A Model of TeV Scale Physics for Neutrino Mass, Dark Matter and Baryon Asymmetry and its Phenomenology

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Abstract

We discuss some details of the model proposed in Ref. \cite{1}, in which neutrino oscillation, dark matter, and baryon asymmetry of the Universe would be simultaneously explained by the TeV-scale physics without introducing very high mass scales. An exact discrete $Z_2$ symmetry is introduced, under which new particle contents (a real singlet scalar field, a pair of charged singlet scalar fields and TeV-scale right-handed neutrinos) are assigned to have odd quantum number, whereas ordinary gauge fields, quarks and leptons, and two Higgs doublets are even. Tiny neutrino masses are generated at the three loop level due to the exact $Z_2$ symmetry, by which stability of the dark matter candidate is also guaranteed. The extra Higgs doublet is required not only for the tiny neutrino masses but also for successful electroweak baryogenesis. We discuss phenomenological properties of the model, and find that there are successful scenarios in which above three problems are solved simultaneously under the constraint from current experimental data. We then discuss predictions in such scenarios at ongoing and future experiments. It turns out that the model provides discriminative predictions especially in Higgs physics and dark matter physics, so that it is testable in near future.

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I. INTRODUCTION

While the standard model (SM) for elementary particles has been successful in describing high energy phenomena at colliders, today we have definite motivation to consider a model beyond the SM. First of all, observed data for neutrino oscillation indicate that neutrinos have tiny masses and mix with each other [2]. Second, cosmological data have revealed that the density of dark energy and dark matter (DM) in the Universe dominates that of baryonic matter [3]. The essence of DM would be weakly interacting massive particles (WIMPs). Finally, asymmetry of matter and anti-matter in our Universe has been addressed for a long time as a serious problem regarding existence of ourselves [4]. They are all beyond the scope of the SM, so that a new model is required to explain these phenomena.

A simple scenario to generate tiny neutrino masses ($m_\nu$) would be based on the seesaw mechanism with heavy right-handed (RH) Majorana neutrinos [5]. Then, $m_\nu$ can be described as $m_\nu \simeq m_D^2/M_R$, where $M_R$ ($\sim 10^{13-16}$ GeV) is the Majorana mass of RH neutrinos and $m_D$ is the Dirac mass of at most the electroweak scale. This scenario would be compatible with the framework with large mass scales like grand unification. The heavy RH Majorana neutrinos would generate lepton number asymmetry in their CP violating decays that could be transferred into baryon asymmetry [6]. In a supersymmetric model with such heavy RH neutrinos, one might find a DM candidate of lightest supersymmetric particle [7]. Introduction of such large scales, however, causes a problem of hierarchy. In addition, the decoupling theorem [8] makes it far from experimental tests.

An alternative approach is a quantum mechanical generation of small neutrino masses. The original idea of radiatively generating neutrino masses due to TeV-scale physics has been proposed by Zee [9]. Adding an isospin doublet scalar field and a charged singlet field to the SM, the neutrino masses are generated at the one-loop level. Phenomenology in the Zee model has been studied in Ref. [10, 11]. Although the original Zee model has been excluded by the neutrino data, lots of studies for its extensions and variations have been proposed [12]. Another possibility for generating neutrino masses via the new scalar particles is the Zee-Babu model [13, 14], where the neutrino masses arise at the two-loop level. Some extensions of these models are discussed for the purpose of the explanation for DM and baryon asymmetry [15, 16]. The extension with a TeV-scale RH neutrino has been discussed in Ref. [17], where the neutrino masses are generated at the three-loop due to the
exact $Z_2$ symmetry which forbids the Dirac neutrino mass at the tree level unlike the tree-level seesaw mechanism mentioned above, and the $Z_2$-odd RH neutrino is a candidate of DM. This model has been extended with two RH neutrinos for the description of the neutrino data [18], which however cannot include a mechanism of baryon number generation. The idea of simultaneous explanation for radiative generation of neutrino masses and stability of DM by introducing various exact discrete symmetries with right-handed neutrinos has been used in several models [19]. Some other models including mechanisms for baryon asymmetry have been considered in the leptogenesis scenario [20] and also in the scenario of electroweak baryogenesis [1, 21].

In this paper, we investigate some details of a model proposed in Ref. [1], in which neutrino oscillation, origin of DM and baryon asymmetry would be simultaneously explained by the TeV-scale physics. As we try to built a renormalizable TeV scale model to explain these phenomena simultaneously without fine-tuning, we do not impose unnatural hierarchy among the mass scales. The model contains an extended Higgs sector with TeV-scale RH neutrinos in addition to the SM particle contents. Tiny masses of left-handed (LH) neutrinos are generated at the three-loop level under an exact $Z_2$ symmetry. The lightest neutral $Z_2$-odd state is a candidate of DM. Baryon asymmetry can be generated at the electroweak phase transition (EWPT) by the non-decoupling property [22] and additional CP violating phases in the Higgs sector [23, 24]. In this framework, a successful model can be built without contradiction of the current data. Notice that in this model we do not intend to solve the so-called hierarchy problem. The model predicts quadratic divergences in quantum corrections to masses of the scalar bosons as in the SM. This model, therefore, has to be considered as an effective theory below the cut-off scale of a more fundamental theory around at most 10 TeV, below which the self-coupling constants for additional scalar bosons do not become larger than the acceptable values for a perturbation calculation.

We show that there are several possible scenarios, in which the data for neutrinos, lepton flavor violation as well as the WMAP data are satisfied. The collider data from experiments at LEP, Fermilab Tevatron and B factories at KEK and SLAC are also taken into account. In addition to the original scenario discussed in Ref [1], we mention another scenario where the recent data from PAMELA and ATIC would also be included with the relatively heavy DM candidate. Furthermore, we also discuss the other scenario where the anomaly in the data from DAMA/LIBRA would be explained by a light DM candidate whose mass is around 5
GeV. It turns out that in these scenarios all the masses of additional physical particles are between $\mathcal{O}(10)$ GeV and $\mathcal{O}(1)$ TeV. We find that the model has discriminative features in Higgs phenomenology, lepton flavor physics and DM physics, so that it is testable at current and future experiments.

In Sec. II, we define the model with introducing new particle entries and symmetries, and discuss the physical states and their masses and coupling constants. In Sec. III, we calculate the neutrino mass matrix in our model, which is induced at the three loop level, and discuss parameter sets in which all the neutrino data are reproduced under current experimental bounds. Sec. IV is devoted to the discussion on the possibility that the lightest $Z_2$ odd scalar boson is a candidate of DM. The thermal relic abundance is evaluated in several parameter sets, and implication for the physics at direct and indirect search experiments are also discussed. In Sec. V, we study the allowed region where the strong first order electroweak phase transition is realized. This is required for a successful scenario of electroweak baryogenesis. In Sec. VI, we summarize the constraints from the current experimental data on the model, and discuss phenomenological predictions at ongoing and future collider experiments and at direct/indirect searches for DM. Some formulas are summarized in the Appendices.

II. MODEL

A. Particle contents and symmetries

In addition to the known SM fields, new particle entries in our model are

$$\Phi_1, \Phi_2, S^\pm, \eta, N^\alpha_R,$$

where $\Phi_1, \Phi_2$ are scalar isospin doublet fields with the hypercharge 1/2, $S^\pm$ are charged isospin singlet fields, $\eta$ is a real scalar singlet field, and $N^\alpha_R$ is the $\alpha$-th generation isospin-singlet RH neutrinos. It turns out that at least two generations are necessary for $N^\alpha_R$ to reproduce the neutrino data. In the following, we mainly consider the minimum model with two generation $N^\alpha_R$ ($\alpha = 1, 2$). We give a comment for the case of more than three generations later. We here only note that the mass scale of new particles derived in the following sections is not so sensitive against the number of $N_R$ generation. This means that the model has high predictability for the mass scale of new particles.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
 & $Q^i$ & $u_R^i$ & $d_R^i$ & $L^i$ & $e_R^i$ & $\Phi_1$ & $\Phi_2$ & $S^\pm$ & $\eta$ & $N_R^\alpha$ \\
\hline
$Z_2$ (exact) & + & + & + & + & + & + & + & + & + & + \\
\hline
$\tilde{Z}_2$ (softly broken) & + & − & + & + & + & + & + & + & + & + \\
\hline
\end{tabular}
\caption{Particle properties under the discrete symmetries.}
\end{table}

In order to generate tiny neutrino masses in the three loop level, in other words to forbid the tree-level Dirac neutrino mass term, and at the same time in order to have a stable DM candidate, we impose a new (exact) $Z_2$ symmetry as in Ref. \cite{17}, which we refer to as $Z_2$. We assign the $Z_2$ odd charge to $N_R^\alpha$, $S^\pm$ and $\eta$, while ordinary gauge fields, quarks and leptons and Higgs doublets are $Z_2$ even.

It has been well known that introduction of the extra Higgs doublet causes a problem of dangerous flavor changing neutral current (FCNC). To avoid FCNC in a natural way, we further introduce the (softly-broken) $Z_2$ symmetry in the model, which can be softly broken \cite{25} in the Higgs potential. We refer to this symmetry as the $\tilde{Z}_2$ symmetry in this paper. Under the $\tilde{Z}_2$ symmetry, there can be four independent types of the Yukawa interaction, depending on the assignment of $\tilde{Z}_2$ charges for quarks and leptons. From a phenomenological reason discussed later, we employ the so-called Type-X Yukawa coupling where $\tilde{Z}_2$ charges are assigned such that only $\Phi_1$ couples to leptons whereas $\Phi_2$ does to quarks \cite{19, 20, 21, 23}. We summarize the particle properties under $Z_2$ and $\tilde{Z}_2$ in TABLE II.

Under these discrete symmetries, the Yukawa interaction is given by

$$L_Y = -y_{\ell_i} \overline{\ell_i} \Phi_1 \ell_R^i - y_{u_i} \overline{u_R^i} \tilde{\Phi}_2 u_R^i - y_{d_i} \overline{d_R^i} \Phi_2 d_R^i + \text{h.c.},$$

where $Q^i$ ($L^i$) is the ordinary $i$-th generation LH quark (lepton) doublet, and $u_R^i$ and $d_R^i$ ($e_R^i$) are RH-singlet up- and down-type quarks (charged leptons), respectively. Notice that the Type-X Yukawa coupling defined in Eq. (2) \cite{26, 27, 28, 29} is different from that in the Type-I or Type-II two-Higgs-doublet model (THDM) \cite{30}. Our $\tilde{Z}_2$ charge assignment for quarks is the same as that in Type I, but that for leptons is the same as of Type II. The charged Higgs boson couplings to leptons are multiplied by tan $\beta$, while those to quarks are by cot $\beta$ in a universal way, where tan $\beta = \langle \Phi_2^0 \rangle / \langle \Phi_1^0 \rangle$. Therefore, phenomenology of the Higgs sector in our model is completely different from that in Type I and Type II.
The scalar potential is then given by

\[
V = -\mu_1^2 |\Phi_1|^2 - \mu_2^2 |\Phi_2|^2 - (\mu_{12} \Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
+ \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \left\{ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\} \\
+ \mu_s^2 |S|^2 + \lambda_s |S|^4 + \frac{1}{2} \mu_s^2 \eta^2 + \frac{1}{2} \mu_\eta^2 \eta^4 + \xi |S|^2 \eta^2 + \sum_{a=1}^2 \left( \rho_a |\Phi_a|^2 |S|^2 + \sigma_a |\Phi_a|^2 \eta^2 \right) \\
+ \sum_{a,b=1}^2 \left\{ \kappa \epsilon_{ab} (\Phi_a^\dagger \Phi_b S^- \eta + \text{h.c.}) \right\},
\]

where \( \epsilon_{ab} \) is the anti-symmetric tensor with \( \epsilon_{12} = 1 \). The mass term and the interaction for \( N^a_R \) are given by

\[
\mathcal{L}_Y = \sum_{a=1}^2 \left\{ \frac{1}{2} m_{N^a_R} N^a_R N^a_R + h_{eR}^\alpha (e_R^\alpha c N^a_R S^+ + \text{h.c.}) \right\}.
\]

The parameters \( \mu_{12}^2, \lambda_5 \) and \( \kappa \) as well as \( h_{i\alpha}^e \) are generally complex. The phases of \( \lambda_5 \) and \( \kappa \) can be eliminated by re-phasing \( S^\pm \) and \( \Phi_1 \). The remaining phase of \( \mu_{12}^2 \) is physical and causes CP violation in the Higgs sector, which is necessary for generating baryon asymmetry at the EWPT \cite{23, 24}. Although the CP violating phase is crucial for successful baryogenesis, it does not much affect in the following discussions on neutrino masses, DM and the strong first order EWPT required for electroweak baryogenesis. Thus, in the following, we neglect the phase of \( \mu_{12}^2 \) (and \( h_{i\alpha}^e \)) for simplicity. We later give a comment on how the phenomenology could change with non-zero CP-violating phase.

