Strings from IIB Matrices

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D-string action is constructed from IIB matrices, a spacetime commutator is essential in this construction. This hints at the central role of the spacetime uncertainty relation in a unified formulation of strings. Vertex operators of fundamental strings are also discussed.
1. Introduction

The recently proposed matrix model of M-theory [1] may represent the beginning of our understanding of the new geometry required for a complete formulation of strings and branes. A number of issues in the BFSS matrix model have been studied in [3]-[10]. Although there are problems, this matrix model has successfully passed several consistency tests. Another matrix model, modeling the IIB strings, has also been constructed [2]. There is no Hamiltonian for this matrix model. There are some advantages to work with an action principle in which both space and time are dynamically generated. Lorentz invariance is manifest, also problems hard to treat with the IMF Hamiltonian can be addressed.

Here we shall study a couple of simple problems in the IKKT matrix model. First, we wish to see how the action of multiple D-strings can be recovered in this model. In doing so, we hit upon a highly suggestive commutation relation concerning space and time, whose physics is discussed in [11] and [12]. Second, we discuss the relation between the operators of the Wilson line type and the vertex operators of the fundamental strings. We point out that something crucial is missing in this strategy.

2. D-string Action

We start with the action proposed in [2]:

\[
S = \frac{1}{2\pi(\alpha')^2 g_s} \text{Tr} \left( \frac{1}{4} [X^\mu, X^\nu]^2 + \frac{1}{2} \bar{\psi} \gamma^\mu [X^\mu, \psi] \right),
\]

(2.1)

where we have dropped the chemical potential term proportional to Tr1, which is important to obtain a Born-Infeld action. We also corrected the sign error in [2] (it is possible that the authors of [2] work with the Euclidean signature). Also the normalization of the action is different from that of [2]. This normalization is justified by the correct D-string action derived in this section, and also by consideration of toroidal compactifications [13]. The above action plus the chemical potential term, as explained in [2], can be viewed as an effective action derived from the large N reduced 10 dimensional $\mathcal{N} = 1$ super Yang-Mills. Therefore, there are two ways to interpret this action. One is that of the effective action, so everything derived from it is not to be treated quantum mechanically. Another is that it is just the large N reduced model, so one also considers quantum fluctuation with this action directly. We shall adopt both view-points here, whichever is appropriate in the context of the special problem being concerned.
A large N solution satisfying $[X^\mu, [X^\nu, X_\mu]] = 0$, adapted from the ansatz for a transverse membrane in the BFSS matrix model, is interpreted as a D-string configuration in \[2\]. The interaction properties between two D-strings are calculated using (2.1) and shown to agree with what expected \[2\]. The ansatz is

$$\begin{align*}
X_0 &= -Tp, \quad X_1 = Lq, \\
X_i &= 0, \quad i \geq 2, \quad [q, p] = i \frac{2\pi}{N},
\end{align*}$$

(2.2)

where the spectrum of $q$ and $p$ is $(0, 2\pi)$. We shall use the following identification of the world-sheet coordinates with $p$ and $q$: $\sigma = Tp$, $\tau = Lq$. It appears strange that we have identified the world-sheet space with $X_0$ and the world-sheet time with $X_1$. The reason will become clear later. With this identification, the world-sheet coordinates become noncommutative, and unlike $[q, p]$ which tends to 0 in the large N limit, the commutator

$$[\tau, \sigma] = i \frac{2\pi LT}{N} = 2\pi i \alpha'$$

(2.3)

is always finite, where we have used the result $LT/N = \alpha'$ obtained in \[3\]. Thus, the space and time viewed on the D-string world-sheet become noncommutative. The commutator (2.3) is precisely the form needed to formulate the spacetime uncertainty relation advocated in \[11\] \[12\] in a more rigorous fashion. From (2.3), one may write $X_0 = 2\pi i \alpha' \partial_\tau$, $X_1 = 2\pi i \alpha' \partial_\sigma$.

