QUICKSILVER: Modeling and Parameterized Verification for Distributed Agreement-Based Systems

NOURALDIN JABER, Purdue University, USA
CHRISTOPHER WAGNER, Purdue University, USA
SWEN JACOBS, CISPA Helmholtz Center for Information Security, Germany
MILIND KULKARNI, Purdue University, USA
ROOPSHA SAMANTA, Purdue University, USA

The last decade has sparked several valiant efforts in deductive verification of distributed agreement protocols such as consensus and leader election. Oddly, there have been far fewer verification efforts that go beyond the core protocols and target applications that are built on top of agreement protocols. This is unfortunate, as agreement-based distributed services such as data stores, locks, and ledgers are ubiquitous and potentially permit modular, scalable verification approaches that mimic their modular design.

We address this need for verification of distributed agreement-based systems through our novel modeling and verification framework, QUICKSILVER, that is not only modular, but also fully automated. The key enabling feature of QUICKSILVER is our encoding of abstractions of verified agreement protocols that facilitates modular, decidable, and scalable automated verification. We demonstrate the potential of QUICKSILVER by modeling and efficiently verifying a series of tricky case studies, adapted from real-world applications, such as a data store, a lock service, a surveillance system, a pathfinding algorithm for mobile robots, and more.

CCS Concepts: • Theory of computation → Program verification; Distributed computing models; Automated reasoning; Verification by model checking; Abstraction; Concurrency; Program analysis.

Additional Key Words and Phrases: Parameterized Verification, Modular Verification, Distributed Systems

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1 INTRODUCTION

Modern distributed services such as data stores, logs, caches, queues, locks, and ledgers heavily rely on distributed agreement to perform their higher-level functions—processes in these distributed services need to agree on a leader, on the members of a group, on configurations, or on owners of locks. Notable instances of such distributed agreement-based services include the Chubby lock service [Burrows 2006] and RedisRaft key-value store [RedisRaft 2021], which are built on top of the Paxos [Lamport 1998] and Raft [Ongaro and Ousterhout 2014] consensus algorithms, respectively. The importance of agreement protocols as a key building block in distributed services has sparked significant verification efforts for these protocols [Chand et al. 2016; Cousineau et al.]

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2012; Drăgoi et al. 2014; Drăgoi et al. 2016; García-Pérez et al. 2018; Lamport 2002; Liu et al. 2012; Marić et al. 2017; Padon et al. 2017b; Woos et al. 2016]. Intriguingly, with rare exceptions, these efforts restrict their attention to the core protocols and do not consider the distributed services that build on those protocols. This is unfortunate because there are arguably more distributed systems that build on agreement protocols than there are implementations of these core protocols. Moreover, such agreement-based systems are more likely to be developed by non-experts who can benefit from verification. In this paper, we ask can we develop modular modeling and verification frameworks for distributed agreement-based systems by (1) assuming that the underlying agreement protocols are verified separately and (2) encapsulating their complexities within cleanly-defined abstractions? Such an approach would both allow us to leverage the heroic efforts towards verifying agreement protocols as well as ease the burden of modeling the distributed systems that rely on those protocols. We note that existing verification efforts for agreement-based systems that go beyond core protocols [Gleissenthall et al. 2019; Hawblitzel et al. 2015; Liu et al. 2012; Padon et al. 2016], with the exception of [Griffin et al. 2020; Sergey et al. 2017], do not leverage the availability of verified agreement artifacts through systematic agreement abstractions.

We further ask: can our agreement abstractions enable fully automated, parameterized verification for interesting classes of agreement-based systems? This second question is an open one. The parameterized model checking problem (PMCP)—the problem of algorithmically verifying correctness of systems parameterized by the number of processes—is well-known to be undecidable in its full generality [Apt and Kozen 1986; Suzuki 1988]. While decidability has been shown for some restricted classes of distributed systems, it is unclear whether agreement-based systems allow for a decidable parameterized verification procedure at all. Past verification efforts for agreement protocols/implementations as well as agreement-based systems sidestep the decidability issue by preferring the use of interactive or semi-automated deductive verification over model checking. The appeal of push-button verification that does not require a user to provide inductive invariants or manipulate a theorem prover, however, remains undeniable. We argue that abstracting away and separately verifying the intricate details of agreement (using deductive techniques) should yield simpler models of agreement-based systems that may now become amenable to decidable and scalable model checking.

In this paper, we propose the QuickSilver framework for modeling and parameterized model checking of distributed agreement-based systems. QuickSilver advances a brand new verification strategy for agreement-based systems that is not only modular, but also fully automated.

The QuickSilver Framework

In our design of QuickSilver, we address several questions:

(1) How should we abstract agreement? The primitives we develop to abstract agreement must be sufficiently general to capture the essential characteristics of a wide variety of agreement protocols, while still permitting decidable parameterized model checking of distributed systems with such primitives.

(2) How should we model our systems? The modeling language we use for distributed agreement-based systems should match the manner in which system designers build their programs.

(3) How should we identify systems that enable decidable and scalable verification? The fundamental obstacle we need to tackle is the undecidability of PMCP. Thus, we must find easily-checkable conditions under which the verification of systems we model (including their use of agreement primitives) is decidable. Further, because our goal is to model fairly complex systems, we must endeavor to find scalable approaches for verifying these systems.

In particular, QuickSilver makes the following contributions.
**MERCURY: A Modeling Language with Agreement Primitives.** We carefully examined a range of agreement protocols in the literature, such as consensus and leader election, and observed that while the protocol internals differed substantially, their externally-observable behavior could be captured with two agreement primitives (namely, **Partition** and **Consensus**) that have simple semantics and abstract away the protocols’ implementation details. The **Partition** primitive allows a set of participant processes to divide themselves into groups (e.g., leaders and followers). The **Consensus** primitive allows its participants, with each proposing a value, to agree on a finite set of decided values. Sec. 3 presents a new, intuitive modeling language, **MERCURY**\(^1\), that allows designers to model finite-state distributed systems using these agreement primitives, and hence design systems without worrying about the internals of the core agreement protocols.

**Parameterized Verification of MERCURY Systems.** With **MERCURY**’s primitives abstracting away the messy details of distributed agreement, we observe that the resulting higher-level systems can be more amenable to automated verification. Sec. 4 identifies a broad class of **MERCURY** systems that permit decidable and efficient parameterized verification. In particular, we present two key results. First, we identify syntactic conditions on **MERCURY** systems that yield decidability of PMCP. Second, we identify additional syntactic conditions that, for a given class of safety/reachability properties, enable *practical* parameterized verification by providing cutoffs: a number \(k\) of processes such that verifying the correctness of a fixed-size \(k\)-process system implies the correctness of arbitrary-sized systems. This result means that *non-parameterized* model checkers can be leveraged to provide parameterized verification.

We prove both results by (1) defining **MERCURY CORE**, a novel extension of the decidable and cutoff-yielding fragments of a recently proposed abstract model for distributed systems [Jaber et al. 2020] and (2) showing that **MERCURY** systems satisfying our syntactic conditions are simulation equivalent to systems in **MERCURY CORE**.

The class of safety/reachability properties to which these results apply include properties forbidding the reachability of global states where *more than a fixed number* of processes are simultaneously in some pivotal local states. An example of such a property is mutual exclusion of a certain critical local state, i.e., *no more than two* processes can reach the critical state simultaneously in any execution. These results currently do not apply to liveness properties, e.g., a leader is eventually elected, or to arbitrary safety properties, e.g., there exists *at least one leader* at all times, or, *no more than half of the processes* can simultaneously be leaders.

**Implementation, MERCURY Benchmarks, and Evaluation.** With our decidability and cutoff results for **MERCURY** programs in hand, we have an approach that enables scalable, parameterized verification of distributed agreement-based systems in theory. Sec. 5 instantiates the theoretical results of **QUICKSILVER** by presenting an implementation of a cutoff-driven parameterized verification procedure for **MERCURY** systems. Crucially, **QUICKSILVER** automatically checks the syntactic conditions that yield practical parameterized verification. When a system is not practically verifiable, **QUICKSILVER** provides best-effort feedback suggesting modifications to the system that may make it so. We show that complex distributed agreement-based systems including a data store, a lock service, a surveillance system, a pathfinding algorithm for mobile robots, the Small Aircraft Transportation System (SATS) protocol [NASA 2021], and several other interesting applications can be naturally and succinctly modeled in **MERCURY**, and can then be efficiently verified.

1.1 Related Work

We first compare with the most related lines of work.

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\(^1\)Modeling Event Reaction and Coordination Using symmetRY
Global Synchronization Protocols (GSPs). In recent work, Jaber et al. [2020] propose a new model, GSP, for crash- and failure-free distributed systems and present decidability and cutoff results for parameterized verification of systems in the model. This model supports global transitions associated with global guards which can be used by multiple processes to synchronize collectively and simultaneously. Such global transitions and guards can be used, in theory, to carefully encode abstractions of agreement protocols. However, the GSP model is an abstract, theoretical model based on counter abstraction and does not provide an intuitive, accessible interface for system designers; for instance, processes in the GSP model cannot use local variables and are specified as low-level state-transition systems with manually-inferred guards. Additionally, users are required to manually check if their GSP system models fall within the decidable, cutoff-yielding fragment.

In contrast, QuickSilver (i) provides a user-friendly modeling language for distributed systems with inbuilt primitives that are designed to abstract agreement protocols, (ii) supports process crash-stop failures, (iii) pushes the boundaries of decidable parameterized verification by expanding the GSP decidability fragment, and (iv) includes a fully-automated implementation for checking if Mercury programs belong to the expanded decidable, cutoff-yielding fragment.

Modular Verification with Abstract Modules. Disel [Sergey et al. 2017] and TLC [Griffin et al. 2020] leverage the same observation we do—that distributed applications build on standard protocols—and enable users to incorporate abstractions of such protocols to provide modular verification using the Coq theorem prover. The user is responsible for providing both the high-level descriptions of the underlying protocols as well as the inductive invariants needed to link protocols to their clients and/or enable horizontal composition with other protocols. The TLC framework could potentially reason about agreement-based systems as it supports vertical composition, but the user would need to manually incorporate abstractions of the underlying agreement protocols. In contrast, QuickSilver is equipped with intuitive, inbuilt primitives that abstract agreement protocols and facilitate vertical composition and fully-automated parameterized verification.

In what follows, we discuss other broad themes of verification approaches for distributed systems.

Semi-Automated, Deductive Verification. Approaches for semi-automated, deductive verification of distributed protocols and implementations expect a user to specify inductive invariants [Andersen and Sørensen 2019; Doenges et al. 2017; Feldman et al. 2019; Krogh-Jespersen et al. 2020; Padon et al. 2016; Rahli 2012; Sergey et al. 2017; Wilcox et al. 2017, 2015; Woos et al. 2016]. Some approaches [Damian et al. 2019; Padon et al. 2017a,b; Taube et al. 2018] enable more (but not full) automation by translating the user-provided system and inductive invariants into a decidable fragment of first-order logic (e.g., effectively propositional logic (EPR) [Piskac et al. 2010]) or a model with a semi-automatic verification procedure (e.g., the Heard-Of model [Charron-Bost and Schiper 2009]). Recent work [Gleissenthall et al. 2019; Kragl et al. 2020] proposes the use of Lipton’s reduction [Lipton 1975] to reduce reasoning about asynchronous programs to synchronous and sequential programs, respectively, thereby greatly simplifying the invariants needed. Our approach builds on deductive verification for agreement protocols to enable modeling and automated parameterized verification of systems built on top of verified agreement protocols.

Model Checking. Prior work on PMCP identifies decidable fragments based on restrictions on the communication primitives, specifications, and structure of the system [Aminof et al. 2018; Außerlechner et al. 2016; Delzanno et al. 2002; Emerson and Kahlon 2003b; Esparza et al. 1999; German and Sistla 1992; Jaber et al. 2020]. To enable efficient parameterized verification, prior work additionally identifies cutoff results for various classes of systems, e.g., cache coherence protocols [Emerson and Kahlon
2003a], guarded protocols [Jacobs and Sakr 2018], consensus protocols [Marić et al. 2017], and self-stabilizing systems [Bloem et al. 2016]. Unfortunately, no existing decidability and cutoff results, except for those in [Jaber et al. 2020], extend to agreement-based systems.

There has also been some work on model checking and synthesis of distributed systems with a fixed number of finite-state processes [Alur et al. 2014, 2015; Alur and Tripakis 2017; Damm and Finkbeiner 2014; Liu et al. 2012; Yang et al. 2009]. However, these frameworks are not naturally extendable for parameterized reasoning and do not consider abstractions of agreement protocols for improving scalability of verification in the fixed-size setting.

2 QUICKSILVER OVERVIEW

This section presents an illustrative example of a complex system that leverages multiple instances of distributed agreement for its high-level function. It then provides an overview of the key building blocks of our modeling language and verification approach using the example. We begin the section with a brief review of distributed agreement protocols.

Distributed Agreement Protocols. Distributed agreement protocols enable a set of distributed participants, each proposing one value, to collectively decide on a set of proposals in the presence of failures and asynchrony. There are many variants of agreement protocols with small differences in their decision objectives. For instance, the participants may wish to decide on a single proposal [Lamport 1998, 2006; Mao et al. 2008], an infinite sequence of proposals [Chandra et al. 2007; Ongaro and Ousterhout 2014], or a finite set of leaders amongst themselves [Arghavani et al. 2011; Garcia-Molina 1982]. Despite these variations, any correct agreement protocol is characterized by the following three guarantees [Lynch 1996]: (i) agreement—all participants decide on the same set of proposals, (ii) validity—every proposal in the decided set of proposals must have been proposed by a participant, and (iii) termination—all participants eventually decide. Accordingly, recent work in verification of agreement protocols and/or their implementations focuses on guaranteeing agreement, validity, and termination (or, some reasonable variant of these properties).

2.1 Illustrative Example: Distributed Store

Suppose a system designer wants to model and verify a distributed store where multiple processes consistently replicate and update a piece of stored data in response to client requests. The clients may request various operations on the stored data including read and update. To ensure that the data is consistent across all replicas, the designer decides to use distributed agreement protocols to determine which operation these replicas should execute next. For efficiency, they use a leader election protocol to pick a leader that acts as a single point of contact to handle requests from the clients. These requests are replicated to all other processes using a consensus algorithm to maintain consistent stored data throughout the system. This design pattern is common in distributed services like fault-tolerant key-value stores (e.g., RedisRaft [2021]). The safety properties for this system are: (1) there is at most one leader at any given time and (2) all processes agree on the stored data.

Notice that the designer’s scheme for the distributed store uses different agreement protocols as building blocks and is inherently modular. Moreover, the safety properties are about the high-level design of the distributed store and do not refer to the internals of the leader election and consensus protocols used. Hence, it is sensible to also adopt a modular approach to reasoning about the correctness of the design. Specifically, instead of reasoning about the distributed store as a monolithic program with all agreement protocols modeled explicitly, one can assume that the underlying agreement protocols are verified separately and verify the system where these protocols are replaced with simpler abstractions that capture their behaviors.
Our framework, QUICKSilver, and its modeling language MERCURY (presented in Sec. 3) enables the designer to utilize such a modular verification approach with the agreement protocols represented using special primitives (denoted Partition and Consensus for leader election and consensus, respectively) that soundly abstract the semantics of agreement protocols.

For instance, the designer may model their distributed store in MERCURY as shown in (Fig. 1). Processes start in the Candidate location (Line 12) and coordinate with each other (Line 13) to elect one leader to move to the Leader location (Line 17), while the remaining processes become replicas and go to the Replica location (Line 38). The leader can receive requests from clients (via the doCmd message on Line 18), while the replicas wait for the leader to replicate requests to them.

When the leader receives a request from a client (which could be one of several potential commands), it handles the request on lines 19–25. A command payload consists of either a directive to set the value to 1 or 2 (cmd <= 2); to read the stored value (cmd = 3); or to increment (cmd = 4) or decrement (cmd = 5) the stored value. On a read request, the leader responds by returning its stored data to that client (Line 22). When the leader receives any update request, such as a request to set, increment, or decrement its stored data, the leader moves to the RepCmd location to initiate

Safety Property: In every reachable state, there is at most one leader.

Safety Property: In every reachable state, all processes in locations Replica and Leader agree on the value of the variable stored.

Fig. 1. MERCURY Representation of a Distributed Store Process Definition. A process in a MERCURY program consists of a collection of variables, communication actions, and locations with associated event handlers. Each event handler consists of an event and a reaction to that event. An event is an empty event, a receive of a communication action, or one of the two agreement primitives: Partition and Consensus. Reactions typically consist of a block of update statements, control statements, and/or sends of communication actions.
a round of consensus to replicate the operation to all replicas in the system (Line 27 for the leader, Line 39 for the replicas). When consensus is complete, all processes update their stored data by executing the operation on which they have agreed, and the leader returns an acknowledgment message to the requesting client. In the event that the leader process has crashed (modeled by the environment sending a special LeaderDown message, not shown in Fig. 1), all processes in the Replica location receive that message (Line 47) and return to the Candidate location so a new leader may be elected.

Note that the key feature of Mercury’s design arises from the encapsulation of the two distributed agreement operations that occur: choosing a leader (Line 13), captured by the Partition primitive, and that leader’s replicating commands to the replicas (lines 27 and 39), captured by the Consensus primitive. These primitives have carefully-designed semantics that capture the essence of agreement protocols, and allow Mercury programs to be built without considering how that agreement is implemented. Sec. 2.2 describes these primitives and motivates their design in more detail.

Having represented the distributed store in Mercury, the designer can now utilize QuickSilver’s push-button parameterized verifier to check the correctness of any distributed system consisting of one or more such identical processes with respect to the two safety properties. In particular, QuickSilver can automatically verify that this Mercury distributed store satisfies the safety properties regardless of the number of processes, in less than a minute. Moreover, any refinement of this program that instantiates the Partition and Consensus primitives with some verified leader election or consensus protocol, respectively, is also guaranteed to satisfy the safety properties.

