Finite-temperature spectral function of the vector mesons in an AdS/QCD model

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We use the soft-wall AdS/QCD model to investigate the finite-temperature effects on the spectral function in the vector channel. The dissociation of the vector meson tower onto the AdS black hole leads to the in-medium mass shift and the width broadening in a way similar to the lattice QCD results for the heavy quarkonium at finite temperature. We also show the momentum dependence of the spectral function and find it consistent with screening in a hot wind.

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INTRODUCTION — The intrinsic properties of hot and dense matter out of quarks and gluons in quantum chromodynamics (QCD) have been drawing our attention to a deeper understanding of strongly-correlated systems. Such matter has been created at the Relativistic Heavy-Ion Collider (RHIC) in Brookhaven and widely called the strongly-correlated quark-gluon plasma or sQGP [1]. It is generally challenging to attack non-perturbative physics problems and there is no systematic strategy established. Recently powerful techniques have developed based on the recognition of the gauge/string duality or the bulk/boundary correspondence [2, 3, 4], with which one can treat the strong-coupling regime in the gauge field theory on the boundary by solving the weak-coupling (or classical in some limit) string theory in the bulk Anti de Sitter (AdS) space.

In sQGP physics the most striking and well-known achievement from the gauge/string duality is the analytical computation of the shear viscosity to the entropy ratio, αs, in the N = 4 supersymmetric Yang-Mills plasma [5]. Although the theory is different from QCD, the obtained result is valuable under the present circumstance that the non-perturbative QCD calculation of the shear viscosity is cumbersome [6, 7]. The smallness of αs is an important indication of the sQGP because a large reaction cross-section implies small αs in gaseous states. Another suggestive hint to the sQGP has come from the lattice QCD study on the J/ψ spectral functions at high temperature [8, 9, 10]. There, it has been found that the mesonic correlation survives even above 2Tc where Tc is the deconfinement critical temperature.

This melting temperature for the J/ψ meson inferred from the lattice QCD simulation is not consistent with the potential model estimate with the Debye screening [11, 12], though the spectral enhancement after melting may explain the lattice observation [13]. We have not yet reached a consensus on the right interpretation of the J/ψ spectral functions above Tc. It is most likely that the heavy-quarkonium spectral property reflects the sQGP nature. Hence, it would be intriguing to take advantage of the dual model to see what spectral shape would transpire non-perturbatively in strongly-correlated systems.

We remark that the spectral functions and also the meson dissociation have been nicely discussed by means of the D3/D7 setup [14, 15]. The correspondence between the J/ψ spectral function and the holographic outcome is not transparent, however, because the embedding of the D7 brane in an AdS-Schwarzschild background leads to a first-order phase transition at which the dissociation takes place.

In this Letter we shall make use of a rather phenomenological approach to QCD inspired by the success of the AdS/CFT correspondence, which is generally referred to as the AdS/QCD models or the holographic QCD models [16]. Particularly, since J/ψ is a vector meson, we adopt the “soft-wall” model [17] (see Ref. [18] for a related idea) which is designed to reproduce the Regge trajectory of the vector mesons (i.e. vector meson tower). Here, we remark on some related works along the similar line. In Ref. [19] the meson spectrum has been discussed in the “hard-wall” holographic QCD model at finite temperature. The hard-wall model is, however, not quite appropriate to look into the spectral functions because the infrared (IR) boundary condition has ambiguity, while the soft-wall model is smooth in the IR side. Although the meson mass shift has been investigated in Ref. [20] in the soft-wall framework, its relevance to the lattice QCD results is rather indirect.

Our aim is to derive the finite-temperature spectral function in the vector channel from the holographic QCD model in a way that we can make a direct comparison with the lattice QCD. As a matter of fact, as we will discuss, the resulting spectral shape and the associated mass shift and width broadening are all in qualitative agreement with the lattice calculations.

MODEL SETUP — We make a brief review on the formalism and equations that we are using. The soft-wall model proposed in Ref. [17] is composed from the gauge fields A_{\mu \nu} in SU_L(N_f) and A_{\mu \nu} in SU_R(N_f) and the bifundamental matter X whose ultraviolet (UV) behavior controls the chiral symmetry breaking.

