Nuclear matter fourth-order symmetry energy in the relativistic mean field model

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Within the nonlinear relativistic mean field model, we derive the analytical expression of the nuclear matter fourth-order symmetry energy $E_{\text{sym},4}(\rho)$. Based on two accurately calibrated interactions FSUGold and IU-FSU, our results show that the value of $E_{\text{sym},4}(\rho)$ at normal nuclear matter density $\rho_0$ is generally less than 1 MeV, confirming the empirical parabolic approximation to the equation of state for asymmetric nuclear matter at $\rho_0$. On the other hand, we find that the $E_{\text{sym},4}(\rho)$ may become nonnegligible at high densities. Furthermore, the analytical form of the $E_{\text{sym},4}(\rho)$ provides the possibility to study the higher-order effects on the isobaric incompressibility of asymmetric nuclear matter, i.e., $K_{\text{sat}}(\delta) = K_0 + K_{\text{sat},2}\delta^2 + K_{\text{sat},4}\delta^4 + O(\delta^6)$ where $\delta = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry, and we find that the value of $K_{\text{sat},4}$ is generally small compared with that of the $K_{\text{sat},2}$. In addition, we study the effects of the $E_{\text{sym},4}(\rho)$ on the proton fraction $x_p$ and the core-crust transition density $\rho_t$ and pressure $P_t$ in neutron stars. Interestingly, we find that, compared with the results from the empirical parabolic approximation, including the $E_{\text{sym},4}(\rho)$ contribution can significantly enhance the $x_p$ at high densities and strongly reduce the $\rho_t$ and $P_t$ in neutron stars, demonstrating that the widely used empirical parabolic approximation may cause large errors in determining the $x_p$ at high densities as well as the $\rho_t$ and $P_t$ in neutron stars within the nonlinear relativistic mean field model, consistent with previous nonrelativistic calculations.

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I. INTRODUCTION

One of fundamental issues in nuclear physics is the equation of state (EOS) of isospin asymmetric nuclear matter, which plays a central role in understanding not only the structure of radioactive nuclei, the reaction dynamics induced by rare isotopes, and the liquid-gas phase transition in asymmetric nuclear matter, but also many critical issues in astrophysics \[\text{\cite{1,2}}\]. For symmetric nuclear matter with equal fractions of neutrons and protons, its EOS is relatively well-determined from analyses of the giant monopole resonances of finite nuclei \[\text{\cite{3}}\] as well as collective flows \[\text{\cite{2}}\] and subthreshold kaon production \[\text{\cite{10,11}}\] in relativistic nucleus-nucleus collisions. On the other hand, the EOS of asymmetric nuclear matter, especially the density dependence of the nuclear symmetry energy $E_{\text{sym}}(\rho)$, is poorly known. During the last decade, significant progress has been made both experimentally and theoretically on constraining the behavior of the symmetry energy around and below normal nuclear matter density \[\text{\cite{12,13}}\] (See, e.g., Refs. \[\text{\cite{14,20}}\] for review of recent progress) while its super-normal density behavior remains elusive and largely controversial \[\text{\cite{21-24}}\]. Theoretically, all many-body theory calculations to date have demonstrated that the nuclear symmetry energy essentially characterizes the isospin dependent part of the EOS of asymmetric nuclear matter and the higher-order terms in isospin asymmetry are unimportant, at least for densities up to moderate values \[\text{\cite{27}}\], leading to the well-known empirical parabolic law.

When the empirical parabolic law itself provides a good approximation to the EOS of asymmetric nuclear matter and thus allows one to extract the symmetry energy from the energy difference between pure neutron matter and symmetric nuclear matter, it may cause large errors when it is applied to determine some physical quantities under special conditions. For example, the higher-order terms in isospin asymmetry presented in the EOS of asymmetric nuclear matter at supra-normal densities can significantly modify the proton fraction in $\beta$-equilibrium neutron-star matter and the critical density for the direct Urca process which can lead to faster cooling of neutron stars \[\text{\cite{25,26}}\]. In addition, recent studies \[\text{\cite{27}}\] indicate that the higher-order terms in isospin asymmetry are very important for determining the transition density and pressure at the inner edge separating the liquid core from the solid crust of neutron stars where the matter is extremely neutron-rich. Furthermore, the higher-order effects on the incompressibility of asymmetric nuclear matter have also been studied recently \[\text{\cite{28}}\]. These studies about the higher-order effects are essentially performed within the nonrelativistic models since the analytical expressions of the higher-order terms in isospin asymmetry, e.g., the nuclear matter fourth-order symmetry energy $E_{\text{sym},4}(\rho)$, can be relatively easily obtained in such nonrelativistic models. It is thus interesting to see if the same conclusion can be obtained within the relativistic models.

One of very popular relativistic models is the relativistic mean field (RMF) model which is generally based on effective interaction Lagrangians involving nucleon and meson fields \[\text{\cite{29}}\]. As a phenomenological ap-
II. THEORETICAL FORMULISM

A. Characteristic parameters of asymmetric nuclear matter

The EOS of isospin asymmetric nuclear matter, defined by its binding energy per nucleon, can be expanded to 4th-order in isospin asymmetry $\delta$ as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + E_{\text{sym},4}(\rho)\delta^4 + \mathcal{O}(\delta^5),$$  

(1)

where $\rho = \rho_n + \rho_p$ is the baryon density with $\rho_n$ and $\rho_p$ denoting the neutron and proton densities, respectively; $\delta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is the isospin asymmetry; $E_0(\rho) = E(\rho, \delta = 0)$ is the binding energy per nucleon in symmetric nuclear matter; the nuclear matter symmetry energy $E_{\text{sym}}(\rho)$ and the 4th-order symmetry energy $E_{\text{sym},4}(\rho)$ are expressed, respectively, as

$$E_{\text{sym}}(\rho) = \left. \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \right|_{\delta=0},$$

(2)

$$E_{\text{sym},4}(\rho) = \left. \frac{1}{4!} \frac{\partial^4 E(\rho, \delta)}{\partial \delta^4} \right|_{\delta=0}. $$

(3)

In Eq. (1), the absence of odd-order terms in $\delta$ is due to the exchange symmetry between protons and neutrons in nuclear matter when one neglects the Coulomb interaction and assumes the charge symmetry of nuclear forces. The higher-order (including the 4th-order) coefficients in $\delta$ are usually very small. For example, the magnitude of the $\delta^4$ term at normal nuclear matter density $\rho_0$ is estimated to be less than 1 MeV in microscopic many-body approaches and also in phenomenological nonrelativistic models as well as relativistic models as will be shown in this work. Neglecting the contribution from higher-order terms in Eq. (1) leads to the well-known empirical parabolic law, i.e.,

$$E(\rho, \delta) \simeq E_0(\rho) + E_{\text{sym}}(\rho)\delta^2$$

for the EOS of asymmetric nuclear matter and the symmetry energy $E_{\text{sym}}(\rho)$ can thus be extracted from $E_{\text{sym}}(\rho) \simeq E(\rho, \delta = 1) - E(\rho, \delta = 0)$.

