Does our universe conform with the existence of a universal maximum energy-density $\rho_{\text{uni}}^{\text{max}}$?

*Hujeirat A.A.

IWR, Universität Heidelberg, 69120 Heidelberg, Germany

April 14, 2021

Abstract

Recent astronomical observations of high redshift quasars, dark matter-dominated galaxies, mergers of neutron stars, glitch phenomena in pulsars, cosmic microwave background and experimental data from hadronic colliders do not rule out, but they even support the hypothesis that the energy-density in our universe most likely is upper-limited by $\rho_{\text{uni}}^{\text{max}}$, which is predicted to lie between 2 to 3 the nuclear density $\rho_0$.

Quantum fluids in the cores of massive NSs with $\rho \approx \rho_{\text{uni}}^{\text{max}}$ reach the maximum compressibility state, where they become insensitive to further compression by the embedding spacetime and undergo a phase transition into the purely incompressible gluon-quark superfluid state. A direct correspondence between the positive energy stored in the embedding spacetime and the degree of compressibility and superfluidity of the trapped matter is proposed.

In this paper relevant observation signatures that support the maximum density hypothesis are reviewed, a possible origin of $\rho_{\text{uni}}^{\text{max}}$ is proposed and finally the consequences of this scenario on the spacetime’s topology of the universe as well as on the mechanisms underlying the growth rate and power of the high redshift QSOs are discussed.

**Keywords:** General Relativity: black holes, pulsars, neutron stars, superfluidity–superconductivity–incompressibility–QCD–quantum fluids, condensed matter physics Cosmology:big bang physics, dark matter, dark energy, cosmic microwave background, high redshift quasars, first generation of stars.

1 Introduction

Very recently we have witnessed a historical breakthrough in observational astronomy: the beginning of the Multi-Messenger era. The accurate and successful calibration of instrumentations operating remotely at different frequency regimes has successfully enabled us to detect astronomical events with unprecedented accuracy. Thanks to modern communication technologies that enable pairing remote...
telescopes and form global networks of virtual telescopes that provided the highest resolution imaging currently possible at any wavelength in astronomy [see 1, 2, and the references therein]. Based thereon, observation of highly energetic astrophysical events, such as mergers of neutron stars and black holes, GRBs, Kilonova resulting from NS-mergers, imaging the event horizons of black holes [see 1, 3, 4, and the references therein] as well as quasars at extremely high redshift [3] delivered a lot of data that enabled us to gain deeper insight of the underlaying physics. Other breakthroughs have been recorded on the micro-scales, such as the Higgs bosons at the Large Hadronic collider (LHC), and the fluidity character of quark-gluon plasmas at the RHIC [see 5, 6, 7, and the references therein].

On the other hand, while modern observations are continuously shedding light on the huge diversity of the state of matter both on the micro and macro-scales, they also raise new questions related to the validity of solutions of problems that have been thought to have settled.

Based on the recent theoretical and observational studies, it is argued in this paper that:

- If $\rho_{\text{uni max}}^{-1}$ indeed exist, then the remnant of the merger of the two neutron stars in GW170817 should be a massive neutron stars (NS) whose core is made of an incompressible quark-gluon superfluid that obeys the laws of quantum field theories [see Fig. 1 and reference: 8]. However, the ambient media remain compressible and dissipative, though these properties are doomed to disappear at the end of the luminous lifetime of NSs, rendering them invisible.

These cores appear to be generic for almost all massive NSs, whose masses and dimensions are set to grow as NSs age. Within the context of general relativity, a physically consistent and causality-preserving treatment of the two totally distinct fluids: incompressible-superfluid and compressible-dissipative normal fluid, is possible, if the embedding spacetime is of a bimetric type: a Minkowski flat at the background of the core surrounded by a Schwarzschild curved one.

![Figure 1: A schematic description of the internal structure of a neutron star and the topology of the embedding spacetime. There are three different regions that are relevant for the present discussion: the central core, which is made of incompressible superfluid and imbedded in a Minkowski-type spacetime. The surrounding shell, inside which the matter is compressible, dissipative and embedded in a Schwarzschild-type spacetime. The geometrically thin boundary layer between the core and the surrounding shell, where matter and spacetime repeatedly undergo phase transitions.](image-url)
The above argument is extended to show that the history and large-scale structures of our universe may indeed tolerate the existence of \( \rho_{\text{uni}}^{\text{max}} \). Also, I address and discuss several relevant observational signatures both on the micro and macroscopic scales that support the maximum energy density hypothesis.

If such a density upper-limit does exist, then what could be the underlying physical mechanisms that may lead to the existence of \( \rho_{\text{uni}}^{\text{max}} \)?

2 Observational signatures

2.1 The origin of power of high redshift QSOs and their growth problem

Early large-scale structures are considered to have formed during the first two billion years after the big bang. Indeed, the three UV-emission lines from the luminous star-forming galaxy GN-z11 which has been recently analysed using Hubble Space Telescope (HST) imaging data, most likely correspond to emission at \( z = 10.957 \pm 0.001 \), i.e. to roughly 420 Myr after the big bang [see 9, 10, and the references therein]. Taking into account that GN-z11 is roughly 100 less massive than our Galaxy, then GN-z11 is expected to host a SMBH at its center.

Similarly, recent observations of the quasar ULAS J1342+0928 reveal that its redshift is \( z = 7.54 \) [see 11 and the references therein], which implies that the object must have been there already when the universe was 700 Myr old. Such rapid growths cannot be explained, unless the seeds are massive BHs of at least \( 10^6 M_\odot \) [see 12, 13 and the references therein]. Therefore primordial black holes that were formed either within the first \( 10^{-23} \) s or during the first \( 10^{-5} \) s after the big bang should be ruled out, due to lacking observational evidence for their existences [see 14 and the references therein]. Irrespective of the underlying growths mechanisms, their mass increase should follow the equation of mass conservation:

\[
\frac{dM}{dt} = \sum \int_S \mathbf{r} \cdot d\mathbf{s} = a_0 \left( \frac{\beta_v}{\beta_c} \right) \rho M^2 + \dot{M}_{\text{ext}},
\]

where \( a_0 = 16\pi G^2/c^3 \approx 10^{-6} \) and \( \beta_v, \beta_c, \rho \) correspond to the relative fluid velocity, \( v/c \), sound speed, \( V_s/c \) and the density of the inflowing matter through the boundary in units of \( 10^{-10} \) g/cc, respectively. Here \( M/t \) are in units of \( 5 \times 10^7 M_\odot \) per year. \( \dot{M}_{\text{ext}} \) denotes other growth mechanisms, such as direct collapse of primordial gas clouds, runaway collisions of dense star in clusters, hyper-Eddington accretion as well as episodic galaxy mergers [see 13 and the references therein]. Assuming \( d(\beta_v \rho/\beta_c^4)/dM \ll 1 \), then the time-dependent mass should evolve as:

