Inception and evolution of coherent structures in under-expanded supersonic jets

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Abstract. The purpose of this paper is to examine the generation and nature of the large coherent structures observed experimentally in an under-expanded supersonic impinging jet. More specifically, the questions to answer are: What mechanisms govern the receptivity process at the nozzle lip?, how does the underlying flow field affect the evolution of the large-scale coherent structure generated from the initial instability? and what are the interactions between the large-scale (forced) coherent structures and the developing turbulence in the jet shear layer? In order to answer some of these questions both alternatives, that these structures come from global modal flow instabilities or from convective instabilities, the latter, are considered in this work.

The stability analysis considered in the former case is performed in this work near the nozzle around the temporal average of the flow obtained by using an in-house LES (Large Eddy Simulation) code. The flow in this region is considered laminar, steady and without non-linear effects. The well known feedback loop in the impinging jet, according to which acoustic waves propagate upstream and excite the jet shear-layer (see Figure 2), advises against some of the hypothesis considered previously in the global stability analysis (ie. non-linear approximation). However the acoustic waves are orders of magnitude smaller than the hydrodynamic waves and should be smoothed out in the temporal average used in the calculation of the mean flow.

The results show that both, axisymmetrical ($m = 0$) and azimuthal modes ($m \geq 1$) are stable to global modal analysis and only convective instability could justify the instabilities observed in experiments in the shear layer. A study on the receptivity problem confirms that external disturbances may enter and excite the shear layer, being responsible of the instabilities observed in both experiments and direct numerical simulations.

1. Introduction

The flow structure of an under-expanded supersonic impinging (USI) jet is presented in Figure 1. Here the flow exits from the nozzle at Mach number $Ma = 1$ towards an impingment wall placed normal to the jet axis at a distance of $z/d = 5$ vertically above the nozzle, where $d$ is the nozzle diameter.

USI jets characteristically produce large-scale structures at discrete frequencies due to the development of an acoustic feedback mechanism. In this mechanism acoustic waves force an instability at the nozzle lip which grows within the shear layer as it travels downstream. These structures interact with the stand-off shock which produces a disturbance in the wall jet. An
Introduction (cont.)

Complexities and multi-stage character of under-expanded supersonic impinging jets: coherent acoustic waves structures forced shear layer instabilities Mach Disk oblique shock second shock cell plate shock recirculating flow region expansion waves

Figure 1: Instantaneous flow obtained from experiment with the physical description of regions and structures presented in a typical under-expanded supersonic impinging jet (LTRAC) [1].

An acoustic wave is then generated in the wall jet which travels upstream and forces a new jet shear layer instability. This process, seen in Figure 2, is often referred to as an acoustic feedback loop and it is a multi-physical complex problem which is not presently understood sufficiently. As impinging jets provide an effective way to transfer mass and energy in industrial applications, the prediction and control of the instabilities that arise in this configuration is of both fundamental and practical interest.

Figure 2: Feedback loop mechanism: Shear layer instability grows and interacts with the standoff shock to produce an acoustic wave. Then acoustic wave reflects and travels back upstream and excite the shear layer.

In this problem the jet emerges from a nozzle with a velocity and a temperature profile, that depends on the nozzle configuration, which in turn determine the characteristics of the flow upstream. This flow presents a characteristic pattern of shock waves, expansion waves, coherent
structures, etc (see Figure 1). Finally the flow impinges against the wall which is perpendicular to the driving direction. At high Reynolds numbers, the shear layer generates flow instabilities that look like classical Kelvin–Helmholtz waves. Theses instabilities have been observed in experiments and numerical simulations but their nature is not sufficiently well understood. For example, Powell [5] identified four frequencies associated with different instability modes in a jet, naming them A, B, C and D. Later, Davies & Oldfield [6] identified the nature of these modes: A1 and A2 are axisymmetric, B is sinuous or lateral, C is helical and D is another lateral mode. Dimensionless parameters important to these flows are: the impingement wall stand-off distance \( z/d \), the nozzle exit Mach number \( M_0 \) and the jet Reynolds number \( \text{Re} \) defined in section 3.

Investigation of the hydrodynamic and acoustic instability mechanisms in these types of flows are essential for the understanding of the transition process from a stable steady laminar flow to transitional and turbulent flow. A modal linear stability analysis, focusing on the spectrum of the flow, and a non-modal analysis, for short-time perturbations, can provide a complete insight of the physical mechanisms responsible of the transition to turbulent flows.

