Non-Quantized Edge Channel Conductance and Zero Conductance Fluctuation in Non-Hermitian Chern Insulators

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The quantized conductance plateaus and zero conductance fluctuation are the general consequence of electron transport in chiral edge states of Hermitian Chern insulators. Here we show that the physics of electron transport through chiral or helical edge channels becomes much richer when the non-Hermicity is allowed. In the presence of an unbalanced non-Hermicity where the chiral edge states have finite-lifetime, the conductance of the edge channels is not quantized, but its conductance fluctuation is zero. For the balanced non-Hermicity, however, the chiral edge states have infinite-lifetime, and the conductance is quantized as in the case of Hermitian Chern insulators. Both non-quantized and quantized conductance plateaus of zero conductance fluctuation are robust against disorders. We present a theory that explains the origin of the non-quantized conductance plateaus. The physics revealed here should also be true for the chiral and helical surface states in other topological materials such as quantum anomalous Hall systems, Weyl semimetals, and topological superconductors.

Introduction.—Searching various topological insulators [1–9], topological semimetals [10–19], and topological superconductors [20–26] has attracted enormous attention in recent years because of their fundamental interest and possible applications of the exotic properties of topologically-protected surface states. The topological states can exist not only in the electronic systems, but also in other classical or quantum systems such as electromagnetic waves [27–31], mechanical waves [32], and spin waves [33–42]. The topological band theory [43–45] is believed to play a crucial role in understanding the physics of these newly found quantum phases. As a well-accepted paradigm, the quantized conductance $e^2/h$ observed in magnetic doped Chern insulators (CIs), known as the quantum anomalous Hall effect, is the consequence of the impurity-immune chiral edge states [46, 47] in Hermitian CIs.

The topological band theory is based on the Hermitian Hamiltonian for closed quantum systems with conserved particle probability. Strictly speaking, all interesting systems that one wants to understand and to probe are not closed. The Hermicity of an open system is universally lost, and the system always involves certain degrees of gain and loss. For example, the inevitable electron-electron, electron-impurity, and electron-photon scatterings in an electronic system lead to complex self-energies [48–51] of single electron states such that the lifetime of a single electron state is always finite. Recently, motivated by this consideration, the non-Hermitian topological nontrivial systems have been intensively investigated [52–64]. A generalization of the topological band theory to the non-Hermitian systems has been done, which highlights the breakdown of the general bulk-boundary correspondence [65–67]. While most of the theoretical studies focused on static properties such as particle spectrum and wavefunctions, the transport properties of the non-Hermitian topological states are less studied, although they are directly measured in experiments.

In this letter, we compute the conductance of a piece of non-Hermitian CI Hall bar. In the case of a balanced non-Hermicity, an electron injected into a chiral edge channel from one electrode will eventually reach the other electrode without losing its quantum coherence, and the conductance is quantized to the integer of $e^2/h$, the same as that of the Hermitian CIs, and robust against disorders. The quantized conductance disappears and diminishes only in very strong disorders when the edge channels are completely destroyed by the Anderson localizations. More interestingly, the conductance of systems with an unbalanced non-Hermicity is non-quantized due to the finite-lifetime of a single electron state. In this situation, conductances vary continuously with the non-Hermicity and the Hall bar length. We call this phase the anomalous CI phase. By adding weak on-site disorders, the anomalous CI phase is then identified by the non-quantized conductance plateaus with zero conductance fluctuation. We develop an effective theory to explain the appearance of non-quantized conductance plateaus. The theory explains perfectly the simulation results.

