Effective Action for Multi-Regge
Processes in QCD

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We construct the effective Lagrangian describing QCD in the multi-Regge kinematics. It is obtained from the original QCD Lagrangian by eliminating modes of gluon and quark fields not appearing in this underlying kinematics.
1 Introduction

Two types of evolution equations for partonic distributions in the small $x$ deep-inelastic ep-scattering are used now. The GLAP equation [1] is applied for finding the $Q^2$-dependence of structure functions and the BFKL equation [2] is applied for fixing their $x$ dependence. These equations correspond to summing the leading logarithmic terms $\sim (g^2 \ln Q^2)^n$ and $\sim (g^2 \ln \frac{1}{x})^n$ correspondingly. Their solutions grow rapidly in the limit $x \to 0$ and violate the Froissart bound for total cross-sections [2]. The violation is a consequence of the fact that the scattering amplitudes in the leading logarithmic approximation (LLA) do not satisfy $s$-channel unitarity constraints. Equations for the unitarity corrections have been investigated in [3]. The treatment of the multi-Reggeon exchanges turns out to be difficult besides of special cases [4] or of the limit of large gauge group ($N_c \to \infty$) [5]. A new method for unitarizing the results of LLA was suggested by one of the authors [6]. It is based on an effective field theory coinciding with QCD in the case, when all particles in the intermediate states of $s$- and $u$- channels have the multi-Regge kinematics [2] (see Fig. 1)

$$
\begin{align*}
    s &= (p_A + p_B)^2, \quad s_i = (k_i + k_{i-1})^2, \quad i = 1, \ldots, n + 1 \\
    k_0 &= p_A', \quad k_{n+1} = p_B', \quad k_i = q_i - q_{i+1}, \quad t_i = q_i^2, \\
    k^\mu &= \frac{(p_A k)}{(p_A p_B)} p_B^\mu + \frac{(p_B k)}{(p_A p_B)} p_A^\mu + k_\perp^\mu, \quad k_\perp^2 = -\vec{k}_\perp^2 \\
    s &\gg s_1, \sim s_2, \sim \ldots, \sim s_{n+1} \gg |t_1| \sim |t_2| \sim \ldots \sim |t_{n+1}|
\end{align*}
$$

$$
\begin{align*}
    s_1 s_2 \ldots s_{n+1} &= s \prod_{i=1}^n \vec{k}_\perp^2_1 \\
    k_1 p_A &\ll k_2 p_A \ll \ldots \ll k_n p_A \\
    k_1 p_B &\gg k_2 p_B \gg \ldots \gg k_n p_B
\end{align*}
$$

The corresponding effective action was constructed in ref. [6] only for pure gluodynamics and for gravity. Recently another effective theory for the high energy scattering in the Yang-Mills model was suggested by H. and E. Verlinde [7]. The effective action for the gravity, suggested in [6], was used by D. Amati, M. Ciafaloni and G. Veneziano for calculation of the scattering amplitudes at super-Plankian energies [8].

The multi-Regge effective action can be viewed as the action reproducing the multi-particle tree amplitudes in the kinematics (1) in the most simple way. This can be taken as a guiding principle to obtain the action as discussed in [3] and in [9].

In this paper we present the direct way from the original QCD action to the effective action. We start from the QCD action in the light-cone gauge. The gluon and quark fields are separated according to regions in the momentum space determined by the kinematics (1). Modes with highly virtual momenta are excluded by means of equations of motion. We distinguish the modes with momenta corresponding to the kinematics of the quanta scattered (and produced) or exchanged in the peripheral scattering process. A discussion of the effective action emphasizing symmetry properties will be given in a further publication [10].
2 The QCD action in the light-cone gauge

For simplicity we consider massless quarks and suppress the flavour indices for quark fields \( \psi(x) \). In this case the QCD Lagrangian takes the form

\[
\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + i \bar{\psi} \gamma^\mu (\partial_\mu - ig t^a A_\mu^a) \psi
\]

where \( f^{abc} \) are the structure constants, \( t^a \) is the generator of the gauge group in quark representation \( ([t^a, t^b] = i f^{abc} t^c) \) and \( g \) is the QCD coupling constant.

To derive the effective action for multi-Regge processes it is convenient to fix the gauge for the vector potential \( A_\mu^a \). In the center-of-mass system, where

\[
p_0^A = p_0^B = p_A^3 = -p_B^3 = \frac{\sqrt{s}}{2}, \quad p_{A\perp} = p_{B\perp} = 0
\]

for initial mass-less particles with momenta \( p_A \) and \( p_B \), we chose the light-cone gauge (cf. \[6\])

\[
A_\perp \equiv A_0 - A_3 = p_B^\mu A_\mu = 0
\]

In this gauge action (2) depends on \( A_+ \equiv A_0 + A_3 \) only quadratically and therefore one can exclude \( A_+ \) from (2) using equations of motion

\[
A^a_+ = -\frac{1}{\partial_x} (\partial_\rho A^\rho) - g \frac{1}{\partial^2} \left[ i A_\rho T^a_\rho (\partial_\perp A^\rho) - \frac{1}{2} \bar{\psi}_- \gamma_- t^a \psi_+ \right]
\]

where \( \rho = 1, 2 \), \( \partial_\perp = \frac{\partial}{\partial x^\perp} \), \( x^\perp = x^0 - x^3 \) and \( T^a \) denotes the generator of gauge group in the adjoint representation \( ((T^a)^{bc} = -i f^{abc}) \).

