Emergence and Expansion of Cosmic Space in Bionic system

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Recently, Padmanabhan [arXiv:1206.4916] argued that the expansion rate of the universe can be thought of as the emergence of space as cosmic time progresses and is related to the difference between the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom inside. The main question arises as to what is origin of emergence of space in 4D universe? We answer to this question in Bionic system. The BIon is a configuration in flat space of a D-brane and a parallel anti-D-brane connected by a thin shell wormhole with F-string charge. We propose a new model that allows all degrees of freedom inside and outside the universe are controlled by the evolutions of BIon in extra dimension and tend to degrees of freedom of black F-string in string theory or black M2-brane in M-theory.

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I. INTRODUCTION

Intrigued by the holographic principle, Padmanabhan recently proposed a novel idea, saying that our cosmic space is emergent as cosmic time progresses. The emergence is governed by the basic relation that the increase rate of Hubble volume is linearly determined by the difference between the number of degrees of freedom on the horizon surface and the one in the bulk [1]. Until now, this idea has been discussed in many papers [2–7]. For example, following Padmanabhan proposal, some authors generalized the basic relation to derive the Friedmann equations of an (n + 1)-dimensional Friedmann-Robertson-Walker universe corresponding to general relativity, Gauss-Bonnet gravity, and Lovelock gravity [2]. Some other authors generalized Padmanabhan paradigm to brane world, scalar-tensor gravity, and f(R) theory, respectively, and found that in the cosmological setting the Friedmann equations can be successfully derived [3]. In another scenario, by modification Padmanabhan idea, the Friedmann equation of the Friedmann-Robertson-Walker (FRW) Universe with any spatial curvature was derived and the study to higher dimensional spacetime is extended [4]. In another paper, researchers applied this method to a non-flat universe, and modified the evolution equation to lead to the Friedmann equation [5]. On the other hand, some investigators discussed that Padmanabhans conjecture holds for the flat Friedmann-Robertson-Walker universe in Einstein gravity but does not hold for the non-flat case unless one uses the aerial volume instead of the proper volume [6] and in more recent investigations, using emergence of cosmic space framework and by considering a generic form of the entropy as a function of area, the general dynamical equation of FRW universe filled with a perfect fluid was derived [7].

Now, the main question arises as to what is origin of emergence of space in 4D universe? We answer to this question in Bionic system. It might be reasonable to ignore the wormhole in the ultraviolet where the branes and antibranes are well separated and the brane’s spike is far from the antibrane’s spike, it is likely that one BIon forms and grows where the spikes of brane and antibrane meet each other [8, 9]. In this condition, there exists many channels for flowing energy from extra dimensions into our universe and as a result, degrees of freedom inside the universe increase and tend to degrees of freedom of black F-string in string theory or black M2-brane in M-theory.

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The outline of the paper is as the following. In section II, we construct Padmanabhan proposal in ten dimensional Bionic system and obtain degrees of freedom inside and outside of universe in terms of wormhole parameters in extra dimension. In section III, we consider Padmanabhan idea in eleven dimensional M2-M5-brane. The last section is devoted to summary and conclusion.

**II. THE PADMANABHAN IDEA IN TEN DIMENSIONAL BIONIC SYSTEM**

In this section, we will search the role of wormholes in emergency of cosmic space and show that they are the main causes of expansion of universe. To this end, we will follow Padmanabhan’s approach in thermal BIonic and discuss that the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom inside the universe depend on the temperature of BIonic, number of branes and the distance between branes.

To illustrate the BIonic we focus to an embedding of the D3-brane world volume in 10D Minkowski space-time having line element:

\[ ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \sum_{i=1}^{6} dx_i^2. \] (1)

without background fluxes. We choose the world volume coordinates of the D3-brane as \( f^a; a = 0 \cdots 3 \) and also define \( f^0 = 0; f^1 = 1 \) as the embedding of the three-brane which is given by [8, 9]:

\[ t(\sigma^a) = \tau, r(\sigma^a) = \sigma, x_1(\sigma^a) = z(\sigma), \theta(\sigma^a) = \sigma^2, \phi(\sigma^a) = \sigma^3 \] (2)

and the remaining coordinates \( x_i = 2, \ldots, 6 \) are constant. Note that here only one non-trivial embedding function \( z(\sigma) \) that expresses the bending of the brane. Assume that \( z \) be a transverse coordinate to the branes and \( r \) be the radius on the world-volume. The flat branes is thus defined as \( z(\sigma) = 0 \). Then the induced metric on the branes will get the form as

\[ \gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + (1 + z'(\sigma)^2) d\sigma^2 + \sigma^2 (d\theta^2 + \sin^2\theta d\phi^2) \] (3)

so that the spatial volume element is \( dV_3 = \sqrt{1 + z'(\sigma)^2}\sigma^2 d\Omega_2 \).

