Abstract

We analyze the flavor structure of the trilinear couplings in the different theoretical models of SUSY breaking. We generically obtain $A_{ij} = A_{ij}^L + A_{ij}^R$ in all the models examined. In fact, this is the rigorous form when SUSY breaking effects appear through the Kähler metric of chiral fields or through wave–function renormalization. Indeed, the low–energy phenomenological requirements from the absence of charge and color breaking minima and the measurements in flavor changing neutral current (FCNC) observables strongly favor this restricted form of the trilinear matrices. As a straightforward consequence the number of unknown parameters associated with the trilinear couplings is decreased in a factor of 2.
1 Introduction

The large popularity of Supersymmetry (SUSY) as an ingredient of many extensions of the Standard Model (SM) is due both to its ability to deal with several theoretical problems that the SM faces when it is stretched up to high energies and to the fact that it naturally appears in most of the theories that attempt to include gravity at the quantum level. For these reasons, it is generally believed that global Supersymmetry must be discovered in the neighborhood of the electroweak scale with the new hadronic colliders.

However, from the phenomenological point of view, even the Minimal Supersymmetric extension of the SM (MSSM), the simplest supersymmetrization of the SM with no additional particle content, contains a host of free parameters related to the unknown SUSY soft–breaking terms. These soft–breaking terms, or more exactly, the trilinear couplings $Y_{ij}^a$ and scalar mass matrices $m_{ij}$, have additional flavor structures besides the usual Yukawa matrices $Y_{ij}$. In fact, the low–energy phenomenology is strongly dependent on these new flavor structures. In the presence of strict flavor–universality, only observables with a dominant chirality changing contribution (i.e. electric dipole moments (EDM), $b \rightarrow s \gamma$ . . . ) are sensitive to new supersymmetric contributions \cite{1, 2}. Still, it has been recently shown that non–universality of the $A$–matrices is very relevant in the low energy observables \cite{3, 4, 5, 6} and it can raise a large contribution to $\varepsilon'/\varepsilon$ without conflicting with EDM constraints \cite{5}. From this point of view, the effects of generic non–universal soft–terms on indirect searches both in the kaon and the B systems was considered in Ref. \cite{6}.

On the other hand, regarding model building, several SUSY breaking and mediation mechanisms, e.g. supergravity–mediation \cite{7} which includes string–inspired supergravity theories\cite{1}, gauge–mediation \cite{10} and anomaly–mediation \cite{11, 12, 13} have been proposed and each mechanism predicts characteristic flavor structures and sparticle spectrum. In this paper, we take advantage of this complementary information to improve our knowledge of the soft–breaking terms, and therefore of the complete MSSM. In particular we concentrate on the flavor structure of the $A$–terms. In a purely phenomenological counting, the quark sector has in total $2 \times 3 \times 3 (= 18)$ complex free parameters for the $A$–matrices, $A_{ij}^u$ and $A_{ij}^d$ associated with the up and down sector Yukawa matrices. However, we show that in quite generic models of SUSY breaking $A_{ij} = A_{ij}^L + A_{ij}^R$. That is, the quark sector has only $3 + 3 + 3 (= 9)$ complex parameters for $A_{ij}^u$ and $A_{ij}^d$. In fact, this structure of the $A$–matrices guarantees the proportionality

\footnote{For example, see Ref. \cite{8} for heterotic models and Ref. \cite{9} for type I models.}

\footnote{The application of pure anomaly mediation to the supersymmetric standard model has the tachyonic slepton mass problem. This problem can be solved by adding a universal soft scalar mass \cite{11}. Another solution is $D$–term contributions to scalar masses which is quite interesting because it does not change the renormalization group flow \cite{14}, one of the characteristic features of anomaly mediation.}

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of the different trilinear couplings to the mass of one of the fermionic partners of the squarks involved. Furthermore, the magnitudes of the $A$-parameters are order of the gaugino and soft scalar masses. Due to these features, many of the phenomenological bounds on the so–called LR Mass Insertions (MI) \cite{13} are naturally avoided, as well as charge and color breaking minima and directions unbounded from bellow \cite{16}.

