Cosmic Strings At the Electroweak Phase Transition

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Abstract

If cosmic strings are present at the electroweak phase transition, they can act as seeds on which bubbles of true vacuum nucleate. We explore the nature of such a phase transition, in particular the wall velocity and thickness of the bubbles. From the viewpoint of electroweak baryogenesis, adiabatic conditions exist in the expanding bubble walls, and such models of baryogenesis can be successfully applied. In the present mechanism, the nature of the electroweak phase transition is insensitive to the other details of the model, thus reducing the uncertainties in the estimate of net baryon asymmetry.

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It is considered as established [1] that $B+L$ anomaly of the standard electroweak theory is unsuppressed in the high temperature phase in the early universe. Given the conditions of approximate equilibrium, this implies zero value for net $B + L$ number. Various possibilities still remain open for obtaining the presently observed residue of $n_B/n_\gamma \simeq 10^{-10\pm0.5}$, but the most promising among these is that of generating $B$-asymmetry in the course of electroweak phase transition. Achieving this possibility requires that the electroweak phase transition should be first order [2]. Investigations of the electroweak temperature dependent effective potential indicate the phase transition to be indeed first order. But in considering any given particle physics model for electroweak $B$ generation, one needs details of the phase transition, such as the thickness of the bubble walls and their speed. These questions has been extensively investigated, for instance, in [3], [4].

Here we investigate the effect on the phase transition of the presence of cosmic strings. Many extensions of the Standard Model involve new gauge forces. Spontaneous breaking of such symmetries generically leads to defects such as cosmic strings. It has been shown that cosmic strings can significantly affect the nature of a first order phase transition. This comes about by the nucleation of bubbles of true vacuum on the strings [5]. The bubble nucleation occurs at a precise temperature on all seeding strings and much sooner than is possible by spontaneous tunneling process. In this paper we shall determine the temperature $T_1$ at which bubble nucleation begins on the strings, and the thickness and the speed of the bubble walls. These parameters are determined as a function of the Higgs mass. Top quark mass is unimportant in this mechanism. Using these parameters, we comment on the viability of some of the scenarios proposed for baryogenesis at the electroweak phase transition.
We begin with a brief discussion of the string induced phase transition \[3\]. Let us assume that the Standard Model gauge group \(SU(3)_c \times SU(2)_L \times U(1)_Y\) is the group of the residual symmetry after the breakdown of some larger group \(G\), \(e.g.\) an \(SO(10)\) or supersymmetric \(SU(5)\) unification group, or a Left-Right symmetric model. The breakdown to the Standard Model may involve more than one stage of symmetry breakdown and involve several scalar multiplets. Let us generically designate one of these higgs fields by \(\chi\). For string induced phase transition to occur, it will be sufficient that one such multiplet satisfies the following conditions. Firstly, \(\chi\) must be nontrivially involved in the semiclassical ansatz for the strings, and the strings remain topologically stable upto, but not necessarily through, the electroweak phase transition. And secondly, the quantum numbers of \(\chi\) must allow interaction terms with \(\phi\), the standard Higgs. Let the translation invariant vacuum value \(< \chi >\) be \(M_\chi\). Since \(\chi\) is involved in the semiclassical string configuration in a sector with nonzero winding number, \(< \chi >\) is a function of the radial distance from the string core. Now by the usual device of maintaining hierarchy, the \(\phi - \chi\) coupling terms have to be fine tuned, so that \(< \phi >\) remains zero in the translation invariant vacuum. However in the vicinity of the strings, this hierarchy arrangement will break down and \(< \phi >\) also will be a function of radial distance from the string core, and \(< \phi > \sim M_\chi\) in the string core. This is the key requirement for the occurrence of string induced phase transition. Further details of how this may come about is discussed in \[3\].

Let \(\phi_1\) be the trivial and \(\phi_2\) the nontrivial minima of the temperature dependent effective potential \(V^T[\phi]\) in the electroweak theory. Let \(T_c\) be the temperature at which \(V^T[\phi_1] = V^T[\phi_2]\). We shall define below a dimensionless parameter \(\epsilon \propto (T - T_c)\). We can prove that below a small negative value \(\epsilon_{cr}\) of \(\epsilon\), there is no solution of the
effective action that is time translation invariant and approaches $\phi_1$ large distance away from the string. If such a solution is given as an initial condition for $\epsilon > 0$, it is rendered unstable for $\epsilon < \epsilon_{cr}$. One finds time dependence setting in and obtains solutions representing expanding bubbles of true vacuum. From numerical solutions one can determine $\epsilon_{cr}$, and the velocity of expanding bubbles under ideal conditions, i.e., ignoring the out of equilibrium interaction with the plasma. We here determine these physical parameters as functions of the Higgs mass. In order to do this, we need to cast the temperature dependent effective potential in a convenient form.

