Polarised parton densities from the fits to the deep inelastic spin asymmetries on nucleons

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Abstract

We have made next to leading order QCD fit to the deep inelastic spin asymmetries on nucleons and we have determined polarised quark and gluon densities. The functional form for such distributions was inspired by the unpolarised ones given by MRST (Martin, Roberts, Stirling and Thorne) fit. In addition to usually used data points (averaged over $Q^2$) we have also considered the sample containing points with the same $x$ and different $Q^2$. Our fits to both groups of data give very similar results. For the integrated quantities we get rather small gluon polarisation. For the non averaged data the best fit is obtained with vanishing gluon contribution at $Q^2 = 1 \text{GeV}^2$. For comparison models with alternative assumptions about quark sea and in particular strange sea behaviour are discussed.
One has quite a lot of data on deep inelastic spin asymmetries for different nucleon targets. The data (recent and old) come from experiments made at SLAC [1-9], CERN [10-15] and DESY [16, 17]. The newest data on proton [8, 15, 17] and deuteron targets [8, 9, 15] have smaller statistical errors and hence dominate in $\chi^2$ fits.

Since the analysis of the EMC group results [10] an enormous interest were started in studying polarised structure functions. It was suggested [18] that polarised gluons may be responsible for the little spin carried by quarks. Determination of polarised gluon distribution is particularly interesting in this context. After calculation of two loop polarised splitting functions [19] several next to leading (NLO) order QCD fits were performed [20-24] and polarised parton distributions (i.e. for quarks, antiquarks and gluons) were determined. Many groups obtained rather highly polarised gluon contribution (however this number is determined with a big error). The aim of this paper is to extend our next to leading order QCD analysis given in [25] by taking into account, in addition to all previously considered data, also proton data from Hermes (DESY) and deuteron data from E155 experiment (SLAC). We will also use more recent MRST fit for partons [24]. In [25] we got negligible gluon contribution at $Q^2 = 1$ GeV$^2$. We will see that our result about gluon polarisation does not change very much. The method of choosing basic functions for fitting, used in this paper, differs from the other groups. As in [25] we will divide the data into two groups. Let us remind that many experimental groups published data [8, 15] sets for the near values of $x$ and different $Q^2$ in addition to the "averaged" data where one averages over $Q^2$ (the errors are smaller and $Q^2$ dependence is smeared out). In principle when we take into account $Q^2$ evolution of polarised and unpolarised functions (in NLO analysis) the first group of data points, i.e. non averaged, should be considered. In most of the fits to experimental data only second group of data, namely with averaged $Q^2$ dependence, was used. We will make fits to the both sets of data (the first group contains 417 points and the second 159 points). The results for both groups of data are very similar (the same conclusion was already drawn in [25]). We will also compare those results with the fits without $Q^2$ evolution taken into account (in other words assuming that asymmetries do not depend on $Q^2$, as in our previous fits [25]). One should add that many experimental groups have not succeeded in finding $Q^2$ dependence for approximately the same value of $x$ and different $Q^2$ [4, 13, 14]. In our analysis we limit ourselves, as one usually does, to
the data with $Q^2 \geq 1\text{GeV}^2$. As was already mentioned in our earlier papers [27] making a fit to spin asymmetries and not directly to $g_1(x, Q^2)$ enables to avoid the problem with higher twist contributions which are probably less important in such case (see for example [28]). Experiments on unpolarised targets provide information on the unpolarised quark densities $q(x, Q^2)$ and $G(x, Q^2)$ inside the nucleon. These densities can be expressed in term of $q^\pm(x, Q^2)$ and $G^\pm(x, Q^2)$, i.e. densities of quarks and gluons with helicity along or opposite to the helicity of the parent nucleon. The unpolarised quark densities are given by the sum of $q^+, q^-$ and $G^+, G^-$, namely:

$$q = q^+ + q^-, \quad G = G^+ + G^-.$$  \hspace{1cm} (1)

On the other hand the polarised DIS experiments give information about polarised parton densities, i.e. the difference of $q^+, q^-$ and $G^+, G^-$:

$$\Delta q = q^+ - q^-, \quad \Delta G = G^+ - G^-.$$  \hspace{1cm} (2)

We will try to determine $q^\pm(x, Q^2)$ and $G^\pm(x, Q^2)$, in other words, we will try to connect unpolarised and polarised data. In principle the asymptotic $x$ behaviour of $q^+$ and $q^-$ will be taken from the unpolarised data (up to the modifications when some leading order terms vanish). We will use the fit given by MRST [26]. In comparison with their previous fit (called R2) [29] there is different behaviour at small $x$ for quark and gluon distributions. It is of course very restrictive assumption that $\Delta q$ and $\Delta G$ have the same behaviour as $q$ and $G$. On the other hand the small $x$ behaviour of unpolarised structure functions is determined from the $x$ values of Hera much smaller than in the polarised case. In our further analysis we will consider the integrals over the region measured in the experiments with polarised particles with the hope that in this case the behaviour of $q^\pm$ and $G^\pm$ could be more plausible. The values of integrals in the whole region ($0 \leq x \leq 1$), involving asymptotic behaviour taken from the unpolarised structure functions, may be not as reliable as in the measured region. As an alternative we also use Regge type behaviour.

