Electromagnetic waves propagating in the string axiverse

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It is widely believed that axions are ubiquitous in string theory and could be the dark matter. The peculiar features of the axion dark matter are coherent oscillations and a coupling to the electromagnetic field through the Chern-Simons term. In this paper, we study consequences of these two features of the axion with the mass in a range from $10^{-13}$ eV to $10^{-20}$ eV. First, we study the parametric resonance of electromagnetic waves induced by the coherent oscillation of the axion. As a result of the resonance, the amplitude of the electromagnetic waves is enhanced and the circularly polarized monochromatic waves will be generated. Second, we study the velocity of light in the background of the axion dark matter. In the presence of the Chern-Simons term, the dispersion relation is modified and the speed of light will oscillate in time. It turns out that the change of speed of light would be difficult to observe. We argue that the future radio wave observations of the resonance can give rise to a stronger constraint on the coupling constant and/or the density of the axion dark matter.

I. INTRODUCTION

According to string theory, axions are ubiquitous in the universe, dubbed the string axiverse [1,2]. Remarkably, the axions could be a dark component of the universe and might be a dominant piece of the dark matter [3,4]. In fact, it is difficult to discriminate between the axion dark matter and the cold dark matter on large scales. Therefore, it is important to find a method for proving the existence of the axions.

The key feature of the axion dark matter is its coherent oscillation. In particular, if the axion has the mass $10^{-23}$ eV, the time scale of the oscillation is a few years and the oscillation produces the oscillation in the gravitational potential. Hence, one can use pulsar timing arrays to observe oscillating gravitational potential [13-19]. There are other methods proposed for detecting the axion dark matter, for example, the super-radiance instability of the axion field in the rotating black holes constraining the mass range $10^{-20} \sim 10^{-10}$ eV [2,17-19], gravitational wave interferometers for probing the axion with mass $10^{-22} \sim 10^{-20}$ eV [20], the dynamical resonance of the binary pulsars probing the mass range $10^{-23} \sim 10^{-21}$ eV [21], and cosmological axion oscillations for exploring a wide mass range [22,23].

Recently, we have studied the gravitational waves in dynamical Chern-Simons gravity in the axion dark matter background [24]. Then, we found that there occurs the parametric resonance of gravitational waves with parity-violation, that is, circularly polarized gravitational waves which allows us to probe the axions with the mass range $10^{-14} \sim 10^{-10}$ eV.

Apparently, we can expect the same phenomena for electromagnetic waves. Since electromagnetic waves are often used to explore the universe, it is worth studying the phenomena in detail. The electrodynamics in the presence of the axion is called the axion electrodynamics [25] which has the Chern-Simons coupling between the axion and the gauge field. We see this interaction induces the parametric resonance of electromagnetic waves and also yields to the oscillation of the speed of light in time. In this paper, we study these two effects to gives rise to a new way to explore the axion dark matter in a mass range $10^{-13} \sim 10^{-1}$ eV corresponding to the observable frequency range of electromagnetic waves $10$ Hz $\sim 10^9$ THz. Note that the axions with the mass above $10^3$ eV are unstable against decaying into photons [3,12].

The organization of the paper is as follows. In Sec. II, we introduce the axion electrodynamics. In Sec. III, we derive wave equations in the oscillating axion background. In Sec. IV, we study the parametric resonance in the axion background. In Sec. V, we investigate the speed of light. The final section is devoted to conclusion.

II. AXION ELECTRODYNAMICS

The action of the axion electrodynamics is given by

$$S = S_{EM} + S_{\Phi} + S_{int},$$

where each part of this action reads

$$S_{EM} = \int dx^4 \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right),$$

$$S_{\Phi} = \int dx^4 \sqrt{-g} \left( -\frac{1}{2} (\nabla_{\mu} \Phi) (\nabla^\mu \Phi) - U(\Phi) \right),$$

$$S_{int} = \int dx^4 \sqrt{-g} \left( -\frac{\lambda}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \right).$$

Here $\lambda$ is a coupling constant, $U(\Phi)$ is a potential function for an axion field $\Phi$, and $A^\mu = (A^0, A)$ is a gauge field with the field strength $F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu$. The dual of the field strength $\tilde{F}^{\mu\nu}$ is defined by

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma},$$
where the anti-symmetrical epsilon tensor $\epsilon^{\mu\nu\rho\sigma}$ is given by
\[
\epsilon^{\mu\nu\rho\sigma} \equiv \frac{1}{\sqrt{-g}} \tilde{\epsilon}^{\mu\nu\rho\sigma} \quad \text{and} \quad \epsilon^{0123} = +1.
\] (4)

Here, $\tilde{\epsilon}^{\mu\nu\rho\sigma}$ is the Levi-Civita symbol.

