LIMITS ON ASSOCIATED PRODUCTION OF VISIBLY AND INVISIBLY DECAYING HIGGS BOSONS FROM Z DECAYS

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Abstract

Many extensions of the standard electroweak model Higgs sector suggest that the main Higgs decay channel is "invisible", for example, \( h \rightarrow JJ \) where \( J \) denotes the majoron, a weakly interacting pseudoscalar Goldstone boson associated to the spontaneous violation of lepton number. In many of these models the Higgs boson may also be produced in association to a massive pseudoscalar boson (HA), in addition to the standard Bjorken mechanism (HZ). We describe a general strategy to determine limits from LEP data on the masses and couplings of such Higgs bosons, using the existing data on acoplanar dijet events as well as data on four and six \( b \) jet event topologies. For the sake of illustration, we present constraints that can be obtained for the ALEPH data.
1 Introduction

The Higgs particle remains one of the missing links in the otherwise well tested standard electroweak model (SM) \[1\]. Its mass is not fixed theoretically and experimental efforts to search for the higgs boson, notably at LEP, have produced a lower bound $\sim 63$ GeV \[2\] on its mass. This bound applies to the standard model (SM) Higgs.

There exist well motivated extensions of the SM which are characterized by a more complex Higgs structure than that of the SM \[3, 4\]. These include (i) the minimal supersymmetric standard model (MSSM) (ii) generic two or more Higgs doublet models and (iii) the majoron type models characterized by a spontaneously broken global symmetry. This symmetry could either be a lepton number \[5, 6\], a combination of family lepton numbers or an R symmetry in the context of supersymmetry \[7\]. These extensions contain one or more parameters in addition to the Higgs mass. Nevertheless, a considerable region of the parameter space has already been ruled out in these models using the LEP data \[8\].

Among the three extensions mentioned above, the majoron type models are qualitatively different compared to the other two as well as to the SM. These models contain a massless goldstone boson, called majoron. There are many types of majoron models \[6\]. Many of such models are characterized by the spontaneous violation of a global $U(1)$ lepton number symmetry close to the electroweak scale. This could also have important implications for the structure of the electroweak phase transition and the generation of the electroweak baryon asymmetry \[9\].

In such models the normal doublet Higgs is expected to have sizeable invisible decay modes to the majoron, due to the strong higgs majoron coupling \[10 - 14\]. This can have a significant effect on the Higgs phenomenology at LEP. In particular, the invisible decay could contribute to the standard signal looked for at LEP namely two acoplanar jets and missing momentum. This feature of the majoron model allows one to strongly constrain the Higgs mass \[15, 16, 17\] in spite of the occurrence of extra parameters compared to the standard model. In particular, the limit on the predominantly doublet Higgs mass can be seen to be close to the SM limit irrespective of the decay mode of the Higgs boson \[15, 16, 17\].

Apart from giving an interesting twist to the Higgs search strategy, the majoron
models are quite well motivated theoretically [8]. They allow the possibility of spontaneously generating lepton number violation and hence neutrino masses. The Higgs couplings to majorons in these models were discussed at length in [18].

As far as the neutral Higgs sector is concerned, all the models fall in two categories. (A) Minimally extended SM characterized by the addition of a singlet Higgs: Such models are typical examples of mechanisms where the neutrinos obtain their mass at the tree level [19]. (B) Models containing two Higgs doublets and a singlet under the $SU(2) \times U(1)$ group: These models though more complex arise naturally in trying to generate the neutrino mass radiatively. The Zee type [20] model modified to obtain spontaneous lepton number violation is a typical example [9] in this category. These models have the virtue of explaining smallness of neutrino masses without invoking any high scale. Moreover, type B models are different from the type A ones as far as the Higgs phenomenology is concerned since they contain a massive pseudoscalar which can be produced in association with the Higgs as in MSSM. All the existing analysis of the invisible Higgs search [15, 16, 17] have concentrated on type A models and hence on the Bjorken process.