2.2 Agreement Primitives
The design of Mercury’s agreement primitives is driven by our goal of automated, parameterized verification of agreement-based systems. Thus, the granularity of abstraction in the agreement primitives was carefully chosen to strike a balance between (a) capturing the essence of most practical agreement protocols without modeling protocol-specific behavior and (b) facilitating decidability of PMCP (when combined with additional syntactic conditions). To meet these objectives, we propose two agreement primitives, Partition and Consensus, that can individually model two common variants of agreement that we refer to as partition-and-move agreement and value-consensus agreement, respectively. The two primitives can further be composed together to model other variants of agreement. We informally explain these primitives here, and provide a more formal treatment of their semantics in Sec. 3.3.

The Partition Primitive. The Partition agreement primitive is used to model partition-and-move agreement, where a set of participants wishes to partition itself into groups. Instances of partition-and-move agreement include variants of leader election protocols which partition the participants into two groups: leaders (or, winners) and non-leaders (or, losers). Note that each participant essentially proposes their process index (PID), which, in a parameterized distributed system with an unbounded number of processes, is drawn from an infinite domain. To enable decidability of PMCP for systems that use partition-and-move agreement protocols, the cardinality of exactly one group must be unbounded (e.g., non-leading), while that of all other groups must be finite (e.g., leaders). This disallows, for instance, partitioning the participants into two equal sets of winners and losers. Fortunately, we observe that most partition-and-move agreement protocols pick a finite number of winners that is independent of the number of participants.
Each Partition agreement primitive has an identifier, and takes two parameters: the set of participants and the desired number of winners. The reaction of each Partition primitive contains a win (resp. lose) handler that indicates how the process behaves upon winning (resp. losing).

Example. Distributed Store (Fig. 1) uses partition-and-move agreement in Line 13, modeled using a handler in the Candidate location with the agreement event: Partition<elect>(All, 1). This event uses the Partition primitive with identifier elect and is used to pick 1 process out of the set of all processes (All). The winners (resp. losers) of this agreement instance move to location Leader (resp. Replica).

The Consensus Primitive. The Consensus agreement primitive is used to model value-consensus agreement, where a set of participants, each proposing one value, wishes to decide on (a set of) values. Instances of value-consensus agreement include protocols such as Paxos [Lamport 1998], Fast Paxos [Lamport 2006], and Mencius [Mao et al. 2008]. To enable decidability of PMCP for systems that use value-consensus agreement, we restrict the domain of the proposed values to be finite. We note that many distributed systems aim to solve coordination-like problems rather than compute a function over their data. Hence, infinite concrete data domains can soundly be treated as finite abstract data domains using, for example, predicate abstraction.

Each Consensus primitive has an identifier, and takes three parameters: the set of participants, the number of proposals to be decided and (an optional) variable that a process uses to propose a value.

Example. Distributed Store (Fig. 1) uses value-consensus agreement in Lines 27 and 39 to allow the processes to decide which operation should be executed next. In particular, the processes use two handlers: one in the RepCmd location with the agreement event Consensus<vc>(All, 1, cmd), and another in the Replica location with the agreement event Consensus<vc>(All, 1, _). Both primitives have the identifier vc and allow all processes to participate (All). In the former, the leader proposes a value from the variable cmd, while in the latter, the replicas propose no value (denoted by _). The primitive decides on one value that can be accessed by all participants using the expression vc.decVar[1].

Composition of Primitives. The Partition and Consensus primitives can be composed to model agreement protocols like Multi-Paxos [Chandra et al. 2007] and Raft [Ongaro and Ousterhout 2014], where a set of participants wish to decide on a potentially infinite sequence of values. Instead of invoking agreement on every value of the sequence individually, such protocols enhance practicality by first electing a leader that proposes the values, while the rest of the processes accept such values. Such protocols can be modeled by using a Partition primitive to elect a leader, and then using Consensus primitives to have the leader propose values in subsequent rounds. Our Distributed Store example uses such a composition: upon receiving a doCmd request, the leader uses the Consensus primitive to agree with the replicas. Note that the replicas pass an empty proposal (denoted _) as only the leader should be proposing values.

2.3 Parameterized Verification in QUICKSILVER

Since PMCP is a well-known undecidable problem, it is not immediately obvious if parameterized verification is even decidable for MERCURY programs with agreement primitives. To this end, we first identify an expanded decidable fragment, MERCURY CORE, of an existing abstract model of distributed systems. Furthermore, we present two additional theoretical results that enable decidable and efficient parameterized verification for MERCURY programs. These results are based on establishing a correspondence between MERCURY programs and programs in the more abstract MERCURY CORE model, and appealing to MERCURY CORE’s decidability and cutoff results.
**Decidable Parameterized Verification.** We identify syntactic conditions, called *phase-compatibility conditions*, on systems with agreement primitives that yield decidability of PMCP. Informally, the phase-compatibility conditions capture systems that proceed in *phases*: each process is always in the same phase as every other process and all processes, simultaneously, move from one phase to the next using some global synchronization such as synchronous broadcasts or agreement. The phase-compatibility conditions ensure that the system’s ability to move between phases is independent of the number of processes, thereby paving the way towards decidable parameterized verification.

*Example.* Distributed Store (Fig. 1) is phase-compatible and hence enables decidable parameterized verification. The system starts in a phase where all processes are in the Candidate location, then uses a *Partition* primitive to move to the second phase where all processes are in locations Leader and Replica where the system is ready to receive requests from the clients.

**Practical Parameterized Verification.** Unfortunately, the decision procedure for PMCP in the M/e.r/c/u/y.M/e.r/c/u/y fragment has non-primitive recursive complexity [Schmitz and Schnoebelen 2013]. Hence, we identify additional syntactic conditions, called *cutoff-amenability conditions*, that, for a given class of safety properties, enable reducing the parameterized verification problem for systems with phase-compatible processes to verification of a system with a small, fixed number of processes. This small, fixed number of processes is called a cutoff, and essentially entails a small model property: if there exists a counterexample to a safety property in a system with a certain, possibly large, number of processes, then there exists a counterexample to the property in a system with a cutoff number of processes.

*Example.* The cutoff for Distributed Store (Fig. 1) and its safety properties is 3. Essentially, due to the nature of the safety properties and the structure of the system, any violation of the property in a system with more than 3 actors can still be reproduced in a system with 3 process—any additional actors will not prevent the 3 process from potentially reaching an error state.

While the cutoff may seem obvious here, in general, cutoff arguments are non-trivial and deriving cutoff results requires a deep understanding of the underlying machinery for decidable parameterized verification.

We note that some MERCURY programs may not be practically verifiable. In this event, QUICKSILVER provides best-effort feedback suggesting program modifications to the designer that can help fit their design into the desirable fragment of MERCURY programs.

### 3 THE MERCURY MODELING LANGUAGE

We present MERCURY, a language for modeling distributed agreement-based systems. We focus on systems in which the uncertainties and intricacies of their behavior in the presence of asynchronous communication and failures are essentially encapsulated within the underlying agreement protocols. Thus, while the agreement protocols may use asynchronous communication and tolerate network and process failures, we impose some simplifying assumptions on the system model outside of agreement. We assume that non-communicating processes can operate asynchronously, but communication is synchronous (i.e., sending and receiving processes must block until they can communicate). We further assume that processes may crash (i.e. exhibit crash-stop failures), but the network is reliable. These assumptions enable our initial exploration of the boundaries of decidable parameterized reasoning for agreement-based systems.

While MERCURY includes standard features like communication actions, events, and event handlers, its distinguishing feature is the availability of *special primitives for encapsulating different*...
agreement protocols. MERCURY also includes some design choices to facilitate decidable parameterized verification. We define the syntax and semantics of MERCURY programs, with a detailed treatment of the semantics of its agreement primitives.

3.1 MERCURY Syntax and Informal Semantics

**Programs.** A MERCURY program is a collection of an unbounded number $n$ of identical\(^2\) system processes $P_1, P_2, \ldots, P_n$ and an environment process $E$, communicating via events. Each system process has a unique process index (PID) drawn from the set $I_n = \{1, 2, \ldots, n\}$.

**Processes.** The syntax for a MERCURY system program is shown in Fig. 2. A process definition begins with a declaration of typed variables and communication actions, and is followed by a sequence of locations with a designated initial location. The variable type idSet corresponds to sets of process indices; the domain of this type is unbounded as the number of system processes is, in general, unbounded. The variable type int has a fixed, finite range that is specified in the declaration—this is one of the inbuilt restrictions in MERCURY to facilitate automated parameterized verification. Communication actions either represent communication between system processes or communication between the environment process and system processes; further, communication actions are either broadcast actions (denoted br) involving communication from one process to all other processes or rendezvous actions (denoted rz) involving communication between a pair of processes. Each communication action act has an optional (finite) integer-valued payload field that can be retrieved via the expression act.payload.

Each location contains a set of event handlers that consists of an event and a reaction to that event. An event can be the empty event (_), a receive of a communication action (recv), or one of two agreement primitives. A handler for an empty event corresponds to a non-reactive action a process may initiate, i.e., an internal computation or a send of a communication action. A Partition primitive part has two parameters: the set of participants and the number of winners to be chosen. The set of winners (resp. losers) is retrieved via the expression part.winS (resp. part.loseS). A Consensus primitive cons has three parameters: the set of participants, the number of proposals to be chosen, and an optional variable, $\langle$optIntVar$\rangle$, from which a process proposes its value. The $k^{th}$ value from the set of decided values, cons.decVar, is retrieved via the expression cons.decVar[$k$].

A reaction to an event consists of a block of update statements, control statements, and/or sends. Update statements include assignments to integer variables and statements to add or remove a PID from a set of PIDs. Control statements include conditionals and goto statements used to switch between locations. A sendrz statement is a rendezvous send that transmits a message with an action identifier act and an optional payload $\langle$optIntVar$\rangle$ to a process with index given by $\langle$idExp$\rangle$. A sendbr statement is a broadcast send that transmits a message with an action identifier act and an optional payload $\langle$optIntVar$\rangle$ to all other processes.

Reactions for empty, receive and Consensus events all begin with do. Additionally, a guarded reaction of the form where($\langle$bExp$\rangle$) do $\langle$stmt$\rangle$ can be used for empty and receive events to ensure that the handler is only enabled if some Boolean predicate evaluates to true. Finally, the reaction for a Partition event is of the form win: $\langle$stmt$\rangle$ lose: $\langle$stmt$\rangle$, indicating how a process should react if it wins and if it loses.

An environment process is a simpler version of a system process with variable types restricted to int and event types restricted to empty and receive events.

\(^2\)This can be relaxed to allow a finite number of distinct process definitions.
Fig. 2. Syntax for MERCURY. In the grammar, non-terminals are enclosed in ⟨ ⟩, keywords are in boldface, and all other terminals are monospaced.

**Expressions.** The syntax for all expressions in MERCURY processes is shown in Fig. 3. An ⟨idExp⟩ expression evaluates to a PID—self retrieves the PID of the current process and, in a receive handler for communication action act, the expression act.sID retrieves the PID of the corresponding sender. An ⟨idSetExp⟩ expression evaluates to a set of PIDs as shown and includes the expressions part.winS and part.loseS introduced earlier. An ⟨intExp⟩ expression evaluates to an integer...
Fig. 3. Syntax of *Mercury* Expressions.

\[
\begin{align*}
\langle \text{idExp} \rangle & \ ::= \text{self} & \text{PID of current process} \\
& \quad | \ \text{act.sID} & \text{PID of sender of action act} \\
\langle \text{idSetExp} \rangle & \ ::= \text{All} | \text{Empty} | \text{idSetV} & \text{Set of winners of Partition primitive part} \\
& \quad | \ \text{part.winS} & \text{Set of losers of Partition primitive part} \\
\langle \text{intExp} \rangle & \ ::= \text{intConst} | \text{intV} | \langle \text{intExp} \rangle \langle \text{arithOp} \rangle \langle \text{intExp} \rangle & \text{Payload of action act} \\
& \quad | \ \text{act.payld} & \text{Selecting some decided value of Consensus primitive cons} \\
\langle \text{bExp} \rangle & \ ::= \text{True} | \text{False} | \langle \text{bExp} \rangle \\
& \quad | \ \langle \text{bExp} \rangle \langle \text{boolOp} \rangle \langle \text{bExp} \rangle \\
& \quad | \ \langle \text{intExp} \rangle \langle \text{cmpOp} \rangle \langle \text{intExp} \rangle \\
& \quad | \ \langle \text{idExp} \rangle \langle \text{eqOp} \rangle \langle \text{idExp} \rangle \\
\langle \text{cmpOp} \rangle & \ ::= < | > | \leq | \geq | \langle \text{eqOp} \rangle \\
\langle \text{eqOp} \rangle & \ ::= = | != 
\end{align*}
\]

Fig. 4. *Mercury* Syntactic Sugar.

and includes the expressions \text{act.payld} and \text{cons.decVar[intConst]}, introduced earlier. A *Mercury* arithmetic expression, \langle \text{arithOp} \rangle, is standard and is not shown. A *Mercury* Boolean expression, \langle \text{bExp} \rangle, constrains comparison of \langle \text{idExp} \rangle expressions to equality and disequality checks; we expand on this restriction at the end of Sec. 3.2 and emphasize that this is a common syntactic restriction used to facilitate the use of *structural symmetries* for scalable verification (cf. [Emerson and Sistla 1996; Emerson and Wahl 2003; Gleissenthall et al. 2019; Ip and Dill 1996; Wahl 2007]).

**Syntactic Sugar.** *Mercury* provides syntactic sugar (Fig. 4) to simplify expressing some common idioms. A \text{passive} handler specifies events a process should *not* react to. A \text{reply} reaction sends a rendezvous reply to the last sender.

### 3.2 Agreement-Free *Mercury* Program Semantics

The semantics of *Mercury* processes and programs is best described using state-transition systems. We first define the semantics of *Mercury* programs without agreement primitives, then extend the definition to *Mercury* programs with agreement primitives in Sec. 3.3. Intuitively, the semantics allows non-communicating processes in *Mercury* programs to operate asynchronously while ensuring that communication and agreement is synchronous and consistent.
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\[
\begin{align*}
(iStmt) & \ ::= \ (uStmts); \ goto \ loc \\
(sStmt) & \ ::= \ (sendStmt); \ (iStmt) \\
(uStmts) & \ ::= \ e \mid (updateStmt) \mid (uStmts); \ (uStmts)
\end{align*}
\]  

(a)

location loc
on where \((\langle bExp\rangle)\) do
(iStmt)

(b)

location loc
on where \((\langle bExp\rangle)\) do
(sStmt)

(c)

location loc
on recv(act) where \((\langle bExp\rangle)\) do
(iStmt)

(d)

location loc
on Partition<part>(\langle idSetExp\rangle, intConst)
win: goto loc lose: goto loc

(e)

location loc
on Consensus<cons>(\langle idSetExp\rangle, intConst, \langle optIntVar\rangle)
goto loc

(f)

Fig. 5. Syntax of Core MERCURY. (a) Core Statements. Core handler for (b) Internal, (c) Send, (d) Receive, (e) Partition, and (f) Consensus.

Core Fragment of MERCURY. To enable a succinct description of MERCURY programs’ semantics, we rewrite process definitions into a core fragment of the language with the event handlers and statements depicted in Fig. 5. The handlers may contain two types of statements (shown in Fig. 5a): a statement \((iStmt)\) that consists of a (possibly empty) sequence of update statements, followed by a \(\text{goto}\) statement; and a statement \((sStmt)\) that consists of a send statement, followed by an \((iStmt)\) statement. The \textit{internal} core handler in Fig. 5b embodies computations that the process does without communication with other processes; the \textit{send} core handler in Fig. 5c embodies a send of some action by a process; and the \textit{receive} core handler in Fig. 5d embodies the reaction of a process to a receive of some action. Note that all three handlers are guarded by a predicate that dictates when they are enabled. The core handlers in Fig. 5e and Fig. 5f are for agreement primitives and only contain \(\text{goto}\) statements as shown. For the rest of this paper, let \(P\) be a process in the core fragment of MERCURY. With some abuse of notation, we use \(vars, acts,\) and \(locs\) to refer to the sets of variables, actions, and locations in their eponymous sequences in Fig. 2.

Process Semantics. The semantics of a process \(P\) is defined as a labeled state-transition system \((S, s_0, s_{cr}, acts, T)\), where \(S\) is the set of (local) states, \(s_0\) is the initial state, \(s_{cr}\) is a special “crashed” state, \(acts\) is the set of actions, and \(T \subseteq S \times \{sendrz, sendbr, recvbr, recvz, crash\} \times acts \times I_n \times S\) is the set of (local) labeled transitions of \(P\). A state \(s \in S\) is a pair \((loc, \sigma)\) where \(loc \in locs\) is a location and \(\sigma\) is a valuation of the variables in \(vars\). We let \(\sigma(var)\) denote the value of the variable \(var\) according to \(\sigma\). For a state \(s = (loc, \sigma)\), we let \(s.loc\) denote the location \(loc\) in \(s\), and \(s.\sigma(var)\) denote the value \(\sigma(var)\) of variable \(var\) in \(s\). Similarly, we use \(\sigma(expr)\) \((s.\sigma(expr))\) to denote the value of expression \(expr\) evaluated under \(\sigma\) (in state \(s\)). The initial state \(s_0 = (loc_0, \sigma_0)\), where \(loc_0\)
denotes the initial location and $s_0$ denotes the initial variable valuation. The crash state $s_{cr}$ is a special state that the process is assumed to enter upon exhibiting a crash-stop failure.

