Momentum-position realization of the Einstein-Podolsky-Rosen paradox

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We report on a momentum-position realization of the EPR paradox using direct detection in the near and far fields of the photons emitted by collinear type-II phase-matched parametric downconversion. Using this approach we achieved a measured two-photon momentum-position variance product of 0.01\hbar^2, which dramatically violates the bounds for the EPR and separability criteria.

In 1935 Einstein, Podolsky and Rosen (EPR) 1 wrote one of the most controversial and influential papers of the twentieth century. They proposed a gedanken experiment involving two particles entangled simultaneously over a continuum of position and momentum states. By measuring either the position or momentum of one of the particles, the position or momentum of the other could be inferred with complete certainty. Under the assumptions of EPR, the ability to make such an inference meant that the position and momentum of the unmeasured particle were simultaneous realities, in violation of Heisenberg’s uncertainty relation. This thought experiment became known as the EPR paradox. In 1951 Bohm 2 cast the EPR paradox into a simpler, discrete form involving spin entanglement of two spin-1/2 particles, such as those produced in the dissociation of a diatomic molecule of zero spin. From Bohm’s analysis sprang Bell’s inequalities 3 4 and much of what is now the field of discrete quantum information 5 6 7 8 9 10.

In recent years, however, there has been a movement toward the study of entanglement of continuous variables as originally discussed by EPR 11 12 13 14 15 16 17 18 19 20 21 22 23 24. Of particular interest was the early work of Reid and Drummond 14. They derived an EPR criterion and showed how it could be implemented with momentum-like and position-like quadrature observables of squeezed light fields. Shortly thereafter, the experiment was realized by Ou et al. 12. Later Duan et al. 16 and Simon 17 derived necessary and sufficient conditions for the inseparability (entanglement) of continuous variable states. A flurry of experimental activity ensued in both atomic ensembles 19 and squeezed light fields 20 21 22.

Here we report on a demonstration of the EPR paradox using position- and momentum-entangled photon pairs produced by spontaneous parametric downconversion. We find that the position and momentum correlations are strong enough to allow the position or momentum of a photon to be inferred from that of its partner with a product of variances \( \leq 0.01\hbar^2 \), which violates the separability bound by two orders of magnitude.

In the idealized entangled state proposed by EPR,

\[
|\text{EPR}\rangle = \int_{-\infty}^{\infty} |x, x\rangle \, dx = \int_{-\infty}^{\infty} |p, -p\rangle \, dp,
\]

the positions and momenta of the two particles are perfectly correlated. This state is non-normalizable and cannot be realized in the laboratory. However, the state of the light produced in parametric downconversion can be made to approximate the EPR state under suitable conditions. In parametric downconversion, a pump photon is absorbed by a nonlinear medium and re-emitted as two photons (conventionally called signal and idler photons), each with approximately half the energy of the pump photon. Considering only the transverse components, the momentum conservation of the downconversion process requires \( p_1 + p_2 = p_p \) where 1, 2, \( p \) refer to the signal, idler, and pump photons. Provided the uncertainty in the pump transverse momentum is small, the transverse momenta of the signal and idler photons are highly anti-correlated. The exact degree of correlation depends on the structure of the signal+idler state. In the regime of weak generation, this state has the form

\[
|\psi\rangle_{1,2} = |\text{vac}\rangle + \int A(p_1, p_2)|p_1, p_2\rangle \, dp_1 dp_2
\]

where |\text{vac}\rangle denotes the vacuum state and the two-photon amplitude \( A \) is

\[
A(p_1, p_2) = \chi E_p(p_1 + p_2) \frac{\exp(i\Delta k_z L) - 1}{i\Delta k_z},
\]

Here \( \chi \) is the coefficient of the nonlinear interaction, \( E_p \) is the amplitude of the plane-wave component of the pump with transverse momentum \( p_1 + p_2 \), \( L \) is the length of the nonlinear medium, and \( \Delta k_z = k_{p,z} - k_{1,z} - k_{2,z} \) (where \( k = p/\hbar \) is the longitudinal wavevector mismatch, which generally increases with transverse momentum and limits the angular spread of signal and idler photons. The vacuum component of the state makes no contribution to photon counting measurements and may be ignored. Also, there is no inherent difference between different transverse components; so without loss of generality, we consider scalar position and momentum. The narrower the angular spectrum of the pump field and the wider the angular spectrum of the generated light, the more closely the integral 20 approximates \( \int_{-\infty}^{\infty} \delta(p_1 + p_2)|p_1, p_2\rangle \, dp_1 dp_2 = |\text{EPR}\rangle \) and the stronger the correlations in position and momentum become.

