Modelling uncertainties with TOPSIS and GRA based on q-rung orthopair m-polar fuzzy soft information in COVID-19

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Abstract
Fuzzy hybrid models are strong mathematical tools to address vague and uncertain information in real-life circumstances. The aim of this article is to introduce a new fuzzy hybrid model named as of q-rung orthopair m-polar fuzzy soft set (q-RO-m-PFSS) as a robust fusion of soft set (SS), m-polar fuzzy set (m-PFS) and q-rung orthopair fuzzy set (q-ROFS). A q-RO-m-PFSS is a new approach towards modelling uncertainties in the multi-criteria decision making (MCDM) problems. Some fundamental operations on q-RO-m-PFSSs, their key properties, and related significant results are introduced. Additionally, the complexity of logistics and supply chain management during COVID-19 is analysed using TOPSIS (technique for ordering preference through the ideal solution) and GRA (grey relational analysis) with the help of q-RO-m-PFS information. The linguistic terms are used to express q-RO-m-PFS information in terms of numeric values. The proposed approaches are worthy efficient in the selection of ventilator's manufacturers for the patients suffering from epidemic disease named as COVID-19. A practical application of proposed MCDM techniques is demonstrated by respective numerical examples. The comparison analysis of the final ranking computed by proposed techniques is also given to justify the feasibility, applicability and reliability of these techniques.

KEYWORDS
GRA, linguistic terms, MCDM, operations on q-RO-m-PFSSs, q-rung orthopair m-polar fuzzy soft sets, TOPSIS

1 | INTRODUCTION

The multi-criteria decision making (MCDM) techniques have been rigorously investigated by many researchers around the sphere of the real world. This pursuit gave rise to many resourceful techniques to deal with real world problems. The methodologies developed for this objective essentially rely on the description of the problem under contemplation. The problems of imperfect, uncertain and vague information have been focused by many researchers in the last decades. To deal with uncertainties and vagueness, Zadeh (1965) instigated fuzzy sets (FSs), Atanassov (1986) established intuitionistic fuzzy sets (IFSs), Molodtsov (1999) inaugurated soft sets (SSs), and Yager (2013, 2014) and Yager and Abbasov (2013) presented Pythagorean fuzzy set (PFS).

Yager (2017) introduced the idea of a q-rung orthopair fuzzy sets (q-ROFSs) which is superior to IFSs and PFSs. A q-rung orthopair fuzzy number (q-ROFN), suggested by Zhang and Xu (2014b), is superior to intuitionistic fuzzy number (IFN) and Pythagorean fuzzy number (PFN). In this way, every IFN and PFN is always a q-ROFN but not conversely. Liu and Wang (2018) introduced MADM method for q-ROFNs using q-rung orthopair fuzzy aggregation operators. Ali (2018) proposed some new aspects of q-ROFSs in terms of L-fuzzy sets and orbits of q-ROFNs. Zhang (1994) introduced the notion of a bipolar fuzzy set (BFS) and its relations. Dey et al. (2016) discussed the TOPSIS (technique for ordering
Algorithm 2 is devoted to cover rudimentary concepts of P-m-PFS, P-m-IFs, and their basic operations. Chen et al. (2021) introduced m-polar fuzzy soft sets (m-PFSSs). Akram (2019) introduced m-polar fuzzy graphs: theory, methods and applications to decision making.

The existing IFNs, PFNs, BFNs and q-ROFNs have been explored by many researchers. Akram, Dudek, and Dar (2019) and Akram, Dudek, and Ilyas (2019) introduced decision-making methods based on Pythagorean fuzzy TOPSIS method and Pythagorean Dombi fuzzy aggregation operators. Garg (2021a, 2021b) and Garg and Rani (2021a, 2021b) introduced various aggregation operators for MCDM problems. Hashmi et al. (2020) m-polar neutrosophic topology with applications to MCDM in medical diagnosis and clustering analysis. Karaaslan (2015) introduced the notion of neutrosophic soft sets and their applications in decision making. Karaaslan and Hunu (2020) introduced TOPSIS method with type-2 single-valued neutrosophic sets and their applications in MCDM. The authors in Chen (2000) and Chen and Tsao (2008) presented the TOPSIS method under fuzzy environment.

Peng et al. (2017) and Peng and Yang (2015) introduced some results on Pythagorean fuzzy information measures and their applications in MCDM. Saha et al. (2021) presented the concept of probabilistic linguistic q-rung orthopair fuzzy sets for group decision-making problems. Riaz, Pamucar, et al. (2020) proposed the concept of q-rung orthopair fuzzy information aggregation using Einstein operations with application to sustainable energy planning decision management. They defined q-Rung orthopair fuzzy prioritized aggregation operators and their application towards green supplier chain management. Riaz and Hashmi (2019) introduced the concept of linear Diophantine fuzzy set (LDFS) and its applications towards MADM. Kumar and Garg (2018) introduced TOPSIS method based on the connection number of set pair analysis under interval-valued intuitionistic fuzzy set environment. Li and Nan (2011) presented an extension of the TOPSIS for multi-attribute group decision making under Atanassov IFS environments. Selvachandran and Peng (2018) developed a modified TOPSIS method based on vague parameterized vague soft sets and its application to supplier selection problems. Yang et al. (2021) investigate the decision-making problems using multiple heterogeneous relationships of q-rung orthopair.

A rank reversal phenomenon in grey relational analysis (GRA) based network selection method is presented by Huszak and Imre (2010). Wei (2011a) introduced the GRA method for intuitionistic fuzzy MADM. Wei (2011a) further introduced GRA model for dynamic hybrid MCDM. Wang et al. (2021) studied assessment of express service quality with Pythagorean fuzzy interactive Hamacher power aggregation operators using entropy weight. Wang and Li (2020) presented multiple attribute decision making (MADM) based on Pythagorean fuzzy interaction power Bonferroni mean aggregation operators. Wang and Garg (2021) introduced a new algorithm for MADM with interactive Archimedean norm operations under Pythagorean fuzzy uncertainty. Recently, Naeem et al. (2019) presented the idea of Pythagorean m-polar fuzzy set (P-m-PFS) and TOPSIS method for the selection of advertisement mode. Riaz, Naeem, and Afzal (2020) established the concept to Pythagorean m-polar fuzzy soft set (P-m-PFSS). They developed an extended TOPSIS method for multi-criteria group decision making (MCGDM). Riaz et al. (2021) proposed the notion of q-rung orthopair m-polar fuzzy set (q-RO-m-PFS) and its application towards agri-robotic farming.

The main objectives and advantages of the manuscript are listed as follows:

1. The main objective of this paper is to introduce q-rung orthopair m-polar fuzzy soft set (q-RO-m-PFSS) as a hybrid model of SS, m-PFS and q-ROFS. A q-RO-m-PFSS is a new approach towards uncertainty which is superior to existing fuzzy models like IFS, PFS, SS, m-PFS, and hybrid models like intuitionistic m-polar fuzzy set (I-m-PFS), Pythagorean m-polar fuzzy soft set (P-m-PFSS), and q-rung orthopair m-polar fuzzy set (q-RO-m-PFS). Moreover, these existing models, IFS, PFS, q-ROFS, SS, m-PFS, I-m-PFS, P-m-PFSS, and q-RO-m-PFS become special cases of proposed model.
2. A secondary objective of this paper is to deal with uncertainties in the real-life problems with parametrization, multi-polarity, and pairs of MGs and NMGs which become necessary in some MCDM problems.
3. We introduce some fundamental operations on q-RO-m-PSSs, their key properties, and related significant results. Suggested operations are helpful to study q-rung orthopair fuzzy soft set theory. Moreover, we define the notion of q-rung orthopair m-polar fuzzy numbers (q-RO-m-PFNs) to investigate various real-life MCDM problems.
4. In order to find an optimal decision, we inaugurate robust extensions of TOPSIS and GRA using q-RO-m-PFS information. We develop Algorithm 1 and Algorithm 2 for modelling uncertainties in MCDM problems. We present a practical application in the selection of ventilator’s manufacturers for the patients suffering from epidemic disease named as COVID-19. The selection of ventilators is a very sensitive issue that may involve vague and uncertain information which can be easily handled by the proposed hybrid model of q-RO-m-PFSS.
5. Proposed MCDM techniques are demonstrated by respective numerical illustrations. The comparison analysis of the final ranking computed by proposed techniques is also given to justify the feasibility, applicability and reliability of these techniques.

To bring smoothness in this study, this paper is managed as follows: Section 2 is devoted to cover rudimentary concepts of P-m-PFS, P-m-PFSS, and q-RO-m-PFS, and so forth, that would be helpful in analysis in the remaining part of the paper. Section 3 introduces the novel concept of q-rung orthopair m-polar fuzzy soft sets along with associated fundamental operations and their related results. Extension of TOPSIS method towards q-RO-m-PFS information for MCDM is introduced in Section 4. In Section 5, an extension of grey relational analysis (GRA) method under q-RO-m-PFS information is established for MCDM. The utilization of proposed MCDM methods is supported with the assistance of numerical
ALGORITHM 1

(Extension of TOPSIS to q-RO-m-PFSS)

Step 1: Analyse the real situation of COVID-19. This means that the real data is collected, summarized, analysed and ranking of feasible alternatives is determined. The final ranking of alternatives and optimal alternative(s) are affected due to pandemic situation of the world. This is so because the DMs have assigned membership grades and non-memberships grades of the objects according to the real situation of spread of COVID-19. We suppose that $V = \{ q_i : i = 1, 2, \ldots, n \}$ is the finite set of alternatives, $D = \{ d_j : j = 1, 2, \ldots, m \}$ are decision makers (DMs) and $E = \{ e_j : j = 1, 2, \ldots, k \}$ is a set of attributes. Then the $(i, j)^{th}$ entry of the q-RO-m-PFSS matrix illustrates a set of $m$ polar fuzzy numbers $m$ PFNs allocated to $j^{th}$ alternative related to $k^{th}$ attribute. Furthermore, $i^{th}$ q-ROFN in the set at $(j, k)^{th}$ position are the membership and non-membership functions, independently.

