We discuss recent developments in gravitational fields with sources, regular black holes, quasiblack holes, and analogue black holes, related to the talks presented at the corresponding Parallel Session AT3 of the 13th Marcel Grossmann Meeting.

Keywords: Regular black holes; Quasiblack holes; Analogue black holes; Modified gravity; Gravitational fields with sources

The AT3 Session of the 13th Marcel Grossmann Meeting covered a variety of topics which are related to the search of viable gravitational solutions in the presence of sources containing normal or exotic matter, and to solutions that take into account corrections to general relativity. Most of the talks were devoted to the analysis of black holes (BHs) in various scenarios. Given the interdisciplinary nature of the subject some overlap exists with other sessions of the meeting, in particular with AT2 (Extended Theories of Gravity), AT4 (Modified Gravity), BH3 (Black Holes) and GT4 (Exact Solutions (Physical Aspects)).

In view of the multitude of covered topics, we find it useful to divide this report in separate sections, which also reflect an internal schedule that was adopted during the meeting.

1. Regular black holes, quasiblack holes and wormholes

A central topic of the session was related to BH-like solutions of general relativity and other theories in the presence of (standard and nonstandard) matter fields, such as regular BHs, quasiBHs and wormholes.

Reinhard Meinel gave an overview of quasiBHs solutions in Einstein-Maxwell gravity. In the static and electrically neutral case, Buchdahl showed that self-gravitating objects cannot reach the BH limit. However, the situation is different...
in the case of spinning metrics or in the presence of electromagnetic fields. In the latter cases the exterior spacetime can approach that of a Kerr-Newman BH in the quasiBH limit, whereas the interior is a regular, nonasymptotically flat spacetime with a Kerr-Newman near-horizon geometry at spatial infinity. Meinel presented rotating models with a perfect-fluid, static models with electric charge and solutions of rotating discs of charged dust.

José P. Sande Lemos reported on a series of studies coauthored by Oleg Zaslavskii and Vilson Zanchin. Lemos presented the properties that distinguish a quasiBH from a classical BH, even though for external far away observers the two spacetimes are virtually indistinguishable. In brief, these properties are: the existence of whole regions of infinite redshift, degeneracy of the spacetime, existence of outer and inner regions which are impenetrable and dynamically (rather than causally) disjoint. Despite these peculiar properties, the curvature invariants remain perfectly regular everywhere in the quasiBH limit. Lemos concluded by presenting mass and entropy formulae for these objects and the distinctive features of their Carter-Penrose diagrams.

In the context of quasiBHs, Vilson Zanchin explored some extremal limits of charged, polytropic perfect-fluid spheres in general relativity. He constructed numerical solutions which describe a Reissner-Nordström exterior and a charged, perfect-fluid star in the interior. When these solutions are extremal, the Buchdahl bound can be made arbitrarily close to the BH limit, \( M = \frac{Rc^2}{G} \), \( M \) and \( R \) being the object’s mass and radius, respectively. However, as Zanchin’s results showed, in the case of polytropic stars this requires an infinite polytropic index, i.e., \( \gamma \to \infty \), where the fluid equation of state is defined as \( P = k \rho^\gamma \), where \( P \) is the fluid pressure and \( \rho \) is its energy density.

Another important class of BH mimickers is described by Mazur’s and Mottola’s gravastars. Emil Mottola gave a review of the properties of this solution. Gravastars are cold, dark compact objects with an interior de Sitter dark-energy condensate and an exterior Schwarzschild or Kerr geometry of arbitrary total mass \( M \). The compactness of these solutions can be arbitrarily close to the BH limit, but they do not possess event horizons, nor singularities. Mottola presented the entropy formula for a gravastar (which scales as \( M^{3/2} \) rather than as \( M^2 \) as in the BH case) and discussed the role of trace anomalies in generating the quantum fluctuations necessary to gravastar formation.

Two contributions were on regular BHs. Stefano Ansoldi discussed a work coauthored by Lorenzo Sindoni in which they present a general procedure to generate a class of (everywhere regular) solutions of Einstein equations with arbitrary number of horizons. The general structure of these solutions can be interpreted as a nested sequence of anisotropic vacua, with necessarily nonincreasing energy density. Ansoldi discussed current work in progress to find a suitable classification scheme for the maximal extension of these multi-horizon regular BHs and to what extent these solutions can arise in modified gravity scenarios or from higher-dimensional compactifications. Another important issue to understand is the stability of the
solutions. Nami Uchikata discussed the stability against radial perturbations of the regular BH solutions found by Lemos and Zanchin and their generalization. These solutions are composed of a single de Sitter core whose surface boundary is a timelike charged massive thin-shell, whereas the exterior spacetime is Reissner-Nordström, with the thin-shell being inside the Cauchy horizon and without matter pressure. Uchikata showed that nearly extremal regular BHs only exist in a certain range of the ratio $GM/(Lc^2)$ (where $M$ is the BH mass and $L$ is the de Sitter horizon radius) and that some solutions are unstable below a certain critical value of this ratio.

