Mass of the Lowest Scalar \((0^{++})\) Glueball in the QCD Sum Rules

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Abstract

The mass of the lowest scalar glueball is discussed by using QCD sum rules. We find that the glueball mass is sensitive to the choice of moments and slightly depends on the radiative corrections. With the help of suitable moments and stability criteria, we get the scalar glueball mass: \(1710 \pm 110\) MeV without radiative corrections and \(1580 \pm 150\) MeV with radiative corrections.

1 Introduction

The existence of bound gluon states, glueballs, is a direct consequence based on the QCD self-interactions among gluons. Although there are several glueball candidates experimentally, there is no conclusive evidence on them. Recently, one pays particular attention to three states: \(f_0(1500)(J=0)\), \(f_2(1710)\) \((J=0 \text{ or } 2)\), and \(\xi(2230)\) \((J \geq 2)\). They seem like glueball candidates of \(0^{++}\) or \(2^{++}\) states.

The properties of glueballs have been investigated in the lattice gauge theory and in many models based on the QCD theory. Even in the lattice gauge calculation, there are different predictions for the \(0^{++}\) glueball. IBM group \([\text{I}]\) predicts the lightest \(0^{++}\) glueball mass, \((1740 \pm 71)\) MeV, and UK QCD group \([\text{III}]\) gives the estimated mass, \((1550 \pm 50)\) MeV respectively. They give the slightly different predictions for the \(2^{++}\) glueball: \((2259 \pm 128)\) MeV (IBM group) and \((2270 \pm 100)\) MeV (UK QCD group). The mass of the \(0^{++}\) glueball can not be predicted consistent at present. It encourage us to restudy the mass of the lowest \(0^{++}\) glueball using QCD sum rules in this paper.

V. A. Novikov et al \([\text{VIII}]\) first tried to estimate the scalar glueball mass by using QCD sum rules, but they only took the mass to be 700 MeV by hand because of uncontrolled instanton contributions. Since then, P. Pascual and R. Tarrach \([\text{IV}]\), S. Narison \([\text{V}]\) and J. Bordes et al \([\text{VI}]\) presented their calculation on the scalar glueball mass in the framework of QCD sum rules. They all got a lower mass prediction around 700 MeV-900 MeV when they neglected the radiative corrections in their calculation of the correlator. Namely, they only considered the perturbative and the leading condensates in the correlator. E. Bagan and T. Steele \([\text{VII}]\) first
took account of the radiative corrections in the correlator calculation, they got the higher
mass prediction around 1.7 GeV since the one-loop $\langle \alpha_s G^2 \rangle$ correction played a important
role. It seems that the radiative corrections make a big difference on the prediction of the
scalar glueball mass.

In order to calculate the predicted mass for the lowest scalar glueball in the QCD sum
rules, we re-study the correlator without radiative corrections and correlator with radiative
corrections, respectively. After Borel transformation of the correlator weighted by different
powers of $Q^2$, we get different moments. It is the moments we choose which make the main
difference of the glueball mass. We give the criteria to choose the continuum threshold that
represents the maximum energy for which a duality exists between resonance physics and
QCD, and give some comments on a reasonable choice of the moments also. The radiative
corrections shift the mass scale a little. Our predicted mass for $0^{++}$ glueball is in agreement
with the result of UK QCD group.

The paper is organized as follows. In sect. 2 a brief review about the calculation of scalar
 glueball mass from QCD sum rules was given. In sect. 3 we give the criteria to choose
$s_0$ and the moments, and present numerical results. Finally, the last section is reserved for a
summary.

\section{Scalar glueball sum rules}

Let us consider the correlator

$$\Pi(q^2) = i \int e^{i q x} \langle 0 | T \{ j(x), j(0) \} | 0 \rangle dx, \quad (1)$$

where the current $j(x)$ is defined as

$$j(x) = \alpha_s G^a_{\mu \nu} G^a_{\mu \nu}(x). \quad (2)$$

$G^a_{\mu \nu}$ in Eq.(2) stands for the gluon field strenth tensor and $\alpha_s$ is the quark-gluon coupling
constant. The current $j(x)$ is the gauge-invariant and non-renormalization \cite{8} (to two loops
order) scalar current for the $0^{++}$ glueball in QCD without quarks. We will keep all of the
calculations in QCD without quarks.

