Global shockwaves of the Hunga Tonga-Hunga Ha’apai volcano eruption measured at ground stations

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Highlights
Dispersion equation of global spherical shockwaves from a point source derived
Air pressure of 3–21 s recorded the shockwaves from the Tonga volcano eruption
Shockwaves and ringing with higher energy analyzed using data from 191 stations
Shockwaves up to 6th pass from 191 stations consistent with global spherical waves

Spherical Shockwave Arrival Times (hr)

First pass (186 stations)

R²=0.993

band-pass ringing spectrum

background noise spectrum
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SUMMARY

The eruption of the Tonga volcano created globally propagating spherical shockwaves in the atmosphere. Analyses are done to data from two southern U.S. stations of the author sampling at 3–21 s intervals and 189 weather stations at 1–5 min intervals. The shockwaves arrived from two routes in the atmosphere: the shortest spherical arc and the longer spherical arc through the antipole. In most stations, signals up to the 6th path of shockwaves were recorded as the waves traveled around the globe multiple times. The speed of shockwaves is estimated to be $309.5 \pm 2.9 \text{ m/s}$, consistent with the speed of sound at the top of the troposphere where a waveguide exists. Discussion is made on the post-shockwave ringing of 4–8 min as higher amplitude oscillations above the level of pre-shockwaves background noise. A theoretical wave dispersion is derived which verifies that the spherical shockwave’s phase speed is the same as the speed of sound.

INTRODUCTION

On 15 January 2022, a catastrophic eruption of the Hunga Tonga-Hunga Ha’apai underwater volcano in the southern Pacific Ocean (Figure 1A) occurred after 0400 UTC (Terry et al., 2022). It was reported by the World Bank (2022) that the eruption occurred at 04:14:45 UTC (17:14:45, 15 January, local time). Yuen et al. (2022) reported that the eruption started at 0402 UTC and drastically increased at 0408 UTC; others reported that it started at 0415 UTC (Burt, 2022), or before 0410 UTC (Smart, 2022) according to the satellite images (Bachmeier, 2022). This volcano was in the southern Pacific at (20.55°S, 175.38°W), approximately 1900 km north-northeast of New Zealand, 3200 km east of Australia, and 600 km southeast of Fiji. The nearby islands of Hunga Tonga and Hunga Ha’apai with an area of 1 km² and a maximum altitude of 149 m were blown apart. Eruption-induced tsunami waves were observed in the Pacific Ocean, damaging at least 600 structures including 300 residential buildings. The economic damage was estimated greater than 90M US dollars (World Bank, 2022). The explosion is believed to be a once-in-1000-year’s event for the Hunga caldera. The eruption reached at least 35- to 45-km altitude (Bachmeier, 2022) and transiently 58 km (Yuen et al., 2022). It created what appeared to be spherical shockwaves propagating around the globe (Smart, 2022; Amores et al., 2022), as captured by the geostationary satellites including NOAA’s GEOS-17 and Japanese Himawari-8 (Bachmeier, 2022), and by barometric pressure at ground stations up to 127 h measured in Britain and Ireland (Burt, 2022). The eruption energy (4–18 megatons, pending confirmation, World Bank, 2022) was estimated to be below that of the explosion at Mount St. Helens (US) in 1980 (with 24 megatons of energy) and the Krakatoa (Indonesia) explosion in 1883 (200 megatons of energy, Symons, 1888; Yokoyama, 1981; Gabrielson, 2010). The 1883 Krakatoa eruption (Symons, 1888; Gabrielson, 2010) produced globally propagating shockwaves lasting for at least 5 days. Preliminary analysis puts the Tonga eruption as the largest volcanic eruption of this century, and since 1991, the Mount Pinatubo eruption (Smith and Kilburn, 2010). On a scale of 0–8 for the volcanic explosive index (VEI), the Hunga Tonga-Hunga Ha’apai underwater volcano eruption is tentatively ranked at 5 (World Bank, 2022), indicating that the total volume ejection was greater than 1 km³. These assessments, however, are preliminary and may be updated as studies continue.

Explosive volcanic eruptions typically generate shockwaves (Nairn, 1974), acoustic waves (Wouff and McGetchin, 1958) including infrasound waves propagating in the atmosphere (Morrissey and Chouet, 1997) detectable by microbarographs and electric signals in the atmosphere measurable by lightning mapping array (LMA) systems. Earthquakes can also produce infrasound waves measurable in the atmosphere...
Seismic waves from volcanic eruptions transmit through the ground whereas infrasound waves from volcanic eruptions propagate in the atmosphere (Smith et al., 2020). It has been known for a long time that volcanic eruptions can produce measurable atmospheric pressure waves (Yokoo et al., 2006) at weather stations, and air pressure variations have been used to estimate the total energy from the eruptions (Gorshkov, 1960).

Atmospheric (barometric) pressure data from meteorological stations and data from infrasound sensors are useful for studying eruption/explosion (including manmade explosions especially nuclear explosions, e.g., Perttu et al., 2020; Pichon et al., 2019). Early observations and studies of infrasound and atmospheric waves caused by volcanic eruptions used analog instruments. Data from different stations can be combined to make inferences about the events. Using such data, some parameters of a mysterious Siberian explosion in 1908 were determined (Ben-Menahem, 1975): the atmospheric shockwave was propagating at a speed of 285–324 m/s, and the total energy was estimated at \( 1.25 \times 10^{12} \) megatons (1 megaton = 4.18 \( \times \) 10\(^{15}\) joules) from an unknown source. The difference in the wave speed estimates was believed to be caused by the different meteorological conditions at different stations. Another study analyzing

**Figure 1. Study site and shockwave routes**

The Hunga Tonga-Hunga Ha’apai underwater volcano site P in the southern Pacific at \( 20^\circ 33'0.00'' S, 175^\circ 23'6.00'' W \), near the International Date Line (approximately along the red dashed line in (A)). In (B), point A is the antipole at \( 20^\circ 33'0.00'' N, 4^\circ 36'54'' E \). The green line is the equator. The thin red line indicates the great circle of 90° from P and the red arrows indicate the directions of the shockwave propagations. The blue arc shows Route 1, the shortest arc on the great circle to reach the sampling site marked as Station. The red arc is Route 2, the longer arc on the great circle to reach S through the antipole A. The stations are shown in (C), except the stations at Guam (Figure 8). The diamond shows the stations in Louisiana where high-resolution data (with 3-s and 21-s intervals) are collected.

(Shani-Kadmiel et al., 2021). Seismic waves from volcanic eruptions transmit through the ground whereas infrasound waves from volcanic eruptions propagate in the atmosphere (Smith et al., 2020). It has been known for a long time that volcanic eruptions can produce measurable atmospheric pressure waves (Yokoo et al., 2006) at weather stations, and air pressure variations have been used to estimate the total energy from the eruptions (Gorshkov, 1960).
eruption movies concluded that the pressure waves induced by the eruptions had phase speeds ranging between 342 m/s and 574 m/s (Yokoo and Ishihara, 2007).

Mathematical models have been developed to study explosion processes especially near-field dynamics, to quantify motions and wave propagations (Clarke et al., 2002). Shockwaves from volcanic eruptions have

| No. | Name                          | Longitude | Latitude | Distance from Tonga Volcano (km) |
|-----|-------------------------------|-----------|----------|---------------------------------|
| 1   | Ridge, Louisiana              | −91.0912  | 30.3695  | 10627                           |
| 2   | Russell, Louisiana            | −91.1794  | 30.4116  | 10621                           |
| 3   | O'Hare Airport, Chicago,      | −87.9319  | 41.9875  | 11316                           |
| 4   | San Francisco, California     | −122.2207 | 37.7213  | 8534                            |
| 5   | Baton Rouge, Louisiana        | −91.1469  | 30.5372  | 10629                           |
| 6   | Boston, Massachusetts         | −71.0097  | 42.3606  | 1287                            |
| 7   | Phoenix, Arizona              | −112.0116 | 33.4343  | 9004                            |
| 8   | Tacoma Airport, Seattle       | −122.3144 | 47.4447  | 9229                            |
| 9   | Dickinson, North Dakota       | −102.8019 | 46.7973  | 10416                           |
| 10  | Westhampton Beach, Long Island| −72.6318  | 40.8436  | 12532                           |
| 11  | Norfolk, Virginia             | −76.1922  | 36.9033  | 12153                           |
| 12  | Newark, New Jersey            | −74.1693  | 40.6827  | 12401                           |
| 13  | Ann Arbor, Michigan           | −83.7397  | 42.224   | 11656                           |
| 14  | Barrow, Alaska                | −156.7922 | 71.2826  | 10311                           |
| 15  | Los Angeles, California       | −118.3865 | 33.9382  | 8548                            |
| 16  | Savannah, Georgia             | −81.2021  | 32.1276  | 11581                           |
| 17  | Groton, Connecticut           | −72.05    | 41.33    | 12588                           |
| 18  | Adak Island, Alaska           | −176.646  | 51.878   | 8055                            |
| 19  | Anchorage, Alaska             | −149.8573 | 61.2163  | 9375                            |
| 20  | Barter Island, Alaska         | −143.5819 | 70.134   | 10388                           |
| 21  | Mobile, Alabama               | −88.0681  | 30.6268  | 10910                           |
| 22  | Little Rock, Arkansas         | −92.2357  | 34.7273  | 10698                           |
| 23  | Denver, Colorado              | −104.6575 | 39.8328  | 9928                            |
| 24  | Dallas, Texas                 | −96.8518  | 32.8471  | 10224                           |
| 25  | Honolulu, Hawaii              | −157.9224 | 21.3187  | 5027                            |
| 26  | Caldwell, Idaho               | −116.6358 | 43.6419  | 9310                            |
| 27  | Miami, Florida                | −80.3169  | 25.788   | 11468                           |
| 28  | San Antonio, Texas            | −98.4711  | 29.337   | 9926                            |
| 29  | Brownsville, Texas            | −97.4231  | 25.9146  | 9866                            |
| 30  | Memphis, Tennessee            | −89.985   | 35.0611  | 10904                           |
| 31  | Salt Lake City, Utah          | −111.97   | 40.78    | 9446                            |
| 32  | Ithaca, New York              | −76.4584  | 42.491   | 12247                           |
| 33  | Raleigh, North Carolina        | −78.7819  | 35.8922  | 11902                           |
| 34  | Reno, Nevada                  | −119.7711 | 39.4839  | 8822                            |
| 35  | Riverton, Wyoming             | −108.4598 | 43.0642  | 9826                            |
| 36  | Havre, Montana                | −109.7633 | 48.5428  | 10053                           |
| 37  | St. Paul, Minnesota           | −93.06    | 44.9345  | 11027                           |
| 38  | Salisbury, Maryland           | −75.5103  | 38.3405  | 12243                           |
| 39  | Frenchville, Maine            | −68.3127  | 47.2855  | 12941                           |
| 40  | Sanford, Maine                | −70.708   | 43.3939  | 12724                           |
Table 2. Locations of Stations with 1-min data (Part I). Shown here are the names, coordinates and distances

