Diffusion as a leading dissipative mechanism in superconducting neutron stars

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ABSTRACT

Despite the fact that different particle species can diffuse with respect to each other in neutron star (NS) cores, the effect of particle diffusion on various phenomena associated with NS oscillations is usually ignored. Here we demonstrate that the diffusion can be extremely powerful dissipative mechanism in superconducting NSs. In particular, it can be much more efficient than the shear and bulk viscosities. This result has important implications for the damping times of NS oscillations, development and saturation of dynamical instabilities in NSs, and for the excitation and coupling of oscillation modes during the late inspiral of binary NSs.

Key words: asteroseismology — dense matter — diffusion — hydrodynamics — stars: neutron

1 INTRODUCTION

Neutron stars (NSs) are unique astrophysical objects offering an opportunity to probe the properties of the superdense matter under the most extreme conditions far beyond those reproduced in terrestrial experiments. These properties can be constrained, in particular, by confronting various observational manifestations of stellar oscillations with the theoretical models (Steiner & Watts 2009; Strohmayer & Mahmoodifar 2014; Andersson et al. 2014; Sotani et al. 2018; Maselli et al. 2020; Kantor et al. 2020).

An NS can start oscillating either as a result of an internal instability or external perturbation. Whether an oscillation mode can actually be excited depends on the interplay between the excitation rate and on how efficiently dissipative mechanisms in the stellar matter counteract this excitation. Shear and bulk viscosities are the most paid attention to dissipative agents, operating in the NS matter (Cutler & Lindblom 1987; McDermott et al. 1988). Generally, at temperatures $T \lesssim 5 \times 10^8$ K, it is the shear viscosity, that appears to be the strongest one, while the effects due to bulk viscosity are comparatively weak and can be ignored (Cutler et al. 1990; Gusakov et al. 2005; Glampedakis & Gualtieri 2018; but see Ofengeim et al. 2019).

Although the chemical composition of the stellar matter is rather complex and includes different particle species, such as neutrons ($n$), protons ($p$), electrons ($e$), and muons ($\mu$) in the simplest case, and also such “exotic” particles as hyperons and quarks in the more sophisticated models, the effect of particle diffusion on NS oscillations, as far as we know, has not yet been investigated. In this Letter we take a first look into the role of the diffusion as a dissipative agent that can damp NS oscillations (in analogy to how the diffusion dissipates the stellar magnetic field, see Goldreich & Reisenegger 1992). Below we restrict ourselves to the case of npe/npe$\mu$ matter and ignore exotica. It is believed that neutrons and protons in NS interiors can be in the superfluid state (see Lombardo & Schulze 2001; Page et al. 2013). According to microscopic calculations (Gezerlis et al. 2014; Ding et al. 2016; Sedrakian & Clark 2019), the maximum critical temperature $T_{cn}$ of neutron superfluidity onset is substantially lower than the proton one, $T_{cp}$. Therefore, for simplicity, neutrons in this initial study are treated as normal, while for protons we consider two possibilities, assuming that they are either normal (normal matter) or completely superfluid (superconducting matter; all protons form Cooper pairs). As we demonstrate, in superconducting matter diffusion becomes the leading dissipative mechanism that strongly accelerates the dissipation of various oscillation modes and thus makes questionable our current vision of a number of aspects of NS life. We expect that our basic conclusions will remain unaffected for not too cold NSs even in the presence of neutron superfluidity, $T \lesssim T_{cn}$ (see a comment in the end of Sec. 4).

