Coherence manipulation of quantum bits through cross-correlations

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In terms of the exact quantum master equation solution for open electronic system, we prove that the coherence of a charge qubit described by a double-dot nanostructure can be manipulated through the cross-correlations without requiring tunable inter-dot coupling. This greatly simplifies the current technology for coherence manipulation of quantum bits in nanostructures. The simplicity of precision coherence controls also makes it very promising for large-scale quantum integrated circuits.

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The great achievement made in the last four decades for classical computers is mainly due to the development of the large-scale integrated circuit (IC). The building blocks of IC are the MOSFET, an electronic semiconductor transistor in which the simple relation of turning on and off a current through the bias and gate voltage controls plays the crucial role. Certainly, to build a practically useful quantum computer relies deeply on whether one can develop a large-scale quantum integrated circuit with a simple and precision control of quantum bit (qubit) coherence and entanglement. Coherence and entanglement manipulation in various qubit systems has been demonstrated experimentally in the past decade [1] but a simple realization of qubit coherence and entanglement in nanoscale devices, as simple as turning on and off a current in MOSFET, is still lacking.

Quantum information and quantum computation implementation in terms of electron charges and spins has attracted tremendous attention for the development of large-scale quantum information processing [2–6]. Current experimental technology in nanofabrications allows to design various nanostructures with tunable couplings and energy levels through external gate voltages [7–11]. The tunability of couplings and energy levels in quantum-dot-based nanostructures becomes a promising technology for large-scale solid-state quantum computing [4, 5]. However, the manipulation of charge qubits in quantum dots, although it can be realized through a series of high-speed voltage pulses [7], is rather difficult for integrated precision controls, mainly due to the inevitable decoherence arising from the dissipative tunneling processes with the coupled electrodes [12, 13]. In fact, to physically isolate a qubit system from various contacts in nanostructures is almost impossible because of the leakage effect induced by higher-order electron tunneling processes.

In this Letter, we shall provide an alternative scheme for the coherence manipulation of charge qubits through the cross-correlations of the dot-lead couplings in nanostructures, where the tunable inter-dot coupling is not required. We analyze the coherence dynamics of electrons by exactly solving the master equation of the corresponding reduced density matrix [12, 13]. The result indicates that the qubit can stay in a decoherence-free coherence state, in which the coherence phase between the two qubit states can be precisely controlled through the cross-correlation. In other words, the qubit is effectively isolated from its contacts and the coherence manipulation is largely simplified. This will greatly simplify the current technology for coherence manipulation of electrons in nanostructures.

We begin with a nanostructure of a double quantum dot coupled in parallel with a lead, each dot has one excess electronic state, shown schematically by Fig. 1 as a specific example. The lead should be half-metallic ferromagnetic [14] or a mesoscopic Stern-Gerlach spin filter [15] so that only fully polarized spin electrons pass into the dot through the lead. The general Hamiltonian of the nanostructure consists of three parts: \( H = H_B + H_S + H' \). \( H_B = \sum_k \epsilon_k c_k^\dagger c_k \) is the lead Hamiltonian, where \( c_k^\dagger, c_k \) are the creation and annihilation electron operators of the lead. The double dot Hamiltonian \( H_S = \sum_{ij} \epsilon_{ij} a_i^\dagger a_j \), where \( a_i^\dagger (a_i) \) creates (destroys) an electron on the dot \( i (i = 1, 2) \) with the energy \( \epsilon_j = \epsilon_{i1} \) and \( \epsilon_{21} = \epsilon_i^* \) is the inter-dot coupling. \( H' = \sum_{k[i]} [P_k e^{i\phi_k} a_i^\dagger c_k + H.c.] \) describes the electron tunneling between the dots and the lead, in which \( t_{ki} \) is a real coupling, and \( \phi_k \) is a phase induced, for instance, by Rashba spin-orbit (SO) interaction during the electron tunneling between the lead and dots [16]. The spin index of electrons is dropped since all spins are polarized in one direction.

Quantum coherence dynamics of a charge qubit is described by the reduced density matrix \( \rho(t) \) which can be obtained by tracing over all the degrees of freedom of the lead from the total density matrix \( \rho_T(t) \) of the nanostructure (the double dot plus the lead),

\[
\rho(t) = \text{tr}_B[\rho_T(t)] = \text{tr}_B[e^{-iH(t-t_0)}\rho_T(t_0)e^{iH(t-t_0)}].
\]

This reduced density matrix can be solved from the exact master equation we developed recently [12, 13]. Explicitly, we may denote the singly occupied states by \( |1\rangle \) and
leakage effects of the charge qubit. Of empty and double occupation states that account the dot.

