On the Deconfinement Phase Transition in Heavy-Ion Collisions

Abdelnasser M. Tawfik

FB Physik, Marburg University, Renthof 5, D-35032 Marburg, Germany

Abstract

The factorial moments (FM) of multiplicity distribution are used to study the deconfinement phase transition in heavy-ion collisions. The relation between FM and the partition number, $M$, results positive intermittency exponents, $\phi_q$. According to the signatures suggested from certain statistical models, the two-dimensional results of the dependence of $\phi_q/\phi_2$, anomalous fractal, $d_q/d_2$ and Rényi dimensions, $R_q/R$, and the normalized exponents, $\zeta_q$, on the orders of FM evidently supply evidence for the quark-hadron phase transition in Pb+Pb collisions at 158 AGeV.

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1 Introduction

The non-statistical fluctuations in the final state of particle production are suggested as an experimental tool to diagnostically confirm the elusive quark-hadron phase transition [1, 2, 3, 4]. Ever since the observations of spike-events first observed in the cosmic ray interactions [5] and later re-produced in the laboratory [6] and since the pioneer works of Bialas and Peschanski [7], the intermittent behavior has attracted a lot of attention. Therefore, it has been examined in different interacting systems. If the quark-hadron phase transition is to be produced in heavy-ion collisions, its critical aftereffects (like soft and hard collisions, clustering and resonance decay, showering processes, critical exponents, etc.) are expected to survive even until the freeze-out [8]. They reflect themselves in form of non-statistical (e.g. dynamical) fluctuations, power-scaling behavior, self-similar branching, mono-fractal density fluctuations, etc. In this letter, we study FM of the multiplicity distribution through successive partitions in one- and in two-dimensions (pseudo-rapidity, $\eta$, and/or azimuthal angle, $\phi$). Using these investigations, we shall try to access the deconfinement
phase transition\cite{4}. For this destination, we will utilize the attitude of intermittency exponents, anomalous fractal, and Rényi dimensions on the orders of FM. QGP signatures principally suggested according to certain statistical models will be read off by means of the relation between intermittency and multi-fractality.

The data sample used for this work is retrieved from some of the Pb-chambers irradiated at CERN-SPS during 1996 for the EMU01 collaboration. Specifically, the data sample employed here is that we have completely analyzed by our automatic measuring system MIRACLE Lab\cite{9} at Marburg university. For details about the emulsion irradiation, we refer to \cite{10}. Due to their high resolution and geometrical acceptance, the nuclear emulsions are effectively capable to measure the charged particles, the angular distribution, and the density fluctuations of high multiplicity collisions. The scanning efficiency in emulsion chambers is $\sim 0.75 \pm 0.05$\cite{11}. The sensitivity for singly charged particles is as good as 30 grains per 100 $\mu$m. It is therefore, possible to locate the track positions to an accuracy $< 2 \mu$m. Depending on the topology of the microscope’s field of view, the produced particles with space angles, $\theta < 30^\circ$ (pseudorapidity, $\eta = -\ln \tan(\theta/2) > 1.32$) can be acquired.

The produced particles are expected to be mixed with contamination of electron-pairs from Dalitz decays and undetected $\gamma$-conversions. The possible overestimation of particle density has been determined as $\sim 2\%$\cite{12}. As a reason of the reconstruction algorithm applied for MIRACLE Lab, the tracks of these electrons are completely disregarded or percolated. The efficiency of MIRACLE Lab is estimated as $\sim 96\%$. From manual/automatic comparison we noticed that the automatic reconstruction underestimates the multiplicity, especially the particles with relatively wide angles ($\eta < 2$). Besides these missing measurements, the frequent scattering, unresolved close-pairs, nearest-neighbor particles, and pair production represent an additional source of this 4%-discrepancy\cite{9}. Many of the extra tracks in the manual measurement are close pairs separated by $< 1 \mu$m. The two track resolution, close neighbors, and the split track recognition are discussed in \cite{9}. In the automatic measuring facility, the lake of close-pairs within $< 1 \mu$m can be referred to the limited instrumental pair resolution. Also if the particle trajectory is split into two tracks, which can result a huge intermittency signal, both of them are rejected, since their own vertex definitely will not be coincident with the common one\cite{9}. Then we can summarize that the data sample grabbed by MIRACLE Lab is evidently liberated from the double counting, measuring bias, and from any contaminations from particle decays or secondary interactions. We could therefore, consciously suggest to disregard the effects of Dalitz decays and $\gamma$-conversions on FM \cite{10} (see Sect. 2.2.1 below).

