Competition, Disclosure and Optimal Information Resource Allocation

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ABSTRACT
This paper proposes a unified framework to explain why firms compete in a product market differ from each other in resource endowments about information. It is proved that the difference may be resulted from a state-contingent optimal information resource allocation scheme carried out by a central planner. Information possessed by the planner will be assigned to only one of the firms when level of competition is above the threshold, and will be assigned to all firms when the level is below the threshold. A new determinant factor is employed to bridge the gap in existing literature.

Keywords: Competition, disclosure, central planner, information resource, optimal allocation

1. INTRODUCTION

1.1. Related Work
Existing literature has studied how a firm possesses private information about uncertain market demand will make its decision of voluntary disclosure under the settings of product market competition [1-2]. The optimal decision usually gives a rule of whether information should be disclosed or not, left an unsolved problem why one of the firm is assumed informed and its rival is uninformed in one work, but all the firms' initial information endowment sets are not empty and the sets are extracted stochastically from the uncertain market demand information set. Thus assumptions about the informed firms should be one or all of the two firms in previous wisdoms are quite different.

It is known that the resource a firm possesses at first will lead to strategy behaviour in the followed duopoly competition game, together with a certain updating of belief inference and behaviour adjustment from the peer firm, and finally leads to different return after the game is over. That is, a different assumption of information endowment always leads to a different proposition and then results concluded by different works cannot be compared directly. So it is necessary for us to clarify why different assumptions are introduced in papers to solve similar problems.

How the discrepancy of the assumptions about information endowment is determined by? Previous works point out that difference from research and development (R & D), political connections, geographic proximity, distribution channels between firms may be the possible factors [3-8]. Obviously, answers in these works are more related to characteristics or actions of a firm itself than similar ones from its rivals.

2. DETAILS OF THE MODEL

For a symmetric differentiated Cournot duopoly: there are two risk-neutral firms which produce similar products in a market and their reverse demand functions are:

\[ p_i = \theta - q_i - q_j, \quad i, j \in \{1, 2\} \]  \hspace{1cm} (1)

This paper proposes a new explanation framework with a differentiated Cournot duopoly model under the perspective of economic efficiency. The main results show that information endowments vary with allocation plan adopted by the central planner whose object is maximizing sum of all firms' ex ante payoffs in an industry. It is not the individual firm's payoff but total of the all firm's payoff is considered in the paper, and how much information a firm could possess is a function of tense of competition between firms, expectation and variance of the stochastic market demand. Unlike works aims to show equilibrium existence and robustness of a differentiated Cournot duopoly model under settings of asymmetric information [9-10], and works provides detail steps of how to solve the Nash Equilibrium solution [11], this paper mainly shows why the initial information structure of one firm is different to the other, and shows that different assumptions about firms' endowment may be just due to different parameters characterize the production market.

The model introduced in our paper is a generalized version of a previous work [12], tense of competition between two firms in this paper is no longer a fixed value and will be proved is a key factor when the optimal information resource allocation plan is proposed. What's more, the model shows that the exogenous information resource may be assigned to none, to one or two of the firms endogenously, a theoretical portrait closer to the reality is provided, and it is also compatible to explanations proposed by the previous paper.
3.1. Strategy Combination 1: All the Firms are Informed

In this case, all the two firms regard $\hat{\theta}$ as a constant $\theta$ and they play a standard differentiated Cournot duopoly game at time $T=3$, each of the two firms chooses his quantity of output $q_i(\theta, \hat{q})$ to maximize

$$\arg\max_{q_i} p_i q_i = (\theta - q_i - t\hat{q}_i)q_i$$

The F.O.C. conditions are

$$q_i = (\theta - q_i - t\hat{q}_i)/2 \quad q_j = (\theta - q_j - t\hat{q}_j)/2$$

