Abstract

The production of \( J/\Psi \)'s in nuclei is re-examined and the critical role of Feynman x distributions in the study of photoproduction and hadronic production of both \( J/\Psi \)'s and Drell-Yan dimuons is expatiated. The need to consider both initial and final state interactions and the key effect of initial state interactions on the total \( J/\Psi \) cross section are demonstrated.

We first present a theoretical study of the expected functional form for the projectile and target atomic number (A,B) dependence of the \( J/\Psi \) suppression, S, and demonstrate why it cannot fall exactly exponentially with \( A^{1/3} + B^{1/3} \), since such a term is multiplied by a weak enhancement factor. We use a Woods-Saxon Monte Carlo simulation to obtain distributions of the numbers of collisions prior and subsequent to the charmonium production. With those results we relate the mean number of nucleon-nucleon collisions, \( <n> \), to \( A^{1/3} + B^{1/3} \), and give a new analytic functional form for the A dependence of the suppression.

Finally we carry out a full Monte Carlo calculation of S, including an initial state prior energy loss in keeping with the measured Feynman \( x_F \) distributions in \( \pi\)-A and p-A interactions. We use an energy loss parameterization which is consistent with the energy loss from minimum bias data fitted in ISAJET. We also use an open charm absorption cross section, \( \sigma_{oc} \), constrained from photoproduction of \( J/\Psi \)'s, to analyze the latest data on the \( J/\Psi \) suppression.

The role of color screening, the time evolution of off-shell charmonium to the on-shell \( J/\Psi \), and the energy dependence of the charmonium-nucleon inelastic and absorption cross sections are discussed. A program of related measurements, necessary for better understanding the \( J/\Psi \) suppression, is presented.

We find no anomalous suppression when comparing our results to published data.
I. Review

Shortly after the proposal \cite{1} of the possibility of seeing evidence for deconfinement of charmonium in nuclear reactions, use was made of studies of soft processes in relativistic nuclear reactions \cite{2} to examine the rich Feynman x distributions of J/Ψ’s that had been measured in production of J/Ψ’s in pion and proton interactions in nuclei. From the outset \cite{3}, charmonium absorption, as the only source of J/Ψ suppression, was ruled out by a glance at the many examples of the strong $x_F$ dependence of the p-A/p-p and $\pi$-A/$\pi$-p ratios of the J/Ψ yields. In that first work, only the energy loss of incoming hadrons before the charmonium production and energy loss due to rescattering of charmonium off nucleons in the final state were considered. A more careful examination of many examples of $x_F$ distributions for both J/Ψ and Drell-Yan production was then carried out \cite{4}.

A direct determination of the charmonium absorption cross section, $\sigma_{oc}$, from photoproduction of J/Ψ’s was then obtained \cite{5}. In photoproduction there are no initial state interaction effects to confuse the interpretation, as in production from hadron probes. We obtained the result, $\sigma_{oc} = 6.6 +/- 2.2 \text{ mb}$.

It is important to understand that there is, presently, no way to calculate the non-perturbative effect on the nucleon structure functions resulting from prior soft scatters before the high $Q^2$ production of vector mesons. Thus the “energy loss” must be treated as a parameter that can be obtained, in principle, from analysis of the Feynman x distributions.

To carry out these early calculations we used the well-known ISAJET model of Frank Paige to arrive at an expression for the approximate functional form of the $\sqrt{s}$ dependence of the energy loss in soft collisions prior to the J/Ψ production. The energy loss per collision is found in low $p_t$ interactions to vary approximately as $d\sqrt{s}/dn = a + b\sqrt{s}$. The functional form is useful but the values of $a$ and $b$ obtained from ISAJET are only a guide. For many calculations of the $x_F$ distributions it is sufficient to use $d\sqrt{s}/dn = a$ alone. We studied $d\sqrt{s}/dn$ in the range .4 to 1.0 in our early work. (Ref. \cite{4})

In Ref. \cite{4}, we presented two key examples of our earlier calculations of the Feynman
x distributions for a pion- and a proton-induced reaction. Two samples of data are reproduced in Fig. 1, since they clearly illustrate the many effects that are responsible for $J/\Psi$ suppression:

1) Loss of $J/\Psi$ 's due to absorption: It is generally assumed, since there are no data to test the hypothesis, that $\sigma_{oc}$ is energy independent. Thus the charmonium absorption alone can only reduce $S(x_f)$ independent of $x_F$. A flat reduction independent of $x_F$ is not what the data show. All measured reactions show falloff of the ratio of the nuclear to nucleon production with $x_F$. Further, suppressions at $x_F = 0$ are not the same for pion and proton induced reactions, additional proof that final state absorption cannot
be the only mechanism for the suppression.