It is known [20] that the behaviour of the quark and gluon distributions in small $x$ region is extremely important in extrapolation of integrals over whole $0 \leq x \leq 1$ range. It could happen that in $\Delta q = q^+ - q^-$ (when we assume that $q^+, q^-$ and $q = q^+ + q^-$ have similar $x$ dependence) most singular $x$ terms cancel (that is especially important in case of valence quarks
and sea contributions where the $x$ behaviour is relatively singular). We will see later how such description infers the fits and calculated parameters. For the less singular distributions for $\Delta u_v, \Delta d_v$ and $\Delta M$ (total sea polarisation) there is no strong dependence of calculated quantities on the extrapolation in an unmeasured region but the fits have higher $\chi^2$. One of the main tasks of considering NLO evolution in $Q^2$ is the determination of the gluon contribution $\Delta G$. In $\overline{MS}$ scheme $\Delta G(x, Q^2)$ comes in through the higher order corrections. In our fits we obtain $\Delta G$ relatively small contrary to some expectations. When we use the non-averaged sample of data the actually the best fit is with vanishing $\Delta G$ contribution.

Let us start with the formulas for unpolarised quark parton distributions gotten (at $Q^2 = 1$ GeV$^2$) from the one of recent fits performed by Martin, Roberts, Stirling and Thorne [26]. For the valence quarks one get (in this fit one uses $\Lambda_{\overline{MS}}^{n_f=4} = 0.3$ GeV and $\alpha_s(M_Z^2) = 0.120$):

$$u_v(x) = 0.6051x^{-0.5911}(1 - x)^{3.395}(1 + 2.078\sqrt{x} + 14.56x),$$

$$d_v(x) = 0.0581x^{-0.7118}(1 - x)^{3.874}(1 + 34.69\sqrt{x} + 28.96x),$$

(3)

and for the antiquarks from the sea (the same distribution is for sea quarks):

$$2\bar{u}(x) = 0.4M(x) - \delta(x),$$

$$2\bar{d}(x) = 0.4M(x) + \delta(x),$$

(4)

$$2\bar{s}(x) = 0.2M(x).$$

In eq.(4) the singlet contribution $M = 2[\bar{u} + \bar{d} + \bar{s}]$ is:

$$M(x) = 0.2004x^{-1.2712}(1 - x)^{7.808}(1 + 2.283\sqrt{x} + 20.69x),$$

(5)

whereas the isovector part ($\delta = \bar{d} - \bar{u}$) reads:

$$\delta(x) = 1.290x^{0.183}(1 - x)^{9.808}(1 + 9.987x - 33.34x^2).$$

(6)

For the unpolarised gluon distribution one gets:

$$G(x) = 64.57x^{-0.6829}(1 - x)^{6.587}(1 - 3.168\sqrt{x} + 3.251x).$$

(7)

We assume, in an analogy to the unpolarised case, that the polarised quark distributions are of the form: $x^\alpha(1 - x)^\beta P(\sqrt{x})$, where $P(\sqrt{x})$ is a
polynomial in $\sqrt{x}$ and the asymptotic behaviour for $x \to 0$ and $x \to 1$ (i.e.
the values of $\alpha$ and $\beta$) are the same (except for $\Delta M$, see a discussion below)
as in the unpolarised case. Our idea is to split the numerical constants
(coefficients of $P$ polynomial) in eqs.(3, 5, 6 and 7) in two parts in such a
manner that the distributions $q^\pm(x, Q^2)$ and $G^\pm(x, Q^2)$ remain positive. At
the end of the paper we will discuss the consequences of relaxing the positivity
conditions. Our expressions for $\Delta q(x) = q^+(x) - q^-(x)$ ($q(x) = q^+(x) + q^-(x)$)
are:

$$\Delta u_v(x) = x^{-0.5911}(1 - x)^{3.395}(a_1 + a_2\sqrt{x} + a_4x),$$
$$\Delta d_v(x) = x^{-0.7118}(1 - x)^{3.874}(b_1 + b_2\sqrt{x} + b_3x),$$
$$\Delta M(x) = x^{-0.7712}(1 - x)^{7.808}(c_1 + c_2\sqrt{x}),$$
$$\Delta \delta(x) = x^{-0.183}(1 - x)^{9.808}c_3(1 + 9.987x - 33.34x^2),$$
$$\Delta G(x) = x^{-0.0829}(1 - x)^{6.587}(d_1 + d_2\sqrt{x} + d_3x).$$

