Towards baryon-baryon scattering in manifestly Lorentz-invariant formulation of SU(3) baryon chiral perturbation theory

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(Dated: 6 May, 2019)

Abstract

We study baryon-baryon scattering by applying time-ordered perturbation theory to the manifestly Lorentz-invariant formulation of SU(3) baryon chiral perturbation theory. We derive the corresponding diagrammatic rules paying special attention to complications caused by momentum-dependent interactions and propagators of particles with non-zero spin. We define the effective potential as a sum of two-baryon irreducible contributions of time-ordered diagrams and derive a system of integral equations for the scattering amplitude, which provides a coupled-channel generalization of the Kadyshevsky equation. The obtained leading-order baryon-baryon potentials are perturbatively renormalizable, and the corresponding integral equations have unique solutions in all partial waves. We discuss the issue of additional finite subtractions required to improve the ultraviolet convergence of (finite) loop integrals on the example of nucleon-nucleon scattering in the \(^3P_0\) partial wave. Assuming that corrections beyond leading order can be treated perturbatively, we obtain a fully renormalizable formalism which can be employed to study baryon-baryon scattering.

PACS numbers: 11.10.Gh,12.39.Fe,13.75.Cs
I. INTRODUCTION

Nuclear systems with non-vanishing strangeness provide an important connection between nuclear, particle and astrophysics. Hypernuclei emerging from replacing one or several nucleons in a nucleus by hyperons serve as a testing ground for effects of strange quarks in nuclear matter. In addition to the nuclear forces, hyperon-nucleon (YN) interactions play a crucial role for understanding hypernuclear binding. Experiments with hypernuclei aiming at the determination of the YN, hyperon-hyperon (YY) and cascade-nucleon (ΞN) interactions are carried out at various laboratories world-wide such as CERN, DAΦNE, GSI, JLab, J-PARC, KEK, MAMI, RHIC and will also be performed at the future FAIR facility, see Refs. [1–4] for review articles of hypernuclear physics.

Given that experiments involving hypernuclei are rather challenging, a particularly valuable source of information on YN and YY interactions is provided by lattice QCD. Several lattice-QCD groups including HAL QCD [5–7] and NPLQCD [8–12] Collaborations, Yamazaki et al. [13, 14] and the Mainz group [15] have already produced interesting results on baryon-baryon systems, and more results will become available in the near future. Although some lattice simulations are already approaching the physical values of the light quarks, most of the available lattice QCD calculations still correspond to unphysically large values of quark masses and require a reliable theoretical framework to perform extrapolations to their physical values.

Chiral effective field theory (ChEFT) for few-baryon systems offers a natural approach to analyze low-energy properties of (hyper)nuclei and to perform chiral extrapolations. It goes back to the seminal papers by Weinberg, who proposed a way of extending chiral perturbation theory to systems involving two and more nucleons [16, 17]. In the resulting ChEFT approach the power counting rules are applied to the effective baryon-baryon potential defined as a sum of contributions of two-baryon-irreducible diagrams. The scattering amplitude is then obtained by solving the Lippmann-Schwinger (LS) or Schrödinger equations. For reviews of ChEFT in the few-body sector see Refs. [18–22].

The Bonn-Jülich and later Munich groups have pioneered chiral EFT for baryon-baryon interactions in the strange sector using the non-relativistic formulation [23–36]. In these studies the standard Weinberg power counting for nucleon-nucleon (NN) interactions extended to the strangeness-$S = -1$ (i.e. ΛN, ΣN) and strangeness-$S = -2$ (i.e. ΛΛ, ΣΣ and ΞN) systems is utilized. At leading order (LO) in the Weinberg power counting, the two-baryon potentials consist of four-baryon contact interactions without derivatives and the one-pseudoscalar-meson exchange. At next-to-leading order (NLO) one has to take into account the contributions from two-pseudoscalar-meson exchange diagrams and from four-baryon contact interactions with two derivatives or a single insertion of the quark masses. To regularize the ultraviolet divergences appearing from iterations of the LS equation already at leading order, a finite-cutoff regularization has been employed using exponential cutoffs in the range of $500\ldots700$ MeV.

A modified approach to nucleon-nucleon scattering has been proposed in Ref. [37]. This novel framework employs time-ordered perturbation theory (TOPT) and relies on the manifestly Lorentz-invariant effective Lagrangian. It offers a perturbatively renormalizable modification of Weinberg’s approach and has already been explored in the non-strange sector.
Dirac spinors have been kept in their full form and an alternative power counting has been suggested in Ref. [41]. First applications of the formalism based on the Lorentz-invariant Lagrangian to baryon-baryon systems with non-zero strangeness can be found in Refs. [42–46]. In these exploratory studies, the baryon-baryon scattering amplitudes were obtained by solving the (generalized) Kadyshevsky equation [48], however, a systematic approach to the SU(3) sector using TOPT, which would allow for a straightforward generalization beyond the leading order, has not been formulated yet. In this paper we fill this gap by generalizing the modified Weinberg approach of Ref. [37] to the SU(3) sector and work out in detail the TOPT diagrammatic rules for particles with non-zero spin and interactions involving time derivatives. To achieve this goal, we start with the Lorentz-invariant effective Lagrangian and derive the diagrammatic rules of TOPT by integrating over zeroth components of loop momenta in Feynman diagrams for baryon-baryon scattering. Special care is taken to deal with complications caused by momentum-dependent interactions and propagators of particles with non-zero spin. We provide details which are not given in Ref. [37] and following papers. The obtained rules of TOPT can be applied systematically to all orders in the loop expansion. Using the standard Weinberg power counting for diagrams contributing to baryon-baryon scattering, one has to take into account an infinite number of graphs already at LO. We define the effective potential as a sum of all possible two-baryon-irreducible TOPT diagrams and obtain the scattering amplitudes by solving the corresponding integral equations. The resulting formulation permits a systematic investigation of few-baryon systems using both a renormalizable approach, which relies on a perturbative treatment of corrections beyond LO and allows one to completely eliminate the ultraviolet cutoff, and a conventional scheme based on iterating a truncated potential to all orders, which requires the ultraviolet cutoff to be chosen of the order of the hard scale of the problem [49–56]. In the latter case, the manifestly Lorentz-invariant formulation of chiral EFT is expected to permit a larger cutoff variation as compared to the conventional non-relativistic framework, which is especially important for the SU(3) sector.

Our paper is organized as follows: in section II we work out the rules of TOPT for a system of baryons interacting with pseudoscalar mesons including momentum-dependent vertices. A system of integral equations for baryon-baryon scattering is derived in section III. Next, in section IV we discuss the LO baryon-baryon potential and the renormalization. The results of our work are summarized in section V.

