Learning to predict requires Integrated Information

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Embodied agents are regularly faced with the challenge to learn new tasks. In order to do that they need to be able to predict their next sensory state by forming an internal world model. We theorize that agents require a high value of information integration to update that world model in light of new information. This can be seen in the context of Integrated Information Theory, which provides a quantitative approach to consciousness and can be applied to neural networks. We use the sensorimotor loop to model the interactions among the agent’s brain, body and environment. Thereby we can calculate various information theoretic measures that quantify different information flows in the system, one of which corresponds to Integrated Information. Additionally we are able to measure the interaction among the body and the environment, which leads to the concept of Morphological Computation. Previous research reveals an antagonistic relationship between Integrated Information and Morphological Computation. A morphology adapted well to a task can reduce the necessity for Integrated Information significantly. This creates the problem that embodied intelligence is correlated with reduced conscious experience. Here we propose a solution to this problem, namely that the agents need Integrated Information to learn. We support our hypothesis with results from a simple experimental setup in which the agents learn by using the em-algorithm.

keywords: Embodied Artificial Intelligence, Information Theory, Information Geometry, em-Algorithm, Morphological Computation, Integrated Information

1 Introduction

1.1 Objective

For every embodied agent, whether it is an animal, a human or a robot, learning new tasks and adapting to changes in its environment is a challenge. An important aspect of this task is to be able to predict what is happening next and especially what the outcomes of the own actions would be. We illustrate this aspect in the following example.

Consider a child who tries to learn to ride a bike. Nearly every task the child has learned up to this point, e.g. walking, speaking or drawing, becomes harder when one tries to do it fast. So the child expects that moving slowly would lead to the best outcome. According to its understanding of the world, its world model, riding a bike slowly is easier than doing it fast. Unfortunately in this case, speed stabilizes a bike and is
therefore beneficial for learning. The child is working with an inaccurate world model. So before the child can learn to ride a bike it has to observe and understand that faster sometimes can mean easier. It has to update its world model in order to learn and to be able to use the world in an optimal way.

In this work we are going to closely examine this process of updating the world model. There we are especially interested in how the information is integrated in the controller during learning. We theorize that in order to update the world model correctly, the agent has to integrate its available information.

We use simple simulated agents and observe how the complexity of the controller develops during the learning process. There we observe that agents, that succeed at learning the task, have first a high Integrated Information and then this value decreases. We theorize that this is because the agents first have to learn the correct world model, before they are able to optimally utilize the interaction of their bodies with the environment, measured by Morphological Computation, which in turn leads to a lower integrated information. This theory is supported by the result that agents who are not as successful have constant high Integrated Information and a lower Morphological Computation compared to the successful agents. Therefore our first main result is the following.

1. Learning to predict requires Integrated Information.

Additionally, we analyze agents that are not able to integrate information in the controller and call them “limited” agents. Those agents perform significantly worse. By calculating three different measures for the world model, we discover that the few limited agents that are successful combine their different information sources directly in the world model, in order to predict the next sensory state. Note that this means that there is an information integration in the world model. To avoid confusion between this and the Integrated Information value in the controller, we will refer to the former as “combining” of information. This leads to our second result.

2. Without Integrated Information in the controller the process of predicting needs to combine the information from different sources and therefore becomes more complex.

Note that we call a system complex, if it is more than the sum of its parts.

1.2 Theoretical Background

The agents are faced with a task that is described in Section 2. We model the agents in their environment by a sensori-motor loop. This reflects the interactions among the sensors \( S \), the actuators \( A \) and the controller nodes \( C \) and can be translated to probability distributions that define the behavior of the agents. This is discussed in Section 3. In addition to its sensor, actuator and controller nodes the agent also has an internal prediction \( S' \). We call the mechanism that generates a prediction the “internal world model”. Such an internal world model was also used in [6] and [7]. This approach allows us to analyze the information flows among the different parts of the agents, and especially its prediction, in detail.

We define the learning algorithm by using an adapted em-algorithm that alternates between optimizing the agent’s behavior and updating its world model. The first part, optimizing the behavior, is done by maximizing the likelihood of a goal variable. This follows the reasoning of the approach described by Attias in [5], and further analyzed in [23] and [22], called “Planning as Inference”. We also use this optimization in [16], here it is described in Section3.1 and in more detail in the Appendix.

