Microwave surface resistance in superconductors with grain boundaries

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I. INTRODUCTION

High-temperature superconductors contain many grain boundaries (GBs), where the order parameter is locally suppressed due to the short coherence length. GBs have attracted much interest for their basic physics as well as for their applications in superconductors and play a crucial role in microwave response and surface resistance \( R_s \) of high-temperature superconducting films.

Electrodynamics of GB junctions can be described using the Josephson-junction model, and one of the most important parameters that characterize GB junctions is the critical current density \( J_{c2} \) for Josephson tunneling current across GBs. The \( J_{c2} \) strongly depends on the misorientation angle of GBs and \( J_{c2} \) can be enhanced and \( R_s \) reduced by Ca doping. The investigation of the relationship between \( R_s \) and \( J_{c2} \) is needed to understand the behavior of GBs.

In this paper, we present theoretical investigation on the microwave field and dissipation in superconductors with laminar GBs. Theoretical expressions of the surface impedance \( Z_s = R_s + iX_s \) of superconductors with GBs are derived as functions of \( J_{c2} \) at GB junctions.

II. BASIC EQUATIONS

A. Superconductors with grain boundaries

We consider penetration of a microwave field (i.e., magnetic induction \( B = \mu_0 H \), electric field \( E \), and current density \( J \)) into superconductors that occupy a semi-infinite area of \( x > 0 \). We investigate linear response for small microwave power limit, such that the time dependence of the microwave field is expressed by the harmonic factor, \( e^{-i\omega t} \), where \( \omega/2\pi \) is the microwave frequency that is much smaller than the energy-gap frequency of the superconductors. Magnetic induction \( B \) is assumed to be less than the lower critical field, such that no vortices are present in the superconductors. (See Ref. [20] for microwave response of vortices.)

The GBs are modeled to have laminar structures as in Ref. [21] the laminar GBs that are parallel to the \( xz \) plane are situated at \( y = ma \), where \( a \) is the spacing between grains (i.e., effective grain size) and \( m = 0, \pm 1, \pm 2, \cdots, \pm \infty \). The thickness of the barrier of GB junctions, \( d_j \), is much smaller than both \( a \) and the London penetration depth \( \lambda \), and therefore, we investigate the thin-barrier limit of \( d_j \rightarrow 0 \), namely, GB barriers situated at \( ma - 0 < y < ma + 0 \).

B. Two-fluid model for intragrain current

We adopt the standard two-fluid model for current transport in the grain at \( ma + 0 < y < (m + 1)a - 0 \). The intragrain current \( J = J_s + J_n \) is given by the sum of the supercurrent \( J_s = i\sigma_s E \) and the normal current \( J_n = \sigma_n E \), where \( \sigma_s = 1/\omega\mu_0\lambda^2 \) and \( \sigma_n \) is the normal-fluid conductivity in the grains. The displacement current \( J_d = -i\omega\varepsilon E \) with the dielectric constant \( \varepsilon \) can be neglected for a microwave range of \( \omega/2\pi \sim \) GHz. Ampère’s law \( \mu_0^{-1}\nabla \times B = (\sigma_n + i\sigma_s)E \) is thus reduced to

\[
E = -i\omega\Lambda_g^2\nabla \times B, \tag{1}
\]

where \( \Lambda_g \) is the intragrain ac field penetration depth defined by

\[
\Lambda_g^{-2} = \omega\mu_0(\sigma_s - i\sigma_n) = \lambda^{-2} - i\omega\mu_0\sigma_n. \tag{2}
\]

Combining Eq. (1) with Faraday’s law, \( \nabla \times E = i\omega B \), we obtain the London equation for magnetic induction \( B = B_z(x,y)\hat{z} \) for \( y \neq ma \) as

\[
B_z - \Lambda_g^2\nabla^2 B_z = 0. \tag{3}
\]

