Entanglement of arbitrary spin modes in expanding universe

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Abstract Pair particle creation is a well-known effect in the domain of field theory in curved space–time. The behavior of the generated entanglement due to expanding universe is very different for spin-0 and spin-1/2 particles. We study spin-1 particles in Friedmann–Robertson–Walker (FRW) space–time using Duffin–Kemmer–Petiau equation and spin-3/2 particles in FRW space–time using Rarita–Schwinger equation. We find that in expanding universe, the behavior of the generated entanglement for spin-1 particles is the same as the behavior of the generated entanglement for spin-0 particles. Also, we find that spin-3/2 and spin-1/2 particles have the same behavior for the generated entanglement in expanding universe. We conclude that the absolute values of spins do not play any role and the differences in the behavior of the generated entanglement in expanding universe are due to bosonic or fermionic properties.

Keywords Entanglement · Expanding universe · Spin-1 · Spin-3/2 · Fermion · Boson

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1 Introduction

Because of the importance and the fundamental role of entanglement in quantum information processing and quantum computing, a lot of researches have been done on entanglement generation, variation and degradation in various domains. In nonrelativistic limits, entanglement has been extensively studied [1]. Recently, relativistic quantum information processing has attracted a lot of interests [2–4]. The world is fundamentally relativistic; therefore, understanding entanglement in space–time is ultimately important. Relativity plays a significant role in quantum entanglement and nonclassical correlation behaviors and relating quantum protocols [4–14]. It has been shown that entanglement of bosonic and fermionic modes degrades by acceleration [15,16].

Experiments have been proposed to test the effect of gravity and acceleration on quantum entanglement in space-based setups [17,18]. It has been shown that the entanglement between excitations of two Bose–Einstein condensates degrades after that one of them undergoes a change in the gravitational field strength [19]. There are already achieved experimental results and advanced plans to implement quantum technologies in space [20–22]; Space-QUEST [23] and space optical clock projects [24,25] are intended to implement quantum communications and quantum clocks at regimes where relativity starts to kick in. To include and exploit these effects, some techniques have been introduced for applications of metrology to quantum field theory. Quantum field theory properly incorporates quantum theory and relativity, in particular, at regimes where space-based experiments take place [26].

Recently, scalar and spinor field modes have been investigated in an expanding universe. It is shown that in both cases, the expanding universe generates entanglement between field modes. A separable vacuum state in the distant past time appears as an entangled state in far future time because of the expanding universe. However, there are differences between entanglement of scalar boson fields and spinor fermion fields [27]. The entanglement of massive boson modes is a monotonically decreasing function with respect to the momentum of the modes, while for the fermion modes, there is an optimum point for the momentum of the modes. For spin-1/2 case, there is no entanglement between zero-momentum modes, while for spin-0 case, the maximum entanglement is related to zero-momentum modes. As a common behavior between spin-0 and spin-1/2 modes, it is shown that there is no entanglement for massless bosons or fermions [28].

Investigation of entanglement generation due to expansion of universe provides useful information about the cosmic microwave background. Although cosmological particle creation typically occurs in extremely large length scale, it is one of the very few examples for such fundamental effects where we may have observational evidences. Besides that, one may experimentally study the phenomenon of cosmological particle creation by means of suitable laboratory analogues [29,30]. Also, it is

\[ \text{For a density matrix } \rho \text{ of a composite bipartite system } AB \text{, a separable state can be written as } \rho_{\text{sep}} = \sum_i w_i \rho_A^i \otimes \rho_B^i, \text{ where } w_i \text{'s are positive weights, and } \rho_A^i \text{'s and } \rho_B^i \text{'s are local states belonging to } A \text{ and } B, \text{ respectively. An entangled state is a state that is not separable.} \]
shown that one can create emergent curved space–times by locally tuning the coupling constant of condensed matter systems [31]. Some new ideas have been presented on how to design analogue model of quantum fields living in curved space–times using ultracold atoms in optical lattices [32].

