Effect of compressibility of ground on bearing capacity of foundation under moment loading

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ABSTRACT

Foundations of structures like earth retaining structures, abutments, waterfront structures, machines, oil/gas platforms in offshore areas, etc. are subjected to eccentric and/or inclined loading. Response of a rectangular foundation on the surface of the ground modeled as non-linear Winkler model is evaluated for its dependence on eccentricity of load, stiffness and ultimate stress of the ground. The ultimate bearing capacity of eccentrically loaded foundation is not only a function of width of the footing and eccentricity of load but also depends on the compressibility of the foundation soil. The ultimate load ratio of eccentrically loaded foundation with respect to concentrically loaded foundation is estimated based on both Meyerhof’s method and the proposed approach. The predicted values of ultimate load ratio compare well with the measured ones.

Keywords: moment loading, footing, ultimate load, subgrade, nonlinear

1 INTRODUCTION

Tall structures are often subjected to moment due to wind or other lateral forces which can cause them to tilt or rotate as a whole. Meyerhof (1953) proposed a method to evaluate effective width of the footing under eccentric loading. Most of the relationships for determining rotation of rigid footing due to moment are obtained on the basis of Winkler (one parameter) or the elastic continuum models based on Boussinesq’s or Mindlin’s expressions, e.g., Veletsos and Verbic (1974). Very limited data exist in the literature regarding the settlements and/or rotations of foundations beyond the linear response (Georgiadis and Butterfield, 1987; Montrasio and Nova, 1991, 1997., Negro et al., 2000). Existing solutions for effective footing width and stress distribution for eccentrically loaded footings do not consider the effects of compressibility of the ground. This paper presents an analysis of footing behavior under moment loading and re-examines the effective width concept.

2 STATEMENT OF THE PROBLEM

A rigid footing of length, L, and width, B, resting on ground surface and subjected to load, P, at an eccentricity, e, from the centre of the footing is considered (Fig. 1a). The foundation - ground response
is represented by a series of independent springs as in Winkler Model (Fig. 1b). The footing undergoes settlements, δl and δr at the left and the right edges respectively and rotates through an angle θ due to the moment caused by the eccentric load (Fig. 1c). The schematic distribution of contact pressure below the footing is shown in Figure 1(d). The modulus of subgrade reaction and the ultimate bearing stress of the foundation soil are respectively kc and qmax.

3 ANALYSIS

The stress, q, deformation, δ, response of the ground idealized as hyperbolic relation (Kondner, 1963) (Fig. 2) is

\[
q = \frac{k_c \delta}{1 + \frac{k_c \delta}{q_{\text{max}}}}
\]

where qmax - is the ultimate bearing stress of soil, k_c - the initial tangent subgrade modulus. For an applied eccentric vertical load, P, the footing rotates through an angle, θ, and the corresponding displacements are δl and δr at the left and the right edges of the footing respectively (Figure 1c). The settlement at any distance x, from the left edge of the footing is δx = δl + θ.x. The contact pressure, qx, at any point distance, x, due to settlement δx, is

\[
x = \frac{k_c \delta_x}{1 + \frac{k_c \delta_x}{q_{\text{max}}}}
\]

Substituting for δx in Eq. 2,

\[
x = \frac{k_c (\delta_l + \theta x)}{1 + \frac{k_c (\delta_l + \theta x)}{q_{\text{max}}}}
\]

The normalized contact pressure, q*, is defined as

\[
q^* = \frac{q_x}{q_{\text{max}}} = \frac{k_c \delta_x}{1 + \frac{k_c \delta_x}{q_{\text{max}}}} \times \frac{1}{q_{\text{max}}}
\]

(4)

Substituting for δx in Eq. 2, and normalizing with L, one gets

\[
q^* = \frac{k_c L (\delta_l + \theta X)}{1 + \frac{k_c L (\delta_l + \theta X)}{q_{\text{max}}}} \times \frac{1}{q_{\text{max}}}
\]

(5)

where \( \delta_l/L \) = normalized displacement below the left edge of the footing, X = x/L = normalized distance, \( \mu = k_c L/q_{\text{max}} \) = relative stiffness factor. Integrating the contact pressure, qx, (Fig. 1d) over the length of the footing, L and equating it to the vertical downward force, P, the force equilibrium equation is obtained as

\[
P = \int_0^L q_x \, B \, dx
\]

(6)

Substituting for qx from Eq. (3) in Eq. (6),

\[
P = \int_0^L \frac{k_c (\delta_l + \theta x)B}{1 + \frac{k_c (\delta_l + \theta x)}{q_{\text{max}}}} \, dx
\]

(7)