**B. Higgs states and coupling constants**

Because \( Z_2 \) is exact, \( Z_2 \) even and odd fields cannot mix. The Higgs doublet fields \( \Phi_i \) \((i = 1, 2) \) can be parameterized as

\[
\Phi_i = \begin{pmatrix} \omega_i^+ \\ \frac{1}{\sqrt{2}} (v_i + h_i + i z_i) \end{pmatrix},
\]

where \( v_i \) are vacuum expectation values and satisfy \( \sqrt{v_1^2 + v_2^2} = v \approx 246 \text{ GeV} \), and \( \tan \beta = v_2/v_1 \). The \( Z_2 \) even states are diagonalized in mass as a usual THDM by introducing the
mixing angles $\alpha$ and $\beta$, where $\alpha$ is that between CP even neutral Higgs bosons $w$.

$$\begin{bmatrix} w_1^+ \\ w_2^+ \end{bmatrix} = \begin{bmatrix} \cos \beta - \sin \beta \\ \sin \beta \cos \beta \end{bmatrix} \begin{bmatrix} w^+ \end{bmatrix}, \quad \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos \beta - \sin \beta \\ \sin \beta \cos \beta \end{bmatrix} \begin{bmatrix} z \end{bmatrix},$$  \hspace{1cm} (6)

and

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha - \sin \alpha \\ \sin \alpha \cos \alpha \end{bmatrix} \begin{bmatrix} H \end{bmatrix},$$  \hspace{1cm} (7)

where $w^+$ and $z$ are Nambu-Goldstone bosons eaten by $W_L^\pm$ and $Z_L$, $H^\pm$ and $A$ are charged and CP-odd scalar states. CP-odd state can be a mass eigenstate only when CP is conserved in the Higgs sector. When CP is conserved $h$ and $H$ are mass eigenstates. Consequently, the $Z_2$ even physical scalar states are two CP-even ($h$ and $H$), a CP-odd ($A$) and charged ($H^\pm$) states, like in usual THDMs. Throughout this paper, $\sin(\beta - \alpha) = 1$ is taken, under which we define $h$ and $H$ such that $h$ is the SM-like Higgs boson $^1$.

The mass formulas for the scalar fields are calculated as $^2$

$$m_h^2 = \sin^2(\alpha - \beta)M_{11}^2 + \sin 2(\alpha - \beta)M_{12}^2 + \cos^2(\alpha - \beta)M_{22}^2,$$  \hspace{1cm} (8)

$$m_H^2 = \cos^2(\alpha - \beta)M_{11}^2 + \sin 2(\alpha - \beta)M_{12}^2 + \sin^2(\alpha - \beta)M_{22}^2,$$  \hspace{1cm} (9)

$$m_{H^\pm}^2 = M^2 - \frac{\lambda_4 + \lambda_5}{2}v^2,$$  \hspace{1cm} (10)

$$m_A^2 = M^2 - \lambda_5v^2,$$  \hspace{1cm} (11)

$$m_{S^\pm}^2 = \mu_5^2 + (\rho_1 \cos^2 \beta + \rho_2 \sin^2 \beta)\frac{v^2}{2},$$  \hspace{1cm} (12)

$$m_{\eta}^2 = \mu_5^2 + (\sigma_1 \cos^2 \beta + \sigma_2 \sin^2 \beta)\frac{v^2}{2},$$  \hspace{1cm} (13)

where the mass matrix elements for the CP-even bosons are given by

$$M_{11}^2 = 2(\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \lambda \cos^2 \beta \sin^2 \beta)v^2,$$  \hspace{1cm} (14)

$$M_{12}^2 = M_{21}^2 = (-2\lambda_1 \cos^2 \beta + 2\lambda_2 \sin^2 \beta + \lambda \cos 2\beta)\sin \beta \cos \beta v^2,$$  \hspace{1cm} (15)

$$M_{22}^2 = M^2 + \frac{1}{4}(\lambda_1 + \lambda_2 - \lambda)(1 - \cos 4\beta)v^2,$$  \hspace{1cm} (16)

where $\lambda = \lambda_3 + \lambda_4 + \lambda_5$. The parameter $M$ relates to the invariant mass scale $\mu_{12}$ in the Higgs potential by $M = |\mu_{12}|/(\sin \beta \cos \beta)$ and has the meanings of the soft breaking parameter for the $\tilde{Z}_2$ symmetry.

$^1$ Notice that this is different from the definition that $h$ is always the lighter CP even Higgs boson when the SM-like Higgs is heavier than the other one.

$^2$ For the expressions in the case of $\sin(\beta - \alpha) = 1$, see Appendix A.
In terms of mass eigenstates, the scalar interaction terms are given by

\[ \mathcal{L}_\rho = \]
\[- (\rho_1 \cos^2 \beta + \rho_2 \sin^2 \beta) \left( \omega^- \omega^+ + \frac{z^2}{2} \right) S^+ S^- - (\rho_1 \sin^2 \beta + \rho_2 \cos^2 \beta) \left( H^+ H^- + \frac{A^2}{2} \right) S^+ S^- - \sin \beta \cos \beta (-\rho_1 + \rho_2) (\omega^+ H^- + \omega^- H^+ + zA) S^+ S^- \]
\[- (\rho_1 \cos \alpha \cos \beta + \rho_2 \sin \alpha \sin \beta) v H S^+ S^- - (\rho_1 \sin \alpha \cos \beta + \rho_2 \cos \alpha \sin \beta) v h S^+ S^- - \frac{1}{2} (\rho_1 \cos^2 \alpha + \rho_2 \sin^2 \alpha) H H S^+ S^- - \frac{1}{2} (\rho_1 \sin^2 \alpha + \rho_2 \cos^2 \alpha) h h S^+ S^- - \cos \alpha \sin \alpha (-\rho_1 + \rho_2) h H S^+ S^- , \] \tag{17} \]

\[ \mathcal{L}_\sigma = \]
\[- (\sigma_1 \cos^2 \beta + \sigma_2 \sin^2 \beta) \left( \omega^- \omega^+ + \frac{z^2}{2} \right) \eta^2 - (\sigma_1 \sin^2 \beta + \sigma_2 \cos^2 \beta) \left( H^+ H^- + \frac{A^2}{2} \right) \eta^2 - \sin \beta \cos \beta (-\sigma_1 + \sigma_2) (\omega^+ H^- + \omega^- H^+ + zA) \eta^2 \]
\[- (\sigma_1 \cos \alpha \cos \beta + \sigma_2 \sin \alpha \sin \beta) v H \eta^2 - (\sigma_1 \sin \alpha \cos \beta + \sigma_2 \cos \alpha \sin \beta) v h \eta^2 \]
\[- \frac{1}{2} (\sigma_1 \cos^2 \alpha + \sigma_2 \sin^2 \alpha) H H \eta^2 - \frac{1}{2} (\sigma_1 \sin^2 \alpha + \sigma_2 \cos^2 \alpha) h h \eta^2 \]
\[- \cos \alpha \sin \alpha (-\sigma_1 + \sigma_2) H H \eta^2 , \] \tag{18} \]

and

\[ \mathcal{L}_\kappa = -\sqrt{2} \kappa \left[ v H^+ - \sin(\alpha - \beta) H \omega^+ - \cos(\alpha - \beta) h \omega^+ \right. \]
\[ + \cos(\alpha - \beta) H H^+ - \sin(\alpha - \beta) h H^+ \] \left. S^- \eta + \text{h.c.} . \right. \tag{19} \]

The Yukawa interactions with quarks and leptons in Eq. [2] can then be written as

\[ \mathcal{L}_Y^{\text{Quarks}} = -m_{u_i} \bar{u}^i u^i - \frac{m_{u_i}}{v} \sin \beta H \bar{u}^i u^i - \frac{m_{u_i}}{v} \cos \alpha \bar{u}^i u^i - m_{d_i} \bar{d}^i d^i - \frac{m_{d_i}}{v} \sin \beta H \bar{d}^i d^i - \frac{m_{d_i}}{v} \cos \alpha \bar{d}^i d^i \]
\[ + \frac{m_{u_i}}{v} z \bar{u}^i i \gamma_5 u^i - \frac{m_{u_i}}{v} \cos \beta \bar{u}^i i \gamma_5 u^i + \frac{m_{d_i}}{v} z \bar{d}^i i \gamma_5 d^i - \frac{m_{d_i}}{v} \cos \beta \bar{d}^i i \gamma_5 d^i \]
\[ + \omega^+ \bar{d}^i \left[ \frac{\sqrt{2} m_{u_i}}{v} \left( \frac{1 + \gamma_5}{2} \right) - \frac{\sqrt{2} m_{d_i}}{v} \left( \frac{1 - \gamma_5}{2} \right) \right] u^i + \text{h.c.} \]
\[ + H^- \bar{d}^i \left[ \frac{\sqrt{2} m_{u_i}}{v} \cot \beta \left( \frac{1 + \gamma_5}{2} \right) - \frac{\sqrt{2} m_{d_i}}{v} \cot \beta \left( \frac{1 - \gamma_5}{2} \right) \right] u^i + \text{h.c.} . \tag{20} \]
\( \mathcal{L}_{\text{Leptons}}^{i} = \frac{1}{\sqrt{2}} m_{\ell} \bar{\nu}^{i} \left( \frac{1 + \gamma_{5}}{2} \right) \ell^{i} \omega^{+} - \frac{\sqrt{2} m_{\ell} \tan \beta}{v} \bar{\nu}^{i} \left( \frac{1 + \gamma_{5}}{2} \right) \ell^{i} H^{+} + \text{h.c.} \)

\[-m_{\ell} \bar{\ell}^{i} \ell^{i} - \frac{m_{\ell}}{v} \cos \alpha \cos \beta \bar{H} \ell^{i} \ell^{i} + \frac{m_{\ell}}{v} \sin \alpha \cos \beta \bar{h} \ell^{i} \ell^{i} - \frac{m_{\ell}}{v} z \ell^{i} i \gamma_{5} \ell^{i} + \frac{m_{\ell}}{v} \tan \beta A \ell^{i} i \gamma_{5} \ell^{i}. \]  

(21)

The coupling constants are constrained by the conditions of theoretical consistencies such as perturbative unitarity \([31]\) or vacuum stability and triviality as a function of the cutoff scale \(\Lambda\) of the model \([11, 32]\). We will take into account these conditions and work in the allowed parameter region for \(\Lambda \gtrsim 10 \text{ TeV}\).

### III. NEUTRINO MASS AND MIXING

In this section we calculate the mass matrix in our model, which is induced at the three loop level, and study the parameter regions which satisfy the current data. The mass term for LH neutrinos

\[ \mathcal{L}^{\text{eff}} = \bar{\nu}_{L}^{i} M_{ij}^{\nu} \nu_{L}^{j} \]  

(22)

is naturally generated from dimension five operators

\[ \mathcal{O}_{5} = \frac{\xi_{ij}}{\Lambda_{N}} \bar{\nu}_{L}^{i} \nu_{L}^{j} \phi \phi, \]  

(23)

in the low energy effective theory, where \(\phi\) represents the neutral component of the Higgs doublet, \(\xi_{ij}\) are dimensionless coefficients, and \(\Lambda_{N}\) is a dimensionful scale of new dynamics. In ordinary models based on the seesaw mechanism, the tiny neutrino mass is essentially realized by taking a very large scale for \(\Lambda_{N}\) \([5]\): \(\Lambda_{N} = 10^{13-15} \text{ GeV}\) is required for \(\xi_{ij} = \mathcal{O}(1)\) to obtain the tiny mass scale comparable to the neutrino data. When \(\Lambda_{N}\) is at most TeV scales, very small values for \(\xi_{ij}\) are required to describe the data. It would be possible that such small \(\xi_{ij}\) would be generated at loop level without making fine tuning on the coupling constants in the Lagrangian.

