Demonstration of vortical beams spectral stability formed in non-zero diffraction orders

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Abstract. We provide a comparative analysis of vortical beams forming by a spiral phase plate and a binary fork diffraction grating. Intensity distributions formed by a diffractive optical element (DOE) have distortions whereas DOE’s relief height is not corresponds to the wavelength of an incident beam. These distortions are numerically studied in this work. Different vortical beams are experimentally created and studied. The natural research is conducted with a spatial light modulator and with binary DOE. We experimentally show that fork binary DOEs are resistant to small deviation of incident beam wavelength and vortical beams are formed without any significant distortions.

1. Introduction

A vortical beam former is an in-demand diffractive optical element [1-3]. For this problem, a balance between an energy efficiency and accuracy of required intensity distribution is very critical. This balancing is very common for DOE manufacturing. The reason of these troubles is close tolerance requirements for a phase transmission function. But as usual high accuracy cannot be achieved because of the manufacturing specificity.

Spiral phase plate is one of the simplest devices of singular optics. It was theoretically offered in 1984 [4]. For the first time the binary phase plate was manufactured in 1992 [5]. To this time multilevel DOE manufacturing has been a significant problem. Nevertheless, DOEs cannot be replaced with spatial beams modulators (SLM) because of weak diffraction efficiency of SLM. So a multilevel phase is often coded into a binary-level relief.

If we carefully consider two main ways of phase modulation with DOE, we can find that these methods have absolutely different requirements to the accuracy of manufacturing. Thus, the element for zero-order diffraction phase modulation should have a relief, each part of that produces certain required phase delay.

For vortical phase modulation DOE relief is a helical surface. Integrating the phase of the field around a path enclosing a vortex yields an integer multiple of 2π. However, fabricating inaccuracies of a helical surface causes a similar deviation of formed phase transmission, in contrast to diffractive lenses, which manufacturing deviation only decreases diffraction efficiency but quality degree of point spread function remains on the same level.

However, modern technology level allows us to avoid the problem of close tolerance DOE manufacturing requirements. Obviously, another unavoidable trouble of phase modulation can be a deviation of incident light from the monochromatic wave. This problem plays a main role in data...
transmission multiplexing [6-8], because the pulse character of utilized light causes a widening of the spectrum. All of these reasons make a DOE extremely difficult to use for optical vortices forming in zero diffraction order. But this problem can be solved with using a DOE, which works in non-zero order of diffraction. The fundamental distinguishing difference is that phase modulation is produced by grating bars shifting, instead of phase delaying by optical thickness whereas a DOE works in zero-order. Technologically, optical thickness controlling is more difficult than grating bars shifting because the bars have a constant height. But we should pay with diffraction efficiency for the simpler manufacturing of non-zero DOE, which can provide 40% of efficiency in the plus and minus first order of diffraction. Moreover, fortunately, related to spectrum widening errors cause only a decreasing the energy efficiency. Indeed, formed wave front shape is kept in good quality whereas incident beam wavelength deviates from the main meaning. Wavelength deviation causes some scale changing because of changing of diffraction angles. However, vortical beams possess unique quality to be invariant to scale-changing. It means that the phase of the vortical beam remains a vortical character whereas the wavelength deviates, just the amplitude distribution changes.

2. Numerical simulation
In this section we show the results of numerical simulation for evaluating of distortions, which arise whereas a DOE is illuminated with deviated wavelength λ. We should mention that the thickness of element’s relief corresponds to the main value of the wavelength λ0.

\[ h(x, y) = \frac{\text{mod}_{2\pi} \left[ \frac{\varphi(x, y)}{2\pi(n-1)} \right]}{\lambda_0}, \]

where \( \varphi(x, y) \) is phase of the element, \( n \) is refractive index of DOE’s material.