II. DIAGRAMMATIC RULES IN TIME-ORDERED PERTURBATION THEORY

To formulate the theoretical framework describing baryon-baryon scattering in SU(3) baryon chiral perturbation theory (BChPT) by applying the rules of TOPT we start with the manifestly Lorentz-invariant effective Lagrangian. It consists of the purely mesonic, single-baryon, two-baryon, \ldots parts,

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_\phi + \mathcal{L}_\phi B + \mathcal{L}_{BB} + \cdots. \]  

\[ \text{\footnotesize (1)} \]  

\[ \text{\footnotesize 1} \]  

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\[ \text{\footnotesize The first application of the modified Winberg approach to hadronic molecules can be found in Ref. [47].} \]
The effective Lagrangian is organised as an expansion in powers of the quark masses and derivatives. The lowest-order mesonic Lagrangian reads \[^{57}\]

\[ L^{(2)}_{\phi} = \frac{F_0^2}{4} \text{Tr} \{ u_\mu u^\mu + \chi^+ \}, \]  

where

\[ u_\mu = iu^\dagger \partial_\mu U u^\dagger, \quad u^2 = U = \exp \left( \sqrt{2}iP/F_0 \right), \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B_0 s, \]  

\[ P \] is the irreducible octet representation of SU(3)_f for the Goldstone bosons,

\[ P = \begin{pmatrix} \pi^0 & \eta \sqrt{6} & K^0 & K^- & K^- \\ \eta \sqrt{6} & \pi^- & \sqrt{2} \Lambda & \Sigma^- & \Xi^- \\ K^0 & \sqrt{2} \Lambda & \Sigma^+ & \Sigma^+ & p \\ K^- & \Xi^- & p & \Xi^0 & n \\ K^- & \Sigma^- & n & -2\sqrt{6} \Lambda & -2\sqrt{6} \Lambda \end{pmatrix}, \]  

while \( F_0 \) is the meson decay constant in the chiral limit. Further, \( s \) stands for the scalar source giving rise to the quark masses and the constant \( B_0 \) is related to the quark condensate. We restrict ourselves to the isospin-symmetric case of \( m_u = m_d \neq m_s \).

The lowest-order Lagrangian in the single-baryon sector is given by

\[ L^{(1)}_{\phi B} = \text{Tr} \left\{ B \left( i\gamma_\mu D^\mu - m \right) B \right\} + \frac{D/F}{2} \text{Tr} \left\{ B \gamma_\mu \gamma_5 [u^\mu, B]_\pm \right\}, \]  

where \( D \) and \( F \) are coupling constants, \( B \) is the irreducible octet representation of SU(3)_f,

\[ B = \begin{pmatrix} \Sigma^0 & \Lambda \sqrt{6} & p \\ \Lambda \sqrt{6} & \Sigma^- & n \\ p & n & \Xi^0 \end{pmatrix}, \]  

and \( D_\mu B = \partial_\mu B + [[u^\dagger, \partial_\mu u], B] \) denotes the covariant derivative.

The effective baryon-baryon (BB) Lagrangian contains terms with an increasing number of derivatives acting on the baryon field. Using field redefinitions and re-organizing certain terms one can achieve that the effective Lagrangian contributing to the LO BB potential involves only the following terms without derivatives \[^{25}\]:

\[ \mathcal{L}_{BB} = C_i^1 \text{Tr} \left\{ \bar{B}_\alpha \bar{B}_\beta (\Gamma_i B)_\beta (\Gamma_i B)_\alpha \right\} + C_i^2 \text{Tr} \left\{ \bar{B}_\alpha (\Gamma_i B)_\alpha \bar{B}_\beta (\Gamma_i B)_\beta \right\} \]
\[ + C_i^3 \text{Tr} \left\{ \bar{B}_\alpha (\Gamma_i B)_\alpha \right\} \text{Tr} \left\{ \bar{B}_\beta (\Gamma_i B)_\beta \right\}, \]  

where \( C_i^1, C_i^2 \) and \( C_i^3 \) are coupling constants, \( \alpha \) and \( \beta \) are the Dirac spinor indices and

\[ \Gamma_1 = 1, \quad \Gamma_2 = \gamma^\mu, \quad \Gamma_3 = \sigma^{\mu\nu}, \quad \Gamma_4 = \gamma^\mu \gamma_5, \quad \Gamma_5 = \gamma_5. \]  

\[^2\] For the purposes of the current work we switch off the external sources with the exception of the scalar one that accounts for the explicit breaking of the chiral symmetry in QCD.
FIG. 1: One-loop diagrams contributing to the nucleon-nucleon scattering. The first two rows represent eight time-ordered diagrams corresponding to one Feynman diagram shown in the third row. The solid and dashed lines correspond to the nucleons and pions, respectively. The contact interaction with filled blob represents the additional vertex due to the momentum-dependent pion-nucleon interaction as explained in the text.

Notice that the $\Gamma_5$-term yields a vanishing contribution at LO, i.e. it starts contributing at NLO.

Our aim is to obtain diagrammatic rules of old-fashioned TOPT. Notice that the systematic technique of Ref. [58] leads to the TOPT rules, which are not directly applicable to vertices involving time components of momenta. Additional complications emerge from treating particles with non-zero spin. In particular, zeroth components of momenta in the numerators of propagators of particles with non-zero spin cannot be substituted by their on-shell values. To our surprise, we were unable to find in the literature a more complete treatment of TOPT applicable to the case at hand.

To obtain the rules of TOPT we follow the usual procedure by first drawing all possible Feynman diagrams (in principle, an infinite number of them) relevant for the process of interest, assigning the momenta to propagators associated with internal lines and performing the trivial momentum integrations using the delta functions appearing at the vertices. The remaining overall delta function ensures momentum conservation for the external legs of a diagram. Next, we perform integrations over the zeroth components of the loop momenta. This leads to a decomposition of each Feynman diagram into a sum of time-ordered diagrams.

