Compact Binary Mergers and Accretion-Induced Collapse: Event Rates

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Abstract. This paper is a brief review of the topic of binary systems as sources of gravitational-wave emission for both LIGO and LISA. In particular I review the current estimates of the associated Galactic event rates and their implications for expected detection rates. I discuss the estimates for (i) the coalescence of close binaries containing neutron stars or black holes, (ii) white dwarfs going through accretion-induced collapse into neutron stars, and (iii) detached but close binaries containing two white dwarfs. The relevant uncertainties and robustness of the estimates are addressed along with ways of obtaining conservative upper limits.

INTRODUCTION

An important factor in the design and development of gravitational wave observatories is the prospect for detection of astrophysical systems known or expected to be sources of gravitational radiation. At an initial level, even qualitative knowledge of the source properties dictates the frequency range of operation and the desired sensitivity levels of the instruments. More detailed understanding of signal characteristics, such as waveforms and polarization, allows the development of optimized data analysis techniques, followed by early testing and calibration of the system based on model data. Hence, studies of several different astrophysical sources of gravitational radiation is an integral part of the collective effort for the direct detection of gravitational waves in the near future.

The inspiral of close binary compact objects, neutron stars (NS) or black holes (BH), driven by gravitational wave emission is considered one of the major sources for ground-based laser interferometers, such as LIGO, VIRGO, GEO600, and TAMA300. At present only systems containing two neutron stars (NS) have been detected, PSR B1913+16 being the prototypical NS–NS system [1]. This binary radio pulsar has provided striking empirical confirmation of general relativity with the measurement of orbital decay due to gravitational radiation [2]. As this decay proceeds, both the amplitude and characteristic frequency of the gravitational-wave signal increase. Although for PSR B1913+16 the frequency will not enter the LIGO
window for another $\sim 3 \times 10^8$ yr, the expectation is that similar systems in other galaxies, well ahead in their inspiral phase, should be detectable now. In addition to NS–NS systems, and based on theories of binary evolution, BH–NS and BH–BH binaries are also expected to exist in galaxies, although they have not been discovered yet.

Another type of gravitational-wave source is provided by hot, young NS formed through the collapse of massive stars or the collapse of white dwarfs driven beyond the Chandrasekhar limit by accretion from a close binary companion (accretion-induced collapse). A variety of physical phenomena (e.g., rotational instabilities) can induce a quadrupole moment in proto-neutron stars and cause them to emit gravitational waves (see contributions in these proceedings by S. Hughes and J. Houser).

An assessment of detection prospects for the various sources requires estimates of (i) the signal strength, and hence the maximum distance out to which sources could be detected given the instrument sensitivity, and (ii) the source formation rate out to that maximum distance, extrapolated from Galactic formation rate estimates. In this paper I review current estimates of Galactic event rates for the coalescence of binary compact objects (NS–NS, BH–NS, BH–BH) and the formation of young NS in accretion-induced collapse of white dwarfs. A critical discussion of the various uncertain factors involved in these estimates is presented. At the end of the paper, I review briefly current predictions for close binaries as gravitational-wave sources expected to be detected by the future space laser interferometer LISA.

COALESCING BINARIES AND LIGO

Estimates of formation rates for coalescing binary compact objects (systems with tight enough orbits that will merge due to gravitational radiation within a Hubble time) can be predicted theoretically, based on our current understanding of binary evolution models. For NS–NS systems, we can also obtain empirical rate estimates based on the observed sample. In what follows I critically review the current coalescence-rate estimates, addressing the various uncertainties involved. A discussion of ways to obtain limits on the NS–NS coalescence rate is also included.

Given the expected strength of the gravitational-wave signal of double NS coalescence, the maximum distance out to which it could be detected by the LIGO-II interferometers has been estimated to be $\sim 450$ kpc [3]. A Galactic NS–NS coalescence rate of $\sim 10^{-6}$ yr$^{-1}$ is then required for a detection rate of 2–3 events per year. The corresponding estimates for the coalescence of two $10 M_\odot$ BH are $\sim 2000$ kpc and $\sim 10^{-8}$ yr$^{-1}$ (these distance estimates take into account cosmological corrections for a flat universe and $H_0 = 65$ km s$^{-1}$ Mpc$^{-1}$ [4]).
Theoretical Estimates

Theoretical calculations of the formation rate of coalescing binaries are possible, given a sequence of evolutionary stages followed by primordial binaries. Over the years a relatively standard picture has been formed based on the consideration of NS–NS binaries [5], although more recently variations of it have also been discussed [6]. In all versions of their formation path the main picture remains the same. The initial binary progenitor consists of two binary members massive enough to eventually collapse into NS or BH. Its evolution involves multiple phases of stable or unstable mass transfer, common-envelope evolution, and accretion onto neutron stars, as well as two supernova explosions.