While the agents learn we calculate various information theoretic measures that we define in Section 4. One of these measures assesses how much information is integrated in the controller. This can be seen in the context of Integrated Information Theory (IIT), originally proposed by Tononi. The core idea of IIT is that the level of consciousness of a system can be equated with the amount of information integration among
different parts of it. This theory developed rapidly from a measure for brain complexity \cite{21} towards a thorough theory of consciousness \cite{20, 19, 9}. Hence there exist various types of integrated information measures depending on the version of the theory they are referring to and the setting they are defined in. Here we make use of the information geometric measure that we propose in \cite{15}.

In \cite{16} we compare the integrated information of an agent with its Morphological Computation. The term “Morphological Computation” describes the reduction of computational cost for the controller that results from the interaction of the agent’s body with its environment. One example where Morphological Computation is used is the field of soft robotics. There the softness of the robots’ bodies leads to a lower computational cost when they, for example, grab fragile objects, \cite{13}. The concept of Morphological Computation is discussed in more detail in, for example, \cite{18} and \cite{11}.

The comparison between Integrated Information and Morphological Computation leads to the result that they have an antagonistic relationship. On the one hand this is intuitive, since the more the agent relies on the interaction of its body with the environment to solve a task, the less Integrated Information is needed. On the other hand this leads to the problem that now embodied intelligence is correlated with reduced conscious experience.

Here we want to present one possible solution, given by the challenge of learning. In \cite{10}, the authors conclude that Integrated Information increases with the fitness of evolving agents. The authors of \cite{2} increase the complexity of the environment, which leads to higher Integrated Information and in \cite{1} high Integrated Information benefited rich dynamical behavior. All these results are clear indications that high Integrated Information is beneficial for an embodied agent that is faced with a task. As discussed in the introduction, learning to perform a task entails updating an internal world model in order to predict the outcome of ones actions. We hypothesize that this process requires the agent to highly integrate the information that it gets, hence that learning to predict requires Integrated Information. Using our toy experiment we are able to reproduce the results from our previous work and additionally confirm this hypothesis. The results are discussed in Section 5.

In Section 5 we also take a close look at the dynamics of the prediction. We calculate three different information theoretic measures on the world model in order to analyze it thoroughly. Additionally, we compare the agents with agents that are not able to generate Integrated Information. We see that the latter are barely, if at all, able to learn to perform the desired task. The ones that do learn to perform the task have a complex prediction mechanism that combines information there, instead of in the controller nodes.

2 Setting of the Experiment

In our experiment, we analyze the information flows of very simplistic, 2-dimensional, acting agents. The agents consist of a round body and two binary sensors. These sensors are visualized as lines that are green when they detect a wall and black otherwise. In Figure 1 five of these agents are depicted in a racetrack. This racetrack is their environment in which they have to move. The agents have four different movements, fast forward (approx. 0.6 unit length per step), slow forward (approx. 0.2 unit length per step), left and right (with approx. $\approx 14^\circ$ and a speed of 0.4 unit length per step). Whenever the body of an agent touches a wall, the agent gets stuck. This means that it can turn on the spot, but will not move away unless both sensors do not detect a wall. A video of this movement can be found at \cite{14}.

Varying the length of the sensors can directly influence the amount of information an agent receives about its environment and hence it influences the quality of the interaction of the agent with its environment. Therefore this has an impact on the potential Morphological Computation. In \cite{18} this is called “Morphology facilitating control”. This is discussed in more detail in \cite{16}.
3 The Agents and the World Model

The agents are modeled by discrete multivariate, time-homogeneous Markov process

$$(X_t)_{t \in \mathbb{N}} = (S_t, A_t, C_t)$$

with the state space $X = S \times A \times C$. Here the variable $S_t$ describes the two binary sensors that detect a wall and a binary variable that encodes whether the agent is touching a wall. The node $A_t$ includes the two binary actuators and $C_t$ the two binary controller nodes. Additionally, we introduce another variable here $S'_t$ that describes the internal prediction of the next sensor state and hence consists of three binary variables.

The elements of the agents are connected according to the graph in Figure 2 which leads to the distribution

$$P(x_t, x_{t+1}, s'_{t+1}) = P(s_t, a_t, c_t)P(s'_{t+1}|a_t, c_t)P(s_{t+1}|s_t, a_t)P(c_{t+1}|c_t, s_{t+1})P(a_{t+1}|s_{t+1}, c_{t+1}).$$

Here we only depict one node for each $S, S', A$ and $C$ in the figures in order to increase clarity.