The field \( \psi \) can be written as the sum of two components \( \psi_\pm \)

\[
\psi = \psi_+ + \psi_- \quad \psi_\pm = P_\pm \psi, \quad P_\pm = \frac{1}{4} \bar{\gamma}_\pm \gamma_\pm,
\]

where \( P_\pm \) are projectors into corresponding subspaces. In the gauge (4) equations of motion for \( \psi_+, \bar{\psi}_+ \) are easily solved

\[
\psi_+ = -\frac{1}{4} \frac{1}{\partial_-} \gamma_- \left( \partial - ig A_\perp \right) \psi_-
\]

\[
\bar{\psi}_+ = -\frac{1}{4} \frac{1}{\partial_-} \left( (\partial_\rho \bar{\psi}_-) \gamma_\rho + ig \bar{\psi}_- \hat{\imath} \right) \gamma_-
\]

\[
\hat{A} \equiv A^a_\sigma \gamma^a \sigma, \quad \hat{\imath} \equiv \gamma_\sigma \partial^\sigma
\]

Putting \( A_+, \psi_+, \bar{\psi}_+ \) from (5), (7) in Lagrangian (2) we obtain

\[
\mathcal{L}^{QCD} = \frac{1}{2} A^a_\sigma \Box A^{a\sigma} - ig(\partial_\perp A_\sigma) T^a_\sigma A_\rho \frac{1}{\partial_-} \partial_\rho A^{a\rho} - \frac{g^2}{2} (\partial_\perp A_\rho) T^a_\rho A^\rho T^a_\perp A_\sigma - ig(\partial_\sigma A_\rho^a) A^{a\rho} T^a_\perp A^\sigma
\]
\[ + \frac{i}{4} \bar{\psi}_- \gamma_\mu \Box \psi_- - \frac{g}{2} \bar{\psi}_- \gamma_\mu t^\rho \bar{\psi}_- \left( \frac{1}{\partial_\nu} \partial_\nu A^{a\rho} \right) + \]
\[ + \frac{g^2}{8} \bar{\psi}_- t^a \psi_- \frac{1}{\partial_\nu} \bar{\psi}_- t^a \psi_- + \frac{i g^2}{2} \bar{\psi}_- t^a \psi_- \frac{1}{\partial_\nu} (\partial_\nu A^a) T^a A^\rho \]
\[ - \frac{g}{4} \bar{\psi}_- \delta_\mu \gamma_- \dot{A} \psi_- - \frac{g}{4} \bar{\psi}_- \delta_\mu \gamma_- \dot{A} \psi_- + \]
\[ + \frac{g^2}{4} \bar{\psi}_- \gamma_- \dot{A} \partial_\nu \psi_- + \frac{g^2}{4} A^a_\rho A_\sigma A^\rho A^\sigma \]

where \( \Box = 4 \partial_+ \partial_- + \partial_\rho \partial^\rho \) is the d’Alembertian.

For high-energy processes the last term in the Lagrangian (8) is not essential (Ref. [2]). It does not give also any contribution to the effective action. This fact is in accordance with Ref. [7]. However, it turns out that the contributions from the transverse part \( \sim \partial_\sigma \partial^\sigma \) of the d’Alembertian \( \Box \) in the first term and from the three-linear fourth term of Lagrangian (8) are needed to reproduce correctly the effective action of Ref. [7]. This seems to be in conflict with the arguments of Ref. [7] about the possibility to neglect at high energies the term \( -\frac{1}{4} G_{\rho \sigma} G^{a\rho \sigma} \) with all transverse indices.

### 3 Exclusion of heavy modes in the multi-Regge kinematics

It is known, that to obtain the generating functional for scattering amplitudes in the tree approximation one has to calculate the Lagrangian on the solutions \( \hat{A}_\sigma \) of equations of motion having a prescribed asymptotic behaviour at \( t \to \pm \infty \) [11]. In this section we use the equations of motions in order to exclude from the action (4) the fields describing strongly virtual gluons and quarks (heavy modes) appearing in the Feynman diagrams for production amplitudes in the multi-Regge kinematics (1). One diagram of this type is shown in Fig. 2 (where \( k^2 \approx k_1^2 = s_2 \gg |k_1^2| \)). We solve the equations of motion for heavy modes within the perturbation theory. The following analysis is based on the first perturbative order. At the end of this chapter we comment on possible improvements of this approximation.

Let us decompose each of \( A_\sigma, \psi_- \) and \( \bar{\psi}_- \) in the sums of two fields for the strongly (s) and moderately (m) virtual particles:

\[ A_\sigma = A_\sigma^{(s)} + A_\sigma^{(m)} \ , \ \psi_- = \psi_-^{(s)} + \psi_-^{(m)} \ , \ \bar{\psi}_- = \bar{\psi}_-^{(s)} + \bar{\psi}_-^{(m)}. \]

In the kinetic terms of Lagrangian (8) one can neglect the interference contributions between s- and m-fields with a good accuracy:

\[ \mathcal{L}^{\text{kin}} \approx 2 A_\sigma^{(s)} \partial_+ \partial_- A_\sigma^{(s)} + \frac{1}{2} A_\sigma^{(m)} \Box A_\sigma^{(m)} + i \bar{\psi}_-^{(s)} \gamma_- \partial_+ \psi_-^{(s)} + \frac{i}{4} \bar{\psi}_-^{(m)} \gamma_- \Box \psi_-^{(m)} \]

where we took into account, that the s-field virtuality is large only due to the presence of the large longitudinal momenta of external particles (\( \Box \approx 4 \partial_+ \partial_- \))(see Fig. 2).

The interaction terms for s fields are obtained from Lagrangian (8) by keeping (after decomposition (9)) the most important (in the underlying kinematics) terms. These terms
are enhanced by the operator \( \frac{1}{\partial_-} \) acting on the \( m \) field with the smallest \( k_- \) momentum component. Up to the first order in \( g \) the essential interaction terms involving \( s \) fields are the following (cf. (10))

\[
\mathcal{L}^{(s)} = 2 A_{\alpha}^{(s)\alpha} \partial_+ \partial_- A^{(s)\alpha} + i \bar{\psi}^{(s)}_m \gamma_\mu \partial_+ \psi^{(s)}_m + g[i A_{\alpha} T^a \partial_- A^a - \frac{1}{2} \bar{\psi}_m \gamma_\mu \partial_- \psi_m](\frac{1}{\partial_-} \partial_\mu A^{(m)\alpha} + \frac{g}{4}(\partial_\mu \bar{\psi}^{(m)}_m) \gamma_\mu \frac{1}{\partial_-} \gamma_\mu \hat{A}_m \psi_m - \frac{g}{4} \bar{\psi}_m \hat{A}_m \gamma_\mu \hat{\partial}_\mu \bar{\psi}^{(m)}_m)
\]

where the fields \( A_\sigma, \psi_- \) and \( \bar{\psi}_- \) are decomposed according to (9). Let us note that because of the absence of the above mentioned enhancement factor a contribution to (11) of fourth term in Lagrangian (8) as well as all the remaining terms can be omitted in our approximation (they involve only \( m \) fields, see eq. (13)).