Step 2: Establishing the weighted parameterized matrix $F$ as

$$ F = [w_{jk}]_{m \times s} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1s} \\ w_{21} & w_{22} & \cdots & w_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ w_{m1} & w_{m2} & \cdots & w_{ms} \end{bmatrix} $$

Step 3: Constructing the normalized weighted matrix

$$ \tilde{F} = \left[ \frac{w_{jk}}{\sqrt{\sum_{i=1}^{s} w_{ij}^2}} \right]_{m \times s} $$

Here $\tilde{w}_{jk} = \frac{w_{jk}}{\sqrt{\sum_{i=1}^{s} w_{ij}^2}}$ and getting a weighted row matrix $\mathcal{V} = (w_1, w_2, \ldots, w_s)$, where $w_j = \frac{w_j}{\sum_{i=1}^{m} w_{ij}}$ and $w_k = \frac{1}{s} \sum_{i=1}^{s} \tilde{w}_{jk}$.

Step 4: Processing for Q-RO-M-PFSS decision matrix

$$ \mathcal{Y} = \left[ \xi_{jk} \right]_{m \times s} = \begin{bmatrix} \xi_{11} & \xi_{12} & \cdots & \xi_{1s} \\ \xi_{21} & \xi_{22} & \cdots & \xi_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \xi_{m1} & \xi_{m2} & \cdots & \xi_{ms} \end{bmatrix} $$

Here $(j, k)^{th}$ element of the q-RO-m-PFSS matrix that is $\xi_{jk}$ denotes a collection of $m$ PFNs designated to $j^{th}$ alternative w. r. to $k^{th}$ attribute.

Step 5: Formulate a weighted q-RO-m-PFSS decision matrix $\mathcal{Y}$ by multiplying every entry in the $k^{th}$ column by $k^{th}$ weight that we have obtained in step 3, for all $k$ varying from 1 to $s$.

Step 6: Processing ahead for q-RO-m-PFSS, $+$-ve ideal solution (PIS) and $-$-ve ideal solution (NIS), by putting in order

$$ q - \text{ROFSV} - \text{PIS} = \{ y_1^+, y_2^+, \ldots, y_n^+ \} $$

$$ = \{ (\vee_k \mu_{y_{k1}}, \wedge_k \nu_{y_{k1}}) : k = 1, 2, \ldots, n \} $$

and

$$ q - \text{ROFSV} - \text{NIS} = \{ y_1^-, y_2^-, \ldots, y_n^- \} $$

$$ = \{ (\wedge_k \mu_{y_{k1}}, \vee_k \nu_{y_{k1}}) : k = 1, 2, \ldots, n \} $$
examples and also well justified by a comparison analysis with some existing techniques. We summarize this research work with a concrete conclusion in Section 6.

2 | PRELIMINARIES

This section is allocated to review some basics of Pythagorean m-polar fuzzy set (P-m-PFS), Pythagorean m-polar fuzzy soft set (P-m-PFSS), and q-rung orthopair m-polar fuzzy set (q-RO-m-PFS) that are helpful in establishing the new ideas in this article. For Pythagorean fuzzy numbers and q-rung orthopair fuzzy numbers the readers are suggested to follow Liu and Wang (2018), Peng and Yang (2015), Yager (2013, 2014, 2017), Yager and Abbasov (2013) and Zhang and Xu (2014b). Some basic concepts of soft set theory and m-polar fuzzy set theory can be seen in Akram (2019), Chen et al. (2014) and Molodtsov (1999).

Definition 2.1. (Naeem et al., 2019) A P-m-PFS over the universal set $X$ is a set of mappings $\mu_j^P : X \to [0,1]$ generally known as membership functions (MFs) and $\nu_j^P : X \to [0,1]$ known as non-membership functions (NMFs) with the condition that $0 \leq \mu_j^P(\xi) + (1 - \mu_j^P(\xi))^2 \leq 1$, $j = 1, 2, \ldots, m$, where $\mu_j^P(\xi)$ and $\nu_j^P(\xi)$ denote the membership grades (MDs) and non-membership grades (NMDs), respectively.

A P-m-PFS may be expressed as

$$P = \{ \langle \mu_j^P(\xi), \nu_j^P(\xi) \rangle : \xi \in X, j = 1, 2, \ldots, m \}$$  \hspace{1cm} (1)

where

$$\langle \mu_j^P(\xi), \nu_j^P(\xi) \rangle = (\langle \mu_1^P(\xi), \nu_1^P(\xi) \rangle, \langle \mu_2^P(\xi), \nu_2^P(\xi) \rangle, \ldots, \langle \mu_m^P(\xi), \nu_m^P(\xi) \rangle)$$  \hspace{1cm} (2)

represents $m$ ordered pairs of Pythagorean fuzzy numbers.
ALGORITHM 2

(Extension of GRA to q-RO-m-PFSSs)

Step 1: Perceive the situation, it mean that the data we have and how to utilize this data: We suppose that \( V = \{ q_j : j = 1, 2, \ldots, n \} \) is the finite set of alternatives, \( D = \{ d_j : j = 1, 2, \ldots, m \} \) are decision makers (DMs) and \( E = \{ e_j : j = 1, 2, \ldots, k \} \) is a set of attributes. Then the \((j,k)\)th entry of the q-RO-m-PFSS matrix illustrate a set of \( m \) polar fuzzy numbers \( m \) PFNs allocated to \( j \)th alternative related to \( k \)th attribute. Furthermore, \( j \)th q-ROFN in the set at \((j,k)\)th position are the membership and non-membership functions, independently.

Step 2: Establishing the weighted parameterized matrix \( F \) as

\[
F = [w_{jk}]_{m \times s} = \begin{bmatrix}
W_{11} & W_{12} & \cdots & W_{1s} \\
W_{21} & W_{22} & \cdots & W_{2s} \\
\vdots & \vdots & \ddots & \vdots \\
W_{m1} & W_{m2} & \cdots & W_{ms}
\end{bmatrix}
\]

Step 3: Constructing the normalized weighted matrix

\[
\tilde{F} = [\tilde{w}_{jk}]_{m \times s} = \begin{bmatrix}
\tilde{W}_{11} & \tilde{W}_{12} & \cdots & \tilde{W}_{1s} \\
\tilde{W}_{21} & \tilde{W}_{22} & \cdots & \tilde{W}_{2s} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{W}_{m1} & \tilde{W}_{m2} & \cdots & \tilde{W}_{ms}
\end{bmatrix}
\]

Here \( \tilde{w}_{jk} = \frac{w_{jk}}{\sqrt{\sum w_{jk}^2}} \) and getting a weighted row matrix \( W = (w_1, w_2, \ldots, w_k) \), where \( w_j = \frac{w_{jk}}{\sum w_{jk}} \) and \( w_k = \frac{w_{jk}}{\sum w_{jk}} \).

Step 4: Processing for q-RO-m-PFSS decision matrix

\[
\mathbf{T} = [\tilde{s}_{ik}]_{n \times s} = \begin{bmatrix}
\tilde{s}_{11} & \tilde{s}_{12} & \cdots & \tilde{s}_{1s} \\
\tilde{s}_{21} & \tilde{s}_{22} & \cdots & \tilde{s}_{2s} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{s}_{n1} & \tilde{s}_{n2} & \cdots & \tilde{s}_{ns}
\end{bmatrix}
\]

Here \((j,k)\)th element of the q-RO-m-PFSS matrix that is \( \tilde{s}_{jk} \) denotes a collection of \( m \) PFNs designated to \( j \)th alternative w. r. to \( k \)th attribute.

Step 5: Here we have to compute grey relational coefficient (GRC): Before finding the GRC, we have to normalize the data. Supposing that we have \( n \) ventilator manufacturing companies \( V = \{ q_j : j = 1, 2, \ldots, n \} \) to be discussed and each company having \( m \) parameters, \( D = \{ d_j : j = 1, 2, \ldots, m \} \). The highest value \( h_j \) is represented as high\( \{d_j : j = 1, 2, \ldots, m\} \) and the least value \( l_j \) is represented as least\( \{d_j : j = 1, 2, \ldots, m\} \). We have now two options to choose least-the-better or highest-the-better by using these formulas:

\[
\Gamma^- (j) = \frac{h_j - d_j}{h_j - l_j} \in [0, 1]
\]

and

\[
\Gamma^+ (j) = \frac{d_j - l_j}{h_j - l_j} \in [0, 1]
\]

The value of (GRC) can be calculated by using the equation

\[
\text{GRC} = \frac{1}{\sum w_j |d_j - \bar{l}_j| + 1}
\]

Step 6: Rank the choices according to values of GRC. The company with the highest GRC is the most valuable manufacturing company.
Definition 2.2. (Riaz, Naeem, & Afzal, 2020) Assume that \( m \) is a natural number. Let \( E \) be a set of attributes and \( A = \{e_1, e_2, \ldots, e_n\} \) be a subset of \( E \). A P-m-PFSS over a classical set \( X \) is defined by

\[
\varphi_A = (\varphi; A) = \left\{ \left( e, (\xi, (\mu^{(1)}_{e}(\xi), \nu^{(1)}_{e}(\xi)), \ldots, (\mu^{(m)}_{e}(\xi), \nu^{(m)}_{e}(\xi))) \right) : e \in A, \xi \in X; j = 1, 2, \ldots, m \right\}
\]

where \( \varphi: A \rightarrow PmPFS(X) \) is a set-valued mapping and \( PmFS(X) \) is the family of all Pythagorean \( m \)-polar fuzzy sets over \( X \).

