We study under which conditions a scalar particle is a viable WIMP Dark Matter candidate with Higgs and dilaton interactions. The theory is a composite Higgs model with top partial compositeness where both the Higgs and the Dark Matter candidate arise as pseudo Goldstone boson of the coset $SO(6)/SO(5)$ from a new physics sector. We highlight the role of the dilaton in direct and indirect searches. We find that a Dark Matter particle with a mass around 200-400 GeV and a relatively light dilaton are a fair prediction of the model.

I. INTRODUCTION

Light scalars are believed to be unlikely in Nature, unless there is a fine tuning or there exists an underlying dynamics screening the quadratic ultraviolet sensitivity. Indeed the Standard Model (SM) suffers from the hierarchy problem because of the Higgs boson: an interesting possibility is that the Higgs boson, rather than an elementary particle, is a composite object, a bound state of a new, yet undiscovered, interacting theory which gets strong at the TeV scale. In particular the idea that the Higgs is not only a composite object but a pseudo Nambu Goldstone boson (pNGB), like pions in QCD, is especially appealing, because of the approximate built in shift symmetry.

From a different perspective, also the Dark Matter (DM) density in the Universe could be accounted for by a scalar particle, again subject to the same naturalness issue, and if it is a weakly interacting massive particle (WIMP), its mass should be broadly in the TeV range. Therefore a very compelling picture emerges if a single new strongly interacting sector is responsible for both the Higgs and the DM. We pursue this approach in a next to minimal pNGB Composite Higgs Model (CHM), based on the symmetry breaking coset $SO(6)/SO(5)$: it includes a custodial $SO(4)$ and it is exactly described by five Goldstone modes, a doublet $H$ and a singlet $\eta$. This coset, or the isomorphic $SU(4)/Sp(4)$, can be formulated in an underlying theory of fundamental techni-quarks and it has already received some attention [1–6]. If $\eta$ is sufficiently stable it is a perfect DM candidate: this is achieved if the theory respects a global $Z_2$ symmetry under which $\eta$ is odd. The main difference with the case of elementary scalars is in the form of the interactions. This very predictive setup has already been explored [7, 8]. We want to extend the analysis assuming that the strong sector provides a second DM portal to SM particles: on top of Higgs exchange the dilaton could play an important role, if the strong sector is an approximate Conformal Field Theory (CFT) and it features a light dilaton. A light dilaton is also a rare phenomenon in spontaneously broken CFTs in the sense that it requires fine tuning, [9–13], but if present it affects the DM phenomenology, if it is a different state than the Higgs scalar. We will show how in our model the light dilaton affects the DM phenomenology, mainly fixing a lighter DM mass; moreover it gives the dominant contribution to Sommerfeld enhanced processes. The dilaton portal in composite DM models has been studied in [14], but neglecting Higgs effects. A complete picture including both is the main object of our present work. In [15] a similar interplay was studied, but without the pNGB structure.

The rest of the paper is organized as follows. After defining an effective Lagrangian in section II, including the other composite resonances typically considered in CHM, we introduce the dilaton field $\sigma$ and we derive the interactions between the light scalars, $h$, $\eta$ and $\sigma$, and the SM fermions and vectors in section III. We move to DM properties, starting from the computation of the relic density, section IV, to direct and indirect constraints, in section V and VI respectively. We take into account collider constraints in section VII. Finally we summarize and we draw our conclusions in section VIII.
II. THE SO(6)/SO(5) MODEL

A. Scalar Sector

The new physics sector, behaving as a CFT, is perturbed by a deformation, which becomes strong at an energy scale around the TeV. It possesses, in isolation, an approximate global SO(6) symmetry, spontaneously broken to SO(5). As a result five pseudo Goldstone bosons are introduced as a CFT, which becomes strong at an energy scale around the TeV. It possesses, in isolation, an approximate global SO(6) symmetry, spontaneously broken to SO(5). As a result five pseudo Goldstone bosons are introduced as a CFT, which becomes strong at an energy scale around the TeV.

\[ V(h, \eta, \chi) = \frac{\mu_0^2}{2} h^2 + \frac{\mu_1^2}{2} \eta^2 + \frac{\lambda_1}{2} \eta h^4 + \frac{\lambda_2}{4} \eta^4 + \frac{\lambda_3}{4} \eta^2 h^2. \]  

We limit to models in whose vacuum the ElectroWeak (EW) symmetry is broken

\[ h = \langle h \rangle + \sqrt{1 - \xi h_{\text{phys}}}, \quad \eta = 0, \]  

where \( \langle h \rangle = v = f \sqrt{\xi} \simeq 246 \text{ GeV} \) and we work in the assumption of \( v \ll f \).

B. Composite Resonances

1. Fermion Resonances

In order to generate fermion Yukawa couplings and the effective potential of the composite Higgs and the composite DM, we adopt the partial compositeness scenario [18]. Additionally, when we formally embed the SM fermions in SO(6) representations, the embedding should preserve the Z_2 symmetry stabilizing the DM. According to [7, 8], we embed the left and right handed fermions in the fundamental representation of SO(6):

\[ \xi_L^u = \frac{1}{\sqrt{2}} \left( \begin{array}{c} b_L & t_L & 0 \end{array} \right)_{2/3}, \]  

where we focus on the top quark and the subscript is the X charge assignment necessary to reproduce the top hypercharge. Other quarks and leptons can be embedded in a similar way, or could receive their mass from a different mechanism, as bilinear Yukawa-like interactions [19–21]. Partial compositeness is introduced as

\[ \mathcal{L} \simeq e\psi \bar{\psi} M \phi + h.c. \] 

According to the CCWZ formalism, at low energy, \( O \psi \) can be represented as a function of \( U(x) \) and \( \Psi \), where \( U \) is the NGB matrix and \( \Psi \) is a collection of SO(5) fields. We focus for definiteness and for simplicity on cases of \( \Psi \) resonances \( S_L \) and \( F_L \) transforming in the trivial and in the fundamental representation of SO(5). Details on the Lagrangian can be find in Appendix A, where we also show how the effects of the heavy resonances can be encoded in form factors.