In this letter we extend this analysis to include the type (B) models and particularly the associated production of the pseudoscalar Higgs boson. We describe a general strategy to determine limits from LEP data on the H and A masses and couplings using the existing data on acoplanar dijet events. We also show how one can set decay mode-independent limits on H and A masses and couplings by including data on four and six b jet events. For the sake of illustration, we display the constraints that can be obtained for the published ALEPH data.

## 2 Higgs Boson Production and Decay

Let us first recall the salient features of the type (B) models. The Higgs sector is characterized by two doublets $\phi_{1,2}$ and a singlet $\sigma$. The part of the scalar potential containing the neutral Higgs fields is given in this case by

\[
V = \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \sigma^\dagger \sigma + \lambda_i (\phi_i^\dagger \phi_i)^2 + \lambda_\sigma (\sigma^\dagger \sigma)^2 + \\
\lambda_{12} (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_{13} (\phi_1^\dagger \phi_1) (\sigma^\dagger \sigma) + \lambda_{23} (\phi_2^\dagger \phi_2) (\sigma^\dagger \sigma) + \\
+ \delta (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{1}{2} \kappa [(\phi_1^\dagger \phi_2)^2 + \text{h. c.}] \quad (1)
\]
where a sum over repeated indices $i=1,2$ is assumed. Here $\phi_{1,2}$ are the doublet fields and $\sigma$ corresponds to the singlet carrying nonzero lepton number.

In writing down the above equation, we have imposed a discrete symmetry $\phi_2 \to -\phi_2$ needed to obtain natural flavour conservation in the presence of more than one Higgs doublet. For simplicity, we assume all the couplings and VEVs to be real.

After minimizing of the above potential one can work out the mass matrix for the Higgs fields. To this end we shift the fields as $(i=1,2)$

$$\phi_i = \frac{v_i}{\sqrt{2}} + \frac{R_i + iI_i}{\sqrt{2}}$$

$$\sigma = \frac{v_3}{\sqrt{2}} + \frac{R_3 + iI_3}{\sqrt{2}}.$$  

The masses of the CP even fields $R_a$ ($a=1\ldots3$) are obtained from

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} R^T M_R^2 R$$

with

$$M_R^2 = \begin{pmatrix} 2\lambda_1 v_1^2 & (\kappa + \lambda_{12} + \delta)v_1 v_2 & \lambda_{13} v_1 v_3 \\ (\kappa + \lambda_{12} + \delta)v_1 v_2 & 2\lambda_2 v_2^2 & \lambda_{23} v_2 v_3 \\ \lambda_{13} v_1 v_3 & \lambda_{23} v_2 v_3 & 2\lambda_3 v_3^2 \end{pmatrix}.$$  

The physical mass eigenstates $H_a$ are related to the corresponding weak eigenstates as

$$H_a = O_{ab} R_b$$

where, $O$ is a $3\times3$ matrix diagonalizing $M_R^2$

$$O M_R^2 O^T = \text{diag} (M_1^2, M_2^2, M_3^2).$$

It is convenient to parametrize the matrix $O$ as

$$O = \begin{pmatrix} c_\alpha c_\theta & -s_\alpha c_\omega - c_\alpha s_\omega s_\theta & s_\alpha s_\omega - c_\alpha c_\omega s_\theta \\ s_\alpha c_\theta & c_\alpha c_\omega - s_\alpha s_\omega s_\theta & -c_\alpha s_\omega - c_\alpha c_\omega s_\theta \\ s_\theta & s_\omega c_\theta & c_\omega c_\theta \end{pmatrix}$$

where $c_\alpha \equiv \cos \alpha$ etc.
In addition, there exists also a massive CP odd state $A$, related to the doublet fields as follows

$$A = \frac{1}{V}(v_2 I_1 - v_1 I_2).$$

(9)

Its mass is given by

$$M_A^2 = -\kappa V^2$$

(10)

where $V = (v_1^2 + v_2^2)^{1/2}$. Thus, after spontaneous $SU(2) \times U(1) \times U(1)_L$ breaking, we have a total of three massive CP even scalars $H_i$ ($i=1,2,3$), plus a massive pseudoscalar $A$ and the massless majoron $J$, simply given as $J = I_3$.

