Charmonium-hadron interactions from QCD

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Abstract. The heavy quark system is an excellent probe to learn about the QCD dynamics at finite density. First, we discuss the properties of the $J/\psi$ and $D$ meson at finite nucleon density. We discuss why their properties should change at finite density and then introduce an exact QCD relation among these hadron properties and the energy momentum tensor of the medium. Second, we discuss attempts to calculate charmonium-hadron total cross section using effective hadronic models and perturbative QCD. We emphasize a recent calculation, where the cross section is derived using QCD factorization theorem. We conclude by discussing some challenges for SIS 200.

1. $J/\psi$ and $D$ mesons at finite density

In the heavy quark system, the heavy quark mass $m_H$ provides a natural normalization scale $\mu$, which makes the perturbative QCD approach possible even for calculating some bound state properties of the heavy quarks. This implies that the heavy quark system is an excellent probe to learn about the QCD dynamics at finite density and/or temperature. Here, we will highlight some calculations which explicitly demonstrate the relations between the changes of the properties of the heavy quark system and the QCD dynamics at finite density.

1.1. $J/\psi$ mass at finite density

Let us first start with the $J/\psi$ meson mass at finite density. The interaction between the $J/\psi$ and a hadron $h$ can be perturbatively generated by multiple gluon exchanges. This implies that the mass shift of the $J/\psi$ in nuclear medium can be estimated if the gluon distribution is known in nuclear medium [1]. In fact, in the heavy quark mass limit $m_c \to \infty$, the mass shift can be calculated exactly to leading order in QCD [2] and is given as

$$\Delta m_{J/\psi} = c_1 \times \Delta \langle E^2 \rangle_{\text{matter}},$$

where $c_1$ is a calculable constant and the matrix element is taken with respect to the nuclear matter. To the leading order in density, the matrix element is known from the trace anomaly relation and the gluon distribution function of the nucleon [3]. For a more realistic charm quark mass, there are some model dependence on how to treat the bound state part. So far, there are potential model approaches [3] and QCD sum rule approaches [4, 5]. In all cases, similar relation to Eq. (1) holds with slightly different expressions for $c_1$, such that the mass shift at nuclear matter density ranges...
from $-4$ to $-7$ MeV. The mass shift is small at nuclear matter density and one doubts whether such a small mass shift can be observed. However, one can turn the argument around and claim that if any mass shift for $J/\psi$ is observed, it will tell us about the changes of gluon field configuration at finite density through Eq. (1).

1.2. $D$ meson mass at finite density

In the $D$ meson, the heavy quark acts as a source for the light quark, which surrounds the heavy quark and probes the QCD vacuum. This is the basic picture of the constituent quark in the heavy-light meson system within the formulation based on the heavy quark symmetry [7]. Therefore, if the vacuum properties are changed at finite temperature or density, the light quark should be sensitively affected and be reflected in the changes of the $D$ meson properties in matter. For the $D$ meson, there does not exist a formal limit where one can calculate the mass change in matter. Nevertheless, model calculations suggest [8, 9, 10] that it is dominantly related to the change in the chiral condensate. In fact, in both the quark-meson coupling model [8] and in the QCD sum rule approach [10], the average mass shift of the $D^\pm$ mesons at normal nuclear matter density were found to be around $-50$ MeV.

The $D$ meson mass shift has several importance in the $J/\psi$ suppression phenomena in relativistic heavy ion collisions (RHIC). First, it leads to subthreshold production and the change of the $D\bar{D}$ threshold for the $J/\psi$ system in matter [11]. Second, a change of the averaged $D^\pm$ mass might lead to level crossings of the $\bar{D}D$ threshold with the $\psi'$ and $\chi$ mass [11, 12], which leads to a step-wise $J/\psi$ suppression [10, 13]. Unfortunately, for the $D$ meson mass shift, there are large model dependence and nontrivial splitting between the $D^+$ and $D^-$ mesons [14].

