Fast Radio Bursts and cosmological tests

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ABSTRACT

We consider future cosmological tests based on observations of Fast Radio Bursts (FRBs). We use Illustris Simulation to realistically estimate the scatter in the dispersion measure (DM) of FRBs caused by the inhomogeneous distribution of ionized gas in the Intergalactic Medium (IGM). We find \(\sim 13\%\) scatter in DM to a source at \(z = 1\) and \(\sim 7\%\) at \(z = 3\) (one sigma). The distribution of DM is close to Gaussian. We simulate samples of FRBs and examine their applicability to simple cosmological tests. Our calculations show that using a sample of 100 FRBs and fixing cosmological model one can find the redshift and sample averaged fraction of ionized gas with \(\sim 1\%\) uncertainty. Finding the ionized fraction with \(\sim 1\%\) accuracy at few different epochs would require \(\sim 10^4\) FRBs with known redshifts. Because DM is proportional to the product of ionized fraction, baryon density and the Hubble constant it is impossible to constrain these parameters separately with FRBs. Constraints on cosmological densities are possible in a flat \(\Lambda\)CDM model but give uninterestingly low accuracy. Using FRBs with other type of data improves the constraints, but the role of FRBs is not crucial. Thus constraints on the distribution of ionized gas are probably the most promising application of FRBs which allow for “tomography” if sources redshifts are known, as opposed to measuring \(\tau_e\) or \(y\) parameters with CMB observations.

Key words: radio continuum: transients – cosmological parameters

1 INTRODUCTION

The discovery of fast radio bursts (FRBs) (Lorimer et al. 2007), large dispersion measures (DM) of some of them (Tendulkar et al. 2017), and measured redshifts (Chatterjee et al. 2017; Tendulkar et al. 2017) make them a class of phenomena which (at least in part) may be happening at cosmological distances. Their application to cosmological tests has been proposed by several authors (Zhou et al. 2014; Gao, Li & Zhang 2014; Lorimer 2016; Yu and Wang 2017), to cite few.

In a recent article (Ravi 2017) the properties of known FRBs are discussed. They seem to be a nonuniform class of objects with complicated characteristics and the extragalactic origin of DM is likely only in a part of the sources. The number of known FRBs is far too low to study their distribution on the sky or make a meaningful \(\log F – \log N\) test, which could disprove the cosmological hypothesis (compare Paczynski 1995 for the case of gamma ray bursts and Katz 2017 for FRBs). We assume, that there is a cosmological population of FRBs and that the improved instruments (e.g. Square Kilometer Array) will allow to study a large number of these events.

Locally the observed DM is proportional to the column density of free electrons along the line of sight (LOS) to the source. It is precisely measured for pulsars (see e.g. Bilous et al. 2016). It is used to find distances to the sources independently of any cosmic ladder. DM measurements are based on the fact that lower frequency radio signals travel with lower velocities in plasma, so they are delayed as compared to a higher frequency signals. The delay can be measured. In an expanding universe the contributions from free electron column density at small redshift interval \(z - z + \Delta z\) scale with the redshift factor as \(\propto 1/(1+z)\). On the other hand one can expect the density of free electrons \(n_e \propto (1+z)^3\) which makes the contributions from high redshift dominating. Thus there is no direct proportionality between DM and the proper distance to the source even in a uniform universe.

Shull & Danforth (2018) model the distribution of free electrons in space based on the observations of absorption lines in QSO spectra and the relations between the neutral and ionized fractions of the gas at different redshifts. They also estimate the redshifts of the known FRBs using a uniform flat universe model, showing that in five of twenty five considered cases \(z > 1\).

We are simulating cosmological tests based on possible future samples of FRBs with known dispersion measures and redshifts. In our approach we calculate DM along many LOS to distant sources employing the results of the Illustris Simulation (Vogelsberger et al. 2014a,b) which are publicly available (Nelson et al. 2015) and give the spatial distribution of ionized gas for a discrete set of redshifts. Cosmological simulations were used in the past by McQuinn (2014) to estimate the variance of the dispersion measure due to the inhomogeneity of the gas distribution in space and by Dolag et al. (2015), who considered various aspects of FRBs including the large scale structure influence on the derived properties of sources population.