2. Theory
2.1. Governing equations

The dimensionless Navier-Stokes equations representing the conservation of mass, momentum and energy that govern the dynamics of such jet flows are defined respectively as

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{1}
\]

\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + p I - \mu \mathbf{T}) = 0, \tag{2}
\]

and

\[
\frac{\partial \rho e}{\partial t} + \nabla \cdot (\mathbf{u} (\rho e + p) - k \nabla T - \mu \mathbf{T} \cdot \mathbf{u}) = 0. \tag{3}
\]

where the deviatoric stress tensor \( \mathbf{T} \) is defined as

\[
\mathbf{T} = \left[ \nabla \mathbf{u} + (\nabla \mathbf{u})^T + \left( \frac{\mu_b}{\mu} - \frac{2}{3} \right) (\nabla \cdot \mathbf{u}) I \right]. \tag{4}
\]

\( \rho \) is the density, \( \mathbf{u} \) is the velocity vector, \( e \) is the total energy per unit mass, \( p \) is the pressure, \( T \) is the temperature, \( \mu \) is the viscosity, \( \mu_b \) is the bulk viscosity, \( k \) is the thermal conductivity and \( I \) is the identity tensor. In these equations length, density, velocity, time, temperature, energy and viscosity have been non dimensionalised by \( L_0, \rho_0, u_0, t_0 = L_0/u_0, T_0, \rho_0 u_0^2 \) and \( \mu_0 \) respectively. This results in the dimensionless Reynolds number

\[
\text{Re}_0 = \frac{\rho_0 u_0 L_0}{\mu_0} \tag{5}
\]

and Mach number

\[
M_0 = \frac{u_0}{\sqrt{RT_0}}. \tag{6}
\]

where \( R = 287 \text{Jkg}^{-1}\text{K}^{-1} \) for air. For an ideal gas the total energy per unit mass and equation of state are defined as

\[
e = \frac{1}{2} ||\mathbf{u}||^2 + \frac{T}{M_0^2 (\gamma - 1)} \tag{7}
\]

and

\[
p = \frac{\rho T}{M_0^2} \tag{8}
\]
respectively. The viscosity and thermal conductivity are determined using the Power law [11]

\[ \mu = \frac{1}{Re_0} T^{0.76} \]  

(9)

and

\[ k = \mu \left( \frac{\gamma}{\gamma - 1} \right) M_2^2 Pr \]  

(10)

respectively, where for air \( \gamma = 1.4 \) and \( Pr = 0.7 \).

Locally one-dimensional inviscid compressible boundary conditions defined in [10] are used for the adiabatic walls and outflow regions. The jet inlet velocity profile was modeled using the hyperbolic-tangent function found in [12] while the temperature profile was determined using the Crocco-Busemann relationship [13]. No inlet turbulence has been applied.

2.2. Linear instability analysis

The three-dimensional compressible Navier-Stokes equations can be written in a compact form as,

\[ \frac{\partial \mathbf{q}}{\partial t} = \mathbf{F}(\mathbf{q}), \]  

(11)

where \( \mathbf{q} = (\rho, u, v, w, T) \) and \( \mathbf{F} \) is the right hand side of the system of equations (1-3) when these are written in primitive variables.

Linear stability theory studies the temporal evolution of small amplitude disturbances \( \mathbf{q}' \) superimposed upon a steady base flow \( \overline{\mathbf{q}} \),

\[ \mathbf{q}(x, y, z, t) = \overline{\mathbf{q}}(x, y, z) + \epsilon \mathbf{q}'(x, y, z, t), \]  

(12)

where \( \epsilon \ll 1 \) is an small parameter. A linearized problem can be written by introducing this decomposition in the Navier-Stokes equations (11) and retaining the terms up to \( O(\epsilon) \). Then two approaches could be considered: (a) Modal linear instability theory where modal perturbations \( \mathbf{q}'(x, r, \theta, t) = \mathbf{q}(x, r)e^{i(m\theta - \omega t)} \) are assumed. In this case a generalized eigenvalue problem must be solved,

\[ A(Re, Ma, \overline{\mathbf{q}}, m) \hat{\mathbf{q}} = \omega B(Re, Ma, \overline{\mathbf{q}}, m) \hat{\mathbf{q}}, \]  

(13)

where \( Re \) and \( Ma \) are the Reynolds and Mach number based on the nozzle conditions, respectively, while \( m \) represent additional parameters that arise from the modal analysis approach considered. The other possibility is, (b) non-modal linear instability analysis, where a initial value problem must be solved,

\[ B(Re, Ma, m, \overline{\mathbf{q}}) \frac{d\mathbf{q}'}{dt} = A(Re, Ma, m, \overline{\mathbf{q}})\mathbf{q}', \]

by computing the singular value decomposition of the evolution operator \( \Phi = e^{(B^{-1}A)t} \).