In section 2 we analyze the structure of the $A$–matrices in several models of SUSY breaking. We then discuss its phenomenological implications in section 3 through discussions on Yukawa matrices and trilinear couplings. Finally, in section 4 we present our conclusions.

2 Generic form of $A$-matrices

In this section, we analyze the form of the $A$-matrices in the main models of SUSY breaking, i.e. supergravity mediation, gauge mediation and anomaly mediation. We show that the generic form of these matrices is,

$$A_{ij} = A^L_i + A^R_j,$$

which implies at the SUSY breaking scale,

$$Y^A_{ij} = Y_{ij} A_{ij} = \begin{pmatrix} Y_{ij} \end{pmatrix} \cdot \begin{pmatrix} A^R_1 & 0 & 0 \\ 0 & A^R_2 & 0 \\ 0 & 0 & A^R_3 \end{pmatrix} + \begin{pmatrix} A^L_1 & 0 & 0 \\ 0 & A^L_2 & 0 \\ 0 & 0 & A^L_3 \end{pmatrix} \cdot \begin{pmatrix} Y_{ij} \end{pmatrix},$$

in matrix notation. This corresponds to a reduction in a factor of 2 of the number of free parameters in the trilinear couplings in the quark sector: from 18 to 9 complex parameters (something similar happens in the leptonic sector). We discuss the supergravity–mediation in section 2.1. In section 2.2 we analyze models where SUSY breaking effects are induced through the spurion formalism. This mechanism includes both gauge–mediation and anomaly–mediation.

2.1 Supergravity mediation

We consider a supergravity model including, on one hand, chiral matter fields, $\Phi^L_i$, $\Phi^R_i$ and the Higgs field $H$, which remain light after SUSY breaking and we denote altogether as $\Phi^i$, and on the other hand some moduli fields $\Phi^a$, which have large vacuum expectation values breaking SUSY and then decouple from the low–energy physics. The supergravity Lagrangian is written in terms of the Kähler potential, the superpotential and the gauge kinetic functions. The Kähler potential provides kinetic terms of chiral fields while the superpotential includes both the Yukawa couplings and the $\mu$-term as well as a nonperturbative superpotential leading SUSY breaking. We start with the following Kähler potential
$K$ and the superpotential $W$,

$$K = \tilde{K}(\Phi^a, \bar{\Phi}^a) + K_i^i(\Phi^a, \bar{\Phi}^a)|\Phi^i|^2,$$

$$W = \tilde{W}(\Phi^a) + \hat{Y}_{ij}\Phi^i_L\Phi^j_RH,$$

with $\tilde{W}(\Phi^a)$ the nonperturbative superpotential leading to SUSY breaking. The scalar potential is obtained as

$$V = F^IF^JF^K - \frac{3}{2}m^2\partial_a(\ln|W|^2),$$

where $\Phi^I$ denotes any of the fields $\Phi^i$ and $\Phi^a$, $F^I$ are the $F$-terms of the field $\Phi^i$ and $G = K + \ln|W|^2$. The fields $\Phi^a$ develop their vacuum expectation values and their $F$-terms contribute to SUSY breaking. Then, taking the flat limit, we can expand the scalar potential around non–vanishing $\Phi^a$ and $F^a$,

$$V = V_0 + \frac{|\partial W_{eff}|^2}{\partial \Phi_a} + \frac{1}{2}m^2_i|\phi^i|^2K_i^i + (Y_{ij}^A\phi^i_L\phi^j_Rh\prod_\ell(K^\ell_\ell)^{1/2} + h.c.) + \cdots,$$

where $\phi^i(h)$ are the scalar component of $\Phi^i(H)$, and $\prod_\ell K^\ell_\ell$ denotes the product of normalization constants of the fields, $\Phi^i_L$, $\Phi^i_R$ and $H$. The first term in the RHS of Eq. (6) is the vacuum energy obtained as $V_0 = F_aF^bK^{ab}_0 - \frac{3}{2}m^2_{3/2}$ with the gravitino mass $m^2_{3/2} \equiv e^G$. The second term corresponds to globally supersymmetric contributions to the scalar potential. Notice that we use $W_{eff}$, the effective superpotential in the global SUSY basis in terms of the canonically normalized fields. In this basis the Yukawa couplings $Y_{ij}^A$ are obtained as,

