The timing of scaffolding characteristics in mathematics learning

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Abstract. The concept of scaffolding in the learning process refers to teacher support of all students in the classroom in solving the problem. The heterogeneity of the students' academic ability in the classroom affects the teacher's consideration in providing scaffolding to mathematics learning. Scaffolding is given when the student needs support. This qualitative research aims to describe the timing of scaffolding characteristics in mathematics learning based on the academic ability of students. This case study approach presents the investigation of the timing of scaffolding implementation based on students' academic ability in learning integrals. Subjects in this study were two experienced teachers of 12th graders from two schools in Malang, Indonesia. Their students consisted of different levels of academic ability students; low, moderate, and high. The results showed that there were three characteristics of the timing of scaffolding based on the academic ability students; immediate scaffolding for low-ability students, partly delayed scaffolding for moderate-ability students, and fully delayed scaffolding for high-ability students. This study complements the existing scaffolding characteristics.

1. Introduction
Scaffolding is an effective learning method [1] for students during the learning process in the classroom [2] in which they have different levels of ability [3]. This difference needs to get the teacher's attention in guiding the students. Students with different academic abilities will need different assistance, both in duration and means of scaffolding. Each teacher's choice in providing scaffolding has the goal of improving students' understanding. Scaffolding is a form of support for students to reach learning goals that they might not achieve by themselves [4]. Researchers have expanded the ideas about the meaning of scaffolding. In this study, scaffolding means the support provided by teachers to their students in the process of solving problems in learning mathematics.

The researchers stated that scaffolding was very beneficial to support students in learning mathematics [5, 6, 7]. One of the important subjects in learning mathematics is integrals, with problem-solving as its main activity. Students often perceive integrals material as a difficult subject [8]; during learning activities in a classroom with diverse levels of student academic ability, not all students can solve the given problems. Thus, scaffolding is a major component that students need in learning integrals, as the heterogeneous students’ ability deeply influences the teacher in providing scaffolding. This paper will present the examples of scaffolding moments during integrals study session, with a focus on the problem of finding the volume of the object.

The timing of scaffolding is one of the teacher's important decisions in mathematics learning. The decision about the precise timing of when scaffolding is provided to students is called the timing of scaffolding. The available research on time scaffolding arrangement that has been done is generally based on the duration of time. Study of delayed scaffolding was performed by [9] in mathematics learning by giving a period and pausing a few seconds before giving feedback to the student, which is one of the scaffolding means. The teacher's goal to delay feedback is to assess students' understanding. [4] have shown the effects of the timing of scaffolding provision, namely immediate and delayed, in supporting the online student question in a science class. The results of [10] suggested that delayed
feedback will increase the knowledge transfer compared with providing immediate feedback on weekly homework assignments. On the other hand, immediate scaffolding had a better effect than delayed scaffolding in supporting students’ online question generation. Research on immediate scaffolding, which includes [11], showed that student learning outcomes in completing homework assignments were better when given immediate feedback than if the same feedback was given the next day.

The research indicated that the timing of scaffolding affected students’ learning outcomes. Differences in the results of the studies led to the researchers’ curiosity to explore more deeply about the timing of scaffolding in instructional practice, especially in mathematics learning.

There are different definitions in the previous research; in this article, immediate scaffolding is scaffolding that is given straight away when the teacher identifies the student's mistake in solving a problem. In contrast, delayed scaffolding is scaffolding that is indirectly given when the teacher identifies the student's mistake in solving the problem.

There has been no particular study discussing the characteristics of the timing of scaffolding in the learning process. The limited studies about the timing of scaffolding [4] and scaffolding practices in class that support students adaptively [12] give rise to the need for further study. This study presents the characteristics of the timing of scaffolding in mathematics learning based on students’ academic ability. The results of this study will complement the characteristics of scaffolding and the findings of previous studies related to the implementation of scaffolding in mathematics learning.

2. Research method

This qualitative study was conducted using the case study approach [13, 14] that explores the behavior of two teachers in providing scaffolding in mathematics learning. The case study approach presents the facts gathered from observations that could serve as grounds for further research [15]. The steps in this study shown as the diagram in Fig. 1 (adapted from [14]). The subjects were selected using purposive sampling. This research involved two experienced mathematics teachers and 101 students from four classes of 12th graders from two schools in Malang, Indonesia. The teachers’ experience was determined by the length of their engagement with students in the class discourse [16]. In this case, researchers chose teachers with more than 10 years of teaching experience. The classification of the students’ levels of academic ability—low, moderate, and high—was based on the results of the mathematics quiz and the level of student’s participation in mathematics class.

