Pruning Deep Neural Networks using Partial Least Squares

Artur Jordao, Ricardo Kloss*, Fernando Yamada and William Robson Schwartz
Smart Sense Laboratory, Computer Science Department
Universidade Federal de Minas Gerais, Brazil
Email: {arturjordao, rbk, fernandoakio, william}@dcc.ufmg.br

Abstract

To handle the high computational cost in deep convolutional networks, recent approaches have proposed to find and remove unimportant filters in these networks. Despite achieving remarkable results, these approaches demand a high computational cost mostly because the pruning is performed layer-by-layer, which requires many fine-tuning iterations. In this work, we propose a novel approach to efficiently remove filters in deep convolutional neural networks based on Partial Least Squares (PLS) and Variable Importance in Projection (VIP) to measure the importance of each filter, removing the unimportant (or least important) ones. These techniques allow us to estimate the filter importance based on its contribution in predicting the class label, which we show to be an adequate indicator to remove filters. We validate the proposed method on ImageNet, CIFAR-10 and Food-101 datasets, where it eliminates up to 65% of the filters and reduces 88% of the floating point operations (FLOPs) without penalizing the network accuracy. Additionally, sometimes the method is even able to improve the accuracy compared to the network without pruning. We show that employing PLS+VIP as the criterion for detecting the filters to be removed is better than recent feature selection techniques that have been employed by state-of-the-art pruning methods. Finally, we show that the proposed method is more efficient and achieves a higher reduction in FLOPs than existing methods.

1. Introduction

Convolutional neural networks have been an active research topic in Computer Vision mostly because they have achieved state-of-the-art results in numerous tasks [27, 25, 15]. Recent works explore the development of architectures, which is a key point for improving the performance in convolutional neural networks. It has been demonstrated that deeper architectures achieve better results. However, they are computationally expensive, present a large number of parameters and consume a considerable amount of memory. To handle these problems, there exist three groups of approaches: (i) $1 \times 1$ convolutional filters, which reduce the dimensionality of the input feature map by squeezing the depth variables, decreasing the number of parameters [27, 9]; (ii) binarization of weights and activations, which replaces arithmetic operations with bitwise operations, improving speed-up and memory requirements [14, 22, 18]; and (iii) pruning approaches, which remove neurons (i.e., filters) from a neural network, providing all the benefits of (i) and (ii) to deep architectures [12, 17, 13]. Based on these advantages, most efforts in the context of neural networks optimization have focused on pruning methods.

According to Han et al. [7], a pruning method can be defined as a three-step iterative process: locate potential neurons to be removed, rebuild the network without them and perform fine-tuning at the end of each iteration. At the step of finding neurons to eliminate, different criteria have been proposed, such as removing neurons with small L1-norm [7, 17] or zero activation [12], or learning a combination of neurons to be discarded [13]. At the rebuilding step, a new network is built (without the discarded neurons) and the weights of the kept neurons are transferred to it. It has been shown that transferring the weights is better than reinitializing them [12, 17, 13]. Finally, in the fine-tuning step, the new network is fine-tuned to compensate the neurons that have been removed. As demonstrated by Han et al. [7], this step is the most important and the one with the highest computational cost. Therefore, it is desirable for pruning methods to execute as few fine-tuning iterations as possible.

Despite being simple and presenting considerable results, existing approaches for pruning neural networks are done layer-by-layer (i.e., each layer is pruned individually at each pruning iteration), which requires many fine-tuning stages. Moreover, the criteria used to eliminate neurons either require human effort or demand a high computational cost. For instance, the method proposed by Li et al. [17]
The idea behind our method is to model the problem of finding and removing filters from convolutional networks as a feature selection problem. Our method works as follows. First of all, we represent the convolutional filters of the network as features. To this end, we present the training data to the network and interpret the output of each convolutional filter as a feature vector (or a set of features), as illustrated in Figure 1. After this stage, we create a high dimensional feature space, representing all convolutional filters of the network at once. Then, we project this high dimensional space onto a latent space using Partial Least Squares (PLS), a discriminative dimensionality reduction method that is ideal to very high-dimensional space and invariant to the number of training samples available [29, 26, 1]. Next, we employ the Variable Importance in Projection technique to estimate the contribution of each feature in generating the latent space, enabling PLS to operate as a feature selection method. The idea behind this process is that, since the filters are represented as features, we are estimating the filter importance w.r.t its contribution in predicting the class label (PLS criterion). Finally, we eliminate filters associated to the features with low importance. This process can be iteratively repeated until a specific number of iterations.

