Double white dwarfs and \textit{LISA}

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Abstract

Close pairs of white dwarfs are potential progenitors of type Ia supernovae and they are common, with the order of 100–300 million in the Galaxy. As such they will be significant, probably dominant, sources of the gravitational waves detectable by \textit{LISA}. In the context of \textit{LISA}'s goals for fundamental physics, double white dwarfs are a source of noise, but from an astrophysical perspective, they are of considerable interest in their own right. In this paper I discuss our current knowledge of double white dwarfs and their close relatives (and possible descendants) the AM CVn stars. \textit{LISA} will add to our knowledge of these systems by providing the following unique constraints: (i) an almost direct measurement of the galactic merger rate of DWDs from the detection of short period systems and their period evolution, (ii) an accurate and precise normalization of binary evolution models at shortest periods, (iii) a determination of the evolutionary pathways to the formation of AM CVn stars, (iv) measurements of the influence of tidal coupling in white dwarfs and its significance for stabilizing mass transfer, and (v) discovery of numerous examples of eclipsing white dwarfs with the potential for optical follow-up to test models of white dwarfs.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

In the early 1980s it was suggested that type Ia supernovae might come from close pairs of white dwarfs merging under the action of gravitational radiation losses (Iben and Tutukov 1984, Webbink 1984). It was later realized that the large number of systems needed to sustain the type Ia rate within the Galaxy under these models meant that double white dwarfs (henceforth DWDs) are likely to be a dominant source of gravitational waves for space-based interferometry, to the extent that over some frequency intervals of interest in the context of \textit{LISA}, DWDs may define \textit{LISA}'s noise floor (Evans \textit{et al} 1987, Hils \textit{et al} 1990).
Early searches for DWDs produced meagre returns, and predictions that 10% of all ‘single’ white dwarfs might in fact be double (Paczynski 1985) seemed wide of the mark. Robinson and Shafter (1987) found no DWDs amongst 44 targets, Foss et al. (1991) found none amongst 25, and Bragaglia et al. (1990) found one certain DWD together with a few candidates from 54 targets. Together with the system L870-2, discovered by Saffer et al. (1988), by the early 1990s only two DWDs had measured periods. This changed when advances in our understanding of white dwarf atmospheres led to the identification of white dwarfs of too low mass for a single star evolution (Bergeron et al. 1992). Optical spectroscopy showed that a large fraction of these objects are DWDs (Holberg et al. 1995, Marsh 1995, Marsh et al. 1995, Maxted et al. 2000, Moran et al. 1997). Since the turn of the millennium, further discoveries have followed from the SPY survey (Napiwotzki et al. 2004) and SDSS as detailed later.

These discoveries have established the presence of large number of DWDs and their importance as gravitational wave sources. There have been numerous studies of the likely impact of DWDs on LISA. These find that below a cutoff frequency of about 2 to 6 mHz, there are many systems whose signals are unresolved, while above this frequency individual systems are resolved, with the odd nearby system rising above the noise at somewhat lower frequencies (Hils and Bender 2000, Hils et al. 1990, Liu et al. 2010, Nelemans et al. 2001, Ruiter et al. 2010, Yu and Jeffery 2010).

Most of these studies have been concerned with predicting the gravitational wave (GW) signal from DWDs in LISA. My interest here is more what we can learn about DWDs from LISA that is hard to deduce from electromagnetic (EM) observations. The potential is great, with direct measurement of tidal coupling between white dwarfs and the first detections of DWDs in globular clusters where their numbers are expected to be dynamically enhanced (Shara and Hurley 2002), likely to come from LISA data.

2. The two types of double white dwarfs

DWDs split into two groups which have different properties, from both the EM and GW perspectives, although in each case the two groups, while physically distinct, can be difficult to distinguish observationally. The two groups are the detached and semi-detached DWDs. Detached DWDs are simple pairs of white dwarfs evolving towards shorter periods under the action of gravitational wave losses. The semi-detached systems, observationally identified as the AM CVn stars (see Solheim 2010 for a recent review), are systems in which stable mass transfer takes place from a Roche-lobe filling hydrogen-deficient star to a more massive companion white dwarf. From now on I will refer to all such systems as AM CVn stars. Hydrogen deficiency is necessary to reach short orbital periods; the hydrogen-rich counterparts to AM CVn stars are the cataclysmic variable stars which reach a minimum orbital period of around 80 min; I do not consider these further here. The Roche-lobe filling stars in the AM CVn stars must be at least partially degenerate to reach very short orbital periods. White dwarfs fit the bill, and although these systems are not necessarily ‘double white dwarfs’ for simplicity I will continue to use the umbrella term ‘DWDs’ for both classes.

