Wiretap Channels with Causal and Non-Causal State Information: Revisited

Te Sun Han, Life Fellow, IEEE, Masahide Sasaki

Abstract—The coding problem for wiretap channels (WTCs) with causal and/or non-causal channel state information (CSI) available at the encoder (Alice) and/or the decoder (Bob) is studied, particularly focusing on achievable secret-message secret-key (SM-SK) rate pairs under the semantic security criterion. One of our main results is summarized as Theorem 3 on causal inner bounds for SM-SK rate pairs, which follows immediately by leveraging the unified seminal theorem for WTCs with non-causal CSI at Alice that has been recently established by Bunin et al. [25]. The only thing to do here is just to re-interpret the latter non-causal scheme in a causal manner by restricting the range of auxiliary random variables appearing in non-causal encoding to a subclass of auxiliary random variables for the causal encoder. This technique is referred to as “plugging.” Then, we are able to dispense with the block-Markov encoding scheme used in the previous works by Chia and El Gamal [12], Fujita [13], and Han and Sasaki [14] and then extend all the known results on achievable rates. The other main results include the exact SM-SK capacity region for WTCs with non-causal CSI at “both” Alice and Bob (Theorem 2), a “tighter” causal SM-SK outer bound for state-reproducing coding schemes with CSI at Alice (Proposition 4), and the exact SM-SK capacity region for degraded WTCs with causal/non-causal CSI at both Alice and Bob (Theorem 4).

I. INTRODUCTION

In this paper we address the coding problem for a wiretap channel (WTC) with causal/non-causal channel state information (CSI) available at the encoder (Alice) and/or the decoder (Bob). The intriguing concept of WTC and secret message (SM) transmission through the WTC originates in Wyner [11] (without CSI) under the weak secrecy criterion. This was then extended to a wider class of WTCs by Csiszár and Körner [2] to provide the more tractable framework. Indeed, these landmark papers have offered the fundamental basis for a diversity of subsequent extensive researches.

Early works include Mitrpanit, Vinck and Luo [9], Chen and Vinck [6], and Liu and Chen [7] that have studied the capacity-equivalence tradeoff for degraded WTCs with non-causal CSI to establish inner and/or outer bounds on the achievable region. Subsequent developments in this direction with non-causal CSI can be found also in Boche and Schaefer [9], Dai and Luo [18], etc., which are mainly concerned with the problem of SM transmission over the WTC.

On the other hand, Khisti, Diggavi and Wornell [11] and Zibaeenjad [29] addressed the problem of secret key (SK) agreement over the WTC with non-causal CSI at Alice (and Bob), and tried to give the exact key-capacity formula.

Prabhakaran et al. [17] studied an achievable tradeoff between SM and SK rates over the WTC with non-causal CSI, deriving a benchmark inner bound on the SM-SK capacity region under the weak secrecy criterion. Subsequently, heavily based on the work of Goldfeld et al. [23], Bunin et al. [24], [25] have improved [17] by explicitly leveraging the superposition coding to obtain a unifying formula (cf. Theorem 1) for inner bounds on the SM-SK capacity region under the semantic secrecy (SS) criterion for WTCs with non-causal CSI at Alice, from which “all” the typical previous results can be derived. Thus, [24], [25] are regarded currently as establishing the best known achievable rate pairs with non-causal CSI at Alice.

The key idea in [24], [25] (which are substantially due to [23]) is to invoke the likelihood encoder (cf. Song et al. [20]) together with the soft-covering lemma (cf. Cuff [22]) on the basis of two layered superposition coding scheme (cf. [17], [25]), which makes it possible to guarantee the semantically secure (SS) information transmission. This is one of the strongest ones among various security criteria.

In contrast to extensive studies on WTCs with “non-causal” CSI mentioned above, there have been less number of literatures on WTCs with “causal” CSI. To our best knowledge, we can list typically a few causal papers including Chia and El Gamal [12], Fujita [13], and Han and Sasaki [14]. They are concerned only with SM rates but not with SK rates.

A prominent feature common in these papers is to leverage the block-Markov encoding to invoke the Shannon cipher [3] (Vernam’s one-time pad cipher). Although there still remain many open problems, possible extensions/generalizations in this direction do not seem to be very fruitful or may be even formidable.

Fortunately, however, to solve these problems we can fully exploit, as they are, all the non-causal techniques/concepts as developed in Bunin et al. [25] to derive the causal version of it. The only thing to do here is simply to restrict the range of auxiliary random variables \((U, \tilde{V})\)’s intervening in [25, Theorem 1] (said to be non-causally achievable) to a subclass of auxiliary random variables \((U, V)\)’s (said to be

*This is the notion to denote the achievability part of resolvability [23].
causally achievable). Then, it suffices to notice only that the encoding scheme given in [25] can be carried out, as it is, in a causal way. This process may be termed “plugging” of causal WTCs into non-causal WTCs.

Thus, it is not necessary to give a separate proof to establish the causal version (Theorem 3) in this paper. The merits of this approach for proof are to inherit all the advantages in [25] to our causal version. For example, the first one is to inherit the SS property as established in [25]; the second one is to enable us, without any extra arguments, to interpret regions of SM-SK achievable rate pairs in [25] as those valid also in Theorem 4 (for degraded WTCs), which is the first solid result causal/non-causal with Proposition 2 (inner bound).

Alice) to derive an SM-SK outer bound, which is paired with Theorem 3. Furthermore, in this section we give Proposition 2 for state-reproducing coding schemes (with causal CSI at Alice) to demonstrate the general formula for the exact “non-causal” SM-SK capacity region for WTCs with non-causal CSI available at both Alice and Bob (Theorem 2).

Furthermore, the arguments that have been used to derive Theorems 2 and 3 can be further exploited to solve harder problems such as deriving a “tighter” causal outer bound (Theorem 3) and finding the causal/non-causal SM-SK capacity region for degraded WTCs (Theorem 4).

The present paper is organized as follows.

In Section II we give the problem statement as well as the necessary notions and notation, all of which are borrowed from [25] along with Theorem II with non-causal CSI at Alice. They are used in the next sections.

In particular, in Section III we give Theorem 2 to demonstrate the general formula for the exact “non-causal” SM-SK capacity region when the state information is available at both Alice and Bob.

In Section V we give the proof of Theorem 3 for WTCs with causal CSI at Alice by using the argument of “plugging,” which is to put the causal scenario into the non-causal scenario, thereby enabling us to produce a diversity of causal inner bounds in Section VII.

In Section V we develop Theorem 3 (for each of Case 1) ~ Case 4) to obtain a new class of inner bounds of SM-SK achievable rate pairs for WTCs with causal CSI at Alice. Here, it is also shown that all the results as established in [12], [13], and [14] can be derived as special cases of Theorem 3. Furthermore, in this section we give Proposition 3 for state-reproducing coding schemes (with causal CSI at Alice) to derive an SM-SK outer bound, which is paired with Proposition 2 (inner bound).

In Section VI we establish the exact SM-SK capacity region with causal/non-causal CSI available at both Alice and Bob (Theorem 4 for degraded WTCs), which is the first solid result from the viewpoint of “causal” SM-SK capacity regions.

In Section VII we conclude the paper with several remarks.

Finally, in Appendix A we give an elementary proof of the soft-covering lemma that plays the key role in [23, 25]. Also, the proof of Remark 8 on typical causal inner bounds is given in Appendix B.

II. WIRETAP CHANNEL WITH NON-CAUSAL CSI

In this section, we recapitulate the seminal work for wiretap channels with “non-causal” channel state information (CSI) available at the encoder (Alice) as in Fig. 1 which was recently established by the group of Bunin, Goldfeld, Permuter, Shamai, Cuff and Piantanida [25]. For the reader’s convenience, we repeat here their notions and key result as they are. Leveraging them, we derive the “causal” counterparts in Section IV.

In Section II, we establish the exact SM-SK capacity region for degraded WTCs (Theorem 4) and finding the causal/non-causal SM-SK capacity

- Proposition 2 (inner bound).

- Proposition 3 (outer bound).

- Proposition 4 (state-reproducing coding scheme).

Let $S, X, Y, Z$ be finite sets and $S^n, X^n, Y^n, Z^n$ be the $n$ times product sets. We let $(S, X, Y, Z, W_S, W_{YZ|SX})$ denote a discrete stationary and memoryless WTC with “non-causal” stationary memoryless CSI $S$ available at the encoder, where $W_{YZ|S} : S \times X \rightarrow P(Y \times Z)$ is the transmission probability distribution (under state $S$) with input $X$ at Alice, and outputs $Y$ at Bob and $Z$ at Eve, while $W_S$ is the probability distribution of state variable $S$. A state sequence $s \in S^n$ is sampled in an i.i.d. manner according to $W_S$ and revealed in a non-causal fashion to Alice. Independently of the observation of $s$, Alice chooses a message $m$ from the set $[1 : 2^{nR_M}]$ ($R_M \geq 0$) and maps the pair $(s,m)$ into a channel input sequence $x \in X^n$ and a key index $k \in [1 : 2^{nR_K}]$ ($R_K \geq 0$; the mapping may be stochastic). The sequence $x$ is transmitted over the WTC under state $s$. The output sequences $y \in Y^n$ and $z \in Z^n$ are observed by the legitimate receiver (Bob) and the eavesdropper (Eve), respectively. Based on $y$, Bob produces the pair $(k, \hat{m})$ as an estimate of $(k, m)$. Eve maliciously attempts to decipher the SM-SK rate pair from $z$ as much as possible. The random variables corresponding to $s, x, y, z, m, k$ may be denoted by $S^n, X^n, Y^n, Z^n$ (or also $S, X, Y, Z$), $M, K$; respectively.

The following Definitions I ~ 6 are borrowed from [25].

Definition 1 (Non-causal code): An $(n, R_M, R_K)$-code $c_n$ for the WTC with “non-causal” CSI at Alice and message set $M_n \triangleq [1 : 2^{nR_M}]$ and key set $K_n \triangleq [1 : 2^{nR_K}]$ is a pair of functions $(f_n, \phi_n)$ such that

1. $f_n : M_n \times S^n \rightarrow P(Y^n \times K_n)$,
2. $\phi_n : Y^n \rightarrow M_n \times K_n$,

where $f_n$ is a stochastic function.
The performance of the code \( c_n \) is evaluated in terms of its rate pair \((R_M, R_K)\), the maximum decoding error probability, the key uniformity and independence metric, and SS metric as follows:

**Definition 2 (Error Probability):** The error probability of an \((n, R_M, R_K)\)-code \( c_n \) is

\[
e(c_n) \tri Equivalent \max_{m \in \mathcal{M}_n} e_m(c_n),
\]

where, for every \( m \in \mathcal{M}_n \),

\[
e_m(c_n) \tri Equivalent \Pr\{ \hat{M}, \hat{K} \neq (m, K) | M = m \}
\]

with the decoder output \((\hat{M}, \hat{K}) \tri Equivalent \phi_n(Y^n)\).

**Definition 3 (Key Uniformity and Independence Metric):** The key uniformity and independence (from the message) metric under \((n, R_M, R_K)\)-code \( c_n \) is

\[
\delta(c_n) \tri Equivalent \max_{m \in \mathcal{M}_n} \delta_m(c_n),
\]

where, for every \( m \in \mathcal{M}_n \),

\[
\delta_m(c_n) \tri Equivalent \| p_{K|M=m}^{(U)} - p_{K}^{(U)} \|_{TV},
\]

and \( p_{K}^{(U)} \) denotes the joint probability distribution over the WTC induced by the code \( c_n \); \( p_{K|M=m}^{(U)} \) is the uniform distribution over \( K \), and \( \| \cdot \|_{TV} \) denotes the total variation.

**Definition 4 (Information Leakage and SS-Metric):** The information leakage to Eve under \((n, R_M, R_K)\)-code \( c_n \) and message distribution \( p_M \in \mathcal{P}(\mathcal{M}_n) \) is

\[
\ell(p_M, c_n) \tri Equivalent \ell_p^{(U)}(M, K; Z),
\]

where \( \ell_p^{(U)}(M, K; Z) \) denotes the mutual information with respect to the joint probability \( p^{(U)}(c_n) \). The SS-metric with respect to \( c_n \) is

\[
\ell_{Sem}(c_n) \tri Equivalent \max_{p_M \in \mathcal{P}(\mathcal{M}_n)} \ell(p_M, c_n).
\]

**Definition 5 (Achievability):** A pair \((R_M, R_K)\) is called an SM-SK achievable rate pair for the WTC with non-causal CSI at Alice, if for every \( \epsilon > 0 \) and sufficiently large \( n \) there exists an \((n, R_M, R_K)\)-code \( c_n \) with

\[
\max[e(c_n), \delta(c_n), \ell_{Sem}(c_n)] \leq \epsilon.
\]

**Definition 6 (Non-causal SM-SK capacity region):** Throughout this paper we use the following notation. The SM-SK capacity region of the WTC with non-causal CSI at Alice, denoted by \( C_{NCSI-E} \) is the set of all SM-SK achievable rate pairs. Furthermore, the supremum of the projection of \( C_{NCSI-E} \) on the \( R_M \)-axis, denoted by \( C_{NCSI-E}^{M} \), is called the SM capacity, whereas the supremum of the projection of \( C_{NCSI-E} \) on the \( R_K \)-axis is called the SK capacity, denoted by \( C_{NCSI-E}^{K} \).