Where $\hat{q}_1$ is firm 1’s inference to firm 2’s quantity and $\hat{q}_2$ is firm 2’s inference to firm 1’s quantity. According to Linear Supply Function Equilibrium (LSFE) [11], the two firms’ equilibrium outputs can be set to linear forms as follows:

$$q_i = m_i + n_i \theta, \quad q_j = m_j + n_j \theta$$

Given that firm 1 and firm 2 are all informed to $\theta$, $\hat{q}_1$ and $\hat{q}_2$ can be rewritten by new forms of:
\[ \hat{q}_i = \text{E}[q_i = m_1 + n_i \theta | \theta] = q_i, \quad \hat{q}_2 = \text{E}[q_2 = m_2 + n_2 \theta | \theta] = q_2 \]

Taking the expressions of \( \hat{q}_1 \) and \( \hat{q}_2 \) to F.O.C conditions and the equilibrium output can be calculated as follows

\[ q_i = q_2 = \theta / (2 + t) \quad (6) \]

Then equilibrium prices of the products can be drawn by replace \( q_i \) and \( q_2 \) in the inverse demand function with the results in the above equations:

\[ p_i = p_2 = \theta / (2 + t) \quad (7) \]

Now, because the margin cost is assumed equal to 0, equilibrium net returns of the firms are:

\[ p_i q_i = p_2 q_2 = \left[ \theta / (2 + t) \right]^2 \quad (8) \]

### 3.2. Strategy Combination 2 or 3: One of the Two Firms is Informed

As it has been pointed out, the total payoffs or the object of the central planners is identical in case of strategy combination (II) and (III). Take firm 1 is informed and firm 2 is uninformed, just a case of strategy combination (II). Firm 1 knows the exact value of \( \theta \) and firm 2 only knows \( \text{E}[\theta] \) and \( \text{Var}[\theta] \). Now, the two firms’ decisions of outputs are given by the following optimization problem:

\[
\begin{align*}
\arg\max_{q_i} & \quad \text{E}[p_i q_i | s_i(\theta) = 1] = \text{E}((\theta - q_i - \hat{q_i})q_i | s_i(\theta) = 1) = (\theta - q_i - t\hat{q_i})q_i \\
\arg\max_{q_2} & \quad \text{E}[p_2 q_2 | s_2(\theta) = 0] = \text{E}((\theta - q_2 - \hat{q_2})q_2 | s_2(\theta) = 0) = \text{E}[\theta] - q_2 - t\hat{q_2})q_2 
\end{align*}
\]

The F.O.C conditions are:

\[ q_i = (\theta - q_i - t\hat{q_i}) / 2, \quad q_2 = (\text{E}[\theta] - q_2 - t\hat{q_2}) / 2 \quad (9) \]

Similarly, under the case of firm 1 is informed and firm 2 is uninformed, now:

\[ q_i = m_1 + n_i \theta, \quad q_2 = m_2 + n_2 \theta \quad (11) \]

\( \hat{q}_1 \) and \( \hat{q}_2 \) can be rewritten as:

\[ \hat{q}_1 = m_1 + n_1 \text{E}[\theta] s_1(\theta) = 0 = m_1 + n_1 \text{E}[\theta] \]

\[ \hat{q}_2 = m_2 + n_2 \text{E}[\theta] s_1(\theta) = 1 = m_2 \quad (12) \]

Replacing \( \hat{q}_1 \) and \( \hat{q}_2 \) in the above equation to the F.O.C conditions and two equations in which \( \hat{q}_1 \) and \( \hat{q}_2 \) is unknown are carried out, solve the equations and then \( q_i = q_i(m_1, n_i, m_2) \) and \( q_2 = q_2(m_1, n_i, m_2) \) are worked out, comparing coefficients in \( q_i(m_1, n_i, m_2) \) and \( q_2(m_1, n_i, m_2) \) to coefficients in the original linear forms, \( m_1, n_i, m_2 \) can be solved. Replacing variables \( m_1, n_i, m_2 \) in the original linear forms with the solved \( m_1, n_i, m_2 \), \( q_i \) and \( q_2 \) could be shown as:

\[ q_i = \frac{2 \theta + t(\theta - \text{E}[\theta])}{2(2 + t)}, \quad q_2 = \frac{\text{E}[\theta]}{2 + t} \quad (13) \]

