Soliton-mode proliferation induced by cross-phase modulation of harmonic waves by a dark-soliton crystal in optical media

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Abstract
The generation of high-intensity optical fields from harmonic-wave photons, interacting with dark solitons via a cross-phase modulation coupling, both propagating in a Kerr nonlinear medium, is examined. The focus is on a pump consisting of time-entangled dark-soliton patterns, forming a periodic waveguide along the path of the harmonic-wave probe. It is shown that an increase of the strength of cross-phase modulation coupling respective to the self-phase modulation coupling, favors soliton-mode proliferation in the bound-state spectrum of the trapped harmonic-wave probe. The induced soliton modes, which display structures of periodic soliton lattices, are not just rich in numbers, they also form a great diversity of population of soliton crystals with a high degree of degeneracy.

Keywords
cross-phase, modulation, dark-soliton, crystals, Lamé, equation, photon, trapping

1 INTRODUCTION

Optical wave trapping, cloning, reconfiguration, duplication, parametric amplification, and recompression are certain physical processes associated with wave interactions in some nonlinear optical media. These processes are usually controlled by nonlinear phenomena that originate from the intensity-dependent index of refraction of propagation media, and are responsible for cross-phase or induced-phase modulations determining shape profiles of the propagating optical fields. Such processes find widespread applications in modern communication technology, and particularly in the processings of relatively low-power fields (such as harmonic fields) interacting with fields of sufficiently high intensity (such as optical solitons), leading to their cloning and reconfiguration into optical fields of higher powers.

Since soliton cloning and reconfiguration can involve a sizable energy cost from the pump field, these processes have most often been envisaged between two solitons of slightly different powers. Thus in Refs. 5, 20, a reconfiguration scheme was proposed in which an intense pump beam with soliton features induces the focusing of a weaker probe beam of different wavelength, but also with soliton features, via cross-phase modulation. The underlying mechanism is simply understood by recalling that an optical soliton propagating in a Kerr nonlinear medium, creates a local distortion of the refractive index that travels with the soliton down the nonlinear propagation medium. As a result of this refractive index distortion a waveguide can be induced, that acts like a local potential by trapping and reshaping another much weaker pulse, different from the soliton pump in both frequency and polarization. However, in Ref. 10, Steiglitz and Rand suggested the possibility to use optical solitons propagating for instance in an optical fiber, to trap an reshape continuous-wave photons by means of their cross-phase modulation with the optical solitons. Thus, by considering a localized bright soliton pump interacting with a harmonic photon field, they established that the probe field was reshaped into new modes the eigenstates of which were described by a linear eigenvalue equation with a reflectionless potential. Subsequent to the study of Steiglitz and Rand, the phenomenon of harmonic-wave trapping and reconfiguration by bright solitons was extended to the context of a waveguide created by a periodic train of bright solitons forming a bright-soliton crystal. This later study led to a linear eigenvalue problem of the Lamé type for the...
probe field, and its bound states were shown to form spectra of rich and abundant soliton modes. Namely in Ref. 9 it was established that increasing the strength of cross-phase modulation relative to the self-phase modulation, favors an increase of the population and the degeneracy of soliton modes composing bound-state spectra of the trapped probe.

While the dynamics of bright solitons as well as their stability under mutual collisions are relatively well understood, dark solitons have remained a curiosity for some reasons. Most importantly dark solitons are odd-symmetry structures, and for this reason they can only propagate in specific media. Nevertheless it is well established that dark solitons have simpler collision dynamics than their bright counterparts, are generally more stable against various perturbations, and hence may offer some important advantages in optical field processing applications. Based on this later features Steiglitz considered using a localized dark soliton pump to trap and reshape harmonic-wave probes. He found that probe modes induced by cross-phase modulation in the waveguide of the single dark soliton, were determined by a linear eigenvalue equation with a reflecting scattering potential. Because of this the groundstate was not a Goldstone translation of the pump as in the case involving a bright soliton pump, but instead a localized sech-type pulse soliton.