Different levels of modelling can be considered in the modal analysis (see Table 1). So for example, the basic flow deepens only on one variable in the ODE model, two variables in the BiGlobal model and three variables in the TriGlobal case, while the rest of homogeneous variables, including time, are considered in a exponential function (ansatz). In our case, we used the temporal BiGlobal stability model which \( \beta \) is the wavenumber in the azimuthal direction and \( \omega = \omega_r + i \cdot \omega_i \) is a complex number which real part is the frequency of the perturbation and the imaginary part the amplification or damping rate of this. Therefore, a large-scale eigenvalue problem is solved by an iterative approach [4].
| Assumptions | Base | Amplitude | Phase Function $\Theta (\text{Exp}(i\Theta))$ |
|-------------|------|-----------|-----------------------------------------------|
| TriGlobal   | $q(x, r, \theta)$ | $\dot{q}(x, r, \theta)$ | $-\omega t$ |
| PSE-3D      | $\partial_\theta \mathbf{q} \ll \partial_x \mathbf{q}, \partial_r \mathbf{q}$ | $\mathbf{q}(x, r, \theta^*)$ | $\dot{q}(x, r, \theta^*)$ | $\int m(\theta')d\theta' - \omega t$ |
| BiGlobal    | $\partial_\theta \mathbf{q} = 0$ | $\mathbf{q}(x, r)$ | $\dot{q}(x, r)$ | $m\theta - \omega t$ |
| PSE         | $\partial_x \mathbf{q} \ll \partial_r \mathbf{q}$ | $\mathbf{q}(x^*, r)$ | $\dot{q}(x^*, r)$ | $\int \alpha(x')dx' + m\theta - \omega t$ |
| Local       | $\partial_x \mathbf{q} = \partial_r \mathbf{q} = 0$ | $\mathbf{q}(r)$ | $\dot{q}(r)$ | $\alpha x + m\theta - \omega t$ |

Table 1: Levels of modeling can be considered in the modal analysis for problems defined in cylindrical coordinates. The asterisk denotes a slowly-varying spatial direction.

### 3. Numerical Implementation

#### 3.1. Mean flow computation

Datasets used in the following analyses were computed using an in-house compressible LES solver implementing a non-uniform three-dimensional cylindrical finite difference, time-accurate scheme with hybrid spatial discretisation. For the hybrid spatial discretisation the spatial fluxes are calculated in smooth regions by utilising a 6th order dispersion relation preserving (DRP) scheme and a 5th order weighted essentially non-oscillatory scheme with local Lax-Friedrichs flux splitting in discontinuous regions. Temporal integration is performed using a fourth order five step Runge-Kutta scheme. The sub-grid scale terms were computed using Germano’s dynamic model with the adjustments made by Lilly [9]. The domain consists of approximately 30 million nodes with the spatial extent of $15d$ the radial direction.

The mean flow components, temperature, stream-wise and radial velocity are illustrated in Figure 3. These fluid variables are scaled by using the flow properties at the nozzle exit. The values of the dimensionless parameters used in the simulations are: $Re = u_jd/\nu_j = 50000$, $Ma = u_j/\sqrt{\gamma RT_j} = 1$, the ratio between the stagnation pressure measured in the jet plenum and the ambient pressure $NPR = 3.4$ and $z/d = 5$, where $u_j$ and $T_j$ are the velocity and temperature in the nozzle at $r = 0$. The velocity gradients in the jet, near to the nozzle lip, create a lateral transfer momentum, the jet loses energy and the velocity profile expands laterally and decreased in magnitude. On the other hand, the temperature of the supersonic jet decreases as it exits the nozzle due to the expansion and increases at the shocks. The temperature is higher near the impinging wall and over the shear layer.