Definition 2.3. (Riaz et al., 2021) A q-rung orthopair \( m \)-polar fuzzy set (q-RO-m-PFS) over the classical set \( X \) is characterized by the mappings \( \mu^{(j)}_{e}: X \rightarrow [0, 1] \) and \( \nu^{(j)}_{e}: X \rightarrow [0, 1] \) representing MFs and NMFs, respectively, and it can be written as:

\[
\mathcal{R} = \left\{ \xi, (\mu^{(1)}_{e}(\xi), \nu^{(1)}_{e}(\xi)), (\mu^{(2)}_{e}(\xi), \nu^{(2)}_{e}(\xi)), \ldots, (\mu^{(m)}_{e}(\xi), \nu^{(m)}_{e}(\xi)) \right\} : \xi \in X; j = 1, 2, \ldots, m
\]

with the condition that \( 0 \leq \left( \mu^{(j)}_{e}(\xi) \right)^q + \left( \nu^{(j)}_{e}(\xi) \right)^q \leq 1 \), \((j = 1, 2, \ldots, m)\) and \( \xi \in X \), where

\[
\left( \mu^{(1)}_{e}(\xi), \nu^{(1)}_{e}(\xi) \right) = \left( \left( \mu^{(1)}_{e}(\xi), \nu^{(1)}_{e}(\xi) \right), \left( \mu^{(2)}_{e}(\xi), \nu^{(2)}_{e}(\xi) \right), \ldots, \left( \mu^{(m)}_{e}(\xi), \nu^{(m)}_{e}(\xi) \right) \right)
\]

represents \( m \) ordered pairs of q-rung orthopair fuzzy numbers.

In other words, a q-RO-m-PFS can be written as

\[
\mathcal{R} = \left\{ \xi, (\mu^{(1)}_{e}(\xi), \nu^{(1)}_{e}(\xi)), (\mu^{(2)}_{e}(\xi), \nu^{(2)}_{e}(\xi)), \ldots, (\mu^{(m)}_{e}(\xi), \nu^{(m)}_{e}(\xi)) \right\} : \xi \in X
\]

3 q-RUNG ORTHOPAIR m-POLAR FUZZY SOFT SETS

This section is allocated to introduce the novel concepts of q-rung orthopair \( m \)-polar fuzzy soft sets. We define some fundamental operations on q-rung orthopair \( m \)-polar fuzzy soft sets and investigate their key properties.

Definition 3.1. Let \( E \) be the set of attributes and \( A \subseteq E \). A q-rung orthopair \( m \)-polar fuzzy soft set (q-RO-m-PFSS) over \( X \) is expressed by the mapping \( \varphi: A \rightarrow qROmPFS(X) \), where qROmPFS(X) represents the family of all q-RO-m-PFSSs over \( X \).

Thus, a q-RO-m-PFSS can be written as follows:

\[
\varphi_A = \left\{ \left( e, \left( (\mu^{(1)}_{e}(\xi), \nu^{(1)}_{e}(\xi)), (\mu^{(2)}_{e}(\xi), \nu^{(2)}_{e}(\xi)), \ldots, (\mu^{(m)}_{e}(\xi), \nu^{(m)}_{e}(\xi)) \right) \right) : e \in A, \xi \in X; j = 1, 2, \ldots, m \right\}
\]

\[
= \left\{ \left( e, \left( (\mu^{(1)}_{e}(\xi)), (\nu^{(1)}_{e}(\xi)), (\mu^{(2)}_{e}(\xi)), (\nu^{(2)}_{e}(\xi)), \ldots, (\mu^{(m)}_{e}(\xi)), (\nu^{(m)}_{e}(\xi)) \right) \right) : e \in A, \xi \in X; j = 1, 2, \ldots, m \right\}
\]

Table 1: Tabular form of \( \varphi_A \)

| \( \varphi_A \) | \( e_1 \) | \( e_2 \) | \( \ldots \) | \( e_n \) |
|---|---|---|---|---|
| \( \xi_1 \) | \( \left\{ (\mu^{(1)}_{e_1}(\xi_1), (\nu^{(1)}_{e_1}(\xi_1)) \right\} \) | \( \left\{ (\mu^{(1)}_{e_2}(\xi_1), (\nu^{(1)}_{e_2}(\xi_1)) \right\} \) | \( \ldots \) | \( \left\{ (\mu^{(1)}_{e_n}(\xi_1), (\nu^{(1)}_{e_n}(\xi_1)) \right\} \) |
| \( \xi_2 \) | \( \left\{ (\mu^{(1)}_{e_1}(\xi_2), (\nu^{(1)}_{e_1}(\xi_2)) \right\} \) | \( \left\{ (\mu^{(1)}_{e_2}(\xi_2), (\nu^{(1)}_{e_2}(\xi_2)) \right\} \) | \( \ldots \) | \( \left\{ (\mu^{(1)}_{e_n}(\xi_2), (\nu^{(1)}_{e_n}(\xi_2)) \right\} \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( \xi_n \) | \( \left\{ (\mu^{(1)}_{e_1}(\xi_n), (\nu^{(1)}_{e_1}(\xi_n)) \right\} \) | \( \left\{ (\mu^{(1)}_{e_2}(\xi_n), (\nu^{(1)}_{e_2}(\xi_n)) \right\} \) | \( \ldots \) | \( \left\{ (\mu^{(1)}_{e_n}(\xi_n), (\nu^{(1)}_{e_n}(\xi_n)) \right\} \) |
If $\mathcal{A} = \{e_1, e_2, \ldots, e_n\} \subseteq E$ and $X$ contains $k$ objects then tabular form of $\varphi_\mathcal{A}$ is as given in Table 1.

We can write it in matrix form also as

$$\varphi_\mathcal{A} = \begin{bmatrix}
\{\mu_\mathcal{A}^{(j)}(e_1)(\xi_1), \nu_\mathcal{A}^{(j)}(e_1)(\xi_1)\} & \{\mu_\mathcal{A}^{(j)}(e_2)(\xi_1), \nu_\mathcal{A}^{(j)}(e_2)(\xi_1)\} & \cdots & \{\mu_\mathcal{A}^{(j)}(e_n)(\xi_1), \nu_\mathcal{A}^{(j)}(e_n)(\xi_1)\} \\
\{\mu_\mathcal{A}^{(j)}(e_1)(\xi_2), \nu_\mathcal{A}^{(j)}(e_1)(\xi_2)\} & \{\mu_\mathcal{A}^{(j)}(e_2)(\xi_2), \nu_\mathcal{A}^{(j)}(e_2)(\xi_2)\} & \cdots & \{\mu_\mathcal{A}^{(j)}(e_n)(\xi_2), \nu_\mathcal{A}^{(j)}(e_n)(\xi_2)\} \\
\vdots & \vdots & \ddots & \vdots \\
\{\mu_\mathcal{A}^{(j)}(e_1)(\xi_k), \nu_\mathcal{A}^{(j)}(e_1)(\xi_k)\} & \{\mu_\mathcal{A}^{(j)}(e_2)(\xi_k), \nu_\mathcal{A}^{(j)}(e_2)(\xi_k)\} & \cdots & \{\mu_\mathcal{A}^{(j)}(e_n)(\xi_k), \nu_\mathcal{A}^{(j)}(e_n)(\xi_k)\}
\end{bmatrix}
$$

This $k \times n$ matrix is known as q-rung orthopair $m$-polar fuzzy soft matrix. The family of all q-RO-m-PFSSs over $X$ may be represented by q-ROmPFSS($X$).

**Example 3.1.** We consider a classical set $X = \{d, f, g, h\}$ and $\mathcal{A} = \{e_1, e_2\} \subseteq E$, then the q-RO-m-PFSS with $m = 3$ can be written as

$$\varphi_\mathcal{A} = \begin{bmatrix}
ed_1 & \begin{bmatrix}d & f \\
ed_2 & \begin{bmatrix}d & f \end{bmatrix}
\end{bmatrix} & \begin{bmatrix}0.210,0.694],[0.321,0.549],[0.104,0.847] & [0.432,0.492],[0.570,0.458],[0.880,0.171] \\
0.732,0.200],[0.569,0.354],[0.102,0.810] & [0.413,0.521],[0.100,0.799],[0.777,0.122]
\end{bmatrix}
\end{bmatrix}
$$

The matrix form of $\varphi_\mathcal{A}$ is

$$\varphi_\mathcal{A} = \begin{bmatrix}
\{(0.210,0.694],[0.321,0.549],[0.104,0.847]\} & \{(0.732,0.200],[0.569,0.354],[0.102,0.810]\} \\
\{(0.432,0.492],[0.570,0.458],[0.880,0.171]\} & \{(0.413,0.521],[0.100,0.799],[0.777,0.122]\}
\end{bmatrix}
$$

**Definition 3.2.** We consider a universe set $\mathcal{X}$ and $\varphi_\mathcal{A}$ as a q-ROmPFSS over $X$. Let $e \in \mathcal{A} \subseteq E$.