The key difference between these two classes from an EM point of view is the presence of accretion in the AM CVn stars which can produce x-rays, atomic line emission and photometric variability. This is both a blessing and a curse: a blessing as it makes these systems, which are rare, easier to find, and a curse because we do not understand accretion well enough to estimate selection effects with certainty. From a GW standpoint, the key differences are (a) the system masses which in the case of the AM CVn stars can reach very low values (< 0.1 M☉) inaccessible to the detached DWDs, and (b) the time derivatives of the gravitational wave
frequencies which on the whole will be negative for the AM CVn stars but always positive for the detached DWDs.

3. Detached DWDs

For LISA, detached DWDs (for this section I will drop the ‘detached’ qualifier) will probably be the single dominant source class, the ‘main sequence’ of space-based gravitational wave astronomy. They are a class of huge current interest as they are the candidate progenitors of type Ia supernovae usually referred to as the ‘double degenerate’ (DD) scenario (Iben and Tutukov 1984, Webbink 1984). The fortunes of DWDs as type Ia progenitors have waxed and waned over the years when squared up against the ‘single degenerate’ (SD) model which supposes accretion from a hydrogen-rich companion (Nomoto 1982, Whelan and Iben 1973). Recent papers continue to show a lack of consensus (Di Stefano 2010, Gilfanov and Bogdán 2010) and it is of course possible that there are multiple progenitor classes, as suggested by evidence for bimodality in the delay time distribution of type Ias (Mannucci et al 2006, Ruiter et al 2009).

The early failures to find many DWDs have often been raised to argue against DDs as potential type Ia supernova progenitors (Branch et al 1995, Hachisu et al 1999), indeed, this perception remains current (Parthasarathy et al 2007). My view is that, within the admittedly rather large margins of error, this is not a huge problem given that DWDs with short merger times and ones with total masses close to the Chandrasekhar limit have been discovered (Karl et al 2003a, Moran et al 1997, Napiwotzki et al 2002). It is perhaps not often realized that the current sample of DWDs remains strongly biased towards low mass systems because these were specifically targeted in the searches that started in the 1990s as well as in more recent searches. Similarly, the enormous difference in our ability to find DD versus SD progenitors should not be underestimated: while it is possible to see potential SD type Ia progenitors in other galaxies (although not necessarily to recognize them as such), it is hard to follow DWDs using EM observations if they are more than a few hundred parsecs away: finding DWDs is hard work. The best prospect for an observational calibration of DWD numbers is offered by the SPY survey (Napiwotzki et al 2004) that did not target particular mass ranges, although even it suffers unavoidable selection biases with respect to both mass and temperature that need allowing for.

To understand what LISA can bring to the study of DWDs, it is important to know first what EM observations can tell us. Table 1 lists the periods and masses of DWDs with published orbital periods. The mass of the brighter component can usually be measured by modelling its optical spectrum. Sometimes both components are visible and then both masses can be measured, but often one can only deduce a lower limit to the mass of the unseen component from the orbital motion of its companion. One can sometimes measure the temperatures of both components and thus the difference between the formation times of each component, a strong discriminator of the prior evolution (van der Sluys et al 2006). The number of detached DWDs in the Galaxy can approximately be estimated from the fraction of systems observed to be DWD and the total number of white dwarfs in the Galaxy. This approach gives a number of systems ranging from 20 to 200 million (Holberg et al 2008, Maxted and Marsh 1999). Binary population synthesis studies have given numbers from around 100 to 400 million (Han 1998, Liu et al 2010, Nelemans et al 2001, Yu and Jeffery 2010).

In comparison with the best EM observations, LISA will give us comparatively limited information on individual systems, yet there are several ways in which LISA can provide greatly superior information on DWDs as a whole, as I now discuss.
Table 1. Detached double white dwarfs ordered by the orbital period. The references are of the discovery papers. Objects starting with ‘J’ are SDSS white dwarfs.

