Polar Coding for Efficient Transport Layer Multicast

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Abstract—In this paper, we shed light on how an adaptive, efficient error coding in the transport layer helps ensure the application’s requirements. We re-cap the use of MDS codes and show that binary coding can significantly reduce the complexity and hence increase the applicability also for embedded devices. We exploit the persymmetric structure of the generator matrix in polar codes to establish a duality of dispersion over channels (the polarization effect) and over packets (the generality required for multicast transmission), thereby constructing systematic polar codes for incremental redundancy whose performance, despite a much lower complexity, is near to MDS codes for medium-range residual loss rates.

Index Terms—Multicast, Forward Error Coding, transport protocols, energy efficiency.

I. INTRODUCTION

Fog computing brings large computational resources closer to the end devices, thereby enabling applications demanding low latency [1]. Networked Control Systems (NCSs) are a paradigmatic example of these applications as they require predictably low latency, as well as reliability [2]. The transport layer is the only one in the protocol stack with an end-to-end perspective of the communication and thus it is fundamental to achieve timeliness and reliability guarantees [3]. This layer can monitor the lower layers and adapt to them to ensure the quality of service by means of a time-aware error control function. The most common error control schemes, namely Automatic Repeat ReQuest (ARQ) and Forward Error Coding (FEC), have inherently different timing characteristics—i.e., one is reactive whereas the other is proactive. Hybrid ARQ (HARQ) can combine them in order to achieve optimal performance in information-theoretical terms under timing constraints [4], [5].

Maximum Distance Separable (MDS) codes have traditionally been used at the transport layer for FEC [5], [8] because the number of correctable losses equals the number of transmitted parity packets. However, MDS codes perform computationally expensive operations in high order Galois Fields, resulting in unacceptable delay and energy consumption in embedded devices. Binary codes [9]–[11] have been designed to reduce the coding complexity, but they have the disadvantage of requiring an excess of parity packets and do not perform well in multicast scenarios. In this paper, we analyze the recently developed polar codes [12] and exploit the persymmetric structure of their generator matrix to construct binary codes that are more efficient than previous binary codes for the transport layer while approaching the performance of MDS in multicast at the same time. The contribution of this paper is threefold:

• To the best of our knowledge, this paper provides the first analysis of polar codes at the transport layer from a joint energy and multicast perspective.

• A code construction mechanism is proposed that exploits polar codes’ dual dispersion over channel and receivers to enable binary codes in multicast.

• We show that the proposed code approaches MDS codes for multicast with much lower complexity, while the excess packets are kept low for short block lengths.

The remainder of the paper is organized as follows: Section II presents related work and some background is discussed in Section III. The impact of the transport layer in HARQ is analyzed in Section IV. Section V discusses energy and multicast aspects of error coding and Section VI describes the proposed code construction, which is then evaluated in Section VII. Section VIII concludes the paper.

II. RELATED WORK

There have been many proposals to complement lower layer error coding with coding at the transport layer in order to improve reliability without prohibitively increasing the delay. MDS codes have been used to provide predictable reliability under time constraints [5], improve video streaming quality [7] and mitigate feedback implosion in multicast [13]. Windowed random linear codes (RLC) are an alternative to block codes that make the end-to-end delay independent of the block length by evenly spreading the parity packets over the source packets. Huang et al. [4] implement RLC in the transport layer together with ARQ, Roca et al. [14] show that RLC reduces the FEC delay at the expense of lower code rates than block codes and Karzand et al. [15] use it to reduce the delay in in-order delivery. Although we have opted to use block codes that in principle induce a larger delay, our proposed error coding is delay-aware, which ensures that the delay does not exceed the application requirements. Other approaches have focused on simplicity and solely apply exclusive or (xor) to all the packets in the block [16]–[18], thereby allowing the recovery of a single loss. Those codes are a subset of the proposed polar coding in this paper because the parity transmission always starts with the xor of all packets. Michel et al. [8] implemented the three aforementioned approaches in QUIC and show that RLC achieves the best performance in terms of delay.
III. BACKGROUND