Around normal nuclear matter density $\rho_0$, the $E_0(\rho_0)$ can be expanded, e.g., up to 4th-order in density, as

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!} \chi^2 + \frac{J_0}{3!} \chi^3 + \frac{I_0}{4!} \chi^4 + \mathcal{O}(\chi^5),$$

(4)

where $\chi$ is a dimensionless variable characterizing the deviations of the density from normal nuclear matter density $\rho_0$ and it is conventionally defined as

$$\chi = \frac{\rho - \rho_0}{3\rho_0}. $$

(5)

The first term $E_0(\rho_0)$ on the right-hand-side (r.h.s) of Eq. (4) is the binding energy per nucleon in symmetric nuclear matter at normal nuclear matter density $\rho_0$ and the coefficients of other terms are,

$$K_0 = 9\rho_0^2 \left. \frac{\partial^2 E_0(\rho)}{\partial \rho^2} \right|_{\rho=\rho_0},$$

(6)

$$J_0 = 27\rho_0^3 \left. \frac{\partial^3 E_0(\rho)}{\partial \rho^3} \right|_{\rho=\rho_0},$$

(7)

$$I_0 = 81\rho_0^4 \left. \frac{\partial^4 E_0(\rho)}{\partial \rho^4} \right|_{\rho=\rho_0}. $$

(8)

The linear $\chi$ term on the r.h.s of Eq. (4) vanishes according to the definition of the saturation density $\rho_0$. The coefficient $K_0$ is the well-known incompressibility coefficient of symmetric nuclear matter and it characterizes the curvature of the $E_0(\rho_0)$ at $\rho_0$. The coefficients $J_0$ and $I_0$ are the 3rd-order and 4th-order incompressibility coefficients of symmetric nuclear matter, respectively.

Similarly, around normal nuclear matter density $\rho_0$, the symmetry energy $E_{\text{sym}}(\rho)$ and the 4th-order symmetry energy $E_{\text{sym},4}(\rho)$ can be expanded, e.g., up to 4th-order in $\chi$, as

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L\chi + \frac{K_{\text{sym}}}{2!} \chi^2$$

$$+ \frac{J_{\text{sym}}}{3!} \chi^3 + \frac{I_{\text{sym}}}{4!} \chi^4 + \mathcal{O}(\chi^5),$$

(9)

and

$$E_{\text{sym},4}(\rho) = E_{\text{sym},4}(\rho_0) + L_{\text{sym},4}\chi + \frac{K_{\text{sym},4}}{2!} \chi^2$$

$$+ \frac{J_{\text{sym},4}}{3!} \chi^3 + \frac{I_{\text{sym},4}}{4!} \chi^4 + \mathcal{O}(\chi^5).$$

(10)
respectively, where the $L$, $L_{sym}$, $J_{sym}$, $I_{sym}$ and $L_{sym, 4}$, $J_{sym, 4}$, $I_{sym, 4}$ are the slope parameter, curvature parameter, 3rd-order and 4th-order density coefficients of the $E_{sym}(\rho)$ and $E_{sym, 4}(\rho)$ at $\rho_0$, respectively, whose definitions are similar to Eq. (3) - Eq. (8). In general, these characteristic parameters can be written as,

$$W_{ij} = (3\rho_0)^2 \frac{\partial^2 E_{sym, 2}(\rho)}{\partial \rho^2} \bigg|_{\rho = \rho_0} , \quad i, j = 1, 2, \cdots$$

(11)

for example, $W_{11} = L$, $W_{12} = K_{sym}$, $W_{23} = J_{sym, 4}$, $W_{24} = I_{sym, 4}$, and so on.

In the above Taylor’s expansions, we have kept all terms up to 4th-order in $\delta$. The 14 characteristic parameters, namely, $E_0(\rho_0)$, $K_0$, $J_0$, $I_0$, $E_{sym}(\rho_0)$, $L$, $K_{sym}$, $J_{sym}$, $E_{sym, 4}(\rho_0)$, $L_{sym, 4}$, $J_{sym, 4}$, $I_{sym, 4}$ and $L_{sym, 4}$ are well-defined, and they characterize the EOS of an asymmetric nuclear matter and its density dependence at normal nuclear matter density $\rho_0$. Among these parameters, $E_0(\rho_0)$, $K_0$, $E_{sym}(\rho_0)$, $L$ and $K_{sym}$ have been extensively studied in the literature and significant progress has been made over past few decades.

The incompressibility of asymmetric nuclear matter is an important quantity to characterize its EOS. Conventionally, the incompressibility coefficient is defined at the saturation density where the pressure $P(\rho, \delta) = \rho^2 \partial E(\rho, \delta)/\partial \rho = 0$, and it is called the isobaric incompressibility coefficient $K_{sat}(\delta)$ given by

$$K_{sat}(\delta) = \frac{9\rho_{sat}^2}{4} \frac{\partial^2 E(\rho, \delta)}{\partial \rho^2} \bigg|_{\rho = \rho_{sat}}.$$  

(12)

The isobaric incompressibility coefficient $K_{sat}(\delta)$ thus only depends on the isospin asymmetry $\delta$. One can show that up to 4th-order in $\delta$, the $K_{sat}(\delta)$ can be expressed as

$$K_{sat}(\delta) = K_0 + K_{sym} \delta^2 + K_{sym, 4} \delta^4 + O(\delta^6),$$  

(13)

with

$$K_{sym} = 9L - \frac{J_0 L}{K_0},$$

(14)

$$K_{sym, 4} = \frac{6L_{sym, 4}}{K_0} - \frac{J_0 L_{sym, 4}}{K_0} + \frac{9L^2}{2K_0} - \frac{J_0 L^2}{K_0},$$

(15)

If we use the parabolic approximation for the EOS of symmetric nuclear matter, i.e., $E_0(\rho) = E_0(\rho_0) + \frac{1}{2} K_0^2 \chi + O(\chi^3)$, then the $K_{sym, 2}$ parameter is reduced to

$$K_{asy} = K_{sym} - 6L,$$

(16)

and this expression has been extensively used in the literature to characterize the isoscalar dependence of the incompressibility of asymmetric nuclear matter in the literature.