We aim at understanding the origin of the differences between the generated entanglement in an expanding universe for spin-0 and spin-1/2 modes, by investigating higher spin modes. Firstly, we consider spin-1 modes using Friedmann–Robertson–Walker (FRW) equation in Duffin–Kemmer–Petiau (DKP) space–time and work out a measure for the generated entanglement. The same procedure is applied on spin-3/2 modes using Rarita–Schwinger equation. We find similar behavior of the generated entanglement for spin-1 mode and spin-3/2 mode. Therefore, we conclude that the absolute value of spin does not play any role and the differences are due to the bosonic or fermionic properties.

This paper is organized as follows: In Sect. 2, we consider the DKP equation in FRW expanding universe specially in two dimensions. In Sect. 3, we calculate entanglement entropies of spin-1 and spin-0 particles and compare them to each other. Also, we investigate the variations of entanglement with respect to the parameters of expansion and the momentum of the modes. In Sect. 4, we consider spin-3/2 particles using Rarita–Schwinger equation and work out the generated entanglement and compare it with the entanglement of spin-1/2 modes. Conclusions are presented in Sect. 5.

2 DKP equation in Friedmann–Robertson–Walker space–time

Klein–Gordon and Dirac equations describe particles with spin-0 and spin-1/2 in flat Minkowski space–time, respectively. Scalar and spinor fields have been considered in curved space–time. There are various ways of formulating a relativistic wave equation describing the dynamical states of a massive vector boson, such as Proca equation, DKP equation and Weinberg–Shay–Good equation [33–36]. We employ DKP equation for considering vector bosons in curved space–time. Before starting the study of the DKP in curved space–time, we notice that it is similar to the Dirac equation in Minkowski space–time as follows

$$(i\beta^\mu \partial_\mu - m) \Psi = 0,$$

where \(\beta^\mu\) matrices are generalizations of Dirac gamma matrices, satisfying an algebra ring, which for spin-1 is

$$\beta^\lambda \beta^\mu \beta^\nu + \beta^\nu \beta^\mu \beta^\lambda = \eta^{\lambda\mu} \beta^\nu + \eta^{\mu\nu} \beta^\lambda. \quad (2)$$

Generally, \(\beta^\mu\)'s represent four \(16 \times 16\) reducible matrices, which decompose into three separate representations, a one-dimensional trivial, a five-dimensional spin-0 and a ten-dimensional spin-1 representations [37–40]. For the ten-dimensional spin-1 representations, \(\beta^\mu\) matrices are given by
\[\beta^0 = \begin{pmatrix} 0 & 0_{1 \times 3} & 0_{1 \times 3} & 0_{1 \times 3} \\ 0_{3 \times 1} & 0_{3 \times 3} & 1_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 1} & 1_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 1} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{pmatrix}, \quad (3)\]

and

\[\beta^j = \begin{pmatrix} 0 & 0_{1 \times 3} & K^j & 0_{1 \times 3} \\ 0_{3 \times 1} & 0_{3 \times 3} & 0_{3 \times 3} & -iS^j \\ -K^j & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 1} & -iS^j & 0_{3 \times 3} & 0_{3 \times 3} \end{pmatrix}, \quad (4)\]

where \(S^j\)’s are standard \((3 \times 3)\) spin-1 matrices and \(K^j\)’s denote \((1 \times 3)\) matrices with elements \(K^j_k = \delta^j_k\).

In curved space–time, we can use the tetrad formalism to obtain a generalized DKP equation

\[\left(i\bar{\beta}^\mu \left(\partial_\mu + \frac{1}{2} \omega_{\mu ab} S^{ab}\right) - m\right) \Psi = 0, \quad (5)\]

where \(S^{ab} = [\beta^a, \beta^b]\) and \(\bar{\beta}^{\mu j}\)’s are Kemmer matrices in curved space–time and they are related to Minkowski space–time \(\bar{\beta}^\mu = e_a^\mu \beta^a\) with the following tetrad relation

\[e^\mu_a e^\nu_b \eta^{ab} = g^{\mu \nu}, \quad e^\mu_a e_{b \mu} = \eta_{ab}, \quad e^\mu_a e^b_\mu = \delta^b_a. \quad (6)\]

