Gauged $D = 7$ Supergravity on the $S^1/Z_2$ Orbifold

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Abstract

We construct the most general couplings of a bulk seven-dimensional Yang-Mills-Einstein $N = 2$ supergravity with a boundary six-dimensional chiral $N = (0,1)$ theory of vectors and charged hypermultiplets. The boundary consists of two brane worlds sitting at the fixed points of an $S^1/Z_2$ compactification of the seven-dimensional bulk supergravity. The resulting 6D massless spectrum surviving the orbifold projection is anomalous. By introducing boundary fields at the orbifold fixed points, we show that all anomalies are cancelled by a Green-Schwarz mechanism. In addition, all couplings of the boundary fields to the bulk are completely specified by supersymmetry. We emphasize that there is no bulk Chern-Simons term to cancel the anomalies. The latter is traded for a Green-Schwarz term which emerges in the boundary theory after a duality transformation implemented to construct the bulk supergravity.
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1 Introduction

The idea that we live on a brane in a higher-dimensional spacetime has led recently to new possibilities for physics beyond the standard model. In the brane world, where the geometry of extra dimensions can naturally account for hierarchies \[1, 2, 3, 4\], one hopes to find constraints which will specify the matter content on the brane. For example, we know that the cancellation of anomalies is a crucial guiding principle for the construction of consistent theories. We are familiar with the cancellation of gauge anomalies in four dimensions, whereas anomaly cancellation in higher dimensions \[5\] may also lead to powerful constraints, especially in those circumstances where gravitational and mixed anomalies on top of gauge ones are present as in the ten-dimensional heterotic/Type I and Type IIB string theories. In particular, anomaly cancellation has turned out to be of great significance in the context of effective field theories arising as low-energy limits of fundamental theories whose detailed high-energy structure is not known. In such a case, anomaly cancellation conditions enable us to infer information about the high-energy theory by studying low-energy physics.

In the brane world context, the prototype model combining the above ideas has been constructed by Hoˇ rava and Witten \[6, 7\]. The goal was to provide an 11-dimensional interpretation of the $E_8 \times E_8$ heterotic string. To obtain a chiral $D = 10$ spectrum from M-theory, compactification on the $S^1/Z_2$ orbifold had been considered. The fixed points of the $Z_2$ action model two brane worlds of zero tension. The orbifold projection eliminates certain bosonic fields and imposes chirality conditions on the spinors, thereby resulting in a $D = 10$ chiral spectrum. However, the price to pay is the appearance of anomalies, whose cancellation requires the presence of extra boundary fields, i.e. vector multiplets, which are the only ones available in ten dimensions. The anomaly cancellation conditions uniquely determine the gauge group on each fixed plane to be $E_8$, and anomalies are cancelled by inflow from Chern-Simons and Green-Schwarz terms. Moreover, the gauge coupling constant which seems to enter as a new free parameter in the theory turns out to be related to the gravitational coupling. Finally, the dynamics of the resulting theory in the limit of small compactification radius are believed to correspond to a strongly coupled version of the $E_8 \times E_8$ heterotic string. The work of Hoˇ rava and Witten provided a missing link in the impressive chain of string dualities and triggered much recent interest in brane-world scenarios.

Anomaly cancellation then on brane worlds in eleven dimensions severely constrains, and in fact, completely specifies the gauge structure. This should not be anticipated for the brane worlds constructed in five dimensions \[8, 9\], where the boundary gauge group is not restricted by any local anomaly, although global anomalies may impose some constraints \[10, 11, 12\]. However, gravitational anomalies also exist in six dimensions \[13\], and this places nontrivial constraints on six-dimensional (6D) theories \[14–21\]. Therefore, it is expected that the matter on six-dimensional brane worlds in seven dimensions will also be restricted, and may even be uniquely specified, by anomaly cancellation. This expectation has been studied in \[18, 19\] where it was shown that in seven-dimensional (7D) brane worlds the gauge group structure and matter content on the boundaries will
be similarly restricted. Of course, the dimensional reduction of the Hořava-Witten (HW) model automatically gives rise to brane worlds that are anomaly-free [22, 23, 24]. However, this is not the only possibility. The starting point of the construction of [18, 19], which we summarize, is $\mathcal{N} = 2$ 7D gauged supergravity with a 3-form potential. The ungauged theory is obtained from the compactification of M-theory on K3 or, equivalently, from the compactification of strongly coupled heterotic theory on $T^3$ [28]. The compactification produces twenty two vectors resulting from expanding the eleven-dimensional 3–form on the $b_2 = 22$ 2–cycles of the K3. Three of these vectors are members of the gravity multiplet, whereas the remaining nineteen fill vector multiplets of the $\mathcal{N} = 2$ 7D theory. Each vector multiplet also contains three scalars, and the 57 total scalars parameterize the coset space $SO(19, 3)/SO(19) \times SO(3)$, for which an $SO(3) \times H$ or $SO(3, 1) \times H$ subgroup of $SO(19, 3)$ can be gauged. A supersymmetric gauged theory can be obtained after introducing an appropriate potential for the scalar field (corresponding to the K3 volume). The scalar potential has two extrema, leading to either a supersymmetric or non-supersymmetric vacuum [29]. The supersymmetric vacuum has a negative cosmological constant implying that the vacuum in the gauged theory is not Minkowski spacetime but rather anti-de Sitter, $AdS_7$. The $AdS_7$ vacuum with $\mathcal{N} = 2$ supersymmetry has been considered in the context of the AdS/CFT correspondence [25], and was shown to be the supergravity dual of the 6D $\mathcal{N} = (1, 0)$ SCFT [26, 27].

The minimal $\mathcal{N} = 2$ 7D gauged supergravity may be compactified down to six dimensions on $S^1$, even in the presence of a cosmological constant as was shown in [33], resulting in a non-chiral $\mathcal{N} = (1, 1)$ 6D theory. However, we are interested in the chiral $\mathcal{N} = (0, 1)$ 6D theory because in this case vector, tensor and hypermultiplets can couple to gravity in a way that is restricted by anomalies. In particular, the possibility of vector multiplets on the 6D boundaries allows one to construct theories which contain the standard model gauge group. Thus, we need to find a way to obtain the chiral 6D theory from the 7D gauged supergravity.

An immediate way to obtain the 6D chiral theory is to compactify on an orbifold $S^1/\mathbb{Z}_2$. Besides the localized gravity multiplet there will also be a localized tensor and hypermultiplet in the resulting 6D $\mathcal{N} = (0, 1)$ chiral theory. However, the spectrum found was anomalous because in six dimensions there are gravitational anomalies. These anomalies were then cancelled by appropriate boundary fields such as vector, tensor and hypermultiplets. The locally supersymmetric bulk-boundary couplings were derived for the case of boundary vector and neutral hypermultiplets. In the HW model, the Bianchi identity for the four-form field strength had to be modified in order to obtain a consistent coupling between the boundary gauge couplings and the bulk. In the scenario of [18, 19] described above, where only the gravity multiplet was considered in the bulk and the boundary fields were restricted to neutral hypermultiplets, a similar modification for the Bianchi identity occurs as well.

In this paper, we construct the gauged $D = 7$ minimal $\mathcal{N} = 2$ supergravity on a manifold with boundaries. The boundaries are the fixed points of the $S^1/\mathbb{Z}_2$ orbifold compactification of the 7D supergravity. The starting point is the 7D minimal $\mathcal{N} = 2$
supergravity theory containing the gravity multiplet and \( N \) vector multiplets, gauged with respect to a subgroup of the holonomy group \( SO(N) \times SO(3) \) of the scalar manifold. There are two versions of this theory. The first one is based on the gravity multiplet with the 3-form potential and has been constructed in \([29, 30, 33]\). This is also the case considered in \([18, 19]\) with no vectors however. There is also the possibility for the 3-form to be traded for a 2-form (since 2- and 3-forms have field strengths which are Poincaré duals in 7D) and the 2-form version of the minimal 7D supergravity has been constructed in \([31, 32]\). We present the version of the theory with a 3–form potential \( A_3 \) which can be dualized (in the absence of a topological mass term) to an equivalent theory phrased in terms of a 2–form potential \( B_2 \). The dualization can be performed according to familiar methods but in our case, care should be given to boundary terms occurring during the dualization procedure. We need to retain such boundary terms, which are usually ignored, as in our case we have boundaries. These boundary contributions give rise to a Green-Schwarz term, necessary for the cancellation of anomalies.

The dimensional reduction of the resulting 2–form version of the theory on the \( S^1/Z_2 \) orbifold entails the assignment of a \( Z_2 \) parity on the various fields as dictated by invariance of the action and supersymmetry. After modding out \( Z_2 \), the odd fields are projected out and the surviving fields fit into multiplets of chiral \( D = 6, \mathcal{N} = (0, 1) \) supergravity. The boundary theory arising from dimensional reduction can be constructed in a straightforward way and it is what one obtains by truncating \( D = 6, \mathcal{N} = 4 \) supergravity according to the \( Z_2 \) projection.

On the other hand, the resulting theory is not self-consistent at this stage. First, the supersymmetry transformation laws contain a certain term that breaks local supersymmetry on the fixed planes. The resulting variations can be cancelled by adding certain boundary potential terms for the scalars, which are interpreted as brane tensions. These terms enter with opposite signs for the two branes and therefore, we have a positive– and a negative-tension brane. Second, since the \( D = 6 \) theories on the fixed planes have a chiral spectrum, they also have gravitational anomalies. To cancel these anomalies, one must follow the Hořava-Witten recipe and introduce additional vector multiplets, tensor multiplets and hypermultiplets living on the fixed planes. As illustrated in \([18, 19]\) for the \( Sp(1) \)–gauged theory, there are many possibilities regarding the choice of the boundary gauge groups. The anomaly cancellation conditions are similar to those presented there and can be solved for many choices of the boundary gauge group.

Our final task is to construct the action of the boundary theory. We will consider the case where (i) no additional boundary tensor multiplets are introduced, (ii) the gauge group contains the \( Sp(n_H) \times Sp(1) \) holonomy group of the hyperscalar manifold and (iii) the boundary hypermultiplets are charged under the gauge group. Starting from the vector multiplets, the appropriate action can be constructed along the lines of \([6, 7]\) starting from the globally supersymmetric theory and introducing additional couplings with the bulk fields. In particular, as in the HW case, local supersymmetry dictates that the 7–component of the 3–form field strength tensor \( G_3 \) should acquire a boundary value proportional to \( * \text{tr} F^2 \), where \( F \) is the Yang-Mills field strength. This introduces a \( B_2 \wedge \text{tr} F^2 \) interaction,
which is well-known to exist in $D = 6$ supergravity theories coupled to tensors and plays the role of the Green-Schwarz term.

Passing on to the hypermultiplets, we can initially construct the locally supersymmetric action for the case where they are inert under the gauge group and then gauge the multiplets by defining appropriate covariant derivatives. Consistency of the supersymmetry transformations then demands that the covariant derivative acting on the spinor $\epsilon$ parameterizing the SUSY transformations should also involve the boundary gauge fields and this in turn implies that the variation of the gravitino kinetic term acquires an extra term. To cancel this term, we follow the Noether method and we introduce additional couplings.

The outline of this paper is as follows. In Section 2 we present the general formalism for the consistent coupling of 7D vector multiplets to the graviton multiplet. The 3-form version of the 7D minimal $\mathcal{N} = 2$ supergravity is presented and the 2-form version is obtained by duality transformation keeping boundary terms which will be used later. In section 3 we discuss the $\mathbf{S}^1/\mathbb{Z}_2$ orbifold compactification of the 2-form version and the resulting effective 6D theory. In section 4 we present couple the bulk supergravity to the 6D boundary theory consisting of vectors and charged hypermultiplets and no tensor multiplets. Without loss of generality and for notational simplicity, the $4n_H$ scalars of the hypermultiplet are taken to parameterize the coset $Sp(n_H,1)/Sp(n_H) \times Sp(1)$, while the gauge group contains the $Sp(n_H) \times Sp(1)$ holonomy group of the hyperscalar manifold. In Section 5 we consider the total anomaly of the theory which is localized at the boundaries and explain the anomaly cancellation through the Green-Schwarz mechanism. In Section 6 we present the complete total bulk and boundary Lagrangian invariant under supersymmetry. It is the most general Lagrangian with no tensor multiplets on the boundary up to four-fermi terms. Finally, in Appendix A, B we summarize our notation and conventions.

2 7-dimensional $\mathcal{N} = 2$ supergravity

Our starting point is the gauged $D = 7$, $\mathcal{N} = 2$ (minimal) supergravity theory. The supergravity multiplet can be described either in terms of a 3–form potential or in terms of its dual 2–form potential and has a $Sp(1)$ rigid symmetry that can be gauged. The resulting theories were first constructed in \cite{31, 32} (2–form version) and \cite{29, 30, 33} (3–form version). The theory can also be coupled to $N$ vector multiplets, in which case the scalars parameterize a coset space and a subgroup of its $SO(N) \times SO(3)$ holonomy group can also be gauged. These theories were constructed in \cite{34, 35}.

In this section, starting with the 3–form version of the gauged $D = 7$, $\mathcal{N} = 2$ supergravity, we construct the 2–form version via a duality transformation of the standard type \cite{37, 38}. The difference in our case is that, we allow the 7D manifold on which we perform the duality transformation to have boundaries. Then, in addition to standard practice, we need also retain a certain boundary term that emerges during the duality transformation. This term will come into play when we will discuss anomaly cancellation in the 6D supergravity living on the boundary of the 7D space.
2.1 General formalism

The field content of the massless representations of the minimal $D = 7$, $\mathcal{N} = 2$ supersymmetry algebra consists of the following multiplets

Gravity multiplet : $(g_{MN}, A_{MNP}, A_M^i j, \phi, \psi^i_M, \chi^i)$,

Vector multiplet : $(A_M, \phi^i j, \theta^i)$, \hspace{1cm} (2.1)

where all spinors are symplectic Majorana and the index $i = 1, 2$ takes its values from the $Sp(1)$ R-symmetry group of the algebra. Thus, the gravity multiplet contains the graviton $g_{MN}$, an antisymmetric 3–form $A_{MNP}$, an $Sp(1)$ triplet of vectors $A_M^i j$, a scalar $\phi$, the gravitinos $\psi^i_M$ and spinors $\chi^i$, whereas the vector multiplet contains a vector $A_M$, an $Sp(1)$ triplet of scalars $\phi^i j$ and an $Sp(1)$ symplectic Majorana spinor $\theta^i$. The $Sp(1)$ R-symmetry can be gauged and the resulting $\mathcal{N} = 2$ 7D gauged supergravity has been constructed in [29, 30, 33].

The coupling of $N$ vector multiplets to the 7D $\mathcal{N} = 2$ supergravity leads to the reducible multiplet

$$\mathcal{V}_3 = (g_{MN}, A_{MNP}, A_I^M, \phi^a, \phi, \psi_M^i, \chi^i, \theta^a), \hspace{1cm} (2.2)$$

where $a = 1, \ldots, N$ labels the individual vector multiplets, $I = 1, \ldots, N + 3$ labels the vector fields resulting from the combination of $A_M^i j$ and $A_M^a$ and $\alpha = 1, \ldots, 3N$ labels the scalars which parameterize the coset space $SO(N, 3)/SO(N) \times SO(3)$. Trading the 3–form $A_{MNP}$ for a dual 2-form $B_{MN}$, we obtain the reducible multiplet

$$\mathcal{V}_2 = (g_{MN}, B_{MN}, A_I^M, \phi^a, \phi, \psi_M^i, \chi^i, \theta^a). \hspace{1cm} (2.3)$$

The action of the theory was first found in [34] and [35], in the 2–form and 3–form version respectively. An $(N + 3)$–parameter subgroup of the isometry group $SO(N, 3)$ of the scalar manifold can be gauged. An important consequence of this gauging is the emergence of a scalar potential, which is however indefinite.

