Is the resonance $X_0(2900)$ a ground-state or radially excited scalar tetraquark $[ud][cs]$?

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We investigate properties of the ground-state and first radially excited four-quark mesons $X_0$ and $X_0'$ with a diquark-antidiquark structure $[ud][cs]$ and spin-parities $J^P = 0^+$ and $J^P = 1^-$, respectively. Our aim is to reveal whether or not one of these states can be identified with the resonance $X_0(2900)$, recently discovered by the LHCb collaboration. We model $X_0$ and $X_0'$ as tetraquarks composed of either axial-vector or scalar diquark and antidiquark pairs. Their spectroscopic parameters are computed by employing the QCD two-point sum rule method and including into analysis vacuum condensates up to dimension 15. For an axial-axial structure of $X_0$, we find partial widths of the decays $X_0^{(i)} \rightarrow D^+ K^+$ and $X_0^{(i)} \rightarrow D^0 K^0$, and estimate full widths of the states $X_0^{(i)}$. To this end, we calculate the strong couplings at the vertices $X_0^{(i)} D K$ in the framework of the light-cone sum rule method. We use also technical approaches of the soft-meson approximation necessary to analyze tetraquark-meson-meson vertices. Obtained results $m = (2545 \pm 160)\text{ MeV}$ and $m' = (3320 \pm 120)\text{ MeV}$ for a scalar-scalar current for the masses of the particles $X_0$ and $X_0'$, as well as estimates for their full widths $\Gamma_0 = (140 \pm 29)\text{ MeV}$ and $\Gamma_0' = (110 \pm 25)\text{ MeV}$ allow us to interpret none of them as the resonance $X_0(2900)$. At the same time, these predictions provide important information about ground-state and radially excited diquark-antidiquark structures $X_0$ and $X_0'$, which should be objects of future experimental and theoretical studies.

I. INTRODUCTION

One of important achievements of last years in physics of multiquark hadrons is observation of structures $X_0(2900)$ and $X_1(2900)$ by the LHCb collaboration. These resonance-like peaks were discovered in the invariant mass distribution $D^+ K^+$ of the decay channel $B^+ \rightarrow D^+ D^- K^+$. The LHCb measured masses and widths of these structures and fixed also their spin-parities. It turned out, that $X_0(2900)$ and $X_1(2900)$ are the scalar and vector resonances with quantum numbers $J^P = 0^+$ and $J^P = 1^-$, respectively.

Appearance of the mesons $D^-$ and $K^+$ at the final state of their decays implies that $X_0(2900)$ and $X_1(2900)$ are composed of quarks $\bar{u}d$, and may be considered as particles containing four quarks of different flavors. In other words, $X_0(2900)$ and $X_1(2900)$ are presumably new evidences for exotic mesons with open-flavor structures. This is important fact, because existence of the resonance $X(5568)$, presumably built of $s\bar{d}u$ quarks and considered as a first candidate to fully open-flavor four-quark state, was not confirmed by other collaborations. Of course, this analysis is correct in the context of the four-quark model of $X_0(2900)$ and $X_1(2900)$, because there are theoretical analyses which claim to explain the LHCb data by hadronic rescattering effects. The LHCb collaboration also did not exclude such interpretation of the observed structures.

New experimental information triggered intensive theoretical activities aimed to reveal internal organization of these resonances, calculate their parameters and study processes in which $X_0(2900)$ and $X_1(2900)$ can be produced. In overwhelming majority of investigations, the resonances $X_0(2900)$ and $X_1(2900)$ were modeled as diquark-antidiquark states or hadronic molecules. In fact, as a scalar tetraquark $[sc][\mu d]$ the resonance $X_0(2900)$ was explored in Refs. [4, 5] using a phenomenological model and the sum rule method, respectively. Predictions for the mass $(2863 \pm 12)\text{ MeV}$ and $(2910 \pm 120)\text{ MeV}$ obtained in these papers allowed the authors to interpret $X_0(2900)$ as the ground-state scalar tetraquark $[sc][\mu d]$. An interesting assumption about nature of $X_0(2900)$ was made in Ref. [6], where it was studied as a radially excited state $[ud][cs]$. In the articles [7, 10] the resonance $X_0(2900)$ was examined as $S$-wave molecule $D^* K^+$. The tetraquark and molecule models were used for the resonance $X_1(2900)$, as well [6, 7, 11]. But two resonance-like peaks in the $D^+ K^+$ mass distribution may have alternative nature and emerge due to triangle singularities in the rescattering diagrams $\chi_{c1} D^* K^+$ and $D_{sJ} D_0^0 K^0$ [12].

In Ref. [27], we investigated $X_0(2900)$ as a molecule $D_{sJ}^0 K^+$ and evaluated its spectroscopic parameters and width. Comparing our results for the mass $(2866 \pm 198)\text{ MeV}$ and width $(49.6 \pm 9.3)\text{ MeV}$ of $D_{sJ}^0 K^+$ with corresponding LHCb data $m = (2866 \pm 7)\text{ MeV}$ and $\Gamma = (57 \pm 12)\text{ MeV}$, we decided a molecule model is acceptable for the resonance $X_0(2900)$. The vector resonance $X_1(2900)$ was considered in the context of the diquark-antidiquark model in our article [28]. We studied it as a vector tetraquark built of a diquark $u^T C \gamma_5 d$ and an antidiquark $\sigma \gamma_{\mu} \gamma_5 C \sigma^T$, and com-
computed relevant parameters. Though predictions for the mass \((2890 \pm 122)\) MeV and width \((93 \pm 13)\) MeV of this tetraquark are smaller than the relevant LHCB data, we interpreted it as the resonance \(X_1(2900)\) by keeping in mind that theoretical and experimental investigations suffer from certain errors.

During last few years diquark-antidiquark states containing four quarks (antiquarks) \(c, s, u, a, d\) in different configurations were objects of investigations. Thus, a scalar tetraquark \(X_c = [su\overline{ud}]\) was considered in our article \cite{29}, where it was modeled as an exotic meson made of scalar-scalar and axial-axial diquarks with \(C\gamma_5 \otimes \gamma_5 C\) and \(C\gamma_\mu \otimes \gamma^\mu C\) type interpolating currents, respectively. The mass of \(X_c\) found using these two structures is \((2634 \pm 62)\) MeV and \((2590 \pm 60)\) MeV, respectively. The result \((2.55 \pm 0.09)\) GeV for the mass of \(X_c\) was obtained also in Ref. \cite{30}.

Though \(X_c\) and \(X_0 = [ud\overline{cd}]\) have similar content, there are two differences between them: \(X_c\) is built of a relatively heavy diquark \([su]\) and heavy antidiquark \([\overline{ud}]\), whereas \(X_0\) has a light diquark \([ud]\)-heavy antidiquark \([\overline{cd}]\) structure. The second difference is decay channels of these particles. While dominant decay mode of \(X_c\) is \(X_c \rightarrow D_s^+ \pi^0\), in the case of \(X_0\) we have \(X_0 \rightarrow D^- K^+\). Nevertheless, as we shall see below, masses and widths of \(X_0\) and \(X_c\) are close to each other mainly due to their quark contents.

In the current work, we explore the scalar tetraquark \(X_0 = [ud\overline{cd}]\) in a detailed form. Thus, we compute masses of the ground-state \(1S\) and radially excited \(2S\) tetraquarks \(X_0\) and \(X'_0\), using the QCD two-point sum rule method, and two interpolating currents. The widths of \(X_0\) and \(X'_0\) are calculated in the framework of the light-cone sum rule (LCSR) method. This is necessary to find strong couplings at vertices \(X_0^{(i)} \rightarrow D^- K^+\) and \(X_0^{(i)} \rightarrow D^0 \overline{K}^0\) which determine partial widths of the decay channels \(X_0^{(i)} \rightarrow D^- K^+\) and \(X_0^{(i)} \rightarrow D^0 \overline{K}^0\). Because aforementioned strong couplings correspond to tetraquark-meson type vertices, the LCSR method is supplied by technique of a soft-meson approximation.

