Characterizing coupled MEMS resonators with an electrical resonator

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Abstract. Rapid development in micro/nano fabrication has enabled the shrinking of MEMS devices and the ability to fabricate them in large arrays. However, process variations and device mismatch have also raised testability issues in the MEMS industry. MEMS resonators have been coupled to simplify the characterization of the fabrication process and device performance using their collective behaviour. Perturbation analysis using eigenvalues can therefore be applied to extract the system matrix of coupled resonators. We propose a new way of perturbation analysis by coupling an electrical resonator to an array of MEMS resonators. The electrical resonator is simple in structure and easy to readout. It can also precisely control the amount of perturbation based on two available techniques. Coupling between MEMS resonators and electrical resonator opens a new window for process characterization, device testing, material characterization, as well as large sensors array actuation.

1. Introduction
Process induced variabilities have been widely seen in MEMS fabrication [1, 2], which are caused by environment factor, temperature difference, as well as uncertainty during lithography or etching. Variabilities would degrade sensor performance, increase device mismatch, reduce yield and add cost to characterization. Device characterization has become a growing concern for the MEMS industry as it can consume up to 70% of the manufacturing cost. Hence there is a need for an accurate and inexpensive way to monitor fabrication processes and characterize device properties. One way of enhancing testability is to optimise MEMS design at the beginning considering the presence of variability [1]. This method, however, is prone to device mismatch, even if they are on the same wafer with identical shape and geometry design. Therefore, it is important to regularly monitor these fabricated devices or test structure. Another solution is to test MEMS devices at the wafer level and predict their performance at the package level [2].

MEMS resonators have been used for extensive applications spanning from mass sensing [3] to acceleration sensing [4]. With the continuing reduction in feature size and advancement in micro/micro fabrication techniques, two or even more resonators can be fabricated on the same substrate thereby providing multi degree-of-freedom or multi function sensing. Individual sensors hence need to be fully characterized before packaging. However, for a large array of sensors, testing each element individually would significantly increase connection wires, bond pads, and the bandwidth for signal processing. Coupling these individual resonators emerges as a simple and economical way to record the collective behaviour of all sensors and hence
Figure 1. Simulated responses of 2 coupled resonators with/without process variabilities.

characterize them [5]. Resonators can be coupled intentionally through mechanical string or electrostatic force. They can also be coupled by simply using the inherent parasitic effect [6].

2. System matrix reconstruction using inverse eigenvalue analysis

2.1. System matrix, eigenvalue and eigenvector

An array of \( n \) nearest-neighbour coupled resonators can be modelled by differential equations

\[
\ddot{x} + B_n \dot{x} + S_n x = F, \tag{1}
\]

with \( x \) being the displacement vector, \( F \) the driving force and \( B_n \) the damping matrix. \( S_n \) denotes the system matrix of the array. In the ideal situation without variabilities, all resonators have identical spring constant \( k_s \), mass \( m \) and coupling strength \( k_c \) in between, hence

\[
S_n = \begin{bmatrix}
\frac{(k_s + k_c)}{m} & -\frac{k_c}{m} & \cdots & 0 \\
-\frac{k_c}{m} & \frac{(k_s + 2k_c)}{m} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{(k_s + k_c)}{m}
\end{bmatrix}, \tag{2}
\]

whose eigenvalues \( \mu_i \) represent the eigenfrequencies (i.e. resonance frequencies) \( f_i \) as \( \mu_i = (2\pi f_i)^2 \). The eigenvectors are the amplitude ratio of all resonators at different modes.

Figure 1 simulates 2 coupled resonators, with each having a nominal natural resonance frequency of 15 KHz (\( Q = 10000, 1\% \) weak coupling). In practice, the inevitable process variations and device mismatch would modify \( S_n \) slightly, hence the actual responses are always different, as can be seen from the responses in figure 1 when considering variabilities.

Eigenvector change has been observed in figure 1 due to device mismatch. However, since the coupling strength can hardly be measured, it is impossible to know the amount of change through eigenvectors. Eigenvectors are also very unstable and do not provide frequency information. Furthermore, they require amplitudes measured from every resonator in order to calculate their ratio, hence increasing the complexity and cost of readout.