Normalizing Eq. 7 with L, one gets

\[
P^* = \frac{1}{k_c L^2 B} \int_0^L (\delta_l + \theta x) \, dx
\]

(8)

Integrating and simplifying Eq. 8 above, one gets

\[
P^* = \frac{1}{\mu^2 \theta} \left[ \mu \theta - \ln \left( \frac{1 + \mu (\delta_l + \theta)}{1 + \mu \delta_l} \right) \right]
\]

(9)

where \( P^* \) = Normalized load = P/k_c BL^2. The normalized load, P*, can be represented in terms of ratio of applied load to ultimate load, P** as

\[
P^* = P/k_c BL^2 = (P/P_{\text{max}}) \times (P_{\text{max}}/ k_c B L^2) = P^{**}/\mu
\]

(10)

where Pmax is maximum possible load on the footing = qmax (L×B) and \( P^{**} = P/ P_{\text{max}} \) = ratio of applied load to ultimate load. Similarly the moment, M, on the footing due to load, P, applied eccentrically leads to

\[
M^* = \frac{1}{2 \mu^2 \theta^2} \left[ (\mu \theta)^2 - 2 \mu \theta + 2(1 + \mu \delta_l) \ln \left( \frac{1 + \mu (\delta_l + \theta)}{1 + \mu \delta_l} \right) \right]
\]

(11)
where $M^* = M/ k_c B L^3 = P^* (e^*)$.

The normalized load, $P^*$ becomes critical load, $P^*_{cr(e^*)}$ for which the displacement below the left edge of the footing ($X=0$) becomes stationary. The load at which the foundation experiences stationery/limiting displacement at $X=0$, is defined as the critical load, $P^*_{cr(e^*)}$. The critical load, $P^*_{cr(e^*)}$ for which $\delta^*$ is stationary is given as

$$P^*_{cr(e^*)} = \frac{P^*}{k_c B L^3} = \frac{1}{\mu^2 \theta} \left[ \mu \theta - \ln \left( \frac{1 + \mu (\delta^* + \theta)}{1 + \mu \delta^*} \right) \right]$$

(12)

The two unknown parameters, $\theta$ and $\delta^*$ can be determined using $P^*$ from Eq. 9 and $M^*$ from Eq. 11 for an applied eccentricity, $e^*$, $\mu$ and $P^{**}$. The critical load, $P^*_{cr(e^* = 0)}$ is obtained for $e^* = 0$ by substituting $\theta = 0$ in Eq. 9.

$$P^*_{cr(e^* = 0)} = \frac{1}{\mu}$$

(13)

The critical load ratio is obtained by dividing Eq. 12 with Eq. 13 as

$$\frac{P^*_{cr(e^*)}}{P^*_{cr(e^* = 0)}} = \frac{1}{\mu} \left[ \mu \theta - \ln \left( \frac{1 + \mu (\delta^* + \theta)}{1 + \mu \delta^*} \right) \right]$$

(14)

Meyerhof (1953) proposed effective area method for eccentrically loaded footing of width, $B$ and length, $L$ and eccentricity, $e$. The effective area, $A'$ is given by $A' = B' L'$, where $B' = B - 2e$ and $L' = L - 2e$. The critical load ratio based on Meyerhof’s theory is given by

$$\frac{P^*_{cr(e^*)}}{P^*_{cr(e^* = 0)}} = 1 - 2e^*$$

(15)

where $e^*$ is normalized eccentricity $= e/B$.

4 RESULTS AND DISCUSSION

A parametric study is carried out to analyze the effect of foundation soil properties on load-displacement and moment-rotation responses of eccentrically loaded rectangular footing. The ranges of different parameters used in the study are: normalized eccentricity, $e^* (= e/B)$ is 0 to 0.3. The relative stiffness factor, $\mu$ is taken as 100.

Figure 3 depicts the variation of normalised displacement, $\delta^*$, with normalised distance, $X$, for different values of $P^{**}$ for $\mu$ of 100 and $e^* = 0.05$. The normalised displacements, $\delta^*$, ($\delta^* = \delta^* + 0.0X$), are 0.0016 and 0.0049 at $X=0$ and $X=1$ respectively for applied normalised load, $P^{**}$ equal to 20% of the ultimate load. The displacement at $X=1$ increases to 0.017 for $P^{**} = 50\%$ of ultimate load, an increase of 364% while the value at the left end ($X=0$) increases to 0.0034 i.e., by about 149%. The displacement at $X=1$ reaches 0.029 while that at $X=0$ remains almost constant with further increase of $P^{**}$ to 60% of ultimate load with no perceptable increase of $\delta$, at $X=0$. The displacement at the left end, $X=0$ becomes stationary for $P^{**}$ equal to 65% of the ultimate value while that at the right edge, $X = 1$ increases to 0.0378.

