THE CORE-CUSP PROBLEM

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ABSTRACT

This paper gives an overview of the attempts to determine the distribution of dark matter in low surface brightness disk and gas-rich dwarf galaxies, both through observations and computer simulations. Observations seem to indicate an approximately constant dark matter density in the inner parts of galaxies, while cosmological computer simulations indicate a steep power-law-like behaviour. This difference has become known as the “core/cusp problem”, and remains one of the unsolved problems in small-scale cosmology.

Subject headings:
models is quite pronounced, and, in principle, it should therefore be possible to identify CDM haloes in real galaxies by measuring their rotation curves.

Over the last 15 years or so, much effort has been put into determining their mass distribution in galaxies using their rotation curves, and comparing them with the outcomes of ever more sophisticated numerical simulations. To first order, one can summarize this work as observational determinations yielding slopes \( \alpha \sim 0 \), while simulations produce \( \alpha \sim -1 \) slopes. This persistent difference is known as the "core/cusp controversy", sometimes also described as "the small-scale crisis in cosmology". The attempts to reconcile the observations and simulations, either by trying to improve them, or by trying to quantify systematic effects or missing physics, are the subjects of this paper. I give a brief overview of past and present work dealing with the determination of the central dark matter density distribution in galaxies, with an emphasis on the observational efforts. An overview like this, touching on many different topics in galaxy evolution, cosmology and computational astrophysics, is never complete, and only a small (but hopefully somewhat representative) fraction of the many papers relevant to this topic can be referred to in the limited space available. The rest of this paper is organised as follows: Sect. 2 gives a description of the results that numerical simulations have produced over the years. Section 3 deals with the observational determinations of the dark matter density distribution. Section 4 discusses physical scenarios that have been proposed to reconcile the core and cusp distributions. Section 5 briefly summarizes the work discussed.

2. Cold Dark Matter Cusps

The presence of a cusp in the centers of CDM halos is one of the earliest and strongest results derived from cosmological N-body simulations. Dubinski & Carlberg (1991) were among the first to investigate the density profiles of CDM halos and found that the inner parts of these simulated halos could be characterized by a power-law slope \( \alpha = -1 \). They did not rule out the existence of central cores, but noted that these would have to be shallower than the resolution of their simulations (~1.4 kpc). Subsequent simulations, at higher and higher resolutions, have made the presence of cores in simulated CDM haloes increasingly unlikely.

A systematic study by Navarro et al. (1996b, 1997) of simulated CDM halos, derived assuming many different sets of cosmological parameters, found that the innermost dark matter mass density distribution could be well described by a characteristic \( \alpha = -1 \) inner slope for all simulated halos, independent of mass, size or cosmology. A similar general result was found for the outer mass profile, with a steeper slope of \( \alpha = -3 \). Navarro et al. (1997) called this the "universal density profile" and it is described by

\[
\rho_{\text{NFW}}(r) = \frac{\rho_\text{i}}{(r/R_\text{s})/(1+r/R_\text{s})^2},
\]

where \( \rho_\text{i} \) is related to the density of the universe at the time of the time of halo collapse and \( R_\text{s} \) is the characteristic radius of the halo. This kind of profile is also known as the "NFW profile".

The corresponding rotation curve is given by

\[
V(r) = \sqrt{\frac{\ln(1+cx) - cx/(1+cx)}{x[\ln(1+c) - c/(1+c)]}},
\]

with \( x = r/R_{200} \). This curve shape is parameterized by a radial distance \( R_{200} \) and a concentration parameter \( c = R_{200}/R_\text{s} \). Here \( R_{200} \) is the radius at which the density contrast with respect to the critical density of the universe exceeds 200, roughly the virial radius; \( V_{200} \) is the circular velocity at \( R_{200} \) (Navarro et al., 1996b). The parameters \( c \) and \( V_{200} \) are tightly related through the assumed cosmology. Indeed, one can be expressed as a function of the other, with only a small scatter (Bullock et al., 2001). That is, the range of \((c,V_{200})\) combinations that describes "real" CDM rotation curves is tightly constrained by the ΛCDM cosmology.

Simulations by Moore et al. (1999) indicated an even steeper inner slope. They found that their simulated halos could be best described by a function

\[
\rho_{\text{M09}}(r) = \frac{\rho_\text{i}}{(r/R_\text{s})^{1.9}(1+r/R_\text{s})^{1.5}},
\]

i.e., with an inner slope \( \alpha = -1.5 \) and an outer slope \( \alpha = -3 \).

The difference between these two results indicated that issues such as numerical convergence, initial conditions, analysis or interpretation could still play a role in defining the inner slope. As more powerful computers and increasingly higher resolution simulations became available, the value and behavior of the inner slope of CDM halos has therefore been extensively discussed in the literature. For example, to give but an incomplete listing of the many papers that have appeared on this topic, Klypin et al. (2001) derived slopes \( \alpha = -1.5 \) for their simulated halos. From phase-space density arguments, Taylor & Navarro (2001) argue that the density profile should resemble an NFW profile, but converging to an inner slope \( \alpha = -0.75 \), instead of the \( \alpha = 1 \) value. Colín et al. (2004) investigated low-mass haloes and found that they were best described using NFW profiles (i.e., \( \alpha = -1 \)). Diemand et al. (2005) found that CDM haloes have cusps with a slope \( \alpha \approx -1.2 \).

Many studies assumed that the central cusp consisted of a region where the mass density behaved as a power-law with a constant slope. Navarro et al. (2004) and Hayashi et al. (2004) suggested that this did not have to be the case. They did not find evidence for an asymptotic power-law slope, but instead noted that the slope kept getting shallower towards smaller radii without converging to a single asymptotic value. At the smallest resolved radii they derive slopes of \( \sim -1.2 \) for "galaxy-sized" halos (as measured at \( \sim 1.3 \) kpc), and \( \sim -1.35 \) for "dwarf galaxy" halos (as measured at \( \sim 0.4 \) kpc). These values are significantly steeper than the original NFW slope, but not as steep as the Moore et al. (1999) value.

