Development of a wall jet model dedicated to 1D combustion modelling for CI engines

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Abstract
Diesel engines are becoming smaller as technology advances, which means that the fuel spray (or jet) interacts with the cylinder walls before combustion starts. Most fuel injection 1D models (especially for diesel fuel) do not consider this interaction. Therefore, a wall-jet sub-model was created on an Eulerian 1D diesel spray model. It was calibrated using data from the literature and validated with experimental data from a fuel spray impacting a plate in a constant volume combustion chamber. Results show that the spray moving along the wall has a higher mixing rate but less penetration as an equivalent free jet, therefore they show a similar volume. Spray-wall interaction creates a stagnation zone right before the impact with the wall, and friction of the jet with the wall is relatively low. All these phenomena are well captured by the wall-jet sub-model.

Keywords
1D model, diesel spray, wall jet interaction, IC engine, CI combustion

1. Introduction
Diesel Engines are widely used all over the world, either for heavy-duty applications such as truck or marine propulsion, off-road and power generation, or for light-duty automotive propulsion. Due to Diesel-gate the share of Diesel car has dropped but was still at 30% in Europe in 2020.1 In the medium term Diesel engines appear as promising solutions for heavy duty applications since they can be easily decarbonized if they use biofuels or synthetic fuels.2,3 However, in this context, a further increase in engine efficiency and keeping pollutant emission (mainly NOx and particles) under control is even more challenging.4,5 Therefore, it is of major importance for engine design and calibration to have simulation tools available which combine good accuracy and limited computational effort so that a lot of options can be tested. In particular the modelling of the spray is critical.6 In the past years, the authors have developed a 1D combustion model that was successfully validated against measurement in bombs with free Diesel jets.7–10 However, combustion chambers of Diesel engines are quite confined so that the jet interacts with the wall well before the end of combustion. Consequently, the development of a wall jet model was undertaken and is presented in this paper. In a first section, a literature review is performed to analyse existing studies on wall/jet interaction. Then, after a short description of 1D combustion model previously developed by the authors, the wall/jet model is detailed, with main assumptions, equations and calibration process. Finally, results are discussed and compared with experiments.

2. Literature Review
2.1. Early Works
When a fluid jet impacts (or impinges on) a surface, the outward flow is termed a “wall jet” (Figure 1). This term was first introduced by Glauert in 1956,11 and later used (with variations such as “partially open jet”, “surface jet” and “submerged jet) by Poreh12 and others. Neglecting compressibility, Glauert divided the wall jet section profile in two overlapping parts: an inner part (in contact with the wall) and an outer part (in contact with the surrounding environment), as seen in Figure 2. The inner part is dominated by friction forces with the wall (like an ordinary boundary layer flow) and the outer part is dominated by free mixing with the surrounding media (like a free jet). The solutions for the velocity of both parts were
matched at the position of the maximum velocity; this way an overall similarity was not achieved, but the solution of each part was self-similar. This result was confirmed experimentally by Bakke\textsuperscript{13} and others.

Since both the velocity at the wall \((z = 0)\) and at the outer edge of the wall jet \((z = \infty)\) is zero (for static surrounding air), and the velocity profile must be continuous, there must be a maximum in between these points. Vlachopoulos\textsuperscript{14} used a power law to model the inner jet, shown below.

\[
\frac{V_r}{V_{re}} = \left( \frac{z}{d_e} \right)^{1/7} \tag{1}
\]

Where \(z\) is the distance perpendicular to the wall, \(d_e\) is the thickness of the boundary layer (where the velocity is the maximum), \(V_r\) is the velocity at coordinate \(z\) and \(V_{re}\) is the velocity at coordinate \(d_e\). However, he observed that the experimental data was closer to a \(1/14\)-power law profile. This correlation was used by Myers,\textsuperscript{15} but with a \(1/7\)-power law profile. The outer jet was modelled with a Gaussian distribution, similar to the free jet (Schlichting profile considering the inner and outer boundary layer thicknesses), shown below.

\[
\frac{V_r}{V_{re}} = \left[ 1 - \left( \frac{z - d_e}{d_{bv} - d_e} \right)^{1.57} \right]^2 \tag{2}
\]

Where \(d_{bv}\) is the thickness of the outer boundary layer (the distance from the wall to the point where the jet velocity is equal to the surrounding velocity).

### 2.2. Penetration And entrainment rate

Song and Abraham\textsuperscript{16} presented theoretical deductions for round jets (free jets), radial jets (free jets propagating in radial direction) and wall-impinging jets (viewed as a special case of radial jet), valid only when the velocity profile is fully developed.

The air entrainment rate for a round (free) jet is calculated by:

\[
\frac{m_{air\_entrained}}{m_{initial}} = k \left( \frac{z^*}{D_e} \right) \tag{3}
\]

Where \(z^*\) is the distance traveled by the jet from the injector along the axial axis \(z\). The entrainment rate is proportional to the axial penetration and inversely proportional to the effective diameter \(D_e = D(\rho_a / \rho_i)^{1/2}\). According to Abraham,\textsuperscript{17} the proportionality constant \(k\) can vary from 0.2 to 0.457.

For the wall impinging jet, theoretical expressions that were proposed by Glauert\textsuperscript{11} were fitted with empirical constants from measured data by Poreh et al.,\textsuperscript{12} Witze and Dwyer\textsuperscript{18} and Tanaka and Tanaka.\textsuperscript{19} The maximum radial velocity \(V_{re}\) is proportional to the wall jet penetration \(x^m\) and the wall jet half width \(Z_{1/2}\) (place in the outer region where the radial velocity is half of the maximum) is proportional to \(x^n\). The constant “\(m\)” is close to \(-1\) and the constant “\(n\)” is close to \(1\). The total entrained mass with respect to time (after start of injection) obtained for the wall-impinging jet is close to \(t^{3/2}\) obtained also for both the round and radial jet, with a small dependence on the impinging distance.

Approximating the wall-impinging jet as a half-radial jet allowed a theoretical comparison with the round jet. The half-radial jet penetration is \(37\%\) of that of the round (free) jet and the entrained mass in the half-radial jet is \(137\%\) of that of the round (free) jet. In this case, the half-radial jet penetration is determined after start of impingement (that means that the impinging distance is disregarded in this case).
The main difference between half-radial jets and wall-impinging jets is that wall-impinging jets have a momentum loss due to the interaction with the wall, which can be described via the chosen velocity profile. Using the theoretical equations from Song and Abraham\textsuperscript{16} with an entrainment constant \( k = 0.32 \)\textsuperscript{17,19,20,21} and the empirical parameters from Poreh et al.\textsuperscript{12} and Witze and Dwyer,\textsuperscript{18} both show a reduction in both the penetration and entrained mass with respect to the half-radial jet, due to the loss of momentum to the wall. The region where the penetration is less than half of the impingement distance (\( x_{0p}/L = 0.5 \)) was not considered, since below this value the relative penetration and entrained mass increase sharply and do not represent the behavior of the overall jet (this is the stagnation zone). When the entrainment in the free jet is increased (\( k = 0.457 \)), the entrainment in the wall jet will be even smaller (there will be a greater loss due to momentum loss, of about 30%).

