Research on flutter reliability of nonlinear binary-wing

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Abstract. In this paper, the aeroelastic analysis of the nonlinear binary wing model under unsteady aerodynamic conditions, the critical flutter speed was calculated by using the v-g method. Based on the critical flutter speed, the implicit limit state function of the nonlinear flutter reliability of the wing was obtained, and the response surface method was used for function fitting. The flutter reliability index was calculated by the checking point method, the design checking point and failure probability were obtained, and the flutter reliability sensitivity was analyzed. Finally, through the analysis of an example, it was proved that this method is feasible.

1. Introduction

With the development of China's aviation industry, the aeroelastic problem of aircraft has received more and more attention. The research focus on wing flutter analysis has also extended from classical linear flutter analysis to binary wing flutter analysis with nonlinear links.

The research on the reliability of wing flutter is mainly based on the wing parameters. The traditional flutter reliability analysis is performed on the determined wing parameters, but in practical applications, the wing parameters are uncertain due to various factors. Therefore, the uncertainty of the parameters should be considered in the reliability analysis of the nonlinear flutter of wings. Pitt [1] studied the influence of structural parameter uncertainty on wing flutter with Monte Carlo Simulation (MCS) method, and proposed that the randomness of parameters should be considered in the analysis of wing flutter. Scarth [2] used the Polynomial Chaos Expansion (PCE) method to deal with the uncertainty of the layup direction of composites, and gave the influence of different layup directions on the flutter probability. Song [3-5] analyzed the probability characteristics of random parameters and flutter reliability sensitivity of wings in transonic flow by using the improved linear sampling technique combined with Monte Carlo Simulation method.

MCS method is suitable for the reliability analysis of implicit limit state function, but has the disadvantages of large sample size and large computational workload. The response surface method has the characteristics of small calculation, high calculation accuracy and easy implementation. Therefore, the response surface method was adopted in this paper to carry out reliability analysis and obtain the nonlinear flutter limit state function of the wing. It provided an efficient and feasible calculation method for the flutter analysis and reliability design of the wing structure.

2. Nonlinear flutter analysis of binary wing

Figure 1 shows the physical model of a binary airfoil, which oscillates in the pitch and plunge directions. The symbol $\alpha$ denotes the pitch angle; the plunge deflection is denoted by $h$; the elastic axis...
is located at a distance $ab$ from the mid-chord, while the mass center is located at a distance $x, b$ from the elastic axis, $b = c/2$ is the half chord of the wing.

![Airfoil Diagram](image)

**Figure 1.** Airfoil model with pitch and plunge degrees-of-freedom.

The motion differential equation of binary wing can be obtained according to the Lagrange equation:

$$\egin{aligned}
 m \ddot{h} + S_0 \dot{\alpha} + c_h \dot{h} + K_a h &= Q_h \\
 S_0 \ddot{\alpha} + I_a \ddot{\alpha} + c_{\alpha} \alpha + K_{\alpha} \alpha + eK_a \alpha^3 &= Q_{\alpha}
\end{aligned}$$

(1)

where: $m$ is the wing mass of the unit length; $S_0$ is the mass static moment of the unit length wing to the elastic axis; $I_a$ is the mass inertia moment of the unit length wing to the rotating shaft; $c_h$ and $c_{\alpha}$ are the structural damping coefficients; $K_a$ and $K_{\alpha}$ are the structural stiffness coefficients; $e$ is the nonlinear stiffness factor; $Q_h$ and $Q_{\alpha}$ are the aerodynamic and aerodynamic moment acting on the wing.

According to Theodorsen's unsteady aerodynamic force theory [6], the aerodynamic force $L$ and the aerodynamic moment $T_{\alpha}$ of unit span with the binary wing can be obtained:

$$\egin{aligned}
 L &= -\pi \rho b^2 \left( v \dot{\alpha} + \dot{h} - ab \ddot{\alpha} \right) - 2 \pi \rho b C(k) \left[ v \alpha + \dot{h} \left( \frac{1}{2} - a \right) \dot{b} \dot{\alpha} \right] \\
 T_{\alpha} &= \pi \rho b^2 \left[ ab (v \ddot{\alpha} + \dot{h} - ab \ddot{\alpha}) \frac{1}{2} v \dot{b} \ddot{\alpha} \frac{1}{8} - \frac{1}{8} v \ddot{b} \ddot{\alpha} \right] + 2 \pi \rho v \ddot{b} \left( \frac{1}{2} + a \right) C(k) \left[ \alpha \ddot{h} \frac{1}{v} \left( \frac{1}{2} - a \right) \dot{b} \ddot{\alpha} \right]
\end{aligned}$$

(2)

where: $k = b \omega/v$ is the reduction frequency, the dimension is 1, and $C(k)$ is the Theodorsen function.

