Application of the conditional gradient method to a network resource allocation problem with several classes of users

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1. Introduction

Despite the existence of powerful processing and transmission devices, increasing demand of different telecommunication services and its variability lead to serious congestion effects and inefficient utilization of network resources. This situation forces one to replace the fixed allocation rules with more flexible mechanisms, which are based on proper mathematical models; see e.g. \[1\]–\[3\]. Most papers in this field are devoted to game-theoretic models and implementation of decentralized iterative methods for finding the Nash equilibrium points or their generalizations; see e.g. \[4, 5\]. Within a telecommunication network with one provider, centralized rules can be used and various optimization based mechanisms are also suggested; see e.g. \[6, 7, 5, 3\].

In \[8\]–\[13\], several optimal resource allocation problems in telecommunication networks and proper zonal decomposition based methods were suggested. They suggest to solve proper constrained convex optimization problems for some selected time period. However, these models do not take into account possible differentiation of users with respect to service levels, which yields different service costs and somewhat different optimization problems. Such a formulation of the optimal resource allocation problem with different service classes was proposed in \[14\], however, it involved separable network expense functions, which enable one to apply various decomposition methods directly to the initial problem.

In this paper, we also consider problems of optimal allocation of a homogeneous resource in a telecommunication network with the differentiation of users in accordance with required service levels. Unlike \[14\], the model deals with arbitrary convex network expense functions, which makes
the model suitable for broader classes of applications. We suggest to solve this problem with a two-level iterative method where the conditional gradient method is used at its upper level despite the fact that its feasible set is not polyhedral in general. Nevertheless, then we obtain separable problems at the lower level and can solve them with simple decomposition algorithms. The computational experiments show rather satisfactory convergence of the suggested method.

2. Problem description

Let us consider a single telecommunication network with nodes (users). A network manager offers users \( m \) levels of network service (classes), which is reflected by prices. Within some selected time period, the network manager has limited total amount of a homogeneous resource of the network \( C \) (for example, total bandwidth). An amount of resource allocated to the \( i \)-th class of service is supposed to be equal to \( x_i \), where \( x_i \) is an unknown traffic volume at this level \( (0 \leq x_i \leq \beta_i) \). Thus, the vector \( x = (x_1, \ldots, x_m)^T \) gives the unknown traffic load profile of the network, which requires the implementation network expense \( \mu(x) \). Each user can choose only one level of service. Let \( N = \{1, \ldots, n\} \) denote a set of users, and \( N_i \) a set of users of the \( i \)-th class (level) for \( i = 1, \ldots, m \). Let \( y_j \) denote the unknown traffic volume offered to the \( j \)-th user with \( 0 \leq y_j \leq \alpha_j \) and \( \eta_j(y_j) \) is the fee (incentive) value paid by the \( j \)-th user for this traffic. If all the users are attributed to the classes, we can establish the traffic volume balance for each \( i \)-th level as follows:

\[
x_i = \sum_{j \in N_i} y_j.
\]

The general problem of the network manager is to find an optimal allocation of a limited homogeneous resource among the users in order to maximize the total payment received from the users and to minimize the total network implementation expenses. This problem is now formulated as follows:

\[
\max_{u=(x,y) \in U} f(x,y) = \sum_{i=1}^{m} \sum_{j \in N_i} \eta_j(y_j) - \mu(x), \tag{1}
\]

where

\[
U = \{ (x,y) \in W : \sum_{i=1}^{m} \varphi_i(x_i) \leq C \}, \tag{2}
\]

\[
W = \{ (x,y) : x_i = \sum_{j \in N_i} y_j, 0 \leq y_j \leq \alpha_j, j \in N_i, 0 \leq x_i \leq \beta_i, i = 1, \ldots, m \}. \tag{3}
\]

We suppose that all the functions \( \mu(x) \), \( \varphi_i(x_i) \) and \( -\eta_j(y_j) \) are convex and differentiable. In this case (1)–(3) is a smooth convex optimization problem.