### B. The 4th-order symmetry energy in the nonlinear RMF model

In the present work, we use the interacting Lagrangian density of the nonlinear RMF model supplemented with couplings between the isoscalar and the isovector mesons, i.e.,

$$\mathcal{L}(\psi, \sigma, \omega, \bar{\rho}_\mu) = \bar{\psi} \left[ \gamma_\mu \left( i \partial^\mu - g_\omega \omega^\mu \right) - (M - g_\sigma \sigma) \right] \psi$$

$$+ \frac{1}{2} \left( \partial^\mu \sigma \partial^\nu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{3} b_\sigma M (g_\sigma \sigma)^3$$

$$- \frac{1}{4} c_\sigma (g_\sigma \sigma)^4 + \frac{1}{2} m_\omega^2 \omega^\mu \omega^\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \frac{1}{4} g_\omega \left( g_\omega^2 \omega^\mu \omega^\nu \right)^2 + \frac{1}{2} m_\rho^2 \bar{\rho}_\mu \cdot \bar{\rho}_\mu$$

$$- \frac{1}{4} G_{\mu\nu} \cdot G^{\mu\nu} - \vec{g}_{\rho} \bar{\rho}_\mu \cdot \vec{\omega}$$

$$+ \frac{1}{2} L_{\phi}^2 \bar{\rho}_\mu \cdot \bar{\rho}_\mu \left[ \Lambda_S (g_\sigma^2 \sigma^2 + \Lambda_V g_\omega^2 \omega^\mu \omega^\nu) \right]$$

(17)

where $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $G_{\mu\nu} \equiv \partial_\mu \bar{\rho}_\nu - \partial_\nu \bar{\rho}_\mu$ are strength tensors for $\omega$ field and $\rho$ field, respectively, $\psi, \sigma, \omega, \bar{\rho}_\mu$ are nucleon field, isoscalar-scalar field, isoscalar-vector field and isovector-vector field, respectively, and the arrows denote the vector in isospin space. The $\Lambda_S$ and $\Lambda_V$ represent coupling constants between the isovector $\rho$ meson and the isoscalar $\sigma$ and $\omega$ mesons, respectively, which are important for the description of the density dependence of the symmetry energy. In addition, $M$ is the nucleon mass and $m_\sigma, m_\omega, m_\rho$ are masses of mesons.

In the mean field approximation, after neglecting effects of fluctuation and correlation, meson fields are replaced by their expectation values, i.e., $\bar{\sigma} \rightarrow \sigma, \bar{\omega}_0 \rightarrow \omega_\mu, \bar{\rho}_0^{(3)} \rightarrow \bar{\rho}_\mu$, where subscript “0” indicates zeroth component of the four-vector, superscript “(3)” indicates third component of the isosip, Furthermore, we also use in this work the non-sea approximation which neglects the effect due to negative energy states in the Dirac sea. The mean field equations are then expressed as

$$m_\sigma^2 \sigma = g_\sigma \left[ \rho_S - b_\sigma M (g_\sigma \sigma)^2 - c_\sigma (g_\sigma \sigma)^3 + (g_\rho \rho_0^{(3)})^2 \Lambda_S g_\sigma \sigma \right]$$

(18)

$$m_\omega^2 \omega_\mu = g_\omega \left[ \rho - c_\omega (g_\omega \omega_\mu)^3 - \Lambda_V g_\omega \omega_\mu \left( g_\rho \rho_0^{(3)} \right)^2 \right]$$

(19)

$$m_\rho^2 \rho_0^{(3)} = g_\rho \left[ \rho_p - \rho_n - \Lambda_S g_\rho \rho_0^{(3)} (g_\sigma \sigma)^2 - \Lambda_V g_\rho \rho_0^{(3)} (g_\omega \omega_\mu)^2 \right]$$

(20)

where

$$\rho = \langle \bar{\psi} \gamma^0 \psi \rangle = \rho_n + \rho_p, \quad \rho_S = \langle \bar{\psi} \gamma^i \gamma^0 \sigma \gamma^i \psi \rangle = \rho_{Sn} + \rho_{Sp},$$

(21)

are the baryon density and scalar density, respectively,
with the latter given by
\( \rho_{S,j} = \frac{2}{(2\pi)^3} \int_0^{k_F^j} \frac{dK}{\sqrt{|K|^2 + M_0^2}} \)
\( = \frac{M_0^j}{2\pi^2} \left[ k_F^j E_F^* - M_0^2 \ln \frac{k_F^j + E_F^*}{M_0^j} \right], \quad J = p, n \) (22)

In the above expression, we have \( E_F^* = \sqrt{k_F^j \gamma + M_0^2} \) with \( M_0^j = M - g_\sigma \bar{\sigma} \) being the nucleon Dirac mass and the Fermi momentum \( k_F^j = k_F^p(1 + \tau_3) \gamma^{1/3} \) with \( \tau_3 = 1 \) for neutrons and \( \tau_3 = -1 \) for protons, and \( k_F = (3\pi^2 \rho/2)^{1/3} \) being the Fermi momentum for symmetric nuclear matter.