It is useful to assemble the $3N$ scalars into a $N \times 3$ matrix $\Phi$ and define the $(N + 3) \times (N + 3)$ matrix $L$ with components

$$L^A_I = \left[ \exp \left( \begin{array}{cc} 0 & \Phi \\ \Phi^T & 0 \end{array} \right) \right]^A_I, \hspace{1cm} A, I = 1, \ldots, N + 3. \hspace{1cm} (2.4)$$

which satisfies the $SO(N, 3)$ orthogonality condition

$$\eta_{AB} L^A_I L^B_J = \eta_{IJ}; \quad \eta_{AB} = \text{diag}(-, -, -, +, \ldots, +). \hspace{1cm} (2.5)$$

One needs also consider the inverse matrix $L^{-1}$ with components given by

$$L^I_A = \eta^{IJ} \eta_{AB} L^B_J, \hspace{1cm} (2.6)$$

which satisfies

$$L^A_I L^B_I = \delta^A_B. \hspace{1cm} (2.7)$$
We may gauge now a subgroup of the isometry group $SO(N, 3)$ of the scalar manifold $SO(N, 3)/SO(N) \times SO(3)$. For this, a subgroup $G \subset SO(N, 3)$ is needed, whose dimension should equal the number of vectors of the theory, i.e., $\text{dim } G = N + 3$. Let $f_{IJ}^K$ be the structure constants of $G$ and let $D = d + i A$ be the gauge-covariant derivative (here, we have absorbed the gauge coupling(s) into the structure constants). Then, from $L$, we can construct the Maurer-Cartan form $L^{-1} DL$ with components

$$L^I_A D_M L^B_I = L^I_A \left( \partial_M \delta^I_K + f_{IJ}^K A_M^I \right) L^K_B. \quad (2.8)$$

For the construction of the action, it is convenient to decompose the index $A$ in $L^I_A$ in terms of $SO(N)$ and $SO(3) \cong SU(2)$ indices according to

$$L^I_A = (L^i_n, L^a_i) = (L^i_{j,}, L^a_i). \quad (2.9)$$

where $n$ and $i$ are triplet and doublet indices for $SU(2)$ respectively. The constraint (2.5) then is written as

$$L^a_i L_J a - L^i_{j,} L^j_l = \eta_{lj}, \quad (2.10)$$

while the Maurer-Cartan form (2.8) decomposes into components as

$P_{Ma}^n = L_a^I D_M L^I_a, \quad Q_{Ma}^b = L^I_a D_M L^I_b, \quad Q_{Ma}^m = L^I_m D_M L^I_n. \quad (2.11)$

or, employing the $SU(2)$ doublet notation as in [34],

$$P_{Maj}^i = L^I_a D_M L^I_j, \quad Q_{Ma}^b = L^I_a D_M L^I_b, \quad Q_{M}^i = L^I_k D_M L^I_j. \quad (2.12)$$

Demanding that $P$ and $Q$ transform as the corresponding quantities in the ungauged theory [34], the following restriction on the structure constants is obtained

$$f_{IJ}^L \eta_{KL} = f_{[IJ}^L \eta_{K]} L. \quad (2.13)$$

Any solution of the above equation specifies a consistent gauging of a subgroup of the isometry group of the scalar manifold. We may easily find solutions of (2.13) by taking $\eta_{lj}$ to be the Cartan-Killing metric of the gauged algebra. In that case, the gauge group can either be $SO(3, 1) \times H$ with $\text{dim } H = N - 3$ or $SO(3) \times H$ with $\text{dim } H = N$. Alternatively, we may consider $\eta_{lj} = (\eta_{mn}, \delta \eta \bar{n})$, where $m, n = 1, \ldots, p, \bar{m}, \bar{n} = p + 1, \ldots, N + 3$ and take $\eta_{mn}$ to be the Cartan-Killing metric of $SO(3) \times H$, with $\text{dim } H = p - 3$, or $SO(3, 1) \times H$, with $\text{dim } H = p - 6$. It should be noted that there are $N + 3 - p U(1)$ factors of the gauge group in this case.

We define also, for later use, the following projections of the structure constants $f_{IJ}^K$,

$$C = if_{IJ}^K L^I_i L^I_j L^I_k, \quad C_{ai}^j = if_{IJ}^K L^I_i L^I_k L^a_j, \quad (2.14)$$

$$C_{ab}^j = f_{IJ}^K L^I_a L^I_b L^I_j,$$

which can be proven to be the only nonvanishing projections.

\(^{1}\)Passing from $SU(2)$ triplet indices to doublet indices is accomplished by $L^i_j = \frac{1}{2} \varepsilon_{ij}^m (\sigma_m)_j^i$ and $Q^i_j = \frac{1}{2} \varepsilon_{mnp} (\sigma^n)_j^p Q^m n p$ etc.
2.2 The 3–form theory

The Lagrangian for the reducible multiplet $\mathcal{V}_3$ in (2.2) which contains the 3–form is

$$E^{-1} \mathcal{L}_7 = \frac{1}{2} R - \frac{1}{48} \sigma^{-4} F_{MNPQ} F^{MNPQ} - \frac{1}{4} \sigma^2 (L^i_j L^j_i + L^i_a L^a_j) F^I_{MN} F^{MNJ}$$

$$- \frac{1}{2} P^a_{M} \Gamma^a_{M} - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{1}{2} \bar{\psi}_M \Gamma^{MNP} D_N \psi_P i - \frac{1}{2} \bar{\chi}^i \Gamma^M D_M \chi_i$$

$$- \frac{1}{2} \bar{\theta}^a \Gamma^M D_M \theta_{ai} - \sigma^{-2} \left[ \frac{1}{8 \sqrt{2}} \left( \bar{\psi}_M \Gamma^{NP} \psi_P i + \frac{1}{12} \bar{\psi}_L \Gamma^{LMNPQR} \psi_R i \right) \right.$$  

$$- \frac{1}{6 \sqrt{10}} \left( \bar{\chi}^i \Gamma^{MNP} \psi_P i - \frac{1}{4} \chi^j \Gamma^{LMNPQ} \psi_{Li} \right) \left. \right]$$

$$- \frac{1}{160 \sqrt{2}} \chi^i \Gamma^{MNP} \chi_i + \frac{1}{96 \sqrt{2}} \bar{\theta}^a \Gamma^{MNP} \theta_{ai} \right] F_{MNPQ}$$

$$- \sigma \left\{ \left[ \frac{i}{2 \sqrt{2}} \left( \bar{\psi}_M \psi_j + \frac{1}{2} \bar{\psi}_L \Gamma^{LMNP} \psi_P j \right) + \frac{i}{2 \sqrt{10}} \left( \bar{\chi}^i \Gamma^{LMN} \psi_{Li} \right) \right. \right.$$  

$$- \sqrt{10} \bar{\theta}^a \Gamma^{MN} \psi_{Li} - 2 \bar{\theta}^a \Gamma^M \psi_{Ni} \left\} \right. L^j_i$$

$$+ \left\{ \frac{1}{4} \left( \bar{\theta}^a \Gamma^{MN} \psi_{Li} - 2 \bar{\theta}^a \Gamma^M \psi_{Ni} \right) - \frac{1}{2} \bar{\chi} \Gamma^{MNP} \psi_{Ni} \partial_M \phi \right.$$  

$$\left. - \frac{i}{\sqrt{2}} \left( \bar{\theta}^a \psi_{M} \psi_{j} - \bar{\theta}^a \Gamma^M \psi_{Ni} \right) P_{Ma} \right. \left. + \frac{1}{2} \bar{\chi} \Gamma^{MNP} \psi_{Ni} \partial_M \phi \right.$$  

$$+ \frac{i}{\sqrt{2}} \sigma^{-1} \bar{\theta}^a \psi_{M} \Gamma^{M} \partial a \psi_{i} - \frac{i}{2} \sigma^{-1} \left( \bar{\psi}_M \Gamma^{M} \partial a \psi_{i} + \frac{2}{\sqrt{5}} \bar{\chi} \Gamma^{a} \psi_{i} \right) C_{a} \right.$$  

$$- \frac{1}{60 \sqrt{2}} \sigma^{-1} \left( \bar{\psi}_M \Gamma^{MNP} \psi_{Ni} + 2 \sqrt{5} \bar{\psi}_M \Gamma^M \chi_i + 3 \bar{\chi} \chi_i - 5 \bar{\psi} \chi_i \right) \right.$$  

$$+ \frac{1}{36} \left. \sigma^{-2} \left( C^2 - 9 C_{a} \bar{C}_{a} \right) \right.$$  

$$+ \frac{1}{48 \sqrt{2}} E^{-1} \epsilon^{MNPQRST} F_{MNPQ} \Omega_{Y, RST} + \text{(Fermi)}^4. \quad (2.15)$$

Here, $E^A_M$ is the siebenbein, $\sigma$ is the following function of the scalar $\phi$

$$\sigma = \exp \left( \frac{-\phi}{\sqrt{5}} \right), \quad (2.16)$$

the spinor covariant derivative $D_M$ is defined as

$$D_M \chi_i = \partial_M \chi_i + \frac{1}{4} \omega_{ABM} \Gamma^{AB} \chi_i + \frac{1}{2} Q_{Mi} ^j \chi_j \quad (2.17)$$
and the two field strengths $F_4$ and $F_2^I$ are given by

$$F_{MNPQ} = 4\partial_{[M}A_{NPQ]}, \quad F^I_{MN} = 2\partial_{[M}A^J_N + f_{JK}^LA^I_MA^K_N.$$  \hspace{1cm} (2.18)

Also, $\Omega_{Y,MNP}$ is a shorthand for the Chern-Simons form of the vector multiplets

$$\Omega_{Y,MNP} = \eta_{IJ}F^I_{[MN}A^J_P] - \frac{1}{3}f_{IK}^LA^I_MA^J_NA^K_P.$$  \hspace{1cm} (2.19)

The Lagrangian (2.14) is invariant under the following set of local supersymmetry transformations

$$\delta E^A_M = \frac{1}{2}\bar{\epsilon}^i\Gamma^A\psi_{Mi},$$

$$\delta \phi = \frac{1}{2}\bar{\epsilon}^i\chi_i,$$

$$\delta A_{MNP} = \frac{3}{2\sqrt{2}}\sigma^2\bar{\psi}^i_{[M}\Gamma_{NP]}\epsilon_i + \frac{1}{\sqrt{10}}\bar{\chi}^i\Gamma_{MNP}\epsilon_i,$$

$$L^i_I\delta A^I_M = \frac{i}{\sqrt{2}}\sigma^{-1}\left(\bar{\psi}^i_{M}\epsilon_j - \frac{1}{2}\bar{\delta}^i_{jk}\bar{\psi}^k_{M}\epsilon_k\right) - \frac{i}{\sqrt{10}}\sigma^{-1}\left(\bar{\chi}^iM\epsilon_j - \frac{1}{2}\bar{\delta}^i_{jk}\bar{\chi}^kM\epsilon_k\right),$$

$$L^a_I\delta A^I_M = \frac{1}{2}\sigma^{-1}\bar{\epsilon}^i\Gamma^A\theta^a_i,$$

$$\delta L^i_I = -\frac{i}{\sqrt{2}}\left(\bar{\epsilon}^i\theta^a_j - \frac{1}{2N}\bar{\delta}^i_{jk}\theta^a_k\right)L^a_I,$$

$$\delta L^a_I = -\frac{i}{\sqrt{2}}\bar{\epsilon}^i\Theta^a_jL^j_I,$$

$$\delta \psi_{Mi} = D_M\epsilon_i + \frac{1}{80\sqrt{2}}\sigma^{-2}\left(\Gamma^M_{NPQR} - \frac{8}{3}\delta^M_{NPQR}\delta^N_{QP}\right)F_{NPQR}\epsilon_i + \frac{i}{10\sqrt{2}}\sigma\left(\Gamma^M_{NP} - 8\delta^M_{NP}\Gamma^P\right)F_{NP_i}^j\epsilon_i - \frac{1}{30\sqrt{2}}\sigma^{-1}CT_M\epsilon_i,$$

$$\delta \chi_i = \frac{1}{2}\Gamma^M\partial_M\phi\epsilon_i + \frac{1}{24\sqrt{10}}\sigma^{-2}\Gamma_{MNPQ}F_{NPQR}\epsilon_i - \frac{i}{2\sqrt{10}}\sigma\Gamma_{MN}F_{MN_i}^j\epsilon_j + \frac{1}{6\sqrt{10}}\sigma^{-1}C\epsilon_i,$$

$$\delta \theta^a_i = -\frac{1}{4}\sigma\Gamma^{MN}F^I_{MN}L^a_i\epsilon_i + \frac{1}{\sqrt{2}}\Gamma^M\partial_M\epsilon_i + \frac{1}{2\sqrt{10}}\Gamma^M\partial_M\epsilon_i - \frac{i}{2}\sigma^{-1}C\epsilon_i.$$  \hspace{1cm} (2.20)

where $\epsilon_i$ is a symplectic Majorana spinor.

We should here remark that the action of the theory may in principle include the topological mass term

$$S_m = \frac{h}{36}\int d^7x\epsilon^{MNPQRST}F_{MNPA_{RST}}.$$  \hspace{1cm} (2.21)
which is present in the pure supergravity case (no vector multiplets). This term has important implications for the 6-dimensional theory obtained by reduction on $S^1$ [33] while, in the context of $S^1/\mathbb{Z}_2$ compactification, its variation contributes to the anomaly and the anomaly cancellation conditions lead to a relation fixing the boundary Yang-Mills coupling in terms of the gravitational coupling and $h$ [18,19]. In addition, this term explicitly depends on $A_3$ and thus it is not possible to perform a duality transformation to obtain an equivalent theory with a 2–form potential. However, in the presence of vector multiplets, this term should vanish ($h = 0$) by supersymmetry and thus, it does not exist for the 7D $\mathcal{N} = 2$ supergravity coupled to vector multiplets [35].

### 2.3 Duality transformation in the presence of boundaries

It is well-known that the ambiguity in the representation of antisymmetric tensor fields, emanating from the fact that a $p$–form potential and a $(D - p - 2)$–form potential contain exactly the same degrees of freedom, often allows us to express a given theory involving such a field in two dual formulations. In such a case, the two dual theories can be obtained one from the other by means of a duality transformation [37,38].

In the context of $D = 7$, $\mathcal{N} = 2$ supergravity, there exist two such formulations, so that the 3–form theory considered above has a dual formulation in terms of a 2–form potential. The equivalence of the two theories under a duality transformation was demonstrated in [35], where the 2–form theory was obtained by dualizing the 3–form one. Here, we shall repeat the same construction, this time on a manifold with boundary. The difference in this case is that the duality transformation yields an extra boundary term that will play a particular role in the boundary theory.

