The equilibrium classical scatter spectrum of waves

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Regardless of the unspecific notions of photons as light complexes, radiation bundles or wave packets, the radiation from a single state transition is at most a single continuous wave train that starts and ends with the transition. The radiation equilibrium spectrum must be the superposition sum of the spectra of such wave trains. A classical equipartition of wave trains cannot diverge since they would be finite in number, whereas standing wave modes are by definition infinite, which had doomed Rayleigh’s theory, and concern only the total radiation. Wave trains are the microscopic entities of radiation interacting with matter, that correspond to molecules in kinetic theory. Their quantization came from matter transitions in Einstein’s 1917 derivation of Planck’s law. The spectral scatter of wave trains by Doppler shifts, which cause the wavelength displacements in Wien’s law used for the frequency dependence in Einstein’s derivation, is shown to yield the shape of the Planck spectrum. A Lorentz transform property of Doppler shifts discovered by Einstein is further shown equivalently necessary and sufficient to have corrected Rayleigh’s theory.

I. INTRODUCTION

Presented below is the first-ever treatment of the classical spectral scatter of electromagnetic wave trains in a cavity, due to classical Doppler shifts by simple reflection from atoms or cavity walls and their thermal motions or vibrations. As the number and energy of wave trains from a finite set of atoms would be finite, the divergence of Rayleigh’s theory cannot occur. The spectral scatter is shown to bear the form of the Planck spectrum classically due to proportional energy increases as a Lorentz transform effect in Doppler frequency shifts, incidentally discovered by Einstein [1], who ascribed it to “light complexes” [1] or “bundles” [2] in deference to the quantum hypothesis [3–5]. Other traditional notions of photons as wave packets having distinct phase and group velocities (cf. [6, §31]) refer to Fourier spreads also possessed by the wave trains. There was also a recent attempt to identify photons with the individual undulations in sinusoidal waves [7], which makes partial sense in harmonic families of standing wave modes [8]. The present focus is more simply on arbitrary classical travelling waves constrained only by finiteness of length and energy.

The Lorentz property holds analytically for classical electromagnetic waves of arbitrary amplitudes between phase boundaries [7], and thus specifically the wave trains. Einstein’s notions of radiation energy and momentum in detailed balance with state transitions in matter [2] extended known classical principles of these quantities to radiation, and its only other input concerning radiation was Wien’s law, also classical in origin. His treatment was thus a rigorous classical derivation of Planck’s law that traced quantization to matter transitions, despite his own apparent attribution of the Lorentz property to inherent quantumness.

The present result proves this contrarian insight correct by showing that the same physical mechanisms do reproduce the Planck form without quantization in the absence of matter transitions. It also implies that Einstein’s treatment was incomplete, despite reproducing Planck’s law, in overlooking the Doppler shifts from non-transitional reflections, and that radiation quantization is an over-reaching premise though empirically correct at optical frequencies.

The result complements emerging reports of entanglement obtainable classically via fluid mechanics [9] with relation to electromagnetic theory [11], and from Maxwell’s equations under various conditions [11, 12], even as loophole-free confirmations of non-locality are achieved for the quantum result [13]. The quantumness implication of entanglement itself is now questioned [14, 15], echoing general anticipation of entanglement in classical waves in [16].

The result is straightforward, and could be overturned only by an error in assuming that any single state transition of matter emits at most a single continuous wave train of radiation. If we assumed, like Planck and Einstein, that matter comprises a dynamical structure of particles, some with charge, and its internal states comprise specific arrangements and motions, then classically, a state transition must involve a continuous change from one arrangement and motions to the next, implying a continuous variation of electric and magnetic forces at any point of observation, which qualifies as a continuous wave train. Emission from a single state transition cannot be a multitude of whole wave trains, nor a more general combination of sinusoidal wave components amounting to more than a single wave train, as also inferred by Einstein [2] for concurrently acting “bundles”.

