A Technique of Time Synchronization in Pseudolite System Based on Single-difference Method

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Abstract. Aiming at the time synchronization problem of pure pseudolite system in high precision positioning, a technique of time synchronization in pseudolite system based on single-difference method is proposed. The reference receiver with dual-antenna and the carrier phase observation equation are used and phase integer ambiguity is adjust-ed by the least-squares ambiguity decorrelation adjustment (LAMBDA) method. High precision synchronization of the system clock can be realized through the clock error broadcast or centralized processing. Both theoretical derivation and simulation show that the system is able to provide high accuracy time synchronization and the precision of synchronization can reach 0.02 ns.

1. Introduction

Pseudolites are "satellites" that broadcast GPS-like signals. Global Navigation Satellite Systems (GNSS) almost solve the navigation problem of most open spaces, but in urban canyons, deep wells, underground or indoors, the number and geometric layout of visible satellites cannot meet GNSS positioning services, while pseudolite can solve this problem which is low cost, easy maintenance and flexible deployment. Different pseudolite systems have been proposed [1,2,3,4], most of them using GNSS hardware with minor modifications.

Unlike GPS satellites, for independent distributed pure pseudolite systems, since there is no unified distribution of clocks, each pseudolite clock is independent of each other and is unsynchronized. And if a clock source with insufficient stability, such as TCXO clock, large clock drift errors will occur. Therefore, independent distributed pure pseudolite systems cannot achieve accurate time synchronization due to system clock errors. In pseudolite positioning systems, all pseudolites must be synchronized [5,6].

There are many different technologies for this problem, but the most important thing is to build a reliable synchronization method that is applicable to different environments. In [7], a new pseudolite time synchronization method is proposed, which uses the optical network to achieve time synchronization. The synchronization accuracy is better than 1 ns. In [8], SDS-TWR-URT (symmetric double side two-way ranging with unequal reply time) is proposed to improve the accuracy of pseudolite time synchronization. SCPA(Self-Calibrating Pseudolite Array) is a pseudolite and the transceiver array can realize autonomous calibration positioning [9]. However, these technologies are either costly or technically complex and difficult to implement. Considering some more complicated
situations, such as field monitoring, it is impossible to use direct clock synchronization method of pseudolite, which includes the laying of cables and fiber optic cables.

This paper presents a time synchronization method for independent distributed pure pseudolite systems using a reference receiver with dual-antenna and carrier phase observation equations. Based on the carrier phase observation equation, the clock error model of the system is established, the ambiguity of the carrier phase is eliminated by using dual-antenna switch, the clock error of the pseudolite system is solved by using single-difference method, and the high-precision synchronization of the system clock can be realized by using clock error broadcast or centralized processing.

2. Scheme and Model

2.1. Pseudolite System Architecture

The pseudolite system consists of several pseudolites, a reference receiver with dual-antenna and the several observation receivers. The reference receiver can be quickly switched between the two antennas via an electronic switch. The spatial geometric relationship between the pseudolite $i, j$ and the reference receiver with dual-antenna is shown in Figure 1.

![Figure 1. Geometry of pseudolites and the reference receiver with dual-antenna](image)

According to actual needs, a number of pseudolites can be deployed to form a pseudolite network. An independent local coordinate system is used in this study. It is required that the feeders of the antenna $a$ and the antenna $b$ must be equal in length. By switching the switch, the receiver can respectively obtain the carrier phase measurement values corresponding to the antenna $a$ and the antenna $b$.

2.2. Algorithm Scheme

Firstly, the reference receiver can obtain the corresponding two sets of carrier phase values through the fast switching and measurement of the two antennas, and perform the single difference operation between pseudolites respectively. Then, the pseudolite relative clock difference and the integer ambiguity of pseudolites are initialized. The integer ambiguity of carrier phase is fixed by the LAMBDAA method [10], and finally the relative clock error between pseudolites is calculated by the fixed ambiguity.

The dual-antenna receiver connected to the antenna $a$ captures the signal of the pseudolite $i$ and outputs the carrier phase observation value $\phi_{ai}$ with respect to the antenna $a$. When the electronic switch quickly turns to the antenna $b$, the receiver can continuously track the signal of the pseudolite $i$ by means of the carrier loop technology, and obtain the phase observation value $\phi_{bi}$ relative to the antenna $b$. Using the carrier phase observation equation, $\phi_{ai}$ and $\phi_{bi}$ can be expressed as:

\begin{align}
\phi_{ai} &= \lambda^{-1}R_a^i + \lambda^{-1}c(\delta t - \delta T^i) - N^i + \epsilon_{\phi,a} \\
\phi_{bi} &= \lambda^{-1}R_b^i + \lambda^{-1}c(\delta t - \delta T^i) - N^i + \epsilon_{\phi,b} 
\end{align}

Where $i$ is the pseudolite constellation number, $\lambda$ is the wavelength of the pseudolite carrier signal, $N^i$ represents the carrier phase ambiguity of the pseudolite $i$ signal captured by the antenna which is unknown, $c$ represents the propagation speed of the positioning signal, $R_a^i$ and $R_b^i$ represent the
geometric distances of antenna a and antenna b to the i-th pseudolite, δt is the receiver clock error, δT is the clock error of pseudolite i, εφa and εφb is the measurement noise of the carrier phase observation.

