Vacuum Structure of Softly Broken $N = 1$ Supersymmetric QCD

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Abstract

We study softly broken $N = 1$ supersymmetric QCD with the gauge group $SU(N_c)$ and $N_f$ flavors of quark pairs. We investigate vacuum structure of the theory with generic soft supersymmetry breaking terms. Trilinear soft breaking terms play an essential role in determining vacua. For $N_f = N_c + 1$, chiral symmetry is broken for a sufficiently large magnitude of trilinear couplings, while it is unbroken in the case with only soft masses. In the case where appearance of trilinear coupling terms is allowed, i.e. for $N_f \geq N_c + 1$, we have two possible vacua, the trivial and non-trivial ones. Otherwise, we only have the non-trivial vacuum, which corresponds to the non-trivial vacuum in the $N_f \geq N_c + 1$ theory.
1 Introduction

Recently, nonperturbative aspects of \( N = 1 \) supersymmetric (SUSY) QCD have been understood \cite{1, 2}. It is very important to extend such analyses to non-SUSY QCD and study its nonperturbative aspects, confinement and chiral symmetry breaking. To this end, it is an interesting trial to study softly broken \( N = 1 \) SUSY QCD. Actually in Refs.\cite{3, 4, 5}, \( N = 1 \) SUSY QCD with soft scalar masses as well as gaugino masses has been discussed and interesting results have been obtained. Also softly broken \( N = 2 \) SUSY QCD has been studied in Ref.\cite{6}. In addition, non-supersymmetric QCD is recently discussed from the viewpoint of brane dynamics \cite{7}.

In particular, the vacuum structure of \( N = 1 \) SUSY QCD broken by adding soft masses has been clarified for the theory with \( SU(N_c) \) gauge group and \( N_f \) flavors of quark pairs in Ref.\cite{3}. For \( N_f < N_c \), there is a nontrivial stable vacuum, while there is no vacuum in the SUSY limit \cite{1}. For \( N_f = N_c \) we can have two nontrivial vacua and there is no trivial vacuum as in the SUSY limit. In one vacuum, only the meson fields \( T \) develop their expectation values (VEVs) and in the other only the baryon fields \( B \) and \( \bar{B} \) develop their VEVs. Which vacuum is realized depends on the soft mass ratio between \( T \) and \( B \ (\bar{B}) \). In both vacua chiral symmetry is broken and this situation is the same as the SUSY limit, where we have chiral symmetry breaking as well as confinement. On the other hand, for \( N_f = N_c + 1 \) we have only the trivial vacuum and chiral symmetry is not broken, while in the SUSY limit we have confinement without chiral symmetry breaking, i.e. s-confinement. For \( N_f > N_c + 1 \), we have only the trivial vacuum and the presence of the Seiberg duality is suggested even in SUSY QCD broken by soft mass terms. Furthermore, an attempt to relate soft masses between dual theories has been done in Ref.\cite{8}.

For our purpose it is useful to add all the allowed soft SUSY breaking terms. Because we do not know which region of the whole soft SUSY breaking parameter space corresponds to the real non-SUSY QCD. However, we expect that generic study on softly broken \( N = 1 \) SUSY QCD, i.e. with generic soft SUSY breaking terms, can present the real non-SUSY QCD. For that purpose, in Ref.\cite{9} softly broken SUSY QCD with generic soft SUSY breaking terms has been discussed for \( N_f > N_c + 1 \). It has been shown that trilinear soft SUSY breaking terms are important to determine a vacuum in the dual theory and we have found the nontrivial vacuum for a sufficiently
large magnitude of the trilinear coupling. Furthermore, the presence of duality between non-SUSY theories has been suggested even in the broken phase.

In this paper, we extend such analyses to the case \( N_f \leq N_c + 1 \), which might correspond more to a real world with the \( SU(3) \) colour symmetry below the GeV scale when studying the confinement region with an effective \( N_f \) without heavy quarks. Moreover, in this case we obtain confinement in the SUSY limit. We investigate the vacuum structure of softly broken SUSY QCD with generic soft SUSY breaking terms. Further, we study relations among non-trivial vacua corresponding to different flavour numbers.

