The role of the elemental nature of $A=3$ nuclei in neutron rich nuclei

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Abstract

The idea of treating the trinucleon systems as elementary entities in the elementary particle model (EPM) as an Effective Field Theory has been a success in explaining the weak charge-changing processes in nuclei. The EPM results are found to be as good as those obtained from nuclear microscopic models using two- and three-body forces. We extend this concept to investigate the validity of the elemental nature of $A = 3$ nuclei through studies of nuclear structure of neutron-rich nuclei. By treating neutron-rich nuclei as primarily made up of tritons as its building blocks, we extract one- and two-triton separation energies of these nuclei. Calculations have been performed here within relativistic mean field (RMF) models with latest interactions. Clear evidence arises of a new shell structure with well-defined predictions of new magic nuclei. These unique predictions have been consolidated by standard one- and two-neutron separation energy calculations. The binding energy per nucleon plots of these nuclei also confirm these predictions. We make unambiguous prediction of six magic nuclei: $^{24}_{8}O_{16}$, $^{60}_{20}Ca_{40}$, $^{105}_{35}Br_{70}$, $^{123}_{41}Nb_{82}$, $^{189}_{63}Eu_{126}$ and $^{276}_{92}U_{184}$.

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I. INTRODUCTION

The Elementary Particle Model (EPM), [1–6] first developed by Kim and Primakoff [3] is a phenomenological approach, which parametrizes the nuclear charge-changing currents in terms of the trinucleon form factors in analogy with the corresponding nucleon weak currents. Here, the pair \((^3\text{He},^3\text{H}) \simeq (h, t)\) is treated as elementary, and this is found to give as good results as those obtained with much more complicated composite structures for the ground state in nuclear microscopic models [1–4]. We carry this concept forward by studying the role of the elemental nature of \((h, t)\) in nucleus in general, and in neutron-rich nuclei in particular, in order to explore some of the pertinent questions that arise in our mind: (i) Can triton be understood as a building block for the structures of neutron rich nuclei? (ii) Are there effective symmetries at \(N = 2Z\)? (iii) Are there new magicities and new properties supported by experimental and theoretical evidences which may demand new shell structure? We, therefore, study the \(N = 2Z\) neutron-rich nuclei and try to find evidences for triton clustering based on binding energies. First of all, we explore available experimental binding energies of such nuclei and extract one- and two-triton separation energies. Supporting our model, a clear evidence of a magic nucleus, \(^{24}_8\text{O}_{16}\) \((N = 2Z = 16)\) appears from the data. This one is also supported by the direct experimental evidence of a magic number, \(N = 16\), near the neutron drip line [7]. This may well be interpreted as \(N = 2Z = 16\), a bound nuclear system of 8 tritons. Unfortunately, the data, as in atomic mass compilations [8, 9] for \(N = 2Z\) neutron-rich nuclei, are not available beyond \(^{51}_{17}\text{Cl}_{34}\) (17 triton bound system).

We employ very successful relativistic mean field (RMF) models with latest interactions like NL3* [10], NL3 [11] and TM1 [12] to calculate binding energies of nuclei with \(N = 2Z\) for \(Z\) ranging from 5 to 120. We then extract one- and two-triton separation energies for a wide spectrum of neutron-rich nuclei and explore for new magicities and evidences for new shell structures. The experimental data exhibit even-odd effects, and so do the RMF results in this region. However, for the nuclei with large neutron numbers, those odd triton number nuclei which carry magic neutron numbers become magic. For example, \(^{105}_{35}\text{Br}_{70}, ^{123}_{41}\text{Nb}_{82}\) and \(^{189}_{63}\text{Eu}_{126}\). Here, \(N = 70\) is a harmonic oscillator magic number. We predict six prominent magic nuclei: \(^{24}_8\text{O}_{16}, ^{60}_{20}\text{Ca}_{40}, ^{105}_{35}\text{Br}_{70}, ^{123}_{41}\text{Nb}_{82}, ^{189}_{63}\text{Eu}_{126}\) and \(^{276}_{92}\text{U}_{184}\). Note that in \(^{60}_{20}\text{Ca}_{40}\), both the numbers 20 and 40, are each magic in nature. Hence, it is heartening to point out that indeed this nucleus has been found to be magic in other mean field calculations [13, 14]. Though, in those calculations, the shell gap at harmonic oscillator magic number \(N = 40\) is not as pronounced as at \(N = 28\) or 50. We also obtain, in
standard conventional manner, one- and two-neutron separation energies for the isotopes of these newly identified magic nuclei in order to understand the role played by neutron and proton magic numbers, and to investigate if these magicities are being translated into triton magic numbers for \( N = 2Z \) nuclei. The standard binding energy per particle plot for all the \( N = 2Z \leq 240 \) nuclei too predicts the same magic nuclei as obtained by extracting one- and two-triton separation energies. The structural properties of these magic nuclei and nearby isotopes have been studied to understand the staggering. We present several interesting results, like the six new magic nuclei as stated above. However, given the significance of superheavy nuclei in current research, this new prediction of a superheavy nucleus \(^{276}_{92}\text{U}_{184}\) stands out.

