A method of sharing secrecy using star graphs

G Prasad¹, R Anandan² and G Uma Maheswari³

¹ Department of Mathematics, Prakasam Engineering college, kandukur prakasam, Andhra prades - 523105, India.
² Department of ECE, Dhanalakshmi Srinivasan College of Engineering and Technology, Mamallapuram, ECR, Chennai-603104, Tamil Nadu, India.
³ Department of Mathematics, Dhanalakshmi Srinivasan College of Engineering and Technology, Mamallapuram, ECR, Chennai-603104, Tamil Nadu, India.

E-mail: uma.vellore@gmail.com

Abstract. This article introduces a new technique of picture coding method for a secret text messages in the form of matrix using star related graphs by applying the subset of super mean labeling, sub super mean labeling is one of the important topics in graph theory. Let \( G \) be a \((p, q)\) graph and \( \alpha : V(G) \rightarrow \{1, 2, 3, \cdots, p+q\} \) be an injection.

For each edge \( l = uv \), let

\[
\alpha^*(e) = \begin{cases} 
\frac{\alpha(u) + \alpha(v)}{2} & \text{if } \alpha(u) + \alpha(v) \text{ is even;} \\
\frac{\alpha(u) + \alpha(v) + \text{one}}{\text{two}} & \text{if } \alpha(u) + \alpha(v) \text{ is odd}
\end{cases}
\]

then \( \alpha \) is called Sub Super Mean Labeling if

\[
\alpha(V) \cup \{\alpha^*(l) : l \in E(G)\} \subset \{\text{one, two, three, \cdots, } p+q\}.
\]

A graph that admits a Sub Super Mean Labeling is called a Sub Super Mean Graph.

1. Introduction for graph theory and coding theory

There are many different types of codes and ciphers. A code is a system where a symbol, picture or group of letters represents a specific alphabetical letter or word. A cipher is where a message is made by substituting one symbol for a letter. The coded matrix is a
shift cipher is a substitution method where the letters are replaced by the numbers with a fixed clue.

1.1. Literature Review:
Conversion of a secret messages is too less from the sender and the decoder but it should not be understand by the next person. Labeling two star graph using Mean Label was given by S. Somasundaram and R. Ponraj [3], [9]. The star graph $K_{1,m} \cup K_{1,n}$ with an edge in common is a mean graph if and only if $|m - n| \leq 4$ which was proved by Maheshwari and Ramesh [2]. The idea of super mean labeling was given and studied by D. Ramya et al., (11). Labeling on super meanness property of the subdivision of the $H$-graph and slanting ladder was developed by R. Vasuki and A. Nagarajan [10] and the development of super meanness of subdivision graph of $K_{1,3}$, some caterpillars and some duplicate graphs was given by R. Vasuki and A. Nagarajan [11]. Method of Coding through star related graphs with super mean labeling is structured by Uma Maheswari et al., [4], [5] and coding with Fibonacci web is established by Uma Maheswari et al., [6]. Secret Coding Technique on Two Star Graphs and Coding Technique through Graph Labelings are given by [7] and [8].

Inspired by the above works, we have put some interest on star related graphs by applying new labeling and GMJ (Graph Message Jumbled) coding method and we presented this article.

2. Definitions

**Definition 2.1.** Let $G$ be a $(a, b)$ graph and $\mu : V(G) \rightarrow \{\text{one, two, three, \cdots, a + b}\}$ be an injection. For each edge $e = uv$, let

$$\mu^*(e) = \begin{cases} 
\frac{\mu(u) + \mu(v)}{\text{two}} & \text{if } \mu(u) + \mu(v) \text{ is even;} \\
\frac{\mu(u) + \mu(v) + 1}{\text{two}} & \text{if } \mu(u) + \mu(v) \text{ is odd.}
\end{cases}$$

then $\mu$ is called SSML if $\mu(V) \cup \{\mu^*(e) : e \in E(G)\} \subset \{\text{one, two, three, \cdots, a + b}\}$. A graph that admits a SSML is called a Sub Super Mean Graph.