This paper is organized as follows. In the next section, we review the vacuum structure of softly broken \( N = 1 \) SQCD for \( N_f > N_c + 1 \). Also we give a brief review on deformation of the flavour number in the SUSY limit. In section 3, we study the vacuum structure of the case with \( N_f = N_c + 1 \) and show that the nontrivial vacuum leading to chiral symmetry breaking can be realized beside the trivial vacuum with chiral symmetry. In section 4, we study the vacuum structure of the case with \( N_f = N_c \). We show the vacuum structure is the same as the case with only soft masses added, that is, we find only the nontrivial vacuum. We consider the nontrivial vacuum for \( N_f \leq N_c \) relating it to the vacuum structure for \( N_f = N_c + 1 \). Section 5 is devoted to conclusions and discussions.

\section{\( N_f > N_c + 1 \)}

At first we give a brief review of softly broken \( N = 1 \) supersymmetric QCD for \( N_f > N_c + 1 \). We concentrate on the case with \( N_c \geq 3 \). We consider the \( N = 1 \) supersymmetric QCD with the gauge symmetry \( SU(N_c) \) and \( N_f \) flavours of quark supermultiplets, \( \hat{Q}^i \) and \( \hat{\bar{Q}} \). This theory has the flavour symmetry \( SU(N_f)_Q \times SU(N_f)_{\bar{Q}} \) and no superpotential. In the case with \( N_f > N_c + 1 \), the dual theory is described by the \( N = 1 \) SUSY theory with the gauge group \( SU(N_f - N_c) \), \( N_f \) flavours of dual quark pairs \( \hat{q}_i \) and \( \hat{\bar{q}}_i \), and \( N_f \times N_f \) singlet meson supermultiplets \( \hat{T}^i \). The dual theory has the same flavour symmetry as the electric theory and the dual theory has the superpotential,

\[ W = \hat{q}_i \hat{T}^i \hat{\bar{q}}^i. \]
In the dual theory, all the symmetries except $R$-symmetry allow the following soft SUSY breaking terms,

$$
\mathcal{L}_{SB} = -m_q^2 \text{tr}|q|^2 - m_{\tilde{q}}^2 \text{tr}|\tilde{q}|^2 - m_T^2 \text{tr}|T|^2 + (h_q T_j^i \tilde{q}_j + h.c.),
$$

(2)

where $q_i$, $\tilde{q}^i$ and $T_j^i$ denote scalar components of $\hat{q}_i$, $\hat{\tilde{q}}^i$ and $\hat{T}_j^i$, respectively. Also the gaugino mass terms are added. For the kinetic term, we assume the canonical form with normalization factors $k_q$ and $k_T$ for $q$, $\tilde{q}$ and $T$. Then we write the following scalar potential:

$$
V(q, \tilde{q}, T) = \frac{1}{k_T} \text{tr}(qq^\dagger \tilde{q}\tilde{q}^\dagger) + \frac{1}{k_q} \text{tr}(qTT^\dagger q^\dagger + \tilde{q}^\dagger T^\dagger T \tilde{q})
$$

$$
+ \frac{\tilde{g}^2}{2} (trq^\dagger \tilde{r}a q - tr\tilde{q}^\dagger \tilde{r}a \tilde{q}^\dagger)^2 + m_q^2 \text{tr}q^\dagger q + m_{\tilde{q}}^2 \text{tr}\tilde{q}\tilde{q}^\dagger
$$

$$
+ m_T^2 \text{tr}T^\dagger T - (h_q T_j^i \tilde{q}_j + h.c.),
$$

(3)

where the third term is the $D$-term and $\tilde{g}$ denotes the gauge coupling constant of the dual theory. In Ref. [4] it has been shown that the trilinear coupling terms $hqT\overline{\psi}$ play a crucial role in determining the minimum of the potential.