Motivated by results of a previous study in which it was established that a waveguide consisting of a crystal of bright solitons favors relatively more diverse and abundant soliton modes in the probe spectrum, in the present study we wish to examine the problem of harmonic-wave reconfiguration by a waveguide consisting of a periodic train of dark solitons. This aspect of the problem has not been considered in any previous study. In Section 2 the model is presented, and the periodic dark-soliton solution to the pump equation is obtained. With this solution the probe equation is formulated in terms of a Lamé-type eigenvalue problem, and in Section 3 some exact bound-state solutions to this eigenvalue problem are found analytically provided specific conditions. These bound-state solutions are represented graphically in order to provide more insight onto their specific profiles. Concluding remarks are presented in Section 4.

2 | THE PUMP-PROBE EQUATIONS AND DARK-SOLITON-CRYSTAL SOLUTION TO THE PUMP EQUATION

The propagation equations for the system composed of a nonlinear pump field, coupled to an harmonic-wave field via a cross-phase modulation and propagating together in a Kerr nonlinear optical medium, are given by:

\[ i \frac{\partial v}{\partial z} + v \frac{\partial^2 v}{\partial t^2} - 2 \zeta |v|^2 v = 0, \]  

(1)

In the first equation which is precisely the cubic nonlinear Schrödinger equation with self-defocusing nonlinearity, the quantity \( v \) is the pump envelope, \( z \) is the propagation distance, \( t \) is the propagation time and \( \zeta \) is the coefficient of self-phase modulation. The quantity \( u \) in the second equation is the probe envelope, \( k_1 \) is the coefficient of group-velocity-dispersion and \( k_2 \) is the coefficient of cross-phase modulation.

In Equation (1) the nonlinear coefficient is effectively negative, that is, \(-\zeta\) with \( \zeta > 0 \), corresponding to a Kerr optical medium with self-defocusing nonlinearity. In Ref. 32 the problem of harmonic-wave trapping and reshaping by a waveguide created by a single dark soliton, was considered. Here we are interested in the context when the waveguide is created by a period train of single dark solitons, forming a dark-soliton crystal. In this purpose we consider a solution to Equation (1) describing a stationary wave, \( v(z,t) = A(t) \exp\left[-i(kz - \omega t)\right] \), where \( k \) is the wave number and \( \omega \) is the frequency. Substituting this in Equation (1), we find that the wave amplitude \( A(t) \) must obey the first-integral equation:

\[ \frac{\partial A}{\partial t} = \zeta \left( \sqrt{A^4 - \frac{s}{\kappa}A^2 + \rho_1} \right), \]  

(3)

with \( \rho_1 \) an energy constant determining shape profile of \( A(t) \). Solving Equation (3) with periodic boundary conditions, the pump amplitude \( A(t) \) is found to be the following nonlocalized periodic pattern of time-entangled dark solitons:

\[ A(t) = \frac{Q}{\sqrt{\zeta}} \text{sn}[Q(t - t_0)], \]  

(4)

where \( \text{sn}() \) is a Jacobi elliptic function of modulus \( \kappa \) (with \( 0 \leq \kappa \leq 1 \)), and:

\[ Q = \sqrt{\frac{s}{1 + \kappa^2}}, \quad s = k - \omega^2. \]  

(5)

The amplitude \( A(t) \) of the periodic dark soliton (4) is represented in Figure 1, for \( \kappa = 0.98 \) (left graph) and \( \kappa = 1 \) (right graph). Note that when \( \kappa \rightarrow 1 \) the Jacobi elliptic function \( \text{sn}(\cdot) \rightarrow \text{tanh}(\cdot) \), corresponding to the dark soliton pump obtained in Ref. 32.

