Algebraic Fermi liquid from phase fluctuations: “topological” fermions, vortex “berryons” and QED\textsubscript{3} theory of cuprate superconductors

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Within the phase fluctuation model for the pseudogap state of cuprate superconductors we identify a novel statistical “Berry phase” interaction between the nodal quasiparticles and fluctuating vortex-antivortex excitations. The effective action describing this model assumes the form of an anisotropic Euclidean quantum electrodynamics in (2+1)-dimensions (QED\textsubscript{3}) and naturally generates non-Fermi liquid behavior for its fermionic excitations. The doping axis in the $x$-$T$ phase diagram emerges as a quantum critical line which regulates the low energy fermiology.

Perhaps the most intriguing property of high temperature superconductors is the anomalous character of their normal state \cite{1}. This “strange metal” stands in stark contrast to the relatively benign features of the superconducting phase which can be understood rather accurately within the framework of a $d$-wave BCS-like phenomenology with well defined quasiparticle excitations \cite{2}.

In this Letter we propose a theory of the pseudogap phase in cuprate superconductors based on the following premise: a successful phenomenology of the strange metal should be built by starting from a comprehensive understanding of the adjacent superconducting state and its excitations. The spirit of our approach is the traditional one \cite{3, 4} but turned upside down. Usually, the strategy is to first understand the normal state before we can understand the superconductor. In the cuprates, however, it is the superconducting state that appears “conventional” and its quasiparticles “less correlated” and better defined. Having adopted this “inverted” paradigm, we proceed to study the interactions of the quasiparticles with the collective modes of the system, i.e., fluctuating (anti) vortices (our strategy here is similar to that of Ref. \cite{5}). We show that in $d$-wave superconductors these interactions take a form of a gauge theory which shares considerable similarity with the quantum electrodynamics in (2+1)-dimensions (QED\textsubscript{3}).

In the superconducting state, where vortices are bound, the gauge fields of the theory are massive and the low energy quasiparticles remain well defined excitations. This is the mundane Fermi liquid state in our inverted paradigm. In the normal state, however, as vortices unbind, our QED\textsubscript{3} like theory enters its massless phase and it abandons this “inverted Fermi liquid” protectorate in favor of a weakly destabilized Fermi liquid characterized by a power law singularity in the fermion propagator which we call algebraic Fermi liquid. We compute the spectral properties of fermions in our theory and find that it captures some key qualitative aspects of the available experimental data.

We concentrate on the portion of the pseudogap phase above the shaded region and below $T^*$ in Fig. 1. We assume that Cooper pairs are formed at or somewhat below $T^*$ but the long-range phase coherence sets in only at the superconducting transition temperature $T_{sc} \ll T^*$. Between $T_{sc}$ and $T^*$ the phase order is destroyed by unbound vortex-antivortex excitations of the Cooper pair field \cite{6, 7, 8}. In this pseudogap regime the $d$-wave superconducting gap is still relatively intact \cite{6, 7, 8} and the dominant interactions are those of nodal quasiparticles with fluctuating vortices. There are two components of this interaction: first, vortex fluctuations produce variations in superfluid velocity which cause Doppler shift in quasiparticle energies \cite{9}. This effect is classical and already much studied \cite{10, 11}. Second, there is a purely quantum “statistical” interaction, tied to a geometric “Berry phase” effect that winds the phase of a quasiparticle as it encircles a vortex \cite{12, 13}. It is this quantum mechanical interaction that ultimately causes the destruction of the Fermi liquid in the pseudogap phase.

Our starting point is the partition function

$$Z = \int D\Psi^\dagger(r, \tau) D\Phi(r, \tau) D\Phi^*(r, \tau) \exp \{-S\},$$

$$S = \int d\tau \int d^2r \{ \Psi^\dagger \partial_\tau \Psi + \Psi^\dagger \hat{H} \Psi + (1/g) \Delta^* \Delta \},$$

where $T^*$ is the phase critical temperature. The interaction is diffeomorphism-invariant and does not possess a Landau order parameter.