2. Vector Resonances

Vector resonances are generically expected as well as fermion resonances. For simplicity we present one adjoint vector resonance \( \rho_{\mu} \) and one fundamental vector resonance \( a_{\mu} \), introduced following [22]: again we refer to Appendix A for detailed expressions.

III. DILATON EXTENSION OF THE COMPOSITE HIGGS MODEL

As we previously stated the strong sector in isolation is a CFT enjoying a global SO(6) symmetry. In the vacuum both the conformal and the global symmetry are spontaneously broken. In this section we want to specify the general relations given in section II including the dilaton field. The dilaton dependence is introduced promoting \( f \) to be a dynamical field \( \chi = f e^{\sigma/f} \) and dressing composite fields with the appropriate powers of \( \chi/f \). Notice that for simplicity we identify the scale associated to the dilaton \( f_{\sigma} \) with \( f \). The Goldstone kinetic term becomes

\[ \mathcal{L} \simeq \frac{\chi^2}{4} \text{Tr}[d_\mu d^\mu]. \]
\( \chi/f \). At energies below the masses of the resonances the effective Lagrangian is

\[
\mathcal{L}_{\text{eff}} \supseteq \sum_{I} \Pi_{tI} \Phi_L + \sum_{I} \Pi_{tI} \Phi_R - (\Pi_{tI} t_R \Phi_{t R} + \text{h.c})
\]

\[
+ \frac{\mu}{2} (\Pi_{\theta} \Phi L + \Pi_{\theta} \Phi^* L)
\]

\[
+ \frac{\mu}{4} (\Pi_{\theta} \Phi L \Phi_r L + \Pi_{\theta} \Phi^* L \Phi_r L)
\]

where the form factors are modified by the presence of the model, given its unpredictability.

The scalar potential \( V(h, \eta, \chi) \) is obtained integrating out the SM top and vector bosons with a standard interaction loop computation. We briefly review the results of this computation in the following.

### A. The Scalar Potential

The gauge contribution to the scalar effective potential \( V_g(h, \eta, \chi) \) is

\[
V_g = \frac{3}{2} \int \frac{d^4 p_E}{(2\pi)^4} \left[ 2 \log[\Pi_{WW}(-p_E^2)] \right.
\]

\[
+ \log[\Pi_{BB}(-p_E^2)] - \Pi_{WW}(-p_E^2)
\]

Notice that as a result no potential is generated for \( \eta \). Fermion loops in principle all the possible terms containing Higgs and \( \eta \) fields, but the case \( N_F = N_S = 1 \) leads to the unsatisfactory prediction \( \mu_\eta = \lambda_\eta = 0 \). Therefore we move to the next to minimal case, namely \( N_F = 1, N_S = 2 \). The fermion contribution to the effective potential \( V_f(h, \eta, \chi) \) is computed from

\[
V_f = -2N_c \int \frac{d^4 p_E}{(2\pi)^4} \log(p_E^2 \Pi_{tL} \Pi_{tR} + \Pi_{tL}^2) .
\]

We impose the generalized Weinberg sum rules [23] and in order to get unsuppressed \( \mu_\eta \) and \( \lambda \), we assume \( m_{1S} >> m_F >> m_{1S} \sim f \).

There is one subtlety: loops of top quarks, due to the large top Yukawa, induce a mixing between the Higgs and the dilaton field. Indeed the most general Lagrangian takes the form

\[
V_f(h, \eta, \chi) = \frac{\chi^2}{4} \sum_{i+j<3} \lambda_{ij} \chi^2 h^2 \eta^{2i}
\]

where \( \gamma \) is the top anomalous dimension [13]. Therefore

\[
\frac{<\partial_i \partial_j V>}{<\partial_i \partial_j V>} \approx \gamma_v
\]

and we get that the mixing is proportional to the top anomalous dimension: since \( \gamma \approx 0 \) we safely neglect it. Similarly the Higgs radion mixing has been studied in a warped extra dimensional background and argued to be small for a pNGB Higgs [24]. We refer to Appendix A3 for a discussion on the dilaton potential. In the following we are going to treat the dilaton mass as a free parameter of the model, given its unpredictability in an effective description.

### B. Interactions with Massless Gauge Bosons

The precise determination of interaction couplings between scalars such as dilaton, DM, and Higgs and gauge bosons is of primary importance in order to study LHC phenomenology and various aspects of DM detection. We therefore proceed in analyzing them.

First, we study the dilaton. It couples to gauge bosons via trace anomaly terms, which depend on the beta functions of the theory, and via triangle diagrams generated by loops of charged fields [9, 14, 25–28]:

\[
\mathcal{L} \supseteq \frac{\alpha_s}{8\pi} (b_{IR}^3 - b_{UV}^3 + \frac{1}{2} F_{1/2}(x)) \sqrt{f} \mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu}
\]

\[
+ \frac{\alpha_m}{8\pi} (b_{IR}^m - b_{UV}^m + \frac{4}{3} F_{1/2}(x)) - F_1(xW) \mathcal{G}_{\mu\nu} \mathcal{F}_{\mu\nu}^W
\]

where \( x_i = 4m_i^2/m_\sigma^2 \). \( F_{1/2} \) and \( F_1 \) are loop functions defined as

\[
F_{1/2}(x) = 2x(1 + (1 - x)f(x)),
\]

\[
F_1(x) = 2 + 3x + 3x(2 - 1)f(x),
\]

\[
f(x) = \begin{cases} \arcsin(1/\sqrt{x}) & \text{if } x \geq 1 \\ - \frac{1}{2} \log(1 + \frac{1}{x} - \frac{1}{x^2}) & \text{if } x < 1. \end{cases}
\]

The loops of heavy top partners cancel with the IR beta function of the same in the limit of masses larger than \( m_\sigma/2 \), as we discuss in Appendix B. Therefore the top partners decouple and the only effects from the IR are from the light degrees of freedom.