From the above action, we get the equations of motion for the electromagnetic waves
\[
\nabla_\mu F^{\alpha\mu} + \frac{1}{2} \epsilon^{\alpha\mu\nu\lambda} (\nabla_\nu \Phi) F_{\mu\lambda} = 0
\] (5)
and the equation for the axion field
\[
\nabla_\mu \nabla^\mu \Phi - \frac{d}{d\Phi} U(\Phi) = \frac{\lambda}{4} F_{\mu\nu} F^{\mu\nu}.
\] (6)

Now, we can study electromagnetic wave propagation in the axion background.

### III. WAVE EQUATIONS IN THE AXIVERSE

We assume the background spacetime is the Minkowski spacetime, because the dynamics of the cosmic expansion can be neglected on inter-galactic scales \([26]\). Then, the metric reads
\[
ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2.
\] (7)

Now, the covariant derivative is simply reduced to a partial derivative $\partial_\mu$. We are interested in the time-evolution of the gauge field in the axion background. The gauge field is considered as the perturbed field $A_\mu = \delta A_\mu$. Next, we consider a homogeneous axion background
\[
\Phi(t,x) = \Phi(t).
\] (8)

Then, the equation of motion of axion is given by
\[
(\partial_t^2 + m^2) \Phi(t) \simeq 0.
\] (9)

Here, we assumed the potential of the axion as
\[
U(\Phi) = \frac{1}{2} m^2 \Phi^2.
\] (10)

It is easy to obtain the solution
\[
\Phi(t) = \Phi_0 \cos(m t),
\] (11)

where $\Phi_0$ is determined by the density of the dark matter $\rho$ and the mass of the axion $m$ as
\[
\Phi_0 = \frac{\sqrt{2\rho}}{m}.
\] (12)

The equations of motion of the axion electrodynamics can be deduced as
\[
\partial_\mu \delta F^{\mu\nu} = 0,
\]
\[
\partial_\mu \delta F^{\mu\nu} - \lambda \epsilon^{ijk} (\delta_0 \Phi) \partial_j \delta A_k = 0.
\] (13)

Here, the epsilon tensor in this coordinate system is defined as
\[
\epsilon^{ijk} \equiv \epsilon^{ijk}.
\] (14)

The time-component of the modified Maxwell equation is the same as the conventional Maxwell equation.

This modified Maxwell theory is invariant under the gauge transformation,
\[
A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \Lambda.
\] (15)

So, we can adopt the radiation gauge for the electromagnetic field,
\[
\delta A^0 = 0, \quad \nabla \cdot \delta A = 0,
\] (16)

and we get the wave equations of the axion electrodynamics,
\[
\square \delta A + \lambda \left(\partial_0 \Phi\right) \left(\nabla \times \delta A\right) = 0,
\] (17)

where we defined the derivative operators $\square \equiv \nabla \mu \nabla^\mu$ and $\nabla \equiv \left(\partial_x, \partial_y, \partial_z\right)$.

We can diagonalize the wave equations with the circular polarization basis. In Fourier space, the vector field $\delta A$ is expressed by
\[
\delta A \equiv \int a(t) e^{ik \cdot x} dk,
\] (18)

where $k$ is the wave number vector. The transverse gauge condition can be written as
\[
k \cdot a(t) = 0.
\] (19)

We can take polarization basis vectors, $e_{(1)}$, $e_{(2)}$, satisfying the following conditions
\[
e_{(I)} \cdot k = 0,
\]
\[
e_{(I)} \cdot e_{(J)} = \delta_{IJ}, \quad \text{for } I, J = (1,2)
\]
\[
e_{(1)} \times e_{(2)} = \frac{k}{k}.
\] (20)

Here, we defined $k = |k|$. Thus, the Fourier coefficient $a(t)$ is expanded as
\[
a(t) = \sum_{I=1,2} a_I(t) e_{(I)}.
\] (23)