This work is organized in the following way: In Section \(\text{II}\) we calculate masses and couplings of the ground-state and radially excited tetraquarks \(X_0^{(i)}\). To this end, we use both the scalar-scalar and axial-axial type interpolating currents. The sum rule computations are carried out by including effects of vacuum condensates up to dimension 15. In Section \(\text{III}\) we compute the strong couplings \(g^{(i)}\) and \(C^{(i)}\) that describe strong interaction of particles at the vertices \(X_0^{(i)} \rightarrow D^- K^+\) and \(X_0^{(i)} \rightarrow D^0 \overline{K}^0\). Here, we evaluate also partial widths of the decays \(X_0^{(i)} \rightarrow D^- K^+\) and \(X_0^{(i)} \rightarrow D^0 \overline{K}^0\), and find full widths of the tetraquarks \(X_0^{(i)}\). Section \(\text{IV}\) is devoted to discussions and conclusions.

\section{The Mass and Current Coupling of 1S and 2S Tetraquarks \(X_0\) and \(X'_0\)}

The mass and current coupling of tetraquarks \(X_0\) and \(X'_0\) are among their important parameters. The masses of these states are necessary to compare them with the LHCB data and fix whether one of these particles may be interpreted as the resonance \(X_0(2900)\). The current couplings of \(X_0\) and \(X'_0\) in conjunctions with their masses are required to calculate partial widths of the decay channels \(X_0^{(i)} \rightarrow D^- K^+\) and \(X_0^{(i)} \rightarrow D^0 \overline{K}^0\), and hence to evaluate full width of these tetraquarks.

We compute the mass and coupling of \(X_0\) and \(X'_0\) in the framework the QCD two-point sum rule method, which is one of effective nonperturbative approaches in the high energy physics \cite{31,32}. It rests on fundamental principles of QCD and leads to reliable predictions using as input parameters only few universal vacuum condensates. Remarkably, sum rules derived by means of this method are applicable to investigate not only ordinary, but also multiquark hadrons \cite{33,34}.

We start our study from consideration of the following two-point correlation function

\[ \Pi(p) = i \int d^4x e^{ipx} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle, \quad (1) \]

where \(T\) means the time-ordered product, and \(J(x)\) is the interpolating current for the tetraquarks \(X_0\) and \(X'_0\). In general, tetraquarks \(X_0\) and \(X'_0\) with required quantum numbers \(J^P = 0^+\) can be built of different diquarks: It may be composed of scalar diquark and antidiquark pair \(u^T C\gamma_5 d\) and \(\overline{c} \gamma_5 \overline{C} \gamma_5 \overline{d}\) or made of an axial-vector diquark \(u^T C\gamma_\mu d\) and an axial-vector antidiquark \(\overline{c} \gamma^\mu \overline{C} \gamma_5\), where \(C\) is the charge conjugation matrix. Interpolating currents that correspond to these structures have the following forms

\[ J_S(x) = \epsilon \overline{c} u^T(x) C\gamma_5 d(x) | \overline{C} \gamma_5 C \overline{d}(x) \rangle, \quad (2) \]

and

\[ J(x) = \epsilon \overline{c} u^T(x) C\gamma_\mu d(x) | \overline{C} \gamma_\mu C \overline{d}(x) \rangle, \quad (3) \]

where \(\epsilon = \epsilon_{abc} \overline{c} \gamma_5 d, a, b, c, d\) are color indices.

In Eqs. (2) and (3) \(c(x), s(x), u(x)\) and \(d(x)\) are corresponding quark fields. In what follows, we consider in a detailed manner the interpolating current \(J(x)\), and provide only final results obtained while employing \(J_S(x)\).

To derive required sum rules, the correlation function \(\Pi(p)\) has to be expressed in terms of \(X_0\) and \(X'_0\) tetraquarks’ physical parameters. The function \(\Pi^{phys}(p)\) obtained after relevant manipulations constitutes the physical (phenomenological) side of the sum rules. We analyze a ground-state and first radially excited particles, therefore include contributions of these states to the correlation function explicitly. As a result, we obtain

\[ \Pi^{phys}(p) = \frac{\langle 0 | J(X_0) \langle X_0 | J^\dagger(0) \rangle}{m^2 - p^2} + \frac{\langle 0 | J(X'_0) \langle X'_0 | J^\dagger(0) \rangle}{m^2 - p^2} + \cdots, \quad (4) \]
where \( m \) and \( m' \) are the masses of the tetraquarks \( X_0 \) and \( X'_0 \). The formula (4) is derived by saturating the correlation function \( \Pi(p) \) with a full set of scalar four-quark states and performing integration over \( x \) in Eq. (4). Dots in Eq. (4) stand for effects of higher resonances and continuum states in the \( X_0 \) channel.

Equation (4) contains two simple-pole terms, which in the case of multiquark hadrons have to be used with some caution. The reason is that the physical side may contain also two-meson reducible contributions. Indeed, the current \( J(x) \) couples not only to the tetraquarks \( X_0 \) and \( X'_0 \), but also interacts with conventional two-meson states [37, 38]. These two-meson contributions modify a quark propagator in Eq. (4)

\[
\frac{1}{m^2 - p^2} \rightarrow \frac{1}{m^2 - p^2 - i\sqrt{2}\Pi(p)}.
\]

(5)

where \( \Pi(p) \) is the finite width of the tetraquark generated by two-meson effects. They should be subtracted from the sum rules, or taken into account in parameters of the pole terms. For tetraquarks the second method was applied in articles [39, 41], and it was demonstrated that these contributions can be absorbed into the current coupling keeping, at the same time, stable the mass of the tetraquark. Detailed analyses proved that two-meson effects are small, and do not exceed theoretical errors of the sum rule method itself [38, 41]. Therefore, the physical side of the sum rules is written down above by applying the zero-width single-pole approximation.

Using the matrix elements

\[
\langle 0 | J(x_0) | x_0^{(j)} \rangle = f^{(j)}m^{(j)},
\]

(6)

it is possible to simplify the function \( \Pi^{\text{phys}}(p) \). Simple operations lead for \( \Pi^{\text{phys}}(p) \) to the expression

\[
\Pi^{\text{phys}}(p) = \frac{f^2m^2}{m^2 - p^2} + \frac{f'^2m^2}{m'^2 - p^2} \cdots
\]

(7)

The function \( \Pi^{\text{phys}}(p) \) has a simple Lorentz structure \( \sim I \), and, depending on a problem under consideration, one or a sum of two terms may form the corresponding invariant amplitude \( \Pi^{\text{phys}}(p^2) \).

The second component of the sum rules \( \Pi^{\text{OPE}}(p) \), should be computed in the operator product expansion (OPE) with certain accuracy. It can be found by employing the expression of the interpolating current \( J(x) \), and replacing contracted quark fields by relevant propagators. After these operations, we obtain for \( \Pi^{\text{OPE}}(p) \)

\[
\Pi^{\text{OPE}}(p) = \frac{1}{16\pi} \int d^4xe^{ipx} \bar{c}_c\bar{c}_d\bar{c}_e\bar{c}_f T \left[ \gamma^\mu S_c^\nu\gamma(x)\gamma^\mu \right] \times S_c^d(-x)\gamma^\nu \left[ \gamma^\rho S_d^b(x)\gamma^\rho S_c^c(x)\gamma^\mu \right],
\]

(8)

where

\[
S_{c(q)}(x) = CS_{c(q)^T}(x)C.
\]

(9)

Here, \( S_c(x) \) and \( S_q(x) \) are the heavy c- and light quark propagators, respectively. Their explicit expressions are collected in Appendix. The correlation function \( \Pi^{\text{OPE}}(p) \) has a simple Lorentz structure: We use for a corresponding invariant amplitude a notation \( \Pi^{\text{OPE}}(p^2) \).