Such theoretical modeling has been undertaken by various authors by means of population syntheses. This provides us with \textit{ab initio} predictions of the coalescence rate. The evolution of an ensemble of primordial binaries with assumed initial properties is followed through specific evolutionary stages until a coalescing binary is formed. The changes in the properties of the binaries at the end of each stage are calculated based on our current understanding of the various processes involved: wind mass loss from massive hydrogen- and helium-rich stars, mass and angular-momentum losses during mass transfer phases, dynamically unstable mass transfer and common-envelope evolution, effects of highly super-Eddington accretion onto neutron stars, and supernova explosions with kicks imparted to newborn neutron stars. Given that several of these phases are not very well understood, the results of population synthesis are expected to depend on the assumptions made in the treatment of the various processes. Therefore, exhaustive parameter studies are required by the nature of the problem.

Recent studies of the formation of binary compact objects and calculations of coalescence rates (see [7], [8], [9], [10]) have explored the model parameter space and the robustness of the results at different levels of (in)completeness. Almost all have studied the sensitivity of the coalescence rate to the average magnitude of the kicks imparted to newborn neutron stars. The range of predicted Galactic NS–NS rates from \textit{all} these studies obtained by varying the kick magnitude within reasonable ranges is \( < 10^{-7} - 5 \times 10^{-4} \text{yr}^{-1} \). This large range indicates the importance of supernovae (two in this case) in the evolution of binaries. Variations in the assumed mass-ratio distribution for the primordial binaries can \textit{further} change the predicted rate by about a factor of 10, while assumptions of the common-envelope phase add another factor of about \( 10 - 100 \). Variation in other parameters typically affects the results by factors of two or less. Results for BH–NS and BH–BH binaries lie in the ranges \( < 10^{-7} - 10^{-4} \text{yr}^{-1} \) and \( < 10^{-7} - 10^{-5} \text{yr}^{-1} \), respectively when the kick magnitude to both NS and BH is varied. Other uncertain factors such as the critical progenitor mass for NS and BH formation lead to variations of the rates by factors of \( 10 - 50 \).

It is evident that recent theoretical predictions for the coalescence rates cover a very wide range of values (typically 3-4 orders of magnitude). We note, however, that binary properties other than the coalescence rate, such as orbital sizes, ec-
centricities, center-of-mass velocities, are much less sensitive to the various input parameters and assumptions; the latter affect more severely the absolute normalization (birth rate) of the population. Given these results it seems fair to say that population synthesis calculations have quite limited predictive power and provide fairly loose constraints on coalescence rates.

**Empirical Estimates**

In the case of NS–NS binaries, there is another way to estimate their coalescence rate, using the properties of the observed coalescing NS–NS (two systems: PSR B1913+16 and PSR B1534+12) and models of selection effects in radio pulsar surveys. For each observed object, a scale factor is calculated based on the fraction of the Galactic volume within which pulsars with properties identical to those of the observed pulsar could be detected, in principle, by any of the radio pulsar surveys, given their detection thresholds. This scale factor is a measure of how many more pulsars like those detected in the coalescing NS–NS systems exist in our galaxy. The coalescence rate can then be calculated based on the scale factors and estimates of detection lifetimes summed up for the observed systems. This basic method was first used by Phinney [11] and Narayan et al. [12] who estimated the Galactic rate to be $\sim 10^{-6}\text{yr}^{-1}$.

Since then, estimates of the coalescence rate have decreased significantly primarily because of (i) the increase of the Galactic volume covered by radio pulsar surveys with no additional coalescing NS–NS binaries discovered [13], (ii) the increase of the distance estimate for PSR B1534+12 based on measurements of post-Newtonian parameters [14] (iii) changes in the lifetime estimates for the observed systems [15], [16]. On the other hand, in these recent studies an upward correction has been added to account for the population of pulsars too faint to be detected by the surveys. The most recently published study [16] gives a lower limit of $2 \times 10^{-7}\text{yr}^{-1}$ and a “best” estimate of $\sim 6 - 10 \times 10^{-7}\text{yr}^{-1}$ which agrees with other recent estimates of $2 - 3 \times 10^{-6}\text{yr}^{-1}$ [14], [17]. Additional uncertainties (typically by factors $\lesssim 10$) arise from the estimates of pulsar ages and distances, the pulsar beaming fraction, the spatial distribution of NS–NS binaries in the Galaxy, the form of the faint end of the luminosity function and the small number of objects in the observed sample.

