Generalized Cramer-Rao Bound for Joint Estimation of Target Position and Velocity for Active and Passive Radar Networks

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Abstract—In this paper, we derive the Cramer-Rao bound (CRB) for joint target position and velocity estimation using an active or passive distributed radar network under more general, and practically occurring, conditions than assumed in previous work. In particular, the presented results allow nonorthogonal signals, spatially dependent Gaussian reflection coefficients, and spatially dependent Gaussian clutter-plus-noise, which are cases of great practical interest. In fact, one could argue that all of these conditions are true in any real system. The initial CRBs we present are applicable to both active and passive radar systems, provided the signals of opportunity in the passive systems are assumed to be perfectly estimated from, for example, the direct path signal reception. These results further assume all the parameters of the observations model are known, including the covariance matrices of the zero-mean Gaussian noise-plus-clutter and reflection coefficients. The mismatched CRB results given in this paper even allow cases where the model assumed by the estimation algorithm is incorrect, including cases where the model for the direct path signal may involve unmodeled noise, interference, or some other imperfection. Similarly, the reflection coefficients, noise, and/or interference may be incorrectly modeled and the mismatched CRB will still provide a lower bound on performance.

While both active and passive radar systems are of great interest, passive radar systems may have attracted even greater attention over the past few years due to the tremendous advantages they provide from using existing communication signals to implement a radar, essentially borrowing the already existing transmitter infrastructure and providing no electronic evidence that a radar is operating in a given area. Passive radar, as the name implies, is a radar system which receives signals to implement a radar, essentially borrowing the already existing communication signals or very simple analytical models. The factors that a detection performance of passive coherent location radar systems or very simple analytical models. The factors that a detection performance of passive coherent location radar systems are discussed in [14]. The ambiguity functions of a set of off-air measurements of signals that may be used for passive coherent location (PCL) radar systems are presented and...
analyzed in [15]. The problem of target detection in passive MIMO radar (PMR) networks comprised of non-cooperative transmitters and multichannels is addressed in [16].

As described later, the CRB is a lower bound, in a certain sense, on the covariance matrix of all unbiased estimators. It is a useful tool for evaluating the best possible estimation performance of a radar system. A derivation of the stochastic CRB is provided in [17]. The CRB expressions for the estimation of range (time delay), velocity (Doppler shift) and direction of a point target using an active radar or sonar array are given in [18]. The CRB of DOA estimation of a non-stationary target for a MIMO radar with colocated antennas for a general time division multiplexing (TDM) scheme is computed in [19]. The CRB for bistatic radar channels is derived in [20], which also exploits the relationship between the ambiguity function and the CRB. Cramer-Rao-like bounds for the estimation of a deterministic parameter in the presence of random nuisance parameters are derived in [21], [22].

For the case of multiple transmit and receive antennas employed in a distributed active radar setting, [24] describes the CRB under the assumption of orthogonal signals, spatially independent reflection coefficients, and spatially independent clutter-plus-noise. For estimation of the position and velocity of a single target using a passive radar, the CRB and ambiguity functions are considered in [25] for a multiple transmitter and receiver radar, but only for the case where a single transmitter and receiver pair is selected from among a much larger set of possible pairs. This work does not consider the effect of signal nonorthogonality or spatially dependent reflection coefficients or noise. Under the same assumptions employed in [24], the CRB has been derived for passive radar settings with well estimated signals of opportunity in [26], [27].

Thus, none of the published work has given the CRB for the important and practical case of nonorthogonal signals, spatially dependent reflection coefficients, and spatially dependent noise for joint target position and velocity estimation performance using a distributed passive or active radar network employing all signals available from the multiple transmit and receive antenna paths in an optimum manner. This result is extremely useful since it describes the best achievable performance for some important cases for the first time. Knowing this best achievable performance allows designers to compare the performance of their developed approaches, to these bounds. If their developed approaches lead to performance close to the bounds, these developed approaches can be deemed "good enough" while these developed approaches are typically constrained to have acceptable complexity. The very recent work in [23] provides an excellent example where these results can be extremely useful. In [23], a very practical scenario is considered where a number of transmitters of opportunity send digital TV signals that can not be accurately modeled by assuming the transmitters send a signal of nonorthogonal signals. The work in [23] presents an interesting suboptimum algorithm for implementing a radar employing these nonorthogonal signals. However, it is not known how far the performance of the suggested approach in [23] is from the optimum achievable performance. Such information would be extremely useful in judging if the approach suggested in [23] provides a good tradeoff in terms of performance and complexity. Similar questions arise in many related practical applications, some of which involve active radars.

In this paper, we consider these more general cases and derive a generalized CRB and mismatched CRB for joint location and velocity estimation in passive and active distributed radar networks. The presented results do not assume the approximate location of the target is known from previous target detection signal processing, unlike the previous results employing optimum processing using all available antennas [24], [26], [27]. A closed-form Fisher information matrix (FIM) is presented. In a few representative cases, the generalized or mismatched CRB is numerically compared with the mean-squared error (MSE) from maximum likelihood (ML) estimation to show consistency at higher signal-to-noise ratio (SNR). We use GSM signals as illuminators for our numerical passive radar investigations. The rest of this paper is organized as follows. The signal model for active and passive distributed radar networks is presented in Section II. The ML estimate is analyzed in Section II-A. In Section III, the generalized CRB is derived. In Section IV, we derive the mismatched CRB. Performance analysis and numerical examples are presented in Section V. Finally, conclusions are drawn in Section VI.

Throughout this paper, the notation for transpose is $(\cdot)^T$, while that for complex conjugate is $(\cdot)^H$. The symbol $\text{Diag} \{ \cdot \}$ denotes a block diagonal matrix with the matrices in the braces being the diagonal blocks, $CN(\mu, \mathbf{R})$ denotes a complex Gaussian distribution with mean vector $\mu$ and covariance matrix $\mathbf{R}$. $\mathbb{E}_{r|\theta, \alpha} \{ \cdot \}$ implies taking expectation with respect to the probability density function (pdf) $p(r|\theta, \alpha)$. $\text{Tr}(\cdot)$ denotes the trace of a matrix, $\otimes$ represents the Kronecker product, $\mathbb{R}(\cdot)$ means taking the real part, $\circ$ represents the Hadamard product, and $\text{vec}(\cdot)$ denotes the column vectorizing operator which stacks the columns of a matrix in a column vector.

