Refractive index of a transparent liquid measured with a concave mirror

Amitabh Joshi\textsuperscript{1} and Juan D Serna\textsuperscript{2}

\textsuperscript{1} Department of Physics, Eastern Illinois University, Charleston, IL 61920, USA
\textsuperscript{2} School of Mathematical and Natural Sciences, University of Arkansas at Monticello, Monticello, AR 71656, USA

E-mail: ajoshi@eiu.edu and serna@uamont.edu

Abstract
This paper describes the spherical concave mirror method for measuring the index of refraction of transparent liquids based on locating the actual and apparent radii of curvature of the mirror. We derived the refractive index equation using Snell’s law and the small-angle approximation.

Introduction
Measuring the refractive index $n$ of a substance or medium is part of every introductory physics course. Various approaches to determine this index have been developed over the years based on the different ways light reflects and transmits in the medium.

With the introduction of lasers in basic physics courses, a number of these methods have become accessible to the undergraduate physics laboratory. Several of these techniques are highly accurate and use specific optical equipment, like spectrometers, interferometers or microscopes. In the laboratory, the method instructors choose to measure the refractive index of a liquid depends on different factors, such as the physical and optical properties of the substance they want to study, or simply the resources available at hand, that, in many cases, are very limited.

Among all the different methods, a spherical concave mirror filled with a liquid is an excellent alternative to measure the refractive index of the liquid, especially when no fancy apparatus is available and the accuracy of the measurements is not critical.

In this paper, we would like to present a simple geometrical derivation of the refractive index of a transparent liquid that is obtained using a spherical concave mirror. This derivation relies mostly on Snell’s law and the small-angle approximation.

This method is based on measurements of the actual and apparent position of the centre of curvature of the mirror, when it is empty and filled with a liquid, respectively. The laws of reflection and refraction are essential to understand the physics behind this method. Students measuring indices of refraction of liquids using this technique are introduced gradually to the concepts of reflection and refraction of light, Snell’s law and image formation by spherical mirrors.

Revisiting spherical mirrors
Let us first consider the spherical concave mirror shown in figure 1. The optical axis is the radial line through the centre of the mirror that intersects
its surface at the vertex point \( V \). Some relevant points on the optical axis are the centre of curvature \( C \) and the focal point \( F \). The centre of curvature coincides with the centre of the sphere of which the mirror forms a section. At the focal point, rays parallel to the optical axis and incident on the concave mirror intersect after being reflected by the mirror’s surface [1]. The distance \( CV \) between the centre of curvature and the vertex is equal to the radius of the sphere and is called the radius of curvature \( R \). Similarly, the distance \( FV \) between the focal point of the mirror and the vertex is known as the focal length \( f \).

In general, when the rays are close to the optical axis, that is, for the small-angle approximation, the focal length can be shown to be half of the radius of curvature [2].

\[
f = \frac{R}{2} \tag{1}
\]

The location and nature of the image formed by a spherical mirror can be determined by graphical ray-tracing techniques [3]. To find the conjugate image point \( I \) of an object point \( O \) located at the centre of curvature \( C \), the paths of any two rays leaving \( O \) are sufficient [4]. We first use the so-called parallel ray 1 that is incident along a path parallel to the optical axis, strikes the mirror at point \( H \) and is reflected through the focal point \( F \) as ray \( 1' \). Next, we use the so-called focal ray 2 that passes through the focal point and is reflected parallel to the optical axis as ray \( 2' \). The image point \( I \) is formed where the two rays \( 1' \) and \( 2' \) intersect. This image is real, inverted, located at the centre of curvature \( C \), and has the same size as the object, as shown in figure 1.

The experiment
Measuring the refractive index \( n \) of a transparent liquid, like water, using a spherical concave mirror is based on locating the actual and apparent centres of curvature of the mirror, when it is empty and filled with a thin layer of water, respectively.

The typical experimental setup (see figure 2) consists on a mirror, a support with a vertical rod and a clamp holding a lamp (source of light) and a screen horizontally [5]. When there is no water in the mirror, the centre of curvature is located at \( C \), and the radius of curvature is \( R = CV \). From (1), the focal length of the concave mirror is \( f = FV = R/2 \). When the mirror is filled with water, the apparent centre of curvature \( C' \) moves down, and the apparent radius of curvature becomes \( R' = C'V \). The new focal length is given by \( f' = F'V = R'/2 \).

To obtain the refractive index of water \( n_w \), the clamp that holds the lamp and the screen is moved to position \( C \) until a sharp image of
Refraction of light through a transparent liquid measured with a concave mirror

The lamp is formed by the empty mirror. The radius of curvature $R$ is then measured. Next, the mirror is filled with a thin layer of water, and the lamp and screen are moved down until a new sharp image of the lamp is formed on the screen. This position corresponds to the apparent centre of curvature $C'$. The new radius of curvature $R'$ is then measured. The refractive index can be obtained by using the equation

$$n_w = \frac{R}{R'}.$$  \hspace{1cm} (2)

**Geometrical derivation of the refractive index**

When the point object $O$ is placed at the centre of curvature $C$, and the mirror is empty (no water has been poured in), we may use the same graphical ray-tracing methods as figure 1 to locate the conjugate image point $I$. Ray $1_a$ leaves point $O$ parallel to the optical axis, strikes the mirror at point $H$ and then reflects. The reflected ray refracts from water into air bending away from the surface normal at the point of incidence $P$, as the refractive index of water $n_w$ is bigger than that of air $n_a$. The refracted ray $1_b'$ intersects the optical axis at the new focal point $F'$. The new focal length becomes $f' = F'V$. If the object is placed at the apparent centre of curvature $C'$ and labelled object point $O'$, the conjugate image point $I'$ is found at the apparent centre of curvature $C'$. The image is real, inverted and has the same size as the object (see figure 3).