The energy-momentum density tensor for the interacting Lagrangian density \(^7\) can be written as
\[ T^{\mu\nu} = \bar{\psi}i\gamma^\mu \gamma^5 \psi + \partial^\mu \phi \partial^\nu \phi - F^{\mu\nu} \omega_\eta - G^{\mu\nu} \rho_q - L g^{\mu\nu}, \] (23)
where \( g_{\mu\nu} = (+, -, -, -) \) is the Minkovski metric. In the mean field approximation, the mean value of time (zero) component of the energy-momentum density tensor is the energy density of the nuclear matter system, i.e.,
\[ \varepsilon = (T^{00}) = \varepsilon_{kin}^0 + \varepsilon_{kin}^p + \frac{1}{2} \left[ m_0^2 \bar{\sigma}^2 + m_2^2 \bar{\omega}_0^2 + m_0^2 \left( \rho_0^{(3)}(\bar{\sigma}) \right)^2 \right] + \frac{1}{3} b_\sigma \left( g_\sigma \bar{\sigma} \right)^3 + \frac{1}{4} c_\sigma \left( g_\sigma \bar{\sigma} \right)^4 + \frac{3}{4} \omega_\omega \left( g_\omega \bar{\omega}_0 \right)^4 + \frac{1}{2} \left( g_\rho \rho_0^{(3)} \right)^2 \left[ \Lambda_0 \left( g_\sigma \bar{\sigma} \right)^2 + 3 \Lambda_0 \left( g_\omega \bar{\omega}_0 \right)^2 \right], \] (24)
where
\[ \varepsilon_{kin}^j = \frac{2}{(2\pi)^3} \int_0^{k_F^j} \frac{dK}{\sqrt{|K|^2 + M_j^2}} \]
\( = \frac{1}{2\pi^2} \int_0^{k_F^j} k^2 dk \sqrt{k^2 + M_j^2} \]
\( = \frac{1}{4} \left[ 3E_F^* \rho_j + M_0^j \rho_{S,j} \right], \quad J = p, n \) (25)
is the kinetic part of the energy density. Similarly, the mean value of space components of the energy-momentum density tensor corresponds to the pressure of the system, i.e.,
\[ P = \frac{1}{3} \sum_{j=1}^{3} (T^{jj}) = P_{kin}^n + P_{kin}^p \\
- \frac{1}{2} \left[ m_0^2 \bar{\sigma}^2 + m_2^2 \bar{\omega}_0^2 + m_0^2 \left( \rho_0^{(3)}(\bar{\sigma}) \right)^2 \right] + \frac{1}{3} b_\sigma \left( g_\sigma \bar{\sigma} \right)^3 + \frac{1}{4} c_\sigma \left( g_\sigma \bar{\sigma} \right)^4 + \frac{1}{4} \omega_\omega \left( g_\omega \bar{\omega}_0 \right)^4 + \frac{1}{2} \left( g_\rho \rho_0^{(3)} \right)^2 \left[ \Lambda_0 \left( g_\sigma \bar{\sigma} \right)^2 + 3 \Lambda_0 \left( g_\omega \bar{\omega}_0 \right)^2 \right], \] (26)
where the kinetic part of pressure is given by
\[ P_{kin}^j = \frac{1}{3\pi^2} \int_0^{k_F^j} dk \frac{k^4}{\sqrt{k^2 + M_j^2}}, \quad J = p, n \] (27)

The binding energy per nucleon of the asymmetric nuclear matter can be calculated through the energy density by
\[ E(\rho, \delta) = \frac{\varepsilon(\rho, \delta)}{\rho} - M. \] (28)

Furthermore, the symmetry energy \( E_{\text{sym}}(\rho) \) can be obtained as
\[ E_{\text{sym}}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \bigg|_{\delta=0} = \frac{k_F^p}{6} \frac{g_\sigma^2 \rho}{Q_{\sigma}} + \frac{2k_F^p}{Q_{\rho}}, \] (29)
while the 4th-order symmetry energy \( E_{\text{sym},4}(\rho) \) can be expressed as
\[ E_{\text{sym},4}(\rho) = \frac{1}{4!} \frac{\partial^4 E(\rho, \delta)}{\partial \delta^4} \bigg|_{\delta=0} = E_{\text{kin}}^{\text{sym},4}(\rho) + E_{\text{sym},4}^M(\rho), \] (30)
where
\[ E_{\text{kin}}^{\text{sym},4}(\rho) = \frac{k_F^p}{648} \frac{4M_0^4 + 11M_0^2k_F^p + 10k_F^p}{E_F^p}, \] (31)
\[ E_{\text{sym},4}^M(\rho) = \frac{g_\sigma^2 \rho^3}{2Q_{\rho}^4} \left( \frac{\Lambda_0^2 - \Lambda_0^2 g_\sigma^2 \bar{\sigma}^2}{Q_{\sigma}} \right) \]
\[ + \frac{g_\sigma^2 \rho M_0^2 k_F^p}{24Q_{\sigma} E_F^3} \left( \frac{4A_0 g_\sigma^2 \bar{\sigma}^2}{Q_{\rho}^2} - \frac{M_0 k_F^p}{3E_F^p} \right), \] (32)
with \( E_{\text{kin}}^{\text{sym},4}(\rho) \) representing the kinetic part (including the interactions due to the nucleon effective mass) while \( E_{\text{sym},4}^M(\rho) \) the other part due to the interaction in the 4th-order symmetry energy, and \( E_F^p = \sqrt{M_0^2 + k_F^p} \). The coefficients \( Q_{\sigma}, Q_{\omega} \) and \( Q_{\rho} \) are defined as
\[ Q_{\sigma} = m_0^2 - g_\sigma \left( \frac{3\rho_0}{M_0^3} \right) + 2b_\sigma M_0 g_\sigma^3 \bar{\sigma} + 3c_\sigma g_\sigma^4 \bar{\sigma}^2, \] (33)
\[ Q_{\omega} = m_0^2 + 3c_\omega g_\omega^4 \bar{\omega}_0^2, \] (34)
\[ Q_{\rho} = m_0^2 + \Lambda_0 g_\sigma g_\sigma^2 \bar{\sigma}^2 + \Lambda_0 g_\omega g_\omega^2 \bar{\omega}_0^2. \] (35)

In the above expressions, all the fields are calculated in the case of symmetric nuclear matter, i.e., at \( \delta = 0 \).

The analytical expression of the symmetry energy \( E_{\text{sym}}(\rho) \), i.e., Eq. (29), is a well-known result firstly given in Ref. \[38\]. To our best knowledge, the formulas \[30-33\] give, for the first time, the analytical expression of the 4th-order symmetry energy \( E_{\text{sym},4}(\rho) \) in the RMF model, which are the main results of the present work. These analytical expressions allow us to evaluate accurately the 4th-order symmetry energy \( E_{\text{sym},4}(\rho) \) and thus study the higher-order corrections to the empirical parabolic approximation within the framework of the RMF model. Before presenting numerical results, it is instructive to analyze firstly the low density behavior of the \( E_{\text{sym},4}(\rho) \). When \( \rho \to 0 \), the magnitude of all fields will approach to zero and both \( E_F^p \) and \( M_0^j \) will
approach to $M$, leading to $E_{\text{sym},4}^M(\rho) \to 0$ from Eq. (32) and $E_{\text{sym},4}^{\text{kin}}(\rho) \to \frac{k_F^2}{162 M}$ from Eq. (31). Therefore, in the low density limit, we have

$$\lim_{\rho \to 0} E_{\text{sym},4}(\rho) \to \frac{1}{162} \frac{k_F^2}{M},$$

which is exactly the result from the free Fermi gas model as expected.

