Evolution Properties of a Partially Coherent Flat-topped Vortex Hollow Beam Propagating in Uniaxial Crystals Orthogonal to the Optical Axis

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(Received June 20, 2016 : revised October 18, 2016 : accepted October 27, 2016)

The analytical expressions for a partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals orthogonal to the optical axis are derived, and the intensity and coherent vortex properties of partially coherent flat-topped vortex hollow beam propagation in uniaxial crystals orthogonal to the optical axis are analyzed by numerical examples. The influence of beam order parameter $N$, topological charge $M$, the coherence length and the ratio of refractive indices $n_e/n_o$ of uniaxial crystals on the normalized intensity distribution and coherent vortex of a partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals are discussed in detail.

Keywords: Partially coherent flat-topped vortex hollow beam, Uniaxial crystal, Laser propagation, Propagation properties

OCIS codes: (260.1180) Crystal optics; (260.1960) Diffraction theory; (350.5500) Propagation

I. INTRODUCTION

Uniaxial crystals have been widely studied in the design of wave plates, polarizers, compensators, and optical modulation devices [1]. The propagation properties of laser beams in uniaxial crystals have been treated by solving the boundary value problems of Maxwell’s equations in uniaxial crystals [2-4]. Since the vectorial theory of laser beam propagation in uniaxial crystals was constructed, propagation of various laser beams [5-18], such as Laguerre-Gauss and Bessel-Gauss beams, Hermite-Gauss beams, dark hollow beams, flat-topped beams, elliptical Gaussian beams, elliptical Gaussian vortex beams and beams generated by a Gaussian mirror resonator in uniaxial crystals, partially coherent flat-topped beams, partially polarized and partially coherent beams, Laguerre-Gaussian correlated Schell model beams, coherent and partially coherent four-petal Gaussian vortex beams, has been widely investigated.

On the other hand, optical vortex beams have been widely studied due to their applications in free-space optical communication. Among the optical vortex beams, the properties of the elliptical Gaussian vortex beam [12], partially coherent four-petal Gaussian vortex beam [18], four-petal Gaussian vortex beam [19], vortex beam carried by an Airy beam [20], vortex beams [21], and partially coherent hollow vortex Gaussian beam [22], have been studied. However, to our knowledge, there have been no reports about partially coherent flat-topped vortex hollow beams propagating in uniaxial crystals orthogonal to the optical axis. And the studies of partially coherent flat-topped vortex hollow beams propagating in birefringent crystals have the potential application in the design of electro-optic devices and in trapping of particles. In this work, we mainly investigate the evolution of the beam’s intensity and vortex distributions of a partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals orthogonal to the optical axis.

II. THEORY ANALYSIS

In our analysis, assume that the beam propagation direction in uniaxial crystals orthogonal to the optical axis...
is along the z axis, and the optical axis of uniaxial crystals coincides with the x-axis, then the relative dielectric tensor of uniaxial crystals can be expressed as

\[
\varepsilon = \begin{pmatrix}
  n_o^2 & 0 & 0 \\
  0 & n_e^2 & 0 \\
  0 & 0 & n_e^2
\end{pmatrix}
\] (1)

with \(n_o\) and \(n_e\) represent the ordinary and extraordinary refractive indices, respectively. In this case, we mainly consider a laser beam propagating in uniaxial crystals orthogonal to the optical axis. Within the framework of paraxial approximation, the components of the laser beam propagating in uniaxial crystals orthogonal to the optical axis. Within the framework of paraxial approximation, the components of the laser beam propagating in uniaxial crystals orthogonal to the optical axis can be treated by the following formulas [2-4]:

\[
E_x(r, z) = \exp(ikn_x z) \frac{kn_x}{2\pi i z} \times \int dx_d dy_d \exp \left\{ - \frac{k}{2zn_x} \left[ n_o^2 (x-x_0)^2 + n_e^2 (y-y_0)^2 \right] \right\} E_i(r_x, 0)
\] (2a)

\[
E_y(r, z) = \exp(ikn_y z) \frac{kn_y}{2\pi i z} \times \int dx_d dy_d \exp \left\{ - \frac{k}{2zn_y} \left[ (x-x_0)^2 + (y-y_0)^2 \right] \right\} E_i(r_y, 0)
\] (2b)

with \(k=2\pi/\lambda\) is the wave number, \(r=(x, y)\) and \(r_0=(x_0, y_0)\) represent the position vectors at the output and input planes, respectively; \(E_i(r, z)\) and \(E_o(r_0, 0)\) \((\alpha=x, y)\) represent the electric fields in the output and input planes, respectively. It can be seen from Eqs. (2) that the x component of the laser beam (Eq. (2a)) undergoes an asymmetry diffraction spreading its shape in x and y directions, and the y component of the laser beam (Eq. (2b)) is the same as that which propagates in isotropic media. In this work, we focus on the propagation properties of an x-polarized laser beam in uniaxial crystals orthogonal to the optical axis.