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In the next section we describe our method to calculate $DM$ to a source at given redshift using Illustris data. In Sec. 3 we describe a possible cosmological test based on a simulated sample of a hundred FRBs with measured hosts redshifts, concentrating on possible estimates of the fraction of ionized gas in space. In Sec. 4 we use the SN Ia Union sample Kowalski et al. 2008 to investigate the role of other data on the applicability of FRBs to cosmological tests. Discussion follows in the last section.

2 CALCULATION OF THE DISPERSION MEASURE IN A NONUNIFORM UNIVERSE MODEL

2.1 The method

We are using the results of Illustris Simulation to describe the evolving ionized gas distribution in space. The Simulation provides the history of the structure formation in a cube of 75 Mpc/h edge length, including the distribution of free electrons. To save the storage space and the calculations time we use Illustris-3, which has the lowest spatial resolution.

The dispersion measure for a source at the redshift $z_s$, neglecting the contribution from our Galaxy, the source itself, and its host galaxy, can be calculated as an integral along the path from the source to the observer (compare Deng & Zhang 2014, Zhou et al. 2014, Gao, Li & Zhang 2014):

$$DM(z_s) = \int_0^{z_s} n_e(z) \frac{dl_{prop}}{dz} dz = \int_0^{z_s} \frac{c/H_0}{(1+z)^2} H(z) dz$$

(1)

where $n_e$ is the concentration of free electrons along the LOS, $dl_{prop}$ is the proper distance differential, $z$ - the redshift, $c$ - the speed of light, and $H_0 = 100$ km/s/Mpc - the Hubble constant. In an ΛCDM model:

$$h(z) = \sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{\Lambda}}$$

(2)

where $\Omega_M$ is the matter density parameter (baryons and dark matter), $\Omega_k$ - dark energy density parameter, $\Omega_k \equiv 1 - \Omega_M - \Omega_{\Lambda}$ - the curvature parameter, and $h(z) \equiv H(z)/H_0$ gives the rate of expansion as a function of $z$. $DM$ here represents its inter-galaxy medium part ($DM_{IGM}$) but we omit this subscript for compactness. In Illustris the cosmological parameters have the following values (denoted by an extra “T” in the subscripts): $H_0 = 70.4$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_M = 0.2726$, $\Omega_{\Lambda} = 0.7274$, and the baryon density parameter $\Omega_B = 0.0456$.

rm The characteristic present number density of electrons in a uniform universe model with hydrogen and helium mass fractions $X, Y$, assuming complete ionization is given as:

$$n_{e0} = \left(1 + \frac{Y}{2}\right) \Omega_B \frac{2H_0^2}{8\pi G m_H} = 2.22 \times 10^{-7}$ cm$^{-3}$

(3)

The numeric value is calculated for the Illustris model parameters with the primordial abundances $X = 0.75$ and $Y = 0.25$.

The averaged free electron concentration $rm$ in the IGM $\langle n_e(z) \rangle$ depends on $rm$ the fraction of gas mass belonging to the IGM $f_{IGM}$, on the the gas chemical composition, and the ionization state of all species. $rm$ We use a redshift dependent factor $f(z) \equiv f_{IGM}(z) f_{ion}(z)$, where $f_{ion}$ describes the effects of chemical composition and ionization state of the gas. For a primordial chemical composition and complete ionization $f_{ion} = 1$ by definition. In general the averaged electron concentration in the IGM can be expressed using $f(z)$ as:

$$\langle n_e(z) \rangle \equiv f(z) n_{e0}(1+z)^3$$

(4)

It is convenient to rewrite Eq. 1 in the form, which separates the terms related to the concentration fluctuations from terms depending on the uniform cosmological model:

$$DM(z_s) = \int_0^{z_s} n_{e0} f(z_e(z)) \frac{c/H_0}{(1+z)^2} dz$$

(5)

where $\delta_3$ is the 3D electron density fluctuation $(n_e = (n_e(z)(1 + \delta_3))$ - the only variable under the integral which depends on the spatial coordinates.

