Spherically Collapsing Matter in AdS, Holography, and Shellons

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Abstract

We investigate the collapse of a spherical shell of matter in an anti-de Sitter space. We search for a holographic description of the collapsing shell in the boundary theory. It turns out that in the boundary theory it is possible to find information about the radial size of the shell. The shell deforms the background spacetime, and the deformed background metric enters into the action of a generic bulk field. As a consequence, the correlators of operators coupling to the bulk field are modified. By studying the analytic structure of the correlators, we find that in the boundary theory there are unstable excitations ("shellons") whose masses are multiples of a scale set by the radius of the shell. We also comment on the relation between black hole formation in the bulk and thermalization in the boundary.

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1 Introduction

One of the many exciting aspects of the AdS/CFT correspondence is that it contains the seeds for solving the black hole information puzzle in a holographic way. So far, the understanding of black holes in the context of the AdS/CFT correspondence is mostly limited to the properties of static solutions. For example, one can give a boundary theory interpretation for the Bekenstein-Hawking entropy, properties of different vacua and their thermodynamic interpretation. However, the static solutions have some rather special features, such as the appearance of multiple asymptotic regions. To address the issue of the fate of information, one would like to study black holes which form from collapsing or colliding matter, and their subsequent evolution to the end. These issues are much less clear as discussed in [20]. In this paper we take a first step towards understanding the collapse of a spherical shell of matter in AdS space.

The first problem to confront is how to account for the presence of a shell of matter in the bulk of an AdS space. In particular, how does one obtain information about the radial size of a shell? For a pointlike probe, the scale-radius duality converts its radial distance in the bulk into a transverse scale of its image at the boundary. However, in the case of a shell, the radial scale is a bit trickier to find because of spherical symmetry. What is the holographic image of a shell at the boundary? Since a pure AdS space corresponds to a vacuum state in the boundary, and the end result of the shell collapse (if it is a stable black hole with positive specific heat) corresponds to a boundary theory at finite temperature, a first guess is that adding a shell to the AdS space corresponds to an off-equilibrium configuration in the boundary theory, which then evolves towards thermal equilibrium as the shell collapses to a black hole in the bulk (see e.g. [12, 25] for discussion). Further, the scale-radius duality of the AdS/CFT would suggest that the initial configuration is somehow peaked at higher energies than the temperature of the black hole. In other words, one might hope to associate an initial average occupation number spectrum with the initial configuration, with a peak at energies corresponding to the initial radius of the shell, and study how the time evolution rearranges the spectrum to a thermal one, with the peak moving down in energies to stabilize at the Hawking temperature.

To test the above idea, one would need some way of detecting or computing e.g. the average occupation number spectrum in the boundary theory. One would then need to study operators which have access to the excited states. For example,

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4 However, see [10].
5 Black hole creation in AdS$_3$ has been studied in [17, 18, 19]. In these cases the resulting black holes are stable.
the shell deforms the background spacetime, and the deformation couples to the stress-energy tensor of the boundary theory, giving it an expectation value. The expectation value gives information only about the total energy, not about the details of the distribution. However, one could study higher n-point correlators of the stress tensor; these involve more detailed information about the spectrum which one can then try to extract out\footnote{In general, higher n-point correlation functions are needed to have more complete information about a quantum state. For example, in \cite{20} it was discussed how higher n-point functions are relevant for understanding the causal structure in AdS/CFT.}.

Another, more direct, way to detect the shell would be to assemble it from matter associated with a bulk field coupling to an operator $O_{\text{shell}}$ in the boundary. Then, the n-point functions of $O_{\text{shell}}$ would provide information of the shell itself, without a detour to its backreaction to the metric. However, this route obviously depends on what the shell is made of and how.

In this paper, we shall follow a third possible strategy. Since the shell deforms the background spacetime, the action for any bulk field in the deformed background is modified as well. Since the bulk action is the generating function for the correlation functions of the corresponding operators in the boundary theory, the correlation functions will reflect the presence of the shell. Even if the expectation values of the operators (the one point functions) vanish, the 2-point functions can be different from those evaluated in the vacuum. For example, in the end of the collapse, when the boundary theory thermalizes, a two-point function will become periodic in imaginary time, even if the operator has a zero expectation value in the thermal background. So our strategy is that we shall not try to directly probe the states associated to the shell, but we investigate their effect on other operators in the boundary theory, and use their correlation functions to obtain information about the shell. A related possibility would be to study the properties of a hanging string \cite{26, 27, 28, 29}. In the case of a black hole the boundary theory will be at finite temperature, and the force between two quarks will be screened at sufficiently large separation. In the picture with the quarks corresponding to endpoints of a hanging string, screening occurs when the string breaks and one obtains a configuration with a string from each of the quarks going straight down through the horizon of the black hole. With a shell the situation is different since the boundary state is not thermal and screening is not expected. From the bulk point of view there is no horizon and the string between the quarks can therefore not break. Even though the distance dependence of the force will be affected by the presence of the shell, there is still a long range contribution.

