Triple absorbing Markov chain model to study the flow of higher education students

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Abstract. Triple absorbing Markov chain has been applied to estimate the probability of students in different levels for graduating without delaying, the probability of academic dismissal and dropping out of the system before attaining the maximum qualification over the batches. Thus, the possibility of graduation increases all over the batches while academic dismissal and male students dropping out are decreasing. Whereas, dropping out of students is higher in the second year and zero in the third year. Relatively female students are more academically dismissed and dropped out of the faculty than male students in their first and second year.

1. Introduction
One of the main aims of education is to develop the capacity of human power. This ability developed through education at deferent levels, including primary, secondary and higher education. Specifically, higher education contributes a lot to the economic development and national modernization systems [1]. As a result, higher education develops an individual’s capacity to manufacture technology and be a responsible nation.

To plan and make a proper decision to improve the quality of the higher education system: student enrollment trend, completion rates, dropout rates, retention rates per class and the expected duration of training is essential. In Ethiopia, female students enrolled in higher education with a lower score than male students [2]. This increased female students’ attrition and dismissal rate [2, 3].

States of the education system partitioned into two categories; transient (non-absorbing) states which correspond to various classes within the education system and absorbing states which also correspond to the group of all successful graduates, academically dismissed and dropouts.

In higher education, students may either passed from one grade to the next or may repeat or may complete or withdraw from the system due to academic dismissal (AD) or other problems before attaining the maximum qualification. Previously, Musiga [4] modeled a Hierarchical system with a single absorbing state, where dropouts (DO) and graduates (G) grouped. Then, Musiga [5] and Nyandwaki [6] modeled double absorbing states, in which case students who dropped outs of the system either successfully or unsuccessfully were separated, but students left out of the system due to AD and other problems grouped.

Thus, this paper designed to explore the flow of Mettu University undergraduate students in which case students who DO of the system either effectively or ineffectively, AD or DO due to other problems, are separately grouped into triple absorbing states.

The model established will help the stakeholders in the distribution of resources to higher education when the year of study put into consideration and, planning and budgeting for the expected future
enrollment for quality and equality in education. Equally the model will be used to predict the probability of AD and DO by sex at each class level of study. This result will assist interested parties to make an informed decision about higher education management in view to decrease AD, prevent DO and reducing the gender disparity in Mettu University faculty of natural and computational science.

2. Literature review
Markov chain is a stochastic process which used to describe what happens in the future depends only on the current state of the system [7]. Let a sequence $X_n, n \geq 0$ be a discrete-time stochastic process with a finite set of states $S = \{i_0, i_1, \ldots, i_j\}$. If the chain is currently in state $i$, then it moves to state $j$ with a probability denoted by $p_{ij}$, or remain in the state with $p_{ii}$. Hence, for all integers $n \geq 0$ and all states $S$ the stochastic process defined as:

$$P(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \ldots, X_0 = i_0) = P(X_{n+1} = j | X_n = i) \quad \cdots (1)$$

represent a Markov chain [8]. The matrix

$$P = \begin{bmatrix} p_{11} & p_{12} & 0 & p_{14} & p_{15} & 0 \\ p_{21} & 0 & p_{23} & p_{24} & p_{25} & 0 \\ 0 & 0 & p_{33} & p_{34} & p_{35} & p_{36} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Thus, if the chain has $s$ non-absorbing and $r$ absorbing states, by putting the absorbing states first, the canonical form of the transition matrix is [9]:

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix} \quad (3)$$

where $I$ is an $r \times r$ unitary matrix corresponding to the absorbing states, $0$ is an $r \times s$ zero matrix, $R$ is an $s \times r$ rectangular sub matrix, corresponding to transition probabilities from non-absorbing to absorbing states, and $Q$ is transition matrix of non-absorbing states.

