Evolution Effects in
$Z^0$ Fragmentation into Charmonium

P. Ernström, L. Lönnblad and M. Vänttinen

NORDITA
Blegdamsvej 17, DK-2100 Copenhagen Ø

Abstract

In $Z^0$ decay into prompt charmonium, i.e. charmonium not originating from $B$-meson decays, the most important contribution is expected to come from colour-octet mechanisms. However, previous fixed-order calculations of the colour-octet contribution contain large logarithms which, in a more complete treatment, should be resummed to all orders. We study this resummation by using a Monte Carlo QCD cascade model and find that the fixed-order colour-octet result is diminished by 15%. We compare the Monte Carlo calculations with results obtained by using analytical evolution equations.
1 Introduction

The production of charmonium and bottomonium states in various processes, especially at high-energy colliders \[1\], has recently received considerable experimental and theoretical interest. New data have become available from \(p\bar{p}\) \[2\], \(ep\) \[3\] and \(e^+e^-\) \[4–6\] colliders. Theoretically, it has been realized that quarkonium production at colliders is dominated by parton fragmentation \[7\]. For the factorization of the hard dynamics of \(Q\bar{Q}\) production and the soft dynamics of quarkonium bound state formation, a new framework is provided by non-relativistic QCD (NRQCD) \[8\]. This new theoretical approach has had initial successes and is now approaching a stage where its predictive power can really be judged by testing relations between various observables.

Within NRQCD, quarkonium is produced either through colour-singlet mechanisms, where a perturbatively produced colour-singlet \(Q\bar{Q}\) pair evolves non-perturbatively into a quarkonium state, or through colour-octet mechanisms, where the intermediate \(Q\bar{Q}\) pair is in a colour-octet state and the non-perturbative transition involves the exchange of soft gluons with the environment. Before the advent of NRQCD, quarkonium production was usually calculated taking into account only the colour-singlet channels. Contributions from colour-octet channels are usually suppressed by powers of \(v^2\), the intrinsic velocity squared of the bound state, which apparently justifies the colour-singlet model. However, there are reactions where this suppression is either absent or compensated by an enhancement of colour-octet channels by powers of the strong coupling constant \(\alpha_s\).

A well-known example is the production of prompt \(\psi'\) or direct prompt \(J/\psi\) in \(p\bar{p}\) collisions at large \(p_{\perp}\). (By “prompt” we mean those charmonium states which are not produced in \(B\)-meson decays, and by “direct” those
which are not produced in the decays of other charmonium states.) The production rates measured by the CDF Collaboration [9] are more than an order of magnitude above the colour-singlet-model prediction. This anomaly can be explained in the framework of NRQCD [10], which includes a colour-octet production channel whose contribution is proportional to the probability of a non-perturbative transition, to be treated as a free parameter of the theory and fitted to the CDF data.

The fact that NRQCD comes with a number of free parameters (although there are scaling rules which determine the order of magnitude of the transition probabilities in terms of $v^2$) necessitates cross checks between different experiments. Colour-octet contributions to e.g. photoproduction [11] and fixed-target hadroproduction [12] have indeed been calculated, partly in order to provide such cross checks, partly as an attempt to explain discrepancies between the colour singlet model and experimental data [13].

An interesting laboratory of quarkonium production is provided by $e^+e^-$ annihilation at the $Z^0$ pole. Experimental branching ratios or upper limits for $Z^0$ decay into prompt quarkonium have recently been published by the OPAL [4,5] and DELPHI [6] Collaborations. Theoretical calculations [7,14–19] show that the largest contribution is expected to come from the same non-perturbative transition that in the NRQCD framework explains the $\psi'$ anomaly. Using the value of the NRQCD matrix element from Tevatron fits, the magnitude of this contribution is consistent with the experimental numbers [18,19]. However, the existing calculations have been done at a fixed order in $\alpha_s$, and the result includes large logarithms, $\ln(M_{Z}^2/M_{J/\psi}^2)$, which call for a resummation. It is the purpose of this paper to do this resummation and see how much the fixed-order results are changed.

Our principal tool in studying the evolution effects is a Monte Carlo QCD
cascade model, implemented in the ARIADNE program \cite{20}. This approach was used in earlier work \cite{21} on the large \( p_{\perp} \) hadroproduction of quarkonium.