$$|Y_{ij}^A|^2 = e^{\tilde{K}}|\hat{Y}_{ij}|^2\prod_\ell(K^\ell_\ell)^{-1}.$$

The last two terms in Eq. (6) are SUSY breaking terms, that is, the soft scalar mass $m^2_i$ and the trilinear scalar coupling $Y_{ij}^A$ which are obtained [17, 18]

$$m^2_i = m^2_{3/2} - F^aF_b\partial_a\partial_b(\ln K^i_i) + V_0,$$

$$Y_{ij}^A = Y_{ij}^A\partial_a(\tilde{K} - \ln \prod_\ell(K^\ell_\ell)) + \Delta_{ij},$$

where $\partial_a$ denotes $\partial/\partial\Phi^a$. In this expression we obtain diagonal soft scalar masses. This results from the fact that we started with diagonal Kähler metric. Generic supergravity theories can lead to non–vanishing off–diagonal Kähler metric $K^j_i$ ($i \neq j$) for different flavors. However, such mixing does not appear in superstring models at leading order. These models have additional continuous and discrete symmetries other than the SM gauge symmetry, and under these symmetries each massless state has definite charges. Different flavors are distinguished by these extra charges and so, off–diagonal metric is forbidden. When these extra symmetry are broken, non–vanishing off–diagonal Kähler metric can appear. In the
following, we assume that such off–diagonal Kähler metric induced by symmetry breaking is suppressed enough [8].

In Eq. (9) we have included a term \( \Delta_{ij} \) which denotes the term proportional to derivatives of \( \hat{Y}^{ij} \) by the fields \( \Phi_a, F_a \partial \hat{Y}^{ij} / \partial \Phi_a \). Although these terms are perfectly possible from the point of view of supergravity, indeed a large class of string–inspired supergravity theories satisfy the condition of field-independent Yukawa couplings. For instance, Yukawa couplings \( \hat{Y}_{ij} \) are constants in heterotic Calabi–Yau models in the large \( T \) limit and type–I models [9]. In these theories, the Yukawa couplings are not functions of the fields \( \Phi_a \) with non–vanishing F–terms \( F_a \neq 0 \), and so the \((i, j)\) entry of the \( A \)-matrix is always written in the form

\[
A_{ij} = A_i^L + A_j^R + A_0, \tag{10}
\]

where

\[
A_i^L = -F_a \partial a \ln K_i^i, \tag{11}
\]

\[
A_j^R = -F_a \partial a \ln K_j^j, \tag{12}
\]

\[
A_0 = F_a \partial a (\tilde{K} - \ln K^H_H). \tag{13}
\]

If we redefine \( A_i^L \) and \( A_j^R \) absorbing \( A_0 \), then the resultant form exactly corresponds to eqs. (4), (5). For example, we redefine \( A_j^R = A_j^R + A_0 \) and then the generic form of \( A \)-matrix is

\[
A_{ij} = A_i^L + A_j^R, \tag{14}
\]

under the condition that the Yukawa couplings do not depend on the fields \( \Phi_a \) with non–vanishing F–terms \( F_a \neq 0 \).

More generally, in some models the Yukawa couplings can be field–dependent (although it is often difficult to calculate field–dependent parts of Yukawa couplings and it is believed that such parts are suppressed compared with the constant term). One of the examples where an explicit calculation can be done is heterotic orbifold models. There are two types of closed string sectors, untwisted and twisted strings. In addition, each twisted sector is assigned to a fixed point \( f_i \) on the orbifold. Yukawa couplings of untwisted sectors are constants, and Yukawa couplings of twisted sectors associated to the same fixed point are also constants. Thus, these two couplings do not lead to non–vanishing \( \Delta_{ij} \). Yukawa couplings of twisted sectors with different fixed points are obtained by the world–sheet instanton action and they depend on the moduli field \( T, Y_{ij} \sim e^{-a_{ij} T} \), where a vacuum expectation value of \( T \) corresponds to the compactification size [10, 20]. Combinations of fixed points for allowed Yukawa couplings are unique, and so the coupled fixed point is determined uniquely with the other two points chosen. Thus we can write the coefficient \( a_{ij} \propto f_{Li} - f_{Rj} \) [20], where \( f_{Li} (f_{Rj}) \) is the fixed point associated to the field \( \Phi^i (\Phi^R) \). So, in this case, the contribution due to the field–dependent Yukawa coupling \( \Delta_{ij} \) also takes the form \( \Delta_{ij} = \Delta_i^L + \Delta_j^R \).
Note that even if the Yukawa couplings depend on the moduli fields the corresponding $A$-matrix has still the form in Eqs. (12) in two cases: $F$-terms of $T$ are not dominant or the dependence of the Yukawa couplings can be factorized as in the case of heterotic orbifold models.

