Scattering by Atomic Spins and Magnetoresistance in Dilute Magnetic Semiconductors

M. Foygel and A. G. Petukhov

Physics Department, South Dakota School of Mines and Technology, Rapid City, SD 57701

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Abstract

We studied electrical transport in magnetic semiconductors, which is determined by scattering of free carriers off localized magnetic moments. We calculated the scattering time and the mobility of the majority and minority-spin carriers with both the effects of thermal spin fluctuations and of spatial disorder of magnetic atoms taken into account. These are responsible for the magnetic-field dependence of electrical resistivity. Namely, the application of the external magnetic field suppresses the thermodynamic spin fluctuations thus promoting negative magnetoresistance. Simultaneously, scattering off the built-in spatial fluctuations of the atomic spin concentrations may increase with the magnetic field. The latter effect is due to the growth of the magnitude of random local Zeeman splittings with the magnetic field. It promotes positive magnetoresistance. We discuss the role of the above effects on magnetoresistance of non-degenerate semiconductors where magnetic impurities are electrically active or neutral.

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I. INTRODUCTION

Dilute magnetic semiconductors (DMS) are materials of great promise in modern technology because they combine semiconductor transport and magnetic properties allowing reach and physically meaningful interplay between them. \[1, 2\] Basically, there are two types of wide bandgap semiconductor alloys in which cations are substituted by randomly distributed magnetic atoms, such as Mn \[1, 3\]. In some of them, such as (II,Mn)VI (Cd\(_{1-x}\)Mn\(_x\)Te, for instance) the substituting magnetic impurity Mn is isoelectronic. We will call these materials type I DMS. Here, the magnetic impurities are not electrically active. However, they can effectively modify electronic transport \[4, 5\] and magnetism \[6, 7\] due to exchange coupling of the free carriers spins to the spins of magnetic atoms. As a result, scattering of the free carriers by the localized magnetic moments determines their mobility that is substantially spin and, therefore, magnetic-field dependent. It leads to giant magnetoresistance (MR), positive or negative \[4, 5, 8\]. (If carriers are localized, the above coupling causes spin-polaron effects that substantially affect the magnetotransport properties of type I DMS \[9, 10\].)

More complex situation occurs in type II DMS, such as (III,Mn)V or (IV,Mn) alloys where Mn atoms serve as acceptors. Examples are Ga\(_{1-x}\)Mn\(_x\)As \[11, 12\] and Ge\(_{1-x}\)Mn\(_x\) \[13, 14, 15\] magnetic semiconductors. Here, contrary to the type I DMS, atomic-spin scattering essentially involves charged impurities a substantial fraction of which are magnetic interstitials. In these materials, both the effects of the atomic-spin scattering and the scattering off the charged impurities are related to each other. They cannot be treated by means of a simple Matthiessen’s rule \[16\].

In this paper we will concentrate mainly on spin-dependent scattering which determines mobility of free carriers in DMS taking into account the spin-disorder effects that are intrinsic for these materials. There are two sources of the spin-disorder effects in question: (a) the thermodynamic fluctuations of atomic spins \[17, 18, 19\], which are present even in the ordered type I magnetic materials with \(x = 1\) \[17, 18\], and (b) the built-in spatial fluctuations of local concentrations of the magnetic impurities \[20\], which are substantial for type II DMS even in the absence of magnetic field due to the long-range nature of Coulomb interaction.

The theory of spin-disorder scattering off the thermodynamic fluctuations of the local magnetization due to atomic moments of the magnetic atoms has been developed by P. G. de Gennes and J. Friedel for magnetic metals \[17\] and for ordered magnetic semiconductors by
C. Haas [18]. In particular, they showed the application of an external magnetic field freezes out the above fluctuations thus leading to negative MR. C. Michel et al. [20] ignored such effects but took into account scattering off the built-in fluctuations of the local concentration of magnetic atoms only. They demonstrated that the field-induced decrease in the mobility associated with these inhomogeneities can be responsible for positive MR in DMS. It is evident, however, that both the above effects should be treated on an equal footing because they have a common source - magnetic atoms. Such a treatment is especially important because, as has been just mentioned, these spin-disorder effects usually give competing contributions to MR of DMS. In this paper we develop an approach that allows us to consistently tackle the problem of the mobility of the majority and minority spins carriers by taking into account the exchange, Coulomb, and deformation effects in scattering by the very same magnetic atoms.

