ESTIMATION OF HYDRODYNAMICAL MODEL PARAMETERS FROM
THE INVARIANT SPECTRUM AND THE BOSE-EINSTEIN CORRELATIONS OF
\(\pi^-\) MESONS PRODUCED IN \((\pi^+/K^+)p\) INTERACTIONS AT 250 GeV/c

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**Abstract:** The invariant spectra of \(\pi^-\) mesons produced in \((\pi^+/K^+)p\) interactions at 250 GeV/c are analysed in the framework of the hydrodynamical model of three-dimensionally expanding cylindrically symmetric finite systems. A satisfactory description of experimental data is achieved. The data favour the pattern according to which the hadron matter undergoes predominantly longitudinal expansion and non-relativistic transverse expansion with mean transverse velocity \(\langle u_t \rangle = 0.20 \pm 0.07\), and is characterized by a large temperature inhomogeneity in the transverse direction: the extracted freeze-out temperature at the center of the tube and at the transverse rms radius are \(140 \pm 3\) MeV and \(82 \pm 7\) MeV, respectively. The width of the (longitudinal) space-time rapidity distribution of the pion source is found to be \(\Delta \eta = 1.36 \pm 0.02\). Combining this estimate with results of the Bose-Einstein correlation analysis in the same experiment, one extracts a mean freeze-out time of the source of \(\langle \tau_f \rangle = 1.4 \pm 0.1\) fm/c and its transverse geometrical rms radius, \(R_G(rms) = 1.2 \pm 0.2\) fm.
1 Introduction

Recent investigations of interference correlations of identical mesons (Bose-Einstein correlations, BEC) give evidence for a collective nature of the evolution of hadronic matter created in high-energy heavy ion collisions [1–4]. The hadronic matter flow (developed predominantly in the longitudinal direction) gives rise to dynamical correlations between space-time and phase-space coordinates of produced particles. As a consequence, the 'interferometric' radius, determining the shape of the two-particle correlation function at small relative momenta, turns out to be dependent on the kinematical variables of the particle pair. It does not measure the genuine geometrical size of the meson-emitting source, but the effective size of the source segment radiating mesons with sufficiently small relative momentum. The effective size reflects both the geometrical size and the 'homogeneity' length of the source. The latter is determined by macroscopic properties as temperature and expanding velocity profiles of the hadronic matter at freeze-out or hadronization time.

These expectations are based on hydrodynamical models (see [5–7] and references therein) and are qualitatively confirmed by experimental data concerning interferometric radii and their dependence on the di-meson rapidity and transverse mass. It is interesting, that this observation not only holds for heavy-ion collisions [1–4], but also for meson-proton collisions [8,9] for which the formation of hadronic matter with properties of macroscopic (hydrodynamical) systems is less evident.

One should stress, however, that correlation measurements do not contain the complete information on the geometrical and dynamical parameters characterizing the evolution of the hadronic matter. More comprehensive information can be provided by a combined analysis of data on two-particle correlations and single-particle inclusive spectra [5–7].

This work is, therefore, devoted to an analysis, in the framework of the hydrodynamical model for three-dimensionally expanding cylindrically symmetric systems [7], of the invariant spectra of $\pi^-$ mesons in the central rapidity region of $(\pi^+ / K^+ ) p$ interactions at 250 GeV/c and its combination with earlier results [8,9] on two-particle correlations. The measurements are performed with the help of the European Hybrid Spectrometer (experiment NA22) at the CERN SPS. The first NA22 data on the pion inclusive spectra [10] (with about half the statistics used in the present paper) show their similarity in $\pi^+$ and $K^+$ induced reactions. The present analysis, therefore, is based on the combined $\pi^+$p and $K^+$p data. Earlier results on BEC in the same data can be found in [8,9,11-14].

The hydrodynamical model parametrizations are discussed in Sect.2. Experimental data and the results of their analysis are presented and combined with the results from two-particle correlations in Sect.3. Conclusions are summarized in Sect.4.