Here, spin connections \(\omega_{\mu ab}\) are given by

\[\omega_{\mu ab} = e_{al} e^j_b \Gamma^l_{j \mu} - e^j_b \partial_\mu e_{aj}, \quad (7)\]

where \(\Gamma^l_{j \mu}\)’s are affine connections that are obtained by space–time metric elements. Specifically, we consider a two-dimensional FRW expanding space–time with line element

\[ds^2 = C^2(\eta) \left(d\eta^2 - dx^2\right), \quad (8)\]

where \(\eta\) is the conformal time. The conformal scale factor, \(C\), is given by

\[C(\eta) = (1 + \epsilon (1 + \tanh \rho \eta))^{1/2}, \quad (9)\]

with positive real parameters \(\epsilon\) and \(\rho\), controlling the total volume and rapidity of the expansion, respectively. Primarily, entanglement between modes of a quantum field in a curved space–time can be investigated in special states where the space–time has at least two asymptotically flat regions. According to Eq. (9), in the distant past time and in the far future time, the space–time becomes Minkowskian, since \(C(\eta)\) tends to
$1 + 2\epsilon$ for $\eta \to +\infty$ and it tends to 1 for $\eta \to -\infty$. In an intermediate region, the concept of particle breaks down.

For simplicity, we restrict ourselves to solve $(1 + 1)$-dimensional DKP equation in FRW space–time. The DKP equation can be obtained in the following form [41].

$$\left[ \beta^0\partial_\eta + ik\beta^1 - \frac{\dot{C}}{C} \left( \beta^1 \right)^2 \beta^0 + iCm \right] \tilde{\Psi} = 0,$$

(10)

where $\Psi(\eta, x) = e^{ikx} \tilde{\Psi}(\eta)$ and $\tilde{\Psi}(\eta)$ has ten components as follows

$$\tilde{\Psi}(\eta) = \begin{pmatrix} \varphi \\ P \\ Q \\ R \end{pmatrix},$$

(11)

where $P$, $Q$ and $R$ are $3 \times 1$ vectors and $\varphi$ denotes a scalar. The Kemmer matrices representation for a suitable arrangement of the components of these vectors and $\varphi$ leads to the following equations

$$iCm \Phi = - \left( \partial_\eta + \frac{\dot{C}}{C} \right) \mathcal{X} + ik \Theta,$$

$$iCm \mathcal{X} = -\partial_\eta \Phi,$$

$$iCm \Theta = -ik \Phi,$$

(12)

where

$$\Phi = \begin{pmatrix} P_2 \\ P_3 \\ Q_1 \end{pmatrix}, \quad \mathcal{X} = \begin{pmatrix} Q_2 \\ Q_3 \\ P_1 \end{pmatrix}, \quad \text{and} \quad \Theta = \begin{pmatrix} R_3 \\ -R_2 \\ \varphi \end{pmatrix}.$$ (13)

The third component of $R$ is vanished. We can obtain an independent equation for $\Phi(\eta)$ as follows

$$\left( \frac{d^2}{d\eta^2} + k^2 + C^2 m^2 \right) \Phi(\eta) = 0.$$ (14)

$\mathcal{X}(\eta)$ and $\Theta(\eta)$ satisfy the following equations

$$\mathcal{X} = \frac{i}{Cm} \partial_\eta \Phi, \quad \Theta = -\frac{k}{Cm} \Phi.$$ (15)