In Section II we will calculate the relaxation time due to scattering by magnetic impurities, which determines the mobility of free carriers to be analyzed in Section III. In Section IV we will calculate the MR of different types DMS, which is in a substantial part defined by the spin-disorder effects associated with scattering of the free carriers off magnetic impurities.

II. RELAXATION TIME

Let us consider a charge carrier in an extended state $|\phi_k(r) X^\alpha\rangle$, where $\phi_k(r) = |k\rangle$ is the Bloch function of the band state of energy $\epsilon_k$ and of wave vector $k$, $X^\pm$ is the electron spin up (+) or down (-) function, and $\alpha$ is an eigenfunction of the atomic-spins Hamiltonian with a temperature dependent probability $w_\alpha$ for the state $\alpha$ to occur. (For the sake of simplicity we will ignore a complex nature of the angular momentum structure of the energy spectrum of such free carriers, like holes [6, 16] in semiconductors, just assuming that the carriers possess spin $s = \pm 1/2$.) For magnetic impurities randomly located at points $\mathbf{R}_i$, the probability of their given configuration

$$dF(\mathbf{R}_1, ..., \mathbf{R}_M) = \prod_{i=1}^M \frac{d\mathbf{R}_i}{\Omega},$$

(1)

where $\Omega$ is the volume of the system. At a given temperature, the magnetization of the system
\[ M = xN g \mu_B \langle J_z \rangle = M_{\text{sat}} \langle J_z \rangle / J \] (2)

is expressed in terms of the average projection of the component of the atomic spin \( J_i \), located at point \( \mathbf{R}_i \), along the direction \( z \) of the magnetization. Here \( N \) is the concentration of the sites in the (sub)lattice that contains magnetic atoms of the fractional concentration \( x = N_m / N \leq 1 \), \( g \) is the Lande-factor of the magnetic-atom spin, \( \mu_B \) is the Bohr magneton, \( M_{\text{sat}} = xN g \mu_B J \) is the saturation magnetization; the brackets represent thermal averaging while the bar represents the averaging over the spatial configurations of magnetic atoms:

\[ \langle J_z \rangle = \int dF(\mathbf{R}_1, ..., \mathbf{R}_M) \sum_{\alpha} w_{\alpha} \langle \alpha | J_{iz} | \alpha \rangle = J B_J(y). \] (3)

Here \( B_J(y) \) is the Brillouin function \[16\] of the atomic spin \( J \) and of the argument \( y \) to be found from the mean-field equation \[1, 2\], which depends on the absolute temperature \( T \) and external magnetic field \( H \). (For the DMS well into paramagnetic phase, \( y = g \mu_B H J / T \).)

Throughout this paper, the temperature \( T \) is measured in the energy units \((k_B = 1)\).

We will start our consideration of a scattering time from the simplest case of a free carrier coupled to randomly distributed non-magnetic impurities by means of the non-exchange interaction:

\[ U_{n-m}(r) = -\sum_i V_{n-m}(r - \mathbf{R}_i). \] (4)

The probability, per unit time, of a free-carrier transition from a state with the wave vector \( \mathbf{k} \) to a state \( \mathbf{k}' \), which is averaged over all possible configurations of impurities, is given by:

\[ P_{n-m}(\mathbf{k}, \mathbf{k}') = \frac{2\pi}{\hbar} |\langle \mathbf{k} | U_{n-m} | \mathbf{k}' \rangle|^2 \delta (\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'}) . \] (5)

Assuming the isotropy of the dispersion law \( \epsilon_{\mathbf{k}} \) one can express the corresponding relaxation time that appears in the Boltzmann transport equation in terms of the above transition probability \[5\] as follows \[16\]:

\[ \frac{1}{\tau_{n-m}} = \frac{\Omega}{(2\pi)^3} \int d\mathbf{k}' \left( 1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}' \right) P_{n-m}(\mathbf{k}, \mathbf{k}'). \] (6)

Then ignoring a spatial dependence of the periodic parts of the Bloch functions, it is easy to show that \[21\]
\[
\frac{1}{\tau_{k}^{(n-m)}} = \frac{1}{(2\pi)^2 \hbar} \int d\mathbf{k}' \left(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}'\right) \Psi_{n-m} (|\mathbf{k} - \mathbf{k}'|) \delta (\epsilon_k - \epsilon_{k'}), \tag{7}
\]

where

\[
\Psi_{n-m} (\mathbf{k}) = \int d\mathbf{r} \exp (-i\mathbf{k} \cdot \mathbf{r}) U_{n-m} (\mathbf{r}) U_{n-m} (\mathbf{0}), \tag{8}
\]

is the Fourier transform of a pair correlation function of the non-exchange part (4) of the random impurity potential. For a simple isotropic dispersion law \( \epsilon_k = \hbar^2 k^2 / 2m \) with an effective mass \( m \), Eq. (7) yields

\[
\frac{1}{\tau_{k}^{(n-m)}} = \frac{m}{4\pi (\hbar k)^3} \int_0^{2k} dzz^3 \Psi_{n-m} (z). \tag{9}
\]