Then the prices of the products can be solved in a similar way, and expectation value of the firms’ net returns are shown as follows:

\[ \begin{align*}
\text{E}[p_i q_i] &= \frac{2\theta + t(\theta - \text{E}[\theta])}{2(2 + t)} \\
\text{E}[p_2 q_2] &= \frac{\text{E}[\theta](4\theta - 2\text{E}[\theta] - t'\text{E}[\theta])}{2(2 + t)^2}
\end{align*} \quad (14) \]

### 3.3. Strategy Combination (4): None of the Firms is Informed

in case of strategy combination (IV), all firms know only value of \( \text{E}[\theta] \) and \( \text{Var}[\theta] \), the exact value of \( \theta \) is left uninformed. Now the firms can just estimate prices of their products by using value of \( \text{E}[\theta] \) in the reverse demand function, this is a case similar to case under strategy combination (I) and the firms’ expectation of equilibrium outputs are:

\[ q_i = q_2 = \text{E}[\theta] / (2 + t) \quad (15) \]

Now the firms’ expectation of net return can be given by:

\[ \text{E}[p_i q_i | s_i(\theta) = 0] = \text{E}[p_2 q_2 | s_i(\theta) = 0] = \left[ \text{E}[\theta] / (2 + t) \right]^2 \quad (16) \]

### 4. THE CENTRAL PLANNER’S OPTIMAL SCHEME

Having solved the firms’ payoffs under all possible settings, the central planner’s optimal plan can now be characterized by some propositions.

**Proposition 1**: It is better for the central planner to assign information resources to all the firms(strategy combination (I)) than to assign nothing to all the two firms(strategy combination (IV)).

**Proof**: Because the planner’s is optimal in the sense that the total of the payoffs of the firms is maximized, according to equation (II) and (IV), the planner’s object function under case of strategy combination (I) is:

\[ p_i q_i + p_2 q_2 = 2\left[ \text{E}[\theta] / (2 + t) \right]^2 = k_i \theta^2 \quad (17) \]

Where \( k_i = 2 / (2 + t)^2 \), and object function under strategy combination (IV) is:

\[ \text{E}[p_i q_i + p_2 q_2 | s_i(\theta) = 0, s_i(\theta) = 0] = \left[ \text{E}[\theta] / (2 + t) \right]^2 = k_i \text{E}^2[\theta] \]

(18)

Form the view of ex ante expectation value, the following inequality holds always:

\[ \text{E}[k_i \text{E}^2[\theta]] = k_i \text{E}[\theta] = k_i \text{E}^2[\theta] > k_i \text{E}^2[\hat{\theta}] = \text{E}[k_i \text{E}^2[\hat{\theta}]] \quad (19) \]

The first equality holds due to the identity relation links expectation to variance of a stochastic variable, and the inequality holds just because variance of a random variable is always greater than 0. That is, strategy
combination (I) is always better than strategy combination (IV) ex ante.

Proposition 1 is proved.

Proposition 1 shows that if the planner disclose value of \( \theta \) at time \( T = 1 \), then the informed firm could make a perfect decision of output quantity at time \( T = 2 \) and the total payoffs of the industry will be greater. On the other hand, if all firms make decisions themselves without any information subsidy, they can only set up their outputs quantities under the rule of maximizing their expectation payoffs by using public knowledge of \( E[\hat{\theta}] \) and \( \text{Var}[\hat{\theta}] \). For the latter case, the value of information has not been taken full advantage of and then a deadweight loss occurs. Obviously, it is not the planner's case that leaves all firms uninformed.

Proposition 1 supports the viewpoint that government intervention and macroeconomic control are required in real economy. In fact, when the market firms face is not a perfect market described in classic general equilibrium theory, market mechanism functions in a way which is different from the one described in perfect competition market and a loss of efficiency is inevitable, as a supplementary mechanism for market price rule in imperfect competition market, government intervention enhances efficiency of resource allocation.

Obviously, strategy combination (I) is proved a better one than strategy combination (IV), but more have to be done to show which is the best one among strategy combination (I), (II) and (III).

**Proposition 2.** If the level of competition between firms is above some threshold, the planner chooses strategy combination (II) or (III) rather than (I), and vice versa.

