Towards an understanding of the deuteron-like dibaryon state $d_{N\Omega}$

Cheng-Jian Xiao$^{1,2,*}$, Yu-Bing Dong$^{1,2,3,‡}$, Thomas Gutsche$^4$§, Valery E. Lyubovitskij$^{1,5,†}$ and Dian-Yong Chen$^{6,**}$

$^1$Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
$^2$Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, China
$^3$School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 101408, China
$^4$Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany
$^5$Departamento de Física y Centro Científico Tecnológico de Valparaíso-CCTVal, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile
$^6$School of Physics, Southeast University, Nanjing 210094, People’s Republic of China

(Dated: April 29, 2020)

Recent lattice QCD calculations showed that a $d_{N\Omega}$ dibaryon state similar to the deuteron with small binding energy may exist. In this work we propose a hadronic molecular approach to study the dynamical properties of this exotic state. We use a phenomenological Lagrangian approach to describe the coupling of the $d_{N\Omega}$ state to its constituents and the decays into conventional hadrons, $d_{N\Omega} \rightarrow \Lambda\Xi$ and $d_{N\Omega} \rightarrow \Sigma\Xi$. Predictions for the sum of the decay rates are in the range of a few hundred keV. In addition, we find that the $d_{N\Omega} \rightarrow \Lambda\Xi$ mode is dominant, preferably searched for in a future RHIC experiment.

PACS numbers:

I. INTRODUCTION

Since the discovery of the $X(3872)$ state in 2003 [1] the study of exotic resonances with heavy flavors, like $X(3872)$, $Z_c(3900)$ or the $P_c$ states, turns out to be extremely important in unravelling their unusual internal structure both in theoretical and experimental investigations [2]. In particular, many experimental efforts at world-wide facilities (BEPCII, BELLE, CERN, JLab, etc.) have been carried out for hunting and identifying these exotics [2–7]. Numerous theoretical calculations were devoted to the understanding of these unusual hadron states with respect to their composite structure, mass spectrum and decay properties [for a detailed list of references and reviews, see, e.g., Refs. [2, 8–17]]. Different interpretations have been proposed and developed in the literature: hadronic molecular scenarios, multiquark states - tetraquark or pentaquark configurations, kinematic triangle singularities, scattering cusps, among others.

Multiquark states can be realized as four-quark (meson sector) and five-quark (baryon sector) systems, but also as dibaryons with six quarks in the minimal configuration and a baryon number of two. The deuteron, discovered in 1931, is the prototype of a dibaryon, mainly residing in a proton-neutron configuration with a weak binding energy of $E_b \approx 2.22$ MeV. Dyson and Xuong were the first to study non-strange two-baryon systems in terms of $SU(6)$ even before the quark model was established [18]. The H-particle, originally proposed by Jaffe [19], and other candidates like the $d^*$, were searched for in experiment for a very long time. Recently, the non-strange dibaryon resonance $d^*(2380)$ was observed and confirmed by the WASA@COSY collaboration [20–23]. So far, the understanding of the nature of the $d^*(2380)$ dibaryon resonance is not conclusive. A compact 6-quark state [24–28] or a hadronic molecule structure [29] are two possible interpretations (see the review article [30]).

Possible nucleon-hyperon dibaryons have also been studied in the literature [19, 31–35]. The $N\Omega$ dibaryon state is a typical example among them as it is believed to be bound. The first investigation of a six-quark system with strangeness $S = -3$ was done by Goldman et al. using the relativistic quark model [36]. They proposed a bound $S$-wave $N\Omega$ dibaryon state with total angular momentum $J = 1$ or 2. Later on, in Ref. [37] it was pointed out that the treatment with a single $N\Omega$ channel cannot lead to a bound dibaryon state, since there is no quark exchange effect in this channel. When considering the coupled channels, like $N\Omega \rightarrow \Lambda\Xi^-\Sigma\Xi^-\Sigma^-\Xi^-$, a bound state might exist. The $N\Omega$ system was also studied in a quark delocalization and color screening model (QDCSM), where the bound state can be obtained both for the single $N\Omega$ or coupled channel configurations [38]. The predicted masses are $M = 2566$ MeV and $M = 2549$ MeV for the two cases, respectively. A further analysis of the QDCSM was recently performed in Ref. [39], and the updated results were consistent with the previous ones. A bound $N\Omega$ dibaryon state is also supported by chiral quark model calculations, where the binding energy varies from ten to around one hundred MeV depending on the specific approach [39, 40]. Moreover, Ref. [41] found a quasibound $N\Omega$ state with a pole at $E_{pole} = 2611.3 - 0.7i$ MeV based on a meson exchange model.

