Cosmological Density Perturbations From
A Quantum Gravitational Model Of Inflation

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ABSTRACT

We derive the implications for anisotropies in the cosmic microwave background following from a model of inflation in which a bare cosmological constant is gradually screened by an infrared process in quantum gravity. The model predicts that the amplitude of scalar perturbations is $A_S = (2.0 \pm .2) \times 10^{-5}$, that the tensor-to-scalar ratio is $r \approx 1.7 \times 10^{-3}$, and that the scalar and tensor spectral indices are $n \approx .97$ and $n_T \approx -2.8 \times 10^{-4}$, respectively. By comparing the model’s power spectrum with the COBE 4-year RMS quadrupole, the mass scale of inflation is determined to be $M = (.72 \pm .03) \times 10^{16}$ GeV. At this scale the model produces about $10^8$ e-foldings of inflation, so another prediction is $\Omega = 1$.

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1 Introduction

The view that the very early universe underwent a period of inflation at some
large mass scale $M$ is strongly supported by the homogeneity and isotropy
of the cosmic microwave background, and by the absence of relics such as
magnetic monopoles [1]. An enormous variety of models have been proposed
to implement inflation [2, 3, 4, 5, 6, 7, 8, 9], all of which involve a dynamical
scalar degree of freedom in some form. Another common feature of these
models is that the cosmological constant must be fine tuned so that inflation
can end. Many models require additional fine tuning in order to make infla-
tion last long enough and in order that quantum fluctuations near the end of
inflation can generate a plausible spectrum of primordial density fluctuations.

Recently a model has been proposed in which fundamental scalars play
no role and for which the cosmological constant is not fine tuned to zero
[10]. Indeed, inflation begins in this model for no other reason than that the
cosmological constant is not unreasonably small. It ends due to the secular
accumulation of gravitational binding energy between virtual gravitons
which have become trapped in the superluminal expansion of spacetime and
are therefore unable to recombine. This effect is unique to particles that are
effectively massless and yet not conformally invariant, the only definitively
known example of which is the graviton [11]. The process is slow because
gravity is a weak interaction, even at GUT scales. However, it must even-
tually null the bare cosmological constant since the effect is coherent and
persists for as long as inflation does.

Because the mechanism operates in the far infrared, it can be studied
perturbatively using quantum general relativity:

$$L = \frac{1}{16\pi G} (R - 2\Lambda) \sqrt{-g} + \text{counterterms} ,$$

without regard to ultraviolet divergences or modifications at the Planck scale.
We did this on the manifold $T^3 \times \mathbb{R}$, in the presence of a homogeneous and
isotropic state for which the expectation value of the metric has the form:

$$\langle 0 | g_{\mu\nu}(t, \vec{x}) dx^\mu dx^\nu | 0 \rangle = -dt^2 + e^{2b(t)} d\vec{x} \cdot d\vec{x} ,$$

with initial conditions $b(0) = 0$ and $\dot{b}(0) = H \equiv \sqrt{\frac{\Lambda}{3}}$. The result is [12]:

$$b(t) = Ht (1 + \ldots) + \frac{1}{2} \ln \left( 1 - \frac{172}{9} \epsilon^2 (Ht)^3 + \ldots \right) .$$
The small parameter is $\epsilon \equiv \frac{GA}{3\pi}$ and the neglected terms turn out to be irrelevant up to and including the breakdown of perturbation theory. The effect is two-loop because it requires one loop to produce 0-point energy through superadiabatic amplification $[13]$ and another loop for it to self-interact.

Perturbation theory breaks down when the argument of the logarithm in (3) approaches zero, at which time higher loop effects are still negligible $[12]$. We accordingly estimate the number of e-foldings of inflation as:

$$N_{\text{pert}} = \left( \frac{9}{172} \right)^{\frac{1}{3}} \epsilon^{-\frac{2}{3}} .$$

One can also use the perturbative result to show that inflation ends suddenly over the course of about five e-foldings $[12]$.

