IMAGE ENHANCEMENT ALGORITHM USING ADAPTIVE FRACTIONAL DIFFERENTIAL MASK TECHNIQUE

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Abstract. This paper addresses a novel adaptive fractional order image enhancement method. Firstly, an image segmentation algorithm is proposed, it combines Otsu algorithm and rough entropy to segment image accurately into the object and the background. On the basis of image segmentation and the knowledge of fractional order differential, an image enhancement model is established. The rough characteristics of each average gray value are obtained by image segmentation method, through these features, we can determine the optimal fractional order of image enhancement. Then image will be enhanced using fractional order differential mask, from which fractional order is obtained adaptively. Several images are used for experiments, the proposed model is compared with other models, and the results of comparison exhibit the superiority of our algorithm in terms of image quality measures.

1. Introduction. Image enhancement is a meaningful and important task in digital image processing, which can be widely used in many fields, such as medical imaging processing, pattern recognition, traffic safety, machine vision, robotics, and remote sensing [16, 15, 14, 37, 41]. Basic algorithms of image enhancement are divided into two categories: the spatial domain method and the frequency domain method. The spatial domain method directly processes the image pixels, including point processing, template processing and some other methods. The frequency domain processing technology is based on improving the Fourier transform of image, the common processing methods are high-pass, low-pass filtering, homomorphic filtering and so on. These traditional algorithms of image enhancement use integer-order differential, which can enhance the image to some extent. Nevertheless, in the process of image enhancement, some details of the image may be lost or noise may be generated by these integer algorithms. It is necessary to improve conventional algorithms of image enhancement to reflect image information more clearly and accurately.

In recent years, the introduction of fractional calculus into image processing has become a novel direction, and good results have been achieved [17, 32, 4, 1, 38, 2, 25, 7]. Fractional calculus is also gradually used in image enhancement. Hungenahally and Suresh [13] introduce and outline generalized fractional discriminant functions,
these functions can extract transitional information under noisy conditions and retain obvious perceptual details, so that the image enhancement effect is good. Then Liu [20] proposes an image enhancement algorithm based on fractional mask, which uses image convolution to realize fractional differential operation. Based on two-dimensional digital fractional-order Savitzky-Golay differentiator, Chen and Xue [5] propose an image enhancement algorithm, which uses an unsupervised optimization algorithm to select fractional parameters. Gao and Zhou [8] also generalize fractional differential to quaternion, and propose a new concept of fractional directional differential of quaternion, which is used in image enhancement. In particular, Pu et al. [29, 12, 30] propose fractional mask, in which the fractional order is constant and is determined by human. In the process of image enhancement, fractional differential operator not only can enhance image edge, but also preserves weak texture. The fractional order enhancement algorithms improve the enhancement effect compared with the integer order algorithms, but the enhancement results are still unsatisfactory because they use the same fractional order to process images.

By way of getting better application of fractional order in practical image enhancement, some adaptive fractional order models have been developed. The adaptive fractional order enhancement algorithms vary with regional characteristics, and they get more accurate results in actual image processing [18, 11, 6]. However, some adaptive fractional order functions have limitations in image processing. When dealing with different kinds of images, the enhancement results are sometimes good or bad. Therefore, it is still a problem to select the appropriate adaptive fractional order function.

The rough set is a mathematical tool for describing incompleteness and uncertainty. It can analyze and infer data to discover hidden knowledge and reveal potential laws. The rough set is a promising method, which is widely used in image processing [34, 23, 21]. Therefore, the authors construct fractional order adaptive function by using rough set theory.

In the interest of enhancing images effectively, a new image segmentation algorithm is proposed based on Otsu algorithm and the rough set, this segmentation algorithm makes the segmentation results accurate. Then, the rough entropy of each gray gradient calculated by image segmentation is taken as the fractional order of each gray value, and image is enhanced by fractional mask. The method of adaptive fractional order construction proposed in this paper uses rough entropy, which has adaptability to images with different characteristics. More importantly, the enhancement model proposed in this paper can enhance the image to a certain extent, and the display effect of image information is better than the traditional image enhancement model.

The rest of this paper is organized as follows. In Section 2, the related theories of fractional differential and rough set theory are introduced. Section 3 proposes image segmentation and the selection of adaptive fractional order for image enhancement based on rough entropy. In Section 4, experiments and comparisons are discussed. Finally, the Section 5 draws a conclusion.

