Beating Lensing Cosmic Variance with Galaxy Tomography

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ABSTRACT

I discuss the use of cross correlations between galaxies with distance information and projected weak lensing dark matter maps to obtain a fully three-dimensional dark matter map and power spectrum. On large scales \( l \lesssim 100 \) one expects the galaxies to be biased, but not stochastic. I show that this allows a simultaneous solution of the full 3-D evolving galaxy bias and the dark matter power spectrum simultaneously. Within the photo-z information of the CFH lensing legacy survey, this allows a threefold reduction of statistical error, while a cross correlation with CLAR or other deep spectroscopic surveys allows a tenfold improvement in dark matter power accuracy on linear scales. This makes lensing surveys more sensitive to the cosmic equation of state and the neutrino masses.

Key words: Cosmology-theory-simulation-observation: gravitational lensing, dark matter, large scale structure

1 INTRODUCTION

The great success of modern physical cosmology has been driven by precision measurements of the cosmic microwave background (CMB), whose properties can be computed to high accuracy from first principles. Current CMB fluctuations have been measured to high statistical accuracy, which describes the universe at recombination. To constrain fundamental cosmological parameters, such as the matter density of the universe, one requires measurements at low redshift as well.

The low redshift measurements are difficult. Galaxy distributions can be measured in three dimensions and to high accuracy, but their formation is not understood in any detail. This leaves a systematic error in using their power spectrum to infer cosmological parameters. While the CMB power spectrum has been measured to statistical accuracies better than 1%, galaxy power spectra are not reliable tracers of matter at any accuracy close to that. It is known that galaxies of different morphologies are biased relative to each other. To complicate matters further, it is now known that the bias depends on scale, and that their distribution is stochastic on non-linear scales [Tegmark & et al. 2003; Fan 2003]. This bias and stochasticity is expected to evolve with redshift. The interpretation of their distribution is difficult to quantify, and galaxy power spectra can only constrain fundamental parameters to limited precision. The WMAP team [Spergel et al. 2003] combined the CMB power spectrum with the 2dF galaxy power spectrum. For the latter, the power is measured to high statistical accuracy, but with an unknown bias, so its interpretation is limited by systematic errors and was not used in their analysis. The WMAP team instead used only the slope of the galaxy power spectrum, which is hoped to be a less biased indicator of the underlying dark matter distribution. But it is not independently known how large an error that might have, so the error on the error bar is very difficult to quantify. Using no external data, WMAP data allows even closed cosmological models with no cosmological constant.

One intrinsic limitation of gravitational lensing studies is the broad redshift distribution of lenses. The observed shear signal integrates over all the matter distribution along the line of sight. Using multiple source planes gives some handle on the distance, but even in the best of cases only very crude redshift resolution can be extracted [Hu 2003]. Sample variance of lensing surveys, and fundamentally the cosmic variance of an all sky survey, are limited to the number of two dimensional modes on the sky. If one could beat...
this cosmic variance, one could significantly improve on the results.

Galaxies, on the other hand, allow measurements of accurate redshifts. One can measure their redshifts from optical or radio spectroscopy. Their sample variance is smaller by the volume to surface area ratio of a survey. But as mentioned previously, the distribution of galaxies is known to be biased, and this bias is likely to depend on scale and cosmic time. Their evolution is unlikely to provide a direct measure of the cosmic equation of state. Jain & Taylor (2003) have proposed a purely geometric procedure to measure the geometry of the universe. Our proposal differs by actually solving for the matter distribution, which allows much higher potential accuracy.

In this paper I show how one can combine the redshift resolved distribution of galaxies to significantly improve the information of lensing surveys. It is possible to simultaneously measure the evolution of bias as a function of scale and time, and thus the 3-D distribution of dark matter. The only intrinsic limit is stochasticity between galaxies and dark matter.

## 2 STRATEGY

Perhaps this sounds like a free lunch. Just counting degrees of freedom, the evolution of bias as a function of scale and time is a two dimensional function. The observable dark matter power spectrum is a one dimensional function of angular scale. The cross correlation, however, does have two dimensions as well. The map data set is also two dimensional on the sky, so the number of degrees of freedom adds up that one could in principle measure the dark matter evolution using galaxy cross correlations.