A transition of process $P$ without agreement primitives corresponds to the execution of one of the three core event handlers in Fig. 5b, Fig. 5c, and Fig. 5d; a transition is labeled either with a send/receive of a communication action in $acts$ or an empty label $\epsilon$ denoting an internal transition. For each core handler, let $1oc$ denote the current location and $1oc'$ denote the target location of the $goto$ statement. Then, the transitions in $T$ are defined as follows:

(a) For each broadcast send handler as shown in Fig. 5c with $\langle sendStmt \rangle$ given by $sendbr(\text{act}, \langle optIntVar \rangle)$, $T$ contains a transition $(1oc, \sigma) \xrightarrow{\text{sendbr}(\text{act})} (1oc', \sigma')$ for each $\sigma$ such that $\sigma(b\text{Exp}) = true$ and $\sigma'$ is obtained from $\sigma$ by applying the sequence of updates $\langle u\text{Stmts} \rangle$. Note that if $\langle optIntVar \rangle$ is $\epsilon$, the payload is empty and if $\langle optIntVar \rangle$ is a variable, denoted by $var_{act}$, the payload is $\sigma(var_{act})$.

(b) For each broadcast receive handler as shown in Fig. 5d with broadcast action $\text{act}$, $T$ contains a transition $(1oc, \sigma) \xrightarrow{\text{recvbr}(\text{act})} (1oc', \sigma')$ for each $\sigma$ such that $\sigma(b\text{Exp}) = true$ and $\sigma'$ is obtained from $\sigma$ by applying the sequence of updates $\langle u\text{Stmts} \rangle$. Note that $\langle u\text{Stmts} \rangle$ may access the received value using the expression $\text{act.payld}$.

(c) To model process crash-stop failures, $T$ contains a transition $(1oc, \sigma) \xrightarrow{\text{crash}} s_{cr}$ for each $(1oc, \sigma)$.

Local transitions corresponding to internal and rendezvous send and receive can be formalized similarly. We use $E$ to denote the environment process and $A_0, A_0', \ldots$ etc. to denote its set of states, initial state etc., respectively.

**Distributed Program Semantics.** The semantics of a MERCURY program consisting of $n$ identical system processes $P_1, \ldots, P_n$ and the environment process $E$ is defined as a state-transition system $M(n) = (Q, q_0, R)$, parameterized by the number of processes $n$, where:

1. $Q = S^n \times S_E$ is the set of global states,
2. $q_0 = (s_0, \ldots, s_0, s_0, E)$ is the initial global state, and
3. $R \subseteq Q \times Q$ is the set of global transitions where (i) all processes synchronize on a broadcast communication action, (ii) two processes synchronize on a rendezvous communication action, (iii) one process makes an asynchronous internal move, or (iv) one process crashes. Formally, a global transition $(q, q')$ based on a broadcast action act is in $R$ iff there exists a process $P_i$ with $s_i \xrightarrow{\text{sendbr}(\text{act})} s_i'$, and every other process $P_j$ with $j \neq i$ has a transition $s_j \xrightarrow{\text{recvbr}(\text{act})} s_j'$ such that $q' = q[s_i \leftarrow s_i', \forall j \neq i : s_j \leftarrow s_j']$ and if $\langle optIntVar \rangle$ is the variable $var_{act}$, $s_i.\sigma(var_{act}) = \text{act.payld}$. Here, $q[s_i \leftarrow s_i']$ indicates that process $P_i$ moves from state $s_i$ to $s_i'$. A global crash transition $(q, q')$ is in $R$ iff there exists a process $P_i$ with a local crash transition $s_i \xrightarrow{\text{crash}} s_{cr}$ and $q' = q[s_i \leftarrow s_{cr}]$. Global transitions corresponding to rendezvous actions or internal transitions can be formalized similarly.

An execution of a global transition system $M(n)$ is a (possibly infinite) sequence of states, $q_0, q_1, \ldots$, in $Q$ such that for each $j \geq 0$, $(q_j, q_{j+1}) \in R$. A state $q$ is reachable if there exists a finite execution of $M(n)$ that ends in $q$.

In what follows, we use $M$ and $M(n)$ as well as system process and process, interchangeably.

**Correctness Specifications.** In this work, we focus on a broad class of invariant properties of systems modeled in MERCURY. In particular, our correctness specifications are Boolean combinations of universally quantified formulas over locations, $\text{int}$ variables, and a finite number of variables with distinct valuations over $I_n$. 

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For example, one can specify that a location c is a critical section (of size 1) as: \( \forall i, j \in I_n. \neg(q[i].loc = c \land q[j].loc = c) \); Distributed Store (Fig. 1) uses a specification of this form to ensure that at most 1 process can be in Leader. As another example, one can specify that all processes in some location d must have the same value in their local variable \( v \) as: \( \forall i, j \in I_n.q[i].loc = d \land q[j].loc = d \Rightarrow q[i].\sigma(v) = q[j].\sigma(v) \); Distributed Store uses specifications of this form to ensure the stored data is consistent.

The program \( M(n) \) is safe if it has no reachable states that violate its correctness specification. Given a specification \( \phi(n) \), also parameterized by \( n \), we use the standard notation \( M(n) \models \phi(n) \) to denote that \( M(n) \) is safe.

**Symmetry for Efficient, Parameterized Verification.** Performing automated parameterized verification for systems with an arbitrary number of processes hinges on the number of different types of processes being bounded (in MERCURY, there are two types: system and environment). Thus, parameterized systems naturally exhibit many similar global behaviors that are independent of specific process indices. The symmetric nature of such global behaviors offers another advantage: it is possible to greatly improve the verification time of symmetric systems through symmetry reduction [Emerson and Sistla 1996]. In particular, a (global) state-transition system \( M \) is fully-symmetric if its transition relation \( R \) is invariant under permutations over the set \( I_n \) of PIDs. As noted in Sec. 3.1, MERCURY syntactically constrains comparison of \( \langle idExp \rangle \) expressions to (dis)equality checks. This is a sufficient condition to ensure MERCURY processes are fully-symmetric and, hence, enable parameterized verification and symmetry reduction.

### 3.3 Semantics of MERCURY Agreement Primitives

We now extend the process and program semantics defined in Sec. 3.2 to MERCURY programs with agreement primitives. Furthermore, we show that our definition of the semantics of agreement primitives provides a sound abstraction of agreement protocols and enables symmetry reduction.

To simplify the presentation of the semantics, we expand the set \( vars \) of variables as follows. For each Partition event \( part \), we add variables \( part\_winS \) and \( part\_loseS \) for storing the sets of winners and losers, respectively. Similarly, for each Consensus event \( cons \), we add variable \( cons\_decVar \) for storing the decided values.

**Process-level Semantics of Agreement Primitives.** For the Partition event handler (Fig. 5e), let \( loc \) be the current location and \( loc_w \) (resp. \( loc_c \) be the target location of the goto statement in the \( \text{win:} \) (resp. \( \text{lose:} \)) block. For the Consensus event handler (Fig. 5f), let \( loc \) denote the current location and \( loc_c \) denote the target location of the goto statement. Then, the set \( T \) of transitions is extended as follows:

(a) For each Partition handler with event Partition <part> \((\langle idSetExp \rangle, \text{intConst})\) in Fig. 5e, \( T \) contains transitions \( (loc, \sigma) \xrightarrow{\text{win:}PC_{\text{part}}(pcpt,k)} (loc_w, \sigma') \) and \( (loc, \sigma) \xrightarrow{\text{lose:}PC_{\text{part}}(pcpt,k)} (loc_c, \sigma') \) for each \( \sigma \) such that \( pcpt \), matched by \( \langle idSetExp \rangle \), denotes the set of participants, \( k \), given by \( \text{intConst} \), denotes the number of winners to be decided, and \( \sigma' \) is obtained from \( \sigma \) by updating variables \( part\_winS \) and \( part\_loseS \) to the sets of winners and losers in the global invocation of \( part \), respectively.

(b) For each Consensus handler with event Consensus <cons> \((\langle idSetExp \rangle, \text{intConst}, \langle \text{optIntVar} \rangle)\) in Fig. 5f, \( T \) contains a local transition \( (loc, \sigma) \xrightarrow{VC_{\text{cons}}(pcpt,k,pVar)} (loc_d, \sigma') \) for each \( \sigma \) such that \( pcpt \) and \( k \) are as before, \( pVar \) denotes the variable from which a process proposes its

---

3While such sets are usually predefined, we allow more flexibility by permitting processes to communicate and construct them.
value, matched by \(\langle\text{optIntVar}\rangle\) if \(\langle\text{optIntVar}\rangle\) is not \(\epsilon\), and \(\sigma'\) is obtained from \(\sigma\) by updating the variable \(\text{cons}\_\text{decVar}\) to the decided values in the invocation of \(\text{cons}\).

**Program-level Semantics of Agreement Primitives.** The local transitions corresponding to agreement primitives are essentially modeling invocation of verified agreement protocols that enable a set of participants to decide on a finite set of winners/values in a globally consistent way. As stated in Sec. 2, verified agreement protocols typically entail agreement, validity, and termination. Thus, to ensure that agreement primitives provide a sound abstraction of verified agreement protocols, the global behavior of these primitives must satisfy a set of conditions entailed by agreement, validity, and termination. We represent this set of conditions on the global transitions corresponding to agreement primitives as a precondition-postcondition pair, stated informally as:

- **C1:** Consistent Participants Precondition. The participants agree on with whom to invoke agreement, and,
- **C2:** Consistent Decisions Postcondition. Upon termination of agreement, all non-crashed participants concur on winners/values.

In what follows, we present the global transitions and specialization of the precondition-postcondition pair \((C_1,C_2)\) for each type of agreement primitive.

**Partition.** Consider an instance of a Partition agreement primitive with identifier \(\text{part}\) and local transitions \((1\text{oc}^c, \sigma)\xrightarrow{\text{win}:\text{PC}_\text{part}(\text{pcpt},k)}(1\text{oc}^c_W, \sigma')\) and \((1\text{oc}^c, \sigma)\xrightarrow{\text{lose}:\text{PC}_\text{part}(\text{pcpt},k)}(1\text{oc}^c_L, \sigma')\). Let \(\text{loc}_{\text{part}}\) be the set of all locations \(1\text{oc}^c\) from which the participants of this instance may invoke Partition (i.e., all locations where the above two transitions originate).

We extend the global transition relation \(R\) of \(\mathcal{M}\) with a Partition agreement transition from global state \(q_{\text{start}W}\) to global state \(q_{\text{end}W}\) encoding a selected set \(W\) of \(k^5\) non-crashed winners, and a set \(F\) of participants that have crashed during agreement if:

**C1(\text{PC}):** There exists a set \(S \subseteq I_n\) of processes in \(q_{\text{start}}\) in appropriate locations for invoking this instance of the Partition primitive and with a consistent view of each other. Formally:

1. \(\forall i \in S : q_{\text{start}}[i].\text{loc} \in \text{loc}_{\text{part}}\) and
2. \(\forall i, j \in S : q_{\text{start}}[i].\sigma(\text{pcpt}) = q_{\text{start}}[j].\sigma(\text{pcpt}) = S\) and,

**C2(\text{PC}):** The non-crashed processes of \(S\) move to their appropriate target locations in \(q_{\text{end}}\) based on whether they win or lose and their \(\text{part}\_\text{winS}\) and \(\text{part}\_\text{LoseS}\) variables in \(q_{\text{end}}\) are updated to reflect the partition while the set \(F \subset S\) of crashed processes move to the crash state \(s_{\text{cr}}\). As explained in Remark 1 below, we assume that if all the participants fail, then no valid \(q_{\text{end}}\) exists. Formally: Let \(N\) be the set \(S \setminus F\) of non-crashed participants, then:

1. \(\forall i \in N : i \in W \land q_{\text{start}}[i].\text{loc} = 1\text{oc}^c \Rightarrow q_{\text{end}}[i].\text{loc} = 1\text{oc}_{\text{win}}^c\),
2. \(\forall i \in N : i \notin W \land q_{\text{start}}[i].\text{loc} = 1\text{oc}^c \Rightarrow q_{\text{end}}[i].\text{loc} = 1\text{oc}_{\text{lose}}^c\),
3. \(\forall i \in N : q_{\text{end}}[i].\sigma(\text{part}\_\text{winS}) = W\),
4. \(\forall i \in N : q_{\text{end}}[i].\sigma(\text{part}\_\text{LoseS}) = N \setminus W\),
5. \(\forall i \in F : q_{\text{end}}[i] = s_{\text{cr}}\), and,
6. \(\forall i \in I_n \setminus S : q_{\text{end}}[i] = q_{\text{start}}[i]\).

**Consensus.** Consider an instance of a Consensus agreement primitive with identifier cons and local transition \((1\text{oc}^c, \sigma)\xrightarrow{\text{VC}_{\text{cons}}(\text{pcpt},k,\text{pVar})}(1\text{oc}_{\text{cons}}^c, \sigma')\). As before, let \(\text{loc}_{\text{cons}}\) be the set of locations \(1\text{oc}^c\) from which the participants of this cons instance may start.

---

\(^4\)Systems in which all processes intend to reach agreement trivially satisfy \(C_1\). The more general form of \(C_1\) enables systems to invoke agreement protocols with only a subset of processes.

\(^5\)All non-crashed participants act as winners if the number of non-crashed participants is less than \(k\).
We extend the global transition relation $R$ of $M$ with a Consensus agreement transition from a global state $q_{\text{start}}$ to a global state $q_{\text{end}}^W$ encoding a selected set $W$ of $k$ decided values and a set $F$ of participants that have crashed during agreement if:

$C_1(VC)$: The state $q_{\text{start}}$ is as defined for Partition, and,

$C_2(VC)$: The set $N := S \setminus F$ such that $|N| > |F|$ of non-crashed processes move to their target locations while in $q_{\text{end}}^W$ and their cons_decVar variables are updated to reflect the decided values while the set $F$ of crashed processes move to the crash state $s_{cr}$. As explained in Remark 1 below, we assume that if a majority of participants fail (i.e., $|N| \leq |F|$), then no valid $q_{\text{end}}^W$ exists. Formally:

1. $\forall i \in N : q_{\text{start}}[i].loc = 1oc^c \Rightarrow q_{\text{end}}^W[i].loc = 1oc^c$,
2. $\forall i \in N : q_{\text{end}}^W[i].\sigma(\text{cons\_decVar}) = W$,
3. $\forall i \in F : q_{\text{end}}^W[i] = s_{cr}$, and,
4. $\forall i \in I_n \setminus S : q_{\text{end}}^W[i] = q_{\text{start}}[i]$.

**Remark 1: Failure Assumptions for Valid Termination.** Common agreement protocols have assumptions about process failures under which they guarantee the validity of results upon termination. For instance, leader election protocols [Garcia-Molina 1982] require the elected leader to not fail, but can tolerate the failures of the losing processes and consensus protocols like Paxos [Lamport 1998] and Raft [Ongaro and Ousterhout 2014] require a simple majority of the participants to not fail and to agree on a proposed value. While, in general, the semantics of Mercury’s agreement primitives can be parameterized over specific failure assumptions, our default definitions encode these common assumptions. Thus, when using the Partition primitive, any global state $q_{\text{end}}^W$ where all the participants have failed is assumed to be not valid. When using the Consensus primitive, any global state $q_{\text{end}}^W$ where a majority of the participants have failed is assumed to be not valid.

**Soundness.** Mercury’s agreement primitives are sound:

**Lemma 3.1.** Our proposed abstraction of verified agreement protocols, as defined using the syntax and semantics of Mercury agreement primitives, is sound. In other words, if an agreement protocol satisfies agreement, validity, and termination, then the agreement protocol satisfies the semantics of agreement primitives captured by the precondition-postcondition pair $(C_1, C_2)$.

**Proof:** We prove this by contradiction. Assume that an agreement protocol satisfying agreement, validity, and termination begins in a state $q_{\text{start}}$ that satisfies precondition $C_1$ but ends in a state $q_{\text{end}}^W$ that violates postcondition $C_2$. A violation of postcondition $C_2$ (i.e., participants not agreeing on the same winner/value or agreeing on a winner/value that was not in the set of participants) contradicts agreement and validity. Finally, a violation due to the absence of a transition between a state $q_{\text{start}}$ satisfying $C_1$ and a state $q_{\text{end}}^W$ satisfying $C_2$ directly contradicts termination.

Note that the statement of Lemma 3.1 implicitly assumes that the failure assumptions encoded in the semantics of Mercury agreement primitives hold. If the failure assumptions do not hold, neither our primitives nor the agreement protocols they abstract provide any guarantees.

**Symmetry.** In a state-transition system, $M_{\text{agree}} = (Q, q_0, R)$, capturing the semantics of a Mercury program, let $R_{\text{agree}}$ be the set of all transitions corresponding to agreement primitives in $R$.

---

6Note that, a proposed value of a crashed process can still be chosen as the decided value.

7A violation of precondition $C_1$ (i.e., participants do not have a consistent view of each other or are in invalid local states to participate in the agreement protocol) indicates an invalid global state to invoke agreement.
Let $\mathcal{M} = (Q, q_0, R \setminus R_{\text{agree}})$ be the state-transition system without the agreement transitions of $\mathcal{M}_{\text{agree}}$.\footnote{We do not make any symmetry-related assumptions about the specific agreement protocol that an agreement primitive encapsulates. In particular, the underlying agreement protocol could employ non-symmetric strategies such as "the process with maximum PID wins".}

**Lemma 3.2.** If $\mathcal{M}$ is fully-symmetric, then $\mathcal{M}_{\text{agree}}$ is fully-symmetric.

Intuitively, the proof (ref. App. A.2) is based on the observation that agreement transitions are oblivious to the identities of the participants and are hence invariant under permutations over $I_n$.

## 4 Verification of Mercury Programs

We now formalize the parameterized verification problem for Mercury programs and present our theoretical results for enabling decidable and efficient parameterized verification.