| Name          | $P$ (days) | $M_1$ ($M_\odot$) | $M_2$ ($M_\odot$) | Ref.  | Name          | $P$ (days) | $M_1$ ($M_\odot$) | $M_2$ ($M_\odot$) | Ref.  |
|---------------|------------|-------------------|-------------------|-------|---------------|------------|-------------------|-------------------|-------|
| J1053+5200    | 0.043      | 0.20              | $>0.26$           | 1     | PG1713+332    | 1.127      | 0.35              | $>0.18$           | 5     |
| J1436+5010    | 0.046      | 0.24              | $>0.46$           | 1     | WD1428+373    | 1.157      | 0.35              | $>0.23$           | 15    |
| WD0957-666    | 0.061      | 0.37              | 0.32              | 2     | WD1022+050    | 1.157      | 0.39              | $>0.28$           | 15    |
| J0849+0445    | 0.079      | 0.17              | $>0.64$           | 3     | WD0136+768    | 1.407      | 0.47              | 0.37              | 16    |
| WD1704+481    | 0.145      | 0.39              | 0.56              | 4     | WD1202+608    | 1.493      | 0.3               | $>0.25$           | 17    |
| PG1101+364    | 0.145      | 0.36              | 0.31              | 5     | WD0135-052    | 1.556      | 0.47              | 0.52              | 18    |
| PG2331+290    | 0.166      | 0.39              | $>0.32$           | 6     | WD1204+450    | 1.603      | 0.46              | 0.52              | 16    |
| J1257+5428    | 0.190      | 0.20              | $>0.95$           | 7, 8  | WD0326-273    | 1.875      | 0.51              | $>0.59$           | 12    |
| NLTT 11748    | 0.236      | 0.15              | 0.71              | 9     | WD1349+144    | 2.209      | 0.44              | 0.44              | 19    |
| J0822+2753    | 0.244      | 0.17              | $>0.76$           | 3     | HE1511-0448   | 3.222      | 0.48              | $>0.46$           | 12    |
| HE2209-1444   | 0.277      | 0.58              | 0.58              | 10    | PG1241-010    | 3.347      | 0.31              | $>0.37$           | 6     |
| J0917+4638    | 0.316      | 0.17              | $>0.28$           | 11    | PG1317+453    | 4.872      | 0.33              | $>0.42$           | 6     |
| WD1013-010    | 0.437      | 0.44              | $>0.38$           | 12    | WD2032+188    | 5.085      | 0.41              | $>0.47$           | 6     |
| HE1414-0848   | 0.518      | 0.71              | 0.52              | 13    | WD1824+040    | 6.266      | 0.43              | $>0.52$           | 15    |
| WD1210+140    | 0.642      | 0.23              | $>0.38$           | 12    | WD1117+166    | 30.09      | 0.7               | 0.7               | 20    |
| LP 400-22     | 1.010      | 0.19              | $>0.41$           | 14    |               |            |                   |                   |       |

1. Mullally et al (2009), 2. Moran et al (1997), 3. Kilic et al (2010), 4. Maxted et al (2000), 5. Marsh (1995), 6. Marsh et al (1995), 7. Badenes et al (2009), 8. Kulkarni and van Kerkwijk (2010), 9. Steinfadt et al (2010), 10. Karl et al (2003a), 11. Kilic et al (2007), 12. Nelemans et al (2005), 13. Napierwotzki et al (2002), 14. Kilic et al (2009), 15. Morales-Rueda et al (2005), 16. Maxted et al (2002), 17. Holberg et al (1995), 18. Safron et al (1988), 19. Karl et al (2003b), 20. Maxted et al (2002).

3.1. Population statistics

At high enough frequencies, LISA will be sensitive to DWDs throughout the Galaxy and will give us a view of the whole population with relatively little selection. Several studies have predicted that $\sim 10,000$ DWDs should have high enough frequencies to be resolvable by LISA (Nelemans et al 2001, Ruijter et al 2010). These will be the shortest period systems, which are those of most relevance to the merger rate of DWDs, a quantity of great interest in the context of type Ia supernovae. EM observations, which are only sensitive out to a limited distance, will always be handicapped in comparison. Assuming a single chirp mass, $M_c = M_1^{1/3}M_2^{1/3}(M_1 + M_2)^{-1/5}$, the flux of DWDs crossing orbital period $P$ at a time $t_0$ since the formation of the Galaxy is given by

$$F(P, t_0) = -n(P, t_0) \dot{P} = \int_{P'}^{P_m} B(P', t_0 - \tau(P' \rightarrow P)) \, dP',$$

where $\tau(P' \rightarrow P)$ is the time taken for a system to the change period from $P'$ to $P$, $n(P, t)$ is the orbital period distribution and $B(P, t)$ is the birth rate period distribution at time $t$. A more realistic model would require integration over the distribution of chirp masses as well. The upper period limit $P_m$ is set by the maximum period that is able to evolve to period $P$ within the lifetime of the galaxy, i.e.