It is very important what assumptions one makes about the sea contribution. From the MRST fit for unpolarised structure functions the natural
assumption would be: $\Delta \bar{s} = \Delta \bar{d}/2 = \Delta \bar{u}/2$. If we add the condition that
SU(3) combination: $a_8 = \Delta u + \Delta d - 2\Delta s$ should be equal to the value
determined from the semileptonic hyperon decays, $\Delta s$ is pushed into negative
values and so is nonstrange sea. Instead of connecting $\Delta s$ in some way to
non-strange sea value we introduce additional free parameters for the strange
sea contribution namely

$$\Delta M_s = x^{-0.7712}(1 - x)^{7.808}(c_{1s} + c_{2s}\sqrt{x}).$$

For the strange quarks we have additional independent parameters. Hence, in
our fits we will start with fourteen parameters. Comparing the expression (5)
with (8) and (9) we see that in $\Delta M$ (and $\Delta M_s$) there is no term behaving like
$x^{-1.2712}$ at small $x$ (hence, we assume that $\Delta M$ and hence all sea distributions
have finite integral) which means that in $\Delta M$ coefficient in front of this term
have to be splitted into equal parts in $\Delta M^+$ and $\Delta M^-$ (the most singular
term in the sea contribution drops out). Hence, in the fitting procedure we
are using functions that are suggested by the fit to unpolarised data. Maybe
not all of them are important in the fit and it could happen that some of
the coefficients in eqs.(8,9) taken as free parameters in the fit are small or in
some sense superfluous. Putting them to zero or eliminating them increase
χ² only a little but makes χ²/N_{DF} smaller. We will see that that is the case with some parameters introduced in eqs. (7,8). On the other hand we have still relatively strong singular behaviour of ∆u_v and ∆M for small x values. For comparison we will also consider later the model in which most singular terms are put equal to zero namely a_i = c_i = c_{is} = 0, which means that plus and minus components have the same coefficients for this power of x. In the less singular models the dependence of calculated parameters in the unobserved region (below x ≤ 0.003) is weak. In the earlier papers we considered the extrapolation of various calculated integrals below x = 0.003 up to 0 assuming Regge type of behaviour for small x values. As will be discussed later the less singular models give significantly higher χ².

In order to get the unknown parameters in the expressions for polarised quark and gluon distributions (eqs.(8,9)) we calculate the spin asymmetries starting from initial Q^2 = 1 GeV^2 for measured values of Q^2 and make a fit to the experimental data on spin asymmetries for proton, neutron and deuteron targets. The asymmetry A_{1}(x, Q^2) can be expressed via the polarised structure function g_{1}(x, Q^2) as

\[ A_{1}(x, Q^2) \equiv \frac{(1+\gamma^2)g_{1}(x, Q^2)}{F_{1}(x, Q^2)} = \frac{g_{1}(x, Q^2)}{F_{2}(x, Q^2)}[2x(1+R(x, Q^2))] \] (10)

where R = [F_{2}(1+\gamma^2) - 2xF_{1}]/2xF_{1} whereas F_{1} and F_{2} are the unpolarised structure functions and γ = 2Mx/Q. We will take the new determined value of R from the [30]. The factor (1 + γ^2) plays non negligible role for x and Q^2 values measured in SLAC experiments. Polarised structure function g_{1}(x, Q^2) in the next to leading order QCD is related to the polarised quark and gluon distributions ∆q(x, Q^2), ∆G(x, Q^2), in the following way:

\[ g_{1}(x, Q^2) = \frac{1}{2} \sum_{q} e_{q}^{2} \{ \Delta q(x, Q^2) + \frac{\alpha_{s}}{2\pi}[\delta c_{q} * \Delta q(x, Q^2) + \frac{1}{3}\delta c_{g} * \Delta G(x, Q^2)] \} \] (11)

with the convolution * defined by:

\[ (C * q)(x, Q^2) = \int_{x}^{1} \frac{dz}{z} C(z)q(z, Q^2) \] (12)

The explicit form of the appropriate spin dependent Wilson coefficient δc_{q} and δc_{g} in the MS scheme can be found for example in ref. [19]. The NLO
expressions for the unpolarised (spin averaged) structure function is similar to the one in eq.(11) with $\Delta q(x, Q^2) \to q(x, Q^2)$ and the unpolarised Wilson coefficients are given in [31, 32].

The $Q^2$ evolution of the parton densities is governed by the DGLAP equations [33]. For calculating the NLO evolution of the spin dependent parton distributions $\Delta q(x, Q^2)$, $\Delta G(x, Q^2)$ and spin averaged $q(x, Q^2)$, $G(x, Q^2)$ ones we will follow the method described in [20, 32]. We will calculate Mellin n-th moment of parton distributions $\Delta q(x, Q^2)$ and $\Delta G(x, Q^2)$ according to:

$$\Delta q^{(n)}(Q^2) = \int_0^1 dx x^{n-1} \Delta q(x, Q^2)$$

and then use NLO solutions in Mellin n-moment space in order to calculate evolution in $Q^2$ for non-singlet and singlet parts.

In calculating evolution of $\Delta \Sigma^{(n)}(Q^2)$ and $\Delta G^{(n)}(Q^2)$ with $Q^2$ we have mixing governed by the anomalous dimension 2x2 matrix [32]. Having evolved moments one can insert them into the n-th moment of eq.(11).

$$g_1^{(n)}(Q^2) = \frac{1}{2} \sum_q e_q^2 \{ \Delta q^{(n)}(Q^2) + \frac{a_s}{\pi \alpha_s} [\delta c_q^{(n)} \cdot \Delta q^{(n)}(Q^2) + \frac{1}{3} \delta c_g^{(n)} \cdot \Delta G^{(n)}(Q^2)] \}$$

and then numerically Mellin invert the whole expression. In this way we get $g_1(x, Q^2)$. The same procedure is applied for the unpolarised structure functions. Having calculated the asymmetries according to equation (10) for the measured in experiments value of $Q^2$ we can make a fit to a measured asymmetries on proton, neutron and deuteron targets. We will take into account all together 417 points (193 for proton, 171 for deuteron and 53 for neutron. We will use also the ”experimental” value of $a_8 = 0.58 \pm 0.1$ with enhanced (to 3$\sigma$) error.

