Production of $J/\psi + c\bar{c}$ through two photons in $e^+e^-$ annihilation

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Abstract

We study the production of $J/\psi + c\bar{c}$ in $e^+e^-$ annihilation through two virtual photons. The cross section is estimated to be 23 fb at $\sqrt{s} = 10.6$ GeV, which is smaller by a factor of six than the calculated cross section for the same process but through one virtual photon. As a result, while the annihilation into two photons may be important for certain exclusive production processes, the big gap between the inclusive production cross section $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c}) \simeq 0.9$ pb observed by Belle and the current nonrelativistic QCD prediction of $\simeq 0.15$ pb still remains very puzzling. We find, however, as the center-of-mass energy increases ($\sqrt{s} > 20$ GeV) the production through two virtual photons $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ will prevail over that through one virtual photon, because in the former process the photon fragmentation into $J/\psi$ and into the charmed quark pair becomes more important at higher energies.

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Charmonium production is one of the important processes to test quantum chromodynamics (QCD) both perturbatively and non-perturbatively. Because of the simpler parton structure involved, which will reduce the theoretical uncertainty, charmonium production in $e^+e^-$ annihilation is expected to be more decisive in clarifying the production mechanisms of heavy quarkonia. The two B factories with BaBar and Belle are collecting huge data samples of continuum $e^+e^-$ annihilation events, which will allow us to have a fine data analysis for charmonium production. Recently, Charmonium production in $e^+e^-$ annihilation has become more interesting and puzzling, because of the large gap between the Belle measurements\(^1\)\(^2\) and the theoretical calculations for both inclusive\(^3\)\(^4\)\(^5\)\(^6\) and exclusive\(^7\)\(^8\) charmonium production via double $c\bar{c}$ pairs based on nonrelativistic QCD (NRQCD).

For the inclusive processes, Belle has reported a measurement on the $J/\psi$ production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6$ GeV\(^1\)\(^2\), and found that a very large fraction of the produced $J/\psi$ is due to the double $c\bar{c}$ production in $e^+e^-$ annihilation\(^1\)

$$\sigma(e^+e^- \rightarrow J/\psi c\bar{c})/\sigma(e^+e^- \rightarrow J/\psi X) = 0.59^{+0.15}_{-0.13} \pm 0.12,$$  \hspace{1cm} (1)
which corresponds to [1, 2]
\[ \sigma(e^+e^- \rightarrow J/\psi c\bar{c}) \approx 0.9 \text{pb}. \] (2)

In contrast, the predicted values for the cross section in NRQCD (the color-octet contribution is negligible for this process) are much smaller than the data[3, 4, 5]. In a recent analytical calculation and numerical estimation for the inclusive octet contribution is negligible for this process) are much smaller than the data[3, 4, 5]. In order to solve the problem, calculations for the exclusive double-charmonium production including all S-wave, P-wave and D-wave states via double \( c\bar{c} \) in NRQCD[6, 7], we find
\[ \sigma(e^+e^- \rightarrow \gamma^* \rightarrow J/\psi c\bar{c}) \approx 0.15 \text{pb}. \] (3)

This value is consistent with other previous calculations, including those obtained based on the quark-hadron duality hypothesis[9], but smaller than the Belle data by about a factor of six\(^1\).

For the exclusive processes, the Belle measurement[1] for the cross section of \( e^+e^- \rightarrow J/\psi + \eta_c \) is about an order of magnitude larger than the NRQCD calculation for \( e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + \eta_c \)[1, 8]. In order to solve the problem, calculations for the exclusive double-charmonium production from \( e^+e^- \) annihilation into two virtual photons are performed[10], and it is pointed out that the cross section for \( e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + J/\psi \) is larger than that for \( e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + \eta_c \) by about a factor of 3.7, despite of possible uncertainties due to the choice of input parameters[11]. This is an interesting result since it indicates that the \( e^+e^- \) annihilation through two virtual photon fragmentation may make important contributions to certain processes, and it might substantially reduce the discrepancy between experiment and theory for the exclusive process \( e^+e^- \rightarrow J/\psi + \eta_c \) (note that it is essential to check experimentally whether \( e^+e^- \rightarrow J/\psi + J/\psi \) is largely misidentified with \( e^+e^- \rightarrow J/\psi + \eta_c \)).

In this situation, it is necessary to examine the contribution of \( e^+e^- \) annihilation through two virtual photons to the inclusive production of \( J/\psi \). In the following we will calculate the complete \( O(\alpha^4) \) color-singlet inclusive cross section for \( e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi c\bar{c} \), and compare the production rate through two virtual photons with that through one virtual photon, to see whether the annihilation through two virtual photons can decrease the discrepancy between the Belle measurement on \( J/\psi c\bar{c} \) production and the calculations based on NRQCD.