Note that we have assumed the universe in brane-antibrane system + wormhole that connect these branes. Therefore, the shape function comprising the induced metric assumes the form [13]

\[ b(\sigma) = \frac{\sigma z^2}{1 + z^2} \]

Here, we demand two boundary conditions as \( z(\sigma) \rightarrow 0 \) for \( \sigma \rightarrow \infty \) and \( z'(\sigma) \rightarrow -\infty \) for \( \sigma \rightarrow 0 \), where \( \sigma_0 \) is the minimal two-sphere radius of the configuration i.e. throat radius. Now, we use \( k \) units of F-string charge along the radial direction to yield [8, 9]:

\[ z(\sigma) = \int_0^{\infty} d\hat{\sigma} \left( \frac{F(\hat{\sigma})^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}} \] (4)

BIonic \( F(\sigma) \) in finite temperature can be expressed as

\[ F(\sigma) = \sigma^2 \frac{4 \cosh^2 \alpha - 3}{\cosh^4 \alpha} \] (5)

where \( \cosh \alpha \) is defined as

\[ \cosh^2 \alpha = \frac{3 \cos \frac{\delta}{4} + \sqrt{3} \sin \frac{\delta}{4}}{2 \cos \delta} \] (6)

with the definitions:

\[ \cos \delta \equiv T^4 \sqrt{1 + \frac{k^2}{\sigma^4}}, T \equiv \left( \frac{9\pi^2 N}{4\sqrt{3} T_{D_3}} \right), \kappa \equiv \frac{k T_{F_1}}{4\pi T_{D_3}} \] (7)
In equation (7), T and N are the finite temperature of BIon and number of D3-branes respectively. \( T_{D3} \) and \( T_{F1} \) are tensions of brane and fundamental strings respectively. We will construct wormhole configurations by attaching a mirror solution to Eq. (4). To measure the separation distance \( \Delta = 2z(\sigma_0) \) between the N D3-branes and N anti D3-branes for a given brane-antibrane wormhole configuration, one has to define the four parameters N, k, T and \( \sigma_0 \).

Now, we have:

\[
\Delta = 2z(\sigma_0) = 2\int_{\sigma_0}^{\infty} d\sigma \left( \frac{F(\sigma)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}}
\]  

(8)

For small temperature limit, we obtain:

\[
\Delta = 2\sqrt{\frac{\pi T(5/4)}{T(3/4)}} \sigma_0 \left( 1 + \frac{8}{27} \frac{k^2}{\sigma_0^4} T^4 \right)
\]  

(9)

Let us now construct the Padmanabhan idea in thermal BIon. For this, we need to compute the contribution of the BIonic system to the degrees of freedom on the holographic horizon and the bulk degrees of freedom inside the universe. To this end, we write the following relations between these degrees of freedom and the entropy of BIon and also, the mass density along the transverse direction,

\[
N_{\text{sur}} + N_{\text{bulk}} = N_{\text{BIon}} = N_{\text{brane}} + N_{\text{anti-brane}} + N_{\text{wormhole}}
\]

\[
\simeq 4L_P^2 S_{\text{BIon}} = \frac{4T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha}
\]

\[
N_{\text{sur}} - N_{\text{bulk}} \simeq \int d\sigma \frac{dM_{\text{BIon}}}{d\sigma} = \frac{2T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 A \cosh^2 \alpha + 1
\]  

(10)

Solving these equations simultaneously, we obtain:

\[
N_{\text{sur}} \simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha} + \frac{2T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 A \cosh^2 \alpha + 1
\]

\[
N_{\text{bulk}} \simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha} - \frac{2T_{D3}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 A \cosh^2 \alpha + 1
\]  

(11)

This equation indicates that with the increase of temperature in BIonic system, the bulk degrees of freedom inside the universe will increase. A possible reason for this is when spikes of branes and antibranes are well separated, wormhole would not be formed and there is no channel for flowing energy from extra dimensions to our universe. On the other hand, when two branes are close to each other and connected by a wormhole, the bulk degrees of freedom gain to large values.