$$\tau(P_m \rightarrow P) = t_0.$$

For the short period systems that we are interested in for LISA, $P_m$ is around 5 to 15 h. As we approach short periods ($P \rightarrow 0$) $P_m$ will tend to a constant and thus the flux $F = -n \dot{P}$
will tend to a constant, i.e. the DWD merger rate. The numbers per unit period then scale as

\[ n \propto 1/P, \] 

or equivalently

\[ n \propto \tau_m/P \propto P^{5/3} \]

where \( \tau_m \) is the merger time at period \( P \)

\[ \tau_m = 1.00 \times 10^7 (M_\odot/M_c)^{-5/3} (P/1\text{h})^{8/3} \text{yr}, \]

assuming that we can neglect the effect of tides, although these are likely to be significant at these short periods (Willems et al 2007). The rapid reduction in lifetime with period makes it hard for EM-based searches to probe the short-period end of the birth rate distribution directly; this was realized by Robinson and Shafter (1987) as the major caveat on their null result. Looking at table 1, EM observations are unlikely to provide strong constraints upon the integrand of equation 1 for periods much below 1 h. At such periods the merger times are of the order 10 to 100 million years, depending upon mass, i.e. at least 100 times shorter than the age of the Galaxy, and still 20 times shorter than the time for which white dwarfs display strong spectral features. It is probably no coincidence that the shortest period systems known have low masses since this increases their survival time.

3.2. DWD sub-types

For some fraction of the resolved \textit{LISA} sources, it will be possible to detect not just the frequency \( f \), but its time derivative \( \dot{f} \). For detached DWDs this is a function of period and chirp mass only. The best EM observations can return \( M_1 \) and \( M_2 \) separately, but \textit{LISA} wins through the very large number of likely detections, sensitivity to those of the high mass and short period, and well-understood selection effects. The outcome of DWD mergers depends upon their masses and their composition. For instance the canonical type Ia model for DWDs involves the merger of two carbon–oxygen white dwarfs. To a large extent, the bulk composition of white dwarfs is thought to map into their mass. If so then, as figure 1 illustrates, chirp mass measurements have good potential when combined with population synthesis to discriminate the fraction of potential type Ia supernovae, pairs of helium white dwarfs, etc. Example numbers are around 10 000 resolvable DWDs \textit{LISA} (Liu et al 2010, Nelemans 2003, Ruiter et al 2010), around 600 of which will have detectable frequency changes within 1 year (Nelemans 2003, Ruiter et al 2010), and presumably many more over longer intervals. The different types (He+He etc) lead to a very strongly structured chirp mass distribution illustrated in figure 7 of Liu et al (2010) supporting the potential that chirp masses hold for probe DWD evolution.

4. AM CVn stars

In an AM CVn star, degenerate or semi-degenerate donor stars lose mass to white dwarf companions (note that there are similar systems, the ultra-compact x-ray binaries in which the accretors are neutron stars). As they do so, they expand, and the orbital periods lengthen. \textit{LISA} sources with \( \dot{f} < 0 \) are therefore likely AM CVn stars. The known systems have orbital periods that range from just over 5 to 65 min. Periods this short require the mass donors to be largely or entirely hydrogen-deficient in order to be dense enough to fit within their Roche lobes. Most known examples do indeed lack hydrogen, the exception being HM Cnc (Reinsch et al 2007). A key unsolved issue for these systems is how they form. Attention has been focused on three types of progenitor: (i) detached DWDs, (ii) evolved cataclysmic variable stars, and (ii) white dwarf/helium star accreting binaries. At the long periods of most known systems, these three models lead to rather subtle differences in the mass transfer rate and other parameters (Deloye et al 2007) which even the best constrained systems are not yet capable of distinguishing (Copperwheat et al 2011). We know DWDs of short enough period to merge
within a Hubble time, while there are no clear progenitors of the other two routes. However, it is not obvious that DWDs will survive the onset of mass transfer because of the instability that can set in if the two white dwarfs are of a similar mass (Marsh et al 2004). Indeed, until recently all DWDs of a known mass ratio were candidates for merging (figure 2).
The differences are more marked at short periods. For instance, only DWDs are thought to be able to reach periods well below 10 min. HM Cnc ($P = 321$ s) is the only such system known (Ramsay et al 2002, Roelofs et al 2010). HM Cnc is optically faint and has a soft x-ray spectrum that could easily be absorbed if it were more distant. LISA’s sensitivity to short period systems holds great potential for finding more such systems and for elucidating the nature of their donors. This can first be carried out in a statistical manner through the frequency distribution of those systems with $f < 0$. This may tell us the nature of the progenitors that survive the onset of mass transfer.