**Definition 2.2.** The original message before encoding is called as Plain text.

**Definition 2.3.** The coded message after coding is called as Cipher text.

**Definition 2.4.** A procedure for converting the text form of a given message into different form by decryption and encryption methods is known as Cipher.
Definition 2.5. A Caesar cipher changes the alphabets in to three and is therefore called a shift cipher. Caesar used a clue three for his decryption and it is called as Caesar Cipher.

3. Description for SSML \((V_4, E_2)\) on two star:

Discussion of SSML \((V_4, E_2)\) for

\[ G = K_{1, p} \cup K_{1, q} \]

for all values of \(p\) and \(q\), for \(p \leq q\) is given, which is used for the following proof.

Let \( G = K_{1, p} \cup K_{1, q} \)

\[ a = \text{one} + p + \text{one} + q = \text{two} + p + q \]

\[ b = p + q \]

\[ a + b = 2p + 2q + 2 \]

In First star \( G = K_{1, p} \cup K_{1, q} \) the labeling is done as follows:

\[ \phi(u) = \text{one} \]

\[ \phi(u_{\text{one}}) = \text{three} \]

\[ \phi(u_{i}) = \text{three} + \text{four}(i - \text{one}), \quad 2 \leq i \leq p \]

\[ \phi(u_{p}) = \text{three} + \text{four}(p - \text{one}) = 4p - \text{one} \]

\[ \phi^{*}(u_{u_{\text{one}}}) = \text{two} \]

\[ \phi^{*}(u_{u_{i}}) = \text{two} + \text{two}(i - \text{one}), \quad \text{two} \leq i \leq p \]

\[ \phi^{*}(u_{u_{p}}) = \text{two}(p) \]

The second star \( G = K_{1, p} \cup K_{1, q} \), allotment i:

consider \( \phi(v) = \phi(u_{p}) + \text{four} = \text{four}(p) + 3 \)

\[ \phi(v_{1}) = 5 \]

\[ \phi(v_{j}) = 5 + 4(j - 1), \quad 2 \leq j \leq J, \]

where \( J = \left\lfloor \frac{2p + 2q - 3}{4} \right\rfloor \)

\[ \phi(v_{J+1}) = 2p + 2q + 2 \]

\[ \phi(v_{J+2}), \phi(v_{J+3}) \text{ etc are assigned the remaining odd integers referred by } [1]. \]

An example is shown in Figure 1.

3.1. Method of Coding

Coding a text message and procedure for encoding a given message is refereed by [6]. An algorithm for transforming a text form into code by encryption and decryption methods is known as Cipher. The letters \( f \) and \( s \) are used to refer to the first and second star respectively. \( t, p_{i} \) and \( l_{j} \) denote the top point, the \( i^{th} \) pendant point and the \( j^{th} \) line value in order.
3.2. Arranging the alphabets:

SVCC

Clue: SVCC denotes vowels taken the number square of positions in order start with 1 and then numbering of consonants.

As there are five vowels, let them be represented by $a_i$, ($a_1 = a$) ($a_2 = e$) and so on.

Here $\varphi(a_i) = i^2$, $i = one, two, three, four, five$.

Consonants takes the number continued is defined by $b_j$, $\varphi(b_j)$ takes value from 1 to 26. The numbering is continued up to 39 ($39 = a + b$).

In this process some letters receive two numbers. For decoding we reverse the process.

The table 4.1 gives an example for communicating some secret messages using sub super
mean labeling graphs through matrix coding. Those have knowledge of graph labeling only can decode the given messages. The matrix coding is the cipher text that is coded message.

4. Illustration on two star

The below Table 1 shows the transformation of text message into coded message

| plain text | cipher text |
|------------|-------------|
| **Message** | Bravo zulu go a head. |
| clue | Twinkling twins with five and nine. (Twinkling represents star) |
| **Graph** | $K_{1.5} \cup K_{1.9}$, $p + q = 30$. fig 1. |
| **Coding (word wise)** | **Letter Codes** |
| Bravo | $f(l_1)s(l_3)f(t)s(p_5)s(l_2)$ |
| zulu | $s(l_1)s(p_6)s(l_6)s(l_8)$ |
| go | $f(p_2)s(l_2)$ |
| a | $f(t)$ |
| head | $f(l_4)f(l_2)f(t)f(p_1)$ |

Presenting the letter codes

The functions $f$ and $s$ are used to denote the first and second star respectively.

