Super-Penrose process and rotating wormholes

O. B. Zaslavskii

Department of Physics and Technology,
Kharkov V.N. Karazin National University,
4 Svoboda Square, Kharkov 61022, Ukraine and
Institute of Mathematics and Mechanics, Kazan Federal University,
18 Kremlyovskaya St., Kazan 420008, Russia

We consider collision of particles in a wormhole near its throat. Particles come from the opposite mouths. If the lapse function is small enough there, the energy $E$ of debris at infinity grows unbounded, so we are faced with the so-called super-Penrose process. This requires the existence of the ergoregion, so a wormhole should be rotating.

PACS numbers: 04.70.Bw, 04.20.-q

In recent years, essential interest revived to high energy collisions in a strong gravitation field. This concerns the behavior of two different characteristics - the energy $E_{c.m.}$ in the center of mass frame of colliding particles and/or the Killing energy $E$ of debris measured at infinity. The first quantity can become arbitrarily large in the test particle approximation. This was found in [1] (the so-called BSW effect) for the extremal Kerr metric that provoked a huge series of works in which the BSW effect was generalized. In spite of unbounded $E_{c.m.}$, the quantity $E$ remains quite modest after collisions near black holes because of strong redshift [2] - [5].

This stimulates search for other types of objects such that $E$ (in the test particle approximation) could be formally unbounded after collision (this is called the super-Penrose process). First of all, this includes wormholes. For the first time, high energy collisions in wormhole space-times were considered in [6] where it was shown for a particular model (the Teo wormhole [7]) that unbounded $E_{c.m.}$ is possible. It turned out that unbounded $E$ are possible as well [8]. It was revealed that there is a general underlying reasons both for

*Electronic address: zaslav@ukr.net
unbounded $E_{c.m.}$ and $E$ for the Teo wormhole. As is shown and extended to more general wormhole metrics in [9], [10], it is connected with extremely rapid rotation.

Recently, a work appeared in which a qualitatively new scenario is realized in static wormholes, so rotation for getting unbounded $E_{c.m.}$ is not required at all [11]. One particle comes from the left region, the other one comes from the right region, so particles experience head-on collision. Such a type of collision gives rise to unbounded $E_{c.m.}$, if the lapse function near the throat is very small. Formally, scenarios with head-on collisions would give unbounded $E_{c.m.}$ near black holes as well but the problem there consists in that near the black hole horizon a particle moves towards the horizon, not away from it, so it is difficult to realize the head-on scenario (see Sec. IV A of [9] for details). One way to resolve this problem and achieve unbounded $E_{c.m.}$ due to head-on collisions consists in considering white holes [12]. The scenario proposed by Krasnikov, enables one to find qualitatively different way to form an initial state needed for head-on collision.

It is worth stressing that enhancement of energy requires the existence of the ergoregion where $E$ can be negative, as usual in the Penrose process [13]. The conservation of energy entails that a particle with large negative energy compensates high positive energy of debris detected at infinity. As the Schwarzschild-like wormhole considered in [11] does not posses the ergoregion, the class of wormholes considered by Krasnikov is not suitable for our purposes. To achieve our goal of making the super-Penrose process possible, we combine the Krasnikov’s type of scenario with the presence ergoregion. It can exist if rotation of a wormhole is rapid enough. Now, we arrange head-on collision simply due to the wormhole character of geometry only. In other realizations of the super-Penrose process (without wormholes) it was assumed that there is a potential barrier from which a particle can bounce back [14], [15], [16].

Let us consider the metric

$$ds^2 = -N^2 dt^2 + g_{\phi}(d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2.$$  \hspace{1cm} (1)

We will consider all the processes in the plane $\theta = \frac{\pi}{2}$ (which is supposed to be a plane of symmetry), so our metric effectively reduces to the 2+1 dimensional one. We also assume that the metric coefficients do not depend on $t$ and $\phi$, so the energy $E = -mu$ and angular momentum $L = mu_\phi$ are conserved, $u^\mu = \frac{dx^\mu}{dt}$ being the four-velocity, $\tau$ the proper time. We suppose that $r_0 \leq r < \infty$, where $r_0$ is the throat radius. In terms of the so-called shape
function \( b(r) \) the quantity \( A = 1 - \frac{\ell}{r} \). We make additional assumptions that \( N \) has a nonzero minimum at \( r = r_0 \) and, moreover, \( N_0 = N(r_0) \ll 1 \).