Motivated by the limitations in current pruning methods [12, 17, 13], we propose a novel approach to efficiently eliminate filters in convolutional neural networks. The core idea in our method is to model the problem of finding and removing filters from convolutional networks as a feature selection problem. Our method works as follows. First of all, we represent the convolutional filters of the network as features. To this end, we present the training data to the network and interpret the output of each convolutional filter as a feature vector (or a set of features), as illustrated in Figure 1. After this stage, we create a high dimensional feature space, representing all convolutional filters of the network at once. Then, we project this high dimensional space onto a latent space using Partial Least Squares (PLS), a discriminative dimensionality reduction method that is ideal to very high-dimensional space and invariant to the number of training samples available [29, 26, 1]. Next, we employ the Variable Importance in Projection technique to estimate the contribution of each feature in generating the latent space, enabling PLS to operate as a feature selection method. The idea behind this process is that, since the filters are represented as features, we are estimating the filter importance w.r.t its contribution in predicting the class label (PLS criterion). Finally, we eliminate filters associated to the features with low importance. This process can be iteratively repeated until a specific number of iterations.

It is important to observe that the proposed method eliminates filters coming from different layers simultaneously, thereby, it is extremely efficient in terms of the number of fine-tuning necessary. Different from the existing approaches, where the number of fine-tuning stages is equal to the number of layers (e.g., to VGG16 and ResNet56 with 16 and 18 convolutional layers, respectively), the number of fine-tuning stages in our method is equal to the number of iterations, which is more efficient. In general, each stage of fine-tuning consists in retraining the network on $n$ epochs. Therefore, to prune all layers in previous works [12, 17, 13] the number of epochs necessary is equal to $n \times \text{layers}$, while in our method the number of epochs is equal to $n \times \text{iterations}$. Specifically, with few iterations (i.e., 1-5) we achieve superior pruning performance.

We evaluate our method by pruning the VGG16 [19] and ResNet18 [9] architectures on ImageNet [4], CIFAR-10 [16] and Food-101 [2] datasets, where we are able to reduce up to 88% of the floating point operations (FLOPs) without penalizing network accuracy. With a negligible drop in accuracy, we can reduce up to 92% of FLOPs. Furthermore, sometimes, the method improves network accuracy. Experimental results show that using filter importance based on the PLS projection as the criterion for detecting the filters to be removed is extremely effective and better than recent feature selection techniques [24, 23], which have been used to prune neural networks.

2. Related Work

The idea in pruning networks is to find neurons that could be removed without degrading its original performance. To this end, several works have proposed different approaches to identify the potential neurons to be removed. In particular, most existing pruning approaches focus on eliminating neurons (filters) in convolutional layers since 99% of overall FLOPs are concentrated on these layers [7].

To find unimportant filters, Hu et al. [12] proposed to observe the percentage of zeros activations (APoZ) in the resulting feature map from a filter (coupled with ReLU activation). The idea behind the method is that filters which yield feature maps with large APoZ are redundant and could be removed. In their method [12], once APoZ from all filters have been computed, the ones larger than one standard deviation are eliminated and the resulting network is fine-tuned. Hu et al. [12] demonstrated that measuring the APoZ considering all layers, decreases network accuracy considerably and it is hard to recover its original performance, therefore, their method is executed layer-by-layer. Similar to [12], Li et al. [17] proposed to discard filters based on its...
magnitude. For this purpose, the authors compute the L1-norm from each filter and remove the filters with small magnitudes. Since filters coming from different layers present different magnitude orders, their L1-norms cannot be compared directly to decide which filters to remove. As a consequence, the approach of Li et al. [17] prunes the network considering one layer at the time followed by a fine-tune of the entire network. A notable drawback in their work is the requirement of human efforts to test different tradeoffs between the pruning rate and the network performance.