At an $e^+e^-$ collider there are two main production mechanisms that allow the production of the higgs boson through their couplings to $Z$. The relevant couplings for their production through Bjorken process (HZ) are given as follows ($a=1...3$)

$$L_{HZZ} = (\sqrt{2}G_F)^{1/2}M_Z^2Z_{\mu}Z_{\nu}\frac{v_1}{V}O_{1a} + \frac{v_2}{V}O_{2a}]H_a.$$ 

(11)

As long as the mixing appearing eq. (11) is $O(1)$, the Higgs bosons can have significant couplings and hence appreciable production rates through the Bjorken process.

In addition, the $H_a$ can also be produced in association with the CP odd field $A$ through the HA coupling

$$L_{HAZ} = -\frac{g}{\cos\theta_W}Z_{\mu}\left[\frac{v_2}{V}O_{1a} - \frac{v_1}{V}O_{2a}\right]H_a \partial_{\mu}A.$$ 

(12)

In what follows we assume that at LEP only the the lightest of the CP even Higgs boson, denoted by $H_1 \equiv H$ can be accessible. Using the matrix $O$ of eq. (8) in eq. (11) and eq. (12) one gets the couplings of $H$ to $ZZ$ and HA as

$$L_{HZZ} = (\sqrt{2}G_F)^{1/2}M_Z^2Z_{\mu}Z_{\nu}\cos(\beta - \alpha)\cos\theta H$$

(13)

where $\tan\beta \equiv \frac{v_2}{v_1}$ and

$$L_{HAZ} = -\frac{g}{\cos\theta_W}Z_{\mu}\sin(\beta - \alpha)\cos\theta H \partial_{\mu}A.$$ 

(14)

As expected, one recovers the well known expressions for the two doublets case in the limit $\theta \to 0$. In particular, note that for sizeable value of $\cos\theta$, one cannot simultaneously suppress the production of Higgs through eq. (13) and eq. (14) if it is allowed kinematically.
Now we turn to Higgs boson decay. For Higgs boson masses accessible at LEP energies the main CP even Higgs decay modes are into $bb$ and $JJ$. The coupling of the physical Higgses to $J$ follows from eq. (1). One can express this coupling entirely in terms of the masses $M_a^2$ and the mixing angles in the matrix $O$

$$\mathcal{L}_J = \frac{1}{2} J^2 (2\lambda_3 v_3 R_3 + \lambda_{13} v_1 R_1 + \lambda_{23} v_2 v_3 R_2)$$

(15)

$$= \frac{J^2}{2v_3} (M_R^2)^{3a} R_a$$

(16)

$$= \frac{1}{2} (\sqrt{2} G_F)^{1/2} \tan \gamma (O^T)_{3a} M_a^2 H_a J^2$$

(17)

where $\tan \gamma \equiv \frac{\nu}{v_3}$. We have made use of eq. (6) and eq. (7) in writing the last line.

Note from eq. (15) that the CP even scalar Higgs bosons $H_i$ couple strongly to a pair of majorons leading to the invisible decay signature. In contrast to $H_i$, the pseudoscalar $A$ does not decay into one or three majorons since the couplings $AJ^3$ or $AHJ$, although possible in general, do not exist at tree level in our simplest model described above. Because of the form of the scalar potential given in eq. (1) only even powers of the majoron field $J \equiv R_3$ appear once the expansions in eq. (2) and eq. (3) are used. As a result the couplings $AJ^3$ and $AHJ$ are absent from the potential and therefore the $A$ decays visibly to fermion antifermion pair $^\star\star$. The branching fraction $B_A$ for $A \to b\bar{b}$ is nearly one. Deviation from unity of this branching ratio is model dependent. If all fermions obtain their mass by coupling to only one higgs boson, as in the models considered in ref. [13, 9], then one has

$$B_A = \frac{1}{1 + r}$$

(18)

where

$$r \approx \sum_f \frac{m_f^2 (1 - 4m_f^2 / m_A^2)^{1/2}}{m_b^2 (1 - 4m_b^2 / m_A^2)^{1/2}}$$