Using Eq. (14), we obtain the following two different solutions, which are analytic in the in region, $\Phi_{\text{in}}$, for distant past, and in the out region, $\Phi_{\text{out}}$, for far future.
\[ \Phi_{\text{in}}(\eta) = \left( \frac{1}{2} (1 + \tanh \rho \eta) \right)^{i \omega_{\text{in}} \rho} \left( \frac{1}{2} (1 - \tanh \rho \eta) \right)^{i \omega_{\text{out}} \rho} \times \Phi_{\text{in}}(\eta) = \alpha \Phi_{\text{out}}(\eta) + \beta \Phi_{\text{out}}^* \]  

where \[ \alpha = 1 + \beta, \quad \beta = \frac{i}{2 \rho} (\omega_{\text{in}} + \omega_{\text{out}}), \quad \gamma = 1 + \frac{i \omega_{\text{in}} \rho}{\omega_{\text{out}} \rho}, \]  

\[ \omega_{\text{out}}^2 = k^2 + m^2 (1 + 2 \epsilon), \quad \omega_{\text{in}}^2 = k^2 + m^2. \]  

\( \Phi_{\text{in}} \) is a constant vector of dimension 3 \( \times 1 \). \( \mathcal{X}(\eta) \) and \( \Theta(\eta) \) can be evaluated easily from (15). \( \Phi_{\text{in}}(\eta) \) is a constant vector of dimension 3 \( \times 1 \). Using properties of these functions, the above solutions will be related to each other by Bogoliubov transformation technique as

\[ \Phi_{\text{in}} = \alpha_k \Phi_{\text{out}} + \beta_k \Phi_{\text{out}}^*, \]  

where \( \alpha_k \) and \( \beta_k \) are Bogoliubov coefficients. Since \( \mathcal{X}(\eta) \) and \( \Theta(\eta) \) are given by \( \Phi(\eta) \), one can relate the \textit{in} and the \textit{out} modes of the wavefunction by a similar equation as follows

\[ \Psi_{\text{in}} = \alpha_k \Psi_{\text{out}} + \beta_k \Psi_{\text{out}}^*. \]  

Using properties of hypergeometric functions [42], we evaluate the Bogoliubov coefficients

\[ \beta_k = \frac{\Gamma \left( 1 + \frac{i \omega_{\text{in}} \rho}{\omega_{\text{out}} \rho} \right) \Gamma \left( i \omega_{\text{out}} \rho \right)}{\Gamma \left( 1 + \frac{i \omega_{\text{in}} \rho}{\omega_{\text{out}} \rho} \right) \Gamma \left( i (\omega_{\text{in}} - \omega_{\text{out}}) \right) \frac{2 \rho}{\omega_{\text{out}} \rho}}, \]  

and

\[ \alpha_k = \frac{\Gamma \left( 1 + \frac{i \omega_{\text{in}} \rho}{\omega_{\text{out}} \rho} \right) \Gamma \left( i \omega_{\text{out}} \rho \right)}{\Gamma \left( 1 + \frac{i \omega_{\text{in}} \rho}{\omega_{\text{out}} \rho} \right) \Gamma \left( i (\omega_{\text{in}} + \omega_{\text{out}}) \right) \frac{2 \rho}{\omega_{\text{out}} \rho}}. \]  

Now, we can work out a relation between annihilation and creation operators in the \textit{in} and the \textit{out} regions as follows

\[ a_{\text{out}}^i = \alpha_k^* a_{\text{out}}^i + \beta_k^* b_{\text{out}}^i, \]  

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where $a$ and $a^\dagger$ ($b$ and $b^\dagger$) are the annihilation and the creation operators of particles (antiparticles) and satisfy the following relations

$$
\begin{bmatrix}
 a_{k}^{\text{in(out)}}, a_{k'}^{\text{in(out)}}
\end{bmatrix} = \delta(k - k'),
$$

and

$$
\begin{bmatrix}
 b_{k}^{\text{in(out)}}, b_{k'}^{\text{in(out)}}
\end{bmatrix} = \delta(k - k').
$$