For a single D-string, any large $N$ matrix can be viewed as a function of $p$ and $q$, or as a function of $\tau$ and $\sigma$. Thus, a matrix appearing in the action (2.1) becomes a world-sheet field. In the large world-sheet limit, the range of both $\tau$ and $\sigma$ is infinite, and a $N$ large matrix such as $X^i$ can be written as

$$X^i = \int d^2kX^i(k)e^{i(k_0\tau + k_1\sigma)},$$

(2.4)

this is the continuous analogue of the discrete Fourier series on a quantum torus \[1\]. A commutator such as $[A, B]$ is now replaced by

$$[A, B] = 2\pi i \alpha' \{A, B\}$$

(2.5)

with the Poisson bracket $\{A, B\} = \partial_\tau A \partial_\sigma B - \partial_\sigma A \partial_\tau B$. A trace is replaced by

$$\text{Tr} \rightarrow \int \frac{d^2\sigma}{(2\pi)^2 \alpha'},$$
To reproduce the D-string action for a single string, we use the following identification between matrices and fields

\[ X_0 = -\sigma + 2\pi\alpha' A_0(\tau, \sigma), \quad X_1 = \tau + 2\pi\alpha' A_1(\tau, \sigma), \]
\[ X_i = X_i(\tau, \sigma), \quad \psi = \psi(\tau, \sigma). \]  

(2.6)

Use the rule (2.5), it is straightforward to calculate commutators:

\[ [X_0, X_1] = 2\pi i\alpha'(1 + 2\pi\alpha' F_{01}), \quad [X_\alpha, X_i] = 2\pi i\alpha' D_\alpha X_i, \]
\[ [X_\alpha, \psi] = 2\pi i\alpha' D_\alpha \psi, \]

where \( F_{01} = \partial_0 A_1 - \partial_1 A_0 + 2\pi\alpha'\{A_0, A_1\} \) and \( D_\alpha A = \partial_\alpha A + 2\pi\alpha'\{A_\alpha, A\} \). We see the \( U(1) \) field strength comes out right. This is one of the reasons to use the identification (2.6). Of course, to reproduce the field strength, it is equally good to use the identification \( X_0 = -\tau + 2\pi\alpha' A_1 \) and \( X_1 = \sigma + 2\pi\alpha' A_0 \), so that the world-sheet time is identified with \( X_0 \) and the world-sheet space is identified with \( X_1 \). But with this scheme, the time derivative terms of other fields will have the wrong sign in the action derived from (2.1).

One remarkable feature of the above discussion is that what we have obtained is not a usual \( U(1) \) gauge theory. The field strength \( F_{01} \) receives correction at the order \( O(\alpha') \), and the covariant derivatives also receive corrections at the same order. This is due to the fact that we have started with a large \( N \) gauge theory. The gauge transformation, for instance \( \delta X_0 = i[\epsilon, X_0] \) becomes \( \delta A_0 = \partial_0 \epsilon + 2\pi\alpha'\{A_0, \epsilon\} \). This is just the remanent of the Schild action as a starting point in the derivation of (2.1) [2]. As we shall see, in a more careful treatment, there are more higher order corrections in \( \alpha' \). It is straightforward to write down the D-string action now. We just need to take a look at the bosonic part

\[ S_B = \frac{1}{2\pi\alpha' g_s} \int d^2\sigma \frac{1}{2} (1 + (2\pi\alpha')^2 F_{01}^2 - D_\alpha X^i D^\alpha X^i). \]

(2.7)

This is the correct action, at least in the leading order in \( \alpha' \). The coefficient of the term \( F^2 \) matches the standard one.

We turn to the case of multiple D-strings. The \( N \) parallel D-string configuration is given by

\[ X_0 = (-\sigma\delta_{ab}), \quad X_1 = (\tau\delta_{ab}), \]
\[ X^i = \text{diag}(X^i_0), \]

3
where we have used the block diagonalized matrices, so \(a, b = 1, \ldots, N\), each \(X^i_a\) is a large 
N matrix and is proportional to the large N identity matrix. In this spirit, we define \(N \times N\) matrix-valued fields 

\[
X_0 = (-\sigma \delta_{ab} + 2\pi \alpha' A_0^{ab}(\tau, \sigma)), \quad X_1 = (\tau \delta_{ab} + 2\pi \alpha' A_1^{ab}(\tau, \sigma)), \\
X_i = (X_i^{ab}(\tau, \sigma)), \quad \psi = (\psi^{ab}(\tau, \sigma)).
\]  

(2.8)