The fit with all fourteen parameters from eqs.(8,9) gives $\chi^2 = 340.4$. It seems that some of the parameters of the most singular terms are superfluous and we can eliminate them. We will put $d_1 = d_2 = 0$ (such assumption gives that $\delta G/G \sim x$ for small $x$), $b_1 = 0$ (the most singular term in $\Delta d_v$) and assume $c_{1s} = c_1$ (i.e. the most singular terms for strange and nonstrange sea contributions are equal). Fixing these four parameters in the fit practically does not change $\chi^2$ but improves $\chi^2/N_{DF}$. The resulting $\chi^2$ per degree of freedom is better than in the previous fit and one gets $\chi^2/N_{DF} = \frac{341.1}{418-10}$.
=0.84. In this case we get the following values of parameters from the fit to all existing (above mentioned) data for \( Q^2 \geq 1\text{GeV}^2 \) for spin asymmetries:

\[
\begin{align*}
a_1 &= 0.61 \pm 0.00, & a_2 &= -6.1 \pm 0.19, & a_4 &= 15.7 \pm 0.42, \\
b_2 &= -1.56 \pm 0.20, & b_3 &= -0.43 \pm 0.49, \\
c_1 &= -0.40 \pm 0.03, & c_2 &= 4.15 \pm 0.00, \\
c_{1s} &= c_1, & c_{2s} &= -0.28 \pm 0.83, \\
c_3 &= -1.29 \pm 2.53, \\
d_3 &= 2.01 \pm 11.2.
\end{align*}
\] (15)

Actually the parameter \( d_3 \) could be put equal to zero without increasing \( \chi^2/N_{DF} \). We get in this case the smallest \( \chi^2/N_{DF} = \frac{341.418}{418} = 0.83 \). That means that \( d_3 \) is not well determined in the fit and the best \( \chi^2/N_{DF} \) is without gluonic contribution.

The obtained quark and gluon distributions lead for \( Q^2 = 1 \text{GeV}^2 \) to the following integrated (over \( x \)) quantities:

\[
\begin{align*}
\Delta u &= 0.80 \pm 0.02, & \Delta d &= -0.65 \pm 0.03, & \Delta s &= -0.21 \pm 0.05, \\
\Delta u_v &= 0.67 \pm 0.02, & \Delta d_v &= -0.59 \pm 0.02, & 2\Delta \bar{u} &= 0.14 \pm 0.03, & 2\Delta \bar{d} &= -0.07 \pm 0.03.
\end{align*}
\]

These numbers yield the following predictions:

\[
\begin{align*}
a_0 &= \Delta u + \Delta d + \Delta s = -0.06 \pm 0.07, & a_3 &= \Delta u - \Delta d = 1.45 \pm 0.02, & \Delta G &= 0.04 \pm 0.19, & \Gamma_p^u &= 0.111 \pm 0.006, & \Gamma_1^u &= -0.096 \pm 0.006, & \Gamma_1^d &= 0.007 \pm 0.005.
\end{align*}
\]

We have positively polarised sea for up and negatively for down quarks and very strongly negatively polarised sea for strange quarks. Because of the big negative value of \( \Delta s \) the quantity \( a_0 \) is negative. The gluon polarisation is small. The value of \( a_3 \) was not assumed as an input in the fit (as is the case in nearly all fits [23]) and comes out slightly higher than the experimental value. \( \Delta \delta \), comes out relatively big from the fit (coefficient in front of \( \Delta \delta \) is equal to that in \( \delta \)).

The asymptotic behaviour at small \( x \) of our polarised quark distributions is determined by the unpolared ones and these do not have the expected theoretically Regge type behaviour or pQCD which is also used by some groups, to extrapolate results to small values of \( x \). Some of the quantities in our fit change rapidly for \( x \leq 0.003 \).

Hence, we will present quantities integrated over the region from \( x=0.003 \) to \( x=1 \) (it is practically integration over the region which is covered by the experimental data, except of non controversial extrapolation for highest \( x \)). The corresponding quantities are \( \Delta u = 0.85 \) (\( \Delta u_v = 0.56, 2\Delta \bar{u} = 0.29 \),
\[ \Delta d = -0.48 \ (\Delta d_v = -0.57, \ 2\Delta \bar{d} = 0.09), \ \Delta s = -0.12, \ \alpha_0 = 0.25, \ \Delta G = 0.04, \ \Gamma_1^p = 0.123, \ \Gamma_1^n = -0.056, \ \Gamma_1^d = 0.036, \ \alpha_3 = 1.32. \text{ In this region the obtained values of sea contributions are relatively high and those of valence quarks relatively small. Gluon contribution practically vanishes. There is relatively strong dependence of different quantities in the unmeasured region (0 \leq x \leq 0.003). May be the unpolarised MRST parton distributions (with the above mentioned modifications) do not describe quite correctly the small x behaviour of polarised parton distributions. On the other hand the fit to the data is very good. So, the values of integrated quantities in the measured region we consider as more reliable then in the whole region. With the value of } \Delta s = -0.12 \text{ in the measured region of } x \text{ we have } \alpha_0 = 0.25 \text{ and with } \Delta s = -0.21 \text{ in the whole region of } x \text{ } \alpha_0 \text{ becomes negative (-0.06).}