The accuracy of measured $\eta$ and $\phi$ is practically depending on determination of the event axis and on the emulsion plates, into which the corresponding track penetrates until it leaves the field of view. The $x$- and $y$-coordinates are ideally to be measured with respect to some reference points whose positions are well-determined. The location of track in such transverse plane has a statistical uncertainty of $\sim 0.3 \mu$m. On the average, the tracks leave the field of view at
distance $\sim 75 \, \mu m$ from the event exists. Then the resulting uncertainty in $\eta$ is $\sim 0.004$. Due to small air gaps, non-uniformity of the spacer between the plates, variations in emulsion thickness, the $z$-coordinates are also uncertain. For most of tracks, the last measured emulsion plate is located 2.5 cm downstream. The uncertainty in $\phi$ is then $\sim 0.136 \, \text{mrad}$.  

![Figure 1: Pseudorapidity distribution of central Pb+Pb events measured by MIRACLE Lab (solid line). The dotted line represents the distribution of FRITIOF, meanwhile the dashed line gives the Gaussian fit. The FRITIOF sample is simulated with zero impact parameter and default configurations.](image)

Fig. 1 shows the pseudorapidity distribution of Pb+Pb central events with multiplicity $\geq 1200$. In order to compare the experimental data to the expectations based on incoherent models, we have simulated a sample of Pb+Pb collisions using FRITIOF 7.02 Monte Carlo code with very small impact parameters. FRITIOF code has been run in its default configuration. The dotted Line shows the pseudorapidity distribution of such sample. For $\eta$ greater than $\sim 1.8$, the two distributions are in good agreement. There is a small flattening in the central peak of experimental distribution. The flattening is expected, if QGP had been produced.

If a phase-space of width $\Delta$ is split into $M$ equal bins of size, $\delta = \Delta/M$, the scaled factorial moments of multiplicity distribution are defined as:

$$F_q(M) = M^{\phi_q - 1} \sum_{m=1}^M \frac{\langle n_m(n_m - 1) \cdots (n_m - q + 1) \rangle}{\bar{n}^q}.$$  

$n_m$ is the multiplicity in the $m$-th bin and $\bar{n}$ is the average multiplicity in the whole $\Delta$-window. According to the self-similar density fluctuations (e.g. QCD parton cascading), the successive partitions lead to the following power-law:

$$F_q(M) \propto M^{\phi_q}.$$  

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The term "intermittency" used for the turbulence in the theory of chaos to describe the development of a hydro-dynamical system from stable to chaotic state can analogously be used in the particle production [15, 16]. The exponents, \( \phi_q \), called "intermittency exponents", can be determined from the asymptotic behavior. Their behavior interprets the interpretation of the intermittency as aftereffects of quark-hadron phase transition [2, 17, 18, 19]. Based on 2D Ising model [20, 22], the same interpretation is obtained. In addition to these statistical results, it is proposed that the intermittency parameters possibly contain signatures for the phase transition in heavy-ion collisions [2, 3]. In the contrast, assuming that the self-similarity dominates the particle production, it was difficult to concretely suggest their intermittency as signature of QGP [3, 14]. This present work distinguishes between all these scenarios.

\( \phi_q \) are related to the "anomalous fractal dimensions" though the following relation [23]:

\[
d_q = \phi_q \cdot (q - 1)^{-1}.
\]  

At the critical point of Ising model [20, 22], \( d_q \) can be given in terms of critical exponents. Also at this point, \( d_q \) are independent on the orders, \( q \). Generally, the \( q \)-dependency of \( d_q \) interdepends on the nature of processes taking place in the interacting system. Therefore, \( d_q \) are liable to the branching processes that precipitate an intermittent behavior. For this reason, \( d_q \) can be successful candidates to give a further signature for the QGP formation [3].

The fractal Rényi dimensions, which ordinarily, are used to measure the randomization in particle production, are, in turn, depending on \( d_q \) [2, 24, 25],

\[
R_q = R \cdot (1 - d_q),
\]  
The constant, \( R \), represents the topological Rényi dimension. For multi-fractal processes, \( d_q \) are linearly depending on \( q \). For mono-fractal density fluctuations, \( d_q \) are constant.

