Quantum Evaporation of a Bose-Einstein Condensate

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We show that a Bose-Einstein condensate emits atoms, if either the condensate wave function, or the scattering length of the atoms depends strongly on time. Moreover, the emission process is coherent and atoms can oscillate back and forth between the condensate and the excited states. Inspired by recent experimental results, we present results of simulations of the response of a Bose-Einstein condensate to a very rapid change in the scattering length. The possibility of molecule formation is also discussed.

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Introduction. — There are several examples where the coupling of a classical field to a quantum field leads to the production of quanta of the latter. One of the best known examples is Hawking radiation, where the curved spacetime provides the classical ‘background’ that couples to a quantum field. Another example is bremsstrahlung, where an electron scattering with a nucleus, and emits a quantum of the electromagnetic field, i.e., a photon. From a quantum mechanical point of view, the emission of radiation by an antenna is also described by the coupling between a classical current, and the photon field.

An example of a classical, albeit complex field, which is subject to a high level of experimental control, is the macroscopic wave function of a Bose-Einstein condensate. Recently, this experimental control could even be extended to the interactions between the atoms [1–3], which are at the temperatures and densities of interest solely determined by the s-wave scattering length. Making use of a so-called Feshbach resonance [1], one is able to vary the scattering length of the atoms to any possible value, by tuning the magnetic bias field. This experimental degree of freedom was recently exploited to study the condensate collapse [2], which was first observed in the experiment by Bradley et al. [1]. Roughly speaking, such a collapse occurs if the condensate contains so many atoms that the mean-field interactions exceed the energy-level splitting of the external trapping potential [1]. Clearly, the wave function of a collapsing condensate is an example of a time-dependent classical field, since it undergoes very rapid and violent dynamics [2–5].

Another type of time dependence arises if the scattering length changes on a time scale that is fast compared to the collective modes of the condensate, i.e., fast compared to the inverse of the frequencies of the external trapping potential.

It is the main purpose of this Letter to point out that both kind of time dependences, can cause the transfer of condensate atoms to excited states. Theoretically, this is described by the coupling between a classical field, i.e., the condensate wave function, and a quantum field which describes the atoms in the excited states, similar to the examples mentioned above. Moreover, due to the fact that this quantum evaporation process also contains a coherent part, the atoms can oscillate back and forth between the condensate and the excited states, in a kind of multimode Rabi oscillation. This condensate loss mechanism has therefore the peculiar feature that at later times less atoms are lost from the condensate. This behavior is completely different from conventional loss mechanisms, such as dipolar relaxation and three-body recombination, which are characterized by a rate coefficient and therefore always lead to more atom loss after a longer time evolution. In a very recent experiment by Clausen et al. [6], the number of condensate atoms was measured after a very rapid change in the scattering length. In this experiment, it was found that the number of condensate atoms indeed increases with time in some regimes, and therefore we believe that our theory might offer an explanation for these experiments.

Quantum evaporation. — A convenient starting point for our discussion is the second-quantized Hamiltonian for the system. It is given in terms of the Heisenberg creation and annihilation operators, denoted by \( \hat{\psi}^\dagger(x,t) \) and \( \hat{\psi}(x,t) \), that create and annihilate an atom at position \( x \) and time \( t \), respectively, and obey the usual Bose commutation relations at equal time. The Hamiltonian reads

\[
\hat{H} = \int dx \ \hat{\psi}^\dagger(x,t) \mathcal{H}_0 \hat{\psi}(x,t) + \frac{T^{2B}(t)}{2} \int dx \ \hat{\psi}^\dagger(x,t) \hat{\psi}^\dagger(x,t) \hat{\psi}(x,t) \hat{\psi}(x,t) .
\]