FIG. 1: Phase diagram of a cuprate superconductor.
where \( \tau \) is the imaginary time, \( \mathbf{r} = (x, y) \), \( g \) is an effective coupling constant, and \( \Psi^\dagger = (\tilde{\psi}_\tau, \psi_\tau) \) are the standard Grassmann variables. The Hamiltonian \( \mathcal{H} \) is given by:

\[
\mathcal{H} = \left( \mathcal{H}_e + \Delta \hat{\delta} - \mathcal{H}_e^* \right)
\]

with \( \mathcal{H}_e = \frac{1}{2m}(\hat{\mathbf{p}} - \hat{\mathbf{A}})^2 - \mathcal{E}_F \), \( \hat{\mathbf{p}} = -i \nabla \) (we take \( \hbar = 1 \)), and \( \Delta \) the d-wave pairing operator \( \hat{\psi}_{\tau} \Delta \hat{\psi}_{\tau}^\dagger \), where \( \Delta(\mathbf{r}, \tau) = |\Delta| \exp(i\varphi(\mathbf{r}, \tau)) \) is the center-of-mass gap function. \( \int D\varphi(\mathbf{r}, \tau) \) denotes an integral over smooth ("spin wave") and singular (vortex) phase fluctuations. Amplitude fluctuations are suppressed below \( T^* \).

It is convenient to eliminate the phase \( \varphi(\mathbf{r}, \tau) \) from the pairing term \( \hat{\psi}_\tau \) in favor of \( \partial_\mu \varphi \) terms \( |\mu = (x, y, \tau) \rangle \) in the fermionic action. In order to avoid dealing with non-single valued wavefunctions we employ the singular gauge transformation devised in Ref. [12]:

\[
\tilde{\psi}_\tau \rightarrow \exp(i\varphi_A)\tilde{\psi}_\tau, \quad \tilde{\psi}_\tau \rightarrow \exp(i\varphi_B)\tilde{\psi}_\tau,
\]

where \( \varphi_A + \varphi_B = \varphi \). Here \( \varphi_{A(B)} \) is the singular part of the phase due to \( A(B) \) vortex defects: \( \nabla \times \nabla \varphi_{A(B)} = 2\pi \delta(\mathbf{r} - \mathbf{r}_{A(B)}^I) \), with \( \delta = \pm 1 \) denoting the topological charge of \( i \)-th vortex and \( \mathbf{r}_{A(B)}^I \) its position. The labels \( A \) and \( B \) represent some convenient but otherwise arbitrary division of vortex defects (loops or lines in \( \varphi(\mathbf{r}, \tau) \)) into two sets. As discussed in [12] this transformation guarantees that the fermionic wavefunctions remain single-valued and the effect of branch cuts is incorporated directly into the fermionic part of the action:

\[
\mathcal{L}' = \tilde{\psi}_\tau [\partial_\tau + i(\partial_\tau \varphi_A)]\tilde{\psi}_\tau + \tilde{\psi}_\tau [\partial_\tau + i(\partial_\tau \varphi_B)]\tilde{\psi}_\tau + \overline{\Psi} \mathcal{H}' \Psi,
\]

where the transformed Hamiltonian \( \mathcal{H}' \) is:

\[
\left( \frac{1}{2m}(\hat{\mathbf{p}} + \mathbf{v})^2 - \mathcal{E}_F \right) D - \frac{1}{2m}(\hat{\mathbf{p}} - \mathbf{v})^2 + \mathcal{E}_F \right),
\]

with \( \mathbf{v} = \mathbf{a} \). The transformation \( \tilde{\psi}_\tau \) generates a "Berry gauge potential" \( a_\mu = \frac{1}{2}(\partial_\mu \varphi_A - \partial_\mu \varphi_B) \) which describes half-flux Aharonov-Bohm scattering of quasiparticles on vortices and mimics the effect of branch cuts in quasiparticle-Aharonov-Bohm scattering of quasiparticles on vortices