Assuming that the intermittency and the mono-fractal density fluctuations survive the further phases until the freeze-out [8], different proposals are suggested to indicate the deconfinement phase transition via the interplay between intermittency and mono-fractality. From statistical models, it is expected that at \( T_c \) the intermittency are related to only one particular combination of the critical exponents. In Eq. 3 \( \phi_q \) have been squeezed out in \( d_q \) and then in \( R_q \) (Eq. 4). Hence, the parameter governing the intermittency at a thermal phase transition can be represented by constant \( R_q \). Ordinarily, it is supposed that the mono-fractality measures the random sets of intermittent clusters of the confined phase inside the deconfined one [20, 27]. In a second-order QCD phase transition, it is expected that intermittent fluctuations take place and therefore, are characterized by unique anomalous fractal dimension. In 2D Ising model, the same behavior with constant \( d_q \) has been proven [21, 22, 26]. The validity of Eq. 8 is proposed as a signature of QGP [2], i.e. the validity of \( d_q/d_2 = 1 \).
according to Eq. \[3\]. The expectations given in this section represent the model used in this letter to study the quark-hadron phase transition.

2 Analysis and Results

2.1 Scaled Factorial Moments

**Figure 2:** The \( q \)-order FM are given as functions of the partition number, \( M \), and drawn in a log-log scale. In the top picture (A) the partition is performed in \( \phi \)—dimension only. The results of the partition in \( \eta \)—dimension are given in picture B. The two-dimensional partition (\( \eta \) and \( \phi \)) is illustrated in the bottom part (C). We notice that the interacting system is obviously intermittent in one- as well as in two-dimensions.

FM are calculated for the Pb+Pb events with the highest particle multiplicity (\( \geq 1200 \)). The events with lower multiplicities are sampled, incompletely. The smallest one has a multiplicity of 410. As a result of this selection process, events at a rate \( \sim 0.002 \) per incoming primary beam are chosen for FM. Based on parameterization of charge-changing cross-section for heavy-ion interactions, we obtain \( \sigma_{\text{Pb+Pb}} \approx 6.71 \pm 0.21 \) barn. According to the distribution shape
in the forward cone, the centrality of analyzed events can be estimated. The central ones contain two or fewer projectile fragmentations and have the highest multiplicity. The other events are classified as peripheral or semicentral. Furthermore, the central events are characterized by total break-up into singly charged particles \[29\] and supposed to be more favorable to produce QGP, as a result of the more nuclear matter they include. Besides the definition of interaction centrality, the multiplicity restriction can be employed to avoid any possible mechanical correlations \[10\] and to determine the kind of FM \[12\]. From these events, only the particles emitted within a predefined \(\eta\)-interval, \(2 < \eta < 6\), are taken into account. This interval obviously contains the region of central rapidity. Due to the chiral symmetry breaking, the produced particles are mainly pions. Therefore, the restriction on \(\eta\) is significant to study the deconfinement phase transition, which, in turn, is restricted to the produced particles. Here we employ an additional restriction on the considered phase-space. Its influence to get flat FM distribution has been discussed elsewhere \[10\]. On the azimuthal space, there is no restriction, i.e. for each \(\Delta\eta\) group, \(\delta\phi\)-bins are allowed to take any real value within the available spectrum \(\{0, 2\pi\}\).

From the analyzed 380 events, 84 central events are selected according the criteria given above. Using the calculated cross section, this sample represents \(\sim 22 \pm 4\%\) of all scanned events. The average multiplicity of their charged particles is \(\sim 1350 \pm 50\). From the multiplicity distribution per unit \(\eta\), we get average particle density \(\langle dn/d\eta \rangle_{\text{max}} \approx 540 \pm 32\). This results, according to the frequently used Bjorken formula \[13\], energy density of \(3.743 \pm 0.324\) GeV/fm\(^3\). This value is evidently larger than the density required for the quark-gluon plasma formation \[30\].

2.1.1 One-dimensional Analysis

The pseudorapidity interval, \(2 < \eta < 6\), is successively divided into \(M\) equal bins. The multiplicity in each such bins is counted and then the corresponding \(q\)-order FM are calculated according to Eq. \[1\]. In Fig. 2A, FM are given in a log-log scale as functions of \(M_\phi\) for the orders, \(q = \{2, 3, 4, 5, 6\}\). The underscore in \(M_\phi\) refers to the partitioned phase-space. The results of \(\eta\)-partition are given in Fig. 2B. We notice that the dependence of FM on \(M_\phi\) shows almost the same characteristics as in \(\eta\)-dimension. We also notice that the experimental points can be fitted as straight lines. All lines have positive intermittency exponents, \(\phi_q\), which evidently increase with increasing \(q\). Generally, the slopes in \(\phi\)- are larger than the slopes in \(\eta\)-dimension.