The essential point in the soft-wall setup is that the five-dimensional action contains the dilaton field potential so that the spectrum realizes the linear confinement.
That is, the Lagrangian density is multiplied by the smooth IR cutoff function, $e^{-\Phi(z)}$, where $z$ is the fifth coordinate and $\Phi(z) = cz^2$. Here, $c$ is the only mass dimensional parameter in the model, and we will later make all other quantities dimensionless in the unit of $\sqrt{c}$.

We can decompose the model action into one piece involving the vector field, $V = (A_L + A_R)/2$, and the other involving the axial-vector field, $A = (A_L - A_R)/2$. Then, as long as the vector $SU(N_f)$ symmetry is left unbroken, the (linearized) equation of motion for $V$ decouples from $X$ and $A$. Thus, we need to solve the following equation of motion in the $V_z = 0$ gauge,

$$
\partial_z \left[ e^{-\Phi(z)} \sqrt{-g} g^{\lambda \nu} \partial_z V_\lambda \right] + e^{-\Phi(z)} \sqrt{-g} g^{\lambda \nu} \partial_\mu \partial_\nu V_\lambda = 0.
$$

(1)

Because the physical degrees of freedom for the massive vector field have three polarizations, we eliminated one of four components by imposing $\partial^\mu V_\mu = 0$. Let us now write down the metric representing the AdS black hole;

$$
g_{\mu \nu} dx^\mu dx^\nu = \frac{L^2}{z^2} \left[ -f(z) dt^2 + dz^2 + \frac{1}{f(z)} d\Omega^2 \right],
$$

(2)

where $f(z) = 1 - z^4/z_h^4$ is the horizon denoted by $z_h$ and $L$ denotes the radius of the AdS space. The Hawking temperature is read as $T = 1/(\pi z_h)$, which translates into the temperature of a QGP medium.

It should be mentioned that the AdS-Schwarzschild metric undergoes a first-order phase transition to the AdS metric and the critical temperature is $T_c \approx 0.492\sqrt{c}$ in the soft-wall model case. The temperature of our interest is, as we will see later, up to $\sim 0.15\sqrt{c}$. This metric transition is not a serious flaw in our present setup, however, by the following twofold reasons. One is that what we really want to know is simply the effect of the presence of a medium on the spectrum and not the spectrum in the ground state determined self-consistently. For this limited purpose it makes sense to accommodate (transient) matter by introducing the meta-stable black hole metric. The second reasoning is a more physical one. If we are interested in $J/\psi$ specifically, it is possible to construct the U(1) soft-wall model in the charm sector alone because the charm current is conserved. The relevant scale $\sqrt{c} = \sqrt{\frac{m_{\psi}}{1\, \text{GeV}}}$. Should be about four times bigger than that for $\rho(770)$, which we shall write $\sqrt{c}$ for the moment. Then, $0.15\sqrt{c}$ might be a higher temperature than $T_c$ which is characterized by $\sqrt{c}$. We will go into a more quantitative argument when we show our numerical results. To fully justify this hand-waving argument, one should break $SU(N_f)$ symmetry and carry out the analysis in Ref. 22 for three light and one heavy flavors. This would be an important check to correctly relate $T_c$ to the heavy-quarkonium dissociation temperature. Here we postpone resolving this quantitatively and let us concentrate on demonstrating qualitatively that the holographic QCD model works well to elucidate the vector spectral function.

**SOLUTION** — Before solving (1), it would be helpful to pursue the analogy to the Schrödinger equation in quantum mechanics. We take a plane-wave form in four dimensional space-time as usual. Then, using dimensionless energy $\omega$ and momentum $q$ in the unit of $\sqrt{c}$ and changing the fifth coordinate by $\xi = \sqrt{c} z$, we can get rid of $c$ from the equation of motion.

Since the presence of matter breaks Lorentz symmetry, the transverse ($\lambda = 1, 2, 3$) and the longitudinal ($\lambda = 0$) components in Eq. (1) lead to distinct differential equations. The difference is, nevertheless, not qualitative but quantitative, and so we shall show the solution of the transverse $V_i$ field only.

