Randomly fluctuating potential-controlled multistable resonant tunneling current through a quantum dot

PEI WANG¹, GAO XIANLONG² and SHAOJUN XU³

¹ Institute of Applied Physics, Zhejiang University of Technology - Hangzhou 310023, China
² Department of Physics, Zhejiang Normal University - Jinhua 321004, China
³ School of Economics and Management, Zhejiang Sci-Tech University - Hangzhou 310018, China

received 21 October 2013; accepted in final form 22 January 2014
published online 17 February 2014

PACS 73.63.-b – Electronic transport in nanoscale materials and structures
PACS 05.60.Gg – Transport processes: Quantum transport
PACS 73.63.Kv – Electronic transport in nanoscale materials and structures: Quantum dots

Abstract – There is a long-standing controversy about the existence of more than one steady state in the transport through quantum dots and molecules. In this letter, we present a theory of the multiple steady states driven by randomly fluctuating potentials simulated by the autoregressive model. We find two general conditions under which the steady states strongly depend on the history of the fluctuating potential: First, the fluctuating potential has a nonzero correlation time. Second, it is applied before the steady states are built up under the decoupled initial condition. Furthermore, we provide a way of controlling the stationary current by the correlation time and strength of the fluctuation.

Copyright © EPLA, 2014

Introduction. – Modeling a quantum nonequilibrium steady state requires at least two infinite reservoirs which are coupled to a scattering regime. As the two reservoirs are in different temperatures or chemical potentials, a stationary charge, spin or heat current goes through the scattering regime [1]. Whether the stationary current is uniquely determined by the temperatures and chemical potentials of the reservoirs is of particular interest in the quantum theory of nonequilibrium states.

Stefanucci and Almbladh [2] proved two theorems of unique stationary current for the simple noninteracting resonant level model. First, if the initial state is in equilibrium with the localized level coupled to the reservoirs, the stationary current is unique, losing memory of the history of perturbation (the memory loss theorem). Second, the same stationary current is reached even when the initial reservoirs are decoupled to the localized level but the coupling is quenched with no perturbation afterwards (the theorem of equivalence). However, the uniqueness of the stationary current under the decoupled initial condition for an arbitrary perturbation is still an open problem.

The situation involving interactions is more complicated. Some authors supposed a unique nonequilibrium steady state and constructed it by the variational principles [3] or the Bethe Ansatz [4], while there is no consensus on whether these approaches lead to the state observed in experiments. Others proposed the theories of multistable nonequilibrium steady states [5–16], which were stimulated by the experiments that discovered the hysteresis loops of the $I$-$V$ curves in the tunneling devices of semiconductors and molecules [17–21].

The multistability of currents in experiments is attributed to the inelastic scattering by the electron-phonon or electron-electron interaction, since the transport through static noninteracting levels leads to a unique stationary current unless the levels are outside the bands of the reservoirs [22] which is not the case in experiments. Theoretical calculations verify that the multiple stationary currents through quantum dots [6,7], molecules [8–13], and disordered bulk systems [23], depend on the initial conditions. However, the validity of the theoretical results is still under debate [24,25] because, approximations used in the theoretical methods to address the strong electron-electron and electron-phonon interactions have not been proved to be effective in the nonequilibrium systems [6–10]. Furthermore, the present numerically exact methods are limited to the transient period [12].

It is then necessary to develop new approaches for a universal understanding of the multistability of the tunneling currents. A way of studying the inelastic scattering is by...
driving the electrons through a fluctuating potential [26] which is the result of tracing out the environmental degrees of freedom in the case of electron-phonon interaction [27]. In this letter, we present a theory of the fluctuating-potential–driven multistability in the tunneling currents. By employing the autoregressive model [28,29] in time series analysis to simulate the randomly fluctuating potential with a Gaussian autocovariance, we find the conditions under which the stationary current is driven multistability in quantum dots.