To illustrate the procedure, we will consider a simplified case where the three dimensional quantities are projected along a Cartesian axis, say $z$. The analogy of this axis is distance along the line of sight, which also corresponds to the direction along which we expect cosmological evolution to be important. One generically expects the bias to depend on cosmic time. If bias did not depend on the length scale tangential to the line of sight, the procedure is straightforward. We assume that we know the three dimensional distribution of galaxies up to an unknown bias function $b(z)$, $\delta_g(x, y, z) = b(z)\delta(x, y, z)$. Any spatial dependence of the bias will be captured by the cross correlation coefficient, which we will discuss later. The two dimensional distribution of dark matter is projected by a known lensing kernel $w_L(z)$, so the lensing surface density $\kappa$ is $\kappa(x, y) = \int \delta(x, y, z)w_L(z)dz$. For this example, we neglect the change of angular scales with distance. We can form a galaxy projection weighted by an unknown weight function $w(z)$. For practical purposes, this weight function will be smooth, so it suffices to sample it at a number of points $n_z$. The galaxy surface density in terms of this yet to be determined function is

$$\Sigma_g(x, y) = \sum_{i=1}^{n_z} \delta_g(x, y, z_i)b(z_i)w(z_i)\Delta z_i. \quad (1)$$

We define a residual function $e = \int (\Sigma_g - \kappa)^2dx dy$. One can then solve for the weight function by differentiating for weights at redshift $z_i$. This results in a coupled system of equations

$$\int dx dy \delta_g(x, y, z_i)\Sigma_g(x, y) = \int dx dy \kappa(x, y)\delta_g(x, y, z_i). \quad (2)$$

The unknowns $w_L(z_i)$ appear implicitly linearly in equation (2) through the surface density $\Sigma_g$ and equation (1).

One can solve this system of equations to obtain $w(z_i) = w_L(z_i)/b(z_i)$ if the intrinsic distributions of galaxies and dark matter are the same, there is no noise, and the number of pixels $n_{pix}$ in the map is larger than the number of unknowns in the redshift dimension. Typically, the bias is a slowly evolving function of time. One would bin in coarse intervals, for example a dozen photometric redshift bins. The image has many effective pixels. With the exact solution, the weighted galaxy map will be identical to the dark matter map.

In the presence of noise, we can write Equation (2) as a linear system

$$Aw = \kappa + \sigma \quad (3)$$

where $A$ is an $n_{pix}$ by $n_z$ matrix. We define a noise matrix $N$. If the primary source of noise is shot noise in the lensing map, this matrix is diagonal with entries being the variance at each pixel $\sigma^2(x, y)$. One solves the minimum variance solution to (3) as

$$w = (A'N^{-1}A)^{-1}A'N^{-1}\kappa. \quad (4)$$

In general one has $n_{pix} \gg n_z$, so the inverse operation is well defined.

One can in a similar fashion compute scale dependent bias evolution. We define the scale dependent bias as the ratio of the power spectrum of galaxies to that of the dark matter,

$$b^2(k, z) = \frac{P_{gal}(k, z)}{P_{dm}(k, z)}. \quad (5)$$

The power spectrum is a two point statistic, which depends on the scale of the bias. In terms of the density distribution, this relates the Fourier transform of the galaxy density field linearly through $b(k, z)$ to that of the dark matter:

$$\delta_g(\vec{k}, z) = b(|\vec{k}|, z)\delta(\vec{k}), \quad (6)$$

or equivalently states that the galaxy density field is a convolution over the dark matter distribution $\delta_g(\vec{x}, z) = \int \delta(\vec{x}, z)b(|x|, z)d^3x$. A bit of care needs to be taken in the interpretation of this Fourier transform. We have represented the three spatial coordinates $(x, y, z)$ by a vector $\vec{x}$. The radial direction $z$ is now the third component of the $\vec{x}$. The three dimensional expression in equation (6) assumes that $b(z, r)$ varies sufficiently slowly as a function of $z$ that one can apply a transform at each interval $z_i$. This is reasonably accurate assumption except for the largest scale, where one would want to solve a general linear parametrized system analogous to (4) instead of Fourier transforming.