$T$, $P_j$ and $L_j$ denote the top vertex, the $j^{th}$ pendant vertex and the $j^{th}$ edge value in order

Horizontal string

$$f(p_1)f(t)f(l_2)f(l_3)(1,1)f(t)$$

$$s(l_2)f(p_2)s(l_8)s(l_6)s(p_6)s(l_1)$$

$$(1,1)s(l_2)s(p_5)f(t)s(l_3)f(l_1)$$

Result: Matrix coding

$$\begin{pmatrix}
  f(p_1) & f(t) & f(l_2) & f(l_3) & (1,1) & f(t) \\
  s(l_2) & f(p_2) & s(l_8) & s(l_6) & s(p_6) & s(l_1) \\
  (1,1) & s(l_2) & s(p_5) & f(t) & s(l_3) & f(l_1)
\end{pmatrix}$$

Illustration Table 1

5. Description for sub super mean labeling for three star graph:

The discussion of SSML($V_4, E_2$) on $G = K_{1.o} \cup K_{1.p} \cup K_{1.q}$, $o \leq p \leq q$ for every $o, p$ and $q$ is given, for which the proof is given.

Consider $G = K_{1.o} \cup K_{1.p} \cup K_{1.q}$, $a = 3 + o + p + q$, $b = o + p + q$, $a + b = 3 + 2o + 2p + 2q$. 
In first star of \( G = K_{1, o} \cup K_{1, p} \cup K_{1, q} \) the integer allotment as follows: \( \mu(u) = one, \mu(u_i) = three \) + four \((i - 1)\), one \(\leq i \leq o\).

Then the line label is \( \mu^*(wu_i) = twoi \), one \( \leq i \leq o \). In second star of \( G = K_{1, o} \cup K_{1, p} \cup K_{1, q} \) then the allotment as given as:

Define: \( \mu(v) = f(u_o) + four = (fouro - one) + four = fouro + three \);

\[ \mu(v_j) = five + four(j - 1), \text{ one } \leq j \leq p \]

Then the line allotment is

\[ \mu^*(vv_j) = \frac{(fouro + three) + five + four(j - one)}{two} = two + twaj + two, \text{ one } \leq j \leq p; \]

\( FEIO = two + two \). The end dots of the 1\(^{st}\) and 4\(^{th}\) copy differ by 2 accordingly, since the \( o^{th}\) end vertex of the star = \( 4o - one \), the \( o^{th}\) end dot of the next star is \( 4o + one \), the \( o^{th}, (o + one)^{st}, (o + to)^{nd}, \cdots \) end dots of the second star are \( 4o + one, 4o + five, 4o + nine, \cdots \), up to the \( o^{th}\) place, no odd integers are omitted and \( 4o + 3 \) is allotted to \( \mu(v) \).

Therefore first odd integer omitted is \( 4o + even \), that is the 1\(^{st}\) end dot of the 3\(^{rd}\) star of \( K_{1, o} \cup K_{1, p} \cup K_{1, q} \).

The 3\(^{rd}\) star of \( G = K_{1, o} \cup K_{1, p} \cup K_{1, q} \) the labeling is done:

Define \( \mu(w) = f(v_p) + 4 = (4p + 1) + 4 = 4p + 5 \):

\[ \mu(w_k) = (4o + seven) + four(k - one), \text{ one } \leq k \leq K. \]

Calculation of value \( K \):

we have \( a + b = 2o + 2p + 2q + 3, (4o + 7) + 4(k - 1) \leq 2o + 2p + 2q + 3, 4k \leq 2p + 2p - 2o, \)

\( k \leq \left[ \frac{q + p - o}{two} \right], K = \left[ \frac{q + p - o}{two} \right], \mu(w_{K+1}) = 2o + 2p + 2q + three \text{ or fouro + nine}, \)

function of \( (w_{K+2}) \) to function of \( (w_q) \) are given to all first and second even integer omitted if required, then the line is obtained as,

\[ \mu^*(ww_k) = \frac{(4p + 5) + (4o + 7) + four(k - 1)}{two} = 2o + 2p + 2k + four, \text{ 1 } \leq k \leq K. \]

