OPTIMAL ORDERING POLICY FOR A TWO-WAREHOUSE INVENTORY MODEL USE OF TWO-LEVEL TRADE CREDIT

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ABSTRACT. In today’s competitive markets, the supplier let the buyer to pay the purchasing cost after receiving the items, this strategy motivates the retailer to buy more items from the supplier and gains some benefit from the money which they did not pay at the time of receiving of the items. However, the retailer will be unable to pay off the debt obligations to the supplier in the future, so this study extends Yen et al. (2012) to consider the above situation and assumes the retailer can either pay off all accounts at the end of the delay period or delay incurring interest charges on the unpaid and overdue balance due to the difference between interest earned and interest charged. We will discuss the explorations of the function behaviors of the objection function to demonstrate the retailer’s optimal replenishment cycle time not only exists but also is unique. Finally, numerical examples are given to illustrate the theorems and gained managerial insights.

1. Introduction. Permissible delay in payments is one of the important factors in present day competitive marketing strategy. It is an interesting strategy for the retailer which motivates him/her to buy more items form the supplier and gain some benefit from the money which him/her did not pay at the time of receiving the items. In that case, the supplier offers a certain credit period to their retailer. For this period, interest is charged by the supplier. However, after this period, a higher rate of interest is charged by the supplier under certain terms and conditions by an agreement between retailer and supplier. It is clear that the retailer will delay the payment up to the last moment of the permissible period in order to gain capital, materials and service without any payment during the trade credit period. Consequently, trade credit can play a major role in inventory control for both the supplier as well as the retailer.

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Delayed payment has been discussed with various assumptions in the literature. Goyal [14] developed an EOQ model for the buyer when the supplier offers a fixed permissible delay period. Teng [35] assumed that the selling price is not equal to the purchasing price to modify Goyal’s model [14]. Chung and Liao [9] dealt with the problem of determining EOQ for exponentially deteriorating items under permissible delay in payments depending on the ordering quantity. Huang and Liao [16] explored a simple method to locate the optimal solution for exponentially deteriorating items under trade credit financing. Chung and Liao [10] revealed the optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach. Many related articles can be found in [2, 28, 11, 46, 29, 51, 5, 44, 33, 34, 12] and their references.

The passing of credit period from the customers to the buyer who is getting from the vendor is termed as the two-level credit policy. Jaber and Osman [17] developed a two-level supply chain system in which the retailer’s permissible delay in payment offered by the supplier is considered as a decision variable in order to coordinate the order quantity between the two levels. Liao [20] developed an EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit financing. Huang and Hsu [15] developed an inventory model under two-level trade credit policy by incorporating partial trade credit option at the customers of the retailer. Teng and Chang [37] analyzed two-level trade credit policy when production rate is finite. Liao and Huang [21] explored an inventory model for deteriorating items with two levels of trade credit taking account of time discounting. Chang et al. [4] presented the optimal manufacturer’s replenishment policies in a supply chain with up-stream and down-stream trade credits. Teng and Lou [38] established the seller’s optimal credit period and cycle time in a supply chain with up-stream and down-stream trade credits. Chen et al. [5] developed an EPQ model for deteriorating items with up-stream full trade credit and down-stream partial trade credit. Feng et al. [13] revealed inventory games with permissible delay in payments. Ting [43] discussed an EPQ model with deteriorating items under two levels of trade credit in a supply chain system. Researchers in this area include [18, 42, 36, 27, 8, 40, 48, 41] and their references.

During the past few years, many articles dealt with various inventory models under a variety trade credits have appeared such as [31] generalized the model for deteriorating items with ramp-type demand and permissible delay in payments. Teng et al. [39] explored an EOQ model with trade credit financing for non-decreasing demand. Chern et al. [7] discussed Nash equilibrium solution in a vendor-buyer supply chain with trade credit financing. Recently, Chen and Teng [6], Sarkar et al. [30], Wang et al. [45] and Wu and Chan [47] considered the deteriorating items with the expiration date under trade credit financing.

The classical inventory models are developed with the single storage facility mainly. This is unrealistic in most business transaction, because in real practice when a production of large amount of units of an items that cannot be stored in its existing own warehouse (OW) at the market place due to limited capacity is made, then excess units are stocked in a rented warehouse (RW) which is located at some distance away from OW. That is, management goes for large purchase at a time when either it gets an attractive price discounts or acquisition cost is higher than the holding cost in RW, i.e., the capacity of rented warehouse can be adjusted according to requirement in real life situation. Yang and Chang [49] developed a two-warehouse partial backlogging inventory model for deteriorating
items with permissible delay in payment under inflation. Liao and Huang [22] explored an inventory model for deteriorating items with trade credit financing and capacity constraints. Yen et al. [50] showed the optimal retailer’s ordering policies with trade credit financing and limited storage capacity in the supply chain system. Liao et al. [23] extended Goyal’s EOQ model to allow for deteriorating items with two warehouses under an order-size-dependent trade credit. Later, Liao et al. [24] revealed an optimal pricing and ordering policy for perishable items with limited storage capacity and partial trade credit. Liao et al. [26] derived optimal strategy for deteriorating items with capacity constraints under two-level trade credit. The other researchers revealed extensions of two-warehouse inventory models such as [32, 25, 19, 1] and their references.

The main purpose of this paper is to generalize the paper of Yen et al. [50] with a view of making their model more relevant and so applicable to practice, so we take into account that the retailer can either pay off all accounts at the end of the credit period or delay incurring interest charges on the unpaid and overdue balance due to the difference between interest earned and interest charged. Under these conditions, this study determines the optimal ordering policy to maximize the retailer’s profit.

2. Model formulation. The mathematical models presented in this study are based on the following notation and assumptions:

Notation:
- $D$: the annual demand rate;
- $p$: selling price per unit in dollars;
- $C$: purchasing cost per unit in dollars, $C < p$;
- $h$: holding cost per unit per year in dollars for the item in own warehouse (OW) excluding interest charge;
- $k$: holding cost per unit per year in dollars for the item in the rented warehouse (RW) excluding interest charges, $(k \geq h)$;
- $A$: ordering cost per order in dollars;
- $I_k$: interest charged per dollar per year;
- $I_e$: interest earned per dollar per year;
- $M$: the retailer’s credit period in years offered by the supplier;
- $N$: the customer’s credit period in years offered by the retailer;
- $W$: the storage capacity of OW;
- $T$: replenishment cycle time in years;
- $Q$: the ordering quantity per replenishment;
- $T_w$: the time in years at which the inventory level reaches zero in RW;
- $Z(T)$: annual total profit function;

Assumptions:
1. Replenishment is instantaneous with a known, constant lead time.
2. The time horizon of the inventory system is infinite.
3. Shortages are not allowed to occur.
4. The owned warehouse (OW) has a fixed capacity of $W$ units.
5. The rented warehouse (RW) has unlimited capacity.
6. The items of OW are consumed only after consuming the items kept in RW.
7. The inventory costs in the RW are higher than those in the OW.
8. The supplier proposes a certain credit period and sales revenue generated during the credit period is deposited in an interest bearing account with rate $I_e$. At the end of this period, the credit is settled and the remaining balance will be
paid off by loan from the bank with interest charges in stock with rate $I_k$ if necessary.

(9) After the credit period, the retailer makes payment to the bank immediately after the sales of the items until the retailer pays off the remaining balance.