The basic equation for the binary wing flutter analysis using the $\nu$-$g$ method is:

$$-\omega^2 \mathbf{M} + i \omega \mathbf{D} + (1 + ig) \mathbf{K} - q_0 Q(k, M_{\alpha}) \mathbf{g} = 0$$

(3)

According to the calculation formula of the equivalent frequency, equation (3) can also be rewritten into:

$$-\left[ \mathbf{M}^\nu + \frac{\rho C(k, M_{\alpha})}{2} \right] \mathbf{g} + \frac{\nu}{1+ig} \mathbf{D} + \frac{\nu}{\sqrt{1+ig}} \mathbf{K} \mathbf{g} = 0$$

(4)

Solved the equation (4) and obtained the system eigenvalue:

$$\rho^\nu = \frac{\nu}{1+ig} = -\nu^2 \frac{1-ig}{1+g} = c + id$$

(5)

Further the damping $g$, the velocity $V$ and the frequency $\omega$ were respectively obtained:

$$g = -\frac{d}{c}, V = \sqrt{\frac{c^2 + d^2}{c}}, \omega = \frac{kv}{\pi b}$$

(6)

### 3. Reliability analysis

The implicit limit state function of binary wing flutter reliability can be expressed as follows:

$$Z = g(x) = 0.85V^* - V_f$$

(7)
where: $V_f^*$ is the deterministic flutter critical speed obtained by the $v$-$g$ method, $V_f$ is the actual flight speed and $V_f = 0.85V_f^*(\mu)$, where $V_f^*(\mu)$ is the flutter critical speed when the random parameter takes the mean, so $Z=g(x)$ is the implicit limit State function.

Basic random parameters: $x = (\mu, a, b, x_0, r, \omega_0, \rho_a)^T$, where: $\mu$ is the mass ratio; $a_b$ is the dimensionless distance from the stiffness center of the wing to the midpoint of the string; $b$ is the half chord length; $x_a$ is the dimensionless distance from the wing’s gravity center to the stiffness center; $r_a$ is the dimensionless radius of gyration of the wing to the stiffness center; $\omega_0$ is the torsional mode frequency of the wing; $\rho_a$ is the bending mode frequency of the wing.

The steps to study the flutter reliability of binary wing using the quadratic response surface method without crossing terms and the deterministic flutter analysis are as follows:

- A quadratic analytical expression (limit state function) was fitted by regression to replace the real response surface:

$$
\hat{g}(x) = a_0 + \sum_{j=1}^a a_j x_j^2
$$

- Bucher test design method was used to extract the sample points which were needed to determine the pending coefficients in the response surface function. In the k-th iteration, $x_m^{(k)} = (x_{m1}^{(k)}, x_{m2}^{(k)}, \ldots, x_{mk}^{(k)})$ was used as the test center, and the $2n+1$ test points were extracted as follows:

$$
x_m^{(k)} = (x_{m1}^{(k)}, x_{m2}^{(k)}, \ldots, x_{mk}^{(k)} \pm f \sigma_j, \ldots, x_{mk}^{(k)}), i = 1, 2, \ldots, 7
$$

The mean value point $\mu = (\mu_1, \mu_2, \ldots, \mu_v)$ (deterministic flutter analysis) was taken as the first iteration test center point $x_m^0$. Where $\sigma_i$ is the standard deviation of the basic random variable $f$ is the interpolation coefficient, and this paper is taken as 3.

- The sample point $x_m^{(i)} = (x_{m1}^{(i)}, x_{m2}^{(i)}, \ldots, x_{mk}^{(i)} \pm \sigma_i, \ldots, x_{mk}^{(i)}), i = 1, 2, \ldots, 7$ which was extracted from Bucher test design was called by $x = (\mu, a, b, x_0, r, \omega_0, \rho_a)^T$ (uncertainty flutter analysis) to iteratively solve the flutter critical velocity $V_f^*$ and obtain the implicit limit state function $Z = g(x) = 0.85V_f^* - V_f$ of wing flutter reliability.

