Genetic Algorithm based Optimization of Uniform Circular Array

Vinod Kumar
Department of ECE
Guru Jambheshwar University of Science & Technology
Hissar, India
vinodspec@yahoo.co.in

Sanjeev Kumar Dhull
Department of ECE
Guru Jambheshwar University of Science & Technology
Hissar, India
sanjeevdhull2011@yahoo.com

Abstract—Signal estimation at the antenna is a major challenge of the antenna array structure because the received signals have different directions. Therefore, in this paper, a Genetic Algorithm (GA) is applied to the uniform circular array for the optimization of array structure in regard to its geometry. On the optimized array structure, four different algorithms (Estimation of Signal Parameter via Rotational Invariance Technique – ESPRIT, First Order Forward Prediction - FOFP, Beamscan, and Multiple Signal Classification - MUSIC) have been implemented in order to estimate the signal direction accurately with quick estimation time. The accuracy has been calculated with Root Mean Square Error (RMSE) indices. From the experimental analysis, it has been found that the performance of the ESPRIT algorithm is better than the others in terms of accuracy and estimation time.

Keywords—computational complexity; FOFP; genetic algorithm; RMSE; uniform circular array

I. INTRODUCTION

When an incoming signal wave is detected and measured at a sensor array, it is processed and information is extracted from it (e.g.: Direction Of Arrival - DOA). The algorithms that are used to find DOA in the antenna array are generally used in wireless communication systems to increase the system capacity and the throughput of the wireless network. The antenna array consists of several antennas for finding the direction of incident waves and suppressing interference signals. The estimation problem is of significant importance in the array signal processing. Parameter estimation is an area where research is conducted for performance improvement. A major topic of interest is the accurate discovery of the signal direction. Sensor array field processing is an area for research focused in accurate signal estimation. As improvements in the geometry of the antenna array occur, the DOA estimation approach will become an important part of smart antenna systems. The antenna array collects data from all the elements and then combines them for spatial information and detects the incoming signal direction based on signal processing algorithms.

The main aim of the antenna array is to find the signal directions from signals that come from different directions. The estimation of the signal may be obtained with the help of the array’s geometry and DOA algorithms. DOA estimation is an interesting field and many DOA algorithms were proposed including spatial spectrum estimation. The first method based on the spatial spectrum was proposed in [1]. This method is used for estimating the direction of the incoming signal by computing and observing the spatial spectrum. After that, a local maxima point is decided. Author in [2] presented a new method based on Minimum Variance Distortionless Response (MVDR). This MVDR is used to conduct spectrum estimation with the help of Maximum Likelihood (ML) method [3] which maximizes the log-likelihood function among a set of arrays. In [4] a subspace based algorithm was proposed which scans all the angles between the signal subspace and noise subspace. Author in [5] improves the solution ability of the MUSIC algorithm and gives good estimation results by introducing a variant of MUSIC algorithm. Authors in [6] propose a new algorithm named ESPRIT which has higher computation efficiency. Authors in [7-8] proposed the weighted subspace fitting method for better estimation results. Authors in [9] proposed a new method for the signal estimation in two dimensions. Authors in [10] improved the MUSIC algorithm. Authors in [11] gave an improvement in the static performance of the MUSIC algorithm. Authors in [12] proposed a new scheme for the increase of resolution ability of the MUSIC algorithm. Authors in [13] gave advancement in the ESPRIT algorithm for two dimensions. Table I gives the detailed description of the existing DOA algorithms. These algorithms are based on spectral search and subspace methods.

| Algorithm       | Comments                                      |
|-----------------|-----------------------------------------------|
| Bartlett [1]    | Estimate DOA by computing spatial spectrum and finalizing local maxima. The noise signal is excluded. |
| MVDR [2]        | Spectrum estimation with maximum likelihood method. |
| MUSIC [4]       | Scans all the angles from signal and noise subspace. |
| ESPRIT [6]      | Increased computation efficiency with the help of two identical sub arrays. |
| Weighted subspace fitting [7-8] | Unified approach for signal estimation. |
| MUSIC [14]      | Discussion on maximum likelihood and Cramor Rao bound |
| MUSIC [9]       | A method for signal detection on two dimensions. |
| MUSIC [11]      | Static performance of MUSIC algorithm. |
| MUSIC [12]      | Improved the resolution ability of MUSIC algorithm |
| Root MUSIC [5]  | Does not find the spectral peaks, it solves the rooting problem of a polynomial with good resolution ability. |

