Backstepping Control of Rewinding Systems with Flexible Couplings

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Abstract Roll-to-roll (R2R) system can be found and applied in various fields, such as paper manufacture and printing process. The matter regarding tension control is still a challenge because this tends to rely on elasticity of web materials with its elasticity and roll inertia which change. Additionally, another issue is limited stiffness shafts connecting the web handling system and driving motor. Due to non-rigid nature of the shafts, unwanted vibration can be released, and this impact to the system. Therefore, the tension control is more complicated. In this paper, a control solution for a roll-to-roll web-transport system with flexible couplings will be performed. The connection of this system coupled with motors through non-rigid driving shaft is formed as a two-mass system. Moreover, the paper shows that the thickness of web material is assumed to be conscious in order to achieve a simple calculation of roll inertia momentum. By using backstepping control technique, control laws for unwinding and rewinding roll are defined. The control schemes ensure a linear velocity and tension tracking performance of the system. Extensive numerical simulations are carried out to verify the feasibility of the proposed control strategy. The simulation results indicate the effectiveness of the proposed control, web tension and position are driven to desired values.

Keywords Backstepping, Flexible Couplings, Roll-to-roll System, Two Mass System

1. Introduction

Roll-to-roll processing, or R2R, is a fabrication method used in manufacturing that embeds, coats, prints or laminates varying applications onto a flexible rolled substrate material [1], [2], and [3]. The material is a substrate-based manufacturing technology involving continuous processing of a flexible and extensible substrate (web) as it is transferred from one roller on to another [4]. General structure of system contains two main parts which are an unwinder and a rewinder [5]. For R2R structure, the control of web tension and web transport velocity are vitally important and needs to be taken into account [6]. In order to create a suitable controller for this system, a mathematical model and a proper control need to be obtained.

Tension control is essential in web-handling systems. Low tension lead to web winkle and non-compact rewinding web roll. Meanwhile, over-tension cause web breakage resulting in serious production efficiency and financial problems. Classical PID control is widely used in tension regulation [7] and [8], however the approach presents several drawbacks in dealing with nonlinear systems with varying parameters such as the web-handling system. Nonideal effects on web tension were studied, based on the models developed, methods for the control of tension. Recently, many researchers have focused on the control law for this drive system [9] and [10]. One of typical control laws based on back-stepping technique for R2R systems is proposed in [11] and [12].

A number of new ideas, approaches, and results have been extensively developed during the past few years related to nonlinear models have been employed for R2R systems [13]–[15] however, a wide range of system nonlinearities are ignored [16] and [17]. Normally, the shaft connecting a drive motor and a load machine is considered to be rigid when designing the industrial drive of R2R system [18]–[20]. This omission is acceptable in case of the small roll inertia and the short shaft, but not in the cases that characteristics of mechanical parts are considered. In fact, a torsional vibration is often generated when the motor and roll are connected with the flexible shaft. Neglecting shaft elasticity in control designs might lead to dramatic oscillations [21] and [22]. As a result, the stability of the R2R systems will be affected or the
mechanical coupling between the driven and loading machine could be destroyed.

The work aims at developing a model of the R2R web-transport system with flexible couplings. The obtained mathematical model used to design the control law for web position in a comprehensive procedure. Based on the backstepping method, this paper shows a simple approach that can be separately applied to the unwinder and rewinder.

2. Dynamics of R2R System with Flexible Couplings

As shown in Fig. 1, a typical structure of R2R system contains an unwinder, a rewinder, a load cell subsystem with idle rollers and two dancer subsystems.

![Figure 1. Structure of rewinding system](image)

The idle rollers guide the moving web around the load cell in a fixed angle. The unwinder and rewinder are controlled with the different motors. By controlling $M_u$ and $M_r$ applied to the unwind and rewind rolls, the web tension, angular velocity and rotational angular of the system can be controlled. By applying Newton’s law and the principle of mass conservation [5], the non-linear dynamic equations of rewinding system can be given as follows:

$$\dot{\omega}_u = \frac{1}{J_u} M_u + \frac{R_u}{J_u} T - \frac{c_u}{J_u} \omega_u$$ (1)

$$\dot{\omega}_r = \frac{1}{J_r} M_r - \frac{R_r}{J_r} T - \frac{c_r}{J_r} \omega_r$$ (2)

$$\ddot{T} = \frac{(T + ES)R_r}{L} \omega_r - \frac{(2T + ES)R_u}{L} \omega_u$$ (3)

