Fast Reliability Assessing Method for Distribution Network with Distributed Renewable Energy Generation

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Abstract. This paper proposes a fast reliability assessing method for distribution grid with distributed renewable energy generation. First, the Weibull distribution and the Beta distribution are used to describe the probability distribution characteristics of wind speed and solar irradiance respectively, and the models of wind farm, solar park and local load are built for reliability assessment. Then based on power system production cost simulation probability discretization and linearization power flow, a optimal power flow objected with minimum cost of conventional power generation is to be resolved. Thus a reliability assessment for distribution grid is implemented fast and accurately. The Loss Of Load Probability (LOLP) and Expected Energy Not Supplied (EENS) are selected as the reliability index, a simulation for IEEE RBTS BUS6 system in MATLAB indicates that the fast reliability assessing method calculates the reliability index much faster with the accuracy ensured when compared with Monte Carlo method.

1. Introduction

As distributed renewable energy generation is widely connected to the distribution network, the one way radiation network has turned into a network with consumers interconnected and power sources scattering[1,2]. Considering the intermittency and randomness of new energy generation-such as wind power and solar power, the security and stability of power system is significantly affected. Therefore, to study power supply reliability of distribution grid with distributed new-energy generation appears to be particularly important.

Previous work has made large number of achievements. Ref[3] adopts the clustering algorithm and establishes the reliability analysis model based on analytical method. Ref[4] addresses the reliability evaluation issue by using Monte Carlo method based on Latin hypercube sampling. Ref[5] utilizes the Copula theory to build joint probability distribution model of output power of wind farm and solar park, and set up the model of wind-solar hybrid power system on that basis. Ref[6] makes use of the mapping relationship between estimation points of distributed generation and power load and power supply reliability, and implements points estimation for reliability of distributed network. Ref[7] proposes the reliability probability distribution model of distribution grid with DG through random function for reliability indexes of distributed network in system level and node layer.

The Monte Carlo method is frequently used to assess the power supply reliability of new energy generation[8,9], due to the advantage of processing random problems and high accuracy. However this method is very time-consuming on the other side. During the planning stages of power system with new energy generation, power supply reliability is evaluated by comparing among several alternative plans.
Generally this type of evaluation has no use for high accuracy and requires a small time cost, as the system network topology can be adjusted. Hence, the traditional Monte Carlo method appears inefficient under that circumstance.

This paper proposes a fast reliability assessing method for distribution network with distributed renewable energy generation. First, the Weibull distribution and the Beta distribution are used to describe the probability distribution characteristics of wind speed and solar irradiance respectively, and the models of wind farm, solar park and local load are built for reliability assessment. Then based on power system production cost simulation probability discretization and linearization power flow, a optimal power flow objectied with minimum cost of conventional power generation is to be resolved. Thus a reliability assessment for distribution grid is implemented fast and accurately. The Loss Of Load Probability (LOLP) and Expected Energy Not Supplied (EENS) are selected as the reliability index, a simulation for IEEE RBTS BUS6 system in MATLAB indicates that the fast reliability assessing method calculates the reliability index much faster with the accuracy ensured when compared with Monte Carlo method.

2. Models of Wind Farm, Solar Park and Powr Load

2.1. Probability Models of Wind Speed and Solar Irradiance

Using Weibull distribution with double parameters to describe the probability distribution characteristics of wind speed[8,10], the probability density function is given as follows:

\[ f(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp \left( -\left( \frac{v}{c} \right)^k \right) \]  

(1)

where \( v \) is wind speed, \( k, c \) is shape parameter and scale parameter of Weibull distribution respectively.

Using Beta distribution to describe the probability distribution characteristics of solar irradiance in a short period[9], the probability density function can be given as

\[ f(r) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left( \frac{r}{r_{max}} \right)^{\alpha-1} \cdot (1 - \frac{r}{r_{max}})^{\beta-1} \]  

(2)

where \( \Gamma(\cdot) \) is the Gamma function, \( r \) is the actual irradiation intensity, \( r_{max} \) is the max irradiation intensity, \( \alpha \) and \( \beta \) are shape parameters of Beta distribution.

\( \alpha \) and \( \beta \) can be calculated as follows:

\[ \alpha = \mu \left[ \frac{\mu(1-\mu)}{\sigma^2} - 1 \right] \]  

(3)

\[ \beta = (1-\mu) \left[ \frac{\mu(1-\mu)}{\sigma^2} - 1 \right] \]  

(4)

where \( \mu \) and \( \sigma^2 \) are the mean value and variance of irradiation intensity in a certain period. \( \alpha \) and \( \beta \) have different values in different seasons and climates.