**Formalism.** — We consider a quantum dot with a fluctuating potential $V_{\phi}(t)$ applied to the level, which is coupled to two semi-infinite leads labeled by $L$ and $R$, with the chemical potentials $\mu_L$ and $\mu_R$, respectively. The voltage bias is $V = \mu_L - \mu_R$. The total Hamiltonian is written as

$$H(t) = \hat{H}_L + \hat{H}_R + \hat{H}_d(t) + \hat{H}_V(t),$$

where $\hat{H}_L = -g \sum_{j=-\infty}^{\infty} (\hat{c}^\dagger_j \hat{c}_{j+1} + \text{h.c.})$ and $\hat{H}_R = -g \sum_{j=1}^{\infty} (\hat{c}^\dagger_j \hat{c}_{j+1} + \text{h.c.})$ describe the left and right leads, respectively, with $g$ the hopping amplitude and $\hat{c}_j$ the electron annihilation operator at the site $j$. $\hat{H}_d(t) = V_{\phi}(t) \hat{c}^\dagger_0 \hat{c}_0$ the fluctuating level and $\hat{H}_V(t) = g_c(t)(\hat{c}^\dagger_0 \hat{c}_0 + \hat{c}^\dagger_0 \hat{c}_0 + \text{h.c.})$ the coupling between the level and two leads. The coupling $g_c(t) = g_c(t)$ is switched on at the initial time $t = 0$.

At $t < 0$, two leads are at their own equilibrium states, and the potential $V_{\phi}(t < 0) = \infty$ so that there is no electron at the level. And $V_{\phi} = 0$ denotes the resonant position, where the level broadening is known to be $\Gamma = 2g^2_c/g$ [31]. We take the wide band approximation by setting $g = 10g_c$.

At $t = 0$, the fluctuating potential is switched on, simulated by the famous autoregressive model [28] of a stochastic process:

$$V_{\phi}(t + \Delta t) = \phi V_{\phi}(t) + RW_{t},$$

where $0 \leq \phi < 1$ is a parameter, $W_t$ a white noise with the standard normal distribution and $R$ the strength of the fluctuation. $\Delta t$ denotes the time step and is set to be much smaller than the time scale over which the system changes significantly. The autoregressive model has been intensively studied in the time series analysis with the average of $V_{\phi}(t)$ being zero and the autocovariance being $\langle V_{\phi}(t_1) V_{\phi}(t_2) \rangle = R^2/(1 - \phi^2) \exp(-|t_2 - t_1|/\tau)$, where the correlation time is defined as $\tau = -\Delta t/\ln \phi$. The fluctuating potential satisfying eq. (2) is featured by its stationarity and exponentially decaying two-time correlation.

The main quantity we are interested in is the tunneling current,

$$I(t) = -g_c \left( \text{Im} \langle \hat{c}^\dagger_{-1}(t) \hat{c}_0(t) \rangle + \text{Im} \langle \hat{c}^\dagger_0(t) \hat{c}_1(t) \rangle \right),$$

which is the averaged current of the left and right leads and the expectation value is done to the initial state. Due to the presence of a time-dependent potential, it is impossible to solve it analytically. We solve it numerically by the excitation operator method [32,33], which is accurate and efficient in obtaining both the transient and stationary currents with a time-dependent Hamiltonian.

The current depends on the potential $V_{\phi}(t)$, a stochastic time series, in which the noise term $W_t$ is created by a random number generator. For the given parameters $R$ and $\tau$, the potential $V_{\phi}(t)$ is generated for statistically significant numbers of times (more than one thousand times) to study the average and the standard deviation of the current over trajectories of the fluctuating potential. A stationary current $I_i$ in the $i$-th simulation is calculated for each simulation of $V_{\phi}(t)$. The averaged current is obtained as a path ensemble average on the total number of simulations $N$, i.e., $I = \lim_{N \to \infty} \langle 1/N \rangle \sum_{i=1}^{N} I_i$, and similarly the standard deviation of current $\sigma_I = \lim_{N \to \infty} \langle 1/N \rangle \sqrt{\sum_{i=1}^{N} (I_i - I)^2}$.

Next, we shortly introduce the excitation operator method for calculating $I(t)$. At the initial time $t = 0$, the two leads are in their own equilibrium states respectively and they are both decoupled to the level which is empty. To obtain $I(t)$, we first calculate the field operators $c^\dagger_0(t), \hat{c}_0(t)$ and $\hat{c}_1(t)$ in the Heisenberg picture and then their expectation values.