Where angular velocities and radiuses are calculated by the following equations:

$$\theta_u = \int_0^t \omega_u$$ (4)

$$\theta_r = \int_0^t \omega_r$$ (5)

$$R_u = R_{u0} - h \frac{\theta_u}{2\pi}$$ (6)

$$J_u = J_{u0} + \pi \rho \frac{(R_u^4 - R_{u0}^4)}{4}$$ (7)

$$J_u = J_{u0} + \pi \rho \omega_u$$ (8)

$$J_r = J_{r0} + \pi \rho \omega_r$$ (9)

Detailed explanation of the system parameters is given in Table 1 as follows.

| Parameters | Units |
|------------|-------|
| $R_u$ | Operating radius of the unwind roll | m |
| $R_{u0}$ | Initial radius of the unwind roll | m |
| $R_r$ | Operating radius of the rewinding roll | m |
| $R_{r0}$ | Initial radius of the rewinding roll | m |
| $J_u$ | Total moment of inertia of the unwind roll | kgm$^2$ |
| $J_{u0}$ | Initial total moment of inertia of the unwind roll | kgm$^2$ |
| $J_r$ | Total moment of inertia of the rewinding roll | kgm$^2$ |
| $J_{r0}$ | Initial total moment of inertia of the rewinding roll | kgm$^2$ |
| $\omega_u$ | Angular velocity of the unwind roll | rad/s |
| $\theta_u$ | Rotational angular of the unwind roll | rad |
| $\omega_r$ | Angular velocity of the rewinding roll | rad/s |
| $\theta_r$ | Rotational angular of the rewinding roll | rad |
| $c_u$ | Coefficient of viscous friction of the unwind roll | Nms |
| $c_r$ | Coefficient of viscous friction of the rewinding roll | Nms |
| $M_u$ | Torque applied to the unwind roll | Nm |
| $M_r$ | Torque applied to the rewind roll | Nm |
|------|----------------------------------|----|
| $T$  | Web tension                      | N  |
| $L$  | Total length of web              | m  |
| $E$  | Elasticity of web                | N/m² |
| $S$  | Cross sectional area of web      | m² |
| $h$  | The thickness of web             | m  |
| $w$  | The width of web                 | m  |
| $\rho$ | The density of web               | kg/m³ |

As shown in Fig. 2, a schematic of a two-mass resonant system consisting of two lumped inertias $J_{mu}$ and $J$ for motor and load, respectively, which are coupled via a shaft with finite stiffness $k_s$.

![Diagram of R2R system with flexible coupling](image)

**Figure 2.** A R2R system with flexible coupling

Since the unwinder and rewinder are driven with different flexible coupling, the following relationships can be obtained according to (1) and (2) by applying the law of energy conservation along with the assumption of the motors are ideal torque generating actuators:

\[
\dot{\omega}_{mu} = \frac{1}{J_{mu}} M_{mu} + \frac{1}{J_{mu}} M \mu - \frac{c_{mu}}{J_{mu}} \omega_{mu} \quad (10)
\]

\[
\dot{\omega}_{mr} = \frac{1}{J_{mr}} M_{mr} - \frac{1}{J_{mr}} M \mu - \frac{c_{mr}}{J_{mr}} \omega_{mr} \quad (11)
\]

\[
M_u = k_{mu} (\theta_{mu} - \theta_u) + c_{mu} (\omega_{mu} - \omega_u) \quad (12)
\]

\[
M_r = k_{sr} (\theta_{mr} - \theta_r) + c_{sr} (\omega_{mr} - \omega_r) \quad (13)
\]

Where the system parameters are given in Table 2.

| Table 2. Flexible coupling system parameters
|---------------|--------------|------------|
| Parameters    | Units        |
| $k_{mu}$      | Shaft stiffness of the unwind motor | Nm/rad    |
| $k_{sr}$      | Shaft stiffness of the rewind motor | Nm/rad    |
| $c_{mu}$      | Coefficient of vicious friction of the unwind motor | Nms/rad    |
| $c_{sr}$      | Coefficient of vicious friction of the rewind motor | Nms/rad    |
| $\theta_{mu}$ | Unwind motor Rotational angular | rad |
| $\omega_{mu}$ | Rotational angular of the rewinder | rad/s |
| $\theta_{mr}$ | Rotational angular of the rewinder | rad/s |
| $\omega_{mr}$ | Angular velocity of the rewinder | rad/s |
| $M_{mu}$      | Control torque generated by the motor at unwinder | Nm |
| $M_{mr}$      | Control torque generated by the motor at rewinder | Nm |

The aforementioned problems will impact on the web tension directly. If the web tension does not meet the requirements, the paper will not be stretched, and the printing work cannot be implemented. In addition, this might cause wrinkles on the paper. By contrast, if the web tension is excessive, the paper might be broken and create a crack. Several parameters that will impact the web tension are the speed and the angle of rotation. If the speed is too slow, the working time takes longer, consuming more energy. On the contrary, the fast speed will cause the very high kinetic energy which might damage the systems. If the accuracy of the angle of rotation is not sufficient, printing errors will occur. Therefore, in order to maintain the stability of the system, the web tension, the speed and the angle of rotation have to track the reference values with short transient time and small overshoot.
3. Backstepping Controller Design

