Non-Abelian tensor multiplet in four dimensions

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Abstract. The long-standing problem with a non-Abelian tensor with non-trivial consistent
couplings in four dimensions has been solved. The key technique is double-fold: (1) Adding
extra Chern-Simons terms for the field strength of non-Abelian tensor, and (2) employing a
compensator mechanism. We generalize this mechanism to supersymmetric system. Our system
has three multiplets: (i) The usual non-Abelian vector multiplet (VM) (A_µ^I, \lambda^I), (ii) A non-
Abelian tensor multiplet (TM) (B_µν^I, \chi^I, \phi^I), and (iii) A compensator vector multiplet (CVM)
(C_µ^I, \rho^I). The indices I, J, ··· are for the adjoint representation of a non-Abelian group
G. All of our fields are propagating with kinetic terms. The C_µ^I-field plays the role of a compensator
absorbed into the longitudinal component of B_µν^I. We give both the component lagrangian
and a corresponding superspace reformulation, reconfirming the total consistency of the system.

1. The Conventional Problem with Non-Abelian Tensors

First, the conventional problem with non-Abelian tensors will be discussed. A solution will be
presented. Also presented will be a supersymmetrized physical non-Abelian field with consistent
interactions [1], both in component language and superfield language.

The long-standing problem with a non-Abelian tensor is described as follows. Let I be
the adjoint index of a non-Abelian group G gauged by the a non-Abelian vector field
A_µ^I, minimally coupled to the antisymmetric tensor B_µν^I with the coupling constant g.
Consider the conventional field strength

\[ G_μρ^I ≡ +3D_μ B_ρ^I ≡ +3(∂_μ B_ρ^I) + gf^{IJK} A_μ^J B_ρ^K , \]

(1.1)

where D_μ is the usual gauge-covariant derivative with the structure constant f^{IJK} of the
group G. Consider an action I_0 ≡ \int d^4x L_0 with the lagrangian

\[ L_0 ≡ -\frac{1}{12} (G_μρ^I)^2 - \frac{1}{4} (F_μ^I)^2 , \]

(1.2)

with F_μ^I ≡ 2∂_μ A_I^J + gf^{IJK} A_μ^J A_ν^K. Obviously, the B-field equation is

\[ \frac{δL_0}{δB_μ^I} = +\frac{1}{2} D_μ G^{(0)μρ^I} ≜ 0 . \]

(1.3)

3 The formulation in this section is the so-called ‘tensor hierarchy’ [2, 3, 4, 5], but we re-formulate their general
expressions in terms of our objectives here.

4 We use the signature (−, +, +, +) for four dimensions (4D) as in [1].

5 The symbol \(\triangleq\) stands for a field equation, to be distinguished from an algebraic identity. We also use the
symbol \(\equiv\) for an equality under question.
The problem is that the divergence of this $B$-field equation does not vanish:

$$0 \neq D_\nu \left( \frac{\delta L_0}{\delta B_{\mu \nu}^I} \right) = + \frac{1}{4} g f^{IJK} F_{\nu \rho}^J G^{(0)\mu \rho \gamma K} \neq 0 ,$$  \hspace{1cm} (1.4)

unless $F_{\mu \nu}^I$ or $G_{\mu \nu \rho}^{(0)I}$ vanishes trivially. This inconsistency arises already at the classical level. This is also one of the reasons, why topological formulations with vanishing field strengths $F_{\mu \nu}^I \equiv 0$, $G_{\mu \nu \rho}^{(0)I} \equiv 0$ such as in [6] are easier to formulate for non-Abelian tensors.

An additional problem is related to the so-called local tensorial gauge transformation of the $B$-field:

$$\delta_\beta B_{\mu \nu}^I = + D_{[\mu}^\beta B_{\nu]}^I - D_{[\nu}^\beta B_{\mu]}^I ,$$  \hspace{1cm} (1.5)

because the field strength $G_{\mu \nu}^I$ is not invariant under $\delta_\beta$:

$$\delta_\beta G_{\mu \nu}^{(0)I} = + 3 g f^{IJK} F_{\mu \nu}^J \gamma^K \neq 0 .$$  \hspace{1cm} (1.6)

This further implies the non-invariance of the action: $\delta_\beta I_0 \neq 0$. These two problems are mutually related, because the non-vanishing of (1.4) is also re-casted into $\delta_\beta I_0 \neq 0$.