II. SIGNAL MODEL

Consider a distributed radar system with $M$ widely spaced single antenna transmit stations and $N$ widely spaced single antenna receive stations, located at $(x_m', y_m')$, $m = 1, \ldots, M$ and $(x_n', y_n')$, $n = 1, \ldots, N$ in a two-dimensional Cartesian coordinate system, respectively. The lowpass equivalent time-sampled version of the signal transmitted from the $m$th transmit station at time instant $kT_s$ is $\sqrt{E_m} s_m(k, \alpha_m)$, where $T_s$ is the sampling period, $k$ ($k = 1, \ldots, K$) is an index running over the different time samples, $\alpha_m$ denotes a vector of parameters needed to describe the waveform, and the waveform is normalized using $\sum_{k=1}^{K} |s_m(k, \alpha_m)|^2T_s = 1$. Let $E_m$ denote the energy transmitted by the $m$th transmit antenna. Then the received waveform at the $m$th receiver at time $kT_s$ is

$$r_n(k) = \sum_{m=1}^{M} \sqrt{\frac{E_m P_0}{d_{nm}^2 d_{ln}^2}} \text{Diag}(s_m(kT_s - \tau_{nm}, \alpha_m)) e^{j2\pi f_{0m}kT_s} + w_n(k),$$

(1)
where $\tau_{nm}$, $f_{nm}$, and $\zeta_{nm}$ represent the time delay, Doppler shift, and reflection coefficient corresponding to the $nm$th path, respectively. The variable $d_{nm}$ denotes the distance between the target and the $nm$th transmitter, while $d_r$ denotes the distance between the target and the $nr$th receiver. The term $w_n(k)$ denotes clutter-plus-noise at the $nr$th receiver at time $kT_s$. The received signal strength at $d_{nm}=d_r=1$ is $\sqrt{E_{mn}P_0}$, so $P_0$ denotes the ratio of received energy at $d_{nm}=d_r=1$ to transmitted energy. The reflection coefficient $\zeta_{nm}$ is assumed to be constant over the observation interval and to have a known complex Gaussian statistical model [28]. Assume the position $(x, y)$ and velocity $(v_x, v_y)$ of the target are deterministic unknowns. The distances $d_{nm}$ and $d_r$ are expressed in terms of $(x, y)$ as

$$d_{nm} = \sqrt{(x_m - x)^2 + (y_m - y)^2},$$  

(2)

$$d_r = \sqrt{(x_r - x)^2 + (y_r - y)^2}. $$  

(3)

The time delay $\tau_{nm}$ is also a function of the unknown target position $(x, y)$

$$\tau_{nm} = \frac{\sqrt{(x_m - x)^2 + (y_m - y)^2} - \sqrt{(x_r - x)^2 + (y_r - y)^2}}{c},$$  

(4)

where $c$ denotes the speed of light. The Doppler shift $f_{nm}$ is a function of the unknown target position $(x, y)$ and velocity $(v_x, v_y)$ given by

$$f_{nm} = \frac{v_x(x_m - x) + v_y(y_m - y)}{\lambda d_{nm}} + \frac{v_x(x_r - x) + v_y(y_r - y)}{\lambda d_r},$$  

(5)

where $\lambda$ denotes the wavelength. Define an unknown parameter vector $\theta$ that collects the parameters to be estimated

$$\theta = [x, y, v_x, v_y]_T.$$  

(6)

The observations from the $nr$th receiver can be expressed as

$$r_n = [r_n(1), r_n(2), \ldots, r_n(K)]_T = U_n\zeta_n + w_n,$$  

(7)

where $U_n$ is a $K \times M$ matrix that collects the time delayed and Doppler shifted signals at the $nr$th receiver as

$$U_n = [u_n(1), u_n(2), \ldots, u_n(K)]_T,$$  

(8)

where

$$u_n(k) = [u_n1(k), u_n2(k), \ldots, u_nM(k)]_T,$$  

(9)

and

$$u_{nm}(k) = \sqrt{\frac{E_{mn}P_0}{d_{mn}d_{rn}^2}} \exp(j2\pi f_{nm}kT_s).$$  

(11)

The $M \times 1$ reflection coefficient vector $\zeta_n$ can be expressed as

$$\zeta_n = [\zeta_{n1}, \ldots, \zeta_{nM}]_T.$$  

Denote the vector of noise samples at the $nr$th receiver as $\omega_n = [\omega_n(1), \ldots, \omega_n(K)]_T$. The observations from the set of all receivers can be written as

$$r = [r_1, r_2, \ldots, r_N]_T = S\zeta + \omega,$$  

(12)

where $S$ collects the time delayed and Doppler shifted signals from all paths

$$S = \text{Diag}(U_1, U_2, \ldots, U_N).$$  

(13)

The $\zeta$ in (12) collects reflection coefficients for all paths

$$\zeta = [\zeta_1^T, \ldots, \zeta_N^T],$$  

(14)

and it is assumed that $\zeta$ is a complex Gaussian distributed vector with zero mean and covariance matrix $R = \mathbb{E}(\zeta\zeta^H)$, i.e. $\zeta \sim \mathcal{CN}(0, R)$. The $w$ in (12) denotes the clutter-plus-noise vector

$$w = [w_1^T, \ldots, w_N^T],$$  

(15)

which is assumed to be complex Gaussian distributed with zero mean and covariance matrix $Q = \mathbb{E}(ww^H)$, i.e., $w \sim \mathcal{CN}(0, Q)$. Assume that the noise vector $w$ is independent from the reflection coefficient vector $\zeta$.

### A. Maximum Likelihood Estimation

In this and the next section (Sections II and III), we assume $S$ (and thus $\alpha$), $Q$, and $R$ are known to the estimation algorithm. We address other cases later. Using the signal model in (12) and the fact that the linear combination of two Gaussian vectors is also Gaussian, the likelihood function conditioned on the waveform parameter vector

$$\alpha = [\alpha_1, \ldots, \alpha_M]^T,$$  

(16)

is independent

$$\pi(\alpha, \theta) = \frac{1}{\pi^{KN} \text{det}(C)} \exp(-r^H C^{-1} r),$$  

(17)

where $C$ denotes the covariance matrix

$$C = \mathbb{E}\left[(S\alpha + w)(S\alpha + w)^H\right] = \mathbb{E}\left[S\alpha\alpha^H + w w^H\right] = SRS^H + Q.$$  

(18)

The log-likelihood function can be written as

$$L(r|\theta, \alpha) = \ln \pi(\alpha, \theta, \alpha) = -r^H C^{-1} r - \ln(\text{det}(C)) - KN \ln(\pi) .$$  

(19)

Neglecting the last constant term of the second line in (19) and assuming known or perfectly estimated $\alpha$, the (ML) estimate of the unknown parameter vector $\theta$ can be calculated as

$$\hat{\theta}_{\text{ML}} = \arg\max_{\theta} L(r|\theta, \alpha) \equiv \arg\max_{\theta} \left\{ -r^H C^{-1} r - \ln(\text{det}(C)) \right\}.$$  

