Dissipative Processes in an Expanding Massive Gluon Gas

J. Cleymans\textsuperscript{1}, S.V. Ilyin\textsuperscript{2}, S.A. Smolyansky\textsuperscript{2}, G.M. Zinovjev\textsuperscript{3}

Abstract

The temperature dependence of the kinetic coefficients is obtained in the non-perturbative region with the help of Green-Kubo-type formulae in the model of massive gluon gas motivated by numerical results from simulations of lattice QCD. The entropy production rate is estimated using scaling hydrodynamics. It is shown that the increase in the viscosity coefficients leads to entropy generation in heavy ion collision processes which could be big, especially for temperatures close to the critical one.

\textsuperscript{1}Department of Physics, University of Cape Town, Rondebosch 7700, South Africa
\textsuperscript{2}Department of Physics, Saratov State University, 410071 Saratov, Russia
\textsuperscript{3}Bogolubov Institut for Theoretical Physics, Academy of Science of Ukraine, 252143 Kiev, Ukraine.
Discussions of the forthcoming projects for ultrarelativistic heavy ion collisions at RHIC and LHC require, with necessity, comprehensive estimates of the space-time development in those reactions. Nowadays, it is commonly believed that central heavy ion collisions pass through several stages. Describing these in terms of equilibrium processes when all memory of prior history has been destroyed, it is natural to consider the entropy to set up the equilibration time scale and to separate a pre-equilibrium stage. In fact, that stage is producing not only the maximum entropy attained [1,2], but a lot of uncertainties in the initial and boundary conditions for the subsequent hydrodynamic expansion. It has usually been taken as isentropic, however it became clear that this stage may be complicated by the dissipative processes generating entropy, together with a possible phase transition (or transitions) when it happens to be of first order [3,4,5]. The aim of the present study is therefore to explore the entropy generation just at this stage although we understand that the freeze-out stage, where the system is made up of free-streaming final particles, could also add to the entropy [6,7].

In order to estimate properly the dissipative effects as well as the dynamics [8] of the QCD phase transition we need to know the behaviour of the kinetic coefficients (KC) over a wide range of temperatures, including the ones close to the phase transition point where non-perturbative effects are dominant. The calculations done previously suffered from the unjustified extrapolation of the asymptotic behaviour, found perturbatively, to the critical region [9,10,11]. The recent estimate of the shear viscosity coefficient, using a model for the contribution of the nonperturbative region, has demonstrated that the behaviour in the critical region is very different from the standard $T^3$ one and the amount of entropy generated in the region close to this temperature appears to be substantial [12,13].

In those calculations we have exploited the so-called momentum “cut-off model” motivated by a special analysis of the numerical results of lattice Yang-Mills field thermodynamics [14,15]. In spite of the fact of providing a good fit to the data, this model is very much phenomenological and doesn’t interpret why the low-momentum modes are removed nor does it answer questions in terms of conventional conceptions of spin systems which are very indicative and conclusive at least for the Monte Carlo analysis of lattice pure gluodynamics. Here we are dealing with the model of an ideal gas of excitations where the effect of interactions in the plasma is provided by the temperature dependence of their effective mass [16]. In a sense it is rather similar to the “cut-off model” as now the medium (plasma) properties suppress low momentum excitations too, since for $M(T)$ increasing $\exp(-\sqrt{M^2 + p^2}/T) \to \exp(-M/T)$ at small momenta $p \to 0$. But it has more advantages as was argued in a recent analysis [17], the most important of them is that it leads to a perfectly detailed description of high precision SU(2) pure gluodynamics lattice data just in the weak coupling limit.

However, as to the KC calculations, this model is more involved as in what follows we need to deal with massive scalar $\lambda \phi^4$ theory. By the way, this latter approximation is also in line with the model of “massive” gluons where these waves are nevertheless considered to be only transversal [18]. Combining our calculations of KC’s in $\lambda \phi^4$-theory with the temperature dependence of the gluon mass extracted from lattice
Monte Carlo data, in particular for the mass gap in pure gluodynamics [19], we are able to estimate the entropy generated based on linear hydrodynamics and to explore the applicability of this approach to the evolution of gluon systems.

As our basic hydrodynamical equation we take

\[
\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + p}{\tau} - \frac{\chi}{\tau^2} = 0.
\]  

(1)

with Bjorken initial and boundary conditions [20]. We fix the equation of state \(p(p(\tau_0), \varepsilon = \varepsilon(\tau_0))\) are the initial pressure and energy density respectively), and taking the initial conditions at a time \(\tau_0 \sim 1 \text{ fm}\). The dissipative term in Eq.(1) contains the factor \(\chi = (4/3\eta_s + \eta_v)\) with \(\eta_s\) and \(\eta_v\) as transport coefficients of shear and bulk viscosities. We should as well take into account that \(\tau > \chi/(\varepsilon + p)\), otherwise we deal with the unrealistic picture of gluon gas contraction (see [5,11]).

