Leptogenesis and low energy observables in left–right symmetric models

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Abstract

In the context of left–right symmetric models we study the connection of leptogenesis and low energy parameters such as neutrinoless double beta decay and leptonic CP violation. Upon imposition of a unitarity constraint, the neutrino parameters are significantly restricted and the Majorana phases are determined within a narrow range, depending on the kind of solar solution. One of the Majorana phases gets determined to a good accuracy and thereby the second phase can be probed from the results of neutrinoless double beta decay experiments. We examine the contributions of the solar and atmospheric mass squared differences to the asymmetry and find that in general the solar scale dominates. In order to let the atmospheric scale dominate, some finetuning between one of the Majorana phases and the Dirac CP phase is required. In this case, one of the Majorana phases is determined by the amount of CP violation in oscillation experiments.

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1 Introduction

In the last few years, there has been mounting evidence for physics beyond the standard model coming from the leptonic sector of the model. In particular, the muon up–down asymmetry, as declared by the SuperKamiokande collaboration, has given compelling evidence for neutrino mass and mixing. Measurements of the solar neutrino fluxes by several experiments have also provided convincing experimental signatures for oscillations. Recently, the first results from the SNO experiment have substantiated the existence of neutrino oscillations among active flavors involving $\nu_e$ from the Sun. In parallel, an interesting problem in cosmology — which could have its solution from the particle physics sector — is the issue of resolving a tiny baryon asymmetry in the universe. To recapitulate, the explanation of this asymmetry requires satisfying the three Sakharov conditions, one of them being the presence of $CP$ violation. As it is well known, within the standard model, $CP$ violation is explained through a phase in the CKM matrix and turns out to produce a baryon asymmetry far too below the observed value and thus additional inputs are required. For instance, the leptogenesis mechanism can produce a baryon asymmetry through the out–of–equilibrium decay of heavy right–handed Majorana neutrinos in the early universe. Courtesy of the see–saw mechanism, these right–handed Majorana neutrinos also produce small masses for the light left–handed neutrinos as indicated by neutrino experiments. In addition, most viable neutrino models with large mixings produce Majorana mass terms which break the $B−L$ quantum number by two units and it is to be noted that if the conservation of this quantum number is assumed, the explanation of the baryon asymmetry is hard.

Therefore, the presence of heavy right–handed Majorana masses can be useful to explain both, the smallness of neutrino masses in oscillation experiments and the baryon asymmetry of the universe. This connection has been analyzed in several recent papers. It would be fair to say, that, from all of these observations, there seems to be a definite indication of new physics interplay between cosmology and particle physics, and leptogenesis could be one viability.

A relevant issue to this subject is to examine possible low energy signatures of leptogenesis. Among them are $CP$ violation in oscillation experiments and the value of the effective electron neutrino mass, $\langle m \rangle$, as measured in neutrinoless double beta decay ($0\nu\beta\beta$). In fact, given that in most models the heavy Majorana neutrinos are too heavy to be produced at realistic collider energies, observation of low energy $CP$ violation and lepton number violation might be the only possibility to validate leptogenesis. It might even be possible to distinguish different models through these additional observables. In this paper, we consider leptogenesis in left–right symmetric (LR) models. We are motivated by the simplicity of the model which offers us to relate the left– and right–handed sectors of the theory due to the symmetry. This choice reduces the ambiguities which arise

\footnote{See e.g. \cite{15} for the possibility of baryogenesis in left–right models.}
due to the unknown right–handed sector of the theory. In this model, one finds that for a specific choice of the Dirac mass matrix, the baryon asymmetry is proportional to the lightest mass eigenvalue. When we impose a unitarity constraint on this mass, we observe that rather stringent constraints on the low energy parameters follow, especially regarding the yet unknown phases in the leptonic mixing matrix. Subsequently, in a limiting two flavor case, we can relate one of the Majorana phases to the solar mass squared difference and also set useful lower bounds on the neutrino parameters.

Our paper is organized as follows. In the next section, we give the basic formalism of the see–saw mechanism in LR models. Section III deals with the leptogenesis mechanism and the results for the baryon asymmetry in the LR model. In Section IV, we apply a unitarity constraint to the light Majorana mass, which is used to constrain the low energy parameters. A lot of our analysis can be complemented with future solar neutrino experiments which will try to pin down the specific solar solution. We conclude in Section V by summarizing the main results.

2 Basic formalism in left–right symmetric theories

We begin by reviewing the known results for neutrino mass in LR models [12, 13]. The see–saw mechanism follows in models where the fermionic sector of the standard model is extended with massive right–handed singlet (under SU(2)\textsubscript{L} group action) neutrinos with mass of the order of 10^{10} GeV or heavier. The decoupling of such heavy mass states from the active left–handed sector can result in a small Majorana mass. In LR symmetric theories this decoupling results in a mass term of the form

\[ m_\nu = M_L - \tilde{M}_D M_R^{-1} \tilde{M}_D^T. \]

In (1), \( \tilde{M}_D \) and \( M_R \) denote the Dirac and the heavy right–handed Majorana neutrino mass matrices, respectively. This is to be contrasted with the conventional form where the left–handed mass matrix \( M_L \) is absent and hence does not contribute to the light neutrino mass. The presence of \( M_L \) is required in order to maintain the LR symmetry. The matrix in (1) can be diagonalized in the usual way with a unitary mixing matrix \( U_L \):

\[ U_L^T m_\nu U_L = \text{diag}(m_1, m_2, m_3), \]

where \( m_i \) are the light neutrino mass eigenvalues which determine the solar and atmospheric mass squared differences, \( \Delta m^2_\odot \) and \( \Delta m^2_A \), respectively. \( M_R \) can be diagonalized by a unitary mixing matrix \( U_R \) leading to

\[ U_R^T M_R U_R = \text{diag}(M_1, M_2, M_3). \]