The change of the field, $v = (e^{-\Phi} \sqrt{-g} g^{ii'} g^{zz'})^{1/2} V_i$ (no sum over $i'$), simplifies the equation of motion in the following form: $v'' - U(\xi)v = 0$ with the potential,

$$
U(\xi) = \xi^2 + \frac{3}{4\xi^2} - \frac{f'}{2f}(2\xi + \frac{1}{\xi}) - \frac{(f')^2}{4f^2} + \frac{f''}{2f} - \frac{1}{f}(\omega^2 - q^2).
$$

(3)

![Figure 1: Potential $U(\xi)$ for the dimensionless temperatures, $t = 0.05$, 0.10, and 0.15 at $\omega^2 = 4$ and $q^2 = 0$. The thin vertical lines represent $\xi_h = 1/(\pi t)$, that is the location of the black hole horizon.](image)

Figure 1 shows this potential for various dimensionless temperatures.In the $t = 0$ case the downward-convex potential, $\xi^2 + 3/(4\xi^2)$, yields the discrete spectrum, $\omega^2 = 4n$ ($n = 1, 2, \ldots$) for $q^2 = 0$, only for which the wave-function is normalizable. We see that the higher $t$ or smaller $\xi_h = 1/(\pi t)$ makes the potential less convex and eventually it becomes monotonic for $t \approx 0.15$. With a monotonic potential we cannot expect a remnant of the original spectrum any more. In other words we should anticipate dissociation then.

At finite temperature the potential is no longer rising in the large $\xi$ side and the normalizability does not quantize the spectrum. We can easily extract the asymptotic solutions of Eq. (1) near the horizon as

$$
V_i(\xi) \rightarrow \phi_\pm = (1 - \xi/\xi_h)^{\pm i\omega \xi_h/4},
$$

(4)

as $\xi \rightarrow \xi_h$. Here $\phi_+$ represents the out-coming solution
and $\phi_-$ the in-falling solution into the black hole. Because the imaginary part of the retarded Minkowskian Green’s function gives the spectral function, the right IR boundary condition should pick only $\phi_-$ up near the horizon.

**SPECTRAL FUNCTIONS** — Following the procedure elucidated in Ref. [23] in details, we can numerically get two independent solutions starting from the UV limit, that is, $\Phi_1(\xi)$ and $\Phi_2(\xi)$ satisfying $\Phi_1(\xi) \approx \xi^2$ and $\Phi_2(\xi) \approx 1$ in the vicinity of $\xi = 0$. Then we can make an appropriate linear combination,

$$V_i(\xi) = A(\omega, q)\Phi_2(\xi) + B(\omega, q)\Phi_1(\xi) \rightarrow \phi_-(\xi), \quad (5)$$

as $\xi \to \xi_+$, with the commonly adopted normalization $A = 1$ (i.e. $V_i(0) = 1$).

From the prescription for Minkowskian correlators in Ref. [23] we can relate the UV property of $V_i(\xi)$ to the retarded Green’s function by

$$D^R(\omega, q) = -C \lim_{\xi \to 0} \left(\frac{1}{\xi} V_i^* \partial_\xi V_i\right) = -2CB(\omega, q), \quad (6)$$

where $C$ is a constant given as $N_c/(24\pi^2)$. The spectral function is, by its definition, $\rho(\omega, q) = -(\text{Im}D^R(\omega, q))/\pi = (2C/\pi) \text{Im}B(\omega, q)$. To be free from the normalization convention we plot $\text{Im}B(\omega, q)$ as a function of $\omega$ and $q$ for various temperatures. We also refer to $\text{Im}B(\omega, q)$ the spectral function discarding the overall factor.

We show our numerical results in Fig. 2 for $q = 0$. It is obvious at a glance that, at low temperature, sharp peaks stand in accord with the spectrum $\omega^2 = 4n$ with $n = 1, 2, \ldots$ known at $t = 0$. Since the eigenvalues for not $\Phi_1(\xi)$ but $\Phi_2(\xi)$ are relevant to our prescription, the peak positions are slightly shifted from $\omega^2 = 4n$, as seen in the data for $t = 0.07$ whose lowest-lying peak is located at $\omega^2 = 3.92$. We can identify this peak as the $J/\psi$ mass squared, from which we get our dimension unit, $\sqrt{\rho_{J/\psi}} = 1.56$ GeV. We find that the peak completely vanishes for $t$ as high as 0.15. This clearly corresponds to the temperature where the potential $U(\xi)$ in Fig. 1 loses convexity. The deconfinement transition is, on the other hand, $T_c = 0.492, \sqrt{\rho_{J/\psi}} = 0.19$ GeV. This means that in our non-perturbative soft-wall QCD model the spectral peak melts around $T \sim 0.15, \sqrt{\rho_{J/\psi}} \sim 1.2T_c$. We note, however, that we should not take this melting temperature in terms of $T_c$ too seriously because we just borrowed $T_c$ from the estimate in Ref. [22]. Our patched-together (decoupled U(1) charm sector plus light quarks for $T_c$) argument is valid only approximately and does not go beyond qualitative estimate. In principle we can improve the holographic deconfinement scale provided in the pure gluonic sector by mimicking QCD [23] and the heavy quarkonium is to be put on top of it. Such self-consistent treatment will be reported elsewhere.