Based on the theory of coherence, the second-order correlation properties of the x-polarized laser beam can be characterized by the cross-spectral density function [23]:

\[
W(r_x, r_y, z) = \langle E(r_x, z) E^* (r_y, z) \rangle
\] (3)

Then the cross-spectral density function of the x-polarized laser beam propagating in uniaxial crystals orthogonal to the optical axis can be obtained as:

\[
W(r_1, r_2, z) = \langle E(r_1, z) E^* (r_2, z) \rangle
= \frac{k^4 n_o^4}{4\pi^2 z^2} \int W(r_1, r_2, 0) \exp \left\{ - \frac{k}{2zn_o} \left[ n_o^2 (x_1 - x_0)^2 + n_e^2 (y_1 - y_0)^2 \right] \right\}
\] (4)

where \(W(r_1, r_2, z)\) and \(W(r_1, r_2, 0)\) represent the cross-spectral density functions of the laser beam in the output and input planes, respectively.

Assume that the flat-topped vortex hollow beam considered in this work is x-polarized and is incident into uniaxial crystals at the plane \(z=0\), then the flat-topped vortex hollow beam at the source plane \(z=0\) can be written as [24]:

\[
E(r_0, 0) = \sum_{m=0}^{N} (-1)^{m-1} \binom{N}{m} \exp \left[ -m \left( \frac{x_0^2}{w_x^2} + \frac{y_0^2}{w_y^2} \right) \right] \left( \frac{x_0 + i \lambda_0}{w_x} \right)^m
\] (5)

where \(N\) is the order of the flat-topped vortex hollow beam, \(M\) is the topological charge, \(w_x\) and \(w_y\) are the beam waist width in x and y directions, respectively. \(\binom{N}{m}\) denotes the binomial coefficient; and if \(w_x = w_y\) the beam represents the circular flat-topped vortex hollow beam.

The cross-spectral density function for a partially coherent flat-topped vortex hollow beam by using the coherence theory can be obtained as:

\[
W(r_1, r_2, 0) = \sum_{m=0}^{N} \sum_{n=0}^{N} \left( -1 \right)^{m+n} \frac{(-1)^m \binom{N}{m}}{N^2} \left( \frac{x_1}{w_x} \right)^m \left( \frac{y_1}{w_y} \right)^n
\] (6)

with \(\sigma\) is the coherence length.

By utilizing the following formulas [25]:
with

\[ a_s = \frac{n}{w_z} + \frac{kn^2}{2inz} + \frac{1}{\sigma^2} \] (12a)

\[ b_s = \frac{m}{w_z} - \frac{kn^2}{2iz} + \frac{1}{\sigma^2} - \frac{1}{a_s} \left( \frac{1}{\sigma^2} \right)^2 \] (12b)

\[ c_s = \frac{1}{\sigma^2} + \frac{kn^2}{2iz} y_1 - \frac{kn^2}{2iz} y_2 \] (12c)

Eqs. (10)-(13) are the main analytical results for a partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals orthogonal to the optical axis.

Based on the theory of coherence, the degree of coherence for the laser beam can be expressed as [23]:

\[ \mu(r_1, r_2, z) = \frac{W(r_1, r_2, z)}{W(r_1, r_2, z)W(r_1, r_2, z)} \] (14)

The position of coherent vortices for the partially coherent flat-topped vortex hollow beam at the propagation distance \( z \) can be expressed as [26]:

\[ \text{Re} \left[ \mu(r_1, r_2, z) \right] = 0 \] (15a)

\[ \text{Im} \left[ \mu(r_1, r_2, z) \right] = 0 \] (15b)

where \( \text{Re} \) and \( \text{Im} \) are the real and imaginary parts of \( \mu(r_1, r_2, z) \), respectively.

### III. Numerical Examples

In this section, we study the evolution properties of a partially coherent four-petal Gaussian vortex beam propagating in uniaxial crystals orthogonal to the optical axis. The parameters \( w_z, n_o \) and \( \lambda \) in the whole paper are chosen as \( w_z = 30 \mu m, n_o = 2.616 \) (rutile crystal) and \( \lambda = 532 nm \).
Figs. 1 and 2 show the contour graphs of the normalized intensity for a partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals orthogonal to the optical axis with $N = 3$, $M = 1$ and $\sigma = 15 \mu m$ for the different $n_e/n_o$, respectively. From Figs. 1 and 2, it is shown that the beam can keep its original dark hollow center pattern in the short propagation distance (Figs. 1(a) and 2(a)); and the beam will lose its initial dark hollow center pattern as the propagation distance increases (Figs. 1(b) and 2(b)), then the beam will evolve into the desired flat-topped beam (Figs. 2(c) and 2(b)); finally, the beam will evolve into the elliptical Gaussian-like beam (Figs. 1(d) and 2(d)) due to the influence of uniaxial crystals; it is also found that the beam propagating in uniaxial crystals with $n_e/n_o = 1.5$ spreads faster along the x direction than the y direction due to the refractive index $n_e > n_o$ of uniaxial crystals.