| No. | Name  | Longitude | Latitude | Distance from Tonga Volcano (km) |
|-----|-------|-----------|----------|----------------------------------|
| 41  | TJSJ  | −66.0021  | 18.4394  | 12670                            |
| 42  | TIST  | −64.9733  | 18.3373  | 12772                            |
| 43  | PHTO  | −155.0485 | 19.7203  | 4995                             |
| 44  | PHOG  | −156.4305 | 20.8986  | 5049                             |
| 45  | PHNL  | −157.9202 | 21.3178  | 5027                             |
| 46  | PHNG  | −157.7679 | 21.4505  | 5047                             |
| 47  | PHMK  | −157.0963 | 21.1529  | 5045                             |
| 48  | PHLI  | −159.3390 | 21.9760  | 5039                             |
| 49  | PHJR  | −158.0703 | 21.3074  | 5020                             |
| 50  | PGUM  | 144.7971  | 13.4840  | 5767                             |
| 51  | PGSN  | 145.7300  | 15.1202  | 5811                             |
| 52  | PAWD  | −149.4166 | 60.1299  | 9274                             |
| 53  | PATK  | −150.0927 | 62.3214  | 9482                             |
| 54  | PASO  | −151.7050 | 59.4439  | 9153                             |
| 55  | PASI  | −135.3611 | 57.0468  | 9398                             |
| 56  | PASC  | −148.4652 | 70.1948  | 10310                            |
| 57  | PAOR  | −141.9281 | 62.9612  | 9737                             |
| 58  | PAOM  | −165.4444 | 64.5126  | 9497                             |
| 59  | PANN  | −149.0739 | 64.5473  | 9729                             |
| 60  | PANC  | −149.9981 | 61.1741  | 9367                             |
| 61  | PAMR  | −149.8447 | 61.2135  | 9375                             |
| 62  | PAKW  | −133.0760 | 55.5792  | 9357                             |
| 63  | PAKT  | −131.7112 | 55.3541  | 9393                             |
| 64  | PAIN  | −134.5785 | 58.3547  | 9542                             |
| 65  | PAIL  | −154.9178 | 59.7556  | 9122                             |
| 66  | PAHO  | −151.4858 | 59.6450  | 9179                             |
| 67  | PAHN  | −135.5235 | 59.2438  | 9587                             |
| 68  | PAFA  | −147.8567 | 64.8154  | 9780                             |
| 69  | PABR  | −156.7686 | 71.2849  | 10312                            |
| 70  | KWLD  | −97.0375  | 37.1686  | 10399                            |
| 71  | KVIH  | −91.7695  | 38.1274  | 10869                            |
| 72  | KVAY  | −74.8457  | 39.9429  | 12331                            |
| 73  | KUTS  | −95.5872  | 30.7469  | 10243                            |
| 74  | KTYC  | −83.9941  | 35.8111  | 11446                            |
| 75  | KTYR  | −95.4030  | 32.3535  | 10328                            |
| 76  | KTVR  | −91.0277  | 32.3516  | 10710                            |
| 77  | KTUL  | −95.8881  | 36.1984  | 10451                            |
| 78  | KTTD  | −122.4013 | 45.5494  | 9086                             |
| 79  | KTLH  | −84.3509  | 30.3968  | 11240                            |
| 80  | KTKI  | −96.5888  | 33.1771  | 10261                            |
| 81  | KTW  | −122.5781 | 47.2679  | 9201                             |
| 82  | KTHV  | −76.8730  | 39.9170  | 12161                            |
| 83  | KSOF  | −85.7365  | 38.1741  | 11371                            |
| 84  | KRST  | −92.5000  | 43.9083  | 11031                            |
| 85  | KPAH  | −88.7730  | 37.0603  | 11079                            |

(Continued on next page)
been simulated using 3D numerical models for near-field dynamics (Saito and Takayama, 2005). In an early article (Pekeris, 1939), a mathematical solution from Lamb (1932) was used to study sound wave propagation in the atmosphere due to a disturbance that is applicable to volcanic eruptions, although the model ignored the curvature of the Earth (not spherical waves) when considering wave propagation in the atmosphere. According to numerical model computations, the exit velocity during a volcanic eruption can be as high as 300 m/s, close to the speed of sound (Turcotte et al., 1990). Another study using a mathematical model showed that when a volcano erupts, the exit silicic magma and volatiles can increase the air pressure by 10–100 times of the atmospheric pressure (Woods and Bower, 1995). The volcanic eruption exit speed rapidly decreases as the materials exiting the volcano are decompressed with reduced pressure in the air but at 0–1 km above the crater rim, the velocity can still be comparable to the speed of sound or even supersonic (Self et al., 1979). Large volcano explosions can release a tremendous amount of energy which generates atmospheric waves and infrasound waves traveling on global scales and the signals can be detected more than 10,000 km away (Dabrowa et al., 2011).

Air pressure data from meteorological stations (Automated Surface Observing Systems, or ASOS) in the U.S. and similar systems elsewhere typically report data at 1-min, 5-min to 1-h intervals. The hourly data are too sparse to capture shockwave events. The 5-min interval data are marginally useful if used alone. When these data are combined with high-resolution data; however, they can corroborate the findings. In this article, we report the use of data from air pressure sensors sampling at 3 and 21 s intervals, making it possible to resolve the signals without aliasing (Proakis and Manolakis, 1992), allowing an accurate computation of the arrival times of the maximum disturbance, the evolution of the signal, and the propagation speed of the waves. In addition, we also use 1-min and 5-min interval data from 189 additional weather stations (Figure 1 and Tables 1, 2, 3, and 4). When data from multiple stations are combined, it allows us to have a holistic view of the shockwaves to reconstruct a global picture of the shockwaves traveling around our planet. We have already seen publications reporting findings about the characteristics of shockwaves and wave propagation speed (Burt, 2022; Harrison, 2022) and other related physical processes (Yuen et al., 2022). In this study, we extend the study to cover different regions with a combination of high (3-s to 1-min) and low (5-min) temporal resolutions. In addition, we also provide a theoretical analysis of the spherical shockwaves in terms of the dispersion relationship.

RESULTS
The time series and signals of shockwaves at Sites 1 and 2
The high-resolution observations at 3–21 s intervals of air pressure from the author’s two stations show two abrupt variations (the circled fluctuations in Figure 2) after the catastrophic eruption on 15 January. Although there are fluctuations in the time series data, these two peaks are unique such that they are sharp (short period) and relatively large, making them stand out. The first signal shows an abrupt increase in air pressure of approximately 1.31 hPa in ~12 min, followed by a rapid decrease at about the same rate. The duration of the first signal is approximately 24.3 min (Figure 3A). The second signal (Figure 3B) has a similar rapid increase but with a smaller magnitude (~0.9 hPa), followed by a rapid drop in pressure by 2.18 hPa. The duration of the second signal is longer (54.5 min, Figure 3B). It is noted that after each of the signals, there appears to be some “ringing”: waves at higher frequencies (~ a few minutes in period).

| No. | Name | Longitude  | Latitude  | Distance from Tonga Volcano (km) |
|-----|------|------------|-----------|---------------------------------|
| 86  | KORH | 71.8756    | 42.2671   | 12615                           |
| 87  | KONO | 117.0130   | 44.0194   | 9311                           |
| 88  | KNKT | 76.8808    | 34.9032   | 12045                           |
| 89  | KMWL | 98.0602    | 32.7816   | 10118                           |
| 90  | KMRH | 76.6604    | 34.7338   | 12061                           |
| 91  | KMLS | 105.8882   | 46.4269   | 10188                           |

*Note: The stations with their names starting with T are those in the tropical Atlantic; those with P are the Pacific stations (Hawaii, Alaska, and Guam); and those starting with K are those in the contiguous U.S.*
Table 3. Locations and Distance of Stations with 1-min data (Part II). Shown here are the names, coordinates and distances

| No. | Name | Longitude | Latitude | Distance from Tonga Volcano (km) |
|-----|------|-----------|----------|---------------------------------|
| 92  | KMIA | -80.2901  | 25.7954  | 11471                           |
| 93  | KMEB | -79.3659  | 34.7922  | 11822                           |
| 94  | KMBS | -84.0796  | 43.5329  | 11665                           |
| 95  | KLVM | -110.4480 | 45.6994  | 9843                            |
| 96  | KNS  | -76.2944  | 40.1224  | 12213                           |
| 97  | KLGB | -118.1519 | 33.8179  | 8557                            |
| 98  | KJST | -78.8347  | 40.3156  | 12006                           |
| 99  | KITR | -102.2854 | 39.2425  | 10079                           |
| 100 | KHZY | -80.6968  | 41.7778  | 11888                           |
| 101 | KUHF | -87.3070  | 39.4506  | 11283                           |
| 102 | KHHR | -118.3351 | 33.9229  | 8551                            |
| 103 | KFTW | -97.3624  | 32.8198  | 10179                           |
| 104 | KFOE | -95.6636  | 38.9509  | 10586                           |
| 105 | KELZ | -77.9900  | 42.1095  | 12115                           |
| 106 | KDWK | -95.5528  | 30.0618  | 10216                           |
| 107 | KDSV | -77.7133  | 42.5705  | 12147                           |
| 108 | KDEW | -117.4286 | 47.9671  | 9549                            |
| 109 | KDAW | -70.9295  | 43.2842  | 12705                           |
| 110 | KCVG | -84.6678  | 39.0488  | 11488                           |
| 111 | KCUB | -80.9952  | 33.9705  | 11654                           |
| 112 | KCTB | -112.3762 | 48.6084  | 9895                            |
| 113 | KCPS | -90.1551  | 38.5704  | 11018                           |
| 114 | KCNM | -104.2634 | 32.3374  | 9573                            |
| 115 | KCLE | -81.8547  | 41.4094  | 11785                           |
| 116 | KCHS | -80.0405  | 32.8986  | 11709                           |
| 117 | KCHA | -85.2036  | 35.0352  | 11316                           |
| 118 | KCNR | -103.0954 | 42.8376  | 10197                           |
| 119 | KCAG | -107.5217 | 40.4952  | 9750                            |
| 120 | KBZN | -111.1503 | 45.7772  | 9802                            |
| 121 | KBYG | -106.7218 | 44.3811  | 10020                           |
| 122 | KBWG | -86.4197  | 36.9645  | 11275                           |
| 123 | KBUR | -118.3587 | 34.2007  | 8567                            |
| 124 | KBTV | -73.1533  | 44.4720  | 12543                           |
| 125 | KBTM | -112.4975 | 45.9548  | 9725                            |
| 126 | KBPK | -92.4705  | 36.3689  | 10743                           |
| 127 | KBOI | -116.2229 | 43.5644  | 9332                            |
| 128 | KBLI | -122.5375 | 48.7927  | 9315                            |
| 129 | KBLF | -81.2075  | 37.2959  | 11729                           |
| 130 | KBJJ | -81.8882  | 40.8748  | 11769                           |
| 131 | KBIS | -100.7457 | 46.7727  | 10555                           |
| 132 | KBHM | -86.7523  | 33.5639  | 11131                           |
| 133 | KBFL | -119.0577 | 35.4339  | 8598                            |
| 134 | KBDE | -94.6111  | 48.7302  | 11062                           |

(Continued on next page)
The arrival time of the first signal at Site 1 was 13:37:26 UTC on 15 Jan., or 9 h, 22 min, and 41 s (or 9.378 h, Table 5) after the eruption. If the peak of the signal corresponds to the maximum eruption, given the distance between P and Site 1 of 10627 km (Table 1), the propagation speed of the wave is estimated at 314.8 m/s. This is consistent with the shockwave speed reported in previous studies (e.g., Ben-Menahem, 1975) and recent reports for the Tonga event (306–315 m/s by Harrison, 2022).