2.1.2 Two-dimensional Analysis

The multi-dimensional FM \((\phi, \eta, \cdots)\) \[31\] are suggested in order to study the interaction dynamics, to study the sources of multiplicity fluctuations, and con-

\[1\] The average multiplicity within \(m\)-th bin of size \(d\eta\) centered around the peak position, \(\eta_m\), of \(dn/d\eta\)-distribution.
sequently to clearly explain the power-law scaling. For the multi-dimensional partition, we should first utilize either *isotropic* or *anisotropical* method. If $\Delta \eta$, for example, is divided into the same number of bins as $\Delta \phi$, this method called self-affine partition (isotropical). Clearly, it leads to total partition number, $M^2$. In Fig. 2C, $F_q$ are drawn in dependence on $M_\phi \cdot M_\eta$ for the orders, $q = \{2, 3, 4, 5, 6\}$. In this picture, the 2D partition is performed isotropically.

We notice that the experimental points can be fitted as straight lines. All lines have positive slopes, $\phi_{\phi\eta}$. These slopes increase with increasing $q$. Obviously, $\phi_{\phi\eta}$ are larger than the 1D ones ($\phi_\phi$ and $\phi_\eta$).

2.2 Intermittency Phenomenon

To study the intermittent behavior observed in Fig. 2, we should first check whether this phenomenon can be understood by means of known physics, like conventional *short-range correlations*. Elsewhere [10], we discussed other interpretations, like Bose-Einstein correlation (BEC), multi-particle cascade, randomization, hadronic Čerenkov-radiation, measurement bias, etc. It is known that the intermittent behavior is inconsistent with $\alpha$-, Lund hadronization, geometrical branching model, etc.

2.2.1 BEC and Coulomb Reactions

The contributions of BEC to the particle correlation have to be considered, especially for identical particles. Furthermore, it is claimed that the intermittency is completely controlled by BEC and certain quantum statistical mechanism [13, 12]. Including BEC in the FRITIOF Monte Carlo code, is was possible to simulate the like-charged two-particle correlations [13]. In spite of these results, one should notice that the Dalitz decays and $\gamma$-conversations dominate the correlations, especially the lower-order ones of unlike-charged particles. Using the emulsion technique, one has *almost* no chance to directly determine neither the charges nor the momenta of produced particles. These measurements are essentially required to estimate the BEC. Nevertheless, I have invented three different methods to estimate BEC in the emulsion [12]. It has been found that BEC *nearly* dominate the interparticle correlations especially within *very* small phase-space intervals $q^2$, i.e. *very* small relative momentum, $q^2$. The intermittent behavior in *very* small phase-space intervals has been discussed elsewhere [12], where not only a linear upwards trend was observed but a steep *exponential* one. This exponential increase can partially be understood according to

\[ q^2 = M^2 - (im)^2 = [\vec{q}_1^2 + \vec{q}_2^2 + 2\vec{q}_1 \cdot \vec{q}_2 \cos(\phi_1 - \phi_2)] + [M_{1t}^2 + M_{2t}^2 + 2M_{1t} M_{2t} \cosh(\eta_1 - \eta_2)] \]

$M$ is the invariant mass.
BEC. The 2D partition method represents another source for this exponential upwards increasing [34, 35]. For relatively small $M$, as that in Fig. 2, both of BEC and 2D partition method play a neglected role in FM [36].

The Coulomb reactions represent an additional effect on FM. These final state interactions take place, when the produced particles are closely emitted. Depending whether the particles are bosons or fermions, these reactions increase or decrease the relative momenta, $Q$. Coulomb final state interactions have therefore, non-neglectable effects on BEC and then on FM, especially for small $Q$. The reactions between the identical points and between the fragments represent about 10% for $Q \leq 1$ MeV/c [37]. For increasing $Q$, the reaction rate decreases, exponentially [12]. It can be neglectable for few MeV/c. Therefore, for small $M$, as in Fig. 2, the contributions of Coulomb final state interaction can be neglected.

2.2.2 Physics of Intermittent behavior

Here, we introduce another physics for the intermittent behavior. We will examine whether it is able to describe the experimental data. In such a way, we simultaneously estimate the responsibility of the critical transition on the intermittent behavior, as introduced in Sect. 1.

