EQUIVALENCE THEOREM AS A CRITERION FOR PROBING THE ELECTROWEAK SYMMETRY BREAKING MECHANISM *

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Abstract  
Based upon our recent study on the Lorentz non-invariance ambiguity in the longitudinal weak-boson scatterings and the precise conditions for the validity of the Equivalence Theorem (ET), we further examine the intrinsic connection between the longitudinal weak-boson scatterings and probing the electroweak symmetry breaking (EWSB) mechanism. We reveal the profound physical content of the ET as being able to discriminate processes which are insensitive to probing the EWSB sector. With this physical content as a criterion, we analyze the sensitivities to various effective operators for probing the mechanism of the EWSB.

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1. Introduction

Despite the astonishing success of the Standard Model (SM) over the years, its scalar part, the electroweak symmetry breaking (EWSB) sector, remains as the greatest mystery. Due to the screening theorem, the current low energy data, allowing the SM Higgs boson mass to range from 60 GeV to about 1 TeV, tell us little about the EWSB mechanism. With the light Higgs particle(s) in the SM and SUSY-like theories remaining un-detected, it is important to probe all possible EWSB mechanisms: either weakly or strongly interacting.

While the transverse components $V_T$ of $W^\pm, Z^0$ are irrelevant to the EWSB mechanism, the longitudinal weak-bosons ($V_L = W_L^\pm, Z_L^0$), as the products of the Higgs mechanism, are expected to be sensitive to probing the EWSB sector. However, even for the strongly coupled case, studying the $V_L$-scatterings does not guarantee probing the EWSB sector in a sensitive and unambiguous way unless certain general conditions are satisfied to avoid the Lorentz non-invariance ambiguity of the $V_L$-amplitudes $^\text{[1]}$. We note that the spin-0 Goldstone bosons (GB’s) are invariant under the proper Lorentz transformations, in contrast, both $V_L$ and $V_T$ are Lorentz non-invariant (LNI). After a Lorentz transformation, the $V_L$ component can mix with or even turn into a pure $V_T$. Thus a conceptual and fundamental ambiguity arises: How can we use the LNI $V_L$-amplitudes to probe the EWSB sector of which the physical mechanism should clearly be independent of the choices of the Lorentz frames? This motivated our precise formulation of the electroweak Equivalence Theorem (ET) in Ref.\textsuperscript{[1]}.

The ET provides a quantitative relation between the $V_L$-amplitude and the corresponding GB-amplitude in the high energy region ($E \gg M_W$) $^\text{[2, 3, 1]}$: the former is physically measurable while the latter carries information of the EWSB sector. Hence, as a bridge, the ET naturally connects the $V_L$-scattering experiments to probing the EWSB sector in a precise way. As shown further below, the difference between the $V_L$- and GB-amplitudes is intrinsically related to the ambiguous LNI part of the $V_L$-scattering which has the same origin as the $V_T$-amplitude, and thus is insensitive to probing the EWSB sector. When the LNI contributions can be safely ignored and the Lorentz invariant (LI) scalar GB-amplitude dominates the experimentally measured $V_L$-amplitudes, the physical $V_L$-scatterings can therefore sensitively and unambiguously probe the EWSB mechanism. Furthermore, in our precise formulation of the
ET, we show that the ET is not just a technical tool in computing $V_L$-amplitudes via GB-amplitudes, as a criterion, it has an even more profound physical content for being able to discriminate processes which are insensitive to probing the EWSB sector.[1]

2. The Precise Formulation of the ET for Probing the EWSB

Starting from the Ward identity[2, 3] $<0|F_0^a(k_1)\cdots F_0^a(k_n)\Phi_\alpha|0> = 0$ and making a rigorous LSZ reduction for the external $F^a$-lines, we derived the following general identity for the renormalized $S$-matrix elements:

$$T[V_L^{a_1}, \cdots, V_L^{a_n}; \Phi_\alpha] = C \cdot T[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] + B,$$