The \( s \) field solutions of equations of motions following from Lagrangian (11) can be easily found in the perturbation theory

\[
\hat{A}_\sigma = -\frac{i g}{2} A_{\alpha}^{(m)\alpha} T^a \left( \frac{1}{\partial_+ \partial_-} \partial_\alpha A^{(m)\alpha} \right) - \frac{g}{16} \left[ (\frac{1}{\partial_+ \partial_-} \bar{\psi}^{(m)}_m) \hat{\partial}_\mu \gamma_\mu \frac{1}{\partial_-} \psi^{(m)}_m - (\frac{1}{\partial_-} \bar{\psi}^{(m)}_m) \hat{\partial}_\mu \gamma_\mu \frac{1}{\partial_+} \psi^{(m)}_m \right] + \mathcal{O}(g^2)
\]

\[
\bar{\psi}^{(s)}_m = -\frac{i g}{2} \left\{ t^a \bar{\psi}^{(m)}_m \left( \frac{1}{\partial_+ \partial_-} \partial_\alpha A^{(m)\alpha} \right) + \frac{1}{2} \hat{\partial}_\alpha \bar{\psi}^{(m)}_m \hat{A}_m \right\} + \mathcal{O}(g^2)
\]

\[
\hat{\psi}^{(s)}_m = \frac{i g}{2} \left\{ (\frac{1}{\partial_+ \partial_-} \partial_\alpha A^{(m)\alpha}) \bar{\psi}^{(m)}_m t^a - \frac{1}{2} \left( \frac{\partial_\alpha}{\partial_-} \bar{\psi}^{(m)}_m \right) \gamma_\mu \hat{A}_m \hat{\partial}_\mu \bar{\psi}^{(m)}_m \right\} + \mathcal{O}(g^2)
\]

where we took into account that in the multi-Regge kinematics (1), for each pair of fields, the field having a smaller value of \( \partial_- \), should have a bigger value of \( \partial_+ \).

We substitute the fields \( A_\sigma, \psi^{(s)}, \bar{\psi}^{(s)} \) in eq. (11) by their classical counterparts (12), and replace the part \( \mathcal{L}^{(s)} \) (11) of the Yang-Mills action (8) by this result. We obtain a modified action involving only the \( m \)-fields:

\[
\mathcal{L}^{mod} = \mathcal{L}^{QCD}_{A, \psi, \bar{\psi}} |_{A, \psi, \bar{\psi} \rightarrow A^{(m)}, \psi^{(m)}, \bar{\psi}^{(m)}} + \Delta \mathcal{L}, \quad (13)
\]

where (cf. (11))

\[
\Delta \mathcal{L} = -2 \hat{A}_\sigma \hat{\partial}_+ \hat{\partial}_- \hat{A}_\sigma - i \hat{\psi}_- \gamma_- \hat{\partial}_+ \psi^{(s)} + \mathcal{O}(g^3). \quad (14)
\]

Performing the differentiations after substitution of eqs. (12) in eq. (14) we have to take into account which field carry small \( k_- \) (large \( k_+ \)) momentum components. Thus, the differentiations \( \hat{\partial}_+ \) and \( \hat{\partial}_- \) in (14) give an essential contribution only when they are applied to the fields with the factor \( \frac{1}{\partial_-} \) and without this factor, correspondingly. Taking into account also the freedom of integrating by parts, the expression (14) can be represented
as follows (cf. (11)).

\[
\Delta \mathcal{L} = \frac{g}{4} [A^{(m)}_\rho T^a (\partial_- A^{(m)}_\rho)] + \frac{i}{2} \bar{\psi}^{(m)} \gamma_- \psi^{(m)} \left( \frac{1}{\partial_+ \partial_-} - \partial_\sigma A^{(m)\sigma} \right) T^a \left( \frac{1}{\partial_-} \partial_\eta A^{(m)\eta} \right) + \\
+ \frac{ig^2}{8} \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^{(m)\sigma} \right) \left[ (\partial^e \bar{\psi}^{(m)}) \gamma_\rho \gamma_- t^a \hat{A} \psi^{(m)} - \bar{\psi}^{(m)} \hat{A} \psi^{(m)} \right] 
\]

All other terms, including those from four quark fields are negligible in the multi-Regge kinematics. In particular, we neglect the contribution of the term symmetric to the one written in eq. (15) in which both fermions carry small \(k_-\) momentum components since the matrix element involving fermions is small.

In the next section we shall simplify the action (13) for the \(m\) fields by the use of their equations of motion. Above we excluded the heavy fields from the action approximately up to the order \(g^2\) of the perturbation theory (see (15)), but it can be done in all orders because \(\mathcal{L}^{(s)}\) in eq.(11) depends only quadratically on them. Instead of eq. (12) we would obtain expressions, containing in particular the path ordered exponentials \(P \exp(\frac{i}{\hbar} \int_L A^a_+ T^a dx^+ )\) with \(A_+\) given by eq. (5) and therefore \(\mathcal{L}^{mod}\) in eq.(13) generally is nonlocal and contains strongly nonlinear interaction terms. Nevertheless, the effective action, which is constructed in the next sections, remains to be three-linear in fields even after taking into account the higher order corrections to the classical solutions (12) for heavy fields. We hope to discuss this point elsewhere.