We discuss the EPM model and triton clustering in the next section, followed by the RMF formalism. Thereafter, we present our results with detailed discussion. Finally, we present a general conclusion.

II. THE EPM MODEL AND TRITON CLUSTERING

The phenomenological EPM, \([1–6]\) treats the pair \(^3\text{He}, ^3\text{H} \simeq (h, t)\) as elementary. In analogy with the corresponding nucleon weak currents, EPM parametrizes the nuclear charge-changing currents in terms of the trinucleon form factors. Amazingly this is found to give as good a result as those obtained with more complicated composite structures for the ground state in nuclear microscopic models \([1–4]\).

For example, EPM has been successful in understanding \(\mu^-\) weak capture on \(^3\text{He}\), \(\mu^- + ^3\text{He} \rightarrow ^3\text{H} + \nu_\mu\). It matches the experimental results as well as the more elaborate and extensive microscopic calculations where full nuclear wave function which arise from realistic two- and three-body interaction terms \([1–4]\) are used. Using EPM, Mintz \([5, 6]\) studied the reaction, \(\mu^- + ^6\text{Li} \rightarrow ^3\text{H} + ^3\text{H} + \nu_\mu\). Taking cue from the Glashow-Salam-Weinberg model, one writes the matrix element for the above process as,

\[
<^3\text{H}(1),^3\text{H}(2),\nu|H_{W}^{(0)}|^6\text{Li},\mu^- > = \frac{G \cos \theta_c}{\sqrt{2}} \bar{u}_\nu(1 - \gamma_5)u_\mu <^3\text{H}(1),^3\text{H}(2)|J^\dagger_\lambda(0)|^6\text{Li}>,
\]

where \(\cos \theta_c = 0.98\) with \(\theta_c\) is the Cabbibo angle and the weak coupling constant \(G = 1.02 \times 10^{-5} m_p^{-2}\) and \(J^\dagger_\lambda(0) = V_\lambda(0) - A_\lambda(0)\). Here, \(V\) and \(A\) are the vector and axial vector part of the
hadronic weak current. Next, one draws a parallel between the reactions $\mu^- + ^6\text{Li} \rightarrow ^3\text{H} + ^3\text{H} + \nu_\mu$ and $\mu^- + d \rightarrow n + n + \nu_\mu$. The current matrix elements $<nn|A_\lambda(0)|d>$ and $<nn|V_\lambda(0)|d>$ thus have the same structure as $<^3\text{H}^3\text{H}|A_\lambda(0)|^6\text{Li}>$ and $<^3\text{H}^3\text{H}|V_\lambda(0)|^6\text{Li}>$, respectively. These are needed to evaluate the above matrix element in Eq. (1).

Relevant physically measurable quantities are determined in terms of four form factors which are obtained from data from reactions $\gamma + ^6\text{Li} \rightarrow ^3\text{H} + ^3\text{He}$, $^3\text{H} + ^3\text{He} \rightarrow ^6\text{Li} + \gamma$ and $\pi^- + ^6\text{Li} \rightarrow ^3\text{H} + ^3\text{He}$ by using CVC and PCAC [6]. This model is very successful in fitting the data, thus confirming the validity of the EPM.

Phenomenologically, various Effective Field Theoretical (EFT) models, motivated by or based on QCD are known. These have acquired more acceptability and hence more respectability [15] in recent years. Appelquist-Cerazzone theorem determines whether a particular EFT would be renormalizable or not. However, the non-renormalizable EFT are no less basic. Essentially, the parameters of the EFT Lagrangian carry in them information of the underlying more fundamental field theory. It is for this reason that even the non-renormalizable EFT’s have become important phenomenological tool in theoretical physics. Hence, we emphasize the basic significance of the EPM model as a rich and useful EFT of the Standard Model.