Among the light composite states we count the Higgs boson doublet, which enters the beta function coefficients with

\[
b_{IR}^3 = - \frac{1}{6}, \quad b_{IR}^1 = - \frac{1}{6}.
\]

In case the right handed top is fully composite then

\[
b_{IR}^3 = - \frac{1}{3}, \quad b_{IR}^1 = - \frac{8}{27} N_c,
\]

while it does not contribute to the composite beta functions if it is elementary. As a result the IR beta function coefficients are

\[
b_{IR}^3 \approx 0, \quad b_{IR}^m = - \frac{1}{3},
\]

or

\[
b_{IR}^3 = - \frac{1}{3}, \quad b_{IR}^m = - \frac{11}{9}
\]

if also \( t_R \) belongs to the composite fields. The UV coefficients \( b_{UV}^3, b_{UV}^m \) are model dependent and we cannot specify them in our effective construction. Since they enter the couplings of the dilaton in the following discussion we will focus on simple benchmark values.

We now turn to Higgs couplings. According to [28] the effect of composite fermion loops is expected to be negligible and the main contribution is given by top loops, closely resembling the SM result:

\[
\begin{align*}
\mathcal{L} & \supseteq \frac{\alpha_s}{8\pi} (b_{IR}^3 - b_{UV}^3 + \frac{1}{2} F_{1/2}(x)) \sqrt{f} \mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} \\
& \quad + \frac{\alpha_m}{8\pi} (b_{IR}^m - b_{UV}^m + \frac{4}{3} F_{1/2}(x)) - F_1(xW) \mathcal{G}_{\mu\nu} \mathcal{F}_{\mu\nu}^W
\end{align*}
\]
at one loop. Given the coupling of \( \eta \) fermions, we have DM to gauge bosons interactions

\[ \mathcal{L} \supset \frac{\alpha_{em}}{8\pi} \frac{1 - 2\xi}{\sqrt{1 - \xi}} m_q^2 + \sqrt{1 - \xi} F_1(\frac{4m_W^2}{m_h^2}) \frac{h}{\sqrt{v}} F_{\mu\nu}F^{\mu\nu} \]

\[ + \frac{\alpha_s}{12\pi} \frac{1 - 2\xi}{\sqrt{1 - \xi}} h G^{\mu\nu}_{\mu\nu} . \tag{20} \]

Similarly, since DM couples at tree level to SM fermions, we have DM to gauge bosons interactions at one loop. Given the coupling of \( \eta \) to fermions [65]

\[ \mathcal{L} \supset \frac{\xi}{2(1 - \xi)} m_\psi \overline{\psi}_\psi \eta^2 \frac{1}{v^2} \tag{21} \]

we easily read the couplings to gauge bosons

\[ \mathcal{L} \supset - \frac{\alpha_{em}}{32\pi} F_{1/2}(\frac{4m_q^2}{m_\eta^2}) \frac{\xi}{1 - \xi} \frac{\eta^2}{v^2} G^{a\mu\nu} G^{a\mu\nu} \]

\[ - 3\alpha_{em} \frac{\xi}{16\pi} \frac{\eta^2}{1 - \xi} v^2 F_{1/2}(\frac{4m_q^2}{m_\eta^2}) F_{\mu\nu} F^{\mu\nu} . \tag{22} \]

We neglect possible couplings of \( \eta \) to pair of gauge bosons arising from the Wess-Zumino-Witten term, they could be computed in principle given the details of the fundamental underlying theory, as done in [29].

C. Effective Lagrangian

An effective Lagrangian for the SM fields, the DM candidate \( \eta \) and the dilaton \( \sigma \) is obtained, expanding the scalars around their VEV

\[ h = v + \sqrt{1 - \xi} h_{phys}, \quad \eta = \eta_{phys}, \quad \sigma = \sigma_{phys}. \tag{23} \]

The resulting Lagrangian, the starting point of our phenomenological analysis, has the following form:

\[ \mathcal{L} \supset \frac{1}{2} (\partial_\mu h)^2 (1 + a_{hh} \frac{h}{v^2} + b_{hh} h^2 + b_{\eta h} \eta^2) e^{2\sigma/\mathcal{F}} \]

\[ + \frac{1}{2} (\partial_\mu \eta)^2 (1 + b_{\sigma \eta} \eta^2) e^{2\sigma/\mathcal{F}} \]

\[ + (\partial_\mu h \partial_\nu \eta)(c_{\eta h} \eta^2 + d_{\eta h} \eta^2) e^{2\sigma/\mathcal{F}} \]

\[ + \sum_{\nu} \frac{\nu^2}{2} V^{\nu\nu} V_{\mu}^\nu (1 + \sqrt{1 - \xi \frac{h}{v^2}} e^{2\sigma/\mathcal{F}} \]

\[ - \sum_{\nu} \frac{\nu}{2} \psi \psi_\nu (1 + a_{\nu h} h + b_{\nu h} h^2 + c_{\nu \eta} \eta^2) e^{\sigma/\mathcal{F}} \]

\[ + \frac{\alpha_{em}}{8\pi} (\psi_1^3 - b_{1\nu}^e) + \frac{1}{2} F_{1/2}(x_1) \frac{\sigma}{f} C^{\mu\nu} \]

\[ + \frac{\alpha_{em}}{8\pi} (b_{1\rho}^m - b_{1\nu}^m) + \frac{4}{3} F_{1/2}(x_\rho) - F_1(x_W) \frac{\sigma}{f} F_{\mu\nu} \]

\[ + \frac{\alpha_{em}}{8\pi} \frac{1 - 2\xi}{\sqrt{1 - \xi}} \frac{3}{4} \xi^2 \]

\[ - \frac{\alpha_s}{12\pi} \frac{1 - 2\xi}{\sqrt{1 - \xi}} \frac{3}{4} \xi^2 \]

\[ - 3\alpha_{em} \frac{\xi}{16\pi} \frac{\eta^2}{1 - \xi} v^2 F_{1/2}(\frac{4m_q^2}{m_\eta^2}) F_{\mu\nu} F^{\mu\nu} \]

\[ - V_{eff}(h, \eta, \chi) . \tag{24} \]