\[
\frac{M(\tau)}{M_0} = \frac{1}{1 - a_0 (\beta_v/\beta_c^4) \rho \tau M_0},
\]

where \( \tau \) corresponds to the elapsed time for a SMBH of initial mass \( M_0 \) to reach \( M(\tau) \). For \( M(\tau)/M_0 = 100 \) and \( M_0 = 1 \), we obtain roughly:

\[
\tau_{100} \approx 10^6 \rho \times \left( \frac{\beta_c^4}{\beta_v} \right) = \begin{cases} > 10^9 & R_A = R_{\text{LSO}} \\ (\beta_v = \beta_c \approx 1) & \mathcal{O}(10^9) \quad \text{Super-Eddington Accr.} \\ \text{collision of clouds,} & \text{collision and merger.} \end{cases}
\]
$R_{LSO}$ here is the last stable orbit, where $\beta_v = \beta_c \approx 1$. The Eddington accretion is used to estimate the density $\rho$. In the case of super-Eddington accretion caused by direct collapse of clouds, star collision and mergers, the matter is expected to be increasingly hotter as the center of mass is approached, hence more difficult to compress, thereby giving rise to slow and episodic accretion.

The conclusion here is that current scenarios are incapable of providing viable solution to the rapid growth of SMBHs within the first 600 Myr after the big bang. This problem will soon become more prominent, when advanced observations shed the light on the existence of QSOs at $z \geq 10$, which would logically imply that their existence of QSOs at $z \gg 10$ cannot be ruled out. Moreover, high redshift QSOs may have been there already before the big bang, and that their nature must be more complicated than being objects with mathematical singularities at their centers.

2.2 Compact objects and the mass-gap

Astronomical observations reveal a gap in the mass spectrum of compact objects: neither black holes nor NSs have ever been observed in the mass range $[2.5M_\odot \leq M_{NS} \leq 5M_\odot]$. Even the nature of the remnant of the well-studied NS-merger event, GW170817, whose mass is predicted to be around $2.79M_\odot$, is still not conclusively determined on whether it is a massive NS or a stellar-mass BH [see 8, 15, and the references therein]. Recently it was suggested that when massive NSs cooldown during their cosmic evolution, their incompressible superfluid cores should grow both in mass and volume to finally fade away as dark energy objects [DEOs; see 16, 17, 18 for further details]. Based on observational data of glitching pulsars, and specifically of the Crab and Vela pulsars, this scenario appears to successfully explain various aspects of the glitch phenomena, such as the growth of mass and volume of their cores, as well as of the mechanisms underlying the under- and overshootings observed to accompany their prompt spin-up [see 18, 19 and the references therein]. Recalling that the Crab and Vela are considered to the most well-studied pulsars in our galaxy, the remarkable agreement of the scenario with observations was possible only, because of the assumption that their cores are made of incompressible quark-gluon superfluids. Based thereon, isolated massive NSs either in the local or early universe should metamorphose into DEOs, whose embedding spacetimes is of Minkowski type.

2.3 Dark matter in the early universe

Dark matter (DM) in cosmology is an essential pillar for understanding the entire cosmic web: starting from structure formation, clusters of galaxies and galaxy formation down to stellar-mass objects. Except for gravitational interaction with normal matter, the physics of dark matter continues to be a conundrum in understanding our universe.

For the present discussion, we select three properties
that support the constant density hypothesis: When a massive ultra-compact object (UCO) run out of all their secondary energies, the embedding spacetime should have compressed the matter inside their interiors up to the maximum limit, at which matter undergo a phase transition into incompressible gluon-quark superfluid, which corresponds to the lowest possible quantum energy state. Objects of this type are both electromagnetically and gravitationally passive and are classified here as DEOs.

Indeed, the ultra diffuse galaxy, Dragonfly 44, is considered to be a dark matter dominated galaxy [see 20, 21, 22 and the references therein]. Its dwarf-like shape and the extremely low surface brightness indicate that it might be the relic of an old massive galaxy, in which the stars have run out of their secondary energies during the course of their cosmic evolution [21]. Also, it was argued that the lack of X-ray emission is an indication that most of the compact objects in the galaxy are relatively old and therefore turned dark by now. Assuming star formation rates and supernovae statistics to apply for this galaxy during its active phase in the past, then roughly 1% of its population must be in the form of NSs and 0.1% stellar black holes. While BHs need to swallow matter in order to be detected, NSs may still eject highly relativistic particles from their polar caps and therefore at least 0.001% of the NS population should be detectable. However, such signatures appear to be missing in the D44 galaxy.

In the context of our scenario, the old and massive NSs in D44 must have metamorphosed into DEOs that subsequently migrated either outwards into the halo or inwards where they conglomerate to form a cluster of massive DEOs. The time duration required for these objects to randomly migrate from central region of a galaxy to the surrounding halo or vise versa may in random manner is of order \( \tau_{\text{diff}} \sim L^2/\nu_{\text{eff}} \), where \( L \) is the average radius of the galaxy and \( \nu_{\text{eff}} \) is a measure of their gravitational scattering, \( \sigma_{\text{DNS-\text{Star}}} \), times the duration of their momentum exchange with other objects \( \tau_{\text{DNS-\text{Star}}} \).

Assuming the masses of both types of objects to be of order the solar mass, then \( \sigma_{\text{DNS-\text{Star}}} \approx 10^{-4} R_\odot^2 \).

On the other hand, \( \tau_{\text{DNS-\text{Star}}} \) can be set to be of order \( \sqrt{\sigma_{\text{DNS-\text{Star}}} / V_{\text{rel}}} \), where \( V_{\text{rel}} \) is the relative velocity of the two arbitrary encountering objects[see 23 for further details]. Setting a maximum relative velocity of 100 km/s, we obtain \( \tau_{\text{diff}} \) of order \( 10^3 \) times the age of the universe.

Consequently, massive NSs that run out of their secondary energies and became DEOs either prior to or post the big bang may be considered as DM-candidates. Their poor collisions and mergers may be attributed to the topology of the bimetric spacetimes embedding DEOs. This picture is in line with recent observational and theoretical studies indicating that DM may have been present already in the pre-big bang era and/or during cosmic inflation [see 24, 25 and the references therein]. However, unlike our present macroscopic approach, the underlying
assumption of the former scenario is that DM should be made of microscopic particles of unknown origin that emerged out of scalar fields.