We further seek for more evidences from other studies in nuclear physics, which treat the $(h, t)$ pair as elementary. Are there any? Well, indeed there are! Within the sphere of low energy nuclear structure studies a new group $\text{SU}_A(2)$ called nusospin has been proposed [16–19]. Just as one takes the pair $(p, n)$ as forming the fundamental representation of the nuclear $\text{SU}(2)$ isospin group, in the same manner one hypothesizes that the pair $(h, t)$ forms the fundamental representation of the new nusospin $\text{SU}_A(2)$ group. The physical justification of this new model are also discussed in detail [16–19]. In support of the nusospin group, we have found strong empirical evidences favouring $A = 3$ clustering in nuclei [20]. So the EPM model in particle physics, finds unequivocal support from the nuclear structure successes of the nuclear $\text{SU}_A(2)$ nusospin group. Both of these justify the treatment of the pair $(h, t)$ as a fundamental entity.

Now, having provided justification for the nusospin group $\text{SU}_A(2)$, here for the sake of completeness and clarification of these points, we would like to emphasize a few simple supporting points and evidences. We hope that this will act as a simple reminder of what we already know about $A = 3$ clustering in nuclei.

First, say as to what is the justification of treating $^{24}\text{O}$ as being made up of eight tritons? We argue as follows. Just as light $N = Z$ nuclei with $A = 4n$, $n = 1, 2, 3, 4...$ may be treated as being
composed of \( n\alpha \) cluster, \([21, 22]\) in Table 1, we show several neutron-rich nuclei which may be treated as being composed of \( n \)-clusters of \(^3\!^1\!^1\!^1\)H\(_2\). We write the binding energy of these nuclei as

\[ E_B = 8.48n + Ck, \]

where 8.48 MeV is the binding energy of \(^3\!^1\!^1\!^1\)H\(_2\). We take these \( n \)-cluster of tritons as forming \( k \) bonds and with \( C \) as inter-triton-bond energy. We are assuming here the same geometric structure of clusters in these nuclei as conventionally done for \( \alpha \)-clusters in \( A = 4n \) nuclei \([23]\). So all numbers arise from similar configurations. Thus, the model seems to hold out well with inter-triton cluster bond energy of about 5.3 MeV. However, note that the \( C \) value of 21\( ^1\!^2\!^8\)N is somewhat on the lower side, but the same was true of the corresponding alpha cluster 28\( ^1\!^4\!^8\)Si nucleus with respect to the other \( \alpha \)-cluster nuclei (see Fig. 4 of Ref. \([23]\)).

| Nucleus | \( n \) | \( k \) | \( E_B-8.48n \)(MeV) | \( C \)(MeV) |
|---------|--------|--------|----------------|----------|
| \(^9\)Li | 3 | 3 | 19.90 | 6.63 |
| \(^{12}\)Be | 4 | 6 | 34.73 | 5.79 |
| \(^{15}\)B | 5 | 9 | 45.79 | 5.09 |
| \(^{18}\)C | 6 | 12 | 64.78 | 5.40 |
| \(^{21}\)N | 7 | 16 | 79.43 | 4.96 |
| \(^{24}\)O | 8 | 19 | 100.64 | 5.30 |

As we stated in 2001, Ref. \([18]\), “we notice that the value seems to work for even heavier neutron-rich nuclei. For example for \(^{42}\!^{14}\!^{28}\)Si the inter-triton cluster energy is still 5.4 MeV.” Note that in our model \(^{42}\!^{14}\!^{28}\)Si is taken to be made up of an even number of 14-tritons. Note the important fact that this nucleus has a large 12-neutron excess over the heaviest stable silicon nucleus. In the triton picture, this nucleus is found to be extra stable compared to the neighbouring \( N = 2Z \) nuclei empirically as well as theoretically though it does not appear as a well defined magic number. In complete variance with theoretical model predictions, \([24-30]\) Fridmann et. al. \([31]\) showed that \(^{42}\!^{14}\!^{28}\)Si was a spherical and a highly magic nucleus. Later, a collapse of \( N = 28 \) shell closure in \(^{42}\!^{14}\!^{28}\)Si was reported by Bastin et. al. \([32]\). They found it to be a well deformed oblate rotor, but its extra binding still indicated a magic character \([8, 9]\). That is what we notice here in the triton picture.

