The Isoscalar Mesons and Exotic States in Light Front Holographic QCD

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Abstract

In this article a systematic quantitative analysis of all isoscalar bosonic states is performed in the framework of supersymmetric light front holographic QCD. It is shown that the spectra of the $\eta$ and $h$ mesons can be well described if one additional parameter, which corresponds to a hard breaking of chiral $U(1)$ symmetry in standard QCD, is introduced. The mass difference of the $\eta$ and $\eta'$ isoscalar mesons is determined by the strange quark mass content of the $\eta'$. Several very probable candidates for isoscalar tetraquarks, the existence of which is a consequence of the theory, are identified. In particular, the $\eta(1475)$ and $f_0(1500)$ are identified as isoscalar tetraquarks; their predicted mass values 1.51 GeV and 1.52 GeV, respectively, agree with the experimental values within the model uncertainties.

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I. INTRODUCTION

In a series of recent papers [1–7] it was shown that Light-Front Holographic QCD (LFHQCD)[8–10], especially after implementation of superconformal algebra [1, 2] (supersymmetric LFHQCD) can explain many spectroscopic and dynamical features of the observed hadrons, thus providing nontrivial connections between the hadron spectrum and structures of form factors and quark distributions [11, 12].

LFHQCD, as other holographic approaches to QCD, is inspired by the Maldacena conjecture [13]: a weakly coupled classical 5-dimensional gravitational theory in a space with anti-de Sitter (AdS) metric is equivalent to a strongly coupled 4-dimensional quantum gauge field theory defined at the asymptotic space-time boundary of AdS\(_5\). This 4-dimensional field theory is a superconformal gauge theory in the limit of \(N_C \to \infty\) colors. A crucial feature of LFHQCD is the remarkable correspondence between the field of the 5-dimensional theory and those of the 4-dimensional theory at fixed light-front time \(\tau = x^+ = x^0 + x^3\). It is based on the observation that the classical equations of motion, derived from the action of the 5-dimensional theory, have the form of bound-state equations for two massless constituents in light front (LF) quantization. This correspondence is also realized dynamically, such as the analytic equality of the AdS and light-front expressions for electromagnetic and gravitational form factors of the composite states [14, 15].

Therefore in [9] it was proposed to consider Light-Front Holography as a first approximation to QCD by mapping the equations of motion, derived from the 5-dimensional action, to the wave equations of a system of two massless constituents in QCD quantized on the light front; the 5th coordinate in AdS\(_5\), the holographic variable \(z\), is identified with the boost-invariant transverse LF separation \(\zeta\) (See the Appendix A). In this approach, the “dictionary” between the bound-state wave functions of the classical five-dimensional theory and the four-dimensional theory quantized on the light front is fixed: a bound state consisting of a quark and an antiquark with orbital and total angular momentum zero, for instance, must be a pseudoscalar field.

The implementation of the superconformal algebra in LFHQCD determines uniquely the form of the interaction. It also explains quantitatively the striking similarities between baryon and meson spectra [2–5] and fixes the modification of the AdS action. A striking success of LFHQCD is the prediction of a massless pion [9, 10] in the chiral limit. It is a consequence of the implementation of the superconformal algebra and not of the Goldstone mechanism in a theory with a degenerate vacuum.

Isospin is not introduced explicitly in LFHQCD; therefore massless particles do not only occur
in the isovector, but also in the isoscalar sector—a sector which has proven to be particularly challenging, and an explicit breaking of the chiral $U(1)$ symmetry of standard QCD is required. We will show in this article that with the introduction of a one-parameter modification of the LF Hamiltonian the spectroscopy of $\eta$- and $h$-meson states can be quantitatively described by LFHQCD. A remarkable feature of the present approach is that the numerical value of the additional parameter coincides numerically with the LFHQCD confinement scale $\lambda$; this could point out to a deeper connection since in the chiral limit there is only one available scale in LFHQCD, the hadronic mass scale $\lambda$.

A consequence of the introduction of supersymmetry is the occurrence of tetraquarks as the second partners of the baryons [3, 6, 16]. Since extra hadronic states appear to be particularly abundant in the isoscalar channels, it is promising to look specifically for possible tetraquarks in those channels. There is a tremendous literature on tetraquarks and one can easily be lost in the possibilities. Supersymmetric LFHQCD, however, has the advantage that it makes quantitative and well defined predictions of the masses of these states. We will therefore perform a quantitative analysis of all of the tetraquark candidates among the isoscalar bosons. In this respect the present analysis is complementary to a previous analysis [6] which gave a general overview over all possible tetraquark states predicted by supersymmetric LFHQCD, whereas in this paper we give predictions for the masses of bosonic hadrons which can be identified as tetraquarks within the extended LFHQCD scheme.

The paper is organized as follows: In Sec. II we briefly review the main theoretical ingredients of LFHQCD. We then extend the approach to $\eta$- and $h$-mesons and discuss the implications of the Pauli principle on the quantum numbers of tetraquarks. In Sec. III we compare theory with experiment in all bosonic channels and predict the candidates which are most likely isoscalar tetraquarks. The possible relation of this modification of the LF Hamiltonian to chiral $U(1)$ breaking in standard QCD is shortly discussed in the last section, Sec. IV.

II. PRINCIPAL RESULTS OF LFHQCD AND SUPERSYMMETRIC LFHQCD

In this section we will review the main theoretical results for the hadron spectroscopy predicted by supersymmetric LFHQCD; a more detailed treatment is given in Refs. [2, 3, 10, 16]. Some intermediate steps, which are most important for this paper, are given in Appendix A.

As other holographic “bottom-up” models, LFHQCD starts from an invariant action in a five-dimensional space with the metric of AdS$_5$. Due to the maximal symmetry of the AdS$_5$ action, the
corresponding 4-dimensional theory is invariant under the conformal group. This symmetry has to be broken by introducing a mass scale. After such a modification, the resulting classical equations of motion of the 5-dimensional theory have the form of Hamiltonian equations for bound states of two massless quarks, where the $q\bar{q}$ interaction is determined by the assumed modification of the invariant AdS$_5$ action. The unique form of the modified AdS$_5$ action can in fact be completely determined by a symmetry principle: the resulting Hamiltonian of our semiclassical theory must be contained within the superconformal algebra $^1$. This requirement completely determines the form of the color-confining $q\bar{q}$ interaction and consequently the modification of the AdS$_5$ action, both for mesons and baryons [1, 2]. It also explains the observed approximate degeneracy between baryon and meson spectra and predicts the masses of the tetraquark states [3].

