Angle Dependence of the Transverse Thermal Conductivity in YBa$_2$Cu$_3$O$_7$ single crystals: Doppler Effect vs. Andreev scattering

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We have measured the transverse thermal conductivity $\kappa_{xy}$ of twinned and untwinned YBa$_2$Cu$_3$O$_7$ single crystals as a function of angle $\theta$ between the magnetic field applied parallel to the CuO$_2$ planes and the heat current direction, at different magnetic fields and temperatures. For both crystals we observed a clear twofold variation in the field-angle dependence of $\kappa_{xy}(\theta) = -\kappa_{xy}^0(T, B) \sin(2\theta)$. We have found that the oscillation amplitude $\kappa_{xy}^0$ depends on temperature and magnetic field. Our results show that $\kappa_{xy}^0 = aB\ln(1/(bB))$ with the temperature- and sample-dependent parameters $a$ and $b$. We discuss our results in terms of Andreev scattering of quasiparticles by vortices and a recently proposed theory based on the Doppler shift in the quasiparticle spectrum.

To the main effects produced by the $d$-wave symmetry of the superconducting order parameter one includes the influence of the extended quasiparticle (QP) states - associated with a gapless structure (nodes) - on the mixed state of the superconductor in a magnetic field. In a semiclassical way, the effect of a magnetic field can be taken into account by introducing a Doppler shift (DS) in the energy spectrum of the QP due to the superfluid flow around vortices. This idea was later developed to calculate the density of states, specific heat and thermal conductivity in the mixed state of $d$-wave superconductors. Within this framework it is possible to explain the specific heat and the thermal conductivity behavior in a magnetic field perpendicular to the CuO$_2$ planes at low enough temperatures.

However, experiments on the field-angle dependence of the thermal conductivity with magnetic field applied parallel to the CuO$_2$ planes on YBa$_2$Cu$_3$O$_7$ single crystals (Y123) were interpreted assuming Andreev scattering (AS) of QP by vortices taking into account the $d$-wave symmetry of the order parameter. We think that this issue needs to be reconsidered experimentally to check whether there is clear evidence from thermal transport supporting either a DS effect on the energy spectrum of QP and/or AS. Furthermore, we note that there are no direct measurements published on the transverse thermal conductivity $\kappa_{xy}$ for magnetic fields applied parallel to the CuO$_2$ planes, although the pioneer experiment by Yu et al. provides an idea about the possible field-angle dependence of the transverse component. Nowadays there is consent that $\kappa_{xy}$ is a suitable property to check theoretical expressions of electronic thermal transport since it is free from non-electronic contributions. In this letter we report on the magnetic field $B$, field-angle $\theta$ and temperature $T$ dependence of $\kappa_{xy}$ for fields applied parallel to the CuO$_2$ planes.

The magnetic field, field-angle and temperature dependence of $\kappa_{xy}$ were studied on two Y123 single crystals. One sample was an optimally doped twinned single crystal with dimensions (length $\times$ width $\times$ thickness) $8.33 \times 0.6 \times 0.045$ mm$^3$ and critical temperature $T_c = 93.4$ K ($\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow$). The second sample was an untwinned Y123 single crystal with dimensions $2.02 \times 0.68 \times 0.14$ mm$^3$ and $T_c = 88$ K. The heat current $J$ was along the longest axis of the crystal (in case of the untwinned (twinned) crystal it was parallel to its $a$-axis ($a/b$-axes)). The magnetic field was applied parallel to the CuO$_2$ planes within a misalignment angle of $\pm 0.5^\circ$. An in-situ rotation system enabled us the measurement of the conductivity as a function of the angle $\theta$ defined between the applied field and the heat current direction. The used temperature gradient $\nabla_y T \leq 300$ K/m was kept constant along the heat current direction $x$ during all scans in order to calculate the transverse component of the thermal conductivity using the relation

$$\kappa_{xy}(T, B, \theta) = \kappa_{xx}(T, B, \theta) \frac{\nabla_y T}{\nabla_x T} \simeq \kappa_{xx}(T, B) \frac{\nabla_y T}{\nabla_x T} \propto -\frac{J\nabla_y T}{(\nabla_x T)^2}. \quad (1)$$

In this equation we used the experimental fact that the oscillation amplitude of the longitudinal component of the thermal conductivity $\kappa_{xx}$ - which has been shown to have fourfold symmetry at $T/T_c < 0.26$ - is much smaller than the longitudinal temperature gradient $|\nabla_x T|$. Therefore $\kappa_{xy}$ shows the same field-angle dependence as $\nabla_y T$.