Through operator products expansion, the correlator without radiative corrections be-
comes

$$\Pi(q^2) = a_0 (Q^2)^2 \ln(Q^2/\nu^2) + b_0 \langle \alpha_s G^2 \rangle + c_0 \langle g G^3 \rangle Q^2 + d_0 \langle \alpha_s^2 G^4 \rangle (Q^2)^2, \quad (3)$$

with $Q^2 = -q^2 > 0$, and

$$a_0 = -\frac{(\alpha_s)^2}{\pi}, \quad b_0 = 4 \alpha_s, \quad c_0 = 8 \alpha_s^2, \quad d_0 = 8 \pi \alpha_s.$$

After taking into account radiative corrections, the correlator is

$$\Pi(q^2) = \left( a_0 + a_1 \ln(Q^2/\nu^2) \right) (Q^2)^2 \ln(Q^2/\nu^2) + \left( b_0 + b_1 \ln(Q^2/\nu^2) \right) \langle \alpha_s G^2 \rangle + \left( c_0 + c_1 \ln(Q^2/\nu^2) \right) \frac{\langle g G^3 \rangle}{Q^2} + d_0 \frac{\alpha_s^2 G^4}{(Q^2)^2}, \quad (4)$$
where

\[ a_0 = -2 \left( \frac{\alpha_s}{\pi} \right)^2 \left( 1 + \frac{51 \alpha_s}{4 \pi} \right), \]
\[ b_0 = 4 \alpha_s \left( 1 + \frac{49 \alpha_s}{12 \pi} \right), \]
\[ c_0 = 8 \alpha_s^2, \quad d_0 = 8 \pi \alpha_s, \]
\[ a_1 = \frac{11}{2} \left( \frac{\alpha_s}{\pi} \right)^3, \quad b_1 = -11 \alpha_s^2, \quad c_1 = -58 \alpha_s^3. \]

For the non-perturbative condensates we use the following notations and estimate

\[ \langle \alpha_s G^2 \rangle = \langle \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a \rangle, \]
\[ \langle gG^3 \rangle = \langle gf_{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle, \]
\[ \langle \alpha_s^2 G^4 \rangle = 14 \langle (\alpha_s f_{abc} G_{\mu\nu}^a G_{\mu\nu}^b) \rangle^2 - \langle (\alpha_s f_{abc} G_{\mu\nu}^a G_{\rho\lambda}^b) \rangle^2. \]

Now, we can use the standard dispersion representation for the correlator

\[ \Pi(Q^2) = \Pi(0) - \Pi'(0) + \frac{(Q^2)^2}{\pi} \int_0^{+\infty} \frac{Im\Pi(s)}{s^2(s+Q^2)} ds \quad (5) \]

to connect the QCD calculation with the resonance physics. From the low energy theorem [3] follows that

\[ \Pi(0) = \frac{32}{11} \pi \langle \alpha_s G^2 \rangle. \quad (6) \]

For the physical spectral density \( Im\Pi(s) \), one can divide it into two parts, low energy part and high energy part. Fortunately, its high-energy behavior is known as trivial,

\[ Im\Pi(s) \to \frac{2}{\pi} s^2 \alpha_s^2(s), \quad (7) \]

while at low energy, \( Im(s) \) can be expressed in the narrow width approximation. The single resonance model for \( Im\Pi(s) \) leads

\[ Im\Pi(s) = \pi f^2 M^4 \delta(s - M^2), \quad (8) \]

where \( M, f \) are the mass and coupling of the state. With these in hand, we can proceed the following calculation.

### 3 Moments and numerical results

To construct the sum rules, we define the moments \( R_k \),

\[ R_k(\tau, s_0) = \frac{1}{\tau} \tilde{L}[(Q^2)^k \{ \Pi(Q^2) - \Pi(0) \} ] \]
\[ - \frac{1}{\pi} \int_{s_0}^{+\infty} s^k e^{-sr} Im\Pi^{(pert)}(s) ds, \quad (9) \]
where $\tau$ is the Borel transformation parameter, $\hat{L}$ is the Borel transformation operator and $s_0$ represents the maximum energy for which a duality exists between resonance physics and QCD calculation with condensates.

The standard dispersion relation is transformed into

$$R_k(\tau, s_0) = \frac{1}{\pi} \int_0^{s_0} s^ke^{-s\tau} Im\Pi(s)ds,$$

(10)

and from Eq.(9) we have (for $k \geq -1$)

$$R_k(\tau, s_0) = (-\frac{\partial}{\partial\tau})^{k+1}R_{-1}(\tau, s_0).$$

(11)