1. Self-similar processes [7, 14, 38] (e.g. QCD parton cascading). At the critical point of phase transition, the correlation length will be on the increase and under the scale transformation the fluctuations are expected to display self-similarity. If the gaussian approximation can be utilized to describe the particle production and from Eq. 3, we get

\[
\frac{\phi_q}{\phi_2} = \frac{d_q}{d_2} \cdot (q - 1).
\]

(5)

As a particular case of this general description, we refer to the monofractal behavior (see Eq. 8 below). The implementation of Lévy index $\mu$ [33, 40, 11, 42] reads

\[
\frac{\phi_q}{\phi_2} = \frac{\mu - q}{2\mu - 2},
\]

(6)

$\mu$ has a continuous spectrum within the region of stability {0, 2}. The index, $\mu$, allows an estimation of the cascading rate [29]. Obviously, Eq. 5 cannot be not applied in the tails of these distributions. Whereas, Eq. 6 is more effective. The two boundaries of Lévy index, drawn in Fig. 4D, are corresponding to the degree of fluctuations in particle production as follows:

\[\text{In different experiments, such as that in [12, 43], it has been found that the index, } \mu, \text{ can be even outside this stability region.}\]
(a) $\mu = 2$, minimum fluctuations from self-similar branching processes. This is a suitable condition to apply the scaling rule (7).

\[ \frac{\phi_q}{\phi_2} = \left( \frac{q}{2} \right) \]  

Therefore, from Eq. (7) the anomalous fractal ratios, $d_q/d_2$, are equal to $q/2$. For these conditions, $R_q$ are corresponding to the multifractal processes. Later, we will realize that the last scaling rule is not able to describe the experimental results given in Fig. 4A,B,C.

(b) $\mu = 0$, maximum fluctuations. Meanwhile beneath the critical point and according to QCD, the correlation length rapidly grows, it diverges suddenly according to the statistical quantum physics. Therefore, near the critical point, i.e. beneath the total randomization, it is expected that $\{d_q|R_q\} \to \text{const.}$ and $\phi_q \propto q - 1$. At this point, the characteristics of interacting system can be compared with that of mono-fractal one [44, 45]. This should not lead to the conclusion that the mono-fractal behavior alone is sufficient for the phase transition. In Fig. 4, we will distinguish between these quantities and ascertain the mono-fractal behavior.

Between these two limits, the approximation, Eq. (5), is no longer valid and should be replaced by Eq. (6).

2. **Critical phase transitions** [2, 46] (e.g. quark-hadron phase transition). If QGP indeed is to be produced, the interacting system is supposed to suffer from thermal phase transition during its space-time evolution. At the critical point, $\phi_q \propto q - 1$, $d_q = d_2$, and therefore, $R_q/R = 1$. In addition to the model introduced in Sect. 1, there are auxiliary argumentations about utilizing the mono-fractality as signatures of the deconfinement. As the correlation length diverges at $T_c$ and if there are no long-range correlations, then $d_q$ are expected to be equal. Therefore, $d_q$ can be used to indicate the phase transition as follows: since the hadronization is supposed to occur, only if the QGP-matter suffers from a phase transition, then the final hadron-matter is expected to be intermittent with constant $d_q$. In the other case, if the hadronization is ordinary a result of cascading processes, $d_q$ are linearly depending on $q$. Therefore, at the phase transition, the ratio of $q$-th intermittency slope to the second one is depending on the orders, $q - 1$, (Eq. (8)).

\[ \frac{\phi_q}{\phi_2} = (q - 1). \]  

$\phi_2$ represents the dimension of the fractal sets in which the observed intermittent behavior occurs [33]. In Sect. 2.3.2, we will get $\phi_2 = 1$ for vanishing $\mu$. Obviously, this value is an evidence for mono-fractality. These patterns are also expected during the second-order phase transition from QGP to hadron gas [44].
Then, we conclude that the dependence of \( \phi_q / \phi_2 \) on \( q - 1 \) reflects essential information about the reaction dynamics and about the physics of intermittency patterns. Fig. 4 visualizes the differences between the possible sources of intermittent phenomenon discussed above. Meanwhile Fig. 5 is used to encourage the conclusion, posted in this letter.

In next sections, the \( q \)-dependency of the quantities, \( \phi_q / \phi_2 \), \( d_q / d_2 \), \( \mathcal{R}_q / \mathcal{R} \), and \( \zeta_q \) will be examined and it will be shown how the intermittency and the fractal structure of multiplicity fluctuations are used as signatures for the deconfinement phase transition [4].

