An interpretation of the scalar $S(151)$ decaying into two photons seen at LHC

François Richard$^1$

Université Paris-Saclay, CNRS/IN2P3, IJCLab$^2$, 91405 Orsay, France

Ongoing work presented at the ILC Workshop on Potential Experiments (ILCX2021), October 2021

Abstract: Following an investigation of several indications for new physics from LHC data, an extended version of the Georgi-Machacek (GM) model was elaborated in a previous publication. This framework is used here to interpret the findings of Crivellin et al., which was obtained by combining the 2 photon mass spectrum from ATLAS and CMS, adding extra conditions, the most efficient one being the presence of >90 GeV missing transverse energy. The claim is a ~5 s.d. excess at 151 GeV. The GM model naturally explains this as coming from the cascade $A(400) \rightarrow H(151)Z$, where $H(151)$ is the missing isosinglet of this model and where the accompanying signal comes from the decays of the $Z$ boson, in particular in neutrino pairs. Consequences of these findings are explored for what concerns the GM Higgs potential and eventual future findings at HL-LHC. A spectacular occurrence of signals is predicted at an $e^+e^-$ collider operating up to 1 TeV, an energy sufficient to cover the whole scenario.

Introduction

In my interpretation of the various LHC indications [1] within the Georgi Machacek model, GM, there was an essential missing element: an isosinglet $H$, partner of $h(125)$, as predicted within the Georgi-Machacek model (GM).

Since then, a new scalar has been convincingly observed with a mass of 151 GeV in its two photons decay mode [2]. This signal was not apparent in the genuine spectra but was obtained by requiring

---

$^1$ richard@lal.in2p3.fr

$^2$ Laboratoire de Physique des 2 Infinis Irène Joliot-Curie
additional features like transverse missing energy, presence of two additional jets, leptons, b jets. This analysis combines ATLAS and CMS data. This is a “premiere” since this work, performed outside the two official collaborations, includes completely the integrated luminosities recorded at 13 TeV.

This search originates from a detailed analysis of LHC data [3], which revealed various topological excesses comprising same sign leptons, leptons + b jets etc... These are interpreted as coming from cascades in which a heavy scalar decays into a lighter one with a mass of order 150 GeV. This constitutes a strong point for this result, which does not suffer from the “look elsewhere” syndrome and can therefore claim a global evidence, at the 4.8 s.d. level, the highest so far observed at LHC, excluding the SM h(125). Figure 1 summarizes this result where one also observes an additional ~2 s.d. contribution from the Z\gamma channel.

![Figure 1: Combined local p-value as a function of the scalar mass from [2]](image)

The question to which the present note attempts to answer is twofold:

- Is this scalar the missing isosinglet H predicted by GM?
- Is it produced through the cascade predicted by this model A(400)\rightarrow HZ?

Recall that in [1] the main conclusion was that there are two heavy scalar candidates. One, a CP-even, H5(660), identified in the ZZ mode into four leptons, observed in the VBF mode. Another one, a CP-odd, A(400), identified in top pairs, tau pairs and Zh(125), the two latter being accompanied by b-jets. The Zh mode, which justifies the CP-odd interpretation, is most clearly seen by requesting an additional b-jet. A natural expectation is the process A\rightarrow Zh(151) which would constitute a perfect candidate to explain the findings of [2]. The following picture summarizes this interpretation:

![Diagram](image)

In the present note, I will argue that this candidate fulfils all the criteria of the GM model.
Recall the observation of ATLAS: A(400)bb→h(125)+Z+b-tag, with a cross section of a few 100 fb. GM predicts that A(400)→H(151)+Z should be enhanced by a factor ~20 with respect to h(125)Z as shown in the following table from [1]. This solution needs to be modified to be compatible with the cross section for A(400)→h(125)+Z found by ATLAS and therefore a different choice of parameters has also been tried, as indicated in the last column of the table below. Noticeable is the significant deviation for the coupling h(125)bb from the SM. This aspect belongs to section V.

| Type       | coupling/SM                  | sα=0.15 sH=0.5 | sα=0.05 sH=0.6 |
|------------|------------------------------|----------------|----------------|
| h(125)WW/ZZ| cαcH- 1.63sαsH              | 0.99           | 0.85           |
| H(151)WW/ZZ| sαcH+1.63cαsH               | 0.68           | 0.94           |
| h(125)tt,bb| cα/cH                       | 1.14           | 1.25           |
| H(151)tt,bb| sα/cH                       | 0.17           | 0.06           |
| Zh(125)A   | 1.63(sαcH+0.6cαsH)          | 0.28           | 0.65           |
| ZH(151)A   | 1.63(cαcH- 0.6sαsH)         | 1.48           | 1.33           |
| Wh(125)H3+ | 1.63(sαcH+0.6cαsH)          | 0.28           | 0.65           |
| WH(151)H3+ | 1.63(cαcH- 0.6sαsH)         | 1.48           | 1.33           |

The table predicts that H(151) should have a narrow width, ~10 MeV and a BR into 2γ and into Zγ of a few per mill.

