Global Structure of the Colliding Bubble Braneworld Universe

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Abstract

The one-brane Randall-Sundrum model offers an example of a model with an “infinite” extra dimension in which ordinary gravity is recovered at large distances and the usual (3+1)-dimensional cosmology at late cosmic times. This is possible because the “bulk” has the geometry of anti de Sitter space, the curvature length $\ell$ of which delineates the (3+1)-dimensional behavior at large distances from the (4+1)-dimensional behavior at short distances. This spacetime, however, possesses a past Cauchy horizon on which initial data must be specified in a natural and convincing way. A more complete story is required that singles out some set of initial conditions to resolve the “bulk” smoothness and horizon problems. One such complete story is offered by the colliding bubble braneworld universe, where bubbles filled with $AdS^5$ nucleate from $dS^5$ or $M^5$ through quantum tunnelling. A pair of such colliding bubbles forms a Randall-Sundrum-like universe in the future of the collision. Because of the symmetry of bubbles produced through quantum tunnelling, the resulting universe is spatially homogeneous and isotropic at leading order, and the perturbations at the next order are completely well defined and calculable. In this contribution we discuss the possible global structure of such a spacetime.

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I. INTRODUCTION

The Randall-Sundrum model [1] demonstrates how a braneworld cosmology with an “infinite” extra dimension may be achieved in such a way that at large distances gravity behaves as ordinary (3+1) dimensional gravity [2] and in the late time limit the ordinary FRW cosmology is recovered [3]. This is achieved by postulating a bulk with the geometry of $AdS^5$ into which our brane is embedded. This scenario, however, is incomplete because of the usual Randall-Sundrum coordinates are bounded in the past by a Cauchy horizon. Some more complete story establishing well-defined initial conditions on this horizon is required.

Unlike de Sitter space, anti de Sitter space lacks the “no hair” property. Perturbations in AdS forever retain their initial amplitude. Therefore some sort of a beginning that resolves the bulk horizon and smoothness problems [4] is required to provide a complete story.

The colliding bubble braneworld universe [5], in which a (3+1)-brane surrounded by $AdS^5$ arises from the collisions of two bubbles filled with $AdS^5$ nucleating in $dS^5$, or $M^5$, offers one such possible beginning.

In this contribution we discuss some issues relating to the global structure of the spacetime in the colliding bubble braneworld universe. It has been pointed out that an isolated bubble filled with anti de Sitter space is likely to generate a spacelike singularity in its interior due to an instability that occurs during the collapse phase. We show how a collision may partially avert the formation of such a singularity. The collisions of bubbles arising from quantum tunnelling were considered by Hawking, Moss, and Stewart [6] and later in the thin-wall approximation with gravity taken into account by Wu [7]. Some of the issues considered in this paper (e.g., Cauchy horizons, the consequence of a small perturbation on the global structure) parallel those arising in the determination of the global structure of a Reissner-Nordstrom black hole produced from a realistic collapse, as discussed for example in Poisson and Israel [8].

II. INTERIOR STABILITY OF ADS BUBBLES

In Coleman and de Luccia and later in Abbott and Coleman [9], it was pointed out that the interior of a bubble filled with AdS is likely to be unstable toward the formation of a spacelike singularity. Unlike most instabilities, which result from small perturbations
For idealized thin-wall bubbles, in which a brane of vanishing thickness and given tension mediates between Minkowski space [panel (a)] or de Sitter space [panel (b)] on the one hand and anti de Sitter space on the other, the conformal diagrams showing the global structure of the spacetimes are indicated. Curiously, an initially globally hyperbolic spacetime evolves into a spacetime that ceases to be so in the interior of the bubble, because beyond a Cauchy horizon, which coincides with the past lightcone of $N'$, additional boundary data is required from the edge of the AdS infinite vertical strip. However, for more realistic AdS bubbles considered to lowest order in the semi-classical ($\hbar \to 0$) expansion, the global structures are modified to those shown in panels (c) and (d) where a spacelike singularity forms owing to an instability of the perfect symmetric solution.

from a symmetric solution that progressively grow and eventually blow up, in this case the singularity arises rather from the absence, or the smallness, of perturbations from the symmetric model solution.