The total entropy of the system is defined as [5,10]

\[
S = \int d\sigma s^\mu s^\mu = \int d\tau s(\tau) \tau,
\]  

(2)

where \(s^\mu = s u^\mu\), \(s(\tau) = (\varepsilon(\tau) + p(\tau))/T\) is the local entropy density and \(y\) is the hydrodynamic rapidity \((\tanh y = x/t)\). Then Eqs.(1) and (2) give us a simple formula to estimate the entropy production in expanding gluon gas

\[
\frac{dS}{dy} = \int d\tau \frac{\chi(\tau)}{\tau T(\tau)}
\]  

(3)

here \(\chi = \chi[T(\tau)]\), \(T(\tau)\) being the solution of Eq.(1). In order to solve Eq.(2) we need the temperature dependence of the KC’s and the equation of in the whole temperature interval, including the nonperturbative critical region.

For pressure and energy density calculations we use the familiar expressions [21]

\[
p(T) = g \frac{T^4}{6\pi^2} \int_0^\infty dk \frac{k^4}{E(k)}
\]  

(4)

\[
\varepsilon(T) = g \frac{T^4}{2\pi^2} \int_0^\infty dk k^2 E(k) n(E(k)),
\]  

(5)

where \(E(k) = \sqrt{k^2 + m_g^2}\) is the relativistic energy; \(n(x) = [\exp(\beta x) - 1]^{-1}\) is Bose’s distribution function; \(\beta\) is the inverse temperature and \(g\) is the degeneracy factor. In the ultrarelativistic case \((T >> m_g)\) it leads to the Stephan-Boltzmann (SB) law (see Fig.(1)). However, if the temperature is close to \(T_c\), the gluon mass \(m_g\) is increasing, (at least for SU(2)-gluodynamics) and becomes too large to be negligible and Bose’s distribution function may be replaced by Boltzmann’s \((n(x) = \exp(-\beta x))\). Then eqs (6),(7) may be taken in the following form

\[
p(T) = \frac{g T^4}{6\pi^2} \int_\alpha^\infty dz [z^2 - \alpha^2]^{3/2} e^{-z} = \frac{g T^4}{2\pi^2} \alpha^2 K_2(\alpha),
\]  

(6)
\[ \varepsilon(T) = \frac{g T^4}{2\pi^2} \int_{\alpha}^{\infty} dz \, z^2 \sqrt{z^2 - \alpha^2} \, e^{-z} = \frac{g T^4}{2\pi^2} \alpha^2 \left( 3K_2(\alpha) + \alpha K_1(\alpha) \right), \]

where \( K_i(\alpha) \) are modified Bessel function and \( \alpha \equiv \alpha_0 \beta \). It is a well-known fact that the asymptotic expansion of the modified Bessel functions is given by \( K(\alpha) \sim \exp(-\alpha) \sqrt{\pi/2} \alpha \) for large \( \alpha \), i.e. for \( T \sim T_c \), this way both pressure and energy become small close to \( T_c \). In the case when \( \alpha \to 0 \) we recover the Stephan-Boltzmann law. This temperature dependence can fit even the SU(3) Monte-Carlo lattice data with constant gluon mass [18] or including finite jump of mass around \( T_c \). As to SU(2)-gluodynamics, for which a much more elaborated Monte Carlo analysis exists [17], we used the following parametrization

\[ m_g = m_0 T_c \left( \frac{T_c}{T - T_c} \right)^q \]

with \( m_0 = 1.83 \) and \( q = 0.4 \). An important difference just reflects our understanding in behaviours of first- and second order phase transitions as seen in lattice Monte-Carlo simulations [19]. To calculate the KC’s of shear and bulk viscosities we use well-known relations obtained within a formalism based on Kubo - type formulae for the \( \lambda \phi^4 \) - thermofield theory [10,11] (The analogous expression for \( \eta_s \) was also obtained for the case of vector fields [22])

\[ \eta_s = \frac{\beta}{15} I_{2,1}, \]  \( \eta_v = \frac{\beta}{9} \left\{ I_{2,1} - 6c_s^2 I_{1,0} + 9c_s^4 I_{0,-1} \right\}, \]

where \( c_s^2 = \partial p/\partial \varepsilon \) is the square of sound velocity, and the integrals \( I_{m,n} \) are defined as

\[ I_{m,n} = 2 \int \frac{d^3\tilde{p}}{E^{2n}(p)\Gamma(p)} \frac{p^m}{n(p)} \left[ 1 + n(p) \right], \]

here \( \Gamma(p) \) is the damping rate of quasiparticle excitation (it is assumed that \( \Gamma \beta << 1 \)). For the scalar theory in one-loop approximation is

\[ \Gamma(p) = \frac{\lambda^2(2\pi)^4}{24E(p)n(p)} \int d^3\tilde{p}_1 d^3\tilde{p}_2 d^3\tilde{p}_3 \delta(p + p_1 - p_2 - p_3)(1 + n_1) n_2 n_3 \]

with the following designations \( d^3\tilde{p} = [2(2\pi)^3 E(p)]^{-1} d^3p \), in both Eqs.(11) and (12), \( n_i = [\exp(\beta E(p_i)) - 1]^{-1} \).