The arrival time of the second signal at Site 1 was 06:39:00 UTC on 16 January (or 16.2771 days in Jan., Table 5) and about 26.405 h after the major eruption. The distance traveled by the wave through Route 2 (Figure 1B) is approximately 29,403 km and the estimated propagation speed is estimated to be 309.3 m/s (Table 5), a value consistent with the speed of the first signal, although slightly smaller.

**Shockwave speed estimate at 189 weather stations**

The 1-min interval data (from 151 stations) are examined to identify the possible shockwave signals. We use the peak signal to determine the time of signal for the wave propagation speed computation. The 5-min interval data from the rest 38 stations have much lower resolution than the Sites 1 and 2 data and the 1-min interval data. For these 5-min interval data, we first smooth the time series using a 4-point digital finite impulse response (FIR) filter (Proakis and Manolakis, 1992), which allows a better resolution of the signals from the shockwaves.

The time series data from most of the 189 stations show at least two signals (Tables 5, 6, 7, and 8) that are consistent with the arrival of the first and second shockwave signals from the eruption through Routes 1 and 2 (Figure 1B), respectively. Figures 4 and 5 show some examples from the 1-min data, while Figure 6 shows some examples from the smoothed 5-min data. Based on the timings of these signals, and the locations of the stations, we computed the propagation speed for both signals at each station (Tables 5, 6, 7, and 8). The method is consistent for all stations: the peak time is used for the computation of the signal arrival time. For all stations, the average speed for the first signal is 309.7 ± 4.2 m/s, and that for the second signal is 309.2 ± 1.5 m/s. The second signals are consistently greater in amplitude. Many of the stations show negative changes in the second signal. The standard deviation (1.5 m/s) for the second signal of the computed speed is only about 1/3 of that of the first signal (4.2 m/s). These velocity values are consistent with the sound wave speed in the upper troposphere and stratosphere (more discussion later). These results demonstrate that it is almost certain that these signals are from eruption-induced shockwaves propagating around the globe. The vertical lines in Figures 4, 5, and 6 are the arrival times of the first two signals using the averaged propagation speed of 309.5 m/s (Table 8). Figure 7 shows the regressions of the computed arrival time and distance from the eruption site. The $R^2$ values are as high as 0.993 and 0.995 for the first and second signals, respectively.

**A global view of the arrival time**

Using the average propagation speed of 309.5 m/s for the first and second passes of the shockwaves, the global distribution of arrival times is computed assuming a spherically propagating wave for the first (Figure 8A) and second (Figure 8B) signals. The observed arrival time (Figure 9A) of the first signal among the 191 stations roughly ranges between 4.5 and 11.7 h after the eruption; while that for the second signal (Figure 9B) ranges roughly between 24.3 and 31.7 h after the eruption. All these show consistent results that the recorded fluctuations of air pressure time series data at the 191 sites are from the eruption of the Hunga Tonga-Hunga Ha’apai underwater volcano.

### Table 3. Continued

| No. | Name  | Longitude  | Latitude  | Distance from Tonga Volcano (km) |
|-----|-------|------------|-----------|---------------------------------|
| 135 | KBCE  | -112.1458  | 37.7064   | 9248                            |
| 136 | KAUW  | -89.6270   | 44.9263   | 11283                           |
| 137 | KATY  | -97.1547   | 44.9140   | 10726                           |
| 138 | KATL  | -84.4279   | 33.6367   | 11339                           |
| 139 | KASK  | -90.9190   | 46.5485   | 11243                           |
| 140 | KAST  | -123.8786  | 46.1580   | 9045                            |
| 141 | KASE  | -106.8682  | 39.2219   | 9729                            |
Table 4. Locations and Distance of Stations with 1-min data (Part III). Shown here are the names, coordinates and distances

| No. | Name | Longitude | Latitude | Distance from Tonga Volcano (km) |
|-----|------|-----------|----------|----------------------------------|
| 142 | KASD | −89.8208  | 30.3463  | 10741                            |
| 143 | KART | −76.0194  | 43.9918  | 12309                            |
| 144 | KARR | −88.4757  | 41.7719  | 11266                            |
| 145 | KARB | −83.7457  | 42.2229  | 11655                            |
| 146 | KAQM | −73.1706  | 42.6963  | 12516                            |
| 147 | KAPN | −83.5603  | 45.0781  | 11747                            |
| 148 | KAPF | −81.7756  | 26.1524  | 11340                            |
| 149 | KAPC | −122.2807 | 38.2132  | 8565                             |
| 150 | KAPA | −104.8493 | 39.5701  | 9900                             |
| 151 | KAOO | −78.3200  | 40.2964  | 12048                            |
| 152 | KAHO | −84.0271  | 40.7075  | 11590                            |
| 153 | KANJ | −84.3684  | 46.4792  | 11723                            |
| 154 | KAND | −85.8581  | 33.5882  | 11211                            |
| 155 | KANB | −85.8581  | 33.5882  | 11211                            |
| 156 | KAMW | −93.6218  | 41.9920  | 10871                            |
| 157 | KAMG | −82.5066  | 31.5361  | 11445                            |
| 158 | KAMA | −101.7059 | 35.2194  | 9928                             |
| 159 | KALW | −118.2841 | 46.0925  | 9371                             |
| 160 | KALS | −105.8679 | 37.4351  | 9709                             |
| 161 | KALO | −92.4010  | 42.5584  | 10987                            |
| 162 | KILI | −98.0269  | 27.7409  | 9893                             |
| 163 | KALB | −73.8020  | 42.7491  | 12466                            |
| 164 | KAKR | −81.4669  | 41.0375  | 11807                            |
| 165 | KAKQ | −77.0011  | 36.9872  | 12084                            |
| 166 | KAKO | −103.2220 | 40.1756  | 10054                            |
| 167 | KAKH | −81.1499  | 35.2026  | 11676                            |
| 168 | KAIL | −102.8037 | 42.0532  | 10180                            |
| 169 | KAHN | −83.3259  | 33.9486  | 11447                            |
| 170 | KAGS | −81.9645  | 33.3699  | 11550                            |
| 171 | KAGC | −79.9290  | 40.3544  | 11916                            |
| 172 | KAPF | −97.3194  | 32.9904  | 10191                            |
| 173 | KAFN | −72.0030  | 42.8051  | 12612                            |
| 174 | KAEX | −92.5486  | 31.3274  | 10535                            |
| 175 | KACY | −74.5772  | 39.4576  | 12345                            |
| 176 | KACV | −124.1085 | 40.9778  | 8646                             |
| 177 | KACT | −97.2303  | 31.6122  | 10137                            |
| 178 | KACK | −70.0599  | 41.2530  | 12752                            |
| 179 | KABY | −84.1945  | 31.5355  | 11292                            |
| 180 | KABR | −98.4224  | 45.4468  | 10658                            |
| 181 | KABQ | −106.6083 | 35.0389  | 9523                             |
| 182 | KABI | −99.6819  | 32.4113  | 9963                             |
| 183 | KABE | −75.4404  | 40.6524  | 12295                            |
| 184 | KAAT | −120.5654 | 41.4829  | 8909                             |

(Continued on next page)
Signals for the 3rd – 6th passes
 Although data recorded at Sites 1 and 2 with the shortest ensemble sampling intervals do not show any sign of the signal after the second pass, many stations did show signals for the 3rd through the 6th passes. This is clear from the 1-min data but not the 5-min data. Figure 10 shows some examples of the time series with the anticipated timings of the shockwaves and the numbers of passes indicated by the vertical lines. As we can see that there are great variabilities in the amplitude among stations but there is one thing in common, that is the signals of the shockwaves at many stations appeared multiple times as the waves traveled around the globe. The timings are quite good approximations using the averaged speed (309.5 m/s, Table 8).

DISCUSSION
Reliability of the signals
 From the analysis, the air pressure data from these 191 stations show double and/or multiple signals that are remarkably consistent with spherical shockwave propagation on a global scale. Is it possible that all these were coincidental? Assume that there is an $a$ probability for a signal to randomly match the timing of the arrival of the first or second signal at a time consistent with the arrival of a shockwave propagating around the globe. To have both signals matching the times of the two passes of the shockwaves, it would have a probability of $a^2$. For all the $n$ stations to match them by chance, the probability is

| No. | Name   | Longitude | Latitude | Distance from Tonga Volcano (km) |
|-----|--------|-----------|----------|----------------------------------|
| 185 | KAAO   | -97.2211  | 37.7476  | 10409                            |
| 186 | KAAF   | -85.0274  | 29.7275  | 11155                            |
| 187 | K79J   | -86.3922  | 31.3084  | 11085                            |
| 188 | K12N   | -74.7380  | 41.0086  | 12360                            |
| 189 | KBD3   | -96.9936  | 45.6695  | 10770                            |
| 190 | K6R6   | -102.2132 | 30.0465  | 9632                             |
| 191 | K1J0   | -85.6017  | 30.8439  | 11141                            |

Figure 2. Shockwave signals from the two routes
Time series of air pressure at Station 1 (Table 1) between 10 and 18 January 2022 UTC. The red line indicates the time (T) of the major eruption that produced the air pressure fluctuations propagated globally. The dashed green lines indicate the times with sharp fluctuations of pressure, corresponding to the first and second arrivals of the shockwave signal through Routes 1 and 2 (Figure 1), respectively.
In our case, there are a few stations that did not record data showing both signals (for the first and second pass) due to data gaps. Considering that, $n = 186$ (instead of 191), the probability of match is $p_{186} = \alpha^{372}$. Even if there was a high probability of 0.5 chance of one match, having all 186 stations match by chance for both signals for this event would have a negligible probability of $\sim 10^{-112}$. This estimate does not even consider the fact that over 70% of the stations with 1-min data show signals from the 3rd, 4th, 5th, or even 6th passes. Of course, the real probability of one match can be reasonably assumed to be much smaller than 0.5, and the combined event being a coincidence is essentially impossible.