FIG. 1: DM relic density at \( f = 1000 \) GeV (right) and \( f = 1500 \) GeV (left). We contour \( \log_{10}(\Omega h^2) < \log_{10}(0.12) \).
IV. RELIC ABUNDANCE

A. Introduction to WIMPs

WIMP is one of the most compelling paradigm for DM. In case of scalar DM fundamental and composite singlet scalar WIMPs have been extensively studied, see e.g. [7, 8, 36, 37].

In order to implement the WIMP scenario, we need to assume that the DM candidate is in thermal equilibrium since the very early universe. In case of composite DM there exists an energy threshold above which DM particles are resolved in their constituents. Since we have $f \gg v$ we can safely assume thermal equilibrium; moreover heavy degrees of freedom of the strong theory are irrelevant being, indeed, heavy. As a result we can use the standard picture of WIMPs [38].

We recall that the measured DM relic density is $\Omega h^2 = 0.1199 \pm 0.002$ [39]. The current relic density is predicted using the Weinberg-Lee equation [38]

$$\frac{dn}{dt} + 3Hn = <\sigma v> (n^2_{eq} - n^2)$$

where $<\sigma v>$ is the thermal average of cross sections times relative speed, and $H$ is the Hubble constant. Expanding $\sigma v$ for small velocities as $\sigma v = a + b/x$ we get $<\sigma v> = a + 6b/x$, where $x = m/T$. We use this expansion because s-wave processes are dominant in our model. By solving the above equation, we get the freeze out temperature

$$x_F = \ln \left( 5 \sqrt{\frac{45}{8}} \frac{M_{pl}m_\eta(a + 6b/x_F)}{2\pi^3 g_* x_F} \right)$$

where $g_*$ is the number of degrees of freedom of the DM and $g_*$ is the effective relativistic degrees of freedom in thermal equilibrium.

As a result, the DM relic abundance is given by

$$\Omega h^2 \simeq \frac{1.07 \times 10^9}{\text{GeV} M_{pl} \sqrt{g_*} a + 3(b - a/4)/x_F}$$

where $g_*$ is the number of degrees of freedom of the DM and $g_*$ is the effective relativistic degrees of freedom in thermal equilibrium.

B. Annihilation Cross Sections

In our model the DM candidate is the fifth pseudo Goldstone boson of the coset $SO(6)/SO(5)$, $\eta$. Its effective potential is determined by the underlying theory and can be reliably computed using an effective IR Lagrangian, as we outlined before: the form of this Lagrangian depends on the details of the theory, as the number of top partners $N_F$ and $N_S$. If $N_F = N_S = 1$ the mass is fixed to be $m_\eta \approx m_h/2$ and the predicted relic density is too small to be a viable option. Therefore we focus on the next to minimal case $N_F = 1$, $N_S = 2$, where the $\eta$ mass varies as a free parameters over an interval. We fix the portal coupling $\lambda_{\eta h} \approx 0.13$, following [8].

We computed the annihilation channels including $\eta \eta \rightarrow WW, ZZ, hh, h\sigma, \sigma\sigma, AA, GG$, and $\psi\psi$, where $\psi$ runs over the SM fermions. Note that the above processes are dominated by s-wave exchange since $p$ and higher order terms are suppressed by $v^2$. Full expressions are reported in Appendix D. We present here asymptotic forms valid in certain limits. We focus on $m_\sigma, m_\eta \gg m_Z$: as a result $\eta\eta \rightarrow VV$ dominates the annihilation cross section.

First we take $m_\eta \gg m_\sigma$. If this is the case we obtain

$$<\sigma v>_{AA} \approx \frac{m_\eta^2}{c_{AA} F^4}$$

where $c_{ZZ} = 16$, $c_{WW} = 8$, $c_{\sigma\sigma} = 4$ and $c_{hh} = 16$. The $\eta\eta \rightarrow h\sigma$ process is controlled by $\eta h^0 \sigma^0$ and suppressed by $\xi^4/(1 - \xi)$.

The total thermally averaged cross section is then

$$<\sigma v> \approx \frac{m_\eta^2}{2\pi F^2} \approx 3 \times 10^{-26} \left( \frac{8.5 \text{ TeV} m_{\eta}}{F^2} \right)^2 \text{cm}^3/s.$$  

Note that $<\sigma v>$ should be equal to or larger than $3 \times 10^{-26} \text{cm}^3/s$ in order to reproduce a relic density equal to or smaller than the observed one.