2. Formulation of related theories.

2.1. Related theories of fractional differential. The main purpose of this section is to introduce the basic contents of fractional differential. The theory of fractional differential in Euclidean space is more developed than its in Hausdorff space, and the definitions of fractional differential based on Euclidean measure are
widely used in mathematical research. The classical fractional different definitions are the G-L definition, the R-L definition, and the Caputo definition [28]. The G-L definition can be converted into convolution form in numerical implementation, so it is very suitable for signal processing.

The definition of the G-L fractional differential of order \( v \) is defined by

\[
D^v_{G-L}s(x) = \frac{d^v}{dx^v} s(x)_{G-L} = \lim_{n \to \infty} \left( \frac{z}{n} \right)^{-v} \sum_{k=0}^{n-1} \frac{\Gamma(k-v)}{\Gamma(k+1)} s(x - k(x/n)), \tag{1}
\]

where \( s(x) \) is the signal under consideration, \([a,x]\) is the duration of \( s(x) \), \( v \) is a real number, and \( \Gamma(\cdot) \) is the Gamma function.

Based on the basic theory of signal processing and fractional differential, the Fourier transform [35, 36] of \( s(x) \) is defined by

\[
\text{FT} \left(D^v s(x)\right) = (i\omega)^v \text{FT}(s(x)) - \sum_{k=0}^{n-1} (i\omega)^k \frac{d^v - k}{dx^v - k} s(0), \tag{2}
\]

where \( i \) is the imaginary unit, and \( \omega \) is the digital frequency. If \( s(x) \) is a causal signal, \( \text{FT}(D^v s(x)) = (i\omega)^v \text{FT}(s(x)) \).

We can get amplitude-frequency figure shown in Fig. 1 based on different orders according to fractional Fourier transform. In the figure, we can see that the magnitude of different fractional order increases with the increase of frequency, and the increase rate is sharply strengthened nonlinearly with the increase of frequency and differential order. It reveals that the fractional differential operator has the ability to enhance signal. In addition, the fractional differential operator has the property of weak derivative, it enhances the high frequency component of the signal while preserving the very low frequency component of the signal nonlinearly.

For a digital image, it has different regions such as boundary, weak texture and smooth areas, the corresponding signal frequencies of the relevant regions are also different. It can be seen from Fig. 1, if the integer order image enhancement operator is used, the texture and smooth region may be weakened when the image edge is enhanced. Fractional order differential mask just overcomes this problem, while enhancing the high frequency, it can also retain the low frequency part.

Based on G-L definition, we introduce the basic knowledge of fractional mask. The duration \([a,t]\) of the signal \( f(t) \) is divided by equal intervals \( h = 1 \), than \( n = \lfloor \frac{t-a}{h} \rfloor = \lfloor t-a \rfloor \), \( \lfloor \cdot \rfloor \) represents the integer part. On the Basis of G-L Definition and [31], we can get the \( v \)-order fractional differential of \( f(t) \):

\[
\frac{d^v f(t)}{dt^v} \approx f(t) + (-v)f(t-1) + \frac{(-v)(-v+1)}{2} f(t-2) + \cdots + \frac{\Gamma(-v+1)}{n!\Gamma(-v-n+1)} f(t-n). \tag{4}
\]

The numerical expressions of fractional partial differential operators defined by \( G-L \) along \( x- \) and \( y- \)coordinate can be obtained:

\[
\frac{\partial^v f(x,y)}{\partial x^v} \approx \sum_{i=0}^{n} q_i^{(v)} f(x-i,y), \tag{5}
\]
Figure 1. Amplitude - frequency characteristic curves of fractional differential operators (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Table 1

| $q_2 \nu$ | 0 | $q_2 \nu$ | 0 | $q_2 \nu$ |
|-----------|---|-----------|---|-----------|
| 0         | $q_1 \nu$ | $q_1 \nu$ | $q_1 \nu$ | 0         |
| $q_2 \nu$ | $q_1 \nu$ | $8 \times q_0 \nu$ | $q_1 \nu$ | $q_2 \nu$ |
| 0         | $q_1 \nu$ | $q_1 \nu$ | $q_1 \nu$ | 0         |
| $q_2 \nu$ | 0 | $q_2 \nu$ | 0 | $q_2 \nu$ |

Figure 2. The superposition of partial differential mask by 8 directions

$$\frac{\partial^\nu f(x,y)}{\partial y^\nu} \approx \sum_{j=0}^{n} q_0^{(v)} f(x, y - j).$$  \hspace{1cm} (6)

According to the formulas (2) and (3), the first three coefficients are $q_0^\nu = 1$, $q_1^\nu = -\nu$, $q_2^\nu = \frac{\nu^2 - \nu}{2}$. (5) and (6) are extended to the other six directions of image, using the coefficients above, than an eight-based image enhancement mask is obtained in Fig. 2.