For the independent distributed pure pseudolite system is asynchronous, the pseudolite clock error δTi is different. But for the same epoch receiver δt is the same, so a single difference to the pseudolite i, j can eliminate the effect of δt:

\[
\varphi^{ij} = \lambda^{-1} R^{ij} + \lambda^{-1} c \delta T^{ij} - N^{ij} + \varepsilon^{ij}_\phi
\]

(3)

Where \(\varphi^{ij}\) is carrier phase single difference of the pseudolite i and j, \(R^{ij}\) is the geometric distance difference between the pseudolite and the receiver, \(\delta T^{ij}\) is the relative clock difference between the pseudolite i and j, \(N^{ij}\) is the relative ambiguity difference between the pseudolites i and j. The fixed position coordinates of the two antennas at the reference station and the pseudolite position coordinates are known, \(R^{ij}\) is a known amount. Rewrite equation (3) as a variable on one side:

\[
\lambda \varphi^{ij} - R^{ij} = c \delta T^{ij} - \lambda N^{ij} + \lambda \varepsilon^{ij}_\phi
\]

(4)

Assuming that the system consists of four pseudolites, with the reference of pseudolite constellation 1, for antenna a, there are:

\[
\begin{align*}
\lambda \varphi^{21}_a - R^{21}_a &= c \delta T^{21} - \lambda N^{21} + \lambda \varepsilon^{21}_\phi \\
\lambda \varphi^{31}_a - R^{31}_a &= c \delta T^{31} - \lambda N^{31} + \lambda \varepsilon^{31}_\phi \\
\lambda \varphi^{41}_a - R^{41}_a &= c \delta T^{41} - \lambda N^{41} + \lambda \varepsilon^{41}_\phi
\end{align*}
\]

(5)

Similarly, for antenna b there are:

\[
\begin{align*}
\lambda \varphi^{21}_b - R^{21}_b &= c \delta T^{21} - \lambda N^{21} + \lambda \varepsilon^{21}_\phi \\
\lambda \varphi^{31}_b - R^{31}_b &= c \delta T^{31} - \lambda N^{31} + \lambda \varepsilon^{31}_\phi \\
\lambda \varphi^{41}_b - R^{41}_b &= c \delta T^{41} - \lambda N^{41} + \lambda \varepsilon^{41}_\phi
\end{align*}
\]

(6)

It can be known from equation (5) and equation (6) that the equations are linear equations, so equations (5-6) can be directly written as a matrix product:

\[
HX = b
\]

(7)

We have to define \(H\), \(X\) and \(b\) in (7). For \(H\), we first define the matrix as:

\[
H = \begin{bmatrix}
\lambda & 0 & 0 & -\lambda & 0 & 0 \\
0 & \lambda & 0 & 0 & -\lambda & 0 \\
0 & 0 & \lambda & 0 & 0 & -\lambda \\
\lambda & 0 & 0 & -\lambda & 0 & 0 \\
0 & \lambda & 0 & 0 & -\lambda & 0 \\
0 & 0 & \lambda & 0 & 0 & -\lambda
\end{bmatrix}, \quad \text{and:} \quad X = \begin{bmatrix}
\delta T^{21} \\
\delta T^{31} \\
\delta T^{41} \\
N^{21} \\
N^{31} \\
N^{41}
\end{bmatrix}, \quad b = \begin{bmatrix}
\lambda \varphi^{21}_a - R^{21}_a \\
\lambda \varphi^{31}_a - R^{31}_a \\
\lambda \varphi^{41}_a - R^{41}_a \\
\lambda \varphi^{21}_b - R^{21}_b \\
\lambda \varphi^{31}_b - R^{31}_b \\
\lambda \varphi^{41}_b - R^{41}_b
\end{bmatrix}.
\]

The condition number of the coefficient matrix is calculated by the MATLAB function, the condition number of the coefficient matrix is \(\text{cond}(H) = 7.6429 \times 10^{24}\), which indicates that the equation \(HX = b\) is seriously ill-conditioned, so the general direct method and the iterative method will have large errors and cannot be solved correctly. Therefore, it is necessary to improve the conditional number of the coefficient matrix first when solving the ill-conditioned equations, that is, to pretreat and reduce the conditional number of the matrix by pretreatment. By multiplying each row (or column) of \(H\) by an appropriate constant, i.e. finding reversible diagonal matrices \(D_1\) and \(D_2\), the system of equation \(HX = b\) is turned into

\[
D_1HD_2X = D_1b, X = D_2X
\]

(8)

This is called the balance problem of the matrix. In theory, it is best to choose the diagonal matrix \(\overline{D_1}, \overline{D_2}\) to satisfy
\[ \text{cond}(\overline{D}_1H\overline{D}_2) = \min(\text{cond}(D_1HD_2)) \quad (9) \]

\( \overline{D}_1 \) in equation (9) is to balance each equation, and \( \overline{D}_2 \) is to balance the unknown. For the sake of simplicity, let \( \overline{D}_2 = I \), \( D_1 = \text{diag}(\frac{1}{x_1}, \frac{1}{x_2}, ..., \frac{1}{x_n}) \), \( s_i = \max|h_{ij}|(i = 1, 2, ..., n) \). Therefore, \( HX = b \) is equivalent to \( DHX = Db \).

