Phenomenology of quarkyonic matter in heavy-ion collisions

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Abstract. We give an overview of the possible phase structure of dense hadronic matter with $T \leq T_c$ and $\mu_Q \geq \Lambda_{QCD}$, arguing for the possibility of a percolating transition in the $T, \mu_Q, N_c$ space. We show that matter after this phase transition has many properties associated with “quarkyonic” matter. Finally, we show how the existence of this phase can be experimentally investigated using electromagnetic signatures. The details of these calculations are in \cite{1, 2, 3, 4}.

1. Quarkyonic matter
The study of nuclear matter at moderate ($T \sim 0 - 180$ MeV) temperature and large baryochemical potential ($\mu_Q = \mu_B/3 \sim \Lambda_{QCD} = 250$ MeV) has recently enjoyed new vigorous theoretical and experimental interest.

From the experimental side, this is due to the start of programs specifically aimed at exploring low-energy collisions with the latest detector technology \cite{5, 6, 7}.

From the theoretical side, it was realized that this regime presents both potential for very interesting physics and unique challenges. The latter come from the breakdown of most of the techniques used to study QCD: lattice gauge theory presents the well-known sign problem at finite chemical potential. Several approaches have been invented to deal with this, but the results are neither conclusive nor precise enough to draw any conclusions about the property of matter in the experimentally interesting region. Effective field theory is also problematic, since the typical momentum exchange is $\sim \mu_Q$, while the “fundamental scale of the theory” is $\Lambda_{QCD} \sim \mu_Q$. Hence, we expect effective field theories to be unreliable. The effect on deconfinement, a non-perturbative phenomenon, on chiral symmetry in the critical region adds an extra complication which is little understood \cite{8}.

This ambiguity leaves room for qualitatively new phenomena, and even new phases of matter, to arise. A recent proposal of this kind is quarkyonic matter \cite{9, 10}. The reason for conjecturing the existence of a new phase boils down to comparing the quark-hole screening with the gluon-gluon antiscreening at large chemical potential.

The “bag model phase diagram”, positing that deconfinement occurs when there is one hadron per hadronic size ($T \sim \Lambda_{QCD}$ or $\mu_Q \sim \Lambda_{QCD}$, small box in Fig. 1) is incompatible with the expected running in momentum space of the gluon self-energy (right side of Fig. 1). If confinement is broken when the screening by quark-hole pairs, proportional to $\mu_Q^2 N_c N_f$ at the Fermi surface (which decreases the effective coupling), overpowers anti-screening by gluon loops.
\( \sim N_c^2 \), then the low-temperature deconfinement point scales at least as \( \sim \Lambda_{QCD} \sqrt{N_c/N_f} \). (The bottom-right diagram of Fig. 1 shows higher loops yielding a \( (N_c/N_f)^{2 > 1/2} \) scaling [2].)

Thus, in contrast to bag model intuition, the phase diagram at \( N_c \to \infty \) looks like the one in Fig. 1, where as \( N_c \to \infty \) the deconfinement line becomes flat. At the same time, the transition to “nuclear matter”, with the baryonic density as order parameter, becomes infinitely sharp since the baryon mass is proportional to \( N_c \Lambda_{QCD} \). Therefore baryons drop out of the confined vacuum partition function entirely, but continue to be present at \( \mu_Q \geq \Lambda_{QCD} \).

\[
\begin{align*}
T & \quad \text{Bag model} \\
\mu_f \mu_b / N_c \sim 1 & \quad \text{Large } N_c \\
\mu_q \mu_h / N_c & \quad \text{perturbative } \beta \text{ Function}
\end{align*}
\]

**Figure 1.** Left: Expected QCD phase diagram in the large-\( N_c \) limit, with the dashed deconfinement line parametrically located at \( \mu_Q \propto \sqrt{N_c/N_f} \) (see text). In the insert, the phase diagram resulting from the bag model. Right: \( N_c \) and \( N_f \)-dependences in the gluon self-energy (top diagrams), and some higher-order contributions (bottom).

Hence unless there are non-perturbative contributions to the running of quark-quark interactions, which could in principle bring the critical \( \mu_Q \) for deconfinement down to \( N_c^0 \Lambda_{QCD} \), “nuclear matter” at \( \mu_Q \sim \Lambda_{QCD} \) should, at large \( N_c \), be in the confined phase. In configuration space, however, inter-quark distance is proportional to \( N_c^{-1/3} \): for large enough \( N_c \), then, one should be in the confinement regime yet somehow neighboring quarks should be so close that asymptotic freedom applies. The authors of [10] proposed a solution to this seeming contradiction by postulating matter in this regime exhibits quark-like degrees of freedom deep inside the Fermi surface (and hence a scaling \( \sim N_c^1 \) for the pressure) but baryonic excitations on the surface. Dense matter at \( \Lambda_{QCD} < \mu_Q < \sqrt{N_c/N_f} \Lambda_{QCD} \), with features of asymptotic freedom in configuration space but features of confinement in momentum space is known as “quarkyonic”.