3. Solution methods

To solve (1)–(3) we suggest to first apply the known conditional gradient method (CGM); see e.g. [15] and references therein.

(CGM) Take an arbitrary initial point \( u^0 \in U \) and numbers \( \alpha \in (0,1), \gamma \in (0,1), \) and \( \delta > 0 \). At the \( k \)-th iteration, \( k = 0,1, \ldots \), we have a point \( u^k \in U \) and calculate \( v^k \in U \) as a solution of the auxiliary problem

\[
\min_{\tilde{v} \in U} \langle f'(u^k), \tilde{v} \rangle. \tag{4}
\]

Then we set \( p^k = v^k - u^k \). If \( \| p^k \| \leq \delta \) (or \( \delta_k = -\langle f'(u^k), p^k \rangle \leq \delta \), stop, we have an approximate solution. Otherwise we find \( t \) as the minimal non-negative integer such that

\[
f(u^k + t p^k) \leq f(u^k) + \alpha t \langle f'(u^k), p^k \rangle,
\]

and set \( u^{k+1} = u^k + \delta_k p^k \).
set \( \sigma_k = \gamma' \) and \( u^{k+1} = u^k + \sigma_k p^k \).

The basic problem (4) can be re-written as follows:

\[
\max_{i=1}^{m} \rightarrow \sum_{j=1}^{m} \sum_{i \in N_j} \eta_j \gamma'(y_j^i) y_j - (\mu'(x^i), x) = \sum_{i=1}^{m} \left[ \sum_{j \in N_j} \eta_j \gamma'(y_j^i) y_j - \mu'(x^i) x_j \right],
\]

where

\[
\mu'_i(x) = \frac{\partial \mu(x)}{\partial x_i}.
\]

For finding a solution of this decomposable problem we use the duality approach with respect to the total capacity bound, i.e. we take the dual problem

\[
\min_{\lambda \geq 0} \rightarrow \psi(\lambda),
\]

where

\[
\psi(\lambda) = \max_{(x, y) \in W} \left\{ \sum_{i=1}^{m} \left[ \sum_{j \in N_j} \eta_j \gamma'(y_j^i) y_j - \mu'_i(x^i) x_j \right] - \lambda \left( \sum_{i=1}^{m} \varphi_i(x_i) - C \right) \right\}
\]

\[
= \lambda C + \max_{(x, y) \in W} \left\{ \sum_{i=1}^{m} \left[ \sum_{j \in N_j} \eta_j \gamma'(y_j^i) y_j - (\mu'_i(x^i) x^i + \lambda \varphi_i(x_i)) \right] \right\}.
\]

By duality (see e.g. [16]), problems (5) and (6) have the same optimal value. But one can find a solution of (6) by one of well-known single-dimensional optimization algorithms; see e.g. [16]. Next, in order to calculate the value of \( \psi(\lambda) \) we have to solve the inner problem:

\[
\max_{(x, y) \in W} \rightarrow \left\{ \sum_{i=1}^{m} \left[ \sum_{j \in N_j} \eta_j \gamma'(y_j^i) y_j - (\mu'_i(x^i) x^i + \lambda \varphi_i(x_i)) \right] \right\}.
\]

This inner problem decomposes into \( m \) independent problems

\[
\max \rightarrow \sum_{j \in N_i} \eta_j \gamma'(y_j^i) y_j - (\mu'_i(x^i) x^i + \lambda \varphi_i(x_i)),
\]

\[
x_i = \sum_{j \in N_i} y_j, 0 \leq y_j \leq \alpha_j, j \in N_i, 0 \leq x_i \leq \beta_i,
\]

for \( i = 1, \ldots, m \). Each problem (7)–(8) can be solved by simple ordering algorithms; see e.g. [11]. We observe that the approach described enables us to solve (1)–(3) with (CGM) although its feasible set in (2)–(3) is not polyhedral in general.

