A fuzzy feedback linearization scheme applied to vibration control of a smart structure

Roberta Varela de Albuquerque Herôncio, João Deodato Batista dos Santos, Wallace Moreira Bessa, Aline Souza de Paula, Marcelo Amorim Savi

Resumo

Smart structures are usually designed with a stimulus-response mechanism to mimic the autoregulatory process of living systems. In this work, in order to simulate this natural and self-adjustable behavior, a fuzzy feedback linearization scheme is applied to a shape memory two-bar truss. This structural system exhibits both constitutive and geometrical nonlinearities presenting the snap-through behavior and chaotic dynamics. On this basis, a nonlinear controller is employed for vibration suppression in the chaotic smart truss. The control scheme is primarily based on feedback linearization and enhanced by a fuzzy inference system to cope with modeling inaccuracies and external disturbances. The overall control system performance is evaluated by means of numerical simulations, promoting vibration reduction and avoiding snap-through behavior.

I. INTRODUCTION

The term smart structures and systems has been used to identify mechanical systems that are capable of changing their geometry or physical properties with the purpose of performing a specific task. They must be equipped with sensors and actuators that induce such controlled alterations. Several applications in different fields of sciences and engineering have been developed with this innovative idea, employing some of the so-called smart materials. Shape memory alloys (SMAs), piezoelectric materials and magneto-rheological fluids are some of the smart materials largely employed in structural systems.

Specifically, shape memory alloys are being used in situations where high force, large strain, and low frequency structural control are needed. SMA actuators are easy to manufacture, relatively lightweight, and able of producing high forces or displacements. Self-actuating fasteners, thermally actuator switches and several bioengineering devices are some examples of these SMA applications. Aerospace technology are also using SMAs for distinct purposes as space savings achieved by self-erectable structures, stabilizing mechanisms, non-explosive release devices, among others. Micromanipulators and robotics actuators have been built employing SMAs properties to mimic the smooth motions of human muscles. Moreover, SMAs are being used as actuators for vibration and buckling control of flexible structures.

SMA thermomechanical behavior is related to thermoelastic martensitic transformations. The shape memory effect is a phenomenon where apparent plastically deformed objects may recover their original form after going through a proper heat treatment. The pseudelastic behavior is characterized by complete strain recovery accompanied by large hysteresis in a loading-unloading cycle (Otsuka and Ren (1999)). Fibers of shape-memory alloys can be used to fabricate hybrid composites exhibiting these two different but related material behaviors. Detailed description of the shape memory effect and other phenomena associated with martensitic phase transformations, as well as examples of applications in the context of smart structures, may be found in references (Lagoudas, 2008; Paiva and Savi, 2006; Machado and Savi, 2003; Rogers, 1995; Shaw and Kyriakides, 1995). The investigation of SMA structures has different approaches. The finite element method is an important tool to this aim. Auricchio and Taylor (1998) proposed a three-dimensional finite element model, Lagoudas et al. (1997) considered the thermomechanical response of a laminate with SMA strips, La Cava et al. (2004) considered SMA bars and Bandeira et al. (2006) treated truss structures. The response of SMA beams was treated by Collet et al. (2001), which analyzes the dynamical response, as well as Auricchio and Sacco (1999), Auricchio and Petrin (2004a,b). Presented a solid finite element to describe the thermo-electro-mechanical problem that is used to simulate different SMA composite applications. Dual kriging interpolation has been employed with finite element method in order to describe the shape memory behavior in different reports (Trocha et al. (1999), Masud et al. (1997), Bhattacharyya et al. (2000), Liew et al. (2002) are other contributions in this field.

The two-bar truss, also known as the von Mises truss, is an important archetypal model, largely employed to evaluate stability characteristics of framed structures as well as of flat arches, and of many other physical phenomena associated with bifurcation buckling (Hataz and Cedolin, 2010). The nonlinear dynamics of this system may exhibit a number of interesting, complex behaviors. The snap-through behavior, represented by a displacement jump, is a classical example of the complexity behind this simple structure.

The dynamic behavior of the two-bar truss is even richer when material nonlinearities are considered. In particular, the present contribution deals with two-bar trusses made from shape memory materials. Savi and Nogueira (2010) and Savi et al. (2002) presented numerical investigations of this kind of structure by considering different constitutive models to describe the thermomechanical behavior of the SMAs.

Due to its simplicity, feedback linearization scheme is commonly applied in industrial control systems, specially in the field of industrial robotics. The main idea behind this control method is the development of a control law that allows the transformation of the original dynamical system into an equivalent but simpler one (Slotine and Li, 1991). Although feedback linearization represents a very simple approach, an important handicap is the requirement of a perfectly known dynamical system, in order to ensure the exponential convergence of the tracking error.