The simplest two-point function is given by a bulk scalar field $\phi$ coupling to an operator $O$ in the boundary theory. For example, $\phi$ could be the five dimensional
dilaton in AdS$_5$, and $\mathcal{O}$ the operator $\text{Tr} F^2$ in the 3+1 dimensional $N = 4$ SYM theory. Also, we will concentrate on the simplest possible situation. We consider a spherically symmetric thin shell of matter, and focus on the initial stage of the collapse when the shell is moving very slowly. In that case, we can use a quasistatic approximation and take the shell radius to be fixed. We solve the field equation of $\phi$ on both sides of the shell, impose appropriate boundary conditions, and join the solutions across the shell. We will then derive the two point function

$$\langle \mathcal{O}(t, \vec{x}) \mathcal{O}(t', \vec{x}') \rangle$$

for the corresponding boundary operator $\mathcal{O}$. Actually, our calculation will give the momentum space Fourier transform of the two-point function,

$$G(\omega, \vec{k}) = \int dt \int d^d \vec{x} \ e^{-i\omega t + i\vec{k} \cdot \vec{x}} \langle \mathcal{O}(t, \vec{x}) \mathcal{O}(0, 0) \rangle .$$

The analytic structure of the propagator gives information on the spectrum of excitations of the theory. In the presence of the shell, the two point function will be identified as the retarded propagator $G_{\text{ret}}(\omega, \vec{k})$. For a fixed value of $\vec{k}$, the retarded propagator is analytic in the upper complex $\omega$ plane. It can be written in an integral representation form

$$G_{\text{ret}}(\omega, \vec{k}) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} A(\omega', \vec{k}) \frac{\omega - \omega' + i\epsilon}{\omega - \omega' + i\epsilon} ,$$

where $A(\omega, \vec{k})$ is the spectral density function. It is the imaginary part of the retarded propagator,

$$A(\omega, \vec{k}) = -2 \cdot \text{Im} G_{\text{ret}}(\omega, \vec{k}) .$$

(See e.g. [30] for discussion.) The spectral density function can be interpreted as the probability that an excitation created by the operator $\mathcal{O}$ has an energy $\omega$ and a momentum $\vec{k}$. Let us now recall some possible analytic features of the retarded propagator in a generic QFT. If we try to do an analytic continuation to the lower half-plane, and take the limit $\epsilon \to 0$, we may find that the propagator has poles at the real $\omega$ axis. In the boundary theory, these correspond to elementary excitations or stable bound states (the latter being possible in an interacting theory). The poles of the propagator correspond to delta function peaks of the spectral density function. There may also be cuts, signaling multiparticle states. We may also find poles of the propagator in the lower complex plane, corresponding to unstable bound states.

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7Note that in the case of the static shell, the two-point function will depend only on the differences ($t - t'$) and ($\vec{x} - \vec{x}'$).

8For bosonic excitations, one needs $\omega > 0$.

9Of course, this issue played an important part in studies of absorption by branes in the path which led to the AdS/CFT correspondence. See [1] for more discussion and references.
(resonances). If we denote the location of such a pole by $\omega_i(\vec{k}) - i\Gamma_i$, the imaginary value $\Gamma_i$ is the width of the unstable bound state. The closer to the real axis the pole is, the longer lived the resonance is. In the spectral density function, the resonances appear smeared peaks with finite width. (See e.g. [32, 30] for discussion).

Thus, the analytic structure of the propagator gives information on the stable and unstable excitations present in the boundary theory. In general, the operators $\mathcal{O}$ are composite operators built out of the elementary fields of the boundary theory. Hence, we obtain only information about a certain subset of particles propagating in the boundary, the stable and unstable composite objects that can be created and annihilated by the operator $\mathcal{O}$.

The key fact is that this gives us enough information to detect the radial size of the shell. We discover unstable resonances in the boundary theory, whose masses are multiples of a mass scale set by the radial size of the shell. The boundary theory is conformal, but it is in an off-equilibrium state characterized by a dimensional parameter, so the massive resonances can exist.

The resonances are discussed in Section 2. In section 3 we discuss the results.