The above method can be clearly applied in the completely separable case where

\[
\mu(x) = \sum_{i=1}^{m} \mu_i(x_i)
\]

and

\[
\mu'_i(x) = \mu'_i(x_i).
\]

This case was considered in [14], where the dual decomposition approaches were suggested for the initial problem (1)–(3) since its cost function is now re-written as follows:

\[
f(x, y) = \sum_{i=1}^{m} \left[ \sum_{j \in N_i} \eta_j(y_j^i) - \mu_i(x_i) \right].
\]

Utilizing the same duality approach we replace problem (9), (1)–(3) with its dual:

\[
\min_{\lambda \geq 0} \rightarrow \xi(\lambda),
\]

where
\[ \xi(\lambda) = \max_{(x,y)\in W} L(x,y,\lambda) = \lambda C + \max_{(x,y)\in W} \sum_{i=1}^{m} \left( \sum_{j\in N_i} \eta_j(y_j) - (\mu_i(x_i) + \lambda \phi_i(x_i)) \right). \]

In order to solve the single-dimensional optimization problem (10) we can use again one of the standard algorithms. In order to calculate the value of \( \xi(\lambda) \) we have to solve the inner problem:

\[ \max_{(x,y)\in W} \sum_{i=1}^{m} \left( \sum_{j\in N_i} \eta_j(y_j) - (\mu_i(x_i) + \lambda \phi_i(x_i)) \right), \]

which decomposes into \( m \) independent problems

\[ \max_{j\in N_i} \sum_{j\in N_i} \eta_j(y_j) - (\mu_i(x_i) + \lambda \phi_i(x_i)), \]

\[ x_i = \sum_{j\in N_i} y_j, 0 \leq y_j \leq \alpha_j, j \in N_i, 0 \leq x_i \leq \beta_i, \]

for \( i = 1, \ldots, m \). In particular, it was suggested in [14] to solve (11)–(12) with either the conditional gradient method (CGDM for brevity) or the combined duality and bisection method (BS for brevity). Both these methods showed rather rapid convergence. However, they are not applicable in the general case.

4. Numerical experiments

In order to evaluate the performance of the new method denoted as (CGMD) we made several series of computational experiments. We compared it with (CGDM) and (BS) and for this reason chose the completely separable case. The methods were implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for choosing the dual variable \( \lambda \) were taken as \([0,1000]\). Values of \( \beta_i (i = 1, \ldots, m) \) were chosen by trigonometric functions in \([1,51]\), values of \( \alpha_i (j \in N_i, i = 1, \ldots, m) \) were chosen by trigonometric functions in \([1,2]\). Value \( C \) were taken equal 1000. The number of tariffs was varied from 3 to 15, the number of users was varied from 210 to 1010. Users were distributed in tariffs either uniformly or according to the normal distribution. We used the following classes of functions in problem (9), (1)–(3):

[Case EQ] All the functions \(-\eta_j(y_j)\) are convex quadratic, all the functions \(\mu_i(x_i)\) and \(\phi_i(x_i)\) are convex exponential;

[Case Q] All the functions \(-\eta_j(y_j)\) are convex quadratic, all the functions \(\mu_i(x_i)\) and \(\phi_i(x_i)\) are convex quadratic;

[Case E] All the functions \(-\eta_j(y_j)\), \(\mu_i(x_i)\) and \(\phi_i(x_i)\) are convex exponential;

[Case L] All the functions \(-\eta_j(y_j)\), \(\mu_i(x_i)\) and \(\phi_i(x_i)\) are convex logarithmic.

Their coefficients were chosen as values of different trigonometric functions. For all the methods the accuracy of the upper problem solution was varied from \(10^{-3}\) to \(10^{-4}\). Let \( e_{\text{top}} \) denote this value.

The accuracy of the lower level problems solution was fixed and equal to \(10^{-2}\). For each set of the parameters we made 50 tests. In what follows, \( J \) denote the total number of users and \( T_e \) denotes the total processor time in seconds. The averaged results of computations for Case Q are given in Tables 1–3, for Case EQ in Tables 4–6, for Case E in Tables 7–9, for Case L in Tables 10–12.

