Equatorial geodesics in ergoregion of dirty black holes and zero energy observers

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We consider equatorial motion of particles in the ergoregion of generic axially symmetric rotating black holes. We introduce the notion of zero energy observers (ZEOs) as counterparts to known zero angular observers (ZAMOs). It is shown that the trajectory of a ZEO has precisely one turning point that lies on the boundary of the ergoregion for photons and inside the ergoregion for massive particles. As a consequence, such trajectories enter the ergosphere from the white hole region under horizon and leave it crossing the horizon again (entering the black hole region). The angular velocity of ZEO does not depend on the angular momentum. For particles with \( E > 0 \) this velocity is bigger than for a ZEO, for \( E < 0 \) it is smaller. General limitations on the angular momentum are found depending on whether the trajectory lies entirely inside the ergoregion, bounces back from the boundary or intersects it. These results generalize the recent observations made in A. A. Grib, Yu. V. Pavlov, arXiv:1601.02592 for the Kerr metric. We also show that collision between a ZEO and a particle with \( E \neq 0 \) near a black hole can lead to the unbound energy in the centre of mass thus giving a special version of the Bañados - Silk - West effect.

PACS numbers: 04.70.Bw, 97.60.Lf

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I. INTRODUCTION

Motion of particle in the ergoregion possesses some peculiarities that are absent in the outer region. Especially interesting is that the negative and zero values of the Killing energy $E$ become possible. Some general properties of trajectories with $E < 0$ were considered in [1], [2] for the Kerr metric and in [3] in much more general background. Recently, the results of [1] for the Kerr metric were extended [4] to include the case $E = 0$. In the present paper, we generalize the analysis of [4] to generic axially symmetric rotating black holes. Real astrophysical black holes are surrounded by matter (they are ”dirty” in this sense) or electromagnetic fields, so their metric can deviate from the Kerr form. Even if in practical astrophysics environmental effects are small [5], the notion of dirty black holes represents conceptual issue important for thermodynamics including the problem of black hole entropy, properties of the event horizons, hairy black holes, etc [8] - [12].

We derive some general inequalities relating the energy and angular momentum depending on whether a trajectory intersects the boundary of the ergoregion, lies entirely inside or bounces from the boundary back into the inner region. We make main emphasis on trajectories with $E = 0$ and argue that they can be considered as counterparts of well known zero angular momentum observers (ZAMOs) [17] adapted to motion in the ergoregion.

Apart from general properties of individual trajectories, we consider collisions of two particles near a black hole one of which has $E \approx 0$. It turns out that this can give rise to the unbounded energy in the centre of mass frame. The corresponding scenario was absent from [7] and extends the list of possibilities of getting high $E_{c.m.}$.

Below, we put fundamental constants $G = c = 1$.

II. METRIC AND EQUATIONS OF MOTION

Let us consider the stationary axially symmetric metric

$$ds^2 = -N^2 dt^2 + R^2 (d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_{\theta \theta} d\theta^2,$$

where all coefficients do not depend on $t$ and $\phi$. The form [11] implies that we consider only spacetimes invariant to the simultaneous inversion of the time and the azimuthal angle (see Ch. 2, Sec. 11 of [13] for details). We are interested in equations of motion for test geodesic
particles. In the Kerr metric, the variables can be separated [14] and this simplifies analysis greatly. In a general case, the separation of variables is impossible. To avoid complication, we assume that the spacetime is symmetric with respect to the equatorial plane and we restrict ourselves by consideration of motion in the equatorial plane \( \theta = \frac{\pi}{2} \) only. Then, the equations of geodesics give us

\[
m \dot{t} = \frac{X}{N^2}, \tag{2}
\]

\[
m \dot{\phi} = \frac{L}{R^2} + \frac{\omega X}{N^2}, \tag{3}
\]

where dot denotes derivative with respect to the proper time \( \tau \). Here, \( m \) is the mass,

\[
X = E - \omega L, \tag{4}
\]

\( E = -mu_0 \) is the energy, \( L = mu_\phi \) being the angular momentum, \( u^\mu = \frac{dx^\mu}{d\tau} \) the four-velocity. One can also use the canonical parameter \( \lambda \) along geodesics according to \( \lambda = \tau/m \) that is convenient for consideration of a massless limit.

