Short-Lived $\phi$ Mesons

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Abstract. We use effective hadronic field theory to study in-medium properties of the $\phi$ meson. The dominant decay channels $\phi \rightarrow K^+K^-$ and $\phi \rightarrow \pi\rho$ are modeled using an SU(3) chiral Lagrangian with normal and abnormal parities. The $\phi$ self-energy is approximated to one-, two-, and even three-loop order in the strong coupling. Effects of modified spectral functions for $\pi$, $K$, $\rho$ and $\phi$ are also included in the calculation. This allows us to study in-medium decay of phi mesons into (in-medium) kaon-pair daughters and (in-medium) $\pi\rho$ pairs. The results point to the possibility of rather short-lived $\phi$'s, short enough to decay inside the fireball in relativistic heavy ion collisions. Implications relevant for the NA49 and NA50 experimental results, recently dubbed the “$\phi$ puzzle”, are discussed.

1. Introduction

Vector mesons are expected to undergo significant changes in heated and compressed nuclear matter near the QCD phase boundary as compared to their vacuum properties[1]. At the very minimum, one expects that spectral distributions will be modified by collisions with light mesons[2]. This type of effect has already been seen in the CERN experiments looking at the low mass dilepton signals[3]. The data seem to support a picture where, for example, the rho meson is nearly completely melted into the background[4]. The omega meson scattering rates are roughly the same as the rho’s, and so it too is distorted beyond recognition in an invariant mass plot. The phi meson has up to now been considered to be a bit different. Its vacuum lifetime of 45 fm/c puts the decays in all likelihood outside the hot reaction zone generated in heavy ion collisions. Any decays, both hadronic and electromagnetic, would therefore again be governed by vacuum physics. And yet, its collision rate has been shown to be significant[2, 5]. Consequently, decay rates could be affected by the medium.

Meanwhile, there are very puzzling experimental results from CERN which are beyond description. First, the NA49 experiment measured the momentum distribution for the hadronic channel $K^+K^-$[6]. The inverse slope parameter suggests an effective temperature around 305 MeV. Next, the NA50 experiment reports a dimuon signal whose momentum distribution carries an inverse slope parameter of 228 MeV[7]. Given that there are slightly different kinematical ranges covered by the two experiments, there is some possibility of different effects playing roles. But we don’t expect the effects are different enough to warrant different physics. Our goal is therefore to study a hadronic fireball in a common framework and to look for a reasonable suggestion for

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such apparent temperatures. If one says that the kaon results are probably indicating strong flow, then there is no reason for the dilepton results to be free from flow. Since the lifetime of the phi is long, the flow should be affecting both signals consistently.

But this is not what the data are showing, which is indeed a challenge for theory that one is calling the phi puzzle. Model calculations taking medium effects into account have begun to appear[8, 9].

2. $\phi$ Meson Self Energy

The essential quantity for studying finite temperature effects on the vector mesons is the self energy, which is related to the inverses of the bare and full propagators. Complete spectral behavior is also obtainable from the components of the self energy.

To begin, we must identify the relevant hadronic degrees of freedom as well as a model for their interactions. The strangeness content of the $\phi$ dictates that kaons be included in the model, and also $K^*(892)$. Of course, the $\phi$ will interact with light unflavored mesons $\pi$ and $\rho$, as well. This argues for an SU(3) chiral Lagrangian. We thus start with the nonlinear sigma model

$$\mathcal{L} = \frac{F^2}{8} \partial_{\mu}U \partial^{\mu}U^\dagger,$$

where $U = \exp(2i\phi/F_\pi)$, $\phi$ is the three-flavor pseudoscalar mesons nonet and $F_\pi = 135$ MeV is the pion decay constant. Vector and axial vector mesons are introduced through the chiral covariant derivative $\partial_{\mu}U \rightarrow D_{\mu}U = \partial_{\mu}U - igA_{\mu}^L U + igU A_{\mu}^R$, and the left- and right-handed vector fields $A_{\mu}^R$ and $A_{\mu}^L$ are linear combinations of physical vector and axial vector fields. Kinetic energy terms for spin-1 fields are added as well as generalized mass terms for $A_{\mu}^L$ and $A_{\mu}^R$. Then, the axial vector fields are gauged away leaving to lowest order a set of interactions for light plus flavor ed pseudoscalar and vector mesons. The interactions are compactly written as[11]