Next, we point out a strong experimental evidence of the possible existence of helion and triton clusters in \(^6\)Li nuclei. Indeed, the same has been very convincingly demonstrated through
direct trinucleon knockout - both triton and helion, from $^6\text{Li}$ via exclusive electron reaction $[33]$. Mirror reactions $^6\text{Li}(e,e'\text{He})^3\text{He}$ and $^6\text{Li}(e,e'\text{H})^3\text{He}$ were measured. The momentum transfer dependence was found to be in complete disagreement with the fundamental spectrum of a direct-single nucleon knockout. On the other hand, the momentum dependence was in good agreement with a direct $A = 3$ knockout mechanism. This clearly demonstrated that $h$- and $t$-clusters existed as primary entities in $^6\text{Li}$.

The $N \sim Z$ nuclei are very well explained by the SU(2) isospin group with ($p$, $n$) pair providing basis for a description for these nuclei $[34]$. As the nuclear SU(2) isospin generates and validates the shell model structure of $N \sim Z$ nuclei, we extrapolate this logic to find a suitable shell model structure generated by and validated by the SU$_A$($2$) nusospin group. We have already seen how the nusospin group predicts new magic numbers of the ($Z$, $N$) pair of (4,8), (6,12), (8,16), (10,20) $[16]$. So it is logical to assume that there may be a different shell structure associated with this new group.

One-proton and one-neutron (as well as two-proton and two-neutron) separation energies play an important role in determining magic numbers $[35, 36]$ and we have pursued similar ideas $[16]$. This is mostly within the framework where SU(2) symmetry based on elementarity of the ($p$, $n$) pair is basic. We extrapolate this idea to the SU$_A$($2$) nuspin with the ($h$, $t$) pair forming elemental entities. Thus, we should be able to talk of one- and two-triton separation energies in neutron-rich nuclei while treating these as made up of tritons as elemental entities.

III. THE RMF FORMALISM

The relativistic mean field theory has been successful in reproducing the experimental observations throughout the periodic table, near as well as far from the stability line $[35, 37–50]$. It has also been pursued to examine cluster structures inside the nuclei $[51–54]$. The relativistic Lagrangian for a many-body system contains all the information of nucleon-nucleon interaction, via
exchanges of $\sigma$, $\omega$- and $\rho$-mesons. It is written as [35, 40–44]

\[
\mathcal{L} = \bar{\psi}_i \{ i \gamma^\mu \partial_\mu - M \} \psi_i + \frac{1}{2} (\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - g_\sigma \bar{\psi}_i \gamma^\mu \psi_i \sigma - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu}
+ \frac{1}{2} m_\omega^2 V^\mu V_\mu - g_\omega \bar{\psi}_i \gamma^\mu \psi_i V_\mu + \frac{1}{2} c_4 (V_\mu V^\mu)^2
- \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} m_\rho^2 \bar{R}^\mu \bar{R}_\mu - g_\rho \bar{\psi}_i \gamma^\mu \bar{\tau}_i \psi_i \bar{R}_\mu
- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \bar{\psi}_i \gamma^\mu \frac{(1 - \tau_3)}{2} \psi_i A_\mu ,
\]

(3)

where $\psi$ represents Dirac spinors for nucleons with mass $M$. The quantities $m_\sigma$, $m_\omega$, $m_\rho$ are the masses assigned to $\sigma$, $\omega$, $\rho$-mesons, respectively. The $\sigma$, $V_\mu$ and $R_\mu$ are the fields of $\sigma$-meson, $\omega$-meson and $\rho$-meson, respectively. The quantities $g_\sigma$, $g_\omega$, $g_\rho$ and $e^2/4\pi = 1/137$ are the coupling constants for $\sigma$, $\omega$, $\rho$-mesons and photon fields, respectively. The $g_2$ and $g_3$ are the self-interaction coupling constants for the $\sigma$-mesons. The quantity $c_4$ is the self-interaction coupling constant for $\omega$-meson, and this term is used in the TM1 potential. The field tensors of the vector, isovector mesons and of the electromagnetic field are given by

\[
\begin{align*}
\Omega^{\mu\nu} &= \partial^\mu V^\nu - \partial^\nu V^\mu , \\
B^{\mu\nu} &= \partial^\mu R^\nu - \partial^\nu R^\mu , \\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu .
\end{align*}
\]

(4)

The classical variational principle is used to solve the field equations for bosons and fermions. The Dirac equation for the nucleon is written as:

\[
[-i \alpha \cdot \nabla + \beta (M + S(r)) + V(r)] \psi_i = \epsilon_i \psi_i .
\]

(5)