The first step in the duality transformation of the 3–form potential $A_3$ is to consider the terms of (2.14) and (2.19) that involve the field strength $F_4$ and replace the latter by a new unconstrained field $S_4$ whose SUSY transformation is taken to be the same as that for $F_4$. The modified Lagrangian may be written as

$$
\mathcal{L}_S = -\frac{1}{48} E\sigma^{-4} S_{MNPQ} S^{MNPQ} - \frac{1}{8\sqrt{2}} E\sigma^{-2} S_{MNPQ} J^{MNPQ} \\
+ \frac{1}{48\sqrt{2}} \epsilon^{MNPQRST} S_{MNPQ} \Omega_{Y,RST} + \ldots
$$

(2.22)

where the dots stand for the rest of the terms in (2.15) and where we defined

$$
J_{MNPQ} = \bar{\psi}_{[MNP} \Gamma_{Q]} \psi_i + \frac{1}{12} \bar{\psi}^L \Gamma_{LMNPQR} \psi_i^R - \frac{4}{3\sqrt{5}} \left( \chi^L \Gamma_{MNPQ} \psi_i^R \right) - \frac{1}{4} \chi^L \Gamma_{LMNPQ} \psi_i^R - \frac{1}{20} \chi^L \Gamma_{MNPQ} \chi_i + \frac{1}{12} \bar{\theta}^{ai} \Gamma_{MNPQ} \theta_a.
$$

(2.23)

The next step is to add the Lagrange-multiplier term

$$
\mathcal{L}_C = -\frac{1}{48} \epsilon^{MNPQRST} B_{MN} \partial_P S_{QRST},
$$

(2.24)
where $B_2$ is a new unconstrained field. Varying $\mathcal{L}_C$ with respect to $B_2$ would simply enforce the Bianchi identity $dS_4 = 0$, taking us back to the original theory. Since in that case, $\mathcal{L}_S$ would be SUSY-invariant, the SUSY variation of $\mathcal{L}_S$ for arbitrary $S_4$ is a term proportional to $\partial_M S_{NPQR}$ arising from the variations of the fermion kinetic terms. Writing this term as

$$\delta \mathcal{L}_S = \frac{1}{48} \epsilon^{MNPQRST} X_{MN} \partial_P S_{QRST},$$

and taking $B_2$ to transform according to

$$\delta B_{MN} = X_{MN},$$

we see that the “intermediate” Lagrangian,

$$\mathcal{L}_I \equiv \mathcal{L}_S + \mathcal{L}_C,$$

is locally supersymmetric since

$$\delta \mathcal{L}_I = -\frac{1}{48} \epsilon^{MNPQRST} B_{MN} \partial_P \delta S_{QRST}$$

$$= -\frac{1}{12} \epsilon^{MNPQRST} B_{MN} \partial_P \partial_Q \delta A_{RST} = 0.$$ (2.28)

We also note here that if $F_4$ were to satisfy, instead of $dF_4 = 0$, a modified Bianchi identity of the type $dF_4 = I_5$ for some 5–form $I_5$, (2.24) would have to be modified to

$$\mathcal{L}_C = -\frac{1}{48} \epsilon^{MNPQRST} B_{MN} \left( \partial_P S_{QRST} - \frac{1}{5} I_{PQRST} \right),$$ (2.29)

so as to enforce this Bianchi identity on $S_4$ when varied with respect to $B_2$.

The next step is to integrate the Lagrange multiplier term (2.24) by parts. After that, $\mathcal{L}_I$ can be written in the form

$$\mathcal{L}_I \equiv \mathcal{L}_{I,\text{bulk}} + \mathcal{L}_{I,\text{bdy}},$$ (2.30)

where $\mathcal{L}_{I,\text{bulk}}$ contains the bulk terms

$$\mathcal{L}_{I,\text{bulk}} = -\frac{1}{48} E \sigma^{-4} S_{MNPQ} S^{MNPQ} - \frac{1}{8 \sqrt{2}} E \sigma^{-2} J^{MNPQ} S_{MNPQ}$$

$$+ \frac{1}{144} \epsilon^{MNPQRST} S_{MNPQ} G_{RST} + \ldots$$ (2.31)

and $\mathcal{L}_{I,\text{bdy}}$ is the surface term

$$\mathcal{L}_{I,\text{bdy}} = -\frac{1}{48} \epsilon^{MNPQRST} \partial_M (B_{NP} S_{QRST}),$$ (2.32)
which, in the presence of boundaries, does not a priori vanish. In (2.31), \( G_3 \) stands for the (modified) field strength of \( B_2 \), defined by

\[
G_{MNP} = 3 \left[ \partial_M B_{NP} + \frac{1}{\sqrt{2}} \left( \eta_{IJ} F^I_{MN} A^J_P - \frac{1}{3} f_{IJ}^K A^I_M A^J_N A^K_P \right) \right].
\]

(2.33)

The intermediate SUSY transformation laws for \( \psi_{Mi} \) and \( \chi_i \) are given by those in (2.20) with \( F_4 \) replaced by \( S_4 \). Hence, we have

\[
\delta \psi_{Mi} = \frac{1}{80 \sqrt{2}} \sigma^{-2} \left( \Gamma_M^{NPQR} - \frac{8}{3} \delta_M^{NP} \Gamma^{PQR} \right) S_{NPQR} \epsilon_i + \ldots
\]

(2.34)

and

\[
\delta \chi_i = \frac{1}{24 \sqrt{10}} \sigma^{-2} \Gamma^{MNQP} S_{MNQP} \epsilon_i + \ldots
\]

(2.35)

where the dots correspond to the rest of the terms in (2.20).

The final step in the duality transformation is to integrate out \( S_4 \). On the bulk, this can be accomplished by using its (algebraic) equation of motion

\[
S_{MNQP} = \frac{1}{6} \epsilon^{MNQRST} G_{RST} - \frac{3}{\sqrt{2}} \sigma^2 J^{MNQP}
\]

(2.36)

Substituting this result in the Lagrangian (2.15) and the SUSY transformation rules (2.20), one obtains a dual theory, phrased in terms of the 2–form potential \( B_2 \), whose Lagrangian and transformation rules will be stated shortly. In the absence of spacetime boundaries, this theory would be equivalent to the 2–form version of \( D = 7, \mathcal{N} = 2 \) supergravity whose special cases were constructed in [31, 34]. However, in the presence of a boundary, it is not: the former theory contains the surface term (2.32) not present in the latter. Although one could think of invoking the Bianchi identity of \( F_4 \) to set the value of this term to zero, this is not correct: in the 3–form version, anomaly and supersymmetry considerations result in a modified Bianchi identity for \( F_4 \), as in the HW case. This in turn induces a nonvanishing boundary value for \( S_4 \) so that the surface term gives rise to a 6D boundary interaction.

### 2.4 The 2–form theory

The gauged 2–form version of \( D = 7, \mathcal{N} = 2 \) supergravity obtained by the duality transformation discussed above constitutes a generalization of the theory constructed in [34] for any subgroup of the holonomy group. The resulting Lagrangian for the multiplet \( V_2 \) in (2.3) is given by

\[
E^{-1} \mathcal{L}_7 = \frac{1}{2} R - \frac{1}{12} \sigma^4 G_{MNP} G^{MNP} - \frac{1}{4} \sigma^2 (L^i_{\phantom{i}j} L^j_{\phantom{j}i} + L^a_{\phantom{a}i} L^j_{\phantom{j}a}) F^I_{MN} F^{MNI} - \frac{1}{2} P_M a_i \phi^M a_i \phi - \frac{1}{2} \phi^M \phi \Delta M NP \psi_{Pi}
\]
We note that the (Fermi)⁴ terms are not those in (2.20), but receive an additional contribution from a \( J_4 \wedge * J_4 \) term arising from the second term of (2.30).

The SUSY transformation rules for this theory are given by

\[
\begin{align*}
\delta E^A_M & = \frac{1}{2} \bar{\epsilon}^i \Gamma^A \psi_{Mi}, \\
\delta \phi & = \frac{1}{2} \bar{\epsilon}^i \chi_i, \\
\delta B_{MN} & = \sigma^2 \left( -\frac{1}{\sqrt{2}} \bar{\epsilon}^i \Gamma_{[M} \psi_{N]} i - \frac{1}{\sqrt{3}} \bar{\epsilon}^j \Gamma_{MN} \chi_i \right) + \frac{1}{\sqrt{3}} A_{[M}^I \delta A_{N]}^I \eta_{IJ}, \\
L_I^i \delta A_M^I & = \frac{i}{\sqrt{2}} \sigma^{-1} \left( \bar{\psi}_M^i \epsilon_j \chi_i \right) - \frac{i}{\sqrt{10}} \sigma^{-1} \left( \bar{\psi}_M^i \Gamma_{M} \chi_i \right), \\
L_I^a \delta A_M^I & = \frac{1}{2} \sigma^{-1} \bar{\epsilon}^i \Gamma_{M} \theta_i^a, \\
\delta L_I^i & = -\frac{i}{\sqrt{2}} \left( \bar{\epsilon} \theta_{ai} - \frac{1}{2N} \delta_i^j \epsilon_k \theta_{ak} \right) L_I^a, \\
\delta L_I^a & = -\frac{i}{\sqrt{2}} \bar{\epsilon}^a \theta_{ai} L_I^i, \\
\delta \psi_{Mi} & = D_M \epsilon_i - \frac{1}{120 \sqrt{2}} \sigma^2 \left( \Gamma_M \Gamma^{NPQ} + 5 \Gamma^{NPQ} \Gamma_M \right) G_{NPQ} \epsilon_i 
\end{align*}
\]

(2.38)
3 Orbifold compactification

The potential of the $3N + 1$ scalars of the theory (2.37) is given by

$$V(\phi^a, \phi) = \frac{1}{36} \sigma^{-2} \left( 9 C^{ai} j C_{ai j} - C^2 \right)$$

where the projections $C$ and $C^{ai} j$, introduced in (2.14), are functions of the scalars $\phi^a$ and $\phi$ and depend on the structure constants of the gauge group. Although a general result for the critical points of the potential (3.1) does not exist, it is not difficult to see that the theory possesses a seven-dimensional Minkowski vacuum. Indeed, we may consider the following scalar configuration

$$L_{\bar{n}}^n = \delta_{\bar{n}}^n \quad \text{for} \quad n = 1, 2, 3 \quad \text{and} \quad L_{\bar{a}}^a = \delta_{\bar{a}}^a \quad \text{for} \quad a = 1, \ldots, N,$$

where we have split the index $I = 1, \ldots, N + 3$ as $I = (\bar{n}, \bar{a})^2$. It is not hard to see that (3.2) satisfies (2.10) and that, for an appropriate choice of the gauge group, we have

$$C = C \bar{i} \bar{j} = C a b \bar{i} \bar{j} = 0.$$  

Then, the potential and its derivatives with respect to $\phi^a$ vanish for the choice (3.2) so that $\phi^a, \phi = \text{const.}$ is a solution, which gives rise to a 7D Minkowski vacuum.

The theory can be dimensionally reduced to six dimensions along an $S^1$ of radius $R$ parameterized by $x_7$. On $S^1$, the various 7-dimensional fields of the reducible multiplet (2.3) decompose à la Kaluza-Klein according to

$$g_{MN} \rightarrow g_{\mu \nu}, A_M, \tilde{\xi} \quad \text{,} \quad B_{MN} \rightarrow B_{\mu \nu}, B_\mu \quad \text{,} \quad A_M^I \rightarrow A_M^I, A^I \quad \text{,} \quad \phi^a, \phi \rightarrow \phi^a, \tilde{\phi} \quad \text{.}$$

$$\psi_M^i \rightarrow \psi_\mu^i, \tilde{\psi}^i \quad \text{,} \quad \chi^i \rightarrow \tilde{\chi}^i \quad \text{,} \quad \theta^{ai} \rightarrow \theta^{ai}.$$  

The detailed reduction procedure was presented in [33]. In the reduction presented therein, the Kaluza-Klein ansatz for the 7D metric reads

$$ds^2 = e^{-\xi/\sqrt{3}} ds_6^2 + e^{\xi/\sqrt{3}} (dx_7 + A_\mu dx^\mu)^2,$$

In the doublet notation, the first of (3.2) translates to $L_{\bar{n}}^i \sigma_{\bar{n}} = \frac{1}{\sqrt{2}}(\sigma_{\bar{n}})^i$. 

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the various 7D spinors reduce according to
\[ \psi_{\mu i}^{(7)} = e^{-\xi/\sqrt{5}} \left[ \psi_{\mu i} - \frac{1}{2\sqrt{5}} \left( \Gamma_\mu \Gamma_7 + 4\sqrt{2} A_\mu \right) \tilde{\psi}_i \right], \]
(3.6)
\[ \psi_{7i}^{(7)} = \frac{2}{\sqrt{5}} e^{\xi/\sqrt{5}} \tilde{\psi}_i, \quad \chi_i^{(7)} = e^{\xi/\sqrt{5}} \tilde{\chi}_i, \quad \theta_{ai}^{(7)} = e^{\xi/\sqrt{5}} \theta_{ai}, \]
(3.7)
while antisymmetric tensor fields reduce in the usual way. In the above, \( \psi_\mu \) is identified with the 6D gravitino. As for the tilded 6D fields \((\tilde{\psi}_i, \tilde{\chi}_i)\) and \((\tilde{\xi}, \tilde{\phi})\), it helps to trade them for the fields \((\psi_i, \chi_i)\) and \((\xi, \phi)\) defined through the linear combinations
\[ \tilde{\psi}_i = \frac{1}{\sqrt{5}} (2\psi_i - \Gamma_7 \chi_i), \quad \tilde{\chi}_i = \frac{1}{\sqrt{5}} (2\chi_i + \Gamma_7 \psi_i), \]
(3.8)
and
\[ \tilde{\xi} = \frac{1}{\sqrt{5}} (2\xi - \phi), \quad \tilde{\phi} = \frac{1}{\sqrt{5}} (2\phi + \xi). \]
(3.9)
The Lagrangian of the theory on \( S^1 \) is then obtained by substituting (3.5–3.9) into (2.37). The final result is a \( D = 6, N = 4 \) supergravity model, which can be consistently truncated to a model with \( N = 2 \) supersymmetry.

In what follows, we will consider compactification of our theory on the \( S^1 / \mathbb{Z}_2 \) orbifold, in order to obtain a model with \( N = 1 \) supersymmetry. In contrast to common practice in bulk-brane theories, we will make the somewhat unconventional choice of writing the 7D bulk fields in the “Kaluza-Klein basis” described by the preceding equations. This particular choice is dictated, in our case, by the fact that we intend to explore in detail the structure of the boundary theory, which is most conveniently done in a basis adapted for 6D fields. To avoid notational confusion, subsequent references to bulk fields will be always accompanied by a “(7)” superscript.