Different from the hand-crafted pruning criteria employed in [12, 17], Huang et al. [13] proposed a data-driven pruning method. Their method eliminates redundant filters by learning agents, which are encouraged to remove a large number of filters while preserving the network performance above a threshold. According to results presented by Huang et al. [13], for each agent (which is responsible for pruning a single layer) around 150 training iterations are necessary for convergence. In addition, after pruning a layer, some stages of fine-tuning are performed on the entire network. As a consequence, their method is unfeasible in deep networks. Similar to Huang et al. [13], He et al. [10] proposed a pruning strategy based on agents that learn to penalize accuracy loss and encourage model compression. These agents can employ different policy search protocols: (i) latency, where resulting network has the best accuracy given an amount of hardware resources and (ii) quality, where the output network is the smallest as possible with no loss of accuracy. We show that our method is able to achieve both criteria and is also more accurate.

To address the problem of eliminating neurons layer-by-layer, Yu et al. [30] proposed to remove the filters coming from all layers at once. To this end, the authors applied the Infinity Feature Selection [24] method on the final layer output to obtain the importance of each neuron. Then, this importance is recursively propagated throughout the network using a proposed neuron score propagation algorithm. Finally, neurons with low importance are removed and the network is fine-tuned.

In contrast to most existing pruning approaches [12, 17, 13], our method is able to remove filters from different layers at once. Therefore, it performs few fine-tuning stages, which makes the method extremely efficient. Additionally, since the PLS can be learned using few samples (e.g., 10% of training samples), our method is computationally efficient. We emphasize that this fact is an advantage of the PLS regarding the other dimensionality reduction and feature selection techniques applied to prune, such as Singular Value Decomposition (SVD) [5, 6], Principal Component Analysis (PCA) [28] and infinity feature selection [30]. Finally, due to our effective pruning criterion, we achieve a higher reduction in FLOPs (without degrading the network performance) than existing pruning techniques [12, 17, 13, 30].

3. Proposed Approach

This section defines the proposed method to eliminate filters in deep convolutional neural networks. We start by describing the representation of convolutional filters as feature vectors. Then, we explain the Partial Least Squares and Variable Importance in Projection techniques, which project the filter representation onto a low dimensional space and measure the filter importance, respectively. Finally, we describe how to remove filters with low importance. An overview of the proposed method is presented in Figure 2.

**Filter Representation.** The first step in our method is to represent the filters that compose the network as feature vectors. For this purpose, let us consider we have \( m \) data samples (i.e., the training samples), which are forwarded on the network to obtain the feature maps provided by each convolutional filter. Since these feature maps are high dimensional, we apply a pooling operation (i.e., max-pooling) to reduce their dimension. In this work, we consider the following pooling operations: global max-pooling, global average pooling and max-pooling \( 2 \times 2 \). Finally, the output of the pooling operation is interpreted directly as one feature (when using the global pooling operations) or a set of features (when using the max-pooling \( 2 \times 2 \)). Specifically, each filter is represented by its feature map followed by the pooling operation. Figure 1 illustrates this process.

The intuition for using the feature map as a feature is that we are able to measure its contribution to discriminate the classes (via PLS). This way, a filter associated with a feature with low discriminability might be removed.

**Dimensionality Reduction.** After executing the previous step, we have generated a high dimensional space \( \mathbb{R}^d \) that represents all filters of the convolutional network. The second step of our method is to project this high dimensional space onto a low dimensional space \( \mathbb{R}^c (c \ll d) \), referred to as latent space. To this end, we employ Partial Least Squares, a dimensionality reduction technique widely employed to predict a set of dependent variables from a (large) set of independent variables, that works as follows. Let \( X \subset \mathbb{R}^{m \times d} \) and \( y \subset \mathbb{R}^{m \times k} \) be a matrix of independent and dependent variables, respectively. In our method, the matrix \( X \) is the representation of the filters we have generated (first step of the proposed method) and \( y \) is the class label matrix, where \( k \) denotes the number of categories.
Algorithm 1: NIPALS Algorithm.