## 4 Equations of motion for Coulomb fields

After eliminating the heavy modes we separate the remaining modes \(A^{(m)}, \psi^{(m)}\) of the gluon and quark fields into a part involving modes \(A', \psi'\) with the momenta obeying \(|k_+ k_-| \ll |k^2_\perp|\) and a part \(A, \psi\) with the momenta obeying \(|k_+ k_-| \simeq |k^2_\perp|\). We call the modes \(A', \psi'\) Coulombic; in their kinetic terms the longitudinal derivatives can be omitted: \(\Box \to \partial_\sigma \partial^\sigma\) and therefore these modes describe the instantaneous Coulomb interactions.

The additional terms (15) in the Lagrangian obtained by elimination of the heavy modes can be factorized into triple vertices connected by Coulombic propagators. In this way we see that the terms (15) are reproduced if we add to the original action (8) the following induced terms

\[
\tilde{\mathcal{L}}^{mod} = \mathcal{L}^{QCD} + \Delta \mathcal{L}^{ind} 
\]

where

\[
\Delta \mathcal{L}^{ind} = \frac{ig}{4} (\partial_- \partial_\rho A^{a\rho}) \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^{\sigma} \right) T^a \left( \frac{1}{\partial_-} \partial_\eta A^{\eta} \right) + \\
+ \frac{g}{8} \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^{a\sigma} \right) \left[ (\partial^e \bar{\psi}'_\perp) \gamma_\rho \gamma_- t^a (\hat{\partial} \bar{\psi}_\perp) + \right. \\
+ \left. \left( \frac{\partial^e}{\partial_-} \bar{\psi}'_\perp \right) \gamma_\rho \gamma_- t^a \hat{\partial} \psi' \right] 
\]
Let us verify, that the quartic terms (15) can be obtained from (16) by the use of equations of motions for Coulomb fields:

\[
\begin{align*}
\partial_\sigma \partial^\sigma A^{a_0} &= ig \frac{\partial^e}{\partial_-}((\partial_- A_\sigma) T^a A^\sigma) + \frac{g}{2} \frac{\partial^e}{\partial_-} (\bar{\psi}_- \gamma^- t^a \psi_-) \\
&- \frac{ig}{4} \partial_- \partial^2 \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^\sigma \right) T^a \left( \frac{1}{\partial_-} \partial_\eta A^\eta \right) \\
\partial_\sigma \partial^\sigma \bar{\psi}'_- &= ig \frac{\partial}{\partial_-} (\hat{A} \bar{\psi}_-) + ig \frac{\partial}{\partial_-} \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^\sigma \right) T^a \left( \frac{1}{\partial_-} \partial_\eta A^\eta \right) \\
\partial_\sigma \partial^\sigma \psi'_- &= -ig \left( \frac{\partial^\sigma}{\partial_-} \bar{\psi}_- \hat{A} \right) \gamma_\sigma - \frac{ig}{2} \partial^\sigma \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^\sigma \right) \frac{\partial^\eta}{\partial_-} \left( \gamma_\eta \right) t^a \gamma_\sigma 
\end{align*}
\]

(18)

In (18) we write down only the terms, which arise from varying the fields containing factors \( \frac{1}{\partial_-} \) in \( \mathcal{L}^{QCD} \) (8) and nonsingular ones for \( \partial_- \to 0 \) in \( \Delta \mathcal{L}^\text{ind} \) (16). These fields describe correspondingly the emission and absorption of Coulomb particles in the crossing channel.

Putting solutions \( \hat{A}, \bar{\psi}_-, \bar{\psi}'_- \) of eqs. (18) back in \( \mathcal{L}^\text{mod} \) we obtain several four-linear terms:

\[
\begin{align*}
-\frac{1}{2} \bar{A}^a_\sigma \partial_\sigma \partial^e \bar{A}^{a_0} &- \frac{i}{4} \bar{\psi}'_- \gamma^- \frac{\partial_\sigma \partial^e}{\partial_-} \bar{\psi}'_- = \\
&= -\frac{g^2}{2} \left[ i(\partial_- A_\sigma) T^a A^\sigma + \frac{1}{2} \bar{\psi}_- \gamma^- t^a \psi_- \right] \left( \frac{1}{\partial_-} \partial^\eta \left[ i(\partial_- A_\sigma) T^a A^\sigma + \frac{1}{2} \bar{\psi}_- \gamma^- t^a \psi_- \right] \right) \\
&- \frac{ig}{4} \bar{\psi}_- \hat{A} \gamma^- \hat{A} \psi_- + \frac{ig}{2} \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^{a\sigma} \right) \left( \frac{\partial^\eta}{\partial_-} \bar{\psi}_- \right) \gamma_\eta \gamma_\tau \hat{A} \psi_- \\
&- \frac{ig}{8} \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^{a\sigma} \right) \bar{\psi}_- \hat{A} \gamma^- t^a \left( \frac{\partial}{\partial_-} \psi_- \right) + \\
&+ \frac{ig}{4} \left[ i(\partial_- A_\sigma) T^a A^\sigma + \frac{1}{2} \bar{\psi}_- \gamma^- t^a \psi_- \right] \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^\sigma \right) T^a \left( \frac{1}{\partial_-} \partial_\eta A^\eta \right) \\
&+ \ldots 
\end{align*}
\]

(19)

