Traffic model by braking capability and response time

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Abstract. We propose a microscopic traffic model where the update velocity is determined by the deceleration capacity and response time. It is found that there is a class of collisions that cannot be distinguished by simply comparing the stop positions. The model generates the safe, comfortable, and efficient traffic flow in numerical simulations with a reasonable values of the parameters, and this is analytically supported. Our approach provides a new perspective in modeling traffic-flow safety and worrying situations like lane changing.

Keywords: traffic and crowd dynamics, traffic models
1. Introduction

The modeling of traffic flow has been an intensive research topic for more than a half century in the engineering and science communities [1–11], of which results are summarized in the reviews [12–15]. In the progress of information technology, various traffic models are required for the control and/or the automation of traffic flow. The need is basically for the credible modeling of safety and mobility, the two categorical but conflicting goals in driving. Thus, the natural driving behaviors have been modeled, for example, for more (less) acceleration for larger (smaller) spacing. However, this is usually based on the trial-function approach, as criticized in [12].

There are a few seminal works that do not use trial function; one is the Gipps (collision-avoidance) model [7,12] and another one is the Nagel–Schreckenberg (minimal collision-free) model [8,13]. In the Nagel–Schreckenberg model, any magnitude of deceleration is applied when required to prevent a collision. This means the deceleration capacity is actually unbounded (the so-called intelligent-braking-behavior suggested in [11] also belongs to this case). Meanwhile, in the Gipps model, a collision-avoidance in bounded deceleration capacity was suggested. We consider this approach is more physical, and thus we adopt it in the present work.

In this work, we examine how collisions can be understood within the physical constraints of deceleration capacity and response time. We propose a microscopic traffic model where the update velocity is determined in the safety criterion by these constraints. It is found that there is a class of collisions that cannot be identified by the usual...
Figure 1. Two types of safety criterion in an emergency when the vehicles decelerate by their own braking capacities until they stop: the Follower’s deceleration as a response to the emergency is delayed by the response time $\tau_n$ (the dotted segment from $t$ and $t + \tau_n$ is not the part of the emergency). The curvature of the trajectory is given by the associated braking capacity. In criterion (a), the two stop positions are compared to demonstrate a collision. In criterion (b), the blue and black solid trajectories meeting though the follower’s stop position do not exceed the leader’s (this is possible only when the follower’s braking capability is stronger than the leader’s). Note this kind of collision cannot be distinguished by comparing the stop positions (see the red dashed curve).

safety criterion comparing emergency-stop positions. The resultant model generates in a numerical test the practical and appealing traffic flow of safety, efficiency, and comfort with reasonable parameter values, and this is analytically supported. Our model also provides a new perspective in modeling traffic-flow safety and the worrying situations like lane changing.

2. Modeling

Since the safety and mobility conflict with each other, a compromise between them is necessary. A natural one is a condition where the driver can marginally avoid collision against the leader’s full stop. Let $x_n(t)$ and $v_n(t)$ be the (front-end) position and velocity at time $t$, respectively, of vehicle $n$. The safety criterion is asking, in the presence of response time $\tau_n$, what is the marginal $x_n(t + \tau_n)$ and $v_n(t + \tau_n)$, which does not result in a collision with the maximum deceleration $D_n$ from $t + \tau_n$, if the front vehicle at $x_{n+1}(t)$ and $v_{n+1}(t)$ begins to decelerate with its maximum deceleration $D_{n+1}$ from $t$ to stop. In short, this asks whether the worst situation is manageable within the physical constraint of braking capability and response time. Obviously, such a worst case scenario may not happen. But it is necessary to check whether the follower can keep safe in that situation with its braking capacity and response time.

One of the safety criteria is shown in figure 1(a), where $x_n(t + \tau_n)$ and $v_n(t + \tau_n)$ are adjusted so that the two trajectories become tangential as the two vehicles stop. It basically compares the two vehicles’ stop positions to demonstrate a collision. This is
same to that considered in the Gipps model [7] believed so far to provide a safe enough dynamics. Here, we point out that this safety criterion only is incomplete. This is because there is another kind of collision that cannot be discerned by comparing the stop positions, as follows.

The other kind is shown in figure 1(b), which is possible only when \( D_n > D_{n+1} \).

In this case, since the follower’s trajectory is bent more strongly than the leader’s, the match of the stop positions (see the red-dashed curve) necessarily brings about a collision before stopping. This collision is, however, not distinguished by simply comparing the stop positions. It is thus necessary to reconsider the configuration at \( t + \tau_n \). The follower’s blue solid trajectory in figure 1(b) is the alternative, which is adjusted to be tangential to the trajectory of the leader still in movement. We remark that, even for \( D_n > D_{n+1} \), there is a situation where the criterion of figure 1(a) should still apply, for example, if the follower is not so close to the leader at time \( t \).

\[
x_n(t + \tau_n) \text{ and } v_n(t + \tau_n) \text{ are related by the position-update rule. When the scheme of constant acceleration between the responses is used, the position update reads}
\]

\[
x_n(t + \tau_n) = x_n(t) + \frac{\tau_n}{2} [v_n(t) + v_n(t + \tau_n)].
\]

(1)

Considering this in the two tangential conditions explained above, as the two marginal velocities at \( t + \tau_n \), one can obtain

\[
v_n^d(t + \tau_n) = -\frac{\tau_n D_n}{2} + \sqrt{\left(\frac{\tau_n D_n}{2}\right)^2 + D_n \left(2s_n(t) - \tau_n v_n(t) + \frac{v_{n+1}^2(t)}{D_{n+1}}\right)},
\]

(2)

\[
v_n^d(t + \tau_n) = v_{n+1}(t) - \frac{\tau_n}{2} (D_n + D_{n+1}) + \sqrt{\left(\frac{\tau_n \Delta D_n}{2}\right)^2 - \Delta D_n (2s_n(t) + \tau_n \Delta v_n(t))},
\]

where \( \Delta D_n \equiv D_{n+1} - D_n, \Delta v_n(t) \equiv v_{n+1}(t) - v_n(t), \) and \( s_n(t) = x_{n+1}(t) - x_n(t) - L_{n+1} \) for the leading vehicle’s length \( L_{n+1} \).