\[\text{Input} : X \subset R^{n \times d}, y \subset R^{m \times k}\]
\[\text{Input} : \text{Number of components } c\]
\[\text{Output} : \text{Projection matrix } W\]

\begin{algorithm}
\begin{algorithmic}[1]
\State \textbf{for } \(i = 1\) \textbf{to } \(c\) \textbf{do}
\State \text{randomly initialize } u \in R^{n \times 1}
\State \(w_i = \frac{X^T u}{\|X^T u\|}\), where \(w_i \in W\)
\State \(t_i = Xw_i, q_i = \frac{y^T t_i}{\|y^T t_i\|}\)
\State \(u = y q_i\)
\State \text{Repeat steps 3–5 until convergence}
\State \(p_i = X^T t_i\)
\State \(X = X - t_i p_i^T, y = y - t_i q_i^T\)
\EndFor
\end{algorithmic}
\end{algorithm}

PLS estimates a projection matrix \(W\) \((w_1, w_2, ..., w_c)\) which projects the high dimensional space \(R^d\) onto a low dimensional space \(R^c\) \((c \text{ is a parameter})\). In this new space, each component \(w_i \in W\) represents the maximum covariance between the \(X\) and \(y\), as shown in Equation 1.

\[w_i = \arg\max(\text{Cov}(Xw, y)), \text{a.t. } \|w\| = 1. \quad (1)\]

To solve Equation 1, we can use Nonlinear Iterative Partial Least Squares (NIPALS) \([1]\) or SVD. In this work, we use the NIPALS algorithm since it is faster than SVD. In addition, it allows us to find only the first \(c\) components, while SVD finds all \(d\) components, spending more computational resources. Algorithm 1 introduces the steps of NIPALS to obtain the first \(c\) components. The convergence step \(6)\) is achieved when no changes occur in \(w_i\). In addition, we might define a finite number of steps \((e.g., 2)\) as convergence criterion, to ensure that the method stops.

It should be noted that, in this step of our method, other dimensionality reduction methods could be employed, for instance, PCA or linear discriminant analysis (LDA). However, we believe that the idea behind PLS, which is to capture the correlation between the feature \((\text{in our context a filter})\) and its class label, is more suitable. In particular, when compared to LDA, PLS has not the restriction that the number of samples needs to be smaller or equal to the number of features. \([20]\). Moreover, as shown in our experiments, PLS can be learned using few samples, not requiring all the data to be available in advance. These advantages make PLS more flexible and efficient than traditional dimensionality reduction techniques, mainly for huge datasets.

Filter Importance. The next step in our method is to measure the filter importance score to remove the ones with low importance. To this end, once we have found the projection matrix \(W\) using Algorithm 1, we estimate the importance of each filter based on its contribution to yield the latent space. Recall that, following the modeling performed in the first step of our method, a feature (or a set of features when using the max-pooling representation) corresponds to a filter.

To achieve the aforementioned goal, we employ the Variable Importance in Projection (VIP) technique \([21]\), where the importance score of a filter, \(f_k\), can be computed by

\[f_k = \sqrt{\frac{d}{\sum_{i=1}^{c} SS_i \frac{W_{ik}}{\|W_{ik}\|^2}}}, \quad (2)\]

where \(SS_i\) is the sum of squares explained by the \(i\)-th component, which can alternatively be expressed as \(q_i^2 t_i^T t_i\) \((\text{defined in Algorithm 1})\) \([21]\). Note that when using the max-pooling operation as filter representation, we have a set of features for each filter; therefore, the final score to a filter on this representation is the average of its \(f_k\).

Prune and Fine-tuning. By computing the importance of all filters that compose the convolutional network, we have generated a set of scores, \(F(f_1, f_2, ..., f_k)\). Then, given a pruning ratio \(p\) \((e.g., 10\%)\), we remove \(p\%\) of the filters based on its scores. The removal stage consists of creating a new network, without the discarded filters, and transferring the weights of the kept filters \([7, 12, 17, 13]\).

