Calculation of SWR spectra in two- and three-layer magnetic films

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Abstract. In the present work, the calculation and experimental studies of the spin-wave resonance (SWR) spectra in two- and three-layer magnetic films with dissipative or mixed spin pinning mechanisms are carried out. The calculation showed that the SWR spectra for two-layer ferrite-garnet films have approximately twice the number of spin-wave (SW) modes compared to three-layer films on the same field interval, which is confirmed by experiment. An increase in the mismatch of the dispersion dependences is shown for perpendicular and parallel orientations for a three-layer film compared to a two-layer film. The SW-mode linewidth $2\Delta H_n$ in three-layer films is approximately twice as wide in two-layer films for identical values of the wave number.

1. Introduction
Interest in studying the spectra of spin-wave resonance in multilayer magnetic film structures is associated with recent active research of such structures (see, for example, [1-2]), as well as the possibility of using magnetic films in logic elements on spin waves [3-4].

As follows from the theory of ferromagnetic resonance and spin waves [5], the spin-wave mode linewidth $2\Delta H_n$ excited in a microwave field of a constant frequency does not depend on the mode number $n$. The relaxation processes in the excitation layer do not lead to a change in the SW-mode linewidth with a change in their number. However, as follows from the results of papers [6-8], the experiment shows a dependence of the SW-mode linewidth on $n$, and in all known cases $2\Delta H_n$ increases with increasing $n$.

2. Calculation
The purpose of this work was to calculate the SWR spectra in two- and three-layer films with strongly differing values of the damping parameter in layers; in such films, a dissipative or mixed spin pinning mechanism is realized.

The calculation was based on obtaining the expression for high-frequency (HF) susceptibility in two- and three-layer films. In this case, exchange boundary conditions on the free and interlayer boundaries of the film were used. The spin-wave mode linewidth $2\Delta H_n$ was also calculated. The calculation was carried out for the perpendicular and parallel orientation of the external field relative to the film plane (the normal to the film plane coincides with the axis of uniaxial magnetic anisotropy). In this case, the direction of the constant magnetic field $H_i$ and the constant magnetization $M_i$ coincide with each other, the $z$ axis is always directed along the constant magnetic field. The high-
frequency field \( h \) was assumed to be perpendicular to the constant magnetic field \( H_s \). The equations for the variable components of the magnetization \( m_x \) and \( m_y \) had the form:

\[
\frac{2A_i}{M_{oi}} \frac{\partial^2 m_x}{\partial z^2} + \left( H_0 \cos(\theta_h - \theta_M) + H_{1x} \cos^2 \theta_M + i \alpha_i \frac{\omega}{\gamma_i} \right) m_x + i \frac{\omega}{\gamma_i} m_y = -M_{0i} h_i
\]

\[
\frac{2A_i}{M_{oi}} \frac{\partial^2 m_y}{\partial z^2} + \left( H_0 \cos(\theta_h - \theta_M) + H_{1x} \cos(2\theta_M) + i \alpha_i \frac{\omega}{\gamma_i} \right) m_y - i \frac{\omega}{\gamma_i} m_x = -M_{0i} h_i
\]

where \( \theta_h \) and \( \theta_M \) are the angles between the normal to the film plane and the magnetization vector \( M_0 \) and constant magnetic field vector \( H_s \), respectively, \( \gamma_i \) are the gyromagnetic ratio, \( \alpha_i \) is the Hilbert damping parameter, \( H_{oi} \) is the effective field of uniaxial magnetic anisotropy, \( A_i \) is the exchange interaction constant of the \( i \)-th layer. The solution of the system of equations (1) was represented as [9]:

\[
m_i(z) = B_i \exp(ikz) + C_i \exp(-ikz) + \frac{M_0}{D_1 D_2 - \left( \frac{\omega}{\gamma} \right)^2} \left[ \frac{D_2}{D_1} \frac{i \omega}{\gamma} \right] h_i
\]

where \( D_1 = H_0 \cos(\theta_h - \theta_M) + H_{1x} \cos^2 \theta_M + i \alpha_i \frac{\omega}{\gamma_i} \), \( D_2 = H_0 \cos(\theta_h - \theta_M) + H_{1x} \cos(2\theta_M) + i \alpha_i \frac{\omega}{\gamma_i} \), \( B_i, C_i \) - amplitude coefficients of variable magnetization, \( k_i = k_i - ik_i^* \) is the wavenumber of the SW-mode in the \( i \)-th layer.