**Mercury Parameterized Verification Problem (MPVP).** Given a Mercury system process $P$, an environment process $E$, and a parameterized safety specification $\phi(n)$ as defined in Sec. 3, MPVP asks if $\forall n. \mathcal{M}(n) \models \phi(n)$.

Our first result (Sec. 4.1) identifies conditions on Mercury programs and the specification $\phi(n)$ for enabling decidability of MPVP. Our second result (Sec. 4.2) identifies additional conditions for which this problem is efficiently decidable, based on cutoff results. Cutoff results reduce the parameterized verification problem to a verification problem over a fixed number of processes. Formally, a cutoff for a parameterized system $\mathcal{M}$ and correctness specification $\phi$ is a number $c \in \mathbb{N}$ such that:

$$\forall n \geq c. (\mathcal{M}(c) \models \phi(c) \iff \mathcal{M}(n) \models \phi(n)).$$

In particular, our second result identifies conditions on Mercury programs for small cutoffs, reducing MPVP to verification of a Mercury program with a small number of processes. The latter problem is decidable for any Mercury program, as the corresponding semantics can be expressed as a finite-state machine.

For the rest of this section, we fix a Mercury process $P$ with a set of process-local states $S$, initial state $s_0$, and process-local transitions $T$, and refer to the corresponding global state-transition system, $\mathcal{M}$, as a (Mercury) system.

### 4.1 Decidable Parameterized Verification

**Phases.** To enable decidable parameterized verification, we view Mercury systems as proceeding in phases. A phase of a Mercury system is a set of process-local states, characterizing the set of events that can occur when all processes co-exist in that set of local states. In any global execution, all processes simultaneously move from one phase to the next, where a new set of events may occur. Processes move between phases strictly via globally-synchronizing events, i.e., broadcasts or agreement primitives; within a phase, processes can use any type of communication. While two phases may share some local states, their associated events are disjoint. We note that any Mercury system can be viewed as proceeding in phases, by identifying phases with appropriate sets of events—so phases do not constrain the applicability of our approach.

In what follows, we refer to the set of globally-synchronizing events as $E_{\text{global}}$ and the set of rendezvous actions in $P$ as $E_{\text{rend}}$. For each event $e$, we define its source set, denoted $\text{src}_e$, as the set of states in $S$ from which there exists a transition in $T$ labeled with $e$. Similarly, we define the destination set of each event $e$, denoted $\text{dst}_e$, as the set of states in $S$ to which a transition in $T$ labeled with $e$ exists. For instance, if $e$ is a broadcast action $\text{act}$, $\text{src}_{\text{act}} = \{s \mid s \xrightarrow{\text{sendbr}(\text{act})} \}$.
\[ s' \in T \lor s \xrightarrow{recvb} s' \in T \] and \( \text{dst}_{\text{act}} = \{s' \mid s \xrightarrow{sendb} s' \in T \lor s \xrightarrow{recvb} s' \in T \} \). The source and destination sets for rendezvous actions and instances of Consensus and Partition can be defined similarly. Finally, we define the relation \( R \subseteq S \times S \) to denote pairs of states related via internal or rendezvous transitions as follows:

\[ R = \{(s, t) \mid s \neq t \land (s \xrightarrow{e} t \lor t \xrightarrow{e} s) \in T \lor (\exists e \in E_{\text{rend}} \{s, t\} \subseteq src_e \cup dst_e)\} \].

We now present a constructive definition for the set of phases of \( S \). Intuitively, two states are in the same phase if they are part of the same source or destination set, or, their phases are connected by internal or rendezvous transitions.

**Definition 4.1 (Phases).** The set of phases is constructed as follows:

1. **Initialization:** The set of phases is initialized to the set of source sets and destination sets of each globally-synchronizing event:

   \[
   \text{inPhases} = \bigcup_{e \in E_{\text{global}}} \{\text{src}_e, \text{dst}_e\}.
   \]

   Informally, the source set of a globally-synchronizing event \( e \) is a subset of the local state space where all processes need to co-exist for \( e \) to occur. The destination set of \( e \) characterizes the set of states in which all processes co-exist after event \( e \) has occurred.

2. **Expansion:** Each initial phase is then expanded such that if a state \( s \) is in a phase, then every state \( t \) such that \( R(s, t) \) holds is in the phase too:

   \[
   \text{exPhases} = \bigcup_{X \in \text{inPhases}} \{X \cup \{t \mid s \in X \land R^+(s, t)\}\},
   \]

   where \( R^+ \) is the transitive closure of \( R \). Informally, this step ensures that any local state that is reachable from or can be reached by a state in an initial phase via internal or rendezvous transitions, is added to that phase.

3. **Merge:** Finally, expanded phases that contain distinct states \( s, t \) with \( R(s, t) \) are merged:

   \[
   \text{phases} = \{\bigcup_{W \in X} W \mid X \subseteq \text{exPhases} \land \forall Y, Z \in X. R^+_p(Y, Z)\},
   \]

   where \( R_p = \{(X, Y) \mid 3s \in X, t \in Y. R(s, t)\} \) and \( R^+_p \) is the transitive closure of \( R_p \). Informally, if processes can move via internal or rendezvous transitions between two expanded phases, then the two phases are merged to ensure that the processes always co-exist in the same phase.

This definition ensures that all the processes in the system are in the same phase at any given time in a program execution.

**Phase-compatibility Conditions.** Our phase-compatibility conditions ensure that all processes in a Mercury system move in phases such that the set of available events within a phase as well as the system's ability to move between phases (through globally-synchronizing events) is independent of the number of processes. Such independence is critical for decidability of MPVP, which needs to reason about an arbitrary number of processes. In particular, since processes can only move between phases using globally-synchronizing events, these conditions ensure that such events behave in a way that is independent of the number of processes. A process that satisfies these conditions is called phase-compatible. In what follows, we present the phase-compatibility conditions.

We first define a classification of local transitions corresponding to globally-synchronizing events into acting and reacting transitions. For broadcasts, sending transitions are acting while receiving transitions are reacting. For the Partition primitive, winning transitions are acting while losing
transitions are reacting. For the Consensus primitive, transitions with winning proposals are acting while other transitions are reacting. For each globally-synchronizing event \( e \), let \( s \xrightarrow{A(e)} s' \) (resp. \( s \xrightarrow{R(e)} s' \)) denote a local acting (resp. reacting) transition of \( e \). Additionally, for some event \( e \) and some subset \( X \) of the local state space \( S \), we say that \( e \) is initiable in \( X \) if some state in \( X \) has an acting transition of \( e \).

**Definition 4.2 (Phase-Compatibility Conditions).**

1. Every state \( s \in P \) which has an acting transition \( s \xrightarrow{A(e)} s' \) must also have a corresponding reacting transition \( s \xrightarrow{R(e)} s'' \).
2. For each internal transition \( s \rightarrow s' \) that is accompanied by a reacting transition \( s' \xrightarrow{R(f)} s'' \) and for each state \( t \) in the same phase as \( s \), if event \( f \) is initiable in that phase, then \( t \) must have a path to a state with a reacting transition of event \( f \).
3. For each acting transition \( s \xrightarrow{A(e)} s' \) that is accompanied by a reacting transition \( s' \xrightarrow{R(f)} s'' \) such that \( f \) is initiable in the set \( dst_e \) of destination states of event \( e \), (i) if there are other acting transitions \( t \xrightarrow{A(e)} t' \) for event \( e \), all of them must transition to a state \( t' \) with a reacting transition \( t' \xrightarrow{R(f)} t'' \) of event \( f \) and (ii) for every reacting transition \( u \xrightarrow{R(e)} u' \) of \( e \), there must be a path from \( u' \) to a state with a reacting transition of event \( f \).

Intuitively, the conditions ensure that if a MERCURY system with a given number of processes can move in phases, then any additional processes can go along with the existing ones by always taking a corresponding reacting transition. In particular, condition (1) ensures that processes in the same state as the process taking the acting transition on some event \( e \) have a way to react to \( e \). Condition (2) ensures that, if a process can reach a state with a reacting transition on an initiable event in that phase, all other processes can also reach a state where they can react to that event. Condition (3) ensures that once a process takes an acting transition of event \( e \) and moves to a state \( s' \) where a reacting transition of event \( f \) can be taken, if \( f \) is initiable in the phase of \( s' \), all processes move in a way that ensures they can take a reacting transition of \( f \) as well.

**Permissible Safety Specification.** We target safety specifications which forbid the reachability of any global state where some number \( m \) (or more) of processes are in some set of local states; simple instantiations of such specifications include mutual-exclusion and process-local safety properties. Let \( f \) be a Boolean formula over locations and int variables of a MERCURY process. Let \( f_i \) be \( f \) indexed by the PID \( i \). For instance, \( f = s.loc \neq c \land s.\sigma(v) < 1 \) has the indexed formula \( f_i = q[i].loc \neq c \land q[i].\sigma(v) < 1 \). Let \( i_1, \ldots, i_m \) represent distinct valuations over \( I_n \). Then, we define \( \phi_{m,f}(n) \) as:

\[
\phi_{m,f}(n) = \forall i_1, \ldots, i_m. - (f_{i_1} \land \ldots \land f_{i_m})
\]

Intuitively, the formula \( f \) encodes a set \( st(f) = \{ s \in S \mid f = true \} \) of process-local states (where \( f \) holds) and the property \( \phi_{m,f}(n) \) forbids the reachability of a global state where \( m \) or more processes are in the set of local states \( st(f) \). We call formulas of the form \( \phi_{m,f}(n) \) permissible safety specifications and note that all specifications in this paper can be expressed using this form. For example, the Distributed Store specification asserting that no more than 1 process is in location Leader is expressed as \( \phi_{2, s.loc = \text{leader}}(n) \), i.e., \( \forall i_1, i_2. - (q[i_1].loc = \text{leader} \land q[i_2].loc = \text{leader}) \).

Examples of specifications that are not permissible include: "there exists at least one process in location Leader at all times", and "no more than half of the processes can be in location Leader". The former forbids \( m \) or less processes to be in a given set of local states (as opposed to \( m \) or more) while the latter forbids the reachability of a possibly unbounded number of processes to a given
set of states (as opposed to a fixed number \( m \) of processes). For the remainder of this section, we will focus on permissible safety specifications of the form \( \phi_{m,f}(n) \), but we note that our results extend to conjunctions and disjunctions of permissible safety specifications (see App. D.1).

On a high-level, if a MERCURY process is phase-compatible, then the behavior of the corresponding MERCURY system is independent of its number of processes. Hence, the reachability, or the lack thereof, of an error state corresponding to a violation of a permissible safety specification is consistent across different “sizes” of the system. In other words, if an error state is reachable in a system with a given number of phase-compatible processes, adding additional processes will not render such an error state unreachable. Decidability follows from a similar argument in the opposite direction: if an error state is reachable in a system with some number of phase-compatible processes, then we can compute a number of processes sufficient for reaching the error state.

The following theorem identifies the decidable fragment for MPVP.

**Theorem 4.3.** MPVP is decidable for MERCURY system process \( P \) and permissible safety specification \( \phi(n) \) if:

1. \( P \) is phase-compatible.
2. The state space of \( P \) is fixed and finite\(^9\).
3. There exists at most one rendezvous-receive transition per action per phase\(^10\).

**Proof Intuition.** The proof leverages a new fragment of an existing abstract model, the GSP model [Jaber et al. 2020], for which MPVP is decidable. The decidability result for the GSP model utilizes the framework of well-structured transition systems (WSTSs). This entails defining a well-quasi-ordering (WQO) over the global state space of a GSP system as well as a set of sufficient “well-behavedness” conditions over the local GSP process definition to ensure that global transitions are “compatible” with the well-quasi-ordering. To admit a larger decidable fragment, we designed a novel WQO (WSTSs) for the most well-behaved transitions. Further, we model process crash-stop failures. We defer the intricacies of this extension (which we refer to as the MERCURY CORE), as well as the formal definitions of WSTSs, WQOs, and compatibility to App. B. We note that, without this extension, the phase-compatibility conditions will not be initiability-aware (e.g., phase-compatibility condition (2) above will need to hold regardless of \( e \) being initiable or not).

We show that for any MERCURY process \( P_{\text{MERC}} \) that satisfies the three conditions of Theorem 4.3, one can construct a corresponding process \( P_{\text{CORE}} \) in MERCURY CORE such that there exists a simulation equivalence between their respective global state-transition systems and \( P_{\text{CORE}} \) belongs to the decidable fragment of MERCURY CORE. We refer the reader to App. C for the full proof.

Recall that, in Sec. 1.1, we discuss reasons why neither the decidable fragment of GSP model nor MERCURY CORE is directly suitable for designing agreement-based decidable systems.

### 4.2 Cutoffs for Efficient Parameterized Verification

We define additional conditions on MERCURY programs to obtain small cutoffs and enable efficient parameterized verification. These cutoff-amenability conditions ensure that any global error state, where \( m \) processes are in local states \( s(t) \) violating a permissible safety specification \( \phi_{m,f}(n) \), can be reached in a system with exactly \( m \) processes iff it can be reached in a system of any size.

---

\(^9\)We note that this condition restricts the way participant sets of agreement primitives are built to the constant set \( \text{All} \) or the result of a previous Partition instance \( \text{part wins or part loses} \), hence ensuring the precondition of agreement is naturally met. In general, this condition can be relaxed to include some systems with an unbounded state space where such sets are built through communication.

\(^10\)Under full symmetry, this condition ensures that abstracting the receiver PID (in any rendezvous-send transition \( s_{\text{send}}(\text{act}, \text{PID}) \rightarrow s' \)) does not introduce spurious behaviors.
larger than \( m \). Thus, programs satisfying these conditions enjoy a small model property: \( \phi_{m,f}(n) \) is satisfied in \( M(n) \) for all \( n \in \mathbb{N} \) if \( \phi_{m,f}(m) \) is satisfied in a system \( M(m) \) with a fixed number of processes \( m \). This requires the conditions to ensure that the reachability of a global state violating \( \phi_{m,f}(m) \) in \( M(m) \) does not depend on the existence of more than \( m \) processes.

**Cutoff-Amenability Conditions.** We first define a notion of independence of transitions and paths of a process. Informally, independent transitions do not require the existence of other processes in certain states. For instance, in Partition agreement, the winning transition \( s \xrightarrow{\text{win:} \text{PC}_{\text{part}}(\text{pcpt},k)} s' \) is independent since a winning process does not require the existence of a losing one to take that transition, but the losing transition \( s \xrightarrow{\text{lose:} \text{PC}_{\text{part}}(\text{pcpt},k)} s' \) is not independent since the losing process requires the existence of a winning process to take that transition. Note that acting transitions of globally-synchronizing events as well as internal transitions are independent while reacting transitions are not independent. A path is independent if it consists of independent transitions.

**Definition 4.4 (Cutoff-Amenability Conditions).** Let \( P \) be a phase-compatible process, \( \phi_{m,f}(n) \) a permissible specification, and \( \mathcal{F} \) the set of independent simple paths from \( s_0 \) to a state \( s \in st(f) \). We require either of the following to hold.

1. All paths from \( s_0 \) to \( st(f) \) are independent, or,
2. For every transition \( s_s \rightarrow s_d \) such that \( s_s \neq s_d \) and \( s_s \) is a state in some path \( p \in \mathcal{F} \), either
   
   a. the state \( s_d \) is in \( p \) and the transition \( s_s \rightarrow s_d \) is independent, or,
   b. the state \( s_d \) is not in \( p \) and all paths out of \( s_d \) lead back to \( s_s \) via independent transitions.

The conditions ensure that the processes required to enable a path to an error state are available in \( M(m) \). Condition (1) ensures that, if \( m \) processes were to reach the error states, they can do so without requiring additional processes, since all paths to the error states are independent. Condition (2) allows for some processes to “diverge” from the independent paths as long as they return independently. We note that the QUICKSILVER tool implements a more advanced version of this lemma that allows for more systems to have cutoffs.

We refer to the pair \( \langle P, \phi_{m,f}(n) \rangle \) as amenable if \( P \) is a phase-compatible process that satisfies the cutoff-amenable conditions w.r.t. permissible safety specification \( \phi_{m,f}(n) \).

On a high-level, an amenable pair \( \langle P, \phi_{m,f}(n) \rangle \) identifies systems where the minimum number of processes to trigger an error (i.e., \( m \) process existing simultaneously in \( st(f) \)) is, in fact, exactly \( m \). This is achieved by ensuring that any path a process may take to an error state is independent and hence if a process may reach an error state, it can do so without the help of other processes.

**Lemma 4.5.** For an amenable pair \( \langle P, \phi_{m,f}(n) \rangle, c = m \) is a cutoff for MPVP.

**Proof Intuition.** We utilize the cutoff results of MERCURY CORE. Using the construction in the proof of Theorem 4.3 to obtain a process \( P_{\text{CORE}} \) in the MERCURY CORE from a process \( P_{\text{MERC}} \) in MERCURY, we show that if cutoff-amenable holds for \( P_{\text{MERC}} \), then \( P_{\text{CORE}} \) will be cutoff-amenable and the resulting cutoff for \( \langle P_{\text{CORE}}, \phi_{m,f}(n) \rangle \) is also a cutoff for \( \langle P_{\text{MERC}}, \phi_{m,f}(n) \rangle \). We refer the reader to App. D for the full proof.

**Automation and Feedback.** While the phase-compatibility and cutoff-amenable conditions are somewhat intricate, we emphasize that our QUICKSILVER tool automatically checks these conditions and additionally gives the system designer feedback on how to make a process phase-compatible and amenable. This allows the designer to proceed without being caught up in the details of the exact conditions. The feedback varies depending on the failed condition and mainly aims to capture the root cause of the failure and to provide heuristically-ranked suggestions to fix it.
A failure of a phase-compatibility condition can be succinctly captured by a phase, a set of local states in that phase, and set of acting/reacting transitions from these states over one or two events. This localization is valuable for the user to pin-point what changes are needed to render the system phase-compatible. **QUICKSILVER** suggests edits that would eliminate the current violation and the user gets to pick which edit to implement.