Following the NRQCD factorization formalism, color-singlet scattering amplitude of the process \( e^-(p_1) + e^+(p_2) \rightarrow \gamma^* \rightarrow c\bar{c}(2S+1L_J)(p) + c(p_3) + \bar{c}(p_4) \) in Fig. 1 is given by
\[
\mathcal{A}(e^-(p_1) + e^+(p_2) \rightarrow c\bar{c}(2S+1L_J)(p) + c(p_3) + \bar{c}(p_4)) = \sqrt{C_L} \sum_{L_sS_z} \sum_{s_1s_2} \sum_{jk} \times \langle s_1; s_2 \mid SS_z; LL_z \mid J; J_J \rangle \langle 3j; 3k \mid 1 \rangle \times \mathcal{A}(e^-(p_1) + e^+(p_2) \rightarrow c_j(p_1; s_1) + \bar{c}_k(p_2; s_2) + c_l(p_3; s_3) + \bar{c}_i(p_4; s_4))(L = S). \] (4)

where \( c\bar{c}(2S+1L_J) \) is the intermediate \( c\bar{c} \) pair which is produced at short distances and then evolves into a specific charmonium at long distances. \( \langle 3j; 3k \mid 1 \rangle = \delta_{jk}/\sqrt{N_c} \),

\(^1\)The numerical value obtained for this process in[3] should be multiplied by a factor of 3.
\[ e^+ e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}. \]

\[ \langle s_1; s_2 \mid SS_z \rangle, \langle LL_z; SS_z \mid JJ_z \rangle \] are respectively the color-SU(3), spin-SU(2), and angular momentum Clebsch-Gordon coefficients for QQ pairs projecting out appropriate bound states. For J/ψ the coefficient \( C_L \) can be related to the radial wave function of the bound state and reads

\[ C_S = \frac{1}{4\pi} | R_S(0) |^2. \] (5)

The spin projection operator can be defined as

\[ P_{SS_z}(p; q) \equiv \sum_{s_1s_2} \langle s_1; s_2 | SS_z \rangle \bar{v}(p) (\frac{p}{2} + q) s_1 \bar{u}(\frac{p}{2} - q; s_2). \] (6)

We write the spin projection operator which will be used in the calculation as

\[ P_{1S_z}(p, 0) = \frac{1}{2\sqrt{2}} \hat{S}_z (\hat{\phi} + M), \] (7)

where \( M \) is the mass of the charmonium, which equals to \( 2m_c \) in the nonrelativistic approximation.

The amplitude for the upper Feynman graph in Fig. 1 can be written as

\[ M_1 = \frac{4\sqrt{3}}{9} \delta_{kl} \sum_{LS\hat{S}_z} \langle LL_z; SS_z \mid JJ_z \rangle \bar{v}(p) \gamma_\mu (\frac{p_1 - \hat{\phi} + m_e}{(p_1 - p)^2 - m^2_e})^\alpha u(p_1) \]

\[ Tr[P_{1S_z}(p, 0) \gamma_\alpha] \bar{u}(p_3) \gamma_\mu v(p_4) \frac{1}{p^2(p_3 + p_4)^2}, \] (8)
Figure 2: Differential cross sections of $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ (solid line) and $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}$ (dotted line) as functions of the scattering angle of $J/\psi$.

and the amplitude for the corresponding flipped graph is denoted as $M_2$. The amplitude for the lower Feynman graph in Fig. 1 reads

$$M_3 = \frac{4}{9\sqrt{3}} \delta_{kl} \sum_{L_z S_z} \langle LL_z; SS_z | JJ_z \rangle \bar{v}(p_2) \gamma_\mu \frac{\not{p}_3 + \not{p}_2 + m_e}{(p_3 + p_e - p_2)^2 - m_e^2} \gamma_\alpha u(p_1)$$

and the amplitude for the flipped graph is denoted as $M_4$.

The calculation of cross section for $e^- + e^+ \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ is straightforward. The differential cross section can be written in the form

$$\frac{d\sigma(e^+ + e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c})}{d\Omega} = \frac{3C_S \alpha^4}{4m_e} | \vec{M} |^2,$$

where $d\Omega$ represents the elements of the quadruple integral related to the final state phase space (see, e.g. Ref. [3]), and $| \vec{M} |^2 = \frac{1}{4} \sum_{pol, color} | M_1 + M_2 + M_3 + M_4 |^2$ is the unpolarized module squared of the amplitude. For simplicity we will not write down their lengthy expressions here.