The forward-in-time condition \( \dot{t} > 0 \) implies that

\[
X \geq 0. \tag{5}
\]

According to (2), the equality is possible on the horizon, where \( N = 0 \). Outside it, \( X > 0 \).

Condition (5) has important physical consequences in our context. In particular, for the value \( E = 0 \) which we are interested in (see below) it restricts the allowed range of \( L \) forbidding positive \( L \).

The quantities \( E \) and \( L \) are conserved due to independence of the metric of \( t \) and \( \phi \), respectively. From the normalization condition \( u_\mu u^\mu = -1 \), one obtains that

\[
m \dot{r} = \pm \frac{\sqrt{A}}{N} Z, \tag{6}
\]

where

\[
Z^2 = X^2 - N^2 (m^2 + \frac{L^2}{R^2}). \tag{7}
\]

It follows from

\[
Z^2 \geq 0 \tag{8}
\]

and (4) that

\[
E \geq \omega L + N \sqrt{m^2 + \frac{L^2}{R^2}}. \tag{9}
\]
It is worth noting that in passing from (8) to (9), we rejected the inequality with the opposite sign since it would violate condition (5).

Within the plane $\theta = \frac{\pi}{2}$ under consideration, we can always redefine the radial coordinate in such a way that $A = N^2$. Then, we have

$$m\dot{r} = \sigma Z,$$

(10)

$\sigma = \pm 1$.

Also, it follows from (2), (3) and (10) that

$$\frac{d\phi}{dt} = \omega + \frac{LN^2}{R^2 X},$$

(11)

$$\frac{dr}{dt} = \sigma ZN^2 \frac{X}{R^2}.$$  

(12)

In the metric (1) we imply that $\omega$ changes sign nowhere. In particular, this happens for the Kerr and Kerr-Newman metrics. Then, one can always achieve $\omega > 0$ (if $\omega < 0$, it is sufficient to make substitution $\phi \to -\phi$). The situation with $\omega$ changing sign is not forbidden, in principle, but we do not discuss such more involved situations.

III. ERGOREGION, ENERGY AND ANGULAR MOMENTUM

For the metric (1),

$$g_{00} = -N^2 + R^2 \omega^2.$$  

(13)

By definition, the boundary of the ergoregion is located where

$$g_{00} = 0, \ N^2 = R^2 \omega^2.$$  

(14)

Inside the ergosphere,

$$g_{00} > 0, \ N^2 < R^2 \omega^2.$$  

(15)

Outside it,

$$g_{00} < 0, \ N^2 > R^2 \omega^2.$$  

(16)

The properties of a trajectory depend strongly on where it is located.

It follows from (12), (16) that outside the ergoregion $E > 0$. Inside the ergoregion all cases $E > 0$, $E = 0$ and $E < 0$ are possible. General properties of geodesics with $E < 0$ are described in [1] for the Kerr metric and, for equatorial motion, are generalized in [3]. Now, we make main emphasis on the trajectories with $E = 0$. 

IV. PROPERTIES OF TRAJECTORIES WITH $E = 0$

Now, we prove the following statements concerning trajectories with $E = 0$ that generalize those of [4].

1) The turning point lies on the boundary of the ergoregion for massless particles and inside it for massive ones.

2) There is exactly one turning point.

3) There are no circular orbits.

By substitution into (7), we have

$$Z^2 = \frac{L^2}{R^2}(\omega^2 - \frac{N^2}{R^2}) - m^2 N^2 = \frac{L^2 g_{00}}{R^2} - m^2 N^2.$$

(17)

In the turning point, $\dot{r} = 0$, so $Z = 0$ according to (10). If $m = 0$, we see that $Z > 0$ everywhere inside the ergosphere due to (15), so there are no turning points there. Only on the boundary (14) $Z = 0$. If $m > 0$, there is a turning point inside where $\frac{L^2 g_{00}}{R^2} = m^2 N^2$. Thus statement 1) is proved.