The correlation function \( \Pi^{\text{phys}}(p) \) corresponds to the "ground-state+excited particle+continuum" scheme, and encompasses contributions of two particles. At the first stage of studies, we employ a familiar "ground-state+continuum" scheme, and find the mass and coupling of the ground-state tetraquark \( X_0 \). This means that, we include the second term in \( \Pi^{\text{phys}}(p) \) into a list of "higher resonances and continuum states", and get the standard expression for the correlation function. Following operations are well known, and were discussed repeatedly in the literature including our papers. Therefore, we skip further details and provide final formulas for \( m \) and \( f \):

\[
m^2 = \frac{\Pi(M^2, s_0)}{\Pi(M^2, s_0)},
\]

(10)

\[
f^2 = \frac{e^m^2/M^2\Pi(M^2, s_0)}{m^2},
\]

(11)

where \( M^2 \) and \( s_0 \) are the Borel and continuum threshold parameters, respectively. Here, \( \Pi(M^2, s_0) \) is the Borel transformed and subtracted invariant amplitude \( \Pi^{\text{OPE}}(p^2) \), and \( \Pi^\prime(M^2, s_0) = d\Pi(M^2, s_0)/d(-1/M^2) \).

At this stage, one should fix the working windows for the parameters \( M^2 \) and \( s_0 \), which are auxiliary quantities of sum rule computations and should obey some important restrictions. The dominance the pole contribution (PC), convergence of OPE, and stability of physical quantities against variations of the Borel parameter are main constraints imposed on the correlation function \( \Pi(M^2, s_0) \). Fulfillment of these constraints can be established using expressions

\[
PC = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)},
\]

(12)

and

\[
R(M^2) = \frac{\Pi^{\text{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)},
\]

(13)

and numerical limits on PC, \( R(M^2) \), as well as fixing acceptable variations of \( m \) and \( f \). Let us note, that in Eq. (13) \( \Pi^{\text{DimN}}(M^2, s_0) \) is a last term or a sum of last few terms in the correlation function. In the present paper, we employ last three terms in the OPE, and hence \( \Pi^{\text{DimN}}(M^2, s_0) = \Pi^{\text{Dim}(13+14+15)}(M^2, s_0) \).

Having fixed working regions for \( M^2 \) and \( s_0 \), one can extract the mass and coupling of the 1S tetraquark \( X_0 \). The quantities \( m \) and \( f \), strictly speaking, should not
depend on the Borel parameter. But there are residual effects of working regions on extracted parameters, which nevertheless have to stay within acceptable limits. On the contrary, the continuum threshold parameter \( s_0 \) bears physical information about the mass of the excited tetraquark \( X'_0 \). In fact, the parameter \( s_0 \) separates contribution of the ground-state particle from ones due to higher resonances and continuum states. This means, that masses of \( X'_0 \) and \( X'_0 \) must obey restrictions \( m < \sqrt{s_0} < m' \).

After calculating the mass and coupling of the \( X'_0 \), we can find parameters of the excited state \( X'_0 \). For these purposes, we treat \( m \) and \( f \) as input parameters and look for new working regions for \( M^2 \) and \( s^*_0 \) which have to satisfy not only Eqs. (12) and (13), but also obey \( s^*_0 > s_0 \). Necessity of last constraint is evident, because in the "ground-state+excited particle+continuum" scheme the parameter \( s^*_0 \) separates two states from remaining higher resonances. The mass of the \( X'_0 \) extracted from a new sum rule is bounded by conditions \( \sqrt{s_0} \leq m' < \sqrt{s^*_0} \). If regions for \( M^2 \) and \( s^*_0 \), and extracted mass \( m' \) comply with these regulations, performed analysis is self-consistent and gives reliable predictions.

The sum rules for \( m' \) and \( f' \) obviously differ from ones for \( m \) and \( f \). For the mass \( m' \), we derive the following expression

\[
m'^2 = \frac{\Pi'(M^2, s^*_0) - f'^2 m^4 e^{-m^2/M^2}}{\Pi(M^2, s^*_0) - f'^2 m'^2 e^{-m'^2/M^2}}, \tag{14}
\]

whereas for \( f' \) get

\[
f'^2 = \frac{e^{m'^2/M^2}}{m'^2} \left[ \frac{\Pi(M^2, s^*_0) - f'^2 m'^2 e^{-m'^2/M^2}}{m'^2} \right]. \tag{15}
\]

It is evident, that parameters \( m' \) and \( f' \) of the excited particle \( X'_0 \) depend explicitly on the mass and current coupling of the ground-state tetraquark \( X_0 \). Such dependence is natural, because Eq. (7) contains two terms, and \( m \) and \( f \) appear as inputs when calculating \( m' \) and \( f' \). In its turn, the excited state \( X'_0 \) also affects the mass \( m \) and coupling \( f \) of the ground-state particle, but its effect is implicit and encoded in a choice of the continuum threshold parameter \( s_0 \). In fact, the parameters \( m \) and \( f \), extracted from sum rules depend on the correlation function \( \Pi(M^2, s_0) \) at \( s_0 \), which is limited by the mass \( m' \) of the excited state \( \sqrt{s_0} \leq m' \). Because two sets \((m, f)\) and \((m', f')\) are determined by the same correlation function at different \( s_0 \) and \( s^*_0 \), one may consider a difference of \( \Pi(M^2, s_0) \) at \( s_0 \) and \( s^*_0 \) as a "measure" of this effect.

The correlation function \( \Pi(M^2, s_0) \) has the following form

\[
\Pi(M^2, s_0) = \int_{\mathcal{M}} ds \rho^{OPE}(s)e^{-s/M^2} + \Pi(M^2), \tag{16}
\]

where \( \mathcal{M} = m_c + m_s \). In current work, we neglect masses of the quarks \( u \) and \( d \), and terms \( \sim m_s^2 \), but take into account contributions of \( m_s \). The spectral density \( \rho^{OPE}(s) \) is calculated as an imaginary part of the correlator \( \Pi^{OPE}(p) \). The function \( \Pi(M^2) \) is the Borel transformation of terms in \( \Pi^{OPE}(p) \) derived directly from their expressions. Computations are performed by including into analysis vacuum condensates till dimension 15. In Appendix, for the sake of brevity, we provide analytical expressions of \( \rho^{OPE}(s) \) and \( \Pi(M^2) \) up to dimension 11.

Our analytical results contain nonperturbative terms up to dimension 15, which makes necessary to explain treatment of higher dimensional vacuum condensates. The propagator \( S_q(x) \) contains various quark, gluon and mixed condensates of different dimensions, terms proportional to \( g_s^2 G^2 \) and \( g_s^3 G^3 \) are taken into account in \( S_q(x) \). Some of terms in the propagator \( S_q(x) \), for instance, ones proportional to \( \langle \bar{q}g_s \sigma G q \rangle \), \( \langle \bar{q}q \rangle \), and \( \langle \bar{q}q \rangle \langle \bar{q}g_s^2 G \rangle \) are obtained using the factorization hypothesis of higher dimensional condensates. These terms and their products with condensates from other light quark propagators, as well as with relevant components of \( S_c(x) \) enter to \( \rho^{OPE}(s) \) and \( \Pi(M^2) \). We carry out computations by taking into account all contributions up to dimension 15 obtained by this way. But factorization of higher dimensional condensates is not precise and generates uncertainties \([42]\), which sometimes are difficult to estimate. Because contributions of higher dimensional terms are numerically very small, we neglect impact of such uncertainties on extracted quantities.