**Propagation speed of shockwaves in the atmosphere**

Infrasound and shockwaves are acoustic or pressure waves propagating at the speed of sound in the atmosphere (Fee and Matoza, 2013). The speed of sound in an idealized gas (Salomons, 2001) is:

$$c = \sqrt{\frac{\gamma}{\rho}} = \sqrt{\gamma RT}$$

(Equation 2)

where $\rho$ is the air density, $\gamma \sim 1.4$ is the specific heat ratio, $R = 287 \text{ J/kgK}$ is the universal gas constant, and $T$ is the air temperature in Kelvin. Because the estimated shockwave propagation speed is $\sim 309 \text{ m/s}$, this corresponds to the sound speed in the upper troposphere where the air temperature is
Table 5. Arrival time of shockwaves, time of travel, and propagation speed. No. 1 & 2 are from the high-resolution stations; No. 3–40 are for those stations with data sampled at 5-min intervals

| Station No. | Arrival T1 (days) | Arrival T2 (days) | Δt1 (hr) | Δt2 (hr) | V1 (m/s) | V2 (m/s) |
|-------------|------------------|------------------|----------|----------|---------|---------|
| 1           | 15.5677          | 16.2771          | 9.378    | 26.405   | 314.8   | 309.3   |
| 2           | 15.5678          | 16.2765          | 9.387    | 26.390   | 314.5   | 309.5   |
| 3           | 15.6007          | 16.25            | 10.171   | 25.754   | 309.0   | 309.7   |
| 4           | 15.5             | 16.3576          | 7.754    | 28.337   | 305.7   | 308.7   |
| 5           | 15.5694          | 16.2778          | 9.420    | 26.421   | 313.4   | 309.1   |
| 6           | 15.6493          | 16.191           | 11.337   | 24.338   | 310.8   | 312.1   |
| 7           | 15.5104          | 16.3403          | 8.004    | 27.921   | 312.5   | 308.7   |
| 8           | 15.5243          | 16.334           | 8.337    | 27.770   | 307.5   | 308.1   |
| 9           | 15.5729          | 16.2889          | 9.504    | 26.688   | 304.4   | 308.2   |
| 10          | 15.6424          | 16.2035          | 11.172   | 24.638   | 311.6   | 310.0   |
| 11          | 15.6354          | 16.2188          | 11.004   | 25.005   | 306.8   | 309.7   |
| 12          | 15.6389          | 16.2083          | 11.088   | 24.753   | 310.7   | 310.0   |
| 13          | 15.6146          | 16.2396          | 10.505   | 25.505   | 308.2   | 309.0   |
| 14          | 15.566           | 16.3056          | 9.338    | 27.089   | 306.7   | 304.8   |
| 15          | 15.4965          | 16.3611          | 7.670    | 28.421   | 309.5   | 307.7   |
| 16          | 15.6007          | 16.2361          | 10.171   | 25.421   | 316.4   | 310.9   |
| 17          | 15.6493          | 16.2014          | 11.337   | 24.588   | 308.4   | 310.0   |
| 18          | 15.4757          | 16.3958          | 7.171    | 29.253   | 312.0   | 303.6   |
| 19          | 15.5278          | 16.3333          | 8.421    | 27.753   | 309.2   | 306.8   |
| 20          | 15.5729          | 16.2986          | 9.504    | 26.921   | 303.6   | 305.9   |
| 21          | 15.5785          | 16.2639          | 9.638    | 26.088   | 314.4   | 310.1   |
| 22          | 15.5694          | 16.2743          | 9.420    | 26.337   | 315.4   | 309.4   |
| 23          | 15.5451          | 16.3056          | 8.837    | 27.089   | 312.1   | 308.7   |
| 24          | 15.5556          | 16.2917          | 9.089    | 26.755   | 312.5   | 309.5   |
| 25          | 15.3681          | 16.4965          | 4.589    | 31.670   | 304.4   | 307.0   |
| 26          | 15.5313          | 16.3306          | 8.505    | 27.689   | 304.1   | 308.2   |
| 27          | 15.6007          | 16.2396          | 10.171   | 25.505   | 313.2   | 311.1   |
| 28          | 15.5451          | 16.3056          | 8.837    | 27.089   | 312.0   | 308.7   |
| 29          | 15.5451          | 16.3056          | 8.837    | 27.089   | 310.1   | 309.3   |
| 30          | 15.5764          | 16.2639          | 9.588    | 26.088   | 315.9   | 310.1   |
| 31          | 15.5382          | 16.3264          | 8.671    | 27.588   | 302.6   | 308.0   |
| 32          | 15.6319          | 16.2153          | 10.920   | 24.921   | 311.5   | 309.7   |
| 33          | 15.6285          | 16.2257          | 10.838   | 25.171   | 305.1   | 310.4   |
| 34          | 15.5139          | 16.3507          | 8.0878   | 28.171   | 303.0   | 307.7   |
| 35          | 15.5486          | 16.3056          | 8.921    | 27.089   | 306.0   | 309.7   |
| 36          | 15.559           | 16.3021          | 9.170    | 27.005   | 304.5   | 308.4   |
| 37          | 15.5938          | 16.25            | 10.005   | 25.754   | 306.2   | 312.8   |
| 38          | 15.6424          | 16.2118          | 11.172   | 24.837   | 304.4   | 310.8   |
| 39          | 15.6632          | 16.184           | 11.671   | 24.170   | 308.0   | 311.3   |
| 40          | 15.6458          | 16.1944          | 11.253   | 24.420   | 314.1   | 310.6   |

approximately –35°C and –40°C. The air temperature profile is not a constant but variable as a function of time, location, height, season, and in general, atmospheric dynamics. Although the actual dynamical processes and propagation of the shockwaves need to be illustrated by a proper mathematical model, it is well known that the vertical profile of sound speed in the atmosphere has a minimum around the top of the
Table 6. Arrival time of shockwaves, time of travel, and propagation speed. For stations with data sampled at 1-min intervals, Part I

| Station No. | Arrival T1 (days) | Arrival T2 (days) | Δt1 (hr) | Δt2 (hr) | V1 (m/s) | V2 (m/s) |
|-------------|------------------|------------------|---------|---------|---------|---------|
| 41          | 15.651           | 16.193           | 11.371  | 24.389  | 309.5   | 311.6   |
| 42          | 15.656           | 16.190           | 11.489  | 24.305  | 308.8   | 311.5   |
| 43          | 15.365           | 16.494           | 4.505   | 31.603  | 308.0   | 307.9   |
| 44          | 15.369           | 16.494           | 4.620   | 31.603  | 303.6   | 307.5   |
| 45          | 15.368           | 16.496           | 4.589   | 31.653  | 304.3   | 307.2   |
| 46          | 15.369           | 16.494           | 4.603   | 31.603  | 304.6   | 307.5   |
| 47          | 15.369           | 16.494           | 4.603   | 31.620  | 304.5   | 307.3   |
| 48          | 15.368           | 16.497           | 4.589   | 31.670  | 305.1   | 306.9   |
| 49          | 15.368           | 16.496           | 4.589   | 31.653  | 303.9   | 307.2   |
| 50          | 15.392           | 16.438           | 5.155   | 30.254  | 310.8   | 314.6   |
| 51          | 15.393           | –                | 5.189   | –       | 311.1   | –       |
| 52          | 15.526           | 16.337           | 8.388   | 27.837  | 307.1   | 306.9   |
| 53          | 15.534           | 16.331           | 8.570   | 27.689  | 307.3   | 306.5   |
| 54          | 15.518           | 16.342           | 8.189   | 27.955  | 310.5   | 306.8   |
| 55          | 15.530           | 16.329           | 8.472   | 27.655  | 308.2   | 307.7   |
| 56          | 15.568           | 16.301           | 9.389   | 26.988  | 305.0   | 305.9   |
| 57          | 15.540           | 16.317           | 8.705   | 27.355  | 310.7   | 307.6   |
| 58          | 15.528           | 16.336           | 8.421   | 27.821  | 313.3   | 304.9   |
| 59          | 15.543           | 16.322           | 8.789   | 27.470  | 307.5   | 306.4   |
| 60          | 15.530           | 16.334           | 8.472   | 27.770  | 307.1   | 306.7   |
| 61          | 15.526           | 16.333           | 8.388   | 27.753  | 310.5   | 306.8   |
| 62          | 15.527           | 16.320           | 8.405   | 27.437  | 309.3   | 310.5   |
| 63          | 15.527           | 16.319           | 8.405   | 27.420  | 310.5   | 310.4   |
| 64          | 15.535           | 16.324           | 8.604   | 27.537  | 308.1   | 307.5   |
| 65          | 15.521           | 16.344           | 8.253   | 28.005  | 307.0   | 306.6   |
| 66          | 15.519           | 16.340           | 8.205   | 27.921  | 310.7   | 306.9   |
| 67          | 15.537           | 16.324           | 8.637   | 27.521  | 308.3   | 307.3   |
| 68          | 15.551           | 16.319           | 8.971   | 27.403  | 302.8   | 306.6   |
| 69          | 15.568           | 16.306           | 9.389   | 27.105  | 305.1   | 304.6   |
| 70          | 15.567           | 16.286           | 9.372   | 26.621  | 308.2   | 309.2   |
| 71          | 15.581           | 16.269           | 9.705   | 26.205  | 311.1   | 309.1   |
| 72          | 15.638           | 16.211           | 11.054  | 24.821  | 309.9   | 310.0   |
| 73          | 15.554           | 16.292           | 9.055   | 26.755  | 314.2   | 309.3   |
| 74          | 15.594           | 16.246           | 10.020  | 25.653  | 317.3   | 309.5   |
| 75          | 15.557           | 16.289           | 9.120   | 26.688  | 314.6   | 309.2   |
| 76          | 15.570           | 16.274           | 9.437   | 26.337  | 315.3   | 309.2   |
| 77          | 15.562           | 16.284           | 9.237   | 26.570  | 314.3   | 309.2   |
| 78          | 15.518           | 16.339           | 8.189   | 27.888  | 308.2   | 308.2   |
| 79          | 15.590           | 16.251           | 9.921   | 25.788  | 314.7   | 310.1   |
| 80          | 15.554           | 16.291           | 9.055   | 26.738  | 314.8   | 309.3   |
| 81          | 15.518           | 16.335           | 8.189   | 27.804  | 312.1   | 308.0   |
| 82          | 15.632           | 16.218           | 10.920  | 24.989  | 309.3   | 309.8   |
| 83          | 15.605           | 16.250           | 10.272  | 25.754  | 307.5   | 309.1   |
| 84          | 15.596           | 16.254           | 10.053  | 25.855  | 304.8   | 311.6   |
| 85          | 15.583           | 16.261           | 9.737   | 26.021  | 316.1   | 309.1   |

(Continued on next page)
troposphere and lower stratosphere (Piece, 1989; Fee and Matoza, 2013). This provides an idealized environment for a waveguide or sound channel within which sound can propagate to a far distance without major dissipation.