2.2. The nth-Step Transition Matrix
The probability of transitioning from state $i$ to state $j$ in n-steps is the $ij$-entry of $Q^n$. Summing this for all $k$ (from 0 to $\infty$) gives the fundamental matrix, denoted by $N$ and defined as:

$$N = \sum_{k=0}^{\infty} Q^k = (I - Q)^{-1} \quad (4)$$
This definition of n-step transition probability leads us to Chapman-Kolmogorov equations which takes the frequency relation for computing the probability transition after n-steps. If \( X_n : n = 0,1,2,\cdots \) is Markov chain with limit m-states and transition probability matrix \( p = (p_{ij}) \) then:

\[
p^{(n)}_{ij} = \sum_{k=1}^{m} p_{ik}^{(n-r)} p_{kj}^{(r)}, \quad \forall r = 1,2,\cdots,n-1
\]

Thus, in canonical form n-step transition probability matrix is:

\[
P^n = \begin{bmatrix} I & 0 \\ R^n & Q^n \end{bmatrix}
\]

Where, \( R^n = (I + Q + Q^2 + \cdots + Q^{n-1}) R \) is sxr matrix gives completion ratio, and \( Q^n \) is sxr matrix gives probability of a student who is in grade \( i \) will be in grade \( j \), after n-years. Because of the largest value of \( q \) is less than 1, and \( q^n \to 0 \) as \( n \to \infty \);

\[
P^n = \begin{bmatrix} I & 0 \\ NR & 0 \end{bmatrix}
\]

2.3. Absorption Time

Let \( u \) is a column vector all of its components are 1’s, the according to [9], the expected number of steps starting from each of the transient states \( i \) before the chain is absorbed is given by:

\[
t = Nu
\]

For different groups and the same issue, the smaller \( t \), indicates the fastest group’s during the process. Let \( B \) stands for the matrix with entries \( b_{ij} \), probability of the chain starting in transient \( i \) will end up in the absorbing state \( j \), and then according to [9], the absorbing rate is defined as:

\[
B = NR
\]

3. Model fitting

3.1. Initial transition matrix

The information of this study collected from Mettu University faculty of Natural and Computational Science undergraduate students from the academic year 2013/14 to 2016/2017. Students’ enrollments and flow in the year I, II and III in the academic year 2015/16 shown in Table 1. Some of the notations used in the subsequent sections are defined as: Subscript \( u \) represents MeU faculty of NCS students; Subscript \( m \) represents MeU faculty of NCS male students and Subscript \( f \) represents MeU faculty of NCS female students.

| Status | E(2015/16) | P(2016/17) | R | AD | DO | G |
|--------|------------|------------|---|----|----|---|
| Gender | M          | F          | M | F  | M  | F |
| Year I | 199        | 216        | 186| 68 | 2  | 6 |
| Year II| 123        | 102        | 112| 81 | 6  | 9 |
| Year III| 118       | 68         | 118| 174| 0  | 0 |

E—Enrolment; P-Pass; R-Repeat; M-Male and F-Female.
Thus, from table 1, using the above notations and assuming time homogeneity the Q and R component of students are;

$$Q_u = \begin{pmatrix} 0.0193 & 0.8675 & 0.0000 \\ 0.0000 & 0.0667 & 0.8576 \\ 0.0000 & 0.0000 & 0.0000 \end{pmatrix}$$

and

$$R_u = \begin{pmatrix} 0.0975 & 0.0337 & 0.0000 \\ 0.0311 & 0.0444 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

With the same procedure, from table 1 and assuming time homogeneity for male students;

$$Q_m = \begin{pmatrix} 0.0101 & 0.9347 & 0.0000 \\ 0.0000 & 0.0488 & 0.9106 \\ 0.0000 & 0.0000 & 0.0000 \end{pmatrix}$$

and

$$R_m = \begin{pmatrix} 0.0302 & 0.0251 & 0.0000 \\ 0.0163 & 0.0244 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Similarly, for female students;

$$Q_f = \begin{pmatrix} 0.0278 & 0.8056 & 0.0000 \\ 0.0000 & 0.0882 & 0.7941 \\ 0.0000 & 0.0000 & 0.0000 \end{pmatrix}$$

and

$$R_f = \begin{pmatrix} 0.1250 & 0.0417 & 0.0000 \\ 0.0490 & 0.0686 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

### 3.2. Completion Rates under triple absorbing states

**Table 2. Completion Rates in 2015/16 academic years using the triple Absorbing States**

| Entry | 2015/16 | 2014/15 | 2013/14 |
|-------|---------|---------|---------|
| Level | Year I  | Year II | Year III|
|       | M       | F       | T       |
|       | M       | F       | T       |
|       | M       | F       | T       |
| AD    | 0.0302  | 0.125   | 0.0795  |
|       | 0.0163  | 0.049   | 0.0311  |
|       | 0.0000  | 0.0000  | 1.0000  |
| DO    | 0.0251  | 0.0417  | 0.0337  |
|       | 0.0244  | 0.0686  | 0.0444  |
|       | 0.0000  | 0.0000  | 1.0000  |
| G     | 0.0000  | 0.0000  | 0.0000  |
|       | 0.0000  | 0.0000  | 1.0000  |
|       | 0.0000  | 0.0000  | 1.0000  |