In magnetic semiconductor, the free carrier is coupled to the randomly distributed magnetic atoms by the following interaction:

\[
U_m (\mathbf{r}) = - \sum_i [s J_i U_{ex} (\mathbf{r} - \mathbf{R}_i) + V (\mathbf{r} - \mathbf{R}_i)], \tag{10}
\]

where \( U_{ex} (\mathbf{r}) \simeq \beta_{ex} \delta (\mathbf{r}) \) is the exchange coupling potential strongly localized within the unit cell containing a magnetic atom (\( \beta_{ex} N \simeq 1 \text{eV} \)) \[2,6\], \( s \) is the electron spin; \( V (\mathbf{r}) \) is a non-exchange part of the magnetic-impurity potential of the Coulomb and/or deformation nature. The probability per unit time for an electron from a state with the wave vector \( \mathbf{k} \) and with spin up (+) or down (-) to get transferred to a state with \( \mathbf{k}' \) and with spin up (+) or down (-) while the state of the atomic spins undergoes transition from \( \alpha \) to \( \alpha' \) is \[18\]

\[
P_m (\mathbf{k} \pm \alpha, \mathbf{k}' \pm \alpha') = \frac{2\pi}{\hbar} |\langle \phi_{\mathbf{k}} (\mathbf{r}) X^{\pm \alpha} | U_m | \phi_{\mathbf{k}'} (\mathbf{r}) X^{\pm \alpha'} \rangle|^2 \delta \left( \epsilon_{k}^{\pm} + \epsilon_{\alpha} - \epsilon_{k'}^{\pm} - \epsilon_{\alpha'} \right). \tag{11}
\]

Here for the simple isotropic conduction band,

\[
\epsilon_{k}^{\pm} = \epsilon_0^{\pm} + \frac{\hbar^2 k^2}{2m} \tag{12}
\]

and \( \Delta = \epsilon_0^- - \epsilon_0^+ \) is the Zeeman splitting of the electron spin-split conduction sub-bands.

For the scattering processes that go without spin flip, thermal averaging over the initial spin states, summation over the final spin states, and averaging over the impurity configurations yield
\[
\sum_{\alpha'} \langle P_m (k + \alpha, k' + \alpha') \rangle = \sum_{\alpha'} \sum_{\alpha} w_\alpha \int dF (R_1, ..., R_M) P_m (k + \alpha, k' + \alpha') = \frac{2\pi}{\hbar \Omega^2} \int dF (R_1, ..., R_M) \sum_{\alpha'} \sum_{\alpha} w_\alpha \times \\
\sum_i e^{i(k' - k) R_i} \left\{ \frac{1}{2} U_{ex} (k - k') \langle \alpha | J_{iz} - \langle J_{iz} \rangle | \alpha' \rangle + \left[ V (k - k') + \frac{1}{2} U_{ex} (k - k') \langle J_{iz} \rangle \right] \delta_{\alpha \alpha'} \right\} \times \sum_j e^{i(k' - k) R_j} \left\{ \frac{1}{2} U_{ex}^* (k' - k) \langle \alpha' | J_{jz} - \langle J_{jz} \rangle | \alpha \rangle + \left[ V^* (k' - k) + \frac{1}{2} U_{ex}^* (k' - k) \langle J_{jz} \rangle \right] \delta_{\alpha \alpha'} \right\} \delta \left( \epsilon_k^+ + \epsilon_\alpha - \epsilon_{k'}^+ - \epsilon_{\alpha'} \right)
\]
\[
= \frac{2\pi}{\hbar \Omega^2} \sum_{ij} e^{i(k' - k) (R_i - R_j)} \left\{ \frac{1}{4} \left| U_{ex} (k - k') \right|^2 (\langle J_{iz} J_{jz} \rangle - \langle J_{iz} \rangle \langle J_{jz} \rangle) + \left[ V (k - k') + \frac{1}{2} U_{ex} (k - k') \langle J_{jz} \rangle \right] \delta \left( \epsilon_k^+ - \epsilon_{k'}^+ \right) \right\} \delta \left( \epsilon_k^+ - \epsilon_{k'}^+ \right),
\]