1.3. Exact sum rule from QCD

The $D$ meson mass and the $D\bar{D}$ threshold have important phenomenological consequences. Unfortunately, only model-dependent calculation could be made. Here, we will introduce an exact QCD equality, which can be used to link the $D\bar{D}$ threshold to $J/\psi$ suppression. The exact sum rules in QCD at finite temperature and/or density were first introduced to the correlation functions between light hadrons [15] and the derivation goes as follows. Let us consider the correlation function between two meson currents and its dispersion relation,

$$\Delta \Pi(Q^2, T, \rho) = i \int d^4 x e^{iqx} \Delta \langle \bar{q} \Gamma q(x) \bar{q} \Gamma q(0) \rangle$$

$$= \frac{1}{\pi} \int ds \frac{\Delta \text{Im} \Pi(s)}{s + Q^2}. \quad (2)$$

Here, we have assumed $q = (iQ, 0, 0, 0)$ and $\Delta \langle \cdot \rangle = \langle \cdot \rangle_{\text{medium}} - \langle \cdot \rangle_{\text{vacuum}}$ means the difference between the medium and the vacuum expectation values. $\Gamma$ is a Dirac gamma matrix that will be determined by what meson we want to study. At $Q^2 \to \infty$, the real part of Eq. (2) can be determined from the operator product expansion (OPE) in QCD,

$$\Delta \Pi(Q^2, T, \rho) = \sum_n \frac{1}{(Q^2)^{d_n+2}} \left( \frac{g^2(Q)}{g^2(\mu)} \right)^{2\gamma_n - \gamma_n} C_n \Delta \langle O_n(\mu) \rangle. \quad (3)$$

Here, $\gamma_n$ ($\gamma_J$) is the anomalous dimension of the operator (current). Equation (3) is an asymptotic expansion in $1/\log(Q^2)$ and $1/Q^2$. For $\gamma_n = \gamma_J = 0$, the leading term
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Figure 1. Charmonium spectral density.

is proportional to $1/Q^2$ and can be related to the first moment of the spectral density as \[ \int ds \frac{s}{\pi} \Delta \text{Im} \Pi(s) = c_4 \Delta \langle O_4 \rangle, \] (4)

where $O_4$ is a dimension 4 operator and $c_4$ its corresponding Wilson coefficient.

One can generalize this formula for the heavy (charm) vector current $\bar{c} \gamma^\mu c$ at rest and obtain the following constraint:

\[ \int ds \frac{s}{\pi} \Delta \text{Im} \Pi(s) = -\frac{1}{12} \Delta \langle \alpha_s G^2 \rangle + \frac{8}{16 + 3n_f} \Delta \langle T_{00} \rangle, \] (5)

where $T_{\mu\nu}$ is the traceless energy momentum tensor and $\langle \alpha_s G^2 \rangle$ is the gluon condensate, which is related to the trace of the energy momentum tensor. Lattice gauge theory provides the temperature dependence of the operators both below and above the phase transition temperature. From this, one can investigate what kind of changes in the spectral density, which in the vacuum has a generic form given in Fig. 1, is consistent with the constraint equation.

Now let us look at the changes at finite density. Using the linear density approximation $\Delta \langle \cdot \rangle = \frac{\rho}{\rho_n} \langle \cdot \rangle |p\rangle$, one has

\[ -\frac{1}{12} \Delta \langle \alpha_s G^2 \rangle \approx 6.25 \times 10^{-5} \times x \text{ GeV}^4, \]

\[ \frac{8}{16 + 3n_f} \Delta \langle T_{00} \rangle \approx 3 \times 10^{-4} \times x \text{ GeV}^4, \] (6)

where $x = \rho/\rho_{n.m.}$ with $\rho_{n.m.} = 0.17/(\text{fm})^3$. In comparison with this, when neglecting the contribution from the $\psi'$, the spectral density in Fig. 1 can be parameterized as

\[ \frac{1}{\pi} \text{Im} \Pi(s) = f_V^2 \delta(s - m_{J/\psi}^2) + \frac{1}{4\pi^2} \frac{\alpha_s}{4\pi} \theta(s - s_0), \] (7)

where $f_V^2 = \frac{3m_{J/\psi}^2}{4\pi^2 Q_e^2} \Gamma(J/\psi \rightarrow e^+ e^-) \approx 0.16 \text{ GeV}^2$. Hence, we have

\[ \int ds \frac{s}{\pi} \Delta \text{Im} \Pi(s) = \Delta (f_V^2 m_{J/\psi}^2) - \frac{1}{8\pi^2} (1 + \frac{\alpha_s}{4\pi}) \Delta (s_0^2). \] (8)