(20)
III. Generalized Cramer-Rao Bound

In this section, we provide the CRB for jointly estimating the target location \((x, y)\) and velocity \((v_x, v_y)\) for the case where \(S\) (and thus \(\alpha\)), \(Q\), and \(R\) are known to the estimation algorithm. The first step in finding the CRB is to compute the FIM, which is a \(4 \times 4\) matrix related to the second order derivatives of the log-likelihood function

\[
J(\theta|\alpha) = \mathbb{E}_{r|\theta, \alpha} \left[ \nabla_{\theta} L(r|\theta, \alpha) \left\{ \nabla_{\theta} L(r|\theta, \alpha) \right\}^\dagger \right].
\]  

(21)

Considering the likelihood is a function of \(r_m, f_m, d_m,\) and \(d_n\) \((n = 1, \ldots, N, m = 1, \ldots, M),\) which depend on \(\theta = [x, y, v_x, v_y]^\dagger,\) we define an intermediate parameter vector

\[
\vartheta = [\tau^\dagger, f^\dagger, d^\dagger, d^\dagger]^\dagger
\]

\[
\tau = [\tau_{11}, \tau_{12}, \cdots, \tau_{NM}], f = [f_{11}, f_{12}, \cdots, f_{NM}],
\]

\[
d = [d_{11}, d_{12}, \cdots, d_{NM}], d_r = [d_{11}, d_{12}, \cdots, d_{NM}]^\dagger
\]

where \(\tau = [\tau_{11}, \tau_{12}, \cdots, \tau_{NM}], f = [f_{11}, f_{12}, \cdots, f_{NM}],\) \(d = [d_{11}, d_{12}, \cdots, d_{NM}]\) and \(d_r = [d_{11}, d_{12}, \cdots, d_{NM}]^\dagger\) collect the unknown time delays, Doppler shifts, and distance parameters, respectively. According to the chain rule, the FIM can be derived by

\[
J(\theta|\alpha) = \left( \nabla_{\vartheta} \theta^\dagger \right) J(\vartheta|\alpha) \left( \nabla_{\vartheta} \theta^\dagger \right)^\dagger,
\]

(23)

where \(J(\vartheta|\alpha) = \mathbb{E}_{r|\theta, \alpha} \left[ \nabla_{\vartheta} L(r|\theta, \alpha) \left\{ \nabla_{\vartheta} L(r|\theta, \alpha) \right\}^\dagger \right].\)

A. Calculation of \(\nabla_{\vartheta} \theta^\dagger\)

Recalling (6) and (22), we have

\[
\nabla_{\vartheta} \theta^\dagger = \begin{bmatrix} F & G & D_t & D_r \end{bmatrix},
\]

(24)

where

\[
F = \begin{bmatrix} \frac{\partial \tau_{11}}{\partial x} & \frac{\partial \tau_{11}}{\partial y} & \cdots & \frac{\partial \tau_{NM}}{\partial y} \\ \frac{\partial \tau_{12}}{\partial x} & \frac{\partial \tau_{12}}{\partial y} & \cdots & \frac{\partial \tau_{NM}}{\partial y} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \tau_{NM}}{\partial x} & \frac{\partial \tau_{NM}}{\partial y} & \cdots & \frac{\partial \tau_{NM}}{\partial y} \end{bmatrix},
\]

(25)

\[
G = \begin{bmatrix} \frac{\partial f_{11}}{\partial x} & \frac{\partial f_{11}}{\partial y} & \cdots & \frac{\partial f_{NM}}{\partial y} \\ \frac{\partial f_{12}}{\partial x} & \frac{\partial f_{12}}{\partial y} & \cdots & \frac{\partial f_{NM}}{\partial y} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{NM}}{\partial x} & \frac{\partial f_{NM}}{\partial y} & \cdots & \frac{\partial f_{NM}}{\partial y} \end{bmatrix},
\]

(26)

\[
H = \begin{bmatrix} \frac{\partial d_{11}}{\partial x} & \frac{\partial d_{11}}{\partial y} & \cdots & \frac{\partial d_{NM}}{\partial y} \\ \frac{\partial d_{12}}{\partial x} & \frac{\partial d_{12}}{\partial y} & \cdots & \frac{\partial d_{NM}}{\partial y} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial d_{NM}}{\partial x} & \frac{\partial d_{NM}}{\partial y} & \cdots & \frac{\partial d_{NM}}{\partial y} \end{bmatrix},
\]

(27)

\[
D_t = \begin{bmatrix} \frac{\partial \tau_{11}}{\partial y} & \frac{\partial \tau_{12}}{\partial y} & \cdots & \frac{\partial \tau_{NM}}{\partial y} \\ \frac{\partial \tau_{11}}{\partial y} & \frac{\partial \tau_{12}}{\partial y} & \cdots & \frac{\partial \tau_{NM}}{\partial y} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \tau_{NM}}{\partial x} & \frac{\partial \tau_{NM}}{\partial y} & \cdots & \frac{\partial \tau_{NM}}{\partial y} \end{bmatrix},
\]

(28)

and

\[
D_r = \begin{bmatrix} \frac{\partial d_{11}}{\partial y} & \frac{\partial d_{12}}{\partial y} & \cdots & \frac{\partial d_{NM}}{\partial y} \\ \frac{\partial d_{11}}{\partial y} & \frac{\partial d_{12}}{\partial y} & \cdots & \frac{\partial d_{NM}}{\partial y} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial d_{NM}}{\partial x} & \frac{\partial d_{NM}}{\partial y} & \cdots & \frac{\partial d_{NM}}{\partial y} \end{bmatrix},
\]

(29)

Using calculations drawing on (2)-(5), the elements of the matrices in (25)-(29) will be described as

\[
a_{nm} = \frac{\partial \tau_{nm}}{\partial x} = \frac{1}{c} \left( x - x_m^r \right),
\]

(30)

\[
b_{nm} = \frac{\partial \tau_{nm}}{\partial y} = \frac{1}{c} \left( y - y_m^r \right),
\]

(31)

\[
C = \begin{bmatrix} \frac{\partial f}{\partial \tau} \frac{\partial \tau}{\partial \theta} \frac{\partial \theta}{\partial \sigma} \end{bmatrix}^T \begin{bmatrix} \frac{\partial C}{\partial \tau} \frac{\partial \tau}{\partial \theta} \frac{\partial \theta}{\partial \sigma} \end{bmatrix},
\]

(40)

Using the following identities, [30]

\[
\text{Tr}(ABXY) = \left( \text{vec}(Y^\dagger) \right)^T \left( X^\dagger \right) \text{vec}(B)
\]

(41)

we can rewrite (40) as

\[
\text{Tr}(AB) = \text{Tr}(BA),
\]

(42)
where $C_{\text{vec}} = \text{vec}(C)$. Calculation of the derivatives and further simplification of (43) are provided in Appendix A. Then we can get the final equation

$$
J(\theta|\alpha) = \begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} \\
A_{21} & A_{22} & A_{23} & A_{24} \\
A_{31} & A_{32} & A_{33} & A_{34} \\
A_{41} & A_{42} & A_{43} & A_{44}
\end{bmatrix},
$$