2 Propagators in the shell background

We shall now proceed, step by step, to compute the two point function of a boundary operator in the case that a (quasi)static shell of matter is present in the bulk. Although this is an unphysical situation, it can be used as an approximate description of a slowly moving shell. In [4, 3], it was described how to compute boundary correlators of an operator $\mathcal{O}(x, t)$ using classical solutions for a related bulk field $\phi(r, x, t)$ using the AdS/CFT correspondence. The computation method we will use is a variant of the one described in the appendix of [33].

In all cases, the spacetime is asymptotically $AdS_{d+1}$ ($d = 2, 4$), and near the boundary the bulk metric reduces to the approximate form

$$ds^2 \approx_{r \to \infty} - \frac{r^2}{R^2} dt^2 + \frac{R^2}{r^2} dr^2 + \frac{r^2}{R^2} d\vec{x}^2$$

(4)

where the boundary is at $r \to \infty$. Let $\phi$ be a scalar field in the bulk, with a mass $m$. In Minkowski signature, the classical equation of motion

$$\frac{1}{\sqrt{g}} \partial_\mu \left( \sqrt{g} g^{\mu\nu} \partial_\nu \phi(r, \vec{x}, t) \right) - m^2 \phi(r, \vec{x}, t) = 0$$

(5)

has two kinds of solutions: normalizable and non-normalizable ones. We denote the former by $\phi^{(+)}(r, \vec{x}, t)$ and the latter by $\phi^{(-)}(r, \vec{x}, t)$; they have an asymptotic behavior

$$\phi^{(\pm)}(r, \vec{x}, t) \to r^{-2h_{\pm}} \quad (r \to \infty)$$

(6)
where
\[ h_{\pm} = \frac{1}{4}(d \pm \sqrt{d^2 + 4m^2}) \equiv \frac{d}{4} \pm \frac{\nu}{2}. \] (7)
The starting point is to impose appropriate boundary conditions for the bulk field \( \phi \) at the interior of the AdS space. In the asymptotic region, the field will in general become a superposition of the normalizable and non-normalizable solutions, with an asymptotic behavior
\[ \phi(r, \vec{x}, t) \approx r^{-2h_+}\phi_+(t, \vec{x}) + r^{-2h_-}\phi_-(t, \vec{x}). \] (8)
In the above, \( \phi_-(t, x) \) is the boundary data which acts as a source for an operator \( O(t, x) \) in the boundary theory. Next, we Fourier transform the variables \( t, x \) and write the equation (8) in the form
\[ \phi(r, \omega, \vec{k}) \approx (r^{-2h_-} + r^{-2h_+}G(\omega, \vec{k}))\phi_-(\omega, \vec{k}), \] (9)
where
\[ G(\omega, \vec{k}) = \frac{\phi_+(\omega, \vec{k})}{\phi_-(\omega, \vec{k})}. \] (10)
One can check that the two point function in the boundary will be
\[ \langle O(\omega, \vec{k})O(\omega', \vec{k}') \rangle = (h_+ - h_-)\delta(\omega + \omega')\delta(k + \vec{k}')G(\omega, \vec{k}). \] (11)
Thus, the essential steps of the computation are as follows: one imposes interior boundary conditions for the bulk field, writes it as a superposition of the normalizable and non-normalizable modes, then the ratio of the superposition coefficients gives the two point function in the boundary. To obtain the answer with a correct overall coefficient \( (h_+ - h_-) \), one needs to perform the calculations with more subtlety, as discussed in [33]. However, overall coefficients will not be important here, so they will be suppressed in what follows. The field \( \phi(r, \vec{x}, t) \) must also obey time boundary conditions related to the fact that one can compute, for example, retarded, advanced or Feynmann propagators. In the absence of interactions, and with respect to a vacuum, all propagators in momentum space are equal. Otherwise, the propagators can be identified by examining their analytic structure.

As a warm-up, we consider the propagator in the absence of a shell in the bulk. The full AdS\(_{d+1}\) manifold is covered by global coordinates, with the metric
\[ ds^2 = -\frac{R^2}{r^2}dt^2 + \frac{R^2}{r^2}dr^2 + \frac{\vec{r}^2}{R^2}d\Omega^2_{d-1} \] (12)
where the radial coordinate \( r \) extends from 0 to \( \infty \) and \( d\Omega^2_{d-1} \) is the metric of a \((d-1)\)-sphere of radius \( R \). The interior boundary condition is that \( \phi \) must be
regular at the origin $r = 0$. The solution is expressed as a linear superposition of normalizable and nonnormalizable modes in Ref. [11]. The propagator (13) is simply the ratio of the relative coefficients, which are found in equation (37) of Ref. [11].