where \( U_{ex} (k) \) and \( V (k) \) are the Fourier transforms of the exchange and non-exchange parts of the magnetic impurity potential. Similarly, for the scattering accompanied by the double spin-flip processes when both the electron and magnetic atom flip their spins simultaneously,

\[
\sum_{\alpha'} \langle P_m (k + \alpha, k' - \alpha') \rangle = \sum_{\alpha'} \sum_{\alpha} w_\alpha \int dF (R_1, ..., R_M) P_m (k + \alpha, k' - \alpha') = \frac{2\pi}{\hbar \Omega^2} \int dF (R_1, ..., R_M) \sum_{\alpha'} \sum_{\alpha} w_\alpha \times \\
\sum_i e^{i(k' - k) R_i} \left\{ \frac{1}{2} U_{ex} (k - k') \langle \alpha | (J_{ix} + J_{iy}) \rangle \alpha' \rangle \right\} \times \sum_j e^{i(k' - k) R_j} \left\{ \frac{1}{2} U_{ex}^* (k' - k) \langle \alpha' | (J_{ix} + J_{iy}) \rangle \alpha \rangle \right\} \delta \left( \epsilon_k^+ + \epsilon_\alpha - \epsilon_{k'}^+ - \epsilon_{\alpha'} \right)
\]
\[
= \frac{\pi}{\hbar \Omega^2} \sum_{ij} e^{i(k' - k) (R_i - R_j)} \left| U_{ex} (k - k') \right|^2 (\langle J_{ix} J_{jx} \rangle + \langle J_{iy} J_{jy} \rangle) \times \delta \left( \epsilon_k^+ - \epsilon_{k'}^+ \right) \delta \left( \epsilon_k^+ - \epsilon_{k'}^+ \right)
\]

Similar expressions can be derived that describe the averaged probabilities for the (-) \( \rightarrow \) (-) and (-) \( \rightarrow \) (+) processes.

As a result, one can calculate, in the first Born approximation, the inverse relaxation time for an electron with the wave vector \( k \) and the spin up (+) or down (-) as follows:
\[
\frac{1}{\tau_k^\pm} = \Omega \frac{1}{(2\pi)^3} \int \frac{dk'}{(2\pi)^3} \left( 1 - \hat{k} \cdot \hat{k}' \right) \sum_{\alpha'} \left[ \langle P_m(k \pm \alpha, k' + \alpha') \rangle + \langle P_m(k \pm \alpha, k' - \alpha') \rangle \right]
\]

\[
= \frac{1}{(2\pi)^2 \hbar \Omega} \int \frac{dk'}{(2\pi)^3} \left( 1 - \hat{k} \cdot \hat{k}' \right) \sum_{ij} e^{i(k' - k)(R_i - R_j)} \left\{ \frac{1}{4} |U_{ex}(k - k')|^2 \left( \langle J_{iz} J_{jz} \rangle - \langle J_{iz} \rangle \langle J_{jz} \rangle \right)
+ \left| V(k - k') \pm \frac{1}{2} U_{ex}(k - k') \langle J_z \rangle \right|^2 \delta\left( \epsilon_k^+ - \epsilon_{k'}^+ \right) + \frac{1}{4} |U_{ex}(k - k')|^2 \left( \langle J_{ix} J_{jx} \rangle + \langle J_{iy} J_{jy} \rangle \right) \delta\left( \epsilon_k^+ - \epsilon_{k'}^+ \right) \right\}. \tag{15}
\]

Here the terms in the right part, which involve the atomic spins correlation functions, are responsible for scattering off the thermodynamic fluctuations of atomic spins. By virtue of the fluctuation-dissipation theorem\cite{22}, the first of them can be expressed in terms of the static value of the \(z\)-component (longitudinal component) of the magnetic susceptibility \(\chi_z = \chi_\parallel\):

\[
\Psi_\parallel(k - k') = \frac{1}{4\Omega} \sum_{i,j} e^{i(k' - k)(R_i - R_j)} |U_{ex}(k - k')|^2 \left( \langle J_{iz} J_{jz} \rangle - \langle J_{iz} \rangle \langle J_{jz} \rangle \right)
= \frac{T |U_{ex}(k - k')|^2 \chi_\parallel(k - k')}{4 (g\mu_B)^2}. \tag{16}
\]