For ideal homogeneous superconductors without GBs, Eq. (3) is valid for \( -\infty < y < +\infty \) and the solution is simply given by

\[
B_z(x) = \mu_0 H_0 e^{-x/\Lambda_s}, \tag{4}
\]

where type II superconductors with laminar GBs.
the electric field is obtained from Eq. (1) as $E_y(x) = -i\omega\mu_0\lambda_y H_0 e^{-x/\Lambda_s}$. The surface impedance $Z_{s0} = R_{s0} - iX_{s0}$ for homogeneous superconductors is given by $Z_{s0} = E_y(x = 0)/H_0 = -i\omega\mu_0\Lambda_g$. The surface resistance $R_{s0} = \text{Re}(Z_{s0})$ and reactance $X_{s0} = -\text{Im}(Z_{s0})$ of ideal homogeneous superconductors without GBs are given by

$$R_{s0} = \mu_0^2 \omega^2 \lambda^3 \sigma_n/2, \quad (4)$$

$$X_{s0} = \mu_0 \omega \lambda \quad (5)$$

for $\sigma_n/\sigma_s \ll 1$ well below the superconducting transition temperature $T_c$.

C. Josephson-junction model for intergrain current

We adopt the Josephson-junction model\textsuperscript{14,15,16} for tunneling current across GBs at $y = ma$. Behavior of the GB junctions is determined by the gauge-invariant phase difference across GBs, $\varphi_j(x)$, and the voltage induced across GB, $V_j(x)$, is given by the Josephson’s relation,

$$\int_{y=ma-0}^{y=ma+0} E_y dy = V_j = \frac{\phi_0}{2\pi} (-i\omega \varphi_j), \quad (6)$$

where $\phi_0$ is the flux quantum. The tunneling current parallel to the $y$ axis is given by the sum of the superconducting tunneling current (i.e., Josephson current) $J_{sj} = J_{sj}\sin \varphi_j$ and the normal tunneling current (i.e., quasiparticle tunneling current) $J_{nj} = \gamma_{nj} V_j$. The critical current density $J_{cJ}$ at GB junctions is one of the most important parameters in the present paper, and the resistance-area product of GB junctions corresponds to $1/\gamma_{nj}$. We neglect the displacement current across GBs, $J_{dj} = -i\omega C_j V_j$ where $C_j$ is the capacitance of the GB junctions.

Here we define the Josephson length $\lambda_J$ and the characteristic current density $J_0$ as

$$\lambda_J = (\phi_0/4\pi\mu_0 J_{cJ}\lambda)^{1/2}, \quad (7)$$

$$J_0 = \phi_0/4\pi\mu_0 \lambda^3. \quad (8)$$

The ratio $J_{cJ}/J_0 = (\lambda/\lambda_J)^2$ characterizes the coupling strength of GB junctions.\textsuperscript{22} For weakly coupled GBs, namely, $J_{cJ}/J_0 = (\lambda/\lambda_J)^2 \ll 1$ (e.g., high-angle GBs), electrodynamics of the GB junctions can be well described by the weak-link model.\textsuperscript{14,15,16} For strongly coupled GBs, namely, $J_{cJ}/J_0 = (\lambda/\lambda_J)^2 \geq 1$ (e.g., low-angle GBs), the Josephson-junction model is still valid but requires appropriate boundary conditions at GBs, as given in Eq. (4) in Ref.\textsuperscript{22} as pointed out by Gurevich; see also Refs.\textsuperscript{21} and\textsuperscript{24}.

In the small-microwave-power limit such that $\sin \varphi_j \simeq \varphi_j = 2\pi V_j/(-i\omega \phi_0)$ for $|\varphi_j| \ll 1$, the $J_{cJ}$ is reduced to

$$J_{sj} \simeq J_{cJ}\varphi_j = i\gamma_{sj} V_j, \quad (9)$$

where $\gamma_{sj} = 2\pi J_{cJ}/\phi_0 = 1/2\mu_0 \lambda^2 \gamma_{nj}$. The total tunneling current across GB is thus given by

$$-\frac{1}{\mu_0} \frac{\partial B_z}{\partial x} \bigg|_{y=ma} = J_{sj} + J_{nj} = (i\gamma_{sj} + \gamma_{nj}) V_j. \quad (10)$$

