The Limiting Curve of Leading Particles from Hadron-Nucleus Collisions at Infinite A

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Abstract

We argue that as the atomic number of the target nucleus $A \to \infty$, the multiplicity of leading particles in hadron-nucleus collisions tends to a finite limit. The limiting multiplicities for various particle production are computed for both proton and pion projectiles. Signatures at finite A are discussed. Data from 100 GeV/c central hadron-nucleus collisions are analyzed and found to be in qualitative agreement with this picture.

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The production of leading hadrons in high-energy hadron-nucleus scattering has been studied for many years (for a recent review, see [1]). The inclusive data indicate a strong suppression of leading particles in central hadron-nucleus (h-A) collisions. Even for heavy nuclei, the main contribution to the inclusive spectra comes from the interactions near the nuclear edge and not the center. More recent attempts to enhance the contribution of collisions at small impact parameters by selecting events with multiple nucleon emission confirm that the leading hadron multiplicity rapidly decreases with increasing A.

There are few theoretical results for the leading particle multiplicity from central h-A collisions. The earliest phenomenological results were based on the applications of various models of soft strong interactions, notably Gribov reggeon calculus. There are some clear problems with applying this approach to the projectile fragmentation region (which we will discuss below) and in fact very few explicit treatments of the leading particle multiplicity problem have been made. An alternative approach is to apply perturbative QCD (pQCD), in which parton correlators are the elementary building blocks, see e.g. [3]. There have been theoretical arguments [4], and more recently experimental evidence from nuclear size dependence tests [5], which show that hadronization of high-energy partons occurs outside the nucleus.

The new observation of this paper is that for large nuclei, and a sufficiently high energy projectile, the origin of leading particles may have a very simple explanation based on the underlying parton dynamics and may be calculable through the pQCD ingredients of the strong interaction dynamics.

Before addressing our proposed pQCD mechanism, let us first review the expectations of the Gribov-Glauber theory of hadron-nucleus interactions [6]. This theory is known to describe quantitatively the total and elastic cross sections of high-energy hadron-nucleus scattering, see [7] and references therein. Corresponding reggeon calculus diagrams include multipomeron exchanges with a hadron, which are expressed through the vertices where the hadron couples to n pomerons. These vertices involve complicated hadron intermediate states and hence are not known independently. The use of the Abramovskii, Gribov,
Kancheli (AGK) cutting rules [8] allows one to describe also the production of hadrons in the central rapidity range, for a review see e.g. [9]. However, the production of particles in the projectile fragmentation region requires additional information because the AGK cutting rules are not applicable for this kinematic region [8]. A generic Gribov-Glauber approach does not have predictive power here. One has to know the structure of the vertex (so called Mandelstam cross) for coupling of the hadron with \( n \) pomerons, including information on how the energy of the projectile is split between several pomerons. One example of such a model is an eikonal type model [10] where it is assumed that the energy is split equally between \( n \) interactions. Another is the additive quark model where the eikonal is applied to the interactions of constituent quarks [11]. One common feature of these models is that in the limit of \( A \to \infty \) the leading particle multiplicity tends to zero.

At the same time according to QCD-parton model concepts, leading particles originate from the emerging fast partons of the collision debris. Here a leading or fast parton is defined as one carrying a large fraction of the projectile’s longitudinal momentum. In particular there is a wide rapidity separation between a fast parton and the sea partons. A reasonable criterion is that \( x > 0.2 \), where \( x \) is its momentum fraction. In addition, the absolute momentum of the fast parton should be sufficiently large so that it cannot easily interact softly with the medium. According to the space-time picture of strong interactions [12] a parton fluctuates into other hadronic states in a time governed by the uncertainty principle. Thus a parton of three-momentum \( p \) fluctuates into a state of mass \( m \) in a time,

\[
t = \frac{2p}{m^2}.
\]