The triplet induced Majorana mass matrices in (1) have the same coupling matrix \( f \) in the flavor basis. Therefore, we have a simple relation between the left– and right–handed
masses,
\[ M_L = f v_L \quad \text{and} \quad M_R = f v_R . \]  
(4)

In (4), \( v_{L,R} \) are the vevs of the left– and right–handed Higgs triplets, whose existence ensures the left–right symmetry. Generically, we can translate this vevs to an approximate equality \([19]\),
\[ v_L v_R \simeq \gamma v^2 , \]  
(5)

where \( v \simeq 174 \text{ GeV} \) is the weak scale and the constant \( \gamma \) is a model dependent parameter of \( \mathcal{O}(1) \). Using (4) and (5) in (1), the light neutrino mass matrix can be written as
\[ m_\nu = v_L \left( f - \frac{\tilde{M}_D}{\gamma v^2} \tilde{M}_D^T \right) . \]  
(6)

An interesting property of the mixing matrices \( U_L \) and \( U_R \), which arises due to the LR symmetry, has been found in \([13]\). If we assume that \( \tilde{M}_D \) is not identified with the up quark mass matrix, then the second term in (3) can be neglected and \( m_\nu \simeq M_L \). Under this circumstance, we have \( U_R \simeq U_L \), where the approximation is true up to \( \mathcal{O}(M_D^2/v^2) \) in \( m_\nu \).

Furthermore, the approximate equality of \( U_L \) and \( U_R \) leads to an interesting and simple connection between the light and heavy mass eigenvalues. From \( m_\nu \simeq M_L \), it follows due to (4) that \( m_\nu \simeq \frac{v_L}{v_R} M_R \). Therefore, one arrives at a very simple connection between the left– and right–handed sectors:
\[ m_i = M_i \frac{v_L}{v_R} . \]  
(7)

Note that in (4), the light neutrino masses are proportional to the heavy right–handed masses. In other words, the low energy spectrum is directly correlated to the spectrum at the see–saw scale. As we shall see in the next section, due to (2), the baryon asymmetry turns out to be proportional to the lightest mass eigenvalue, \( m_1 \).

In the following, we specify the strengths of \( v_{L,R} \) which determine the corresponding size of the light neutrino masses. From terrestrial neutrino experiments, the scale of the mass matrix is \( m_\nu = v_L f \simeq (10^{-2} \ldots 10^{-3}) \text{ eV} \), which for not too small \( f \) is only compatible with \( v_L v_R \simeq \gamma v^2 \) for \( v_R \simeq (10^{14} \ldots 10^{15}) \text{ GeV} \). This implies that \( v_R \) is probably close to the grand unification scale and \( v_L \) is of the order of the neutrino masses. This situation is expected since under our assumption for the Dirac mass, \( M_L \) is the dominating contribution to \( m_\nu \). We shall work with (7) and explore its consequences on leptogenesis and low energy observables.

3 Leptogenesis in left–right symmetric models

The observed baryon asymmetry, usually given as a ratio of the baryon to photon number density in the universe requires physics beyond the standard model. This asymmetry
can be generated by the leptogenesis mechanism through the mediation of sphalerons in the intermediate states \[20\]. Within the framework of the see–saw mechanism, the heavy right–handed fields can produce a lepton asymmetry in an out–of–equilibrium decay. A lepton asymmetry is caused by the interference of tree level with one–loop corrections to the decays of the lightest Majorana states, \(N_1 \to \Phi l^c\) and \(N_1 \to \Phi^\dagger l\). The resulting decay asymmetry reads

\[
\varepsilon = \frac{\Gamma(N_1 \to \Phi l^c) - \Gamma(N_1 \to \Phi^\dagger l)}{\Gamma(N_1 \to \Phi l^c) + \Gamma(N_1 \to \Phi^\dagger l)} = \frac{1}{8 \pi v^2} \left(\frac{1}{M_D M_D^\dagger} \sum_{j=2,3} \text{Im}(M_D^\dagger M_D)_{1j}^2 f(M_2^j/M_1^2)\right). \tag{8}
\]

Here, \(\varepsilon\) is now a function of \(M_D = \tilde{M}_D U_R\) and the function \(f\) represents the terms arising from vertex and self–energy contributions and is given to be

\[
f(x) = \sqrt{x} \left(1 + \frac{1}{1-x} - (1+x) \ln\left(1 + \frac{x}{1-x}\right)\right) \approx -\frac{3}{2\sqrt{x}}. \tag{9}
\]

The approximation in (9) holds for \(x \gg 1\) with \(x \equiv M_2^j/M_1^2\). It is worth mentioning that the hierarchical assumption \((x \gg 1)\) is also favored in order to produce large lepton mixings within the see–saw mechanism \[21\]. As a result of (7), when the see–saw spectrum is hierarchical, so is the low energy spectrum. The decay asymmetry \(\varepsilon\) is related to the baryon asymmetry \(Y_B\) through the relation

\[
Y_B = c \kappa \frac{\varepsilon}{g^*}. \tag{10}
\]

In (10), \(c \simeq -0.55\) is the fraction of the lepton asymmetry converted into a baryon asymmetry via sphaleron processes \[20\], \(\kappa\) is a suppression factor due to lepton number violating wash–out processes and \(g^* \simeq 110\) is the number of massless degrees of freedom at the time of the decay. In supersymmetric models, \(g^*\) and \(\varepsilon\) are roughly twice as large, therefore the results are rather unaffected by the presence of supersymmetry. We shall work with the non–supersymmetric version of the theory. Phenomenologically, the preferred range for the baryon asymmetry is \(Y_B \simeq (0.6 \ldots 1) \cdot 10^{-10}\). 