Of course perturbation theory cannot be trusted past the time when loop effects become comparable to the classical result. One way to evolve beyond this point is by using effective field equations for the expectation value of the metric $g_{\mu\nu}$. These can always be written as the classical field equations plus a quantum-induced stress tensor $T_{\mu\nu}[g]$:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}[g] .$$

Computing $T_{\mu\nu}[g]$ for an arbitrary metric is as difficult as solving quantum gravity. However, for the purposes of cosmology one loses nothing by restricting to the stress tensor of an effective scalar $\phi[g]$ which is itself a non-local functional of the metric:

$$T_{\mu\nu}[g] = \partial_\mu \phi[g] \partial_\nu \phi[g] - g_{\mu\nu} \left( \frac{1}{2} g^{\rho\sigma} \partial_\rho \phi[g] \partial_\sigma \phi[g] + P(\phi[g]) \right) .$$

When specialized to a homogeneous and isotropic metric $[2]$, the evolution equation is independent of the potential:

$$\ddot{b} = -4\pi G \left( \frac{d\phi}{dt} \right)^2 .$$

The induced stress tensor is therefore completely specified by giving the effective scalar as a functional of the metric. The potential can be reconstructed as a function of time from the solution $b(t)$ $[14]$:

$$P = \frac{1}{8\pi G} \left( \dot{b}(t) + 3b^2(t) - 3H^2 \right) .$$
and then expressed as a function of the scalar.

Careful consideration of the physical mechanism, plus general principles such as coordinate invariance and causality, along with the requirement of reproducing the known perturbative result (3), have led us to the following ansatz for the effective scalar [14]:

$$\phi = \frac{1}{\sqrt{8\pi G}} \ln \left[ 1 - \frac{43}{48} e^2 \left( \frac{1}{\Box} \right) \right] .$$

(9)

Here $\Box^{-1}$ is the retarded Green’s function associated with the scalar covariant d’Alembertian:

$$\Box \equiv \frac{1}{\sqrt{-g}} \partial_{\mu} \left( g^{\mu\nu} \sqrt{-g} \partial_{\nu} \right).$$

(10)

Our ansatz for the induced scalar $\phi[g]$ is not unique. However, it can be shown that the behavior before the breakdown of perturbation theory is universal and that the post-inflationary evolution depends only upon how many factors of $R$ stand immediately to the right of the outermost $\Box^{-1}$ [14]. For one such factor of $R$, the asymptotic late time behavior of the effective Hubble constant is:

$$\dot{b}(t) = \frac{1}{2(t - t_z)} - \frac{\alpha \ln[H(t - t_z)]}{H(t - t_z)^2} + \ldots ,$$

(11)

where $\alpha \approx 0.25$ and $t_z \approx \frac{N_{pert}}{H}$ is the co-moving time when inflation ceases.

We emphasize that the effective scalar $\phi[g]$ is not a fundamental particle but rather a convenient parametrization of quantum deformations to the field equations on the largest scales. However, this does not mean that it is devoid of physical content. In particular it seems reasonable to interpret the simple form of the induced stress tensor as signaling the formation of a scalar bound state at cosmological scales. The physical picture of such scalars is just the virtual graviton pairs, ripped apart by superadiabatic amplification, whose collective gravitational binding eventually nulls inflation. The reason terrestrial experiments reveal no such particles is that none have ever formed at less than cosmological scales.

A tremendous advantage of this interpretation for the effective scalar is that we can analyze the cosmological implications of our model using the standard methods of scalar-driven inflation. The “scalar potential” of these
methods subsumes the bare cosmological constant:

\[ V(\phi) = P(\phi) + \frac{\Lambda}{8\pi G} \]  \hspace{1cm} (12)

We use it to compute the standard parameters of inflationary cosmology, \( A_s \), \( r \), \( n \) and \( n_T \) \[13\]. We then compare with the COBE RMS quadrupole \[16\] to fix the one free parameter of our model, namely the mass scale of inflation:

\[ M = M_{Pl} \left( \frac{\Lambda G}{8\pi} \right)^{\frac{1}{4}} = M_{Pl} \left( \frac{3}{8} \epsilon \right)^{\frac{1}{4}} = (0.72 \pm 0.03) \times 10^{16}\text{GeV} \] \hspace{1cm} (13)

One prediction which is independent of the bound state interpretation for the scalar is that the enormous period of inflation (\( N_{pert} \approx 10^8 \)) associated with this scale drives any reasonably sized initial spatial curvature to zero. Our model accordingly entails \( \Omega = 1 \).