3 | PUMP-INDUCED TRAPPING, RESHAPING, AND PROBE-MODE PROLIFERATION

Using the periodic dark soliton solution to the pump equation obtained in Equation (4), we will now seek solutions to the probe Equation (2). Proceeding with it is useful to start
by the important remark that Equation (2) is a linear
Chrödinger equation, but with a time-dependent “external”
potential represented by the norm squared of the pump en-
velope \(q(z,t)\). Substituting (4) in the probe equation given by
(2), and expressing the probe envelope as a stationary wave,
that is, \(A(t) = u(t)e^{-i\omega z}\), where \(u(t)\) is the core of
the probe envelope and \(q\) is its wave number, we obtain the
Lamé Equation\(^{25}\):

\[
\frac{\partial^2 u}{\partial t^2} + \left( P(q) - l(l+1)k^2 \right) u = 0, \quad (6)
\]

\[
P(q) = \frac{q}{k_1 Q^2}, \quad \tau = Q(t-t_0), \quad l(l+1) = \frac{2k_2}{k_k^2 k_1 \zeta}. \quad (7)
\]

It is worth stressing that when \(\kappa = 1\), the Lamé Equation (7)
becomes the Associated Legendre equation obtained in
Ref. 32.

The Lamé equation possesses a rich spectrum with a
great variety of eigenmodes.\(^{25}\) However the most relevant to
us are its eigenmodes that display a permanent profile typical
of solitons. Precisely these later modes are bound states of
the Lamé equation, and because their formation through the
cross-phase modulation with the dark-soliton crystal
involves energy cost (momentum transfer) from the pump,
they can be looked out as low-energy states of the probe
spectrum created by the pump-induced periodic potential
(4). Discrete states of Lamé’s equation form a spectrum of
finite orthogonal modes, whose population depends on the
integer quantum number \(l\).\(^{25}\) According to Equation (7),
values of the integer quantum number \(l\) will be determined
by the competition between the self-phase modulation
responsible for the fiber nonlinearity, and the cross-phase
modulation exerted by the pump field on the harmonic
probe. For a given value of \(l\), the discrete spectrum of Lamé
equation possesses \(2l+1\) modes some of which can be
degenerate.\(^{9,15,25}\)

We start with the lowest value of \(l\); \(l = 1\) corresponding
to the case when the cross-phase modulation and the self-

phase modulation coefficients are related by \(k_2 = k_1 \zeta k^2\). In
this case the Lamé equation possesses three distinct localized
modes, namely:

\[
u_{11}(\tau) = u^{(11)}cn(\tau), \quad q = q_{11} = \frac{k_2 Q^2}{k^2 \zeta}, \quad (8)
\]

\[
u_{12}(\tau) = u^{(12)}dn(\tau), \quad q = q_{12} = \frac{k_2 Q^2}{\zeta}, \quad (9)
\]

\[
u_{13}(\tau) = u^{(13)}sn(\tau), \quad q = q_{13} = \frac{(1+k_2^2)k_2 Q^2}{k^2 \zeta}, \quad (10)
\]

where \(u^{(11)}\) are normalization constants. The three modes are
represented in Figure 2, for \(\kappa = 0.98\) (left column) and \(\kappa = 1\)
(right column).

Taking \(l = 2\), or equivalently \(k_2 = 3k_1 \zeta k^2\), leads to five
distinct localized modes for the probe which are listed below:

\[
u_{21}(\tau) = u^{(21)}cn(\tau)dn(\tau), \quad q = q_{21} = \frac{1+k_2^2}{3k_2^2} \frac{k_2 Q^2}{\zeta}, \quad (11)
\]

\[
u_{22}(\tau) = u^{(22)}sn(\tau)dn(\tau), \quad q = q_{22} = \frac{1+4k_2^2}{3k_2^2} \frac{k_2 Q^2}{\zeta}, \quad (12)
\]

\[
u_{23}(\tau) = u^{(23)}sn(\tau)cn(\tau), \quad q = q_{23} = \frac{4+4k_2^2}{3k_2^2} \frac{k_2 Q^2}{\zeta}, \quad (13)
\]

\[
u_{24}(\tau) = u^{(24)} \left[ sn^2(\tau) - \frac{1+k_2^2 + \sqrt{1-k_2^2(1-k_2^2)}}{3k_2^2} \right], \quad q = q_{24} = \frac{2(1+k_2^2) - \sqrt{1-k_2^2(1-k_2^2)}}{3k_2^2} \frac{k_2 Q^2}{\zeta}, \quad (14)
\]

