Methods for Setting up a Three-Dimensional Industrial Surveying System of “Building Blocks Type”

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ABSTRACT This paper is to advance some relevant techniques to set up a three-dimensional industrial surveying system of “building blocks type”, making use of the electronic theodolite, standard ruler and portable computer.

KEY WORDS three-dimensional industrial surveying system; initial direction; inner target; collimator method; standard ruler; accuracy

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1 Methods for orientating initial direction in industrial survey

Different from conventional engineering survey, the distance between the two angle-measurement devices in industrial survey is no more than ten meters. The accuracy for the measurement of object points is required to be between ±(0.02~0.05~0.20) mm, so the accuracy for the orientation of the initial direction ought to be high. The existing methods for orientating the initial direction between the two angle-measurement devices include method of using inner target, collimator method, method of revolving the targeted equipment, method of symmetric objects, method of orientation by back sight and method of using additional target.

2 Conventional methods for error analysis based on the displacement of points

The surveying accuracy of the coordinate of the third dimension (height Z) in the three-dimensional industrial surveying system is basically influenced by plane accuracy of point \( M_P \). So this paper is not to discuss the survey of Z.

According to forward intersection accuracy estimation formula, if the initial direction error is ignored, \( M_P \) of the displacement of the unknown point \( P \) is:

\[
M_P = m \frac{\sqrt{a^2 + b^2}}{\sin \gamma}
\]

If \( a \approx b \), \( \gamma = 45^\circ \), then

\[
M_P = \frac{2a \cdot m}{\rho} (2)
\]

The accuracy estimation of unknown point \( M_P \) for different angle-measurement error \( m \) and \( a \), the different distance of unknown point, are shown by Table 1.

| \( m \) | \( 2^" \) | \( 1^" \) | \( 0.5^" \) |
|---|---|---|---|
| \( 2^" \) | 0.038 | 0.097 | 0.194 | 0.388 | 0.970 |
| \( 1^" \) | 0.019 | 0.048 | 0.097 | 0.194 | 0.485 |
| \( 0.5^" \) | 0.010 | 0.024 | 0.048 | 0.097 | 0.242 |

It can be seen that if the initial point error is ignored to meet the surveying accuracy ±0.05 mm, when angle-measurement is ±0.5", the object distance cannot exceed 10 m; if the angle-measurement accuracy is ±1", the
object distance is not beyond 5 m; if the angle measurement accuracy is ±2", the object distance cannot exceed 2.5 m.

In industrial survey, the RMSE (root mean square error) of the initial direction \( m_k \) will lead to extra \( M_{AM} \) of the displacement of points:

\[
M_{AM} = \frac{m_k \sqrt{a^2 + b^2}}{\rho \sin \gamma}
\]

That is say, the RMSE \( m_k \) of the initial direction will lead to extra point position RMSE of the same amount\(^{(1)}\). If \( m_k = \pm 2" \), for the object 5 meters away, then \( M_{AM} = \pm 0.1 \text{ mm} \), if \( m_k = \pm 1" \), for the object 10 meters away, then \( M_{AM} \approx \pm 0.1 \text{ mm} \).

### 3 Displacement of points and deformation of figure resulting from orientation error with the same amount and equal direction

The orientation error with the same amount and equal direction refers to the orientation method for making initial directions parallel but not mixed. In conventional long-distance engineering survey, nothing is discussed about the above-mentioned initial direction error and its influence, but in short-distance (e.g. no more than 10 meters) industrial survey, if collimator method is used to orientate the initial direction, the orientation accuracy will be improved and the operation is made possible. The following deduction can show that the initial direction error with the same amount and equal direction will lead to the rotation and deformation of the whole object to be surveyed.

#### 3.1 Displacement of points resulting from initial direction error with the same amount and equal direction

According to the forward intersection formula of conventional engineering survey, as is shown in Fig. 1, we have an equation of plane coordinates for point \( P \) to be measured:

\[
\begin{align*}
X_P &= X_A + b \cos \alpha \cos \beta \frac{\delta_2}{\rho} + b \sin \alpha \left( - \sin \alpha \sin \beta \right) \\
Y_P &= Y_A + b \sin \alpha \cos \beta \frac{\delta_2}{\rho} + b \sin \beta \sin \alpha \cos \beta
\end{align*}
\]

(4)

In operation, if there is error with the same amount and equal direction \( \delta \) in the initial direction, this direction error \( \delta \) should be related to the resulted corresponding inner angle error \( \delta_1 \) and \( \delta_2 \):

\[
\delta = - \delta_1 = \delta_2 = \delta_0
\]

(5)

If there is ±1" change in direction angle \( \alpha \), angle \( \alpha \) will decrease by 1", and angle \( \beta \) will increase by 1".