Another example of field–dependent Yukawa couplings is given by Calabi–Yau models without the large $T$ limit. We consider the case with one moduli field $T$. Here Yukawa couplings are obtained as $Y_{ij} \sim C + d e^{-aT}$, where $C$, $d$, $a$ are constants, and Yukawa couplings are universal. Hence, this situation simply provides a flavor–universal correction $\Delta_{ij}$. With only these couplings of $O(1)$ one can not lead to realistically hierarchical Yukawa matrices, either. We need higher dimensional operators like the Froggatt–Nielsen mechanism [21] to derive realistic Yukawa matrices, which we discuss in the next section. However, coupling strengths of higher dimensional operators have not been calculated completely for Calabi–Yau models or orbifold models and possible corrections given by these operators would be suppressed, in any case, although it certainly deserves further analysis.

2.2 Gauge–mediation and anomaly–mediation: spurion formalism

We consider a global SUSY model with chiral matter fields, $\Phi^i_L$ and $\Phi^i_R$ and the Higgs field $H$. We start with the following renormalized Lagrangian at the scale $\mu$,

$$
\mathcal{L} = \int d^4 \theta \ Z(\Lambda_M, \mu)^i \Phi^i_v \Phi^i_L e^V + \int d^2 \theta \ S(\Lambda_M, \mu) \ W^\alpha W_\alpha + \int d^2 \theta \ W(\Phi) + \text{h.c.},
$$

$$
W(\Phi) = Y_{ij} \Phi^i_L \Phi^j_R H,
$$

where $Z(\Lambda_M, \mu)^i$ and $S(\Lambda_M, \mu)\alpha$ are the wave–function renormalization constant of $\Phi^i$ and the renormalized coupling for the gauge multiplet $V$ respectively. Moreover $W^\alpha$ is the field strength superfield of $V$. In this equation, $\Lambda_M$ is simply a threshold energy scale, where some matter fields become heavy and decouple, and the $\beta$-function changes. Alternatively we can consider $\Lambda_M$ as a non–dynamical field determined by a vacuum expectation value of a superfield $\Lambda_M$. Then, the superfield $\Lambda_M$ acquires a vacuum expectation value along the scalar and auxiliary components,

$$
<\Lambda_M> = \Lambda_M + \theta^2 F_M,
$$

and so, SUSY is broken by $F$–term of $M$. We assume that the effective Lagrangian is still valid after replacing $\Lambda_M^2 \rightarrow |M|^2$ in $Z(\Lambda_M, \mu)^i$ and $S(\Lambda_M, \mu)\alpha$ even with non–vanishing $F_M$. Thus, SUSY breaking effects due to $M$ appear through the $\Lambda_M$–dependence of the wave–function renormalization $Z$ as well as $S$. One can
consider the case with more than one threshold in a similar way. This is the mechanism used in gauge–mediated SUSY breaking [10]. Hence, the expression for the soft SUSY breaking A–term is,

\[ A(\mu)_{ij} = \sum_{\ell} \frac{\partial \ln Z_\ell(M, \bar{M}, \mu)}{\partial \ln M} F_M \Lambda_M, \quad (17) \]

where the summation is taken for the wave–functions of \( \Phi^i_L, \Phi^j_R \) and \( H \). Thus, within this framework, the A–matrix \( A_{ij} \) is the summation of the three parts, i.e. \( \Phi^i_L \)–dependent part, \( \Phi^j_R \)–dependent part and the Higgs part. The Higgs part is universal for any family. So, it is straight–forward to obtain the form in Eqs. (1,2) after we absorb the Higgs part into the \( \Phi^j_R \)–dependent part, for instance.