In computing commutators of matrices, there are two types of product to deal with. One type is associated to the matrix product with indices \(a, b\), this will give rise to the \(U(N)\) gauge theory. Another type is the product of entries of the matrices, each is by itself a large N matrix parametrized as a function of \(\tau\) and \(\sigma\). As the rule (2.3) shows, this product is noncommutative too. As we shall see later, this noncommutativeness will introduce terms in higher powers of \(\alpha'\). For the moment, we consider only the leading order. Thus 

\[
[X_0, X_1] = 2\pi i \alpha'(1 + 2\pi \alpha' F_{01}), \quad [X_\alpha, X^i] = 2\pi i \alpha' D_\alpha X^i, \\
[X^i, X^j] = [X^i(\tau, \sigma), X^j(\tau, \sigma)], \quad [X_\alpha, \psi] = 2\pi i \alpha' D_\alpha \psi,
\]  

(2.9)

where the field strength and the covariant derivatives are the standard ones for the \(U(N)\) gauge theory. Subtituting the above result with the rule \(\text{Tr} = \int d^2 \sigma / (4\pi^2 \alpha')\text{tr}\), \(\text{tr}\) is the trace taken for \(N \times N\) matrices, the 1+1 dimensional \(N = 8\) super Yang-Mills theory is obtained.

Now, consider \(\alpha'\) corrections. These are sources for the unusual gauge theory for a single D-string discussed above. To systemacally compute these corrections, we need to introduce some “normal representation” for a large N matrix. Let \(f\) a large N matrix as a function of \(\tau\) and \(\sigma\). We introduce its normal representation as

\[
f(\tau, \sigma) = \int d^2 \sigma f(k)e^{i(k_0 \tau + k_1 \sigma)}.
\]  

(2.10)

Apparently, due to the commutator (2.3), \(fg \neq gf\). The commutator in the leading order in \(\alpha'\) is given by (2.3). In order to distinguish between the large N matrix product and the usual product of functions, we introduce \(f \ast g\) to denote the large N matrix product. By virtue of the formula \(e^A \ast e^B = e^{1/2[A,B]}e^{A+B}\) valid for a constant \([A, B]\),

\[
e^{i(k_0 \tau + k_1 \sigma)} \ast e^{i(k_0' \tau + k_1' \sigma)} = e^{i\alpha' \pi (k_1 k_0' - k_0 k_1')}e^{i[(k_0 + k_0')\tau + (k_1 + k_1')\sigma]}.
\]
Substituting the above relation to \( f \ast g \) and using the normal representation (2.10), we find

\[
f \ast g(\tau, \sigma) = fg(\tau, \sigma) + \sum_{n \geq 1} (i\alpha' \pi)^n \{ f, g \}_n,
\]

where we introduced the higher Poisson brackets

\[
\{ f, g \}_n = \frac{1}{n!} \left( \partial^1_\tau \partial^2_\sigma - \partial^1_\sigma \partial^2_\tau \right)^n fg,
\]

where \( \partial^1_\tau \) is \( \partial_\tau \) acting only on the first function, etc.

From (2.11) we see that the first term in the commutator \( f \ast g - g \ast f \) is just what given in (2.5). There are infinitely many more terms in this commutator, all in odd powers of \( \alpha' \). Also, there are corrections to a \( N \times N \) matrix commutator in the multiple D-string theory. For instance

\[
X^i \ast X^j - X^j \ast X^i = [X^i, X^j] + \sum_{n \geq 1} (i\alpha' \pi)^n [X^i, X^j]_n,
\]

where

\[
[X^i, X^j]_n = \frac{1}{n!} \left( \partial^1_\tau \partial^2_\sigma - \partial^1_\sigma \partial^2_\tau \right)^n [X^i, X^j].
\]

To compute \((X^i \ast X^j - X^j \ast X^i) \ast (X^i \ast X^j - X^j \ast X^i)\), just apply (2.11) one more time. It is not necessary to do this in the action, since the correction terms are total derivatives. This reflects the fact that in taking trace, \( \text{Tr} A \ast B = \text{Tr} B \ast A \).

In defining the normal representation (2.10), one may replace \( \exp(i(k_0 \tau + k_1 \sigma)) \) by, say \( \exp(i k_0 \tau) \exp(i k_1 \sigma) \). This amounts to a redefinition of field \( f \). The resulting commutators such as the one in (2.12) will not respect the world-sheet Lorentz invariance. We also should mention that high order corrections exist in the BFSS matrix model when a certain configuration is considered. However, the Planck constant there is \( 2\pi/N \) instead of \( \alpha' \), so higher orders are supressed in the large \( N \) limit. It is possible that for certain problems, higher orders in that context will become important.