For the simulation we use integral transform, which describes the diffraction on thin DOE with a spherical lens:

\[ G(u, v) = \frac{2\pi}{\lambda f} \int \int A(x, y)g(x, y) \exp \left[ -\frac{i2\pi}{\lambda f}(xf + yv) \right] dx dy, \]

where \( A(x,y) \) is an incident beam with wavelength \( \lambda \), \( g(x,y) \) is a complex transmission function of the DOE, \( f \) is focal length. The incident beam has Gaussian energy distribution.

We study to types of diffraction optical elements for optical vortices forming. The first type is a multilevel spiral phase plate [5, 9], which is used for optical vortex generation in zero diffraction order. The binary spiral phase plate was considered in the paper [10]. Technology of fabricating multilevel DOE requires is very difficult. In some cases they can be replaced by spatial light modulators whereas diffraction efficiency does not play any significant role. A spiral phase plate which produces m-order optical vortex, has following complex transmission function:

\[ g_m^s(x, y) = \exp \left[ i \text{arctg} \left( \frac{y}{x} \right) \right]. \]

Another type of DOE is a fork diffractive grating [11, 12] which forms two optical vortices with topological charges of \( \pm 1 \) in \( \pm 1 \) orders of diffraction respectively. Complex transmission function of the fork grating is

\[ g_m^b(x, y) = \exp \left[ i \arg \left[ \cos \left( ax + \text{arctg} \left( \frac{y}{x} \right) \right) \right] \right]. \]
where $a$ is a factor corresponding to the carrying frequency of the grating. Gaussian beam transverse intensity distribution (figure 1a), the phase of the spiral phase plate for order $m=1$ (figure 1b), and the phase of the fork grating for order $m=1$ (figure 1b) are shown in the figure 1.

![Figure 1. Gaussian beam transverse intensity distribution (a), phase of the spiral phase plate for order $m=1$ (white color means $2\pi$ radian, black color means 0 radian) (b), and phase of the fork grating for order $m=1$ (white color means $\pi$ radian, black color means 0 radian) (c).](image)

If the DOE with relief thickness $h(x,y)$ is illuminated by an incident beam with the wavelength $\lambda$, suppose that the phase of the incident beam is changed as follow:

$$\psi'(x,y) = \frac{\lambda_0}{\lambda} \psi(x,y).$$  \hspace{1cm} (5)

In table 1 we show the results of simulation of vortical beams forming with phase helical plates for $m=1,2,3$, in case of the incident beam has different wavelength.

From the results we can see that if the incident beam wavelength $\lambda$ deviates from the base wavelength $\lambda_0$ (which corresponds to the element thickness), formed beam has very bad quality, and actually, it is not a vortical beam. Moreover, beams are formed with worse quality for greater phase topological charge orders.

**Table 1.** Vortical beams forming by means of helical phase plates with orders $m=1, 2, 4$. The incident Gaussian beam has different wavelengths (negative intensity distributions are shown).

| Wavelength   | $m=1$ | $m=2$ | $m=4$ |
|--------------|-------|-------|-------|
| $\lambda=432$ nm | ![image] | ![image] | ![image] |
| $\lambda_0=532$ nm | ![image] | ![image] | ![image] |
| $\lambda=632$ nm   | ![image] | ![image] | ![image] |

Notice, that wavelength deviation causes separating of $m$-order vortical phase singularity into $m$ shifted first-ordered vortices. The same situation was mentioned in the works [11, 13] for adjustment errors of DOE and the incident beam. Similar picture arise whereas spiral phase plates of fractional orders are used in [14] and if focusing element has periodical angle dependence as well as in [15].

In table 2 we show the results of simulation of vortical beams forming with fork diffractive gratings in $\pm1$ orders for $m=2, 4$, in case of the incident beam has different wavelength.

From the tables 1, 2 we can see that incident beam wavelength deviation from the base meaning does not cause the destroying formed vortical beam. Moreover, produced optical vortex quality does not depend on singularity topological charge $m$.

The main negative effect lies in that fact that the greater the departure of the incident wavelength from the base meaning, the greater the central peak. It means that only diffraction efficiency becomes less, but vortical beam structure does not undergo disturbance.
In addition, we should notice, that wavelength mismatch results to shifting of diffraction orders in the focal plane. Obviously, in that case, a diffraction grating displays its dispersive behavior.