In the case of anomaly–mediation, we consider \( \Lambda_M \) as the cut–off scale of the MSSM, above which our 4-dimensional field theory is not valid, that is, the cut-off \( \Lambda_M \) could be the string scale, compactification scale or the breaking scale of conformal symmetry. All the expressions above remain exactly the same. Thus, also in anomaly–mediation, we have the same form of Eqs. (1,2). Furthermore, we always have this form of the \( A \)–matrices in a generic case where SUSY breaking effects appear only in wave–function renormalization through loop effects. Moreover, natural magnitudes of \( A_{Lj} \) and \( A_{Rj} \) are obtained as \( A_{Lj}, A_{Rj} \lesssim \mathcal{O}(M_\alpha) \), with \( M_\alpha \) the gaugino mass.

### 3 Yukawa textures and phenomenological implications

In the previous section, we have seen that the form of A–matrices, (1), (2) is obtained in quite generic case. If we apply this form to the MSSM, the \( A_{ij}^u \) and \( A_{ij}^d \) matrices are written as,

\[ A_{ij}^u = A_{ij}^Q + A_{ij}^U, \quad A_{ij}^d = A_{ij}^Q + A_{ij}^D. \quad (18) \]

This structure has very important phenomenological effects. In first place, it implies an important reduction on the number of free parameters associated to the trilinear coupling matrices. We fix the eighteen complex matrix elements in the quark sector with only nine complex parameters. Secondly, the low energy flavor change (FCNC) phenomenology sets very stringent bounds on generic LR MI [15] and moreover, the absence of charge and color breaking minima and directions unbounded from bellow [16] constrains strongly the LR sfermion

\[ ^3 \text{See for example Ref. [22]. As a further example, this class of SUSY breaking mediation mechanisms also includes the case where SUSY breaking appears through loop effects due to Kaluza–Klein modes in extra dimensional models [23]. Even in such case we have the form of A–matrices in Eqs. (1,2).} \]
mixing. All these phenomenological requirements imply that the structure of the trilinear couplings, $Y_{ij}^{A}$, goes beyond the usually assumed form $Y_{ij}^{A} = A_{ij}Y_{ij}$. Indeed, we show below that the trilinear couplings in Eqs. (14) obtained in generic SUSY breaking models naturally fit in the low energy scenario as required by the available phenomenological constraints.

To do this, we first make some general remarks on the Yukawa matrices in these models because these Yukawa matrices are the additional ingredient in the flavor structure of the whole trilinear couplings, $Y_{ij}^{A}$. In a model independent way, we can write the Yukawa matrices in the basis of diagonal sfermion masses as follows,

$$v_1 Y_d = K^{UL\dagger} \cdot V_{CKM} \cdot M_d \cdot K^{DR}, \quad v_2 Y_u = K^{UL \dagger} \cdot M_u \cdot K^{UR} \cdot$$

(19)

where $v_1$ ($v_2$) is the vacuum expectation value of the down (up) sector Higgs field, $M_d$ and $M_u$ are diagonal quark mass matrices, $V_{CKM}$ the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix and $K^{UL}$, $K^{DR}$, $K^{UR}$ general $3 \times 3$ unitary matrices. These matrices measure the flavor misalignment among, $u_L^c Q_L^c$, $d_R^c d_R^c$ and $u_R^c \bar{u}_R^c$ respectively and their structure depends on the particular theory of supersymmetry breaking and Yukawa flavor that we consider. Hence, we can not give the final texture of these matrices, but we can still discuss several generic aspects of their flavor structures. Gauge interactions are completely flavor-blind and the existence of three different generations must be understood in terms of additional symmetries. In underlying theory, as for instance superstring inspired models, all allowed Yukawa couplings (at the string scale) are naturally order 1. Other couplings can only be obtained through higher dimensional operators and hence are suppressed hierarchically through a mechanism analogous to the well-known Froggatt–Nielsen mechanism [21]. That is, within this mechanism, some extra symmetries, e.g. $U(1)$ symmetries and/or discrete symmetries, are assumed and extra charges are assigned to the MSSM fields and extra Higgs fields $\chi_k$. Then, after $\chi_k$ develop their vacuum expectation values, these higher dimensional operators generate effective Yukawa couplings, $y_{ij}(\chi_k/M)^{n_u} Q_i U_j H_2$ and $y_{ij}(\chi_k/M)^{n_d} Q_i D_j H_1$. In a similar way, stringy selection rules of Yukawa couplings and higher dimensional operators\(^4\) can be understood in terms of discrete symmetries. In fact, these symmetries determine completely the flavor structure both in the Yukawa and soft-breaking sectors within the framework of string-inspired supergravity.