(44)

where

$$
A_{11} = a_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{12} = b_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{13} = \beta_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{14} = k_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{22} = b_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{23} = \beta_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{24} = k_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{33} = \beta_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{34} = k_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

$$
A_{44} = k_{pq}a_{lm}(m_T, r) + e_{nmm}(f_T, r) + v_{m}(J_{d_T}) + m_d N + \eta N(J_{d_T}) + d/M,
$$

(51)

where $c = M(n - 1) + m$ and $d = M(p - 1) + q$. $J_T$, $J_f$, $J_{d_T}$, $J_{d_f}$, and $J_{d_d}$ are defined in (72). It should be noted that, the results obtained here, say (44)-(54), are a highly nontrivial extension of the previous results in [24]. Unfortunately, they are, as one might expect, considerably more complicated but they describe the best possible estimation performance in non-ideal scenarios that are of great practical interest in the following sense. Given any unbiased estimator \(\hat{\theta}\) of an unknown parameter \(\theta\) based on an observation vector \(r\), when \(\alpha\) is assumed known and fixed, we have [29]

$$
\text{MSE} = \mathbb{E}_{\theta|\alpha}\left((\hat{\theta} - \theta)(\hat{\theta} - \theta)^T\right) \geq \text{CRB}(\theta|\alpha) = J^{-1}(\theta|\alpha),
$$

(55)

which is the standard CRB for vector parameters where $A \geq B$ means $A - B$ is positive semidefinite, and MSE is the mean squared error matrix of the unbiased estimator.

IV. CRAMER-RAO BOUND FOR MISMATCHED CASE

In order to find an ML estimate or use the CRB result in (55), now called the generalized CRB (GCRB), we must know the actual values of the signal matrix $S$ (and thus \(\alpha\)) from (13), the reflection coefficients covariance matrix $R$ described near (14), and the noise covariance matrix $Q$ described near (15). Here, we assume the estimation algorithm employs incorrect values for these matrices denoted by $S_0$, $R_0$, and $Q_0$ respectively. The incorrect values $S_0$, $R_0$, and $Q_0$ might be obtained from some inaccurate estimation. Given the estimation algorithm uses these incorrect values $S_0$, $R_0$, and $Q_0$, we find a lower bound on the estimation performance using some recently published work [31]. In the case described, the assumed likelihood function is

$$
p_\theta(r|\theta, \alpha) = \frac{1}{\pi^{KN} \det C_0} \exp(-r^HC_0^{-1}r),
$$

(56)

where $C_0 = S_0R_0S_0^T + Q_0$. To avoid confusion with the GCRB, we denote the actual values of the signal matrix from (13), the reflection coefficients covariance matrix described near (14), and the noise covariance matrix described near (15) by $S_1$, $R_1$, and $Q_1$.

Thus, the actual likelihood function is

$$
p_\theta(r|\theta, \alpha) = \frac{1}{\pi^{KN} \det C_1} \exp(-r^HC_1^{-1}r),
$$

(57)

where $C_1 = S_1R_1S_1^T + Q_1$. According to [31], we know that

$$
\text{MSE}_{\text{mis}} \geq \text{CRB}_{\text{mis}}(\theta|\alpha) = J_{\text{mis}}^{-1}(\theta|\alpha),
$$

(58)

where $\text{MSE}_{\text{mis}}$, $\text{CRB}_{\text{mis}}(\theta|\alpha)$ and $J_{\text{mis}}(\theta|\alpha)$ denote the MSE, CRB and FIM matrices under mismatched situation, and

$$
J_{\text{mis}}(\theta|\alpha) = \mathbb{E}_{p_\theta(r|\theta, \alpha)}\left\{ \left( p_\theta(r|\theta, \alpha)^2 \right) \right\}^T
$$

(59)

Next note that $\nabla_\theta \log p_\theta(r|\theta, \alpha) = (\nabla^T \theta), \nabla_\theta \log p_\theta(r|\theta, \alpha)$ and $p_\theta(r|\theta, \alpha) = p_\theta(r|\theta, \alpha)$ so that

$$
J_{\text{mis}}(\theta|\alpha) = (\nabla^T \theta)J_{\text{mis}}(\theta|\alpha)(\nabla \theta)^T,
$$

(60)
where

\[ J_{n,m}(\theta|\alpha) = \mathbb{E}_{p_1(r|\theta, \alpha)} \left( \frac{p_0(r|\theta, \alpha)}{p_1(r|\theta, \alpha)} \right)^2 \times \nabla_{\theta} \log p_0(r|\theta, \alpha) \left\{ |\nabla_{\theta} \log p_0(r|\theta, \alpha)|^2 \right\} \]  

(61)

Calculation of the derivatives and further simplification of (61) is omitted due to similarity to the case without mismatch.

V. Numerical Examples

In this section, examples are presented which demonstrate the use of the GCRB and the mismatched CRB presented in the previous section to bound the performance of distributed radar networks which employ multiple widely spaced transmitters and receivers to jointly estimate target position and velocity. For brevity, we focus on examples which employ signals that are more applicable for passive radar. Initially, we describe performance when the transmitted signals are either known or where the transmitted signals of opportunity are estimated perfectly from a direct path reception. Later we consider cases where this is not true. We also assume that the positions of the transmitters and receivers are exactly known. For passive radar cases, these assumptions allow us to describe the best possible performance that can be obtained under the best circumstances.

It is easy to employ our bounds for cases where all parameters to be included in the vector \( \alpha \) are known and thus the bound in (55) is applicable. However, the vector \( \alpha \) might include a random bit sequence which contains information being transmitted. In order to avoid presenting a CRB for every possible bit sequence \((\alpha)\), we quote the expected CRB averaged over all bit sequences (ECRBOB), assuming each bit sequence to be perfectly estimated. From (55),

\[ \text{ECRBOB}(\theta) = \mathbb{E}_\alpha \left[ \text{CRB}(\theta|\alpha) \right] \]  

(62)

clearly bounds the corresponding covariance matrix averaged over all bit sequences. For the best case, when the bit sequence in \( \alpha \) is perfectly estimated, the ECRBOB is a good indicator of performance. For example, it describes how the system parameters, such as the number of antennas, the geometry, and the waveforms impact performance, assuming accurate estimation of \( \alpha \). One can use the ECRBOB to optimize any parameter of interest.