A finite wave train is the simplest and most general description of an electromagnetic waveform associable with any single, discrete state transition in matter, and is sufficient, as each such wave train would not only bear a full energy quantum but also satisfy directionality and momentum balance in interactions with matter, as shown by Einstein [2], and its spatial extent suffices to explain quantum non-locality, as also proved here. The traditional broader allowance for complexes or bundles thus did not serve for generality except in analogy to “complexions” or microstates in view of their arbitrary amplitude and polarization distributions and wavefronts, and has instead inadvertently denied the
only possible functional form for representing discrete interactions with matter. Related historical difficulties, leading to a belief in the inadequacy of classical physics itself, can be attributed to standing wave modes without the Lorentz property being the only classical model in Rayleigh and Planck theories \cite{17,19}, as well as in recent ideas of a classical origin of Planck’s law arguing zero-point energy and Casimir forces are classical (cf. \cite{20,22}).

The modes are favoured as a stationary representation of the equilibrium steady state, and also for their similarity to the vibrational modes of molecules and to cyclic coordinates associated with stationary states in classical equipartition \cite{20,27}. However, standing wave modes by definition concern the total radiation in the cavity, and the role of molecular vibrations more precisely concerns spin and orbital angular momenta, as contained in the polarization and wavefront distributions, respectively which are now well understood \cite{28}. Further, modes are defined by geometry and inherently infinite in number, as wavelengths are infinitely divisible real valued quantities, so Rayleigh’s theory was doomed to divergence by this choice of entities, in treating classical equilibration without the Lorentz property. As shown later by Bose \cite{29}, Planck’s oscillator hypothesis also obviates Wien’s law, and thereby all dynamical considerations. This leaves Einstein’s work the only historical consideration of the microscopic dynamics of radiation.

Correspondingly, the present theory is the radiative analogue of the treatment of molecular motions in the kinetic theory but got skipped in radiation theory due to the success of macroscopic thermodynamic principles, using standing waves, in yielding the analogous result of radiation pressure in Stefan-Boltzmann and Wien’s laws. The wavelength displacements in Wien’s law are Doppler shifts upon reflection at walls subject to macroscopic motions \cite{19}, so wall Doppler shifts provided a critical physical mechanism for dispersing the radiant energy across frequencies in treatments including Einstein’s, as modes, by definition, cannot mutually interact and atomic spectra are narrow. However, as all considerations in Wien’s law are macroscopic, it constrained rather than explain the spectral equilibration. Brownian motions due to radiation reactions, also considered by Einstein \cite{2}, would be a weak mechanism at very low pressures.

It could be argued, especially on the basis of Fermi-Pasta-Ulam theory (cf. \cite{30–33}) and related ideas of fluctuations, that reflections by stationary walls suffice to explain the spectral equilibration, as they would spatially disperse wave fronts much like molecules. However, the premise of stationary confining walls in both kinetic and radiation theories is itself another traditionally overlooked defect, since wall impacts, whether by molecules or photons, should also set real walls of finite mass into vibration, much as the photon interactions would set the molecules into Brownian motions in Einstein’s treatment \cite{2}, until the wall vibration modes reach equilibrium with the confined gas or radiation. The wall vibrations do not alter equilibrium energy, temperature or pressure, so their neglect did not affect the overall result, but their absence must denote walls at absolute zero temperature by the third law, which was only realized later \cite{34}, and would thereby violate the premise of equilibrium of a confined gas or radiation at a steady non-zero temperature. Their inclusion is thus required for completeness of the dynamical considerations, and would also resolve Loschmidt’s paradox (cf. \cite{35,36,58}), besides providing a general mechanism for Doppler shifts at the microscopic level.