2.3 Dependency on the orders of FM

Figure 3: The relations between \( q \)-order FM and the second-order ones are drawn in log-log charts. The top picture, A, depicts the analysis in \( \phi \)-dimension. The relations in \( \eta \)-dimension are given in picture B. The bottom picture, C, illustrates the analysis in two-dimensions. In all pictures, the experimental points are linearly fitted. We notice that the slope ratios, \( \phi_q / \phi_2 \), increase with increasing \( q \) and they are in \( \phi \)-space larger than in \( \eta \)-space. Obviously, the 2D slope ratios are larger than the 1D ones.

In Fig. 2, we noticed that the intermittency exponents, \( \phi_q \), are independent on \( M_{\{\eta,\phi\}} \). Therefore, the ratios, \( \phi_q / \phi_2 \), can directly be deduced from the relation \( F_q(M_{\{\eta,\phi\}}) \) vs. \( F_2(M_{\{\eta,\phi\}}) \).
\[ \log F_q(M_\{\eta\}|\phi) = \frac{\phi_q}{\phi_2} \log F_2(M_\{\eta\}|\phi) + c_q. \]  

(9)

This power-law is also valid for the multi-dimensional partition method [47]. In a log-log scale, such relation in \(\phi\)-dimension is depicted in Fig. 3A for the orders, \(q = \{3, 4, 5, 6\}\). Fig. 3B illustrates the results in \(\eta\)-dimension. We notice that the experimental points can be fitted as straight lines. Also, the slopes clearly increase with increasing \(q\). The results from 2D analysis are given in Fig. 3C. For the comparison between \(\phi_q/\phi_2\) in 1D and 2D, one should first use anisotropical methods, in order to overcome the additional effects of bin superposition especially in \(\eta\)-dimension [10, 34, 35].

2.3.1 Dependency of \(\phi_q/\phi_2\) on \(q\)

Relation (9) can be read as a power-law,

\[ F_q \propto F_2^{\beta_q} \]  

(10)

In Fig. 3A, it is clear to recognize that the powers, \(\beta_q \equiv \phi_q/\phi_2\), increase with increasing \(q\). Also the 1D \(\beta_q\) in the rapidity dimension are smaller than that in the azimuthal space. The 2D \(\beta_q\) are apparently greater than the 1D ones.

Using a specific form of Ginzburg-Landau model to simulate the deconfinement phase transition in heavy-ion collisions led to the conclusion that \(\beta_q\) are independent on the temperature \(T < T_c\). This guides the use of \(\beta_q\) as a signature for the quark-hadron phase transition [45].

\[ \beta_q = (q - 1)^\nu \]  

(11)

The powers, \(\nu\), describe the aftereffects of the phase transition from chaotic to coherent state. In our case, the aftereffects are the dimension ratios or basically the fluctuations in the final state of particle production. Generally, \(\nu\) are independent on the details of interacting system. Therefore, \(\nu\) can be used to characterize the behavior of the measurable quantities at \(T\) beneath \(T_c\) (e.g. \(\phi_q/\phi_2\), \(d_q/d_2\), \(R_q/R\), \(\zeta_q\), etc.). In this regard, \(\nu\) are not a set of critical exponents in a conventional sense. Furthermore, they have a universal relevance [45].

In Fig. 3A,B the relation \(\beta_q\) vs. \(q - 1\) are illustrated in \(\eta\)- and \(\phi\)-dimension, respectively. As discussed above, \(\beta_q\) can directly be determined from the slopes of the relation \(\log F_q\) vs. \(\log F_2\) (Eq. 9). We notice that the experimental results cannot be predicted by any of the lines given by Eq. 8 (\(\nu = 1.0\) [24]) or Eq. 11 (\(\nu = 1.304\) [15]). Comparing our results with previous experiments of KLM- and EMU01-collaboration [48, 49], we notice that even though all results can not be fitted by any of these \(\nu\)-values, Pb+Pb results are relatively closer to the first line. Although, the other line is obtained from the Ginzburg-Landau description of a second-order phase transition [45], it is unable to fit any experimental data. As given in Sect. 2.2.2, \(\nu = 1.0\) characterizes the critical phase transition. This
The dependences of the ratios, $\beta_q \equiv \phi_q / \phi_2$ on the orders, $q-1$, for 1D are drawn in A and B for $\phi$- and $\eta$-dimension, respectively, where the experimental results (solid points) cannot be predicted neither by Eq. 8 nor by Eq. 11. In A, the triangles represent O-AgBr and p+AgBr at 200 AGeV [48] and the squares are for S+Au at 200 AGeV [49]. Pb+Pb data are lower than the line with $\nu = 1$, meanwhile the previous results are over the line with $\nu = 1$.304. The 2D analysis is given in C. $d_q/d_2$ are also studied in dependence on $q-1$ and drawn in D (solid circles). The solid lines represent the two boundaries of Lévy stable law. The dash-line is the implementation of Eq. 6 for $\mu = +0.1$. The open triangles represent the 1D results.

