Orbiting globular clusters formed in dark matter mini-halos

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ABSTRACT
We seek to differentiate dynamical and morphological attributes between globular clusters that were formed inside their own dark matter mini-halo, and those who were not. For that, we employ high resolution $N$-body simulations of globular clusters with (and without) an enveloping dark matter mini-halo, orbiting a host galaxy. We set the same prescriptions of the Fornax dwarf spheroidal galaxy and its main five globular clusters, and use $N$-body particles for all components (i.e., stars and dark matter, for both Fornax and its clusters). For clusters embedded in dark matter, we observe that the increment of mass from the extra dark component triggers a tidal radius growth that allows the mini-halo to work as a protective shield against tidal stripping, being itself stripped beforehand the stars. Consequently, tidal effects such as inflation of the stellar velocity dispersion, development of prominent tidal tails, ellipticity increase and diffusion of the stellar distribution profile are generally much milder in clusters originally embedded in dark matter. However, this shielding effect becomes negligible after an important amount of dark matter has been stripped, which happens faster for clusters having simultaneously short orbital periods, low typical orbital radii and relatively high eccentricities. Finally, we notice that even for clusters that retain a large amount of dark matter at redshift zero, their inner regions are still predominantly composed of stars, with the typical density ratio of dark matter to cluster stars being of the order of 1% up to roughly 10 pc away from the clusters’ centre.

Key words: globular clusters: general – dark matter – galaxies: kinematics and dynamics – galaxies: formation – galaxies: dwarf – software: simulations

1 INTRODUCTION
There is no doubt that the nature of dark matter (DM) is one of the most elusive concepts in modern day physics. However, the existence of this astrophysical component has been used and requested to explain a vast range of phenomena for a considerable amount of time: Back when Zwicky (1933, 1937) proposed that some sort of non-luminous matter could compose the amount of mass needed to explain the discrepancy between mass measurements of the Coma cluster based on the Virial theorem (e.g., Binney & Tremaine 2008) and based on brightness and number of galaxies, up to recent measurements from the Planck Collaboration et al. (2016, 2020) that yield very robust fits of the cosmic microwave background (CMB) using a $\Lambda$CDM model accounting for the existence of DM. In between, other important confirmations of this mysterious dark component were provided: Rubin & Ford (1970); Rubin, Ford & Thonnard (1980) showed that the rotation curves of outer stars in nearby galaxies needed an extra amount of mass (compared to observable, luminous matter) to explain their high velocity values and finally, gravitational lensing studies (e.g., Taylor et al. 1998) have also confirmed that the total amount of mass in many galaxy clusters correspond to the dynamical measurements accounting for DM.

Such findings point to the DM as a fundamental component in galaxy formation, present in most galaxies as an enveloping halo, from the smallest to the highest scales. Thus, it seems in principle curious that dense collections of stars spanning from $\sim 10^5 – 10^6 \, M_\odot$ such as globular star clusters (GCs), with some of them thought to be accreted dwarf galaxies (e.g., Majewski et al. 2000; Bekki & Freeman 2003; Boldrini & Bovy 2021; Pechetti et al. 2021), do not seem to require any amount of DM to explain their dynamical mass (e.g., Meylan & Mayor 1986; van de Ven et al. 2006; Vitral & Mamon 2021). In fact, Peebles (1984) proposed a formation scenario where GCs are formed inside their own DM mini-halo1, and further studies defended that if formed before re-ionisation2, GCs could be smaller counterparts of galaxies (e.g., Bromm & Clarke 2002, Figure 2 from Mamon et al. 2012 and Silk & Mamon 2012 for a review on galaxy formation). On the other hand, different formation scenarios, where GCs are not necessarily embedded in DM mini-halos also exist. For instance, GCs could be formed as bound gas clouds (Peebles &Dicke 1968), as galaxy DM-free fragments after accretion events (e.g., Searle & Zinn 1978; Abadi, Navarro & Steinmetz 2006), as relics of young massive clusters (YMC, Portegies Zwart, McMillan & Gieles 2010; Longmore et al., 2014) formed in the high-redshift Universe (Kruijssen 2014, 2015), or as debris from the galactic disc after merger events (in-situ scenario). Moreover, recent cosmological simulations

1 Those DM mini-halos could have between $10^6 – 10^8 \, M_\odot$, such as general DM sub-structures or subhalos (Zavala &Frenk 2019).
2 Which is consistent with GCs having typical ages up to ~ 13 Gyr Marín-Franch et al. (2009).

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DM mini-halos are expected to produce much less dense and much smaller DM mini-halos. As a consequence, GCs originally embedded in DM mini-halos can still keep an inner protective DM shield. Even though the DM inside mini-halos might experience a tidal field (because of the lack of a DM shield), the DM shield survives, also forming a diffuse stellar envelope of considerably smaller dimensions than the stellar component, so that its dynamical detectability might only be possible beyond several scale radii (Peñarrubia et al. 2017), where GCs stars are usually confused with galactic field stars.

With the astrometric revolution brought by the Gaia mission (Gaia Collaboration et al. 2018b,a, 2021), and the promising future discoveries of the James Webb Space Telescope (JWST, Gardner et al. 2006) and the Euclid mission (Laureijs et al. 2011; Lançon et al. 2021), the need to better constrain the expected differences between the multiple GC formation scenarios is a priority, so that these rich data sets can be fully exploited to better understand the many long-sought questions regarding GC formation. As a matter of fact, although many robust attempts to better understand the observational implications of the DM mini-halo scenario have been made, the high computational cost of simulating a GC+DM system in a Milky Way (MW) type of galaxy forced these attempts to be placed in idealised scenarios: for instance, isolated GCs not experiencing tidal forces (Peñarrubia et al. 2017), or orbiting GCs in a static potential (Mashchenko & Sills 2005). Preferably, simulations with clusters experiencing tidal forces, in a host galaxy fully composed of particles (and not just a static potential) would allow to take into account more correctly dynamical friction and tidal effects between globular clusters and their host galaxy.