The overall result is thus not only that the classical electromagnetic wave is also the most general notion of a photon given by quantum theory, in all aspects of matter interactions and thermodynamics as well as non-locality, but more so that the current ideas fundamentally derive from ignorance and flaws in the classical attempts, and misperception of a key classical electromagnetic wave property by Einstein. Complementary ideas of the thermodynamics of information representation and erasure in physical observers \cite{37–40}, and application of the third law to observer state transitions in process of physical observations, classically impose an inherent probabilistic character to all observations, so that even the quantum probabilities cannot remain a matter of postulate.

The next section revisits Einstein’s 1905 and 1917 papers in the present view. Sufficiency of the Lorentz property to correct Rayleigh’s theory is shown in Section III. The core result for wave trains follows in Section IV.

II. BACKGROUND OF THE CORE RESULT

In the 1917 paper \cite{2}, Einstein considered that the equilibrium with radiation could not change the molecular energy distribution, \( W_n = n \rho e^{-E_n k_B T} \) from the kinetic theory, where \( W_n \) denoted the relative frequency of a state of energy \( E_n \). This led, via considerations of energy and momentum exchanges with radiation, to the condition

\[
p_n e^{-E_n k_B T} B_m^n \rho = p_m e^{-E_m k_B T} (B_m^n + A_m^n) \quad \text{whence} \quad \rho = \frac{A_m^n B_m^n}{e^{(E_m - E_n) k_B T} - 1} \tag{1}
\]

where \( A_m^n \) was a probability coefficient to represent spontaneous emission, and \( B_m^n \) and \( A_m^n \) were coefficients to denote absorption and stimulated emission, respectively. The emission and absorption mechanisms were the only assumptions new in the 1917 paper, and no inherent dependence on frequency was assumed.

The connection to frequencies was obtained by equating to Wien’s law in the form \( \rho = \alpha \nu e^{-h \nu k_B T} \), which led to the relations \( A_m^n B_m^n = \alpha \nu^3 \) and \( E_m - E_n = h \nu \), where the latter relates to Bohr’s model. In this form of Wien’s law, \( h \) is a scale factor relating time and energy and signifies no quantization, as pointed out by Einstein. The wavelength displacements providing the frequency \( \nu \) result from the Doppler shifts of standing wave modes caused by macroscopic
motions of cavity walls during adiabatic compression or expansion [13, 41]. The only assumptions of discreteness were therefore that molecules had discrete energy levels \( E_n \), so only the energy levels enter eq. (1), and that their radiative interactions occurred as discrete events, limiting the duration or length of the wave trains emitted in each transition.

The additional arguments of momentum were necessary for completeness of the dynamical picture of interaction, including the implication of induced Brownian motion from the radiative reactions. Those ideas refined over his 1905 photoelectricity paper [11], which had concerned energy quanta but not momentum. The arguments used the relation \( p = Ec \) from classical electromagnetics [59], and the Lorentz property,

\[
E' = E\sqrt{\frac{1-vc}{1+vc}} \quad \text{along with} \quad \nu' = \nu \sqrt{\frac{1-vc}{1+vc}},
\]

which had been derived from a standpoint of energy conservation in his 1905 relativity paper [1], without involving any specific wave representation, as opposed to its derivation in [7]. The approach avoided convergence issues historically encountered in Fourier theory [12, 13], as well as problems of complexity and completeness of representation.

Einstein had been expressly concerned with “the theory of light which operates with continuous spatial functions” in his photoelectricity paper [11], but his perception was classical and deterministic, just as the action-angle formalism of quantum mechanics to which he made key contributions [44]. Eq. (2) did not require or prove the irreducibility of quantization to classical laws, despite its similarity to Planck quantization. The dissociation from wave representations would have impeded application of eq. (2) to standing wave modes following Rayleigh and Planck, however, just as the strong notion of localization of radiant energy in deriving eq. (2) went against the probabilistic quantum notions in [15]. In any case, every input in Einstein’s derivation was thus classical except the discreteness of molecular states and their transitions (eq. (1)), so the derivation already implicated the discreteness of matter for quantization.