The answer is

$$G(\omega, k) = \frac{\Gamma(-\nu) \Gamma(h_+ + \frac{R}{2}(\omega + k)) \Gamma(h_+ + \frac{R}{2}(-\omega + k))}{\Gamma(\nu) \Gamma(h_- + \frac{R}{2}(\omega + k)) \Gamma(h_- + \frac{R}{2}(-\omega + k))},$$

(13)

where $k = \text{integer}/R$. The above result is valid when $\nu$ is not an integer. If $\nu$ is an integer then the propagator can be written in terms of derivatives of $\Gamma$ functions.

In this case it is easier to compute first the spectral function part and then use the integral representation (2) to reconstruct the full propagator. It is easy to see that the propagator (13) has poles at $\omega \pm k = \mp(2n + h_+)^{\frac{1}{R}}$, $n = 1, 2, \ldots$ corresponding to the normalizable modes in AdS space. From the boundary point of view the discrete spectrum is possible since the spatial directions are compactified to a sphere $S^{d-2}$. Accordingly, note that the dispersion relation is not Lorentz invariant. Between successive poles, the propagator (13) has also zeroes at $\omega \pm k = \mp(2n + h_-)^{\frac{1}{R}}$.

In the limit $R \to \infty$, keeping $\omega$ and $k$ fixed, the poles and zeros accumulate and they degenerate to cuts in the propagator for $\omega > k$ and $\omega < -k$. Further, in the limit $R \to \infty$ the boundary decompactifies. Applying the limit to (13), we find that the propagator reduces to the form

$$G(\omega, k) \sim (\omega^2 - k^2)^\nu \quad (\omega^2 \gg k^2).$$

(14)

Thus, we recover the expected result for a free propagator in an infinite boundary. From the bulk point of view one has to take $R \to \infty$ with $r/R^2$ fixed or equivalently $r \to \infty$. In this limit the metric reduces to (4). Using this metric the propagator (14) follows immediately. The only subtlety is the boundary condition to use in the interior since in the large $R$ limit, the point $r = 0$ corresponds to the coordinate horizon of a Poincare patch. Near the horizon any solution can be written as a linear combination of an ingoing and an outgoing wave. If $\omega$ has a small imaginary part it is easily seen (see Appendix) that one has to use an ingoing wave for $\text{Im}(\omega) > 0$ and an outgoing one for $\text{Im}(\omega) < 0$. The use of different boundary conditions for $\text{Im}(\omega) > 0$ and $\text{Im}(\omega) < 0$ produces a cut on the real axis corresponding to switch from the retarded to the advanced one.

Let us now turn to the case of a black hole of radius $r_+ > R$ in AdS space. Now we must impose a boundary condition at the horizon. Again, both solutions of the wave equation are regular at the horizon. The general solution is a superposition of an outgoing and an ingoing wave:

$$\phi \approx A(r - r_+)^{i\frac{\omega}{R}} + B(r - r_+)^{-i\frac{\omega}{R}} \quad (r \to r_+).$$

(15)
Imposing the condition that $\phi$ be regular at $r = r_+$ implies that if $\text{Im}\omega > 0$ we take the solution with $A = 0$ and with $B = 0$ if $\text{Im}\omega < 0$. Since the solutions with $A = 0$ and with $B = 0$ are one complex conjugate of the other, the imaginary part of the propagator changes sign when $\omega$ crosses the real axis. In the case of a BTZ black hole of radius $r_+$, the propagator (10) follows from \cite{13, 36, 37, 38, 39, 40} and is

\[
G_{\text{ret}}(\omega, k) = \lim_{\epsilon \to 0} G(\omega + i\epsilon, k) = \frac{\Gamma(1 - \nu) \Gamma(h_+ - i\frac{\nu}{2}(\omega + k)) \Gamma(h_+ - i\frac{\nu}{2}(\omega - k))}{\Gamma(1 + \nu) \Gamma(h_- - i\frac{\nu}{2}(\omega + k)) \Gamma(h_- - i\frac{\nu}{2}(\omega - k))}
\]

(16)

As before, if $\nu$ is integer then the calculation must be done more carefully. For example, for $\nu = 1$ the result is

\[
G(\omega, k) = \frac{(\omega^2 - k^2)}{T^2} \frac{1}{e^{\frac{\omega}{2T} - 1}} \frac{1}{e^{\frac{k}{2T} - 1}}
\]