In order to estimate the baryon asymmetry, we can insert in the neutrino mass matrix (4) any of the solar solutions, i.e. the small angle (SMA), large angle (LMA) or quasi–vacuum (QVO) solution. Following this, the baryon asymmetry is obtained using (5) and (10). As a passing remark, we wish to mention that within the context of the left–right models, this procedure can also be useful to extract the possible structure of the high scale theory based on the available phenomenological information at the low scale. This also relaxes the need to make, sometimes unavoidable, assumptions on the various neutrino parameters in order to satisfy the observed baryon asymmetry \[11\]. In addition, as we shall see, the contributions due to the solar and atmospheric sectors to the asymmetry can be analyzed individually.
By performing a numerical analysis of the allowed oscillation parameters \[22\] and the three unknown phases in \(U_L\), it is found that if \(M_D\) is a down quark or lepton mass matrix, \(m_1\) should not be too small \[13\]. Furthermore, the LMA solution gives a better fit to the baryon asymmetry and is thus slightly favored over SMA and QVO. It is interesting to note that current neutrino data also prefers the LMA solution \[16\] over the other possible solutions. If \(M_D\) is an up quark mass matrix, some fine tuning of the parameters is required \[14\].

Let us parameterize the mixing matrix \(U_L\) to be of the form

\[
U_L = U_{\text{CKM}} \cdot P = U_{\text{CKM}} \cdot \text{diag}(1, e^{i\alpha_1}, e^{i(\beta+\delta)})
\]

where \(c_i = \cos \theta_i, s_i = \sin \theta_i\) and the two Majorana phases are factored out in a matrix \(P\). Within this parameterization, \(CP\) violation in neutrino oscillations is governed by the Dirac phase \(\delta\) and the \(ee\) element of \(m_\nu\) \((\langle m \rangle)\), as measurable in neutrinoless double beta decay, depends on the Majorana phases \(\alpha\) and \(\beta\). The presence and relevance of these additional phases was introduced in \[17\]. If \(s_3 = 0\), then there is no \(CP\) violation in oscillations and \(\langle m \rangle\) is only a function of \(\alpha\). In oscillation experiments, any \(CP\) violation will depend on the Jarlskog invariant \[18\]

\[
J_{CP} = \frac{1}{8} \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \cos \theta_3 \sin \delta \lesssim \frac{1}{4} s_3 c_3^2 \sin \delta . \tag{12}
\]

For approximately bimaximal mixing, with \(c_1^2 = c_2^2 = 1/2\) and keeping the leading order in \(s_3\), the baryon asymmetry is given to be \[13\]

\[
Y_B \cdot 10^{10} \simeq \frac{4m_1}{1 - 2s_3c_3} \left( \frac{m}{\text{GeV}} \right)^2 \left\{ \frac{s_{2\alpha} + 4 s_3 s_4 c_{2\alpha}}{\sqrt{\Delta m^2_{\odot}}} + \frac{2s_{2(\beta+\delta)} - 4 s_3 s_2s_{\beta+\delta}}{\sqrt{\Delta m^2_{A}}} \right\} . \tag{13}
\]

Here \(c_\delta = \cos \delta, s_{2\alpha} = \sin 2\alpha\) and so on. The largest entry in \(M_D\) is denoted by \(m\). A few remarks are in order from \[13\]. This form holds for both, the LMA and the QVO solution and clearly separates out the contributions due to the solar and atmospheric sectors. Also, the Majorana phases \(\alpha\) and \(\beta\) do not mix and are related to the solar and atmospheric sector, respectively. This feature will help us to individually analyze the phases depending on the scales, \(\Delta m^2_{\odot}\) and \(\Delta m^2_{A}\). It is explicitly seen that the baryon asymmetry vanishes if \(CP\) conservation holds, which is the case when all the phases are zero or \(\pi\). The asymmetry is proportional to the square of the heaviest entry in \(M_D\), which in our case is either the tau or bottom quark mass. Due to the mass relation in \[1\], \(Y_B\) is proportional to the lightest neutrino mass eigenstate \(m_1\) and has a lower limit of \(O(10^{-7} \ldots 10^{-8})\) eV \[13\]. Choosing the Dirac mass matrix to be the up quark mass matrix will erase this simple proportionality of the baryon asymmetry. In the following section, we use this proportionality,
\( Y_B \propto m_1 \) to impose a restriction following from unitarity. We then discuss the implications of this for leptonic \( CP \) violation and \( 0\nu\beta\beta \). To derive (13), we assumed that the wash–out factor \( \kappa \) is approximately 0.1.

