The numerical study of energy separation in a two-cascade Leontiev tube

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Abstract. This paper presents results of numerical modeling of energy separation for helium-xenon gas mixture flow in a two-cascade Leontiev tube with central and outer (annular) supersonic nozzles. The Mach number and stagnation temperature distributions in longitudinal section and the heat-transfer intensity from the subsonic to supersonic flow have been obtained. The dependences of the cooling effect, the temperature efficiency factor and the adiabatic efficiency on a stagnation pressure in the receiver have been investigated.

1. Introduction

It is known that various physical effects lead to energy separation in the gas flow without heat transfer between the gas and external heat-sources (to temperature stratification). The most famous energy-separation effect is the Ranque-Hilsch effect in the vortex tube [1–3]. This effect has been used for building the aircraft air-conditioning system, cooling systems with a swirl gas flow, and small refrigerators [4, 5].

In the Leontiev tube, the heat flux appears due to the temperature differences between the subsonic and supersonic parts of the flow when the expanding supersonic part of the flow is cooled. The dissipation of kinetic energy of the stagnating supersonic flow near the wall increases its temperature but for the low-Prandtl gas mixture the recovery temperature on the wall is lower than the temperature of the subsonic part of the flow. As a result, the heat transfers across the heat-conducting separating wall from the supersonic part of the flow to the subsonic part even if the stagnation parameters for both parts of the flow are equal.

The energy-separation method in a compressible flow was suggested and theoretically analyzed in [6, 7]. There it was shown theoretically that the heat transfer across the heat-conducting separating wall depended on the recovery temperature on the wall from the side, flown by the supersonic part of gas under the adiabatic conditions. To decrease the recovery temperature and increase the energy separation effect, it was suggested to use the low-Prandtl gas mixtures (helium-xenon, hydrogen-xenon etc.) [8–10], to blow some part of gas into the high-speed flow, to suck some part of gas from the low-speed flow [7, 11–14], or to use the wall with various surface reliefs [15].

The experimental investigations of heat transfer in the single Leontiev tube with a central cylindrical channel [16, 17] show the energy separation in the airflow with Prandtl number equal to 0.71. The authors obtain the static pressure distribution on the nozzle wall, and temperature and Mach number distributions in the outer cross section of the tube for the subsonic and supersonic parts of the flow. The results of numerical modeling of energy separation in the trans- and supersonic external flows are presented in monograph [18]. The study was performed for various gas mixtures with
Prandtl number from 0.2 to 0.8 and adiabatic factor from 1.3 to 1.67. It was shown that the subsonic part of the flow was cooled more effectively if the Mach number of the supersonic part of the flow varied from 1.9 to 2.4. For the transonic flows in the flat channel it was obtained that the adiabatic efficiency of the Leontiev tube, working on the mixture of helium-xenon with Prandtl number equal to 0.23, is higher than the adiabatic efficiency of the vortex tube, working on air with Prandtl number equal to 0.71 if other conditions being equal [19].

This paper presents the results of numerical modeling of energy separation for the helium-xenon gas mixture flow in the two-cascade Leontiev tube with central and outer (annular) supersonic nozzles. This configuration leads to an increase in the area of the cooled wall for the subsonic part of the flow and allows increasing the mass flow rate of the cooled gas.

2. Statement of the problem and numerical approach

A steady turbulent heat transfer is considered in the air (Pr=0.71) flowing through the nozzle with central cylindrical channel, used to test the numerical approach (see figure 1(a)), and in the helium-xenon mixture (Pr=0.23) flowing through the two-cascade Leontiev tube with central and outer (annular) supersonic nozzles, used in this study (see figure 1(b)).

![Figure 1](image-url)

**Figure 1.** Channel configuration for single Leontiev tube (a) and two-cascade Leontiev tube (b):
1 – supersonic annular nozzle, 2 – supersonic central nozzle, 3 – external heat-conducting wall, 4 – internal heat-conducting wall.