2.2. Model of Wind Farm

The active power output of wind turbine generation is closely related to wind speed, which can be presented in Figure 1.
The piecewise function between output power of wind turbine generation and wind speed can be given as

\[
P = \begin{cases} 
0 & (V \leq V_{ci} \text{ or } V \geq V_{co}) \\
\frac{V - V_{ci}}{V_{co} - V_{ci}} P_{ci} & (V_{ci} \leq V \leq V_{co}) \\
P_{tr} & (V_{r} \leq V < V_{co})
\end{cases}
\]

where \(V\) is the wind speed at wind turbine hub’s height, \(V_{ci}\) is the cut-in wind speed, \(V_{co}\) is the cut-out wind speed, \(V_{r}\) is the rated wind speed, and \(P_{tr}\) is the rated output power of wind turbine generation.

A steady-state simplified model is adopted in this paper (namely the PZ model)[8]. Taking wind turbine generators as PQ buses in power flow calculation, and considering active power of wind turbine generators equals to mechanical power of wind turbine and only relates to wind speed, we can get the whole wind farm’s output active power as

\[
P_f = n \cdot P_w
\]

where \(P_f\) is the output active power of wind farm, \(P_w\) is the output active power of a single wind turbine generator, and \(n\) is the amount of wind turbine generators in operation.

The probability density function of wind farm output power can be described as follows combined with the probability density function of wind speed[10]:

\[
f(P) = k \frac{(P - nk_2)^{k-1}}{nk_c^{k+1}} \exp\left(-\frac{(P - nk_2)^k}{nk_c}\right).
\]

Noticing that equation (7) is Weibull distribution with triple parameters, where \(k_2 = -k_{V_{ci}}, k_1 = P_{f}/(V_{r} - V_{co})\).

### 2.3. Model of Solar Park

Assuming that photovoltaic modules works at maximum power point, the solar irradiance of installation site is \(r\) at the certain time point \(t\), the conversion efficiency of photovoltaic module equals to \(\eta\), the area of a single photovoltaic cell panel is \(A\), and the amount of photovoltaic cell panels of whole solar park is \(n\), we can calculate the output power \(P_m\) of solar park at the certain time point \(t\) as follows[9]:

\[
P_m = n \cdot \frac{rA\eta}{860.4} (kW).
\]

The solar park can be addressed in this manner in power flow calculation[9]: solar park can be calculated as an ordinary power plant, and solar park buses are regarded as PV buses during the daytime while solar park buses are regarded as PQ buses when the output equals to 0 (considering that active power is 0).
According to the probability density function of solar irradiance, we can see that the probability density function of solar park output active power also appears Beta distribution, which can be described as

$$f(P_m) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \left(\frac{P_m}{P_{\text{max}}}\right)^{\alpha - 1} \cdot \left(1 - \frac{P_m}{P_{\text{max}}}\right)^{\beta - 1}. \quad (9)$$

where $P_{\text{max}} = \frac{r_{Ap}}{860.4}(kW)$, namely the maximum output power of solar park.

2.4. Model of Power Load

Considering that power load forecasting is somewhat uncertain during the power system planning stage, this paper adopts the normal distribution to signify load forecasting results (only consider active power), which can well reflect the uncertainty of load forecasting[11].

$$f(P) = \frac{k}{\sqrt{2\pi\sigma_p^2}} \exp\left(-\frac{(P - \mu_P)^2}{2\sigma_p^2}\right) \quad (10)$$

where $\mu$ is the expectation, $\sigma^2$ is the variance, and $P$ is the active power.

3. Fast Reliability Assessing Method

3.1. Power System Production Cost Simulation Probability Discretization

The power supply elements in power system can be called resources, such as conventional power generator, wind farm and solar park. While the power-consuming elements can be called demands, such as power load. As Ref[12,13] indicate, we can deduce the distribution law by taking available capacity of resources and demands elements as random variables.

Assuming that the discretization step of common capacity of all elements is $\Delta C$, the capacity of element $m$ is $C_m$, we can discretize element $m$ as follows.

Let the equation be written as

$$S_m = \left[\frac{C_m}{\Delta C}\right]. \quad (11)$$

where $[x]$ denotes the largest integer less than $x$.

Therefore, element $m$ has $(S_m + 1)$ states, where the available capacity of the $i$ th state can be given as

$$C_i = i\Delta C, \quad 0 \leq i \leq S_m. \quad (12)$$

Every state of element has some probability. Taking the $i$ th state of element $m$ in the initial circumstances as $PR_m^{(0)}(i)$, hence $PR_m^{(0)}(i) (i = 0, 1, 2...S_m)$ is to be a sequence of probability discretization. Assuming that wind farm, solar park and power load are mutually independent, we can calculate every state’s probability of every single element by using equations (7), (9) and (10).

