A line spectrum detector based on improved coherent power spectrum estimation

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Abstract. The line spectrum detection method based on coherent power spectrum estimation is one of the important research directions of weak signal line spectrum detection. For the purpose of resolving the problem of phase compensation inaccurate introduced by limited frequency search accuracy with the existing coherent power spectral estimate method, a modified line spectrum detector is proposed. The zero padding is used to improve the line spectrum frequency search accuracy in discrete Fourier transform (DFT) for phase compensation calculations, thereby obtaining more accurate line spectrum frequencies and phase compensation, which further improves line spectrum detection performance. The detection performances of three detections based on Welch's method, coherently averaged power spectral estimate(CAPSE) and modified method are compared each other by computer simulations. The results show modified line spectrum detection can obtain a gain of 2-3dB, which verifies the effectiveness of the modified method.

1. Introduction

The acoustic stealth technology has developed rapidly in recent years. It is difficult for passive sonar to effectively detect ship radiated noise through conventional broadband detection methods. The use of the line spectrum higher than the continuous spectrum by more than 10dB can increase the range of passive sonar [1]. Nevertheless, for quiet submarines, the detection of the line spectrum in background noise is still a relatively difficult thing.

The essence of line spectrum detection is to detect one or more discrete spectral components contained in broadband noise. In order to complete signal detection under low signal-to-noise ratio(SNR), the classic FFT-based power spectrum estimation method (PSE), which does not require any additional assumptions on the data, is undoubtedly the best choice. Directly uses the periodogram method when detecting weak signals, will require a relatively large analysis time to ensure sufficient frequency resolution to improve the line spectrum detection performance. However, the non-stationary of the signal or background, the storage capacity of the signal processing equipment, the increase in false alarms caused by the increase in the length of the FFT, and the increase in the variance of the background noise all limit the FFT length, so the segmented average method is usually used, such as the Welch's method [2], for reduce the amount of calculation and the variance of the spectrum estimation. But the Welch's method is a non-coherent estimation method, which does not use the phase information between the pieces of data, so the line spectrum detection ability is not as good as the periodogram method based on coherent estimation.
Regarding how to use the phase information of the frequency spectrum in the segmented average method, most domestic and foreign studies are focus on time domain average [3]. Later, Wang [4] proposed to use the maximum likelihood method to estimate the phase difference between segments and compensate for it. It is more complicated and not conducive to real time estimation. Lan [5] proposed the coherent average power spectrum estimation (CAPSE) based on the Welch's method to obtain more accurate line spectrum frequency search and phase compensation by performing the discrete Fourier transform (DFT) on the time series of DFT values, this improves line spectrum detection performance. However, the frequency search accuracy of this method is limited by the number of FFT points of the second DFT, which is the accumulated number of segments, and the accuracy of the line spectrum frequency estimation is still not high enough.

Based on the CAPSE method, this paper uses zeros padding method to the time series of DFT values in the same frequency points when performing the second DFT, which further improves the accuracy of the line spectrum frequency search and obtains more accurate phase compensation, thereby further improving line spectrum detection performance.

2. Coherent averaging line spectrum detection

2.1 Line spectrum detection model and Welch’s method

The line spectrum detection model is usually described as a binary hypothesis testing problem:

\[
\begin{align*}
H_0 : x(n) &= z(n) \\
H_1 : x(n) &= s(n) + z(n)
\end{align*}
\]  

(1)

where a single sinusoidal signal of \( s(n) \) can be expressed as \( A \sin(\omega_0 n + \varphi) \), with amplitude \( A \), angular frequency \( \omega_0 \), and \( \varphi \) is the initial phase obeys uniform distribution on \((0, 2\pi)\); \( z(n) \) is a stationary additive Gaussian distribution with a mean of 0 and a variance of \( \sigma_z^2 \), \( z(n) \sim N(0, \sigma_z^2) \).

