Quantum discord dynamics in the presence of initial system–cavity correlations

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Abstract

We analyse the roles of initial correlations between a two-qubit system and a dissipative cavity on quantum discord dynamics of two qubits. Considering two initial system–cavity states, we find that the initial system–cavity correlations not only initially increase the two-qubit quantum discord but also lead to a larger long-time quantum discord asymptotic value. Moreover, quantum discord due to initial correlations is more robust than the case of the initial factorized state. Finally, we show the initial correlations’ importance for dynamic behaviour of mutual information and classical correlation.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Recently, a lot of interest has been devoted to the definition and understanding of correlations in quantum systems. Entanglement is a kind of quantum correlation that has been playing a central role in quantum information and communication theory [1]. However there are other nonclassical correlations apart from entanglement [2–4] that can be of great importance to these fields. In order to characterize all nonclassical correlations, Ollivier and Zurek introduced what they called quantum discord. They marked the beginning of a new line of research shifting the attention from the entanglement versus separability dichotomy to the quantum versus classical paradigm. The quantum discord measures quantum correlations of a more general type than entanglement; there exist separable mixed states having nonzero discord [5]. Interestingly, it has been proven both theoretically and experimentally that such states provide computational speedup compared to classical states in some quantum computation models [5, 6].

Understanding of the quantum dynamics of an open system is a very important task in many areas of physics ranging from quantum optics to quantum information processing and to quantum cosmology. Therefore, the study of quantum and classical correlations’ dynamical behaviour in the presence of both Markovian [7, 8] and non-Markovian [9, 10] decoherence has attracted much attention in recent years. It is believed that the quantum correlations measured by the quantum discord, in the Markovian case, decay exponentially in time and vanish only asymptotically [11], contrary to the entanglement dynamics where sudden death may occur [12]. In particular, the authors of [13, 14] have discovered that quantum discord can be completely unaffected by Markovian depolarizing channels or non-Markovian depolarizing channels for long intervals of time, and this phenomenon has been observed experimentally [15]. In these above studies, quantum discord dynamics of an open system in which system and environment are initially separable has been analyzed in detail. As everyone knows it is possible, and often unavoidable in experiment, to create correlations between system and environment, therefore such correlations will play important roles in the time evolution of the system [16, 17]. So the study of quantum correlation dynamics due to the initial system–environment correlations is certainly necessary.
The influence of initial correlations on the open system dynamics has recently been intensively studied [18–25]. In order to clarify what happens to the quantum discord in this situation, we study an exactly solvable model for the time evolution of two atoms interacting with a lossy cavity. In this model where the coupling strength effects are considerable, the presence of system–cavity correlations invalidates the initial state in which system and cavity are independent. We investigate quantum discord dynamics for two initial states of two atoms and the lossy cavity. Not only are the reduced density matrices for the atomic system in these two different initial states the same, but also the reduced density matrices for the lossy cavity in them are equivalent. Under a certain condition, the initial system–cavity correlations not only can initially increase the atomic quantum discord but also would lead to a larger long-time quantum discord asymptotic value. For another condition, correlations between two atoms and lossy cavity can more effectively restrain the reduction of quantum discord than the case of the factorized state. Then we also analyze the different dynamic behaviour of mutual information and classical correlation due to the initial system–cavity correlations. These findings obtained from our paper show that dynamics of mutual information, quantum discord and classical correlation not only depend on the system degrees of freedom, the initial system–cavity correlations must also be properly taken into account.