1. The collection of the points $\xi$ in $\mathcal{X}$ for which $\mu_\mathcal{A}^{(j)}(e)(\xi) \neq 0$ or $\nu_\mathcal{A}^{(j)}(e)(\xi) \neq 1$, for at least one $j = 1, 2, \ldots, m$, is said to be support of $\varphi_\mathcal{A}$ may be shown as

$$\text{supp}(\varphi_\mathcal{A}) = \{\xi \in \mathcal{X} : \mu_\mathcal{A}^{(j)}(e)(\xi) \neq 0 \text{ or } \nu_\mathcal{A}^{(j)}(e)(\xi) \neq 1 \text{ for atleastone } j = 1, 2, \ldots, m\}.$$

2. A collection of the points $\xi$ in $\mathcal{X}$ for which $\mu_\mathcal{A}^{(j)}(e)(\xi) = 1$ and $\nu_\mathcal{A}^{(j)}(e)(\xi) = 0$, for at least one $j = 1, 2, \ldots, m$, is said to be core of $\varphi_\mathcal{A}$ may be shown as

$$\text{core}(\varphi_\mathcal{A}) = \{\xi \in \mathcal{X} : \mu_\mathcal{A}^{(j)}(e)(\xi) = 1 \text{ and } \nu_\mathcal{A}^{(j)}(e)(\xi) = 0 \text{ for atleast one } j = 1, 2, \ldots, m\}.$$

3. The maximum value of membership grade is known as height of $\varphi_\mathcal{A}$ and is termed as $\text{ht}(\varphi_\mathcal{A})$. A q-RO-m-PFSS $\varphi_\mathcal{A}$ is known as normal if $\text{ht}(\varphi_\mathcal{A}) = 1$ and is subnormal on the other hand.

**Example 3.2.** Let $\mathcal{X} = \{f, g, h\}$ and $\mathcal{A} = \{e_1, e_2\}$. Consider a q-RO-m-PFSS $\varphi_\mathcal{A}$ is given by

$$\varphi_\mathcal{A} = \begin{bmatrix}
\{(0.1330,0.600],[0.354,0.444],[0.357,0.370]\} & \{(0.555,0.322],[0.109,0.711],[0.222,0.689]\} \\
\{(0.000,1.000],[0.000,1.000],[0.000,1.000]\} & \{(0.000,1.000],[0.000,1.000],[0.000,1.000]\} \\
\{(1.000,0.000],[0.617,0.222],[0.334,0.600]\} & \{(0.277,0.455],[0.223,0.587],[0.700,0.277]\}
\end{bmatrix}
$$

then it is easy to calculate the support, core and height of q-RO-m-PFSS as follows:
1. \(\text{supp}(\varphi_A) = \{f, h\}\).
2. \(\text{core}(\varphi_A) = \{h\}\).
3. \(\text{ht}(\varphi_A) = 1\). So that the q-RO-m-PFSS \(\varphi_A\) is normal.

**Definition 3.3.** We consider two sets \((\varphi_1, A_1)\) and \((\varphi_2, A_2)\), where both are q-RO-m-PFSSs upon \(X\) with \(A_1, A_2 \subseteq \mathcal{E}\). We recognize them as \((\varphi_1, A_1)\) is a subset of \((\varphi_2, A_2)\), represented by \((\varphi_1, A_1) \subseteq (\varphi_2, A_2)\) if

1. \(A_1 \subseteq A_2\)
2. \(\mu^{\varphi_1}_x(e)(\xi) \leq \mu^{\varphi_2}_x(e)(\xi)\), and
3. \(\nu^{\varphi_1}_x(e)(\xi) \geq \nu^{\varphi_2}_x(e)(\xi)\)

for all \(e \in A, \xi \in X\) and permissible values of \(j\).

To illustrate the Definition 3.3, we provide a numerical example as below.

**Example 3.3.** Consider a universe set \(X = \{f, g, h\}\) and \(\varphi_A\) be a q-RO-m-PFSS upon \(X\). Let \(A_1 = \{e_1, e_2\}, A_2 = \{e_2\} \subseteq \mathcal{E}\) and \((\varphi_1, A_1), (\varphi_2, A_2)\) be q-RO-m-PFSSs. The rating values of the q-RO-m-PFSSs are given in Table 2. Then, it can be easily obtain from Definition 3.3 that \((\varphi_2, A_2) \subseteq (\varphi_1, A_1)\).

**Definition 3.4.** A q-RO-m-PFSS over \(X\) is called a null q-RO-m-PFSS if \(\mu^{\varphi}_x(e)(\xi) = 0\) and \(\nu^{\varphi}_x(e)(\xi) = 1\), for all \(e \in \mathcal{E}, \xi \in X\) and \(j = 1, 2, \ldots, m\). It is denoted by \((\Phi, E)\) or \(\Phi\).

**Definition 3.5.** A q-RO-m-PFSS over \(X\) is called an absolute q-RO-m-PFSS if \(\mu^{\varphi}_x(e)(\xi) = 1\) and \(\nu^{\varphi}_x(e)(\xi) = 0\), for all \(e \in \mathcal{E}, \xi \in X\) and \(j = 1, 2, \ldots, m\). It is denoted by \((X, \mathcal{E})\) or \(X\).

**Proposition 3.1.** If \((\varphi, A)\) is any q-RO-m-PFSS over \(X\), then \((\varphi, E) \subseteq (\varphi, A) \subseteq (X, \mathcal{E})\).

**Remark 3.1.** From the above proposition it is clear that \((\varphi, E)\) is the smallest and \((X, \mathcal{E})\) is the greatest q-RO-m-PFSS upon \(X\).

**Definition 3.6.** 1. The union of two q-RO-m-PFSSs \((\varphi_1, A_1)\) and \((\varphi_2, A_2)\) upon the same universal set \(X\) is defined as

\[
(\varphi_1, A_1) \cup (\varphi_2, A_2) = \left\{ e, \left( \frac{\xi}{\max(\mu^{\varphi_1}_x(e)(\xi), \mu^{\varphi_2}_x(e)(\xi))}, \min(\nu^{\varphi_1}_x(e)(\xi), \nu^{\varphi_2}_x(e)(\xi)) \right) : e \in A_1 \cup A_2, \xi \in X, j = 1, 2, \ldots, m \right\}
\]

**TABLE 2** Rating values of the q-RO-m-PFSSs \((\varphi_1, A_1)\) and \((\varphi_2, A_2)\)

| Rating value of q-RO-m-PFSS \((\varphi_1, A_1)\) | \(e_1\) | \(e_2\) |
|---|---|---|
| \(f\) | \((0.623, 0.231),(0.500, 0.337),(0.433, 0.410)\) | \((0.536, 0.358),(0.430, 0.463),(0.520, 0.444)\) |
| \(g\) | \((0.747, 0.200),(0.519, 0.252),(0.498, 0.324)\) | \((0.611, 0.219),(0.628, 0.300),(0.549, 0.350)\) |
| \(h\) | \((0.800, 0.102),(0.636, 0.179),(0.582, 0.226)\) | \((0.783, 0.188),(0.704, 0.179),(0.649, 0.229)\) |

| Rating value of q-RO-m-PFSS \((\varphi_2, A_2)\) | \(e_2\) |
|---|---|
| \(f\) | \((0.309, 0.516),(0.403, 0.551),(0.533, 0.466)\) |
| \(g\) | \((0.329, 0.542),(0.404, 0.556),(0.534, 0.467)\) |
| \(h\) | \((0.362, 0.600),(0.421, 0.561),(0.000, 1.000)\) |
2. The intersection of two $q$-RO-$m$-PFSSs ($\phi_1, A_1$) and ($\phi_2, A_2$) upon the same universal set $X$ is defined as

$$(\phi_1, A_1) \cap (\phi_2, A_2) = \left\{ e \left| \xi \in X : j = 1, 2, \ldots, m \right\} : \min(\Omega_1(e)(\xi), \Omega_2(e)(\xi)) \geq \max(\Omega_1^b(e)(\xi), \Omega_2^b(e)(\xi)) \right\}$$

Example 3.4. Consider a classical set $X = \{d, f, g\}$ and a set of attributes $E$, $A_1 = \{e_1, e_2\}$ and $A_2 = \{e_2, e_3\} \subseteq E$, then we may represent $q$-RO-$m$-PFSSs ($\phi_1, A_1$) and ($\phi_2, A_2$) by Table 3.

Based on this information, we compute the union ($\phi_1, A_1) \cup (\phi_2, A_2$) and intersection ($\phi_1, A_1) \cap (\phi_2, A_2$) of ($\phi_1, A_1$) and ($\phi_2, A_2$) and the results are represented in Table 4.

**Proposition 3.2.** Let ($\phi, A$), ($\phi_1, A_1$), ($\phi_2, A_2$) and ($\phi_3, A_3$) be $q$-RO-$m$-PFSSs upon the universe $X$, then

1. ($\phi, A) \cup (\phi, A) = (\phi, A)$.
2. ($\phi, A) \cap (\phi, A) = (\phi, A)$.
3. ($X, E) \cup (\phi, A) = (X, E)$.
4. ($X, E) \cap (\phi, A) = (\phi, A)$.
5. ($\phi, E) \cap (\phi, A) = (\phi, A)$.
6. ($\phi, E) \cap (\phi, A) = (\phi, E)$.
7. ($\phi_1, A_1) \cup (\phi_2, A_2) = (\phi_2, A_2) \cup (\phi_1, A_1)$.
8. ($\phi_1, A_1) \cap (\phi_2, A_2) = (\phi_2, A_2) \cap (\phi_1, A_1)$.
9. $\{ \{(\phi_1, A_1) \cup (\phi_2, A_2) \} \cup (\phi_3, A_3) = (\phi_1, A_1) \cup (\phi_2, A_2) \cup (\phi_3, A_3) \}$.
10. $\{ \{(\phi_2, A_2) \cup (\phi_3, A_3) \} \cap (\phi_1, A_1) \cap (\phi_2, A_2) \cap (\phi_3, A_3) \} = (\phi_1, A_1) \cup (\phi_2, A_2) \cup (\phi_3, A_3)$.
11. $\{ (\phi_1, A_1) \cup (\phi_2, A_2) \} \cup (\phi_1, A_1) \cup (\phi_2, A_2)$.
12. $\{ (\phi_2, A_2) \cup (\phi_3, A_3) \} \cap (\phi_1, A_1) \cup (\phi_2, A_2)$.

**Proof.** Their proofs are very simple and we can prove them easily by definition.

**Corollary 3.1.**

1. $\Phi_X \cap X = X$.
2. $\Phi_X \cap X = \Phi_X$.