In this work, we aim to clearly separate the observational behaviours of GCs which are formed inside a DM mini-halo and which are devoid of it, both orbiting a host galaxy and thus permanently experiencing a tidal field. We do it by performing N-body simulations of a GC system with and without a DM embedding mini-halo, alongside with a host galaxy. In order to bypass the high computational cost mentioned above while still keeping a high resolution, we place our GCs in a dwarf spheroidal (dSph) galaxy, following the same prescriptions as the Fornax dSph, in a similar manner than done in Boldrini, Mohayaeae & Silk (2020). This allows one to consider much less stars than it would be needed if the satellites were orbiting a MW-like galaxy, and also to avoid using a potential instead of particles, which brings unwanted numerical effects (see section 4 from Boldrini & Vitral 2021). Besides, by taking a real galaxy (i.e., Fornax) as our model, we are likely better exploring the dynamics and orbital evolution of different parameters, and thus reaching more realistic conclusions.

Among our findings, we observe that the increment of mass from the extra DM component tends to protect the GCs from tidal stripping by increasing the tidal radius of the system. This tidal radius growth allows the DM to behave as a shield, which is stripped beforehand GC stars, thus leaving the latter almost intact (while the shield is not considerably stripped) with respect to the case without a DM mini-halo. As a consequence, GCs originally embedded in DM mini-halos are expected to produce much less dense and much less long tidal tails while the DM shield survives, also forming a diffuse stellar envelope of considerably smaller dimensions than their respective tails. In addition, DM-free GCs tend to have higher ellipticities (or flattening) and higher half-number radii, being therefore less dense or compact. Interestingly, the inflation of stellar velocity dispersion profiles of GCs devoid of DM seems to be more effective than in the case with DM, given that the former are more exposed to dynamical heating from the host tidal field (because of the lack of a protective DM shield).

We show that GCs formed inside DM mini-halos can still keep an important amount of DM if their orbital parameters do not strongly push the cluster to the inner regions of the host galaxy, but these DM particles would be located in the outskirts of the clusters, such that the inner 10 pc of surviving GCs have a typical density ratio of DM to cluster stars of ~ 1%. This is consistent with the fact that dynamical mass measurements of GCs do not seem to require DM, since the data used to perform these studies is often within this ~ 10 pc region. We describe our methods and lay a few theoretical predictions in sections 2 and 3, while our results and main DM signatures are presented in section 4. We discuss the implications of our work and conclude it in sections 5 and 6. Throughout the rest of the paper, we reference the stellar component of the system of GC stars without DM as ⋆, and the stellar component of the system of GCs formed inside DM mini-halos as ⋆⋆. The dark matter is labelled as ⋆.

Figure 1. Globular clusters in Fornax: Image of the five globular clusters we simulate in the Fornax dwarf spheroidal galaxy, labelled as throughout this paper, according to Boldrini et al. (2020). The original image (without the orange circles and labels) is a composite from Giuseppe Donatiello, constructed with data from ESO and the Digitized Sky Survey 2 (Credit: ESO/Digitized Sky Survey 2, under the CC BY 4.0 licence).
2 METHODS

2.1 Simulations

The initial conditions for the Fornax-GC system are taken from Boldrini et al. (2020), see their section 2 and Table 1. We consider the scenario in which the GCs were accreted recently at $z = 0.36$ by Fornax. It ensures that at $z = 0$, the GCs embedded in DM are still orbiting and no star clusters form in the centre of Fornax in accordance with observations. Figure 1 illustrates the GC system we study and the labels we use throughout the paper.

To generate our $N$-body objects, we use the initial condition code MAGI (Miki & Umemura 2018). Adopting a distribution-function-based method, it ensures that the final realization of the galaxy is in dynamical equilibrium (Miki & Umemura 2018). We perform our simulations with the high performance collisionless $N$-body code gorics (Miki & Umemura 2017). This gravitational octree code runs entirely on GPU and is accelerated by the use of hierarchical time steps in which a group of particles has the same time step (Miki & Umemura 2017). We evolve the Fornax-GC system over 4 Gyr in each scenario, i.e. for GCs with and without a DM mini-halo. We set the particle resolution of all the live objects to 50 $M_\odot$ and the gravitational softening length to 0.1 pc. Numerical convergence tests have been performed in Boldrini et al. (2020).

2.2 Data analysis

Using the instantaneous orbital radius $r_{\text{orbit}}$ and the satellite mass $M_{\text{sat}}$, we calculate the theoretical tidal radius of each GC (or GC+DM system) at each snapshot from the simulation as derived by Bertin & Varri (2008) as

$$r_t = \left( \frac{GM_{\text{sat}}}{\Omega^2 v} \right)^{1/3},$$

(1)

where the orbital frequency $\Omega$ at $r_{\text{orbit}}$, the epicyclic frequency $\kappa$ at $r_{\text{orbit}}$, and a positive dimensionless coefficient $\nu$ related to the orbit’s eccentricity, are defined as:

$$\Omega^2 = \frac{(d\Phi_{\text{host}}(r)/dr)_{r_{\text{orbit}}}}{r_{\text{orbit}}},$$

(2)

$$\kappa^2 = 3\Omega^2 + \frac{(d^2\Phi_{\text{host}}(r)/dr^2)_{r_{\text{orbit}}}}{r_{\text{orbit}}},$$

(3)

$$\nu = 4 - \kappa^2/\Omega^2,$$

(4)

where the suffix “host” stands for the Fornax dSph, in our case. For the system composed of GC stars plus DM particles (i.e., GC+DM), we measure the total tidal radius in our simulations by taking into account the mass of both DM and stars in our calculation. In order to determine the bound particles of the DM and stellar components of GCs, we follow the procedure described in van den Bosch & Ogiya (2018), see their section 2.3. We performed most of the data analysis with internal routines of the astrometry modelling code BALRoGO\footnote{Code repository: https://gitlab.com/eduardo-vitral/balrogo.} (Vitrá 2021). This includes fits of the surface density (and ellipticity), construction of dispersion profiles and sky projection. Whenever targeting these themes, we mention in detail how we proceeded to perform the analysis.

3 THEORETICAL PREDICTIONS

First, we detail the properties expected from a globular cluster formed inside a dark matter mini-halo. Due to the overall mass increase of the system (GC+DM), some dynamical behaviours can be observed in such a cluster (notably, their orbits become considerably different than the DM-free case). These trends can be whether disruptive or protective, regarding the GC tidal disruption. They can be summarised as:

- Dynamical heating from the DM mini-halo and subsequent increase of the stellar velocity dispersion (disruptive).
- Faster orbital decay of the cluster (disruptive).
- Increase of the tidal radius of the system (protective).

