Anisotropy effects on Baryogenesis in $f(R)$-Theories of Gravity

A. Aghamohammadi,1‡ H. Hossienkhani,2† and Kh. Saaidi3

1 Islamic Azad University, Sanandaj Branch, Sanandaj Iran.
2 Hamedan Branch, Islamic Azad University, Hamedan, Iran
3 Department of Physics, Faculty of Science, University of Kurdistan, Sanandaj, Iran

(Dated: April 9, 2018)

We study the $f(R)$ theory of gravity in an anisotropic metric and its effect on the baryon number to entropy ratio. The mechanism of gravitational baryogenesis based on the CPT-violating gravitational interaction between derivative of the Ricci scalar curvature and the baryon-number current is investigated in the context of the $f(R)$ gravity. The gravitational baryogenesis in the Bianchi type I (BI) Universe is examined. We survey the effect of anisotropy of the Universe on the baryon asymmetry from point of view the $f(R)$-theories of gravity and its effect on $n_b/s$ for radiation dominant regime.

I. INTRODUCTION

In recent decades, it was indicated that our Universe is in a positive accelerating expansion phase [1–3], and the standard model of gravity could not explain this phenomena. This is a shortcoming for Einsteinian theory of gravity and several attempts have been accomplished to solve it. One way for explaining this positive accelerating expansion is modified gravity. In fact some people have believed this shortcoming of Einsteinian theory of gravity is coming from the geometrical part of Hilbert-Einstein action and try to modify it by replacing a function of Ricci scalar, $f(R)$, instead of $R$ in the Hilbert-Einstein action and so-called $f(R)$ model of gravity. The reason relies on the fact that they allow to explain, via gravitational dynamics, the observed accelerating phase of the Universe, without invoking exotic matter as sources of dark matter. There are another modified gravity formalism which are completely different with $f(R)$ models of gravity. The various aspect of $f(R)$ models of gravity is investigated in [4–18]. The unification of dark energy and early time inflation with late time acceleration from $f(R)$ theory to all Lorentz non-invariant theories is discussed by Nojiri and Odintsov [19]. In addition to this model, a singular analog of $R^2$ inflation is studied in [20–23]. It is well known, the most of researches about the dynamical evolution of the Universe have been done in a homogenous and isotropic space-time background such as FLRW. But tiny deviation from isotropy at the level of $10^{-5}$, has also been suggested by Bennett et al (1996) and afterwards this suggestion was confirmed by high resolution WMAP data. Although, by considering the present Universe, it is known that the anisotropy is small. The possible effects of anisotropy in the early Universe have investigated with Bianchi I type (BI) models from different point of view [24–23]. The Kasner-type is a special class of BI model, for which cosmological scale factors evolve as a power law in time. In GR the vacuum Kasner solutions [30] and their fluid filled counterparts, the BI models, proved useful as a starting point for the investigation of the structure of anisotropic models. Barrow and Clifton [31, 32] have recently shown that it is also possible to find solutions of the Kasner type for $R^n$-gravity models. Recently, Hossienkhani et al. [33] discussed the effects of the anisotropy on the evolutionary behavior DE models and compare with the results of the standard FRW, ΛCDM and $w$CDM models. Also, they shown that the anisotropy is a non-zero value at the present time although it is approaching zero, i.e. the anisotropy will be very low after inflation.

The main purpose of this paper is that show $f(R)$-theories of gravity provide a framework in which the gravitational baryogenesis may occur in the anisotropic background and lead to the observed baryon asymmetry in the Universe. The cause of the baryon number asymmetry is still an open problem of the particle physics and cosmology. The measurements of cosmic microwave background [34], the absence of $\gamma$ ray emission from matter-antimatter annihilation [35] and the theory of Big-Bang nucleosynthesis [36, 37] imply that there is matter in excess of antimatter in the Universe. The observational results indicates that the ratio of the baryon number to entropy density is approximately $n_b/s \sim 10^{-10}$. In [38], it was pointed out that a process generating, baryon, antibaryons and those different rates may be satisfied by the following three conditions

*Electronic address: a.aqamohamadi@gmail.com; a.aghamohamadi@iausdjk.ac.ir
†Electronic address: Hossienhossienkhani@yahoo.com
‡Electronic address: ksaaidi@uok.ac.ir
1. Baryon number non-conservation.
2. C- and CP-symmetry violation.
3. Deviation from thermal equilibrium.