A failure of a cutoff-amenability condition for correctness properties $\phi_{m,f}(n)$ can be succinctly captured by a non-independent path from the initial state $s_0$ to a state $s \in states(f)$. This path indicates a scenario where an error state could be unreachable in a system with $m$ processes but can be reachable in a bigger system; hence $m$ is not a valid cutoff. In these cases, **QUICKSILVER** presents the non-independent paths but does not suggest edits, and the user is responsible for eliminating the non-independent transitions from these paths.

**Example.** Consider a system where a set of processes wish to select up to two processes that can then perform an action one after the other. The designer starts with the process definition shown in Fig. 6. All processes start in location Start, and coordinate to pick up to two processes to move to the Selected location while the rest move to Idle. From Selected, the chosen processes send the getReady broadcast and move to Prepare. In Prepare, they attempt to move to the Target location one after the other using a sequencer message. The correctness property for this system is $\phi_{2,\text{loc}=\text{Target}}(n)$ indicating that at most one process can be at the Target location at any time.

```plaintext
1 process SelectiveSerializer
2 actions
3  br getReady : unit
4  br sequencer : unit
5
6 initial location Start
7   on Partition<select>(All,2)
8     win: goto Selected
9     lose: goto Idle
10
11 location Idle
12 passive getReady, sequencer
13 location Selected
14   on _ do
15     sendbr(getReady)
16     goto Prepare
17
18 location Prepare
19   on recv(sequencer) do
20     goto Target
21
22 location Target
23 // perform action
```

Fig. 6. An Initial **MERCURY** Process Definition for Distributed Coordination for Serializing Access.

When **QUICKSILVER** in run on this process definition, it reports that the system is not phase-compatible with the following feedback suggesting adding a receive handler of event getReady from the Selected location:

```plaintext
(Selected,\{\}) needs a corresponding reacting transition on getReady
Suggestions to solve this:
- add transition (Selected,\{\}) —R(getReady)——> (Prepare,\{\})
- add transition (Selected,\{\}) —R(getReady)——> (Anywhere!,\{\})
```

The designer accepts the first heuristically-ranked suggestion and adds the following handler to the Selected location:

```plaintext
on recv(getReady) do goto Prepare
```

With this edit, the system is now phase-compatible. However, the system is not cutoff-amenable. **QUICKSILVER** returns the following feedback:
Cutoff computation failed: on path
(Start,{})—A(select)—>(Selected,{})—A(getReady)—>(Prepare,{})—R(sequencer)—>(Target,{}).

the following transition(s) are not independent:
(Prepare,{})—R(sequencer)—>(Target,{}).

Based on this feedback, the designer realizes that processes in Prepare need to send the sequencer broadcast to move to the Target location, which is an independent transition. Learning from the previous phase-compatibility violation, the user additionally adds the corresponding reacting transition. Thus, the designer replaces the receive handler from location Prepare with the following two handlers:

```
on _ do sendbr(sequencer) goto Target
on recv(sequencer) do goto Prepare
```

The system is now phase-compatible and cutoff-amenable, and QUICKSILVER reports a cutoff value of two.

**Modular Verification.** Recall that Lemma 3.1 shows that any verified agreement protocol (i.e., one proven to satisfy agreement, validity, and termination) also meets the pre- and post-condition pair of our agreement primitives. So, by verifying an MERCURY program with agreement primitives w.r.t. to a safety specification, we can conclude that the program, when instantiated with any agreement protocol that satisfies agreement, validity, and termination, also satisfies the safety specification.

## 5 IMPLEMENTATION AND EVALUATION

We describe the implementation of QUICKSILVER\textsuperscript{11} and evaluate the performance of its automated parameterized verification procedure on various benchmarks encoded in MERCURY.

### 5.1 Implementation

QUICKSILVER performs automated, parameterized verification of MERCURY programs in three steps:

1. **Parsing.** QUICKSILVER compiles MERCURY processes into the core fragment by rewriting all non-core handlers (e.g., handlers with if-statements or multiple send statements) into core handlers and expanding syntactic sugar.
2. **Analysis.** From the core fragment, QUICKSILVER creates a labeled graph representing the process-level semantics including transitions that model crash-stop failures. QUICKSILVER checks the phase-compatibility and cutoff-amenability conditions against this graph, and if the conditions are met, computes a cutoff to verify the system.
3. **Verification.** QUICKSILVER’s verification engine is built on top of Kinara [Alur et al. 2015], a verification tool for distributed systems with a fixed number of processes. QUICKSILVER extends Kinara to support the **Partition** and **Consensus** primitives as well as their global behaviors. QUICKSILVER translates the core fragment of MERCURY into the input representation accepted by the extended version of Kinara using the cutoff number of processes, as computed during the analysis step. Permissible safety specifications $\phi_{m,f}(n)$ are encoded in QUICKSILVER as $\text{atmost}(m-1,\{\text{loc: (bExp)}\})$ where the Boolean expression $f$ is such that $\forall s \in \text{states}(f) : s.\text{loc} = \text{loc} \land s.\sigma(bExp) = \text{True}$. For example, the property $\phi_{3,\text{loc=Replica, stored=1}}(n)$ is encoded as $\text{atmost}(2,\{\text{Replica: stored} = 1\})$. The environment process is automatically generated to nondeterministically send/receive all environmental communication actions that the MERCURY process expects. The specifications as well as the environment process are translated to Kinara’s representation similarly. QUICKSILVER reports successful parameterized verification of $\phi_{m,f}(n)$ when a cutoff is determined.

\textsuperscript{11}A virtual machine containing QUICKSILVER is publicly available [QuickSilver 2021].
iff Kinara reports successful verification for the system consisting of the cutoff number of processes.

**User Feedback.** *QUICKSILVER* helps the user obtain a phase-compatible and cutoff-amenable process by providing heuristically ranked suggestions to handle any violation of the phase-compatibility and cutoff-amenable conditions during the analysis step. For instance, the phase-compatibility conditions do not hold if a local state has an acting transition but not its corresponding reacting transition. In this case, *QUICKSILVER* returns the violated condition and suggests adding a handler that corresponds to the reacting transition from that state.

### 5.2 Evaluation

The research questions we tackle in this evaluation are:

- **RQ1** Can interesting agreement-based systems be modeled concisely in *MERCURY*?
- **RQ2** Can interesting agreement-based systems be modeled in the decidable fragment of *MERCURY* with relative ease?
- **RQ3** Can *QUICKSILVER* perform automated parameterized verification of agreement-based systems in *MERCURY* in a reasonable amount of time?
- **RQ4** Do *QUICKSILVER*’s cutoffs enable efficient verification?

In what follows, we first present our *MERCURY* benchmarks and address RQ1 and RQ2. We then analyze the performance of *QUICKSILVER* on these benchmarks and address RQ3 and RQ4. All experiments were performed on an Intel Xeon machine with E5-2690 CPU and 32GB of RAM. We report the mean run time for 10 runs as well as the 95% confidence interval for each benchmark.

**MERCURY Benchmarks.** Our benchmarks are briefly described below.\(^\text{12}\)

1. **Distributed Store** is the illustrative example from Sec. 2.1.
2. **Consortium** is a distributed system where a set of actors wants to reach a decision based on information the actors gather individually. A subset of the actors is elected and trusted with making a decision that is then announced to the rest of the actors. This resembles scenarios where a trade-off between trust and performance is needed (e.g., a consortium blockchain [Amsden et al. 2020; Hyperledger 2021]). The safety property for this system is (1) at most two actors are elected to decide on a value for all processes and (2) that all actors agree on the decided value.
3. **Two-Object Tracker** is a system for collaborative surveillance based on leader election and is inspired by an example in [Chang and Tsai 2016]. Upon detecting an object, a leader is elected to be responsible for monitoring it along with its followers. The system can additionally fork another set of processes to monitor a second object simultaneously. The safety property for Two-Object Tracker is that there can be at most two leaders at a time, and when a second object is spotted, each of the leaders is tracking a distinct object.
4. **Distributed Robot Flocking** is a distributed system where processes follow a common leader as a flock and is inspired by an example in [Canepa and Potop-Butucaru 2007]. Processes can disperse into various locations where they can elect a leader. The leader then issues directions to the rest of the flock. This is especially useful in self-stabilizing systems. The safety property for this system is that the system stabilizes by ensuring there can be at most one leader at a time making direction decisions.
5. **Distributed Lock Service** is a distributed lock service similar to Chubby [Burrows 2006] for coarse-grained synchronization with an elected leader handling clients requests. Clients can interface with this lock service as a file system where they send reads and writes to an elected leader, and have their requests replicated safely on different servers. The leader periodically times out, sends a step down signal to the rest of the servers, and a new round of election is used to pick a

\(^\text{12}\)MERCURY code for benchmarks available at: https://tinyurl.com/m3zx7jxs

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new leader. The safety property for this system is that at most one server is elected as a single point-of-contact for the clients.

6. **Distributed Sensor Network** is a sensor network application that elects a subset of processes, who have sensed an environmental signal, to report to a centralized monitor. In this application, the set of sensors that have detected the environmental signal (and hence need to coordinate) is dynamically built before invoking the agreement protocol. The safety property here is that the environmental signal is reported by no more than two sensors.

7. **Sensor Network with Reset** is a variant of the Distributed Sensor Network benchmark that uses a “reset” signal to resume monitoring for the environmental signal, thereby requiring an unbounded number of rounds of agreement. The safety property is as before.

8. **Small Aircraft Transportation System (SATS) Landing Protocol** is the landing protocol of SATS proposed by NASA [2021]. The goal of SATS is to increase access to small airports without control towers by allowing aircraft to coordinate with each other to operate safely upon entering the airport airspace. For the landing protocol, the aircraft coordinate to choose successive subsets of aircraft to progress to the next phase of landing, until just one aircraft is chosen to land at a time. The desired safety properties for the SATS landing protocol, provided by NASA, are as follows: (1) there are a total of at most four aircraft across the airport vicinity; (2) there are a total of at most two aircraft across each left (right) holding zone of the airport; and (3) there is at most one aircraft that can attempt a final approach (i.e., attempt landing) at a time.

9. **SATS Landing Protocol II** is a version of the SATS landing protocol where aircrafts communicate explicitly to build a participant set when nearing the final approach, and return to a specific holding zone if they miss landing. The safety property is as before.

10. **Mobile Robotics Motion Planning** is a distributed system based on an existing benchmark [Desai et al. 2017] where a set of robots share a workspace with obstacles, and need to coordinate their movements. The robots coordinate to create a motion plan by successively choosing each robot to create a plan while taking into account the previous robots’ plans. The targeted property for this system is collision avoidance; this is achieved by allowing the robots to create their motion plans consecutively one-at-a-time.

11. **Mobile Robotics with Reset** is a variant of the Mobile Robotics Motion Planning benchmark that allows all the robots to return to their initial state upon receiving a signal to do so. The safety property is as before.

12. **Distributed Register** is a data store à la Atomix’s AtomicValue [Atomix 2021] which gives a consistent view of a stored value under concurrent updates. Updates that do not change the stored data in the register are ignored. The safety property for the Distributed Register system is that no two replicas that are about to serve user requests may have different values of the register; hence ensuring the clients have a consistent view of the data.

**RQ1.** Our benchmarks are models of distributed agreement-based systems commonly found in the literature and have all been encoded in MERCURY with relative ease by the authors of this work. Further, as can be seen in Column 2 of Table 1, the corresponding MERCURY process definitions are fairly compact, i.e., within 100 lines of code (LoC). Thus, our answer to RQ1 is Yes.

**RQ2.** We found two factors valuable in addressing RQ2. **Value of User Feedback.** It is not always easy for a system designer to ensure that their initial model of a MERCURY process is phase-compatible. For example, when modeling the Distributed Lock Service benchmark, we made assumptions about the behavior of the system, causing us to omit reacting transitions on some events and, consequently, our initial model was not phase-compatible. However, the feedback provided by QUICKSILVER helped identify the missing transitions that needed to be added.
Table 1. QuickSilver Performance.

| Benchmark                          | LoC | Phases | Cutoff | Time(s)       |
|-----------------------------------|-----|--------|--------|---------------|
| Distributed Store                 | 64  | 2      | 3      | 45.079 ± 0.621|
| Consortium                        | 58  | 9      | 3      | 6.953 ± 0.022 |
| Two-Object Tracker                | 69  | 3      | 3      | 0.641 ± 0.006 |
| Distributed Robot Flocking        | 78  | 7      | 2      | 0.105 ± 0.002 |
| Distributed Lock Service          | 38  | 2      | 2      | 0.059 ± 0.002 |
| Distributed Sensor Network        | 55  | 3      | 3      | 1.041 ± 0.003 |
| Sensor Network with Reset         | 63  | 3      | 3      | 1.662 ± 0.012 |
| SATS Landing Protocol             | 90  | 3      | 5      | 638.393 ± 0.872|
| SATS Landing Protocol II          | 99  | 5      | 5      | 736.417 ± 3.659|
| Mobile Robotics Motion Planning   | 71  | 5      | 2      | 0.114 ± 0.004 |
| Mobile Robotics with Reset        | 83  | 4      | 2      | 0.166 ± 0.003 |
| Distributed Register              | 32  | 1      | 2      | 0.329 ± 0.006 |

**Value of Mercury Core.** Prior decidability results did not encompass all of the benchmarks we evaluate; in particular, those marked with * in Table 1 fall outside prior known fragments. Mercury Core’s extension of decidability results, on the other hand, enables decidable parameterized verification for all of our benchmarks.

Thus, with the help of QuickSilver’s user feedback and the Mercury Core decidable fragment, our answer to RQ2 is Yes.

**RQ3.** In Table 1, for each benchmark we provide the number of phases, the cutoff used for verification, and the mean run time of QuickSilver with its 95% confidence intervals. Notice that the cutoffs computed by QuickSilver for all benchmarks are small (under 6 processes). Overall, QuickSilver performs efficient parameterized verification for all benchmarks, taking less than 2 seconds to verify most benchmarks, and about 12 minutes for the largest benchmark, SATS Landing Protocol II. Thus, our answer to RQ3 is also Yes.

**RQ4.** To examine the contribution of cutoffs in enabling efficient verification, we performed experiments studying the effect of varying the number of processes on the run time of QuickSilver. As expected, increasing the number of processes causes the run time to grow exponentially. For instance, the time to verify the Consortium benchmark jumps from 9 seconds to about 8 minutes when verifying a system with 5 processes instead of the 3-process cutoff. Fortunately, QuickSilver is able to detect small cutoffs to sidestep the exponential growth caused by increasing the number of processes, enabling practical parameterized verification. Thus, our answer to RQ4 is Yes.

**Remark.** QuickSilver additionally reports the number of phases, which correspond to global guards in the Mercury Core, that QuickSilver automatically generates. This shows the value of automation as designing such guards manually is tedious and error-prone.

## 6 CONCLUDING REMARKS

We presented a framework, QuickSilver, for modeling and efficient, automated parameterized verification of agreement-based systems. The framework supports a modular approach to the design and verification of distributed systems in which systems are (i) modeled using sound abstractions of complex distributed components and (ii) verified using model checking-based techniques assuming that the complex components are verified separately, presumably using deductive techniques.
In ongoing work, we focus on extending QUICKSILVER to handle non-blocking communication and network failures using “channels” that can buffer or drop messages, to support infinite variable domains using abstract interpretation, and to help system designers synthesize amenable processes. Eventually, we hope to see this framework generalized by us or our readers to other verified distributed components and richer properties such as liveness. We also hope to see more conversations and verification frameworks, in particular layered ones, that cut across the deductive verification and model checking communities.

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A SYMMETRY FOR PARAMETERIZED CORRECTNESS

Since parameterized systems are designed to work for an arbitrary number of processes, their behaviors should be independent of a specific PID (as such PID may not even exist in every instantiation of the system). As a result, such parameterized systems naturally exhibit many similar global behaviors. In this section, we define the notion of full symmetry, how to check if a system is fully-symmetric, and the effect of full symmetry on verification.

A.1 Symmetry Reduction

Full Symmetry. Let \( \pi : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\} \) be a permutation acting on the set \( I_n \) of process indices. Let \( PG \) denote the set of all permutations over \( I_n \). A permutation of a global state \( q = (s_0, s_1, \ldots, s_n) \) can then be defined as: \( \pi(q) = (\pi(s_0), \pi(s_1), \ldots, \pi(s_n)) \), where \( \pi(s_i) = (\text{loc} \cdot \pi(i), \pi(s_i)) \) for \( s_i = (\text{loc}_i, \sigma_i) \). Note that \( \pi(\sigma_i) \) depends on the type of the local variable being permuted: if it is of type \( I_n \), then \( \pi(\sigma_i) = \sigma_{\pi(i)} \); otherwise \( \pi(\sigma_i) = \sigma_i \).

Definition A.1 ([Emerson and Wahl 2003; Wahl 2007]). A global transition system \( M \) composed of identical processes with index set \( I_n \) is fully-symmetric if its transition relation \( R \) is invariant under permutations in \( PG \): \( \forall \pi \in PG : \pi(R) = R \), where \( \pi(R) = \{(\pi(q_1), \pi(q_2)) : (q_1, q_2) \in R\} \).

MERCURY syntax enforces sufficient syntactic constraints on manipulation of PIDs to ensure full symmetry. Such constraints are similar to these in [Emerson and Wahl 2003, 2005; Wahl 2007] where they prove that limiting predicates over PIDs to equality and disequality checks yields full-symmetry of \( M \).