When comparing the parameters with respect to time of round and wall jets (assuming that the wall jet begins at the impingement time \( t_0 \)), both penetration and entrained mass are lower for the wall jet, due to the momentum loss of the wall jet. However, if the wall jet is assumed to be already developed at start time \( t_0 \), the entrainment is greater in the wall jet for a period of time (until about \( t = 17 \) ms), then it becomes lower. This difference in entrainment estimation is due to the stagnation region.

Tomita et al.\textsuperscript{22} showed that for a period of time after impingement the increase rate of the entrained volume for the wall-impinging jet was greater than that of the free jet, but this difference was reduced after impingement, and finally this trend was reversed.

From the experiments of Fujimoto et al.,\textsuperscript{23} the penetrations found were approximately proportional to \( t^{0.15} \) but smaller than that of the free jet. For computational models (k-\( \varepsilon \) or RNG k-\( \varepsilon \) model), Launder and Spalding\textsuperscript{24} proposed wall functions to model momentum and heat fluxes at the walls. The momentum loss to the wall reduced the jet penetration by about 12%.

Song and Abraham\textsuperscript{25} compared their theoretical deductions with experimental and computed results. The penetration of the wall-impinging jet was defined as the summation of the impinging distance \( L \) and the radial distance \( X \), as shown in Figure 1. In the developed wall jet velocity profile, the location of the maximum radial velocity (\( d_0 \)) is computed at a non-dimensional distance (\( z / Z_{1/2} \)) of 0.25 from the wall,\textsuperscript{25} and it agrees closely with the measured\textsuperscript{13} and analytical results\textsuperscript{11,16}. There is also close agreement between the computed, analytical and measured results from (\( z / Z_{1/2} \)) of 0.25 to 1.25. In the inner region of the jet, the difference may be due to the more important laminar diffusivity with respect to the turbulent diffusivity, which is the one considered in the turbulence modeling. In the outer region, extrapolation of the computed results shows that at (\( z / Z_{1/2} \)) between 1.75 and 2 the radial velocity becomes zero.\textsuperscript{25} The analytical value\textsuperscript{11} at this location results in a radial velocity below 0.1\( V_{re} \). Therefore, the location of the outer boundary layer (dbv) can be defined as 2 \( Z_{1/2} \).

With respect to the entrainment and mixing rate, it is seen that in the wall jet, the flammable fraction of the mixture is greater than in the free jet (initially it is lower, but after some time it becomes greater), and the lean fraction is lower. This leads to a difficulty in interpretation: if the mixing rate is the rate of appearance of lean mixture, then the wall jet mixes slower; however, if the mixing rate is the appearance of flammable mixture, then the wall jet mixes faster. In this case, the flammability limits were considered as 0.5 < \( \Phi \) ≤ 2.0 for \( \text{C}_3\text{H}_4 \) fuel.

2.3. Spreading Rate modeling

The jet half-width (\( z \)) with respect to the penetration (\( x \)) is referred to as the “spreading rate”. Launder and Rodi\textsuperscript{26} suggested a linear rather than a power relationship for the growth rate of the half-width of the wall jet with penetration, which appears reasonable as the exponent in the power relationship is close to unity. They report a spreading rate of 0.09. The computed spreading rate can be approximated by the linear relationship: \( Z_{1/2} = 0.10x + 0.06 \) (cm); the difference with Launder and Rodi’s theoretical value of 0.09 is of 11%, and with the measured value of 0.95 is of 6%. However, the biggest difference is in the jet origin, which may be related to the description of the impingement zone. There is also a major difference in the maximum radial velocity at the impingement zone, mainly because the correlations for the measured values are not valid in the impingement zone.

Knowles and Myszko\textsuperscript{27} used the k-\( \varepsilon \) turbulence model to calculate radial wall jets and carried out experiments to measure the radial wall jets originating from impingement of a round, compressible jet.\textsuperscript{28} Distance of the free jet origin to the wall (\( L \)) was between 2 and 30 nozzle diameters (\( D_n \)) and nozzle pressure ratios (NPR, the ratio of nozzle stagnation pressure to ambient static pressure) from 1.05 to 4.

A correlation of the relative wall jet half-thickness (\( Z_{1/2}/D_n \)) with respect to the relative impingement distance and the pressure ratio indicates that for low distances to the wall (\( L/D_n < 10 \)) the wall-jet thickness increases, but for high distances it essentially remains constant.

Different computational approaches were compared with the measured wall jet growth of Poreh et al.\textsuperscript{12} (\( Z_{1/2}/L \) versus relative radial distance \( x/D_n \)). The k-\( \varepsilon \) model under-predicts the wall thickness. The Rodi correction\textsuperscript{29} makes an even greater under-prediction, due to the smaller angle of the free jet that impacts the wall. The Malin correction\textsuperscript{30} improves the wall thickness prediction but seems worse at initial radial distances (in the impingement zone). Comparing different free jet and wall jet
descriptions it is possible to see that there is an influence of the free jet on the wall jet.

Knowles and Bray\textsuperscript{31} showed that the wall jet momentum and thickness increase with increasing nozzle height, while increasing nozzle pressure ratio (NPR) causes a reduction in wall-jet thickness.\textsuperscript{28} Measurements by Knowles and Myszko\textsuperscript{32} show that the wall jet half-thickness increases linearly with radial distance, as well as the overall thickness with respect to the nozzle height. However, at a height of 4 nozzle diameters, the thickness is the minimum, which is due to turbulent kinetic energy being the maximum at this condition resulting from a greater radial velocity gradient along the jet wall and therefore a lesser jet thickness.

Lin\textsuperscript{33} makes a summary of different spreading rate correlations found, using the correlation shown in Table 1.

\begin{equation}
Z_1/2 = Bx^a
\end{equation}

### 2.4. Axial Velocity profile

From the measurements of Knowles and Myszko,\textsuperscript{32} while the free jet reaches a fully developed profile at a distance of about 5 nozzle diameters from the nozzle, the wall jet mean velocity reaches a fully developed profile at a radial distance of about 3 nozzle diameters from the axis. Before this point, the wall jet goes through the edge of the free jet. However, the turbulent velocity reaches a developed profile at about 4.5 nozzle diameters from the axis (this being the definition of the impingement zone, confirmed also by the stabilization of the Reynolds stress profiles). The peak radial velocity $V_m$ in the wall jet increases as the height of the nozzle decreases, up to a radial distance of 4.5 nozzle diameters; after this point, the peak velocities do not change with nozzle height. The peak in turbulent velocity is reached at a radial distance of 2 nozzle diameters, inside of the impingement zone.