2) Initial state energy loss: This has two effects: a) the yield is reduced because of the exponential dependence of the $J/\Psi$ yields on $1/\sqrt{s}$, $e^{-M/\sqrt{s}}$, and b) events are shifted to lower $x_F$. Both effects appear in all the measured $x_F$ distributions.

3) The outgoing charmonium interacts with nucleons inelastically, losing further energy by soft collisions: $J/\Psi + n \rightarrow J/\Psi + X$. This will affect the shape of the $x_F$ distributions as well, forcing events to lower $x_F$.

(In Fig. 1 the absorption cross section was taken as 9 mb and $d\sqrt{s}/dn$ taken as .4 but a smaller cross section and somewhat larger $d\sqrt{s}/dn$ would provide a similar fit. Also, an energy loss of the outgoing $J/\Psi$ of $\Delta E/E = .1$ resulted in a need for an inelastic cross section, $\sigma_{inel}$, of about 11 mb.)

Clearly the Feynman $x$ distributions are complicated, but they are rich in information. Fortunately the inelastic $J/\Psi$ interactions will not affect the total $J/\Psi$ suppression ratios. (However, they will affect the rapidity distributions and thus affect the rapidity acceptance, producing a possible experimental A dependent effect on S.)
II. Bases for Suppression Calculations

We digress to emphasize some of the constraints that time dilation places on multiple scatterings in nuclei: Immediately after a low $p_t$ proton collision on a nucleon in a nucleus, the proton is in an “excited” state. It is “off-shell”. The asymptotic final state of the proton is a real “on-shell” proton. Real pions and other produced mesons appear later in the asymptotic state. The time for this evolution in the proton center of mass is usually assumed to be roughly set by the uncertainty relation, $\Delta E \Delta t = \hbar$. It is then time dilated so that the final “asymptotic” state of on-shell proton and pions occurs much later.

(One would like to believe this argument is quantitative, but the time evolution is not presently understood, as recent studies of color screening have indicated [6].)

For light nuclei it is known that the hadronization occurs when the hadrons have left the nucleus [7]. Since the measured multiplicity distributions in p-D, p-He, and He-He interactions are well fit from the p-p multiplicity distributions and the probabilities of making n wounded nucleons, neglecting any final state effects is reasonable for light nuclei. However there may be final state corrections needed for the multiplicity or $E_t$ distributions in heavy nuclei. Thus, one cannot use the p-p measured $E_t$ distributions directly in predicting the nuclear $E_t$ distributions.

In this picture it is the excited off-shell proton, $p^*$, which collides with nucleons to produce the charmonium in the following scatterings. We have made, in all our calculations, the assumption that the $p^*$ - nucleon cross section is little different than the p-nucleon cross section, on the possible ground that high energy nucleon-nucleon cross-sections are essentially geometric, as the energy independence of the p-p cross sections suggest. However, if the cross sections were to increase as the protons become excited, calculations of the number of soft collisions, prior and subsequent to the charmonium production, would be enhanced, increasing the energy loss effect. It is useful to appreciate this difference in the low $p_t$ collisions in p-A and A-B collisions.

However, in A-B interactions, the charmonium may be interacting with nucleons struck by other incoming nucleons, so the A-B absorption is not on ground state nucleons but sometimes on excited nucleons. This too is ignored in the model but it points out that $J/\Psi$ production in p-A reactions cannot strictly be treated identically with
A-B interactions.

The high $Q^2$ charmonium state, however, has a short lifetime, so that $\Delta t$ is much smaller than in low $p_t$ collisions. Therefore it is believed to be mainly made within the nucleus where it can interact with nucleons further along the path and be absorbed by reactions leading to open charmed particles.

Because the pions and kaons made in the soft interactions mainly materialize outside the nuclei, these “co-movers” should not appreciably affect the suppression. We do not consider them and we shall see that they are not needed to account for the measured suppressions, although they might have a small effect on $S$. (Aspects of this subject are treated theoretically, and data on pion production by muons has been examined in a study of the $x$ dependence of pion production in nuclei.)

“Dependence of Hadron production in Inelastic Muon Scattering and Dimuon Production by Protons”, S. Frankel and W. Frati, Phys. Rev. D51 4783 (1995).