Similarly,

\[
\Psi_\perp(k - k') = \frac{1}{4\Omega} \sum_{i,j} e^{i(k' - k)(R_i - R_j)} |U_{ex}(k - k')|^2 \langle J_{iz} J_{jz} \rangle
= \frac{1}{4\Omega} \sum_{i,j} e^{i(k' - k)(R_i - R_j)} |U_{ex}(k - k')|^2 \langle J_{iz} J_{jz} \rangle
= \frac{T \chi_\perp(k - k') |U_{ex}(k - k')|^2}{4 (g\mu_B)^2}, \tag{17}
\]

where \(\chi_\perp = \chi_x = \chi_y\) are transversal components of the magnetic susceptibility in a direction perpendicular to magnetic field. The second term in the right part of Eq. \(15\) represents scattering off a random built-in potential of magnetic atoms due to the spatial fluctuations of their local concentrations. Similarly to Eqs \(7\) and \(8\) describing scattering by non-magnetic atoms, it can be expressed in terms of the Fourier transform of the corresponding correlation function:
\[
\Psi_m (|k - k'|) = \frac{1}{\Omega} \sum_{i,j} e^{i(k' - k)(R_i - R_j)} \left| V (k - k') + \frac{1}{2} U_{ex} (k - k') \langle J_z \rangle \right|^2.
\] (18)

It should be mentioned here that in the case of non-degenerate semiconductors, where the
typical wave numbers of scattered free carriers are small, one can usually ignore the disper-
sion of the Fourier transforms of the correlation functions described by Eqs (8) and (16) -
(18). The exclusion is to be made for the charged impurities, magnetic and/or non-magnetic,
when the corresponding Fourier transforms \( V_{n-m}(k) \) and \( V(k) \) involved diverge at small \( k \).

A. Type I DMS

We can further simplify expression (15) for the relaxation time in the type I magnetic
semiconductor compounds where the magnetic centers are isoelectronic. In this case, the
potential of such a center consists of the exchange and deformation components, both short-
range ones,

\[
U_m^{(i)}(r) = -[sJ_i U_{ex}(r - R_i) + V(r - R_i)] = -[sJ_i \beta_{ex} + \beta_{def}] \delta(r - R_i),
\] (19)

where \( \beta_{ex} \) is the exchange coupling constant and, for ternary solid solutions, \( \beta_{def} =
N^{-1}dE_C/dx \) is the deformation potential constant of the relevant band edge \( E_C \). So,
extactly like for any mixed semiconductor compound (see Ref. [21]), the Fourier transform
of the potential correlation function

\[
\Psi_m (k) = Nx (1 - x) \left| \beta_{def} \pm \frac{1}{2} \beta_{ex} \langle J_z \rangle \right|^2,
\] (20)

where \( \langle J_z \rangle \) is given by Eq. (3). The magnetic-field dependence \( \langle J_z \rangle \) follows that of the
magnetization (2). As a result, using simple isotropic dispersion law (12) and relations
similar to (9), Eqs (3), (16) - (19) yields

\[
\frac{1}{\tau_T^\pm (k)} = \frac{m^2 \beta_{ex}^2 k}{\pi \hbar^3} \left\{ \frac{T}{(2g\mu_B)^2} \left[ \chi_\parallel + 2\chi_\perp F^\pm \left( 1 \mp \frac{2m\Delta}{\hbar^2 k^2} \right)^{1/2} \right] + \right.

\left. Nx (1 - x) \left| \frac{\beta_{def}}{\beta_{ex}} \pm \frac{JM}{2M_{sat}} \right| \right\}^{-2}. \] (21)

Here \( F^\pm = 1 \) if the average Zeeman band splitting [9]

8
\[ \Delta (H) = xN \beta_{ex} J B_j (y) + g^* \mu_B H = \frac{\beta_{ex} M (H)}{g \mu_B} + g^* \mu_B H \]  

(22)

is less than \( h^2 k^2 / 2m \), otherwise \( F^+ = 0 \) and \( F^- = 1 \). (In Eq. (22) \( g^* \) is the electron \( g \)-factor.)

In the above expression (21) for the relaxation time of the type I DMS, the term in the square brackets is responsible for scattering off the thermal fluctuations of atomic spins. In particular, the term with \( \chi_\parallel \) describes the scattering processes that go without spin flip. The term with \( \chi_\perp \) takes into account the scattering accompanied by the double spin-flip processes. For the majority-spin carriers (+), the latter processes \((+) \rightarrow (-)\) gradually disappear with the increase of the applied magnetic field because the energies of the final-state sub-band progressively exceed that of the initial-state subband. As a result, the corresponding transitions become energetically less favorable.