(17)

This propagator can be understood as a thermal propagator of a composite operator. Acting on a vacuum, the operator $\mathcal{O}$ creates states with right- and leftmoving particles of two-momentum $k_1 = (\omega_1, \omega_1)$ and $k_2 = (\omega_2, -\omega_2)$. These two-particle states have total energy $\omega = \omega_1 + \omega_2$ and total momentum $k = \omega_1 - \omega_2$. With this notation the propagator reads

\[
G(\omega, k) = \frac{1}{T^2} \frac{\omega_1}{e^{\frac{\omega_1}{2T} - 1}} \frac{\omega_2}{e^{\frac{\omega_2}{2T} - 1}}
\]

(18)

Now it is obvious that this is a thermal propagator for a composite operator propagating two particle states. The Bose occupation numbers arise from the gamma functions \cite{40}. In fact this propagator follows simply from conformal invariance, so it does not give us any information about interactions in the boundary theory, only that it is at temperature $T \sim r_+/R^2$. A similar calculation can be done for the black hole in $AdS_5$ using the function obtained in \cite{14}. The result is similar to the case of $AdS_3$ although it cannot be expressed in terms of special functions as in (16).

After this lengthy introduction, we are prepared to compute the propagator in the presence of a shell of radius $r_s$. In the interior of the shell, the metric is the AdS metric in global coordinates, given by (12). The exterior metric is the AdS black hole metric,

\[
ds^2 = -(1 - \frac{\mu}{r^{d-2}} + \frac{r^2}{R^2})dt^2 + \frac{dr^2}{(1 - \frac{\mu}{r^{d-2}} + \frac{r^2}{R^2})} + \frac{r^2}{R^2}d\Omega_{d-1}^2,
\]

(19)

where the parameter $\mu$ is related to the ADM mass of the black hole and determines the horizon radius $r_+$. The interior boundary condition for the field $\phi$ is the same.

\footnote{For coordinate space expressions including the correct overall coefficient, see also \cite{34, 35}.}
as in pure AdS space: regularity at $r = 0$. The equation (3) reduces to:

$$\frac{1}{r^{d-1}} \partial_r \left( r^{d-1} f(r) \partial_r \phi(r, \omega, k) \right) + \left( \frac{\omega^2}{f(r)} - \frac{k^2}{r^2} - m^2 \right) \phi(r, \omega, k) = 0$$

with

$$f(r) = \begin{cases} f_1(r) = 1 + \frac{r^2}{2} & \text{if } r < r_s \\ f_2(r) = 1 - \frac{\mu}{r_{d-2}} + \frac{r^2}{2R^2} & \text{if } r > r_s \end{cases}$$

and $\frac{k^2}{R^2} = l(l + d - 2), l \in Z_{\geq 0}$ is the eigenvalue of the laplacian on the sphere $S^{d-1}$. The solutions of the field equation, $\phi_1$ for $r < r_s$ and $\phi_2$ for $r > r_s$, are the known solutions in the global and black hole coordinates. The matching conditions at $r = r_s$ are

$$\phi_1|_{r=r_s} = \phi_2|_{r=r_s}$$

$$f_1(r) \partial_r \phi_1|_{r=r_s} = f_2(r) \partial_r \phi_2|_{r=r_s}$$

where the last condition follows from integrating the equation between $r_s - \epsilon$ and $r_s + \epsilon$ with $\epsilon \to 0$. It also ensures current conservation. Using the matching conditions the propagator follows as

$$G(\omega, k) = \frac{f_2 \phi_1 \partial_r \phi_2^{(-)} - f_1 \partial_r \phi_1 \phi_2^{(-)}}{f_2 \phi_1 \partial_r \phi_2^{(+)} - f_1 \partial_r \phi_1 \phi_2^{(+)}}$$

where $\phi_2^{(\pm)}$ are the normalizable and non-normalizable modes in the black hole background (in three dimensions, they can be found in \[13\]), and $\phi_1$ is the mode in global coordinates that is regular at $r = 0$. We have performed this calculation, and have found a tower of resonances in the boundary theory. However, it will be much simpler and more illustrative to discuss a computation which uses some additional approximation methods, which are good for a large shell with radius $r_s \gg r_+ \gg R$.