2. Solution to the Problem

The solution to the problem above is to introduce a non-trivial Chern-Simons (CS) term into the $G$-field strength:

$$G_{\mu \nu \rho}^I \equiv + 3(\partial_{[\mu} B_{\nu \rho]}^I + g f^{IJK} A_{[\mu}^J B_{\nu \rho]}^K) - 3 f^{IJK} C_{[\mu}^J F_{\nu \rho]}^K$$

$$= + 3 D_{[\mu}^\beta B_{\nu \rho]}^I - 3 f^{IJK} C_{[\mu}^J F_{\nu \rho]}^K \equiv + G_{\mu \nu \rho}^{(0)I} - 3 f^{IJK} C_{[\mu}^J F_{\nu \rho]}^K ,$$  \hspace{1cm} (2.1)

where $C_{\mu}^I$ is a ‘compensator’ vector field, also carrying the adjoint index. The field strength for $C$ is defined by

$$H_{\mu \nu}^I \equiv + D_{[\mu} C_{\nu]}^I - D_{[\nu} C_{\mu]}^I + g B_{\mu \nu}^I .$$  \hspace{1cm} (2.2)

Now these field strengths $G$ and $H$ are invariant under the $\delta_\beta$-transformation:

$$\delta_\beta B_{\mu \nu}^I = + D_{[\mu}^\beta B_{\nu]}^I - D_{[\nu}^\beta B_{\mu]}^I$$  \hspace{1cm} (2.3a)

$$\delta_\beta C_{\mu}^I = - g \beta_{\mu}^I ,$$  \hspace{1cm} (2.3b)

which is the ‘proper’ gauge transformation for $B_{\mu \nu}^I$, and $\gamma$-transformation

$$\delta_\gamma B_{\mu \nu}^I = - f^{IJK} F_{\mu \nu}^J \gamma^K ,$$  \hspace{1cm} (2.4a)

$$\delta_\gamma C_{\mu}^I = D_{\mu}^\gamma I ,$$  \hspace{1cm} (2.4b)

which is the ‘proper’ gauge transformation for $C_{\mu}^I$. As (2.3b) shows, $C_{\mu}^I$ is a compensator field for the $\delta_\beta$-transformation.

The role played by the $C \wedge F$-term in (2.1) is to cancel the unwanted term in (1.6). The $C$-field itself should have its own ‘gauge’ transformation as the covariant gradient (2.4b). The contribution of $\delta_\gamma (2 D_{[\mu} C_{\nu]}^I)$ in (2.2) is cancelled by the contribution of $\delta_\gamma (g B_{\mu \nu}^I)$. In other words, we have the total invariances

$$\delta_\beta (G_{\mu \nu \rho}^I, H_{\mu \nu}^I) = (0, 0) , \ \delta_\gamma (G_{\mu \nu \rho}^I, H_{\mu \nu}^I) = (0, 0) .$$  \hspace{1cm} (2.5)
Accordingly, we also have the consistency problem (1.4) solved, by considering the lagrangian
\[ \mathcal{L}_1 = \left( G_{l\mu l} \right)^2 - \frac{1}{3} \left( H_{\mu l} \right)^2 - \frac{1}{3} \left( F_{\mu l} \right)^2 \].
(2.6)

The total action is also invariant \( \delta \beta I_1 = \delta \gamma I_1 = 0 \). The field equations for \( B \) and \( C \)-fields are
\[ \frac{\delta \mathcal{L}_1}{\delta B_{\mu l}^I} = + \frac{1}{2} D_\rho G_{\mu l}^\rho I - \frac{1}{2} g H_{\mu l} - \delta B_{\mu l}^I = 0 \],
(2.7a)
\[ \frac{\delta \mathcal{L}_1}{\delta C_{\mu l}^I} = - D_\rho H_{\mu l}^I + \frac{1}{2} f^{IJK} F_{\rho l}^J G_{\mu l}^\rho K - \delta C_{\mu l}^I = 0 \],
(2.7b)

The divergence of the \( B \)-field equation vanishes now:
\[ 0 \equiv D_\rho \left( \frac{\delta \mathcal{L}_1}{\delta B_{\mu l}^I} \right) = + \frac{1}{2} g \left( \frac{\delta \mathcal{L}_1}{\delta C_{\mu l}^I} \right) = 0 \],
(2.8)
where the last equality holds because of the \( C \)-field equation. In other words, the unwanted \( FG \)-term in (1.4) is now cancelled by the contribution of the \( C \)-field equation.