To find \( B_i, C_i, k_i \) besides the dispersion relation, we used boundary conditions on free and interlayer boundaries:

\[
\left. \frac{\partial m_x}{\partial z} \right|_{z=d_1} = 0, \quad \frac{m_x}{M_1} = \frac{m_z}{M_2}, \quad \frac{A_i}{M_1} \frac{\partial m_x}{\partial z} = \frac{A_i}{M_2} \frac{\partial m_z}{\partial z} \quad \left|_{z=0} \right.
\]

\[
\frac{m_x}{M_2} = \frac{m_z}{M_3}, \quad \frac{A_i}{M_3} \frac{\partial m_x}{\partial z} = \frac{A_i}{M_2} \frac{\partial m_z}{\partial z} \quad \left|_{z=-d_2} \right., \quad \frac{\partial m_x}{\partial z} \left|_{z=-d_2,d_3} \right. = 0
\]

where \( d_1, d_2, d_3 \) is the thickness of the first, second and third layers, respectively. The excitation layer of harmonic SW-modes was the second (middle) layer. The joint solution of the system of equations (1) written for each layer, dispersion relations, and boundary conditions (3) allowed us to find the value of high-frequency magnetization averaged over the thickness of the film:

\[
\langle m \rangle = \frac{1}{d_1 + d_2 + d_3} \times \left( \int_{-d_1}^{0} m_x(z) \, dz + \int_{-d_2}^{d_2} m_x(z) \, dz + \int_{d_2}^{d_2 + d_3} m_x(z) \, dz \right) = \bar{\chi} h,
\]

where \( \bar{\chi} \) is the tensor of high-frequency susceptibility of a three-layer film.

The high-frequency field power absorbed by the film (\( P \)) is determined by the anti-Hermitian part of the high-frequency susceptibility \( \bar{\chi}^p = \bar{\chi} - i \bar{\chi}^\alpha \). If we assume that the linearly polarized field \( h \) is directed along the axis \( X \), then the absorbed power is determined by the component \( P \sim \bar{\chi}^\alpha \).

The verification of the proposed model for calculating the SWR spectra was carried out by comparing the calculated and experimental spectra of two- and three-layer films.
3. Experiment

The measurements of the parameters of the SWR spectra were carried out on two- and three-layer single-crystal films of ferrite garnets with different thicknesses \( d \), the value of the Gilbert damping parameter \( \alpha \), and the magnetization \( 4\pi M \) in the layers. Films were obtained by the method of liquid phase epitaxy on gadolinium gallium garnet substrates with the (111) orientation by successive immersion in various solutions in the melt.

In two layer films the first layer (close to the substrate) of the composition \((SmEr)_1(FeGa)_2O_4\) had thickness \( d_1 = 1.2 \) \( \mu m \), saturation magnetization \( 4\pi M_1 = 1330 \) Gs, Gilbert damping parameter \( \alpha_1 = (\Delta H\gamma/\omega) = 0.2 \), gyromagnetic ratio \( \gamma_1 = 1.38 \times 10^{-7} \) Oe s \(^{-1} \), exchange interaction constant \( A = 3.7 \times 10^{-7} \) erg/cm, effective uniaxial anisotropy field \( H_{41} = (2K_s/M) - 4\pi M_1 = 96 \) Oe. Here \( \Delta H \) - the half-width of the absorption line, \( \omega \) - is the circular frequency of the microwave field. The second layer (the excitation layer of harmonic standing spin-wave (SW) modes) of the composition \( Y_{12.98}Sm_{0.02}Fe_2O_4 \) had \( d_2 = 0.72 \) \( \mu m \), \( 4\pi M_2 = 1740 \) Gs, \( \alpha_2 = (\Delta H\gamma/\omega) = 0.003 \), \( \gamma_2 = 1.76 \times 10^{-7} \) Oe s \(^{-1} \), \( A_2 = 3.7 \times 10^{-7} \) erg/cm, \( H_{42} = (2K_s/M) - 4\pi M_2 = -1715 \) Oe.