The classical derivation of eq. (2) needed for the present result is adequately treated in [8], but the further notion of quanta in [6] as half-wavelength segments of travelling waves is not endorsed, as it contradicts the uncertainty principle in single photon observations. The only physical basis for that notion is the conservation of energy of any sequence of such segments, i.e., of whole wave trains, under propagation and Lorentz transformations, already assured classically without quantum significance. Such half-wavelength segments of standing wave modes do correspond to quanta under second quantization because harmonically related modes differ in geometry only by multiples of half-wavelengths, and in energy by whole quanta. Along an internal dimension \( L \) in a cavity, a standing wave mode of wavelength \( \lambda \) would have \( N = 2L\lambda = 2Lec \) such segments, whence \( N \) is proportional to frequency \( \nu \). Planck’s quantization rule \( E = \hbar \nu \) then promises \( EN = h\nu c2L = \hbar c2L \), independent of wavelength or mode, for segment energy, and the quantum \( \hbar \nu \) is the energy of a whole mode of frequency \( \nu \), and not of an individual half-wavelength segment.

### III. DOPPLER-LORENTZ CORRECTION TO RAYLEIGH’S THEORY

The further attempt in [8] to identify families of harmonically related standing wave modes with Planck oscillators, consistent with second quantization, was incomplete, as it needed mode energies to correspond to integrals over phase increments in multiples of \( \pi 2 \), instead of the cavity dimension \( L \). The ordinary classical proportionality of standing wave energies to \( L \) would leave the modes equiprobable under a Boltzmann probability distribution, and thus reduce to Rayleigh’s theory and its problem of divergence. In hindsight, eq. (2) also addresses that problem since the premise of Doppler shifts due to wall vibrations as the mechanism of mode interactions implies that a transition from a mode of frequency \( Nc2L \) to frequency \( (N+1)c2L \) would also raise the energy by a ratio \( (N+1)/N \), taking up energy and momentum from the wall vibrations, and the vice versa. The energy of a mode of \( N \) segments is then proportional to \( N \), as required for Planck quantization per the reasoning above, instead of being invariant of frequency, as Rayleigh had assumed in ignorance of eq. (2). The corrected theory then yields the same cutoff at higher frequencies as Planck’s, as the higher frequencies would be less probable by the same probability factors because of the higher energies.

Since Planck’s theory did not concern matter states, and the standing wave modes, though discrete like the molecular states, led to divergence in Rayleigh’s theory, Planck’s inference of quantization derived from, and thereby depended on, his hypothesis of oscillators with discrete energy levels. This dependence is especially clear from the Bose derivation [29], which uses a dynamical phase space for the radiation energy and momentum discretized under Planck quantization to eliminate Wien’s law. In his 1901 paper [18], Planck introduced a model of \( N \) discrete resonators that could bear only exact multiples of a small amount of energy \( \epsilon \), to compute the probability \( W \) in Boltzmann’s equation \( S = k_B \log W \), from the number of ways the total energy \( U_N = Pe \) could be shared among the \( N \) resonators. This gave

\[
S = k(1 + Ue) \log(1 + Ue) - (Ue) \log(Ue)
\]