Using Snell’s law at surface point $P$, we get

$$n_w \sin \theta = n_a \sin \phi,$$  \hspace{1cm} (3)

where $n_w$ and $n_a$ are the refractive indices of water and air, respectively. From this point on, we will assume $n_a \approx 1.00$.

Figure 3 shows that from triangles $FHG$ and $F'H'G'$

$$HG = FH \sin \theta,$$  \hspace{1cm} (4a)

$$H'G' = F'H' \sin \phi.$$  \hspace{1cm} (4b)

The backward extension of the refracted ray $1_b'$ strikes the mirror at point $H'$. If we assume that the focal lengths $F$ and $F'$ are large compared with the thickness of the water layer in the mirror, points $G$ and $G'$, as well as $H$ and $H'$, are very close to each other. Therefore, we can make the following approximations for the geometrical lengths:

$$H'G' \approx HG,$$  \hspace{1cm} (5)

$$F'H' \approx F'H.$$  \hspace{1cm} (6)

Physically, points $H$ and $H'$, as well as $G$ and $G'$, are not coinciding but are located nearby. Using (5) in (4b), we may write

$$HG = F'H \sin \phi.$$  \hspace{1cm} (7)

Since the left-hand sides of (4a) and (4b) are the same, we may equate the two equations to obtain

$$FH \sin \theta = F'H \sin \phi.$$  \hspace{1cm} (8)

Using (3) in (7), we obtain

$$\frac{FH}{F'H} = \frac{\sin \phi}{\sin \theta} = n_w.$$  \hspace{1cm} (9)

If the angles $\theta$ and $\phi$ are small, then these angles can be replaced by their tangents (small-angle approximation). Furthermore, the
distance $GV$ in figure 3, the sagittal depth of the surface [6], is also small, and we may neglect it. Therefore, we can write

$$n_w = \frac{\tan \phi}{\tan \theta} = \frac{f}{f'}.$$ (9)

Finally, using (1) we can express the refractive index of water in terms of the actual and apparent radii of curvature as follows:

$$n_w = \frac{R}{R'}.$$ (10)

**Conclusion**

The experiment described in this paper gives a simple and effective method of measuring the refractive index of water (or any transparent liquid) using a spherical concave mirror.

A valuable pedagogical consideration of this method is that students may observe that a real image of the object is formed at the centre of curvature of the empty mirror when the object is located at the same point. Similarly, a real image is formed at the apparent centre of curvature when the mirror is filled with water, if the object is located at the same place. This provides an easy way of finding the refractive index of the liquid by measuring the ratio between the actual and apparent radii of curvature. In addition, by using Snell’s law and reflection of rays in a concave spherical mirror, they should notice that the images are not only real, but inverted and have the same size as the object.

As shown in the geometrical derivation, with the concave mirror method we can get an expression for the refractive index of a liquid using merely Snell’s law and the small-angle approximation (paraxial approximation), and avoiding complications introduced by the thin-lens equation. At this point, we would like to mention that, to the best of our knowledge, we have not seen in the literature a geometrical derivation such as the one we present in this paper.

This experiment is suitable for the undergraduate physics laboratory. It is easy to set up and execute. It has been carried out successfully in the laboratory, providing reasonably accurate numerical values of the refractive index of water. For example, with a 40 cm radius of curvature mirror, we obtained a refractive index within a 5% error. With a larger 60 cm radius of curvature mirror, the error decreased to within 3%. It is pertinent to mention that the main sources of error come from the experimenter’s ability to estimate the exact position where the image of the object is focused on the screen and his/her interpretation of the instrumental readings (e.g., with a ruler). This method can also be used to determine refractive indices of organic liquids, which are usually higher than that of water.

**Acknowledgment**

AJ gratefully acknowledges funding support from RCSA.

Received 10 February 2012, in final form 3 March 2012
doi:10.1088/0031-9120/47/5/559

**References**

[1] Wilson J D, Buffa A J and Lou B 2010 *College Physics* 7th edn (San Francisco, CA: Addison-Wesley) p 782
[2] Hecht E 2002 *Optics* 4th edn (Reading, MA: Addison-Wesley) p 183
[3] Pedrotti F L, Pedrotti L M and Pedrotti L S 2007 *Introduction to Optics* 3rd edn (Upper Saddle River, NJ: Pearson) p 31
[4] Suppapittayaporn D, Paniipan B and Emarat N 2010 Can we trace arbitrary rays to locate an image formed by a thin lens? *Phys. Teach.* 48 256–7
[5] Rachna S 2009 *Physics Lab Manual-XII (Together With)* 2nd edn (New Delhi: Rachna Sagar) p 86
[6] Blaker J W and Rosenblum W M 1993 *Optics—An Introduction for Students of Engineering* (New York: Macmillan) p 15