The field operators satisfy the Heisenberg equation $\frac{d}{dt} \hat{c}^\dagger_j(t) = i[\hat{H}, \hat{c}^\dagger_j(t)]$. Its solution is supposed to be

$$\hat{c}^\dagger_j(t) = \sum_k W_{jk}(t) c^\dagger_k.$$

Because $\hat{c}^\dagger_j(t + \Delta t) = e^{-i\hat{H}t}\hat{c}^\dagger_j(0)e^{-i\hat{H}\Delta t}$, we obtain

$$\sum_k W_{jk}(t + \Delta t) c^\dagger_k = \sum_k W_{jk}(t)\left(\hat{c}^\dagger_k + i\Delta t [\hat{H}, \hat{c}^\dagger_k] + \frac{(\Delta t)^2}{2} [\hat{H}, [\hat{H}, \hat{c}^\dagger_k]] + O(\Delta t^3)\right).$$

The small time step $\Delta t$ is taken, and thus, the terms $O(\Delta t^3)$ can be neglected. $W_{jk}(t + \Delta t)$ can also be expressed as a linear function of $W_{jk}(t)$:

$$W_{jk}(t + \Delta t) = W_{jk}(t) + i\Delta t \sum_l W_{jl}(t) G_{lk} - \frac{\Delta t^2}{2} \sum_{l,m} W_{jl}(t) G_{lm} G_{mk},$$

where the coefficients $G_{kl}$ are defined by the commutators $[\hat{H}, \hat{c}^\dagger_k] = \sum_l G_{kl} \hat{c}^\dagger_l$. Then the propagators $W_{jk}(t)$ at an arbitrary time are worked out by an iterative algorithm starting from $t = 0$ when $W_{jk}(0) = \delta_{jk}$ and moving forward $\Delta t$ at each step. The error caused by a finite $\Delta t$ is of the order $O(\Delta t^3)$, and then can be made negligible by

37004-p2

Pei Wang et al.
setting $\Delta t$ very small. One can also keep the higher-order terms of $O(\Delta t^2)$ in eq. (5). But we find that keeping the terms in order of $O(\Delta t^2)$ is efficient enough for obtaining the stationary current.

At each step, the nonzero propagators $W_{jk}(t)$ are stored and used to calculate the propagators at next step. The truncation scheme is then applied so that only a fixed number of nonzero propagators (the number is denoted by $M$) with the largest magnitudes are kept. This truncation scheme is critical for obtaining the stationary current within a reasonable computation time. The error caused by a finite $M$ can be made negligible by setting $M$ large enough. To obtain the current at the longer time, a larger $M$ should be chosen. The value of $M$ depends on the relaxation time of the current. In this letter, setting $M$ to several thousands is enough for obtaining the high-precision stationary current (see fig. 1).

**History-dependent stationary current.** – After the coupling between leads and the level is switched on at $t = 0$, the current experiences a transient period before relaxing to its stationary value. The turn-on and turn-off times of the fluctuating potential are denoted by $t_i$ and $t_f$ in the following, respectively. We find that applying a fluctuating potential during the transient period will drive the system into the nonequilibrium steady states (NESS) with suppressed stationary currents, distinguished from that of no fluctuating potentials (i.e., $V_g(t \geq 0) \equiv 0$). And the suppressed current survives even after the fluctuating potential is closed after a certain time (see fig. 2).

The suppressed stationary current depends on two critical conditions. First, the fluctuating potential must be applied before the system relaxes to the steady state, i.e., $t_i$ must be smaller than the relaxation time of the currents.

The current is inactive to any fluctuating potentials once the steady correlation is built between the level and leads (see the solid circle in fig. 2), which coincides with the $I$-$t$ curve for $V_g = 0$. We find that, to control the NESS, one should drive the system before it reaches the steady state but not after. Second, the correlation time of $V_g$ must be finite. As $\tau \to 0$, i.e., $V_g$ becomes a white noise, the stationary current is totally ignorant to the fluctuating potential (see the empty circle in fig. 2), which can be understood as follows. The evolution operator for a given path of $V_g(t)$ from $t_i$ to $t_f$ can be factorized into a series of unitary operators,

\[ \hat{U}[V_g(t)] = \prod_{j=1}^{n} \hat{U}(t_j, t_{j-1}), \]

where $t_0 = t_i$, $t_n = t_f$, and $t_j - t_{j-1} = \Delta t$. If $V_g(t)$ is a white noise, the values of $V_g(t)$ at different times are independent of each other and then the average of the evolution operator over different paths of $V_g(t)$, defined as $\hat{U} = \int D[V_g(t)] p[V_g(t)] [\hat{U}[V_g(t)]]$, is found to be

\[ \hat{U} = \prod_{j=1}^{n} dV_g(t_j) p(V_g(t_j)) \hat{U}(t_j, t_{j-1}), \]

where $p[V_g(t)]$ denotes the probability of a path. As $\Delta t \to 0$, we have $\hat{U}(t_j, t_{j-1}) \approx 1 - i \hat{H}(t_j) \Delta t$. Then eq. (8) gives $\hat{U} = \hat{U}[V_g(t) = 0]$, indicating that the evolution of