for the per-resonator entropy in terms of the resonator energy \( U \). Separately, Wien’s law in the form \( E d\lambda = \theta^5 \gamma (\lambda \theta) d\lambda \)
was shown to imply, via Kirchhoff-Clausius law for blackbody emissivity, the energy density
\[
u = \frac{\nu^3}{c^3} f \left( \frac{\theta}{\nu} \right),
\]
(4)
where \(\gamma\) and \(f\) are functions whose precise form is not significant in the result, and \(\theta\) is the thermodynamic temperature. The energy \(U\) of a resonator had been independently related to the intensity of an applied linearly polarized oscillating field as \(I = U\nu^2 c^2\), which then led, via the relation \(u = 8\pi Ic\), to \(u = 8\pi\nu^2 Ic^3\), so \(U\) could be obtained independently of \(c\), and related to temperature and frequency, by combining with eq. (4) as
\[
U = \nu f \left( \frac{\theta}{\nu} \right) \quad \text{or, equivalently,} \quad \theta = \nu f \left( \frac{U}{\nu} \right).
\]
(5)
The statistical definition of temperature \(dSdU = 1/\theta\) then led to \(S = f(U\nu)\) with eqs. (3), and to the proportionality \(\epsilon \propto \nu\) on comparing with eq. (3). Replacing \(\epsilon\) with \(h\nu\) in eq. (3) and evaluating the derivative \(dSdU\) then led to
\[
U = \frac{h\nu}{e^{h\nu/k_BT} - 1} \quad \text{and, via the } u-I-U \text{ relations above,} \quad u = \frac{8\pi^2 h\nu^3}{c^3} \frac{h\nu}{e^{h\nu/k_BT} - 1}.
\]
(6)
Eqs. (5–6) are directly reproduced from [18] but using symbol \(I\) for the field intensity, instead of Planck’s \(K\), and \(k_B\) for Boltzmann’s constant, to avoid confusion with wave numbers in the present treatment.

The resonator thermodynamic relations in eqs. (5) could not have implied discreteness of energy, since eq. (4) comes from Wien’s law whose considerations are strictly macroscopic, and the only other consideration is the analogue field intensity \(I\). The discreteness and quantization in the result, eqs. (6), thus seem to originate from the oscillator model in eq. (4). The proportionality implication is that the same factor \(h\) must hold for all resonators in any cavity, and also in all sets of cavities allowed to interact via radiation, so \(h\) must be a universal constant, uniquely determined by the empirical energy density spectra, eqs. (6), at different temperatures.

Yet, dimensionally, \(h\) is a scale factor linking energy and frequency scales, much as Boltzmann’s constant \(k_B\) relates energy and temperature and is also a universal constant, and Planck’s model clearly followed Boltzmann’s treatment of molecular energies in equal increments of \(\epsilon\) [46, 47], which does not lead to a quantization of the molecular energies. The result \(\epsilon = h\nu\) does not hold without eqs. (4) from Wien’s law, however, so the cause of quantization in Planck’s theory is again the atomic transitions producing the radiation in the observed spectra. Bose’s derivation showed that the nature of the frequency domain makes \(h\) irreducible to zero, echoed in the quantum condition [48–50], but that aspect is implicit in the inherent anti-commutation of Poisson brackets [51] [6, 21] and confers no more “experimental authority” than say Hamilton already had for anticipating quantization (cf. [52, §10-8]).

To verify this, consider that in standing wave modes, discreteness is assured by definition, and further, any energy \(\epsilon\) transferred from a mode of frequency \(\nu\) to another of frequency \(\nu’ \neq \nu\), by simple reflection and Doppler shift from a moving wall or molecule, or indirectly by its absorption by a molecule and subsequent re-emission with Doppler shift, would be amplified to \(\epsilon \nu’\nu\) by eq. (2), the difference \(\epsilon(\nu’ – \nu)\) coming from the reflecting wall or molecule. Equality of energy in the half-wavelength segments of a mode is assured by wave propagation. Assuming all frequency transitions are governed by eq. (2), if the excitation energy \(\epsilon\) at frequency \(\nu\) transits to a harmonically related mode of frequency \(N\nu\), by eq. (2), the excitation at the harmonic would be \(N\epsilon\), and the \textit{vice versa}. If the transitions are due to classical interactions with molecules or the walls, the harmonic excitation should occur with probability \(e^{-Nk_BT}\). We should find the initial excitation appears as energy \(\epsilon\) at \(\nu\) with probability (or relative frequency) \(e^{-ek_BT}\), as \(2\epsilon\) at \(2\nu\) with probability \(e^{-2k_BT}\), and so on, so the mean energy across harmonically related modes becomes
\[
\langle \epsilon \rangle = \frac{\epsilon e^{-ek_BT} + 2\epsilon e^{-2k_BT} + \ldots}{e^{-ek_BT} + e^{-2k_BT} + \ldots} = \frac{\epsilon}{e^{ek_BT} - 1},
\]
(7)
same as the expression for \(U\) in Planck’s result, eqs. (6), but with no reason to assume \(\epsilon\) is quantized. Eq. (7) implies that harmonic families of standing wave modes behave as Planck’s harmonic oscillators under equilibration by Doppler shifts. The \(-1\) in the denominator implies the same cutoff as Planck’s law, so Rayleigh’s theory stands corrected.