Non-Hermitian Hamiltonian.—We consider the following two-dimensional (2D) tight-binding Hamiltonian on
a square lattice,
\[
H = \sum_i \left[ c_i^\dagger (m\sigma_z + i\gamma \cdot \vec{\sigma} + V_i^1\sigma_0 + V_i^2\sigma_z) c_i \right] - \sum_i \frac{t}{2} \left[ c_i^\dagger (\sigma_z + i\sigma_x) c_i + c_i^\dagger (\sigma_z + i\sigma_y) c_i + H.c. \right],
\]
where \(c_i^\dagger = (c^\dagger_i, c^\dagger_{i+y})\) and \(c_i\) are the single electron creation and annihilation operators on lattice site \(i = (x_i, y_i)\) with \(x_i\) and \(y_i\) being integers and \(a\) the lattice constant. \(m\) and \(t > 0\) are respectively the Dirac mass and the hopping energy. \(\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z, \sigma_0)\) with \(\sigma_x, y, z, 0\) being Pauli matrices and \(\sigma_0\) identity matrix, acting on spin (or pseudospin) space. Hamitonian is Hermitian in the absence of \(\gamma\)-terms, \(i\gamma \cdot \vec{\sigma}\), where \(\gamma = (\gamma_x, \gamma_y, \gamma_z, \gamma_0)\) with \(\gamma_x, y, z, 0\) being small real numbers that measure the degrees of non-Hermiticity. This form of non-Hermicity has been widely used in literature and was derived from the electron-electron, electron-photon, and electron-impurity interactions. Disorders are introduced through \(V_i^1/t\) and \(V_i^2/t\) that distribute randomly and uniformly in the range of \([-W/2, W/2]\).

In the clean limit of \(W = 0\), Eq. (1) can be block-diagonalized in the momentum space as \(H = \sum_{\vec{k}} c_{\vec{k}}^\dagger h(\vec{k}) c_{\vec{k}}\) with
\[
h(\vec{k}) = (t \sin k_x + i\gamma_x)\sigma_x + (t \sin k_y + i\gamma_y)\sigma_y
\]
\[
+ (m - t \cos k_x - t \cos k_y + i\gamma_z)\sigma_z + i\gamma_0\sigma_0.
\]
The Hermitian part of Eq. (2) is the Qi-Wu-Zhang model. For \(0 < |m| < 2t\), this model supports the quantum anomalous Hall phase with chiral edge states that form a chiral edge channel. The non-Hermiticity \(i\gamma_j\sigma_j\) (here \(j = x, y\), acting on the eigenstates of the Hermitian part, adds a pre-factor \(\exp[i\gamma_j x_j/t]\) to the Bloch wavefunction of the Qi-Wu-Zhang model. (This can be seen either from the differential equation form of the continuum model of (2) near \(\vec{k} = 0\) by replacing \(k_j\) with \(-\partial/\partial x_j\) or from the perturbation theory by treating the non-Hermitian part as the perturbation.) Thus, for finite-size systems with the open boundary condition, the non-Hermitian potential causes the Bloch states with well-defined momentum to exponentially localize at sample boundaries. This notable phenomenology is referred as the non-Hermitian skin effect.

Anomalous Chern insulator.—To reveal the electron transport properties of non-Hermitian CIs, we calculate the conductance of a rectangle Hall bar of length \(L\) and width \(M\) connected to two semi-infinite electrodes at two ends along the \(x\) direction. Electrons in the Hall bar are governed by Eq. (1) while electrons in the leads are assumed to be normal free Fermi gas. Thus, the non-Hermiticity exists only in the disordered Hall bar and leads are Hermitian. The conductance \(G\) of the two-terminal setup is numerically computed by the Landauer-Büttiker formalism. Without losing generality, we consider three different non-Hermiticities: \(\gamma_y = (0, \gamma, 0, 0)\), \(\gamma_z = (0, 0, \gamma, 0)\), and \(\gamma_0 = (0, 0, 0, \gamma)\). We will show below that the first two non-Hermitians are fundamentally different from that of the last case.

The dimensionless conductance \(g = \langle G \rangle/(e^2/h)\) and the conductance fluctuation \(\Delta g = \sqrt{\langle G^2 \rangle - \langle G \rangle^2}/(e^2/h)\), where \(\langle \cdots \rangle\) denotes the ensemble average, are calculated by varying the disorder strength \(W\) and the system width \(M\). We fix the Dirac mass \(m = t\) and the Fermi energy \(E = -0.01t\) to focus on the edge channel transport. Figure 1 shows \(g\) (each point is averaged over more than 6000 different configurations) and \(\Delta g\) as a function of \(W\) for \(\gamma_y\), \(\gamma_z\), and \(\gamma_0\) with \(\gamma = 10^{-3}t\) and \(L = 400a\). For the first two cases \((\gamma_y\) and \(\gamma_z)\), we observed the width-independent quantized plateaus exactly at \(g = 1\) and the vanishing conductance fluctuations \(\Delta g = 0\) as long as \(W\) is below a critical value \(W_{cr}\), see Figs. (a–d). The calculated results of the non-Hermitian CIs thus accord quantitatively with the expectation for Hermitian CIs: \(g\) is quantized at 1 within a certain range of disorder \((W < W_{cr})\). Beyond
$W_c$ ($W > W_c$), $g$ is not quantized and $\Delta g$ is non-zero until all conducting channels disappear and $g$ becomes zero. The quantized conductance plateau is a direct consequence of the existence of the backscattering-immune edge channels.