Note that in (13), the baryon asymmetry is predominantly governed by the solar scale. However, if there are any accidental cancellations in the first term in (13), then \( Y_B \) will depend on the atmospheric scale. Alternatively, both the solar and atmospheric sectors can contribute to the asymmetry when the two terms in (13) are of the same order, which requires

\[
\frac{2\sqrt{\Delta m^2_A}}{s_{2\alpha} + 4s_3s_2\delta c_{2\alpha}} \gg 1 .
\]

This is possible when \( s_{2\alpha} + 4s_3s_2\delta c_{2\alpha} \approx 0 \) or equivalently, \( t_{2\alpha} \approx -4s_3s_\delta \), where \( t_{2\alpha} = \tan 2\alpha \). Given the strong constraints from reactor based experiments like CHOOZ [23] and Palo Verde [24] we have \( 0 \leq s_3 \lesssim 0.28 \). Hence the atmospheric scale can contribute only if \( \alpha \) is in the range such that, \(-1.12s_\delta \leq t_{2\alpha} \leq 0 \). Clearly, this relation is not valid for values of \( \alpha \approx (2n + 1)\pi/4 \). For example, with \( \alpha \approx n\pi/2 \), we require \( \delta \approx n\pi \) in order to let the second term in (13) dominate, which implies \( J_{CP} \approx 0 \). This would be identical to a two flavor scenario where we can set \( \delta = 0 \). An interesting outcome is that the value of \( \alpha \) is determined by the amount of \( CP \) violation in oscillation experiments, \( J_{CP} \). Therefore, in terms of \( J_{CP} \) from (12), the atmospheric scale contributes to (13) only if \( \alpha \) satisfies the relation

\[
t_{2\alpha} \approx \frac{16J_{CP}}{(s_3^2 - 1)} \approx -16J_{CP} .
\]

The approximation in (13) is assuming that \( s_3^2 \ll 1 \) and this allows us to directly probe \( \alpha \), up to \( O(s_3^2) \), by measuring the amount of \( CP \) violation in oscillation experiments. In the next section, we independently analyze the contributions of both the solar and atmospheric sector for (13). We find that given the large mass scale involved for the atmospheric sector, it is hard to make a direct estimate of its contribution to the baryon asymmetry.

### 4 Unitarity bound, neutrinoless double beta decay and \( CP \) violation

In the presence of any new physics originating at some scale \( M_X \) above the electroweak scale, one should consider the standard model as an effective theory. An upper limit for \( M_X \) can be determined by examining the high energy behavior of the lepton number violating reactions like \( \nu \nu \rightarrow WW \) or \( ZZ \), which can occur because of a Majorana mass term. It was noted that a stringent bound for \( M_X \) is obtained by considering the following linear combination of the zeroth partial wave amplitudes [25]:

\[
a_0 \left( \frac{1}{2}(\nu_+\nu_- - \nu_-\nu_+) \rightarrow \frac{1}{\sqrt{3}}(W^+W^- + Z^0Z^0) \right),
\]
where $\nu_{\pm}$ are helicity components of the neutrino mass eigenstate and the final state bosons are longitudinally polarized. This amplitude to obey unitarity requires $|a_0| \leq 1/2$. In terms of the mass eigenvalue $m_1$, this can be translated to

$$m_1 = \frac{4\pi v^2}{\sqrt{3}M_X}.$$  \hfill (16)

In the left–right symmetric model, the Higgs triplet $\Delta_L = (\Delta^0, \Delta^+, \Delta^{++})$, which generates the light left–handed neutrino masses, also induces lepton number violating processes like $e^+ e^+ \rightarrow W^+ W^+$. The $t$ and the $u$ channels of this process exhibit a unitarity violating high energy behavior, since the amplitude grows with energy. It has been explicitly shown that the presence of the doubly charged Higgs boson ($\Delta^{++}$), which mediates the same process via exchange in the $s$ channel, restores unitarity in the high energy limit \[26\]. Furthermore, it follows from (5), that for a light $m_1$, the scale $M_X$ is dependent on $\gamma$ and for $\gamma \simeq 1$ we require $M_X \sim v_R$. However, in the following, for our analysis, it is sufficient if we maintain the requirement that $M_X$ be lower than the Planck scale.

Using (13), we have an equality relating the scale $M_X$ to $Y_B$:

$$M_X \simeq \frac{16\pi v^2}{\sqrt{3}Y_B \cdot 10^{10}(1 - 2s_3 \delta)} \left( \frac{m}{\text{GeV}} \right)^2 \left\{ \frac{s_{2\alpha} + 4 s_3 s_\delta c_{2\alpha}}{\sqrt{\Delta m^2_\odot}} + \frac{2 s_{2(\beta+\delta)} - 4 s_3 s_{2\beta+\delta}}{\sqrt{\Delta m^2_\odot}} \right\}. \hfill (17)$$

As observed in the previous section, there are contributions due to the solar and atmospheric sector. We analyze them separately.

### 4.1 Effects due to the solar scale

In the following, we neglect the contribution due to the atmospheric scale, and examine (17) for the LMA and QVO solar solutions. In this case, the Majorana phase $\beta$ is a free parameter in the theory. Depending on the solar solution, constraints on $\alpha$ and $\delta$ are obtained, which could reflect in low energy observables such as $\langle m \rangle$ and in the $CP$ violating parameter, $J_{CP}$. In order to satisfy the baryogenesis requirement, we set $Y_B = 10^{-10}$ and (17) can be rewritten as

$$M_X \simeq \left( \frac{m}{\text{GeV}} \right)^2 \frac{16\pi v^2}{\sqrt{3\Delta m^2_\odot}} A, \text{ where } A = \frac{s_{2\alpha} + 4 s_3 s_\delta c_{2\alpha}}{1 - 2s_3 \delta c_{2\alpha}}. \hfill (18)$$