The resulting hadronic spectrum has the form of a supersymmetric 4-plet of $q\bar{q}$ mesons (M), quark+diquark baryons (B) and diquark+antidiquark tetraquarks (T), where in $SU(3)_C$ the diquark cluster has color $\bar{3}_C$. The predicted hadron masses can in the limit of massless quarks be summarized in the following formulae [3]:

\begin{align}
M^2_M &= 4\lambda(n + L_M) + 2\lambda S, \\
M^2_B &= 4\lambda(n + L_B + 1) + 2\lambda S, \\
M^2_T &= 4\lambda(n + L_T + 1) + 2\lambda S.
\end{align}

Here $n$ denotes the radial excitation quantum number, $L_M$ denotes the LF orbital angular momentum between the quark and antiquark in the meson, $L_B$ that between the diquark cluster and the quark in the baryon, and $L_T$ that between the two diquark clusters in the tetraquark. $S$ is for mesons the total quark spin, for baryons and tetraquarks the minimal possible quark spin of a diquark cluster inside the hadron. In supersymmetric LFHQCD only mesons with $J = L + S$, $S = 0$ or 1, can be considered. The scale $\lambda$ is the only free constant of supersymmetric LFHQCD in the limit of massless quarks.

As mentioned above, the multiplets of supersymmetric LFHQCD contain only mesons and tetraquarks with total spin of the hadron $J = L + S$, $S = 0$ or 1. But once the modification of the AdS$_5$ action is fixed by the superconformal implementation, one can apply this modification also in normal LFHQCD and derive the Hamiltonian for mesons with quark spin $S = 1$ and $J = L$ or $J = L - 1$. In this way we can also compare theory with the observed mesons with $J^{PC} = 0^{++}$ and $1^{++}$. In this case the quantity $S$ in (1) is replaced by $(J - L)$, see [17].

$^1$ The quantum field theory underlying the Maldacena conjecture is a superconformal theory.
TABLE I. Mass corrections according to (4) in GeV$^2$.

| Mass correction | Value |
|-----------------|-------|
| $\Delta M^2[m_q, m_q]$ | $(0.14)^2$ |
| $\Delta M^2[m_s, m_s]$ | $(0.773)^2$ |
| $\Delta M^2[m_q, m_q, m_q]$ | $(0.344)^2$ |
| $\Delta M^2[m_q, m_s, m_q, m_s]$ | $(0.959)^2$ |
| $\Delta M^2[m_s, m_s, m_s, m_s]$ | $(1.53)^2$ |

In order to incorporate the effects of quark masses in LF theory, at least at lowest order, one can include the invariant mass term $\sum_i m_i^2 x_i$ to the LF Hamiltonian – the contribution of quark masses to the LF kinetic energy. To first approximation, this leaves the confining LF potential unchanged. For a state containing $N$ quarks with masses $m_1 \ldots m_N$ one then obtains the quadratic mass shift [3]:

$$\Delta M^2[m_1, \ldots, m_N] = \lambda^2 \frac{\partial}{\partial \lambda} \log F,$$

with

$$F[\lambda] = \int_0^1 dx_1 \cdots \int_0^1 dx_N e^{-\frac{1}{\lambda} \left( \sum_{i=1}^N \frac{m_i^2}{x_i} \right)} \delta \left( \sum_{i=1}^N x_i - 1 \right).$$

The quark mass corrections lead to a modified mass spectroscopy for the bosons:

$$M^2_M = 4\lambda(n + L_M) + 2\lambda(J - L) + \Delta M^2[m_1, m_2],$$

$$M^2_T = 4\lambda(n + L_T + 1) + 2\lambda S + \Delta M^2[m_1, m_2, m_3, m_4].$$

The value of $\lambda$ has been fitted previously [3] to the full hadron spectrum with the result: $\sqrt{\lambda} = \kappa = 0.523 \pm 0.025$ GeV. The effective masses of the light and strange quarks were determined in [10] from $m^2_\pi = \Delta M^2[m_q, m_q]$ and $m^2_K = \Delta M^2[m_s, m_q]$, which yields the values $m_q = 0.046$ and $m_s = 0.357$ GeV. In Table I the numerical values of the mass corrections for non-strange and strange quark masses according to (4) are collected. The numerical results for the boson masses $M_M$ and $M_T$ according to (6) and (7) are given in Table II.

A. $\eta$-and $h$ mesons

As can be seen from (1) the ground states ($n = 0$) of pseudoscalar mesons with angular momentum $J = L = 0$ in LFHQCD have zero mass in the limit of massless quarks. This prediction of a massless pseudoscalar meson $qq$ bound state is a remarkable success of LFHQCD for the isovector
channel. But since LFHQCD does not treat flavor explicitly, this result also applies to the isoscalar channel. In this case, however, the least massive observed hadron is the $\eta$ meson, which has a mass of 0.548 GeV, much heavier than the pion.

In standard QCD the difference between the isovector and isoscalar sector is generally attributed to a hard breaking of the chiral $U(1)$ symmetry of the classical QCD Lagrangian by nonperturbative effects [18–25]. Since flavor is not treated explicitly in LFHQCD we will treat that breaking phenomenologically by ensuring that the lowest $I = 0, J = S = 0$ meson has the correct mass $m_\eta = 0.548$ GeV [26]. This can be achieved by adding to the LF Hamiltonian the constant term

$$\Delta_\eta^2 \equiv \lambda_\eta \delta_{S0} \delta_{I0} = \delta_{S0} \delta_{I0} (m_\eta^2 - \Delta M^2[m_q, m_\eta]),$$  

(8)

with

$$\lambda_\eta = m_\eta^2 - m_\pi^2 = (0.53 \text{ GeV})^2.$$  

(9)

