On the Stability of Rapidity Gap Analysis

Liao Hongbo  Wu Yuanfang
Institute of Particle Physics, Huazhong Normal University, Wuhan 430079 China

Abstract

It is argued that the newly introduced moments of rapidity gaps for the event-by-event fluctuations depends on the number of events and multiplicity. The interesting ones of them are unstable under ISR energies of h-h collisions. The instability get well improved when multiplicity increases.
In recent decade, the event-by-event fluctuations become more and more important in multiparticle production and relativistic heavy ion collisions. This is partly due to the fact that the large local multiplicity fluctuations have been observed in all kind of collisions [1] and that the appearance of new state of matter, Quark Gluon Plasma (QGP), predicted by QCD, will definitely associate with large energy density fluctuations [2]. How to measure the fluctuations turn to be a powerful tool in probing the QGP phase transition. However, in the market at present, there is no good suggested measure for the purpose. In order to measure first the event-by-event fluctuations of low multiplicity samples, such as those in h-h collisions under ISR energies, R.C.Hwa and Q.Zhang newly introduced the rapidity gap analysis [4] after a long exploration for the end [5]. Though it has been used to analyze experimental data [3], the efficiency of the method for low energies of h-h collisions has not been seriously checked. In this letter, we are going to study the efficiency of the method.

The rapidity gap is defined by the difference of rapidity of two neighbor particles in an event,

\[ x_i = X_{i+1} - X_i, \quad i = 0, \ldots, N, \quad (1) \]

where \( X_i \) is the cumulant variable of rapidity of \( i \)th particle, which is free of the influence of energy conservation in rapidity distribution [6] and is uniformly distributed from 0 to 1. \( N \) is the total number of particles in the event. The set \( S_e \) of \( N + 1 \) number: \( S_e = \{ x_i \mid i = 0, \ldots, N \} \) provides information of rapidity distribution of all particles in the event. A quantitative character of single event therefore can be estimated by the moments of its rapidity gaps:

\[ G_q = \frac{1}{N+1} \sum_{i=0}^{N} x_i^q, \quad (2) \]
or:
\[ H_q = \frac{1}{N+1} \sum_{i=0}^{N} (1 - x_i)^{-q}, \]  

(3)

where moment order \( q \) is an integer. \( G_q \) and \( H_q \) vary from event to event. R. C. Hwa and Q. Zhang suggest the simplest moments of them:

\[ s_q = -\langle G_q \ln G_q \rangle, \]  

(4)

and

\[ \sigma_q = \langle H_q \ln H_q \rangle \]  

(5)

as the descriptions of event-by-event fluctuations of the sample. Here, \( \langle \ldots \rangle \) is the average over all the events in the sample. However, according to this description, the statistical fluctuations, which we are not interested in, are included. In order to reduce the statistical fluctuations, they further suggested the same estimation for pure statistical sample as

\[ s_{q}^{st} = -\langle G_{q}^{st} \ln G_{q}^{st} \rangle, \]  

(6)

and

\[ \sigma_{q}^{st} = \langle H_{q}^{st} \ln H_{q}^{st} \rangle \]  

(7)

and defined

\[ S_q = \frac{s_q}{s_{q}^{st}}, \]  

(8)

and

\[ \Sigma_q = \frac{\sigma_q}{\sigma_{q}^{st}} \]  

(9)

as interesting measure of event-by-event fluctuations of the sample, \( i.e., \) so called the measure of erraticity in terms of rapidity gaps.
From the definition of rapidity gaps above, it is clear that the rapidity gaps will be large (small) and vary violently (smoothly) from event to event if the multiplicity of the event is very low (high), such as h-h collisions under (above) ISR energies. Since rapidity gap $x_i$ is less than 1, event moments $G_q \ll 1$ and $H_q \gg 1$. The measure of $\Sigma_q$ is simply the amplification of $S_q$. The higher the moment order is, the bigger are the time of the amplification. In this case, a stable measure of $\Sigma_q$ requires much larger number of events than that of $S_q$ so that the fine structure of $H_q$ in the sample can be completely demonstrated. Moreover, for $H_q$, only those events with multiplicity $N \geq q + 1$ are available for the average. It makes the measure of $\Sigma_q$ even more unstable in comparison to $S_q$ in the same sample. How the measures of $S_q$ and $\Sigma_q$ depend on the number of events and multiplicity and how to get the stable measures of $S_q$ and $\Sigma_q$ are the questions that we are going to answer in this letter.