To summarize, the action of the multiple D-strings is reproduced from the IIB matrix model, with high order corrections in \( \alpha' \). The scheme used here to construct the effective D-string action as well as a similar scheme used to construct brane actions in the matrix model of M theory [9] reproduces only the lowest mass states. Excited stringy states are absent. This may be due to the fact that we have assumed that the fluctuations such as those in (2.8) are more or less smooth functions of \( \tau \) and \( \sigma \), so highly excited open string states are to be found in singular fluctuations.
It also remains to incorporate the Dirac-Born-Infeld action into this scheme, in which there are high order corrections too. We shall discuss this in the next section. Of course, it should be interesting to check these corrections against direct string calculations. The most suggestive of the derivation in this section is the commutator \((2.3)\), which should be a universal feature in a complete formulation of string theory. This commutator may form the mathematical foundation for the spacetime uncertainty relation discussed in [11] [12].

3. Born-Infeld Action

To derive the Born-Infeld action, we will work with a small area cell \(\Delta^2\sigma\) of the worldsheet. We will also work with the Euclidean signature. The reason for this is that the Lagrangian for a single D-string as written in (2.7) is not equal to

\[
\text{det}(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + 2\pi\alpha' F_{\alpha\beta}),
\]

since the constant term has the wrong sign. We will see that rewriting this part as a determinant is an important step in deriving the Born-Infeld action.

We assume that the area element \(\Delta^2\sigma\) is built from \(N\) D-instantons. Following [3], we shall let \(N\) be a variable, so the ratio \(\Delta^2\sigma/(4\pi^2 N) = \tilde{\alpha}\) is not equated to \(\alpha'\). In this area element, we have \([\tau, \sigma] = 2\pi i \tilde{\alpha}\), and the trace \(\text{Tr} \to \Delta^2\sigma/(4\pi^2 \tilde{\alpha})\). The Euclidean action serving as our starting point is

\[
S = -\frac{1}{2\pi(\alpha')^2 g_s} \text{Tr} \left( \frac{1}{4} [X^\mu, X^\nu]^2 + \frac{1}{2} \bar{\psi} \gamma^\mu [X_\mu, \psi] + \frac{\pi}{g_s} \text{Tr} 1. \right) (3.1)
\]

We still use the ansatz (2.6) to define fluctuations on the D-string. The relevant commutators are

\[
[X_0, X_1] = 2\pi i \tilde{\alpha}(1 + 2\pi \alpha' F_{01}), \quad [X_\alpha, X_i] = 2\pi i \tilde{\alpha} D_\alpha X_i, \quad [X_\alpha, \psi] = 2\pi i \tilde{\alpha} D_\alpha \psi,
\]

we ignored other terms with higher powers of \(\tilde{\alpha}\). Substituting the above result into the action (3.1), we find

\[
\frac{\Delta^2\sigma}{4\pi(\alpha')^2 g_s} \left( \tilde{\alpha}(1 + (2\pi \alpha')^2 F_{01} + D_\alpha X_i D_\alpha X_i) - \frac{i}{2\pi} \bar{\psi} \gamma^\alpha D_\alpha \psi + (\alpha')^2/\tilde{\alpha} \right). (3.2)
\]

Note that the fermionic term is independent of \(\tilde{\alpha}\), and this is important in order to preserve SUSY in the resulting Born-Infeld action. The term linear in \(\tilde{\alpha}\) is just the following determinant

\[
\text{det} = \text{det}(\delta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + 2\pi\alpha' F_{\alpha\beta}). (3.3)
\]
Taking variation of (3.2) with respect to \( \tilde{\alpha} \) determines
\[
\tilde{\alpha} = \frac{\alpha'}{\sqrt{\det}}.
\]
Thus, we obtain the Born-Infeld action
\[
S = \Delta^2 \sigma \left( \sqrt{\det} - \frac{i}{4\pi\alpha'} \bar{\psi} \gamma^\alpha D_\alpha \psi \right).
\] (3.4)
This action is invariant under the supersymmetry transformation
\[
\delta A_\alpha = \bar{\psi} \gamma_\alpha \epsilon, \quad \delta X_i = \bar{\psi} \gamma_i \epsilon,
\]
\[
\delta \psi = \frac{2\pi i\alpha'}{\sqrt{\det}} \left( \frac{1}{2} \gamma^{\alpha\beta} F_{\alpha\beta} + \gamma^\alpha \gamma^i D_\alpha X_i \right) \epsilon.
\] (3.5)
Of course, both the action and the SUSY transformation make more sense in the Minkowski space.
\( \tilde{\alpha} \) is equal to \( \alpha' \) when the world-sheet fluctuation is small, in which case \( \sqrt{\det} \sim 1. \) \((\tilde{\alpha})^{-1}\) is the local density of D-istantons. Since the area element is corrected by a factor \( \sqrt{\det} \), our formula for \( \tilde{\alpha} \) says that the real D-instanton density is always a constant \( 1/\alpha' \).
The Born-Infeld action contains high order \( \alpha' \) corrections. As we discussed in the previous section, the noncommutativeness of the world-sheet coordinates also introduces high order \( \alpha' \) corrections. To treat all these corrections in a uniform manner, one has to replace \( \alpha' \) in formulas such as (2.11) by \( \tilde{\alpha} \), and then take variation of the whole action with respect to \( \tilde{\alpha} \). Although it is tedious to carry out this procedure, the method is rather systematic.

