Isotope effect on superconductivity in Josephson coupled stripes in underdoped cuprates

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Inelastic neutron scattering data for YBaCuO as well as for LaSrCuO indicate incommensurate neutron scattering peaks with incommensuration $\delta(x)$ away from the $(\pi, \pi)$ point. $T_c(x)$ can be plotted as a linear function of the incommensuration for these materials. This linear relation implies that the constant that relates these two quantities, one being the incommensuration (momentum) and another being $T_c(x)$ (energy), has the dimension of velocity we denote $v^*$: $k_B T_c(x) = h v^* \delta(x)$. We argue that this experimentally derived relation can be obtained in a simple model of Josephson coupled stripes. Within this framework we address the role of the $O^{16} \rightarrow O^{18}$ isotope effect on the $T_c(x)$. We assume that the incommensuration is set by the doping of the sample and is not sensitive to the oxygen isotope given the fixed doping. We find therefore that the only parameter that can change with O isotope substitution in the relation $T_c(x) \sim \delta(x)$ is the velocity $v^*$. We predict an oxygen isotope effect on $v^*$ and expect it to be $\simeq 5\%$.

INTRODUCTION

The isotope effect has played an important role in the understanding of the underlying pairing mechanism in superconductors. Historically it was used to identify the role of the electron-lattice interaction for the superconductivity. Experimental evidence as to the nature of interaction that causes superconductivity were first provided in 1950 by Maxwell [1] and by Reynolds et al. [2]. They showed that $T_c \propto M^{-\alpha}$, where $\alpha = 0.5 \pm 0.05$ and $M$ is the mean mass of different isotopes of the superconductor. These findings indicated that the ion mass, and therefore lattice vibrations, phonons, are important to the mechanism of superconductivity.

Frohlich [3], who was unaware of the experiments on the isotope effect, and Bardeen [4], the same year, have provided theories of the phonon-electron interaction, which in turn led to models of superconductivity dependent on the phonon energies.

For the high-$T_c$ superconductors the study of the isotope effect does not paint a simple and straightforward picture. The role of the electron-lattice interactions in the mechanism of superconductivity was initially ruled out, and the pairing mechanism was ascribed to antiferromagnetic exchange and fluctuations [5, 6]. As time goes by we witness the growing acknowledgement that interactions of the lattice with the carriers in high-$T_c$ might be important. In the discussion, the role of lattices and phonons is today gaining importance again [7, 8, 9, 10, 11, 12, 13, 14]. The isotope effect on $T_c$ is believed to be small at optimal doping, but increases to the BCS value in the underdoped regime [15]. What complicates the discussion on the isotope effect is the fact that underdoped LSCO [16] is electronically inhomogeneous. Inhomogeneity is also well established in a Bi2212 superconductor [9]. The situation is different for the YBCO compounds that are believed to be more homogeneous. In both the LSCO and the YBCO case the incommensuration, that possibly is related to stripes, is certain.

For the purposes of this discussion, we would like to point out a distinction of the isotope effect we consider here versus the notion of an isotope effect in the inhomogeneous systems. In the case of conventional homogeneous superconductors, a discussion of the isotope effect is centered on an exponent that describes the effect of ionic isotope substitution on the superconducting $T_c$. In the case of spatially inhomogeneous systems and materials with more than one energy scale, e.g. superconducting gap vs pseudogap energy scale, the notion of an isotope effect has to be expanded to address the difference in changes that could be caused by isotope substitution on different energy scales [17, 18, 19, 20, 21, 22, 23, 24]. Similar arguments can be made about the effect isotope substitution can have on pairing gap vs the superfluid stiffness. The very notion of a single exponent for the isotope effect in the presence of an electronic inhomogeneity that characterizes the whole sample by a single exponent, has to be viewed at best as a very crude average description of what is really happening in these materials.

We will take the view that there are stripes in the underdoped cuprates and address how they modify the isotope effect. The discussion on the precise real space shape of the stripes in the presence of disorder has revealed a variety of complicated patterns [25, 26]. We are not concerned here with the specific form of stripe order since we assume some typical stripe-stripe distance.

Recent scanning tunneling microscopy data on two lightly hole-doped cuprates, Ca$_{1.85}$Na$_{0.12}$CuO$_2$Cl$_2$ and
BiiSr2Dy0.2Ca0.8Cu2O8+x by Kohsaka et al. 27, reported the presence of a cluster glass with a large pairing amplitude of the localized pairs on the oxygen sites that form a real space glass-like pattern. We therefore assume that superconductivity in the underdoped regime develops through the onset of phase coherence between superconducting regions that communicate with each other via Josephson coupling 28. The precise real space arrangement of these regions is not crucial for our analysis except for the fact that the SC regions look like quasi-1D clusters with broken orientational symmetry.