Figure 2: The sensori-motor loop of the learning agents.

Note that since $S'_t$ is a prediction of $S_t$ it is made of the same substrate as $S_t$, hence the state space of $S'_t$ is $S$. The difference between $S_t$ and $S'_t$ lies in the mechanism with which they are generated. The node $S_t$
is influenced by the information from $S_{t-1}$ and $A_{t-1}$. These are indirect influences, since the information flows through the environment here. The role of the environment is discussed in more detail in [16]. The conditional distribution, $P(S_{t+1}|S_t, A_t)$, is called world model in [24] and [17].

The internal prediction $S'_t$ is generated by $P(S'_{t+1}|A_t, C_t)$ and was also named “world model” in [6], [7] and [8]. To prevent confusion we refer to $P(S'_{t+1}|A_t, C_t)$ as internal world model and to $P(S'_{t+1}|S_t, A_t)$ as empirical world model. We chose the term empirical world model, since the agents gain this distribution by sampling their experience.

We sample the distributions $P(S_t, A_t, C_t)$ and $P(S_{t+1}|S_t, A_t)$ as described in more detail in [24] and we denote the sampled distributions by $\tilde{P}(S_{t+1}|S_t, A_t)$ and $\tilde{P}(S_t, A_t, C_t)$.

Additionally we analyze the behavior of two different types of agents. The first one is fully connected, meaning that both controller nodes depend on both controller nodes in the previous point in time. The second one is not able to generate Integrated Information in its controller. This is done by ensuring that the controller node $C_{i+1}$ only receives information from $C_i$ and not from $C_j$, $i, j \in \{1, 2\}$, $i \neq j$, as depicted in Figure 3. We refer to them as fully connected agents and limited agents.

![Figure 3: The connections of the fully connected agents on the left and the limited agents on the right.](image)

**3.1 Learning**

In this setting the agent learn while it is inside the racetrack. Therefore at each step $t$, the realized states $s_{t-1}, a_{t-1}, c_{t-1}$ are known. Hence the agent use these certainties. To that end we need the following definitions

Let $P_{a_t}(C_{t+1}|S_{t+1})$ be the probability distribution of $C_{t+1}$ conditioned on $S_{t+1}$ and a fixed state $a_t$:

$$P_{a_t}(c_{t+1}|s_{t+1}) := P(c_{t+1}|s_{t+1}, a_t), \forall s_{t+1}, c_{t+1} \in S_t \times C_t.$$ 

From an internal, agent-centric perspective, the predictive process is as depicted in Figure 3. Note that the $s_t$ is the actual realized last sensor state, it is not $s'_t$. 


Hence, the agent can optimize the following three distributions

\[ P(c_{t+1}|s'_{t+1}), \quad P(a_{t+1}|s'_{t+1}, c_{t+1}), \quad P(s'_{t+2}|a_{t+1}, c_{t+1}) \]

and \( P(s_t,a_t|s'_{t+1}) \) can be gained from \( P(s'_{t+2}|a_{t+1}, c_{t+1}) \).

In our previous publication [16], we used the concept of “Planning as Inference” in order to optimize the behavior of the agent. In this method the conditional distributions are optimized with respect to a goal variable by using the em-algorithm. This is a well-known information geometric algorithm that is guaranteed to converge, but might converge to a local minimum [3], [4].

In this setting we have two goals. We want to optimize \( P(c_{t+1}|s'_{t+1}) \) and \( P(a_{t+1}|s'_{t+1}, c_{t+1}) \) such that the probability of touching the wall after the next movement is as low as possible, while keeping the internal world model \( P(s'_{t+2}|c_t, a_t) \) close to the empirical world model \( P(s_{t+2}|s_t, a_t) \). The second goal is important, because otherwise the optimization of the behavior would use faulty assumptions leading to a failure of the agent. In the example in the introduction this would be the child trying to learn to ride a bike while going as slow as possible. Hence, both of the world models should result in similar predictions. These are highlighted in Figure 5.

Hence we alternate between optimizing \( P(c_{t+1}|s'_{t+1}) \) and \( P(a_{t+1}|s'_{t+1}, c_{t+1}) \) with respect to the goal on one hand and with respect to the difference between \( P(s'_{t+2}|c_t, a_t) \) and \( P(s_{t+2}|s_t, a_t) \) on the other hand.

