Analysis of Wave Groups in Crossing Seas Using Hilbert Huang Transformation

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Abstract

The random ocean waves have a natural tendency to form groups of waves that produced due to the interaction of lower and higher frequency wave components. In this paper Hilbert Huang Transformation is considered, which is the result of the empirical mode decomposition (EMD) and the Hilbert spectral analysis (HSA). HHT uses the EMD method to decompose a signal into so-called intrinsic mode functions, and uses the HSA method to obtain instantaneous frequency data. Using EMD method, any non-stationary data set can be decomposed into a finite and often small number of components, which are intrinsic mode functions (IMF). This decomposition method operating in the time domain is adaptive and highly efficient. Since the decomposition is based on the local characteristic time scale of the data, it can be applied to nonlinear and non stationary processes. This method is applied to the field data collected in the Natural Ocean Engineering Laboratory (NOEL, Reggio Calabria, Italy) and results on the wave groups in crossing sea are discussed.

Keywords: Wave groups; HHT; IMF; EMD; Groupiness factor.
1. Introduction

Johnson et al. (1978) and Bruun (1985) found that wave groups can have significant effect on the stability and behaviour of rubble mound structures. Medina and Hudspeth (1987) showed that wave grouping characteristics are strongly related to the statistical parameters of the sea state and the uncertainty of wave climates estimated from wave records can be influenced by wave groupiness. Burchart (1979) and Longuet-Higgins (1984) insisted to consider the concept of that wave grouping characteristics in random seas for designing coastal and ocean structures. Tucker (1950) demonstrated the importance of the groups of high waves in the generation of coastal long waves and harbor resonance. Goda (1970) was the first to give a simple methodology for analyzing the presence of wave groups in random seas. Long et al. (1995) proposed that, at any given time, the data may involve more than one oscillatory mode so the simple Hilbert transform cannot provide the full description of the frequency content for the general data. It is necessary to decompose the data into IMF components.

A different approach to analyse wave groups was proposed by Boccotti (see 2014 for a complete review), with the Quasi-Determinism theory. This theory shows that in a Gaussian process, when a very large wave occurs (very large with respect to the standard deviation of the free surface displacement), the wave elevation tends to a deterministic profile, which is proportional to the covariance function. Second-order effects were investigated by Fedele and Arena (2005), Arena et al. (2008), Romolo et al. (2014) for unimodal spectra and by Arena & Guedes Soares (2009) for bimodal one in undisturbed wave field. Some verification in field of the mechanics of wave groups and on marine structures have been executed in the Natural Ocean Engineering Laboratory, where it is possible to operate in the field with the same techniques used in laboratories equipped with wave tanks, thanks to some favourable natural conditions of the site. The facility is located on the waterfront of Reggio Calabria (Italy) in the East coast of the Strait of Messina, where a wind from NNW blows many days per week, generating wind waves with significant wave height within $0.20 \text{ m} < H_s < 0.60 \text{ m}$ and a peak period within $2.0 \text{ s} < T_p < 2.6 \text{ s}$, in regular operating conditions. Moreover, in situ some exceptional events were recorded with significant wave heights of $0.80 – 1.00 \text{ m}$ and peak periods ranging in $3.8 – 4.3 \text{ s}$. The wave spectra are very close to the JONSWAP spectra (Hasselmann et al., 1973), confirming the occurrence of wind wave sea states. Sometimes these wind waves are superimposed on swells propagating from the South of the Straits, with the possibility to test in situ models under the action of crossing sea states.

In the paper the Hilbert–Huang transform (HHT) is used, starting from data of small scale field experiments carried out in the Natural Ocean Engineering Laboratory of Reggio Calabria (Arena & Barbaro, 2013 – www.noel.unirc.it). The data used for this analysis is from Boccotti (2011). The schematic diagram of the data acquisition scheme is as shown in Fig.1.
2. Methodology

HHT is used to deal with both non-stationary and nonlinear data by decomposing the signal and to verify the existence of wave groups. In contrary to the methods like Smoothed Instantaneous Wave Energy Method (SIWEH) (Funk and Mansard, 1979), Wave group based on run length (Goda, 1970), Wavelet Analysis (Dong et al., 2008), this method is instinctive, direct and adaptive, with the basis of the decomposition based on the derived form of data. According to the HHT, the time series $X(t)$ can be defined as follows

$$X(t) = \sum_{i=1}^{n} c_i + r_n$$  \hspace{1cm} (1)

where, $n$ is the number of empirical modes and $r$ is residue which can be mean trend or constant.

To get the physical meaning, the zeroth spectral moment of the time series $X(t)$ and $n$ empirical modes of the decomposition of $X(t)$ is calculated and the above definition is verified. The values of zeroth spectral moments are shown in Table 1. The pressure head series of the measured data $X(t)$, and its decomposed components are shown in Fig.2. As per the definition of HHT and based on the concept of EMD, the zeroth spectral moments of the measured pressure head and its decomposed components are calculated and verified as per the equation (1).

Fig.2 Measured Pressure Head series and its decomposed components.
Table 1. Spectral moments.

| Data used      | Zeroth spectral moment ($m_0$) |
|----------------|-------------------------------|
| Measured data  | 0.0026                        |
| $c_1$          | 0.0021                        |
| $c_2$          | 0.000223                      |
| $c_3$          | 0.00012                       |
| $c_4$          | 0.000121                      |
| $c_5$          | 0.00001                       |
| Sum (from $C_1$ to $C_5$) | 0.002574 (~ 0.0026) = Actual zeroth moment from measured values |

3. Identification of wave groups

The presence of wave group is identified based on the theory mentioned in (Dean and Darlympe, 1984). The group celerity for the pressure signal is calculated from the identified wave group and also theoretically as follows.