#### A. Evaluation of the three-loop induced neutrino mass matrix

In our model, the LH neutrino mass matrix \(M_{ij}^{\nu}\) is generated by the three-loop diagrams in FIG. 1. The absence of lower order loop contributions is guaranteed by \(Z_{2}\). The charged Higgs boson \(H^{\pm}\) from the doublets and the charged leptons \(e_{R}^{i}\) play a crucial role to connect
FIG. 1: The diagrams for generating tiny neutrino masses.

LH neutrinos with the one-loop (box) diagram by the $Z_2$-odd particles. The resulting LH neutrino mass matrix is obtained as

$$M_{ij}^\nu = \sum_{\alpha=1}^{2} C_{ij}^\alpha F(m_{H^\pm}, m_S, m_{N^\alpha_R}, m_\eta),$$  \hspace{1cm} (24)$$

where

$$C_{ij}^\alpha = 4\kappa^2 \tan^2 \beta (y_{\ell i}^{SM} h_{i}^\alpha)(y_{\ell j}^{SM} h_{j}^\alpha),$$  \hspace{1cm} (25)$$

and the loop integral function $F$ is given by

$$F(m_{H^\pm}, m_S, m_N, m_\eta) = \left(\frac{1}{16\pi^2}\right)^3 \frac{(-m_N)}{m_N^2 - m_\eta^2} \frac{v^2}{m_{H^\pm}^4} \times \int_0^\infty x dx \left\{ B_1(-x, m_{H^\pm}, m_S) - B_1(-x, 0, m_S) \right\}^2 \left( \frac{m_N^2}{x + m_N^2} - \frac{m_\eta^2}{x + m_\eta^2} \right),$$  \hspace{1cm} (26)$$

with $m_f$ representing the mass of the field $f$ and $y_{\ell i}^{SM} = \sqrt{2m_e}/v$. The function $B_1$ is the tensor coefficient in the formalism by Passarino-Veltman for one-loop integrals \[33\]. The detailed calculation of $F$ is shown in Appendix B.

In Fig. 2, we show the magnitude of the integral function $F$ as a function of $m_N$ for several values of $m_{S^\pm}$ and $m_\eta$. It can be seen that $F$ becomes smaller for larger values of $m_{S^\pm}$. For $m_{S^\pm} \lesssim 500$ GeV $F$ decreases monotonically as $m_{N^\alpha_R}$ grows, and for greater values of $m_{S^\pm}$ it mildly increases but finally turns to decrease as $m_{N^\alpha_R}$ grows. The dependences on $m_{N^\alpha_R}$ are however not very sensitive. Numerically, the magnitude of $F$ is of order $10^4$ eV in the wide range of parameter regions of our interest. We note that the dependence on
the mass of the charged Higgs boson $H^\pm$ is qualitatively similar to that on $m_{S^\pm}$, so that $F$ becomes smaller for larger values of $m_{H^\pm}$.

Magnitudes of $h^\alpha_\alpha$, $\kappa$ and $\tan \beta$ as well as $F(m_H, m_S, m_{N_R}, m_\eta)$ determine the universal scale of $M_\nu^{ij}$, whereas variation of $h^\alpha_i$ ($i = e, \mu, \tau; \alpha = 1,2$) reproduces the mixing pattern indicated by the neutrino data [2].

**B. Parameters to reproduce the data of neutrino masses and mixings**

The generated mass matrix $M_\nu^{ij}$ in Eq. (24) of LH neutrinos can be related to the neutrino oscillation data by

$$M_\nu^{ij} = U_{is} (M_\nu^{\text{diag}})_{st} (U^T)_{tj}, \quad (27)$$

where $M_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$. For the case of the normal hierarchy we identify the mass eigenvalues as $m_1 = 0$, $m_2 = \sqrt{\Delta m^2_{\text{solar}}}$ and $m_3 = \sqrt{\Delta m^2_{\text{atm}}}$, while for inverted hierarchy $m_1 = \sqrt{\Delta m^2_{\text{atm}}}$, $m_2 = \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{solar}}}$ and $m_3 = 0$ are taken. The Maki-Nakagawa-
Sakata matrix \[34\] is parameterized as

\[
U = \begin{bmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{bmatrix}
\begin{bmatrix}
c_{13} & 0 & s_{13}e^{i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{-i\delta} & 0 & c_{13}
\end{bmatrix}
\begin{bmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & e^{i\alpha} & 0 \\
0 & 0 & e^{i\beta}
\end{bmatrix},
\]  

(28)

where \(s_{ij}\) and \(c_{ij}\) represent \(\sin \theta_{ij}\) and \(\cos \theta_{ij}\) respectively with \(\theta_{ij}\) to be the neutrino mixing angle between the \(i\)th and \(j\)th generations, and \(\delta\) is the Dirac phase while \(\alpha\) and \(\beta\) are Majorana phases. For simplicity, we neglect the effects of these phases in the following analysis. Current neutrino oscillation data give the following values \[2\] :

\[
\Delta m_{\text{solar}}^2 \simeq 7.65 \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{\text{atm}}^2| \simeq 2.4 \times 10^{-3} \text{ eV}^2, \\
\sin^2 \theta_{12} = 0.3, \quad \sin^2 \theta_{23} \approx 0.5, \quad \sin^2 \theta_{13} < 0.04.
\]  

(29)

(30)

In order to study parameter sets that satisfy the current experimental data, we need to discuss constraints from the \(\mu \to e\gamma\) results. This process is induced by one loop diagram of a right handed neutrino \(N_R\) and a charged scalar boson \(S^\pm\) through Yukawa coupling \(h_i^a\) \((i = e, \mu\). The branching ratio is given by

\[
B(\mu \to e\gamma) \simeq \frac{3\alpha_{em}v^4}{32\pi} \left| \frac{m_{N_R}^2}{m_{S^\pm}^2} \right|^2 \left\{ F_2 \left( \frac{m_{N_R}^2}{m_{S^\pm}^2} \right) \right\}^2 \left\{ F_2 \left( \frac{m_{S^\pm}^2}{m_{S^\pm}^2} \right) \right\}^2, 
\]  

(31)

where \(F_2 (x) \equiv (1 - 6x + 3x^2 + 2x^3 - 6x^2 - \ln x)/6(1 - x)^4\). Comparing this to the current experimental bound, \(B(\mu \to e\gamma) < 1.2 \times 10^{-11}\) \[35\], the mass of \(N_R\) or \(S^\pm\) is strongly constrained from below.

Under the natural requirement that \(h_e^a = O(1),\) and taking into account the \(\mu \to e\gamma\) search results \[35\], we find that \(m_{N_R}^a \gtrsim O(1)\) TeV, \(m_{H^\pm} = 100-200\) GeV and \(\kappa \tan \beta = O(10)\). In addition, with the LEP direct search data for Higgs bosons and precision measurement data \[2\], possible values for the scalar masses uniquely turn out to be \(m_{H^\pm} \simeq m_H \simeq 100\) GeV and \(m_{S^\pm} \sim O(100)\) GeV for \(\sin (\beta - \alpha) = 1\). As we see in later sections, relatively heavier \(S^\pm\) \((m_{S^\pm} \gtrsim 400\) GeV\) is favored from the discussion on DM and electroweak baryogenesis. It is known that in the Type X Yukawa interaction a light \(H^\pm\) \((m_{H^\pm} \lesssim 300\) GeV\) is not excluded by the \(b \to s\gamma\) data \[36\]. This is the reason that the Yukawa coupling in Eq. \([2]\) has been employed in our model.

For values of \(h_i^a\) \((i = \mu, \tau)\), we only require that \(h_\nu^ay_i \sim O(y_e) \sim 10^{-5}\), since we cannot avoid to include the hierarchy among \(y_i^{\text{SM}}\). Several sets for \(h_i^a\) which satisfy the neutrino
| Set               | Mass (TeV) | Coupling constants | LFV                  |
|------------------|------------|--------------------|----------------------|
|                  | $m_{\eta}$ | $m_{S}$ $m_{N_{R}}$ | $\kappa \tan \beta$ | $h_{e}^{1}$ | $h_{e}^{2}$ | $h_{\mu}^{1}$ | $h_{\mu}^{2}$ | $h_{\tau}^{1}$ | $h_{\tau}^{2}$ | $B(\mu\rightarrow e\gamma)$ |
| (hierarchy, sin$^{2}\theta_{13}$) |            |                    |                      |            |            |            |            |            |            |                          |
| A (normal, 0)    | 0.05       | 0.4                | 3                    | 29         | 2.0        | 2.0        | 0.041      | -0.020      | 0.0012      | -0.0025      | 6.8 × 10^{-12}           |
| B (normal, 0.14) | 0.05       | 0.4                | 3                    | 34         | 2.2        | 2.1        | 0.0087     | 0.037       | -0.0010     | 0.0021       | 5.3 × 10^{-12}           |
| C (inverted, 0)  | 0.05       | 0.4                | 3                    | 66         | 3.8        | 3.7        | 0.013      | -0.013      | -0.00080    | 0.00080      | 4.2 × 10^{-12}           |
| D (inverted, 0.14)| 0.05      | 0.4                | 3                    | 66         | 3.7        | 3.7        | -0.016     | 0.011       | 0.00064     | -0.00096     | 4.2 × 10^{-12}           |

**TABLE II:** Values of $h_{i}^{a}$ which satisfy neutrino data and the constraint from $\mu \rightarrow e\gamma$ for $m_{\eta} = 50$ GeV, $m_{H^{\pm}} = 100$GeV and $m_{N_{R}^{1}} = m_{N_{R}^{2}}$.

| Set               | Mass (TeV) | Yukawa couplings | LFV                  |
|------------------|------------|------------------|----------------------|
|                  | $m_{\eta}$ | $m_{S}$ $m_{N_{R}}$ | $\kappa \tan \beta$ | $h_{e}^{1}$ | $h_{e}^{2}$ | $h_{\mu}^{1}$ | $h_{\mu}^{2}$ | $h_{\tau}^{1}$ | $h_{\tau}^{2}$ | $B(\mu\rightarrow e\gamma)$ |
| (hierarchy, sin$^{2}\theta_{13}$) |            |                    |                      |            |            |            |            |            |            |                          |
| E (normal, 0)    | 0.7        | 0.9               | 3                    | 33         | 3.0        | 2.0        | -0.014     | 0.057       | -0.0028     | 0.0021       | 7.2 × 10^{-12}           |
| F (normal, 0)    | 0.005      | 0.4               | 3                    | 29         | 2.0        | 2.0        | -0.019     | 0.041       | -0.0024     | 0.0012       | 6.6 × 10^{-12}           |

**TABLE III:** Values of $h_{i}^{a}$ which satisfy neutrino data and the constraint from $\mu \rightarrow e\gamma$ for $m_{\eta} = 700$ GeV (Set E) and $m_{\eta} = 5$ GeV (Set F). The other parameters are taken to be $m_{H^{\pm}} = 100$GeV and $m_{N_{R}^{1}} = m_{N_{R}^{2}}$ for the normal hierarchy.

data are shown in TABLE II with the predictions on the branching ratio of $\mu \rightarrow e\gamma$ for $m_{\eta} = 50$ GeV, $m_{H^{\pm}} = 100$ GeV and $m_{N_{R}^{1}} = m_{N_{R}^{2}} = 3$ TeV. Set A and Set B are rather standard choices in our model assuming the normal hierarchy, while C and D are those assuming the inverted hierarchy. We note that larger values are required for $\kappa \tan \beta$ in the case of the inverted hierarchy. On the other hand, small $\tan \beta$ is favored by the realization of electroweak baryogenesis as we will discuss later$^{3}$. Therefore, our model turns out to be better compatible with the normal hierarchy scenario$^{[37]}$.