**II.B: Wiretap Channel with Non-causal CSI at Alice**

We can now describe the unifying key theorem of [25]. Let \( U, V \) be finite sets and let \( U, V \) be random variables taking values in \( U, V \), respectively, where \( U, V, S, X \) may be correlated. Define joint probability distributions \( p_{YZXSUV} \) on

\[
Y \times Z \times X \times S \times U \times V
\]

(said to be non-causally achievable) so that \( UV \rightarrow SX \rightarrow YZ \) forms a Markov chain and

\[
p_S = W_S, \quad p_{YZ|SX} = W_{YZ|SX}.
\]

Notice here that, in view of [23], such a distribution \( p_{YZXSUV} \) is specified by giving the marginal \( p_{SU} \) (input), so we may use \( p_{SU} \) in short instead of \( p_{YZXSUV} \). Define \( R_{in}(p_{SU}) \) to be the set of all nonnegative rate pairs \((R_M, R_K)\) satisfying the rate constraints:

\[
R_M \leq I(UV; Y) - I(UV; S), \quad R_M + R_K \leq I(V; Y|U) - I(V; Z|U) - |[I(U; S) - I(U; Y)]| + ,
\]

where \([x]^+ = \max(x, 0)\) and \( I(\cdot; \cdot), I(\cdot|\cdot)\) denotes the (conditional) mutual information.

With these definitions, Bunin et al. [25] have established the following non-causal inner bound:

**Theorem 1 (Non-causal SM-SK inner bound):**

\[
\mathcal{C}_{NCSI-E} \supset R_{in}^N \tri Equivalent \bigcup_{\mathcal{N}_{PSUV}} R_{in}(p_{SU}),
\]

where the union is taken over all “non-causally” achievable probability distributions \( p_{SU} \)'s. Here, the cardinalities of \( U, V \) may be restricted to \(|U| \leq (|X| - 1)|S| + 3\) and \(|V| \leq (|X| - 1)^2|S| + 2\).

**Remark 1:** In particular, in Section III the inner bound given by Theorem 1 is shown to be optimal when the state information is available at both Alice and Bob.

**Remark 2:** It should be emphasized also that the technical crux of the paper [25] (due to [23]) is based on the soft covering lemma, which is summarized as

**Lemma 1 ([23, Lemma 4]):** Let \( W : U \times V \rightarrow S \) be the memoryless channel induced by joint probability distribution \( p_{SU} \), and set, with \( L_n = 2^{nR_1} \) and \( N_n = 2^{nR_2} \),

\[
q_n^U(s) = \frac{1}{L_nN_n} \sum_{i=1}^{L_n} \sum_{j=1}^{N_n} W(s|u_i, v_{ij}).
\]

Then, for any small \( \varepsilon > 0 \) and for all sufficiently large \( n \), it holds that

\[
E(D(q_n^U||p^U_n)) \leq \varepsilon,
\]

provided that rate constraints \( R_1 > I(U; S), R_1 + R_2 > I(UV; S) \) are satisfied, where \( D(Q||P) \) denotes the Kullback-Leibler divergence between \( Q \) and \( P \), and \( p_n^U(s) \) indicates the probability of i.i.d. \( s = (s_1, s_2, \ldots, s_n) \) and \( E \) denotes the expectation over all random codewords \( u_i, v_{ij} \) of Codebook \( B_n \) as given later in Section IV.

Although in this paper we do not use explicitly this lemma, in view of its importance, it would be worthy of giving a separate elementary proof, which is stated in Appendix A.

\[3\] We may use \( UV, SX, UV \) instead of \( (U, V), (S, X), (U, V) \), and so on, for notational simplicity.

**Appendix A:** A “stronger” version of the soft covering lemma is given in [23], although it is actually not necessary to prove Theorem 1.
III. CAPACITY REGION WITH NON-CAUSAL CSI AT ALICE AND BOB

In this section, we address the problem of converse part (outer bound) for Theorem 1 (inner bound). Specifically, we establish the exact SM-SK capacity region for WTCs with non-causal CSI available at “both” Alice and Bob as in Fig. 2. To do so, let the corresponding non-causal SM-SK capacity region be denoted by $C_{\text{CSI-ED}}^{\text{NC}}$. Moreover, let $\mathcal{X}_{\text{in}}(\mathcal{P}_{\text{SU}})$ denote the set of all nonnegative rate pairs $(R_M, R_K)$ satisfying the rate constraints:

$$R_M \leq I(UV; Y|S),$$
$$R_M + R_K \leq I(V; Y|SU) - I(V; Z|SU) + H(S|ZU),$$

where $UV$ may be dependent on $S$, and $H(\cdot), H(\cdot|\cdot)$ denote the (conditional) entropy. Then, we have

Theorem 2 (Non-causal SM-SK capacity region):

$$C_{\text{CSI-ED}}^{\text{NC}} = \mathcal{X}_{\text{in}} \Delta \mathcal{X}_{\text{in}}(\mathcal{P}_{\text{SU}}),$$

where the union is taken over all “non-causally” achievable probability distributions. Here, the cardinalities of $U, V$ may be restricted to $|U| \leq (|X| - 1)|S| + 2$ and $|V| \leq (|X| - 1)^2|S|^2 + 2(|X| - 1)|S| + 2$, so that the right-hand side of (15) is a compact set.

Remark 3: Theorem 2, in particular, means the “optimality” of the non-causal inner bound (Theorem 1) given by Bunin et al. [25] when the CSI $S$ is available at both Alice and Bob. $\blacksquare$

![Fig. 2. WTC with the same CSI available at Alice and Bob ($t = 1, 2, \ldots, n$).](image)

Proof of achievability for Theorem 2:

The achievability immediately follows from Theorem 1 with $SV, SY$ instead of $V, Y$ in (3) and (9), that is,

$$R_M \leq I(UV; SY) - I(UV; S),$$
$$R_M + R_K \leq I(SV; SY|U) - I(SV; Z|U) + H(S|ZU),$$

where we have noticed that $I(U; SY) \geq I(U; S)$ and hence $[I(U; S) - I(U; SY)]^+ = 0$, and also that $I(U SV; S) = H(S)$.

Proof of converse for Theorem 2

Suppose that $(R_M, R_K)$ is achievable, and set $Y^n = S^n Y^n$. It suffices here to assume that $M$ is uniformly distributed on $\mathcal{M}_n$.

1) We first show (13). Observe that $H(M|Y^n) \leq n \varepsilon_n$, holds by Fano inequality, where $\varepsilon_n \to 0$ as $n$ tends to $\infty$. Then, noting that $S^n$ and $M$ are independent, we have

$$nR_M = H(M) \leq H(M) - H(M|Y^n) + n \varepsilon_n \leq I(M; Y^n) + n \varepsilon_n \leq I(MS^n; Y^n) - I(S^n; Y^n|M) + n \varepsilon_n$$
$$\leq I(MS^n; Y^n) - H(S^n|M) + 2n \varepsilon_n \leq I(MS^n; Y^n) - H(S^n) + 2n \varepsilon_n$$

$$\leq \sum_{t=1}^{n} I(MS^n; Y^n_l Y^{n-1}_l) - \sum_{t=1}^{n} H(S_l) + n H(S^n) + 2n \varepsilon_n$$
$$\leq \sum_{t=1}^{n} I(MS^n Y^{-1}_t; Y^{n-1}_t) - \sum_{t=1}^{n} H(S_l) + 2n \varepsilon_n$$

$\leq \sum_{t=1}^{n} I(MS^n Y^{-1}_t Z_{t+1}^n; Y^{n-1}_t) - \sum_{t=1}^{n} H(S_l) + 2n \varepsilon_n$

$$= \sum_{t=1}^{n} I(U_t S_t V_t; Y^{n-1}_t) - \sum_{t=1}^{n} H(S_l) + 2n \varepsilon_n$$

$$= \sum_{t=1}^{n} I(U_t S_t V_t; Y^{n-1}_t) - \sum_{t=1}^{n} H(S_l) + 2n \varepsilon_n$$

where we have set

$$U_t = Y^{t-1} Z_{t+1}^n, \quad V_t = M K S^{t-1} S_{t+1}^n.$$

Let us now consider the random variable $J$ such that $\Pr\{J = t\} = 1/n$ ($t = 1, 2, \ldots, n$). Then, (19) is written as

$$R_M \leq I(UJ S_j V_j; S_j Y_j J) - H(S_j J) + 2\varepsilon_n$$
$$\leq I(U J S_j V_j; S_j Y_j J) - H(S_j J) + 2\varepsilon_n$$
$$= I(U J S_j V_j; S_j Y_j J) - H(S_j J) + 2\varepsilon_n$$
$$= I(U V; S Y) - H(S) + 2\varepsilon_n$$

$$= I(U V; Y S) + 2\varepsilon_n,$$

where, noting that $S^n$ is stationary and memoryless and hence $H(S_j J) = H(S_j) = H(S)$, we have set $U = U J J, \quad V = V J, \quad S = S J, \quad Y = Y J, \quad Z = Z J$. (22)

Thus, by letting $n \to \infty$ in (21), we obtain (13). It is obvious here that $UV \to XS \to YZ$ forms a Markov chain, where we have similarly set $X = X J$.

2) Next, we show (14). First observe that, in view of Definitions 3 ~ 5 in Section II as well as the uniform

**ED denotes Encoder=E and Decoder=D.**
Since continuity of entropy (cf. [26, Lemma 2.7]), we have
\[ H(K|M = m) - H(U_K) \leq n\varepsilon_n \]
for all \( m \in M_n \), where \( U_K \) denotes the random variable uniformly distributed on \( K_n \). In addition, recall that \( M \) is uniformly distributed on \( M_n \), and therefore
\[ nR_M = H(M), \quad nR_K = H(U_K) \leq H(K|M = m) + n\varepsilon_n \]
for all \( m \in M_n \), which yields
\[ nR_M = H(M), \quad nR_K \leq H(K|M) + n\varepsilon_n. \]
Since \( I(M; Z^n) \leq n\varepsilon_n \) by assumption and \( H(M|Y^n) \leq n\varepsilon_n \) by Fano inequality, we obtain
\[ n(R_M + R_K) \leq H(M) + H(K|M) + n\varepsilon_n \]
\[ = H(M) + n\varepsilon_n \]
\[ \leq H(MK) - H(M|Y^n) + 2n\varepsilon_n \]
\[ = I(M; Y^n) + 2n\varepsilon_n \]
\[ \leq I(M; Y^n) - I(M; Z^n) + 3n\varepsilon_n. \]

On the other hand,
\[ I(MK; Y^n) \]
\[ = I(MKS^n; Y^n) - I(S^n; Y^n|MK) \]
\[ = I(MKS^n; Y^n) - H(S^n|MK) \]
\[ + H(S^n|MKY^n) \]
\[ = \sum_{t=1}^{n} I(MKS^n_t; Y^n_t|Y^{t-1}) \]
\[ - \sum_{t=1}^{n} I(MKS^n_t; Z^n_t|Y^{t-1}) + 4n\varepsilon_n \]
\[ \equiv \sum_{t=1}^{n} I(S_tV_t; Y^n_t|U_t) - \sum_{t=1}^{n} I(S_tV_t; Z^n_t|U_t) + 4n\varepsilon_n \]
\[ = \sum_{t=1}^{n} I(S_tV_t; Y^n_t) - \sum_{t=1}^{n} I(S_tV_t; Z^n_t) + 4n\varepsilon_n \]
\[ = \sum_{t=1}^{n} I(V_t; Y^n_t) - \sum_{t=1}^{n} I(V_t; Z^n_t) + 4n\varepsilon_n \]
\[ + \sum_{t=1}^{n} H(S_t|Z^n_t) + 4n\varepsilon_n, \]
where \((c)\) and \((d)\) follow from Csiszár identity (cf. [19]); \((c)\) comes from \((20)\).

Therefore, using \((22)\), we have
\[ R_M + R_K \leq I(V; Y|SU) - I(V; Z|SU) + H(S|ZU) + 4\varepsilon_n. \]

Thus, letting \( n \to \infty \) in \((31)\), we conclude \((14)\), thereby completing the proof of Theorem 2.

An immediate consequence of Theorem 2 is the following two corollaries, where we let \( C_{NCSI-ED}^M \) (called the SM capacity) denote the supremum of the projection of \( C_{NCSI-ED} \) on the \( R_M \)-axis, and \( C_{NCSI-ED}^K \) (called the SK capacity) denote the supremum of the projection of \( C_{NCSI-ED} \) on the \( R_K \)-axis.