Details of this optimization are described in the Appendix 7.

Furthermore, we add gaussian noise to \( P(a_{t+1}|s'_{t+1}, c_{t+1}) \), because if the em-algorithm reaches a point where for some action \( a_{t+1} \) \( P(a_{t+1}|s'_{t+1}, c_{t+1}) = 0 \), then it can not gain a positive value again.

Note that the controller has only two binary nodes whereas the sensors consist of 3 binary nodes. Therefore merely copying the information from the sensors is not a viable strategy for the agents. This is also a natural
setting, because we are not able to consciously perceive every detail from our environment that our sensors are able to pick up, but we learn to distinguish between important and irrelevant information.

4 Measures of the Information Flow

We measure the importance of an information flow by calculating the difference between the actual distribution and the closest distribution without said information flow. The set of distributions without this information flow is called a split system. More precisely, the measures for the different information flows in the system are defined in the following way.

**Definition 1 (Measure Ψ).** Let $M$ be a set of positive probability distributions on $Z$. Then we define the measure $Ψ$, by minimizing the KL-divergence between the split system $M$ and the full distribution $P$

$$Ψ_M = \inf_{Q \in M} D_Z(P || Q) = \inf_{Q \in M} \sum_z P(z) \log \frac{P(z)}{Q(z)}.$$ 

All of the discussed measures have a closed form solution and can be written in the form of sums of (conditional) mutual information terms. The mutual information $I(Z_1; Z_2)$ and the conditional mutual information, $I(Z_1; Z_2|Z_3)$, are defined as:

$$I(Z_1; Z_2) = \sum_{z_1, z_2} P(z_1, z_2) \log \frac{P(z_1, z_2)}{P(z_1)P(z_2)},$$

$$I(Z_1; Z_2|Z_3) = \sum_{z_1, z_2, z_3} P(z_1, z_2, z_3) \log \frac{P(z_1, z_2|z_3)}{P(z_1|z_3)P(z_2|z_3)}.$$ 

These formulas can be interpreted in the following way. If $I(Z_1; Z_2|Z_3) = 0$, then the variable $Z_1$ is independent of $Z_2$ given $Z_3$. Hence this quantifies the connection between $Z_1$ and $Z_2$, given the influence of $Z_3$. In the following section we emphasize the connection that we are measuring by a dashed arrow.

Additionally, when $Ψ_M$ is the measure for the fully connected agents, then we denote the measure in case of the limited agents by $Ψ_M^0$. 

4.1 Integrated Information

There exist various types of Integrated Information measures, as discussed in the introduction. The measure for Integrated Information we are using here was defined in our previous publication [15], also applied in [16]. It measures how much information gets integrated among different nodes across different points in time. The graph corresponding to the split system is depicted in Figure 6. The minimization described in the previous section then results in the closed form solution below, as shown in [15].

$$\Phi_T = \sum_j I(C_j^{t+1}; C_{t} \setminus \{j\} | C_j^t, S_{t+1})$$

In our case we only have two binary controller nodes, hence $J = \{1, 2\}$. In [16] we discuss that the importance of the Integrated Information for the behavior of the agent also depends on the following measures.

Additionally, we calculate the importance of the information flow from the sensory nodes to the controller nodes. This measure is called “sensory information”

$$\Psi_{SI} = \sum_j I(C_j^{t+1}; S_{t+1} | C_t).$$

The strength of the connection from the controller nodes to the actuator nodes is measured by

$$\Psi_C = \sum_i I(A_i^{t+1}; C_{t+1} | S_{t+1})$$

and called “control”.

4.2 Morphological Computation

The concept of Morphological Computation describes the reduction of the necessary computation in the controller that results from the interaction of the agent’s body with its environment. There exist various types of Morphological Computation, [18], and different measures for it. We are going to use the following formulation

$$\Psi_{MC} = I(S_{t+1}; S_t | A_t).$$

This measures the information flow through the world going from one sensory state to the next one, given the actuator state. In [12] this was introduced as a measure for Morphological Computation and in [11] in comparisons with other measures $\Psi_{MC}$ shows desirable properties.
4.3 Prediction

We analyze the prediction, defined by the conditional distribution $P(S_{t+1}'|A_t, S_t)$, by calculating three different measures.