2 THE EFFECT OF DIFFUSION

Let us first discuss the effect of diffusion on the damping of sound waves perturbed in the initially homogeneous unmagnetized npe-matter. Each particle species $\alpha$ ($\alpha = n, p \; or \; e$) is characterized by the electric charge $e_\alpha$, number density $n_\alpha$, relativistic chemical potential $\mu_\alpha$ (including the rest mass), and the velocity $v_\alpha$. In superconducting matter $v_p$ is the velocity of proton Bogoliubov thermal excitations. Proton superconductivity adds one more velocity field to the system — the proton superfluid velocity $v_{sp}$. Thanks to the electromagnetic interaction protons and electrons move together, so that, to a very high precision, their number densities and cur-
rents coincide (see Braginskii 1965; Mendell 1991; Gusakov & Dommes 2016; Dommes et al. 2020)

\[ n_e = n_p, \quad n_e v_e = n_p v_p + (n_p - n_p e) v_{ap}, \quad (1) \]

where \( n_p e \) is the number density of proton thermal excitations; in normal matter \( n_p e = n_p \). An unperturbed matter is supposed to be in \( \beta \)-equilibrium, \( \delta \mu_\beta = \mu_p - \mu_p - \mu_p = 0 \) (hereafter index ‘0’ refers to the unperturbed quantities). Beta-processes are too slow (Yakovlev et al. 2001) to affect oscillations in sufficiently cold matter and will be neglected in what follows.

Within the framework developed in Braginskii (1965), the friction force between the species \( \alpha \) and \( \beta \) is \( J_{\alpha \beta} w_{\alpha \beta} \), where \( w_{\alpha \beta} \equiv w_{\alpha} - w_{\beta} \) and \( J_{\alpha \beta} \) is the momentum transfer rate. The energy leakage due to diffusion equals the work done by the friction force per unit time and can be written as an integral over the stellar volume \( V \) (see Braginskii 1965; Gusakov et al. 2017)

\[ \dot{E}_{\text{diff}} = -\frac{1}{2} \sum_{\alpha \beta} J_{\alpha \beta} w_{\alpha \beta}^2 \, dV. \quad (2) \]

Although Braginskii (1965) discussed Eq. (2) in application to the normal matter, the same expression, in principle, is valid in the superfluid/superconducting matter provided that the proton velocity in Eq. (2) is that of the proton thermal excitations (because only thermal excitations can scatter from other particle species; see, e.g., Aronov et al. 1981).

Relative velocities in Eq. (2) depend on the efficiency of particle collisions (more effective collisions correspond to smaller relative velocities) and should be expressed through other variables describing perturbation. When particle collisions are much more frequent than the oscillation frequency \( \omega \), the system is in the hydrodynamic regime. In this regime the linearized Euler-like equations governing perturbations in the normal matter take the form (Braginskii 1965; Goldreich & Reisenegger 1992; Dommes et al. 2020) \((\alpha = n, p, e)\)

\[ \frac{n_\alpha n_\beta \partial \delta \mu_\alpha}{c^2} = e_\alpha n_\alpha \mathbf{E} - n_\alpha \nabla \delta \mu_\alpha - \sum_\beta J_{\alpha \beta} w_{\alpha \beta}, \quad (3) \]

where \( \mathbf{E} \) is the electric field and \( c \) the speed of light. The hydrodynamic limit corresponds to large values of \( J_{\alpha \beta} \) such that \( w_{\alpha \beta} \) is much smaller than a typical hydrodynamic velocity \( v \). Thus, in what follows \( w_{\alpha \beta} \) can be set to zero unless it is multiplied by \( J_{\alpha \beta} \).

Eqs. (3) are widely known in the neutron-star and plasma literature (e.g., Gusakov et al. 2017 and references therein) and can be derived from the Boltzmann transport equations for each particle species \( \alpha \) in a manner, completely analogous to the derivation in Yakovlev & Shalybkov (1991). Note that, in these equations only diffusion dissipation is allowed for, while other dissipative mechanisms (e.g., viscosity) are ignored. This is justified in the hydrodynamic regime, in which effects of different dissipation mechanisms can be studied separately (e.g., Landau & Lifshitz 1987).