Alternatively, we can use the circular polarization basis
\[
e_R = \frac{e_{(1)} + ie_{(2)}}{\sqrt{2}} \quad \text{and} \quad e_L = \frac{e_{(1)} - ie_{(2)}}{\sqrt{2}}.
\] (24)

Now, the Fourier coefficient $a(t)$ is expanded as
\[
a(t) = \sum_{B=L,R} a_B(t) e_B.
\] (25)

Note that the components are related as
\[
a_R = a_{(1)} - i a_{(2)}, \quad a_L = a_{(1)} + i a_{(2)}.
\] (26)
This basis is useful for studying the parity violation. Using the relation

\[ e^{ijk} k^j \frac{k_i}{k} e^k_{R/L} = \mp i e^i_{R/L} \]  

(27)

we can diagonalize the wave equations as

\[ \dot{a}_B + k^2 \left( 1 + \epsilon_B \frac{m}{\lambda} \Phi_0 \sin(mt) \right) a_B = 0 \]  

(28)

where

\[ \epsilon_B = \begin{cases} 1 & : B = R, \\ -1 & : B = L. \end{cases} \]  

(29)

This equation is nothing but the Mathieu equation describing the parametric resonance. Therefore, the growth rate is given by

\[ \Gamma = \frac{1}{4} \lambda m \Phi_0. \]  

(30)

Since the axion has a non-trivial profile, the parity symmetry is violated in the equation of motion. Thus, the circular polarization should be generated. To be more precise, it is useful to define the polarization-rate of the electromagnetic field

\[ \text{parity}(t) = \frac{|\dot{a}_R|^2 - |\dot{a}_L|^2}{|\dot{a}_R|^2 + |\dot{a}_L|^2}. \]  

(31)

Due to the parametric amplification, the growth of one of the modes is larger than the other mode. In that case, we should have parity(t) ≃ ±1. Moreover, since the dispersion relation is modified by the axion, the speed of light is oscillating. We study the effects of these phenomena on electromagnetic waves in the following.

### IV. PARAMETRIC RESONANCE

We assume that a lot of clumps whose sizes are about the Jeans length \( L_J \) exist in the core of Galaxy and the axion is coherently oscillating there. These fuzzy objects have the interaction with the electromagnetic fields through the Chern-Simons coupling. Thus, the coherent oscillations of the axion induce the parametric resonance of electromagnetic waves.

From the general theory of the parametric resonance, the resonance wave number \( k_r \) is given by

\[ k_r = \frac{m}{2}. \]  

(32)

It is convenient to convert \( k_r \) into the resonance frequency \( f_r \) of the waves as

\[ f_r = 1.2 \times 10^4 \text{ Hz} \times \left( \frac{m}{10^{-10} \text{ eV}} \right). \]  

(33)

This frequency corresponds to VLF (very low frequency) band, 3 ∼ 30 kHz. The existing FAST (Five-hundred-meter Aperture Spherical radio Telescope) has the frequency band from 70 MHz to 3 GHz in [27]. Hence, this detector can survey the mass range from \( 10^{-7} \text{ eV} \) to \( 10^{-5} \text{ eV} \). The SKA (Square Kilometre Array) has the frequency from 50 MHz to 350 MHz (SKA-low) and from 350 MHz to 14 GHz [28]. Now, this detector will survey the mass range from \( 10^{-7} \text{ eV} \) to \( 10^{-4} \text{ eV} \). If we consider the heavier axion with mass \( m \sim 1 \text{ eV} \), the resonance frequency is that of the visible light around 10² THz.

On halo scales of the Galaxy, the energy density of the axion dark matter is about 0.3 GeV/cm³. Hence, the growth rate can be estimated as

\[ \Gamma_{\text{max}} = 5.4 \times 10^{-29} \text{ eV} \times \left( \frac{\lambda}{10^{16} \text{ GeV}} \right) \sqrt{\frac{\rho}{0.3 \text{ GeV/cm³}}}. \]  

(34)

Notice that this quantity is independent of the mass of the axion. In fact, the growth rate is determined by the coupling constant and the energy density of the axion dark matter. From this growth rate, we can estimate the time scale, \( t_{\times 10} \), for the amplitude to become ten times, as

\[ t_{\times 10} = 4.3 \times 10^{28} \text{ eV}^{-1} \times \left( \frac{10^{16} \text{ GeV}}{\lambda} \right) \sqrt{\frac{0.3 \text{ GeV/cm³}}{\rho}}. \]  

(35)

Note that the time corresponding to 1pc is given by \( t_{1\text{pc}} \approx 1.6 \times 10^{23} \text{ eV}^{-1} \). Thus, after the 10 Mpc propagation, the amplitude will be enhanced by \( 10^{102} \) times. Therefore, we can obtain a stringent constrain on the coupling constant and/or the fraction of the axion dark matter in the universe.