\( v_n^s(t + \tau_n) \) is enough to tell a collision when \( D_n \leq D_{n+1} \), while it is not when \( D_n > D_{n+1} \). Thus, in the latter case, one of \( v_n^s(t + \tau_n) \) or \( v_n^d(t + \tau_n) \) should be selected depending on the situation. Although figure 1(b) shows a situation where \( v_n^d(t + \tau_n) \) should be selected, this is not always the case. If the follower is not so close to the leader, one can easily argue that \( v_n^s(t + \tau_n) \) is instead the proper choice. This way, considering a few conditions, one knows the candidate of the update velocity is given by

\[
v_n^{\text{cand}}(t + \tau_n) = \begin{cases} 
v_n^d(t + \tau_n) & \text{if } D_n > D_{n+1}, \frac{v_n^d(t + \tau_n)}{D_n} \text{ is real, and } v_n^d(t + \tau_n) < \frac{v_{n+1}(t)}{D_{n+1}}, \\
v_n^s(t + \tau_n) & \text{else if } v_n^s(t + \tau_n) \text{ is real}, \\
v_n(t) - \tau_n D_n^+ & \text{otherwise},
\end{cases}
\]

(3)

where the last case is introduced to cover such a situation allowing no physically meaningful \( v_n^d(t + \tau_n) \) and \( v_n^s(t + \tau_n) \). For example, a careless cutting-in may bring it about. This means there is no way to avoid a collision if the one cutting in brakes maximally until stopping. \( D_n^+ \) is introduced as an indicator of such an emergency that requires a deceleration larger than \( D_n \), which is not possible by the definition of \( D_n \). In order to simply cover that situation in the model, one may assign a value larger than \( D_n \) to \( D_n^+ \).

We anticipate that the third case is crucial in modeling (in)tolerable perturbations.
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\( v_{n}^{\text{cand}}(t + \tau_n) \) in equation (3) is still the candidate velocity of the next step because its realizability from the current velocity is not taken into account yet. The realizability is determined in the vehicular performance represented by the deceleration and acceleration capacities. Thus, when an acceleration capacity \( A_n \) is additionally introduced, the realizability corresponds to \( -D_n \leq \frac{(v_{n}^{\text{cand}}(t + \tau_n) - v_n(t))/\tau_n \leq A_n} \), named as ‘mechanical restriction’ [16]. When a velocity change exceeding this range is required, only \( -\tau_n D_n \) or \( \tau_n A_n \) is possible by the definition of \( D_n \) and \( A_n \). Finally, considering the traffic regulation also, we arrive at

\[
v_n(t + \tau_n) = \max\{v_n(t) - \tau_n D_n, 0, \min\{v_n(t) + \tau_n A_n, v_{\text{max}}, v_{n}^{\text{cand}}(t + \tau_n)\}\},
\]

where zero stands for the directionality and \( v_{\text{max}} \) is the speed limit. This gives the velocity update with equations (2) and (3), and then the position update is followed in equation (1). The update rule is applied in parallel to all the vehicles in the system.

3. Manageability and potential collision

Before analyzing our new model, we discuss a few implications of equation (4). The interest is in the case when

\[
v_{n}^{\text{cand}}(t + \tau_n) \geq v_n(t + \tau_n).
\]

We below call a vehicle holding equation (5) manageable at time \( t \). The inequality says the realization of \( v_{n}^{\text{cand}}(t + \tau_n) \) is possible in the braking capacity \( D_n \). This again implies, if a deceleration is required for safety, it is realizable. Interestingly, the manageability at \( t \) lasts thereafter unless perturbed later. This is attributed to the fact that \( v_{n}^{\text{cand}}(t + \tau_n) \) are constructed in a way to keep safe against the leader’s worst behavior; the consecutive maximal braking to stop (if already stopped, it is assumed not to be moving). Thus, once all the vehicles in a system are manageable, it lasts forever and the traffic flow remains collision-free, as long as no perturbation is applied. A closed system composed of vehicles initially at rest is such an example.

As a perturbation, one may consider the insertion of a vehicle into a gap between two vehicles. When the insertion takes place, it is reasonable to examine the manageability of the follower and that of the new comer. One may call it manageable insertion when the two vehicles are manageable at that instant. We consider the notion of manageable insertion is crucial in modeling on-ramp and/or lane-change. Also, a strategy to the dilemma zone by the traffic signal can be examined as considering the insertion of a standing object. This way, the manageability may shed light on designing and/or modeling (in)tolerable traffic perturbations.

From the other perspective, the non-manageability (violation of equation (5)) can give a measure for safety indicating a possible collision. The situation of non-manageability results in a collision if the leader really applies the maximal brake to stop. Thus the statistics on the non-manageability can be a reasonable measure for the traffic-flow safety. Note the flow including non-manageable configurations does not necessarily result in a collision. Therefore, the flow without a collision can be regarded as dangerous in our approach. We believe this viewpoint should be applied to real traffic. It is necessary to discern a traffic flow with potential collisions so as to prevent a traffic accident in advance.
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Figure 2. Three phases. (a) The location-speed snapshots for $\tau = 1$ section of hS flow ($\rho = 30 \text{veh km}^{-1}$, $m_D = 5 \text{m s}^{-2}$), fS flow ($\rho = 68$, $m_D = 5$), and jam phase ($\rho = 97$, $m_D = 10$). (b) The phase diagram in the parameter space ($\rho, m_D$) for $\tau = 1$. Dotted (solid) lines are numerically (analytically) obtained phase boundaries.