The first one will be called “full prediction” and measures the total information flow from $C_t$ and $A_t$ to $S_{t+1}'$. The measure results in the mutual information between $S_{t+1}'$ and $(A_t, C_t)$

$$\Psi_{FP} = I(S_{t+1}'; A_t, C_t).$$

The corresponding split system is depicted in Figure 9.

The next two measures allow us to further differentiate the two information flows, from the actuators to the prediction and from the controller nodes to the prediction.

The influence the actuator has on the prediction can be calculated by the following conditional mutual information

$$\Psi_{AP} = I(S_{t+1}'; A_t|C_t).$$

We call this measure “actuator prediction”.

Equivalently, we define the “controller prediction” by

$$\Psi_{CP} = I(S_{t+1}'; C_t|A_t).$$

This describes how dependent the prediction of the next sensory state is on the controller nodes, given the actuators. The graphical representation is shown in Figure 10.

5 Results

The success rate is calculated by sampling how long the agents are stuck at a wall. Hence a success rate of 10% means that the agents were stuck 90% of the time. When the agents perform random movements, then they achieve a success rate of roughly 7.92% after 20 000 steps. We define agents as successful if their success rate is roughly double the rate of the agents that perform random movement, meaning above 16% and we refer to agents below 16% as unsuccessful. Dividing the agents at 16% allows us to call only the agents successful, for which the success rate increased significantly during learning, while still having some limited agents that can also be called successful.

We took 500 random input distributions, meaning that we trained 500 fully connected and 500 limited agents for 20 000 steps and calculated the measures after every 100 steps.

In Figure 11 we can see the Integrated Information and Morphological Computation for the successful, fully connected agents after 20 000 steps. These two measures behave antagonistically. Therefore the results confirm our previous observation published in [16]. Note that when the sensors are too long, so that the agents almost always detect a wall, then this additional information is no longer beneficial for the agents and the Morphological Computation no longer increases, while Integrated Information increases again.
This leads to the question, why agents with a well-adapted morphology would need Integrated Information. Wouldn’t it be possible to build agents that are so well adapted to their environment, that any Integrated Information is irrelevant? There might be several reasons why Integrated Information is necessary, despite our results as we discuss further in Section 6. Here we are going to give one reason by analyzing the behavior of the agents during learning.

The top row of Figure 12 depicts on the left the success rate of the fully connected agents, that are not successful. In the middle there is the Integrated Information for these agents and on the right there are the 2-dimensional results for Integrated Information, meaning firstly the Integrated Information depicted with the x-axis of sensor length and secondly with the x-axis denoting the steps. These unsuccessful agents have an Integrated Information value around 0.3 and 0.4 and it slightly decreases with the sensor length. There is no significant decrease or increase in the Integrated Information with the step size.

Now we compare these results to the Integrated Information value of the successful agents. The bottom row of Figure 12 shows the success rate of the fully connected, successful agents on the left, the Integrated Information in full in the middle and the 2-dimensional perspectives on the right. There we can observe that the Integrated Information decreases with the sensor length as discussed in the context of Figure 11. Additionally, there is a first increase in the Integrated Information value in the first 400 steps and then a strong decrease. After 20 000 steps the Integrated Information value is roughly between 0.05 and 0.15. Hence in the case of the successful agents, the Integrated Information value goes down to a significantly lower value after time, compared to the unsuccessful agents.

Figure 11: Comparison of Morphological Computation and Integrated Information for the agents that have a success rate above 16% after 20 000 steps.
This could lead to two different interpretations.

On the one hand, a high Integrated Information value could be important as long as the agents have not been able to learn the correct world model. Without a correct world model the agents are not able to find a policy that would allow them to optimally use their interaction with the environment. Hence they learn the world model and then find a policy that uses high Morphological Computation, which leads in turn to a low Integrated Information. In Figure 13 we see that the Morphological Computation for the successful agents is higher than the value for the unsuccessful agents. This results supports the first possible interpretation.
On the other hand, high Integrated Information could also prevent the agents from performing well, by over complicating such a simple task. In theory the agents only need to steer away every time they touch a wall and decide on a direction in which to turn if they are stuck. Hence one could argue, that since this task could be performed by simply reacting, without planning, a highly integrated controller is counterproductive.