Attention should be paid to this relation. Taking \( \alpha \) and \( \beta \) as variable and differentiating Eq. (1), there is a displacement of point coordinates:

\[
\begin{align*}
dX_P &= \frac{S}{\sin(\alpha + \beta)} \left[ \cos \beta \sin \alpha - \sin \beta \left( - \sin \alpha \right) \right] \\
dY_P &= \frac{S}{\sin(\alpha + \beta)} \left[ \cos \beta \cos \alpha - \sin \beta \sin \alpha \cos \beta \right]
\end{align*}
\]

Suppose the direction from station \( A \) and station \( B \) is the coordinate axis \( Y \), as \( \sin \alpha = \cos \beta \), \( \cos \alpha = \sin \beta \), so the above formula can be simplified as:

\[
\begin{align*}
dX_P &= \frac{S}{\rho} \left( \frac{\delta}{\rho} \right) \\
dY_P &= \frac{S}{\rho} \left( \frac{\delta}{\rho} \right)
\end{align*}
\]

(6)

and the general displacement \( dP \) of point:

\[
dP = (dX_P^2 + dY_P^2)^{\frac{1}{2}} = \frac{\delta}{\sin \gamma \cdot \rho}
\]

(7)

Eq. (7) and Eq. (8) represent, respectively, the point coordinate error \( (dX_P, dY_P) \) and the general displacement \( dP \) that results from initial direction error \( \delta \) with the same amount and equal direction.

#### 3.2 Deformation of figures resulting from initial direction error with the same amount and equal direction

In industrial object surveying, e.g. the survey of large military antenna, attention is only paid to the shape and size of the antenna, and its absolute location to the angle-measurement device is ignored.

Suppose there is a square object whose side
S, the locations of its four angle points \((P_1, P_2, P_3, P_4)\) are shown in Fig. 1. Common industrial objects should be within the square object, and the objects are more likely to be closer to the surveying stations \((A, B)\) to have a better mixing angle. As is shown by analysis, if there is an error \(\delta\) in the initial direction, the figure is shifted to \((P'_1, P'_2, P'_3, P'_4)\). For the absolute position of \((P_1, P_2, P_3, P_4)\) is not to be studied, it is only necessary to analyze the difference between the figure \((P_1, P_2, P_3, P_4)\) and the figure \((P'_1, P'_2, P'_3, P'_4)\).

The coordinate change of the four points are shown in Table 2, and \(a = \frac{\delta}{\rho}\).

| \(dX\) | \(dY\) |
|---|---|
| \(-a\) | \(-a\) |
| \(+a\) | \(-a\) |
| \(+a\) | \(-2a\) |
| \(-a\) | \(-2a\) |

On the basis of analysis, the figure after deformation \((P'_1, P'_2, P'_3, P'_4)\) is a rhombus, as is shown in Fig. 2.

1) There is no obvious difference between side \(d\) of rhombus (e.g. \(P'_1 P'_2\)) and side \(S\) of square.

\[ d = \sqrt{S^2 + a^2} \approx S \] (9)

2) The long diagonal \(t_1 (= P'_1 P'_2)\) and the short diagonal \(t_2 (= P'_3 P'_4)\) are:

\[
\begin{align*}
    t_1 &= \sqrt{2S^2 + 2Sa + 5a^2} \approx \sqrt{2S(S + a)} \\
    t_2 &= \sqrt{2S^2 - 2Sa + 5a^2} \approx \sqrt{2S(S - a)}
\end{align*}
\] (10)

The difference \(|dl|\) between them is:

\[ |dl| = \left| \sqrt{2S^2 + 2Sa + 5a^2} - \sqrt{2S^2 - 2Sa + 5a^2} \right| \\ 
  \approx \left| \sqrt{2a} \right| \] (11)

3) For the sides of the rhombus are already known, the formula for acute angle and obtuse angle can be known accordingly:

\[
\begin{align*}
    \cos \varphi &= -\frac{\delta}{\rho} \\
    \cos \eta &= \frac{\delta}{\rho}
\end{align*}
\] (12)

When \(\delta = \pm 2''\), then \(\eta = 90^\circ00'02''\), \(\varphi = 89^\circ59'58''\). Therefore, the change in the internal angle of the rhombus is the same as the initial direction error \(\delta\).

As a summary, when \(\delta\) is a small angle, as is shown in Fig. 2, the square whose side is \(S\) will change into a rhombus whose side is still \(S\). The change in the internal angle is the same as the initial direction error. The difference between the lengths of the two diagonals \(|dl|\) is \(\sqrt{2a}\), that is \(\sqrt{2a}\).

The coordinate change of the four points are shown in Table 2, and \(a = \frac{\delta}{\rho}\).

On the basis of analysis, the figure after deformation \((P'_1, P'_2, P'_3, P'_4)\) is a rhombus, as is shown in Fig. 2.