To employ the results of Illustris we imagine that our line of sight goes through several simulation cubes corresponding to different epochs (redshift ranges). The matter distribution is correlated on scales of tens of megaparsecs (Geller et al. 1987) and modeling the line of sight as a sum of sections, each belonging to a single cube these correlations into account. On the other hand there should be no strong correlations on larger scales and choosing at random photon paths through each cube guarantees that. For simplicity we consider photon paths parallel to one of the simulation cube edges, which saves programming and calculation time. (Compare Carbone et al. 2008 about shifting and rotating simulation cubes at different epochs to get better approximation of statistical uniformity and isotropy of the simulated matter distribution.)

We construct a sequence of simulation cubes on the photon path using the condition:

$$\int_{z_{i+1}}^{z_i} (1+z) \frac{dl_{prop}}{dz} dz = \int_{z_{i+1}}^{z_i} \frac{c/H_0}{h(z)} dz = 75$ Mpc/h$$

(6)

where the integral gives the comoving distance along the path. We start from $z_0 = 0$ and calculate the other $z_i$ recursively. The redshift intervals corresponding to the photon travel through a single cube are small ($z_i - z_{i+1} \ll 1 + z_i$) so we can neglect the effects of cosmic expansion/evolution and use distribution of electrons given by $\delta_3$ corresponding to the averaged redshift $z_{av}(i) \equiv (z_i + z_{i+1})/2$. Projecting the 3D distribution of the electron density fluctuations $\delta_{3}(z_{av}(i))$ along one of the simulation cube edges we get the 2D distribution of the fluctuations in surface electron density $\delta_{2}(z_{av}(i))$.

The dispersion measure to a point at the redshift $z_i$ can now be expressed as:

$$DM(z_i) = \sum_{j=1}^{3} (1 + \delta_2(z_{av}(i))) \int_{z_{i-1}}^{z_i} n_{e0} f(z_e(z)) \frac{c/H_0}{h(z)} dz$$

(7)

For other source redshifts we get $DM(z_s)$ by interpolation. The averaged dispersion measure $\langle DM(z_s) \rangle$ can be calculated by substituting $\delta_2(z_{av}, j) = 0$ into Eq. 7.

In Fig. 1 we show the redshift dependence of the dispersion measure in a model with uniform electron density and cosmological parameters corresponding to the Illustris simulation. The solid line shows the relation for averaged electron density in the IGM taken from Illustris (see the next subsection). The dashed line shows the dependence in a case, when all free electrons, regardless of their position relative to dark matter haloes, are included. Calculation with the same cosmological parameters but assuming $f_{ion} = 1$ gives the dotted line. The scatter in the dispersion measure $\sigma_{DM}$ resulting from the nonuniform electron distribution is shown in the lower panel. For $z = 1$ we get $\sigma_{DM} = 115$ pc cm$^{-3}$ about two times smaller than the result of McQuinn (2014) (his Fig. 1, lower panel) based on simulations. On the other hand our result based on the distribution of all electrons is in agreement with his value.

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2.2 Distribution of ionized gas from Illustris simulation

We have downloaded the 3D maps of the distribution of matter in the Illustris-3 simulation cube corresponding to the epochs \(z = 0, 0.5, 1, 2, 3, 4, 5, 5.5, \) and 6. The simulation gives the positions, masses, densities, and ionization states of \(\approx 455^3\) gas cells present at the beginning of calculations. Some of them become stars/wind particles during the evolution. The distribution of dark matter is given similarly as positions and masses of another \(455^3\) dark matter particles.

Haider et al. (2016) analyze large-scale mass distribution in the Illustris simulation, dividing the space into regions of voids, filaments, and haloes based on the local dark matter density. A region with \(\rho_{DM} \geq 15\rho_c\) (dark matter density fifteen times higher than critical) is treated as belonging to a halo. The haloes occupy a tiny fraction of the volume (0.16% today), but 49% of the dark matter and 23% of the baryons can be found there.