2.2. Rough set theory. Rough set theory was first proposed by Pawlak Z in 1982 [26]. The rough set is a kind of effective mathematical tool for dealing with vague descriptive objects, its advantage is that it does not need any additional relevant information data. The combination of the rough set and other theories can improve the mining ability of data, and they performance well in the practice of image processing [23, 27]. For information systems $S = (U, C)$, letting $R$ be an
equivalent relation on universe $U$, $x_i$ represents an object in $U$, $[x_i]_R$ represents a set of objects that are not distinguishable from $x_i$.

For arbitrary $X \subseteq U$, the upper and lower approximations of set $X$ are the following respectively:

$$\overline{R}(X) = \{x_i | x_i \in U, [x_i]_R \subseteq X\}, \quad (7)$$
$$\underline{R}(X) = \{x_i | x_i \in U, [x_i]_R \cap X \neq \emptyset\}. \quad (8)$$

The boundary region of rough set theory is $BN_R(X) = \overline{R}(X) - \underline{R}(X)$. If the upper and the lower approximations are equal, the boundary region is empty, then such a set is called an exact set.

Roughness of the rough set is used to express the degree of roughness of set, which is defined as

$$\rho_R(X) = 1 - d_R(X) = 1 - \frac{|R(X)|}{|\overline{R}(X)|}, \quad (9)$$

roughness reflects uncertainty of set to some extent. Rough entropy defined by roughness is obtained:

$$E = - \sum \rho_R \ln \rho_R. \quad (10)$$

Rough Entropy is a statistical form of features, which reflects the average amount of information in an image. On the premise of containing the information of image, the constructed image entropy highlights the comprehensive features of the gray information of the pixel position and the gray distribution in the neighborhood of the pixel. Thus, the introduction of rough entropy into image processing has attracted more and more scholars’ attention [10, 3, 33, 9].

3. Proposed methods.

3.1. Image segmentation based on rough entropy. Otsu algorithm is an efficient algorithm proposed by Japanese scholar Otsu in 1979 [22], it determines the optimal segmentation threshold according to maximum criterion distance amongst class. In order to improve the effect of image segmentation, a new image segmentation algorithm is proposed by combining Otsu algorithm and rough entropy [24, 39].

Reference [40] applies Monte-Carlo method and rough entropy standard to segment images. It uses rough set theory to divide the image into sub-blocks according to certain rules, then calculates the rough entropy with the sub-blocks as samples, finally uses the gray value corresponding to the maximum rough entropy to segment the image. The rough set method makes image segmentation more precise, and this segmentation method improves the speed of segmentation when smaller image sub-blocks are used to achieve better segmentation effect. Inspired by this algorithm, this paper combines rough entropy with Otsu algorithm to make image segmentation clearer.

In order to make the boundary and texture regions better separated by the algorithm, replacing the traditional gray of each pixel $f(i, j)$ with the average gradient $M(i, j)$,

$$M(i, j) = |f(i, j) - \frac{1}{8}(\sum_{k=j-1}^{j+1}(f(i, k) + f(i + 1, k)) + f(i, j - 1) + f(i, j + 1))|, \quad (11)$$

the matrix $M$ is composed of the above average gradients.
Then the image is divided into blocks of suitable size, and their optimal segmentation thresholds are obtained by using Otsu algorithm for each sub-block. The optimal thresholds are combined with the knowledge of rough entropy to calculate the upper and the lower approximations of the object and the background of sub-blocks $\overline{O}_\tau$, $\underline{O}_\tau$, $\overline{B}_\tau$, $\underline{B}_\tau$. The use of optimal thresholds reduces the range of the upper and the lower approximations, which determines the segmentation results within more precise scope, thus it makes the result of segmentation more ideal to a certain extent.

The segmentation process is as follows:

**Step 1.** For an image $f(i, j)$, its size is $[m, n]$, calculate the average gradient $M(i, j)$ of eight directions of every pixel by using the formula (11).