The moment $R_{-1}(\tau, s_0)$ with radiative corrections has been given\cite{7},

$$R_{-1}(\tau, s_0) = -\frac{a_0}{\tau^2}[1 - \rho_1(s_0\tau)]$$

$$+ 2\frac{a_1}{\tau^2}(\gamma_E + E_1(s_0\tau) + \ln s_0\tau + \exp(-s_0\tau) - 1)$$

$$- [1 - \rho_1(s_0\tau)] \ln \frac{s_0}{\nu^2} + \Pi(0)$$

$$- \{b_0 - b_1[\gamma_E + \ln \tau\nu^2 + E_1(s_0\tau)]\}\langle G^2 \rangle$$

$$- \{c_0 + c_1(1 - \gamma_E - \ln \tau\nu^2 - E_1(s_0\tau) + \frac{\exp(-s_0\tau)}{s_0\tau}\}\langle G^3 \rangle$$

$$- \frac{1}{2}d_0\langle \alpha_s^2 G^4 \rangle \tau^2,$$

(12)

where

$$\rho_1(s_0\tau) = (1 + s_0\tau)e^{-s_0\tau}, \quad E_1(x) = \int_x^\infty \frac{e^{-y}}{y}dy,$$

and $\gamma_E = Euler's constant \approx 0.5772$. Renormalization-group improvement of the sum rules amounts to the substitution:

$$\nu^2 \rightarrow \frac{1}{\tau},$$

$$\langle g G^3 \rangle \rightarrow \left[ \frac{\alpha_s}{\alpha_s(\nu^2)} \right]^{7/11} \langle g G^3 \rangle.$$

$R_{-1}(\tau, s_0)$ without radiative corrections can be obtained as the coefficients $a_1, b_1$, and $c_1$ are set to zero and $a_0, b_0, c_0$ from Eq. (3).

Complete knowledge of $\Pi(Q^2)$ would allow us to fix the mass and width of the glueball, but we are far from this idea. One can only choose some suitable moments at appropriate $s_0$ to derive the prediction. As shown in Ref.\cite{8}, the $R_{-1}$ sum rule leads to a much smaller mass scale due to the anomalously large contribution of the low-energy part $\Pi(0)$ of the sum rule and it violates asymptotic freedom at large energy. They claimed that $R_{-1}$ is not reliable to predict the $0^{++}$ glueball mass. They employed the $R_0$ and $R_1$ sum rules to predict the $0^{++}$ glueball mass by fitting the stability criteria with the radiative corrections considered. Their approach shows that the $R_0$ and $R_1$ sum rules with the radiative corrections can obtain a higher mass scale compared to the previous approaches, thus one would ask: which factor (to choose an appropriate moments $R_k$ or to consider the radiative corrections) is crucial to get the higher mass prediction? In order to answer it, we re-examine the $R_k$ sum rules.
To improve the convergence of the asymptotic series, we study the ratio \( R_{k+1}/R_k \), such as \( R_0/R_{-1} \) and \( R_0/R_6 \). In the narrow width approximation, we have

\[
M^{2k+4} f^2 \exp(-\tau M^2) = R_k(\tau, s_0),
\]
and (with \( k \geq -1 \))

\[
M^2(\tau, s_0) = \frac{R_{k+1}(\tau, s_0)}{R_k}.
\]  \( (13) \)

To proceed calculation, we choose the following parameters

\[
\langle \alpha_s G^2 \rangle = 0.06 \text{GeV}^4,
\]
\[
\langle g G^3 \rangle = (0.27 \text{GeV}^2) \langle \alpha_s G^2 \rangle,
\]
\[
\langle \alpha_s^2 G^4 \rangle = \frac{9}{16} \langle \alpha_s G^2 \rangle^2,
\]
\[
\Lambda_{MS} = 200 \text{MeV},
\]
\[
\bar{\alpha}_s = \frac{-4\pi}{11 \ln(\tau \Lambda_{MS}^2)}.
\]

\( M^2 \) and \( f^2 \) are the functions of \( s_0 \) which is the starting point of the continuum threshold, \( s_0 > M^2 \). Since the glueball mass \( M \) in Eq. \( (13) \) depends on \( \tau \) and \( s_0 \), we take the stationary point of \( M^2 \) versus \( \tau \) at an appropriate \( s_0 \) as the square of the glueball mass.

To determine the appropriate \( s_0 \), the following stability criteria are employed: (1), \( s_0 \) should be a little higher than the physical mass and approaches it as near as possible due to the continuum threshold hypothesis and the narrow width approximation; (2), The choice of a suitable \( s_0 \) should lead to not only a widest flat portions of the plots of \( M^2 \) versus \( \tau \) but also an appropriate parameter region of \( \tau \) with the parameter region comparable to the value of the glueball mass.