Furthermore the main decay modes of H(151) are ZZ with BR~10% and WW with BR~90%, while almost negligible into bb and ττ, as expressed in the same table. Its coupling to tt being also suppressed, the gg production cross section is 50 times smaller than for h(125), hence non observable in the ZZ and WW analyses performed at LHC.

Technically, this low mass allows fulfilling the unitary constraints of GM as will be argued in section II, while this would not be the case for heavier scalars. One therefore concludes that H(151) fulfils the criteria to be identified as the missing neutral isosinglet of the GM model.

H(151)→Zγ is suggested by the same reference, which also fits in this interpretation.

Recall that A→h(125)Z has been observed by requiring a spectator b-jet, a trick which also applies to H(151) to further reduce the background.

For what concerns future e+e- colliders, the process e+e- →ZH(151) is produced with half the cross section of a genuine Higgs and is fully accessible at ECM=250 GeV, a centre of mass energy provided by all projects under study, which could therefore act as factories for the two scalars h(125) and H(151).
Section I Quantitative interpretation of the S(151) signal

In [1], the following diagram was summarizing the particle content of the GM models and the transitions expected between 5-plets and 3-plets and 3-plets and singlets. In this work, I will identify the scalar found in [2] with the missing singlet H. In green are shown identified particles and processes.

An obvious comment is the absence of charged Higgs indications, which I will interpret in section III.

Section I.1 Branching ratios of H(151)

All rates are calculable, once the masses are fixed, in terms of two parameters:

- The mixing angle $\alpha$ between h and H which needs to be small to preserve h SM properties
- A mixing angle $\theta_H$ which fixes the vacuum expectation of the triplets

These two parameters were chosen as $\sin \alpha = -0.15$ and $\sin \theta_H = 0.5$ in [1], but some flexibility remains which is used to better reproduce the ATLAS observation of $A \rightarrow hZ$ (see the table of the introduction).

An extra input was needed in [1] to explain the fermionic couplings observed in the decays of A(400). I therefore have added two new ingredients:

- An extra doublet, to allow the usual flexibility on fermion couplings, well known from MSSM
- The concept of Aligned-Two-Higgs-Doublet-Scheme [4] mechanism A2HDS, which allows to vary separately the couplings for top, b and $\tau$.

In [4] it becomes possible to adjust these couplings by factors called $\zeta_f$. This new scheme is called EGM, for Extended Georgi Machacek model.

The boson H decays primarily into WW and ZZ as shown in the table below. This is due to the low mixing angle between h and H.
Solution II shown in the next table is motivated by the cross section for A(400)-Zh(125).

| Solution | $\Gamma^{\text{Tot}}$ MeV | BR(VV) % | BR(ZZ) % | BR(bb) % | BR($\tau\tau$) % | BR($\gamma\gamma$) % | BR(Z$\gamma$) % | BR(gg) % |
|----------|-----------------|----------|----------|----------|-----------------|-----------------|-----------------|--------|
| I        | 7               | 89       | 10       | 0.4      | 0.046           | 0.3             | 0.3             | 0.2    |
| II       | 13              | 90       | 10       | ~0       | ~0              | 0.3             | 0.3             | ~0     |

Solution I: $\alpha_s = -0.15$ $sH=0.5$  Solution II: $\alpha_s = -0.05$ $sH=0.6$

Section I.2 Branching ratios of A(400)

In the present interpretation, one assumes that A(400) cascades into ZH(151) accompanied by two spectator b-jets. The spectator Z produces events with transverse missing energy, $E_T^{\text{miss}}$, best observed in [2], by decaying into neutrinos in 20% of the cases. The scalar H can decay into $\gamma\gamma$ and Z$\gamma$.

To achieve the rates seen at LHC, one can adjust the three following parameters from EGM: $\zeta_b$, $\zeta_t$ and $\zeta_{\tau\tau}$. This is summarized in the picture below:

---

Figure 2: Cross section at LHC for the production of a SM Higgs boson associated to a pair b$\bar{b}$ from [5].

| Channel                                      | Cross sec. [fb] | Obs. lim | Exp. lim |
|----------------------------------------------|-----------------|----------|----------|
| $S(\gamma\gamma)$                           | 4.4±2.6         | 9.0      | 5.4      |
| $S(\gamma\gamma)+E_T^{\text{miss}}>60\,\text{GeV}$ | 0.42±0.13       | 0.65     | 0.22     |
| $S(\gamma\gamma)+V\rightarrow j\bar{j}$     | 0.28±0.28       | 0.80     | 0.57     |
| $S(\gamma\gamma)+b$-jets                     | 0.08±0.08       | 0.22     | 0.16     |
| $S(\ell\ell)+V\rightarrow b\nu$ or $\ell\ell$ | 1.3±0.7         | 2.8      | 1.7      |
| $S(b\bar{b})+150<E_T^{\text{miss}}<250\,\text{GeV}$ | 0.90±0.79       | 2.2      | 1.6      |
| $S(4\ell)$                                   | 0±0.16          | 0.28     | 0.33     |

TABLE 1: Extracted cross sections in fb for each final state considered here (see main text for details). The observed and expected limits on fiducial cross-section at 95% confidence level are also provided. For the second and the last category total cross sections are quoted while for the associate production channels fiducial ones are given. The $S\rightarrow 4\ell$ is not included in the fit but rather given as a constraint.
The previous table summarizes the results from [2].