The gluon gas viscosities may be obtained from Eqs.(8)-(11) by the standard procedure of changing [11,23]

\[ \lambda^2 \rightarrow c^* 32\pi^2 \alpha_s^2 \ln \alpha_s^{-1}, \quad c^* = 20 \div 60, \]

where (for the value of \( c^* \), see also [24])

\[ \alpha_s = 6\pi \left[ 11/2 \, N \ln(M^2/\Lambda^2) \right]^{-1}, \]

\( \lambda \)
and $N$ is a number of colours, $M^2 = \frac{4}{3} < p^2 >$ and $< p^2 >$ is the thermodynamically averaged squared momentum of the gluon field [21]. The degeneracy factor $g = (d - 1)(N^2 - 1)$ is absent in final result, for the numerator of Eq.(12) must contain it as well as the damping rate in the denominator. This phenomenological estimate can also be justified by the fact that in the lowest order of interaction the cross sections for gluons and scalar particles have a similar momentum dependence. This procedure (Eq.(13)) is fair only for small enough interaction constant. The model under consideration brings us to the following temperature dependence for the $M^2$ factor from Eq.(15)

$$M^2 = \frac{3}{4} \frac{\int d^3 p p^2 n(E(p))}{\int d^3 p n(E(p))} = 4T^2 \alpha \frac{K_3(\alpha)}{K_2(\alpha)}.$$  

(14)

In the ultrarelativistic case where $\alpha \to 0$ we obtain the conventional result $M \sim 4T$. When $T \to T_c$ and $\alpha \to \infty$ we have $M^2 \sim 4T^2 \alpha$. This means that the coupling constant $\alpha_s$ remains still small even when the temperature is close to $T_c$. This lucky fact has been met already in the so-called cut-off model [14,15] that interprets Monte-Carlo data as well as our model does. It can be explained evidently by the fact that the application of both gluodynamic models takes the contribution of long wavelength excitations away.

We now proceed to calculate the damping rate (Eq.(11)). After some straightforward integrations using the $\delta$ - function we have

$$\Gamma(p) = \frac{\lambda^2(2\pi)^{-4}}{192E(p)n(E(p))} \int d^3 p_1 \frac{(1 + n_{1})}{E(p_1)} I_1,$$  

(15)

where

$$I_1 = 2\pi\beta^{-1} \int dy \exp(-\beta\Omega) \Theta(z_0(y) - 1) \Theta(z_0(y) + 1),$$

$$z_0(y) = \frac{K^2 - \Omega^2 + 2\Omega y}{2K\sqrt{y^2 - m_g^2}},$$  

(16)

here we need to solve an inequality

$$-1 \leq z_0(y) \leq 1,$$  

(17)

where

$$\Omega = \sqrt{p_1^2 + m_g^2} + \sqrt{p^2 + m_g^2}$$  

(18)

and $K = \sqrt{p^2 + p_1^2 + 2pp_1 \cos(\theta)}$, $\theta$ being the angle between $p$ and $p_1$. It then leads us to the expression

$$y_- \leq y \leq y_+; \quad y_\pm = \frac{\Omega}{2} \pm \frac{K\Lambda}{2}; \quad \Lambda = \sqrt{1 - \frac{4m_g^2}{\Omega^2 - K^2}},$$  

(19)
with the values of $\Omega$ and $K$ satisfying the inequality

$$\Omega^2 - K^2 > 4m_g^2.$$  \hfill (20)

After analysis of Eqs. (19) and (20) we obtain for Eq.(15)

$$\Gamma(p) = \frac{\lambda^2 (2\pi)^{-3}}{\beta^2 192 E(p)} \int_0^\infty dx \frac{x^2 [1 + \exp(-\sqrt{x^2 + \alpha^2})]}{\exp(-\sqrt{x^2 + \alpha^2})\sqrt{x^2 + \alpha^2}} \int_0^\pi d\theta \sin(\theta) K \Lambda.$$  \hfill (21)