Another more striking result is the non-quantized conductance plateaus (insensitive to disorder) in the case of $\gamma_0$. As a representative example, Fig. 3(e) shows a non-quantized conductance plateau of $g = 0.449$ for $\gamma = 10^{-3}t$ and various $M$ range from $80a$ to $200a$. For $W < W_c = t$, the non-quantized conductance plateaus do not show any conductance fluctuation, as shown in Fig. 3(f). Interestingly, the value of the non-quantized conductance depends neither on the width of the bar nor on disorder $W < W_c = t$, a behavior which, together with the emergence of the conductance plateaus, is reminiscent of the chiral edge channel transport in Hermitian CIs. However, one cannot simply tie our results to the non-Hermitian topological band theory [65–67] that claims generalized Chern numbers for non-Hermitian CIs, in contrast to non-quantized $g$ plateaus observed here.

We thus name the state as the anomalous Chern insulators that can have an arbitrary value of conductance with zero conductance fluctuation. This is highly non-trivial because the fluctuation is an intrinsic nature of quantum physics under the orthodox statistical interpretation of quantum mechanics. The only case of a zero conductance fluctuation is the single 1D-unidirectional channel including the quantum Hall systems that ties conductance quantization with zero conductance fluctuation.

![Fig. 2: $g$ vs $\gamma$ for $\gamma_0$ (black), $\gamma$ (red), and $\gamma_0$ (blue). For each non-Hermicity, the system lengths are $L/a = 400$ (triangles), 200 (squares), and 100 (circles). Green lines are Eq. (10) without any fitting parameter. Inset: $g$ (blue stars from numerical calculations) and the green line is Eq. (10) as a function of $L$ for $\gamma_0$ with $\gamma = 10^{-3}t$. Here $M = 80a$ and $W = 0.5t$.](image)

To further reveal the nature of this anomalous CI phase, we investigate how $g$ depends on the bar length $L$ and the non-Hermicity strength $\gamma$. Figure 2 shows $g(\gamma, L)$ for $\gamma_0$, $\gamma_z$, and $\gamma_0$ at a fixed randomness $W = 0.5t < W_c$ and sample width $M = 80a$. Clearly, the quantized conductance plateaus is independent to $L$ and $\gamma$ for the cases of $\gamma_y$ and $\gamma_{yz}$. However, for the cases of $\gamma_0$ where the anomalous CI phase is observed, an exponential decay of the conductance with both $L$ and $\gamma$, $g = \exp[-c_1L]$, is observed for either fixed $\gamma$ or fixed $L$ as shown in Fig. 2 and its inset. Here $c_1$ is a constant derived later. Again, zero conductance fluctuations always accompany with the conductance plateaus, see the Supplemental Materials [73].

**Origins of anomalous CI**—One crucial question arises immediately: Why are the non-quantized conductance plateaus of zero fluctuation possible in the anomalous CI phase? From the standard topological band theory [8], the chiral edge state of Hamiltonian (1) is the linear superposition of both spin-up and spin-down orbits with the equal weight. For the quantized conductance with the non-Hermicity of $\gamma_y$ or $\gamma_z$, the gain and loss of the two orbits compensate with each other. Thus, the local electron density in the chiral edge channel is over all conserved (real eigenenergies in the edge channel discussed below and Fig. 3). As a result, each edge channel contributes one conductance quanta, leading to a quantized conductance plateau as a Hermitian edge channel does. For the case of $\gamma_0$, both spin-up and spin-down orbits decay, thus an electron in the chiral channel has a finite-lifetime (complex eigenenergies in the channel), and electron density in the edge channel decays, leading to a non-quantized conductance plateau whose value decreases exponentially with the length of the Hall bar, but does not depend on the bar width.

