TILTED BIANCHI TYPE V BULK VISCOUS COSMOLOGICAL MODELS IN GENERAL RELATIVITY

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Abstract. Conformally flat tilted Bianchi type V cosmological models in presence of a bulk viscous fluid and heat flow are investigated. The coefficient of bulk viscosity is assumed to be a power function of mass density. Some physical and geometric aspects of the models are also discussed.

Keywords: cosmology; Bianchi type V universe; tilted models.

1. Introduction

A considerable interest has been shown to the study of physical properties of spacetimes which are conformal to certain well known gravitational fields. The general theory of relativity is believed by a number of unknown functions - the ten components of $g^{ij}$. Hence there is a little hope of finding physically interesting results without making reduction in their number. In conformally flat spacetime the number of unknown functions is reduced to one. The conformally flat metrics are of particular interest in view of their degeneracy in the context of Petrov classification. A number of conformally flat physically significant spacetimes are known like Schwarzschild interior solution and Lemaître cosmological universe.

The study of Bianchi type V cosmological models create more interest as these models contain isotropic special cases and permit arbitrarily small anisotropy levels at any instant of cosmic time. This property makes them suitable as model of our universe. Also Bianchi type V models are more complicated than the simplest Bianchi type models e.g. the Einstein tensor has off diagonal terms so that it is more natural to include tilt and heat conduction. Spacetimes of Bianchi type I, V and

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IX universes are the generalizations of FRW models and it will be interesting to construct cosmological models of these types which are of class one. Roy and Prasad (1994) have investigated Bianchi type V universes which are locally rotationally symmetric and are of embedding class one filled with perfect fluid with heat conduction and radiation. Bianchi type V cosmological models have been studied by other researchers (Farnsworth, 1967; Maarteins and Nel, 1978; Wainwright *et al*., 1979; Collins, 1974; Meena and Bali, 2002).

The general dynamics of tilted models have been studied by King and Ellis (1973) and Ellis and King (1974). The cosmological models with heat flow have been also studied by Coley and Tupper (1983, 1984); Roy and Banerjee (1988). Ellis and Baldwin (1984) have shown that we are likely to be living in a tilted universe and they have indicated how we may detect it. Beesham (1986) derived tilted Bianchi type V cosmological models in the scale-covariant theory. A tilted cold dark matter cosmological scenario has been discussed by Cen, Nickolay, Kofman and Ostriker (1992). Recently Bali and Meena (2002) have investigated two tilted cosmological models filled with disordered radiation and heat flow. Tilted Bianchi type I cosmological model for perfect fluid distribution in presence of magnetic field is investigated by Bali and Sharma (2003). Tilted Bianchi type I cosmological models filled with disordered radiation in presence of a bulk viscous fluid and heat flow are obtained by Pradhan and Rai (2003).

Most cosmological models assume that the matter in the universe can be described by ‘dust’ (a pressureless distribution) or at best a perfect fluid. Nevertheless, there is good reason to believe that - at least at the early stages of the universe - viscous effects do play a role (Israel and Vardalas, 1970; Klimek, 1971; Weinberg, 1971). For example, the existence of the bulk viscosity is equivalent to slow process of restoring equilibrium states (Landau and Lifshitz, 1962). The observed physical phenomena such as the large entropy per baryon and remarkable degree of isotropy of the cosmic microwave background radiation suggest analysis of dissipative effects in cosmology. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Gron *) for a review on cosmological models with bulk viscosity). The model studied by Murphy (1973) possessed an interesting feature in that the big bang type of singularity of infinite spacetime curvature does not occur to be a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large
density. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity (Pavon, 1991; Padmanabhan and Chitre, 1987; Johri and Sudarshan, 1988; Maartens, 1995; Zimdahl, 1996; Santos et al., 1985; Pradhan, Sarayakar and Beesham, 1997; Kalyani and Singh 1997; Singh, Beesham and Mbokazi, 1998; Pradhan et al., 2001, 2002, 2003). This motivates to study cosmological bulk viscous fluid model. Recently Meena and Bali (2002) have investigated two conformally flat tilted Bianchi type V cosmological models filled with a perfect fluid and heat conduction. In this paper, we propose to find tilted Bianchi type V cosmological models in presence of a bulk viscous fluid and heat flow.