4. Numerical result and analytic support

The present model is a consequence of the physical meaning of $D_n$ and $\tau_n$. We thus examine the flow property while varying them in a numerical study. For the other model parameters, we use the typical values; $v_{\text{max}} = 110 \text{km h}^{-1}$, $A_n = 1.5 \text{m s}^{-2}$, and $L_n = 7.5 \text{m}$ [16,17]. A system of randomly distributed vehicles initially at rest on a 100 km-long circular road is tested for various vehicular density $\rho$ (the total number of vehicles divided by the road length). For $D_n$, a random value out of the interval $(m_D - w_D, m_D + w_D)$ is assigned for various $m_D$ and fixed $w_D = 1.5 \text{m s}^{-2}$ ($w_D$-value turns out not to change the results qualitatively). Below, we will use $\tau_n = \tau$ for all $n$ for a simplicity of the numerical implementation.

We observe that there emerge three kinds of steady state traffic flows depending on $m_D$, $\tau$, and $\rho$. Figure 2(a) shows the three flows with the position-velocity snapshots, where each dot represents a car. One of the snapshots (circles) shows an almost flat velocity profile, and we name it ‘homogeneous steady’ (hS) flow. Another snapshot (squares) exhibits fluctuating velocities, named ‘fluctuating steady’ (fS) flow. Meanwhile, the last one (crosses) shows a traffic jam (J) where the vehicles can hardly move. Figure 2(b) shows the phase diagram on $\rho$-$m_D$ plane for the $\tau = 1$ section, where the dotted (solid) curves are the numerically (analytically) obtained phase boundaries. We observe that the boundary between hS and fS is robust, while the region for the J phase depends on the initial configuration. In the following, we analytically argue that the observation above is the intrinsic feature of our model.

The hS flow is a homogeneous-velocity solution (HVS) of the model. Considering a constant velocity $v$ in equation (3) for all $n$, one can find (see appendix A)

$$\frac{\rho^{-1} - L}{\tau^2 \eta} = \begin{cases} \frac{v}{\tau \eta} & \text{for } \frac{v}{\tau \eta} \leq 1, \\ f \left( \frac{v}{\tau \eta} \right) & \text{for } \frac{v}{\tau \eta} > 1 \end{cases}$$

(6)
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Figure 3. Density-velocity relation and deceleration. (a) Scaled density-velocity relation for $m_D = 5$ m s$^{-2}$ and various $\tau = 1, 1/2, 1/4, 1/8, 1/16$ section. The solid line is the analytic curve by equation (6). Each point is the 10 000-section average of the vehicles’ velocities in steady state for a given density (all the points in figures 3 and 4 are obtained in the same way). The bar shows the fluctuation of the velocity. (b) Average, rms (root-mean-square) fluctuation, and maximum of the decelerations in the data giving (a) (each deceleration is measured in the ratio to the deceleration capacity).

for $\eta^{-1} \equiv (m_D - w_D)^{-1} - (m_D + w_D)^{-1}$, where $f(z) = \left[ \int_{1/z}^{1} (z - z^2 y/2) + \int_{1/z}^{1} 1/2y \right] p(y) dy$ for $p(y)$ the probability distribution of the random variable $y$ standing for $\eta(1/D_n+1 - 1/D_n)$ (equation (A22) is the details of $f(z)$ for the $D_n$s we use). As a general property indifferent to the statistics of $D_n$s, $f(z)$ is increasing and convex downward, and $f(z) \to z$ for $z \to 1$ while $f(z) \sim z^2$ for $z \gg 1$ (see the two limiting behaviors of the red curve (and also data points) in the upper-right part of figure 3(a)).

The solid curve in figure 3(a) is equation (A22) (a realization of equation (6) for the $D_n$s we use). The numerical data for hS are perfectly on it above $v/\tau \eta = 1$. Therein, the circles and bars are, respectively, the velocity averages and fluctuations (note the latter is small enough to be covered in the data points for average). Interestingly, the average velocity of the numerical data for fS (diamonds) are also on the curve on the other side, even though there are considerable fluctuations as indicated by the bars. This suggests fS can also be understood with HVS. Below, we demonstrate that the dynamic property of HVS can explain these observations.

We performed the linear stability analysis [18] on HVS and find this is linearly stable, including marginal stability, regardless of density and model parameters (see appendix B). When $v/\tau \eta > 1$, there are at most two marginally stable modes out of the total $2N$ stable eigenmodes, where $N$ is the number of vehicles. Otherwise with $v/\tau \eta \leq 1$, a half of the total modes are marginally stable. Thus, when HVS is realized with $N \gg 1$ with $v/\tau \eta > 1$, it readily shows an almost uniform velocity over the whole system, while, with $v/\tau \eta \leq 1$, it may exhibit the fluctuations attributed to the macroscopic number of marginal modes.
This dynamic property is consistent with the numerical observation on hS and fS shown in figure 2(a).

It is worth noting that the stability boundary of \( v/\tau \eta = 1 \) is identical to that of the numerical phase boundary shown in figures 2(b) and 3(a). When \( v \) is replaced with \( \rho \) using equation (6), \( (\rho^{-1} - L)/\tau^2 \eta = 1 \) is immediate. This is the phase boundary (solid curve) shown in figure 2(b), for \( \tau = 1 \). The macroscopic number of marginal modes in fS can explain the observation of J in the fS-region (see figure 2(b)). Since the findings so far hold for any \( \tau \) and the statistics of \( D_n \), we conclude that hS and fS are the dynamic phases of our model. We finally emphasize the phase boundary condition of \( v/\tau \eta = 1 \) is the same as the condition where at least one vehicle follows \( v^d \). The flow established in the presence of such a vehicle is the very hS that is much more stable than fS. This indicates that \( v^d \) unrecognized in the earlier models plays a significant role in stabilizing the whole system.