Table 6. Continued

| Station No. | Arrival T1 (days) | Arrival T2 (days) | Δt₁ (hr) | Δt₂ (hr) | V₁ (m/s) | V₂ (m/s) |
|------------|------------------|------------------|----------|----------|----------|----------|
| 86         | 15.646           | 16.199           | 11.253   | 24.537   | 311.4    | 310.4    |
| 87         | 15.531           | 16.331           | 8.505    | 27.689   | 304.1    | 308.2    |
| 88         | 15.615           | 16.221           | 10.521   | 25.053   | 318.0    | 310.3    |
| 89         | 15.550           | 16.296           | 8.954    | 26.853   | 313.9    | 309.4    |
| 90         | 15.616           | 16.219           | 10.538   | 25.020   | 317.9    | 310.5    |
| 91         | 15.564           | 16.296           | 9.288    | 26.853   | 304.7    | 308.7    |

Table 7. Arrival time of shockwaves, time of travel, and propagation speed. For stations with data sampled at 1-min intervals, Part II

| Station No. | Arrival T1 (days) | Arrival T2 (days) | Δt₁ (hr) | Δt₂ (hr) | V₁ (m/s) | V₂ (m/s) |
|------------|------------------|------------------|----------|----------|----------|----------|
| 92         | 15.604           | 16.240           | 10.255   | 25.505   | 310.7    | 311.1    |
| 93         | 15.608           | 16.230           | 10.337   | 25.272   | 317.7    | 310.1    |
| 94         | 15.615           | 16.237           | 10.505   | 25.437   | 308.5    | 309.7    |
| 95         | 15.549           | 16.310           | 8.921    | 27.204   | 306.5    | 308.2    |
| 96         | 15.633           | 16.216           | 10.953   | 24.938   | 309.7    | 309.8    |
| 97         | 15.499           | 16.359           | 7.737    | 28.370   | 307.2    | 308.2    |
| 98         | 15.626           | 16.224           | 10.788   | 25.121   | 309.1    | 309.9    |
| 99         | 15.557           | 16.299           | 9.120    | 26.937   | 307.0    | 308.9    |
| 100        | 15.622           | 16.229           | 10.687   | 25.238   | 309.0    | 309.7    |
| 101        | 15.601           | 16.253           | 10.188   | 25.821   | 307.6    | 309.3    |
| 102        | 15.498           | 16.358           | 7.704    | 28.353   | 308.3    | 308.4    |
| 103        | 15.552           | 16.294           | 9.005    | 26.803   | 314.0    | 309.4    |
| 104        | 15.578           | 16.280           | 9.621    | 26.472   | 305.6    | 309.0    |
| 105        | 15.630           | 16.219           | 10.872   | 25.003   | 309.5    | 310.1    |
| 106        | 15.554           | 16.294           | 9.055    | 26.803   | 313.4    | 309.0    |
| 107        | 15.630           | 16.218           | 10.872   | 24.989   | 310.4    | 310.0    |
| 108        | 15.535           | 16.322           | 8.587    | 27.470   | 308.9    | 308.2    |
| 109        | 15.649           | 16.196           | 11.321   | 24.453   | 311.7    | 310.4    |
| 110        | 15.592           | 16.245           | 9.972    | 25.637   | 320.0    | 309.3    |
| 111        | 15.602           | 16.237           | 10.205   | 25.437   | 317.2    | 309.9    |
| 112        | 15.548           | 16.308           | 8.904    | 27.153   | 308.7    | 308.3    |
| 113        | 15.592           | 16.264           | 9.972    | 26.088   | 306.9    | 308.9    |
| 114        | 15.531           | 16.317           | 8.505    | 27.372   | 312.6    | 309.1    |
| 115        | 15.619           | 16.233           | 10.603   | 25.337   | 308.8    | 309.7    |
| 116        | 15.605           | 16.233           | 10.272   | 25.353   | 316.6    | 310.3    |
| 117        | 15.590           | 16.251           | 9.921    | 25.788   | 316.8    | 309.3    |
| 118        | 15.563           | 16.288           | 9.254    | 26.654   | 306.1    | 310.9    |
| 119        | 15.546           | 16.302           | 8.853    | 27.005   | 305.9    | 311.5    |
| 120        | 15.549           | 16.311           | 8.937    | 27.221   | 304.6    | 308.5    |
| 121        | 15.558           | 16.303           | 9.137    | 27.021   | 304.6    | 308.5    |
| 122        | 15.589           | 16.254           | 9.888    | 25.838   | 316.7    | 309.1    |

(Continued on next page)
Ringing after the shockwaves

In the air pressure time series data from Sites 1 and 2, it is apparent that higher-frequency ringing (oscilla-
tions) occurred immediately after the shockwave signals (Figure 3). Previous studies on great explosions

| Station No. | Arrival T1 (days) | Arrival T2 (days) | Δt₁ (hr) | Δt₂ (hr) | V₁ (m/s) | V₂ (m/s) |
|-------------|------------------|------------------|---------|---------|---------|---------|
| 123         | 15.499           | 16.358           | 7.721   | 28.353  | 308.2   | 308.2   |
| 124         | 15.644           | 16.204           | 11.220  | 24.638  | 310.5   | 309.9   |
| 125         | 15.547           | 16.315           | 8.870   | 27.305  | 304.5   | 308.3   |
| 126         | 15.572           | 16.274           | 9.470   | 26.321  | 315.1   | 309.1   |
| 127         | 15.532           | 16.330           | 8.520   | 27.672  | 304.2   | 308.2   |
| 128         | 15.525           | 16.332           | 8.354   | 27.720  | 309.7   | 307.8   |
| 129         | 15.620           | 16.235           | 10.637  | 25.404  | 306.3   | 309.5   |
| 130         | 15.617           | 16.233           | 10.572  | 25.353  | 309.2   | 309.6   |
| 131         | 15.575           | 16.280           | 9.554   | 26.472  | 306.9   | 309.3   |
| 132         | 15.584           | 16.257           | 9.770   | 25.920  | 316.5   | 309.7   |
| 133         | 15.501           | 16.352           | 7.771   | 28.205  | 307.3   | 309.6   |
| 134         | –                | 16.262           | –       | 26.037  | –       | 309.0   |
| 135         | 15.529           | 16.333           | 8.438   | 27.737  | 304.5   | 308.3   |
| 136         | 15.606           | 16.246           | 10.303  | 25.653  | 304.2   | 311.3   |
| 137         | 15.585           | 16.267           | 9.804   | 26.172  | 303.9   | 311.0   |
| 138         | 15.591           | 16.249           | 9.938   | 25.721  | 316.9   | 309.9   |
| 139         | 15.605           | 16.254           | 10.272  | 25.838  | 304.0   | 309.5   |
| 140         | 15.516           | –                | 8.138   | –       | 308.7   | –       |
| 141         | 15.544           | 16.313           | 8.820   | 27.271  | 306.4   | 308.6   |

(Continued on next page)
also indicated ringing after the arrival of the initial disturbance (e.g., Whipple, 1930, as one of the earliest examples). To examine this ringing, we performed an additional analysis of the filtered data and spectrum of the data before and after the main shockwaves. To better visualize the ringing, a band-pass filter is used on the original time series data (Figure 11): it is a Fourier filter (O’Haver, 2022) with cutoff periods of 30 s and 10 min.

Using the arrival time of the signal, the time series was divided into two parts: one before and one after the arrival of the signal (Figure 11). Indeed, it is obvious that after the arrival of the shockwaves, identifiable

Table 8. Continued

| Station No. | Arrival T1 (days) | Arrival T2 (days) | Δt1 (hr) | Δt2 (hr) | V1 (m/s) | V2 (m/s) |
|-------------|-------------------|-------------------|-----------|-----------|---------|---------|
| 158         | 15.542            | 16.304            | 8.755     | 27.038    | 315.0   | 309.3   |
| 159         | 15.529            | 16.312            | 8.455     | 27.621    | 307.9   | 308.3   |
| 160         | 15.540            | 16.314            | 8.705     | 27.288    | 309.8   | 308.7   |
| 161         | 15.592            | 16.260            | 9.955     | 26.004    | 306.6   | 310.2   |
| 162         | 15.544            | 16.307            | 8.820     | 27.120    | 311.6   | 308.7   |
| 163         | 15.641            | 16.206            | 11.138    | 24.689    | 310.9   | 310.1   |
| 164         | 15.620            | 16.233            | 10.637    | 25.337    | 308.3   | 309.4   |
| 165         | 15.616            | 16.220            | 10.538    | 25.037    | 318.5   | 310.1   |
| 166         | 15.558            | 16.300            | 9.153     | 26.954    | 305.1   | 308.9   |
| 167         | 15.602            | 16.235            | 10.205    | 25.204    | 317.8   | 310.0   |
| 168         | 15.562            | 16.290            | 9.237     | 26.705    | 306.1   | 310.5   |
| 169         | 15.594            | 16.244            | 10.020    | 25.620    | 317.3   | 309.9   |
| 170         | –                 | 16.240            | –         | 25.521    | –       | 310.0   |
| 171         | 15.624            | 16.228            | 10.721    | 25.221    | 308.8   | 309.6   |
| 172         | –                 | 16.294            | –         | 26.803    | –       | 309.2   |
| 173         | 15.645            | 16.199            | 11.237    | 24.537    | 311.8   | 310.4   |
| 174         | 15.565            | 16.281            | 9.305     | 26.489    | 314.5   | 309.3   |
| 175         | 15.638            | 16.210            | 11.071    | 24.804    | 309.7   | 310.1   |
| 176         | 15.506            | 16.356            | 7.905     | 28.289    | 303.8   | 308.2   |
| 177         | 15.551            | 16.296            | 8.971     | 26.853    | 313.9   | 309.2   |
| 178         | –                 | 16.194            | –         | 24.420    | –       | 310.3   |
| 179         | 15.591            | 16.250            | 9.938     | 25.754    | 315.6   | 310.0   |
| 180         | 15.581            | –                 | 9.689     | –         | 305.6   | –       |
| 181         | 15.529            | 16.320            | 8.455     | 27.437    | 312.9   | 308.9   |
| 182         | 15.544            | 16.302            | 8.820     | 27.005    | 313.8   | 309.3   |
| 183         | 15.636            | 16.213            | 11.021    | 24.854    | 309.9   | 310.0   |
| 184         | –                 | 16.346            | –         | 28.053    | –       | 308.2   |
| 185         | 15.566            | 16.288            | 9.338     | 26.654    | 309.6   | 308.7   |
| 186         | 15.588            | 16.255            | 9.871     | 25.872    | 313.9   | 310.0   |
| 187         | 15.584            | 16.258            | 9.770     | 25.953    | 315.2   | 309.8   |
| 188         | 15.639            | 16.210            | 11.088    | 24.804    | 309.6   | 309.9   |
| 189         | 15.585            | 16.274            | 9.787     | 26.321    | 305.7   | 308.8   |
| 190         | 15.531            | 16.313            | 8.503     | 27.271    | 314.7   | 309.6   |
| 191         | 15.586            | 16.255            | 9.821     | 25.872    | 315.1   | 310.2   |

Average: 309.7  
Standard deviation: 4.2  
Overall average: 309.5  
Standard deviation: 2.9
Oscillations appear to be greater than the background noise before the event (Figures 11B and 11D). For convenience, we converted the Fourier transform coefficients to Fourier series coefficients so that the unit is the same as the data (hPa, Figure 12). The spectrum of the air pressure before the shockwave’s arrival (blue lines in Figure 12) has a much lower magnitude than those after (red lines in Figure 12). This is true for both signals, confirming that the ringing of the shockwaves has increased energy. The ringing after the first signal appears to have a broader spectrum (Figure 12A) than that after the second (Figure 12B), indicating the dissipation of high-frequency oscillations with a longer distance for the second signal.