2. The metric and field equations

We consider the Bianchi type V metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} \left(dy^2 + dz^2\right),$$

where $A$, $B$ are function of $t$ only.

The Einstein’s field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R^i_j - \frac{1}{2}Rg^i_j = -8\pi T^i_j,$$

where $R^i_j$ is the Ricci tensor; $R = g^{ij}R_{ij}$ is the Ricci scalar; and $T^i_j$ is the stress energy-tensor in the presence of bulk stress given by

$$T^i_j = (\rho + \bar{p})v^i v^j + \bar{p}g^i_j + q_i v^j + v_i q^j,$$

and

$$\bar{p} = p - \xi v^i_i.$$

Here $\rho$, $p$, $\bar{p}$ and $\xi$ are the energy density, isotropic pressure, effective pressure and bulk viscous coefficient respectively and $v_i$ is the flow vector satisfying the relations

$$g_{ij}v^i v^j = -1,$$

$$q_i q^i > 0,$$

$$q_i v^i = 0,$$

where $q_i$ is the heat conduction vector orthogonal to $v_i$. The fluid flow vector has the components ($\sinh \lambda$, 0, 0, $\cosh \lambda$) satisfying Eq. (5) and $\lambda$ is the tilt angle.
The Einstein’s field equations (2) for the line element (1) has been set up as

\[-8\pi[(\rho + \bar{\rho}) \sinh^2 \lambda + \bar{\rho} + 2Aq_1 \sinh \lambda] = \frac{2B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{1}{A^2}, \quad (8)\]

\[-8\pi\bar{\rho} = \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2}, \quad (9)\]

\[-8\pi[(-\rho + \bar{\rho}) \cosh^2 \lambda + \bar{\rho} - 2Aq_1 \sinh \lambda] = \frac{2A_4B_4}{AB} + \left(\frac{B_4}{B}\right)^2 - \frac{3}{A^2}, \quad (10)\]

\[-8\pi[(\rho + \bar{\rho}) A \sinh \lambda \cosh \lambda + A^2q_1(\cosh \lambda + \sinh \lambda \tanh \lambda)] = \frac{2A_4}{A} - \frac{2B_4}{B}, \quad (11)\]

where the suffix 4 at the symbols A, B denotes ordinary differentiation with respect to t.

3. Solution of the field equations

Equations (8) - (12) are four independent equations in seven unknowns A, B, ρ, p, ξ, q and λ. For the complete determinacy of the system, we need three extra conditions. First we assume that the spacetime is conformally flat which leads to

\[C_{2323} = \frac{1}{3} \left[ \frac{A_{44}}{A} - \frac{B_{44}}{B} - \frac{A_4B_4}{AB} + \frac{B_4^2}{B^2} \right] = 0 \quad (12)\]

and secondly, we assume

\[A = B^n, \quad (13)\]

where n is any real number. Eqs. (12) and (13) lead to

\[\frac{B_{44}}{B} + (n - 1)\frac{B_4^2}{B^2} = 0. \quad (14)\]

From Equations (8), (10) and (13), we have

\[-4\pi[(\rho + \bar{\rho}) \cosh 2\lambda + 4B^nq_1 \sinh \lambda] = \frac{B_{44}}{B} - n\frac{B_4^2}{B^2} + \frac{1}{B^{2n}}, \quad (15)\]

and

\[4\pi(\rho - \bar{\rho}) = \frac{B_{44}}{B} + (n + 1)\frac{B_4^2}{B^2} - \frac{2}{B^{2n}}. \quad (16)\]

Equations (9), (16) and (12) lead to

\[(n - n^2 + 1)\frac{B_4^2}{B^2} - n\frac{B_{44}}{B} - \frac{1}{B^{2n}} = 4\pi(\rho + \bar{\rho}). \quad (17)\]
Equations (11) and (13) lead to

\[-16\pi q_1 B^n \sinh \lambda = \frac{2(n-1)B_4 \tanh 2\lambda}{B^{n+1}} + 4\pi (\rho + \bar{\rho}) \sinh 2\lambda \tanh 2\lambda. \tag{18}\]