### 3.1 Compactification on \( S^1 / \mathbb{Z}_2 \)

To obtain a chiral 6-dimensional theory from our model, we consider compactification of the \( x_7 \) coordinate on the \( S^1 / \mathbb{Z}_2 \) orbifold. The \( \mathbb{Z}_2 \) action is as usual \( x_7 \to -x_7 \) and the two fixed points are at \( x_7 = 0 \) and \( x_7 = \pi R \). We should note that we may consider two different, but equivalent, approaches. In the “downstairs” approach, one considers the 7D spacetime manifold as the product \( M_6 \times I \) of 6D spacetime times the interval \( I = [0, \pi R] \) obtained by modding out the \( \mathbb{Z}_2 \) symmetry. Since this is a manifold with boundary, possible surface terms resulting from partial integrations need to be retained. In the “upstairs” approach, which is the one originally used by Hořava and Witten, one regards spacetime as the smooth manifold \( M_6 \times S^1 \) subject to \( \mathbb{Z}_2 \) invariance. On this orbifold, no surface terms arise from partial integration; however, the fact that \( \mathbb{Z}_2 \) has fixed points requires that certain \( \mathbb{Z}_2 \)-odd functions have step-function discontinuities. In what follows, we shall mainly work in the “downstairs” approach.
To begin the construction of the theory, we have to mod out \( S^1 \) by \( \mathbb{Z}_2 \). In order for the reduction to be consistent, we must first make certain parity assignments to the fields so that the Lagrangian and the supersymmetry transformation rules stay invariant under \( x_7 \to -x_7 \). Then, when we mod out \( \mathbb{Z}_2 \), only fields of even parity survive on the two orbifold fixed planes. By the same considerations as in [36, 18, 19], it is easy to see that, after the \( \mathbb{Z}_2 \) projection, the surviving bosonic fields are

\[
g_{\mu \nu}, B_{\mu \nu}, A_I, \phi^\alpha, \phi, \xi,
\]

while the spinors are subject to the chirality constraints

\[
\Gamma_7 (\psi^i, \epsilon^i) = - (\psi^i, \epsilon^i), \quad \Gamma_7 (\psi^i, \chi^i, \theta^{ai}) = (\psi^i, \chi^i, \theta^{ai}).
\]

The surviving fields can be then arranged into multiplets that furnish representations of the \( \mathcal{N} = (0, 1) \), \( D = 6 \) supersymmetry algebra. To perform the decomposition, we split \( B_{\mu \nu} \) into a self-dual and an anti-self-dual part

\[
B_{\mu \nu} = B^+_{\mu \nu} + B^-_{\mu \nu},
\]

and we group all scalars except from \( \phi \) and all spin 1/2 fermions except from \( \chi^i \) according to

\[
\Phi^Y_z = (A^I, \phi^\alpha, \xi), \quad Z^{i+} = (\psi^i, \chi^i, \theta^{ai}),
\]

where the index \( Y \) runs from 1 to \( N + 1 \) while \( z \) runs from 1 to 4. Then, the fields surviving the \( \mathbb{Z}_2 \) projection can be arranged as follows

\[
\begin{align*}
\text{Gravity multiplet} & : (g_{\mu \nu}, B^+_{\mu \nu}, \psi^-_{\mu}), \\
\text{Tensor multiplet} & : (B^-_{\mu \nu}, \phi, \chi^i), \\
N + 1 \text{ hypermultiplets} & : (\Phi^Y_z, Z^{i+}).
\end{align*}
\]

where the \( \pm \) superscripts on spinors indicate six-dimensional chirality. Thus, the massless spectrum of the \( S^1/\mathbb{Z}_2 \) compactification consists of the graviton multiplet, a tensor and \( N + 1 \) hypermultiplets.

In the rest of this paper, we shall study the theory pertaining to a single fixed plane which we may take to be \( x_7 = 0 \). Appropriate modifications that apply to the other fixed plane will be indicated when necessary.

### 3.2 The dimensionally reduced bulk Lagrangian

The Lagrangian and the supersymmetry transformation rules of the \( D = 6 \) theory arising from the dimensional reduction of the bulk theory can be obtained in a straightforward way by substituting (3.5–3.9) (with \( \Gamma_7 = 1 \) and \( A_\mu = 0 \)) in (2.37) and (2.38), neglecting all \( \mathbb{Z}_2 \)-odd fields and taking account of the chirality constraints satisfied by the spinors. To keep things relatively simple, we will neglect the terms involving the bulk hypermultiplets.
Although we will discuss some of their consequences in what follows.

Let us start from the Lagrangian. By the reduction rules, it is clear that all kinetic terms retain their canonical form, apart from a different scalar function in front of $G_3 \wedge \ast G_3$.

The only interactions that survive when we ignore the hypermultiplets are $\bar{\chi} \Gamma \psi \partial \phi, \bar{\psi} \Gamma \psi G, \bar{\chi} \Gamma \psi G$ and $\bar{\chi} \Gamma \chi G$. The former two retain the same form as in the original action, while the latter two reduce to

\[ \frac{1}{48 \sqrt{2}} e^\phi \left( -\frac{4}{\sqrt{5}} \bar{\psi} \Gamma \mu \nu \rho \psi \right) + \frac{8}{\sqrt{5}} \bar{\chi} \Gamma \mu \nu \rho \psi \chi \left( \frac{1}{24 \sqrt{2}} \chi^{i} \Gamma \mu \nu \rho \chi G_{\mu \nu} \right) \]

and

\[ \frac{1}{48 \sqrt{2}} e^\phi \left( -\frac{1}{5} \bar{\psi} \Gamma \mu \nu \rho \Gamma \sigma \psi + \frac{16}{5} \bar{\chi} \Gamma \mu \nu \rho \psi \chi \right) \]

\[ \frac{1}{48 \sqrt{2}} e^\phi \left( -\frac{6}{25} + \frac{32}{25} + \frac{24}{25} \right) \chi^{i} \Gamma \mu \nu \rho \chi G_{\mu \nu} \]

\[ \frac{1}{24 \sqrt{2}} e^\phi \chi^{i} \Gamma \mu \nu \rho \chi G_{\mu \nu} \]

Therefore, the dimensionally reduced Lagrangian reads

\[ e^{-1} L_{\text{bulk}} = \frac{1}{2} R - \frac{1}{12} e^{2 \phi} G_{\mu \nu \rho} G^{\mu \nu \rho} - \frac{1}{2} \partial \phi \partial \phi - \frac{1}{2} \bar{\psi} \Gamma \mu \nu \rho \psi \psi + \frac{1}{2} \bar{\chi} \Gamma \mu \nu \rho \chi \chi \]

\[ = \frac{1}{24 \sqrt{2}} e^\phi \left( -\frac{1}{\sqrt{2}} \bar{\psi} \Gamma \mu \nu \rho \psi \right) + \frac{1}{2} \bar{\chi} \Gamma \mu \nu \rho \chi \chi \]

\[ \frac{1}{24 \sqrt{2}} e^\phi \left( -\frac{1}{\sqrt{2}} \bar{\psi} \Gamma \mu \nu \rho \psi \right) + \frac{1}{2} \bar{\chi} \Gamma \mu \nu \rho \chi \chi \]

\[ + \frac{1}{2} \bar{\chi} \Gamma \mu \nu \rho \chi \chi \]

(3.18)

Passing on to the supersymmetry transformation laws, one has again to substitute (3.5–3.9) in the transformation laws of the 7D theory. We easily find

\[ \delta e^a_{\mu} = \frac{1}{2} e^\phi \Gamma^a \psi_{\mu i}, \]

\[ \delta \phi = \frac{1}{2} e^\phi \chi_i, \]

\[ \delta B_{\mu \nu} = e^{-\phi} \left( -\frac{1}{\sqrt{2}} \bar{\psi} \Gamma_{\mu \nu} \psi \right) + \frac{1}{2} \bar{\chi} \Gamma_{\mu \nu} \chi, \]

\[ \delta \psi_{\mu i} = D_{\mu} \epsilon_i - \frac{1}{24 \sqrt{2}} e^\phi \Gamma_{\mu \nu} \Gamma_{\mu \nu} \epsilon_i, \]

\[ \delta \chi_i = \frac{1}{2} \Gamma_{\mu \nu} \partial_{\mu} \epsilon_i - \frac{1}{12 \sqrt{2}} e^\phi \Gamma_{\mu \nu} G^{\mu \nu \rho \epsilon_i}. \]

(3.18)

The Lagrangian (3.17) and the transformation rules (3.18) are exactly the same as those appearing in the fully truncated theory presented in [33].
Although we have not explicitly included the contributions of the bulk hypermultiplets in our discussion, there is a particular effect that has to be discussed. In particular, it turns out that the combination $\sigma^{-1}C$ appearing in the supersymmetry variation of $\psi^M_M$ in (2.38) is $\mathbb{Z}_2$-odd and thus gives rise to delta-function terms on the orbifold fixed points. On the $x^7 = 0, \pi R$ fixed planes, these can be cancelled if we introduce the boundary action

$$S_0 = -\frac{\sqrt{2}}{3} \int d^7 x \, E \sigma^{-1} C \left( \delta(x^7) - \delta(x^7 - \pi R) \right).$$

(3.19)

It is easy to see then that the boundary value of $C = C(\phi^\alpha)$ and $\phi$ specifies $S_0$, which is nothing else that the tension of the branes at $x^7 = 0, \pi R$.

4 Boundary multiplets

As we have seen in the previous section, the bulk fields surviving the orbifold projection arrange themselves into a gravity multiplet, a tensor multiplet and $N + 1$ hypermultiplets of the $D = 6, \mathcal{N} = 1$ supersymmetry algebra. Since the above spectrum is chiral the theory suffers from gravitational anomalies which render it inconsistent at the quantum level. In order to arrive at an anomaly-free theory, we have to follow the Hořava-Witten recipe by adding boundary fields whose contribution to the anomalies will cancel those of the gravitational theory. In the 6D case, the available types of boundary multiplets are tensor multiplets and hypermultiplets such as those appearing in (3.14) plus vector multiplets whose field content is given by

$$\text{Vector multiplet} : (A_\mu, \lambda^-).$$

(4.1)

The $D = 6, \mathcal{N} = (0, 1)$ supergravity theory coupled to vector and tensor multiplets has been constructed in [43] and the inclusion of hypermultiplets has partially been obtained in [44]. The most general up to date supergravity coupled to vectors, tensors and hypermultiplets has been given in [45]. All three types of multiplets give extra contribution to the gravitational anomaly of the theory. Moreover, the theory has now gauge and mixed anomalies coming from the vector multiplets and the hypermultiplets. As we shall see in §5 the inclusion of these multiplets makes it possible to cancel all anomalies of the theory via the Green-Schwarz mechanism.

Considering the fixed plane $x^7 = 0$, we introduce $n_V$ vector multiplets, $n_H$ hypermultiplets and $n_T$ tensor multiplets. We take the Yang-Mills group $\mathcal{G}$ to be a product of simple factors, $\mathcal{G} = \prod_z \mathcal{G}_z$ and we further assume that $\mathcal{G}$ is semisimple (no $U(1)$ factors). The vector multiplets transform in the adjoint of $\mathcal{G}$, so we have

$$n_V = \dim \mathcal{G} = \sum_z \dim \mathcal{G}_z.$$  

(4.2)

The hypermultiplets can also be charged under the gauge group. We will let $n_{z,k}$ be the number of hypermultiplets in $\mathcal{R}_{z,k}$ and $n_{zz',ij}$ be the number of hypermultiplets in the
representation \((R_{z}, i, R'_{z'})\) of the product group \(G_{z} \times G'_{z'}\). Finally, the tensor multiplets are not charged under \(G\).

Having introduced new boundary multiplets in order to achieve anomaly cancellation, we must now determine the appropriate action that describes these fields and their interactions with the bulk fields. For its construction, some well-known facts about \(D = 6, \mathcal{N} = (0, 1)\) supergravity turn out to be useful. First of all, the form of the Lagrangian is not determined by supersymmetry alone but, instead, there are some constant factors which are determined by anomaly cancellation conditions. Second, unlike its counterpart in \(D = 10, \mathcal{N} = 1\) supergravity, the \(D = 6\) Green-Schwarz term \(B_{2} \wedge \text{tr} F^{2}\) is not a higher-derivative correction and hence it must be present in the low-energy action in the first place; as we shall see, this is indeed the case. Third, it is known that no invariant Lagrangian exists for the case of antisymmetric tensor fields subject to self-duality projections, and thus an action can be written down only when \(n_{T} = 0\); for simplicity, in this section only this case will be considered.

In the remainder of this section, we will use the above insights to construct, by the Noether method, the locally supersymmetric action (up to (Fermi)\(^{4}\) terms) and the supersymmetry transformation rules (up to (Fermi)\(^{3}\) terms) required to describe the boundary multiplets and their interactions with the bulk fields.

4.1 Boundary vector multiplets

The construction of the boundary vector multiplet Lagrangian proceeds by starting from the globally supersymmetric theory and coupling it to gravity through the Noether method, so as to restore local supersymmetry.

For the construction, some experience with HW theory is useful. In that construction, it was found that the coupling of boundary vector multiplets to the bulk supergravity leaves out certain uncanceled supersymmetry variations, whose cancellation requires that the field-strength \(G_{4}\) of M-theory acquire (in the downstairs approach) a boundary value proportional to \(\text{tr} F^{2}\), where \(F\) is the Yang-Mills field strength. In the present context, we have the 3–form field strength \(G_{3}\) obtained by dualizing \(F_{4}\). On these grounds, we expect that \(G_{3}\) must acquire a boundary value proportional to \(* \text{tr} F^{2}\). It turns out that, due to some subtleties related to the duality transformation in the presence of a boundary, this will induce the Green-Schwarz term of the theory.

4.1.1 The action

We start by determining the action describing the boundary vector multiplets and their interactions with the bulk theory, along the lines of [18, 19]. Our starting point is the globally supersymmetric Lagrangian

\[
e^{-1} \mathcal{L}^{(0)}_{YM} = v_{z} \text{tr}_{z} \left( -\frac{1}{4} e^{-\phi} F_{\mu \nu} F^{\mu \nu} - e^{\phi} \bar{\chi} \Gamma^{\mu} D_{\mu} \lambda_{i} \right).
\]  \hspace{1cm} (4.3)
Here, the index $i$ labels the simple factors of the gauge group, $\text{tr}_z$ denotes the relevant traces normalized with respect to the fundamental representation, and $v_z$ are some numerical constants that will be determined in the next section. The Lagrangian is invariant under the rigid supersymmetry transformations

$$\delta A_\mu = e^\phi \bar{\psi} \Gamma^\mu \lambda_i, \quad \delta_0 \lambda_i = -\frac{1}{4} e^{-\phi} \Gamma^{\mu\nu} \epsilon_i F_{\mu\nu}. \quad (4.4)$$

Our first step towards obtaining a locally supersymmetric theory is to introduce the usual Noether coupling of the gravitino to the supercurrent of the multiplet. The required term is

$$L_{YM}^{(1)} = e v_z \text{tr}_z \left( -\frac{1}{2} \bar{\psi}_i \Gamma^\mu \Gamma^\nu \lambda_i F_{\nu\rho} \right). \quad (4.5)$$

Next, we must cancel the $\bar{\lambda} \Gamma F \partial \phi \epsilon$ variation of $L_{YM}^{(0)}$. This variation is found to be

$$\Delta_{YM}^{(1)} = e v_z \text{tr}_z \left( \frac{1}{4} \bar{\lambda}_i \Gamma^{\mu\nu} \Gamma^\rho \epsilon_i F_{\mu\rho} \partial_\phi \right) \quad (4.6)$$

and can be cancelled by the $\delta \chi \sim \Gamma \partial \phi \epsilon$ variation of the additional term

$$L_{YM}^{(2)} = e v_z \text{tr}_z \left( -\frac{1}{2} \bar{\lambda}_i \Gamma^{\mu\nu} \chi_i F_{\mu\nu} \right). \quad (4.7)$$

The introduction of these new interactions results in additional uncancelled terms of the form $\bar{\lambda} \Gamma FG_3 \epsilon$ coming from the $\delta \psi$ and $\delta \chi$ variations of $L_{YM}^{(1)}$ and $L_{YM}^{(2)}$ respectively. The first one vanishes by the 6D identity $\Gamma^\mu \Gamma^{\nu\rho\sigma} \Gamma_\mu = 0$, while the second one is given by

$$\Delta_{YM}^{(2)} = e v_z \text{tr}_z \left( \frac{1}{24\sqrt{2}} e^{2\phi} \bar{\lambda}_i \Gamma^{\mu\nu\rho\sigma} \epsilon_i G_{\rho\sigma} F_{\mu\nu} \right). \quad (4.8)$$

This can be cancelled by introducing the additional interaction

$$L_{YM}^{(3)} = e v_z \text{tr}_z \left( \frac{1}{12\sqrt{2}} e^{2\phi} \bar{\lambda}_i \Gamma^{\mu\rho\sigma} \lambda_i G_{\mu\rho\sigma} \right) \quad (4.9)$$

What remains is to cancel the $\bar{\psi} \Gamma F^2 \epsilon$ and $\bar{\chi} \Gamma F^2 \epsilon$ terms coming from the $\delta \lambda$ variations of $L_{YM}^{(1)}$ and $L_{YM}^{(2)}$. These terms are given by

$$\Delta_{YM}^{(3)} = e v_z \text{tr}_z \left( \frac{1}{8} e^{-\phi} \bar{\psi}_i \Gamma^{\mu\nu\rho\sigma} \epsilon_i F_{\nu\rho} F_{\sigma\tau} \right), \quad (4.10)$$

and

$$\Delta_{YM}^{(4)} = e v_z \text{tr}_z \left( \frac{1}{8} e^{-\phi} \bar{\chi}_i \Gamma^{\mu\nu\rho} \epsilon_i F_{\mu\nu} F_{\rho\sigma} \right). \quad (4.11)$$
It turns out that these terms can be cancelled by a mechanism that will be shortly described. Provided that this happens, we can write the locally supersymmetric Lagrangian describing the vector multiplets and their bulk interactions in the form

\[ e^{-1} \mathcal{L}_Y = v_z \text{tr} \left( \frac{1}{4} e^{-\phi} F_{\mu\nu} F^{\mu\nu} - e^{\phi} \bar{\lambda}^i \Gamma^\mu D_\mu \lambda_i - \frac{1}{2} \bar{\psi}^i \Gamma^\mu \Gamma^\nu \lambda_i F_{\mu\nu} - \frac{1}{2} \bar{\lambda} \Gamma^\mu \chi_i F_{\mu\nu} + \frac{1}{12} \sqrt{2} e^2 \phi \bar{\lambda}^i \Gamma^\mu \Gamma^\nu \lambda_i G_{\mu\nu} + \frac{1}{2} \sqrt{2} \bar{\lambda}^i \Gamma^\mu \Gamma^\nu \lambda_i G_{\mu\nu} \right). \] (4.12)

### 4.1.2 The boundary value of $G_3$ and the Green-Schwarz term

What remains is to discuss the mechanism by which the supersymmetry variations (4.10) and (4.11) can be cancelled, which proceeds in analogy to the HW case. There are two ways in doing that, the “downstairs” and “upstairs” approach.