(19)

and the sum is over all the fermions except $b$. The rate for $H \to b\bar{b}$ can be expressed as

$$\Gamma(H \to b\bar{b}) = \frac{3\sqrt{2} G_F}{8\pi} M_H m_b^2 (1 - 4m_b^2 / M_H^2)^{3/2} \cos^2 \alpha \cos^2 \theta$$

(20)

On the other hand the width for the invisible $H$ decay can be parametrized by

$$\Gamma(H \to JJ) = \frac{\sqrt{2} G_F}{32\pi} M_H^2 (\tan \gamma O_{13})^2$$

(21)

**Note that $A$ could decay invisibly at the tree level through the process $A \to Z^* H \to \nu \bar{\nu} JJ$. However the branching ratio in this mode is quite small.**
For many choices of parameters the invisible decay mode can be rather important and, in fact, provide the strongest limits. Finally, note that additional decay modes of $H$ to AA may also exist.

### 3 Search Strategy and Analysis

Since $A$ can decay only visibly, one expects dijets + missing momentum as a signal of the Higgs production in our simplest model. This signal arises from three processes – i.e. $Z^* \rightarrow \nu\bar{\nu}$ with $H \rightarrow b\bar{b}$, or $Z^* \rightarrow q\bar{q}$ with $H \rightarrow JJ$ or $H \rightarrow JJ$ with $A \rightarrow b\bar{b}$. For each process one has a sizeable missing momentum which is aligned neither along the beam nor along the jets. In contrast, for the SM background, the missing momentum arises from i) jet fluctuation (including decay $\nu$ from $b, c$ and $\tau$ jets) in which case it is aligned along the jets, and ii) initial state radiation (ISR) or the two-photon process $e^+e^- \rightarrow (e^+e^-)\gamma\gamma$ in which case it is aligned along the beam direction. This enables one to eliminate the SM background in the dijet + missing momentum channel by a suitable combination of kinematic cuts without depleting the signals seriously. In fact, this procedure has been extensively used to search for the SM Higgs signal (first process) in the LEP data and obtain the corresponding mass limit [2]. More recently the analysis has been extended to the invisibly decaying Higgs signal (second process) and obtain the corresponding mass limits for the A type majoron models [13, 16, 17]. In this section we shall extend the analysis further to include the Higgs signal from the third process of associated production (HA) and study the resulting mass limits for the B type models.

We shall use a parton level Monte Carlo event generator, which has been shown [16] to reproduce the signals obtained with the full Monte Carlo program [8] quite well.

For our illustrative purposes, below we will determine the corresponding Higgs boson mass limits which can be obtained from the ALEPH data sample of ref. [2] based on a statistics of $\sim 1.23$ million hadronic $Z$ events. A detailed account of the experimental cuts can be found in [8]. We shall only summarize the main features. One starts with a visible mass cut, $M_{jj} < 70$ GeV, to ensure a sizeable missing energy ($E^\gamma$) and momentum ($\vec{p}$). A low angle cut, requiring the energy deposit within $12^\circ$ of the beam axis to be $< 3$ GeV and that beyond $30^\circ$ to be $> 60\%$ of the visible energy, removes jets close to the beam pipe where measurement errors can simulate a large $E^\gamma$. An acollinearity cut, requiring the
angle between the two jets to be $< 165^\circ$, suppresses the $Z \rightarrow q\bar{q}$ and $\tau^+\tau^-$ background where the $E$ can come from jet fluctuation (including escaping $\nu$). Moreover an isolation cut on $\vec{p}$ removes the $E$ background from the fluctuation of any one of the jets. A cut on the angle ($\alpha$) of $\vec{p}$ with respect to the beam axis, $\tan\alpha > 0.4$, suppresses the background from ISR and two photon processes. The $E$ background coming from the jet fluctuations along with an ISR are removed by an acoplanarity cut, requiring the azimuthal angle between the two jets to be $< 175^\circ$. An acoplanarity cut for 3-jet like events, requiring the sum of the 3 dijet angles to be $< 350^\circ$, removes the $E$ background arising from the fluctuation of these jets. The remaining few events are residual two-photon events, which are removed by a total $p_T$ cut. The cuts remove all the events from the data sample $[8]$, while retaining $\gtrsim 50\%$ of the signal for each of the three processes mentioned above.