At this stage, we can calculate the relation between some of cosmological parameters like Hubble parameter and energy density and some properties of BIon. For example, the number of degrees of freedom on the spherical surface of apparent horizon with radius \( r_A \) is proportional to its area and is given by:

\[
N_{\text{sur}} = \frac{4\pi r_A^2}{L_P^2}
\]  

(12)

where \( r_A = \sqrt{H^2 + \frac{k}{a^2}} \) is the apparent horizon radius for the FRW Universe, \( H = \frac{\dot{a}}{a} \) is the Hubble parameter and a is the scale factor. Using this equation, we can estimate the Hubble parameter for flat universe:

\[
H \simeq \frac{18\pi k^4 N_{12} T_{F1}^{14} T_{D3}^{12}}{T_{D3}^{14} \sigma_0^8} + \frac{8\pi k^2 N_{24} T_{F1}^{12} T_{D3}^{12}}{T_{D3}^{14} \sigma_0^8}
\]  

(13)

This equation has some interesting results which can be used to explain the reasons for occurrence of expansion in present era of universe. According to these calculations, the expansion of universe is controlled by the number of branes, F-string and brane tensions, the location of throat of wormhole and temperature of BIon.

On the other hand, using the Friedmann equation of the flat FRW Universe, we can calculate the universe energy density:

\[
\rho = \frac{3}{8\pi L_P^2} H^2 \simeq \frac{27\pi k^8 N^{24} T_{F1}^{28} T_{D3}^{28}}{4L_P^2 T_{D3}^{28} \sigma_0^{16}} + \frac{3\pi k^4 N^{20} T_{F1}^{24} T_{D3}^{20}}{L_P^2 T_{D3}^{20} \sigma_0^{16}}
\]  

(14)
As is obvious from this equation, the energy density depends on the temperature of BIon and any enhancement or
decline in this density can be a signature of some interactions between two universes in extra dimension. The reason
of this is that with the moving of two branes in the directions to each other, radiation of energy from wormhole raises
and obtains the large values near the colliding point.

Now, the main question arises that what is the fate of universe at the end of expansion? To search this question, we
match the finite temperature BIon and black F-string at corresponding point[9]. We have the supergravity solution
for k coincident non-extremal black F-strings lying along the z direction as

$$
\frac{dM_{F_k}}{dz} = \frac{3^2 T_{D3}^2 (1 + 6 \cosh^2 \alpha)}{2 T_{D3}^2 \sinh \alpha}$$

$$
k^2 = \frac{3^2 T_{D3}^2 (1 + \cosh^2 \alpha)}{2^2 T_{D3}^2 T_{F1} k^2 \sinh \alpha}, T_{F1} = \frac{1}{2 \pi l_s^2}
$$

For small temperatures one can expand the mass density as follows:

$$
\frac{dM_{F_k}}{dz} = T_{F1} k + \frac{16 (T_{F1} k^2)^{3/2} T^3}{81 T_{D3}^2} + \frac{40 T_{D3}^2 T_{F1} k^2 \pi^2 T^6}{729 T_{D3}^2}
$$

On the other hand, for small temperature BIon, we have [9]:

$$
\frac{dM_{BIon}}{dz} = T_{F1} k + \frac{3 \pi T_{D3}^2 k^2 T^4}{32 T_{D3}^2 \pi^2} + \frac{7 \pi T_{D3}^2 k^3 T^8}{512 T_{D3}^4 \pi^4}
$$

If one compares the mass densities for BIon to the mass density for the F-strings, one will see that the thermal D3-F1
configuration at \( \sigma = \sigma_0 \) behaves like k F-strings and \( \sigma_0 \) should have the following dependence on the temperature as :

$$
\sigma_0 = \left( \frac{\sqrt{C_0}}{T_{D3}} \right)^{1/2} \sqrt{T} \left[ C_0 + C_1 \frac{\sqrt{C_0}}{T_{D3}} T^3 \right]
$$