$$\mathcal{L}_{\text{int}} = ig \text{Tr}(\rho_{\mu} \{\partial^{\mu}\phi, \phi\}) - \frac{g^2}{2} \text{Tr}\left(\{\phi, \rho^{\mu}\}^2\right) + ig \text{Tr}\left(\partial_{\mu}\rho_{\nu} \{\rho^{\mu}, \rho^{\nu}\}\right) + \frac{g^2}{4} \text{Tr}\left(\{\rho^{\mu}, \rho^{\nu}\}^2\right),$$

where $\rho^{\mu}$ is the nonet of vector mesons. We use the SU(3) symmetry to dictate the form of the interactions, but then allow individual coupling strengths to be fixed by appealing to data. Since the $\phi$ has a nonnegligible decay branch to $\pi\rho$, we must also model the abnormal parity interactions. We write these as

$$\mathcal{L}_{\phi\pi\rho} = g_{\phi\pi\rho} \epsilon_{\mu\nu\alpha\beta} \partial^{\mu}\phi^{\ast} \partial^{\nu}\rho^{\alpha} \partial^{\beta}\pi, $$

where $\bar{\rho}^{\mu}$ and $\bar{\pi}$ are the complex rho meson and pion fields. Empirical constraints are again used to fix the coupling constant.

One-loop topologies contribute to the self energy in ways that correspond physically to thermal adjustments to the pole mass and to on-shell decays. We include kaon bubble and tadpole graphs, and a $\pi\rho$ loop (see Fig. 1, but for now remove the higher order effects which are indicated by blobs). These one-loop effects have been thoroughly studied in the literature and turn out to be rather small[10]. At two-loop order, the contributions are again thermal adjustments to the pole mass, off-shell particle decays, and most importantly, scattering processes such as $\phi + K \rightarrow \phi + K$, $\phi + K^* \rightarrow \pi + K$, and others. If one of the internal kaon lines in the kaon-bubble graph of Fig. 1 is dressed with a $K^* - \pi$ loop, this would then correspond to a two-loop
contribution which has dominant influence on the imaginary part of the self energy. It corresponds physically to the scattering process $\phi + \pi \rightarrow K + K^*$, as well as other possibilities for 2→2 body scattering. Then at three-loop order there are many possibilities. Both internal lines could be dressed, vertex corrections can be made, etc. Depending on the specific kinematics, the diagrams correspond to off-shell decays, off-shell scattering, and even three-body decays. The cases of particular interest here are the off-shell decays $\phi \rightarrow K^+ K^-$ and $\phi \rightarrow \pi \rho$, where all particles are off shell.

3. Spectral Functions

The important quantity which is needed for an assessment of a particle’s response to the medium is the spectral function. It depends on the real and imaginary parts of the self energy as follows (for the $\phi$)

$$\rho(M) = \frac{1}{\pi} \frac{-\text{Im} \Pi}{(M^2 - m_\phi^2 - \text{Re} \Pi)^2 + (\text{Im} \Pi)^2}. \quad (4)$$

Notice that transverse and longitudinal excitations have not been distinguished. Since we are interested in two-, and even three-loop effects, we will make some simplifying assumptions. We will suggest that to a good approximation it is appropriate to absorb the real part of the self energy into the mass, hence $\text{Re} \Pi \approx 0$. The imaginary part can be shown to be directly related to the rate of decays plus collisions, plus various absorption rates[12]. The rate which dominates in the hot and dense system is the collision rate. We will therefore use

$$\text{Im} \Pi = -\omega \Gamma_{\text{coll}}. \quad (5)$$

For the general scattering process $\phi + b \rightarrow 1 + 2$, the scattering rate from kinetic theory is

$$d \Gamma_{\text{coll}} = \frac{g_a g_b}{n_\phi} \frac{d^3 p_\phi}{(2\pi)^3 2 E_\phi} f_\phi \frac{d^3 p_b}{(2\pi)^3 2 E_b} f_b \frac{d^3 p_1}{(2\pi)^3 2 E_1} (1 + f_1)$$

$$\times \frac{d^3 p_2}{(2\pi)^3 2 E_2} (1 + f_2)|M|^2 (2\pi)^4 \delta^4(p_\phi + p_b - p_1 - p_2). \quad (6)$$