We denote the number of signal events for the three processes, after the cuts, by $N_{SM}$, $N_{JJ}$ and $N_A$ respectively, assuming no suppression due to the mixing angles or branching fractions in each case. Then the expected number of signal events, after incorporating these effects, is given by

$$N_{\text{expt}} = \epsilon_B^2 [BN_J + (1 - B)N_{SM}] + \epsilon_A^2 B_A B N_A$$

(22)

where

$$B = BR(H \rightarrow JJ)$$

(23)

is the branching fraction for the $H$ decaying into the invisible mode and $\epsilon_A \equiv \sin(\beta - \alpha) \cos \theta$ and $\epsilon_B \equiv \cos(\beta - \alpha) \cos \theta$.

The number of signal events for the three processes that pass the cuts are shown in Fig. 1. As we can see $N_A \gg N_{SM}, N_{JJ}$ thus implying that, if kinematically open and not suppressed by mixing angles, associated production tends to give the strongest limits.

We shall now consider the limits which can be obtained by comparing eq. (22) with the 95% CL limit of 3 events, corresponding to 0 events in the data sample $[8]$ after the cuts.

First, we assume that $B=0$. In this case we get the limits corresponding to the SM decay mode. Note that this is the weakest possible limit for the Bjorken coupling strength $\epsilon_B^2$. This is in complete agreement with the results of ref. $[15, 16]$ as seen in Fig. 2. Note that in this case we can set no limits on $\epsilon_A^2$.

We can also obtain limits assuming $B=1$. From Fig. 1 one can see that, except for the region very close to the edge of the phase space, $m_A + m_H \sim m_Z$, the major contribution
comes from the associated production \((N_A)\) allowing the corresponding coupling strength parameter \(\epsilon_A^2\) to be strongly constrained, as seen in Fig. 3. It is easily seen that the limits close to the edge of the phase space are typically at the level \(\epsilon_A^2 \sim 0.05\). However, very close to the kinematical limit the Bjorken process becomes important and could further strengthen the limits we have obtained. Outside the region where associated production is kinematically possible, the Bjorken process can still take place and the limits on \(\epsilon_B^2\) are the same as obtained in ref. [13, 16]. The limits on the coupling strengths \(\epsilon_A^2\) and \(\epsilon_B^2\) can be improved further by using more recent data from LEP.

In the case \(B=0\) considered earlier one does not get any limit on \(\epsilon_A^2\) since in this case the associated production does not lead to the dijets + missing momentum signal, we have considered so far. However one could derive limits on the \(\epsilon_A^2\) using data on four and six b jets topologies, assuming good b identification. As an illustration of what can be achieved we assume some typical value for the corresponding branching ratios \(BR(e^+e^- \rightarrow H A \rightarrow 4b) \leq 5 \times 10^{-4}\). Using this value we can determine the excluded region for different values of \(\epsilon_A^2\) as a function of \(m_A\) and \(m_H\) from

\[
BR_{HA} = \frac{1}{2} (BR)_{\nu\nu} \lambda^3 \epsilon_A^2 ,
\]

where \(BR_{\nu\nu}\) is the Z decay branching ratio into one generation of neutrinos in the SM, the factor 1/2 refers to the scalar HA decay mode and \(\lambda\) is the corresponding phase space factor

\[
\lambda(s, m_H, m_A) = \sqrt{(s + m_H^2 - m_A^2)^2 - 4 s m_H^2} / s
\]