Wood\textsuperscript{34} used a wall jet to model a thunderstorm downburst (air jet impacting with the ground). The velocity profiles were measured at different radial locations of the wall jet. Normalizing the curves with the parameters $V_{re}$ (maximum radial velocity at a given radial location) and $Z_{1/2}$ (half-thickness, where the velocity is half of $V_{re}$), it was seen that the profiles become stable (approximately self-similar) after a distance of 1.5 $D_n$ from the central axis of the jet. Equation (5) was found to adequately fit the normalized data from.\textsuperscript{34} It was valid also for the data obtained at 0.5 $D_n$ and 5 $D_n$.

\begin{equation}
\frac{V}{V_{re}} = 1.55 \left( \frac{z}{Z_{1/2}} \right)^{1/2} \left[ 1 - \text{erf} \left( 0.70 \frac{z}{Z_{1/2}} \right) \right]
\end{equation}

Where $V$ is the velocity at height $z$ and “erf” is the error function, defined by:

\begin{equation}
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \! e^{-t^2} \, dt
\end{equation}

### 2.5. Wall Jet visualization

Most experimental work describe a tendency to increase the mixing of fuel with the surrounding air during the jet wall interaction, decreasing the local fuel/air ratio\textsuperscript{35,36} and increasing the jet volume compared to a free jet.\textsuperscript{(22)} However, numerical calculations appear contradictory,\textsuperscript{(16,25,37)} due to the incapacity in reproducing the turbulent nature of this jet wall interaction (jet wall vortex\textsuperscript{38}). Additionally, the uncertainties during the measurements, such as aerodynamic uncertainties in defining a control surface\textsuperscript{(23,39,40)} and non-quantitative jet volume calculation\textsuperscript{22} cast doubts on the previous conclusions.

Bruneaux\textsuperscript{41} has visualized a high pressure high temperature jet, both free and interacting with a wall in a constant volume combustion chamber, using a Laser Induced Exciplex Fluorescence (LIEF) technique.\textsuperscript{42} This allowed him to make a quantitative measurement of the volume and thus estimate the global air entrainment with little uncertainties. A volume change of 15% was the minimum required to be considered significant, according to the LIEF calibration and normalization.

For an injection pressure of 1500 bar, the jet volume was not significantly different (10%) from the free jet condition, even with the nozzle at a different distance normal to the wall. However, for an injection pressure of 2000 bar, there was a volume increase of 20% at $t > 1$ ms, indicating an increase in entrained air. This is due to the higher momentum of the jet with a higher injection pressure.

For the fuel mass concentration profiles of both the free jet and the wall jet were almost identical far from the wall. At a location closer to the wall and along the axis of the jet, the fuel mass concentration increased considerably with respect to the free jet (5 kg/m\textsuperscript{3} for the wall jet, 3 kg/m\textsuperscript{3} for the free jet).

A volume histogram for the free and wall jets showed a similar global air entrainment. However, the free jet had a higher portion with high fuel mass concentration (2–5 kg/m\textsuperscript{3}), while the wall jet has a higher portion with low fuel

| Authors            | Data Range | Jet Spread | | | |
|--------------------|------------|------------|---|---|
| Bakke              | 5 to 10.7  | 6.4E4      | 0.53 | 0.94 | -- |
| Cooper et al.      | 3 to 7     | 2.3E4      | 2   | 1   | 0.073 |
| Cooper et al.      | 3 to 6     | 10         | 1   | 0.083 |
| Knowles & Myszko   | 2 to 9     | 9.0E4      | 2   | 1   | 0.091 |
| Knowles & Myszko   | 3 to 10    | 10         | 1   | 0.109 |
| Bradshaw & Love    | 3.2 to 20  | 1.5E5      | 18  | 1   | 0.088 |
| Arithmetic mean    |            |            |     |     | 0.089 |
| Sample standard deviation |       |            |     |     | 0.013 |
mass concentration (1–2 kg/m³). At the wall jet tip the fuel concentration (1.3 kg/m³) was lower than an equivalent point at the free jet tip (1.8 kg/m³). This indicates that the loss in mixing due to stagnation at the wall is compensated with the increased mixing at the jet vortex.

Following Bruneaux’s results and assuming that injection pressures in real engines are rarely higher than 1600 bar, Bordet⁴³ proposed a model to represent the spray-wall interaction previously described. His main assumptions were that the wall jet volume and the free jet volume remain the same and that the overall entrained air also remains the same. Only the local mixing near the wall is changed.

For this model, Bordet stated that for each time step “dt”, the volume of the free jet that would go past the wall is relocated along the wall. The thickness of this zone is determined by assuming an elastic collision of particles in the wall, conserving the incidence angle. Knowing this thickness, the extension along the radial direction of the new volume is determined.

The wall jet penetration and the fuel vapor mass concentration are fairly well approached with respect to the results from Bruneaux.⁴² However, the jet thickness along the wall is considerably overestimated; the vortex impact cannot be considered and there are concentration discontinuities at the boundary of the modified zones with the upstream jet.

Le et al.⁴⁴ visualized a diesel jet inside an engine cylinder using Planar Laser-Induced Fluorescence imaging of hydroxyl (OH-PLIF), both online and offline;⁴⁵–⁴⁹ fuel-PLIF, which the same PLIF as with OH but at a timing where there is no high temperature reaction, therefore the emissions are from aromatic hydrocarbons and additives from the fuel;⁵⁰ and line-of-sight chemiluminescence.

The images showed that the combustion reaction inside the cylinder starts at the wall jet first, due to the enhanced mixing in the wall jet (low temperature reactions seen with fuel-PLIF, as described in.⁵¹–⁵³) Then the high temperature reactions appear, seen with OH-PLIF and chemiluminescence, also in the wall jet. This high temperature reaction starts almost at the time of the end of injection (EOI). The combustion reaction seems to not reach the free jet, possibly due to excessive lean mixtures right after EOI.

### 3. Model Description

#### 3.1. Existing Spray model

One dimensional approach seems a good compromise for Diesel spray combustion modelling. Indeed, 0D thermodynamic combustion models do not bring any spatial information on the spray, while 3D CFD, obviously the most detailed approach, is not practical when extensive studies in which thousands of configurations are assessed. In the literature some 1D models are available, either with Lagrangian⁵⁴–⁵⁷ or Eulerian⁵⁸,⁵⁹ approach; though, they rarely describe spray wall interaction. The authors have been developing for a few years a 1D Eulerian spray and combustion model.⁷–¹⁰,⁶⁰ It is based on Musculos and Kattke inert spray model⁶¹ which is largely completed to include a pseudo 2D species repartition and to describe combustion process.