Then, we have, with \( UV \) and \( S \) that may be correlated,

**Corollary 1 (Non-causal SM capacity):**
\[ C_{NCSI-ED}^M = \max_{p_{SU}} \min(I(V; Y|SU) - I(V; Z|SU) + H(S|ZU), I(UV; Y|S)). \]  

**Corollary 2 (Non-causal SK capacity):**
\[ C_{NCSI-ED}^K = \max_{p_{SU}} (I(V; Y|SU) - I(V; Z|SU) + H(S|ZU)). \]  

**Remark 4:** The variable \( U \) in \((33)\) appears to play the role of “time-sharing” parameter, so one may wonder if this \( U \) can be omitted as in Khisti et al. [11, Theorem 3] who have, instead of \((33)\), given the following formula with the time-sharing parameter \( U \) omitted:
\[ C_{NCSI-ED} = \max_{p_{SU}} (I(V; Y|S) - I(V; Z|S) + H(S|Z)). \]

It is evident that the achievability in formula \((33)\) subsumes that of formula \((34)\) in that we can set \( U = \emptyset \) in \((33)\) to get \((34)\). We notice here also that, as will be seen from the proof of Theorem 4 if the WTC in consideration is a degraded one (\( Z \) is a degraded version of \( Y \)), then the right-hand sides of both \((33)\) and \((34)\) boil down to the right-hand side of \((120)\) in Corollary 8. Nevertheless, the \( U \) cannot be omitted in general, because in maximizing \((33)\) the “time-sharing” parameter \( U \) cannot necessarily be selected so as to be independent of the given “state” \( S \) (see “technical flaws” in the converse proofs
of [11, Theorem 3] and [10, Theorem 1]). We are thus tempted to think about the following conjecture:

**Conjecture:** There exists a WTC with non-causal CSI $S$ at both Alice and Bob such that

$$\max_{p_{SU}} (I(V; Y | SU) - I(V; Z | SU) + H(S | Z))$$

$$> \max_{p_{SV}} (I(V; Y | S) - I(V; Z | S) + H(S | Z)),$$

(35)

which then means that formula (34) is not tight in general.

**IV. WIRETAP CHANNEL WITH CAUSAL CSI**

The encoding scheme in [23] used to prove Theorem 1 is based on the soft covering lemma as well as the “non-causal” likelihood encoding. [20]. Since the re-interpretation of this scheme from the “causal” viewpoint is the very point to be invoked in this section, we here summarize the (non-causal) encoding scheme given by [25].

**Codebook $B_n$.** Define the index sets $I_n \triangleq [1 : 2^{nR_1}]$ and $J_n \triangleq [1 : 2^{nR_2}]$. For each $i \in I_n$, generate $u_i \in U^n$ of length $n$ that are i.i.d. according to probability distribution $p_{U}^n$. Next, given $i \in I_n$, for each $(j, k, m) \in J_n \times K_n \times M_n$ generate $v_{ijkm} \in V^n$ that are i.i.d. according to conditional probability distribution $p_{V | U}^n(\cdot | u_i)$.

**Likelihood encoder $f_n$:** Given $m \in M_n$ and $s \in S^n$, the encoder “randomly” chooses $(i, j, k) \in I_n \times J_n \times K_n$ according to the conditional probability ratio “proportional” to

$$f_{LE}(i, j, k | m, s) \triangleq p_{S | UV}(s | u_i, v_{ijkm}),$$

(36)

where $p_{SU}^n$ is the conditional probability distribution induced from $p_{SVUX}$. The encoder declares the chosen index $k \in K_n$ as the key. Given the chosen $(u_i, v_{ijkm})$, the channel input sequence $x \in X^n$ is generated according to conditional probability distribution $p_X^n(V | U) | U \triangleq f_{LE}(\cdot | u_i, v_{ijkm})$.

**Decoder $\phi_n$:** Upon observing the channel output $y \in Y^n$, the decoder searches for a unique $(i, j, k, m) \in I_n \times J_n \times K_n \times M_n$ such that

$$(u_i, v_{ijkm}, y) \in T^n_{\epsilon}(p_{UY}),$$

(37)

where $T^n_{\epsilon}(p_{UV})$ denotes the set of jointly $\epsilon$-typical sequences (cf. [26]). If such a unique quadruple is found, then set $\phi_n(y) = (m, k)$. Otherwise, $\phi_n(y) = (1, 1)$.

**Remark 5:** Roughly speaking, the likelihood encoder $f_n$ can be regarded as a *smoothed* joint typicality encoder (cf. Gelfand and Pinsker [21]) that, given $s$, picks up “at random” sequences $(u_i, v_{ijkm})$ with larger weights on jointly typical (with $s$) sequences and smaller weights on jointly atypical sequences.

Theorem 1 is of crucial significance in the sense that this provides the “best” inner bound to subsume, in a unifying way, all the known results in this field for WTCs with “non-causal” CSI available at Alice. As such, on the other hand, at first glance Theorem 1 does not appear to give any insights into WTCs with “causal” CSI. However, for the region $R_{in}(p_{SU}^n)$ with a class of some simple but relevant UVSs, it is possible to re-interpret $R_{in}(p_{SU}^n)$ as inner bounds for WTCs with “causal” CSI at Alice. This operation is called *plugging*, which is developed hereafter.

The “causal code” that we consider in this section is the following, which is the causal counterpart of the non-causal code defined as in Definition 1.

**Definition 7 (Causal code):** An $(n, R_M, R_K)$-code $c_n$ for the WTC with “causal” CSI at Alice and message set $M_n$ and key set $K_n$ is a triple of functions $(f_n^{(1)}, f_n^{(2)}, \phi_n)$ such that

1) $f_n^{(1)} : M_n \times S^n \rightarrow \mathcal{P}(X^n)$ ($t = 1, 2, \cdots, n$);
2) $f_n^{(2)} : M_n \times S^n \rightarrow \mathcal{P}(K_n)$,
3) $\phi_n : Y^n \rightarrow M_n \times K_n$,

where $f_n^{(1)}, f_n^{(2)}$ are stochastic functions.

**Remark 6:** One may wonder if $f_n^{(2)}$ in the above should be $f_n^{(2)} : M_n \rightarrow K_n$ because we are here considering “causal” encoders but $f_n^{(2)}$ here looks to require $S^n$ at once before the beginning of encoding at Alice. However, actually, the operation $f_n^{(2)} : M_n \times S^n \rightarrow \mathcal{P}(K_n)$ can be carried out by Alice at the end of the current block (of length $n$). This is possible with causal codes.

**Definition 8 (Causal SM-SK capacity region):** The SM-SK capacity region of the WTC with “causal” CSI at Alice, denoted by $C_{CSLE}$, is the set of all causally SM-SK achievable rate pairs with CSI at Alice, and the supremum of the projection of $C_{CSLE}$ on the $R_M$-axis, denoted by $C_{CSLE}^M$, is called the SM capacity, whereas the supremum of the projection of $C_{CSLE}$ on the $R_K$-axis is called the SK capacity, denoted by $C_{CSLE}^K$.

**Definition 9 (Causal achievability):** We now consider the following special class of random variables $UV$’s such that there exists some $UV$ independent of $S$ ($\hat{U} \text{ and } \hat{V}$ may be correlated) for which

$$\begin{align}
\text{Case 1)} : & \quad V = \hat{V}, \quad U = \hat{U}; \\
\text{Case 2)} : & \quad V = (S, \hat{V}), \quad U = \hat{U}; \\
\text{Case 3)} : & \quad V = \hat{V}, \quad U = (S, \hat{U}); \\
\text{Case 4)} : & \quad V = (S, \hat{V}), \quad U = (S, \hat{U}).
\end{align}$$

(38-41)

We say that the probability distribution $p_{YZSUXV}$ (or the corresponding random variable $YZSXUV$) is *causally achievable* if, in addition to (27) and the independence of $S$ and $UV$, one of conditions (38-41) is satisfied.

![Fig. 3. Causal SM-SK achievable rate region.](image-url)
With these preparations, we have the following causal version of Theorem 1 (cf. Fig. 5), where $\mathcal{R}_m^N$ as in Section II is replaced here by the causally achievable region $\mathcal{R}_m^C$.

**Theorem 3 (Causal SM-SK inner bound):**

\[
\mathcal{C}_{CSLE} \supseteq \mathcal{R}_m^C \equiv \bigcup_{C_{PSUV}} \mathcal{R}_m(p_{PSUV}),
\]

where the union is taken over all “causally” achievable probability distributions $p_{PSUV}$’s and $\mathcal{R}_m(p_{PSUV})$ is the same one as in Theorem 1.

**Proof:** In this proof too, under all Definitions 1 ~ 6 with Definition 7, we invoke the same Codebook $B_n$ and the likelihood encoder $f_n$ as in Section II.

The point here is to show that the likelihood encoder $f_n$ can in fact be implemented in a causal way for causally achievable probability distributions $p_{PSUV}$’s.

Although it may look to be necessary to give the proofs for each of Case 1) ~ Case 4), the ways of those proofs are essentially the same, so it suffices, without loss of generality, to show that the likelihood encoder $f_n$ can actually be implemented for Case 2) in a causal way.

First, recall that, in Case 2), $p_{S|U,V}$ is the conditional distribution of $S$ given $UV = USV$ and hence, irrespective of $u, v$,

\[
p_{S|U,V}(s|u, s', v) = \begin{cases} 1 & \text{if } s = s', \\ 0 & \text{if } s \neq s'. \end{cases}
\]

Then, since $p^n$ is a product probability distribution (i.e., memoryless) of $p$, setting as $v_{ijkm} = (s_{ijkm}, \tilde{v}_{ijkm})$, the conditional probability ratio in (56) can be evaluated as follows.

\[
f_{LE}(i, j, k|m, s) = p^n_{S|U,V}(s|u_i, v_{ijkm}) = p^n_{S|U,V}(s|u_i, s_{ijkm}, \tilde{v}_{ijkm}) = \prod_{t=1}^n p_{S|U,V}(s^{(t)}|u^{(t)}_i, s^{(t)}_{ijkm}, \tilde{v}^{(t)}_{ijkm}),
\]

where we have set

\[
s = (s^{(1)}, s^{(2)}, \ldots, s^{(n)}),
\]

\[
u_i = (u^{(1)}_i, u^{(2)}_i, \ldots, u^{(n)}_i),
\]

\[
s_{ijkm} = (s^{(1)}_{ijkm}, s^{(2)}_{ijkm}, \ldots, s^{(n)}_{ijkm}),
\]

\[
\tilde{v}_{ijkm} = (\tilde{v}^{(1)}_{ijkm}, \tilde{v}^{(2)}_{ijkm}, \ldots, \tilde{v}^{(n)}_{ijkm}).
\]

Now, in view of (43), it turns out that $p^n_{S|U,V}(s^{(t)}|u^{(t)}_i, s^{(t)}_{ijkm}, \tilde{v}^{(t)}_{ijkm})$ in (44) is equal to 1 if $s^{(t)} = s^{(t)}_{ijkm}$; otherwise, equal to 0 ($t = 1, 2, \ldots, n$), so that we have, irrespective of $(u, \tilde{v})$,

\[
p^n_{S|U,V}(s|u_i, s_{ijkm}, \tilde{v}) = \begin{cases} 1 & \text{if } s_{ijkm} = s, \\ 0 & \text{if } s_{ijkm} \neq s. \end{cases}
\]

Therefore, in particular,

\[
p^n_{S|U,V}(s|u_i, s_{ijkm}, \tilde{v}) = 1 \text{ for all } (i, j, k) \in \mathcal{I}_n \times \mathcal{J}_n \times \mathcal{K}_n,
\]

so that, given $(m, s)$, the stochastic (non-causal) likelihood encoder $f_n$ as specified in Section II chooses $(u_i, s, \tilde{v}_{ijkm})$ uniformly over the set

\[
\mathcal{L}(m, s) \equiv \{(u_i, s, \tilde{v}_{ijkm})|(i, j, k) \in \mathcal{I}_n \times \mathcal{J}_n \times \mathcal{K}_n\}. (51)
\]

We notice here that, since $UV$ and $S$ are independent and hence $(u_i, \tilde{v}_{ijkm})$, $s_{ijkm}$ and $s$ are also mutually independent, the set

\[
\mathcal{L}(m) \equiv \{(u_i, \tilde{v}_{ijkm})|(i, j, k) \in \mathcal{I}_n \times \mathcal{J}_n \times \mathcal{K}_n\}
\]

can actually be generated in advance of encoding, not depending on $(s_{ijkm}, s)$.

Up to here, it was assumed that the full state information $s$ is non-causally available at the encoder, so the point here is how this non-causal encoder $f_n$ can be replaced by a causal encoder. This is indeed possible, because $s_{ijkm} = s$ can be written componentwise as $s^{(t)}_{ijkm} = s^{(t)}$ ($t = 1, 2, \ldots, n$) and therefore the encoder can set $s^{(t)}_{ijkm}$ to be $s^{(t)}$ at each time $t$ using the state information $s^{(t)}$ available at time $t$ at the encoder, which clearly can be carried out in the “causal” way. Moreover, $(u_i, \tilde{v}_{ijkm})$ can also be fed in the causal way (componentwise) according as $(u_i^{(t)}, \tilde{v}_{ijkm}^{(t)})$ ($t = 1, 2, \ldots, n$), because $(u_i, \tilde{v}_{ijkm})$ was generated in advance of encoding.