In this paper we discuss the effect of isotope substitution on Josephson coupled stripes. In doing so our starting point will be the linear relation between the incommensurate peak splitting and \( T_c \) observed in YBCO and LSCO 29, 30. In case of YBCO this relation does not extend as far as for LSCO as a function of doping.

Inelastic neutron scattering data for YBCO as well as for LSCO indicate incommensurate neutron scattering peaks with incommensuration \( \delta(x) \) away from the \((\pi, \pi)\) point. It is also known that \( T_c(x) \) taken as a function of doping \( x \) can be replotted as a linear function of the incommensuration for these materials, a so called Yamada plot. This proportionality implies that the constant that relates these two quantities, one being the incommensuration (momentum) and another being \( T_c(x) \), or energy, has the dimension of velocity and is denoted \( v^* \):

\[
k_B T_c(x) = \hbar v^* \delta(x),
\]

This experimentally derived relation can be obtained in a simple model of Josephson coupled stripes. We address the role of the \( O^{16} \rightarrow O^{18} \) isotope effect on the \( T_c(x) \) within this framework. We argue that the incommensuration is set by the doping of the sample and is not sensitive to the oxygen isotope given the fixed doping. We find therefore that the only parameter that can change in the relation \( T_c(x) \sim \delta(x) \) is the velocity \( v^* \). We estimate that the effect of isotope substitution on \( v^* \) is on the order of 5% for both LSCO and YBCO materials.

**DISCUSSION**

Progress in neutron scattering has allowed for a multitude of inelastic neutron scattering data to be gathered for the high-\( T_c \) superconductor YBa2Cu3O6+x. If one follows the off-resonance spectrum to lower energies one finds incommensurate peaks with an incommensuration \( \delta \) that is doping dependent. \( \delta \)From the neutron data for YBCO for oxygen concentration \( x \), where \( 0.45 \leq x \leq 0.95 \), with max \( T_c(x) = 93 \) K, a simple linear relation between \( T_c \) and \( \delta \) for the doping range \( x \leq 0.6 \) was found to follow Eq. 1 with \( \hbar v^* = 37 \) meV \( \AA \), see 30.

Another well-studied system is LaSrCuO. Inelastic neutron scattering on La214 compounds show incommensurate peaks at \((\pi \pm \delta, \pi)\) and \((\pi, \pi \pm \delta)\), see 29. It was found that \( T_c \) was a linear function of \( \delta \) up to the optimal Sr doping value 29 and Eq. 1 holds as well. Here the constant of proportionality for LSCO is \( \hbar v^* = 20 \) meV \( \AA \). For both materials the velocity \( \hbar v^* \) is two orders of magnitude smaller than the Fermi velocity of nodal quasiparticles \( \hbar v_F \approx 1 \) eV \( \AA \), see 31. Further the velocity \( \hbar v^* \) is one order of magnitude smaller than the spin-wave velocity \( \hbar v_{sw} \approx 0.65 \) eV \( \AA \) of the parent compound 29.

Similarly, for La214 the inferred velocity is much smaller than the spin-wave velocity \( \hbar v_{sw} \approx 0.85 \) eV \( \AA \), see 32.

Does the relation in Eq. 1 imply the existence of an ex-
citation with such a velocity? An interpretation of this relation is to connect the superconductivity mechanism to the existence of fluctuating stripes. The simple relation above gives an inverse proportionality between $T_c(x)$ and the doping dependent length $\ell(x)$ determined from neutron scattering, $\ell(x) = 1/\delta(x)$. Josephson tunneling of pairs between stripe segments can produce such a relation.

A model Hamiltonian of random stripe separation and inter- and intra-stripe random Josephson coupling is

$$\mathcal{H} = \sum_{ij} J_{ij} \exp\left(i \phi_i - \phi_j \right),$$

where the summation is taken over coarse-grained regions $i$ with well-defined phases, and where $J_{ij} = J(r_{ij}) = t_0/r_{ij}^{\beta}$. It is here assumed that $J(r)$ has an exponential cutoff at lengths much larger than the stripe-stripe distance in order to have a well-defined thermodynamic limit. The stripe-stripe distance $r$ is given by a probability distribution $P(r, \theta)$. For simplicity we will, as was done in [28], assume a uniform distribution in 2D with $P(r, \theta) = C$ for $\ell - a \leq r \leq \ell + a$ and $0 \leq \theta < 2\pi$, otherwise $P(r, \theta) = 0$. Here $a = n\ell$ where $\nu$ is a bounded parameter. This gives

