Theoretical methods for definition of the emission area of multi-tip cathodes and their experimental validation

E O Popov¹, A G Kolosko¹, S V Filippov¹ and S A Poniap¹

¹Ioffe Institute, ul. Politekhnicheskaya 26, St.-Petersburg, 194021, Russia

E-mail: e.popov@mail.ioffe.ru

Abstract. The processing of the current-voltage characteristics (IVC) taken from multi-tip field emitters (LAFEs) requires a streamlined and objective processing method for obtaining the main emission parameters. This paper considers different approaches for determining the effective emission surface. Methods for determining the area are applied to the analysis of experimental IVCs for single-walled (SW) and multi-walled (MW) LAFEs. The dependence of the effective emission area on the applied voltage was confirmed.

1. Introduction

Amongst field emission parameters, the "emission area" is one of the most difficult to determine. The difficulty lies both in the conceptual apparatus and in the choice of the method for estimating its value. The concept of "emission area" does not have an accurate and unambiguous description, and there are different views on how to best define this parameter [1-3]. For single-tip systems, determining the emission area seems a simpler task. The geometrical parameters of the tip apex are considered known and can be observed in various microscopes and emission projectors. However, the number of actually operating emission sites on a multi-tip emitter (LAFE) is, as a rule, an unknown parameter.

One can distinguish several approaches to estimating the effective emission area of LAFEs consisting of multiple points:

1) extraction of the effective emission area [1] by processing the experimental IVC using different approximations for the current density function;
2) determination of the emission area by multiplying the number of emission sites (for example, as seen by FEM [4]) by the calculated value of the emission area for a single tip;
3) calculating of the emission area from a simulated array of tips, with a predetermined distribution of geometric parameters [5], by fitting the IVC of the model emitter to the experimental IVC;
4) calculating of the effective and formal area of a single tip with the geometric parameters of a carbon nanotube by simulation methods in the COMSOL software package.

As was shown in [3], a LAFE involves a set of FEFs and therefore a set of IVCs. When the number of emission sites, and the distribution of their geometrical characteristics, are unknown, the definition of the emission surface is much more complicated. Hence, a comparison of approaches for estimating LAE emission area, and the development of computerized IVC processing methods, are important for the understanding and development of practical devices.
2. Theoretical approaches to determining emission area

As already indicated, there are several different parameters used to describe different types of emission area [2,3]. These include notional area \( A_n \), formal area \( A_f \), active area \( A_s \), local area \( A_L \), characteristic area \( A_C \), effective area \( A_{\text{eff}} \), and macroscopic area \( A_M \).

In connection with the emission area of multi-tip systems, a certain macroscopic parameter \( J_M \) is often used. This is defined as the ratio of the measured current to the area ("footprint") of the entire emitter. This parameter defines an empirical "average current density". The macroscopic area represents the size (or "footprint") of the whole LAFE.

The effective area in a general sense is a certain parameter or estimated area, which in principle is determined from the experimental or model IVC, as a result of FN-plot processing. That is, to find it, measurements are needed at various voltages, and detailed IVC is required.

The characteristic area is the equivalent area, or the area of the apex of the tip, which provides the experimentally measured current \( I = NA_C J_C \) (where \( N \) is the number of emitters, including \( N = 1 \)) provided knowledge of the field strength and local current density at the top of the emitter \( J_C \) (characteristic local current density). According to various estimations, it takes about 1/3 of the geometric area of the tip apex [6]. This is the most commonly used method for estimating the emission area, although the exact relationship between \( A_C \) and the effective emission area \( A_{\text{eff}} \) is not yet been known.

The local area is the area of integration over the emitter surface (generally speaking, curvilinear), taking into account the functional dependence of the field distribution. In some works, it is considered as the area of the base of the tip [3]. In this case, we can assume that the sum of all local areas yields the macroscopic area of the emitter \( A_M = NA_L \).

The active area, \( A_s \) is the part of the local area where the highest emission occurs. For the estimates, the value of \( A_s = \sigma A_L \) is taken, where \( \sigma = 10^{-8} \) [7]. For a single emitter, it is most logical to equate the effective area and the notional area \( A_f = A_s = 10^{-8} A_L \) [3] (see below).

Formal area \( A_f \) is a parameter defined as the ratio \( (A_f = I/J_k) \) of the measured current \( I \) to the so-called kernel current density \( J_k \), where

\[
J_k = a \varphi^{-1} F^2 \exp(-v_F b \varphi^{3/2} / F).
\]

Here, \( a, b \) are the FN constants, \( \varphi \) is the local work function, \( F \) is the local field strength, and \( v_F \) ("num") is the barrier-form correction factor appropriate to the values \( \varphi, F \) and the barrier under discussion. For the Schottky-Nordheim barrier used in the Murphy-Good [8] version of Fowler-Nordheim theory, \( v_F \) is replaced by \( v_v \), where \( v \) is a special mathematical function used in field emission [9].

