Security trade-offs in ancilla-free quantum bit commitment in the presence of superselection rules

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Abstract. Security trade-offs have previously been established for one-way bit commitment. We study this trade-off in two superselection settings. We show that for an ‘abelian’ superselection rule (exemplified by particle conservation), the standard trade-off between sealing and binding properties still holds. For the non-abelian case (exemplified by angular momentum conservation), the security trade-off can be more subtle, which we illustrate by showing that if the bit commitment is forced to be ancilla-free, an asymptotically secure quantum bit commitment is possible.

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1. Introduction

The question of whether the no-go result for quantum bit commitment (BC) [1]–[3] remains valid in the presence of superselection rules has been addressed recently [4]–[6]. The most general result is described in [5], where the authors prove that secure quantum BC is impossible even with general superselection rules. The authors of this paper distinguish security in the case of ‘abelian’ superselection rules (such as particle conservation) and ‘non-abelian’ superselection rules in which the imposed symmetry is described by a non-abelian group. In both cases, the authors prove the impossibility of establishing a secure BC protocol. In this paper, we point out that the no-go result for the non-abelian case is fairly non-trivial; we show that if the cheating strategies are forced to be ancilla-free, an asymptotically secure protocol can be found. In contrast, for the abelian case (or in the absence of superselection rules) the ‘no-ancilla’ enforcement does not alter the security trade-offs for a one-way BC (called a ‘purification BC’ in [7]).

2. One-way BC protocol

We repeat the definition in [7] of this important class of BC protocols.

**Definition 1. (One-way BC protocol [7]).** In this protocol, Bob begins with no quantum state of his own, Alice begins with a two-part Hilbert space $\mathcal{H}_p \otimes \mathcal{H}_t$ (‘proof’ and ‘token’). Alice chooses to commit to bit $b$. Alice prepares one of two orthogonal states $|\chi_b\rangle$ in her total Hilbert space. In the commit phase, Alice transmits to Bob the state in $\mathcal{H}_t$ which we denote by $\rho_b$; in the unveiling phase, Alice transmits the state in $\mathcal{H}_p$ to Bob; Bob determines the committed bit by projectively measuring the state using orthogonal projectors $\{\Pi_0, \Pi_1, \Pi_{\text{fail}}\}$.

This is certainly not the most general quantum BC protocol, which would permit more than one round of communication. The security trade-offs of one-way BC protocols have been described as follows, see [7]. Two scenarios are considered: (i) Alice is honest, but Bob tries to cheat by learning the bit in the commit phase. (ii) Bob is honest, but Alice tries to cheat by changing her committed bit after the commit phase. In case (i), Bob is trying to make his ‘information gain’ $G(S^B)$ non-zero, defined as the difference between his probability of estimating Alice’s commitment correctly in the commit phase when he employs cheating strategy $S^B$, and when he is honest,

$$G(S^B) = P_E(S^B) - \frac{1}{2}. \quad (1)$$

In case (2), Alice is trying to make her ‘control’ $C(S^A)$ non-zero, defined as the difference between her probability of unveiling whatever bit she desires when she implements $S^A$, and when she is honest,

$$C(S^A) = P_U(S^A) - \frac{1}{2}. \quad (2)$$

Then, the security of any given protocol can be characterized by the maximum of these two quantities:

$$G^{\text{max}} \equiv \max_{S^B} G(S^B),$$
$$C^{\text{max}} \equiv \max_{S^A} C(S^A). \quad (3)$$
For one-way BC protocols it has been established [7] that

\[ G_{\text{max}} = \frac{1}{2} D(\rho_0, \rho_1) = \frac{1}{4} \text{tr} |\rho_0 - \rho_1|. \]  

[4]

\[ C_{\text{max}} = \frac{1}{2} F(\rho_0, \rho_1) = \frac{1}{2} \text{tr} |\sqrt{\rho_0} \sqrt{\rho_1}|, \]  

[5]

where \( F(., .) \) is the fidelity function and \( D(., .) \) the trace-distance function. Notably, the cheating strategies that achieve these optima do not make use of ancillas: Alice’s cheating strategy is the creation of a \( b \)-independent state potentially followed by unitary rotation, whereas Bob’s cheating strategy is a complete von Neumann measurement projecting in the eigenbasis of \( \rho_0 - \rho_1 \), achieving the trace distance. Let us define what we mean by an ancilla-free BC.