$$\int_{0}^{2\pi} \int_{0}^{\infty} P(r, \theta) \, dr \, d\theta = 2\pi C \int_{-\ell}^{\ell} r \, dr = 4\pi C \ell a = 1$$

so that $C = (4\pi \ell a)^{-1}$. This gives the expected $\langle r \rangle$

$$\langle r \rangle = \int_{0}^{2\pi} \int_{0}^{\infty} r P(r, \theta) \, dr \, d\theta = 2\pi C \int_{-\ell}^{\ell} r^2 \, dr = \ell + \frac{\alpha^2}{3\ell} \propto \ell$$

and the expected $\langle J(r) \rangle$

$$\langle J(r) \rangle = \langle t_0/r^\beta \rangle = \int_{0}^{2\pi} \int_{0}^{\infty} t_0 r^{-\beta} P(r, \theta) \, dr \, d\theta = 2\pi C t_0 \int_{-\ell}^{\ell} r^{1-\beta} \, dr = \frac{2\pi C t_0}{2 - \beta} \left( (\ell + a)^{2-\beta} - (\ell - a)^{2-\beta} \right)$$

which for $\beta = 1$ gives $\langle J(r) \rangle = t_0/\ell$ so that one recovers the experimentally observed relation [28]

$$T_c(x) \simeq \langle J(r) \rangle \propto (r)^{-1} \propto \delta(x).$$

The velocity $v^*$ cannot be determined for this simple model without any further assumptions. We suggest that $v^*$ is related to the phase dynamics of the superconducting regions (stripes).

From the simple relation in Eq. [1] we can now investigate the effect of isotope substitution on $T_c$. Because the hole concentration is not changing and since $\delta(x)$ is not changing with isotope substitution, the only parameter left to be isotope dependent is $h v^*$. Since $v^*$ is related to the phase dynamics of the stripes it is natural to expect that $v^*$ will not change much by isotope substitution, because of its slight effect on the band structure. From the measured oxygen isotope effect on $T_c$ of YBCO [33], we predict $v^*_{18}/v^*_{16}$ to be at least 0.95, where the velocity $v^*$ has been indexed by the isotope mass, in agreement with our expectation.

The prediction on the change of $v^*$ with isotope is made as follows. The isotope effect parameter $\alpha$ is calculated as

$$\alpha = \frac{\ln \left( 1 - (T_c^{16} - T_c^{18})/(T_c^{16}) \right)}{\ln (m_{16}/m_{18})},$$

where $m_{16}$ and $m_{18}$ are the oxygen isotope masses. Further, due to Eq. [1] $T_c^{16}/T_c^{18} = v^*_{18}/v^*_{16}$. This leads to

$$\frac{v^*_{18}}{v^*_{16}} = \left( \frac{16}{18} \right)^{\alpha}.$$

For YBCO $\alpha = 0.27$ ($T_c^{16} = 60$ K) [33] so we get $v^*_{18}/v^*_{16} = 0.969$, for LSCO $\alpha = 0.38$ ($T_c^{16} = 38.3$ K) [33] and so $v^*_{18}/v^*_{16} = 0.956$.

We find that if the doping level is kept the same in isotope substitution then the typical stripe-stripe distance, controlled by doping $x$, does not change with $x$. Therefore the only parameter that can change with isotope substitution is the coefficient that relates $T_c$ to $\delta(x)$: $k_B T_c(x) = h v^* \delta(x)$. This coefficient $v^*$ has dimension velocity and describes the phase dynamics in the Josephson coupled superconductors. Since $v^*$ is related to an electronic degree of freedom, it is hardly surprising that it is only weakly dependent on O isotope substitution. We estimated the isotope effect on $v^*$, or equivalently on $T_c$, to be less than 5%. We can therefore predict the change of $v^*$ to be of the order of a few percent in a wide doping range. This estimate is consistent with the isotope effect observed for the superfluid stiffness $\rho_s$ for underdoped LCSO [34, 35].

**CONCLUSION**

In conclusion, we have considered the role of the $O^{16} \rightarrow O^{18}$ isotope effect on the $T_c$ of the Josephson coupled stripes and its implications for $v^*$. We find that the effect is small and is on the order of 5% at most. We argue that the effect on $v^*$ is small because the underdoped materials enter into a superconducting state due to phase fluctuations and therefore the main effect that controls $T_c$ is a Josephson coupling between superconducting regions. If these phase fluctuations are due to electronic
degrees of freedom, lattice dynamics has a small but observable effect on superfluid stiffness \( \rho \), an estimate that is consistent with other experiments [34].

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