To demonstrate the above procedure we consider a Feynman diagram contributing to the nucleon-nucleon scattering amplitude and depicted in the third line of Fig 1. Omitting SU(3) coefficients, the corresponding expression has the form

$$ g_A^2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}_\alpha(p_3) [\gamma_5 \slashed{k}]_{\alpha\lambda}(p_3 + k + m_N)_{\lambda\beta} C_{\beta\delta,\mu\nu} \bar{u}_\gamma(p_4) [\gamma_5 \slashed{k}]_{\gamma\sigma}(p_4 - k + m_N)_{\sigma\delta} u_\mu(p_1) u_\nu(p_2)}{(k^2 - M_\pi^2 + i\epsilon)((p_3 + k)^2 - m_N^2 + i\epsilon)((p_4 - k)^2 - m_N^2 + i\epsilon)}, $$

where $g_A = D + F$ is the axial coupling, $p_1$ and $p_2$ ($p_3$ and $p_4$) are the four-momenta of incoming (outgoing) nucleons and $C_{\beta\delta,\mu\nu}$ stands for the four-nucleon interaction with the Dirac-spinor indices $\beta\delta, \mu\nu$. All external momenta are on-shell i.e. $p_i^2 = m_N^2$. 

\[ (9) \]
In our calculations we use the Dirac spinors with the four-momentum $q$

\[ u(q) = \sqrt{\frac{\omega(q, m_N) + m_N}{2 m_N}} \left( \frac{\chi}{\omega(q, m_N) + m_N} \right), \quad (10) \]

where $\omega(q, M) := (q^2 + M^2)^{1/2}$ is the particle energy and $\chi$ - the two-component spinor. By using the following identity for the nucleon propagator

\[ (\not{q} + m_N)_{\mu\nu} = 2 m_N \sum u_{\mu}(p) \bar{u}_{\nu}(p) + [p_0 - \omega(p, m_N)] [\gamma_0]_{\mu\nu}, \quad (11) \]

where the summation is done over polarizations, we obtain from Eq. (9)

\[ \frac{g^2}{4 F^2} \int \frac{d^4k}{(2\pi)^4} C_{\delta\mu\nu} \bar{u}_\alpha(p_3) u_\gamma(p_1) u_{\mu}(p_1) u_{\nu}(p_2) \]

\[ \times \left[ 4 m_N^2 [\gamma_5 k]_{\alpha\lambda} \sum u_{\lambda}(p_3 + k) u_{\beta}(p_3 + k) [\gamma_5 k]_{\gamma\sigma} \sum u_{\sigma}(p_4 - k) \bar{u}_{\delta}(p_4 - k) \right. \]

\[ \frac{(k^2 - M_N^2 + i\epsilon) \left( (p_3 + k)^2 - m_N^2 + i\epsilon \right) \left( (p_4 - k)^2 - m_N^2 + i\epsilon \right)}{(k^2 - M_N^2 + i\epsilon) \left( (p_3 + k)^2 - m_N^2 + i\epsilon \right) \left( p_4^0 - k^0 + \omega(p_4 - k, m_N) - i\epsilon \right) \left( p_4^0 - k^0 + \omega(p_4 - k, m_N) - i\epsilon \right)} \]

\[ \left. + \frac{2 m_N [\gamma_5 k]_{\gamma\alpha} \sum u_{\alpha}(p_3 + k) u_{\beta}(p_3 + k) [\gamma_5 k]_{\gamma\delta} \sum u_{\delta}(p_4 - k) \bar{u}_{\delta}(p_4 - k) \right] \right] \quad (12) \]

Next, we make use of the following identities for the propagator with the momentum $k$

\[ \frac{1}{k^2 - M^2 + i\epsilon} = \frac{1}{2 \omega(k, M)} \left( \frac{1}{k^0 - \omega(k, M) + i\epsilon} + \frac{1}{-k^0 - \omega(p, M) + i\epsilon} \right), \]

\[ \frac{k^2}{k^2 - M^2 + i\epsilon} = \frac{1}{2 \omega(k, M)} \left( \frac{\omega(k, M)}{k^0 - \omega(k, M) + i\epsilon} + \frac{-\omega(k, M)}{-k^0 - \omega(p, M) + i\epsilon} \right), \]

\[ \frac{k^0}{k^2 - M^2 + i\epsilon} = \frac{1}{2 \omega(k, M)} \left( \frac{\omega(k, M)^2}{k^0 - \omega(k, M) + i\epsilon} + \frac{(-\omega(k, M))^2}{-k^0 - \omega(p, M) + i\epsilon} \right). \quad (13) \]

These relations turn out to be particularly useful for deriving the diagrammatic rules of TOPT. Then, we simplify the expression for the amplitude by collecting terms with equal denominators. Those terms which have all poles on the same side of the real axis in the complex $k_0$-plane vanish after the integration over $k_0$ as a consequence of the Cauchy theorem. For the remaining terms, closing the contour of integration on the sides with single poles and picking up the corresponding residues we obtain eight contributions which can be represented as time-ordered diagrams shown in first two lines in Fig. [1]. We stress that due to the cancellation of the $k_0$-dependent vertices with the pion propagator, see the first term (i.e. the unity) on the right-hand side of the last line of Eq. (13), the Feynman diagram shown in Fig. [1] contains also purely short-range TOPT contributions. For the sake of compactness, we do not provide the explicit expressions corresponding to these diagrams but rather formulate the general rules of TOPT for processes with baryons and pseudoscalar
mesons in the initial and final states.\textsuperscript{3} In the absence of fermion fields and interaction terms with time derivatives, the rules given below reduce to the ones of Ref. \textsuperscript{59}.

The $S$ matrix for a process $\alpha \rightarrow \beta$ can be written as

$$
S_{\beta \alpha} = \delta_{\beta \alpha} - (2\pi)^4 i M_{\beta \alpha} \delta^4(P_\beta - P_\alpha)
\times \Pi_B^{\alpha,\beta} \left( \frac{(2\pi)^{-3/2}}{2\omega(p_B, m_B)} \right)^{1/2}
\times \Pi_F^{\alpha,\beta} \left( \frac{m_F}{\omega(p_F, m_F)} \right)^{1/2},
$$

(14)

where $P^\mu$ is the total four-momentum and $\Pi_B^{\alpha,\beta}$ denotes a product of the expressions following these symbols over all bosons (labelled by B)/fermions (labelled by F) in the initial and final states. The invariant amplitude $M_{\beta \alpha}$ is obtained by using the following diagrammatic rules:

- Draw all possible time-ordered diagrams for the process $\alpha \rightarrow \beta$.\textsuperscript{4} That is, draw each Feynman diagram with $N$ vertices $N!$ times while ordering the vertices in every possible way in a sequence running from right to left. Label each line with a four-momentum $p = (p_0, \vec{p})$ as prescribed by the corresponding Feynman diagram.

- Include a factor

$$
(2\omega(p_i, M_i))^{-1}
$$

(15)

for every internal line corresponding to a pseudoscalar meson with the mass $M_i$ and four-momentum $p_i$.

- For every internal baryon line with the momentum $p_j$ and mass $m_j$ include a factor

$$
\frac{m_j}{\omega(p_j, m_j)} \sum u(p_j)\bar{u}(p_j),
$$

(16)

where summation is done over polarizations.

- For every internal anti-baryon line (i.e. the line with baryon number flowing opposite to the time direction) with the momentum $p_i$ and mass $m_i$ include a factor

$$
\frac{m_i}{\omega(p_i, m_i)} \sum u(p_i)\bar{u}(p_i) - \gamma_0,
$$

(17)

where summation is done over polarizations.

- For every incoming (outgoing) external fermion with momentum $p$ ($p'$) include $u(p)$ ($\bar{u}(p')$).