In contrast, eigenvalues can be measured from the response of any one single resonator due to the collective behaviour of the coupled system. They can be used to extract full information.
of the system matrix after a perturbation has been introduced to the array, and hence fully characterize coupled MEMS resonators. In this paper, we propose two inverse eigenvalue techniques to perform perturbation analysis using an electrical resonator.

2.2. Perturbation analysis
With variabilities taken into account, the system matrix in equation 2 then has 2n − 1 unknown parameters, among which n are the diagonal elements containing spring constant and mass information, whereas n − 1 are off-diagonal elements related to the coupling strength. Measuring one set of n eigenvalues is therefore impossible to extract all 2n − 1 elements. Another set of eigenvalues is hence needed by perturbing the array slightly. There are two techniques to obtain a second set of eigenvalues and reconstruct the system matrix.

The first technique perturbs one diagonal element of the system matrix, by changing the spring constant or mass of the related resonator. This change would give another n eigenfrequencies that could be measured from the response of any one of the resonators, and hence n new eigenvalues \( \mu_i^* \). The eigenvector \( x_{i,n} \) for the nth eigenvalue is calculated by:

\[
x_{i,n}^2 = \frac{\mu_i^* - \mu_i}{s^*} \prod_{j=1,j\neq i}^{n} \frac{\mu_i - \mu_j^*}{\mu_i - \mu_j} \quad i = 1, \cdots, n.
\]

where \( s^* \) is the difference between the traces of the initial and the changed system matrix. The initial \( S_n \) can be extracted from the eigenvalues and eigenvectors using Lanczos’ algorithm [5].

The second technique couples an additional resonator to the array and hence adds a new element to the existing array of resonators. Another set of eigenvalues \( \lambda \) can be measured from such new array. This technique first computes the weights \( \omega_i \) from \( \lambda \) and \( \mu \) [7]:

\[
\omega_i = \frac{\lambda_i - \mu_j}{\lambda_i - \mu_{j+1}} \\
i = 1, \cdots, n + 1.
\]

A set of orthogonal polynomials are then calculated, the first two of which are

\[
p_0(\lambda_i) = 1, \quad p_1(\lambda_i) = \lambda_i - \frac{\sum_{j=1}^{n+1} w_j \lambda_j}{\sum_{j=1}^{n+1} w_j} \\
i = 1, \cdots, n + 1.
\]

Three-term recurrence relationships are then used to derive the diagonal elements \( d_i \) and off-diagonal elements \( o_i \) of \( S_n \), as well as the subsequent orthogonal polynomials \( p_k(\lambda_i) \):

\[
d_{n+2-k} = \frac{\sum_{i=1}^{n+1} \lambda_i w_i p_{k-1}^2(\lambda_i)}{\sum_{i=1}^{n+1} w_i p_{k-1}^2(\lambda_i)}, \quad o_{n+1-k}^2 = \frac{\sum_{i=1}^{n+1} w_i p_{k-2}^2(\lambda_i)}{\sum_{i=1}^{n+1} w_i p_{k-1}^2(\lambda_i)},
\]

\[
p_k(\lambda_i) = (\lambda_i - d_{n+2-k}) p_k(\lambda_i) - o_{n+2-k}^2 p_{k-2}(\lambda_i) \quad k = 2, \cdots, n - 1.
\]

3. Characterization method
3.1. Electrical resonator
Inside the dotted box of figure 2 shows a simple electrical resonator that can be represented by

\[
\ddot{x}_e + \frac{1}{R_2 C} \dot{x}_e + \frac{1}{R_1 R_3 C^2} x_e = \frac{1}{R_3 R_f C^2} F_e,
\]

where \( F_e \) is the driving voltage and \( x_e \) is the output voltage. The centre frequency and quality factor can be easily adjusted. The above equation shares the similar form as equation 1, making it
Figure 2. Coupling an electrical resonator to an array of MEMS resonators.

possible to couple the electrical resonator to MEMS resonators. To electrically couple them, the driving mechanism of the MEMS resonators needs to be voltage controlled, e.g. via electrostatic force or converse piezoelectric effect. Conversely, the sensor readout needs to be converted to voltage signal to feed back to the circuit.