2. Quantum discord and classical correlation

We now present a brief review of the classical correlation and quantum discord. In classical information theory, the information can be quantified by Shannon entropy $H(X) = -\sum_i p_i \log p_i$, where $p_i$ is the probability with $X$ being $i$. Similarly, the joint entropy, which measures the total uncertainty of a pair of random variables $X$ and $Y$, is defined as $H(X, Y) = -\sum_{i,j} p_{ij} \log p_{ij}$, with $p_{ij}$ being the probability in the case of $X = i$ and $Y = j$. Then the total correlation between $X$ and $Y$ can be measured by the mutual information which is defined as $I(X : Y) = H(X) + H(Y) - H(X, Y)$, whose quantum version can be written as [26]

$$I(X : Y) = S(1) + S(2) - S(1,2),$$

where $S(1) = -\text{Tr}(\rho \log \rho)$ is the von Neumann entropy of $\rho$, and $S(1) + S(2)$ and $S(1,2)$ are the reduced density matrix of $\rho_{1Y}$ by tracing out $Y(X)$. By introducing the conditional entropy $H(X|Y) = H(X) - H(Y)$, we can rewrite the mutual information as

$$I(X : Y) = H(X) - H(X|Y),$$

where $H(X|Y) = -\sum_{i,j} p_{ij} \log p_{ij}$ is the conditional entropy of the random variables $X$ and $Y$ for the average uncertainty about the value of $X$ given that the value of $Y$ is known. In order to generalize the above equation to the quantum domain, we measure the subsystem $Y$ by a complete set of projectors $|\prod_i \rho_{XY}|\prod_i p_i$, with $p_i = \text{Tr}_{XY} (|\prod_i \rho_{XY}|\prod_i)$. Then the quantum mutual information can alternatively be defined by

$$\mathcal{J}_{\prod_i}(X : Y) = S(1) - S(1,2),$$

where $S(1) = \sum_i p_i S(\rho_{X|i})$ is the conditional entropy of the quantum state. The above quantity strongly depends on the choice of the measurements $\{|\prod_i\}$. By maximizing $\mathcal{J}_{\prod_i}(X : Y)$ over all $\{|\prod_i\}$, we define the classical correlation between $X$ and $Y$

$$C(X : Y) = \max_{\prod_i} \mathcal{J}_{\prod_i}(X : Y),$$

and the quantum discord as

$$D(X : Y) = I(X : Y) - C(X : Y),$$

which is interpreted as a measure of the quantum correlation [2–4]. It is zero only for states with classical correlations and nonzero for states with quantum correlations. In particular, quantum discord is equal to the entanglement of formation for pure states; it is not true for mixed states, since some states present finite quantum discord even without entanglement [2].

3. The model

We consider two atoms $A$ and $B$ interacting with a dissipative cavity. The Hamiltonian of such a total system in the rotating-wave approximation is given by $H = H_0 + H_{\text{int}}$, which, in the basis $|gg\rangle, |eg\rangle, |ge\rangle, |ee\rangle$, reads

$$H_0 = \omega_0 (\sigma_a^+ \sigma_a^- + \sigma_b^+ \sigma_b^-) + \omega_a a^+ a,$$

$$H_{\text{int}} = \Omega (\sigma_a^+ \sigma_a^- + \sigma_b^+ \sigma_b^-) + h.c.;$$

here, $\sigma_a^+, \sigma_a^-$ are, respectively, the Pauli raising and lowering operators for atoms $A$ and $B$, $\omega_0$ is the Bohr frequency of two atoms, $a$ and $a^+$ are the annihilation and creation operators for the cavity mode, respectively, which is characterized by the frequency $\omega_c$ and the coupling constant $\Omega$. For the sake of simplicity, in the following we assume that two atoms interact resonantly with the dissipative cavity mode.