2.4 Merger of NSs and glitching pulsars

Observations reveal that NSs occupy a relatively narrow mass range that lies between $1.2M_\odot$ and $2.2M_\odot$, though the error bars in many cases are relatively large. Despite the use of modern instrumentations and global coordination of multimessenger observations, their radii, which should decrease with increasing mass, are still rather uncertain.

Both theory and numerical calculations indicate that the central density of neutron stars must be larger than the nuclear $\rho_0$. However probing the state of ultracold supranuclear dense matter under terrestrial conditions is unfeasible and the governing physics remains rather uncertain. Therefore, when modeling the internal structure of NSs, the regularity condition usually imposed at the centers of UCO is actually identical to the incompressibility condition, which is a limiting case of:

$$\frac{\Delta \varepsilon}{\Delta r} \xrightarrow{\Delta r \to 0} \nabla \varepsilon = 0.$$  (4)

Microscopically $\Delta r$ is limited from below by the average separation between two arbitrary baryons $\Delta_{bb}$. Using $\Delta r = \Delta_{bb}$ and $\Delta \varepsilon \Delta t \approx \hbar$, we obtain that $\Delta r \gg 10^{10} \times \Delta_{bb}$. Among others, this implies that incompressibility is a macroscopic property, but that quantum fluids must fulfill on the microscopic scale at $r = 0$. In this case, the pressure here ceases to behave as a local thermodynamical quantity and it becomes solely a Lagrangian multiplier: a mathematical term that dictates the global dynamics of the quantum fluid, which in turn depends strongly on the topology of the embedding spacetime. This pseudo-pressure should be carefully constructed, so that its transition into the ultrabaric regime remains forbidden, $dp/\Delta \varepsilon \geq 1$, and therefore causality condition is grossly violated [see Fig. 7 in 16]. Indeed, we consider the following observational signatures to support our scenario:

- Isolated NSs with $M_{NS} \geq 2.5M_\odot$ are practically missing and theoretically difficult to model, irrespective of EOSs used.
- Based on the multi-messenger observations of the NS-merger event GW170817, the remnant should have a mass of about $2.69 M_\odot$, though its nature is still an open question. In particular, observations did not conclusively rule out that the remnant might be a massive NS. In fact recent numerical calculations of merging two incompressible superfluid cores trapped in a potential that mimics an embedding curved spacetime show that such cores are capable of merging without decaying [Fig.2 see 26]. Combining this numerical observation with the possibility that the merger of the overlying compressible and dissipative ambient media of both objects may switch off the dynamo action of the remnant, thereby diminishing most magnetic activity not a short period.
- The discrete events of glitches observed in pul-
By means of numerical solving the modified time-dependent Gross-Pitaevskii equation in 2D, the merger of two superfluid cores trapped in a symmetric external potential have been calculated. The energy loss due to merger of the two cores was found to decrease with decreasing the numerical diffusion, i.e. with increasing the spatial and temporal accuracies.

Glitches are considered to be the responses of their cores to the long-term cooling of the objects. Moreover, the majority of glitching pulsars appear to have the following common features:

- Glitches are associated with the evolution of pulsars and young NSs, they occur instantly but the corresponding relaxation time increases as they age. Recently, accurate observations revealed certain association of under and overshootings with the prompt spin-up events of glitching pulsars [see 18, 19, and the references therein].

- In the $\dot{P} - P$ diagram, almost all observed glitching pulsars appear to be younger than $10^7$ yr and their magnetic fields are stronger than $10^{11}$ Gauss.

- The reoccurrence of glitches decreases with pulsar’s age [27, 28].

- UCOs, with magnetic fields larger than the quantum critical value $B > B_c = m^2c^3/eh \approx 4.4 \times 10^{13}$ G, are found to occupy a narrow range in $B$—age diagram, where the majority of magnetars are found to be younger than $10^5$ yr [Fig.3 29, 30, 31, and the references therein].

As it was shown in [8, 16, 32], these features may easily be explained by invoking the bimetric spacetime scenario. Accordingly, cores of massive NSs are made of incompressible superfluids embedded in Minkowski spacetimes, whereas the surrounding compressible and dissipative media is embedded in Schwarzschild-type spacetimes. This scenario is consistent with general relativity as the mass-energy inside the cores dictates the topology of the embedding spacetime and vice versa.

### 2.5 The missing first generation of massive neutron stars

Within the framework of the big bang scenario, the first generation of stars must have formed approximately 500 Myr after the big bang [see 33 and the references therein]. The progenitor clouds were made mainly of atomic hydrogen with an average temperature of about 1000 K. This relatively high temperature enables cloud to collapse under their
Figure 3: The derivative of the period time, $\dot{P}$, versus period $P$ of the population of UCOs and their tendency to migrate (marked with arrows) toward the lower-right region, where they end up as DEOs. There are three distinct classes that can be identified: millisecond pulsars (gray squares/orange region), glitching and normal pulsars (triangles & squares/blue region) and magnetars (pentagons/red region). All types of UCOs should migrate into the lower right region, where they end up as DEOs. While migration of strongly magnetized isolated and massive UCOs should proceed relatively fast, those with intermediate masses in binaries and/or in accretion mode generally need much longer time to migrate.

own self-gravity, if the mass content is larger than the critical Jeans mass amounting to $10^2$ up to $10^4$ solar masses. Hence the resulting stars must have been extraordinary luminous and therefore short-living, and expected to subsequently collapse into BHs or massive NSs. While BHs are generally accepted to be the remnants of collapsed population III stars, NSs however have been neither investigated nor observed. Moreover, given the relatively large dimensions and long time scales characterizing the progenitor clouds in combination with rotation, it is not at clear, why the centrally collapsing portion of the cloud with its relatively short dynamical time scale should proceed self-similar and fail to blow away the outer slowly contracting shells or forbids fragmentation of the collapsing cloud. We note that the effect of numerical diffusion in the simulations of the collapsing clouds is to enhance coupling of the different parts of the cloud and facilitate their direct collapse. In real life, however, the cores of collapsing clouds will be much hotter and the advected magnetic fields via ambipolar diffusion will soon be strong enough to fragment the cloud. Such sophisticated and robust numerical solvers are still to be developed, as important mechanisms, such as radiation transfer, ambipolar diffusion, multi-component fluids, large scale magnetic fields, implicit time-stepping that respect causality are still not accounted for.