The longitudinal and transverse temperature gradients were measured using a previously field- and temperature-calibrated type E thermocouples with a dc-picovoltmeter. We obtained similar results in both crystals for the field-angle and field dependence of $\kappa_{xy}$. The temperature dependence of $\kappa_{xy}^0$ as well as its absolute value depends, however, on the sample. This result appears to be related to different impurity scattering rates. The measured behavior of the longitudinal thermal conductivity as a function of temperature indicates a stronger impurity scattering for the untwinned crystal probably related to its oxygen deficiency.
The heat current direction is along +\(\hat{x}\), then there is a positive transverse thermal gradient for magnetic fields parallel to the CuO\(_2\) planes. Due to the enhancement of the inelastic scattering rate of QP by vortices screening currents using the 2D expression of the Bardeen-Richer-zen-Tewordt model and a fourfold one expected for the field-angle dependence of the longitudinal thermal conductivity\(\kappa_{xx}(T, B)\) for the twinned crystal at \(B = 8\) T and \(T = 13.8\) K, \(\nabla_y T = 2.1\) K/m for a longitudinal gradient of \(\nabla_z T = 241\) K/m; the amplitude of the oscillation of the longitudinal component remains of the order of one percent or less of \(\nabla_z T\). This is the reason for the results obtained in the experiment of Yu et al.\(\) where only a twofold pattern was measured, not the fourfold one expected for the field-angle dependence of the longitudinal thermal conductivity\(\kappa_{xx}(T, B)\). A comparison between these two effects will be reported elsewhere\(\).

The observed angular dependence of the thermal conductivity in Refs.\(\)\cite{AS} was interpreted in terms of AS of QP by vortices screening currents using the 2D expression of the Bardeen-Richer-zen-Tewordt model and assuming a \(d\)-wave order parameter\(\)\

\[
\kappa_{\alpha\beta} = \frac{1}{2\pi^2 c_B T h^2} \int_{pF} d^2p \frac{v_{\alpha 0} v_{\beta 0} E_z^2}{\Gamma(B, p, T)} \text{sech}\left(\frac{E_p}{2k_B T}\right),
\]

where, in general, \(\Gamma(B, p, T)\) is the relaxation rate given by the sum of scattering rates of QP by impurities \(1/\tau_0(p)\), by phonons \(1/\tau_P(p, T)\), by QP \(1/\tau_B(p, T)\) and AS by vortex currents \(1/\tau_{\alpha\beta}(B, p, T)\). \(\alpha\) and \(\beta\) denote the \(x\) or \(y\) directions on the plane of the sample and \(v_{\alpha 0}\) is the group velocity along the \(\alpha\) direction. All other symbols have the conventional meaning and are described in Ref.\(\)\cite{AS} in detail. With similar parameters as used in Ref.\(\)\cite{AS}, e.g. \(B_{c2}^0 = 500\) T, Ginzburg-Landau parameter \(\kappa = 100\) and a zero-temperature energy gap \(\Delta_0(0) = 20\) meV we obtain a \(\sin(2\theta)\) angle dependence for \(\kappa_{xy}\) as the experiments show. In what follows we compare our experimental results and those from literature - for magnetic fields applied parallel to the CuO\(_2\) planes - and discuss to which extent the available data can help to differentiate between the AS and DS contributions proposed by the existing models.

FIG. 1. (a) Angle dependence of the transverse temperature gradient at a fixed field of 8 T and at different temperatures below \(T_c\) for the twinned crystal. Continuous lines are fits to the function \(\sin(2\theta)\). (b) Angle dependence of the transverse conductivity at \(T = 29.8\) K for the twinned crystal. If the heat current direction is along +\(\hat{y}\) and a positive 90° angle is along +\(\hat{y}\), then there is a positive transverse thermal gradient for \(B\) at +45°.