- For interaction vertices, use the ones of the standard Feynman rules. Care has to be taken of zeroth components of momenta appearing in the vertices. Indeed, as follows from Eq. (13), for interactions containing one power of the zeroth component of a

\textsuperscript{3} Modifications are required for processes involving anti-baryons in initial and final states.

\textsuperscript{4} Do not include diagrams with closed baryon lines - for low-energy processes their contributions are taken into account by redefining coupling constants of the effective Lagrangian.
FIG. 2: Time-ordered diagrams contributing to the LO baryon-baryon potential. The solid and
dashed lines correspond to baryons and pseudoscalar mesons, respectively. The contact interaction
with filled blob represents the additional vertex due to the momentum-dependent pseudoscalar-
meson–baryon interaction as specified in the diagrammatic rules (see the text).

momentum, this zeroth component has to be replaced by the energy of the particle
carrying this momentum if it corresponds to an particle and by $-1$ times the energy
if the line corresponds to an antiparticle. If a Feynman diagram involves two vertices
containing zeroth component of a momentum corresponding to the same line, the rule
of the previous sentence applies to each of the vertices. However, in addition, one
has to include also the TOPT diagrams where the two vertices and the connecting
propagator cancel each other thus resulting in an effective contact contribution – see
the unity in the very last line of Eq. (13).

• For every intermediate state, i.e. a set of lines between any two vertices corresponding
to particles enumerated with $\gamma$, include an energy denominator

$$\left[ E - \sum_{\gamma} \omega(p_{\gamma}, m_{\gamma}) + i \epsilon \right]^{-1},$$

where $E$ is the total energy of the system, i.e. the sum of the energies of all particles
in initial/final state.

• Integrate over all internal momenta ($\int d^3 \vec{k}_i/(2\pi)^3$ for each $k_i$), and add together the
contributions from all time-ordered diagrams.

III. INTEGRAL EQUATIONS FOR BARYON-BARYON SCATTERING

The baryon-baryon scattering amplitude $M_{BB}$ is obtained from the four-point vertex
function $\tilde{\Gamma}_{4B}$ by applying the standard LSZ formula

$$M_{BB} = Z_B^2 \bar{u}(p_3)\bar{u}(p_4)\tilde{\Gamma}_{4B} u(p_1)u(p_2) \equiv Z_B^2 \tilde{M},$$

where $Z_B$ is the residue of the dressed baryon propagator and $u, \bar{u}$ are Dirac spinors cor-
responding to the incoming and outgoing baryons. The on-shell amplitude $\tilde{M}$ is given as
a sum of an infinite number of TOPT diagrams. Notice that it does not include diagrams
with corrections on external legs.

For baryon-baryon scattering, the purely two-baryon intermediate states are enhanced
\[16\]. Therefore, it is convenient to define the effective potential as a sum of all possible
two-particle irreducible TOPT diagrams. The amplitude \( \tilde{M} \) is then given by an infinite series

\[
\tilde{M} = \tilde{V} + \tilde{V}G\tilde{V} + \tilde{V}G\tilde{V}G\tilde{V} + \ldots
\]

\[
= \tilde{V} + \tilde{V}G\tilde{V} + \tilde{V}G[V + VG\tilde{V} + \ldots]G\tilde{V} = \tilde{V} + \tilde{V}G\tilde{V} + \tilde{V}G\mathcal{M}G\tilde{V},
\]

where \( G \) is the two-baryon Green’s function and \( \tilde{M}, \mathcal{M}, \tilde{V}, \tilde{V} \) and \( V \) are the on-shell amplitude, the off-shell amplitude, the on-shell potential, the half-off-shell potential and the off-shell potential, respectively. The on-shell potential \( \tilde{V} \) does not include diagrams with corrections on external legs. The half-off-shell potential \( \bar{V} \) does not include diagrams with corrections on external legs with on-shell momenta while the off-shell potential \( V \) also includes diagrams with corrections on external legs.

The off-shell amplitude \( \mathcal{M} \) can be obtained by solving the following equation:

\[
\mathcal{M} = V + VG\mathcal{M}.
\]  

(21)

To cover all different processes with strangeness \( S = 0, -1, -2 \) in the isospin limit, Eq. (21) has to be understood as a \( 7 \times 7 \) matrix equation, (a generalization of the Kadychevskiy equation [48]) where

\[
\mathcal{M} = \begin{pmatrix}
M_{NN,NN} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & M_{\Lambda\Lambda,\Lambda\Lambda} & M_{\Lambda\Lambda,\Sigma\Sigma} & M_{\Lambda\Lambda,\Xi\Xi} & M_{\Lambda\Lambda,\Sigma\Xi} & M_{\Lambda\Lambda,\Xi\Sigma} & M_{\Lambda\Lambda,\Sigma\Sigma} \\
0 & 0 & M_{\Sigma\Sigma,\Sigma\Sigma} & M_{\Sigma\Sigma,\Xi\Xi} & M_{\Sigma\Sigma,\Sigma\Xi} & M_{\Sigma\Sigma,\Xi\Sigma} & M_{\Sigma\Sigma,\Sigma\Sigma} \\
0 & 0 & 0 & M_{\Lambda\Sigma,\Lambda\Sigma} & M_{\Lambda\Sigma,\Xi\Xi} & M_{\Lambda\Sigma,\Sigma\Xi} & M_{\Lambda\Sigma,\Xi\Sigma} & M_{\Lambda\Sigma,\Sigma\Sigma} \\
0 & 0 & 0 & 0 & M_{\Xi\Xi,\Xi\Xi} & M_{\Xi\Xi,\Xi\Sigma} & M_{\Xi\Xi,\Xi\Sigma} & M_{\Xi\Xi,\Xi\Xi} \\
0 & 0 & 0 & 0 & 0 & M_{\Xi\Xi,\Xi\Xi} & M_{\Xi\Xi,\Xi\Sigma} & M_{\Xi\Xi,\Xi\Sigma} & M_{\Xi\Xi,\Xi\Xi} \\
0 & 0 & 0 & 0 & 0 & 0 & M_{\Xi\Xi,\Xi\Xi} & M_{\Xi\Xi,\Xi\Sigma} & M_{\Xi\Xi,\Xi\Sigma} & M_{\Xi\Xi,\Xi\Xi}
\end{pmatrix},
\]