Nevertheless, to form a BH, the spacetime embedding the progenitor should have compressed the central core and increased its central density up to the critical density $\rho_{cr}$, beyond which normal repulsive forces fail to oppose further compression. In this case, the event horizon $r_H$ should surpass the actual radius of the central core, and the enclosed average density must surpass the critical value:

$$\rho_{cr} \approx 1.6 \times 10^{27} \frac{1}{r_H^2}. \quad (5)$$

For $r_H$ of order $10^6$ cm, one obtains $\rho_{cr} \approx 6 \rho_0$, where $\rho_0$ is the nuclear density.

Obviously, the reaction time scale of the core at the verge of collapse is of order micro-seconds, which is
comparable to the light crossing time. Hence the core is well-equipped to oppose all types of external forces including compressional forces. In this case, the accumulated matter on the surface most likely would bounce rather than triggering a collapse of the object into a BH. Indeed, most sophisticated numerical modeling of core-collapse supernovae give rise to bouncing rather than to direct collapse into a stellar BH [see 34, and the references therein], provided the domain of calculations does not contain an apparent horizon, which is manually created by changing the EOSs long before $\rho_{\text{uni}}^\text{max}$ has been reached and/or surpassing the maximum compressibility limit [see 35, and the references therein]. Moreover, most of today numerical solvers are still neither capable of modeling incompressible supranuclear dense matter inside the cores of massive objects that are on the verge of collapsing into BHs nor treating the topology of the embedding spacetime in a self-consistent manner and therefore are irrelevant for the present discussion.

On the other hand, the multi-messenger observations of NS merger event, GW170817, suggest that violent merger of NSs may not necessarily lead to BH formation, but that forming a dynamically stable massive NS with $M_{\text{NS}} > 2.79 M_\odot$ should not be ruled out either [8]. The luminous lifetime of such a massive NS would be extremely short and would soon turn invisible DEO.

Consequently, as the first generation of stars must have been massive, their collapse should have generated a numerous number of massive NSs. Their luminous lifetime would be relatively short and therefore they should have entered their final evolutionary phase as DEOs long ago. On the other hand, supernovae statistics predict that at least 1% of star populations in star-forming clouds should be NSs. Although this rate is expected to be even higher in the early universe, yet isolated NSs older than one Gyr have not been observed. The topology of the spacetime embedding DEOs is of a bimetric type, which in turn prohibits their merger and makes them to excellent BHs and DM-candidates.

2.6 Hadronic collisions: LHC and RHIC

Among many recent explorations related to particle collisions in labs, there are several discoveries that should be relevant for the present discussion:

- Fluidity of quark-gluon plasmas

Quark-gluon-plasmas emerging from smashed nuclei both at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) were verified to behave nearly as perfect liquids [see 6, 7, and the references therein]. This was a turning point in physics, as the entropy of such colliding particles under normal laboratory conditions is extremely high and the outcome is expected to be a dilute gas: an assembly of loosely connected particles that running out of the center of collisions almost in spherical symmetric patterns.

Recalling that QGPs are imperfect due to unitarity, the cross-sections of colliding particles ought to be bounded from below. Indeed, based on infinitely coupled supersymmet-
ric Yang-Mills gauge theory (SYM) and using anti-de Sitter space/conformal field theory (ADS/CFD) duality conjecture, the lower-limit was verified to be:

\[
\left( \frac{\eta}{s} \right)_{\text{SYM}} = \frac{1}{4\pi},
\]

which is roughly equal to the lower-limit obtained using the uncertainty principle [36].

While the physics governing particle collisions under terrestrial conditions are totally different from those at the center of massive NSs, the emergence of quarks and plasma out of nucleons as liquids can be considered as a strong evidence for the here-proposed hypothesis. Indeed, let the velocity of particles immediately after collisions be of order the sound speed:

\[ V \approx V_s = c/\sqrt{3}. \]

This yields a dynamical time scale \( \tau_d = \lambda/V_s = \alpha \lambda r_b/c. \) On the other hand, the viscous time scales as:

\[
\tau_{vis} \approx \frac{\lambda^2}{\nu_{vis}} = \frac{\alpha_v^2 V_s}{\alpha_v \nu_{vis} \ell_{vis}} = \frac{\alpha_v^2}{\alpha_v \alpha_{\ell}} \tau_d, \quad (7)
\]

where \( \alpha_{vis} \) denotes the effective viscous velocity in units of the sound speed and \( \alpha_{\ell} \) the corresponding viscous length scale in units of \( r_b \).

Given that elliptic flows start shaping already during the initial expansion, then \( \frac{\alpha_v^2}{\alpha_v \alpha_{\ell}} \) should be of order unity, which yields the Reynolds number:

\[
Re = \frac{\tau_{diff}}{\tau_{dyn}} \approx 1. \quad (8)
\]

Going back in time, this result would imply that the confined QGPs, inside the sample of nucleons under lab conditions, are dissipative liquids with non-vanishing viscosity over entropy, i.e. \( 0 < \eta/s < 1. \)

Therefore the energy state of the QGPs here should decay into a lower energy state to finally hardonize into stable baryons.

However, in low Reynolds number flows, the effect of viscosity is to smooth out irregularities and prohibits the formation of distinct flow patterns, such as the micro-elliptical flows of deconfined QGPs, whose patterns considerably deviate from a pure spherical expansion. This implies that the dynamical time scale, on which particle collisions at the LHC/RHIC occur, are still too long compared to the time scales on which gluon effects are operating during the hardonization of the QGP.

In the case of NSs, as they cooldown on the cosmic time, their heat contents decrease and the supranuclear dense matter inside their cores approach the limit of maximum compressibility. Here both the temperature and entropy should go to the zero limit. From statistical thermodynamical point of view, the number of possible microstates of constituents of incompressible superfluid reduces to just one single quantum energy state, where all constituents are forced to occupy. This corresponds to the zero-degree of freedom and therefore to vanishing entropy and viscosity:

\[
\lim_{T \to 0} \eta = \lim_{T \to 0} s = \frac{k_B}{4\pi} \sum_{n=1}^{\infty} \ln(\Omega_n) = \frac{k_B}{4\pi} \ln(1) = 0 \quad (9)
\]

This implies that \( \frac{\eta}{s}, \eta \) should be more sensitive
to temperature’s variation than entropy, i.e.,
\[
\lim_{T \to 0} \left( \frac{\eta}{s} \right) = \lim_{T \to 0} \left( \frac{d\eta}{dT} \right) = \lim_{T \to 0} T^\gamma = 0,
\]
where \( \gamma \) is an arbitrary positive constant. This convergence behaviour is in line with energy conservation, as viscous energy extraction from an incompressible fluid is upper-limited by the availability of secondary energies: \( dQ^{vis} < TdS \).