It is evident that the retailer can accumulate revenue and earn interest during the period $N$ to $T$ when $T \geq M$ and that during the period $N$ to $M$ when $T < M$. On the other hand, the account is settled at time $M$, the retailer pays off loan owed to the supplier whenever he/she has money from sales, and starts paying for the higher interest charges on the items in stock with rate $I_k$ when $T \geq M$. The account is settled at time $M$ and the retailer needs not to pay any interest charge when $T < M$. On the basis of above assumptions, the total annual profit of the inventory system for the retailer is computed using the following various components:

1. Annual net sales revenue = $(p - C)D$
2. Annual ordering cost = $A/T$
3. Annual stock-holding cost (excluding interest charged) is obtained as follows:
   - Case 1: $\frac{W}{T} < T$
     \[ \text{Annual stock-holding cost} = \frac{k(DT-W)^2+hW(2DT-W)}{2DT} \]
   - Case 2: $\frac{W}{T} \geq T$
     \[ \text{Annual stock-holding cost} = \frac{DTh}{2} \]

According to the assumption of (8), we get that the retailer buys $DT$ units at time 0, she/he owes the supplier $CDT$ at time $M$. On the other hand, the retailer has $pDM + pI_eD(M^2 - N^2)/2$ in the account at time $M$. Furthermore, based on the difference between the purchasing cost and the money in the account, we let

\[ W^* = \frac{2pM + pI_e(M^2 - N^2)}{2C}, \]

then $pDM + \frac{pI_eD(M^2 - N^2)}{2} < CDT$ if and only if $W^* < T$.

Regarding interests charged and earned, based on the length of the order cycle, there are four possible cases: (1) $0 < T \leq N$, (2) $N < T \leq M$, (3) $M < T \leq W^*$ and (4) $W^* < T$.

4. The cost of interest payable per year can be obtained as follows:
   - Case 1: $0 < T \leq N$
     In this case, there is no interest payable.
   - Case 2: $N < T \leq M$
     In this case, there is no interest payable.
   - Case 3: $M < T \leq W^*$
     In this case, the money in the account at time $M$ is greater than or equal to the purchasing cost, so there is on interest payable.
   - Case 4: $W^* < T$
     In this case, the money in the account at time $M$ is less than the purchasing cost, so the retailer needs to finance the difference $L = CDT - [pDM + \frac{pI_eD(M^2 - N^2)}{2}]$ with rate $I_k$ at time $M$. Thereafter, the retailer gradually reduces the amount of financed loan from constant sales and revenue received. The graphical representation of this case is shown in Fig. 1, and the interest charged per cycle
can be obtained as follows:

$$I_kL\left(\frac{L}{pD}\right) = \frac{I_k}{2T} \left\{ CDT - \left[ pDM + \frac{pLDM^2 - N^2}{2pD} \right] \right\}^2$$

(5) The interest earned per year can be obtained as follows:

Case 1: $0 < T \leq N$

In this case, the graphical representation is shown in Fig. 2, then

annual interest earned = $\frac{pLDT(M - N)}{T} = pIcD(M - N)$

Case 2: $N < T \leq M$

In this case, the graphical representation is shown in Fig. 3, then

annual interest earned = $\frac{pLDM(M^2 - N^2)}{2T}$

Case 3: $M < T \leq W^*$

In this case, the money in the account at time $M$ is greater than or equal to the purchasing cost, so the retailer pays off the total amount owed to the supplier $CDT$ and has amount $pDM + \frac{pLDM^2 - N^2}{2} - CDT$ on hand at time $M$. In addition, the retailer continuously sells items and uses the revenue to earn interest after the time point $M$ to $T$. The graphical representation of this case is shown in Fig. 4, then

annual interest earned = $\frac{pIcD(M^2 - N^2)}{2T}$

$$+ \frac{Ic}{T} \left[ pDM + \frac{pIcD(M^2 - N^2)}{2} - CDT \right] (T - M)$$

$$+ \frac{pIcD(T - M)^2}{2T}$$

Case 4: $W^* < T$

In this case, the money in the account at time $M$ is less than the purchasing cost, so the retailer gradually reduces the financed loan from constant sales and revenue received until the time point $M + \frac{L}{pD}$. After that time point, the retailer continuously sells items and uses the revenue to earn interest until time $T$. The graphical representation of this case is shown in Fig. 5, then

annual interest earned = $\frac{pLDM^2 - N^2}{2T} + \frac{pIcD(M - \frac{L}{pD})^2}{2T}$

From the above arguments, the annual total profit for the retailer can be expressed as

$$Z(T) = \text{Net sales revenue} - \text{ordering cost} - \text{stock-holding cost}$$

$$- \text{interest changed} + \text{interest earned}.$$

Moreover, we consider the following results:

(1) Suppose $\frac{W}{pD} < N < M < W^*$

In this case, $Z(T)$ is given by

$$Z(T) = \begin{cases} Z_1(T) & \text{if } 0 < T < W/D \\ Z_2(T) & \text{if } W/D \leq T < N \\ Z_3(T) & \text{if } N \leq T < M \\ Z_4(T) & \text{if } M \leq T < W^* \\ Z_5(T) & \text{if } W^* \leq T \end{cases}$$

(1a) (1b) (1c) (1d) (1e)
where

\[ Z_1(T) = (p - C)D - \frac{A}{T} - \frac{DT_h}{2} + pI_eD(M - N) \] (2)

\[ Z_2(T) = (p - C)D - \frac{A}{T} - \frac{k(DT - W)^2}{2DT} - \frac{hW(2DT - W)}{2DT} + pI_eD(M - N) \] (3)

\[ Z_3(T) = (p - C)D - \frac{A}{T} - \frac{k(DT - W)^2}{2DT} - \frac{hW(2DT - W)}{2DT} + pI_eD(2MT - N^2 - T^2) \] (4)

\[ Z_4(T) = (p - C)D - \frac{A}{T} - \frac{k(DT - W)^2}{2DT} - \frac{hW(2DT - W)}{2DT} + pI_eD(M^2 - N^2) \]
\[ + \frac{I_e}{T} \left[ pDM + \frac{pI_eD(M^2 - N^2)}{2} - CDT \right] (T - M) \]
\[ + \frac{pI_eD(T - M)^2}{2T} \] (5)

and

\[ Z_5(T) = (p - C)D - \frac{A}{T} - \frac{k(DT - W)^2}{2DT} - \frac{hW(2DT - W)}{2DT} \]
\[ - \frac{I_kL^2}{2pDT} + \frac{pI_eD(M^2 - N^2)}{2T} + \frac{pI_eD(T - M - \frac{L}{pD})^2}{2T} \] (6)

Since

\[ Z_1\left(\frac{W}{D}\right) = Z_2\left(\frac{W}{D}\right) \] (7)

\[ Z_2(N) = Z_3(N) \] (8)

\[ Z_3(M) = Z_4(M) \] (9)

and

\[ Z_4(W^*) = Z_5(W^*) \] (10)

Furthermore, \( Z(T) \) is continuous and well-defined on \( T > 0 \).

(2) Suppose \( N < W/D < M < W^* \)

In this case, \( Z(T) \) is given by

\[
Z(T) = \begin{cases} 
Z_1(T) & \text{if } 0 < T \leq N \\
Z_6(T) & \text{if } N < T \leq W/D \\
Z_3(T) & \text{if } W/D < T \leq M \\
Z_4(T) & \text{if } M < T \leq W^* \\
Z_5(T) & \text{if } W^* < T 
\end{cases} \] (11a)

(11b)

(11c)

(11d)

(11e)

where

\[ Z_6(T) = (p - C)D - \frac{A}{T} - \frac{DT_h}{2} + \frac{pI_eD(2MT - N^2 - T^2)}{2T} \] (12)

Since

\[ Z_1(N) = Z_6(N) \] (13)

and

\[ Z_6\left(\frac{W}{D}\right) = Z_3\left(\frac{W}{D}\right) \] (14)

Furthermore, \( Z(T) \) is continuous and well-defined on \( T > 0 \).
(3) Suppose $N < M < \frac{W}{D} < W^*$
In this case, $Z(T)$ is given by

$$Z(T) = \begin{cases} 
    Z_1(T) & \text{if } 0 < T \leq N \\
    Z_6(T) & \text{if } N < T \leq M \\
    Z_7(T) & \text{if } M < T \leq W/D \\
    Z_4(T) & \text{if } W/D < T \leq W^* \\
    Z_5(T) & \text{if } W^* < T 
\end{cases}$$

where

$$Z_7(T) = (p-C)D - \frac{A}{T} - \frac{DT_h}{2} + \frac{pL_cD(M^2 - N^2)}{2T}$$
$$+ \frac{I_t}{T} \left[ pDM + \frac{pI_cD(M^2 - N^2)}{2} - CDT \right] (T - M)$$
$$+ \frac{pL_cD(T - M)^2}{2}$$

Since

$$Z_6(M) = Z_7(M) \quad (17)$$

and

$$Z_7\left(\frac{W}{D}\right) = Z_4\left(\frac{W}{D}\right) \quad (18)$$

Furthermore, $Z(T)$ is continuous and well-defined on $T > 0$.