**Figure 2.** The percolation (lines only) and deconfinement (lines and points) transition lines as a function of the baryon density \( \rho_B \) and the number of colors \( N_c \). \( F_T,S \) are two quark propagators, and \( \theta = T/T_c(\mu_Q = 0) \).
This is an interesting idea, but how much of quarkyonic dynamics survives at $N_c = 3$ and $N_f = 2, 3$ is an open question. It has long been known [4] that there are significant \textit{qualitative} differences between the $N_c \to \infty$ limit and $N_c = 3$. Baryons in the $N_c \to \infty$ regime have an excitation energy $\ll \Lambda_{\text{QCD}}$ and strong binding (binding energy scales as the baryon mass). The critical point for the nuclear liquid-gas phase transition is at $T, \mu_B \sim \Lambda_{\text{QCD}}$. Neither of these holds true in the real world, by at least an order of magnitude.

As argued in [1, 2], this indicates that the large-$N_c$ limit is separated from the real world by a percolation-type phase transition. The quarkyonic matter transition line is therefore bound to be curved in $N_c$ as well as $T, \mu_B$ space, the former being accessible only on the lattice. Indeed, some aspects of quarkyonic matter (the “Skyrme crystal” phase) are likely to be captured by the liquid-gas phase transition in the physical world. Other aspects, most importantly the appearance of quark degrees of freedom, could manifest themselves in our world [1] provided the deconfinement phase transition is far enough on the baryochemical potential axis.

In [1] it has been shown that, for a wide variety of reasonable propagators at a fixed baryonic number density of $\rho_B = \Lambda_{\text{QCD}}^3/8$, a percolation transition is found as $N_c$ is varied. If one identifies the percolation transition with the quarkyonic phase, deconfinement and percolation are indeed separate, and they cover different regions not only in $T$ and $\mu_B$, but also along $N_c$. Reference [2] further demonstrated that, as shown in Fig. 2, the critical density of percolation generally drops as $N_c$ is raised, while that of deconfinement rises. Hence, in the “low-$N_c$” limit deconfinement occurs before percolation, and hence the percolating phase, which requires baryonic wave functions, does not occur. In the “large-$N_c$” limit, on the other hand, percolation is physically realized at densities of $\mathcal{O}(1)$ baryons per baryonic size, below deconfinement. The critical regime separating these two is at $N_c \sim \mathcal{O}(2–8)$, and hence it is far from clear whether the quarkyonic phase occurs in our world. This makes the development of a quarkyonic matter \textit{phenomenology} of considerable interest.

2. Phenomenology of quarkyonic matter

Looking for quarkyonic matter in experiment presents some of the same difficulties inherent in looking for Quark-Gluon plasma. The system is dynamically evolving, and includes both a high-density phase (which could be quarkyonic) and a low-density phase which should be identical to a weakly interacting hadron gas.

In addition, the quarkyonic phase should include both “hadronic” characteristics (since hadrons continue to exist, and dynamics at “slow” scales should be hadronic), and “quark characteristics”, emerging via pQCD-like quark-hole interactions.

Perhaps the most specific signature of quarkyonic matter is given by electromagnetic interactions: Quark-hole scattering diagrams should, by the quarkyonic hypothesis, be essentially the same as quark-antiquark scatterings in asymptotically free pQCD. However, as shown in Fig. 3, quark wavefunctions should not be the same as free particle wavefunctions, but should contain a form factor which reflects the fact that quarks are still localized in baryons (there is no Fermi surface for antiquarks and gluons, and hence they should not exist as asymptotically free states in the confined phase).

Photon [3] and dilepton [2] spectra are both sensitive the the earliest, densest phase of the evolution of the system. They are also amenable to a rigorous calculation in the quarkyonic Ansatz (defined by the above assumptions), and details of the spectra can be used to distinguish between the “trivial” form factor of asymptotically free QGP and the quarkyonic form factor.

In Fig. 4 [3] we show the radial distribution, and the angular distribution (parametrized in azimuthal harmonic coefficients $v_2$) of Bremsstrahlung photons (the reaction hole-$q \to \gamma q$ hole). As can be seen, the photons exhibit a steeper $p_T$ spectrum than QGP, due to the localization of quarks in “slow” baryons. In addition, $v_2$ oscillates chaotically event-by-event around zero, since it is generated not by collective flow but by the anisotropies of the baryonic wavefunction.
Figure 3. The form factor for $\gamma q$ terms in scattering diagrams, encoding the baryon distribution, originating from the presence of a quark Fermi sphere.

Figure 4. The $p_T$ distribution and $v_2$ of Bremsstrahlung photons calculated with pQCD diagrams and the form factor in Fig. 3. The left panel shows the transverse momentum spectrum while the right panel shows the elliptic flow. While the effect of the hadronic phase and resonance decays have to be considered (see [11] for the background), these plots show the way to a possible experimental distinction between a quarkyonic-dominated system and other types of matter.

In conclusion, we have argued that, in addition to the deconfinement and chiral phase transition, the formation of a Fermi sphere of quarks could be associated with a percolation phase transition which curves on the number-of-colors axis in addition to energy- and number-density, effectively separating a large-$N_c$ from a small-$N_c$ limit at finite chemical potential. We argued that it is difficult to say conclusively where the physical $N_c = 3$ is located w.r.t. the percolation transition, but that experimentally electromagnetic signatures are our best bet in distinguishing quarkyonic matter from the more conventional states of QCD matter.

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