TABLE I. DIFFERENT DOA ALGORITHMS

Corresponding author: Vinod Kumar

www.etasr.com Kumar & Dhull: Genetic Algorithm based Optimization of Uniform Circular Array
II. ESTIMATION OF SIGNAL PARAMETERS VIA ROTATIONAL INVARIANCE TECHNIQUES

Authors in [6] proposed an algorithm named Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT) for DOA estimation. In ESPRIT, array doublets are used and formed by \( N/2 \) pairs which further form a displacement vector, where \( N \) is the number of array elements. The starting two elements of the doublet are separated and grouped to make two \( \frac{N}{2} \) sub arrays. The vectors \( x \) and \( y \) are the data vectors corresponding to each of the sub arrays. The output of the sub arrays \( x \) and \( y \) can be expressed as:

\[
x_k[n] = \sum_{i=0}^{r-1} x_i[n]u_k(\theta_i) + v_k^{(x)}[n] \quad (1)
\]

\[
y_k[n] = \sum_{i=1}^{r-1} y_i[n]e^{j2\pi\delta\sin\theta_i}a_k(\theta_i) + v_k^{(y)}[n] \quad (2)
\]

where similar notation has been used and \( \delta \) is the displacement magnitude in wavelengths. The estimated angle by ESPRIT algorithm is relative to the displacement vector. The output of sub arrays, \( x \) and \( y \), in matrix form is given as:

\[
x_n = A_s x_n + v_n^{(x)} \quad (3)
\]

\[
y_n = A_p s_n + v_n^{(y)} \quad (4)
\]

The matrix \( \phi \) is a diagonal \( r \times r \) matrix where the diagonal elements are:

\[
\{ \exp(j2\pi\delta\sin\theta_0), \exp(j2\pi\delta\sin\theta_1), ..., \exp(j2\pi\delta\sin\theta_{r-1}) \}
\]

The phase delay may be represented by the complex exponentials between the \( r \) signals and the doublet pair. The data vectors may be concatenated from sub arrays to make a single \( 2N-2 \) data vector like:

\[
z_n = \begin{bmatrix} x_n \\ y_n \end{bmatrix} = A_0 s_n \quad (5)
\]

\[
A_0 = \begin{bmatrix} A & A_\rho \end{bmatrix}, \quad V_n = \begin{bmatrix} v_n^{(x)} \\ v_n^{(y)} \end{bmatrix} \quad (6)
\]

The columns of \( A_j \) occupy the signal subspace of the new array. Let \( V_j \) be the column matrix depending upon the signal subspace and \( A_s, A_\rho, V_0 \) are related with \( r \times r \) transformation \( T \) and can be written as:

\[
V_j = A_j T \quad (7)
\]

and can be portioned as follows:

\[
V_j = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} A T \\ A_\rho T \end{bmatrix} \quad (8)
\]

From this step, \( A \) will be equal to \( E_x, E_y \) and have the same range. The rank \( r \) of matrix \( E_{xy} \) is:

\[
E_{xy} = [E_x \quad E_y] \quad (9)
\]

To find the \( r \times 2r \) rank \( r \) matrix having null space of \( E_{xy} \) to form the matrix \( F \) is written as:

\[
\begin{bmatrix} E_x & E_y \end{bmatrix} F = E_x F_x + E_y F_y = A T F_x + A_\rho T F_y \quad (10)
\]

Assuming \( \Psi \) is:

\[
\Psi = -F_x [F_x]^\dagger \quad (11)
\]

by reshuffling the above equations, we get:

\[
E_x \Psi = E_y \quad (12)
\]

Now, by substituting we get:

\[
A T \Psi = A_\rho T \Rightarrow A T \Psi T^{-1} = A \rho \Rightarrow T \Psi T^{-1} = \phi \quad (13)
\]

The given equations mean that the eigenvalues of \( \Psi \) are the same as the diagonal elements of \( \phi \). Once the eigenvalues \( \lambda \) of \( \phi \) have been calculated, the angle of arrival is calculated as:

\[
\lambda_k = e^{j2\pi\sin\theta_k} \quad (14)
\]

\[
\theta_k = \arcsin \left( \frac{\text{arg}(\lambda_k)}{2\pi\delta} \right) \quad (15)
\]

If \( A \) is the full rank matrix, then the eigenvalues of the matrix \( \Psi \) are the diagonal elements of \( \phi \) and the eigenvectors of \( \Psi \) are the columns of \( T \). Practically, the signal subspace is not known exactly, the only estimate is taken from sample covariance matrix \( R_n \) or from a sub space tracking algorithm. Therefore, \( E_x \psi = E_y \) will not be exactly satisfied and we will have to resort to a least square solution to compute \( \Psi \). The least square process assumes that the columns in \( E_x \) are known exactly whereas the data in \( E_y \) are noisy. If the assumption that \( E_x \) and \( E_y \) are equally noisy is made, then the total least square criteria is used to solve to above problem with better results.