The state of lowest mass gives the characteristic time required for the point-like parton to become spatially extent. Conservatively taking \( m \sim m_\rho \) (versus \( m_\pi \)) one finds that the condition,

\[
t = \frac{2p}{m^2} > 2R_A
\]

is satisfied at \( A=200 \) for \( p \geq 15 \) GeV/c, where \( R_A \) is the radius of a nucleus of atomic number \( A \). This requires projectiles with energy \( E > 75 \) GeV.
In the ideal parton model limit, such fast partons would interact rarely with the surrounding nuclear medium. In this ideal description, the interaction of the hadron projectile with the nuclear target would be primarily through the wee partons in the former. As such, a fast parton or a coherent configuration of fast partons would filter through essentially unaltered. This would imply that the leading particle spectrum for a given hadronic projectile on any nuclear target would be the same [13]. Such universality in the spectra is qualitatively inconsistent with experiment [1].

QCD also predicts that fast partons will lose a negligible fraction of their longitudinal momentum if $p_{\text{inc}} \to \infty$ and $A = \text{const}$. This expectation for fast partons to retain their longitudinal momentum can be tested from the nuclear size dependence of the Drell-Yan cross section. Recall that Drell-Yan production depends kinematically only on the longitudinal momentum fractions of the two impinging partons. Thus if there is no substantial attenuation of longitudinal momentum for a fast parton, the Drell-Yan cross section will scale linearly with $A$. This is what experiment finds [14].

In addition, QCD predicts [15] that fast partons undergo quasi-elastic rescatterings which give them a transverse momentum $< p^2_T >$ that increases linearly with the distance traveled in the nuclear medium. Up to geometrical corrections the dependence goes approximately as $A^{1/3}$. This QCD prediction has been confirmed by the Drell-Yan experiment [14]. The $p_T$ - broadening in the dimuon spectrum for hadron projectile $h_p$ and target nucleus $A$, $\Delta \langle p^2_T \rangle_{h_pA}$, is given by the transverse momentum of the initiating partons in the projectile. The effect of coherence between collisions of the fast parton with the nuclear media does not change the linear dependence on distance for the average $\langle p^2_T \rangle_{h_pA}$, as discussed recently in [16].

In pQCD transverse broadening and energy loss are related as [16]

$$\Delta E \approx \frac{\alpha_s N_c}{8} \Delta \langle p^2_T \rangle L/2,$$

(3)

†This relation depends weakly on the approximations used in [16], see discussion in [17].
where \( L \) is the path in the nuclear matter. Transverse momentum smearing implies that a type of limiting curve is expected. This limiting behavior arises because an increase in the relative transverse momentum between two leading partons decreases the probability for two such partons to coalesce. In particular one expects a high coalescence probability for relative transverse momenta \( p_{T_R} \equiv \sqrt{2 \langle p_T^2 \rangle} \) that are of typical hadronic scale, e.g. \( p_{T_R} < 0.420 \text{ GeV}/c \). Increasing the nuclear volume, thus increasing \( p_{T_R} \), decreases the likelihood of coalescence at least as \( \propto 1/p_T^4 \) for large \( p_T \), see equation (8) below. So in the limit of infinite nuclear volume, \( A \to \infty \), the coalescence probability goes to zero, and the leading partons should fragment independently and so produce independent jets. At the same time the fractional energy loss tends to zero for a fixed large value of \( A \). Thus the \( z \)-distribution of these jets will not depend on \( A \), although the transverse momentum will increase with \( A \). Obviously one can chose the double limit of \( E \propto A^k \), \( A \to \infty \) so that \( p_T^2 \geq cA^n \) while both condition (2) and the inequality \( \Delta E/E_h \to 0 \) are satisfied. This corresponds to

\[
 k > n + 1/3. \tag{4}
\]

