Noise Sensitivity in Continuum Percolation

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In critical bond percolation on \( \mathbb{Z}^2 \) (i.e., with \( p = 1/2 \)), consider the event that there is a horizontal crossing of the box \([n]^2\). Suppose that this event occurs for a particular (random) configuration \( x \in \{0, 1\}^E \), and let \( x^\varepsilon \) be obtained by re-randomizing each edge with probability \( \varepsilon > 0 \). What is the probability that there is a crossing in \( x^\varepsilon \)?

This question was first asked by Benjamini, Kalai and Schramm [1], who proved that the probability converges to 1 as \( n \to \infty \), and termed this phenomenon noise sensitivity. Their proof used techniques from discrete Fourier analysis, and built on ideas introduced in the famous paper of Kahn, Kalai and Linial [3]. In recent years much more precise results have been obtained about the Fourier spectrum of percolation, and about dynamical percolation on the triangular lattice (see [2, 5]).

In this talk we consider the corresponding question for Continuum Percolation, and in particular for the Poisson Boolean model (also known as the Gilbert disc model). Let \( \eta \) be a Poisson process of density \( \lambda \) in the plane, and connect two points of \( \eta \) by an edge if they are at distance at most 1. We prove that, at criticality, the event that there is a crossing of the box \([n]^2\) is noise sensitive. The proof is based on two extremely general tools: a version of the BKS Theorem for product measure, and a new extremal result on hypergraphs. The former result was first proved in [4]; we shall describe how it may be easily deduced from the uniform version.

This is joint work with Daniel Ahlberg, Erik Broman and Simon Griffiths.

References

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