Assumptions made for this 1D model include:

- The model is Eulerian, meaning that the jet is modelled with a fixed frame of reference (grid), using troncone sections (the jet is treated as a cone, with the control volumes being sections of this cone cut perpendicular to the jet axis).
- The fluid is supposed incompressible, even in the gaseous phase.
- Viscous forces are neglected, since these are small compared with pressure forces, and shear forces at the spray outer edge are also small, due to the low velocity of the spray at this location.
- Turbulent velocities and diffusion are not considered in the momentum transfer balance. Only axial momentum transfer is considered.
- The jet angle is considered constant, since only small variations in this angle are observed during injection transient phases.⁶¹
- The evaporation rate of fuel is calculated using a single evaporation constant proportional to the liquid fuel mass, as in.⁶⁰
- The jet tip penetration is calculated as a frontier that moves within the last control volume that the jet occupies, according to the control volume average velocity and the differential time “dt” used for the calculation.
- The radial distribution of velocity and fuel fraction is calculated using an analytical equation introduced in.⁶² Both variables are assumed to have the same distribution, which is equivalent to assuming that the Schmidt number is equal to 1.⁵⁹
- The model assumes that the jet expands in a sufficiently large combustion chamber, therefore there is no entrainment of jet mass (recirculation of gases). This means that the entrainment air is only that of the surroundings. This does not affect the calculation of an inert gas, but it could have an effect when combustion of the fuel is considered in the model.

In each control volume of the grid, the total mass, liquid fuel mass and momentum transfer are calculated. A gaseous mass differential calculated from the differential ideal gas equation, and then air mass entrainment is calculated by adding all the mass differentials. A mean velocity for each control volume is calculated from their momentum and mass at a given time, and this mean velocity is used to determine the penetration of the jet tip.

Full details can be found in.⁹
3.2. Wall Jet model

A gas-only wall jet is modelled, since in the current simulation conditions the liquid phase does not reach the wall. Therefore, the gas-jet hypotheses can be adopted in this model (as described in section 2). The physical assumptions for this wall jet model are the same as those for the free jet, with some changes as detailed hereafter:

- The area and volume of the control volume are no longer those of a section of a cone, but a section of a torus.
- The jet angle (or spreading angle) is still considered constant. However, the angle for the wall jet is different from that of the free jet. This aspect is different from the wall jet model described in, where there was no jet angle included.
- Viscous forces are still not considered directly. However, the effect of the friction of the fluid against the wall is taken into account by the velocity profile, which shows zero velocity at the location of the wall, and with coefficients to compensate the loss of energy in the momentum transfer equation.
- Turbulent velocities and diffusion is still not considered. However, the zone where the free jet and wall jet overlap (impingement zone) has a high content of these. The modelling is done by the geometry of the control volume of this zone and by introducing a coefficient to account for the loss of energy in this zone.

The development of these aspects of the model are described in next sub-sections. The model assumes that the wall jet expands along a wall perpendicular to the free jet axis. Therefore, small combustion chambers where the curvature of the wall is small with respect to the free jet length might not be well simulated, since there might be interactions between the wall and free jet when the wall jet circles back close to the injection point.

![Figure 3. Schematic of the volume of one zone of the wall jet.](image)

### 3.2.1. Area And Volume

The cross-sectional area of the wall jet corresponds to a cylindrical surface, as shown in Figure 3.

\[
A = 2\pi xz
\]  

(7)

Where \( x \) is the radial distance from the central axis of the free jet (impinging point) and \( z \) corresponds to the thickness at radial distance \( x \). The differential of this area is then:

\[
dA = 2\pi xdz
\]  

(8)

When the thickness variable is normalized by the half-thickness, the differential of area becomes:

\[
dA = 2\pi x \frac{Z_{1/2}}{Z_{1/2}} d\xi, \quad d\xi = \frac{z}{Z_{1/2}} dz
\]  

(9)

The volume is calculated by considering a trapezoidal cross-section in revolution, as shown in Figure 3.

The equation obtained for the volume is:

\[
Vol = 2\pi(b-a) \left[ \tan \alpha \left( a^2 + ab + b^2 \right) + (r_e - a\tan \alpha) \frac{(b+a)}{2} \right]
\]  

(10)

It can be rewritten, in terms of \( r_s, r_e \) and \( x_{ab} \) (with \( b = a + x_{ab} \) and \( \tan \alpha = \frac{r_e - r_s}{x_{ab}} \)) as:

\[
Vol = \pi \left[ \frac{2r_s + r_e}{3} \right] x_{ab}^2 + a(r_s + r_e)x_{ab}
\]  

(11)

The differential of this volume is:

\[
dVol = \pi \left[ \left( a(r_s + r_e) + 2x_{ab} \frac{2r_s + r_e}{3} \right) dx + x_{ab} \frac{x_{ab}}{3} dr_e + x_{ab} \frac{2x_{ab}}{3} + a \right] dr_e
\]  

(12)

The terms “\( dr_e \)” and “\( dx \)” are left in the expression, to consider dilatation of the wall spray.

### 3.2.2. Spreading Ratio

All the authors reviewed propose a linear spreading ratio for the jet half-thickness. Initially it was proposed as a power law, but the exponent is close to unity; therefore, a linear ratio is used (equation (13)). The origin of the linear expression is taken to be at the impinging point, as shown by the experimental data. The computational data showed a different origin, but due to the lack of modelling of the impingement zone.

\[
Z_{1/2} = (\tan \alpha)x
\]  

(13)

Table 2 shows the proposed values for \((\tan \alpha)\).
3.2.3. Radial Velocity profile

From the studies reviewed, there are two authors that model the velocity profile of the wall jet (Figure 4):

\[
\frac{V}{V_{\text{max}}} = \left( 1 - \left[ \frac{z - d_e}{d_{by} - d_e} \right]^{1.5} \right) \quad \text{for outer zone} \tag{15}
\]

Wood (34): \[
\frac{V}{V_{\text{max}}} = 1.55 \left( \frac{z}{Z_{1/2}} \right)^{0.5} \left[ 1 - \text{erf} \left( \frac{0.70 \frac{z}{Z_{1/2}}}{} \right) \right] \quad \text{for outer zone} \tag{16}
\]

Since Vlachopoulos\textsuperscript{14} does not give a fixed value of the inner boundary layer, it is taken as 0.25\*Z\textsubscript{i/2} (one-quarter of the half-thickness), as described by Song\textsuperscript{16} as the coordinate with the highest velocity. Also from Song\textsuperscript{16} the outer boundary layer (d\textsubscript{by}) is taken as 2*Z\textsubscript{i/2}, according to computed results using CFD. The equation proposed by Wood results in the location of the maximum velocity closer to the wall, since his data includes velocity profiles close to the impinging point.

To obtain the average velocity, the following equation is used:

\[
\bar{V}_v = \frac{\int V dA}{\int dA} = V_{\text{max}} \left[ 1 - \frac{\int V(\xi) d\xi}{2 \pi (2Z_{1/2})} \right] = V_{\text{max}} A \left[ 1 - \frac{\int V(\xi) d\xi}{2 \pi (2Z_{1/2})} \right] = V_{\text{max}} B_1 \tag{17}
\]

Where B\textsubscript{1} replaces the term multiplying V\textsubscript{max}. Integrating numerically, the value of B\textsubscript{1} obtained for the Vlachopoulos’ equation is 0.502 and for Wood’s equation is 0.527 (difference of 4.7%).