This additional term encodes a hard chiral $U(1)$ breaking in standard QCD. In the isoscalar vector channel current conservation in QCD forbids an anomaly; accordingly, the effective light front Hamiltonian should not be modified in this sector. The scale associated with the $\eta$ mass, $\lambda_\eta$, numerically nearly coincides with the confinement scale $\lambda = (0.523 \pm 0.025 \text{ GeV})^2$ [3]. It is therefore tempting to speculate there is a deeper connection behind this numerical equality and that the two scales are related. We will, however, not discuss this issue further in this article.
The resulting mass formulæ in channels with \( I = 0, J = L, \mathcal{S} = 0 \) are:

\[
M_{M}^{2} = 4\lambda(n + L_{M}) + \Delta M_{1}[m_{1}, m_{2}] + \lambda_{\eta}, \tag{10}
\]

\[
M_{T}^{2} = 4\lambda(n + L_{T} + 1) + \Delta M_{1}[m_{1}, m_{2}, m_{3}, m_{4}] + \lambda_{\eta}, \tag{11}
\]

where \( \lambda_{\eta} \simeq \lambda \). As shown below, it leads to a very satisfactory description of all \( \eta \) and \( h \) mesons.

**B. Isospin and spin of tetraquarks**

The two quarks of a diquark cluster in a tetraquark are antisymmetric in color; the spin-statistics theorem therefore demands that they are symmetric in the remaining quantum numbers. Therefore a diquark cluster with specific isospin and relative orbital angular momentum zero must either have isospin and spin both equal to zero, or both equal to 1. This implies that the lowest lying tetraquark with isospin 0 must have total angular momentum \( J = 0 \) and the one with isospin 1 must have \( J = 1 \); as a result, the squared masses of the two states differ by \( 2\lambda \).

These arguments do not apply to tetraquarks containing constituents which are not related by isospin symmetry. The lowest state consisting of the type \( (\bar{q}sqs) \) is isospin degenerate and has in supersymmetric LFHQCD the mass 1.42 GeV. Since diquark clusters are bosons, the parity and C-parity of a tetraquark with \( \mathcal{S} = 0 \) is \((-1)^{L}\). For a tetraquark with \( \mathcal{S} = 1 \) both C-parities are possible.

For a tetraquark consisting only of strange and antistrange quarks, the spin-statistics theorem requires that the total spin of each diquark cluster is 1. Therefore such tetraquarks are not predicted in the scheme of supersymmetric LFHQCD. We will therefore restrict our general discussion to tetraquarks, where only one diquark cluster has spin 1. But at least tentatively, we suggest that the higher lying states of \( f_{2} \) mesons might contain admixtures of tetraquarks \( (\bar{q}sqs) \) and extend \((7)\) to \( \mathcal{S} = 2 \). This is motivated by the appearance of two \( \phi \) mesons in the decay channels of these mesons.

**III. COMPARISON WITH EXPERIMENT**

The augmented LFHQCD theory presented here contains four parameters. Three of them, the scale \( \sqrt{\lambda} = \kappa = 0.523 \text{ GeV} \) and the quark masses \( m_{q} = 0.045, m_{s} = 0.357 \text{ GeV} \), are taken from previous analyses of the hadron spectrum [3]. For this analysis the shift term \( \Delta_{\eta}^{2} = \lambda_{\eta} \delta_{S0} \delta_{I0} \) has been introduced in the LF Hamiltonian for mesons \((10)\) and tetraquarks \((11)\) in the isoscalar sector. As discussed above, the scale \( \lambda_{\eta} \) is fixed by the \( \eta \) mass, \( \lambda_{\eta} \equiv M_{\eta}^{2} - \Delta_{S}^{2}[m_{q}, m_{q}] \simeq \lambda \).
A. \( \eta \) and \( h \) mesons

As can be seen from Fig. (1), the mass difference \( \sqrt{M^2_x - M^2_y} \) agrees reasonably well with the corresponding mass differences between other hadrons \( x \) and \( y \) with the same external quantum numbers. This quantity is determined by the mass difference between the strange and the light quarks: two light quarks are replaced by two strange quarks in baryons, and for mesons the comparison is made for isoscalar mesons with the same quantum numbers. If the hadrons \( x \) and \( y \) are mesons one can conclude from their decays that the heavier meson \( x \) is predominantly an \( \bar{s} - s \) pair, whereas the lighter hadron \( y \) is a \( \bar{q} - q \) pair, both with the same quark spin \( S \). If the hadrons \( x \) and \( y \) are baryons, then they both have the same quantum numbers \( J^P \), but the strangeness of the heavier one is larger by two units; \( i.e. \), the \( q - q \) diquark cluster of the lighter baryon \( y \) is replaced by an \( s - s \) cluster in the heavier baryon \( x \). The mean value of the mass difference \( M_d = \sqrt{M^2_x - M^2_y} \) is 0.84 GeV with a standard deviation of 0.09 GeV; the theoretically predicted value is \( \sqrt{\Delta^2[m_s, m_s] - \Delta^2[m_q, m_q]} = 0.76 \text{ GeV} \) (see Table (I)).

Therefore in the present analysis the \( \eta - \eta' \) mass difference, sometimes referred to as the \( \eta - \eta' \) puzzle, is determined by the strange quark mass contribution

\[
M^2_{\eta'} = \lambda_{\eta} + \Delta M^2[m_s, m_s] = M^2_{\eta} - \Delta^2[m_q, m_q] + \Delta^2[m_s, m_s]. \tag{12}
\]

Using the results of Table (I), Eq. (12) leads to \( M_{\eta'} = 0.937 \text{ GeV} \), in good agreement with the experimental value \( M_{\eta'} = 0.958 \pm 0.06 \text{ GeV} \) [26].

In Fig. 2 the theoretical trajectories for the \( \eta, \eta' \) and \( h \) mesons and their radial and orbital
excitations are shown. Since not very many $\eta$ mesons, and even fewer $h$ mesons, have been confirmed experimentally, we have also considered unconfirmed states listed in [26]. As can be seen from the figure, the agreement between theory and experiment is indeed satisfactory – the worst difference between theory and experiment is 100 MeV for the first radial excitation of the $\eta$, the $\eta(1295)$; even the unconfirmed states (in gray) fit nicely. For a more detailed discussion, see the next subsection.