Below, we briefly describe them from a theoretical point of view. Next, in the following sections, we balance the effects of each of these trends with the help of our simulations, and explore under which conditions each of these trends prevails over the others.

3.1 Dynamical heating from the dark matter

For a system in equilibrium, densification due to the increment of DM mass will increase the velocity dispersion along the stellar distribution, causing dynamical heating. This is a consequence of the handling of the Virial theorem and has been observed for numerical simulations of isolated GCs embedded in DM minihalos (see Figure 2 from Peñarrubia et al. 2017).

For isolated systems, the increase of velocity dispersion is likely counterbalanced by the higher gravitational pull of the new more massive system, or in other words, the stars with higher velocity can still be bound to the system due to an increase of the overall escape velocity. However, for orbiting systems experiencing tidal effects, this dynamical heating can help higher velocity stars to escape from the satellite, since the gravitational pull from the main body contributes to decrease the escape velocity of the satellite’s stars. Therefore, for orbiting GCs embedded in DM mini-halos, one could, in
principle, expect dynamical heating from the DM mass to contribute to the tidal stripping of the cluster.

### 3.2 Faster orbital decay

The exchange of energy between the satellite and its host galaxy will lead to the drag of the satellite, also referred to as dynamical friction. This will lead the satellite to lose energy and sink towards the centre of mass of the system. The dynamical friction time of the cluster is the timescale needed for the satellite to reach the centre of mass of the host galaxy. It has been defined in Binney & Tremaine (2008, eq [7-26]), and follows the relation

$$ t_{\text{fric}} \propto \frac{1}{M}, $$

where $M$ is the satellite’s total mass.

Applying this relation to our simulations, where in one case we have a GC system alone of $10^6$ M$_\odot$ and in the other case the total system mass is that of the GC plus the DM mini-halo (i.e., $2.1 \times 10^7$ M$_\odot$), we find that the system with DM is supposed to sink to the centre roughly ten times faster than the system without the DM mini-halo. Indeed, when looking at Figure 2, one can notice that the systems with DM (solid green) occupy much shorter radii than the systems without DM (dashed red) throughout their orbits.

As a consequence, systems with a DM mini-halo tend to be located close to their host centres sooner, and thus feel a stronger dynamical heating coming from the host galaxy. This dynamical heating, on its turn, can potentially work to remove more GC stars (along with their DM envelope) than in the case without DM.

### 3.3 Tidal radius growth

As presented in section 2.2, the tidal radius of a satellite of mass $M$, in a given potential, follows the relation

$$ r_t \propto M^{1/3}. $$

Therefore, for our GC system with DM, the respective tidal radius will be roughly 2.7 times greater than the case without dark matter, considering the same distance and enclosed mass from the host. This means that the region where the stars are better protected against tidal stripping is larger in the case with DM than the case without it, and thus one could argue that GCs with DM can be more resilient to tidal forces from their host galaxy. In Figure 3, we illustrate this idea with a scheme where the cluster embedded in DM has a broader tidal radius, and we demonstrate this effect on our simulations in Figure 4.

### 4 RESULTS: DARK MATTER SIGNATURES

In this section, we analyse the main implications of the presence of a DM mini-halo in the overall GC dynamics and morphology, according to our simulations. We remind that our analysis is modelled on the specific case of the Fornax dSph, in order to probe a realistic scenario. In addition, we keep in mind the points discussed in the previous section concerning disruptive and protective effects on the GC stellar component.

#### 4.1 Dark matter shield

In the beginning of our simulations with DM mini-halos, all GCs presented a DM envelope massive ($M_\text{DM} = 2 \times 10^7$ M$_\odot$) and concentrated enough ($r_{\text{NFW}} \approx 300$ pc, where $r_{\text{NFW}}$ is the scale radius of the Navarro, Frenk & White 1996 profile), so the tidal radius growth explained in section 3.3 and illustrated in Figure 3 was observed. In fact, we observed that whenever we had such tidal radius increase, the DM particles are stripped beforehand GC stars, so the DM envelope works effectively as a shield against tidal stripping of the stellar component, being gradually removed as the system experiences stronger tidal forces from the host galaxy. Indeed, as depicted by Figure 4, we notice that a certain tracer (stars or dark matter) is more affected once the ratio $r_{\text{bound}}/r_t$ is greater or comparable to unity, what happens first for the dark matter envelope, for the plots on the left.

In order to visualise this effect, we created a velocity dispersion map (see appendix A1 for details) of DM particles, which are much more spread than GC stars, and therefore cover better the spatial extent of the tidal radius. Such map helps us to spot the transitory region where the DM particles start to effectively feel tidal effects.
of the host galaxy, which is characterised by a steep increase of the velocity dispersion produced by tidal heating of the system.

In Figure 5, we show this velocity dispersion map for GCS, one of the clusters where this DM shield seemed most effective. We display the last six pericentres of its orbit, when the tidal effects are stronger. The extension of bound GC stars and bound DM particles are highlighted as dotted and dashed green lines, respectively, while the theoretical tidal radius, calculated according to section 2.2 (eq. [1]), is displayed as a solid green circle. The centres of Fornax and of the GC are represented as a thick green cross and a plus sign, respectively.

We also observe (still in Figure 5) a gradual decrease in the extension of the blue region, characterised by a low velocity dispersion, where the DM shield is effective. This means that at first, the DM shield is highly protective, and with time, as DM particles are stripped from the cluster, the system mass decreases (and so does the tidal radius), and the shield becomes weaker. The red colours, on the other hand, point to a region of high dynamical heating, which becomes more intense in the centre of Fornax and favours tidal stripping of the DM shield.

This protective DM shield is a key mechanism to explain most of the DM signatures we observed in GCs embedded in DM. Although this phenomenon is ubiquitous in our GCs formed in DM mini-halos, the survival of such shield depends mostly on the orbital parameters of each cluster, and a specific discussion on this matter is addressed in section 5.1. In the following, we focus on describing the main impacts of such shield with respect to the case where GCs are not embedded in DM mini-halos.

4.2 Velocity dispersion profile
Peñarrubia et al. (2017) simulated isolated GCs embedded in DM mini-halos and showed that due to the extra DM mass, an inflation of the radial velocity dispersion profile towards outer radii is to be expected. In this work, we are able to test such predictions for a more realistic scenario, where the GC+DM subsystem is orbiting a host galaxy, and therefore experiencing tidal effects. For that, we measured the radial velocity dispersion profiles (see section A2 for details) of the bound stars in our clusters in both the cases with and without a DM mini-halo, through the course of the clusters’ evolution in the Fornax tidal field.