To obtain an explicit expression that can be compared with the numerical results, we use the linear response theory [74] to derive the electron conductance in a decayed chiral edge channel. In the two-terminal configuration without disorder, the current $I_{n,k}$ of a transverse mode (label by $n$) with momentum $k_x = k$ is given by

$$I_{n,k} = \rho_{n,k}v^f n,k f(Re [E_n])T(E_n),$$

where $\rho_{n,k}$ is the electron charge density, $T(E_n)$ is the transmission coefficient of state $E_n$, $f$ and $v$ are respectively the Fermi-Dirac distribution and the group velocity, $E_n(k)$ is the energy dispersion relation of the transverse mode. We assume the reflectionless contacts so that $+k$ electrons in the edge channel come from the left lead and the $-k$ electrons from the right lead with chemical potentials of $\mu_1$ and $\mu_2$, respectively. In the linear response region, we set $|\mu_2|, |\mu_1| < \Delta$ so that no bulk states contribute to the conductance, where $2\Delta$ is the bulk gap, as shown in Fig. 3(a). At the zero temperature, $f_{1,2}(Re [E_n]) = \theta(\mu_1, 2 - Re [E_n])$ for the $+k$ and the $-k$ states, respectively. Here $\theta(x)$ is the Heaviside step function.

Since the non-Hermitian Hamiltonian (1) breaks the parity-time-reversal symmetry [75], the energies of eigen-
modes are complex, see Fig. 3. The dispersion relation of the real part relates to the electron group velocity and imaginary part is the lifetime of the electrons in the modes so that the particle density in the non-Hermitian CIs is

$$\rho_{n,k} = \frac{e}{L} \exp \left[ -\frac{4\pi \text{Im}[E_n]}{\hbar} \tau_{n,k} \right], \quad (4)$$

where $e$ and $\hbar$ are respectively the electron charge and the Plank constant. $\tau_{n,k} = L/v_{n,k}$ is the travel time for an electron through the Hall bar from one lead to the other. $\hbar/(4\pi \text{Im}[E_n])$ is the single electron lifetime in state $(n,k)$. In general, the non-Hermiticity adds a correction to the Hellmann-Feynman theorem such that the group velocity differs from that of the Hermitian system as discussed in References [76, 77]. However, for the constant non-Hermiticity considered here, the group velocity is

$$v_{n,k}^{\gamma} = \frac{2\pi}{\hbar} \frac{d\text{Re}[E_n]}{dk}, \quad (5)$$

providing that $\gamma \ll t$ [73].

Using Eqs. (3), (4), and (5), the net current flowing from one lead to the other is [73]

$$I = \frac{e}{\hbar} (e^{i2\pi(k_x/L)(ta)}\mu_1 - e^{-i2\pi(k_x/L)(ta)}\mu_2) + (I_{\text{bulk}}^+ - I_{\text{bulk}}^-). \quad (6)$$

The first term is the contribution from the edge channels where $e_\pm$ denote the edge-state energy for $k > 0$ and $k < 0$, respectively. The second term is the currents to the right (+) and to the left (-) due to bulk states,

$$\sum_{\text{bulk}} \sum_{k>0(k<0)} \frac{2\pi e}{L} \exp \left[ -\frac{4\pi \text{Im}[E_n]}{\hbar v_{n,k}} \right] \frac{d\text{Re}[E_n]}{dk} T(E_n). \quad (7)$$

The dimensionless conductances $g$ can then be derived from Eq. (6) for both balanced non-Hermiticities of $\gamma_y$ and $\gamma_z$ and unbalanced non-Hermiticity of $\gamma_0$.

For balanced non-Hermiticities, as shown in Figs. 3[a-d], both $\text{Re}[E_n]$ and $\text{Im}[E_n]$ of bulk eigenenergies are the even functions of $k$,

$$\text{Re}[E_n(k)] = \text{Re}[E_n(-k)], \quad (8)$$

and

$$\text{Im}[E_n(k)] = \text{Im}[E_n(-k)], \quad (9)$$

such that the bulk currents $I_{\text{bulk}}^+$ and $I_{\text{bulk}}^-$ cancel each other. On the other hand, the eigenenergies of the chiral edge states are real ($\text{Im}[\epsilon(k)] = 0$) [73] so that the electrons in chiral edge states are conserved, and the conductance is always quantized.