Two further contributions were devoted to the study of traversable wormholes in modified gravities. In the standard approach, general relativity is assumed and wormhole solutions require the presence of exotic matter that violates some energy condition. An alternative approach makes use of some modified gravity theory to find traversable wormholes without invoking any exotic matter. In this context, Francisco Lobo reviewed some recent work by himself and collaborators on traversable wormholes supported by dark gravity. As he discussed, in alternative theory scenarios higher-order curvature corrections can be interpreted as a gravitational fluid, which supports wormhole geometries. The latter are peculiarly different from their general relativity counterpart. Along similar lines, Remo Garattini’s work discusses self-sustained traversable wormholes in noncommutative theories of gravity and in gravity’s rainbow models. In a semiclassical approach, Garattini computed the energy density of the graviton one-loop contribution to the energy in a wormhole background and considered it as a self-consistent source. A comparison among various models with respect to effective traversability was also made.

Finally, Eduardo Guendelman presented explorable horned-particle geometries, which are indirectly related to wormhole solutions. More precisely, horned particle spacetimes have a tube-like structure and, in order to be fully explorable (i.e., not to possess surfaces of infinite redshift), they require a shell sitting at the throat with negative surface energy density. The condition of finite energy of the system (or of asymptotic flatness) implies that the charged object sitting at the throat completely expels the flux it produces towards the other side of the horned particle. The geometry turns out to have a tube-like structure, i.e., the electric flux is hidden from an outside observer. One important issue to understand is the origin of the negative energy density at the throat of the horned particle which can have quantum-mechanical origin.

2. Black holes as probes of strong gravity

This part was devoted to the study of BHs in modified theories of gravity (see also Session AT2 chaired by Salvatore Capozziello and Session AT4, chaired by Fawad Hassan and Shinji Mukohyama).
2.1. Scalar-tensor and quadratic gravity theories

Among the most studied extensions to general relativity are theories which describe fundamental scalar fields nonminimally coupled to gravity. These theories are well-studied in cosmology. Among other possibilities, they can be tested against Einstein’s gravity in the strong-curvature regime, e.g., using current X-ray binary observations and near-future gravitational-wave signals. The inspiral of two massive BHs is a perfect testbed, because the system is relatively clean and it probes regions of strong curvature. Unfortunately, in one of the most studied classes of alternatives to general relativity, the class of scalar-tensor theories, it is well-known that BH binaries do not emit scalar radiation to the first Post Newtonian orders. During the session, Kent Yagi, Michael Horbatsch, and Caio Macedo presented two complementary approaches to circumvent this obstacle.

Kent Yagi presented a Post-Newtonian formalism to study the evolution of a compact binary in quadratic gravity. The latter is an extension of general relativity in which the Einstein-Hilbert action is supplemented by all independent curvature invariants to second order in the curvature and coupled to a single scalar field. In this class of theories, at variance with scalar-tensor gravity, scalar radiation is emitted from the binary if at least one of the objects is a BH. This is used to constrain specific deviations from Einstein’s gravity using current electromagnetic observations and future gravitational-wave interferometers. Projected bounds from gravitational-wave observations may place constraints that are more than six orders of magnitude stronger than those obtained from current solar system observations.

Michael Horbatsch presented a work coauthored by Cliff Burgess in which they show that scalar dipole radiation can be actually emitted by a BH binary in scalar-tensor theories if the scalar field is slowly evolving in time, e.g., due to a cosmological evolution. The analysis is based on a previous result by Jacobson who showed the existence of BHs with time-dependent scalar hairs. In this case, the scalar dipole emission can be constrained using quasar observations, putting bounds on the cosmological evolution of light scalar fields.