Consider a target moving with velocity \((50,30)\) m/s is present at \((15,15,10,1275)\) km. To define a general test set up that is easy to describe for general \( M \) and \( N \), each transmit and receive (single antenna) station is located 7 km from the reference point \((15,10)\) km. The \( M \) transmit stations are uniformly distributed in angle over the range \([0,2\pi]\), i.e., the angle of the \( m \)-th transmitter is \( \varphi_m = 2\pi(m-1)/M, m = 1, \ldots, M \). The \( N \) receive stations are also uniformly distributed in angle over the range \([0,2\pi]\), i.e., the angle of the \( n \)-th receive station is \( \varphi_n = 2\pi(n-1)/N, n = 1, \ldots, N \), where the angles are measured with respect to the horizontal axis originated at the reference point as illustrated in Figure 1. Suppose \( E_1 = E_2 = \ldots = E_M = E \). Fix \( \text{SNCR} = 10 \log\left( (\sum_{m=1}^M \sigma_{mm}^2 + \sigma_{mn}^2) / E \right) \), called the signal-to-clutter-plus-noise ratio (SCNR), where \( \sigma_{nn} = \mathbb{E}_w \left[ \left| \xi_n \right|^2 \right] \) and \( \sigma_{mn} = \mathbb{E}_w \left[ \left| \xi_m \xi_n \right| \right] \).

Fig. 1: Parameter set up for a distributed radar network with \( M = 2 \) and \( N = 3 \).

and \( \sigma_{nn}^2 = \mathbb{E}[w_n(1)w_n(1)^H] = \cdots = \mathbb{E}[w_n(K)w_n(K)^H] \). We set \( \sigma_{nn}^2 = 1 \) for all \( n \) and \( m \), and \( P_0 = 1 \).

To be relevant to a passive radar system, the signals considered are those employed by the popular Global System for Mobile (GSM) Communications system. The baseband transmitted waveforms are Gaussian minimum shift keying (GMSK) signals [13]

\[ s_m(k,\alpha_m) = A_m \exp \left\{ \sum_{i=1}^N c_{mi} \sum_{j=1}^k z(kT_s - iT_p) Y_j \right\} e^{2\pi i \cdot j \cdot kT_s} \]  

(63)

where \( \alpha_m = \left[ \alpha_m^1, \ldots, \alpha_m^N \right] \),

\[ z(t) = \frac{\pi}{2T_p} \left\{ \theta_\tau \left( \frac{2\pi B}{\sqrt{2}} \right)^t - \theta_\tau \left( \frac{2\pi B}{\sqrt{2}} \right)^t \right\} \]  

(64)

\( \theta_\tau(t) = \sqrt{2\pi} \int_0^t e^{-\tau^2/2} d\tau \). \( T_p \) is the bit duration, \( B \) denotes the 3 dB bandwidth of the Gaussian prefiler used in the GMSK modulators, \( c_{mi} \in [-1,1] \) is the \( i \)-th \((i = 1, \ldots, N_m)\) binary data bit of the \( m \)-th transmitted waveform, \( N_m \) denotes the number of bits contained in the observation interval, \( A_m \) is the normalization factor, and \( \Delta f = f_{k+1} - f_k \) is the frequency offset between different signals of opportunity with neighboring frequencies. In the simulations, we generate \( c_{mi} = -1 \) or 1 randomly with the same probability of 0.5. To model a GSM system, assume \( T_p = 577\mu s, B_T = 0.3, N_c = 16 \), the carrier frequency \( f_c = 900 \) MHz and \( \Delta f = 3 \) KHz (orthogonal signals) or \( \Delta f = 300 \) Hz (nonorthogonal signals). It should be noticed that the bandwidth is only 520Hz and \( N_c = 16 \) in the simulation because of the huge calculation complexity.

Figure 2 shows the cases with \( M = 2, N = 3 \) and \( M = 5, N = 4 \) for spatially independent reflection coefficients, spatially independent noise, and nonorthogonal signals. The solid and dashed curves show the root ECRBOB (RECRBOB) and the root mean squared error (RMSE) of the ML estimation, respectively, in the cases investigated. It is seen that all curves show that the RMSE decreases as the signal-to-clutter-plus-noise ratio (SCNR) is increased. In support of the correctness of our derived CRBs, all RMSE curves show the existence of a
In this section, we focus on the effects of the nonorthogonality of the different transmitted signals. We consider the situation of spatially independent reflection coefficients, spatially independent noise, and nonorthogonal signals. The other factors are the same as in Figure 2. The system considered in Figure 4 has \( M = 2 \) transmitters and \( N = 3 \) receivers. The red and blue curves in this figure correspond to the cases with orthogonal and nonorthogonal signals, respectively. We see that the threshold obtained using the orthogonal signals is 15 dB while the threshold obtained using the nonorthogonal signals is 20 dB. Thus the threshold for the nonorthogonal signals tested is higher than the threshold for the orthogonal signals tested. It is also seen that the RECRBOB of the orthogonal signals tested is smaller than that for the nonorthogonal signals tested over the whole region of SCNR shown. So both the RMSE and RECRBOB indicate that the radar can achieve better performance if the waveforms are closer to being orthogonal in the case considered.

2) Spatially Dependent Reflection Coefficients: In this section, we consider the situation of spatially dependent reflection coefficients, spatially independent noise, and orthogonal signals. The elements of the covariance matrix \( R \) describing the correlation between the different reflection coefficients are generated with [24]

\[
R = R' \otimes R'
\]

where

\[
R' = \begin{bmatrix}
\rho_{11} & \cdots & \rho_{1N} \\
\vdots & \ddots & \vdots \\
\rho_{N1} & \cdots & \rho_{NN}
\end{bmatrix},
\]

\[
\rho_{\text{me}} = \exp(-\sigma_{\Delta\phi_{\text{me}}}),
\]

\[
R' = \begin{bmatrix}
\rho_{11}' & \cdots & \rho_{1M}' \\
\vdots & \ddots & \vdots \\
\rho_{M1}' & \cdots & \rho_{MM}'
\end{bmatrix}
\]
The symbol $\Delta \phi_{mn}$ denotes the separation angle between the $n$th and $m$th transmitter-to-target paths, $\Delta \phi'_{mn}$ denotes the separation angle between the $n$th and $m$th target-to-receiver paths, and $\sigma$ sets the exponential decay in correlation with angle. From the model, it is easy to see that larger $\sigma$ implies less dependency for fixed $\Delta \phi_{mn}$ and $\Delta \phi'_{mn}$. We consider $\sigma = 0.01, 0.1, \text{ and } \infty$ in the figures. Here $\sigma = \infty$ implies that the reflection coefficients are independent.

Figure 5 shows the comparison of RECRBOB and RMSE for different $\sigma$ when all of the other parameters are the same as in Figure 2. We can see that the thresholds for the cases with $\sigma = \infty, 0.1, \text{ and } 0.01$ are 15 dB, 20 dB and 25 dB, respectively. Thus, less dependency leads to a more favorable threshold such that the RECRBOB is achievable at lower SCNR. Above threshold, all the curves are relatively close. The results imply that the dependency of the reflection coefficients does not have tremendous impact on the radar estimation performance, provided we operate above threshold. However, with less dependency the radar can operate at lower SCNR while still achieving an acceptable performance level.