First, we focus on the case in which the total system contains only one excitation. The exact dynamics of two atoms is contained by the set of ordinary differential equations [27, 28]

$$\frac{dc_1}{dt} = \omega_c c_1 + \Omega b_1,$$

$$\frac{dc_2}{dt} = \omega_c c_2 + \Omega b_1,$$

$$\frac{db_1}{dt} = (\omega_c - i \Gamma/2)b_1 + \Omega c_1 + \Omega c_2,$$

where $\Gamma$ is the decay rate of the cavity mode, and $c_1, c_2$ and $b_1$ are the complex amplitudes for the states with only one excitation in the first atom, one excitation in the second atom and one excitation in the cavity mode, respectively. According to the initial state of the two atoms and their common lossy cavity ($c_1(0)$, $c_2(0)$ and $b_1(0)$), we can acquire the exact solutions ($c_1(t)$, $c_2(t)$ and $b_1(t)$) of the differential equations (8) easily through a computer program. Then, tracing out the cavity degree of freedom, we obtain the reduced density matrix
of the atomic system for the total system which only contains one excitation.

Then we treat another case where the total system contains two excitations. In this case the dynamics of two atoms can be effectively described by a four-state system in which three states are coupled to the cavity mode in a ladder configuration, and one state is completely decoupled from the other states [28]. In the basis $|0\rangle = |gg\rangle, |+\rangle = (|eg\rangle + |ge\rangle)/\sqrt{2}, |−\rangle = (|eg\rangle − |ge\rangle)/\sqrt{2}, |2\rangle = |ee\rangle$, the Hamiltonians (6) and (7) can be rewritten as

$$H_0' = 2\omega_0|2\rangle\langle 2| + \omega_0(|+\rangle\langle +| − |−\rangle\langle −|) + \omega_0 a^\dagger a, \quad (9)$$

$$H_{int}' = \sqrt{2}\Omega(|+\rangle\langle 0|a + |2\rangle\langle +|a|a⟩ + \text{h.c.} \quad (10)$$

From the total Hamiltonian given by equations (9) and (10), the subradiant state does not decay, and the superradiant state is coupled to states $|0\rangle$ and $|2\rangle$ via the cavity mode. The transitions $|0\rangle \rightarrow |+\rangle$ and $|+\rangle \rightarrow |2\rangle$ have the same frequencies and are identically coupled with the cavity. In the interaction picture, the dynamics of two atoms interacting with a lossy cavity can be treated in the following differential equations:

$$i\frac{dC_1}{dt} = 2\omega_0C_1 + \sqrt{2}\Omega B_1,$n

$$i\frac{dB_1}{dt} = (\omega_e − i\Gamma/2)B_1 + \omega_0B_1 + 2\Omega B_2 + \sqrt{2}\Omega|C_1\rangle\langle 2|,$n

$$i\frac{dB_2}{dt} = 2(\omega_e − i\Gamma/2)B_2 + 2\Omega B_1,$n

where $C_1, B_1$ and $B_2$ are the complex amplitudes for the states with two excitations in the two atoms, one excitation in the two atoms and one excitation in the cavity mode, and two excitations in the cavity mode, respectively. When the initial state of the two atoms and their common structured reservoir $(C_1(0), B_1(0)$ and $B_2(0))$ is given, we can also calculate the exact solutions $(C_1(t), B_1(t)$ and $B_2(t))$ of the differential equations (11) easily. Finally, we obtain the density matrix of the reduced atomic system by tracing out the cavity degree of freedom.

To illustrate the roles of the initial correlations between two atoms and cavity on the quantum discord dynamics of two atoms, we consider an initial condition $\rho_{ABc}^{(1)} = |\Psi\rangle_{ABc}\langle \Psi|$, with $|\Psi\rangle_{ABc} = a|ge0\rangle + b|eg0\rangle + \sqrt{2}\Omega|gg1\rangle$ (having correlations), and here $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$ $(\alpha, \beta, \gamma \neq 0)$, $|e\rangle$ and $|g\rangle$ are the excited and ground states of atoms and $|0\rangle$ and $|1\rangle$ are the vacuum and single-photon states of the lossy cavity, respectively. Obviously, two atoms and the cavity in this initial state are

$$\rho_{AB} = \begin{pmatrix} |\gamma|^2 & 0 & 0 & 0 \\ 0 & |\alpha|^2 & a\beta^* & 0 \\ 0 & a\beta & |\beta|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (12)$$

$$\rho_c = \begin{pmatrix} |\alpha|^2 + |\beta|^2 & 0 \\ 0 & |\gamma|^2 \end{pmatrix} \quad (13)$$