Set E and Set F are chosen to potentially include the results from PAMELA (and ATIC) and those from DAMA, respectively, as discussed later. The parameters describing the data are listed in Table III.

---

$^{3}$ In the THDM, $\tan \beta \gtrsim 10$ would not be favored for successful electroweak baryogenesis under the EDM constraints$^{[24]}$. Although we do not discuss CP violating effects in our present analysis, we keep in mind this constraint and just impose the condition that $\tan \beta$ is not larger than about 10.
We also examined the case with three generations for RH neutrinos. Unlike the case with two generations, the mass of the smallest mass eigenvalue for LH neutrinos can be nonzero. It is however found that larger values for $\kappa \tan \beta$ are preferred to reproduce the neutrino data, so that it would be rather unnatural as compared to the case with two generations. Therefore, we concentrate on the model with two generation RH neutrinos in the rest of the paper.

In Fig. 3 the contour plots for the branching ratio of $\mu \rightarrow e\gamma$ as a function of $m_{N_R}$ and $m_{S^\pm}$ for values of $h_1^a$ to be of Set A - Set F in Table II and III at $m_{H^\pm} = 100$ GeV. The dependence of the branching ratio on the mass of $S^\pm$ is rather mild. On the other hand, the branching ratio is strongly dependent on the mass of $N_R$, so that the lower bound on $m_{N_R}$ is obtained for a mass of $S^\pm$.

IV. DARK MATTER

Since $Z_2$ is exact, the lightest $Z_2$-odd particle is stable and can be a candidate of DM if it is neutral. In our model, $N_R^a$ must be heavy ($m_{N_R^a} \sim$ a few TeV), so that the DM candidate is identified as $\eta$. Since $\eta$ is a singlet under the SM gauge group, the interactions with $Z_2$ even particles are only through the Higgs coupling. It is, however, worth noticing the presence of interactions through $\kappa$ coupling, which is absent in the simplest gauge singlet scalar DM [38].

A. Thermal relic abundance

In principle, $\eta$ has various annihilation processes into $\gamma\gamma, f\bar{f}, W^+W^-$ and so on. The annihilation into a $W$ boson pair or a Higgs pair leads to too rapid annihilation. Thus, relatively light $\eta$ would be favored in order to kinematically close these annihilation modes. When $\eta$ is lighter than the $W$ boson, $\eta$ predominantly annihilates into two photons through one-loop diagrams by $H^\pm$ and $S^\pm$, as well as into $b\bar{b}$ and $\tau^+\tau^-$ through s-channel Higgs ($h$ and $H$) exchange diagrams: see Fig. 4.
FIG. 3: The contour plot of $\mu \rightarrow e\gamma$ branching ratio in the $m_N - m_S$ plane for the values of $h_i^\nu$ given in Set A to Set F.

The relevant annihilation rate is evaluated for $\sin(\beta - \alpha) \simeq 1$ as

$$w(s) \simeq \frac{1}{16\pi} \left( \frac{e^2}{16\pi^2} \right)^2 (\kappa v)^4 \left( \frac{8}{m_S^2} \right)^2 + \frac{3m_H^2 v^2}{16\pi} \left( \frac{\sigma_1 \cos^2 \beta + \sigma_2 \sin^2 \beta}{s - m_h^2 + i m_h \Gamma_h} - \frac{\sigma_1 + \sigma_2}{s - m_H^2 + i m_H \Gamma_H} \right)^2$$

$$+ \frac{m_H^2 v^2}{16\pi} \left( \frac{\sigma_1 \cos^2 \beta + \sigma_2 \sin^2 \beta}{s - m_h^2 + i m_h \Gamma_h + \frac{s - m_h^2 + i m_H \Gamma_H}{s - m_H^2 + i m_H \Gamma_H}} \right)^2,$$

where $\Gamma_h$ and $\Gamma_H$ are the decay widths of $h$ and $H$ respectively, and $s$ is a usual Mandelstam variable. The second and third term correspond to the annihilation into fermion and anti-fermion. The annihilation into two gammas is given by the first term whose expression was

15
FIG. 4: Feynman diagrams for the DM annihilation. For the one-loop diagrams for $\eta\eta \rightarrow \gamma\gamma$ only those which contain the coupling constant $\kappa$ are shown because they give dominant contributions in our parameter choices.

obtained under the s-wave approximation. From their thermal averaged annihilation rate $\langle \sigma v \rangle$, the relic mass density $\Omega_\eta h^2$ is evaluated for $\eta\eta \rightarrow \gamma\gamma, b\bar{b}$ and $\tau^+\tau^-$ as

$$\Omega_\eta h^2 = 1.1 \times 10^9 \left( \frac{m_\eta/T_d}{\sqrt{g_* M_P \langle \sigma v \rangle}} \right) |_{T_d} \text{GeV}^{-1},$$

(33)

where $M_P$ is the Planck scale, $g_*$ is the total number of relativistic degrees of freedom in the thermal bath, and $T_d$ is the decoupling temperature [39].

Except for the region near the resonance annihilation by s-channel Higgs ($h$ and $H$) exchange where $m_\eta \approx m_h/2$ and $m_H/2$, the DM relic density is basically determined by the annihilation into $\gamma\gamma$ whose cross section essentially depends on only two free parameters; $\kappa$ and $m_{S^\pm}$. Since we already know $\kappa = O(1)$ from neutrino masses discussed in the previous section, we obtain $m_{S^\pm} \gtrsim 400$ GeV from the $\eta$ DM abundance result.

FIG. 5 shows $\Omega_\eta h^2$ as the function of $m_\eta$ for $\sin(\beta - \alpha) = 1$. The parameters are chosen as $(\sigma_1, \sigma_2, \kappa \tan \beta, m_h, m_H) = (0.05, 0.03, 30, 120\text{GeV}, 100\text{GeV})$. Strong annihilation can be seen near 50 GeV $\simeq m_H/2$ (60 GeV $\simeq m_h/2$) due to the resonance of $H$ ($h$) mediation. The $\Omega_\eta h^2$ is proportional to $m_\eta^2$ for the dominant annihilation into $\gamma\gamma$. The data ($\Omega_{\text{DM}} h^2 \sim 0.11$ [3]) indicate that $m_\eta$ is around 50 – 65 GeV for $m_{S^\pm}$ of 400 GeV. If $S$ is much lighter than 400 GeV, the resultant $\Omega_\eta h^2$ is below 0.1 for most of $m_\eta$ range. For a heavier $m_{S^\pm}$, only the resonance annihilation would provide the desired $\Omega_\eta h^2$. 
FIG. 5: [Left] The thermal relic abundance of η and contributions from tree and one-loop processes. [Right] The thermal relic abundance of η and its $m_S \pm$ dependence. In both figures, parameters are taken as $(\kappa, \sigma_1, \sigma_2, m_h, m_H, \tan \beta) = (3, 0.05, 0.03, 120 \text{GeV}, 100 \text{GeV}, 10)$.

B. Constraints from DM direct searches

Generally speaking, WIMP DM is detectable by direct DM searches such as CDMS \[40]. Scatterings off of WIMP DM with nuclei take place in two different ways, depending on the nature of WIMP \[7]. One is spin-dependent scattering whose cross section depends on the total spin of target nuclei, while the other is spin-independent(SI). η is a scalar particle, therefore the relevant process is a SI scattering through t-channel Higgs ($h$ and $H$) exchange \[38]. The SI cross section for a proton is given as \[7]

$$\sigma_{SI}^p = \frac{m_p^2}{\pi (m_\eta + m_p)^2} f_p^2,$$

with

$$f_p = \sum_{q=u,d,s} f_{Tq}^{(p)} f_q + \frac{2}{27} \sum_{q=c,b,t} f_{TGq}^{(p)} f_q,$$ \hspace{2cm} (35)

where $m_p$ and $m_q$ are the proton and each flavor quark masses. The hadronic matrix elements $f_{Tq}^{(p)}$ and $f_{TGq}^{(p)}$ are defined as

$$m_p f_{Tq}^{(p)} \equiv \langle p | m_q \bar{q} q | p \rangle, \quad f_{TGq}^{(p)} = 1 - \sum f_{Tq}^{(p)},$$ \hspace{2cm} (36)

and we adopted the values in Ref. \[41] for the following estimation. In our model, the coupling constant of the effective interaction between η and quark, $L_{int} \supset f_q \bar{q} q \eta$, is given as

$$\frac{f_q}{m_q} = \frac{(-\sigma_1 \sin \alpha \cos \beta + \sigma_2 \cos \alpha \sin \beta) \cos \alpha}{2m_h^2} \frac{\sin \beta}{2m_H^2} + \frac{(\sigma_1 \cos \alpha \cos \beta + \sigma_2 \sin \alpha \sin \beta) \sin \alpha}{2m_H^2} \frac{\sin \beta}. \hspace{2cm} (37)$$
Until now, only null results are reported from all experiments of direct DM search. The current most stringent bounds are given by XENON 10 \cite{42} and CDMS II as \( \sigma_{SI}^p \lesssim 5 \times 10^{-8} \) pb. We find the coupling constants \( \sigma_i \lesssim \mathcal{O}(10^{-2}) \) are consistent.

Remember the most of parameter region of the minimal singlet scalar DM is excluded by null results of DM direct search and only resonance annihilation region, which would mean certain fine-tuning between DM mass and Higgs mass, is consistent \cite{38}. Hence, in contrast with this, it is worth emphasizing that such a tuning to realize large resonance is not necessarily required in our \( \eta \) DM, because the one-loop processes give sizable contributions to annihilation for \( m_S \simeq 400 \) GeV. Another singlet scalar DM model without necessity of a resonance due to different mechanism is also recently proposed \cite{43}.

C. Variations with a particular choice of parameters

So far, we have discussed properties of \( \eta \) DM on a standard basis according to parameter sets in Table III. However, there are a few recent observational results which would imply that the nature of DM could differ from the standard WIMP.

The first example is anomalous excesses in the observed positron flux. The PAMELA data shows that observed positron flux exceeds the normally expected astrophysical background flux for energy range above a few hundred GeV with a rising spectrum in \( \Phi^+ / (\Phi^- + \Phi^+) \) \cite{44}. In addition, ATIC also reported a bump around \( E \sim 600 \) GeV in the spectrum \cite{45}. These excesses could be interpreted as a signal from annihilation \cite{46} or decay \cite{47} of DM particles. As mentioned above, we have argued that the mass of \( \eta \) is most likely around 50 GeV in order to avoid its annihilation into \( W^- \) or Higgs boson. However, if we accept negligibly tiny \( \sigma_i \) coupling constants to suppress these annihilation via the s-channel Higgs exchange, it is possible to realize that \( \eta \) is as heavy as several hundreds GeV in order to avoid its annihilation into \( W^- \) or Higgs boson. However, if we accept negligibly tiny \( \sigma_i \) coupling constants to suppress these annihilation via the s-channel Higgs exchange, it is possible to realize that \( \eta \) is as heavy as several hundreds GeV in order to avoid its annihilation into \( W^- \) or Higgs boson.

\[ \text{We estimated this from Eq. (32), which might be too simple for a heavy } \eta. \text{ However, we suppose that the error would be just a factor difference.} \]
FIG. 6: A large enough scattering cross section of light DM with a proton to account for the DAMA/LIBRA claim.

might be account for a heavy \( \eta \). The detailed calculation, paying attentions to the spectrum, will be presented elsewhere [48].