We assume $h$ is real. The minimum of potential can be obtained along the following diagonal direction,

$$
q = \begin{pmatrix}
q_{(1)} & 0 \\
0 & q_{(2)} \\
& \ldots \\
0 & 0 & \ldots & q_{(N_c)}
\end{pmatrix},
$$

(4)

$$
\tilde{q} = \begin{pmatrix}
\tilde{q}_{(1)} & 0 \\
0 & \tilde{q}_{(2)} \\
& \ldots \\
0 & 0 & \ldots & \tilde{q}_{(N_c)}
\end{pmatrix},
$$

(5)

$$
T = \begin{pmatrix}
T_{(1)} & 0 \\
0 & T_{(2)} \\
& \ldots \\
0 & 0 & \ldots & T_{(N_c)}
\end{pmatrix},
$$

(6)
where all the entries, $q(i)$, $\bar{q}(i)$ and $T(i)$, can be made real. Along the $D$-flat direction,

$$q(i) = \bar{q}(i) = X_i,$$

the potential is written as

$$V(X,T) = \frac{1}{kT_i} \sum_{i=1}^{N_f} [\frac{1}{kT} X_i^4 + (m_q^2 + m_{\bar{q}}^2) X_i^2 + m_T^2 T_i^2]
+ 2 \frac{k_T}{k_q} T_i^2 X_i^2 - 2 h T_i X_i^2].$$

This potential always has the nontrivial vacuum with $X_i \neq 0$ and $T_i \neq 0$ if

$$h \gg \frac{m_q^2 + m_{\bar{q}}^2}{2k_q}.$$ 

In addition, we have the nontrivial vacuum with $X_i \neq 0$ and $T_i \neq 0$ for a certain region of intermediate values of $h$, if $m_T^2/k_T$ is sufficiently large compared with $(m_q^2 + m_{\bar{q}}^2)/2k_q$. Otherwise, in particular for a sufficiently small value of $h$ we have the trivial vacuum with $T = 0$ and $q = 0$.

Next we give a brief review on deformation of the flavour number [4]. We add a mass term of one flavour, e.g. $m\tilde{Q}N_f\tilde{Q}N_f$, in the superpotential of the \(N = 1\) supersymmetric QCD, so that one flavour becomes massive and the flavour symmetry breaks into $SU(N_f - 1)Q \times SU(N_f - 1)\bar{Q}$. That corresponds to add the term $m\tilde{T}^N_{N_f}$ in the superpotential of the dual theory, i.e.

$$W = \tilde{q}T^i\tilde{q}^j + m\tilde{T}^N_{N_f}.$$ 

If we add only soft scalar masses, we have the following scalar potential,

$$V(q, \tilde{q}, T) = \frac{1}{kT} \sum_{i,j} [|q_i \tilde{q}^j + \delta_{i,j} m_i|^2 + \frac{1}{k_q} \text{tr}(T^i q^j + \tilde{q}^j T^i)]
+ \frac{g^2}{2} (\text{tr}^i \tilde{q}^j q - \text{tr}^j \tilde{q}^i q)^2 + m_q^2 \text{tr}^i q + m_{\bar{q}}^2 \text{tr} \tilde{q} \tilde{q}^i
+ m_T^2 \text{tr} T^i T.$$ 

In the SUSY limit the dual squark pair develops its VEV,

$$q^N_{N_f} = -m.$$
Therefore, the gauge symmetry and the flavour symmetry break into $SU(N_f - N_c - 1)$ and $SU(N_f - 1) \times SU(N_f - 1)$. This breaking takes place even in the case with nonvanishing SUSY breaking parameters if SUSY breaking parameters are small enough compared with the mass $m$. Hence, it is obvious that through this breaking the trilinear coupling $h$ in the $N_f$ flavour theory corresponds to one in the $(N_f - 1)$ flavour theory.

Now we consider the $N_f$ flavour theory which has the superpotential (1) and the trilinear SUSY breaking term only for the $N_f$-th flavour $h q N_f T_{N_f}^{N_f} \bar{q}^{N_f}$ as well as soft scalar mass terms. Its scalar potential is the same as eq.(3) except for replacing $q_i T_i j q_j$ by $q N_f T_{N_f}^{N_f} \bar{q}^{N_f}$. Thus, if $h$ is large enough, the gauge and flavour symmetry are broken as $SU(N_c) \rightarrow SU(N_c - 1)$ and $SU(N_f) \rightarrow SU(N_f - 1)$. Let us compare such scalar potential with the scalar potential (11). If we fix $T_{N_f}^{N_f}$, we find that the term $h q N_f T_{N_f}^{N_f} \bar{q}^{N_f}$ works effectively in a way similar to the mass term $m T_{N_f}^{N_f}$ in the scalar potential (11).