From Eqs. (15) and (18), we obtain

\[\frac{B_{44}}{B} - \frac{nB_4^2}{B} + \frac{1}{B^{2n}} = -\frac{4\pi (\rho + \bar{\rho})}{\cosh 2\lambda} + \frac{2(n-1) \tanh 2\lambda}{B^n}. \tag{19}\]

Equations (17) and (19) lead to

\[\frac{B_{44}}{B} - \frac{nB_4^2}{B} + \frac{1}{B^{2n}} = \frac{nB_{44}}{B} + \frac{(n^2 - n - 1)B_4^2}{B^2} + B^{2n} \sech 2\lambda + \frac{2(n-1) B_4 \tanh 2\lambda}{B^{n+1}}. \tag{20}\]

Equation (14) can be rewritten as

\[\frac{B_{44}}{B_4} + \frac{(n-1)B_4}{B} = 0, \tag{21}\]

which on integration leads to

\[B = n^{\frac{1}{n}} (\alpha t + \beta)^{\frac{1}{n}}. \tag{22}\]

where \(\alpha, \beta\) are constants of integration. Hence we obtain

\[A^2 = n^2 (\alpha t + \beta)^2, \tag{23}\]

\[B^2 = n^{\frac{2}{n}} (\alpha t + \beta)^{\frac{2}{n}}. \tag{24}\]

Hence the geometry of the spacetime (1) reduces to the form

\[ds^2 = -dt^2 + n^2 (\alpha t + \beta)^2 dx^2 + [n(\alpha t + \beta)]^{\frac{2}{n}} e^{2x}(dy^2 + dz^2). \tag{25}\]

After the suitable transformation of coordinates, the metric (25) takes the form

\[ds^2 = -\frac{dT^2}{2} + n^2 T^2 dX^2 + n^{\frac{2}{n}} T^{\frac{2}{n}} e^{2X} (dY^2 + dZ^2). \tag{26}\]

The effective pressure and density of the model (26) are given by

\[8\pi \bar{p} = 8\pi (p - \xi \theta) = -\frac{(\alpha^2 - 1)}{n^2 T^2}, \tag{27}\]

\[8\pi \rho = \frac{3(\alpha^2 - 1)}{n^2 T^2}, \tag{28}\]
where $\theta$ is the scalar of expansion calculated for the flow vector $v^i$ and given is as

$$\theta = \frac{2K + (n + 2)k\alpha}{nT}. \quad (29)$$

The tilt angle $\lambda$ is given by

$$\cosh^2 \lambda = k^2, \quad (30)$$

$$\sinh^2 \lambda = K^2, \quad (31)$$

where $k$ and $K$ are constants given by

$$k^2 = \frac{(n\alpha^2 - 1)^2}{(\alpha^2 - 1)(2n^2\alpha^2 - n\alpha^2 - 1) + 2\alpha^2(n - 1)\sqrt{(n^2 - n)(1 - \alpha^2)}},$$

$$K^2 = \frac{(n - 1)(2n\alpha - n\alpha^4 - \alpha^2) - 2\alpha^2(n - 1)\sqrt{(n^2 - n)(1 - \alpha^2)}}{(\alpha^2 - 1)(2n^2\alpha^2 - n\alpha^2 - 1) + 2\alpha^2(n - 1)\sqrt{(n^2 - n)(1 - \alpha^2)}}. \quad (32)$$

If we put $\xi = 0$ in (27), we get the solutions as obtained by Meena and Bali (2002).

Thus, given $\xi(t)$ we can solve the system for the physical quantities. Therefore to apply the third condition, let us assume the following adhoc law (Maartens, 1995; Zimdahl, 1996)

$$\xi(t) = \xi_0 \rho^m \quad (34)$$

where $\xi_0$ and $m$ are real constants. If $m = 1$, Eq. (34) may correspond to a radiative fluid (Weinberg, 1972), whereas $m = \frac{3}{2}$ may correspond to a string-dominated universe. However, more realistic models (Santos, 1985) are based on lying the regime $0 \leq m \leq \frac{1}{2}$.