To satisfy two later conditions, the conventional approach has been to introduce interactions which violate C and CP in vacuo and a period in which the Universe is out of thermal equilibrium. Referring to Lorentz’s and CPT symmetries, the more general setting in which they have been studied is the Standard Model Extension (SME) \cite{39, 40}. However, a dynamically violation of CPT may be lead to generation the baryon number asymmetry also in regime of thermal equilibrium \cite{41}. They have introduced an interaction between Ricci scalar curvature and any current that give rise to net $B-L$ charge in equilibrium ($L$ is lepton number) which dynamically violates CPT symmetry in expanding Friedmann Robertson Walker (FRW) Universe. As a consequence, in this work, the baryon number asymmetry can not be directly generated in radiation dominated epoch. But in \cite{42}, in the framework of modified theories of gravity, it was shown that the baryon asymmetry may be generated even in the radiation dominated era. We explicitly calculate the asymmetry in our scenario and compare it to the baryon asymmetry of the Universe. We follow the work of Ref. \cite{42}. Ref. \cite{43} studied the gravitational baryogenesis scenario, generated by an $f(T)$ theory of gravity. But in \cite{44}, it was calculated the baryon to entropy ratio for the Gauss-Bonnet term and by using the observational constraints. Moreover, in the context of modified it is possible to generalize the gravitational baryogenesis mechanism, in the $f(R)$ theory of gravity \cite{45} and the Loop Quantum Cosmology \cite{46}. In a less symmetric background spacetime, however, some possibility of the enhancement of the baryon asymmetry was argued in \cite{47, 48}. They showed that the baryon asymmetry will increase with the anisotropy of the Universe. In this way more general cosmological evolutions can be considered and the resulting baryon-to-entropy ratio is compatible to the observational data. This paper is organized as follows. In the next section we first review the field equation of $f(R)$ gravity theory in the BI Universe. In Sect. III, we study the gravitational baryogenesis in the anisotropic Universe. At last, we summarize our results in last section.

II. DESCRIPTION AND GENERAL PROPERTIES OF THE MODEL

A. $f(R)$ Theory

The action $S$ of $f(R)$ gravity with general matter is given by

$$S = \int \sqrt{-g} \, d^4 x \, \left[ \frac{f(R)}{2} + L_m(\psi, g_{\mu\nu}) \right],$$

where $f(R)$ is an arbitrary function of Ricci scalar, $R$, $L_m = L_m(\psi, g_{\mu\nu})$, $\psi$, $g_{\mu\nu}$ and $g$ are the matter Lagrangian, the matter field, the metric of space-time and the determinant of metric respectively. Variation of (1) with respect of $g^{\mu\nu}$ gives

$$R_{\mu\nu} f' - \frac{1}{2} f g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f' = \kappa^2 T^m_{\mu\nu},$$

where prime represents the derivative with respect to the scalar curvature $R$, $\Box \equiv \nabla_\alpha \nabla^\alpha$ and $T^m_{\mu\nu}$ are the covariant d’Alembert operator and the stress-energy tensor of matter respectively and $T^m_{\mu\nu}$ is defined by

$$T^m_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(-gL_m)}{\delta g^{\mu\nu}}.$$ (3)

B. Solutions on the anisotropic Universe

The simplest model of the non-isotropic Universe is BI model, which exhibit a homogeneity and special flatness and is the straightforward generalization of the flat FRW. Therefore we introduce the line element of the BI metric as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2,$$ (4)

where the metric functions, $A$, $B$, $C$, are only functions of time, $t$. It is assumed that the matter is perfect fluid, then the energy momentum tensor is given by

$$T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu},$$ (5)
where \( \rho, p, h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu \) and \( u^\mu \) are the total energy density of a perfect fluid, the corresponding total pressure, the projection tensor and the flow vector respectively, and the latter satisfying the following relation

\[
u_\mu u^\mu = -1. \tag{6}\]