Navarro et al. (2004) introduce a new fitting formula to quantify their results. For reasonable choices of its input parameters, this formula yields an extrapolated slope of \( \alpha \sim -0.7 \) at \( r \sim 0.01 \) kpc. Stoehr (2006) also finds a gradual turn-over in slope towards smaller radii. Though his simulations formally resolve only radii \( \sim 1 \) kpc (where a slope of \( \alpha \sim -1 \) is measured), an extrapolation of his favoured fitting function towards smaller radii results in a decreasing slope ending up as a flat slope \( \alpha = 0 \) around \( r \sim 0.01 \) kpc.

Merritt et al. (2005) and Graham et al. (2006) showed that the density distribution presented in Navarro et al. (2004) could be equally well described by a Sérsic function. In the context of CDM halos they refer to this function as an Einasto model. For completeness, this profile is given by

\[
\rho_{\text{Ein}}(r) = \rho_\text{i} \exp \left( -d_n \left( \frac{r}{r_\text{c}} \right)^{1/n} - 1 \right),
\]

with \( d_n = 5.765 \).
where $n$ determines the shape of the profile, and $d_r$ is a function of $n$ which enables the use of the density $\rho_e$ measured at the effective radius $r_e$. The latter is defined as the radius of the volume containing half of the total mass. In terms of observationally more accessible quantities this can be written as

$$\rho_{\text{Ein}}(r) = \rho_{-2, \text{Ein}} \exp \left( -2n \left( \frac{r}{r_{-2, \text{Ein}}} \right)^{1/n} - 1 \right), \quad (6)$$

where $r_{-2, \text{Ein}}$ is the radius at which the logarithmic derivative of the density profile equals $-2$, and $\rho_{-2, \text{Ein}}$ is the density at that radius. The two versions of radius and density are related by $\rho_{-2, \text{Ein}} = \rho_e \exp \left( d_r - 2n \right)$ and $r_{-2, \text{Ein}} = \left( 2n / d_r \right)^{1/n} r_e$.

A further discussion of this model is beyond the scope of this paper, except to note that for typical parameterisations of CDM galaxy halos one derives a slope of $\alpha \sim -1.3 \pm 0.2$ at a radius of 1 kpc and of $\alpha \simeq -0.9 \pm 0.2$ at a radius of 0.1 kpc. The precise values depend on the exact values of $n$ and $r_{-2}$; the values just listed assume $4 < n < 8$ and $r_{-2} = 10$ kpc, as shown in Graham et al. (2008).

Amongst the highest resolution measurements of the inner slope of CDM halos so far are those by Navarro et al. (2008) and Stadel et al. (2008). The former were done as part of the Acquarius project (Springel et al. 2008). Navarro et al. (2008) found no convergence to a single asymptotic slope. Rather, as before, the slope keeps decreasing with decreasing radius. At $r = 0.1$ kpc, which is approximately the smallest reliably resolved radius, the slope has a value $\alpha \simeq 0.85$. At $r = 1$ kpc the value is $\alpha \simeq 1.4$. An Einasto profile with $n = 5.9$ provides a good fit to the change in slope with radius.

In an independent, but equally detailed simulation, Stadel et al. (2008) find a similar behavior, as well as comparable slope values. They quote a slope $\alpha = -0.8$ at 120 pc, and $\alpha = -1.4$ at 2 kpc.

Even though the details of the simulations, the analytical fits and the interpretation and analysis differ, we can still draw some conclusions from the above discussion. All simulations and fitting functions considered here produce slopes $\alpha \lesssim -1$ at a radius of 1 kpc. At radii less than 1 kpc the most recent simulations tend to produce slightly more shallow slopes where a typical value seems to be $\alpha \simeq -0.8$ at 0.1 kpc. From an observer’s perspective, all models described here produce slopes $\alpha \sim -0.8$ or steeper at the smallest observationally accessible radii, and will thus all produce results very similar to those derived using a “standard” NFW profile. In the simulations, radii less than 0.1 kpc cannot yet be reliably resolved, and the values of the slope derived there depend on the validity of the assumed analytical fitting function.

3. OBSERVATIONS

3.1. Early measurements

The first comparisons of the HI rotation curves of gas-rich dwarf galaxies with those predicted by CDMPH profiles were presented in Moore (1994) and Flores & Primack (1994). The dynamics of these galaxies are dominated by dark matter, and they are therefore thought to be good probes of its distribution. Both studies note a large discrepancy between the observed rotation velocities and those predicted, especially in the inner parts. They show that the PI model gives a superior description, implying that the halos of these late-type dwarf galaxies are best characterized by an approximately constant-density core. Moore (1994) briefly addresses some of the observational uncertainties that might affect the data, such as resolution, projection effects due to inclination, and the effects of pressure support, and concludes they are not significant enough to affect the results. He also notes that it is conceivable that, during the galaxy formation process, gas settling in the halo will have affected the dark matter distribution. Usually this is thought to take place in the form of a process called “adiabatic contraction” (Blumenthal et al. 1986), which has the effect of contracting the inner dark matter distribution (increasing the density). If currently derived halo properties are the result of this process, then the initial halos must have been of even lower density, exacerbating the discrepancy.

Navarro et al. (1996a) argue that baryonic processes might be the cause of the observed core distribution. They use N-body simulations to model the effect of star formation on the baryons and the dark matter, and find that a central dark matter core can be created if a large fraction of the baryons is suddenly expelled into the halo. They estimate that star formation rates of up to 10 $M_\odot$ yr$^{-1}$ are needed over a dynamical timescale of a galaxy for the process to have the desired impact.