The central $2 \times 2$ block matrix is just the density matrix of the charge qubit, and $\rho_{ij}^{(1)}(t) = \langle \phi_i \sigma_j \rho(t) \rangle$ is the single particle reduced density matrix of the double dot. $\rho_{00}(t)$ and $\rho_{33}(t)$ are respectively the probabilities of empty and double occupation states that account the leakage effects of the charge qubit.

If we prepare the double dot in the empty state $|0\rangle$ at $t = t_0$. Then $\rho^{(1)}(t)$ can be explicitly solved from the exact master equation \cite{17}:

$$\rho^{(1)}(t) = \int \frac{d\omega}{2\pi} u(t, \omega) f(\omega) \Gamma(\omega) u^\dagger(t, \omega), \quad (3)$$

where $u(t, \omega) = \int_{t_0}^{t} d\tau e^{i\omega(t-\tau)} u(t, \tau)$, $u(t, \tau)$ is the retarded Green function of electrons in the double dot and obeys the equation of motion:

$$\frac{d}{d\tau} u(t, \tau) + i e u(t, \tau) + \int_{t_0}^{t} d\tau g(t-\tau) u(t, \tau) = 0 \quad (4)$$

with the time-correlation function $g(t) = \int \frac{d\omega}{2\pi} \Gamma(\omega) e^{-i\omega t}$, where $\Gamma = \{\Gamma_{ij}\}$ is the spectral density defined by

$$\Gamma_{ij}(\omega) = 2\pi \sum_k t_{kj} t_{kj}^* e^{i(\phi_i - \phi_j)} \delta(\omega - \epsilon_k), \quad (5)$$

and $f(\omega) = 1/(e^{\beta(\omega-\mu)}+1)$ is the initial electron distribution function in the lead at the temperature $\beta^{-1} = k_B T$. For an initial empty dot state, we have also shown \cite{17} that $\rho_{00}(t) = \det[I - \rho^{(1)}(t)]$ and $\rho_{33}(t) = \det[\rho^{(1)}(t)]$. Thus the coherence dynamics of the charge qubit is completely determined by the single particle reduced density matrix $\rho^{(1)}(t)$ through the retarded Green function $u(t, t_0)$ of Eq. \(4\). Obviously, the coherence manipulation of the charge qubit can be realized via the changes of the Fermi distribution of the lead (see Eq. \(3\)), the inter-dot coupling $\epsilon_{12}$ (see Eq. \(4\)), and/or the spectral density matrix of Eq. \(5\).

Conventionally, the coherence manipulation of a lateral double dot charge qubit is performed with the bias and gate voltages controlling the electron tunneling between the leads and dots, the dot energy levels, and the inter-dot coupling \cite{4, 5}. However, precision controls of the inter-dot coupling and the dot-lead potential barriers through high-speed gate voltage pulses are rather difficult and inevitably involve the charge noise, such as the 1/f noise. Here we shall show that a precise external field control of the qubit coherence through the cross-correlations in a parallel double dot is much more reliable and simpler for coherence manipulation in nanostructures. In particular, the present manipulation scheme is rather insensitive to the charge noise \cite{18, 20} with the condition of no inter-dot coupling, a low temperature, and a small bias voltage as demonstrated below.

The cross-correlations are defined by the off-diagonal matrix element of the spectral density matrix arising from the dot-lead coupling. They satisfy the relations of $\Gamma_{12}(\omega) = \Gamma_{21}(\omega)$, and $|\Gamma_{12}(\omega)| = \sqrt{\Gamma_{11}(\omega) \Gamma_{22}(\omega)}$. The spectral density matrix of Eq. \(5\) can be reduced to

$$\Gamma(\omega) = \Gamma(\omega) \begin{pmatrix} 1 & e^{i\phi} \\ e^{-i\phi} & 1 \end{pmatrix}, \quad (6)$$

where we have assumed a symmetric coupling: $t_{1k} = t_{2k}$, and $\phi = \phi_1 - \phi_2$, see Eq. \(5\). We will show that the cross-correlations of the dot-lead coupling, $\Gamma_{12}/\Gamma_{21}$, totally determine the coherence manipulation of the charge qubit, especially the phase $\phi$ in the cross-correlations is just the coherent phase of the charge qubit without decoherence.