First, we start the \( R_k \) sum rules without radiative corrections to see the influence of different moments. According to the criteria above, the acceptable region of \( s_0 \) is chosen between \( s_0 = 3.0 \text{ GeV}^2 \) and \( s_0 = 4.3 \text{ GeV}^2 \). Let us start with the \( R_{-1} \), the ratio \( R_0/R_{-1} \) results in a much smaller mass scale which can’t be comparable to the parameter region(see Fig. 1) and it is not acceptable. This result is similar to that as pointed out in Ref. [7]. The numerical results of \( R_0/R_6 \) and \( R_2/R_1 \) without radiative corrections are illustrated in Fig. 2 and Fig. 3, respectively. In the figures, the optimum parameter of \( s_0 \) is chosen as \( s_0 = 3.6 \text{ GeV}^2 \). The ratio \( R_0/R_6 \) can get a higher mass scale, but the parameter region of the \( \tau \) can’t be comparable to the mass scale(the parameters corresponding to the stationary point are too low ), so it doesn’t satisfy the criteria above. We won’t take it for the mass prediction. The ratio \( R_2/R_1 \) in Fig. 3 gives an excellent shape and it satisfies all of the criteria. The curve shows that the 0\(^{++}\) glueball mass is 1710 MeV. In the acceptable region of \( s_0 \), the 0\(^{++}\) glueball mass is 1710 ± 110 GeV. The moments with higher \( k \) can’t stress the resonance contribution in the sum rule, and the higher dimension condensates will contribute to the sum rule(but we learn little about higher dimension condensates at present), we think they are unsuitable for the mass prediction either.

As the radiative corrections are taken into account, the predicted mass from ratio \( R_2/R_1 \) only shifts the glueball mass a little, and the values are a little lower (17% ∼ 9%) than those predicted from the \( R_0/R_6 \) without radiative corrections in the acceptable region of \( s_0 \). The
curve in Fig. 4 shows that the glueball mass is 1580 MeV with radiative corrections. In the acceptable region of $s_0$, the $0^{++}$ glueball mass is $1580 \pm 150$ MeV.

4 Summary

In this paper we re-study the scalar glueball mass based on the duality among resonance physics and QCD. The modified Borel transformation has been employed, it make the calculation more convenient and reasonable.

We find that the predicted mass is sensitive to the choice of the moment, and it is the moment which makes different predictions. Not all the moments are suitable for predicting the glueball mass, the moments $R_{-1}$ and $R_k$ with higher $k$ aren’t suitable. The moment $R_{-1}$, the contribution of low energy part of the correlator is large; The moments with higher $k$, the contribution of higher dimension condensates will come into play.

To stress the contribution of resonance in sum rules, we give our criteria on the choice of the continuum threshold, these criteria also make it possible to choose a suitable moment for the calculation of the glueball mass. We conclude that the ratio $R_2/R_1$ is the best one for the calculation.

The radiative corrections shift the mass a little which is lower than that without radiative corrections. The numerical results are obtained: $1710 \pm 110$ MeV without radiative corrections, and $1580 \pm 150$ MeV with radiative corrections. The predicted mass is in agreement with the result of UK QCD group and consistent with the glueball candidate $f_0(1500)(J=0)$.

References

[1] D. Weingarten, Nucl. Phys. B (Proc. Suppl.) 34, 29 (1994).
[2] G. Bali et al. (UKQCD), Phys. Lett. B 309, 378 (1993).
[3] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and Zakharov, Nucl. Phys. B 165, 67 (1980).
[4] P. Pascual and R. Tarrach, Phys. Lett. B 113, 495 (1982).
[5] S. Narison, Z. Phys. C 26, 209 (1984).
[6] J. Bordes, V. Giménez and J. A. Peñaarcho, Phys. Lett. B 223, 251 (1989).
[7] E. Bagan and T. G. Steele, Phys. Lett. B 243, 413 (1990).
[8] R. Tarrach, Nucl. Phys. B 196, 45 (1982).
Figure caption

Figure 1: $\frac{R_{0}}{R_{-1}}$ versus $\tau$ at $s_{0} = 3.6 \text{ GeV}^2$ without radiative corrections.

Figure 2: $\frac{R_{1}}{R_{0}}$ versus $\tau$ at $s_{0} = 3.6 \text{ GeV}^2$ without radiative corrections.

Figure 3: $\frac{R_{2}}{R_{1}}$ versus $\tau$ at $s_{0} = 3.6 \text{ GeV}^2$ without radiative corrections.

Figure 4: $\frac{R_{3}}{R_{1}}$ versus $\tau$ at $s_{0} = 3.6 \text{ GeV}^2$ with radiative corrections.
Figure 1: $\frac{R_0}{R_{-1}}$ versus $\tau$ at $s_0 = 3.6 \text{ GeV}^2$ without radiative corrections.
Figure 2: $\frac{R_1}{R_0}$ versus $\tau$ at $s_0 = 3.6$ GeV$^2$ without radiative corrections.
Figure 3: $\frac{R_2}{R_1}$ versus $\tau$ at $s_0 = 3.6$ GeV$^2$ without radiative corrections.
Figure 4: \( \frac{R_2}{R_1} \) versus \( \tau \) at \( s_0 = 3.6 \text{ GeV}^2 \) with radiative corrections.