The five-dimensional representations of Kemmer matrices are given by

$$
\beta^0 = \begin{pmatrix} 0_{2\times3} \\ \sigma_x \end{pmatrix}, \quad \beta^i = \begin{pmatrix} \rho_{2\times2}^{i\dagger} \\ 0_{3\times3} \end{pmatrix},
$$

where $\sigma_x$ is the Pauli $X$ matrix and $\rho_{jk}^i = -\delta_{1j}\delta_{ik}$. In this case, $\tilde{\Psi}(\eta)$ has five $\tilde{\Psi}_i$ components, where $i = 1, \ldots, 5$. Therefore, DKP equation reduces to the following equations

$$
i Cm \tilde{\Psi}_1 = -\left( \partial_\eta - \frac{\dot{C}}{C} \right) \tilde{\Psi}_2 + ik\tilde{\Psi}_3,
$$

$$
i Cm \tilde{\Psi}_2 = -\partial_\eta \tilde{\Psi}_1,
$$

$$
i Cm \tilde{\Psi}_3 = -ik\tilde{\Psi}_1,
$$

and the components $\tilde{\Psi}_4 = \tilde{\Psi}_5 = 0$. $\tilde{\Psi}_1$ satisfies Eq. (14). $\tilde{\Psi}_2$ and $\tilde{\Psi}_3$ satisfy Eqs. (15). Therefore, solutions for the above equations with respect to the $\text{in}$ modes and to the $\text{out}$ modes are the same as the solutions for spin-1 case and they are related to each other by the same Bogoliubov coefficients. This is a significant point achieved in the rest of the paper.

### 3 Entanglement generation due to expansion

We consider the vacuum state in the $\text{in}$ region as a separable state

$$
|0\rangle^{\text{in}} = \prod_{k \in R^+} |0_k\rangle^{\text{in}}|0_{-k}\rangle^{\text{in}}.
$$

For simplicity, we focus on a specific momentum, $k$, mode as follows

$$
|0_k\rangle^{\text{in}} = |0_k\rangle^{\text{in}}|0_{-k}\rangle^{\text{in}}.
$$

In other words, we disregard all other modes of vacuum by tracing out the total density matrix over them. Using Eq. (23), we write the above bipartite separable state as a linear combination of the excited states in mode $k$ by an observer in the $\text{out}$ region.
\[|0_k\rangle^{\text{in}}|0_{-k}\rangle^{\text{in}} = \sum_{n=0}^{\infty} A_n |n_k\rangle^{\text{out}}|n_{-k}\rangle^{\text{out}}. \tag{30}\]

Obviously, the last relation is the Schmidt decomposition of a pure state of a bipartite system. The Schmidt coefficients are obtained by the normalization condition and by applying the annihilation operator on the state

\[a_k^{\text{in}}|0_k\rangle^{\text{in}} = \left(\alpha_k^* a_k^{\text{out}} - \beta_k^* b_k^{\text{out}}\right) \sum_{n=0}^{\infty} A_n |n_k\rangle^{\text{out}}|n_{-k}\rangle^{\text{out}} = 0. \tag{31}\]

Therefore, one finds the following relation for the coefficients

\[A_n = \left(\frac{\beta_k^*}{\alpha_k^*}\right)^n A_0, \quad A_0 = \sqrt{1 - \left|\frac{\beta_k}{\alpha_k}\right|^2}. \tag{32}\]

Thus, a vacuum state in the \textit{in} region corresponds to a state with particle excitations in the \textit{out} region. The particle creation via the expansion is a well-known concept \[43\]. In the following, we concentrate on the entanglement of the corresponding generated particles.

We construct the density matrix in the \textit{in} region by using Eq. (30) in the \textit{out} region

\[\rho_{k,-k} = |0_k\rangle^{\text{in}}|0_{-k}\rangle^{\text{in}}\langle 0_k|^{\text{in}}\langle 0_{-k}|^{\text{in}}. \tag{33}\]

Because the Schmidt coefficients in Eq. (30) are nonzero, the \textit{in} vacuum is entangled from the point of view of an observer in the \textit{out} region.