It is notable in Fig. 2 that the second lowest-lying state near $\omega^2 \sim 8$ melts far earlier than the $\omega^2 = 4$ state. This is quite natural because higher excited states are less stable generally. In terms of the potential a larger $\omega^2$ causes stronger absorption into the black hole by the term, $-\omega^2/f^2$ in Eq. 4, which is negative large near the horizon. Furthermore, the lowest-lying state moves only slightly to a smaller mass, while the excited states shift more as seen in the second peak in the $t = 0.07$ curve.

In Fig. 3, we plot the mass shift $\Delta m$ and the width $\Gamma$ associated with the lowest-lying peak in the spectral function. We have found that a functional form, $a\omega^b/[(\omega - \omega_0)^2 + \Gamma^2]$, fits the spectral shape precisely, from which we extracted the peak position $\omega_0(t)$ (leading to the mass shift $\Delta m(t) = \omega_0(0) - \omega_0(t)$) and the width $\Gamma(t)$ as a function of the temperature. Interestingly enough, our numerical results show that $(\Delta m(t))^2 \propto \Gamma(t)$.
seems to hold as long as the spectral peak is sharp. This relation seemingly makes a contrast to the linear proportionality predicted in the QCD sum rule [26] but could be interpreted as consistent [27]. The mass shift is saturated with increasing \( t \), while the width broadening goes consistently. The saturation behavior above \( t = 0.14 \), as depicted in Fig. 3, can define where the vector meson melts quantitatively in terms of the spectral function. Finally we shall discuss the momentum dependence of the spectral functions. We plot the numerical results in Fig. 4 for \( q^2 \) ranging from 0 to 12 with \( t = 0.10 \) fixed. It is apparent that the spectral peak is gradually collapsed as \( q \) increases. This is highly non-trivial. In the perturbative regime, typically, a larger \( q \) makes the spectral peak less sensitive to the medium effect [28]. In strongly-correlated matter, as discussed in Ref. [29] (see also Ref. [30]), \( J/\psi \) is more screened and melts at higher \( q \), or in a frame where \( J/\psi \) is at rest, it melts under the hot wind of matter. Figure 1 clearly illustrates that the \( J/\psi \) suppression by the hot wind is also the case in our calculation. It is intriguing that the screening is evident at the momentum scale \( q \sim \sqrt{t} \) which corresponds to about 5 GeV, which is not far from the transverse momentum of particles observed at RHIC.

**SUMMARY** — We derived the spectral function of the vector meson states at finite temperature using the soft-wall AdS/QCD model. The qualitative properties of the obtained spectral pattern are in good agreement with the lattice observation for the heavy quarkonium.

We numerically found that the mass shift squared is approximately proportional to the width broadening. Another interesting finding is that the spectral peak diminishes at high momentum. We can interpret this as consistent with the \( J/\psi \) suppression under the hot wind.

In our future publication we will report not only the vector channel addressed here but also the axial vector channel. It would also be an important extension to investigate the medium modification with finite baryon number. In that case the Chern-Simons coupling plays an intriguing role in the vector-axial-vector mixing [31] [32].

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**FIG. 4:** Spectral functions \( \text{Im} B(\omega, q) \) as a function of \( \omega \) and \( q \) for a fixed temperature \( t = 0.10 \).
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[33] The longitudinal part has the solution near the horizon as $V_0(\xi) \to (1 - \xi/\xi_h)^{1/2+i\sqrt{(\omega \xi_h)^2-1/4}}$, which is different from Eq. (4), implying the existence of a threshold frequency at $\omega \xi_h = 2$ or $\omega = 2\pi t$. 

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