At this point, we can obtain the the degrees of the surface, degrees of freedom on the holographic horizon and
the bulk degrees of freedom inside the universe as ( here, we have assumed infinitely thin shell wormhole that connect
these branes ) :

$$
N_{surf} - N_{bulk} \simeq \int_{\sigma_0}^{\sigma_0 + \epsilon} d\sigma \frac{dM_{BIon}}{dz} \rightarrow
$$

$$
N_{surf} - N_{bulk} = T_{F1} k \epsilon + \frac{3 \pi T_{D3}^2 k^2 T^4 \epsilon}{32 T_{D3}^2 \pi^2} + \frac{(7 \pi T_{D3}^2 k^3 T^8) (\epsilon^2 - 2 \sigma_0 \epsilon)}{512 T_{D3}^4 \pi^4 (\sigma_0 + \epsilon)^2}
$$

$$
\lim_{\epsilon \rightarrow 0} [N_{surf} - N_{bulk}] = 0 \rightarrow
N_{surf} - N_{bulk} \rightarrow
N_{surf} + N_{bulk} = 2 N_{surf} = N_{black F-string}
$$

This equation indicates that the bulk degrees of freedom inside the universe will be equal to the degrees of freedom
of black F-string at the end of universe expansion. This means that universe evolves by the the phenomenological
events and wormholes in extra dimension and ends up in one black F-string.

III. THE PADMANABHAN IDEA IN ELEVEN DIMENSIONAL M2-M5 BIONIC SYSTEM

In this section we will enter the effects of evolution in M2-M5 Blonic system on the surface and the bulk degrees of
freedom in FRW model of cosmology. We will show that contrary to previous section, universe expands and ends up
in black M2-brane.
To describe the BIon we specialize to an embedding of the M5-brane world volume in 11D Minkowski space-time with metric [11, 12]:

\[ ds^2 = -dt^2 + (dx^1)^2 + dr^2 + r^2 d\Omega_3^2 + \sum_{i=6}^{10} dx_i^2. \]  

without background fluxes. Using the standard angular coordinates \((\psi, \phi, \omega)\) to express the round three-sphere metric

\[ d\Omega_3^2 = -d\psi^2 + \sin^2 \psi (d\phi^2 + \sin^2 \phi d\omega^2). \]  

We choose the static gauge:

\[ t(\sigma) = \sigma^1, \quad x^1(\sigma) = \sigma^1, \quad r(\sigma^a) = \sigma^2 \equiv \sigma \]
\[ \psi(\sigma^a) = \sigma^3, \quad \phi(\sigma^a) = \sigma^4, \quad \omega(\sigma^a) = \sigma^5, \quad \phi_0(\sigma^a) = z(\sigma) \]  

With this ansatz the induced metric on the effective fivebrane world volume is

\[ \gamma_{ab} d\sigma^a d\sigma^b = -(d\sigma^3)^2 + (d\sigma^1)^2 + (1 + z'(\sigma)^2) d\sigma^2 + \sigma^2 (-d\psi^2 + \sin^2 \psi (d\phi^2 + \sin^2 \phi d\omega^2)) \]  

We impose the two boundary conditions that \(z(\sigma) \to 0\) for \(\sigma \to \infty\) and \(z'(\sigma) \to -\infty\) for \(\sigma \to \sigma_0\), where \(\sigma_0\) is the minimal two-sphere radius of the configuration. After some algebra, we obtain [11, 12]:

\[ z_{\pm}(\sigma) = \int_{\sigma}^{\infty} ds \left( \frac{F_{\pm}(s)^2}{F_{\pm}(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}} \]  