The last three terms in eq. (19) reproduce the contribution (15) of heavy modes and first two are cancelled with the four-linear singular terms in eq. (8). Neglecting the last nonsingular term in (8) one can write instead of (13) the following comparatively simple expression for the effective lagrangian:

\[
\begin{align*}
\mathcal{L}^\text{eff} &= \frac{1}{2} A^a_\sigma \square A^{a_0} + \frac{i}{4} \bar{\psi}_- \gamma^- \square \psi_- \\
&- g \bar{A}_+^a \left[ i A_\sigma T^a (\partial_- A^\sigma) - \frac{1}{2} \bar{\psi}_- \gamma^- t^a \psi_- \right] \\
&+ g \left( \bar{\psi}'_- \hat{A} \psi_- + \bar{\psi}_- \hat{A} \psi'_- \right) - ig \left( \bar{\psi}'_- \hat{A} \psi_- \right) \gamma^- t^a A^\sigma \\
&- \frac{ig}{8} \left( \partial_\sigma \partial^\sigma A^{a\sigma} \right) \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^\sigma \right) T^a \left( \frac{1}{\partial_-} \partial_\eta A^\eta \right) \\
&- \frac{g}{2} \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^{a\sigma} \right) \left[ (\partial^e \bar{\psi}'_-) \gamma_\tau \bar{\psi}^a \psi_+ + \bar{\psi}_+ t^a \hat{A} \psi_-' \right] \\
&- \frac{g}{2} \left( \frac{1}{\partial_+ \partial_-} \partial_\sigma A^{a\sigma} \right) \left[ (\partial^e \bar{\psi}'_-) \gamma_\tau \bar{\psi}^a \psi_+ + \bar{\psi}_+ t^a \hat{A} \psi_-' \right] \\
&+ \ldots
\end{align*}
\]
where we introduced the following notations (cf. (5),(7)) for the Coulomb fields (in the corresponding kinematics)

\[
\begin{align*}
A'_+ &= -\frac{1}{\sigma_-} \partial_\sigma A^\sigma - A'_- = -\frac{2}{\sigma_\sigma} \partial_\sigma A^\sigma \\
\psi'_+ &= -\frac{1}{4} \gamma_\gamma_\gamma_\gamma \hat{\sigma}_\sigma \psi'_{\bar{\sigma}} - \bar{\psi}'_+ = -\frac{1}{4} (\hat{\gamma}_\gamma_\gamma_\gamma) \gamma_\sigma \gamma_- 
\end{align*}
\]

(21)

Let us note that the role of \( A'_- \) field in the fermionic case is played by \( \psi'_- \). We want also to emphasize that fields appearing in (20) without prime contain all modes of moderately virtual fields, i.e. they are in fact \( A + A'_- \) or \( \psi + \psi'_- \).

Although \( A'_+, \psi'_+, \bar{\psi}'_+ \) according to eqs. (21) are expressed through \( A'_-, \psi'_-, \bar{\psi}'_- \), we shall consider them in the following discussion as independent fields, supposing in particular, that

\[
0 = \langle A'_+(x) A'_+(0) \rangle = \langle A'_-(x) A'_-(0) \rangle = \langle \psi'_-(x) \bar{\psi}'_-(0) \rangle = \langle \psi'_+(x) \bar{\psi}'_+(0) \rangle 
\]

(22)

for the virtual (Coulomb) fields. The vanishing of the Green functions for the same fields (22) is needed to put effectively to zero the two superfluous terms in the expression (19) in the second order of perturbation theory, which are cancelled with the corresponding nonlinear terms in eq. (8).

In accordance with eqs. (21) we should take the following free actions for the Coulomb fields:

\[
L_{\text{free}}^{\text{Coul}} = \frac{1}{2} A'^a_\sigma \partial_\sigma \partial^a A'^\sigma + i \bar{\psi}'_+ \hat{\sigma}_\sigma \psi' + i \bar{\psi}'_+ \hat{\sigma}_\sigma \psi' 
\]

(23)

Note that to reproduce correctly the Coulomb field propagators the coefficients in (23) are chosen to be different from what one would obtain by formal substitution of (21). In writing eq. (23) we took also into account that field \( A'_+ \) contains only the longitudinal part \( \sim \partial_\sigma \) of field \( A'_\sigma \) (eq. (21)).

In the next sections the action (20) will be transformed to a simpler form by introducing separately two sorts of fields for the produced and virtual particles.

### 5 Interacting vertices for virtual and real particles

The three-linear terms in Lagrangian (20) describe the various processes of particle production and scattering. The real (produced) and virtual (Coulomb) particles have completely different kinematics and therefore it is natural to introduce for them different notations \[ \mathbf{[3]} \]. Temporarily we leave for the fields, corresponding to produced particles the old notations \( A_\sigma, \psi_- \), \( \bar{\psi}_- \) and use the notations (21) for fields, describing Coulomb-like forces.

All vertices for the ‘forward’ scattering off right particles linear in \( A'_+, \psi'_+ \) and \( \bar{\psi}'_+ \) can be written down from (20):

\[
\begin{align*}
L_{\text{scat}}^{(R)} &= -g [i A_\sigma T^a (\partial_\sigma A^a) - \frac{1}{2} \bar{\psi}_- \gamma_\gamma_\gamma_\gamma \psi_- ] A'^a_+ \\
&\quad + g (\bar{\psi}'_+ \hat{A}_+ \psi_- + \bar{\psi}_- \hat{A}_+ \psi' '_+) 
\end{align*}
\]

(24)

In the dominant kinematics these vertices describe the scattering of particles close in momentum to \( p_A \) off the right particles close in momentum to \( p_B \).
Since $A_t^a$ contains $\partial_\xi A^{ao}$ and do not contain $\varepsilon_{\rho\sigma} \partial^r A^{a\sigma}$ one can use one of the following substitutions for the fields describing the Coulomb particles:

$$A^{a_\rho} \rightarrow \frac{1}{\partial_\eta \partial^\rho} \partial_\rho A^{a_\sigma} = \frac{1}{2} \frac{\partial^r}{\partial_-} A^a, \quad -\frac{1}{\partial_\eta \partial^\rho} \partial_- \partial^r A^a_+$$  \hspace{1cm} (25)