**Definition 2.** (Ancilla-free one-way bit-commitment protocol, AFBC). A one-way BC protocol in which we restrict the cheating strategies to ones which have no access to additional quantum systems, i.e. ancillas. Thus, cheating strategies consist of local unitary transformations and complete von Neumann measurements.

### 3. One-way BC with superselection rules

To proceed with this analysis, we must stipulate how the one-way BC protocol is constrained by superselection rules [8]. We also refer to [10] for a detailed description of how superselection rules and their associated symmetry groups impose constraints.

We assume that Alice’s and Bob’s actions have to obey a local superselection rule which can be given by a Hermitian operator \( K_A \) and \( K_B \). We will discuss two examples here: (i) \( K_{A/B} \) is the local particle number \( N_{A/B} \) and (ii) \( K_{A/B} \) is a local total angular momentum operator \( J_{A/B}^2 \); this is a ‘non-abelian’ case (the symmetry group is that of space rotations).

Here then are the restrictions that we will require for the one-way BC. These restrictions include the possible cheating strategies of the parties:

1. The states in the token + proof Hilbert space in Alice’s lab are eigenvectors of \( K_A \) with the same eigenvalue, say \( k \). Thus, \( K_A |\chi_0\rangle = k |\chi_0\rangle \) and \( K_A |\chi_1\rangle = k |\chi_1\rangle \). If Alice cheats and she chooses to create other states, then these should also be eigenstates of \( K_A \).

2. Any action that Alice performs before the commit phase must involve unitary transformations that leave the eigenvalue \( k \) unchanged, i.e. \([U, K_A] = 0\).

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1 The distinction between selection rules and superselection rules is not always very clear. At an informal level, one may think that superselection rules are due to fundamental laws in nature and thus never to be violated, whereas selection rules are of a more relative nature, valid for the particular energy scales at hand, or due to technological/practical constraints. A different and more precise definition was given in the original paper [9]. There superselection rules are defined as selection rules, i.e. dynamical conservation laws, with the additional restriction that off-diagonal matrix elements in the conserved quantum number basis cannot be distinguished by measurement. These definitions of selection and superselection rules seem to leave open the possibility for an initial condition that is a superposition of different quantum numbers. In this section, we specify in detail what we mean by BC constrained by the superselection rules.

2 In fact, the security of both our schemes does not change if we permit Alice to create any state \( \chi \).
3. After the commit phase, any unitaries performed by Alice must satisfy \([U, K_A] = 0\). Similar
for Bob when he is given the token: \([U, K_B] = 0\).

4. Any measurements that Bob performs must respect the superselection rule. Thus, in the commit
phase (when Bob might wish to cheat), any measurement projector \(\Pi\) of Bob on the token
space must satisfy \([\Pi, K_B] = 0\). In the unveiling phase, the same must be true for the joint
token + proof Hilbert space in Bob’s possession: \([\Pi, K_B] = 0\).

If the cheating strategy is not required to be ancilla-free, Bob could create a state of definite
quantum number \(k_B\) in his lab before he receives the token. After he has received the token, he
should respect the conservation rule on his total token + ancilla space, but the ancilla may still
help him to do a better measurement.

Let us use the following notation: since a one-way BC protocol is completely specified
once the orthogonal pair \(|\chi_0, 1\rangle\) is agreed upon, we will refer to this protocol as \(\text{BC}(|\chi_0, 1\rangle)\). If
the protocol has the above superselection rule restrictions imposed involving operator \(K\), we
will refer to this protocol as \(\text{BC}_K(|\chi_0, 1\rangle)\). If the protocol is forced to be ancilla-free we write
\(\text{AFBC}_K(|\chi_0, 1\rangle)\).