Figure 4 shows the complex growth/decay dynamics of the acoustically forced instabilities. The left Figure is the temporal spectrum of the turbulent kinetic energy (integrated in the azimuthal direction) as a function of position along the shear layer. The central Figure shows the same information particularized for a given position in the shear layer, $x = 0.05d$ (marked with a discontinuous white line in the left figure), where two peaks corresponding a two forced modes are obtained: $St_\infty = 0.12$ (red line) and $St_\infty = 0.54$ (green line). Finally, the right Figure shows how the peak broadband turbulence frequency corresponding to the previous modes moves from high ($St 3-5$) to low ($St 0.25$) along the shear layer.
3.2. Global instability analysis

Global stability analysis near the nozzle on supersonic impinging jet is performed in this section. This analysis was used by Theofilis and Colonius [3] in the case of compressible flows and it is described in detail by Theofilis [4]. The two-dimensional generalized eigenvalue problem for temporal three-dimensional perturbations defined in Equation (13) is solved near to the nozzle by using an in-house code [7] based on a FD-q scheme, where $x \in [0, 1.5]$ and $r \in [0, 3]$. The variables were dimensionless with the displacement thickness of the shear layer at $x = 0.5$. The Reynolds number based on this length and the nozzle conditions is approximately equal to 6800.

Homogeneous Dirichlet at the wall and nozzle and axial boundary conditions at $r = 0$ are considered. The later is described in detail by Malik et al. [2], and depends on the value for the azimuthal wave number,

$m = 0 \quad : \quad \partial_r u' = v' = \partial_r T' = 0$

$|m| = 1 \quad : \quad u' = T' = \partial_r v' = 0 \text{ and } v' + imw' = 0$

$|m| > 2 \quad : \quad u' = v' = w' = T' = 0$

where prime denotes perturbation variables. The density is not imposed at the boundaries in the analysis, instead this is solved by using the continuity equation in the inner points. Homogeneous Dirichlet boundary condition for the perturbations at $r_{max}$ and homogeneous Neumann at $x_{max}$ are considered in the other boundaries.

Figures 5-7 shown the convergence of the spectrum for different values of the azimuthal wavenumber $m$ where $\omega_i$ is the growth/damping rate of the temporal disturbances and $\omega_r$ is its frequency (see third row of Table 1 for more details). In all the cases, left Figure shows the convergence of the spectrum as function of the resolution for a given order of the method (4 and 8) while right Figure shows the convergence of the spectrum as function of the order of the method for a given resolution. In both cases the leading mode is reasonably well converged.
The amplification function of this mode is represented in the following figures. Figure 8 shows the real and imaginary part of the amplitude functions of the leading mode for $m = 0$. In this case the azimuthal component is negligible compared to other components and then is not represented. The same information for the leading mode for $m = 1$ and 2 is shown in Figures 9 and 10, respectively. As it can been see in these two cases the leading mode has qualitatively the same structure. On the other hand, in all the cases the system is stable to linear global modes.

![Figure 5: Spectrum for $m = 0$. (Left:) Convergence of the solution in function of the order of the discretization, 4 and 8, (Right:) Convergence of the solution in function of the resolution: $n_x \times n_r = 200 \times 300$ and $250 \times 375$.](image1)

![Figure 6: Spectrum for $m = 1$. (Left:) Convergence of the solution in function of the order of the discretization, 4 and 8, (Right:) Convergence of the solution in function of the resolution: $n_x \times n_r = 200 \times 300$ and $250 \times 375$.](image2)

4. Receptivity

In this section a receptivity analysis of a solution of the linearised compressible Navier-Stokes (LCNS) equations applied to the jet is presented. The receptivity analysis tries to quantify the system response to external forcing of the shear layer at the nozzle lip by acoustic waves. The
Figure 7: Spectrum for $m = 2$. (Left:) Convergence of the solution in function of the order of the discretization, 4 and 8, (Right:) Convergence of the solution in function of the resolution: $n_x \times n_r = 200 \times 300$ and $250 \times 375$.

Figure 8: Amplitude functions of the leading mode at $m = 0$. The flow goes from left to right. $r = 0$ correspond with the jet center-line.

