Stall Warning of Axial Compressor Using Spatial FFT and Combined Analysis of Multiple Statistical Parameters

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Abstract. In order to study an effective algorithm for the pre-stall warning of axial compressor, a pre-stall warning algorithm based on Spatial Fast Fourier Transform (Spatial FFT) and combined analysis of multiple statistical parameters is proposed to solve the problems of insufficient accuracy of stall judgment and short warning time margin in view of the existing pre-stall warning research. Taking the dynamic pressure signal at the tip of the first stage stator of a multi-stage axial compressor as the research object, the circumferential multi-channel signals which are reconstructed from the single-channel sensor signal by using the signal translation method is decomposed to obtain the multi-order modal amplitudes through the spatial FFT. Analyzing the 1st-order modal amplitude coefficient by using a variety of statistical parameter methods, several conclusions were obtained. Compared with the one-dimensional analysis methods, the spatial FFT analysis method has better warning time margin; Compared with the two statistical parameters of kurtosis and autocorrelation coefficient, the parameter analysis method combining autocovariance and autocorrelation function is less computationally intensive, has a larger difference between stable and stall conditions, lower misjudgment rate, better early warning effect, and can achieve a warning time margin of 0.04~0.137s.

Keywords: Axial compressor, stall warning, Spatial Fast Fourier Transform, statistical parameter analysis.

1. Introduction
As a key component of aero-engines, the study of compressors stability has always been a hot topic at home and abroad [1]. The rotating stall of the compressor causes the aero engine to hover, posing a serious threat to flight safety. At present, there are many researches on the rotating stall problem of axial compressors [2,3]. Day [4] and McDougall [5] et al. found different stall precursor characteristics such as spike-type and progressive-type stall precursors through experiments. On this basis, different stall warning methods have been developed. Zhang Pu et al. [6,7] used one-dimensional methods such as probability density function and average numerical difference method to analyze the compressor stall problem. Because the flow field pressure changes dynamically during the rotating stall and cannot reflect the circumferential disturbance change, it is easy to apply. There was a misjudgment. Wu Yanhui et al. [8] analyzed the cause of the stall based on the frequency of the unsteady pressure by analyzing the
frequency spectrum of the unsteady pressure, but the result of the Fourier transform was the frequency characteristic of the entire time range, and could not reflect the essential characteristics of the signal at a certain moment. Cheng Xiaobin et al. [9,10], Lin et al. [11], Fu Lao et al. [12] and Zhang et al. [13] used wavelet analysis to identify stall signals. The stall flow field can be analyzed in the time-frequency domain, but it is affected by the time domain and frequency domain. Constraints and compromises of domain resolution cannot achieve ideal results.

Aiming at the above problems, this paper takes the imported circumferential dynamic pressure signal obtained from the compressor surge experiment with different speed modes as the research object, and uses the signal translation method [14] to reconstruct the multi-channel signal to solve the limited number of sensors and acquisition errors in engineering applications. The problem of spatial FFT on the reconstructed signal [15]. The amplitude coefficient after spatial FFT is jointly determined by two parameters of autocovariance [16] and auto-correlation function [16] to realize a stall warning algorithm for axial compressors with low misjudgment rate and good warning time margin, compared with the one-dimensional stall warning algorithm and other statistical parameter determination methods, its effect is more excellent, so as to provide a more optimized solution for the implementation of active stability control.

2. Processing Method of Circumferential Dynamic Pressure Signal
In practical engineering applications, the number of installed dynamic pressure signal acquisition sensors is limited. If multiple signal sensors are used for acquisition, due to uncontrollable external factors and the influence of the loss of different sensors, acquisition will occur during the signal acquisition process. Errors increase the difficulty of data preprocessing.

According to the rotation characteristics of the compressor stall precursor disturbance [20], both the progressive stall and the spike-type stall precursor have the characteristic of rotating around the circumference at a certain speed. Using the signal translation method [14], the dynamic pressure signal of the compressor's single-channel sensor is reconstructed in the circumferential direction to solve the problem of the limited number of sensors and the acquisition error. At the same time, it can also reproduce the stall disturbance in the compressor circumferential direction. The characteristics of change.

The single-channel reconstruction method is as follows:

At the circumferential position $\theta$, the dynamic pressure signal at time $t$ can be expressed as:

$$ p = p(\theta, t) $$

(1)

The dynamic pressure signal $p(\theta_i)$ at the circumferential toroidal position $\theta_i$ is translated in the same circumferential direction, and the dynamic pressure signal $p(\theta_{i+1})$, $p(\theta_{i+2})$, …, $p(\theta_{i+n})$ at different angular positions $\theta_{i+1}$, $\theta_{i+2}$, …, $\theta_{i+n}$ in the circumferential direction is obtained.