Let us assume that

$$N' > 0, \quad \omega' < 0, \quad R' > 0,$$

(18)

$$(\omega R')' < 0.$$  

(19)

Here, prime denotes derivative with respect to $r$. One can check that all the assumptions (18), (19) are valid for the Kerr and Kerr-Newman metrics. They do not have a universal meaning but look quite natural physically since they state that the metric approaches its flat spacetime limit at infinity rapidly enough and in a quite "natural" way. Namely, the assumptions (18) state that the lapse function, areal radius and rotation of spacetime monotonically change everywhere from the horizon to infinity. Condition (19) is more strong assumption according to which fall-off of rotation dominates over growth of the areal radius, so the metric approaches its static limit more closely even before it approaches the Minkowski form.

Now, we may take advantage of the approach of [3] where it was used for negative energies. For $E = 0$ it somewhat simplifies.

Taking the derivative we obtain

$$(Z^2)' = -m^2 (N^2)' + L^2(\omega^2)' - L^2(\frac{N^2}{R^2})'.$$  

(20)
It follows from (14), (15) that
\[- \left( \frac{N^2}{R^2} \right)' \leq - \frac{2 NN'}{R^2} + \frac{2 \omega^2 R'}{R}, \]
whence
\[(Z^2)' \leq -m^2(N^2)' + 2 \frac{L^2 \omega(\omega R)'}{R} - 2 NN'R^2 L^2 < 0 \quad (22)\]
due to (18).

Thus $Z^2$ is a monotonically decreasing function of $r$. Therefore, equation $Z = 0$ has precisely one root, so there is only one turning point. As is established above, it is located inside the ergoregion (for massive particles) or on its boundary (for massless ones). Thus statement 2) is also proved.

The existence of circular orbits requires $Z = 0$ and $(Z^2)' = 0$. However, this is also impossible because of (22). Thus circular orbits are absent, so statement 3) is proved as well.

Properties 2 and 3 have an important consequence if we consider the complete behavior of geodesics. As there are no circular orbits, and oscillating trajectories between two turning points are also excluded, geodesics with $E = 0$ (similarly to the case $E < 0$) enter the ergosphere from the inner region under the horizon radius (i.e. from the white hole region) and leave it again entering the region under the horizon (into the black hole region). This generalizes the corresponding result for the Kerr metric [4]. See also [1] and [6].

It is also worth noting that (5) entails that for $E = 0$ the angular momentum $L < 0$.

V. GENERAL INEQUALITIES ON E AND L

The expression (7) can be rewritten as
\[Z^2 = \frac{g_{00}}{R^2} (L - L_+)(L - L_-), \quad (23)\]
where
\[L_\pm = \frac{\omega ER^2}{g_{00}} \pm \frac{NR}{g_{00}} \sqrt{E^2 + m^2g_{00}} = \frac{\omega ER^2}{g_{00}} \pm \frac{R\sqrt{\omega^2 R^2 - g_{00}}}{g_{00}} \sqrt{E^2 + m^2g_{00}}. \quad (24)\]
A. Outside the ergoregion

Now, \( g_{00} < 0 \), it follows from (9) and (16) that \( E > 0 \), as it should be. It is seen from (23) and (24) that now

\[
L_+ \leq L \leq L_-. \tag{25}
\]

It is also necessary that the expression inside the square root be nonnegative. Otherwise, \( Z^2 \) would be negative. Thus we obtain

\[
E \geq m\sqrt{-g_{00}}. \tag{26}
\]

This is in agreement with eq. (88.9) of [15]. It is easy to check that \( L_+ < \frac{E}{\omega} \), so if (25) is satisfied, eq. (5) is satisfied as well.

B. Inside the ergoregion

Now, \( g_{00} > 0 \). Then, (8) entails \( L \geq L_+ \) or \( L \leq L_- \). However, condition (5) excludes the first variant since it is easy to check that now \( L_+ \geq \frac{E}{\omega} \). Thus the only possible variant is

\[
L \leq L_- = \frac{\omega ER^2}{g_{00}} - \frac{NR}{g_{00}} \sqrt{E^2 + m^2 g_{00}}, \tag{27}
\]

where equality is achieved at the turning point.