Besides these model calculations, recently, a lattice calculation for the $d_{N\Omega}$ system was performed by the HAL QCD Collaboration [42, 43]. As a result they reported that an $S$-wave $d_{N\Omega}$ dibaryon with $J^P = 2^+$ and with deuteron-like binding energy of $E_b = 2.46$ MeV indeed does exit. The HAL QCD Collaboration performed their lattice simulations for nearly...
physical quark masses corresponding to pseudoscalar masses of $m_u \approx 146$ MeV and $m_K \approx 525$ MeV. The possible strong short range attraction in the proton-$\Omega$ system can also be accessed by the momentum correlation of $p\Omega$ emission in relativistic heavy ion collisions [44]. The corresponding measurement has been carried out by the STAR Collaboration at RHIC using the Au+Au collision [45]. The results slightly favor a bound $d_{\Xi N}$ dibaryon with a binding energy of about 27 MeV. Besides the first work in Ref. [44] the authors extended their analysis on the pair momentum correlation functions in [46].

To check for the existence of a $d_{\Omega \pi}$ dibaryon, a direct search for a signal in the invariant mass of the final decay channels is necessary. Therefore, also a theoretical calculation on the decay properties of the $d_{\Xi N}$ dibaryon state is needed. In this work we consider the $d_{\Xi N}$ dibaryon as a loosely bound state of a nucleon and a $\Omega$ with a value for the binding energy set by the lattice calculation. Then we employ an effective Lagrangian approach to calculate the strong decays of the dibaryon. The phenomenological Lagrangian approach is a reasonable method to describe the properties of weakly bound states. We applied it to a wide range of exotic resonances like $D_{s0}^*(2317)$, $X(3872)$, $Z_c(3900)$, and $Y(4260)$ in the meson sectors [47–57], and for $\Lambda_c(2940)$, $\Sigma_c(2800)$, $\Omega(1820)$, and $P_c$ in the baryon sector [58–63]. We also applied this method to study deuteron properties [64, 65]. Since the binding of this dibaryon can be similar to that of the deuteron, the approach will result in reasonable predictions for the strong decay properties of the $d_{\Xi N}$ dibaryon.

This paper is organized as follows. In Sec. II, we discuss the set-up of the hadronic structure of the $d_{\Xi N}$ bound state and follow up with the formalism of the strong decay modes in the context of an effective Lagrangian approach. Sec. III is devoted to the numerical evaluation and discussion of the strong decays of the $N\Omega$ dibaryon. Finally, a short summary will be given in Sec. IV.

II. STRONG DECAYS OF THE $d_{\Xi N}$ DIBARYON

In the following we assume that the $d_{\Xi N}$ dibaryon is a loosely bound state of a nucleon and a $\Omega^-$ hyperon. Following the results of the recent lattice calculations [42], the quantum numbers of the $d_{\Xi N}$ are chosen as $I(J^P) = \frac{1}{2}(2^+)$. The bound state has two isospin components, $p\Omega$ for $I_3 = 1/2$ and $n\Omega$ for $I_3 = -1/2$. To set up a framework for the treatment of a bound state of two hadrons we construct a phenomenological Lagrangian describing the interaction of the $d_{\Xi N}$ with its constituents as

$$\mathcal{L} = g_{d_{\Xi N}} \bar{\psi}^{\alpha\beta}_{d_{\Xi N}} \int dy \Phi(y^2) \hat{\Sigma}_{\alpha\beta}^\dagger(x + \omega_{\Xi N}y)\gamma_\nu N(x - \omega_{\Xi N}y) + \text{h.c.},$$  \hspace{1cm} (1)

where $\psi^c = C\bar{\psi}^T$, $\bar{\psi} = \psi^T C$, and $\bar{\psi}^\gamma_1\gamma_\nu\psi = \bar{\psi}^\gamma_1\gamma_\nu\psi$. Here $C = i\gamma^2\gamma^0$ is the charge-conjugation matrix, superscript $T$ denotes the transposition and $\omega_{\Xi N} = m_i/(m_i + m_j)$ is the hadron mass fraction parameter, where $m_i$ is the mass of the $i$-th particle. To describe the distribution of the constituents in the hadronic molecular system, we introduce the correlation function $\Phi(y^2)$, which, in addition, plays the role to render the Feynman diagrams ultraviolet finite. $\Phi(y^2)$ is related to its Fourier transform in momentum space $\tilde{\Phi}(-p^2)$ as:

$$\Phi(y^2) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot y} \tilde{\Phi}(-p^2),$$  \hspace{1cm} (2)

where $p = \omega_{\Xi N}p_\Omega - \omega_{\Xi N}p_N$ is the Jacobi momentum. Here $\Phi(-p^2)$ is the correlation function describing the distribution of constituents in the molecular state. It was widely and successfully used in the investigation of hadronic molecules [47–50, 54, 56, 64]. For simplicity $\tilde{\Phi}$ is chosen as a Gaussian-like form $\tilde{\Phi}(-p^2) = \exp(p^2/\Lambda^2)$, where $\Lambda$ is the model parameter, which has dimension of mass and defines a scale for the distribution of the constituents inside the molecule. All calculations are performed in Euclidean space after Wick transformation for loop and all external momenta: $p^\mu = (p^0, \vec{p}) \rightarrow p^\mu_E = (p^0, \vec{p})$ with $p^0 = -ip^0$. In Euclidean space the Gaussian correlation function provides that all loop integrals are ultraviolet finite.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Mass operator of the $d_{\Xi N}$.}
\end{figure}