The observation was done 14 times with 90 minutes duration for each study session. The focus of each observation was the teachers’ scaffolding moment in guiding students to solve problems. Data in the form of pictures, sound recordings, and documents were obtained during observation, as well as the transcript of interviews with each teacher. Interviews with the teachers were conducted to explore more in-depth their awareness of the timing of scaffolding and aim related to the timing of scaffolding.
based on their students' ability academic. The collected data were synthesized and reduced to obtain coherent information. The next step was interpreting and constructing conclusions from the findings.

3. Results and discussion

Based on the diversity of the students’ levels of academic and observation results, the timing of scaffolding can be classified into two categories: immediate and delayed. Immediate scaffolding is suitable for low-ability students, whereas delayed scaffolding is more effective for moderate and high-ability students. Both categories will be explained in the problem-solving process during the exercise session, i.e., find the volume of the solid if the area bounded by the curves is rotated around the y-axis.

Class activities were started by the teacher’s explanation of the basic concept of the volume of a rotating object and how to find it, followed by an exercise session. In this session, the teacher gave a problem to be solved. The steps in which students should take were: drawing the curves, drawing the object formed by a rotated bounded area, determining the mathematical model, and finding the volume of a solid of revolution. During this session, the teacher controlled the students' work and gave scaffolding if needed. The teachers gave high attention to the level of academic ability, works, errors, and responses of the student in answering the teachers' questions. Each teacher carried out activity control by observing each student’s work on problem-solving. The following subsections explain the examples of timing of scaffolding moments. There are teacher's strategies that explain the timing of scaffolding characteristics based on the students’ academic ability.

3.1. Immediate scaffolding

Immediate scaffolding is given straight away when the teacher identifies the student's mistake in solving the problem. The following example shows teacher identification when she looked at low-ability student’s (S1) work. The teacher noticed the obstacles faced by the student, as identified in his work. Fig. 2a shows the work of a low-ability student (S1) before being given scaffolding. S1 could not draw the curve; he only drew the coordinate axis and could not determine the value in the auxiliary point table. In the table, S1 was wrong in determining the value of y, i.e., \( y = 0 \) for the value \( x = 0 \) where \( y = x^2 + 1 \) (red mark), and could not continue to set the other points. So S1 had difficulty in drawing the curve.

Figure 2b is an immediate scaffolding flowchart done by the teacher. S1 experienced an obstacle in drawing the curve (red lined box). This obstacle was causing S1’s inability to continue to the next step (red arrow), which is drawing the object formed by a rotated bounded area, modeling the mathematical form, and finding the volume of the solid of revolution (black lined box). Scaffolding was given (blue arrow) straight away when the teacher noticed S1’s mistake in drawing the curve without waiting for S1 to proceed to the next step (red arrow).

Figure 2. (a) S1’s work before being given scaffolding; (b) Flowchart of immediate scaffolding

The first scaffolding was given to guide S1 to draw the correct curve. For S1 to draw the curve correctly, the teacher assisted in the form of instructions to fill in the points table. The transcript of the dialogue between the teacher (T) and S1 is shown in Table 1.
Table 1. Excerpt of the Transcript of Dialogue Between T and S1

| Subject | Transcript |
|---------|------------|
| T       | To draw curve $y = x^2 + 1$, the first thing to do is to make the points table. If $x = 0$, then the value of $y$ is… |
| S1      | Um……… |
| T       | Squared of zero… what does it mean? |

The scaffolding started with the explanation of the basic concept on how to draw the curve. Because S1 was a low-ability student, the teacher provided him with easily understandable assistance in the form of remembering the meaning of a squared number. The teacher understood the ability of S1, so he chose a scaffolding tool that could be understood easily. Scaffolding involves the actions of the teacher by the needs of students, which is to simplify the task by the students’ ability [3]. The teacher’s purpose of giving immediate scaffolding was to make S1 able to solve the problem and not face the failures [1, 17], which could result in S1’s reluctance to learn math. The teacher believed that S1 did not have sufficient knowledge to solve the problem; thus, she decided to immediately provide scaffolding so that S1 could follow the lesson well. Immediate scaffolding affects students’ motivation in solving math problems posed by the teacher.