Thus, given the chosen $(u_i, s, \tilde{v}_{ijkm})$, the encoder generates the channel input sequence

\[
x = (x^{(1)}, x^{(2)}, \ldots, x^{(n)}) \in \mathcal{X}^n
\]

according to the conditional probability:

\[
p^n_{X|S,U,V}(x|s, u_i, s, \tilde{v}_{ijkm}) = \prod_{t=1}^n p_{X|S,U,V}(x^{(t)}|s^{(t)}, u^{(t)}_i, s^{(t)}, \tilde{v}^{(t)}_{ijkm}),
\]

which implies that the $x$ can also be generated in the causal way according as $x^{(t)}$ ($t = 1, 2, \ldots, n$), thereby completing the proof of Theorem 3.

So far in this section we have invoked, as a crucial step, the argument of plugging, the logical core of which is schematically summarized as follows:

**Proposition 1 (Principle of plugging):** Consider a channel coding system (memoryless but not necessarily WTCs) with CSI $S$ and auxiliary random variables $U_1, U_2, \ldots, U_n$ together with rate tuple $(R_1, R_2, \ldots, R_b)$ to be used for generation of the random code

\[
\mathcal{C} = \{(u_{i_1}, u_{i_2}, \ldots, u_{i_a})\}_{i_1 \in [1; 2^n R_1], i_2 \in [1; 2^n R_2], \ldots, i_a \in [1; 2^n R_b]} \]

(54)

where each $R'_k$ ($k = 1, 2, \ldots, a$) is a partial sum of $R_1, R_2, \ldots, R_b$ (for example, $R'_1 = R_1 + R_3, R_2 = R_2$, etc.) and each codeword $(u_{i_1}, u_{i_2}, \ldots, u_{i_a})$ is generated according to product probability distribution $p^n_{U_1, U_2, \ldots, U_a}$ (or its marginal conditional distributions). Given message $m$ and state sequence $s$, the non-causal (likelihood) encoder $f_n$ stochastically picks an element of $\mathcal{C}$ and maps it “componentwise”

\[^{11} f_n \text{ may also be a joint typicality encoder (cf. Example 1).} \]
to a channel input $x$ according to conditional probability distribution $p^n_{X|SU_0U_1:U_{n}}(s,u_{i_1},u_{i_2},\ldots,u_{i_n})$. Now suppose that any rate tuple $(R_1,R_2,\ldots,R_k)$ satisfying the rate constraints
\begin{align*}
F_1(R_1,R_2,\ldots,R_k;U_1,U_2,\ldots,U_n;S) &\geq 0, \\
F_2(R_1,R_2,\ldots,R_k;U_1,U_2,\ldots,U_n;S) &\geq 0, \\
F_c(R_1,R_2,\ldots,R_k;U_1,U_2,\ldots,U_n;S) &\geq 0
\end{align*}
is “non-causally” SM-SK achievable. Then, any rate tuple $(R_1,R_2,\ldots,R_k)$ satisfying the rate constraints $F_1,F_2,F_c$ with

$$U_1 = \tilde{U}_1 \text{ or } (S,\tilde{U}_1); U_2 = \tilde{U}_2 \text{ or } (S,\tilde{U}_2); \ldots; U_n = \tilde{U}_n \text{ or } (S,\tilde{U}_n)$$

is “causally” SM-SK achievable, where $\tilde{U}_1,\tilde{U}_2,\ldots,\tilde{U}_n$ (may be correlated) are independent of $S$. \hfill \Box

**Example 1:** A simple example (with $Z = \emptyset$ (constant variable)) is the relation of the Gelfand-Pinsker (non-causal) coding [21] and the Shannon strategy (causal) coding [4]. The former gives the formula
\begin{equation}
C_{\text{CSI-E}}^M = \max_{p_{SU}}(I(U;Y) - I(U;S)) \quad \text{(59)}
\end{equation}
while the latter gives the formula
\begin{equation}
C_{\text{CSIE}}^M = \max_{p_{pSU}} I(U;Y) \quad \text{(60)}
\end{equation}

Principle of plugging applied to (59) claims that, given independent $S$ and $\tilde{U}$, rates $R' = I(\tilde{U};Y) - I(\tilde{U};S) = I(\tilde{U};Y)$ and $R'' = I(\tilde{U};S) - I(\tilde{U};Y) - H(S)$ are “causally” achievable. It is easy to check that $R' \geq R''$, so in this case $R''$ is redundant. Thus, the achievability part of (59) is concluded from that of (59) without a separate proof. \hfill \Box

V. APPLICATIONS OF THEOREM 3

Having established Theorem 3 on WTCs with causal CSI at Alice, in this section we develop it for each of Case 1) – Case 4) to demonstrate that, via Theorem 3, we can uniformly derive the previously known causal “lower” bounds such as in [12, 13] and [14]. In addition, we also demonstrate that a new class of causal “inner” bounds directly follow from Theorem 3, which could not have been easily obtained without Theorem 3. They are largely classified into Propositions 2 and 3. In particular, we emphasize that in this section we are concerned solely with “two-dimensional” inner/outer bounds of causally achievable rate pairs $(R_M,R_K)$, which are derived in this paper for the first time.

VA: Causal inner bounds:

Let us now scrutinize the claim of Theorem 3. For the convenience of discussion, we record again here the rate constraints (8) and (9) as
\begin{align*}
R_M &\leq I(UV;Y) - I(UV;S), \\
R_M + R_K &\leq I(V;Y|U) - I(V;Z|U) - [I(U;S) - I(U;Y)]^+.
\end{align*}

which is specifically developed according to Cases 1) – 4) as follows.

**Case 1):** Since $U = \tilde{U}, V = \tilde{V}$ and $\tilde{U}\tilde{V}$ is independent of $S$, (61) and (62) reduce to
\begin{equation}
R_M \leq I(\tilde{U}\tilde{V};Y),
\end{equation}
\begin{equation}
R_M + R_K \leq I(V;Y|\tilde{U}) - I(V;Z|\tilde{U}),
\end{equation}
where we have used $I(\tilde{U}\tilde{V};S) = 0$ and $I(\tilde{U};S) - I(\tilde{U};Y)^+ = 0$. Clearly, (63) is redundant, so only (63) remains. Hence, removing tilde to make the notation simpler, we have
\begin{equation}
R_M + R_K \leq I(V;Y|U) - I(V;Z|U).
\end{equation}

It is not difficult to check that replacing (63) by
\begin{equation}
R_M + R_K \leq I(V;Y) - I(V;Z)
\end{equation}
does not affect the inner region. Thus,
\begin{equation}
C_{\text{CSI-E}} \supset \bigcup_{p_{pSU}} \{ \text{rate pairs } (R_M,R_K) \text{ satisfying (69)} \},
\end{equation}
which implies, in particular, the non-causal SM achievability ($R_K = 0$) of Dai and Luo [18, Theorem 3].

**Case 2):** Since $U = \tilde{U}, V = \tilde{V}$ and $\tilde{U}\tilde{V}$ is independent of $S$, (61) and (62) are computed as
\begin{align*}
R_M &\leq I(\tilde{U}\tilde{V};Y) - I(\tilde{U}\tilde{V};S) = I(\tilde{U}\tilde{V};Y) - H(S), \\
R_M + R_K &\leq I(\tilde{S}\tilde{V};Y|\tilde{U}) - I(\tilde{S}\tilde{V};Z|\tilde{U}) - [I(\tilde{U};S) - I(\tilde{U};Y)]^+ \overset{(a)}{=} I(\tilde{S}\tilde{V};Y|\tilde{U}) - I(\tilde{S}\tilde{V};Z|\tilde{U}),
\end{align*}
where (a) follows from $I(\tilde{U};S) = 0$. Therefore, removing tilde again to make the notation simpler, we have the rate constraints for Case 2),
\begin{align*}
R_M &\leq I(USV;Y) - H(S), \\
R_M + R_K &\leq I(SV;Y|U) - I(SV;Z|U),
\end{align*}
where $UV$ and $S$ are independent. Therefore, any nonnegative rate pair $(R_M,R_K)$ is achievable if rate constraints (70) and (71) are satisfied. Thus, we have the following fundamental inner bound:

**Proposition 2 (Causal SM-SK inner bound: type I):**

\begin{equation}
C_{\text{CSI-E}} \supset \bigcup_{p_{pSU}} \{ \text{rate pairs } (R_M,R_K) \text{ satisfying (70) and (71)} \}.
\end{equation}

An immediate by-product of (72) is the following corollary:

**Corollary 3 (Causal lower bound (1) at Alice):**

\begin{align*}
C_{\text{CSI-E}}^M &\geq \max_{p_{pSU}} \min(I(SV;Y|U) - I(SV;Z|U), \\
C_{\text{CSI-E}}^K &\geq \max_{p_{pSU}} \min(I(SV;Y|U) - I(SV;Z|U), \\
&\geq I(USV;Y) - H(S), \\
&\geq I(USV;Y) - I(SV;Z|U),
\end{align*}

where $UV$ and $S$ are independent. \hfill \Box
Proof: Setting $R_K = 0$ in (72) yields (73), while setting $R_M = 0$ in (72) yields (74).

Let us now consider two special cases of (72).

A: Let $U = \emptyset$ (constant variable), then (70) and (71) reduce to

$$R_M \leq I(SV;Y) - H(S);$$

(75)

$$R_M + R_K \leq I(SV;Y) - I(SV;Z)$$

(76)

with independent $V$ and $S$. Consequently, any nonnegative rate pair $(R_M, R_K)$ is achievable if rate constraints (75) and (76) are satisfied. Thus, we have

$$C_{CSl-E} \supseteq \bigcup_{p \in PV} \{\text{rate pairs} (R_M, R_K) \text{ satisfying (75) and (76)}\}.$$  

(77)

Remark 7: Setting $R_K = 0$ in (77) yields the SM lower bound:

$$C_{CSl-E}^\mu \geq \max_{p \in PV} \{\min(I(SV;Y) - I(SV;Z), I(SV;Y) - H(S))\}.$$  

(78)

On the other hand, setting $R_M = 0$ in (77) yields the SK lower bound:

$$C_{CSl-E}^\xi \geq \max_{I(SV;Y) \geq H(S)} \{I(SV;Y) - I(SV;Z)\},$$

(79)

which was leveraged, without the proof, in Han and Sasaki [14, Remark 5].

Next, in order to compare formula (78) with the previous result, we develop it in the sequel. First, (75) is rewritten as

$$R_M \leq I(SV;Y) - H(S)$$

(80)

$$= I(VY) + I(SY|V) - I(VZ|S)$$

$$= I(VY) + H(SV) - H(SVY) - H(S)$$

$$+ H(SZ) - I(VZ|S)$$

$$= I(VY) - I(VSZ) + I(VS) + H(SV)$$

$$- H(SVY) - H(S) + H(SZ)$$

$$= I(VY) - I(VSZ) + H(SZ) - H(SVY).$$

(81)

where (b) follows from the independence of $V$ and $S$. On the other hand, (76) is evaluated as follows:

$$R_M + R_K \leq I(SV;Y) - I(SV;Z)$$

$$= I(VY) + I(SY|V) - I(VZ|S)$$

$$= I(VY) + H(SV) - H(SVY) - H(S)$$

$$+ H(SZ) - I(VZ|S)$$

$$= I(VY) - I(VSZ) + I(VS) + H(SV)$$

$$- H(SVY) - H(S) + H(SZ)$$

$$= I(VY) - I(VSZ) + H(SZ) - H(SVY).$$

(82)

Summarizing, we have, with independent $V$ and $S$,

$$R_M \leq I(VY) - H(SVY),$$

(83)

$$R_M + R_K \leq I(VY) - I(VSZ)$$

$$+ H(SZ) - H(SVY).$$

(84)

which is equivalent to (77). Now, setting $R_K = 0$ in (84), it turns out that formula (78) is rewritten as

$$C_{CSl-E}^\mu \geq \max_{p \in PV} \{\min(I(VY) - I(VSZ) + H(SZ) - H(SVY),$$

$$I(VY) - H(SVY))\}$$

(85)

with independent $V$ and $S$, which was given as $R_{CSl-1}$ by Han and Sasaki [14, Theorem 1] (also cf. Fujita [13, Lemma 1]).

B: Let $V = \emptyset$, then (70) and (71) reduce to

$$R_M \leq I(UY) - H(S),$$

(86)

$$R_M + R_K \leq I(SY|U) - I(SZ|U)$$

(87)

with independent $U$ and $S$. It is easy to check that (86) and (87) are rewritten equivalently as

$$R_M \leq I(UY) - H(SUY),$$

(88)

$$R_M + R_K \leq H(SUZ) - H(SUY).$$

(89)

Consequently, any nonnegative rate pair $(R_M, R_K)$ is achievable if constraints (88) and (89) are satisfied. Thus,

$$C_{CSl-E} \supseteq \bigcup_{p \in PUV} \{\text{rate pairs} (R_M, R_K) \text{ satisfying (88) and (89)}\}.$$  

(90)

Remark 8: Setting $R_K = 0$ in (80) yields the lower bound with independent $U$ and $S$:

$$C_{CSl-E}^\mu \geq \max_{p \in PVU} \{H(SUY) - H(SUY),$$

$$I(UY) - H(SUY))\}$$

(91)

which was given as $s R_{CSl-2}$ by Han and Sasaki [14, Theorem 1].