The evolution effects can also be studied using analytical evolution equations \cite{22} to determine the scale dependence of fragmentation functions. Since the analytic fragmentation formalism fails in the threshold region where the colour-octet fragmentation process peaks, we derive our final results using the Monte Carlo method. Above the threshold region we use the analytical evolution equations as a check of the Monte Carlo implementation.

In Section 2 below, we discuss in detail the colour-singlet and colour-octet mechanisms of parton fragmentation into charmonium and review the earlier work of other authors. We also describe the analytical fragmentation formalism used as a check of our Monte Carlo simulation. In Section 3 the Monte Carlo simulation of evolution effects with the ARIADNE program is discussed. The Monte Carlo method is then applied to the colour-octet fragmentation process to obtain our main results. These are presented in Section 4, where we show that the evolution effects diminish the theoretical prediction of \( Z^0 \) branching ratio into prompt charmonium by about 15%. Finally, the results are discussed in Section 4.

2 QCD production mechanisms

In \( Z^0 \) decay into \( ^3S_1 \) charmonium (which we denote generically as \( \psi \)) the dominant colour-singlet contribution comes from charm fragmentation, i.e. the process where \( Z^0 \) decay into \( c\bar{c} \) is followed by the perturbative QCD process \( c \rightarrow c\bar{c} \left[ ^3S_1^{(1)} \right] + c \) and then the non-perturbative process \( c\bar{c} \left[ ^3S_1^{(1)} \right] \rightarrow \psi \).

There is an equal contribution from \( \bar{c} \) fragmentation. The cross section is

\[
\frac{d\Gamma}{dz}(Z^0 \rightarrow \psi + X) = 2 \Gamma(Z^0 \rightarrow c\bar{c}) D^{\psi}_{\bar{c}}(z, \mu),
\]

\(1\)
where $z = E_\psi/(M_Z/2)$ is the fraction of the initial charm quark energy taken by the $\psi$, $D_\psi^c(z,\mu)$ is the charm fragmentation function into $\psi$, and $\mu = O(M_Z/2)$ is the fragmentation scale \[14\]. The normalization of this contribution is related to the leptonic width of the $\psi$.

In \[14\] large logarithms $\ln(M_Z^2/M_{J/\psi}^2)$ were resummed using Altarelli-Parisi evolution equations \[23\] for the scale dependence of $D_\psi^c(z,\mu)$. Here we will use both Monte Carlo simulations and improved analytical evolution equations \[22\] to study the evolution effects.

The dominant colour-octet contribution comes from fragmentation mechanisms where $Z^0$ decay into $q\bar{q}g$ is followed by the perturbative splitting $g \to c\bar{c} \; [^3S_1^{(8)}]$ and then the non-perturbative process $c\bar{c} \; [^3S_1^{(8)}] \to \psi + X$. A leading contribution to the non-perturbative process comes from the emission of two soft gluons. The probability of this non-perturbative transition is proportional to the NRQCD matrix element $\langle 0|O_8^\psi(^3S_1)|0 \rangle$, i.e. the same element whose value has been fit to reproduce the CDF data. The normalization of the colour-octet component can therefore be obtained from Tevatron fits.

In the following, all numbers were derived using the colour-octet matrix element $\langle 0|O_8^{J/\psi}(^3S_1)|0 \rangle = 0.0066 \text{ (GeV)}^3$ obtained from a fit \[24\] of the CDF data and the colour-singlet matrix element $\langle 0|O_1^{J/\psi}(^3S_1)|0 \rangle = 0.73 \text{ (GeV)}^3$ \[14\] related to the wavefunction at the origin of coordinate space $R_S(0)$ through $|R_S(0)|^2 = (2\pi/9)\langle 0|O_1^{J/\psi}(^3S_1)|0 \rangle = 0.512 \text{ (GeV)}^3$. Furthermore, $m_c = 1.5 \text{ GeV}$, $M_\psi = 2m_c$, and $\alpha_s(2m_c) = 0.253$ \[15\] were used.