The 3D electron density fluctuations \(\delta_3\) are defined by the distribution of gas elements. Each gas cell has defined mass, mean density, and electron abundance, so their characteristic size and electron number can be calculated. The total mass density \(\rho\) at the position of a gas cell is also given. We use the criterion \(\rho \geq 15\rho_c\) to distinguish gas elements belonging to haloes. Since the dark matter is overabundant in haloes and its density dominates anyway, this criterion is close to the one used by Haider et al. (2016). We neglect gas cells belonging to haloes when studying gas distribution in the IGM. We project the electron distribution onto three mutually perpendicular walls of the simulation cube, using top hat filter for each cell. We choose 2D maps of 16 k × 16 k resolution. The pixel size (~5/h kpc) corresponds to the comoving size of the densest gas cells at \(z = 0\). (All cells belonging to the IGM have much larger sizes.) The resulting maps of projected free electron distribution give probability distributions of 2D electron density fluctuations \(f_\delta(\delta_2)\) based on \(3 \times 16384^2\) “measurements” for chosen redshifts. By definition \((\delta_3) = 0\). We find the range of fluctuations on all considered maps to be \(-0.86 \leq \delta_2 \leq 20\). We use histograms with logarithmic bins of width 0.01 dex for storing and plotting probability distributions as functions of \(-1 \leq \log(1 + \delta_2) \leq 1.5\).

Using Illustris data we find the values of \(f_{IGM}(\zeta), f_{ion}(\zeta),\) and \(f_\delta(\zeta)\) for a set of redshifts (compare the upper part of Table 2). Almost all the baryons belong to the IGM at \(z = 5\) (not shown in the Table) and then their fraction decreases monotonically to \(f_{IGM} \approx 0.84\) at \(z \approx 1\), to increase slightly at \(z = 0\) due to the gas ejection from haloes. Since \(z = 3\) IGM gas is almost completely

### Table 1. Distribution of \(DM\) [pc cm\(^{-3}\)] for \(z = 1, 2, 3, 4,\) and 5

| \(z\) | (DM) \(\sigma_{DM}\) | \(\gamma_1\) | \(\gamma_2\) |
|------|----------------|----------|----------|
| 1    | 90.5           | 0.64     | 3.79     |
| 2    | 1803.          | 0.48     | 3.44     |
| 3    | 2649.          | 0.39     | 3.30     |
| 4    | 3431.          | 0.34     | 3.23     |
| 5    | 4136.          | 0.31     | 3.19     |
ionized with $f_{\text{ion}} \geq 0.99$, so $f_\gamma$ is very close to $f_{\text{IGM}}$. At $z = 5$
$f_{\text{ion}} \approx 0.93$ due to incomplete helium ionization. The electron fraction parameter changes
only by few per cent in the range $0 \leq z \leq 5$, and we linearly interpolate its value between $z = 0$ and $z = 1$ and
2, etc where we know $f_\gamma(z)$ based on Illustris data. At $z > 5$ electron concentration given by Illustris
becomes nonphysically low and we do not exploit this range. On the other hand the results of Illustris
for $2 < z < 4$ are in good agreement with the observations of Ly-$\alpha$ forest (Vogelsberger et al. 2014a)
and there the data can be trusted. The plots of $d$ ($DM$) / $dz$ in Fig. 1 show the changes in Illustris
electron density (solid line) as compared to the model assuming full ionization of all baryonic matter (dotted).

We approximate the probability distributions at any $z$ interpolating between the distributions given by Illustris. For any $z$ $z_1 \leq z \leq z_2$ we assume

$$f_\gamma(\delta_2) = \frac{z_2 - z}{z_2 - z_1} f_\gamma(\delta_1) + \frac{z - z_1}{z_2 - z_1} f_\gamma(\delta_2)$$

where $z_1 \in \{0, 1, 2, 3, 4\}$ and $z_2 = z_1 + 1$. We check our assumption comparing $f_{\text{IGM}}(\delta_2)$ based on Illustris data with interpolated
$f_{\text{IGM}}(\delta_2) = 0.5 f_\gamma(\delta_2) + 0.5 f_\gamma(\delta_2)$. We show both distributions in
Fig. 2. We also perform Kolmogorov-Smirnov test comparing the cumulative probability distributions:

$$F_\gamma(\delta_2) = \int_{-\infty}^{\delta_2} f_\gamma(x)dx$$

$$\text{max}|F_{\text{IGM}}(\delta_2) - F_{\text{IGM}}(\delta_2)| = 0.0097 \text{ at } \delta_2 = -0.45$$

The maximal difference between cumulative probability functions is much lower than the critical value of K-S test ($\approx 1/\sqrt{N}$) for
$N = 250$ bins.