It is clear that the soft energy loss of the pertinent constituents, gluons and quarks, which produce $J/\Psi$’s and Drell-Yan pairs, and which should be different for proton and pion projectiles, will have to be parameters obtained from the Feynman $x$ distributions.

In our work Ref. [4], studying $d\sqrt{s}/dn$ for $J/\Psi$’s, we observed empirically that the value of $d\sqrt{s}/dn$ was smaller, $\approx .2$, for Drell-Yan pairs. This latter value is very crude but provided a rough fit to our analyses of various early experiments [9].

As shown in Fig 1, reasonable fits to both 800 GeV proton data and the 150 Gev pion data with a charmonium-nucleon inelastic cross section of about 11 millibarns were obtained, assuming no color transparency effects were present. (It was assumed in these analyses that no color screening was present and that $\sigma_{oc}$ was $x_F$ independent.)

It was abundantly clear that energy loss in the initial state as well as the absorption into open charm played important roles. The initial state energy loss was crucial to the $J/\Psi$ analyses and was small but not zero in the Drell-Yan data.

Thus, in this paper we return to examining the new data on $J/\Psi$ suppression in heavy nuclei since the recent analyses [10] make assumptions which the Feynman $x$ data contradict.
III. Analytic Study of the A Dependence of the $J/\Psi$ Suppression

In this section we see what insights can be obtained from collision of a 2 lines of interacting nucleons to obtain simple formulas for the effects of energy loss and open charm absorption on $J/\Psi$ and dimuon production. As shown in Fig. 2. $N$ and $M$ are the number of nucleons in each nucleus along a collision line and $n$ and $m$ are the nucleon positions in the line. In a nucleus-nucleus collision there will be a distribution of such line-on-line collisions and the final result will require averaging over the $N + M$ distribution. We shall describe such Monte Carlo results later in this paper but the analytic calculation for the line-on-line expression for the $J/\Psi$ suppression contains interesting information.

We shall assume for this calculation that all the measurements are taken at the same projectile energy so that $e^{-\alpha}$ represents the reduction in yield per soft collision due to energy loss prior to the $J/\Psi$ production and $e^{-\beta}$ represents the absorption loss in a single $J/\Psi$ -nucleon collision subsequent to the production. (No difference in any of the cross sections due to proton-neutron differences appears in this simple analysis, although it will affect yields at high $A^{1/3} + B^{1/3}$.)

Consider the $J/\Psi$ production in the scattering of nucleon $n = 2$ on nucleon $m = 3$ as shown in Fig. 2: $n = 2$ is slowed down by soft interactions with $m = 1$ and $m = 2$, while $m = 3$ has been slowed by interaction with $n = 1$. Our calculation sums over the $n, m$ variables. Similarly, nucleons 3-6 in $N$ and 4-7 in $M$ interact with the produced charmonium, these collisions absorbing the charmonium into free charmed particles, with an absorption cross-section denoted as $\sigma_{oc}$. Because of the high $Q^2$
(low cross-section) nature of charmonium production, the number of \(J/\Psi\)'s produced is proportional to \(NM\), all nucleons being equally likely to interact to make a \(J/\Psi\).

The energy loss effect comes from the strong \(\sqrt{s}\) dependence of the \(J/\Psi\) cross section which is known to be given by:

1) \(\sigma_{J/\Psi} = e^{-\gamma M/\sqrt{s}}\)

with \(\gamma = 14.5\) and \(M = 3.1\) GeV.

It is important to note that this expression is \(\sqrt{s}\) dependent and the measurements cover a wide variety of \(\sqrt{s}\).

Taking into account prior energy loss, this becomes:

2) \(e^{-\gamma M/(\sqrt{s_0} - A_n)} = e^{-\gamma M/(\sqrt{s_0} \times [1 - A_n/\sqrt{s_0}])}\)

where \(A = d\sqrt{s}/dn\) and \(n\) is the number of collisions prior to the \(J/\Psi\) production. Expanding the denominator, it can be approximated by \(e^{-(\gamma M/\sqrt{s})(1 + A_n/s_0)}\). Thus we have the simple linear approximation:

2') \(e^{-\alpha n} \approx e^{-(\gamma MA/s_0)n}\).

(Note that the energy loss effect depends on \(s\) and not \(\sqrt{s}\) as in the cross section for \(J/\Psi\) production and that the lower the energy of the reaction, the larger the suppression.)

As we shall see, this is a fair approximation and we shall use it in our analytic studies. But we shall use the exact formulation in our Monte Carlo calculations reported later in this paper.