- The transition from a maximally compressible into a purely incompressible fluid-phase should be associated with instantaneous input of free energy. This is a consequence of global energy conservation: The gravitational field, or equivalently, the energy stored in the curved spacetime embedding the ambient compressible media may be calculated using the surface integral [see [37] for further details]:
\[
\Delta E^g_{tot} = \frac{1}{16\pi} \int_{S^{n+1}} d^2S j^i (\frac{\partial}{\partial x^k} g_{jk} - \frac{\partial}{\partial x^j} g_{kk}),
\]
where the domain of integration, \( D^{n+1} \), is bounded by the surfaces \( S^n \) and \( S^{n+1} \), and \( g_{jk} \) is the spacetime metric. As UCOs cooldown, the topology of the manifold \( D^{n+1} \) changes from a curved into flat one, and therefore \( \Delta E^g_{tot} \) becomes free. As the matter has reached the maximum compressibility limit at zero-entropy, \( \Delta E^g_{tot} \) cannot be absorbed locally, and therefore it is conjectured that this energy goes into enhancing the surface tension, i.e. the ”bag energy” confining the enclosed continuum of gluon-quark medium.

For \( \rho = \rho^{uni}_{max} \), the energy per particle may be estimated as follows: since the separation between baryons vanishes, i.e. \( \Delta_{bb} = 0 \) (see Fig. 4). In this state the distance between quarks initially belonging to different baryons reduces to \( \Delta_{qq} = \ell_{qq} \), which is assumed to be the smallest possible distance between quarks at \( T = 0 \). By reducing \( \Delta_{qq} \) to \( \ell_{qq} \), an energy of order \( \Delta\varepsilon^+ \approx c\hbar/(2\ell_{qq}) \approx 1/3 \) GeV becomes available, but which cannot be absorbed locally\(^3\). \( \Delta\varepsilon^+ \) may be viewed as an excited energy state of the carriers of the rest nuclear forces between baryons, which is subsequently converted into tensorial surface tensions that keep the enclosed quarks and gluons inside super-baryons stably confined.

When integrating \( \Delta\varepsilon^+ \) over all baryons that reached \( \rho = \rho^{uni}_{max} \), one should obtain an energy that is equal to the rest energy of the individual and infinitely-separated baryons. For \( \rho = \rho^{uni}_{max} \), this extra-energy is necessary for facilitating the phase transition of the fermionic QGPs inside individual baryons into the Bosonic superfluid phase, where the mass-energy of the constituents is uniformly distributed with restored Chiral symmetry.

The process here is reversible, as once the symmetry is broken, the super-baryon starts decaying, thereby liberating \( \Delta\varepsilon^+ = \Delta E^g_{tot} \) in a
run-away hadronization procedure.

A straightforward consequence of this scenario would be that the first generation of massive NSs should have metamorphosed into invisible dark energy objects (DAOs), to end up doubling their initial mass. At a certain point in their cosmic history, the cores, would have to decay, thereby hardonizing their entire contents and giving rise to a monstrous explosion through which each DEO would liberate roughly $10^{-57}$ GeV. Here the entire content of baryons is jettisoned into space almost with the speed of light. Recalling that the lifetime of neurons in free space is extremely short compared to protons, then the present scenario may nicely explain the dominant abundant of hydrogen in our universe.

As DEOs are made of SuSu matter, which is characterized by a single characteristic speed: the speed of light and as they are embedded in purely flat spacetime, then the decaying front should propagate almost with the speed of light and would turn the object apart in less than $10^{-5}$ s. The straightforward consequence here is that the ultra-relativistic outward-propagating baryons would leave the center sufficiently fast, before the embedding spacetime starts changing its curvature significantly. At such high speeds, the baryons are too energetic and elements heavier than hydrogen are unlikely to form, which in turn, may explain the dominant abundance of hydrogen atoms in our universe.

### 3 A possible origin of $\rho_{cr}^{uni}$

Given the fundamental constants, such as the Planck constant $\hbar$, the speed of light $c$ and the gravitational constant $G$, one may construct the following scales:

$$\ell_p = \sqrt{\frac{\hbar c}{G}} = \mathcal{O}(10^{-33}) \text{ cm},$$

$$m_p = \sqrt{\frac{\hbar c}{G}} = \mathcal{O}(10^{-5}) \text{ g}.$$  \hspace{1cm} (12)

The inclusion of $G$ in these two expressions is purely ad hoc and supported by neither astronomical observations nor experimental data. However, it suggests a range of scales, where the effects of both GR and quantum fields may overlap theoretically. On the other hand, the spacetime embedding an incompressible supranuclear dense superfluid, such as inside the cores of massive NSs, is conformally flat [32], gravity-effects diminish, and therefore the characteristic length and time scales governing such fluids are independent of $G$.

Moreover, the incompressibility condition, i.e. $d\varepsilon/d\text{Vol} = 0$, implies that the separation between the massive constituents making the mass-energy of the fluid should saturate around a minimum length scale $\ell_{uni}^{\text{min}}$. Recalling that the effective masses of baryons are endowed by quarks and gluons, $\ell_{uni}^{\text{min}}$ is expected to mainly be related to quarks. Indeed, the separation between nucleons in atomic nuclei, $\Delta_{bb}$, is generally larger than the radius of a single nucleon, which can be reduced through compression exerted either by the curvature of the embedding
spacetime or by momentum-exchange during collisions with other nuclei.

Based on experimental and theoretical studies as well as observations of NSs, we argue that $\ell_{\text{uni}}^{\text{min}}$ should lie in the interval:

$$\left[ \frac{1}{4} \leq \frac{\ell_{\text{uni}}^{\text{min}}}{r_b} \leq 1 \right],$$

(13)

where $r_b$ is the radius of the a baryon at zero-temperature.

Under normal terrestrial conditions, the nuclear density is roughly $n_0 \approx 0.16/\text{cm}^3$, where the radius of the proton lies within the range: $[0.84 \leq r_p \leq 0.87]$ fm [see Ref. and the references therein].