Now, all cases \( E < 0 \), \( E = 0 \) and \( E > 0 \) are possible. In particular, for \( E = 0 \), we obtain from (27) that

\[
L \leq -\frac{NRm}{\sqrt{g_{00}}}. \tag{28}
\]

C. On the boundary

Using (14) on the boundary, we obtain from (9)

\[
E \geq \omega_0(L + \sqrt{m^2 R_0^2 + L^2}) \geq 0. \tag{29}
\]

Let us consider the limit \( g_{00} \to 0 \) for \( L_\pm \) taken, say, in the outer region. It is seen from (24) that in this case

\[
L_+ \to -\infty, \quad L_- \to \frac{E}{2\omega_0} - \frac{m^2 \omega_0 R_0^2}{2E}, \tag{30}
\]

where subscript "0" refers to the boundary \( r = r_0 \) of the ergoregion.
Thus

\[ L \leq \frac{E}{2\omega_0} - \frac{m^2\omega_0 R_0^2}{2E}. \]  

(31)

One can also consider the same limit taken from inside and obtain the same result (31). It can be obtained from (7) directly with (14) taken into account. Then,

\[ Z^2 = E^2 - 2\omega EL + \frac{L^2 g_{00}}{R^2} - m^2 N^2 \]  

(32)

and (8) gives us (31).

Impossibility of \( E < 0 \) that follows from (29) can be noticed also from (27) since \( E < 0 \) for a trajectory approaching the boundary would lead to \( \lim_{\omega_{00} \to +0} L_- = -\infty \) thus violating (27).

The trajectories with \( E > 0 \) are possible for massive and massless particles. The case \( E = 0 \) can be realized for \( m = 0 \) only (for the Kerr metric, this was noticed in [4]). Then, \( L < 0 \) is arbitrary. Formally, (31) for massless particles admits also \( E = 0, L = 0 \) but this is inconsistent with (5) outside the horizon although on the horizon itself such a trajectory is possible [16].

VI. ANGULAR VELOCITY

Now, we will find some general properties of geodesics depending on the energy.

A. \( E = 0 \)

The angular velocity of any particle with \( E = 0 \) (massive or massless) does not depend on the angular momentum. This is valid inside the ergoregion and on its boundary, where it vanishes.

Proof. By substitution \( E = 0 \) into (14) and (11), we obtain

\[ \left( \frac{d\phi}{dt} \right)_{E=0} = \frac{g_{00}}{\omega R^2}, \]  

(33)

where (13) was taken into account. We see that indeed \( L \) drops out from the formula. Everywhere inside the ergoregion, \( \left( \frac{d\phi}{dt} \right)_{E=0} > 0 \) as it should be. On the boundary,

\[ \left( \frac{d\phi}{dt} \right)_{E=0} = 0 \]  

(34)

according to (14).
B. $E \neq 0$

The sign of the difference $(d\phi/dt)_E - (d\phi/dt)_0$ is defined by that of $E$.

Proof. Subtracting (33) from (11), we obtain

$$
(d\phi/dt)_E - (d\phi/dt)_{E=0} = \frac{N^2 E}{R^2 \omega X}.
$$

(35)

For $E < 0$ the angular velocity is smaller than for $E = 0$ and for $E > 0$ it is bigger. This generalizes observation made in [4] for the Kerr black hole.

C. Zero energy versus zero angular momentum observers

Bearing in mind all properties described above, it is natural to introduce the zero-energy observer (ZEO) by analogy with zero angular momentum (ZAMO) ones [17]. For a ZAMO, $d\phi/dt = \omega$ is determined by the metric entirely and does not depend on the energy. Then, the difference $d\phi/dt - (d\phi/dt)_{ZAMO}$ is determined by the sign of $L$ completely according to (11) since always $X > 0$ [11]. This is direct counterpart of the corresponding properties of ZEO discussed in this Section, with $E$ replaced with $L$.

Meanwhile, there is some difference between two groups of these observers. ZAMOs are not geodesics since they imply $r = const$, so (6) is, generally speaking, not satisfied. By contrary, the trajectories with $E = 0$ under discussion are geodesics. Moreover, circular orbits $r = const$ are impossible for them. In this respect, ZEO and ZAMO can be considered as complimentary to each other. From the other hand, ZEOs are possible in the ergoregion only whereas there is no such a restriction on ZAMOs.