The ringing in the first signal has peak frequencies at \( \frac{1}{0.125}, \frac{1}{0.175}, \) and \( \frac{1}{0.217} \) cycles per minute. These frequencies correspond to periods of 8, 5.7, and 4.6 min, respectively. The ringing in the second signal has a narrower band with low frequencies and lower magnitude for all frequencies. The first few major frequency peaks are at \( \frac{1}{0.12}, \frac{1}{0.13}, \frac{1}{0.15}, \) and \( \frac{1}{0.17} \) cycle per minute. These frequencies correspond to periods 5.9 to 8.3 min. The overall spectra however are continuous. The results are consistent with those from Britain and Ireland where oscillations at 6–8 min intervals were reported by Burt (2022).

The 5-min interval data are too sparse to show the ringing. The 1-min interval data are sufficient to resolve the ringing. Among the 151 stations that provided the 1-min interval data, about 110 (~73%) of them showed clear ringing and significant contrast in oscillation energy between the data before and after the arrival of the shockwaves. The rest 27% do not show obvious differences. To gain a general view of this, we have computed the spectra for the bandpass filtered time series data for all the 151 stations for the first pass of the shockwaves with a 2-h length before and after the shockwaves, respectively. The bandpass filter is the same as the one applied to the high-resolution data from Sites 1 and 2. The averaged spectra are shown in Figure 12C, together with 1 standard deviation below and above the mean, respectively. It is
confirmed that the air pressure after the arrival of the shockwaves had significantly higher (>100%) energy oscillations compared to that before the shockwaves, for the majority of the stations. This is particularly true for periods between 4 and 8 min. The spectra, however, are continuous and the oscillations are not simple sinusoidal variations with discrete frequencies. The spectra comparisons for the second pass of the shockwaves and those for Site 2 are similar and the figures are omitted here.

Spherical waves
Spherical waves (different from the cylindrical Lamb waves, Pekeris, 1939) are those propagating on the spherical surface of the Earth in the atmosphere. The satellite images (Bachmeier, 2022) are perhaps the first visual evidence and direct observations of the catastrophic volcano eruption-induced shockwaves being remarkable spherical waves. The numerical model simulations (Amores et al., 2022) provide additional support through the dynamics framework that the shockwaves are essentially spherical in nature. The early study of the Krakatoa eruption-induced shockwaves already implied that the waves traveled around the world multiple times must have been spherical waves (Symons, 1888; Gabrielson, 2010), although there was no direct visual evidence. Observations of the Tonga eruption induced shockwaves reverberating around the Earth multiple times (e.g., Burt, 2022 and resulted presented here) are consistent with the satellite images. This means that the shockwaves, after being generated by the enormous explosion, must be bending their rays on the Earth’s surface and travel in a spherical form (rather than expanding in a cylindrical way like the Lamb wave). This can be verified by considering a linear sound wave model in the atmosphere. The small aspect ratio of the atmosphere (thin layer) and the small curvature (relatively large radius of the Earth) make the wave spherical on the Earth’s “surface” (in the atmosphere). The method section derives the wave propagation dispersion relationship which indicates that (1) the propagation speed or celerity \( U \) is not exactly the same as the speed of sound \( c \) because of the curvature of the Earth (or the radius of the Earth \( r \) being finite, not infinity); (2) the waves are non-dispersive—meaning that the wave propagation speed is not dependent on the frequency which allows the waves to travel long distance without major dissipation.
a st hem multipleg pass sof the wave sdemonstrated. Wecanse from the method section (Equation 9) that the celerity of the spherical wave is the speed of sound multiplied by a factor $\alpha$:

$$\alpha = \sqrt{1 - \frac{1}{r^2} \cot \theta} = \alpha_r + i\alpha_i$$  \hspace{1cm} (Equation 3)

where $\alpha_r$ is the real part of $\alpha$ and $\alpha_i$ is the imaginary part of $\alpha$. The actual spherical wave propagation speed is $\alpha_r$ multiplied by $c$ while $\alpha_i$ multiplied by $c$ gives the exponent for the change in the amplitude of the wave. To provide an intuition for this, we plotted the functions (Figure 13). At $1^\circ$ and $179^\circ$ polar angles, the real

Figure 6. Examples of the 5-min data showing the shockwave signals
The 5-min interval air pressure times series at 9 stations (Table 1). The raw data were treated by a 4-point or 15 min moving average filter twice (back and forth, to eliminate the phase shift). The red bars indicate the timings of the first and second arrivals of the shockwave signals. Panels A–I are for different stations.

as the multiple passes of the waves demonstrated. We can see from the method section (Equation 9) that the celerity of the spherical wave is the speed of sound multiplied by a factor $\alpha$:

$$\alpha = \sqrt{1 - \frac{1}{r^2} \cot \theta} = \alpha_r + i\alpha_i$$  \hspace{1cm} (Equation 3)

where $\alpha_r$ is the real part of $\alpha$ and $\alpha_i$ is the imaginary part of $\alpha$. The actual spherical wave propagation speed is $\alpha_r$ multiplied by $c$ while $\alpha_i$ multiplied by $c$ gives the exponent for the change in the amplitude of the wave. To provide an intuition for this, we plotted the functions (Figure 13). At $1^\circ$ and $179^\circ$ polar angles, the real

Figure 7. Shockwave speed computation
Regression of the spherical distance of the stations to the volcano and the arrival times of the first signal (A) and second signal (B).
The real part is $1.0000000000108$, a value very close to $1$ (Figure 13A). Thus, it can be seen that the real part is essentially $1$ except at two singular points: the pole and antipole (Figure 13A), which means that the shockwave propagation speed is essentially the speed of sound. The imaginary part has the physical meaning of the exponent, which influences the amplitude of the wave. The factor is

$$f = e^{i(\pi - cu_t)}e^{iat} \sim e^{i(\pi - ct)}e^{iat}$$

(Equation 4)

The average radius of the Earth $r = 6,371,000$ m, and the second term in the square root of Equation 3 is much smaller than $1$ (except at the singular points of $0$ and $180^\circ$):

$$\frac{1}{r}\cot \theta \ll 1, \quad (\theta \neq 0, \quad \theta \neq 180^\circ)$$

(Equation 5)

Using Taylor series expansion, Equation 3 gives:

$$\alpha = 1 - \frac{1}{2r}\cot \theta$$

(Equation 6)

This gives an expression of the imaginary part

$$\alpha_i = -\frac{1}{2r}\cot \theta$$

(Equation 7)

The imaginary part $\alpha_i$ is negative when the azimuthal angle is less than $90^\circ$ but positive when it is greater. This is consistent with the computation using (Equation 3), as shown in Figure 13B. This is expected because when the azimuthal angle is less than $90^\circ$, the radius of the wave increases and the amplitude of the wave should decrease even without dissipation, whereas after passing the $90^\circ$ azimuthal angle, the radius starts to decrease thereby causing the amplitude to increase again under an idealized situation (no dissipation). Note that the plots in Figure 13 are obtained by specifying the azimuthal angle to be between $1^\circ$ and $179^\circ$ to avoid singular points at the pole ($0^\circ$) and antipole ($180^\circ$). In mathematics, an impulse signal from a point source is a delta function that is infinite in magnitude at that point, but an integration over an arbitrarily small region encompassing the point gives a finite value (Bracewell, 2000).

Even though our analysis shows that the shockwave propagation speed is consistent with the speed of sound, the data do not show a clear amplitude variation (i.e. decreasing before the azimuthal angle.
reaching 90° and increasing after 90°). This may indicate that the actual mechanism is more complicated than the linear model if the shockwaves propagate in an atmospheric waveguide, given the minimum air temperature at the top of the troposphere and the lower stratosphere. A numerical model can be a useful tool to illustrate the mechanism better.

Conclusions

The 15 January 2022 eruption of the Hunga Tonga-Hunga Ha’apai underwater volcano generated globalscale spherical shockwaves in the atmosphere recorded by air pressure sensors at meteorological stations. The present study included stations that are more than 5000–12000 km away in the subtropical Pacific (Hawaii and Guam), Alaska, the contiguous U.S., and Puerto Rico. Most of the 191 stations recorded the first two signals with the timing consistent with the arrival of the shockwaves for the first and second passes through Routes 1 and 2. This preliminary analysis allows us to conclude that:

1. The probability of the data randomly having the two peaks at the “right time” for the two signals of shockwaves from the Tonga underwater volcano eruption through the two routes at 191 different
stations thousands of kilometers apart is almost impossible. These signals must have been caused by the eruption on January 15, 2022.

2. The estimated shockwave propagation speed is 309.5 ± 2.9 m/s. The small standard deviations and consistency in speed for both signals indicate that the shockwaves propagated radially and symmetrically as spherical (not cylindrical or Lamb) waves at the first-order approximation.

3. More than 70% of the stations with 1-min interval data also recorded multiple passes of the shockwaves, at least for the third to 6th passes.

4. These wave speed values are consistent with the acoustic wave speed in the upper troposphere and lower stratosphere, although our data and analysis are not sufficient to determine more specific dynamics, which would be suitable subjects for study with numerical modeling.

5. Ringing occurred after the arrival of the major wave disturbances. These ringings had periods of 4–8 min that are resolvable by the high-resolution data and the 1-min interval data. A comparison of the spectra before and after the peak signal indicated that the ringings have continuous spectra. The spectrum of the second signal is narrower, possibly due to the longer traveling distance-related dissipation of higher frequency components.

6. The analysis of the dispersion relationship for spherical wave propagation indicates that the spherical shockwaves is non-dispersive and should propagate essentially at the same speed as sound in the air because of the small aspect ratio and Earth’s curvature. This is consistent with the findings of the data analysis.

Limitation of the study
The dispersion equation of the simple model predicts a change in amplitude: before the azimuthal angle reaches 90°, the amplitude should decrease, and it should increase after 90°. However, this has not been confirmed, which might be because of the noise in the data but also could be a result of complex dynamics not resolvable by the linear model. This may require more high-resolution data to better quantify the magnitude of the signals at different sites, or numerical modeling of the shockwaves.

ETHICS STATEMENT
This study does not involve animal or human.
FUNDING STATEMENT
This research is not funded.

STAR METHODS
Detailed methods are provided in the online version of this paper and include the following:

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Figure 11. The post-arrival ringing of signals at Site 1
(A) the first signal - before and after its arrival; the blue and red lines are low-pass filtered versions of the raw data with 30-s and 10-min cutoff periods, respectively; (B) is the band-pass filtered results using the Fourier filter (O’Haver, 2022) for periods between 30 s and 10 min, showing the ringing after the arrival of the wave; (C) and (D) are similar, except that they correspond to the second signal period. Data from Site 2 are similar (figures omitted).
Figure 12. Spectrum comparison to show the shockwave ringing
The blue lines are the spectra before the arrival of the shockwaves while the red lines are the spectra after the arrival of the shockwaves: (A) for the first signal for Site 1 (Table 1); (B) for the second signal for Site 1; (C) averaged spectrum for all 151 stations with 1-min interval data. The dashed blue lines show 1 standard deviation above and below the mean before the arrival of the first shockwaves; while the dashed red lines show 1 standard deviation above and below the mean after the arrival of the shockwaves. Results for Site 2 are similar.