III. OPTIMIZATION USING GENETIC ALGORITHM

The Genetic Algorithm (GA) is a population search algorithm inspired by the evolutionary process of natural selection [15]. GA is a simple algorithm that has been proven to be very powerful and widely applicable. The algorithm arrives at a solution by performing operations on a population of solutions referred to as chromosomes over several iterations, which are referred to as generations. The operations used are borrowed from genetic concepts such as mutation, reproduction, fitness, etc. The algorithm uses fitness evaluation to rank solutions and randomness in creating new generations of solutions in order to promote diversity. Authors in [16] used the GA to optimize the relative position in an antenna system. Authors in [17] used the GA in a linear array with infinite SNR in order to extract the amplitude and DOA of various signals. Authors in [18] introduced the use of GA in ML based parameter and spatial spectrum estimation. Authors in [19] introduced a new refined GA for accurate and reliable DOA estimation with the help of a sensor array based on the ML function. Authors in [20] introduced a new genetic based estimation method with the help of fourth order cumulant. A new genetic firefly hybrid algorithm based on the neural network was designed in [21] for finding the best position in...
the data cube. A new algorithm was designed in [22] for finding
the best path in all the clusters based on the generalized
travelling salesman problem. This algorithm was based on the
minimum cost tour within a clustered set of cities. The
application of GA on beam forming of correlated and
uncorrelated static sources was discussed. When the target is
moving in CDMA systems by knowing the spreading code, the
LMS algorithm received the signal at zero rates. The SNR was
as low as -9dB for uncorrelated static sources but when the
signals were correlated, the LMS gave unstable beam forming.
Hence, GA gives good results for all sources which are static
with good error rate [23]. Authors in [24] proposed a multiple
source localization method for DOA estimation at each sensor
mode. The GA based proposed algorithm was compared with
conventional sequential search techniques. The computational
load and inter node communication burden was reduced, so
the algorithm was considered suitable for use in IT based
applications having large numbers of sensor nodes and sources.
The GA is most commonly used as an optimization algorithm
and has efficiently solved various optimization problems across
many disciplines in proficient manners. Alternating projection
maximum likelihood, stimulated annealing grid based search,
and data based grid search are used in the GA. The GA has
widely been used in the DOA estimation by using the spatial
spectrum estimation. The ML technique gives an optimal
solution when compared to other methods. Based on the
likelihood function, the ML is superior to other techniques [3,
14, 25]. A new optimized array structure for DOA estimation
was designed which can estimate both azimuth and elevation
angles. The array is designed with the help of the GA that has
as a fitness function the optimization of the array structure.
The array aperture size increases when the number of elements
in the array is increased. After that, various DOA algorithms have
been applied to the proposed structure and simulation results
were obtained. The proposed array was compared with other
arrays in terms of RMSE of azimuth and elevation angles.

A. Structure of the New Optimized Array

Based on the GA, the proposed design array structure is
given in Figures 1 and 2. In this structure, the population size is
taken as 100 with 1000 iterations and a randomly generated
initial array structure of 10 elements on the XY plane. The
signal generated from various directions is received at each
array element and combined to produce the output.

The optimized new array structure is formed and the total
spacing between array elements is 314.1593 for 10 array
elements and 157.0796 for 20 array elements. Total spacing
reduces when the number of array elements increases along
with the complexity of the structure. The formed structure is
the optimized structure that can be used to detect the signal in
two dimensions. The optimized structure is designed with the
help of GA after several iterations.

B. Steps Involved in Array Optimization

Figure 3 gives the flowchart of the used algorithm. The steps
involved in the optimization of the array are:

Input: Config (PQ, distance, Cost Function), Population,
Crossover 90%, and Mutation 10%, iterations
for Number of Iterations
for population
Selection Cost( Config) ==> MSE ==> Small MSE 10%
Crossover= offsprings are created in hope of producing
better Config
Mutation =Changing the configuration PQ in hope of
producing better Config
Population for next iteration
end
end
C. Execution Time of DOA Estimation Algorithms

In terms of execution time (the time taken by the algorithm to estimate the signal), the ESPRIT algorithm requires the least time when compared to the other algorithms, as can be seen in Figure 4.