Therefore, the initial correlations must illustrate clear effects on the dynamics of the atomic quantum discord. In the following, by comparing to the second initial condition that the two atom system and the cavity are in the factorized state $\rho_{ABc}^{(2)} = \rho_{AB} \otimes \rho_c$, we mainly study the different atomic quantum discord dynamical behaviour due to these two initial atom–cavity states, which contain the identical states of subsystems.

For $\rho_{ABc}^{(1)}$, it is composed by the basis $|ge0\rangle, |eg0\rangle$ and $|gg1\rangle$, that is to say the initial correlation state $\rho_{ABc}^{(1)}$ satisfies the first case in which the total system only contains one excitation. So the evoulutional state of the total system at time $t$ can be calculated by equations (8). However, not only the basis $(|ge0\rangle, |eg0\rangle$ and $|gg1\rangle)$ but also the bases $(|ge1\rangle$ and $|eg1\rangle)$ are contained in the factorized state, so $\rho_{ABc}^{(2)}$ actually consists of two parts, one satisfying one excitation, and the other meeting two excitations. Hence the evoulutional density matrix of $\rho_{ABc}^{(1)}$ can be acquired by equations (8) and (11). Tracing out the cavity degree of freedom, we obtain the density matrix of the reduced atomic system for these two different initial states. In the basis $(|gg\rangle, |eg\rangle, |ge\rangle, |ee\rangle)$, we measure the atom $B$ from the matrix $\rho_{AB}(t)$ by projecting on $|\cos\theta|e\rangle_B + e^{i\phi}\sin\theta|g\rangle_B, e^{-i\phi}\sin\theta|e\rangle_B − \cos\theta|g\rangle_B$. Then the quantum discord and classical correlation could be calculated numerically using equations (4) and (5).

4. Numerical results and discussions

In this paper we study a system whose dynamics is described by the well-known damped Tavis–Cummings model. To see the effects of initial system–cavity correlations explicitly, we consider two distinct initial states, the system–cavity correlated state $\rho_{ABc}^{(1)}$ and the factorized state $\rho_{ABc}^{(2)}$. The reduced density matrices for the atomic system or the lossy cavity in these two initial states are the same. For simplicity, we choose the parameters $\alpha = \sin\theta\cos\varphi, \beta = \sin\theta\sin\varphi$ and $\gamma = \cos\theta$ $(\theta \in [0, \pi/2]$ and $\varphi \in [0, \pi])$. In figure 1, by choosing $\rho_{ABc}^{(1)}$ and $\rho_{ABc}^{(2)}$ as the initial states of the total system, we plot the time evolution of quantum discord for two atoms as a function of the dimensionless quantity $\Omega t$, with $\theta = \pi/3$ and $\varphi = 3\pi/4$. We observe that the atomic quantum discord can be initially increased to a maximum in the case $\rho_{ABc}^{(1)}$ (as shown in figure 1(a)), which contains system–cavity correlations at $t = 0$. While in the factorized initial state $\rho_{ABc}^{(2)}$, the initially decreased atomic quantum discord to a minimum would occur (as shown in figure 1(b)). Hereafter, how much would the coupling strength of our regime influence the quantum discord? Since $\Gamma$ is connected to the lossy cavity correlation time $\tau_B$ by the relation $\tau_B \approx 1/\Gamma$, and $\Omega$ is related to the time scale $\tau_B$ over which the state of the system changes, $\tau_R \approx 1/\Omega$. So $\Gamma/\Omega < 2$ characterizes the strong coupling regime, and $\Gamma/\Omega > 2$ means the weak coupling regime. Figure 1(a) presents the case for the initial system–cavity correlated state $\rho_{ABc}^{(1)}$; it can be seen that the initially increased quantum discord maximum increases with the decrease of the $\Gamma/\Omega$ corresponding to the enhancement of the coupling strength. We also clearly see that for the factorized initial state $\rho_{ABc}^{(2)}$ in figure 1(b), the initially decreased quantum discord minimum decreases with the decrease of $\Gamma/\Omega$.