Relevantly, the divergence of (2.7b) also vanishes, as it should without any inconsistency:
\[ 0 \equiv D_\rho \left( \frac{\delta \mathcal{L}_1}{\delta C_{\mu l}^I} \right) = + f^{IJK} F_{\rho l}^J \left( \frac{\delta \mathcal{L}_1}{\delta B_{\mu l}^K} \right) = 0 \].
(2.9)

We emphasize repeatedly that these invariances have never been accomplished without the peculiar CS terms both in (2.1) and (2.2) [2, 3, 4, 5].

### 3. Component Formulation of \( N=1 \) TM

The supersymmetrization of the purely bosonic system (2.6) has been accomplished in our recent paper [1]. We need three multiplets: (i) A tensor multiplet (TM) \((B_{\mu l}^I, \chi^I, \varphi^I)\), (ii) A Yang-Mills vector multiplet (YMVM) \((A_{\mu l}^I, \lambda^I)\), and (iii) A compensating vector multiplet (CVM) \((C_{\mu l}^I, \rho^I)\). Our total action \( I \equiv \int d^4 x g^2 \mathcal{L} \) has the lagrangian [1]
\[ \mathcal{L} = - \frac{1}{12} \left( G_{l\mu l} \right)^2 + \frac{1}{2} \left( \chi^I D^{I} \chi^I \right) \left( \frac{1}{2} D_\mu \varphi^j \right)^2 - \frac{1}{2} g^2 \left( \varphi^j \right)^2 - g \left( \chi^I \rho^I \right) \]
\[ - \frac{1}{4} \left( H_{\mu l} \right)^2 + \frac{1}{2} \left( \rho^I D_\mu \varphi^j \right)^2 - \frac{1}{4} \left( F_{\mu l} \right)^2 + \frac{1}{2} \left( \chi^I D^{I} \chi^I \right) \]
\[ - \frac{1}{2} g f^{IJK} \left( \chi^I \chi^J \right) \varphi^K + \frac{1}{2} f^{IJK} \left( \chi^I \gamma^\mu \varphi^j \right) D_\mu \varphi^K + \frac{1}{12} f^{IJK} \left( \chi^I \gamma^\mu \gamma^\nu \gamma^\rho \right) G_{\mu l}^\rho K \]
\[ + \frac{1}{4} f^{IJK} \left( \rho^I \gamma^\mu \gamma^\nu \gamma^\rho \right) F_{\mu l}^J - \frac{1}{4} f^{IJK} \left( \chi^I \gamma^\mu \varphi^j \right) H_{\mu l}^J - \frac{1}{2} f^{IJK} F_{\mu l}^I H_{\nu l}^J \varphi^K \].
(3.1)

up to quartic-order terms \( \mathcal{O}(\varphi^4) \), and the coupling constant \( g \) has the dimension of mass.

The scalar \( \varphi^I \) has its mass \( g \), while there is a mixture between \( \chi^I \) and \( \rho^I \) with the same mass \( g \). As has been mentioned after (2.4), \( C_{\mu l}^I \) is a compensator field [7], absorbed into the longitudinal component of \( B_{\mu l}^I \). The kinetic term of the \( C \)-field becomes the mass term of \( B_{\mu l}^I \). Accordingly, the degrees of freedom (DOF) for the massive TM fields are \( B_{\mu l}^I \) (3), \( \chi \) with \( \rho^I \) (4) and \( \varphi^I \) (1).
Our action $I$ is invariant under global $N = 1$ supersymmetry [1]

$$
\delta_Q B_{\mu\nu}^I = \frac{1}{2} \epsilon_{\mu
u\rho} \chi^I - 2 f^{IJK} C_{[\mu}^J (\delta_Q A_{\nu]}^K) ,
$$  