3 $N_f = N_c + 1$

For $N_f = N_c + 1$, the $N = 1$ supersymmetric QCD is described in terms of $(N_c + 1) \times (N_c + 1)$ meson fields $\hat{T}^i_j$ and $(N_c + 1)$ baryon fields $\hat{B}_i$ and $\hat{\bar{B}}^i$. They have the superpotential,

$$W = \frac{1}{\Lambda^{2N_c-1}}(\hat{B}_i T^i_j \hat{\bar{B}}^j - \det \hat{T}).$$

(13)

The flavour symmetry allows the SUSY breaking trilinear coupling

$$h' B_i T^i_j \bar{B}^j.$$  (14)

Thus we consider here the following SUSY breaking terms,

$$L_{SB} = -m_B^2 \text{tr}|B|^2 - m_{\bar{B}}^2 \text{tr}|\bar{B}|^2 - m_T^2 \text{tr}|T|^2 + (h' B_i T^i_j \bar{B}^j + h.c.).$$  (15)

The above theory can be obtained from the dual theory with the flavour number $N_f = N_c + 2$ by deforming the flavour number, i.e. by adding the mass term $m_{N_c+2} T_{N_c+2}^{N_c+2}$ into the superpotential (4). In the SUSY limit the
dual gauge symmetry $SU(2)$ is broken completely and the nonperturbative superpotential appears due to instanton contribution,

$$\frac{1}{\Lambda^{2N_c-1}} \det \hat{T}, \quad (16)$$

where $\det \hat{T}$ denotes the determinant of the $(N_c + 1) \times (N_c + 1)$ part of $\hat{T}$. We rescale $B_i = \Lambda^{N_c-1} \sqrt{m_{N_c+2}} q_i$ and $\overline{B}_i = \Lambda^{N_c-1} \sqrt{m_{N_c+2}} q_i$. Furthermore, SUSY breaking terms (2) for $N_f = N_c + 2$ correspond to those for $N_f = N_c + 1$ (15) except the $(N_c + 2)$-th flavour. In particular, the trilinear coupling term $h q_i T_i j q_j$ corresponds to $h \prime B_i T_i j \overline{B}_j$.

Here we consider the minimum of the potential. The flavour symmetry allows the possibility of the further SUSY breaking term $h \prime \overline{T} j \partial T_j$ corresponding to the nonperturbative superpotential (16). Such possibility can be included in the following discussions, as shall be shown later. We assume the canonical kinetic terms with the normalization factors $k_B$ and $k_T$ for $B$, $\overline{B}$ and $T$. We have the scalar potential,

$$V = \lambda_T^2 \sum_{i,j} |B_i \overline{B}_j| - (\det' T)^i j |^2 + \lambda_B^2 \text{tr}(\overline{B} T |^2 + |B T|^2)$$

$$+ m_B^2 \text{tr} B \overline{B} + m_B^2 \text{tr} \overline{B} B + m_T^2 \text{tr} T \overline{T} - (h' \overline{B}_j T_i j \overline{B}_j + h.c.), \quad (17)$$

where $\lambda_T = 1/(k_T \Lambda^{2N_c-1})$, $\lambda_B = 1/(k_B \Lambda^{2N_c-1})$ and $(\det' T)^i j \equiv \partial \det T/\partial T_i j$. Here we consider the case where the SUSY breaking terms are small compared with $\lambda_T$ as well as with $\lambda_B$. In Ref. [3] the potential with $h' = 0$ has been discussed. In this case we have only the trivial vacuum with $B = \overline{B} = T = 0$ and chiral symmetry is unbroken.