Thus, for the interacting nucleons shown in Fig. 2, we have:

3) energy loss: \(e^{-\alpha[(n-1)+(m-1)]}\)

Similarly we can parameterize the loss into open charm where \(\beta\) is proportional to the open charm cross section, \(\sigma_{OC}\), assuming that \(\sigma_{OC}\) is energy independent.

4) open charm: \(e^{-\beta[(N-n)+(M-m)]}\)
This product can be re-factored as
open charm: $= e^{-\beta[(N-1)+(M-1)]} \times e^{+\beta[(n-1)+(m-1)]}$

5) Including both energy loss and open charm we obtain:

$$S_p = e^{-\beta[(N-1)+(M-1)]} \times e^{-(\alpha-\beta)[(n-1)+(m-1)]} \times \frac{1}{NM}$$

Summing over the nucleons, this can be rewritten as:

6) $S_p = e^{-\beta(N+M)} \times \sum_1^N \sum_1^M e^{-(\alpha-\beta)(i-1)}e^{-(\alpha-\beta)(j-1)}$

Carrying out the sums and rearranging, we get the final result:

7) $S_p = e^{-(\alpha+\beta)(N-1+M-1)/2} \times \frac{\sinh(\alpha-\beta)N/2}{N\sinh(\alpha-\beta)/2} \times \frac{\sinh(\alpha-\beta)M/2}{M\sinh(\alpha-\beta)/2}$

$(N-1 + M-1)/2$ can be considered the mean number of prior or subsequent collisions in the row. We shall see later how this is related to the mean number of collisions calculated from a Woods-Saxon distribution which we relate to $A^{1/3} + B^{1/3}$.

We denote the bracketed factor in 7) as $C$. Since it increases with $N$ and $M$, $C$ is an enhancement factor as opposed to the exponential suppression factor it multiplies.

Note that the cancellation of $\alpha$ and $\beta$ in $C$ helps in making $C$ small and the exponential fall-off a better approximation.

We will return to this effect later when we discuss central $A-B$ collisions, where the enhancement effect can be seen to be an appreciable contribution to the $A$ dependence of $S$.

A useful approximation for $C$ is:

8) $C \approx [1 + (\alpha - \beta)^2(N^2 - 1)/6][1 + (\alpha - \beta)^2(M^2 - 1)/6]$

Thus we have demonstrated that the cross section does indeed fall off exponentially with $\alpha + \beta$ except for the enhancement factor, $C$, given by the sinh expression. The enhancement effect is understood by recognizing that the interactions at the very front of the nuclei have only open charm absorption while the interactions at the very rear of the nuclei have only energy loss. However, at other positions the effects are both present.
We now turn to a comparison of $J/\Psi$ production to Drell-Yan production. In this case there is no open charm contribution for the Drell-Yan production, only energy loss entering into the ratio, denoted as $S_{dy}$. The result is:

$$9) \ S_{dy} = e^{-(\alpha - \alpha')(N-1+M-1)/2} \times C_{dy},$$

where

$$C_{dy} = \frac{\sinh(\alpha - \beta)N/2}{N\sinh(\alpha - \beta)/2} \frac{\sinh(\alpha - \beta)M/2}{M\sinh(\alpha - \beta)/2}.$$ 

In this expression $\alpha'$ is the Drell-Yan energy loss parameter which is considerably smaller than that for $J/\Psi$ production. We have estimated $d\sqrt{s}/dn = 0.2$ in our prior studies of Drell-Yan production in nuclei. \[4\] \[5\]

Prediction: The “slopes” of the logarithmic plots of $S_p$ vs the mean number of scatters should be somewhat larger than that for $S_{dy}$.

We now turn to an extension of the analytic calculation, which depends on the mean number of scatters in a line of interactions, to integration of the line interactions over the spatial configurations of the nuclei with the goal of relating $(N-1)/2 + (M-1)/2$ to $A^{1/3} + B^{1/3}$. We have shown that the $J/\Psi$ suppression that would take place if a string of $N$ nucleons interacted with a string of $M$ nucleons along a line is given approximately by

$$10) \ S = e^{-(\alpha + \beta)(N-1+M-1)/2}.$$

The $C$ of equations 7, 8 is a function of $\alpha$, $\beta$, $N$ and $M$ but is close to unity so we will ignore it for the present discussion. In this form $\beta$ represents the loss due to $J/\Psi$ absorption but $\alpha$ represents an approximation of the effect of prior energy loss of the incoming nucleons on the $J/\Psi$ yield.