Fig. 4. Execution time of algorithms on the proposed configuration.

In the optimized array, we have applied simulations of various DOA algorithms in order to compare their performance in terms of execution time. The simulation results are obtained with different signal directions at various angles. Figures 4-9 give the simulation results of the different DOA algorithms on the optimized array structure. Figure 4 gives the simulation results of the different algorithms in terms of execution time. It is evident that the ESPRIT algorithm shows better results than the other algorithms.

Figures 5 and 6 show the simulation results of the ESPRIT and beam-forming algorithm at different angles (60°, 80°, 90°, 100°, 120°). The angles are accurately estimated through the ESPRIT algorithm but the estimation through beam-forming is poor. Also, in Figures 7-9 (estimation through MUSIC, FOFA, and Capon algorithms) the estimation is accurate and clear. The simulation results of the discussed algorithms depend on the number of sources, the number of antenna elements, the spacing between the elements, and the SNR. When the number of sources is bigger than the antenna elements, it is difficult to estimate the signal directions. When there are more of antenna elements, the obtained results are better. The spacing between the elements is taken as 0.5λ.

Fig. 5. ESPRIT histogram corresponding to 5 signals directions.

Fig. 6. Beam-forming spatial spectrum corresponding to 5 signals.

Fig. 7. MUSIC spatial spectrum corresponding to 5 signals.

Fig. 8. FOFP spatial spectrum corresponding to 5 signals.

D. Comparison with Existing Arrays

The proposed array is compared with Circular, Rectangular, Concentric, and IASA arrays in terms of RMSE of azimuth and elevation angles. The results of the GA based array have been observed to be better when compared with the other arrays. Figures 9-21 give the detailed comparison and show that the RMSE of the proposed design offers better results. Figures 10-11 depict the azimuth and elevation angle comparison with RMSE of the proposed GA based array. The circular array is compared with the GA based array in Figure 10. The result shows that the RMSE of the GA based array is lower for both azimuth and elevation angles.
Figures 12 and 13 show the azimuth and elevation angle comparison with RMSE of the proposed GA based array with the rectangular array. The result shows that the RMSE of the GA based array is lower for both angles. Figures 14 and 15 show the comparison results of azimuth and elevation angle with RMSE of the proposed GA based array with the concentric array. The result shows that the RMSE of the GA based array is lower for both angles. Figures 16 and 17 illustrate the comparison of azimuth and elevation angle with RMSE of the proposed GA based array with the IASA array. The result shows that the RMSE of the GA based array is lower for both angles.

Fig. 9. Capon algorithm spatial spectrum corresponding to 5 signals.

Fig. 10. RMSE of the azimuth angles of circular and GA based arrays.

Fig. 11. RMSE of the elevation angle of circular and GA based arrays.

Fig. 12. RMSE of the azimuth angle of rectangular and GA based arrays.

Fig. 13. RMSE of the elevation angle of rectangular and GA based arrays.

Fig. 14. RMSE of the azimuth angle of concentric and GA based arrays.

Fig. 15. RMSE of the elevation angle of concentric and GA based arrays.
Fig. 16. RMSE of the azimuth angle of IASA and GA based arrays.

Fig. 17. RMSE of the elevation angle of IASA and GA based arrays.

Fig. 18. RMSE error of the azimuth angle of circular, rectangular, concentric, and IASA arrays.

Figures 18 and 19 show the azimuth and elevation angle comparison with RMSE of the four arrays. The result shows that the RMSE of the IASA array is lower for both azimuth and elevation angles. Figures 20 and 21 show the azimuth and elevation angle comparison with RMSE of the proposed GA based array with all the considered existing arrays. The result shows that the RMSE of the GA based array is lower for both azimuth and elevation angles. This type of GA based array is generally used in the communication field for the estimation of the signal and radar and sonar field for signal estimation. The only limitation of this type of optimized GA based array is that it basically estimates the signal direction of static targets.

Concluding, this optimized array structure detects the signal accurately in terms of RMSE.

Fig. 19. RMSE of the elevation angles of circular, rectangular, concentric, and IASA arrays.

Fig. 20. RMSE error of the azimuth angle of circular, rectangular, concentric, IASA, and GA based arrays.

Fig. 21. RMSE error of the elevation angle of circular, rectangular, concentric, IASA, and GA based arrays.