For simplicity, we set the double dot in degenerate: $\epsilon_1 \simeq \epsilon_2 = \epsilon_0$ and the dot-lead coupling in the wide band limit: $\Gamma(\omega) \to \Gamma$. Since the inter-dot coupling is not required, we can let $\epsilon_{12} \simeq 0$. Then, the retarded Green function of Eq. \(4\) can be explicitly solved (let $t_0 = 0$)

$$u(t, 0) = \frac{e^{-i\epsilon_0 t}}{2 \sqrt{1 + e^{-\Gamma t} - (1 - e^{-\Gamma t}) (\sigma_x \cos \phi - \sigma_y \sin \phi)}}, \quad (7)$$

with $\sigma_x/y$ being the Pauli matrix. The solution of the single particle reduced density matrix can be analytically obtained from Eq. \(3\):

$$\rho^{(1)}(t) = n(t) \begin{pmatrix} 1 & e^{i\phi} \\ e^{-i\phi} & 1 \end{pmatrix}, \quad (8)$$
where
\[
 n(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \epsilon_0)^2} f(\omega) \times \left\{ 1 - 2e^{-\Gamma t} \cos[(\omega - \epsilon_0)t] + e^{-2\Gamma t} \right\},
\]
is the electron population in the dot 1, which is also the same for the dot 2 due to the degeneracy of the double dot. It is easy to show that the double occupation state \( \rho_{33}(t) = \det[\rho^{(1)}(t)] = 0 \). In other words, with the initial empty state for the degenerate double dot, the double occupation state is never excited. The full solution of the reduced density matrix of the double dot thus becomes
\[
\rho_{00}(t) = 1 - 2n(t), \quad \rho_{11}(t) = \rho_{22}(t) = n(t), \quad \rho_{33}(t) = 0, \quad \rho_{12}(t) = \rho_{21}(t) = n(t)e^{i\phi}.
\]

The above solution shows that when the double occupation state vanishes, the qubit state is fully determined by the single-particle reduced density matrix of Eq. (8) which can be rewritten in terms of a pure state:
\[
\rho_{\text{qubit}}(t) = \rho^{(1)}(t) = |\psi(t)\rangle \langle \psi(t)|
\]
with \(|\psi\rangle = \frac{c(t)}{\sqrt{2}}((1) + e^{-i\phi} |2\rangle)\). The probability of the parallel double dot in such a qubit coherent state is \(|c(t)|^2 = 2n(t) < 1 \) (see Eq. (9)) due to the leakage effect given by \(\rho_{00}(t)\). While the coherence phase of the charge qubit is totally immunity from the intrinsic fluctuation of the lead. This unusual result shows that the charge qubit can be kept in a complete decoherence-free coherent state, and the qubit coherence is totally controlled by changing the phase \(\phi\) in the cross-correlations between the dots and the lead.

To further demonstrate the coherence manipulation, we plot the electron population and the coherence dynamics in Fig. 2. It shows that \(n(t)\) soon grows to its steady values \(\bar{n}\) within a very short time scale \((\sim 2/\Gamma)\) and without further decay. The steady electron population is given by
\[
\bar{n} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Gamma f(\omega)}{\Gamma^2 + (\omega - \epsilon_0)^2} \simeq \frac{1}{4} + \frac{1}{2\pi} \text{arctan} \left( \frac{\epsilon V}{\Gamma} \right),
\]
where the second identity is for \(\mu = \mu_F + \epsilon_0\) with \(\mu_F = eV\) at zero temperature. If we apply the bias to the lead such that \(eV > \epsilon_0 \sim \Gamma\), then \(2\bar{n} \simeq 1\), as also shown numerically in Fig. 2. Then the double dot can stay almost in the perfect coherent state: \(|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{-i\phi} |2\rangle)\), where the coherent phase \(\phi\) is just the phase in the cross-correlation which can be tuned, for example, by an external electric field through the Rashba SO interaction [12, 23, 24–27]. Thus a decoherence-free manipulation of the qubit coherence via the cross-correlations is demonstrated in nanostructures.

The physical picture of the coherence manipulation via the cross-correlations \(\Gamma_{12/21}\) is rather clear. There are two tunneling channels in this devices, one is the electron tunneling from lead \(\rightarrow\) dot 1 (called channel 1) and the other is from \(\text{lead} \rightarrow \text{dot 2}\) (called channel 2). For zero cross-correlation (\(\Gamma_{12/21} = 0\)), the electrons transfer through the tunneling channel 1 is independent from the electrons via the channel 2. No electron coherence between the two channels can be generated in this case. When \(\Gamma_{12/21} \neq 0\), the electrons transfer through the tunneling channel 1 is indistinguishable from the electrons via the channel 2. It is such an indistinguishable electron tunneling induces the coherence of the two charge states in the double dot. Thus the coherence of qubit can be developed and manipulated purely through the cross-correlation between the lead and the dots.