**Proposition 3.3.** Let ($\phi_1, A_1$) and ($\phi_2, A_2$) be $q$-RO-$m$-PFSSs upon $X$, then any one of them may be sandwiched between ($\phi_1, A_1) \cap (\phi_2, A_2$) and ($\phi_1, A_1) \cup (\phi_2, A_2$), that is

1. $\{ (\phi_1, A_1) \cup (\phi_2, A_2) \} \subseteq (\phi_1, A_1) \cup (\phi_2, A_2)$.
2. $\{ (\phi_1, A_1) \cap (\phi_2, A_2) \} \subseteq (\phi_1, A_1) \cup (\phi_2, A_2)$.

**Table 3**

| ($\phi_1, A_1$) | ($\phi_2, A_2$) | ($\phi_3, A_3$) |
|-----------------|-----------------|-----------------|
| ($d$)           | ($f$)           | ($g$)           |
| ($0.210,0.694,0.321,0.549,0.104,0.847$) | ($0.432,0.492,0.570,0.358,0.780,0.171$) | ($0.000,1.000$) |
| ($0.732,0.200,0.569,0.354,0.102,0.810$) | ($0.413,0.521,0.100,0.799,0.777,0.122$) | ($0.000,1.000,0.000,1.000,0.000,1.000$) |
| ($f$)           | ($g$)           | ($d$)           |
| ($0.000,1.000$) | ($0.000,1.000$) | ($0.000,1.000$) |
| ($0.000,1.000$) | ($0.000,1.000$) | ($0.000,1.000$) |
| ($0.000,1.000$) | ($0.000,1.000$) | ($0.000,1.000$) |
| ($0.413,0.521,0.100,0.799,0.777,0.122$) | ($0.413,0.521,0.100,0.799,0.777,0.122$) | ($0.000,1.000$) |
### TABLE 4  Union and intersection of q-RO-m-PFSSs ($\varphi_1, A_1$) and ($\varphi_2, A_2$)

#### Union ($\varphi_1, A_1 \cap \varphi_2, A_2$) of q-ROmPFSSs

|    | $e_1$                                                                 | $e_2$                                      | $e_3$                                      |
|----|-----------------------------------------------------------------------|--------------------------------------------|--------------------------------------------|
| $d$| {(0.210,0.694),(0.321,0.549),(0.104,0.847)}                            | {(0.732,0.200),(0.569,0.354),(0.110,0.810)}| {(0.745,0.222),(0.555,0.335),(0.145,0.834)}|
| $f$| {(0.432,0.492),(0.570,0.358),(0.780,0.171)}                            | {(0.413,0.521),(0.100,0.799),(0.777,0.122)}| {(0.000,1.000),(0.000,1.000),(0.000,1.000)}|
| $g$| {(0.000,1.000),(0.000,1.000),(0.000,1.000)}                            | {(0.432,0.492),(0.570,0.358),(0.780,0.171)}| {(0.413,0.521),(0.100,0.799),(0.777,0.122)}|

#### Intersection ($\varphi_1, A_1 \cap \varphi_2, A_2$) of q-ROmPFSSs

|    | $e_2$                                                                 |                                           |
|----|-----------------------------------------------------------------------|--------------------------------------------|
| $d$| {(0.231,0.594),(0.333,0.449),(0.102,0.822)}                            |                                           |
| $f$| {(0.000,1.000),(0.000,1.000),(0.000,1.000)}                            |                                           |
| $g$| {(0.000,1.000),(0.000,1.000),(0.000,1.000)}                            |                                           |
Proof. 1. As from the definition we know the fact, that $\max\left\{ \mu_1^{(0)/e}(e)(\xi), \nu_2^{(0)/e}(e)(\xi) \right\} \geq \mu_2^{(0)/e}(e)(\xi) \geq \min\left\{ \mu_1^{(0)/e}(e)(\xi), \nu_2^{(0)/e}(e)(\xi) \right\}$ and $\min\left\{ \mu_1^{(0)/e}(e)(\xi), \nu_2^{(0)/e}(e)(\xi) \right\} \leq \mu_1^{(0)/e}(e)(\xi) \leq \max\left\{ \mu_1^{(0)/e}(e)(\xi), \nu_2^{(0)/e}(e)(\xi) \right\}$.

2. The proof is obvious.

**Definition 3.7.** The complement of a q-RO-m-PFSS

$$\varphi_E = \left\{ e, \left\{ \frac{\xi}{\left( \mu_1^{(0)/e}(e)(\xi), \nu_2^{(0)/e}(e)(\xi) \right)} \right\} : e \in E, \xi \in X; j = 1, 2, \ldots, m \right\}$$

upon $X$ is represented as

$$(\varphi_E)^c = \left\{ e, \left\{ \frac{\xi}{\left( \nu_2^{(0)/e}(e)(\xi), \mu_1^{(0)/e}(e)(\xi) \right)} \right\} : e \in E, \xi \in X; j = 1, 2, \ldots, m \right\}$$

It is interesting to note that $(\Phi_E)^c = \bar{X}_E$ and $(\bar{X}_E)^c = \Phi_E$. Also, $(\varphi_E)^c = \varphi_E$.

**Example 3.5.** Consider $X = \{d, f, g\}$ and $A = \{e_1, e_2\} \subseteq E$, and a q-RO-m-PFSS $(\varphi_1, A_1)$ is defined in Table 3. Then, the complement of $(\varphi_1, A_1)$ is computed by using Definition 3.7 and the result is expressed in Table 5.

**Proposition 3.4.** Let $(\varphi_1, A_1)$ and $(\varphi_2, A_2)$ be q-RO-m-PFSSs upon $X$. As in classical sets De Morgan laws hold but here do not holds, that is

1. $((\varphi_1, A_1) \cap (\varphi_2, A_2))^c \neq (\varphi_1, A_1)^c \cap (\varphi_2, A_2)^c$.
2. $((\varphi_1, A_1)^c \cap (\varphi_2, A_2))^c \neq (\varphi_1, A_1)^c \cap (\varphi_2, A_2)^c$.

**Proposition 3.5.** If $\varphi_A$ is a q-RO-m-PFSS on $X$ and $A \subseteq E$, then

1. $\varphi_A \cup \varphi_A^c = \bar{X}_E$.
2. $\varphi_A \cap \varphi_A^c = \Phi_E$.

**Definition 3.8.** Let $(\varphi_1, A_1)$ and $(\varphi_2, A_2)$ be q-RO-m-PFSSs on the same universe $X$. The difference of two $(\varphi_1, A_1)$ and $(\varphi_2, A_2)$ may be defined as

$$(\varphi_1, A_1) \setminus (\varphi_2, A_2) = \left\{ e, \left\{ \frac{\xi}{\left( \max\left\{ \mu_1^{(0)/e}(e)(\xi), \nu_2^{(0)/e}(e)(\xi) \right\}, \min\left\{ \nu_2^{(0)/e}(e)(\xi), \mu_1^{(0)/e}(e)(\xi) \right\} \right)} \right\} : e \in A_1 \setminus A_2, \xi \in X; j = 1, 2, \ldots, m \right\}$$

**Example 3.6.** To illustrate the concept of difference by using Definition 3.8, we consider two q-RO-m-PFSSs as given in Example 3.4. Then their difference $(\varphi_1, A_1) \setminus (\varphi_2, A_2)$ is expressed in the Table 6.

---

**Table 5** Complement of q-RO-m-PFSS

| $e_1$            | $e_2$            |
|------------------|------------------|
| $(0.694,0.210), (0.549,0.321), (0.847,0.104)$ | $(0.200,0.732), (0.354,0.569), (0.810,0.102)$ |
| $(0.492,0.432), (0.358,0.570), (0.171,0.780)$ | $(0.521,0.413), (0.799,0.100), (0.122,0.777)$ |
| $(1.000,0.000), (1.000,0.000), (1.000,0.000)$ | $(1.000,0.000), (1.000,0.000), (1.000,0.000)$ |
Definition 3.9. Let \((\varphi_1, A_1)\) and \((\varphi_2, A_2)\) be q-RO-m-PFSSs on the same universe \(X\).

1. The sum of two \((\varphi_1, A_1)\) and \((\varphi_2, A_2)\) is defined as

\[
(\varphi_1, A_1) \oplus (\varphi_2, A_2) = \left\{ e, \left( \frac{\xi}{\sqrt{\left( \mu_{\varphi_1}(\xi) \mu_{\varphi_2}(\xi) \right)^a + \left( \nu_{\varphi_1}(\xi) \nu_{\varphi_2}(\xi) \right)^a - \left( \mu_{\varphi_1}(\xi) \nu_{\varphi_2}(\xi) \right) \cdot \nu_{\varphi_1}(\xi) \mu_{\varphi_2}(\xi) \cdot \nu_{\varphi_1}(\xi) \mu_{\varphi_2}(\xi) \cdot \nu_{\varphi_1}(\xi) \mu_{\varphi_2}(\xi)}} \right) \right\} : e \in A_1 \cup A_2, \xi \in X; j = 1, 2, \ldots, m).
\]

2. The product of two \((\varphi_1, A_1)\) and \((\varphi_2, A_2)\) is defined as

\[
(\varphi_1, A_1) \otimes (\varphi_2, A_2) = \left\{ e, \left( \frac{\xi}{\sqrt{\left( \mu_{\varphi_1}(\xi) \mu_{\varphi_2}(\xi) \right)^a + \left( \nu_{\varphi_1}(\xi) \nu_{\varphi_2}(\xi) \right)^a - \left( \mu_{\varphi_1}(\xi) \nu_{\varphi_2}(\xi) \right) \cdot \nu_{\varphi_1}(\xi) \mu_{\varphi_2}(\xi) \cdot \nu_{\varphi_1}(\xi) \mu_{\varphi_2}(\xi) \cdot \nu_{\varphi_1}(\xi) \mu_{\varphi_2}(\xi)}} \right) \right\} : e \in A_1 \cup A_2, \xi \in X; j = 1, 2, \ldots, m).
\]

Example 3.7. For two q-RO-m-PFSSs \((\varphi_1, A_1)\) and \((\varphi_2, A_2)\) as given in the Example 3.4. The sum \((\varphi_1, A_1) \oplus (\varphi_2, A_2)\) and product \((\varphi_1, A_1) \otimes (\varphi_2, A_2)\), for \(q = 3\), are computed by using Definition 3.9 and the results are expressed in Table 7.