B. General comparison of isoscalar bosons

In Table III we show all of the confirmed isoscalar bosons. In the first 4 columns we show the experimental results; the letter $d$ in the column “Decay” indicates that decay channels with open or hidden strangeness are dominant, and $ss$ indicates that there are decay channels with four-fold hidden strangeness, such as the $\phi\phi$, for the $\eta'$ decay (see the discussion below). In the four columns listed under “Theory” we show below $M_{nL,J−L}$ and $T_{nL,J−L}$ the quark content, the radial excitation number $n$, the light front angular momentum $L$, and the difference $J−L$, as indices. For the $\eta$ like mesons with $J^{PC} = 0^{++}$ or $1^{+-}$ we denote by an upper index $\eta$ the fact
that the shift given by $\Delta^2_\eta$, see Sect. II, has to be performed; the theoretical masses are calculated according to (6, 7) and (10, 11), respectively.

We will leave out from this comparison the extremely broad $f_0(500)$ which will be discussed later. The overall fit from theory to experiment is satisfactory – the standard deviation between theory and experiment is $SD = 93$ MeV, well inside the model uncertainty of $\approx 100$ MeV, as expected from the $N_C \to \infty$ expansion [27]. The discrepancy between theory and experiment is no more than 3 standard deviations for any of the considered 27 states, and only two states deviate more than 2 standard deviations. Therefore we accept only as probable a tetraquark assignment for states where the difference between the experimental masses and LF holographic predictions is less than 3 SD $\approx 280$ MeV.

We now start a detailed discussion of the states:

a. $\eta$ and $h$ mesons, $J^{PC} = 0^{-+}, 1^{+-}, 2^{-+}$ In our approach the $\eta$ is predominantly $\bar{q}q$ and the $\eta'$ is predominantly $\bar{s}s$. Since the $\eta$ is slightly below the $4\pi$ threshold and the $\eta'$ below the $KK$ threshold we cannot test this assignment by the decays, but the width of the $\eta'(958)$ meson is very small (196 keV). The dominant decay $\eta\pi\pi$ is not forbidden by any selection rule or spin and statistics. This is an additional argument that $\eta$ and $\eta'$ have a different quark content, with the $\eta'$ having an important hidden strangeness. As mentioned in sect. II A, the agreement between theory and experiment is very satisfactory. Our interpretation of the $\eta(1475)$ as a tetraquark in the $\eta'$ family is strongly favored: its predicted mass 1.52 GeV agrees well with the experimental value 1.48 GeV and the nearby $\eta(1405)$ fits well to the first radial excitation of the $\eta'$ $^1S_0$ meson state.

b. $f_0$ states The assignment of $f_0(500)$ and the $f_0(980)$ as members of a nonet of tetraquarks, with and without hidden strangeness, appears very plausible [6, 28–32]; however, quantitative predictions from LFHQCD do not add support to this assignment: The lightest tetraquark consisting of non-strange quarks, $(\bar{q}qqq)_{000}$ has a mass of 1.10 GeV, compatible with the $f_0(980)$, but the lightest tetraquark with hidden strangeness, $(\bar{s}sqq)_{000}$ has a mass of 1.42 GeV. The conventional meson in a $qq$ ($^3P_0$) state, $\bar{q}q_{01-1}$, has in LFHQCD the mass 0.75 GeV which is also in this mass range. Unfortunately, the quantitative predictions from LFHQCD do not contribute to the solution of this interesting situation.

The other $f_0$ states fit very well into the LFHQCD theoretical scheme. The $f_0(1500)$ is a very good candidate for a radially excited tetraquark; its mass fits nicely, and its baryonic partner is the Roper resonance $N(1440)$, see Table IV.
TABLE III. All confirmed light isoscalar bosonic states. The data are from the PDG [26], and the theoretical results from (6, 7) and (10, 11). The label \( d \) signifies that channels with open or hidden strangeness are dominant; for the \( \eta' \) decay see the note in the text. If a state with \( I = 1 \), but equal residual quantum numbers occurs, it is indicated in the last column. For further explanation see the text.

| \( J^{PC} \) | Experiment | Theory | \( M_{nL,J−L} \) [GeV] | \( T_{nL,S} \) [GeV] | \( I = 1 \) partner |
|-----------|------------|---------|------------------------|------------------------|----------------------|
| 0−+ \( \eta \) | 548 | \( \bar{q}q_{000} \) | (0.548) |
| 0−+ \( \eta'(958) \) | 958 | \( \bar{s}s_{000} \) | 0.94 |
| 0−+ \( \eta(1295) \) | 1294 ± 4 | \( \bar{q}q_{100} \) | 1.18 |
| 0−+ \( \eta(1405) \) | 1409 ± 2 | \( \bar{s}s_{100} \) | 1.41 |
| 0−+ \( \eta(1475) \) | 1476 ± 4 | \( (\bar{s}sqq)_{000} \) | 1.51 |
| 0++ \( f_0(500) \) | 475 ± 75 | \( \bar{q}q_{01−} \) | 0.75 |
| 0++ \( f_0(980) \) | 990 ± 20 | \( (\bar{q}qqq)_{000} \) | 1.10 |
| 0++ \( f_0(1370) \) | 1350 ± 150 | \( \bar{q}q_{11−} \) | 1.29 |
| 0++ \( f_0(1500) \) | 1504 ± 6 | \( (\bar{q}qqq)_{100} \) | 1.52 |
| 0++ \( f_0(1710) \) | 1723 ± 6 | \( \bar{s}s_{21−} \) | 1.82 |
| 1−− \( \omega \) | 783 | \( \bar{q}q_{001} \) | 0.75 | \( \rho \) |
| 1−− \( \omega(1420) \) | 1425 ± 25 | \( \bar{q}q_{011} \) | 1.29 |
| 1−− \( \omega(1650) \) | 1670 ± 30 | \( \bar{q}q_{201} \) | 1.66 |
| 1−− \( \phi \) | 1019 | \( \bar{s}s_{001} \) | 1.07 |
| 1−− \( \phi(1680) \) | 1680 ± 20 | \( \bar{s}s_{101} \) | 1.49 |
| 1−− \( \phi(2170) \) | 2188 ± 10 | \( \bar{s}s_{301} \) | 2.10 |
| 1−+ \( h_1(1170) \) | 1170 ± 20 | \( \bar{q}q_{010} \) | 1.18 | \( \rho(1450) \) |
| 1++ \( f_1(1285) \) | 1282 | \( \bar{q}q_{110} \) | 1.49 | \( a_1(1260) \) |
| 1++ \( f_1(1420) \) | 1426 | \( \bar{s}s_{010} \) | 1.30 |
| 2−− \( \eta_2(1645) \) | 1617 ± 5 | \( \bar{q}q_{020} \) | 1.58 | \( a_2(1320) \) |
| 2++ \( f_2(1270) \) | 1276 | \( \bar{q}q_{101} \) | 1.29 | \( a_2(1320) \) |
| 2++ \( f_2'(1525) \) | 1525 ± 5 | \( \bar{s}s_{011} \) | 1.49 |
| 2++ \( f_2(1950) \) | 1944 ± 12 | \( \bar{s}s_{111} \) | 1.82 |
| 2++ \( f_2(2010) \) | 2011 ± 70 | \( \bar{s}s_{211} \) | 2.10 |
| 2++ \( f_2(2300) \) | 2297 ± 28 | \( \bar{s}s_{311} \) | 2.35 |
| 2++ \( f_2(2340) \) | 2345 ± 50 | \( (\bar{s}sss)_{102} \) | 2.37 |
| 3−− \( \omega_3(1670) \) | 1667 ± 4 | \( \bar{q}q_{021} \) | 1.66 | \( \rho_3(1690) \) |
| 3−− \( \phi_3(1850) \) | 1854 ± 7 | \( \bar{s}s_{021} \) | 1.82 |
| 4++ \( f_4(2050) \) | 2018 ± 11 | \( \bar{q}q_{031} \) | 1.96 | \( a_4(2040) \) |
TABLE IV. A compilation of the tetraquark states and their partners. States omitted from the summary table of PDG are marked by a question mark ?. The indices of the quark content denote $n, L, S$, the radial excitation, LF momentum and the quark or diquark spin. The differences of the theoretical masses, $M_{\text{theo}}$ shows the amount of SUSY breaking due to the additional mass terms. Assignments with remark C do not explain the occurrence of isovector states with the same residual quantum numbers and and similar masses.