Next, the results at the DAMA/LIBRA experiment claim a significant annual modulation in their DM detection rate which also could be caused by WIMPs [50], whereas all other experiments do not find any signal yet. These two results appear to conflict each other, but a few compatible scenarios are possible, e.g., inelastic scattering DM [51], nevertheless. One of them is a light WIMP with the mass of \( 3 \text{ GeV} \lesssim m_\eta \lesssim 8 \text{ GeV} \) and a large scattering cross section with nuclei of \( \mathcal{O}(10^{-4}) \) pb [52] (as in Set F in Table IIII). Such a large \( \sigma_{SI}^p \) can be available for \( \sigma_2 \sim 0.4 \) as seen in FIG. 6. With a slightly heavier charged scalar \( S \) to suppress one-loop contributions, the desired thermal abundance \( \Omega_\eta h^2 \simeq 0.1 \) can be obtained almost independently from \( m_\eta \) in the relevant mass range.

\[\text{5 Within the framework of three loop generation of neutrino masses, the explanation of positron excess was already examined in Ref. 49 by extending the other model in Refs. 17, 18.}\]
V. STRONG FIRST ORDER PHASE TRANSITION FOR SUCCESSFUL ELECTROWEAK BARYOGENESIS

The model satisfies the Sakharov’s conditions for baryogenesis \[4\]: baryon number violation occurs via sphaleron effect at high temperatures, C and CP violation is automatic in the electroweak theory and additional CP violating phases are in the Higgs sector and in the Yukawa interaction, and the condition of departure from thermal equilibrium can be realized by the strong first order EWPT in the electroweak baryogenesis scenario. In this section, we examine the region in the parameter space where the strong first order EWPT is realized.

In this paper, we take the SM-like limit for the mixing between \(h\) and \(H\) \((\sin(\alpha - \beta) = -1)\), where \(h\) is the SM-like Higgs boson while \(H\) does not have VEV. In such a case, the one-loop (both zero and finite temperature) effective potential is described in terms of a unique order parameter \(\varphi = \langle \phi \rangle^6\). All extra scalar bosons \((H, A, H^\pm, S^\pm, \eta)\) approximately behave just like additional particles running in the loop in the effective potential. The following relations and equations do not depend on the value of \(\tan \beta\), so that we apply our result here to the case of \(\tan \beta = \mathcal{O}(1) - \mathcal{O}(10)\).

A. Effective potential

We first consider the one-loop effective potential at zero temperature \(V_{\text{eff}}[\varphi] = V_{\text{tree}}[\varphi] + \Delta V[\varphi]\). The one-loop contribution \(\Delta V[\varphi]\) is given by

\[
\Delta V[\varphi] = \frac{1}{64\pi^2} \sum_f N_{c_f} N_{s_f} (1)^{2s_f} (M_f[\varphi])^4 \left\{ \ln \left( \frac{M_f[\varphi]^2}{Q^2} \right) - \frac{3}{2} \right\} ,
\]

where \(\varphi = \langle \phi \rangle = v + \langle h \rangle\), \(N_{c_f}\) is the color factor, \(s_f (N_{s_f})\) is the spin (degree of freedom) of the field \(f\) in the loop, \(M_f[\varphi]\) is the field dependent mass of \(f\), and \(Q\) is an arbitrary scale.

\[\text{The effect of the two stage phase transition in the case with a large } \tan \beta \gg 1 \text{ is neglected in our analysis}\].

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6 The effect of the two stage phase transition in the case with a large \(\tan \beta \gg 1\) is neglected in our analysis \[53\].
For $\sin(\beta - \alpha) = 1$, the field dependent mass functions are given by

\[
\tilde{m}_h^2(\varphi) = \frac{3m_h^2}{2} \left( \frac{\varphi^2}{v^2} - \frac{1}{3} \right),
\]

(39)

\[
\tilde{m}_H^2(\varphi) = \left( m_H^2 - M^2 + \frac{m_h^2}{2} \right) \frac{\varphi^2}{v^2} + M^2 - \frac{m_h^2}{2},
\]

(40)

\[
\tilde{m}_{H\pm}^2(\varphi) = \left( m_{H\pm}^2 - M^2 + \frac{m_h^2}{2} \right) \frac{\varphi^2}{v^2} + M^2 - \frac{m_h^2}{2},
\]

(41)

\[
\tilde{m}_A^2(\varphi) = \left( m_A^2 - M^2 + \frac{m_h^2}{2} \right) \frac{\varphi^2}{v^2} + M^2 - \frac{m_h^2}{2},
\]

(42)

\[
\tilde{m}_S^2(\varphi) = \left( m_S^2 - \mu_S^2 \right) \frac{\varphi^2}{v^2} + \mu_S^2,
\]

(43)

\[
\tilde{m}_\eta^2(\varphi) = \left( m_\eta^2 - \mu_\eta^2 \right) \frac{\varphi^2}{v^2} + \mu_\eta^2.
\]

(44)

The renormalized mass of the SM like Higgs boson $h$ and the one-loop corrected $hhh$ coupling are given by the conditions

\[
\frac{\partial}{\partial \varphi} V_{\text{eff}}[\varphi] \bigg|_{\varphi=v} = 0,
\]

(45)

\[
\frac{\partial^2}{\partial \varphi^2} V_{\text{eff}}[\varphi] \bigg|_{\varphi=v} = m_h^2,
\]

(46)

\[
\frac{\partial^3}{\partial \varphi^3} V_{\text{eff}}[\varphi] \bigg|_{\varphi=v} = \lambda_{hhh}^{\text{eff}},
\]

(47)

when $\sin(\beta - \alpha) = 1$.

### B. Electroweak phase transition

The additional part of the one-loop effective potential at finite temperature is expressed by

\[
\Delta V_T[\varphi, T] = \frac{T^4}{2\pi^2} \sum_f N_{c_f} N_{s_f} (-1)^{2s_f} I_f(a_f),
\]

(48)

where $I_f(a_f)$ is

\[
I_B(a_f) = \int_0^\infty dx \; x^2 \log[1 - e^{-\sqrt{x^2 + a_f^2}}], \quad \text{(boson)},
\]

(49)

\[
I_F(a_f) = \int_0^\infty dx \; x^2 \log[1 + e^{-\sqrt{x^2 + a_f^2}}], \quad \text{(fermion)},
\]

(50)
for the loop particle \( f \) with \( a_f = \tilde{m}(\varphi)_f/T \). For a given mass parameter set, the critical temperature \( T_c \) and the critical expectation value \( \varphi_c (\neq 0) \) are obtained as solutions of

\[
V_{\text{eff}}[\varphi, T_c] = 0, \tag{51}
\]
\[
\frac{\partial}{\partial \varphi} V_{\text{eff}}[\varphi, T_c] = 0. \tag{52}
\]

The expression by using the high temperature expansion \([54]\) (see Appendix D) is useful to see the structure in an analytic way. When \( \sin(\beta - \alpha) = 1, m_H^2, m_A^2, m_{H^\pm}^2 \gg M^2, m_{S^\pm}^2 \gg \mu_{S^\pm}^2 \) and \( m_1^2 \gg \mu_n^2 \), the effective potential at finite temperatures \( V_{\text{eff}}[\varphi, T] = V_{\text{tree}}[\varphi] + \Delta V[\varphi] + \Delta V_T[\varphi, T] \) can be approximately expressed as

\[
V_{\text{eff}}[\varphi, T] = D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + ..., \tag{53}
\]

where

\[
D \simeq \frac{1}{24v^2} \left( 6m_W^2 + 3m_Z^2 + 6m_t^2 + \frac{7}{2}m_h^2 + m_H^2 + m_A^2 + 2m_{H^\pm}^2 + 2m_{S^\pm}^2 + m_n^2 \right), \tag{54}
\]
\[
T_0^2 \simeq \frac{1}{2D} \left( \frac{m_h^2}{2} - 4Bv^2 \right), \tag{55}
\]
\[
E \simeq \frac{1}{12\pi v^3} (6m_W^2 + 3m_Z^2 + 2m_{H^\pm}^2 + m_H^2 + m_A^2 + 2m_{S^\pm}^2 + m_n^2), \tag{56}
\]
\[
\lambda_T \simeq \frac{m_h^2}{2v^2} \left[ 1 - \frac{1}{8\pi^2 v^2 m_h^2} \left\{ +6m_W^4 \log \frac{m_W^2}{\alpha_B T^2} + 3m_Z^4 \log \frac{m_Z^2}{\alpha_B T^2} - 12m_t^4 \log \frac{m_t^2}{\alpha_B T^2} + \frac{9}{4}m_h^4 \log \frac{m_h^2}{\alpha_B T^2} + 2m_{H^\pm}^4 \log \frac{m_{H^\pm}^2}{\alpha_B T^2} + m_H^4 \log \frac{m_H^2}{\alpha_B T^2} + m_A^4 \log \frac{m_A^2}{\alpha_B T^2} + 2m_{S^\pm}^4 \log \frac{m_{S^\pm}^2}{\alpha_B T^2} + m_n^4 \log \frac{m_n^2}{\alpha_B T^2} \right\} \right], \tag{57}
\]

where \( \log \alpha_B = 2 \log 4\pi - 2 \gamma_E \) and \( \log \alpha_B = 2 \log \pi - 2 \gamma_E \), and

\[
B \simeq \frac{1}{64\pi^2 v^4} \left\{ 6m_W^4 + 3m_Z^4 - 12m_t^4 + \frac{9}{4}m_h^4 + m_{H^\pm}^4 + m_H^4 + m_A^4 + 2m_{H^\pm}^4 + 2m_{S^\pm}^4 + m_n^4 \right\}. \tag{58}
\]

We analytically obtain the critical temperature \( T_c \) and the order parameter \( \varphi_c \) at \( T_c \) as

\[
T_c = T_0 \frac{1}{\sqrt{1 - \frac{E^2}{\lambda_T D^2}}}, \quad \varphi_c = \frac{2ET_c}{\lambda_T}. \tag{59}
\]

In order to satisfy the sphaleron decoupling condition in the broken phase, it is required that \([55]\)

\[
\frac{\varphi_c}{T_c} \gtrsim 1. \tag{60}
\]
FIG. 7: The region of strong first order EWPT is shown in the $m_A$-$m_{S\pm}$ plane. Shaded (yellow colored) area is excluded by the condition in Eq. (60). The invariant mass parameters $M$ and $\mu_S$ are taken as $M = 100$ GeV and $\mu_S = 200$ GeV.

In terms of high temperature description of the effective potential, Eq. (60) implies the condition

$$\frac{2E}{\lambda T_c} \gtrsim 1.$$  \hspace{1cm} (61)

A large value of $E$ is important for the strongly first-order EWPT \cite{22}, for which only the bosonic loop effects can contribute. In the case of the SM, the value of the coefficient $E$ mainly comes from loop contributions of weak gauge bosons, so that the above relation gives the upper bound on $m_h$ which is much below the lower bound from the LEP experiment. However, in some new physics models, the situation can be improved due to the contributions of additional bosonic fields which couple to $h$. In our model, there are many additional scalars running in the loop so that a larger $E$ can be easily realized. Consequently, the first order EWPT is possible without contradiction with the LEP data in a wide region of the parameter space.

In FIG. 7, the allowed region under the condition of Eq. (61) is shown for $m_h \simeq 120$ GeV, $m_H \simeq m_{H\pm}(\simeq M) \simeq 100$ GeV and $\sin(\beta - \alpha) \simeq 1$. The invariant mass parameters $M$ and $\mu_S$ in Eqs. (74) and (75) are taken as $M = 100$ GeV and $\mu_S = 200$ GeV. The condition is satisfied when $m_{S\pm} \gtrsim 360$ GeV for $m_A \simeq 100$ GeV and when $m_A \gtrsim 340$ GeV.
for $m_S \simeq 200 \text{ GeV}$. If we take larger values for $M (< m_A)$ and $\mu_S < m_{S^\pm}$, then the area of the allowed regions becomes smaller: in order to obtain similar magnitude of non-decoupling effects a heavier $A$ or $S^\pm$ is favored. The result is not sensitive to $\tan \beta$. We note that the approximation with high temperatures becomes worse for larger $M (M \gg T)$ but the conclusion here is not qualitatively changed for our parameter set even when we evaluate it without high temperature expansion.

VI. PHENOMENOLOGY

A. Constraints on the parameters from the current data

Before going to the discussion on the prediction of the model, we here summarize the constraints on the model from the following current experimental data and the theoretical requirements.