Intelligent control, on the other hand, has proven to be a very attractive approach to cope with uncertain nonlinear systems (Bessa 2005; Bessa and Bessa 2005; Bessa et al. 2008, 2012, 2014, 2017, 2018, 2019; Dos Santos and Bessa, 2019; Lima et al., 2018, 2020, 2021). By combining nonlinear control techniques, such as feedback linearization or sliding modes, with adaptive intelligent algorithms, for example fuzzy logic or artificial neural networks, the resulting intelligent control strategies can deal with the nonlinear characteristics as well as with modeling imprecisions and external disturbances that can arise.
On this basis, much effort has been made to combine feedback linearization with intelligent algorithms in order to improve the trajectory tracking of uncertain nonlinear systems. On this basis, in order to enhance the tracking performance, Janaka et al. (2013) used a fuzzy inference system with the state variables in the premise of the rules to approximate the unknown system dynamics. However, the adoption of all state variables in the premise of the rules is a drawback of this approach. As for instance, for higher-order systems the number of fuzzy sets and fuzzy rules becomes incredibly large, which compromises the applicability of this technique.

In this work, to reduce the number of fuzzy sets and rules and consequently simplify the design process, only on variable (an error measure) instead of the state variables, is adopted in the premise of the fuzzy rules. A polynomial constitutive model is assumed to describe the behavior of the shape memory bars. Although this model is simple and does not present a proper description of the hysteretic behavior, it can qualitatively represent the general SMA behavior. This system has a rich dynamic response and can easily reach a chaotic behavior even at moderate loads and frequencies (Savi et al. 2002). Numerical simulations are carried out in order to demonstrate the control system performance.

The main goal is the vibration reduction, avoiding some critical responses as snap-through behavior. A linear actuator is employed to help this control procedure and therefore, the SMA actuation is not employed for the control purposes. In this regard, we are investigating an SMA structure that needs an appropriate control using external actuators. It is important to highlight that SMA properties are being used to achieve other goals than control. This situation is common in distinct applications that include aerospace systems as self-erectable structures.

II. DYNAMIC MODEL

The two-bar truss is depicted in Figure 1. This plane, framed structure, is formed by two identical bars, free to rotate around their supports and at the joint.

In the present investigation, we consider a shape memory two-bar truss where each bar presents the shape memory and pseudoelastic effects. The two identical bars have length \( L \) and cross-sectional area \( A \). They form an angle \( \varphi \) with a horizontal line and are free to rotate around their supports and at the joint, but only on the plane formed by the two bars (Figure 1). The critical Euler load of both bars is assumed to be sufficiently large so that buckling will not occur in the simulations reported here.

We further assume that the structure’s mass is entirely concentrated at the junction between the two bars. Hence, the structure is divided into segments without mass, connected by nodes with lumped mass that is determined by static considerations. We consider only symmetric motions of the system, which implies that the concentrated mass, \( m \), can only move vertically. The symmetric, vertical displacement is denoted by \( X \). Under these assumptions, the dynamic behavior is expressed through the following equation of motion

\[
-2F \sin \varphi - cX + P = m\ddot{X}
\]  

(1)

where \( F \) is the force on each bar, \( P \) is an external force and \( cX \) is a linear viscous damping term used to represent all dissipation mechanisms.

There are several works dedicated to the constitutive description of the thermomechanical behavior of shape memory alloys (Lagoudas 2008; Paiva and Savi 2006). In this article, we employ polynomial constitutive model to describe the thermomechanical behavior of the SMA bars (Falk 1980; Muller 1991). Despite the simplicity of this model, it allows an appropriate qualitative description of the dynamical response of the system. Its major drawback is the hysteresis description. In this regard, for control purposes, it should be appropriate with robust controllers that could deal with unmodelled dynamics. Here, dissipation process is represented by an equivalent viscous damping term.

Polynomial model is concerned with one-dimensional media employing a sixth degree polynomial free energy function in terms of the uniaxial strain, \( \varepsilon \). The form of the free energy is chosen in such a way that its minima and maxima are respectively associated with the stability and instability of each phase of the SMA. As it is usual in one-dimensional models proposed for SMAs (Savi and Braga 1993), three phases are considered: austenite (A) and two variants of martensite (M+, M-). Hence, the free energy is chosen such that for high temperatures it has only one minimum at vanishing strain, representing the equilibrium of the austenitic phase. At low temperatures, martensite is stable, and the free energy must have two minima at non-vanishing strains. At intermediate temperatures, the free energy must have equilibrium points corresponding to both phases. Under these restrictions, the uniaxial stress, \( \sigma \), is a fifth-degree polynomial of the strain (Savi and Braga 1993), i.e.

\[
\sigma = a_1(T - T_M)\varepsilon - a_2\varepsilon^3 + a_3\varepsilon^5
\]  

(2)

where \( a_1, a_2 \) and \( a_3 \) are material constants, and \( T \) the temperature, while \( T_M \) is the temperature below which the martensitic phase is stable. If \( T_A \) is defined as the temperature above which austenite is stable, and the free energy has only one minimum at zero strain, it is possible to write the following condition,

\[
T_A = T_M + \frac{1}{4} \frac{a_2^2}{a_1a_3}
\]  

(3)
Therefore, the constant \( a_3 \) may be expressed in terms of other constants of the material. Now, the following strain definition is considered,

\[
\varepsilon = \frac{L}{L_0} - 1 = \frac{\cos \varphi_0}{\cos \varphi} - 1
\]

with \( L_0 \) and \( \varphi_0 \) representing the nominal values of \( L \) and \( \varphi \), respectively.