---

Fig. 1: The currents calculated by the numerical excitation operator method at different $M$. The potential is $V_g(t) = 0$ and the voltage bias is $V = 5\Gamma$. Taking $M = 5000$ is enough for obtaining the high-precision stationary current, since increasing $M$ further to 9000 will not significantly change the result. The relative difference of the currents at $M = 5000$ and $M = 9000$ is found to be less than $10^{-5}$.

Fig. 2: The evolution of the current after the coupling between leads and the level is switched on in the presence of different fluctuating potentials, compared with that of no fluctuating potentials (solid line). We illustrate the case with a potential of a white noise ($\tau = 0$) applied in arbitrary (shortened as arb.) time (solid circle), and the one with an arbitrary potential switched on at $t_i = 4/\Gamma$ larger than the relaxation time of currents (empty circle). They both coincide with the $I$-$t$ curve of $V_g(t) = 0$. However, a fluctuating potential with a finite correlation time switched on during $[0, 2/\Gamma]$ significantly suppresses the current. The correlation time is $\tau = 0.002/\Gamma$ (short-dashed line), and $\tau = 0.008/\Gamma$ (long-dashed line), respectively. And the voltage bias and the strength of fluctuation are set to $5\Gamma$ in all of the above simulations except for $V_g(t) = 0$. 

---

37004-p3
the system in the presence of a white noise is exactly the same as that without any noise.

The stationary current in the presence of a fluctuating potential satisfying the above two conditions depends upon the history of $V_g(t)$. We then estimate the statistics of the stationary current by simulating $V_g(t)$ for the given $R$ and $\tau$ for many times. In fig. 3, we show the scatter plot of the fluctuations of both the current and the electron density $n_d = \lim_{t \to \infty} \langle \hat{c}_0^\dagger(t) \hat{c}_0(t) \rangle$ at the level around their means. It is obvious that the stationary current distributes in a wide range with an upper limit equal to the current without fluctuating potentials. The current is substantially related to the electron density, indicated by the bell shape of the cloud. The more the electron density $n_d$ deviates from 0.5, the less the current is. The suppressed current in the presence of a fluctuating potential is then attributed to the temporary deviation of the level from the resonant position, which causes the off-resonance effect accompanied by $n_d$ deviating from 0.5.

The nonequilibrium stationary current depends on the history of the fluctuating potential with a finite correlation time applied during the transient period, i.e., the tunneling current is multistable. The maximal current is obtained when there is no fluctuating potential or the fluctuating potential is a white noise. We emphasize that the two theorems of Stefanucci and Almbladh [2] do not apply to our model due to the existence of both a fluctuating potential and the decoupled initial condition. The memory loss theorem stands under the coupled initial condition, but our model supposes the decoupled initial condition. The current in the coherent transport depends on the correlation between the leads and the localized level. Under the decoupled initial condition, the correlation is established sequentially in the evolution process, and then depends on the history of perturbation. In fact, we show in the above that if the correlation has been established before the fluctuating potential is applied, the stationary current is then unique. Thus, our result is an extension to the memory loss theorem. The theorem of equivalence demands the absence of perturbation to the localized level after the quench of coupling, but in our model the perturbation is critical to the multiple stationary currents. It is also worth mentioning that the multistability found here should be distinguished from the nonuniqueness of the NESS in the dc transport [22], resulting from the bound states outside the conduction bands of the leads which under the coupled initial condition will cause an everlasting oscillation of currents [34,35]. There is no bound state in our model, since the central level is at the resonant position, and the fluctuation of the level is always inside the conduction bands.