5. Comfort and flux

In the following, we examine our model generating a practically appealing traffic flow. If the traffic flow is safe, one of the next concerns is driving comfort, which is required for autonomous driving systems [19,20]. For this, we measure the decelerations each vehicle’s experience in the simulation for figure 3(a). Each deceleration is measured in the ratio to the deceleration capacity. The results are shown in figure 3(b). We obtain three statistical observables; the average, fluctuation (root-mean-square), and maximum of the ratios. The average is approximately 0.1 and 0.001 \( \sim 0.01 \) in the hS and fS flows, respectively, and the root-mean-square shows similar values. The maximum is around 0.01 \( \sim 0.1 \) and 0.2 \( \sim 0.7 \), respectively. We remark the deceleration strength is a characteristic of the flow phase as observed, and thus driving comfort can be considerably improved by promoting hS with smaller \( \tau \) (see figure 4).

The other practical interest is probably the flow efficiency, which can be represented by vehicular flux. For a homogeneous-velocity solution \( v \), the flux is \( \rho v \) by the hydrodynamic relation [14]. Since \( 2f(z) > zf'(z) \) as a general property of \( f(z) \) (appendix A), the steady-state velocity \( v \) increases as \( \tau \) decreases. The velocity-increase for smaller \( \tau \) is drawn in figure 4(a) along each constant-\( \rho \) curve. This gives the flux-increase, as shown in figure 4(b). Since \( f(z) \sim z^2 \) for large \( z \), the flux converges to

\[
C\sqrt{\rho(1/L - \rho)}
\]

in the \( \tau \rightarrow 0 \) limit (see the upper-bounding solid curve in figure 4(b)), where \( C \) is a constant by the statistics of \( D_n \) (see equation (A26) for the detail). We consider this result is also appealing because (i) the flux for \( \tau = 1 \) section (the typical response time of drivers [21]) is comparable to the empirical maximum value of around 2500 veh/h [22–24], (ii) the flux enhancement is more sensitive for larger \( \tau \), (iii) the flux becomes considerable for \( \tau \) of the 0.1-section-order, and (iv) all these are achieved in the manageable

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\(^5\) Whether a marginal property in linearized system will be maintained in the original nonlinear system is nontrivial. In fact, however, fS data reveals the period of \( N\tau \) (not shown here). This strongly suggests that fS phase is the combination of the periodic orbits deformed from the periodic marginal modes in the linearized system [18].
Figure 4. Effect of response time. (a) Average speed as a function of the density $\rho$ and the response time $\tau$ when $m_D = 5\text{ ms}^{-2}$. The plateau in the upper-left corner is the speed limit $v_{\text{max}}$. The solid curve separating the data points type is the phase boundary between $hS$ and $fS$, and the dashed curve below is its projection onto the $\rho$-$\tau$ plane. (b) The projection of (a) onto the density-flux plane for $\tau = 1, 1/2, 1/4, 1/8, \text{ and } 1/64$ section The upper-bounding curve is obtained in the $\tau \to 0$ limit (see equation (7)). All the curves in (a) and (b) are analytic results.

condition of equation (5) guaranteeing safety. It is worth noting that the research field of autonomous driving systems has already been treating the processing time down to 0.1 section [19,20].

6. Final remarks

We finally remark that an extension of our model, in order to cover the other features of traffic flow (lane change, on-ramp flow, traffic signal, and so forth), is straightforward. This is because the problem is still a compromise between safety and mobility in consideration of the positions, velocities, and deceleration capacities of the related objects. Also, our model provides a new perspective to the study of traffic-flow safety through the interpretation of the non-manageable events and its statistics. Besides, we expect an autonomous driving system based on our model can be possible in the solid safety criterion and in the appealing traffic-flow quality of comfort and flux.

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Appendix A. Homogeneous solution

Let velocity $v$ be the homogeneous solution of our model. Substituting it for all the velocities in equation (2), we obtain the optimal spacing as

$$S_n = \begin{cases} \tau v - \frac{1}{2} \left( \frac{1}{D_{n+1}} - \frac{1}{D_n} \right) v^2 & \text{for } \frac{1}{D_{n+1}} - \frac{1}{D_n} \leq \frac{\tau}{v}, \\ 0 & \text{for } \frac{1}{D_{n+1}} - \frac{1}{D_n} > \frac{\tau}{v}. \end{cases}$$

(A1)

Then the average spacing is $S = \sum_{n=1}^{N} S_n / N$, where $N$ is the total number of the vehicles. The (global) vehicular density is simply given by

$$\rho = \frac{1}{S + L},$$

(A2)

and the average flux is

$$q = \rho v.$$

(A3)

Equation (A1) can be written as

$$s(v, y) = \begin{cases} s^s(v, y) = \tau v - \frac{1}{2} y v^2 & \text{for } y \leq \tau/v, \\ s^d(v, y) = \frac{\tau}{2} & \text{for } y > \tau/v, \end{cases}$$

(A4)

where $y$ stands for $\Delta d_n \equiv 1/D_{n+1} - 1/D_n$. When $D_n$ is randomly assigned out of the interval $(m_D - w_D, m_D + w_D)$, the range of $\Delta d_n$ is $(-1/\eta, 1/\eta)$ where

$$\frac{1}{\eta} = \frac{1}{m_D - w_D} - \frac{1}{m_D + w_D}.$$

(A5)

Note that $v^d$ (and accordingly $s^d$) does not appear when $v < \tau \eta$.

If there are sufficiently many vehicles and their $D_n$'s are uncorrelated, the average spacing can be obtained by the integral

$$S = \int_{-1/\eta}^{1/\eta} s(v, y) P_{\Delta d}(y) dy = \int_{-1/\eta}^{\tau/v} \left( \tau v - \frac{v^2 y}{2} \right) P_{\Delta d}(y) dy + \int_{\tau/v}^{1/\eta} \frac{\tau^2 y}{2} P_{\Delta d}(y) dy,$$

(A6)

where $P_{\Delta d}(y)$ is the probability density function of $\Delta d_n = 1/D_{n+1} - 1/D_n$. Thus, the average spacing of homogeneous solution with $v$ is completely determined by the distribution of $\Delta d_n$, which can be obtained directly from the distribution of $D_n$. Let us denote the probability density function of $D_n$ as $P_D(y)$. Then $P_d(y)$ of $1/D_n$ is given by $P_d(y) = (1/y^2) P_D(1/y)$ and $P_{\Delta d}(y)$ is given by the convolution $P_{\Delta d}(y) = \int_{-\infty}^{\infty} P_d(x) P_d(x - y) dx$. Note that $P_{\Delta d}(y)$ is an even function, $P_{\Delta d}(-y) = P_{\Delta d}(y)$.