The resulting excluded regions are shown in Fig. 4. On the other hand if H has a mass bigger than \(2m_A\) then eq. (24) would give a weaker limit since it should now be multiplied by the branching ratio for H into 2b. But now additional information can be obtained from the study of events with six b jet topologies. One may obtain a similar limit using an illustrative reference limit for the corresponding branching \(BR(e^+e^- \rightarrow H A \rightarrow 6b) \leq 4 \times 10^{-4}\).

4 Discussion

Here we have shown that the production of invisibly decaying Higgs bosons in Z decays through the associated channel HA can lead to substantial limits on Higgs boson masses
and couplings. Such invisibly decaying Higgs bosons arise in a wide class of $SU(2) \otimes U(1)$ majoron-type models, such as the majoron extension of the Zee model for radiatively induced neutrino masses, as well as extensions of the minimal supersymmetric standard model with spontaneously broken R parity [4].

Note that it is also possible to derive a truly decay mode independent limit on the H mass in the case of associated HA production by allowing B to vary from 0 to 1 and by combining the data on dijet + missing momentum, with those from four and six b jet searches described above.

It is important to update the limits obtained here by using the full statistics provided by the four LEP experiments and take them into account in designing the strategies to search for the Higgs boson at higher energies, such as at the LHC [22] and NLC [21].

Finally we note that a similar analysis may be performed in the context of the minimal supersymmetric standard model, where the invisible decay of the Higgs bosons into neutralinos may take place. A major difference in the latter case arises from the fact that in the MSSM case the pseudoscalar boson A is the one most likely [23] to decay invisibly, whereas in the case discussed in the present paper the pseudoscalar A always decays into $b\bar{b}$.

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Figure Captions

Figure 1
Numbers of expected dijet+missing momentum events for the processes $e^+e^- \to Z^*H \to \nu\bar{\nu}\ b\bar{b}\ (N_{SM})$, $e^+e^- \to Z^*H \to q\bar{q}\ JJ\ (N_{JJ})$ and $e^+e^- \to H\ A \to J\ Jb\bar{b}\ (N_{HA})$ after imposing ALEPH cuts [8, 9].

Figure 2
Illustrative limits on $\epsilon_B^2$ for visibly decaying H (B=0) from the $Z^*H \to \nu\nu + \text{two jets}$ channel as a function of $m_H$ using the ALEPH data of ref. [2].

Figure 3
Limits on $\epsilon_A^2$ in the $m_Am_H$ plane, based on the $e^+e^- \to H\ A \to J\ Jb\bar{b}$ production channel. We have assumed BR ($H \to J\ J$) = 100% and a visibly decaying A with branching ratio into $b\bar{b}$ given in eq. (18)

Figure 4
Limits on $\epsilon_A^2$ for the B=0 case based only on $e^+e^- \to H\ A \to b\bar{b}b\bar{b}$ channel. We have assumed a hypothetical sensitivity for the 4b channel of BR($e^+e^- \to H\ A \to 4b$) $\leq 5 \times 10^{-4}$.
References

[1] P. W. Higgs, *Phys. Lett.* **12**, 132 (1964).