By executing all the above steps, we have executed one iteration of the proposed method, as illustrated in Figure 2. It is important to note that the input network to the next iteration is the pruned network of the previous iteration.

After each iteration, we execute fine-tuning on the entire remaining network. According to previous pruning methods \([7, 17, 13]\), fine-tuning is recommended to compensate the removed filters. However, even though important, this represents one of the most expensive steps in the pruning approaches. Therefore, it is desirable to execute as few as possible fine-tuning stages. We emphasize that different from most of previous works, the proposed method executes a single fine-tuning step for each pruning iteration (see Figure 2). Furthermore, with few iterations \((e.g., 3\) to \(5)\) of the proposed method, we are able to achieve a higher pruning ratio and a smaller drop in accuracy than existing pruning techniques.

4. Experimental Results

In this section, we first introduce the experimental setup, the datasets and the employed convolutional networks. Then, we present the experiments regarding the parameters of our method and how to employ it in large datasets. Next, we show the advantages of executing the method iteratively. Finally, we compare our method with existing pruning criteria and approaches, and present qualitative results.
Experimental Setup and Datasets: We conduct experiments using a single NVIDIA GTX 1080 on a machine with 64GB of RAM. Following previous works [12, 13, 30], we validate the method using CIFAR-10 [16] and ImageNet [4] datasets. However, for the ImageNet dataset, we also use its 32 × 32 version since it has been demonstrated to be more challenging than the original version (224 × 224), therefore, we can evaluate the methods in a harder scenario. In addition, we use the Food-101 [2] dataset, which consists of 101,000 images of 101 different food types. The idea in using this dataset is to evaluate the methods on an intermediate dataset between CIFAR-10 and ImageNet in terms of the number of samples. We report the drop in accuracy using Top-1 accuracy on CIFAR-10 and Top-5 accuracy on ImageNet and Food-101 datasets. Finally, we compute FLOPs following the work of [17].

Convolutional Networks. Throughout the experiments, we prune two well-known deep networks, VGG16 [19] and ResNet56 [9]. We select these networks because they are widely employed in computer vision tasks such as object detection [15, 8] and face verification [3]. Similarly to previous works [17, 13], we examine some aspects and parameters of our method by considering VGG16 only on CIFAR-10, to simplify the experiments. Finally, we set the pruning rate of 10% in all experiments.

Number of Components. As explained in Section 3, the only parameter of PLS is the number of components, c. To estimate its best value, we create a set of possible c by varying it from 1 to 15 (step of 1) and evaluate the network accuracy for each value after executing one iteration of our method. To measure the network accuracy, we use a validation set with 10% of the training data. By using this process, we found that the value that resulted in the highest accuracy was $c = 2$, thus it was used in the remaining experiments (including the pruning on ResNet56).

Influence of the Filter Representation. One of the most important issues in our method is the pooling operation, referred to as filter representation, employed on the feature map provided by a filter. This experiment aims at validating this issue. For this purpose, we execute ten pruning iterations using different pooling operations. As illustrated in Figure 3, accuracy decreases slower when global max-pooling is employed. On the contrary, by using the max-pooling $2 \times 2$ accuracy drops faster, where at the 10th iteration the method drops 26 percentage points (p.p.) compared to the network without pruning. In addition, this representation has the drawback of consuming additional memory compared to the global operations (global and average) which reduce the feature map to one dimension.

Besides playing an important role in the pruning performance, the filter representation has an interesting aspect regarding scores assigned by VIP. Note that, to select a filter to be removed means that VIP assigned a low score to it. In particular, by modifying the filter representation, we drastically alter the selection of filters to be removed. Figure 4 illustrates this idea, where we show the relation between the pruning iteration and the number of removed filters per layer. According to Figure 4, the max-pooling representation eliminates a larger number of filters from layers 3 to 7, while the global average pooling has a similar distribution of the VIP scores, since it removes filters from all layers uniformly (except for layers 3, 10 and 11). On the other hand, the global max-pooling representation removes a larger number of filters from layers 3 to 9 and keeps the filters from layers 1 and 2. Finally, VIP assigns high scores for the filters from the layers 10 and 11. Based on the results, we used the global max-pooling as filter representation in the remaining experiments.