Once the application requirements are known, a time-aware error control could adapt its performance to meet them with a minimal resource footprint. Monitoring the residual loss rate and delay of the lower layers, the transport layer can react only when strictly necessary to guarantee the end-to-end performance—i.e., only add redundancy if the lower layers do not provide enough reliability. This adaptability can be achieved by dynamically changing the partitioning between ARQ, whose delay is dominated by the round-trip time (RTT) as packets are retransmitted upon the detection of losses with some acknowledgement mechanism, and FEC, whose delay is dominated by the inter-packet time (IPT) that is required to collect packets before encoding. ARQ only retransmits redundancy when required, thereby increasing efficiency. On the other hand, FEC is a better alternative under tight delay constraints as the sender does not need to wait for feedback, as well as in multicast, where collecting feedback from all of the receivers is infeasible [19] and the generality of repair packets is important. Therefore, under delay constraints, an adaptive HARQ mechanism is optimal in information-theoretical terms. Figure 1 shows the delay budget of a HARQ scheme. Although the coding process entails complex operations, the exponential increase in processing power in the last decades made it possible to run this operation in most machines with negligible delay. However, this is not the case when the protocol is executed on embedded devices, resulting in a large delay and energy consumption that should be minimized.

IV. THE COMPLEXITY DILEMMA REVISITED

Block codes encode a message $m$ with $k$ symbols into a codeword $c$ via multiplication with a $k \times n$ generator matrix $G (c = m \cdot G)$. At the receiving end, the original message can be recovered by solving $m = \hat{c} \cdot G^{-1}$, where $\hat{c}$ is a subset of $k$ symbols of $c$ and $G$ is a $k \times k$ submatrix of $G$. The encoding process entails a matrix-vector multiplication with complexity $O(kn)$, whereas the matrix inversion has complexity $O(k^3)$ and therefore dominates the overall process. In the following sections, we introduce two types of codes that use different approaches to create $G$: MDS and binary codes.

A. Maximum Distance Separable Codes

The principal characteristic of MDS codes is that they fulfill the Singleton Bound with equality—i.e., $d_{\text{min}} = e + 1$ where $d_{\text{min}}$ is the minimum distance between codewords and $e = n - k$ the number of correctable erasures [20]. They are defined on $GF(2^q)$ where $q$ is the number of bits per symbol. A usual configuration is $GF(2^8)$ such that each symbol is a byte. In order to correct $e$ erasures with any $k$ received packets, every $k \times k$ submatrix of $G$ should be invertible. From all the available mechanisms to construct such a matrix [9], [21], this paper focuses on systematic Vandermonde MDS codes, which does not limit the analysis because the basic operations remain the same. The particular characteristic of systematic codes is that a verbatim copy of the original message is present in the codeword. In other words, the generator matrix has a systematic part, consisting of a $k \times k$ identity matrix, and a $k \times (n - k)$ parity part ($G = [I | P]$). Although the code properties are unchanged, systematic codes have a better residual loss rate as some of the original symbols can still be recovered even when more than $n - k$ erasures occur. The complexity of systematic MDS codes is $O(kp)$ for encoding and $O(kp^2)$ for decoding [9]—$p$ is the number of parity symbols—reducing the complexity if $k >> p$, which is usually the case in multimedia applications, as shown in Section VII.

B. Binary Codes

Binary codes are defined in $GF(2)$ and they perform encoding and decoding with simple xors instead of arithmetic operations in higher-order fields. As a result, they can use graph-based decoding algorithms [20], thereby reducing the complexity as no explicit matrix inversion is needed. The complexity of binary codes depends on the average degree of $G$ and they achieve quasilinear complexity when sparse matrices are employed. Despite this complexity reduction, using $GF(2)$ does not guarantee the invertibility of every $k \times k$ submatrix. Consequently, binary codes require excess symbols to approach the channel capacity, giving place to the “complexity dilemma”: the computational complexity is reduced via basic xor operations and sparse matrices, whereas the network complexity increases due to the redundancy excess. It can be shown that the excess portion in the transmitted redundancy decreases when large block lengths are used—e.g., $10,000$ [9]. Although these block lengths are common in the physical layer, they are infeasible in the transport layer.

C. The Role of the Transport Layer

IP networks “see” packetized erasure channels—e.g., erroneous packets are not forwarded to the upper layers and complete packets are dropped due to congestion. Therefore,
although HARQ at the physical layer uses bits as symbols, at the transport layer it should recover complete packets—we assume a 1500-byte MTU throughout the paper. Consequently, k packets need to be collected before encoding, resulting in very short codes under time constraints. For example, a video conference stream at 10 Mbit/s with a target delay of 100 ms can only collect 84 packets before the time constraint expires, a difference of 3–4 orders of magnitude with the physical layer. This blocklength difference makes the matrix inversion no longer dominate the complexity! While a single matrix inversion is required per block, the encoding step is iterated throughout the whole packet. Under time constraints, MTU >> k and in networks with small erasure rates k >> p.