In Figure 6 (right plots), we show the values of the maximum radial velocity dispersion as a function of time, colour-coded by the distance to the centre of Fornax (i.e., r_{\text{ ещё}}) in kpc. One of the most important realisations of our analysis is the fact that the overall shape of the velocity dispersion is much more impacted by the host galaxy’s tidal field than by an eventual DM mini-halo: The amplitude of the dispersion, traced by its value at the peak follows a

\[ r_{1/2} \approx 20 r_{1/2} \]

Their models presented an inflated structure at roughly \( r \approx 20 r_{1/2} \), where \( r_{1/2} \) is the half-mass radius.
periodic variation, with same period than the GC orbit, and has values almost uniquely dependent on the ongoing tidal forces.

In the plot, we can clearly observe that at pericentres (blueish), the velocity dispersion inflates as a whole: the tidal heating from the host galaxy is effectively felt more intensively, leading to a higher velocity dispersion. In contrast, at apocentres (reddish or blackish), the cluster is closer from an ideal isolated scenario, and tidal heating is less effective, leading to low velocity dispersion peaks.

To illustrate the much stronger dependence on tidal forces than on possible DM mini-halos, we selected two snapshots for GC5 (one of the cases where the DM shield seemed most effective) for the case with and without a DM mini-halo (lower and upper plots, respectively). In these snapshots (see Figure 6, four left plots), we can verify that the radial velocity dispersion profile of both the DM and DM-free scenarios assume forms similar to both the isolated cases with no DM mini-halo (left), and with a massive DM mini-halo (right), as presented in Figure 2 of Peñarrubia et al. (2017).

As a general trend, all the clusters had an increasing velocity dispersion close to pericentre, with multiple points of velocity dispersion inflation throughout the GC radial extension. In apocentres, as mentioned above, the clusters resembled better to an isolated case (with one or two inflation points), specially for the case with DM mini-halo, where the shapes retrieved by Peñarrubia et al. (2017) could be better spotted. The reason behind the best resemblance in apocenter for the case with DM mini-halos is directly connected to the protective DM shield and the increase of tidal radius, which disassembles less the GC stars and approaches better the ideal isolated framework.

4.3 Compactness of the stellar distribution

An important general characteristic of GCs is that they display a very compact and dense stellar distribution. In our simulations, we observed that the distribution of stars in GCs formed in DM mini-halos is in general more compact and dense than in systems devoid of DM, as a consequence of the protective DM shield. To quantify this trend, we measured the ellipticity (also referred as flattening) and half-number radii for the stellar component of our clusters throughout their evolution, and compared to observational data from GC catalogues. We describe this results below.

4.3.1 Ellipticity (or flattening)

In order to recreate a two-dimensional ellipticity of the projected density distribution of stars, such as we observe in true data, we placed, for each snapshot, the GC centre in the position \((X_\alpha, Y_\alpha) = \)
Figure 6. Stellar velocity dispersion: Series of plots for GC5. The upper plots relate to the simulations devoid of dark matter mini-halos (●), while the lower plots indicate the results for the globular clusters formed in such mini-halos (★★). The two columns on the left display hand-picked snapshots where the radial velocity dispersion profile (i.e., $\sigma_r(r)$) resembled better to an isolated case without a dark matter mini-halo (left), and with a massive dark matter mini-halo (right), according to Figure 2 from Peñarrubia et al. (2017). The column on the right presents the evolution with time of the maximum value of the radial velocity dispersion profile (i.e., $\sigma_{r,\text{max}}$), for each scenario concerning the dark matter mini-halo, colour-coded according to the distance of the cluster to the centre of the host galaxy (i.e., $r_{\text{orbit,}⋆}$), with two vertical dashed lines corresponding to the instants from the two columns on the left. These plots argue that the tidal field from the host galaxy tends to have a much greater impact on inflating the velocity dispersion than the presence of a dark matter mini-halo. In fact, such mini-halos help to protect the cluster from tidal effects, rather than contributing to it.

Figure 7. Ellipticity and half-number radius evolution: We display the evolution of the mean ellipticity of the projected density distribution of stars of our five simulated globular clusters, in the left, and the mean 3D half-number radius evolution of our five simulated globular clusters, in the right. The mean of the five globular clusters formed in a dark matter mini-halo (★★) is in green, while the mean of the globular clusters without dark matter (●) is in red. As the values fluctuated considerably from one snapshot to another (up to a factor $\sim 2$ for the ellipticity and $\sim 5$ for the half-number radius), we actually display the running mean over the ten closest snapshots (time-wise). We compared our results of ellipticity with the Harris (1996, 2010) catalogue and our half-number radii with the catalogue of Vitral (2021), by colour-coding their $x$ or $(100 - x)$ percentiles from white to dark purple. Since the tidal interactions that Milky Way globular clusters suffered can be considerably different from the ones in the Fornax dwarf spheroidal, no strong conclusions should be derived from such a comparison, which rather aims to give a general comparative sample of typical ellipticities and half-number radii values in globular clusters. As a general trend, globular clusters embedded in dark matter mini-halos present more compact shapes (smaller ellipticities and half-number radii) than the case without dark matter, attesting the efficacy of the dark matter shield illustrated in Figures 4 and 5.
(0, 0), at five kpc away from the observer\(^7\). Next, we fitted the distribution of bound stars with a Sérisc (Sérisc 1963; Sersic 1968) asymmetric model, using the recipe described in appendix B. The fits yielded a semi-major (\(a\)) and semi-minor axis (\(b\)), from which the ellipticity (or flattening) could be calculated as

\[ e = \sqrt{1 - \frac{a^2}{b^2}}. \]  

\(7\)

Next, we constructed Figure 7 (left), by computing, at each instant, the mean ellipticity of the five simulated clusters with (green, \(\bullet\)) and without (red, \(\bullet\)) DM. The error bars were calculated as \(\left\langle (e)^2 + \sigma_n^2 \right\rangle\), where \(\langle e \rangle\) is the mean uncertainty of the ellipticity fit, and \(\sigma_n\) is the standard deviation of the values from the five clusters. As the values fluctuate (up to a factor \(\sim 2\)) from one snapshot to another, we choose to display the running mean over the closest ten points, in order to better observe the evolution of \(e\) with time. We then compare it with the values of ellipticity from the Harris (1996, 2010) catalogue\(^8\), by plotting the \(x\) and \((100 - x)\) percentiles of their catalogue colour-coded from white to dark purple. Since the tidal interactions that MW GCs suffered can be considerably different from the ones in the Fornax dSph, no strong conclusions should be derived from such a comparison, which rather aims to give a general comparative sample of typical ellipticity values in GCs.