[2] ALEPH Collaboration, *Phys. Lett.* **B313**, 312 (1993); *Phys. Lett.* **B313**, 299 (1993); DELPHI Collaboration, Marseile conference 93; L3 Collaboration, *Phys. Lett.* **B303**, 391 (1993); OPAL Collaboration, *Phys. Lett.* **B327**, 397 (1994). For a recent review see A. Sopczak, CERN-PPE/94-73, proceedings of the Lisbon Fall School 1993

[3] J. F. Gunion, H. Haber, G. L. Kane, S. Dawson, *The Higgs Hunters Guide*, Addison Wesley, 1990

[4] A. G. Cohen, D. Kaplan, A. Nelson, *Ann. Rev. Nucl. Part. Sci.* **43** (1993) 27

[5] Y. Chikashige, R. N. Mohapatra and R. D. Peccei, *Phys. Lett.* **98B**, 265 (1980).

[6] J. W. F. Valle, *Prog. Part. Nucl. Phys.* **26**, 91 (1991) and references therein.

[7] A. Masiero and J. W. F. Valle, *Phys. Lett.* **B251**, 273 (1990). J. C. Romao, C. A. Santos, and J. W. F. Valle, *Phys. Lett.* **B288**, 311 (1992).

[8] ALEPH collaboration, *Phys. Rep.* **216** (1992) 253

[9] J. Peltoniemi, and J. W. F. Valle, *Phys. Lett.* **B304** (1993) 147

[10] J. D. Bjorken, SLAC Report, SLAC-PUB-5673 (1991). R. Barbieri, and L. Hall, *Nucl. Phys.* **B364**, 27 (1991).

[11] R. E. Scrock and M. Suzuki, *Phys. Lett.* **10B**, 250 (1982); L. F. Li, Y. Liu, L. Wolfenstein, *Phys. Lett.* **B159**, 45 (1985); See also D. Chang and W. Keung, *Phys. Lett.* **B217**, 238 (1989); S. Bertolini, A. Santamaria, *Phys. Lett.* **B213** (1988) 487; *Phys. Lett.* **B220** (1989) 597

[12] J. C. Romao, F. de Campos, and J. W. F. Valle, *Phys. Lett.* **B292** (1992) 329

[13] A. S. Joshipura, S. Rindani, *Phys. Rev. Lett.* **69** (1992) 3269

[14] E. D. Carlson and L. B. Hall, *Phys. Rev.* **D40**, 3187 (1985); G. Jungman and M. Luty, *Nucl. Phys.* **B361**, 24 (1991).
[15] A. Lopez-Fernandez, J. Romao, F. de Campos and J. W. F. Valle, *Phys. Lett.* B312 (1993) 240; F. de Campos et al, FTUV/94-28, to appear in the proceedings of Moriond 94.

[16] B Brahmachari, A S Joshipura, S Rindani, D P Roy, K Shridar, *Phys. Rev.* D48 (1993) 4224; D.P. Roy, CERN TH.7221/94, To appear in the Proc of the XXIXth rencontres the Moriond on QCD and High Energy Hadronic Interactions, Editions Frontiers (1994).

[17] J. F Grivaz, private communication; see also the 1st paper of ref. [2]

[18] A. S. Joshipura, J. W. F. Valle *Nucl. Phys.* B397 (1993) 105

[19] M. C. Gonzalez-Garcia and J. W. F. Valle, *Phys. Lett.* B216, 360 (1989). R. Mohapatra and J. W. F. Valle, *Phys. Rev.* D34, 1642 (1986).

[20] A. Zee, *Phys. Lett.* B93, 389 (1980).

[21] O. Eboli, etal. Valencia report FTUV/93-50; *Nucl. Phys.* B (1994) , in press; see also proc. of workshop on $e^+e^-$ collisions at 500 GeV, the physics potential, ed. P. Zerwas etal, pag. 55.

[22] D. Choudhury, D. P. Roy, *Phys. Lett.* B322 (1994) 368; J. C. Romao, F. de Campos, L. Diaz-Cruz, and J. W. F. Valle, *Mod. Phys. Lett.* A9 (1994) 817; S. Frederiksen, N. Johnson, G. Kane, and J. Reid, SSCL-preprint-577 (1992)

[23] P. Zerwas et al, *Zeit. fur Physik* C57 (1993) 569
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