\[
\int D\varphi(\mathbf{r}, \tau) \text{ in which the half-flux-to-minus-half-flux } (\mathbb{Z}_2) \text{ symmetry of the singular gauge transformation } \tilde{\psi}_\tau \text{ is manifest: }
\]

\[
\mathcal{L} = \overline{\Psi} [(\partial_\tau + ia_\tau)\sigma_0 + iv_\tau \sigma_3] \Psi + \overline{\Psi} \mathcal{H} \Psi + \mathcal{L}_0[\nu_\mu, a_\mu],
\]

where \( \mathcal{L}_0 \) is the 'Jacobian' of the transformation given by:

\[
e^{-\int d\tau \mathcal{L}_0} \left( \int d^2r \mathcal{L}_0 \right)^{-1/2} \int D\varphi(\mathbf{r}, \tau) \mathcal{L}_0[\mathbf{v}, \mathbf{a}],
\]

Here \( \sigma_\mu \) are the Pauli matrices and \( \hat{\mathcal{H}} = \mathcal{H}' \). We call the quasiparticles \( \tilde{\psi}_\tau = (\tilde{\psi}_\tau^+, \tilde{\psi}_\tau^-) \) appearing in \( \tilde{\mathbf{H}} \) "topological fermions" (TF's). TF's are the natural fermionic excitations of the pseudogapped normal state. They are electrically neutral and are related to the original quasiparticles by the inversion of transformation \( \tilde{\psi}_\tau \).

To proceed we must extract the low energy, long distance properties of the Jacobian (4). This is done by focusing on the fluctuations of two gauge fields \( \nu_\mu \) and \( a_\mu \) in the fluid of vortex excitations. We use the saddle-point approximation to compute the leading (quadratic) terms in \( \mathcal{L}_0 \) for two cases of interest: i) the thermal vortex-antivortex fluctuations in 2D layers and ii) the space-time vortex loop excitations relevant for low temperatures (\( T \ll T^* \)) in the underdoped regime (but still above the shaded region in Fig. 1). The computation is straightforward but the algebra is laborious and will be presented elsewhere [13]. Here we quote only the final results whose form is ultimately dictated by the symmetries of the problem. For the case i):

\[
\mathcal{L}_0 \rightarrow \frac{T}{2\pi^2 n_l} \left[ (\nabla \times \mathbf{v})^2 + (\nabla \times \mathbf{a})^2 \right],
\]

where \( n_l \) is the average density of free vortex defects. Both \( \mathbf{v} \) and \( \mathbf{a} \) have a Maxwellian bare stiffness and are massless in the normal state. As one approaches \( T_{sc} \), \( n_l \approx \xi_{sc}^{-2} \rightarrow 0 \), where \( \xi_{sc}(x, T) \) is the superconducting correlation length, and \( \mathbf{v} \) and \( \mathbf{a} \) become massive. Similarly, for the case ii), the quantum fluctuations of unbound vortex loops result in [14]:

\[
\mathcal{L}_0 \rightarrow \frac{1}{2\pi^2} \left[ K_\tau(\partial \times a)^2 + \sum_i K_i(\partial \times a)^2 \right],
\]

where \( K_\tau, K_i(i = x, y) \) are functions of \( x \) and \( T \): \( K_i \sim \xi_{sc}, K_\tau \sim \xi_{sc}, z \) being the dynamical exponent. The Maxwell form of \( \mathcal{L}_0 \) is dictated by symmetry: the bare propagators for \( \nu_\mu \) and \( a_\mu \), \( D^0_{\nu}(q, i\omega) \) and \( D^0_{a}(q, i\omega) \), are massless in the normal state and massive within a superconductor. Note that we dropped \( \nu_\mu \) from (3) – the reason for this is made apparent below.