According to Li et al. [17], filters from the first layer should not be pruned since their removal degrades network performance significantly. According to Figure 4, our method is able to identify this because either it does not remove or it removes few filters from this layer, which indicates the suitability of PLS to identify the relevant filters. It is important to mention that in [17], the conclusion that the filters from the first layer are important was done by a human analyzing the accuracy drop when removing these filters. However, this is performed automatically in our work.

Number of samples to learn the PLS. A basic requirement in deep learning approaches is a large number of training samples to avoid overfitting and provide a good generalization. Based on this statement, large datasets have been proposed, e.g., ImageNet [4] and VGGFaces [3] with 1.2 and 3.31 million images. On these datasets, our method could be impracticable due to memory constraints since Ni-PALS requires the training samples ($X$ in Algorithm 1) be in memory. However, an advantage of PLS is that it can be learned with a very small number of samples. Therefore, we can subsample $X$ before executing Algorithm 1, enabling our method to operate on large datasets.

In this experiment, we intend to demonstrate that the
proposed method is robust when fewer samples are used to learn the PLS projection. To this end, we vary the percentage of training samples (using a uniform subsampling) used to compose $X$ in Algorithm 1. Table 1 shows the results obtained after one pruning iteration and it is possible to observe that the network accuracy is slightly changed as a function of the number of samples used to learn the PLS. In particular, sometimes, the accuracy is the same as employing 100% of the samples (e.g., using 20% or 60%). In addition, the difference between using 100% and 10% of the samples is only 0.1 p.p. Therefore, to conduct the experiments on ImageNet, we used only 10% of the samples.

Iterative pruning vs. single pruning. In this experiment, we show that it is more appropriate to execute our method iteratively, as illustrated in Figure 2, with a low pruning ratio (i.e. 10%) instead of using a single pruning iteration with a high pruning ratio. In other words, if we want to remove i.e. 40% of filters, it is better to execute some iterations of our method with a low pruning ratio instead of setting a pruning ratio of 40% and execute only a single iteration. To this end, we first execute five iterations of the proposed method with a pruning ratio of 10%. Then, after each iteration, we compute the percentage of removed filters, $p_i$. Finally, we use each $p_i$ as the pruning ratio to execute a single iteration of the method. According to the results showed in Table 2, performing our proposed method iteratively with a low pruning ratio is more effective than using it with a large pruning ratio, which led to a higher drop in accuracy. For instance, by executing five iterations of the method with a pruning ratio of 10%, we are able to remove 40% of filters while improving the network accuracy (indicated by negative values in Table 2). On the other hand, by applying a single iteration with a pruning ratio of 40%, the accuracy decreased 1.76 p.p.

Comparison with other pruning criteria. The idea behind this experiment is to compare the criterion employed by our method, which is the contribution of each filter on the latent space yielded by PLS, with the criteria proposed by Li et al. [17] and Hu et al. [12], the L1-norm and the APoZ, respectively. To perform the comparison, we use one iteration of pruning and follow the process suggested in [30], which consists of setting the same pruning ratio and modifying only the criteria to select the filters to be removed. Also, we compare PLS+VIP against the feature selection methods proposed in [24, 23] using the global max-pooling filter representation as input for the methods.

Table 3 shows the results obtained by these criteria, where we call our criterion PLS+VIP, using a pruning ratio of 10% on the CIFAR-10 and ImageNet datasets. According to the results, our criterion to define the filter importance is more suitable than L1-norm and APoZ, where it is the one with the lower drop in accuracy. While the results on CIFAR-10 dataset are similar, our method achieves a notable difference on the ImageNet dataset.

Comparison with existing pruning approaches. This experiment was the one where the methods achieved the best results in CIFAR-10 (validation set).