Verification Advantages of Full Symmetry. Emerson and Sistla [Emerson and Sistla 1996] show that it is possible to exploit the symmetries present in a global transition system \( M \) (of a system with many similar processes) to improve scalability of model checking by constructing a compressed quotient structure \( \overline{M} \) such that \( M \models \phi \iff \overline{M} \models \phi \), where \( \phi \) is any (\( \mathcal{LTL} \)) specification. It follows from their result that \( M \) can be constructed for any \( M \) that is fully-symmetric and can enable symmetry reduction for model checking w.r.t. any \( \mathcal{LTL} \) specification. We refer the interested reader to Emerson and Sistla [Emerson and Sistla 1996] for further details.

A.2 Symmetry of MERCURY Programs

Since we introduced the agreement primitives \texttt{Partition} and \texttt{Consensus}, we need to show that full symmetry is preserved. In a global transition system, \( M_{\texttt{AGREE}} = (Q, q_0, R) \) of a MERCURY program, let \( R_{\texttt{AGREE}} \) denote the set of all agreement transitions in \( R \). Let \( M = (Q, q_0, R \setminus R_{\texttt{AGREE}}) \) denote the global transition system without the agreement transitions of \( M_{\texttt{AGREE}} \).

Lemma A.2. If \( M \) is fully-symmetric, then \( M_{\texttt{AGREE}} \) is fully-symmetric.

Proof. Essentially, we need to show that \( R_{\texttt{AGREE}} \) is also invariant under permutations in \( PG \), i.e.,

\[
\forall \pi \in PG, (q_{\text{start}}, q_{\text{end}}^W) \in R_{\text{AGREE}} : (\pi(q_{\text{start}}), \pi(q_{\text{end}}^W)) \in R_{\text{AGREE}} \iff (\pi(q_{\text{start}}), \pi(q_{\text{end}}^W)) \in R_{\text{AGREE}}
\]

We first examine \texttt{Consensus} transitions. Recall that a \texttt{Consensus} transition is created between \( q_{\text{start}} \) (that encodes the set of participants \texttt{pcpt}, the desired number of values to agree on \( k \), and proposal variable \texttt{pVar}) and a possible \( q_{\text{end}}^W \) (that encodes a winning set of values \( W \in W^* \)). We prove this by contradiction. Assume:

\[
\exists \pi \in PG : (q_{\text{start}}, q_{\text{end}}^W) \in R_{\text{AGREE}} \land (\pi(q_{\text{start}}), \pi(q_{\text{end}}^W)) \notin R_{\text{AGREE}}
\]

13In case the variable was of an enumerated type (e.g., set, array, or record) containing values of type \( I_n \), then the permutation is applied recursively to all elements. If the array has an index type \( I_n \), then we permute the array elements themselves, too.

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Let \( q'_{\text{start}} = \pi(q_{\text{start}}) \). Note that we obtain a new participant set \( \text{pcpt}' = \pi(\text{pcpt}) \). Since constructing \( W' \) is invariant to permutations, we have \( W'' = \pi(W') \). Because \( q'_{\text{start}} \) is also reachable for any \( \pi (M) \) is fully symmetric, we have a Consensus transition \( (q'_{\text{start}}, q'_{\text{end}}) \in R_{\text{AGREE}} \) out of that state for every \( W' \in W'' \). Hence, we have \( (q'_{\text{start}}, q'_{\text{end}}) \in R_{\text{AGREE}} \) for \( W' = \pi(W) \) where \( q'_{\text{end}} = \pi(q'_{\text{end}}) \). So, we have a contradiction, and:

\[
\forall \pi \in PG : (q_{\text{start}}, q_{\text{end}}) \in R_{\text{AGREE}} \iff (\pi(q_{\text{start}}), \pi(q_{\text{end}})) \in R_{\text{AGREE}}
\]

For Partition transitions, the essence of the argument is identical. Since both \( R_{\text{AGREE}} \) is invariant under permutations in \( PG \), we conclude that \( M_{\text{AGREE}} \) is fully-symmetric. \( \square \)

## B THE MERCURY CORE MODEL

In this section, we present, MERCURY CORE, our extension to the GSP model [Jaber et al. 2020]. Both models share the same representations of local and global semantics, but use different conditions to classify systems for which the parameterized verification problem is decidable. We first present the GSP model in App. B.1 and MERCURY CORE in App. B.2.

### B.1 The GSP Model [Jaber et al. 2020]

The GSP model generalizes synchronization-based models including models based on rendezvous and broadcasts. In the GSP model, each global transition synchronizes all processes, where multiple processes act as the senders of the transition, while the remaining processes react as receivers. The model supports two types of transitions: (i) a k-sender transition, which can fire only if the number of processes available to act as senders in the transition is at least \( k \) and is fired with exactly \( k \) processes acting as senders, and, (ii) a k-maximal transition, which can fire only if the number \( m \) of processes available to act as senders is at least one and is fired with \( k \) processes acting as senders if \( m \geq k \), or, with \( m \) processes acting as senders, otherwise. Additionally, each transition can be equipped with a global guard which identifies a subset of the local state space. A transition is enabled whenever it can fire and the local states of all processes are in the transition guard.

#### Processes.

A GSP process is defined as \( P_{\text{GSP}} = \langle A, S, s_0, T \rangle \), where \( A \) is a set of local actions, \( S \) is a finite set of states, \( s_0 \in S \) is the initial state, and \( T \subseteq S \times A \times S \) is the transition relation. The set \( A \) of local actions corresponds to a set \( A \) of global actions. Each global action \( a \in A \) has an arity \( k \) with local send actions \( a_1, \ldots, a_k \) and a local receive action \( a?? \), and is further associated with either a k-sender or a k-maximal global transition as well as a global guard \( G_a \subseteq S \) of the transition.

#### Composition of Processes.

Given a process \( P_{\text{GSP}} \), the parameterized global transition system is defined as \( M_{\text{GSP}}(n) = \langle Q, q_0, R \rangle \) where \( Q \) is the set of states, \( q_0 \) is the initial state and \( R \) is the global transition relation. Thus, a global state \( q \in Q \) is a vector of natural numbers, representing the number of processes that are in any given local state \( s \in S \). The global transition relation \( R \subseteq Q \times A \times S \times Q \) defines how processes synchronize using a k-sender or a k-maximal transition to move between global states. In a global transition based on a global action \( a \in A \) with arity \( k \), each of the local send actions \( a_1, \ldots, a_k \) is taken by at most one process. A transition \( q \xrightarrow{a,G_a} q' \) based on a k-sender action is in \( R \) if (i) the transition is enabled (i.e. all the processes in \( q \) are in the subset of the local state space defined by the transition guard \( G_a \)), and (ii) in \( q \), there are at least \( k \) senders each taking a local sending transition \( s \xrightarrow{a_1!!} s' \). The remaining processes take the local transition \( s \xrightarrow{a??} s' \). The global state \( q' \) is obtained by all processes moving accordingly. Transitions \( q \xrightarrow{a,G_a} q' \) based on a k-maximal action behave similarly, except that at least one (instead of \( k \)) sender is required, and \( q' \) is obtained by maximizing (up to \( k \)) the number of processes that can act as senders. For each global action \( a \), the GSP model defines a synchronization matrix.
that describes how receivers move and a (set of) vector-pairs \((v_a, v_a')\) that describe how senders move. In particular, \(M_a(s, s') = 1\) if there is a local transition \(s \xrightarrow{a??} s'\), \(v_a\) dictates the expected senders each state \(t \in S\): \(v_a(t) = |\{s \xrightarrow{a??} s' \mid s = t\}|\), and the vector \(v_a'\) describes the number of senders that will be in each state \(t \in S\) after the transition: \(v_a'(t) = |\{s \xrightarrow{a??} s' \mid s = t\}|\). Then, a global state \(q'\) resulting from transition \(q \xrightarrow{a} q'\) can be computed as: \(q' = M_a \cdot (q - v_a) + v_a'\).

**Well-Structured Transition Systems (WSTSs).** Finkel [1987] introduces the WSTSs framework used to prove decidability for reachability problems. A WSTS \(M(n) = (Q, R, \leq)\) is a transition system with a set of infinite states \(Q\), a transition relation \(R\) over \(Q\), and a well-quasi-ordering (WQO) \(\preceq\) over \(Q\) that is compatible with the transition relation \(R\). Compatibility is defined as follows: for every \(q, q', p \in Q\) with \(q \leq p\) and \(q \xrightarrow{a} q'\) there exists \(p' \in Q\) with \(q' \leq p'\) and \(p \xrightarrow{a} p'\).

**Upwards-Closed Sets.** A possibly infinite subset \(U \subseteq Q\) is called upwards-closed w.r.t. \(\preceq\) if \(q \in U\), then every \(p\) such that \(q \leq p\), \(p \in U\). Every upwards-closed set \(U\) has a finite set of minimal elements w.r.t. \(\preceq\) which are called the basis of \(U\).

**Effective Computability of Predecessors.** For \(U \subseteq Q\), let \(\text{Pred}(U)\) denote the predecessor states of \(U\) with respect to \(R\). We say \(\text{Pred}(U)\) is effectively computable if there exists an algorithm that computes a finite basis of \(\text{Pred}(U)\) from any finite basis of any upwards-closed \(U\).

The following theorem states the decidability result of WSTSs.

**Theorem B.1** ([Finkel and Schnoebelen 2001]). In a WSTS with effectively computable \(\text{Pred}\), reachability of any upwards-closed set is decidable.

**Parameterized Verification in the GSP Model.** Jaber et al. [2020] define a set of well-behavedness conditions over the process definition \(P_{\text{GSP}}\) as well as a WQO over the global states to ensure decidability of parameterized verification; the decidability result itself is based on a reduction to WSTSs [Finkel 1987]. Jaber et al. [2020] uses the following WQO:

\[
q \leq p \iff (q \leq p \land \forall G \in \mathcal{G} : (\text{supp}(q) \subseteq G \iff \text{supp}(p) \subseteq G)),
\]

where

1. \(\leq\) is the component-wise order on global state vectors \(q, p\): \(q \leq p \iff q(s) \leq p(s)\) for all \(s \in S\),
2. \(\mathcal{G}\) is the set of all guards in the system, and,
3. \(\text{supp}(q) = \{s \in S \mid q(s) > 0\}\) is the support of a global state \(q\), i.e., the set of local states that appear at least once in \(q\).

Intuitively, \(p\) is greater than \(q\) if \(p\) has at least as many processes as \(q\) in any given state, and for every transition \(q \xrightarrow{a} q'\) that is enabled in \(q\), a transition on action \(a\) is also enabled in \(p\).

An example of a well-behavedness condition that Jaber et al. [2020] impose on the process definition is the following. For a \(k\)-sender action \(a\) with local sending transitions \(s_i \xrightarrow{a_{i,G}} s_i'\) for \(i \in \{1, \ldots, k\}\), let \(s'_a\) be the set of all states \(s_i\), \(s'_a\) the set of states \(s'_i\) and \(M_a\) the synchronization matrix. We say that action \(a\) is strongly guard-compatible if the following holds for all \(G' \in \mathcal{G}\):

\[
\tilde{s}'_a \subseteq G' \Rightarrow \forall s \in G: M_a(s) \in G'
\]

and weakly guard-compatible if the following condition holds:

\[
\tilde{s}'_a \subseteq G' \Rightarrow \forall s \in G: (M_a(s) \in G' \land \exists s' \in S : (s' \neq \tilde{s}'_a \land M_a(s) \sim s')).
\]

Jaber et al. [2020] present additional conditions for phase-compatibility of \(k\)-maximal transitions, which we omit here for brevity.
B.2 Mercury Core

In this section we present a novel firability-aware WQO which we define over the global states. We start with some definitions. We define a relation assoc to connect actions to their relevant guards: \( \text{assoc}(a, G) \iff \exists s : a \in G \). Additionally, let \( \mathcal{AG} \) be the set of all associated actions and guards: \( \mathcal{AG} = \{ (a, G) \in \mathcal{A} \times \mathcal{G} \mid \text{assoc}(a, G) \} \). Moreover, we define the following function to capture when an action \( a \) can fire from global state \( q \) using the send-vector \( v_a \): \( \text{canFire}(q, v_a) \iff \forall s \in S. q[s] \geq v_a[s] \). For simplicity, we also lift this existentially to actions instead of their associated send-vectors: \( \text{canFire}(q, a) \iff \exists v_a. \text{canFire}(q, v_a) \).

We define a function below to capture the set of actions which are enabled and can fire in a global state. We say that these action-guard pairs are "ready" in the global state \( q \).

\[
\text{ready}(q) = \{ (a, G) \in \mathcal{AG} \mid \text{canFire}(q, a) \land \text{supp}(q) \subseteq G \}
\]

Using this function, we can define the firability-aware WQO\(^{14} \) as follows:

\[
q \preceq p \iff (q \preceq p \land \text{ready}(q) \subseteq \text{ready}(p))
\]

Intuitively, we say that \( q \preceq p \), \( p \) has as many processes in every local state in \( q \), and if every enabled firable action in \( q \) is enabled and firable in \( p \).

With firability-awareness in mind, we relax condition (C1) as follows. Let \( M_a(G) := \{ M_a(s) \mid s \in G \} \), i.e., the set of all states where the receivers of \( a \) that started in \( G \) transition to. Then, condition (C1) becomes:

\[
\forall (a1, G1), (a2, G2) \in \mathcal{AG} : \left( (s_{a1}' \subseteq G2) \land (\{ (s_{a1}' \cup M_{a1}(G1)) \cap s_{a2}' \} \neq \emptyset) \right) \Rightarrow \forall s \in G1 : M_{a1}(s) \in G2
\]

In a nutshell, the state of guard \( G2 \) after \( a1 \) happens is irrelevant if all the processes performing \( a1 \) move outside of the sending set of \( a2 \) as, at that point, \( a2 \) is not firable anyway. Similarly, Condition (C1w) can be relaxed as follows:

\[
\forall (a1, G1), (a2, G2) \in \mathcal{AG} : \left( (s_{a1}' \subseteq G2) \land (\{ (s_{a1}' \cup M_{a1}(G1)) \cap s_{a2}' \} \neq \emptyset) \right) \Rightarrow
\]

\[
\{ \forall s \in G1 : (M_{a1}(s) \in G2 \lor \exists s' : (s' < s_{a}' \land M_{a1}(s) \leadsto s')) \}
\]

Other conditions in \cite{Jaber} can be relaxed similarly.

B.3 Parameterized Verification in the Mercury Core

In this section, we show that, under the relaxed guard-compatibility conditions and the new WQO \( \preceq \), we obtain the following theorem. Let \( M_{\infty} \) denote the global transition system composed of an infinite number of \( P_{\text{core}} \) processes.

**Theorem B.2.** If \( M_{\infty} \) is based on a well-behaved Mercury Core process \( P_{\text{core}} \), then \( M_{\infty} \) is a WSTS and we can effectively Compute Pred.

**Proof.** We show compatibility of transitions w.r.t. \( \preceq \), i.e., if \( q \preceq p \) and \( q \rightarrow q' \), the \( \exists p' \) with \( q' \preceq p' \) and \( p \rightarrow^* p' \). We consider the case of \( k \)-sender transitions here. \( k \)-maximal transitions are handled similarly.

---

\(^{14}\)We show that \( \preceq \) is a WQO by proving that every infinite sequence of global states \( q_1, q_2, \ldots \) contains \( q_i, q_j \) with \( i < j \) and \( q_i \preceq q_j \). To this end, consider an arbitrary infinite sequence \( q = q_1, q_2, \ldots \). Then there is at least one set \( A \) of pairs \( (a, G) \) such that infinitely many \( q_i \) have ready\( (q_i') = A \) (since there are infinitely many guards and infinitely many actions). Let \( q' \) be the infinite subsequence of \( q \) where all elements have ready\( (q'_i) = A \). Since \( \preceq \) is a WQO, there exist \( q_i', q_j' \) with \( i < j \) and \( q_i' \preceq q_j' \), and since ready\( (q'_i) = \text{ready}(q'_j) = A \), clearly ready\( (q_i') \subseteq \text{ready}(q'_j) \), so we also get \( q_i' \preceq q_j' \). Since \( q_i' = q_k \) and \( q_j' = q_l \) for some \( k < l \), we get \( q_k \preceq q_l \) for \( k < l \), and thus \( \preceq \) is a WQO.
Suppose \(a1\) is a \(k\)-sender action. Let \(q \xrightarrow{a1,G1} q'\) be a transition and \(q \preceq p\). Since \(q \xrightarrow{a1,G1} q'\), \((a1, G1) \in \text{ready}(q)\), since \(q \preceq p\), this implies that \((a1, G1) \in \text{ready}(p)\), we know that transition \(p \xrightarrow{a1,G1} p''\) is possible and \(p''\) can reach \(p'\) with zero or more internal transitions (i.e., \(p'' \rightarrow^* p'\)), and by the proof of Theorem 2 [Jaber et al. 2020] we know that \(q' \preceq p'\). To prove compatibility with respect to \(\preceq\), it remains to show that \(\forall (a2, G2) \in \mathcal{AG} : (a2, G2) \in \text{ready}(q') \implies (a2, G2) \in \text{ready}(p')\).