After analysing the same four dwarf galaxies that Moore (1994) investigated, Burkert (1995) comes to the conclusion that their rotation curves, after appropriate scaling, are self-similar (see also Salucci & Burkert 2000). He notes that it is unlikely that baryonic blow-outs and mass-flows can cause this kind of behaviour, unless fine-tuned, and attributes the slow rise of the rotation curve to the intrinsic properties of the dark matter (i.e., a dark matter core). He also finds tentative evidence that the mass density in the outer parts of these dwarfs drops off as $r^{-3}$ (consistent with CDM), and not with $r^{-2}$ (as suggested by the asymptotically flat rotation curves of spiral galaxies). Based on this, he introduces what has since become known as the “Burkert-profile”:

$$\rho_{\text{Bur}} = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)^{3/2}}, \quad (7)$$

with $\rho_0$ the central density, and $r_0$ the scale radius, similar to the core radius $R_C$ of the PI model. This model is thus characterised by $\alpha = 0$ in the inner parts, and $\alpha = -3$ in the outer parts.

A paper by Gelato & Sommer-Larsen (1999) presents a more detailed analysis of the baryonic blow-out process and its impact on the halo structure. They attempt to reproduce the observed rotation curve of DDO 154 by simulating NFW halos, and subjecting them to the effect of violent gas outflows, which are simulated by suddenly changing the disk potential in the simulations. In order for the final rotation curve to resemble the observed curve, they need to suddenly blow out 33 to 75 percent of the initial disk material. They note that they can reproduce the rotation curve of DDO 154 for a rather wide range of blow-out scenarios, and suggest that the fine-tuning argument put forward by Burkert (1995) may not be applicable.

The blow-out process immediately gives rise to a number of observational consequences. Firstly, a period of star formation intense enough to blow out the majority of the baryons should leave behind a substantial (by now) old stellar population. In practice, however, these dwarfs are seen to be dominated by a young stellar population. Is there a discrepancy here? Secondly, what happens to the baryons that are blown out? Do they stay in the halo? Presumably, they will be in the

1 Less steep slope values listed in the same paper are derived assuming $r_{-2} = 100$ kpc and are thus more appropriate for giant or group-sized CDM halos.
form of hot gas. Is this hot gas detectable? Or, if the gas cools down, can we see it raining back on the disk? Discussing these questions in detail is beyond the scope of this paper, but note that there are a number of starbursting dwarf galaxies in our local universe where one can attempt to study these phenomena directly. As an example, Ott et al. (2005) analyse the properties of the hot and cold gas in 8 dwarf galaxies that are in a starburst phase. They show that outflows of hot gas are possible, but also that the presence of (tidal) cold gas can again confine the hot gas. In the galaxies in their sample, approximately 1 percent of the total ISM is in the form of hot, coronal gas. The outflows are found to be efficient in removing hot, metal-rich gas. Whether these processes can, in the early universe, also remove the bulk of the ISM is still an open question. It is clear that with this kind of analysis we are no longer in the realm of cosmology, but are dealing with “messy” astrophysics.

Fortunately, there is an alternative way to study the dark matter distribution. Navarro et al. (1996a) already note that the blow-out process can only be effective in dwarf galaxies. In more massive galaxies, such as spiral galaxies, the potential well is too deep to efficiently remove the gas. Finding and investigating more massive dark-matter dominated galaxies may therefore be a more effective way to explore the core/cusp issue. These galaxies, fortunately, do exist, and are called Low Surface Brightness (LSB) galaxies.

**3.2. LSB Galaxies**

The term LSB galaxies is used here to indicate late-type, gas-rich, dark-matter-dominated disk galaxies. Their optical component is well-described by an exponential disk with an (extrapolated) inclination-corrected central surface brightness fainter than $I_{2600} \sim 23$ mag arcsec$^{-2}$ (McGaugh & Bothun 1994; McGaugh et al. 1995; de Blok et al. 1995). Despite their low surface brightness, their integrated luminosity is a few magnitudes brighter than that of late-type dwarf galaxies ($M_B \sim -18$ to $-20$ for LSB galaxies, as opposed to $M_B \gtrsim -16$ for the dwarf galaxies). As noted, they are gas-rich ($M_{HI}/L_B \gtrsim 1$; Schombert et al. 1992; McGaugh & de Blok 1997; Schombert et al. 2001), and their interstellar medium has a low metallicity (McGaugh 1994; de Blok & van der Hulst 1998). Their optical appearance is dominated by an exponential disk with a young, blue population, with little evidence for a dominant old population. Additionally, these galaxies do not have large dominant bulges, and seem to have had a star formation history with only sporadic star formation (van der Hulst et al. 1993; van den Hoek et al. 2000; Gerritsen & de Blok 1999). Central light concentrations, if present at all, tend to be only fractionally brighter than that of the extrapolated exponential disk. In terms of their spatial distribution, they are found on the outskirts of the large scale structure filaments (Bothun et al. 1993; Mo et al. 1994). In short, most observational evidence indicates that these galaxies have had a quiescent evolution, with little evidence for major merging episodes, interactions, or other processes that might have stirred up the baryonic and dark matter (see also Bothun et al. 1997; Impey & Bothun 1997).

As for the term “LSB galaxies”, there is some confusion in the literature about what type of galaxies it applies to. The type of LSB galaxies most commonly studied, in particular with regards to the core/cusp controversy, are the late-type LSB galaxies whose properties are described above. The other type of LSB galaxies often discussed in the literature are the massive, early-type, bulge-dominated LSB galaxies. These galaxies have properties entirely different from the late-type LSB galaxies (Sprayberry et al. 1995; Pickering et al. 1997). The massive LSB galaxies are a lot more luminous and their optical appearance is dominated by a bright central bulge with a clearly detectable old population (Beijersbergen et al. 1995). Many of them have low-level AGN activity (Schombert 1998). All indications are that the evolution of these galaxies has been entirely different from that of late-type LSB galaxies: if anything, they resemble S0 galaxies with extended disks, rather than late-type galaxies. The presence of the dominant bulge also indicates that their central dynamics are likely to be dominated by the stars, rather than dark matter. In the following, the term “LSB galaxies” therefore refers to late-type LSB galaxies only.

