trying to combine the value of $\rho_A$ as close to one as possible with a nonzero $\phi_A$, the scenario S2 is considered. In addition, the scenario S2 is comparable with the scenario S3 of Ref.[23], where the “universal annihilation” parameters $\rho_A = 1$ and $\phi_A = -45^\circ$ are used.

Using such inputs, we list the QCDF results for $\mathcal{B}(B_d \rightarrow K^+K^-)$ and $\mathcal{B}(B_s \rightarrow \pi^+\pi^-)$ with and without considering the $\Phi_{B2}$ contributions in Table II, in which the theoretical predictions of scenario S3 of Ref.[23] and experimental data are also listed for convenience of comparison. In order to show the effects of $\Phi_{B2}$ much more clearly, we collect the numerical results of $A^k_i$ in Table III.

From Table II, it can be found that: (i) The experimental data for both $B_d$ and $B_s$ decays can not be well explained simultaneously by QCDF approach without considering the $\Phi_{B2}$ contributions; (ii) The numerical difference between the case for $A^{i,B2}_k = 0$ of scenario S2 and scenario S3 of Ref.[23] arises from different inputs, such as decay constants, the CKM parameters and so on, besides parameters $\rho_A$ and $\phi_A$. (iii) With the scenario S2, the $\Phi_{B2}$ contributions present about 60% and 20% corrections to $\mathcal{B}(B_d \rightarrow K^+K^-)$ and $\mathcal{B}(B_s \rightarrow \pi^+\pi^-)$, respectively, which significantly improve the QCDF predictions and can explain the data within uncertainty.

The results in Table II show that $\Phi_{B2}$ contributions to nonfactorizable WA amplitude building blocks $A^i_k$ are small, due to the small moment $\langle \xi \rangle_{B2}$. In addition, according to the conventions of Refs.[8, 23], building block $A^3_3$ is always accompanied by the small value of Wilson coefficient $C_5$. Hence, on one hand, the dominant contributions to WA amplitudes come from $\Phi_{B1}$ part; on the other hand, to some certain extent, the $\Phi_{B2}$ contributions present un-negligible correction to the amplitude especially for the pure annihilation decay modes and can improve the performances of the QCDF approach.

In summary, the improvements of measurement precision with the running Belle-II and LHCb experiments call for the refinements of theoretical calculation on hadronic $B$ weak decays. For the $B$ mesons, there are two scalar DAs $\Phi_{B1}$ and $\Phi_{B2}$. The $\Phi_{B2}$ contributions to formfactors and branching ratios can be significant for some cases with the pQCD approach. In this paper, we study the $\Phi_{B2}$ contributions with the QCDF approach, and find that for the $B \rightarrow PP$ decays, they can be safely neglected in the spectator scattering amplitudes, and
contribute to only the nonfactorizable WA amplitudes. The $\Phi_{B2}$ contributions to WA amplitudes are small compared with the dominant $\Phi_{B1}$ contributions, due to the small moment $\langle \xi \rangle_{B2}$. However, the participation of $\Phi_{B2}$ plays a positive role in accommodating the pure WA decays $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ to data with the universal annihilation parameters $\rho_A$ and $\phi_A$. The values of annihilation parameters $\rho_A$ and $\phi_A$ with the QCDF approach have been under discussion for a long period. More information about WA parameters $\rho_A$ and $\phi_A$ could be obtained by a comprehensive study on nonleptonic $B$ decays.

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[1] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D 98, 030001 (2018).
[2] Ed. A. Bevan, B. Golob, Th. Mannel *et al.*, Eur. Phys. J. C 74, 3026 (2014).
[3] K. Kou *et al.*, arXiv:1808.10567.
[4] I. Bediaga *et al.* (LHCb Collaboration), arXiv:1808.08865.
[5] Ed. A. Cerri, V. Gligorov, S. Malvezzi *et al.*, arXiv:1812.07638.
[6] M. Beneke, G. Buchalla, M. Neubert, C. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999).
[7] M. Beneke, G. Buchalla, M. Neubert, C. Sachrajda, Nucl. Phys. B 591, 313 (2000).
[8] M. Beneke, G. Buchalla, M. Neubert, C. Sachrajda, Nucl. Phys. B 606, 245 (2001).
[9] D. Du, D. Yang, G. Zhu, Phys. Lett. B 488, 46 (2000).
[10] D. Du, D. Yang, G. Zhu, Phys. Lett. B 509, 263 (2001).
[11] D. Du, D. Yang, G. Zhu, Phys. Rev. D 64, 014036 (2001).
[12] Y. Keum, H. Li, Phys. Rev. D 63, 074006 (2001).
[13] Y. Keum, H. Li, A. Sanda, Phys. Lett. B 504, 6 (2001).
[14] Y. Keum, H. Li, A. Sanda, Phys. Rev. D 63, 054008 (2001).
[15] C. Lü, K. Ukai, M. Yang, Phys. Rev. D 63, 074009 (2001).
[16] A. Grozin, M. Neubert, Phys. Rev. D 55, 272 (1997).
[17] M. Beneke, Th. Feldmann, Nucl. Phys. B 592, 3 (2001).
[18] S. Descotes-Genon, C. Sachrajda, Nucl. Phys. B 625, 239 (2002).
[19] Z. Wei, M. Yang, Nucl. Phys. B 642, 263 (2002).
[20] T. Huang, X. Wu, Phys. Rev. D 71, 034018 (2005).
[21] C. Lü, M. Yang, Eur. Phys. J. C 28, 515 (2003).
[22] T. Kurimoto, Phys. Rev. D 74, 014027 (2006).
[23] M. Beneke, M. Neubert, Nucl. Phys. B 675, 333 (2003).
[24] D. Du, H. Gong, J. Sun, D. Yang, G. Zhu, Phys. Rev. D 65, 074001 (2002).
[25] D. Du, H. Gong, J. Sun, D. Yang, G. Zhu, Phys. Rev. D 65, 094025 (2002).
[26] M. Beneke, J. Rohrer, D. Yang, Nucl. Phys. B 774, 64 (2007).
[27] Data updated in April 2019 by HFLAV group, see https://hflav.web.cern.ch.
[28] J. Sun, Z. Xiong, Y. Yang, G. Lu, Eur. Phys. J. C 73, 2437 (2013).
[29] K. Wang, G. Zhu, Phys. Rev. D 88, 014043 (2013).
[30] Q. Chang, J. Sun, Y. Yang, X. Li, Phys. Rev. D 90, 054019 (2014).
[31] Q. Chang, J. Sun, Y. Yang, X. Li, Phys. Lett. B 740, 56 (2016).