The physical picture advanced in this Letter rests on the following observations: \( \nu_\mu \) couples to the TF "charge" in the same way as the real electromagnetic gauge field. Consequently, if we integrate out TF's in (3) to obtain...
the renormalized (or dressed) gauge field propagators \( \mathcal{D}_v(q, i\omega) \) and \( \mathcal{D}_a(q, i\omega) \), we find that \( \mathcal{D}_v^{-1}(q \to 0, i\omega = 0) \to \text{const.} \), i.e. the Doppler gauge field \( v_\mu \) is massive.

This is a consequence of the Meissner response of TFs. Physically, this means that the integration over the quasiparticles leads to the familiar long range interactions between vortices. In contrast, the “Berry” gauge field \( a_\mu \) couples to the TF spin. This implies that any contribution of TF’s to the stiffness of \( a_\mu \) must be massless: a singlet superconductor retains the global SU(2) spin symmetry ensuring that \( \mathcal{D}_a^{-1}(q = 0, i\omega = 0) = 0 \).

When we combine this with the bare propagators implied by Eqs. (2,3) the following physical picture emerges. In the superconducting state, both \( v_\mu \) and \( a_\mu \) are massive by virtue of vortex excitations being bound in finite loops. The massive character of \( v_\mu \) and \( a_\mu \) protects the coherent TF excitations from being smeared by vortex fluctuations. The coupling of TF’s to the gauge fields \( v_\mu \) and \( a_\mu \) is irrelevant. This is our inverted Fermi liquid phase.

In the normal (pseudogap) state, the situation changes dramatically. The bare propagators for \( v_\mu \) and \( a_\mu \) are now massless but the renormalization by the medium of TF’s screens these bare propagators and still keeps \( v_\mu \) massive. Thus, TF coupling to the Doppler shift and “spin-waves” remains irrelevant even in the normal state. The Berry gauge field \( a_\mu \), however, is now truly massless since the spin polarization in the medium of TF’s cannot fully screen the massless bare propagator. Instead, by computing the TF polarization bubble, we find \( \mathcal{D}_a^{-1} \propto 1/ (\omega^2 + q^2) \) for \( (q, i\omega) \to 0 \) [13]; stiffer than the Maxwellian form [24,12], but still massless. The massless gauge field \( a_\mu \) produces strong scattering at low energies and affects qualitatively the spectral properties of TF’s.

The low energy quasiparticles are located at the four nodal points of the \( d_{xy} \) gap function: \((\pm k_F, 0)\) and \((0, \pm k_F)\), hereafter denoted as (1,1) and (2,2) respectively. Linearizing the fermionic spectrum in the proximity of these nodes leads to the effective Lagrangian

\[
\mathcal{L}_D = \sum_{\alpha=1,2} \Psi^\dagger_\alpha \left[ D_{\tau} - i v_F D_\sigma \sigma_3 - i \xi \Delta D_y \sigma_1 \sigma_3 \right] \Psi_\alpha + \mathcal{L}_0[a_\mu],
\]

where \( \Psi^\dagger_\alpha \) is a two-component nodal spinor, \( \alpha \) is a node index, \( D_{\tau} = \partial_{\tau} + i a_\mu \), and \( \mathcal{L}_0 \) is given by (3). We have dropped the Doppler gauge field \( v_\mu \) since it is massive both below and above \( T_{sc} \) and irrelevant for our purposes.