In the massive dilaton limit, $m_\sigma \gg m_\eta$, the dilaton exchanging processes are suppressed by $m_\sigma^2/m_\eta^2$ and Higgs exchanging processes have a similar asymptotic form as before. Consequently we get larger annihilation cross section, parametrized as in (28) where now $c_{ZZ} = 4$, $c_{WW} = 2$, and $c_{hh} = 4$. As a result, the total thermally averaged cross section is

$$<\sigma v> \approx \frac{m_\eta^2}{\pi F^4} \approx 3 \times 10^{-26} \left( \frac{12 \text{ TeV} m_{\eta}}{F^2} \right)^2 \text{cm}^3/s.$$  

In Fig. 1 we present the predicted relic density of DM particles in the $m_\sigma - m_\eta$ plane. We clearly distinguish a depletion of $\Omega h^2$ in correspondence of the points with $m_\eta = m_h/2 \approx 63 \text{ GeV}$ and $m_\eta = m_\sigma/2$. If Fig. 2 we present the value of the scale $f$ which is necessary to reproduce the observed relic density, in the same plane.
V. DIRECT DETECTION

Null results from direct detection experiments, as LUX [40, 41], put limits on the nucleon-DM scattering cross section. The interactions in (24) relevant in this regard are the vertices between the scalars \( h, \eta \) and \( \sigma \) with the fermion bilinears \( \bar{\psi}\psi \) and the field strength operator \( G_{\mu\nu}G^{\mu\nu} \) of colored interactions. From those we derive an effective theory for nucleons

\[
\mathcal{L} \supseteq \sum_{i=n,p} \bar{\psi}_i \psi_i (y_{s,i}\sigma + y_{h,i}h + y_{\eta,i}\eta^2),
\]

where

\[
y_{s,i} = \sum_\psi \frac{1}{2} (i|m_\psi \bar{\psi}_i \psi_i) - \frac{C_s}{8\pi f_i} (i|\alpha_s G_{\mu\nu}^a G^{a\mu\nu}|i),
\]

\[
y_{h,i} = \frac{1}{2} \frac{1}{v} \sqrt{1 - \xi} \sum_\psi (|m_\psi \bar{\psi}_i \psi_i|) - \frac{1}{12\pi} (i|\alpha_s G_{\mu\nu}^a G^{a\mu\nu}|i),
\]

\[
y_{\eta,i} = - \frac{1}{2\eta^2} \frac{1}{2\xi} \sum_\psi (i|m_\psi \bar{\psi}_i \psi_i) + \frac{1}{32\pi^2 \eta^2} \frac{\xi}{1 - \xi} F_{1/2}(\frac{4m_\eta^2}{m_\eta^2}) (i|\alpha_s G_{\mu\nu}^a G^{a\mu\nu}|i),
\]

where \( i \) stands for neutron and proton and \( \psi \) stands for SM quarks [66]. Integrating out the dilaton and the Higgs we obtain

\[
\mathcal{L}_{\text{eff}} \supseteq -a_\eta \bar{n} n \eta^2 - a_\eta \bar{p} p \eta^2
\]

where

\[
a_\eta \approx y_{\eta,i} = \frac{2m_\eta^2 y_{s,i}}{f m_\sigma^2} = \frac{\lambda_{h\eta} v \sqrt{1 - \xi} y_{h,i}}{2m_h^2}.
\]

For the matrix elements, we take the values for up and down quarks from [42], and for s, c, t quarks from [43]:

\[
f_\psi = (i|\bar{\psi}\psi|i) \frac{m_\psi}{m_i},
\]

\[
f_u^a \simeq 0.016, f_d^a \simeq 0.018, f_u^a \simeq 0.038, f_d^a \simeq 0.034,
\]

\[
f_u^V \simeq f_d^V \simeq 0.043, f_c \simeq 0.0814, f_b \simeq 0.0785, f_t \simeq 0.0820, 
\]

\[
\alpha_s (n|G_{\mu\nu}^a G^{a\mu\nu}|n) \simeq -2.4 \text{ GeV}.
\]

We then derive the nucleon-DM cross section

\[
\sigma_{n,i} \simeq \frac{a_i^2 m_i^4}{m_\eta^2}.
\]

By comparing with the LUX data we get the allowed parameter region, shown in Fig. 3. For the points for which the model predicts a relic density lower than the observed one we rescale the bound.

VI. INDIRECT DETECTION

A. Sommerfeld Enhancement

To correctly evaluate the signals searched by indirect detection experiments we take into account Sommerfeld enhancement, following [44, 45]. To this end we need the three fields interaction vertices of DM \( \eta \) with dilaton and Higgs, which are respectively of the form

\[
\frac{1}{2} \langle \partial \eta \rangle^2 2\sigma = \frac{1}{2} m_\eta^2 \eta^2 2\sigma
\]

and

\[
\frac{\xi}{1 - \xi} \langle \partial_\eta \partial^\mu \eta \rangle \frac{\eta^4}{v} - \sqrt{1 - \xi} \frac{\lambda_{h\eta}}{2} \eta^2 h
\]
FIG. 4: Excluded parameter region with $f = 1000$ GeV (left) and $f = 1500$ GeV (right), using the informations on antiproton fluxes from the Galactic gas.

where $\lambda_{h\eta} = 0.013$. These lead to interaction potential, in momentum space, of the form

$$V(p-q) = \frac{1}{4m_h^2} \left( \frac{2}{f} \right)^2 \Pi_i \frac{(p_0 + (-1)^{i+1} q_i)^2 + 2m_h^2}{(p-q)^2 - m_h^2 - \Pi_i(4m_h^2)}$$

and

$$V(p-q) = \frac{1}{4m_h^2} \Pi_i=1 \frac{(p_0 + (-1)^{i+1} q_i)p_i(p-q) - \nu\sqrt{1 - \xi \frac{\lambda_{h\eta}}{2}}}{(p-q)^2 - m_h^2}$$

for dilaton and Higgs respectively, where $p_1$ and $p_2$ are the momenta of the incoming particles and $p = (p_1 - p_2)/2$. In the non-relativistic limit, in the instant interaction limit and in the CM frame the above expressions reduce to

$$V(p-q) = -\frac{1}{4m_h^2} \frac{2}{f} \frac{m_h^4}{(p-q)^2 + m_h^2}$$

and

$$V(p-q) = -\frac{1}{4m_h^2} \frac{\nu^2(1 - \xi)\lambda_{h\eta}^2}{4(p-q)^2 + 4m_h^2}.$$ 