3) Gaussian Spatially Dependent Noise: In this section, we consider the situation of spatially independent reflection coefficients, spatially dependent noise, and orthogonal signals. The elements of the noise covariance matrix $Q$ are generated with the following model

$$Q = \sigma_n^2 \tilde{Q} \otimes I_K,$$

where the $nn$th element of $\tilde{Q}$ is assumed to be

$$\tilde{Q}_{nn'} = \exp(-d_{nn'}\gamma),$$

where $d_{nn'} = \sqrt{(x_n' - x_n)^2 + (y_n' - y_n)^2}$, $\gamma$ sets the exponential decay in correlation with distance, and $I_K$ denotes a $K \times K$ identity matrix. From the model we can see larger $\gamma$ results in less dependency for fixed $d_{nn'}$. We consider the situations of $\gamma = 0.000005, 0.00001, \text{ and } \infty$ and assume all the other parameters are the same as in Figure 2. Here $\gamma = \infty$ implies that the noise components are independent.

Figure 6 shows the comparison of the RECRBOB and RMSE for different values of $\gamma$. Case 1, Case 2 and Case 3 respectively represents $\gamma = \infty, 0.00001, \text{ and } 0.000005$. It is observed that the thresholds for cases with $\gamma = \infty, 0.00001, \text{ and } 0.000005$ are 15 dB, 10 dB, and 5 dB, respectively. Thus, more dependency leads to a more favorable threshold such that the RECRBOB is achievable at lower SCNR. Above the threshold, we see that $\gamma = 0.000005$ has the smallest RECRBOB while $\gamma = \infty$ has the largest RECRBOB, which means larger dependency can lead to lower RECRBOB. In the cases considered in Figure 6, correlated noise leads to better performance.

4) Inaccurate Signal Estimation: Now consider the case where the transmitted signals are not estimated perfectly, possibly from the direct path receptions. Let $n_{mn}(k), n = 1, \ldots, N, m = 1, \ldots, M, k = 1, \ldots, K$ denote an independent and identically distributed sequence of complex Gaussian noise samples, each with zero mean and variance 0.1 which models the estimation error in the signal using

$$u_{nn}(k) = \frac{E_m P_0}{d_{mn}^2} (s_n(kT_s - \tau_m, \alpha_m) + n_{mn}(k)) e^{j2\pi f_s kT_s}.$$ (72)

Then (72) is used to form $S$ with the equations (9), (10), (13) already given in the paper, and we call this mismatched $S$. $S_0$. The undistorted $S$ obtained this way, but without additive noise, is called $S_T$. This is exactly a case where the model we employ in our estimation algorithm is mismatched so the RECRBOB$_{mis}$ results from Section IV become applicable. The resulting average RECRBOB$_{mis}$ and RMSE$_{mis}$, after averaging over the noise using a Monte Carlo simulation, are plotted in Figure 7. From the figure, we can see that the RECRBOB$_{mis}$ provides an informative lower bound on RMSE$_{mis}$ in this case.

\footnote{We have verified that the unaveraged values of RECRBOB$_{mis}$ also provide a lower bound to the unaveraged values of RMSE$_{mis}$.}
Note that all the other details of the system analyzed in Figure 2, except for this signal mismatch, are the same as in Figure 7.

VI. CONCLUSIONS

In this paper, we studied the performance of joint target position and velocity estimation using a distributed radar network under more general conditions than assumed in previous work. A received signal model has been developed for active and passive radar with $M$ transmit and $N$ receive stations. The ML estimate and the exact CRB expression are derived for possibly nonorthogonal signals, spatially dependent Gaussian reflection coefficients, and spatially dependent Gaussian clutter-plus-noise. For cases in which some parameters (for example the transmitted signal from direct path reception) are not estimated correctly, we also derive the mismatched CRB. Numerical results are given to illustrate the use of the CRB and mismatched CRB. The numerical results show various cases with signals of opportunity taken from a GSM wireless communication system. It was shown that in the particular cases investigated, the nonorthogonality of signal degraded the estimation performance both in terms of RECRBOB and in terms of the threshold above which the RMSE starts to become close to the RECRBOB in value and slope. Decreasing the dependency between the different reflection coefficients led to a more favorable threshold such that the radar can operate at lower SCNR while still achieving an acceptable performance level. Above threshold the dependency of the reflection coefficients had little impact on the estimation performance, provided a well performing estimation approach (nearly CRB achieving) is employed. In some specific examples, it was also shown that an increase in the dependency between the noise samples at different antennas led to better estimation performance in terms of the threshold and RECRBOB.

The work here can be generalized in several directions. The CRB, a tight bound only for the high SCNR region and being limited to unbiased estimators, is incapable of characterizing the threshold value or accurately describing the low-SCNR estimation performance of estimators. In this regard, we need better analytical tools which can predict estimation performance for low SCNR. The Ziv-Zakai bound is one promising approach, which will be studied in our future work. The Ziv-Zakai bound also allows prior information to be incorporated into the estimation.

APPENDIX A

CALCULATION OF $J(\vartheta|\alpha)$

According to (23) and (43), we can obtain the FIM of the vector $\vartheta$ as

$$J(\vartheta|\alpha) = \begin{pmatrix} J_{\tau\tau} & J_{\tau f} & J_{\tau d_1} & J_{\tau d_r} \\ J_{f\tau} & J_{ff} & J_{fd_1} & J_{fd_r} \\ J_{d_1\tau} & J_{d_1 f} & J_{d_1 d_1} & J_{d_1 d_r} \\ J_{d_r\tau} & J_{d_r f} & J_{d_r d_1} & J_{d_r d_r} \end{pmatrix}, \quad (73)$$

where

$$J_{\tau\tau} = J_{\tau\tau}^{\mathbb{H}} J_{\tau\tau}, \quad J_{ff} = J_{ff}^{\mathbb{H}} J_{ff}, \quad J_{fd_1} = J_{fd_1}^{\mathbb{H}} J_{fd_1}, \quad J_{fd_r} = J_{fd_r}^{\mathbb{H}} J_{fd_r},$$

$$J_{d_1\tau} = J_{d_1\tau}^{\mathbb{H}} J_{d_1\tau}, \quad J_{d_1 f} = J_{d_1 f}^{\mathbb{H}} J_{d_1 f}, \quad J_{d_1 d_1} = J_{d_1 d_1}^{\mathbb{H}} J_{d_1 d_1}, \quad J_{d_1 d_r} = J_{d_1 d_r}^{\mathbb{H}} J_{d_1 d_r},$$

$$J_{d_r\tau} = J_{d_r\tau}^{\mathbb{H}} J_{d_r\tau}, \quad J_{d_r f} = J_{d_r f}^{\mathbb{H}} J_{d_r f}, \quad J_{d_r d_1} = J_{d_r d_1}^{\mathbb{H}} J_{d_r d_1}, \quad J_{d_r d_r} = J_{d_r d_r}^{\mathbb{H}} J_{d_r d_r}.$$

Then we reformulate the $J(\vartheta|\alpha)$ in a somewhat more explicit matrix form.