Within one year, i.e., in the academic year, 2016/17 faculty of natural and computational science student’s completion rates will be given by;

$$(I + Q_u)*R_u = \begin{pmatrix} 0.1080 & 0.0729 & 0.0000 \\ 0.0332 & 0.0474 & 0.8576 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Within two years it will be;

$$(I + Q_u + Q_u^2)*R_u = \begin{pmatrix} 0.1104 & 0.0762 & 0.7441 \\ 0.0333 & 0.0476 & 0.9150 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

From this, it is clear that by academic year 2015/16, 7.95% and 3.37% of the students who were in the first year had AD and DO of the faculty respectively. By the year 2020/21, 81.30% of the students who were in the first year in the first form are expected to graduate successfully whereas 11.05% and 7.64% of the same students will be AD and DO respectively. Similarly, by the year 2020/21, 91.91% of students who were in the second year in the second form are expected to graduate from the University, and 3.33% and 4.76% of the same students will AD and DO respectively. From students in the third year form, 100% of them are expected to graduate successfully by the year 2020/21.

Completion rates for male students within one year are:
Within two years it will be:

\[
(I + Q_m + Q^2_m) R_m = \begin{pmatrix}
0.0466 & 0.0495 & 0.8511 \\
0.0171 & 0.0256 & 0.9550 \\
0.0000 & 0.0000 & 1.0000 \\
\end{pmatrix}
\]

Similarly, in summery by 2020/21, 4.67% and 4.96% of male students who were in the first year form in 2015/16 had AD and DO of respectively. By the year 2020/21, 90.39% of the same year students who were in the same year will be expected to graduate successfully. For the male students in the second year in 2015/16, 95.73% of them are expected to graduate successfully by the year 2020/21, and 1.71% and 2.57% of the same students will AD and DO of respectively. Similarly, a completion rate for female students within one year is:

\[
(I + Q_f) R_f = \begin{pmatrix}
0.1679 & 0.0981 & 0.0000 \\
0.0533 & 0.0747 & 0.7941 \\
0.0000 & 0.0000 & 1.0000 \\
\end{pmatrix}
\]

Within two years it will be

\[
(I + Q_f + Q^2_f) R_f = \begin{pmatrix}
0.1726 & 0.1046 & 0.6397 \\
0.0537 & 0.0752 & 0.8641 \\
0.0000 & 0.0000 & 1.0000 \\
\end{pmatrix}
\]

Similarly, in summery by the year 2020/21, 17.31% and 10.52% of female students who were in the first year form in the year 2015/16 will AD and DO respectively. At the same time, 72.16% of the same year female students who were in the first year form will be expected to graduate from successfully by the year 2020/21. Furthermore, for the students in the second year form of the year in 2015/16, 87.09% of female students are expected to graduate successfully and while 5.37% and 7.52% of female students will AD and DO by the year 2020/21 respectively.

3.3. The expected duration of the study

Given the specified starting state $S_i$, the expected number of periods students spends in non-absorbent states before absorption, $t$, is determined by equation number (9). Thus; $t = Nu = (I - Q)^{-1} u$ and we get:

\[
t_u = \begin{pmatrix}
2.7805 \\
1.9906 \\
1.0000 \\
\end{pmatrix}
\]

From the result, the expected duration of study till completion for students from the year I, II and III is 2.7805, 1.9906 and one years respectively. Similarly, we can found the expected duration of the study by sex using the fundamental matrix N for both cases and equation (9). Thus; for male students the expected length of study till completion, $t_m$, is determined by; $t_m = N_m u = (I - Q_m)^{-1} u$ and we get:
\[ t_m = \begin{pmatrix} 2.9068 \\ 2.0086 \\ 1.0000 \end{pmatrix} \]

While for female students, \( t_f \), is:

\[ t_f = N_f u = \left( I - Q_f \right)^{-1} u \]

\[ t_f = \begin{pmatrix} 2.6591 \\ 1.9676 \\ 1.0000 \end{pmatrix} \]

From the above results, the expected duration of study till completion for a male student in the year I, II and III is 2.9068, 2.0086 and one years respectively while 2.6591, 1.9676 and one years for female student in the same order.