3.1. Backstepping Controller Design of Rewinding System

The widening role is to control the web position, the system with backstepping control structure (BSC) of rewinder is demonstrated in Fig. 3. A backstepping control strategy to control the position of roll, each step includes the dynamics of the previous subsystems for backstepping technique:

\[
\begin{align*}
\dot{x}_4 &= \theta_{nr} \\
x_3 &= M_r \\
x_2 &= \omega_r \\
x_1 &= \omega_t
\end{align*}
\]

First step

Second step

Third step

Fourth step

**Figure 3.** Backstepping control structure for the rewinder

The main objective of this step is to design the control law to ensure position tracking performance in the face of flexible couplings. The following equations are derived according to the step-by-step fashion in formulating the BSC of the system. This can be written by the set of equations as follows [8].

\[
\begin{align*}
\dot{\theta}_r &= \omega_r \\
\dot{\omega}_r &= \left( \frac{R}{J_r} T - \frac{c_r}{J_r} \omega_r \right) + \frac{1}{J_r} M_r \\
M_r &= -k_{sr} \omega_r + c_{sr} (\omega_{nr} - \omega_r) + k_r \omega_{nr} \\
\dot{\omega}_{nr} &= \left( -\frac{1}{J_{nr}} M_r - \frac{c_{nr}}{J_{nr}} \omega_{nr} \right) + \frac{1}{J_{nr}} M_{nr}
\end{align*}
\]

Based on a virtual control laws derived in each step for the rewinding system, the equations (14) can be rewritten as follows:

\[
\begin{align*}
\dot{\omega}_r^* &= -c_1 (\theta_r - \theta_{nr}) \\
M_r^* &= J_r (-c_2 (\omega_r - \omega_r^*) + \frac{R}{J_r} T + \frac{c_r}{J_r} \omega_r) \\
&+ \dot{\omega}_r^* - \frac{1}{b_2} (\theta_r - \theta_{nr}) \\
\omega_{nr}^* &= \frac{1}{k_{sr}} (-c_3 (M_r - M_r^*) + k_r \omega_r) \\
+c_2 (\omega_{nr} - \omega_r^*) + M_r^* - \frac{1}{b_3 J_r} (\omega_r - \omega_r^*)) \\
M_{nr}^* &= J_{nr} (-c_4 (\omega_{nr} - \omega_r) + \frac{1}{J_{nr}} M_{nr} \\
&+ \frac{c_{nr}}{J_{nr}} \omega_{nr} - \frac{k_r}{b_4} (M_r - M_r^*))
\end{align*}
\]

In the case that \( \omega_r^* = -c_1 (\theta_r - \theta_{nr}) < \omega_{nr} \), control functions for rewinding system can be written as follows:

\[
\begin{align*}
\dot{\omega}_r &= -c_1 \omega_r \\
\dot{\omega}_r^* &= -c_2 \omega_r \\
\dot{\omega}_{nr}^* &= -c_3 \omega_r \omega_{nr}
\end{align*}
\]
In the case that \( > \), control laws for rewinding system can be derived as follows:

\[
\begin{align*}
M^*_{mr} &= J_m (-c_4 (\omega^*_{mr} - \omega_r) + \frac{1}{J_m} M^*), \\
\dot{M}^*_{mr} &= J_m \left[ -c_2 (\omega_r - \omega_{mr}) + \frac{1}{J_m} \frac{T}{J_r} + c_0 \omega_r \right] \\
\dot{\omega}^*_{mr} &= \frac{1}{k_{sr}} (-c_3 (M_r - M^*) + k_{sr} \omega_r) \\
-\dot{c}_4 & (\omega^*_{mr} - \omega_r) + \dot{M}^* + \frac{1}{J_m} (\omega_r - \omega^*_r) \\
\dot{M}^* &= J_m (-c_4 (\omega^*_{mr} - \omega_r) + \frac{1}{J_m} M_r) \\
+ \frac{c_{mr}}{J_m} \omega_{mr} + \dot{\omega}_{mr} - \frac{k_{sr}}{b_4} (M_r - M^*).
\end{align*}
\]

Where \( c_1, c_2, c_3, c_4 \) are the positive gains.

