Hadronic Shower Energy Scale Uncertainty in the MINOS Experiment

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Abstract
In this paper we determine the model uncertainty in the calorimetric response of the MINOS detector to hadronic showers produced by neutrino interactions.

1 Introduction
The uncertainty in the incoming neutrino energy plays an important role in long-baseline neutrino oscillation experiments, in particular the MINOS experiment [1, 2]. The determination of $|\Delta m^2_{23}|$ from a reconstructed charged current energy spectrum involves model dependencies, even in the presence of a near detector. One reason is that in order to determine the neutrino energy for a given event, one needs a model for how much energy goes ‘missing’ in the nuclear environment. This missing energy is strongly detector dependent and includes nuclear binding, charged pion rest masses, and intranuclear absorption of produced hadrons. The absorption of pions can dramatically change the detector response since the pion energy is typically redistributed to a sizable number of nucleons, which are often below the detection threshold. In this note we will survey the components of a particular neutrino interaction model that contribute to this uncertainty and tabulate contributions to the hadronic shower energy scale uncertainty. The model used for this study is version v3.5.5 of NEUGEN3 [3], the event generator version used for the production of the 2008 round of Monte Carlo simulations by the MINOS experiment. Details of the MINOS analysis can be found elsewhere [4].

2 The NEUGEN3 Neutrino Event Generator
NEUGEN3 is a widely-used neutrino event generator that produces complete final states for neutrino-nucleus interactions for energies from 100 MeV to 100 GeV. It incorporates a
Fermi Gas Model as the basic nuclear model with some modifications for nucleon-nucleon correlations [5]. The cross section model includes quasi-elastic interactions [6], resonance production [7], coherent neutrino-nucleus scattering [8], and non-resonant inelastic scattering. The model for non-resonant inelastic scattering is a modified DIS model which can describe electron scattering structure function data down to very low $Q^2$ and was designed for use by neutrino experiments in the few-GeV energy range [9]. Although the events from this model often do not fall in the canonical DIS regime of $Q^2 > 1 \text{ GeV}^2$ and $W > 2 \text{ GeV}/c^2$, we will nonetheless in this document refer to the class of events generated by this model as “DIS events”. The most important model aspects for the hadronic shower scale uncertainty are the hadronization model, which determines the set of particles produced from a particular DIS event, the formation zone, and the intranuclear rescattering model, which determines how this set of hadrons is altered as altered by final-state interactions (fsi) as they exit the target nucleus. We will describe the hadronization, formation zone, and intranuclear rescattering models in somewhat more detail.

### 2.1 Hadronization Model

NEUGEN uses the AGKY model to describe the hadronization process from events produced by DIS interactions [10]. This model has two components. The low invariant mass component is called the modified-KNO model and is largely based on empirical parametrizations of hadronic system features like Koba-Nielsen-Olesen scaling [3, 11]. The high invariant mass component is JETSET [12]. The modified-KNO model is used for all DIS events below $W = 2.3 \text{ GeV}/c^2$ and JETSET is used for all DIS events above $W = 3.0 \text{ GeV}/c^2$. A linear transition is made between the two models for events with $2.3 \text{ GeV}/c^2 < W < 3.0 \text{ GeV}/c^2$.

### 2.2 Formation Zone

Within the past few years results on the production of hadrons in a nuclear environment have been released by HERMES [13], CLAS [14], and JLab Hall C [15]. It is now well established that it takes the hadrons time to ‘form’ in the nuclear medium and that during this time they interact with a reduced cross section. The modeling of this effect in INTRANUKE is based on a study from the SKAT neutrino experiment [16]. We utilize a very simple model where it takes the hadron a time $\tau_0$ to form during which its interaction cross section in the nucleus is set to zero. The value of $\tau_0$ is based on [16] and is taken to be 0.342 fm/c.