3.2. Delayed scaffolding

Delayed scaffolding is given when the teacher realizes that the student is experiencing an obstacle in solving the problem but delays the assistance. Delayed scaffolding consists of partly delayed scaffolding for moderate-ability students and fully delayed scaffolding for high-ability students. The explanations of these two delayed scaffoldings are presented in subsections 3.2.1 and 3.2.2.

3.2.1. Partly delayed scaffolding

Partly delayed scaffolding was performed when the teacher was aware of the moderate-ability student’s (S2) obstacle identified from his work (red mark in Fig. 3a). Fig. 3b shows the partly delayed scaffolding flowchart done by the teacher. S2 drew and found the area bounded by the curves correctly (blue lined box). The obstacles (red lined box) identified by the teacher were the error of drawing the object formed by a rotated bounded area and determining the mathematical model. Due to these errors, S2 could not determine a correct mathematical model to find the volume of the solid of revolution. In this situation, the teacher gave partly delayed scaffolding to S2. The scaffolding moment (blue arrow) was given after S2 chose the wrong mathematical model, without waiting for him to do the next step (red arrow), which was finding the volume of a solid of revolution (black lined box).

![Figure 3](image)

(a) (b)

**Figure 3.**

(a) S2’s work before being given scaffolding;
(b) Flowchart of partly delayed scaffolding

S2’s errors in drawing the object formed by a rotated bounded area and finding the volume of the tube, which was not using an integral method, were used as grounds to select the appropriate scaffolding means as stated in Table 2, which shows a transcript of teacher’s dialogue (T) in giving scaffolding to S2.
Table 2. Excerpt of the Transcript of Dialogue Between T and S2

| Subject | Transcript |
|---------|------------|
| T       | The area bounded the curves is correct. Imagine when the shaded area rotated around the y-axis. What shape of the object will we get? |
| S2      | Tube       |
| T       | What is the volume of a shaded area? |
| S2      | The volume of the tube minus volume of the rotating parabolic |
| T       | See your drawing. If the curve is rotated, then the shape is a spatial object, right? Try to fix the drawing so that you can find the volume of a rotating of the shaded area |

Scaffolding was given to guide S2 as he drew the geometric area by asking him to imagine the resulting shape of the shaded area when rotated around the y-axis. This scaffolding helped S2 in visualizing [18] the area of the rotating object so that he could determine the mathematical model and calculate the volume the object correctly. Furthermore, the teacher reduced the aid by asking him to determine the volume of a solid object according to his drawing of the rotated shaded area.

The teacher deliberately delayed giving scaffolding when noticing S2’s initial mistake in drawing the spatial area. When teacher identified that S2 was unable to determine the mathematical model in integral form, the teacher believed S2 should be immediately given scaffolding so that he could realize his mistake; thus, he could continue to correctly find the volume of a solid object as a final step of solving the problem.

3.2.2. Fully delayed scaffolding. Fully delayed scaffolding was given when the teacher identified the errors of the high-ability student (S3) and let him finish the problem until he obtained the result, which was the volume of a solid of revolution (red mark in Fig. 4a). The initial error occurs when S3 determines the volume formula of the solid of revolution obtained by rotating the bounded area. S3 is not careful in reading the image of the volume by rotating the curves he made. He does not pay attention to the boundaries of the intended area on the given problem.

Fig. 4b shows the flowchart of a fully delayed scaffolding in which the errors (red lined box) occurred when S3 chose the mathematical model and found the volume of a solid, although he drew the curves and spatial area correctly. Scaffolding (blue arrow) was given after S3 reached the final step of finding the volume of a solid of revolution.