where $\pi^{a}$'s are GB fields, and the finite constant modification factor $C_{mod}^{a_i}$ has been systematically studied in Ref.[3]. For clarity, let us assume that $\Phi_\alpha$ contains either no field or some physical scalars and/or photons. From (1), the LNI $V_L$-amplitude can be decomposed into two parts: the 1st part is $C \cdot T[-i\pi; \Phi_\alpha]$ which is LI; the 2nd part is the $v^\mu$-suppressed $B$-term which is LNI because of the external spin-1 $V^\mu$-field(s). Such a decomposition shows the essential difference between the $V_L$- and the $V_T$-amplitudes since the former contains a LI GB-amplitude which is the intrinsic source causing a large $V_L$-amplitude in the case of strongly coupled EWSB sector. We note that only the LI part of the $V_L$-amplitude is sensitive to probing the EWSB sector, while its LNI part contains a significant Lorentz-frame-dependent $B$-term and therefore is not sensitive to the EWSB mechanism. Thus, for a sensitive and unambiguous probe of the EWSB, we must find conditions for ignoring the $B$-term such that the LI GB-amplitude dominates the $V_L$-amplitude. This physical content is essentially independent of how to compute the $V_L$-amplitude. It is the LI GB-amplitude that matters. It is clear that one can technically improve the prediction of the $V_L$-amplitude from the RHS of (1) by including the complicated $B$-term (or part of $B$)[4], but this is not an improvement of the equivalence for $V_L$- and GB-amplitudes and thus irrelevant to the physical content of the ET as a criterion for probing the EWSB mechanism.

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1 Here $F_0^a$ is the bare gauge fixing function and $\Phi_\alpha$ denotes other possible physical in/out states.
2 See the 2nd paper by H.-J. He, Y.-P. Kuang and X. Li in Ref.[3].
From a detailed analysis on the LNI $V_L$-amplitude, we estimate the $B$-term as

$$B \approx \mathcal{O}\left(\frac{M^2}{E_j^2}\right) T[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + \mathcal{O}\left(\frac{M_W}{E_j}\right) T[V_T^{a_1}, -i\pi^{a_2}, \ldots, -i\pi^{a_n}; \Phi_\alpha].$$

(2)

We emphasize that the condition $E_j \sim k_j \gg M_W$, ($j = 1, 2, \ldots, n$) for each external longitudinal weak-boson is necessary for making the $B$-term (and its Lorentz variation) to be much smaller than the GB-amplitude. This also precisely defines the safe Lorentz frames in which the LNI $B$-term can be ignored (cf. (3)). In conclusion, we give our general and precise formulation of the ET as follows:

$$T[V_L^{a_1}, \ldots, V_L^{a_n}; \Phi_\alpha] = C \cdot T[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + \mathcal{O}(M_W/E_j - \text{suppressed}),$$

(3)

$$E_j \sim k_j \gg M_W, \quad (j = 1, 2, \ldots, n);$$

$$B \ll C \cdot T[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha],$$

(3a, b)

where (3a,b) are the precise conditions for ignoring the LNI $B$-term to validate the equivalence in Eq. (3). The amplitude $T$, to a finite order, can be written as $T = \sum_{\ell=0}^{N} T_\ell = \sum_{\ell=0}^{N} \tilde{T}_\ell \alpha^\ell$ in the perturbative calculation. Let $T_0 > T_1, \ldots, T_N \geq T_{\text{min}}$, where $T_{\text{min}} = \{T_0, \ldots, T_N\}_{\text{min}}$, then the condition (3b) implies

$$\mathcal{O}\left(\frac{M^2}{E_j^2}\right) T_0[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha] + \mathcal{O}\left(\frac{M_W}{E_j}\right) T_0[V_T^{a_1}, -i\pi^{a_2}, \ldots, -i\pi^{a_n}; \Phi_\alpha]$$

$$\ll T_{\text{min}}[-i\pi^{a_1}, \ldots, -i\pi^{a_n}; \Phi_\alpha].$$

(4)

We note that the above formulation of the ET discriminates processes which are insensitive to probing the EWSB sector when either (3a) or (3b) fails. Furthermore, as a physical criterion, the condition (4) determines whether or not the $V_L$-scattering process of interest is sensitive to probing the EWSB sector to the desired precision in perturbative calculations.

From (2) or the LHS of (4) and the precise electroweak power counting rules, we can easily estimate the $B$-term to be

$$B = \mathcal{O}(g^2) f_\pi^{4-n}$$

for theories with strongly coupled EWSB sector (i.e., the heavy Higgs SM or the chiral Lagrangian formulated electroweak theories (CLEWT)). It is of the same order in magnitude as the leading $V_T$-amplitude $T_0[V_T^{a_1}, \ldots, V_T^{a_n}]$. Since both the $B$-term and the leading $V_T$-amplitude are of order $g^2$, they are therefore insensitive to the EWSB sector in accordance with the above general analysis. If we want to probe the leading
new physics contributions in the CLEWT at the $E^4$-level, of \( \mathcal{O}(E^2 f^2 \Lambda^2 f^{-n}) \) \(^3\) then Eq. (5) and the criterion (4) yield \( \frac{M^2_{\Delta}}{E^2} \ll \frac{1}{4} \frac{E^2}{\Lambda^2} \), or \( (0.70 \text{TeV}/E)^4 \ll 1 \). This shows that in order to sensitively probe the strongly coupled EWSB sector, up to the order of $E^4$, we must measure the $V_L$ production rates in the energy region above 1TeV.