The minimum of the potential can be obtained along the diagonal direction (18). Here we consider the following direction,

$$B_i \overline{B}_j - (\det' T)^i j = 0. \quad (18)$$

Along this direction, the first term in the scalar potential (17) vanishes. For simplicity, we consider the case where

$$B_i = \overline{B}_i = T, \quad T_{(i)} = T, \quad \text{for all } i \text{'s}, \quad (19)$$

and $B$ and $T$ are real. In this case we have the potential,

$$\frac{V}{N_c + 1} = 2\lambda_B^2 T^{N_c+2} - 2h' T^{N_c+1} + (m_B^2 + m_T^2)T^{N_c} + m_T^2 T^2, \quad (20)$$
along the direction (18). Here the direction (18) means $T \geq 0$. If we add the SUSY breaking term $h'_T \det T$ corresponding to the nonperturbative superpotential, we have the extra term $h'_T T^{N_c+1}$. That corresponds to only the shift,

$$h' \rightarrow h' + h'_T,$$

in the scalar potential (20).

For a sufficiently large value of $h'$ the nontrivial global minimum with $T \neq 0$ can appear. As an example, Figs. 1 and 2 show the potential (20) for $N_c = 3$.

Fig.1: The scalar potential for $\lambda_B = 1$, $m_B = m_{\overline{B}} = m_T = 0.1$ and $h' = 0, 0.4$ and 0.8.
Fig. 2: The scalar potential for $\lambda_B = 1$, $h' = 0.8$ and $m = m_B = m\overline{B} = m_T = 0.1, 0.2$ and 0.3.

We have the nontrivial vacuum with nonvanishing $B$, $\overline{B}$ and $T$, i.e. the flavour symmetry breaking, if $h'$ is sufficiently large compared with the soft scalar masses. Thus, the trilinear coupling term $h'BT\overline{B}$ plays an important role in the chiral symmetry breaking for the case with $N_f = N_c + 1$ like the term $hqT\overline{q}$ for the case with $N_f > N_c + 1$. In both cases with $N_f = N_c + 1$ and $N_f > N_c + 1$, the same trilinear term appears to be important.

Here we give a comment on the case where SUSY breaking terms break the flavour symmetry $SU(N_f = N_c + 1) \rightarrow SU(N_f = N_c)$. That will be useful to understand the case with $N_f = N_c$. As an example we consider only nonvanishing trilinear coupling term $h'B_{N_c+1}T_{N_c+1}B_{N_c+1}$. In the following discussion only the soft mass terms of $B_{N_c+1}$, $\overline{B}_{N_c+1}$ and $T_{(i)}$ for $i \neq N_c + 1$ are relevant and only these soft mass terms are considered. Now we have

\[
V = \lambda_T^2 \sum_{i,j} |B_i\overline{B}_j - (\det' T)^{ij}|^2 + \lambda_B^2 \text{tr}(|BT|^2 + |BT|^2) + m_{B(N_c+1)}^2 |B_{N_c+1}|^2 + m_{\overline{B}(N_c+1)}^2 |\overline{B}_{N_c+1}|^2 + m_T^2 \sum_{i=1}^{N_c} |T|^2 - (h'B_{N_c+1}T_{N_c+1}^{N_c+1}\overline{B}_{N_c+1} + h.c.)).
\]

(22)
We consider this potential for a fixed value of $T_{N_c+1}$, which is taken to be large enough here. In addition, we take the case where SUSY breaking terms are sufficiently large. If $m_T^2 \ll m_B^2, m_T^2, \lambda_B^2$, we have the potential minimum at $B = 0$. The relevant part is expanded near $B = 0$:

$$V = \lambda_T^2 \sum_{i,j} |(\text{det}'T)_{ij}|^2 + m_T^2 \sum_{i=1}^{N_c} |T|^2 - (h_T' \text{det}T + h.c.) + \cdots. \quad (23)$$

In this case $T$ develops its nonvanishing finite vacuum expectation value, which is determined by $\lambda_T$ and $h_T'$ as well as $m_T^2$. On the other hand, if $m_T^2 \gg m_B^2, m_B^2, \lambda_B^2$, we have the potential minimum at $B \neq 0$ and $T = 0$.

## 4 $N_f \leq N_c$

In Ref.\[3\] the $N_f = N_c$ and $N_f < N_c$ supersymmetric QCD are broken softly by adding soft scalar masses. It has been shown that in both cases with $N_f = N_c$ and $N_f < N_c$ we have only the nontrivial vacua leading to the chiral symmetry breaking.