The $\mu \rightarrow e\gamma$ results:

The constraint from the data for $\mu \rightarrow e\gamma$ and the natural generation of the tiny neutrino masses require that $m_{N_R^\alpha} > \mathcal{O}(1) \text{ TeV}$ and $m_{S^\pm}$ is several hundred GeV.

Tiny neutrino masses with $\mathcal{O}(1)$ coupling constants:

We require no unnatural fine tuning on the coupling constants $\kappa$ and $h_e^\alpha$; i.e. $\kappa \sim h_e^\alpha \sim \mathcal{O}(1)$. In this case, the data prefer $m_{H^\pm} \lesssim 100 \text{ GeV}$, $m_{S^\pm} = \mathcal{O}(100) \text{ GeV}$ and $\kappa \tan \beta \simeq 30$. We do not explain the mass hierarchy among charged leptons, so that we consider scales of $h_i^\alpha$ to satisfy $\mathcal{O}(h_i^\alpha y_i^i) \sim 10^{-5} \sim \mathcal{O}(y_e)$.

The data from the LEP and from the Tevatron:

Throughout this paper, we take

$$m_{H^\pm} = m_H, \quad \text{and} \quad \sin(\beta - \alpha) = 1. \quad (62)$$

In this case, the Higgs potential has the global custodial $SU(2)_V$ symmetry. The bound from the LEP precision measurement especially those for electroweak rho parameter is easily satisfied. The scalar $h$ plays the same role as the SM Higgs boson. The other scalar
bosons do not contribute to the rho parameter. Therefore, we have similar mass bound on $h$ to that on the SM Higgs boson from the LEP data; i.e., $m_h = 114$-$160$ GeV. The LEP lower bound for charged Higgs boson mass should be $m_{H^\pm} \gtrsim 100$ GeV, which must also be respected. Combining this with the requirement of natural generation of tiny neutrino mass, we obtain $m_{H^\pm} = m_H \sim 100$ GeV. A little bit larger values are also allowed when we take small fine tuning for the coupling constants.

The $b \to s\gamma$ results:

As discussed in previous section, the results from $b \to s\gamma$ strongly constrain the mass of charged Higgs bosons in the Type-II Yukawa interaction in THDMs. It gives the lower bound as $m_{H^\pm} \gtrsim 295$ GeV [56]. In order to avoid this constraint and in order to have a compatible value of $m_{H^\pm}$ with the neutrino data, we choose Type-X Yukawa interaction in our model.

The $g - 2$ results:

The predicted muon $g - 2$ in this model with the above parameter sets of new particle mass and Yukawa couplings to reproduce neutrino mass is far below the current experimental bound [57]. In addition, the electron $g - 2$ is also below the current bounds even for order unity couplings $h_e$ because it is suppressed by the electron mass [58]. Thus, there are no significant constraints on the model from $g - 2$.

The dark matter data:

The DM thermal relic abundance $\Omega_\eta h^2 \simeq 0.1$ (WMAP) requires $m_\eta \lesssim 65$ GeV for $m_S \gtrsim 400$ GeV, or $m_\eta \simeq m_h/2$ or $m_H/2$. The null result of direct DM searches corresponds to couplings $\sigma \lesssim 10^{-2}$ for $m_\eta \gtrsim 20$ GeV.

Strong first order phase transition:

We need non-decoupling property in the Higgs sector to realize the strong first order phase transition of electroweak symmetry. As, the mass of $H$ and $H^\pm$ are constrained to be around 100 GeV, these fields cannot give such strong non-decoupling loop effects. We consider the case where $A$ and $S^\pm$ are relatively heavy so that the sufficient non-decoupling loop effects can be generated. Concretely, we may consider $m_A = 100 - 200$ GeV, and
FIG. 8: Branching ratios of the SM like Higgs boson $h$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $\sin(\beta - \alpha) = 1$ and $m_\eta = 48$ GeV. The coupling constants are taken to be $\sigma_1 = 0.05$, $\sigma_2 = 0.03$, $\rho_1 = 1.0$ and $\rho_2 = 1.0$.

$m_{S^\pm} \gtrsim 400$ GeV.

B. Predictions

We now show phenomenological predictions in the scenario in (63) in order. Here we mainly work on the scenario which can simultaneously solve the above three issues under the data \cite{2, 35, 36}:

\begin{align}
\sin(\beta - \alpha) &= 1, \\
\kappa \tan \beta &\simeq 30, \\
m_h &= 120 \text{GeV}, \\
m_H &\simeq m_{H^\pm} (\simeq M) \simeq 100 \text{GeV}, \\
m_A &\sim 100 - 200 \text{GeV}, m_{S^\pm} \sim 400 \text{GeV}, \\
m_\eta &\sim 40 - 65 \text{GeV}, \\
m_{N_1^R} &= m_{N_2^R} = 3 \text{TeV}. 
\end{align}

(63)

By this scenario, the model can explain the origin of tiny neutrino mass and mixing, the origin of DM, and the strongly first-order phase transition which is necessary for successful electroweak baryogenesis. This can be realized without assuming unnatural hierarchy among the coupling constants. All the masses are between $\mathcal{O}(100)$ GeV and $\mathcal{O}(1)$ TeV. They are indicated by the data, so that the model has a predictive power.

Invisible decays of the Higgs boson to a DM pair:

The scalar boson $h$ is the SM-like Higgs boson, whose coupling constants to the SM
FIG. 9: The cross section for the scattering process with a proton at the direct search experiment. The upper (lower) curve corresponds to the parameter $\sigma_2 = 0.03$ (0.01). The other parameters are taken to be $m_h = 120$ GeV, $m_H = 100$ GeV, $\sigma_1 = 0.05$ and $\tan \beta = 10$.

particles coincide with that of the SM Higgs boson at the tree level. Therefore, its production mechanisms are the same as those in the SM. However, when $m_\eta < m_h/2$ it decays into a DM pair $\eta \eta$, namely the invisible decay of Higgs as a common prediction of singlet scalar DM (such as Higgs portal model) $[59]$, whose decay rate is given by

$$
\Gamma(h \to \eta \eta) = \frac{v^2}{32 \pi m_h} \sqrt{1 - \frac{4m_\eta^2}{m_h^2}} \sigma_1 \cos^2 \beta + \sigma_2 \sin^2 \beta |^2. \tag{64}
$$

The branching ratio in our model is shown in Fig. 8. When $m_h = 120$ GeV and $m_\eta = 48$ GeV, the branching ratio of $h \to \eta \eta$ amounts to about 36 % (34%) for $\tan \beta = 3$ (10). For $m_h = 120$ GeV and $m_\eta = 55$ GeV, it is about 25 % (22 %) for $\tan \beta = 3$ (10). For heavier $h$ than 120 GeV, the branching ratio of $h \to \eta \eta$ is smaller because the mode $h \to WW^*$ becomes larger. The decay rate is strongly related to the DM abundance: see Eq. (32) we may be able to reconstruct at least the annihilation into $f \bar{f}$, so that our DM scenario can be testable at the Large Hadron Collider (LHC).

DM searches:

Our DM candidate $\eta$ is potentially detectable by direct and indirect DM searches. For the coupling constants $\sigma_i \sim 10^{-2}$ and the mass $m_\eta \sim 50$ GeV, the typical SI cross section
is of order of $10^{-8}$ pb and within the reach of near future direct search experiments such as superCDMS and XMASS [60]. In Fig. 9, the SI cross section of DM for a proton $\sigma_p^{SI}$ is plotted. In addition, indirect searches by observing cosmic rays which would be a signal of DM annihilation is also promising. It is regarded that a line gamma is the smoking gun of DM annihilation. $\eta$ has a large cross section of annihilation into two photon. Although this is a one-loop process, the cross section is not so much suppressed because of a large coupling $\kappa$ and light particles inside loops ($S^\pm$ and $H^\pm$). As a similar case, one may recall the inert Higgs doublet model where the $W^\pm$-boson loop gives a sizable contribution [61]. The FERMI satellite [62], used to be called GLAST, sensitivity with the solid angle of detector $\Delta \Omega \sim 10^{-5}$ is expected to be $\gtrsim 10^{-9}\text{GeVcm}^{-2}\text{s}^{-1}$ for $\mathcal{O}(10)$ GeV energy. Line gamma-ray emissions could be observed for the Navarro, Frenk, and White (NFW) density profile [63] with the astrophysics dependent dimensionless function $J \sim 10^4$, because the flux originated from $\eta$ annihilation is expected to be as large as

$$E^2\Phi = 5 \times 10^{-8} \left( \frac{50\text{GeV}}{m_\eta} \right) \left( \frac{\sigma v}{10^{-9}\text{GeV}^{-2}} \right) J\Delta \Omega \text{[GeVcm}^{-2}\text{s}^{-1}]$$

with $E = m_\eta$.

The non-decoupling effect of $S^\pm$ on the $hhh$ coupling:

As we have seen the condition for the first order electroweak phase transition requires non-decoupling property for $S^\pm$ or $A$. It is known that such non-decoupling property affects physics of the renormalized $hhh$ coupling. In terms of the renormalized mass parameter $m_h$, the one-loop corrected tri-linear coupling is obtained from the effective potential [64],

$$\lambda_{hhh}^{\text{eff}} = \frac{3m_h^2}{v} \left\{ 1 + \frac{m_H^4}{12\pi^2m_h^2v^2} \left( 1 - \frac{M^2}{m_H^2} \right)^3 + \frac{m_A^4}{12\pi^2m_H^2v^2} \left( 1 - \frac{M^2}{m_A^2} \right)^3 \right. $$

$$+ \frac{m_{H^\pm}^4}{6\pi^2m_h^2v^2} \left( 1 - \frac{M^2}{m_{H^\pm}^2} \right)^3 + \frac{m_{S^\pm}^4}{6\pi^2m_h^2v^2} \left( 1 - \frac{\mu_{S^\pm}^2}{m_{S^\pm}^2} \right)^3 $$

$$+ \frac{m_\eta^4}{12\pi^2m_h^2v^2} \left( 1 - \frac{\mu_\eta^2}{m_\eta^2} \right)^3 - \frac{N_c m_h^4}{3\pi^2m_h^2v^2} + \mathcal{O} \left( \frac{p_t^2m_\Phi^2}{m_h^2v^2}, \frac{m_A^4}{m_h^2v^2}, \frac{m_{H^\pm}^4}{m_h^2v^2}, \frac{m_{S^\pm}^4}{m_h^2v^2}, \frac{m_\eta^4}{m_h^2v^2} \right) \right\}. \quad (66)$$

The deviation from the SM prediction

$$\lambda_{hhh}^{\text{eff}}(\text{SM}) = \frac{3m_h^2}{v} \left\{ 1 - \frac{N_c m_h^4}{3\pi^2m_h^2v^2} + \mathcal{O} \left( \frac{p_t^2m_\Phi^2}{m_h^2v^2}, \frac{m_A^4}{m_h^2v^2} \right) \right\}, \quad (67)$$

is defined by $\Delta \lambda_{hhh}/\lambda_{hhh}^{\text{eff}}(\text{SM})$, where $\Delta \lambda_{hhh} = \lambda_{hhh}^{\text{eff}} - \lambda_{hhh}^{\text{eff}}(\text{SM})$. The quantum effect on the $hhh$ coupling amounts to more than 10-20 % (see FIG. [10], due to the non-decoupling
FIG. 10: The contour plots represent deviations in the $hhh$ coupling from the SM value. For the invariant mass parameters, $M = 100$ GeV and $\mu_S = 200$ GeV are taken. The region of the strongly first order phase transition at this parameter choice is also shown.

property of $A$ and $S^\pm$, which is required for successful baryogenesis \cite{22,24}. Thus, it should be testable at the International Linear Collider (ILC) \cite{65,66,67}.

We note that in addition to the non-decoupling effect on the $hhh$ coupling the non-decoupling property of the charged singlet scalars $S^\pm$ can be further tested by measuring $\gamma\gamma h$ vertices \cite{11}.