The question we address here is how $(N-1+M-1)/2$ depends on the quantity $K(A^{1/3} + B^{1/3} - 2)$ since this is the variable that many workers use in plotting the suppression data. (We choose to keep the -2 in these expressions since, for p-p collisions, these quantities are zero, so $S$ is unity.)

It is clear that equation 10) would have to be integrated over the probability that
one gets $N$ and $M$ nucleons in a line, averaged over the impact parameter in a nucleus-nucleus collision.

Another question is whether the needed average is some simple function of $(A^{1/3} + B^{1/3} - 2)$.

To answer these questions we do not (at first) go to a full scale Monte Carlo calculation of the $J/\Psi$ suppression but first consider only the distribution of prior and subsequent collisions, $n_{\text{prior}} = n_{\text{subs}} = n$, before and after the $J/\Psi$ production.

These are needed to examine eq. 7. We obtained these distributions from a full Monte Carlo, using Woods-Saxon representation of the nuclei.

The averages for p-A and A-A interactions are shown in Fig. 3. In this figure $n_A + n_B$ is the sum of the prior scatters in both nuclei.

![Figure 3:](image)

(For deuterium and helium we have used the best spatial distributions, as was done in our search for deconfinement in p, d, and $\alpha$ experiments at the ISR citeakesson. This shows up in the slight curvatures at very low A.)
The calculations show that the slope of $< n >$ vs $(A^{1/3} + B^{1/3})$ is essentially the same for p-A and A-A interactions, and a little reflection will allow the reader to realize that this is what is to be expected. We can then ask what value of $K$ is needed to calculate $< n >$. What we find from the slopes in the figure is that $K \approx .46$.

The fits to a straight line, $< n > = a + b(A^{1/3} + B^{1/3} - 2)$, are given in Table 1.

|            | b  | a  |
|------------|----|----|
| p-A average (prior) | .46 | .05 |
| A-A average (prior)  | .46 | .00 |
| p-A central (prior)   | .63 | .00 |
| A-A central (prior)   | .51 | -.10 |

We also note from the curves for central, i.e., zero impact parameter, collisions, Fig 4, that for a central $AB$ collision the values of $< n >$ are somewhat different than those of an $AA$ collision with the same $(A^{1/3} + B^{1/3})$ value. However, as expected, $A-B$ and $A-A$ average collisions should fall on the $A-A$ curve at the appropriate $A^{1/3} + B^{1/3}$.

One might guess that a good approximation for the averaging over all $n$ might be to replace $(N - 1)/2$ by $< n >$ in eq. 7. Instead, we will carry out the averaging to test this hypothesis.

If one examines the distribution of prior collisions, $n$, when the average number is $< n >$, the normalized distribution is given approximately [12] by:

11) $F(n) = e^{-n/<n>}/<n>$

Therefore we can average $S(n)$ over the $n$ distribution to obtain $S(< n >)$
12) \( S(\langle n \rangle) = \int e^{\alpha n} e^{-n/\langle n \rangle} / \langle n \rangle \, dn \)

This is just

13) \( S(\langle n \rangle) = \frac{1}{(\alpha+1/\langle n \rangle)\langle n \rangle} = \frac{1}{1+\alpha\langle n \rangle} \)

Using

14) \( 1/S(\langle n \rangle) = 1 + \alpha < n > \)

To verify this formula, we can plot \( 1/S(\langle n \rangle) \) vs \( \alpha \), which should then be a straight line whose slope is the mean number of collisions. To do this we have used our complete Monte Carlo calculation for \( S(\alpha) \) for average Pb-Pb collisions to determine the validity of the approximate calculation of eq. 14. We have chosen \( \alpha \)'s corresponding with hypothetical absorption cross-sections from 4.08 to 9.4 mb.

Fig 5 shows this plot. It is indeed linear with a slope of 0.44, which is, within error, the same as the value determined from the Monte Carlo calculation of \( \langle n \rangle \) for a Pb-Pb collision, shown in Fig 3.

Thus the theoretical assumption for an exponential shape of the \( n \) distribution is apparently a good approximation.
As a second check, we can examine, for fixed $\alpha$, how the Monte Carlo calculation of $< n >$ and the approximate calculation of $< n >$ from eq. 14 depend on $A^{1/3} + B^{1/3} - 2$ for p-A collisions. This plot is shown in Fig. 6 showing that both methods agree for a range of A’s from P-C to p-U.