The production process [5] depends on ζb: σ(gg→Abb)=10(ζb)^2 fb. The two dominant BR of A are H(151)Z and tt. The width Γ(A→tt) depends on ζt, which is taken as 0.7 to reproduce the cross section of gg→A→tt measured by CMS. The choice ζb=15 is adjusted to the 4.1 fb cross section seen in [2].

| ΓAtot GeV | BR(HZ) % | BR(hZ) % | BR(tt) % | ζt | BR(ττ) % | ζτ | BR(bb) % | σAbb fb | σHZBR(H→2γ) fb | σhZ fb |
|-----------|---------|---------|---------|----|---------|----|---------|------|--------------|-------|
| 31        | 62      | 12      | 19.5    | 0.7| 1.3     | 22 | 5.3     | 15   | 2300         | 4.1   | 270       |

Applying ETMiss>90 GeV, [2] measures 0.42±0.13 fb, that is ~10% of the total cross section 4.1 fb. Given that BR(Z→invis)=20%, this corresponds to a ~50% efficiency. This result is qualitatively understood given the large pZ*→157 GeV for the decay A(400)→H(151)Z.

To get a result reproducing the ATLAS indication on h(125)Z, a further adjustment was needed: sin α = -0.05 and sin θH = 0.6. This modification increases by a factor 4.6 the A→hZ BR, decreasing by only 20% the A→H(151)Z BR.

Section I.3 ZWW and WWW final states

These states are naturally produced in the process A→ZH, which has a cross section ~1200 fb. This process produces in ~90% of the cases a ZWW final state. This final state is often accompanied by visible b-jets. In the analysis presented in section IV, there is veto against b-jets, meaning that a large fraction of these events is lost. This not the case for the Higgsstrahlung process q̅q’→ZH(151) which has a smaller cross section, ~200 fb.

The process q̅q’→WH(151) gives a larger cross section, ~450 fb. It produces WWW and should contribute to a larger excess with respect to the SM prediction, which is of the same order. I shall come back to this in section IV.

Cascades like H5→AZ and H5+ → AW+ will produce even more exotic topologies like ZZWW and ZWWW, which have so far not been identified. The production cross section for H5 is about 250 fb and the branching ratio in AZ ~50%, meaning that this sort of cascade contribution is not entirely negligible.

Section II Unitary constraints

In this section, I will indicate how the EGM model allows constraining the mass of H(151). As already mentioned in [1], unitarity arguments allow to put stringent upper limits on the masses m5 and m3. Here I will show that knowing m5 and m3 it is possible to predict a narrow interval on the mass of the second singlet mH.

In the appendix are given the detailed formulae for this derivation in the convention of [14]. The main equation needed for this derivation reads:

$$8(3\lambda_3+2\lambda_4)v^2\chi^2=m5^2+2(mH^2cos\alpha^2+mh^2sin\alpha^2)-3cos\theta H^2m3^2$$

where v=246 GeV and m3 and m5 are the masses of the 5-plets and 3-plets, taken as m5=660±20 GeV and m3=420±20 GeV.
Note that this equation is independent of the two other parameters, M1 and M2, which enter in the GM scalar potential (see Appendix).

Figure 3: Regions allowed by unitarity for the couplings $\lambda 3/\pi$ and $\lambda 4/\pi$ of the GM potential. The thick black line corresponds to the 150 GeV solution for an isosinglet, the red one to 250 GeV. The green area gives the lower part of the uncertainty on the GM parameters.

From this equation, one can draw in the plane $\lambda 3/\pi$ and $\lambda 4/\pi$, straight lines for each value of $mH$. Figure 3 shows that the line $mH=150$ GeV is compatible, within errors, to the unitarity band, while, for instance, the line 250 GeV is excluded by this criterion. Large values of $mH$, above 250 GeV, are therefore excluded, which comforts the interpretation of H as a valid isosinglet candidate.

Note also that the preferred values for $\lambda 3/\pi$ is positive and between 1/2 and 4/5, the later limit comes from unitarity. $\lambda 4/\pi$ should be negative and above the unitarity limit -16/25.

Note also that this method does not allow to set a limit against $mH<150$ GeV, for instance against the CMS 96 GeV candidate.

One can predict that the $e^+e^-\rightarrow h(96)Z$ cross section should be smaller than for the SM Higgs, as observed in LEP2 but, at LHC, A(400) should abundantly decay into Zh(96), which needs also to be searched for, in order to exclude or establish this scenario.