The $N_f = N_c$ supersymmetric QCD can be described in terms of the baryon pair $\hat{B}$ and $\overline{B}$ and $N_c \times N_c$ meson fields $\hat{T}^i_j$. We have the quantum constraint \[1, 2\],

$$\hat{B}\overline{B} - \text{det} \hat{T} = \Lambda^{2N_c}. \quad (24)$$

The flavour symmetry allows the SUSY breaking term $h_B B \overline{B}$, i.e. the mixing mass term of $B$ and $\overline{B}$. Thus we consider the following SUSY breaking terms here,

$$\mathcal{L}_{SB} = -m_B^2 |B|^2 - m_B^2 |\overline{B}|^2 - m_T^2 \text{tr}|T|^2 + (h_B B \overline{B} + h.c.). \quad (25)$$

Following Ref.\[3\] we consider the direction,

$$B = -\overline{B} = \frac{b}{\Lambda^{N_c}}, \quad T_{(i)} = t/\Lambda^2. \quad (26)$$

Hence, we study the minimum of the potential,

$$V = 2m_B^2 |b|^2 + m_T^2 N_c |t|^2 - (h_B^2 b^2 + h.c.), \quad (27)$$
where $m_B' = (m_B^2 + m_T^2)/\Lambda^{2N_c}$, $m_T^2 = m_T' / \Lambda^{2N_c}$ and $h_B' = h_B / \Lambda^{2N_c}$, taking into account the constraint,

$$t^{N_c} + b^2 = 1. \quad \text{(28)}$$

There are two stable points of the potential. One point corresponds to $b = 0$ and $|t| = 1$. The other corresponds to $t = 0$ and $b = \pm 1$, where $V$ takes the value $V = 2m_B'^2 - 2h_B'$ if $h_B'$ is real. Thus the effect of the SUSY breaking term $h_B' \overline{B} \overline{B}$ is the shift $2m_B'^2 \rightarrow 2m_B'^2 - 2h_B'$ in determining the minimum. Determining the global minimum among the two stable points depends on the ratio, $r \equiv (2m_B'^2 - 2h_B')/m_T'^2$. If $r$ is sufficiently large, the vacuum with $T = 1$ is realized and the flavour symmetry is broken. If $r$ is sufficiently small or negative, the vacuum with $b = 1$ is realized and the baryon number symmetry is broken. This situation is very similar to the case without the SUSY breaking term $h_B' \overline{B} \overline{B}$. Even without $h_B' \overline{B} \overline{B}$, we always have a nontrivial vacuum with $T \neq 0$ or $b \neq 0$ for the $N_f = N_c$ case with soft scalar masses $[3]$. Thus, the SUSY breaking term $h_B' \overline{B} \overline{B}$ is not so important as the term $h' B T \overline{B}$ in the $N_f = N_c + 1$ case.

Furthermore, there is the possibility of adding the SUSY breaking term $h_T \det T$. However, that just leads to the shift $h_B' \rightarrow h_B' + h_T$ under the constraint (24).

For $N_f < N_c$ the $N = 1$ supersymmetric QCD has the nonperturbative superpotential,

$$W = (N_c - N_f) \left( \frac{\Lambda^{2N_c - N_f}}{\det T} \right)^{1/(N_c - N_f)}. \quad \text{(29)}$$

This potential has no stable point for a finite value of $T$. However, if we add the soft SUSY breaking scalar mass term,

$$\mathcal{L}_{SB} = -m_T^2 \tr |T|^2, \quad \text{(30)}$$

we have a stable vacuum for a finite value of $T$ as already shown in Ref.[1]. The soft mass terms are all the SUSY breaking terms allowed by the symmetries.