Figure 2 shows an example of coupling an electrical resonator to an array of electrostatically driven MEMS resonators. These resonators are capacitive type with opposing comb structures. The electrical circuit is coupled to the array of MEMS resonators by adding its output voltage signal to electrostatic force to drive one of the coupled MEMS resonators. Conversely, the mechanical output of the same MEMS resonator, in this case a current signal, is converted to a voltage signal to feed back to the circuit. Once coupled to an array of $n$ MEMS resonators, readouts from both the electrical circuit and the mechanical resonators contain $n+1$ resonant peaks. These can be measured from the response of the mechanical resonator or simply from the electrical resonator. The amount of the feed in and feed back signals can be any different values, as long as the resonance peaks are detectable. It can be proven mathematically that unequal feed in/back amount would not affect the diagonal elements of the system matrix [7]. Only the diagonal elements contain useful information.

Perturbation can be carried out using both techniques mentioned in section 2.2. For the first technique, perturbation can be performed by coupling the electrical resonator and changing its centre frequency. For the second technique, the array can be perturbed by connecting/disconnecting the electrical resonator.

3.2. Characterizing an example of ten coupled MEMS resonators

We simulated ten nearest-neighbour coupled MEMS resonators to demonstrate both techniques using an electrical resonator. Each resonator has a nominal natural resonance frequency of 15 KHz and 4% coupling ratio in between. We intentionally modified $S_n$ by adding a Gaussian distributed variability ($\sigma = 5\%$) to the spring constants. The additional electrical resonator to be coupled to such array has a centre frequency of 15 KHz. The feed in and feed back amount for the electrical resonator are intentionally set to be different to simulate real situation.

We designed a series of experiments to characterize such MEMS array using both techniques. First, before coupling the electrical resonator, ten eigenfrequencies were recorded from any one of the ten coupled MEMS resonators. Second, the electrical resonator was coupled to the last element of the ten MEMS resonators, forming a new eleven-element array. Eleven new eigenfrequencies were hence recorded. Third, the centre frequency of the electrical resonator was then changed from 15 to 16 KHz, thereby providing another set of eleven eigenfrequencies.

The acquired data for the above three steps are shown in table 1. From the eigenfrequencies
Table 1. Eigenfrequencies recorded for different modes.

| Mode | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Step 1 | 13231 | 13839 | 14626 | 15139 | 15320 | 15351 | 15844 | 16492 | 16914 | 17871 | –   |
| Step 2 | 13231 | 13839 | 14626 | 14959 | 15139 | 15320 | 15351 | 15844 | 16492 | 16946 | 17875 |
| Step 3 | 13231 | 13839 | 14626 | 15139 | 15320 | 15351 | 15844 | 15928 | 16492 | 16975 | 17877 |

Table 2. Nominal, actual and calculated natural resonance frequencies.

| Resonator | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------|---|---|---|---|---|---|---|---|---|----|
| Nominal   | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 |
| Actual    | 15403 | 16375 | 13306 | 15647 | 15239 | 14019 | 14675 | 15257 | 17684 | 17077 |
| 1st technique | 15403 | 16365 | 13313 | 15652 | 15239 | 14019 | 14675 | 15257 | 17684 | 17077 |
| 2nd technique | 15403 | 16375 | 13307 | 15648 | 15239 | 14019 | 14675 | 15257 | 17684 | 17077 |

obtained through step 1 and step 2, the second technique can be applied, giving a set of natural resonance frequencies of the ten MEMS devices. From the eigenfrequencies obtained through step 2 and step 3, the first technique can be applied, giving a similar set of natural resonance frequencies. These two sets of frequencies are compared with the actual natural resonance frequencies, as shown in table 2, with the standard deviations of frequency errors as low as 0.03% and 0.005% for the first and second technique respectively. These values match well with the actual natural resonance frequencies. The slight difference is caused by numerical errors.

4. Conclusions
A new inverse eigenvalue analysis approach based on an electrical resonator has been revealed to characterize coupled MEMS resonators. With the help of the electrical resonator, perturbation analysis can be easily achieved. We have proposed two different techniques using such electrical resonator, both of which can provide full information about the system matrix. Coupling an additional electrical resonator to the MEMS array can be used for a wide range of applications, including process monitoring, device testing, material characterization, and single-input-single-output sensors actuation. Future work will be dedicated to the fabrication of a large array of coupled resonators to experimentally verify both techniques. Comparison of both techniques in terms of accuracy and sensitivity will also be performed.

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