First we derive $J_{\tau\tau}$, let $s_i$ and $z_i$ denote the $i$th column of $S$ and $R$, respectively, such that $S = [s_1, \ldots, s_{MN}]$ and $R = [z_1, \ldots, z_{MN}]$. Note that $R$ is a Hermitian matrix, i.e., $R = [z_1^\mathbb{H}, \ldots, z_{MN}^\mathbb{H}]^\mathbb{T}$. Then, we have

$$\frac{\partial C}{\partial \tau_{im}} = \frac{\partial \left( SRS^\mathbb{H} + Q \right)}{\partial \tau_{im}} = \frac{\partial S}{\partial \tau_{im}} \bar{S}^\mathbb{H} + S \frac{\partial \bar{S}}{\partial \tau_{im}} \bar{S}^\mathbb{H} = s_i^* \bar{z}_i^\mathbb{H} S^\mathbb{H} + S z_i (s_i^*)^\mathbb{H}, \quad (78)$$

where $i = M(n-1) + m$ for $n = 1, \ldots, N$ and $m = 1, \ldots, M$, and

$$s_i = \frac{\partial s_i}{\partial \tau_{im}} = e_n \otimes \left[ \frac{\partial u_{im}(1)}{\partial \tau_{im}}, \ldots, \frac{\partial u_{im}(K)}{\partial \tau_{im}} \right]^\mathbb{T}, \quad (79)$$

where $e_n$ is an $N \times 1$ column vector with zero everywhere except for a 1 in the $n$th entry and

$$\frac{\partial u_{im}(k)}{\partial \tau_{im}} = \frac{\partial s_i}{\partial \tau_{im}} = \left. \frac{\partial s_i}{\partial \tau_{im}} \right|_{\tau_{im} = \tau_{mn}} = \frac{E_m P_0}{d_m^2 d_n^2} \frac{\partial s_i(kT_i - \tau_{mn}, \alpha_m)}{\partial \tau_{im}} e^{j2\pi f_m kT_i}. \quad (80)$$
According to the following identity [30]

\[ (X^\top \otimes A) \vec{v}(B) = \vec{v}(ABX), \]

we can obtain

\[
J_{\tau_m} = \left( C^{-1/2} \otimes C^{-1/2} \right) \frac{\partial \vec{v}(C)}{\partial \tau_m} \\
= \vec{v}\left( C^{-1/2} \frac{\partial C}{\partial \tau_m} C^{-1/2} \right) \\
= \vec{v}\left( C^{-1/2} \left\{ \left( s_i^t z_i^H S^H + S z_i(s_i^t)^H \right) C^{-1/2} \right\} \right) \\
= \vec{v}(V_t + V_i^H),
\]

where \( V_t = C^{-1/2} s_i^t z_i^H S^H C^{-1/2} \). Using (82) and the following identity [30]

\[
\vec{v}(A) \otimes \vec{v}(B) = \text{Tr}(AB) = \text{Tr}(BA),
\]

we can derive the \( j \)th element of \( J_{\tau} \) as follows

\[
[J_{\tau}]_{ij} = \vec{v}(V_t + V_i^H) \vec{v}(V_j + V_j^H) \\
= \text{Tr}\left( (V_t + V_i^H) (V_j + V_j^H) \right) \\
= 2\Re(\text{Tr}(V_i V_j + V_j V_i)) \\
= 2\Re \left( s_i^t S^H C^{-1} s_j^t \right) + \left( s_i^t \right)^H C^{-1} s_j^t \left( s_i^t \right)^H C^{-1} S^H z_i z_i^H,
\]

where \( j = M(n' - 1) + m' \) for \( n' = 1, \ldots , N \) and \( m' = 1, \ldots , M \). Then, according to (84), we can reformulate \( J_{\tau} \) in the form of a matrix

\[
J_{\tau} = 2\Re \left( \bar{Y} S^T \otimes (\bar{Y} S^T)^\top + (S^T)^H C^{-1} \bar{S}^T \otimes (\bar{Y} S \bar{R})^\top \right),
\]

where \( \bar{S}^T = [s_1^T, \ldots , s_M^T] \) and \( \bar{Y} = \bar{R} S^H C^{-1} \). Similarly, we can obtain

\[
J_{\tau f} = 2\Re \left( \bar{Y} S^f \otimes (\bar{Y} S^f)^\top + (S^f)^H C^{-1} \bar{S}^f \otimes (\bar{Y} S \bar{R})^\top \right),
\]

and

\[
J_{\tau f} = 2\Re \left( \bar{Y} S^f \otimes (\bar{Y} S^f)^\top + (S^f)^H C^{-1} \bar{S}^f \otimes (\bar{Y} S \bar{R})^\top \right),
\]

where \( S^f = [s_1^f, \ldots , s_M^f] \),

\[
s_i^f = \frac{\partial s_i}{\partial f_n} = \epsilon_n \otimes \left[ \frac{\partial u_m(1)}{\partial f_{nm}}, \ldots , \frac{\partial u_m(K)}{\partial f_{nm}} \right],
\]

and

\[
\frac{\partial u_m(k)}{\partial f_{nm}} = j2\pi k T_s u_m(k).
\]

Next we derive \( J_{d} \),

\[
\frac{\partial C}{\partial d_{im}} = \frac{\partial (S R S^H + Q)}{\partial d_{im}} \\
= \frac{\partial S}{\partial d_{im}} R S^H + S R \frac{\partial S^H}{\partial d_{im}} \\
= (s_i^t z_i^H + s_m^t z_m^H + \ldots + s_{m+N-1} z_{m+N-1} z_m^H) S^H + S(z_m(s_m^t)^H + z_m+M(s_m+M)^H + \ldots + z_m+N-1 M(s_m+N-1 M)^H)^H),
\]

where

\[
s_{m}^t = \frac{\partial s_{m}^{(n-1)M}}{\partial d_{im}}, \quad n = 1, \ldots , N
\]

and

\[
\frac{\partial u_m(k)}{\partial d_{im}} = -\frac{\sqrt{E_m P_0}}{d^2 d_m n_s s(kT_s - \tau_m, \alpha_m)e^{j2\pi f_s m T_s}}.
\]