Summing up Eq. (3) divided by \( n_\alpha \) for protons and electrons and subtracting Eq. (3) for neutrons divided by \( n_\alpha \), we, using Eq. (1) and the \( \beta \)-equilibrium condition \( \delta \mu_\beta = 0 \), arrive at the equation

\[ \frac{\mu_\alpha}{c^2} \frac{\partial \delta \mu_\alpha}{\partial t} = \nabla \delta \mu_\alpha + \left[ \frac{J_{np}}{n_\rho} + \frac{J_{en}}{n_\alpha} + \frac{J_{np} + J_{en}}{n_\alpha} \right] w_{np}. \quad (4) \]

The inertial term in the left-hand side of this equation, which can be estimated as \(-\mu_\alpha \omega w_{np} / c^2\), is much smaller than the last term in the right-hand side if \( \mu_\alpha \omega / c^2 \ll J_{np} / n_\alpha \). Taking into account that \( J_{np} \sim (10^{24} - 10^{30}) (T / 10^8 \text{K})^2 \times \text{g cm}^{-3} \text{s}^{-1} \) (see, e.g., Section VII in Dommes et al. 2020), this inequality is always satisfied for typical neutron-star conditions, so that we can neglect the inertial term in (4). This allows us to express the relative velocities through \( \delta \mu \) and transform the energy loss rate due to diffusion (2) into

\[ \dot{E}_{\text{diff}} = -\frac{1}{2} \sum_{\alpha \beta} J_{\alpha \beta} w_{\alpha \beta}^2 \, dV, \quad (5) \]

where \( n_\alpha = n_\alpha + n_p \) is the baryon number density. To derive (5) we neglected \( J_{en} \) compared to \( J_{np} \) since in normal matter collisions between protons and neutrons are extremely efficient due to strong interactions and \( J_{np} / J_{en} \sim 10^5 \) (see Dommes et al. 2020).

In what follows, we consider the case of strong proton superconductivity, \( T \ll T_\text{cp} \), suitable for not too young NSs. At \( T \ll T_\text{cp} n_p e \rightarrow 0 \) and Eq. (3) for protons should be replaced with the Josephson equation for the superconducting proton component (Putterman 1974)

\[ \frac{\mu_\alpha}{c^2} \frac{\partial \delta \mu_\alpha}{\partial t} = e_p E - \nabla \mu_p. \quad (6) \]

Moreover, in this limit superconductivity strongly suppresses scattering processes involving protons (the number of proton thermal Bogoliubov excitations is exponentially suppressed), so that the proton-related momentum transfer rates tend to zero, \( J_{np} = 0 \). In these conditions Eq. (2) reduces to

\[ \dot{E}_{\text{diff}} = -\frac{1}{2} \sum_{\alpha \beta} J_{\alpha \beta} w_{\alpha \beta}^2 \, dV. \quad (7) \]

To derive (7) we make use of Eq. (1) with \( n_p e \rightarrow 0 \), Eq. (3) for neutrons and electrons, Eq. (6), and the equality \( \delta \mu_\beta = 0 \).

When dissipation is weak, the imbalance \( \delta \mu \) in Eqs. (5) and (7) can be calculated using the equations of nondissipative hydrodynamics (see, e.g., Section 3.1), which are the same in normal and superconducting matter. Then Eqs. (5) and (7) imply that dissipation in superconducting matter is by a factor of \( \sim J_{np} / J_{en} \sim 10^5 \) more efficient than in normal matter.

It can be shown that Eqs. (5) and (7) are equally applicable to the inhomogeneous npe-matter of Newtonian stars. Similar equations valid in General Relativity (GR) can be derived within the framework of relativistic multi-fluid dissipative hydrodynamics developed in Dommes et al. (2020); Dommes & Gusakov (2021).

### 3 Oscillation damping times

To estimate the effect of diffusion on damping of sound waves, as well as on damping of global f-, p-, g-, and r-modes, we confront here the damping (e-folding) times due to diffusion,

\[ \tau_{\text{diff}} = -2E / \dot{E}_{\text{diff}}, \]

with those due to shear viscosity,

\[ \tau_\eta = -2E / \dot{E}_\eta, \]

where \( \dot{E}_\eta \) and \( \dot{E}_{\text{diff}} \) are the corresponding dissipation rates averaged over the oscillation period and \( E \) is the oscillation energy (Landau & Lifshitz 1987).