To reproduce correctly the observed hierarchy of masses and mixings, the $Y_{33}^u$ must be given as a 3-point coupling with $n_{33}^u = 0$, because experimentally the top Yukawa coupling is of $O(1)$. All other couplings are suppressed by $\epsilon_{i,j,d}^{u,d} = (\chi_k/M)^{n_{ij}}$ depending on the charges $n_{ij}^{u,d}$ with $(\chi_k/M) << 1$. The up Yukawa

\(^4\) See e.g. Ref. [24] for stringy selection rules of higher dimensional operators in heterotic orbifold models.
matrix would be then,

\[
Y^u \simeq Y_t \begin{pmatrix}
\epsilon_{1,1}^u & \epsilon_{1,2}^u & \epsilon_{1,3}^u \\
\epsilon_{2,1}^u & \epsilon_{2,2}^u & \epsilon_{2,3}^u \\
\epsilon_{3,1}^u & \epsilon_{3,2}^u & 1
\end{pmatrix}.
\tag{20}
\]

At this point, it is interesting to comment the possibility of obtaining a maximal mixing \cite{23}. For instance, in the left–handed sector, the maximal (3, 3) mixing could be realized for \( \epsilon_{i,3}^u = 1 \). The condition \( n_{33}^u = 0 \) fixes uniquely the extra charges of \( Q_3 \) once the charges of \( U_3 \) and \( H_2 \) are chosen definitely. Thus, the condition \( \epsilon_{i,3}^u = 1 \) i.e., \( n_{33}^u = 0 \), requires that both \( Q_1 \) and \( Q_2 \) should have the same extra charges as \( Q_3 \). Hence, nothing can distinguish the three \( Q_i \). Under such situation, it is clear that both \( A_i^Q \) and soft scalar masses of \( Q_i \) are universal and there is no new flavor structure in the left–sector. We can rotate freely this democratic couplings to a single entry in the (3, 3) element and we have Eq. (24) with \( \epsilon_{i,j}^u << 1 \). So, in this final basis we have again a hierarchical structure and the observable mixing angles will be small.

Once we assume a hierarchical structure, there is a simple relation among Yukawa elements and mixing angles at first order: \( K_{32}^{U_L} = \epsilon_{2,3}^u, K_{31}^{U_L} = \epsilon_{1,3}^u \), \( K_{32}^{U_R} = \epsilon_{3,2}^u \) and \( K_{31}^{U_R} = \epsilon_{3,1}^u \). This suppression is then transmitted to the \( K_{i3}^{U_{L,R}} \) and \( K_{DL} \) by unitarity of the mixing matrices and by the CKM mixing respectively. In principle, the \( K_{DL} \) mixings are not strongly constrained except in the large tan \( \beta \) regime, where only \( Y_{33}^u \simeq 1 \) and all other elements must be suppressed. In summary, we have discussed the form (20) with suppressed factors \( \epsilon_{i,j}^u \) and found \( K_{3i}^{(U,D)_L}, K_{i3}^{(U,D)_L}, K_{3i}^{U_R}, K_{i3}^{U_R} << 1 \). However, explicit values of \( \epsilon_{i,j}^u \) are model–dependent and it is difficult to estimate them for generic case. Henceforth we use the following approach in our estimates: we do not expect an accidental cancellation to obtain the CKM matrix, \( V_{CKM} = K^{U_L}K^{D_L} \). So, this means for instance, \( \text{Max}(K_{13}^{U_L}, K_{13}^{D_L}) \sim \lambda^3 \), with \( \lambda \) the Cabibbo angle. Notice that this assumption implies a further step which could be easily circumvented in some models, but we consider it a natural feature in most of the models that we analyze here. Still, we comment our results with and without this assumption.