In Section 2 we derive a simple approximate form for the scalar potential which is valid until about the last five e-foldings of inflation. This is used in Section 3 to compute the scalar and tensor amplitudes and spectral indices employing the standard formulae of scalar-driven inflation. Our conclusions comprise Section 4.

## 2 The scalar potential

The density perturbations relevant to the cosmic microwave background experienced their first horizon crossing in the period from about 60 to 40 e-foldings before the end of inflation. This is the region in which our approximations must work. Figure 1 shows the result of a direct numerical computation of the effective Hubble constant for the last 100 e-foldings of inflation at a scale only slightly lower than that of expression (13). Superimposed in dots is the perturbative result obtained from differentiating (13):

\[ \dot{b}(t) = H \left\{ 1 - \frac{3}{2N_{pert}} \frac{(Ht/N_{pert})^2}{1 - (Ht/N_{pert})^3} \right\} \] \hspace{1cm} (14)

\(^1\)The “end of inflation” was determined by fitting the parameter \( t_z \) in \[11\] to the asymptotic results. It comes within four e-foldings of the value predicted by perturbation theory.
Figure 1: The effective Hubble constant, $\dot{b}(t)$, versus $t$ for $\epsilon = 2.45 \times 10^{-13}$ (solid) and the perturbative approximation (dots).

The end of inflation is obviously sudden. It is also clear that perturbation theory remains valid until about the last 10 e-foldings. In fact, only a small error results, in the region of interest, from setting $\dot{b}(t) \approx H$. Substituting the perturbative result (3) into (9) and ignoring terms which are irrelevant for $1 \ll Ht < \sim N_{\text{pert}}$ gives the following relation for the effective scalar:

$$\phi_{\text{pert}}(t) = -\frac{1}{\sqrt{8\pi G}} \ln \left(1 - \left(\frac{Ht}{N_{\text{pert}}}\right)^3\right).$$

(15)

A similar substitution into (8) and (12) gives the scalar potential as a function of co-moving time:

$$V_{\text{pert}} = \frac{A}{8\pi G} \left\{1 - \frac{3}{N_{\text{pert}}} \left(\frac{Ht}{N_{\text{pert}}}\right)^2 \right\} \left(1 - \frac{1}{4N_{\text{pert}}} \frac{\left(\frac{Ht}{N_{\text{pert}}}\right)^2}{1 - \left(\frac{Ht}{N_{\text{pert}}}\right)^3}\right).$$

(16)

\footnote{We only need the correction in order to get the first non-zero contribution to the spectral index for gravitons.}
Figure 2: $V(\phi)$ versus $\phi$ for $\epsilon = 2.45 \times 10^{-13}$ (solid) and our approximation (dots). The top scale shows the number of e-foldings to the end of inflation.

Inverting (15) and substituting gives $V_{\text{pert}}(\phi)$ as the following function of the scalar field:

$$\frac{\Lambda}{8\pi G} \left\{ 1 - \frac{3e^{\sqrt{8\pi G}\phi}}{N_{\text{pert}}} \left( 1 - e^{-\sqrt{8\pi G}\phi} \right)^{\frac{3}{4}} \left[ 1 - \frac{e^{\sqrt{8\pi G}\phi}}{4N_{\text{pert}}} \left( 1 - e^{-\sqrt{8\pi G}\phi} \right)^{\frac{3}{4}} \right] \right\} . \quad (17)$$

The asymptotic form (17) is actually considerably more accurate than necessary. Figure 2 demonstrates that the following approximation is quite good enough:

$$V(\phi) \approx \frac{\Lambda}{8\pi G} \left\{ 1 - \frac{3e^{\sqrt{8\pi G}\phi}}{N_{\text{pert}}} \right\} . \quad (18)$$

This expression is sufficiently simple that we can obtain analytic results.

3 Parameters of inflationary cosmology

Cosmological perturbations derive from the 0-point motion of particles which are not conformally invariant and whose masses are substantially smaller than
the expansion rate. In a spacetime which undergoes superluminal expansion these particles experience a phenomenon known as **superadiabatic amplification**. When a mode of such a particle redshifts beyond the causal horizon, the 0-point energy it contains becomes vastly enhanced with respect to $\frac{1}{2} \hbar \omega$. A simple way to understand this is that virtual pairs become trapped in the expansion of spacetime and are unable to recombine.