In all numerical calculations below we employ the BSk24 equation of state (Goriely et al. 2013), allowing for muons, and adopt shear viscosity coefficients and momentum transfer...
rates from Shternin (2018) and Dommes et al. (2020). In the case of superconducting matter we use equation (28) of Shternin (2018) for the shear viscosity and neglect the effect of proton superconductivity on $J_{\text{en}}$.

### 3.1 Sound waves

To calculate $\tau_{\text{diff}}$ for sound waves, we expand $\delta\mu$ in Eqs. (5) and (7) [which is, generally, a function of all particle number densities, $\delta\mu = \delta\mu(n_\alpha, n_\beta, n_\gamma)$] as

$$\delta\mu = \sum_\alpha \frac{\partial \delta\mu}{\partial n_\alpha} n_\alpha = \sum_\alpha \frac{\partial \mu_\alpha n_\alpha v k}{\omega},$$

where $k$ is the wave number, and we used the continuity equations, $-\omega \delta n_\alpha + kn_\alpha v = 0$, to express the particle number density perturbations $\delta n_\alpha$ ($\alpha = n, p, e$) through the fluid velocity $v = v_0 \cos(kx - \omega t)$ (which is the same for all particle species in the nondissipative limit). For sound waves, the oscillation energy per unit volume is $E = \sum_\alpha \mu_\alpha n_\alpha v^2 k^2 / (2c^2)$, thus

$$\tau_{\text{diff}} = J \frac{2\omega}{k^4} \left( \frac{n_{\mu 0}}{n_0 n_{\mu 0}} \right)^2 \left( \sum_\alpha \frac{\partial \mu_\alpha}{\partial n_\alpha} n_{\mu 0} \right)^2 \sum_\alpha \frac{\mu_\alpha n_{\mu 0}}{c^2},$$

where $J$ stands for $J_{\text{en}}$ in normal matter and for $J_{\text{en}}$ in superconducting matter.

In order to proceed to higher densities, where muons are present, we generalize the results derived above to the case of $\text{npe}$ matter. The ratio $\tau_{\text{diff}}/\tau_\eta$ is plotted in Fig. 1 as a function of $n_b$ for normal (dashes) and strongly superconducting ($T \ll T_{\text{sp}}$, solid line) matter. In superconducting matter $\tau_{\text{diff}}$ and $\tau_\eta$ obey almost the same temperature dependence ($\propto T^2$) and their ratio is almost temperature-independent ($\tau_{\text{diff}}$ is approximately $\propto T^2$ because $J_{\text{en}}$ and $J_{\text{en}}$ are $\propto T^2$, see Dommes et al. 2020; small deviation from this scaling is caused by the coefficient $J_{\text{en}}$, which has a more complicated behavior, but whose contribution is small). This is not the case in normal matter for which we present $\tau_{\text{diff}}/\tau_\eta$ for $T = 10^7$ and $10^8$ K. The curves do not vary with $k$ or $\omega$, since both damping times scale as $k^{-2} \propto \omega^{-2}$ (see Landau & Lifshitz 1987 and note that $\omega \propto k$ for sound waves). In normal npe-matter particles are locked to each other: neutrons are locked to protons due to frequent collisions caused by strong interaction, while electrons are locked to protons by electromagnetic interaction. As a result, diffusion is inefficient. Appearance of muons allows charged particles to move with respect to each other, and the efficiency of diffusion increases strongly. However, our results imply that, anyway, diffusion is less effective in normal matter in the whole range of densities than the shear viscosity. At the same time, in superconducting matter neutrons are free to move with respect to protons and it is the diffusion, that becomes the dominant channel of energy losses. As we demonstrate below, this conclusion remains correct also for global oscillatory modes of NSs.