In this limit, when the leading partons (mostly quarks in a nucleon projectile or \( q \) and \( \bar{q} \) in a pion) fragment independently, it is possible to calculate the leading parton production cross section integrated over the transverse momentum \( p_T \). Let \( z d\sigma_{h/p}^{h/p}(z)/dz \) denote the one-particle inclusive differential cross section, from collisions of a hadronic projectile \( h_p \) on a nucleus \( A \), for the production of a hadron \( h \) that carries a longitudinal momentum fraction \( z \), integrated over its transverse momentum. The differential leading particle multiplicity is then defined as

\[
 z dN_{h/p}^{h/p}(z) = \frac{1}{\sigma_{h/p}^{\text{inel}}(A)} d\sigma_{h/p}^{h/p}(z)/dz \tag{5},
\]

where \( \sigma_{h/p}^{\text{inel}}(A) \) is the inelastic cross section for \( h_p - A \) scattering. In the limit of interest, \( z dN_{h/p}^{h/p}(z)/dz \) takes on the asymptotic form:

\[
 z \frac{dN_{h/p}^{h/p}(z)}{dz} = \sum_{a=q,g} \int_{1}^{\infty} \frac{dD_{h/a}(\zeta, Q^2)}{x} f_{a/h_p}(x, Q^2), \tag{6}
\]
involving the convolution of \( f_{a/h_p}(x, Q^2) \), the distribution of parton \( a \) in the hadron projectile \( h_p \), and \( D^{h/a}(z, Q^2) \), the fragmentation function for parton \( a \) into hadron \( h \). Eq. (3) leads to a steep decrease of the hadron spectrum at large \( z \) because at \( Q^2 \sim 1 \) GeV\(^2\) the structure functions drop as \( x f_{q/h_p}(x, Q^2) \sim (1 - x)^n \) with \( n \sim 2 - 3 \) and the dominant fragmentation functions \( z D^{q/p}(z, Q^2) \sim (1 - z) \) based on quark counting rules or at most are constant in low virtuality approximations [18,19]. The hard interactions that transfer transverse momenta to the fast partons are at low virtuality. We set the virtuality at \( Q^2 = 1 \) GeV\(^2\).

For all our calculations, we used the parton distributions of GRV [20] since they are determined down to the low-virtuality range \( Q^2 \sim 1 \) GeV\(^2\). Two types of fragmentation functions were used. One was determined by the EMC collaboration [21] at \( < Q^2 > = 20 \) GeV\(^2\) and the other was calculated [18] for low virtuality from the quark-gluon string model (QGSM) [18]. The EMC fragmentation functions set a lower bound on our multiplicity estimates, since scaling violations will enhance fragmentation at low virtuality and large \( z \).

As a cross check, we fit the EMC data for \( D^{\pi+/p}(z) \) to the higher twist form calculated in [19]. We then evolved this down to \( Q^2 = 1 \) GeV\(^2\). In this region in particular the Berger form also becomes nonzero at \( z = 1 \) which is consistent with the QGSM form. Numerically we found that the fitted form of [19] was a factor of one to two times bigger than the QGSM form. We consider this discrepancy acceptable.

All three fragmentation functions suppress light quark fragmentation into protons. The EMC data indicate a factor of 4 to 8 depletion of protons versus \( \pi^+ \) in the largest multiplicity region of leading particles \( z \sim 0.2 - 0.3 \). The higher twist mechanism in [19] is inapplicable for proton fragmentation. The QGSM results [18] give zero fragmentation into protons.