In this limit, the metric in the interior reduces to the Poincare metric, so the solution in the interior will be a Poincare mode. Correspondingly, the interior boundary condition will be mapped to those in a Poincare patch, as we discussed before in the context of the large $R$ limit. We will relegate the details in the Appendix, and quote just the end result. By studying the propagator in $d$ dimensions, we find that it has an infinite number of poles at the complex values

$$\omega_n \approx \frac{\pi r_s}{R^2} \left( n + \frac{3}{4} + \frac{\nu}{2} \right) - \frac{ir_s}{2R^2} \ln \left( \frac{4\pi n}{(d-1) r_+^d} r_s^d \right)$$

As we discussed in the Introduction, these poles are interpreted as unstable excitations (resonances) in the boundary theory that appear due to the shell background. The real part (the mass) is proportional to the radius of the shell. We call the
unstable excitations as ”shellons”. Note also that the width is also of the order of
the shell radius, hence the shellons have a short lifetime. This is good, since then a
slowly moving shell is static compared to the lifetime of the shellons, and the results
are consistent with the quasistatic approximation.

3 Discussion

Our calculation shows the existence of unstable excitations, shellons, with properties
like mass and lifetime depending on the radius of the shell. Thus, in particular, one
can detect a difference between spherical shells of different radii but with the same
mass. It would be very interesting to have a more detailed understanding of how
the shellons and their properties are to be understood from the boundary point of
view.

Another important problem is to let the shell fall freely and follow its evolution
into a black hole. Let us make some comments on what to expect, but first we need
a more detailed understanding of the composition of the shell from the boundary
point of view. We begin with a static shell consisting of a large number of elementary
particles. A single particle with mass of the order $1/R$ at rest at radius $r_s$, will
through holography show up as a blob in the boundary with size $R^2/r_s$, and total
energy given by $E \sim r_s/R^2$. The shell will therefore be a superposition of such blobs,
and correspond to a state that is out of thermal equilibrium with a homogeneous
energy density. If we turn to the falling shell, each blob will turn into a bubble
expanding at the speed of light [14]. and a falling dust shell containing several
particles will then correspond to a superposition of such expanding bubbles. How
will they interact?

To understand this, it is useful to consider first a particle falling into a black
hole. In [14] the shape of a bubble produced by a particle falling into a BTZ black
hole was given. If one considers the energy momentum tensor as in [25] then the
shape is given by [14]:

$$T_{00} = \frac{1}{(a^2 + (1 + a^2) \sinh^2 \frac{r_+(x+t)}{2})^2} + \frac{1}{(a^2 + (1 + a^2) \sinh^2 \frac{r_-(x-t)}{2})^2} \quad (26)$$

where $a$ is a parameter related to the initial position of the particle and $r_+$ is the
radius of the black hole. We see then that in this case the bubble still propagates at
the speed of light. However, for fixed $t$, the energy density decays exponentially for

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11This follows simply by performing the conformal transformation that corresponds to change
from global to BTZ coordinates in the bulk
large $x$ with a characteristic length of $1/T$ where $T \sim r_+/R^2$ is the temperature of the black hole, while if no thermal bath is present, it decays with a power law. If the particles consist of supergravity modes, their mass is of order $\sim 1/R$ while that of a black hole is $\sim N^2/R$ and so we need order $N^2$ of them. In the large $N$ limit the gas of bubbles in the boundary will be very dense and we expect that each bubble will see a mean field created by the rest. Each bubble will then expand similarly as in eq. (20). Alternatively, we can have a dilute gas of bubbles in the boundary if the shell consists of $k$ dust particles, each containing of the order $N^2$ elementary particles of mass $1/R$. This could be thought of as a shell consisting of $k$ black holes (of course in this case it is not a thin shell). In the boundary, one now finds a dilute gas of $k$ expanding bubbles. Even when the falling dust approaches $r_+$, it is easy to see that the bubbles still remain dilute, unless the boundary equivalent of the bulk gravitational interactions are taken into account. The interactions will, as the final black hole forms, merge all the bubbles into a homogeneous background. Another way to understand this is to note that the holographic scale/radius relation is changed when the deviation from a pure AdS metric becomes important, making the bubbles even larger. Does this mean that the $k$ small black holes have merged into one big black hole? In this context it is important to know the strength of the interactions. The interactions between the black holes that make up the shell are not suppressed by powers of $N$, and are therefore part of the supergravity description. In units where $R = 1$, the Newton’s constant is $G \sim 1/N^2$, but since the mass of each of the black holes that make up the shell is $M \sim N^2$, the corrections to the metric around one of the black holes are $GM \sim 1$. However, we do not expect these collective interactions to result in a final thermal state, since this will require interactions between the individual particles that make up the black holes and the shell. An interaction between any two elementary particles will be suppressed by powers of $N$, and therefore thermalization is expected to be a $1/N$ effect. We expect this to be a general result, not depending on the detailed composition of the collapsing shell. It is intriguing that similar results have been obtained previously in studies of thermalization in large $N$ field theories in a different context [11, 12, 13]. In fact the collapse of the falling shell gives interesting information about the non-equilibrium processes they have studied. However a precise relation is certainly lacking.