The production vertices $L_{\text{prod}}$ are obtained with the use of (25) from the terms giving contributions to $L^{(R)}_{\text{scat}}$ (24) and from the three last terms in eq. (20)

$$L_{\text{prod}} = gA^a_+[iA_\sigma T^a (\partial^\sigma A_-') + \frac{1}{2} \bar{\psi}_-^o \gamma_- t^o \psi_- + \frac{1}{2} \bar{\psi}_-^o \gamma_- t^o \psi_-']$$

$$+ g[\bar{\psi}_+^o \hat{A} \psi_- + \bar{\psi}_+^o \hat{A} \psi_- - \frac{1}{2} \bar{\psi}_+^o (\hat{\partial}_- A^a) \psi_- - \frac{1}{2} \bar{\psi}_-^o (\hat{\partial}_- A^a) \psi_-']$$

$$+ \frac{ig}{4} (\partial_\rho \partial^\rho A^a_0) (\frac{1}{\partial_+ \partial_-} \partial_\sigma A^{a_\sigma}) T^a A_+$$

$$- \frac{g}{2} [\bar{\psi}_+^o \gamma_+ T^o A^a_+ \hat{\partial}_+ \psi_-'] - \bar{\psi}_-^o \gamma_- t^o \psi_-'] \frac{1}{\partial_+}$$  \hspace{1cm} (26)

We shall transform $L_{\text{prod}}$ to a simpler form in the next section. Now, let us consider the forward scattering off left particles. Here one should take into account that according to our definitions (21) $A_-', \psi_-'$ and $\bar{\psi}_-$ do not contain the singularities $\frac{1}{\partial_-}$ and therefore for the forward scattering off left particles all three-linear terms of $L^{(L)}_{\text{eff}}$ (20) give contributions of the necessary order:

$$L^{(L)}_{\text{scat}} = \frac{g}{2} \left( \frac{1}{\partial_\xi} \partial^a A^{a_\rho} \right) [i(\partial_\rho A_-') T^a A^\rho + i(\partial_- A_\rho) T^a (\frac{\partial^\rho}{\partial_-} A_-') - \bar{\psi}_-^o \gamma_- t^o \psi_-]$$

$$+ g[\bar{\psi}_+^o \hat{A} \psi_- + \bar{\psi}_+^o \hat{A} \psi_- - \frac{1}{2} \bar{\psi}_+^o (\hat{\partial}_- A^a) t^o \psi_- - \frac{1}{2} \bar{\psi}_-^o (\hat{\partial}_- A^a) t^o \psi_-']$$

$$- \frac{ig}{8} (\partial_\eta \partial^\rho A^a_0) (\frac{1}{\partial_+ \partial_-} \partial_\sigma A^{a_\sigma}) T^a (\frac{1}{\partial_-} \partial_\rho A^a \rho)$$

$$- \frac{g}{2} [(\partial^o \bar{\psi}_-) \gamma_- t^o \frac{1}{\partial_+ \partial_-} \partial_\sigma A^{a_\sigma} \psi_+ + \bar{\psi}_-^o \gamma_- t^o (\frac{1}{\partial_+} \partial_- A^{a_\sigma}) \hat{\partial}_- \psi_-']$$

$$+ \frac{ig}{2} \left( (\partial_\rho A^a_0) - (\partial_\rho A^{a_\rho}_0) \right) \frac{\partial^\rho}{\partial_-} A_-') T^a A^\rho$$  \hspace{1cm} (27)

We stress that the last term in expression (27) appears from the nonsingular Yang-Mills vertex which disappears in Verlinde scaling limit \footnote{[7]}. One can transform the expression (27) by integrating by part to the form:
The first half of expression (31) is obtained from the terms in eq. (20) containing fields $A'_+, \psi'_+, \bar{\psi}'_+$. The second half of it should be obtained from induced terms in eq. (20) after a more accurate calculation of them (earlier it was assumed that derivatives $\partial_+$ from the fields $A'_-, \psi'_-, \bar{\psi}'_-$ are negligible which is not the case now).
6 Effective action for multi-Regge processes in QCD

It is convenient to introduce the complex and light-cone coordinates in the two-dimensional transverse and longitudinal subspaces, correspondingly

\[
\varrho = x^1 + ix^2, \quad \varrho^* = x^1 - ix^2, \quad \partial = \frac{\partial}{\partial \varrho}, \quad \partial^* = \frac{\partial}{\partial \varrho^*}
\]  

and the analogous components for transverse fields and \(\gamma\)-matrices:

\[
A = A^1 + iA^2, \quad A^* = A^1 - iA^2
\]

\[
\gamma = \gamma^1 + i\gamma^2, \quad \gamma^* = \gamma^1 - i\gamma^2
\]

\[
\gamma_\pm = \gamma_0 \pm \gamma_3
\]

In accordance with ref. [6] we describe the produced particles by the complex scalar fields related with \(A\) by the equation:

\[
A = i\partial^* \phi, A^* = -i\partial \phi^*
\]  

(34)

For the Coulomb fields we use instead of \(A'_\pm\) the old denotions \(A_\pm\):

\[
A'_\pm \to A_\pm
\]  

(35)

Let us introduce the following basis for the spinors:

\[
u_{-} = \frac{1}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \quad u_{-} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad u_{+} = \frac{1}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad u_{++} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}
\]  

(36)

satisfying the relations:

\[
u_{-} = \frac{1}{4} \gamma^*_+ \gamma^- u_{-} \quad \gamma^*_+ u_{-} = 0,
\]

\[
u_{+} = \frac{1}{4} \gamma^- \gamma^+ u_{+} \quad \gamma^- u_{+} = 0,
\]

\[
u_{i+} = \frac{1}{4} \gamma^* \gamma^+ u_{i+} \quad \gamma^* u_{i+} = 0,
\]

\[
u_{i-} = \frac{1}{4} \gamma^* \gamma^- u_{i-} \quad \gamma^* u_{i-} = 0,
\]

\[
u_{+} = -\frac{1}{2} \gamma^- u_{+} - \frac{1}{2} \gamma^* u_{++},
\]

\[
u_{-} = \frac{1}{2} \gamma^+ u_{+-} = -\frac{1}{2} \gamma^* u_{++}
\]