(4) Suppose $N < M < W^* < \frac{W}{D}$
In this case, $Z(T)$ is given by

$$Z(T) = \begin{cases} 
    Z_1(T) & \text{if } 0 < T \leq N \\
    Z_6(T) & \text{if } N < T \leq M \\
    Z_7(T) & \text{if } M < T \leq W^* \\
    Z_8(T) & \text{if } W^* < T \leq W/D \\
    Z_5(T) & \text{if } W/D < T 
\end{cases}$$

where

$$Z_8(T) = (p-C)D - \frac{A}{T} - \frac{hDT}{2} - \frac{I_kL^2}{2pDT} + \frac{pI_cD(M^2 - N^2)}{2T}$$
$$+ \frac{pL_cD(T - M - \frac{L}{pD})^2}{2T}$$

since

$$Z_7(W^*) = Z_8(W^*) \quad (21)$$

and

$$Z_7\left(\frac{W}{D}\right) = Z_5\left(\frac{W}{D}\right) \quad (22)$$

Furthermore, $Z(T)$ is continuous and well-defined on $T > 0$. 
3. The concavities of $Z_i(T)$ ($i = 1, 2, \ldots, 8$). Eqs. (2)~(6), (12), (16) and (20) yield the first and second derivatives with respect to $T$ as follows:

\[
Z'_1(T) = \frac{A}{T^2} - \frac{Dh}{2} \\
Z''_1(T) = \frac{-2A}{T^3} < 0
\]

\[
Z'_2(T) = \frac{A}{T^2} - \frac{k(D^2T^2 - W^2)}{2DT^2} - \frac{hW^2}{2DT^2}
\]

\[
Z''_2(T) = \frac{-2A}{T^3} - \frac{(k-h)W^2}{DT^3} < 0
\]

\[
Z'_3(T) = \frac{A}{T^2} - \frac{k(D^2T^2 - W^2)}{2DT^2} - \frac{hW^2}{2DT^2} + \frac{pI_eD(N^2 - T^2)}{2T^2}
\]

\[
Z''_3(T) = \frac{-2A}{T^3} - \frac{(k-h)W^2}{DT^3} - \frac{pI_eDN^2}{T^3} < 0
\]

\[
Z'_4(T) = \frac{A}{T^2} - \frac{k(D^2T^2 - W^2)}{2DT^2} - \frac{hW^2}{2DT^2} + \frac{pI_eD [(M^2 - N^2)MI_e + N^2]}{2T^2}
\]

\[
- \frac{2}{D(2C - p)I_e}
\]

\[
Z'_4(T) = \frac{-2A}{T^3} - \frac{(k-h)W^2}{DT^3} - \frac{pI_eD [(M^2 - N^2)MI_e + N^2]}{T^3} < 0
\]

\[
Z'_5(T) = \frac{A}{T^2} - \frac{k(D^2T^2 - W^2)}{2DT^2} - \frac{hW^2}{2DT^2} + \frac{pD}{8T^2} \left\{ M^2(I_k - I_e) + I_e \left[ (M^2 - N^2)MI_k + N^2 \right] \right\}
\]

\[
Z'_5(T) = \frac{-2A}{T^3} - \frac{(k-h)W^2}{DT^3} - \frac{pD}{4T^3} \left\{ M^2(I_k - I_e) + I_e \left[ (M^2 - N^2)MI_k + N^2 \right] \right\} < 0
\]

\[
Z'_6(T) = \frac{A}{T^2} - \frac{Dh}{2} + \frac{pI_eD(N^2 - T^2)}{2T^2}
\]

\[
Z''_6(T) = \frac{-2A}{T^3} - \frac{pI_eDN^2}{T^3} < 0
\]

\[
Z'_7(T) = \frac{A}{T^2} - \frac{Dh}{2} + \frac{pI_eD [(M^2 - N^2)MI_e + N^2]}{2T^2} - \frac{D(2C - p)I_e}{2}
\]

\[
Z''_7(T) = \frac{-2A}{T^3} - \frac{pI_eD [(M^2 - N^2)MI_e + N^2]}{T^3} < 0
\]

\[
Z'_8(T) = \frac{A}{T^2} - \frac{Dh}{2} + \frac{pD}{8T^2} \left\{ M^2(I_k - I_e) + I_e \left[ (M^2 - N^2)MI_k + N^2 \right] \right\}
\]

\[
+ \frac{D}{2} \left\{ \frac{C^2(I_e - I_k)}{p} - (2C - p)I_e \right\}
\]
and

\[ Z_8''(T) = \frac{2A}{T^3} - \frac{pD I_e^2 (M^2 - N^2)^2 (I_k - I_e)}{4T^3} \]
\[ - \frac{pD \left\{ M^2 (I_k - I_e) + I_e \left[ MI_k (M^2 - N^2) + N^2 \right] \right\}}{T^3} < 0 \]  

(38)

Eqs.(24), (26), (28), (30), (32), (34), (36) and (38) imply \( Z_i(T) (i = 1, 2, \ldots, 8) \) are concave on \( T > 0 \) and we have

\[ Z_i'(T) \begin{cases} 
  > 0 & \text{if } 0 < T < T_i^* \\
  = 0 & \text{if } T = T_i^* \\
  < 0 & \text{if } T_i^* < T < \infty
\end{cases} \]  

(39a) 

(39b) 

(39c)

Eqs.(39a)~(39c) imply that \( Z_i(T) \) is increasing on \((0, T_i^*] \) and decreasing on \([T_i^*, \infty) \) for \( i = 1, 2, \ldots, 8 \).

4. **Decision rule of the optimal replenishment cycle time** \( T^* \) when \( \frac{W}{T^*} < N < M < W^* \). Obviously, eqs.(23), (25), (27), (29) and (31) yield that

\[ Z_1' \left( \frac{W}{D} \right) = Z_2' \left( \frac{W}{D} \right) = \frac{1}{2(\frac{W}{D})^2} \left\{ 2A - \frac{hW^2}{D} \right\} \]  

(40)

\[ Z_2'(N) = Z_3'(N) = \frac{1}{2N^2} \left\{ 2A - \frac{k}{D} (D^2 N^2 W^2) - \frac{h}{D} W^2 \right\} \]  

(41)

\[ Z_3'(M) = \frac{1}{2M^2} \left\{ 2A - \frac{k}{D} (D^2 M^2 - W^2) - \frac{h}{D} W^2 + pI_e D (N^2 - M^2) \right\} \]  

(42)

\[ Z_4'(M) = \frac{1}{2M^2} \left\{ 2A - \frac{k}{D} (D^2 M^2 - W^2) - \frac{h}{D} W^2 + pI_e D [(M^2 - N^2) MI_e + N^2] - DM^2 (2C - p) I_e \right\} \]  

(43)

and

\[ Z_4'(W^*) = Z_5'(W^*) = \frac{1}{2W^*^2} \left\{ 2A - \frac{k}{D} (D^2 W^*^2 - W^2) - \frac{h}{D} W^2 \right\} \]  

(44)

\[ + pI_e D [(M^2 - N^2) MI_e + N^2] - DW^*^2 (2C - p) I_e \]

Let

\[ \Delta_1 = 2A - \frac{hW^2}{D} \]  