Here, $V(r)$ and $S(r)$ represent the vector and scalar potential is, defined as

\[
V(r) = g_\omega V_0(r) + g_\rho \tau_3 R_0(r) + e \frac{(1 - \tau_3)}{2} A_0(r) ,
\]

(6)

and

\[
S(r) = g_\sigma \sigma(r) ,
\]

(7)
where subscript \( i \) stands for neutron(n) and proton(p), respectively. The field equations for bosons are

\[
\begin{align*}
\{ - \nabla^2 + m_\sigma^2 \} \sigma(r) &= -g_\sigma \rho_s(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r), \\
\{ - \nabla^2 + m_\omega^2 \} V_0(r) &= g_\omega \rho_v(r) + c_4 V_0^3(r), \\
\{ - \nabla^2 + m_\rho^2 \} R_0^3(r) &= g_\rho \rho_3(r), \\
- \nabla A_0(r) &= e \rho_c(r).
\end{align*}
\]

Here \( \rho_s \) and \( \rho_v \) are the scalar and vector density for \( \sigma \)- and \( \omega \)-fields in nuclear system which are expressed as

\[
\begin{align*}
\rho_s(r) &= \sum_{i=n,p} \bar{\psi}_i(r) \psi_i(r), \\
\rho_v(r) &= \sum_{i=n,p} \psi_i^\dagger(r) \psi_i(r). \tag{8}
\end{align*}
\]

The vector density \( \rho_3(r) \) for \( \rho \)-field and charge density \( \rho_c(r) \) are expressed by

\[
\begin{align*}
\rho_3(r) &= \sum_{i=n,p} \psi_i^\dagger(r) \gamma^0 \tau_3 \psi_i(r), \\
\rho_c(r) &= \sum_{i=n,p} \psi_i^\dagger(r) \gamma^0 \left( 1 - \tau_3 \right) \frac{1}{2} \psi_i(r). \tag{9}
\end{align*}
\]

The quadrupole deformation parameter \( \beta_2 \) is extracted from the calculated quadrupole moments of neutrons and protons through

\[
Q = Q_n + Q_p = \sqrt{\frac{16\pi}{5} \frac{3}{4\pi} AR^2 \beta_2}, \tag{10}
\]

where \( R = 1.2A^{1/3} \) fm.

The various rms radii are defined as

\[
\begin{align*}
\langle r_p^2 \rangle &= \frac{1}{Z} \int r_p^2 d^3 r \rho_p, \\
\langle r_n^2 \rangle &= \frac{1}{N} \int r_n^2 d^3 r \rho_n, \\
\langle r_m^2 \rangle &= \frac{1}{A} \int r_m^2 d^3 r \rho,
\end{align*}
\]

for proton, neutron and matter rms radii, respectively. The quantities \( \rho_p \), \( \rho_n \) and \( \rho \) are their corresponding densities. The charge rms radius can be found from the proton rms radius using the
relation \( r_e = \sqrt{r_p^2 + 0.64} \) taking into consideration the finite size of the proton. The total energy of the system is obtained by

\[
E_{\text{total}} = E_{\text{part}}(N) + E_\sigma + E_\omega + E_\rho + E_c + E_{\text{pair}} + E_{\text{c.m.}},
\]

where \( E_{\text{part}}(N) \) is the sum of the single-particle energies of the nucleons. \( E_\sigma, E_\omega, E_\rho, E_c, E_{\text{pair}} \) and \( E_{\text{c.m.}} \) are the contributions of meson fields, Coulomb field, pairing energy and the center-of-mass energy, respectively.

It is worth mentioning that pairing correlations play an important role for open shell nuclei. It has significant impact on binding mechanism as well as shape of the nuclei \([35, 55, 56]\). There are three possibilities of pairing for example, neutron-proton, proton-proton and neutron-neutron correlation. We account the pairing into consideration in a way similar to Refs. \([35, 56]\). To take care of pairing effect, the constant gap BCS approximation is used throughout the calculations. In case of simple BCS prescription, the expression of pairing energy is written by

\[
E_{\text{pair}} = -G \left[ \sum_{i > 0} u_i v_i \right]^2,
\]

where \( G \) is the pairing force constant, and \( v_i^2 \) and \( u_i^2 = 1 - v_i^2 \) are the occupation probabilities. The variation with respect to the occupation numbers, \( v_i^2 \), is expressed by the well-known BCS equation

\[
2\epsilon_i u_i v_i - \Delta (u_i^2 - v_i^2) = 0,
\]

with \( \Delta = G \sum_{i > 0} u_i v_i \). The occupation number \( n_i \) is given by

\[
n_i = v_i^2 = \frac{1}{2} \left[ 1 - \frac{\epsilon_i - \lambda}{\sqrt{(\epsilon_i - \lambda)^2 + \Delta^2}} \right],
\]

where \( \epsilon \) is the single-particle energy for the state \( i \). The chemical potential \( \lambda \) for protons (neutrons) is obtained requiring

\[
\sum_i n_i = Z(N).
\]