It is very useful, therefore, to note that one can use the analytical expression of equation 1) and the parameter $K = .46$ to calculate effects of absorption whether they are due to disappearance of the J/$\Psi$ or due to an exponential decrease in yield as the result of prior energy loss.

Thus we conclude that an effective $L$ for plotting the suppression should be

$$15) \ L_{eff} = .46(A^{1/3} + B^{1/3} - 2)$$
IV. Complete Monte Carlo Calculations of the $J/\Psi$ Suppression

A. Charmonium Absorption Only

We first turn to a calculation of the suppression on the assumption that initial state energy loss can be completely neglected. Fig 7 shows the effect of varying the absorption cross section obtained from our full Monte Carlo and based on the same Woods-Saxon distribution used by M Nardi \[ \text{[13]} \].

From this figure we see that the cross section that fits the data is greater than 9 mb and thus is in disagreement with the cross section obtained from photoproduction. This alone points to the need for energy loss. However Ref. \[ \text{[13]} \] has done what is presumably the same calculation and obtain a cross section of 7.6 mb. We believe that this might be due to the following difference in the details of the calculation: In calculating absorption effects it is important to recognize that the nucleus is composed of finite nucleons and that there is a counting problem when one attempts calculations
using a smooth density distribution. If one integrates along a path up to a point within the nucleus and then asks for the probability of a collision on the way out of the nucleus, the struck nucleon is included in the absorption path, overestimating the absorption and thus requiring a smaller absorption cross section. One cannot have the struck nucleon absorb the charmonium without including this effect in the p-p cross section as well. This effect also occurs in the calculation of the effects of color screening in high \( p_t \) e-p and p-p collisions. There the overcounting resulted in a claim for observation of color screening. The difference in the two methods is easily demonstrated \([3]\).
B. Energy Loss Only

Figure 8:

In this subsection we show the effects of energy loss with and without absorption. Fig 8 shows a plot of average collisions for several values of $\sqrt{s}$, corresponding with ones used in present experiments. The upper lines show the effects of energy loss alone. The lower points show the effects of changing $\sqrt{s}$ when both absorption and energy loss are computed.
C. Combining Absorption and Energy Loss

In this final section we turn to a complete study of both the energy loss and absorption effects without the exponential energy loss approximation. It is obvious from eqs 1) and 2) that energy loss will produce deviations from pure exponential behavior of $S$ vs $A^{1/3} + B^{1/3}$. If the energy loss were given by the approximate relation, namely $e^{-\alpha n}$, we have seen that $S$ falls off almost exponentially with $\alpha$. However, as we have seen previously, the actual energy dependence is more complicated. Unlike the so-called “scaling factor” of eq. 1, which depends on $\sqrt{s}$, the energy loss factor is more sensitive and depends on $s$, as shown in eq. 2. The effect of using the true dependence of the $J/\Psi$ cross section on energy is to introduce curvature into the plot of $S$ vs the mean number of collisions. The direction of the curvature is to depress $S$ at high $A$ and low $\sqrt{s}$ which, as we shall see from our Monte Carlo calculations is in agreement with what the data show.
Fig 9 shows the results using equation 2) with the parameters $d\sqrt{s}/dn = .5 + .018 \sqrt{s}$ and $\sigma_{oc} = 6.3$ and 7.9 mb. These are reasonable choices of the parameters and are consistent with the photoproduction data on $\sigma_{oc}$. However, other choices, within limits, give equivalent results. It is worth pointing out, however, that the data are taken at different energies and with different experimental arrangements and no systematic errors have been provided. We have marked the different $\sqrt{s}$ points so the reader can decide whether slightly larger $\sigma_{oc}$ with a smaller $d\sqrt{s}/dn$ will give a “better fit”, but we will not do so, in view of the need for three parameters which are not accurately known.

This simple counting model, incorporating energy loss, accounts for the present data without the need to invoke any “new physics”.
VI. Central Collisions

We have seen in section II. that there is an enhancement factor $C$ that multiplies the exponential dependence $S$ on $A^{1/3} + B^{1/3}$. This was shown for the special case of interactions of two lines of nucleons. One can verify this, of course, using the full exact Monte Carlo. The largest number of scatters would occur in measurements that trigger on high transverse energy, so this type of measurement would find deviations from true exponential behavior. Since high $E_t$ triggers favor central, zero impact parameter, collisions, we choose to examine central A-A collisions. Fig 10 shows a plot of central $A - A$ collisions for $\sigma_{\text{oc}} = 7.9$ mb. The energy loss is set equal to zero to demonstrate the effect. It is clear from the full MC calculation that $S$ turns up at large $A$. Once again we see how simple exponential extrapolations to large $A$ can be misleading.
VII. Examination of the $\sqrt{s}$ Approximation

In section III. we carried out an analytic calculation based on the approximation in the energy loss equation, going from eq 2 to eq 2'. To see this effect in more detail, we show in Fig. 11 a comparison of the two results. It appears that the approximation is noticeable but produces only a small effect on the suppression even at high $A^{1/3} + B^{1/3}$. 