Evaluation of the expectation over arbitrary sets of modes would yield the same result, since both molecular motions and wall vibrations would equilibrate across all modes. This too is easy to prove with the hindsight of past work. The detailed balance between an arbitrary pair of modes of frequencies \(\nu\) and \(\gamma\nu\) then requires \(e^{-\gamma k_BT} = \gamma e^{-\gamma k_BT}\) for a non-zero real ratio \(\gamma\) due to eq. (2), whence \(\gamma = e^{(\gamma - 1)k_BT}\), or equivalently \(\gamma + 1 = e^{\gamma k_BT}\), so that \((\gamma + 1)\gamma = e^{\gamma k_BT}\), whereby the energy expectation is again \(\langle \epsilon \rangle = \epsilon e^{\gamma k_BT} - 1\), along the lines of the Bose-Einstein derivation of Planck’s law (cf. [53, III-4-5]), without depending even on the discreteness of the modes. The constancy of \(h\) in Planck’s result, eqs. (6), thus indeed came from the atomic interactions leading to Wien’s law.
IV. EQUILIBRIUM AND SPECTRA OF WAVE TRAINS

Whereas Wien’s law was a classical constraint in obtaining the blackbody spectrum, physicists have long followed the inverse position epitomized in the correspondence principle, that classical physics is the long wavelength approximation of quantum mechanics. The view reflects the traditional belief that classical physics cannot possibly explain quantum behaviour, whereas extrapolating the internal structure and properties of matter revealed by quantum mechanics to explain macroscopic classical phenomena look like a mere computational exercise. While fundamental limits on such computations are also being discovered [54], our preceding result implies more than a mere equivalence of the power of classical physics, since its reasoning involves inherently fewer assumptions.

The only new premise, that frequency transitions are governed by the Lorentz condition, eq. (2), had been already argued by Einstein in 1917 as a condition for dynamical consistency with the equilibrium of matter. Only the further notion of equilibrating Doppler shifts provided by wall vibrations is new, but is more an intuition than a requirement, because eq. (2) also holds with molecular absorptions and emissions in Einstein’s reasoning. The idea instead allows us to correct the defect of in Boltzmann’s and Planck’s theories of neglecting wall vibrations, which would be available for equilibrating radiation even in a cavity with a perfect vacuum.

Also by Einstein’s argument, both molecular motions and wall vibrations should retain their classical equipartitions requiring a distribution of $k_B T^2$ energy per degree of freedom, but the standing wave modes are spectral components of the total radiation, as well as macroscopic in geometry, and for that reason, cannot exhibit mean energies of $k_B T^2$. As wave trains emitted or absorbed by matter transitions were pointed out as the real microscopic entities of radiation in thermal equilibrium with matter, it might be thought that the wave trains must then possess $k_B T^2$ energies, which too is impossible since the wave train energies must correspond initially to atomic transitions and do not change other than via atomic transitions, in Einstein’s treatment. Radiation is merely a means of exchange of energy and momentum between the material constituents of the classical equipartition. Like the stiffness of massless springs in a mechanical lattice, a constraint on wave train energies would affect the equilibration rate but not the final equilibrium distribution. All initial quantized energies and frequency spreads of the wave trains are lost in the overall equilibrium, so even wave trains, though particle-like in dynamical interactions, are thermodynamically not at all particulate.