The single-particle Hamiltonian \( \mathcal{H}_0 \) contains the kinetic energy of the atoms and the external trapping potential. Its eigenstates and eigenvalues are denoted by \( \chi_n(x) \) and \( \epsilon_n \), respectively. The strength and sign of the interactions are fully determined by the two-body \( T \) (transition) matrix, \( T^{2B}(t) = 4\pi a(t) \hbar^2 / m \), where \( m \) is the mass of one atom. Note that the s-wave scattering length \( a(t) \) is explicitly allowed to depend on time, which is experimentally realized by tuning the magnetic field near the Feshbach resonance.
We split the operators in a condensate part, denoted by \( \phi(x,t) \), which will be treated as a complex classical field, and a part that describes the fluctuations, denoted by \( \hat{\psi}'(x,t) \). We insert the separation \( \hat{\psi} = \phi + \hat{\psi}' \) into the Hamiltonian in Eq. (1), and keep terms up to quadratic order in the fluctuations. This is known as the Bogoliubov approximation, and means physically that we assume that most of the atoms are in the condensate. This assumption is reasonable for the zero-temperature applications under consideration here. The Hamiltonian now consists of three parts, \( H_{\text{GP}} + H_{\Omega} + \hat{H}_{\text{int}} \), where \( H_{\text{GP}} \) is the usual Gross-Pitaevskii energy functional, and \( \hat{H}_{\Omega} \) contains the terms that are quadratic in the fluctuations. The interaction between the condensate and the quantum fluctuations is described by \( \hat{H}_{\text{int}} \), which, in first instance, is given by the sum of

\[
\int dx \, \phi^*(x,t) \left[ H_0 + T^{2B}(t) |\phi(x,t)|^2 \right] \hat{\psi}'(x,t),
\]

and the hermitian conjugate expression.

Physically, we want the classical field \( \phi(x,t) \) to describe the low-energy part of the system, i.e., the low-energy single-particle states, whereas the fluctuations are the high-energy excited states. Depending on the physics of the specific application, we introduce a cut-off between these two parts of the system. This implies that we drop the terms containing \( H_0 \) in \( \hat{H}_{\text{int}} \), which now takes the form of a coupling between the classical 'current density' \( J(x,t) = T^{2B}(t)|\phi(x,t)|^2 \phi(x,t) \), and the quantum field \( \hat{\psi}'(x,t) \). Because of this coupling, a perturbation of the classical field can result in the production of quanta of the quantum field, i.e., atoms can be transferred from the condensate to the excited states. To study this process, and to derive a rate equation for the number of atoms in the condensate, we have to solve the Heisenberg equation of motion for the quantum field operators, given by

\[
\left[ i\hbar \frac{\partial}{\partial t} - H_0 - 2T^{2B}(t)|\phi(x,t)|^2 \right] \hat{\psi}'(x,t) = J(x,t), \tag{2}
\]

where the hermitian conjugate expression holds for the creation operator \( \hat{\psi}'(x,t) \). In this equation of motion, we have neglected the anomalous parts of the hamiltonian \( H_{\Omega} \), since these describe the collective motion of the condensate, and are supposed to be included in the condensate wave function \( \phi(x,t) \).

The Heisenberg equation of motion in Eq. (2) is most easily solved by introducing the retarded Green’s function \( G^{(+)}(x,t;x',t') \) by means of

\[
\left[ i\hbar \frac{\partial}{\partial t} - H_0 - 2T^{2B}(t)|\phi(x,t)|^2 \right] G^{(+)}(x,t;x',t') = \hbar \delta(x - x') \delta(t - t'), \tag{3}
\]

with the boundary condition \( G^{(+)}(x,t;x',t') = 0 \) for \( t < t' \). Physically, this Green’s function describes the propagation of the atoms in the excited states, in the absence of the interaction \( \hat{H}_{\text{int}} \). Therefore, we have that

\[
iG^{(+)}(x,t;x',t') = \theta(t - t') \langle \hat{\psi}'(x,t), \hat{\psi}'(x',t') \rangle_{J=0}. \tag{4}
\]

