Linear Superposition Compensation Strategy to Optimize Transient Response for Critical Conduction Mode Boost Converter

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Abstract. For boost converter operating in the Critical Conduction Mode (CRM), linear compensation strategy is widely used to improve the control performance. However, conventional linear compensation is designed for limited input, since transfer functions for different input are often incompatible. In this paper, transient responses with respect to input/reference voltage and load variations are optimized simultaneously by a linear superposition compensation strategy. The objective closed-loop transfer functions for each kind of variations are investigated, which contribute to the overall compensation loop design. According to linear superposition principle, the compensation results are summed as reference current, based on which the switching on/off time is modulated. Although the summed results are convergence, separate compensation loop that includes integral terms can lead to divergence in calculation process. To eliminate the potential divergence, the overall compensation loop is re-designed so as to suit digital implementation. Experimental results show that the linear superposition compensation strategy achieves great transient performance to different kind of variations.

1. Introduction
Converters operating in critical conduction mode (CRM) operation have potentials to degrade system order and achieve high efficiency, which are widely used in low to medium power conditions, such as on-board charger, LED driver, and power factor correction [1, 2]. In addition to smaller inductor size brought by CRM operation, its zero-voltage/valley switching feature decreases turn-on switching loss and reverse recovery loss of the diode [3]. In order to achieve better transient performance to suit different working conditions a number of interesting and practical controls have been proposed based on different methods [4, 5].

The commonly-used practice for optimal transient respond is to make improvements to conventional linear-mode controls, such as voltage and current mode controls. For example, a current-mode controller that enhances the load transient response is proposed in [6] by increasing the system bandwidth. The transient response is improved due to the gain-boosting effect around the compensator’s mid-band. Although the transient response to load variation can be improved by current mode control, current sensors or detecting tequeniques are often required. Aimed at this issue, sensorless current mode controllers are investigated [7, 8]. Furthermore, various control methods investigate real-time monitoring of circuit state, since switching-mode converter system are naturally non-linear. According to the sampled peak and valley value of inductor current, load change is estimated immediately [9]. By bringing the monitored resistance value back to the controller in real
time, the load dependent adaptive controller is obtained to improve the load response speed. Another
method to acquire the circuit state is the implementation of algebraic parameter identification [10]. In
above methods, only one compensator is applied to optimize load or input voltage transient
performance, where reference voltage transient is not considered.

Different from the analysis for load or input voltage transients, detailed discussions on reference
voltage transient are relatively insufficient, since these three transients involve different dynamics [11].
In [12], voltage-mode control, current-mode control and \( V^2 \)-control are analyzed in both small-signal
and large-signal domains for comparing their reference tracking speed. It is found that by end-point
prediction, reference tracking speed under \( V^2 \)-control can be greatly enhanced. In [13], a charge-
recycling technique is implemented to decrease the output voltage and the stored energy can be sent
back to the output node for rapidly increasing the voltage level. In aforementioned literatures,
compensators for input/reference voltage and load transient are incompatible. Nevertheless, these
linear-mode control methods are investigated for pulse width modulation (PWM), their applicability
for CRM operation should always be considered.

In this paper, a linear superposition compensation strategy is proposed to optimize the transient
responses with respect to input/reference voltage and load variations simultaneously. Firstly, the
objective closed-loop transfer functions for each kind of variations are investigated, which gives direct
design rules for conventional closed-loop design. According to the linear superposition principle, the
closed-loop results are summed as reference current, based on which the switching on/off time is
modulated. Although the summed results are convergence, separate compensation loop that includes
integral terms can lead to divergence in calculation process. To eliminate the potential divergence, the
overall compensation loop is revised so as to suit digital implementation.

This paper is organized as follows. In Section II, the small-signal model of CRM boost converter is
given. The objective closed-loop transfer functions for each kind of variations input are investigated,
based on which the linear superposition compensation is derived. In Section III, the potential
divergence in the calculation process is analysed. The proposed controller is then re-designed for
digital implementation. In Section IV, experimental results are given to verify effectiveness of the
proposed strategy. Finally, a brief conclusion is given.