The requirement of the absence of charge and color breaking minima and directions unbounded from below sets strong constraints on off–diagonal elements of the \( Y_{ij}^A \) matrix. In fact from charge and color breaking minima the following bounds are required \cite{16},

\[
|Y_{ij}^{Au}|^2 \leq Y_{uk}^2 \left( m_{U_{Li}}^2 + m_{U_{Rj}}^2 + m_2^2 \right),
\tag{21}
\]

\[
|Y_{ij}^{Ad}|^2 \leq Y_{dk}^2 \left( m_{D_{Li}}^2 + m_{D_{Rj}}^2 + m_1^2 \right), \quad k = \text{Max}(i, j),
\]

and similarly from directions unbounded from below,

\[
|Y_{ij}^{Au}|^2 \leq Y_{uk}^2 \left( m_{U_{Li}}^2 + m_{U_{Rj}}^2 + m_{E_{Lp}}^2 + m_{E_{Rp}}^2 \right), \quad k = \text{Max}(i, j), \quad p \neq q
\tag{22}
\]

\[
|Y_{ij}^{Ad}|^2 \leq Y_{dk}^2 \left( m_{D_{Li}}^2 + m_{D_{Rj}}^2 + m_{\nu_m}^2 \right), \quad k = \text{Max}(i, j), \quad m \neq i, j
\]

\[9\]
in the basis where Yukawa couplings, \( Y_{uk}, Y_{dk}, \) are diagonal. It is important to notice that these bounds are indeed competitive and in many cases more stringent than the corresponding FCNC bounds \([15, 16]\). However, from Eqs. (21, 22) it is evident that for \( A_{ij} \) elements of the same order of the scalar masses, the main condition these bounds require is precisely that the masses of the fermionic partners of the squarks involved set the scale of the coupling, which we naturally find in the SUSY breaking models that we analyze. Note that, as can be seen below, the LHS’s in Eqs. (21) and (22) usually include additional suppression factors due to the diagonalizing \( K–\)matrices in the Yukawa–diagonal basis.

Similarly FCNC processes set very stringent bounds on generic LR squark mixing matrices \([15]\). Nevertheless, when we take into account the proportionality to the fermion masses these constraints are largely relieved. Following references \([5, 6]\), and using Eq. (18), an estimate\(^5\) of the off–diagonal LR MI at \( M_W \) can be obtained as,

\[
(\delta_{LR}^{d})_{ij} = \frac{1}{m_{\tilde{q}}} \left( m_j (A_2^Q - A_1^Q) K_{i2}^{D_L} K_{j2}^{D_L*} + m_j (A_3^Q - A_1^Q) K_{i3}^{D_L} K_{j3}^{D_L*} \right) + m_i (A_2^D - A_1^D) K_{i2}^{D_R} K_{j2}^{D_R*} + m_i (A_3^D - A_1^D) K_{i3}^{D_R} K_{j3}^{D_R*},
\]

where \( m_{\tilde{q}} \) is the average squark mass. The value \( (\delta_{LR}^{d})_{ij} \) depends on \( K_{i2}^{D_L} \) and \( K_{i3}^{D_R} \) and hereafter we neglect small masses as \( m_d/m_s \) or \( m_s/m_b \).