The sum is taken over proton (neutron) states. The value of constant gap (pairing gap) for proton and neutron are determined from the phenomenological formula of Madland and Nix \([57]\) which are given as

\[
\Delta_n = \frac{r}{N^{1/3}} \exp(-sI - tI^2),
\]

and

\[
\Delta_p = \frac{r}{Z^{1/3}} \exp(-sI - tI^2),
\]
where \( I = (N - Z)/A \), \( r = 5.73 \text{MeV} \), \( s = 0.117 \), and \( t = 7.96 \). In particular, for the solution of the RMF equations with pairing, we never calculate the pairing force constant \( G \) explicitly. But the occupation probabilities are directly calculated using the gap parameters (\( \Delta_n \) and \( \Delta_p \)) and the chemical potentials (\( \lambda_n \) and \( \lambda_p \)) for neutrons and protons, whereas chemical potentials are determined by the particle numbers for protons and neutrons. And now, the expression of pairing energy is simplified to

\[
E_{\text{pair}} = -\Delta \sum_{i > 0} u_i v_i .
\]  

(19)

The centre-of-mass correction is included by non relativistic expression i.e \( E_{\text{c.m.}} = -\frac{2}{41} A^{-1/3} \) MeV.

The RMF calculations are simplified by taking the various symmetries into consideration, like conservation of parity, time-reversal symmetry and no-sea approximation which eliminates all spatial components of the meson fields and Dirac anti-particle contribution of the physical observables \[35\]. Moreover, it is not an easy task to compute the binding energy and quadrupole moment of odd-\( N \) or odd-\( Z \) or both odd-\( N \) and odd-\( Z \) systems. In RMF calculations with the effect of time-reversal symmetry, the spatial components of vector fields are eliminated which are odd under time-reversal and parity. However, spatial components of vector fields play an important role in determination of magnetic moment \[58\] but these have very little impact on the bulk properties of the nucleus such as binding energy, quadrupole deformation and radii \[59\]. We pursued our calculation in this context. For dealing with odd-\( Z \) nuclei, we employ the Pauli blocking approximation, which restores the time-reversal symmetry and as a result reveals the even-odd staggering very nicely \[60, 61\]. A pair of nucleons with spin up/down or spin down/up has a mirror image and therefore time reversal symmetry is obeyed. But in case of odd-\( A \) or odd-odd nuclei, lone odd nucleon fills a quantum state with spin up but its corresponding conjugate spin down state remains empty, which violates the time reversal symmetry. To take care of this, Pauli blocking approximation is used. First, we carry out free calculations without any blocking and obtain the level of maximum occupancy of the lone odd nucleon for minimum energy configuration and then block it in that level either with spin up or down. In this approach, the odd nucleon stays in one of the conjugate states, ±m, which is taken out from the pairing scheme. Thus, the nucleon of this state is not allowed to fluctuate to other levels. The rest of the system has even number of nucleons, which obeys time reversal symmetry. This blocking scheme known as Pauli blocking almost restores the time reversal symmetry. But, it doubles our effort as we need to perform our
calculations twice.

IV. RESULTS AND DISCUSSION

A nucleus defined as bound state of $Z$ number of tritons may be written as $^{A=3Z\times N}_{Z}X_{N=2Z} \equiv ^{A}_{Z}X_{N}$. First of all, we look for the available experimental binding energies for such nuclei and extract one- and two-triton separation energies, which may respectively be obtained as,

$$s_{1t} = B(^{A}_{Z}X_{N}) - B(^{A-3}_{Z-1}Y_{N-2}) - B(^{3}_{1}H_{2})$$

and

$$s_{2t} = B(^{A}_{Z}X_{N}) - B(^{A-6}_{Z-2}Y_{N-4}) - 2B(^{3}_{1}H_{2})$$

where, $B(^{A}_{Z}X_{N})$ is the binding energy of the nucleus $^{A}_{Z}X_{N}$. The experimental binding energies are not available beyond $N_{t} = 17$ bound systems. We, therefore, resort to RMF theory to extend our calculation for a wide spectrum of $N = 2Z$ nuclei, $5 \leq N_{t} \leq 120$. The RMF binding energies as well as $s_{1t}$ and $s_{2t}$ agree with experimental data to a great extent in their overlap region. We plot $s_{1t}$ and $s_{2t}$ as a function of triton numbers ($N_{t}$) in Fig. 1.