2 The parametrizations of invariant pion spectra

The experimental data are analyzed in the framework of the hydrodynamical model for three-dimensionally expanding cylindrically symmetric systems [7]. There, the invariant spectrum of pions in rapidity $y$ and transverse mass $m_t$ is approximated by

$$f(y, m_t) = \frac{1}{N_{ev}} \frac{dN_{\pi^-}}{dy dm_t^2} = C m_t^\alpha \cosh \eta_s \exp \left( \frac{\Delta \eta^2}{2} \right) \exp \left[ -\frac{(y - y_0)^2}{2\Delta y^2} \right] \exp \left( -\frac{m_t}{T_0} \right) \times$$

$$\times \exp \left\{ \frac{\langle u_t \rangle^2 (m_t^2 - m^2)}{2T_0[T_0 + \langle u_t \rangle^2 + (\Delta T^2)m_t]} \right\},$$

with

$$\Delta y^2 = \Delta \eta^2 + \frac{T_0}{m_t}$$

$$\frac{1}{\Delta \eta^2} = \frac{1}{\Delta \eta^2} + \frac{m_t}{T_0} \cosh \eta_s,$$

$$\eta_s = \frac{y - y_0}{1 + \Delta \eta^2 \frac{m_t}{T_0}}.$$
The width \( \Delta y \) of the rapidity distribution given by (2) is determined by the width \( \Delta \eta \) of the longitudinal space-time rapidity \( \eta \) distribution of the pion emitters and by the thermal smearing width \( \sqrt{\Delta \eta^2/m_t} \), where \( T_0 \) is the freeze-out temperature (at the mean freeze-out time \( \tau_f \)) at the axis of the hydrodynamical tube, \( T_0 = T_f(\tau_f = 0) \). For the case of a slowly expanding system one expects \( \Delta \eta \ll T_0/m_t \), while for the case of a relativistic longitudinal expansion the geometrical extension \( \Delta \eta \) can be much larger than the thermal smearing (provided \( m_t > T_0 \)).

Except for the inhomogeneity caused by the longitudinal expansion, (1) also considers the inhomogeneity related to the transverse expansion (with the mean radial component \( \langle u_r \rangle \) of hydrodynamical four-velocity) and to the transverse temperature inhomogeneity, characterized by the quantity
\[
\left\langle \frac{\Delta T}{T} \right\rangle = \frac{T_0}{T_{\text{rms}}} - 1,
\]
where \( T_{\text{rms}} = T_f(r_T = r_T(\text{rms})) \) is the freeze-out temperature at the transverse rms radius \( r_T(\text{rms}) \) and at time \( \tau_f \).

The exponential parameter \( \alpha \) in (1) is related [7] to the number \( k \) of dimensions in which the expanding system is inhomogeneous. For the special case of the one-dimensional inhomogeneity \( (k = 1) \) caused by the longitudinal expansion, \( \alpha = 1 - 0.5k = 0.5 \) (provided \( \Delta \eta^2 \gg T_0/m_t \)). The transverse inhomogeneity of the system leads to smaller values of \( \alpha \). The minimum value of \( \alpha = -1 \) is achieved at \( k = 4 \) for the special case of a three-dimensionally expanding system with temporal change of local temperature during the particle emission process.

The parameter \( y_0 \) in (1) denotes the midrapidity in the interaction c.m.s. and can slightly differ from 0 due to different species of colliding particles. The parameter \( C \) is an overall normalization coefficient.

Note that (1) yields the single-particle spectra of the core (the central part of the interaction that supposedly undergoes collective expansion). However, long-lived resonances also contribute to the single-particle spectra through their decay-products. Their contribution can be determined in the core-halo picture [20] by the momentum dependence of the intercept parameter \( \lambda(y, m_t) \) of the two-particle Bose-Einstein correlation function. We determined this quantity in [8] in two different \( m_t \) windows for the reaction investigated in the present paper, and found that within three standard deviations the \( \lambda(m_t) \) parameter is approximately independent of \( m_t \). Hence this correction can be absorbed in the overall normalization.