The limiting curve for proton (solid) and pion (dashed) projectiles are shown in Fig. 1 using the QGSM fragmentation functions. One typically finds the leading behavior as \( z \to 1 \) to be \( z dN^{h/h_p}(z)/dz \sim (1 - z)^{\alpha_{h/h_p}} \). Here \( \alpha_{h/h_p} \) will be referred to as the leading exponent for produced species \( h \) from projectile \( h_p \) with \( \alpha_{T/h_p} \) denoting the exponent for the total leading particle multiplicity. For the proton projectile with QGSM fragmentation functions, we find \( \alpha_{\pi+/p} \sim 4.4, \alpha_{\pi-/p} \sim 5.3 \) and \( \alpha_{T/p} \sim 4.5 \). For the EMC fragmentation functions
\[ \alpha_{X/p} \sim 6 \] for all species \( X \) except for the proton, where \( \alpha_{p/p} \sim 7 \), and \( \alpha_{T/p} \sim 6 \). For the pion projectile with QGSM fragmentation functions, we find \( \alpha_{\pi^+/p} \sim 1.6 \), \( \alpha_{\pi^-/p} \sim 2.8 \) and \( \alpha_{T/\pi^+} \sim 1.8 \); for the EMC fragmentation functions \( \alpha_{T/\pi^+} \sim 3.0 \). In all cases, multiplicity distributions for \( \bar{p} \) and \( \pi^- \) projectiles are obtained by applying charge conjugation to those for the \( p \) and \( \pi^+ \) projectiles, respectively.

Let us now turn to what we might expect for the leading particle multiplicities. In a large nucleus, say \( A = 200 \), the relative transverse momentum that two leading partons acquire can be estimated following [22] through the \( p_T \)-broadening of the Drell-Yan spectrum. Using data [14] for \( \Delta \langle p_T^2 \rangle_{pA} \approx 0.114 \text{ GeV}^2 \) and the average number of struck nucleons in the Drell-Yan events \( \bar{n}(A=200) = 5.3 \) [23]:

\[
p_{TR} A \approx 2 \sqrt{\Delta \langle p_T^2 \rangle_{pA} \bar{n}_A / (\bar{n}_A - 1)}|_{A=200} \approx 0.75 \text{ GeV}/c. \tag{7}
\]

This number should be considered as a lower limit since in QCD \( p_T^2 \)-broadening is larger for the interaction of partons with smaller virtualities [24] and in our case the virtuality is much smaller than in the Drell-Yan processes studied in [14] for \( \langle M_{\mu^+\mu^-}^2 \rangle = 30 \text{ GeV}^2 \).

Within the \( z > 0.2 \) region, we estimate the coalescence probability into a pion, \( P_c^{\pi}(p_T^2) \), for a \( q\bar{q} \) pair with relative momentum \( p_{TR} \). It is easy to show in the constituent quark model that

\[
P_c^{\pi}(p_T^2) = |F_\pi(p_T^2)|^2 \tag{8}
\]

where \( F_\pi(p^2) \) is the pion form factor. Using the Vector-Dominance-Model fit for \( F_\pi(p^2) \), one finds for Eq. (7) \( P_c^{\pi}(0.5) \approx 0.25 \). Further suppression comes from the presence of gluons in the pion wave function which on average carry half of the pion momentum.

For large \( A \), the above mechanism for leading particle production goes as \( A^{-2/3} \). One finds a similar \( A \)-dependence for the fragmentation of a diquark to a nucleon. Another mechanism for the production of leading particles which is similar to [25] is the filtering of color singlet small transverse size clusters of the projectile. This is analogous to the propagation of ultrarelativistic positronium through say a piece of lead. Using equations
derived in [26], one finds for large $A$ that this mechanism leads to a multiplicity of leading pions (protons) $\propto A^{-2/3}(A^{-4/3})$.

Another "background" to our mechanism is the coherent diffractive production of leading particles. Its contribution to $zdN_A^{h/p}/dz \propto A^{-1/3}$ for $A \to \infty$ and somewhat slower for $A = 12 - 200$ [27].

Although we see no expectation for the asymptotic form in Eq. (6) even for $A = 200$, we feel large nuclei may still inhibit coalescence. To test this claim, since the leading correction to the asymptotic behavior is due to diffractive events, it is preferable to perform data analyses which would explicitly exclude diffractive events by selecting the events where the nucleus breaks up. For this these asymptotic expectations can be compared with data from central hadron-nucleus collisions from Fermilab experiment E597 [28, 229].