At this point, we can use the constitutive equation (2) together with kinematic equation (4) into the equation of motion (1), obtaining the governing equation of the SMA two-bar truss:

\[
m\ddot{x} + c\dot{x} + \frac{2A}{L_0} X\left\{a_1(T - T_M) - 3a_2 + 5a_3\right\} + \left[-a_1(T - T_M) + a_2 - a_3\right] L_0 (x^2 + B^2)^{-1/2} + \] 
\[+ \left[3a_2 - 10a_3\right] \frac{1}{L_0} (x^2 + B^2)^{1/2} + \left[-a_2 + 10a_3\right] \frac{1}{L_0} (x^2 + B^2) + \] 
\[\left[-5a_3\right] \frac{1}{L_0} (x^2 + B^2)^{3/2} + \] 
\[- \frac{a_3}{L_0} (x^2 + B^2)^2 = P(t) \]

where \( B \) is the horizontal projection of each truss bar (Figure 1).

Considering a periodic excitation \( P = P_0 \sin(\omega t) \), equation (5) may be written in non-dimensional form as

\[
x' = y
\]
\[y' = \gamma \sin(\Omega t) - \xi y + x\left\{-(\theta - 1) - 3a_2 + 5a_3\right\} + \] 
\[+ [(\theta - 1) - \alpha_2 + \alpha_3] (x^2 + B^2)^{-1/2} - \left[3a_2 - 10a_3\right] (x^2 + B^2)^{1/2} + \] 
\[+ [-\alpha_2 + 10a_3] (x^2 + B^2) + \] 
\[- 5a_3 (x^2 + B^2)^{3/2} - a_3 (x^2 + B^2)^2 \]

where \( \xi \) is a non-dimensional viscous damping coefficient. The dissipation due to hysteretic effect may be considered by assuming an equivalent viscous damping related to this parameter. Moreover, the following non-dimensional parameters are considered:

\[
x = \frac{X}{L}, \quad \gamma = \frac{P_0}{mL_0\omega_0^2}, \quad \omega_0^2 = \frac{2Aa_1T_M}{mL_0}, \quad \Omega = \frac{\omega}{\omega_0}, \quad \tau = \omega_0 t,
\]
\[\theta = \frac{T}{T_M}, \quad \alpha_2 = \frac{a_2}{a_1T_M}, \quad \alpha_3 = \frac{a_3}{a_1T_M}, \quad b = \frac{B}{L_0} \text{ and } (\cdot)' = \frac{d(\cdot)}{dt} \]

III. CONTROLLER DESIGN

A. Feedback linearization

Consider a class of \( n^{th} \)-order nonlinear systems:

\[
x^{(n)} = f(x, t) + b(x, t)u + d
\]

where \( u \) is the control input, the scalar variable \( x \) is the output of interest, \( x^{(n)} \) is the \( n \)-th time derivative of \( x \), \( x = [x, \dot{x}, \ldots, x^{(n-1)}] \) is the system state vector, \( f, b : \mathbb{R}^n \to \mathbb{R} \) are both nonlinear functions and \( d \) is assumed to represent all uncertainties and unmodeled dynamics regarding system dynamics, as well as any external disturbance that can arise.

In respect of the disturbance-like term \( d \), the following assumption will be made:

**Assumption 1.** The disturbance \( d \) is unknown but continuous and bounded, i.e., \( |d| \leq \delta \).

Let us now define an appropriate control law based on conventional feedback linearization scheme that ensures the tracking of a desired trajectory \( \hat{x}_d = [\hat{x}_d, \dot{\hat{x}}_d, \ldots, \dot{x}^{(n-1)}_d] \), i.e., the controller should assure that \( \hat{x} \to 0 \) as \( t \to \infty \), where \( \hat{x} = x - \hat{x}_d = [\hat{\dot{x}}, \ldots, \dot{x}^{(n-1)}] \) is the related tracking error.

On this basis, assuming that the state vector \( x \) is available to be measured and system dynamics is perfectly known, i.e., there is no modeling imprecision nor external disturbance \( (d = 0) \) and the functions \( f \) and \( b \) are well known, with \( |b(x, t)| > 0 \), the following control law:

\[
u = b^{-1}(-f + x^{(n)} - k_0\ddot{x} - k_1\dot{x} - \ldots - k_{n-1}\dot{x}^{(n-1)})
\]

guarantees that \( x \to x_d \) as \( t \to \infty \), if the coefficients \( k_i \) \( (i = 0, 2, \ldots, n - 1) \) make the polynomial \( p^n + k_{n-1}p^{n-1} + \ldots + k_0 \) a Hurwitz polynomial [Slotine and Li 1991].