(45)

\[ \Delta_2 = 2A - \frac{k}{D} (D^2 N^2 - W^2) - \frac{h}{D} W^2 \]  

(46)

\[ \Delta_3 = 2A - \frac{k}{D} (D^2 M^2 - W^2) - \frac{h}{D} W^2 + pI_e D (N^2 - M^2) \]  

(47)

\[ \Delta_4 = 2A - \frac{k}{D} (D^2 M^2 - W^2) - \frac{h}{D} W^2 + pI_e D [(M^2 - N^2) MI_k + N^2] \]
\[ - DM^2 (2C - p) I_e \]  

(48)

and

\[ \Delta_5 = 2A - \frac{k}{D} (D^2 W^*^2 - W^2) - \frac{h}{D} W^2 + pI_e D [(M^2 - N^2) MI_k + N^2] \]
\[ - DW^*^2 (2C - p) I_e \]  

(49)

Moreover, we have

\[ \Delta_4 - \Delta_3 = pI_e D [(M^2 - N^2) MI_k + M^2] - \frac{D(2C - p) I_e}{2} > pI_e D [(M^2 - N^2) MI_k] > 0 \]
Hence, eqs.(45)∼(49) yield that \(\Delta_1 > \Delta_2 > \Delta_3, \Delta_3 < \Delta_4\) and \(\Delta_4 > \Delta_5\).

Theorem 1: Suppose \(\frac{W}{D} < N < M < W^*\). Hence

(A) If \(\Delta_1 < 0, \Delta_2 < 0, \Delta_3 < 0\) and
   (A1) if \(\Delta_4 < 0\) and \(\Delta_5 < 0\), then \(T^* = T_1^*\).
   (A2) if \(\Delta_4 \geq 0\) and \(\Delta_5 < 0\), then \(T^* = T_1^*\) or \(T_4^*\) associated with the most profit.
   (A3) if \(\Delta_4 \geq 0\) and \(\Delta_5 \geq 0\), then \(T^* = T_1^*\) or \(T_5^*\) associated with the most profit.

(B) If \(\Delta_1 \geq 0, \Delta_2 < 0, \Delta_3 < 0\) and
   (B1) if \(\Delta_4 < 0\) and \(\Delta_5 < 0\), then \(T^* = T_2^*\).
   (B2) if \(\Delta_4 \geq 0\) and \(\Delta_5 < 0\), then \(T^* = T_2^*\) or \(T_4^*\) associated with the most profit.
   (B3) if \(\Delta_4 \geq 0\) and \(\Delta_5 \geq 0\), then \(T^* = T_2^*\) or \(T_5^*\) associated with the most profit.

(C) If \(\Delta_1 \geq 0, \Delta_2 \geq 0, \Delta_3 < 0\) and
   (C1) if \(\Delta_4 < 0\) and \(\Delta_5 < 0\), then \(T^* = T_3^*\).
   (C2) if \(\Delta_4 \geq 0\) and \(\Delta_5 < 0\), then \(T^* = T_3^*\) or \(T_4^*\) associated with the most profit.
   (C3) if \(\Delta_4 \geq 0\) and \(\Delta_5 \geq 0\), then \(T^* = T_3^*\) or \(T_5^*\) associated with the most profit.

(D) If \(\Delta_1 \geq 0, \Delta_2 \geq 0, \Delta_3 \geq 0\) and
   (D1) if \(\Delta_4 \geq 0\) and \(\Delta_5 < 0\), then \(T^* = T_4^*\).
   (D2) if \(\Delta_4 \geq 0\) and \(\Delta_5 \geq 0\), then \(T^* = T_5^*\).

Proof. It immediately follows from above discussion.

5. Decision rule of the optimal replenishment cycle time \(T^*\) when \(N < \frac{W}{D} < M < W^*\). Likewise, eqs.(23), (33), (27), (29) and (31) yield that

\[ Z_1'(N) = Z_6'(N) = \frac{1}{2N^2} (2A - DhN^2) \]  (50)

and

\[ Z_6'(\frac{W}{D}) = Z_3'(\frac{W}{D}) = \frac{1}{2(\frac{W}{D})^2} \left\{ 2A - \frac{h}{D} W^2 + pI_eD \left[ N^2 - (\frac{W}{D})^2 \right] \right\} \]  (51)

Let

\[ \Delta_6 = 2A - DhN^2 \]  (52)

and

\[ \Delta_7 = 2A - \frac{h}{D} W^2 + pI_eD \left[ N^2 - (\frac{W}{D})^2 \right] \]  (53)

Hence, we have \(\Delta_6 > \Delta_7 > \Delta_3\). Based on the above arguments, we have the following results.

Theorem 2: Suppose \(N < \frac{W}{D} < M < W^*\)

(A) If \(\Delta_6 < 0, \Delta_7 < 0, \Delta_3 < 0\) and
   (A1) if \(\Delta_4 < 0\) and \(\Delta_5 < 0\), then \(T^* = T_1^*\).
   (A2) if \(\Delta_4 \geq 0\) and \(\Delta_5 < 0\), then \(T^* = T_1^*\) or \(T_4^*\) associated with the most profit.
   (A3) if \(\Delta_4 \geq 0\) and \(\Delta_5 \geq 0\), then \(T^* = T_1^*\) or \(T_5^*\) associated with the most profit.
(B) If \( \Delta_6 \geq 0, \Delta_7 < 0, \Delta_3 < 0 \) and
(B1) if \( \Delta_4 < 0 \) and \( \Delta_5 < 0 \), then \( T^* \) is \( T_6^* \).
(B2) if \( \Delta_4 \geq 0 \) and \( \Delta_5 < 0 \), then \( T^* \) is \( T_6^* \) or \( T_4^* \) associated with the most profit.
(B3) if \( \Delta_4 \geq 0 \) and \( \Delta_5 \geq 0 \), then \( T^* \) is \( T_6^* \) or \( T_5^* \) associated with the most profit.
(C) If \( \Delta_6 \geq 0, \Delta_7 \geq 0, \Delta_3 \leq 0 \) and
(C1) if \( \Delta_4 < 0 \) and \( \Delta_5 < 0 \), then \( T^* \) is \( T_7^* \).
(C2) if \( \Delta_4 \geq 0 \) and \( \Delta_5 < 0 \), then \( T^* \) is \( T_3^* \) or \( T_4^* \) associated with the most profit.
(C3) if \( \Delta_4 \geq 0 \) and \( \Delta_5 \geq 0 \), then \( T^* \) is \( T_3^* \) or \( T_5^* \) associated with the most profit.
(D) If \( \Delta_6 \geq 0, \Delta_7 \geq 0, \Delta_3 \geq 0 \) and
(D1) if \( \Delta_4 \geq 0 \) and \( \Delta_5 < 0 \), then \( T^* \) is \( T_7^* \).
(D2) if \( \Delta_4 \geq 0 \) and \( \Delta_5 \geq 0 \), then \( T^* \) is \( T_7^* \).

Proof. It immediately follows from above discussion.

\[ \text{\( \Delta_8 \)} = 2A - DhM^2 + pI_eD((M^2 - N^2)MI_e + N^2) - D(2C - p)M^2I_e \]  \hspace{1cm} (57)

\[ \text{\( \Delta_9 \)} = 2A - DhM^2 + pI_eD((M^2 - N^2)MI_e + N^2) - D(2C - p)M^2I_e \]  \hspace{1cm} (58)

\[ \text{\( \Delta_{10} \)} = 2A - \frac{h}{D}W^2 + pI_eD((M^2 - N^2)MI_e + N^2) - D(2C - p)(\frac{W}{D})^2I_e \]  \hspace{1cm} (59)

Additionally,
\[ \Delta_9 - \Delta_8 = pI_eD((M^2 - N^2)MI_e + M^2) - D(2C - p)M^2I_e > 0 \]

then we obtain \( \Delta_6 > \Delta_8, \Delta_9 > \Delta_8 \) and \( \Delta_9 > \Delta_{10} > \Delta_5 \). From the above arguments, we get the following results.