Table 2. Vortical beams forming with fork diffractive gratings in \( \pm 1 \) orders for \( m=2, 4 \). The incident Gaussian beam has different wavelengths (negative intensity distributions are shown).

| Wavelength \( \lambda \) | \( m=2 \) | \( m=4 \) |
|--------------------------|----------|----------|
| \( \lambda=533 \text{ nm} \) | ![Image](image1.png) | ![Image](image2.png) |
| \( \lambda_0=633 \text{ nm} \) | ![Image](image3.png) | ![Image](image4.png) |
| \( \lambda=733 \text{ nm} \) | ![Image](image5.png) | ![Image](image6.png) |

3. Experimental study

For experimental studying vortical beams forming we have provided two series of experiments. In the first series we investigate vortical beams producing with spiral phase plates. The spiral phase plate was obtained with the spatial light modulator SLM PLUTO-VIS. Incident laser beam has the wavelength of 532 nm. The display of the spatial beam modulator represents the phase distributions of spiral phase plates with different relief thickness. After illuminating the display by laser we take photographs of formed vortical beams intensity distributions. DOE phases is calculated with equation (6). In that way, we investigate DOE relief thickness influence to quality of formed vortical beams. In table 3 we show experimentally received intensity distributions of vortical beams with topological singularity charges of 1, 2 and 4. We vary the relief thickness of the DOE. We can see quite good agreement between experimentally received distributions and simulation results from table 1.

Table 3. Experimental forming of vortical beams with topological singularity charges of 1, 2 and 4, which are formed with the SLM representing spiral phase plates. Laser wavelength is 532 nm.

| Wavelength \( \lambda \) | \( m=1 \) | \( m=2 \) | \( m=4 \) |
|--------------------------|----------|----------|----------|
| \( \lambda=432 \text{ nm} \) | ![Image](image7.png) | ![Image](image8.png) | ![Image](image9.png) |
| \( \lambda_0=532 \text{ nm} \) | ![Image](image10.png) | ![Image](image11.png) | ![Image](image12.png) |
| \( \lambda=632 \text{ nm} \) | ![Image](image13.png) | ![Image](image14.png) | ![Image](image15.png) |

Meanwhile, we investigate forming of \( \pm 2 \) and \( \pm 4 \)-orders vortical beams by a multi-order DOE. The relief thickness is about 690 nm. According to the expression (1) initial wavelength should be 633 nm because of refractive index of silica glass is 1.46. For demonstration of DOE resistance to spectrum shifting we use laser EKSPLA NT200 with tunable wavelength. In table 4 we show experimentally received intensity distributions of vortical beams with topological singularity charges of \( \pm 2 \) and \( \pm 4 \), which are formed with silica glass multi-order DOE. We vary the wavelength of incident light from 533 nm to 733 nm.
Table 4. Experimental forming of vortical beams with topological singularity charges of 2 and 4 in the \(+1\) diffractive order. Initial laser wavelength is varying from 533 nm to 733 nm.

| Wavelength | \(m=2\) | \(m=4\) |
|------------|---------|---------|
| \(\lambda=533\) nm | ![Image](image1) | ![Image](image2) |
| \(\lambda_0=633\) nm | ![Image](image3) | ![Image](image4) |
| \(\lambda=733\) nm | ![Image](image5) | ![Image](image6) |

4. Conclusions
We provide the comparative analysis of vortical beams forming by spiral phase plates and binary fork diffraction grating. We show that spiral phase plate does not work whereas relief thickness does not correspond to the incident beam wavelength. The greater the vortex order, the worse the quality of formed beam. However, fork-type multi-order diffractive optical elements which produce optical vortices in \(\pm 1\)-orders are resistant to spectrum shifting. Incident beam wavelength deviations only cause energy losing and shifting of image in the focal plane. Experimental study with the SLM and the binary fork-type multi-order DOE verifies received numerical simulation results.

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