III. RESULTS AND DISCUSSIONS

A. The 4th-order symmetry energy and higher-order effects on the isobaric incompressibility of asymmetric nuclear matter

![Graph](image)

FIG. 1: (Color online) Density dependence of the 4th-order symmetry energy $E_{\text{sym},4}(\rho)$ as well as its kinetic part $E_{\text{sym},4}^{\text{kin}}(\rho)$ and interacting part $E_{\text{sym},4}^M(\rho)$ from two accurately calibrated interactions, i.e., FSUGold (a) and IU-FSU (b).

Shown in Fig. 1 is the density dependence of the 4th-order symmetry energy $E_{\text{sym},4}(\rho)$ as well as its kinetic part $E_{\text{sym},4}^{\text{kin}}(\rho)$ and interacting part $E_{\text{sym},4}^M(\rho)$ using two accurately calibrated interactions, i.e., FSUGold and IU-FSU. The FSUGold has been accurately calibrated to the ground-state properties of closed-shell nuclei, their linear response, and the structure of neutron stars while the IU-FSU is a recently developed effective interaction that improves the FSUGold by incorporating some of the recent constraints on properties of neutron stars. One can see from Fig. 1 that the $E_{\text{sym},4}(\rho)$ is quite small (less than 0.7 MeV) at normal nuclear matter density while it increases with density and can reach to about 7 MeV at $\rho = 1$ fm$^{-3}$. Furthermore, one can see that the kinetic part $E_{\text{sym},4}^{\text{kin}}(\rho)$ dominates over the interacting part $E_{\text{sym},4}^M(\rho)$ with the latter is generally negative.

In order to investigate higher-order $E_{\text{sym},4}(\rho)$ effects on the EOS of asymmetric nuclear matter, it may make more sense to calculate the ratio of the 4th-order symmetry energy to the symmetry energy, i.e., $E_{\text{sym},4}(\rho)/E_{\text{sym}}(\rho)$. In Fig. 2 we show this ratio as a function of density with interactions FSUGold and IU-FSU. It is seen that the ratio $E_{\text{sym},4}(\rho)/E_{\text{sym}}(\rho)$ has a very small value of about 2% around normal nuclear matter density $\rho_0$, but it can reach to about 6% $\sim 7\%$ at high densities (e.g., 1.0 fm$^{-3}$ or 6 $\sim 7\rho_0$). This result is essentially consistent with the nonrelativistic calculations in some phenomenological models. These features imply that the $E_{\text{sym},4}(\rho)$ may become important at higher densities in some extreme physical conditions such as in neutron star where the isospin asymmetry $\delta$ can be close to unity. As an example, in the next subsection, we shall study effects of the 4th-order symmetry energy on the proton fraction in $\beta$-stable neutron star matter. In addition, one can see from Fig. 2 that in the low density limit, we have $\lim_{\rho \to 0} E_{\text{sym},4}(\rho)/E_{\text{sym}}(\rho) = 1/27$ as expected from the free Fermi gas model.

The analytical expression of the 4th-order symmetry energy $E_{\text{sym},4}(\rho)$ allows us to calculate accurately the density slope and curvature parameters of $E_{\text{sym}}(\rho)$, i.e., $L_{\text{sym},4}$ and $K_{\text{sym},4}$, and thus obtain the accurate value of the higher-order isobaric incompressibility $K_{\text{sat},4}$ of asymmetric nuclear matter according to Eq. (10). Table I displays the characteristic parameters of asymmetric nuclear matter, namely, $\rho_0$, $E_0(\rho_0)$, $E_{\text{sym}}(\rho_0)$, $E_{\text{sym}4}(\rho_0)$, $K_0$, $I_0$, $L$, $K_{\text{sym}}$, $I_{\text{sym}}$, $L_{\text{sym}4}$, $K_{\text{sym}4}$, $J_{\text{sym}4}$, $K_{\text{asy}}$, $K_{\text{sat}2}$, $K_{\text{sat}4}$ and the ratios $K_{\text{sat}2}/K_{\text{asy}}$ and $K_{\text{sat}4}/K_{\text{sat}2}$, for the two accurately calibrated interactions FSUGold and IU-FSU. To see the variation of the higher-order characteristic parameters with the density dependence of the symmetry energy, we also include in Table I the results from 5 interactions denoted as FSU-I, FSU-II, FSU-III, FSU-IV and...
TABLE I: Characteristic parameters of asymmetric nuclear matter, namely, $\rho_0$ (fm$^{-3}$), $E_0(\rho_0)$ (MeV), $E_{sym}(\rho_0)$ (MeV), $E_{asy}(\rho_0)$ (MeV), $K_0$ (MeV), $J_0$ (MeV), $L$ (MeV), $K_{asy}$ (MeV), $K_{sat,2}$ (MeV), $K_{sat,4}$ (MeV), and the ratios $K_{sat,2}/K_{asy}$ and $K_{sat,4}/K_{sat,2}$, for different interactions.

|               | FSUGold | IU-FSU | FSU-I | FSU-II | FSU-III | FSU-IV | FSU-V |
|---------------|---------|--------|-------|--------|---------|--------|-------|
| $\rho_0$      | 0.148   | 0.155  | 0.148 | 0.148  | 0.148   | 0.148  | 0.148 |
| $E_0(\rho_0)$ | -16.3   | -16.4  | -16.3 | -16.3  | -16.3   | -16.3  | -16.3 |
| $E_{sym}(\rho_0)$ | 32.5  | 31.3   | 37.4  | 35.5   | 33.9    | 31.4   | 30.9  |
| $E_{asy}(\rho_0)$ | 0.66  | 0.67   | 0.66  | 0.66   | 0.66    | 0.66   | 0.78  |
| $K_0$         | 229.2   | 232.3  | 229.2 | 229.2  | 229.2   | 229.2  | 229.2 |
| $J_0$         | -521.6  | -288.5 | -521.6| -521.6 | -521.6  | -521.6 | -521.6|
| $I_0$         | 2815.7  | 4541.9 | 2815.7| 2815.7 | 2815.7  | 2815.7 | 2815.7|
| $L$           | 60.4    | 47.3   | 109.5 | 87.4   | 71.7    | 52.1   | 49.4  |
| $K_{sym}$     | -51.4   | 29.0   | 2.7   | -68.4  | -74.4   | -16.7  | 5.5   |
| $J_{sym}$     | 426.5   | 363.9  | 101.4 | 157.6  | 390.0   | 251.2  | 80.4  |
| $-I_{sym}$    | 6331.8  | 11346.5| 285.8 | 1364.1 | 4211.9  | 6136.2 | 4020.4|
| $L_{sym,4}$   | 1.9     | 1.8    | 1.9   | 1.9    | 1.9     | 2.3    |       |
| $K_{sym,4}$   | 0.5     | 0.1    | 0.5   | 0.5    | 0.5     | 0.1    |       |
| $J_{sym,4}$   | 5.0     | 6.3    | 4.8   | 4.8    | 5.2     | 5.1    |       |
| $K_{asy}$     | -413.8  | -255.1 | -654.3| -509.6 | -504.4  | -329.4 | -290.9|
| $K_{sat,2}$   | -276.4  | -196.3 | -405.1| -393.8 | -341.4  | -210.8 | -178.5|
| $K_{sat,4}$   | 3.0     | 47.6   | 338.6 | 183.4  | 50.0    | 12.9   | 32.4  |
| $K_{sat,2}/K_{asy}$ | 67%  | 77%    | 62%   | 66%    | 68%     | 64%    | 61%   |
| $-K_{sat,4}/K_{sat,2}$ | 1%   | 24%    | 84%   | 47%    | 15%     | 6%     | 18%   |