$\ell_{\text{uni}}^{\text{min}} < 1/4$ should be excluded as the resonance energy would largely exceed the energy required for deconfining quarks inside individual baryons. Also, it would correspond to matter density $\rho \gg 10 \times \rho_0$; much larger than the critical density beyond which intermediate massive NSs collapse into BHs (see Eq. 3).

On the other hand, using $\ell_{\text{uni}}^{\text{min}} \approx r_b$, the resulting resonance energy amounts to $\Delta \varepsilon \approx 0.23$ GeV. Summing over all possible numbers of quark-bonds, would yield roughly the rest energy of an individual baryon.

Based thereon, the volume of a massive NS at the end of its luminous lifetime would be the sum of the volumes of the enclosed baryons at zero-temperature, i.e.

$$V_{\text{NS}}^{t=\infty} = \sum_{n=1}^{N} V_n^b = N \times V_0^b,$$

(14)

where $N, V_0^b$ denote the number of merged baryons and the volume of a single baryon at zero-temperature, respectively.

For an intermediate massive NS with $N = 10^{57}$ baryons and $\ell_{qq} = \Delta qq = r_b = 0.85$ fm, we obtain a radius $R_{\text{NS}}^{t=\infty} = N^{1/3} r_b = 1.0168 \times 10^6$ cm. While $R_{\text{NS}}^{t=\infty}$ here is larger than the corresponding Schwarzschild radius, it is still smaller than the last stable orbit, though these comparisons are irrelevant, as the spacetime inside $R_{\text{NS}}^{t=\infty}$ is Minkowski flat and not curved Schwarzschild.

Recalling that the action of compression of baryons by the embedding curved spacetime under zero entropy conditions, is to mainly transform the positive energy $\Delta E_{\text{tot}}^{r}$ (see Eq. 11) into storable energy $dW$.  

Figure 4: A schematic description of arbitrary two baryons in the maximally compressible state on the verge of merging to form incompressible gluon-quark superfluid. Here the optimal separation between three quark flavour inside a baryon at $\rho_{\text{uni}}^{\text{max}}$ and $T = 0$ is $\ell_{qq} = \Delta qq = r_b = 0.85$ fm.
From global energy conservation, one finds:

$$\Delta E_{\text{tot}}^g = dW = -PdV = -P \left( d \sum_{i=1}^{N} \Delta V_i \right), \quad (15)$$

where $V$ consists of the sum of compressible space between baryons, $\Delta V_{bb}$, and those of the incompressible gluon-quark plasma inside baryons at zero-temperature, $V_{b0}^g$. For a given number $N$ of baryons, we obtain $V = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \Delta V_i + NV_{b0}^g$, where $\Delta V_i(r)$ is a radius-dependent quantity which is expected to increase as the surface of the object is approached.

Once the neighboring baryons are compressed together and came into direct contact with each other (see Fig. 4), $\Delta V_i$ reaches the lower limit:

$$\Delta V_i \xrightarrow{\Delta_{qq} \to \ell_{qq}} V_{b0}^g, \quad (16)$$

which corresponds to the maximum compressibility state of matter. Under these critical conditions, the available free energy due to spacetime compression cannot be stored locally. One reasonable possibility would be that the quantum fluid undergoes a phase transition into a pure incompressible state, and the free energy goes into enhancing the pairing processes of quarks and energize the confining force of the quarks inside the super-baryon.

As a consequence, the enhanced energetics of the super-baryon implies that the embedding curved spacetime must flatten. The limiting case here would be a vanishing gravitational energy, $\Delta E_{\text{tot}}^g$. This energy state corresponds to the end cosmic phase of NSs, where they become invisible DEOs embedded in Minkowski spacetimes. In this case, the total mass-energy of a single DEO consists of:

$$E_{\text{tot}}(t = \infty) = E_{\text{rest}} + \Delta E_{\text{tot}}^g|_{t=0}$$

$$= N \times 0.931 \text{ GeV} + E_g|_{t=0} \quad (17)$$

$$= 2 \times N \times 0.931 \text{ GeV},$$

where $\Delta E_{\text{tot}}^g|_{t=0}$ is the initial gravitational energy stored in the curved spacetime embedding a newly born pulsar (see Fig. 5). In the absence of destructive external forces, the gravitational and thermodynamical properties of DEOs suggest that
these should be long-living objects with a lifetime most likely much longer than the current age of the universe. Once a fully evolved DEO starts decaying, its hadronization process goes instantly, thereby liberating its total $\Delta E_{\text{tot}}^{\text{sp}}|_{t=0}$. Recalling that the DEOs are governed by one single speed, the speed of light, its hadronization process would be associated with a giant explosion through which the entire content of the SB is jettisoned into almost a flat spacetime with ultra-relativistic speeds, i.e. sufficiently fast, before the embedding spacetime starts re-curving.

Due to the relatively short lifetime of neutrons in free space, the hadronization process of the entire DEO would give rise to creating hydrogen and light elements.

Finally, there are another two issues that support the present scenario: The hyperon puzzle in NS and the predicted radii of the pre-merged NSs in GW170817. In the former case, it was shown that the inclusion of hyperons production in the EOSs reduces the mass of the NSs significantly and make them incompatible with observations, whereas hybrid stars with cores made of deconfined quark would reverse this reduction tendency [see 39 for further details]. Also, noting that the onset of hyperon production occurs for $\rho \geq 2 \rho_0$, the formation of incompressible gluon-quark superfluid in their cores is a viable scenario. In the latter case, however recent studies of the internal structure of the NSs in GW170817 using sophisticated EOSs with tidal deformability, predict their radii to be roughly 11 km. Our scenario predicts that isolated NSs would end up their luminous lifetime as DEOs having radii of order $R_{NS}^{t=\infty} \approx 10$ km, which is a reasonable prediction, when noting that the volume of isolated NSs should shrink as they evolve on the cosmic time.

3.0.1 Asymptotic freedom of SuSu-matter

Although the density and entropy regimes of quantum fluids inside the cores of ultra-compact objects are totally different from those of particle collisions under terrestrial conditions, the QCD properties in the former case are expected to be even simpler than in the latter case. Indeed, the spatial variation of local thermodynamical quantities both in the maximally compressible and purely incompressibility fluid phases would vanish. The incompressible superfluid core is characterized by one single length scale: the radius of the core, $R_{\text{core}}$, and one single time scale $\tau_{\text{core}} = R_{\text{core}}/c$. As all constituents occupy the same quantum state, the momentum transfer $Q^2$ between them saturates around the maximum possible value, where the divergence of the tensorial momentum, $Q_{\mu\nu}$ are forbidden, i.e.

$$Q_{\mu\nu} = 0$$

In terms of the renormalization group equation for the running coupling constant $\alpha_s$ [see 40 and the references therein]:

$$Q^2 \frac{\partial \alpha_s(Q^2)}{\partial Q^2} = \beta(\alpha_s(Q^2)),$$ (18)

the conditions to be imposed on $Q^2$ at both boundaries are identical and therefore $\alpha_s(Q^2)$ must remain
constant. In fact the Chiral symmetry is fully re-
stored ($\langle \bar{q}q \rangle = 0$) in this regime, which grants the
core dynamical stability on cosmic timescales.