\[
u_{25}(\tau) = u^{(25)} \left[ sn^2(\tau) - \frac{1+k_2^2 - \sqrt{1-k_2^2(1-k_2^2)}}{3k_2^2} \right], \quad q = q_{25} = \frac{2(1+k_2^2) + \sqrt{1-k_2^2(1-k_2^2)}}{3k_2^2} \frac{k_2 Q^2}{\zeta}, \quad (15)
\]
The five bounded modes are plotted versus time in Figure 3, for $\kappa = 0.98$ (left graphs) and $\kappa = 1$ (right graphs). $u_i$ in the graphs mean $u_{l1}$ in Equations (8)-(10), with $i = 1, 2, 3$ [Color figure can be viewed at wileyonlinelibrary.com]

\[
q = q_{25} = \left[ \frac{2(1 + \kappa^2) + \sqrt{1 - \kappa^2(1 - \kappa^2)}}{3\kappa^2} \right] k_2 Q^2 \zeta. \tag{15}
\]

The third and last case considered is $l = 3$, corresponding to $k_2 = 6k_1 \zeta \kappa^2$. In this case the bound-state spectrum of the probe comprises seven distinct localized modes that is:

\[
u_{31}(\tau) = u_{(31)}^{(31)}sn(\tau)cn(\tau)dn(\tau),
q = q_{31} = \left( 2 + 2\kappa^2 \right) \frac{k_2 Q^2}{\zeta}, \tag{16}
\]

\[
u_{32}(\tau) = u_{(32)}^{(32)} \left[ sn^3(\tau) - \frac{2(1 + \kappa^2) - \sqrt{4 - 7\kappa^2 + 4\kappa^2}}{5\kappa^2} sn(\tau) \right],
q = q_{32} = \left[ \frac{5(1 + \kappa^2) + 2\sqrt{4 - 7\kappa^2 + 4\kappa^2}}{6\kappa^2} \right] \frac{k_2 Q^2}{\zeta}. \tag{17}\]
FIGURE 3  Temporal profiles of amplitudes of the five bounded modes given by Equations (11)–(15), for $\kappa = 0.98$ (left column) and $\kappa = 1$ (right column). $u_i$ in the graphs correspond to $u_{1i}$ in Equations (11)–(15), with $i = 1, 2, 3, 4, 5$ [Color figure can be viewed at wileyonlinelibrary.com]
Table 1: Fundamental components (solutions with \( \kappa = 1 \)) of the \( l = 1 \) eigenmodes

| Eigenfunction                | Eigenvalue     |
|------------------------------|----------------|
| \( u_{11}(\tau) = u_{12}(\tau) \times \text{sech}(\tau) \) | \( q_{11} = q_{12} = k_2 Q^2 / \zeta \) |
| \( u_{13}(\tau) \times \tanh(\tau) \)     | \( q_{13} = 2k_2 Q^2 / \zeta \) |

Table 2: Fundamental components (solutions with \( \kappa = 1 \)) of the \( l = 2 \) eigenmodes

| Eigenfunction                | Eigenvalue     |
|------------------------------|----------------|
| \( u_{21}(\tau) \times \text{sech}^2(\tau) \) | \( q_{21} = 2k_2 Q^2 / (3\zeta) \) |
| \( u_{22}(\tau) = u_{23}(\tau) \times \text{sech}(\tau)\tanh(\tau) \) | \( q_{23} = 5k_2 Q^2 / (3\zeta) \) |
| \( u_{24}(\tau) \times -\text{sech}^2(\tau) \)     | \( q_{24} = k_2 Q^2 / \zeta \) |
| \( u_{25}(\tau) \times 2 - 3\text{sech}^2(\tau) \) | \( q_{25} = 5k_2 Q^2 / (3\zeta) \) |