**Downstairs:** In this approach, one considers possible variations of the bulk action which can be written as total derivatives with respect to $x_7$ (see [39, 40] for a clear discussion of this point). For reasons that will soon become clear, we start by using the gamma-matrix duality relation (A.3) to write the uncancelled variations (4.10) and (4.11) as

\[ \Delta^{(3)}_{Y_M} = -\frac{1}{8} \epsilon^{\mu\nu\rho\sigma\tau\upsilon} e^{-\phi} \bar{\psi}^i \Gamma_\nu \epsilon_i \text{tr} (F_{\rho\tau} F^{\mu\upsilon}), \] (4.13)

and

\[ \Delta^{(4)}_{Y_M} = \frac{1}{16} \epsilon^{\mu\nu\rho\sigma\tau\upsilon} e^{-\phi} \chi^i \Gamma_{\mu\nu} \epsilon_i \text{tr} (F_{\rho\tau} F^{\upsilon}). \] (4.14)

Next, we consider the $\bar{\psi} \Gamma^i \psi G_3$ and $\bar{\psi} \Gamma^L G_3$ interactions present in the bulk Lagrangian (2.37). Starting with the former, it is easy to see that it contains the term

\[ -\frac{1}{24 \sqrt{2}} \int d^7 x \sigma^2 \bar{\psi}^i \Gamma_{[L} \Gamma_{MNP} \Gamma_{Q]} \psi^Q_i G_{MNP} \rightarrow -\frac{1}{6 \sqrt{2}} \int d^7 x \sigma^2 \bar{\psi}^i \Gamma^N \psi^P_i G_{MNP}. \] (4.15)

Under $\delta \psi \sim \partial \epsilon$, the variation of this term is given by

\[ -\frac{1}{3 \sqrt{2}} \int d^7 x \epsilon \sigma^2 \bar{\psi}^i \Gamma^N \partial^P \epsilon_i G_{MNP}, \] (4.16)

and its $P = 7$ part contributes a total $x_7$ derivative term which, after integration over $x_7$, results in the boundary variation

\[ -\frac{1}{3 \sqrt{2}} \int d^6 x \sigma^2 \bar{\psi}^i \Gamma^N \partial^7 \epsilon_i G_{\mu\nu\tau} = -\frac{1}{3 \sqrt{2}} \int d^6 x \sigma^2 \bar{\psi}^i \Gamma^N \partial^7 \epsilon_i G_{\mu\nu\tau}. \] (4.17)

Repeating the same procedure for the $\bar{\chi} \Gamma G_3$ interaction, we find that it contains the term

\[ \frac{1}{6 \sqrt{10}} \int d^7 x \sigma^2 \bar{\chi}^i \Gamma^L \Gamma_{MNP} \psi_{Li} G_{MNP} \rightarrow \frac{1}{2 \sqrt{10}} \int d^7 x \sigma^2 \bar{\chi}^i \Gamma^L \Gamma_{MNP} \psi^P_i G_{MNP}; \] (4.18)
which results in the boundary variation

\[ \frac{1}{2\sqrt{10}} \int d^7x \partial^7 \left( E\sigma^2 \chi^{(7)i} \Gamma^{\mu\nu} \epsilon_i G_{\mu\nu7} \right) = \frac{1}{2\sqrt{10}} \int d^6x E\sigma^2 \chi^{(7)i} \Gamma^{\mu\nu} \epsilon_i G_{\mu\nu7}. \] (4.19)

The above are expressed in terms of the 7D fields \( E, \sigma, \psi^{(7)}_\mu, \chi^{(7)}_i \). To pass over to the basis of 6D fields we use the Kaluza-Klein ansatz (3.5–3.9). Then, it is easy to see that (4.17) gives rise to the 6D \( \bar{\psi}^\mu i \Gamma^\nu \epsilon G_{\mu\nu7} \) variation

\[ \Delta^{(1)}_B = -\frac{1}{3\sqrt{2}} ee^{-1/2} \left( E\sigma^2 \chi^{(7)i} \Gamma^{\mu\nu} \epsilon_i G_{\mu\nu7} \right). \] (4.20)

while the combination of (4.17) and (4.19) results in a 6D \( \bar{\chi}^\mu i \Gamma^\nu \epsilon G_{\mu\nu7} \) variation, given by

\[ \Delta^{(2)}_B = \left( -\frac{1}{3\sqrt{2}} \cdot \frac{1}{10} + \frac{1}{2\sqrt{10}} \cdot \frac{2}{\sqrt{5}} \right) ee^{-1/2} \bar{\chi}^\mu i \Gamma^{\mu\nu} \epsilon_i G_{\mu\nu7} = \frac{1}{6\sqrt{2}} ee^{-1/2} \bar{\chi}^\mu i \Gamma^{\mu\nu} \epsilon_i G_{\mu\nu7}. \] (4.21)

Comparing (4.20) and (4.21) with the uncancelled variations (4.13) and (4.14) respectively, we see that we can cancel both of them by requiring that \( G_{\mu\nu7} \) attain the boundary value

\[ G_{\mu\nu7} \big|_{\partial M} = -\frac{3}{4\sqrt{2}} ee^{-1/2} \bar{\chi}^\mu i \Gamma^{\mu\nu} \epsilon_i G_{\mu\nu7}. \] (4.22)

Moreover, we are now in a position to make sense out of our remnant from the duality transformation of Section 2.3, namely the surface term

\[ S_{I,bdy} = -\frac{1}{48} \int d^7x \epsilon^{MNPQRT} \partial_T (S_MNPQ B_{RS}). \] (4.23)

Its integration over \( x_7 \) gives rise to the nonvanishing boundary term

\[ S_{I,bdy} = -\frac{1}{48} \int d^6x \epsilon^{\mu\nu\rho\sigma\tau} S_{\mu\nu\rho\sigma} B_{\tau\nu}. \] (4.24)

for the boundary at \( x_7 = 0 \). This implies that, for this boundary, the algebraic equation of motion (2.36) of the spacetime components \( S_{\mu\nu\rho\sigma} \) of the auxiliary field \( S_4 \) is now modified to

\[ S_{\mu\nu\rho\sigma} = \frac{1}{6} \sigma^4 E^{-1} \epsilon_{\mu\nu\rho\sigma\tau\upsilon} \left[ G_{\tau\upsilon7} - 3 B_{\tau\upsilon} \delta(x_7) \right] = \frac{3}{\sqrt{2}} \sigma^2 J^{\mu\nu\rho\sigma}. \] (4.25)

To incorporate this modification in our theory, we must first replace \( G_{\mu\nu7} \) by

\[ \tilde{G}_{\mu\nu7} = G_{\mu\nu7} - 3 B_{\mu\nu} \delta(x_7) \] (4.26)

everywhere in the 7D Lagrangian (2.37) and transformation rules (2.38) and then substitute the solution (4.25) in the surface term (4.24).
Let us examine the new boundary terms that arise in this way. First, it is easy to see that the $\mathbb{Z}_2$ chirality projections forbid the appearance of possible boundary couplings of fermions to $B_{\mu\nu}$. Second, we notice that there also occurs a coupling

$$-\frac{1}{4}\delta(0) \int d^6x e^2 \phi B_{\mu\nu}B^{\mu\nu},$$

which looks like a singular mass term for $B_{\mu\nu}$. The appearance of such a singularity is somewhat surprising since the theory we started with was perfectly regular; however, it is an effect that is known to occur in dual formulations of supergravities on orbifolds. After suitable regularization (see [42]), it turns out that this term does not affect the mass spectrum of the theory and that $B_2$ stays massless at tree level. Third and more important, there appears the surface term

$$S_{GS} = \frac{1}{6} \int d^6x e^2 \phi B_{\mu\nu}G^{\mu\nu7},$$

which, on account of (4.22), results in the interaction

$$S_{GS} = -\frac{1}{8\sqrt{2}} \int d^6x \epsilon_{\mu\nu\rho\sigma\tau\upsilon} B_{\mu\nu}v_z \text{tr}_z (F_{\rho\sigma}F_{\tau\upsilon}) \delta(x_7).$$

This is the Green-Schwarz term of our theory. Supplemented by a suitable gravitational contribution and given an appropriate gauge/Lorentz transformation law for $B_2$, this term can completely cancel all anomalies of the theory.

**Upstairs:** To complete the discussion and to provide a consistency check for our method, let us also briefly describe how things work out in the “upstairs” approach. In this approach, we have to go back to the 3-form theory considered in [18, 19], where it was found that $F_4$ should satisfy a modified Bianchi identity whose generalization to our case reads

$$\partial_7 F_{\mu\nu\rho\sigma} = -3\sqrt{2} v_z \text{tr}_z (F_{\mu\nu}F_{\rho\sigma}) \delta(x_7).$$

According to (2.29), this implies that the Lagrange-multiplier action entering the duality transformation should contain the additional term

$$S'_C = -\frac{1}{48} \int d^7x \epsilon_{\mu\nu\rho\sigma\tau\upsilon\eta} B_{\mu\nu} \left[3\sqrt{2} v_z \text{tr}_z (F_{\rho\sigma}F_{\tau\upsilon}) \delta(x_7)\right],$$

besides the usual $B_2 \wedge dS_4$ term. In the “upstairs” approach, the $B_2 \wedge dS_4$ term can be integrated by parts without the emergence of surface terms so that the duality transformation works in the usual way. However, we are still left with the extra term (4.31) which, after the trivial $x_7$ integration, exactly reproduces the Green-Schwarz interaction (4.29).
4.2 Hypermultiplets

To determine the action describing the boundary hypermultiplets and their interactions with the bulk fields, we proceed as follows. Initially, we set up the basic formalism required to describe $D = 6$ hypermultiplets and their gauging under Yang-Mills groups. Next, we consider the simple case where the hypermultiplets are inert under the gauge group and we construct the appropriate locally supersymmetric action. Finally, we consider gauging these multiplets and we determine all extra terms required to maintain local supersymmetry.

4.2.1 General formalism

The construction of the gauged theory describing the hypermultiplets was discussed in detail in [46]. To give a brief review, we write our $n_H$ boundary hypermultiplets as $\left(\varphi^\alpha, \zeta^a + \right)$ where $\alpha = 1, \ldots, 4n_H$ and $a = 1, \ldots, 2n_H$. The $4n_H$ hyperscalars $\varphi^\alpha$ parameterize a quaternionic manifold, i.e. a manifold whose holonomy group is a subgroup of $Sp(n_H) \times Sp(1)$. We pick this manifold to be the coset space $Sp(n_H,1)/Sp(n_H) \times Sp(1)$ and we denote its metric by $g_{\alpha\beta}(\varphi)$. A representative of this space can be parameterized by a matrix $L$ whose Maurer-Cartan form decomposes as

$$L^{-1} \partial_{\alpha}L = \omega_{\alpha}^{ab}T_{ab} + \omega_{\alpha}^{ij}T_{ij} + V_{\alpha}^{ai}T_{ai}, \quad (4.32)$$

where $T_{ab}$ and $T_{ij}$ are the $Sp(n_H)$ and $Sp(1)$ generators, $\omega_{\alpha}^{ab}$ and $\omega_{\alpha}^{ij}$ are the associated connections and $T_{ai}$ and $V_{\alpha}^{ai}$ are the coset generators and vielbeins. The $Sp(n_H)$ and $Sp(1)$ curvatures are denoted by $\Omega_{\alpha\beta}^{ab}$ and $\Omega_{\alpha\beta}^{ij}$ respectively and the latter is expressed in terms of the vielbeins as

$$\Omega_{\alpha\beta}^{ij} = 2 \left(V_{\alpha}^{ai}V_{\beta}^{aj} + V_{\alpha}^{aj}V_{\beta}^{ai}\right). \quad (4.33)$$

To gauge the hypermultiplets, we must make a choice for the Yang-Mills group $G = \prod_z G_z$ and the representations in which the hypermultiplets transform. One can also take some of the $G_z$'s to be subgroups of the $Sp(n_H) \times Sp(1)$ holonomy group whose $Sp(1)$ factor is identified with the R-symmetry group. In this section, we will take $G_1 = Sp(n_H)$ and $G_2 = Sp(1)$, without further specification of the remaining factors. The corresponding gauge fields are denoted by $A_{\mu}^{ab}$ and $A_{\mu}^{ij}$ and their field strengths are defined in the usual way. Under a $Sp(n_H) \times Sp(1)$ gauge transformation, the gauge fields and the hyperscalars transform according to

$$\delta A_{\mu}^{ab} = D_{\mu} \Lambda^{ab}, \quad \delta A_{\mu}^{ij} = D_{\mu} \Lambda^{ij}, \quad (4.34)$$

and

$$\delta \varphi^{\alpha} = \Lambda^{ab} \xi_{ab}^{\alpha} + \Lambda^{ij} \xi_{ij}^{\alpha}, \quad (4.35)$$

where $\xi_{ab}^{\alpha}$ and $\xi_{ij}^{\alpha}$ are $Sp(n_H)$ and $Sp(1)$ Killing vectors, given by

$$\xi_{ab}^{\alpha} = T_{ab} \varphi^{\alpha}, \quad \xi_{ij}^{\alpha} = T_{ij} \varphi^{\alpha}. \quad (4.36)$$
The covariant derivative acting on the hyperscalars is then
\[ \mathcal{D}_\mu \varphi^\alpha = \partial_\mu \varphi^\alpha - A_{\mu}^{ab} \xi_{ab}^\alpha - A_{\mu}^{ij} \xi_{ij}^\alpha. \]  
\[(4.37)\]