The coupling constant $g_{d_{\Xi N}}$ in Eq. (1) is determined using the Weinberg-Salam compositeness condition [66–69]. This condition means that the probability to find the dressed bound state as a bare (structureless) state is equal to zero. It also means that the corresponding wave function renormalization constant $Z$ is set to be zero. In case of the $d_{\Xi N}$ state the compositeness condition reads:

$$Z_{d_{\Xi N}} = 1 - \frac{\partial \Sigma_{d_{\Xi N}}^{(1)}(m^2_{d_{\Xi N}})}{\partial m^2_{d_{\Xi N}}} = 0,$$  \hspace{1cm} (3)

where $\Sigma_{d_{\Xi N}}^{(1)}(m^2_{d_{\Xi N}})$ is the nonvanishing part of the mass operator of the $d_{\Xi N}$ state having spin-parity $2^+$. In Fig. 1 we display the diagram contributing to the mass operator of the $d_{\Xi N}$ state. Note that the respective mass operator of the $2^+$ hadron is given by the rank-4 tensor $\Sigma_{\mu\nu\rho\lambda}$ sandwiched by the polarization vectors $\epsilon_{\mu\nu}^{(d_{\Xi N})}(p)$ for the spin $2^+$ hadron:

$$\hat{\Sigma}(p) = \epsilon_{\mu\nu}^{(d_{\Xi N})}(p) \Sigma_{\mu\nu\rho\lambda}(p) \epsilon_{\rho\lambda}^{(d_{\Xi N})}(p).$$  \hspace{1cm} (4)

The polarization vector $\epsilon_{\mu\nu}^{(d_{\Xi N})}(p)$ obeys the conditions of symmetry $\epsilon_{\mu\nu}^{(d_{\Xi N})}(p) = \epsilon_{\nu\mu}^{(d_{\Xi N})}(p)$, transversality $p^\mu \epsilon_{\mu\nu}^{(d_{\Xi N})}(p) = 0$, and tracelessness $g^{\mu\nu} \epsilon_{\mu\nu}^{(d_{\Xi N})}(p) = 0$. 
The expression for the mass operator $\Sigma_{\mu\nu\rho}$ reads
\[
\Sigma_{\mu\nu\rho} = \frac{\alpha_0}{g} \int \frac{d^4q}{(2\pi)^4} \bar{\psi}^\dagger (-\not{q} - m_N p) \psi^\dagger \gamma_\mu S(p - m_N),
\]
where $S$ and $S_{\mu\nu}$ are the free fermion propagators for spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ particles with
\[
S(p, m) = (\not{p} - m)^{-1}, \quad S_{\mu\nu}(p, m) = (\not{p} - m)^{-1}
\]
\[
\times \left( -\gamma_\mu + \gamma_\nu \gamma_5 \gamma_\mu \right) / 3 + \frac{2p_\mu p_\nu}{3m^2} + \frac{\gamma_\mu p_\nu - \gamma_\nu p_\mu}{3m}.
\]
Using properties of the polarization vector $\epsilon^{(i)}_{\mu\nu}(p)$ mentioned above $\Sigma_{\mu\nu\rho}$ can be decomposed into the Lorentz structures $I^{(i)}_{\mu\nu\rho}$ ($i = 1, \ldots, 5$) multiplied by the scalar functions $\Sigma^{(i)}(p^2)$ with:
\[
\Sigma_{\mu\nu\rho}(p) = \sum_{i=1}^{5} I^{(i)}_{\mu\nu\rho} \Sigma^{(i)}(p^2),
\]
where
\[
I^{(1)}_{\mu\nu\rho} = \frac{1}{2} \left[ g_{\mu\nu} g_{\rho\beta} + g_{\mu\rho} g_{\nu\beta} \right],
\]
\[
I^{(2)}_{\mu\nu\rho} = g_{\mu\nu} g_{\rho\beta},
\]
\[
I^{(3)}_{\mu\nu\rho} = \frac{1}{2} \left[ g_{\mu\rho} P_\nu + g_{\nu\rho} P_\mu + g_{\mu\nu} P_\rho \right],
\]
\[
I^{(4)}_{\mu\nu\rho} = \frac{1}{4} \left[ g_{\mu\rho} P_\nu + g_{\nu\rho} P_\mu + g_{\mu\nu} P_\rho \right],
\]
\[
I^{(5)}_{\mu\nu\rho} = p_\mu P_\nu p_\rho.
\]
As already mentioned, due to the properties of the polarization vector $\epsilon^{(i)}_{\mu\nu}(p)$ only the first term in the sum of Eq. (8) contributes, while the others vanish. The scalar function $\Sigma^{(1)}(p^2)$ contributing to the isospin of $p^2$ is obtained from the full mass operator $\Sigma_{\mu\nu\rho}(p)$ when acting with the following Lorentz projector
\[
T^{\mu\nu\rho}_\dagger = \frac{1}{10} \left( p^\mu p^\nu + p^\nu p^\rho + p^\rho p^\mu \right) - \frac{1}{15} P^\mu P^\nu P^\rho.
\]
The projector $P^\mu$ is defined as $P^\mu = g^\mu - p^\mu p^\nu / p^2$ and satisfies the conditions:
\[
g^\mu p^\nu = p^\mu p^\nu = 3, \quad p^\mu P^\nu = p_\nu P^\mu = 0.
\]
The full projector $T^{\mu\nu\rho}_\dagger$ satisfies the following conditions
\[
i T^{\mu\nu\rho}_\dagger = 0, \quad I^{(i)}_{\mu\nu\rho} T^{\mu\nu\rho}_\dagger = 1, \quad I^{(j)}_{\mu\nu\rho} T^{\mu\nu\rho}_\dagger = 0, \quad j = 2, 3, 4, 5.
\]
Finally, the required scalar function $\Sigma^{(1)}(p^2)$ is fixed using the identity
\[
\Sigma^{(1)}(p^2) = T^{\mu\nu\rho}_\dagger \Sigma_{\mu\nu\rho}(p).
\]
metry relations [71, 75]:

\[ f_{\lambda pK^*} = -\frac{1}{\sqrt{3}} g_{NN\pi}(1 + 2\alpha_{BBP}), \]
\[ f_{\psi pK^*} = g_{NN\pi}(1 - 2\alpha_{BBP}), \]
\[ g_{\lambda pK^*} = -\frac{1}{\sqrt{3}} g_{NN\pi}(1 + 2\alpha_{BBV}), \]
\[ g_{\psi pK^*} = g_{NN\pi}(1 - 2\alpha_{BBV}). \]

The remaining parameter \( \kappa \) in the BBV coupling is fixed using the relation between vector and tensor couplings \( f_{NK^*} = g_{YNK^*} k_{YNK^*} \), and the relation of the tensor couplings \( f_{NK^*} \) to the \( f_{NN\omega} \) and \( f_{NN\rho} \) couplings [71]:

\[ f_{\lambda pK^*} = -\frac{1}{2\sqrt{3}} f_{NN\omega} - \frac{\sqrt{3}}{2} f_{NN\rho}, \]
\[ f_{\psi pK^*} = -\frac{1}{2} f_{NN\omega} + \frac{1}{2} f_{NN\rho}. \]

For the couplings between the baryon decuplet and the pseudoscalar/vector meson octets \( g_{\Delta\pi} \) and \( g_{\Delta\rho} \), we use \( SU(3) \) symmetry constraints [71]:

\[ g_{\Omega\pi K^*} = g_{\Omega\pi K^0} = g_{\Delta\pi}, \]
\[ g_{\Omega\pi K^*} = g_{\Omega\pi K^0} = g_{\Delta\rho}. \]

TABLE I: Meson-baryon coupling constants.

| Coupling | Set I | Set II |
|----------|-------|--------|
| \( g_{NN\pi} \) | 0.989 [70, 71] | |
| \( g_{NN\pi} \) | 2.12 [70, 71] | |
| \( f_{NN\omega} \) | 0 [70, 71] | 3.25 [70, 75] |
| \( \alpha_{BBP} \) | 0.4 [71, 74] | 0 [71, 74] |
| \( \alpha_{BBV} \) | 1.15 [71, 74] | 1.15 [71, 74] |
| \( g_{NN\rho} \) | 3.1 [72] | 3.25 [70, 75] |
| \( \kappa_{\rho} \) | 1.825 [72] | 6.1 [70] |
| \( g_{NN\rho} \) | -6.08 [72] | -16.0 [70] |

In Table I, we present the values for the meson-baryon coupling constants used in our calculations [see Eqs. (15)-(22)]. Note that \( g_{NN\pi} = 0.989 \) was determined in Ref. [70] based on \( \pi N \) scattering, where the authors found that the \( \pi N \) phase shift, scattering length, and the \( \pi N \Sigma \) term were in agreement with the experimental data. We also use \( \alpha_{BBP} = 0.4 \) and \( \alpha_{BBV} = 1.15 \), taken from an analysis of elastic \( \pi N \) scattering [71]. For the coupling \( g_{NN\pi} \) we take the value 2.12 [70] determined from the \( \Lambda \to N\pi \) decay rate. Besides these well determined parameters, the value of \( \kappa_{\rho} \) can vary in a wider range, e.g., from \( \kappa_{\rho} = 1.825 \) in Ref. [72] to \( \kappa_{\rho} = 6.1 \) in Ref. [70]. The values for the \( g_{NN\rho} \) coupling cited in these two references do not vary too much and are also presented in Table I. The difference in values for \( \kappa_{\rho} \) and \( g_{NN\rho} \) consequently has an impact on the coupling constant \( g_{\Delta\rho} \) [72]:

\[ g_{\Delta\rho} = -\sqrt{\frac{72}{25}} g_{NN\pi}(1 + \kappa_{\rho}) \frac{1}{2m_N} m_N, \]

with \( g_{\Delta\rho} = -6.08 \) in Ref. [72] and \( g_{\Delta\rho} = -16.0 \) in Ref. [70]. Since there is no way to distinguish the cases of parameter values for \( g_{NN\rho} \), \( \kappa_{\rho} \), and \( g_{NN\rho} \) we use both sets in the present paper. Further details will be discussed in the next section.

Starting from our total effective Lagrangian we generate matrix elements corresponding to the diagrams of Fig. 2. Their expressions read as:

\[ M_i = \bar{u}(p_4) \Lambda_i(p_3, p_4) C \bar{u}^T(p_3) \epsilon_{\rho\rho}(p) \quad i = 1, 2, \]

where

\[ \Lambda_1(p_3, p_4) = -g_{d_{\rho\pi}} \frac{f_{\rho\pi K} f_{\lambda pK}}{m_{\pi}^2} \int \frac{dq}{(2\pi)^4} g_{\rho\pi q} \cdot q \times D(q, m_K) \frac{S_{\rho\pi}(p_1, m_\rho)}{y^\rho} S(-p_1, m_\rho) y^\rho \times \hat{F}(-p_1 - w_{\rho\pi} p_E^2) F(m_\rho, q), \]

\[ \Lambda_2(p_3, p_4) = -g_{d_{\rho\pi}} \frac{g_{\Omega\pi K^*} g_{\lambda pK^*}}{m_{\rho}^2} \int \frac{dq}{(2\pi)^4} \left[ g_{\rho\pi q} + g_{\rho\pi q} \right] \times \gamma^\rho y^\rho \frac{S_{\rho\pi}(p_2, m_\rho)}{y^\rho} S(-p_1, m_\rho) D_{\rho\pi}(q, m_K) \times \left[ \gamma_\rho - i\sigma_{\rho\pi} q^\perp \frac{K^*_{\rho\pi}}{2m_{\rho}^2} \right] \times \hat{F}(-p_1 - w_{\rho\pi} p_E^2) F(m_\rho, q). \]

\[ D(q, m_K) = (q^2 - m_{\rho}^2)^{-1} \quad \text{and} \quad D_{\rho\pi}(q, m_{\Sigma}) = (g_{\rho\pi} + q_\rho q_{\rho\pi}/m_{\rho}^2) \left( q^2 - m_{\rho\pi}^2 \right)^{-1} \text{are the propagators of the } K \text{ and } K^* \text{ mesons, respectively.} \]

A phenomenological dipole form factor

\[ F(m_\rho, q) = (m_\rho^2 - \Lambda^2_{\pi\rho}) / (q^2 - \Lambda^2_{\pi\rho}) \]

is introduced to take into account off-shell effects and the nonlocal structure of the interacting particles [76]. Here \( \Lambda_{\pi\rho} = m_\rho + \alpha_{\pi\rho} q \) is a cut-off parameter with \( m_\rho \) being the mass of the exchange particle and the QCD scale parameter \( \alpha_{\pi\rho} = 0.22 \text{GeV} \). The other two transition amplitudes corresponding to the diagrams Figs. 2(3) and 2(4) are generated from the underlying phenomenological Lagrangian in analogy.

Finally, the total contribution to the matrix element of the \( d_{\Sigma\Lambda} \to \Xi\Lambda \) process is:

\[ \mathcal{M}_{\text{tot}}(d_{\Sigma\Lambda} \to \Xi\Lambda) = M_1 + M_2 \]

and for the \( d_{\Sigma\Lambda} \to \Xi\Sigma \) transition

\[ \mathcal{M}_{\text{tot}}(d_{\Sigma\Lambda} \to \Xi\Sigma) = M_3 + M_4. \]

The expression for the decay widths of \( d_{\Sigma\Lambda} \to \Xi\Lambda/\Xi\Sigma \) is evaluated as

\[ \Gamma = \frac{1}{2 J + 1} \frac{|\vec{\beta}|^2}{8\pi m_{\Sigma}^2} \frac{|\mathcal{M}_{\text{tot}}|^2}{2}, \]

where \( J \) is the total angular momentum of the initial state \( d_{\Sigma\Lambda} \), \( \vec{\beta} \) is the relative 3-momentum of the final states in the rest frame of the initial state and the overline denotes the sum of spin polarizations for initial/final states.
TABLE II: Masses of the relevant particles (in units of GeV)\[2\].