Now, since the mass shift $\Delta m_{J/\psi}$ is very small, the change in the first term is coming from the change in $f_V^2$, which is related to the change of the coupling of the $J/\psi$ to the electromagnetic current. This is precisely related to the amount of $J/\psi$ suppression in matter. At linear density approximation, as can be seen from Eq. (6), the expected
change in the OPE is of $10^{-4}$ GeV$^4$ in the nuclear matter and can be neglected compared to a fractional change in the phenomenological side of Eq. (8). Therefore, in this case, the suppression in $f^2_V$ is related to the decrease of $s_0$. In an order of magnitude estimate, we arrive at

$$\frac{1}{2} \Delta s_0^{1/2} = \frac{m^2_{J/\psi} f^2_V \pi^2}{(1 + \frac{g_s}{4\pi})^{3/2}} \times x \sim 280 \times \frac{\Delta f^2_V}{f^2_V} \text{ MeV},$$  

(9)

where $\Delta f^2_V/f^2_V$ quantifies the amount of $J/\psi$ suppression seen through the dilepton signal. Assuming that $s_0^{1/2} = m_D + m_{\bar{D}}$, Eq. (9) implies that 10% suppression is related to 28 MeV mass decrease of the $D$ meson in nuclear matter.

The relation in Eq. (4) can be used in the other ways. Namely, from the measurement of the changes in the mass of the $D$ mesons and the suppression of the $J/\psi$ peak, one is able to learn about the changes in the energy momentum tensor and gluon condensate at finite temperature or density, where the measurement would be made.

2. $J/\psi$-hadron cross section

$J/\psi$ suppression [16] seems to be one of the most promising signal for QGP formation in RHIC. Indeed the recent data at CERN [17] show an anomalous suppression of $J/\psi$ formation, which seems to be a consequence of QGP formation [18]. However, before coming to a concrete conclusion, one has to estimate the amount of $J/\psi$ suppression due to other non-QGP mechanism, one of which is the hadronic final state interactions. Indeed, there are model calculations that show large suppression by hadronic final state interactions [19, 20, 21]. The typical values for $J/\psi$ + hadron dissociation cross sections one uses for these calculations are $\sigma_{J/\psi + N} \sim 5$ mb and $\sigma_{J/\psi + comover} \sim 1$ mb. However, there is no direct experimental data from which we can determine or confirm these cross sections. In fact, the existing model calculations vary greatly in their energy dependence and magnitude near the threshold. In Fig. 2, we show the latest typical results of model calculations based on perturbative QCD [22, 23, 24, 25, 26], the meson exchange model [27] and the quark exchange model [28]. To understand the discrepancies and to obtain a consistent result, it is crucial to probe each model calculations further so as to spell out the limitations and range of validity of each model calculations. Here, we will have a closer look at the perturbative QCD result.

2.1. Perturbative QCD methods

More than two decades ago, Peskin [22] and Bhanot and Peskin [23] showed that one could apply perturbative Quantum Chromodynamics (pQCD) to calculate the interactions between the bound state of heavy quarks and the light hadrons. Such calculation was feasible because in the large quark mass limit one could consistently obtain the leading order OPE of the correlation function between two heavy meson currents in the light hadron state. The justification of such calculation stems from the fact that there exist two relevant scales in the bound state [29] in the large quark mass limit, namely the binding energy, which becomes Coulomb-like and scales as $mg^4$, and the typical momentum scale of the bound state, which scales like $mg^2$, where $m$ is the heavy quark mass and $g$ the quark-gluon coupling constant. Hence, taking the separation scale of the OPE to be the binding energy, it is consistent to take into account the bound state property, which is generated by the typical momentum scale
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3.5 4.0 4.5 5.0
\(\sqrt{s}\) (GeV)

\[10^{-2}\]

\[10^{-1}\]

\[10^{0}\]

\[10^{1}\]

\(\sigma_{\pi^{+}J/\psi}\) (mb)

\[\text{quark exchange model}\]

\[\text{meson exchange model}\]

\[\text{pQCD}\]

Figure 2. \(\sigma_{\pi^{+}J/\psi}\) cross section from meson exchange model [27], quark exchange model [28] and perturbative QCD [23] calculations.