VII. COMPARISON TO THE KERR METRIC

For the Kerr metric in the Boyer-Lindquiste coordinates in the plane $\theta = \pi/2$ one has

$$
R^2 = r^2 + a^2 + \frac{2M}{r} a^2,
$$

(36)

$$
\omega = \frac{2Ma}{R^2 r},
$$

(37)

$$
N^2 = \frac{\Delta}{R^2}, g_{00} = -(1 - \frac{2M}{r}),
$$

(38)
\[ \Delta = r^2 - 2Mr + a^2. \]  

Taking these quantities on the boundary of the ergoregion \( r = 2M \), one obtains from (31) that

\[ L \leq E(a + \frac{2M^2}{a}) - \frac{m^2a}{2E}. \]  

This coincides with eq. (22) of [4] if one puts there \( \theta = \frac{\pi}{2} \) and \( Q = 0 \) (where \( Q \) is the Carter constant).

Eq. (33) turns into

\[ \left( \frac{d\phi}{dt} \right)_{E=0} = \frac{r - 2M}{2Ma} \]  

that coincides with eq. (35) of [4].

Eq. (28) gives us

\[ L \leq -\frac{m\sqrt{\Delta}}{\sqrt{\frac{2M}{r} - 1}}, \]  

where we took into account (38). Eq. (42) coincides with eq. (33) of [4] for \( \theta = \frac{\pi}{2} \) and \( Q = 0 \).

VIII. PROPERTIES OF PHOTON ORBITS

Here, we describe some properties of trajectories of massless particles (photons). It follows from (11), (12) that

\[ \frac{d\phi}{dr} = \sigma \frac{E\omega R^2 - Lg_{00}}{ZN^2R^2}. \]  

This expression admits the zero mass limit \( m \to 0 \). Let also \( E = 0 \). Then, \( L = -|L| \), \( X = \omega|L| \), it is seen from (17) that

\[ Z = \frac{|L|}{R} \sqrt{g_{00}}. \]  

Then, we obtain

\[ \frac{d\phi}{dr} = \sigma \frac{\sqrt{g_{00}}}{RN^2}, \]  

so the angular momentum drops out from the formula similarly to the Kerr case [4].

Now, the upper point of the trajectory with \( E = 0 \) is on the boundary of the ergoregion \( r = r_0 \). Near it, \( g_{00} \approx B(r_0 - r) \), where \( B > 0 \) is some constant, so

\[ \phi \approx \frac{2}{3} \frac{\sigma}{R(r_0)N^2(r_0)} \sqrt{B(r_0 - r)^3} + \text{const}. \]  

(46)
Thus there is a point of inflexion. One can check that this is in agreement with the properties of eq. (39) of [4].

IX. COLLISIONS

It turns out that consideration of particles with zero (or extremely small) energy suggests a new option in the theory of high energy collisions that remained unnoticed before. Let two particles 1 and 2 collide. Then, in the point of collision, one can define the energy in the centre of mass frame according to

\[
E_{\text{c.m.}}^2 = -(m_1 u_{1\mu}^\mu + m_2 u_{2\mu}^\mu)(m_1 u_{1\mu}^\mu + m_2 u_{2\mu}^\mu) = m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \tag{47}
\]

where

\[
\gamma = -u_{1\mu} u_{2\mu} \tag{48}
\]

is the Lorentz factor of their relative motion. Using the geodesic equations (2) - (7) and substituting them directly into (48), one can obtain after summation over indices that

\[
m_1 m_2 \gamma = X_1 X_2 - Z_1 Z_2 - \frac{L_1 L_2}{R^2}. \tag{49}
\]

Under certain conditions, \( \gamma \) becomes unbounded. This is the Bañados, Silk and West (BSW) effect [7]. This happens, if collisions occurs near the horizon (\( N = 0 \)) and (i) one of particles has such fine-tuned parameters that \( X_H = 0 \), (ii) the second particle is not fine-tuned, \( X_H > 0 \) for it (this is so-called usual particle) [18]. Here, subscript ”H” means that the corresponding quantity is calculated on the horizon. Now we will see that inclusion of trajectories with \( E = 0 \) into consideration extends this scheme.