While it is satisfying that we can find information in the boundary theory about a collapsing shell in the bulk, it is also obvious that this is just a step towards understanding the formation of black holes as a hologram. One of the steps to follow would be to go beyond the quasistatic approximation and study what happens as the shell is falling rapidly, or what happens when it crosses its Schwarzschild radius.
It will also be interesting to study what happens when the resulting black hole is small and has a negative specific heat. Another interesting issue would be to add charge to the collapsing matter, to make contact with the recently discussed charged AdS black hole solutions \[44, 45\].

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**Appendix**

We are interested in finding the solutions in the region near the radius of the shell, where we match the interior solution with the exterior solutions. If the shell is large, this region is at \(r \gg R\), where the interior metric takes the approximate form (11). Correspondingly, the interior solutions are equal to the Poincare modes, which can be expressed using Bessel functions, as discussed e.g. in [11]. The situation is now analogous to the large \(R\) limit which we discussed in section 2. The continuum of Poincare modes will correspond to a cut in the propagator. Correspondingly, we have a choice of two interior boundary conditions: the solution must look like an ingoing or outgoing wave as \(r \to 0\). Which one is appropriate follows by taking the unique solution in global coordinates and obtaining its behaviour for large frequencies. Alternatively one can reason as follows: an ingoing wave behaves as

\[
\phi_1(r) \approx r^{1-d/2} e^{i \sqrt{\omega^2 - k^2} r}, \quad (r \to 0)
\]

where we set \(R = 1\) as in the rest of the Appendix. If \(\omega\) has a small positive imaginary part the function is well behaved for \(r \to 0\) but it blows up if the imaginary part is negative. So an ingoing wave is appropriate for the region above the cut, and an outgoing wave for the region below. We choose the former condition, so in the end we will obtain a retarded propagator. We denote the interior solution by \(\phi_1\), as in section 2.

Outside the shell the solution to equation (20) cannot be expressed in terms of known functions (except for the BTZ black hole). so we use a WKB approximation. Again we consider the region \(r \gg r_+ \gg R\). To start with, we rewrite the equation
in an alternative form. First, we rescale the field as

\[ \phi(r, \omega, k) = \frac{1}{\sqrt{r^{d-1}f(r)}} \chi(r, \omega, k) \]  

(28)

where \( f(r) = f_2(r) = 1 + r^2 - \mu/r^{d-2} \). Then, the equation for the function \( \chi \) can be written in a form which resembles a Schrödinger equation

\[ -\partial_{rr} \chi^{(\pm)} + V(r) \chi^{(\pm)} = 0 \]  

(29)

with a potential

\[ V(r) = \frac{(d-1)(d-3)}{4r^2} + \frac{\partial_r^2 f(r)}{2f(r)} - \frac{1}{4} \left( \frac{\partial_r f(r)}{f(r)} \right)^2 + \frac{1}{2} \frac{(d-1)}{r} \frac{\partial_r f(r)}{f(r)} \]

\[ -\frac{\omega^2 - (k^2 / r^2 + m^2)f(r)}{f^2(r)} \]  

(30)

with \( f(r) \) as defined above.

Outside the shell, we want to find the normalizable and non-normalizable solutions \( \chi^{\pm} \), which satisfy the boundary conditions

\[ \chi^{(\pm)} \approx r^{\frac{1}{2} \mp \nu}, \quad (r \to \infty) \]  

(31)

Now \( r \gg r_+ \), and the potential reduces to

\[ V(r) = \frac{\nu^2 - 1}{r^2} - \frac{\omega^2 - k^2}{r^4}, \]  

(32)

where \( \nu = \sqrt{d^2/4 + m^2} \) and \( k^2 = l(l + d - 2) \) (where \( l \) is a nonnegative integer). In this case the equation can be solved in terms of Bessel functions, and we find the normalizable and non-normalizable modes to be

\[ \chi^{(\pm)}(r, \omega, k) \approx 2^\pm \nu \Gamma(1 \pm \nu)(\omega^2 - k^2)^{\frac{1}{2} \mp \frac{d-2}{2}} \sqrt{r} J_{\pm \nu} \left( \frac{\sqrt{\omega^2 - k^2}}{r} \right) \]  