On the other hand, setting $R_M = 0$ in (90), we have, for independent $U$ and $S$,

$$C_{CSl-E}^\xi \geq \max_{I(UY) \geq H(SUY)} \{H(SUZ) - H(SUY),$$

(92)

which is a new type of lower bound. We notice here that either (75) or (91) does not always outperform the other. Similarly, we can check that either (76) or (92) does not always outperform the other. The proof of them is given in Appendix B.

We now have the following two corollaries for WTCs with causal CSI available at “both” Alice and Bob.

Corollary 4 (Causal inner bound (2) at Alice and Bob): Let us consider the WTC with causal CSI at both Alice and Bob, as depicted in Fig. 2. Then, a pair $(R_M, R_K)$ is achievable if the following rate constraints are satisfied:

$$R_M \leq I(V;YS);$$

(93)

$$R_M + R_K \leq I(V;YS) - I(VZS)$$

$$+ H(SZ),$$

(94)

where $V$ and $S$ are independent. Thus,

$$C_{CSl-ED} \supseteq \bigcup_{p \in PV} \{\text{rate pairs} (R_M, R_K) \text{ satisfying (93) and (94)}\},$$

(95)
where ED denotes that the causal CSI $S$ is available at both Alice and Bob.

Proof: It is sufficient to replace $Y$ by $SY$ in (75) and (76).

Remark 9: As far as we are concerned with “degraded” WTCs ($Z$ is a degraded version of $Y$), the inclusion $\supset$ in (95) can be replaced by $=$, so that in this case (95) actually gives the causal SM-SK capacity region, as will be explicitly stated later in Theorem 4.

Remark 10: Setting $R_M = 0$ in (95) yields one more new lower bound:

$$C^K_{\text{CSI-ED}} \geq \max_{p_{SV}} (I(V;Y|S) - I(V;Z|S) + H(S|Z)).$$

(96)

where $V$ and $S$ are independent, and $C^K_{\text{CSI-ED}}$ denotes the causal SK capacity.

On the other hand, setting $R_K = 0$ in (95) yields the lower bound given by Chia and El Gamal [12, Theorem 1]:

$$C^M_{\text{CSI-ED}} \geq \max_{p_{SV}} \min_{p_{SU}} (I(V;Y|S) - I(V;Z|S) + H(S|Z)).$$

(97)

with independent $V$ and $S$, where $C^M_{\text{CSI-ED}}$ denotes the causal SM capacity.

Corollary 5 (Causal inner bound (3) at Alice and Bob): Let us consider the WTC with causal CSI at both Alice and Bob, as depicted in Fig. 2. Then, a pair $(R_M, R_K)$ is achievable if the following rate constraints are satisfied:

$$R_M \leq I(U;Y|S)$$

(98)

$$R_M + R_K \leq H(S|UZ)$$

(99)

where $U$ and $S$ are independent, Thus,

$$C_{\text{CSI-ED}} \supset \bigcup_{p_{SV}} \{ \text{rate pairs } (R_M, R_K) \text{ satisfying } (98) \text{ and } (99) \}.$$ 

(100)

Proof: It is sufficient to replace $Y$ by $SY$ in (88) and (89).

Remark 11: Setting $R_K = 0$ in (100) yields the lower bound given by Chia and El Gamal [12, Theorem 3]:

$$C^M_{\text{CSI-ED}} \geq \max_{p_{SV}} \min_{p_{SU}} (H(S|UZ), I(U;Y|S)).$$

(101)

On the other hand, setting $R_M = 0$ in (100) yields $C^K_{\text{CSI-ED}} \geq H(S|UZ)$. Also, we can set $U = \emptyset$ to obtain

$$C^K_{\text{CSI-ED}} \geq \max_{p_{SH}} H(S|Z),$$

(102)

which is obviously attained without transmission coding at the encoder, because in this case sharing of common secret key at Alice and Bob is enough without extra transmission of secret message (cf. Ahlswede and Csiszár [16]). Here, in view of (102) and (111, Corollary 1), it is easy to see that, for reversely degraded ($Y$ is a degraded version of $Z$) WTCs, $C^K_{\text{CSI-ED}} = C^K_{\text{NSCSI-ED}} = \max_{p_{SH}} H(S|Z).$

(103)

Remark 12: Comparing (96) and (102), we see that either one does not necessarily subsume the other, which depends on whether $I(V;Y|S) \geq I(V;Z|S)$ or not. Specifically, in the case of $I(V;Y|S) \geq I(V;Z|S)$ coding helps, otherwise coding does not help. Notice that, for example, if $Z$ is a degraded version of $Y$, then $I(V;Y|S) \geq I(V;Z|S)$ always holds and so coding helps. □

Case 3: Since $U = S \bar{U}, V = \bar{V}$ and $\bar{U} \bar{V}$ is independent of $S$, (61) and (62) are computed as

$$R_M \leq I(\bar{U}S\bar{V};Y) - I(US\bar{V};S)$$

(104)

$$= I(\bar{U}S\bar{V};Y) - H(S);$$

$$R_M + R_K \leq I(\bar{V};Y|SU) - I(\bar{V};Z|SU)$$

(105)

$$- [I(SU;S) - I(SU;Y)]^+$$

$$= I(\bar{V};Y|SU) - I(\bar{V};Z|SU)$$

$$- [H(S) - I(SU;Y)]^+. $$

As a consequence, removing tilde “,” we have the rate constraints, with independent $UV$ and $S$,

$$R_M \leq I(SUB;Y) - H(S);$$

(106)

$$R_M + R_K \leq I(V;Y|SU) - I(V;Z|SU)$$

(107)

$$- [H(S) - I(SU;Y)]^+. $$

Therefore, any nonnegative rate pair $(R_M, R_K)$ is achievable if rate constraints (106) and (107) are satisfied. Thus, we have the following one more fundamental inner bound (type II), which is paired with Proposition 2 (type I):

Corollary 6 (Causal lower bound (4) at Alice):

$$C_{\text{CSI-ED}} \supset \bigcup_{p_{SVU}} \{ \text{rate pairs } (R_M, R_K) \text{ satisfying } (106) \text{ and } (107) \}.$$ 

(108)

Remark 13: We observe here that (106) and (107) remain invariant under replacement of $Z$ by $SZ$. This implies that the achievable due to Case 3) is invulnerable to the leakage of state information $S^n$ to Eve, which is in notable contrast with Case 2). □

An immediate consequence of (108) is the following corollary:

Corollary 6 (Causal lower bound (4) at Alice):

$$C^M_{\text{CSI-ED}} \geq \max_{p_{SVU}} \min_{p_{SU}} (I(V;Y|SU) - I(V;Z|SU)$$

(109)

$$- [H(S) - I(SU;Y)]^+, I(USV;Y) - H(S)), $$

$$C^K_{\text{CSI-ED}} \geq \max_{I(USV;Y) \geq H(S)} \min_{p_{SVU}} (I(V;Y|SU) - I(V;Z|SU)$$

(110)

$$- [H(S) - I(SU;Y)]^+),$$

where $UV$ and $S$ are independent. □

Proof: Setting $R_K = 0$ in (108) yields (109), while setting $R_M = 0$ in (108) yields (110). □

Remark 14 (Comparison of Case 2) and Case 3): We first notice that (105) is the same as (70), and moreover, noting that

$$H(S) - I(SU;Y) = H(S|Y) - I(U;Y|S)$$

$$= H(S|Y) - I(U;SY)$$

$$= H(S|Y) - I(U;Y) - I(U;S|Y)$$

$$= H(S|UY) - I(U;Y)$$

(111)
and summarizing (106), (107) and (111), we have for Case 3):

\[ R_M \leq I(U_SV; Y) - H(S); \quad (112) \]
\[ R_M + R_K \leq I(V; Y|SU) - I(V; Z|SU) - [H(S|UY) - H(S|UZ)]. \quad (113) \]

In order to compare this with that for Case 2), we rewrite (70) and (71) as

\[ R_M \leq I(U_SV; Y) - H(S); \quad (114) \]
\[ R_M + R_K \leq I(SV; Y|U) - I(SV; Z|U) + I(V; Y|SU) - I(V; Z|SU) - [H(S|UY) - H(S|UZ)]. \quad (115) \]

Thus, for Case 2),

\[ R_M \leq I(U_SV; Y) - H(S); \quad (116) \]
\[ R_M + R_K \leq I(V; Y|SU) - I(V; Z|SU) - [H(S|UY) - H(S|UZ)]. \quad (117) \]

Comparing (115) and (116), we see that the difference consists in that of the terms \([H(S|UY) - I(U; Y)]^+ + [H(S|UY) - H(S|UZ)],\) so either one does not necessarily subsume the other, which depends on the choice of achievable probability distributions \(p_{YZSVUV}.\)

**Remark 15**: As such, to get more insight, let us consider the WTC with causal CSI available at both Alice and Eve, as depicted in Fig. 3. Then, since \([H(S|UY) - I(U; Y)]^+ \leq H(S|UY) + [H(S|UY) - H(S|UZ)] = H(S|UY),\) in this case Case 3) outperforms Case 2), where \(Z\) was replaced by \(SZ\) as the state \(S\) is available also at Eve (cf. Remark 13). This means that Case 3) is preferable to Case 2) when Eve have full access to \(S^n.\)

On the other hand, consider an opposite case with CSI available at both Alice and Bob as n Fig. 2. Then, since \(H(S|UY) = 0\) with \(SY\) instead of \(Y\) and hence \([H(S|UY) - I(U; Y)]^+ = 0\) and \([H(S|UY) - H(S|UZ)] = -H(S|UZ),\) we see that, in this case, Case 2) outperforms Case 3).

**Remark 16**: As is seen from the proof of Theorem 1 in Bunin et al. [24, 25], in both cases of Case 2) and Case 3) the state information \(S^n\) is to be reliably reproduced at Bob, while the crucial difference between Case 2) and Case 3) is that in Case 2) the \(S^n\) is used to carry on secure transmission of message and/or key between Alice and Bob, whereas in Case 3) the \(S^n\) is not used to convey secure message and/or key but simply to help reliable (secured or unsecured) transmission. On the other hand, in Case 1) the \(S^n\) is not to be reproduced at Bob. As was illustrated in Remark 13, favorable choices of these three cases depend on the probabilistic structure of WTCs.

**Case 4)**: Since \(U = S\bar{U}, V = S\bar{V}\) and \(\bar{U}\bar{V}\) is independent

![Fig. 4. WTC with the same CSI available at Alice and Eve (t = 1, 2, \ldots, n).](image)

of \(S, \{61\}\) and \(\{62\}\) are computed as

\[ R_M \leq I(U_S\bar{V}; Y) - I(U_S\bar{V}; S) = I(U_S\bar{V}; Y) - H(S); \quad (117) \]
\[ R_M + R_K \leq I(V; Y|SU) - I(V; Z|SU) - [H(S|UY) - H(S|UZ) - I(SU; Z) - I(U; Y)]^+, \quad (118) \]

which is nothing but (104) and (105) in Case 3), and therefore Case 4) reduces to Case 3).

**V.B: Causal outer bound:**
So far we have discussed a diversity of causal SM-SK inner bounds, but not about outer bounds. This is because, in general, it is much harder with the problem of *causal* outer bounds, in contrast with non-causal outer bounds. However, we can show an example of causal “tighter” outer bound, which is a rare case (from the causal viewpoint) and is paired with Proposition 2 (achievability part). In passing this section we consider this problem.

To do so, we first notice that the coding scheme used to prove Proposition 2 required the CSI \(S^n\) to be reliably reproduced at Bob, i.e., \(H(S^n|Y^n) \leq n\varepsilon_n.\) This kind of coding scheme is said to be state-reproducing (cf. Han and Sasaki [14]). Then, one may ask what happens if we confine ourselves to within such state-reproducing coding schemes. An answer is:

**Proposition 4 (Causal/non-causal outer bound):** With state-reproducing coding schemes, we have the following outer bound:

\[ C_{\text{CSI-E}} \subseteq C_{\text{NCSI-E}} \subseteq \bigcup_{PSUV} \{\text{rate pairs } (R_M, R_K) \text{ satisfying (70) and (71)}\} \quad (119) \]

Notice that the difference between Proposition 3 (outer bound) and Proposition 2 (inner bound) is that the union in the former is taken over all probability distributions \(p_{PSUV}\)’s, while in the latter the union is taken over all product probability distributions \(p_{PSUV}\)’s.

**Proof:** It suffices only to literally parallel the converse part of Theorem 2 with \(Y^n = S^nY^n\) replaced by \(Y^n,\) while using \(H(S^n|Y^n) \leq n\varepsilon_n\) (due to the state-reproducibility) in inequality (27) of Section III which together with (18) (with \(Y_i\) instead of \(\tilde{Y}_i\)) brings about the required outer bound. \(\square\)
An immediate consequence of (19) is the following corollary, which is paired with Corollary 3.