Let us introduce the dimensionless speed $z \equiv v / \tau \eta$ and the rescaled probability function $p(y)$ for $\eta(1/D_{n+1} - 1/D_n)$. Then equation (A6) can be written as

$$\frac{S}{\tau^2 \eta} = \int_{-1}^{1/z} \left( z - \frac{z^2 y}{2} \right) p(y) dy + \int_{1/z}^{1} \frac{1}{y} p(y) dy.$$

(A7)

Since $p(y)$ has the normalization $\int_{-1}^{1} p(y) dy = 1$ and the symmetry property $p(-y) = p(y)$, we have $\int_{-1}^{1/z} p(y) dy = 1 - \int_{1/z}^{1} p(y) dy$ and $\int_{1/z}^{1} y p(y) dy = - \int_{1}^{1/z} p(y) dy$. Therefore we obtain

$$\frac{S}{\tau^2 \eta} = z \left[ 1 - I_0(z) \right] + \frac{z^2}{2} I_1(z) + \frac{1}{2} L_1(z) \equiv F(z),$$

(A8)

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where

\[ I_k(z) \equiv \int_{1/z}^{1} y^k p(y) \, dy. \]  \hspace{1cm} (A9)

This provides the scaling relation between \( S \) (or \( \rho \)) and \( v \),

\[ x \equiv \frac{S}{\tau^2 \eta} = \frac{\rho^{-1} - L}{\tau^2 \eta} = F\left(\frac{v}{\tau \eta}\right) = F(z). \] \hspace{1cm} (A10)

Now we find the general properties of \( F(z) \). Since \( p(y) = 0 \) for \( |y| > 1 \), we obtain \( I_k(z) = 0 \) for \( z \leq 1 \), leading to \( F(z) = 0 \) for \( z \leq 1 \), or, equivalently, \( q = (1 - \rho L)/\tau \) for \( \rho \geq 1/(\tau^2 \eta + L) \). Therefore we arrive at

\[ F(z) = \begin{cases} 1 & \text{for } z \leq 1, \\ f(z) & \text{for } z > 1, \end{cases} \] \hspace{1cm} (A11)

where the function \( f(z) \) is determined by the distribution \( p(y) \). Since \( I_k(z = 1) = 0 \), we obtain \( \lim_{z \to 1} f(z) = 1 = F(1) \). Thus \( F(z) \) is a continuous function.

We have \( f(z) = z/2 + \left[ \int_0^{1/z} z + \int_{1/z}^{1} (z^2 y/2 + 1/2 y) \right] p(y) \, dy \) from \( I_0(z) = 1/2 - \int_0^{1/z} p(y) \, dy \). Since \( \int_0^1 p(y) = 1/2 \) and the integrand function has the minimum \( z \) and the maximum \((z^2 + 1)/2\) over \([0, 1]\), the integral is bounded between \( z/2 \) and \((z^2 + 1)/4\). Therefore we obtain

\[ z \leq F(z) \leq \frac{1}{4} (z + 1)^2. \] \hspace{1cm} (A12)

The lower bound \( F(z) \geq z \) leads to \( v \leq (\rho^{-1} - L)/\tau \), which gives the upper bound of the average flux \( q \leq (1 - \rho L)/\tau \).

For \( z \leq 1 \), we have \( F'(z) = 1 \). Using

\[ I_k'(z) = \frac{d}{dz} \int_{1/z}^{1} y^k p(y) \, dy = \frac{1}{z^{k+2}} p\left(\frac{1}{z}\right), \] \hspace{1cm} (A13)

we obtain \( f'(z) = 1 - I_0(z) + z I_1(z) = 1/2 + \left[ \int_0^{1/z} 1 + \int_{1/z}^{1} z y \right] p(y) \, dy \). Since the integrand has the minimum \( 1 \) and the maximum \( z \) for \( z > 1 \), the integral is bounded between \( 1/2 \) and \( z/2 \). Thus we have

\[ 1 \leq f'(z) \leq \frac{z + 1}{2}. \] \hspace{1cm} (A14)

Since \( F'(z) > 0 \) for all \( z > 0 \), there exists the inverse function \( G = F^{-1} \) such that \( G(F(z)) = G(x) = z \). Therefore we obtain the inverse relation of (A10) as

\[ z = \frac{v}{\tau \eta} = G\left(\frac{\rho^{-1} - L}{\tau^2 \eta}\right) = G(x) = \begin{cases} x & \text{for } x \leq 1, \\ g(x) & \text{for } x > 1. \end{cases} \] \hspace{1cm} (A15)

From (A12) and (A14), we obtain \( 2\sqrt{x} - 1 \leq G(x) \leq x \). Since \( G'(x) = 1/F'(z) \), equation (A14) leads to \( 1/\sqrt{x} \leq g'(x) \leq 1 \). From \( dv/d\rho = -G'(x)/\tau \rho^2 \), we obtain

\[-1/\tau \rho^2 \leq dv/d\rho < 0.\]
From (A14) and (A13), we obtain
\[ F''(z) = I_1(z) - I'_0(z) + zI'_1(z) = I_1(z) \geq 0, \]  \tag{A16}
leading to \( G''(x) = -F''(z)/(F'(z))^3 \leq 0 \). This implies the average flux \( q \) is a non-concave function of \( \rho \) because \( d^2q/d\rho^2 = G''(x)/\tau^3\eta\rho^3 \leq 0 \).