Integration of Faraday’s law, $\partial E_y/\partial x - \partial E_z/\partial y = i\omega B_z$, yields

$$E_z(x, y = ma + 0) - E_z(x, y = ma - 0) = \int_{y=ma-0}^{y=ma+0} dy \left[ \frac{\partial E_y(x, y)}{\partial x} - i\omega B_z(x, y) \right] = \frac{\partial V_j(x)}{\partial x} \quad (11)$$

where we used Eq. (10). The static version (i.e., $\omega \to 0$) of Eq. (11) corresponds to Eq. (4) in Ref.\textsuperscript{22} Substitution of Eqs. (10) and (11) into Eq. (11) yields the boundary condition for $B_z$ at $y = ma$,

$$-\frac{\partial B_z}{\partial y} \bigg|_{y=ma+0} + \frac{\partial B_z}{\partial y} \bigg|_{y=ma-0} = \frac{a\lambda_J^2 \partial^2 B_z}{\Lambda_g^2 \partial x^2} \bigg|_{y=ma}, \quad (12)$$

where $\Lambda_j$ is the characteristic length for ac field penetration into GBs defined by

$$\Lambda_j^{-2} = \omega \mu_0 a (\gamma_{sj} - i\gamma_{nj}) = \mu_0 a (2\pi J_{cJ}/\phi_0 - i\omega \gamma_{nj}). \quad (13)$$

III. SURFACE IMPEDANCE

A. Microwave field and surface impedance

Equations (3) and (12) are combined into a single equation for $x > 0$ and $-\infty < y < +\infty$ as

$$B_z - \lambda_g^2 \nabla^2 B_z = a\lambda_J^2 \sum_{m=-\infty}^{+\infty} \frac{\partial^2 B_z}{\partial x^2} \delta(y - ma), \quad (14)$$

whose solution is calculated as

$$B_z(x, y) = e^{-x/\Lambda_s} + \frac{2}{\pi} \int_0^\infty dk \frac{\cosh[K(y - a/2)]}{\Lambda_g^2 K^2 \sinh[Ka/2]} \times \frac{k \sin kx}{(2\lambda_g^2/a\lambda_J^2) + k^2 \coth[Ka/2]} \quad (15)$$

for $0 < y < a$, where $K = (k^2 + \lambda_g^{-2})^{1/2}$. The right-hand side of Eq. (15) and the second term of the right-hand side of Eq. (15) reflect the GB effects. See Appendix A for the derivation of Eq. (15) from Eq. (14).

Electric field in the grains is obtained from Eq. (11) as $E_y = i\omega \lambda_g^2 \partial B_z/\partial x$, and voltage induced across GB is obtained from Eq. (11) as $V_j = i\omega \Lambda_J^2 \partial B_z/\partial x \bigg|_{y=0}$. The mean electric field $\bar{E}_S$ at the surface of the superconductor is thus calculated as

$$\bar{E}_S = \frac{1}{a} \int_{-a}^{a} dy E_y(x = 0, y)$$
Substitution of Eq. (14) into Eq. (16) yields the surface impedance \( Z_s = R_s - iX_s = \tilde{E}_s/H_0 \) as
\[
\frac{Z_s}{-i\omega \mu_0 \Lambda_g} = 1 + \frac{2}{\pi} \int_0^\infty \frac{dk}{k^3} \frac{1}{(\Lambda_g^2/k^2) + (k^2\sigma_s/2) \coth(Ka/2)}.
\]
(17)
The surface resistance and reactance are given by \( R_s = \text{Re}(Z_s) \) and \( X_s = -\text{Im}(Z_s) \), respectively.