The Lagrangian (7) and the physics it embodies are our main results. In the normal state \( a_\mu \) becomes massless and the problem of quasiparticle interactions with vortex fluctuations takes the form of topological fermions interacting with massless “berryons”, i.e. quanta of the Berry gauge field \( a_\mu \). We recognize the above theory as equivalent (apart from the intrinsic anisotropy) to the Euclidean quantum electrodynamics of massless Dirac fermions in \( (2+1) \)-dimensions (QED$_3$). The remarkable feature of QED$_3$ is that it naturally generates a non-Fermi liquid phenomenology for its fermionic excitations. This property of QED$_3$ has led to previous suggestions that it should be in some form relevant to cuprate superconductivity [9,10]. However, the physical content of QED$_3$ as an effective low energy theory in this Letter is entirely different from those earlier works.

We now discuss the low energy phenomenology governed by the TF propagator: \( G_{\alpha}^{-1}(k, \omega) = G_{\alpha 0}^{-1}(k, \omega) - \Sigma_\alpha(k, \omega) \), where \( G_{\alpha 0}^{-1} \) is a free Dirac propagator at node \( \alpha \). We first consider the \( T = 0 \) case in the isotropic limit \( (v_F = v_\Delta) \) where explicit results are readily obtained. In this case \( G_{\alpha 0}^{-1} = \omega - v_F k_x \sigma_3 - v_\Delta k_y \sigma_1 \) and we find

\[
\Sigma_\alpha = \frac{8}{3\pi^2 N} (-\omega + v_F k_x \sigma_3 + v_\Delta k_y \sigma_1) \ln(\Lambda/p), \quad (8)
\]

with \( p = (-\omega^2 + v_F^2 k_x^2 + v_\Delta^2 k_y^2)^{1/2} \), \( \Lambda \) a high energy cutoff, and \( N = 2 \) is the number of pairs of nodes.

The essential feature of the TF propagator is the singular behavior of the self-energy \( \Sigma_\alpha(k, \omega) \) which arises from the massless nature of the dressed beryron propagator \( \mathcal{D}_a(q, i\omega) \) and is logarithmic in the leading order. This result can be formalized as the leading term in a large \( N \) expansion. Ultimately, the resummation of such an expansion [10] yields a low energy singularity \( G_\alpha \propto p^{\eta - 1} \), with a small exponent \( \eta = -8/3\pi^2 N \) [13]. For our purposes, having to deal with both the anisotropy and finite \( T \), the leading order form (3) is more convenient since it allows for explicit computation of various quantities. Once we move beyond the leading order, the vertex corrections to \( \Sigma_\alpha \) are necessary and the algebra becomes impenetrable. Furthermore, the available experiments are unlikely to distinguish between \( \eta = 0^+ \) and a small finite exponent.

The singularity in \( \Sigma_\alpha \) heralds the breakdown of the Fermi liquid behavior in the normal state. To see this consider Eq. (6) for \( E_k \equiv (v_F^2 k_x^2 + v_\Delta^2 k_y^2)^{1/2} \ll |\omega| \). We find \( \Sigma_\alpha \propto -(8/3\pi^2 N) \omega \ln(\Lambda/\sqrt{|\omega|}) \). The residue of the fermion pole vanishes as \( \omega \to 0 \), \( Z(\omega) \sim 1/\ln|\omega| \), while its width goes as \( \Sigma''_\alpha = -(4/3\pi^2 N)|\omega| \). This behavior is reminiscent of the MFL expression for the self-energy, assumed on phenomenological grounds by Varma, Abrahams and collaborators [10]. Note, however, that our QED$_3$ TF propagator implied by Eq. (4) remains qualitatively different from the MFL Ansatz [13], both by the fact that \( \ln(\Lambda/p) \) is replaced by a weak power law (thus algebraic Fermi liquid) and by the momentum dependence of \( \Sigma_\alpha(k, \omega) \). As shown below it is this combined momentum-frequency dependence that provides a natural explanation for some of the remarkable features of the fermionic spectral function in cuprates observed in the ARPES experiments [13]. Also, we emphasize that our results apply to the pseudogap phase below \( T^* \). The physics of the normal state at higher temperatures is beyond the scope of our present theory.