The single Leontiev tube geometry experimentally studied in [16, 17] is used for the test configuration. The supersonic flow is formed in the outer annular Laval nozzle with outer diameter at the inlet $d_{i1} = 30$ mm, critical diameter $d_{c1} = 12.5$ mm and outer diameter at the outlet $d_{o1} = 19$ mm. The inner diameter of the annular nozzle is equal to the outer diameter of the cylindrical channel, which is installed on the nozzle axis, $d_s = 10.4$ mm. The expanding part of the Laval nozzle has the conical form ($L_4 = 150$ mm) and cubic conjugation in the critical section with converging part ($L_5 = 50$ mm). The expanding angle of the conical tube is $1.15^\circ$. The converging part of the Laval nozzle has the form of sinus half-period ($L_3 = 20$ mm). The length of the cylindrical inner part of the nozzle $L$ is equal to 30 mm. Thus, the whole length of the single Leontiev tube is 250 mm.

The two-cascade Leontiev tube has outer diameter $d_3 = 27.2$ mm and heat-conducting walls with thickness of 1 mm. The inner diameter of the central nozzle at the inlet and outer section $d_i$ is equal to...
12.1 mm. The critical diameter \(d_{cr1} = 9.8\) mm. The diameter of the critical section of the annular nozzle is 20 mm. The outer diameter of the subsonic channel \(d_2 = 16.5\) mm. The expanding parts of the nozzles have conical form \((L_1 = 150\) mm\) and cubic conjugation in the critical section with converging parts \((L_3 = 50\) mm\). The converging parts and the cylindrical inner part of the nozzles have the same forms and lengths as in the single Leontiev tube. The whole length of the two-cascade tube is 250 mm.

We solved the problem by the implicit numerical method, based on the 2D axisymmetric RANS equations for the steady compressible flow with heat transfer. ANSYS Fluent solver licensed for SSCC SB RAS and self-made code were used for mesh generation and data analysis. The S-A turbulence model and the Kays-Crawford model [20] for the turbulent Prandtl number were used for modeling. As it was obtained in [18], this numerical model allows solving the convective heat transfer problems for the turbulent low-Prandtl gas flows. The turbulent intensity was 2\%, and the turbulent length scale was 1 m at the channel inlet. The adhesion conditions for motion equations, the temperature and heat flux coupling for energy equation were set as boundary conditions on the heat-conducting walls. The meshes contained from 90000 to 300000 nodes. The convergence for all variables and mass-flow rates in each channels were provided with maximum error \(10^{-5}\).

3. Result and discussion

3.1. Validation of numerical approach

The figure 2 shows the static pressure distribution on the outer wall, obtained by numerical modeling, in comparison with experimental data. In the test configuration, the air flows through the nozzle with central ebonit or cuprum cylindrical channel. The stagnation pressure at the inlet \(P_{00}\) is equal to 7.5 atm, and the stagnation pressure \(T_{00} = 21.8\) ºC.

\[\text{Figure 2. Static pressure distribution at the outer wall (line, points) and at the longitudinal section (color field) of the supersonic part of the flow: square is experimental data for low-heat-conducting wall (ebonit), circle is experimental data for high-heat-conducting wall (cuprum), and line is numerical data.}\]

As we can see, the numerical data are agreed with the experimental data. The static pressure is decreased down to 0.3 atm. Near the outlet section the system of pseudo-shocks is generated, and the supersonic flow stagnates with the pressure increase up to \(P_0 = 1\) atm. The longitudinal coordinate of the first shock attachment that is obtained by numerical modeling lies 10 mm lower downstream compared with the experiment. We suppose that this disagreement is explained by the inaccuracy in an expanding angle of the conical part of the nozzle. It may be noted, that the energy separation has a weak influence on the flow dynamic in the Laval nozzle.

\[\text{Figure 3. Recovery temperature distribution for the single Leontiev tube at the 7 mm distance from the outer cross section: vertical line is stagnation temperature.}\]
Figure 3 shows the experimental data for temperature, measured by thermo-probe at a distance of 7 mm from the nozzle outlet section. As it is noted in [16], the used thermo-probe allowed measuring the slightly less value of temperature than the stagnation temperature. The temperature obtained by the used numerical approach is indeed higher than the experimental data. For comparison with the experiment we calculate the recovery temperature distributions with recovery factor $r = 0.94$ based on numerical results. As we can see on figure 3, the numerical data for recovery temperature agree with the experimental data both on the form of distribution across the section and on the influence of heat-conductivity of the separated wall.