It can be derived that

\[
J_{d_{im}} = \left( C^{-1/2} \otimes C^{-1/2} \right) \frac{\partial \vec{v}(C)}{\partial d_{im}} \\
= \vec{v}(C^{-1/2} \frac{\partial C}{\partial d_{im}} C^{-1/2}) \\
= \vec{v}(C^{-1/2} ((s_i^t z_i^H + s_m^t z_m^H + \ldots) + s_i^t z_m^H + z_m+M(s_m+M)^H + \ldots + z_m+N-1 M(s_m+N-1 M)^H) C^{-1/2}) \\
= \vec{v}(c_m + \ell_i^H),
\]

where \( c_m = C^{-1/2} (s_i^t z_i^H + s_m^t z_m^H + \ldots) + s_i^t z_m^H + z_m+M(s_m+M)^H + \ldots + z_m+N-1 M(s_m+N-1 M)^H) S^H C^{-1/2} \). Then, we obtain

\[
[J_{d_{iM}}]_{mn} = \vec{v}(\ell_m + \ell_i^H) \vec{v}(\ell_m + \ell_i^H) \\
= \text{Tr}(\ell_m + \ell_i^H)(\ell_m + \ell_i^H) \\
= 2\Re(\text{Tr}(\ell_m + \ell_i^H) \ell_i^H) \\
= 2\Re \left( \sum_{n=1}^{N} \sum_{n'=1}^{N} ((s_m+n-1 M)^H s_m n-1 M (s_m+n-1 M)^H s_m n-1 M) S^H C^{-1} s_i^t z_i^H + (s_m+n-1 M)^H C^{-1} s_i^t z_i^H \right),
\]

Refomulate \( J_{d_{iM}} \) in the form of a matrix

\[
J_{d_{iM}} = 2\Re \left( \sum_{n=1}^{N} \sum_{n'=1}^{N} (N_s^H C^{-1} S_{n}^t \otimes (N_s^H C^{-1} S_{n})^t) + (3_s^H C^{-1} S_{n}^t \otimes (3_s^H C^{-1} S_{n})^t) \right),
\]

where \( N_s = (z_1 + \ldots + z_m)^t, \ldots , (z_{M-1} + \ldots + z_{M-1})^t, \) \( F = (s_1^t + \ldots + s_{m-1}^t)^t \). Similarly, we can derive

\[
J_{d_{iF}} = 2\Re \left( \sum_{n=1}^{N} (N_s^H C^{-1} S_{n}^t \otimes (N_s^H C^{-1} S_{n})^t) + (3_s^H C^{-1} S_{n}^t \otimes (3_s^H C^{-1} S_{n})^t) \right),
\]

\[
J_{d_{iF}} = 2\Re \left( \sum_{n=1}^{N} (N_s^H C^{-1} S_{n}^t \otimes (N_s^H C^{-1} S_{n})^t) + (3_s^H C^{-1} S_{n}^t \otimes (3_s^H C^{-1} S_{n})^t) \right).
\]
To derive $d_m$, we employ

$$\frac{\partial C}{\partial d_m} = \frac{\partial (SR^H + Q)}{\partial d_m}$$

$$= \frac{\partial S}{\partial d_m} R^H + \frac{\partial R}{\partial d_m} S^H$$

$$= (s_{1 + (n-1)M} z_{1 + (n-1)M}^H + s_{2 + (n-1)M} z_{2 + (n-1)M}^H + \cdots + s_{M + (n-1)M} z_{M + (n-1)M}^H) S^H$$

$$+ S((z_{1 + (n-1)M} s_{1 + (n-1)M}^H + z_{2 + (n-1)M} s_{2 + (n-1)M}^H + \cdots + z_{M + (n-1)M} s_{M + (n-1)M}^H) H)$$

$$= \left(s_{m(n+1)-1} + \cdots + s_{m(n+1)-M} \right) S^H$$

where

$$s_{m(n+1)-1} = \frac{\delta s_{m(n+1)-1}}{\partial d_m} = e_n \otimes \left[ \frac{\partial u_{m1}}{\partial d_m}, \frac{\partial u_{m2}}{\partial d_m}, \ldots, \frac{\partial u_{MK}}{\partial d_m} \right], \quad m = 1, \ldots, M,$$

and

$$\frac{\partial u_{mk}(k)}{\partial d_m} = -\frac{\sqrt{E_m P_0}}{d_m d_m} s(kT_s - \tau_{nm}, \alpha_m)e^{j2\pi fm_kT_s},$$

We can then derive

$$J_{d_m} = (C^{-1/2} \otimes C^{-1/2}) \frac{\partial C_{vec}}{\partial d_m}$$

$$= vec(C^{-1/2}) \frac{\partial C}{\partial d_m}$$

$$= vec(C^{-1/2})(s_{1 + (n-1)M} z_{1 + (n-1)M}^H + s_{2 + (n-1)M} z_{2 + (n-1)M}^H + \cdots + s_{M + (n-1)M} z_{M + (n-1)M}^H) S^H$$

$$+ S((s_{1 + (n-1)M}^H z_{1 + (n-1)M} + (s_{2 + (n-1)M})^H z_{2 + (n-1)M} + \cdots + (s_{M + (n-1)M})^H z_{M + (n-1)M})) C^{-1/2}$$

$$= vec(w_n + w_H),$$

where

$$w_n = C^{-1/2} (s_{1 + (n-1)M} z_{1 + (n-1)M}^H + s_{2 + (n-1)M} z_{2 + (n-1)M}^H + \cdots + s_{M + (n-1)M} z_{M + (n-1)M}^H) S^H C^{-1/2},$$

Then, we can obtain

$$[J_{d_m}]_{mn} = vec(w_n + w_H^T) vec(w_m + w_H^T)$$

$$= Tr\{(w_n + w_H^T)(w_m + w_H^T)\}$$

$$= 2\Re\{Tr[w_n w_m^T + w_H w_H^T]\}$$

$$= 2\Re\{\sum_{m=1}^{M} (z_{m + (n-1)M})^H S^H C^{-1} s_{m + (n-1)M}^T (z_{m + (n-1)M})^H S^H$$

$$- z_{m + (n-1)M})^H (z_{m + (n-1)M})^H S^H C^{-1} S (z_{m + (n-1)M})\}.$$

The result of (102) can be reformulated as

$$J_{dv} = 2\Re\{\sum_{m=1}^{M} (\tilde{J}_m S^H C^{-1} s_{m}^T \otimes \tilde{J}_m S^H C^{-1} s_{m})^T$$

$$+ (\tilde{J}_m^H C^{-1} s_{m}) \otimes \tilde{J}_m S^H C^{-1} S (\tilde{J}_m^H)\},$$

where $\tilde{J}_m = (z_m, \ldots, z_m, z_m, \ldots, z_m)$. Similarly, we can obtain

$$J_{d_r} = 2\Re\{\sum_{m=1}^{M} (\tilde{J}_m S^H C^{-1} s_{m}) \otimes (\tilde{J}_m S^H C^{-1} s_{m})^T$$

$$+ (\tilde{J}_m^H C^{-1} s_{m}) \otimes (\tilde{J}_m S^H C^{-1} S (\tilde{J}_m^H))\}.$$

$$J_{d_r} = 2\Re\{\sum_{m=1}^{M} (\tilde{J}_m S^H C^{-1} s_{m}) \otimes (\tilde{J}_m S^H C^{-1} S (\tilde{J}_m^H))\}.$$
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