A finite element method based on modified Alonso unsaturated soil constitutive model

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Abstract. Based on the revised ALONSO model, combined with the ABAQUS software platform, this paper compiled the USDFLD subroutine and obtained the stress-strain curve and the axial strain-volume deformation by simulating the triaxial shear test of the unsaturated soil to control the suction and confining pressure. The curve verifies the accuracy and reliability of the algorithm. The research results show that: (1) When the suction is the same, the higher the confining pressure is, the higher the yield point of the soil sample is. According to the comparison between experiment and simulation, the modified Alonso model can reflect the nonlinearity of the stress-strain relationship of the reshaped unsaturated soil sample and its plastic flow characteristics; (2) Compared with saturated soil, unsaturated soil has a higher yield point as a result of the suction. And the larger the suction is, the higher the yield point is, which proves that the model can simulate the influence of unsaturated soil liquid relative to the solid phase.

Keywords: Modified Alonso constitutive model; USDFLD Subroutine; finite element method; triaxial shear test.

1. Introduction

The earliest elastoplastic constitutive model of unsaturated soil was proposed by Alonso. This model was established based on the modified Cambridge model in saturated state and also considered the relationship between suction, soil strength, and yield surface. It has a research history of nearly 20 years and has been widely used and recognized [1-5].

The predecessors tried to explore the accuracy of the Alonso model through experiments and numerical methods. Chen Zhenghan [6] analyzed the model and believed that the model has inaccuracies such as the inaccuracy of the SI yield surface, and improved it through experimental data; Lu Zaihua [7] improved the model based on the unsaturated three-dimensional test under different stress paths; Zhan Liangtong [8] proved the yield characteristics of soil samples through unsaturated tests; Wu Zhouli [9] improved the LC yield surface and CSL expression of the model through
experimental analysis; Chen Yong [10] obtained model parameters through experiments, and derived the stress-strain increment equation based on the Alonso model, and compiled a finite element program; Li Xikui [11] put forward a numerical model of finite element analysis on the deformation and seepage of unsaturated soil and the numerical verification was carried out; Yang Gengyu [12] used the plastic increment theory to derive the calculation formula of the elastoplastic constitutive matrix of unsaturated soil based on the Alonso model, which was used in finite element analysis.

In this paper, based on the previous research, combined with the modified Alonso unsaturated soil elastoplastic constitutive model, and using the large-scale finite element software ABAQUS as the platform, the USDFLD subroutine is compiled in Fortran language. Moreover, based on the results of the unsaturated triaxial test of Nanyang expansive soil, the test results are compared with the stress-strain curve obtained by the numerical calculation to verify the accuracy and reliability of the algorithm.

2. Modified Alonso Model

The Alonso elastoplastic constitutive model is based on the modified Cambridge model and extends from the saturated field to become an unsaturated soil constitutive model. But the model is still insufficient [9]: (1) The parameter M is considered to be a constant in the Alonso model. But according to previous experimental data analysis, its value will increase nonlinearly with the increase of suction; (2) In Alonso model description, a tensile stress area appears. But in the triaxial test, the soil sample cannot truly achieve the tensile stress state, and the tensile properties of the soil are different from the compressive properties.

Based on the shortcomings of the above Alonso model, Wu Lizhou [9] corrected it. The correction takes into account the influence of suction on the slope of CSL. It is assumed that the M of different suctions passes through the origin coordinates in the (p, q) plane, ensuring that the yield surface does not appear the tensile stress zone in the p-q-s stress space. In this paper, programming is based on the modified model. The content of the model is as follows:

2.1. LC Yield Surface

In the (p-q-s) three-dimensional stress space, the LC yield surface equation is as follows:

\[ q^2 - M^2(s)p(p_0 - p) = 0 \]  \hspace{1cm} (1)

\[ \frac{p_e}{p_c} = \left\{ \begin{array}{ll} \frac{p_0}{p_c} & \text{if} \quad \sigma_s < 0 \\ \frac{p_0 \frac{\sigma_s}{\lambda(s) - k}}{p_c} & \text{if} \quad \sigma_s \geq 0 \end{array} \right. \]  \hspace{1cm} (2)

\[ p_0 = p_c \exp \left( \frac{1 + \frac{\sigma_s}{\lambda(s) - k}}{\lambda(s) - k} \right) \]  \hspace{1cm} (3)