In finite temperature BIon, \(F(\sigma)\) is given by

\[ F_{\pm}(\sigma) = \sigma^3 \left( \frac{1 + \frac{\kappa^2}{\sigma^2}}{1 \pm \sqrt{1 - \frac{4q^2}{\sigma^2}(1 + \frac{\kappa^2}{\sigma^2})}} \right)^{3/2} \left( -2 + \ln \frac{1 \pm \sqrt{1 - \frac{4q^2}{\sigma^2}(1 + \frac{\kappa^2}{\sigma^2})}}{2q} \right) \]  

where

\[ \beta = \frac{3}{4\pi T}, \quad q_2 = \sigma^3 \frac{r_0^3}{2} \sin \theta \sinh 2\alpha, \quad q_5 = \frac{r_0^3}{2} \cos \theta \sinh 2\alpha \]  

with the definitions:

\[ \cosh \alpha_{\pm} = \frac{\beta^3}{\sqrt{2q_5}} \frac{1 \pm \sqrt{1 - \frac{4q^2}{\sigma^2}(1 + \frac{\kappa^2}{\sigma^2})}}{\sqrt{1 + \frac{\kappa^2}{\sigma^2}}} \]
\[ r_{0,\pm} = \left( \frac{\sqrt{2q_5}}{\beta^2} \right)^{1/2} \frac{1 \pm \sqrt{1 - \frac{4q^2}{\sigma^2}(1 + \frac{\kappa^2}{\sigma^2})}}{\sqrt{1 + \frac{\kappa^2}{\sigma^2}}} \]
\[ \tan \theta = \frac{k}{\sigma^3}, \quad q_2 = q_5 = -4\pi \frac{N_2}{N_5} \frac{r_0^3}{q_5} \]  

In above equation, \(N_2\) and \(N_5\) are the number of M2 and M5-branes and \(q_2\) and \(q_5\) are the charges of M2 and M5-branes respectively and \(T\) is temperature of BIon. Attaching a mirror solution to Eq. (28), we construct thin shell wormhole configuration. The separation distance \(\Delta = 2z(\sigma_0)\) between the N M5-branes and N anti M5-branes for a given brane-antibrane wormhole configuration is obtained by:

\[ \Delta = 2z(\sigma_0) = 2 \int_{\sigma_0}^{\infty} ds \left( \frac{F(s)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}} \]  

Let us now construct the Padmanabhan idea in M2-M5 BIon. Similar to previous section, we need to compute the contribution of the M2-M5 system to the degrees of the surface degrees of freedom on the holographic horizon and
the bulk degrees of freedom inside the universe. To this end, we write the following relations between these degrees of freedom and the entropy of M2-M5 and also, the mass density along the transverse direction,

\[ N_{\text{sur}} + N_{\text{bulk}} = N_{M2-M5} = N(M5 - \text{brane}) + N(anti - M5 - \text{brane}) + N(M2 - \text{brane}) \]

\[ \simeq 4L_P^2 S_{M2-M5} = \frac{\Omega_3(\Omega'_4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^4 \sigma^3 \frac{1}{\cosh^2 \alpha} \]

\[ N_{\text{sur}} - N_{\text{bulk}} \simeq \int d\sigma \frac{dM_{M2-M5}}{dz} = \frac{\Omega_3(\Omega'_4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^3 \sigma^3 3 \cosh^2 \alpha + 1 \cosh^4 \alpha \]  

Solving these equations simultaneously, we obtain:

\[ N_{\text{sur}} \simeq \frac{\Omega_3(\Omega'_4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^4 \sigma^3 \frac{1}{\cosh^2 \alpha} + \frac{\Omega_3(\Omega'_4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^3 \sigma^3 3 \cosh^2 \alpha + 1 \cosh^4 \alpha \]

\[ N_{\text{bulk}} \simeq \frac{\Omega_3(\Omega'_4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^4 \sigma^3 \frac{1}{\cosh^2 \alpha} \]

This equation implies that any increase or decrease in the bulk or surface degrees of freedom can be a signature of some evolutions in M2-M5 brane. Also, similar to the results of previous section, with decreasing the distance between two branes, wormhole will be formed and as a result, the bulk degrees of freedom increase.

Similar to previous section, we can obtain Hubble parameter and energy density in terms of the number and charges of M2 and M5 branes. Using equation (12), we can estimate the Hubble parameter for flat universe:

\[ H \simeq \frac{16\pi Gk^4 q_0^6}{3\Omega_3(\Omega'_4)q_0^2 \sigma_0^6} T^3 + \frac{8\pi Gk^2 q_0^4}{\Omega_3(\Omega'_4)q_0^2 \sigma_0^6} T^2 \]  

This equation indicates that Hubble parameter depends on the charges of M2 and M5 branes and also temperature of M2-M5 system. Comparing this equation to (13), we find that the Hubble parameter in M2-M5 system is less sensitive to temperature respect to Bionic system.