The three-layer films differed from the two-layer films by the presence of another (upper) layer similar to the first one. In these samples the dominant mechanism for pinning the spins was the mixed mechanism of spin pinning (dissipative and dynamic).

In films with different fields of homogeneous resonance, the dynamic mechanism of spin pinning realize in the layers. With a dynamic mechanism, a microwave field excites localized modes that are harmonic in one layer (a layer with a large value of the field of a homogeneous resonance \( H_{40} \) ) and exponentially fall off in another layer, which in external fields larger than its homogeneous resonance field \( H_{42} \) is for spin waves by a reactive medium and thereby ensures the pinning of the spins. The interval of fields in which spin waves are intensively excited, with dynamic pinning mechanism is limited by the values \( H_{40}, H_{42} \).

The dissipative pinning mechanism arises in multilayer films with widely differing values of the Gilbert damping parameter in the layers. The presence of an exchange coupling between the layers leads to the appearance of a node of a standing spin wave at the interface of the layers or near it. One of the qualitative differences between the dissipative mechanism for the spin pinning from the dynamic one lies in the fact that it does not depend on the orientation of the external magnetic field relative to the film, which relate to the isotropy of the damping parameter. For any orientation, the excitation region of standing harmonic spin waves is localized in the layer with a small damping parameter.

In our films different fields of homogeneous resonance are realized and the damping parameters in the layers vary greatly; therefore, both the dissipative and the dynamic mechanism of spin pinning appear, i.e. a mixed mechanism for the spin pinning is realized.

To measure and control parameters on pure substrates, single-layer analogs of each of the layers of two- and three-layer films were grown. The thickness of the layers was determined by the thickness of single-layer analogs, determined by the time of complete etching, and also measured by the interference method. The thickness of the pinning layer in all samples was 1.2 \( \mu m \), which, as the calculation and experimental results show [10-13], significantly exceeds the path length \( l \) (penetration depth) of spin waves in this layer. The SWR spectra were recorded at room temperature on a radiospectrometer with a microwave field frequency \( \omega/2\pi \) equal to \( 9.34 \times 10^{9} \) Hz. Samples were placed in crossed constant and microwave magnetic fields. The magnetic field was measured using an NMR magnetometer.

4. Discussion
In figure 1 shows the calculated (dependences $dP/dH$) SWR spectra for a two- (a) and three-layer (b) ferrite garnet film with a perpendicular orientation of the constant magnetic field relative to the film plane. It can be seen that the calculated spectra for a two-layer ferrite-garnet film have approximately twice the number of SW-modes compared with a three-layer film in the same field interval, which was observed in experiments.

The difference of the fields $H_0 - H_n$ is calculated from the dispersion relation, which in turn is obtained from the equation of motion of magnetization. In our case, this difference has the form

$$H_0 - H_n = (2A_2/M_0)[(k'_{1/2})^2 - (k_0^2)]$$

for a two-layer film and

$$H_0 - H_n = (2A_2/M_0)(2n + 1)^2(2k_{1/2})^2$$

for a three-layer film. The values $k'_{1/2} = (n + 1/2)(\pi/d_z)$ and $k_0 = (2n + 1)(\pi/d_z)$ are obtained from the boundary conditions, that is, the function $H_0 - H_n = f((n + 1/2)^2)$ and $H_0 - H_n = f((2n + 1)^2)$ has a linear relationship (figures 2, 3). On the

**Figure 1.** Calculated SWR spectra for two-layer (a) and three-layer (b) films with a perpendicular orientation of the external field relative to the film plane.