LCNS equations are

\[
\frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho u)' = 0
\]

\[
\frac{\partial (\rho u)'}{\partial t} + \nabla \cdot (\rho u' \otimes u + \rho u + \rho u' \otimes \bar{u}') + \nabla p' = \bar{\mu} \nabla \cdot \bar{T}'
\]

\[
\frac{\partial (\rho e)'}{\partial t} + \nabla \cdot \left( (\rho e + p) u' (\rho e + p) \bar{u}' \right) = \kappa \nabla^2 T' + \bar{\mu} V_{KE}
\]

where

\[
V_{KE} = \left( \left( \nabla \cdot T' \right) \cdot \bar{u} + \left( \nabla \cdot \bar{T}' \right) \cdot u' + T' : \nabla \bar{u} + \bar{T}' : \nabla u \right) ,
\]
Figure 9: Amplitude functions of the leading mode at $m = 1$. The flow goes from left to right. $r = 0$ correspond with the jet center line.

Figure 10: Amplitude functions of the leading mode at $m = 2$. The flow goes from left to right. $r = 0$ correspond with the jet center line.
\[ T = \nabla u + (\nabla u)^T + \left( \frac{\mu_B}{\mu} - \frac{2}{3} \right) (\nabla \cdot u) I, \quad (18) \]

\[ (\rho e)' = \frac{1}{2} \rho \|u\|^2 + \frac{p'}{\gamma - 1}, \quad (19) \]

and the linearised version of the equation of state

\[ p' = \frac{\rho T' + \rho T}{M_0^2}. \quad (20) \]

Finally, viscosity and thermal conductivity are given by,

\[ \mu = T_{0.76} Re \] and \[ k = \mu \frac{1}{\gamma - 1} M_0^2 Pr \] (21)

where the non dimensional quantities are

\[ Re = \frac{\rho_\infty d u_j}{\mu_\infty} \] and \[ M_0 = \frac{a_\infty}{\sqrt{RT_\infty}}. \quad (22) \]

The mean fields input into the linear simulation are calculated by taking the mean of the jet time series data generated by the LES solver detailed in section 3.

To establish the effect of the receptivity the transfer function between the input fluctuating acoustic velocity and output fluctuating hydrodynamic axial and radial velocities are determined. An initial acoustic wave is generated impulsively at the beginning of the linear simulation as a Gaussian pressure disturbance in the free-stream and is defined as

\[ p' = A p_\infty e^{-\frac{(x-R_a \cos(\theta_a))^2 + (r-R_a \sin(\theta_a))^2}{2 \sigma^2}} \] and \[ R_a = R_a + R'_a \cos(2\pi k_0 \theta) \]

where \( A \) is the amplitude ratio between the initial pulse peak and free-stream pressure, \( x \) is the axial position from the nozzle outlet, \( r \) is the radial position from the jet axis, \( \theta \) is the azimuthal position, \( \sigma \) controls the initial width of the pulse, \( \theta_a \) is the angle between the initial pulse and the x-axis taken from the nozzle lip, \( R_a \) is the distance between the nozzle lip and the pulse centre, \( \bar{R}_a \) the azimuthal mean position of \( R_a \), \( R'_a \) is the amplitude of the variation of \( R_a \) in the azimuthal direction and \( k_0 \) is the azimuthal mode of the initial pulse. Each of these variables as they relate to the geometry of the jet may be found in Figure 11(a). The parameter values chosen for the analysis are \( R_a = 2 \), \( R'_a = 0.06 \), \( \sigma = 0.05 \), \( \theta_a = \{15^\circ, 25^\circ, 35^\circ, 45^\circ\} \), and \( k_0 = \{0, 2\} \). \( \sigma \) was chosen to be as small as possible in order to increase the initial spread of energy in the wavenumber domain while maintaining numerical accuracy.

Locations of the sample points for the incoming acoustic wave and outgoing hydrodynamic wave are \( s_{in} \) and \( s_{out} \) respectively. The transfer function is then defined in the frequency domain as

\[ F (St, A) = \frac{\hat{A} (St)}{\hat{u'}_{a} (St)} \] where \( u'_a \) is the fluctuating velocity at the input sample point in the direction of \( n_a \) (seen in Figure 11(b)), \( A \) represents either the fluctuating axial velocity \( u' \) or fluctuating radial velocity \( v' \) at the output sample point, and the hat ascent denotes the Fourier transform of the variable as \( \hat{\phi} (St) = \int_{-\infty}^{\infty} \phi (t) e^{-2\pi i t St} dt \).
To obtain a representative form of the transfer function for the receptivity process then $s_{in} \to 0$ and $s_{out} \to 0$. However the minimum values of $s_{in}$ and $s_{out}$ are limited by the ability to distinguish the input and output signals at the sample points which in turn is limited by the spatial width of the initial acoustic wave and grid resolution in the simulation. For this analysis $s_{in} = 0.8d$ and $s_{out} = 0.3d$. The evolution of a single case from the initial acoustic wave, its interaction with the nozzle lip and the generation of the hydrodynamic wave in the shear layer may be seen in the contour plot sequence of Figure 12.