As for the spinor covariant derivatives, they are defined by adding the appropriate composite connections of the form \( \mathcal{D}_\mu \varphi^\alpha \omega_\alpha \) plus the gauge field terms. In particular, the covariant derivative of the spinor \( \epsilon^i \) parameterizing the SUSY transformations is modified to
\[ \mathcal{D}_\mu \epsilon^i = \mathcal{D}_\mu \epsilon^i + (\mathcal{D}_\mu \varphi^\alpha) \omega_\alpha^{ij} \epsilon_j + A_{\mu}^{ij} \epsilon_j. \]  
\[(4.38)\]

As a result, the commutator of two covariant derivatives on \( \epsilon^i \) is given by
\[ [\mathcal{D}_\mu, \mathcal{D}_\nu] \epsilon^i = \frac{1}{4} R_{\mu\nu\rho\sigma} \Gamma^{\rho\sigma} \epsilon^i + \mathcal{D}_\mu \varphi^\alpha \mathcal{D}_\nu \varphi^\beta \Omega_{\alpha\beta}^{ij} \epsilon_j - \text{tr}_z (F_{\mu\nu} \mathcal{C}^{ij} \epsilon_j), \]
\[(4.39)\]
where \( \mathcal{C}^{ij} \) denotes the following triplet of \( Sp(n_H) \times Sp(1) \) matrices
\[ \mathcal{C}^{ij} = \omega^{ij}_{\alpha} \xi_{cd}^\alpha T^{cd} + \omega^{ij}_{\alpha} \xi_{kl}^\alpha T^{kl} - T^{ij}. \]
\[(4.40)\]
These matrices satisfy the identity \[46\]
\[ \mathcal{D}_\mu \mathcal{C}^{ij} = (\mathcal{D}_\mu \varphi^\alpha) \Omega_{\alpha\beta}^{ij} \xi_{\beta}^\beta, \]
\[(4.41)\]
where
\[ \xi^\alpha = \xi^{\alpha ab} T_{ab} + \xi^{\alpha ij} T_{ij}. \]
\[(4.42)\]

The above construction is purely six-dimensional. To implement it in the context of our bulk-brane theory, we must find a 7D explanation for the quaternionic structure of the scalar manifold, or else our approach will not be consistent with local supersymmetry in 6D. We recall that, in the 6D case, the quaternionic structure is a result of the fact that the gravitino is charged under the \( Sp(1) 6D \) R-symmetry so that the supersymmetry variation of its kinetic term gives rise to a term involving the \( Sp(1) \) curvature. However, in our 7D theory, the gravitino is a bulk field which does not couple to the boundary \( Sp(1) \) connection and the quaternionic structure is not a priori imposed. However, we will soon see that the required structure does indeed arise by a boundary condition on the bulk gauge field.

### 4.2.2 The action for neutral hypermultiplets

To construct the action, let us begin from the ungauged theory. Our starting point is the globally supersymmetric Lagrangian
\[ e^{-1} L^{(0)}_H = -g_{\alpha\beta} (\varphi) \partial_\mu \varphi^\alpha \partial^{\mu} \varphi^\beta - \bar{\zeta}^a \Gamma^\mu D_\mu \zeta_a, \]
\[(4.43)\]
which is invariant under the transformations
\[ \delta \varphi^\alpha = V_{ai}^{\alpha} \bar{\zeta}^a \epsilon^i, \quad \delta \zeta^a = V_{ai}^{\alpha} \Gamma^\mu \partial_\mu \varphi^\alpha \epsilon^i. \]
\[(4.44)\]
As before, we introduce again the appropriate interaction of the gravitino with the supercurrent,
\[
\mathcal{L}_H^{(1)} = 2e\bar{\psi}_\mu^i \Gamma^\nu \Gamma^\mu \zeta a V_{aai} \partial_\nu \varphi^a ,
\]
which, under \( \delta \psi \sim \Gamma G \epsilon \), yields the uncancelled term
\[
\Delta_H^{(1)} = -\frac{1}{6\sqrt{2}} ee^{\phi} \bar{\psi}_\mu^i \Gamma^\nu \Gamma^\mu \zeta^a V_{aai} \partial_\nu \varphi^a G_{\nu \rho \sigma} .
\]
This term cancels if we introduce the interaction
\[
\mathcal{L}_H^{(2)} = -\frac{1}{12\sqrt{2}} ee^{\phi} \tilde{\zeta} a \Gamma^{\mu \nu \rho} \zeta^a G_{\mu \nu \rho} .
\]
Finally, there is also a term arising from the \( \delta \zeta \) variation in \( \mathcal{L}_H^{(1)} \), which is given by
\[
\Delta_H^{(2)} = -\frac{1}{2} e\bar{\psi}_\mu^i \Gamma^{\mu \nu \rho} \partial_\nu \varphi^a \partial_\rho \varphi^\beta \Omega_{\alpha \beta}^{ij} \epsilon_j .
\]
To cancel this term, we note, as in [18, 19], that the bulk theory contains a \( \bar{\psi} \Gamma \psi F_2 \) interaction. Under \( \delta \psi \sim \partial \epsilon \), this term has the variation
\[
-\frac{i}{2\sqrt{2}} F_{\sigma \mu \nu \rho} \bar{\psi}_{LMNP} \partial_\nu \varphi^a \partial_\rho \varphi^a G_{\sigma \mu \nu \rho} .
\]
where \( F_{MN \ i} = F_{MN}^{T} L_{i} \). Following the same reasoning as in Section 4.1, we take the total-derivative contribution of the \( P = 7 \) part, integrate over \( x_7 \) and express the result in the 6D basis. The result is
\[
\Delta_H^{(3)} = -\frac{i}{2\sqrt{2}} e^{\phi} \bar{\psi}_{\mu \nu \rho} F_{\mu \rho} \partial_\nu \varphi^a \partial_\rho \varphi^a G_{\mu \nu \rho} .
\]
This can exactly cancel \( \Delta_H^{(2)} \) provided that \( F_{\mu \rho} \partial_\nu \varphi^a \partial_\rho \varphi^a \Omega_{\alpha \beta}^{ij} \epsilon_j \) is subject to the following boundary condition, in the downstairs approach
\[
F_{\mu \rho} \partial_\nu \varphi^a \partial_\rho \varphi^a \Omega_{\alpha \beta}^{ij} \epsilon_j = 0 .
\] or the corresponding Bianchi identity in the upstairs approach. As explained in detail in [19], this does indeed induce the required quaternionic structure on the scalar manifold as a result of the Bianchi identity \( D_{\lambda} F_{\mu \rho}^{ij} = 0 \).

Collecting all terms, we arrive at the locally supersymmetric action for neutral hypermultiplets,
\[
e^{-1} \mathcal{L}_H = -g_{\alpha \beta} (\varphi) \partial_\mu \varphi^a \partial^\mu \varphi^a - \tilde{\zeta} a \Gamma^\mu D_\mu \zeta a + 2\bar{\psi}_\mu^i \Gamma^\nu \Gamma^\mu \partial_\nu \varphi^a V_{aai} \zeta^a -\frac{1}{12\sqrt{2}} ee^{\phi} \tilde{\zeta} a \Gamma^{\mu \nu \rho} \zeta a G_{\mu \nu \rho} .
\]
4.2.3 Gauging

Next, we shall extend our results for the case where the hypermultiplets are charged under the gauge group. The first step in the construction of the theory is to replace all derivatives by covariant ones with respect to the gauge group, that is, replace the hypermultiplet Lagrangian and transformation rules by

\[
e^{-1} \mathcal{L}_H = -g_{\alpha\beta}(\phi) D_\mu \varphi^\alpha D^\mu \varphi^\beta - \bar{\zeta}^a \Gamma^\mu D_\mu \zeta_a + 2 \bar{\psi}_i^j \Gamma^\mu D_\mu \varphi^a V_{ai} \zeta^a
- \frac{1}{12 \sqrt{2}} e^\phi \bar{\zeta}^a \Gamma^\mu \Gamma^\nu \zeta_a G_{\mu\nu},
\]

(4.53)

and

\[
\delta \varphi^\alpha = V_{ai}^\alpha \bar{\zeta}^a \epsilon^i, \quad \delta \zeta_a = V_a^i \Gamma^\mu D_\mu \varphi^a \epsilon^i.
\]

(4.54)

After this modification, there arise additional uncancelled terms. First of all, the variation \( \Delta^{(2)}_H \) is replaced by its covariant version. To cancel it, we could think of modifying the boundary condition (4.51) to involve covariant derivatives as well. However, upon applying this naive modification, we find that \( D_{\mu} F_{ij}^\alpha \) contains a term proportional to \( \text{tr}_z (\Omega_{ij}^\alpha \delta \varphi^\alpha F_{\mu}^\alpha \zeta^\beta) \) and thus \( F_{ij}^\alpha \) fails to satisfy the 6D Bianchi identity required for the quaternionic structure. The correct modification to (4.51) is instead given by

\[
F_{ij}^\alpha \big|_{\partial M} = i \sqrt{2} e^{\phi/2} \left[ D_\mu \varphi^\alpha D_\nu \varphi^\beta \Omega_{ij}^{\alpha\beta} - \text{tr}_z (F_{\mu}^\alpha C_{ij}^\beta) \right].
\]

(4.55)

and, by virtue of (4.41), the extra term on the RHS restores the Bianchi identity maintaining the quaternionic structure; as we shall see below, this extra term is actually necessary for local supersymmetry. After imposing (4.55), (4.50) is replaced by the additional variations

\[
\Delta_G^{(1)} = \frac{1}{2} e \bar{\psi}_i^j \Gamma^\mu \Gamma^\nu D_\mu \varphi^\alpha D_\nu \varphi^\beta \Omega_{ij}^{\alpha\beta} \epsilon^i,
\]

(4.56)

and

\[
\Delta_G^{(2)} = \frac{1}{2} e \text{tr}_z (C_{ij} \bar{\psi}_i^j \Gamma^\mu \Gamma^\nu \epsilon^i F_{\nu\rho}),
\]

(4.57)

the first of which exactly cancels the covariant version of \( \Delta^{(2)}_H \). Meanwhile, the hyperino kinetic term also gives rise to the extra variation

\[
\Delta_G^{(3)} = -e \text{tr}_z \left( \bar{\zeta}^a \Gamma^\mu F_{\mu} \epsilon_i V_{ai} \zeta^a \right).
\]

(4.58)

To cancel \( \Delta_G^{(2)} \), we may either modify the gaugino SUSY transformation law by adding the extra term

\[
\delta_1 \lambda_i = a \nu^{-1}_z C_{ij} \epsilon^j,
\]

(4.59)

(summation over \( z \) implicit) or introduce the additional interaction

\[
\mathcal{L}_G^{(1)} = be^\phi \text{tr}_z \left( C_{ij} \bar{\psi}_i^j \Gamma^\mu \lambda^j \right).
\]

(4.60)
Here, $a$ and $b$ are two coefficients, which can be determined by considering the $\bar{\psi} \Gamma F \epsilon$ terms. The variations of this type arising from the $\delta_1 \lambda$ variation of $L_{YM}$ is

$$\Delta_G^{(4)} = \frac{a}{2} e \text{tr}_z \left( C_{ij} \bar{\psi}_i \Gamma_{\mu \rho} e^j F_{\mu \rho} \right) - a e \text{tr}_z \left( C_{ij} \bar{\psi}_i \Gamma^\nu e^j F_{\mu \nu} \right),$$

(4.61)

while the $\delta_0 \lambda$ variation of $L_G^{(1)}$ gives

$$\Delta_G^{(5)} = -\frac{b}{4} e \text{tr}_z \left( C_{ij} \bar{\psi}_i \Gamma_{\mu \rho} e^j F_{\mu \rho} \right) - \frac{b}{2} e \text{tr}_z \left( C_{ij} \bar{\psi}_i \Gamma^\nu e^j F_{\mu \nu} \right).$$

(4.62)

We observe that the requirement for cancellation of the $\bar{\psi}_i \Gamma_{\mu \rho} e^j F_{\mu \rho}$ and $\bar{\psi}_i \Gamma_{\mu \nu} e^j F_{\mu \nu}$ terms fixes the coefficients $a$ and $b$ to

$$a = -\frac{1}{2}, \quad b = 1.$$

(4.63)

Next, let us consider the $\delta_1 \lambda$ variation of the gaugino kinetic term in $L_{YM}$. Performing an integration by parts and using (4.41), we obtain

$$\Delta_G^{(6)} = e e^\phi \text{tr}_z \left( C_{ij} \bar{\lambda}_i \Gamma_{\mu} e^j \right) + e e^\phi \text{tr}_z \left( \lambda_i \Gamma_{\alpha \beta} \partial_{\mu} \phi^{\alpha} \xi^{\beta} \xi^j \right)$$

$$- \frac{1}{2} e e^\phi \text{tr}_z \left( C_{ij} \bar{\lambda}_i \Gamma_{\mu} e^j \partial_{\mu} \phi \right).$$

(4.64)

On the other hand, using $\Gamma_{\mu} \Gamma_{\mu} \Gamma_{\mu} = 0$, we find that the $\delta \psi$ variation of $L_G^{(1)}$ is given by

$$\Delta_G^{(7)} = -e e^\phi \text{tr}_z \left( C_{ij} \bar{\lambda}_i \Gamma_{\mu} e^j \right),$$

(4.65)

and it exactly cancels the first term of $\Delta_G^{(6)}$. The second term can be cancelled by the $\delta \zeta$ variation of the new term

$$L_G^{(2)} = -4 e e^\phi \text{tr}_z \left( \bar{\lambda}_i \zeta_{\alpha} V_{\alpha}^{ai} \xi^j \right).$$

(4.66)

Taking the $\delta_0 \lambda$ variation of this term, we see that $\Delta_G^{(3)}$ cancels as well. We thus confirm the fact that the boundary condition (4.55) that induces the variation $\Delta_G^{(6)}$ and necessitates the addition of $L_G^{(2)}$ is necessary for local supersymmetry; this serves to emphasize that local 6D supersymmetry requires the scalar manifold to be quaternionic. Two other uncancelled terms are the $\delta_1 \lambda$ variations of the $\bar{\lambda} \Gamma \chi F_2$ and $\bar{\lambda} \Gamma \lambda G_3$ terms of $L_{YM}$, given by

$$\Delta_G^{(8)} = \frac{1}{4} e \text{tr}_z \left( C_{ij} \bar{\lambda}_i \Gamma_{\mu \nu} e^j F_{\mu \nu} \right),$$

(4.67)

and

$$\Delta_G^{(9)} = -\frac{1}{12 \sqrt{2}} e e^{2 \phi} \text{tr}_z \left( C_{ij} \bar{\lambda}_i \Gamma_{\mu \rho} e^j G_{\mu \rho} \right),$$

(4.68)

27
respectively. $\Delta_G^{(8)}$ is cancelled by the $\delta_0 \lambda$ variation of yet another new term

$$\mathcal{L}_G^{(3)} = e e^\phi \text{tr}_z \left(C_{ij} \bar{\chi}^i \lambda^j\right),$$

(4.69)

whose $\delta \chi$ variation is given by

$$\Delta_G^{(9)} = -\frac{1}{2} e e^\phi \text{tr}_z \left(C_{ij} \bar{\lambda}^i \Gamma^\mu e^\mu \partial_\mu \phi \right) + \frac{1}{12 \sqrt{2}} e e^{2\phi} \text{tr}_z \left(C_{ij} \bar{\lambda}^i \Gamma^{\mu \nu \rho} \epsilon_{\mu \nu \rho}\right),$$