(3.2a)

$$
\delta_Q \chi^I = \left( \frac{1}{6} \epsilon_{\mu
u\rho} \right) G_{\mu\nu\rho}^I - \epsilon_{\mu\nu\rho} \phi^I
\left( \frac{1}{2} f^{IJK} [ + \epsilon (X^{I} \rho^J) - (\gamma_5 \gamma^\mu \epsilon)(X^{I} \gamma_5 \gamma_5 \gamma_0^J) - (\gamma_5 \gamma_5 \gamma_0 \epsilon)(X^{I} \gamma^\mu \gamma_5 \gamma_5 \gamma_0^J) - (\gamma_5 \gamma_5 \gamma_0 \epsilon)(X^{I} \gamma_5 \gamma_5 \gamma_0 \gamma_5) ] \right) ,
$$  

(3.2b)

$$
\delta_Q \phi^I = \frac{1}{2} \epsilon_{\mu\nu\rho} \chi^I ,
$$  

(3.2c)

$$
\delta_Q A_{\mu}^I = \frac{1}{2} \epsilon_{\mu\nu\rho} \phi^I + 2 f^{IJK} (\gamma_5 \epsilon)^{IJK} (\gamma_5 \epsilon) \phi^K ,
$$  

(3.2d)

$$
\delta_Q \beta^I = \frac{1}{2} \epsilon_{\mu\nu\rho} \phi^I + 2 f^{IJK} (\gamma_5 \epsilon)^{IJK} (\gamma_5 \epsilon) \phi^K ,
$$  

(3.2e)

$$
\delta_Q \gamma^I = \frac{1}{2} \epsilon_{\mu\nu\rho} \phi^I + 2 f^{IJK} (\gamma_5 \epsilon)^{IJK} (\gamma_5 \epsilon) \phi^K ,
$$  

(3.2f)

up to $O(\phi^3)$-terms.

Our tensorial gauge transformation $\delta_\beta$, and $\delta_\gamma$-transformation are exactly the same as (2.3) and (2.4), while all the fermionic fields transform only under the usual non-Abelian gauge transformation $\delta_\phi$, as the $B$ and $C$-fields do, so that there is no problem with the $\delta_\beta I = 0$ and $\delta_\gamma I = 0$ of the field strengths as in (2.1) and (2.2), via (2.5).

The $\delta_Q$-transformations of the field strengths reflect their CS terms:

$$
\delta_Q G_{\mu\nu\rho}^I = 3 (\epsilon_{\mu\nu\rho} D_\rho^I \chi^I) + 3 f^{IJK} (\delta_Q A_{\mu}^J) H_{\nu\rho}^K - 3 f^{IJK} (\delta_Q C_{\mu}^J) F_{\nu\rho}^K ,
$$  

(3.3a)

$$
\delta_Q H_{\mu\nu}^I = - 2 (\epsilon_{\mu\nu\rho} D_\rho^I) + g (\epsilon_{\mu\nu\rho} \chi^I) + 2 f^{IJK} D_{[\mu}^I [ (\delta_Q A_{\nu]}^J) \phi^K ] ,
$$  

(3.3b)

$$
\delta_Q F_{\mu\nu}^I = - 2 (\epsilon_{\mu\nu\rho} D_\rho^I) ,
$$  

(3.3c)

Since we have not added the $D$-auxiliary field, our YMVM and CVM have on-shell DOF 2+2, while off-shell DOF 3+4. However, our TM is in the off-shell formulation with the total off-shell DOF 4+4, because the off-shell DOF of each field are $(4 - 1) \cdot (4 - 2)/2 = 3$ for $B_{\mu\nu}$, 4 for $\chi$ and 1 for $\phi$.

The field equations for all of our fields are\footnote{These equations are fixed up to $O(\phi^3)$-terms, due to the quartic fermion terms in the lagrangian.}

$$
+ \phi \lambda^I - \frac{1}{2} g f^{IJK} \chi^J \phi^K + \frac{1}{2} f^{IJK} (\gamma_5 \gamma_0 \epsilon)^{IJK} D_\mu \phi^K - \frac{1}{4} f^{IJK} (\gamma_5 \gamma_0 \epsilon)^{IJK} H_{\mu\nu}^K + \frac{1}{12} f^{IJK} (\gamma_5 \gamma_0 \epsilon)^{IJK} G_{\mu\nu\rho}^K = 0 ,
$$  