The sum rules for \( m^{(v)} \) and \( f^{(v)} \) contain universal quark, gluon and mixed vacuum condensates listed below

\[
\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3, \quad \langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle,
\]

\[
\langle \bar{q}g_s \sigma G q \rangle = m_0^2 \langle \bar{q}q \rangle, \quad \langle \bar{q}g_s \sigma G s \rangle = m_0^2 \langle \bar{s}s \rangle,
\]

\[
m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2
\]

\[
\langle \frac{\alpha_s G^2}{\pi} \rangle = (0.012 \pm 0.004) \text{ GeV}^4,
\]

\[
m_s = 93^{+11}_{-5} \text{ MeV}, \quad m_c = 1.27 \pm 0.02 \text{ GeV}. \tag{17}
\]

The masses of \( c \) and \( s \) quarks are also included into Eq. (17).
FIG. 1: The mass $m$ of the tetraquark $X_0$ as a function of the Borel parameter $M^2$ [left panel], and as a function of $s_0$ [right panel].

FIG. 2: The same as in Fig. 1, but for the mass $m'$ of the excited tetraquark $X'_0$.

the working regions

$$M^2 \in [2, 4] \text{ GeV}^2, \ s_0 \in [9, 10] \text{ GeV}^2,$$  \hspace{1cm} (18)

satisfy aforementioned restrictions. Thus, at $M^2_{\text{max}} = 4 \text{ GeV}^2$ the pole contribution is equal to 0.23, whereas at $M^2_{\text{min}} = 2 \text{ GeV}^2$ it equals to 0.7. At $M^2_{\text{min}} = 2 \text{ GeV}^2$, we get $R(M^2_{\text{min}}) < 0.01$, hence the convergence of the sum rules is ensured. Mean values of $m$ and $f$ averaged over the regions (18) read

$$m = (2545 \pm 160) \text{ MeV},$$
$$f = (3.0 \pm 0.5) \times 10^{-3} \text{ GeV}^4.$$

(19)

Uncertainties of the results in Eq. (19) are within acceptable limits: for the mass and coupling they form $\pm 6.3\%$ and $\pm 16.7\%$ of the corresponding central values, respectively. Theoretical uncertainties of $m$ are smaller, because the relevant sum rule Eq. (10) is given as a ratio of correlation functions, whereas $f$ is determined by the expression with the correlation function in the numerator of Eq. (11). On the Fig. 1 we depict the sum rule’s prediction for $m$ as functions of $M^2$ and $s_0$ in which one can look at dependence of $m$ on the Borel and continuum threshold parameters.

To find parameters of the first radially excited tetraquark $X'_0$, we start our analysis from Eqs. (14) and

| Tetraquarks | $X_S$ | $X'_S$ |
|-------------|-------|--------|
| $M^2$ (GeV$^2$) | 2 – 4 | 2.5 – 4.5 |
| $s_0(s'_0)$ (GeV$^2$) | 9 – 10 | 12 – 13 |
| $m_S$ (MeV) | 2663 ± 110 | 3325 ± 85 |
| $f_S \cdot 10^3$ (GeV$^4$) | 2.2 ± 0.3 | 2.7 ± 0.4 |

TABLE I: The mass and current coupling of the tetraquarks $X_S$ and $X'_S$, and parameters $M^2$ and $s_0$ used in their computations.
and explore regions for $M^2$ and $s^*_0$ bearing in mind that $s^*_0 > s_0$. It is not difficult to see that working windows

$$M^2 \in [2.5, 4.5] \, \text{GeV}^2, \quad s^*_0 \in [12, 13] \, \text{GeV}^2,$$

obey necessary constraints. In these regions the pole contribution to $\Pi(M^2, s^*_0)$ changes inside of interval

$$0.75 \geq PC \geq 0.34. \quad (21)$$

The mass and coupling of the radially excited tetraquark are

$$m' = (3320 \pm 120) \, \text{MeV},$$
$$f' = (3.7 \pm 0.6) \times 10^{-3} \, \text{GeV}^4, \quad (22)$$

respectively. Dependence of $m'$ on the parameters $M^2$ and $s^*_0$ is shown on Fig. 2. Comparing figures 1 and 2 one sees, that theoretical ambiguities for the mass of the tetraquark $X'_0$ are smaller than that for $m$.

With these final predictions in hand, one can check self-consistency of performed analysis. Using mean values of the parameters $\sqrt{s_0} = 3.54 \, \text{GeV}$ and $\sqrt{s_0} = 3.08 \, \text{GeV}$ it is easy to be convinced that all regulations discussed above are correct.

The mass and coupling of the ground-state and excited tetraquarks $X_S$ and $X'_S$ extracted from the sum rules by employing the interpolating current $J_S(x)$ are shown in Table I. We plot also the masses $m_S$ and $m'_S$ in Figs. 3 and 4 as functions of the Borel and continuum threshold parameters.

**FIG. 3:** Dependence of the mass $m_S$ on the Borel parameter $M^2$ at some fixed $s_0$ [left panel] and on the continuum threshold parameter $s_0$ at fixed Borel parameter [right panel].

**FIG. 4:** The same as in Fig. 3, but for the mass $m'_S$ of the excited state.
Results obtained for the masses of the states $X_0$ and $X_0'$ are either smaller than the LHCb data for the resonance $X_0(2900)$, as in the case of the ground-state tetraquark $X_0$, or exceed it. These conclusions are valid for both currents $J(x)$ and $J_5(x)$, and even ambiguities of calculations, taken into account in $m_0^{(4)}$ and $m_0^{(5)}$, do not solve the problem. It seems, that the diquark-antidiquark structure of $X_0$ and its radial excitation $X_0'$ are new exotic mesons not yet seen in experiments. To gain detailed information on their properties, we consider decays of the tetraquarks $X_0$ and $X_0'$, and estimate their full widths in the next section.

III. PROCESSES $X_0^{(r)} \rightarrow D^- K^+$ AND $X_0^{(r)} \rightarrow \overline{D}^0 K^0$

Masses of the tetraquarks $X_0^{(r)}$ calculated in the previous section, as well as their quark content allow us to specify their decay channels. It is not difficult to see that thresholds $\approx 2364$ MeV for production of conventional meson pairs $D^- K^+$ and $\overline{D}^0 K^0$ are smaller than masses of $X_0^{(r)}$. Moreover the modes $X_0^{(r)} \rightarrow D^- K^+$ and $X_0^{(r)} \rightarrow \overline{D}^0 K^0$ are S-wave decay channels for the tetraquarks $X_0^{(r)}$, and decay to mesons $D^- K^+$ is dominant process for the resonance $X_0(2900)$.

In this section, we consider in a rather detailed form decays $X_0^{(r)} \rightarrow D^- K^+$, and provide final information about channels $X_0^{(r)} \rightarrow \overline{D}^0 K^0$. Partial widths of the processes $X_0 \rightarrow D^- K^+$ and $X_0' \rightarrow D^- K^+$ are determined by strong couplings at corresponding tetraquark-meson-meson vertices $X_0D^- K^+$ and $X_0'D^- K^+$, respectively. We denote relevant strong couplings as $g$ and $g'$, and use for their calculations the QCD sum rules on the light-cone [43, 44], and techniques of the soft-meson approximation [45].

The strong couplings $g$ and $g'$ are defined by the on-mass-shell matrix element

$$
\langle K(q) D(p) |X_0^{(r)}(p') \rangle = g^{(r)} p \cdot p'.
$$

In the framework of the LCSR method the vertex $X_0D^- K^+$ can be investigated by means of the correlation function

$$
\Pi(p, q) = i \int d^4 x e^{ipx} \langle K(q) | T{\{J^D(x) J^I(0)\}} |0\rangle, \quad (24)
$$

where mesons $K^+$ and $D^-$ are shortly denoted by $K$ and $D$, respectively. In Eq. (24) $J(x)$ and $J^D(x)$ are the interpolating currents for the tetraquarks $X_0^{(r)}$ and meson $D^-$. First of them is defined by Eq. (3), and for $J^D(x)$, we employ

$$
J^D(x) = \tau_j(x) i \gamma_5 d_j(x), \quad (25)
$$

with $j$ being the color index.