The two-dimensional distribution (1) can be simplified for one-dimensional slices [7,15]:

1. At fixed \( m_t \), the rapidity distribution reduces to the approximate parametrization
\[
f(y, m_t) = C_m \exp \left[ -\frac{(y - y_0)^2}{2\Delta y^2} \right],
\]
where \( C_m \) is an \( m_t \)-dependent normalization coefficient and \( y_0 \) is defined above. The width parameter \( \Delta y^2 \) extracted for different \( m_t \)-slices is predicted to depend linearly on \( 1/m_t \), with slope \( T_0 \) and intercept \( \Delta \eta^2 \) (cf. (2)).

Note, that for static fireballs or spherically expanding shells (6) and (2) are satisfied with \( \Delta \eta = 0 \) [15]. Hence the experimental determination of the \( 1/m_t \) dependence of the \( \Delta y \) parameter can be utilized to distinguish between longitudinally expanding finite systems versus static fireballs or spherically expanding shells.

2. At fixed \( y \), the \( m_t^2 \)-distribution reduces to the approximate parametrization
\[
f(y, m_t) = C_y m_t^\alpha \exp \left( -\frac{m_t}{T_{\text{eff}}} \right)
\]
where \( C_y \) is a \( y \)-dependent normalization coefficient and \( \alpha \) is defined as above.

The \( y \)-dependent "effective temperature" \( T_{\text{eff}}(y) \) can be approximated as
\[
T_{\text{eff}}(y) = \frac{T_s}{1 + \alpha(y - y_0)^2},
\]
where \( T_s \) is an overall normalization.
where \( T_\ast \) is the maximum of \( T_{\text{eff}}(y) \) achieved at \( y = y_0 \), and

\[
a = \frac{T_0 T_\ast}{2m_π^2 (\Delta \eta^2 + \frac{T_0}{m_π})^2}
\]

with \( T_0 \) and \( \Delta \eta^2 \) as defined above.

The approximations (6) and (7) explicitly predict a specific narrowing of the rapidity and transverse mass spectra with increasing \( m_t \) and \( y_t \), respectively (cf. (2) and (8)). The character of these variations is expected [15] to be different for the various scenarios of hadron matter evolution.

3 The results

3.1 The data

The experimental setup and the data-handling procedure are described in [10,16]. The results of this paper are based on the analysis of 104145 events of \((\pi^+/K^+)p\) interactions containing at least one negative track with momentum resolution better than 4\% (depending on the momentum, the average momentum resolution varies from 1\% to 2.5\%). All negative particles are assumed to have the pion mass. The contamination from other particles is estimated to be \((7 \pm 3)\% \) [10]. Candidates for single diffraction dissociation with charge multiplicity \( n \leq 6 \) are excluded. For each event, a weight is introduced in order to normalize to the non-single diffractive topological cross sections [16].

The analysis is restricted to centrally produced \( \pi^- \)-mesons with cms rapidity \(|y| < 1.5\). The total number of pions used in the analysis is about 397 k.

3.2 The rapidity distribution

The rapidity distributions fitted by (6) are given in Fig. 1 for 23 \( m_t \) slices and \( m_t \) values ranging from 0.16 to 0.63 GeV. The fit quality is satisfactory for all slices \((0.7 < \chi^2/NDF < 1.5)\). The fitted \( \Delta \eta^2 \) values plotted in Fig. 2 demonstrate the widening of the \( y \)-distribution with increasing \( 1/m_t \). According to (2), the linear dependence of \( \Delta \eta^2 \) on \( 1/m_t \) is described by intercept \( \Delta \eta^2 = 1.91 \pm 0.12 \) and slope \( T_0 = 159 \pm 38\text{MeV} \). Thus, the width of the \( y \)-distribution is dominated by the spatial (longitudinal) distribution of pion emitters (inherent to longitudinally expanding systems) and not by the thermal properties of the hadron matter, as would be expected for static or radially expanding sources.