4. Our results

We find that in the presence of particle number superselection rules, the security of one-way BC
is rigorously unchanged, see section 5:

\[ \forall |\chi_{0, 1}\rangle \quad \text{such that} \quad N|\chi_{0, 1}\rangle = n|\chi_{0, 1}\rangle, \]

\[ G_{\text{max}}(\text{BC}_N(|\chi_{0, 1}\rangle)) = G_{\text{max}}(\text{BC}(|\chi_{0, 1}\rangle)) = G_{\text{max}}(\text{AFBC}(|\chi_{0, 1}\rangle)), \]

\[ C_{\text{max}}(\text{BC}_N(|\chi_{0, 1}\rangle)) = C_{\text{max}}(\text{BC}(|\chi_{0, 1}\rangle)) = C_{\text{max}}(\text{AFBC}(|\chi_{0, 1}\rangle)). \]  

However, in the presence of angular momentum superselection rules (operator \(J^2\); a
‘non-abelian’ case), a one-way BC with arbitrarily high security can be devised if we force the
scheme to be ancilla-free

\[ \exists |\chi_{0, 1}(j)\rangle, \quad \text{such that} \quad J^2|\chi_{0, 1}(j)\rangle = j(j + 1)|\chi_{0, 1}\rangle, \]

\[ G_{\text{max}}(\text{AFBC}_{J^2}(|\chi_{0, 1}(j)\rangle)) = 0, \]

\[ C_{\text{max}}(\text{AFBC}_{J^2}(|\chi_{0, 1}(j)\rangle)) \to 0 \quad \text{for} \quad j \to \infty. \]  

We do not have a rigorous proof of the statement about the asymptotic behaviour of
\(C_{\text{max}}(\text{AFBC}_{J^2}(|\chi_{0, 1}\rangle))\), but we conjecture a formula for the fidelity function which coincides
with numerical data up to \(j = 11\), and which implies the asymptotic security just stated,
see section 6.

5. Particle conservation

We now prove equation (6). Mayers [6] first proved the simplest case in which the fact that
the protocol is completely sealing makes it completely unbinding, i.e. Alice can always change
her commitment. The arguments here are a straightforward extension of this simple case. The number operator is additive over tensor product Hilbert spaces, so we can write

\[ N = N_t + N_p. \]  

(8)

This means that if, as we assume, \( N|\chi_{0,1} \rangle = n|\chi_{0,1} \rangle \), then these states must have the form

\[ |\chi_b \rangle = \sum_{i_p,i,m} c_b(i_t,i_p,m)|i_p,n - m \rangle_p|i_t,m \rangle_t, \]  

(9)

i.e. if the token system has \( m \) particles, the proof system must have \( n - m \) particles. The labels \( i_p \) and \( i_t \) denote other quantum numbers characterizing the states. It is understood that the range of the \( i_t \) and \( i_p \) sums can depend on \( m \) (i.e. they depend on the local particle number).

The security parameters defined above depend only on the reduced density operators on the token subsystem of the two states. For the states of the form of equation (9), these can be written as

\[ \rho_b = \sum_{i_p,i,m=0}^n c_b(i_t,i_p,m)c^*_b(j_t,i_p,m)|i_t,m \rangle \langle j_t,m| = \bigoplus_{m=0}^n p_{b,m}\hat{\sigma}_{b,m}, \]  

(10)

where \( p_{b,m} \) is the probability of each \( m \) and \( \hat{\sigma}_{b,m} \) a normalized density operator in each \( m \) sector: \( \text{tr}\, \hat{\sigma}_{b,m} = 1 \).

Consider what happens if Bob cheats. The ideal optimal measurement for Bob that he can do to achieve equation (4) is a complete von Neumann measurement in the eigenbasis of \( \rho_0 - \rho_1 \). However, because of equation (10), this eigenbasis is also an eigenbasis of the particle number of the token system and given the superselection rule, Bob is allowed to do this optimal measurement. So, Bob can gain exactly the same amount of information in the protocol constrained by superselection rules,

\[ G^\text{max}(\text{BC}_N(|\chi_{0,1} \rangle)) = G^\text{max}(\text{BC}(|\chi_{0,1} \rangle)). \]