The current $J(x)$ couples to both the ground-state and radially excited tetraquarks $X_0$ and $X_0'$, therefore in the function $\Pi^{\text{Phys}}(p, q)$ we should take into account contribution of these particles explicitly. We are interested in terms which have poles at variables $p^2$ and $p'^2$, where $p$ and $p' = p + q$ are the momenta of the meson $D^-$ and tetraquarks $X_0^{(r)}$, and $q$ is momentum of the $K^+$ meson. The terms in $\Pi^{\text{Phys}}(p, q)$ necessary for our analysis have the following forms

$$
\Pi^{\text{Phys}}(p, q) = \frac{f_D m_D^2}{m_c (p^2 - m_c^2)} \left[ \frac{g f m}{(p'^2 - m_f^2)} + \frac{g' f' m'}{(p'^2 - m'^2)} \right] \times p \cdot p' + \cdots, \quad (26)
$$

where $m_D$ and $f_D$ are the mass and decay constant of the $D^-$ meson. To derive Eq. (26) we use the vertex function given by Eq. (24), well known matrix elements of the tetraquarks $X_0^{(r)}$, Eq. (6), and new matrix element of the $D^-$ meson

$$
\langle 0 | J^D | D(p) \rangle = \frac{f_D m_D^2}{m_c}, \quad (27)
$$

The terms presented explicitly in Eq. (26) correspond to ground-state meson in $D^-$ channel, and ground-state and radially excites tetraquarks in $X_0$ channel. Contributions of remaining higher resonances and continuum states in the $D^-$ and $X_0$ channels are denoted by dots.

An expression of the same correlation function obtained using quark-gluon degrees of freedom forms the second component $\Pi^{\text{QCD}}(p, q)$ of the sum rule analysis. Calculations carried out using quark propagators give

$$
\Pi^{\text{QCD}}(p, q) = \int d^4x e^{ipx} \bar{c}i\gamma_\mu \tilde{S}_d(x) \gamma_\nu K(q) \tau_\alpha \bar{c} \langle 0 | s_\beta(0) s_\gamma(0) |0 \rangle, \quad (28)
$$

with $\alpha$ and $\beta$ being the spinor indices. The correlator $\Pi^{\text{QCD}}(p, q)$ contains quark propagators, which determine a hard-part of this function. But it depends also on $\bar{c}c$ operator’s local matrix elements: this is soft factor in $\Pi^{\text{QCD}}(p, q)$.

The matrix elements $\langle K|\bar{s}s|0\rangle$ bear spinor and color indices, and are inconvenient for further usage. To recast them into color-singlet form and factor out spinor indices, we expand $\bar{c}c$ over the full set of Dirac matrices $\Gamma^J$

$$
\Gamma^J = 1, \gamma_5, \gamma_\mu, i\gamma_5\gamma_\mu, \sigma_{\mu\nu}/\sqrt{2}, \quad (29)
$$

and project them onto the colorless states

$$
\bar{s}_\alpha(0) s_\beta(0) \rightarrow \frac{1}{12} \delta_{\beta\alpha} \Gamma^J \beta_\alpha \left[ \bar{s}(0) \Gamma^J s(0) \right], \quad (30)
$$

Obtained operators placed between the $K$ meson and vacuum give rise to local matrix elements of the $K$ meson.

When considering the tetraquark-meson-meson vertices $X_0^{(r)} D^- K^+$, we encounter the correlation function containing only local matrix elements of quark operators. Let us note that such behavior of $\Pi^{\text{QCD}}(p, q)$ is typical
for all vertices built of one tetraquark and two conventional mesons. The reason is actually very simple: The tetraquark current \( J_0(0) \) is composed of four quark fields at the same space-time position. Contractions of relevant fields from interpolating currents \( J^D(x) \) and \( J^J(0) \) leave two free quark fields at the space-time point \( x = 0 \). As a result, local matrix elements of the \( K \) meson appear in the correlation function as overall normalization factors.

It is instructive to compare this situation with three-meson vertices, in which contractions of quark fields with different space-time coordinates generate \( \Pi^{OPE}(p, q) \) containing non-local operators. Then manipulations performed in accordance with Eqs. (29) and (30) lead to operators, matrix elements of which are distribution amplitudes (DAs) of a final-state meson. In other words, for a three-meson vertex a correlation function depends on integrals over DAs of a meson. A situation described above in the LCSR method emerges in the kinematical limit \( q \to 0 \) known as a soft-meson approximation [41]. In this approximation instead of a light-cone expansion, one gets expansion in terms of local matrix elements of a final meson. Because in the soft limit phenomenological and QCD sides of the light-cone sum rules acquire distinctive features, they have to be treated in accordance with elaborated methods [44, 45]. It is important that strong couplings at three-meson vertices calculated using the full version of the LCSR method and soft-meson approximation lead to predictions, which are numerically very close to each other [44].

The soft-meson approximation were applied to explore tetraquark-meson-meson vertices in Ref. [46], and used later in numerous similar studies [39]. It is worth emphasizing, that in exclusive processes with two tetraquarks and an ordinary meson correlation functions contain integrals over DAs of a meson, and their treatment does not differ from standard LCSR analysis [47].

Here, we employ this technique to analyze the vertices \( X_0(9) \to D^-\bar{K}^+ \). As is seen from Eq. (28), the soft-meson approximation considerably simplifies the QCD side of the sum rule: There are only local matrix elements of the \( K \) meson in \( \Pi^{OPE}(p^2) \), and only a few of them contribute at the limit \( q = 0 \). On the contrary, the physical side of the sum rule has more complicated structure than in the case of the full version of the LCSR method. The soft limit implies fulfilment of the equality \( p = p' \), hence in the limit \( q \to 0 \) invariant amplitudes \( \Pi^{phys}(p^2, p'^2) \) and \( \Pi^{OPE}(p^2, p'^2) \) are functions of a variable \( p^2 \). Therefore, in Eq. (26) one should take into account that \( p^2 = p'^2 \), and gets

\[
\Pi^{phys}(p^2) = \frac{f_D m_D^2}{m_c} \left[ g f m \frac{\tilde{m}^2}{(p^2 - \tilde{m}^2)^2} + g' f' m' \frac{\tilde{m}'^2}{(p^2 - \tilde{m}'^2)^2} \right] + \cdots , \tag{31}
\]

where \( \tilde{m}^2 = (m^2 + m_D^2)/2 \) and \( \tilde{m}'^2 = (m'^2 + m_D^2)/2 \), respectively. Remaining problems are connected with the Borel transform of the amplitude \( \Pi^{phys}(p^2) \), which due to double poles at \( p^2 = \tilde{m}^2 \) and \( p^2 = \tilde{m}'^2 \) has the following form

\[
\Pi^{phys}(p^2) = \frac{f_D m_D^2}{m_c} \left[ g f m \frac{\tilde{m}^2 e^{-\tilde{m}^2/M^2}}{M^2} + g' f' m' \frac{\tilde{m}'^2 e^{-\tilde{m}'^2/M^2}}{M^2} \right] + \cdots . \tag{32}
\]

In general, the Borel transformation applied to a correlation function suppresses contributions of higher resonances and continuum states. This allows one to subtract these terms from the QCD side of the sum rule using an assumption about quark-hadron duality. In the soft approximation, after the Borel transformation there are still unsuppressed terms in the physical side of the sum rule, which contribute to \( \Pi^{phys}(p^2) \) on an equal footing with ground-state term. Because we are interested in analysis of both the ground-state \( X_0 \) and excited \( X'_0 \) particles, it is necessary to clarify a nature of these unsuppressed terms. The main contribution to \( \Pi^{phys}(p^2) \) comes from the vertex \( X_0 D^-\bar{K}^+ \), where the tetraquark and mesons are ground-state particles. Unsuppressed terms correspond to vertices, in which \( X_0 \) is on its excited state. While considering the vertex \( X_0 D^-\bar{K}^+ \) such contributions should be treated as contaminations and removed applying some procedures. Such prescriptions are well known and were described in Refs. [44, 45]. To eliminate contaminations from \( \Pi^{phys}(p^2) \), one has to apply the operator

\[
\mathcal{P}(M^2, m^2) = \left( 1 - M^2 \frac{d}{dM^2} \right) M^2 e^{m^2/M^2}, \tag{33}
\]

to both sides of the sum rule equality, and subtract remaining conventional terms in a standard manner.