The geodesic equations read
\[
m \frac{dt}{d\tau} = \frac{X}{N^2}, \tag{2}
\]
\[
m \frac{d\phi}{d\tau} = \frac{L}{g_\phi} + \frac{\omega X}{N^2}, \tag{3}
\]
\[
\frac{N}{\sqrt{A}} \frac{dr}{d\tau} = \sigma Z, \tag{4}
\]
\( \sigma = \pm 1 \) depending on direction of motion,

\[
X = E - \omega L, \tag{5}
\]
\[
Z = \sqrt{X^2 - N^2(m^2 + \frac{L^2}{g_\phi})}. \tag{6}
\]

It follows from the forward-in-time condition \( \frac{dr}{d\tau} > 0 \) that
\[
X > 0. \tag{7}
\]

Let particles 1 and 2 come from the right and left infinities, respectively, and collide near the wormhole throat. Then, for the energy in the centre of mass frame we have
\[
E_{c.m}^2 = -(m_1 u_1^\mu + m_2 u_2^\mu)(m_1 u_1^\mu + m_2 u_2^\mu) = m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \tag{8}
\]
where \( \gamma = -u_1^\mu u_2^\mu \) is the Lorentz gamma factor of relative motion.

The conservation laws read
\[
E_1 + E_2 = E_3 + E_4, \tag{9}
\]
\[
L_1 + L_2 = L_3 + L_4. \tag{10}
\]
We assume that particle 3 escapes to the right infinity and particle 4 so does to the left infinity. Thus \( \sigma_1 = -1, \sigma_2 = +1, \sigma_3 = +1, \sigma_4 = -1 \). Then, the conservation of the radial momentum gives us from (11)
\[
Z_2 - Z_1 = Z_3 - Z_4. \tag{11}
\]

It follows from (9) and (10) that
\[
X_1 + X_2 = X_3 + X_4. \tag{12}
\]
Let collision happen just in the region near the throat where \( N \ll 1 \). We suppose that \( X(r_0) \neq 0 \) and is not small for all particles. It means that characteristics of all particles are not fine-tuned. This makes the problem of escaping much more easy than in the black hole case (see, e.g. [3]).

Then,

\[
Z \approx X - N^2 z, \quad z = \frac{1}{2X} \left( m^2 + \frac{L^2}{g_\phi} \right).
\]

(13)

In this region, \((11)\) can be written as

\[
X_2 - X_1 \approx X_3 - X_4 + bN^2,
\]

(14)

\[
b = z_4 - z_3 + z_2 - z_1.
\]

(15)

It follows from \((12), (14)\) that near the throat (denoted by subscript "th")

\[
(X_3)_{th} \approx (X_2)_{th} - \frac{bN_0^2}{2},
\]

(16)

\[
(X_4)_{th} \approx (X_1)_{th} + \frac{bN_0^2}{2}.
\]

(17)

The quantities \( X_1, X_2 \) are given by the initial conditions. Then, in the region under discussion, one finds \( X_3, X_4 \) from \((16), (17)\). For a given value \((X_3)_{th}\), there is an infinite set of pairs \((E_3, L_3)\). They should obey \((10), (12)\). Fixing, say, \( L_4 \) one finds \( L_3 \) and

\[
E_3 = (X_3)_{th} + \omega_{th} L_3.
\]

(18)

Taking \( L_3 \) large, positive and unbounded, one obtains \( E_3 \) large, positive and unbounded. This implies that, according to \((9)\), for fixed finite \( E_1, E_2, L_1, L_2 \) the energy \( E_4 \to -\infty, L_4 \to -\infty \) formally. As, by assumption, there exists the ergoregion, negative energies are admissible.

Thus we obtain simultaneously not only the analogue of the BSW effect but also the super-Penrose process without requiring fine-tuning typical of the BSW effect near black holes [1]. This means that rotating wormholes can be considered as legitimate candidates for such high-energy processes.