In the above formula, \( p_0 \) is the pre-consolidation pressure when \( s=0 \). \( p_0 \) is the pre-consolidation pressure of unsaturated soil. \( M(s) \) varies with suction. The modified model no longer shows tensile stress. Figure 1 shows the projection of the LC yield surface of the Alonso model and the modified LC yield surface in the p-q plane. The center of the yield surface of the modified model is always in the first quadrant of the p-q plane, and p will not appear negative.
2.2. CSL Expression

The experimental results of Futai [13] show that the value of M is not fixed and it will change with the suction. Toll [14] believes that the critical state of unsaturated soil needs to be determined by five relevant parameters, and the determination of these parameters requires a large number of unsaturated shear tests. The expression of the critical state parameter of unsaturated soil is modified as:

\[ M(s) = M_0 + \frac{\alpha(u_0 - u_{sat})}{1 + b(u_0 - u_{sat})} \]  
\[ \lambda(s) = \lambda(0)[(1 - r)\exp(-\beta s) + r] \]

In the above formula, \( M_0 \) is the slope of the CSL curve undersaturation. \( a \) and \( b \) are regression parameters, which are determined by experiments. When the suction gradually increases from 0, \( M(s) \) will also increase rapidly. As the suction increases, the increase of \( M(s) \) will gradually slow down. When the suction tends to infinity, which means under very dry conditions, \( M(s) \) is infinitely close to a certain value, and at this time \( M = M_0 + \alpha/b \). The projection of the improved CSL critical state line on the \( p-q \) plane is shown in Figure 2.

![Figure 1](image1.png)  
(a) the Alonso model  
(b) the modified Alonso model

**Figure 1.** Projection map of LC yield surface in \( p-s \) plane

![Figure 2](image2.png)  
**Figure 2.** Projection diagram of CSL critical state line on \( p-q \) plane
2.3. SI Yield Surface

According to the triaxial shear test of unsaturated soils, under a fixed confining pressure, when the suction exceeds a certain level, the sample will quickly reach yield, and the yield surface is the SI yield surface. To realize this characteristic in the numerical simulation, the LC yield surface decreases rapidly when S>SI is set in the program so that the corresponding yield effect of the soil can be described. The shape of the yield surface in the p-q-s three-dimensional stress space is shown in Figure 3.

![Figure 3. Morphology of SI yield surface in p-q-s three-dimensional stress space](image)

3. USDFLD Secondary Development

The calling steps of the USDFLD field variable subroutine in ABAQUS are as follows:

In the first step, the ABAQUS main program gives stress, strain, strain increment, saturation, and time increment before the time increment step starts.

The second step is to update the position and size of the yield surface based on the constitutive model of the modified Alonso unsaturated soil, and then these model parameters are modified in time through the field variable subroutine.

The third step is to calculate the elastic trial stress, load or unload through the constitutive integration algorithm to check and update the elastic strain, plastic strain, stress, and consistent tangent stiffness matrix.

The fourth step is to return to the ABAQUS main program to perform balance iteration. If the calculation result converges, the next increment is proceeded. Otherwise, it needs to shorten the time increment step and return to the first step until the calculation result converges.

The subroutine flowchart is shown in Figure 4:
Figure 4. Subroutine flow chart

4. Calculation Example Verification
A USDFLD subroutine based on the modified Alonso unsaturated soil constitutive model was developed. After compilation and debugging, the finite element analysis of the indoor unsaturated soil triaxial test was carried out. Compared with the results of the unsaturated triaxial shear test conducted by Lu Zaihua [7] to test the computing power, accuracy, and efficiency of the USDFLD program.