For the Welch’s method, we divide the signal \( s(n) \) into \( K \) segments of length \( M \), and overlap as \((M-D)\). so that the \( k \)th segment signal is expressed as:

\[ x_k(m) = x(m+kD), \quad m=0,1,\ldots,M-1, k=0,1,\ldots,K-1. \]

(2)

The Fourier transform of \( x_k(m) \), \( X_k(\omega) \), is defined as

\[
X_k(\omega) = \sum_{m=0}^{M-1} w(m) x_k(m) e^{-j\omega m},
\]

(3)

where \( w(m), \quad m=0,1,\ldots,M-1 \) is a spectral window function. The periodogram of the \( k \)th segment is given by

\[
I_k(\omega) = \frac{1}{MU} |X_k(\omega)|^2,
\]

(4)

\[
U = \frac{1}{M} \sum_{m=0}^{M-1} w^2(m).
\]

(5)

the power spectrum obtained by averaging the periodograms of all segments can be defined as,

\[
P_{\text{welch}}(\omega) = \frac{1}{K} \sum_{k=0}^{K-1} I_k(\omega).
\]

(6)
2.2 phase compensation

Similar to the principle of beamforming, the principle of CAPSE compensates for the phase difference introduced by the delay difference between the segments and then superimposes them.

For the line spectrum detection model, as defined in (1), the power spectrum in (6) can be rewritten as

$$P_{\text{welch}}(\omega) = \frac{1}{KMU} \sum_{k=0}^{K-1} \left( \left| S_k(\omega) \right|^2 + \left| Z_k(\omega) \right|^2 + 2 \Re \{ S_k(\omega) \ast Z_k(\omega) \} \right),$$

where $$S_k(\omega)$$ and $$Z_k(\omega)$$ are the Fourier transforms of the line spectrum and noise components of $$x_k(m)$$ respectively. When the noise is unrelated to the line spectrum, the cross term tends to zero as the value of K increases. Assuming that the frequency spectrum of the line spectrum of the 1th segment is $$S_0(\omega)$$ and the center frequency is $$\omega_0$$, the frequency spectrum of the line spectrum of the $$k^{\text{th}}$$ segment of data can be expressed as

$$S_k(\omega) = S_0(\omega) e^{j\Phi_k}$$

where

$$\Phi_k = \omega_0 kD$$

The $$\Phi_k$$ represents the phase difference between the $$k^{\text{th}}$$ segment signal and the 1th segment signal. Then, after the phase compensation of the $$k^{\text{th}}$$ signal is coherently superimposed,

$$\bar{X}(\omega) = \frac{1}{K} \sum_{k=0}^{K-1} X_k(\omega) e^{j\Phi_k}$$

$$= \frac{1}{K} \sum_{k=0}^{K-1} X_k(\omega) e^{-j\omega_0 kD}$$

$$= S_0(\omega) + \frac{1}{K} \sum_{k=0}^{K-1} Z_k(\omega) e^{-j\omega_0 kD}.$$  

After signal phase compensation and coherent averaging, the signal remains unchanged, and the noise power becomes $$1/K$$ before averaging, then the SNR of the line spectrum is improved by $$10\log K$$. This is a coherent average line spectrum detector which the theoretical limit of SNR improvement relative to an incoherent detector.

2.3 Frequency estimation and improvement

The DFT of the $$k^{\text{th}}$$ segment data is expressed as

$$X_k(\omega_n) = \sum_{m=0}^{M-1} x_k(m) e^{-j\omega_n m}.$$  

Discrete frequency $$\omega_n$$ can be expressed as

$$\omega_n = 2\pi \frac{m}{M}, m = 0,1,\ldots,M-1.$$  

The difference $$\delta\omega_0$$ between the line spectrum frequency $$\omega_0$$ and the nearest $$\omega_n$$ in the DFT values is expressed as

$$\delta\omega_0 = \omega_0 - \omega_n = \frac{2\pi \delta_0}{M}, |\delta_0| \leq \frac{1}{2}.$$
where $\delta_0$ is the frequency deviation of a sine wave from the closest frequency point $\omega_n$ in the DFT. In the absence of noise, the relationship between the DFT value of the $k^{th}$ segment and the DFT value of the 1st segment can be expressed as

$$X_k(\omega_n) = X_0(\omega_n)e^{j\delta_0 k D}.$$  \hspace{1cm} (14)