3.1 The bimetric universe and CMB ra-
diation

While the existence of a universal maximum energy
density promotes the hypothesis that our universe is
ininitely large and old, it suggest that it went nei-
ther through an inflationary phase nor creating BHs.
The model is still conforms with the smooth and
uniform temperature of the cosmic microwave back-
ground radiation (CMB). Here the instant decay of
a finite and perfectly symmetric cluster of DEOs,
whose contents are made of quark-gluon superfluids
with restored Chiral symmetry may have the poten-
tial of creating the CMB we observe today. In this
case, our location in the embedding spacetime must
have been pretty close to the center of a spherically
symmetric gigantic explosion, which is termed here
as a local big bang (LBB). The progenitor of the
LBB is a giant cluster of DEOs embedded in flat
spacetimes. Have the cluster started decaying, its
whole content undergo a rapid hadronization pro-
cess, and the energy stored in the system undergo
a reversible process: part of this energy goes back
into curving the embedding spacetime, while the rest
goes into forming a soup of highly energetic particles
and radiation.

Here a race starts between the outward-oriented
ultra-relativistically propagation of fluid flows and
topology fitting of the embedding spacetime. Hence
two fronts were created: The hardonization front
that must go through the entire cluster, and the
topological front, “$f_1$” at the interface between the
flat the curved spacetimes (see Fig.6). Although the
material front propagates at ultra-relativistic speed,
the topological front “$f_1$” propagates with the speed
of light. This generates a time delay that increases
with the cosmic age. Note that the hardonization
front is associated with the creation of interacting
particles, generating entropy, and turning the matter
into compressible dissipative media. In the course of
this process, the embedding spacetime ought to un-
dergo a topological change from Minkowski flat into
a curved Schwarzschild. However, as the amount of
energy that goes into curving the embedding space-
time is finite, the curvature should decrease as the
expansion goes on, thereby asymptotically converg-
ing into a perfectly flat spacetime (see Fig. 6).

Mathematically, let the rest mass-energy of the clus-
ter of DEOs on the verge of an LBB explosion be $E_{cl}$.
As the cluster is embedded in a flat spacetime, the
gravitational energy vanishes, i.e., $E_g = 0$. Assuming
mass-energy conservation, then the total energy shortly before and after LBB should remain con-
stant, i.e.:

$$\left\{ [E_{cl}^b + E_{cl}^{vac}] + E_g \right\}_{t \leq 0} = \left\{ E_{cl}^b + [E_{cl}^{vac} + E_g] \right\}_{t > 0}, \quad (19)$$

where $E_{cl}^b$ is the rest energy of the total baryons at
the nuclear density, $\rho_0$, and zero-entropy, $E_{cl}^{vac}$ is the
work needed for compressing the constituents in the
system from $\rho_0$, up to the maximum compressibility
limit, where $\rho = \rho_{\text{uni, max}}$. $E_g$ is the gravitational energy,
which is calculated from the integral:

$$\mathcal{E}_g = \frac{1}{16\pi} \int_{S^{n+1}} \frac{d^2 S_j}{(\partial_{x^k} g_{jk} - \partial_{x^j} g_{kk})}$$

\[
\begin{cases} 
0 & \text{flat spacetime} \\
\mathcal{E}_{cl}^b & \text{Schwarzschild spacetime.} 
\end{cases}
\]

(20)

Based thereon, the total mass involved in the big bang may be predicated as follows: for an average density of $\rho_{\text{now}} \approx 10^{-29} \text{ g/cc}$, and a radius of $R_{\text{uni}} > c \times t_{\text{age}} \approx 10^{28} \text{ cm}$, we obtain a total mass of $\mathcal{E}_{cl}^b \approx 5 \times 10^{22} \text{ M}_\odot$. Inside the DEO cluster the density is roughly uniform and has the value $\rho_{\text{max}}^{\text{uni}}$. This yields a radius of approximately $2 \text{ AU}$ for the DEO cluster. Note that a SMBH with $R_S = 2 \text{ AU}$ yields an enclosed mass of roughly $10^8 \text{ M}_\odot$ which falls in the lower mass-regime of currently observed quasars. Alternatively, a SMBH of $10^{22} \text{ M}_\odot$ would yields Schwarzschild radius of order $10^{27}$, which should be ruled as well.

Indeed, the cluster of DEOs presented here has a uniform density and is embedded in a Minkowski spacetime, so that the necessity for an exponential growth to smooth out density irregularities becomes it unnecessary.

3.2 Summary & Discussion

In this paper, we have discussed the possibility that our universe may permit the existence of a universal maximum energy density $\rho_{\text{uni}}^{\text{max}}$, which characterizes the state of incompressible superfluids at the center of massive NSs. Based on theoretical and observational studies of pulsars and neutron stars, $\rho_{\text{max}}^{\text{uni}}$ is predicted to lie between $2 \times \rho_0$ and $3 \times \rho_0$. Under these conditions and at zero-temperature, the irreducible distance between quarks is predicated to be of order: $\ell_{\text{qq}} \approx 0.85 \text{ fm}$, i.e. to the experimentally revealed value of the radius of a stable baryon.

The underlying assumption here is that UCOs should be embedded in bimetric spacetimes: their incompressible quark-gluon superfluid cores are embedded in Minkowski-type spacetimes, whereas the ambient compressible and dissipative media are...
embedded in curved spacetimes. Based on this bimetric spacetime scenario and on the universal maximum density hypothesis, solutions to several debates both in physics and astrophysics could now be provided:

- Incompressible fluids are treatable using the field equations of General Relativity and may be modeled in a self-consistent manner, without violating the causality condition or creating physically unrealistic ultrabaric regimes [see Fig. 7 in [16]].

- The quantum mechanisms underlying the glitch phenomena in pulsars, the increasing intervals between successive glitches as the objects age, the origin of under and over-shootings of the rotational frequencies observed to associate the glitch events of the Vela pulsar can be well explained.