The subsequent history of the perturbations is characterized by linear evolution until long after the end of inflation. That is, no mixing occurs between perturbations of different co-moving wavenumber. Of course the physical wavenumber of each mode redshifts with the general expansion of spacetime. Since the scale factor of astrophysics is conventionally normalized to unity at current time ($t_0$), the physical wavenumber of a perturbation at any other time can be expressed, with our metric (2), as:

$$k_p(t, k) = e^{b(t_0) - b(t)} k,$$

where $k$ is the current wavenumber.

An important event in the evolution of a perturbation is **horizon crossing**. This is when the perturbation’s physical wavenumber equals the Hubble constant:

$$\dot{b}(t) = e^{b(t_0) - b(t)} k.$$  

The perturbations we observe today have all experienced two horizon crossings: the first during inflation, as they redshifted below the nearly constant expansion rate; and the second time afterwards as the expansion rate slowed. The amplitude of a perturbation approaches a time independent constant during the period between first and second horizon crossings. The square of this constant is known as the perturbation’s **power spectrum**. It is this quantity and simple combinations of it that are usually reported for models of inflation, even though it is not directly observable.

The observable quantity is the perturbation’s imprint in anisotropies of the cosmic microwave background. This entails evolving from second horizon crossing to the time of recombination, when the cosmic microwave background decoupled. Tensor perturbations simply redshift during this period, so they can be important only at the largest scales. Scalar perturbations that re-enter the horizon before the time of recombination experience acoustic oscillations as a result of the competition between their self-gravitation
and their pressure. The fluctuating density and the special relativistic velocity redshift is what causes the so-called, “Doppler Peaks.” The pressure disappears at recombination, allowing gravitational collapse to produce the various compact structures we observe today.

Although much work has been done since the first studies of inflationary density perturbations [17], conventions are still in the process of crystallizing. We have decided to follow those of the recent review article by Lidsey, et. al. [15]. To leading order in the slow roll approximation they give the following formulae for the power spectra of scalar and tensor perturbations:

\[
A_s^2(k) \approx \frac{512\pi}{75} G^3 \left[ \frac{V^3(\phi)}{V''(\phi)} \right]_{1\text{st crossing}},
\]

\[
A_T^2(k) \approx \frac{32}{75} G^2 [V(\phi)]_{1\text{st crossing}}.
\]

From these they compute the tensor-to-scalar ratio:

\[
r \equiv 12.4 \frac{A_T^2(k)}{A_S^2(k)},
\]

and the scalar and tensor spectral indices:

\[
n \equiv 1 + \frac{d \ln (A_S^2)}{d \ln (k)},
\]

\[
n_T \equiv \frac{d \ln (A_T^2)}{d \ln (k)}.
\]

The parameters \(r\), \(n\), and \(n_T\) are all technically dependent upon \(k\) but are typically reported at a particular value.

\[\text{It is worth noting how these normalizations relate to those employed in some recent reviews. Mukhanov, Feldman and Brandenberger [18] compute the following power spectra:}
\]

\[|\delta(k)|^2 = \frac{9}{4} A_s^2(k), \quad |\delta_h(k)|^2 = \frac{25}{9} A_T^2(k).
\]

The power spectra of Liddle and Lyth [19] are:

\[P_R = \frac{25}{4} A_s^2(k), \quad P_g = 100 A_T^2(k),
\]

but their quantity \(\delta_H^2(k)\) is exactly \(A_S^2(k)\).
Although we are mostly concerned with describing the unobserved, primordial spectra, some contact must be made with the measured multipole moments of the cosmic microwave anisotropy in order to fix the initial scale of inflation. Suppose we knew the time dependent scalar power spectrum after second horizon crossing. Its contribution to the variance of the $\ell$-th multipole moment of the cosmic microwave anisotropy would be [19]:

$$C_\ell = \pi \int_0^\infty \frac{dk}{k} j_\ell^2 \left( 2H_0^{-1}k_p(t_{\text{rec}}, k) \right) A_S^2(t_{\text{rec}}, k) ,$$