Then, by investigating the subsystem of two atoms, we compareAT. Mol. Opt. Phys. 44 (2011) 035503
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the roles of different initial system–cavity states on the atomic quantum discord dynamics. Although the
initial reduced density matrices for the atomic system in $\rho^{(1)}_{ABC}$ and $\rho^{(2)}_{ABC}$ are the same as in the full calculation, figure 2 shows that the presence of the system--cavity correlations in the initial state changes the atomic quantum discord dynamics dramatically. In figures 2(a) and (b), quantum discord dynamics is given for the conditions $\theta = \pi/3$ and $\phi = 3\pi/4$ in the strong coupling regime and weak coupling regime, respectively. For the case $\rho^{(1)}_{ABC}$, the atomic quantum discord can first increase to a maximum, then periodically decrease to a long-time asymptotic value in the strong coupling regime. While in the weak coupling regime, quantum discord would reach a maximum at first and then gradually decrease to the asymptotic fixed value. For the other case $\rho^{(2)}_{ABC}$, quantum discord of two atoms initially reduces to a minimum, and eventually periodically increases to another long-time asymptotic value in the strong coupling regime. Because the subradiant state $|\rangle$ in two initial states does not decay in the evolution process, the long-time asymptotic quantum discord would be acquired. The final long-time asymptotic value due to the system--cavity correlations is much larger than the case of the factorized state. So we can conclude that the initial system--cavity correlations not only can initially increase the atomic quantum discord but also lead to a larger long-time quantum discord asymptotic value.

Figures 2(c) and (d) give the atomic quantum discord dynamics for the conditions $\theta = \pi/3$ and $\phi = \pi/4$ in the strong coupling regime and weak coupling regime, respectively. When $\phi = \pi/4$, the initial atomic state does not contain the subradiant state $|\rangle$; the asymptotic stationary state of two atoms is $\rho_{AA}(t \to \infty) = |gg\rangle\langle gg|$, so the long-time asymptotic value of the atomic quantum discord is zero. For both the initial system--cavity correlated state or the factorized state, in the strong coupling regime the atomic quantum discord presents damped oscillations while in the weak coupling regime quantum discord decays only asymptotically to zero. A comparison between the red solid curve and the dark dash-dotted curve in figures 2(c) and (d) reveals that for the same initial atomic state, the atomic quantum discord dynamics due to the initial system--cavity correlations is more robust than the case of the factorized state. This suggests that correlations between two atoms and the lossy cavity effectively restrain the reduction of atomic quantum discord.

To further understand the roles of different initial system--cavity states ($\rho^{(1)}_{ABC}$ and $\rho^{(2)}_{ABC}$) on dynamics of the total correlations, we study the atomic mutual information $I$ and
It is clearly found that for the initial correlated state time evolution of the atomic mutual information, quantum $\rho(\text{a, c and e})$ as the total initial state, and (ii) the factorized state decays to zero (figure 3(b)). Comparing to the case of the at first, then reduce asymptotically to zero. This finding value. While for the initial factorized state value at first, and then reduce to a long-time asymptotic value. The correlations between the qubits and a lossy cavity are found to have an important effect on the time evolution of quantum discord when two qubits couple with a common lossy cavity. Finally, we also analyse the different dynamic behaviour of mutual information and classical correlation due to the initial system–cavity correlations. In comparison with some recent work on the initial factorized state between the system and environment, our present work might be more practical to explain the total correlation dynamic behaviour of the system.

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