The float integer ambiguity can be obtained by pre-processing, and the integer ambiguity can be fixed by the LABMDA algorithm. In the next epoch, the fixed ambiguity is brought in and initialization is completed. The next epoch only needs a group of antennas in a or b to calculate the clock error.

Taking antenna a as an example, in the epoch \( k \), there is:

\[
\begin{align*}
& c \delta T_{k}^{21} + \lambda \varphi_{o,n,k}^{21} = \lambda \varphi_{h,k}^{21} - R_{a}^{21} + \lambda N^{21} \\
& \left(\begin{array}{c}
\delta T_{k}^{21} + \lambda \varphi_{o,n,k}^{21} = \lambda \varphi_{h,k}^{21} - R_{a}^{21} + \lambda N^{21} \\
\delta T_{k}^{31} + \lambda \varphi_{o,n,k}^{31} = \lambda \varphi_{h,k}^{31} - R_{a}^{31} + \lambda N^{31} \\
\delta T_{k}^{41} + \lambda \varphi_{o,n,k}^{41} = \lambda \varphi_{h,k}^{41} - R_{a}^{41} + \lambda N^{41}
\end{array}\right) \quad (10)
\]

Similarly, by using matrix multiplication, the equation (10) is written as:

\[ X = H^{-1}b \quad (11) \]

\[ H = \begin{bmatrix}
 c & 0 & 0 \\
 0 & c & 0 \\
 0 & 0 & c
\end{bmatrix}, X = \begin{bmatrix}
 \delta T^{21} \\
 \delta T^{31} \\
 \delta T^{41}
\end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix}
 \lambda \varphi_{h,k}^{21} - R_{a}^{21} + \lambda N^{21} \\
 \lambda \varphi_{h,k}^{31} - R_{a}^{31} + \lambda N^{31} \\
 \lambda \varphi_{h,k}^{41} - R_{a}^{41} + \lambda N^{41}
\end{bmatrix}. \]

3. Simulation and Discussions

3.1. Simulation Scheme

In order to simulate and verify the correctness and feasibility of the clock difference algorithm between slave pseudolites and the master pseudolite, four pseudolites are used in the system, with No. 1 main pseudolite as the reference, and the remaining 2-4 are the slave pseudolites. The carrier wavelength of the radio frequency signal used by the pseudolite is 0.32m. In the simulation process, the carrier phase measurement value of the receiver is superimposed with a maximum random noise of 0.01 cycle.

3.2. Simulation Results and Discussions

The statistical values of the error between the estimated clock error and the real clock error between the slave pseudolites and the main pseudolite are shown in Table 1. Figure 2 is the error between the estimated clock error and the real clock difference, and Figure 3 is the ranging error corresponding to the estimation error of the clock difference.

| Item       | Pseudolite2-1 | Pseudolite3-1 | Pseudolite4-1 |
|------------|---------------|---------------|---------------|
| Maximum/ns | 7.14\times10^2 | 7.50\times10^2 | 8.51\times10^2 |
| Minimum/ns | 4.74\times10^3 | 5.01\times10^5 | 4.26\times10^5 |
| Mean/ns    | 2.04\times10^2 | 2.05\times10^2 | 2.01\times10^2 |
Through the 1000 epochs simulation above, it can be seen from Table 1 and Figure 2 that the relative clock error (synchronization accuracy) obtained by the method proposed in this paper can reach 0.02 ns on average. As can be seen from Figure 3, the estimation error of the above clock error brings the ranging error less than 3 cm.

From the above analysis, it can be seen that using the reference receiver with dual-antenna and the carrier phase observation equation, the integer ambiguity of carrier phase is fixed by the LAMBDA algorithm. The method of calculating the clock error of single difference pseudolite system is feasible. The synchronization accuracy can reach 0.02 ns, and the ranging error is less than 3 cm, which can realize the high precision of clock synchronization in pseudolite network.

4. Conclusion

In this paper, the problem of clock synchronization in high-precision positioning of pure pseudolite networks is studied. A method based on dual antennas is proposed to realize the clock synchronization of pseudolite networks. The simulation results show that the method can realize the high-precision clock synchronization of the pseudolite network, and the synchronization accuracy can reach 0.02 ns, and the ranging error is less than 3 cm. If you want to achieve higher precision requirements, consider using the cesium atomic clock as the clock for the main pseudolite.

5. Acknowledgments

This work was financially supported by the 2018 Guilin University of Electronic Technology Graduate Education Innovation Program No. 2018YJCX27 fund.
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