Figure 13. Spherical wave dispersion relation
The real and imaginary parts of the factor $a$ in the dispersion relationship: (A) is the real part, which is essentially 1, except when it is at the singular points 0 or 180°; and (B) is the imaginary part as functions of the azimuthal angle $\theta$. 
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AUTHOR CONTRIBUTIONS
The author conceived the study, collected the high-resolution data, analyzed all data, conducted the study, and wrote the article.

DECLARATION OF INTERESTS
The author declares no competing interests.

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REFERENCES
Amores, A., Monserrat, S., Marcos, M., Arqueso, D., Villalongo, J., Jordà, G., and Bosch, D.G. (2022). Numerical simulation of atmospheric lamb waves generated by the 2022 Hunga-Tonga volcanic eruption. Geophys. Res. Lett. 49, e2022GL098240. https://doi.org/10.1029/2022GL098240.

Arfken, G.B., Weber, H.J., and Harris, F.E. (2013). Mathematical Methods for Physicists - A Comprehensive Guide, 7th edition (Elsevier), p. 1205.

Bachmeier, S. (2022). Explosive eruption of the Hunga Tonga volcano. https://cimss.ssec.wisc.edu/satellite-blog/archives/44252.

Ben-Menahem, A. (1975). Source parameters of the siberian explosion of June 30, 1908, from analysis and synthesis of seismic signals at four stations. Phys. Earth Planet. Inter. 11, 1–35.

Bevington, P.R., and Robinson, D.K. (2003). Data Reduction and Error Analysis for the Physical Sciences, 3rd edition (McGraw-Hill), p. 320.

Bosch (2021). BME280 - combined humidity and pressure sensor. In BME28- Data Sheet, p. 60. Document number BST-BME280-D5001-22.

Bosch, R.N. (2000). The Fourier Transform and its Applications, 3rd edition (McGraw Hill), p. 616.

Bronshtein, I.N., Semendiyayev, K.A., Musiol, G., and Muehlig, H. (2015). Handbook of Mathematics, Sixth edition (Springer), p. 1207.

Burt, S. (2022). Multiple airwaves crossing Britain and Ireland following the eruption of Hunga Tonga-Hunga Ha’apai on 15 January 2022. Weather 77, 76–81.

Clarke, A.B., Voight, B., Neri, A., and Macedonio, G. (2002). Transient dynamics of volcanic explosions and column collapse. Nature 415, 897–901.

Dabrowsa, A.L., Green, D., Rust, A., and Phillips, J. (2011). A global study of volcanic infrasound characteristics and the potential for long-range monitoring. Earth Planet Sci. Lett. 310, 369–379.

Fee, D., and Mataza, R.S. (2013). An overview of volcano infrasound: from Hawaiian to plinian, local to global. J. Volcanol. Geotherm. Res. 249, 123–139.

Feynman, R.P., Leighton, R.B., and Sands, M. (2010). The Feynman Lectures on Physics, II, New Millennium Edition (Basic Books), p. pp1096.

Gabrielson, T.B. (2010). Krakatoa and the royal society: the Krakatoa explosion of 1883. Acoust. Today 6, 14–19.

Gorshkov, G.S. (1960). Determination of the explosion energy in some volcanoes according to barograms. Bull. Volcanol. 23, 141–144.

Harrison, G., 2022. Pressure anomalies from the January 2022 Hunga Tonga-Hunga Ha’apai eruption. Weather, 77, 87-90.Lamb, H. (1932) The Earth in History. VOL. I. Cambridge University Press, Cambridge.

Harrison, G., 2022: Pressure anomalies from the January 2022 Hunga Tonga-Hunga Ha’apai eruption. Weather, 77, 87-90. Lamb, H. (1932) The Earth in History. VOL. I. Cambridge University Press, Cambridge.

M. (1979). Transient dynamics of vulcanian explosions and column collapse. Nature 415, 897–901.

Nairn, I.A. (1976). Atmospheric shock waves and condensation clouds from Ngauruhoe explosive eruptions. Science 259, 1290–1293.

Perttu, A., Taisne, B., De Angelis, S., Assink, J.D., Taillepied, D., and Williams, R.A. (2020). Estimates of plume height from infrasound for regional volcano monitoring. J. Volcanol. Geotherm. Res. 402, 106977.

Pichon, A.L., Blanc, E., and Hauhocene, A. (2019). Infrasound Monitoring for Atmospheric Studies - Challenges in Middle Atmosphere Dynamics and Societal Benefits, 2nd edition (Springer), p. 1167.

Pierce, A.D. (1989). Acoustics: An Introduction to its Physical Principles and Applications, 3rd Edition (ASA Press), p. 678.

Proakis, J.G., and Manolakis, D.G. (1992). Digital Signal Processing - Principles, Algorithms, and Applications, 2nd Edition (Macmillan Publishing), p. pp969.

Saito, T., and Takayama, K. (2005). Applying shock-wave research to volcanology. Comput. Sci. Eng. 7, 30–35.

Salomons, E.M. (2001). Computational Atmospheric Acoustics (Springer Science+Business Media), p. 335.

Self, S., Wilson, L., and Nairn, I.A. (1979). Vulcanian eruption mechanisms. Nature 277, 440–443.

Shani-Kodmiel, S., Averbuch, G., Smets, P., Assink, J., and Evers, L. (2021). The 2010 Haiti earthquake revisited: an acoustic intensity map from remote atmospheric infrasound observations. Earth Planet Sci. Lett. 560, 116795.

Smart, D. (2022). The first hour of the paroxysmal phase of the 2022 Hunga Tonga-Hunga Ha’apai volcanic eruption as seen by a geostationary meteorological satellite. Weather 77, 81–82.
Smith, C.M., Thompson, G., Reader, S., Behnke, S.A., McNutt, S.R., Thomas, R., and Edens, H. (2020). Examining the statistical relationships between volcanic seismic, infrasound, and electrical signals: a case study of Sakurajima volcano, 2015. J. Volcanol. Geotherm. Res. 402, 106996.

Smith, R., and Kilburn, C.R.J. (2010). Forecasting eruptions after long repose intervals from accelerating rates of rock fracture: the June 1991 eruption of Mount Pinatubo, Philippines. J. Volcanol. Geotherm. Res. 191, 129–136.

Stone, M., and Goldbart, P. (2009). Mathematics for Physics - A Guided Tour for Graduate Students (Cambridge University Press), p. 806.

G.J. Symons, ed. (1888). The eruption of Krakatoa and subsequent phenomena. Report of the Krakatoa committee of the Royal Society (Royal Society), p. 494.

Terry, J.P., Goff, J., Winspear, N., Bongolan, V.P., and Fisher, S. (2022). Tonga volcanic eruption and tsunami, January 2022: globally the most significant opportunity to observe an explosive and tsunamigenic submarine eruption since AD 1883 Krakatau. Geosci. Lett. 9, 24. https://doi.org/10.1186/s40562-022-00232-z.

Turcotte, D.L., Ockendon, H., Ockendon, J.R., and Cowley, S.J. (1990). A mathematical model of vulcanian eruptions. Geophys. J. Int. 103, 211–217.

Vermeille, H. (2002). Direct transformation from geocentric coordinates to geodetic coordinates. J. Geodes. 76, 451–454.

Whipple, F.J.W. (1930). The great Siberian meteor and the waves, seismic and aerial, which it produced. Q. J. R. Meteorol. Soc. 60, 505–522.

Woods, A.W., and Bower, S.M. (1995). The decompression of volcanic jets in a crater during explosive volcanic eruptions. Earth Planet Sci. Lett. 131, 189–205.

World Bank, T. (2022). The January 15, 2022 Hunga Tonga-Hunga Ha’apai Eruption and Tsunami, Tonga, Global Rapid Post Disaster Damage Estimation (Grade) Report (The World Bank), p. 41.

Wouff, G., and McGetchin, T.R. (1958). Acoustic noise from volcanoes: theory and experiment. Geophys. J. Int. 1, 601–616.

Yokoo, A., Ichihara, M., Goto, A., and Taniguchi, H. (2006). Atmospheric pressure waves in the field of volcanology. Shock Waves 15, 295–300.

Yokoo, A., and Ishihara, K. (2007). Analysis of pressure waves observed in Sakurajima eruption movies. Earth Planets Space 59, 177–181.

Yokoyama, J. (1981). A geophysical interpretation of the 1883 Krakatoa eruption. J. Volcanol. Geotherm. Res. 9, 359–378.

Yuen, D.A., Scruggs, M.A., Spera, F.J., Zheng, Y., Hu, H., McNutt, S.R., Thompson, G., Mandli, K., Keller, B.R., Wei, S.S., et al. (2022). Under the surface: pressure-induced planetary-scale waves, volcanic lightning, and gaseous clouds caused by the submarine eruption of Hunga Tonga-Hunga Ha’apai volcano. Earthq. Res. Adv. 2, 100134.
STAR★METHODS

KEY RESOURCES TABLE

| REAGENT or RESOURCE | SOURCE | IDENTIFIER |
|---------------------|--------|------------|
| Deposited data      |        |            |
| High-resolution air pressure data | This paper. | Li, C. (2022a). High-Resolution Air Pressure Measured from Ground Stations. https://doi.org/10.31390/oceanography_coastal_wavcis.02 |
| Low-resolution air pressure data | NOAA | N/A |
| Software and algorithms |        |            |
| MATLAB              | MathWorks | https://www.mathworks.com |
| TECPLLOT            | Tecplot | https://www.tecplot.com |
| Filtering & spectrum | Li (2022b) | https://doi.org/10.1017/9781108697101 |
| Analysis code       | This paper | N/A |

RESOURCE AVAILABILITY

Lead contact
Information and requests for resources should be directed to and will be fulfilled by the Lead Contact, Chunyan Li (cli@lsu.edu).

Materials availability
N/A.

Data and code availability
- The original data of the high-resolution air pressure measured by the author using his lab-made sensor package is available at the DOI listed in the key resources table.
- The low-resolution data from weather stations are open source from NOAA and the author does not re-post third party’s data but can provide them individually if requested.
- Any additional information about the paper and MATLAB scripts for the analysis can be available from the lead contact upon request.