If C_{mul} = MTU × O(kp) is the complexity of the matrix-vector multiplication and C_{inv} = O(kp^2) the complexity of the matrix inversion, at the transport layer C_{mul} >> C_{inv}. Using basic xor, binary codes implement a much simpler matrix-vector multiplication than MDS codes. However, the most efficient binary codes use sparse binary matrices that require large block lengths to reduce the redundancy excess [9]. [11]. Fully (pseudo-)random fountain codes [10] and polar codes [12] are an interesting alternative, as they would benefit from the complexity reduction of binary codes with a lower transmission overhead due to their non-sparse matrices.

In random fountain codes the parity packets include each source packet or not with equal probability, resulting in a k/n column degree. Therefore, the coding complexity of systematic random fountain codes is O(2k/p). On the other hand, polar codes exploit the polarization effect of binary codes, which can be seen as a combination of n independent binary channels whose capacity polarizes to either almost perfect or almost fully noisy. Polar codes build n × n generator matrices where the information symbols are complemented by frozen symbols—typically set to 0—located in the worst channel positions. If the channels are the columns of the generator matrix, then the row with the most 1s disperses the source symbol over the most channels, thereby achieving the highest channel capacity. The Bhattacharyya parameter has been proposed to measure the channel quality [12], which for the binary erasure channels (BEC) considered in this paper exactly calculates the erasure probability for each channel. The deterministic generator matrix construction in polar codes allows to i) efficiently en-/decode with complexity O(kp), and ii) construct codes that perform well in multicast, something impossible with fountain codes due to their randomness.

V. ENERGY-EFFICIENT, MULTICAST HARQ

The dominant operation in the HARQ complexity at the transport layer is the matrix-vector multiplication—see Section IV—and therefore the efforts to reduce the energy consumption should focus on it. Processing a column in MDS codes entails the multiplication of k symbols and the results of these multiplications are then xored. All these operations can be implemented as the summation of 8-bit unsigned integers [6], resulting in 2k − 1 operations per column. This process is iterated 1500 times to en-/decode all the bytes in the MTU, although several instructions can concurrently run when executed on CPUs supporting SIMD instruction sets. On the other hand, binary codes only require on average \( \frac{k−1}{2} \) xors per column, as no multiplications are needed due to the binary matrix. The binary nature of these codes also allows the grouping of several bits into a single instruction, thereby reducing the required instructions to process a packet and hence the delay and energy consumption. For example, 32-bit CPUs can reduce the required instructions per packet xoring 32 bits together. Although increasing the field order results in a similar effect in MDS codes, this comes at the expense of large memory usage as the complete field is stored in memory to avoid modulo operations [6].

Unlike MDS codes, binary codes do not include information from all source packets in each parity packet. This is a consequence of operating in \( GF(2) \), as \( G \) includes 0s to produce linearly independent columns. Although this reduces the complexity, it is undesirable for multicast, where the receivers may experience different erasure patterns. The column degree is a measure of parity packet generalizability: while the most general column has all bits to one, the degree one columns are the least general ones. In a multicast deployment, the average degree of the generator matrix should be maximized—as long as linearly independent columns can still be generated—and each new column should maximize the number of correctable erasure patterns. Therefore, for binary codes to be suitable for energy-efficient, multicast-enabled HARQ at the transport layer, they should fulfill that i) the number of excess packets is low for short block lengths, and ii) a mechanism can be found to maximize the number of satisfied receivers in each transmission round.

VI. MULTICAST POLAR CODES

It can be observed that the 4 × 4 polar generator matrix \( G \) below has a persymmetric structure—i.e., the matrix is symmetric with respect to the upper-right to the lower-left diagonal. While the last row corresponds to the largest temporal dispersion, conversely, the first column is the column with the largest dispersion over receivers in a multicast group—a.k.a. the most generalizable parity packet. Therefore, the same Bhattacharyya parameter used to calculate the reliability of each channel can merely be flipped to measure the quality of each column for multicast. We use this property to generate incremental redundancy for multicast. In order to do so, a mechanism is required that transforms the code into systematic form such that the parity part \( P \) and the original matrix \( G \) share those columns with the lowest Bhattacharyya parameter.

\[
G = \begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1
\end{bmatrix}
\]