The origin of the instability is as follows. For tunnelling described by a scalar field order parameter, the Euclidean instanton never takes one all the way to the true minimum, but rather at best very close but slightly displaced from there. Said another way, even very thin-wall bubbles always have some tail of their wall that extends into the forward lightcone of the bubble nucleation center. The evolution of the field in the lightcone interior is governed by
A collision with a second bubble spoils the symmetry that led to the perfect focusing of the scalar field. It was this perfect focusing that ensured the formation of a singularity. For two colliding bubbles, a shock wave of debris emanates from the collision, perturbing the perfect AdS bulk. If this perturbation is sufficiently strong, it is plausible that a hole opens up into the would-be singularity, allowing the spacetime to be extended into the full vertical AdS strip. Because of causality, the singularity from $S_N$ to $C_S$ cannot be averted. However, the rest of the singularity above the null curve from $C$ to $C_S$ can be avoided. A Cauchy horizon $H_C$ emanates from $C_S$ upward along the diagonal. This horizon indicates the boundary beyond which the initial value problem is ill-defined without additional data from the AdS boundary and from the “other side of the singularity”. The globally hyperbolic spacetime bounded by the horizon, however, suffices to provide a ‘complete story’ for what happens on our brane, because this brane propagates to the conformal boundary of the AdS strip, to $LB_\infty$, presumably before the intersection of $H_C$ with the conformal boundary. Because of the divergence of the conformal factor on the boundary, a infinite proper time elapses on our brane prior to $LB_\infty$. Panel (b) shows the entire proposed conformal diagram for the two colliding bubbles. The diagram includes two parallel AdS strips side by side. These strips, however, are not connected to each other because of the divergence of the conformal factor on the line separating them running from $LB_\infty$ upward. From $C$ to $LB_\infty$, the worldline of our local brane, the conformal factor is finite and a $Z_2$ symmetry across the brane is present at lowest order.

the equation $\ddot{\phi} + 4(\dot{a}/a)\dot{\phi} = -V,_{\phi}$, where the derivatives are with respect to the proper time $\tau$ from the nucleation center. While the bubble interior is expanding, $(\dot{a}/a)$ is positive, and this term dampens any oscillations about the true minimum. For a Minkowski or de Sitter interior, $(\dot{a}/a)$ is always positive. However, when the bubble interior has the geometry of anti de Sitter space, $a(\tau) = \ell \sin[\tau/\ell]$ where $\ell$ is the AdS curvature length, and for $\tau \geq (\pi\ell/2)$ the aforementioned dampening turns into anti-dampening during this collapse phase, causing the oscillations to blow up near $\tau \approx \pi\ell$ (except for the implausible case where the phase of the oscillations in finely tuned with infinite precision by proper choice of the potential). So far we have ignored gravitation backreaction, but the gravity of the scalar field only
hastens the formation of a singularity, turning what would simply be a divergence in the energy density into a spacetime singularity, of the same sort as the initial singularity of a hyperbolic FRW universe with the arrow of time reversed.

The spacetime singularity results because the Euclidean instanton has finely-tuned the wave front of the bubble wall tail, aiming it exactly toward the antipodal point of the nucleation center $N$. There is nothing mysterious about this singularity. The AdS space inside the bubble simply acts as a sort of perfect lens, with absolutely no aberration and of infinite size so that diffraction does not cut off the energy density at the focus. In the language of geometric optics, the ‘rays’ of the scalar field, which here are the timelike geodesics emanating from the nucleation center $N$, are re-focused at the conjugate point $N’$.

A scalar field in flat space with similarly perfectly focused initial conditions, as contrived as they may be, would behave exactly the same.

Having explained the nature of the singularity in the symmetric solution, we now consider how it might be avoided. The obvious way is to spoil the perfect focusing. As Abbott and Coleman pointed out, in this instance the Hawking singularity theorems do not pose an obstacle to avoiding the singularity altogether, because the relevant singularity theorem demonstrates that a singularity must arise if the spacetime is globally hyperbolic. However, a thin-wall solution of a bubble nucleating from Minkowski or $dS$ space and tunnelling to $AdS$, whose spacetime conformal diagram is shown in Fig. 1, is not globally hyperbolic. Consequently, the singularity theorems cannot be applied. The discussion above assumed the lowest order of the $\hbar \to 0$ semi-classical limit. What happens at finite $\hbar$ remains an open question.

Of primordial importance for the colliding bubble model is understanding the consequences of the perturbation presented by the collisions of two AdS bubbles. In the idealized case, as considered in refs. [3, 10, 11], upon colliding, the bubbles deposit all their excess energy (beyond that required to form in its unexcited state the intermediate brane on which we live) on the brane in the form of radiation-matter. This idealization of no energy escaping into the bulk is a caricature, just as that of an infinitely thin bubble wall with no tail. There will always be some, if not a lot of leakage from the collision into the bulk, which spoils the perfect focusing symmetry in the causal future of the collision surface. It is highly plausible that the perturbation from this wave into the bulk disrupts the formation of the singularity.

The previous analysis tacitly presupposes that a bubble wall, to the extent that it is not
perfectly thin-wall, can be represented as a kink in a scalar order-parameter field. It is not entirely evident that the “tail” of a “brane” bubble wall would behave in the same way. However, we expect that branes, even if they are “fundamental,” are dressed with some sort of tail similar to that of a scalar field kink because of couplings to other fields and radiative corrections.

To determine the global structure of the spacetime resulting from a realistic collision requires numerical simulations, which are currently in progress \cite{12}. It is nevertheless interesting to speculate on the possible outcome. In Fig. 2 we indicate a possible conformal diagram. Each point in this diagram represents a hyperboloid with the geometry of $H^3$. The shock wave of debris emanating from the collision disrupts the singularity. Our local brane reaches the conformal infinity of AdS before the Cauchy horizon. It remains to be seen whether this picture will be confirmed numerically.

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