From a similar handling of equation (17), the volumetric flow rate can be calculated:

\[
\bar{V}_v = \frac{\int V dA}{\int dA} = V_{\text{max}} \left[ 1 - \frac{\int V(\xi) d\xi}{2 \pi (2Z_{1/2})} \right] = V_{\text{max}} \frac{2}{V_{\text{max}}} A \left[ 1 - \frac{\int V(\xi) d\xi}{2 \pi (2Z_{1/2})} \right] = V_{\text{max}} A (B_2) \tag{18}
\]

The values obtained for B\textsubscript{2} are: 1.004 for Vlachopoulos and 1.054 for Wood (difference of 4.7%). In fact, B\textsubscript{2} = 2*B\textsubscript{1}. This value can also be compared to the value stated by Song, of 1.092 (8.1% difference with respect to Vlachopoulos and 3.8% difference with respect to Wood). This difference is due to the type of equation used to approximate the velocity profile of the wall jet.

To obtain the momentum flux, the following equation is used:

\[
M = \bar{p} \int V^2 dA = \bar{V}_{\text{max}}^2 \int V^2(\xi) d\xi = \bar{p} \int V^2(\xi) d\xi = \bar{V}^2_{\text{max}} A (B_1) \tag{19}
\]

The values obtained for B\textsubscript{3} are: 2.959 for Vlachopoulos and 2.705 for Wood (difference of 8.6%). It is important to note that these values are significantly different from the value of 2.019 indicated by Musculus\textsuperscript{61} for the free jet. It illustrates that a wall jet going at the same average velocity as a free jet has more momentum than the free jet, needed to overcome the friction resistance of the wall.

To obtain the fuel flow, a calculation of the fuel fraction distribution must be introduced. In the literature there is no information on the fuel fraction distribution for wall jets, and if the velocity distribution is used for the fuel mass fraction distribution it would yield a fuel mass fraction of zero at the wall. Therefore, the same Gaussian profile that was used in the free jet\textsuperscript{(9,60,61)} is used now.

\[
Y_f = \frac{Y_{f, \text{max}}}{Y_{f, \text{max}}} = \left[ 1 - \left( \frac{z}{2Z_{1/2}} \right)^{1.5} \right] ^2 \tag{20}
\]

**Figure 4.** Wall jet axial velocity and fuel fraction profiles along the axial direction for any given radial position. Both axes are in a relative scale, the vertical axis is (value / maximum value) and the horizontal axis is (axial position / half width).
This way the highest fuel concentration is in contact with the wall and then reduces gradually to zero at the outer edge of the wall jet, which is the same behavior used to define the fuel fraction distribution in the free jet (with the jet axis replacing the wall). Figure 4 shows the fuel fraction profile in relation to the velocity profiles.

The average fuel fraction for a given axial position can be calculated by:

\[ Y_{f,av} = \frac{\int_{0}^{\xi} Y_f dA}{\int_{0}^{\xi} A} = \frac{\int_{0}^{\xi} Y_f(\xi) \cdot 2\pi x z_{1/2} d\xi}{2\pi(x2Z_{1/2})} = Y_{f,max} \left( B_4 \right) \]  \hspace{1cm} (21)

The value obtained for \( B_3 \) is 0.450. It is different from the free jet value of 0.257, due to the different area distribution (torus instead of a cone).

The fuel flow rate is given by:

\[ \dot{m}_f = \bar{\rho} Y_f \dot{V} dA \]

\[ = \bar{\rho} Y_{f,av} \dot{V}_{max} \int_{0}^{\xi} Y_f(\xi) \cdot 2\pi x z_{1/2} d\xi \]

\[ = \bar{\rho} \int_{0}^{\xi} \frac{Y_{f,av} \cdot \dot{V}_{av}}{B_4} A \int_{0}^{\xi} Y_f(\xi) \cdot 2\pi x z_{1/2} d\xi = \bar{\rho} Y_{f,av} \dot{V}_{av} A \left( B_5 \right) \]  \hspace{1cm} (22)

The values obtained for \( B_4 \) are: 2.996 for Vlachopoulos and 2.868 for Wood (difference of 4.3%). It is important to see that these values are different than the values obtained for momentum, since the fuel fraction profile and the velocity profile are not equal (as in the free jet described by Musculus\(^{61}\)).

Vlachopoulos\(^{14}\) also mentioned that from the experimental data he observed that the inner part of the velocity profile (the boundary layer flow) was more likely behaving according to a 1/14\(^{th}\) power law rather than a 1/7\(^{th}\) power law (although he stayed with the latter due to the little relevance and influence it had for his work). Table 3 summarizes the constants calculated, and gives the constants obtained for a 1/14\(^{th}\) power law in the Vlachopoulos equation.

For the 1D code, the constants to be used are the average \( B_3 \) (momentum) and average \( B_5 \) (fuel flow).

### Table 3. Wall jet model constants.

| Vlachopoulos (1/14\(^{th}\) power law) | Wood | Vlachopoulos (1/14\(^{th}\) power law) | Average |
|----------------------------------------|------|----------------------------------------|---------|
| B\(_1\) | 0.502 | 0.527 | 0.509 | 0.513 |
| B\(_2\) | 1.004 | 1.054 | 1.017 | 1.025 |
| B\(_3\) | 2.959 | 2.705 | 2.970 | 2.878 |
| B\(_4\) | 0.450 | 0.450 | 0.450 | 0.450 |
| B\(_5\) | 2.956 | 2.868 | 3.014 | 2.959 |

### 3.2.4. Impingement Zone

The impingement zone is modelled as shown in Figure 5. It has an entry area of a disk of radius \( r_z(nzf-1) \) and an exit area of a ring of radius \( r_z(nzf) \) and height of \( d_0w \), where \( nzf \) is the number of zones of the free jet. Since there is no accumulation of mass at the impingement zone, the mass flux out of this zone is the same as if the jet were free. This allows the calculation of a relationship between the outlet free-jet velocity and outlet wall-jet velocity, shown in equation (23).

\[ \dot{m}_{out,free} = \dot{m}_{in,wall} \rightarrow V_{out} = \frac{r_{nzf}}{2d_0w} V_{in} = \frac{1}{2 \tan \alpha} V_{in} \]  \hspace{1cm} (23)

The overlapping of both jets is not directly modelled, due to the complexity of the phenomena that occurs in this region. However, a “momentum flux transmission coefficient” is introduced to account for energy losses at impingement. It is included in the incoming momentum flux of the zone following the impingement section (the first wall jet zone). It is expressed as follows:

\[ \dot{M}_{in,i} = \dot{M}_{out,i-1} \cdot C_{momentum} \]  \hspace{1cm} (24)

Where \( C_{momentum} \) represents the fraction of momentum flux that arrives to the first wall jet zone from the last free jet zone (impinging zone). The value \((1-C_{momentum})\) can be associated to the dissipation in the overlapping zone\(^{(63,64)}\).