The lower limit of $s_3 = 0$ is identical to a two flavor system, where one can set the Dirac $CP$ phase to zero. The fact that in a two flavor limit there is still a $CP$ violating phase ($\alpha$) reflects the Majorana nature of the neutrinos involved. Choosing $M_D$ to be the charged lepton mass matrix, therefore $m = m_\tau = 1.77$ GeV, we have

$$M_X \simeq 2.8 \cdot 10^{15} \frac{A}{\sqrt{\Delta m^2_\odot}} \text{ GeV} \leq M_{Pl} = 1.2 \cdot 10^{19} \text{ GeV}, \hfill (19)$$
where $\Delta m^2_\odot$ is given in eV$^2$ and $M_{Pl}$ denotes the Planck scale. This sets an upper bound on $A$ for a given $\Delta m^2_\odot$. Furthermore, it could restrict the values in $\langle m \rangle$ and $J_{CP}$. This approach can be useful to probe the possible value for $M_X$ based on our chosen low energy observables. Future terrestrial solar experiments like BOREXINO $[27]$, which will identify the preferred $\Delta m^2_\odot$, can also correlate to the scale in our scheme. However, we can still make an estimate of the size of $\Delta m^2_\odot$, depending on the parameters. To see this, we rewrite the result in (19) as

$$ A \leq 4.3 \cdot 10^3 \sqrt{\Delta m^2_\odot} / \text{eV} \, . $$

(20)

Thus, depending on the values for $\alpha$, $\delta$ and for a given $s_3$, a lower bound on $\sqrt{\Delta m^2_\odot}$ is possible. For example, the maximum value that $A$ can take is for the case when $\delta = 0$ and $s_3 = 0.28$, which is its maximally allowed value. For this choice, we have $\max(A) \simeq 2.3$. Correspondingly, this sets a lower bound of

$$ \Delta m^2_\odot \geq 2.7 \cdot 10^{-7} \text{eV}^2 \text{ for } A = 2.3 \, . $$

(21)

Note that from (21), in order to incorporate the QVO solar solution, we need to restrict the value of $A$ much below its upper limit. As we shall see in the following, the allowed region of parameter space for the LMA solution clearly covers $A \geq 2.3$ and for the QVO solution the parameter space is restricted with $A \ll 2.3$.

4.1.1 LMA solution

In this case, we choose $\Delta m^2_\odot \simeq 5 \cdot 10^{-5} \text{eV}^2$ and for this value we have $M_X \simeq 3.9 \cdot 10^{17} \text{A GeV}$. Following (19) we can have a closed bound

$$ 0 \leq \frac{s_{2\alpha} + 4s_3s_\delta c_{2\alpha}}{1 - 2s_3c_\delta} \lesssim 30.6 \, . $$

(22)

Clearly, there are no restrictions in the various angles, $\alpha$ and $\delta$ in order to satisfy the bound in (22). Note that for the two flavor limit, the bound in (21) gives a consistent upper bound for one of the Majorana phases,

$$ s_{2\alpha} \leq 4.3 \cdot 10^3 \cdot \sqrt{\Delta m^2_\odot} / \text{eV} \text{ or } \Delta m^2_\odot \geq 5.4 \cdot 10^{-8} \text{eV}^2 \text{ for } \alpha = \pi/4 \, . $$

(23)

This bound can however be revised if we lower the scale where unitarity may break down. For LMA, one requires $m_1 \lesssim 10^{-3} \text{eV}$ in order to have a hierarchical scheme, which corresponds to $M_X \gtrsim 2.2 \cdot 10^{17} \text{GeV}$. Then, (22) is modified to the range

$$ 0.6 \leq \frac{s_{2\alpha} + 4s_3s_\delta c_{2\alpha}}{1 - 2s_3c_\delta} \lesssim 30.6 \, , $$

(24)

which can be used to set bounds on the phases. On the other hand, for the hierarchical scheme to hold in the QVO solution, one needs values of $M_X$ close to the Planck scale. In Fig. 4, we show the area in $\alpha-\delta$ space which is allowed for $\Delta m^2_\odot \simeq 5 \cdot 10^{-5} \text{eV}^2$ and
\( s_3^2 = 0.08 \) and 0.001, respectively. As expected, the allowed region for the Majorana phase \( \alpha \) is strongly constrained while the \( CP \) violating phase \( \delta \) remains unbounded. This is an indication of possible effects in \( \langle m \rangle \) while \( J_{CP} \) could still not be sensitive to our constraints.

From Fig. 1 one observes that the phase \( \alpha \) is basically around \( \pi/4 \) or \( 5\pi/4 \), which incidentally, from (15), are the values disallowed when the atmospheric scale contributes to the asymmetry. If \( \alpha \) is fixed, then \( \langle m \rangle \) is a function of the second phase \( \beta \). As known, the LMA solution provides the highest value for \( \langle m \rangle \) in the hierarchical scheme. For \( \alpha = \pi/4 \) we show in Fig. 2 the expected \( \langle m \rangle \) for different \( \Delta m^2_\odot \) and \( s_3^2 \). For \( \alpha = 5\pi/4 \) the situation is basically the same. A measurement of \( \langle m \rangle \) could probe the second phase \( \beta \), which drops out of the baryon asymmetry in our scenario. The limiting values are \( 0.001 \text{ eV} < \langle m \rangle < 0.01 \text{ eV} \), depending on the values for \( \Delta m^2_\odot \) and \( s_3 \). Large part of this range is well within the sensitivity of the GENIUS experiment \([28]\). If \( s_3^2 \) is too small, the dependence on \( \beta \) vanishes, as does the presence of \( CP \) violation in oscillation experiments.

We remark that one could in principle obtain all phases by measuring the other entries of the light neutrino mass matrix, e.g. the element \( m_{\mu\nu} \), which triggers the decay \( K^+ \rightarrow \pi^- \mu^+ \mu^+ \). However, this and other analogue processes have far too low branching ratios to be observed \([29]\).