![Fig. 3 Characteristics of wave group (Dean and Darlympe, 1984)](image_url)

The group celerity is calculated based on the following theory (Dean and Darlympe, 1984)

$$C_g = \frac{\Delta \sigma}{\Delta k}$$  \hspace{1cm} \text{(2)}

$$C_g = nC_g, \text{ where, } n = \frac{1}{2} \left[ 1 + \frac{2kh}{\sinh 2kh} \right]$$  \hspace{1cm} \text{(3)}

where, $C_g$ is the group celerity,  
$\sigma$ is the wave frequency,  
$k$ is the wave number and $h$ is the water depth.

Fig. 3 shows the wave profile, with respect to space. Similarly, the wave group is identified from the decomposed components obtained by applying HHT technique for the measured pressure head series. The typical wave group obtained by analysing the pressure head in a water depth of 5m is shown in Fig.4(a), with respect to time. Then the difference between the peaks is considered as the wave period and corresponding wave length is calculated by using the dispersion relationship which is shown in Fig.4(b). The instantaneous frequency denotes the frequency at the particular instant of time. The peaks in the instantaneous frequency denotes that the origin or end point of the crest or trough, which can be confirmed by comparing the time series of the wave groups and instantaneous frequency and is presented in Fig.4(c). Similar plots for other water depths (4m, 3m and 2m) are presented in Figs. 5(a-c), 6(a-c) and 7(a-c) respectively.

Then the group celerity is calculated based on the equation (3). In other way, the wave envelope is considered as a single wave and its velocity is computed. The velocity of the wave envelope is equal to the group celerity, which confirms the existence of wave group. The group celerity obtained from both the methods are tabulated in Table 2.
Fig. 4. Wave Envelope superposed over time series (a) temporal and (b) transformed wave length (at depth = 5m)

![Graph](image1)

Fig. 4(c). Instantaneous frequency (at depth = 5m)

![Graph](image2)

Fig. 5. Wave Envelope superposed over time series (a) temporal and (b) transformed wave length (at depth = 4m)

![Graph](image3)

Fig. 5(c). Instantaneous frequency (at depth = 4m)

![Graph](image4)

Fig. 6. Wave Envelope superposed over time series (a) temporal and (b) transformed wave length (at depth = 3m)

![Graph](image5)
Fig. 6(c). Instantaneous frequency (at depth = 3m)

Fig. 7. Wave Envelope superposed over time series (a) temporal and (b) transformed wave length (at depth = 2m)

Table 2. Comparison of Group Celerity to identify the existence of wave groups

| Depth (m) | Group Celerity (m/s) [from IMF] | Group Celerity (m/s) [Theoretical value] |
|-----------|---------------------------------|-----------------------------------------|
| 5         | 3.924                           | 3.916                                   |
| 4         | 3.913                           | 3.898                                   |
| 3         | 3.364                           | 3.834                                   |
| 2         | 3.500                           | 3.614                                   |

3.1 Wave Groupiness Factor

The Smoothed Instantaneous Wave Energy History (SIWEH) was defined by Funk and Mansard (1979) to identify and control the wave groupiness in time series of water surface elevation for the generation of grouped waves in tank. The SIWEH(m²) is computed as

\[
SIWEH(t) = \frac{1}{T_p} \int_{-\infty}^{\infty} \eta^2(t + \tau)Q(\tau)\,d\tau
\]  

(4)

Where Q(t) is the bartlett window.
The level of groupiness in a given record is then characterized by the groupiness factor $G_{SIWEH}$ defined as

$$G_{SIWEH} = \frac{\sigma_{SIWEH}}{SIWEH(t)}$$

(6)

where, the bar denotes the time average and $\sigma_{SIWEH}$ denotes the standard deviation of $SIWEH(t)$. The Hilbert spectrum $H(\omega, t)$ represents the distribution of the amplitude of each IMF against frequency and time. By integrating the spectrum against frequencies, the instantaneous energy signal of that mode $IMF(t) (m^2)$ is here calculated as

$$IMF(t) = \sum_j H^2(\omega_j(t), t)$$

(7)

Similarly to the SIWEH, the groupiness factor $GF_{IMF}$ may be formed as the ratio

$$GF_{IMF} = \frac{\sigma_{IMF}}{IMF(t)}$$

(8)

where, $\sigma_{IMF}$ denotes the standard deviation of $IMF(t)$.

The wave groupiness factor is estimated based by using the IMFs as given specified in Equation (8) is tabulated in Table 3

| Depth(m) | Groupiness factor(GF) | Mean Instantaneous frequency (Hz) |
|----------|-----------------------|-----------------------------------|
| 5        | 0.74                  | 0.5                               |
| 4        | 0.60                  | 0.46                              |
| 3        | 0.57                  | 0.51                              |
| 2        | 0.60                  | 0.39                              |

The wave groupiness factor is gradually decreasing, because of the steep slope and almost the waves break in the shoreline.

4. Summary and Conclusion.

The use of Hilbert Huang transformation to estimate is the best method to define the instantaneous variance function of a record. The Groupiness factor is used to study the cross-shore variations of the groupiness degree of irregular waves, for the experiments conducted in Natural Ocean Engineering Laboratory, Reggio Calabria, Italy with an approximate slope of 1:5, and considering irregular waves with spectra very close to the JONSWAP. The cross-shore variations of the GF are presented. Observations from the groupiness factor show that, there is no effect of breaking, because the waves break very near to the shoreline.
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