In the context of quadratic gravity theory considered by Yagi, Macedo presented analytical solutions describing slowly-rotating BHs in this theory. These solutions include, as particular cases, nonKerr BHs that arise as consistent solutions of some modified gravity and that are known in closed form. Macedo presented parametric corrections to the geodesic quantities (e.g., the frequency of the innermost stable circular orbit) which can be useful to constrain the theory via electromagnetic observations. Furthermore, having an analytical expression for the BH metric greatly facilitates other types of analysis, e.g., computing the linear response of spinning BHs and the corresponding gravitational-wave emission.

2.2. Spinning BHs and tests

The interaction of spinning BHs and matter fields might give rise to very interesting effects. This is the case of the so-called BH bombs, first studied by Press and
Teukolsky In a further contribution, Paolo Pani presented a new framework to study perturbations of slowly-rotating BHs and applied it to the study of vector fields around Kerr BHs. As it turns out, massive spin-1 fields are subject to BH superradiance precisely as scalar fields do, giving rise to a BH bomb instability. Pani used this result to constrain the mass of possible spin-1 particles from current observations of spinning supermassive BHs.

Furthermore, Dinesh Singh described the dynamics of a spinning test particle in a BH spacetime, taking into account spin-curvature coupling effects through the Mathisson-Papapetrou-Dixon equations. As Singh discussed, it is in principle possible to use this framework as a diagnostic tool to probe the structure of curved spacetime in suitable astrophysical contexts, for example when the test particle is taken to be a solar mass compact object orbiting an intermediate-mass BH. As an illustration, Singh considered the inspiral around a Kerr BH and that around a nonspinning but dynamical BH during its late-time ringdown phase (the latter modelled by a Vaidya metric). The gravitational-wave signal emitted by the system in the two cases carries unique information on the nature of the central object, providing another means to search for signatures of BH mergers and to test the no-hair theorem in general relativity.

2.3. Singularities in alternative theories
The contributions were also devoted to the singularity problem, namely, how corrections to general relativity can avoid the singularities that quite generically appear in Einstein’s gravity. Two classes of theories that resolve curvature singularities under certain circumstances are Palatini $f(R)$ gravity and the Eddington-inspired Born-Infeld gravity. A common feature of these theories is that they are equivalent to general relativity in vacuum, but introduce nonlinear couplings to the matter fields. The latter are eventually responsible of resolving the singularities in early-time cosmology and in the stellar collapse.

Térence Delsate presented a work coauthored by Jan Steinhoff, in which they uncover the mechanism responsible of the singularity resolution in this class of theories. They also show that there exists a degeneracy between corrections due to the nonminimal couplings and the matter equation of state. Delsate showed that this degeneracy must be disentangled in order to constrain these theories. Interestingly enough, the very same mechanism that prevents the appearance of singularities seems to give rise to other kinds of pathologies.

Gonzalo Olmo presented spherically-symmetric, charged BH solutions in Palatini gravity, a work coauthored by Diego Rubiera-Garcia. They found that charged BHs in this theory have a central core whose area is proportional to the Planck area times the number of charges, whereas far from the core they approach the standard Reissner-Nordström metric. However the causal structure of these solutions is very different from their general relativity counterpart. Indeed, several interesting solutions exist, including regular BHs and solutions with a single nondegenerate
Finally, Vincenzo Vitagliano discussed the cosmological appearance of dynamical horizons in an inhomogeneous universe in the context of Brans-Dicke theory, a particular subclass of scalar-tensor gravity theories. Vitagliano presented a two-parameter family of spherically symmetric and time-dependent solutions of Brans-Dicke cosmology, which should describe central objects embedded in a spatially flat universe. There exist multiple dynamical apparent horizons, both BH horizons covering a central singularity and cosmological ones. In some cases, these horizons can dynamically merge, leaving a naked singularity enclosed in a cosmological horizon.

3. Nonasymptotically flat gravitational fields

While most of the session was devoted to asymptotically flat spacetimes in the presence of sources, also solutions with matter sources containing a cosmological constant have been discussed. Two contributions of the session were devoted to black branes, i.e., black objects whose horizon topology is planar – in the presence of a negative cosmological constant, and another one was devoted to cylindrically symmetric spacetimes.

Kengo Maeda presented static, inhomogeneous charged black-brane solutions of Einstein-Maxwell gravity in asymptotically anti-de Sitter spacetime. Such solutions are constructed perturbatively (both numerically and analytically) starting from Reissner-Nordström black branes. Interestingly, Maeda shows that the Cauchy horizon disappears in these solutions, a result that supports the strong cosmic censorship conjecture. In the extremal limit, even if the curvature invariants are small close to the horizon, a free-fall observer would experience infinite tidal forces at the horizon for any long wavelength perturbation.