### 4.1.2 QVO solution

In this case, requiring \( M_X \leq M_{Pl} \) and for \( \Delta m^2_\odot \approx 5 \cdot 10^{-10} \text{ eV}^2 \), this corresponds to \( M_X \approx 1.2 \cdot 10^{20} \text{ A GeV} \). Similar to the bound in (22), we now have

\[
0 \leq \frac{s_{2\alpha} + 4s_3 s_\delta c_{2\alpha}}{1 - 2s_3 c_3} \lesssim 0.1 .
\]

This is a stronger limit than the one obtained for the LMA solution. It also imposes restrictions on the values for \( \alpha, \delta \) and \( s_3 \), and requires the value of \( A \) to be lower than the upper limit suggested in (21). In Figs. 3 and 4, we plot the allowed areas in \( \alpha-\delta \) space for different values for \( s_3 \) and \( \Delta m^2_\odot \). As can be seen from these figures, in contrast to the LMA solution, the phase \( \alpha \) is basically around \( \pi/2 \) or \( \pi \). For these two choices of \( \alpha \) we show in Figs. 5 and 6 the value of \( \langle m \rangle \) for different \( \Delta m^2_\odot \) and \( s_3^2 \). The limiting values are now \( 10^{-5} \text{ eV} \leq \langle m \rangle \leq 0.01 \text{ eV} \), depending on the values for \( \Delta m^2_\odot \) and \( s_3 \). Some part of this range is well within the sensitivity of the GENIUS experiment. As known, for the QVO solution large \( s_3^2 \) is required in order to give accessible \( \langle m \rangle \). We note that the cases \( \alpha = \pi \) or \( \pi/2 \) together with \( \beta = \pi \) or \( \pi/2 \) are situations in which one can not distinguish \( CP \) violation from \( CP \) conservation in \( 0\nu\beta\beta \) \([30]\).

A common feature for both solar solutions is that the case \( \delta = 0 \) is allowed, which can be seen from Figs. 1, 3 and 4. This means that vanishing \( CP \) violation in oscillation experiments does not mean that leptogenesis is disfavored. Also, since the atmospheric scale decouples in our framework, the presence of only one non–vanishing Majorana phase
is sufficient to generate the observed baryon asymmetry. These features have also been observed in the model presented in [31].

4.2 Effects due to the atmospheric scale

As mentioned earlier, for the contributions from the atmospheric scale to be significant, we require to satisfy (15). This leads to two possibilities: case (i) with \( CP \) violation or \( \text{Im}(U_{e3}) \neq 0 \) and case (ii) with no \( CP \) violation or \( \text{Im}(U_{e3}) = 0 \). Note that case (ii) is also identical to the two flavor scenario where we can set \( \delta = 0 \). However, as already hinted, due to the largeness of the atmospheric scale, regardless of the unitarity bound, we do not expect to have strong constraints on the Majorana phases, unlike the situation for the solar sector.

We first analyze the possibility where \( \text{Im}(U_{e3}) \neq 0 \). For this analysis, we take the largest value of \( s_3 = 0.28 \), for which case we can express the scale \( M_X \) as

\[
M_X \simeq \frac{16\pi v^2}{\sqrt{3} \Delta m^2_\Lambda} Y_B \cdot 10^{10} \left( \frac{m}{\text{GeV}} \right)^2 \tilde{A} \leq M_{Pl}, \quad \text{where} \quad \tilde{A} = \frac{2s_2(\beta + \delta) - 1.12 s_{2\beta + \delta}}{1 - 0.56c_{\delta}}. \tag{26}
\]

As in the previous cases, setting \( Y_B = 10^{-10} \) and for \( m = m_\tau = 1.77 \text{ GeV} \), we have the lower bound

\[
\sqrt{\Delta m^2_\Lambda} \geq 2.29 \cdot 10^{-4} \tilde{A} \text{ eV}, \tag{27}
\]

which is easily satisfied for any value of the angles, \( \beta \) and \( \delta \). Furthermore, setting \( \Delta m^2_\Lambda \simeq 3 \cdot 10^{-3} \text{ eV}^2 \), we have the closed bound

\[
0 \leq \frac{2s_2(\beta + \delta) - 1.12 s_{2\beta + \delta}}{1 - 0.56c_{\delta}} \leq 187.6 \tag{28}
\]

and there are no restrictions on the angles from (28). The reason for this uninteresting situation is that the atmospheric scale is too large to set any useful limits on \( \tilde{A} \). As a result, the bounds have no impact on either \( \langle m \rangle \) or \( J_{CP} \). In case (ii), where we have no \( CP \) violation, we set \( \delta = 0 \) in all the results obtained above for the \( CP \) violating scenario. The only Majorana parameter \( \beta \), as expected, satisfies all the bounds derived above and remains unconstrained. As a further check, we briefly address the question: could there be any restrictions on \( \beta \) if we relate the estimates from \( 0\nu\beta\beta \) and \( J_{CP} \)? To see this, in the hierarchical scheme of neutrino masses, we have

\[
\langle m \rangle \simeq \left[ \Delta m^2_\odot s_1^4 + \Delta m^2_\Lambda s_3^4 + 2s_1^2s_3^2\sqrt{\Delta m^2_\odot \Delta m^2_\Lambda} \cos \phi \right]^{1/2}, \tag{29}
\]

where \( \phi = 2(\alpha - \beta) \). As a numerical illustration, we choose \( \Delta m^2_\odot = 5 \cdot 10^{-5} \text{ eV}^2 \), \( \Delta m^2_\Lambda = 5 \cdot 10^{-3} \text{ eV}^2 \), \( s_3 = 0.28 \) and \( s_1 = \pi/4 \), for which case

\[
\langle m \rangle \simeq 6.3 \cdot \sqrt{1 + c_{\phi}} \cdot 10^{-3} \text{ eV}. \tag{30}
\]
Using (15), we can rewrite (30) as

\[ \langle m \rangle \simeq 6.3 \left( 1 + \cos \left[ \tan^{-1} 16J_{CP} + 2\beta \right] \right)^{1/2} \cdot 10^{-3} \text{ eV}. \]  