As a result, the following Yukawa potential arises

$$V(r) = -\frac{\alpha_{\sigma}}{r} e^{-m_{\sigma} r} - \frac{\alpha_h}{r} e^{-m_h r}$$

where $\alpha_{\sigma} = \frac{9m_{\sigma}^2}{8\pi F}$ and $\alpha_h = \frac{(1 - \xi)\lambda_{h\eta}^2}{10\sigma m_{h}^2}$. Notice that $\alpha_{\sigma} \gg \alpha_h$, and DM is in non relativistic regime, thus Sommerfeld enhancement is dilaton dominated. According to [45–47], an analytic approximate formula for dilaton mediated Sommerfeld enhancement is

$$S = \frac{\pi}{\epsilon_v \cosh(\frac{2\epsilon_v}{\epsilon_{\sigma}})} \frac{\sinh(\frac{2\epsilon_v}{\epsilon_{\sigma}})}{\cos(2\pi \nu_{\sigma} - \frac{\epsilon_{\sigma}}{\epsilon_{\sigma}})^2}$$

where $\epsilon_v = v/\alpha_{\sigma}$ and $\epsilon_{\sigma} = m_{\sigma}/(\alpha_{\sigma} m_{h})$.

FIG. 5: Differential antiproton spectrum per DM annihilation, computed for $m_{\eta} = 300$ GeV, $m_{\sigma} = 1000$ GeV and $f = 1500$ GeV.

B. Antiproton Flux

DM annihilation can produce antiprotons in various ways and we take into account the AMS-02 [48, 49] measure to constraint the parameter space of the model, demanding that the predicted antiproton flux does not exceed the observed one. Following [14, 50], we derive a bound on the antiproton flux produced by DM annihilation by imposing that the amount of anti-antiprotons produced by the DM annihilation in the Galactic disk is smaller than the antiproton flux due to primary cosmic rays colliding with interstellar medium in the disc [51].

We followed [52, 53] to compute antiproton spectrum, and [54] to evaluate cascade annihilation processes initiated by $\eta \eta \rightarrow \sigma \sigma, hh$, including the Sommerfeld enhancement (44).

The injection rate density of antiprotons produced
by DM annihilation is
\[ Q_\bar{p}(E) = \frac{1}{2} n_\eta^2 \langle \sigma v \rangle \frac{dN_\bar{p}}{dE} \]
\[ \simeq 5 \times 10^{-36} \text{cm}^{-3} \text{s}^{-1} \text{GeV}^{-1} \left( \frac{\rho_\eta}{0.4 \text{ GeVcm}^{-3}} \right)^2 \left( \frac{3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}}{\langle \sigma v \rangle} \right) \left( \frac{m_\eta}{1 \text{ TeV}} \right)^{-3} \left( \frac{m_\sigma}{dN_\bar{p}}/dE \right) \]
where \( \rho_\eta = m_\eta n_\eta \) and \( dN_\bar{p}/dE \) is the differential antiproton spectrum per annihilation event. According to [54] dilaton and higgs contributions to antiproton flux is given by
\[ \frac{dN_\bar{p}}{dx} = 2 \int_{t_{\min}}^{t_{\max}} \frac{dx_0}{x_0\beta_\sigma} \frac{dN_{\bar{p},S}}{dx_0} \]
where \( S = h, \sigma, \beta_\sigma = \sqrt{1 - \gamma_\sigma^2}, x = E/m_\eta, t_{\min} = 2x\gamma_\sigma^2(1 - \beta_\sigma), t_{\max} = \min[1, 2x\gamma_\sigma^2(1 + \beta_\sigma)] \) and \( \gamma_\sigma = m_\eta/m_\sigma \). By including cascade effects, we obtain the full differential antiproton spectrum, following [53]. Fig. 5 shows a typical spectrum at \( f = 1500 \text{ GeV} \) and for \( m_\eta = 300 \text{ GeV} \) and \( m_\sigma = 1000 \text{ GeV} \).

In order to impose our condition we use a propagation model independent injection rate [51] given by
\[ Q_{\bar{p},CR}(E) \simeq 8.4 \times 10^{-33} \text{cm}^{-3} \text{s}^{-1} \text{GeV}^{-1} \left( \frac{E}{100 \text{ GeV}} \right)^{-2.8} \left( 1 - 0.22 \log_{10} \left( \frac{E}{500 \text{ GeV}} \right) \right) J_{p,0}(1 \text{ TeV}) \]
where \( J_{p,0}(1 \text{ TeV}) \) is the local proton flux at \( E = 1 \text{ TeV} \) and scaled to measured value \( J_{p,0}(1 \text{ TeV}) \simeq 8 \times 10^{-9} \text{ GeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \). Due to uncertainty in the derivation of the injection rate, it varies within a factor of 2 [51].

The results of our analysis are shown in Fig. 4. Also in this case for the points predicting a too low relic density we assume that our DM candidate is the only source of antiprotons.

Furthermore, by adopting the Cosmic Rays (CR) grammage given in [51], we compute the antiproton flux and compare antiproton to proton flux to measured \( \bar{p}/p \) data reported by AMS-02 [48, 49, 55].