2. Linear superposition compensation strategy

For a CRM boost converter, the output voltage is influenced by input/output voltage and load, which
should be regulated to track reference voltage, as shown in Figure 1. Inductor current of a CRM boost
converter is given by Figure 2. Different from conventional linear compensation strategy in Figure
3(a), the proposed strategy consists of three individual compensators, as shown in Figure 3(b). Since
there is only one PI compensator under conventional linear compensation strategy, the transient
performance cannot be optimized simultaneously towards input/reference voltage and load variations.
Comparatively, the adopted feedback compensator and feedforward compensators make it achievable
to optimize the transient performance simultaneously.
In the followings, the small-signal modelling (SSM) for CRM boost converter and the closed-loop SSMs of the proposed strategy is first given, based on which the objective closed-loop transfer functions for each kind of variations input are investigated.

2.1. SSM for CRM boost converter and the Closed-loop SSMs of the proposed strategy

![Closed-loop analysis of CRM boost converter based on (a) conventional linear compensation strategy and (b) linear superposition compensation strategy.](image)

The inductor current of a CRM boost converter is shown in Figure 2. The peak current is regulated as \( i_{\text{ref}} \) in the overall system, where the switching on/off time is given by:

\[
T_{\text{off}} = \frac{i_{\text{ref}} L}{v - v_{\text{in}}}, \quad T_{\text{on}} = \frac{i_{\text{ref}} L}{v_{\text{in}}}
\]  
(1)

Under CRM operation, the average value of inductor current equals half the peak current, which gives the average output current \( i_o \) as:

\[
i_o = \frac{i_{\text{ref}} T_{\text{off}}}{2(T_{\text{on}} + T_{\text{off}})} = \frac{i_{\text{ref}} L}{v_{\text{in}} - v_{\text{in}}}/2\left(\frac{i_{\text{ref}} L}{v_{\text{in}}} + \frac{i_{\text{ref}} L}{v_{\text{in}} - v_{\text{in}}}\right) = \frac{i_{\text{ref}} v_{\text{in}}}{2v_{\text{o}}}
\]  
(2)

According to the transfer function of the output RC network, the \( s \)-domain function of the converter is given by:

\[
i_o = \frac{i_{\text{ref}} v_{\text{in}}}{2v_{\text{o}}}, \quad v_o = \frac{1}{sC}(i_o - \frac{v_o}{R})
\]  
(3)

Differentiating equation (3) gives a linear equation by:

\[
i_o = \frac{v_{\text{in}}}{2v} = \frac{i_{\text{ref}} v_{\text{in}}}{2v} = \frac{i_{\text{ref}} v_{\text{in}}}{2v} \hat{i} + \frac{i_{\text{ref}} v_{\text{in}}}{2v} \hat{v}_{\text{in}}, \quad \hat{v}_o = \frac{1}{sC}(\hat{i}_o - \frac{1}{R} \frac{v_o}{R} \hat{R})
\]  
(4)

Since \( i_{\text{ref}} v_{\text{in}}/2v_o \approx \frac{i_{\text{ref}} R v_{\text{in}}}{2v_o} = 1 \), solving equation (4) gives the SSM for CRM boost converter by:

\[
\hat{v}_o = G_v(s) \hat{i}_{\text{in}} + G_{vg}(s) \hat{v}_{\text{in}} + G_v(s) \hat{R}
\]  
(5)

where

\[
G_v(s) = K_v, \quad G_{vg}(s) = \frac{K_v}{s + \omega_p}, \quad G_v(s) = \frac{K_v}{s + \omega_p}, \quad K_v = \frac{v_{\text{in}}}{2vC}, \quad K_i = \frac{v_{\text{in}}}{RC v_{\text{in}}}, \quad K_3 = \frac{v}{CR^2}, \quad \omega_p = \frac{2}{RC}
\]  
(6)

According to Figure 3(b), the closed-loop SSMs of the linear superposition compensation strategy is derived as:

\[
\Phi_{\text{r}}(s) = \frac{G_v(s)}{1 - G_v(s) H_v(s)}, \quad \Phi_{\text{ref}}(s) = \frac{H_v(s) G_v(s)}{1 - G_v(s) H_v(s)}, \quad \Phi_{\text{p}}(s) = \frac{G_v(s) + H_{vg}(s) G_v(s)}{1 - G_v(s) H_v(s)}
\]  
(7)

In the followings, objective closed-loop transfer functions for each variations input is given based on the derived SSMs.