In this way, no matter there are several single frequency signals, phase compensation can be done. And the frequency of the phase compensation is $\delta_0$. It can be seen from (14) that the DFT value sequence formed by $X_k(\omega_n)$ along the time series (K series) is denoted as $X_{ak}(k)$, and its frequency sequence is $\omega_n$. It performs DFT again according to the time series (K series) as follows:

$$X(\omega_n) = \sum_{k=0}^{K-1} X_{ak}(k)e^{-j\omega_n k D}. \hspace{1cm} (15)$$

defines the average value of the $k^{th}$ phase compensation. When $\omega_v = \delta_0$, the phase difference is fully compensated. However, this step is to perform a frequency search with a search accuracy $2\pi/KM$ near $\omega_n$, due to the limited estimation accuracy of segment number K, the estimation accuracy of $\omega_v$ is limited, the phase compensation accuracy is affected. Add K zero points after $X_k(\omega_n)$ in (15), and then perform a DFT operation of 2K points, so that the search accuracy will increase to $\pi/KM$:

$$X(\omega_n) = \sum_{k=0}^{2K-1} X_{ak}(k)e^{-j\omega_n k D}.$$  \hspace{1cm} (16)

Because there are generally not many segments, the amount of calculation added is small. (16) defines the average value of the K phase compensation corresponding to zero padding. For each DFT value $\omega_n$, take the maximum value of $X(\omega_n)$ in (16), so as to ensure that the signal information is retained to the greatest extent. The corresponding frequency $\omega_d$ can be expressed as:

$$\omega_d = \arg \max_{\omega_n} |X(\omega_n)|^2. \hspace{1cm} (17)$$

The power spectrum of ZP-CAPSE can be defined as:

$$P_{ZP-CAPSE}^{xx}(\omega_d) = \frac{1}{UM}|X(\omega_d)|^2.$$  \hspace{1cm} (18)

2.4 Performance analysis

We consider the mean and variance of $P_{\text{Welch}}^{xx}(\omega)$ and $P_{\text{CAPSE}}^{xx}(\omega)$ based on a rectangular window, which means U=1.

Welch [2] has shown that the mean and variance of $P_{\text{Welch}}^{xx}(\omega)$ are as follows:

$$E(P_{\text{Welch}}^{xx}(\omega)) = MS_0^2 + S_s^2. \hspace{1cm} (19)$$

$$\text{Var}(P_{\text{Welch}}^{xx}(\omega)) = \frac{K}{K-1}S_s^2.$$  \hspace{1cm} (20)

Lan [5] has shown that the mean and variance of $P_{\text{CAPSE}}^{xx}(\omega)$ are as follows:
\[ E(P_{\text{CAPSE}}^{\text{AVG}}(\omega)) = MS_0^2 + \frac{1}{K} \sum_{k=1}^{K} \frac{1}{k} S_k^2. \]  

\[ \text{Var}(P_{\text{CAPSE}}^{\text{AVG}}(\omega)) = \frac{1}{K^2} \left( 2K + 2K \sum_{k=1}^{K-1} (-1)^k \left( K - 1 \right) \left( k+1 \right)^{-1} \right) S_k^2. \]

The mean value of coherent methods is significantly greater than that of non-coherent methods. If we define the difference between the output SNR and the input SNR as the gain of the line spectrum detector, then the gain of the averaging power spectrum detector (AVGPR) is \( M \), the gain of the line spectral detection based on coherent method is \( MK \sum_{k=1}^{K} \frac{1}{k} \), and the gain of the periodogram method is \( MK \).

Table (1) is the comparison of the theoretical value and the simulated value of the SNR obtained by the simulation value under different segments and the number of DFT points after adding the rectangular window with the mean value \( U=1 \) to the total length of 131072 points of the independent data sample. The gain obtained the line spectral detection based on the ZP-CAPSE (ZP-CAVGPR) is 2-3dB higher than the line spectral detection based on the CAPSE (CAVGPR), and is closer to the theoretical value, which verifies the effectiveness of the zero padding method.

| segments of data | theoretical SNR of incoherent line spectral detection | rhetorical SNR of incoherent line spectral detection | input SNR | output SNR of AVGPR | output SNR of CAVGPR | output SNR of ZP-CAVGPR |
|------------------|---------------------------------------------|-----------------------------------------------|---------|-------------------|-------------------|---------------------|
| \( M=2048, K=64 \) | 33                                           | 44                                           | -25     | 7                 | 14                | 16                  |
| \( M=4096, K=32 \) | 36                                           | 45                                           | -25     | 10                | 15                | 18                  |