| Particle | \(d_{NN}\) | deuteron | \(p\) | \(\Lambda^0\) | \(\Sigma^0\) | \(\Xi^-\) | \(\Omega^-\) |
|----------|--------|---------|-----|---------|---------|-------|-------|
| Mass     | 2.563  | 1.876   | 0.9396 | 1.116   | 1.193   | 1.315  | 1.672  |

III. NUMERICAL RESULTS

First in Table II we summarize the mass values used in the present calculation [2]. Although different masses were predicted for the \(d_{NN}\), we choose a mass for the \(d_{NN}\) state as reliably determined in the recent lattice calculation, where the binding energy is about 2.46 MeV.

TABLE III: Dependence of the coupling \(g_{d_{NN}}\) on \(\Lambda\).

| \(\Lambda\) (GeV) | 0.20 | 0.30 | 0.40 | 0.50 |
|-----------------|-----|-----|-----|-----|
| \(g_{d_{NN}}\)  | 2.38| 2.11| 1.97| 1.88|

Values for the coupling \(g_{d_{NN}}\) of the \(d_{NN}\) bound state to the constituents are generated by the compositeness condition and are listed in Table III. The values depend on the model parameter \(\Lambda\), which is introduced in the correlation function of Eq. (2) and phenomenologically represents the distribution of the \(N\) and \(\Omega\) baryons in the \(d_{NN}\) state. In Ref. [64], utilizing the same approach for the rather weakly bound deuteron, the parameter \(\Lambda\) was deduced to be less than 0.5 GeV. The numerical results for the deuteron electromagnetic form factors of Ref. [64] are in fairly good agreement with data. Based on the similarity between the \(d_{NN}\) and the deuteron, we choose four typical values for the phenomenological cutoff parameter \(\Lambda = 0.2, 0.3, 0.4\) and 0.5 GeV. The resulting values for \(g_{d_{NN}}\) are 2.38, 2.11, 1.97 and 1.88, respectively (see Table III).

TABLE IV: Two-body decay widths of the \(d_{NN}\) dibaryon in keV for different values of \(\Lambda\). The uncertainties of results for a fixed \(\Lambda\) reflect the variation in \(\alpha\) from 0.9 to 1.1. Coupling constants are taken from Set I.

| Parameters | \(\Lambda\) (GeV) | 0.20 | 0.30 | 0.40 | 0.50 |
|------------|-----------------|-----|-----|-----|-----|
| \(\Lambda^0\Xi^-\) mode | 154–275 | 253–455 | 321–582 | 355–646 |
| \(\Sigma^0\Xi^-\) mode | 4.00–6.61 | 6.00–10.0 | 6.75–11.4 | 7.03–12.0 |
| \(\Sigma^+\Xi^-\) mode | 4.00–6.61 | 6.00–10.0 | 6.75–11.4 | 7.03–12.0 |
| Total \(\Lambda\) | 162–288 | 265–475 | 335–605 | 369–670 |

We have a remaining parameter \(\alpha\) in the phenomenological form factor of Eq. (27). The parameter \(\alpha\) cannot be fixed from first principles, instead we choose \(\alpha = 0.9 – 1.1\) previously determined from an extended analysis of decay data on possible baryon-antibaryon bound states (see, e.g., the detailed discussion in Ref. [77]).

In Table IV, the partial strong decay widths of the \(d_{NN}\) \(\rightarrow\) \(\Lambda^0\Xi^-\), \(d_{NN}\) \(\rightarrow\) \(\Sigma^0\Xi^-\) and \(d_{NN}\) \(\rightarrow\) \(\Sigma^+\Xi^-\) transitions together with their dependence on \(\Lambda\) are displayed. For a fixed value of \(\Lambda\) the range in results corresponds to a variation of the parameter \(\alpha\) from 0.9 to 1.1 entering in the transition form factor.

Using the values for the coupling constants of Set I we find that the partial strong decay width for \(d_{NN}\) \(\rightarrow\) \(\Lambda^0\Xi^-\) varies from 154–275 keV to 355–646 keV, and that for \(d_{NN}\) \(\rightarrow\) \(\Sigma^0\Xi^-\) from 4.0–6.61 keV to 7.03–12.0 keV. Therefore, the mode \(d_{NN}\) \(\rightarrow\) \(\Lambda^0\Xi^-\) dominates over the \(d_{NN}\) \(\rightarrow\) \(\Sigma^0\Xi^-\) decay. From the relations of Eqs. (15) and (16) for the couplings it should be clear that \(g_{d_{NN}}\) is much larger than \(g_{d_{NN}}\), therefore resulting in a dominant branching fraction of the \(\Lambda\Xi\) mode. In addition, the partial width of the charged \(\Sigma^+\Xi^-\) mode was obtained by isospin symmetry, where isospin breaking effects, like mass differences of charged and neutral baryons, are not considered. Assuming that the sum of the three partial decay widths results in the total decay width, we can conclude that the total decay width of the \(d_{NN}\) is in the range of 162–670 keV.