### 3.4. The Absorbing Rates

In the long run, the absorbing rate under triple absorbing states is given by equation (9). Accordingly, the absorbing rate of MeU faculty of NCS is:

\[
(I - Q_u)^{-1} R_u = \begin{pmatrix} 0.1105 & 0.0764 & 0.8130 \\ 0.0333 & 0.0476 & 0.9191 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}
\]

From these results, in the long run, starting from the first year, there is a probability of 0.1105 absorptions in academic dismissal state, 0.0764 completions in dropout state and 0.8130 completions in the successfully graduate state. The second row tells us that starting from the second year, there is a probability of 0.0333 absorptions in academic dismissal, 0.0476 of completion in dropout state and a probability of 0.9191 completions in the successfully graduate state. The third row also tell us that starting from the third year, there is a probability of one completion in the successfully graduate state. The absorbing rate of a male student is:

\[
(I - Q_m)^{-1} R_m = \begin{pmatrix} 0.0467 & 0.0496 & 0.9039 \\ 0.0171 & 0.0257 & 0.9573 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}
\]

Similarly, female students absorbing rate is:

\[
(I - Q_f)^{-1} R_f = \begin{pmatrix} 0.1731 & 0.1052 & 0.7217 \\ 0.0537 & 0.0752 & 0.8709 \\ 0.0000 & 0.0000 & 1.0000 \end{pmatrix}
\]

From the above results, in the long run, one can see that, from male students who were in the first, second and third year form in the year 2015/16; 4.67%, 1.71% and 0%, and 4.96%, 2.57% and 0% respectively AD and DO while for female students in the same order 17.31%, 5.37% and 0%, and 10.52%, 7.52% and 0% respectively AD and DO respectively. Students starting from the first, the second and the third-year form in the year 2015/16 were 90.39%, 95.73% and 100% of male students and 72.17%, 87.09% and 100% of female students respectively graduated successfully.
3.5. Retention Rate
The retention rates for students in the first, second and third-year form in 2015/16 were established to be 86.75%, 85.78%, and 100% respectively. Thus, students in the second year form had the lowest retention rate while students in the third year form had the highest. Based on their gender, male student’s retention rates were 93.47%, 91.06% and 100% for year one, two and three respectively. Similarly, female students’ retention rates were 80.56%, 79.41% and 100% in the same order.

4. Discussion and Conclusions
The overall analysis of the results can be summarized as follows. An increased probability of graduation and decrease of AD of students observed as the student moves to the higher levels. The rate of dropping out of Mettu University natural and computational science faculty before getting their degree is highest in the second year and zero in the third year for female students and as whole students whereas highest in the first year and zero in the third year for male students. Comparing gender difference in both first year and second-year level male student has more probability of graduation than female students at each level. The more remarkable point is that for the third year both gender students probability of graduation is one, which ensures once a student reaches third-year he/she is certain of completing successfully his/her course of study.

This implies the probability of academic dismissal and male students dropping out of Mettu University natural and computational science faculty before attaining their degree is decreasing from lower class level to higher class level. In line with this, comparatively female students are more academically dismissed and dropped out than male students in their first year and second year in the faculty.

The completion rates and expected duration of the study for male students are higher than that of female students in the first year and second-year students. The transition rates from students in the first year and the second year form are higher for male students than female students and the same for a third year.

In the long run, the predicted academic dismissal before getting their degree in the first and second year increased from 7.95% to 11.05% and from 3.11% to 3.33% respectively. Dropout of the faculty before attaining their degree in the first and second year increased from 3.37% to 7.64% and from 4.44% to 4.47% respectively. Students in the second year form have lower retention rate, and moreover, comparatively male retention rate is higher than female in the first and second year. This may be due to the relatively high repetition rate in the second year level. Similarly, in the long run, the predicted graduates from first-year form increased from 74.41% to 81.30% and from second-year from 85.78% to 91.91%.

Hence, we recommend that the faculty should give more emphasis for first and second year students, especially for female students to increase their graduation rate. This may be because of female students admitted with lower GPA than male students in higher education; the first year students do not have university experience, the university environment differs from their home environment and punishment of breaking university rule. Moreover, research should be done to get the root cause of female students’ academic dismissal, dropouts, and gender disparity in graduation rates.

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