Up to now, we have studied the minimum of the potential in the softly broken $N = 1$ supersymmetric QCD for different flavour numbers. It is suggestive to compare the theory with different flavour numbers. For $N_f >
\(N_c + 1\) and \(N_f = N_c + 1\), it is possible to add the trilinear SUSY breaking terms \(hqT\bar{q}\) and \(h'BT\bar{B}\). Furthermore, in this case two types of vacua are possible. One is the trivial vacuum with \(T = 0\) and the other is the nontrivial vacuum with \(T \neq 0\) as well as \(q \neq 0\) and \(B \neq 0\). Which vacuum is realized depends on the magnitude of trilinear couplings \(h\) or \(h'\). If they are large enough, the nontrivial vacuum is realized.

On the other hand, there does not appear such trilinear terms for \(N_f = N_c\) and \(N_f < N_c\). In this case we always have the nontrivial vacua, i.e. \(T \neq 0\) or \(B \neq 0\) for \(N_f = N_c\) and \(T \neq 0\) for \(N_f < N_c\).

It is interesting to go further in this comparison. For that purpose we consider deformation of softly broken \(N = 1\) supersymmetric QCD with \(N_f = N_c + 1\) into the theories with \(N_f = N_c\) and \(N_f < N_c\). Here we add the mass term \(m\bar{T}N_{c+1}\) in the superpotential of the \(N_f = N_c + 1\) theory (13),

\[
W = \frac{1}{\Lambda^{2N_c - 1}}(\bar{B}_i\bar{T}^j_i \bar{B}^j - \text{det} \bar{T}) + m\bar{T}^{N_{c+1}}. \tag{31}
\]

In the SUSY limit we obtain the constraint (24) after we integrate out \(T^{N_{c+1}}\) and take \(m = \Lambda\). Here we add only the soft scalar masses as SUSY breaking terms. Then the corresponding scalar potential is obtained as

\[
V = \lambda_T^2 \sum_{i,j=1} |B_i\bar{B}^j| - (\text{det}' T)_i^j + \delta_i^{N_c+1}\delta_j^{N_{c+1}} m_i^2 + \lambda_B^2 \text{tr}(|BT|^2 + |BT|^2) \\
+ \ m_{B(N_c+1)}^2 |B_{N_c+1}|^2 + m_{\bar{B}(N_c+1)}^2 |\bar{B}^{N_c+1}|^2 + m_T^2 \sum_{i=1}^{N_c} |T_i|^2. \tag{32}
\]

This scalar potential is very similar to eq.(22) and the term \(mB_{N_c+1}\bar{B}^{N_c+1}\) works effectively similar to the SUSY breaking trilinear term \(h'\bar{B}_{N_c+1}T^{N_{c+1}}\bar{B}^{N_{c+1}}\) in eq.(22). Thus the vacuum of the potential (22), which corresponds to the \(N_f = N_c\) flavour theory in the SUSY limit with large \(m\), corresponds to the nontrivial vacuum of the \(N_f = N_c + 1\) flavour theory. Similarly, the vacuum of the case with \(N_f < N_c\) corresponds to the nontrivial vacuum of the case with \(N_f = N_c + 1\).

5 Conclusions

We have studied the broken \(N = 1\) supersymmetric QCD with all the possible SUSY breaking terms. For \(N_f > N_c + 1\) and \(N_f = N_c + 1\), we have two types
of vacua, namely one is trivial and the other is nontrivial vacuum. Which vacuum is realized depends on the magnitude of the trilinear SUSY breaking terms. If we add only the soft mass terms, we have only the trivial vacuum and chiral symmetry is unbroken [3]. Thus, the trilinear SUSY breaking terms are very important in determining the vacuum structure.

For $N_f = N_c$ and $N_f < N_c$, we can not have the trilinear SUSY breaking term and we always have the nontrivial vacua.

Reduction of the flavour number is realized by adding a suitable trilinear SUSY breaking term. If we have still trilinear SUSY breaking terms for massless modes, i.e. for $N_f \geq N_c + 1$, we have the possibility of having the trivial vacuum only for the massless modes. Otherwise, i.e. for $N_f \leq N_c$, we have only nontrivial vacua. The vacua with $N_f = N_c$ and $N_f < N_c$ correspond to the nontrivial vacuum of the $N_f = N_c + 1$ theory with a large trilinear coupling for the ($N_c + 1$)-th flavour.

**Acknowledgments**

This work was partially supported by the Academy of Finland under Project no. 37599.

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