Table 3: Fundamental components (solutions with \( \kappa = 1 \)) of the \( l = 3 \) eigenmodes

| Eigenfunction                | Eigenvalue     |
|------------------------------|----------------|
| \( u_{31}(\tau) \times \text{sech}^2(\tau)\tanh(\tau) \) | \( q_{31} = 4k_2 Q^2 / (3\zeta) \) |
| \( u_{32}(\tau) \times (2 - 5\text{sech}^2(\tau))\tanh(\tau) \) | \( q_{32} = 2k_2 Q^2 / \zeta \) |
| \( u_{33}(\tau) \times -\text{sech}^2(\tau)\tanh(\tau) \) | \( q_{33} = 4k_2 Q^2 / (3\zeta) \) |
| \( u_{34}(\tau) \times (4 - 5\text{sech}^2(\tau))\text{sech}(\tau) \) | \( q_{34} = 11k_2 Q^2 / (6\zeta) \) |
| \( u_{35}(\tau) \times -\text{sech}^2(\tau) \)     | \( q_{35} = k_2 Q^2 / (2\zeta) \) |
| \( u_{36}(\tau) \times \text{sech}(\tau)\tanh(\tau) \) | \( q_{36} = 11k_2 Q^2 / (6\zeta) \) |
| \( u_{37}(\tau) \times (1 - 5\text{sech}^2(\tau))\text{sech}(\tau) \) | \( q_{37} = k_2 Q^2 / (2\zeta) \) |

Although the analytical expressions of the seven modes seem to suggest complex combinations of Jacobi elliptic functions, we can convince ourselves of the contrary by examining their expressions for \( \kappa = 1 \), a value for which these analytical expressions are fundamental components composing the soliton trains in the seven distinct modes. This remark also holds for the cases \( l = 1 \) and \( l = 2 \). To gain a better understanding of shape profiles of these fundamental components, in Tables 1, 2, and 3 their eigenfunctions \( u(\tau) \) are listed together with corresponding eigenvalues \( q \).

As it is apparent the three tables feature very rich and varied spectra of bounded states for the three values of \( l \) considered. Also remarkable, Table 1 suggests that the \( l = 1 \) spectrum possesses two modes which are nearly degenerate, whereas the \( l = 2 \) spectrum has three nearly degenerate modes (see 2) and the spectrum for \( l = 3 \) possesses three distinct double-degenerate modes as one can see in Table 3, where only \( u_{32} \) out of the seven bounded states is non-degenerate.

### 4 Conclusion

We examined profiles of an harmonic wave propagating in the waveguide structure created by a periodic lattice of single dark solitons, obtained as a periodic solution to the self-defocusing cubic nonlinear Schrödinger equation describing the propagation of a pump field along an optical fiber. In Ref. 32 the same problem was discussed assuming that the harmonic probe is coupled to a single dark soliton. We obtained that due to the coupling of the pump and probe via the cross-phase modulation, the probe equation can be transformed into a general family of eigenvalue equation called Lamé equation. Considering bounded states of this eigenvalue problem, we found that the population of the associated discrete spectrum was determined by an integer quantum number which in turn was determined by the competition between the self-phase modulation and the cross-phase modulation. We obtained analytical expressions of these bounded modes of the trapped probe for \( l = 1, 2, 3 \), and observed that as \( l \) increases the number of bounded modes...
was higher and higher and the spectra more and more degenerate.

Our study demonstrates unambiguously that waveguides induced by a periodic train of dark solitons, can be used to control weak probes in the same way bright solitons can. Dark solitons offer real advantages over bright soliton collisions in controlling light waves: Firstly dark solitons are well known to be more stable in the presence of noise, and are generally more robust than bright solitons. Secondly the probe, which is of much lower intensity and here an harmonic wave, is expected to peak at the dip in the intensities of its host single-dark soliton components, thus increasing the signal-to-noise ratio and making it easier, in principle, to detect. The present study finds its most important application in quantum communication and cryptographic systems, we think in particular of experimental verification of photon capture and transport in an optical fiber. The problem of detecting the probe in the presence of a pump, and tailoring physical parameters to realize virtual devices, is also of current interest in quantum computing applications involving entangled photon emitters and photon qubits.\textsuperscript{37–40}

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CONFLICT OF INTEREST

The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

Data supporting the present study are all included in the manuscript.

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