Definition 3.10. Let \((\varphi, A)\) be a q-RO-m-PFSS over the universe \(X\), then the necessity operator \(\square\) on \((\varphi, A)\) is defined as

\[
\square (\varphi, A) = \left\{ e, \left( \mu_{\varphi}(e)(\xi) \right)^a \cdot \nu_{\varphi}(e)(\xi) \right\} : e \in A, \xi \in X; j = 1, 2, \ldots, m \right\}
\]

Definition 3.11. Let \((\varphi, A)\) be a q-RO-m-PFSS over the universe \(X\), then the possibility operator \(\Diamond\) on \((\varphi, A)\) is defined as

\[
\Diamond (\varphi, A) = \left\{ e, \left( \mu_{\varphi}(e)(\xi) \right)^a \cdot \nu_{\varphi}(e)(\xi) \right\} : e \in A, \xi \in X; j = 1, 2, \ldots, m \right\}
\]

Now we express these operators with the following example.

Example 3.8. Let us consider \(X = \{f, g, h\}\) and \(A = \{e_1, e_2\}\). Then a q-RO-m-PFSS \((\varphi, A)\) can be written as

\[
\begin{array}{|c|c|}
\hline
\varphi & \epsilon_1 \\
\hline
d & \{(0.210, 0.694), (0.321, 0.549), (0.104, 0.847)\} \\
f & \{(0.432, 0.492), (0.570, 0.358), (0.780, 0.171)\} \\
g & \{(0.000, 1.000), (0.000, 1.000), (0.000, 1.000)\} \\
\hline
\end{array}
\]
A Table 7: Ratings of the q-RO-m-PFSSs \((\varphi_1, A_1)\boxplus(\varphi_2, A_2)\) and \((\varphi_1, A_1)\boxdot(\varphi_2, A_2)\)

| q-RO-m-PFSS \((\varphi_1, A_1)\boxplus(\varphi_2, A_2)\) | \(e_1\) | \(e_2\) |
|----------------|----------------|----------------|
| \(d\) | \((0.308,0.412),(0.450,0.247),(0.151,0.696)\) | \((0.891,0.044),(0.729,0.119),(0.177,0.676)\) |
| \(f\) | \((0.432,0.492),(0.570,0.358),(0.780,0.171)\) | \((0.413,0.521),(0.100,0.799),(0.777,0.122)\) |
| \(g\) | \((0.432,0.492),(0.570,0.358),(0.780,0.171)\) | \((0.413,0.521),(0.100,0.799),(0.777,0.122)\) |

| q-RO-m-PFSS \((\varphi_1, A_1)\boxdot(\varphi_2, A_2)\) | \(e_1\) | \(e_2\) |
|----------------|----------------|----------------|
| \(d\) | \((0.049,0.779),(0.107,0.662),(0.011,0.938)\) | \((0.545,0.266),(0.316,0.431),(0.015,0.929)\) |
| \(f\) | \((0.000,1.000),(0.000,1.000),(0.000,1.000)\) | \((0.000,1.000),(0.000,1.000),(0.000,1.000)\) |
| \(g\) | \((0.000,1.000),(0.000,1.000),(0.000,1.000)\) | \((0.000,1.000),(0.000,1.000),(0.000,1.000)\) |

\((\varphi, A) = \left\{ \begin{array}{l}
\{ (0.1330.600),(0.354.0.444),(0.357.0.370)\} \\
\{ (0.000.1.000),(0.000.1.000),(0.000.1.000)\} \\
\{ (1.000.0.000),(0.617.0.222),(0.334.0.600)\}
\end{array} \right. \}
\{(0.555.0.322),(0.109.0.711),(0.222.0.689)\} \)

\((\varphi, A) = \left\{ \begin{array}{l}
\{ (0.1330.991),(0.354.0.935),(0.357.0.934)\} \\
\{ (0.000.1.000),(0.000.1.000),(0.000.1.000)\} \\
\{ (1.000.0.000),(0.617.0.787),(0.334.0.943)\}
\end{array} \right. \}
\{(0.555.0.832),(0.109.0.994),(0.222.0.975)\} \)

The relation between necessity operator and possibility operator is expressed in the next proposition.

**Proposition 3.6.** For a q-RO-m-PFSS \((\varphi, A)\) defined on \(X\), \(\square(\varphi, A) \subseteq \diamond(\varphi, A)\).

**Proof.** As we are taking each \(\xi \in X\), \(e \in A\) and for all permissible values of \(j\), so

\[
\left( \nu_{\mu}^{(j)}(e)(\xi) \right)^{q} + \left( \nu_{\phi}^{(j)}(e)(\xi) \right)^{q} \leq 1
\]

As well as

\[
\nu_{\mu}^{(j)}(e)(\xi) \leq \sqrt[2]{1 - \left( \nu_{\phi}^{(j)}(e)(\xi) \right)^{q}}
\]

**Corollary 3.2.** For any q-RO-m-PFSS \(\varphi_A\), we have the following results

1. \(\square\varphi_A \subseteq \diamond\varphi_A\)
2. \(\varphi_A \subseteq \diamond\varphi_A\)
Definition 3.12. If \((\varphi_1,A_1) = (\varphi_2,A_2)\), then we express \((\varphi_1,A_1) \circ (\varphi_2,A_2)\) that is \((\varphi_1,A_1) \circ (\varphi_1,A_1)\) by \((\varphi_1,A_1)^2\). Thus,

\[
(\varphi,A)^2 = \left\{ \left( e, \frac{\xi}{\left( \sqrt{\mu_{\varphi}(e)(\xi^2)}, \sqrt{1 - \left( \sqrt{\mu_{\varphi}(e)(\xi^2)} \right)^2} \right)} \right) : e \in A, \xi \in X; j = 1,2,\ldots,m \right\}
\]

This \((\varphi,A)^2\) is known as concentration of \((\varphi,A)\), abbreviated as \(\text{con}(\varphi,A)\).

Generally we say, if \(k \in [0,\infty)\), then

\[
(\varphi,A)^k = \left\{ \left( e, \frac{\xi}{\left( \sqrt{\mu_{\varphi}(e)(\xi^2)}, \sqrt{1 - \left( \sqrt{\mu_{\varphi}(e)(\xi^2)} \right)^2} \right)} \right) : e \in A, \xi \in X; j = 1,2,\ldots,m \right\}
\]

The set

\[
(\varphi,A)^d = \left\{ \left( e, \frac{\xi}{\left( \sqrt{\mu_{\varphi}(e)(\xi^2)}, \sqrt{1 - \left( \sqrt{\mu_{\varphi}(e)(\xi^2)} \right)^2} \right)} \right) : e \in A, \xi \in X; j = 1,2,\ldots,m \right\}
\]

is said to be dilution of \((\varphi,A)\), abbreviated as \(\text{dil}(\varphi,A)\).

Example 3.9. Let \(X = \{d,f,g,\} \) and \(A = \{e_1,e_2\} \subseteq E\), then we consider a q-RO-m-PFSS \(\varphi_A\) given by

\[
\varphi_A = \left\{ \{(0.210,0.694),(0.321,0.549),(0.104,0.847)\}, \{(0.732,0.200),(0.569,0.354),(0.102,0.810)\} \right\}
\]

\[
\{(0.432,0.492),(0.570,0.358),(0.780,0.171)\}, \{(0.413,0.521),(0.100,0.799),(0.777,0.122)\} \right\}
\]

\[
\{(0.000,1.000),(0.000,1.000),(0.000,1.000)\}, \{(0.000,1.000),(0.000,1.000),(0.000,1.000)\} \right\}
\]

The \(\text{con}(\varphi_A)\) and \(\text{dil}(\varphi_A)\) are given as follows

\[
\text{con}(\varphi_A) = \left\{ \{(0.044,0.855),(0.103,0.715),(0.011,0.959)\}, \{(0.536,0.280),(0.324,0.485),(0.010,0.939)\} \right\}
\]

\[
\text{dil}(\varphi_A) = \left\{ \{(0.458,0.529),(0.567,0.405),(0.323,0.684)\}, \{(0.855,0.142),(0.754,0.255),(0.319,0.643)\} \right\}
\]

\[
\{(0.657,0.359),(0.755,0.257),(0.883,0.121)\}, \{(0.643,0.383),(0.316,0.631),(0.881,0.086)\} \right\}
\]

\[
\{(0.000,1.000),(0.000,1.000),(0.000,1.000)\}, \{(0.000,1.000),(0.000,1.000),(0.000,1.000)\} \right\}
\]

4 | EXTENSION OF TOPSIS TOWARDS q-RO-m-PFS INFORMATION

This section is established for modelling uncertainties in MCDM using TOPSIS and GRA under q-RO-m-PFSS. The proposed model of q-RO-m-PFSS is used to express uncertain situation into a mathematical model and to seek a best possible solutions by means of positive (+ve) ideal solution and negative (−ve) ideal solution. Then a compromise solution is obtained which is nearer to +ve ideal solution and farthermost from the −ve ideal solution. For these objectives, the extension TOPSIS to q-RO-m-PFSS is established and justified by an application.
At the end of 2019, the world was progressing in every field of life that may be electronics, automobiles, industrialization, agriculture, computers, health and pharmaceutical advancements, space technology, and so forth, very smoothly and working day and night for the betterment of humanity. But what happened suddenly, that the whole mankind fell in a pandemic situation. This pandemic named as COVID-19 started in December 2019, and attacked the whole world and created a public unrest. COVID-19 is a viral epidemic of respiratory disease, as the virus infect the lungs and damage the immune system of the body. This infection may be in any one or may be in both lungs. The infection caused by this virus is known as pneumonia. Due to this infection the air sacs of the lungs become inflammable called alveoli. Fluid (pus) will fill up the alveoli, which produces a difficulty in breathing. The virus that produces pneumonia is much contagious, so it spreads from person to person. The pneumonia also spread to the people through inhalation by airborne droplets from sneezing or coughing. This pneumonia also spreads by touching a surface or object that has been already contaminated with corona virus. The major difference between viral and bacterial pneumonia is in treatment of such patients, as the infection produced by the virus do not respond to antibiotics. According to World Health Organization (WHO), the confirmed of COVID-19 from March 2020 to September 2020 are expressed in Figure 1.