A similar strategy can be followed for $\lambda 2$ and $\lambda 5$, which, after eliminating the parameter M1, reads:

$$4\lambda 2-\lambda 5= \sin2\alpha(mH^2-mh^2)/2\sqrt{3v\chi\phi+2m^23/v^2}$$

Together with the combination of LHC results on $h\rightarrow2\gamma$, which gives $\chi_2=1.11\pm0.10-0.09$ (PDG 2020), one sees that it is already possible to have a determination of $\lambda 2$ and $\lambda 5$. The central value found $\lambda 5/\pi=-1.24$ is consistent with unitarity. One expects a gain of an order of magnitude in precision with HL-LHC.
\[ \lambda_1 = \frac{(M_H^2 \sin^2 \alpha + m_h^2 \cos^2 \alpha)}{8 v^2} \]

Which gives \( \lambda_1 = 0.07 \), fully in agreement with unitarity which predicts \(-1/3 < \lambda_1 / \pi < 1/3\).

In conclusion, this analysis comforts the interpretation of \( H(151) \) as a GM isosinglet.

Section III Expectations for LHC

What else can be expected after this interpretation of \( S(151) \) as a GM singlet?

III.1 Immediate consequences

One can try to use this interpretation to further improve the analysis presented in [2]. The large production cross section of \( A(400)bb \) seem to offer various additional opportunities. In particular, one can:

- Require in all cases the presence of b-jets and see if the ratio signal/background is improved
- Require that events with leptons or 2 jets be compatible with the Z boson mass

These selections have an obvious cost in efficiency but their effects could provide a useful check of the GM interpretation.

When the Z boson mass can be reconstructed, one can see if the \( ZH(151) \) mass is compatible with \( A(400) \). To improve the mass resolution, one can impose the Z mass constraint, of help for decays into jets.
III.2 WWW and ZWW channels

ATLAS and CMS have searched for processes like ZWW and WWW. In the SM, the WWW process receives a contribution from $h(125)$ which can decay into WW. The presence of $H(151)$ should add a contribution of $\sim 400$ fb, not at all negligible in comparison the expected SM cross section $\sim 500$ fb.

Recently ATLAS\cite{6}, using a BDT selection process, has measured the following cross section:

$$\sigma(pp \rightarrow WWW) = 850 \pm 100 \text{ (stat.)} \pm 80 \text{ (syst.) fb}$$

with a 2.4 s.d. excess ($\mu=1.66$), which is compatible with the GM contribution.

The cleanest channel for this analysis is the 3 leptons channel shown below:

![Figure 5: Results showing, in yellow, the estimated contribution of the WWW given by ATLAS vs. the BDT output for the 3 leptons final states.](image)

In the analysis performed by CMS\cite{7}, no excess in WWW was observed noting however, that errors bars are such that the results of the two experiments are not incompatible.
When one considers the ZWW final, the Higgstrahlung process is smaller ~200 fb but one also expects a contribution from:

with a cross section ~1000 fb. This channel, when accompanied by two b-quark jets, is vetoed in ATLAS and CMS analyses in order to reduce the top contamination. It is however likely that such a veto cannot be effective enough to reduce by much more than 50% this contribution.

From this, I tend to conclude that there is some tension with CMS for both channel, worrying but not allowing yet a definite conclusion. One should also not forget that a BDT method takes into account some features, which may quite distinct from SM distributions when H(151) gives a substantial contribution.

A suggestion would be to try a ZWW analysis requesting the presence of a b-quark jet.

Remark: H5, H5+, which can decay into AZ and AW, give small extra contributions, which can be neglected at the present level of this analysis. Bearing in mind that H(151) mostly decays into WW, these contributions will feed into the 4-boson final states, perhaps detectable in the future.

In conclusion, the presence of H(151) should reflect into the WWW and ZWW final states, which offers important prospects for an independent approach of these GM searches.

III.3 Search for H3+

As mentioned in in the introduction, it is intriguing that searches for a charged Higgs have been sterile so far. In an MSSM scenario, these particles have been searched in the tb and τν modes. There are two mechanisms to produce a single H3+:
Within GM, \( H_3^+ \) can decay into \( H(151)W^+ \), therefore going mainly into \( WWW \) accompanied by a top and a b, which allows a clear distinction from the previous case.

Taking into account the \( \zeta_t \) and \( \zeta_b \) parameters defined in section I, the cross sections are small, which reflects the need to produce a top quark: \(~100 \text{ fb for the first process} \) 60 fb for the second one. These figures and a dominant decay mode of \( H_3^+ \) into \( WWW \), could explain the absence of evidence in present searches, where only \( tb \) and \( \tau \nu \) final states were considered.

III.4 Searches for \( H_5^+ \) and \( H_5^{++} \)

The scalars of the 5-plet are produced through the VBF mechanism. The absence of signal in like-sign \( W \) [8], which seems to eliminate the present scenario, is easily understood given that:

- \( \sin \theta_H=1 \) was assumed by [8], meaning that the cross section should be divided by four for \( \sin \theta_H=0.5 \)
- \( \sin \theta_H=1 \) also implies that the decays are 100\% into \( ZW^+ \) and \( W^+W^+ \), while for \( \sin \theta_H=0.5 \) \( ZH_3^+,W+A \) and \( H_3^++W^+ \) contribute to about 50\% of the BR.