Despite all these uncertainties the empirical estimates of the NS–NS coalescence rate appear to span a range smaller than two orders of magnitude, which is relatively narrow compared to the range covered by the theoretical estimates.

**Small-Number Sample**

One important limitation of empirical estimates of the coalescence rates is that they are derived based on only two observed NS–NS systems, under the assumption that the observed sample is representative of the true population, particularly in terms of their radio luminosity. Therefore, assessing the effect of small-number
statistics on the results of the above studies is necessary. Assuming that NS–NS
pulsars follow the radio luminosity function of young pulsars and that therefore their
population is dominated in number by low-luminosity pulsars, it can be shown that
the current empirical estimates most probably underestimate the true coalescence
rate. If a small-number sample is drawn from a parent population dominated by
low-luminosity (hence hard to detect) objects, it is statistically more probable that
the sample will actually be dominated by objects from the high-luminosity end of
the population. Consequently, the empirical estimates based on such a sample will
tend to overestimate the detection volume for each observed system, and therefore
underestimate the scale factors and the resulting coalescence rate.

This effect can be clearly demonstrated with a Monte Carlo experiment [18] using
simple models for the pulsar luminosity function and the survey selection effects.
As a first step, the average observed number of pulsars is calculated given a known
“true” total number of pulsars in the Galaxy (thick-solid line in Figure 1). As
a second step, a large number of sets consisting of “observed” (simulated) pulsars
drawn from a Poisson distribution of a given mean number ($<N_{\text{obs}}>$) with assigned
luminosities according to the assumed luminosity function are realized using Monte
Carlo methods. Based on each of these sets, one can estimate the total number of
pulsars in the Galaxy using empirical scale factors, as is done for the real observed
sample. The many (simulated) ‘observed’ samples can then be used to obtain the
distribution of the estimated total Galactic numbers ($N_{\text{est}}$) of pulsars. The median
and 25% and 75% percentiles of this distribution are plotted as a function of the
assumed number of systems in the (fake) ‘observed’ samples in Figure 1 (thin-solid
and dashed lines, respectively).

It is evident from Figure 1 that, in the case of small-number observed samples
(less than $\sim 10$ objects), it is highly probably that the estimated total number,
and hence the estimated coalescence rate, is underestimated by a significant factor.
For a two-object sample, for example, the true rate maybe higher by more than a
factor of ten. This correction factor associated with the faint-end of the luminosity

![FIGURE 1. Bias of the empirical estimates of the NS–NS coalescence rate because of the small-number observed sample. See text for details.](image-url)
function should be applied to the estimated NS–NS coalescence rate in place of the factor of $\sim 10$ used so far from a direct extrapolation of the luminosity function.

**Limits on Coalescence Rates**

One way to circumvent the uncertainties involved in the estimates of the NS–NS coalescence rate is to focus on obtaining upper or lower limits to this rate. Depending on how their value compares to the value of $\sim 10^{-6}$ yr$^{-1}$ needed for a few LIGO-II events per year, such limits can provide us with valuable information about the prospects of gravitational-wave detection.

Bailes [19] used the absence of any young pulsars detected in NS–NS systems and obtained a rough upper limit to the rate of $\sim 10^{-5}$ yr$^{-1}$, while recently Arzoumanian et al. [16] reexamined this in more detail and claimed a more robust upper limit of $\sim 10^{-4}$ yr$^{-1}$.

An upper bound to the rate can also be obtained by combining our theoretical understanding of orbital dynamics (for supernovae with neutron-star kicks occurring in binaries) with empirical estimates of the birth rates of other types of pulsars related to NS–NS formation [20]. Binary progenitors of NS–NS systems experience two supernova explosions when the neutron stars are formed. The second supernova explosion (forming the neutron star that is not observed as a pulsar) provides a unique tool for the study of NS–NS formation, since the post-supernova evolution of the system is simple, driven only by gravitational radiation. There are three possible outcomes after the second supernova: (i) a coalescing binary is formed (CB), (ii) a wide binary (with a coalescence time longer than the Hubble time) is formed (WB), or (iii) the binary is disrupted (D) and a single pulsar similar to the ones seen in NS–NS systems is ejected. Based on supernova orbital dynamics we can calculate the probability branching ratios for these three outcomes, $P_{\text{CB}}$, $P_{\text{WB}}$, and $P_{\text{D}}$. For a given kick magnitude, we can calculate the maximum ratio $(P_{\text{CB}}/P_{\text{D}})_{\text{max}}$ for the complete range of pre-supernova parameters defined by the necessary constraint $P_{\text{CB}} \neq 0$ (Figure 2). Given that the two types of systems have a common parent progenitor population, the ratio of probabilities is equal to the ratio of the birth rates ($BR_{\text{CB}}/BR_{\text{D}}$).