\[
V = \begin{pmatrix}
V_{NN,NN} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & V_{\Lambda\Lambda,\Lambda\Lambda} & V_{\Lambda\Lambda,\Sigma\Sigma} & V_{\Lambda\Lambda,\Xi\Xi} & V_{\Lambda\Lambda,\Sigma\Xi} & V_{\Lambda\Lambda,\Xi\Sigma} & V_{\Lambda\Lambda,\Sigma\Sigma} \\
0 & 0 & V_{\Sigma\Sigma,\Sigma\Sigma} & V_{\Sigma\Sigma,\Xi\Xi} & V_{\Sigma\Sigma,\Sigma\Xi} & V_{\Sigma\Sigma,\Xi\Sigma} & V_{\Sigma\Sigma,\Sigma\Sigma} \\
0 & 0 & 0 & V_{\Lambda\Xi,\Lambda\Xi} & V_{\Lambda\Xi,\Xi\Xi} & V_{\Lambda\Xi,\Sigma\Xi} & V_{\Lambda\Xi,\Sigma\Sigma} \\
0 & 0 & 0 & 0 & V_{\Xi\Xi,\Xi\Xi} & V_{\Xi\Xi,\Xi\Sigma} & V_{\Xi\Xi,\Xi\Sigma} & V_{\Xi\Xi,\Xi\Xi} \\
0 & 0 & 0 & 0 & 0 & V_{\Xi\Xi,\Xi\Xi} & V_{\Xi\Xi,\Xi\Sigma} & V_{\Xi\Xi,\Xi\Sigma} & V_{\Xi\Xi,\Xi\Xi} \\
0 & 0 & 0 & 0 & 0 & 0 & V_{\Xi\Xi,\Xi\Xi} & V_{\Xi\Xi,\Xi\Sigma} & V_{\Xi\Xi,\Xi\Sigma} & V_{\Xi\Xi,\Xi\Xi}
\end{pmatrix},
\]

\[
G = \begin{pmatrix}
G_{NN} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & G_{\Lambda\Lambda} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & G_{\Sigma\Sigma} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & G_{\Xi\Xi} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & G_{\Xi\Xi} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & G_{\Xi\Xi} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & G_{\Xi\Xi}
\end{pmatrix},
\]  

(22)

and the two-body Green functions read

\[
G^{IJ}(E) = \frac{1}{\omega_I \omega_J} \frac{m_im_j}{E - \omega_I - \omega_J + i\epsilon},
\]

(23)

where \( m_I \) and \( \omega_I \) are the mass and energy of the \( I \)-th baryon.
The baryon-baryon scattering amplitude can be conveniently calculated in the center-of-mass system (CMS). We denote the three-momenta of the incoming and outgoing baryons in the CMS by \( \vec{p} \) and \( \vec{p}' \), respectively. In the partial wave basis, Eq. (21) leads to the following coupled-channel equations with the partial wave projected potential \( V_{IJ,KL}^{IJ,KL}(p', p) \),

\[
T_{v_{I},s_{I},j}^{I,I,J,K,L}(E, p', p) = V_{v_{I},s_{I},j}^{I,I,J,K,L}(E, p', p) + \sum_{v''_{I},s''_{I},P,Q} \int_{0}^{\infty} \frac{dk}{2\pi^{2}} V_{v''_{I},s''_{I},j}^{I,I,J,K,L}(E, p', k) G_{PQ}^{PQ}(E) T_{v''_{I},s''_{I},j}^{I,I,J,K,L}(E, k, p),
\]

where \( IJ, KL \) and \( PQ \) denote initial, final and intermediate particle channels, \( l, l'' \), \( l' \) and \( s, s'' \), \( s' \) correspond to their orbital angular momentum and their spin, respectively, while \( j \) refers to the total angular momentum of BB states. A standard UV counting shows that in the limit of large integration momenta, considered for the same potential, Eq. (24) with the Green functions of Eq. (23) has a milder UV behaviour than the corresponding Lippmann-Schwinger equation. Therefore, its solutions are expected to show less sensitivity to the variation of the cutoff parameter.

### IV. LO BARYON-BARYON POTENTIAL AND RENORMALIZATION

#### A. Perturbative renormalization of LO baryon-baryon amplitude

We apply the standard Weinberg power counting to the baryon-baryon interaction potential, thus representing it as an expansion with a finite number of TOPT diagrams at any given order. Diagrams contributing to the LO effective potential \( V_{LO} \) for BB scattering are shown in Fig. 2. The potential consists of the short-range contact interaction part \( V_{0,C}^{IJ,KL} \), the long-range part generated by two one-pseudoscalar-meson-exchange time-ordered diagrams and the additional short-range contact term due to the momentum-dependent interactions as explained in the diagrammatic rules. Adding together the last three contributions (i.e. all but \( V_{0,C}^{IJ,KL} \)) and performing simplifications, up to higher-order effects, we obtain the following expression for the one-meson exchange contribution

\[
V_{0,Mp}^{IJ,KL} = \frac{f_{IKP} f_{ILP} I_{IJ,KL}}{2 \omega(q, M_P)} \left[ \frac{\omega(q, M_P) + \omega(p_K, m_K) + \omega(p_I, m_I) - E - i\epsilon}{\omega(q, M_P) + \omega(p_L, m_L) + \omega(p_I, m_I) - E - i\epsilon} \right] \times \frac{1}{(\sigma_1 \cdot \vec{p}_I (\omega(p_K, m_K) + m_K) - \sigma_1 \cdot \vec{p}_K (\omega(p_I, m_I) + m_I))} \times \frac{(\sigma_2 \cdot \vec{p}_J (\omega(p_L, m_L) + m_L) - \sigma_2 \cdot \vec{p}_L (\omega(p_J, m_J) + m_J))}{\sqrt{\omega(p_I, m_I) + m_I \sqrt{\omega(p_J, m_J) + m_J} \sqrt{\omega(p_K, m_K) + m_K} \sqrt{\omega(p_L, m_L) + m_L}}}
\]

\[
+ \frac{1}{\omega(q, M_P) + \omega(p_L, m_L) + \omega(p_I, m_I) - E - i\epsilon} \times \frac{1}{4 \sqrt{m_I m_J m_K m_L}} \times \frac{(\sigma_1 \cdot \vec{p}_I (\omega(p_K, m_K) + m_K) - \sigma_1 \cdot \vec{p}_K (\omega(p_I, m_I) + m_I))}{\sqrt{\omega(p_I, m_I) + m_I \sqrt{\omega(p_J, m_J) + m_J} \sqrt{\omega(p_K, m_K) + m_K} \sqrt{\omega(p_L, m_L) + m_L}}}
\]

\[
+ \frac{1}{\omega(q, M_P) + \omega(p_L, m_L) + \omega(p_I, m_I) - E - i\epsilon} \times \frac{1}{4 \sqrt{m_I m_J m_K m_L}} \times \frac{(\sigma_1 \cdot \vec{p}_I (\omega(p_K, m_K) + m_K) - \sigma_1 \cdot \vec{p}_K (\omega(p_I, m_I) + m_I))}{\sqrt{\omega(p_I, m_I) + m_I \sqrt{\omega(p_J, m_J) + m_J} \sqrt{\omega(p_K, m_K) + m_K} \sqrt{\omega(p_L, m_L) + m_L}}}
\]

\[
+ \frac{1}{\omega(q, M_P) + \omega(p_L, m_L) + \omega(p_I, m_I) - E - i\epsilon} \times \frac{1}{4 \sqrt{m_I m_J m_K m_L}} \times \frac{(\sigma_1 \cdot \vec{p}_I (\omega(p_K, m_K) + m_K) - \sigma_1 \cdot \vec{p}_K (\omega(p_I, m_I) + m_I))}{\sqrt{\omega(p_I, m_I) + m_I \sqrt{\omega(p_J, m_J) + m_J} \sqrt{\omega(p_K, m_K) + m_K} \sqrt{\omega(p_L, m_L) + m_L}}}
\]

\[(25)\]
where \( q = p_I - p_K = p_L - p_J \). The isospin factors \( I_{IJ,KL} \) and the values of \( f_{IKP} \) can be found in Refs. \([24, 27]\).