We have performed $10^9$ trial simulation calculating dispersion measure for sources at the redshifts $0 < z < 5$, which in our approach corresponds to crossing up to $75 (z \approx 5)$ simulation cubes. The dispersion measure is calculated using Eq. 5 with $\delta_2$ at each
cube chosen at random using probability distributions like these shown in Fig. 2.

The relative probability distributions of the simulated $DM$ to sources at the chosen redshifts is shown in Fig. 3. In Table 1
we give the expected $DM$ value expressed in units of pc cm$^{-3}$ and its standard deviation $\sigma_{DM}$ for chosen source redshifts. The values of higher moments of the probability distributions $y_n \equiv \langle (DM - \langle DM \rangle)^n \rangle / \sigma_{DM}^n$ show that they are not far from being Gaussian (in which case $y_1 = 0$ and $y_2 = 3$).

3 SIMULATING FRBS OBSERVATIONS AND FITTING COSMOLOGICAL PARAMETERS

We simulate observations of FRBs with known redshifts. The source redshifts are drawn at random from the distribution $f(z) \propto \exp(-z)$ (Zhou et al. 2014, Walters et al. 2018) limited to the range $0.5 \leq z \leq 4.5$. We also consider smaller redshift ranges $0.5 \leq z < 2.5$ and $0.5 \leq z < 1.5$. (The redshift distribution $f(z)$ describes gamma ray bursts (GRBs). We do not assume any physical relation between GRBs and FRBs. The difficulty in measuring redshifts for both kinds of phenomena may be similar, and FRBs may be related to stellar evolution as are GRBs. We use the distribution of observed redshifts of a known class of sources only as an example.)

Following Deng & Zhang (2014), Gao, Li & Zhang (2014) we assume, that the observations give the dispersion measure which is a sum of contributions from inter-galaxy medium, host galaxy, and our Galaxy:

$$DM_{\text{obs}} = DM_{\text{IGM}} + DM_{\text{host}} + DM_{\text{Galaxy}}$$

The contribution of our Galaxy to the dispersion measure can be estimated based on local measurements and subtracted from the observed value. As shown by simulations of Yang and Zhang (2016) the averaged value of the host galaxy contribution can be estimated based on observations of close FRBs. This result is based on the assumption that $DM_{\text{host}}$ does not significantly evolve which implies its $\propto 1/(1 + z)$ contribution to $DM_{\text{obs}}$, while for the IGM part we have roughly $DM_{\text{IGM}} \propto z$ (compare Fig. 1, Table 1). Different redshift dependencies allow fits of both parts for a large enough sample of close bursts. In the following we assume, that the average host contribution can be subtracted from the observed $DM$, giving the approximate value of the IGM part:

$$DM_{\text{IGM,obs}} = DM_{\text{obs}} - \langle DM_{\text{host}} \rangle / (1 + z) - DM_{\text{Galaxy}}$$

According to Tendulkar et al. (2017), who analyze the repeating FRB 121102 and Yang et al. (2017), who statistically investigate 21 FRBs, the value of $DM_{\text{host}}$ is high ($\sim 200$ pc cm$^{-3}$) and has large scatter. Following Yang and Zhang (2016), Walters et al. (2018) we assume $DM_{\text{host}}$ to have normal distribution. We adopt the value $\sigma_{\text{host}} = 50$ pc cm$^{-3}$ for its standard deviation. Thus our simulated $DM_{\text{host}}$, which represents its IGM part after subtracting the Galaxy and averaged host contributions, includes also an extra term which is normally distributed:

$$DM_{\text{Galaxy}} \sim DM_{\text{host}} + \mathcal{N}(0, 1)$$

$$\sigma_{\text{host}} = \sigma_{\text{host}}/(1 + z)$$

where $DM_{\text{host}}$ is given by Eq. 7 and $\mathcal{N}(0, 1)$ is the normal distribution with zero mean and unit variance.

We fit model parameters denoted $\Theta$ looking for the minimum
one can see that to fit assuming other model parameters (2). According to this study (2) 2
the expected values of DM for all sources at z > 1 do depend on this parameter. Examining Table 2 one can see that to fit \( f_e \) with the accuracy of one per cent samples with \( N \sim 10^5 \) are needed.