3.1. Model I: \hspace{1cm} ($\xi = \xi_0$)

When $m = 0$, Equation (34) reduces to $\xi = \xi_0 = \text{constant}$ and hence Equation (27) with the use of (29) leads to

$$p = \frac{\xi_0\{2K + (n + 2)k\alpha\}}{nT} - \frac{(\alpha^2 - 1)}{8n^2T^2}. \quad (35)$$

3.2. Model II: \hspace{1cm} ($\xi = \xi_0\rho$)

When $m = 1$, Equation (34) reduces to $\xi = \xi_0\rho$ and hence Equation (27) with the use of (29) leads to

$$p = \frac{1}{8n^2T^2} \left[ 1 - \alpha^2 + \frac{3\xi_0(\alpha^2 - 1)\{2K + (n + 2)k\alpha\}}{nT} \right]. \quad (36)$$
It is observed from Equations (28), (35) and (36) that $\rho$ and $p$ vary as $\frac{1}{T}$. The models are singular at $T = 0$ and as they evolve, the pressure and density decrease.

4. Some Physical and Geometric Properties of the Models

The weak and strong energy conditions, we have, in Model I

$$\rho + p = \frac{(\alpha^2 - 1)}{4\pi n^2 T^2} + \frac{\xi_0(2K + (n + 2)k\alpha)}{nT},$$  \hspace{1cm} (37)

$$\rho - p = \frac{(\alpha^2 - 1)}{2\pi n^2 T^2} - \frac{\xi_0(2K + (n + 2)k\alpha)}{nT},$$  \hspace{1cm} (38)

$$\rho + 3p = 3\xi_0(2K + (n + 2)k\alpha),$$  \hspace{1cm} (39)

$$\rho - 3p = \frac{(3\alpha^2 - 1)}{4\pi n^2 T^2} - \frac{3\xi_0(2K + (n + 2)k\alpha)}{nT}.$$  \hspace{1cm} (40)

In Model II, we have

$$\rho + p = \frac{1}{8\pi n^2 T^2} \left[ 2(\alpha^2 - 1) + \frac{3\xi_0(\alpha^2 - 1)(2K + (n + 2)k\alpha)}{nT} \right],$$  \hspace{1cm} (41)

$$\rho - p = \frac{1}{8\pi n^2 T^2} \left[ 4(\alpha^2 - 1) - \frac{3\xi_0(\alpha^2 - 1)(2K + (n + 2)k\alpha)}{nT} \right],$$  \hspace{1cm} (42)

$$\rho + 3p = \frac{9\xi_0(\alpha^2 - 1)(2K + (n + 2)k\alpha)}{8\pi n^3 T^3},$$  \hspace{1cm} (43)

$$\rho - 3p = \frac{3}{8\pi n^2 T^2} \left[ 2(\alpha^2 - 1) + \frac{3\xi_0(\alpha^2 - 1)(2K + (n + 2)k\alpha)}{nT} \right].$$  \hspace{1cm} (44)

The reality conditions $\rho \geq 0$, $p \geq 0$ and $\rho - 3p \geq 0$ impose further restrictions on both of these models.

The flow vector $v^i$ and heat conduction vector $q^i$ for the models (26) are obtained by Meena and Bali (2002)

$$v^1 = \frac{K}{nT},$$  \hspace{1cm} (45)

$$v^4 = k,$$  \hspace{1cm} (46)

$$q^1 = -\frac{k((\alpha^2 - 1)kK + (n - 1)\alpha)}{4\pi n^3 T^3(k^2 + K^2)},$$  \hspace{1cm} (47)
The rate of expansion $H_i$ in the direction of $X$, $Y$, $Z$-axe are given by

$$H_1 = \frac{\alpha}{T},$$

$$H_2 = H_3 = \frac{\alpha}{nT}. \tag{50}$$

The non-vanishing components of shear tensor $(\sigma_{ij})$ and rotation tensor $(\omega_{ij})$ are obtained as

$$\sigma_{11} = \frac{2}{3} nk^2 T[n(1)k\alpha - K], \tag{51}$$

$$\sigma_{22} = \sigma_{33} = \frac{(nT)^\frac{2}{n}-1}{3} e^{2X}[(1 - n)k\alpha - 2K], \tag{52}$$