The expressions for the LO contact interactions relevant for our TOPT approach \((V_{0,C}^{IJ,KL})\) are derived straightforwardly from the Lagrangian of Eq. (7) and can be found in Refs. \([41, 42, 45]\). It is important to emphasize that both the one-meson exchange of Eq. (25) and the contact interaction potentials contain also higher-order contributions according to Weinberg’s power counting. For example, for the contact interaction those emerge from the relativistic energy-dependent normalization factors of the nucleon spinors, as given by Eq. (10). To single out LO contact interactions, we rewrite the relativistic expressions with baryon energies as

\[
\sqrt{\omega (p, m) + m} = 2m + \left[ \sqrt{\omega (p, m) + m - 2m} \right] = 2m + \mathcal{O}(p^2/m), \quad (26)
\]

and shift the term in the square brackets to the higher-order baryon-baryon potential. By doing so we obtain contact interactions which are identical to those of the non-relativistic approach, see Refs. \([23, 25]\). We also perform such splitting for the terms in the numerator of the expression in Eq. (25), that is we replace the particle energies by their masses up to higher-order effects. However, in full analogy to the treatment of the Green function (see Eq. (23)), we keep the full energy expressions in the denominator in Eq. (25), because performing their non-relativistic expansion is not commutative with loop integration \([37]\). While after renormalization relativistic effects as such are not expected to play a significant role at low energies, keeping the full unexpanded expressions in the denominators, as they come out from TOPT, allows one to carry out renormalization of the LO baryon-baryon amplitude by obtaining cutoff independent results – see further discussion below. Thus, for the one-meson-exchange contribution to the LO potential we finally obtain

\[
V_{LO,Mp}^{IJ,KL} = \frac{f_{IKP} f_{JLP} I_{IJ,KL}}{2 \omega(q, Mp)} \left[ \frac{1}{\omega(q, Mp) + \omega(p_K, m_K) + \omega(p_J, m_J) - E - i \epsilon} \right] \frac{1}{\omega(q, Mp) + \omega(p_L, m_L) + \omega(p_I, m_I) - E - i \epsilon} \frac{1}{\sqrt{m_I m_J m_K m_L}} \left( m_I + m_K \right) \left( m_J + m_L \right) \left( m_I \bar{\sigma}_1 \cdot \vec{p}_I - m_J \bar{\sigma}_1 \cdot \vec{p}_K \right) \left( m_L \bar{\sigma}_2 \cdot \vec{p}_L - m_J \bar{\sigma}_2 \cdot \vec{p}_J \right) \sqrt{\omega(p_I, m_I) + m_I \sqrt{\omega(p_J, m_J) + m_J \sqrt{\omega(p_K, m_K) + m_K \sqrt{\omega(p_L, m_L) + m_L}}}} \right] \quad (27)
\]

The difference between \( V_{0,Mp}^{IJ,KL} \) and \( V_{LO,Mp}^{IJ,KL} \) (cf. Eqs. (25) and (27)) is included in the higher-order corrections. The resulting LO long-range potential has a milder ultraviolet behaviour than its analog obtained by using the LO approximation for Dirac spinors as done in Ref. [37]. A distinctive feature of the potential of Eq. (27) as compared to its non-relativistic analog is that its iterations within the integral equations (24) lead to ultraviolet finite diagrams. To demonstrate this feature, consider e.g. the one-loop integral

\[
\sum_{Q,R} \int \frac{d^3 \vec{k}}{(2\pi)^3} X_{QG} \equiv \sum_{Q,R} \int \frac{d^3 \vec{k}}{(2\pi)^3} V_{LO,Mp}^{IJ,QR} \mathcal{G}_{QR}(E) V_{LO,Mp}^{QR,KL}, \quad (28)
\]
where \( \vec{p}_Q = \vec{p}_I - \vec{k} \) and \( \vec{p}_R = \vec{p}_J + \vec{k} \). For \( k \equiv |\vec{k}| \to \infty \), we obtain for the integrand

\[
X_{VGV} = \sum_{Q,R} \int_{Q,R} f_{Q,P} f_{P,R} \mathcal{I}_{I,I,Q,R} f_{Q,K,P} f_{R,L,P} \mathcal{I}_{Q,R,K,L} \frac{(m_I + m_Q)(m_J + m_R)(m_Q + m_K)(m_R + m_L)}{4 m_Q m_R \sqrt{m_I m_J m_K m_L}} \left( m_I \sigma_1 \cdot (\vec{p}_I - \vec{k}) \right) \left( m_K \sigma_1 \cdot (\vec{p}_I - \vec{k}) - m_Q \sigma_1 \cdot \vec{p}_K \right) \left( m_R \sigma_2 \cdot (\vec{p}_J + \vec{k}) \right) \left( m_L \sigma_2 \cdot (\vec{p}_J + \vec{k}) - m_R \sigma_2 \cdot \vec{p}_L \right)
\]

where two factors of \( 1/k^3 \) stem from the denominators of the one-meson exchange potentials of Eq. (27) and an additional factor of \( 1/k^3 \) represents the UV behavior of the Green function of Eq. (23). The integrand \( X_{VGV} \) behaves as \( \sim 1/k^5 \) and thus leads to an UV convergent one-loop integral. Analogously, it can be easily shown that all iterations of the one pseudoscalar meson-exchange potential lead to UV finite diagrams. Then, the full LO potential, which also contains the contact interactions, is perturbatively renormalizable since all divergences appearing from its iterations can be absorbed in the coupling constant of the contact interaction. As a consequence, the ultraviolet cutoff can be safely removed (set to infinity) at LO which allows one to avoid finite-cutoff artefacts inherent to the conventional non-relativistic framework. Also, in this approach, one does not face a well-known issue of the integral equation having non-unique solution for singular attractive potentials (this is e.g. the case for the sufficiently strong \( 1/r^2 \) attractive potential substituted in the Lippmann-Schwinger equation) – see, e.g., Ref. [37] for more details. For our LO potential the integral equations (24) for the baryon-baryon scattering amplitudes have unique solutions for all partial waves.