3.3 The \( m_t \) distribution

The \( m_t \)-distributions fitted by (7), are given in Fig. 3 for 11 \( y \)-slices for \( y \)-values between -1.5 to +1.5. For all slices \( \chi^2/NDF < 1.4 \). The fitted values of \( T_{\text{eff}}(y) \) (Fig. 4) tend to decrease with increasing \(|y|\) and approximately follow parametrization (8) with \( T_\ast = 160 \pm 1 \text{MeV}, a = 0.083 \pm 0.007 \) and \( y_0 = -0.065 \pm 0.039 \). Note, however, that the fit of \( T_{\text{eff}}(y) \) by (8) has low confidence level: \( \chi^2/NDF = 29/11 \). This large value is due to an asymmetry in the \( T_{\text{eff}} \) distribution with respect to \( y = 0 \). Except for the last point, \( T_{\text{eff}} \) is higher in the meson than in the proton hemisphere.

Note, furthermore, that the small value of the parameter \( a \) is the result of the comparatively large longitudinal extension \( \Delta \eta \) of the pion source (cf. (9)). Note also the consistency between the quoted value of \( a \) and the value \( a \approx 0.07 \) calculated from (9) at values of the parameters \( \Delta \eta^2, T_0 \) and \( T_\ast \) extracted from two different fits of the one-dimensional spectra (6) and (7).

The fitted values of the exponential parameter \( \alpha \) in (7) are near zero (varying between \(-0.02 \pm 0.02\) and \(0.14 \pm 0.02\) for different \( y \)-slices). These small values of \(|\alpha|\) correspond to a two-dimensional inhomogeneity of the expanding system \((\alpha = 1 - 0.5k)\). One concludes, therefore, that apart from a longitudinal inhomogeneity caused by the relativistic longitudinal flow, the hadron matter can also possess a transverse inhomogeneity (caused by transverse expansion or a transverse temperature gradient) or undergoes a temporal change of local temperature during the particle emission process.
3.4 The transverse direction

Further information on hadron-matter evolution in the transverse direction can be extracted from a more detailed analysis using the two-dimensional parametrization (1) with parameters \((u_t)\) and \((\Delta T^\parallel)\) characterizing the strength of the transverse expansion and temperature inhomogeneity. The results of the fit are presented in Table 1.

The fitted value of the exponential parameter of \(\alpha = 0.26 \pm 0.02\) corresponds to an effective number \(k_{\text{eff}}\) of dimensions in which the expanding system is inhomogeneous, \(k_{\text{eff}} \approx 1.5\), definitely in excess of \(k = 1\) corresponding to a one-dimensional (longitudinal) inhomogeneity. This can be at least partly attributed to a transverse inhomogeneity of the system. The moderate value of the mean transverse four-velocity \(\langle u_t \rangle = 0.20 \pm 0.07\) (Table 1) indicates that the transverse inhomogeneity is only to small extent caused by a (non-relativistic) transverse expansion. It is mainly stipulated by a rather large temperature inhomogeneity characterized by the fit result of \((\Delta T^\parallel) = 0.71 \pm 0.14\). Using (5), one infers that the freeze-out temperature decreases from \(T_0 = 140 \pm 3\) MeV at the central axis of the hydrodynamical tube to \(T_{\text{rms}} = 82 \pm 7\) MeV at a radial distance equal to the transverse rms radius of the tube.

3.5 Combination with two-particle correlations

As already mentioned in the introduction, more comprehensive information on geometrical and dynamical properties of the hadron matter evolution are expected from a combined consideration of two-particle correlations and single-particle inclusive spectra [5–7].

The two-particle correlation function \(K_2(p_1,p_2)\) at small momentum difference \(q = p_1 - p_2\) is often approximated by a Gaussian function [17]:

\[
K_2(p_1,p_2) \sim \exp[-R^2_{1q} - R^2_{0q} - R^2_{s quadratic}],
\]

where the three orthogonal components \(q_L, q_{out}, q_{side}\) of the vector \(q\) are oriented, respectively, along the collision axis, along and perpendicular to the pair transverse momentum; \(R_L, R_o, R_s\) are, respectively, the longitudinal, 'out' and 'side' effective dimensions of the source segment radiating the BE correlated pion pairs. Due to the non-static nature of the source, these effective sizes vary with the average transverse mass \(M_t = \frac{1}{2}(m_{t1} + m_{t2})\) and the average rapidity \(Y = \frac{1}{2}(y_1 + y_2)\) of the pion pair. In the 'longitudinal c.m.s.' (LCMS) [18], where \(Y = 0\), the effective radii can be approximately expressed as \([7,15,19]\):