**Proof:** For the planner, difference between strategy combination (I) and (II) or (III) is:

\[
E[\{p_1q_1 + p_2q_2|s_1(\theta) = 1, s_2(\theta) = 1\}] - E[\{p_1q_1 + p_2q_2|s_1(\theta) = 1, s_2(\theta) = 0\}] 
\]

\[
= 2E(\hat{\theta}^2) + 2t 
\]

\[
= E(\{2(\theta - E[\hat{\theta}])\}^2 + E(\{4\theta - 2E[\hat{\theta}] - t'\theta - tE[\hat{\theta}]\}) 
\]

\[
= (t^2 + 4t - 4)\text{Var}[\hat{\theta}] 
\]

(20)

Note that \( \theta \) and \( \theta^2 \) are constants for an informed firm and are stochastic variables for an uninformed firm, moreover, the realization of \( \hat{\theta} \) is unobserved to all the firms, that means the two expressions \( E[\hat{\theta}] \) and \( \text{Var}[\hat{\theta}] \) holds.

Now the sign of the difference value between two strategy combinations is in line with the sign of \( t^2 + 4t - 4 \), given \( t \in [0,1] \):

\[
t^2 + 4t - 4 > 0, \ t \in (-2 + 2\sqrt{2}, 1] 
\]

\[
t^2 + 4t - 4 < 0, \ t \in [0, -2 + 2\sqrt{2}) 
\]

(21)

Where \(-2 + 2\sqrt{2} \approx 0.8284\) is a threshold above which the planner's best plan is strategy combination (II) or (III) and below which the optimal plan is strategy combination (I). Proposition 2 is proved.

**Proposition 3:** Total of firms' payoffs in case of strategy combination (I), (II), (III) is negative related to the level of competition between firms.

**Proof:** Let \( EPQ_1, EPQ_2, EPQ_3 \) are ex ante total of firms' payoffs when the planner chooses strategy combination (I), (II), (III), their values can be solved as follows:

\[
EPQ_1 = \frac{2}{2 + t} E[\theta]_t \quad \text{or} \quad 0 < t < 0.8284 
\]

\[
EPQ_2 = EPQ_3 = \left( \frac{2\theta + t(\theta - E[\hat{\theta}])}{2(2 + t)} \right)^2 + \frac{E[\hat{\theta}](4\theta - 2E[\hat{\theta}] - t'\theta - tE[\hat{\theta}])}{2(2 + t)^2} 
\]

(22)

And then:

\[
dEPQ_1/dt = -4E'[\hat{\theta}] + \text{Var}[\hat{\theta}] < 0 
\]

(23)

and

\[
dEPQ_2/dt = dEPQ_3/dt = -\frac{4E'[\hat{\theta}]}{(2 + t)^3} < 0 
\]

(24)

Proposition 3 is proved.

Though the total count of firms in the model is fixed and much less than the count a complete competition market should consist of, and the tense between firms is not a constant. Proposition 3 shows that the market mechanism plays a more important role and contribution of policies implemented by the planner becomes less significant when the tense of competition increases.

The value of \( EPQ_2 \) and \( EPQ_3 \) are influenced by the level of competition \( t \) through channel \(-4E'[\hat{\theta}] / (2 + t)^3\), but \( EPQ_1 \) is also influenced by channel \(-4\text{Var}[\hat{\theta}] / (2 + t)^3\) and \( EPQ_3 \) decreases in a faster speed while \( t \) increases than \( EPQ_2 \) and \( EPQ_3 \) do. As \( t \) is small, \( EPQ_3 > EPQ_2 \) and difference between them decreases while \( t \) increases, finally, \( EPQ_3 < EPQ_2 \) when \( t \) is large enough.

As it will be shown in proposition 4 and 5, besides \( t \), difference between strategy combinations are influenced by expectation and variance of market demand.

**Proposition 4:** For the central planner, sum of firms' payoffs in case of strategy combination (I), (II), (III) is positive related to \( E[\hat{\theta}] \) and \( \text{Var}[\hat{\theta}] \).