Controlling of the nonequilibrium stationary current. – Finally, we address the effects of the strength of the fluctuation ($R$), the correlation time ($\tau$), and the turn-on ($t_i$) and turn-off time ($t_f$), on the average and standard deviation of currents. By studying the relation between the parameters of the stochastic time series and currents, we find different ways in controlling the nonequilibrium stationary current.

The strength of fluctuation $R$ represents how far the level deviates from the resonant position temporarily, since the variance of $V_g(t)$ is $R^2/(1 - \phi^2)$. As $R$ increases, the central level temporarily moves to a position further away from the resonant point, suppressing the current more due to the off-resonance effect (see fig. 4).

The standard deviation of current, approximately equal to the deviation of the averaged current from its maximum ($\langle I \rangle = 0$), however, increases with $R$. The standard deviation is zero if and only if the current reaches its maximum, i.e., there is no fluctuating potential. The scatter plot of $(I, n_d)$ has a bell shape (see fig. 3). For small $R$, the deviation of the current from its resonant value is small, so that the points in the scatter plot accumulate around the cross (tested, but not shown in the figure). Then the averaged current is close to its maximum and its standard deviation is small. As $R$ increases, the points in the scatter plot diffuse into two wings of the cloud, then the standard deviation increases, while the averaged current is reduced.

The correlation time $\tau$ represents how fast the fluctuating potential oscillates. In the white noise limit, the potential oscillates too fast so that it cannot be felt by the electrons moving in it, and then the current does not reduce as we analyzed above. Increasing $\tau$ will increase the life of a temporary level that deviates from the resonant position and then suppress the current by the off-resonance effect. The averaged current should then decrease as the correlation time $\tau$ of $V_g(t)$ increases, as we see in fig. 4. Increasing either $R$ or $\tau$ will strongly suppress the averaged current. At $R = 20\Gamma$ for $\tau = 0.002/\Gamma$ or $\tau = 0.01/\Gamma$ for $R = 10\Gamma$, the current reduces only to be

$$I(t) = \frac{1}{2} I_{\text{on}} + \frac{1}{2} I_{\text{off}} \left(1 - \frac{t}{t_f} \right) \left(1 - \frac{t - t_i}{t_f - t_i} \right)$$

Fig. 3: A scatter plot of the fluctuations of the stationary current $I$ and the electron density $n_d$ around their means. The voltage bias is set to $V = 5\Gamma$. The fluctuating potential $V_g(t)$ is applied during $[0, 1/\Gamma]$ with the correlation time being $\tau = 0.002/\Gamma$ and the strength being $R = 5\Gamma$. The cross “x” represents the stationary current and electron density at $V_g(t) = 0$. 

37004-p4
A fluctuating potential can thus be a candidate for suppressing the tunneling current.

Our results then provide a possible explanation on the reduced current in the transport through quantum dots due to the temporary deviation of the level from the resonant position by a fluctuating potential. The suppression of the Kondo resonance [36] provides another example, in which the fluctuating potential comes from the fluctuations of the charges in the dot exerting forces to the electrons moving through it.

Finally, we clarify the effects of the turn-on time $t_i$ on the current. In addition to the first condition for the suppressed current, that is, the fluctuating potential must be applied before the system relaxes to the steady state, we find that the most efficient way of changing the non-equilibrium stationary current is to apply the fluctuating potential at the time that the correlation between leads and the level begins to build. Increasing the turn-on time will gradually raise the averaged current to its maximum, at the same time decreasing the standard deviation of the current (see fig. 5).

The averaged current as a function of voltage bias is studied and compared with that of no fluctuating potentials in fig. 5. In the absence of fluctuating potentials, the $I-V$ curve coincides well with the function $I = (Γ/π) \arctan[V/(2Γ)]$ in the wide band limit [37]. A fluctuating potential significantly suppresses the current, but keeps the shape of the $I-V$ curve invariant. The percent of the current reduction keeps finite in the range of the voltage bias (see the bottom right panel of fig. 5), showing that the fluctuating potential suppresses the current both in the linear response regime and beyond it.

The averaged current is not uniquely determined by the temperatures and chemical potentials of the reservoir, but depends on the strength, correlation time and turn-on and turn-off time of the fluctuating potential. We provide a robust evidence of the existence of the multistability in the tunneling currents, which at present is under debate in the interacting models without randomly fluctuating potentials [6–13,23–25].