Below, we shall speak of $Z^0$ decay into direct prompt $J/\psi$. The production of $\chi_c$ and $\psi'$ from $c\bar{c} \; [^3S_1^{(8)}]$ intermediate states and their decays into $J/\psi$ can be included by multiplying the octet contribution by a factor 2.1, derived from the octet matrix elements of Ref. \[24\] and the branching ratios
given in [25]. The production of \( \psi' \) from \( c\bar{c} \) \( ^3S_1^{(1)} \) and its decay into \( J/\psi \) can be included by multiplying the singlet contribution by a factor 1.3, derived using the colour-singlet-model relation

\[
\frac{|R_{2S}(0)|^2}{|R_{1S}(0)|^2} \left( \frac{M_{\psi'}}{M_{J/\psi}} \right)^2 \frac{\Gamma(\psi' \rightarrow \ell\ell)}{\Gamma(J/\psi \rightarrow \ell\ell)} \quad (2)
\]

with experimental leptonic widths [25].

This far, the calculation of the colour-octet contribution has only been done at a fixed order in perturbative QCD. The process is then represented by the Feynman diagram shown in Fig. 1a and another diagram where the gluon is emitted by the antiquark line. The resulting charmonium energy spectrum is [18, 19]

\[
d\Gamma \left( Z^0 \rightarrow \psi + X \right) = \frac{\alpha_s^2(2m_c)}{18} \Gamma(Z^0 \rightarrow q\bar{q}) \frac{\langle 0|O_{8}^\psi (^3S_1)|0 \rangle}{m_c^3} \times \left[ \frac{1 + (1 - z)^2}{z} + 2 \left( \frac{M_\psi}{M_Z} \right)^2 \frac{2 - z}{z} + \left( \frac{M_\psi}{M_Z} \right)^4 \cdot \frac{2}{z} \right] \times \ln \left( \frac{z + \sqrt{z^2 - 4(M_\psi/M_Z)^2}}{z - \sqrt{z^2 - 4(M_\psi/M_Z)^2}} - 2 \sqrt{z^2 - 4(M_\psi/M_Z)^2} \right), \quad (3)
\]

which leads to an integrated width

\[
\Gamma(Z^0 \rightarrow \psi + X) = \Gamma(Z^0 \rightarrow q\bar{q}) \cdot \frac{\alpha_s^2(2m_c)}{36m_c^3} \frac{\langle 0|O_{8}^\psi (^3S_1)|0 \rangle}{m_c^3} \times \left[ 5 - \frac{\pi^2}{3} + 3 \log \left( \frac{M_\psi^2}{M_Z^2} \right) + \log^2 \left( \frac{M_\psi^2}{M_Z^2} \right) + \mathcal{O} \left( \frac{M_\psi^2}{M_Z^2} \right) \right], \quad (4)
\]

giving \( \text{Br}^{(8)}(Z^0 \rightarrow \text{direct prompt } J/\psi) = 0.68 \cdot 10^{-4} \) for the \( J/\psi \). Hence this contribution is clearly larger than the colour-singlet charm fragmentation contribution \( \text{Br}^{(1)}(Z^0 \rightarrow \text{direct prompt } J/\psi) = 0.22 \cdot 10^{-4} \) [16]. The \( z \) distribution is also markedly different, peaking around \( z = 0.1 \) whereas the singlet component peaks around \( z = 0.8 \).
Figure 1: The Feynman diagrams which describe the dominant colour-octet and colour-singlet mechanisms of $Z^0$ fragmentation into prompt $J/\psi$. (a) Colour-octet gluon fragmentation. There is another diagram where the gluon is emitted by the antiquark line. (b) Colour-singlet charm fragmentation. There is (in an axial gauge) another diagram which represents $\bar{c}$ fragmentation.

Far from the threshold ($z \gg M_\psi/M_Z$) the differential decay width can be written as

$$
\frac{d\Gamma}{dz} = \frac{\alpha_s^2(2m_c)}{18} \Gamma(Z^0 \to q\bar{q}) \frac{\langle 0|O_8^{\psi}(3S_1)|0\rangle}{m_c^3} \times \left[ 1 + \frac{(1 - z)^2}{z} \ln \left( \frac{z^2}{M_\psi^2/M_Z^2} \right) - 2z + O\left( \frac{M_\psi^2}{z^2M_Z^2} \right) \right].
$$

The large logarithm arises from the nearly collinear splitting of an initial quark into a quark and a virtual gluon, the virtuality $M_\psi^2$ being small on the scale set by the $Z^0$ mass. The double logarithm in Eq. (4) arises from the soft gluon emission represented by the factors $1/z$ in Eq. (3). In a complete calculation such logarithms should be resummed to all orders in perturbation theory.