Figure 11:
VIII. Discussion and Future Analyses

Because the soft processes that take place in nuclei are not able to be handled by perturbative QCD, it is essential to examine how $J/\Psi$'s are made in a large variety of experiments. It is especially important to study the energy loss effect since it appears in all particle production in nuclei.

1) Photon and electron probes, having no initial state interactions, should be used to determine both the $x_F$ dependence of the open charm cross section and the energy loss of the outgoing $J/\Psi$. Both are presently unknown, as is the $J/\Psi$-nucleon inelastic cross section. Several incident energies will be needed to unravel these three quantities.

2) Feynman $x$ distributions of Drell-Yan dimuons are poorly known. This reaction studies only the initial state interactions and is the companion to the photo-production experiments. The Drell-Yan data are the only data that determine $d\sqrt{s}/dn$ uniquely, since there are no final state interactions. This information is needed in comparing $J/\Psi$ production relative to Drell-Yan production. The Feynman $x$ distributions at $x_F = 1$ are constrained by a knowledge of the Glauber coefficients; in fact, $R$ is given by the probability that the pair is made in the first collision. Thus the $x_F = 1$ measurements are a check on Glauber coefficients and the accuracy of the nucleon spatial distributions. The $x_F A$ dependence directly checks this information, which enters into every nuclear reaction.

3) Finally, the Feynman $x$ distributions for the nuclear $J/\Psi$ cross section relative to p-p and relative to the Drell-Yan can be examined and compared for internal consistency.

Even these measurements may not allow one to get a full understanding of the processes for the following reasons:

1) As pointed out by Hufner et al. [14], that part of charmonium that is in the singlet state should be color screened and therefore $\sigma_{oc}$ can easily be $x_F$ dependent. It cannot be a major effect, since the fall in the ratio $R(x_F)$ with $x_F$ seems to be dominated by energy loss, but any color screening will contribute to an increase in the values of $S$.

2) The time evolution of the off-shell charmonium to the on-shell $J/\Psi$ configuration
is poorly known. There are, in fact, no estimates other than $\Delta E \Delta t$ arguments and no calculations of the time evolution of the transition of a $c-\bar{c}$ color-screened configuration to the $J/\Psi$ on-shell configuration of charmed quarks and gluons, especially since this is surely dependent on the density of gluons in normal nuclear matter. This must be known since the final state interactions of charmonium and the $J/\Psi$ are not necessarily identical.

We conclude that careful study of $J/\Psi$ production in nuclei can teach us about the interplay between the non-perturbative and perturbative processes of the $J/\Psi$, but that claims for “new physics” from total yields have little bearing on these interesting problems that, so far, are poorly understood.

**IX. Conclusions**

1) Analytic examination of the $J/\Psi$ suppression, $S$, shows that $S$ cannot be the exactly exponential in $A^{1/3} + B^{1/3}$ and therefore that such an extrapolation to large $A^{1/3} + B^{1/3}$ is invalid.

2) All Feynman $x_F$ p-A data demonstrate that final state absorption of charmonium cannot be the only contributor to the suppression, so calculations of suppression in A-B interactions must include the effects of energy loss in soft scatters prior to charmonium production.

3) Complete calculations using a Woods-Saxon nucleon distribution shows that there appears to be nothing “anomalous” in the suppression observed in Pb-Pb collisions.
Figures

Fig. 1 Measured Feynman x distributions, for the nucleus to proton target ratio, R, of J/Ψ production. (Also shown are fits to the data, including a crude estimate of the J/Ψ - nucleon inelastic cross section in the final state taken from Reference [5].

Fig. 2 Schematic of interactions of nucleons in a line: representation of the collision of a line of N nucleons with a line of M nucleons.

Fig. 3 Results of Monte Carlo Woods-Saxon calculation of the mean number of collisions prior to (or succeeding) an interaction producing charmonium in p-A and A-A average collisions.