Practically, the factors of thermal fluctuation, the finite bandwidth of lead, and the deviation from the degenerate electronic states should be considered. Fig. 3(a) shows the magnitude of the stationary coherence, i.e., \(|\rho_{12}\rangle = \bar{n}\) in Eq. (12), as a function of the initial temperature and the Fermi surfaces of the lead. The result shows that the perfect coherence is developed with an initially relative low temperature, as expected. Almost perfect coherence occurs for \(k_B T < 2\Gamma\), see Fig. 3(a). This can easily be realized in the current experiments, e.g., \(T \simeq 350\text{mK}\) for \(\Gamma = 30\mu\text{eV} [7, 22]\). On the other hand, slightly splitting the degeneracy, e.g., \(\epsilon = \epsilon_1 - \epsilon_2 = 5\%\Gamma\), only causes a very slow decay of the coherence, see Fig. 3(b), which is almost negligible. Also, a small inter-dot coupling, say \(\epsilon_2 \simeq 2.5\%\Gamma\), does not change significantly the coherence dynamics of the charge qubit, see Fig. 3(b). Regardless of the above fluctuations, the coherence phase does not be affected. Furthermore, we also examine the situation beyond the wide band limit. To be specifically, we consider the energy dependence of the spectral density of Eq. (9) be a Lorentzian-type form [12, 23, 24]: \(\Gamma(\omega) = \frac{\Gamma d^2}{(\omega - \mu)^2 + \Gamma^2}\), where \(d\) describes the bandwidth of the lead. Obviously the wide band limit \(d \rightarrow \infty\) leads to \(\Gamma(\omega) \rightarrow \Gamma\). Conventionally, a finite band-
width can induce strong non-Markovian memory effect to the transient electron dynamics. Here, when the Fermi surface of the lead is much higher than the energy level of the double dot, the finite bandwidth effect is also negligible, because the involved tunneling electrons have been restricted to the region near the Fermi surface. In addition, a much larger degenerate splitting \( \epsilon = \epsilon_1 - \epsilon_2 = 5\% \Gamma \). (c) The coherence dynamics and its stationary distribution deviated away from the degenerate electronic states, with energy level difference \( \epsilon = \epsilon_1 - \epsilon_2 = 5\% \Gamma \).

The present scheme is thus experimentally reliable. For the case of the coherence phase induced by the Rashba SO interaction, \( \phi \) is proportional to the strength of the SO interaction. One can tune the SO interaction by applying gate voltage pulses on the top of the tunnel barriers between the lead and the dots [11]. Taking an experimental value of the SO interaction strength \( \alpha \approx 3 \times 10^{-11} \text{eV m} \) for some semiconductor, such as InAs nanowire dots [25, 27] and the typical length between the dots and the lead of 100 nm with the electron effective mass \( m^* = 0.05 m_e \), the phase \( \phi = m^* \alpha L / h^2 > \pi / 2 \). With the fast growing nanotechnology, the full period of the phase control for the charge qubit is expected. Meanwhile, manipulation of two qubit entanglement in such system is also not difficult, see, for example, Ref. [28]. Thus a large-scale quantum IC with such a simple coherence manipulation is possible in the future experiments.

Finally, we should also point out that the present scheme is generic for qubit coherence manipulation through the tunable phase of the cross-correlation in the system-reservoir coupling, as long as the spectral density has the form of Eq. (B). Besides the example of the double dot setup through the Rashba SO coupling specified in this Letter, any other system which has the property of Eq. (B) with a simple tunable phase in the cross-correlation can be a good candidate for a reliable qubit in large-scale quantum IC.

In summary, we have demonstrated a novel scheme via the cross-correlation to manipulate the coherence of solid-state charge qubit in a parallel double-dot nanostructure. By solving the exact master equation of the double dot system, we show that the perfect two charge coherent state, \( \frac{1}{\sqrt{2}} (|1\rangle + e^{-i\phi} |2\rangle) \), can be developed without requiring a turnable inter-dot coupling. The coherence phase \( \phi \) between the two charge states is just the phase in the cross-correlation of the dot-lead coupling which can be easily tuned through, for example, the Rashba SO interaction. This greatly simplifies the current technology for coherence manipulation of qubits in nanostructures. Through investigating the effects of temperature and finite band width of the lead, we show that the present scheme is experimentally very reliable. Also, the simplicity of precision coherence controls in such scheme makes it very promising for large-scale quantum integrated circuits in nanostructures.

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