We named by (CGMD0) and (CGDM0) the version of (CGMD) and (CGDM), respectively, where the zero initial point was taken in each conditional gradient method and by (CGMDB) and (CGDMB) the version of (CGMD) and (CGDM), respectively, where a boundary initial point was taken in each conditional gradient method.

Values of \( \alpha \) and \( \gamma \) in (CGMD) and (CGDM) were taken as 0.7 and 0.2, respectively. This values were taken experimentally. This allows us to get more appropriate time for finding solution.
Table 1. Results of testing Case Q with $J = 510$, $m = 7$.

| $\varepsilon_{\text{pop}}$ | $T_\varepsilon$ (CGMD0) | $T_\varepsilon$ (CGMDB) | $T_\varepsilon$ (CGDM0) | $T_\varepsilon$ (CGDMB) | $T_\varepsilon$ (BS) |
|-----------------------------|--------------------------|--------------------------|--------------------------|--------------------------|-----------------------|
| $10^{-1}$                   | 0.0260                   | 0.0344                   | 0.0182                   | 0.0667                   | 0.0008                |
| $10^{-2}$                   | 0.0520                   | 0.0816                   | 0.0422                   | 0.0901                   | 0.0010                |
| $10^{-3}$                   | 0.0869                   | 0.1218                   | 0.0727                   | 0.1270                   | 0.0040                |
| $10^{-4}$                   | 0.0952                   | 0.1447                   | 0.1008                   | 0.1619                   | 0.0058                |

Table 2. Results of testing Case Q $m = 7$, $\varepsilon = 10^{-2}$.

| $J$ | $T_\varepsilon$ (CGMD0) | $T_\varepsilon$ (CGMDB) | $T_\varepsilon$ (CGDM0) | $T_\varepsilon$ (CGDMB) | $T_\varepsilon$ (BS) |
|-----|--------------------------|--------------------------|--------------------------|--------------------------|-----------------------|
| 210 | 0.0209                   | 0.0269                   | 0.0134                   | 0.0407                   | 0.0005                |
| 410 | 0.0484                   | 0.0765                   | 0.0316                   | 0.0777                   | 0.0015                |
| 610 | 0.0625                   | 0.0974                   | 0.0401                   | 0.1046                   | 0.0010                |
| 810 | 0.0906                   | 0.1400                   | 0.0473                   | 0.1350                   | 0.0031                |
| 1010| 0.1271                   | 0.1861                   | 0.0698                   | 0.1790                   | 0.0047                |

Table 3. Results of testing Case Q with $J = 510$, $\varepsilon = 10^{-2}$.

| $m$ | $T_\varepsilon$ (CGMD0) | $T_\varepsilon$ (CGMDB) | $T_\varepsilon$ (CGDM0) | $T_\varepsilon$ (CGDMB) | $T_\varepsilon$ (BS) |
|-----|--------------------------|--------------------------|--------------------------|--------------------------|-----------------------|
| 3   | 0.0405                   | 0.0568                   | 0.0457                   | 0.0979                   | 0.0015                |
| 6   | 0.0515                   | 0.0880                   | 0.0343                   | 0.0953                   | 0.0010                |
| 9   | 0.0553                   | 0.1063                   | 0.0369                   | 0.0936                   | 0.0025                |
| 12  | 0.0901                   | 0.1352                   | 0.0452                   | 0.1020                   | 0.0021                |
| 15  | 0.1046                   | 0.1462                   | 0.0515                   | 0.1125                   | 0.0036                |

Table 4. Results of testing Case EQ with $J = 510$, $m = 7$.

| $\varepsilon_{\text{pop}}$ | $T_\varepsilon$ (CGMD0) | $T_\varepsilon$ (CGMDB) | $T_\varepsilon$ (CGDM0) | $T_\varepsilon$ (CGDMB) | $T_\varepsilon$ (BS) |
|-----------------------------|--------------------------|--------------------------|--------------------------|--------------------------|-----------------------|
| $10^{-1}$                   | 0.0359                   | 0.0531                   | 0.0318                   | 0.1638                   | 0.0036                |
| $10^{-2}$                   | 0.1072                   | 0.1954                   | 0.0520                   | 0.1936                   | 0.0047                |
| $10^{-3}$                   | 0.2026                   | 0.3021                   | 0.0986                   | 0.2562                   | 0.0093                |
| $10^{-4}$                   | 0.2100                   | 0.3453                   | 0.1078                   | 0.3090                   | 0.0114                |