| Tetraquark Name | quark cont. | $M_{\text{theo}}$ | Baryon Name | quark cont. | $M_{\text{theo}}$ | Meson Name | quark cont. | $M_{\text{theo}}$ | Rem. |
|----------------|-------------|-------------------|-------------|-------------|-------------------|-------------|-------------|-------------------|------|
| $f_0(980)$     | ($\bar{q}qqq)_000$ | 1.10              | N(940)      | ($qqq)_000$ | 1.07              | $h_1(1170)$ | $\bar{q}q_{010}$ | 1.18              |      |
| $\omega(1420)$ | ($\bar{q}qqq)_{010}$ | 1.52              | $N^+ (1535)$ | ($qqq)_{010}$ | 1.50              | $\eta_2(1645)$ | $\bar{q}q_{200}$ | 1.58              | C    |
| $f_0(1500)$    | ($\bar{q}qqq)_{100}$ | 1.52              | $N^{++} (1440)$ | ($qqq)_{100}$ | 1.50              | $h_1(1595)$ | $\bar{q}q_{110}$ | 1.58              |      |
| $\eta(1475)$  | ($\bar{q}sqq)_{000}$ | 1.51              | $\Xi^{++} (1330)$ | ($ssq)_{000}$ | 1.31              | $h_1(1380)$ | $\bar{s}s_{010}$ | 1.41              |      |
| $f_1(1420)$    | ($\bar{q}sqq)_{001}$ | 1.6               | $\Xi^{*} (1690)_2$ | ($ssq)_{001}$ | 1.67              | $f_2(1255)$ | $\bar{s}s_{011}$ | 1.49              |      |
| $\phi(1680)$  | ($\bar{q}sqq)_{010}$ | 1.76              | $\Xi^{*} (1690)_2$ | ($ssq)_{010}$ | 1.67              | $\eta_2(1870)_2$ | $\bar{s}s_{020}$ | 1.75              |      |

C. $f_1$ states: The situation is similar as for the $f_0(980)$. The $f_1(1282)$ is heavier than the $^1P_1$ ground state (1.06 GeV) and lighter than its first radial excitation (1.49 GeV), there is also no tetraquark with fitting mass and the occurrence of the near degenerate isospin partner $a_1(1260)$ make an interpretation as a meson state plausible anyhow. An interpretation of the $f_1(1282)$ and $a_1(1260)$ as tetraquarks with $S = 1$ and hidden strange ($\bar{s}sqq)_{001}$ [6] is not supported by the quantitative LFHQCD analysis, the discrepancy between the theoretical mass value for this assignment and experiment is 315 MeV $\sim 3.6$ SD.

d. $f_2$ states: Our quark assignment in Table III for the $f_2(1270)$ and the $f_2'(1525)$ is compatible with a recent lattice computation above threshold, where the lighter state couples predominantly to $\pi\pi$ and the heavier to $K\bar{K}$ [36]. On the other hand, the profusion of $f_2$ states makes the occurrence of exotic states very plausible. Furthermore the occurrence of two $\phi$-mesons in the decay channels of the heavier $f_2$ mesons strongly suggest the admixture of a tetraquark consisting of two strange and two anti-strange quarks. As mentioned in Sec. II A, a diquark cluster with strangeness $\pm 2$ must have spin 1, so it is plausible that these states contribute to mesons with $J = 2$. Extending (7) also to diquarks with spin 2, we obtain for the lightest tetraquarks with quark content $(\bar{s}sqq)_{002}$ the mass $M_T = 2.13$ GeV, and for the $(\bar{s}sqq)_{102}$ configuration $M_T = 2.37$ GeV; they are just in the mass range of those states which decay into two $\phi$ mesons. We therefore propose that the $f_2(2010), f_2(2300)$ and $f_2(2340)$ are mixtures of normal meson states and tetraquark states, as indicated in Table III. These tetraquarks with aligned spin have positive parity and charge parity $P = C = 1$. 