Phenomenology of the Type-X Yukawa interaction:

Because of the Type-X Yukawa coupling defined in Eq. \cite{28,29}, the phenomenology for the extra Higgs bosons is different from that in Type I or Type II especially for $\tan \beta \gtrsim 2$. In our model $H$ and $A$ ($H^\pm$) can predominantly decay into $\tau^+\tau^-$ ($\tau^\pm\nu$), while in the Type II THDM the main decay modes of $H$ and $A$ ($H^\pm$) are $b\bar{b}$ ($\tau\nu$) when the masses are around 100 GeV and $\sin(\beta - \alpha) = 1$.

Recently, the phenomenology in Type-X THDM has been studied in the similar parameter choice in Ref. \cite{28}. The physics of Type-X Yukawa interaction can be tested at the LHC with the low luminosity (30 fb$^{-1}$) via the single direct (associated) production processes $gg \rightarrow \Phi \rightarrow \tau^+\tau^-$ and $\mu^+\mu^-$ ($pp \rightarrow b\bar{b}\Phi \rightarrow b\bar{b}\tau^+\tau^-$ and $b\bar{b}\mu^+\mu^-$), where $\Phi = A$ and
FIG. 11: Production cross section of a $S^+S^-$ pair via the Drell-Yan process at the LHC ($\sqrt{s} = 14$ TeV) as a function of the mass of $S^\pm$.

\[ H^\pm \text{ at high luminosity (300 fb}^{-1}) \text{, } pp \to W^* \to AH^\pm \text{ and } pp \to W^* \to HH^\pm \] can also be used to discriminate between the types of Yukawa interaction when the masses are not too large. In the minimal supersymmetric standard model (Type II Yukawa interaction) the main signal would be the $b\bar{b}\tau\nu$ final state while that in the Type-X the final main states would be $\tau^+\tau^−\tau^\pm\nu$ or $\mu^+\mu^−\tau^\pm\nu$.

The phenomenology of the charged singlet scalar field $S^\pm$:

The physics of $Z_2$-odd charged singlet $S^\pm$ is important to distinguish this model from the other models. At the LHC, they are produced in pair via the Drell-Yan process $[11]$. In Fig. 11 the cross section for $pp \to S^+S^-$ at the LHC is shown as a function of the mass $m_{S^\pm}$. The cross section amounts to 0.5 fb for $m_{S^\pm}$, so that more than a hundred of the $S^+S^-$ events are produced for the integrated luminosity 300 fb$^{-1}$. The produced $S^\pm$ bosons decay as $S^\pm \to H^\pm\eta$, and $H^\pm$ mainly decay into $\tau^\pm\nu$ due to the Type-X Yukawa coupling when $\tan\beta \gtrsim 2$ $[28]$. The signal would be a high-energy hadron pair $[69]$ with a large missing transverse momentum.

The charged singlet scalar bosons $S^\pm$ in our model can also be better studied at the ILC via $e^+e^- \to S^+S^-$ shown in Fig. 12(Left). The total cross sections are shown as a function of $m_{S^\pm}$ for several values of the center-of-mass energy $\sqrt{s}$ in Fig. 13(Left).
FIG. 12: Feynman diagrams for the processes of $e^+e^- \rightarrow S^+S^-$ (Left) and $e^-e^- \rightarrow S^-S^-$ (Right).

FIG. 13: [Left] Production cross sections for $e^+e^- \rightarrow S^+S^-$ via the s-channel gauge boson ($\gamma$ and $Z$) mediation (dotted curve), the t-channel RH-neutrino ($N^\alpha_R$) mediation (solid curve), and both contributions (dashed curve) for $\sqrt{s} = 300, 500$ and $1000$ GeV. The masses for RH neutrinos are taken to be $m_{N^1_R} = m_{N^2_R} = 3$ TeV. [Right] The angular distribution of the $e^+e^- \rightarrow S^+S^-$ at $\sqrt{s} = 1$ TeV for $m_{N^1_R} (= m_{N^2_R}) = 0.6, 1, 2$ and $3$ TeV. In both figures, $h_1^e = h_2^e = 2.0$ are taken.

other relevant parameters are taken as $m_{N^1_R} = m_{N^2_R} = 3$ TeV and $h_1^e = h_2^e = 2.0$. Both the contributions from the s-channel gauge boson ($\gamma$ and $Z$) mediation and the t-channel RH neutrino mediation are included in the calculation. The total cross section can amount to about 100 fb for $m_{S^\pm} = 400$ GeV at $\sqrt{s} = 1$ TeV due to the contributions of the t-channel RH neutrino-mediation diagrams with $\mathcal{O}(1)$ coupling constants $h_1^e$. The signal would be a number of energetic tau lepton pairs with large missing energies. Although several processes such as $e^+e^- \rightarrow W^+W^-$ and $e^+e^- \rightarrow H^+H^-$ can give backgrounds for this final state, we expect that the signal events can be separated by kinematic cuts. In Fig. [13(Right)], the dependences on the scattering angle in the differential cross section $d\sigma/d(\cos \theta)$ are shown. For $m_{N^1_R} = m_{N^2_R} = 3$ TeV, the special behavior in the angular distribution is more insensitive than the cases with lighter values for $m_{N^R}$.
Finally, there is a further advantage in testing our model at the $e^-e^-$ collision option of the ILC, where the dimension five operator $e^-e^-S^+S^+$, which appears in the sub-diagram of the three loop induced masses of neutrinos in our model, can be directly measured. The production cross section for $e^-e^- \rightarrow S^-S^-$ [t-channel $N^\alpha_R$ mediation: see Fig. 12(Right)] is given by

$$\sigma(e^-e^- \rightarrow S^-S^-) = \int_{t_{\min}}^{t_{\max}} dt \frac{1}{128\pi s} \left| \sum_{\alpha=1}^{2} (|h^\alpha_e|^2 m_{N^\alpha_R}) \left( \frac{1}{t - m_{N^\alpha_R}^2} + \frac{1}{u - m_{N^\alpha_R}^2} \right) \right|^2. \quad (68)$$

Because of the structure of our model that the tiny neutrino masses are generated at the three loop level, the magnitudes of $h^\alpha_e$ ($\alpha = 1, 2$) are of $\mathcal{O}(1)$, by which the cross section becomes very large. Furthermore, thanks to the Majorana nature of the t-channel diagram, we obtain a much larger cross section in the $e^-e^-$ collision than at the $e^+e^-$ collision when $m_{N^\alpha_R}^2 \gg s$. The cross section can be as large as 10 pb for $m_{S^\pm} = 400$ GeV for $\sqrt{s}_{e^-e^-} = 1$ TeV, $m_{N^1_R} = m_{N^2_R} = 3$ TeV and $h^1_e = h^2_e = 2.0$: see Fig. 14. The backgrounds are expected to be much less than the $e^+e^-$ collision.

We emphasize that a combined study for these processes would be an important test for our model, in which neutrino masses are generated at the three loop level.
by the $Z_2$ symmetry and the TeV-scale RH neutrinos\footnote{Unlike our model, in the model in Ref. \cite{17}, the coupling constants corresponding to our $h^\alpha_e$ are small and instead those to $h^\alpha_\mu$ are $\mathcal{O}(1)$, so that its Majorana structure is not easy to test at $e^-e^-$ collisions.}. In the other radiative seesaw models in which the neutrino masses are induced at the one-loop level with RH neutrinos, the corresponding coupling constants to our $h^\alpha_e$ couplings are necessarily one or two orders of magnitude smaller to satisfy the neutrino data, so that the cross section of the $t$-channel RH neutrino mediation processes are small due to the suppression factor $(h^\alpha_e)^4$.

Lepton flavor violation:

Finally, the couplings $h^\alpha_i$ cause lepton flavor violation such as $\mu \rightarrow e\gamma$, depending on $m_{N_R}$. If such a phenomenon is observed at future experiments \cite{70}, we could obtain information on $m^\alpha_{N_R}$. In addition, our model predicts $B(\mu \rightarrow e\gamma) \gg B(\tau \rightarrow e\gamma) \gg B(\tau \rightarrow \mu\gamma)$.

In summary, the possible scenario in this model provides discriminative phenomenological characteristics so that it can be tested at future experiments.

We have discussed the various features of this model neglecting the CP violating phases in the Higgs sector, which are crucial for generating baryon number at the EWPT. We comment on the case with the CP violating phases. Our model includes the THDM, so that the same generation mechanism can be applied in evaluation of produced baryon number asymmetry at the EWPT unless $\tan \beta$ is too large \cite{23, 24}. For a larger value of $\tan \beta$, the constraint from the EDM data would be more serious. The mass spectrum in the Higgs sector would be changed by including the CP violating phases, but most of the phenomenological features discussed above should be conserved with a little modification.

\section{Discussions and Conclusions}

We have discussed the model in which neutrino oscillation, DM, and baryon asymmetry of the Universe can be simultaneously explained by the TeV-scale physics without introducing large fine tuning. Tiny neutrino masses are generated at the three loop level due to the exact $Z_2$ symmetry, by which stability of the DM candidate is also guaranteed. The extra Higgs doublet is required not only for the tiny neutrino masses but also for
successful electroweak baryogenesis. The phenomenology of the model has been discussed, and it has been found that there are several successful scenarios under the constraints from the current experimental data. The predictions have been discussed in these scenarios at the present and future experiments. It turns out that the model provides discriminative predictions especially in Higgs physics and DM physics, so that it is thoroughly testable in future experiments.

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APPENDICES

A. SM-like limit

Especially, for the case of the SM like limit \( \sin(\alpha - \beta) = -1 \) \[71\], namely \( \alpha = \beta - \pi/2 \), (\( \sin \alpha = -\cos \beta \) and \( \cos \alpha = \sin \beta \)), \( h \) becomes the SM-like Higgs boson and \( H \) decouples from the gauge fields (with respect to the three point couplings). In this case, the Yukawa interactions are

\[
L_{Y}^{\text{Quarks}} = -m_t \bar{t}t + \frac{m_t}{v} \cot \beta H \bar{t}t - \frac{m_t}{v} h \bar{t}t \\
- m_b \bar{b}b + \frac{m_b}{v} \cot \beta H \bar{b}b - \frac{m_b}{v} h \bar{b}b \\
+ \frac{m_t}{v} z \bar{t}i\gamma_5 t + \frac{m_t}{v} \cot \beta A \bar{t}i\gamma_5 t \\
- \frac{m_b}{v} z \bar{b}i\gamma_5 b - \frac{m_b}{v} \cot \beta A \bar{b}i\gamma_5 b \\
+ \omega - \bar{b} \left[ \sqrt{2} m_t \cos \beta \frac{1 + \gamma_5}{2} - \sqrt{2} m_b \sin \beta \frac{1 - \gamma_5}{2} \right] t + \text{h.c.} \\
+ H - \bar{b} \left[ \sqrt{2} m_t \sin \beta \frac{1 + \gamma_5}{2} - \sqrt{2} m_b \cos \beta \frac{1 - \gamma_5}{2} \right] t + \text{h.c.}
\]

(69)

and

\[
L_{Y}^{\text{Leptons}} = + \sqrt{2} m_\tau \bar{\nu} \frac{1 + \gamma_5}{2} \nu \tau^+ + \text{h.c.} \\
- \sqrt{2} m_\tau \tan \beta \frac{1 + \gamma_5}{2} \tau H^+ + \text{h.c.} \\
- m_\tau \tau^+ - \frac{m_\tau}{v} \tan \beta H \tau^+ - \frac{m_\tau}{v} h \tau^+ \\
- \frac{m_\tau}{v} z \bar{\tau}i\gamma_5 \nu + \frac{m_\tau}{v} \tan \beta A \bar{\tau}i\gamma_5 \nu.
\]

(70)

In the decoupling limit \( \sin(\alpha - \beta) = -1 \), the masses of scalar fields are expressed as

\[
m_h^2 = M_{11}^2 = (\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + 2\lambda \cos^2 \beta \sin^2 \beta)v^2,
\]

(71)

\[
m_H^2 = M_{22}^2 = M^2 + (-\lambda_1 \cos^4 \beta + \lambda_2 \sin^2 \beta + \lambda \cos 2\beta) \cos \beta \sin \beta v^2,
\]