The derivation of the equilibrium spectrum for wave trains below illustrates another property of the Doppler effect rarely discussed because of the traditional focus on individual frequencies, which correspond to Fourier components, instead of the composite wave trains they represent. As derived in [1] and also explained in [5], the Lorentz transform implies quadratic variation in electromagnetic field strengths with Doppler shifts, but as the shifts themselves signify an inverse linear variation in wavelengths and thereby in the total duration of any sequence of half-wavelength segments, meaning time dilations, the energies delivered by the wave trains vary linearly with the shifts, yielding eq. (2).

The Doppler time dilations have been inconsequential for sinusoidal wave functions and Fourier components because such functions are considered to extend to infinity. They do affect timings in signal modulation, as discovered recently the hard way [53, 54], and admit propagating chirp modes that have been correspondingly overlooked in all of physics for three centuries [57]. The time dilations were not relevant in the reasoning of eq. (7) as standing wave modes are Fourier components extending in time to $\pm \infty$. The mode energies and probabilities were determined from the varying number $N$ of half-wavelength segments under Doppler transitions under a static cavity dimension $L$. The lengths and amplitude distributions of wave trains would be unchanged by reflections at strictly static walls even as the wavefronts are spatially scattered, but both would be affected by the Doppler shifts from vibrating walls.

Three further complications arise in considering wave trains. Firstly, only one standing wave mode is defined to exist at each location within a cavity with given frequency, direction, polarization and angular momentum characteristics, but any number of wave trains can share these properties, analogous to photons in lasing. As the spectral contributions of each wave train in equilibrium must bear the same shape, and therefore follow eq. (4).

Secondly, eq. (7) was derived for the thermalization of modes describing the total radiation, and a similar treatment seems needed for wave train spectra, but eq. (2) is only applicable to finite sequences of half-wavelength segments, whereas a Fourier component of a wave train would have infinite segments. A solution exists in the Dirichlet treatment for Fourier representations over a finite interval, according to which the behaviour of components outside the interval should be immaterial. The component energy beyond the wave train must be irrelevant because the components must cancel out in phase beyond the wave train. This notion is not diminished by the spatial dispersion of the wave fronts by the wall reflections, since the energy delivered by a component at any given location is an integral over time. With wall vibrations, continuity of the wave trains is also not assured because the velocity of a reflecting wall will fluctuate over the duration and expanses of a wave train. The temporal volume of the original wave train is unaffected except as the overall Doppler time dilation, however, so both time dilations and the Lorentz property are meaningful.

The third complication is that the probability factors of the “oscillator expansion” represented in eq. (7), as well as in the corresponding detailed balance of frequencies $\nu$ and $\gamma \nu$, concern different standing wave modes, as in Boltzmann’s original reasoning for molecular energies in the kinetic theory, but must now relate spectral components of the same
wave train. If a wave train occurred with probability \( p \) at overall energy \( \epsilon \), which got boosted to \( \gamma \epsilon \) by a Doppler shift of its spectrum, the probability of observing it at the total energy \( \gamma \epsilon \) should be still \( p \), as Doppler shifts do not create or annihilate waves or wave spectral components.

However, wave trains suffer fragmentation due to the wall vibrations, and the fragments suffer differing, unrelated shifts at random, so the multiplicative law \( p(u) \equiv e^{-u k_B T} = \exp(-\sum_i u_i k_B T) \equiv \prod_i p(u_i) \), holds, where \( u_i \) denote fragment probabilities. If a fragment bears energy \( \nu \) due to the Lorentz increase and only for the shorter period or annihilate waves or wave spectral components.