- Formation of stable massive NSs without collapsing into stellar BHs, such as the remnant of GW170817, may be considered as a reliable evolutionary track. In this case the remnant of the first generation of massive NSs should have fully metamorphosed into invisible DEOs by now.

- The cosmological origin of high redshift QSOs and dark matter in the early universe conform with our scenario that the universe should be infinitely large and old.

- The dominance and origin of chemical abundance of the light elements in the universe is a natural consequence of the collective decay clusters of DEOs that led to LBB explosion through which hadronization operated efficiently.

- The scenario of a bimetric universe provides a simple and reasonable explanation to the flatness problem of the universe, the origin of dark matter and dark energy.

- The present scenario is capable of providing a new robust mechanism for the origin and power of high redshift QSOs. Indeed, the topological front that separating the Schwarzschild from the Minkowski spacetimes carries sufficient energy to significantly deform the locally curved spacetime embedding distant galaxies, thereby turning the well-ordered motions of their contents of objects into turbulent motion. This excited energy state would considerably increase the rate of destruction and collision of stars and would turn the galaxy into an active mode for a considerable cosmic time.

The duration of this activity starts from the moment at which the topological front collides with locally curved spacetime embedding the respective galaxy and would continue until the remaining stellar components have settled back into a significantly lower energy state [see [41] for further details].

One of the far-reaching consequences here is that the big bang should be a recurrent phenomenon in our infinitely large and old universe. Indeed, the existence of a maximum universal density \( \rho_{cr}^{uni} \) would render the inflationary phase unnecessary and would
naturally explain why our universe escaped its collapse into a giant BH.

The scenario here predicts that massive stars should have sufficient time to collapse into massive UCOs, cooldown and to subsequently turn invisible at the end of their luminous lifetimes. Their further collapse into BHs is prohibited by the existence of $\rho_{\text{crit}}^{\text{Uni}}$, at which the topology of the embedding spacetime changes into a Minkowski-type spacetime (see Hujeirat 2021 in preparation).

Nevertheless, our scenario addresses several new problems that need to be answered, such as:

1. What is the physical nature of the well-observed stellar and massive black holes?

2. What are the physical mechanisms that drive DEOs to conglomerate into clusters, how DEOs interact with each other or with other gravitationally bound astrophysical objects?

3. What are the predicated lifetimes of DEOs, and what are the mechanisms underlying their decay?

References

[1] Abbott et al., Ligo & Virgo Collaboration, (2017), ApJL, 848, L12

[2] Falcke, H., Laing, R., Testi, L. & Zensus, A., (2012), ESO Headquarters Astronomical News

[3] Nicholl, M., Margalit, B., Schmidt, P., et al. (2021), arXiv:2102.02229

[4] Psaltis, D., et al. (EHT Collaboration) (2020), Phys. Rev. Lett., 125, 141104

[5] McInnes, B., (2017), Nuclear Physics B921, 39

[6] Pasechnik, R., Šumbera, M., (2017), arXiv:1611.01533v3

[7] Shuryak, E.V., (2004), arXiv: 0405066v

[8] Hujeirat, A.A., Samtaney, R., (2020), Journal of Modern Physics, Vol.11, No.11

[9] Fan, X., Wang, F., Jinyi Yang, J., et al. (2019), ApJL, 870, L11

[10] Oesch, P. A. et al. (2016), Astrophys. J. 819, 129

[11] Yang, J., Wang, F., Fan, X., et al. (2020), arXiv:2006.13452

[12] Shimasaku, K., Izumi, T., (2019), ApJL, 872, L29,

[13] Smith, A., Bromm, V., 1 and Abraham Loeb, A. (2017), arXiv: 1703.03083v2

[14] Carr, B.J., Kohri, K., Sendouda, Y., Yokoyama, J., (2010), arXiv: 0912.5297v2

[15] Abbott et al., 2019, Phys. Review X 9, 011001

[16] Hujeirat, A.A. (2018), Journal of Modern Physics, 9, 532, https://doi.org/10.4236/jmp.2018.94037

[17] Hujeirat, A.A., Samtaney, R., 2020, Journal of Modern Physics, Vol.11 No.11
[18] Hujeirat, A.A., Samtaney, R., (2020), Journal of Modern Physics, Vol.11, No.3

[19] Ashton, G., Lasky, P.D., Graber, V. and Palfreyman, J.,(2019), Astro-ph. 1907.01124

[20] Arbey, A. Ellis, J., Mahmoudi, F., Robbins, G., (2018), arXiv:1811.12764v1

[21] Bogdan, A., (2020), astro-ph 2009.07846v2

[22] Argueelles,C.R., Dieaz, M.I., Krut, A., et al. (2021), MNRAS, Vol. 502, 4227

[23] Gustafsson B., Ross P., Church M.B.D., (2016), A&A, 593, A85

[24] Barkana, R., (2018), Nature, Vol. 555, P 71

[25] Tenkanen, T., (2019), Phys. Rev. Lett. 123, 061302

[26] Sivakumer, A., Muruganandam, P., Hujeirat, A.A., (2021), in preparation

[27] Roy, J., Yashwant Gupta, Y. and Lewandowski, W. (2012) MNRAS, 424, 2213-2221.

[28] Espinoza, C.M., Lyne, A.G., Stappers, B.W. and Kramer, C. (2011) MNRAS, 414, 1679

[29] Gotthelf, E.V., Halpern, J.B., Allen, B., et al. (2013), ApJ, Vol. 773, 2

[30] Ashton,G., Prix, R., Jones, D.I., (2017), Phy. Rev. D, 96, 063004

[31] Ng, C. Champion, D.J., Bailes, M., et al. (2015), astro-ph 1504.08000

[32] Hujeirat, A.A., (2020), Journal of Modern Physics, Vol.11, No.11

[33] Bromm, V., (2013), arXiv:1305.5178

[34] Fischer, T., Wu, M-R., Benjamin Wehmeyer, B., et al., (2021), arXiv:2003.00972

[35] Chon,S., Hosokawa, T., Kazuyuki Omukai, K., et al. (2021), MNRAS, Vol. 502, 700

[36] Mueller, B., (2007), arXiv:0710.3366

[37] Witten, E., (1981), Commun. Math. Phys. 80, 381

[38] Karr, J-P., Marchand, D., (2019), Nature. 575, 61

[39] Bombaci, I., (2016), arXiv:1601.05339v1

[40] J. A. Gracey, J.A., (2020), Phys. Rev. D, 101, 125022

[41] Hujeirat, A.A., (2021), In preparation