(3.4a)

$$
+ \phi \chi^I - g \phi^I + \frac{1}{2} g f^{IJK} \chi^J \phi^K - \frac{1}{4} f^{IJK} (\gamma_5 \gamma_0 \epsilon)^{IJK} H_{\mu\nu}^K + \frac{1}{4} f^{IJK} (\gamma_5 \gamma_0 \epsilon)^{IJK} F_{\mu\nu}^K = 0 ,
$$  

(3.4b)

$$
+ \phi \phi^I - g \phi^I + \frac{1}{2} f^{IJK} (\gamma_5 \gamma_0 \epsilon)^{IJK} D_\mu \phi^K = 0 .
$$  

(3.4c)
In deriving each of these equations, we have also used other field equations.

\[ -\frac{1}{12} f_{IJK} (\gamma^{\mu \rho} \chi^J) G_{\mu \rho}^I + \frac{1}{4} f_{IJK} (\gamma^{\mu} \chi^J) F_{\mu \nu}^I \geq 0 \]  

\[ + D_\nu F_{\mu \nu}^I + g f_{IJK} \varphi^I D_\mu \varphi^K + \frac{1}{2} g f_{IJK} (\bar{\chi}^I \gamma_\mu \lambda^K) + f_{IJK} H_{\mu \rho}^J D_\nu \varphi^K \]

\[ - \frac{1}{2} f_{IJK} G_{\mu \rho \sigma}^J H^{\sigma \rho \kappa} + \frac{1}{2} f_{IJK} (\bar{\chi}^I D_\mu \rho^K) + \frac{1}{2} f_{IJK} (\bar{p}^I D_\mu \chi^K) \geq 0 \]  

\[ + D_\rho G^{\mu \nu \rho I} - g H^{\mu \nu I} - \frac{1}{2} f_{IJK} D_\rho (\bar{\chi}^J \gamma^{\mu \rho} \rho^K) \]

\[ + g f_{IJK} F^{\mu \nu \rho I} \varphi^K - \frac{1}{2} g f_{IJK} (\bar{\chi}^I \gamma^{\mu \rho} \rho^K) \geq 0 \]  

\[ + D_\mu \varphi^I - g f_{IJK} (\bar{\chi}^I \gamma^K) - g^2 \varphi^I - \frac{1}{2} f_{IJK} F_{\mu \nu}^J H^{\mu \nu \rho I} \geq 0 \]  

\[ + D_\nu H^{\mu \nu I} - \frac{1}{2} f_{IJK} F_{\rho \sigma}^J G_{\mu \rho \sigma}^I - \frac{1}{2} f_{IJK} (\bar{\chi}^I D_\mu \lambda^K) - \frac{1}{2} f_{IJK} (\bar{\chi}^I D_\mu \rho^K) \]

\[ + \frac{1}{2} g f_{IJK} (\bar{\chi}^I \gamma^{\mu} \rho^K) - f_{IJK} E^{\mu \nu \rho} D_\rho \varphi^K \geq 0 \]  

In deriving each of these equations, we have also used other field equations.

4. Superspace Reformulation of N=1 TM

We can re-formulate our theory in superspace, as an independent consistency-reformulation.

Our superspace BIds for the superfield strengths \( F_{AB}^I \), \( G_{ABC}^I \) and \( H_{AB}^I \) are\(^7\)

\[ + \frac{1}{6} \nabla (AG_{BCD})^I - \frac{1}{4} T_{[AB]} G_{E(CD)}^I - \frac{1}{4} f_{IJK} F_{[AB]} H_{C(D)}^K \equiv 0 \]

\[ + \frac{1}{2} \nabla (AH_{BC})^I - \frac{1}{2} T_{[AB]} D_{D(C)}^I - g G_{ABC}^I \equiv 0 \]

Our relevant superspace constraints at the mass dimensions \( 0 \leq d \leq 1 \) are

\[ T_{\alpha \beta}^c = 2 (\gamma^c)_{\alpha \beta} , \quad G_{\alpha \beta c}^I = 2 (\gamma_c)_{\alpha \beta} \varphi^I \]

\[ G_{abc}^I = - (\gamma_{bc})_\alpha , \quad H_{ab}^I = - (\gamma_{b})_\alpha \varphi^I - f_{IJK} (\gamma_\beta \chi^I) \varphi^K \]

\[ F_{ab}^I = - (\gamma_{b})_\alpha \varphi^I = - (\chi_\alpha)^I \]