**3.3. Early HI observations of LSB galaxies**

The first detailed studies of large samples of LSB galaxies soon led to the picture of them being unevolved, gas-rich disk galaxies, as described above. The observation that they followed the same Tully-Fisher relation as normal galaxies (Zwaan et al. 1992) was intriguing, as this implied they had to be dark-matter dominated. Follow-up radio synthesis observations in HI (de Blok et al. 1996) soon confirmed this. Though the resolution of these early observations was limited, the derived rotation curves clearly resembled those of late-type dwarf and ”normal” disk galaxies: a slow rise, followed by a gradual flattening. When expressed in terms of scale lengths, the rotation curves of LSB and HSB galaxies of equal luminosity turned out to be very similar, indicating that LSB galaxies are in general low density objects (de Blok & McGaugh 1996).

Mass models derived using the rotation curves clearly showed that for reasonable assumptions for the stellar mass-to-light ratio, $\Upsilon_\star$, the dynamics of LSB galaxies had to be dominated by dark matter (de Blok & McGaugh 1997). Assuming that the stars had to dominate the dynamics in the inner parts (the so-called maximum disk solution) led to unrealistically high $\Upsilon_\star$ values, and, even when taken at face value, still showed a need for a moderate amount of dark matter at small radii (see also McGaugh & de Blok 1998).

The distribution of the dark matter at first sight seemed similar to that in gas-rich dwarf galaxies (de Blok & McGaugh 1997). Because of the limited resolution of the data, de Blok & McGaugh (1997) did not attempt fits with the NFW model, but noted that the halos had to be extended, diffuse and low density.

A first attempt at comparing the HI data with CDM predictions was made by McGaugh & de Blok (1998). Rather than making fits to the rotation curve, they simply assumed that the typical velocity $V_{200}$ of the halo had to equal the outer (maximum) rotation velocity of the galaxy. The strict cosmological relation between $c$ and $V_{200}$ then automatically yields a value of $c$ compatible with $\Lambda$CDM. Adopting these values, the resulting halo rotation curve turned out to be very different from the observed curve, in a similar way as the Moore (1994) analysis: the NFW curve is too steep and rises too quickly in the inner parts. The only way the halo curve could be made to resemble the observed curve, was by abandoning the cosmological ($c$, $V_{200}$) relation. Similar conclusions were derived by Côté et al. (2000). They presented high-resolution HI observations of dwarfs in the nearby Centaurus and Sculptor groups, and noted that the derived rotation curves did not agree with the NFW model.

A possible explanation that was soon put forward was...
that there were still unrecognized systematic effects in the data, that would give the false impression of a core-like behaviour. Initially, attention focussed on the resolution of the de Blok et al. (1996) HI observations. These had beam sizes of ~ 15'', resulting in the HI disks of the LSB galaxies investigated having a diameter of between 3 and 18 independent beams. This limited resolution can potentially affect the shapes of the rotation curves through a process called "beam smearing", as also mentioned in de Blok et al. (1996). In observations with limited resolution, the beam smearing process decreases the observed velocities (compared to the true velocities), and in extreme cases can turn any steeply rising rotation curve into a slowly rising solid-body one. This would therefore give the impression of a core being present in the data, while the true distribution could still be cuspy. In their paper, de Blok & McGaugh (1997) argued, through modelling of these beam smearing effects, as well as the direct detections of steeply rising rotation curves in the data, that while some beam smearing was indeed present, the effect was not strong enough to completely "hide" the dynamical signature of a cusp, and concluded that the data were consistent with the existence of dark matter cores.

An alternative interpretation was given in van den Bosch et al. (2000) who used the de Blok & McGaugh (1997) data, along with high-resolution literature rotation curves of a number of late-type "normal" and dwarf galaxies, such as DDO154 and NGC 247. They derived and applied explicit analytical corrections for beam smearing and concluded that the LSB galaxy HI data were consistent with both cored and cuspy halos. In their analysis of the other gas-rich dwarf and late-type galaxies, they found evidence for cores in the dwarf galaxies, but detected a steep mass-density slope — consistent with a cusp — in NGC 247, the one late-type disk galaxy that met their sample selection criteria.

In a related paper, van den Bosch & Swaters (2001) derive similar conclusions for a different, larger sample of dwarf galaxies observed in HI as part of the Westerbork HI Survey of Irregular and Spiral Galaxies (WHISP) van der Hulst et al. (2001). They attempt to correct for adiabatic contraction and resolution and conclude that, while in a majority of the galaxies they investigate a core is somewhat preferred in terms of fit quality, they cannot exclude halo models that have a steep inner mass-density slope.

Clearly, with this wide range of (sometimes contradictory) conclusions, obtaining higher resolution data is the only way to put stronger constraints on the exact distribution of dark matter in galaxies.

3.4. Hα observations

After the initial HI observations, the most efficient way to improve the resolution was by obtaining observations in the Hα line. These usually took the form of long-slit spectra taken along the major axes of the galaxies, and resulted in an order of magnitude improvement in resolution. Typical observations now had resolutions of a few arcseconds, instead of a few tens of arcseconds.

Some of the first Hα long-slit rotation curves of LSB galaxies were presented by Swaters et al. (2000). They found that the Hα curves indeed did rise somewhat more steeply in the inner parts than the HI curves, and for one or two galaxies very much so, but they noted that on the whole the HI and Hα curves were consistent when the beam smearing effects were taken into account. The Hα curves and corresponding mass models did still indicate that LSB galaxies had to be dominated by dark matter. The curves also still rose less steeply than those of higher surface brightness late-type galaxies of a similar luminosity, as indicated by the agreement between the curves when they were radially scaled using their exponential disk scale lengths.

A large set of high-resolution, Hα long-slit rotation curves was published and analysed by McGaugh et al. (2001), de Blok et al. (2001a,b) and de Blok & Bosma (2002). Many of these galaxies had been part of the de Blok & McGaugh (1997) sample, and a direct comparison between the Hα and HI rotation curves showed that in the majority of cases, beam smearing effects were present, but not significant enough to alter the previous conclusions regarding the dark matter distribution. In the few cases where there were large differences this could be explained by inclination effects or elliptical beam shapes in the HI observations.

The Hα curves thus showed that the inner, slowly rising slopes could not be caused by resolution effects. This meant that for reasonable (stellar population synthesis inspired) T⋆ values, LSB galaxies were still dark matter dominated throughout their disk, even though the Hα curves formally allowed maximum disk fits with even higher T⋆ values than the HI observations did. These high values are, however, completely at odds with the observed colours and star formation histories, and still imply a significant dark matter fraction.