3.2. Energy separation in Leontiev tubes

Figure 4 presents the Mach number and stagnation temperature distributions for the helium-xenon gas mixture flow with the stagnation pressure in receiver 7.5 atm, stagnation temperature in receiver 21.8 °C for the single Leontiev tube and 327 °C for two-cascade Leontiev tube. The figure demonstrates the heating of supersonic flows by subsonic flow cooling.

![Figure 4. Mach number and stagnation temperature distributions at the longitudinal section of single (a) and two-cascade Leontiev tubes (b) for helium-xenon gas mixture.](image)

Figure 5 presents the cooling effect $(\bar{T}_2 - T_{\infty})$, the temperature efficiency factor $\eta_T$ and adiabatic efficiency $\eta$ depending on the stagnation pressure in the receiver for the single and two-cascade Leontiev tubes:

$$\eta_T = \frac{\bar{T}_2 - T_{\infty}}{\Delta T_s^*}; \eta = \frac{G_2}{G} \frac{\bar{T}_2 - T_{\infty}}{\Delta T_s^*}; \Delta T_s^* = T_{\infty} \left( \frac{P_{\infty}}{\bar{P}_2} \right)^{\frac{1}{\gamma}} - 1.$$
\( G_z \) is the mass flow rate of the cooled gas (in subsonic flow). \( G \) is the total mass flow rate through the tube. \( P_{00} \) and \( T_{00} \) are the stagnation pressure and temperature in the receiver. \( \bar{P}_z \) and \( \bar{T}_z \) are the mass average total pressure and total temperature of cooled gas at the outlet section. \( \gamma \) is the adiabatic exponent of gas.

![Graphs](https://via.placeholder.com/150)

**Figure 5.** Cooling effect (a), temperature efficiency factor (b) and adiabatic efficiency (c) depending on the stagnation pressure in receiver for single Leontiev tube (circle points) and for two-cascade Leontiev tube (square points).

For the two-cascade Leontiev tube it is shown that the maximum of temperature efficiency factor is achieved for the lesser value of stagnation pressure in the receiver (about 4.5 atm), and for the higher value it does not change. The adiabatic efficiency for the two-cascade Leontiev tube is higher compared to the single tube configuration if the stagnation pressure is less than 4.5 atm and achieves 25%. If the stagnation pressure is higher than 4.5 atm the single Leontiev tube is more effective and \( \eta_p \) achieves 40%. The temperature efficiency factor for the studied configurations of Leontiev tube reaches 5 – 6%. In whole the efficiency of the two-cascade Leontiev tube is less than the efficiency of the single tube, but the cooling power of the two-cascade configuration is 2.5 time higher as a result of 6 time increase in the mass flow rate of the cooled gas. It is noted, that the efficiency of energy separation in the Leontiev tube does not depend on the stagnation temperature in the receiver. The obtained results for \( T_{00} = 21.8 \) °C and 327 °C are very close to each other.

**4. Conclusions**

Energy separation of the compressible gas flow in the single and two-cascade Leontiev tube has been analyzed. It is found that the parameters of energy-separation efficiency (temperature efficiency factor
and adiabatic efficiency) for the two-cascade Leontiev tube are less than the same factors for the single Leontiev tube if the stagnation pressure in the receiver is near to maximum. If pressure in the receiver is less than 4.5 atm (for the studied configurations), the two-cascade Leontiev tube turns out to be a more effective energy separation device. For the same geometry parameters (length of tube and outer diameter) the two-cascade Leontiev tube is more powerful due to the high mass flow rate of the cooled part of the gas mixture.

Acknowledgments
The reported study was funded by RFBR according to the research project № 15-08-04203.

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