The edge values \( \mu^*(ww_{K+1}) \text{ to } \mu^*(ww_q) \) consider the integer values 1,3,5,..., either the pendant vertices are not even or first or second even integers omitted \[2\]. An example for case B is shown in Figure 2. The table 6.1 gives an example for transforming plain text into cipher text through sub super mean labeling here matrix code plays an important role for coded message. This method is applicable in the field of Army, Navy, governmental and to mention a few where ever the we are maintaining secrecy it is possible.
5.1 Arranging the alphabets: EBEF

Integers from one to thirteen are given to the even places of alphabets and going backward from Y to W to U and so on and the integers from fourteen to twenty six are given to the even places of alphabets going forward starting from B to D to F and continue and hence it is called as EBEF (Evens backward, Evens forward).

The expression for the numbering of alphabets in the form of a function for encoding, the function \( \tau \) is done by,

\[
\tau(2k) = (13 + k) \text{ for } k = 1, 2, \cdots, 13.
\]

For odd positioned alphabets the function \( g \) is given by,

\[
\tau(2k + 1) = (13 - k) \text{ for } k = 1, 3, 5, \cdots, 13.
\]

A triplet is used to code a letter. Corresponding to the letter in the message, the integer is taken and it is found out in the graph. The top vertices of the three star are denoted by \((1,1,1)\), \((2,2,2)\) and \((3,3,3)\).

6. Illustration on three star

The following Table 2 shows the transformation of text message into coded message.

![Figure 2](image-url)
| **plain text**  | **cipher text**                                |
|----------------|-----------------------------------------------|
| Message        | Finish it and have Eight day skate.          |
| clue           | Twinkling thrice with four, five and six.    |
| (Twinkling represents star) |                                           |
| Graph          | $K_{1,4} \cup K_{1,5} \cup K_{1,6}$.        |
| Coding (word wise) | Letter Codes                               |
| Finish         | $(2,0,3)(2,2,0)(2,0,5)(2,2,0)(1,0,2)(2,0,4)$ |
| it             | $(2,2,0)(3,1,0)$                             |
| and            | $(2,3,0)(2,0,5)(1,4,0)$                      |
| have           | $(2,4,0)(2,3,0)(3,6,0)(1,3,0)$               |
| eight          | $(1,3,0)(2,2,0)(2,1,0)(2,4,0)(3,1,0)$        |
| day            | $(1,4,0)(2,3,0)(1,1,1)$                      |
| skate           | $(1,0,2)(1,0,4)(2,3,0)(3,1,0)(1,3,0)$        |
| **Letter codes presentation** | single digits defining letters which is got by as in coding a letter are pictured in a horizontal string |
| Horizontal string | 5474364457565944466455335544          |
| **Result: Matrix coding** | $$
\begin{bmatrix}
5 & 4 & 7 & 4 & 3 & 6 & 4 \\
4 & 5 & 7 & 5 & 6 & 5 & 9 \\
4 & 4 & 4 & 6 & 6 & 4 & 5 \\
5 & 3 & 3 & 5 & 5 & 4 & 4
\end{bmatrix}$$ |

Illustration Table 2

7. Algorithm

- **step 1**: Guess the star graph with the code.

- **step 2**: Use sub super mean labeling.

- **step 3**: Divide the alphabets using clues.

- **step 4**: Define method of coding of the text message.
• **step 5:** Write the original message.

• **step 6:** Apply letter codes each letter.

• **step 7:** Converting codes into different form.

• **step 8:** Presented it to order matrix.

Communicator must pass these steps to the decoder:

• code word for star diagram

• SVCC and EBEF without abbreviation.

• string or matrix

8. Conclusion and future work:
Here we have finalized the graph is a star graph with sub super mean labeling. Provided two examples for converting the given text message in to picture matrix code. In this research article we have provided the descriptions for sub super mean labeling on star based graphs. In our future work we would like to apply sub super mean labeling for flower and web based graphs and apply similar techniques.

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