### 2.3 Intranuclear Rescattering

Like all neutrino event generator packages, the MINOS model for intranuclear rescattering of produced hadrons uses a semiclassical Intranuclear Cascade (INC) model. This subpackage of NEUGEN3 is called INTRANUKE and has undergone numerous revisions and updates since it was originally written [17]. The simulation is anchored to a large body of hadron-nucleus scattering data and has been compared with the results of neutrino bubble chamber scattering on neon [18].
The intranuclear cascade simulation proceeds in two steps. In the first step each produced pion and nucleon is stepped through the nucleus to determine if it interacts. The calculation at each point \((r)\) uses the mean free path based on the local density:

\[
\lambda(E, r) = \frac{1}{\rho(r)\sigma_{hN}(E)}
\]

where \(\rho\) is the nuclear matter density and \(\sigma_N\) in the total hadron-nucleon cross section \((\sigma_{tot})\). The density distribution for nuclei heavier than oxygen are modeled with a Woods-Saxon distribution, others are modeled with a Gaussian distribution.

A fundamental problem exists in that we are trying to use a semiclassical model to describe a quantum mechanical phenomenon. The primary advantage is that the highly inelastic processes such as pion absorption and inelastic scattering can be matched to data while quantum mechanical treatments are much more difficult or impossible. The primary disadvantage is that elastic scattering, a large contributor to the total scattering cross section at low energy, is primarily a wave process. The result is that a naive semiclassical model fails to correctly describe the measured hadron-nucleus cross sections at low hadron energy. In \textsc{INTRANUK\textsc{E}} we account for this by increasing the nuclear size in the transport calculation by a fraction of the hadron de Broglie wavelength. We take this fraction to be 0.5 by comparing with \(\pi + ^{56}\text{Fe}\) total cross section data.

In \textsc{INTRANUK\textsc{E}} each neutrino-produced hadron is allowed to reinteract in the nucleus only once. Once an interaction has taken place, a list of particles in the “final state” is determined and these particles are placed outside the nucleus. The choice of final state is derived from data on hadron interactions with iron. Since iron is the principal nucleus in the MINOS detector this approach will, by design, give the correct distribution of final states in iron at the expense of lacking the full generality of a complete INC model where multiple reinteractions are allowed to occur within the nucleus. The reaction types that are considered include charge exchange, elastic scattering, inelastic scattering, secondary pion production, and absorption and are implemented in the code essentially as energy-dependent branching ratios.

3 Methodology

In this analysis we have determined the contribution to the hadronic shower energy scale uncertainty from 15 sources. The general methodology can be described as follows:

1. **Identify Sources of Uncertainty:** The first task was to identify sources of uncertainty. We can view the generator as a set of physics models which are tuned to some external data. These models are developed in a certain theoretical framework and may involve significant assumptions or approximations. Likewise the data which is used for model tuning may possess substantial measurement uncertainty. We can broadly categorize these classes of uncertainties into *external data* uncertainties and *model* uncertainties.
| branching ratios                  | 1σ uncertainty (%) |
|----------------------------------|---------------------|
| parameter                        |                     |
| π charge-exchange                 | 50                  |
| π elastic                         | 10                  |
| π inelastic                       | 40                  |
| π absorption                      | 30                  |
| π secondary π production          | 20                  |
| N absorption                      | 20                  |
| N secondary π production          | 20                  |
| N elastic                         | 30                  |

| cross-sections                   | 1σ uncertainty (%) |
|----------------------------------|---------------------|
| parameter                        |                     |
| π total cross-section             | 10                  |
| N total cross-section             | 15                  |

Table 1: Uncertainties on intranuclear rescattering processes. The N elastic and total cross-section terms are 100% correlated.

2. **Quantify uncertainty**: Once we have enumerated all of the individual sources of uncertainty we have to quantify these uncertainties. For instance, what is the uncertainty on the pion absorption cross section itself? For external data uncertainties these input uncertainties are fairly straightforward to evaluate for one familiar with the external data in question.

3. **Evaluate output uncertainty**: All estimates presented here were determined by comparing the results of 4-vector simulations combined with a parametrized detector response model. The detector response model is a parametrization of the output of full GEANT3 simulations of the response of the MINOS detector to various particles over a range of energies. In each case 500k events were generated in the NuMI low energy beam. A reference sample of events was generated using NEUGEN3 v3.5.5. We then compute the change in the estimated response for a particular model change in bins of true shower energy. The shift in each bin is calculated as \((\text{shifted response} - \text{default response})/\text{(default response)}\).