Comparison of mass models assuming PI and NFW models showed that the data were better described by the PI models. In many cases, NFW models did yield reasonable fits, but usually with very low concentrations and high V200 values, inconsistent with the cosmological (c, V200) relation. Taken at face value these results would imply that the halos of LSB galaxies have barely collapsed, and have typical velocities many times higher than those of the galaxies that inhabit them. The low c-values are, however, due to the intrinsically different shape of the NFW rotation curves compared to the solid-body observed curves. An NFW curve has an inner velocity slope r ∝ r^1/2, and the only way this kind of curve can fit the observed solid-body v ∝ r curve is by stretching, resulting in the low c and high V200 values.

The observational inner mass density slopes were derived by de Blok et al. (2001b) and plotted against the resolution (physical radius of the innermost point) of the rotation curve. A comparison with the expected slopes from various halo models showed that the majority of the data scattered towards the predicted slopes of the PI model. They also showed that for resolutions of ~ 1 kpc the PI and NFW models yielded identical slopes. This would go some way towards explaining why some of the lower resolution HI observations were unable to distinguish between models. The behaviour of the change in slope when going from the lower resolution HI observation to higher resolution Hα observations is consistent with that expected from the PI model, as shown in de Blok & Bosma (2002). Clearly, for unambiguous determinations of the inner mass density slope, resolutions of better than a kpc are needed.

Independent observations and analyses came to similar conclusions. Marchesini et al. (2002) obtained long-slit Hα observations of some of the de Blok & McGaugh (1997) galaxies, and also found little evidence for strong beam smearing, as well as strong evidence for the existence of dark matter cores. Zackrisson et al. (2006) observed solid-body models showed that the majority of the data scattered towards the predicted slopes of the PI model. They also showed that for resolutions of ~ 1 kpc the PI and NFW models yielded identical slopes. This would go some way towards explaining why some of the lower resolution HI observations were unable to distinguish between models. The behaviour of the change in slope when going from the lower resolution HI observation to higher resolution Hα observations is consistent with that expected from the PI model, as shown in de Blok & Bosma (2002). Clearly, for unambiguous determinations of the inner mass density slope, resolutions of better than a kpc are needed.

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of the rotation curves of six extremely blue and bulge-less LSB galaxies and also finds strong evidence for the presence of cores in the dark matter distribution of these galaxies. Salucci (2001) analysed the rotation curves of over a hundred rotation curves of disk galaxies, and found clear signatures for the existence of dark matter cores in these galaxies. This analysis is complementary to the work on LSB galaxies, as it also analyses less dark-matter-dominated galaxies. Borriello & Salucci (2001) analysed Hα rotation curves of a number of dark matter dominated galaxies, and also found strong evidence for core-like dark matter distributions. They also fit Burkert haloes to their data and find that these also provide good fits. Analysis for a sample of ~25 spiral galaxies is presented in Donato et al. (2004), and leads to similar conclusions. They also find that the core radius of the dark matter distribution is related to the disk scale length.

Hayashi et al. (2004) find that the Hα rotation curves are consistent with steep slopes. de Blok (2005) suggests that these conclusions are based on artificial constraints imposed on the fitting functions. Once these constraints are removed, 17 of the 20 galaxies with the highest-quality data are best fitted with cored models. Three are possibly consistent with cuspy models; two of these are high-surface brightness dwarf galaxies that are likely dominated by stars.

3.5. Possible systematic effects

At first glance, the observational data seems to provide good evidence for the presence of cores in LSB galaxies. This does of course not exclude the possibility that systematic effects might be present in the data that could give the (false) impression of cores. A number of studies have therefore focused on these effects and asked whether the detected cores could still be consistent with cuspy halo models.

The systematic effects that were investigated fall into two categories.

1. Pointing problems — If small offsets exist between the central slit position and the true, dynamical center, this will cause the spectrograph slit to miss the cusp. This could be due to inaccurate telescope pointings or, alternatively, if large physical offsets between the dynamical and photometric centers of galaxies exist, the measured slopes will also be biased downwards.

2. Non-circular motions — The fundamental assumption in the observational analyses discussed so far is that the gas moves on circular orbits. If for some reason the orbits are elliptical, or of the gas motions are disturbed, this will also lead to an underestimate of the slope.

These effects are difficult to recognize in long-slit, one-dimensional Hα data without additional information. Many authors therefore, apart from modeling these effects, also emphasized the need for high-resolution, two-dimensional velocity fields. These make pointing problems irrelevant, while non-circular motions can be directly measured. Fortunately, these velocity fields are now available (see Sect. 3.6), largely superseding the results derived from the Hα rotation curves. Nevertheless, for completeness, the further analysis of the Hα curves is briefly discussed here.

In de Blok et al. (2003) a first attempt was made at modeling the observational systematic effects. Their main conclusion was that NFW halos can be made to resemble dark matter cores only if, either systematic non-circular motions with an amplitude of ~20 km s⁻¹ exist in all disks, or systematic telescope pointing offsets of ~3–4″ exist for all observations, or if the dynamical and photometric centers are systematically offset in all galaxies by ~0.5–1 kpc.

Marchesini et al. (2002) and de Blok & Bosma (2002) compare independent sets of observations of the same galaxies, obtained at different telescopes, by independent groups, using independent data sets, and find no evidence for telescope pointing errors. In general, galaxies can be acquired and positioned on the slit with a repeatability accuracy of 0.3″ or so.

Using further modeling, de Blok et al. (2003) find that a halo model with a mildly cuspy slope α = −0.2 ± 0.2 gives the best description of the data in the presence of realistic observational effects. It is interesting to note that this best-fitting slope had already been derived by Kravtsov et al. (1998) on the basis of the de Blok et al. (1996) data.

