Problem solving strategy in the teaching and learning processes of quantitative reasoning

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Abstract. The study presents an analysis of Polya's problem-solving strategy used in the training processes of quantitative reasoning competence in students of the Universidad Simón Bolívar, San José de Cúcuta, Colombia. The research was based on a descriptive design and had an intentional sample of 58 students who were studying the sciences and general competencies elective. For the collection of information, a diagnostic test (pre-test) and a final test (post-test) were applied, in order to check the incidence of the applied strategy. The results showed a significant improvement in the final results obtained by the students in each of the processes formed: interpretation, representation and modeling, and argumentation.

1. Introduction
One of the fundamental purposes of the higher education institutions is precisely the integral formation of the future professionals. Such integrality seeks, among some aspects, the development of the general competences within which the competence of quantitative reasoning lies. In view of the above, the department of social and human sciences, and the faculty of basic and biomedical sciences of the Universidad Simón Bolívar, San José de Cúcuta, Colombia have been developing curricular strategies for the strengthening of these competences within the framework of the project entitled curricular design for strengthening of generic competences in undergraduate students.

In the context of the curricular reflections made from the area of exact sciences of the faculty of basic and biomedical sciences, the present research project is created, which, from the elective of sciences and general competences in the law program, the present study focused on the application of the Polya’s problem solving strategy, for the formation of quantitative reasoning in these students. All this application was strengthened with the use of technological mediation or virtual interaction space of the university known as "extended classroom".

At a theoretical level, categories such as generic competences [1], problem solving [2-4], learning process based on the cognitive theory of meaningful learning [5] and the competence category of quantitative reasoning [6], operationalized according to the theoretical guidelines for the construction of the evidence matrix containing the statements, evidence and associated tasks for each of the competencies associated with quantitative reasoning.

The research was conducted in three phases: The first one was composed by the diagnosis (pre-test) made to students to determine the level and / or development they had of quantitative reasoning, the second phase was made up of the implementation of the Polya’s problem solving strategy with the design and application of learning guides for each process associated with the competence, following the steps of the Polya’s problem solving strategy for the solution of problems in the mathematical - legal context.
The last phase determined the added value of the implementation of this strategy based on a focused interview and a final test (post-test).

It can be concluded that, by solving problem situations and actively participating in the different activities proposed in the extended classroom, (forums, tasks, glossaries, wiki and questionnaires), it generates a sense of identification and understanding in mathematical concepts and competences, at the same time, a stimulus is created for the student to develop aspects that characterize the interpretation, formulation and argumentation competences that are addressed in the aforementioned elective.

2. Materials and methods
The research was based on an empirical analytical paradigm [7] with a quantitative approach and a descriptive design [8], which sought to measure and describe the variables associated with the competence of quantitative reasoning (interpretation, formulation and execution, and argumentation) starting of situation problems in real life mathematical and legal contexts.

A non-probabilistic sample was used for convenience formed by the teachers of the area and the 58 students of the Law program who attended the elective during the 2018-2 academic period. The phases of the study are described below:

2.1. Phase 1
A diagnostic test was applied to the students (pre-test) designed from the logic of the operationalization of the variable "quantitative reasoning" and considering each one of the mathematical thoughts.

2.2. Phase 2
Based on the results obtained in phase 1, three learning guides were designed (one for each process associated with the quantitative reasoning competence) based on Polya's problem-solving strategy: understanding the problem, configuring the plan, executing the plan and look back, developing in this way, the formation process of the competition.

This strategy was alternated with the technological mediation of the institutional platform "extended classroom" in order to strengthen the internal processes developed within the classroom.

2.3. Phase 3
A final test (post-test) was applied to measure the incidence of the strategy used in the formative process of the quantitative reasoning competence, as well as a focused interview to know the perception that the participating students had of the process around Polya’s problem solving.

Finally, the data was systematized and analyzed in a descriptive way (objective tests) and comprehensive (focused interview) to achieve the final interpretation of the applied process.

3. Results
The results obtained with the application of the diagnostic test (pre-test) allowed observing the low performance that students had about the "quantitative reasoning" competition.

From the fraction measured, only 19% of students passed the test with a grade between 3.0 and 3.9 (out of 5.0) compared to 81% who did not pass the pre-test (grades below 2.9 out of 5.0) (Figure 1).

![Figure 1. Diagnostic test (pre-test).](image-url)
At a general level, it was found that interpretive competence was the one that obtained the highest percentage, 41%, followed by argumentative competence with 22% and only 14% of the fraction of students measured was able to respond correctly to the situations concerning problems with the formulation and execution competence (Figure 2).

![Figure 2. Diagnosis by type of competence.](image)

This showed that students have greater difficulty in the processes related to the formulation and execution competence, therefore, difficulties related to the processes that are related to the approach of a mathematical process, design of strategies for the resolution of problems and processes of modeling, whose main dimension is the use of mathematical language [9] for the representation of information that is in a "common language" and is written in a proper mathematic language.

This competence also implies that the student uses different knowledge and concepts related to statistics, arithmetic, metric, geometric and algebraic operations, to strengthen the next step that consists of the development of mathematical argumentation processes [10] and thus solve a situation problem raised.