Fig. 2 Deformation of figure

On the side \(P'_1 P'_2\), the relative error to point \(P'_2\), the maximum deformation of point \(P'_1\), is \(P'_2Q = a\):

\[ a = S \frac{\delta}{\rho} \] (13)

If it is changed into the form of RMSE, then we have

\[ m_a = S \frac{m_{\delta_1}}{\rho} \] (14)

If the RMSE of the initial direction is studied according to the \(m_{\delta_1}\) of the displacement of point \(m_{\delta_1}\), with reference to Eq. (3) and Fig. 1, we have

\[ m_{\delta_1} = M_{\delta_1} \sin \gamma \sqrt{\frac{a^2 + b^2}{P}} \] (15)

On the other hand, if the RMSE of the initial direction is studied according to the RMSE \(m_{\delta_1}\),
of the deformation of figures, with reference to Eq. (14), we have
\[ m_{x_1} = \frac{m_x\rho}{s} \]  \hspace{1cm} (16)

Suppose \( M_{xM_p} = m_x = \pm 0.05 \text{ mm} \), to the unknown point \( P'_4 \) (as is shown by Fig. 1) 10 m away, \( S = 5 \ 000 \text{ mm} \), \( a = 10 \ 000 \text{ mm} \), \( b = \sqrt{5} \times 5 \ 000 \text{ mm} \); \( \sin \gamma = \frac{1}{\sqrt{5}} \), then
\[ m_{x_1} = \frac{0.05 \cdot \frac{1}{\sqrt{5}}}{\sqrt{10 \ 000^2 + 5 \times 5 \ 000^2}} \rho = \pm 0.31'' \] \hspace{1cm} (17)

\[ m_{x_2} = \frac{0.05}{5 \ 000} \rho = \pm 2.06'' \] \hspace{1cm} (18)

Consequently, if \( m_{x_2} \) of the initial direction is calculated according to the error \( M_{xM_p} \) in the displacement, it is difficult to achieve the accuracy \( \pm 0.3'' \), even with the most advanced angle-measurement devices (e. g. 0.5'' equipment). On the contrary, if the \( m_{x_1} \) of the initial direction error with the same amount and equal direction according to the figure deformation theory, the accuracy \( \pm 2.06'' \) is easy to obtain.

If “collimator method” succeeds in making initial directions parallel, then there are numerous orientation results. In order to reduce \( \delta \), the above orientation methods can be used to make \( \delta \) reach the class of \( \pm (1-2)'' \). Then on the basis of the deviation degree and the direction of cross wire, with the employment of “collimator method”, \( \delta \) will be reduced further.

4 Survey of the length of basic lines based on standard rulers

The survey of the length of the basic lines between two surveying stations and the heights of the two stations can be carried out with the help of the standard ruler method.

To survey the distance \( S \) between station \( A \) and station \( B \), as is shown in Fig. 3, there is coordinate system \( A-XYZ \) and standard ruler \( MN \) and there exists the following relation:
\[ s = s' \frac{L}{L'} = s' \frac{B_y}{B_y} = s' \frac{B_x}{B_x} = s' \frac{B_z}{B_z} \] \hspace{1cm} (19)

Fig. 3 Application of standard ruler

Standard rulers can be 3-meter long invar tape, or other length measurement rulers within the tolerance of linear expand coefficient. These standard rulers should be inspected. Horizontal requirement for placing the standard ruler is not very strict, and it is easy to operate. It is suggested that standard ruler \( MN \) should be placed on the circle with \( AB \) as its diameter, basically parallel to the line \( AB \), as is shown in Fig. 4. This operation makes the mixing angle nearly \( 90^\circ \) and reduces the deformation of image within the visual range.

Fig. 4 Approximate position of standard ruler

According to the forward intersection accuracy estimation formula, the RMSE of point \( M \) is:
\[
\begin{align*}
m_{x_M}^2 &= \left( \frac{\sin \alpha \cos \alpha_{AM}}{\sin^2(\alpha + \beta)} \right)^2 \frac{m_x^2}{\rho^2} + \left( \frac{\sin \alpha \sin \alpha_{AM}}{\sin^2(\alpha + \beta)} \right)^2 \frac{m_{y_M}^2}{\rho^2} \\
m_{y_M}^2 &= \left( \frac{\sin \alpha \cos \alpha_{AM}}{\sin^2(\alpha + \beta)} \right)^2 \frac{m_y^2}{\rho^2} + \left( \frac{\sin \beta \sin \beta_{AM}}{\sin^2(\alpha + \beta)} \right)^2 \frac{m_{z_M}^2}{\rho^2}
\end{align*}
\] \hspace{1cm} (20)