The convergence of the closed-loop system could be easily established by substituting the control law (8) in the nonlinear system (7). The resulting dynamical system could be rewritten by means of the tracking error:

\[
\dot{\hat{x}}^{(n)} + k_{n-1}\dot{\hat{x}}^{(n-1)} + \ldots + k_1\dot{x} + k_0\ddot{x} = 0
\]

where the related characteristic polynomial is Hurwitz.

The characteristic polynomial could be assured to be Hurwitz by defining \( k^T\hat{x} = k_{n-1}\dot{\hat{x}}^{(n-1)} + \ldots + k_1\dot{x} + k_0\ddot{x} \), where \( k = [c_0\lambda^n, c_1\lambda^{n-1}, \ldots, c_{n-1}\lambda] \), \( \lambda \) is a strictly positive constant and \( c_i \) states for binomial coefficients, i.e.

\[
c_i = \binom{n}{i} = \frac{n!}{(n-i)!i!}, \quad i = 0, 1, \ldots, n - 1
\]

Since in real-world applications the nonlinear system (7) is often not perfectly known, the control law (8) based on conventional feedback linearization is not sufficient to ensure the exponential convergence of the tracking error to zero.

Thus, we propose the adoption of fuzzy inference system within the control law, in order to compensate for \( d \) and to enhance the feedback linearization controller.
B. Fuzzy inference system

Because of the possibility to express human experience in an algorithmic manner, fuzzy logic has been largely employed in the last decades to both control and identification of dynamical systems.

If $s$ is $S_r$, then $\hat{d} = \hat{D}_r$; $r = 1, 2, \ldots, N$

where $s = k_{n-1} \hat{x}^{(n-1)} + \ldots + k_1 \hat{x} + k_0 \hat{x}$ represents a combined tracking error measure, $S_r$ are fuzzy sets, whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each one of the $N$ fuzzy rules.

At this point, it should be highlighted that the adoption of a combined tracking error measure $s$ in the premise of the rules, instead of the state variables as in Tanaka et al. (2013), leads to a smaller number of fuzzy sets and rules, which simplifies the design process. Considering that external disturbances are independent of the state variables, the choice of a combined tracking error measure also seems to be more appropriate in this case.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}(s) = \hat{D}^\top \Psi(s)$$

where, $\hat{D} = [\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_N]$ is the vector containing the attributed values $\hat{D}_r$ to each rule $r$, $\Psi(s) = [\psi_1(s), \psi_2(s), \ldots, \psi_N(s)]$ is a vector with components $\psi_r(s) = w_r / \sum_{i=1}^{N} w_r$ and $w_r$ is the firing strength of each rule.

C. Fuzzy feedback linearization

Considering that fuzzy logic can perform universal approximation (Kosko, 1994), we propose the adoption of a TSK fuzzy inference system within the feedback linearization controller to compensate for modeling inaccuracies and consequently enhance the trajectory tracking of uncertain nonlinear systems.

Therefore, the control law with the fuzzy compensation scheme can be stated as follows

$$u = b^{-1} [-f + \dot{x}_d^{(n)} - k_0 \ddot{x} - k_1 \dot{x} - \ldots - k_{n-1} \ddot{x}^{(n-1)} - \hat{d}(s)]$$

and the related closed-loop system is:

$$\dot{\hat{x}}^{(n)} + k_{n-1} \hat{x}^{(n-1)} + \ldots + k_1 \dot{x} + k_0 \ddot{x} = \hat{d}$$

Theorem 2. Consider the uncertain nonlinear system (7) and Assumption 1, then the fuzzy feedback linearization with inference system (11) can approximate the disturbance $d$.

Demonstração. Considering the universal approximation feature of fuzzy logic (Kosko, 1994), the output of the adopted inference system (11) can approximate the disturbance $d$ to an arbitrary degree of accuracy, i.e., $|\hat{d}(s) - d| \leq \varepsilon$ for an arbitrary $\varepsilon > 0$. Thus, from (13) one has

$$|\hat{x}^{(n)} + k_{n-1} \hat{x}^{(n-1)} + \ldots + k_1 \dot{x} + k_0 \ddot{x}| \leq \varepsilon$$

From (10), inequality (15) may be rewritten as

$$-\varepsilon \leq \hat{x}^{(n)} + c_{n-1} \lambda \hat{x}^{(n-1)} + \ldots + c_1 \lambda^{n-1} \dot{x} + c_0 \lambda^n \ddot{x} \leq \varepsilon$$