3.2. Linearization Power Flow of Distribution Grid

The DistFlow method is commonly used to calculate power flow of distribution network[14]. We can get the calculation formula of distribution network power flow by linearizing the DistFlow equation[15], as is shown below:
where $P_i$ and $Q_i$ are active power and reactive power from bus $i$ to bus $i+1$, $P_i^D$ and $Q_i^D$ are active load and reactive load of bus $i$, $P_i^G$ and $Q_i^G$ are active supply and reactive supply of bus $i$, $r_i$ and $x_i$ are resistance and reactance between bus $i$ and bus $i+1$.

### 3.3. Principle and Flow of Fast Reliability Assessing Method

Assume that $\Omega_{REW}$ is set of all wind and solar generations, $\Omega_G$ is set of all conventional generations, $\Omega_L$ is set of all power loads, $\Omega_F$ is set of all transmission lines, $\Omega_N$ is set of all buses in power system.

For element $\xi \in \Omega_{REW} \cup \Omega_L$, the probability of operating in state $i_\xi$ is noted as $\Pr(i_\xi)$, $1 \leq i_\xi \leq S_\xi +1$.

Considering constrained boundary conditions of power system, the optimal power flow objected with minimum cost of conventional power generation can be expressed as follows

$$
\begin{align*}
\min \ & \sum_{\xi \in \Omega_G} c_{\xi}(i_\xi) + c_{\omega}(\omega) \\
\text{s.t.} \ & \sum_{\xi \in \Omega_G} i_\xi + \sum_{\xi \in \Omega_{REW}} i_\xi - \sum_{\xi \in \Omega_L} i_\xi + \omega = 0 \\
& f_{i_l}^{\min} \leq f_l \leq f_{i_l}^{\max}, l \in \Omega_F \\
& 0.95 \leq V_l \leq 1.05, l \in \Omega_N \\
& \omega \geq 0
\end{align*}
$$

(14)

where $c_{\xi}(\cdot)$ is cost function of element $\xi (\xi \in \Omega_G)$, $f_l$ is power flow of line $l$, $f_{i_l}^{\min}$ and $f_{i_l}^{\max}$ are the maximum and minimum power flow of line $l$, $V_l$ is per-unit voltage of bus $l$, $\omega$ is slack variable ($\omega \geq 0$) and $c_{\omega}(\cdot)$ is cost function of slack variable. Generally $c_{\omega}(\cdot)$ is far more than $c_{\xi}(\cdot)$.

To improve computation speed, linearization power flow method in section 2.2 is adopted to calculate $f_l$. Considering that power flow of most transmission lines would not cross the boundaries in practical circumstances, we can accelerate the optimal power flow problem (14) in the following way:

i0) Let LOLP=0, EENS=0;

i1) Resolve the optimal power flow problem neglecting constrained conditions of power system, and get the output power of all conventional power generations $i_\xi^{(0)} (\xi \in \Omega_G)$;
\[
\begin{align*}
\min & \sum_{\xi \in \Omega} c_\xi(i_\xi) + c_\omega(\omega) \\
\text{s.t.} & \sum_{\xi \in \Omega_i} i_\xi + \sum_{\xi \in \Omega_{\text{rew}}} i_\xi - \sum_{\xi \in \Omega_L} i_\xi + \omega = 0 \\
& \omega \geq 0 
\end{align*}
\]

(15)

i2) If \( \omega > 0 \), the system will lose power load and then flip to i5); if \( \omega = 0 \), calculate linearization power flow in terms of \( i^{(0)}_\xi (\xi \in \Omega_G) \) and \( i^{(0)}_\xi (\xi \in \Omega_{\text{rew}} \cup \Omega_L) \);

i3) If line power flow is not out of limit, then the system will lose no power load; otherwise, if power flow of line \( h \) is out of limit, add the power flow constraints of line \( h \) to equation (15) as follows (if voltage of \( h \) is out of limit, address as the similar way):

\[
\begin{align*}
\min & \sum_{\xi \in \Omega} c_\xi(i_\xi) + c_\omega(\omega) \\
\text{s.t.} & \sum_{\xi \in \Omega_i} i_\xi + \sum_{\xi \in \Omega_{\text{rew}}} i_\xi - \sum_{\xi \in \Omega_L} i_\xi + \omega = 0 \\
& f^{\min}_h \leq f_h \leq f^{\max}_h, h \in \Omega_F \\
& \omega \geq 0 
\end{align*}
\]

(16)

i4) Replace equation (15) with (16), repeat flow i1) to i3), and get \( i^{(1)}(\xi \in \Omega_G) \). If all lines cross no boundaries and supply and consumption get balanced, then the system will lose no power load in certain states combination of wind power, solar power and load; otherwise flip to i5);

i5) The system will lose power load under this state combination of wind power, solar power and load, and the probability of this state combination can be calculated as 

\[
\prod_{\xi \in \Omega_{\text{rew}} \cup \Omega_L} \Pr(i_\xi),
\]

and the losing power load is to be \( \omega \cdot \Delta C \);

i6) Calculate the LOLP and EENS as the following equation

\[
\begin{align*}
\text{LOLP}^i = & \text{LOLP}^0 + \prod_{\xi \in \Omega_{\text{rew}} \cup \Omega_L} \Pr(i_\xi) \\
\text{EENS}^i = & \text{EENS}^0 + \omega
\end{align*}
\]

(17)

where \( \text{LOLP}^0 \) and \( \text{LOLP}^i \) are the earlier value and updating value of LOLP; \( \text{EENS}^0 \) and \( \text{EENS}^i \) are the earlier value and updating value of EENS.