With Eq. (10) we can evaluate the inclusive cross sections for $J/\psi$ production from $e^+e^-$ through two virtual photons. The input parameters used in the numerical calculations are the same as in ref. [3]:

$$m_e = 0, \quad m_c = 1.5 GeV, \quad \alpha = 1/137, \quad | R_S(0) |^2 = 0.81 GeV^3. $$

(11)
Figure 3: Ratio of the fragmentation contribution to the total cross section as the function of the center-of-mass energy.

Now we give the numerical result at the Belle energy $\sqrt{s} = 10.6$ GeV:

$$\sigma(e^+ e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}) = 23\text{fb}. \quad (13)$$

It is well known that $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ is a pure electromagnetic process (except for the hadronization of the quark pair at long distances), while $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}$ involves both electromagnetic and strong interactions. For a naive order of magnitude estimate, the ratio of the production rate of the former to the latter would be proportional to $\alpha^2/\alpha_s^2$, but the photon fragmentation into $J/\psi$ and into the charmed quark pair can substantially enhance the former. To see the role of photon fragmentation, we plot the differential cross sections of $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ and $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}$ as functions of the scattering angle of $J/\psi$ at $\sqrt{s} = 10.6$GeV in Fig. 2. (Here for the one-photon process we choose $\alpha_s = 0.26$.) One can see that the small angle $J/\psi$ production in $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ is dominant. This indicates that most of the $J/\psi$ production comes from the photon fragmentation (corresponding to the upper graph in Fig. 1 with one photon fragmenting to $J/\psi$ and the other fragmenting to a charm quark pair). Indeed, our calculation shows that at $\sqrt{s} = 10.6$GeV, the contribution from fragmentation graphs is about seventy-two percent of the total cross section. In Fig. 3 we show the ratio of the fragmentation contribution to the total cross section as a function of the $e^+e^-$ center-of-mass energy in $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$. It is clear that
the photon fragmentation becomes more and more dominant as the center-of-mass energy increases. This is in agreement with the observation in the $J/\psi + J/\psi$ exclusive production through two virtual photons\[10].

From the above discussions we have seen the importance of the photon fragmentation to $J/\psi$ as well as to the charm quark pair in the two-photon process $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$. A even more crucial result is that at high $e^+e^-$ center-of-mass energies the contribution through two virtual photons will prevail over that through one virtual photon in the production of $J/\psi c\bar{c}$. The reason lies simply in the fact that the virtuality of the photon in the two-photon process can be as small as $4m_c^2$, whereas it is as large as $s$, the center-of-mass energy squared in the one-photon process. In Fig. 4 we show the cross sections of $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ (solid line) and $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}$ (dotted line) as functions of the center-of-mass energy $\sqrt{s}$. We see clearly that the cross section for $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ decreases very slowly, whereas that for $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}$ decreases rapidly as $\sqrt{s}$ increases. At $\sqrt{s} = 20$ GeV, the two photon process becomes to prevail over the one photon process.

However, unfortunately, at the Belle energy $\sqrt{s} = 10.6$ GeV, since the enhancement effect due to the factor $s/m_c^2$ is not large enough as compared with the suppression factor $\alpha^2/\alpha_s^2$, we find $\sigma(e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}) = 23$ fb, which is still much smaller than $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}) = 148$ fb\[6], and therefore is negligible.  

Figure 4: Cross sections of $e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}$ (solid line) and $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}$ (dotted line) as functions of the center-of-mass energy.
In summary, we have calculated the complete $\mathcal{O}(\alpha^4)$ color-singlet inclusive cross sections for $J/\psi c\bar{c}$ production from $e^+e^-$ annihilation into two photons. Due to the suppression factor of $\alpha^2/\alpha_s^2$, at the $e^+e^-$ center-of-mass energy $\sqrt{s} = 10.6$ GeV the cross section of this process is smaller by about a factor of six than that from $e^+e^-$ annihilation into one photon. We then conclude that while the $e^+e^-$ annihilation into two photons could be helpful in solving the puzzle for the exclusive $J/\psi \eta_c$ production, it can do very little to reduce the big gap between the observed inclusive production cross section of $\sigma(e^+e^- \to J/\psi + c\bar{c}) \approx 0.9$ pb and the current NRQCD predictions of about 0.15 pb. This puzzle still needs to be explained with new theoretical considerations. We find, however, as the center-of-mass energy increases ($\sqrt{s} > 20$ GeV) the production through two photons $e^+e^- \to 2\gamma^* \to J/\psi + c\bar{c}$ will prevail over that through one photon, because in the former case the photon fragmentation into $J/\psi$ and into the charmed quark pair becomes more important at higher energies.

Acknowledgments

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