The mean Hubble parameter \( H \) is obtained by

\[
H = \frac{1}{3} \theta = \frac{\dot{a}}{a}, \tag{7}
\]

where \( \theta = u^j_{\phantom{j}j}, a = (ABC)^{1/3} \) are the scalar expansion and the scale factor respectively. It is defined the shear scalar \( \sigma_{ij} \) as

\[
\sigma_{ij} = u_{i,j} + \frac{1}{2}(u_{i;k}u^{k,j} + u_{j;k}u^{k,i}) + \frac{1}{3} \theta h_{ij}, \tag{8}
\]

by using Eqs. (6) and (8), we obtain

\[
\sigma^2 = \frac{1}{2}\left[ \left( \frac{\dot{A}}{A} \right)^2 + \left( \frac{\dot{B}}{B} \right)^2 + \left( \frac{\dot{C}}{C} \right)^2 \right] - \frac{1}{6} \theta^2. \tag{9}
\]

In the context of BI Universe, the Ricci scalar is given by

\[
R = 6(\dot{H} + 2H^2) + 2\sigma^2. \tag{10}
\]

We assume the Universe filled with perfect fluids and it satisfies the effective equation of state \( \omega = p/\rho \) and \( T^m = T^{\mu\nu}_m. \) Combining Eqs. (2)-(5) gives

\[
\kappa^2 \rho = \frac{f(R)}{2} - \left( 3(\dot{H} + H^2) + 2\sigma^2 \right) f'(R) + 3H\dot{R}f''(R), \tag{11}
\]

\[
\kappa^2 p = -\frac{f(R)}{2} + (3H^2 + \dot{H})f'(R) - (2H\dot{R} + \ddot{R})f''(R) - f'''(R)\dot{R}^2. \tag{12}
\]

Moreover, the Bianchi identities give an another condition on the conservation of the energy

\[
\dot{\rho} + \theta (\rho + p) = 0. \tag{13}
\]

Solving Eqs. (11)-(12), requiring an explicit form from \( f(R) \) gravity. Hence, we take advantage of a \( f(R) \) function proposed in Ref. [50]

\[
f(R) = R + R \ln \left( \frac{R}{R_c} \right)^{\mp \epsilon}, \tag{14}
\]

where \( R_c \) is positive constants. It is clear \( f(R)|_{R=0} = 0, \) at the flat space time and in the \( \epsilon \ll 1, \) Eq. (14) reduced to

\[
f(R) = R (\frac{R}{R_c})^{\mp \epsilon}, \tag{15}
\]

or for convenience, one can write

\[
f(R) = R^{1 \pm \epsilon}, \tag{16}
\]

where \( \epsilon \) is a constant. For obtain stability of the above \( f(R), \) the following condition must be satisfied [15].

\[
\frac{d^2 f}{dR^2} \geq 0. \tag{17}
\]

For determining of \( +\epsilon, \) by employ the above constraint and limit of power-law solution in the expanding anisotropic Universe, the \( \epsilon \) form should satisfy the following relations [8, 51]

\[
\epsilon \geq 0. \tag{18}
\]
In order to gain better insight we assume $A(t) = t^\alpha$ and $B(t) = C(t) = t^{\alpha n}$, where $\alpha$ and $n$ are real constant. For a positive expand Universe, the allowed ranges for the parameter $\alpha$ is $\alpha \geq 1.1$ and against, small deviations from the isotropic background the $n$ parameter is taken $0.9 \leq n \leq 1.1$. Furthermore, one can define a scale factor as $a(t) = (ABC)^{1/3} = t^{\alpha(1+2n)/3}$. So based on this definition, we obtain the Hubble parameter, the shear and Ricci scalar are given by

$$H = \frac{\alpha(1+2n)}{3t},$$

$$\sigma^2 = \frac{\alpha^2(1-n)^2}{3t^2},$$

$$R = \frac{2\alpha^2(1+2n+3n^2) - 2\alpha(1+2n)}{t^2}. \tag{21}$$

By using Eqs. (16), (19), (20) and (21), Eqs. (11), (12) and (13) end up where $\epsilon = \alpha(1+2n)/2 - 1/2$. From [16] it is clearly obvious that for $\alpha = 1/(1+2n)$, the obtained $f(R)$, [16], reduces to $f(R) = R$ which is the standard Einstein general relativity action. From Eq. (21), it follows that

$$\dot{R} = -\frac{4\alpha}{t^3}[\alpha(1+2n+3n^2) - (1+2n)]. \tag{22}$$

If the space time to be isotropic, $\sigma = 0$ ($n = 1$), Eq. (22) reduce to the result of [42].