TABLE V: Two-body decay widths of the \(d_{NN}\) dibaryon in keV for different values of \(\Lambda\). The range of results for \(\Lambda\) corresponds to the variation in \(\alpha\) from 0.9 to 1.1. Coupling constants are taken from Set II.

| Parameters | \(\Lambda\) (GeV) | 0.20 | 0.30 | 0.40 | 0.50 |
|------------|-----------------|-----|-----|-----|-----|
| \(\Lambda^0\Xi^-\) mode | 329–593 | 346–793 | 741–1180 | 842–1550 |
| \(\Sigma^0\Xi^-\) mode | 10.3–17.7 | 16.1–27.9 | 20.4–36.0 | 22.5–40.0 |
| \(\Sigma^+\Xi^-\) mode | 10.3–17.7 | 16.1–27.9 | 20.4–36.0 | 22.5–40.0 |
| Total | 350–628 | 578–1049 | 782–1432 | 881–1630 |

With the other set of coupling constants (Set II), the partial decay widths for both the \(\Lambda\Xi\) and \(\Sigma\Xi\) modes increase by a factor of about two. The obtained partial decay width for the \(\Lambda^0\Xi^-\) mode is from 329 to 1550 keV, and for the \(\Sigma^0\Xi^-\) decay width we have values from 10.3 to 40.0 keV when varying \(\Lambda\) and \(\alpha\) in the allowed range. For Set II of the coupling constant we conclude that the total decay width of \(d_{NN}\) is in the range of 350–1630 keV.

TABLE VI: Decay widths of the processes \(d_{NN}\) \(\rightarrow\) \(\Xi^0\Lambda^0\) and \(d_{NN}\) \(\rightarrow\) \(\Sigma^0\Xi^-\) in units of keV. The binding energy of the dibaryon is fixed at 2.46 MeV. The parameter \(\Lambda\) is chosen to be 0.2 GeV and \(\alpha\) is 0.9.

| Parameters | \(\Lambda\) (GeV) | 0.20 | 0.30 | 0.40 | 0.50 |
|------------|-----------------|-----|-----|-----|-----|
| \(d_{NN}\) \(\rightarrow\) \(\Xi^0\Lambda^0\) | 125 | 0.731 | 0.291 | 125 | 5.42 | 24.8 |
| \(d_{NN}\) \(\rightarrow\) \(\Sigma^0\Xi^-\) | 4.33 | 0.146 | 0.0444 | 4.33 | 1.11 | 3.78 |

To check for the contribution of individual diagrams to the processes \(d_{NN}\) \(\rightarrow\) \(\Xi^0\Lambda^0\) and \(d_{NN}\) \(\rightarrow\) \(\Sigma^0\Xi^-\) as well as the effect of different coupling values, we look at particular results for the partial decay widths as given in Table VI. The detailed results are based on the choice \(\Lambda = 0.2\) GeV and \(\alpha = 0.9\). The entry for \(K\) in Table VI represents the contribution from the \(K\) meson exchange as shown in Fig. 2, while \(K^+\) and \(K^0\) correspond to the vector and tensor parts of the \(K^+\) meson exchange contribution. For the couplings of Set I it is clearly seen that \(K\) exchange plays the essential role in both \(d_{NN}\) \(\rightarrow\) \(\Xi^0\Lambda^0\) and \(d_{NN}\) \(\rightarrow\) \(\Sigma^0\Xi^-\) processes. The contribution of \(K^+\) exchange is at least one order of magnitude smaller, where the tensor part is much smaller than the vector part. For Set II \(K\) meson exchange result in the same values for the partial decay widths.
since the relevant coupling constants $g_{\Lambda\Xi K}$, $f_{\Sigma NK}$ and $f_{\Sigma NK}$ are the same in the two cases. But now the contribution from $K^*$ exchange increases since the coupling constant $g_{\Lambda\Xi K}$ and $\kappa$ are larger, where the tensor contribution dominates over the vector part.

We conclude that for both sets of couplings the $K$ meson exchange contribution is dominant for both $\Lambda\Xi$ and $\Sigma\Xi$ modes, hence the full decay widths do not change dramatically within the two different sets of parameter values. The uncertainties in the parameters $\Lambda$ and $\alpha$ obviously have a sizable impact on the calculated decay widths. The total decay width can reach from a few hundred to above a thousand keV although the transitions of the $d_{\Omega}$ to the possible final states occur via a $D$-wave. The analysis of the partial decay widths indicates that the process $d_{\Omega} \to \Lambda\Xi$ dominates in the $d_{\Omega}$ decays with a branching fraction of around 95% independent of the particular parameter choice.