The experimental setup used to determine position and momentum correlations is portrayed in Fig. 1. The idea is
to measure the positions and momenta by measuring the downconverted photons in the near and far fields respectively [24]. The source of entangled photons is spontaneous parametric downconversion generated by pumping a 2mm thick Type-II BBO crystal with a 30 mW, cw, 390 nm laser beam. A prism separates the pump light from the downconverted light. The signal and idler photons have orthogonal polarizations and are separated by a polarizing beamsplitter. In each arm, the light passes through a narrow 40 μm vertical slit, a 10nm spectral filter, and a microscope objective. The objective focuses the transmitted light onto a multimode fiber which is coupled to an avalanche photodiode single-photon counting module. The spectral filter ensures that only photons with nearly equal energies are detected. To measure correlations in the positions of the photons, a lens of focal length 100 mm (placed prior to the beamsplitter) is used to image the exit face of the crystal onto the planes of the two slits (Fig. 1a). One slit is fixed at the location of peak signal intensity. The other slit is mounted on a translation stage. The photon coincidence rate is then recorded as a function of the displacement of the second slit. To measure correlations in the transverse momenta of the photons, the imaging lens is replaced by two lenses of focal length 100 mm, one in each arm, a distance $f$ from the planes of the two slits (Fig. 1b). These lenses map transverse momenta to transverse positions, such that a photon with transverse momentum $\hbar k_\perp$ comes to a focus at the point $x = f k_\perp / k$ in the plane of the slit. Again, one slit is fixed at the location of peak count rate while the other is translated to obtain the coincidence distribution.

By normalizing the coincidence distributions, we obtain the conditional probability density functions $P(x_1|x_2)$ and $P(p_1|p_2)$ (Fig. 2). These probability densities are then used to calculate the uncertainty in the inferred position or momentum of photon 1 given the position or momentum of photon 2:

$$\Delta x_{\text{inf}}^2 = \int (x_1 - x_2)^2 P(x_1|x_2) dx_1 - \left( \int (x_1 - x_2) P(x_1|x_2) dx_1 \right)^2$$

and

$$\Delta p_{\text{inf}}^2 = \int (p_1 + p_2)^2 P(p_1|p_2) dp_1 - \left( \int (p_1 + p_2) P(p_1|p_2) dp_1 \right)^2.$$  

Because of the finite width of the slits, the raw data in Fig. 2 describe a slightly broader distribution than is associated with the downconversion process itself. By adjusting the computed values of $\Delta x_{\text{inf}}$ and $\Delta p_{\text{inf}}$ to account for this broadening (an adjustment smaller than 10%), we obtain the correlation uncertainties $\Delta x_{\text{inf}} = 0.027 \text{ mm}$ and $\Delta p_{\text{inf}} = 3.7 \text{h}^{-1}$. The measured variance product of the inferred state is

$$\Delta x_{\text{inf}}^2 \Delta p_{\text{inf}}^2 = 0.01 \text{h}^2. \ (6)$$

Also shown in Fig. 2 are the predicted probability densities. These curves contain no free parameters and are obtained directly from the two-photon amplitude $A(p_1, p_2)$, which is determined by the optical properties of BBO and the measured profile of the pump beam. Fig. 2 indicates that the correlation widths we obtained are intrinsic to the downconversion process and are limited only by the degree to which it deviates from the idealized EPR state [1]. The value of $\Delta p_{\text{inf}}$ is limited by the finite width of the pump beam. The pump photons in a Gaussian beam of width $w$ have an uncertainty $\hbar / 2w$ in transverse momentum which, due to conservation of momentum, is imparted to the total momentum $p_1 + p_2$ of the signal and idler photons. The value of $\Delta x_{\text{inf}}$ is limited by the range of angles over which the crystal generates signal and idler photons. If the angular width of emission is $\Delta \phi$, then the principle of diffraction indicates that the photons cannot have a smaller transverse dimension than $\sim (k_{1,i} \Delta \phi)^{-1}$. Careful analysis based on the angular distribution of emission yields $\Delta x_{\text{inf}} = 1.88 (k_{1,i} \Delta \phi)^{-1}$. With the measured beam width of $w = 0.17 \text{ mm}$ and predicted angular width 0.012 rad, the theory predicts $\Delta x_{\text{inf}}^2 \Delta p_{\text{inf}}^2 = 0.0036 \text{h}^2$. This is somewhat smaller than the experimentally calculated value of
This inconsistency was revealed by showing that measurements of one particle could be used (seemingly) to infer the state of an unmeasured particle with greater certainty than is allowed by the uncertainty relation, i.e. \( \Delta x^2 \Delta p^2 \geq h^2 / 4 \) \( [14] \). A key component of EPR’s argument was the assumption of “local reality.” Under this assumption, the statistics of each particle depend only on a (hidden) state parameter \( s \) which is determined while the particles are close enough to interact. This assumption is commonly taken to mean that the joint probability of any pair of observables \( a_1 \) and \( b_2 \) must be expressible as

\[
P(a_1, b_2) = \sum_s P_s P(a_1|s)P(b_2|s) \tag{7}
\]

where \( P_s \) is the probability of state \( s \) and \( P(a_1|s) \) and \( P(b_2|s) \) are the conditional probabilities for particle 1 and particle 2 respectively. This constraint led to the Bell tests of local realism \([3, 4]\) and their subsequent experimental realizations (e.g., \([5]\)). Another consequence of \([1]\) has been the development of the concept of entanglement, which is closely connected to non-locality. A bipartite system is entangled (inseparable) if its density matrix cannot be written in the form

\[
\rho = \sum_s P_s \rho_{1,s} \otimes \rho_{2,s}. \tag{8}
\]

Measurement of either member of an entangled system projects both members into a mixture of states consistent with the result of the measurement. It is now generally accepted that this mutual projection occurs even when the particles are widely separated; hence, within the framework of quantum theory, entanglement and non-local behavior have a one-to-one relationship. This conclusion is confirmed by the fact that any system which satisfies eqn. \( [8] \) also satisfies eqn. \( [4] \), and vice versa. However, entanglement does not always rule out local realism in the context of a theory other than quantum mechanics. For example, the strong position and momentum correlations of the entangled EPR state can be readily explained by local-realistic classical mechanics, which does not impose an uncertainty relation on position and momentum.