### III. GRAVITATIONAL BARYOGENESIS IN ANISOTROPIC UNIVERSE

The existent mechanism in [52] to generate baryon asymmetry is proposed by the presentation a dynamical breaking CPT. The responsible interaction is specified by coupling between the derivative of the Ricci scalar curvature $R$ and the baryon current $J^\mu$ [42]. The Ricci scalar and the baryon number current, $J^\mu$, are given by

$$S_{\text{int}} = \frac{\varepsilon}{M_*^2} \int d^4x \sqrt{-g} \partial_\mu R J^\mu, \tag{23}$$

where $M_*$ is the cutoff scale characterizing the effective theory and $\varepsilon = \pm 1$. The baryon number density in the thermal equilibrium has been worked out in detail in [52]. It lead to

$$n_b = n_B - \bar{n}_B = \frac{g_B T^3}{6\pi^2} \left( \frac{\pi^2 g_B}{T} + \left(\frac{\mu_B}{T}\right)^3 \right), \tag{24}$$

where $\mu_B$ is a chemical potential and $\mu_B = -\mu_B = -\epsilon R/M_*^2$ and $g_b \simeq 1$ is the number of internal degrees of freedom of baryons. The baryon to entropy ratio is [52]

$$\frac{n_b}{s} \simeq -\frac{15g_b}{4\pi^2 g_s} \frac{\dot{R}}{M_*^4 T_D^3}|_{T_D}, \tag{25}$$

where $s = (2\pi^2/45)g_s T^3$, where $g_s \simeq 106$. In an expanding Universe the baryon number violation decouples at a temperature denoted by $T_D$ and a net baryon asymmetry is remaining.

#### A. $f(R) = R$

For the case of $\epsilon = 0$ and using Eqs. (22) and (25), the baryon asymmetry in terms of temperature can be determined by

$$\frac{n_b}{s} \simeq \frac{15g_b}{\pi^2 g_s} \frac{\alpha(1+2n+3n^2) - (1+2n)}{M_*^2 T_D^4 t_D^3}, \tag{26}$$

where $t_D$ is the decoupling time. An important assumption that needs to be taken into account is that the vacuum decays "adiabatically", that means the specific entropy of the massless particles remains constant irrespective of the total entropy, which may increase. In this case, some equilibrium relations are hold but not all [52], for instance the
energy density versus temperature, $\rho_R \propto T^4$, particle number versus temperature, $n_R \propto T^3$, but the temperature does not obey the scaling relation $T_R \propto a^{-1}$. In what follows, by using the following usual expression of the energy density \[\rho = KT^4, \] (27)
where $K = g_s \pi^2/30$ is proportional to the total number of effectively degree of freedom, we get
\[T = \left(\frac{30}{\pi^2 g_s}\right)^{1/4} \left(\frac{\alpha^2 n(n+2)}{t^{1/2}}\right)^{1/4} m_p, \] (28)
where $m_p \approx 1.22 \times 10^{19}$GeV is the Planck mass. Inserting (28) into (26), one obtains
\[\frac{n_b}{S} \approx 2.95 \left(\frac{T_D}{m_p}\right)^5 \left(\frac{m_p}{M_*}\right)^2 G(\alpha, n), \] (29)
where $G(\alpha, n) = \frac{\alpha (1+2n+3n^2)-(1+2n)}{\alpha^2 (1+2n)^2}$. Fig. 1, shows the $\frac{n_b}{S}$ versus $\alpha$ for three different values of the $n$ parameter. It is clear that the curve is shifted to the smaller value of the $\frac{n_b}{S}$ with increasing $n$ as can be seen from the diagram, also, the $\frac{n_b}{S}$ decrease with increasing of the $\alpha$ parameter, in addition, by increasing of the $n$, its slope becomes sharper. Fig. 2, illustrate the $\frac{n_b}{S}$ versus $n$ parameter for three different values of the $\alpha$ parameter. It is clear that the curve is shifted to the smaller value of the $\frac{n_b}{S}$, with increasing $\alpha$ as can be seen from the diagram, also, the $\frac{n_b}{S}$ decrease with increasing of the $n$ parameter, and its slope becomes sharper.