Mariano Cadoni presented black-brane solutions of Einstein-Klein-Gordon theory endowed with a scalar hair. The scalar potential is such that these theories admit anti-de Sitter vacuum. On the other hand, well-known no-hair theorems forbid the existence of hairy black-brane solutions with anti-de Sitter asymptotics in this model. By relaxing the requirement of an asymptotic anti-de Sitter solution, Cadoni showed that these models allow for hairy black-brane solutions with non-anti-de Sitter domain wall asymptotics, whose extremal limit is a scalar soliton. He also discussed several features that make these solutions particularly interesting for holographic applications, for example in the context of hyperscaling violation in critical systems.

As another case of solutions with nonflat asymptotics, Irene Brito presented cylindrically symmetric solutions with a cosmological constant that match a conformally flat interior to an exterior Linet-Tian spacetime. Brito showed that such matching is only possible when the cosmological constant is positive because of the existence of an upper limit on the mass density of these solutions. In the case of vanishing or negative cosmological constant, the mass would exceed such limit.
4. Analogue models of gravity and analogue black holes

In this 13th edition of the Marcel Grossmann Meeting, the contributions about analogue BHs have been merged with the rest of the topics of the AT3 Session.

Among various analogue models, acoustic models have attracted particular attention. Under certain conditions, the motion of sound waves in a fluid is governed by a Klein-Gordon equation propagating on an effective acoustic metric, which is determined by the flow properties.\(^32\,33\)

In this context, Leandro Oliveira presented a work,\(^33\) coauthored by Dolan and Crispino, in which they compute the quasinormal modes of a draining bathtub. The latter is a circulating, draining flow whose acoustic metric shares many key features of a Kerr spacetime. The linear perturbation equations are evolved in the time domain and also solved in the frequency domain. Oliveira showed that these spacetimes present the typical quasinormal ringing\(^35\) and that the characteristic modes can be interpreted geometrically in terms of null geodesic orbits.

A related contribution was presented by Ednilton Oliveira, who discussed the propagation of plane waves on a draining bathtub vortex.\(^36\) The dynamics exhibits an analogue of the Aharonov-Bohm effect in quantum mechanics. As suggested, this effect can be observed in the laboratory using gravity waves in a shallow basin. Oliveira presented a modified version of the effect, which is inherently asymmetric even in the low-frequency limit. This leads to novel interference patterns which carry the signature of both rotation and absorption.

Finally, Christian Cherubini presented a recent study\(^37\) which uncovers a relation between analog gravity and the nonlinear von Mises wave equation of fluid dynamics. Interestingly, such correspondence is valid at any perturbative order. Cherubini also focused on the canonical draining bathtub configuration, discussing the results of some 2+1 numerical simulations. He discussed the acoustic analogue of superradiance and BH bomb effects in these geometries.\(^35\) He concluded by extending the analysis to include compressibility effects.

5. Conclusions

Several interesting issues and ideas have been raised during the discussion.

As for regular BHs, quasiBHs and other BH mimickers, an important issue is related to their stability and formation. Can these objects be formed in viable astrophysical scenarios? Are they (at least linearly) stable? To answer the latter question, recently a linear stability analysis of regular BHs has been performed.\(^38\) Formation in astrophysical viable scenarios remains an interesting open question.

As for modified theories of gravity and their astrophysical imprint, the most relevant question is to what extent such deviations are observable in the near future and how to select viable theories from the plethora of modifications of general relativity that have proliferated during the last decades. Electromagnetic observations of the galactic center\(^39\,40\) (and especially gravitational-wave observations of BH binaries) will certainly be instrumental in the near future and they will provide
complementary information with respect to observations at cosmological scale and those involving the structure of compact stars.\textsuperscript{[31]} The latter suffer from the problem that possible deviations from general relativity are degenerate with different equations of state at nuclear density, whose microphysics is not completely understood.

In the context of anti-de Sitter solutions – beside the stability issue – the most urgent question is to understand whether theories that admit solutions with non-trivial hairs have a solid holographic interpretation and how to characterize the dual theory. Scalar dressed solutions are particularly relevant in the context of the so-called anti-de Sitter/condensed matter correspondence.\textsuperscript{[42]}

As for analogue gravity, in recent years real experiments have been performed to test some of the predictions of analogue models,\textsuperscript{[43]} and other experiments have been designed. In the short term, the experimental challenges will be overcome and the field will witness an exciting interplay between theoretical predictions and accumulating experimental evidences.\textsuperscript{[44]}

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