- The least square method was used to minimize the square sum of the response surface function $g(x)$ and the implicit limit state function $g(x)$ at each sample point, and obtained the response surface function of the k-th iteration.

- JC checking point method was used to calculate the flutter reliability index of wings:

$$
\beta^{(i)} = \frac{\mu_k}{\mu_a} = \frac{g(x_i)}{\sum \sigma_i \frac{\partial g}{\partial x_i}}
$$

- It was judged whether the reliability index $\beta$ satisfied the convergence condition $|\beta^{(i)} - \beta^{(i-1)}| < \epsilon$, $\epsilon$ is a given accuracy requirement, and if the condition was satisfied, the response surface method converged, and the iteration ended, otherwise the execution was performed (2) Steps. At the end of the cycle, the failure probability $P_f$ can be obtained from the reliability index $\beta^0$.

4. Sensitivity analysis of flutter reliability

The sensitivity is the partial derivative of the failure probability $P_f$ to the distribution parameter of the basic random variable $x = (\mu, a, b, x_0, r, \omega_0, \rho_a)^T$. The purpose of reliability sensitivity analysis is to study the influence of the variation of the basic random parameters on the flutter reliability index or failure probability in the wing mechanics model. The reliability sensitivity can be expressed as [7-9]:
\[ e_x = \frac{\sigma_x}{\mu_x} = \left( \frac{1}{\sqrt{\sum_i \sigma_x^2}} \right) \left[ \frac{\sigma_x^2}{\sum_i \sigma_x^2} \right] \left( \frac{\mu_x}{\sum_i \mu_x} \right)^2 \]  

The mean sensitivity of the above-mentioned reflects the degree to which the mean value of the variable affects reliability.

\[ e_x = \frac{\sigma_x}{\mu_x} = \frac{\sum_i \sigma_x^2}{\sum_i \sigma_x^2} \left( \frac{\mu_x}{\sum_i \mu_x} \right)^2 \]  

The standard deviation sensitivity of the above formula reflects the degree of influence of random parameter volatility on reliability.

**5. Example analysis**

The binary wing model shown in figure 1 was used and determined the random parameters \( x = (\mu, a, b, x_0, r, 0, a, \rho) \), the mean value and standard deviation are shown in table 1.

| Random Variable | Mean Value | Standard Deviation |
|-----------------|------------|--------------------|
| \( \mu \)       | 20.000     | 0.4036             |
| \( a \)         | -0.1000    | -0.0060            |
| \( b \)         | 1.0000     | 0.0025             |
| \( x_0 \)       | 0.2500     | 0.0086             |
| \( r \)         | 0.5000     | 0.0146             |
| \( \omega_0 \)  | 62.800     | 0.1842             |
| \( \rho \)      | 28.100     | 0.0734             |

Figure 2 shows the relationship between velocity \( V \) and structural damping coefficient \( g \):

![Figure 2. \( v-g \) figure.](image)

It can be seen from figure 3, that the critical speed of the deterministic flutter of this binary wing is \( V^* = 0.6126 \), and the implicit limit state function describing the wing flutter reliability was established as \( g(x) = 0.85V^* - 0.52071 \) considering the parameter uncertainty. The implicit limit state function \( g(x) \) was fitted to the explicit formula \( \hat{g}(x) \) by using the quadratic response surface method:

\[ \hat{g}(x) = -33605 + 19.1685 \mu - 433.1843 a + 25145 b + 535.4303 x_0 + 368.6466 r + 290.4442 \omega_0 + 823.0207 R_a - 0.4836 \mu^2 - 2212.5a^2 - 12582b^2 - 1079.7x_0^2 - 370.8255r_a^2 - 2.3136 \omega_0^2 - 14.6502 \rho^2 \]  

The flutter reliability index of the binary wing was calculated by the checking point method, and obtained the flutter reliability design checking point and failure probability. The mean value point and design checking point are shown in table 2.
Table 2. The mean point and the design verification point.

| Random Parameters | Mean Value | Design Checking Point |
|-------------------|------------|-----------------------|
| $\mu$             | 20.000     | 20.9319               |
| $a$               | -0.1000    | -0.1113               |
| $b$               | 1.0000     | 1.0037                |
| $x_a$             | 0.2500     | 0.2601                |
| $r_a$             | 0.5000     | 0.5145                |
| $\alpha_a$        | 62.8000    | 62.9595               |
| $\rho_a$          | 28.1000    | 28.1563               |

The flutter reliability calculation results at the mean point and design checking point of the binary wing are shown in Table 3.