Because of the coupling with the classical ‘current density’ \( J(x,t) \), the operator for the fluctuations acquires a nonzero expectation value, given by

\[
\langle \hat{\psi}'(x,t) \rangle = \frac{1}{\hbar} \int dt' \int dx' G^{(+)}(x,t;x',t')J(x',t'). \tag{5}
\]

Assuming that initially all the atoms are in the condensate the density of the noncondensed atoms is given by \( n'(x,t) = |\langle \hat{\psi}'(x,t) \rangle|^2 \), and the rate equation for the number of atoms \( N_c \) in the condensate now reads

\[
\frac{dN_c(t)}{dt} = \frac{1}{\hbar} \int dx \, n'(x,t) \int dt' \int dx' \text{Im} \left[ T^{2B}(t)|\phi(x,t)|^2 \phi^*(x,t) \right] G^{(+)}(x,t;x',t')T^{2B}(t')|\phi(x',t')|^2 \phi(x',t'). \tag{6}
\]

This equation is our most important result, and describes the change in the number of condensate atoms due to a strong time dependence of the condensate wave function, and/or the scattering length of the atoms. Since we are treating the quantum field \( \hat{\psi}' \) as noninteracting, the process is coherent. In particular, oscillations of atoms between the condensate and the thermal cloud can occur. Clearly, the rate equation is nonmarkovian, and on short time scales there is no conservation of energy, due to the Heisenberg uncertainty principle. However, in the markovian limit, where the energy of the ejected atoms is taken much larger than the energy of a condensate atom, the rate equation takes the form of Fermi's Golden Rule for the process of an elastic collision between two condensate atoms, where one atom is ejected from the condensate and one atom is stimulated back into the condensate. In particular, this means that in equilibrium there is no correction to the usual Gross-Pitaevskii equation. Presently, we are mainly interested in the new features that arise due to the nonmarkovian nature of the process under consideration.

**Multimode Rabi oscillations** — As an example, we discuss the response of the system to a change in the scattering length that is too fast for the condensate to react dynamically. More specifically, we do our calculations for the experimental parameters of Claussen et al. (13). In these experiments, \(^{85}\text{Rb} \) atoms are confined in a cigar-shaped trap with radial frequency \( \omega_r/2\pi = 17.4 \) Hz and axial frequency \( \omega_z/2\pi = 6.8 \) Hz, and the Feshbach resonance at 156.9 (G)auss is used to vary the scattering length very rapidly. This is achieved by a trapezoidal pulse in the magnetic field, which means that the field is ramped linearly to a certain value in a time \( t_{\text{rise}} \), then held for a time \( t_{\text{hold}} \), before ramping back to the initial value. The initial and final values of the magnetic
field are chosen such that the scattering length is initially equal to zero, and is large and positive during the hold. The rise time and the hold time are typically of the order of microseconds, and therefore the shape of the condensate wave function hardly changes during the pulse, but remains the ground state of the trap, i.e., a gaussian. However, the phase of the condensate wave function changes considerably, and is in a gaussian approximation given by

$$\theta_0(t) + \frac{m}{2\hbar} \sum_j \frac{x_j^2}{q_j(t)} \frac{dq_j(t)}{dt},$$

with $q_j(t)$ the width of the gaussian in the three spatial directions. Including both global and local phases of the condensate wave function in our calculations is important, since, roughly speaking, these phases determine the energy of a condensate atom. The variational parameters $q_j(t)$ turn out to obey Newton’s equations of motion \[8\]. The equation of motion for the global phase $\theta_0(t)$ is determined by the condensate energy, which includes the effects of the evaporation process.