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In Table IV we have listed those tetraquark states which fit quantitatively into the scheme of supersymmetric LFHQCD. The masses of the first two, the \( f_0(980) \) and the \( \omega(1420) \), fit reasonably well into the LFHQCD scheme; although the isospin degeneracy with the \( a_0(980) \) and the \( \rho(1440) \) cannot be explained by this assignment. The remaining four states are, however, very probably tetraquark states. Note that the additional mass correction term leads to a breaking of the supersymmetry, as can be seen from the different theoretical masses inside one supermultiplet.

IV. SUMMARY AND DISCUSSION

As has been shown in Sec. II A, the addition of a constant term to the \( I = 0, S = 0 \) bosonic sector of the supersymmetric light front holographic Hamiltonian provides an explanation of the entire \( \eta - h \) spectrum. This additional term, which breaks the supersymmetry of the spectra, plays the role of a direct chiral \( U(1) \) breaking term in effective chiral representations of standard QCD. In fact, at the classical level, the QCD Lagrangian with massless up and down quarks is invariant under \( U_L(2) \otimes U_R(2) \) transformations. In conventional effective chiral theory, \( SU_L \otimes SU_R(2) \) is broken spontaneously and leads to an isovector of Goldstone particles which are identified with the \( \pi \) mesons. Since the isoscalar pseudoscalar meson, the \( \eta \)-meson, is considerably heavier than \( \sqrt{3} M_\pi \approx 237 \) MeV [33] there is apparently no Goldstone boson of the remaining chiral \( U(1) \) symmetry and it is most probably broken directly, for instance by instanton solutions or other nonperturbative effects, which modify the effective QCD Hamiltonian [18–25].

In supersymmetric LFHQCD, the implementation of the superconformal algebra is the origin of the vanishing mass of mesons with \( L = 0, S = 0 \), since it predicts a constant term \(-2\lambda\) in the LF potential which exactly cancels the LF kinetic energy. Since the lowest meson state has no supersymmetric partners [2], it plays the role of a zero-energy non-degenerate ground state. This occurrence of massless mesons is conventionally associated with the spontaneous breaking of the chiral \( SU(2) \) symmetry of QCD based on effective meson fields.

But if the chiral \( U(1) \) symmetry is broken directly, also the supersymmetry of LFHQCD needs a hard breaking. Such a symmetry breaking term is given by the additional term (8) in the bosonic sector which modifies the supersymmetric LF Hamiltonian. After this additional term (8) has been incorporated in LFHQCD, all masses are uniquely determined by the modified LFHQCD Hamiltonian and the mass corrections (4) shown in Table I. An astonishing feature of the present approach is the numerical coincidence of the direct breaking scale \( \lambda_\eta \) with the confinement scale \( \lambda \). In the channels with quark spin \( S = 1 \) no symmetry-breaking term is present and no modification
of supersymmetric LFHQCD is necessary.

We have also considered states with \( J \neq L + S \), which are not members of super-multiplets; their masses are predicted by LFHQCD, given the quadratic dilaton profile \( e^{\lambda z^2} \) which is determined by the implementation of the superconformal algebra. The theoretical mass predictions are collected in Fig. 2 and Table III.

The quantitative discussion of tetraquarks in this investigation is complementary to that of Ref. [6], where a general qualitative overview of possible supermultiplets in all channels was given. In this paper we have concentrated on the possible tetraquark states, the mass of which agrees within the model accuracy of \( \approx 100 \) MeV with the theoretical predictions. Candidates fulfilling these criteria are collected in Table IV, together with their partners. The most convincing candidates are the \( f_0(1500) \) and the \( \eta(1475) \). The \( f_0(1500) \) has also been considered as a candidate for a glueball, see e.g., [37, 38]; it should be noted, however, that in LFHQCD there is no sign of valence gluons, and the identification of the \( f_0(1500) \) as a tetraquark is quantitatively very convincing.

In this article we have concentrated on phenomenological aspects which hopefully will lead to further insights into the relation of our approach with symmetry breaking mechanisms in standard QCD, and especially elucidate the remarkable numerical coincidence of the “symmetry breaking” parameter \( \lambda_\eta \) with \( \lambda \), the confinement scale in LFHQCD.

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Appendix A: Summary of LFHQCD results

In this appendix we summarize the relevant equations from LFHQCD which underlie the theoretical foundations of this paper. The equations of motion for a meson with arbitrary spin-\( J \), represented by a fully symmetric tensor field of rank-\( J \) in 5 dimensional AdS space, follows from an AdS5 action with the soft-wall dilaton term \( e^{\phi(z)} \) [17]:

\[
\left(-\frac{d^2}{dz^2} + \frac{4L^2_{AdS}}{4z^2} + U_{AdS}(z)\right) \Phi_J(z, q) = q^2 \Phi_J(z, q).
\]  (A1)
Here $z$ is the fifth coordinate of $\text{AdS}_5$, the holographic variable in the 5-dimensional space, and $q^2$ the momentum in the 4-dimensional (physical) spacetime. The potential $U_{\text{AdS}}(z)$ depends on the dilaton profile $\varphi(z)$:

$$
U_{\text{AdS}}(z) = \frac{1}{2}\varphi''(z) + \frac{1}{4}(\varphi'(z))^2 + \frac{2J-3}{2z}\varphi'(z). \quad (A2)
$$

The quantity $L^2_{\text{AdS}}$ is determined by $J$ and the dimensionless product of the AdS-mass $\mu$ with the space curvature $R$: $L^2_{\text{AdS}} = (\mu R)^2 + (2 - J)^2$.

The form of (A1) is that of a bound-state equation for a hadron consisting of two massless constituents in light-front quantization. The holographic variable $z$ is identified with the boost-invariant LF variable $\zeta = \sqrt{x(1-x)}b_\perp$, where $x$ is the longitudinal momentum fraction of one of the quark constituents and $b_\perp$ is the transverse separation of the constituents (quarks or quark clusters) in the transverse plane. The LF angular momentum $L$ is identified with the quantity $L_{\text{AdS}}$ and the light-front potential $U(\zeta)$ is therefore determined by (A2).