Multiplying (16) by $e^{\lambda t}$ yields

$$-\varepsilon e^{\lambda t} \leq \frac{d^n}{dt^n} (\hat{x} e^{\lambda t}) \leq \varepsilon e^{\lambda t}$$

Integrating (17) between 0 and $t$ gives

$$-\varepsilon \frac{e^{\lambda t}}{\lambda} + \frac{\varepsilon}{\lambda} \leq \frac{d^n}{dt^n} (\hat{x} e^{\lambda t}) \bigg|_{t=0}^{t} \leq \frac{\varepsilon}{\lambda e^{\lambda t}} - \frac{\varepsilon}{\lambda}$$

or conveniently rewritten as

$$-\frac{\varepsilon}{\lambda} e^{\lambda t} + \frac{\varepsilon}{\lambda} \leq \frac{d^n}{dt^n} (\hat{x} e^{\lambda t}) \bigg|_{t=0}^{t} \leq \frac{\varepsilon}{\lambda e^{\lambda t}} + \frac{\varepsilon}{\lambda}$$

The same reasoning can be repeatedly applied until the $n^{th}$ integral of (17) is reached:

$$-\frac{\varepsilon}{\lambda^n} e^{\lambda t} + \left( \frac{d^{n-1}}{dt^{n-1}} (\hat{x} e^{\lambda t}) \bigg|_{t=0}^{t} \right) + \frac{\varepsilon}{\lambda^{n-1}} \leq \frac{\varepsilon}{\lambda^n} e^{\lambda t} + \left( \frac{d^{n-1}}{dt^{n-1}} (\hat{x} e^{\lambda t}) \bigg|_{t=0}^{t} \right) + \frac{\varepsilon}{\lambda^n}$$

Furthermore, dividing (20) by $e^{\lambda t}$, it can be easily verified that, for $t \to \infty$,

$$-\frac{\varepsilon}{\lambda^n} \leq \hat{x}(t) \leq \frac{\varepsilon}{\lambda^n}$$
Considering the \((n - 1)^{th}\) integral of \((17)\)

\[
- \frac{\varepsilon}{\chi^{n-1}} e^{\lambda t} - \left( \frac{d^{n-1}}{dt^{n-1}} (\tilde{x} e^{\lambda t}) \right)_{t=0}^{\infty} + \frac{\varepsilon}{\chi^{n-2}} e^{\lambda t} - \cdots - \left( |\tilde{x}(0)| + \frac{\varepsilon}{\chi^{n-1}} \right) \leq \frac{d}{dt} (\tilde{x} e^{\lambda t}) \leq \frac{\varepsilon}{\chi^{n-1}} e^{\lambda t} +
\]

\[
+ \left( \frac{d^{n-1}}{dt^{n-1}} (\tilde{x} e^{\lambda t}) \right)_{t=0}^{\infty} + \frac{\varepsilon}{\chi^{n-2}} e^{\lambda t} + \cdots + \left( |\tilde{x}(0)| + \frac{\varepsilon}{\chi^{n-1}} \right) \leq \frac{d}{dt} (\tilde{x} e^{\lambda t}) \leq \frac{\varepsilon}{\chi^{n-1}} e^{\lambda t} +
\]

and noting that \(d(\tilde{x} e^{\lambda t})/dt = \dot{\tilde{x}} e^{\lambda t} + \tilde{x} \lambda e^{\lambda t}\), by imposing the bounds \([21]\) to \([22]\) and dividing again by \(e^{\lambda t}\), it follows that, for \(t \to \infty\),

\[
- 2 \frac{\varepsilon}{\chi^{n-1}} \leq \dot{\tilde{x}}(t) \leq 2 \frac{\varepsilon}{\chi^{n-1}}
\]

Now, applying the bounds \([21]\) and \([23]\) to the \((n - 2)^{th}\) integral of \((17)\) and dividing once again by \(e^{\lambda t}\), it follows that, for \(t \to \infty\),

\[
- 6 \frac{\varepsilon}{\chi^{n-2}} \leq \dot{\tilde{x}}(t) \leq 6 \frac{\varepsilon}{\chi^{n-2}}
\]

The same procedure can be successively repeated until the bounds for \(\tilde{x}^{(n-1)}\) are achieved:

\[
- \left[ 1 + \sum_{i=0}^{n-2} \left( \frac{n-1}{i} \right) \tilde{\zeta}_i \right] \frac{\varepsilon}{\chi} \leq \tilde{x}^{(n-1)} \leq \left[ 1 + \sum_{i=0}^{n-2} \left( \frac{n-1}{i} \right) \tilde{\zeta}_i \right] \frac{\varepsilon}{\chi}
\]

where the coefficients \(\tilde{\zeta}_i\) \((i = 0, 1, \ldots, n - 2)\) are related to the previously obtained bounds of each \(\tilde{x}^{(i)}\) and can be summarized as in \([14]\).