The electroweak phase transition can be studied using a high-temperature expansion of the effective potential, which, as shown by Turok and Zadrozny [3] and Anderson and Hall [7] is very reliable in the relevent range of temperatures. The $V_T[\phi]$ has two temperatures of interest

$$T_0^2 = \frac{1}{4D}(m_H^2 - 8Bv_0^2) \quad \text{and} \quad T_c^2 = \frac{T_0^2}{1 - \frac{E^2}{\lambda D}}$$

where, using the known Standard Model data, $D = 0.04 + 0.06(m_t/125GeV)^2$, $E \simeq 0.01$, $v_0 = 246GeV$ and $B \simeq -0.001(m_t/125GeV)^4$, and we assume $\lambda \simeq m_H^2/2v_0^2$.

Details of the parameterisation can be found in [3]. $T_0$ is the temperature at which the symmetric phase becomes an unstable extremum. We rescale:

$$\phi \rightarrow \phi/(\frac{2ET_c}{\lambda}), \quad r \rightarrow (\frac{2ET_c}{\lambda}) r, \quad \text{and} \quad t \rightarrow (\frac{2ET_c}{\lambda}) t \quad (2)$$

Then for any $T$ such that $|T_c - T| \ll T_c$ the equation satisfied by $\phi$ is

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial r^2} - \frac{1}{r} \frac{\partial \phi}{\partial r} + \lambda \phi^3 - \frac{3}{2} \lambda \phi^2 + \frac{\lambda}{2} (1 + \epsilon) \phi = 0 \quad (3)$$

where,

$$\epsilon = \frac{D(T_c^2 - T_0^2)\lambda}{E^2T_c^2} - 1 = \frac{T^2 - T_c^2}{T_c^2 - T_0^2} \quad (4)$$
For our calculation we take $m_t$ to be within the range $100 - 120$ GeV.

We used IMSL subroutine BVPFD for solving the time independent equation. As initial trial, configurations with all $\phi_i = 0$ for grid points with large $r$ were provided. Solutions of class 1 were found to exist for $\epsilon \geq -0.07$. For $\epsilon$ more negative BVPFD fails to return solutions with the correct asymptotic behaviour, as supplied in the initial trial. These configurations are defined unstable. In such cases a solution is always found when a trial with all $\phi_i = \phi_2$ for large $r$ is provided.

The value of $\epsilon_{cr}$ is mildly sensitive to $\lambda[5]$, and hence $m_H$. For $m_H$ in the range 60 to 120 GeV, the critical value varies between $-0.07$ and $-0.09$.

Now to check the nature of time dependence of $\phi$, we solve the time dependent equation (3) using the IMSL subroutine MOLCH. The initial data consisted of the unstable configuration found for each $m_H$ by BVPFD and initial time derivative zero. After initial period of slow evolution, a wall interpolating between the two minima makes its appearance. An example is shown in fig-1.

From above calculations we can obtain wall thickness and velocity. Wall thickness is a parameter that is better understood. When the bubble radius is large, the problem reduces to that of two dimensional soliton theory. Whence thickness

$$\Delta r = \int_{\phi_{in}}^{\phi_{out}} \frac{d\phi}{\sqrt{2V'(\phi)}}$$

(5)

where the limits of integration are the values of $\phi$ inside and outside the bubble. For definiteness we choose the limits $\phi_{in} = 0.75$ and $\phi_{out} = 0.25$. These are the points of inflection of the scaled $V_{\text{eff}}$ curve. We find

$$\Delta r = s(m_H)T^{-1}$$

(6)

where $s(m_H)$ is a dimensionless scaling which we find to vary from $0.7m_H$ /GeV to
$0.5m_H$/GeV as $m_H$ changes from 60 to 120 GeV. This observation is also borne out by the numerically computed graphs of the bubble profile.

The correct estimate of wall velocity is more complicated. We can read it off from a graph, superposing the wall profile at multiple instants of time (fig-1). We find for the entire $m_H$ range 60 - 120 GeV, $v \sim 0.5$. This is in accord with the relativistic detonation bubble wall theory, as discussed by Steinhardt and Turok which predicts relativistic speeds with no significant damping. Since the result is derived from $V_{\text{eff}}^T(\phi)$, this result includes the nondissipative interactions with other particle species. The dissipative interaction that may exist with the surrounding plasma is also subleading in an expansion in $v$ as demonstrated in and . Analytical understanding of the wall velocity in our mechanism is very difficult. In the asymptotic region where the two dimensional soliton theory applies, the forward acceleration of the wall is

$$\dot{v} = \frac{3}{2} \frac{E\epsilon}{\sqrt{2\lambda}} \left| \epsilon \right|$$

where we have restored dimensions to $t$. This is a small acceleration since $E\epsilon \sim 10^{-3}$. Thus the large velocity is the initial condition on the soliton arising from the event during which the bubble was formed. This happens in the region where the $\phi' / r$ term is dominant, making analytic estimates difficult. Any mild source of damping could help the wall to reach a terminal velocity. The terminal velocity is expected to be between 0.1 to 0.5.