(31)

Taking the current allowed range, \( 0 \leq \langle m \rangle \leq 0.35 \text{ eV} \), this translates to a closed bound

\[ 0 \lesssim \cos \left[ \frac{1}{2} \tan^{-1} 16J_{CP} + \beta \right] \lesssim 39.2, \]  

(32)

which, as expected, is easily satisfied for all \( \beta \) and \( J_{CP} \). We see from (31) that almost all of the allowed regions are well within the reach of GENIUS except for \( \phi = \pi \) for which case we arrive at \( \tan 2\beta \simeq -16J_{CP} \). Therefore, for a null 0νββ result, the Majorana phase \( \beta \) is determined by the amount of \( CP \) violation in oscillation experiments. Again, this conclusion requires some finetuning, now in \( \langle m \rangle \), and is equivalent to the requirement in (15).

5 Summary

In left–right symmetric theories one can find a simple formula for the baryon asymmetry, expressing it in terms of the low energy neutrino parameters. In our analysis, we have made a specific choice for the Dirac mass matrix to be the charged lepton mass matrix. This choice results in only the triplet term contributing to the neutrino mass, while, for all practical purpose, the conventional see–saw term gives a negligible contribution. This results in a simple expression for \( Y_B \) which is proportional to the lightest Majorana mass \( m_1 \). On the other hand, if we choose \( M_D \) to be the up–quark mass matrix, such a proportionality is not possible. We find that within this model the imposition of an additional constraint on \( m_1 \) coming from unitarity restricts the allowed parameter space for the \( CP \) violating phases. This could be a distinguishing feature of the choice of the Dirac mass matrix with observable low energy consequences. The ensuing bound helps in narrowing down one of the Majorana phases, thereby reducing the theoretical uncertainty in the prediction for \( \langle m \rangle \). The value of the phase constrained by our approach is different for the LMA and QVO solution. Upon measuring the effective mass, one could obtain the second Majorana phase. In each of the cases, the corresponding limit of a two flavor system is obtained by setting \( s_3 = 0 \). In general, for both cases, most of the allowed parameter space predicts \( \langle m \rangle \) in the measurable range of GENIUS with its sensitivity of \( \langle m \rangle \geq 10^{-3} \) eV. The presence of one single Majorana phase is sufficient to generate the correct baryon asymmetry, especially the case \( \delta = 0 \) is allowed, which corresponds to no \( CP \) violation in oscillation experiments. We examined individually the contributions to the asymmetry due to the solar and atmospheric sectors and found that in general the solar mass scale dominates \( Y_B \). Under a special situation, when \( \langle m \rangle \simeq 0 \), one could relate the phase \( \beta \) to \( CP \) violation in oscillations experiments. This might perhaps indicate the possible atmospheric contribution. However, if there are no positive indications for \( \langle m \rangle \) from GENIUS, we still need to rule out the QVO
solution in order to strengthen our claim for the atmospheric contribution. The reason is that the QVO solution predicts a small though nonzero $\langle m \rangle \leq 10^{-4}$ eV (see Figs. 5 and 6). It is in this context that future experiments like BOREXINO, which can pin down the correct solar solution, will help in better understanding the various $CP$ phases within this scenario. Furthermore, upon correlation with long baseline experiments searching for a nonzero $U_{e3}$ and $CP$ violation, together with a simultaneous measurement of $0\nu\beta\beta$, it might be possible to make a reasonable guess on the Majorana phase contributions to the baryon asymmetry.

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References

[1] Super–Kamiokande Collaboration, Y. Fukuda, Phys. Rev. Lett. 81, 1562 (1998); Phys. Lett. B467, 185 (1999).

[2] GALLEX Collaboration, W. Hampel et al., Phys. Lett. B447, 127 (1999); SAGE Collaboration, J. N. Abdurashitov et al. Phys. Rev. C60, 055801 (1999); Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 77, 1683 (1996); Homestake Collaboration, B.T. Cleveland et al., Astrophys. J. 496 505 (1998); GNO Collaboration, M. Altmann et al., Phys. Lett. B490 16 (2000).

[3] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 87, 071301 (2001).

[4] G. Steigman, Ann. Rev. Astron. Astrophys. 14, 339 (1976).

[5] A.D. Sakharov, JETP Lett. 5, 24 (1967).

[6] P. Huet, hep-ph/9406301 and references therein.

[7] K.A. Olive, G. Steigman, and T.P. Walker, Phys. Rept. 333, 389 (2000).

[8] For recent reviews see, e.g. A. Pilaftis, Int. J. Mod. Phys. A14, 1811 (1999); W. Buchmüller, M. Plüümacher, hep-ph/0007176.

[9] M. Gell–Mann, P. Ramond, and R. Slansky, in Supergravity, P. van Nieuwenhuizen & D. Z. Freedman (eds.), North Holland Publ. Co., 1979 p 315; T. Yanagida, Proc. of the Workshop on Unified Theories and the Baryon Number of the Universe, edited by O. Sawada and A. Sugamoto, KEK, Japan 1979; R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[10] S.M. Barr, I. Dorsner, Nucl. Phys. B585, 79 (2000); Z. Berezhiani, A. Rossi, Nucl. Phys. B594, 113 (2001); G. Altarelli, F. Feruglio, Phys. Rept. 320, 295 (1999); H. Fritzsch, Z.Z. Xing, Prog. Part. Nucl. Phys. 45, 1 (2000).