**Step 2.** Divide the $M$ into $N$ sub-blocks according to the appropriate size, mark the sub-blocks as $Pr$. Then we calculate the maximum gray value $Pr_{\text{max}}$ and the minimum gray value $Pr_{\text{min}}$ of each sub-block. In experiments, it is found that with the complexity of images, the thinner the sub-blocks division, the better the segmentation effect. Therefore, the number of sub-blocks should be suitable for the images as far as possible.

**Step 3.** Calculate the best segmentation threshold $T\tau$ for each sub-block using Otsu algorithm, and the maximum and minimum thresholds $T_{\text{max}}$, $T_{\text{min}}$ are found.

**Step 4.** For each average gradient level $M(i, j)$, we combine rough set theory and calculate the upper and the lower approximations of the object $\overline{O}_\tau$, $\underline{O}_\tau$ and the background $\overline{B}_\tau$, $\underline{B}_\tau$ respectively:

$$\overline{B}_\tau = \{Pr \mid T_{\text{min}} \leq M(i, j) \leq Pr_{\text{max}}, r \in [1, N]\}; \quad (12)$$

$$\underline{B}_\tau = \{Pr \mid T_{\text{min}} \leq M(i, j) \leq Pr_{\text{min}}, r \in [1, N]\}; \quad (13)$$

$$\overline{O}_\tau = \{Pr \mid Pr_{\text{min}} \leq M(i, j) \leq T_{\text{max}}, r \in [1, N]\}; \quad (14)$$

$$\underline{O}_\tau = \{Pr \mid Pr_{\text{max}} \leq M(i, j) \leq T_{\text{max}}, r \in [1, N]\}. \quad (15)$$

**Step 5.** Find out the upper and the lower approximations of the object and the background, and calculate the roughness and entropy by formulas (9)-(10). Then the average gradient is found corresponding to the maximum entropy, it is the best threshold to segment the image.

**Step 6.** We use the best threshold to segment $M$. Finally, the segmentation result is obtained.

The segmentation algorithm proposed in this paper uses Otsu algorithm to calculate the maximum and minimum thresholds of matrix sub-blocks, then they are brought into the upper and the lower approximations of image, so that the image is segmented more accurately into the object and the background. This segmentation model is more accurate than method of [40] and traditional Otsu algorithm, but it takes too long to compute, it needs to be improved in terms of computing time in the future. The segmentation results of images, named ‘Lena’ and ‘Fishing boat’, are shown in Fig. 3 and Fig. 4, respectively. From the comparison of image segmentation results, the segmentation algorithm in this paper achieves better segmentation effects.

### 3.2. Selection of adaptive fractional order for image enhancement.

The results of image enhancement are directly related to the choice of adaptive fractional function. Generally, the selection of adaptive fractional order is based on the position characteristics of gray value or average gradient value of image. By
determining where gray value or average gradient value is located (the boundary, weak texture or smooth areas), the frequency is obtained to determine the fractional order. However, a part of adaptive fractional order functions are conservative, using them to enhance images will appear inadaptable situations such as noise generation.

Introducing knowledge of the rough set will solve this problem well, this paper unites rough entropy to establish adaptive fractional order function. Firstly, the image is segmented by the method of this paper. Then, rough entropy of each average gradient based on the segmentation is got to be as the fractional order of image enhancement, and fractional mask is used to enhance image.

The detailed calculation process is as follows:

**Step 1.** Segmentation of images using the above mentioned method.

**Step 2.** The rough entropy of each average gradient is calculated by the above segmentation algorithm. Then the rough entropy is obtained as the fractional order of the corresponding average gradient.

**Step 3.** Image enhancement using adaptive fractional mask.

The block diagram for our model is shown in Fig. 5.

4. **Experiments and analysis.** In this section, five kinds of images are used to evaluate the effectiveness of the algorithm proposed in this paper, their original images are shown in Fig. 6. The image enhancement effects of this method are compared with that of the AFDA method [19] and traditional fractional differential method at the order 0.2 and 0.8 respectively. The image enhancement results are evaluated by visual analysis, entropy of information and average gradient.
Fig. 7 reflects the experimental results of ‘Lena’. Although the 0.8-order method obviously enhances the edge of image, it also produces significant noise. Contrasted the 0.2-order method and the AFDA method, the proposed method not only enhances image edge, preserves weak texture and smooth areas, but also the quality of image is not affected by noise.

Fig. 8 is a moving head image. The AFDA method acts out inadaptable features in this image, it produces a amount of noise beside human hair, which seriously affects the reading of image information. The method in this paper has better adaptability to the image, and the effect of image enhancement is better than method of the 0.2-order and the 0.8-order methods.