\text{When we use the quantities calculated in the measured region and extend them to the full } x \text{ region using asymptotic Regge behaviour for small } x \text{ we get } \Delta u = 0.86 \ (\Delta u_v = 0.59, \ 2\Delta \bar{u} = 0.27), \ \Delta d = -0.51 \ (\Delta d_v = -0.58, \ 2\Delta \bar{d} = 0.07), \ \Delta s = -0.14, \ \alpha_0 = 0.21, \ \Delta G = 0.04, \ \alpha_3 = 1.37. \text{ We have used } x^{-\alpha} \text{ behaviour for small } x \text{ (with -0.25 \leq \alpha \leq 0.25) and the quantities do not depend strongly on a specific value of } \alpha. \text{ For the given above values } \alpha = 0 \text{ was used.}

![Graph](image.png)

**Figure 1:** The comparison of our predictions for \( g_1^N(x,Q^2)/F_1^N(x,Q^2) \) from basic fit with the recent averaged data for proton and deuteron targets.

Now, we shall calculate \( \Gamma^p, \ \Gamma^n \) and \( \Gamma^d \) in the measured region for \( Q^2 = 5 \text{ GeV}^2 \) and compare them with the quantities given by the experimental groups. We get in the region between \( x = 0.003 \) and \( x = 0.8 \) (covered by the
data) $\Gamma_p = 0.132 \pm 0.006$, $\Gamma_n = -0.051 \pm 0.007$ and $\Gamma_d = 0.037 \pm 0.006$. The experimental group SMC present \cite{22} the following values in such region (for $Q^2 = 5$ GeV$^2$):

$$
\begin{align*}
\Gamma_p &= 0.130 \pm 0.007, \\
\Gamma_n &= -0.054 \pm 0.009, \\
\Gamma_d &= 0.036 \pm 0.005.
\end{align*}
$$

One can see that our results are in good agreement with experimental values. For comparison we have also made fits using formulas of the simple parton model (as in our papers before \cite{27}) neglecting evolution of parton densities with $Q^2$. More detailed result of these fits (integrated densities and so on) will be given later.

In fig.1 as an example we present our fit to the non averaged data in comparison with measured (averaged over $Q^2$) $g_1/F_1$ for new proton (Hermes) and deuteron (E155) data. The curves are obtained by joining the calculated values of asymmetries corresponding to actual values of $x$ and $Q^2$ for measured data points. The curves are not fitted but the difference in fitted asymmetries for averaged and non-averaged data are very small. For asymmetries the curves with $Q^2$ evolution taken into account and evolution completely neglected do not differ very much so we do not present them.

In figs. 2 and 3 we show the comparison of our predictions for $g_1$ from the basic fit with the measured averaged values for proton, deuteron and neutron data. The values of $g_1$ were calculated for the values of $x$ and $Q^2$ measured for averaged data points in different experiments and then joined together. The agreement is good. On the other hand the spread of experimental points is still substantial. Polarised quark distributions for up and down valence quarks as well as non strange, strange quarks and gluons for $Q^2 = 1$ GeV$^2$ are presented in figure 4. Dashed curves represent the $+$ and $-$ components for different parton densities. the solid curves correspond to the difference of $+$ and $-$ components, the sums of components (not shown) correspond to non-polarised parton distributions. We see that especially polarised gluon distribution function is really tiny and does not resemble the distribution function for unpolarised case.

This function is also quite different from the gluon distribution (given in \cite{34}) used to estimate $\Delta G/G$ in COMPASS experiment planned at CERN \cite{35}. For $x = 0.1$ at $Q^2 = 1$ GeV$^2$ $\Delta G/G = 0.01$ and is below a planned
Figure 2: The comparison of our predictions for $g_1^N(x, Q^2)$ with the measured structure functions in different experiments and on different nucleon targets.
Figure 3: The comparison of our predictions for $g_1(x,Q^2)$ calculated from basic fit with the measured structure functions in E142, E155 and Hermes experiments.
Figure 4: Our predictions for spin densities for quark and gluons at $Q^2 = 1 \text{GeV}^2$. We present distributions for partons polarized along $(xq^+(x), xG^+(x))$ and opposite $(xq^-(x), xG^-(x))$ to the helicity of parent proton. The polarized densities (i.e. the differences of above mentioned quantities) are also shown.
experimental error. In fig.5 we present spin densities for $u$ quark. We show $\Delta u(x, Q^2)$ obtained in the basic fit (solid line) as well as the same quantity from the fit with $SU(3)$ symmetric sea (dashed curve) and from the fit where evolution in $Q^2$ was not taken into account (dotted line). In the second plot the same curves for $\Delta u(x, Q^2)$ are presented and one sees from that the values of $\Delta u$ from the basic fit and fit with $SU(3)$ symmetric sea are very close in spite of the differences in the valence values. The similar conclusions are also true for the $d$ quark densities.