Effects of the acoustic reflections from the nozzle wall and radiation of acoustic waves generated by the hydrodynamic instability travelling downstream in the shear layer must be removed from the input acoustic time series. Their effect may be seen in Figure 13 in which the initial acoustic wave reaches the nozzle lip at $tu_j/d = 2$. Removal of these effects is performed by setting $u'_a = 0$ for $t > 2$. The initial acoustic pulse in the output signal, due to the acoustic wave interacting with the shear layer, is ignored as it’s amplitude is approximately 0.07% of the
The resulting transfer function for the fluctuating axial hydrodynamic velocity is presented in Figure 14 and the fluctuating radial hydrodynamic velocity in Figure 15, both as a function of $\theta_a$ and $k_\theta$. Colours represent varying initial pulse angle ($\theta_a$), the solid lines represent the axisymmetric mode ($k_\theta = 0$), and the dashed lines represent the 2nd azimuthal mode ($k_\theta = 2$). For all $\theta_a$ and $k_\theta$ there are two spectral peaks in the magnitude response found for the axial velocity in the low frequency range, $St \approx 1.75$ and mid-frequency range $St \approx 4$ while there is only a single dominant spectral peak at $3 < St < 4$ for the radial velocity with a small peak occurring for $\theta_a = 15^\circ$ and $\theta_a = 25^\circ$ at $St \approx 6.75$. The phase response for all cases are linear with respect to temporal frequency. Increasing the angle of the incoming acoustic wave to the nozzle lip increases the magnitude of the resulting hydrodynamic wave velocities. The 2nd azimuthal mode also exhibits a slight increase in the amplitude of the resulting hydrodynamic velocity with small shifts in the peak spectra positions. Noise exists in the magnitude and phase response for frequencies greater than $St > 7.5$ and is believed to be due to numerical error as the high frequencies relate to smaller scale structures.

5. Conclusions
In conclusion, it has been shown that temporal global disturbances defined in the near region to the nozzle are globally stable near the nozzle and only convective instabilities are present for $m \leq 2$. This suggests that a transient growth analysis based on the BiGlobal approach should be undertaken. Future work on the modal analysis requires the study of the interactions of the shear layer with the turbulence and acoustic forcing present in the jet. This could be performed by using the resolvent model developed by Sharma and McKeon [8]. It is expected the resolvent analysis will produce a more detailed representation of the energy transfer processes within the jet.

An initial analysis of the receptivity process has also been performed. It has shown that the result of the interaction of the shear layer lip with the incoming acoustic wave results in two dominant temporal modes in the axial fluctuating velocity and one dominant mode in the radial fluctuating velocity of the generated hydrodynamic instability. Further analysis must be undertaken to establish how these instabilities are modified by the shear layer as they travel downstream and interact with the stand-off shock to produce a new acoustic wave that acts as the forcing phenomenon for the following cycle in the feedback loop.
Figure 14: Magnitude and phase of the transfer function from the fluctuating acoustic input velocity \(u'_a\) to the fluctuating hydrodynamic output axial velocity \(u'\) as a function of initial pulse angle and azimuthal mode. Colours represent varying initial pulse angle (\(\theta_p\)), the solid lines represent the axisymmetric mode (\(k_\theta = 0\)), and the dashed lines represent the 2nd azimuthal mode (\(k_\theta = 2\)).

Figure 15: Magnitude and phase of the transfer function from the fluctuating acoustic input velocity \(u'_a\) to the fluctuating hydrodynamic output radial velocity \(v'\) as a function of initial pulse angle and azimuthal mode. Colours represent varying initial pulse angle (\(\theta_p\)), the solid lines represent the axisymmetric mode (\(k_\theta = 0\)), and the dashed lines represent the 2nd azimuthal mode (\(k_\theta = 2\)).

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