An analysis by Spekkens et al. (2005) of longslit Hα rotation curves of 165 low-mass galaxies comes to the same conclusion: depending on how they select their sample, they find best-fitting slopes of α = −0.22 ± 0.08 to α = −0.28 ± 0.06. They also model pointing and slit offsets, but come to the conclusion that, after correction, their data are consistent with cuspy haloes. Swaters et al. (2003a) also present extensive modeling of high-resolution long-slit Hα rotation curves, and show that while their data are consistent with α = 0 cores, steeper slopes cannot be ruled out.

Given the difference in interpretation of otherwise similar samples, a double-blind analysis of the modeling performed by the various groups would have been interesting. However, with the availability of high-resolution velocity fields, this is now a moot issue.

Rhee et al. (2004) attempt to model many of the observational effects using numerical simulations. Most of their conclusions apply to long-slit observations. The systematic effects they investigate (dealing with inclination effects, non-circular motions and profile shapes) have now been directly tested on high-resolution velocity fields and do not critically affect the data (e.g., Gentile et al. 2005; Zackrisson et al. 2006; Trachternach et al. 2008; Oh et al. 2008; de Blok et al. 2008; Kuzio de Naray et al. 2008, 2009).

3.6. High-resolution velocity fields

As noted, high-resolution two-dimensional velocity fields provide the context which long-slit observations are missing. With these velocity fields the pointing problem becomes irrelevant, offsets between kinematical and dynamical centers can be directly measured, as can non-circular motions. Following is a brief overview of the various observational studies that have presented and analysed these velocity fields within the context of the core/cusp debate.

Some of the first high-resolution optical velocity fields of late-type dwarf and LSB galaxies were presented by Blais-Ouellette et al. (2001). They analyse Hα Fabry-Perot data of IC 2574 and NGC 3109, two nearby dwarf galaxies, and derive slowly rising rotation curves, consistent with a core. This work was later expanded in Blais-Ouellette et al. (2004), and led to the work presented in Spano et al. (2008), where optical velocity fields of 36 galaxies of different morphological types are presented. All three studies find that the PI model generally provided better fits than NFW models. If NFW models give fits of comparable quality, then this is usually at the cost of an unrealistically low Υ, value and non-cosmological (c, V₂₀) values. A similarly large collection of
Hα velocity fields is presented in [Dicaire et al. 2008]. They do not explicitly address the core/cusp issue, but show that when bars are present, their influence on the velocity fields is very noticeable.

Kuzio de Naray et al. 2008, 2009 present DensePak velocity fields of LSB galaxies, many of them taken from the de Blok & McGaugh (1997) sample. Their conclusions are that NFW models provide a worse fit than PI models, for all values of $Y_\alpha$. Where an NFW model could be fit, the $c$-values generally again do not match the cosmological CDM ($c, V_{200}$) relation. They introduce a “cusp mass excess”: when the predicted ($c, V_{200}$) relation is assumed, and the $V_{200}$ velocities are matched with those in the outermost observed velocities, the inner parts require about $\sim 2$ times more dark matter mass than implied by the observed rotation curves.

Kuzio de Naray et al. 2008, 2009 also explore non-circular motions: they find that random velocities with an amplitude of $\sim 20$ km s$^{-1}$ are needed to bring the observed curves in agreement with the CDM predictions. A comparison of simulated long-slit observations (extracted from the velocity fields) with the original long-slit data from [McGaugh et al. 2001] and de Blok & Bosma (2002) shows good agreement.

Swaters et al. (2003b) present a DensePak velocity field of the late-type dwarf galaxy DDO 39. They derive a rotation curve, and show that its slope is steeper than implied by lower resolution HI data, and also different from earlier long-slit data from de Blok & Bosma (2002). They indicate that measurable non-circular motions are present, but do not explicitly quantify them. They show fit results for NFW halos, but do not show the corresponding results for a core model, so further comparisons are difficult to make.

Weldrake et al. (2003) present the HI velocity field of NGC 6822, a nearby Local Group dwarf galaxy. Their data has a linear resolution of about 20 pc, so beam smearing is definitely not a problem. The rotation curve shows a strong preference for a core-like model, but they do not quantify possible non-circular motions. Salucci et al. (2003) present similarly high-resolution HI data (including VLA B array) of the dwarf galaxy DDO 47. Their analysis shows the dynamics are not consistent with a cusp. Similar results are derived in Gentile et al. (2004) for a number of spiral galaxies.

Simon et al. (2005) present results from CO and Hα velocity fields of 5 low-mass dark-matter-dominated galaxies (see also Simon et al. 2003 and Bolatto et al. 2002). They derive a range of slopes, from core-like ($c = 0.01$ for NGC 2976) to cuspy ($c = 1.20$ for NGC 5963). Note that NGC 5963 has an inner bright disk, and it may therefore not be dark-matter-dominated all the way to the center, making the value of its mass-density slope uncertain (see also Bosma et al. 1988). The average slope they derive is $c = 0.73 \pm 0.44$. Their analysis method differs in a few aspects from the other studies referenced in this paper. Firstly, their inner slope values are derived from a single power-law fit to the entire rotation curve. Most of their models do not take into account the gas component due to a lack of HI observations. In low-mass galaxies the gas can dynamically be more important than the stars (especially in the outer parts), and correcting for this component could potentially change the derived slopes.

More importantly, the mass models in Simon et al. 2005 are derived under the explicit assumption of a constant inclination and position angle for each galaxy. Most velocity fields of nearby galaxies show radial inclination and position angle trends, especially in the outer parts (e.g., de Blok et al. 2008). Simon et al. 2005 attribute these to radial velocities that give the impression of changes in inclination and position angle. Nevertheless, the non-circular velocities derived in this way are typically less than $\sim 20$ km s$^{-1}$, and in a few galaxies less than $\sim 5$ km s$^{-1}$. Whether these velocities are real or whether inclination and position angle changes are preferred would be an interesting topic of further study. Simon et al. (2005) also derive harmonic decompositions of the velocity fields to study non-circular motions, but they do not list the values of the harmonic coefficients. It would be similarly interesting to compare these with results derived for other disk and LSB galaxies.