![Figure 5: Our predictions for $x\Delta u_v(x, Q^2 = 1 \text{ GeV}^2)$ and $x\Delta u(x, Q^2 = 1 \text{ GeV}^2)$ in different models. The solid curve is gotten using the parameters of our basic fit whereas dashed and dotted lines correspond to the quantities calculated in the fit with a $SU(3)$ symmetric sea and a fit without QCD evolution, respectively.](image)

Fixing the value of $a_8$ is very important for the fit. When we relax the condition for $a_8 = 0.58$ we get $\chi^2 = 340.8$, so this number practically does not change. We get the fit with the parameters not very different from our basic fit but with very small $a_8 = 0.03$ and positive $\Delta s = 0.01$. It seems that $\Delta s$ is not well determined from the data on spin asymmetries alone but that does not influence strongly the values of $\Delta u$, $\Delta d$ and $\Delta G$.

In order to check how the fit depends on the assumptions made about the sea contribution we have also made fit with $\Delta \bar{u} = \Delta \bar{d} = 2\Delta s$, the assumption that follows directly from MRST unpolarised fit. The $\chi^2$ value increases significantly and per degree of freedom one gets a number $\chi^2/N_{DF} = \frac{353.2}{418-9} = 0.86$ which is worse than in our basic fit. In this case we have $\Delta u = 0.80$ ($\Delta u_v = 0.87$, $2\Delta \bar{u} = -0.07$), $\Delta d = -0.61$ ($\Delta d_v = -0.40$, $2\Delta \bar{d} = -0.21$), $\Delta s = -0.07$, $a_0 = 0.11$, $\Delta G = 0.07$ and $a_8 = 0.33$. The quantity $\Delta s$ must
be negative in order to get experimental value for $a_s$ and because of our assumption $\Delta \bar{u} = \Delta \bar{d} = 2 \Delta \bar{s}$ (the value of $a_s$ does not come out correctly in the fit because of our assumption $c_{1s} = c_1$) we obtain negative values of non strange sea for up and down quarks. One sees that the values for sea polarisation depend very strongly on assumptions we made (in many papers [20, 21, 34] SU(3) symmetric sea is assumed that also together with fixing of $a_s$ value gives negative non strange sea). On the other hand $\Delta u = \Delta u_v + 2 \Delta \bar{u}$ and $\Delta d = \Delta d_v + 2 \Delta \bar{d}$ practically do not change (however, $\Delta u_v$ and $\Delta d_v$ also change). Also $\Delta G$ does not change and is small. We get that the value of $a_s$ is not coming out correctly from the fit to spin asymmetries. Fixing $a_s$ and making specific assumption about $\Delta s$ introduces shifts in nonstrange sea polarisation (and so in $\Delta u_v$ and $\Delta d_v$) but $\Delta u, \Delta d$ do not change. Because the value of $\Delta s$ is needed to determine $a_0$ we decided to use additional free parameters for strange sea contribution in order to determine it (with fixed value of $a_s$) from the fit to experimental data.

In many papers making fits to experimental data on spin asymmetries the assumption of SU(3) symmetric sea was made. We have also for comparison made fit with this assumption. The value of $\chi^2 = 350.6$ is substantially higher in comparison with our basic fit. In this case we have $\Delta u = 0.81$ ($\Delta u_v = 0.87, 2 \Delta \bar{u} = -0.06$), $\Delta d = -0.57$ ($\Delta d_v = -0.40, 2 \Delta \bar{d} = -0.17$), $\Delta s = -0.11$, $a_0 = 0.13$, $\Delta G = 0.14$, $a_3 = 1.38$ and $a_8 = 0.47$. In this case we also have shifts in values of valence and sea contributions similar to the case discussed above.

Looking at the dependence of unpolarised quark and gluon densities we see that after elimination of most singular term in $\Delta d_v(x)$ the most singular behaviour for small $x$ one has for $\Delta u_v(x)$ and $\Delta M(x)$. For comparison we have investigated the model when in polarised densities these singular contributions are absent. In this case $\Delta u_v$ and $\Delta M$ are $\sqrt{x}$ less singular than in our basic fit. For such a fit we get $\chi^2/N_{DF} = \frac{356.6}{418-8} = 0.87$, i.e significantly higher than in our basic fit. We get in this case: $\Delta u = 0.77$ ($\Delta u_v = 0.57, 2 \Delta \bar{u} = 0.20$), $\Delta d = -0.38$ ($\Delta d_v = -0.63, 2 \Delta \bar{d} = 0.25$), $\Delta s = -0.10$, $a_0 = 0.28$, $\Delta G = 0.22$. In such fit the integrated quantities taken over the whole range of $0 \leq x \leq 1$ and in the truncated one ($0.003 \leq x \leq 1$) differ very little. The quantity $\Delta G$ is positive and different from zero. So it is possible to get the fit with practically no change of integrated quantities in the region between $x = 0$ and $x = 0.003$ but with significantly higher $\chi^2$ value. For $Q^2 = 1$ GeV$^2$ we have $\Gamma^p_1 = 0.122$ and $\Gamma^n_1 = -0.041$. 