$\epsilon_{cr}$ is a measure of the temperature $T_1$ at which the bubble can be considered to have been formed, and the expansion of the wall to have commenced. Using equation (4) we define $T_1$ by

$$\epsilon_{cr} \equiv \frac{T_1^2 - T_c^2}{T_c^2 - T_0^2}$$
Typical numbers for $T_c$, $T_0$ and $T_1$ are $T_c = 125$ GeV, $T_c - T_0 = 1$ GeV and $T_c - T_1 = 0.08$ GeV.

Our general conclusion is that we have bubble formation promptly after $T_c$ is reached and these bubbles have thick walls with relativistic speeds. It can not be overemphasised that although we are considering gauge extensions of the standard model, the bubble shape and velocity are determined by weak scale physics. Similarly it can be seen from equation (3) that the value of $\epsilon_{cr}$ depends if at all, on $\lambda$. And we have checked that $\epsilon_{cr}$ is insensitive to $\lambda$. Thus the promptness of bubble formation (smallness of $|\epsilon_{cr}|$) is also universal. The top quark mass enters parameters that are small compared to those determined by the Higgs mass (in the range in which the high temperature expansion used here is valid), and hence is unimportant to the conclusions presented here.

We have shown that the parameters such as bubble wall thickness and velocity are determined essentially by the Standard Model physics. However the nature of the complete phase transition depends on one additional feature. This is the length per unit volume if the strings that can serve as the nucleation sites. The phase transition begins by the nucleation of the true vacuum bubbles on the strings. If there are sufficiently many such strings present, these bubbles will expand, coalesce and complete the transition. If however the strings are sparse, the transition has to be complemented by the formation of spontaneous bubbles. Given that the strings formed at a temperature $T \sim M_\chi$, there is no simple theory to trace their subsequent evolution and the numerical computations are difficult. For this reason definite predictions along these lines are impossible. However, dimensional analysis combined with intuition regarding collective phenomena suggests that if $M_\chi$ is not greater
than $M_{ew}$ by more than a few orders of magnitude, there will be sufficiently many strings present to complete the phase transition.

From the point of view of baryogenesis, the dynamics of the bubbles is very important. For string induced bubbles to work for baryogenesis, we need a mechanism that will work in thick, fast moving walls. We have seen that the walls are thick $\sim 50$ to $100T_c^{-1}$. These provide adiabatic conditions for baryogenesis. In the criteria of [10] we have

$$\tau_{wall} = \frac{\Delta r}{v} \sim 10^2 - 10^3 T_c^{-1}$$

(9)

whereas equilibrating timescales for Standard Model particles is $\tau_T \sim 10^3 T_c^{-1}$. Thus either the mechanism of McLerran Shaposhnikov Turok and Voloshin [11] or of Cohen Kaplan Nelson [12] can in principle work. The general recipe of spontaneous baryogenesis of Cohen and Kaplan [10], [13] is directly applicable here. We also see that extension of the Higgs sector does not alter the main conclusions reached here so long as at least one Higgs couples to the strings. The extension of the Higgs sector is useful to electroweak baryogenesis for two reasons. Firstly, it provides larger $CP$ violation, as for instance the proposals of [11] and [12] which rely on the two Higgs doublet model. Secondly, the extra Higgs aids the suppression of the washout of baryon asymmetry in the broken symmetry phase. It was shown by Anderson and Hall [7] that simply adding a gauge singlet scalar with a biquadratic coupling with the doublet Higgs substantially weakens the upper bound on the Standard Model Higgs [15]. String induced phase transition easily accommodates such modifications.

In conclusion, the nature and dynamics of string induced bubbles is determined by electroweak physics. We have shown that the conditions in the walls are adiabatic with regard to interaction time scales of the known particles. For electroweak scale
baryogenesis the Standard Model has to be extended to have $B$ or $L$ violation in conjunction with sufficient $CP$ violation. Sphaleronic wash out in the broken phase has to be prevented. The details of such extensions however do not affect the conditions supplied by the walls for baryogenesis. This reduces the uncertainties faced in constructing models of electroweak baryogenesis.

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**Figure caption:**

Figure-1 Time evolution of $\langle \phi \rangle$
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