The variations of normalised load, $P^{**}$, with normalised central (when $X = 0.5$) displacement, $\delta^*$, for different $e^*$ and for $\mu$ of 100 are shown in Figure 4. The curves are plotted up to the respective critical loads, $P^*_{cr}$. The foundation undergoes unloading or may lose its contact with the base for $P^*_{cr} > 0.65$. As the response of the ground is non-linear, the foundation has to rotate significantly to generate larger displacements towards the loading side and consequently larger pressures to counter the increased load and the corresponding moment for a given eccentricity, $e^*$.

The variations of normalised load, $P^{**}$, with normalised central (when $X = 0.5$) displacement, $\delta^*$, ($\delta^* = \delta^* + 0.0X$), for different $e^*$ and for $\mu$ of 100 are shown in Figure 4. The curves are plotted up to the respective critical loads, $P^*_{cr}$ for each normalised eccentricity, $e^*$. The displacement, $\delta^*$, for any given load, $P^*$ increases with increase of $e^*$ reflecting the major effect of eccentricity of applied load on the footing response. $P^*_{cr}$ reduces to about 44.4 % for $e^*$ is 0.1 with respect to concentric ($e^* = 0.0$) load. The critical load decreases with increase in normalised eccentricity, $e^*$, because of the increased rotation induced by the non-linearity of the soil - foundation response to counter the moments generated by the eccentric load. The effect of relative stiffness factor, $\mu$ on normalised load, $P^{**}$ versus displacement, $\delta^*$, response for normalised eccentricity, $e^*$ of 0.05 is presented in Figure 5. The non-dimensional parameter $\mu$ is defined as $k_c L/q_{max}$. 

Fig. 3. Variation of Normalised displacement, $\delta^*$ with normalised distance, $X$ ($\mu = 100, e^* = 0.05$).
Normalised displacement at centre of the footing, $\delta_c^*$

$\mu = \frac{1}{100}$

Fig. 4. Effect of eccentricity ($e'$) on normalised load–normalised displacement ($\mu=100$).

The parameter, $\mu$, increases with increasing value of $k_c$ and/or $L$ and with decreasing ultimate load, $q_{max}$. The normalised displacement, $\delta_c^*$ decreases with increase in $\mu$ value for any given value of $P^{**}$. For a given $q_{max}$, higher value of $\mu$ indicates stiffer ground. The asymptotic value of the ultimate load is reached at much lower normalised displacement, $\delta_c^*$ for higher values of $\mu$ (stiffer subgrade) than that for lower values of $\mu$ (weaker subgrade). At a constant value of $P^{**}$ of 0.4, the displacements, $\delta_c^*$, are 0.01, 0.022 and 0.075 for $\mu$ values of 100, 30 and 10 respectively. The critical load ratio versus normalised eccentricity, $e^*$ predicted based on proposed approach is compared with Meyerhof’s approach and measured data of Georgiadis and Butterfield (1987) in Figure 6. Georgiadis and Butterfield (1987) present results in the form of curves for vertical load versus displacement at centre of the footing, $\delta_c$. The normalised eccentricities, $e^*$ used by Georgiadis and Butterfield (1987) are 0, 1/24, 1/12, 1/6, 1/4, and 1/3 on a footing of 50 mm width. Very good agreement can be noted between predicted values and measured data. The critical load ratio estimated based on Meyerhof’s effective width concept are in the range of about 36 % to 73% higher than the predicted values based on proposed approach for an applied normalised eccentricity, $e^*$ of 1/12 and 1/3 respectively.

Fig. 5. Effect of $\mu$ on load-displacement for $e^* = 0.05$.

6 CONCLUSIONS
Response of a rigid footing subjected to eccentric loading is analysed modelling the response of the ground as a Winkler model incorporating hyperbolic stress – displacement response. Normalised force, $P^{**}$ – normalised displacement, $\delta^*$, responses are obtained in terms of normalised eccentricity, $e^*$ and relative stiffness parametr, $\mu$. The displacements of the near edge, that is on the side of the loading increase rapidly with increasing load for all eccentricities and relative stiffness parameters while those for farthest edge, that is on other side of the footing increase with decreasing rate and reach a stationery value for a particular load. The load at which the displacement at the farthest edge becomes stationery is termed as the critical load. Critical load decreases with increasing values of eccentricity but is not much sensitive to relative stiffness factor. The critical load ratio predicted by the proposed approach compare very well with with those reported by Georgiadis and Butterfield (1987) but are less than those predicted by Meyerhof’s approach. It is thus shown that Meyerhof’s approach based on linear analysis of foundation response results in higher values of ultimate loads as the bearing capacity is effected by the non-linear properties along with load eccentricity.
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