COVID-19 is upsetting more or less 200 nation states and regions around the world including two international conveyances: ‘The Diamond Princess cruise ship’ harboured in Yokohama and the Holland America’s ‘MS Zaandam cruise ship’. This virus banquets predominantly through discharge from the nose or droplets of spittle when a disease-ridden person sneezes or coughs, so it is imperative that one should exercise respiratory etiquettes, for example, by coughing into an arched elbow. Till to date, there are no explicit serums or treatments for COVID-19. Though, there are several ongoing clinical trials assessing latent treatments.

![Confirmed COVID-19 cases details](https://example.com/figure1)

**FIGURE 1**  Confirmed COVID-19 cases details. *Source: WHO*
The World Health Organization (WHO) has reported that up to 80% patients of COVID-19 without admitting in the hospitals, and one out of six become seriously infected with corona virus.

The patients suffer short of breath from mild to critical and feel difficulty in taking in the oxygen and making it harder to breath. To overcome this situation a breathing machine called a ventilator is used to push the oxygen inside the lungs with required pressure to attain the required level. There is a provision of a humidifier in the ventilator that makes an addition of heat and moisture with the oxygen to attain the body temperature of the patient. To make the patient’s respiratory system relax, he is provided medicine so that the breathing by the machine should be regularized. Simply saying that when a patient feels trouble breathing or fails to breath, put him under the ventilator. The patient will get time to fight against the infection and recover. A ventilator is a machine that helps in breathing and is helpful in many life threatening situations. These breathing machines are complicated to run and operate. A trained and qualified professional is required to run these machines. A competent physician must be present to take care of the patients. There are two types of ventilators:

1. Invasive ventilators: This kind of ventilators are used generally in ICU’s, a tube is connected to the windpipe inside the patient’s body under general anaesthesia.
2. Non-invasive ventilators: This kind of ventilators are ordinary, in such ventilators a rubber mask is attached to the nostrils and mouth of the patient externally. Such ventilators having two levels of pressure, also called bi-level positive airway pressure (BiPAP). Again in these ventilators we have two types, that is, pneumatic ventilators and fluidic ventilators. The pneumatic ventilators are going to be replaced with computer-controlled ventilators.

Our working with Non-Invasive computer-controlled mechanical ventilators, which are carefully manufactured and properly designed, not to put the patients in danger. A community corporation requires ventilators for use in their hospitals throughout the country. For this the corporation has selected five companies worldwide for the purchase of such ventilators. A three members Board (DMs) have been announced for the selection of a ventilator manufacturing firm, that will supply the best and comparatively low price ventilators to the corporation. Meticulous Research has announced the top companies in the global ventilators market are Philip, Becton-Dickinson and Company, Koninklijke Philips N.V., Hamilton Medical AG, Fisher-Paykel Healthcare, Limited, Draegerwerk AG CO. KGaA, Medtronic PLC, GE Healthcare, Smiths Group PLC, ResMed Inc., Maquet Holding B.V. & Co. KG, Drager, Getinge, Hamilton Medical, Vyaire, Fisher and Paykel and so on. The demand for these breathing machines are increasing rapidly. A bar graph shows the requirement of ventilators in the coming years.

Extension of TOPSIS under q-RO-m-PFSSs is expressed by Algorithm 1 as given below.

Example 4.1. As an illustration, we present an application of MCDM problem by following the procedural steps of Algorithm 1. The lingual phrases and their weighted fuzzy numbers are given in the Table 8. The lingual phrases are selected for the judgement of alternatives from low preference to high preference and weighted fuzzy numbers can be chosen according to the significance of alternatives.

Step 1: Identify the real life situation.
Let $V = \{s_j : j = 1, 2, \ldots, 5\}$ be the collection of some companies for ventilator’s manufacturers for the patients suffering from epidemic disease of COVID-19. Let $E = \{e_j : j = 1, 2, 3\}$ be the set of features (attributes), and $D = \{d_j : j = 1, 2, 3\}$ be the committee of decision makers (DMs) who will decide the optimal company to purchase of ventilators with the following features (attributes):

$$
e_1 = \text{embedded system and simple user interface}$$
$$e_2 = \text{life – critical system and patients safety}$$
$$e_3 = \text{fine – tuned system and synchrony software}.$$

Step 2: Establishing a matrix $[w_{ij}]_{3 \times 3}$ with weighted parameters as

| Lingual phrases       | Weighted fuzzy numbers |
|-----------------------|------------------------|
| Less valuable (LV)    | [0.000, 0.200]         |
| Valuable (V)          | [0.201, 0.400]         |
| Essentially valuable (EV) | [0.401, 0.600]     |
| Strongly valuable (SV) | [0.601, 0.800]         |
| Absolutely valuable (AV) | [0.801, 1.000]      |
These \( w_k \) are the weights allocated to the characteristics \( e_i \) by the decision makers (DM) \( d_j \) to the trait \( e_k \) with the help of lingual phrases as given in Table 8.

**Step 3:** After normalizing the new weighted matrix \( \tilde{W} = [\tilde{w}_{ij}]_{3 \times 3} \) becomes

\[
\begin{bmatrix}
  0.606 & 0.653 & 0.149 \\
  0.258 & 0.653 & 0.910 \\
  0.753 & 0.385 & 0.325 \\
\end{bmatrix}
\]

and the required weighted row matrix (vector) we obtained \( \mathbf{W} = \{0.340, 0.356, 0.304\} \).

**Step 4:** Here we consider that the committee of three members (DMs) gave the following q-RO-m-PFSS matrix where the \((j,k)^{th}\) entry shows m-polar q-ROFN \( \{\mu, \nu\}_m \). This matrix give the information such that alternatives are along row-wise and traits are along column-wise.

\[
\begin{align*}
\Upsilon &= \begin{bmatrix}
  (0.204,0.165),(0.292,0.095), (0.165,0.192) & (0.200,0.158),(0.164,0.203), (0.262,0.089) & (0.200,0.158),(0.164,0.203), (0.262,0.089) \\
  (0.201,0.119),(0.307,0.097), (0.153,0.183) & (0.195,0.150),(0.270,0.083), (0.223,0.136) & (0.195,0.150),(0.270,0.083), (0.223,0.136) \\
  (0.240,0.111),(0.166,0.116), (0.182,0.109) & (0.200,0.158),(0.164,0.203), (0.262,0.089) & (0.200,0.158),(0.164,0.203), (0.262,0.089) \\
  (0.153,0.145),(0.274,0.084), (0.148,0.181) & (0.290,0.079),(0.197,0.037), (0.011,0.091) & (0.290,0.079),(0.197,0.037), (0.011,0.091) \\
  (0.122,0.123),(0.156,0.119), (0.219,0.152) & (0.203,0.158),(0.198,0.040), (0.166,0.152) & (0.203,0.158),(0.198,0.040), (0.166,0.152) \\
\end{bmatrix}
\end{align*}
\]

**Step 5:** The weighted q-RO-m-PFSS decision matrix is

\[
\begin{align*}
\Gamma &= \begin{bmatrix}
  (0.204,0.165),(0.292,0.095), (0.165,0.192) & (0.200,0.158),(0.164,0.203), (0.262,0.089) & (0.200,0.158),(0.164,0.203), (0.262,0.089) \\
  (0.201,0.119),(0.307,0.097), (0.153,0.183) & (0.195,0.150),(0.270,0.083), (0.223,0.136) & (0.195,0.150),(0.270,0.083), (0.223,0.136) \\
  (0.240,0.111),(0.166,0.116), (0.182,0.109) & (0.200,0.158),(0.164,0.203), (0.262,0.089) & (0.200,0.158),(0.164,0.203), (0.262,0.089) \\
  (0.153,0.145),(0.274,0.084), (0.148,0.181) & (0.290,0.079),(0.197,0.037), (0.011,0.091) & (0.290,0.079),(0.197,0.037), (0.011,0.091) \\
  (0.122,0.123),(0.156,0.119), (0.219,0.152) & (0.203,0.158),(0.198,0.040), (0.166,0.152) & (0.203,0.158),(0.198,0.040), (0.166,0.152) \\
\end{bmatrix}
\end{align*}
\]

**Step 6:** Processing ahead for q-RO-m-PFSS, +ve ideal solution (PIS) and -ve ideal solution (NIS), by putting in order, are

\[ q - \text{ROFSV - PIS} = \{ r_1^+, r_2^+, \ldots, r_5^+ \} \]
\[ = \{ 0.292,0.095, 0.307,0.069, 0.262,0.089, 0.290,0.037, 0.238,0.040 \} \]

and

\[ q - \text{ROFSV - NIS} = \{ r_1^-, r_2^-, \ldots, r_5^- \} \]
\[ = \{ 0.052,0.286, 0.100,0.183, 0.111,0.203, 0.011,0.181, 0.122,0.158 \} \]