In this way, by inspection of the integrals of \((17)\), as well as \([21]\), \([23]\), \([24]\), \([25]\) and the other omitted bounds, it follows that the tracking error exponentially converges to the \(n\)-dimensional box determined by the limits \(|\tilde{x}^{(i)}| \leq \tilde{\zeta}_i \lambda^{n-i} \varepsilon, i = 0, 1, \ldots, n - 1\), where \(\tilde{\zeta}_i\) is defined by \([14]\).

**Corollary 3.** It must be noted that the proposed control scheme provides a smaller tracking error when compared with the conventional feedback linearization controller. By setting the output of the fuzzy inference system to zero, \(\ddot{d}(\tilde{x}) = 0\), Theorem 2 implies that the resulting bounds are \(|\tilde{x}^{(i)}| \leq \tilde{\zeta}_i \lambda^{n-i} \delta, i = 0, 1, \ldots, n - 1\). Considering that \(\varepsilon < \delta\), from the universal approximation feature of \(\tilde{d}\), it can be concluded that the tracking error obtained with the fuzzy feedback linearization controller is smaller than the associated with the conventional scheme.

**IV. NUMERICAL SIMULATIONS**

Numerical simulations are now in focus exploring the controller capability to perform vibration reduction in smart structures. A fourth-order Runge-Kutta scheme is adopted. In all simulations, the material properties presented in Table I are used. These values are chosen in order to match experimental data obtained by \(\text{Sittner et al.} (1995)\) for a Cu-Zn-Al-Ni alloy at 373 K (see Figure 2). For the data in Table I, the parameters defined in equation \(4\) assume the values: \(\alpha_2 = 1.240 \times 10^2\) and \(\alpha_3 = 1.450 \times 10^4\). We further let \(\phi = 0.866\), corresponding to a two-bar truss with an initial position \(\varphi_0 = 30^\circ\). Sampling rates of \(2000\pi / \pi\) for control system and \(10000\pi / \pi\) for dynamical model are assumed.

| \(T_a\) (K) | \(T_A\) (K) | \(\alpha_1\) (MPa/K) | \(\alpha_2\) (MPa) | \(\alpha_3\) (MPa) |
|------------|------------|----------------|----------------|----------------|
| 523.29 | 364.3 | 1.868 x 10^7 | 2.186 x 10^9 | 288 |

---

**Figura 2: Stress-strain curve: experimental and predicted by polynomial model.**

In order to demonstrate that the adopted control scheme can deal with both modeling inaccuracies and external disturbances, an uncertainty of \(\pm 20\%\) over the values of \(\alpha_2\) and \(\alpha_3\) is considered. Moreover, the periodic excitation is treated as an unknown external disturbance. Under this assumption, \(\gamma \sin(\Omega t)\) is not taken into account within the design of the control law. On this basis, it is assumed estimation values as \(\alpha_2 = 10^2\) and \(\alpha_3 = 1.15 \times 10^4\) in the control law.
The other estimates in $f$ are chosen based on the assumption that model coefficients are perfectly known. Concerning the fuzzy system, trapezoidal (at the borders) and triangular (in the middle) membership functions are adopted for $S_r$, with the central values defined respectively as $C = \{ -1.00 ; -0.50 ; -0.02 ; 0.02 ; 0.50 ; 1.00 \} \times 10^{-1}$. The control parameter $\lambda$ is defined as $\lambda = 0.6$.

The stabilization of the state vector in the neighborhood of one of the equilibrium points of the shape memory two-bar truss (Savi et al., 2002) is carried out. This approach shows that the adopted control scheme can significantly reduce the vibration level and also avoid the undesired snap-through behavior. Figures 3 and 4 show the obtained results considering $x_d = [0.68, 0.0]$, $\theta = 0.03$, $\Omega = 0.5$, $\xi = 0.05$ and $\gamma = 0.020$. Note that the chaotic behavior with large amplitudes of the uncontrolled response is replaced by a regular behavior with small amplitudes around the equilibrium point. It should be highlighted that the proposed control law provides a smaller stabilization error, Figure 4, when compared with the conventional feedback linearization scheme, Figure 3.

V. CONCLUDING REMARKS

In this paper, a fuzzy feedback linearization controller is considered for vibration reduction in a shape memory two-bar truss. A polynomial constitutive model is assumed to describe the constitutive behavior of the bars. Despite the deceiving simplicity, this model allows an appropriate qualitative description of system dynamics, which can exhibit chaotic behavior. Numerical simulations show the efficacy of the proposed scheme against modeling inaccuracies and external disturbances. The improved performance over the conventional feedback linearization is also demonstrated. It should be highlighted that the controller robustness to modeling inaccuracies is an important issue that allows the use of a simple constitutive model for control purposes.