(4.70)

so that its first part cancels the third term of $\Delta_G^{(6)}$ and its second part cancels $\Delta_G^{(9)}$. What remains to be cancelled are the $\delta_1 \lambda$ variations of $\mathcal{L}_G^{(1)}$ and $\mathcal{L}_G^{(3)}$, given by

$$\Delta_G^{(10)} = -\frac{1}{2} e e^\phi \text{tr}_z \left(C_{ij} C^{ij} \bar{\psi}^k \Gamma^\mu \epsilon_k\right),$$

(4.71)

and

$$\Delta_G^{(11)} = -\frac{1}{2} e e^\phi \text{tr}_z \left(C_{ij} C^{ij} \bar{\chi}^k \epsilon_k\right),$$

(4.72)

respectively. To cancel them, we introduce the term

$$\mathcal{L}_G^{(4)} = -\frac{1}{2} e e^\phi \text{tr}_z \left(C_{ij} C^{ij}\right),$$

(4.73)

whose SUSY variation is given by

$$\Delta_G^{(12)} = -\frac{1}{4} e e^\phi \text{tr}_z \left(C_{ij} C^{ij} \bar{\psi}^k \Gamma^\mu \epsilon_k\right) - \frac{1}{4} e e^\phi \text{tr}_z \left(C_{ij} C^{ij} \bar{\chi}^k \epsilon_k\right),$$

(4.74)

and the desired cancellation does indeed occur, due to the spinor identity

$$\bar{\psi}^i \chi_j = -\frac{1}{2} \delta_j^i \psi^k \chi_k + \frac{1}{2} (\sigma I)^i_j [\bar{\psi} \chi] I; \quad [\bar{\psi} \chi] I \equiv (\sigma I)^i_j \bar{\psi}^j \chi_i.$$ 

(4.75)

To summarize, the terms that should be added to $\mathcal{L}_{bulk} + \mathcal{L}_0 + \mathcal{L}_{YM} + \mathcal{L}_{GS} + \mathcal{L}_H$ in order to restore local supersymmetry are the following

$$e^{-1} \mathcal{L}_G = e^\phi \text{tr}_z \left(C_{ij} \bar{\psi}^i \Gamma^\mu \lambda^j + C_{ij} \bar{\chi}^i \lambda^j - 4 \bar{\chi}_i \zeta a V_a \lambda^i \xi^a - 1 \sqrt{2} e^\phi \text{tr}_z \left(C_{ij} C^{ij}\right)\right).$$

(4.76)

## 5 Anomaly cancellation

In this section, we shall describe the mechanism by which the anomalies of the bulk theory cancel after introducing additional multiplets living on the boundary. We begin by analyzing the gravitational anomalies of the bulk theory plus the gravitational, gauge and mixed anomalies arising due to the extra boundary multiplets. Next, we present the conditions necessary for the cancellation of anomalies, which result in stringent constraints on the boundary matter content. Finally, we present the Green-Schwarz mechanism employed for local anomaly cancellation and we briefly comment on the issues of global anomaly cancellation and non-perturbative anomalies.
5.1 Anomaly analysis

Here we will analyze the anomaly structure of the bulk-boundary theory and we will derive the anomaly cancellation conditions that must be satisfied by any consistent model. For the sake of simplicity, we restrict to the case where the gauge group is contained in $Sp(n_H)$, i.e. does not include a subgroup of the $Sp(1)$ R-symmetry group of the 6D SUSY algebra.

As remarked earlier on, the bulk theory dimensionally reduced on $S^1/Z_2$ has gravitational anomalies as an obvious consequence of its chiral spectrum. By standard arguments, the anomalies should be equally distributed in the two fixed planes. To determine their form, we first observe that the contributions coming from the self-dual and anti-self-dual parts of $B_2$ cancel each other. Therefore, the anomaly on each fixed plane $i$ is half of that corresponding to a negative-chirality gravitino and $N+2$ positive-chirality spinors. Including another factor of $\frac{1}{2}$ due to the symplectic Majorana-Weyl property of the fermions, we find

$$I_{\text{bulk}}^8(R) = \frac{1}{4} \left[ -I_8^{3/2}(R) + (N + 2)I_{8}^{1/2}(R) \right].$$ (5.1)

or, using the explicit expressions for the anomaly polynomials [13] summarized in Appendix B,

$$I_{\text{bulk}}^8(R) = \frac{1}{960} \left[ (243 - N) \text{tr} R^4 - \frac{5}{4}(45 + N)(\text{tr} R^2)^2 \right].$$ (5.2)

Next, let us consider the gravitational anomaly of the boundary multiplets on a given fixed plane. The inclusion of the boundary multiplets, namely the $n_T$ tensor multiplets, the $n_H$ hypermultiplets and the $n_V$ vector multiplets, leads to the following contribution to the gravitational anomaly

$$I_{\text{bdy}}^8(R) = \frac{1}{2} \left[ n_TI_{8}^A(R) + (n_T + n_H - n_V)I_{8}^{1/2}(R) \right],$$ (5.3)

which has the explicit form

$$I_{\text{bdy}}^8(R) = \frac{1}{960} \left[ (2n_V - 2n_H - 58n_T) \text{tr} R^4 + \frac{5}{4}(2n_V - 2n_H + 14n_T)(\text{tr} R^2)^2 \right].$$ (5.4)

Putting everything together, we find that the total gravitational anomaly is given by

$$I_{\text{total}}^8(R) = \frac{1}{960} \left[ \frac{1}{2} \left( 2n_V - 2n_H - 58n_T + 243 - N \right) \text{tr} R^4 \right. 
+ \left. \frac{5}{4}(2n_V - 2n_H + 14n_T - 45 - N)(\text{tr} R^2)^2 \right].$$ (5.5)

Since $SO(5,1)$ has an independent fourth-order Casimir, the $\text{tr} R^4$ term in the above anomaly is irreducible and its coefficient is required to vanish. Hence, we must require that

$$2n_H + 58n_T - 2n_V = 243 - N,$$ (5.6)
in which case the total gravitational anomaly is given by the expression
\[ I_8(R) = -\frac{3}{8} \left( 1 - \frac{n_T}{4} \right) (\text{tr} \, R^2)^2 \equiv -\frac{3}{8} k (\text{tr} \, R^2)^2. \quad (5.7) \]

After the inclusion of the boundary multiplets, the theory also has gauge and mixed anomalies arising from the couplings of spinors to the gauge field. The relevant contributions come from the negative-chirality gauginos of the vector multiplet and from the positive-chirality hyperinos. To study these anomalies, we will denote the trace in a generic representation \( \mathcal{R}_{z,k} \) of \( G_z \) by \( \text{tr}_{z,k} \) while, as usual, we will reserve the notation \( \text{Tr}_z \) for the trace in the adjoint. Also, following Schwarz [50], we will define
\[ X^{(n)}_z = \text{Tr}_z F^n - \sum_k n_{z,k} \text{tr}_{z,k} F^n, \quad Y_{zz'} = \sum_{i,j} n_{zz',ij} \text{tr}_{z,i} F^2 \text{tr}_{z',j} F^2. \quad (5.8) \]

Starting from gauge anomalies, we find that the anomaly corresponding to fermions transforming in representations of a single \( G_z \) factor is given by
\[ I_{8,z}(F) = \frac{1}{2} \left[ -I_{1/2,z}^{1/2}(F) + \sum_k n_{z,k} I_{1/2,z,k}(F) \right] = \frac{1}{2} \left( \text{tr} \, R^2 - \sum_k n_{z,k} \text{tr}_{z,k} F^4 \right) = \frac{1}{2} X^{(4)}_z, \quad (5.9) \]
while the anomaly corresponding to fermions transforming in representations of \( G_z \times G_{z'} \) is
\[ I_{8,zz'}(F) = -3 \sum_{i,j} n_{zz',ij} \text{tr}_{z,i} F^2 \text{tr}_{z',j} F^2 = -3 Y_{zz'}. \quad (5.10) \]

Summing over the various gauge group factors, we find the gauge anomaly
\[ I_8(F) = \frac{1}{2} \sum_z X^{(4)}_z - \frac{3}{2} \sum_{z \neq z'} Y_{zz'}. \quad (5.11) \]
where the extra \( \frac{1}{2} \) in front of \( Y_{zz'} \) takes care of double counting. Similarly, for the mixed anomaly, the contribution from \( G_z \) is given by
\[ I_{8,z}(F, R) = -\frac{1}{8} \text{tr} \, R^2 \left( \text{Tr}_z F^2 - \sum_k n_{z,k} \text{tr}_{z,k} F^2 \right) = -\frac{1}{8} \text{tr} \, R^2 X^{(2)}_z. \quad (5.12) \]
and the sum over all gauge group factors reads
\[ I_8(F,R) = -\frac{1}{8} \text{tr} \, R^2 \sum_z X^{(2)}_z. \quad (5.13) \]
Collecting all contributions, we finally arrive at the total anomaly

$$I_8 = \frac{1}{2} \left[ \sum_z X^{(4)}_z - 3 \sum_{z \neq z'} Y_{zz'} - \frac{1}{4} \text{tr} R^2 X^{(2)}_z - \frac{3k}{4} (\text{tr} R^2)^2 \right]. \tag{5.14}$$

In order for this anomaly to cancel via the Green-Schwarz mechanism, the polynomial (5.14) must factorize. To determine the conditions under which factorization can occur, it is convenient to express the $F$–dependent terms in terms of traces with respect to the fundamental representation of each $G_z$. We write

$$X^{(4)}_z = \alpha_z \text{tr} z F^4 + \gamma_z (\text{tr} z F^2)^2$$
$$X^{(2)}_z = \beta_z \text{tr} z F^2$$
$$Y_{zz'} = \delta_{zz'} \text{tr} z F^2 \text{tr} z' F^2 \tag{5.15}$$

where $\alpha_z$, $\beta_z$, $\gamma_z$ and $\delta_{zz'}$ are some coefficients which depend on the various groups and representations. Substituting into (5.14), we write our anomaly polynomial as

$$I_8 = \frac{1}{2} \left[ \sum_z \alpha_z \text{tr} z F^4 + \sum_z \gamma_z (\text{tr} z F^2)^2 - 3 \sum_{z \neq z'} \delta_{zz'} \text{tr} z F^2 \text{tr} z' F^2$$
$$- \frac{1}{4} \text{tr} R^2 \sum_z \beta_z \text{tr} z F^2 - \frac{3}{4} (\text{tr} R^2)^2 \right]. \tag{5.16}$$

Since each $\text{tr} z F^4$ term carries a coefficient $\alpha_z$ (generally dependent on the $n_{z,k}$'s), factorization can occur either (i) when every $\text{tr} z F^4$ term is reducible, so that $\alpha_z$ is manifestly zero or (ii) when we set $\alpha_z = 0$ by our choice of boundary multiplets. The first condition is strictly group-theoretical and holds for all representations having no fourth-order Casimirs, while the second one is model-dependent. In any case, our anomaly polynomial can be written as

$$I_8 = -\frac{3}{8} \left[ k (\text{tr} R^2)^2 + \frac{1}{3} \text{tr} R^2 \sum_z \beta_z \text{tr} z F^2 - \frac{4}{3} \sum_z \gamma_z (\text{tr} z F^2)^2$$
$$+ 4 \sum_{z \neq z'} \delta_{zz'} \text{tr} z F^2 \text{tr} z' F^2 \right]. \tag{5.17}$$

In order for this to cancel via the Green-Schwarz mechanism, it must factorize as

$$I_8 = -\frac{3k}{8} \left( c_z \text{tr} z F^2 - \text{tr} R^2 \right) \left( \tilde{c}_z \text{tr} z F^2 - \text{tr} R^2 \right). \tag{5.18}$$

where we reintroduced the summation convention for $z$. This can happen provided that (i) for each $z$,

$$c_z + \tilde{c}_z = -\frac{1}{3k} \beta_z, \quad c_z \tilde{c}_z = -\frac{4}{3k} \gamma_z \tag{5.19}$$
and (ii) for each pair \( z \neq z' \),
\[
c_z \tilde{c}_{z'} + c_{z'} \tilde{c}_z = \frac{4}{k} \delta_{zz'}
\] (5.20)

Eqs. (5.19) and (5.20) are similar to the conditions found in \cite{49} and they are interpreted as follows. The first two conditions of (5.19) can be used to determine \( c_z \) and \( \tilde{c}_z \): real solutions exist when
\[
\beta_z^2 + 48 k \gamma_z \geq 0,
\] (5.21)
in which case they are given by the roots of the equation
\[
3kx^2 + \beta_z x - 4\gamma_z = 0.
\] (5.22)

The third condition (5.20) amounts then to a set of non-trivial relations that must be satisfied by the group-theoretical coefficients \( \beta_z, \gamma_z \) and \( \delta_{zz'} \), which, in turn result to stringent restrictions on the boundary matter content.

The extension of the above in the case where a \( Sp(1) \) or \( U(1) \) R-symmetry subgroup is gauged is straightforward. One has just to supplement the gauge and mixed anomaly polynomials by the contributions from R-charged fermions, i.e. the boundary gauginos (and possibly tensorinos). One again arrives at similar expressions as those presented just above.

So far, we have considered only one single fixed plane. However, having in mind that we intend to cancel the anomalies by a Green-Schwarz mechanism using a single bulk 2–form, we have to ensure that one of the two factors in the factorization equation (5.18) is common to both planes. Taking this to be the second factor, we have thus the additional restriction
\[
\tilde{c}^{(1)}_z = \tilde{c}^{(2)}_z
\] (5.23)
This condition obviously holds when the boundary matter and gauge groups are the same on both fixed planes, as in the HW model.

### 5.2 Green-Schwarz anomaly cancellation

Provided that the local anomaly cancellation conditions described above are satisfied, the application of the Green-Schwarz mechanism is straightforward. Since a Lagrangian formulation of the theory is possible only for \( n_T = 0 \) \( (k = 1) \), we will concentrate on this case. Our starting point is the factorized anomaly polynomial
\[
I_8 = -\frac{3}{8} \left( c_z \text{tr}_z F^2 - \text{tr} R^2 \right) \left( \tilde{c}_z \text{tr}_z F^2 - \text{tr} R^2 \right).
\] (5.24)

The 6D Yang-Mills and Lorentz Chern-Simons forms \( \omega_{3Y,z} \) and \( \omega_{3L} \) satisfy
\[
d\omega_{3Y,z} = \text{tr}_z F^2, \quad d\omega_{3L} = \text{tr} R^2,
\] (5.25)
and their gauge and Lorentz variations are respectively given by the descent equations
\[
\delta\omega_{3Y,z} = d\omega_{2Y,z}^1, \quad \delta\omega_{3L} = d\omega_{2L}^1.
\] (5.26)
It is not hard to see that the resulting variation of the supergravity effective action from fermion loops can be written as

$$\delta \Gamma = -\xi \int \left( \tilde{c}_z \omega_{2Y,z} \omega_{2L}^1 \right) \left( c_z \text{tr}_z F^2 - \text{tr} R^2 \right). \quad (5.27)$$

where we introduced the shorthand

$$\xi \equiv \frac{1}{32(2\pi)^3}. \quad (5.28)$$

On the other hand, our Green-Schwarz term, determined by supersymmetry considerations in Section 4 reads, completed with its gravitational part,

$$S_{GS} = -\frac{1}{\sqrt{2}} \int B_2 \left( v_z \text{tr}_z F^2 - \text{tr} R^2 \right). \quad (5.29)$$

To cancel the anomalies using this term, we set all undetermined coefficients $v_z$ to the values

$$v_z = c_z, \quad (5.30)$$

we endow $B_2$ with the anomalous gauge/Lorentz transformation law

$$\delta B_2 = -\sqrt{2} \xi \left( \tilde{c}_z \omega_{2Y,z}^{1} - \omega_{2L}^{1} \right) \quad (5.31)$$

and we appropriately modify its field strength so that it remains gauge/Lorentz invariant, that is, we set

$$G_3 = dB_2 + \sqrt{2} \xi \left( \tilde{c}_z \omega_{3Y,z} - \omega_{3L} \right). \quad (5.32)$$

After these modifications, it is readily seen that the anomalous variation of $S_{GS}$ under gauge/Lorentz transformations of $B_2$ is equal and opposite from that of $\Gamma$, yielding the desired anomaly cancellation,

$$\delta (\Gamma + S_{GS}) = 0, \quad (5.33)$$

as required. It is expected that anomaly cancellation will still be possible even in the presence of more tensor multiplets by an appropriate modification of the Green-Schwarz mechanism [41].