### B. Application to nucleon-nucleon scattering

Although the LO potential can be properly renormalized as discussed above, in certain channels of baryon-baryon scattering one may still run into the situation that corrections beyond LO are large enough to require their nonperturbative treatment. Below, we address this issue in detail on the example of nucleon-nucleon scattering.

The LO nucleon-nucleon potential consists of two momentum-independent contact interactions contributing to S-waves and the one-pion exchange potential (OPEP) corresponding to Eq. (27), which has the form

\[
V_\pi (\vec{p}', \vec{p}) = -\frac{g_A^2}{4 F_0^2} \frac{\sigma_1 \cdot \sigma_2}{\sqrt{(\vec{p} - \vec{p}')^2 + M_\pi^2}} \left( m_N + \sqrt{\vec{p}^2 + m_N^2} \right) \left( m_N + \sqrt{\vec{p}'^2 + m_N^2} \right) \frac{4 m_N^2}{\sqrt{(\vec{p} - \vec{p}')^2 + M_\pi^2 + \sqrt{\vec{p}^2 + m_N^2} + \sqrt{\vec{p}'^2 + m_N^2}}} \frac{[\sigma_1 \cdot (\vec{p} - \vec{p}')] [\sigma_2 \cdot (\vec{p} - \vec{p}')]}{\sqrt{(\vec{p} - \vec{p}')^2 + M_\pi^2 + \sqrt{\vec{p}^2 + m_N^2} + \sqrt{\vec{p}'^2 + m_N^2}} - E - i \epsilon}
\]

where \( E = 2 \sqrt{m_N^2 + q_{on}^2} \) with \( q_{on} \) being the absolute value of the three-momentum of incoming (and outgoing) nucleons in the center-of-mass frame. The LO potential, substituted
FIG. 3: Nucleon-nucleon phase shifts and mixing angles at LO. Lines correspond to our results while solid dots refer to the Nijmegen partial wave analysis [60].

in the partial wave integral equations of [24], is perturbatively renormalizable and the integral equations have unique solutions. As a general trend, the calculations done at LO provide a reasonable description of the empirical phase shifts as shown in Fig. 3. However, one recognizes large deviations from the Nijmegen partial wave analysis (PWA) for the cases of the $^1S_0$ and $^3P_0$ partial waves. In the $^1S_0$ channel, the observed discrepancy can be traced back to the large (as compared to the inverse range of the OPEP) experimental value of the effective range. In the $^3P_0$ channel, the OPEP is known to become non-perturbative at rather low momenta [61]. Given that in this channel the loop integrals in our Lorentz-invariant formulation are effectively cut off at momenta of the order of the nucleon mass and there is no contact interaction at LO, the observed discrepancy does not come as a surprise. For both the $^1S_0$ and $^3P_0$ partial waves, the large differences between the LO results and the empirical phase shifts suggest that at least a part of the subleading corrections must be treated non-perturbatively. While the non-perturbative inclusion of pion-exchange potentials beyond LO would generally destroy the explicit renormalizability feature of our approach and thus prevent one from eliminating the cutoff, it is still possible to treat the sub-leading contact interactions non-perturbatively within a cutoff-independent approach in the way consistent with the principles of EFT. For the $^1S_0$ channel, it was already demonstrated in Ref. [40] that the proper inclusion of the NLO contact interactions in the considered non-perturbative approach results in the significant improvement for the phase shift, see also Ref. [62] for related discussion in the context of low-energy theorems. The use of the modified OPE potential from Eq. (30) in the $^1S_0$ case is expected to produce results similar to those of Ref. [40].

In the following, we consider in detail the case of the $^3P_0$ partial wave. To improve the
description of the $^3P_0$ phase shift, we add the lowest-order contact interaction term to the potential which is treated non-perturbatively thus obtaining

$$V_{LO}^{^3P_0}(p', p) = C p' p + V_\pi \equiv V_C + V_\pi,$$

(31)

where $V_\pi$ stands for the OPEP of Eq. (30) projected onto the $^3P_0$ partial wave. For the above potential it is possible to write the solution to the integral equation in such a form (analogously to Ref. [63]), which allows one to carry out the subtractive renormalization. For this purpose, we write the integral equations (24) symbolically as

$$T = V + V G T,$$

(32)

and present their solution as [40]

$$T = T_\pi + (1 + T_\pi G) T_C (1 + G T_\pi),$$

(33)

where the amplitude $T_\pi$ satisfies the equation

$$T_\pi = V_\pi + V_\pi G T_\pi.$$  

(34)

For a separable contact interaction potential

$$V_C(p', p) = \tilde{\xi}(p') C \xi(p),$$

(35)

the amplitude $T_C$ is also given in a separable form

$$T_C(p', p) = \tilde{\xi}(p') \mathcal{X} \xi(p),$$

(36)

where

$$\mathcal{X} = [C^{-1} - G \tilde{\xi} - G T_\pi G \tilde{\xi}]^{-1}.$$  

(37)

Thus, the final expression for the amplitude $T$ has the form

$$T = T_\pi + (1 + T_\pi G) \tilde{\xi} \mathcal{X} \xi (1 + G T_\pi).$$  

(38)

In a close analogy to Ref. [40], we apply the subtractive (BPHZ-type) renormalization, i.e. we subtract all divergences in loop diagrams and replace the coupling constants by their renormalized, finite values (see, e.g., Ref. [64] for further details of BPHZ renormalization). Subtractive renormalization of the considered problem corresponds to the inclusion of contributions of an infinite number of counter terms generated by bare parameters of the effective Lagrangian [56].

We need to apply subtractive renormalization to the expression of Eq. (38), where for our contact interaction potential we have

$$C = C, \quad \tilde{\xi}(p') = p', \quad \xi(p) = p.$$  

(39)

Furthermore, by analysing the asymptotic behavior of the OPEP of Eq.(30) in the $^3P_0$ channel,

$$V_\pi(p', p) \big|_{p \to \infty, p' \to \infty} \sim \frac{1}{p^2}, \quad V_\pi(p', p) \big|_{p < \infty, p' \to \infty} \sim \frac{1}{p' p}, \quad V_\pi(p', p) \big|_{p \to \infty, p' \to \infty} \sim \frac{1}{pp'},$$  