VI. ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Brazilian Research Agencies CNPq, CAPES and FAPERJ, and through the INCT-EIE (National Institute of Science and Technology - Smart Structures in Engineering) the CNPq and FAPEMIG. The German Academic Exchange Service (DAAD) and the Air Force Office for Scientific Research (AFOSR) are also acknowledged.
Referências

Auricchio, F. and Petrini, L., 2004a. “A three-dimensional model describing stress-temperature induced solid phase transformations: solution algorithm and boundary value problems”. International Journal for Numerical Methods in Engineering, Vol. 61, No. 6, pp. 807–836.
Auricchio, F. and Petrini, L., 2004b. “A three-dimensional model describing stress-temperature induced solid phase transformations: thermomechanical coupling and hybrid composite applications”. International Journal for Numerical Methods in Engineering, Vol. 61, No. 5, pp. 716–737.
Auricchio, F. and Sacco, E., 1999. “A temperature-dependent beam for shape-memory alloys: Constitutive modelling, finite-element implementation and numerical simulations”. Computer Methods in Applied Mechanics and Engineering, Vol. 174, No. 1–2, pp. 171–190.
Auricchio, F. and Taylor, R.L., 1996. “Shape memory alloy superelastic behavior: 3D finite element simulations”. In Proceedings of the 3rd International Conference on Intelligent Materials. Lyon, France.
Bandeira, E.L., Savi, M.A., Monteiro Jr., P.C.C. and Netto, T.A., 2006. “Finite element analysis of shape memory alloy adaptive trusses with geometrical nonlinearities”. Archive of Applied Mechanics, Vol. 76, No. 3–4, pp. 133–144.
Bazant, Z.P. and Cedolin, L., 2010. Stability of Structures: Elastic, Inelastic, Fracture, and Damage Theories. World Scientific Publishing Company, Singapore.
Bessa, W.M., Kreuzer, E., Lange, J., Pick, M.A. and Solowjow, E., 2017. “Design and adaptive depth control of a micro diving agent”. IEEE Robotics and Automation Letters, Vol. 2, No. 4, pp. 1871–1877. doi:10.1109/LRA.2017.2714142.
Bessa, W.M., Brinkmann, G., Dücke, D.A., Kreuzer, E. and Solowjow, E., 2018. “A biologically inspired framework for the control of mechatronic systems and its application to a micro diving agent”. Mathematical Problems in Engineering, Vol. 2018, pp. 1–16. doi:10.1155/2018/9648126.
Bessa, W.M., 2005. Controle por Modos Deslizantes de Sistemas Dinâmicos com Zona Morta Aplicado ao Posicionamento de ROVs. Tese (D.Sc.), COPPE/UFRJ, Rio de Janeiro, Brasil.
Bessa, W.M. and Barrêto, R.S.S., 2010. “Adaptive fuzzy sliding mode control of uncertain nonlinear systems”. Controle & Automação, Vol. 21, No. 2, pp. 117–126.
Bessa, W.M., De Paula, A.S. and Savi, M.A., 2012. “Sliding mode control with adaptive fuzzy dead-zone compensation for uncertain chaotic systems”. Nonlinear Dynamics, Vol. 70, No. 3, pp. 1989–2001.
Bessa, W.M., De Paula, A.S. and Savi, M.A., 2014. “Adaptive fuzzy sliding mode control of a chaotic pendulum with noisy signals”. Zeitschrift für Angewandte Mathematik und Mechanik, Vol. 94, No. 3, pp. 256–263. doi:10.1002/zamm.201200214.
Bessa, W.M., Dutra, M.S. and Kreuzer, E., 2005. “Thrustor dynamics compensation for the positioning of underwater robotic vehicles through a fuzzy sliding mode based approach”. In COBEM 2005 – Proceedings of the 18th International Congress of Mechanical Engineering. Ouro Preto, Brasil.
Bessa, W.M., Otto, S., Kreuzer, E. and Seifried, R., 2019. “An adaptive fuzzy sliding mode controller for uncertain actuated mechanical systems”. Journal of Vibration and Control, Vol. 25, No. 9, pp. 1521–1535. doi:10.1177/1077546319827393.
Bhattacharyya, A., Faulkner, M.G. and Amalraj, J.J., 2000. “Finite element modeling of cyclic thermal response of shape memory alloy wires with variable material properties”. Computational Materials Science, Vol. 17, No. 1, pp. 93–104.
Collet, M., Follette, E. and Lexcellent, C., 2001. “Analysis of the behavior of a shape memory alloy beam under dynamical loading”. European Journal of Mechanics A – Solids, Vol. 20, No. 4, pp. 615–630.
Dos Santos, J.D.B. and Bessa, W.M., 2019. “Intelligent control for accurate position tracking of electrohydrodynamic actuators”. Electronics Letters, Vol. 55, No. 2, pp. 78–80. doi:10.1049/el.2018.7218.
Falk, E., 1980. “Model free energy, mechanics, and thermodynamics of shape memory alloys”. Acta Metallurgica, Vol. 28, No. 12, pp. 1773–1780.
Kosko, B., 1994. “Fuzzy systems as universal approximators”. IEEE Transactions on Computers, Vol. 43, No. 11, pp. 1323–1339.
La Cava, C.A.P.L., Savi, M.A. and Pacheco, P.M.C.L., 2004. “A nonlinear finite element method applied to shape memory bars”. Smart Materials and Structures, Vol. 13, No. 5, pp. 1118–1130.
Lagoudas, D.C., Mouithy, D., Quddai, M.A. and Reddy, J.N., 1997. “Modeling of the thermomechanical response of active laminates with SMA strips using the layerwise finite element method”. Journal of Intelligent Material Systems and Structures, Vol. 8, No. 6, pp. 476–488.
Lagoudas, D.C., 2008. Shape Memory Alloys: and Engineering Applications. Springer-Verlag, New York.
Liew, K., Kitipornchai, S., Ng, T.Y. and Zou, G.P., 2002. “Multi-dimensional superelastic behavior of shape memory alloys via nonlinear finite element method”. Engineering Structures, Vol. 24, No. 1, pp. 51–57.
Lima, G.S., Bessa, W.M. and Trimpe, S., 2018. “Depth control of underwater robots using sliding modes and gaussian process regression”. In LARS 2018 – Proceedings of the Latin American Robotic Symposium. João Pessoa, Brazil. doi:10.1109/LARS/SBR/WRE.2018.00012.
Lima, G.S. Porto, D.R., de Oliveira, A.J. and Bessa, W.M., 2021. “Intelligent control of a single-link flexible manipulator using sliding modes and artificial neural networks”. Electronics Letters, Vol. 57, No. 23, pp. 869–872. doi:10.1049/el.2021.12300.
Lima, G.S., Trimpe, S. and Bessa, W.M., 2020. “Sliding mode control with gaussian process regression for underwater robots”. Journal of Intelligent & Robotic Systems, Vol. 99, No. 3, pp. 487–498. doi:10.1007/s10846-019-01128-5.
Machado, L.G. and Savi, M.A., 2003. “Medical applications of shape memory alloys”. Brazilian Journal of Medical and Biological Research, Vol. 36, No. 6, pp. 683–691.
Masud, A., Panahandeh, M. and Auricchio, F., 1997. “A finite-strain finite element model for the pseudoelastic behavior of shape memory alloys”. Computer Methods in Applied Mechanics and Engineering, Vol. 148, No. 1–2, pp. 23–37.
Muller, I., 1991. “On the pseudo-elastic hysteresis”. Acta Metallurgica, Vol. 39, No. 3, pp. 263–271.
Otsuka, K. and Ren, X., 1999. “Recent developments in the research of shape memory alloys”. Intermetallics, Vol. 7, No. 5, pp. 511–528.
Paiva, A. and Savi, M.A., 2006. “An overview of constitutive models for shape memory alloys”. Mathematical Problems in Engineering, Vol. 2006, p. 56876.
Rogers, C.A., 1995. “Intelligent materials”. Scientific American, Vol. 273, No. 3, pp. 154–157.
Savi, M.A., Pacheco, P.M.C.L. and Braga, A.M.B., 2002. “Chaos in a shape memory two-bar truss”. *International Journal of Non-Linear Mechanics*, Vol. 37, No. 8, pp. 1387–1395.