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Despite reproducing Planck’s result and reinforcing his own heuristic of radiation quanta in photoelectricity theory with momentum arguments, Einstein’s 1917 derivation was vigorous reasoning of classical dynamics, and its prediction of stimulated emission, for instance, would have been unconvincing otherwise. It traced quantization to the discreteness of atomic states and constancy of their transition energies, which means conversely that the equilibrium should exhibit the Planck form without quantization if matter state transitions were excluded. This converse implication has been proved above for the equilibration both of standing wave modes and of arbitrary sets of wave trains.

That Einstein did not emphasize this fundamental implication of matter interactions in his derivation of Planck’s law, and of the Lorentz property that he himself had discovered, and instead endorsed Bose’s derivation and formulated Bose-Einstein statistics, is consistent with his focus on the internal structure of matter. It is at most a error of omission in not examining the converse implication, and an informal concern against the present result.

More particularly, the physicists’ ignorance of the classical Lorentz property, and not an inability of classical physics, has been shown the real reason for Rayleigh’s failure in obtaining the correct law. Likewise, the inattention to matter interactions, which enter only via Wien’s law and were addressed only later by Einstein, has been shown to have left Planck unable to distinguish the origin of quantization from a computational artefact in estimating entropy also used by Boltzmann. An even older defect, neglect of thermal vibrations of the bounding walls in direct contact with a gas or radiation stipulated to be in equilibrium, has been also identified and corrected with the converse insight.

V. CONCLUSION

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[1] A Einstein. On the electrodynamics of moving bodies. Ann Phys. 17(10):891–921, 1905.
[2] A Einstein. On the quantum theory of radiation. Phys Zu, 18, 1917.
[3] D Redžić and J Strnad. Einstein’s light complex. Fizika A, 13(3):113–120, 2004.
[4] J D Norton. Atoms, entropy, quanta: Einstein’s miraculous argument of 1905. Studies History & Phil Mod Phys, 37:71–100, 2006.
[5] J D Norton. Einstein’s miraculous argument of 1905: The thermodynamic grounding of light quanta. In C Joas, C Lehner, and J Renn, editors, HQ-1: Conf History Quant Phys. Max-Planck-Institut für Wissenschaftsgeschichte, 2008.
[6] P A M Dirac. The Principles of Quantum Mechanics. Cambridge Univ, 4th edition, 1953.
[7] S Popescu and B Rothenstein. Counting energy packets in the electromagnetic wave. 2007. arXiv:0705.2655.
[8] V Guruprasad. The radiation equilibrium is classical. In APS March 2006 Meeting, March 2006. arXiv:physics0003041.
[9] R Brady and R Anderson. Violation of Bell’s inequality in fluid mechanics. 2013. arXiv:1305.6822v1.
[10] C Abelln, W Amay, V Pruneri, M W Mitchell, M Markham, D J Twitchen, D Elkouss, S Wehner, T H Taminiau, and R Hanson. Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. Nature, 526:682–686, 2015. arXiv:1508.05949.
[11] E J Carroll. Classical Maxwellian polarization entanglement. 2015. arXiv:1511.06185.
[12] X-F Qian and J H Eberly. Entanglement is sometimes enough. 2013. arXiv:1307.3772 [quant-ph].
[13] B Hensen, H Bernien, A E Drau, A Reiserer, N Kalb, M S Blok, J Ruitenberg, R F L Vermeulen, R N Schouten, C Abell, W Amay, V Pruneri, M W Mitchell, M Markham, D J Twitchen, D Elkouss, S Wehner, T H Taminiau, and R Hanson. Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. Nature, 526:682–686, 2015. arXiv:1508.05949.
[14] P Ghose and A Mukherjee. Entanglement in classical optics. Rev Theor Sci, 2:1–14, 2014. arXiv:1308.6154 [physics.optics].
[15] X-F Qian, B Little, J C Howell, and J H Eberly. Shifting the quantum-classical boundary: theory and experiment for statistically classical optical fields. Optica, 2(7), 2015.