Just as in the colour singlet charm fragmentation case, such a resummation can be performed by rewriting the decay width in terms of fragmentation functions and then resumming the large logarithms using analytical evolution
equations for the fragmentation functions. Such a treatment does, however, neglect terms of $O(M_\psi^2/(z^2M_Z^2))$, which become important in the threshold region, where the colour octet $J/\psi$–production peaks. We will therefore perform the resummation by means of a Monte Carlo simulation, and use the analytical resummation described below only as check at large $z$. Note that even at the threshold the virtuality of the fragmenting quark can be as low as of the order $M_\psi M_Z$ which is still much smaller than $M_Z^2$, thus motivating a fragmentation approach.

In fragmentation language, the Feynman diagram in Fig. 1a may be viewed either as quark production followed by quark fragmentation into $J/\psi$ (via splitting), or if the quark–gluon invariant mass is large, as gluon production followed by gluon fragmentation into $J/\psi$. Correspondingly, the colour octet contribution can be written as

$$
\frac{d\Gamma}{dz}(Z^0 \to J/\psi + X) = 2 \Gamma(Z^0 \to q\bar{q}) D_{q}(z, \mu^2) + \int_0^1 \frac{dx}{x} \frac{d\Gamma(Z^0 \to q\bar{q}g)}{dz_g}(z/x, \mu^2) D_{g}(x, \mu^2),
$$

(6)

where $\mu$ sets the scale for the fragmentation functions and defines the minimal $qg$ or $\bar{q}g$ invariant mass for the differential three-parton decay width

$$
\frac{d\Gamma(Z^0 \to q\bar{q}g)}{dz_g}(z_g, \mu^2) = \frac{C_F \alpha_s(\mu^2)}{\pi} \Gamma(Z^0 \to q\bar{q}) \times \left( \frac{1 + (1-z)^2}{z} \right) \log \left( \frac{z_g - \mu^2/M_Z^2}{\mu^2/M_Z^2} \right) - z_g + \frac{2\mu^2}{M_Z^2}.
$$

(7)

Inserting LO expressions for the fragmentation functions

$$
D_{q}(z, \mu^2) = \frac{\alpha_s^2(z) O_8^{(3S_1)} |0\rangle}{36m_c^3} \left( \frac{1 + (1-z)^2}{z} \right) \log \left( \frac{z\mu^2}{M_\psi^2} \right) - z + \frac{M_\psi^2}{\mu^2}
$$

(8)

$$
D_{g}(z, \mu^2) = \frac{\pi \alpha_s(z) O_8^{(3S_1)} |0\rangle}{24m_c^3} \delta(1 - z)
$$

(9)
in Eq. (3) the LO large \( z \) result Eq. (3) is reproduced up to corrections \( \mathcal{O}(\mu^2/M_Z^2) \).

At scales \( \mu \) of the order of \( M_Z \), Eq. (3) is totally dominated by the quark fragmentation term. The corrections are \( \mathcal{O}(1) \), but the leading logarithmic terms are still correctly reproduced.

To resum the large logarithms we have used the improved evolution equations \[22\]
\[
\mu^2 \frac{\partial}{\partial \mu^2} D^\psi_i(z, \mu^2) = d^\psi_i(z, \mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} \sum_j \int_z^1 \frac{dy}{y} D^\psi_j(z/y, y \mu^2) P_{ji}(y),
\]
\[
D^\psi_i(z, M^2_\psi/z) = 0.
\]

In the colour singlet case we have used the following charm fragmentation source term extracted from \[16\]
\[
d^c(z, x m_c^2) = \left(\frac{8 \alpha_s^2 |R(0)|^2}{27 \pi m_c^3}\right) \frac{x}{(x-1)^4} \left[ (x^2 - 2x - 47) - z(x-1)(x-9) + \frac{4z(1-z)}{2-z}x(x-1) - \frac{4(8-7z-5z^2)}{(2-z)}(x-1) + \frac{12z^2(1-z)}{(2-z)^2}(x-1)^2 \right] \theta \left( x - \frac{4}{z} - \frac{1}{1-z} \right).
\]

Using this source term we have solved the evolution equation \[16\] numerically and used Eq. (3) to find the colour singlet contribution to \( J/\psi \) production in \( Z^0 \) decays.