METHOD DETAILS

Derivation of dispersion relationship of large scale shockwaves in the atmosphere
The linearized acoustic wave equation is written as (Pierce, 1989),
\[ \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \]
where \( p \) is the air pressure, and \( c \) is the speed of sound, which is determined by
\[ c^2 = \frac{\partial p}{\partial \rho} \]
where \( \rho \) is the air density. Since the geostationary satellites showed spherical waves on the Earth’s surface (Bachmeier, 2022; World Bank, 2022), we can use the spherical polar coordinate system to express the Laplacian (\( \nabla^2 \)). This leads to (Arfken et al., 2013),
\[ \nabla^2 p = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial p}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 p}{\partial \varphi^2} \]
Here, \( r \) is the radial distance from the center of Earth to the point of wave disturbance. The variable \( \theta \) is the polar angle with the location of eruption (\( P \) in Figures 1 and 8) as the pole and \( \varphi \) the azimuthal angle. For a problem with global-scale shockwave propagation, the aspect ratio of the motion is small, that is, the
thickness of the atmosphere within which the waves propagate is much smaller than the lateral scale over which the wave can propagate on the Earth’s surface. If the waves can reach a height of 20–40 km, half of the perimeter of the Earth is about 20 thousand km, so the aspect ratio is ~1/500–1/1000. With this small aspect ratio, the waves propagate in a thin layer on the surface of Earth, and the change in \( r \) is negligible at the first-order approximation. Likewise, the waves expand radially and symmetrically outward with the location of the eruption as the pole (or center); thus, the dependence on the azimuthal angle can be neglected at the first-order approximation. Based on these assumptions, we have

\[
\frac{\partial}{\partial r} = 0, \quad \frac{\partial}{\partial \theta} = 0
\]

Therefore, we have the simplified equations,

\[
\nabla^2 p = \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial p}{\partial \theta} \right)
\]

(Equation 8)

and

\[
\frac{\partial^2 p}{\partial \xi^2} - \frac{c^2}{r^2} \left( \frac{\partial^2 p}{\partial \theta^2} + \cot \theta \frac{\partial p}{\partial \theta} \right) = 0
\]

Equation 8 has singular points at \( \theta = 0^\circ \) and \( \theta = 180^\circ \) (Stone and Goldbart, 2009). This is typical for problems with a point source, such as the electric field induced by an electric particle (Feynman et al., 2010) at an idealized (geometric) point. Considering the spherical waves propagating on the “surface” of the Earth (presumably in the upper troposphere and perhaps also including the lower stratosphere), the wave would be dependent on the variable

\[
\xi = r \theta - Ut
\]

where \( U \) is the celerity (or phase speed of the shockwaves). With the above variable, we can obtain the dispersion relationship of the waves as

\[
U^2 = c^2 - i \frac{c^2}{r} \cot \theta
\]

or

\[
U = c \sqrt{1 - i \frac{1}{r} \cot \theta} = ca
\]

(Equation 9)

Here \( i = \sqrt{-1} \). The above equation is the dispersion relationship for the large scale shockwaves propagating in the atmosphere as spherical waves.

**Instrument and data**

The sensors used in this study were the Bosch Sensortec BME280 digital humidity, pressure, and temperature sensors (Bosch, 2021) for measurements in the air. The pressure sensors had a range of 300hPa to 1100hPa with an RMS noise of 0.2 Pa. The sampling frequency was set at 1 Hz. The sensor was integrated by the author with a microprocessor, a UBXL NEO-6M Global Positioning System (GPS) module, and an SD card for data recording. A total of two sensor packages, one run by an AC power supply and one powered by solar panel charged batteries, are deployed at two locations in Baton Rouge, Louisiana, U.S.A. The two sites are separated by 9.6 km. The first package was deployed at the Ridge station or Site 1 (Table 1, 91.0912\(^\circ\) W, 30.3695\(^\circ\) N); while the second was deployed at the Russell station or Site 2 (Table 1), 9.6 km northwest of the first at (91.1795\(^\circ\) W, 30.4116\(^\circ\) N). The sensors were inside a ventilated weather-shielded box filled with replaceable desiccant and deployed with a free connection to air. The data (Li, 2022a) were validated by air pressure measurements from the closest ASOS weather station at the Baton Rouge airport ~ about 14.2 km north of Site 2 and 19.5 km north-northwest of Site 1.

The first dataset from the Ridge station (Site 1) has 3 s ensemble recording intervals. The sensor has been collecting air-pressure data for more than 10 years. The second dataset from the Russell stations (Site 2) has 21 s ensemble recording intervals. This sensor has been collecting data for 8 years. In this study, the data in January 2022 are used.
In addition to these data, we also used ASOS data from 151 weather stations recording data at 1 min intervals and 38 weather stations (Tables 1, 2, 3, and 4) recording data at 5 min intervals throughout the contiguous U.S., Alaska, Hawaii, tropical Pacific (Guam and Saipan Islands), and tropical Atlantic Oceans (Puerto Rico). There are comparable data around the world, e.g., data from 40 stations in Britain and Ireland (Burt, 2022) were used in analysing the shockwave generated by the volcano eruption of the Hunga Tonga-Hunga Ha’apai.

The time series of high-resolution air pressure data from Sites 1 and 2 are first QA/QCed by excluding invalid data (less than 0.5% of the total data points) followed by interpolation to fill the data points where invalid data are excluded.

Distance and propagation speed estimate

Based on observations from the geostationary satellite GEOS-17 (World Bank, 2022), shockwaves were generated following the eruption on 15 January 2022 and the waves propagated around the globe in the form of (surficial) spherical waves (SW). Conceptually, if we assume that the Earth is a perfect sphere, the SWs should radiate outward with the volcano as a pole on the globe (P in Figure 1). The radius of the SWs increases to 90° from point P (Figure 1B). As the SWs propagate toward the antipole (point A in Figure 1B), the radius decreases until it reaches the antipole of the volcano, after which they propagate back as return SWs with the original antipole (point A in Figure 1B) of the volcano position becoming the pole and point P the antipole for the return waves. Since the volcano is at (20.55°S, 175.385°W), it is situated near the International Date Line and the antipole is at (20.55°N, 4.615°E), which is in southern Algeria. Because of the spherical propagation of the waves, the analysis must calculate the distance between the source location at P and the air pressure observation stations.

Given a location, the spherical shockwaves have two possible routes (Figure 1) on the great circle determined by the location of the source P and location of signal reception S (Station): the first is the shortest spherical arc \( L_1 = PS \) while the second is the longer arc going through the antipole \( A \), \( L_2 = PAS \). If the shockwave is strong enough to allow multiple passes around the world, and if we denote the circumference of the Earth as \( L \), the distances of passes are: first pass: \( L_1 \); second pass: \( L_2 \); third pass: \( L + L_1 \); fourth pass: \( L + L_2 \); fifth pass: \( 2L + L_1 \); sixth pass: \( 2L + L_2 \). In general, the formula is

\[ L = (k - 1)L + \sum_{i=1}^{k} L_i \quad (k = 1, 2, ...) \]

Using the timing of the signal, we can estimate the wave propagation speed to be \( v = \frac{L}{T} \) For example, for the first pass:

\[ v_1 = \frac{L_1}{T_1 - T} \]

where, \( T_1 \) is the time of the first arrival of the signal and \( T \) is the time of the eruption. Likewise, we can estimate the wave propagation speed for the second pass:

\[ v_2 = \frac{L_2}{T_2 - T} \]

in which \( T_2 \) is the time of the second arrival of the shockwaves. The speed values \( v_1 \) and \( v_2 \) for these two passes should be consistent (about the same). At this point, it should be noted that it is a little tricky to choose the exact start time of the eruption for the computation of the shockwave propagation speed. There are several different times reported as discussed earlier, ranging from 0402 UTC to 0415 UTC. In a numerical model study (Amores et al., 2022), the eruption time was defined at 0430 UTC. There were even several smaller eruptions starting from Dec. 20, 2021, which did not seem to have caused any global scale shockwaves. Satellite images (Bachmeier, 2022) suggest that the start time of the catastrophic eruption was before 0410 UTC on 15 January but reached maximum around 0415 UTC and lasted for about 1 h (World Bank, 2022). To allow a globally propagating wave, the wave energy must have exceeded certain threshold. Although we do not know what the threshold value is, in the computation, we reasonably assume that the peak arrival of the shockwave corresponds to the peak eruption. Therefore, we use the reported time of the maximum explosion (World Bank, 2022) as the start time \( T \) in the above equations. This, however, may still have some uncertainties. An error estimate with some sensitivity computation is discussed in the last subsection of the method section.
Spherical distance
The distance between Hunga Tonga-Hunga Ha’apai underwater volcano and a given weather station is computed assuming that the Earth is a perfect sphere. Since the Earth is close to an ellipsoid, the error introduced using this assumption is approximately 0.3% for distance computation (e.g., Vermeille, 2002). With this in mind, and given that the longitude and latitude of two points (P₁ and P₂) on the surface of Earth are

\[ P_1 = (\lambda_1, \varphi_1), \quad P_2 = (\lambda_2, \varphi_2) \]

where \( \lambda_1 \) and \( \lambda_2 \) are the longitudes and \( \varphi_1 \) and \( \varphi_2 \) are the latitudes of the two points, respectively. Spherical trigonometry (Bronshtein et al., 2015) gives the following cosine equation for the shortest distance on a sphere between two points:

\[ \cos a = \sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos (\lambda_2 - \lambda_1) \]

Here, \( a \) is the arc of the great circle on Earth’s surface between the two points. This leads to

\[ a = \arccos(\sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos (\lambda_2 - \lambda_1)) \]

Given that Earth’s average radius \( r = 6371 \) km, the distance between the two points on the sphere is

\[ L_1 = \frac{P_1P_2}{r} = ra \]

The Hunga Tonga-Hunga Ha’apai underwater volcano is located at (20.55° S, 175.385° W). The distance between the volcano and the first (second) sensor location or Site 1 (Site 2) is 10,627 (10,621) km (Table 1). The distances between the volcano and the additional 189 weather stations from the contiguous U.S., Alaska, Hawaii, tropical Pacific (Guam and Saipan Islands), and tropical Atlantic Oceans (Puerto Rico) are also computed in this way (Tables 1, 2, 3, and 4), with the closest station being that in the central Pacific (Hawaii, tropical Pacific (Guam and Saipan Islands), and tropical Atlantic Oceans (Puerto Rico) are also computed in this way (Tables 1, 2, 3, and 4), with the closest station being that in the central Pacific (Hawaii, tropical Pacific (Guam and Saipan Islands), and tropical Atlantic Oceans (Puerto Rico) are also computed in this way (Tables 1, 2, 3, and 4), with the closest station being that in the central Pacific (Hawaii, tropical Pacific (Guam and Saipan Islands), and tropical Atlantic Oceans (Puerto Rico) are also computed in this way (Tables 1, 2, 3, and 4), with the closest station being that in the central Pacific (Hawaii, tropical Pacific (Guam and Saipan Islands), 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(Hawaii, tropical Pacific (Guam and Saipan Islands), and tropical Atlantic Oceans (Puerto Rico) are also comput...
signal indicates that the assumed eruption time is probably too early (so that the first signal appears to have arrived with a “slower” speed while the second signal with a “faster” speed). If the eruption time is selected at 0410 UTC, the speed is $306.0 \pm 4.5$ and $307.9 \pm 1.4$ m/s for the first two signals. The difference of the two speeds is $\approx 1.9$ m/s, which is between 4 m/s and 0.5 m/s. The conclusion is: the use of the reported maximum eruption time (0414 UTC) gives the smallest difference (0.5 m/s) and is a reasonable choice.