That is, the dynamic pressure signal $p(\theta_i, t)$ of the same section and different angular positions in the circumferential direction at a specific time can be expressed as:

$$
\begin{cases}
p(\theta_i, t), & t \leq n \cdot T_{\text{delay}} \\
p(\theta_i, t - n \cdot T_{\text{delay}}), & t > n \cdot T_{\text{delay}}
\end{cases}
$$

(2)

Among them, $p(\theta_i, t)$ is the original signal, $n$ is the $n$-th reconstructed signal, and $T_{\text{delay}}$ is the translation length.

The reconstructed multi-channel signal needs to reproduce every instantaneous change in the stall development process, that is, the reconstruction needs to be dense enough, and according to the Shannon sampling theorem, the value of the translation length $T_{\text{delay}}$ satisfies:
\[ T_{\text{delay}} < \frac{1}{2} t_{\text{disturbance}} \]  

(3)

\( t_{\text{disturbance}} \) is the minimum disturbance period of stall precursor.

In order to verify the above reconstruction method, set \( T_{\text{delay}} \) to \( 0.25 \cdot t_{\text{disturbance}} \) to obtain the reconstructed signal and compare it with the measured signal.

Figure 1. The measured pressure signals change at different circumferential positions at stall state at 70% speed

Figure 2. One-channel reconstruction signals change at different circumferential positions at stall state at 70% speed

As shown in Fig. 2, according to the signal change diagram of the signal Ch. \( \theta \) reconstruction (Ch. \( \theta_0 \) in Fig. 1) and the actual measured position in Fig. 1, it can be found that the reconstructed signal can reproduce the actual measurement well during the development of the stall. The characteristics of the stall precursor signal rotating around the circumference.
It should be noted that, through the signal reconstruction method of formula (2), a single-channel acquisition signal and its reconstructed signal are used as the experimental signal. In order to effectively remove the interference of clutter and extract the real disturbance signal, each experimental signal is decomposed into the superposition of the circumferential average value \( p_{\text{avg}}(t) \) and the circumferential disturbance signal \( p_{\text{disturbance}}(\theta_s, t) \), namely:

\[
p(\theta_s, t) = p_{\text{disturbance}}(\theta_s, t) + p_{\text{avg}}(\theta_s, t)
\]  

By formula (4), the circumferential disturbance signal \( p_{\text{disturbance}}(\theta_s, t) \) can be obtained and used as the decomposition object of the spatial FFT.

3. Spatial Fast Fourier Analysis Method

The spatial fast Fourier transform is simple to calculate, has good real-time performance, and is not affected by the sampling frequency. The circumferential multi-dimensional signal can be decomposed into the superposition of multi-order modes in the time domain, and the spatial disturbance degree of the stall signal can be obtained with time.

In the compressor stall development process, the circumferential dynamic pressure signal is a spatial multi-dimensional signal, and the amplitude and phase information of the multi-order modes can be obtained through spatial FFT decomposition.

Take the measured \( N \)-channel dynamic pressure signal at the inlet as the decomposing object. Similarly, similar to the expression in equation (1), the measured signals from the 1st to the \( N \)-th channel can be expressed as \( p_r(\theta_s, t_r), p_r(\theta_s, t_r), \ldots, p_r(\theta_s, t_r) \). That is, the \( N \) measured data in the circumferential direction at time \( t_r \) are \{ \( p_r(\theta_s, t_r), p_r(\theta_s, t_r), \ldots, p_r(\theta_s, t_r) \) \}, and the Fourier series expansion is described as:

\[
p(\theta_s) = \frac{a_0}{2} + \sum_{j=1}^{\frac{N}{2}} a_j \cos\left(\frac{2\pi j k}{N}\right) + \sum_{j=1}^{\frac{N}{2}} b_j \sin\left(\frac{2\pi j k}{N}\right) \tag{5}
\]

The Fourier coefficients can be solved as:

\[
\begin{align*}
a_0 &= \frac{2}{N} \\
a_j &= \frac{2}{N} \sum_{k=1}^{N} p(\theta_s) \cos\left(\frac{2\pi j k}{N}\right) \\
b_j &= \frac{2}{N} \sum_{k=1}^{N} p(\theta_s) \sin\left(\frac{2\pi j k}{N}\right)
\end{align*}
\]  

Similar to the Fourier transform of the one-dimensional signal in the time domain, \( m \left( m \leq \frac{N}{2} \right) \) modes can be obtained by solving the equations. That is, the spatial Fast Fourier transform decomposes the circumferential \( N \) signals into the complex Fourier coefficients \( C \) corresponding to \( \frac{N}{2} + 1 \) spatial modes. The real and imaginary parts of the coefficient \( C \) can be expressed as:
\[ \begin{align*}
&\text{Re } C_j = \frac{N}{2} a_j \\
&\text{Im } C_j = \frac{N}{2} b_j
\end{align*} \quad (7) \]

The bandwidth of each amplitude is \( \frac{2}{N} \), then the actual amplitude of the signal is:

\[ \text{Amp} = \frac{2}{N} \sqrt{\text{Re } C_j^2 + \text{Im } C_j^2} = \sqrt{a_j^2 + b_j^2} \quad (8) \]

By using formula (5) to (8), \( \frac{N}{2} + 1 \) spatial modal amplitudes of the circumferential signal can be obtained.