\[ \nabla \chi_\alpha^I = - \frac{1}{6} (\gamma^{cde})_{\alpha \beta} G_{cde}^I - (\gamma^c)_{\alpha \beta} \nabla \varphi^I \]

\[ - \frac{1}{2} f_{IJK} [ C_{\alpha \beta} (\chi^J \rho^K) - (\gamma_5)^c_{\alpha \beta} (\chi^J \gamma_5 \gamma_c \rho^K) - (\gamma_5)_{\alpha \beta} (\chi^J \gamma_5 \rho^K) ] \]

\[ \nabla \rho^I_{\alpha \beta} = + \frac{1}{2} (\gamma^{cd})_{\alpha \beta} H_{cd}^I + g C_{\alpha \beta} \varphi^I - \frac{1}{2} f_{IJK} (\gamma^{cd})_{\alpha \beta} F_{cd}^J \varphi^K \]

\[ - \frac{1}{2} f_{IJK} [ C_{\alpha \beta} (\chi^J \chi^K) + (\gamma^c)_{\alpha \beta} (\chi^J \gamma_c \chi^K) - \frac{1}{2} (\gamma^{cd})_{\alpha \beta} (\chi^J \gamma_c \chi^K) \]

\[ - (\gamma_5)^c_{\alpha \beta} (\chi^J \gamma_5 \chi^K) - (\gamma_5)_{\alpha \beta} (\chi^J \gamma_5 \chi^K) \]

\[ \nabla \lambda^I_{\alpha \beta} = + \frac{1}{2} (\gamma^{cd})_{\alpha \beta} F_{cd}^I - \frac{1}{2} (\gamma_5)_{\alpha \beta} f_{IJK} (\bar{p}^J \gamma_5 \chi^K) \]  

\( ^7 \) In this superspace section, we use the indices \( A = (a, a) \), \( B = (b, b) \), ... for superspace coordinates, where \( a, b, ... = 0, 1, 2, 3 \) (or \( a, b, ... = 1, 2, 3, 4 \) are for bosonic (or fermionic) coordinates. In superspace, we use the (anti)symmetrization convention, e.g., \( X_{(AB)} = X_{AB} - (1)4ABX_{BA} \), different from our component formulation.
All other components, such as $G_{\alpha \beta \gamma I}$ or $T_{\alpha \beta \gamma}$ etc. at $d \leq 1$ are zero.

Although most of technical details associated with superspace formulation are skipped here, we presented a rather independent confirmation for the total consistency of our $N = 1$ non-Abelian tensor multiplet in superspace.

5. Concluding Remarks

In this talk, we have explained how to formulate the $N = 1$ supersymmetrization in 4D of a physical non-Abelian tensor with consistent couplings [1]. This is the supersymmetrization of the special case [4] of the minimal tensor hierarchy [5], which, in turn, is a special case of more general hierarchy in [2][3]. Both the component and superspace formulations of our system are given, as the cross-verification of our system. Our CVM $(C_{\mu I}^I, \mu')$ plays the role of a compensator multiplet, absorbed into the TM $(B_{\mu \nu I}, \chi^I, \varphi^I)$, making the latter massive.

There exists certain problem for the quantization of Stueckelberg theory [7] for non-Abelian gauge groups [9]. This is because the longitudinal components of the gauge field do not decouple from the physical Hilbert space, so that the renormalizability and unitarity of the system are spoiled [9]. We take rather an optimistic standpoint about this potential problem for the following reasons. First, we mention that our theory is not renormalizable due to Pauli couplings. This feature is not necessarily a fatal drawback for our theory, because certain theories exist in 4D, such as non-linear sigma models that are not renormalizable, but are not rejected from the outset. Second, $N = 1$ supersymmetry may well improve quantum behavior of our theory, compared with non-supersymmetric systems. There is good chance that supersymmetries solve the quantum problem of non-Abelian Stueckelberg theories.

The importance of our result [1] is double-fold: (i) A new supersymmetric physical system with Stueckelberg mechanism that solves the problem with non-Abelian tensor is presented. (ii) The problem with extra vector fields in the non-singlet representation of a non-Abelian gauge group is now solved. We should also consider the possibility that $N = 1$ supersymmetry may well provide better quantum behavior compared with non-supersymmetric cases.

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