Fig. 6 presents the allowed region by imposing that the computed \( \bar{p}/p \) ratio does not exceed the \( \bar{p}/p \) measured by AMS-02. We found that the points reproducing a nearly exact DM relic density do not give significant antiproton flux, and points fitting the \( \bar{p}/p \) flux predict a too low relic density. Note that the allowed region can be significantly changed by precise determination of CR grammage and proper knowledge on spallation loss, propagation and solar modulation. In addition, we find that parameter points which generate resonant Sommerfeld enhancement factor are excluded by AMS-02 data. For sake of illustration we provide the \( \bar{p}/p \) flux spectra for two points in the Sommerfeld enhanced region in Fig. 7: data points are the
measured $\bar{p}/p$ flux ratio reported by [55], the red line is the secondary prediction as given by [49], the blue area is the deviation of the secondary prediction due to uncertainties. Model predictions are computed at two parameter points, where (1) is $f = 1500$ GeV, $m_\eta = 1866$ GeV and $m_\sigma = 1303$ GeV, and (2) is $f = 1000$ GeV, $m_\eta = 1183$ GeV and $m_\sigma = 746$ GeV.

C. Gamma Ray Flux

As well known, see for instance [56], gamma ray excesses can be a good probe of DM. Since, in our model, DM annihilation produces gamma ray via direct annihilation and Higgs and dilaton mediation, we check whether our model fits the experimental data. Because of the fact that the dwarf spheroidal satellite galaxies (dSphs) of the Milky Way are expected to contain considerable DM amount [57] and have ignorable noise of non-thermal astrophysical gamma ray production, we use the limit on thermally averaged scattering cross sections observed by the Fermi-LAT Collaboration [58] to constrain our model. Note that the analysis is relatively insensitive to the detailed DM distribution inside the dSphs.

Following [53, 54] we compute the gamma ray spectrum per annihilation, and we compare with the SM channels, which we find in [53]. Fig. 9 shows the ratio of gamma ray spectrum at each energy. The spectrum generated by DM annihilation of our model is within a factor of 2 or 3 with respect to the gamma ray spectrum generated by pure $\eta \rightarrow bb$ channel and $\eta\eta \rightarrow WW$ channel, thus we assume that the constraints given by [58] is applicable to our model.

In many points of the parameter space the correct relic density of DM is not reproduced, as we discussed above and we showed in Fig. 1. For those points we assume that $\eta$ only partially accounts for the DM density around the dSphs and the additional DM does not contribute to the CR production.

Under such assumptions the resulting effective J factor contributing to the gamma ray flux is

$$J_{eff} = (\frac{\Omega_\eta}{\Omega_{DM}})^2 J$$

(48)

where $\Omega_{DM} h^2 \approx 0.12$ and $\Omega_\eta$ is the relic density for $\eta$ DM. Consequently we derive a cross section bound much weaker the bound given by [58].

Fixing $m_\sigma = 1000$ GeV we present thermally averaged cross section and bounds given by the Fermi-LAT Collaboration in Fig. 10. Fig. 8 shows the allowed parameter region imposing the constraints from the Fermi-LAT experiment at 95% confidence level. We do not observe any peak in the gamma ray spectrum because $\sigma_{\eta\eta \rightarrow \gamma\gamma}/\sigma_{\text{tot}}$ is negligible in our model.

In the high DM mass region, where $m_\eta \geq 1$ TeV, experimental constraints given by the H.E.S.S Collaboration [59] provide tighter bound though we have more dependence on the propagation model. By assuming that DM distribution follows a cusp distribution such as the Navarro-Frenk-White [60], we could superimpose this additional bound on the constraints given by Fermi-LAT, but that region is already ruled out and this procedure does not provide additional information.

VII. COLLIDER CONSTRAINTS

A. Higgs Measurements

We consider the impact of the measurements of signal strengths reported in [31, 32] on the allowed parameter space of the theory, namely on $\xi$ or equivalently on $f$. We perform a $\chi^2$ analysis using the following channels

$$\mu_V/\mu_F = 1.06^{+0.35}_{-0.27}, \quad \mu_F^{\gamma\gamma} = 1.13^{+0.24}_{-0.21},$$

$$\mu_F^{ZZ} = 1.29^{+0.25}_{-0.25}, \quad \mu_F^{WW} = 1.08^{+0.22}_{-0.19},$$

$$\mu_F^T = 1.06^{+0.35}_{-0.28}, \quad \mu_F^{bb} = 0.65^{+0.37}_{-0.28},$$

(49)

and the result is shown in Fig. 13, from which we read that at 95% CL f larger than 960 GeV is still allowed.

B. Heavy Scalar Searches

Since the dilaton has couplings to SM particles similar to the Higgs’ ones its parameter space is constrained by searches for heavy Higgses [33–35]. A dilaton whose mass lies between 200 and 1000 GeV is probed by such searches, and the experimental measures convert to a lower bound on $f$. In Fig. 12 we report the allowed minimum value for $f$ at 95% CL for each choice of dilaton mass, focusing for definiteness on specific values for the UV beta functions $b_{UVi}$, chosen as representative.

C. Precision Tests

We proceed inspecting the contribution of new physics to the EW precision parameters measured by LEP [62]. The presence of composite resonances is expected to have an impact on EW precision tests. At tree level vector resonances give, imposing the generalized Weinberg sum rules as in [23],

$$\delta S = \frac{8\sin^2 \theta_w m_W^2}{\alpha m_p^2} \left(1 - f_p^2 \frac{f_p^2}{4 f_p^2} \right)$$

(50)

which in turn implies for instance $m_p > 2$ TeV if $f_p = f$. Also modification of Higgs couplings play a role in enhancing EW precision parameters: interestingly enough once we include the dilaton we get vanishing $T$ corrections due to the fact that $c_{\eta h}^2 + c_{\eta \sigma}^2 = 1$. Furthermore, for the same reason, $S$ correction are also suppressed. Higgs and dilaton loops are computed...
FIG. 8: Allowed parameter region at \( f = 1000 \) GeV (left) and \( f = 1500 \) GeV (right), comparing with the Fermi-LAT data at 95\% confidence level. Each contour represents a different level for the value of the ratio of \( \sigma_{bb}\nu \) over the Fermi-LAT bound.

FIG. 9: Gamma ray spectrum ratio for various channels.