The notional area is the area of emission closest to reality, since the equation for the current density takes into account all field \( \lambda_F \) and temperature coefficients \( \lambda_T \) [8] that affect the current density \( I = A_s \lambda_F \lambda_T J_s = A_L \lambda J_s \) or \( I = A_m J_C \). That is, the characteristic and conditional area coincide if all the correction factors are known and taken into account.

From the above definitions it can be seen that only the effective area requires knowledge of the IVC. Also, that the effective area can coincide with the conditional only for an idealized emitter (plane-parallel case), with adequate mathematical processing of the FN-plot.

In general, these definitions of areas can be used for both single-tip and multi-tip systems, depending on the context.

3. Processing Methods

3.1. The Elinson-Shrednik (ES) method

The Elinson-Shrednik (ES) method or the tangent method is described in [10,11]. In [11], a simplified field emission formula was given, which in its current form under the condition \( F/F_v = 0.3 \) will take the form in \([\text{A/m}^2]\):

\[
J = (1.39 \cdot 10^{-6}) \varphi^{-1} \exp(10.17/\sqrt{\varphi}) u^2 a^2 \exp(-6.49 \cdot 10^9 \varphi^{3/2} / (\alpha U)).
\]
\[ I = A_0 J = A_0 A_q U^2 \alpha^2 \exp(-B_\phi / (\alpha U)). \]  

(3)

where the expression \( F = \alpha U \) (where \( \alpha \) is a dimensional coefficient) is being used to relate \( F \) to the applied voltage \( U \), and \( A_q \) and \( B_\phi \) are appropriate coefficients.

If \( S^f \) and \( \ln(R^f) \) are the slope and intercept of a Fowler-Nordheim (FN) plot of the form \( \ln\{I/U^2\} vs 1/U \), then estimates of \( \alpha \) and of formal area \( A_f \) can be obtained from the equations

\[ \alpha = -B_\phi / S^f \]  

(4)

\[ A_f = R^f \{S^f\}^2 / (A_q B_\phi^2) \]  

(5)

where \( A^{ES} \approx 6.54 \text{ nm}^2/\text{A} \) for \( \varphi = 4.60 \text{ eV} \). These estimates are similar to those derived from the so-called tangent method [10,12]. It can be seen that the tangent method [13] uses the equations for the kernel current density \( J_k \), that is, the factor \( r^2 \) is omitted.

3.2. The method of correction functions.

In [13], to find the area on the graph in the FN-plot quadratic coordinates, a method was proposed using correction functions for both the slope (s) and intercept (r). For brevity, let call it the "RS" method.

Consider the FN equation expressed in terms of the kernel current density eq. (1). It is convenient to decompose the special mathematical function of field emission \( v \) into terms, expressing it through other special functions [9]:

\[ \exp(-v(f)\eta/f) = \exp(\eta u) \exp(-s(f)\eta/f) \]  

(6)

where \( f = F/F_k \) - dimensionless field, \( \eta = bc_\phi^2 \varphi^{-1/2}, bc_\phi^2 \approx 9.836239 \text{ eV}^{1/2} \), and \( \eta = 4.586170 \) at \( \varphi = 4.60 \text{ eV} \).

When we process the FN-plot, we are interested: (a) in its slope at a so-called "fitting point" (labeled "t"), where the tangent to the theoretical plot is parallel to the fitted experimental straight line; and (b) in the intersection that this tangent makes with the vertical axis. Let us introduce \( r_t \), and as a result of constructing (17) in FN coordinates, we get:

\[ r_t = \exp(\eta u_{t}/f) \]  

(7)

\[ R^f = A_f a \varphi^{-1} \exp(\eta u_{t}/f) = A_f a \varphi^{-1} R^f_{fit} \]  

(8)

\[ S^f = -s_{fit} b \varphi^{3/2}/\alpha \]  

(9)

\[ A_f = R^f \{S^f\}^2 / (\tau s_{fit}^2 ab^2 \varphi^2). \]  

(10)

As already noted, \( A_f \) is called the formal area (strictly, it's the formal area for the SN barrier). For the approximating functions for \( r_t \) and \( s_{fit} \), we can select [9]:

\[ s_{fit} \approx 1 - f/6. \]  

(11)

\[ u_t \approx (5/6) - (1/6) \ln f. \]  

(12)

\[ r_t = \exp(\eta u_t). \]  

(13)

3.3. Method of dependence of area on voltage

Theoretically, detailed knowledge of \( R^f \) is redundant if the number of emitters does not change with increase or decrease in voltage. This method is called [12] the functional dependence of the emission area. Here we call it "SN", this is determined only from the slope of the FN plot. The basic idea is in a simple relationship:

\[ A_f = I/J_k \]  