Table 5. Results of testing Case EQ with $m = 7$, $\varepsilon = 10^{-2}$.

| $J$ | $T_\varepsilon$ (CGMD0) | $T_\varepsilon$ (CGMDB) | $T_\varepsilon$ (CGDM0) | $T_\varepsilon$ (CGDMB) | $T_\varepsilon$ (BS) |
|-----|--------------------------|--------------------------|--------------------------|--------------------------|-----------------------|
| 210 | 0.0535                   | 0.0804                   | 0.0244                   | 0.0806                   | 0.0015                |
| 410 | 0.0964                   | 0.1489                   | 0.0426                   | 0.1989                   | 0.0041                |
| 610 | 0.1326                   | 0.2213                   | 0.0624                   | 0.2438                   | 0.0041                |
| 810 | 0.2077                   | 0.3312                   | 0.0877                   | 0.3160                   | 0.0051                |
| 1010| 0.3042                   | 0.4370                   | 0.1250                   | 0.4061                   | 0.0109                |

Table 6. Results of testing Case EQ with $J = 510$, $\varepsilon = 10^{-2}$.

| $m$ | $T_\varepsilon$ (CGMD0) | $T_\varepsilon$ (CGMDB) | $T_\varepsilon$ (CGDM0) | $T_\varepsilon$ (CGDMB) | $T_\varepsilon$ (BS) |
|-----|--------------------------|--------------------------|--------------------------|--------------------------|-----------------------|
| 3   | 0.0650                   | 0.0859                   | 0.0612                   | 0.2099                   | 0.0041                |
| 6   | 0.0797                   | 0.1479                   | 0.0480                   | 0.1989                   | 0.0020                |
| 9   | 0.1312                   | 0.2057                   | 0.0443                   | 0.2016                   | 0.0021                |
| 12  | 0.2088                   | 0.2255                   | 0.0458                   | 0.2101                   | 0.0052                |
| 15  | 0.3186                   | 0.3156                   | 0.0769                   | 0.2353                   | 0.0041                |
**Table 7.** Results of testing Case E with \( J = 510 \), \( m = 7 \).

| \( \varepsilon_{top} \) | \( T_c : \) (CGMD0) | \( T_c : \) (CGMDB) | \( T_c : \) (CGDM0) | \( T_c : \) (CGDMB) | \( T_c : \) (BS) |
|---|---|---|---|---|---|
| \( 10^1 \) | 0.3494 | 0.3853 | 0.8499 | 1.0339 | 0.0069 |
| \( 10^2 \) | 0.6918 | 0.6994 | 1.3370 | 1.5047 | 0.0077 |
| \( 10^3 \) | 0.7468 | 0.7583 | 2.2729 | 2.4233 | 0.0114 |
| \( 10^4 \) | 0.8189 | 0.8185 | 2.9617 | 3.2801 | 0.0146 |

**Table 8.** Results of testing Case E with \( m = 7 \), \( \varepsilon = 10^{-2} \).

| \( J \) | \( T_c : \) (CGMD0) | \( T_c : \) (CGMDB) | \( T_c : \) (CGDM0) | \( T_c : \) (CGDMB) | \( T_c : \) (BS) |
|---|---|---|---|---|---|
| 210 | 0.3115 | 0.3056 | 0.6827 | 0.7661 | 0.0030 |
| 410 | 0.5667 | 0.5859 | 1.1483 | 1.3282 | 0.0046 |
| 610 | 0.7688 | 0.8230 | 1.4881 | 1.7088 | 0.0089 |
| 810 | 1.0040 | 1.1307 | 2.2509 | 2.6323 | 0.0093 |
| 1010 | 1.1867 | 1.4454 | 2.7924 | 3.4399 | 0.0130 |