FSU-V for which the parameters ($\Lambda_S, \Lambda_V$) are selected as (0.00, 0.00), (0.00, 0.01), (0.00, 0.02), (0.00, 0.04) and (0.01, 0.03), respectively, while the $\rho_0$ parameter is adjusted accordingly to fix $E_{sym}(\rho_f) = 25.57$ MeV at $\rho_f = 0.1$ fm$^{-3}$ as in the FSUGold interaction. The other parameters for FSU-I, FSU-II, FSU-III, FSU-IV and FSU-V are exactly the same as in FSUGold (Note: the FSUGold corresponds to the case of ($\Lambda_S, \Lambda_V$) = (0.00, 0.03)). This is equivalent to solve a constraint equation about $\rho_0$, $\Lambda_S$ and $\Lambda_V$, i.e., $\frac{\partial E_{asy}}{\partial \rho_0} = 13.47$ MeV where $\hat{\sigma}$ and $\hat{\omega}$ in $Q_\sigma$ (See Eq. (33)) are determined by the properties of symmetric nuclear matter in FSUGold.

From Table I one can see that the $E_{sym,4}(\rho_0)$ is generally less than 1 MeV (about 0.66 MeV for most of the interactions considered here), consistent with that observed in Fig. I. These results about the $E_{sym,4}(\rho_0)$ are further in agreement with the calculations in the nonrelativistic models of MDI and Skyrme-Hartree-Fock [28], nicely verifying the empirical parabolic law around the normal nuclear matter density $\rho_0$.

As pointed out previously, the difference between $K_{sat,2}$ and $K_{asy}$ reflects the contribution from higher-order effects, namely, the value of $J_0L/K_0$, which has been usually neglected in many calculations in the literature [4, 8, 12, 14, 36]. From Table I one can see that neglecting the $J_0L/K_0$ term generally leads to 30-40% relative error for the $K_{sat,2}$ parameter and thus the $J_0L/K_0$ term contribution to the $K_{sat,2}$ parameter cannot be neglected simply, confirming the previous findings in the nonrelativistic studies [28].

Furthermore, it is seen from Table I that the value of higher-order $K_{sat,4}$ is generally small compared with that of $K_{sat,2}$ for most of the interactions considered here. In addition, one can see that the $K_{sat,4}$ becomes more important for the interactions with larger $L$ values and this is consistent with the nonrelativistic studies [28]. It should be noted that the higher-order $K_{sat,4}$ term can be safely neglected in the study of giant resonance of finite nuclei [42] where the isospin asymmetry $\delta$ is usually small, i.e., about 0.2.

B. Effects of $E_{sym,4}$ on the proton fraction in $\beta$-stable nuclear matter

In order to further illustrate the effects of the 4th-order symmetry energy on the EOS of asymmetric nuclear matter, we calculate the proton fraction $x_p$ in $\beta$-stable neutron star matter where the isospin asymmetry $\delta$ is generally close to 1. The chemical composition of the neutron star is determined by the requirement of charge neutrality and equilibrium with respect to the weak interaction ($\beta$-stable matter). From the binding energy per nucleon, i.e., Eq. (11), we can calculate the proton fraction, $x_p = (1 - \delta)/2$, for $\beta$-stable nuclear matter as found in interior of neutron stars. For neutrino free $\beta$-stable nuclear matter, the chemical equilibrium for the reactions
$n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$ requires

$$\mu_e = \mu_n - \mu_p = \frac{\partial E}{\partial \delta} = 4\delta E_{\text{sym}}(\rho) + 8\delta^2 E_{\text{sym},4}(\rho) + O(\delta^5) \quad (37)$$

where $\mu_i = \partial E_i/\partial x_i$ (i.e., $n, p, e, \mu$) is the chemical potential. For relativistic degenerate electrons, we have

$$\mu_e = \left( m_e^2 + k_F^2 \right)^{1/2} = \left[ m_e^2 + (3\pi^2 \rho x_e^2)^{2/3} \right]^{1/2} \simeq (3\pi^2 \rho x_e^2)^{1/3} \quad (38)$$

where $m_e = 0.511$ MeV is the electron mass, and $x_p = x_e$ because of charge neutrality.

Just above a nuclear matter density at which $\mu_e$ exceeds the muon mass $m_\mu = 0.105$ GeV, the reactions $e^- \to \mu^- + \nu_e + \bar{\nu}_\mu$, $p + \mu^- \to n + \nu_\mu$, and $n \to p + e^- + \bar{\nu}_e$ are energetically allowed so that both electrons and muons are present in $\beta$-stable nuclear matter, this alters $\beta$-stability condition to

$$\mu_n - \mu_p = \mu_e, \quad \mu_n - \mu_p = \mu_\mu = \left[ m_\mu^2 + (3\pi^2 \rho x_\mu)^2/3 \right]^{1/2} \quad (39)$$

with $x_p = x_e + x_\mu$.