We can then use (i) the absolute maximum of the probability ratio ($\sim 0.26$ from Figure 2) and (ii) an empirical estimate of the birth rate of single pulsars similar to those seen in NS–NS systems based on the current observed sample to obtain an upper limit to the NS–NS coalescence rate. The selection of this small-number sample involves some subtleties [20], and the analysis shows $BR_{\text{CB}} \lesssim 1.5 \times 10^{-5}$ yr$^{-1}$ [20]. Note that this number could be increased because of the small-number sample and luminosity bias affecting this time the empirical estimate of $BR_{\text{D}}$ by a factor of 2 – 6.

This is an example of how we can use observed systems other than NS–NS to improve our understanding of their coalescence rate. A similar calculation can also be done using the wide NS–NS systems instead of the single pulsars [20].
Conclusions

A comparison of the various results on the NS–NS coalescence rate indicates that theoretical estimates based on modeling of NS–NS formation have a rather limited predictive power. The range of predicted rates exceeds 3 orders of magnitude and most importantly includes the “critical” value of $10^{-6}\text{yr}^{-1}$ required for a LIGO-II detection rate of 2-3 events per year. This means that at the two edges of the range the conclusion swings from no detection to many per month. In other words no firm conclusions can be drawn from these estimates about the detection prospects of NS–NS coalescence. Empirical estimates, on the other hand, derived based on the observed sample appear to be more robust (estimates are all within a factor smaller than 100). Given those we would expect a LIGO-II detection rate of a few events per year up to even a few tens of events per year.

For coalescence rates of BH–NS and BH–BH systems we have to rely solely on our theoretical understanding of their formation. As in the case of double NS, the model uncertainties are significant and the ranges extend to more than 2 orders of magnitude. However, the requirement on the Galactic rate so that the detection rate is a few events ($2 - 3$) per year is less stringent for the BH binaries, only $\sim 10^{-8}\text{yr}^{-1}$. Therefore, even with the pessimistic estimates for BH–BH coalescence rates ($\lesssim 10^{-7}\text{yr}^{-1}$), we would expect at least a few or even up to 10 detections per year, which is quite encouraging. We note that a very recent examination of dynamical BH–BH formation [21] in globular clusters leads to detection rates as high as a few per day.

Our expectations for the detectability of BH–NS coalescence could be improved significantly if we actually detect one or more such systems in the near future. Current pulsar surveys, such as the Multibeam Parkes Survey, are considerably more sensitive than previous searches to distant and faint pulsars in close binaries. This high sensitivity is the combined result of long integration times, rapid sampling, and the incorporation of acceleration searches in the data analysis techniques. A

![FIGURE 2. Maximum probability ratio for the formation of coalescing NS–NS binaries and the disruption of binaries as a function of the kick magnitude at the second supernova.](image-url)
candidate NS–NS system (PSR J1811-1736) has been already discovered [22] and it would not be surprising if in the next few years close BH–NS systems are discovered.

YOUNG NS IN ACCRETION-INDUCED COLLAPSE

Hot, young NS formed in supernovae are susceptible to a range of instabilities each of which has a different physical origin: e.g., convection, bar-mode (dynamical or secular), r-mode instabilities. These instabilities can induce a quadrupole moment to the proto-NS and cause it to emit gravitational waves. The strength of the emission varies significantly with the physical mechanism and its calculation depends on model assumptions made (for a review see [23]). A mechanism which recently has attracted attention is related to the exponential growth of rotational mode (r-mode) instabilities in fast spinning ($\lesssim 10-15$ ms), hot ($>10^9$ K) proto-NS (see contribution in these proceedings by B. Owen, [24], and [25]). Other mechanisms involve the centrifugal hang-up of a rapidly spinning collapsing core at early or late stages of the collapse [23]. Such rapidly rotating proto-NS can be formed either during the collapse of massive stars, provided that the stellar core is not well coupled to the envelope and is spinning fast just prior to collapse (see [26]), or perhaps in accretion-induced collapse (AIC) of fast spinning white dwarfs (WD) in close binaries that accrete at appropriate mass transfer rates.