If one implements superconformal algebra [1, 2] by requiring the LF Hamiltonian to be a superposition of the generators of the superconformal algebra following [39, 40], the form of the LF potential is completely fixed to

$$
U(\zeta) = \lambda^2 \zeta^2 + 2\lambda(L - 1), \quad (A3)
$$

which leads to eigenvalues

$$
M^2 = |\lambda|(4n + 2L + 2) + 2\lambda(L - 1). \quad (A4)
$$

The potential derived from the implementation of the superconformal algebra is only compatible with the holographic approach if the dilaton profile $\phi(z) = \lambda z^2$ and holds for mesons with $J = L$. It is remarkable that this choice of maximal symmetry breaking is the one which had been chosen before in the soft wall model [41] for purely phenomenological reasons, namely to generate linear Regge trajectories for mesons. A zero mass state only occurs if the sign of $\lambda$ is positive:

$$
M^2 = 4\lambda(n + L), \quad (A5)
$$

and therefore the lowest meson state has no baryon partner [2, 35]. One sees immediately that in order to get the harmonic part of the potential from (A2), one has to choose the $\varphi(\zeta) = \lambda \zeta^2$, and one obtains in this case

$$
U_{\text{AdS}}(\zeta) = \lambda^2 \zeta^2 + 2\lambda(J - 1). \quad (A6)
$$
We can thus extend the superconformal approach to mesons with quark spin $S = 1$ and $J = L + 1$ by adding the term $2\lambda S$ to (A3), to recover the result (A6) both for mesons with $S = 0$ and $S = 1$, $J = L + S$. Therefore our final result for the LF potential, valid for mesons with $J = L + S$ is:

$$U(\zeta) = \lambda^2 \zeta^2 + 2\lambda(L + S - 1),$$

(A7)

and the resulting meson spectrum is the one given by (1).

The implementation of the superconformal algebra implies [39, 40] that besides the Hamiltonian for the bosonic wave function, there is also one for a fermionic one, which describes the supersymmetric fermion of the boson described by (A3). Its potential is:

$$U(\zeta) = \lambda^2 \zeta^2 + 2\lambda(L + 1),$$

(A8)

and leads to the eigenvalues [1, 2]

$$M^2 = 4\lambda(n + L + 1).$$

(A9)

Consequently a $q\bar{q}$ meson with angular momentum $L_M$ has the same mass as a baryon with angular momentum $L_B = L_M + 1$ between its quark and diquark cluster components. This relation has been tested and is very well satisfied for many spectra of light and even heavy hadrons [2, 3, 5, 6]. The superconformal baryon potential (A8) can also be obtained from an AdS action for fermion fields if a Yukawa-like term $\bar{\Psi}\gamma^2\Psi$ is added to the Lagrangian. This modification had been introduced earlier for purely phenomenological reasons [42, 43]. The Hamiltonian (A8) in this case applies to the positively aligned chirality component $\psi^+$ of the baryon. There is also the negative chirality component $\psi^-$ of the baryon. The corresponding LF potentials are [1]

$$U_+(\zeta) = \lambda^2 \zeta^2 + 2\lambda(L + 1),$$

(A10)

namely Eq. (A8) for the positive component, and

$$U_-(\zeta) = \lambda^2 \zeta^2 + 2\lambda L,$$

(A11)

for the negative component. The LF potential, together with the term $2\lambda S$ introduced above, leads to the baryon spectrum given in (2).

Finally, there is also a bosonic superpartner of the negative chirality component $\psi^-$ of the baryon, which is interpreted as a tetraquark [3, 6, 16]. Its mass spectrum is given in (3). The meson, positive and negative chiral baryon states $\psi^\pm$, and the tetraquark form a 4-plet supermultiplet.
We can generalize the results (1,2,3) obtained in supersymmetric LFHQCD by going back to normal LFHQCD. Once the modification is fixed, we can use the specific AdS result (A6) and obtain for the meson spectrum Eq. (6).

1. G. F. de Téramond, H. G. Dosch and S. J. Brodsky, Baryon spectrum from superconformal quantum mechanics and its light-front holographic embedding, Phys. Rev. D 91, 045040 (2015) [arXiv:1411.5243 [hep-ph]].
2. H. G. Dosch, G. F. de Téramond and S. J. Brodsky, Superconformal baryon-meson symmetry and light-front holographic QCD, Phys. Rev. D 91, 085016 (2015) [arXiv:1501.00959 [hep-th]].
3. S. J. Brodsky, G. F. de Téramond, H. G. Dosch and C. Lorcé, Universal effective hadron dynamics from superconformal algebra, Phys. Lett. B 759, 171 (2016) [arXiv:1604.06746 [hep-ph]].
4. H. G. Dosch, G. F. de Téramond and S. J. Brodsky, Supersymmetry across the light and heavy-light hadronic spectrum, Phys. Rev. D 92, 074010 (2015) [arXiv:1504.05112 [hep-ph]].
5. H. G. Dosch, G. F. de Téramond and S. J. Brodsky, Supersymmetry across the light and heavy-light hadronic spectrum II, Phys. Rev. D 95, 034016 (2017) [arXiv:1612.02370 [hep-ph]].
6. M. Nielsen and S. J. Brodsky, Hadronic superpartners from a superconformal and supersymmetric algebra, Phys. Rev. D 97, 114001 (2018) [arXiv:1802.09652 [hep-ph]].
7. M. Nielsen, S. J. Brodsky, G. F. de Téramond, H. G. Dosch, F. S. Navarra and L. Zou, Supersymmetry in the double-heavy hadronic spectrum, Phys. Rev. D 98, 034002 (2018) [arXiv:1805.11567 [hep-ph]].
8. S. J. Brodsky and G. F. de Téramond, Light-front hadron dynamics and AdS/CFT correspondence, Phys. Lett. B 582, 211 (2004) [hep-th/0310227].
9. G. F. de Téramond and S. J. Brodsky, Light-front holography: A first approximation to QCD, Phys. Rev. Lett. 102, 081601 (2009) [arXiv:0809.4899 [hep-ph]].
10. S. J. Brodsky, G. F. de Téramond, H. G. Dosch and J. Erlich, Light-front holographic QCD and emerging confinement, Phys. Rep. 584, 1 (2015) [arXiv:1407.8131 [hep-ph]].
11. G. F. de Téramond, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, A. Deur, Universality of generalized parton distributions in light-front holographic QCD, Phys. Rev. Lett. 120 182001 (2018) [arXiv:1801.09154 [hep-ph]].
12. R. S. Sufian, T. Liu, G. F. de Téramond, H. G. Dosch, S. J. Brodsky, A. Deur, M. T. Islam and B. Q. Ma, Nonperturbative strange-quark sea from lattice QCD, light-front holography, and meson-baryon fluctuation models, Phys. Rev. D 98, 114004 (2018) [arXiv:1809.04975 [hep-ph]].
13. J. M. Maldacena, The large-N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. 38, 1113 (1999) [hep-th/9711200].
14. S. J. Brodsky and G. F. de Téramond, Hadronic spectra and light-front wave functions in holographic QCD, Phys. Rev. Lett. 96, 201601 (2006) [hep-ph/0602252]: Light-front dynamics and AdS/QCD.
correspondence: The pion form factor in the space- and time-like regions, *Phys. Rev. D* **77**, 056007 (2008) [arXiv:0707.3859 [hep-ph]].