Table 2. Drop in accuracy (in percentage points) when executing our method with few iterations and a low pruning ratio (Iterative Pruning), and when executing a single iteration with a high pruning ratio (Single Pruning). Results on CIFAR-10 (test set). Negative values denote improvement w.r.t. the original network.

| Percentage of Removed Filters (%) | Iterative Pruning Accuracy ↓ | Single Pruning Accuracy ↓ |
|----------------------------------|-------------------------------|---------------------------|
| 10                               | −0.89 (it=1)                 | −0.89                     |
| 27                               | −1.08 (it=3)                 | −0.03                     |
| 40                               | −0.69 (it=5)                 | 1.76                      |
| 65                               | 1.56 (it=10)                 | 20.21                     |
experiment compares the proposed method with state-of-the-art pruning approaches. Following previous works [17, 13, 30], we discuss the methods based on their drop in accuracy. In addition, we report the results using one and five iterations of our method and the iteration where it achieved the closest drop in accuracy compared to the best method. Table 4 summarizes the results.

On the CIFAR-10 dataset, our method achieved the best tradeoff between the drop/improvement in accuracy and FLOPs reduction. When compared to [12, 17], we achieved around $2 \times$ more FLOPs reduction with superior improvement in accuracy on both networks. Compared to Huang et al. [13], our method decreased $1.5 \times$ more FLOPs with a smaller drop in accuracy on VGG16. In addition, by pruning ResNet56, our method achieved a higher FLOPs reduction than the most recent pruning approach [30], with negligible difference in accuracy drop. Note that, while our method achieved a FLOPs reduction $2$ p.p. smaller than He et al. [10], their drop in accuracy is about $3 \times$ higher. We also compared the proposed method with Li et al. [17] following the procedure of [30], which consists of employing a pruning ratio of $15\%$ and $25\%$ (Li et al. (A)-(B)) on each layer of ResNet56. By performing five iterations of the proposed method, we outperformed Li et al. (A) and (B) on both FLOPs reduction and accuracy improvement.

Similar to the CIFAR-10 dataset, on the Food-101 dataset, our method achieved the best FLOPs reduction without decreasing accuracy. Particularly, our method was able to reduce up to $75.88\%$ of the FLOPs while improving the accuracy of the original network by $3.72$ p.p. On the other hand, Hu et al. [12] and Li et al. [17] reduced only $34\%$ and $26\%$ of the FLOPs, respectively. Finally, on the ImageNet ($224 \times 224$ version) dataset with only three iterations of the proposed method, we were able to achieve the smaller drop in accuracy and $1.80 \times$ more FLOPs reduction than all the methods. Also, with two additional iterations, we decreased about $3 \times$ more FLOPs than all the methods. On the ImageNet $32 \times 32$, we reduce $1.9 \times$ and $1.21 \times$ more FLOPs than Li et al. [17] and Hu et al. [12], respectively, with only three iterations.

It is important to mention that, when evaluated on the original ImageNet, the drop in accuracy of the methods are small. In contrast, on the $32 \times 32$ version the drop is no-

Table 4. Comparison of existing pruning methods. Negative values denote improvement regarding the original network.