where $j_\ell$ is the spherical Bessel function of order $\ell$ and $t_{\text{rec}}$ is the time of recombination. The transfer function between the primordial power spectrum and $A_S^2(t_{\text{rec}}, k)$ is known but there is no point in using it for the lowest $\ell$ values. Except for the factor of $A_S^2(t_{\text{rec}}, k)$, the integrand peaks at $k \approx \ell H_0^2$, and thereafter falls off like $1/k^3$. For small values of $\ell$ the integral is effectively restricted to wavenumbers that have re-entered the horizon too soon to be much affected by subsequent evolution. For the quadrupole we can certainly replace $A_S^2(t_{\text{rec}}, k)$ with the primordial power spectrum. For $A_S^2(k) \sim k^n$, with constant spectral index, the integral can be expressed in closed form:

$$C_2 = \frac{\pi^2}{4} \frac{\Gamma \left( \frac{3-n}{2} \right) \Gamma \left( \frac{n+3}{2} \right)}{\Gamma \left( \frac{4-n}{2} \right) \Gamma \left( \frac{2-n}{2} \right)} A_S^2(H_0/2) .$$

(27)

For $n \approx 1$ (which is the case for this model) we can make the further simplification:

$$C_2 \approx \frac{\pi}{12} A_S^2(H_0/2) .$$

(28)

For small $r$ (which is also the case) we can forget about the tensor contribution and compare this with the RMS quadrupole averaged over the whole Universe:

$$\left\langle \frac{Q_{\text{RMS}}^2}{T_0^2} \right\rangle \approx \frac{5}{4\pi} C_2 \approx \frac{5}{48} A_S^2(H_0/2) ,$$

where $T_0 = 2.728$ K and the best fit to the COBE 4-year maps (assuming $n = 1$) gives $\left\langle Q_{\text{RMS}}^2 \right\rangle^{\frac{1}{2}} = (1.80 \pm 0.16) \times 10^{-5}$ K [10].

It remains to solve for the time $t_k$ of first horizon crossing and evaluate the various parameters. We do not know precisely how many e-foldings have transpired from the end of inflation (at $t_z$) to the present, so this number
must enter as a parameter:

$$\Delta N \equiv b(t_0) - b(t_z) \quad .$$

(30)

We can set $b(t_z) \approx N_{\text{pert}}$, since the end of inflation is quite accurately predicted by (4). Because even galaxy-sized perturbations would have experienced first horizon crossing when perturbation theory is still an excellent approximation, we can re-express (20) as follows:

$$\left(e^{N_{\text{pert}}+\Delta N-Ht_k} \right) k \approx H \quad .$$

(31)

Hence the time of horizon crossing is:

$$t_k \approx H^{-1} \left[N_{\text{pert}} - \ln \left(\frac{H}{k}\right) + \Delta N\right] \quad ,$$

(32)

where the three terms in the square brackets are arranged in order of decreasing magnitude.

The effective scalar is obtained by substituting the time of horizon crossing into (13):

$$\phi(t_k) \approx -\frac{1}{\sqrt{8\pi G}} \ln \left[\frac{3}{N_{\text{pert}}} \left(\ln \left(\frac{H}{k}\right) - \Delta N\right)\right] \quad .$$

(33)

Combining our approximation (13) with the standard formulae (21-22) and then evaluating at (33) results in the following scalar and tensor power spectra:

$$A_S^2(k) \approx \frac{8}{25} \frac{G\Lambda}{3\pi} \left[\ln \left(\frac{H}{k}\right) - \Delta N\right]^2 \quad ,$$

(34)

$$A_T^2(k) \approx \frac{4}{25} \frac{G\Lambda}{3\pi} \left[1 - \frac{1}{\ln \left(\frac{H}{k}\right) - \Delta N}\right] \quad .$$

(35)

The tensor-to-scalar ratio is:

$$r \approx 6.2 \left[\ln \left(\frac{H}{k}\right) - \Delta N\right]^{-2} \quad ,$$

(37)
and the spectral indices are:

\[
\begin{align*}
    n & \approx 1 - 2 \left[ \ln \left( \frac{H}{k} \right) - \Delta N \right]^{-1}, \\
    n_T & \approx - \left[ \ln \left( \frac{H}{k} \right) - \Delta N \right]^{-2},
\end{align*}
\]

In each case we have only carried the expansion far enough to give the first correction to exact scale invariance.