3. Sensitivities to the Effective Operators via Weak-Boson Scatterings

Given the above conclusion, we examine the sensitivities to the next-to-leading order effective operators \([6]\), \( L_1, L_2, L_9 \), \( L_{10} \) and \( L_{\Delta \rho} \), for probing the EWSB mechanism of the CLEWT via high energy weak-boson scatterings. The coefficients of these operators are of \( \mathcal{O}(1) \). The condition (4) and Eq. (5) discriminate which scattering process can sensitively probe the EWSB sector at the next-to-leading order in either hadron and electron collisions. Define \( R_L \) to be the ratio of \( B \approx \mathcal{O}(g^2) \) to \( T_1[\pi \pi \to \pi \pi] \), and \( R_T \) the ratio of \( B \) to \( T_1[V_T \pi \to \pi \pi] \). At a given energy scale \( E \), if \( R_L \) (or \( R_T \)) is much less than one for including the new physics contribution from the operator, say, \( L_1 \), then we expect that this operator (e.g., \( L_1 \)) can be sensitively probed via the scattering process \( V_L V_L \to V_L V_L \) (or \( V_T V_L \to V_L V_L \)). ( We have assumed the coefficient of each operator to be \( \mathcal{O}(1) \), after factorizing out the dimensional-counting factor \( f^2 = \frac{1}{16\pi^2} \).

As summarized in Table 1, for \( V_L V_L \to V_L V_L \) process, the operators \( L_{1,2} \) can be sensitively probed for \( E \geq 1 \text{TeV} \); while \( L_{9L,R} \) and \( L_{\Delta \rho} \) are insensitive even for \( E \approx 3 \text{TeV} \), where the effective Lagrangian \((L_{\text{eff}})\) description becomes invalid. \( L_{10} \) has no contribution to this process at this order. The operators \( L_{9L} \) and \( L_{9R} \) are better probed via \( V_T V_L \to V_L V_L \) (+permutations) than \( V_L V_L \to V_L V_L \). However, \( L_{10} \) and \( L_{\Delta \rho} \) are totally insensitive via the \( V_T \)-processes. A more complete discussion is given in Ref.[5].

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\(^3\)In the CLEWT, \( f_\pi = 246 \text{ GeV} \) and the effective cut-off \( \Lambda \sim 4\pi f_\pi \sim 3.1 \text{ TeV} \).

\(^4\) Due to the stringent experimental bound on \( \Delta \rho \), the coefficient of the dimension-2 operator \( L_{\Delta \rho} \) can also be at most \( \mathcal{O}(1) \).
Table 1. Sensitivities to the Next-to-Leading Order Effective Operators in $\mathcal{L}_{\text{eff}}$.

| Operators | $\mathcal{L}_1$, $\mathcal{L}_2$ | $\mathcal{L}_{9L}$ | $\mathcal{L}_{9R}$ | $\mathcal{L}_{10}$ | $\mathcal{L}_{\Delta\rho}$ (dim = 2) |
|-----------|-------------------------------|--------------------|--------------------|-------------------|----------------------------------|
| $T_1[\pi\pi \to \pi\pi]$ | $\frac{E^2}{f_K^2}$ | $g^2 \frac{E^2}{\Lambda^2}$ | $g'^2 \frac{E^2}{\Lambda^2}$ | / | $\frac{E^2}{\Lambda^2}$ |
| $T_1[V_T\pi \to \pi\pi]$ | $g \frac{E}{f_{\pi}} \frac{E^2}{\Lambda^2}$ | $g \frac{E}{f_{\pi}} \frac{E^2}{\Lambda^2}$ | $g \frac{E}{f_{\pi}} \frac{E^2}{\Lambda^2}$ | $g g'^2 \frac{f_{\pi} E}{\Lambda^2}$ | $g \frac{f_{\pi} E}{\Lambda^2}$ |
| $R_L \equiv \frac{B \approx O(g^2)}{T_1[\pi\pi \to \pi\pi]}$ | $(0.70 T eV)^4$ | $(3.1 T eV)^2$ | $(5.7 T eV)^2$ | / | $(2.0 T eV)^2$ |
| $R_T \equiv \frac{B \approx O(g^2)}{T_1[V_T\pi \to \pi\pi]}$ | $(1.15 T eV)^3$ | $(1.15 T eV)^3$ | $(1.4 T eV)^3$ | $\frac{200 T eV}{E}$ | $\frac{25 T eV}{E}$ |

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5. References

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