| Method      | Filters↓ | FLOPs↓ | Acc↓ |
|-------------|----------|--------|------|
| **VGG16 on CIFAR-10** |          |        |      |
| Li (A) [17] | -        | 10.40  | −0.06|
| Li (B) [17] | -        | 27.60  | −0.02|
| Huang et al. [13] | -        | 64.70  | 1.90 |
| Ours (it=1) | 4.34     | 7.95   | −1.03|
| Ours (it=5) | 17.60    | 31.48  | −0.46|
| Ours (it=8) | 24.49    | 48.01  | 0.34 |
| **ResNet56 on CIFAR-10** |          |        |      |
| Li (A) [17] | -        | 14.60  |      |
| Li (B) [17] | -        | 12.06  | 0.52 |
| Ours (it=1) | 2.04     | 3.66   | 0.46 |
| Ours (it=2) | 4.21     | 7.50   | 0.63 |
| Ours (it=5) | 10.46    | 17.66  | 1.21 |
| **VGG16 on Food-101** |          |        |      |
| Li et al. [12] | -        | 20.00  | 14.60|
| Hu et al. [12] | -        | 19.69  | 0.84 |
| He et al. [11] | -        | 20.00  | 1.7  |
| Wang et al. [28] | -        | 20.00  | 2.00 |
| He et al. [10] | -        | 20.00  | 1.40 |
| Ours (it=1) | 9.99     | 9.31   | −0.98|
| Ours (it=3) | 27.08    | 36.03  | 1.06 |
| Ours (it=5) | 40.93    | 59.27  | 2.2 |
| **VGG16 on ImageNet (224 × 224)** |          |        |      |
| Li et al. [12] | -        | 13.26  | 21.77|
| Li (A) [17] | 13.26    | 21.77  | 9.11 |
| Ours (it=1) | 9.99     | 20.17  | 6.36 |
| Ours (it=2) | 18.99    | 31.64  | 8.26 |
| Ours (it=3) | 27.08    | 41.54  | 10.89|
| **ResNet56 on ImageNet (32 × 32)** |          |        |      |
| Li et al. [17] | -        | 4.21   | 10.30|
| Ours (it=1) | 4.34     | 6.73   | 1.17 |
| Ours (it=5) | 5.87     | 9.62   | −1.03|
| Ours (it=9) | 7.65     | 13.37  | −0.76|
et al. [13] remove a large number of filters from the layers 9 to 13 (Figure 5 (b)), but they remove a small number of filters from other layers. On the contrary, our method eliminates a large number of filters from all layers, as shown in Figure 5 (b). In particular, we eliminate more than 50% of filters from layers 2 to 10, which are the ones with the large number of FLOPs, and more than 25% from the other layers. Hence, we are able to achieve a higher FLOPs reduction than existing state-of-the-art methods, which are biased in eliminating filters of particular layers.

Besides removing more FLOPs, our method is also computationally more efficient in terms of the number of fine-tuning necessary. For instance, the methods of Hu et al. [12], Li et al. [17] and Huang et al. [13] require 16 stages of fine-tuning to prune VGG16. On the other hand, our method achieves better results with only around five fine-tuning stages. Recall that, for each iteration of the proposed method, we execute a single stage of fine-tuning, thereby, the number of iterations define the number of fine-tuning stages. Similar to our method, the approaches of Yu et al [30] and He et al. [10] demand few fine-tuning stages, however, we achieve a better tradeoff between FLOPs reduction and accuracy drop, as seen in Table 4.

**Qualitative Results.** Our last experiment shows that the important regions in the image to predict the class label are preserved after pruning a network with the our method.

Figure 6 illustrates the attention maps of the VGG16 network on images from the ImageNet dataset. It is possible to note that our method preserves the important regions (warmer regions), which are the ones where the object is located. In addition, sometimes, our method locates class-discriminative regions better than the original network, e.g., Figure 6 (a)-(c). This is an effect of the PLS+VIP which focuses on keeping filters with high contribution in predicting the class label.

**5. Conclusions**

This work presented a computationally efficient and accurate pruning method to remove filters from convolutional neural networks. The method interprets each filter as a feature vector and creates a high dimensional space using these features. Then, it projects this space onto a low dimensional space, called latent space, using the Partial Least Squares. Finally, it estimates the importance of each feature (in our modeling a filter) to yield the latent space and removes the ones with low importance. The method is able to eliminate up to 65% of the filters which reduces 88% of the floating point operations without penalizing the network accuracy. In particular, the method is even able to improve the accuracy regarding the original network. In addition, with a negligible drop in accuracy, the method is able to remove 83% of the filters and reduce 90% of the floating point operations. Compared to existing state-of-the-art pruning methods, the proposed method is extremely efficient in terms of the number of fine-tuning necessary, since it eliminates filters coming from different layers simultaneously. Furthermore, our method achieves the highest FLOPs reduction.

Figure 6. Attention maps of the VGG16 network. From top to down. Input images; Attention maps from the original network; Attention maps from the pruned network (best viewed in color).
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