In the colour octet case the gluon fragmentation source term is a double delta function
\[
d^g(z, s) = \frac{\pi \alpha_s}{24 m_c^3} \frac{\langle 0 | \mathcal{O}_8^{S}(S_1) | 0 \rangle}{\delta(1-z) \delta \left( 1 - \frac{s}{M_\psi^2} \right)}.
\]

Equivalently, one may omit this source term and use Eq. (3) as an initial condition at the scale \( 2m_c \).
We have solved the evolution equation (10) numerically and used Eq. (3), performing the convolution of the evolved gluon fragmentation function and the $Z^0 \rightarrow q\bar{q}g$ differential decay width Eq. (7), to find the colour octet contribution to $J/\psi$ production in $Z^0$ decays.

The commonly used AP evolution differs from the improved evolution of Eq. (10) in two ways:

- In the AP evolution equations the source term $d_i^\psi(z, \mu^2)$ is put to zero and instead unphysical initial conditions are chosen at the $M_\psi$ scale such that the AP–evolved fragmentation functions agree with perturbative fragmentation functions at large scales.

- In the AP evolution equations, the fragmentation functions on the right hand side of Eq. (10) are evaluated at the scale $\mu^2$ instead of at the scale $y\mu^2$, and thus, fail to take into account the momentum constraint saying that the maximum virtuality of a parton taking a positive light-cone momentum fraction $y$ from a parent parton with virtuality $\mu^2$, is $y\mu^2$.

Neither of these points is crucial for our comparison with the Monte Carlo simulation at large scales and far from the threshold.

3 The Monte Carlo method

The implementation of charmonium production in the ARIADNE event generator was described in Ref. [21], and here we will only give the main points and present some improvements.

Three mechanisms are included (see below). In all cases, the perturbative splitting into a $c\bar{c}$ pair and the non-perturbative formation of the charmonium state are treated together as one step in the otherwise purely perturbative
QCD shower. The motivation behind including the soft formation of a charmonium state in the perturbative level, before the normal hadronization, is that all hard interactions in principle are already included in the splitting function.

The three implemented mechanisms are treated somewhat differently:

- **Colour octet gluon fragmentation:** implemented as a perturbative splitting of a gluon into a collinear colour octet $c\bar{c} \left[3S_1^{(8)}\right]$ pair, followed by a non-perturbative transition into a $\psi$ and one soft gluon. To correctly conserve quantum numbers, there should be at least two gluons emitted, but since the subsequent string fragmentation is stable w.r.t. addition of soft gluons, emitting only one to conserve colour should give a good description of the final state hadrons. For practical reasons, the delta function in the splitting kernel $\delta(z - 1)$ is given a small width by replacing it with step functions $(1/\epsilon) \theta(z - (1 - \epsilon)) \theta(1 - z)$. It is now possible to study the final-state properties of the octet mechanism as a function of this width which typically is expected to be on the order of the squared velocity $v^2$ of the quarks inside the charmonium.

- **Colour singlet gluon fragmentation:** a gluon is perturbatively split into a collinear colour singlet $c\bar{c} \left[3S_1^{(8)}\right]$ pair and two hard gluons. The $c\bar{c}$ pair directly forms a $\psi$ while the two gluons may continue cascading until hadronization sets in. In Ref. [21] only one gluon was emitted to conserve colour and momentum, but now the full three-body phase space is explored with the two-gluon invariant mass correctly distributed according to Ref. [27]. The final-state distributions discussed in Ref. [21] turned out to be insensitive to this change, and the conclusions therein still hold. The colour singlet gluon fragmentation contribution to $J/\psi$ production at LEP is very small and will not be
discussed further in this report.

- **Colour singlet charm quark fragmentation**: a charm quark is split into a collinear $c\bar{c}^{[3S_1^{(8)}]}$ pair and a charm quark according to Fig. 1b. The $c\bar{c}$ pair directly forms a $\psi$ while the charm quark may continue radiating.