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The obtained results can be compared with the fit when instead of 417 points for different $x$ and $Q^2$ values we take spin asymmetries for only 160 data points with the averaged $Q^2$ values for the same $x$ (one has smaller errors in this case). In such fit the ratio of number of neutron to number of deuteron and proton data points is increased. It seems that the influence of neutron points is stronger than in basic fit $\chi^2/N_{DF} = \frac{118.3}{161-10} = 0.78$ is a little bit better than in our basic fit. The integrated values for quark and gluon densities are: $\Delta u = 0.79$ ($\Delta u_v = 0.65$, $2\Delta \bar{u} = 0.14$), $\Delta d = -0.66$ ($\Delta d_v = -0.60$, $2\Delta \bar{d} = -0.06$), $\Delta s = -0.22$, $a_0 = -0.09$, $\Delta G = 0.31$ and $a_3 = 1.45$. We see that averaging over $Q^2$ and different numbers of data points leads to very similar fit. The values for integrated valence densities and nonstrange sea contribution are only a bit shifted ($\Delta u$ and $\Delta d$ in the whole region of integration do not differ from the basic fit for non averaged data and the same is also true for integrated quantities in the region $0.003 \leq x \leq 1$). Integrated gluon density is relatively small and positive. A little bit higher value for $\Delta G = 0.31 \pm 0.28$ we do not consider as significant difference. $\chi^2/N_{DF}$ is very good and smaller then in [24] where the same experimental data sample and MRST parton distributions (modified for small values of $x$) were used.

As was already mentioned before we have also made for comparison fits neglecting evolution of parton densities with $Q^2$ (formulas from the simple parton model). We get for non averaged data sample $\chi^2/N_{DF} = \frac{349.9}{118-9} = 0.86$ (biger than in our basic fit: $\chi^2/N_{DF} = 0.84$): $\Delta u = 0.66$ ($\Delta u_v = 0.56$, $2\Delta \bar{u} = 0.10$), $\Delta d = -0.49$ ($\Delta d_v = -0.49$, $2\Delta \bar{d} = 0.0$), $\Delta s = -0.20$, $a_0 = -0.03$, $a_3 = 1.14$, $\Gamma_1^{\bar{p}} = 0.108$, $\Gamma_1^{n} = -0.082$. For averaged data points we get $\chi^2/N_{DF} = \frac{125.4}{161-9} = 0.83$ (this number should be compared with $\chi^2/N_{DF} = 0.78$, the corresponding quantity from the NLO fit) and we have: $\Delta u = 0.66$ ($\Delta u_v = 0.58$, $2\Delta \bar{u} = 0.08$), $\Delta d = -0.48$ ($\Delta d_v = -0.48$, $2\Delta \bar{d} = 0.0$), $\Delta s = -0.20$, $a_0 = -0.03$. Hence, $\chi^2$ per degree of freedom is smaller in the case of averaged sample. We see that both fits give very similar results. It means that the averaging of data does not influence the fit when we do not take $Q^2$ evolution into account (the differences are also very small in the $0.003 \leq x \leq 1$ region).

It has been pointed out [21] that the positivity conditions could be restrictive and influence the contribution of polarised gluons. We have also made a fit to experimental data without such assumption for polarised par-
tons. The $\chi^2$ value does not changed much $\chi^2/N_{DF} = \frac{340.7}{118-10} = 0.84$ and we get $\Delta u = 0.84$ ($\Delta u_v = 0.72$, $2\Delta \bar{u} = 0.12$), $\Delta d = -0.74$ ($\Delta d_v = -0.50$, $2\Delta \bar{d} = -0.24$), $\Delta s = -0.24$, $a_0 = -0.13$, $a_3 = 1.57$, $\Delta G = 0.02$. The results are a little bit different but the value of $\Delta G$ is not influenced by the positivity conditions. The same is also true in the case of averaged data. It seems that our positivity conditions are not very restrictive.

We have made fits for two samples of data with averaged $Q^2$ values and non averaged ones (adding neutron data from E154 and Hermes experiments) leading to very similar results for calculated parameters (except small difference in $\Delta G$). The value of $a_3$ was not fixed in the fit and comes out high in comparison with experimental value. In order to check the influence of different assumptions about strange sea we considered fits without fixing $a_8$ value, with $SU(3)$ symmetric sea and with modified sea contribution. The models with less singular behaviour for valence $u$ quark and sea contribution were also discussed. In most of the modifications the $\chi^2$ value increases significantly. For comparison we have also considered fits to the simple parton model neglecting $Q^2$ dependence of parton densities. It seems that splitting of integrated densities $\Delta u$, $\Delta d$ into valence and sea contribution is model dependent ($\Delta u$ and $\Delta d$ do not differ much). The integrated gluon contribution comes out small. The best fits (measured by $\chi^2$ per degree of freedom) we have for zero (for non averaged data points) or rather small (for averaged data) gluon polarisation. It seems that from results of our fits the perspective of measuring in COMPASS $\Delta G/G$ is not very encouraging. The experimental accuracy still must be improved and probably additional experiments are needed in order to make more precise statements about polarised quark and gluon densities.
References

[1] M.J.Alguard et al., Phys. Rev. Lett. 37, 1261 (1976); Phys. Rev. Lett. 41, 70 (1978);