The parameter \( \Delta N \) can be expressed in terms of the reheating temperature \( T_R \):

\[
\Delta N = \ln \left( \frac{T_R}{T_0} \right).
\]

We do not yet know \( T_R \) but it is easy to make some plausible guesses, and the actual number does not depend much on realistic uncertainties. Suppose that about half of the initial energy density of the cosmological constant goes into the energy density of reheating and that this excites \( g \) ultra-relativistic species:

\[
\frac{1}{2} M^4 \approx \frac{\pi^2}{30} g T_R^4.
\]

A reasonable estimate for the number of species is \( g \approx 500 \), which gives:

\[
\Delta N \approx \ln \left( \frac{M}{T_0} \right) - 1.45.
\]

Note that even an order of magnitude change in \( g \) or in the thermalized fraction of \( M^4 \) would only alter \( \Delta N \) by about 0.6. Substituting the stated expression for \( \Delta N \) into (34) and (29) results in a transcendental relation between the COBE RMS quadrupole and the single free parameter of our model, \( \epsilon \):

\[
\frac{\langle Q^2_{\text{RMS}} \rangle}{T_0^2} \approx \frac{\epsilon}{30} \left[ \ln \left( \frac{T_0}{H_0} \right) + \frac{1}{4} \ln(\epsilon) + 2.96 \right]^2 ,
\]

\[
\approx \frac{\epsilon}{30} \left[ 70.12 + \frac{1}{4} \ln(\epsilon) \right]^2.
\]

When one assumes exact scale invariance (i.e., \( n = 1 \)) the 4-year COBE results \((T_0 = 2.728 \text{ K and } \langle Q^2_{\text{RMS}} \rangle^{1/2} = (1.80 \pm .16) \times 10^{-5} \text{ K})\) imply:

\[
\epsilon = (3.3 \pm .6) \times 10^{-13}.
\]
The corresponding inflationary Hubble constant and mass scale are:

\[
H \equiv M_{\text{Pl}} \left(\frac{\pi \epsilon}{2}\right)^{\frac{1}{2}} = (6.3 \pm 0.6) \times 10^{26} \text{ cm}^{-1}, \tag{46}
\]

\[
M \equiv M_{\text{Pl}} \left(\frac{3}{8} \epsilon\right)^{\frac{1}{4}} = (0.72 \pm 0.03) \times 10^{16} \text{ GeV}. \tag{47}
\]

This gives \(\Delta N \approx 64.1\), where the spread in \(M\) has no effect on the first 3 digits. Recall, however, that there are still appreciable uncertainties in \(\Delta N\) arising from lack of knowledge about re-heating.

It is natural to evaluate the various parameters at the horizon scale, \(k = 2\pi \times 10^{-28} \text{ cm}^{-1}\). With this choice we compute:

\[
A_S = (2.0 \pm 0.2) \times 10^{-5}, \tag{48}
\]

\[
r \approx 1.7 \times 10^{-3}, \tag{49}
\]

\[
 n \approx 0.97, \tag{50}
\]

\[
n_T \approx -2.8 \times 10^{-4}. \tag{51}
\]

The spread in \(\epsilon\) engenders no appreciable uncertainty in \(r\) or in the spectral indices, although they are affected by the uncertainty in \(\Delta N\). In view of the small tensor-to-scalar ratio we are amply justified in fixing \(\epsilon\) by comparing the scalar power spectrum with the COBE RMS quadrupole. The proximity of \(n\) to 1 also justifies the assumption of exact scale invariance in making the comparison. Figure 3 shows \(A_S(k)\) for scales between \(10^{22}\) cm (galaxies) and \(10^{28}\) cm (horizon). Note that there is only a small distinction, on these scales, between the true logarithmic form (34) (the solid lines) and the power law approximation (dotted lines). The uncertainty in normalization is far greater. Had we plotted the individual COBE data points, the error bars would cover the vertical scale.