### B. Microwave dissipation and surface resistance

The time-averaged electromagnetic energy passing through the surface of a superconductor at \( x = 0 \) and \( -a < y < a - 0 \) is given by the real part of
\[
\mathcal{E} = \frac{1}{2\mu_0} \int_{-a}^{a} dy \left( E_y B_z \right)_{x=0} = \frac{a}{2} \tilde{E}_s H_0,
\]
(18)
where \( \tilde{E}_s = Z_s H_0 \) is defined by Eq. (10), and \( (B_z)_{x=0} = \mu_0 H_0 \). Poynting’s theorem\(^2\) states that \( \mathcal{E} \) is identical to the energy stored and dissipated in the superconductor, thereby
\[
\mathcal{E} = \frac{1}{2} \int_{0}^\infty dx \left[ \int_{-a}^{a} \left( \sigma_n - i\sigma_s \right) |E|^2 dy + (\gamma_{nj} - i\gamma_{sj}) |V_j|^2 - \int_{-a}^{a} dy \frac{i\omega}{\mu_0} |B_z|^2 \right].
\]
(19)
The real parts of Eqs. (18) and (19) show that the surface resistance \( R_s = \text{Re}(Z_s) \) is composed of two terms:
\[
R_s = R_{sg} + R_{sj}.
\]
(20)
The intragrain contribution \( R_{sg} \) is from the energy dissipation in the grains, and the intergrain contribution \( R_{sj} \) is from the dissipation at GBs:
\[
R_{sg} = \frac{1}{a|H_0|^2} \int_{0}^\infty dx \int_{-a}^{a} dy \sigma_n |E|^2,
\]
(21)
\[
R_{sj} = \frac{1}{a|H_0|^2} \int_{0}^\infty dx \gamma_{nj} |V_j|^2.
\]
(22)
Both the intragrain current \( |J_g| \) around GBs and the intergrain tunneling current \( |J_f| \) across GBs are suppressed by the GBs, and are increasing functions of \( J_{cj} \). With increasing \( J_{cj} \), the intragrain electric field \( |E| = |J_g/(\sigma_n + i\sigma_s)| \) also increases, whereas the intergrain voltage \( |V_j| = |J_f/(\gamma_{nj} + i\gamma_{sj})| \) decreases because \( \gamma_{nj} \propto J_{cj} \). The dissipation in the grains, \( \sigma_n |E|^2/2 \), and the intragrain contribution to the surface resistance, \( R_{sg} \), therefore, tend to increase with increasing \( J_{cj} \). The dissipation at GBs, \( \gamma_{nj} |V_j|^2/2 \), and the intergrain contribution to the surface resistance, \( R_{sj} \), on the other hand, decrease with increasing \( J_{cj} \).

The surface reactance \( X_s = -\text{Im}(Z_s) \) is also divided into two contributions,
\[
X_s = X_{sg} + X_{sj},
\]
(23)
where the intragrain contribution \( X_{sg} \) and the intergrain contribution \( X_{sj} \) are given by
\[
X_{sg} = \frac{1}{a|H_0|^2} \int_{0}^\infty dx \int_{-a}^{a} dy \left( \sigma_n |E|^2 + \frac{i\omega}{\mu_0} |B_z|^2 \right),
\]
(24)
\[
X_{sj} = \frac{1}{a|H_0|^2} \int_{0}^\infty dx \gamma_{sj} |V_j|^2.
\]
(25)
Both \( X_{sg} \) and \( X_{sj} \) decrease with increasing \( J_{cj} \).

### C. Simplified expressions for surface impedance

The following Eqs. (26)–(34) show simplified expressions of the surface impedance \( Z_s \), the surface resistance \( R_s = \text{Re}(Z_s) \), and the surface reactance \( X_s = -\text{Im}(Z_s) \) for certain restricted cases, assuming \( \sigma_n/\sigma_s \ll 1 \) and \( \gamma_{nj}/\gamma_{sj} \ll 1 \) well below the transition temperature.

For small grains of \( a \ll \lambda \) such that \( \coth(Ka/2) \simeq 2/Ka \), Eq. (17) is reduced to
\[
Z_s \simeq -i\omega \mu_0 \left( \Lambda_g^2 + \Lambda_s^2 \right)^{1/2}.
\]
(26) The right-hand side of Eq. (17) is reduced to \( \Lambda_g^2 \partial^2 B_z/\partial x^2 \) for \( a \ll \lambda \), and the effective ac penetration depth is given by \( \Lambda_{\text{eff}} \simeq (\Lambda_g^2 + \Lambda_s^2)^{1/2} \) as in Ref. 21, resulting in the surface impedance given by Eq. (26). The \( R_s \) and \( X_s \) for small grains is obtained as
\[
\frac{R_s}{R_{s0}} \simeq \left( 1 + 2\frac{\lambda}{\lambda_{cj}} \right)^{-1/2} \left( 1 + 4\frac{\gamma_{nj}}{\sigma_n} \left( \frac{J_0}{\lambda_{cj}} \right)^2 \right),
\]
(27)
\[
\frac{X_s}{X_{s0}} \simeq \left( 1 + 2\frac{\lambda}{\lambda_{cj}} \right)^{+1/2},
\]
(28)
where \( R_{s0}, \ X_{s0} \), and \( J_0 \) are defined by Eqs. (14), (5) and (8), respectively. Equation (26) is further simplified when \( a \ll 2\lambda_s^2/\lambda \) for small grain and weakly coupled GBs as
\[
Z_s \simeq -i\omega \mu_0 \Lambda_j,
\]
(29)
and we have

$$\frac{R_s}{R_{s0}} \simeq \frac{2\gamma_{nj}\lambda}{\sigma_n} \left(\frac{2\lambda}{a}\right)^{1/2} \left(\frac{J_0}{J_{cj}}\right)^{3/2},$$