Assume a vector \( \mathcal{A}_i \) containing the indices of the k best channels in decreasing quality order and the complementary vector \( \mathcal{A}_i^C \) with the n − k remaining channels in increasing
quality order. For example, for the (16, 8) code in Section VII-C the indices of the channels in decreasing quality are [16, 15, 14, 12, 8, 13, 11, 10, 7, 6, 4, 9, 5, 3, 2, 1], so that $A_i = [16, 15, 14, 12, 8, 13, 11, 10]$ and $A_i^C = [1, 2, 3, 5, 9, 4, 6, 7]$. The proposed mechanism to turn $G$ into systematic form is as follows: the worst channels are used for the frozen bits, meaning that the rows in $A_i$ contribute a 0 to the xor. However, instead of using the channels in $A_i$ as usual, the precode turns these channels into the systematic part. The $k$ columns in $A_i$ are iteratively substituted by the columns in $I$ the $k \times k$ identity matrix—due to the fact that $G^{-1} = G$ for polar codes this is the same as pre-multiplying with $G(A_i, A_i)$. Finally, the reservoir of parity packets consists of the columns in $A_i^C$. The persymmetric structure of the generator matrix means that ordering the channels in decreasing quality equals ordering the columns in increasing generalizability. Therefore, the parity columns in the reservoir should be transmitted in the order given by $A_i^C$ to maximize the HARQ performance in multicast.

VII. EVALUATION

This section evaluates the proposed polar coding in terms of the required excess packets, encoding and decoding delay on an embedded platform and loss recovery in a multicast group.

A. Methodology

For a fair comparison between the codes, we analyze their performance when meeting a target packet loss rate ($PLR_T$). Assuming an i.i.d. BEC with erasure probability $p_e$, the packet loss rate can be calculated with Eq. 1.

$$PLR_{BEC} = \frac{1}{k} E[I_k] = \frac{1}{k} \sum_{i=1}^{k} i \cdot P(I_k = i)$$ (1)

$P(I_k = i)$ is the probability of $i$ erasures and it depends on the used code. The expression for MDS codes is given in Eq. 2 where $p_d(e)$ is the probability that exactly $i$ out of $e$ erasures in a block fall into the systematic part.

$$P(I_k = i) = \sum_{e = \max(n-k+1,i)}^{n-k+i} \binom{n}{e} \cdot p_e^e \cdot (1-p_e)^{n-e} \cdot p_d(e)$$ (2)

$$p_d(e) = \frac{k \cdot (n-k-e) \cdot \binom{n-e}{e-i}}{\binom{k}{i}}$$ (3)

The same probability for random fountain codes is given in Eq. 2, where $\delta_e$ is the probability that the decoding fails after $e$ erasures. While the decoding always fails when less than $k$ packets are received, fountain codes also have a decoding failure probability when enough packets are received. We have used the upper bound to calculate this probability, which converges quickly to the true value for $k > 10$ [10].

$$P(I_k = i) = \sum_{e=1}^{n-k+i} \delta_e \cdot \binom{n}{e} \cdot p_e^e \cdot (1-p_e)^{n-e} \cdot p_d(e)$$ (4)

| Code $(n,k)$ | (8,4) | (8,6) | (16,8) | (16,10) | (16,12) |
|-------------|------|------|-------|--------|-------|
| Error ($\times 10^{-5}$) | 4.608 | 4.788 | 2.805 | 4.190 |

TABLE I: PLR estimation error for polar coding when using simulated BEC in comparison to the ground-truth. The BEC has been simulated with $R = 50,000$ receivers.

To the best of our knowledge, there is no known analytical expression $P(I_k = i)$ for polar codes. Therefore, we empirically calculated it by constructing the matrix, emulating $R$ receivers and checking how many of them could recover from losses. The ground truth has been calculated for short block lengths to select the number of receivers $R = 50,000$, which achieves an estimation error two orders of magnitude below the used PLR (see Table I).

In the following, both classes of codes are analyzed considering four metrics: i) amount of redundancy, ii) encoding and decoding delay, iii) energy consumption and iv) multicast performance. A Raspberry Pi Zero W running the Raspbian Buster operating system with Linux kernel 4.19 has been used as the evaluation platform and the energy consumed by it has been measured with an LTC2991 board. For the MDS code, we have used the C implementation in [6], whereas binary codes have been implemented in Rust.

B. Redundancy, Delay and Energy Consumption

For the estimation of the transmission overhead, a WiFi-like deployment with $p_e = 0.05$ is considered as well as two different reliability requirements $PLR_T = \{0.01, 0.001\}$. Figure 2 shows that the proposed polar coding clearly outperforms fountain codes for short block lengths. Although for $PLR_T = 0.001$ and $k > 38$ there are some cases in which polar codes require the most parity packets, the validity of the proposed approach still depends on the ratio between transmission delay, and encoding and decoding delay. The latter are shown in the lower two plots in Figure 2, where both codes correct $e = 8$ erasures to consider the worst-case delay in MDS and perform a fair comparison between them. In this experiment, only random fountain codes have been considered because, despite achieving the same $PLR_T$, the proposed polar code fails to decode when the number of erasures is large ($e \geq 4$), which also explains its poor performance for larger block lengths. However, as the underlying coding operations are the same, fountain codes already show the benefits of binary codes. For $PLR_T = 0.01$ the binary code achieves $3.5 \times$ lower delay, whereas $5 \times$ reduction is achieved for $PLR_T = 0.001$. Considering these delay differences, binary codes require at least 15 $Mb/s$ to compensate such an excess—$7.5 \Mb/s$ when both ends are embedded devices—, 2