Figure 3. one-channel reconstruction signals and measured signals amplitude of each order mode at 70% speed

Fig. 3 is the spatial FFT modal decomposition diagram of the reconstructed multiplex signal (only the first four orders are plotted) and the measured signal spatial FFT modal decomposition diagram (only the 1st-order is plotted) during the stall development at the speed of 70%.

Thus, the signal reconstruction method is basically consistent with the measured signal amplitude of the 1st-order mode, which verifies the feasibility of the signal circumferential reconstruction method. In addition, it can be seen that the 1st-order mode amplitude has a better distinguishing degree than other modes, and can better reflect the stall condition. Through analyzing the amplitude of the 1st-order mode with time, we can judge the law of the circumferential uneven distribution with time.

That is to say, the problem of stall warning is transformed into the problem of amplitude analysis main mode.

4. Stall Warning Judgment with Multiple Statistical Parameters

In order to realize the stall warning and enhance the accuracy of stall threshold, two statistical parameters, autocovariance and autocorrelation function which have better real-time performance and less computation, are selected to analyze the amplitude of the 1st-order mode.
When the normal condition develops to stall condition, the autocovariance and autocorrelation function of the 1st-order mode amplitude change greatly and change abruptly in advance, so two statistical parameters are chosen as the evaluation parameters of stall warning threshold.

4.1. Autocovariance
In statistics, the autocovariance of a particular time series is the covariance between the series and its time-shifted series. If each state of the sequence has an average of $\bar{x}$, then the self-covariance is:

$$C_s(t) = \frac{1}{N} \sum_{i=1}^{N} [x(i) - \bar{x}] [x(i + d) - \bar{x}]$$

Among them, $N$ is the length of disposable data, $\bar{x}$ is the mean value of the processed data $x$, and $d$ is the number of data delays. It can be seen from the formula that there are both autocorrelation factors and turbulence degree factors in the autocovariance parameters.

4.2. Autocorrelation Function
The Autocorrelation Function describes the linear relationship or degree of similarity between signals at different times. For a discrete time-series within a given time window. The autocorrelation function is defined as:

$$R_s(t) = E[x(i)x(i + d)] = \frac{1}{N} \sum_{i=1}^{N} x(i)x(i + d)$$

The definitions of variables are the same as the autocovariance.

It should be noted that there is some misjudgment in the single statistical parameter judging, and in order to reduce this error, multiple statistical parameters are combined and monitored simultaneously for stall warning.

The 1st-order mode amplitude is regarded as a special time series, and two statistical parameters of the first mode amplitude are monitored in real time. Setting the threshold $A$, when $|C(t)| + |R(t)| > A$, the compressor is considered to be about to stall.

4.3. Kurtosis
The Kurtosis is a numerical statistic that reflects the distribution characteristics of the signal and is a normalized 4th order central moment, and the discrete kurtosis is defined as:

$$K = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{x(i) - \bar{x}}{\sigma} \right)^4$$

When $K = 3$, the distribution curve is defined as having zero cliff. When $K > 3$, the distribution curve has positive cliff. When $K < 3$, the distribution curve has a negative cliff, and it is generally determined that when the coefficient is negative cliff, the compressor is about to stall.

4.4. Autocorrelation coefficient
By removing the mean of the series at a given time and dividing by the degree of disorder then equation (10) can be evolved as:

$$r_s(t) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{x(i) - \bar{x}}{\sigma} \right) \left( \frac{x(i + d) - \bar{x}}{\sigma} \right)$$
\( \bar{x} \) is the mean value of the sequence \( x \), and \( \sigma \) is the standard deviation of \( x \). It is determined that compressor is about to stall when the value is greater than the set threshold, that is, \( r(t) > r_0 \).

The variation characteristics of the four statistical parameters listed above for stall monitoring are compared and summarized. The dynamic pressure signal obtained from the inlet 1-stage static tip at 75% speed was selected for the comparison experimental data.