In the analysis above we have neglected the uncertainty resulting from using wrong values of cosmological model parameters when fitting \( f_e(z) \). We assume four parameters \( p_i \) \((p_i \in \{H_0, \Omega_B, \Omega_M, \Omega_{\Lambda}\})\) to have values known from other studies with errors \( \sigma_i\). Neglecting correlations between \( p_i \) we would get:

\[
\sigma_{\text{model}}^2 = \sum_{i=1}^{4} (\frac{\partial f_e}{\partial p_i})^2 \sigma_i^2
\]

The recent estimates of cosmological parameters are given in Planck Results 2018 (Aghanim et al. 2018). According to this study \( H_0 \) is weakly anti-correlated with \( \Omega_B \). On the other hand the measured quantity \( DM \approx f_e \times \Omega_B \times H_0 \) (compare Eqs. 3, 5), so Eq. 16 overestimates uncertainty resulting from errors in these two parameters. The universe model is practically flat (ibid.) so \( \Omega_M \) and \( \Omega_{\Lambda} \) are strictly anti-correlated. We examine the influence of their values on electron fractions by repeating calculations for \( \Omega_M = \Omega_M + \Delta \) and \( \Omega = \Omega_M - \Delta \), where \( \Delta = \Omega_{\Lambda} = \Omega_M \). We add the estimated uncertainty in quadrature to uncertainties from the other two parameters getting values shown in the last row of Table 2. Because we have neglected the weak anti-correlation between \( H_0 \) and \( \Omega_B \) the numbers in the table should be treated as upper limits. This shows that the uncertainty of fitted electron fractions resulting from the uncertainty of the used precision cosmology model is of the order of one per cent.

Next we check the possibility of testing both the electron fraction and the geometry of the universe model using FRBs. Since the dispersion measure is directly proportional to the product of \( f_e \), \( H_0 \), and \( \Omega_M \) (Eqs. 3, 5) placing any limits on these parameters separately is impossible. We simplify our approach looking only for the expected electron fraction assuming \( H_0 \) and \( \Omega_B \) to be known independently. The dispersion measure to source at a given redshift \( z \) depends on the electron fraction in the redshift interval \( 0 \leq z \leq z_S \) in proportion to

\[
\langle f_e(z) \rangle = \frac{\int_0^z f_e(z) (1+z)^2 dz}{\int_0^z (1+z)^2 dz}
\]

The averaged electron fraction \( \langle f_e(z) \rangle \) is slowly and monotonically changing from 0.825 at \( z = 0.5 \) to 0.845 at \( z = 4.5 \). Averaging over the source redshift distribution

\[
\langle \langle f_e(z) \rangle \rangle = \frac{\int_{z_{min}}^{z_{max}} \frac{f_e}{f_{e,\text{mean}}} (f_e(z)) f(z) dz}{\int_{z_{min}}^{z_{max}} f(z) dz}
\]

we get the expected \( \langle \langle f_e(z) \rangle \rangle \) values which are almost the same for the three ranges of sources redshifts considered (see Table 3). Despite simplifications we have been unable to place any interesting constraints on \( \Lambda CDM \) model with three free parameters \( (\Omega_M, \Omega_{\Lambda}, f_e) \). The fits give large and irregular confidence regions with a substantial fraction of solutions grouped near the boundaries of the considered parameters region. (We limit the possible val-

| Table 2: Electron fraction parameters |
|--------------------------------------|
| \( z \)  | \( f_{eGM} \) | \( f_{en} \) | \( f_e \) |
| 0   | 0.847  | 0.993  | 0.841  |
| 1   | 0.841  | 0.992  | 0.835  |
| 2   | 0.876  | 0.992  | 0.869  |
| 3   | 0.928  | 0.990  | 0.918  |
| 4   | 0.964  | 0.979  | 0.944  |

\( f_e(z = 4) \) has an impact only on sources with \( z_S \geq 3 \), which are less numerous (0.22 \% on average). Thus the estimate of \( f_e \) at higher redshifts has a larger standard deviation.
ues of the parameters to $\Omega_M \leq \Omega_M \leq \Omega_M + 1$, 0.3 $\leq \Omega_A \leq 1.3$, 0.3 $\leq f_e \leq 1.3$).