IV. CONCLUSIONS

The DOA of signal estimation is a major concern for the communication technologist. The proposed geometry for DOA estimation aims to meet the estimation accuracy requirements. The simulation results of the proposed configuration show that the GA based array with ESPRIT algorithm has better estimation accuracy among all four proposed DOA estimation
algorithms. The results can be further improved with the help of some other optimized algorithms in terms of number of elements and SNR. Other array configurations can also be taken into consideration for the estimation of the signal angles.

REFERENCES

[1] R. T. Lacoss, “Data adaptive spectral analysis methods,” Geophysics, vol. 36, no. 4, pp. 661–675, Aug. 1971, https://doi.org/10.1190/1.1440203.

[2] J. Capon, “High-resolution frequency-wavenumber spectrum analysis,” Proceedings of the IEEE, vol. 57, no. 8, pp. 1408–1418, Aug. 1969, https://doi.org/10.1109/PROC.1969.7278.

[3] M. I. Miller and D. R. Fuhrmann, “Maximum-likelihood narrow-band direction finding and the EM algorithm,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 38, no. 9, pp. 1560–1577, Sep. 1990, https://doi.org/10.1109/29.232276.

[4] R. Schmidt, “Multiple emitter location and signal parameter estimation,” IEEE Transactions on Antennas and Propagation, vol. 34, no. 3, pp. 276–280, Mar. 1986, https://doi.org/10.1109/TAP.1986.1143830.

[5] A. Barabell, “Improving the resolution performance of eigenstructure-based direction-finding algorithms,” in ICASSP '83. IEEE International Conference on Acoustics, Speech, and Signal Processing, Boston, MA, USA, Apr. 1983, vol. 8, pp. 336–339, https://doi.org/10.1109/ICASSP.1983.1172124.

[6] R. Roy and T. Kailath, “ESPRIT-estimation of signal parameters via rotational invariance techniques,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 37, no. 7, pp. 984–995, Jul. 1989, https://doi.org/10.1109/29.19598.

[7] M. Viberg and B. Ottersten, “Sensor array processing based on subspace fitting,” IEEE Transactions on Signal Processing, vol. 39, no. 5, pp. 1110–1121, May 1991, https://doi.org/10.1109/78.90966.

[8] M. Viberg, B. Ottersten, and T. Kailath, “Detection and estimation in sensor arrays using weighted subspace fitting,” IEEE Transactions on Signal Processing, vol. 39, no. 11, pp. 2456–2460, Nov. 1991, https://doi.org/10.1109/78.97999.

[9] H. S. Hung, S. H. Chang, and C. H. Wu, “3-D MUSIC with polynomial rooting for near-field source localization,” in 1996 IEEE International Conference on Acoustics, Speech, and Signal Processing Conference Proceedings, Atlanta, GA, USA, May 1996, vol. 6, pp. 3065–3068 vol. 6, https://doi.org/10.1109/ICASSP.1996.550523.

[10] Z. Ping, S. Haoshan, and S. Kui, “The 3D Location Algorithm Based on Smart Antenna with MUSIC DOA Estimates,” in 2009 WRI World Congress on Computer Science and Information Engineering, Los Angeles, CA, USA, Mar. 2009, vol. 4, pp. 750–753, https://doi.org/10.1016/CISIE.2009.123.

[11] M. Kaveh and A. Barabell, “The statistical performance of the MUSIC and the minimum-norm algorithms in resolving plane waves in noise,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 34, no. 2, pp. 331–341, Apr. 1986, https://doi.org/10.1109/TASSP.1986.1164815.

[12] Q. T. Zhang, “Probability of resolution of the MUSIC algorithm,” IEEE Transactions on Signal Processing, vol. 43, no. 4, pp. 978–987, Apr. 1995, https://doi.org/10.1109/78.376849.

[13] X. Gu and Y. Zhang, “Effects of amplitude and phase errors on 2-D MUSIC and 2-D ESPRIT algorithms in ISAR imaging,” in 2009 2nd Asian-Pacific Conference on Synthetic Aperture Radar, Oct. 2009, pp. 634–638, https://doi.org/10.1109/APSAR.2009.5374274.

[14] P. Stoica and K. C. Sharrman, “Maximum likelihood methods for direction-of-arrival estimation,” IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 38, no. 7, pp. 1132–1143, Jul. 1990, https://doi.org/10.1109/78.257542.

[15] M. Yang, A. M. Haimovich, X. Yuan, L. Sun, and B. Chen, “A Unified Array Geometry Composed of Multiple Identical Subarrays With Hole-Free Difference Coarrays for Underdetermined DOA Estimation,” IEEE Access, vol. 6, pp. 14238–14254, 2018, https://doi.org/10.1109/ACCESS.2018.2813313.