If we analyze the MI which contribute in the Kaon system, we obtain

\[
(\delta_{LR}^{d})_{12} \simeq \frac{m_s}{m_{\tilde{q}}} \frac{(A_2^Q - A_1^Q)}{m_{\tilde{q}}} K_{12}^{D_L} K_{22}^{D_L*},
\]

and,

\[
(\delta_{LR}^{d})_{21} \simeq \frac{m_s}{m_{\tilde{q}}} \frac{(A_2^D - A_1^D)}{m_{\tilde{q}}} K_{12}^{D_R} K_{22}^{D_R*}.
\]

In this expression we can see clearly what is important to obtain a large MI. Both, non–universality of \( A–\)matrices and a sizeable mixing among squark generations are needed. In this case we have \( K_{12}^{D_R} = \mathcal{O}(\lambda) \) and \( (A_2^Q - A_1^Q)/m_{\tilde{q}} = \mathcal{O}(1) \), and this is enough to give rise to a very sizable contribution to \( \varepsilon'/\varepsilon \) \([3, 4]\). As explained above, the right handed mixings are in principle unconstrained but, in any case, it is very difficult to expect a larger mixing in the 1–2 sector given that, due to unitarity, the maximal value for \( K_{12}^{D_R} K_{22}^{D_R*} = 0.5 \) (to be compared with \( K_{12}^{D_L} K_{22}^{D_L*} = 0.22 \)). Hence, can be \( (\delta_{LR}^{d})_{21} \) at most a factor 2 larger than the \( (\delta_{LR}^{d})_{12} \). In summary, thanks to the high sensitivity of \( \varepsilon'/\varepsilon \) and to the presence of a large mixing among the first two generations, these MI can still have an observable effect, even overcoming the large mass suppression \( m_s/m_{\tilde{q}} \).

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\(^5\)The main RGE effects are either flavor–universal or flavor–diagonal in the basis of diagonal Yukawas \([26]\).
Similarly, in the neutral $B$ system, contributions to the $B_d$-$\bar{B}_d$ mixing parameter, $\Delta M_{B_d}$ are controlled by,

$$
(\delta_{LR}^d)_{13} \simeq \frac{m_b}{m_\tilde{q}} \frac{(A^Q_3 - A^Q_1)}{m_\tilde{q}} K^{D_L}_{13} K^{D_L*}_{33},
$$

(26)

and

$$
(\delta_{LR}^d)_{31} \simeq \frac{m_b}{m_\tilde{q}} \frac{(A^D_3 - A^D_1)}{m_\tilde{q}} K^{D_R}_{13} K^{D_R*}_{33}.
$$

(27)

A natural value for the mixing angles is $K^{D_L}_{13} K^{D_R*}_{33} \simeq \lambda^3$, which together with $(A^Q_3 - A^Q_1)/m_\tilde{q} \sim \mathcal{O}(1)$ implies $(\delta_{LR}^d)_{13} \simeq 5 \times 10^{-5}$, much smaller the MI $LR$ bounds for $\tilde{b} - \tilde{d}$ transitions. Even for $K^{D_R}_{13} K^{D_R*}_{33} \lesssim 0.5$ we obtain $(\delta_{LR}^d)_{13} \simeq 3 \times 10^{-3}$, roughly one order of magnitude too small to saturate $\Delta M_{B_d}$.

As a result, we have shown that low energy phenomenology fits nicely with the trilinear couplings in Eqs. (1,2). In fact this structure relieves the strong constraints from charge and color breaking and most of the FCNC constraints. Still, as has been recently shown, observables in the kaon sector are very sensitive to this trilinear couplings [3,4].

4 Conclusions

In this work, we have studied the flavor structure in the soft SUSY breaking trilinear couplings. We have shown that we obtain $A_{ij} = A^L_i + A^R_j$ in a quite generic case, that is, when SUSY breaking effects appear through the Kähler metric of chiral fields or through wave-function renormalization due to loop effects. Furthermore, even in the known examples with the Yukawa couplings depending on the moduli fields with non–vanishing $F$–terms, as for instance in heterotic orbifold models, this form is still maintained. Then, we have investigated the phenomenological implications of this form of the trilinear couplings. We have found that they naturally satisfy the conditions required from the absence of charge and color breaking minima and directions unbounded from below. Similarly, we have found that they are safe with most FCNC constraints with the only remarkable exception of $(\delta_{LR}^d)_{12(21)}$ from $\varepsilon'/\varepsilon$.

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\footnote{Another interesting MI is $(\delta_{LR}^d)_{32}$ which contributes to $b \rightarrow s\gamma$. However, in the case of hierarchical Yukawa structures these MI are still small (see last paper in Ref. [3]).}
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