We show the results for flat $\Lambda$CDM models in Fig. 4 and in Table 3. We have considered three ranges of FRBs redshifts in our simulations (0.5 $\leq z_5 \leq 4.5$, 0.5 $\leq z_6 \leq 2.5$, and 0.5 $\leq z_7 \leq 1.5$). In each case we have simulated $4 \times 10^6$ “observed” FRBs samples. For each sample we have fitted a flat $\Lambda$CDM cosmological model. We have obtained histograms of fitted parameter values $N(\Omega,M,f_e)$ for various ranges of source redshifts. We find, that even a wide redshift range of FRBs (0.5 $\leq z_5 \leq 4.5$) gives only a rough estimate of parameters. The fitted $\Omega_M$ and $f_e$ are strongly correlated with $\rho \approx 0.97$. The averaged result of many fits depends on the sample's redshift range and is biased.

Fixing both density parameters ($\Omega_M = \Omega_M$, $\Omega_A = \Omega_A$) we get estimates of sample averaged electron fraction, which we denote $f'_e$ in Table 3. Their values agree with the analytical prediction $\langle(f_e)\rangle$. This shows that in a fixed cosmological model one can get the redshift and sample averaged electron fraction with the uncertainty of $\sim$ two per cent using a sample of one hundred FRBs (plus one per cent uncertainty of the model).

### 4 TESTS BASED ON FRBS AND OTHER DATA

Limited possibilities of FRBs tests suggest to use also some other data to make useful constrains on the models, obtaining at least cosmological density parameters and electron fraction simultaneously.

As an example we combine simulated FRBs and SNe Ia samples. We use the SCP “Union” SN Ia data (Kowalski et al. 2008) as basis of obtaining our synthetic samples. The “Union” sample contains 307 usable lightcurves. We have downloaded the data in the form $\{z_j, H_j, \sigma_j\}$ redshift – distance modulus – its estimated error. The best fit of a $\Lambda$CDM model to the real SNe data gives cosmological parameters which are within 1-sigma from Illustris parameters but are different. To check whether our tests reproduce cosmological parameters used in simulations we replace the “Union” SN Ia data by synthetic samples $\Omega'$ with the same redshifts and estimated errors but corrected distance moduli: $\mu'_j$ with Gaussian noise:

$$\mu'_j = 5 \log \left( \frac{d_L(z_j; \Omega_M, \Omega_A)}{10 \, \text{pc}} \right) + \sigma_j \mathcal{N}(0,1)$$

where $d_L$ is the luminosity distance and Illustris cosmology parameters are used.

We consider $\Lambda$CDM models with two free parameters ($\Omega_M$ and $\Omega_A$) while the Hubble constant and baryon density are fixed at Illustris values. The averaged electron fraction $f_e$, which does not influence the geometry of the universe model or the SN’ data fits, is necessary to model FRBs. The SN’ part of the $\chi^2$ reads:

$$\chi^2_{SN} = \sum_{j=1}^{N} \frac{(\mu'_j - \mu_o(z_j; \Omega_M, \Omega_A))^2}{\sigma_j^2}$$

$$\mu_o(z_j; \Omega_M, \Omega_A) = 5 \log \left( \frac{d_L(z_j; \Omega_M, \Omega_A)}{10 \, \text{pc}} \right)$$

where $\mu_o$ is a model predicted distance modulus.

Now we look for cosmological parameters which minimize $\chi^2$ for a combined FRB plus SN’ data. For each FRB and SN’ data simulation we find the minimum of:

$$\chi^2 = \chi^2_{FRB}(\Omega_M, \Omega_A, f_e) + \chi^2_{SN}(\Omega_M, \Omega_A)$$

Since there are 307 SN Ia in the SN Union sample, we use simulated FRBs data samples consisting of 300 sources each. We examine many combined sets. In Fig. 5 we show confidence regions for $\Omega_M$, $\Omega_A$, and sample averaged $f_e$ projected into two or one dimension. Table 4 gives the fitted parameter values with 1D 68% confidence regions.

The tests based on combined data allow to constrain both cosmological density parameters ($\Omega_M$, $\Omega_A$) but their accuracy is still far from the present precision cosmology standards. On the other hand our simulation shows that using few hundred of FRBs with another moderate precision test one is able to measure the averaged electron fraction with uncertainty of $\sim$ two per cent.