4th-order symmetry energy is moderately important for the proton fraction, especially at higher densities. For the FSUGold (IU-FSU) interaction and $\rho = 1.0$ fm$^{-3}$, for instance, including the 4th-order symmetry energy in the parabolic approximation to the EOS of asymmetric nuclear matter will increase the proton fraction $x_p$ from 15.37% (13.44%) to 16.43% (14.81%), producing a relative variation of about 7% (10%). These results indicate that the 4th-order symmetry energy may have obvious effects on the proton fraction $x_p$ in $\beta$-stable $npe\mu$ matter and the parabolic approximation to the EOS of asymmetric nuclear matter may significantly underestimate the proton fraction, especially at higher densities. These features are consistent with the nonrelativistic Skyrme-Hartree-Fock calculations.

Furthermore, one can see from Fig. 3 that the difference between the results with the full EOS and with the one containing the terms up to the 4th-order symmetry energy is very small, indicating that the EOS of asymmetric nuclear matter including the terms up to the 4th-order symmetry energy (up to $\delta^4$ in Eq. (1)) could be a good approximation for the determination of the proton fraction in $\beta$-stable $npe\mu$ matter.

C. Effects of $E_{\text{sym},4}$ on core-crust transition density and pressure in neutron stars

The transition density $\rho_c$ is the baryon number density that separates the liquid core from the inner crust in neutron stars and it plays an important role in determining many properties of neutron stars [27, 38, 43–46]. One simple and widely used way to determine the core-crust transition density $\rho_c$ is the so-called thermodynamical method, which requires the system to obey the following intrinsic stability condition [44, 48],

$$-\left( \frac{\partial P}{\partial v} \right)_{\mu_{np}} > 0, \quad (40)$$

$$-\left( \frac{\partial \mu_{np}}{\partial q_c} \right)_{v} > 0, \quad (41)$$

where the $P = P_b + P_e$ is the total pressure of the $npe$ matter system with $P_b$ and $P_e$ denoting the contributions from baryons and electrons respectively, and the $v$ and $q_c$ are the volume and charge per baryon number. The $\mu_{np}$ is defined as the chemical potential difference between neutrons and protons, i.e., $\mu_{np} = \mu_n - \mu_p$. The pressure $P_e$ is only a function of the chemical potential difference $\mu_{np}$ by assuming the $\beta$-equilibrium condition is satisfied, i.e., $\mu_{np} = \mu_e$. By using the relation $\partial E_b(\rho, x_p)/\partial x_p = -\mu_{np}$ with $E_b(\rho, x_p)$ being energy per baryon from the baryons in the $\beta$-equilibrium neutron star matter and $x_p = \rho_p/\rho$, and treating the electrons as free Fermi gas, one can show [27] that the thermodynamical relations Eq. (10) and Eq. (11) are actually equivalent to the following condition

![FIG. 3: (Color online) Density dependence of the proton fraction $x_p$ in $\beta$-stable $npe\mu$ matter with FSUGold (a) and IU-FSU (b). Three cases, i.e., the full EOS of asymmetric nuclear matter (solid lines), its parabolic approximation (up to $\delta^2$ in Eq. (1)) (dotted lines), and further including the 4th-order symmetry energy (up to $\delta^4$ in Eq. (1)) (dashed lines), are considered.](image-url)
\[ V_{\text{thermal}} = 2\rho \frac{\partial E_0(\rho, x_p)}{\partial \rho} + \rho^2 \frac{\partial^2 E_0(\rho, x_p)}{\partial \rho^2} - \left( \frac{\partial^2 E_0(\rho, x_p)}{\partial \rho \partial x_p} \right)^2 \frac{\partial^2 E_0(\rho, x_p)}{\partial x_p^2} > 0, \]  

(42)

which determines the thermodynamical instability region of the \( \beta \)-equilibrium neutron star matter. The baryon number density that violates the condition Eq. (42) then corresponds to the core-crust transition density in neutron stars for the thermodynamical method.

With the EOS of asymmetric nuclear matter including the terms up to the 4th-order symmetry energy (i.e., up to \( \delta^4 \) in Eq. (1)), Eq. (42) is then reduced to

\[ V_{\text{thermal}} = \rho^2 \frac{\partial^2 E_0}{\partial \rho^2} + 2\rho \frac{\partial E_0}{\partial \rho} + \delta^2 \left[ \rho^2 \frac{\partial^2 E_\text{sym}(\rho)}{\partial \rho^2} + 2\rho \left( \frac{\partial E_\text{sym}(\rho)}{\partial \rho} \right) \right] + \delta^4 \left[ \rho^2 \frac{\partial^2 E_\text{sym,4}(\rho)}{\partial \rho^2} + 2\rho \left( \frac{\partial E_\text{sym,4}(\rho)}{\partial \rho} \right) \right] 
- \frac{\rho^2 \delta^2}{E_\text{sym}(\rho) + 6E_\text{sym,4}(\rho)} \left[ \frac{\partial E_\text{sym}(\rho)}{\partial \rho} + 2\delta^2 \left( \frac{\partial E_\text{sym,4}(\rho)}{\partial \rho} \right) \right]^2 > 0. \]  

(43)

The baryon number density that violates the condition Eq. (43) then corresponds to the core-crust transition density \( \rho_i^{4\text{th}} \) in neutron stars for the EOS of asymmetric nuclear matter including the terms up to the 4th-order symmetry energy (up to \( \delta^4 \) in Eq. (1)). The corresponding transition pressure \( P_i^{4\text{th}} \) at \( \rho_i^{4\text{th}} \) for the EOS of asymmetric nuclear matter including the terms up to the 4th-order symmetry energy (up to \( \delta^4 \) in Eq. (1)) is then given by

\[ P_i^{4\text{th}} = P_i^{b,4\text{th}} + P_i^{e,4\text{th}} \]

\[ = \left[ \rho^2 \left( \frac{\partial E_0}{\partial \rho} + \delta^2 \frac{\partial E_\text{sym}}{\partial \rho} + \delta^4 \frac{\partial E_\text{sym,4}}{\partial \rho} \right) \right]_{\rho=\rho_i} + \mu_c^{4\text{th}} \rho_c \]

\[ = \left[ \rho^2 \left( \frac{\partial E_0}{\partial \rho} + \delta^2 \frac{\partial E_\text{sym}}{\partial \rho} + \delta^4 \frac{\partial E_\text{sym,4}}{\partial \rho} \right) \right]_{\rho=\rho_i} + \left. \left\{ 4\rho \delta \cdot \frac{1 - \delta}{2} \cdot [E_\text{sym} + \delta^2 E_\text{sym,4}] \right. \right|_{\rho=\rho_i, \delta=\delta_i} \]  

(44)

where \( \delta_i \) is the isospin asymmetry of the \( \beta \)-equilibrium neutron star matter at the corresponding transition density.