Next, we consider what happens if Alice cheats. We will show that \( C^\text{max}(\text{BC}_N(|\chi_{0,1} \rangle)) = C^\text{max}(\text{BC}(|\chi_{0,1} \rangle)) \). First of all, due to the block-diagonal character of \( \rho_0 \) and \( \rho_1 \) we can write

\[ C^\text{max}(\text{BC}(|\chi_{0,1} \rangle)) = \frac{1}{2} \text{tr}\, |\sqrt{\rho_0}\,\sqrt{\rho_1}| = \frac{1}{2} \sum_{m=0}^n \sqrt{p_{0,m}p_{1,m}} F(\hat{\sigma}_{0,m}, \hat{\sigma}_{1,m}). \]

(11)

On the other hand, Uhlmann’s theorem for the fidelity function \( F \) is also written as

\[ F(\rho_0, \rho_1) = \max_{U_p} |\langle \chi_0| U_p \otimes I \rangle \langle \chi_1 |\rangle| = \max_{U_p} \Re\{\langle \chi_0| U_p \otimes I \rangle \langle \chi_1 |\rangle\}. \]

(12)

We arrive at this last form by recognizing that \( U_p \) can have any arbitrary global phase. In [7] it is shown that \( C = |\langle \chi_0| U_p \otimes I \rangle \langle \chi_1 |\rangle|/2 \) can be achieved if Alice creates the \( b \)-independent state

\[ |\chi \rangle \propto (|\chi_0 \rangle + e^{-i\arg(|\chi_0| U_p \otimes I \rangle \langle \chi_1 |)}) \]  

(13)

prior to the commit phase. If she decides during the commit phase that \( b = 0 \), she leaves that state unchanged and sends it to Bob; if she wants \( b = 1 \), she applies the optimal \( U_p^\dagger \) of equation (12) to the proof system that she still holds.

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However, in the presence of the charge superselection rule, it is not possible to do the unconstrained maximization of equation (12); we must respect constraints 2 and 3 above imposed by the superselection rules, which require that $U_p$ be block diagonal in the charge index. We observe that Alice can create the cheating state $\chi$ of equation (13) while respecting the superselection rule. Thus, we can write $C_{\text{max}}$ in this case as a constrained maximization:

$$C_{\text{max}}(\text{BC}_N(|\chi_{0.1}\rangle)) = \frac{1}{2} \max_{U_p=\hat{\otimes}_{m=0}^{n} U_{m,p}} \Re((\chi_0|U_p \otimes I_1|\chi_1)).$$  \hspace{1cm} (14)

To evaluate this expression, we rewrite the states of equation (9) as

$$|\chi_b\rangle = \sum_{m=0}^{n} \sqrt{p_{b,m}} |\chi_{b,m}\rangle.$$  \hspace{1cm} (15)

Inserting (15) into (14) and working out the expression gives

$$C_{\text{max}}(\text{BC}_N(|\chi_{0.1}\rangle)) = \frac{1}{2} \sum_{m=0}^{n} \sqrt{p_{0,m} p_{1,m}} F(\hat{\sigma}_{0,m}, \hat{\sigma}_{1,m}).$$  \hspace{1cm} (16)

In other words, we obtain the claimed equality $C_{\text{max}}(\text{BC}_N(|\chi_{0.1}\rangle)) = C_{\text{max}}(\text{BC}(|\chi_{0.1}\rangle))$.

We will now find a very different situation for angular-momentum conservation. The idea is to find two states $\chi_b$ for $b = 0, 1$ such that the local density matrices for Bob, $\rho_b$, appear the same given the superselection rule. This implies that the diagonal elements of $\rho_b$ in the local definite angular momentum basis must be the same. The off-diagonal elements can be different however, and we can try to adjust these free parameters so as to limit Alice’s cheating strategies. In the case of particle conservation there is no room to adjust free parameters since the off-diagonal elements of $\rho_b$ in the number basis are always zero, see equation (10). This is the essential difference between the abelian and the non-abelian case.