4. Vertex Operators of Fundamental Strings

The suggestion in this section is highly conjectural.

It is suggested in [2] that the Wilson line operator
\[
W(p(\sigma)) = \text{Tr} P \exp \left( i \int p_\mu(\sigma) X^\mu d\sigma \right)
\] (4.1)
should be interpreted as the creation operator of a closed string state with momentum density \( p(\sigma) \), and some preliminary evidence in supporting this interpretation is provided there. It is well-known that the wave functions are directly related to vertex operators of strings in computing scattering amplitudes. A general correlation function in the matrix model is then
\[
\langle W(p_1) W(p_2) \cdots W(p_n) \rangle.
\] (4.2)
It is natural to conjecture that this correlation function is the one of \( n \) off-shell string states, containing all loops weighted by \( g_s \). Curiously, the appearance of the string coupling in the matrix model action (2.1) is not in \( g_s^2 \), but in \( g_s \). The origin of this feature is that the authors of [2] intended to reproduce D-string interaction in which open string states are the relevant quanta. It is interesting to see how the usual closed string coupling \( g_s^2 \) comes out correctly in the correlation function (4.2).

Here we shall attempt to extract vertex operators from the Wilson line operator (4.1) and similar operators. The first problem we need to solve is to compute the trace in the Wilson lines. To represent a large \( N \) matrix, one can either appeal to the spherical basis or to the toroidal basis [14]. We believe that different bases correspond to contribution of different loops. Of course perturbative expansion of the action (2.1) also contains different loops. The correlation function at a given loop therefore receives contribution not only from different representation of the large \( N \) matrices, but also from perturbative expansion of the action. For simplicity, we use the toroidal representation. Introduce \( p \) and \( q \) with 
\[
[q,p] = i\hbar = i2\pi/N.
\]
In the large \( N \) limit, matrices become functions of \( p \) and \( q \). Use 
\[
(q^1,q^2) = (q,p),
\]
then 
\[
[q^\alpha,q^\beta] = i\hbar \delta^{\alpha\beta}.
\]
In the large \( N \) limit, the Wilson line in the leading order can be written as
\[
W(p) = \int \frac{d^2q}{2\pi\hbar} e^{i P_\mu X^\mu(q,p)}, \tag{4.3}
\]
where \( P_\mu = \int p_\mu(\sigma)d\sigma \). So this leading order term can be interpreted as the tachyon operator. The correlation function (4.2) is then interpreted as the correlation of \( n \) tachyon states starting at the one-loop, since we use the toroidal representation here. The difficulty for this correspondence to be exact lies in the fact that the matrix model action (2.1) is like the action of string field theory, we do not know how the world-sheet action of the fundamental strings can be derived from it. For instance, to compute the one-loop correlation function, the action (2.1) contributes only at the \( O(g_s^0) \) level in the present consideration, since we have already used the toroidal representation for the Wilson line operators. Only classical solution representing the vacuum contribute. This solution is just the Goldstone modes \( X^\mu = x^\mu 1_N \).