As an appropriate measure of entanglement, we calculate von Neumann entropy as follows

\[S(\rho_k) = -\text{tr}(\rho_k \log_2(\rho_k)), \tag{34}\]

where \(\rho_k\) is the reduced density matrix of particles’ subsystems

\[\rho_k = \text{tr}_{-k}(\rho_{k,-k}). \tag{35}\]

The von Neumann entropy for spin-1 modes is then given by

\[S = -\sum_{n=0}^{\infty} |A_n|^2 \log_2 |A_n|^2
\]

\[= \log_2 \frac{x^{\frac{1}{r-1}}}{1-x}, \tag{36}\]
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where \( x = |\frac{\beta_k}{\alpha_k}|^2 \) with \( \alpha_k \) and \( \beta_k \) given by Eqs. (21) and (22), respectively.

Figure 1 shows the contour plot of the von Neumann entropy with respect to the mass and the momentum of spin-1 particle for the specified expanding universe (fixed values of total volume and rapidity of expansion). The generated entanglement is a decreasing function with respect to the momentum. There is no entanglement between massless spin-1 particles. For each value of momentum, there exists a specific value of mass, \( m_{\text{max}} \), at which the entanglement is maximum. \( m_{\text{max}} \) is an increasing function with respect to momentum.

It is obvious from Fig. 2 that for the specified mass and momentum of spin-1 particles, the generated entanglement is an increasing function with respect to the total volume, \( \epsilon \), and the rapidity of expansion, \( \rho \). There is no entanglement generated for \( \epsilon = 0 \) and \( \rho = 0 \), corresponding to a flat Minkowskian space–time.

In addition, one can observe from Fig. 3 that there is a certain value of mass, \( m_{\text{max}} \), wherein the generated entanglement is maximum. Naturally, the entanglement generation is larger for an expanding universe with a larger total volume expansion. In a similar behavior, \( m_{\text{max}} \) tends to larger values for larger total volume expansion.

The above results are presented for spin-1 particles. Since von Neumann entropy is a function with respect to Bogoliubov coefficients and we showed in Sect. 2 that these coefficients are the same for spin-1 and spin-0 particles, the corresponding generated entanglements for spin-0 particles and spin-1 particles are the same.
4 Spin-3/2 particles and entanglement generation

The Rarita–Schwinger equation characterizes the dynamics of massive spin-3/2 particles in flat Minkowski space–time [44]. In supergravity models, the superpartner of graviton field is described by spin-3/2 particles. The complicated form of the Rarita–
Schwinger equation makes it inexplicable to obtain explicit results even in a simple background. It has been shown that when one considers helicity $\pm 3/2$ states propagating in arbitrary homogeneous and isotropic scalar or gravitational backgrounds, the equations can be reduced to a Dirac-like equation in flat Minkowski space–time [45,46] as follows

\begin{equation}
(i \not\partial - m)\psi_\mu = 0, \tag{37}
\end{equation}

with two constraints

\begin{align}
\gamma^\mu \psi_\mu &= 0, \\
\partial^\mu \psi_\mu &= 0. \tag{38}
\end{align}

By replacing the ordinary derivatives by covariant ones, we obtain the equation for FRW metrics as follows [46]

\begin{equation}
(i \not D - m)\psi_\mu = 0, \\
\gamma^\mu \psi_\mu = 0, \\
D^\mu \psi_\mu = 0, \tag{39}
\end{equation}

where $D_\rho \psi_\sigma = \left(\partial_\rho + \frac{i}{2} \Omega_\rho^{\ ab} \Sigma_{ab}\right) \psi_\sigma$ with $\Omega_\rho^{\ ab}$ the spin connection coefficients and $\Sigma_{ab} = \frac{i}{4} [\gamma_a, \gamma_b]$ and $[D_\mu, D_\nu] = -i R^{ab}_{\ \mu \nu} \Sigma_{ab}$. According to Eq. (8) for FRW expanding universe, we obtain the spin connections, Riemann tensor and finally the following equation

\begin{equation}
\left( i C^{-1} \gamma^\mu \partial_\mu - m + \frac{3}{2} \frac{\dot{C}}{C^2} \gamma^0 \right) \psi_\mu = 0. \tag{40}
\end{equation}