https://git.root.uni-saarland.de/open-access/ccnc2022
which is small in comparison to the data rates in 5G and WiFi-6. On top of that, these communication technologies are more energy-efficient than previous generations [22], [23], which emphasizes the relevance of codes that reduce the CPU load.

Assuming a constant CPU power draw, the delay difference should produce an equivalent energy reduction. This hypothesis is tested by measuring the energy consumption for different block lengths, \( PLR_T = \{0.01, 0.001\} \) and \( p_e = 0.05 \) (see Figure 3). Each code, including encoding and decoding, is iterated 100 times to produce visible results above the background energy when the system is idle so that the energy consumed by the codes can be isolated using a smoothing average of 30 samples and discarding the samples below a certain threshold (the middle point between minimum and maximum is typically a good threshold and thus it has been used for the results here presented). In addition to the coding operations, each measurement also considers the transmission overhead in binary codes. The iperf3 tool has been used to transmit over UDP 100 times the parity packet excess. Figure 3 depicts the average energy consumption of 10 executions for each block length, including the standard deviation as error bars to ensure that the achieved differences are statistically significant. The background energy has been subtracted for the calculation of the total energy in order to only represent the energy excess when en-/decoding or transmitting. As expected, the energy consumption follows the same ratio between codes as the delay in Figure 2. As the block length is reduced, so is the coding energy consumption, whereas the transmission overhead stays roughly the same. The implication of such a result is twofold: i) as the transmission overhead accounts for most of their energy consumption, binary codes that reduce it in the short block length regime, such as the proposed polar code, are required, and ii) MDS might potentially be the optimal coding technique in some cases. Therefore, an energy-aware FEC function is required for the selection of the coding mechanism that meets the application’s target loss rate and delay with minimum energy consumption.

C. Multicast Performance

For the comparison between MDS codes and the proposed polar code, a multicast group has been simulated with as many receivers as possible erasure patterns for \( i \in [1, e] \) erasures in a block—\( e \) here is the number of erasures the reference MDS code can correct. After the transmission of each packet, every receiver performs Gaussian elimination to check how many erasures it can correct. Since each erasure pattern has a different probability, each receiver is weighted differently depending on the number of erasures it originally experienced. Figure 4 shows the Cumulative Distribution Function (CDF) for three different coding configurations and two erasure rates \( p_e = \{0.01, 0.05\} \). Fountain codes are not depicted because the amount of correctable patterns always depends on the selected columns due to their randomness—e.g., after the first parity packet is transmitted only 50% of the receivers on average could recover their losses.

Although the performance of polar code is close to MDS, a closer look shows that there is still room for improvement. Ordering the columns according to the Bhattacharyya parameter results in decreasing column degree. For example, for \( n = 16 \), \( k = 10 \) the degrees of the columns are \( [16, 8, 8, 8, 8, 8, 4] \). However, the frozen positions do not contribute any information to the parity packets, so that the column degree is reduced in practice when there is a one in a frozen bit position. For the previous example, the degrees are reduced to \( [10, 6, 6, 6, 7, 7, 3] \). Enforcing decreasing degrees can lead to code-rate dependent reordering that sacrifices the systematic code construction. Since the degree difference is small, so is the expected gain of re-sorting and hence it is not implemented.

VIII. Conclusion and Future Work

The energy aspects of data transmission become fundamental as more NCSs are deployed on embedded devices. Error coding at the transport layer can provide timeliness with predictable reliability and we have shown in this paper that binary codes are an energy-efficient option when their error handling performance is brought close to MDS codes.
The proposed polar coding outperforms other binary codes in the short-blocklength regime and approaches MDS codes in multicast, thereby resulting in an alternative to efficient HARQ for transport layer multicast.

The results presented here show that binary codes fail to achieve good performance when the required code rates are low. In that case, MDS codes will potentially be required since they generate lower transmission overhead. The selection of the most efficient coding technique is still an open problem and an energy-aware decision considering the energy consumption for the redundancy computation and transmission could be an interesting solution.

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