![Figure 4. Curves of autocorrelation coefficient and Kurtosis at 75% speed](image)

**Figure 4.** Curves of autocorrelation coefficient and Kurtosis at 75% speed

**Table 1. Statistical Parameter Evaluation**

| Misjudgement rate | Autocorrelation function | Autocovariance | Autocorrelation coefficient | Kurtosis |
|-------------------|--------------------------|----------------|-----------------------------|----------|
| Low               | low                      | low            | high                        | high     |
| Sudden change     | 7.643s                   | 7.643s         | 7.642s                      | 7.643s   |

Compare Figure 4 and Figure 7, and count the results in Table 1. It can be seen that the autocovariance and the autocorrelation function statistics are more stable and have no jumps than the other two statistical parameters, and the misjudgment rate is low, which is more advantageous to engineering practice.

To clarify, because of the misjudgment of autocorrelation coefficient and kurtosis curve, the value of catastrophe time here is obtained from the angle of known global data, which shows the superiority of the analysis effect of the statistics chosen in this paper.
5. Test and verification

5.1. Steps of the Algorithm

As shown in the flow chart, the circumferential dynamic pressure signal acquired from the inlet 1-stage static tip of a multi-stage compressor in different speed modes (70%, 75%, 80%, 90%, 95%, 98%) is analyzed with a sampling frequency of 6KHz, a one-time processing data volume of $\Delta N = 2000$, a Butterworth low-pass filtering process with a cut-off frequency $f_s$ of 100Hz, then take the pre-processed single-channel compressor acquisition signal as the original signal, and reconstruct the $n$ signal by translation. The reconstruction number $n=10$ to simulate the output of $n+1$ sensors installed evenly distributed along the compressor circumferential direction, and the $n+1$ channel signals are converted into the superposition of the circumferential mean value and the circumferential pressure disturbance, the circumferential pressure disturbance signal is solved, and the spatial Fourier transform is carried out to decompose into the $\frac{n}{2}+1$ modes, and the amplitude of the first-order mode is compared with Analyze the given threshold $A$ of multiple statistical parameters, judge whether it is about to stall, and give a warning time margin.
5.2. Result

Figure 6. Curves of Autocovariance and autocorrelation function at 70% speed

Figure 7. Curves of Autocovariance and autocorrelation function at 75% speed

Figure 8. Curves of Autocovariance and autocorrelation function at 80% speed
Figure 9. Curves of Autocovariance and autocorrelation function at 90% speed

Figure 10. Curves of Autocovariance and autocorrelation function at 95% speed

Figure 11. Curves of Autocovariance and autocorrelation function at 98% speed

Figure 6 to Figure 11 show, respectively, the experimental plots obtained for the compressor speed modes at 75%, 80%, 90%, 95%, and 98%. The upper half of each plot is the measured data of the circumferential dynamic pressure of the inlet 1st stage static tip at the corresponding speed, and the lower half is the 1st order mode amplitude and the mode amplitude autocovariance and autocorrelation function curves obtained based on the modal decomposition of the reconstructed signal.
From the figure, it can be seen that the autocovariance and autocorrelation function curves change obviously during the development of the compressor stall, and the warning moment of each speed mode can be obtained separately by the warning threshold determination rule, and the moment results are recorded in Table 2.

**Table 2. Comparison of Algorithm Warning Time**

| Speed mode | Stall moment | Fluctuating Pressure Change Rate [17] | Short-Time Energy [18] | Frequency Feature Changes [19] | Proposed algorithm |
|------------|--------------|--------------------------------------|------------------------|-------------------------------|-------------------|
| 70%(s)     | 7.860        | 7.842                                | 7.833                  | 7.824                         | 7.820             |
| 75%(s)     | 7.684        | 7.666                                | 7.640                  | 7.656                         | 7.655             |
| 80%(s)     | 7.772        | 7.750                                | 7.700                  | 7.732                         | 7.723             |
| 90%(s)     | 8.775        | 8.700                                | 8.700                  | 8.700                         | 8.698             |
| 95%(s)     | 7.311        | 7.318                                | 7.318                  | 7.318                         | 7.314             |
| 98%(s)     | 8.893        | 8.617                                | 8.608                  | 8.608                         | 8.598             |

From the data in Table 2, it can be seen that the prediction time is 0.04s ~ 0.137s earlier than the stall time, and is better than the Fluctuating Pressure Change Rate method, Short-Time Energy method and Frequency Feature Changes method.

6. Conclusions

The following conclusions were drawn from the experiment:

1. Based on the rotating characteristics of compressor stall signal, the signal reconstruction method is feasible, which solves the problems of sensor installation limitation and acquisition error.
2. For the statistical parameters selected in this paper, compared with kurtosis and autocorrelation coefficient, autocovariance and autocorrelation function have a higher accuracy for amplitude monitoring.
3. Compared with the four one-dimensional early-warning methods such as Fluctuating Pressure Change Rate method, Short-Time Energy method and Frequency Feature Changes method, the algorithm proposed in this paper has better stall early-warning time margin.

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