(30)

$$\frac{X_s}{X_{s0}} \simeq \left(\frac{2\lambda}{a}\right)^{1/2} \left(\frac{J_0}{J_{cj}}\right)^{1/2}.$$  

(31)

Thus, we obtain the dependence of $R_s$ and $X_s$ on the material parameters as $R_s \propto \gamma_{nj}a^{-1/2}J_{cj}^{-3/2}$ and $X_s \propto a^{-1/2}J_{cj}^{-1/2}$, which are independent of $\lambda$. The $R_s$ given by Eq. (30) for the small grain and weakly coupled GBs is mostly caused by intergrain dissipation, $R_s \simeq R_{sj} \gg R_{sg}$. For $X_s$ given by Eq. (31), on the other hand, both intragrain $X_{sg}$ and intergrain $X_{sj}$ contribute to the total $X_s = X_{sg} + X_{sj}$.

For large $J_{cj}$ (i.e., strong-coupling limit) such that $KA_s^2/\Lambda_s^2 \gg (k^2a/2)\coth(Ka/2)$, Eq. (14) for the surface impedance $Z_s$ is simplified as

$$Z_s \simeq -i\omega\mu_0 \left(\Lambda_g + \Lambda_s^2/2\Lambda_g\right),$$

(32)

and we have

$$\frac{R_s}{R_{s0}} \simeq 1 - \frac{\lambda J_0}{\alpha J_{cj}} + 4\frac{\lambda^2\gamma_{nj}}{a\sigma_n} \left(\frac{J_0}{J_{cj}}\right)^2,$$

(33)

$$\frac{X_s}{X_{s0}} \simeq 1 + \frac{\lambda J_0}{\alpha J_{cj}}.$$  

(34)

The first and second terms of the right-hand side of Eq. (33) correspond to the intragrain contribution, $R_{sj}$, whereas the third term corresponds to the intergrain contribution, $R_{sj}$.

**IV. DISCUSSION**

Figure 1(a) and (b) shows $J_{cj}$ dependence of $R_s$. As shown in Fig. 1(a), the intergrain contribution $R_{sj}$ is dominant for weakly coupled GBs (i.e., small $J_{cj}/J_0$ regime), whereas the intragrain contribution $R_{sg}$ is dominant for strongly coupled GBs (i.e., large $J_{cj}/J_0$). The $R_{sj}$ decreases with increasing $J_{cj}$ as $R_{sj} \propto J_{cj}^{-1.5}$ [see Eq. (33)], whereas $R_{sg}$ increases with $J_{cj}$. The resulting surface resistance $R_s = R_{sj} + R_{sg}$ nonmonotonically depends on $J_{cj}$ and has a minimum, because $R_s$ is determined by the competition between $R_{sj}$ and $R_{sg}$. As shown in Fig. 1(c), on the other hand, $X_s$ monotonically decreases with increasing $J_{cj}$ [i.e., $X_s \propto J_{cj}^{-0.5}$ for weakly coupled GBs as in Eq. (34)].

The nonmonotonic dependence of $R_s$ on the grain size $a$ is also seen in Fig. 1(b). For small $J_{cj}/J_0$ the $R_s$ decreases with increasing $a$ as $R_s \propto a^{-0.5}$ [see Eq. (34)], whereas $R_s$ increases with $a$ for large $J_{cj}/J_0$.

The $R_s$ for strongly coupled GBs can be smaller than $R_{s0}$ for ideal homogeneous superconductors without GBs, namely, $R_s/R_{s0} < 1$ for $J_{cj}/J_0 \approx 1$. The minimum surface resistance for $\lambda\gamma_{nj}/\sigma_n = 0.2$ is $R_s/R_{s0} \approx 0.97$ for $a/\lambda = 5$, $R_s/R_{s0} \approx 0.86$ for $a/\lambda = 1$, and $R_s/R_{s0} \approx 0.59$ for $a/\lambda = 0.1$. The minimum $R_s/R_{s0}$ is further reduced when $\lambda\gamma_{nj}/\sigma_n$ is further reduced.