**Corollary 7 (Causal/non-causal upper bound):** With state-reproducing coding schemes, we have the upper bounds:

\[
C_{\text{CSI-E}}^{d,M} \leq C_{\text{NCI-E}}^{d,M} \leq \max_{p_{SV}} \min(I(SV; Y|U) - I(SV; Z|U), I(USV; Y) - H(S)), \quad (120)
\]

\[
C_{\text{CSI-E}}^{d,K} \leq C_{\text{NCI-E}}^{d,K} \leq \max_{p_{SV}} \min(I(SV; Y|U) - I(SV; Z|U)). \quad (121)
\]

VI. SM-SK CAPACITY THEOREMS FOR DEGRADED WTCs

1) Let us now address the problem of SM-SK capacity regions to provide the exact SM-SK capacity region for degraded WTCs with causal/non-causal CSI available at “both” Alice and Bob as in Fig. 2. To do so, let the corresponding causal SM-SK capacity region be denoted by \(C_{\text{CSI-ED}}^{d}\). Similarly, the corresponding non-causal SM-SK capacity region is denoted by \(C_{\text{NCI-ED}}^{d}\). Moreover, let \(\mathcal{R}_{\text{NS}}^{d}\) denote the set of all non-negative rate pairs \((R_M, R_K)\) satisfying the rate constraints:

\[
R_M \leq I(X; Y|S), \quad (122)
\]

\[
R_M + R_K \leq I(X; Y|S) - I(X; Z|S) + H(S|Z), \quad (123)
\]

Then, we have

**Theorem 4 (Causal/non-causal SM-SK capacity region):** Consider a degraded WTC \((Z)\) is a degraded version of \(Y\) with causal/non-causal CSI at Alice and Bob. Then,

\[
C_{\text{CSI-ED}}^{d} = C_{\text{NCI-ED}}^{d} = \mathcal{R}_{\text{NS}}^{d} = \bigcup_{p_{SV}} \mathcal{R}_{\text{NS}}^{d}(p_{SV}), \quad (124)
\]

where the union is taken over all possible probability distributions \(p_{SV}\)’s.

**Remark 17:** Notice, in particular, that Theorem 4 means also that the causal and non-causal capacity regions coincide for degraded WTCs. An immediate consequence of Theorem 4 is

**Corollary 8:**

\[
C_{\text{CSI-ED}}^{d,M} = C_{\text{NCI-ED}}^{d,M} = \max_{p_{SV}} \min(I(X; Y|S) - I(X; Z|S) + H(S|Z), I(X; Y|S)). \quad (125)
\]

\[
C_{\text{CSI-ED}}^{d,K} = C_{\text{NCI-ED}}^{d,K} = \max_{p_{SV}} \min(I(X; Y|S) - I(X; Z|S) + H(S|Z)), \quad (126)
\]

where \(C_{\text{CSI-ED}}^{d,M}, C_{\text{NCI-ED}}^{d,M}\) (resp. \(C_{\text{CSI-ED}}^{d,K}, C_{\text{NCI-ED}}^{d,K}\)) is the supremum of the projection of \(C_{\text{CSI-ED}}^{d,M}, C_{\text{NCI-ED}}^{d,M}\) on the \(R_M\)-axis (resp. \(R_K\)-axis).

**Remark 18:** Formula (125) has earlier been given by [12] Theorem 3] in a quite different manner.

**Proof of achievability for Theorem 4.**

Let \((X, S)\) be arbitrarily given, then the functional representation lemma [19] claims that there exist a random variable \(V\) and a deterministic function \(f : V \times S \to X\) such that \(V\) and \(S\) are independent and \(X = f(V, S)\). Then, Theorem 5 (Case 2): with \(U = \emptyset\) claims that any rate pair \((R_M, R_K)\) satisfying the rate constraints (93) and (94), that is,

\[
R_M \leq I(V; Y|S); \quad (127)
\]

\[
R_M + R_K \leq I(V; Y|S) - I(V; Z|S) + H(S|Z), \quad (128)
\]

is “causally” achievable. Then, it suffices to observe that the right-hand sides of (127) is rewritten as

\[
I(V; Y|S) \leq I(VX; Y|S) \quad (129)
\]

where \((e)\) is because \(X\) is a deterministic function of \((V, S)\); \((g)\) follows from the Markov chain property \(UV \to SX \to YZ\). Similarly, (128) can be rewritten as

\[
I(V; Y|S) - I(V; Z|S) + H(S|Z) = I(SX; Y|S) - I(S; Z|S) + H(S|Z). \quad (130)
\]

**Proof of converse for Theorem 4.**

Theorem 4 claims that any achievable rate pair \((R_M, R_K)\) must satisfy the rate constraints (13) and (14), that is,

\[
R_M \leq I(UV; Y|S), \quad (131)
\]

\[
R_M + R_K \leq I(V; Y|SU) - I(V; Z|SU) + H(S|SU) \quad (132)
\]

with some \(UVSXYZ\). The right-hand sides of (131) and (132) are evaluated as follows:

\[
I(UV; Y|S) \leq I(UVX; Y|S) = I(X; Y|S) + I(UV; Y|SX) \quad (133)
\]

where \((v)\) follows from the Markov chain property \(UV \to SX \to Y\). Hence,

\[
I(UV; Y|S) \leq I(X; Y|S). \quad (134)
\]

On the other hand,

\[
I(V; Y|SU) - I(V; Z|SU) = I(VX; Y|SU) - I(X; Y|SU) \quad (a)
\]

\[
= I(VX; Z|SU) + I(X; Z|SU) \quad (b)
\]

\[
= I(VX; Y|SU) - I(VX; Z|SU) \quad (c)
\]

\[
- I(X; Y|SU) - I(X; Z|SU) \quad (d)
\]

\[
= I(X; Y|SU) - I(X; Z|SU) \quad (e)
\]

where \((a)\) follows from the Markov chain property \(UV \to SX \to Y\). Hence,

\[
I(UV; Y|S) \leq I(X; Y|S). \quad (134)
\]
where (a), (c) follows from the Markov chain property $UV \rightarrow SX \rightarrow YZ$; (b) follows from the assumed degradedness. Moreover, since
\[ H(S|ZU) - H(S|Z) = -I(S;U|Z), \]

it follows that
\[
H(S|ZU) - H(S|Z) - [I(U;Y|S) - I(U;Z|S)]
= -I(S;U|Z) - [I(U;Y|S) - I(U;Z|S)]
= -I(U;Y|S) - I(S;U|Z) - I(U;Z|S)
= I(S;U) - I(U;SY) - [I(S;U) - I(U;Z)]
\]
\[
\leq -I(U;Y) + I(U;Z)
\]
\[
(j)\quad -I(U;Y) + I(U;Z) \leq 0,
\]

(137)

where (j) follows from the assumed degradedness. Therefore,
\[
H(S|ZU) - [I(U;Y|S) - I(U;Z|S)] \leq H(S|Z).
\]

Thus, by virtue of (135) and (138), we obtain
\[
I(V;Y|SU) - I(V;Z|SU) + H(S|ZU)
\leq I(X;Y|S) - I(X;Z|S) + H(S|Z),
\]

which together with (134) completes the proof of Theorem 3.

2) Next let us address the problem of SM-SK capacity region to provide an SM-SK outer bound for degraded WTCs with causal/non-causal CSI available “only” at Alice as in Fig. 1. To do so, let the corresponding causal SM-SK capacity region be denoted by $C^e_{CSIE}$. Similarly, the corresponding non-causal SM-SK capacity region is denoted by $C^e_{NCSIE}$. Moreover, let $R_{\text{out}}(p_{SX})$ denote the set of all nonnegative rate pairs $(R_M, R_K)$ satisfying the rate constraints:
\[
R_M \leq I(X;Y|S),
\]
\[
R_M + R_K \leq I(X;Y|S) - I(X;Z|S) + H(S|Z) - H(S|Y).
\]

(140)

Then, we have
\[
\text{Theorem 5 (Causal/non-causal SM-SK outer bound):}
\]
Consider a degraded WTC ($Z$ is a degraded version of $Y$) with causal/non-causal CSI at Alice. Then,
\[
C^e_{CSIE} \subset C^e_{NCSIE} \subset R_{\text{out}} \Delta \bigcup_{p_{SX}} R_{\text{out}}(p_{SX}),
\]

(142)

where the union is taken over all possible probability distributions $p_{SX}$’s.

Proof: The upper bound (122) for WTCs with CSI at both Alice and Bob must hold also for WTCs with CSI only at Alice, yielding (140).

On the other hand, in order to yield inequality (141), it suffices to parallel the converse proof of Theorem 2 while keeping in mind $H(S^n|M K \bar{Y}^n) \leq H(S^n|M K Z^n)$ with $Y^n$ instead of $\bar{Y}^n = S^n Y^n$ (due to the assumed degradedness) in (26) and skipping (27) to (28) (with $Y_t$ instead of $\bar{Y}_t$), which claims that the achievable rate pair $(R_M, R_K)$ needs to satisfy the rate constraints:
\[
n(R_M + R_K) \leq \sum_{t=1}^{n} I(V_t;Y_t|S_t U_t) - \sum_{t=1}^{n} I(V_t;Z_t|S_t U_t)
+ \sum_{t=1}^{n} H(S_t|Z_t U_t) - \sum_{t=1}^{n} H(S_t|Y_t U_t) + 4n \varepsilon
\]
\[
= nI(V;Y|SU) - nI(V;Z|SU)
+ nH(S|ZU) - nH(S|YU) + 4n \varepsilon.
\]

(143)

Therefore, by dividing by $n$ and letting $n \to \infty$, we have
\[
R_M + R_K \leq I(V;Y|SU) - I(V;Z|SU)
+ H(S|ZU) - H(S|YU).
\]

(144)

Then, in the same manner as in the converse proof of Theorem 4, we can check that (144) yields inequality (141).

Finally, the following corollary follows from Theorem 5.

Corollary 9 (Upper bound on SK rates): For a degraded WTC with causal/non-causal CSI at Alice, we have
\[
C_{\text{CSIE}}^{e,K} \leq C_{\text{NCSIE}}^{e,K} \leq \max_{p_{SX}}(I(X;Y|S) - I(X;Z|S) + H(S|Z) - H(S|Y)).
\]

(145)

VII. CONCLUDING REMARKS

So far, we have studied the coding problem for WTCs with causal/non-causal CSI available at Alice and/or Bob under the semantic security criterion, the key part of which was summarized as Theorem 3 for WTCs with causal CSI at Alice. As is already clear, all the advantages of Theorem 3 are inherited directly from Theorem 1 that had been established by Bunin et al. [25] for WTCs with non-causal CSI at Alice, This suggests that it is sometimes useful to deal with the causal problem as a special class of non-causal problems.

It is rather surprising to see that all the previous results [12], [13], [14] for WTCs with causal CSI follow immediately from Theorem 3 alone. Notice here that the validity of Theorem 1 is based heavily on the superiority of the two layered superposition coding scheme (cf. [17], [23]) along with that of soft covering lemma. It is also pleasing to see that Theorem 2 as a by-product of Theorem 1 gives for the first time the exact SM-SK capacity region for WTCs with non-causal CSI at both Alice and Bob. Theorem 4 is also regarded as one of the key results from the viewpoint of SK-SM capacity regions for degraded WTCs.

Although Theorem 3 treats the WTC with causal CSI available only at Alice, it can actually be effective also for investigating general WTCs with three correlated causal CSIs $S_a, S_b, S_c$ (correlated with state $S$) available at Alice, Bob and Eve, respectively (cf. Fig. 5).

We would like to remind that this seemingly “general” WTCs actually boils down to the so far studied WTC.
with causal CSI available only at Alice simply by replacing channel $W_{Y|Z|S}^{x}(y,z|s,x)$ with $W_{Y|Z|S}^{x}(y,z|s,a)$ and at the same time by replacing $Y,Z$ with $S_{b}Y,S_{a}Z$, respectively. In this connection, the reader may refer, for example, to Khisti, Diggavi and Wornell [11], and Goldfeld, Cuff and Permuter [23].