Direct measurements of non-circular motions are presented in [Gentile et al. 2005]. Using high-resolution HI observations, they make a harmonic decomposition of the velocity field, and show that non-circular motions are only present at the level of a few km s$^{-1}$. This is a factor of $\sim 10$ lower than is needed for the non-circular motions to wipe out the kinematical signature of a cusp and to give the impression of a core.

An analysis by Gentile et al. (2004) of another gas-rich dwarf galaxy, NGC 3741, based on the entire 3D HI data cube, showed non-circular motions around $5-10$ km s$^{-1}$, and a strong preference for a core model. NFW models could be accommodated, but with the usual caveats of the fit parameters not being consistent with the cosmological ($c, V_{200}$) relation.

Trachternach et al. (2008) present harmonic decompositions of the velocity fields of galaxies from the Hi Nearby Galaxy Survey (THINGS; Walter et al. 2008). The THINGS survey covers a large range in galaxy properties, from luminous early-type disk galaxies to late-type dwarfs. Trachternach et al. (2008) find a relation between the median strength of the non-circular motions and the luminosity of the galaxies, indicating that the non-circular motions are associated with the baryons. Luminous disk galaxies have non-circular motions up to $\sim 30$ km s$^{-1}$, mostly associated with bars and spiral arms. These then decrease rapidly to a level of only a few km s$^{-1}$ for dwarf galaxies like DDO 154. They also found that offsets between photometric and kinematic centers were typically $\sim 200$ pc or less.

This low level of non-circular motions seems inconsistent with observations by Pizzella et al. (2008), who find significant non-circular motions in a sample of 4 LSB galaxies. However, their galaxies all contain bright bulges and are therefore probably more representative of the class of giant LSB galaxies, rather than the late-type ones discussed in this paper (in this regard see also Coccato et al. 2008).

Finally, McGaugh et al. (2007) and Gentile et al. 2007b show that the core/cusp issue is not limited to the very inner parts of galaxies. As mentioned before, the shape of the NFW curve is fundamentally different from that of observed curves. One can try and work out the implications in two ways: McGaugh et al. (2007) match the observed outer rotation velocities with those of corresponding cosmological NFW halos (effectively identifying NFW halos that have a similar dark matter density at these outer radii as real galaxies), and find that these halos are too massive. Due to the cusp mass excess their average density is then also a factor of $2-3$ too high (see also de Blok et al. 2008). Gentile et al. 2007b take a different approach and identify halos that have the same enclosed mass as observed galaxies. Again, due to the cusp mass excess, this results in halos that, compared to what can be derived for real galaxies, are more dense in the center and...
less dense in the outer parts. Due to the fundamentally different shapes of the mass distributions, the core/cusp issue is therefore not limited to the inner parts, but is relevant at all radii.

4. EFFECTS OF BARYONS AND TRIAXIALITY

The high-resolution velocity fields thus seem to indicate mass distributions that are cored. As many observational effects such as pointing, center offsets, and even non-circular motions seem to be too small to be the cause of the observed cores, many studies have attempted to explain the apparent presence of cores as the result of processes such as interactions between and merging of dark matter halos, or the effect of baryons on the dark matter distribution. Following is a brief discussion of some of these effects.

4.1. Feedback and merging

It was already noted that violent, large-scale star formation might be able to explain the cores in gas-rich dwarf galaxies, but not in more massive galaxies (Navarro et al. 1996). The implied large bursts of star formation are also not consistent with the quiescent evolution of LSB galaxies. An alternative way to achieve the removal of the cusp was proposed by Weinberg & Katz (2002). They model a rotating rigid bar in a disk which is embedded in a CDM halo and show that this bar creates a wake in the dark matter. This trailing wake slows down the bar, transferring some of the bar angular momentum to the dark matter. This “puffs up” the dark matter distribution thus forming a core. Recent numerical simulation work by Dubinski et al. (2009) does, however, suggest that the bar does not destroy the central cusp, and may even increase the halo density slightly.

A very detailed case-study of the effects of bars and feedback is presented in Valenzuela et al. (2007). They attempt to reproduce the observed rotation curves of NGC 6822 and NGC 3109 using NFW halos and a variety of feedback effects and non-circular motions. They present numerical models including gas dynamics, and tune them to resemble the two target galaxies. Whilst they succeed in matching the rotation curves, it is not clear whether these results can be generalized, due to, e.g., the different assumptions that are made in determining inclinations and position angles of the models, compared to the data. Although the authors present simulated velocity fields, they do not make a direct comparison between observed and simulated velocity fields, which would provide much additional information on the bar and feedback effects.

Dekel et al. (2003ab) argue that merging of cuspy halos inevitably leads to a cusp. The only way to prevent this from happening is to puff up the dark matter distribution of the infalling halos before they merge. This way the halos get disrupted more easily and a core-like distribution can be gradually built up. The authors describe the scenario as speculative, however, and note that it is unlikely that supernovae will be able to cause the puffing up for any galaxy with \( V > 100 \text{ km s}^{-1} \). Boylan-Kolchin & Mul (2004) show that for this process to work, none of the merging halos can be cuspy to start with, as only mergers between cored halos give a cored merger product. Any merger involving a cusp inevitably leads to a cuspy end-product. Dehnen (2005) also shows that the final slope always equals that of the steepest component. Setting up cored halos and ensuring that they remain cored is apparently not trivial.

4.2. Dynamical Friction

El-Zant et al. (2001) propose a different way to make halos cored: they note that merging gas clouds of \( \sim 10^5 M_\odot \) (for dwarfs) to \( \sim 10^8 M_\odot \) (for spirals) can disrupt cusps through dynamical friction. If this happens early enough in the universe (when halos were smaller), this process could be very efficient. Similar scenarios are presented in Tonini et al. (2006), Romano-Díaz et al. (2008) in their study also argue that the effect of the baryons on the halo structure must be significant. They suggest that in the presence of baryons, initially a very steep cusp is formed (with \( \alpha \sim -2 \)), which is then heated by sub-halos through dynamical friction, and, subsequently, from the inside out becomes shallower. According to their analysis, the end result is a density profile that is less steep than \( \alpha = -1 \) in the inner few kpc, and may even be cored in the very center.