Although we observe a decreasing pattern in the mean ellipticities of both scenarios\(^9\), one also notices that clusters embedded in a DM mini-halo show considerably smaller ellipticities than the clusters devoid of DM. Such relatively small ellipticities are indeed predictable for systems less affected by tidal forces (van den Bergh 2008), attesting the efficacy of the DM shield illustrated in Figures 4 and 5.

The comparison with the observed ellipticities in the MW (purple), which is roughly in the midway between GCs embedded and devoid of DM, could also be considered somewhat unfair, since we envisage a considerable contribution from rotation to the increase of ellipticity in GCs (Fabricius et al. 2014; Kamann et al. 2018). However, rotation is likely a remnant from the cluster’s formation or interaction with other systems (Hénault-Brunet et al. 2012; Mapelli 2017) that is not well-accounted in our simulations. Thus, it is reasonable to assume that the ellipticities of GCs embedded in DM would approach even better the ones from the galactic GCs, if rotation was not to affect ellipticity.

4.3.2 Half-number radius

Another way to measure the compactness of the cluster is to compute its half-number radius, i.e. the radius wherein half of the bound particles are. It is a straightforward parameter to look in our simulations as we just need to sort the spherical distances from the GC centre, and find the mid-point of the respective values. As before, we did that for each snapshot of the cases with and without DM.

\(8\) In Harris (1996, 2010), the ellipticity is defined as \(e = 1 - \frac{b^2}{a^2}\), so we convert it to our definition from equation 7.

\(9\) This is expected from orbiting satellites, due to the stripping of outer stars by the tidal field (Akiyama 1991).

In Figure 7 (right), we display the evolution of the mean half-number radius for the five GCs\(^{10}\), with error bars computed as the Poisson uncertainty on the mean\(^{11}\). In addition, we computed the half-number radius of the GCs analysed in Vitral (2021) by assuming spherical symmetry and subsequently converting the projected half-number radii into 3D half-number radii\(^{12}\), and plotting the \(x\) or \((100 - x)\) percentiles of this catalogue colour-coded from white to dark purple.

Once again we notice that GCs embedded in DM mini-halos present a more compact structure than in the DM-free case, which this time is characterised by smaller scale radii of the former. The comparison with Milky Way clusters is now more straightforward, as the half-number radii of the true data lies slightly below the mean radii from both scenarios of our simulations, but still closer to the case where GCs do have a DM mini-halo. In spite of the fact that the tidal fields of the MW and of Fornax are different, and therefore these comparisons with MW GCs should not be taken as a settling argument, we argue that Figure 7 still presents an encouraging propel to further studies concerning the GC formation scenario from Peebles (1984).

4.4 Tidal tails

The study of tidal tails in GCs has been revolutionised by Gaia data and simulations. For instance, a troubling question that arises when simulating GCs on tidal fields is why these simulations usually predict much more prominent tidal tails (e.g. Boldrini & Vitral 2021; Montuori et al. 2007) than what is observed for the majority of MW clusters? The answer to this question is partially given by Babbinot & Gieles (2018), who showed with simulations that there is a preferential bias towards the escaping of low-mass stars, specially in denser clusters. Such trend reduces considerably the visibility of the tails. In addition, Gieles et al. (2021) recently defended this trend by showing that the visible and extended tails of Palomar 5 are well explained by a supra-massive population of stellar-mass black holes, which is a characteristic associated with less dense GCs (Kremer et al. 2020). On the other hand, the presence of a DM mini-halo could also reduce the prominence of tidal tails, since the mini-halo is expected to be stripped beforehand GC stars, thus delaying tail formation (e.g., Bromm & Clarke 2002; Mashchenko & Sills 2005; Saitoh et al. 2006; Bekki & Yong 2012; Boldrini & Vitral 2021).

In any case, the increase of better quality data such as Gaia EDR3 has allowed to go deeper into this question: Although some GCs, such as NGC 1851 and NGC 7089 (M2), were thought not to have tails based on ground-based imaging (Kuzma et al. 2016, 2018), further Gaia studies revealed long tails associated to them (Ibata et al. 2021). However, many clusters with no tidal features, or with only

\(10\) As this value considerably fluctuates (up to a factor \(\sim 5\)) from one snapshot to another, we also display the running mean over the closest ten points (as for the plot on the left), in order to better observe the evolution of \(r_{1/2}\) with time.

\(11\) The uncertainty \(e\) on the mean of \(n\) values is \(e = \sigma / \sqrt{n}\), where \(\sigma\) is the standard deviation of those values.

\(12\) The conversion for the Plummer (Plummer 1911) profile is \(r_{1/2} = 1.305 R_{1/2} = 1.078 R_{1/2}\) for the Sersic profile, it depends on the Sersic index \(n\), so we used the deprojection routine presented in appendix A from Vitral & Mamon (2021), which employs the analytical forms from Lima Neto, Gerbal & Marquez 1999; Simonneau & Prada 2004; Vitral & Mamon 2020) in order to use the bisection method to invert the number density profile.
extended envelopes (without tails) are still present\textsuperscript{13}, and a clear understanding of which mechanisms are behind the lack of very extended tails in these GCs is still not available.

We decide to explore the impact of a DM mini-halo on tidal tails by simply plotting, in Figure 8, the stellar distribution of GCs formed in DM mini-halos (⋆, turquoise) and GCs devoid of DM (⋆, magenta) at the last snapshot of our simulations. We also provide a video of the evolution of the tails in the two kind of clusters in the footnote link\footnote{See Table 3 from Zhang & Mackey 2021.} (we display some snapshots of this video in appendix C), where different outcomes can be clearly spotted for each case. We observe much more prominent and obvious tails in the case without DM, with dense streams measuring up to $\gtrsim 20$ kpc long. In the case where the GCs are formed inside DM mini-halos however, the stellar distribution remains roughly spherical, and with a diffuse stellar envelope of much smaller dimensions than the respective tails. Namely, the extension of the tidal tails around GCs formed in DM mini-halos seems not to exceed $\sim 5-10$ kpc. We connect this difference between two kind of GCs to the protection of the DM shield (as explained in section 4.1, and in Figures 3, 4 and 5), which reduces and delays tidal effects on the GCs where it is present.