4 **Uncertainty due to final state interactions**

4.1 **External Data**

For this part of the study we have evaluated the effect of the ten sources of uncertainty listed in Tab. 1. For this study we shifted each of these inputs by \(+1\sigma\). In the case of specific reaction cross sections, the other cross sections are scaled down in their original proportions so that the total scattering cross section is unchanged. In each case the stated uncertainty refers to the magnitude of the relevant branching ratio or cross-section. The underlying cross-sections and branching ratios are energy dependent as are their uncertainties. The values
adopted here correspond to the maximum value of the energy dependent uncertainties and are therefore overestimates.

The change in the estimated shower energy in MINOS due to $+1\sigma$ changes in each of these inputs is shown in Figures 1, 2, and 3. These results are in good agreement with those obtained using a different technique involving event-by-event reweighting of fully reconstructed MC events. The largest contribution coming from pion scattering is again the absorption process. The contributions from baryon rescattering are also sizable in the first few energy bins. For true shower energies of 0-500 MeV, the dominant process is quasi-elastic scattering. An increase to nucleon absorption produces a 4.5% decrease in the response in this energy bin. Similarly an increase in nucleon elastic scattering results in an increase in the response. This is due to the fact that although an increase in the elastic cross section is correlated with an increase in the total cross section, 71.9% of nucleons already reinteract, so the increase in the total cross section has a relatively small effect on the number of interacted nucleons. The elastic increase is therefore largely offset by a decrease in the fraction of nucleons which are absorbed, which accounts for the increase in response.

Figure 4 shows the contributions from all INTRANUKE external data sources added in quadrature.

4.2 Model Assumptions

In this work we also attempt to explicitly identify key assumptions in the models and evaluate their impact. This task can be difficult for two reasons. Firstly, assumptions in a model can be difficult to identify and may be all but invisible to any but the model creator. Secondly it is not always clear how to ‘undo’ an assumption. The best one can usually hope for is that there are two models, which incorporate different assumptions, which can be compared side-by-side, and that there is abundant external data available to constrain the models.

The first key assumption in INTRANUKE is the way in which the free particle hadron scattering cross sections are modified to account for the increased scattering cross section at low hadron energy. This change constituted the largest change to INTRANUKE for NEUGEN3 v3.5.5. The relevant NEUGEN3 parameter, EFNUCR was tuned to its default value of 0.5 based on a comparison to pion-iron scattering data. This setting was then checked for consistency with neutrino data.

There are numerous effects which one would expect to lead to different overall rescattering rates in pion and neutrino-induced reactions. The most fundamental is that at low energies the deBroglie wavelength of the hadrons is large and overlaps a significant fraction of the nucleus. One would therefore expect differences in the scattering of pions approaching from infinity and pions born in the nuclear environment. The external data (hadron and neutrino) is not sufficient to disentangle these differences since the statistics on the neutrino data are poor. There is therefore a question of which external data one should use to determine the uncertainty on the EFNUCR parameter. If one uses hadron data, the acceptable variation is $\pm 0.10$, while if one uses neutrino data the allowable range is $\pm 0.60$. For this estimate we have taken the most conservative approach and used the 0.60 value.

The left hand panel of Figure 5 shows the effect of changing EFNUCR from 0.50 to
Figure 1: The figures correspond to $+1\sigma$ shifts in $\pi$ charge exchange (top left), $\pi$ elastic (top right), $\pi$ inelastic (lower left) and $\pi$ absorption (lower right).
Figure 2: The results of $+1\sigma$ shifts to pion secondary pion production (left) and the pion scattering cross section (right).

1.10. For this comparison two other changes were also made to NEUGEN3. The first is a technical point - the increase in the nuclear size for low energy hadrons is normally limited to 0.75 times the nominal nuclear radius. For these large changes to the model this cutoff value is removed. The second is that in comparing with the neutrino data it is necessary to increase the pion absorption cross section to reach agreement for any value of EFNUCR. The $\pm 0.60$ allowable change to EFNUCR was determined including a $+0.5\sigma$ increase in the pion absorption cross section. Both of these effects are included in the simulated data set used for this calculation.

The second major assumption has to do with the amount of missing energy produced in pion / nucleon absorption reactions. When an absorption reaction occurs the pion energy is typically redistributed to 2-4 nucleons which carry the total energy of the absorbed particles. These 'cascade' particles typically emerge from the nucleus followed by lower energy evaporation nucleons and de-excitation photons as the nucleus fragments or returns to the ground state. INTRANUKE only simulates the cascade process and gives these nucleons the full pion energy in a phase space decay. The dominant mechanism for missing energy in this model is the production of low momentum nucleons which do not register in the detector.