[2] G.Baum et al., Phys. Rev. Lett. 45, 2000 (1980); 51, 1135 (1983);

[3] E142 Collaboration, P.L.Anthony et al., Phys. Rev. Lett. 71, 959 (1993); Phys. Rev. D 54, 6620 (1996);

[4] E143 Collaboration, K.Abe et al., Phys. Rev. Lett. 74, 346 (1995);

[5] E143 Collaboration, K.Abe et al., Phys. Rev. Lett. 75, 25 (1995);

[6] E154 Collaboration, K.Abe et al., Phys. Rev. Lett. 79, 26 (1997); Phys. Lett. B 405, 180 (1997);

[7] E143 Collaboration, K.Abe et al., Phys. Lett. B 364, 61 (1995);

[8] E143 Collaboration, K.Abe et al., Phys. Rev. D 58, 112003 (1998);

[9] E155 Collaboration, P.L.Anthony et al., Phys. Lett. B 463, 339 (1999);

[10] European Muon Collaboration, J.Ashman et al., Phys. Lett. B 206, 364 (1988); Nucl. Phys. B 328, 1 (1989);

[11] Spin Muon Collaboration, B.Adeva et al., Phys. Lett. B 302, 533 (1993); D.Adams et al., Phys. Lett. B 357, 248 (1995);

[12] Spin Muon Collaboration, D.Adams et al., Phys. Lett. B 329, 399 (1994); B.Adeva et al., Phys. Lett. B 412, 414 (1997);

[13] Spin Muon Collaboration, D.Adams et al., Phys. Rev. D 56, 5330 (1997);

[14] Spin Muon Collaboration, D.Adams et al., Phys. Lett. B 396, 338 (1997);

[15] Spin Muon Collaboration, B.Adeva et al., Phys. Rev. D 58, 112001 (1998);
[16] Hermes Collaboration, K.Ackerstaff et al., Phys. Lett. B 404, 383 (1997);

[17] Hermes Collaboration, A.Airapetian et al., Phys. Lett. B 442, 484 (1998);

[18] A.V.Yefremov, O.V.Teryaev, Dubna Report No. JIN-E2-88-287 (1988);
G.Altarelli, G.G.Ross, Phys. Lett. B 212, 391 (1988); R.D.Carlitz, J.D.Collins, A.H.Mueller, Phys. Lett. B 214, 219 (1988);

[19] R.Mertig, W.L.van Neerven, Zeit. f. Phys. C 70, 637 (1996);
W.Vogelsang, Phys. Rev. D 54, 2023 (1996); Nucl. Phys. B 475, 47 (1996);

[20] M.Glick, E.Reya, M.Stratman, W.Vogelsang, Phys. Rev. D 53, 4775 (1996);

[21] G.Altarelli, R.D.Ball, S.Forte, G.Ridolfi, Acta Phys. Pol. B 29, 1145 (1998);

[22] Spin Muon Collaboration, B.Adeva et al., Phys. Rev. D 58, 112002 (1998);

[23] T.Gehrmann, W.J.Stirling, Phys. Rev. D53, 6100 (1996); G.Altarelli, R.D.Ball, S.Forte, G.Ridolfi, Nucl. Phys. B 496, 337 (1997); C.Bourrely, F.Buccella, O.Pisanti, P.Santorelli, J.Soffer, Prog. Theor. Phys. 99, 1017 (1998); E.Leader, A.V.Sidorov, D.B.Stamenov, Int. J. Mod. Phys. A 13, 5573 (1998);

[24] E.Leader, A.V.Sidorov, D.B.Stamenov, Phys. Rev. D58, 114028 (1998);

[25] S.Tatur, J.Bartelski, M.Kurzela; Acta Phys. Pol. B 31, 647 (2000);

[26] A.D.Martin, W.J.Stirling, R.G.Roberts, R.S.Thorne; Eur. Phys. J. C 4, 463 (1998);

[27] J.Bartelski, S.Tatur, Acta Phys. Pol. B 26, 913 (1994); J.Bartelski, S.Tatur, Zeit. f. Phys. C 71, 595 (1996); J.Bartelski, S.Tatur, Acta Phys. Pol. B 27, 911 (1996); J.Bartelski, S.Tatur Zeit. f. Phys. C 75, 477 (1997);
[28] E. Stein et al., Phys. Lett. B 343, 369 (1995);

[29] A.D. Martin, W.J. Stirling and R.G. Roberts, Phys. Lett. B 354, 155 (1995);

[30] E143 Collaboration, K. Abe et al., Phys. Lett. B 452, 194 (1999);

[31] E.G. Floratos, C. Kounnas, R. Lacaze, Nucl. Phys. B 192, 417 (1981);

[32] M. Glück, E. Reya, A. Vogt, Zeit. f. Phys. C 48, 471 (1990);

[33] V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972), 15, 675 (1972); G. Altarelli, G. Parisi, Nucl. Phys. B 126, 298 (1977); Yu.L. Dokshitzer, Sov. Phys. JETP 46, 647 (1977);

[34] T. Gehrmann, W.J. Stirling, Zeit. f. Phys. C 65, 461 (1995);

[35] J.P. Nassalski, Acta Phys. Pol. B 29, 1315 (1998).