$$\sigma_{44} = \frac{2K^2}{3nT}[(n - 1)k\alpha - K], \tag{53}$$

$$\sigma_{14} = \frac{K}{3}[2(n - 1)k^2 + 2kK - 3n], \tag{54}$$

$$\omega_{14} = n\alpha K. \tag{55}$$

The models, in general, represent shearing and rotating universes. The models start expanding with a Big bang at $T = 0$ and the expansion in the models decreases as time increases and the expansion in the models stops at $T = \infty$ and $\alpha = -\frac{2K}{(n+2)K}$. Both density and pressure in the models become zero at $T = \infty$. For $\alpha = 1$, $n = 1$, we observe that heat conduction vector $q^1 = q^4 = 0$. When $T \to \infty$, $v^1 = 0$, $v^4 = \text{constant}$, $q^1 = q^4 = 0$. Since $\lim_{T \to \infty} \frac{\pi}{2} \neq 0$, the models do not approach isotropy for large values of $T$. There is a real physical singularity in the model at $T = 0$.

5. Particular Models

If we set $n = 2$, then the geometry of the spacetime (26) reduces to the form

$$ds^2 = - \frac{dT^2}{2} + TdX^2 + Te^X(dY^2 + dZ^2). \tag{56}$$

The effective pressure and density for the model (56) are given by

$$8\pi \bar{p} = 8\pi(p - \xi\theta) = -\frac{(\alpha^2 - 1)}{4T^2}, \tag{57}$$
\[ 8\pi \rho = \frac{3(\alpha^2 - 1)}{4T^2}, \quad (58) \]

where \( \theta \) is the scalar expansion obtained as
\[ \theta = \frac{K_1 + 2k_1 \alpha}{T}. \quad (59) \]

The tilt angle \( \lambda \) is given by
\[ \cosh^2 \lambda = k_1^2, \quad (60) \]
\[ \sinh^2 \lambda = K_1^2, \quad (61) \]
where \( k_1 \) and \( K_1 \) are constants given by
\[ k_1^2 = \frac{(2\alpha^2 - 1)^2}{6\alpha^4 - 7\alpha^2 + 1 + 2\alpha^2 \sqrt{2(1 - \alpha^2)}}, \quad (62) \]
\[ K_1^2 = \frac{\alpha^2 \left[(3 - 2\alpha^2 - 2\sqrt{2(1 - \alpha^2)}\right]}{6\alpha^4 - 7\alpha^2 + 1 + 2\alpha^2 \sqrt{2(1 - \alpha^2)}}. \quad (63) \]

Thus, for given \( \xi(t) \) one can solve the system for the physical quantities.

5.1. Model I: \( (\xi = \xi_0) \)

When \( m = 0 \), Equation (34) reduces to \( \xi = \xi_0 = \) constant and hence Equation (57) with the use of (59) leads to
\[ p = \frac{\xi_0 \{K_1 + 2k_1 \alpha\}}{T} - \frac{(\alpha^2 - 1)}{32\pi T^2}. \quad (64) \]

5.2. Model II: \( (\xi = \xi_0 \rho) \)

When \( m = 1 \), Equation (34) reduces to \( \xi = \xi_0 \rho \) and hence Equation (57) with the use of (59) leads to
\[ p = \frac{1}{32\pi T^2} \left[ 1 - \alpha^2 + \frac{3\xi_0 (\alpha^2 - 1) \{K_1 + 2k_1 \alpha\}}{T} \right]. \quad (65) \]

It is observed from Equations (58), (64) and (65) that \( \rho \) and \( p \) vary as \( \frac{1}{T} \). The models are singular at \( T = 0 \) and as they evolve, the pressure and density decrease.
6. Some Physical and Geometric Properties of Particular Models

The weak and strong energy conditions, we have, in Model I

\[ \rho + p = \frac{(\alpha^2 - 1)}{16\pi T^2} + \frac{\xi_0 \{K_1 + 2k_1\alpha\}}{T}, \]  
(66)

\[ \rho - p = \frac{(\alpha^2 - 1)}{8\pi T^2} - \frac{\xi_0 \{K_1 + 2k_1\alpha\}}{T}, \]  
(67)

\[ \rho + 3p = \frac{3\xi_0 \{K_1 + 2k_1\alpha\}}{T}, \]  
(68)

\[ \rho - 3p = \frac{(3\alpha^2 - 1)}{16\pi T^2} - \frac{3\xi_0 \{K_1 + 2k_1\alpha\}}{T}, \]  
(69)