Therefore, GCs embedded in DM mini-halos are expected to have a roughly spherical, extended ($\sim 1$ kpc) stellar-envelope, and tails of mild dimension ($\sim 5-10$ kpc), while GCs devoid of DM seem to develop much longer and well-defined tails extending up to $\gtrsim 20$ kpc. Evidently, once the DM shield is destroyed, which can happen much before a Hubble time for clusters having specific orbital parameters (see section 5.1), the development of tidal tails can be considered similar to the case without DM, and thus even clusters originally embedded in DM could present extended tails by present time, in those conditions. Having said that, the DM shield has the effect of delaying tidal effects, by making them much milder while the shield is present.

5 DISCUSSION

In the previous section, we have consistently showed that, at least during the first orbits of a GC embedded in a DM mini-halo, the DM behaves as a protecting shield that prevents the formation of extended tidal tails and keeps the cluster closer to a compact, spherical shape. Now, we discuss different points such as how long this DM shield is effective and the influence of the DM in the cluster's internal dynamics.

5.1 Survival of the dark matter mini-halo

The DM mini-halo of our simulated GCs was shown to effectively work as a shield against tidal effects. However, not only this mini-halo is gradually removed by being itself stripped beforehand GC stars, as the increase of mass in the GC+DM system (with respect to the GCs devoid of DM) also brings the system closer to the centre of the host galaxy\textsuperscript{15}, where tidal effects are stronger. As a result, the DM mini-halo is expected to become negligible with time, so that the question of interest is: For how long does the DM mini-halo is well preserved in GCs?

The answer to this question is well visible when comparing the five GCs we simulated, and their different orbital parameters. First,

\textsuperscript{13} See Table 3 from Zhang & Mackey 2021.

\textsuperscript{14} Link here:
https://gitlab.com/eduardo-vitral/vitral_boldrini/-/blob/main/movie.mp4.

\textsuperscript{15} Through orbital decay (see section 3.2).
Table 1. Mean of structural parameters from the last ten snapshots, considering only bound particles and stars.

| ID  | $M_*$ [10^5 M$_\odot$] | $M_{**}$ [10^5 M$_\odot$] | $M_\star$ [10^5 M$_\odot$] | $r_{1/2}$, • [pc] | $r_{1/2}$, •• [pc] | $r_{1/2}$, ••• [pc] | $r_{\text{bound}}$, • [pc] | $r_{\text{bound}}$, •• [pc] | $r_{\text{bound}}$, ••• [pc] |
|-----|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| GC1 | 10.2                | 8.7                 | 6.1                 | 415.8               | 12.2                | 11.7                | 1391.0              | 508.5               | 508.1               |
| GC2 | 8.2                 | 9.1                 | 8.0                 | 269.7               | 13.2                | 22.4                | 770.1               | 282.6               | 281.5               |
| GC3 | 14.6                | 9.6                 | 8.8                 | 158.2               | 11.9                | 31.6                | 745.2               | 273.7               | 274.4               |
| GC4 | 11.4                | 9.7                 | 9.0                 | 317.2               | 11.9                | 13.8                | 795.4               | 289.6               | 292.1               |
| GC5 | 9.9                 | 9.8                 | 8.6                 | 164.2               | 14.9                | 45.0                | 766.7               | 280.1               | 282.2               |

Notes: Columns are: (1) Globular cluster ID; (2) Total mass of the surviving dark matter mini-halo (in 10^5 M$_\odot$); (3) Total mass of the surviving globular cluster originally embedded in a dark matter mini-halo (in 10^5 M$_\odot$); (4) Total mass of the surviving globular cluster devoid of dark matter (in 10^5 M$_\odot$); (5) 3D half-number radius of the surviving dark matter mini-halo (in pc); (6) 3D half-number radius of the surviving globular cluster originally embedded in a dark matter mini-halo (in pc); (7) 3D half-number radius of the surviving globular cluster devoid of dark matter (in pc); (8) 3D maximum radius of the bound dark matter mini-halo (in pc); (9) 3D maximum radius of the bound globular cluster originally embedded in a dark matter mini-halo (in pc); (10) 3D maximum radius of the bound globular cluster devoid of dark matter (in pc).

we can separate these five GCs into two categories: (1) The GCs where the DM shield is not effective by the end of our simulation and (2) GCs whose shield still manages to protect them against strong tidal effects. Looking at Figure 9, we can clearly assign GCs 1 and 2 to category (1), while GCs 3, 4 and 5 are better suited to category (2). This assignment is due to the fact that the limit of bound stars in GCs 1 and 2 is comparable or greater than the respective tidal radius of the GC+DM system (as shown in Figure 4), meaning that the DM shield illustrated in Figure 3 is no longer well observed.

We argue that the DM loss in GCs 1 and 2 is accelerated by the many passages in pericentre (i.e., relatively small orbital periods), as well as the relatively small orbital radii ($r_{\text{orbit}}$) throughout the clusters’ orbits. Those factors force these GCs to stay longer next to the centre of the host galaxy, where the tidal interactions are more severe. As a consequence, the DM mini-halos in GCs 1 and 2 suffer from stronger tidal effects and are stripped faster. This can also be checked in Figure 2: GC1 has a short orbital period and small values of $r_{\text{orbit}}$; GC2 has a very short orbital period and small values of...
$r_{\text{orb}}$: GC3 has a long orbital period and higher values of $r_{\text{orb}}$. GC4 has a short orbital period, but not too small values of $r_{\text{orb}}$ (so it still managed to keep the DM shield, although it is more stripped than in GCs 3 and 5); GC5 has an orbital period sufficiently long and higher values of $r_{\text{orb}}$. Hence, the DM mini-halo survives better to worse, in the following order: GC3, GC5, GC4, GC1 and GC2, respectively. In Table 1, we display the mean of the structural parameters of stars and DM particles, over the last ten snapshots of the simulation, and the relation above can be verified by jointly analysing columns (2) and (5).