Regarding the results obtained in the second phase, three learning guides were designed, one for each quantitative reasoning competence [11,12], taking into consideration the procedure of Polya’s four steps (understanding the problem, developing a plan, executing the plan and look back).

These guides were resolved as group workshops based on solving problems in which they were proposed, modeled and solved mathematical problems and contextualized questions that contained concepts of the five types of mathematical thinking (numerical, variational, geometric, metric and random). Furthermore, they were the instrument for systematic monitoring and evaluation of the training process.

In the constructed guides, problems of the juridical and mathematical context were proposed, in view of the theoretical guidelines of the evidence matrix within the framework of the institutional project "curricular design for the strengthening of generic competences in undergraduate students" (see Table 1), in order to guarantee the balance of the evidences worked and the knowledge addressed in the training process. For the last phase, the added value of the implementation of this strategy was determined from a focused interview and a final test. The interviews were systematized and analyzed through three main categories: Attitude towards mathematics, Polya methodology and its impact. The questions addressed were:

a) What attitude have you had towards learning to solve mathematical problems in your academic education?
The students expressed that initially at the university in the subject of reasoning quantitative existed an attitude of conflict, apathy, laziness and fear of failure, since being students of the profile of social sciences had no prior knowledge of mathematics or cognitive structure to be able solve mathematical problems.

They mentioned that due to this lack of mathematics training in secondary education they presented great difficulty to understand a problem, identify it, define it or break it down into simpler tasks.

Some students answered this question that in high school the teaching and learning of mathematics did not address problem solving and those who did saw this strategy as something boring and meaningless, they were not willing to assimilate their learning.

**Table 1. Evidence Matrix Operationalization.**

| Competences          | Affirmations                                | Evidences                                      | Code   | Tasks or indicators / average performance level | Tasks or indicators / high performance level |
|----------------------|---------------------------------------------|-----------------------------------------------|--------|------------------------------------------------|---------------------------------------------|
| Interpretation and representation of data | Understands and transforms representations of quantitative data or mathematical objects, in different formats (texts, tables, graphics, diagrams, schemes) | Realizes the basic characteristics of the information presented in different formats such as series, graphs, tables and diagrams. | 1.1.1  | Selects among several representations the one pertinent to address a contextualized situation. | Recognizes the necessary information within a problem. |
|                      |                                             |                                               | 1.1.2  | Establishes criteria to choose the appropriate representation from a situation. |                                             |
|                      |                                             |                                               | 1.1.3  | Identifies the representation that fits the describing information in a particular situation. |                                             |
|                      |                                             |                                               | 1.2.1  | Extracts information from a graph that contains negative variations. |                                             |
|                      |                                             |                                               | 1.2.2  | Extracts information from a representation that allows to make inferences and process validation. |                                             |
|                      |                                             |                                               | 1.2.3  | Extrapolate data from a graph. |                                             |

b) How did you find the implementation of Polya's problem-solving methodological strategy?

The students mentioned that Polya problem solving is a concrete method where you can ask questions, define and execute an action plan and evaluate the solution of a mathematical problem, which requires as a fundamental step the detailed reading of the problem and its understanding besides having the basic mathematical knowledge to be able to design the solution strategies.
They were motivated by the learning of this methodology because, just as the method has a heuristic approach, that is to say a process formed by a series of steps, they saw the relationship and applicability with the academic formation of a law student because they use heuristics (process-norms) to solve legal cases.

c) What contributions has this strategy made to your professional training?

The students expressed that the strategy made them understand that knowledge should be put into practice and in this way helped them to form specific competencies such as identifying, interpreting and applying legal norms in order to be able to reason and solve cases of the legal context, skills needed to guarantee professional success.

We understood that the competences of quantitative reasoning were very much related to the competences that a lawyer must develop, since in order to solve a legal case a representation or reconstruction process is necessary, this representation can be obtained through auxiliary information and with the Support of other facts it is possible to infer the unknown. In the results of the final test (post-test) it can be observed in a general aspect that the three evaluated competences had a significant increase.

The interpretation competence from 41% of approval in the pre-test went to 81% in the post-test; the competence of formulation and execution, from 14% of approval in the pre-test went to 47% in the post-test and the argumentation competence from 22% of approval in the pre-test reached 52% in the post-test (see Figure 3).

![Figure 3. Results of the comparative analysis.](image)

4. Conclusions

Polya's problem-solving strategy allowed the formation of the competencies associated with quantitative reasoning (interpretation, formulation and execution, and argumentation), as well as the development of other transversal competences in the law program students of the Universidad Simón Bolívar, San José de Cúcuta, Colombia: teamwork and critical and reflective thinking.

Significantly improved in the academic performance of law program students, in the components related to the competence of quantitative reasoning from the Polya’s problem solving strategy and the use mediated used of the institutional platform.

Polya's problem-solving strategy, the development of the evidence matrix based on the operationalization of the quantitative reasoning competence and the construction of the learning guides for the competence teaching and learning that articulate both the strategy and the matrix of evidence, implies a critical, reflective and dedicated work by teachers.
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