The parametric resonance occurs in the frequency band

\[ f_r - \frac{\Delta f}{2} \lesssim f_r \lesssim f_r + \frac{\Delta f}{2}, \]  

(36)

where \( \Delta f \) is given by

\[ \Delta f = 2.6 \times 10^{-14} \text{ Hz} \times \left( \frac{\lambda}{10^{16} \text{ GeV}} \right) \sqrt{\frac{\rho}{0.3 \text{ GeV/cm³}}}. \]  

(37)

Since the band is very narrow, the circularly polarized monochromatic wave grows sharply at the resonance frequency.

If the electromagnetic waves go through near the core of the Galaxy, the energy density of dark matter gets enhanced

\[ \rho \lesssim 0.3 \times 10^6 \text{ GeV/cm³}. \]  

(38)

In this situation, \( t_{\times 10} \) becomes

\[ t_{\times 10} = 4.3 \times 10^{25} \text{ eV}^{-1} \times \left( \frac{10^{16} \text{ GeV}}{\lambda} \right) \sqrt{\frac{0.3 \times 10^6 \text{ GeV/cm³}}{\rho}}. \]  

(39)

From this estimation, the amplitude of waves going through the Galaxy core is further amplified by about
10^2 times. At the resonance frequency, when the amplitudes of waves are highly amplified, the electromagnetic wave should be fully polarized, namely, parity(t) ≈ ±1.

If we detected the resonance signal, we would argue that the axion dark matter exist. If we did not detect the resonance signal, we would be able to give the constraint on the energy density or the coupling constant. Therefore, we can say that the future very long wavelength radio wave observations of this effect can give rise to stronger constraints on the coupling constant and/or the density of the axion dark matter.

V. THE SPEED OF LIGHT

In axion electrodynamics, the dispersion relation in the axion background reads

$$\omega^2 = k^2 \left( 1 + \epsilon_A \lambda \frac{m}{k} \Phi_0 \sin(mt) \right). \quad (40)$$

The phase velocity \( v_p \) is given by

$$v_p \equiv \frac{\omega}{k} = \sqrt{1 + \epsilon_A \lambda \frac{m}{k} \Phi_0 \sin(mt)}.$$

Then, the deviation from the speed of light \( \delta c_p \) is given by

$$\delta c_p \equiv |v_p - 1| \leq \sqrt{1 + \epsilon_A \lambda \frac{m}{k} \Phi_0 - 1} \approx \lambda \sqrt{\rho} \frac{\lambda}{\sqrt{2k}}. \quad (41)$$

For example, if we observe the visible light which is in the wavelength range 380 ~ 750 nm, we find the relative deviation of the speed of light:

$$\delta c_p \approx 4.3 \times 10^{-29} \times \left( \frac{\lambda}{(10^{16} \text{GeV/cm}^3)^{1/4}} \right) \left( \frac{l_{em}}{500 \text{nm}} \right) \sqrt{\frac{\rho}{0.3 \text{GeV/cm}^3}}. \quad (42)$$

Here, \( l_{em} \) is the wavelength of the visible light.

In fact, the group velocity is more relevant to observations. The group velocity \( v_g \) is given by

$$v_g \equiv \frac{\partial \omega}{\partial k} = \frac{1}{2\omega} (2k + \epsilon_A \lambda m \Phi_0 \sin(mt)). \quad (43)$$

The deviation from the speed of light \( \delta c_g \) is given by

$$\delta c_g \equiv |v_g - 1| \approx \left| 1 - \frac{1}{4} \lambda^2 \frac{m^2}{k^2} \Phi_0^2 \sin^2(mt) - 1 \right| \lesssim \frac{\lambda^2 \rho}{2k^2}. \quad (44)$$

Notice that the linear term is canceled out in the above formula \[29\] and the deviation of the group velocity is given by the square of that of the phase velocity

$$\delta c_g = (\delta c_p)^2.$$
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