By rewriting $\psi_\mu$ as a multiplication of spatial part, $\exp(i k \cdot x)$ and time-dependent function, $\kappa(\eta)$, also an appropriate function of $C(\eta)$, we obtain the following equation for time-dependent part [46]

\begin{equation}
\left( \frac{d^2}{d\eta^2} + k^2 - i m \dot{C} + m^2 C^2 \right) \kappa(\eta) = 0. \tag{41}
\end{equation}

Comparison between Eq. (41) and the same evaluation for spin-1/2 particles in curved space–time [47] shows that the two distant past and far future asymptotic solutions are related by the same Bogoliubov coefficients. It is straightforward to show that the von Neumann entropy is given by

\begin{equation}
S = - \sum_{n=0}^{1} |A_n|^2 \log_2 |A_n|^2 \\
= \log_2 \frac{1 + x}{x^{\frac{1}{1+x}}}, \tag{42}
\end{equation}
where $x = \left| \frac{\beta_k}{\alpha_k} \right|^2$, $\alpha_k$ and $\beta_k$ are Bogoliubov coefficients. Since von Neumann entropy is directly related to these coefficients, we can argue that the generated entanglement of spin-3/2 particles due to expanding universe will have the same properties as of spin-1/2 case.

5 Conclusion

DKP equation is employed for considering spin-1 particles in FRW expanding universe. It is shown that there are two asymptotic distant past and far future times and one can connect these solutions to each other by Bogoliubov technique. The separable vacuum state of a distant past time appears as an entangled particle–antiparticle state in a corresponding far future time. In order to measure the entanglement of the generated state, von Neumann entropy is used. The general behavior of the generated entanglement is summarized with respect to the characteristics of an expanding universe, the mass and the momentum of the modes.

It is also shown that the general behavior of the generated entanglement of spin-0 particles is the same as the one for spin-1 particles because of the same Bogoliubov coefficients that relate the asymptotic solutions to each other.

We considered spin-3/2 particles in FRW space–time using the Rarita–Schwinger equation. Comparing the equation that describes the time-dependent part of the field with spin-1/2, one showed that the Bogoliubov coefficients for both spin-3/2 and spin-1/2 will be the same. Therefore, the general behavior of the generated entanglement will be the same for spin-3/2 and spin-1/2 particles.

Nevertheless, investigations of the generated entanglement in an expanding universe for spin-0 and spin-1/2 particles show some deep differences between these modes [27, 28]. Our consideration in this paper proposes that these differences are independent from the absolute values of spins of particles. We argue that the absolute values of spins do not play any role in the behavior of the generated entanglement, whereas the differences are due to the bosonic or fermionic properties of spins.

The general behavior of the entanglement generation in an expanding universe for higher spin scalar and spinor modes is an open question. The procedure, as explained in this paper, turns out to be complicated for higher spins. By employing different approaches, there are very few studies on higher spins in literature [48–50]. The equation of motion for interacting massless fields of all spins in (3+1) dimensions has been considered also in [51]. The classical theory of higher spins has been introduced by Sorokin in [52].

It is an interesting question to ask whether there might be any observation to detect the changes in entanglement values in an expanding universe and to provide an impartial evidence on the results that have been discussed in this paper. While answering this question and running experiments for the purpose of the detection are not anything trivial, there are some feasible proposals. The expansion of the universe depends on two parameters, the total volume and the rapidity of the expansion. It may be possible to consider the cosmic microwave background photons and evaluate the entanglement of them. One can assume that the origin of the cosmic microwave background photons comes back to the expansion of the universe; therefore, the comparison between the
theoretical and the experimental entanglement of these photons may phenomenologi- 
cally fix the expansion parameters. Also, the cosmological constant can be achieved as 
the result of the entanglement and statistically correlated minisuperspace cosmologi- 
cal states, built up by using a minimal choice of observable quantities, which assign 
the cosmic dynamics [53].

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