For the retarded propagator of the ejected atoms, we use the expansion

$$G^{(+)}(x, t; x', t') = -i\theta(t-t') \sum_{n,m \neq 0} a_{n,m}(t, t') \times \chi_n(x) \chi_m(x') e^{-\pi(x-a t - m t')},$$

in which the sum is over all eigenstates, except for the ground state, which is already contained in $\phi(x, t)$. The coupled equations for the expansion coefficients $a_{n,m}(t, t')$ are found from the equation of motion for the Green’s function in Eq. \[3\]. These coefficients clearly obey the initial condition $a_{n,m}(t, t) = \delta_{n,m}$ at the initial time, since all the atoms are then in the condensate.

In Fig. 1 we show the fraction of atoms in the condensate as a function of the hold time, for a fixed rise time of $t_{\text{rise}} = 12.5\mu s$. The initial number of condensate atoms is $N_c(0) = 16500$ atoms, and $N_c(0) = 6100$ atoms, respectively. The pulse is such that the scattering length is equal to $a = 2000a_0$ during hold. Here, $a_0$ is the Bohr radius. A significant fraction of the atoms is transferred to the excited states in both cases, and part of this fraction can come back into the condensate after some time, as seen from Fig. 1. The atoms come back faster in the case of the largest initial number of atoms, which is caused by the fact that the coupling between the condensate and the excited states is proportional to the number of atoms. Note also that there are various frequencies in the curve, which displays the fact that we are dealing with oscillations of atoms between the condensate and several excited states. In Fig. 1 the fraction of atoms in the condensate as a function of the rise time is displayed, for various hold times. Clearly, the number of atoms increases with the rise time, over some range. This can, of course, not be understood from the viewpoint of a loss mechanism characterized by a rate constant, such as three-body recombination, or dipolar decay.

Discussion. — Comparing Fig. 1 to the experimental results of Claussen et al. \[4\], we notice that experimentally the number of atoms always decreases with increasing hold time. An explanation for this behavior is the presence of three-body recombination. The comparison of Fig. 1 with the experimental results leads to the same conclusion. The minima in this figure are seen to shift to the left, as in the experiments, and also occur at the correct value of $t_{\text{rise}}$. However, experimentally they become lower with increasing hold time.
which can also be explained by a background loss mechanism independent of the loss mechanism discussed here. Such an additional loss mechanism leads also to the smoothing of the higher-order oscillations seen in our numerical results, which are not observed in the experiments. Unfortunately, near a Feshbach resonance the behavior of the three-body recombination is not known as a function of the magnetic field. Therefore, it turns out that a more detailed comparison with experiment is impossible at this point.

A popular model for the description of a Feshbach resonance involves the coupling of an atomic field to a molecular field [15,16]. In such a model Rabi oscillations can only be observed if a more detailed comparison with experiment is possible at this point.

In Fig. 3 we present the result of our calculations, for two different values of the magnetic field during hold and for a condensate of initially \( N_c(0) = 16500 \) atoms. Note that for the experimentally relevant case of \( a = 2000a_0 \) essentially no atoms are converted to molecules. This can be understood from the fact that for this value the detuning \( \epsilon \) is much larger than the effective coupling, i.e., \( \epsilon \gg g\sqrt{n} \), with \( n \) the density of the atomic condensate. This means that the amplitude of the multimode Rabi oscillations between the atomic condensate and the molecular gas, which is of order \( \mathcal{O}(g\sqrt{n}/\epsilon^2) \), is very small. In order to convert a significant fraction of atoms to molecules, we have to consider magnetic fields very close to the resonance. In Fig. 3 we also show the results of our calculations for \( a = 4.9 \times 10^4 a_0 \), which corresponds to a magnetic field that is 0.01 G above resonance. Due to the collective motion of the condensate the coupling between the atomic and the molecular trap states decreases with time, which leads to a damping and dephasing of the oscillations. Since we are dealing with an enormous effective scattering length, the decay through three-body recombination, and spin-flip processes becomes important. In our calculations, we have however, for the same reasons as before, not taken these decay processes into account. They will lead to a further dephasing of the Rabi oscillations.

In a future publication we intend to study the two pulse experiments conducted recently [13], and look at the properties of the ejected atoms in more detail.

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