Figure 1(a) shows the transverse temperature gradient \(\nabla_y T\) as a function of the angle \(\theta\) at different temperatures at a constant field \(B = 8\) T. A clear twofold pattern is measured as the magnetic field is rotated parallel to the CuO\(_2\) planes. Due to the enhancement of the inelastic scattering rate of QP with increasing temperature the angle dependence cannot be resolved above 70 K within the experimental error. The background in each curve of Fig. 1(a) is due to the experimental misalignment in the position of the sensors of the thermocouple used to measure \(\nabla_y T\) since its temperature dependence follows roughly that of \(\nabla_z T\) (the transverse thermocouple measures partially the longitudinal gradient on the sample as well). Taking into account the measured longitudinal conductivity \(\kappa_{xx}\) at \(B = 0\) we estimate a maximum misalignment of 80 \(\mu\text{m}\) along the heat current direction between the ends of the transverse thermocouple sensors. Thus, since the length and width of both crystals are larger than the misalignment of the transverse thermocouple, we can consider our system as a probe of the true transverse signal, in contrast to Ref.\(\)\cite{AS}.

A proof of this assumption is that within our experimental resolution the curves keep their background when the magnetic field is changed. This result allows us to subtract it from the curves and to consider only the oscillation amplitude \(\kappa_{xy}^0(T, B)\) which contains the physical information of the transverse response of the system. The angular dependence of \(\kappa_{xy}\) obtained in this way is shown in Fig. 1(b) for different values of the magnetic field. We think that this is the first time that such a symmetry is unambiguously established for \(\kappa_{xy}\); it follows a field-angle dependence proportional to \(\sin(2\theta)\). Based on the published work we assume that the twofold field-angle dependence reflects the symmetry of a \(d\)-wave order parameter. Note that the amplitude of the angle dependence on the transverse gradient is a relatively larger effect than in the longitudinal case (e.g. Fig. 2 of the twinned crystal at \(B = 8\) T and \(T = 13.8\) K, \(\nabla_y T = 2.1\) K/m for a longitudinal gradient of \(\nabla_z T = 241\) K/m; the amplitude of the oscillation of the longitudinal component remains of the order of one percent or less of \(\nabla_z T\). This is the reason for the results obtained in the experiment of Yu et al.\(\) where only a twofold pattern was measured, not the fourfold one expected for the field-angle dependence of the longitudinal thermal conductivity\(\kappa_{xx}\). A comparison between these two effects will be reported elsewhere\(\).
observed angle dependence of $\kappa_{xy}$ indicates a higher amplitude if AS affects the thermal transport. For example, if $B$ is at $+45^\circ$ the measured $\nabla_y T$, see Fig. 1, indicates a higher $T$ at $+90^\circ$, i.e. there is an excess of QP flowing along the field relative to those moving in the $-45^\circ$ nodal direction.

Recent studies pointed out that a DS in the excitation density as well as the scattering rate. This last dominates the field-angle dependence of $\kappa_{xy}$ perpendicular to the magnetic field and the sign in (3) seems to turn out to be the correct one. The temperature dependence takes place between a decreasing inelastic scattering rate due to the opening of the energy gap and the decrease in the number of QP with decreasing temperature. However, the DS affects both the carrier density as well as the scattering rate. This last dominates $\kappa_{xy}$ at high enough temperatures [18], thus the QP move easier parallel to the magnetic field and the sign in (3) turns out to be the correct one. The temperature dependence of $\kappa_{xy}$ in the clean limit is still under development.

Although preliminary results [18] indicate that the main arguments in Eq. (3) may remain, we should take (3) as a trial function to compare with our results.

![Figure 2](image1.png)

**FIG. 2.** Temperature dependence of the normalized oscillation amplitude $\kappa_{xy}^0$ at different applied fields for the untwinned crystal. The two continuous curves are obtained with (2) with the parameters mentioned in the text and normalized by $\kappa_{xy}^0(55 K,8.5 T)$. $\kappa_{xy}^0(34 K,8.5 T) = 0.084 W/Km.$

![Figure 3](image2.png)

**FIG. 3.** Magnetic field dependence of the normalized oscillation amplitude at different temperatures for the twinned (□) and untwinned (●) crystals. The dashed and dotted lines are fits of the data to the function $B \ln(1/(bB))$ with $b$ as a free parameter. The continuous line is obtained from Eq. (3) assuming Andreev and impurity scattering with the same parameters as in Ref. [1].