In order to clarify what interpretation is more likely, we now look at the average success rates of the fully connected agents compared with the limited agents in Table 1.

|                  | random movement | fully connected agents | limited agents |
|------------------|-----------------|------------------------|----------------|
| success rate     | ≈ 7.92%         | ≈ 15.27%               | ≈ 8.04%        |

Table 1: Average success rates of the agents with random movement and the fully connected and limited agents.

There we see, that in average the limited agents barely perform better than the fully connected agents. Note that there is also a strong difference in the number of agents that is successful, as listed in Table 2.

|                  | fully connected agents | limited agents |
|------------------|------------------------|----------------|
| percentage of successful agents | ≈ 35%               | ≈ 2.5% |

Table 2: Percentage of successful fully connected and limited agents.

In summary, the limited agents perform only marginally better than agents that move purely random and only 2.4% of limited agents are successful, compared to 35% of fully connected agents. This strongly supports the hypothesis that Integrated Information is necessary for learning to predict.

We additionally discuss in [16] that the importance of the Integrated Information value for the behavior of the agent depends on the value of control and the sensory information. In Figure 14 we see that these two measures behave similar to the Integrated Information. They decrease with the number of steps and with the length of the sensors.
Now we focus on prediction. Therefore we compare the successful fully connected and the successful limited agents. The results for measures for the full prediction $\Psi_{FP}$ and $\Psi^0_{FP}$ are shown in Figure 15.

While both measures don’t really change after roughly 2500 steps, the full prediction for the fully connected models, $\Psi_{FP}$, is overall higher than the full prediction for limited agents, $\Psi^0_{FP}$, and while $\Psi_{FP}$ increases with the length of the sensors, $\Psi^0_{FP}$ decreases. Without Integrated Information, the next sensory state depends not as much on the controller nodes and the last sensory state, especially when the sensors are long. This directly influences their performance, since they can not reliably optimize the likelihood of not touching a wall after the next time step, if they cannot predict the outcome of their actions.

Comparing these results with the measures for the actuator and controller predictions, $\Psi_{AP}, \Psi^0_{AP}, \Psi_{CP}$ and $\Psi^0_{CP}$, reveals another interesting aspect of the difference between the prediction process in the fully connected.
and limited agents. In Figure 16 we see the range of the values for these measures for the agents between step 10 000 and 20 000. See full results for these measures are depicted in the Appendix in Figure 18.

![Graph showing the range of different prediction measures.](image)

Figure 16: Range of the different prediction measures in the during the steps 10 000 to 20 000.

When we take into account $\Psi_{AP}^{0}$, $\Psi_{CP}^{0}$, and $\Psi_{CP}$, we see that these are much lower for the fully connected agents. To explain further what this means we look at the difference between the full prediction and the actuator and controller prediction respectively.

\[
\Psi_{FP} - \Psi_{AP} = I(S_{t+1}'; A_t)
\]
\[
\Psi_{FP} - \Psi_{CP} = I(S_{t+1}'; C_t)
\]

The same relationship holds for the measures of the limited agents. The mutual information between two variables is a measure of the dependence between these variables. In the case of the fully connected agents $S_{t+1}'$ strongly depends on $A_t$ or $C_t$, leading to high values of $I(S_{t+1}'; A_t)$ and $I(S_{t+1}'; C_t)$, but once one of these are known, then the other does not add a lot of necessary information. Hence $\Psi_{AP}^{0}$, $\Psi_{CP}^{0}$ are low. The information is integrated well beforehand in the controller, so that there is redundant information.

In the case of the limited agents, on the other hand, the mutual information between $S_{t+1}'$ and $A_t$, and $S_{t+1}'$ and $C_t$, is lower compared to the fully connected agents. Here the prediction of the next sensory state does not depend on the actuators or the controller alone, but on their interaction. So without Integrated Information in the controller, the process of predicting itself combines the information from the controller and the actuators, leading to a more complex process. This is the second statement from the introduction.

6 Discussion

In this article we discuss the dynamics of the Integrated Information in the controller of learning, embodied artificial agents. We use an adapted em-algorithm, that alternates between optimizing the behavior to reach a goal and updating the internal world model, to train simulated agents. These agents move inside a racetrack and learn to not touch the walls. Using this simplistic example, we are able to analyze the different information flows inside the agents and especially examine the process of prediction.
The results of our experiment regarding Integrated Information and Morphological Computation support our previous publication [16] and show the antagonistic relationship between them. This leads to the insinuation that agents with a highly adapted morphology could have a less conscious experience. There are many possible ways to solve this problem. One possibility is that our tasks are simply too easy to solve, so that an agent truly only needs Morphological Computation to be successful. Another possibility is that the nature of the task has to require a higher order of understanding its surroundings. Therefore we will develop this approach further in order to apply this to more involved settings.