Inspection of Eq. (8) reveals that \( \Sigma_\alpha \) has imaginary part only inside the cone defined by \( \omega^2 > v_F^2 k_x^2 + v_\Delta^2 k_y^2 \).
FIG. 2: Energy versus momentum distribution curves of $A(k, \omega)$. Left: EDC cut taken for $k = 0$ (coincident with a nodal point), and MDC cut taken for $\omega = 0$ and $k_y = 0$. Both curves have been broadened (by the same amount) to simulate the finite resolution of an ARPES experiment. Right: The corresponding spectral function density plots for isotropic (top) and anisotropic $v_F/v_A = 17$ (bottom) cases; to be compared with Figs. 1 and 2 of Ref. [19].

outside this cone $\Sigma''_a$ vanishes. This implies that TF spectral function plotted as a function of momentum at fixed $\omega$ (MDC) will be very sharp close to the Fermi surface, while the corresponding energy distribution curve (EDC) will be broad. This is illustrated in Fig. 3 where we plot the spectral function $A(k, \omega) = \pi^{-1} \text{Im}[G_a(k, \omega)]_{11}$ deduced from Eq. (8). We note that precisely such striking asymmetry between the EDC and MDC cuts is observed in the ARPES data [19].

These qualitative features of the spectral function survive at finite temperature and away from the isotropic limit. Unfortunately, away from this simple limit the precise form of the TF propagator is not known: as soon as the “relativistic” invariance of the $T = 0$ problem [6] is lost, analytic calculations become intractable. We find that for $T \ll \omega, E_k$ the self energy retains its $T = 0$ form Eq. (3) with a small temperature correction. On the other hand when $T \gg \omega, E_k$ we find $\Sigma''_a \sim T$, qualitatively consistent with the original MFL conjecture $\Sigma''_a \sim \max(\omega, T)$. We note that such $T$-linear scattering rate has been deduced from ARPES experiments [19].

In ARPES one measures the spectral function of real electrons not of TF’s. While the inversion of the transformation [3] after the phases have been coarsely-grained and replaced by the gauge fields is a daunting task, our theory insures the gauge invariance with respect to $a_\mu$ of the true electron propagator. The simplest such gauge invariant propagator is $C_{11}^{\text{lec}}(x, x') = \langle \exp(i \int x^x ds_a a_\mu)[\Psi(x)\Psi^\dagger(x')]|_{11}\rangle$, where $x = (r, \tau)$. By employing a gauge in which the line integral of $a_\mu$ vanishes [20], we have computed the asymptotic behavior of $C_{11}^{\text{lec}}(x, x')$. We find [14] that it exhibits a power law singularity with the exponent $\eta' = 2\eta = -16/3\pi^2 N$. This strongly suggests that the true electron propagator, whose precise form within QED$_3$ is unknown at present, will exhibit a power law with small positive exponent.

In conclusion, we argued that the pseudogap regime in cuprates can be modeled as a phase disordered d-wave superconductor. Such assumption naturally leads to a QED$_3$ theory for the massless Dirac “topological” fermions interacting with a massless gauge field of vortex “berryons”. Coupling to the massless gauge field destroys the Fermi liquid pole in the fermion propagator and generates algebraic Fermi liquid. Lacking any energy or length scale this theory can be thought of as being critical independently of the actual doping level $x$. Below $T^*$ the low energy spectral properties of the fermions are therefore regulated by a quantum critical line. In this regime the low energy fermiology, including thermodynamics, transport, and density and current responses are all controlled by the universal properties of topological fermions and vortex berryons encoded in the anisotropic QED$_3$ Lagrangian [7]. Eventually, this peculiar quantum critical behavior gives way to the actual superconducting phase at $T_c(x)$ and the Fermi liquid character of the nodal quasiparticles is restored as vortices bind into finite loops. At very low doping, hole Wigner crystal, SDW, and other low $T$ phases become possible, reflecting the strong Mott-Hubbard correlations.

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