### 5.2 Detectability of dark matter

If some GCs manage to preserve part of their original DM mini-halo, it is important that we understand the observational limitations that may allow us (or not) to detect this DM component. For instance, as the mini-halo mass distribution is more diffuse than the GC stellar component, one might ask if the DM amount in the GC inner regions is significant. For that purpose, we plot in Figure 10 the mass density ratio between a certain tracer (whether DM or GCs) and the stars from the GCs simulated inside DM mini-halos (i.e., ••) as a function of distance from the cluster’s centre, at the last snapshot of our simulations.

Figure 10 reveals that up to $\sim 10$ pc, the density ratio between the DM component and the stellar component of the GCs (black squares) is of the order of 1%, meaning that the influence of DM in the internal dynamics of these clusters is mostly negligible\(^{16}\). This is consistent with many past studies (e.g., Meylan & Mayor 1986; de Ven et al. 2006; Vitral & Mamon 2021) that do not account for any amount of DM when performing mass modelling of GCs, since these studies generally use cluster’s data from its inner regions, where we show that DM composes only about 1% of the mass. If one intends to investigate the dynamics in regions where the DM mini-halo influence is considerable, our plots suggest that at roughly $\sim 30$ pc, the DM exceeds 10% of the respective cluster’s mass density, and only at $\sim 60$ pc, the DM particles become more numerous, locally.

Finally, Figure 10 also compares the densities of the GCs formed in DM mini-halos and the ones devoid of it (upside down red triangles). Clearly, clusters with DM have inner stellar densities much higher than GCs devoid of DM, with GCs 3 and 5, which managed to retain most of their initial DM mini-halo, having densities up to ten times higher than their DM-free counterparts. Hence, in addition to denser clusters producing an observational bias where tidal structures are more difficultly detected (Balbinot & Gieles 2018), we point that the ones that might be formed in DM mini-halos also tend to have at the same time higher inner densities and less prominent tidal structures (see section 4.4), which can render the search for stellar streams in these GCs particularly hard.

### 6 SUMMARY & CONCLUSIONS

We have used $N$-body simulations of the globular cluster system in the Fornax dwarf spheroidal galaxy to probe the differences from globular clusters originally embedded in a dark matter mini-halo, and those devoid of it. For that, we simulated the two cases, where globular clusters have or not an enveloping dark matter mini-halo, using the initial conditions from Boldrini et al. (2020), where all tracers (stars and dark matter) are assumed as $N$-body particles, for both the globular clusters and the host galaxy. Although the extra mass of the system containing dark matter accelerates orbital decay (see Figure 2), sinking the clusters to regions of stronger tidal...
fields, the same extra mass triggers an increase of the tidal radius, which generally goes well beyond the limiting region of globular cluster bound stars (see Figure 3). As a result, this tidal radius growth causes the dark matter mini-halo to work as a protective shield (see Figure 5), which is itself stripped beforehand globular cluster stars. This process is actually applicable not only to a system composed of a globular cluster plus a dark matter mini-halo, but to any system composed of two components: a compact and dense one, of smaller dimensions (analogue to globular clusters), and a diffuse, more massive one (analogue to dark matter mini-halos). We summarise below the main results of our work, focusing on the difference between globular clusters embedded in dark matter and those devoid of it.

- The impact of a dark matter mini-halo in the stellar velocity dispersion profile of globular clusters is negligible when compared to the effect of the host galaxy tidal field. As a matter of fact, Figure 6 highlights that the presence of a dark matter mini-halo has a rather protective effect, which brings the cluster’s velocity dispersion profile closer to an isolated scenario (specially in regions of low tidal forces), with dark matter footprints more visible. The amplitude of the velocity dispersion however, is still highly (and most uniquely) dependent on the ongoing tidal forces.
- Globular clusters embedded in dark matter have denser and more compact profiles. We verified this by measuring the observed ellipticity (or flattening) and half-number radii of all our simulated clusters throughout their evolution, and further noticing considerably smaller values for clusters with dark matter (see Figure 7).
- Although both globular clusters embedded in dark matter and devoid of it do form tidal features such as stellar streams or tidal tails, the structures from dark matter-free clusters are much longer, more prominent and well-defined. In Figure 8, we display the projection of the stellar distribution of the simulated clusters in the last snapshot of our simulation, and Figure C1 highlights three snapshots of the evolution of one of the clusters, depicted in the movie at the footnote link.\(^{17}\) We are clearly able to differentiate the two cases: globular clusters embedded in dark matter form a diffuse extra-tidal envelope of much smaller dimensions than its tails, which in turn do not surpass \(\sim 5 - 10\) kpc long, while clusters devoid of dark matter loose a considerable amount of mass since the first snapshots, forming obvious streams that are not only denser, but very long, reaching up to \(\gtrsim 20\) kpc long.
- The combination of orbital parameters has a profound impact on how long the dark matter shield depicted in Figure 3 remains effective. Clusters that lie very close to the centre of the host galaxy for longer periods of time will feel a more severe tidal field, which will in turn strip the dark matter mini-halo faster (see Figure 9). This tends to happen for clusters who have simultaneously short orbital periods, low typical orbital radii and relatively high eccentricities.
- Although the dark matter mini-halo is more massive than the stellar component of globular clusters, its more diffuse profile inflicts much lower dark matter mass densities, compared to the stars, in the central regions of the system. Figure 10 shows that up to roughly 10 pc, the dark matter particles represent only \(\sim 1\%\) of the total mass in clusters where the dark matter mini-halo is maintained throughout the simulation. This sets this dark matter mini-halo as a very hard component to be detected in mass-modelling analyses, since the available data used to these sort of studies is usually incomplete or non-existent beyond farther radii. In fact, the dark matter mini-halo from our simulations only becomes denser than the stellar component at around 60 pc.
- Figure 10 also reveals that, at the end of our simulations, clusters formed in dark matter mini-halos have inner densities up to ten times higher than their dark matter-free counterparts, with higher density ratios related to clusters where the dark matter shield is more effective. Beyond \(\sim 30\) pc, however, dark matter-free globular clusters tend to become denser.

The points above represent a valuable and clear compilation of properties expected from globular originally embedded in dark matter mini-halos. With the astrometric revolution brought by \textit{Gaia} and the promises from \textit{JWST} and \textit{Euclid}, this list constitutes an utmost guidance for those wishing to search for dark matter in globular clusters, by means of the rich data sets from these missions.