of the bound state, into the process-dependent Wilson coefficient. The result obtained by Peskin and Bhanot was derived within the OPE [22, 23]. Recently, we have derived anew the leading order pQCD result using the QCD factorization theorem [26]. The equivalence between the OPE and the factorization approach is well established in the deep inelastic scattering and Drell-Yan processes [30]. Our work shows that the equivalence between the two approaches are also true for bound state scattering to leading order in QCD. Similar approach has been used by one of us (SHL) to estimate the dissociation cross section of the \(J/\psi\) at finite temperature [31]. The factorization formula also provides a manageable starting point to calculate higher twist gluonic effects [32, 6], which should be nontrivial for the \(J/\psi\)-hadron scattering. Here, we will sketch the derivation.

We refer to a bound state of heavy quark and its own antiquark as \(\Phi\). According to the factorization formula, the total scattering cross section of \(\Phi\) with a hadron \(h\) can be written as [23]

\[
\sigma_{\Phi h}(\nu) = \int_0^1 dx \sigma_{\Phi g}(x\nu) g(x) \label{eq:sigma_phi_h}
\]

with \(\nu = p \cdot q / M_\Phi\), where \(p \ (q)\) is the momentum of the hadron (\(\Phi\)) and \(M_\Phi\) is the \(\Phi\) mass. Here, \(\sigma_{\Phi g}\) is the perturbative \(\Phi\)-gluon scattering cross section and \(g(x)\) is the leading twist gluon distribution function within the hadron. The separation scale is taken to be the binding energy of the bound state. Hence, the bound state properties have to be taken into account in \(\sigma_{\Phi g}\). This can be accomplished by introducing the Bethe-Salpeter (BS) amplitude \(\Gamma(p_1, -p_2)\), which satisfies [33]

\[
\Gamma_\mu(p_1, -p_2) = iC_{\text{color}} \int \frac{d^4k}{(2\pi)^4} g^2 V(k) \gamma^\nu \Delta(p_1 + k)
\times \Gamma_\mu(p_1 + k, -p_2 + k) \Delta(-p_2 + k) \gamma_\nu, \label{eq:BS_amplitude}
\]

where \(C_{\text{color}} = (N_c^2 - 1)/(2N_c)\) with the number of color \(N_c\). The trivial color indices have been suppressed and \(p_1 (-p_2)\) is the four-momentum of the heavy quark (antiquark).
We write $q = p_1 + p_2$ and $p = (p_1 - p_2)/2$ and work in the $\Phi$ rest frame and introduce the binding energy $\varepsilon$, such that $q_0 = M_\Phi = 2m + \varepsilon$ ($\varepsilon < 0$). Then, in the non-relativistic limit, the BS amplitude reduces to the following form:

$$
\Gamma_{\mu}^i \left( \frac{1}{2} q + p, \frac{1}{2} q + p \right) = - \left( \varepsilon - \frac{p^2}{2m} \right) \sqrt{\frac{M_\Phi}{N_c}} \psi(p) \frac{1 + \gamma_0}{2} \gamma_i \frac{1 - \gamma_0}{2},
$$

(12)

and the corresponding BS equation becomes the non-relativistic Schrödinger equation for the Coulombic bound state, so that $\psi(p)$ is the normalized wave function for the bound state.

With the BS amplitude defined as Eq. (12), we now obtain the $\Phi$-gluon scattering amplitude with the processes depicted in Fig. 3. The scattering matrix elements can be obtained by substituting the BS amplitude for the vertex. To obtain the leading order result, we should pay attention to the counting scheme. First the binding energy $\varepsilon_0 = m \left[ N_c g^2/(16\pi) \right]^2$ is $O(mg^4)$. Combined with the energy conservation $q + k = p_1 + p_2$ in the non-relativistic limit, this implies $|p_1| \sim |p_2| \sim O(mg^2)$ and $k^0 = |k| \sim O(mg^4)$. Within this counting scheme, the final answer reads

$$
M_{\mu
u} = -g \left[ \frac{M_\Phi}{N_c} \right] \left\{ k \cdot \frac{\partial \psi(p)}{\partial p} \delta_{\mu0} + k_0 \frac{\partial \psi(p)}{\partial p^0} \delta_{\nu j} \right\} \delta_{\mu i} \times \bar{u}(p_1) \frac{1 + \gamma_0}{2} \gamma_i \frac{1 - \gamma_0}{2} T^a v(p_2).
$$