Despite the simplicity of our example, we were able to offer an additional solution to the posed problem. We theorize that learning to predict the environment results in a necessity for Integrated Information. The Integrated Information of the successful agents is first high, while the agents learn their world model, and then it decreases. We argue that this decrease could result from a rise in Morphological Computation that is facilitated by the correct world model. Comparing the fully connected agents with the limited ones, that are not able to integrate information, we see that the latter are not able to predict the next sensory state as well. The limited agents perform in average only marginally better than completely random moving agents and there is only a very small percentage of limited agents that are successful.

Additionally we observe that for these limited agents, the process of predicting becomes more complex, meaning that it is more than the sum of its parts. This process itself combines the information from the controller and the actuator in order to form a prediction. This again supports the claim that an agent needs to integrate its available information in order to build an accurate internal world model.

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7 Appendix

Learning the policies and the world model simultaneously

The learning algorithm works by adapting the em-algorithm. This is a well-known information geometric algorithm that iteratively projects to different sets of probability distributions and thereby reduces the KL-divergence between them. [3], [4].

The first of these sets is defined for optimizing with respect to the goal. Let $S^a$ be the variable indicating whether the agent is touching a wall. Then $s^a = 1$ means that the agent is not touching a wall. Now let $X_{t+1} = (S^t_{t+1}, A_{t+1}, C_{t+1})$, then the goal manifold consists of all those probability distributions for which it is certain that the agent will not touch a wall at point $t + 2$:

$$\mathcal{M}^G_{t+2}(s_t, a_t, c_t) = \left\{ Q \in \mathcal{P}(X \times S) | Q(s^a_{t+2} = 1) = 1 \right\}.$$ 

The second set consists of all the distributions that factor according to the agents, meaning that they consist
of all the possible agents, given the current world model:

$$\mathcal{M}_A^P(s_t, a_t, c_t, \mathcal{P}) = \left\{ P \in \mathcal{P}^\circ(X \times \mathcal{S}) | P(x_{t+1}, s'_{t+2}) = \mathcal{P}_{a_t, c_t}(s'_{t+1}) \prod_j P_{c_t}(c^j_{t+1} | s'_{t+1}) \right\},$$

where $\mathcal{P}^\circ$ is the interior of $\mathcal{P}$ and $\tilde{P}$ indicates that this distribution is fixed.

In [16] we iteratively project between these two sets in order to find the distribution in $\mathcal{M}_A^P$ that is closest to $\mathcal{M}_G^P$. This would be the distribution that describes a valid agent and has a high likelihood of achieving the goal. This approach is also called planning as inference, [5], [23], [22]. The approach is guaranteed to converge, but might converge to a local minimum.

In our case we want to adapt this approach in order to simultaneously learn the internal world model. The distribution $P(S'_{t+1} | A_t, C_t)$ predicts the next sensory input and reflects therefore the agent’s understanding of its environment. Hence we want to optimize our world model such that

$$P(S'_{t+2} | S_{t+1}, A_{t+1}) = \hat{P}(S_{t+2} | S_{t+1}, A_{t+1}),$$

where $\hat{P}$ is the sampled, empirical world model. Note that we require the goal to be a requirement on a joint distribution, not a conditional, hence the actual optimization works with

$$\overline{P}(S_{t+1}, A_{t+1}) P(S'_{t+2} | S_{t+1}, A_{t+1}) = \overline{P}(S_{t+1}, A_{t+1}) \hat{P}(S_{t+2} | S_{t+1}, A_{t+1}).$$

The joint distribution $\overline{P}(S_{t+1}, A_{t+1})$ is fixed to the joint distribution resulting from the last step in the algorithm.