(37)

and the normalization conditions:

\[
1 = \bar{u}_{+} \gamma^+ u_{+} = \bar{u}_{-} \gamma^- u_{-} = 2 \bar{u}_{++} u_{+} = -2 \bar{u}_{-} u_{-} = -2 \bar{u}_{-} u_{++} = 2 \bar{u}_{--} u_{+-}
\]  

(38)
Then the fermions are described by anticommuting fields $\chi_\pm, \chi_\pm^*$ and $a_\pm, a_\pm^*$ related with initial fields $\psi_\pm, \psi_\pm'$ written in the above basis as follows:

\[
\begin{align*}
\psi_\pm &= 2i[(\partial \chi_\pm)u_{\pm+} - (\partial^* \chi_\pm^*)u_{\pm-}] \\
\psi_\pm' &= a_\pm^* u_{\pm+} + a_\pm u_{\pm-}
\end{align*}
\] (39)

We remind, that the fields $a_+, a_+^*$ and $a_-, a_-^*$ are considered as independent ones. For the complex conjugated fields one obtains from eqs. (39):

\[
\begin{align*}
\bar{\psi}_\pm &= -2i[(\partial^* \bar{\chi}_\pm)\bar{u}_{\pm+} - (\partial \bar{\chi}_\pm^*)\bar{u}_{\pm-}] \\
\bar{\psi}_\pm' &= a_{\pm}^* \bar{u}_{\pm+} + a_{\pm} \bar{u}_{\pm-},
\end{align*}
\] (40)

where by the definition the complex conjugation is denoted by bar. Only for the Majorana particles one obtains $\bar{\chi} = \chi^*$, $\bar{a} = a^*$.

The newly introduceD fields $\chi_+, \chi_+^*$ are expressed through the fields $\chi_-, \chi_-^*$ (see (7)).

\[
\begin{align*}
\chi_+ &= -\frac{\partial^*}{\partial_-} \chi_+^*, \\
\chi_+^* &= -\frac{\partial}{\partial_-} \chi_-
\end{align*}
\] (41)

From eqs. (20) and (23) using above definitions we obtain the free Lagrangian for the effective theory in the form

\[
\begin{align*}
\mathcal{L}_{\text{free}} &= -2A^a_+ \partial^a \partial^* A^a_- - i[a^a_+ (\partial a^a_+) + a^a_+ (\partial^* a^a_-) + a^a_- (\partial a^a_+)] + \\
&+ \frac{\partial a^a_+ (\partial^* a^a_-)}{2} - \frac{1}{2} (\partial^* \phi^a) \Box (\partial \phi^a) + \\
&+ i[(\partial^* \chi^a_-) \frac{\partial}{\partial_-} (\partial \chi^a_-) + (\partial \chi^a_+) \frac{\partial}{\partial_-} (\partial^* \chi^a_-)]
\end{align*}
\] (42)

From eq.(24) one can find the following Lagrangian for scattering off right particles:

\[
\begin{align*}
\mathcal{L}_{\text{scat}}^{(R)} &= -\frac{i g}{2} A^a_+ [((\partial_- \partial^* \phi) T^a (\partial \phi) + (\partial_+ \partial^* \phi) T^a (\partial^* \phi)] + \\
&+ 2gA^a_+ [(\partial^* \bar{\chi}_-) t^a_+ (\partial \chi_-) + (\partial \chi_-^*) t^a_+ (\partial^* \chi_-^*)] - \\
&- g[(\partial \phi^a_+) \bar{a}^a_+ t^a_+ (\partial^* \chi_-^*) + (\partial^* \phi^a) \bar{a}^a_+ t^a_+ (\partial \chi_-)] + \\
&+ (\partial \phi^a_+) (\partial^* \bar{\chi}_-) t^a_+ a_- + (\partial^* \phi^a) (\partial \chi_-) t^a_+ a_-^*
\end{align*}
\] (43)

Let us consider now the contribution (28) for the scattering off left particles. With the use of mass-shell condition for the real particles $(\partial_+ \partial_- = \partial \partial^*)$ we obtain (cf. (43))

\[
\begin{align*}
\mathcal{L}_{\text{scat}}^{(L)} &= -\frac{i g}{2} A^a_+ [((\partial_+ \partial^* \phi^a) T^a (\partial \phi) + (\partial_+ \partial \phi^a) T^a (\partial^* \phi))] + \\
&+ 2gA^a_+ [(\partial^* \bar{\chi}_+) t^a_+ (\partial \chi_+) + (\partial \chi_+) t^a_+ (\partial^* \chi_+^*)] - \\
&- g[(\partial \phi^a_+) \bar{a}^a_+ t^a_+ (\partial^* \chi_+) + (\partial^* \phi^a) \bar{a}^a_+ t^a_+ (\partial \chi_+)] + \\
&+ (\partial \phi^a_+) (\partial^* \bar{\chi}_+) t^a_- a_- + (\partial^* \phi^a) (\partial \chi_+) t^a_- a_-^*
\end{align*}
\] (44)
The production terms of the effective Lagrangian are obtained from eq. (26):

\[
\mathcal{L}_{\text{prod}} = g[\phi^a(\partial A_-)T^a(\partial^* A_+) - \phi^a(\partial^* A_-)T^a(\partial A_+)] \\
- \frac{ig}{2}[\phi^a(-\bar{a}_+^* t^a(\partial a_-) + (\partial^* a_+^*) t^a a_-) + \\
+ \phi^a(\bar{a}_+^* t^a(\partial^* a_+) - (\partial a_-^*) t^a a_-) + \\
+ \phi^a((\partial a_-^*) t^a a_+^* - \bar{a}_+^* t^a (\partial a_-))] + \\
+ ig[\bar{\chi}_-^* t^a A_-^+(\partial^* A_+^a) - \bar{\chi}_+^* t^a A_-^a] \\
+ \bar{a}_-^* t^a A_-^+(\partial^* A_+^a) - \bar{a}_+^* t^a A_- + \\
+ \bar{\chi}_+^* t^a A_-^+ - \bar{\chi}_+^* t_a^a(\partial A_+^a)] \tag{45}
\]