B. $f(R) = (R/a)^{1+\epsilon}$

As an example, we examine the case that $f(R) = (R/a)^{1+\epsilon}$, where $A = m_p^2 - \frac{2}{\alpha}$. In this case, using Eqs. (11), (19), (20), (21), (22) and (27) and contribute the $\epsilon = \alpha(1+2n)/2 - 1/2$, we can obtain the decoupling temperature $T_D$ as
α

1.4

1.4

1.6

1.8

2.0

1.2

54x225

α

sharper slope is shifted to the smaller value of parameter. 

It is clears that the n diagram with a bit of sharper slope is shifted to the smaller value of n affect the baryon-to-entropy ratio in the same way, and thus the baryon-to-entropy ratio may be used to constrain the parameter, but with decreasing α, the 4b/s close to zero value, in addition, by increasing the parameter, the diagram with sharper slope is shifted to the smaller value of α. Fig. 4, shows the nb/s versus n for three different values of the parameter. It is clear that the nb/s is from order of 10^{-14} for the larger value of n, against the different values of the parameter, but with decreasing n, the nb/s close to zero value, in addition, by increasing the parameter, the diagram with a bit of sharper slope is shifted to the smaller value of n. As it can be seen, both the parameters α and n affect the baryon-to-entropy ratio in the same way, and thus the baryon-to-entropy ratio may be used to constrain the functional form of the f(R) gravity.

C. f(R) = R + \frac{1}{\Lambda} R^2

It would be interesting to analyze another realistic model in f(R) gravity [20–23] and compare its results to the preceding case.

f(R) = R + \frac{1}{H_i} R^2,  \quad (32)
with $H_i \gg 1$. The dynamics presented by this model is found to be in agreement with the data presented by the recent observations of the Planck collaboration [57]. This model could be helpful in the study of non-singular version of the Starobinsky $R^2$ inflation model. The same as preceding case, decoupling temperature for this model becomes

$$T = \left(\frac{m_p^2\alpha^2}{\pi^2 g_s H_i}\right)^{\frac{4}{7}} \frac{1}{t} \left(60[3 + \alpha + n(6 + (n - 2)\alpha)][-1 + \alpha + n(-2 + (2 + 3n)\alpha)] - 30nH_it^2(n + 2)\right)^{\frac{1}{7}}. \quad (33)$$

By substituting $T$ from Eq. (33) into (25), the final expression for the baryon-to-entropy ratio is equal to

$$\frac{n_b}{S} \simeq \frac{15}{M_*^2 \pi^2 t_D^2} \sqrt{\frac{\alpha}{m_p^2 g_s^3}} \left(\frac{H_i}{g_s}\right)^{\frac{4}{7}} \frac{-1 + \alpha + n(-2 + (2 + 3n)\alpha)}{\left(60[3 + \alpha + n(6 + (n - 2)\alpha)][-1 + \alpha + n(-2 + (2 + 3n)\alpha)] - 30nH_it^2(n + 2)\right)^{\frac{1}{7}}}, \quad (34)$$

where $H_i = 6.3 \times 10^{13}s^{-1}$ [22, 57], and $t_D = 3 \times 10^{-13}s$, for $\alpha = 1.1, n = 1$. Fig. 5, shows the $n_b/s$ versus $\alpha$ for three different values of the $n$ parameter. It is clear that the $n_b/s$ is from order of $10^{-11}$, and the $n_b/s$ is increased by increasing of $\alpha$, in addition, by increasing the $n$ parameter, the diagram is shifted to the larger value of $n_b/s$. Fig. 6, shows the evolution of $n_b/s$ versus $n$ is the same as Fig. 5. In addition, it is clear that the $n_b/s$ ratio is in better agreement with observations rather than the prior case.