To account for friction losses of the jet moving along the wall, another momentum flux coefficient is introduced in a similar way as in the impingement section, except that it applies to all the wall jet zones:

\[ \dot{M}_{in,i} = \dot{M}_{out,i-1} \cdot C_{momentum,w} \]  \hspace{1cm} (25)

Values for \( C_{momentum} \) and \( C_{momentum,w} \) have to be calibrated. The calibration procedure is described in the Results and Discussion section.

![Figure 5. Modeling of impingement zone.](image)
3.2.5. Computational Domain

The computational domain used for the wall jet is taken from the existing domain used for the free jet. The total number of zones used for calculation is still nz, as it must be a constant parameter in the code. A new parameter is used (nzf) to determine the number of zones used for the free jet (before the impingement). As it can be seen in Figure 6, nzf must be less than nz.

As in the free jet, the zones are trapezoidal, which form the outline of the jet longitudinal cross-section, assuming a constant angle of expansion (spreading ratio). There is one angle for the free jet and a different angle for the wall jet (as indicated before). Each zone i has an exit area of radius r(i). The entry area for zone i is the exit area of zone i-1. The entry area for the first zone (in the free jet) is defined from the spray radius (given as an entry parameter). The entry area for the first zone of the wall jet (zone nzf + 1) is calculated assuming that the wall jet has its origin at the impingement point of the jet axis. Therefore, the entry radius d0w is calculated by:

\[ d_{0w} = r(nz) \tan \alpha \]  

(26)

With \( \alpha \) being the angle of the wall jet (with a value of 5.38° as indicated before) and \( r(nz) \) being the radius of the free jet at the impinging point.

Both the free jet domain and the wall jet domain are shown one next to the other in the same axis (z). This implies that the wall jet axis is rotated 90° to match the free jet axis, as shown in Figure 7.

4. Results and discussion

The model is tested using the experimental conditions defined in Bruneaux, shown in Table 4. The measured penetration, shown in Figure 8 along with the free jet penetration with 60 zones, shows a fairly good correlation in the free jet zone (axial distance less than 26.6 mm), with a relative error of 2%. The liquid length (LL) for the free jet is also shown, having a maximum extension of 14 mm. This means that the liquid phase never reaches the wall, which validates the model assumption of having only gas phase in the impinging jet.

From Bruneaux, with these injection parameters, the free and the wall jet have approximately the same volume. Therefore, the wall jet parameters (\( C_{\text{momentum}} \) and \( C_{\text{momentum}_w} \)) are calibrated to find a wall jet of the same volume as the free jet. Figure 9 shows different combinations of \( C_{\text{momentum}} \) with \( C_{\text{momentum}_w} = 1 \). Figure 10 shows different combinations of \( C_{\text{momentum}_w} \) with \( C_{\text{momentum}} = 1 \). The best match is found for \( C_{\text{momentum}}: 0.50 \) and \( C_{\text{momentum}_w}: 0.98 \). These values show that there is a large momentum loss at the impingement zone (approximately 50%) and there is less loss at the following zones (2%). This is consistent with the fact that the friction losses along the wall are related to the fluid viscosity, which is relatively low in this case (\( 10^{-4} \) g/cm/s). The losses at the impingement zone are more related to the dissipation due to the collision and stagnation of the fluid during impingement.

4.1. Time Plots

The time-dependent variables analyzed are: penetration, tip velocity, entrainment rate, and flammable fractions (determined by the flammability limits). For the chosen parameters, the jet penetration is shown in Figure 11, along with the free jet and the Bruneaux measurements.

It can be seen that the penetration for the model wall jet is considerably higher than the measured values. This can be interpreted as the extra length that the wall jet has if its vortex was “unrolled”. It is mentioned by different authors that the vortex at the tip of a wall jet increases with time. And so does the difference between the calculated penetration and the measured penetration. This interpretation could also explain the additional entrainment seen in the vortex; in the real jet it is due to turbulent mixing, but in the model it can be seen as extra length of the jet through which additional entrainment flow can enter.

Since the penetration of the calculated wall jet was greater than that of the wall jet measured by Bruneaux,
this additional length could be interpreted as the tip vortex seen for the wall jets (Figure 12). The tip vortex was not considered in previous models; only turbulent effects on the whole jet through the k-ε model.

Having this additional length helps understand some observed behavior of the wall jet. In this particular case of Bruneaux, the wall jet has approximately the same volume (and therefore the same entrained mass) as the equivalent free jet. However, the wall jet entrains less mass than the free jet. The extra entrained mass comes from the tip vortices, where mixing is enhanced. This extra entrained mass would then be modeled as that additional length of the wall jet. It can be seen from the free vs. wall jet penetration plot that the difference between the measured penetration and the calculated penetration increases with time. This is the same behavior of the tip vortex, which increases size with time. Therefore it seems

### Table 4. Experimental parameters.

| Parameter               | Quantity               | Reference          |
|-------------------------|------------------------|--------------------|
| AMBIENT DENSITY:        | 25 kg/m³               | Bruneaux⁴²         |
| AMBIENT TEMPERATURE:    | 800 K                  |                    |
| INJECTION PRESSURE:     | 1500 bar               |                    |
| INJECTION DIAMETER:     | 0.15 mm                |                    |
| IMPINGING DISTANCE:     | 26.6 mm                |                    |
| MASS FLOW RATE:         | 7.2 g/s                | Bruneaux⁴²         |
| AREA COEFFICIENT:       | 0.82                   | Naber-Siebers⁶⁵    |
| DISCHARGE COEFFICIENT:  | 0.56                   | Naber-Siebers⁶⁵    |
| OTHER DERIVED QUANTITIES|                        |                    |
| VELOCITY COEFFICIENT:   | 0.7                    | Naber-Siebers⁶⁵    |
| AMBIENT PRESSURE:       | 57.4 bar               |                    |
| INJECTION VELOCITY:     | 45.32 m/s              |                    |

**Figure 8.** Jet tip penetration for a free jet calculated with the 1D model. The liquid length (LL) is also shown (max. value of 19 mm), which as can be seen is less than the distance to the wall (26.6 mm), therefore indicating that the liquid phase of the jet never reaches the wall. The circles indicate the measured penetration for the experimental wall jet in Bruneaux⁴¹ where the first data point corresponds to the moment where the jet reaches the wall (therefore corresponds also to the free jet penetration).

**Figure 9.** Modeled wall jet volume curves for different values of \(c_{\text{momentum}}\) (with \(c_{\text{momentum, w}} = 1\)), compared to the modeled free jet volume.