\[
R^2_L = \tau^2_\parallel \Delta \eta^2_t
\]

\[
R^2_o = R^2_s + \beta^2_\parallel \Delta \tau^2_t
\]

\[
R^2_s = \tau^2_\parallel \Delta \eta^2_t
\]

with

\[
\frac{1}{\Delta \eta^2_t} = \frac{1}{\Delta \eta^2_t} + \frac{M_t}{T_0}
\]

\[
R^2_G = \frac{R^2_G}{1 + \frac{M_t}{T_0} \langle u_t \rangle^2 + \langle \Delta T^\parallel \rangle^2}
\]

where parameters \(\Delta \eta^2_t, T_0, \langle u_t \rangle\) and \(\langle \Delta T^\parallel \rangle\) are defined and estimated above from the invariant spectra; \(R_G\) is related to the transverse geometrical rms radius of the source as \(R_G(\text{rms}) = \sqrt{2}R_G\); \(\tau_\parallel\) is the mean freeze-out (hadronization) time; \(\Delta \tau_t\) is related to the duration time \(\Delta \tau\) of pion emission and to the temporal inhomogeneity of the local temperature; if the latter has a small strength (as one can deduce from the restricted inhomogeneity dimension estimated above: \(k_{\text{eff}} \approx 1.5\)), an approximate relation \(\Delta \tau \geq \Delta \tau_t\) holds; the variable \(\beta_\parallel\) is the transverse velocity of the pion pair.

Relations (11)–(15), combining the results of the single-particle invariant spectrum and the two-particle correlation function, allow one to extract additional information on the parameters \((\tau_\parallel, \Delta \tau_t, R_G)\) characterizing the space-time evolution of hadron matter.

The interferometric radii \(R_L, R_o, R_s\) were extracted recently [8,9] in the same experiment from the correlations of negative pions. To match with parametrization (10), the radii quoted in [8,9] are divided here by a factor \(\sqrt{2}\).
The effective longitudinal radius $R_L$, extracted for two different mass ranges, $M_t = 0.26 \pm 0.05$ and $0.45 \pm 0.09$ GeV/$c^2$ are found to be $R_L = 0.93 \pm 0.04$ and $0.70 \pm 0.09$ fm, respectively. This dependence on $M_t$ matches well the predicted one. Using (11) and (14) with $T_0 = 140 \pm 3$ MeV and $\Delta \eta^2 = 1.85 \pm 0.04$ (Table 1), one finds that the values of $\tau_1$ extracted for the two different $M_t$-regions are similar to each other: $\tau_1 = 1.44 \pm 0.12$ and $1.36 \pm 0.23$ fm/c. The averaged value of the mean freeze-out time is $\tau_1 = 1.4 \pm 0.1$ fm/c.

Since we find that $\Delta \eta$ is significantly bigger than 0, static fireballs or spherically expanding shells, that were found to be able to describe our two-particle correlation data in [8], fail to reproduce our single-particle spectra.

The transverse-plane radii $R_\perp$ and $R_\parallel$ measured in [8,9] for the whole $M_t$ range are: $R_\perp = 0.91 \pm 0.08$ fm and $R_\parallel = 0.54 \pm 0.07$ fm. Substituting in (12) and (13), one obtains (at $\beta_t = 0.484c$ [8]): $\Delta \tau_T = 1.3 \pm 0.3$ fm/c. The mean duration time of pion emission can be estimated as $\Delta \tau = \Delta \tau_T$, which might be that the radiation process occurs during almost all the hydrodynamical evolution of the hadronic matter produced in meson-proton collisions.

An estimation for the parameter $R_G$ can be obtained from (13) and (15) using the quoted values of $R_\parallel, T_0, \langle u_t \rangle$ and $\langle \Delta \tau^* \rangle$ at the mean value of $\langle M_t \rangle = 0.31 \pm 0.04$ GeV/$c$ (averaged over the whole $M_t$-range): $R_G = 0.88 \pm 0.13$ fm. The geometrical rms transverse radius of the hydrodynamical tube, $R_G(rms) = \sqrt{2}R_G = 1.2 \pm 0.2$ fm, turns out to be larger than the proton rms transverse radius.