**Proof:** Because it has been assumed that \( \hat{\theta} \in [0, \infty) \) and then \( E[\hat{\theta}] > 0 \) holds, then the following results can be verified:
\[ \begin{align*}
\frac{dE[\theta]}{dE[\bar{\theta}]} &= -4E[\bar{\theta}]/(2+t)^2 > 0 \\
\frac{dE[\theta]}{dE[\bar{\theta}]} &= -2/(2+t)^2 > 0
\end{align*} \] (25)
and
\[ \begin{align*}
\frac{dE[\theta]}{d\text{Var}[\theta]} &= -4E[\bar{\theta}]/(2+t)^2 > 0 \\
\frac{dE[\theta]}{d\text{Var}[\theta]} &= -(t^2 + 4t + 4)/(2+t)^2 > 0
\end{align*} \] (26)
can also be verified.

Proposition 4 is proved.

**Proposition 5:** For the planner, difference value between strategy combination (I) or (II) or (III) is independent of \( E[\theta] \), and the correlation between the difference value and \( \text{Var}[\theta] \) is determined by the value of \( t \).

**Proof:** Is can be easily verified that:
\[ \begin{align*}
\frac{dE[\theta]}{dE[\bar{\theta}]} &= -4E[\bar{\theta}]/(2+t)^2 > 0 \\
\frac{dE[\theta]}{d\text{Var}[\theta]} &= -(t^2 + 4t + 4)/(2+t)^2 > 0
\end{align*} \] (27)
Proposition 5 is proved.

From proposition 1-5, \( t \) is a key factor when the central planner makes his optimal plan of information resource allocation, but difference value between the best plan and the second best plan is influenced by variance of stochastic market demand.

It can be concluded from the above propositions that the planner should do something when the market is imperfect. As strategy combination (I) is always better than the 4th one, the first one is the optimal one when \( t \) is small and the second or the third one is best for the planner when \( t \) is large enough, but strategy combination (IV) would never be the planner’s choice in any case.

Intuitively, information resource is valuable in the real world rather than that in the perfectly competitive market. Assignment of resource to all firms may not be the best way of government regulation, but assignment nothing is always the most inefficient plan. A central planner can usually choose his plan to make the market run more efficiently, though regulations by planner could decrease the variance of random information resources, the social total welfare is enhanced by planner’s regulation policy when variance of market demand is larger.

Now linking assumption that only one of the firm possesses private information about the uncertain demand to the case \( t \in (-2+2\sqrt{2},1] \) in this paper, in this cases, the planner’s optimal decision is \( (s_1(\theta) = 1, s_2(\theta) = 0) \) or \( (s_1(\theta) = 1, s_2(\theta) = 0) \), this kind of settings is in line with Verecchia’s works[2]. Though the case \( t \in [0,-2+2\sqrt{2}) \) could not linked to the settings just like that in Clinch and Verecchia’s work[1], the case that \( t = -2+2\sqrt{2} \) may matter. When \( t = -2+2\sqrt{2} \) holds, the planner’s mixed dominant strategies should be considered. In case of \( t = -2+2\sqrt{2} \), strategy combination (I) is indifferent to the 2nd or 3rd one, and any mixed strategy satisfy the expression that \( p(1)p(2)p(3) = p(1) + p(2) + p(3) = 1 \) could be the planner’s optimal decision, this is a case which match assumptions in Clinch and Verecchia’s work[1].

From the above analysis, a new explanation of why information endowments differ from each other in a unified framework, in which tense of competition is crucial. The limit of model in this paper is that when \( t \in (-2+2\sqrt{2},1] \) holds, the information resource is allocated randomly, the problem which should be endowed with the exact value of the demand remains unsolved.

**5. CONCLUSIONS**

In this paper, a unified framework is proposed to classify the contradiction that all firms are partially informed to market demand information in some early works, but only one of the firms is assumed informed in the other works. The paper shows that one possible reason may be that the information resource is allocated by a central planner, and different assumptions can be linked to the planner’s optimal decision related to tense of competition between firms. Results in this paper bridge the gap of different assumptions with a determinant which was not considered in early works.

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