### 5.3 Global anomaly cancellation and non-perturbative anomalies

Apart from the local cancellation of anomalies on the orbifold fixed points, the anomalies of the theory must also cancel globally on our orbifold. Labelling the two orbifold fixed planes and all quantities pertaining to them by an index $i$ taking the values 1 ($x_7 = 0$) and 2 ($x_7 = \pi R$), we find that cancellation of the irreducible tr $R^4$ part of the gravitational anomaly requires that

$$\sum_{i=1}^{2} \left( n_{H}^{(i)} + 29 n_{T}^{(i)} - n_{V}^{(i)} \right) = 243 - N \quad (5.34)$$
Provided that this holds, the remaining gauge, gravitational and mixed anomalies can cancel, in the case \( n_T = 0 \), when the total anomaly polynomial factorizes as

\[
I_8^{(1)} + I_8^{(2)} = -\frac{3}{4} \left( \sum_i u_z^{(i)} \mathrm{tr}_Z F^{(i)2} - \mathrm{tr} R^2 \right) \left( \sum_i \tilde{u}_z^{(i)} \mathrm{tr}_Z F^{(i)2} - \mathrm{tr} R^2 \right),
\]

where \( u_z^{(i)} \) and \( \tilde{u}_z^{(i)} \) are some numerical constants. In the case \( n_T > 0 \), this factorization condition need not be satisfied; the anomaly can cancel by the generalized Green-Schwarz mechanism.

One final issue concerns non-perturbative anomalies of the type first discovered by Witten [52] for the case of an odd number of 4D Weyl fermions coupled to an \( SU(2) \) gauge field. In the present case, such anomalies may appear when one of the gauge group factors \( G_z \) has non-trivial sixth homotopy group since this implies that fermionic path integrals in certain representations may pick up phase factors under “large” gauge transformations and are therefore ill-defined. Among all possible simple groups, the only one that can give rise to such anomalies is \( G_2 \) due to the fact that \( \pi_6(G_2) = \mathbb{Z}_3 \). In such a case, the condition for the absence of such anomalies is determined by the “mod 3” Atiyah-Singer index theorem.

## 6 The complete bulk-boundary action

We are now in a position to write down the locally supersymmetric action resulting from the combination of the bulk action as well as the various boundary contributions constructed above. The full action of our theory reads

\[
S = \int d^7 x \mathcal{L}_7 + \int d^6 x \mathcal{L}_6^{(1)} + \int d^6 x \mathcal{L}_6^{(2)},
\]

where \( \mathcal{L}_7 \) is the bulk Lagrangian given by (2.37), while \( \mathcal{L}_6^{(i)} \) denotes the boundary Lagrangian localized at the fixed plane \( i \). For the fixed plane at \( x_7 = 0 \), we have

\[
e^{-1} \mathcal{L}_6^{(1)} = -\frac{1}{4} e^{-\phi} c_z \mathrm{tr}_Z (F_{\mu\nu} F^{\mu\nu}) - e^{\phi} c_z \mathrm{tr}_Z (\bar{\lambda}^i \Gamma^\mu D_\mu \lambda_i) - g_{\alpha\beta}(\varphi) D_\mu \varphi^\alpha D^\mu \varphi^\beta \\
-\bar{\zeta}^\alpha \Gamma^\mu D_\mu \zeta^\alpha - \frac{1}{12 \sqrt{2}} e^{\phi} \left[ -e^{\phi} c_z \mathrm{tr}_Z (\bar{\lambda}^i \Gamma^{\mu\nu} \lambda_i) + \bar{\zeta}^\alpha \Gamma^{\nu\rho} \zeta^\alpha \right] G_{\mu\nu} \\
-\frac{1}{2} e^{\phi} c_z \mathrm{tr}_Z (\bar{\psi}^i \Gamma^\mu \Gamma^\nu \Gamma^\rho \lambda_i F_{\mu\nu}) - \frac{1}{2} c_z \mathrm{tr}_Z (\bar{\chi}^i \Gamma^{\mu\nu} \lambda_i F_{\mu\nu}) + 2 \bar{\psi}^i \Gamma^\mu \Gamma^\nu \Gamma^\rho \psi^\alpha V_{\alpha i} \zeta^\alpha \\
+e^{\phi} \mathrm{tr}_Z (C_{ij} \bar{\psi}^i \Gamma^\mu \lambda^j) + e^{\phi} \mathrm{tr}_Z (C_{ij} \bar{\chi}^i \lambda^j) - 4 e^{\phi} \mathrm{tr}_Z (\bar{\lambda}_i \zeta^\alpha \zeta^\alpha V_{\alpha i} \zeta^\alpha) \\
-\frac{1}{2} e^{\phi} c_z^{-1} \mathrm{tr}_Z (C_{ij} C^{ij}) - \frac{1}{8 \sqrt{2}} e^{-1} \epsilon^{\mu\nu\rho\sigma\tau} B_{\mu\nu} c_z \mathrm{tr}_Z (F_{\rho\sigma} F_{\tau\upsilon}) \\
-\frac{\sqrt{2}}{3} e^{\phi} C.
\]

(6.2)
On the bulk, the action (6.1) is invariant under the SUSY transformations listed in (2.38). On a given boundary, the action is invariant under the SUSY transformations obtained by combining the transformations of the boundary theory of the surviving bulk fields with those of the additional boundary multiplets. These transformations are listed below.

\[
\begin{align*}
\delta e_a^\mu &= \frac{1}{2} \bar{\epsilon}^i \Gamma_a \psi_{\mu i}, \\
\delta \phi &= \frac{1}{2} \bar{\epsilon} \chi_i, \\
\delta B_{\mu \nu} &= e^{-\phi} \left( -\frac{1}{\sqrt{2}} \bar{\epsilon}^i \Gamma_{[\mu} \psi_{\nu] i} + \frac{1}{2 \sqrt{2}} \bar{\epsilon} \Gamma_{\mu \nu} \chi_i \right), \\
\delta \psi_{\mu i} &= D_{\mu} \bar{\epsilon}_i - \frac{1}{24 \sqrt{2}} e^\phi \Gamma_{\nu \rho \sigma} \Gamma_{\mu} G_{\nu \rho \sigma} \bar{\epsilon}_i, \\
\delta \chi_i &= \frac{1}{2} \Gamma_{\mu} \partial_{\mu} \phi \bar{\epsilon}_i - \frac{1}{12 \sqrt{2}} e^\phi \Gamma_{\mu \nu \rho} G_{\mu \nu \rho} \bar{\epsilon}_i, \\
\delta A_{\mu} &= i e^\phi \bar{\epsilon} \Gamma_{\mu} \lambda_i, \\
\delta \lambda_i &= -\frac{1}{4} e^{-\phi} \Gamma_{\mu \nu} \bar{\epsilon}_i F_{\mu \nu} - \frac{1}{2} e^{-1} C_{ij} \bar{\epsilon}_j, \\
\delta \varphi^\alpha &= V_{a i} \tilde{a}^\alpha \bar{\epsilon}_i, \\
\delta \zeta^a &= V_a \bar{\epsilon} \Gamma^\mu D_{\mu} \varphi^\alpha \bar{\epsilon}_i.
\end{align*}
\] (6.3)

The Lagrangian (6.2) contains all couplings for the case of $D = 6$, $\mathcal{N} = (0, 1)$ supergravity coupled to one tensor multiplet, $n_V$ vector multiplets and $n_H$ hypermultiplets found in [44, 45]. Among the various interaction terms, a particularly important role is played by the Green-Schwarz term $B_2 \wedge \text{tr} F^2$ (and its gravitational counterpart $B_2 \wedge \text{tr} R^2$) that cancel the anomalies of this theory. The 7-dimensional origin of these couplings is now clarified: they appear as a consequence of dualizing the 3–form theory on a manifold with boundary.

## 7 Conclusions

In the present work, we discussed brane worlds in seven-dimensional minimal $\mathcal{N} = 2$ Yang-Mills-Einstein supergravity. The bulk theory is the one of [34], where $N$ abelian vector multiplets are coupled to pure 7D supergravity. In our construction we have employed the 3–form version of [30] from where the 2–form version is obtained by Poincaré duality. The latter produces surface terms, thrown away usually, which we keep however, as the seven-dimensional vacuum spacetime of the theory is of the form $M^6 \times S^1 / \mathbb{Z}_2$ and, thus, it has boundaries at the fixed points of the $\mathbb{Z}_2$ action. These surface terms are nothing else than Green-Schwarz terms, necessary for the cancellation of anomalies in the boundary 6D theory. This should be anticipated with the 3–form version of the 7D theory [18, 19]
and the HW case \[6, 7\], where the existence of a bulk Chern-Simons term is crucial for the anomaly cancellation. This is also the case for 5D theories as well. Here there is no bulk Chern-Simons term to participate in the anomaly cancellation [33], but instead, there exists a boundary Green-Schwarz term to do this.

The \(3N\) scalars of the 7D theory parametrize the coset \(SO(N,3)/SO(N) \times SO(3)\) and an appropriate subgroup \(G \subset SO(N,3)\) of the isometry group \(SO(N,3)\) can be gauged. In that case, supersymmetry is maintained by the introduction of a potential, given in (3.1), for the scalars. Contrary to the 3–form version, the 2–form version of the 7D minimal \(\mathcal{N} = 2\) supergravity admits a 7D Minkowski vacuum. We may then compactify the theory down to six dimensions on the orbifold \(S^1/\mathbb{Z}_2\). The resulting effective 6D theory contains in its massless sector a gravity multiplet, a tensor multiplet and \(N + 1\) vector multiplets. As this spectrum is anomalous, extra matter fields are needed at the fixed planes to cancel the anomalies. This matter fields can be either 6D vector, hyper or tensor multiplets. We are not considering here extra tensor multiplets as there is no Lagrangian in that case and we left this possibility for future work. Here, the matter at the boundaries necessary to cancel the anomalies comes in the form of vector and hypers. Following [14, 15, 16, 17, 18], we considered without loss of generality the case in which the scalars parametrize the symmetric space \(Sp(n_H,1)/Sp(n_H) \times Sp(1)\), whose isometry group is \(Sp(n_H,1)\). The latter is a global symmetry of the supergravity theory and we gauged then its maximal compact subgroup \(Sp(n_H) \times Sp(1)\). All the results obtained here for this particular choice of the gauge group can easily be generalized to other quaternionic hyperscalar spaces. It should be noted that the hypermultiplets are not neutral as in [18, 19], but rather charged under the gauge group. In addition, both the vector and hypermultiplets are coupled to the bulk fields and their couplings have been specified by supersymmetry.

We should also mention that, since the brane worlds constructed here are six dimensional, compactification down to four dimensions is needed. Such a compactification which involves an \(S^2\), much like the monopole compactification [32], will be presented elsewhere.

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**A Conventions**

We closely follow the sign and normalization conventions used in [18, 19]. Our spacetime metric has the “mostly-plus” signature \((-+,\ldots,+)\). The 6D \(8 \times 8\) matrices \(\Gamma^a\) satisfy the Clifford algebra

\[
\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}, \quad a, b = 0, \ldots, 5, \quad (A.1)
\]
while the 7D gamma matrices are $\Gamma^A = (\Gamma^a, \Gamma_7)$ with the latter given by
\begin{equation}
\Gamma_7 = \Gamma^0 \ldots \Gamma^5,
\end{equation}
so that $(\Gamma_7)^2 = 1$. The 6D gamma matrices satisfy the duality relation
\begin{equation}
\Gamma_{\mu_1 \ldots \mu_n} = (-1)^{[n/2]} \frac{1}{(6-n)!} \epsilon_{\mu_1 \ldots \mu_n \mu_{n+1} \ldots \mu_6} \Gamma_{\mu_{n+1} \ldots \mu_6} \Gamma_7.
\end{equation}
Our spinors satisfy the symplectic Majorana condition
\begin{equation}
\chi^i = \epsilon^{ij} \bar{\chi}_j^T, \quad \bar{\chi}_i = \chi^i \Gamma_0
\end{equation}
where $i, j = 1, 2$ are $Sp(1)$ R-symmetry indices and $\epsilon_{ij}$ is the $Sp(1)$–invariant tensor, defined as
\begin{equation}
\epsilon_{ij} = \epsilon^{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\end{equation}
This tensor is used for raising and lowering $Sp(1)$ indices according to the standard NW-SE convention,
\begin{equation}
\chi^i = \epsilon^{ij} \chi_j, \quad \chi_i = \chi^j \epsilon_{ji}.
\end{equation}
The same convention is employed in the contraction of $Sp(1)$ indices in all spinor inner products,
\begin{equation}
\bar{\chi} \Gamma^{M_1 \ldots M_n} \psi = \chi^i \Gamma^{M_1 \ldots M_n} \psi_i.
\end{equation}
We also note that the following identities hold
\begin{equation}
\bar{\chi} \Gamma^{M_1 \ldots M_n} \psi^j = (-1)^{n+1} \psi^j \Gamma^{M_n \ldots M_1} \chi^i.
\end{equation}
and
\begin{equation}
\bar{\chi} \Gamma^{M_1 \ldots M_n} \psi = (-1)^n \psi \Gamma^{M_1 \ldots M_n} \chi.
\end{equation}
In 6 dimensions, a symplectic Majorana spinor decomposes into positive- and negative-chirality parts according to $\chi^i = \chi^{i^+} + \chi^{i^-}$, where $\chi^{i^\pm}$ are symplectic Majorana-Weyl spinors satisfying $\Gamma_7 \chi^{i^\pm} = \pm \chi^{i^\pm}$.

## B Anomaly polynomials

In the discussion of anomaly cancellation, we will use the following normalization for the 8–form anomaly polynomials,
\begin{align*}
I_8^{1/2}(F) &= - \text{tr} \, F^4, \\
I_8^{1/2}(R) &= - \frac{1}{240} \text{tr} \, R^4 - \frac{1}{192} (\text{tr} \, R^2)^2, \\
I_8^{1/2}(F, R) &= \frac{1}{4} \text{tr} \, R^2 \, \text{tr} \, F^2, \\
I_8^{3/2}(R) &= - \frac{49}{48} \text{tr} \, R^4 + \frac{43}{192} (\text{tr} \, R^2)^2, \\
I_8^A(R) &= - \frac{7}{60} \text{tr} \, R^4 + \frac{1}{24} (\text{tr} \, R^2)^2.
\end{align*}
where the superscripts $1/2$, $3/2$ and $A$ refer to a spin $1/2$ fermion, a spin $3/2$ fermion and a 2–form antisymmetric tensor potential respectively. The above anomaly polynomials correspond to Weyl spinors of positive chirality and 2–form potentials with self-dual field strengths. For a Majorana-Weyl spinor, one needs to include a factor of $\frac{1}{2}$, while for a negative-chirality spinor (or an anti-self-dual field strength) the sign of the anomaly must be reversed.

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