Fig. 4 Results of Monte Carlo Woods-Saxon calculation of the mean number of collisions prior to (or succeeding) an interaction producing charmonium in A−A central collisions of zero impact parameter as well as for average A−A collisions. Several A−B collisions show departures from the straight line slopes, but only for central collisions.

Fig. 5 Plot of 1/S = α + < n > from the exact full Monte-Carlo calculation, using the values of < n > for Pb-Pb and varying σoc from 4.1 to 9.4 mb. This plot is used to demonstrate the validity of this new equation which has been derived using the n distribution of equation 11.

Fig. 6 Comparison of the A^{1/3} + B^{1/3} dependence of the mean number of scatters < n >_m c, taken from the full exact Monte Carlo calculation of S with the value, < n >_approx, using the derived formula, 1/S = α + < n >_approx, for the case σoc = 7.4 mb.

Fig. 7 Complete Monte Carlo calculation for pure absorption. The best fit disagrees with the value of σoc from photoproduction.

Fig. 8 Final predicted values of the suppression for different values of √s for a fixed σoc = 6.3 mb, showing how energy loss has different suppressions in experiments at different √s.

Fig. 9 Comparison of Calculations with Data at Various Values of √s . The solid line shows the extrapolation of the lower √s data. Calculations are shown for two values of σoc consistent with the photoproduction data for a reasonable value of d√s/dn (See text.)

Fig. 10 Calculated Suppression in Central A-A Collisions for σoc = 7.9 mb and no
energy loss, showing that the enhancement factor, C, shows up in the full Monte Carlo Calculation.

Fig. 11 Effect of using the exact expression for the $\sqrt{s}$ dependence of the $J/\Psi$ cross section, eq. 2 vs the use of eq 2'. The plot is shown for $d\sqrt{s}/dn = .4 + .012s$ and $\sigma_{oc} = 6.9$ mb.
References

[1] T. Matsui and H. Satz, Phys. Lett. **178B** 416 (1986)

[2] “Predicting Ultrarelativistic Nuclear Interactions from p-p Data”, S. Frankel and W. Frati, Nuclear Physics **B308** (1988) 699

[3] “J/Ψ and Dimuon Production in Ultrarelativistic Nuclear Collisions”, S. Frankel and W. Frati, *Quark Matter in Collision*, World Scientific. (1988)

[4] Dimuon and Vector Meson Production in Nuclei”, S. Frankel and W. Frati, UPR 50 pp unpub. UPR 0342 T (1990)

[5] “The J/Ψ Absorption Cross Section in Nuclei and Color Screening”, S. Frankel and W. Frati, Zeit . Phys. **57** 225-7 (1993)

[6] S. Frankel and W. Frati Physics letters, **B291** 368 (1992); S. Frankel, W. Frati, and N. R. Walet, Nucl. Phys **A580** 595 (1994)

[7] Phys. Lett. **119B** 464 (1982); Phys. Rev **D28** 2334 (1983); These were the experiments that verified, for multiplicity distributions, the validity of the Bialas, Bleszynski and Czyz hypothesis (Nucl. Phys. **B111** 461 (1976) that the number of participants and not the number of scatters determined the event structure. It is now appreciated that the basis for the wounded nucleon counting is the time dilation effect.

[8] “A Dependence of Hadron Production in Inelastic Muon Scattering and Dimuon Production by Protons”, S. Frankel and W. Frati, Phys. Rev. **D51** 4783 (1995).

[9] J. Badier et al. Zeit fur Phys. **20** 201 (1983), S. Katsenevas et al. Phys. Rev. Lett. **60** 2121 (1987); A.S. Ito et al. Phys. Rev. D 23 604 (1981), P. Bordalo et al., Phys. Lett. **193** 368 (1987), D. M. Alde et al., Phys. Rev. Lett. **64** 2479 (1990).

[10] See, for example, D. Kharzeev et al., [hep-ph/9612217](http://arxiv.org/abs/hep-ph/9612217) 30 Nov 1996.

[11] Åkesson et al. Phys. Rev. Lett. **55** 2535 (1985).
[12] Tabulation of Distributions of Scatters in Nucleus-Nucleus Collisions - UPR Report 776T, W. Frati and S. Frankel, 1997.

[13] Private communication. We used the value of $R$, $R = 1.19A^{1/3} - 1.61A^{-1/3}$, reported in D. Kharzeev et al. CERN-TH/96-328 BI-TP 96/53 (1996).

[14] Phys. Lett. **258** 465 (1991).