But the vertex \( X'_0 D^-\bar{K}^+ \) and strong coupling \( g' \) are also interesting for us. Therefore, we keep the following strategy: we determine the strong coupling \( g \) utilizing the "ground-state+continuum" scheme and first term in \( \Pi^{phys}(p^2) \). At this stage we apply the operator \( \mathcal{P}(M^2, \tilde{m}^2) \) that singles out the ground-state term. Afterwards, we use \( g \) as an input parameter in "ground-state+excited-state+continuum" scheme, and by employing full expression of \( \Pi^{phys}(p^2) \) determine the strong coupling \( g' \).

Then the sum rule for \( g \) reads

\[
g = \frac{m_c}{f_D f m_D^2} \mathcal{P}(M^2, m^2) \Pi^{OPE}(M^2, s_0), \tag{34}
\]

whereas for \( g' \), we obtain

\[
g' = \frac{e^{\tilde{m}'^2/M^2}}{f'D^2} \left[ M^2 m_c \Pi^{OPE}(M^2, s_0) - g f m \tilde{m}'^2 e^{-\tilde{m}'^2/M^2} \right]. \tag{35}
\]

The \( K \) meson is characterized by some local matrix elements of different quark-gluon contents and twists.
Having performed numerical computation, we see that the correlator $\Pi^{\text{OPE}}(p, q)$ receives contribution from the two-particle twist-3 element

$$\langle 0 | \bar{c}\gamma_5 s | K \rangle = \frac{f_K m_K^2}{m_s}.$$  \hspace{1cm} (36)

The technical sides of required calculations of the $\Pi^{\text{OPE}}(p, q)$ in the soft limit were described in Refs. [46], hence we omit further details and write down final formula for the Borel transformed and subtracted invariant amplitude, which is computed with dimension-9 accuracy and given by the formula

$$\Pi^{\text{OPE}}(M^2, s_0) = -\frac{\mu_K}{4\pi^2} \int_{M^2}^{s_0} ds (m_c^2 - s)^2 e^{-s/M^2} \frac{1}{s} + \mu_K m_c \Pi_{\text{NP}}(M^2).$$  \hspace{1cm} (37)

The nonperturbative component of the correlation function $\Pi_{\text{NP}}(M^2)$ is determined by the expression

$$\Pi_{\text{NP}}(M^2) = \frac{2(\bar{d}d)}{3} e^{-m_c^2/M^2} + \frac{\alpha_s G^2}{\pi} \frac{m_c^2}{36 M^4} \int_0^1 dx \frac{e^{-m_c^2[1-x]^2}}{x^3(1-x)^4} - \frac{\alpha_s G^2}{\pi} \frac{m_c^2}{36 M^4} \int_0^1 dx \frac{e^{-m_c^2[1-x]^2}}{x^3(1-x)^4} - \frac{\alpha_s G^2}{\pi} \frac{m_c^2}{36 M^4} \int_0^1 dx \frac{e^{-m_c^2[1-x]^2}}{x^3(1-x)^4} - \frac{\alpha_s G^2}{\pi} \frac{m_c^2}{36 M^4} \int_0^1 dx \frac{e^{-m_c^2[1-x]^2}}{x^3(1-x)^4} - \frac{\alpha_s G^2}{\pi} \frac{m_c^2}{36 M^4} \int_0^1 dx \frac{e^{-m_c^2[1-x]^2}}{x^3(1-x)^4}$$

where $\mu_K = f_K m_K^2 / m_s$.

It is worth noting that the limit $q \to 0$ is performed in a hard component of the amplitude. As a result, it does not contain terms $\sim m_K^2$, which nevertheless would be small due to $m_K^2 / m_c^2, m_K^2 / m_c^2, m_K^2 / m_c^2, m_K^2 / m_c^2 \ll 1$. In the soft approximation the mass and decay constant of $K^+$ meson through $\mu_K$ form the nonperturbative soft factor in $\Pi^{\text{OPE}}(M^2, s_0)$.

Parameters of the mesons $D^-$ and $K^+$ necessary to calculate $g$ are removed to Table II. For masses and decay constants of these particles, we use their values from Ref. [45]. In numerical computation of the strong couplings $g$ and $g'$ the Borel and continuum subtraction parameters are chosen as in corresponding mass analysis. Numerical computations yield

$$g = (1.06 \pm 0.27) \text{ GeV}^{-1},$$  \hspace{1cm} (39)

and

$$g' = (0.52 \pm 0.13) \text{ GeV}^{-1}. \hspace{1cm} (40)$$

Partial widths of the processes $X_0^{(*)} \rightarrow D^- K^+$ can be found by means of the expression

$$\Gamma \left[ X_0^{(*)} \rightarrow D^- K^+ \right] = \frac{g'^2 m_D^2 \lambda^{(*)}}{8\pi} \left( 1 + \frac{\lambda^{(*)}}{m_D^2} \right), \hspace{1cm} (41)$$

where $\lambda^{(*)} = \lambda \left( m^{(*)}, m_D, m_K \right)$ and

$$\lambda(a, b, c) = \frac{1}{2a} \left[ a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2) \right]^{1/2}. \hspace{1cm} (42)$$

Now it is easy to get

$$\Gamma \left[ X_0 \rightarrow D^- K^+ \right] = (64.7 \pm 23.3) \text{ MeV}, \hspace{1cm} \Gamma \left[ X_0' \rightarrow D^- K^+ \right] = (53.3 \pm 18.8) \text{ MeV}. \hspace{1cm} (43)$$

Another processes which form the full widths of the tetraquarks $X_0$ and $X_0'$ are decays $X_0 \rightarrow D^0 K^0$ and $X_0' \rightarrow D^0 K^0$. Investigation of these channels runs in accordance with the scheme described above, therefore we write down only final results: for the strong couplings $G$ and $G'$ corresponding to vertices $X_0 D^0 K^0$ and $X_0' D^0 K^0$, we find

$$G = (1.14 \pm 0.18) \text{ GeV}^{-1}, \hspace{1cm} (44)$$

and

$$G' = (0.54 \pm 0.11) \text{ GeV}^{-1}. \hspace{1cm} (45)$$

For partial widths of these decays, we get

$$\Gamma \left[ X_0 \rightarrow D^0 K^0 \right] = (74.8 \pm 16.7) \text{ MeV}, \hspace{1cm} \Gamma \left[ X_0' \rightarrow D^0 K^0 \right] = (56.3 \pm 16.2) \text{ MeV}. \hspace{1cm} (46)$$

Then full widths of the particles $X_0$ and $X_0'$ are equal to

$$\Gamma_0 = (140 \pm 29) \text{ MeV}, \hspace{1cm} \Gamma_0' = (110 \pm 25) \text{ MeV}, \hspace{1cm} (47)$$

respectively.