15. S. J. Brodsky and G. F. de Téramond, Light-front dynamics and AdS/QCD correspondence: Gravitational form factors of composite hadrons, *Phys. Rev. D* **78**, 025032 (2008) [arXiv:0804.0452 [hep-ph]].

16. L. Zou and H. G. Dosch, A very practical guide to light front holographic QCD, [arXiv:1801.00607 [hep-ph]].

17. G. F. de Téramond, H. G. Dosch and S. J. Brodsky, Kinematical and dynamical aspects of higher-spin bound-state equations in holographic QCD, *Phys. Rev. D* **87**, 075005 (2013) [arXiv:1301.1651 [hep-ph]].

18. J. B. Kogut and L. Susskind, Quark confinement and the puzzle of the ninth axial-vector current, *Phys. Rev. D* **10**, 3468 (1974).

19. G. ’t Hooft, Symmetry breaking through Bell-Jackiw anomalies, *Phys. Rev. Lett.* **37**, 8 (1976); How instantons solve the U(1) problem, *Phys. Rept.* **142**, 357 (1986).

20. E. Witten, Current algebra theorems for the U(1) Goldstone boson, *Nucl. Phys. B* **156**, 269 (1979).

21. G. Veneziano, U(1) Without instantons, *Nucl. Phys. B* **159**, 213 (1979).

22. G. Veneziano, Goldstone mechanism from gluon dynamics, *Phys. Lett.* **95B**, 90 (1980).

23. P. Di Vecchia and G. Veneziano, Chiral dynamics in the large N limit, *Nucl. Phys. B* **171**, 253 (1980).

24. M. Engelhardt, Center vortex model for the infrared sector of Yang-Mills theory: Topological susceptibility, *Nucl. Phys. B* **585**, 614 (2000) [hep-lat/0004013].

25. R. Alkofer, C. S. Fischer and R. Williams, U(1)_A anomaly and \(\eta'\)-mass from an infrared singular quark-gluon vertex, *Eur. Phys. J. A* **38**, 53 (2008) [arXiv:0804.3478 [hep-ph]].

26. M. Tanabashi *et al.* [Particle Data Group], Review of Particle Physics, *Phys. Rev. D* **98**, 030001 (2018).

27. G. ’t Hooft, A planar diagram theory for strong interactions, *Nucl. Phys. B* **72**, 461 (1974).

28. G. ’t Hooft, G. Isidori, L. Maiani, A. D. Polosa and V. Riquer, A theory of scalar mesons, *Phys. Lett. B* **662**, 424 (2008) [arXiv:0801.2288 [hep-ph]].

29. D. Black, A. H. Fariborz and J. Schechter, Mechanism for a next-to-lowest lying scalar meson nonet, *Phys. Rev. D* **61**, 074001 (2000) [hep-ph/9907516].

30. R. L. Jaffe and F. Wilczek, Diquarks and exotic spectroscopy, *Phys. Rev. Lett.* **91**, 232003 (2003) [hep-ph/0307341].

31. R. L. Jaffe, Multiquark hadrons. 1. Phenomenology of \(Q^2Q^2\) mesons, *Phys. Rev. D* **15**, 267 (1977).

32. L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, A new look at scalar mesons, *Phys. Rev. Lett.* **93**, 212002 (2004) [hep-ph/0407017].

33. S. Weinberg, The U(1) problem, *Phys. Rev. D* **11**, 3583 (1975).

34. A. A. Belavin, A. M. Polyakov, A. S. Schwartz and Y. S. Tyupkin, Pseudoparticle solutions of the Yang-Mills equations, *Phys. Lett. B* **59B**, 85 (1975).

35. E. Witten, Dynamical breaking of supersymmetry, *Nucl. Phys. B* **188**, 513 (1981).
[36] R. A. Briceno, J. J. Dudek, R. G. Edwards and D. J. Wilson, Isoscalar $\pi\pi, K\bar{K}, \eta\eta$ scattering and the $\sigma, f_0, f_2$ mesons from QCD, Phys. Rev. D 97, 054513 (2018) [arXiv:1708.06667 [hep-lat]].

[37] C. Amsler and F. E. Close, Is $f_0(1500)$ a scalar glueball?, Phys. Rev. D 53, 295 (1996) [hep-ph/9507326].

[38] F. E. Close and A. Kirk, The mixing of the $f_0(1370), f_0(1500)$ and $f_0(1710)$ and the search for the scalar glueball, Phys. Lett. B 483, 345 (2000) [hep-ph/0004241].

[39] V. de Alfaro, S. Fubini and G. Furlan, Conformal invariance in quantum mechanics, Nuovo Cim. A 34, 569 (1976).

[40] S. Fubini and E. Rabinovici, Superconformal quantum mechanics, Nucl. Phys. B 245, 17 (1984).

[41] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Linear confinement and AdS/QCD, Phys. Rev. D 74, 015005 (2006) [arXiv:hep-ph/0602229].

[42] I. Kirsch, Spectroscopy of fermionic operators in AdS/CFT, JHEP 0609, 052 (2006) [arXiv:hep-th/0607205].

[43] Z. Abidin and C. E. Carlson, Nucleon electromagnetic and gravitational form factors from holography, Phys. Rev. D 79, 115003 (2009) [arXiv:0903.4818 [hep-ph]].