The last term in the right part of Eq. (21) describes scattering off the spatial fluctuations of the local concentrations of magnetic atoms (see Fig. 1). For the ordered magnetic semiconductors, such as EuSe or ErAs, where magnetic atoms form a regular sub-lattice, \( x = 1 \), this term disappears. Then Eq. (21) coincides with one obtained by Haas [18]. Meanwhile, in the limit of the saturation magnetic fields, when the thermal fluctuations are frozen out because both \( \chi_\parallel (H) = \partial M / \partial H \) and \( \chi_\perp (H) = M / H \) tend to zero, for \( |\beta_{def} / \beta_{ex}| >> J \) we recover the expression for the scattering time in disordered nonmagnetic alloys [21].

**B. Type II DMS**

In these materials, both the magnetic impurities and the compensating non-magnetic centers are charged, so that the spin-independent components of their potentials are of the screened Coulomb form \( V_C (r) \) for which the Fourier transform [16]

\[ V_C (k) = -\frac{4 \pi Ze^2}{\kappa (k^2 + r_0^{-2})}. \]  

(23)

Here \( Ze \) is the charge of a center, \( \kappa \) is the dielectric constant and \( r_0 \) is the screening length. We will use this dependence while calculating the Fourier transforms of the relevant correlation functions [18] and [8] due to magnetic and non-magnetic impurities, respectively:
FIG. 1: (Color online.) Built-in magnetic-impurity potential in DMS in the absence (a) and in the presence (b) of external magnetic field for the case when \( b = \beta_{def}/\beta_{ex} > 1 \).

\[
\Psi_{n-m}(k) = N_{n-m} \left[ \frac{4\pi Z_{n-m}e^2}{\kappa (k^2 + r_0^2)} \right]^2,
\]

(24)

\[
\Psi_{m}(k) = N_m \left[ \frac{4\pi Z_me^2}{\kappa (k^2 + r_0^2)} \pm \frac{\beta_{ex}JM}{2M_{sat}} \right]^2,
\]

(25)

Here \( Z_ie \) and \( N_i \) are the charge and concentration of the magnetic \((m)\) and non-magnetic \((n - m)\) impurities. Then by employing Eq. (21) it is easy to obtain the following expression for the relaxation time in the type II DMS with large enough screening length \((kr_0 >> 1)\):
last term is due to scattering off charged impurities, magnetic and non-magnetic. (For these materials we obviously have ignored the short-range deformation potential.) It should be noted that the presence of an "interference" term in the second term in the braces violates empirical Matthiesen’s rule \[16\] because the impurity scattering processes involve the Coulomb and magnetic forces that originate from the same atoms. It can be seen that these processes are not independent even within the first Born approximation.

III. CALCULATING MOBILITY

If the relaxation time is known, one can use the standard approach to calculate the mobilities of the majority (+) and minority (-) spin carriers \[18\]

$$\mu^\pm = \frac{q\hbar}{3m^2} \sum_k k^2 \left( \frac{\partial f_k^\pm}{\partial \epsilon_k^\pm} \right) \tau_k^\pm,$$

where $$f_k^\pm$$ are the Fermi distribution functions for the spin-split subbands and $$q = \pm e$$ is the carrier charge. By means of Eq. \[21\] we find that for type I non-degenerate DMS

$$\mu_I^\pm = \mu_0^I \int_0^\infty dt \exp(-t) t \left\{ \frac{T}{(2g\mu_B)^2 N} \left[ \chi_\parallel + 2\chi_\perp F^\pm (1 \mp \delta/t)^{1/2} \right] + x (1-x) \left| \frac{\beta_{def}}{\beta_{ex}} \right| \pm \frac{JM}{2M_{sat}} \right\}^{-1},$$

where

$$\mu_0^I (T) = \frac{2 (2\pi)^{1/2} q\hbar^4}{3m^{5/2} N \beta_{ex}^2 T^{1/2}}.$$  \[29\]