First assume that condition (C1r) holds. Then, let \((a2, G2) \in \mathcal{AG}\) be an arbitrary associated action-guard pair. By this condition, we either have

1. \(s'_{a1} \notin G2\), i.e., at least one sender of \(a1\) moves to a state outside of \(G2\), so we are guaranteed that \(G2\) is disabled after \(a1\) fires. In this case, \(\text{supp}(q') \notin G2\), so \((a2, G2) \notin \text{ready}(q')\); therefore \((a2, G2) \in \text{ready}(q') \implies (a2, G2) \in \text{ready}(p')\) is (trivially) satisfied.
2. \((s'_{a1} \cup M_{a1}(G1)) \cap s_{a2} = \emptyset\), i.e., all senders and receivers of \(a1\) move into states with no sending transition on \(a2\). In this case, \(\text{canFire}(q', a2)\) is false, so \((a2, G2) \notin \text{ready}(q')\); therefore \((a2, G2) \in \text{ready}(q') \implies (a2, G2) \in \text{ready}(p')\) is (trivially) satisfied.
3. \(s'_{a1} \subseteq G2 \land \forall s \in G1 : M_{a1}(s) \in G2\), i.e., all potential senders move into \(G2\), and all receivers move into \(G2\) to arrive in a state where all processes are in \(G2\). Additionally, if \((a2, G2) \in \text{ready}(q')\) it must be the case that \(\text{canFire}(q', v_{a2})\) for some \(v_{a2}\), and since we know that \(q' \preceq p'\) then we have that \(\text{canFire}(p', v_{a2})\); therefore \((a2, G2) \in \text{ready}(p')\).

Hence, because \(q' \preceq p'\) and \(\forall (a2, G2) \in \mathcal{AG} : (a2, G2) \in \text{ready}(q') \implies (a2, G2) \in \text{ready}(p')\), we conclude that \(q' \preceq p'\).

In case condition (C1rw) holds, the argument is the same, except that the receivers in case (3) can take multiple internal transitions to reach \(p'\).

Effective computability of \(\text{Pred}\) follows from the proof of Theorem 2 [Jaber et al. 2020]. The only difference is that we must consider the guards, i.e., a predecessor is only valid if it additionally satisfied the guard of the transition under consideration.

**Cutoff Results.** All the cutoff results from the GSP model naturally extend to MERCURY CORE. In essence, the additional special transitions that model crash-stop failures are independent as they never require additional processes to fire. Furthermore, the special crash state \(s_{cr}\) is never on a path to a state that shows in the specifications. Combined with the extended decidable fragment, this entails a larger cutoff-yielding fragment in MERCURY CORE.

## C DECIDABLE PARAMETERIZED VERIFICATION FOR MERCURY PROGRAMS

In this section, we prove that if a process is phase-compatible, then the MPVP is decidable.

**Theorem C.1.** MPVP is decidable is decidable for MERCURY system process \(P\), and permissible safety specification \(\phi(n)\) if:

1. \(P\) is phase-compatible.
2. The state space of \(P\) is fixed and finite.
3. There exists at most one rendezvous-receive transition per action per phase.

For the remainder of this section, we lay out the proof of Theorem C.1. To show that parameterized verification is decidable for phase-compatible programs in MERCURY, we will utilize decidability and cutoff results of MERCURY CORE by showing that for each process \(P_{\text{MERC}}\) that satisfies the above conditions, there exists a corresponding process \(P_{\text{CORE}}\) in the MERCURY CORE such that there exists a simulation equivalence between \(P_{\text{MERC}}\) and \(P_{\text{CORE}}\).

---

15We know that \(\forall s \in S : p'[s] \geq q'[s] \land q'[s] \geq v_{a2}[s]\) and hence, \(\forall s \in S : p'[s] \geq v_{a2}[s]\).
We now present a mapping procedure (Rewrite) in Alg. 1 and show that there exists a simulation equivalence between a rewriteable phase-compatible process $P_{\text{MERC}}$ and the corresponding $P_{\text{CORE}}$. The rewriting procedure consists of a series of rewriting steps in which the semantics of each type of transition of a MERCURY program is converted into (a set of) transitions in MERCURY CORE.

**Rewritable**. This function checks if $P_{\text{MERC}}$ has a fixed and finite local state space, and that the pairwise transitions are used in a way that does not violate full symmetry when the process IDs are abstracted away. Under these conditions, we show that any phase-compatible $P_{\text{MERC}}$ can be mapped to a well-behaved $P_{\text{CORE}}$.

Recall that the set $S$ represents the local state space of $P_{\text{MERC}}$. We start with a processes definition, $P_{\text{CORE}}$, in MERCURY CORE whose local states are $S$. In $M_{\text{MERC}}$, a global state $q_{\text{MERC}} \in Q_{\text{MERC}}$ of the form $S^n$ is a concatenation of the local states of all processes. In the MERCURYCORE, a global state $q_{\text{CORE}} \in Q_{\text{CORE}}$ of the form $[S]^{[S]}$ is a counter representation recording how many processes are in a given local state. Below, we provide an abstraction function that maps the global state space of $M_{\text{MERC}}$ to the global state space of $M_{\text{CORE}}$. We define the function $\alpha : Q_{\text{MERC}} \rightarrow Q_{\text{CORE}}$ as follows:

$$q_{\text{CORE}}(s) = \sum_{s_i \in q_{\text{MERC}}} I(s_i = s),$$

where $I(\text{cond})$ evaluates to 1 if $\text{cond}$ is true and 0 otherwise. The function counts the local states in $q_{\text{MERC}}$ and encodes that into the counter representation in $q_{\text{CORE}}$.

**RewriteBroadcastTransitions**. Consider an arbitrary broadcast action act. For each broadcast send transition $s \xrightarrow{\text{sendbr}(\text{act})} s'$ in $P_{\text{MERC}}$, we create a 1-sender transition $s \xrightarrow{\text{act}!!\text{src}_\text{act}} s'$ in $P_{\text{CORE}}$, and for each broadcast receive transition $s \xrightarrow{\text{recvbr}(\text{act})} s'$ in $P_{\text{CORE}}$, we create a receiving transition $s \xrightarrow{\text{act}??\text{src}_\text{act}} s'$ in $P_{\text{CORE}}$. Recall that $\text{src}_\text{act}$ is the source set of action act. We add the guard $\text{src}_\text{act}$ to ensure that the broadcast primitives have similar semantics. Hence, it is not hard to see that the following correspondence holds:

**Lemma C.2.** Let act be a broadcast action, and let $G$ be a guard for act obtained by the RewriteBroadcastTransitions procedure described above. Then, $\forall q_1, q_2 \in Q_{\text{MERC}} : q_1 \xrightarrow{\text{act}} q_2 \in M_{\text{MERC}} \iff \alpha(q_1) \xrightarrow{\text{act}, G} \alpha(q_2) \in M_{\text{CORE}}$.

**RewriteRendAndInternalTransitions**. Internal and rendezvous transitions can be supported using 1- and 2-sender actions, respectively. We replace each internal transition $s \xrightarrow{\text{phase}(s)} s'$ in $P_{\text{CORE}}$ with a 1-sender transition $s \xrightarrow{\text{sendrz}(\text{act})} s'$ and $s \xrightarrow{\text{act}!!\text{phase}(s)} s'$ and with $s \xrightarrow{\text{act}??\text{phase}(s)} s'$ in $P_{\text{CORE}}$. While rendezvous and internal transitions do not coordinate with other processes, transitions in MERCURY CORE are assumed to...

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**Algorithm 1: Procedure for rewriting a process $P_{\text{MERC}}$ to a process $P_{\text{CORE}}$.**

```plaintext
1 procedure Rewrite($P_{\text{MERC}}$) 
2 Input : $P_{\text{MERC}}$, a phase-compatible MERCURY process 
3 Output : $P_{\text{CORE}}$, a process in MERCURY CORE 
4 if Rewritable($P_{\text{MERC}}$) then 
5 \hspace{1cm} $P_{\text{CORE}} = \text{RewriteBroadcastTransitions}($$P_{\text{MERC}}$$) 
6 \hspace{1cm} $P_{\text{CORE}} = \text{RewriteRendAndInternalTransitions}($$P_{\text{CORE}}$$) 
7 \hspace{1cm} $P_{\text{CORE}} = \text{RewritePartitionTransitions}($$P_{\text{CORE}}$$) 
8 \hspace{1cm} $P_{\text{CORE}} = \text{RewriteConsensusTransitions}($$P_{\text{CORE}}$$) 
9 \hspace{1cm} $P_{\text{CORE}} = \text{RewriteCrashTransitions}($$P_{\text{CORE}}$$) 
10 return $P_{\text{CORE}}$
```

---

**C.0.1 Simulation Equivalence.** We now present a mapping procedure (Rewrite) in Alg. 1 and show that there exists a simulation equivalence between a rewriteable phase-compatible process $P_{\text{MERC}}$ and the corresponding $P_{\text{CORE}}$. The rewriting procedure consists of a series of rewriting steps in which the semantics of each type of transition of a MERCURY program is converted into (a set of) transitions in MERCURY CORE.

**Rewritable**. This function checks if $P_{\text{MERC}}$ has a fixed and finite local state space, and that the pairwise transitions are used in a way that does not violate full symmetry when the process IDs are abstracted away. Under these conditions, we show that any phase-compatible $P_{\text{MERC}}$ can be mapped to a well-behaved $P_{\text{CORE}}$.
to synchronize with all processes. As such, we create guards on these transitions based on phases, as we can be sure that if a process is in a phase to take such a transition, no process will be outside the phase. No feasible behaviors are removed. Based on the simple translation from internal and rendezvous messages given above, the following correspondence between the global transitions based on those primitive holds:

$$\forall q_1, q_2 \in Q_{MERC} : q_1 \xrightarrow{\text{act}} q_2 \in M_{MERC} \implies \alpha(q_1) \xrightarrow{\text{act}} \alpha(q_2) \in M_{Core}.$$  (1)

where, act is a rendezvous or an internal action. Note that, for rendezvous transitions, we still need to show that the other direction holds. Since the MERCURY Core does not support process indices, a global transition based on rendezvous communications in MERCURY programs between, say, process $p_1$ in state $s_1$ and process $p_2$ in state $s_2$ would correspond to a 2-sender transition involving any process in $s_2$ and any process in $s_2$. However, because $P_{MERC}$ is phase-compatible, the PID-based communications are equivalent to the communication actions without PIDs in the MERCURY Core.

**Rendezvous Transitions under Full Symmetry.** Recall that MERCURY syntax imposes syntactic constraints on how expressions of type $I_n$ can be used. Essentially, the only expressions allowed are equality checks. These constraints ensure that the send statement $sendrz(\text{act}, (optIntVar),(idExp))$ (corresponding to the local send transition $(\elloc, \sigma) \xrightarrow{sendrz(\text{act},i)} (\elloc', \sigma')$ for some $i \in I_n$, then there exists a transition $(\elloc, \sigma[i \leftarrow j]) \xrightarrow{sendrz(\text{act},j)} (\elloc', \sigma'[i \leftarrow j])$ for all $j \neq i \in I_n$. Hence, if a global transition $q_1 \xrightarrow{\text{act}} q_2$ based on a rendezvous action $\text{act}$ between processes $P_i$ and $P_j$ exists in $M_{MERC}$, then $\forall \pi \in PG. \pi(q_1) \xrightarrow{\pi(q_2)}$ between processes $P_{\pi(i)}$ and $P_{\pi(j)}$ also exists in $M_{MERC}$.

**Lemma C.3.** Let $\text{act}$ be an rendezvous action and $P_{MERC}$ be a phase-compatible process. Then, $\forall q_1, q_2 \in Q_{MERC} : \alpha(q_1) \xrightarrow{\text{act}} \alpha(q_2) \in M_{Core} \implies q_1 \xrightarrow{\text{act}} q_2 \in M_{MERC}$.

**Proof.** Since permuting a global state of MERCURY programs does not change the local state of a process but just its PID, we know that: $\forall q_1, q_2 \in Q_{MERC}$ if $q_1 = \pi(q_2)$ for some $\pi \in PG$ then $\alpha(q_1) = \alpha(q_2)$ (since the abstraction function $\alpha$ only captures the local state space, but not the indices). Since $P_{MERC}$ is rewritable, we know that there exists at most one rendezvous-receive transition per action per phase. Under full symmetry, this condition ensures that abstracting the receiver PID (in a rendezvous-send transition $s \xrightarrow{sendrz(\text{act},\text{PID})} s'$) does not introduce spurious behaviors. It then follows that: $\forall q_1, q_2 \in Q_{MERC} : \alpha(q_1) \xrightarrow{\text{act}} \alpha(q_2) \in M_{Core} \implies q_1 \xrightarrow{\text{act}} q_2 \in M_{MERC}$.

By Lemma C.3 and property (1) above, we obtain the following result:

**Lemma C.4.** Let $\text{act}$ be an rendezvous action and $P_{MERC}$ be a phase-compatible process. Then, $\forall q_1, q_2 \in Q_{MERC} : \alpha(q_1) \xrightarrow{\text{act}} \alpha(q_2) \in M_{Core} \iff q_1 \xrightarrow{\text{act}} q_2 \in M_{MERC}$.

**RewriteConsensusTransitions.** We now show that $k$-sender actions in MERCURY Core can be used to model Consensus agreement. Consider a Consensus instance $\text{cons}$ in $P_{MERC}$. Let $p\text{Var}$ have the domain $V$. Let $W = \{w_1, \ldots, w_m\}$ be the set of all sets constructed from elements in $V$ with size between 1 and $k$ (inclusive). Also, for each $s_j \in src_{\text{cons}}$, let $f_{i,j} \in S$ denote the local state into which each participant in $s_j$ transitions after $l$ ends, for a given set of winning values $w_j \in W$. For each such $w_j$, create a $k$-sender $a_j$ action as follows:

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- create a transition $s_i \xrightarrow{\text{local-send} \_\text{state}} f_i \_j$ in $P_{\text{CORE}}$, and,
- create for each element $e \in w_i$ a new local-send action $a\_j\_e$, and for each $s_i$ such that $s_i.\sigma(v) = e$, create a transition $s_i \xrightarrow{\text{local-send} \_\text{state}} f_i \_j$ to $P_{\text{CORE}}$.

**Rewrite Partition Transitions** We now show how $k$-maximal actions in MERCURY Core are used to model Partition agreement. Consider a Partition instance $p$ in $P_{\text{MERC}}$. For the win transition $s \xrightarrow{\text{win}; \text{PC}_p (\text{pcpt,} k)} s' \in P_{\text{MERC}}$, we create the transitions $s \xrightarrow{a\_j\_e, \text{PC}_p (\text{pcpt,} k)} s' \forall i \in \{1 \ldots k\}$ in $P_{\text{CORE}}$, where $a$ is a $k$-maximal action. For each $s \xrightarrow{a\_j\_e, \text{PC}_p (\text{pcpt,} k)} s' \in P_{\text{MERC}}$ we create the receive transition $s \xrightarrow{\text{local-send} \_\text{state}} s' \in P_{\text{CORE}}$.

We show that the Rewrite Partition Transitions and Rewrite Consensus Transitions procedures described above yield an equivalence between agreement transitions and the corresponding $k$-sender and $k$-maximal transitions used to model them.

**Lemma C.5.** Let $q_1 \rightarrow q_2$ be an Partition agreement transition with cardinality $k$, $b$ be a $k$-maximal action, and $G$ be a global guard built as described by the Rewrite Partition Transitions in Algo. 1. Then, $\forall q_1, q_2 \in Q_{\text{MERC}} : q_1 \rightarrow q_2 \in M_{\text{MERC}} \iff \alpha(q_1) \xrightarrow{b, G} \alpha(q_2) \in M_{\text{Core}}$

**Proof.** The semantics of the agreement transition in MERCURY is to pick up to a total of $k$ winning participating processes from multiple possible start states $\{c_1, \ldots, c_m\}$ and switch their states to the corresponding winning states $\{w_1, \ldots, w_m\}$ while all the other participating processes move to the corresponding losing states $\{l_1, \ldots, l_m\}$.

A $k$-maximal transition in MERCURY Core behaves similar to a agreement transition by moving $k$ processes from their respective $c_i$ states to a winning state $w_i$ and all the other processes to a losing state $l_i$. Since we create a $k$-maximal transition with the same number of winning send transitions for each starting start $c_i$, winning processes may be arbitrarily distributed among the starting states, similar to the corresponding MERCURY transition. In both models, the rest of the processes move to the losing states.

Hence, the semantics of $k$-maximal transitions enforces the Consistent Winners post condition of the corresponding agreement transition. The guard $G$ ensures that the broadcast is only enabled when all the processes participating in the agreement round are in the right states, hence, ensuring the Consistent Participants precondition. Since the guard $G$ is a set of local states, it is invariant to the abstraction function $\alpha$ (i.e. $G$ is enabled in $q$ iff it is enabled in $\alpha(q)$).

**Lemma C.6.** Let $q_1 \rightarrow q_2$ be a Consensus transition with cardinality $k$, $b$ be a $k$-sender transition, and $G$ be a global guard built as described by the Rewrite Consensus Transitions in Algo. 1. Then, $\forall q_1, q_2 \in Q_{\text{MERC}} : q_1 \rightarrow q_2 \in M_{\text{MERC}} \iff \alpha(q_1) \xrightarrow{b, G} \alpha(q_2) \in M_{\text{Core}}$

**Proof.** The semantics of the agreement transition in MERCURY is to pick up to $k$ winning values proposed by the participating processes starting from multiple possible start states $\{c_1, \ldots, c_m\}$ and switch the processes’ states to the corresponding next states where agreement is reached on a set of $1$ to $k$ values.

By construction, each Consensus transition is simulated by at least one $k$-sender transition that guarantees agreement on a given set of winning values. The semantics of the generated $k$-sender transition ensures that it is only enabled when there exists at least one participant proposing each value of the set of winning values. This is achieved by placing the local-send transitions from each state $s$ with $s.\sigma(v) \in w$, where $w$ is the winning set. As in Lemma C.5, the guard $G$ ensures that the transition is only enabled in a state satisfying Consistent Participants precondition.

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**RewriteCrashTransitions.** Since process crashes are nondeterministic, they behave like internal transitions, and are rewritten similarly.