Besides the treatment of the soft gluons in the octet model, the main uncertainty in the implementation comes from the fact that the dipole cascade model in the ARIADNE program in each step has all partons on-shell. In order for a gluon to split into a charmonium, some energy has to be borrowed from one of the colour-connected partons. As discussed in Ref. [21], this is a safe procedure as long as the cascade is strongly ordered, i.e. when the transverse momentum $p_{Tg}$ of the gluon to be split is much larger than the transverse mass $m_{\psi T}$ of the charmonium. For high-$p_T$ charmonium production at hadron colliders this is a good approximation. But in $Z^0$ decays there may even be situations where the mass $m_{\psi+g}$ of the formed charmonium-gluon state is larger than $p_{Tg}$, especially for the octet channel at small $z$. We have tried to estimate these uncertainties by e.g. explicitly requiring $m_{\psi+g} < p_{Tg}$, and the results turned out to be insensitive to such modifications. Therefore we assume that the description is good also at small $z$, although further studies are needed.

In this paper we will only use this program to study $J/\psi$ production at LEP, but the implementation is general enough for studying the fragmentation production of any quarkonia at any experiment.
\[ \mu^2 = \frac{m_Z^2}{16} \]

\[ \mu^2 = \frac{m_Z^2}{4} \]

\[ \mu^2 = \frac{m_Z^2}{16} \]

Figure 2: The \( z \) distribution of \( J/\psi \) from colour-singlet charm quark fragmentation. Solid and dashed histograms are from \textsc{Ariadne} in leading order and with full evolution respectively. The dashed line is from an analytical calculation to leading order, and the dotted and dash-dotted line are from analytical calculations with improved evolution using two different scales.

### 4 Results

In this section we present the results of our Monte Carlo simulations. We first compare with the analytical evolution, then we discuss the energy isolation cuts of the experimental analysis in the presence of evolution effects and finally proceed to determine the branching ratio of \( Z^0 \) into prompt \( J/\psi \).

In each case, we also discuss the dependence of our results on the energy carried by the soft gluons emitted in the non-perturbative colour-octet transitions. This is of the order of the quark kinetic energies \( m_c v^2 / 2 \) in the charmonium rest frame and hence negligible in the first approximation.

In Figs. 2 and 3 we compare the results using the analytical fragmentation formalism with the results from the Monte Carlo simulation. The results agree well in the large \( z \) limit. Closer to the threshold region, where
the analytical fragmentation formalism is known to fail, the disagreement becomes large, especially for octet gluon fragmentation. If we introduce a finite width in Eq. (13), corresponding to an energy of the order of $M_\psi v^2$ carried by the soft gluons in the non-perturbative transition $c\bar{c}[3S_1^{(8)}] \rightarrow \psi + gg$, the distribution somewhat softened, as expected.

In the experimental analysis, the separation of prompt $J/\psi$ from the $B$-decay component is primarily based on energy isolation cuts, supplemented by vertex detector information \[3, 5\]. The $J/\psi$ produced in $B$-meson decays are typically accompanied by approximately collinear, energetic light hadrons. The mechanisms of parton fragmentation into prompt $J/\psi$, on the other hand, have a tendency to kick the $c\bar{c}$ pair rather far apart from the original parton direction, resulting in the production of charmonia with little

Figure 3: The $z$ distribution of $J/\psi$ from colour-octet gluon fragmentation. The solid line is from an analytical calculation to leading order. The dotted and dash-dotted lines are from analytical calculations with evolution using two different scales. The histograms are from Ariadne using different widths of the step function in the splitting function: long-dashed corresponds almost to a delta function ($\epsilon = 10^{-4}$), and short-dashed to a width of $\epsilon = 0.2 \approx v^2$. 

accompanying energy. Prompt charmonia can therefore be identified by requiring the accompanying energy $E_{\text{cone}}$ within a cone of half-opening angle $\alpha$ around the charmonium direction to be less than some $E_0$. The LEP groups use $E_0 = 4$ GeV and $\alpha = 20^\circ$ [6] or $30^\circ$ [5].