This means that the cross sections that were assumed for \( H_5^{++} \) and \( H_5^+ \) need to be divided by almost an order of magnitude, implying that the present scenario is not yet covered by these CMS searches.

III.5 Topological excesses

At LHC, \( Abbott \rightarrow ZH(151)bb \) has a large cross section, \(~1200 \text{ fb} \). While exclusive reconstruction may look rather challenging (but perhaps not impossible), one can identify the presence of this channel using various topological selections: like-sign leptons, multi leptons, missing transverse energy and b jets. Note that b jets can come from Z decays but also from the production mechanism itself.

| Selection | Best-fit \( \beta^2 \) | Significance |
|-----------|-------------------|-------------|
| ATLAS Run 1 SS \( \ell \ell \) and \( \ell \ell \ell + \text{b-jets} \) | 6.51 ± 2.99 | 2.37σ |
| ATLAS Run 1 OS \( e\mu + \text{b-jets} \) | 4.09 ± 1.37 | 2.99σ |
| CMS Run 2 SS \( e\mu, \mu\mu \) and \( \ell \ell + \text{b-jets} \) | 1.41 ± 0.80 | 1.75σ |
| CMS Run 2 OS \( e\mu \) | 2.79 ± 0.52 | 5.45σ |
| CMS Run 2 \( \ell \ell + E_T^{\text{miss}} (WZ) \) | 9.70 ± 3.88 | 2.36σ |
| ATLAS Run 2 SS \( \ell \ell \) and \( \ell \ell + \text{b-jets} \) | 2.22 ± 1.19 | 2.01σ |
| ATLAS Run 2 OS \( e\mu + \text{b-jets} \) | 5.42 ± 1.28 | 4.06σ |
| ATLAS Run 2 \( \ell \ell + E_T^{\text{miss}} (WZ) \) | 9.05 ± 3.35 | 2.52σ |
| Combination | 2.92 ± 0.35 | 8.04σ |

To this contribution, one should add the cascades between the 5-plet and 3-plet, which correspond to smaller, but not negligible contributions.

To conclude, a plausible interpretation of the various anomalies recorded by [3] is provided by the present GM scenario, which needs to be worked out in detail for a full confirmation.
Section IV Expectations at lepton colliders

IV.1 Annihilation process

Figure 6 shows the cross sections expected for the SM Higgs boson and the two low mass scalars indicated by LHC data.

In the three cases, given the high rates, it will be possible to achieve “exquisite” accuracies on the parameters of these scalars, as for h(125): for the total width, the invisible width, the couplings to vector bosons and fermions.

![Figure 6: Predicted e+e- cross sections in fb vs. the centre of mass energy ECM for the SM Higgs, the isosinglet H(151) and the scalar h(96) indicated by CMS and LEP2 data.](image)

Interference effects between h(125) and H(150) should exist but be easy to control since the dominant final states are different: bb for h(125), WW for H(151).

Figures 7 and 8 show the cross sections expected for the heavier scalars with the two set of parameters defined in section 1. These cross sections are much lower, meaning a challenge for future e+e- colliders. On top of that, final states are much more complex and also challenge the detectors for a full solid angle coverage, in particular for b-tagging, up to now a weakness of all solenoidal systems built so far. A full study with realistic simulations of the various channels and a realistic evaluation of efficiencies and backgrounds is therefore highly desirable to evaluate the performances of the various detectors on the market.
IV.2 Case for an e-e- collider

The process $e^+ e^- \rightarrow H^+ H^-$ requires reaching $\sim 1.5$ TeV. An alternate is to consider VBF single $H^-$ production at 1 TeV, the process shown above:

- One of the largest backgrounds, shown above, is easily removed by appropriate cuts, e.g. by requiring a large missing transverse momentum of the final state and no spectator lepton[9]
• The decay mode $H^-(660) \rightarrow H^3(400)W$ with $H^3(-t) \rightarrow tb$, is easily separated from most backgrounds using b-tagging and applying a multijet selection

Furthermore:

• $\mathcal{L}e-e\sim70\%Le+e$- seems feasible [10]
• The cross section for this process reaches 3.5 fb at 1 TeV for $\sin\theta_H=0.5$
• With polarized beams, 80% for e-, one can achieve a luminosity gain $1.8^2$ for the W-W-luminosity, hence a cross section above 10 fb
• Switching from e+e- to e-e- will require changing polarity of many magnets, which seems manageable if anticipated
• An ERL scheme [11] seems feasible if anticipated

V Precision measurements

SM couplings of $h(125)$ are modified within the GM model as shown by the table of couplings given in the introduction. At LHC, where the total width cannot be measured, PM accuracies need to be taken with a grain of salt. Nevertheless, it is fair to say that the retained solution significantly differs from the SM predictions and offer good prospects for HL-LHC. A prominent example is the $hbb$ coupling.