Theorem 3: Suppose \( N < M < \frac{W}{D} < W^* \)

(A) If \( \Delta_9 < 0, \Delta_{10} < 0, \Delta_5 < 0 \) and
(A1) if \( \Delta_8 < 0 \) and \( \Delta_6 < 0 \), then \( T^* \) is \( T_7^* \).
(A2) if \( \Delta_8 < 0 \) and \( \Delta_6 \geq 0 \), then \( T^* \) is \( T_6^* \).
Additionally, we obtain $\Delta_9 \geq 0, \Delta_{10} < 0, \Delta_5 < 0$ and

(B1) if $\Delta_8 < 0$ and $\Delta_6 < 0$, then $T^*$ is $T_1^*$ or $T_7^*$ associated with the most profit.

(B2) if $\Delta_8 < 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_6^*$ or $T_7^*$ associated with the most profit.

(B3) if $\Delta_8 \geq 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_7^*$.

(C) If $\Delta_9 \geq 0, \Delta_{10} \geq 0, \Delta_5 < 0$ and

(C1) if $\Delta_8 < 0$ and $\Delta_6 < 0$, then $T^*$ is $T_1^*$ or $T_4^*$ associated with the most profit.

(C2) if $\Delta_8 < 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_6^*$ or $T_4^*$ associated with the most profit.

(C3) if $\Delta_8 \geq 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_4^*$.

(D) If $\Delta_9 \geq 0, \Delta_{10} \geq 0, \Delta_5 \geq 0$ and

(D1) if $\Delta_8 < 0$ and $\Delta_6 < 0$, then $T^*$ is $T_1^*$ or $T_5^*$ associated with the most profit.

(D2) if $\Delta_8 < 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_6^*$ or $T_5^*$ associated with the most profit.

(D3) if $\Delta_8 \geq 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_5^*$.

Proof. It immediately follows from above discussion. \hfill \Box

7. Decision rule of the optimal replenishment cycle time $T^*$ when $N < M < W^* < \frac{W}{D}$. Eqs.(35), (37) and (31) yield

$$Z_i'(W^*) = Z_8'(W^*)$$

$$= \frac{1}{2W^{*2}} \left\{ 2A - DhW^* + pI_eD \left[ (M^2 - N^2)MI_e + N^2 \right] 
- D(2C - p)I_eW^{*2} \right\}$$

and

$$Z_8'(\frac{W}{D}) = Z_6'(\frac{W}{D})$$

$$= \frac{1}{2(\frac{W}{D})^2} \left\{ 2A - \frac{h}{D}W^2 + \frac{pDI_e^2 (M^2 - N^2)^2 (I_k - I_e)}{4} 
+ pD \left\{ M^2(I_k - I_e) + I_e \left[ (M^2 - N^2)MI_k + N^2 \right] \right\} 
\right\}$$

$$+ \frac{W^2}{D} \left[ \frac{C^2(I_e - I_k)}{p} - (2C - p)I_e \right] \}$$

Let

$$\Delta_{11} = 2A - DhM^* + pI_eD \left[ (M^2 - N^2)MI_e + N^2 \right] - D(2C - p)I_eW^{*2}$$

and

$$\Delta_{12} = 2A - \frac{h}{D}W^2 + \frac{pDI_e^2 (M^2 - N^2)^2 (I_k - I_e)}{4} 
+ pD \left\{ M^2(I_k - I_e) + I_e \left[ (M^2 - N^2)MI_k + N^2 \right] \right\}$$

$$+ \frac{W^2}{D} \left[ \frac{C^2(I_e - I_k)}{p} - (2C - p)I_e \right] \}$$

Additionally, we obtain $\Delta_9 > \Delta_{11} > \Delta_{12}$. 
From the above arguments, we get the following results.

Theorem 4: Suppose $N < M < W^* < \frac{W}{D}$

(A) If $\Delta_9 < 0$, $\Delta_{11} < 0$, and $\Delta_{12} < 0$ then

(A1) if $\Delta_8 < 0$ and $\Delta_6 < 0$, then $T^* = T_1^*$

(A2) if $\Delta_8 < 0$ and $\Delta_6 \geq 0$, then $T^* = T_6^*$.

(B) If $\Delta_9 \geq 0$, $\Delta_{11} < 0$, and $\Delta_{12} < 0$ then

(B1) if $\Delta_8 < 0$ and $\Delta_6 < 0$, then $T^*$ is $T_1^*$ or $T_7^*$ associated with the most profit.

(B2) if $\Delta_8 < 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_6^*$ or $T_7^*$ associated with the most profit.

(C) If $\Delta_9 \geq 0$, $\Delta_{11} \geq 0$, and $\Delta_{12} < 0$ then

(C1) if $\Delta_8 < 0$ and $\Delta_6 < 0$, then $T^*$ is $T_1^*$ or $T_8^*$ associated with the most profit.

(C2) if $\Delta_8 < 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_6^*$ or $T_8^*$ associated with the most profit.

(C3) if $\Delta_8 \geq 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_8^*$.

(D) If $\Delta_9 \geq 0$, $\Delta_{11} \geq 0$, and $\Delta_{12} \geq 0$ then

(D1) if $\Delta_8 < 0$ and $\Delta_6 < 0$, then $T^*$ is $T_1^*$ or $T_5^*$ associated with the most profit.

(D2) if $\Delta_8 < 0$ and $\Delta_6 \geq 0$, then $T^*$ is $T_6^*$ or $T_5^*$ associated with the most profit.

Proof. It immediately follows from above discussion. \qed

8. Numerical examples and sensitivity analysis. To illustrate the usefulness of the models developed in Sections 5~7, the parameters needed for analyzing the above inventory situations are given below:

(Insert Tables 1~4 about here)

Based on the computational results as shown in Table 5, we can obtain the following managerial insights:

(Insert Table 5 about here)

(1) In general, when ordering cost per order $A$ increases, the length of replenishment cycle increases but the total relevant profit decreases. The simple economic explanation for this is the larger ordering cost parameter, the lower the total relevant profit will be.

(2) It is obvious that all the values of $T^*$ and $Z(T^*)$ decrease as the revenue parameter $C$ increases. That is, purchasing cost per unit has nonnegative effects on the length of replenishment cycle and the annual total relevant profit.

(3) The values of $T^*$ and $Z(T^*)$ increase as the parameter $W$ increases.

9. Summary and conclusion. This paper presents a modified inventory model which accounts for considering that the retailer can either pay off all accounts at the end of the credit period or delay incurring interest charges on the unpaid and overdue balance due to the difference between interest earned and interest charged when using the EOQ formulae. We have demonstrated that the optimal solution for each possible alternative exists uniquely, which simplifies the search for the global optimal to finding a local solution that are given as Theorems 1~4 under various conditions. Numerical examples are given to illustrate the theoretical results and obtain some managerial insights.
Additionally, in traditional marketing and economic theory, price is a major factor on the demand rate, so one could take pricing strategy into consideration in the future research. One can consider the conditions of the price and trade-credit dependent demand or the effect of inflation rates on the optimal credit period and cycle time simultaneously in the model. Finally, we could generalize the model to allow for shortages, quantity discount, backlogging and inflation rates, etc.