FIG. 3: Bulk spectra of the clean Hall bar of width $M = 20a$. The chemical potentials of the +$k$ electrons and =$k$ electrons are $\mu_1$ and $\mu_2$, respectively. (a) $\text{Re}[E/t]$ for $\gamma_y$ with $\gamma = 0.1t$. (b) $\text{Im}[E/t]$ for $\text{Re}[E] < 0$ with the same parameters as (a). (c,d) Same as (a,b) but for $\gamma_z$. (e,f) Same as (a,b) but for $\gamma_0$. Colors encode $\langle y \rangle/M$.

For unbalanced non-Hermiticities, there exists an anomalous CI phase whose bulk eigenenergies also satisfy Eqs. (8) and (9), as shown in Figs. 3[e,f]. Thus, the bulk states do not contribute any net current. Different from the balanced non-Hermiticity case, the eigenenergies of the edge chiral states are not real any more. The imaginary part $\text{Im}[\epsilon(k)] = \gamma$ destroys the electron conservation and leads to a non-quantized conductance that decays exponentially with $L$ and $\gamma$ [73]:

$$g = \exp[-2\gamma L/(\hbar a)]. \quad (10)$$

Indeed, our analytical formula Eq. (10) accords very well with the simulation results without any fitting parameter, see Fig. 2.

To understand zero $\Delta g$, we recall origin of the universal conductance fluctuation in ordinary disordered metals governed by the Hermitian Hamiltonian. The fluctuation is from the random opening and closing of a conducting channel as disorder configurations vary due to inevitable diffusion of defects and impurities at the finite temperature. Each conducting channel contributes exactly one quantum conductance of $e^2/\hbar$ no matter what its dispersion relation is in the channel [78]. Thus there is no conductance fluctuation in the one-dimensional conduction channel, and this explains conductance plateaus in quantum Hall systems as well as quantum point contacts.
in ballistic region. Our results suggest that zero conductance fluctuation is also true for non-Hermitian conducting channels. Disorders may modify electron dispersion relation in a channel, but will not introduce an uncertainty into the conductance even though the conductance is not quantized, as long as the number of edge channels (1 here) does not change in the presence of moderate disorder $|W < W_c|$ as shown in Fig. [1](f)]. Interestingly, the conductance will eventually go to zero as $W$ increases further beyond $W_c$, indicating the anomalous CI will be destroyed by strong disorders. The critical disorder $W_c$ can be determined by applying the self-consistent Born approximation to the non-Hermitian CI [73].

We would like to make a few remarks before the conclusion. (1) The edge state conductance and conductance fluctuation reported here for both balanced and unbalanced non-Hermicities should be generically applicable to edge states in non-Hermitian topological semimetals. (2) In this study, the randomness is introduced through the Hermitian on-site potential. If the randomness is on (2) In this study, the randomness is introduced through the Hermitian on-site potential. If the randomness is on the non-Hermicity coefficient $\gamma$, we observe that the conductance fluctuation would not be zero any more in the anomalous CI phase [73], and the non-quantized conductance plateau would not be perfect as observed here.

In conclusion, it is shown that the conductance of chiral edges states of non-Hermitian Chern insulators can be quantized (to the integer of $e^2/h$) or non-quantized, depending on whether the non-Hermicities are balanced or unbalanced. However, the conductance in both cases is insensitive to Hermitian disorder potential, leading to the zero conductance fluctuation and conductance plateaus against disorders and the sample width. The conductance plateaus can be destroyed by strong disorders above a critical value $W_c$ through Anderson localization. The non-quantization of conductance is due to the finite lifetime of chiral edge states in non-Hermitian CIs.

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[72] According to the non-Hermitian topological band theory, the Dirac mass is renormalized as $m - |\gamma|^2/2$ [67]. Thus, for the parameters considered here ($m = t, \gamma = 10^{-3}t$), our system is still topological nontrivial with chiral edge channels.

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