In addition, using the Friedmann equation of the flat FRW Universe, we can get the universe energy density:

\[ \rho = \frac{3}{8\pi L_P^2} H^2 \simeq \frac{384\pi G^2 k^2 q_0^6}{3L_P^2 \Omega_3(\Omega'_4)q_0^6 \sigma_0^6} T^6 + \frac{24\pi G^2 k^4 q_0^8}{L_P^2 \Omega_3(\Omega'_4)q_0^2 \sigma_0^6} T^4 \]  

As can be seen from this equation, the energy density is related to the charges of M-branes and the location of throat of wormhole in M2-M5 system. When two M5-branes becomes close to each other and thin shell wormhole is formed, energy flows from extra dimensions to our universe and energy density increases.

Now, we try to know the fate of universe in M2-M5 system. To search this question, we demand the matching of finite temperature Bion and black M2-M5 at corresponding point[12]. The M2 brane has the same charge \( Q_2 \) and the same temperature \( T \) as the M2-M5 system. A perturbative expansion of the tension around the extremal limit give [12]:

\[ \frac{M_{M2-brane}}{L_x^3 L_z} = Q_2 \left( 1 + \sqrt{\frac{q_2}{3\sqrt{2} \beta}} + \frac{5q_2}{2\beta^2 \beta_0^6} \right) \]  

\[ \frac{dM_{\text{Bion}}}{L_x d\sigma} = Q_2 \left( \frac{1 + \frac{\sigma}{k^2}}{k^2} + \frac{5q_2}{6\beta^2} \left( 1 + \frac{\alpha^6}{\sigma_0^6} \right)^{3/2} \right) + \frac{11q_2^2}{8\beta^12} \left( 1 + \frac{\alpha^6}{\sigma_0^{12}} \right)^{5/2} \]  

We now compare the mass densities for M2-M5 system to the mass density for black M2-brane and note that the thermal Bion at \( \sigma = \sigma_0 \) behaves like black M2-brane. \( \sigma_0 \) depends on the temperature as [12]:

\[ \sigma_0 = \frac{q_2^{1/4}}{\beta^{3/2}} \left( 1.234 - 0.066 \frac{q_2^{1/2}}{\beta^3} \right) \]
At this point, we can obtain the the degrees of the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom inside the universe as (as before, we have assumed infinitely thin shell wormhole that connect these branes) ::

\[
N_{\text{sur}} - N_{\text{bulk}} \simeq \int_{\sigma_0}^{\sigma_0 + \epsilon} d\sigma \frac{dM_{2-M_5}}{dz} \\
N_{\text{sur}} - N_{\text{bulk}} \simeq Q_2 \left( \frac{\epsilon^{7/2}}{\sigma_0^2k^2} + \frac{15q_2^2 \epsilon^{21/2}}{84\beta^6 k^2\sigma_0^3} + \frac{55q_2^4 \epsilon^{35/2}}{112\beta^{12} k^2\sigma_0^5} \right) \\
\lim_{\epsilon \to 0} [N_{\text{sur}} - N_{\text{bulk}}] = 0 \Rightarrow \\
N_{\text{sur}} = N_{\text{bulk}} \\
N_{\text{sur}} + N_{\text{bulk}} = 2N_{\text{sur}} = N_{\text{black}M_2-\text{brane}}
\]

This equation indicates that the bulk degrees of freedom inside the universe will be equal to the degrees of freedom of black M2-brane at the end of universe expansion. This result is in contrary with the results of previous section that showed the final state of universe will be a black F-string.

\section*{IV. SUMMARY AND DISCUSSION}

In this research, we construct Padmanabhan in thermal BIon and discuss that the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom inside the universe depend on the temperature of BIon, number of branes and the distance between branes. For large separation distance of branes and antibranes as well as the brane’s spike is far from the antibrane’s spike, the role of wormhole is ignorable, however, when the spikes of brane and antibrane comes closer to each other, one thin shell wormhole would be formed. In this condition, there is many channels for flowing energy from extra dimensions into our universe, the bulk degrees of freedom increase and tend to the degrees of freedom of one black F-string in string theory or black M2-brane in M-theory.

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