**Figure 1.** The interest charged when $W^* < T$

**Figure 2.** The total accumulation of interest earned when $0 < T \leq N$

**Figure 3.** The total accumulation of interest earned when $N < T \leq M$
Table 1. The ordering policy by using Theorem 1.

| D | A   | C   | p  | k   | h   | I_p | N   | M   | W   | W/D | W*  | Δ_1  | Δ_2  | Δ_3  | Δ_4  | Δ_5  | T*  | Z(T*) |
|---|-----|-----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A1 | 100 | 0.010 | 1 | 2 | 1.0 | 0.055 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0141 | 98.6175 |
| A2 | 100 | 0.010 | 1 | 2 | 1.0 | 0.055 | 0.115000 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0141 | 98.6175 |
| A3 | 100 | 0.010 | 5 | 20 | 0.9 | 0.100 | 0.130000 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.9999 | 1521.51 |
| B1 | 100 | 0.002 | 10 | 15 | 0.9 | 0.100 | 0.130000 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0169 | 502.332 |
| B2 | 100 | 0.002 | 5 | 15 | 0.9 | 0.100 | 0.130000 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0169 | 1092.36 |
| B3 | 100 | 0.002 | 5 | 15 | 0.9 | 0.100 | 0.130000 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.3110 | 1192.80 |
| C1 | 100 | 0.010 | 10 | 15 | 0.9 | 0.100 | 0.130000 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0190 | 501.884 |
| C2 | 100 | 0.010 | 5 | 15 | 0.9 | 0.100 | 0.130000 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0300 | 499.948 |
| C3 | 100 | 0.010 | 5 | 15 | 0.9 | 0.100 | 0.130000 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0371 | 1101.50 |
| D1 | 100 | 0.014 | 1 | 2 | 1.0 | 0.055 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0175 | 98.8843 |
| D2 | 100 | 0.090 | 1 | 2 | 1.0 | 0.055 | 0.000115 | 0.01650098 | 0.016501 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | <0 | T* = 0.0420 | 95.8097 |
Table 2. The optimal ordering policy by using Theorem 2.

| D  | A  | C  | p  | k  | h  | \( l_e \) | \( l_p \) | N  | M  | W  | \( W/D \) | \( W^* \) | \( \Delta_6 \) | \( \Delta_7 \) | \( \Delta_1 \) | \( \Delta_4 \) | \( \Delta_2 \) | \( T^* \) | \( Z(T^*) \) |
|----|----|----|----|----|----|---------|---------|----|----|----|---------|---------|----------|----------|----------|----------|----------|----------|----------|
| A1 | 35 | 2  | 1  | 3  |ug. | 0.12    | 0.15    | 0.200| 0.300| 10 | 0.2857  | 0.9090  | <0       | <0       | <0       | <0       | <0       | 0.1952   | 50.7661  |
| A2 | 500| 3  | 10 | 20.8| 1.12| 0.20   | 0.21   | 0.123| 0.300| 100 | 0.2     | 0.6397  | <0       | <0       | <0       | <0       | >0       | 0.1036   | 5716.50  |
| A3 | 300| 3  | 10 | 24  | 1.12| 0.20   | 0.21   | 0.300| 0.400| 100 | 0.3333  | 0.9768  | <0       | <0       | <0       | >0       | >0       | 1.0000   | 4330.40  |
| B1 | 35 | 1  | 5  | 8  | 3.00| 1.00   | 0.12   | 0.15 | 0.200| 0.300| 10 | 0.2857  | 0.4848  | >0       | <0       | <0       | <0       | <0       | 0.2208   | 99.9341  |
| B2 | 35 | 2  | 5  | 8  | 3.00| 1.00   | 0.12   | 0.15 | 0.200| 0.300| 10 | 0.2857  | 0.4848  | >0       | <0       | <0       | >0       | <0       | 0.0169   | 95.9332  |
| B3 | 37 | 1.4| 10 | 20.8| 1.12| 0.20   | 0.21   | 0.120| 0.179| 6   | 0.1622  | 0.3760  | >0       | <0       | <0       | >0       | >0       | 0.3796   | 399.6874 |
| C1 | 250| 10 | 5  | 10 | 3.00| 1.00   | 0.12   | 0.15 | 0.200| 0.300| 50 | 0.2000  | 0.6060  | >0       | >0       | <0       | <0       | <0       | 0.2230   | 1206.30  |
| C2 | 250| 30 | 5  | 10 | 3.00| 1.00   | 0.12   | 0.15 | 0.200| 0.300| 50 | 0.2000  | 0.6060  | >0       | >0       | <0       | <0       | <0       | 0.3510   | 1132.50  |
| C3 | 37 | 1  | 9  | 18 | 1.12| 1.10   | 0.20   | 0.21 | 0.160| 0.179| 6   | 0.1622  | 0.3593  | >0       | >0       | >0       | <0       | >0       | 0.3629   | 330.0850 |
| D1 | 250| 100| 5  | 10 | 3.00| 1.00   | 0.12   | 0.15 | 0.200| 0.300| 50 | 0.2000  | 0.6060  | >0       | >0       | >0       | >0       | >0       | 0.5570   | 782.8170 |
| D2 | 250| 160| 5  | 10 | 3.00| 1.00   | 0.12   | 0.15 | 0.200| 0.300| 50 | 0.2000  | 0.6060  | >0       | >0       | >0       | >0       | >0       | 0.6838   | 881.6120 |

\( T_1^* \) is the optimal ordering policy.
Table 3. The optimal ordering policy by using Theorem 3.

|   | D | A  | C | p | k | h | l_e | l_p | N | M | W | W/D | W⁺ | Δ₉ | Δ₁₀ | Δ₅ | Δ₈ | Δ₆ | T⁺ | Z(T⁺) |
|---|---|----|---|---|---|---|-----|-----|---|---|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A1| 50| 0.2| 5  | 7 | 3 | 1 | 0.12| 0.15| 0.10| 0.15| 10 | 0.2000| 0.2111| <0  | <0  | <0  | <0  | <0  | T₁ = 0.0894 | 144.3954 |
| A2| 50| 0.3| 5  | 7 | 3 | 1 | 0.12| 0.15| 0.10| 0.15| 10 | 0.2000| 0.2111| <0  | <0  | <0  | <0  | >0  | T₂ = 0.1053 | 97.6279  |
| B1| 50| 0.4| 3  | 6 | 3 | 1 | 0.12| 0.15| 0.13| 0.15| 10 | 0.2000| 0.3007| >0  | <0  | <0  | <0  | <0  | T₁ = 0.1265 | 142.3945 |
| B2| 50| 0.7| 7  | 7 | 3 | 1 | 0.12| 0.15| 0.10| 0.15| 10 | 0.2000| 0.2111| >0  | <0  | <0  | <0  | >0  | T₂ = 0.1590 | 93.3635  |
| B3| 50| 0.85| 5 | 7  | 3 | 1 | 0.12| 0.15| 0.10| 0.15| 10 | 0.2000| 0.2111| >0  | <0  | <0  | >0  | >0  | T₂ = 0.1680 | 92.4482  |
| C1| 49| 1  | 5  | 7 | 1.5| 1.2| 0.12| 0.15| 0.19| 0.20| 10 | 0.2041| 0.2803| >0  | >0  | <0  | <0  | <0  | T₂ = 0.1844 | 87.5672  |
| C2| 50| 1.2| 5  | 7 | 3 | 1 | 0.12| 0.15| 0.10| 0.19| 10 | 0.2000| 0.2682| >0  | >0  | <0  | <0  | >0  | T₂ = 0.1751 | 91.8729  |
| C3| 50| 1.5| 5  | 7 | 3 | 1 | 0.20| 0.15| 0.10| 0.15| 10 | 0.2000| 0.2111| >0  | >0  | <0  | >0  | >0  | T₂ = 0.2100 | 89.2023  |
| D1| 50| 0.01| 10 | 20 | 0.25| 0.2| 0.12| 0.15| 0.05| 0.08| 5 | 0.1000| 0.1605| >0  | >0  | <0  | <0  | <0  | T₁ = 0.0447 | 503.1528 |
| D2| 50| 0.1| 10 | 20 | 0.25| 0.2| 0.12| 0.15| 0.05| 0.08| 5 | 0.1000| 0.1605| >0  | >0  | <0  | >0  | >0  | T₁ = 0.1901 | 502.4800 |
| D3| 50| 2  | 5  | 7 | 3 | 1 | 0.12| 0.15| 0.10| 0.15| 10 | 0.2000| 0.2111| >0  | >0  | >0  | >0  | >0  | T₂ = 0.2237 | 86.8978  |
Table 4. The optimal ordering policy by using Theorem 4.