### 3.2. Backstepping Controller Design of Unwinding System

Different from rewinding section, the unwinding roll is in charge of tension regulation. A BSC strategy to control the web position of the R2R, BSC applied to the unwinder is illustrated in Fig. 4.

\[
\begin{align*}
\dot{\omega}^*_u &= -\frac{L}{(2T + ES)R_u} \left[ -c_1 (\omega_u - \omega^*_u) - (2T + ES)R_u \frac{\omega}{L} \right] \\
\dot{\omega}^*_u &= \left[ -c_2 (\omega_u - \omega^*_u) - \frac{R_u T}{J_u} + c_0 \omega_u \right] + \frac{1}{J_u} M_u \\
\dot{\omega}^*_{mu} &= \left[ -\frac{1}{J_{mu}} M_u - \frac{c_{mu}}{J_{mu}} \omega_{mu} \right] + \frac{1}{J_{mu}} M_{mu}
\end{align*}
\]

Where \( c_5, c_b, c_7, c_b \) are the positive gains.

According to the control steps in Fig. 4, the following equations are derived as

\[
\begin{align*}
M^* &= J_u \left[ \frac{L}{(2T + ES)R_u} - \frac{(2T + ES)R_u}{L} \right] \\
\dot{\omega}^*_u &= \left[ \frac{1}{L} \left( \frac{(2T + ES)R_u}{L} \right) \right] \\
\dot{\omega}^*_u &= \left[ \frac{1}{L} \left( \frac{(2T + ES)R_u}{L} \right) \right]
\end{align*}
\]
3.3. Determination of Optimal Controller Parameters

For the same class, parameters b and c can be chosen as follows:

\[ b_2 = 16, \quad b_3 = 4.10^{-6}, \quad b_4 = 2.5.10^4, \]
\[ b_5 = 9.10^7, \quad b_6 = 4.94x10^{-6}, \quad b_7 = 2.3x10^9, \quad c_1 = 8, \]
\[ c_2 = 60, \quad c_3 = 101, \quad c_4 = 137, \quad c_5 = 29, \quad c_6 = 128, \]
\[ c_7 = 157, \quad c_8 = 195. \]

Where \( c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8 \) are the positive gains and determined by the modified genetic algorithm [16].

4. Simulation Results

The simulation reference values of the rewinding control system are shown in Table 3. The goal is to design the precise control algorithm and BSC that is required to keep the web tension, rotational angular and angular velocity at prescribed reference values, to satisfy the performance specifications and to obtain the high stability.

In order to test the proposed algorithm in the operating condition of the model R2R with flexible couplings. The change of angular of the rewind roll is assumed in the Fig.5 corresponding to the web speed position.

| Table 3. Simulation parameters |
|--------------------------------|
| Parameters | Values | Units |
| The width of web | 1 | m |
| The thickness of web | 0.0002 | m |
| Total length of web | 8 | m |
| Elasticity of web | 2.5.10^9 | N/m² |
| Operating radius of the unwind roll | 1.5 | m |
| Operating radius of the rewind roll | 0.5 | m |
| Total moment of inertia of the unwind roll | 6400 | Kgm² |
| Total moment of inertia of the rewind roll | 160 | Kg.m² |
| Coefficient of vicious friction of the unwind roll | 1 | Nms |
| Coefficient of vicious friction of the rewind roll | 50 | Nms |
| Shaft stiffness of the unwind motor | 10⁴ | Nm |
| Shaft stiffness of the rewind motor | 10⁴ | Nm |

![Figure 5. Angular response of the rewind roll](image)
Fig. 6 shows the tracking performance of the web tension, in which the time response of web tension with BSC yields the peak overshoot of 1.2% and the fast response with settling time of 0.15s. Although the overshoot of 2.4% is observed again during the deceleration, it is still in an acceptable range.

By contrast, it can be seen from Fig. 7 that the angular speed of unwinding and rewinding section is kept stable at the reference value without overshoot with respect to the position reference of the web.
Figure 8. Rotational angular responses of the rewind motor and the rewind roll.

Figure 9. Rotational angular responses of the unwind motor and the unwind roll.

Figure 10. Torque responses of the rewind motor and the rewind roll.
Fig. 8 and 9 indicate the angular speed at motor and rewinding-unwinding rolls sides. The speed difference in between the two sides show the effects of the flexible couplings on the system. The flexibility again is clearly demonstrated in viewing applied torques on motor and rolls as can be seen in Figure 10 and 11. The difference in speed and torque responses is eliminated with the system possessing rigid couplings.

5. Conclusions

In this paper, the mathematical model of the roll to roll system by taking flexible couplings into account has been systematically developed. The control of position and tension of the moving web have been considered to enhance the production quality of the system. Notice that the backstepping controller was proposed for the control algorithm of the attained model and verification of the proposed control strategy was examined by simulation results. The results show that the precision and the stability of the system were achieved with high dynamics performance and negligible overshoots.

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