As is seen, parameters of the tetraquarks $X_0$ and $X_0'$ differ considerably from the mass and width of the resonance $X_0(2900)$ measured by the LHCb collaboration.

**IV. DISCUSSION AND CONCLUSIONS**

In the present paper, we have examined the tetraquark $X_0$ and its radial excitation $X_0'$ by calculating their masses and widths. The masses of $X_0$ and $X_0'$ have been
computed using the axial-axial and scalar-scalar type interpolating currents \( J(s) \) and \( J_0(x) \). The widths of these particles have been estimated for the axial-axial structure.

The diquark-antidiquark state \( X_0 \) is built of the four quarks of different flavors \( X_0 = [ud][\bar{cs}] \). Properties of the ground-state scalar tetraquark with similar content \( X_c = [su][\bar{cd}] \) were investigated in Refs. [29-30]. The mass of \( X_c \) found in Ref. [29] using axial-axial and scalar-scalar structures is equal to

\[
m_{X_c} = (2590 \pm 60) \text{ MeV}, \quad \Gamma_{X_c} = (63.4 \pm 14.2) \text{ MeV},
\]

and

\[
\bar{m}_{X_c} = (2634 \pm 62) \text{ MeV}, \quad \bar{\Gamma}_{X_c} = (57.7 \pm 11.6) \text{ MeV},
\]

respectively. The prediction

\[
m_{X_c} = (2550 \pm 90) \text{ MeV}
\]

for the mass of the state \( X_c \) was made also in Ref. [30]. It is worth to emphasize that all of these results were extracted using the QCD two-point sum rule method, and predictions for the mass of \( X_c \) from articles [29] and [30] almost coincide with each other. It is also evident that \( m \) and \( m_g \) in Eq. (19) and in Table I are comparable with predictions for \( m_{X_c} \) and \( \bar{m}_{X_c} \) within uncertainties of computations. Stated differently, the masses of the ground-state tetraquarks with different internal organizations, but composed of \( c, s, u, d \) quarks vary approximately in limits 2550 – 2660 MeV.

Parameters of the tetraquark \([cs][\bar{ud}]\) in the context of the sum rule approach were recently calculated also in Ref. [3]. Here, for the mass of this particle with the scalar-scalar (SS) or axial-axial (AA) structures, the author found

\[
M_{SS} = (3050 \pm 100) \text{ MeV}, \quad M_{AA} = (2910 \pm 120) \text{ MeV},
\]

Because \( M_{AA} \) is compatible with the LHCb data, the resonance \( X_0(2900) \) was interpreted there as a ground-state tetraquark \([cs][\bar{ud}]\). Results of this work differ considerably from our findings, as well as from prediction made in Ref. [30]. The \( X_0(2900) \) was considered as a radially excited state \( X_c(2S) \) with \( X_c \) being the tetraquark \([ud][\bar{cs}]\).

The mass of the state \( X_c(2S) \) was estimated there around of 2860 MeV, which is lower than our results for \( X_0^c \) and \( X_0^s \).

Performed analysis allows us to consider tetraquarks \( X_0^{(t)} \) and \( X_0^{(s)} \) built of the axial-vector and scalar diquarks (antidiquarks), respectively, as states that differ from the resonance \( X_0(2900) \) observed by the LHCb collaboration. Therefore, parameters calculated in the present work are all the more important to search for the tetraquarks \( X_0^{(t)} \) and \( X_0^{(s)} \) in various processes. The masses of states \( X_0^{(t)} \) and \( X_0^{(s)} \) have been extracted with high enough accuracy. Though \( m^{(t)} \) and \( m^{(s)} \) contain uncertainties typical for all sum rule computations, they provide valuable information on these exotic mesons. We evaluated also full widths of the tetraquarks \( X_0^c \) and \( X_0^s \) by considering their decays to pairs of conventional mesons \( D^- K^+ \) and \( \bar{D}^0 K^0 \). For the particles \( X_0^c \) and \( X_0^s \) these two processes are their only \( S \)-wave decay channels. Other possible modes of the tetraquarks \( X_0^{(t)} \), for instance, \( S \)-wave decays \( X_0^{(t)} \to \bar{D}^0(2400)^0 K^* \) are kinematically forbidden processes. Hence, estimates for full widths \( \Gamma_0^{(t)} \) of the four-quark mesons \( X_0 \) and \( X_0^c \) are rather credible.

Our results imply that \( X_0(2900) \) can not be identified with ground-state or radially excited scalar tetraquark \([ud][\bar{cs}]\). It seems interpretation of the resonance \( X_0(2900) \) as hadronic molecules \( \bar{D}^0 K^0 \) and \( D^{*-} K^+ \), or their some admixture was correct and overcame successfully this examination.

**Appendix: The quark propagators and invariant amplitude \( \Pi(M^2, s_0) \)**

In the current article, for the light quark propagator \( S_{q}^{ab}(x) \), we employ the following expression

\[
S_{q}^{ab}(x) = i \delta_{ab} \frac{\not{x}}{2n^2x^4} - \delta_{ab} \frac{m_q}{4\pi^2x^2} - \delta_{ab} \frac{\langle \bar{q}q \rangle}{12} + i \delta_{ab} \frac{fm_q \langle \bar{q}q \rangle}{48} - \delta_{ab} \frac{x^2}{192} \langle \bar{g}_{s} \sigma G q \rangle
+i \delta_{ab} \frac{x^2}{1152} \langle \bar{g}_{s} \sigma G q \rangle - i \frac{g_{s} \sigma G}{32\pi^2x^2} \left[ \not{x} \sigma_{ab} + \sigma_{ab} \not{x} \right] - i \delta_{ab} \frac{x^2}{7176} \langle \bar{g}_{s} \sigma G q \rangle^2
- \delta_{ab} \frac{x^2\langle \bar{q}q \rangle}{27648} G^2 + \ldots \quad \text{(A.1)}
\]
For the heavy quark $Q = c$, we use the propagator $S_Q^{ab}(x)$

$$S_Q^{ab}(x) = i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left\{ \delta_{ab} (\not{k} + m_Q) - \frac{g_s G_{a\beta}^b}{4} \frac{\sigma_{\alpha\beta} (\not{k} + m_Q) + (\not{k} + m_Q) \sigma_{\alpha\beta}}{(k^2 - m_Q^2)^2} \right\} + \frac{g_s^2 G^2}{12} \delta_{ab} m_Q \frac{k^2 + m_Q \not{k}}{(k^2 - m_Q^2)^4} \left[ \frac{3}{48} \delta_{ab} (\not{k} + m_Q) \frac{\left( \not{k} (k^2 - 3m_Q^2) + 2m_Q (2k^2 - m_Q^2) \right)}{(k^2 - m_Q^2)^2} \right].$$

(A.2)

Here, we have used the short-hand notations

$$C_{a\beta}^{ab} = C_A^{a\beta} \lambda^A_{ab}/2, \quad G^2 = G_A^{a\beta} C_A^{a\beta}, \quad G^3 = f^{ABC} G_A^{a\beta} G_B^{b\gamma} G_C^{c\delta},$$

(A.3)

where $G_A^{a\beta}$ is the gluon field strength tensor, $\lambda^A$ and $f^{ABC}$ are the Gell-Mann matrices and structure constants of the color group $SU_c(3)$, respectively. The indices $A, B, C$ run in the range 1, 2, \ldots 8.