Based on the above, the calculation flow of fast reliability assessing method in this paper can be designed in following steps:

j1) Set the discretization step capacity of all elements’ capacity \( \Delta C \) (the greatest common divisor of all elements’ capacity can be selected as \( \Delta C \));

j2) Get every element’s capacity probability–discretized, and calculate the certain probability of every single state;

j3) Implement step i0) to i6) for all the possible combinations of wind power, solar power and power load;

j4) Exit calculation, and get LOLP and EENS.

4. Simulation Analysis

A IEEE RBTS BUS6 test system is adopted to verify the validity of fast reliability assessing method proposed in this paper. Select the radial network topology of A IEEE RBTS BUS6 system, and only consider 30-buses network to study the case[9]. The topology of simulation system is shown in Figure 3.
The conventional power generation is connected to the 33kV bus, wind farm is connected to bus 18, solar park is connected to bus 6. The concerning parameters are adopted with the practical parameters of certain wind farm and solar park in AnHui province.

![Figure 3](image)

**Figure 3** Topology of IEEE RBTS BUS6 system

The parameters of wind farm are given as follows: the total capacity is 5×600kW, the cut-in wind speed, cut-out wind speed and rated wind speed are 3m/s, 25m/s and 16m/s respectively, the impedance of transmission line between wind farm and bus 18 is (12.6+j24.96)Ω, the air density of wind farm is 1.2245kg/m3, and the sweeping area of a wind turbine blade is 1840m².

The parameters of solar park are given as follows: the total capacity of solar park is 600 kW, the latitude of photovoltaic module installation site is 33.38°, the power capacity of a photovoltaic module is 180 wP, the geometry size of a photovoltaic cell panel is 1580mm×793mm×50mm, the cell panels are installed in the south direction and inclined angle is , the revised atmospheric transparency is 0.7, and rated conversion efficiency of photovoltaic module is 13%.

The parameters of power load: according to Ref[8], calculate load forecasting uncertainty by using normal distribution.

Simulation is conducted in MATLAB with a PC of 2.5GHz CPU clock speed and 2GB RAM. To make a good comparison, the Monte Carlo method is also used to calculate the power supply reliability for the case. Table 1 shows the reliability indexes and computation time by using the two methods.

| method                     | LOLP     | EENS/MWh | time/s |
|----------------------------|----------|----------|--------|
| Monte Carlo method         | 0.010141 | 60.7939  | 3321   |
| proposed method (△C=0.5MW) | 0.011137 | 64.7509  | 361    |
| (△C=1.0MW)                 |          |          |        |

The computation efficiency of proposed method and Monte Carlo method is compared in Figure 4 and Figure 5.
We can conclude from the above simulation results as follows:

1) The convergence speed of Monte Carlo method is rather slow, and the calculation consumes 3321s (about 55.35min). In comparison, the proposed method in this paper calculate significantly fast while maintain a rather high accuracy.

2) When discretization step $\Delta C = 0.5$MW, the computation time is 361s, which account for 10.87% of Monte Carlo method's consuming-time, and the calculation errors are 9.82% and 6.51% respectively compared with the one by using Monte Carlo method.

3) When discretization step $\Delta C = 1.0$MW, the computation time is even faster, reducing to 194s and accounting for 5.84% of the one by Monte Carlo method, and the calculation errors get larger to some degree, increasing to 24.95% and 23.94%.

4) A appropriate discretization step is critical to calculation accuracy and computation time. On one side, if $\Delta C$ is selected rather big, the calculation accuracy would greatly decrease although the computation speed would be very fast. On the other side, if $\Delta C$ is selected rather small, the computation time would last much longer although the calculation accuracy would be greatly improved. Hence, a suitable discretization step can get a good balance between the two concerning factors, and would significantly improve the efficiency of power system planning.

5. Conclusion
A fast reliability assessing method for distribution network with distributed renewable energy generation is proposed in this paper in terms of the low efficiency of Monte Carlo method. The method is based on power system production cost simulation probability discretization and linearization power flow, and resolve a optimal power flow objected with minimum cost of conventional power generation. A appropriate discretization step can get high calculation accuracy and fast computation speed, making this method an efficient and practical way to evaluate the power supply reliability of power system.
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7. References
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