**Experimental proposal.** – We now show that the quantum dot device made of the metal surface electrodes on a heterostructure [30] is a potential candidate for observing the predicted suppression of the tunneling current when the dot is subject to a randomly fluctuating potential. Our model describes qualitatively the transport through a quantum dot turned into the Coulomb blockade regime, where the level spacing in the dot is much larger than the level width $Γ$ such that only a single level is used to shuttle the electrons. In experiments, the fluctuating potential $V_g(t)$ can be controlled by the gate voltage $V_g$, satisfying $V_g = αV_g$ with $α$ measured in the experiment [38]. The gate voltage should first be adjusted to be sufficiently negative so that the quantum dot pinches off. Then the gate voltage is tuned to the resonant value corresponding to a conductance peak. At the same time, a sequence of pulses generated according to $V_g(t + Δt) = φV_g(t) + RWg$ is applied to the gate voltage with the parameters $φ = \exp(−Δt/τ)$ and $R = R/α$. 

---

**Fig. 4:** Left panel: the averaged current $I$ and the standard deviation of current $σ_I$ as a function of the strength of fluctuation $R$ in $V_g(t)$. The correlation time of $V_g(t)$ is set to $τ = 0.002/Γ$. Right panel: $I$ and $σ_I$ as a function of the correlation time $τ$ of $V_g(t)$. The strength of fluctuation is set to $R = 5Γ$. In both panels, the voltage bias is $V = 5Γ$, and the potential $V_g(t)$ is switched on during $[0, 1/Γ]$. The solid line connecting the symbols serves as a guide for the eyes.

**Fig. 5:** Left panel: the averaged current and the standard deviation of current as a function of the turn-on time $t_i$ of $V_g(t)$. The voltage bias is set to $V = 5Γ$. The strength, the correlation time, and the lasting time of $V_g(t)$ are set to $R = 5Γ$, $τ = 0.002/Γ$, and $t_f − t_i = 1/Γ$, respectively. Right panel: the top part shows the current as a function of the voltage bias in the presence of a fluctuating potential (solid circle, labeled by $I$) with $τ = 0.002/Γ$ and $R = 5Γ$ switched on during the period $[0, 1/Γ]$, compared with that without fluctuating potentials (empty circle, labeled by $I_{max}$). The latter coincides well with the function $I = (Γ/π) \arctan[V/(2Γ)]$ in the wide band limit (solid line). The bottom part shows the percent of the current reduction, i.e., the ratio of $ΔI = I_{max} − I$ to $I_{max}$. The line connecting the symbols serves as a guide for the eyes.
Conclusions. – In summary, we have investigated the fluctuating-potential–driven multistability of the tunneling currents through a quantum dot under the decoupled initial condition with the help of the autoregressive model. Our results predict that the fluctuating potential with a finite correlation time, when applied before the nonequilibrium steady state is built up, efficiently suppresses the nonequilibrium steady state without changing the temperatures and chemical potentials of the reservoirs.

***

We acknowledge useful discussions with GUY COHEN. PW was supported by the NSF of China under Grant No. 11304280. GX was supported by the NSF of China under Grants Nos. 11374266 and 11174253 and by the Zhejiang Provincial Natural Science Foundation under Grant No. R6110175. SX was supported by the NSF of China under Grant No. 71103161.