(14)

where \( I \) is the measured current at a given voltage. It remains only to determine \( J_k \) at this voltage. As usual, we use the formula for the kernel current (1):

\[ J_k = a \varphi^{-1} U^2 \alpha^2 \exp(-v \varphi^{3/2}/aU) \]  

(15)

To find \( v \) at the point \( U_n \) let us use the approximating formula:

\[ v(f) \approx 1 - f + (f/6) \ln f \]  

(16)

But in order to calculate \( f \) at the point \( t \), let use the obvious relations:

\[ S^f = -s_{fit} b \varphi^{3/2} U / (f \varphi^{1/2}) \]  

(17)
3.4. PKF coordinates method

Perform transformations for the preexponential factor:

\[ r = \exp[(5/6 - \ln f / 6)\eta] = f^{-\eta/6}\exp(\eta/6) \]  

(18)

Let’s take approximate analytical functions:

\[ I = A_nt^{-2}e^{-f^2\eta/6}F_{R}^{2}\exp(\eta)\exp(-\eta/f) \]  

(19)

Introduce new notation:

\[ J_{\text{fit}} = R(\alpha/F_{R})^{2-\eta/6} \]  

(20)

\[ S_{\text{fit}} = -b_{P}\alpha^{3/2}/\alpha \]  

(21)

\[ \ln(1/U^{2-\eta/6}) = \ln(J_{\text{fit}}) + S_{\text{fit}}(1/U) \]  

(22)

Or in full notation:

\[ \ln(1/U^{2-\eta/6}) = \ln[A_nt^{-2}e^{-f^2\eta/6}\exp(\eta)e^{2-\eta/6}] - (\eta/F_{R}/\alpha)(1/U) \]  

(23)

If \( t \) is treated as a constant, the graph constructed in the coordinates \( \ln(1/U^{2-\eta/6}) \) vs \( 1/U \) should be a strictly straight line. (For convenience, we call this a PKF-plot).

4. IVC experimental processing

In this paper, we consider four types of equations: in the Elinson-Shrednik (4) approximation "ES"; the basic equation \( J_{c}=J_{s}/1.1 \) eq. (15) "FP11"; the complete equation using approximate analytical functions eq. (19) "FP". Each equation was processed using the methods for determining the emission area. Some of these had their own modifications. Consider these features and approaches in the processing of graphs FN and PKF plots.

Experimental IVCs, especially near threshold, can have a strong noise dependence. To do this, we have provided for the averaging of the initial IVC using the simple moving average (SMA) method.

The following notations are used in the figure. All areas are "effective" areas (see above), so other indices have been omitted:

- AES - Elinson-Shrednik area (5) or tangential method area.
- ARS is the area obtained by the method of corrective functions by the formula (10), multiplied by 1.1 (\( t^{2}=\text{const}=1.1 \)).
- ASNS - area SN (14) simplified. We took \( s_{i}=0.95 \), and then determined the effective FEF for each \( S_{0} \) (9), and the current density (15) \( J_{c} \) by the formula (24):

\[ J_{c} = aU^{2}(0.95b_{P}/S_{\text{fit}})^{2}\exp(v(f)S_{\text{fit}}/(0.95U)) \]  

(24)

- APKF is the area obtained based on the PKF plot (23) multiplied by 1.1 (\( t^{2}=\text{const}=1.1 \)):

\[ \alpha = -b_{P}^{3/2}/S_{\text{fit}} \]  

(25)

\[ A^{\text{PKF}} = 1.1R_{\text{fit}}(\alpha F_{R})^{2-\eta/6}\exp(-\eta)\sigma^{\eta/6-2}/\alpha \]  

(26)

Some of the methods described above have been applied to LAFE emitters formed by multi-walled and single-walled carbon nanotubes. A research methodology one can find, for example, in [14-16].

Experiments were undertaken that had similar emission current values of 6–9 mA. The interelectrode distances were chosen such as to obtain approximately identical macroscopic: 300 \( \mu \)m for MWCNT and 375 \( \mu \)m for SWCNT. The IVC and the corresponding PKF plot are shown in figure 1. The figure also shows the case where the IVC is approximated according to the FP law.

The IVC processing is shown in figure 2. The previously defined nature of the dependencies [17] for both MWCNT and SWCNT was confirmed. The emission area for the MWCNT has a characteristic bending and rapid growth in the area of high voltages. The SWCNT is characterized by a monotonic increase in area and a fall in FEF.