**Figure 10.** Modeled wall jet volume curves for different values of \(c_{\text{momentum, w}}\) (with \(c_{\text{momentum}} = 1\)), compared to the modeled free jet volume.
reasonable to associate this additional length an equivalent tip vortex length.

The tip velocity of both the free and wall jets is shown in Figure 13. It increases at the impingement zone (due to the flow area reduction) and then becomes slower than that of the free jet. This is an expected result, as the cross-sectional area of the wall jet becomes larger than that of the free jet. Therefore, to maintain a similar spray volume, the tip velocity must be smaller.

The entrainment rate for the free and wall jet with respect to time is shown in Figure 14. It is roughly similar as that of the free jet before reaching the wall. At the impingement point a slight drop in the entrainment rate is seen, consistent with stagnation. After impingement, the entrainment rate of the wall jet has a higher value than that of the free jet. The higher entrainment value can be related to the constraint to keep the volume of the wall jet similar to that of the free jet; since penetration is slower in the wall jet, it needs to entrain more air to maintain volume equality. Towards the end of the simulation time both curves tend to equalize (overlap).

It is also visible that the entrainment rate has dependence to $t^{1/2}$ (which gives it the same shape as free jet penetration - Figure 14) as indicated by Song. The oscillations seen in both free and wall jet entrainment could be related to the discretization scheme, which calculates each finite volume only once in a time step and in ascending order.

Figure 11. Penetration of the calculated model wall jet, compared to the model free jet penetration and the experimental values for the wall jet measured by Bruneaux.

Figure 12. Schematic of the wall jet interpretation. The shaded region represents the model wall jet profile showing the penetration along the wall (x-direction), and the white region represents the actual wall jet development (with tip vortices). The extra penetration of the model wall jet corresponds to the length of the curled tip vortex.

Figure 13. Jet tip velocity of the modeled wall jet compared to the modeled free jet.

Figure 14. Entrainment rate (left axis), and free jet penetration (right axis) of the modeled wall jet compared to the modeled free jet.
The flammability interval for the mixture is considered in the range of \( 0.5 < \Phi < 2.0 \), as in Song.\textsuperscript{16} Therefore, values of \( \Phi \) above 2.0 are considered rich and those below 0.5 are considered lean. The rich, flammable and lean mass fractions of both the free and wall jet are shown in Figure 15, assuming \( O_2 \) content in the ambient air of 23\% by mass.

The dash-dot lines represent the free jet rich, flammable and lean mass fractions in the jet, while the solid lines represent those of the wall jet. After impingement, the rich mass fraction in the wall jet is slightly greater than that of the free jet and then it returns to similar values; this is expected, as the stagnation and reduced entrainment during impingement reduce the mixing of fuel and ambient air. The flammable mass fraction in the wall jet decreases significantly (peak value of 0.69 for the free jet vs a peak value of 0.42 for the wall jet); the lean mass fraction increases in the wall jet (maximum value of 0.28) with respect to the free jet (maximum value of 0.09). This shows that in the wall jet, the overall mixing process is faster than in the free jet, taking the spray mixture from rich to lean faster.

### 4.2. Axial Plots

The axial-dependent variables presented are: velocity, fuel mass fraction, flammable mass fraction and flammability limits. The steady-state jet velocity (average for each zone) is shown in Figure 16. The time is \( t = 1.3 \) ms, with a constant injection rate.

The wall jet velocity decreases at the impingement zone, due to stagnation before contact with the wall; then it increases at the start of the wall jet, due to the flow area reduction (as mentioned before) and then it remains below that of the free jet. For both jets, the velocity is inversely proportional to the axial distance. However, in the wall jet after impingement, the decrease in velocity is greater, due to the greater increase in cross-sectional area and, to a lesser extent, to friction with the wall).

Figure 17 shows the axial fuel mass fraction (averaged for each discrete volume) for the whole jet at \( t = 1.3 \) ms, with a steady state injection rate. The behavior of the fuel mass fraction is equivalent to that of the steady state velocity (seen before) and represents the equivalence ratio trend. Just as the rich mass fraction in Figure 15, it becomes slightly larger at impingement for the wall jet, due to flow stagnation. It then decreases below free jet values, due to the enhanced mixing in the wall jet (mentioned before). Stagnation is observed only on the last free jet discrete volume, since the discretization scheme does not allow for modification of upstream volumes.

Due to the assumed radial distribution of the equivalence ratio\textsuperscript{56,61} these flammability limits (\( 0.5 < \Phi < 2.0 \)) can be located along the radial direction. Figure 18 shows the rich, flammable and lean mass intervals, as cumulative mass fraction, for the steady state injection, for axial positions from the injector (again, assuming an \( O_2 \) concentration of 23\% in air). The intervals are determined by the rich (\( \Phi = 2.0 \)) and lean (\( \Phi < 0.5 \)) limits. The highest equivalence ratio is found along the spray axis, and reduces as the radial position increases towards the spray edge. The rich interval therefore corresponds to the mass concentration between 0 and the rich limit for a given axial position; the flammable interval corresponds to the cumulative mass between the rich and lean limit; and the lean fraction corresponds to the cumulative mass between the lean limit and 1.

The dash-dot lines represent the limits for the free jet and the solid lines represent the limits for the wall jet. The rich

---

**Figure 15.** Rich, flammable and lean mass fractions of the modeled wall jet compared to the modeled free jet.

**Figure 16.** Steady-state velocity at \( t = 1.3 \) ms of the modeled wall jet compared to the modeled free jet.
and lean limits for both the free and wall jet follows the same path, until just before impingement. At impingement, there is an increase of rich and flammable mixture, as stagnation increases the fuel concentration at the last discrete volume. After impingement, both limits rapidly decrease to zero, as the mixing with the ambient air is enhanced. The lean limit of the free jet descends much more gradually. The axial mass fraction between both limits represents the flammable fraction “cross-section” of mixture in the jet, whose value is shown in Figure 19.

These flammability limits can be traced with respect to the real jet cross-section, obtaining a “flammability profile” of the jet, shown in Figure 20. The dash-dot lines represent the free jet and the solid lines represent the wall jet. For the free jet, the flammable cross-section seems to reach a constant thickness, although the jet keeps expanding. In the wall jet, this flammable cross-section continues to reduce until it becomes zero and all the mixture becomes too lean. For both jets, the rich cross-section extends until about 26 mm from the injector (the impingement point). For the wall jet, the increase in rich mass fraction can be seen between 25 mm and 31 mm from the injector (the region within the free jet touching the wall). The relation between the flammable areas illustrates the difference in flammable mass fractions in the jet (mentioned before in Figure 19).
Due to the 1D modelling assumptions, the wall jet cross-section is placed following the free jet cross-section. Here, the beginning of the wall jet cross-section is placed at a distance of about 31 mm from the injector, while the wall is located at 26.6 mm from the injector. This additional distance represents the free jet radius, as shown in section 3.2.5, where the wall jet model was described.