value is also obtained from 2D Ising model with second-order phase transition. Ordinarily, $\nu = 1.0$ refers to the mono-fractality. Experimentally, different values are obtained for $\nu \geq 2/3$. But none of them can be related to the phase transition.

We can conclude that the 1D results in either $\eta$- or $\phi$-dimension do not show clear signature for QGP. On the account of this restriction, one narrows the QGP signatures either in the rapidity or in the location. This restriction is

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On the one hand, the appearance of intermittent patterns during a phase transition has been proven in 2D Ising model long time ago. The intermittent fluctuations are characterized by a constant anomalous fractal dimension. On the other hand, we should utilize such statistical model to study the quark-hadron phase transition, since in lattice QCD at final temperature, which practically is close relevant to QGP, there is no direct calculation for the phase transition. Its characteristics can be obtained by numerical and analytical studies for statistical spin models.
that as it is the reason for this breakdown. For that reason, the search for QGP signatures should be performed through the possible rapidity partitions and simultaneously over the possible locations. In such a way, we hope to disintegrate the fine scale of QGP.

The 2D dependence of $\beta_q$ on $q - 1$ is given in Fig. 4C. We notice that the relation (7), which has been suggested in [7] and successfully utilized in [51], is not able to fit the experimental data. This means that the self-similar branching processes and consequently the minimal fluctuations are not responsible for the power-law scaling (Sect. 2.2.2). The argumentations given in Sect. 2.2.1 are valid, if the intermittency is in fact the self-similar behavior. The dash-lines represent the power-fit according to Eq. 11 with $\nu = 1.304$ [45]. The same process, but with $\nu = 1.0$, is represented by the dotted lines (Eq. 8). We notice that the first power-fit is completely unable to describe the experimental data. Meanwhile, the second one gives a line very close to the experimental data (solid points). As given above, this result can be regarded as a signature of deconfinement phase transition. On the contrary to 1D results, we get $\nu = 1.0$ for the 2D dependency of $\beta_q$ on $q$.

Furthermore, it is important to recognize that Eq. 8 and Eq. 11 are not belonging to Lévy stable region. In the next section, we use Lévy space to draw the $q$-dependency of $d_q/d_2$. In such a way, we try to reconfirm the previous results.

2.3.2 Dependency of $d_q/d_2$ on $q$

From Eq. 3 and Eq. 11 we get

$$\frac{d_q}{d_2} = (q - 1)^{\nu - 1} \tag{12}$$

Fig. 4D illustrates this relation. We notice that the 1D results (open triangles) are completely outside the Lévy stable region (Sect. 2.2.2). In spite of this result, it is a significant finding to realize, that the values of $d_q/d_2$ are apparently independent on $q$ ($d_q/d_2 \approx 0.66 \pm 0.04$). Eq. 8 with $\mu = +0.1$, is only able to describe the 2D results (solid circles). The values of 2D $d_q/d_2$ are also constant ($d_q/d_2 \approx 1.0 \pm 0.03$).

For small but distinctly non-zero $\mu$, the possibility of QGP formation mixed within cascading processes can not be withdrawn [39]. If $\mu = 0$, then $\phi_q/\phi_2 = q - 1$ and correspondingly $d_q/d_2 = 1$ (Eq. 8 and Eq. 3). The predictions of Eq. 8 ($d_q/d_2 = 1$), which, as given above, emphatically supports the conclusion of phase transition [4], are at the low boundary of Lévy space. Once again, the obtained values of $d_q/d_2$ are not consistent with being proportional to $q$ as claimed in [52]. On the contrary, they are equal.
2.3.3 Dependency of $R_q/R$ on $q$

As discussed above, $R_q$, one the one hand, measure the degree of randomization in the final state of particle production. On the other hand, they represent the fractal critical dimensions of the randomized intermittent clusters observed in the confined matter which is entirely bedded inside the deconfined one. As given in Sect. 2.3.4 below, the thermal phase transition can be marked by constant $R_q$. The $q$-dependency of $R_q$ in 1D and 2D is depicted in Fig. 5A. As expected, we notice that $R_q/R \approx 1$. Near the critical point, $d_q/d_2 \to 1$. Then the corresponding Rényi dimensions, $R_q \to R$, and $\phi_q/\phi_2 \propto q - 1$.