From (A15), \( \partial v/\partial \tau \) at a fixed \( \rho \) is given by \( \eta(G(x) - 2xG'(x)) = -\eta(2F(z) - zF'(z))/F'(z) \). For \( z \leq 1 \), we have \( 2F(z) - zF'(z) = z > 0 \). From (A6) and (A14), we obtain \( 2f(z) - zf'(z) = z(1 - I_0(z)) + I_{-1}(z) = z + z \int^{1}_{1/z} (1/zy - 1) p(y) dy \geq z \). Thus we have
\[ 0 < \frac{2F(z) - zF'(z)}{F'(z)} \leq z, \]  \tag{A17}
leading to \( -(\rho^{-1} - L)/\tau^2 \leq \partial v/\partial \tau < 0 \).

There is another lower and upper bound of \( F(z) \). Using \( I_0(\infty) = 1/2 \), we have
\[ f(z) = z^2I_1(\infty)/2 + z/2 + \left[ \int^{1/z}_{0} (z - z^2y/2) + \int^{1/z}_{1/z} (1/2y) \right] p(y) dy. \]
Since the integrand has the maximum \( z \) and the minimum 1/2, the integral is bounded between 1/4 and \( z/2 \). Therefore we obtain
\[ \frac{z}{2} + \frac{1}{4} \leq f(z) - \frac{z^2}{2} I_1(\infty) \leq z. \]
This leads to the asymptotic relation
\[ F(z) \simeq \frac{z^2}{2} I_1(\infty) \quad \text{for} \quad z \gg 1, \]  \tag{A19}
which implies \( q \simeq \sqrt{2\eta I_1(\infty)}/(1 - \rho L) \) for \( \rho^{-1} - L \gg \tau^2\eta \).

If \( D_n \) has uniform distribution on \( (m_D - w_D, m_D + w_D) \equiv (a, b) \), then \( P_D(y) = 1/2w_D \), which leads to \( P_d(y) = 1/2w_Dy^2 \). Thus we obtain
\[ P_{\Delta d}(y) = \begin{cases} R(|y|) & \text{for} \ |y| \leq 1/\eta, \\ 0 & \text{for} \ |y| > 1/\eta, \end{cases} \]  \tag{A20}
where
\[ R(y) = \frac{b - a - \frac{a}{1-ay} + \frac{b}{1+by} - \frac{2}{y} \ln [(1 - ay)(1 + by)]}{4w_D^2y^2}. \]  \tag{A21}

After some algebra, we obtain
\[ f(z) = z + \frac{1}{4w_D^2} \left\{ \frac{2}{3} (1 - z) (abz + w_D^2) - \frac{\eta^2z^3}{3} \left[ \ln \left( 1 - \frac{a}{\eta z} \right) + \ln \left( 1 + \frac{b}{\eta z} \right) \right] \right\} + \frac{\eta z^2}{2} \left[ a \ln \left( \frac{b}{a} - \frac{b}{\eta z} \right) - b \ln \left( \frac{a}{b} + \frac{a}{\eta z} \right) \right] - \frac{1}{6\eta} \left[ a^3 \ln \left( \frac{b}{a} - \frac{b}{\eta} \right) - b^3 \ln \left( \frac{a}{b} + \frac{a}{\eta} \right) \right]. \]  \tag{A22}

In the limit of \( \tau \to 0 \), we have \( s(v, y) = -\frac{1}{2} yv^2 \) for \( y \leq 0 \) and 0 for \( y > 0 \). Then the average spacing is given by
\[ S_{\tau \to 0} = \frac{v^2}{C^2}, \]  \tag{A23}
where

\[ C^2 = \frac{2\eta}{\int_0^1 y p(y) \, dy}. \quad (A24) \]

Then the average speed and flux are given by

\[ v_{r \to 0} = C \sqrt{\left( \frac{1}{\rho} - L \right)}, \quad q_{r \to 0} = C \sqrt{\rho (1 - \rho L)}. \quad (A25) \]

For the uniform distribution of \( D_n \) over \( (m_D - w_D, m_D + w_D) \), we obtain

\[ C^2 = \frac{2w_D^2 m_D \tanh^{-1} \left( \frac{w_D}{m_D} \right)}{m_D - w_D}. \quad (A26) \]

**Appendix B. Linear stability analysis**

We investigate the linear stability of the homogeneous solution with respect to small perturbations. Assuming that the velocity and spacing are very close to those of the homogeneous solution, we can write down

\[ v_n(t) = v + u_n(t), \quad s_n(t) = S_n + \sigma_n(t), \quad \text{(B1)} \]

where the optimal spacing \( S_n \) is given by (A1). By linearizing (3), we obtain

\[ v^e_n(t + \tau) = v + \frac{\sigma_n(t) - \frac{v}{2} u_n(t) + \frac{v}{D_{n+1}} u_{n+1}(t)}{\frac{v}{2} + \frac{v}{D_n}}, \quad (B2) \]

\[ v^d_n(t + \tau) = v + u_{n+1}(t) + \frac{D_n - D_{n+1}}{D_n + D_{n+1}} \left[ \frac{2\sigma_n(t)}{\tau} - u_n(t) + u_{n+1}(t) \right], \quad (B2) \]

up to the first order of \( u_n \) and \( \sigma_n \). Meanwhile, from the integration scheme (1), we obtain

\[ \sigma_n(t + \tau) = \sigma_n(t) + \frac{\tau}{2} [u_{n+1}(t) + u_{n+1}(t + \tau) - u_n(t) - u_{n+1}(t + \tau)]. \quad (B3) \]

The periodic boundary condition \( x_{N+1} = x_1 \) is applied for any quantity \( x \). We introduce the new variable

\[ \psi_n(t) \equiv \frac{\sigma_n(t)}{\tau} + \frac{u_n(t)}{2} - \frac{u_{n+1}(t)}{2}. \quad (B4) \]