4 Conclusions

We have predicted five standard cosmological parameters: \(A_S, r, n, n_T\) and \(\Omega\) for a model of inflation in which a bare cosmological constant is gradually screened by an infrared process in quantum gravity. The process is just the buildup of gravitational interactions between pairs of virtual gravitons which are ripped apart by the superluminal expansion of spacetime. It is
Figure 3: Amplitude of scalar density perturbations $A_S(k)$ for upper and lower values of $\epsilon \equiv G\Lambda/3\pi$. The dotted lines give the power law fit for spectral index $n = .97$. 
very slow because gravity is a weak interaction, even at the GUT scale, but
the effect adds coherently on account of the graviton’s unique combination
of masslessness without conformal invariance. The mechanism acts to slow
inflation because gravity is attractive, and it must continue to build for as
long as inflation persists. Although perturbation theory must break down
when inflation is finally choked off, it can be used to follow the process almost
to its end [12].

The resulting model of inflation contains only one free parameter, \( \epsilon \equiv \frac{G\Lambda}{3\pi} \), which we have determined to obtain agreement with the COBE RMS quadrupole. This essentially absorbs \( A_S \), leaving four genuine predictions. With the possible exception of \( \Omega = 1 \), they are all in good agreement with current data. It is worth emphasizing that this did not have to happen, nor does it have to remain true as the data improves. And the data will improve dramatically when the Microwave Anisotropy Probe (MAP) and the Planck Surveyor are flown [20]. This model is falsifiable. It is perhaps the first result from quantum gravity for which that can be said in anything but a trivial sense.

It might be objected that the model’s non-perturbative extension effectively introduces new parameters in the form of guesswork about the effective field equations [14]. That is not so. The various approximations derived in Section 2 all came from the known results of perturbation theory [12] which are independent of any non-perturbative ansatz. This suffices for the study of perturbations because they experience first horizon crossing some 40 e-foldings before the end of inflation, when perturbation theory is still quite reliable. The only non-perturbative result we have used is that there is an end to inflation.

With the current data, the model’s chief advantage over scalar-driven
inflation is aesthetic. There is no fine tuning beyond the near-universal require-
ment that inflation occur on the GUT scale. Many other models have
free parameters which must be carefully adjusted in order to produce the
correct magnitude of anisotropies in the cosmic microwave background. For
example, in chaotic inflation based on a \( \lambda \phi^4 \) potential, one needs \( \lambda \approx 10^{-14} \)[18]. And the late time cosmological constant must be fine tuned in all scalar-driven models.

Aesthetics aside, the model does have a somewhat distinguishing feature
in the form of a small tensor-to-scalar ratio: \( r \approx .0017 \). This derives ulti-
nately from the fact that the relevant form (18) of the potential obeys:

\[
\frac{V''}{V} \gg \left( \frac{V'}{V} \right)^2 .
\]  

(52)

In contrast, power law inflation [6] characterized by \(a(t) \sim t^p\) produces \(r = \frac{12}{p}\), which is actually greater than unity for powers less than 12. Chaotic inflation based on a potential \(\phi^p\) results in \(r = \frac{\phi}{20}\), which is 10% for \(\phi^2\) and 20% for \(\phi^4\) [19]. On the other hand, “natural inflation” [8] gives much smaller tensor-to-scalar ratios than the .17% of our model [19]. One commonly studied model does overlap ours. That is Starobinsky’s \(R^2\) inflation [2], which results in \(r \approx .004\) [19].

The model can be falsified by either MAP or Planck on the basis of its prediction for the scalar spectral index: \(n \approx .97\). However, one must also consider the possibility of distinguishing it from scalar-driven models whose parameters have been adjusted to give the same value of \(n\). This requires measuring the tensor-to-scalar ratio. Neither MAP nor Planck will be able to distinguish \(r\) from zero at the level we predict, but Planck would detect the tensor contribution from either polynomial chaotic inflation or power law inflation [20]. So our model can certainly be distinguished from these. It is conceivable that a future, very sensitive polarization experiment could detect the tensor-to-scalar ratio we predict, although this depends upon whether or not the curl signal is dominated by foreground emission at this level [20].

What we have not done in this paper is to consider re-heating or the model’s response to late time phase transitions. This is complicated in that one must rely on an ansatz for extending past the breakdown of perturbation theory. One must also come up with a tractable way of incorporating matter. However, there is a rich harvest of observables to motivate the effort. Chief among these is the residual effect of screening on the deceleration parameter.

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