The density can be thought of as a one-point distribution, which depends linearly on the bias. For this redshift $z$ and scale $k$ dependent bias $b(z, k)$ we can Fourier transform from wave number space to separation $r$:

$$b(z, r) = \frac{1}{(2\pi)^2} \int b(z, k)\frac{\sin(kr)}{kr}k^2dk. \quad (7)$$

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Our redshift binned bias function now generalizes to a two dimensional function of redshift and separation. We then convolve each redshift slice by the parametrized \( b(z, r) \). We then solve for the spatial and temporal structure of the bias. In practice, the bias should evolve slowly in space and time. Once we know the evolution of bias as a function of length scale and cosmic time, it is straightforward to obtain the dark matter power spectrum as a function of scale and time. In the linear regime, this measures the growth factor. In the non-linear regime, the PD96 (Peacock & Dodds 1996) approach gives a mapping between linear and non-linear power spectra. This needs to be calibrated against simulations, but as we see below, the inferences are probably systematically limited by other non-linear effects. The growth factor in turn maps directly onto the equation of state of space-time and matter, i.e. dark energy and neutrino mass (Hu 2002).

### 3 LIMITATIONS

The improvement is intrinsically limited on several fronts. It requires that bias be linear and non-stochastic (Pen 1998). On non-linear scales, the power spectrum of galaxies is known to be stochastic (Pen et al. 2003; Hoekstra et al. 2002). In the cross correlation tomography, one can measure the degree of stochasticity which provides a fundamental limit to the accuracy. Other sources of limitations are the noise in the lensing and galaxy power measurement.

One can measure the impact of stochasticity. We use the model from Hoekstra et al. (2002). The power spectrum of galaxies is related to the dark matter by the bias, \( P_b = b^2 P \), and the cross correlation coefficient \( r(k, z) \) can depend on time and scale. It relates \( \delta_g \delta = r \sqrt{P_b} \). At the two point level, the bias and cross correlation completely describe all two point statistics, and parametrizes all effects including non-linearity.

In the presence of stochasticity, the solution using our procedure becomes

\[
w(z_i) = \frac{w_L(z_i) r(z_i)}{b(z_i)}.
\]  

(8)

The problem is that one needs to know the cross correlation parameter \( r \) to infer the true dark matter power spectrum. In the previous section we implicitly assumed \( r = 1 \), which is expected on linear scales.

While we cannot simultaneously measure the stochasticity and bias, we can infer an integrated upper bound. Once we have solved for the weight function \( w \), we define a residual \( \chi^2 \) as

\[
\chi = S^{-1} (A w - \kappa)
\]  

(9)

In the absence of noise, this is the lensing power which was not predicted from the galaxies, and corresponds to the integral of \( (1 - r(z)^2) w_L^2(z) \) over the lensing weight. If this integral is small, it places an upper bound on the variation of \( r \) at any redshift.

With this framework in mind, we can substitute the actual parameters. We will use a linear power spectrum evolution model, \( \Delta^2(z) = \Delta^2(z = 0) D(z)^2/(1 + z)^2 \). The notation \( \Delta^2 \equiv k^3 P(k)/2\pi^2 \) is taken from Peacock (1999).

The comoving angular diameter distance is

\[
\chi(z) = c \int_0^z \frac{dz}{H(z)}
\]  

(10)

where \( H(z) \) is the Hubble constant at redshift \( z \):

\[
H(z) = H_0 \sqrt{(1 + z)^3 / (\Omega_m z + 1 - \Omega_k (z + 2)^{3/2})}
\]  

(11)

For the angular diameter distance \( \chi \) we use the fitting formula from Pen (1997).

For dark matter the lensing weight is

\[
w_L(z) = \frac{3}{2} \Omega_m H_0^2 g(z)(1 + z)
\]  

(12)

where

\[
g(z) = \chi(z) \int_z^{\infty} dz' n_s(z') \frac{\chi(z') - \chi(z)}{\chi(z')}.
\]  

(13)

\( n_s(z) \) is the normalized distribution of source galaxies. The two dimensional dark matter map is then

\[
\kappa(\theta_x, \theta_y) = \int \delta(d_A(z) \theta_x, d_A(z) \theta_y) w_L(z) \frac{dy}{dz} dz.
\]  

(14)

Analogously, we define a weighted galaxy surface density with respect to a yet undetermined weight \( w(z) \)

\[
\Sigma_g(\theta_x, \theta_y) = \int \delta_g(d_A(z) \theta_x, d_A(z) \theta_y, z) w_L(z) dz.
\]  

(15)

In analogy to equation 4 we solve for \( w(z) \) and infer the combination of bias and stochasticity given by equation 5.