**Figure 2.** Dependences of the difference of the resonant fields of the zero and $n$-th modes $H_0 - H_n = f((n + 1/2)^2)$ for two-layer films. Solid lines - calculation, icons - experiment ($\Delta$ - parallel orientation, $\times$ - perpendicular orientation)

**Figure 3.** Dependences of the difference of the resonant fields of the zero and $n$-th modes $H_0 - H_n = f((2n + 1)^2)$ for three-layer films. Solid lines - calculation, icons - experiment ($\Delta$ - parallel orientation, $\times$ - perpendicular orientation).
calculated dependences $H_0 - H_n = f((2n+1)^2)$, as well as in the experiment, an effect was observed of an increase in the mismatch of the dispersion dependences for perpendicular and parallel orientations for three-layer films in comparison with two-layer.

It was previously established that, for any pinning mechanism, a standing spin wave in a multilayer film can be considered consisting of several exchange-related components: harmonic, localized in the excitation layer, and components damping in the pinning layers [8, 10–13]. In this case, both in the case of dissipative and dynamic pinning mechanisms, the penetration depth (path length) $l$ of the spin wave in the pinning layer depends on the wave number of its harmonic spatial component localized in the excitation layer ($l = l_0/k_s = (2A\gamma/c_0M)k_s'$, i.e. $l \sim k_s'$).

In samples with a mixed or dissipative spin pinning mechanism, the width of the absorption line of the $n$-th SW-mode $2\Delta H_n$ increases monotonically with increasing mode number $n$ (figure 4). This is due to the fact that for such samples with an increase in $n$, the wave number in the excitation layer increases $k_s' \sim n$, and hence the penetration depth of the spin wave in the pinning layer also increases, i.e. $l \sim k_s' \sim n$. As a result, the magnitude of the variable magnetic moment in this layer increases, which ultimately leads to an increase in the linewidth $2\Delta H_n$ of the $n$-th SW mode [8, 10-13]. In this case, the broadening of the spin-wave mode lines, caused by the damping region, can many times exceed the intrinsic width of the excitation layer line.

It was also previously established that when the external field is perpendicularly oriented relative to the film plane, the spin wave in the pinning layer is exponentially decaying, and when parallel, it is exponentially decaying, harmonic. In this case, the penetration depth of a spin wave in the pinning layer $l_\parallel$ with a parallel orientation of the external field relative to the film plane exceeds the penetration depth $l_\perp$ at perpendicular orientation, which is associated with large values of the wave number of its harmonic spatial component localized in the excitation layer ($k_{\parallel}' > k_{\perp}'$) at this orientation. As a result, the field difference with parallel orientation exceeds the field difference with perpendicular orientation, which is confirmed in the experiment (figures 2, 3). Also as a result, the magnitude of the variable magnetic moment in the pinning layer with a parallel orientation is greater than the magnitude of the variable magnetic moment with a perpendicular orientation, which
ultimately leads to a larger value of the SW-mode linewidth $2\Delta H_{s1} > 2\Delta H_{s2}$ (figure 4) for the same values of n. Some discrepancy between the calculated and experimental dependences $2\Delta H_{s2}$ on n may be due to the fact that the calculation model does not fully take into account the contribution from the damping regions of spin oscillations in the pinning layer (s) to the energy dissipation of the spin-wave mode. In addition, the calculation of the SW-mode linewidth $2\Delta H_{s2}$ showed that the linewidth $2\Delta H_{s2}$ in three-layer films is approximately twice as large as $2\Delta H_{s1}$ in two-layer films (figure 4). This is due to the additional influence of the damping region of the spin wave in a layer with a large $\alpha$ in three-layer films compared to two-layer films [8, 10-13]. Thus, the approach in this work allows one to obtain a satisfactory agreement between the calculated and experimental characteristics of the SWR spectra in two- and three-layer films.

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