From (B3), we obtain

\[ \psi_n(t + \tau) = \psi_n(t) - u_n(t) + u_{n+1}(t). \quad (B5) \]

Then we can rewrite (B2) as

\[ v^e_n(t + \tau) - v = \frac{\psi_n(t)}{\alpha_n} - \frac{u_n(t)}{\alpha_n} + \frac{\alpha_{n+1}}{\alpha_n} u_{n+1}(t), \quad (B6) \]

\[ v^d_n(t + \tau) - v = \beta_n \psi_n(t) - \beta_n u_n(t) + (1 + \beta_n) u_{n+1}(t), \]

where

\[ \alpha_n \equiv \frac{1}{2} + \frac{v}{\tau D_n}, \quad \beta_n \equiv 2 \frac{D_n - D_{n+1}}{D_n + D_{n+1}}. \quad (B7) \]
Note that $\alpha_n > 1/2$ and $0 < 1/\alpha_n < 2$. The selection criterion $1/D_{n+1} - 1/D_n > \tau/v$ for $v_n^d$ is equivalent to $\alpha_{n+1} - \alpha_n > 1$. Now we combine (B6) into

$$u_n(t + \tau) = \gamma_n \psi_n(t) - \gamma_n u_n(t) + \theta_n u_{n+1}(t),$$

(B8)

where

$$\gamma_n = \gamma_n^s, \quad \theta_n = \theta_n^s \quad \text{for} \quad \alpha_{n+1} - \alpha_n \leq 1,$$

(B9)

and

$$\gamma_n = \gamma_n^d, \quad \theta_n = \theta_n^d \quad \text{for} \quad \alpha_{n+1} - \alpha_n > 1,$$

and

$$\gamma_n^s = \frac{1}{\alpha_n}, \quad \theta_n^s = \frac{\alpha_{n+1}}{\alpha_n},$$

$$\gamma_n^d = \beta_n, \quad \theta_n^d = 1 + \beta_n.$$ (B10)

Here we have assumed that the selection of $v_n^s$ or $v_n^d$ is not changed by $u_n$. Note that $\gamma_n$ and $\theta_n$ are continuous functions of $\alpha_n$ and $\alpha_{n+1}$.

By introducing the vector notation $f(t) \equiv (\psi_1(t), \ldots, \psi_N(t))$, $u(t) \equiv (u_1(t), \ldots, u_N(t))$, and $x(t) \equiv (f(t), u(t))$, we can combine (B5) and (B8) into a Jacobian matrix equation

$$x(t + \tau) = \begin{pmatrix} f(t + \tau) \\ u(t + \tau) \end{pmatrix} = \begin{pmatrix} I_N & B \\ C & T \end{pmatrix} \begin{pmatrix} f(t) \\ u(t) \end{pmatrix} = Jx(t),$$

(B11)

where $I_N$ is the $N \times N$ identity matrix and the lower-left submatrix $C$ is a diagonal matrix with $G_{nn} = \gamma_n$. The upper-right submatrix $B$ and the lower-right submatrix $T$ are given by

$$B = \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{pmatrix}, \quad T = \begin{pmatrix} -\gamma_1 & \theta_1 & 0 & \cdots & 0 \\ 0 & -\gamma_2 & \theta_2 & \cdots & 0 \\ 0 & 0 & -\gamma_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta_N & 0 & 0 & \cdots & -\gamma_N \end{pmatrix}$$

(B12)

The long-time behavior of the perturbation amplitude is determined by the largest magnitude among the eigenvalues. The eigenvalues are given by the zeros of the characteristic polynomial

$$h(\lambda) \equiv \det (J - \lambda I_{2N}) = \det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = 0,$$

(B13)

where $A = (1 - \lambda)I_N$ and $D = T - \lambda I_N$. Using the properties of the determinant, we obtain

$$h(\lambda) = \lambda^N \prod_{n=1}^N \mu_n(\lambda) - \prod_{n=1}^N \omega_n(\lambda) = 0,$$ (B14)

where

$$\mu_n(\lambda) = \lambda - 1 + \gamma_n, \quad \omega_n(\lambda) = \theta_n(\lambda - 1) + \gamma_n.$$ (B15)

For $\lambda = 1$, we have $\mu_n(1) = \omega_n(1) = \gamma_n$, leading to $h(1) = 0$. Therefore at least one eigenvalue is exactly 1. From (B14), we have $h'(1) = \prod_{n=1}^N \gamma_n \sum_{n=1}^N (1 + (1 - \theta_n)/\gamma_n)$. Since the summand is zero for $\gamma_n^d$ and positive for $\gamma_n^s$, we obtain $h'(1) > 0$. Thus, the

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eigenvalue 1 is unique without degeneracy. On the other hand, for \( \lambda = -1 \), we have \( \omega_n(-1)/\mu_n(-1) = (2\alpha_{n+1} - 1)/(2\alpha_n - 1) \), irrespective of whether \( v_n^{s\text{el}} \) is \( v_n^s \) or \( v_n^d \), leading to \( \prod_{n=1}^{N} (\omega_n(-1)/\mu_n(-1)) = 1 \). Thus, we have \( h(-1) = (1 - (1)^N) \prod_{n=1}^{N} (2 - \gamma_n) \), which is zero for even \( N \) and negative for odd \( N \). Thus \( -1 \) is an eigenvalue if and only if \( N \) is even. For even \( N \), we obtain \( h'(-1) = (1)^N (\prod_{n=1}^{N} (2 - \gamma_n)) (-N + \sum_{n=1}^{N} (1/(2 - \gamma_n/\theta_n) - 1/(2 - \gamma_n))) \). The sum is zero when \( v_n^{s\text{el}} = v_n^s \) for all \( n \). Since \( 1/(2 - \gamma_n^d/\theta_n) - 1/(2 - \gamma_n^d) < 1/(2 - \gamma_n^d/\theta_n) - 1/(2 - \gamma_n^d) \) for \( \alpha_{n+1} - \alpha_n > 1 \), the sum is not positive. Thus we obtain \( h'(-1) \neq 0 \) and \(-1 \) is also a unique eigenvalue.