This is not the end of story. To achieve our goal, it is necessary that particle 3 escape to infinity without reflection from the potential barrier back to the vicinity of the throat, so that the expression inside the radical in \((6)\) should be positive everywhere. Then, the
condition under discussion reads $Z^2 > 0$, where $Z$ is given by (6). To simplify formulas, let us assume that $m_3 = 0$ or negligible. We have from (5) that

$$X > N \frac{L}{\sqrt{g_\phi}},$$

(19)

where it is supposed that $L > 0$. With (16), (18) taken into account, it gives us

$$(X_2)_{th} + L_3(\omega_{th} - \Omega_+) > 0,$$

(20)

where $\Omega_+ = \omega + \frac{N}{\sqrt{g_\phi}}$. It has the meaning of the maximum possible angular velocity (for orbits with $r = \text{const}$) compatible with the time-like character of the interval, $ds^2 < 0$, so

$$\Omega_- < \omega < \Omega_+,$$

(21)

$\Omega_- = \omega - \frac{N}{\sqrt{g_\phi}}$. As $(X_2)_{th} > 0$ and, by assumption, $L_3 > 0$, it is sufficient to require

$$\omega_{th} - \Omega_+ > 0$$

(22)

for (19) to be valid. Inequality (22) should be compatible with (21).

For a black hole, in the horizon limit $N \to 0$ and all three velocities $\omega$ and $\Omega_\pm$ tend to the same limit. By contrast, for a wormhole case the lapse function remains small but nonzero. Let in the vicinity of $r_0$,

$$N^2 \approx N_0^2 + \frac{(r - r_0)^2}{r_1^2},$$

(23)

where $r_1$ is some constant. Then, using the Taylor expansion in the immediate vicinity of the throat, we have

$$\Omega_+ \approx \omega_{th} + \omega'(r_0)(r - r_0) + \frac{N_0}{\sqrt{g_\phi(r_0)}}.$$

(24)

Usually, $\omega' < 0$ (in particular, this is valid for the Kerr and Kerr-Newman metrics). Then, we see that for a wormhole under discussion there is a small region $r - r_0 \leq r^*$, in which $0 < \Omega_+ - \omega(r_0) \leq \frac{N_0}{\sqrt{g_\phi(r_0)}}$, $r^* = \frac{N_0}{|\omega'(r_0)| \sqrt{g_\phi(r_0)}}$. There, condition (22) can be violated. However, it simply means that $L_3$ is somewhat bounded according to (20). In the worst case, for $r \to r_0$,

$$L_3 < (L_3)_{\text{max}} = \frac{(X_2)_{th} \sqrt{g_\phi(r_0)}}{N_0}.$$

(25)

For a given $N_0$, this gives some upper bound. However, sending $N_0 \to 0$ we obtain that $(L_3)_{\text{max}}$ grows without bound. According to (18), $E_3$ also becomes unbounded. Thus, near the throat (where collision occurs), large $L_3$ is compensated by small $N$, so $Z^2 > 0$. Far
from the throat, $X_3(r)$ is large due to the term $E_3$ and overcomes the contribution from $L_3$ in (6), so $Z^2 > 0$ again.

For small but nonzero $N_0$, the existence of the bound on $E_3$ follows also from the Wald inequalities \[17\]. If, say, two massless particles appear as a result of collision, we have for such a collisional Penrose process (see, e.g. eq. (4) of \[16\])

$$2E_3 \leq E + \sqrt{E^2 + g_{00}E_{c.m.}^2},$$

(26)

$E = E_1 + E_2$. For very large $E_{c.m.}^2$, the maximum possible energy at infinity

$$(E_3)_{\text{max}} \approx \frac{\sqrt{g_{00}}}{2}E_{c.m.}.$$

(27)

Here, $g_{00} > 0$ since collision is supposed to occur in the ergoregion.

The quantities $\gamma$ and $E_{c.m.}$ \[8\] can be found directly from equations of motion (2) - (4). Then, for head-on collision ($\sigma_1 = -1$, $\sigma_2 = +1$) we obtain the formula (listed in many papers on the BSW effect)

$$2m_1m_2\gamma = \frac{X_1X_2 + Z_1Z_2}{N^2} - \frac{L_1L_2}{g_{\phi}}.$$ 

(28)

When $N_0 \to 0$,

$$E_{c.m.}^2 \approx \frac{E_1E_2}{N_0^2}.$$ 

(29)

Taking $E_1 = E_2 = m$, we have $E_{c.m.} \approx m/N_0$. Thus $(E_3)_{\text{max}} \sim m/N_0$ as well.