### IV. SUMMARY

The dibaryon $d_{\Omega}$ stands for a bound, minimal six-quark configuration with baryon number $B = 2$ and strangeness $S = -3$. Because of its weak binding it is analogous to the deuteron, which is an experimentally confirmed dibaryon. The $d_{\Omega}$ was predicted in many theoretical works, and in particular by Lattice calculations. There are also some hints for the existence of the $d_{\Omega}$ from recent experimental approaches. In the present work, we give an analysis of the strong decays of the $d_{\Omega}$ based on the use of phenomenological Lagrangian approach, in which the $d_{\Omega}$ is assumed to be a loosely bound state. Here, we simply use the Lattice prediction for the binding energy.

All possible strong two-body decay modes of the $d_{\Omega}$ are calculated. In the calculation two sets of coupling parameters are employed. We find that the total decay width of the $d_{\Omega}$ is in the range of a few hundred keV up to just above 1 MeV although the transitions to the two modes proceed through the $D$-wave. Independent of the particular choice of parameters the $d_{\Omega} \to \Lambda\Xi$ process dominates completely and almost captures the total branching fraction. A search for the $d_{\Omega}$ in the $\Lambda\Xi$ invariant mass spectrum can provide direct evidence for its existence. Finally, we would like to point out that more theoretical efforts are needed to understand the structure of the $d_{\Omega}$ exotic state as well as to search for other possible candidates in the dibaryon family [80].

### Acknowledgement

This work is supported, in part, by the National Natural Science Foundation of China (NSFC) under Grant Nos. 11947224, 11975245 and 11775050, by the fund provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD” project by the NSFC under Grant No. 11621131001, by the Key Research Program of Frontier Sciences, CAS, Grant No. Y7292610K1, by the Fundamental Research Funds for the Central Universities, and by the China Postdoctoral Science Foundation under Grant No. 2019M650843. This work was funded by the Carl Zeiss Foundation under Project “Kepler Center für Astro- und Teilchenphysik: Hochsensitive
Nachweistechnik zur Erforschung des unsichtbaren Universums (Gz: 0653-2.8/5812)”, by “Verbundprojekt 05A2017 - CREST-XENON: Direkte Suche nach Dunkler Materie mit XENON1T/nT und CREST-III, Teilprojekt 1 (Förderkennzeichen 05A17VTA)”, by ANID PIA/APOYO AFB180002 (Chile) and by FONDECYT (Chile) under Grant No. 1191103. YBD thanks Institute of Theoretical Physics, Tübingen University for the hospitality and the support of Alexander von Humboldt foundation, and thanks the useful discussion with Jialun Ping.
[60] T. Gutsche and V. E. Lyubovitskij, arXiv:1912.10894 [hep-ph].
[61] Q. F. Lyu and Y. B. Dong, Phys. Rev. D 93, 074020 (2016).
[62] C. J. Xiao, Y. Huang, Y. B. Dong, L. S. Geng, and D. Y. Chen, Phys. Rev. D 100, 014022 (2019).
[63] T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D 100, 094031 (2019).
[64] Y. B. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. C 78, 035205 (2008).
[65] C. Liang, Y. Dong, and W. Liang, Chin. Phys. C 38, 074104 (2014).
[66] A. Salam, Nuovo Cim. 25, 224 (1962).
[67] S. Weinberg, Phys. Rev. 130, 776 (1963).
[68] K. Hayashi, M. Hirayama, T. Muta, N. Seto and T. Shirafuji, Fortsch. Phys. 15, 625 (1967).
[69] G. V. Efimov and M. A. Ivanov, The Quark Confinement Model of Hadrons, (IOP Publishing, Bristol & Philadelphia, 1993).
[70] C. Schutz, J. W. Durso, K. Holinde, and J. Speth, Phys. Rev. C 49, 2671 (1994).
[71] D. Ronchen et al., Eur. Phys. J. A 49, 44 (2013).
[72] A. Matsuyama, T. Sato, and T.-S. H. Lee, Phys. Rept. 439, 193 (2007).
[73] J. He, Phys. Rev. D 95, 074031 (2017).
[74] R. Machleidt, K. Holinde, and C. Elster, Phys. Rept. 149, 1 (1987).
[75] W. Liu, C. M. Ko, and Z. W. Lin, Phys. Rev. C 65, 015203 (2002).
[76] H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005).
[77] Y. Dong, A. Faessler, T. Gutsche, Q. L and V. E. Lyubovitskij, Phys. Rev. D 96, 074027 (2017).
[78] A. W. Chan et al. (E756 Collaboration), Phys. Rev. D 58, 072002 (1998).
[79] L. C. Lu et al. [HyperCP Collaboration], Phys. Lett. B 617, 11 (2005).
[80] M. Bashkanov, “d”(2380) hexaquark”, Talk given at the “24th European Conference on Few-Body Problems in Physics”, Sept. 2-5, 2019, Surrey, UK.