(40)
one is led to conclude that the amplitude $T_{\pi}$ is finite, and so are $\Xi(p') = (1 + T_{\pi} G) \xi$ and $\Xi(p) = \xi(1 + G T_{\pi})$. Therefore, we proceed with the renormalization of $\mathcal{X}$. All (multi-loop) sub-diagrams contained in $\Xi G T_{\pi} G \xi$ are also finite, so that this quantity contains only the overall logarithmic divergence which stems from the regime $p \to \infty$, $p' \to \infty$ in Eq. (40). On the other hand, the term $\xi G \bar{\xi}$ is quadratically divergent and therefore requires additional BPHZ subtractions. Indeed, in the limit when the cutoff $\Lambda$ is much larger than the nucleon mass $\xi G \bar{\xi}$ can be written as

$$
\xi G \bar{\xi} = -\frac{m_N^2}{8\pi^2} \left( \Lambda^2 + E \Lambda + (3m_N^2 - E^2/2) \log(\Lambda/m_N) + f(E) \right),
$$

where $f(E)$ is a finite function. By carrying out the subtractive renormalization, the final renormalized expression reads

$$
T(p', p) = T_{\pi}(p', p) + \frac{\Xi(p') \Xi(p)}{\alpha - (\xi G \bar{\xi})^R - (\xi G T_{\pi} G \xi - \alpha)},
$$

where the counter term $\alpha$ subtracts the overall divergence of $\xi G T_{\pi} G \xi$. The subtracted expression of $(\xi G \bar{\xi})^R$ is given in Ref. [40]

$$
(\xi G \bar{\xi})^R = q_{on}^2 I_0^R(\mu, q_{on}) = \frac{m_N^2 q_{on}^2}{4\pi^2 E} \left( 2q_{on} \left( \sinh^{-1} \frac{q_{on}}{m_N} - i\pi \right) - \pi m_N \right) + \frac{m_N^2 q_{on}^2}{8\pi^2 \sqrt{m_N^2 - \mu^2}} \left( 2\mu \left( \sin^{-1} \frac{\mu}{m_N} - \pi \right) + \pi m_N \right),
$$

where $\mu$ is the renormalization scale. In practice, we fix the bare constant $1/C = 1/C_R + \alpha$ as a function of the cutoff numerically in such a way that it cancels the divergent part of $\xi G T_{\pi} G \xi$ and the resulting cutoff-independent scattering amplitude describes the phase shift for a fixed value of the energy chosen to be 20 MeV. The resulting nucleon-nucleon phase shift in the $3P_0$ channel is plotted in Fig. 4 (right panel) and shows a very good description of the data. The renormalized amplitude shows a weak dependence on the renormalization scale $\mu$. Indeed, the variation of $\mu$ from the pion mass to the nucleon mass results in an effect of $\sim 0.6$ degrees in the phase shift for $E_{\text{lab}} \sim 100$ MeV.

It is instructive to compare the above subtractive renormalization with the so-called “non-perturbative renormalization”, where the contact interaction is tuned to reproduce the empirical value of the scattering amplitude at a given energy without subtracting all ultraviolet divergences as advocated e.g. in Refs. [65–67] within the nonrelativistic framework. Such “non-perturbative renormalization” does remove the quadratic divergence in $\xi G \bar{\xi}$ and the logarithmic divergence in $\xi G T_{\pi} G \xi$ with the energy-independent prefactors, while the linear and logarithmic divergences which scale with energy-dependent coefficients still survive in $\xi G \xi$ (see Eq. (41)). In Fig. 4 we confront the cutoff dependence of the $3P_0$ phase shift for such a “non-perturbatively renormalized” scattering amplitude (left panel) with the residual cutoff dependence in the properly renormalized approach as discussed above (right panel). In line with the reasoning discussed above, one may conclude that the frequently used approach without explicit subtractions of divergences does not lead to a properly renormalized result for the scattering amplitude. This conclusion is fully in line with the considerations in Refs. [56, 68].
FIG. 4: Cutoff-dependence of the calculated $^3P_0$ phase shifts. Left and right panels correspond to the “non-perturbatively renormalized” and subtractively renormalized amplitudes, respectively. Solid dots are the results of the Nijmegen PWA [60].

V. SUMMARY

In this paper we considered the baryon-baryon scattering problem in the framework of manifestly Lorentz-invariant formulation of SU(3) BChPT by applying time-ordered perturbation theory. By integrating over zeroth components of loop momenta in Feynman diagrams we formulated the diagrammatic rules of time-ordered perturbation theory, which can be applied to momentum-dependent interactions and particles with non-zero spin. For the case of baryon-baryon scattering, the importance of time-ordered diagrams can be determined using the Weinberg’s power counting rules [16, 17]. An infinite number of diagrams contributes to the baryon-baryon scattering amplitude at any finite order. To sum up the relevant contributions it is convenient to define the effective potential as a sum of all two-baryon irreducible contributions to the scattering amplitude within TOPT. In a full analogy with the conventional nonrelativistic framework, the scattering amplitudes are obtained as solutions to a system of coupled-channel integral equations with the potentials at the corresponding order. These equations represent a coupled-channel generalization of the Kadyshevsky equation [48] and feature a milder ultraviolet behaviour as compared to their non-relativistic analogs. We obtained new perturbatively renormalizable LO baryon-baryon potential which leads to unique solutions of the integral equations for scattering amplitudes in all partial waves. On the example of nucleon-nucleon scattering we addressed the issue of the non-perturbative inclusion of the leading short-range interaction in the $^3P_0$ partial wave. For this purpose, we carried out subtractive renormalization in a way consistent with EFT. We also considered the “non-perturbative renormalization” approach for the problem at hand as advocated, e.g., in Refs. [65–67] to determine the value of the contact interaction. The resulting cutoff dependence of the amplitude supports the conclusions of Refs. [56, 68] about the incompatibility of such “non-perturbative renormalization” with the principles of EFT.

The established formalism can be used to study baryon-baryon scattering in the SU(3)
sector based on a renormalizable formulation with the corrections beyond LO treated perturbatively. Such a framework permits a complete removal of the ultraviolet cutoff \( \Lambda \) by taking the limit \( \Lambda \to \infty \), see Refs. [37–39] for applications in the non-strange sector. Alternatively, one may follow a more traditional approach by solving the integral equations for a truncated potential without relying on a perturbative treatment of higher-order contributions as it is usually done for nucleon-nucleon scattering, see e.g. [69–72]. In that case the \( \Lambda \to \infty \) limit is not legitimate anymore, but one may still expect to benefit from the milder ultraviolet behavior of the integral equations in the Lorentz-invariant formulation, which should provide more flexibility in the choice of the ultraviolet cutoff. Work along these lines is in progress and will be reported in forthcoming publications.

Acknowledgments

We are grateful to Ulf-G. Meißner for useful comments on the manuscript. This work was supported in part by the Georgian Shota Rustaveli National Science Foundation (Grant No. FR17-354), by DFG and NSFC through funds provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD” (NSFC Grant No. 11621131001, DFG Grant No. TRR110), the BMBF (Grant No. 05P18PCFP1) and the Russian Science Foundation (Grant No. 18-12-00226).

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