In a future work, we will aim at understanding better, and more quantitatively, the mechanisms and timescales related to the stripping of dark matter mini-halos surrounding globular clusters, and we will try to accommodate it to case of the Milky Way. We will also soon provide parametrisations of dark matter mass profiles, to be used in mass-modelling routines wishing to detect dark matter in globular clusters’ outskirts. All in all, our work provides a fairer perspective when studying dark matter and dark matter-free scenarios of globular cluster formation.

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We greatly benefited from the public software 	extsc{Python} (Van Rossum & Drake 2009) packages 	extsc{BALROG} (Vitral 2021), 	extsc{SciPy} (Jones et al. 01), 	extsc{NumPy} (van der Walt et al. 2011) and 	extsc{Matplotlib} (Hunter 2007). We also used 	extsc{GeoGebra} (Hohenwarter 2002), as well as the 	extsc{Spyder} Integrated Development Environment (Raybaut 2009).

DATA AVAILABILITY

The data that support the plots within this paper and other findings of this study are available from the corresponding authors upon reasonable request.

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\(^{17}\) Link here: 
https://gitlab.com/eduardo-vitral/vitral_boldrini/-/blob/main/movie.mp4.
APPENDIX A: VELOCITY DISPERSION

We describe here how we constructed velocity dispersion profiles for the output of our simulations.

A1 Dispersion map

The velocity dispersion maps displayed in Figures 5 and 9 were constructed by first binning the projected X vs. Y map with Python's hexbin routine, setting the argument gridsize= 100. For each bin, we then computed the 1D velocity dispersion of the DM component by summing quadratically the velocity dispersion on X, Y and Z directions and normalising it by the square root of the number of dimensions (i.e., three). Whenever there were more than hundred particles inside the bin, we used all the bin’s particles, otherwise we completed the sample by picking the closest particles to the bin’s centre of mass, until the threshold of a hundred particles was attained.

The process above assured that each bin had a statistically significant number of tracers, which helped us to reach a better spatial resolution, eventually. However, as the results still presented an important amount of statistical noise, we decided to smooth our maps with Python’s guassian_filter routine, with the argument sigma= 3. We finally displayed the outcome of this procedure in a map colour-coded logarithmically from blue (lower dispersion) to red (higher dispersion).

A2 Dispersion radial profiles

To build the radial dispersion profiles from Figure 6, we used the routines angle.cart_to_sph and dynamics.dispersion from the BALROGO Python package, to first convert the data into spherical coordinates and then compute the velocity dispersion of the radial component, as a function of the distance to the cluster’s centre. The way we set BALROGO to compute this dispersion is by first dividing the radial extent into thirty equally spaced logarithmic bins and then calculating the respective dispersion and Poisson error associated to it. Next, it smooths the profile in order to remove statistical noise, by
fitting a 10-th order polynomial to it with the numpy.polyfit routine. 
1−σ regions were obtained by following the recipe presented in the footnote link\(^{18}\). We finally chose to consider the results inside a more restrict spatial extent in order to neglect the potentially bad fits of the polynomial on the borders of our data.

**APPENDIX B: ASYMMETRIC SURFACE DENSITY**

Here we describe how the BALRoGO’s position.ellipse

likelihood routine performs asymmetric Sérsic fits with \((\alpha, \delta)\) data. First, it projects the \((\alpha, \delta)\) data according to classical spherical trigonometry relations, translating it so it is centred at the origin:

\[
x_p = \cos \delta \sin (\alpha - \alpha_0), \quad (B1a) \\
y_p = \sin \delta \cos \delta_0 - \cos \delta \sin (\alpha - \alpha_0), \quad (B1b)
\]

where \((\alpha_0, \delta_0)\) is the centre of the cluster. Next, it rotates the axis so the data can be easily handled:

\[
x = x_p \cos \theta + y_p \sin \theta, \quad (B2a) \\
y = -x_p \sin \theta + y_p \cos \theta, \quad (B2b)
\]

where \(\theta\) is the angle between the original reference frame and the new one. With this new set, we are able to define a likelihood function of the stellar distribution as:

\[
L = \prod_i \frac{\Sigma(m)}{N_{tot}}. \quad (B3)
\]

with \(m = \sqrt{(x/a)^2 + (y/b)^2}\), and where \((a, b)\) are the semi-axis of the ellipse. The surface density \(\Sigma(m)\) and the number of tracers at infinity \(N_{tot}\) are defined such as in van de Ven & van der Wel (2021), so that we have:

\[
\frac{\Sigma(m)}{N_{tot}} = \frac{b_n^n \exp \left[-b_n m^{1/n}\right]}{2\pi a b n \Gamma(2n)}. \quad (B4)
\]

In equation B4, \(n\) is the Sérsic index, \(\Gamma(x)\) is the gamma function of the variable \(x\) and the \(b_n\) is computed with the precise approximation from Ciotti & Bertin (1999). Thus, the fit finds the parameters which maximise \(L\).

Finally, to derive statistical errors of our Bayesian estimates, we use Python’s numdifftools.Hessian method to compute the Hessian matrix of the probability distribution function (i.e., eq [B4]). After, we assign the uncertainties of each parameter as the square root of the respective diagonal position of the inverted Hessian matrix.

**APPENDIX C: STELLAR STREAMS**

We display, in Figure C1, three selected snapshots (0.75 Gyr, 1.5 Gyr and 4 Gyr, which corresponds to \(z = 0\)) from the evolution of GC5 in the tidal field, for the case where the GC was embedded in a DM mini-halo and where it was not. The stellar distribution is logarithmically colour-coded according to the density of points in 3D space. The full movie from where these snapshots were taken can be accessed in the footnote link\(^{19}\).

\(^{18}\) https://stackoverflow.com/questions/28505008/

\(^{19}\) Link here: https://gitlab.com/eduardo-vitral/vitral_boldrini/-/blob/main/movie.mp4.
Figure C1. Stellar streams: Three selected snapshots (0.75 Gyr, 1.5 Gyr and 4 Gyr, which corresponds to $z = 0$) from the evolution of GC5 in the tidal field, for the case where the globular cluster was embedded in a dark matter mini-halo (right) and where it was not (left). The stellar distribution is logarithmically colour-coded according to the density of points in 3D space.