3.6 Estimation of systematic errors

The analytic formulae used to estimate the hydrodynamical model parameters were evaluated under certain approximations. As shown in [21], an accuracy better than 10-20% is reached under the following conditions:

a) 
\[ \frac{\beta_t \langle u_t \rangle R_\pi^2}{r_0 R_G T_0 / M_t} < 0.6 \]  
(16)

With our parameter values, the l.h.s. of (16) turns out to be less than 0.13 for the whole range of $M_t$;

b) Additionally, the validity of the formulae (1)-(9) for the invariant spectra requires:

\[ \frac{|y - y_0|}{1 + \Delta \eta^2 M_t / T_0} < 1 \]  
(17)

With our parameter values, the l.h.s. of (17) turns out to be less than 0.52 for the full ranges of $y$ and $m_t$ considered.

c) Formulae (10)-(15) for the two-particle correlation function are derived under the additional, more stringent condition:

\[ \frac{|Y - y_0|}{1 + \Delta \eta^2 (M_t / T_0 - 1)} < 1 \]  
(18)

At the maximum value of $|Y - y_0|$, (18) is fulfilled, except for a fraction of 8% of pion pairs with $M_t < 0.18$ GeV/$c^2$. In the full range of $|Y - y_0| < 1.5$ considered, this is reduced to less than 4% of pion pairs. Note further that the upper limit of 10-20% relative errors is reached only at the lower limit of inequalities (16-18) and that the squared relative errors increase quadratically with the increase of the left-hand-sides of (16-18).

Hence we estimate an upper limit of order $10-20\% \times (0.13/0.6) \times (0.52/1.0) \times 1.0 = 1-2\%$ for the precision of the analytic formulae used in the fits, except for 4% of the pion pairs where the relative systematic errors may reach 10-20%. This accuracy is well within the mean experimental (statistical) errors of the measured two-dimensional invariant spectra, analyzed in the present work (about 10%) and of the three-dimensional correlation function (about 15%), analyzed in [8,9].

3.7 The space-time distribution of $\pi$ emission

On Figure 5 we show a reconstruction of the space-time distribution of pion emission points, expressed as a function of the cms time variable $t$ and the cms longitudinal coordinate $z$. The momentum-
integrated emission function along the z-axis, i.e., at \( r_1 = (r_x, r_y) = (0, 0) \) is given by

\[
S(t, z) \propto \exp \left( -\frac{(\tau - \tau_0)^2}{2 \Delta \tau^2} \right) \exp \left( -\frac{(\eta - \eta_0)^2}{2 \Delta \eta^2} \right).
\]

It relates the parameters fitted to our data with particle production in space-time. Note that the coordinates \((t, z)\), can be expressed with the help of the longitudinal proper-time \(\tau\) and space-time rapidity \(\eta\) as \((\tau \cosh(\eta), \tau \sinh(\eta))\).

We find a structure looking like a boomerang, i.e., particle production takes place close to the regions of \(z = t\) and \(z = -t\), with gradually decreasing probability for ever larger values of space-time rapidity. Although the mean proper-time for particle production is \(\tau_f = 1.4 \text{ fm/c}\), and the dispersion of particle production in space-time rapidity is rather small, \(\Delta \eta = 1.35 \text{ fm}\), we still see a characteristic long tail of particle emission on both sides of the light-cone, giving a total of 40 fm maximal longitudinal extension in \(z\) and a maximum of about 20 fm/c duration of particle production in the time variable \(t\).

4 Summary

The invariant spectra of \(\pi^-\)-mesons produced in \((\pi^+ / K^+)\) p-interactions at 250 GeV/c are analyzed in the framework of the hydrodynamical model of a three-dimensionally expanding cylindrically symmetric finite system [7].