Despite the above difficulty, we still want to push our strategy to its limit to see how much we can get out of it. Since \( q^\alpha \) are noncommutative, the Wilson line operator (4.1) contains infinitely many terms in higher orders in \( \hbar \). To compute these terms systematically, one may use the method introduced in the previous section. One first uses the
normal representation for $X(p, q)$, then arranges the Wilson line operator in a normal representation. Here we write down only the second term in the expansion in $\hbar$:

$$W(p) = \int \frac{d^2 q}{2\pi \hbar} e^{i p \mu X^\mu} (1 - i \hbar \epsilon^{\alpha\beta} \xi_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + \cdots), \quad (4.4)$$

where

$$\xi_{\mu\nu} = \frac{1}{2} \int_{\sigma_1 < \sigma_2} p_\mu(\sigma_1) p_\nu(\sigma_2) d\sigma_1 d\sigma_2.$$ 

(To compute the expansion (4.4) systematically, one needs the formula

$$A_1 \ast A_2 \ast \cdots \ast A_n = \exp \left( \frac{i}{2} \hbar \sum_{i < j} \epsilon^{\alpha\beta} \partial^i_\alpha \partial^j_\beta \right) A_1 A_2 \cdots A_n,$$

where $\partial^i_\alpha$ is $\partial_\alpha$ acting on the i-th factor.) We see that the vertex operator of the anti-symmetric tensor is reproduced as the second term. The polarization tensor is nonvanishing only when the density $p_\mu(\sigma)$ is not constant. This sounds reasonable, since this mode is an excited state above the tachyon. One more difficulty with this approach appears here. It is impossible to construct the graviton vertex operator from the Wilson line or more complicated operator such as $\text{Tr}[X^\mu, X^\nu] P \exp(i \int p_\lambda(\sigma) X^\lambda d\sigma)$. There is one place we know graviton exists. In computing the interaction between two parallel static D-strings [2], the effect of the bosonic fluctuations cancels that of the fermionic fluctuations. In the closed string channel, this is interpreted as the cancelation between the exchange of graviton and dilaton and the exchange of the quanta of R-R anti-symmetric tensor field. It is interesting to complete the calculation of the Wilson line tapole in the background in a D-string, as suggetsed in [2], to see whether the graviton vertex operator is contained in the Wilson line.

The vertex operator for the R-R tensor field can not be constructed directly from matrices either. The natural candidate $\text{Tr}\bar{\psi}\gamma^\mu\nu\psi P \exp(i \int p_\lambda(\sigma) X^\lambda d\sigma)$ vanishes, since $\psi$ is chiral. Just as in the usual world-sheet approach, only the vertex for the field strength exists, which is $\text{Tr}\bar{\psi}\gamma^\mu\nu p_\sigma\gamma^\sigma\psi P \exp(i \int p_\lambda(\sigma) X^\lambda d\sigma)$. Of course, it is also necessary to generate both the left moving and the right moving world-sheet spinors from a single $\psi$.

One expects from the definition of the Wilson line (4.1) that only a loop structure emerges, not a world-sheet as we have suggested. This puzzle may be resolved by the following interpretation. Upon large N expansion (4.4), there are infinitely many vertex operators, inserted at the puncture $(q, p)$. The whole effect is then to open this puncture.
to a loop. So in a toroidal representation of the large N matrices, a Wilson line really represents an open torus whose boundary is a circle. In the spirit of string field theory, what one expects from action are vertices represented by a Riemann surface with multiple boundaries. The contractions between inserted operators \((1,2)\) and the action, and the contractions between operators contained in the action, will help to form a Riemann surface of higher genus. In any case, it is worthwhile to pursue the matter along this line further, although it seems there are many difficulties.

5. Discussion

We have constructed the action of multiple D-strings from IIB matrices. One nice feature of this construction is the commutation relation for the D-string world-sheet co-ordinates, which reflect time and the space these D-strings are embeded in. This relation implies high order \(\alpha'\) corrections, in terms of the lowest mass states. The open problem is to construct the excited open string states living on D-strings. The spacetime uncertainty relation implied by the commutation relation was discussed some time ago \([1]\) and has also received attention recently in the more general context involving D-objects \([2]\). We believe that this uncertainty principle plays a central role in our future understanding of the foundation of string theory. And here it is the first time for us to see its mathematical underpinning in the IIB matrix model, rather than by analysis of perturbative amplitudes \([1]\).

A suggestion to extract vertex operators of fundamental strings from operators of the Wilson line type is made. We are beginning to see difficulty in this scheme. For one thing, we do not know how to construct a simple vertex operator such as that of the graviton. For another, we are not yet able to recover a world-sheet picture from the matrix model action \((2.1)\). This difficulty is perhaps of technical nature rather than of conceptual one. To conclude, we are just beginning to see the tip of the huge iceberg burried in the formal matrix model.

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