One more reservation is in order. The combination of small $N$ and a wormhole nature of the metric (because of which the horizon is absent) leads to the undesirable behavior of the Kretschmann invariant in the limit $N_0 \to 0$. This gives a low bound on admissible value of $N_0$. Corresponding estimates were made in \[18\] for wormholes having the same mass ($10^5 cm$ in geometric units) as astrophysical black holes. Then, the condition that tidal forces do not destroy atomic matter gives $N_0 > 10^{-13}$. This is a very weak restriction. For rotating wormholes formulas from \[18\] are not applicable directly but they can be used at least for rough estimates.

Thus we showed that there is a way to achieve the super-Penrose process without fine-tuning or specially invented scenario. Rotating wormholes realize this in a quite natural way. It is worth stressing that the results are model-independent and are insensitive to the details of the metric. The key point consists in that due to a wormhole character of the metric, head-on collision is possible in the region of small $N_0$. 

Acknowledgments

This work was funded by the subsidy allocated to Kazan Federal University for the state assignment in the sphere of scientific activities. I also thank for support SFFR, Ukraine, Project No. 32367.

[1] M. Bañados, J. Silk and S.M. West, Kerr black holes as particle accelerators to arbitrarily high energy, Phys. Rev. Lett. 103, (2009) 111102 [arXiv:0909.0169].
[2] M. Bejger, T. Piran, M. Abramowicz, and F. Håkanson, Collisional Penrose process near the horizon of extreme Kerr black holes, Phys. Rev. Lett. 109 (2012) 121101 [arXiv:1205.4350].
[3] T. Harada, H. Nemoto and U. Miyamoto, Upper limits of particle emission from high-energy collision and reaction near a maximally rotating Kerr black hole, Phys. Rev. D 86 (2012) 024027 [Erratum ibid. D 86 (2012) 069902] [arXiv:1205.7088].
[4] O. Zaslavskii, On energetics of particle collisions near black holes: BSW effect versus Penrose process, Phys. Rev. D 86 (2012) 084030 [arXiv:1205.4410].
[5] O. B. Zaslavskii, Is the super-Penrose process possible near black holes? Phys. Rev. D 93 (2016), 024056 [arXiv:1511.07501].
[6] Tsukamoto, N., Bambi, C.: High energy collision of two particles in wormhole spacetimes, Phys. Rev. D 91, 084013 (2015). [arXiv:1411.5778].
[7] E. Teo, Rotating traversable wormholes, Phys. Rev. D 58, 024014 (1998) [arXiv:gr-qc/9803098].
[8] Tsukamoto, N., Bambi, C. Collisional Penrose Process in Rotating Wormhole Spacetime, Phys. Rev. D 91, 104040 (2015). [arXiv:1503.06386].
[9] O. B. Zaslavskii, Rotation as an origin of high energy particle collisions, Mod. Phys. Lett. A, Vol. 31, No. 4 (2016) 1650029 [1506.02638].
[10] O. B. Zaslavskii, Rapidly rotating spacetimes and collisional super-Penrose process, Gen. Relat. and Gravitation 48 (2016) 67 [arXiv:1511.00844].
[11] S. Krasnikov, Schwarzschild-Like Wormholes as Accelerators, [arXiv:1807.00890].
[12] A. Grib and Yu. V. Pavlov, “Are black holes totally black?” Grav. Cosmol. 21, 13 (2015); [arXiv:1410.5736].
[13] R. Penrose, Gravitational Collapse: The Role of General Relativity, Rivista del Nuovo Cimento, Numero Speziale I, 257 (1969).

[14] O. B. Zaslavskii, Ultrahigh energy head-on collisions without horizons or naked singularities: General approach. Phys. Rev. D 88, 044030 (2013) [arXiv:1305.6136].

[15] Patil, M., Harada, T., Nakao, K., Joshi, P.S., Kimura, M.: Infinite efficiency of collisional Penrose process: can over-spinning Kerr geometry be the source of ultra-high-energy cosmic rays and neutrinos? Phys. Rev. D 93, 104015 (2016) [arXiv:1510.08205].

[16] I. V. Tanatarov and O. B. Zaslavskii, Collisional super-Penrose process and Wald inequalities, Gen Relativ Gravit 49, 119 (2017) [arXiv:1611.05912].

[17] R. Wald, Energy limits on the Pentose process, Astrophysical Journal, 191, 233 (1974).

[18] S. V. Sushkov and O. B. Zaslavskii, Horizon closeness bounds for static black hole mimickers. Phys. Rev. D 79, 067502 (2009).