The transition density obtained by neglecting the \( E_\text{sym,4}(\rho) \) term in the condition Eq. (43) corresponds to the core-crust transition density \( \rho_i^{2\text{nd}} \) in neutron stars for the parabolic approximation to the EOS of asymmetric nuclear matter (up to \( \delta^2 \) in Eq. (1)). The corresponding transition pressure at \( P_i^{2\text{nd}} \) for the parabolic approximation to the EOS of asymmetric nuclear matter (up to \( \delta^2 \) in Eq. (1)) is then expressed as

\[ P_i^{2\text{nd}} = P_i^{b,2\text{nd}} + P_i^{e,2\text{nd}} \]

\[ = \left[ \rho^2 \left( \frac{\partial E_0}{\partial \rho} + \delta^2 \frac{\partial E_\text{sym}}{\partial \rho} \right) \right]_{\rho=\rho_i} + \mu_c^{2\text{nd}} \rho_c \]

\[ = \left[ \rho^2 \left( \frac{\partial E_0}{\partial \rho} + \delta^2 \frac{\partial E_\text{sym}}{\partial \rho} \right) \right]_{\rho=\rho_i} + \left. \left\{ 4\rho \delta \cdot \frac{1 - \delta}{2} \cdot E_\text{sym} \right. \right|_{\rho=\rho_i, \delta=\delta_i} \]  

(45)

Due to simplicity, the \( \rho_i^{2\text{nd}} \) and \( P_i^{2\text{nd}} \) have been extensively applied to determine the inner edge of neutron star crusts within the nonrelativistic models \( [41, 48, 51] \) and recently in the RMF model \( [51] \) as well. However, recent studies based on some nonrelativistic models have demonstrated \( [27] \) that the parabolic approximation to the EOS of asymmetric nuclear matter may lead systematically to significantly higher core-crust transition densities and pressures, especially with stiffer symmetry energy functionals. It is thus very interesting to see how the higher-order \( E_\text{sym,4}(\rho) \) affects the transition density \( \rho_i \) and pressure \( P_i \) in the RMF model.

In Table I we show the \( \rho_i^{2\text{nd}}, \rho_i^{4\text{th}}, P_i^{2\text{nd}}, \) and \( P_i^{4\text{th}} \) obtained from the thermodynamical method with different interactions as in In Table II. It is interesting to see that including the 4th-order symmetry energy in the parabolic approximation to the EOS of asymmetric nuclear matter indeed reduces significantly the core-crust transition density \( \rho_i \), which is consistent with the nonrelativistic calculations \( [27] \). Furthermore, one can see that the 4th-order symmetry energy may have even more drastic effects on the core-crust transition pressure \( P_i \), namely, including the 4th-order symmetry energy in the parabolic approximation to the EOS of asymmetric nuclear matter reduces drastically the core-crust transition pressure \( P_i \). Therefore, our results indicate that the empirical parabolic approximation may cause large errors for the determination of the \( \rho_i \) and \( P_i \) in neutron stars in the nonlinear RMF model.
IV. SUMMARY

We have derived for the first time the analytical expression of the nuclear matter fourth-order symmetry energy $E_{\text{sym},4}(\rho)$ within the framework of the nonlinear RMF model. It should be mentioned that the analytical expression of $E_{\text{sym},4}(\rho)$ can be easily generalized to the case of the density dependent RMF model that has similar isospin structure as the nonlinear RMF model (See, e.g., Ref. [52]). This provides the possibility to investigate the higher-order $E_{\text{sym},4}(\rho)$ corrections to the widely used empirical parabolic law for the asymmetric nuclear matter in the RMF model. In the present work, as examples, we have investigated the $E_{\text{sym},4}(\rho)$ effects on the properties of asymmetric nuclear matter, the proton fraction $x_p$ in β-stable $npe\mu$ matter and the core-crust transition density $\rho_t$ and pressure $P_t$ in neutron stars within the nonlinear RMF model with two accurately calibrated interactions, i.e., FSUGold and IU-FSU.

Firstly, our results have indicated that the value of $E_{\text{sym},4}(\rho)$ at normal nuclear matter density $\rho_0$ is generally less than 1 MeV, and thus the empirical parabolic approximation $E(\rho, \delta) \simeq E_0(\rho) + E_{\text{sym}}(\rho)\delta^2$ has been nicely confirmed around $\rho_0$. However, at higher densities such as 1 fm$^{-3}$, the value of $E_{\text{sym},4}(\rho)$ can be about 7 MeV and the ratio of $E_{\text{sym},4}(\rho)/E_{\text{sym}}(\rho)$ can reach to about 7%. These results imply that the $E_{\text{sym},4}(\rho)$ may become nonnegligible at higher densities. Furthermore, the analytical form of the $E_{\text{sym},4}(\rho)$ allows us to study the higher-order effects on the isobaric incompressibility of asymmetric nuclear matter. Our results have indicated that the value of the higher-order $K_{\text{sat},4}$ is generally small compared with that of $K_{\text{sat},2}$, confirming the previous nonrelativistic calculations [25].

Secondly, for the proton fraction $x_p$ in β-stable $npe\mu$ matter, we have found that, compared with the results from the empirical parabolic approximation to the EOS of asymmetric nuclear matter, including the 4th-order symmetry energy $E_{\text{sym},4}(\rho)$ can enhance the proton fraction $x_p$ by about 10% at higher densities. These results indicate that the empirical parabolic approximation to the EOS of asymmetric nuclear matter may cause obvious errors for the determination of the proton fraction in neutron stars within the nonlinear RMF model, which is in agreement with the results from the nonrelativistic models [25].

Finally, we have demonstrated that including the 4th-order symmetry energy $E_{\text{sym},4}(\rho)$ in the parabolic approximation to the EOS of asymmetric nuclear matter can reduce significantly the core-crust transition density $\rho_t$ and furthermore it has even more drastic effects on the core-crust transition pressure $P_t$. Therefore, our results have clearly demonstrated that the extensively used empirical parabolic approximation to the EOS of asymmetric nuclear matter may lead systematically to significantly higher core-crust transition density $\rho_t$ and pressure $P_t$ in neutron stars within the nonlinear relativistic mean field model, confirming the previous finding based on nonrelativistic calculations [27].

Therefore, we conclude that the higher-order $E_{\text{sym},4}(\rho)$ in the EOS of asymmetric nuclear matter may have different effects on different quantities, and generally one cannot simply neglect them, especially under some extreme physical conditions, such as in neutron stars.

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