Table 3. Reliability index.

|                      | Reliability Index $\beta$ | Reliability $R$ | Failure Probability $P_f$ |
|----------------------|----------------------------|-----------------|---------------------------|
| Mean Point           | 1.7289                     | 0.9581          | 0.0419                    |
| Checking Point       | 3.8447                     | 0.9999          | 0.0011                    |

The reliability at the design checking point of the binary wing is 4% higher than that at the mean point. Therefore, through the research of flutter reliability, the design checking point of each parameter of the wing's aeroelasticity can be calculated, which can improve the flutter reliability of the wing and reduce the flutter amplitude.

The mean sensitivity and standard deviation sensitivity of failure probability to basic random parameters $x=\{\mu,a,b,x_a,r_a,\alpha_a,\rho_a\}$ obtained from formulas (12)-(13) are shown in Table 4, figure 3 and figure 4 are corresponding histograms.

Table 4. Basic random variable sensitivity.

| $x_i$   | $\frac{\partial P_f}{\partial \mu x_i}$ | $\frac{\partial P_f}{\partial \sigma x_i}$ |
|---------|----------------------------------------|------------------------------------------|
| $\mu$   | 2.4651                                 | -9.6675                                  |
| $a$     | -136.5169                               | 430.8393                                 |
| $b$     | 252.7235                                | -631.1765                                |
| $x_a$   | 59.7008                                 | -121.6933                                |
| $r_a$   | 29.4352                                 | -50.5179                                 |
| $\alpha_a$ | 2.0187                              | -2.9871                                  |
| $\rho_a$ | 4.5083                                 | -5.8533                                  |

The sign of $\frac{\partial P_f}{\partial \mu x_i}$ and $\frac{\partial P_f}{\partial \sigma x_i}$ indicates the tendency of the influence of the change of the mean and standard deviation of the random parameters on the flutter failure probability of the wing. If the sign is positive, the probability of wing flutter failure tends to be safe with the increase of basic random parameters, and on the contrary, tends to fail. The absolute values of $\frac{\partial P_f}{\partial \mu x_i}$ and $\frac{\partial P_f}{\partial \sigma x_i}$ reflect the speed at which the change of random parameters affects the change of wing flutter failure probability. Therefore, the sign and absolute value of $\frac{\partial P_f}{\partial \mu x_i}$ and $\frac{\partial P_f}{\partial \sigma x_i}$ analysis results are the key to judge the wing flutter stability trend.

It can be seen from the sensitivity $\frac{\partial P_f}{\partial \mu x_i}$ of the failure probability to the mean of the random parameters that the increase of the values of $\mu$, $b$, $x_a$, $r_a$, $\alpha_a$, $\rho_a$ will cause the wing flutter stability to deteriorate and tend to fail; the increase in the value of $a$ causes the wing flutter stability to be better.

From the sensitivity $\frac{\partial P_f}{\partial \sigma x_i}$ of failure probability to standard deviation of random parameters, it can be seen that the increase of values of $\mu$, $b$, $x_a$, $r_a$, $\alpha_a$, $\rho_a$ will lead to better flutter stability of wings. The increase of $a$ value leads to poor flutter stability of the wing and tends to fail.
6. Conclusions
The response surface method combined with deterministic flutter analysis was used to study the reliability of binary wing flutter, which solved the problem that the flutter limit state function of the wing aeroelastic system is difficult to explicitly calculated. The reliability sensitivity design was combined to obtain the influence law of random parameter changes on wing flutter reliability, the influence degree of random parameters on flutter critical speed was quantitatively determined, more comprehensive information was provided for the uncertainty flutter analysis, an important theoretical basis was provided for improving flight stability and reliability of the aircraft, and an efficient and feasible calculation method was provided for flutter analysis and reliability design of wing structures.

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