The simplest way of the investigation is to simulate a statistical sample, where the number of events and multiplicities are all controllable. It is enough for us to estimate the $s_{st}^q$ and $\sigma_{st}^q$ of the sample. The cumulant variables of rapidities of a statistical event with multiplicity $N$ are constructed by $N$ random number $X'_1, \cdots, X'_N$ which is uniformly distributed in $[0, 1]$. Here the distribution of multiplicity is taken from h-h collisions of NA22 experiments as an example. Then according to Eqs.(6)-(7), $s_{st}^q$ and $\sigma_{st}^q$ can be calculated.

Firstly, the dependency of $s_{st}^q$ on the number of events is presented in fig.1, where the number of events are 10,000, 20,000, $\cdots$, 70,000 and moment order is from $q = 1$ to 8. In these range of number of events, the behavior of all orders’ $\ln s_{st}^q$ are very stable. This results tell us that the measure of $S_q$ are stable in experimentally allowed number of events even if the multiplicity is low.

Then, let’s turn to the same dependency of $\sigma_{st}^q$. The results is given in Fig.2(a), (b) and (c) for the orders of moment $q = 1, 2, 3$ respectively. It is clear from the results that the higher order of moment is, the larger are the number of events for a stable measure
of $\sigma_{q}^{st}$. For $q = 1$, a stable measure can be reached at about $N_{\text{event}} = 100,000$, while for $q = 2$, $N_{\text{event}}$ has to go up to 500,000, for $q = 3$, $N_{\text{event}} = 1,500,000$ is out of the range of all experiments. Therefore, the measure of $\sigma_{q}^{st}$ is rather unstable in experimentally reachable number of events if multiplicity is low.

In order to improve the measure of $\sigma_{q}^{st}$, we slightly increase multiplicity from above $1 - 25$ to $11 - 35$ under same multiplicity distribution as NA22 for all corresponding multiplicities. The results are provided in Fig.3. Now for moment order $q$ from 1 to 5, the measures of $\sigma_{q}^{st}$ are all stable at number of events only about 10,000. The stability of $\sigma_{q}^{st}$ get well improved by slightly increasing multiplicity.

From the discussions and MC results of statistical sample above, the suggested measure of $S_{q}$ for rapidity gaps is stable even under ISR collision energies. However, as already pointed out in [4] that $S_{q}$ is essentially 1, and therefore not very interesting. Unfortunately, another suggested measure of rapidity gaps $H_{q}$ is unstable under ISR collision energies. But the instability gets well improved by increasing multiplicity of events in the sample. It means that the measure is applicable for h-h collisions above the ISR energies or for heavy ion collisions.
References

[1] E. A. De Wolf, I. M. Dremin and W. Kittel, *Phys. Rep.* **270**, 1(1996).

[2] L. Van Hove, *Z. Phys.* **C21**, 93 (1984); J. Kapusta and A. Vischer, *Phys. Rev.*, **C52**, 2725 (1995).

[3] Wang Shaoshun and Wu Chong, *Phys. Lett.* **B505**, 43(2001).

[4] Rudolph C. Hwa and Qing-hui Zhang. *Phys. Rev.* **D62**, 014003(2000).

[5] Rudolph C. Hwa, *Acta Phys. polon.* **B27**, 1789–1900(1996); Z. Cao and Rudolph C. Hwa, *Phys. Rev.* **D61**, 074011(2000).

[6] W. Ochs, *Z. Phys. C50* 3391991; A. Białas and M. Gazdzici, *Phys. Lett. B252*, 483(1990).

Figure Captions

**Fig. 1** $\ln s_q^{st}$ vs. $N_{\text{event}}$ for different order of moments $q$.

**Fig. 2** $\ln \sigma_q^{st}$ vs. $N_{\text{event}}$ for different order of moments $q$.

**Fig. 3** $\ln \sigma_q^{st}$ vs. $N_{\text{event}}$ for different order of moments $q$ after shifting multiplicity $n$ to $11 – 35$, where the full circles, open circles, full squares, open squares and full triangles represent $q = 1, 2, 3, 4, 5$ respectively.
Fig. 1
Fig. 2
Fig. 3