The most prominent feature in Fig. 1 is the first peak shown by data and RMF both e.g., for $N_{t}=8$ i.e. for $^{24}_{8}O_{16}$ and an equally sharp dip for $N_{t}=9$ i.e., for $^{27}_{9}F_{18}$. We know that such drops in one-neutron and one-proton separation energies when going from one $Z/N$ number to the next one is a signal of the magicity character of a particular $Z/N$ number. In the context of our discussion here, magicity means a much stronger binding for a particular number of tritons as compared to the adjoining number of tritons. Hence, $N_{t}=8$ is a magic number with respect to different bound states of tritons. Besides limited experimental data the vast RMF results clearly show $N_{t}=8$, 20, 35, 41, 63 and 92 as magic numbers, which correspond to $N =16$, 40, 70, 82, 126 and 184. The experimentally observed magic numbers are 8, 20, 50, 82 and 126. The magic number 184 is predicted to be the next neutron magic number after 126 by many models [62–65] yet to be determined experimentally. In Ref. [65], the magicity at $N = 184$ is associated with $Z = 120$; that is $^{304}_{120}X_{184}$. The $N = 184$ magicity in our work is however true only for $Z = 92$. This $N = 2Z$ link is basic in our work.

The magic nuclei that appear in the triton picture are $^{24}_{8}O_{16}$, $^{60}_{20}Ca_{40}$, $^{105}_{35}Br_{70}$, $^{123}_{41}Nb_{82}$, $^{189}_{63}Eu_{126}$ and $^{276}_{92}U_{184}$. Are these magic nuclei an effective manifestation of proton and neutron magic numbers? We investigate it in Fig. 2, wherein we plot one- and two-neutron separation energies ($s_{1n}$...
FIG. 1: One- and two-triton separation energies. In the lower panel, the region of experimental data has been emphasized.

and $s_{2n}$) for the isotopes of these nuclei. This figure shows that for $N = 40, 70, 82, 126$ and 184 there is a sharp fall indicating magicities at $Z = N/2$ i.e., triton numbers $N_t=20, 35, 41, 63$ and 92. A less significant fall is also seen for $N = 50$ in the plots of $Z = 35$ and $Z = 41$ isotopes (middle row), which do not show up as magic numbers in the triton picture. We also note that $^{24}_{8}O_{16}$ is a magic nucleus, which is in line with the findings of the study [7]. However, no staggering is seen for $N = 16$ in top left panel of Fig. 2. We conclude that for the heavier nuclei with large number of neutrons, it is neutrons which decide the effective magicities in terms of tritons, and for lighter nucleus like $^{24}_{8}O_{16}$, it is protons which play this role. Probably, due to this reason $N = 70$ for $^{105}_{35}Br_{70}$, though less significant, it still turns out to be a magic number. The $N = 40$ too appears to be a magic number in Fig. 1, which is confirmed by the staggering seen at $N = 40$ in case of $^{60}_{20}Ca_{40}$ in the top right panel of Fig. 2. But for this nucleus, $Z = 20$ too is an experimentally observed magic value, and we notice the highest peak for it in Fig. 1. The result is in line with the previous studies [13, 14], wherein $^{60}_{20}Ca_{40}$ is found to be a doubly magic nucleus though the shell gap at $N = 40$ is not as pronounced as at $N = 28$ and 50. Interestingly, $N = 40$ and 70 are magic values for the harmonic oscillator potential.
FIG. 2: One- and two-neutron separation energies ($s_{1n}$ and $s_{2n}$, respectively) for the isotopes of the newly identified magic nuclei in Fig-1.

FIG. 3: B/A for $^{A=3Z}_{Z}X_{N=2Z}$ nuclei
We find that whenever triton number is even, the triton separation energy is significantly higher than the adjoining odd triton numbers except for those odd triton numbers whose neutron number, \( N = 2Z \), is magic. The nuclei, \( ^{105}_{35}\text{Br}_{70}, ^{123}_{41}\text{Nb}_{82} \), and \( ^{180}_{63}\text{Eu}_{126} \) are more bound with respect to their neighbouring even triton number nuclei because of the magicities of the neutrons they carry. Thus, we observe an effective manifestation of nucleon odd-even effect and that the nucleon magicities are being translated to those of tritons.