6. Angular momentum conservation

We introduce a family of one-way BC protocols, one for each total angular momentum quantum number $j (2j \in \mathbb{Z}^+)$ (although we will only discuss the integer case in detail). Alice’s two states $\chi_{0.1}$ are states with total angular momentum $j_{\text{tot}} = j$ and $m_{\text{tot}} = j$. We denote the Clebsch–Gordan coefficients as $C(j_A, m_A, j_B, m_B, j_{\text{tot}}, m_{\text{tot}})$, where $j_A, m_A$ are the quantum numbers of the ‘proof’ spin (kept by Alice, initially) and $j_B, m_B$ are the quantum numbers of the ‘token’ spin (sent to Bob). Unlike the number operator, total angular momentum is not an additive quantity (herein lies the crucial difference between the security of the two cases), i.e. the two local angular momenta $j_A$ and $j_B$ can give rise to total angular momentum between $|j_A - j_B|$ and $j_A + j_B$. We take states for fixed $j$ to be

$$\chi_0(j) = \sum_{j_B=0}^{2j} \sqrt{\beta_{j_B}} \sum_{m_A=-j}^{j} C(j, m_A, j_B, j - m_A, j, j) |j, m_A\rangle_p |j_B, j - m_A\rangle_t.$$  \hspace{1cm} (17)
\[ \chi_1(j) = \sum_{j_B = 0}^{2j} (-1)^{jb} \sqrt{\beta_{jb}} \sum_{m_A = -j}^j C(j, m_A, j_B, j - m_A, j, j) \langle j, m_A \rangle_p \langle j_B, j - m_A \rangle_i. \]  

(18)

Of course, the \( z \)-component of the local spin cannot be larger than the total local spin; we impose this in the above equations by taking the Clebsch–Gordan coefficients \( C(j, m_A, j_B, j - m_A, j, j) \) to be zero if \( j - m_A > j_B \). The \( \beta \)-coefficients obey the following constraints:

\[ \sum_{j_B = 0}^{2j} \beta_{jb} = 1, \]  

(19)

\[ \sum_{j_B = 0}^{2j} (-1)^{jb} \beta_{jb} = 0, \]  

(20)

\[ \forall j_B, \beta_{jb} \geq 0. \]  

(21)

The first equation enforces the normalization and the second equation enforces the orthogonality of \( \chi_0 \) and \( \chi_1 \). Before considering Alice’s strategies, which will give us additional equations for the \( \beta \)-coefficients, let us consider Bob’s cheating strategies. Due to the superselection rule his measurement has to be diagonal in the total angular momentum of his particle which implies that he will not be able to detect any difference in the terms of \( \rho_0 \) and \( \rho_1 \) that are off-diagonal in the \( j_B \) basis. The states \( \chi_b(j) \) are chosen such that the on-diagonal terms of \( \rho_0 \) and \( \rho_1 \) are identical. This implies that for all \( j \), \( G^{\max}_{AFBCJ}(|\chi_{0,1}\rangle) = 0 \).

To analyse Alice’s cheating ability, we work out the expression for the fidelity \( F \) in equation (5), which provides an upper bound on her cheating strategies. Note that the expressions for \( \chi_b \) have the Schmidt form

\[ |\chi_b\rangle = \sum_{m_A} \sqrt{\lambda_{m_A}} |j, m_A\rangle |\phi^{m_A}_b\rangle, \]  

(22)

where \( |\phi^{m_A}_b\rangle \) are normalized orthogonal vectors:

\[ |\phi^{m_A}_b\rangle = \frac{1}{\sqrt{\lambda_{m_A}}} \sum_{j_B = 0}^{2j} (-1)^{jb} \sqrt{\beta_{jb}} C(j, m_A, j_B, j - m_A, j, j) \langle j_B, j - m_A \rangle. \]  

(23)

The Schmidt coefficients in equation (22) are independent of the bit \( b \):

\[ \lambda_{m_A} = \sum_{j_B} \beta_{jb} C^2(j, m_A, j_B, j - m_A, j, j). \]  

(24)

Also note that for these states

\[ \langle \phi^{m_A}_0 | \phi^{m_A}_1 \rangle = \frac{1}{\lambda_{m_A}} K_{m_A} \delta_{m_A m_A'}. \]  

(25)
where

\[ K_{m_A} = \sum_{j_B=0}^{2j} (-1)^{j_B} \beta_{j_B} C^2(j, m_A, j_B, j - m_A, j, j). \]  