On top of tree level effects, there are loop effects as described, for instance, in [12], which gives the two examples of $h\gamma\gamma$ and $hZZ$.

In the Appendix, the contributions from GM charged Higgs to $h(125)\rightarrow2\gamma$ are given and it is shown that one can easily generate effects at the 10% level since the SM process itself is at the loop level. Less trivial will be the case of $hZZ$, but in this case the measurement from e+e- factories is at the few per mill level.

Section VI Prospects

The GM interpretation obviously offers a promising harvest of discoveries for HL-LHC. Is there more to it? An answer to this question is most probably YES. Recall that in [1] it was shown that the fermion couplings to $A(400)$ were requesting an additional doublet, implying additional charged scalar $H^+$, neutral scalar $H^0$ and pseudoscalar $A^0$. This brings more complexity in the definition of the potential and requires an input from theory to define rigorously this extended model. This work will be essential to guide future searches at colliders. In particular, as was done in section II, one can hope to use unitarity to constrain the masses of these new scalars. Naively, one can think that since they should mix with $A(400)$ to produce the right pattern for fermions, their masses should lie nearby.

The experimental status and interpretation of $h(96)$ still need to be clarified. Is it simply an additional isosinglet which, in particular, is expected to mix with $h(125)$? Or a Radion [13]? How can we distinguish between the two hypotheses?

Finally, one may wonder why Nature has been so generous to add this large family of scalars to the abundant family of fermions. Since the SM model is, by itself, unable to explain the primordial
electroweak transition and the baryogenesis process, one is tempted to attribute to these scalars this mission. In particular measuring CP violation in this sector will be a capital but difficult mission for e+e- colliders.

Conclusion

A consistent interpretation of the S(151) candidate found by [2] is provided within the extended GM framework. More than the actual statistical significance of this signal, always disputable, this mere fact contributes to increase our confidence in this finding. The present interpretation reconciles the topological approach [3] and the spectroscopic approach [1], providing a reliable consolidation of the various evidences for BSM physics at LHC discussed in these three references.

Together with the other indications of a CP-odd scalar A(400) and of a CP even H5(660), this new evidence completes the GM spectrum, providing an excellent isoscalar candidate for the GM model, called here H(151). Given its low mass, it opens a wide range of new opportunities, which will be exploited at HL-LHC or, even before, with Run3 and/or by combining the Run2 data of the two collaborations.

From LHC one expects a full confirmation of H(151), A(400), H5(660) and, possibly, S(96) and an exploration of new avenues like WWW, ZWW and even more complex states.

Needless to repeat that an e+e- machine reaching 1 TeV will provide an ideal tool to investigate this new zoology and achieve very precise measurements of the parameters of these particles. H5- - can be singly produced with a 1 TeV collider operating in the e-e- mode with similar luminosity.

There are demanding requirements on detectors, calling for an almost perfect solid angle coverage, needed for efficiently reconstructing events with up to 10 jets, in clear distinction to what is presently achievable for a Higgs factory. This coverage includes b-quark identification and appears very challenging.

An additional input of this work is an exploration of the parameters of the complex scalar GM potential. The present solution appears to satisfy the unitarity constraints in a non-trivial way, which would not be the case if the H particle had been heavier, say with a mass above 250 GeV. Using the process h(125)->2γ measured at LHC, it is shown that even with the present accuracy, one can already tightly constraint some parameters of the Higgs potential.

Precision measurement will also play a critical role in providing the final assessment of the GM interpretation. As an example, this analysis predicts a ~15% deviations on h(125)bb couplings, which is already observable at HL-LHC.

If such GM indications are confirmed, the international initiatives towards future e+e- colliders should be greatly boosted.

Acknowledgements: I am grateful to my colleagues from IJCLab M. Davier, A. Falkowski, R. Poeschl, D. Zerwas and Z. Zhang for kindly encouraging this work. I also thank G. Moultaka from University Montpellier L2C, for providing his expertise on the Georgi-Machacek model.

References:
[1] Global interpretation of LHC indications within the Georgi-Machacek Higgs model, Talk presented at the International Workshop on Future Linear Colliders (LCWS2021), 15-18 March 2021. C21-03-15.1
François Richard (IJCLab, Orsay) (Mar 22, 2021) Contribution to: LCWS 2021
E-Print: 2103.12639

[2] Accumulating Evidence for the Associate Production of a Neutral Scalar with Mass around 151 GeV
Andreas Crivellin (Zurich U. and PSI, Villigen and CERN), Yaquan Fang (Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Oliver Fischer (Liverpool U.), Abhaya Kumar (U. Witwatersrand, Johannesburg, Sch. Phys.), Mukesh Kumar (U. Witwatersrand, Johannesburg, Sch. Phys.) et al. (Sep 6, 2021)
E-Print: 2109.02650