We have studied how much the evolution effects change the distribution of energy around the charmonium. The results are presented in Figs. 4 and 5, where we plot the distribution of events as a function of accompanying energy, $(d\Gamma/dE_{\text{cone}})/\Gamma$. In the majority of events, charmonium states coming from $B$ decays are accompanied with energy $E_{\text{cone}} > 4$ GeV; the prompt component, on the other hand, contains a dominant peak at $E_{\text{cone}} \simeq 0$, representing events where there are no hard partons within $30^\circ$ of the charmonium momentum direction. Although the fixed-order result for prompt production is changed by including the effects of the parton cascade, hadronization and a finite energy of the soft gluons emitted in the octet channel, the fixed-order calculation already gives a good idea of how the isolation cuts work in a realistic situation. Hence we do not expect any large corrections to the results of Refs. [5] and [6].

As an aside we want to point out that the threshold at $z = 2M_\psi/M_Z$ is reproduced in the analytic fragmentation formalism if the distinction between the light cone momentum fraction $y = p_\psi^+/M_Z = (E_\psi + |p_\psi|)/M_Z$ and the energy fraction $z = 2E_\psi/M_Z = y + y^{-1}M_\psi^2/M_Z^2$ is taken into account. This distinction has been neglected in equations such as Eq. (6), since fragmentation functions $D(y, \mu^2)$ are correct only up to corrections of $\mathcal{O}(y^{-2}M_\psi^2/M_Z^2)$.

Our results for the direct prompt $J/\psi$ energy spectrum $d\Gamma/dz$ before and after the cut on $E_{\text{cone}}$, are presented in Figs. 6 and 7. The shapes of the distributions are not significantly changed by the evolution effects.

For the colour-singlet contribution from charm fragmentation (where evo-
Figure 4: The distribution of $J/\psi$ events from charm fragmentation and from $B$ decays as a function of the accompanying energy $E_{\text{cone}}$ within a cone of half-opening angle $\alpha = 30^\circ$ around the $J/\psi$ direction. Solid histogram is charm fragmentation from Ariadne to leading order on the parton level. Long-dashed histogram is the same but with full evolution and hadronization. Short-dashed histogram is $J/\psi$ from $B$ decays. The distributions have been normalized to unity.

Solution effects can easily be included also in the analytical calculation by means of differential equations, see Fig. 2, the total integrated branching ratio is $\text{Br}^{(1)}(Z^0 \rightarrow \text{direct prompt } J/\psi) = 0.27 \cdot 10^{-4}$, which is is the same as the leading order result within the statistical error of 1% in the Monte Carlo calculation. This is consistent with the normal AP evolution which does not change the overall normalization. Also the improved evolution only differs from standard AP in the small-$z$ region, hence there is no large change in the normalization in that case either. Note, however, that for the charm fragmentation we have everywhere used a fixed $\alpha_s = 0.253$ in the splitting function of Eq. (12). By default, Ariadne would use a running $\alpha_s$ also here, and with the $\Lambda_{\text{QCD}} = 0.22$ GeV fitted from global event shapes, the result is
For the the colour-octet gluon fragmentation channel using Eq. (13) we get
\[ \text{Br}^{(8)}(Z^0 \to \text{direct prompt } J/\psi) = 0.58 \cdot 10^{-4}, \]
which is 15% below the fixed-order analytical result of \( 0.68 \cdot 10^{-4} \). Introducing a finite energy of the soft gluons in the octet mechanism lowers the value even further to \( \text{Br}^{(8)}(Z^0 \to \text{direct prompt } J/\psi) = 0.46 \cdot 10^{-4} \). Note, however, that the value used for \( \langle 0|O_8 J/\psi (3S_1) |0 \rangle \) is from a fit [24] to Tevatron data neglecting the energy of the soft gluons. But high-\( p_\perp \) hadroproduction cross sections are approximately proportional to the fifth moment \( \int dz z^4 D(z, \mu) \) of the fragmentation function, so that a trigger bias is introduced and the normalization becomes very sensitive to the exact form of the fragmentation function at large \( z \). It turns out that changing from a delta function in Eq. (13) to
Figure 6: The $z$ distribution of $J/\psi$ from charm fragmentation without (upper curves) and with (lower curves) isolation cut. Solid histograms are from Ariadne in leading order, and dashed histograms are from Ariadne with full evolution and hadronization.