As is shown in Fig. 4, what influences the surveying accuracy of \( MN \) is mainly \( m_{y_M} \), so the influence of \( m_{x_M} \) can be ignored. In consideration with \( \cos(\alpha + \beta) \approx 0 \), \( \sin(\alpha + \beta) \approx 1 \), there is
\[
\begin{align*}
m_{x_M}^2 &= \left( \frac{\sin \alpha \sin \alpha_{AM}}{\sin^2(\alpha + \beta)} \right)^2 \frac{m_x^2}{\rho^2} + \left( \frac{\sin \beta \sin \beta_{AM}}{\sin^2(\alpha + \beta)} \right)^2 \frac{m_{z_M}^2}{\rho^2}
\end{align*}
\] \hspace{1cm} (21)

Suppose \( m = m_x = m_y \), and in consideration with the following formula:
where \( b = \frac{S}{\sin(\alpha + \beta)} \sin\beta \approx S\sin\beta \approx Scosa \)

\[
\sin a_{AM} \approx \cos a \quad \cos a_{AM} \approx \sin a
\]

And \( m_{yM} \) in Eq. (21) can be simplified as

\[
m_{yM} = \frac{\sqrt{2}}{2} \frac{m_{yN}}{\rho} S \sin 2a
\]  

That is, the surveying accuracy of point \( M \) with the standard ruler is corresponding to angle \( a \).

According to this, \( M_L \) of standard ruler \( L \) is influenced by the RMSE of points \( M \) and \( N \):

\[
m_L = \sqrt{m_{xM}^2 + m_{yM}^2} = \frac{\sqrt{2}}{\rho} m_N S \sqrt{\sin^2 2a_N + \sin^2 2a_M}
\]

(23)

For \( a_N = 90^\circ - a_M \), we can get a very simple and clear estimation formula for \( m_L \) of the standard ruler:

\[
m_L = \frac{m_L S \sin 2a_M}{\rho}
\]

(24)

If \( S=2.4 \text{ m}, L=1.2 \text{ m}, m_s = \pm 1^\prime \), on the basis of above formulae, the data for reference are

\[
a_M = \arctan(\frac{\sqrt{2.4^2 - 1.2^2}}{2.4 - 1.2}) = 60.0^\circ
\]

\[
a_N = \arctan(\frac{\sqrt{2.4^2 - 1.2^2}}{2.4 + 1.2}) = 30.0^\circ
\]

\[
m_M = \pm \frac{\sqrt{2}}{2} \frac{\pm 1^\prime}{206.265} \times 2 \times 400 \sin(2 \times 60^\circ) = \pm 0.007 \text{ mm}
\]

\[
m_N = \pm \frac{\sqrt{2}}{2} \frac{\pm 1^\prime}{206.265} \times 2 \times 400 \sin(2 \times 30^\circ) = \pm 0.007 \text{ mm}
\]

\[
m_L = \pm \frac{\sqrt{2}}{2} \frac{\pm 1^\prime}{206.265} \times 2 \times 400 \times \sqrt{\sin^2(2 \times 60^\circ) + \sin^2(2 \times 30^\circ)} = \pm 0.01 \text{ mm}
\]

(25)

The surveying accuracy \( m_s \) for the distance \( S \) is

\[
m_s = nm_L = 2m_L = \pm 0.02 \text{ mm}
\]

Suppose side \( S \) is 6 000 mm or 9 000 mm, and the direction RMSE are 2", 1" and 0.5", respectively, then the \( m_s \) prediction RMSE of side \( S \) is shown in Table 3.

| Side (mm) | Prediction RMSE of sides/mm |
|----------|-----------------------------|
| 6 000    | \pm 2" \pm 0.100 \pm 0.046 |
| 9 000    | \pm 1" \pm 0.050 \pm 0.123 |
|          | \pm 0.5" \pm 0.025 \pm 0.065 |

### 5 Conclusions

1) Instead of making use of the imported three-dimensional industrial surveying system which is expensive, we can take advantage of the existing equipment to set up a three-dimensional industrial surveying system of “building blocks type”. The system includes on-line angle-measurement devices (e.g., total station, electronic theodolite), the standard ruler (e.g., invar tape that has been inspected), portable computer and necessary softwares.

2) Collimator method results in error with the same amount and equal direction in the initial direction of the two surveying stations, which further leads to the displacement and deformation of the whole object which is surveyed. However, the collimator method is a strict method and can be successful applied in short-distance industrial survey. Besides, the deformation of figures measured with this method is acceptable tolerance to industrial objects. The advantage of this method is that its orientation limit error is much more loose than that of conventional methods.

3) Other orientation methods can be used to carry out three-dimensional industrial survey, such as the corresponding estimation formula, which includes the accuracy estimation of the survey of basic lines to predict the accuracy of the result.

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