In order to investigate the influence of coherence length, Fig. 3 shows the contour graphs of the normalized intensity for fully coherent flat-topped vortex hollow beam propagating in uniaxial crystals with $n_e/n_o = 1.1$, $N = 3$, $M = 1$ and $\sigma = \infty$. By comparing Figs. 1-3, it can be seen that the fully coherent beam propagating in uniaxial crystals can keep its initial dark hollow center as the propagating distance increases. And the partially coherent beam evolves into the Gaussian-like beam due to the influence of coherence length.

From Fig. 4, it can be found that the partially coherent flat-topped vortex hollow beam ($N = 2$, $M = 2$) propagating in uniaxial crystals with $n_e/n_o = 1.5$ have similar evolution properties with the beam with the different $M$ and $N$ (Figs. 1 and 2), and the beam will evolve into a Gaussian-like beam as the beam propagation distance increases.

Figs. 5 and 6 give the curves of $\text{Re} \mu(r_1, r_2, z) = 0$ and $\text{Im} \mu(r_1, r_2, z) = 0$ for a partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals orthogonal to the optical axis with $r_2 = (2 \mu m, 5 \mu m)$, $N = 2$ and $M = 1$ for the different $n_e/n_o$, respectively. From Figs. 5(a) and 6(a), it can be seen that the beam propagating in uniaxial crystals at the distance $z = 500 \mu m$ has a coherent vortex, and as the propagation distance increases, the beam at the distance $z = 1000 \mu m$ has more coherent vortices than for the propagation distance $z = 500 \mu m$. Then the partially coherent flat-topped vortex hollow beam will have the changed number of coherent vortices during the beam propagation in uniaxial crystals. And we also found that the beam propagating in uniaxial crystals with higher $n_e/n_o$ becomes narrower in the x direction than in the y direction.

Fig. 7 gives the curves of $\text{Re} \mu(r_1, r_2, z) = 0$ and $\text{Im} \mu(r_1, r_2, z) = 0$ for partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals with $n_e/n_o = 1.1$, $r_2 = (2 \mu m, 5 \mu m)$, $N = 3$ and $M = 2$. We found that the beam has similar evolution properties, the beam will have a changed number of coherent vortices with the propagation
FIG. 2. The contour graphs of normalized intensity for partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals with \( n_e/n_o = 1.5 \), (a) \( z = 500 \mu m \), (b) \( z = 2000 \mu m \), (c) \( z = 6000 \mu m \), (d) \( z = 10000 \mu m \).

FIG. 3. The contour graphs of normalized intensity for fully coherent flat-topped vortex hollow beam propagating in uniaxial crystals with \( n_e/n_o = 1.1 \), (a) \( z = 500 \mu m \), (b) \( z = 2000 \mu m \), (c) \( z = 6000 \mu m \), (d) \( z = 10000 \mu m \).
FIG. 4. Figure 4: The contour graphs of normalized intensity for partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals with $n_e/n_o = 1.5$, (a) $z = 500 \mu m$, (b) $z = 2000 \mu m$, (c) $z = 6000 \mu m$, (d) $z = 10000 \mu m$.

FIG. 5. The curves of $\text{Re} \mu = 0$ and $\text{Im} \mu = 0$ for partially coherent flat-topped vortex hollow beam propagating in uniaxial crystal with $n_e/n_o = 1.1$, (a) $z = 500 \mu m$, (b) $z = 1000 \mu m$.

FIG. 6. The curves of $\text{Re} \mu = 0$ and $\text{Im} \mu = 0$ for partially coherent flat-topped vortex hollow beam propagating in uniaxial crystal with $n_e/n_o = 1.5$, (a) $z = 500 \mu m$, (b) $z = 1000 \mu m$. 
FIG. 7. The curves of $\text{Re} \mu = 0$ and $\text{Im} \mu = 0$ for partially coherent flat-topped vortex hollow beam propagating in uniaxial crystal with $n_e/n_o = 1.1$, (a) $z = 500 \mu m$, (b) $z = 1000 \mu m$.

FIG. 8. The contour graphs of normalized intensity for partially coherent elliptical flat-topped vortex hollow beam propagating in uniaxial crystals with $n_e/n_o = 1.1$, (a) $z = 500 \mu m$, (b) $z = 2000 \mu m$, (c) $z = 6000 \mu m$, (d) $z = 10000 \mu m$.

We found that the elliptical beam has the similar evolution properties with the circular beam with the propagation distance increasing, and the beam will evolve into the elliptical Gaussian-like beam.

IV. CONCLUSION

In this paper, the analytical expressions of a partially coherent flat-topped vortex hollow beam propagating in uniaxial crystals is derived and analyzed. We found that the partially coherent flat-topped vortex hollow beam will keep its original dark hollow center pattern in the short propagation distance, and the beam will evolve into an elliptical Gaussian-like beam with the propagation distance increasing due to the influence of uniaxial and the coherence length; and the beam will spread faster in the $x$ direction than in the $y$ direction with the larger $n_e/n_o$. And the partially coherent flat-topped vortex hollow beam will have a changed number of coherent vortices as the propagation distance increases.

ACKNOWLEDGEMENTS

This work was supported by National Natural Science Foundation of China (11604038, 11404048, 11375034), Natural Science Foundation of Liaoning Province (201602062,
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