As given in Sect. 2.3.4 below, near the critical point, the normalized exponents, $\zeta_q$, show an abnormal behavior. They have an exponential $q$-dependency rather than a linear one. This will be utilized to distinguish between the scaling rules given in Sect. 2.2.2.
2.3.4 Dependency of $\zeta_q$ on $q$

The scaling rule, Eq. 6, is only valid, if the correlated $q$-tuplets are built up from single correlated particle-pairs and $q - 2$ non-correlated ones. Starting from the assumption given in [51] that FM are defined as the integration of multi-particle correlation functions together with the lower-order ones which strongly effect the higher-order ones and defining the normalized exponents as

$$\zeta_q \equiv \phi_q / \left( \frac{q}{2} \right),$$

it is assumed that $\zeta_q$ is linearly depending on $q$ (as given in Fig. 6 of Ref. [51]). In Fig. 5B, we check this relation for 1D as well as for 2D FM. It is clear to notice that $\zeta_q$ exponentially increase with increasing $q$. This result supports, on the one hand, the assumption that Eq. 6 is not able to describe the ratios $\phi_q/\phi_2$. On the other hand, it supports the conclusion that $d_q$ are constant for all orders of FM, as we noticed in Fig. 4.

Using the scaling-rule, Eq. 8, to define the normalized exponents, we get

$$\zeta_q \equiv \phi_q / (q - 1),$$

Eq. 14 has been tested in Fig. 5C. We find that $\zeta_q$ are linearly depending on $q$. This guides to the conclusion that meanwhile Eq. 6 failed to describe the Pb+Pb data, Eq. 8 is obviously able. Otherwise, we notice that $\zeta_q$ in 1D decrease with increasing $q$, whereas in 2D there is a positive increasing.

3 Summary and Conclusions

In this latter, the non-statistical fluctuations in Pb+Pb collisions at 158 AGeV are investigated. First, I would like to sum up the results obtained so far. The analysis of FM in 1D and 2D shows that this interacting system is obviously intermittent. The values of 1D $\beta_q$ are considerably lower than the results obtained by KLM- and EMU01-collaboration. Eq. 7 suggested in [6] and utilized in [51] failed to simulate the dependency of 1D and 2D $\beta_q$ on $q - 1$. Meanwhile, Eq. 8 results a line in 1D more close to Pb+Pb than to the other data, the results of 2D $\beta_q$ can effectively be fitted by this equation. The behavior described by Eq. 8 is predicted, on the one hand, if the interacting system suffers from thermal phase transition during its space-time evolution. On the other hand, the disability of this equation to perfectly describe the 1D ratios leads to the conclusion that the Pb+Pb data are not uniquely mono-fractal. This results should not disturb the main conclusion posted in this letter, since on the basis of the fine scale of QGP, one can expect, that its signatures might be locally oriented. Therefore, the individual analysis in rapidity- or azimuthal-space may not be favorable to detect QGP. The 2D analysis is therefore, more effective than the 1D. In addition to these results, we get $d_q/d_2 = 1$ in 2D investigation.
of the anomalous fractal dimensions. This value confessedly supports the hypothesis of the deconfinement phase transition. Lévy index, $\mu = 0.1$, obtained for this 2D analysis, is an additional confirmation of this transition. We noticed that the fractal Rényi dimensions are constant ($R_q = R$). Also, in Eq. 13 the relation between the normalized exponents, $\zeta_q$, and the orders of FM shows that $\zeta_q$ exponentially increase. This dependence assists the conclusion that the relation (7) is not able to describe the experimental data and therefore, it should be replaced by Eq, 8, as done in Eq. 14.

Finally, we come to the conclusion that the data sample used in this letter is intermittent in 1D as well as in 2D. The intermittency ratios can be given by using simple relations, as that given in Eq. 8. Furthermore, according to certain statistical models, the $q$-dependency of anomalous fractal and Rényi dimensions and the index $\mu$ evidently support the deconfinement phase transition. More data are however, needed to confirm the results and interpretations reported in this letter.

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