Here $$F^\pm = 1$$ if the dimensionless average Zeeman band splitting $$\delta = \left( \epsilon_0^- - \epsilon_0^+ \right) / T = \Delta / T \leq 1$$, otherwise $$F^+ = 0$$ and $$F^- = 1$$. With the well known from the Hall effect theory coefficient $$\gamma_H = \mu_H / \mu = 3\pi / 8$$ for the scattering by isoelectronic impurities \[23\], in the limit of the saturating magnetic fields and for $$|\beta_{def}/\beta_{ex}| > J$$, one can easily obtain by means of Eqs \[28\] and \[29\] the expression for the Hall mobility $$\mu_H$$ in mixed non-magnetic alloys \[21\]. And again, in the limiting case of the ordered magnetic semiconductors ($$x = 1$$) we retrieve the expression for the mobility $$\mu_I^\pm$$ obtained by Haas \[18\].
FIG. 2: (Color online.) Mobility of the minority (a) and majority (b) spin carriers as a function of magnetic field for different values of the parameter $b = \beta_{def}/\beta_{ex}$. 
Fig. 2 shows the mobilities of spin-up and spin-down carriers in the type I DMS calculated by means of Eq. (28) for different values of parameter $b = \beta_{\text{def}}/\beta_{\text{ex}}$. We used $N|\beta_{\text{ex}}| = 1\text{eV}$, $J = 5/2$, $x = 0.02$, and $T = 50\text{K}$ in our calculations. It can be seen that, as a rule, the mobility of spin down carriers drops while that of the spin up carriers rises with the application of the magnetic field. The latter (b) is dominated by scattering off the thermodynamic fluctuations of the atomic spins, which is suppressed by magnetic field. Simultaneously, with the application of magnetic field, the former (a) is more and more determined by scattering off the built-in fluctuations of the local concentration of magnetic atoms because the input from the double spin-flip $(+)$ $\Rightarrow (-)$ processes of scattering by the thermal fluctuations are substantially reduced for these processes are accompanied by the absorption of the increasingly greater amounts of thermal energy. Contrary to that, the spin-up mobility drops and spin-down mobility rises with magnetic field only if the deformation potential constant $\beta_{\text{def}}$ is large and have the same sign as the exchange coupling constant $\beta_{\text{ex}}$. Then in those both cases the zero-magnetic-field mobility is governed by scattering off the build-in fluctuations of the deformation potential of magnetic impurities (Fig. 1(a)). The application of the magnetic field increases the amplitude of the impurity potential fluctuations for the majority-spin carriers and decreases the above amplitude for the minority spin carriers (Fig. 1(b)), which explains the calculated dependencies.

For the type II non-degenerate DMS with large enough screening length $(8mTr_{0}/\hbar^{2} >> 1)$, Eqs (26) and (27) yield

$$\mu_{\pm} = \mu_{(II)}^{\pm} \left\{ \int_{0}^{\infty} dt \exp(-t) \left( \frac{T}{2g\mu_{B}} \right)^{2} N_{m} \left[ \chi_{\parallel} + 2\chi_{\perp} F_{\pm} (1 \mp \delta/t)^{1/2} \right] + \frac{JM}{2M_{\text{sat}}} \left( \frac{JM}{2M_{\text{sat}}} \pm \frac{Z_{m}T_{0}}{Tt} \right) + \frac{N^{*}}{2\pi N_{m}} \left( \frac{T_{0}}{2Tt} \right)^{2} \ln \left( 8mTr_{0}^{2}/\hbar^{2} \right) \right\}^{-1}. \quad (30)$$

Here $\mu_{(II)}^{\pm} \sim T^{-1/2}$ is given by Eq. (29), where $N$ is to be changed for the concentration of magnetic impurities $N_{m}$. In Eq. (30), we introduced parameter $T_{0} = 2\pi e^{2}h^{2}/(\beta_{\text{ex}}\kappa m)$, which is of the order of 1 eV, so that at practically all temperatures $T << T_{0}$. Therefore, the last term in the expression in the braces in the right part of Eq. (30) is much larger than the previous two. As a result, the mobility here is dominated by scattering off charged impurities and is approximately described by [23, 24]
\[ \mu_{II}^\pm \simeq \mu_{CW}(T) = \frac{2^{7/2}\kappa^2 T^{3/2}}{N^* q^3 m^{1/2} \ln \left( \frac{24mTr_0^3}{h^2} \right)}. \] (31)

Eq. (30) allows someone to easily calculate small magnetic-field-dependent corrections to the Conwell-Weisscopf expression (31).