### 3.2 Global oscillation modes

Using the formalism developed in Dommes et al. (2020) and Dommes & Gusakov (2021), we calculate the damping times of relativistic $f$-, $p$-, and $g$-modes, and Newtonian $r$-modes for an NS in the Cowling approximation (e.g., Lindblom & Splinter 1990). We consider a three-layer NS consisting of the barotropic crust, npe outer core, and npe inner core. We assume that protons are strongly superconducting, $T_{\text{sp}} \gg T$, while neutrons are normal. The oscillation eigenfunctions and eigenfrequencies in the absence of dissipation (in particular, the imbalance $\delta\mu$) are calculated with our codes developed in Kantor & Gusakov (2014, 2017).

#### f-, p-, and g-modes

Damping times versus eigenfrequency $\sigma \equiv \omega/(2\pi)$ are shown in Fig. 2 for the first ($l = 2$, $m = 0$) $f$-, $p$-, and $g$-modes of an NS with the mass $M = 1.4M_\odot$ and redshifted internal stellar temperature $T_{\text{ex}} = 10^8$ K (as seen by a distant observer). Upper points represent dissipation due to shear viscosity, $\tau_\eta$, while lower points show diffusion damping times, $\tau_{\text{diff}}$.

One can see that for $p$-modes $\tau_\eta$ exceeds $\tau_{\text{diff}}$ by approximately two orders of magnitude. The same result was obtained for sound waves (Fig. 1), which is not surprising since $p$-modes are their close relatives. In analogy to sound waves, both damping times fall simultaneously with increasing num-
For g-modes the difference between $\tau_\eta$ and $\tau_{\text{diff}}$ is even larger — almost four orders of magnitude. For these low-frequency oscillations $\sigma \propto L$, where $L$ is the lengthscale of the perturbation, while higher harmonics are dominated by the $\theta$-component of the velocity $v_\theta \approx v_\nu \sigma_0/\sigma$, where $v_\nu$ is the radial velocity and $\sigma_0$ is the eigenfrequency of the main harmonic. Moreover, the Eulerian perturbation of the pressure is very small, which leads to $\nabla \delta \mu \propto v_\nu/(\sigma L)$. As a result, $\tau_\eta \propto \tau_{\text{diff}} \propto \sigma^2$ for g-modes. Note that in case of g-modes diffusion remains the most powerful dissipation mechanism even for normal NSs, e.g. $\tau_\eta/\tau_{\text{diff}} \approx 4.8$ for the main g-mode harmonic. In turn, the efficiency of diffusion for the f-mode is strongly suppressed ($\tau_\eta$ and $\tau_{\text{diff}}$ almost coincide) since this mode is almost incompressible and chemical potential imbalances are practically not perturbed in the course of oscillations.

**r-modes:** The rotational, predominantly toroidal modes — r-modes — puzzle NS community since 1998, when they were predicted to be unstable due to gravitational radiation (Andersson 1998; Friedman & Morsink 1998). Confronting the r-mode excitation and dissipation timescales defines the instability window, that is the region on the $\nu - T^\infty$ plane, where dissipation cannot counteract the r-mode growth (Haskell 2015) ($\nu$ is the NS rotation frequency). Despite predictions of the modeling that an NS cannot stay in the instability window for any considerable amount of time (see Levin 1999), numerous sources have been observed there (Gusakov et al. 2014). Revealing some strong dissipative mechanism, that has not yet been identified could reconcile theory and observations. Here we examine whether diffusion could serve as such a mechanism.

We calculate damping times for the most unstable $l = m = 2$ r-mode, treating perturbations within the Newtonian framework in the Cowling approximation, and adopting the same stellar model and microphysics input as above. In our calculations we assume that $\nu$ is small compared to the Kepler frequency, $\nu_K$, and expand all the perturbations in the parameter $\nu/\nu_K$. Fig. 3 shows the resulting instability curves (boundaries of the instability window), where the r-mode excitation is balanced by the shear viscosity only (dashes) and by the combined action of shear viscosity and diffusion (thick solid line). Above the curves the r-mode is unstable.