4.1. Model and Calculation Parameters
The calculation model is established concerning real experiments. It is divided into 820 units. The model is a cylinder with a diameter of 39mm and a height of 80mm. The unit type is a three-dimensional solid unit with complete integration. The three degrees of freedom of the model's bottom surface is constrained. The simulation steps are as follows: in the initial step, the initial consolidation pressure is applied to the model; in the in-suction step, a pore pressure value that increases uniformly with time is added to the top and bottom of the model while keeping the confining pressure constant to simulate the dehumidification process of the soil sample; in the compress step, the confining pressure is set to increase uniformly with time, and fixed pore pressure values are set at the top and bottom of the model to simulate the consolidation process under constant suction; In the shear step, the confining pressure is fixed, the top and bottom pore pressures are fixed, and the compression displacement increment is added to the top to simulate the three-axis shear process under a fixed suction.

The initial calculation parameters of the model are consistent with the actual test. The corrected yield surface parameters are obtained from the triaxial shrinkage test, triaxial expansion test, and shear test done by Lu Zaihua. The parameters are shown in Table 1 and Table 2.
Table 1. Sample initial condition

| test contents | particle density $G_s$ | dry density $\rho_d$ g·cm$^{-3}$ | moisture content $w_0$ % | void ratio $e_0$ | Saturation $S_r$ % |
|---------------|------------------------|-----------------------------|-------------------------|----------------|-------------------|
| shear test    | 2.73                   | 1.50                        | 30.2                    | 0.82           | 100               |

Table 2. Soil sample model parameters

| initial stress state (kPa) | initial hardening parameter (kPa) | CSL parameters |
|----------------------------|----------------------------------|----------------|
| p  | q  | s  | $s_i$ | $s_d$ | $P_0$ | $M_0$ | a  | b  |
| 0  | 0  | 35 | 125   | 25    | 105   | 0.56  | 2.575 | 3.069 |

| LC parameters | SI parameters | Space yield surface parameters |
|---------------|---------------|-------------------------------|
| $k$ | $\lambda(0)$ | $r$ | $U$ | $P_c$ | $k_s$ | $k$ | $G$ |
| 0.03 | 0.08 | 0.85 | 0.013 | 30 | 0.015 | 0.09 | 0.58 | 9801.3 |

4.2. Numerical Simulation of Shear Test

The comparison between the calculated results and the test results under the same suction and different confining pressures is as follows:

![Numerical simulation and experimental data comparison chart](image)

Figure 5. Numerical simulation and experimental data comparison chart

It can be seen from Figure 5 that the stress-strain curves of the soil samples in the twelve cases are all strain hardening. When the suction is the same, the greater the confining pressure, the higher the yield point of the soil sample. According to the comparison, the modified Alonso model can reflect the nonlinearity of the stress-strain relationship of the unsaturated soil sample and its plastic flow characteristics.
Stress-strain diagrams under the same confining pressure and different suction is plotted. Here, only the curve diagram when the confining pressure is 100kPa is given as follows:

![Stress-strain diagram](image)

**Figure 6.** Stress-strain curves of different suction forces when the confining pressure is 100 kPa

It can be seen from Figure 6 that under the influence of suction, the yield point of unsaturated soil is higher than that of saturated soil. The greater the suction, the higher the yield point. This reflects the influence of suction on the yield surface and proves that the model can simulate the influence of unsaturated soil liquid relative to solid phase.

5. Conclusion

Based on the modified ALONSO unsaturated soil elastoplastic constitutive model, combined with the ABAQUS finite element software platform, this paper uses Fortran to compile a suitable USDFLD subroutine and simulates the triaxial shear test of the unsaturated soil on control the suction and confining pressure. The stress-strain curve and the axial strain-volume curve that have been drawn verify the accuracy and reliability of the algorithm. The main relevant conclusions are as follows:

1. When the suction is the same, the larger the confining pressure, the higher the yield point of the soil sample. According to the comparison between experiment and simulation, the modified Alonso model can well reflect the nonlinearity of the stress-strain relationship of the reshaped unsaturated soil sample and its plastic flow characteristics.

2. Compared with saturated soil, unsaturated soil has a higher yield point under the influence of suction. The larger the suction, the higher the yield point, which proves that the model can simulate the influence of unsaturated soil liquid relative to solid phase.

Acknowledgments

This work was financially supported by the national key research and development program (2017YFC1501206), China Postdoctoral Science Foundation (2020M672557).

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