A finite width $\epsilon = 0.2 \approx v^2$ implies that $\langle 0|C_8^{J/\psi} (3S_1)|0 \rangle$ has to be doubled to give the same cross section at the Tevatron. This would mean that the branching ratio becomes $\text{Br}(Z^0 \rightarrow \text{direct prompt } J/\psi + X) = 0.92 \cdot 10^{-4}$ in the finite width case. Also for the octet channel, the default running $\alpha_s$ in the splitting function used in Ariadne gives a 20% higher branching ratio than the fixed $\alpha_s$ used here.

To compare with experimental branching ratio of $Z^0$ into total prompt $J/\psi$, the colour-octet contribution has to be multiplied by a factor of 2.1 and the colour-singlet contribution by a factor of 1.3 to take into account $\chi_c$ and $\psi'$ decays into $J/\psi$. The resulting branching ratios are $1.57 \cdot 10^{-4}$ for $\epsilon = 0$ and $2.28 \cdot 10^{-4}$ for $\epsilon = 0.2$, both consistent with the OPAL measurement \cite{OPAL} $\text{Br}(Z^0 \rightarrow \text{total prompt } J/\psi + X) = (1.9 \pm 0.7 \pm 0.5 \pm 0.5) \cdot 10^{-4}$.
Figure 7: The $z$ distribution of $J/\psi$ from colour-octet gluon fragmentation without (upper curves) and with (lower curves) isolation cut. Solid lines are from an analytical leading order calculation. Dashed histograms are from Ariadne with full evolution and hadronization: long-dashed with a delta function in the splitting function, and short-dashed a finite width of $\epsilon = v^2$.

5 Discussion

We have studied evolution effects in $Z^0$ fragmentation into prompt $J/\psi$ by means of a Monte Carlo simulation. We worked within the framework of non-relativistic QCD (NRQCD), where the production of charmonium proceeds through either colour-octet or colour-singlet $c\bar{c}$ intermediate states.

Evolution effects turn out to be relatively small, diminishing the colour-octet contribution to the decay width by 15% while colour-singlet contribution is unchanged. The shape of the $J/\psi$ energy spectrum also turns out to be stable against the evolution effects. Thus the conclusion drawn by previous authors [18, 19] on the basis of a leading-order calculation – that the colour-octet contribution has larger normalization and a much softer energy spectrum than the colour-singlet contribution – remains valid after the
inclusion of the evolution effects.

In fact, the dominant uncertainties in the comparison between NRQCD predictions and experimentally measured branching ratios are due to uncertainties in the fits of NRQCD matrix elements and to the small statistics of the experimental results.

The Tevatron fit of the $\langle 0|O^{J/\psi}_{8} (3S_{1}) |0 \rangle$ matrix element is subject to a large uncertainty due to the trigger bias effect. The existing fits of Tevatron data neglect the energy of the two soft gluons emitted in the transition $c\bar{c} [3S_{1}^{(8)}] \rightarrow \psi + gg$, thus maximizing the hardness of the fragmentation function. A realistic emitted energy of the order of $M_{\psi} v^{2}$ corresponds to a softening of the fragmentation function which suppresses the cross section by approximately a factor of two. The fit value of the matrix element would then have to be increased by this factor of two.

Apart from this and other theoretical uncertainties, the statistical errors of the Tevatron data imply uncertainties in the matrix elements around 30% [24].

In the bottomonium sector, the approximation of a heavy quark mass is better than in the charmonium sector. On the other hand, there is less data available, and theoretical expressions involve more free parameters than in the charmonium sector. In the Tevatron experiments, contributions from $\chi_{b}$ decays are not resolved from direct $\Upsilon$ production; in $Z^{0}$ decays, even the different radial excitations of the $\Upsilon$ can currently not be resolved from each other. In principle, however, the analysis can be easily extended to the bottomonium sector [19].

The predictions of NRQCD are consistent with the observed quarkonium production in $Z^{0}$ decays. However, both theoretical and experimental uncertainties are large. A precision test of NRQCD may become possible if
uncertainties in the Tevatron fits of NRQCD matrix elements are reduced and if more data become available at the $Z^0$ pole. Evolution effects in the theoretical calculations are smaller than these uncertainties and are under control, as we have shown above.

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