These relations are complicated to integrate, in order to get an approximate analytical form we can calculate the angle integral from Eq. (21) in the $p = 0$ case. This gives a minimal value for the damping rate and a maximum one for the kinetic coefficients. Here we also take into account that the main contribution in Eqs.(8)-(10) is connected with long wavelength excitations. Thus, in this approximation the damping rate looks as

$$\Gamma(p) = \frac{\lambda^2 (2\pi)^{-3}}{96 E(p)} \int_0^\infty dz (z - \alpha) \sqrt{z^2 - \alpha^2} \left( e^{-z} + e^{-2z} \right).$$  \hfill (22)

It is evident that the momentum dependence is very simple in this equation. This allows us to represent the integral $I_{m,n}$ from Eq.(10) in the following form

$$I_{m,n} = \frac{T^{2(n-m)-2}}{2 \pi^2 \Gamma(T)} J_{m,n}$$  \hfill (23)

with

$$J_{m,n} = \int_0^\infty dz z^{-2n} (z^2 - \alpha^2)^{2n+1} \left[ \exp(-z) + \exp(-2z) \right]$$  \hfill (24)

After integration we obtain, for some specific cases

$$J_{2,1} = \frac{15}{8} \left[ (2\alpha)^3 K_3(\alpha) + \alpha^3 K_3(2\alpha) \right],$$  \hfill (25)

$$J_{1,0} = 3\alpha^3 \left[ \frac{5}{8} K_3(2\alpha) + 5 K_3(\alpha) + \frac{\alpha}{4} K_2(2\alpha) + \alpha K_2(\alpha) \right],$$  \hfill (26)

$$J_{0,-1} = \alpha^2 \left[ 60 K_2(\alpha) + \frac{15}{4} K_2(2\alpha) + 27 \alpha K_1(\alpha) + \frac{27}{8} K_1(2\alpha) + 6\alpha^2 K_0(\alpha) + \frac{3}{2} \alpha^2 K_0(2\alpha) + \alpha^3 K_1(\alpha) + \frac{\alpha^3}{2} K_1(2\alpha) \right].$$  \hfill (27)

The result for the damping increment can be written as

$$\tilde{\Gamma}(T) = E(p) T^2 \Gamma(p)$$

$$= \frac{\lambda^2 \pi^{-3}}{32^9} \alpha^2(T) \left[ 2K_2(\alpha) - 2K_1(\alpha) + K_2(2\alpha) - K_1(2\alpha) \right]$$  \hfill (28)
For the calculation of the bulk viscosity we also need to know the expression for the velocity of sound. With the help of Eqs. (6) and (7) we obtain

\[ c_s^2 = \frac{1}{3 + \Delta} \]

with

\[ \Delta = \alpha \frac{4K_1(\alpha) + T \frac{d\alpha}{dT} [K_2(\alpha) - 2K_0(\alpha)]}{4K_2(\alpha) - T \frac{d\alpha}{dT} K_1(\alpha)} \]  

(29)

Here we see that, for \( \alpha \) small, \( \Delta \sim \alpha \), and the sound velocity \( c_s \) vanishes. In the \( \alpha \to 0 \) case Eq.(28) gives the common value of \( c_s^2 = 1/3 \), \( \Delta \to 0 \) (see Fig.(2)).

Eqs.(24-28) together with Eqs.(8,9,10,12-14) determine the temperature dependence of the KC of the gluon gas. The asymptotic behaviour \( \alpha \to 0 \) gives results analogous to Ref. [11] that is somewhat large, for, here we have a maximal estimate of KC (see below Eq.(21)). The temperature dependences of the KC are presented in Fig.(3). It is evident that the KC increase considerably in the temperature region close to \( T_c \). This leads to a big deviation of the solution of Eq. (1) from the scaling one, and to the so-called critical delay of the evolution of the system near \( T_c \).

The rate of entropy production as a function of proper time is depicted in Fig. (5). It is evident that the entropy of the system increases rapidly. The analogous calculations for entropy production rate for \( T^3 \)- dependency of KC brings to approximately 20 per cent increasing of entropy in gluon gas cooling process.

The authors are aware of the fact that the obtained results are model dependent. The increase of the kinetic coefficients makes the application of linear hydrodynamics for the description of quark gluon gas questionable. Evidently, taking into account other dissipative mechanisms will lead to finite values for the KC and diminish somehow the entropy production. However the results obtained indicate that a realistic picture of the evolution of the system under consideration can differ a lot from the scaling one, especially in the phase transition region and demands thorough examination.
Figure Captions

Figure 1. Dependence of the scaled energy density and pressure on the temperature.

Figure 2. Dependence of the speed of sound on the temperature.

Figure 3. Evolution of the temperature with proper time.

Figure 4. Dependence of the shear and bulk viscosities on the temperature.

Figure 5. Entropy generation as a function of proper time.
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