[3] Stefan Buddenbrock (U. Witwatersrand, Johannesburg, Sch. Phys.), Alan S. Cornell (Witwatersrand U.), Yaquan Fang (Beijing, Inst. High Energy Phys.), Abdualazem Fadol Mohammed (Witwatersrand U. and Beijing, Inst. High Energy Phys. and Beijing, GUCAS), Mukesh Kumar (Witwatersrand U.) et al. (Jan 16, 2019)
Published in: JHEP 10 (2019) 157
E-Print: 1901.03500

[4] Antonio Pich (Valencia U. and Valencia U., IFIC), Paula Tuzon (Valencia U. and Valencia U., IFIC) (Aug, 2009)
Published in: Phys.Rev.D 80 (2009) 091702
E-Print: 0908.1554

[5] The Anatomy of electroweak symmetry breaking. I: The Higgs boson in the standard model
Abdelhak Djouadi (Montpellier U. and Orsay, LPT) (Mar, 2005)
Published in: Phys.Rept. 457 (2008) 1-216
E-Print: hep-ph/0503172

[6] Observation of WWW production in pp collisions at sqrt(s)=13 TeV with the ATLAS detector
ATLAS Collaboration
ATLAS-CONF-2021-039 (25 Aug 2021)

[7] Observation of the Production of Three Massive Gauge Bosons at sqrt(s)=13 TeV
CMS Collaboration
Albert M Sirunyan (Yerevan Phys. Inst.) et al. (Jun 19, 2020)
Published in: Phys.Rev.Lett. 125 (2020) 15, 151802
E-Print: 2006.11191

[8] Search for charged Higgs bosons produced in vector boson fusion processes and decaying into vector boson pairs in proton–proton collisions at sqrt(s) = 13TeV
CMS Collaboration
Albert M Sirunyan (Yerevan Phys. Inst.) et al. (Apr 10, 2021)
Published in: Eur.Phys.J.C 81 (2021) 8, 723
E-Print: 2104.04762

[9] Production of weak bosons and Higgs bosons in e-e collisions
Vernon D. Barger (Wisconsin U., Madison), John F. Beacom (Wisconsin U., Madison), King-man Cheung (Northwestern U.), Tao Han (UC, Davis) (Apr, 1994)
Published in: Phys.Rev.D 50 (1994) 6704-6712
E-Print: hep-ph/9404335

[10] Potential BSM searches in e-e collisions at ILC
A. Drutskoy (LPI, Moscow)
ILC Workshop on Potential Experiments (ILCX 2021) October 26-29, 2021
https://agenda.linearcollider.org/event/9211/contributions/49249/

[11] A high luminosity superconducting twin e+e−e^+e^− linear collider with energy recovery
V.I. Telnov (Novosibirsk, IYF and Novosibirsk State U.) (May 23, 2021)
Contribution to: LCWS 2021
E-Print: 2105.11015
Since the conventions used for the GM model greatly vary from author to author, I will recall below the ones used in the present analysis and the derivation of some results derived from LHC indications.

Defining [14] the following doublet and the two triplets of GM:

\[
\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ -\phi^+ & -\phi^0 \end{pmatrix}, \\
X = \begin{pmatrix} \chi^0 & \xi^+ & \chi^{++} \\ -\chi^+ & -\xi^+ & \chi^{*} \\ \chi^{++} & \chi^{*} & \chi^0 \end{pmatrix}
\]

One writes the potential:

\[
V(\Phi, X) = \frac{\mu^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_2^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) \\
+ \lambda_3 \text{Tr}(X^\dagger X^{t\dagger} X^{t\dagger}) + \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b) \\
- M_1 \text{Tr}(\Phi^\dagger \Phi \tau^b) (U X U^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t^a X t^b) (U X U^\dagger)_{ab}.
\]

I have used the following relations:
From LHC, the masses $m_3$ and $m_5$ are known, with some uncertainties. The CP-odd meson has a mass difficult to ascertain from the t\bar{t} analysis given interferences with the QCD background. Abb\texttt{->}Z\texttt{hbb}, which gives the best evidence, prefers 440 GeV, while other indications cluster at \approx 400 GeV. Tentatively, I assume that $m_3=420\pm 20$ GeV. $H(660)$ is a wide resonance and, accordingly, I will assume that $m_5=660\pm 20$ GeV.

$v_\phi$ and $v_\chi$ are the two vacuum expectations for the doublet and the two triplets, which are related to the SM vacuum expectation: $v^2=v_\phi^2+8v_\chi^2$. Redundantly one usually defines a mixing angle $\theta_H$, $\cos\theta_H=v_\phi/v$, with the following choice in [1]: $\sin\theta_H=0.5$

In the minimal version, there are two singlets, $h(125)$ and $H(151)$ with a mixing angle $\alpha$. This mixing angle has to be small to avoid altering the properties of the SM $h(125)$. From [1]: $\sin\alpha=0.15$

One writes:

\[
M_1^2 = 8\lambda_1 v_\phi^2, \\
M_2^2 = \frac{\sqrt{3}}{2} v_\phi \left[ M_1 + 4 (2\lambda_2 - \lambda_0) v_\chi \right], \\
M_3^2 = \frac{M_1 v_\phi^2}{4 v_\chi} - 6 M_2 v_\chi + 8 (\lambda_3 + 3 \lambda_4) v_\chi^2.
\]