From figure 2 it can be seen that the area increases several times with an increase in the applied voltage. The explanation for this behavior is provided by the figure 2a and figure 2b plots plotted for FEF (on inserts). With increasing voltage for LAFE, emission sites with lower FEF make the largest contribution. It can be argued that despite the increase in voltage, there is a general drop in current density. However, a significant increase in the emission area, apparently due to an increase in the number of nanosites, gives an increase in the current – voltage characteristic as a whole. The transition of the emitter from one state with a large FEF and a small area leads to the opposite situation, in which the
predominant emission contribution is provided by several orders of magnitude a larger number of emission sites with a small FEF.

As can be seen from the figures, the effective values of the area and FEF for the full range, or in other words, for a model IVC, are very different from local ones in the sections of the I–V characteristic.

The ES method is the only one that allows to quickly controlling the behavior of the I–V characteristic. Although, it is applicable only for the middle section of the IVC.

![Figure 1](image1.png)

Figure 1. IVC and their corresponding PFK-plot (in the inserts) for MWCNT (a) and SWCNT based (b) emitters with approximation by FP formula

![Figure 2](image2.png)

Figure 2. Processing of experimental IVCs: the dependence of the effective area on the inverse voltage (in the insert is a corresponding dependence for FEF) for (a) MWCNTs and (b) SWCNTs. "APKFt" means a theoretical area dependence plotted on the base of the effective area and FEF obtained from full diapason of voltage for experimental IVC.

5. Conclusion
Modern approaches to the formal determination of the emission area, as well as methods for processing the current-voltage characteristics to find the effective values of the emission area, have been considered.

A comparative analysis of the processing methods of the current-voltage characteristics by parts of the current-voltage characteristic is performed. A good match is obtained for the majority of methods, with the exception of Elinson-Shrednik approximation approaches. However, this method is the best one for quick analysis of IVCs.

The analysis of the IVCs for SWCNTs and MWCNTs confirmed the dependence of the emission area on the applied voltage.
LAFE emitters should be considered as generating a complex set of many individual IV Cs. The total IVC gives the appearance of obeying a Fowler-Nordheim-like equation. However, processing this total IVC in FN coordinates in the traditional way can lead to erroneous FEF and area values. The analysis performed in this paper allowed us to determine the fine structure of the behavior of the IVC of LAFE, as well as the applicability of the FN-type formulas for analyzing such emitters. The emission current at high voltages is provided by a large number of emission centers with low FEF values.

Acknowledgments
The author S.A. Ponyaev acknowledge the support from Presidium RAS Program No. 7: New developments in promising areas of energy, mechanics and robotics

References
[1] Oostrom A G J 1966 Philips Res. Rep. Suppl. 1 1-162
[2] Forbes R G, Deane J H B, Fischer A and Mousa M S 2015 Jordan J. of Phys. 8 125-147
[3] Amorim M V, Dall’Agnol F F, den Engelsen D, de Assis T A and Baranauskas V 2018 J. of Phys.: Condens. Matter 30 385303
[4] Chumak M A, Filippov S V, Kolosko A G and Popov E O 2018 J. of Phys.: Conference Series 1038 012121-1-6
[5] Liu H, Kato S and Saito Y 2009 Nanotechnology 20 275206-1-6
[6] Tang W W, Shiffler D A, Harris J R, Jensen K L, Golby K, LaCour M and Knowles T 2016 AIP Advances 6 095007-1-9
[7] De Assis T A, Dall’Agnol F F and Andrade R F S 2016 J. Phys. D: Appl. Phys. 49 355301
[8] Murphy E L and Good R H 1956 Phys. Rev. 102 1464–1472
[9] Forbes R G and Deane J H B 2007 Proc. R. Soc. Lond. A 463 2907–2927
[10] Forbes R G 1999 J. Vac. Sci. Technol. B 17 526–533
[11] Elinson M I 1974 Cold Cathodes (Moscow: Sovetskoe Radio) p 336
[12] Forbes R G 2015 Technical Digest 28th International Vacuum Nanoelectronics Conf. (Guangzhou, China) pp 70-71
[13] Forbes R G 2017 Technical Digest 30th International Vacuum Nanoelectronics Conf. (Regensburg, Germany) pp 234-235
[14] Popov E O, Kolosko A G, Ershov M V and Filippov S V Technical Digest 25th International Vacuum Nanoelectronics Conf. (Jeju, Republic of Korea) 2012 306-307
[15] Popov E O, Kolosko A G, Filippov S V, Romanov P A and Fedichkin I L Nanomaterials and nanostructures - XXI century 1 2016 14-26.
[16] Popov E O, Kolosko A G, Filippov S V and Romanov P A Materials Today: Proceedings 5 2018 13800–13806
[17] Popov E O, Filippov S V, Kolosko A G, Romanov P A and Forbes R G 2016 Technical Digest 29th International Vacuum Nanoelectronics Conf. (Vancouver, Canada) pp 177-178