(33)

However, in the region \( (d^2 - 1)r^2/4 \ll \omega^2 - k^2 \), namely for large frequencies, better approximate solutions are obtained using the WKB expression

\[ \chi = \frac{1}{V^{1/4}} \left( A \exp(i \int \sqrt{\tilde{V}(r)} \, dr) + B \exp(-i \int \sqrt{\tilde{V}(r)} \, dr) \right) \]  

(34)

with

\[ \tilde{V}(r) = \frac{\omega^2}{(r^d - r_+^d)^2} - \frac{k^2 r^{d-4}}{r^d - r_+^d}. \]  

(35)
Matching the WKB functions with the Bessel functions, the following expressions are obtained for the normalizable and nonnormalizable modes:

\[
\chi^{(\pm)}(r, \omega, k) \approx \sqrt{\frac{2}{\pi (V(r))^{1/4}}} 2^{\pm \nu} \Gamma(1 \pm \nu)(\omega^2 - k^2)^{\mp \frac{\nu}{2}} \times \\
\times \cos \left[ \sqrt{\frac{\omega^2 - k^2}{r}} \left( 1 + \frac{r^d_+ 2 \omega^2 - k^2}{r^d d(\omega^2 - k^2)} \right) \mp \frac{\pi}{2} \nu - \frac{\pi}{4} \right]
\]

in the region \((d - 1)r^2/4 \ll \omega^2 - k^2\) and \(r \gg r_+\). These expressions will be used to find the propagator using (24).

Thus, for large frequencies, we can use the approximate solutions (36) outside the shell. In the large frequency limit, the Poincare mode \(\phi_1\), discussed above, reduces to the form (we can focus on \(k = 0\))

\[
\phi_1 \sim r^{\frac{1-d}{2}} e^{i\frac{\pi}{2}}
\]

up to some constants which cancel in the propagator. Inserting the above expressions to the formula (24), we obtain the propagator (at \(k = 0\))

\[
G(\omega, k = 0) \approx 2^{-2\nu} \frac{\Gamma(1 + \nu)}{\Gamma(1 - \nu)} \omega^{2\nu} \frac{\alpha e^{-i\xi(-)} e^{-i\frac{\pi}{2}} + \beta e^{i\xi(-)} e^{i\frac{\pi}{2}}}{\alpha e^{-i\xi(+)} e^{i\frac{\pi}{2}} + \beta e^{i\xi(+)} e^{-i\frac{\pi}{2}}}
\]

with

\[
\xi^{(\mp)} = \frac{\omega}{r_s} \left( 1 + \frac{r^d_+}{(d + 1)r^d_s} \right) + \frac{\pi}{4}
\]

\[
\alpha = \frac{(1 - d) r^d_+}{2 r^d_s} - \frac{2i \omega}{r_s}
\]

\[
\beta = \frac{(1 - d) r^d_+}{2 r^d_s}.
\]

A useful check is that for \(r_+ = 0\) the last fraction in the propagator is a constant, so the propagator correctly reduces to the expected form in pure AdS. The propagator has poles when the denominator vanishes. For \(r_s \gg r_+\) it follows that there are poles at the values of \(\omega\) which satisfy

\[
\frac{\omega}{r_s} = \frac{3}{4} \pi + \frac{\nu}{2} - \frac{i}{2} \ln \left( 1 + \frac{4i \omega}{(d - 1)r_s r^d_+} \right).
\]

In the leading approximation, we drop the “1” inside the brackets in the last term. We obtain

\[
\omega_n \approx \frac{\pi r_s}{R^2} \left( n + \frac{3}{4} + \frac{\nu}{2} \right) - \frac{ir_s}{2R^2} \ln \left( \frac{4\pi n}{(d - 1) r^d_+} \right),
\]
where $n$ is a (large) positive integer. Notice that the imaginary part is negative since $r_s > r_+$. Finally, we would like to emphasize we have also performed a calculation using the exact solutions in global and black hole coordinates, with regularity at $r = 0$ as the interior boundary condition. In that case, the resonances are found by examining the imaginary part of the propagator, the spectral function. They are seen in the envelope that modulates the amplitudes of the spikes associated with the elementary excitations (the poles at the real values of $\omega$). The envelope has periods of hills and valleys, and by examining the position and the width of the hill one can deduce the position of the complex poles. In this case the calculations were performed numerically. In this way one obtains a better approximation for the location of the poles, but the qualitative results remain the same. The approximations used in the above give accurate enough results to reach the main conclusions in the case of large shells.

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