2.2 Objective closed-loop transfer functions and linear superposition compensation strategy

Instead of using \( \Delta v = v_{\text{ref}} - v_o \) to design the PI controller for conventional linear compensator, variations of \( v_o, v_{\text{ref}} \) and \( v_{\text{in}} \) are compensated separately under the proposed strategy. They are regulated together to improve the transient performance with respect to different variations, and then (7) is adjusted as
objective transfer functions, which are given by:

\[
\Phi_{R,\text{obj}}(s) = \frac{C_1s}{s^2 + 2\xi_R\omega_R + \omega_R^2}, \quad \Phi_{f,\text{obj}}(s) = \frac{\omega_{\text{dc}}}{\omega_{\text{dc}} + s}, \quad \Phi_{g,\text{obj}}(s) = \frac{C_2s}{s^2 + 2\xi_g\omega_g + \omega_g^2}
\]

(8)

where \{\omega_{\text{dc}}, \omega_R, \omega_g\}, \{\xi_R, \xi_g\}, and \{C_1, C_2\} are nature frequencies, corresponding damping factors, and gains of the transfer functions, respectively.

Based on (6) and making the closed-loop SSMs of the proposed strategy equals to the objective transfer functions (8), the three individual compensation transfer functions are derived as:

\[
\begin{align*}
H_v(s) &= \left( \frac{C_1 - K_3s^2}{K_1s} + \frac{C_1\omega_{\text{dc}} - 2K_3s\omega_R}{K_1s^2} \right) \omega_{R,\text{obj}}, \\
H_i(s) &= \frac{K_3}{K_1} \omega_{R,\text{obj}} s^2 + \frac{2\xi_R\omega_R s + \omega_R^2}{sC_1}, \\
H_{ip}(s) &= \frac{C_2}{K_1} s^2 + \frac{2\xi_g\omega_g s + \omega_g^2}{sK_1} - \frac{K_2}{K_1}
\end{align*}
\]

(9)

In order to simplify the transfer functions, the parameters in (9) is designed as \(C_1 = K_3\), \(\omega_{\text{dc}} = \omega_R\) and \(\xi_g = \xi_R\). Therefore, the three individual compensation transfer functions in the proposed linear superposition compensation strategy can be organized as:

\[
H_v(s) = \frac{\omega_{\text{dc}}}{\omega_{R,\text{obj}}} s - \omega_R^2, H_i(s) = \frac{\omega_{\text{dc}}}{\omega_{R,\text{obj}} + s} s^2 + \frac{2\xi_R\omega_R s + \omega_R^2}{sK_1}, H_{ip}(s) = \frac{C_2 - K_2}{K_1}
\]

(10)

3. Digital implementations of the proposed strategy in CRM boost converter

The proposed strategy is implemented in digital processor based on the \(z\)-domain of equation (10). Noted that there are two integral items in \(H_v(s)\) and \(H_i(s)\), where their compensation objects are large-signal, divergence will appear in calculation process. In order to eliminate the potential divergence, the overall compensation loop should be re-designed so as to suit digital implementation.

Firstly, the proposed linear superposition compensation strategy is re-written as:

\[
\begin{align*}
iv_{\text{ref}}(s) &= v_v(s)H_v(s) + v_{\text{ref}}(s)H_i(s) + v_{\text{a}}(s)H_{ip}(s) \\
\end{align*}
\]

(11)

Converting equation (11) to \(z\)-domain by backward difference law according to \(s = (1 - z^{-1})/T\) gives:

\[
\begin{align*}
i_{\text{ref}}(z) &= v_v(z)P + v_{\text{ref}}(z)d_1 z^{-1} + v_{\text{a}}(z) e + (v_{\text{ref}}(z) b - v_v(z)) T \frac{1 - z^{-1}}{1 - az^{-1}} \\
\end{align*}
\]

(12)