![Flowchart of fully delayed scaffolding](image_url)

Figure 4. (a)S3’s work before being given scaffolding; (b) Flowchart of fully delayed scaffolding

Scaffolding began when the teacher asked S3 to find the volume of a rotating object using another method and compared the result with the previous one. Furthermore, S3 was asked to analyze the difference of both methods. The excerpt of a dialogue between teacher (T) and S3 is presented in Table 3.
Table 3. Excerpt of the Transcript of Dialogue Between T and S3

| Subject | Transcript |
|---------|------------|
| T       | Another method to determine the volume of a rotating object is by substracting the tube volume with rotated parabolic volume. Try doing it that way. |
| S3      | The result is different. The volume of the rotating object using the first method is $50 \pi$ unit volume, while the other one is $49.5 \pi$ unit volume. |
| T       | Both methods should give the same result. In this case, why is it not giving the same result? Which one is true? |

In assisting S3, the teacher offered another way of finding the volume of a solid object. The difference in the results of the two ways of solving the problem enabled S3 to understand his own mistakes, which was the integration limit. The teacher’s consideration of delaying scaffolding was that the teacher believed S3 was able to realize the error in determining the mathematical model by himself so that he could find the volume of a solid object correctly. This was done by the teacher with the aim that high-ability students reflect on their own mistakes because they had enough knowledge to dig up their concept of the volume of a solid object. The purpose of this scaffolding is the supporting of the students' metacognition [1].

The explanation of the three characteristics of the timing of scaffolding based on students' academic ability could be summarized as shown in Fig. 5. The teachers' attention to their students during the learning process is very important to achieve learning objectives. Paying careful attention will affect the teachers' sensitivity to student needs [19] so their belief will be productive in giving scaffolding. The teachers’ awareness is needed in giving scaffolding [20], one of which is understanding each student’s ability. The diversity of students' ability in the classroom will affect the teacher's decisions about timing in providing scaffolding.

![Figure 5](image)

**Figure 5.** The timing of scaffolding characteristics based on students' academic ability

The characteristics of the timing of scaffolding based on students’ academic ability are in line with the basic characteristics of scaffolding: contingency, fading, and transfer of responsibility [1]. First, the immediate timing of scaffolding characteristic for low-ability students is in accordance with the contingency characteristic of scaffolding, in which the teacher's assistance is fully given to the students adaptively. There is a close interaction between teacher and students in the process of problem-solving that will affect the students' appreciation of the teacher’s assistance. The long-term implication of this assistance is that it will support students to engage more in subsequent learning [12]. Then, the partly delayed timing of scaffolding characteristic for moderate-ability students is in accordance with the fading characteristic. The teacher gradually withdraws his support to give students the chance to solve the problem. By reducing support, students are given the confidence to become more independent [18] and thinking critically [21]. Finally, the characteristic of the fully delayed timing of scaffolding is in line with the transfer of responsibility characteristic of scaffolding. It means that the teacher withdraws support by letting the students solve the problems independently. By
delaying assistance, teachers have the opportunity to understand their students' thinking [9, 22], which will be used as a consideration for choosing the appropriate scaffolding means for students.

Learning occurs when the teachers assist responsively [5, 18]. The teacher's responses, which are suitable for the students’ ability, will help students solve problems based on their own understanding. The teachers’ responsiveness is one scaffolding characteristic [2], which is essential in determining the proper scaffolding based on students’ ability. Teachers' actions that appropriate for students affect learning achievement[23]. The provision of scaffolding is adaptive [24] with the ability of students. This is based on Vygotsky's (1978) theory of the zone of proximal development (ZPD) [25]. ZPD is the zone where students cannot solve problems independently and need assistance. Learning is successful when teachers can assist according to the student's ZPD. The teacher's response, according to the ZPD of each student, is very important in the learning process [26].

The importance of studying the timing of scaffolding characteristics is for teachers can implement the appropriate scaffolding based on their students' academic ability. Implementation of the suitable timing of scaffolding in mathematics class could help achieve its objective, which is supporting students’ independence and active engagement in solving problems.

4. Conclusion
The timing of scaffolding characteristics based on students’ academic ability in mathematics learning consists of immediate scaffolding, partly delayed scaffolding, and fully delayed scaffolding. Immediate scaffolding is given directly when the teacher identifies the student's error in solving the problem. This is suitable for low-ability students. Delayed scaffolding is given when the teacher identifies the student's error in solving the problem but delays in assisting. Delayed scaffolding is divided into two types; partly delayed scaffolding for moderate-ability students and fully delayed scaffolding for high-ability students.

Appropriate attention to students' ability affects teachers' actions in choosing the suitable timing of scaffolding. Timing scaffolding in accordance with the academic ability of students has a productive impact on students’ understanding in solving problems. Research on what teachers notice in the timing of scaffolding needs to be studied further.

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