(1) **Angular dependence of $\kappa_{xy}(\theta)$:** We note that the observed angle dependence of $\nabla_y T$ agrees with the expected sign if AS affects the thermal transport. For example, if $B$ is at $+45^\circ$ the measured $\nabla_y T$, see Fig. 1, indicates a higher $T$ at $+90^\circ$, i.e. there is an excess of QP flowing along the field relative to those moving in the $-45^\circ$ nodal direction.

The clean-limit expression (3) for the transverse conductivity:

$$\frac{\kappa_{xy}}{\kappa_0} \simeq -\frac{1}{3\pi} \frac{v_F eB}{\Delta} \sqrt{\frac{4\Delta}{\hbar v_F}} \left[ \ln(4\sqrt{\frac{2\Delta}{\hbar v_F}}) \sin(2\theta) \right]$$

$$\ln\left(\frac{2\Delta}{\sqrt{\hbar v_F}eB}\right) - 0.422 \sin(2\theta) \simeq -\frac{2aB \ln(\frac{1}{b}) \sin(2\theta)}{\kappa_0},$$

(3) that applies for $T \ll T_c$, $B \ll B_{c2}$ and in the clean limit $\tau_0 \gg \hbar/\Delta$, where $\Delta(\Gamma) \simeq \Delta_0$, this last an impurity independent order parameter. Here $\Gamma = \tau_0^{-1}/2$ represents the impurity scattering rate, $\kappa_0 = \hbar k_F^2 T \pi/32m$, $v = v_F$, $v' = v_F \lambda_d/\lambda_c$, $n$ the density of QP and $m$ their mass, $b = \hbar v_F e/4\Delta^2$. The clean-limit expression (3) gives a simple field dependence for $\kappa_{xy}^0$ with both, $a$ and $b$, temperature and sample dependent parameters in which the scattering rate $\Gamma$ of the QP is included. Within a simple picture the sign of the measured $\nabla_y T$ seems to disagree with that expected from the DS: an excess of QP should be in the nodal direction perpendicular to the applied field. However, the DS affects both the carrier density as well as the scattering rate. This last dominates $\kappa_{xy}$ at high enough temperatures [18], thus the QP move easier parallel to the magnetic field and the sign in (3) turns out to be the correct one. The temperature dependence of $\kappa_{xy}$ in the clean limit is still under development.

(2) **Temperature dependence of $\kappa_{xy}^0$:** This dependence is presented in Fig. 2 for different magnetic fields. We note that every point in this figure represents the measurement of a complete curve as shown in Fig. 1 and that the experiments below 20 K have to be performed in a field-cooled procedure to avoid pinning effects [1]. The results show a maximum in $\kappa_{xy}^0$ at a field-independent temperature of $T \sim 35 K$. $\kappa_{xy}(T)$ shows a maximum at a similar temperature, also for fields applied parallel to the c-axis of the crystal. These facts indicate a common origin for the observed maxima: a competition takes place between a decreasing inelastic scattering rate due to the opening of the energy gap and the decrease in the number of QP with decreasing temperature [19,20]. Therefore, to fit the observed $T -$dependence we need the rates $1/\tau_P(p,T)$ and $1/\tau(B,p,T)$ to include in $\Gamma$, see Eq. (2). As seen in Fig. 2, with the above mentioned parameters the AS model and without $1/\tau_P(p,T)$
and $1/\tau(B, \mathbf{p}, T)$ does not provide a satisfactory fit of the data. Unfortunately, the exact expressions as well as the values of the necessary parameters of the rates are not known with certainty to allow a confidential result from the fits. Then and since Eq. (3) is strictly valid at low $T$, from the observed $T$–dependence it is not straightforward to conclude which of the proposed mechanisms predominates.