REFERENCES

[1] DATTA S., Electronic Transport in Mesoscopic Systems (Cambridge University Press, Cambridge) 1995.
[2] STEFANUCCI G. and ALMBLADH C.-O., Phys. Rev. B, 69 (2004) 195318.
[3] BOKES P. and GODBY R. W., Phys. Rev. B, 68 (2003) 125414.
[4] MEHTA P. and ANDREI N., Phys. Rev. Lett., 96 (2006) 216802.
[5] STRUKOV D. B., SNIDER G. S., STEWART D. R. and WILLIAMS R. S., Nature, 453 (2008) 80.
[6] KOHSRAVI E., UMOGENE A.-M., STAN A., STEFANUCCI G., KURTH S., VAN LEEUWEN R. and GROSS E. K. U., Phys. Rev. B, 85 (2012) 075103.
[7] WILNER E. Y., WANG H., COHEN G., THOSS M. and RABANI E., Phys. Rev. B, 88 (2013) 045107.
[8] DZHIOEV A. A. and KOsov D. S., J. Chem. Phys., 135 (2011) 174111.
[9] BEVAN K. H., KIENLE D., GUO H. and DATTA S., Phys. Rev. B, 78 (2008) 035303.
[10] GALPERIN M., RATNER M. A. and NITZAN A., Nano Lett., 5 (2005) 125.
[11] STEFANUCCI G. and BRATKOVSKY A. M., Phys. Rev. B, 80 (2009) 115321.
[12] ALBRECHT K. F., WANG H., MEHLBACHER L., THOSS M. and KOMNIK A., Phys. Rev. B, 86 (2012) 081412(R).
[13] STEFANUCCI G. and BRATKOVSKY A. M., Phys. Rev. B, 67 (2003) 235312.
[14] RYNDVK D. A., D’AMICO P., CUNIBERTI G. and RICHTER K., Phys. Rev. B, 78 (2008) 085409.
[15] KURTH S., STEFANUCCI G., KOHSRAVI E., VERDOZZI C. and GROSS E. K. U., Phys. Rev. Lett., 104 (2010) 236801.
[16] MIL’NIKOV G., MORI N., KAMAKURA Y. and EZAKI T., Phys. Rev. Lett., 102 (2009) 036801.
[17] GOLDMAN V. J., TSU D. C. and CUNNINGHAM J. E., Phys. Rev. Lett., 58 (1987) 1256.
[18] CHEN J., REED M. A., RAWLETT A. M. and TOUR J. M., Science, 286 (1999) 1550.
[19] DÖNHAUZER Z. J. et al., Science, 292 (2001) 2303.
[20] LILJEROOTH P., REPP J. and MEYER G., Science, 317 (2007) 1203.
[21] COLLIER C. P., MATTERSTEIG G., WONG E. W., LUO Y., BEVERLY K., SAMPAIO J., RAYMO F. M., STODDART J. F. and HEATH J. R., Science, 289 (2000) 1172.
[22] DIAR A. and SEN D., Phys. Rev. B, 73 (2006) 085119.
[23] BASKO D. M., ALEINER I. L. and ALTSHULER B. L., Phys. Rev. B, 76 (2007) 052203.
[24] MITRA A., ALEINER I. and MILLIS A. J., Phys. Rev. Lett., 94 (2005) 076404.
[25] SPATAU C. D., HYBERTSEN M. S., LOUIE S. G. and MILLIS A. J., Phys. Rev. B, 79 (2009) 155110.
[26] GERMAN A. I., KOVARSKI V. A. and PEREL’MAN N. F., JETP, 79 (1994) 439.
[27] COFFEY W. T., KALMYKOV Y. P. and WALDRON J. T., The Langevin Equation: With Applications to Stochastic Problems in Physics, Chemistry and Electrical Engineering (World Scientific Publishing Company, Singapore) 2004.
[28] TSAY R. S., Analysis of Financial Time Series (John Wiley and Sons, New York) 2005.
[29] KAPLAN D. and GLASS L., Understanding Nonlinear Dynamics (Springer-Verlag, New York) 1995.
[30] HEINZEL T., Mesoscopic Electronics in Solid State Nanostructures (Wiley-VCH, Weinheim) 2003.
[31] HEIDRICH-MEISNER F., FEIGUIN A. E. and DAGOTTO E., Phys. Rev. B, 79 (2009) 235336.
[32] WANG P., Physica E, 47 (2013) 141.
[33] WANG P., arXiv:1209.3881 (2013).
[34] STEFANUCCI G., Phys. Rev. B, 75 (2007) 195115.
[35] KOHSRAVI E., STEFANUCCI G., KURTH S. and GROSS E. K. U., Phys. Chem. Chem. Phys., 11 (2009) 4535.
[36] ROSCH A., KROHA J. and WÖLFELE P., Phys. Rev. Lett., 87 (2001) 156802.
[37] JAUCH J.-P., WINGREEN N. S. and MEIR Y., Phys. Rev. B, 50 (1994) 5528.
[38] GOLDBADER-GORDON D., GÖRES J., KASTNER M. A., SATRIKIAN H., MAHALU D. and MEIRAV U., Phys. Rev. Lett., 81 (1998) 5225.
[39] HANSON R., KOVZHENZHEN L. P., PETTA J. R., TARUCHA S. and VANDERSYPEN L. M. K., Rev. Mod. Phys., 79 (2007) 1217.