Note that for the r-mode $\sigma \propto \nu$, while perturbations of thermodynamic quantities, in particular chemical potentials, are suppressed by a factor of $\nu^2/\nu_K^2$. As a result, $1/\tau_{\text{diff}}$ turns out to be $\propto \nu^2$ [see Eq. (7)], while $1/\tau_{\eta}$ does not depend on $\nu$. Thus, while at high $\nu$ diffusion is as effective as shear viscosity\(^1\), at lower $\nu$ diffusion is negligible. Consequently, in the Newtonian framework the effect on the instability curve is smaller at higher $T^\infty$, since the curve $\nu(T^\infty)$ in Fig. 3 is a decreasing function of temperature.

Notice, however, that in GR perturbations of chemical potentials are $\propto \nu/\nu_K$ (see equation 17 in Lockitch et al. 2003 and also Lockitch et al. 2001), hence $1/\tau_{\text{diff}}$ does not scale with $\nu$. We have not carried out r-mode calculations in full GR yet. However, to get an impression of how efficient diffusion in GR may be, we assumed that Newtonian approach and GR give similar results for $\tau_{\text{diff}}$ at $\nu = \nu_K$ (since chemical potential perturbations are not suppressed at $\nu = \nu_K$).

Note that, while the expansion parameter $\nu/\nu_K$ is not small at $\nu = \nu_K$, we can still formally solve the expanded oscillation equations at $\nu = \nu_K$ and rescale the result to smaller $\nu$. Then we calculate $\tau_{\text{diff}}$ at $\nu = \nu_K$ in the Newtonian framework and assume that in GR $\tau_{\text{diff}}$ equals this value at any $\nu$. The resulting GR instability curve due to diffusion and shear viscosity is shown by thin solid line and noticeably differs from the thick one. We emphasize that this is only an estimate, an accurate calculation will be published elsewhere.

### 4 CONCLUSIONS

We propose that particle diffusion can be a very efficient dissipative mechanism in NSs. In contrast to the mutual friction dissipation (e.g., Haskell 2015), this mechanism does not need superfluid vortices in order to operate. We compare damping of sound waves, as well as of $f$-, $p$-, $g$-, and r-modes due to diffusion and shear viscosity in NSs composed of neutrons, protons, and leptons. We find that, when protons are normal, the effect of diffusion on stellar oscillations is relatively small and can be ignored for all modes except for g-modes.

In contrast, for superconducting protons diffusion leads to the very fast damping of oscillations, especially in the case of sound waves, p- and g-modes, leaving shear viscosity (which is believed to be the key dissipative mechanism) far behind.

Our results imply that damping times of all oscillation modes should be revisited. Every physical phenomenon, dealing with the development of one or another hydrodynamic instability — for example, resonant excitation of modes (Yu & Weinberg 2017) and $p$-g tidal instability (Weinberg 2016; Andersson & Ho 2018; Abbott & et al. 2019) during NS-NS and NS-BH binary inspirals; excitation of r-modes in rapidly rotating NSs (Haskell 2015); saturation of r-modes due to non-linear coupling to the inertial-gravity modes (Bondarescu et al. 2007); instability onset due to precession (Glampedakis et al. 2009); glitch models based upon the excitation of hydrodynamic instabilities in the core (e.g., Glampedakis & Ander-
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B will not be substantially reduced for not too high magnetic fields, hence decrease dissipation. However, our estimates show that the reduction in the number of “normal” neutrons participating in the scattering processes. Moreover, neutron superfluidity modifies the hydrodynamic flows, which can also affect diffusion. A combined study of all these effects is important and deserves a special consideration.

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DATA AVAILABILITY

The data underlying this article are available in the article.

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