APPENDIX A

PROOF OF LEMMA 1

From the manner of generating the random code, we see that the total joint probability of all $(u_i,v_{ij})/s$ is given by $P_{1n}P_{2n}P_{3n}$, where

$$P_{1n} = \prod_{k=2}^{L_n} \prod_{\ell=1}^{N_n} p(u_k) p(v_{k\ell} | u_k),$$

$$P_{2n} = \prod_{\ell=2}^{N_n} p(v_{1\ell} | u_1),$$

$$P_{3n} = p(u_1, v_{11}).$$

We now directly develop $ED(q_{S}^n|p_{S}^n)$ as follows. Here, for simplicity, we set $p(s) = p_{S}^n(s)$.

$$ED(q_{S}^n|p_{S}^n) = \sum_{s \in S^n} \sum_{i=1}^{L_n} \sum_{j=1}^{N_n} \sum_{u_1 \in U^n} \sum_{v_{ij} \in V^n} P_{1n}P_{2n}P_{3n}$$

$$\cdot \left( \frac{1}{L_n N_n} \sum_{k'=1}^{L_n} \sum_{\ell'=1}^{N_n} W(s|u_{k'}, v_{k'\ell'}) \right)$$

$$\cdot \log \left( \frac{1}{L_n N_n p(s)} \sum_{k'=1}^{L_n} \sum_{\ell'=1}^{N_n} W(s|u_{k'}, v_{k'\ell'}) \right)$$

$$= \sum_{s \in S^n} \sum_{i=1}^{L_n} \sum_{j=1}^{N_n} \sum_{u_1 \in U^n} \sum_{v_{ij} \in V^n} P_{1n}P_{2n}P_{3n}$$

$$\cdot W(s|u_1, v_{11}) \log \left( \frac{1}{L_n N_n p(s)} \sum_{k'=1}^{L_n} \sum_{\ell'=1}^{N_n} W(s|u_{k'}, v_{k'\ell'}) \right),$$

where $(a)$ follows from the symmetry of codes. We decompose the quantities in (149) as

$$\sum_{k'=1}^{L_n} \sum_{\ell'=1}^{N_n} W(s|u_{k'}, v_{k'\ell'}) = A_{1n} + A_{2n} + A_{3n},$$

where

$$A_{1n} = \sum_{k'=2}^{L_n} \sum_{\ell'=1}^{N_n} W(s|u_{k'}, v_{k'\ell'})$$

$$A_{2n} = \sum_{\ell'=2}^{N_n} W(s|u_1, v_{1\ell'})$$

$$A_{3n} = W(s|u_1, v_{11}).$$

Again, from the manner of generating the random code, we see that $A_{1n}$ and $(A_{2n}, A_{3n})$ are independent, whereas $A_{2n}$ and $A_{3n}$ are conditionally independent given $u_1$. Thus,

$$ED(q_{S}^n|p_{S}^n) = \sum_{s \in S^n} \sum_{i=1}^{L_n} \sum_{j=1}^{N_n} \sum_{u_1 \in U^n} \sum_{v_{ij} \in V^n} P_{1n}P_{2n}P_{3n}$$

$$\cdot W(s|u_1, v_{11}) \log \left( \frac{A_{1n} + A_{2n} + A_{3n}}{L_n N_n p(s)} \right)$$

$$\leq \sum_{s \in S^n} \sum_{i=1}^{L_n} \sum_{j=1}^{N_n} \sum_{u_1 \in U^n} \sum_{v_{ij} \in V^n} P_{2n}P_{3n}$$

$$\cdot W(s|u_1, v_{11}) \log \left( \frac{\sum^* A_{1n} + A_{2n} + A_{3n}}{L_n N_n p(s)} \right),$$

where $(b)$ follows from the concavity of the function $x \mapsto \log x$ along with the Jensen’s inequality. Here,

$$\sum^* A_{1n} = \sum_{i=2}^{L_n} \sum_{j=1}^{N_n} \sum_{u_1 \in U^n} \sum_{v_{ij} \in V^n} P_{1n}A_{1n}$$

$$= (L_n - 1) N_n p(s).$$

Hence,

$$ED(q_{S}^n|p_{S}^n) \leq \sum_{s \in S^n} \sum_{i=1}^{L_n} \sum_{j=1}^{N_n} \sum_{u_1 \in U^n} \sum_{v_{ij} \in V^n} P_{2n}P_{3n}$$

$$\cdot W(s|u_1, v_{11}) \log \left( 1 + \frac{A_{2n} + A_{3n}}{L_n N_n p(s)} \right).$$

Moreover,

$$ED(q_{S}^n|p_{S}^n) \leq \sum_{s \in S^n} \sum_{i=1}^{L_n} \sum_{j=1}^{N_n} \sum_{u_1 \in U^n} \sum_{v_{ij} \in V^n} P_{3n}$$

$$\cdot W(s|u_1, v_{11}) \log \left( 1 + \frac{A_{2n} + A_{3n}}{L_n N_n p(s)} \right),$$

where

$$\sum^* A_{2n} = \sum_{i=1}^{L_n} \sum_{j=2}^{N_n} \sum_{u_1 \in U^n} \sum_{v_{ij} \in V^n} P_{2n}A_{2n}$$

$$= (N_n - 1) W(s|u_1).$$
so that, with $0 \leq \rho < 1$,

\[
ED(q_2^* || p_2^*) \leq \sum_{s \in \mathcal{S}^n} \sum_{i=1}^{1} \sum_{u \in \mathcal{U}^n} \sum_{v \in \mathcal{V}^n} P_{3n} \cdot W(s|u_1, v_{11}) \cdot \log \left( \frac{1 + W(s|u_1) + W(s|u_1, v_{11})}{L_n p(s)} \right) = \sum_{s \in \mathcal{S}^n} \sum_{u \in \mathcal{U}^n} \sum_{v \in \mathcal{V}^n} \sum_{u_1 \in \mathcal{U}^n} \sum_{v_{11} \in \mathcal{V}^n} p(u_1, v_{11}) W(s|u_1, v_{11}) \cdot \log \left( \frac{1 + W(s|u_1) + W(s|u_1, v_{11})}{L_n p(s)} \right) = \sum_{s \in \mathcal{S}^n} \sum_{u \in \mathcal{U}^n} \sum_{v \in \mathcal{V}^n} \frac{1}{\rho} p(s, u_1, v_{11}) \cdot \log \left( 1 + \frac{W(s|u_1)}{L_n p(s)} + \frac{W(s|u_1, v_{11})}{L_n p(s)} \right) \leq \sum_{s \in \mathcal{S}^n} \sum_{u \in \mathcal{U}^n} \sum_{v \in \mathcal{V}^n} \frac{1}{\rho} p(s, u_1, v_{11}) \cdot \log \left( 1 + \frac{(W(s|u_1))^{\rho} + (W(s|u_1, v_{11}))^{\rho}}{L_n p(s)} \right) + \sum_{s \in \mathcal{S}^n} \sum_{u \in \mathcal{U}^n} \sum_{v \in \mathcal{V}^n} \frac{1}{\rho} p(s, u_1, v_{11}) \cdot \left( \frac{W(s|u_1, v_{11})}{L_n p(s)} \right)^{\rho} \tag{159} \end{align*}

where (c) follows from $(x + y + z)^\rho \leq x^\rho + y^\rho + z^\rho$; (d) follows from $\log(1 + x) \leq x$. For simplicity, we delete the subscripts “1,11” in (159) and (160) to obtain

\[
F_{1n} \triangleq \sum_{s \in \mathcal{S}^n} \sum_{u \in \mathcal{U}^n} \frac{1}{\rho} p(s, u) \left( \frac{W(s|u)}{L_n p(s)} \right)^{\rho}, \tag{161} \]

\[
F_{2n} \triangleq \sum_{s \in \mathcal{S}^n} \sum_{u \in \mathcal{U}^n} \sum_{v \in \mathcal{V}^n} \frac{1}{\rho} p(s, u, v) \left( \frac{W(s|u, v)}{L_n p(s)} \right)^{\rho}. \tag{162} \]

Hereafter, let us show that $F_{1n} \to 0$, $F_{2n} \to 0$ as $n$ tends to $\infty$ if rate constraints $R_1 > I(U; S), R_1 + R_2 > I(UV; S)$ are satisfied. First, let us show $F_{2n} \to 0$. Since $p(s, u, v) = p(u, v)W(s|u, v)$, $F_{2n}$ can be rewritten as

\[
F_{2n} = \frac{1}{\rho (L_n N_n)^\rho} \sum_{s \in \mathcal{S}^n} \sum_{u \in \mathcal{U}^n} \sum_{v \in \mathcal{V}^n} p(u, v)W(s|u, v)^{1+\rho}p(s)^{-\rho}. \tag{163} \]

On the other hand, by virtue of Hölder’s inequality,

\[
\left( \sum_{(u,v) \in \mathcal{U}^n \times \mathcal{V}^n} p(u,v)W(s|u, v)^{1+\rho}p(s)^{-\rho} \right)^{\frac{1}{\rho+1}} \leq \left( \sum_{(u,v) \in \mathcal{U}^n \times \mathcal{V}^n} p(u,v)W(s|u, v) \right)^{\frac{1}{\rho+1}} \left( \sum_{(u,v) \in \mathcal{U}^n \times \mathcal{V}^n} p(s)^{-\frac{\rho}{\rho+1}} \right)^{\frac{\rho+1}{\rho}} \leq \left( \sum_{(u,v) \in \mathcal{U}^n \times \mathcal{V}^n} p(u,v)W(s|u, v) \right)^{\frac{1}{\rho+1}} \left( \frac{1}{\rho} \right)^{\frac{\rho}{\rho+1}} \tag{164} \]

for $0 < \rho < 1$. Therefore, it follows from (163) that

\[
F_{2n} \leq \frac{1}{\rho (L_n N_n)^\rho} \sum_{s \in \mathcal{S}^n} \left( \sum_{(u,v) \in \mathcal{U}^n \times \mathcal{V}^n} p(u,v)W(s|u, v) \right)^{\frac{1}{\rho+1}} \leq \frac{1}{\rho} \exp \left[ -n \rho (R_1 + R_2) + E_0(\rho, p) \right], \tag{165} \]

where

\[
E_0(\rho, p), \tag{166} \]

Then, by means of Gallager [27, Theorem 5.6.3], we have $E_0(\rho, p)|_{p=0} = 0$ and

\[
\frac{\partial E_0(\rho, p)}{\partial \rho} \bigg|_{\rho=0} = -I(p, W) = -I(UV; S) \tag{167} \]

where (e) follows because $(UV; S)$ is a correlated i.i.d. sequence with generic variable $(UV; S)$. Thus, for any small constant $\tau > 0$ there exists a $\rho_0 > 0$ such that, for all $0 < \rho \leq \rho_0$,

\[
E_0(\rho, p) \geq -n \rho(1 + \tau)I(UV; S) \tag{168} \]

which is substituted into (165) to obtain

\[
F_{2n} \leq \frac{1}{\rho} \exp \left[ -n \rho (R_1 + R_2 - (1 + \tau)I(UV; S)) \right]. \tag{169} \]

On the other hand, in view of rate constraint $R_1 + R_2 > I(UV; S)$, with some $\delta > 0$ we can write

\[
R_1 + R_2 = I(UV; S) + 2\delta, \tag{170} \]

which leads to

\[
R_1 + R_2 - (1 + \tau)I(UV; S) = I(UV; S) + 2\delta - I(UV; S) - I(UV; S) = 2\delta - \tau I(UV; S). \tag{171} \]
We notice here that \( \tau > 0 \) can be arbitrarily small, so that the last term on the right-hand side of (171) can be made larger than \( \delta > 0 \). Then, (169) yields
\[
F_{2n} \leq \frac{1}{\rho} \exp[-n\rho \delta],
\]
which implies that with any small \( \varepsilon > 0 \) it holds that
\[
F_{2n} \leq \varepsilon
\]
for all sufficiently large \( n \).
Similarly, \( F_{1n} \leq \varepsilon \) with rate constraint \( R_1 > I(U; S) \) can also be shown.
Thus, the proof of Lemma 1 has been completed.

\[\square\]

**Appendix B**
**Proof of Remark 8**

For simplicity, set the right-hand sides of (78), (79), (91) and (92) as
\[
M_1 = \max_{p_{PV}} \min_{p_{PU}} (I(SV; Y) - I(SV; Z),)
\]
\[
K_1 = \max_{p_{PV}} \min_{p_{PU}} (I(SV; Y) - H(S)),
\]
\[
M_2 = \max_{p_{PU}} \min_{p_{PV}} H(S|UZ) - H(S|UY),
\]
\[
K_2 = \max_{p_{PU}} \min_{p_{PV}} H(S|UZ) - H(S|UY),
\]
which, after some calculation with \( Y \) replaced by \( SY \), leads, respectively, to
\[
M'_1 = \max_{p_{PV}} \min_{p_{PU}} (I(V; Y|S) - I(V; Z|S) + H(S|Z),
\]
\[
K'_1 = \max_{p_{PV}} \min_{p_{PU}} (I(V; Y|S) - I(V; Z|S) + H(S|Z)),
\]
\[
M'_2 = \max_{p_{PV}} \min_{p_{PU}} H(S|UZ),
\]
\[
K'_2 = \max_{p_{PV}} \min_{p_{PU}} H(S|UZ) = \max_{p_{PSX}} H(S|Z).
\]
Notice here that (174) \( \sim \) (177) give lower bounds for \( C_{CSLE} \) (with CSI \( S \) available only at Alice), whereas (178) \( \sim \) (181) give lower bounds for \( C_{CSLED} \) (with CSI \( S \) available at both Alice and Bob).