We have evaluated this assumption by making a dramatic and unphysical change to the model. We produced a sample whereby pion and nucleon absorption produce eight-nucleon final states rather than the known four-nucleon state. The effect of this change is shown in the right panel of Figure 5.

As Figure 5 indicates, the model assumptions in INTRANUKE have a large role in determining the overall energy scale. Their quadrature sum is shown in Figure 6.
Figure 3: The results of $+1\sigma$ shifts to nucleon absorption (top left), secondary pion production by nucleons (top right), nucleon elastic scattering (bottom left), and the nucleon scattering cross section (bottom right).
Figure 4: Total uncertainty from all INTRANUKE external data sources.
Figure 5: The results of a change to EFNUCR (left) and the treatment of pion/nucleon absorption (right). Note scale change from previous plots.

5 Formation Zone

The treatment of the hadron formation zone used in NEUGEN3 is a model presented by the SKAT collaboration to describe their data [16]. Previous studies which look at detailed measurements of hadron attenuation in electron scattering experiments at the Jefferson Lab and HERMES have shown that this model fails to describe some important features, in particular the increase in nuclear attenuation at high $z = E_\pi/\nu$.

Our estimates for formation zone uncertainty thus come from two pieces. The first is the measurement uncertainty on the single parameter in our current model, the formation time, as measured by the SKAT experiment [16]. This uncertainty is taken to be 50%. The effect of a +50% increase in the formation time is shown in the left panel of Figure 7.

The second contribution, the overall model uncertainty, has been evaluated by using a preliminary new model which is in better agreement with more recent data on hadron attenuation. This model incorporates a more sophisticated modeling of the time development of the interaction cross section for hadrons produced in nuclei. This model is currently being compared with Jefferson Lab and HERMES data in preparation for inclusion in the next round of generator improvements. The change produced in going to the new model is shown in the right panel of Figure 7.

The quadrature sum of the two contributions to the formation zone uncertainty is shown in Figure 8.
Figure 6: Total uncertainty from all INTRANUKE model assumptions.
Figure 7: The results of a $+1\sigma$ change to the formation time (left) and use of a more sophisticated formation zone model (right).

6 Hadronization Model Uncertainties

We evaluate the overall hadronization model uncertainty based on the change in going from the carrot to v3.5.5 hadronization model. In this change many aspects of the model were changed which impact shower energy scale. These include particle multiplicities, the use of JETSET for hadronization, and the dynamics of the fragmentation process. The daikon version of NEUGEN3 does a much better job of describing the external data than the models in carrot so this change is conservative and certainly brackets any difference between the model and reality. For the evaluation of this component the evaluation sample was generated using the old hadronization model, but the rest of the simulation, in particular the intranuclear rescattering model, was the same as in v3.5.5. The uncertainty in the shower energy response coming from the hadronization model is shown in Figure 9.

7 Conclusion

The quadrature sum of the contributions to the shower energy scale uncertainty from fifteen independent sources is shown in Figure 10. Also shown are the contributions from each of the categories considered in this paper: hadronization model, INTRANUKE input data, INTRANUKE assumptions, and formation zone. The largest excursion in a single bin is 8.2% and occurs in the lowest energy bin.

There is a strong energy dependence to the uncertainty. The main reason is that the first two energy bins largely populated by quasi-elastic events which are strongly affected by intranuclear rescattering for two reasons - the hadron energies are low and these events are
Figure 8: Total uncertainty from all formation zone sources.
Figure 9: Changes to the hadronization model.
not subjected to the formation zone. At high energies the uncertainty is reduced because the formation zone carries most of the hadrons out of the nucleus before they have a chance to interact. For many MINOS analyses the hadronic energy scale uncertainty is characterized by a single number. When that is done, a conservative approach is advocated where the 8.2% value corresponding to the largest excursion a single energy bin should be used.

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Figure 10: Total uncertainty from all sources (solid black). Contributions from intranuke assumptions (blue), INTRANUKE input (dashed red), hadronization model (solid red), and formation zone (dashed black).
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