The mixing angle is fixed by

\[
\sin 2\alpha = \frac{2 M_2^2}{m_H^2 - m_h^2}, \\
\cos 2\alpha = \frac{M_2^2 - M_1^2}{m_H^2 - m_h^2},
\]

with the masses given by

\[
m_{h,h'}^2 = \frac{1}{2} \left[ M_1^2 + M_2^2 \pm \sqrt{(M_1^2 - M_2^2)^2 + 4 (M_1^2 M_2^2)} \right].
\]

Using these relations, one can eliminate the parameters $M_1$ and $M_2$, writing that:

\[4\lambda_2 - 5\lambda_5 = \sin 2\alpha (m_H^2 - m_h^2)/(2V_3 v_\chi v_\phi) + 2 m^2/3 v^2\]

Similarly, one can write:

\[8(3\lambda_3+2\lambda_4)v^2 v_\chi = m_5^2 + 2(m_H^2 \cos\alpha^2 + m_h^2 \sin\alpha^2) - 3 \cos\theta_H m_3^2\]

Knowing the parameters of the second member of these equalities, one can draw straight lines in the planes $\lambda_2/\lambda_5$ and $\lambda_3/\lambda_4$ as was done in the main text. This allows deciding if these parameters fulfil the unitarity requirements defined in [14]. It turns out that they barely do it for $\lambda_3/\lambda_4$, which allows fixing these values in a narrow window:
To determine $\lambda_2/\lambda_5$, one can use the measured value of $\kappa_\gamma$ for the process $h\rightarrow\gamma\gamma$ given by a recent PDG averaging of ATLAS and CMS:

$$\kappa_\gamma = 1.11 \pm 0.11 - 0.09$$

This measurement depends on $\lambda_2/\lambda_5$, through the charged scalar loops of GM as shown from the formulae of reference [15]:

$$C_{H^+_3 H^0_3} = \frac{1}{\sqrt{3} \alpha} \left\{ \sqrt{3} \alpha \left[ 4(\lambda_2 - \lambda_5) v_\phi^2 + 8(\lambda_1 + \lambda_5) v_\phi v_\chi + 4M_1 v_\phi v_\chi \right] 
- [s_\alpha \left( 8\lambda_3 + 16(\lambda_2 + \lambda_5) v_\phi^2 + 4M_1 v_\phi v_\chi + 6M_2 v_\chi^2 \right) \right\},$$

$$C_{H^+_5 H^0_5} = \frac{1}{\sqrt{3} \alpha} \left\{ \sqrt{3} \alpha \left[ 4(\lambda_2 - \lambda_5) v_\phi^2 + 8(\lambda_1 + \lambda_5) v_\phi v_\chi + 4M_1 v_\phi v_\chi \right] 
+ [s_\alpha \left( 8\lambda_3 + 16(\lambda_2 + \lambda_5) v_\phi^2 + 4M_1 v_\phi v_\chi + 6M_2 v_\chi^2 \right) \right\},$$

$$C_{H^+_3 H^+_5} = C_{H^+_5 H^+_3} = \alpha \left[ (4\lambda_2 + \lambda_5) v_\phi^2 - \sqrt{3} \alpha \left[ 8(\lambda_3 + \lambda_5) v_\phi v_\chi + 2M_2 \right] \right],$$

$$C_{H^+_3 H^+_5} = C_{H^+_5 H^+_3} = \alpha \left[ (4\lambda_2 + \lambda_5) v_\phi^2 + \sqrt{3} \alpha \left[ 8(\lambda_3 + \lambda_5) v_\phi v_\chi + 2M_2 \right] \right].$$

From these formulae, one concludes that:

- The dominant contribution comes from $H3+H3-$, which depends on $4\lambda_2-\lambda_5$ and can therefore be precisely predicted
- The $H5+H5-$ and $H5++H5-$ contributions depend on $4\lambda_2+\lambda_5$, which can vary over a wide range

Measuring $h\rightarrow\gamma\gamma$ and assuming that the fermionic and WW couplings are taken from the table of section I, one can extract $4\lambda_2+\lambda_5$ and therefore each of the two quantities.

The following table summarizes these findings:

| m_3 (GeV) | m_5 (GeV) | s_\alpha | s_H | \lambda_1/\pi | \lambda_2/\pi | \lambda_5/\pi | M_1 (GeV) | \lambda_3/\pi | \lambda_4/\pi | M_2 (GeV) |
|----------|----------|----------|------|--------------|--------------|--------------|----------|--------------|--------------|----------|
| 420      | 660      | -0.15    | 0.5  | 0.02         | 0.13         | -1.24        | 800      | 0.6          | -0.2         | 1000     |

GM loops also affect the process $e^+e^\rightarrow ZH(125)$, which will be very precisely measured at a future Higgs factory, allowing to constrain the Higgs potential [12].