### 4 APPLICATIONS

We consider parameters for the CFHT Legacy Survey (Van Waerbeke & Mellier 2003), and its expected accuracy. We use a source density comparable to the VIRMOS-DESCARTES survey of 30 galaxies per square arcminutes with a median source redshift of \( z_0 = 1 \). The mean ellipticity is taken to be \( \epsilon = 0.3 \). The survey area is 200 square degrees, which is half a percent of the sky. We neglect the Poisson noise from the cross correlation lens plane galaxies, which is much smaller than the lensing noise in each redshift slice.

The LS lensing survey measures the projected two dimensional dark matter distribution. In principle, the three dimensional power spectrum estimation requires inverting the Limber equation, which can be numerically unstable. We will use the half weighted approach of Pen et al. (2003), which leads to mostly uncorrelated error bins in the three dimensional power spectrum. For simplicity, we use the power spectrum at the angular diameter distance of the typical source redshift \( z_0 = 0.4 \). The mapping becomes

\[
k = l/\chi(z_0) \sim (l/1000) h^{-1} \text{ Mpc}^{-1}.
\]

We bin the power spectrum in factors of 2 in \( l \), and use Gaussian errors \( \Delta C_l = (C_l + C_l^H) / \sqrt{(2l + 1)|\Delta f_{\text{sky}}|} \).

The assumption of zero stochasticity is likely to break down when the galaxy clustering is non-linear. We model this conservatively as the stochasticity parameter being inversely proportionate to the variance

\[
r \sim \frac{1}{1 + \Delta^2}.
\]  

(16)

Current data suggests that galaxies and dark matter are probably better correlated than that Pen et al. (2003).
power, for which many more modes can be measured. These non-linear scales in principle encode the power spectrum and its history, but require detailed N-body simulations to calibrate. There are also questions of fundamental limits to the precision of non-linear scale power, since baryons back react on the dark matter. The distribution of baryons is intrinsically harder to predict since they can heat and cool in ways that is not predictable from first principles.

Future lensing surveys in the next five years can cover the whole sky, and can map sources at distances beyond the epoch of reionization at \( z > 7 \) \cite{Pen_2003}. It is also possible to map the redshift distribution of galaxies to similar distances on the whole sky with the Square Kilometer Array. This allows one to beat the normal sample variance limitation, also sometimes called 'cosmic variance'. In an interval of angular wavenumber \( l \sim 2l \) there are \( \sim 3l^2 \) two dimensional modes on the sky. This can in general be increased to \( \sim l^3 \) modes using tomography. But actual accuracy that can be gained will depend on the effects of non-linearity and stochasticity in the distribution of galaxies.

5 CONCLUSIONS

Gravitational lensing is a clean procedure to measure the power spectrum of dark matter at low redshifts. It is already providing competitive constraints on cosmological parameters, and does not make any assumptions about the nature of galaxy biasing nor any other astrophysical process.

Like the cosmic microwave background, gravitational lensing accuracy is limited by sample variance. It can only provide two dimensional power spectra. Some of the most important open cosmological questions involve precision measurements of the cosmic equation of state. For this procedure to be successful, one needs accurate measurements of the dark matter power spectra, ideally in the linear regime and at different redshifts. This is a regime where gravitational lensing is significantly restricted in accuracy.

We have shown in this paper how one can improve on the accuracy by using the cross correlation to galaxies with redshift information. One can reduce the errors without any assumptions on galaxy bias, even if it evolves and depends on scale. The only requirement is a low stochasticity. The total actual stochasticity of the data set is measurable, and can be folded into the error analysis.

We estimated this improvement on two concrete surveys: the Canada-France-Hawaii-Telescope Legacy Survey Weak Lensing Survey, and the Canadian Large Adaptive Reflector deep survey. With the added distance information, one can reduce the error bars on linear scales \( l \sim 50 \) by up to a factor of 3.

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