### IV. CONCLUSION

The main purpose of the present work has been the study of the $f(R)$ theory of gravity in an anisotropic metric and evaluate its effect on the baryon number to entropy ratio. In the context of $f(R)$ baryogenesis, the baryon-to-entropy ratio depends on $\dot{R}$, and we discussed three cases of $f(R)$ theories of gravity, the case $f(R) = R$, $f(R) = R^{1+\epsilon}$ and also $f(R) = R + \frac{1}{M_*^2} R^2$. It has been show that the $f(R)$ theories of gravity provide a natural setting in which the baryon asymmetry in the Universe may be generated through the mechanism of the gravitational baryogenesis. we have shown, the variables $\alpha$ and $n$ plays a crucial role in the calculation of the baryon-to-entropy ratio. These results depend on the exact solution of $f(R)$ field equations. Therefore, by select valid different $f(R)$ functions, one can examined the evolution of the baryon number to entropy ratio to obtained a best agreement to value of $n_b/s$ estimated. Also, we have obtained $\dot{R}$ and $n_b/s$ for radiation dominant regime and the effect of anisotropy of space time obviously was seen.
in it, in addition, the imprint of shear tensor was inevitable and if fix $\sigma = 0$ then $n_b/s = 0$. It is clear, that we have shown the baryon asymmetry in anisotropic Universe is larger than the baryon asymmetry in Friedmann Robertson Walker (FRW) space time. It is shown that deviations from general relativity on the anisotropic Universe in the $f(R)$ gravity may be considerable in the study of Universe. We have considered the plots $n_b/s$ for three case $f(R) = R$ and $f(R) \neq (\frac{\lambda}{R})^{1+\epsilon}$, and $f(R) = R + \frac{\lambda}{R}R^2$, which, those results have been explained in below the relevant figures. It was explicit, procedure of the evolution $n_b/s$ for the case $f(R) = R$ is subtractive but for $\frac{\lambda}{R}R^2$, and $f(R) = R + \frac{1}{2}R^2$, this trend is increasing. Finally, we close the work drawing a possible connection of this scenario with the extended gravity theories. It is well known that the physics of the Universe at early times constitutes a great laboratory to explore physics beyond the standard cosmological model. For instance, torsion in $f(T)$ gravity can be constrained by big bang nucleosynthesis. Thus, it is worth noting that the nonzero ratio of baryon-to-entropy of the Universe could be a potential quantity to constrain extended theories of gravity too.