In Model II, we have

\[ \rho + p = \frac{1}{32\pi T^2} \left[ 2(\alpha^2 - 1) + \frac{3\xi_0 (\alpha^2 - 1) \{K_1 + 2k_1\alpha\}}{T} \right], \]  
(70)

\[ \rho - p = \frac{1}{32\pi T^2} \left[ 4(\alpha^2 - 1) - \frac{3\xi_0 (\alpha^2 - 1) \{K_1 + 2k_1\alpha\}}{T} \right], \]  
(71)

\[ \rho + 3p = \frac{9\xi_0 (\alpha^2 - 1) \{K_1 + 2k_1\alpha\}}{32\pi n^3 T^3}, \]  
(72)

\[ \rho - 3p = \frac{3}{32\pi n^3 T^2} \left[ 2(\alpha^2 - 1) + \frac{3\xi_0 (\alpha^2 - 1) \{K_1 + 2k_1\alpha\}}{T} \right]. \]  
(73)

The reality conditions \( \rho \geq 0, \ p \geq 0 \) and \( \rho - 3p \geq 0 \) impose further restrictions on both of these models.

The flow vector \( v^i \) and heat conduction vector \( q^i \) for the models (56) are obtained as

\[ v^1 = \frac{K_1}{2T}, \]  
(74)

\[ v^4 = k_1, \]  
(75)

\[ q^1 = -\frac{k_1 \{(\alpha^2 - 1)k_1 K_1 + \alpha\}}{32\pi T^3 (k_1^2 + K_1^2)}, \]  
(76)

\[ q^4 = -\frac{K_1 \{(\alpha^2 - 1)k_1 K_1 + \alpha\}}{32\pi T^3 (k_1^2 + K_1^2)}. \]  
(77)

The rate of expansion \( H_1 \) in the direction of \( X, Y, Z \)-axe are given by

\[ H_1 = \frac{\alpha}{T}, \]  
(78)
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\[
H_2 = H_3 = \frac{\alpha}{2T}.
\]  (79)

The non-vanishing components of shear tensor (\(\sigma_{ij}\)) and rotation tensor (\(\omega_{ij}\)) are obtained as

\[
\sigma_{11} = \frac{4}{3} k_1^2 T |k_1 \alpha - K_1|,
\]  (80)

\[
\sigma_{22} = \sigma_{33} = -\frac{e^{2X}}{3} [k_1 \alpha + 2K_1],
\]  (81)

\[
\sigma_{44} = \frac{K_1^2}{3T} [k_1 \alpha - K_1],
\]  (82)

\[
\sigma_{14} = -\frac{K_1}{3} [2k_1^2 - 2k_1 K_1 + 6],
\]  (83)

\[
\omega_{14} = 2\alpha K_1.
\]  (84)

The models in general represent shearing and rotating universes. The models start expanding with a Big bang at \(T = 0\) and the expansion in the models decreases as time increases and the expansion in the models stops at \(T = \infty\) and \(\alpha = -\frac{K_1}{2k_1}\). Both density and pressure in the models become zero at \(T = \infty\). For expansion and rotation, we have \(\alpha \neq 1, K_1 \neq 1\) when \(T \to \infty\) we observe that heat conduction vector \(q^1 = q^4 = 0, v^1 = 0, v^4 = K_1\). Since \(\lim_{t \to \infty} \frac{q^2}{T} \neq 0\), the models do not approach isotropy for large values of \(T\). There is a real physical singularity in the model at \(T = 0\). When \(k_1 = 1\) then \(\lambda = 0\). Thus, the tilted cosmological models lead to non-tilted one for \(k_1 = 1\).

7. Conclusions

We have obtained a new class of conformally flat tilted Bianchi type V magnetized cosmological models with a bulk viscous fluid as the source of matter. Generally, the models are expanding, shearing and rotating. In all these models, we observe that they do not approach isotropy for large values of time \(T\) in the presence of magnetic field.

The coefficient of bulk viscosity is assumed to be a power function of mass density. The effect of bulk viscosity is to introduce a change in the perfect fluid model. We also observe here that the conclusion of Murphy (1973) about the absence of a Big bang type of singularity in the finite past in models with bulk viscous fluid is, in general, not true.
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