**Step 7:** By implementing the Step 7 of the proposed Algorithm, we compute the q-ROFS-Euclidean distances of every q-ROFSV-PIS and q-ROFSV-NIS and hence get

\[
\begin{align*}
  d_1^+ &= 0.4704; &  d_2^+ &= 0.3812; &  d_3^+ &= 0.3106; &  d_4^+ &= 0.4319; &  d_5^+ &= 0.3094 \\
  d_1^- &= 0.5655; &  d_2^- &= 0.4371; &  d_3^- &= 0.3537; &  d_4^- &= 0.4501; &  d_5^- &= 0.2828
\end{align*}
\]

**Step 8:** Based on these distance values, the closeness coefficients \( I_k^* = \frac{d_k^+}{d_k^+ + d_k^-} \) of each alternative \( s_k \)'s are computed as...
\[ \begin{align*}
\Gamma_1 &= 0.5545; \quad \Gamma_2 = 0.5342; \quad \Gamma_3 = 0.5324; \quad \Gamma_4 = 0.6084; \quad \Gamma_5 = 0.4775
\end{align*} \]

Step 9: From the order of the \( \Gamma_k \)'s, the superiority order of the alternatives is given as,

\[ \xi_4 \succ \xi_1 \succ \xi_2 \succ \xi_3 \succ \xi_5 \]

In view of the final ranking of alternative computed by TOPSIS approach with q-RO-m-PFS information, the optimal alternative is the company \( \xi_4 \).

5 | EXTENSION OF GRA TO q-RO-m-PFSSs

The Algorithm 2 is proposed for grey relational analysis (GRA) using q-RO-m-PFSSs for the selection of ventilator's manufacturers.

Note that the first four steps in TOPSIS and GRA are almost same. We are going to illustrate the proposed GRA approach by using Algorithm 2.

**Example 5.1.** We consider the same data which is used in the Example 4.1. Then we give a comparison of TOPSIS and GRA techniques. Step 1, Step 2 and Step 3 of TOPSIS and GRA are almost same. We begin with Step 4 for GRA.

Step 4: We consider that the committee of three members (DMs) gave the following q-RO-m-PFSS matrix where the \((j,k)\)th entry shows \(m\)-polar q-ROFN \(\{(\mu,\nu)_m\}\). This matrix give the information such that alternatives are along row-wise and attributes are along column-wise.

\[
0.9\Gamma = \begin{bmatrix}
(0.357, 0.841), & (0.547, 0.451), & (0.654, 0.326) \\
(0.573, 0.462), & (0.821, 0.268), & (0.462, 0.539) \\
(0.171, 0.452), & (0.621, 0.387), & (0.550, 0.664) \\
(0.590, 0.352), & (0.903, 0.284), & (0.449, 0.537) \\
(0.548, 0.421), & (0.758, 0.232), & (0.627, 0.381) \\
(0.852, 0.337), & (0.641, 0.226), & (0.329, 0.410) \\
(0.707, 0.326), & (0.488, 0.342), & (0.535, 0.321) \\
(0.562, 0.443), & (0.461, 0.571), & (0.737, 0.249) \\
(0.500, 0.451), & (0.548, 0.410), & (0.366, 0.311) \\
(0.449, 0.426), & (0.808, 0.247), & (0.436, 0.533) \\
(0.815, 0.223), & (0.552, 0.104), & (0.031, 0.256) \\
(0.679, 0.213), & (0.891, 0.123), & (0.603, 0.267) \\
(0.358, 0.362), & (0.459, 0.352), & (0.645, 0.447) \\
(0.569, 0.444), & (0.557, 0.111), & (0.468, 0.426) \\
(0.662, 0.356), & (0.733, 0.337), & (0.782, 0.341)
\end{bmatrix}
\]

The matrix of mean values is

\[
\Gamma = \begin{bmatrix}
0.367 & 0.663 & 0.559 \\
0.663 & 0.567 & 0.486 \\
0.590 & 0.499 & 0.546 \\
0.648 & 0.750 & 0.357 \\
0.530 & 0.483 & 0.632
\end{bmatrix}
\]

and hence the normalized matrix is

\[
\tilde{\Gamma} = \begin{bmatrix}
0.024 & 0.946 & 0.493 \\
0.746 & 0.512 & 0.071 \\
0.568 & 0.346 & 0.461 \\
0.710 & 0.959 & 0.000 \\
0.422 & 0.307 & 0.671
\end{bmatrix}
\]

Step 5: Based on these information, the GRCs are computed for each alternative as

\[
\text{GRC}(\xi_1) = 0.2854; \quad \text{GRC}(\xi_2) = 0.2825; \quad \text{GRC}(\xi_3) = 0.2820; \\
\text{GRC}(\xi_4) = 0.2921; \quad \text{GRC}(\xi_5) = 0.2819
\]

Step 6: The preference order with ‘highest-the-better’ of the alternatives is

\[ \xi_4 \succ \xi_1 \succ \xi_2 \succ \xi_3 \succ \xi_5 \]

and hence \( \xi_4 \) is the best one.
The proposed methods of TOPSIS and GRA are compared and same ranking is obtained by these methods. These approaches are also compared with some other existing methods as indicated in the Table 9 as given below listing the results of the comparison in the final ranking of top 5 alternatives.

The superiority and validity of the proposed approach is shown in the Table 10. The proposed model of qRomPFSS is superior to existing models like fuzzy set, IFS, PFS, qROFS, mPFS and PmPFS. In fact each of the model IFS, PFS, qROFS, mPFS and PmPFS are the special cases of qRomPFS. For $q = 2$, qRomPFSS reduces to PmPFS. For $m = 1$, it reduces to qROFSS. For $q = 2 & m = 1$, it reduces to PFSS.

From this analysis, we have seen the features of the proposed set over the existing sets. For instance,

1. The existing models like SS, m-PFS, and q-ROFS have been independently applied by the researchers to address uncertainties in real-life problems while SS deals with uncertainties by means of parameterization.
2. An m-PFS deals with uncertainties by assigning membership grades from $[0,1]^m$ with a natural number $m$, that is, $m$–PFS assign $m$ degrees of memberships to show multi-polar information to access the ratings.
3. A q-ROFS is helpful to address uncertainties by using pair $(\mu, \nu)$ of membership grade $\mu$ and non-membership grades $\nu$ which the constraints $\mu^2 + \nu^2 \leq 1$.
4. The proposed hybrid model q-RO-m-PFSS is more effective to describe vague and uncertain information with multi-polar ordered pairs $(\mu_1, \nu_1), (\mu_2, \nu_2), \ldots, (\mu_m, \nu_m)$ satisfying the condition $\sum_{i=1}^{m} \mu_i^2 + \sum_{i=1}^{m} \nu_i^2 \leq 1$. Each object/alternative is essentially analysed under the influence of parameters and multi-polar characteristic in the form of $q$–RO-m–PFSs. Thus proposed model is superior to existing models.

6 | CONCLUSION

The notions of soft sets (SSs), m-polar fuzzy sets (m-PFSs) and q-rung orthopair fuzzy sets (q-ROFSs) have been focused by numerous researchers to deal with uncertainties by parameterizations, multi-polarity and membership grades (MGs) and non-membership grades (NMGs), respectively.
In order to deal with real life circumstances when it become necessary to consider all these notions in a single hybrid model, we introduced the concept of q-rung orthopair m-polar fuzzy soft sets (q-RO-m-PFSSs) by hybridizing the concepts of SSs, m-PFSs, and q-ROFSs. We defined some fundamental operations and key properties of q-RO-m-PFSSs. We find out crisp sets while determining support, core and height of q-RO-m-PFSSs. A sufficient number of illustrations are also incorporated within this article to apprehend these concepts clearly and productively. The complexity of logistics and supply chain management during COVID-19 using TOPSIS and GRA techniques are analysed under q-RO-m-PFS information. The DMs have assigned membership grades and non-memberships grades to the objects according to the real situation in COVID-19. The proposed MCDM approaches are worthy efficient for the selection of ventilator’s manufacturers for the patients suffering from epidemic disease named as COVID-19 which has been spread all over the world. We have collected, summarized, analysed the real data and ranking of feasible alternatives is determined. Applications of proposed MCDM techniques are demonstrated by respective numerical examples. The comparison analysis of the final ranking computed by proposed techniques is also given to justify the feasibility, applicability and reliability of these techniques.

For future studies, we shall discuss the ideas of q-RO-m-PFS groups, q-RO-m-PFS rings, q-RO-m-PFS topology, q-RO-m-PFS graphs and q-RO-m-PFS aggregation operators. Moreover, we shall discuss the applications of these notions in the different fields like pattern recognition, artificial intelligence, social sciences, economics, human resource management, robotics, transportation, agriculture, soft computing and many other fields (Shahir et al., 2013; Shahir & Naz, 2011; Tehrim & Riaz, 2019; Veeramani & Batulan, 2010; Wei, 2011b; Zhang & Xu, 2014a).

CONFLICT OF INTEREST
The authors declare no potential conflict of interest.

AUTHOR CONTRIBUTIONS
Muhammad Riaz, Harish Garg, Muhammad Tahir Hamid and Deeba Afzal contributed to each part of this paper equally. The authors read and approved the final manuscript.

DATA AVAILABILITY STATEMENT
Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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How to cite this article: Riaz, M., Garg, H., Hamid, M. T., & Afzal, D. (2022). Modelling uncertainties with TOPSIS and GRA based on q-rung orthopair m-polar fuzzy soft information in COVID-19. Expert Systems, 39(5), e12940. https://doi.org/10.1111/exsy.12940