In a Boltzmann approximation and an s-channel resonance picture, the collision rate for the process $a + b \rightarrow 1 + 2$ can be simplified to

$$\Gamma_{a_i}^{\text{coll}} = \frac{T g_b}{8\pi^2 m_a^2 K_2(m_a/T)} \int_{z_{\text{min}}}^\infty dz \lambda(s, m_a^2, m_b^2) K_1(z) \sigma(s), \quad (7)$$
where $g_b$ is the degeneracy of species $b$, $K_i$ is a modified Bessel function of order $i$, 
\[ \lambda(x, y, z) = x^2 - 2x(y+z) + (y-z)^2, \quad z = \sqrt{s}/T \text{ and } z_{\min} = \left[ \min(m_a + m_b, m_1 + m_2) \right]/T. \]

Results for the $\phi$ and for kaons are displayed in Fig. 2. The striking feature is that near $T_c \approx 170$ MeV, the spectra are quite dramatically distorted as compared with the vacuum. Similar pictures can be generated for the pion and for $\rho$. Broadening of $\rho$ has also been well studied in the literature[2, 13].

Next, we make a comment regarding the physical meaning of the width of the spectral function. It is a direct measure of the rate for something dynamical to happen. This could be a decay, but it turns out to be essentially dominated by elastic and inelastic scattering. When one asks about the production rate of $\mu^+ \mu^-$ or the rate to decay into $K^+ K^-$, this width is not the relevant one. However, the in-medium spectral function can be used to estimate these decay rates[14], as we discuss below.

4. Decay Rate

On general field theoretic grounds, a resonant hadronic state $|R\rangle$ decays into a two-body final state $|f\rangle = 1 + 2$ with off-shell daughters at the rate[15]

\[ \frac{dN_f}{d^4x d^4q} = \frac{(2J+1)}{(2\pi)^3} \frac{1}{\exp(\beta q_0) \pm 1} \rho(M) 2M \Gamma_{\text{med}}^R \to f, \]

\[ \text{where} \]

\[ d \Gamma_{\text{med}}^R \to f = d s_1 \rho(s_1) d s_2 \rho(s_2) \Gamma_{\text{vac}}^\text{R} \to f(M^2, s_1, s_2). \]

The spectral functions for the daughters 1 and 2 come from Eq. (4), and $\Gamma_{\text{vac}}^\text{R} \to f(M^2, s_1, s_2)$ is the vacuum decay rate with specific invariant masses. This function is obtainable from knowledge of the interaction Lagrangian. Integrating Eq. (8) over all three momentum and over all off-shell energies gives the number of decays per unit time per unit volume. From this result, we simply divide by the number of $\phi$ mesons per unit volume to arrive at the number of decays per unit time.
This in-medium decay rate is then inversely related to the lifetime through

\[ \tau_\phi = \frac{1}{\Gamma_{\text{decay}}} = \left[ \frac{1}{n_\phi} \frac{dN_f}{d^4x} \right]^{-1}. \]  

(10)

Should the decay lifetime turn out to be on the order of the fireball lifetime or shorter, this would signal that phi mesons indeed decay inside. On the other hand, if the medium has little effect on the lifetime and it remains around 45 fm/c, then most likely the \( \phi \)'s decay outside.

The physical interpretation of the lifetime result, which is plotted in Fig. 3, is the following. If we including broadening effects on the \( \phi \) spectral function due to collisions (~ 40 MeV width near \( T_c \)), but we do not broaden the decay products, we find the short-dashed curve. The lifetime in this case does decrease at finite temperature to something like 20 fm/c near \( T_c \). Again, the broadening here comes physically from two-loop self energy contributions of scattering the \( \phi \) meson with light mesons. The medium does however, have an effect also on the daughters. We therefore next allow the daughters to scatter, which means we allow for the daughters to have spectral functions which are also quite broad due to their respective scattering rates with pions, rho mesons, etc. Results are displayed as the long dashed curve with the off-shell \( K^+K^- \) final state only, and the solid curve when we also include the off-shell \( \pi\rho \) final state. The specific feature to point out is that the lifetime of the \( \phi \) decreases. It decreases quite rapidly with rising temperature, dropping to roughly 10 fm/c by 150 MeV, and 7 fm/c by 200 MeV temperature.