The conditional distribution $P(S'_{t+2} | S_{t+1}, A_{t+1})$ can be calculated as

$$P(s'_{t+2} | s_{t+1}, a_{t+1}) = \sum_{c_{t+1}} P_{a_t, c_t}(s'_{t+1}) P_{c_t}(c_{t+1} | s'_{t+1}) P(a_{t+1} | s_{t+1}, c_{t+1}) P(s'_{t+2} | c_{t+1}, a_{t+1}) /
\sum_{c_{t+1}} P_{a_t, c_t}(s'_{t+1}) P_{c_t}(c_{t+1} | s'_{t+1}) P(a_{t+1} | s_{t+1}, c_{t+1})$$

Then the world goal manifold results in

$$\mathcal{M}_W^G(s_t, a_t, c_t, \overline{P}) = \left\{ Q \in \mathcal{P}(X \times S) | Q(s'_{t+2}, s'_{t+1}, a_{t+1}) = \overline{P}(s_{t+1}, a_{t+1}) \hat{P}(s_{t+2} | s_{t+1}, a_{t+1}) \right\}.$$  

Similar to the agent manifold above, we also define a world agent manifold

$$\mathcal{M}_W^W(s_t, a_t, c_t, \overline{P}) = \left\{ P \in \mathcal{P}^\circ(X \times S) | P(x_{t+1}, s'_{t+2}) = \mathcal{P}_{a_t, c_t}(s'_{t+1}) \prod_j \mathcal{P}_{c_t}(c^j_{t+1} | s'_{t+1}) \overline{P}(a_{t+1} | s_{t+1}, c_{t+1}) \right\}.$$

Note that we can define a full agent manifold by

$$\mathcal{M}_A(s_t, a_t, c_t) = \left\{ P \in \mathcal{P}^\circ(X \times S) | P(x_{t+1}, s'_{t+2}) = P_{a_t, c_t}(s'_{t+1}) \prod_j P_{c_t}(c^j_{t+1} | s'_{t+1}) \overline{P}(a_{t+1} | s_{t+1}, c_{t+1}) \right\}.$$
and then $\mathcal{M}_A^W \subset \mathcal{M}_A$ and $\mathcal{M}_A^G \subset \mathcal{M}_A$. Similarly, we can also define a full world goal manifold

$$\mathcal{M}_G^W(\text{st}_t, \text{at}_t, \text{ct}_t) = \left\{ Q \in \mathcal{P}(X \times S)|Q(s_{t+2}, a_{t+1}, s_{t+1}) = P(s_{t+2}, a_{t+1})\tilde{P}(s_{t+2}|s_{t+1}, a_{t+1}) \right\}.$$ 

Now we are able to define the algorithm depicted in Figure 17.

The first step is to project to $\mathcal{M}_G^P(\text{st}_t, \text{at}_t, \text{ct}_t)$ via an $e$-projection

$$Q^0 = \arg \inf_{Q \in \mathcal{M}_G^P(\text{st}_t, \text{at}_t, \text{ct}_t)} D(Q \parallel P^0).$$

Then we project with an $m$-projection to $\mathcal{M}_A^G(\text{st}_t, \text{at}_t, \text{ct}_t, P^0)$

$$P^1 = \arg \inf_{P \in \mathcal{M}_A^G(\text{st}_t, \text{at}_t, \text{ct}_t, P^0)} D(Q^0 \parallel P).$$

Up to this point, this is the standard em-algorithm. Now instead of projecting to $\mathcal{M}_G^P(\text{st}_t, \text{at}_t, \text{ct}_t)$ again, we update the internal world model by projecting to $\mathcal{M}_G^W(\text{st}_t, \text{at}_t, \text{ct}_t, P^1)$ with an $e$-projection

$$Q^1 = \arg \inf_{Q \in \mathcal{M}_G^W(\text{st}_t, \text{at}_t, \text{ct}_t, P^1)} D(Q \parallel P^1).$$

Afterwards we project, with an $m$-projection, to $\mathcal{M}_A^W(\text{st}_t, \text{at}_t, \text{ct}_t, P^1)$

$$P^2 = \arg \inf_{P \in \mathcal{M}_A^W(\text{st}_t, \text{at}_t, \text{ct}_t, P^1)} D(Q^1 \parallel P).$$

Then we start the process over by projecting to $\mathcal{M}_G^W(\text{st}_t, \text{at}_t, \text{ct}_t)$ as depicted in Figure 17.

Note that this algorithm is not guaranteed to converge. Since we are interested in agents that learn while performing a task, we perform only of the 15 optimization steps, as described above, after each step of the agent. Therefore a convergence is not needed in our scenario.
7.1 Full Prediction Measures

Figure 18: The measures for the actuator prediction and controller prediction for the successful and unsuccessful agents.

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