Although the details of the the growth of the r-mode instability and the processes (hydrodynamic or gravitational radiation) for the removal of the excess angular momentum of rapidly rotating cores are not yet fully understood, it is interesting to consider these possibilities, primarily because the signal strength could be high enough for such sources to be detected by the LIGO-II interferometers out to $\sim 20$ Mpc for the r-mode instability [27] and late centrifugal hang-up at $\sim 20$ km, and out to $\sim 100$ Mpc for early centrifugal hang-up at $\sim 100$ km [23]. Adopting the extragalactic extrapolation used by Phinney [11], we find that for a detection rate of a few events per year Galactic AIC rate of $\sim 10^{-4}$ yr$^{-1}$ and $\sim 10^{-5}$ yr$^{-1}$ are required, respectively.

As in the case of binary compact objects, formation of accreting WD that are expected to go through AIC can be studied via binary population synthesis techniques and theoretically predicted event rates can then be obtained. The accuracy and robustness of these results are actually significantly better than for binary NS and BH because they solely depend on our understanding of WD formation, which is considerably better than that of NS and BH formation with birth kicks. The most recent theoretical study of accreting WD formation [28] includes the most up-to-date picture for the conditions (WD mass and mass transfer rate) necessary for AIC to occur. The predicted AIC rates for a wide variety of models lie in the range $8 \times 10^{-7} - 8 \times 10^{-5}$ yr$^{-1}$.

Recently an alternative way of addressing the question of AIC event rate has been explored by [29]. They use the measured abundances of neutron-rich nuclei (e.g., $^{62}$Ni, $^{66}$Zn, $^{87}$Rb, $^{88}$Sr) to set an upper limit on the AIC rate. This study includes
a detailed investigation of the effect of a number of uncertain factors (equation of state, neutrino transport, etc.) and the conclusion is that the upper limit lies in the range $10^{-7} - 10^{-4}$ yr$^{-1}$.

Both the actual estimates and the empirical upper limits from the above studies appear to be in agreement and this gives us confidence that our understanding of the frequency of AIC events is relatively good. Comparison with the required Galactic rates for a few events detected per year indicates that it may be difficult to actually detect gravitational waves from rapidly rotating proto-NS. However, the upper end of our estimates is high enough to justify further consideration of AIC as candidate sources of gravitational radiation.

**CLOSE BINARIES AND LISA**

The construction of a space laser interferometer, LISA, is also being planned for the next one or two decades. The frequency range of its operation is expected to be $\sim 0.1 - 10^3$ mHz, clearly different than that of the ground-based interferometers that are limited by seismic noise at such low frequencies. As a consequence, LISA will be sensitive to very different types of gravitational-wave sources and allow us to explore different regimes of gravity and astrophysical processes.

Although the primary source-target for LISA are supermassive black holes in the centers of galaxies, LISA will also be sensitive to emission from a large population of relatively close binaries in our Galaxy [30]. For frequencies exceeding $\sim 0.1$ mHz, this emission is dominated by close binaries consisting of two white dwarfs. At frequencies up to $\sim 3 - 4$ mHz the emission from a large collection of WD–WD binaries is blended and appears as a continuous background. For higher frequencies up to $\sim 30$ mHz, individual sources could be detected, and at even higher frequencies the extragalactic background becomes detectable but at a much lower power (a factor of $\sim 10$).

It is only very recently that WD–WD binaries have been discovered [31], despite the large expected number in the Galaxy. Their identification requires challenging optical observations that explain the small number of objects detected. With such a limited observed sample estimates of their true number in the Galaxy, and hence of the strength of the associated GW background, rely on purely theoretical calculations of their formation (population synthesis). As in the case of AIC models, the results of these studies are quite reliable since our understanding of the formation process for WD–WD binaries is better than that of NS and BH. This is clearly suggested by a comparison of the results from a number of different recent studies and for different model assumptions [32], [33]. The formation rate estimates span a relatively narrow range (factor of 2) from $5 \times 10^{-3}$ yr$^{-1}$ to $0.1$ yr$^{-1}$. The predicted GR background is largely insensitive to model variations with the most significant factor being that of mass-ratio distribution in the primordial binaries.
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