Jardel & Sellwood (2009) take the opposite view. They model the dynamical friction process, but argue that it is difficult to find “free-floating” baryon clumps massive enough to make this process happen, as these clumps must not be associated with dark matter (due to the cusp that will then be formed; see above). They note that work by Kaufmann et al. (2006) implies clump masses that are two orders of magnitude too small.

The processes just described are similar to those proposed by Mashchenko et al. (2006, 2008); see also Ricotti & Wilkinson (2004). They make it all happen in the early universe, when mass scales and the required amount of baryons were both smaller. Their numerical simulations suggest that cusps can be erased in the very early universe (\( z \lesssim 10 \)) when (proto-)galaxies had approximately the size of the HI holes and shells observed in the disks of present-day, gas-rich spiral and dwarf galaxies. In objects that small, a random motion of only \( \sim 7 \text{ km s}^{-1} \) (i.e., equal to the HI velocity dispersion observed in local disk galaxies) is sufficient to disrupt the cusp and keep the halos cored until the present day. Note though that in a recent analysis, Ceverino & Klypin (2009) perform similar calculations, but find that halos remain cuspy, and suggest that differences in the simulation approach might be responsible for this.

Chen & McGaugh (2008) also argue against the idea of creating cores at high redshift. Their argument is that cuspy halos cannot explain gravitational lensing results, but demand halos with an even steeper mass distribution (a singular isothermal sphere with slope \( \alpha = -2 \)). This means that if the halos of the elliptical galaxies that do the lensing indeed form with NFW-like profiles, these need to steepen over the course of their evolution, using some process for which the quiescent adiabatic contraction is the most likely candidate. LSB galaxies, on the other hand, need to experience vigorous bursts of early star formation that drive feedback to erase the initial cusp. These scenarios are at odds with what we know about the evolution of these galaxies: ellipticals typically undergo a large amount of merging, with the associated vigorous star formation, as evidenced by their extensive old stellar populations. LSB galaxies show no evidence at all for any kind of violent interactions, nor intense star formation. Dynamical friction and adiabatic contraction thus seem to place demands on the evolution of elliptical and LSB galaxies contrary to what can be derived from their star formation histories.

4.3. Triaxiality

Hayashi & Navarro (2006) and Hayashi et al. (2007) show that introducing an elliptical disturbance in an NFW poten-
tial can lead to systematic (non-circular) motions that, when unrecognized, could be interpreted as evidence for a core. Their arguments particularly apply to long-slit Hα observations, where indeed the context of a velocity field is not available to gauge the validity of the circular motion assumption. As described earlier, work on high-resolution velocity fields by Gentile et al. (2005), Trachternach et al. (2008) and Kuzio de Naray et al. (2009) has put significant observational constraints on the strength of non-circular motions and the ellipticity of the (equatorial) potential. For a large sample of disk and dwarf galaxies, this potential is consistent with being round and the non-circular motions are too small to give the illusion of cores in long-slit observations.

The results from Hayashi & Navarro (2006) and Hayashi et al. (2007) assume massless disks. Bullock et al. (2008) also presents models of “live” disks in tri-axial halos, and can approximate an observed LSB rotation curve by introducing tri-axiality in the halo.

In all these studies it would be interesting to compare simulated velocity fields with the observed ones. As has become clear from the preceding discussion, modeling the long-slit rotation curves leaves too many ambiguities which only studies of the velocity fields can address. In an observational study, Kuzio de Naray et al. (2009) subject model NFW velocity fields to the DensePak observing procedure, and show that even in the presence of observational uncertainties the signature of an NFW velocity field can be observed.

They show that axisymmetric NFW velocity fields are unable to reproduce the observed velocity fields for any combination of inclination and viewing angle. They derive NFW velocity fields in an elliptical potential, and show that the only way these can be made to resemble the observations is by having the observer’s line of sight along the minor axis of the potential for 6 out of the 7 galaxies investigated, inconsistent with a random distribution of the line of sight. Kuzio de Naray et al. (2009) also note that the kind of rapidly varying ellipticity of the potential as proposed by Hayashi et al. (2007) might help in better describing the data, but that the problem of a preferred viewing angle will remain.

For an elliptical potential with a random viewing angle, one also expects, once in a while, to observe a rotation curve that is steeper than the corresponding axisymmetric NFW profile. Rotation curves like that are, however, exceedingly rare, if not absent, in the available observations.

5. SUMMARY

Rotation curves of LSB and late-type, gas-rich dwarf galaxies indicate the presence of constant-density or mildly cuspy cores (α ~ 0.2) dark matter cores, contradicting the predictions of cosmological simulations. The most recent simulations still indicate resolved mass density slopes that are too steep to be easily reconciled with the observations (typically α ~ −0.8 at a radius ~ 0.1 kpc). Claims of shallow slopes at even smaller radii depend on the validity of the analytical description chosen for the mass-density profile.

Whereas early HI observations and long-slit Hα rotation curves still left some room for observational (pointing, resolution) or physical (non-circular motions, tri-axiality) systematic effects to create the illusion of cores in the presence of a cuspy mass distribution, the high-resolution optical and HI velocity fields that have since become available significantly reduce the potential impact of these effects. Measured non-circular motions and potential ellipticities are too small to create the illusion of a core in an intrinsically cuspy halo.

This indicates either that halos did not have cusps to begin with, or that an as yet not understood subtle interplay between dark matter and baryons wipes out the cusp, where the quiescent evolution of LSB galaxies severely limits the form this interplay can take. Adiabatic contraction and dynamical friction yield contradictory results, while models of massless disks in tri-axial halos result in preferred viewing directions. LSB galaxy disks, despite their low T, values, are not entirely massless, and observations and simulations will need to take this into account. Similarly, the difficulties in reconciling a possible underlying triaxial potential with the circularizing effects of the baryons also needs to be investigated. In short, studies which, constrained and informed by the high-quality observations now available, self-consistently describe and model the interactions between the dark matter and the baryons in a cosmological context are likely the way forward in resolving the core/cusp problem.

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