(26)

With these tools, Uhlmann’s fidelity can be written as

\[ F(\rho_0, \rho_1) = \text{tr} \sqrt{\rho_0} \rho_1 \sqrt{\rho_0} = \sum_{m_A} |K_{m_A}| = \sum_{m_A} s_{m_A} K_{m_A}, \]

(27)

where \( s_{m_A} = \text{sign}(K_{m_A}) \) and thus

\[ F = \sum_{m_A=-j}^{j} s_{m_A} \sum_{j_B=0}^{2j} (-1)^{j_B} \beta_{j_B} C^2(j, m_A, j_B, j - m_A, j, j). \]  

(28)

We arrive at a piecewise linear program: determine the vector \( \vec{\beta} \) which minimizes \( F \) under the constraints given by equation (21). For completeness, let us state the Clebsch–Gordan coefficients as they appear in the expression for \( K_{m_A} \):

\[ C^2(j, m_A, j_B, j - m_A, j, j) = \frac{(-m_A+j_B)(m_A+j_B)}{(-2j_B+1)} \frac{(-m_A-j_B)(m_A-j_B)}{(2j_B+1)}, \quad 0 \leq j_B \leq 2j, 1 \leq m_A \leq j, m_A \geq j - j_B. \]  

(29)

(That is, a binomial coefficient outside its usual range should be taken to be zero.)

6.1. Numerical analysis

For each integer \( j \) we have investigated this piecewise linear program numerically. (We do not report the half-integer \( j \) results here, they work out similarly.) We have done the minimization up to \( j = 11 \) and we observe the following patterns. As it turns out, in all solutions we find that \( \beta_k = 0 \) for \( j + 2 \leq k \leq 2j \). Secondly, in every case, the terms in equation (28) for \( m_A > 0 \) are identically zero. This gives us a set of linear equalities for \( \beta_{j_B} \):

\[ \forall m_A: \ 1 \leq m_A \leq j, \ \sum_{j_B=0}^{2j} (-1)^{j_B} \beta_{j_B} C^2(j, m_A, j_B, j - m_A, j, j) = 0. \]  

(30)

Thirdly, in alternating cases (\( j \) even or \( j \) odd), we find that \( s_{-1} \) and \( s_0 \) are \{+1, −1\} and \{−1, +1\}. In addition, the remaining terms in equation (28), for \( m_A < 0 \), are also identically zero. This gives us a set of linear equalities for \( \beta_{j_B} \):

• \( j = 1: \ \beta_{0,1,2} = \{\frac{2}{5}, \frac{1}{7}, \frac{5}{18}\} \), leading to the sign assignments \( s_{-1,0,1} = \{+1, −1, x\} \), where \( x \) implies that \( s_1 \) is irrelevant in the minimization. This leads to \( F = \frac{1}{10} \).

• \( j = 2: \ \beta_{0,1,2,3,4} = \{\frac{3}{20}, \frac{9}{30}, \frac{7}{30}, \frac{7}{30}, 0\} \), leading to the sign assignments \( s_{-2,-1,0,1,2} = \{x, −1, +1, x, x\} \). For these \( \beta \) values, \( F = \frac{1}{10} \).
\[ j = 3: \quad \beta_{0,1,2,3,4,5,6} = \left\{ \frac{4}{25}, \frac{2}{7}, \frac{78}{255}, \frac{3}{14}, \frac{33}{35}, 0, 0 \right\}, \] leading to the sign assignments
\[ s_{-3,-2,-1,0,1,2,3} = \{x, x, +1, -1, x, x, x\}. \]
For these \( \beta \) values, \( F = \frac{1}{35}. \)

Most importantly, we have observed that for all cases up to \( j = 11 \), we find optimal values of \( F \) that agree with the simple formula
\[ F = \left[ \frac{(2j + 1)}{j + 1} \right]^{-1}. \quad (31) \]

As is clear from this formula, which we conjecture to correspond to a feasible solution for the \( \vec{\beta} \)-vector for all \( j \), \( F \) approaches zero exponentially as fast as \( j \to \infty \) which implies the security of the protocol.

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