4.1. Spin–orbit coupling and AM CVn numbers

The degree to which detached DWDs survive mass transfer to live on as gravitational wave sources is highly dependent upon the degree to which the angular momentum accreted onto the more massive component is fed back into the orbit (Marsh et al 2004). Some theoretical studies have indicated that this coupling is strong and stabilizing (Motl et al 2007, Racine et al 2007), while others suggest that much of the angular momentum can be fed back even in the absence of tidal coupling between the stars (Sepinsky et al 2010). However the best observational calibration of the space density of AM CVn stars gives a space density around a factor of 10 lower than previously assumed, and around 250 times lower than that of the detached systems, suggesting perhaps that in fact many detached DWDs do not survive mass transfer (Roelofs et al 2007). This issue is not settled, but it is another that LISA is ideally suited to answer: the ratio of systems with $f > 0$ to those with $f < 0$ as a function of $f$ will be of great interest for addressing this question.

Although one can probably assume that a system with $f < 0$ is an AM CVn star, the reverse is not true, i.e. $f > 0$ does not imply a DWD, even if we expect this to be the case more often than not. AM CVn stars must pass through an initial phase during which $f > 0$. Indeed it is possible that the two shortest period candidate AM CVn stars, HM Cnc and V407 Vul, are in precisely this phase as both have decreasing orbital periods (D’Antona et al 2006, Deloye et al 2007). As first pointed out by Webbink and Han (1998), and further investigated by Nelemans et al (2004) and Stroeer et al (2005), the ‘braking index’ $n = \frac{f \dot{f}}{f^2}$ is an interesting parameter in these cases. For pure GWR-driven evolution, $n = 11/3$; we expect $n < 11/3$ during the turn-on phase of AM CVn stars, and during some phases $n < 0$. The second derivative $\ddot{f}$ leads to a cubic dependence of the binary phase on time, which places a high value on extending LISA’s lifetime as long as possible: without this we will not be able to distinguish detached DWDs from early-phase AM CVn stars except on a statistical basis, or perhaps through the optical follow-up of nearby systems.

The braking index has one further use. The standard $n = 11/3$ value for detached DWDs treats the two stars as point masses, but as they approach the onset of mass transfer we can expect the effects of tidal spin–orbit coupling to become significant (Willems et al 2008), in effect acting as an additional sink of the orbital angular momentum. Scaling as a high inverse power of the separation, tidal losses will act to increase the value of $n$. LISA detections of systems with $n > 11/3$ may therefore provide a direct indication of the significance of tidal coupling effects between white dwarfs.

5. Combined EM and GW observations

A significant number of the DWDs that LISA will see are potentially detectable through optical observations. Several hundred with $V < 24$ are predicted (Nelemans 2009). The bias towards short periods means that many will eclipse (Cooray et al 2004) (although the
first, and at the moment only, eclipsing detached DWD known, NLTT 11748, (Steinfadt et al 2010), has a surprisingly long 5.6 h period. Eclipsing systems allow measurement of the scaled radii, $R_1/a$ and $R_2/a$. Moreover, they permit precise optical timing measurements which are quite capable of determining the conjunction phases to within $< 0.01$ cycles within a single night of observation. Optical follow-up of such systems could add significantly to the numbers of systems for which we know the first and second derivatives, $\dot{f}$ and $\ddot{f}$, and hence the braking index $n$. The first challenge, as recognized by (Cooray et al 2004), will be to locate them once LISA has signalled their presence, but with projects such as the LSST underway, this seems feasible, even given the large-by-optical-standards LISA error boxes. Photometric measurements alone have the capability to determine the orbital inclination $i$ as well as the scaled radii. Using mass–radius relationships this could determine the mass ratio, which combined with the chip mass can lead to the two masses, giving a major insight into the past evolution of the binaries. Spectroscopic observations will be difficult given the faintness and short periods of most of the LISA targets. However, it should be noted that purely photometric observations may well be able to return kinematic information entirely equivalent to spectroscopy through Doppler beaming. This has already been detected in the DWD eclipser, NLTT 11748, (Shporer et al 2010). This effect may even allow the identification of non-eclipsing optical counterparts using standard phase-locked detection techniques. In the best cases there will result a redundant set of constraints which will allow for tests of white dwarf mass–radius models. The result could be a bonanza for white dwarf astrophysics and provide the best LISA calibration sources.

6. Conclusions

Double white dwarfs are predicted to be the dominant source population at LISA frequencies. LISA’s sensitivity to short orbital periods will allow for the best estimates of the merger rates of these stars, tidal coupling of the two stars and answer questions about their evolution that are hard to solve at the longer periods favoured by electromagnetic observations. The dual combination of the GW and EM observations will be a powerful tool for probing white dwarf astrophysics.

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