1) First, consider the reversely degraded binary WTC as in Fig 6 with \( W(y, z|x, s) = W(y, z|x) \) with binary entropy \( H(S) = 1 - h(0.2) < 1 - h(0.1) \). It is easy to check that \( I(SV; Y) - I(SV; Z) \) in (175) can be rewritten as
\[
I(SV; Y) - I(SV; Z) = (I(V; Y) - I(V; Z)) + H(S|UZ) - H(S|UY).
\]
Suppose here that \( I(V; Y) = 0 \), then
\[
I(SV; Y) = I(S; Y|V) = H(S|V) - H(S|VY),
\]
from which, together with the constraint \( I(SV; Y) \geq H(S) \) in (178), it follows that \( H(S|V) - H(S|VY) \geq H(S), \) i.e.,
\[
-H(S|VY) \geq I(S; V) = 0 \quad \text{(owing to the independence of} \ S \text{ and} \ V) \quad \text{and hence} \quad H(S|VY) = 0 \quad \text{on the other hand, in view of the Markov chain property} \ SV \to Z \to Y \quad \text{(due to the reverse degradedness) as well as the independence of} \ S \text{ and} \ V, \text{it must hold that} \ H(S|VY) > 0 \text{ in that we are here considering the causal WC with CSI} \ S \text{ available only at Alice. This is a contradiction. Thus, it should hold that} \ H(V; Y) = 0 \quad \text{for all} \ V \quad \text{satisfying the constraint} \ I(SV; Y) \geq H(S) \quad \text{and hence} \quad H(V; Y) \geq c_0 \quad \text{for some} \ c_0 > 0 \quad \text{and for all} \ V \quad \text{satisfying the constraint. Furthermore, we can show also that} \ I(V; Z) - I(V; Y) \geq I(V; Z|Y) > 0. \text{To see this, assume} \ I(V; Z|Y) = 0 \quad \text{to lead to a contradiction. Then, it is easy to check that} \ I(V; Z|Y) = 0 \quad \text{together with the reverse degradedness implies that} \ V \text{ and} \ ZY \text{ are independent and hence particularly} \ I(V; Y) = 0, \quad \text{which is a contradiction. Thus,} \ I(V; Y) - I(V; Z) \leq -d_0 \quad \text{for some} \ d_0 > 0 \quad \text{and for all} \ V \quad \text{satisfying the constraint, which, together with} \ (175), \quad (177) \quad \text{and} \quad (182), \quad \text{implies that}
\]
\[
K_1 < K_2,
\]
where we have taken account that the constraint in (175) is tighter than that in (177).

![Fig. 6. Reversely degraded binary WTC with causal CSI S available only at Alice.](image)

2) Now consider the reversely degraded binary WTC as in Fig 7 with \( W(y, z|x, s) = W(y, z|x) \). Then, setting \( U = X \) (Pr\{X = 1\} = Pr\{X = 0\} = 1/2) independently of \( S \) with binary entropy \( H(S) = 1 - h(0.1) \) in (180), we have
\[
M'_2 \geq 1 - h(0.1) > M'_1,
\]
where the first inequality in (185) follows directly by observing that in this case \( H(S|UZ) = I(U; Y|S) = 1 - h(0.1) \) and the second inequality in (185) can be verified as follows. Suppose otherwise, i.e.,
\[
1 - h(0.1) \leq M'_1,
\]
which then, together with (178), means that there exists some \( VX \) such that
\[
I(V; Y|S) \geq 1 - h(0.1),
\]
\[
I(V; Y|S) - I(V; Z|S) + H(S) \geq 1 - h(0.1).
\]
On the other hand, (187) implies that it must hold that $V = X$ ($\Pr\{X = 1\} = \Pr\{X = 0\} = 1/2$) independently of $S$ with $H(S) = 1 - h(0.1)$. Thus, in view of (188) with this $V = X$, we must have

$$I(X; Y|S) - I(X; Z|S) + H(S) \geq 1 - h(0.1),$$

which, together with $I(X; Y|S) = 1 - h(0.1), I(X; Z|S) = 1$, means that

$$-h(0.1) + H(S) \geq 1 - h(0.1),$$

that is,

$$H(S) = 1 - h(0.1) \geq 1,$$

which is a contradiction, thus establishing the second inequality in (188).

Fig. 7. Reversely degraded binary WTC with causal CSI $S$ available at Alice and Bob.

3) We next consider the degraded binary WTC as in Fig. 8 with $W(y, z|x, s) = W(y, z|x)$ and $S = \emptyset$. Let $\Pr\{X = 1\} = \Pr\{X = 0\} = 1/2$. Then, since $S = \emptyset$, (178) and (180) are evaluated as

$$M'_1 = \max_{p_V} \min_v \left( I(V; Y) - I(V; Z), (I(V; Y)) \right),$$

$$= \max_{p_V} \left( I(V; Y) - I(V; Z) \right) \geq I(X; Y) - I(X; Z) = h(0.1),$$

$$M'_2 = 0.$$

Hence,

$$M'_1 > M'_2.$$  \hfill (194)

Similarly, again by letting $\Pr\{X = 1\} = \Pr\{X = 0\} = 1/2$ and taking account of $S = \emptyset$, we see that (179) reduces to

$$K'_1 \geq \max_{p_V} \left( I(V; Y) - I(V; Z) \right) \geq I(X; Y) - I(X; Z) = h(0.1) > 0 = K'_2,$$

that is,

$$K'_1 > K'_2.$$  \hfill (195)

Finally, summarizing up (184), (185), (194) and (195), the claim of Remark 8 has been proved.

Fig. 8. Degraded binary WTC without CSI at Alice and Bob.

ACKNOWLEDGMENTS

The authors are grateful to Hiroyuki Endo for useful discussions. Special thanks go to the reviewers for their stimulating comments which have occasioned to largely improve the quality of the earlier version. This work was funded by JSPS KAKENHI Grant Number 17H01281, and partly supported by “Research and Development of Quantum Cryptographic Technologies for Satellite Communications (JPJ007462)” of Ministry of Internal Affairs and Communication (MIC), Japan.

REFERENCES

[1] A. D. Wyner, “The wire-tap channel,” Bell Syst. Tech. J., vol.54, pp.1355-1387, 1975
[2] I. Csiszár and J. Körner, “Broadcast channels with confidential messages,” IEEE Transactions Information Theory, vol.24, no.3, pp.339-348, 1978
[3] C. E. Shannon, “Communication theory of secrecy systems,” Bell Syst. Tech. J., vol. 28, pp.656-715, 1949
[4] C. E. Shannon, “Channels with side information at the transmitter,” IBM J. Tech. Develop., vol. 2, no. 4, pp. 289-293, 1958
[5] C. Mirpant, A. J. H. Vink and Y. Luo, “An achievable region for the Gaussian wiretap channel with side information,” IEEE Transactions on Information Theory, vol. 52, no. 5, pp. 2181-2190, 2006
[6] Y. Chen and A. J. H. Vink, “Wiretap channel with side information,” IEEE International Symposium on Information Theory, Seattle, USA, July 2006; IEEE Transactions on Information Theory, vol. 54, no. 1, pp. 395-402, 2008
[7] W. Liu and B. Chen, “Wiretap channel with two-sided channel state information,” 41st Asilomar Conference on Signals, Systems and Computation, November, 2007
[8] B. Dai, Z. Zhuang and A. J. H. Vink, “Some new results on the wiretap channel with causal side information,” Proc. IEEE. ICCT, Chengdu, China, pp. 609-614, Nov. 2012
[9] H. Boche and R. F. Schaefer, “Wiretap channels with side information-strong secrecy capacity and optimal transceiver design,” IEEE Transactions on Information Forensics and Security, vol. 8, no. 8, pp. 1397-1408, 2013
[10] A. Khisti, S. Diggavi and G. Wornell, “Secret Key Agreement Using Asymmetry in Channel State Knowledge,” Proc. IEEE International Symposium on Information Theory, pp. 2286-2290, Seoul, 2009
[11] A. Khisti, S. Diggavi and G. Wornell, “Secret-key agreement with channel state information at the transmitter,” IEEE Transactions on Information Forensics and Security, no.3, vol.6, pp.672-681, 2011
[12] Y. K. Chia and A. El Gamal, “Wiretap channel with causal state information,” IEEE Transactions on Information Theory, vol.IT-50, no.5, pp.2838-2849, 2012
[13] H. Fujita, “On the secrecy capacity of wiretap channels with side information at the transmitter,” IEEE Transactions on Information Forensics and Security, vol.11, no.11, pp.2441-2452, 2016
T. S. Han and M. Sasaki, “Wiretap channels with causal state information: strong secrecy,” IEEE Transactions on Information Theory, vol. 65, no. 10, pp. 6750-6765, 2019

T. S. Han, H. Endo and M. Sasaki, “Wiretap channels with one-time state information: strong secrecy,” IEEE Transactions on Information Forensics and Security, vol. 13, no. 1, pp. 224-236, 2018

R. Ahlswede and I. Csiszár, “Common randomness in information theory and cryptography–Part II: CR capacity,” IEEE Transactions on Information Theory, vol. 44, no. 1, pp. 225-240, 1998

V. M. Prabhakaran, K. Eswaran and K. Ramchandran, “Secrecy via Sources and Channels,” IEEE Transactions on Information Theory, vol. 58, no. 11, pp. 6747-6765, 2012

B. Dai and Y. Luo, “Some new results on the wiretap channel with side information,” Entropy, vol. 14, no. 9, pp. 1671-1702, 2012.

A. El Gamal and Y. H. Kim, Network Information Theory, Cambridge University Press, New York, 2011.

E. Song, P. Cuff and V. Poor, “The likelihood encoder for lossy compression,” IEEE Transactions on Information Theory, vol. 62, no. 4, pp. 1836-1849, 2016

S. I. Gelfand and M. S. Pinsker, “Coding for channel with random parameters,” Problems of Control and Information Theory, vol. 9, no. 1, pp. 19-31, 1980

P. Cuff, “Strong soft-covering and applications,” arXiv:1508.01603v1, 2015

Z. Goldfeld, P. Cuff, and H. H. Permuter, “Wiretap channel with random states non-causally available at the encoder,” IEEE Transactions on Information Theory, vol. 66, no. 3, pp. 1497-1519, 2020

A. Bunin, Z. Goldfeld, H. Permuter, S. Shamai, P. Cuff and P. Piantanida, “Semantically-secured message-key trade-off over wiretap channels with random parameters,” Proc. of the 2nd Workshop on Communication Security, pp. 33-48, 2018.

A. Bunin, Z. Goldfeld, H. Permuter, S. Shamai, P. Cuff and P. Piantanida, “Key and message semantic-security over state-dependent channels,” IEEE Transactions on Information Forensics and Security, vol. 15, pp. 1541-1556, 2020

I. Csiszár and J. Körner, Information Theory: Coding Theorems for Discrete Memoryless Systems, 2nd ed., Cambridge University Press, 2011

R. Gallager, Information Theory and Reliable Communication, John Wiley & Sons, NJ, 1968

T. S. Han and S. Verdú, “Approximation theory of output statistics,” IEEE Transactions on Information Theory, vol. 39, no. 3, pp. 752-772, 1993

A. Zibaeenejad, “Key Generation Over Wiretap Models With Non-Causal Side Information,” IEEE Transactions on Information Forensics and Security, vol. 10, no. 7, pp. 1456-1471, 2015

Syed Ali Jafar, “Capacity with causal and non-causal side information: A unified view,” IEEE Transactions on Information Theory, vol. 52, no. 12, pp. 5468-5474, Dec. 2006

Te Sun Han (M’80-SM’88-F’90-LF’11) received the B.Eng., M.Eng., and D.Eng. degrees in mathematical engineering from the University of Tokyo, Japan, in 1964, 1966, and 1971, respectively. Since 1993, he has been a Professor with the University of Electro-Communications, where he has been a Professor Emeritus since 2007. He has published papers on information theory, most of which appeared in the IEEE TRANSACTIONS ON INFORMATION THEORY. Also, he has published two books: one of them is Information-Spectrum Methods in Information Theory (Springer Verlag, 2003). Especially, this book was written to try to demonstrate part of the general logics latent in information theory. His research interests include basic problems in Shannon theory, multi-user source/channel coding systems, multiterminal hypothesis testing and parameter estimation under data compression, large-deviation approach to information-theoretic problems, and especially, information spectrum theory. He is a member of the Board of Governors for the IEEE Information Theory Society. He has been an IEICE Fellow and an Honorary Member since 2011. He was a recipient of the 2010 Shannon Award.

Masahide Sasaki received the B.S., M.S., and Ph.D. degrees in physics from Tohoku University, Sendai Japan, in 1986, 1988 and 1992, respectively. During 1992–1996, he worked on the development of semiconductor devices in Nippon-Kokan Company (currently JFE Holdings). In 1996, he joined the Communications Research Laboratory, Ministry of Posts and Telecommunications (since 2004, National Institute of Information and Communications Technology (NICT), Ministry of Internal Affairs and Communications). He has been working on quantum optics, quantum communication and quantum cryptography. He is presently Distinguished Researcher of Advanced ICT Research Institute, and NICT Fellow. Dr. Sasaki is a member of Japanese Society of Physics, and the Institute of Electronics, Information and Communication Engineers of Japan.