(13)

The scattering cross section $\sigma_{\Phi g}$ can now be obtained from

$$
\sigma_{\Phi g} = \int \frac{1}{4M_\Phi k_0} |M|^2 (2\pi)^3 \delta^4(p_1 + p_2 - k - q) \frac{d^3p_1}{4p_1^0} \frac{d^3p_2}{4p_2^0} (2\pi)^6.
$$

(14)

With the amplitude $|M|^2$ we obtain

$$
|M|^2 = \frac{4g^2 m^2 M_\Phi k_0^2}{3N_c} |\nabla \psi(p)|^2.
$$

(15)

Substituting Eq. (14) into Eq. (10) and using the known gluon distribution function in the pion, we obtain the dashed line in Fig. 2.

2.2. Corrections to leading order formula

The counting of this formalism implies that for the charmonium system $\alpha_s = 0.84$ and the separation scale $\varepsilon_0 = 0.78$ GeV. Hence, both the $\alpha_s$ and higher twist effects
should be important. However, calculating higher $\alpha_s$ corrections in this formalism is intricately related to the relativistic corrections including $1/m$ corrections, which is a formidable future task.

Here, we will only look at the simple but important relativistic correction coming from the relativistic calculation of the phase space integral in Eq. (14). The difference after a full calculation is shown in Fig. 4 for $\sigma_{\Phi g}$, which shows larger suppression of the cross sections at higher energy. However, we find that this relativistic correction has almost no effect on the cross section $\sigma_{\Phi h}$. This follows because $\sigma_{\Phi g}$ is highly peaked at small energy region and the gluon distribution function increases towards small $x$.

Through Eq. (10), this implies that $\sigma_{\Phi h}$, at all energies, are dominated by the low energy behavior of $\sigma_{\Phi g}$, which is reliably calculated in the non-relativistic limit. This implies that the full relativistic corrections to the scattering amplitude $\sigma_{\Phi g}$ would have little effects on $\sigma_{\Phi h}$.

The remaining important and interesting correction is the higher twist-effect, which contributes as $A/\varepsilon^2$ in our formalism. Since $A$ is related to some hadronic scale of the hadron $h$, it should be of the order $\varepsilon^2$ itself, hence should be non-negligible [34]. It is also important to extend this calculation to investigate the absorption cross section for the $\chi$ states. This will influence the amount of $J/\psi$ production coming from the decay of the $\chi$’s in a p-A or A-A reaction [34].

3. Conclusion and challenges for SIS200

As we have discussed, observation of the changes in the properties of the heavy quark system will provide us useful information about the QCD dynamics both in the vacuum and in the medium. Important experimental observation would be mass shifts of the charmonium and open-charm system in nuclear medium. Also important would be the quantitative observation of the suppression of the $J/\psi$ and $\psi'$ signals in the medium through the dileptons.

$\bar{p}$-A experiments to probe the $J/\psi$ in nuclear matter would also provide useful informations on the $J/\psi$-nucleon cross section at low energy. This is precisely where there is huge discrepancy between existing theoretical predictions. Experimental investigation will provide useful information on the higher twist effects in the pQCD result for the $J/\psi$-nucleon cross section. Here, the higher twist effect comes solely...
from gluon operators and hence this in turn would be a unique place to learn about the higher twist gluon effect of the nucleon and nuclear matter.

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References

[1] Brodsky S J, Schmidt I A and de Teramond G F 1990 Phys. Rev. Lett. 64 1011
[2] Luke M E, Manohar A V and Savage M J 1992 Phys. Lett. B 288 355
[3] Voloshin M and Zakharov V 1980 Phys. Rev. Lett. 45 688
[4] Wasson D A 1991 Phys. Rev. Lett. 67 2237
[5] Klingl F, Kim S, Lee S H, Morath P and Weise W 2001 Nucl. Phys. A 679 517
[6] Huang S-Z and Lissia M 1995 Phys. Lett. B 348 571
[7] Arleo F, Gossiaux P-B, Gousset T and Aichelin J 2002 Phys. Rev. D 65 014005