**Table 4. Fits of $\Lambda$CDM and electron fraction models to combined synthetic samples of SNe and FRBs**

| model         | $\Omega_M$ | $\Omega_A$ | $f_e$ |
|---------------|------------|------------|-------|
| Illustris     | 0.2726     | 0.7274     |       |
| SN            | 0.345±0.099| 0.863±0.245|       |
| SN’ & FRBs    | 4.5        | 0.64±0.06  | 0.832±0.013 |
| SN’ & FRBs    | 2.5        | 0.70±0.09  | 0.831±0.013 |
| SN’ & FRBs    | 1.5        | 0.74±0.12  | 0.828±0.014 |
| SN’ & FRBs flat| 4.5        | 0.825±0.013|       |
DISCUSSION

We have analyzed the evolution of the distribution of ionized gas in the IGM using the results of the Illustris simulation (Vogelsberger et al. 2014a,b). According to our calculations the fraction of gas in the IGM changes from $f_{\text{IGM}} \approx 1$ at $z = 5$ to $0.85$ at $z = 0$. This result is different from Haider et al. (2016), who obtain $f_{\text{IGM}} \approx 0.77$ at $z = 0$. For $z < 3$ the IGM gas is almost completely ionized, and free electron fraction $f_e \approx f_{\text{IGM}}$. The estimate of Shull & Danforth (2018) who claim that the diffuse baryons constitute $0.6 \pm 0.1$ of all gas is based on different reasoning.

The spatial distribution of ionized gas and its evolution given by Illustris allows us to simulate calculations of the dispersion measure along any LOS and obtain its standard deviation as a function of the source redshift. Our results on averaged DM to a source at given redshift (Fig. 1, Table 1) are in rough agreement with other authors (e.g. Zhou et al. 2014, Deng & Zhang 2014), and with the approximate formula of Zhang (2018):

$$DM_{\text{IGM}} \approx 855 \text{ pc cm}^{-3} z$$

At $z = 1$ we get $\sigma_{DM}$ which is about two times lower as compared to the results of McQuinn (2014) based on simulations. Only if all gas
from Illustris except stellar cells is included in calculations we get the agreement with his $\sigma_{DM}(z = 1) = 1$ value. The shape of our $\sigma_{DM}(z)$ dependence is different and resembles his analytical results.

The dispersion measure to a given source is directly proportional to the density of free electrons along LOS. In our approach we model it using the electron fraction parameter $f_e(z)$ (Eq. 4). In simulations we use its values at $z = 0, 1, ..., 5$ and linearly interpolate between them. Following Deng & Zhang (2014), Shull & Danforth (2018) and others we check the possibility of finding the history of ionization using a sample of observed FRBs and assuming cosmological parameters ($H_0$, $\Omega_M$, $\Omega_L$) to be known exactly. We show, that to get the dependence of the electron fraction on the redshift with one per cent accuracy, samples of $N \sim 10^4$ events are needed (compare Table 2). The present errors in cosmological parameters values (e.g. Aghanim et al. 2018) introduce another one per cent uncertainty. On the other hand the redshift and sample averaged electron fraction can be constrained with accuracy better than two per cent based on $N = 100$ FRBs in a fixed cosmological model (compare Table 3). The dependence on cosmological parameter errors is the same as above.

We have shown, that using a sample of three hundred FRBs and a sample resembling SN Ia Union Sample (Kowalski et al. 2008) we are able to constrain the averaged electron fraction parameter with the accuracy of $\sim$ two per cent (compare Table 4).

The cosmological tests based on large future samples of FRBs are possible, but are unlikely to give the accuracy of the present day precision cosmology. This conclusion with more details has already been reached by Walters et al. (2018). There are degeneracies between derived universe model parameters, which can be removed only by using other type of data, as shown by our consideration of joint FRBs and SN Ia tests. The observations of FRBs with measured redshifts may give valuable data on the distribution of free electrons in space also on cosmological scales (compare Shull & Danforth 2018). Since the dispersion measure depends on the electron distribution between the source and the observer, the dependence of the electron fraction on the redshift can, at least in principle, be investigated. In this aspect samples of FRBs with known redshifts may be a better target than Cosmic Microwave Background observations yielding Thompson optical depth $\tau_T$ and Sunyaev-Zeldovich $y$ parameter characterizing the electron population between the observer and the Last Scattering Surface (see e.g. Aghanim et al. 2018).

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