4.3. Injection Pressure

Testing the model with an injection pressure of 2000 bar (fuel flow rate of 8.5 g/s), a good match is seen in the measured and calculated volumes of the jet (Figure 21), with a slight deviation at a time greater than 1 ms. At 2000 bar, the jet volume (Figure 21) and entrainment (Figure 22) are greater than the 1500 bar case, as described by Bruneaux.41 In this case, the entrainment is greater from the beginning and during the early wall jet phase. Approaching the large times, the 2000 bar wall jet still has a larger entrainment and the difference with the 1500 bar jet stabilizes. The drop in entrainment rate due to stagnation is more clearly seen in Figure 22.

The tip velocity (Figure 23) changes in the free jet zone, the one for 2000 bar being slightly greater. The velocity peak at impingement occurs slightly earlier for the 2000 bar case (at t = 0.155 ms, versus t = 0.179 ms for the 1500 bar case), corresponding to a slight difference in penetration. Afterwards, the average tip velocity becomes practically equal for both jets. This indicates that the tip velocity (and therefore the penetration) of the wall jet has little dependence on the injection pressure.

The mass fraction distribution with time (Figure 24) indicates a higher mixing rate for the 2000 bar jet, which is consistent with the higher entrainment rate observed. The rich mass fraction is lower and the flammable and lean fractions are higher. It is interesting to note that after a time of about 0.87 ms, the flammable mass fraction in the 2000 bar jet becomes lower than the 1500 bar jet, again indicating a higher mixing rate. The flammable mass fraction maximum value for the 2000 bar jet is 0.425 and is reached faster than at the 1500 bar jet, whose maximum value is practically the same (0.424).

In the steady-state plots at a time of 1.3 ms, a higher axial velocity (Figure 25) is seen for the 2000 bar jet before impingement. After impingement, both velocities tend to the same values, with the 2000 bar jet having a slightly higher velocity. This is consistent with the increased velocity due to increased injection pressure. The axial fuel mass fraction (Figure 26) is the same for both injection pressures (except for the slight difference in penetration, which is higher for the 2000 bar jet), with the corresponding increase at the impingement point (26.6 - 31 mm) for both jets. In this case, the greater axial velocity is compensated with a greater entrainment to yield the same fuel mass fraction (and therefore, the same equivalence ratio). The rich and lean mass fraction limits and axial profile show no significant difference.

4.4. Other Parameters

To further validate the model, additional simulations have been done, changing the impingement length and changing the chamber pressure. Figure 27 shows the volume of the experimental41 and modeled wall jets, maintaining an injection pressure of 1500 bar but changing the impingement distance from 26.6 mm to 31.5 mm. As reported by
Bruneaux, there is not a significant difference between both wall jet volumes. The model results also show very close values of volume for both cases. The percent difference between the experimental and modeled volumes for the 31.5 mm case is 6.7% (for t < 1 ms) and 8.4% (for t > 1 ms). For the 26.6 mm case, the percent difference is 4.6% (for t < 1 ms) and 17% (for t > 1 ms).

Another validation of the model was done changing the combustion chamber pressure (10 bar, 30 bar and 50 bar) with an injection pressure of 1200 bar, and comparing the wall spray volume with the experimental values from Allocca \cite{66 and 67}, seen in Figure 28. A good qualitative match can be seen, with an average error of 7.9% for the 50 bar case, 7.0% for the 30 bar case and 10.6% for the 10 bar case. These errors are mainly due to the lack of additional information on the experimental conditions, to use as input for the model. Additionally, a deviation is seen in the 10 bar case at a time around 1.1 ms. It seems that at low combustion chamber pressures a radial dilatation (further described in \cite{9}) is seen in the modelled spray volume, indicating a potential limitation for the model to calculate wall jets with low chamber pressures.

Figure 23. Tip velocity of the modeled wall jet for an injection pressure of 1500 bar and 2000 bar.

Figure 24. Rich, flammable and lean mass fractions of the modeled wall jet for an injection pressure of 1500 bar and 2000 bar.

Figure 25. Steady velocity of the modeled wall jet for an injection pressure of 1500 bar and 2000 bar, at t = 1.3 ms.

Figure 26. Average fuel mass ratio of the modeled wall jet for an injection pressure of 1500 bar and 2000 bar, at t = 1.3 ms.
5. Conclusions and future work

A sub model was introduced in an existing 1D Eulerian model to account for the spray-wall interaction. It used a spreading ratio of 0.094 (equivalent to an angle of 5.38°) and included two constants that were calibrated to model the dissipation at the impingement zone and friction losses along the wall that are typical of wall jets and affect the momentum flux balance. These values were \( C_{\text{momentum}} = 0.50 \) and \( C_{\text{momentum, w}} = 0.98 \), obtained by calibrating with available experimental data from Bruneaux.\(^{41}\) to match the wall jet volume to that of the free jet. The model showed good qualitative correlation to the theory developed for wall jets, and allowed for an inclusion of the tip vortex influence on entrainment.

The mixing dynamics which yield the rich, flammable and lean mass fractions for the whole jet were coherent with the wall jet theory, even though there was not a good match to the theoretical data from Song.\(^{25}\) The rich mass fraction behaved as expected, increasing in the wall jet due to stagnation at the impingement zone. The flammable and lean mass fractions had opposite behaviors, but both are explained by a greater mixing in the wall jet. The flammable mass fraction was expected to initially decrease and then increase with respect to the free jet values, but in turn it was always decreasing. The lean mass fraction was expected to decrease with respect to the free jet, but it increased. These differences are probably caused by the radial distribution of equivalence ratio used, which may differ from that used in the literature. With a small difference in the rich mass fraction, a greater mixing in the wall jet will produce more lean mixture than in the free jet (and less flammable mixture), which is the behavior seen in the model results.

An increase in injection pressure showed an increased penetration, spray volume and overall entrainment. The tip velocity increased in the free jet zone, but remained essentially equal in the wall jet. The steady axial parameters (fuel mass fraction, flammable mass fraction) showed little to no variation, indicating that the injection pressure has little effect on the steady-state variables. This means that the axial mass fractions (which are related to air entrainment) are not dependent on injection pressure (only on the spray angle).

The time plot of flammable, rich and lean mass fractions showed a shift in the curves when increasing injection pressure, meaning that the flammable mass will appear faster but so will the lean mass. The injection pressure determines the penetration reached at a given time, and therefore the mass fraction distribution. Since the 2000 bar spray has a higher penetration, it has more lean mass exposed. Therefore, the lean mass fraction will be higher and the flammable mass fraction will be lower.

The model was validated by varying other parameters, such as impingement distance and combustion chamber pressure, and comparing to experimental results reported in the literature. This model can be applied to small engine simulations, where the cylinder pressure is relatively high and the wall is reached by the fuel spray while still reacting, therefore modifying the combustion dynamics of the spray and potentially changing the heat release rate.

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