The data were obtained using the Fermilab 30-inch hybrid bubble chamber spectrometer with associated downstream particle identifiers (DPI). The bubble chamber, in a 2T magnetic field, provided a visual target and vertex detector as well as a spectrometer for the slower produced particles. The faster particles were momentum analyzed using the fringe field of the bubble chamber magnet and seven planes of proportional wire chambers and three drift chambers. In addition to being filled with liquid hydrogen, the bubble chamber also contained thin nuclear foils of Mg, Ag and Au. The beams consisted of 100 GeV/c $\bar{p}, p, \pi^+$ and $\pi^-$. Mass identification for these particles was provided by Čerenkov counters in the beam-line. Additional experimental details are given elsewhere [29].

To make the relevant comparisons, leading particle spectra have been obtained by combining $p/\bar{p}$ and combining $\pi^\pm$ projectile data. To study the nuclear thickness dependence of the data, use is made of the known correlation between the number of nuclear collisions and the number of observed slow protons [30]. The number ($n_p$) of slow protons (with momentum less than 1.3 GeV/c) is determined by performing a visual ionization scan of each interaction occurring in the nuclear foil targets. The selection of $n_p \geq 1$ automatically removes all diffractive events.

In Fig. 2a the differential multiplicity $zdN_A^{\pi/\pi}(z)/dz$ is shown for the data from $\pi^- A \to$
\( h^+ \) (combined with \( \pi^+ A \rightarrow h^- \)) with \( n_p = 1 \) or 2. In these figures, \( h^+(h^-) \) indicates a positively (negatively) charged hadron, mostly pions. The dashed curve is a fit to the form \((1 - z)^\alpha\) and demonstrates that this form is an acceptable representation of the data with \( \alpha = 3.27 \pm 0.48 \). Figs. 2b and 2c show the resulting values of \( \alpha \) as a function of \( n_p \) for the \( \pi \) projectile to \( \pi^+/\pi^- \) and \( p/\bar{p} \), respectively, with \( z > 0.2 \) and for produced hadrons having the same or opposite charge to that of the beam projectile. The solid (dashed) horizontal lines at the right side of the figure are the theoretical predictions using the QGSM fragmentation functions in Fig. 2b and the EMC fragmentation functions in Fig. 2c for the opposite (like) charge leading particle exponent.

We next compare various integrated multiplicities from the experiment to the predictions. Let \( I_{h/h_p}(z_m) \) denote the integrated multiplicity of produced hadrons of type \( h \) with \( z > z_m \) from a projectile \( h_p \) with \( h \) replaced by \( T \) when all the produced hadrons are summed. For the predictions, these are understood to be for \( A \rightarrow \infty \). In table 1 the limiting predictions for the integrated multiplicities are compared with the experimental results for \( n_p \geq 15 \). Fig. 2d shows the E597 data for the integrated multiplicity with \( z_m = 0.2 \) as a function of \( n_p \). The solid (dashed) horizontal lines at the right are the predictions from the QGSM fragmentation functions for \( p (\pi) \) projectiles. The data seem to be consistent with the idea of limiting multiplicity with the magnitude as determined by the model.

It is also interesting to examine multiplicity ratios. Denote the ratio as,

\[
R_{h_p}^{h_1/h_2}(z) = \frac{I_{h_1/h_p}(z)}{I_{h_2/h_p}(z)}. \tag{9}
\]

For the ratio between protons and pions, only the EMC fragmentation functions are available. The calculated values are \( R_{p/\bar{p}/\pi^+ + \pi^-}(0.2) = 0.18 \) and \( R_{p/\bar{p}/\pi^+ + \pi^-}(0.3) = 0.17 \). To examine the depletion of leading protons from a proton projectile, the lower bound estimates

\[ \dagger \text{Using QGSM versus GRV proton distribution functions} \text{[31], the predicted multiplicity is about the same. All the leading exponents are about two smaller, which is consistent with the slower decrease of the former as } x \rightarrow 1. \]
from the EMC fragmentation functions are $I_{p/p}(0.2) = 0.009$ and $I_{p/p}(0.3) = 0.002$. The statistics on experimental data for $p \rightarrow p$ are insufficient to quote. It is worth noting that the forward multiplicity for large $n_p$ is substantially larger than in eikonal models where usually it is assumed that the energy is equally divided between all exchanges, see e.g. [10]. In these models one should expect that almost no particles are produced for $z \geq 0.2$ for the case in which more than 5 nucleons have been struck.