We further investigate the structural properties of these magic nuclei and calculate their
quadrupole deformations. We observe smaller departure from sphericity for these newly identified magic nuclei compared to their nearby isotopes as given by deformation parameter ($\beta_2$) in Table 2. The nucleus gets more deformed and less bound with an extra triton which appears to go in the next higher shell. The staggering at these magic nuclei may be attributed to it. The radii are found to increase with increasing mass number. Our study shows that an effective shell structure of the bound states of tritons seems to be manifesting here.

In Fig. 3, we plot binding energy per nucleon for all the $^{A=3Z}_ZX_{N=2Z}$ nuclei studied here. It too shows clear magicities of the same set of nuclei: $^{24}_8O_{16}$, $^{60}_20$Ca$_{40}$, $^{105}_35$Br$_{70}$, $^{123}_41$Nb$_{82}$, $^{189}_63$Eu$_{112}$ and $^{276}_92$U$_{184}$. This consolidates our assertions above on new magicities. The above new neutron-rich magic nuclei and in particular the superheavy $^{276}_92$U$_{184}$ nucleus, are the most unique predictions of our model here.

V. CONCLUSIONS

What seems to be happening in the neutron-rich nuclei is that the degree of freedom appears to be changing from ($p$, $n$) of SU$_1$(2) isospin structure to ($h$, $t$) of SU$_A$(2) nusospin structure, so much so that for $^{3Z}_ZX_{2Z}$ nuclei the predominant structure is that of $Z$-tritons. As neutron number is always even for the $N = 2Z$ nuclei, the odd triton number (odd proton number) decides the spin and hence the shell model predictions of spin remain unaltered. The zero spin may be assigned to all even triton nuclei. This may appear to be amazing to those who wish to continue treating ($p$, $n$) as being the only degree of freedom relevant for all nuclei: ($N \sim Z$) nuclei and as well as very neutron-rich nuclei.

Against the theoretical predictions that $N = 28$ shell closure will be destroyed and that $^{42}_{14}$Si$_{28}$ will be highly deformed [24–30], the empirical evidence [31] showed that this indeed was spherical and a magic nucleus. However, at variance with it, Bastin et al. [32] reported a collapse of $N = 28$ shell closure, wherein they found it to be a well-deformed oblate rotor. The experimental binding energies of N=2Z nuclei still predicted it to be extra stable, though not a magic nucleus [8, 9]. We observe exactly the same in the triton picture in Fig. 1. As no staggering is seen after this nucleus, it can not be predicted as being magic. The $^{43}_{14}$Si$_{28}$ has 12-neutron excess over the heaviest stable silicon nuclide. Nature appears to be more enterprising. As neutron number increases SU$_1$(2) isospin group leads to an induced SU$_A$(2) nusospin group.

We have already shown in other papers [16–19] as to how SU$_A$(2) nusospin finds justification.
In Ref. [20], we have enumerated several strong empirical evidences in support of triton-helion cluster structure effects in nuclei. The triton picture of $N = 2Z$ nuclei as well as conventional studies of one- and two-neutron separation energies indicate magicity of some special nuclei, e.g., $^{24}_{8}$O$_{16}$, $^{60}_{20}$Ca$_{40}$, $^{105}_{35}$Br$_{70}$, $^{123}_{41}$Nb$_{82}$, $^{189}_{63}$Eu$_{126}$ and $^{276}_{92}$U$_{184}$. The superheavy nucleus, $^{276}_{92}$U$_{184}$, may be easily accessible to experimental confirmation due to its relatively small charge, $Z = 92$.

Thus, what we call nusospin model here and what is called Elementary Particle Model, are essentially talking about the same physical reality of $(h, t)$ being fundamental and elementary, though using them in different framework of nuclear studies; the former one in nuclear strong interaction studies, while the latter in the electro-weak studies.

With this hindsight, one now looks at Fig. 1, and then the wisdom of the $SU_A(2)$ nusospin model dawns upon us with the possibility of a new shell structure of tritons. The magicity of eight-tritons is already confirmed by the empirical study of $^{24}_{8}$O$_{16}$. The binding energy per nucleon plot as in Fig. 3, which predicts the same magic nuclei as in triton picture of Fig. 1, consolidates our assertions. We, therefore, urge the experimentalists to conduct studies for the possible confirmations of these exotic magic nuclei.

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