Fig. 9 shows a medical image. The AFDA method reflects serious inadaptability between the image and enhancement method, the contours and details of medical image produce a great deal of noise, and the quality of image is obviously worse. Compared with the other three methods, the image enhanced by the algorithm proposed in this paper is clearer, and its details are more vivid.

Fig. 10 is an aerial image. It can be evidently seen that the results in this paper is better than the 0.2-order method and the AFDA method. The mountain of the image processed by the enhanced algorithm in this paper is clearer and more vivid, and it does not produce noise as the 0.8-order method do.

Fig. 11 is an airplane image. Compared with the other three algorithms(the 0.2-order method, the 0.8-order method and the AFDA method), the algorithm proposed in this paper has better enhancement effect, which makes the details of the image clear.

Information entropy and average gradient are used as criteria for image evaluation.

The average gradient of the image refers to the change rate of image gray level, which can be used to express the image clarity. It embodies that the bigger the
Figure 6. The original images

![Original Images](image1.png)

(a) Lena  (b) The moving head  (c) The medical image  (d) The aerial image  (e) The airplane image

Figure 7. Enhancement results of Lena

rate value of image minute detail contrast changes, the greater the relative clarity of the representation image. It is defined as follows:

Table 1. The information entropy of images

| Fig. | original | 0.2 order | 0.8 order | AFDA method | our method |
|------|----------|-----------|-----------|-------------|------------|
| 7    | 5.0572   | 5.0803    | 5.2640    | 5.1225      | 15.2047    |
| 8    | 3.5754   | 3.5843    | 3.6526    | 3.6005      | 3.6358     |
| 9    | 4.9163   | 4.9366    | 5.0044    | 4.9196      | 4.9273     |
| 10   | 5.1387   | 5.2031    | 4.9300    | 5.2184      | 5.3338     |
| 11   | 5.5089   | 5.5302    | 6.7563    | 5.6435      | 5.9013     |

Table 2. The average gradient of images

| Fig. | original | 0.2 order | 0.8 order | AFDA method | our method |
|------|----------|-----------|-----------|-------------|------------|
| 7    | 3.0202   | 3.6840    | 33.3047   | 4.6871      | 10.5149    |
| 8    | 2.2055   | 2.3810    | 6.1382    | 3.9755      | 4.0240     |
| 9    | 1.5844   | 1.7256    | 5.3264    | 4.3742      | 2.5745     |
| 10   | 9.3186   | 12.9233   | 50.3865   | 19.8006     | 23.5776    |
| 11   | 4.3996   | 5.7272    | 38.6371   | 7.5458      | 14.5694    |
Figure 8. Enhancement results of the moving head

Figure 9. Enhancement results of the medical image

Figure 10. Enhancement results of the aerial image

\[ \overline{y} = \frac{1}{(M-1)(N-1)} \sum_{i=1}^{M-1} \sum_{j=1}^{N-1} \sqrt{[f(i, j) - f(i + 1, j)]^2 + [f(i, j) - f(i, j + 1)]^2}, \]

where \( M \times N \) is the size of the image.

Information entropy is a key consideration in evaluating information quality in images, the bigger the entropy, the richer the information of the image will be. The
formula for calculating image information entropy is as follows:

\[ H = - \sum_{k=0}^{L-1} p(k) \ln p(k), \]  

(17)

where \( p(k) \) is the frequency of the image gray values \( k \), and \( L \) is the constant 255.

The information entropy and average gradient of colorbluefive images in the experiments are shown in the Table. 1, Table. 2. They embody that our results are better than other researches, which further illustrate the effectiveness of the proposed algorithm in this paper.

5. Conclusion. This paper mainly builds a new image enhancement algorithm, which is combined fractional order differential mask and rough entropy based on the segmentation result. The enhancement results are more accurate than traditional algorithms. The advantages of enhancement algorithm in this paper are as follows: by adopting fractional order differential operator, the image preserves more texture details and sharp edges. By using rough entropy to establish adaptive fractional order, corresponding the adaptive fractional order differential mask can enhance various images very well. It overcomes the selectivity of general fractional order function to images and makes enhancement algorithm more practical. The experimental results show that the proposed algorithm is effective and progressive. In the future, we will strive to improve the speed of this algorithm, and more works need doing in order to enhance the image significantly.

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