Let \( E_1 = 0 \) and \( L_1 \) be extremely small and negative. Namely, we assume that

\[
|L_1| \omega_H = \alpha m_1 N_c, \tag{50}
\]

where \( \alpha = O(1), \alpha > 1 \), subscript ”c” refers to the point of collision near the horizon, \( N_c \ll 1 \).

Then, (49) gives us that

\[
m_1 m_2 \gamma \approx \frac{X_2 m_1}{N_c} (\alpha - \sqrt{\alpha^2 - 1}) \tag{51}
\]
becomes unbounded, \( \frac{E_2}{m_1^2} = O\left(\frac{X_2}{m_1 N_c}\right) \gg 1 \), provided \( m_1 N_c \ll X_2 \). The same is true if \( E_1 \neq 0 \) but \( |E_1| \ll |L_1| \omega_H \). Condition (8) is satisfied in a narrow but nonzero strip where \( 0 < N \lesssim \alpha N_c \).

It is instructive to compare the situation to that with participation of near-critical particles in the standard BSW scenario [7], [18]. In the latter case, let particle 1 be "near-critical" in the sense that \( E_1 = \omega_H L_1 + O(N_c) \). Then, taking into account that \( \omega - \omega_H = O(N_c) \) for extremal black holes [19], we can see that \( X_1 = O(N_c) \). One obtains from (49) that, if particle 2 is usual, \( \gamma = O(N_c^{-1}) \) is unbounded. In doing so, in the limit \( N_c \to 0 \), the energy \( E_1 \to \omega_H L_1 \), \( (X_1)_H \to 0 \), so near-critical particle becomes exactly critical. However, now such near-critical trajectories do not have the critical ones as their limit when \( N_c \to 0 \). Indeed, \( |L_1| \omega_H \gg |E_1| \), so \( E_1 \neq \omega_H L_1 \) and we see that \( X_H \neq 0 \). Thus both particles 1 and 2 are usual. In the scenario under discussion, a black hole can be extremal or nonextremal, so this does not require multiple collisions in contrast to the standard scenarios near nonextremal black holes [20].

A. Kinematic explanation

There is a relation [21]

\[
X = E - \omega L = \frac{mN}{\sqrt{1 - V^2}},
\]

where \( V \) is the velocity measured by a ZAMO. When a point of collision approaches the horizon, \( N \to 0 \) and, in a general case, \( V \to 1 \). However, for a fine-tuned (critical) particle with \( E = \omega_H L + O(N) \), we have \( V < 1 \). Thus we have collision between a rapid and slow particles that leads to the relative velocity \( w \to 1 \), so \( \gamma \) diverges (see [21] for details). Now, \( X \) is also of the order \( N \) but not due to special relation between \( E \) and \( L \) but due to (50) together with \( E = 0 \), hence again \( V < 1 \) and \( w \to 1 \).

X. CONCLUSION

Thus we derived some general restriction on the relation between the angular momentum and energy depending on whether or not a geodesics intersects the boundary of the ergoregion. The results apply to any axially symmetric rotating metrics. In the particular case of the Kerr one the results of [4] are reproduced. It turned out that trajectories with
$E = 0$ contain precisely one turning point inside or on the boundary, the latter case being possible for massless particles only. The full history of geodesics with $E = 0$ is such that they originate from the white hole region under the horizon and return to the black hole region thus realizing behavior typical of a black-white eternal hole.

We showed that zero energy observers (ZEO) can be considered as counterparts to well known zero angular momentum observers (ZAMO) adapted just to motion in the ergoregion. Their angular velocity does not depend on $L$ and in this sense represents a natural reference point for comparison with angular velocities of other particles. The sign of the difference of angular velocities with $E \neq 0$ and $E = 0$ is determined by $E$ entirely.

We also found a new scenario of high energy collisions. It represents modification of the standard BSW effect with participation of ZEOs or particles which differ from it slightly having nonzero but extremely small energy.

It would be of interest to extend all or some of these generic results to nonequatorial motion.

Acknowledgments

This work was funded by the subsidy allocated to Kazan Federal University for the state assignment in the sphere of scientific activities.

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