(72)

\[
m_{H^\pm}^2 = M^2 - \lambda_4 + \lambda_5 v^2,
\]

(73)

\[
m_A^2 = M^2 - \lambda_5 v^2,
\]

(74)

\[
m_S^2 = \mu_S^2 + (\rho_1 \cos^2 \beta + \rho_2 \sin^2 \beta)\frac{v^2}{2},
\]

(75)

\[
m_\eta^2 = \mu_\eta^2 + (\sigma_1 \cos^2 \beta + \sigma_2 \sin^2 \beta)\frac{v^2}{2}.
\]

(76)
B. Neutrino mass matrix

In order to compute the three loop induced neutrino mass matrix $M_{ij}^\nu$ in Eq. (24), we start from the calculation of the effective vertex of $\ell_R(p_1) - \ell_R(p_2) - H^+(p_3) - H^+(p_4)$ comes from the loop by $Z_2$-odd particles, $iT(p_1, p_2, p_3, p_4)$, where $p_1, p_2, p_3$ and $p_4$ are incoming momenta of $\ell_R(p_1), \ell_R(p_2), H^+(p_3)$, and $H^+(p_4)$ respectively;

$$iT_{ij}(p_1, p_2, p_3, p_4)$$

$$= \sum_{\alpha=1}^{2} \int \frac{d^D k}{(2\pi)^D} (-ih_i^\alpha) \left( \frac{1 + \gamma_5}{2} \right) \frac{i(k + m_{N^\alpha})}{k^2 - m_{N^\alpha}^2} \left( \frac{1 + \gamma_5}{2} \right) (-ih_j^\alpha) \frac{i}{(k + p_1)^2 - m_{S^\pm}^2} \times (-i\sqrt{2\kappa v}) \frac{i}{(k + p_1 + p_3)^2 - m_\eta^2} \times (-i\sqrt{2\kappa v}) \frac{i}{(k + p_1 + p_3 + p_4)^2 - m_{S^\pm}^2}$$

(77)

The Majorana mass matrix of the left handed neutrinos is generated by connecting the external lines of $T_{ij}(p_1, p_2, p_3, p_4), \ell_R^i$ and $H^+$ with $\nu_L^i$ (also $\ell_R^i, H^+$ with $\nu_L^i$) by the Yukawa coupling, and integrate all the internal momenta taking into account the momentum conservation. There are two Feynman diagrams which give exactly the same contribution, corresponding to the way of connecting which $H^\mp$ couples to $\nu_L^i$: see Fig. 1. The mass matrix is calculated as

$$iM_{ij}^2 = + \int \frac{d^D p_1}{(2\pi)^D} \int \frac{d^D p_2}{(2\pi)^D} \int \frac{d^D p_3}{(2\pi)^D} \int \frac{d^D p_4}{(2\pi)^D}$$

$$\times (-i\sqrt{2\kappa v}) \frac{i}{(k + p_1)^2 - m_\eta^2} \times (-i\sqrt{2\kappa v}) \frac{i}{(k + p_1 + p_3)^2 - m_{S^\pm}^2} \times (-i\sqrt{2\kappa v}) \frac{i}{(k + p_1 + p_3 + p_4)^2 - m_{S^\pm}^2}$$

$$\times (-i\sqrt{2\kappa v}) \frac{i}{(k + p_1 + p_3 + p_4)^2 - m_{S^\pm}^2}$$

$$\times \left( \frac{1}{(2\pi)^D p_1^2 - m_{N^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_2^2 - m_{L^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_3^2 - m_{H^\mp}^2} \right) \left( \frac{1}{(2\pi)^D p_4^2 - m_{H^\mp}^2} \right)$$

$$\times \left( \frac{1}{(2\pi)^D p_1^2 - m_{N^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_2^2 - m_{L^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_3^2 - m_{H^\mp}^2} \right) \left( \frac{1}{(2\pi)^D p_4^2 - m_{H^\mp}^2} \right)$$

$$\times \left( \frac{1}{(2\pi)^D p_1^2 - m_{N^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_2^2 - m_{L^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_3^2 - m_{H^\mp}^2} \right) \left( \frac{1}{(2\pi)^D p_4^2 - m_{H^\mp}^2} \right)$$

$$\times \left( \frac{1}{(2\pi)^D p_1^2 - m_{N^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_2^2 - m_{L^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_3^2 - m_{H^\mp}^2} \right) \left( \frac{1}{(2\pi)^D p_4^2 - m_{H^\mp}^2} \right)$$

(78)

$$\times \left( \frac{1}{(2\pi)^D p_1^2 - m_{N^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_2^2 - m_{L^\alpha}^2} \right) \left( \frac{1}{(2\pi)^D p_3^2 - m_{H^\mp}^2} \right) \left( \frac{1}{(2\pi)^D p_4^2 - m_{H^\mp}^2} \right)$$

where we used momentum conservation law $p_3 = -p_1$ and $p_4 = -p_2$ ($p_3 = -p_4$ and $p_4 = -p_1$) for the first (the second) diagram, neglecting the invariant mass of neutrinos. By using Passarino-Veltman formalism for one-loop integral functions, we obtain the expression
as
\[
    iM_{ij} = + \left( \frac{1}{16\pi^2} \right)^2 \left( \frac{1 - \gamma_5}{2} \right) \sum_{\alpha=1}^{2} \left[ \frac{4\kappa^2 v^2 \tan^2 \beta m_{N^\alpha}}{m^4_{N^\alpha} - m^2_{\eta}} \right] \left( \frac{y_{i\ell_i}^{SM} h_{i}^\alpha}{m^2_{H^\pm} - m^2_{\ell_i}} \right) \left( \frac{y_{j\ell_j}^{SM} h_{j}^\alpha}{m^2_{H^\pm} - m^2_{\ell_j}} \right)
\]
\[
    \times \left\{ \int \frac{d^Dk}{(2\pi)^D} \left( \frac{m^2_{N^\alpha}}{k^2 - m^2_{N^\alpha}} - \frac{m^2_{\eta}}{k^2 - m^2_{\eta}} \right) (B_1(k^2, m^2_{H^\pm}, m^2_{S^\pm}) - B_1(k^2, m^2_{\ell_i}, m^2_{S^\pm})) \right\}
\]
\[
    \times (B_1(k^2, m^2_{H^\pm}, m^2_{S^\pm}) - B_1(k^2, m^2_{\ell_j}, m^2_{S^\pm})) \right]\right], \quad (79)
\]
where \( B_1(k^2, m_1, m_2) \) is the tensor coefficient by Passarino and Veltman [33]. As \( m^2_{\ell} \ll m^2_{H^\pm} \), we neglect \( m^2_{\ell} \) in the expression and obtain
\[
    iM_{ij} = + \left( \frac{1}{16\pi^2} \right)^2 \left( \frac{1 - \gamma_5}{2} \right) \frac{4\kappa^2 v^2 \tan^2 \beta m_{N^\alpha}}{m^4_{H^\pm}} \sum_{\alpha=1}^{2} \left[ \frac{m^2_{N^\alpha}}{m^2_{N^\alpha} - m^2_{\eta}} \right] (y_{i\ell_i}^{SM} h_{i}^\alpha)(y_{j\ell_j}^{SM} h_{j}^\alpha)
\]
\[
    \times \left\{ \int \frac{d^Dk}{(2\pi)^D} \left( B_1(k^2, m^2_{H^\pm}, m^2_{S^\pm}) - B_1(k^2, m^2_{\ell_i}, m^2_{S^\pm}) \right) \right\}
\]
\[
    \times (B_1(k^2, m^2_{H^\pm}, m^2_{S^\pm}) - B_1(k^2, m^2_{\ell_j}, m^2_{S^\pm})) \right]\right], \quad (80)
\]
As the asymptotic behavior \( k^2 \to \infty \) of the \( B_1 \) function is
\[
    \{ B_1(k^2, m_H, m_S) - B_1(k^2, 0, m_S) \} \sim 1/k^2, \quad (81)
\]
so that the integral over \( d^4k \) is not divergent. For numerical evaluations, we work in the Euclideanized momentum space,
\[
    k^2 = -k^2_E, \quad d^4k = id^4k_E = i\pi^2 k^2_E d^4(k^2_E).
\]
and introduce the cutoff scale \( \Lambda \) which is a very large number as compared to the scale of \( m_H \) or \( m_S \) and so on. Then we obtain the expression in Eq. (24).

C. Line photon flux from DM annihilation

The differential flux of gamma-ray from a DM annihilation near the center of our galaxy is given as
\[
    \frac{d\Phi}{d\Omega}(E, \psi) = \frac{1}{4\pi m^2_{\eta}} \left[ \langle \sigma v(\to \gamma\gamma) \rangle \delta(E - m_{\eta}) + \langle \sigma v(\to f\bar{f}) \rangle \frac{dN}{dE} \right] \int \text{l.o.s.} \, dl(\psi) \rho(l)^2, \quad (83)
\]
where \( \psi \) is the angle to the galactic center direction, \( \rho(l) \) is the mass density distribution for the DM and we will integrate it along the line of sight \( l \) [72]. The first term in the right hand side of Eq. (83) denotes the line spectrum comes from the annihilation into two photons,
and the second term does to the continuous one. The latter with the differential photon spectrum \( dN/dE \) dominantly comes from the decay of pions produced by the fragmentation or decay of DM annihilation final state \( f \) such as b-quark or \( \tau \) lepton.

For the integration around the line of sight axis over the solid angle \( \Delta \Omega \), we introduce a dimensionless function \[ J(\psi) = \frac{1}{8.5\text{kpc}} \left( \frac{1}{0.3\text{GeVcm}^{-3}} \right) \int_{\text{l.o.s.}} d\ell(\psi) \rho(\ell)^2, \tag{84} \]
in which all informations about the DM halo model are encoded. For a given \( \Delta \Omega \sim 10^{-3}(10^{-5}) \), one may find, for instance, \( J \sim 10^3(10^4) \) for NFW density profile \[63\] and \( J \sim 10(10) \) for isothermal \[74\]. With the integration over angle and substitution of Eq. \( \tag{84} \), we obtain Eq. \( \tag{65} \).

**D. High Temperature Expansion**

Let us calculate the integral \( I_f \) by using the high temperature expansion \[54\]. For the case of bosonic contrbutions, we have

\[
I_B(a) = \int_0^\infty dx x^2 \log[1 - e^{-\sqrt{x^2 + a^2}}]
= -\frac{2\pi^2}{T^4} \left\{ -\frac{1}{\beta} \int_0^\infty \frac{d^3p}{(2\pi)^3} \log[1 - e^{-\beta\omega}] \right\}, \tag{85}
\]
where \( \omega = \sqrt{p^2 + m^2} \). The integral is expanded as

\[
-\frac{1}{\beta} \int_0^\infty \frac{d^3p}{(2\pi)^3} \log[1 - e^{-\beta\omega}]
= \frac{\pi^2}{90} T^4 - \frac{1}{24} m^2 T^2 + \frac{1}{12\pi} m^3 T + \frac{m^4}{64\pi^2} \left[ \log \left( \frac{m^2}{16\pi^2 T^2} \right) + 2\gamma_E - \frac{3}{2} \right] + ..., \tag{86}
\]
where \( \gamma_E = 0.5772 \) is the Euler constant. For fermions, we obtain

\[
I_F(a) = \int_0^\infty dx x^2 \log[1 + e^{-\sqrt{x^2 + a^2}}]
= -\frac{2\pi^2}{T^4} \left\{ -\frac{1}{\beta} \int_0^\infty \frac{d^3p}{(2\pi)^3} \log[1 + e^{-\beta\omega}] \right\}
= -\frac{2\pi^2}{T^4} \left\{ \frac{7\pi^2}{720} T^4 - \frac{1}{48} m^2 T^2 - \frac{m^4}{64\pi^2} \left[ \log \left( \frac{m^2}{\pi^2 T^2} \right) + 2\gamma_E - \frac{3}{2} \right] + ... \right\}. \tag{87}
\]
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