IV. APPLICATION TO GMR

An applied magnetic field changes the conductivity of DMS

\[ \sigma = q \left( \mu^+ n^+ + \mu^- n^- \right) \] (32)

because it affects both the mobility \( \mu^\pm \) and the concentration of the majority and minority spin carriers, which for non-degenerated DMS is equal to

\[ n^\pm = \frac{1}{2} N_c \exp \left( -\frac{E_C - \Gamma^\pm - F \mp \bar{\Delta}/2}{T} \right). \] (33)

Here \( F \) is the Fermi energy, \( N_c \) is the density of the conduction band edge \( E_C \) states and

\[ \Gamma^\pm = \frac{x(1-x)m}{2\pi N\hbar^2 a} \left( \beta_{def} \pm \beta_{ex} \frac{JM}{2M_{sat}} \right)^2 \] (34)

is the shift of the bottom of the spin-split sub-bands, which is caused by the renormalization of the energy spectrum by the fluctuating short-range electronic potential in mixed ternary compounds including type I DMS [20, 21]. In Eq. (34), \( a \) is the lattice constant.

Let us, for the sake of simplicity, assume that the total concentration of the free carriers, \( n = n^+ + n^- \), does not depend on magnetic-field. It is usually correct when the concentration is determined by shallow non-magnetic impurities for which the ionization energy does not depend on magnetic field [18]. Then the MR can be calculated as

\[ \frac{\Delta \rho(H)}{\rho(0)} = \frac{\sigma(0)}{\sigma(H)} - 1 = \left[ \frac{\mu^+(H)/\mu(0)}{1 + \exp \left( -\frac{(\bar{\Delta}(H) + \gamma(H))/T}{\mu^+(H)/\mu(0)} \right)} \right]^{-1} - 1, \] (35)

where \( \mu^\pm (H) \) is given by Eq. (28), \( \bar{\Delta}(H) \) is given by Eq. (22), and \( \gamma(H) = \Gamma^+(H) - \Gamma^-(H) \).
FIG. 3: (Color online.) Magnetoresistance of DMR for different values of the parameter $b = \beta_{def}/\beta_{ex}$ with (b) and without (a) band-edge shift taken into account.
Fig. 3 shows the magnetoresistance of DMS calculated by means of Eq. (35) with the same set of parameters we used to generate Fig. 2. The first set of graphs (a) represents the MR calculated without the shifts (34) of the spin-split band edges taken into account ($\gamma(H) = 0$), whereas the second set (b) takes this effect into consideration. (Here we used $m = 0.1m_e$ and $N = 4/a^3_0$ in our calculations). One can see that MR is predominantly governed by the magnetic-field dependence of the mobility of the majority carriers, which itself is dominated by scattering off the thermodynamic fluctuations of atomic spins. As a result, the MR is usually negative because both the magnitude of these fluctuations and related scattering decrease with the application of magnetic field. For the model under discussion the MR becomes positive only if the deformation potential constant $\beta_{def}$ is large and has the same sign as the exchange coupling constant $\beta_{ex}$ ($b = \beta_{def}/\beta_{ex} = 5$). In that case, the zero-magnetic field mobility is dominated by scattering off the large build-in fluctuations of the deformation potential of magnetic impurities. The application of the magnetic field increases the amplitude of these fluctuations for the majority-spin carriers thus leading to positive MR. Taking into account the renormalization of the band edges does not substantially change the MR with an insignificant exclusion for the case of $b = \beta_{def}/\beta_{ex} = -5$ when the MR becomes slightly less negative.

In the recent publication [25] the authors analyzed the observed MR of $Mn : Ge$ DMS by using a modified version of the atomic spin scattering model under consideration. In the paramagnetic phase, these materials reveal large positive MR. It has been demonstrated [25] that this phenomenon is related to superparamagnetic nature of magnetic clusters with enhanced concentration of $Mn$ atoms. In addition, antiferromagnetic coupling between magnetic atoms in the superparamagnetic clusters has been taken into account, which, together with an anisotropy of the magnetic susceptibility, is shown to promote positive MR.

In summary, we analyzed spin-dependent electrical conductivity in DMS where free carriers are scattered off randomly distributed magnetic impurities. The mobility of the minority and majority spin carriers is shown to be governed by competing impurity spin-disorder effects caused by (a) thermodynamic fluctuations of the atomic spins and (b) random impurity potential generated by the spatially fluctuating concentration of the magnetic atoms. The former effect usually dominates the mobility of the majority-spin carriers. It is quenched by external magnetic field leading to giant negative magnetoresistance. Depending on ma-
terial parameters, the spin-dependent scattering rate may be enhanced due the growing
fluctuations of local Zeeman splittings of the expanded electronic states leading to positive
magnetoresistance in such DMS.

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