The invariant amplitude $\Pi(M^2, s_0)$, obtained using the interpolating current $J(x)$ from Eq. (3), after the Borel transformation and subtraction procedures is given by the expression

$$\Pi(M^2, s_0) = \int_{M^2}^{s_0} d\rho^{OPE}(s) e^{-s/M^2} + \Pi(M^2),$$

where the spectral density $\rho^{OPE}(s)$ and the function $\Pi(M^2)$ are determined by formulas

$$\rho^{OPE}(s) = \rho^{\text{pert.}}(s) + \sum_{N=3}^{8} \rho^{\text{DimN}}(s), \quad \Pi(M^2) = \sum_{N=6}^{15} \Pi^{\text{DimN}}(M^2),$$

(A.4)

respectively. The components of $\rho^{OPE}(s)$ and $\Pi(M^2)$ are given by the expressions

$$\rho^{\text{DimN}}(s) = \int_0^1 d\alpha \rho^{\text{DimN}}(s, \alpha), \quad \Pi^{\text{DimN}}(M^2) = \int_0^1 d\alpha \Pi^{\text{DimN}}(M^2, \alpha).$$

(A.5)

In Eq. (A.5) the variable $\alpha$ is the Feynman parameter.

The perturbative and nonperturbative components of the spectral density $\rho^{\text{pert.}}(s, \alpha)$ and $\rho^{\text{DimN}(4,5,6,7,8)}(s, \alpha)$ are given by the following expressions

$$\rho^{\text{pert.}}(s, \alpha) = \frac{[m_c^2 - s(1 - \alpha)]^3}{1536\pi^6(\alpha - 1)^3} \left[ 4m_c m_s + m_c^2 \alpha + 3s \alpha(1 - \alpha) \right],$$

(A.6)

$$\rho^{\text{Dim3}(s, \alpha)} = -\frac{(\bar{s}s) \Theta(L)}{36\pi^2(\alpha - 1)^2} \alpha^2 \left[ m_c^2 - s(1 - \alpha) \right] \left[ m_c^2 + m_c^2 m_s(\alpha - 1) + m_c s(\alpha - 1) + 4m_s s(\alpha - 1)^2 \right]$$

(A.7)

$$\rho^{\text{Dim4}(s, \alpha)} = -\frac{(\bar{s}s) G^2(4\pi)^2 \Theta(L)}{9 \cdot 2^2 \pi^4(\alpha - 1)^3} \alpha^2 \left[ 6s^2(\alpha - 1)^3(5\alpha - 6) + m_c^2 m_s(-9 + 9\alpha - 8\alpha^2) + m_c^4(18 - 33\alpha + 19\alpha^2) + m_c m_s(9 - 22\alpha + 25\alpha^2 - 12\alpha^3) + 3m_c^2 s(-18 + 51\alpha - 50\alpha^2 + 17\alpha^3) \right],$$

(A.8)

$$\rho^{\text{Dim5}(s, \alpha)} = -\frac{3(\bar{s}s) G^3(4\pi)^2 \Theta(L)}{96 \pi^4(\alpha - 1)} \alpha \left[ 3m_c^2 + m_c^2 m_s(\alpha - 1) + 3m_c s(\alpha - 1) + 6m_s(s(\alpha - 1)^2) \right],$$

(A.9)

$$\rho^{\text{Dim6}(M^2, \alpha)} = -\frac{\Theta(L)}{405 \cdot 2 \pi^6(\alpha - 1)^3} \left\{ 27(g_s^2 G^3) m_c^2 \alpha^5 + 34560(\bar{d}d) (\bar{u}u) \pi^4(\alpha - 1)^3 \left[ -2m_c m_s + 2m_c^2 \alpha + 3s \alpha(1 - \alpha) \right] \right\} + 320 g_s^2 (\bar{d}d)^2 \pi^2(\alpha - 1)^3 \left[ -m_c m_s + 4m_c^2 \alpha + 6s \alpha(1 - \alpha) \right] + 320 g_s^2 \pi^2(\alpha - 1)^3 \times \left[ -m_c m_s (\bar{u}u)^2 + (\bar{s}s)^2 + (\bar{u}u)^2 \right] \left( 4m_c^2 \alpha + 6s \alpha(1 - \alpha) \right).$$

(A.10)
\[
\rho^\text{Dim7}(M^2, \alpha) = \frac{\langle \alpha_s G^2/\pi \rangle \langle \bar{s}s \rangle \Theta(L)}{288\pi^2 (\alpha - 1)^2} [3m_s \alpha (\alpha - 1)^2 + m_c (2 - 7\alpha + 5\alpha^2 - 2\alpha^3)] ,
\]

\[
\rho^\text{Dim8}(M^2, \alpha) = \frac{\langle \alpha_s G^2/\pi \rangle^2 \alpha + 96 \langle \bar{g}_s \sigma Gd \rangle \langle \bar{\pi} u \rangle (\alpha - 1)}{1152\pi^2} \Theta(L) ,
\]

Components of the function \(\Pi(M^2)\) are:

\[
\Pi^\text{Dim6}(M^2, \alpha) = -\frac{(g_s^3 G^3) m_c^3}{45 \cdot 2^{10} M^2 \pi^6 (\alpha - 1)^5} \exp \left[ -\frac{m_c^2}{M^2 (\alpha - 1)} \right] [m_c^3 \alpha (2 + \alpha) + 4m_c^2 m_s (2\alpha - 1) - 8m_s M^2 (\alpha - 1)^2],
\]

\[
\Pi^\text{Dim7}(M^2, \alpha) = \frac{\langle \alpha_s G^2/\pi \rangle \langle \bar{s}s \rangle m_c^2 \alpha^2}{288 M^2 \pi^2 (\alpha - 1)^3} \exp \left[ -\frac{m_c^2}{M^2 (\alpha - 1)} \right] [m_c^2 m_s + (m_c - m_s) M^2 (\alpha - 1)],
\]

\[
\Pi^\text{Dim8}(M^2, \alpha) = -\frac{\langle \alpha_s G^2/\pi \rangle^2 m_c \alpha}{9 \cdot 2^9 M^2 \pi^2 (\alpha - 1)^3} \exp \left[ -\frac{m_c^2}{M^2 (\alpha - 1)} \right] [2m_c^2 m_s (\alpha - 1) + m_c^3 \alpha - m_c M^2 \alpha (\alpha - 1)],
\]

\[
\Pi^\text{Dim9}(M^2, \alpha) = \frac{\langle \alpha_s G^2/\pi \rangle \langle \bar{s}s \rangle m_c^2 \alpha^2 M_s^2 (2 - 4\alpha) + 8 M^4 (\alpha - 1) + m_c^3 m_s (2 + \alpha) + 5 \langle \alpha_s G^2/\pi \rangle \langle \bar{g}_s \sigma Gd \rangle M^2 (\alpha - 1)^2 [3m_s^2 M^2 (\alpha - 1)^2 + 3m_s M^4 (\alpha - 1)^3 + 4m_s^3 M^2 (\alpha - 1) + 3m_c M^4 (3 - 4\alpha + 3\alpha^2 - 2\alpha^3)]
\]

and

\[
\Pi^\text{Dim10}(M^2, \alpha) = -\frac{1}{135 \cdot 2^{15} M^6 \pi^4 (\alpha - 1)^5} \exp \left[ -\frac{m_c^2}{M^2 (\alpha - 1)} \right] \left\{ -4 (\bar{u} u)^2 m_c^2 m_s (\alpha - 1) + 8 (\bar{u} u)^2 m_c M^2 (\alpha - 1) + \left[ 2 \langle \bar{s}s \rangle + \langle \bar{u} u \rangle \right] -3 m_s^2 M^2 (\alpha - 1)^2 + 3m_s M^4 (\alpha - 1)^3 + 4m_s^3 M^2 (\alpha - 1) + 3m_c M^4 (3 - 4\alpha + 3\alpha^2 - 2\alpha^3) \right\}.
\]

In expressions above, \(\Theta(z)\) is Unit Step function. We have used also the following short-hand notations:

\[
L \equiv L(s, \alpha) = s\alpha (1 - \alpha) - m_c^2 \alpha.
\]