The close connection between entanglement and non-locality has prompted refinement of EPR’s criterion, resulting in a number of different separability tests and entanglement measures. Of relevance to the original EPR paradox are the tests for separability of continuous-variable systems developed by Duan et al. \([16]\), Simon \([17]\), and Mancini et al. \([18]\). The tests in \([16]\) and \([17]\) involve sums of dimensionless variances and are not scale invariant; hence it is not clear how, or even if, they may be applied to the present EPR experiment, which involves dimensional position and momentum. The criterion derived by Mancini et al. \([18]\) is more useful here. It states that separable systems must satisfy the joint uncertainty product of \(0.01 h^2\), even though the data appears to closely match the theoretical curves. The reason for this discrepancy is that the experimental distributions have small (\( \approx 1\% \) of the peak) but very broad wings. The origin of these coincidence counts is unknown; they are perhaps due to scattering from optical components. If these counts are treated as a noise background and subtracted, the experimentally obtained uncertainties come into agreement with the theoretically predicted values, yielding an uncertainty product of \(0.004 h^2\).

To interpret these results, it is helpful to consider the relationship between the original EPR paradox and the issues of entanglement, non-locality, and quantum signatures, which have been the subjects of more modern studies. Because of lingering doubts about quantum theory, raised in part by the EPR paradox, there has been much interest in “quantum signatures”, i.e. phenomena that confirm the predictions of quantum mechanics and cannot be explained by classical mechanics. However, the intent of EPR was not to reveal a discrepancy between classical and quantum theory, but to reveal an inconsistency, or incompleteness, within quantum theory. This inconsistency was revealed by showing that measurements of one particle could be used (seemingly) to infer the state of an unmeasured particle with greater certainty than is allowed by the uncertainty relation, i.e. \( \Delta x^2 \Delta p^2 \geq h^2 / 4 \) \( [14] \). A key component of EPR’s argument was the assumption of “local reality.” Under this assumption, the statistics of each particle depend only on a (hidden) state parameter \( s \) which is determined while the particles are close enough to interact. This assumption is commonly taken to mean that the joint probability of any pair of observables \( a_1 \) and \( b_2 \) must be expressible as

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relation
\[(\Delta x_{12})^2 (\Delta p_{12})^2 \geq \hbar^2\]  \hspace{1cm} (9)
where \((\Delta x_{12})^2 = ((x_1 - x_2)^2) - ((x_1 - x_2))^2\) and \((\Delta p_{12})^2 = ((p_1 + p_2)^2) - ((p_1 + p_2))^2\). The angle brackets denote expected values over the respective joint probability distributions. In our experiment the widths of the conditional probability distributions \(P(x_1|p_2)\) and \(P(p_1|x_2)\) are essentially independent of \(x_2\) and \(p_2\) over most of their ranges, so that \(\Delta x_{1\text{inf}}^2\) and \(\Delta p_{1\text{inf}}^2\) are nearly equal to \((\Delta x_{12})^2\) and \((\Delta p_{12})^2\). Hence our results constitute a two order-of-magnitude violation of the joint uncertainty relation \[\text{[18]}\], which in this case is both a separability criterion and a local-realism criterion.

Finally, we note that the EPR paradox does not represent a true inconsistency. It is generally accepted that the EPR argument fails because the assumption of local realism is invalid: The position or momentum of the unmeasured particle becomes a reality when, and only when, the corresponding quantity of the other particle is measured. As the measurement involves only one quantity the other, the position and momentum of the unmeasured particle need not be simultaneous realities.

In conclusion, we have reported the experimental realization of Einstein, Podolsky and Rosen’s paradox using momentum-position entangled photons. We have measured position and momentum correlations resulting in a variance product which dramatically violates the original EPR criterion and a modern inseparability criterion. Compared to squeezed-light realizations of the EPR paradox, the momentum-position realization has several attractive features which make it promising for further development. For one, the entanglement is observed using direct photon detection, which is experimentally simpler than homodyne detection. Secondly, the entanglement does not reside in the photon count, which frees this quantity to be used for postselection. Since the position and momentum measurements involve only those photons that are detected, the measured entanglement is not degraded by optical loss which inevitably occurs in real systems. For both of these reasons, systems with very small values of the joint uncertainty product can be readily achieved in practice. This capability has already been used to achieve near- diffractionless coincidence imaging \[\text{[27]}\]. We believe that the work presented here sets the stage for many more interesting applications to come.

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