\begin{align*}
\kappa_{\text{el}}^0 & (T) \approx \kappa_{\text{el}}^0(0) + \kappa_{\text{el}}^0 \left( T, B \right) \left( \frac{1}{T} \right)^n \left( \frac{B^2}{T} \right)^m, \\
\kappa_{\text{ph}} & = \kappa_{\text{ph}}^0(0) + \kappa_{\text{ph}}^0 \left( T, B \right) \left( 1 - e^{-T/\Theta} \right) + \kappa_{\text{ph}}^0 \left( T, B \right) \left( \frac{1}{T} \right)^n \left( \frac{B^2}{T} \right)^m, \\
\kappa_{\text{tot}} & = \kappa_{\text{el}}^0(0) + \kappa_{\text{el}}^0 \left( T, B \right) \left( \frac{1}{T} \right)^n \left( \frac{B^2}{T} \right)^m + \kappa_{\text{ph}}^0 \left( T, B \right) \left( 1 - e^{-T/\Theta} \right) + \kappa_{\text{ph}}^0 \left( T, B \right) \left( \frac{1}{T} \right)^n \left( \frac{B^2}{T} \right)^m.
\end{align*}

(3) Field dependence of $\kappa_{xy}^0$: This dependence is presented in Fig. 4 for the two samples at different constant temperatures. Equation (3) describes satisfactorily the field dependence of $\kappa_{xy}^0$ from the lowest measured temperature $T \sim 13$ K. The temperature dependence of the fit parameters $a$ and $b$ are presented in Fig. 4. We note that $a$ and $b$ show a similar $T$–dependence for the two samples but different absolute values. From the parameter $a$ obtained at $T \leq 20$ K we calculate the scattering time $\tau_0 \approx 0.34(2.9) \text{ ps}$ for the untwinned (twinned) crystal assuming $\Delta = \Delta_0(0) = 20 \text{ meV}$ and $v \approx 5 \times 10^5 \text{ m/s}$. With these values we obtain a reasonable agreement for the $b$ parameter of the untwinned crystal. Note, however, that the predicted scattering-rate dependence in $b$ is not only too small in comparison to the factor 10 difference between samples, but is qualitatively different, i.e. $b$ increases with $\Gamma$.

Assuming Andreev and impurity scattering rates in (3) we cannot reproduce satisfactorily the observed field dependence of Fig. 3 in a broad temperature range even varying generously the fitting parameters. In Fig. 3 we show the results of this calculation using similar parameters as indicated above. At high $T$ both models provide similar good fits to the data.

(4) Field and angular dependence of $\kappa_{xx}$: In the temperature region of our work $\kappa_{xx}$ decreases with field (see for example Fig. 15 in [1]). This effect as well as the angular fourfold pattern [11] appear to be accounted for by the AS mechanism and at odds with the DS predictions since in the clean limit (dirty limit in [3]) a DS increases $\kappa_{xx}$ and gives the incorrect sign for the fourfold pattern [11]. However, two new approaches can provide an explanation of the results within the DS effect. First, high $T$ and the influence of scattering processes revert the field dependence of $\kappa_{xx}$ (as shown already for the superclean limit in [3]) and the sign of the fourfold pattern [11]. Second, we note that experiments measure the total conductivity in which phonons also contribute as heat carriers, i.e. $\kappa_{xx}(T, B) = \kappa_{xx}^c(T, B) + \kappa_{xx}^p(T, B)$. In general it is assumed that $\kappa_{xx}^p$ is field independent. This is not necessarily true. Note that none of the separation methods used in literature is accurate enough to prove that $\kappa_{xx}^p$ is strictly field independent. Due to the phonon-electron interaction a decrease of $\kappa_{xx}^p$ of a few percent, for fields of a few teslas and parallel to the planes is possible. In this case the DS may decrease the total conductivity with field due to its influence to the phonon contribution [11]. This contribution would provide also the correct sign of the fourfold pattern.

(5) Angular dependence of the specific heat: Recent specific heat results indicate the absence of a fourfold symmetry in its angular dependence, expected if the DS changes the QP density of states [22]. It is still unclear, however, how large this effect should be [10] as well as the influence of extra effects that may not allow an accurate test of the expected fourfold symmetry in the specific heat [22].

In summary, a clear twofold angle dependence has been obtained in the whole measured temperature and field range for the transverse thermal conductivity. Its oscillation amplitude $\kappa_{xy}^0$, follows $a(T)B \ln(1/b(T)B)$ with the sample-dependent parameters $a$ and $b$. Rigorously speaking none of the available models explain satisfactorily all the details of the experimental data. The expected influence of the DS casts doubts whether the AS mechanism is the only and appropriate one to understand $\kappa_{xy}(T, B)$. We think that a further development of both models is necessary before a clear answer can be given.

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