The data favour a picture according to which the hadron matter undergoes extensive longitudinal expansion with a space-time rapidity width of \(\Delta \eta = 1.36 \pm 0.02\) and a non-relativistic transverse expansion with the mean transverse four-velocity \(\langle u_t \rangle = 0.20 \pm 0.07\).

The hadron matter possesses a large temperature inhomogeneity in the transverse direction: the freeze-out temperature varies from \(T_0 = 140 \pm 3\ \text{MeV}\) at the central axis of the hydrodynamical tube to \(T_{\text{rms}} = 82 \pm 7\ \text{MeV}\) at a radial distance equal to the transverse rms radius of the tube.

The information from the single-particle invariant spectrum is combined with that from the two-particle correlation function. The transverse mass dependence of the width \(\Delta y\) of the single-particle \(y\)-distribution and that of the interferometric longitudinal radius \(R_L\) are found to be consistent. This allows to extract the mean freeze-out (hadronization) time of the strongly interacting matter: \(\tau_f = 1.4 \pm 0.1\ \text{fm/c}\). The duration time of pion emission is estimated to be \(\Delta \tau \geq 1.3 \pm 0.3\ \text{fm/c}\), i.e. close to \(\tau_f\), indicating that the emission process occurs during almost all the hydrodynamical evolution. As an estimate for the transverse geometrical rms radius of the hydrodynamical tube we obtain: \(R_{G}(\text{rms}) = 1.2 \pm 0.2\ \text{fm}\).

The combined analysis of single-particle spectra and Bose-Einstein correlation functions increases the selective power of the analysis. Static (Kopylov-Podgoretskii) fireballs and spherically expanding shells, that were found to be able to reproduce our interferometric data in [8] fail to reproduce simultaneously the single-particle spectra and the BE correlations in the NA22 experiment.

The systematic errors on the quoted parameters related to the limited accuracy of the approximate analytic formulae (used in the present study) are estimated to be of the order of or smaller than the quoted statistical errors.

As far as we know, this is the first time that a full reconstruction of the space-time distribution of the particle emitting source is obtained in hadron-hadron reactions from a combined analysis of single-particle spectra and Bose-Einstein correlation measurements.

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Table 1: Fit results according to parametrization (1) for $|y| < 1.5$. 

| $\alpha$ | $\Delta \eta$ | $T_0$ (GeV) | $y_0$ | $\langle u_4 \rangle$ | $\langle \frac{\Delta T}{T} \rangle$ | $\chi^2$/NDF |
|--------|-------------|-------------|-------|-----------------|------------------|--------------|
| 0.26 $\pm 0.02$ | 1.36 $\pm 0.02$ | 0.140 $\pm 0.003$ | 0.082 $\pm 0.006$ | 0.20 $\pm 0.07$ | 0.71 $\pm 0.14$ | 642/683 |
Figure Captions

Fig. 1 The rapidity distributions of centrally produced pions (|y| < 1.5) for different \( m_t \)-slices given. The curves are the fit results obtained according to parameterization (6).

Fig. 2 The \( (1/m_t) \)-dependence of \( (\Delta y)^2 \) for inclusive \( \pi^- \) meson rapidity distributions at |y| < 1.5. The straight line is the fit result according to parametrization (2).

Fig. 3 The \( m_t \)-distribution of centrally produced pions for different y-slices, as indicated. The solid lines are the fit results obtained according to parameterization (7).

Fig. 4 \( T_{\text{eff}} \) as a function of y fitted according to parametrization (8).

Fig. 5 The reconstructed \( S(t, z) \) emission function in arbitrary units, as a function of time \( t \) and longitudinal coordinate \( z \). The best fit parameters of \( \Delta \eta = 1.36, y_0 = 0.082, \Delta \tau = 1.3 \text{ fm/c} \) and \( \tau_f = 1.4 \text{ fm/c} \) are used to obtain this plot.
Fig. 1
Fig. 2

\[ \Delta y^2 \] vs. \[ \frac{1}{m_t}, \text{GeV}^{-1} \]
Fig. 3
Fig. 4

\( \frac{\pi^+}{K^+}/p \ 250 \text{ GeV/c (NA22)} \)
Fig. 5