**Table 9.** Results of testing Case E with \( J = 510 \), \( \varepsilon = 10^{-2} \).

| \( m \) | \( T_c : \) (CGMD0) | \( T_c : \) (CGMDB) | \( T_c : \) (CGDM0) | \( T_c : \) (CGDMB) | \( T_c : \) (BS) |
|---|---|---|---|---|---|
| 3 | 0.3465 | 0.4006 | 1.4146 | 1.5447 | 0.0052 |
| 6 | 0.6452 | 0.5858 | 1.2704 | 1.5092 | 0.0061 |
| 9 | 0.8730 | 0.8016 | 1.5476 | 1.7573 | 0.0062 |
| 12 | 0.8409 | 0.9405 | 1.9259 | 2.1004 | 0.0084 |
| 15 | 1.1057 | 1.1031 | 2.1245 | 2.2838 | 0.0088 |

**Table 10.** Results of testing Case L with \( J = 510 \), \( m = 7 \).

| \( \varepsilon_{top} \) | \( T_c : \) (CGMD0) | \( T_c : \) (CGMDB) | \( T_c : \) (CGDM0) | \( T_c : \) (CGDMB) | \( T_c : \) (BS) |
|---|---|---|---|---|---|
| \( 10^1 \) | 0.0655 | 0.0718 | 0.1547 | 0.1928 | 0.0025 |
| \( 10^2 \) | 0.2708 | 0.2141 | 0.2536 | 0.2989 | 0.0030 |
| \( 10^3 \) | 0.5005 | 0.5261 | 0.4145 | 0.4334 | 0.0046 |
| \( 10^4 \) | 0.8778 | 0.6200 | 0.5717 | 0.5948 | 0.0052 |

**Table 11.** Results of testing Case L with \( m = 7 \), \( \varepsilon = 10^{-2} \).

| \( J \) | \( T_c : \) (CGMD0) | \( T_c : \) (CGMDB) | \( T_c : \) (CGDM0) | \( T_c : \) (CGDMB) | \( T_c : \) (BS) |
|---|---|---|---|---|---|
| 210 | 0.0837 | 0.0812 | 0.0618 | 0.0890 | 0.0005 |
| 410 | 0.2150 | 0.1927 | 0.1838 | 0.2243 | 0.0046 |
| 610 | 0.3104 | 0.2808 | 0.3109 | 0.3858 | 0.0033 |
| 810 | 0.4292 | 0.4287 | 0.4375 | 0.5281 | 0.0036 |
| 1010 | 0.5494 | 0.5447 | 0.5406 | 0.6526 | 0.0063 |

**Table 12.** Results of testing Case L with \( J = 510 \), \( \varepsilon = 10^{-2} \).

| \( m \) | \( T_c : \) (CGMD0) | \( T_c : \) (CGMDB) | \( T_c : \) (CGDM0) | \( T_c : \) (CGDMB) | \( T_c : \) (BS) |
|---|---|---|---|---|---|
| 3 | 0.1740 | 0.1427 | 0.3317 | 0.3131 | 0.0007 |
| 6 | 0.2645 | 0.3380 | 0.2855 | 0.3219 | 0.0026 |
| 9 | 0.3682 | 0.4568 | 0.2604 | 0.3151 | 0.0027 |
| 12 | 0.3626 | 0.4624 | 0.2510 | 0.2951 | 0.0025 |
| 15 | 0.4317 | 0.4624 | 0.2307 | 0.2952 | 0.0031 |
As we can see from the results in the tables, in all the cases the suggested method (CGMD) was rather efficient in finding a solution. In general, (CGMD) was slower than (BS), but had almost the same or better estimates than (CGDM). Thus, we find its convergence rather satisfactory.

5. Conclusions
We considered a general problem of optimal allocation of a homogeneous resource (bandwidth) in a wireless communication network with different levels of service. We suggest to apply conditional gradient method with decomposition techniques. The results of computational experiments confirm the applicability of the method.

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