Theoretical results shown above may possibly be observed by measuring $R_s$, $X_s$, and $J_{cj}$ in Ca doped YBa$_2$Cu$_3$O$_{7-\delta}$ films. The enhancement of $J_{cj}$ (Ref. 14) and reduction of $R_s$ (Ref. 15) by Ca doping are individ-
ually observed in YBa$_2$Cu$_3$O$_{7-\delta}$, but simultaneous measurements of $J_{c2}$ and $R_s$ are needed to investigate the relationship between $R_s$ and $J_{c2}$. The nonmonotonic $J_{c2}$ dependence of $R_s$ for strongly coupled GBs may be observed in high quality films with small grains $a < \lambda$ and with large $J_{c2}$ on the order of $J_0 \sim 10^{10}$ A/m$^2$ at low temperatures.

V. CONCLUSION

We have theoretically investigated the microwave-field distribution in superconductors with laminar GBs. The field calculation is based on the two-fluid model for current transport in the grains and on the Josephson-junction model for tunneling current across GBs. Results show that the microwave dissipation at GBs is dominant for weakly coupled GBs of $J_{c2} \ll J_0$, whereas dissipation in the grains is dominant for strongly coupled GBs of $J_{c2} \gg J_0$. The surface resistance $R_s$ nonmonotonically depends on $J_{c2}$; the $R_s$ decreases with increasing $J_{c2}$ as $R_s \propto J_{c2}^{-1.5}$ for $J_{c2} \ll J_0$, whereas $R_s$ increases with $J_{c2}$ for $J_{c2} \gg J_0$. The intragrain dissipation can be suppressed by GBs, and the surface resistance of superconductors with GBs can be smaller than that of ideal homogeneous superconductors without GBs.

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APPENDIX A

Equation (15) is derived by solving Eq. (14) with the boundary condition of $B_z = \mu_0 H_0$ at $x = 0$, as follows. We introduce the Fourier transform of $B_z(x, y)$ and

$$B_z(x, ma) = B_z(x, 0)$$

respectively. The Fourier transform of Eq. (14) leads to

$$\hat{b}(k, q) = \int_0^\infty dx \int_{-\infty}^{+\infty} dy B_z(x, y)e^{-iqy} \sin kx, \quad (A1)$$

$$\hat{b}_0(k) = \int_0^\infty dx B_z(x, 0) \sin kx = \int_{-\infty}^{+\infty} dq \frac{\hat{b}(k, q)}{2\pi}, \quad (A2)$$

where $K = (k^2 + \Lambda_0^2)^{1/2}$ and $\alpha = \alpha^0 k^2 / \Lambda_0^2$. Substituting Eq. (A3) into Eq. (A2), we have

$$\hat{b}_0(k) = \frac{k}{\mu_0 H_0} + \alpha k \sum_m \int_{-\infty}^{+\infty} dq \frac{e^{-imq \Lambda_0^2}}{2\pi [K^2 + q^2]}.$$ 

which is reduced to

$$\frac{\hat{b}_0(k)}{\mu_0 H_0} = \frac{1}{k K \Lambda_0^2} \frac{2}{2K + \alpha k^2 \coth(Ka/2)} \frac{1}{k} \hat{b}_0(k), \quad (A4)$$

$B_z(x, y)$ is calculated from $\hat{b}(k, q)$ given by Eq. (A3) as

$$B_z(x, y) = \frac{2}{\pi} \int_0^\infty dk \int_{-\infty}^{+\infty} dq \frac{\hat{b}(k, q)}{\mu_0 H_0} e^{iqy} \sin kx$$

$$= e^{-x/\Lambda_s} + \frac{2\alpha}{\pi} \int_0^\infty dk k \sin kx \left[ 1 - \frac{k \hat{b}_0(k)}{\mu_0 H_0} \right]$$

$$\times \sum_m \int_{-\infty}^{+\infty} dq e^{iq(y-ma)} \frac{e^{-imq \Lambda_0^2}}{2\pi [K^2 + q^2]}.$$ 

Substitution of Eq. (A5) into Eq. (A6) yields Eq. (16).

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