The following analyzes the detection performance of the coherent average line spectrum detector. Figure 1 shows the detection probability and SNR curve (ROC curve) of AVGPR and ZP-CAVGPR. The total length of independent data samples is 131072 points, the false alarm probability is \( 3 \times 10^{-3} \), and the complex signal SNR is defined as \( \text{SNR} = 10 \log(A^2 / \sigma^2) \). It can be seen that for incoherent average estimation, the more segments there are, the more the detection performance decreases due to incoherent superposition. For the coherent average spectrum estimation, although the segmentation has an impact on its detection effect, the impact is limited. The ROC curves of different segments are basically distributed between the two ROC curves estimated by the coherent average spectrum in Figure 1. From the application point of view, the number of segments is limited by the frequency resolution. Too many segments will result in insufficient frequency resolution. When the SNR is not less than -25dB, the four cases have higher detection probability, and the coherent average line spectrum estimation method still has better detection performance at -30dB. Figure 2 is the ROC curve of AVGPR, CAVGPR, ZP-CAVGPR under the condition of non-integer frequencies. The coherent average spectrum estimation method is better than the incoherent average spectrum estimation method, compared with the CAPSE method, the detection performance based on the ZP-CAPSE method is improved by 2-3dB, and the ROC curve also gives the limit of the line spectrum detection performance.
3. Results
The following will verify the effectiveness of the method through simulation. The simulation has been performed more than 100 statistical running.

3.1 Simulation
Computer simulation analysis was used to compare the detection performance of AVGPR, CAVGPR, and ZP-CAVGPR under different SNR. The data sample is 131072, there is no overlap, based on a hanning window. The two frequency components contained in the signal are \( f_1 = 300.18 \text{Hz} \), \( f_2 = 600.12 \text{Hz} \), the sampling frequency is \( f_s = 2000 \text{Hz} \).

Figure 3 shows the line spectrum detection results of the periodogram, AVGPR, CAVGPR, and ZP-CAVGPR when the SNR is -30dB. Under the condition of low SNR of -30dB, the line spectrum of AVGPR is basically invisible, and the line spectrum of CAVGPR is basically submerged in false alarms, while the performance of ZP-CAVGPR is still excellent, which can be close to the effect of periodogram method. This is very meaningful in the detection of weak signals. At the same time, compared with the improvement before, the false alarm is reduced, and the detection performance is improved by 2-3dB.

(a) Periodogram, \( M=131072, K=1 \) (b) AVGPR, \( M=4096, K=32 \)
Figure 3 shows the line spectrum detection results for scenarios involving two sine waves buried in noise, where the signal frequencies are 300.18 Hz and 600.12 Hz, with a SNR of -30 dB, and using a data length of N=131072 without overlap. The spectral analysis was performed using (a) the periodogram, (b) AVGPR, (c) CAVGPR, and (d) ZP-CAVGPR, each employing different parameters.

Figure 4 demonstrates the line spectrum detection outcomes for the periodogram, AVGPR, CAVGPR, and ZP-CAVGPR methods when the SNR is -20 dB. At this SNR level, all four methods yield visible line spectra. The coherent average power spectrum demonstrates superior performance compared to the incoherent average power spectrum. The ZP-CAVGPR method achieves a gain of 2-3 dB relative to the CAVGPR.
From the theoretical analysis part, we can understand that the ZP-CAPSE method is introduced from Welch, and the periodogram method can be seen to be the full sequence Fourier transform of \( K = 1 \), which completely retains the phase information, so the periodogram method is used for detection performance is the theoretical limit of the ZP-CAPSE method. Comparing these four methods, we can see the superiority of the coherent average spectrum estimation method, which achieves the desired effect while reducing the amount of calculation. And the benefits of the zero padding method are very obvious. The detection performance of the CAPSE method at non-integer frequency points is improved by 2-3dB, which is beneficial to the subsequent line spectrum tracking.

4. Conclusions

The ZP-CAVGPR detector proposed in this paper has a gain of 2-3dB compared with the CAVGPR detector when detecting the line spectrum of non-integer frequency points. The improved ZP-CAPSE method proposed at the same time is simple and effective, suitable for real-time implementation. This improved gain comes from improving the search accuracy of the line spectrum frequency, obtaining more accurate phase compensation, the end result is to improving the performance of the CAPSE method at non-integer frequencies. Detection performance. The ZP-CAVGPR detector in this article can be used as the preprocessing of narrow-band tracking. The data can be in different channels, and the corresponding time period can be selected to estimate the coherent power spectrum to improve the SNR of line spectrum.

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