Savi, M.A. and Braga, A.M.B., 1993. “Chaotic vibration of an oscillator with shape memory”. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 15, No. 1, pp. 1–20.

Savi, M.A. and Nogueira, J.B., 2010. “Nonlinear dynamics and chaos in a pseudoelastic two-bar truss”. *Smart Materials and Structures*, Vol. 19, No. 11, p. 115022010.

Shaw, J.A. and Kyriakides, S., 1995. “Thermomechanical aspects of NiTi”. *Journal of the Mechanics and Physics of Solids*, Vol. 43, No. 8, pp. 1243–1281.

Sittner, P., Hara, Y. and Tokuda, M., 1995. “Experimental study on the thermoelastic martensitic transformation in shape memory alloy polycrystal induced by combined external forces”. *Metallurgical and Materials Transactions A*, Vol. 16A, pp. 2923–2935.

Slotine, J.J.E. and Li, W., 1991. *Applied Nonlinear Control*. Prentice Hall, New Jersey.

Tanaka, M.C., de Macedo Fernandes, J.M. and Bessa, W.M., 2013. “Feedback linearization with fuzzy compensation for uncertain nonlinear systems”. *International Journal of Computers, Communications & Control*, Vol. 8, No. 5, pp. 736–743.

Trochu, F., Sacépé, N., Volkov, O. and Turenne, S., 1999. “Characterization of NiTi shape memory alloys using dual kriging interpolation”. *Materials Science and Engineering: A*, Vol. 273–275, No. 15, pp. 395–399.