**Simulation Equivalence of \(M_{\text{MERC}}\) and \(M_{\text{Core}}\).** Based on Lemmas C.2 - C.6, we obtain the following theorem:

**Theorem C.7.** Given a rewritable phase-compatible \(P_{\text{MERC}}\),
\[
P_{\text{Core}} = \text{Rewrite}(P_{\text{MERC}}),
\]
the respective global transition systems \(M_{\text{MERC}}(n), M_{\text{Core}}(n)\), and permissible specification \(\phi\), we have:
\[
\forall n. M_{\text{Core}}(n) \models \phi \iff M_{\text{MERC}}(n) \models \phi.
\]

We now show that, for any rewritable phase-compatible process \(P_{\text{MERC}}\), the parameterized verification of \(P_{\text{MERC}}\) for a permissible specification \(\phi_{m,f}(n)\) is decidable. We show that \(P_{\text{Core}} = \text{Rewrite}(P_{\text{MERC}})\) falls within the decidable fragment of the \(M_{\text{Core}}\).

**Sufficient Well-Behavedness Condition.** First, we define a condition over \(M_{\text{MERC}}\) processes which generalizes the well-behavedness conditions for \(M_{\text{Core}}\). For a \(k\)-sender or \(k\-maximal\) action \(a\), let \(\hat{s}\) be the set of states \(s_i\) such that \(s_i \xrightarrow{a_1!G} s'_i, 1 \leq i \leq k\), are the local sending transitions, \(\hat{s}'\) the set of all states \(s'_i\), and \(M_1(s_k)\) denote the state \(s_i\) that \(s_k\) is mapped to in a transition \(a_1!G\). We say that action \(a\) is *weakly guard-compatible* if the following holds for all \(G' \in \mathcal{G}\), the set of all guards in the system:

\[
\forall (a_1, G_1), (a_2, G_2) \in \mathcal{AG} (\hat{s}_a' \cap G_2 = \emptyset) \lor \\
((\hat{s}_a' \cup M_1(G_1)) \cap \hat{s}_a = \emptyset) \lor (\hat{s}_a' \not\subseteq G_2) \lor (\forall s \in G_1. \exists t \in G_2. M_a(s) \leadsto t) \quad \text{(wellBehaved)}
\]

where \(M_a(s) \leadsto t\) represents that the receiver must have an unguarded path of internal transitions to the state \(t\) from \(M_a(s)\). Informally, this condition requires that all senders of a transition must end outside of a guard together, all process performing \(a_1\) end outside of the send set of \(a_2\), or all processes must end inside the guard. We point out that this condition implies all of the well-behavedness conditions of \(M_{\text{Core}}\).

**Claim:**
\[
\forall P_{\text{MERC}}. \text{phase-compatible}(P_{\text{MERC}}) \rightarrow \text{wellBehaved}(P_{\text{Core}}), \text{ where } P_{\text{Core}} = \text{Rewrite}(P_{\text{MERC}}).
\]

**Proof:** This will be a proof by contradiction. Assumption: \(\exists P_{\text{MERC}}. \text{phase-compatible}(P_{\text{MERC}}) \land \neg\text{wellBehaved}(P_{\text{Core}})\). More specifically, the following must be true for \(P_{\text{Core}} = \text{Rewrite}(P_{\text{MERC}})\).

\[
\exists (a_1, G_1), (a_2, G_2) \in \mathcal{AG} (\hat{s}_{a_1}' \cap G_2 \neq \emptyset) \land \\
((\hat{s}_{a_1}' \cup M_1(G_1)) \cap \hat{s}_{a_2} \neq \emptyset) \land (\hat{s}_{a_1}' \not\subseteq G_2) \lor (\exists s \in G_1. \neg(\exists t \in G_2. M_a(s) \leadsto t)) \quad (2)
\]

That is, for some guard \(G_2\) and action \(a_1\), some acting transition on \(a_1\) ends in \(G_2\), some process may perform \(a_1\) and end in a state with an acting transition on \(a_2\), and either an acting transition on \(a_1\) ends outside of \(G_2\), or a receiver cannot reach \(G_2\).

**Globally-Synchronizing Actions.** Consider the case in which \(a_1\) is a globally-synchronizing action.

In the scenario where some acting transition ends outside \(G_2\), we examine the two possibilities under which the guard \(G_2\) may have been created.

1. If \(G_2\) is \(src_{a_2}\) for some \(a_2 \in E_{\text{global}}\), we have a contradiction, because phase-compatible \(P_{\text{MERC}}\) ensures that if an acting transition of a globally-synchronizing action (e.g., \(a_1\)) ends in a state with an outgoing reacting action (e.g., \(a_2\)), and \(a_2\) is initiable in \(dst_{a_1}\), then all other acting transitions of \(e\) must end in a state with an outgoing receive action \(a_2\), which is in \(src_{a_2}\).
(2) If $G_2$ is the guard of an internal or pairwise transition, $G_2$ is a union of phases. Since any two states in any destination set of a globally-synchronizing action always appear together in phases, we have a contradiction with the assumption that one of them ends outside of $G_2$. We proved that acting transitions must end in $G_2$ if any of them does. Now, consider the scenario where the acting transitions end inside $G_2$ but some reacting transition cannot reach $G_2$ (via some path of unguarded internal transitions). We again examine the two creation scenarios of $G_2$.

1. If $G_2$ is $\text{src}_{a_2}$ for some $a_2 \in E_{\text{global}}$, we have a contradiction since our restriction on reacting actions imposed by phase-compatible($P_{\text{MERC}}$) ensures that because $a_2$ is initiable in $\text{dst}_{a_1}$, then the reacting transition’s destination state must be able to reach a source state of $a_2$.

2. If $G_2$ is the guard of an internal or pairwise transition, $G_2$ is a union of phases. Since any two states in any destination set of a globally-synchronizing action always appear together in phases, we have a contradiction with the assumption that one receiver ends outside of $G_2$.

Thus, globally-synchronizing actions in $P_{\text{MERC}}$ map to guard-compatible actions in $P_{\text{CORE}}$.

**Internal.** Consider the case in which $a_1$ is an internal action. This is a transition in $P_{\text{CORE}}$ with only one sending transition, so there must be some self-looping receive transition which cannot reach $G_2$. This requires that, for $G_1$ (the guard on the internal transition), $G_1 \neq G_2$. We consider the two possibilities under which the guard $G_2$ may have been created.

1. If $G_2$ is $\text{src}_{a_2}$ for some $a_2 \in E_{\text{global}}$, then the state of the self-looping receive transition (which is in $G_1$) must not have a path of internal transitions to reach a source state of $a_2$. However, our phase-compatible($P_{\text{MERC}}$) conditions on internal actions state that, if $a_2$ is initiable in $G_1$, then every state in $G_1$ has a path to a state with a receive of $a_2$, so we have a contraction.

2. If $G_2$ is the guard of an internal or pairwise transition (i.e. some phase), then the state of the self-looping receive transition (a state in $G_1$, which is also a phase) must not have a path of internal transitions to reach $G_2$. However, the sender begins in $G_1$ and ends in $G_2$, so by our definition of phases, $G_1$ and $G_2$ would be merged, so $G_1 = G_2$, and no such self-looping transition can exist. So, we have a contradiction.

Thus, internal actions in $P_{\text{MERC}}$ map to guard-compatible actions in $P_{\text{CORE}}$.

**Rendezvous.** Consider the case in which $a_1$ is a rendezvous action. There are two senders, so one sender must end in some guard $G_2$ and either the other sender must end outside of $G_2$, or some receiver must not be able to reach $G_2$. Considering the scenario where some sender (i.e. the rendezvous sender or receiver) ends outside $G_2$, we consider the two creation scenarios of $G_2$.

1. If $G_2$ is $\text{src}_{a_2}$ for some $a_2 \in E_{\text{global}}$, we have a contradiction, because phase-compatible($P_{\text{MERC}}$) ensures that if $a_2$ is initiable in $G_1$ and either the rendezvous sender or receiver ends in a state where there is a globally-synchronizing reacting transition on $a_2$, then the other must also be able to reach it, which contradicts our assumption.

2. If $G_2$ is the guard of an internal or pairwise transition, we also have a contradiction, because all source and destination states of a rendezvous action will be in the same phase.

Now that we know the senders of the 2-sender action must both end in $G_2$ if either of them does, consider the scenario where some self-looping receiver ends outside $G_2$. We consider the two creation scenarios of $G_2$.

1. If $G_2$ is $\text{src}_{a_2}$ for some $a_2 \in E_{\text{global}}$, then the state of the self-looping receive transition (which is in $G_1$) must not have a path of internal transitions to reach a source state of $a_2$. However, phase-compatible($P_{\text{MERC}}$) ensures that, because $a_2$ is initiable in $G_1$, this is not the case, so we have a contradiction.

2. If $G_2$ is the guard of an internal or pairwise transition (i.e. some phase), then the assumption requires that the state of the self-looping receive transition (which is in $G_1$) must not have
Thus, rendezvous actions in $P_{MERC}$ map to guard-compatible actions in $P_{CORE}$. Thus, we arrive at a contradiction, because all paths from $s$ corresponding $P_{MERC}$ is well-behaved, contradicting the assumption. □

D CUTOFFS FOR EFFICIENT PARAMETERIZED VERIFICATION

We show that, for any cutoff-amenable process $P_{MERC}$ and permissible specification $\phi_{f,m}(n)$ has a cutoff $c = m$. We show that when any such cutoff-amenable process $P_{MERC}$ is mapped to $P_{GSP} = \text{REWRITE}(P_{MERC})$ that is guaranteed to have the cutoff $c = m$.

The cutoff-amenability conditions (ref. Sec. 4.2) assume permissible specifications of the form $\phi_{m,f}(n)$. For simplicity, we assume the existence of a state $s$ such that each local state in $st(f)$ has an internal transition to $s$. We write $\text{Cutoff}_{MERC}(P_{MERC})$ to denote $P_{MERC}$ satisfies the cutoff amenability conditions and $\text{Cutoff}_{CORE}(P_{CORE})$ to denote that the cutoff conditions in MERCURY CORE hold for process $P_{CORE}$. A transition of a MERCURY CORE process $P_{CORE}$ is free if it is (i) an internal transition, (ii) a sending transition of either a broadcast (i.e., a 1-sender action) or a $k$-maximal action, or (iii) a receiving transition $s \xrightarrow{a?G} s'$ of a broadcast with matching sending transition $s \xrightarrow{a!G}$.

A path from one state to another is free if all transitions on the path are free.

We now show that if a process $P_{MERC}$ satisfies the cutoff-amenability conditions, then the corresponding $P_{CORE}$ satisfies $\text{Cutoff}_{CORE}$, and thus $P_{MERC}$ has cutoff $c = m$.

Claim: $\text{Cutoff}_{MERC}(P_{MERC}) \rightarrow \text{Cutoff}_{CORE}(P_{CORE})$, where $P_{CORE} = \text{REWRITE}(P_{MERC})$.

Proof: This will be a proof by contradiction, assume: $\text{Cutoff}_{MERC}(P_{MERC}) \land \neg \text{Cutoff}_{CORE}(P_{CORE})$.

In order for $\text{Cutoff}_{CORE}(P_{CORE})$ to not hold,

1. there must exist a path from $s_0$ to $s$ which is not free, and,
2. there must exist a transition $s_s \rightarrow s_d$ starting on a free path ($s_s \in \mathcal{F}_{CORE}$) and leaves it ($s_d \notin \mathcal{F}_{CORE}$) and either
   a. there are no paths out of $s_d$,
   b. some path from $s_d$ that does not lead back to $s_s$, or
   c. some path from $s_d$ leads back to $s_s$ and is not free between $s_d$ and $s_s$.

In case (1) where all paths from $s_0$ to $s$ are independent, there must be some independent transition in $P_{MERC}$ which maps to a transition which is not free. This transition must be an acting Consensus transition, because winning Partition, broadcast send, and internal transitions all map directly to free transitions. In the case where the cardinality $k$ is 1, such a transition maps to a broadcast send transition $t$, as well as a corresponding receive $u$, which are free transitions because they form a negotiation. The transition also maps to other broadcast receive transitions corresponding to sends from other states where the decided value is the same as that of $t$. Since taking any of these receive transitions is semantically indistinguishable from taking the $u$ transition, we consider these additional receives to be part of the negotiation and thus are equally free. As such, all independent transitions map to free transitions, so all paths in $\mathcal{F}_{CORE}$ are free if all paths in $\mathcal{F}_{MERC}$ are independent. Thus, we arrive at a contradiction, because all paths from $s_0$ to $s$ are independent, so there cannot be a path from $s_0$ to $s$ which is not free.

If the second cutoff-amenability condition holds, we get a contradiction as follows.

Some transition $s_s \rightarrow s_d$ that starts on a free path ($s_d \in \mathcal{F}_{CORE}$) and leaves it ($s_d \notin \mathcal{F}_{CORE}$) must be the result of rewriting some transition in $P_{MERC}$. According to our cutoff-amenability conditions, all paths out of $s_d$ must lead back to $s_s$ and be independent between $s_d$ and $s_s$, and such a path
must exist. Since every state in $\mathcal{F}_{\text{Mercury}}$ is a state in $\mathcal{F}_{\text{Core}}$ (ref. Lemma D.1), then $s_i$ is in $\mathcal{F}_{\text{Core}}$ and the path from $s_d$ to $s_i$ is free, so we have a contradiction.

Since we arrive at a contradiction on all proof branches, if our cutoff-amenability conditions hold on $P_{\text{Mercury}}$, then the cutoff conditions of Mercury Core hold on $P_{\text{Core}}$.

While the definitions of $\mathcal{F}_{\text{Mercury}}$ and $\mathcal{F}_{\text{Core}}$ differ, it can be shown that the states they include are the same. Note that the local state spaces of $P_{\text{Mercury}}$ and $P_{\text{Core}}$ are the same. Let $\text{states}(\mathcal{F}) = \{ s \in p \mid p \in \mathcal{F} \}$ be the set of states of all the paths in $\mathcal{F}$.

**Lemma D.1.** For any $P_{\text{Mercury}}$ such that $P_{\text{Core}} = \text{REWRITE}(P_{\text{Mercury}})$, $\text{states}(\mathcal{F}_{\text{Mercury}}) = \text{states}(\mathcal{F}_{\text{Core}})$.

**Proof.** For any path $p \in \mathcal{F}_{\text{Mercury}}$, if there are no Consensus transitions in $p$, then all transitions in $p$ are independent and map to free transitions in $P_{\text{Core}}$. Hence, all states in $p$ will be in $\mathcal{F}_{\text{Core}}$.

Consider a scenario where two sets of free paths $b$ and $a$ are connected by a value-consensus transition $t_v$. Each path $b_i \in b$ begins at a state $\text{first}(b_i)$ and ends at a state $\text{last}(b_i)$, which is a source state of $t_v$. Each path $a_j \in a$ begins at a destination state of $t_v$, $\text{first}(a_j)$, and ends at a state $\text{last}(a_j)$.

After rewriting, every source state $\text{last}(b_i)$ of $t_v$ has a 1-sender broadcast-send transition from $\text{last}(b_i)$ to some $\text{first}(a_j)$, and every destination state $\text{first}(a_j)$ of $t_v$ is reached by 1-sender broadcast-send transition from some $\text{last}(b_i)$. Therefore, every $\text{first}(b_i)$ has a free path to some $\text{last}(a_j)$ through the 1-sender broadcast-send from $\text{last}(b_i)$ to $\text{first}(a_j)$, and every source and destination state of $t_v$ is on at least one of these free paths.

The paths in $\mathcal{F}_{\text{Mercury}}$ can be viewed as a series of sets of free paths separated by value-consensus transitions. So, by applying the above logic repeatedly, we can see that every source and destination state of every value-consensus transition in $\mathcal{F}_{\text{Mercury}}$ is, in fact, part of a free path in $\mathcal{F}_{\text{Core}}$. 

**D.1 Reachability of Combinations of States.**

We handle conjunction and disjunction of the permissible specification $\phi_{m,f}(n)$ as follows. Consider the correctness property of the form:

$$\phi_{m_1,f_1}(n) \land \ldots \land \phi_{m_j,f_j}(n)$$

for some finite $j$. Then, we compute the cutoff $m_j$ independently for each $j$, and use the maximum cutoff value $m = \max(m_1, \ldots, m_j)$ to check the correctness of the system. A system that satisfies this property is one that satisfies all conjuncts. Subsequently, a global error state for this property is one that violates any conjunct, so one can check each conjunct individually with the corresponding $m_j$. Hence, it is sufficient to check the correctness of the system with the maximum cutoff since that system has a sufficient number of processes to produce any reachable error states in a system with fewer than $m$ processes.

Next, consider the correctness property of the form:

$$\phi_{m_1,f_1}(n) \lor \ldots \lor \phi_{m_j,f_j}(n)$$

for some finite $j$. Then, we compute the cutoff $m_j$ independently for each $j$, and use the sum of the resulting cutoff values $m = \sum(m_1, \ldots, m_j)$ to check the correctness of the system. A system that satisfies this property is one that always satisfies some disjunct. Subsequently, a global error state for this property is one that violates all disjuncts, so one must check correctness of a system where all the processes needed to reach a global error state are available in the respective local error states (characterized by $f_1, \ldots, f_j$) at the same time. For each $i \in [1, j]$, let $\mathcal{F}_i$ denote the set of paths from the initial state $s_0$ to $s(t(f_i))$. To ensure that one can compute the cutoffs independently,
we require that for any two sets of paths $\mathcal{F}_i$ and $\mathcal{F}_j$, there does not exist any non-internal transition $s \rightarrow s'$ such that $s \in \text{states}(\mathcal{F}_i)$ and $s' \in \text{states}(\mathcal{F}_j)$.

Finally, cutoff values to check correctness properties that are conjunction or disjunction (but not negation) of permissible safety specifications can be computed as follows. First, one can convert the property to CNF form. Then, apply the disjunct rule to the clauses and the conjunct rule to the resulting simplified formula.