[1] A.G. Reiss, A.V. Filippenko, P. Challis et al., Astron. J. 116, 1009 (1998); S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
[2] D.N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003).
[3] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004).
[4] K. Freese and M. Lewis, Phys. Lett. B 540, 1 (2002).
[5] S. Nojiri, S.D. Odintsov, Gen. Rel. Grav. 36, 1765 (2004).
[6] N. Banerjee, D. Pavón, Phys. Lett. B 647, 447 (2007).
[7] N. Banerjee and D. Pavón, Phys. Rev. D 63, 043504 (2001); Class. Quantum Grav. 18, 593 (2001).
[8] S. Capozziello, S. Carloni and A. Troisi, Astron. Astrophys. 1, 625 (2003).
[9] S. Nojiri and S.D. Odintsov, Phys. Rev. D 68, 123512 (2003).
[10] S.M. Carroll, Phys. Rev. D 71, 063513 (2005).
[11] S. Capozziello, S. Nojiri, S.D. Odintsov, A. Troisi, Phys. Lett. B 639, 135 (2006).
[12] M.C.B. Abdalla and S.D. Odintsov, Class. Quantum Grav. 22, L35 (2005).
[13] A. Aghmohammadi et al., Phys. Scr. 80, 065008 (2009).
[14] K. Saaidi, A. Vajdi, A. Aghmohammadi, Gen. Rel. Grav. 42, 2421 (2010).
[15] A. Aghmohammadi, K. saaidi, M.R. Abolhassani, Int. J. Theor. Phys. 49, 709 (2010).
[16] A. Aghmohammadi, K. saaidi, Phys. Scr. 83, 025902 (2011).
[17] K. Saaidi et al., Int. J. Mod. Phys. D 21, 1250057 (2012).
[18] A.A. Starobinsky, JETP Lett. 86, 157 (2007).
[19] S. Nojiri and S.D. Odintsov, Phys. Rept. 505, 59 (2011).
[20] A.A. Starobinsky, Phys. Lett. B 91, 99 (1980).
[21] S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Phys. Rept. 692, 1 (2017).
[22] S.D. Odintsov, V.K. Oikonomou, Phys. Rev. D 92, 124024 (2015).
[23] S. Appleby, R. Battye, A. Starobinsky, JCAP 1006, 005 (2010).
[24] E. Komatsu et al., Astrophys. J. Suppl. Ser. 180, 330 (2009).
[25] S. Kumar, A.K. Yadav, Mod. Phys. Lett. A 26, 647 (2011).
[26] A.K. Yadav, L. Yadav, Int. J. Theor. Phys. 50, 218 (2011).
[27] A.K. Yadav, F. Rahman, S. Ray, Int. J. Theor. Phys. 50, 871 (2011).
[28] B. Saha, Phys. Rev. D 64, 123501 (2001).
[29] B. Saha, T. Boyadjiev, Phys. Rev. D 69, 124010 (2004).
[30] E. Kasner, Trans. Am. Math. Soc. 27, 101 (1925).
[31] J.D. Barrow and T. Clifton, Class. Quantum Grav. 23, L1 (2006).
[32] T. Clifton and J.D. Barrow, Class. Quantum Grav. 23, 2951 (2006).
[33] H. Hossienkhani et al., Physics of the Dark Universe 18, 17 (2017).
[34] C. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003).
[35] A.G. Cohen, A.D. Rujula and S.L. Glashow, Astrophys. J. 495, 539 (1998).
[36] S. Burles, K.M. Nollet and M.S. Turner, Phys. Rev. D 63, 063512 (2001).
[37] M. Hashemi, S. Jalalzadeh and S. Vasheghani Farahani, Gen. Rel. Grav. 47, 139 (2015).
[38] A.D. Sakharov, JETP 5, 24 (1967).
[39] D. Colladay, V.A. Kostelecký, Phys. Rev. D 58, 116002 (1998); Phys. Rev. D 55, 6760 (1997).
[40] S.D. Odintsov and V.K. Oikonomou, EPL 116, 49001 (2016).
[41] A. Cohen, D. Kaplan, Phys. Lett. B 199, 251 (1987).
[42] G. Lambiase and G. Scarpetta, Phys. Rev. D 74, 087504 (2006).
[43] V.K. Oikonomou and E.N. Saridakis, Phys. Rev. D 94, 124005 (2016).
[44] S.D. Odintsov, V.K. Oikonomou, Phys. Lett. B 760, 259 (2016).
[45] V.K. Oikonomou, Int. J. Geom. Meth. Mod. Phys. 13, 1650033 (2016).
[46] S.D. Odintsov and V.K. Oikonomou, Europhys. Lett. 116, 49001 (2016).
[47] Kh. Saaidi and H. Hossienkhani, Astrophys. Space Sci 333, 305 (2011).
[48] V. Fayaz and H. Hossienkhani, Astrophys. Space Sci 344, 291 (2013).
[49] N. Dadhich, (2005) arXiv: gr-qc/0511123.
[50] A. Aghamohammadi, K. Saaidi, Astrophys. Space. Sci. 333, 327 (2011).
[51] S. Nojiri, S.D. Odintsov, Phys. lett. B 657, 238 (2007).
[52] H. Davoudiasl, R. Kitano, G.D. Kribs, H. Murayama and P.J. Steinhardt, Phys. Rev. Lett. 93, 201301 (2004).
[53] J.A.S. Lima, Phys. Rev. D 54, 2571 (1996).
[54] J.A.S. Lima, S. Basilakos and J. Solá, Eur. Phys. J. C 76, 228 (2016).
[55] E.W. Kolb and M.S. Turner, “The Early Universe”, Westview Press, (1994).
[56] K.C. Jacobs, thesis “Bianchi type I cosmological models” California Institute of technology, (1969).
[57] P.A.R. Ade et al, [Planck Collaboration], Astron. Astrophys. 571, A22 (2014); P.A.R. Ade et al. [Planck Collaboration], Astron. Astrophys. 594, A13 (2016).
[58] S. Capozziello, G. Lambiase and E.N. Saridakis, Eur. Phys. J. C 77, 576 (2017).