[11] J. Ellis, S. Lola, and D.V. Nanopoulos, Phys. Lett. B452, 87 (1999); M.S. Berger, B. Brahmachari, Phys. Rev. D60, 073009 (1999); K. Kang, S.K. Kang, and U. Sarkar, Phys. Lett. B486, 391 (2000); H. Goldberg, Phys. Lett. B474, 389 (2000); E. Nezri, J. Orloff, hep-ph/0004227; D. Falcone, F. Tramontano, Phys. Rev. D63, 073007 (2001); H.B. Nielsen, Y. Takanishi, Phys. Lett. B507, 241 (2001); R. Jeannerot, S. Khalil, and G. Lazarides, Phys. Lett. B506, 344 (2001); M. Hirsch, S.F. King, Phys. Rev. D64, 113005 (2001); F. Bucella, D. Falcone, and F. Tramontano, hep-ph/0108172; W. Buchmüller, D. Wyler, Phys. Lett. B521, 291 (2001); M.S. Berger, K. Siyeon, hep-ph/0110001; D. Falcone, hep-ph/0111176.

[12] A.S. Joshipura, E.A. Paschos, hep-ph/9906498; A.S. Joshipura, E.A. Paschos, and W. Rodejohann, Nucl. Phys. B611, 227 (2001).

[13] A.S. Joshipura, E.A. Paschos and W. Rodejohann, JHEP 08, 029 (2001).

[14] W. Rodejohann, Acta Phys. Pol. B32, 3845 (2001).

[15] R. N. Mohapatra, X. Zhang, Phys. Rev. D46, 5331 (1992).

[16] V.D. Barger, D. Marfatia, and K. Whisnant, Phys. Rev. Lett. 88, 011302 (2002); J.N. Bahcall, P.I. Krastev, A.Yu. Smirnov, JHEP 0105, 015 (2001); P.I. Krastev, A.Yu. Smirnov, hep-ph/0108177; J.N. Bahcall, M.C. Gonzalez–Garcia, and C. Peña–Garay, JHEP 0108, 014 (2001); M.C. Gonzalez-Garcia, M. Maltoni, and C. Peña–Garay, Phys. Rev. D64, 093001 (2001) and references therein.

[17] J. Schechter, J.W.F. Valle, Phys. Rev. D22, 2227 (1980); D23, 1666 (1981).

[18] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985).

[19] J.C. Pati, A. Salam, Phys. Rev. D10, 275 (1974); R.N. Mohapatra, J.C. Pati, Phys. Rev. D11, 566, 2558 (1975); G. Senjanovic, R.N. Mohapatra, Phys. Rev. D12, 1502 (1975).

[20] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, Phys. Lett. B155, 36 (1985).

[21] A.Yu. Smirnov, Phys. Rev. D48, 3264 (1993).

[22] M.C. Gonzalez–Garcia et al., Phys. Rev. D63, 033005 (2001).

[23] The CHOOZ collaboration, M. Apollonio et al., Phys. Lett. B466, 415 (1999).

[24] F. Boehm et al., Phys. Rev. Lett. 84, 3764 (2000).

[25] F. Maltoni, J.M. Niczyporuk, and S. Willenbrock, Phys. Rev. Lett. 86, 212 (2000).
[26] T.G. Rizzo, Phys. Lett. B116, 23 (1982); J. Gluza, M. Zralek, Phys. Rev. D52, 6238 (1995).

[27] G. Ranucci et al., for Borexino Collaboration, Nucl. Phys. Proc. Suppl. 91, 58 (2001).

[28] GENIUS Collaboration, H.V. Klapdor–Kleingrothaus et al., hep-ph/9910205.

[29] W. Rodejohann, Phys. Rev. D62, 013011 (2000).

[30] W. Rodejohann, Nucl. Phys. B597, 110 (2001).

[31] G.C. Branco et al., hep-ph/0202030.
Figure 1: Allowed area in $\alpha$–$\delta$ space in the LMA solution for $M_X = 2.2 \cdot 10^{17}$ GeV, $\Delta m^2_\odot = 5 \cdot 10^{-5}$ eV$^2$, $s^2_3 = 0.08$ (dark shaded) and $s^2_3 = 0.001$ (light shaded).

Figure 2: $\langle m \rangle$ as a function of $\beta$ for $\alpha = \pi/4$, different $\Delta m^2_\odot$ and $s^2_3$. 
Figure 3: Allowed area in $\alpha-\delta$ space in the QVO solution for $M_X = M_{Pl}$, $\Delta m^2_{\odot} = 10^{-8}$ eV$^2$, $s_3^2 = 0.08$ (dark shaded) and $s_3^2 = 0.001$ (light shaded).

Figure 4: Allowed area in $\alpha-\delta$ space in the QVO solution for $M_X = M_{Pl}$, $\Delta m^2_{\odot} = 10^{-10}$ eV$^2$, $s_3^2 = 0.08$ (dark shaded) and $s_3^2 = 0.001$ (light shaded).
Figure 5: $\langle m \rangle$ in the QVO solution as a function of $\beta$ for $\alpha = \pi/2$, different $\Delta m^2_{\odot}$ and $s^2_3$.

Figure 6: $\langle m \rangle$ in the QVO solution as a function of $\beta$ for $\alpha = \pi$, different $\Delta m^2_{\odot}$ and $s^2_3$. 