Rearranging the integral items to a group, (12) is derived as:

\[
i_{\text{ref}}(z) = v_v(z)P + v_{\text{ref}}(z)d c z^{-1} + v_{\text{a}}(z) e + (v_{\text{ref}}(z) b - v_v(z)) T \frac{1 - z^{-1}}{1 - az^{-1}}
\]

(13)

where the parameters \(a, b, c, d\) and \(e\) are:

\[
\begin{align*}
a &= \frac{1}{1 + \omega_{\text{dc}} T}, b = \frac{\omega_{\text{dc}} T}{1 + \omega_{\text{dc}} T}, c = 1 + 2\omega_R T, d = \frac{\omega_{\text{dc}} K_1}{1 + \omega_{\text{dc}} T}, e &= \frac{C_2 - K_2}{K_1}
\end{align*}
\]

(14)

For the original linear superposition compensation strategy displayed in (10), three different variations are compensated seperately. After the rearrangement, the integral items are combined together. Based on (13) and (14), the digital calculation process for the proposed linear superposition compensation strategy is given in Figure 4.

Figure 4. Revised digital calculation process for the proposed compensation strategy.

With the low pass filter as shown in Figure 4, \(\text{v}_{\text{err}}\) can be regarded as the error by \(v_{\text{ref}}\) and \(v_o\)
compared with the conventional linear compensation strategy. Therefore, the 3 th branch is the feedback compensator, which is similar to the integral item in conventional PI compensator. Comparatively, the other three branches act as feedforward compensators, which stabilizes the converter towards $v_{in}$ and $v_{ref}$ variations.

4. Experimental results

An experimental prototype is built to compare the performances under conventional PI control strategy and the proposed strategy. Comparisons are given with respect to transient performance of $v_{in}$/load transients and reference tracking. Main specifications of the prototype are as followings: rated input voltage $V_{in}=24V$, output voltage $V_o=48V$, load $R=80\Omega$, output capacitor $C=20\mu F$ while the inductor is $40\mu H$. To verify the effectiveness of the proposed controller, the controller is designed by $\omega_{nr}=0.15/T$, $\omega_{nf}=0.1/T$ and $C_2=0.2K_2$. For PI controller optimizing $R$ and $v_{in}$ variations, the bandwidth of the closed-loop transfer function is set by $\omega_{nr}=0.15/T$. For PI controller optimizing $v_{ref}$ variation, the bandwidth of the closed-loop transfer function is set by $\omega_{nf}=0.1/T$.

The corresponding experimental results are given in Figure 5, Figure 6 and Figure 7, respectively. As shown in Figure 5, $v_o$ restabilizes in 80µs, 70µs and 100µs when $v_{in}$, load and $v_{ref}$ steps, respectively. For PI compensation to optimize $v_{ref}$ variations in Figure 6, $v_o$ restabilizes in 1.2ms, 1.2ms and 120µs when $v_{in}$, load and $v_{ref}$ steps. Although the reference tracking speed under PI controller is similar to that under the proposed strategy, the re-stabilization time for $v_{in}$ and $R$ variations turns much longer. For PI compensation to optimize $v_{in}$ and $R$ variations in Figure 7, $v_o$ restabilizes in 100ms, 80ms and 180µs when $v_{in}$, load and $v_{ref}$ steps. Although the re-stabilization time for $v_{in}$ and $R$ variations under PI controller is close to that under the proposed strategy, there is voltage overshoot of 1V appears at $v_{ref}$ steps.

From the experimental results, the linear superposition compensation strategy achieves great transient performance to different kind of variations. Comparatively, the conventional linear compensation strategy can only compensate one or two variation kind, which restrict its applications.
5. Conclusions
In this paper, a linear superposition compensation strategy is proposed to optimize the transient responses with respect to input/reference voltage and load variations simultaneously. The objective closed-loop transfer functions for each kind of variations are investigated, which contribute the overall compensation loop design. Furthermore, the potential divergence in digital implementation is investigated. The final version of the z-domain control diagram indicates that there is one branch acting as the feedback compensator, which is similar to the integral item in conventional PI compensator. Comparatively, the other three branches act as feedforward compensators, which stabilizes the converter with respect to $v_{in}$ and $v_{ref}$ variations. Experimental results verify the effectiveness of the proposed strategy.

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