Moreover, \( \pm 1 \) are the absolute bounds of real eigenvalues. For \( \lambda > 1 \), we have \( \mu_n(\lambda) > 0 \) and \( \omega_n(\lambda) > 0 \). Similarly, we have \( \mu_n(\lambda) < 0 \) and \( \omega_n(\lambda) < 0 \) for \( \lambda < -1 \) since \( \gamma_n - 2\theta_n < 0 \). For \( \lambda \) to be a solution of (B14), it is necessary to satisfy \( |\prod_{n=1}^{N} \omega_n(\lambda)/\mu_n(\lambda)| = |\lambda|^N \). Meanwhile, it can be easily shown that

\[
\prod_{n=1}^{N} \left| \frac{\omega_n^s(\lambda)}{\mu_n^s(\lambda)} \right| = 1, \tag{B16}
\]

due to the transitivity \( \omega_n^s(\lambda) = \theta_n^s \mu_{n+1}^s(\lambda) \) and \( \prod_{n=1}^{N} \theta_n^s = 1 \). For \( |\lambda| > 1 \), we obtain

\[
\left| \frac{\omega_n^d(\lambda)}{\mu_n^d(\lambda)} \right| < \left| \frac{\omega_n^s(\lambda)}{\mu_n^s(\lambda)} \right| \quad \text{for } \alpha_{n+1} - \alpha_n > 1, \tag{B17}
\]

by using

\[
\left| \omega_n^s(\lambda) \mu_n^d(\lambda) \right| - \left| \omega_n^d(\lambda) \mu_n^s(\lambda) \right| = \frac{(\alpha_{n+1} - \alpha_n)(\alpha_{n+1} - \alpha_n - 1)}{\alpha_n(\alpha_{n+1} + \alpha_n - 1)} (|\lambda|^2 - 1). \tag{B18}
\]

Therefore we obtain \( \prod_{n=1}^{N} |\omega_n(\lambda)/\mu_n(\lambda)| \leq 1 < |\lambda|^N \) for \( |\lambda| > 1 \). Thus there is no real eigenvalues in the range \( \lambda > 1 \) or \( \lambda < -1 \).

The equality (B16) and the inequality (B17) hold even for the complex \( \lambda = re^{i\phi} \) with the magnitude \( r > 1 \) and angle \( \phi \) (\( 0 \leq \phi < 2\pi \)). After some algebra, we have

\[
\left| \omega_n^s(re^{i\phi}) \mu_n^d(re^{i\phi}) \right|^2 - \left| \omega_n^d(re^{i\phi}) \mu_n^s(re^{i\phi}) \right|^2
\]

\[
= \frac{(\alpha_{n+1} - \alpha_n)(\alpha_{n+1} - \alpha_n)^2 - 1}{\alpha_n^2(\alpha_{n+1} + \alpha_n - 1)^2} 4(1 - \cos^2 \phi) + (a_0 + a_1 \cos \phi)(r^2 - 1) + a_2 (r^2 - 1)^2
\]

\[
\equiv M_n(r, \cos \phi), \tag{B19}
\]

where \( a_0, a_1, \) and \( a_2 \) are constants to be determined by \( \alpha_n \) and \( \alpha_{n+1} \). Due to (B17), we have \( M_n(r, \pm 1) > 0 \) for \( r > 1 \), which corresponds to \( \lambda = \pm r \) (\( \phi = 0 \) or \( \pi \)). Since \( M_n(r, \cos \phi) \) is a quadratic function of \( \cos \phi \) with the negative coefficient on the quadratic term. Therefore \( M_n(r > 1, \cos \phi) \) is positive over the entire range of \( \phi \). This leads to

\[
\left| \frac{\omega_n^d(re^{i\phi})}{\mu_n^d(re^{i\phi})} \right| < \left| \frac{\omega_n^s(re^{i\phi})}{\mu_n^s(re^{i\phi})} \right| \quad \text{for } \alpha_{n+1} - \alpha_n > 1, \tag{B20}
\]

for any angle \( \phi \) if \( r > 1 \). Thus, we can conclude that \( \prod_{n=1}^{N} |\omega_n(re^{i\phi})/\mu_n(re^{i\phi})| \leq 1 < r^N \) for \( r > 1 \). Therefore there is no complex eigenvalues in the region \( |\lambda| > 1 \).

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obtain |1 − 1/a_n| < 1, we obtain |λ|^max| = 1. Thus, a v^s-only homogeneous flow is *marginally stable*. Note that N eigenvalues among the total 2N eigenvalues have |λ| = 1.

On the other hand, if v^d is selected for at least one vehicle, the situation changes drastically. For λ = e^{iφ}, we obtain

\[ |ω^s_n(e^{iφ})μ^d_n(e^{iφ})|^2 − |ω^d_n(e^{iφ})μ^s_n(e^{iφ})|^2 = \frac{(α_{n+1} − α_n)[(α_{n+1} − α_n)^2 − 1]}{α_n^2(α_{n+1} + α_n − 1)} 4 (1 − cos^2 φ) \]

by substituting r = 1 in equation (B19). Since −1 < cos φ < 1 for any non-real λ along the unit circle on the complex plane, we have

\[ |ω^s_n(e^{iφ})μ^d_n(e^{iφ})| < |ω^d_n(e^{iφ})μ^s_n(e^{iφ})| \]

for α_{n+1} − α_n > 1,

unless φ is an integer multiple of π. Thus, we have \( ∏_{n=1}^{N} |ω^s_n(e^{iφ})/μ^s_n(e^{iφ})| < 1 \) for all non-real λ = e^{iφ} if at least one v^d is selected. Therefore, all the eigenvalues except for 1 and −1 for even N have a magnitude of less than 1. Consequently, the v^d-mixed flow is much more stable than the v^s-only flow.

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