In our estimate of $p_T$ - broadening we have used experimental data on Drell-Yan production at 400 GeV/c. No estimates of the energy dependence of the $p_T$ - broadening are available at the moment. The model used in the analysis [10] leads to energy independent energy losses. However this model assumes that the soft cut-off does not depend on energy. It seems natural to expect a certain increase of hardness of the soft interaction with energy leading to an increase of $p_T$ - broadening with an increase of energy. So fewer leading particles will be produced in the central collisions as the energy of $hA(AA)$ collisions increases from the SPS energy range to the RHIC or LHC energy range both because of further suppression of the coalescence effect and the scaling violations for the quark fragmentation functions.

An implication of the current analysis is that in high energy central $AA$ collisions very few baryons should be left in the fragmentation region. In fact our analysis indicates that the probability for a baryon to carry more than $z = 0.2$ of the projectile’s momentum fraction is less than 1%. To conserve baryon number, a guess is that nucleons should move on average at least 4 units of rapidity to the central rapidity region for RHIC and beyond.

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FIGURE CAPTIONS

Figure 1: Limiting curves for leading $\pi^\pm$ production from $p - A$ (solid) and $\pi^+ - A$ (dashed) collisions.

Figure 2: Experimental results for $h - A$ collisions at $100 GeV/c$: (a) Differential multiplicity for $\pi^- A \rightarrow \pi^+$ combined with $\pi^+ A \rightarrow \pi^-$ for events with $n_p = 1, 2$ ($n_p$ is the number of slow protons). The dashed curve is a fit to the form $(1 - z)^\alpha$; (b) Leading exponent $\alpha$ for $\pi A \rightarrow \pi$, solid circles (open boxes) when the produced $\pi$ has the same (opposite) charge to that of the beam projectile; the horizontal lines on the right are the theoretical limits from QGSM fragmentation functions to the same (dashed) and opposite (solid) charge leading particle; (c) Leading exponent $\alpha$ for $\pi A \rightarrow p(p\bar{p})$, the horizontal lines on the right are the theoretical results from EMC fragmentation functions to like (dashed) and opposite (solid) charge leading particle; (d) The integrated multiplicity of hadrons with $z_m > 0.2$ for $\pi^\pm$ (solid circles) and proton (open boxes) beam projectiles as a function of $n_p$. The horizontal lines on the right of the figure are the theoretical asymptotic limits for $\pi$ (dashed) and proton (solid) projectiles.
$p + A \rightarrow \pi + X$

$\pi^+ + A \rightarrow \pi + X$
Table 1: The integrated multiplicities, $I_{h/p}(z_m)$, of produced hadrons of type $h$ with $z > z_m$ from a projectile $h_p$. The first two columns give the predictions for the limiting case $A \to \infty$ for the EMC and QGSM fragmentation functions, respectively, and the third column gives the experimental value for $n_p > 15$, each for $z_m = 0.2$. The last three columns correspond to the case with $z_m = 0.3$.

|        | EMC (0.2) | QGSM (0.2) | Exp (0.2) | EMC (0.3) | QGSM (0.3) | Exp (0.3) |
|--------|-----------|------------|-----------|-----------|------------|-----------|
| $I_{T/p}$ | 0.09      | 0.27       | 0.28 ± 0.07 | 0.02      | 0.09       | 0.10 ± 0.04 |
| $I_{T/\pi^+}$ | 0.10      | 0.20       | 0.35 ± 0.09 | 0.04      | 0.10       | 0.15 ± 0.06 |