At last from eqs. (29),(30) we obtain the following contribution to the interaction of pure Coulomb fields:

\[
\mathcal{L}_{\text{Coul}} = \frac{ig}{2}(\partial^* A_-^a)(\frac{1}{\partial a_-^a})T^a(\partial^* A_+) - \frac{g}{2}(\frac{1}{\partial a_+^a})[(\partial a_-^*) t^a a_+^* + \\
+ (\partial^* a_-^*) t^a a_+ + (\partial a_-^*) t^a A_-^a + (\partial^* a_+^*) t^a a_-] + \\
+ \frac{ig}{2}(\partial^* A_-^a)(\frac{1}{\partial a_-^a})T^a(\partial^* A_+) - \frac{g}{2}(\frac{1}{\partial a_+^a})[(\partial a_-^*) t^a a_+ + \\
+ (\partial^* a_-^*) t^a a_+^* + (\partial a_-^*) t^a A_-^a + (\partial^* a_+^*) t^a a_-] \tag{46}
\]

Note, that the terms corresponding to the gluon transition into the quark-anti-quark pair in the t-channel are absent in eq. (46) because they are small.

As we stressed above the fields \(\chi_+\) and \(\chi_-\) in the above action are related by eq. (41) due to equations of motion. However one can modify the kinetic action \(\mathcal{L}^\chi_{\text{free}}\) for \(\chi\) fields (see eq. (42)) in the way corresponding to the initial free fermion contribution \(i\bar{\psi}\gamma^\mu\partial\mu\psi\) in eq. (2)

\[
\mathcal{L}^\chi_{\text{free}} = 4i[(\partial^* \chi_+)(\partial_- \chi_+) + (\partial \chi_+^*)(\partial_- \chi_-^*) + \\
+ (\partial^* \chi_-)(\partial_+ \chi_-) + (\partial \chi_-^*)(\partial_+ \chi_+^*) + \\
+ (\partial^* \chi_+)(\partial \chi_-^*) + (\partial \chi_+^*)(\partial \chi_-) + \\
+ (\partial^* \chi_-)(\partial \chi_-^*) + (\partial \chi_-^*)(\partial \chi_+^*)] \tag{47}
\]

In this case the constraint (41) will be a consequence of the Dirac equation for fields \(\chi_\pm, \chi_\pm^*\) for the quark on its mass-shell. The representation (47) for \(\mathcal{L}^\chi_{\text{free}}\) is local and has the same symmetry for the transformation + \(\leftrightarrow\ -\) as the interaction terms (43), (44), (45) and (46).
7 Discussion

We have obtained the effective action for high-energy multi-Regge processes directly from the original QCD action by separating the gluon and quark fields into parts with momenta obeying conditions imposed by the multi-Regge kinematics. The heavy modes are eliminated by their equations of motion. This induces new interaction terms. Further it is convenient to distinguish the Coulomb modes with the kinematics typical for exchanged particles and the modes corresponding to particles produced or scattered with small momentum transfer.

We have chosen notations such that the two distinct modes appear as different fields. However, we have to keep in mind the momentum ranges appropriate for them. For example, one should not allow the Coulomb fields $(A_\pm, a_\pm)$ to have momenta with $|k_+ k_-| \geq |k_\perp^2|$, because this would violate the conditions of multi-Regge kinematics. Also, one should not allow the scattering fields $(\phi, \chi_\pm)$ to have momenta with $|k_+ k_-| \ll |k_\perp^2|$, because this would double a contribution already accounted for by the Coulomb fields.

In the present form of the action these restrictions are not included and are to be taken as external conditions. We are looking for a modification of the vertices or propagators, which would suppress the forbidden configurations.

It has been checked that the effective action, obtained in this paper, reproduces multi-Regge scattering amplitudes on the tree level. In particular the vertices of production of gluons and quarks reproduce the known effective vertices [2], [12]. The analysis of tree amplitudes provides a straightforward but fairly heuristic way [9] to obtain the effective action up to the triple interaction of Coulomb fields.

The Coulomb fields turn into reggeons by summing over all loops in LLA. The significance of triple Coulomb interaction depends on the interpretation of the scattering fields. One can consider the elimination of the heavy modes as an operation done on the classical action only (on mass-shell). Doing quantization only after this step would not imply any restriction on the virtualness of the scattering fields (besides of avoiding a mixing with Coulomb fields). Reggeization arises by $s$-channel interaction and there is no triple Coulomb interaction.

A different point of view would be to do the elimination of the heavy modes as an operation done on the functional integral, i.e. integrating them out approximately. After this the scattering fields are not allowed to be strongly virtual and $s$-channel iteration does not give the essential contribution to reggeization. Here the reggeization arises from the triple Coulomb interaction.

A triple interaction of negative signature reggeons seems to contradict the Gribov signature conservation rule. A. White showed [13] that nevertheless such reggeon interaction can be treated consistently and that triple-reggeon vertices are very important for understanding the gluon reggeization.

The derivation outlined here can be made more rigorous by avoiding the approximation up to the first order in $g$ in the equations of motion. It provides a deeper understanding of the effective action. Owing to the approximations and external conditions the resulting action can be represented in several forms, some of which emphasize symmetry properties. Note that one can obtain one-loop corrections to above effective action using the results of Ref. [14].
With the effective action we achieved already essential simplifications compared to the original theory. We consider this as a first step on the way leading finally to a simple effective theory from which the leading contributions to the multi-Reggeon Green functions could be calculated.

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\[
\begin{array}{cccccc}
\cdots \\
\end{array}
\]

Fig. 1

\[
\begin{array}{cccccc}
\cdots \\
\end{array}
\]

Fig. 2