We suggest from the model that the in-medium \( \phi \) does decay inside the fireball!

5. Implications for Experiment

A \( \phi \) decaying in the medium into \( K^+K^- \) is unmeasurable since the kaons will most likely rescatter and be lost. Here, lost means that it will not be possible to reconstruct the parent \( \phi \). However, the dilepton branch \( \mu^+\mu^- \), will not rescatter. Those muon
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Figure 4. Inverse slope parameters and flow velocities coming from a minimum \(\chi\)-square fit to the NA49 and NA50 data.

Pairs will escape the system and be identifiable as having come from the $\phi$. Those muon pairs should exhibit the in-medium spectral properties, namely a collision broadened invariant mass distribution of several tens of MeV. Furthermore, the dilepton signal, being an increasing function of temperature, ought to be dominated by the highest temperature, when flow has not had a great deal of time to build up.

The reconstructed kaon pairs, which are identified as having come from $\phi$, will be post-freezeout decays emerging from a stage when the flow has had sufficient time to build up. Experimental indications are that a reasonable value for the flow velocity is roughly half the speed of light, or greater. The in-medium spectral behavior of $\phi$, $\rho$, $K$ and $\pi$ seem to point to this as a possibility. Therefore, we next look at the experimental data from CERN in this picture to extract temperatures and flow values.

To do that we take the Siemens-Rasmussen formula for radial flow[16] and do a minimum \(\chi\)-square fit to extract a effective temperatures and radial flow velocities for the experimental data. In this picture, the local rest frame is assumed to exhibit an equilibrium momentum distribution while collectively moving radially outward with speed $v$. If we Lorentz transform back to the fireball rest frame we find

$$\frac{d^2n}{m_t dm_t dy} = \frac{e^{-\gamma m_t/T}}{(2\pi)^2} \left[ (\gamma m_t + T) \frac{\sinh(\alpha)}{\alpha} - T \cosh(\alpha) \right], \quad (11)$$

where $\gamma = 1/\sqrt{1 - \beta^2}$ and $\alpha = \gamma \beta |\vec{p}|/T$.

The NA49 and NA50 data are then fit to this functional form. Results are shown in Fig. 4 and seem to indicate first that the kaon spectrum is consistent with low temperature ($T = 135$ MeV) but high flow ($\beta = 0.54$). This is consistent with near freezeout behavior. And second, the results indicate the muon pair spectrum is consistent with smaller flow ($\beta = 0.23$, albeit with very large uncertainty) and a higher temperature ($T = 178$ MeV). This scenario fits with the model, and fits with the experimental results.
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6. Conclusions

We have applied an effective Lagrangian to describe the dynamics of $\phi$ mesons in a hot and dense hadronic fireball. The formalism attempted to include effects beyond one-loop (on-shell decays), to include collision broadening effects of the $\phi$ (two-loop contributions) and even three-loop effects (off-shell decays). The higher order effects correspond physically to broadened spectral functions for not only $\phi$, but also for all the daughter particles $K^+K^-$ and $\pi\rho$. We find that the in-medium decay rate jumps from its vacuum value of roughly 4 MeV to nearly 40 MeV at high temperature. This corresponds to an in-medium lifetime at high temperature of roughly 5 fm/c. Therefore we conclude that the $\phi$ meson is likely to decay inside the hadronic fireball.

This suggestion implies the following scenario for possibly resolving the NA49 and NA50 “phi puzzle”. At high temperatures ($T \approx T_c$) the phi lifetime is short, and it will decay, both into $K^+K^-$ and $\mu^+\mu^-$. The daughter kaons will be reabsorbed, while the dileptons will reach the detector. As the system expands and cools, the lifetime for the phi increases as medium effects are diminished. The $\phi$ mesons that decay near the freezeout surface and beyond have returned to vacuum behavior. Therefore the two-kaon distributions are expected to have the free-space width. The prediction in this calculation, if there is one, is that the dilepton signal will show an in-medium spectral function broadened by collisions while the hadronic signal will show free-space behavior. Finally, the apparent branching ratio of the dilepton channel will increase by something like a factor of 2–5. This feature is currently being studied quantitatively[17].

Acknowledgments

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