|   |   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $D$ | $A$ | $C$ | $p$ | $k$ | $h$ | $l_e$ | $l_p$ | $N$ | $M$ | $W$ | $W/D$ | $W^*$ | $\Delta_9$ | $\Delta_{11}$ | $\Delta_{12}$ | $\Delta_8$ | $\Delta_6$ | $T^*$ | $Z(T^*)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A1 | 30 | 0.020 | 3 | 4 | 0.12 | 0.1 | 0.15 | 0.13 | 0.18 | 10 | 0.33 | 0.2412 | <0 | <0 | <0 | <0 | $T_1^* = 0.1140$ | 30.4546 |
| A2 | 30 | 0.030 | 3 | 4 | 0.12 | 0.1 | 0.15 | 0.13 | 0.18 | 10 | 0.33 | 0.2412 | <0 | <0 | <0 | >0 | $T_6^* = 0.1320$ | 30.2946 |
| B1 | 30 | 0.040 | 3 | 4 | 0.12 | 0.1 | 0.15 | 0.15 | 0.18 | 10 | 0.33 | 0.2408 | >0 | <0 | <0 | <0 | $T_1^* = 0.0816$ | 30.1871 |
| B2 | 30 | 0.125 | 3 | 4 | 0.12 | 0.1 | 0.15 | 0.15 | 0.18 | 10 | 0.33 | 0.2408 | >0 | <0 | <0 | >0 | $T_8^* = 0.2408$ | 29.5262 |
| C1 | 50 | 0.010 | 10 | 20 | 0.25 | 0.2 | 0.12 | 0.15 | 0.15 | 80 | 1.6 | 0.3010 | >0 | >0 | <0 | <0 | $T_8^* = 0.3740$ | 504.7746 |
| C2 | 50 | 0.500 | 10 | 20 | 0.25 | 0.2 | 0.12 | 0.15 | 0.15 | 80 | 1.6 | 0.3010 | >0 | >0 | <0 | <0 | $T_8^* = 0.4420$ | 503.5738 |
| C3 | 50 | 1.000 | 10 | 20 | 0.25 | 0.2 | 0.12 | 0.15 | 0.15 | 80 | 1.6 | 0.3010 | >0 | >0 | >0 | >0 | $T_8^* = 0.5030$ | 502.5159 |
| D1 | 50 | 0.050 | 10 | 20 | 0.25 | 0.2 | 0.12 | 0.15 | 0.15 | 18 | 0.36 | 0.3010 | >0 | >0 | <0 | <0 | $T_5^* = 0.3774$ | 504.6673 |
| D2 | 50 | 0.500 | 10 | 20 | 0.25 | 0.2 | 0.12 | 0.15 | 0.15 | 18 | 0.36 | 0.3010 | >0 | >0 | >0 | >0 | $T_5^* = 0.4330$ | 503.5667 |
| D3 | 50 | 0.800 | 10 | 20 | 0.25 | 0.2 | 0.12 | 0.15 | 0.15 | 18 | 0.36 | 0.3010 | >0 | >0 | >0 | >0 | $T_5^* = 0.4663$ | 502.8894 |
Table 5. Sensitivity analysis with respect to parameters $A$, $C$ and $W$

| $D$ | $A$ | $C$ | $p$ | $k$ | $h$ | $I_a$ | $I_p$ | $N$ | $M$ | $W$ | $W/D$ | $W^*$ | $\Delta_1$ | $\Delta_2$ | $\Delta_3$ | $\Delta_4$ | $\Delta_5$ | $T^*$ | $Z(T^*)$ |
|-----|-----|-----|-----|-----|-----|-------|-------|-----|-----|-----|-------|-------|--------|--------|--------|--------|--------|-------|----------|
| 100 | 0.010 | 1 | 2 | 1.00 | 0.955 | 0.000115 | 0.15 | 0.01650098 | 0.01650098 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | $T_1^* = 0.0141$ | 98.6175 |
| 100 | 0.014 | 1 | 2 | 1.00 | 0.955 | 0.000115 | 0.15 | 0.01650098 | 0.01650098 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | $T_2^* = 0.0175$ | 98.3643 |
| 100 | 0.090 | 1 | 2 | 1.00 | 0.955 | 0.000115 | 0.15 | 0.01650098 | 0.01650098 | 1.65 | 0.0165 | 0.0330 | <0 | <0 | <0 | <0 | $T_3^* = 0.0420$ | 95.8097 |
| 100 | 0.002 | 4 | 15 | 0.90 | 0.100 | 0.130000 | 0.15 | 0.01700000 | 0.030000 | 1.65 | 0.0165 | 0.1126 | <0 | <0 | <0 | <0 | $T_4^* = 0.3110$ | 1102.300 |
| 100 | 0.002 | 5 | 15 | 0.90 | 0.100 | 0.130000 | 0.15 | 0.01700000 | 0.030000 | 1.65 | 0.0165 | 0.0901 | <0 | <0 | <0 | <0 | $T_5^* = 0.0169$ | 1002.300 |
| 100 | 0.002 | 10 | 15 | 0.90 | 0.100 | 0.130000 | 0.15 | 0.01700000 | 0.030000 | 1.65 | 0.0165 | 0.0451 | <0 | <0 | <0 | <0 | $T_6^* = 0.0169$ | 502.3318 |
| 50 | 0.500 | 10 | 20 | 0.25 | 0.200 | 0.120000 | 0.15 | 0.12000000 | 0.030000 | 1.65 | 0.0165 | 0.3010 | <0 | <0 | <0 | <0 | $T_7^* = 0.0430$ | 503.5567 |
| 50 | 0.500 | 10 | 20 | 0.25 | 0.200 | 0.120000 | 0.15 | 0.12000000 | 0.030000 | 1.65 | 0.0165 | 0.3010 | <0 | <0 | <0 | <0 | $T_8^* = 0.4390$ | 503.5631 |
| 50 | 0.500 | 10 | 20 | 0.25 | 0.200 | 0.120000 | 0.15 | 0.12000000 | 0.030000 | 1.65 | 0.0165 | 0.3010 | <0 | <0 | <0 | <0 | $T_9^* = 0.4420$ | 503.5738 |
Figure 4. The total accumulation of interest earned when $M < T \leq W^*$

Figure 5. The total accumulation of interest earned when $W^* \leq T$

REFERENCES

[1] A. K. Bhunia, C. K. Jaggi, A. Sharma and R. Sharma, A two-warehouse inventory model for deteriorating items under permissible delay in payment with partial backlogging, *Applied Mathematics and Computation*, 232 (2014), 1125–1137.

[2] L. E. Cárdenas-Barrón, Optimal manufacturing batch size with rework in a single-stage production system-A simple derivation, *Computers and Industrial Engineering*, 55 (2008), 758–765.

[3] L. E. Cárdenas-Barrón, K. J. Chung and G. Trevino-Garza, Celebrating a century of the economic order quantity model in honor of Ford Whitman Harris, *International Journal of Production Economics*, 155 (2014), 1–7.

[4] C. T. Chang, J. T. Teng and M. S. Chern, Optimal manufacturer’s replenishment policies for deteriorating items in a supply chain with up-stream and down-stream trade credits, *International Journal of Production Economics*, 127 (2010), 197–202.

[5] S. C. Chen, C. T. Chang and J. T. Teng, A comprehensive note on “Lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit”, *International Transactions in Operational Research*, 21 (2014), 855–868.

[6] S. C. Chen and J. T. Teng, Retailer’s optimal ordering policy for deteriorating items with maximum lifetime under supplier’s trade credit financing, *Applied Mathematical Modelling*, 38 (2014), 4049–4061.

[7] M. S. Chern, L. Y. Chan, J. T. Teng and S. K. Goyal, Nash equilibrium solution in a vendor-buyer supply chain model with permissible delay in payments, *Computers and Industrial Engineering*, 70 (2014), 116–123.

[8] K. J. Chung and L. E. Cárdenas-Barrón, The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade credit in the supply chain management, *Applied Mathematical Modelling*, 37 (2013), 4653–4660.
[9] K. J. Chung and J. J. Liao, Lot sizing decisions under trade credit depending on the ordering quantity, *Computers and Operation Research*, **31** (2004), 909–928.
[10] K. J. Chung and J. J. Liao, The optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach, *European Journal of Operational Research*, **196** (2009), 563–568.
[11] K. J. Chung, S. D. Lin and H. M. Srivastava, The inventory models under conditional trade credit in a supply chain system, *Applied Mathematical Modelling*, **37** (2013), 10036–10052.
[12] K. J. Chung and P. S. Ting, The inventory model under supplier’s partial trade credit policy in a supply chain system, *Journal of Industrial and Management Optimization*, **11** (2015), 1175–1183.
[13] J. Feng, H. Li and Y. Zeng, Inventory games with permissible delay in payments, *European Journal of Operational Research*, **234** (2014), 694–700.
[14] S. K. Goyal, Economic order quantity under conditions of permissible delay in payments, *Journal of the Operational Research Society*, **36** (1985), 335–338.
[15] Y. F. Huang and K. H. Hsu, An EOQ model under retailer partial trade credit policy in supply chain, *International Journal of Production Economics*, **112** (2008), 655–664.
[16] K. N. Huang and J. J. Liao, A simple method to locate the optimal solution for exponentially deteriorating items under trade credit financing, *Computers and Mathematics with Applications*, **56** (2008), 965–977.
[17] M. Y. Jaber and I. H. Osman, Coordinating a two-level supply chain with delay in payments and profit sharing, *Computers and Industrial Engineering*, **50** (2006), 385–400.
[18] V. B. Kreng and S. J. Tan, The optimal replenishment decisions under two levels of trade credit policy depending on the order quantity, *Expert Systems with Applications*, **37** (2010), 5514–5522.
[19] Y. Liang and F. Zhou, A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment, *Applied Mathematical Modelling*, **35** (2011), 2221–2231.
[20] J. J. Liao, An EOQ model with noninstantaneous receipt exponentially deteriorating items under two-level trade credit, *International Journal of Production Economics*, **113** (2008), 852–861.
[21] J. J. Liao and K. N. Huang, An inventory model for deteriorating items with two levels of trade credit taking account of time discounting, *Acta Applicandae Mathematicae*, **110** (2010), 313–326.
[22] J. J. Liao and K. N. Huang, Deterministic inventory model for deteriorating items with trade credit financing and capacity constraints, *Computers and Industrial Engineering*, **59** (2010), 611–618.
[23] J. J. Liao, K. N. Huang and K. J. Chung, Lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit, *International Journal of Production Economics*, **137** (2012), 102–115.
[24] J. J. Liao, K. N. Huang and K. J. Chung, Optimal pricing and ordering policy for perishable items with limited storage capacity and partial trade credit, *IMA Journal of Management Mathematics*, **24** (2013), 45–61.
[25] J. J. Liao, K. N. Huang and K. J. Chung, A deterministic inventory model for deteriorating items with two warehouses and trade credit in a supply chain system, *International Journal of Production Economics*, **146** (2013), 557–565.
[26] J. J. Liao, K. N. Huang and P. S. Ting, Optimal strategy of deteriorating items with capacity constraints under two-levels of trade credit policy, *Applied Mathematics and Computation*, **233** (2014), 647–658.
[27] Y. Liang and F. Zhou, A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment, *Applied Mathematical Modelling*, **35** (2011), 2221–2231.
[30] B. Sarkar, S. Saren and L. E. Cárdenas-Barrón, An inventory model with trade-credit policy and variable deterioration for fixed lifetime products, *Annals of Operations Research*, 229 (2015), 677–702.

[31] K. Skouri, I. Konstantaras, S. Papachristos and J. T. Teng, Supply chain models for deteriorating products with ramp type demand rate under permissible delay in payments, *Expert Systems with Applications*, 38 (2011), 14861–14869.

[32] X. Song and X. Cai, On optimal payment time for a retailer under permitted delay of payment by the wholesaler, *International Journal of Production Economics*, 103 (2006), 246–251.

[33] A. A. Taleizadeh, S. S. Kalantari and L. E. Cardenas-Barron, Determining optimal price, replenishment lot sizes and number of shipments for an EPQ model with rework and multiple shipments, *Journal of Industrial and Management Optimization*, 11 (2015), 1059–1071.

[34] A. A. Taleizadeh, S. S. Kalantari and L. E. Cardenas-Barron, Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items, *International Journal of Production Economics*, 159 (2015), 285–295.

[35] J. T. Teng, On the economic order quantity under condition of permissible delay in payments, *Journal of the Operational Research Society*, 53 (2002), 915–918.

[36] J. T. Teng, Optimal ordering policies for a retailer who offers distinct trade credits to its good and bad credit customers, *International Journal of Production Economics*, 119 (2009), 415–423.

[37] J. T. Teng and C. T. Chang, Optimal manufacturer’s replenishment policies in the EPQ model under two levels of trade credit policy, *European Journal of Operational Research*, 195 (2009), 358–363.

[38] J. T. Teng and K. R. Lou, Seller’s optimal credit period and replenishment time in a supply chain with up-stream and down-stream trade credits, *Journal of Global Optimization*, 53 (2012), 417–430.

[39] J. T. Teng, J. Min and Q. Pan, Economic order quantity model with trade credit financing for non-decreasing demand, *Omega*, 40 (2012), 328–335.

[40] J. T. Teng, H. L. Yang and M. S. Chern, An inventory model for increasing demand under two levels of trade credit linked to order quantity, *Applied Mathematical Modelling*, 37 (2013), 7624–7632.

[41] A. Thangam, Retailer’s inventory system in a two-level trade credit financing with selling price discount and partial order cancellations, *International Journal of Industrial Engineering Journal*, 11 (2015), 159–170.

[42] A. Thangam and R. Uthayakumar, Two-echelon trade credit financing for perishable items in a supply chain when demand depends on both credit period and selling price, *Computers and Industrial Engineering*, 57 (2009), 773–786.

[43] P. S. Ting, The EPQ model with deteriorating items under two levels of trade credit in a supply chain system, *Journal of Industrial and Management Optimization*, 11 (2015), 479–492.

[44] C. T. Tung, P. Deng and J. Chuang, Note on inventory models with a permissible delay in payments, *Yugoslav Journal of Operations Research*, 24 (2014), 111–118.

[45] W. C. Wang, J. T. Teng and K. R. Lou, Seller’s optimal credit period and cycle time in a supply chain for deteriorating items with maximum lifetime, *European Journal of Operational Research*, 232 (2014), 315–321.

[46] H. M. Wee, W. T. Wang and L. E. Cárdenas-Barrón, An alternative analysis and solution procedure for the EPQ model with rework process at a single-stage manufacturing system with planned backorders, *Computers and Industrial Engineering*, 64 (2013), 748–755.

[47] J. Wu and Y. L. Chan, Lot-sizing policies for deteriorating items with expiration dates and partial trade credit to credit-risk customers, *International Journal of Production Economics*, 155 (2014), 292–301.

[48] J. Wu, K. Skouri, J. T. Teng and L. Y. Ouyang, A note on ‘optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment’, *International Journal of Production Economics*, 155 (2014), 324–329.

[49] H. L. Yang and C. T. Chang, A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation, *Applied Mathematical Modelling*, 37 (2013), 2717–2726.
[50] G. F. Yen, K. J. Chung and Z. C. Chen, The optimal retailer’s ordering policies with trade credit financing and limited storage capacity in the supply chain system, *International Journal of Systems Science*, 43 (2012), 2144–2159.

[51] J. Zhang, Z. Bai and W. Tang, Optimal pricing policy for deteriorating items with preservation technology investment, *Journal of Industrial and Management Optimization*, 10 (2014), 1261–1277.

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