FIG. 10: Thermally averaged cross section in \( \bar{b}b \); the magenta curve is computed at \( f = 1000 \) GeV and the red curve at \( f = 1500 \) GeV, fixing \( m_\sigma = 1000 \) GeV. The black line is the constraint for \( \bar{b}b \) channel determined by the Fermi-LAT Collaboration, and the blue area is the 2\( \sigma \) uncertainty.

Following [63]. From the Lagrangian

\[
\mathcal{L} \supset (2m^2_W W^+ W^- + m^2_\gamma Z \gamma) \left( c_{V,h} \frac{h}{v} + c_{V,\sigma} \frac{\sigma}{v} \right)
\]

we easily read

\[
\alpha \Delta T \simeq - \frac{3g^2_Y}{32\pi^2} (1 - c_{V,h}^2 - c_{V,\sigma}^2) \log(\Lambda/m_Z) = 0,
\]

\[
\alpha \Delta S \simeq \frac{g_L g_Y \log(\Lambda/m_Z)}{48\pi^2 (g^2_L + g^2_Y)} \left( 2g_L g_Y (1 - c_{V,h}^2 - c_{V,\sigma}^2) + 6c_{V,\sigma} (2g_L g_Y c_{\gamma\gamma} + c_{Z\gamma} (g^2_L - g^2_Y)) \right.
\]

\[
\left. + 3 (g_L g_Y (c_{Z\gamma}^2 - c_{\gamma\gamma}^2) - (g^2_L - g^2_Y) c_{\gamma\gamma} c_{Z\gamma}) \right),
\]

\[
\alpha \Delta W \simeq \frac{g^2_L}{4\pi^2} \left( c_{\gamma\gamma} + \frac{g_L}{g_Y} c_{Z\gamma} \right)^2 \log(\Lambda/m_Z),
\]

\[
\alpha \Delta Y \simeq \frac{g^2_L}{4\pi^2} \left( c_{\gamma\gamma} - \frac{g_L}{g_Y} c_{Z\gamma} \right)^2 \log(\Lambda/m_Z),
\]

where \( \Lambda \simeq 4\pi f \) and in our model

\[
c_{V,h} = \sqrt{1 - \xi}, \quad c_{V,\sigma} = \sqrt{\xi},
\]

\[
c_{\gamma\gamma} = - \frac{\alpha_{em}}{2\pi} \left( \frac{b_1^m}{b_1^m} - \frac{b_1^m}{b_1^m} + \frac{4}{3} F_1(x_t) - F_1(x_W) \right) \sqrt{\xi},
\]

\[
c_{Z\gamma} = - \frac{\alpha_{em}}{2\pi \tan \theta_W} \left( b_1^R - b_1^R - b_1^R - b_1^R \right) \sqrt{\xi} + \frac{e g_L}{8\pi^2} (A^Z_1 (\tau_W, \lambda_W) + \sum_f N_f q_f g_f A_{1/2}^Z (\tau_f, \lambda_f)) \sqrt{\xi},
\]

with \( A_1 \) and \( A_{1/2} \) given in [64].

As a result EW precision tests do not significantly constrain the model for \( f \geq 900 \) GeV. Finally note that typical values of \( \alpha W \) and \( \alpha Y \) are \( \sim 10^{-7} \). Fermionic resonances are expected to affect EW parameters as well but in a model dependent way: we
FIG. 11: Relic density of DM, as $\log_{10}(\Omega_\eta h^2)$, fixing $f = 1000$ GeV (left) and $f = 1500$ GeV (right) taking into account all the constraints discussed in the text.

FIG. 12: 95% CL lower bound on the symmetry breaking scale $f$ in GeV varying the dilaton mass and the UV beta functions from searches for heavy scalars.

direct and indirect searches discussed in the previous sections. Fig. 11 shows the predicted density for two given symmetry breaking scales $f = 1000$ GeV and $f = 1500$ GeV. For these plots we use benchmark UV beta functions $b^{\text{UV}}_3 = b^{\text{UV}}_{10^5} = 0$. While for $f = 1000$ GeV the available parameter space, in which our candidate DM scalar entirely accounts for the observed density, shrinks to zero, if we allow for $f = 1500$ GeV we have a region in parameter space starting with $m_\eta \simeq 200$ GeV and $m_\sigma \simeq 500$ GeV; a heavier dilaton requires a heavier DM particle and an asymptotic value of $m_\eta \simeq 300$ GeV is reached at $m_\sigma \simeq 1500$ GeV. Interestingly, according to the scan performed in [8], $\eta$ mass can vary between 100 and 700 GeV for $f = 800 – 1100$ GeV. Notice that $f = 1000$ GeV returns to be a viable option if a fraction of the DM relic density is accounted for by a different particle, as for instance an axion.

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VIII. SUMMARY AND CONCLUSIONS

The presence of additional light scalars, beyond the Higgs, is an expected feature of CHM. We have considered a candidate DM scalar particle in a specific CHM based on the coset SO(6)/SO(5), enlightening the possible role of a light dilaton as a mediator of DM interactions with the SM. To summarize our analysis we combine results from collider constraints, direct and indirect searches discussed in the previous sections. Fig. 11 shows the predicted density for two given symmetry breaking scales $f = 1000$ GeV and $f = 1500$ GeV. For these plots we use benchmark UV beta functions $b^{\text{UV}}_3 = b^{\text{UV}}_{10^5} = 0$. While for $f = 1000$ GeV the available parameter space, in which our candidate DM scalar entirely accounts for the observed density, shrinks to zero, if we allow for $f = 1500$ GeV we have a region in parameter space starting with $m_\eta \simeq 200$ GeV and $m_\sigma \simeq 500$ GeV; a heavier dilaton requires a heavier DM particle and an asymptotic value of $m_\eta \simeq 300$ GeV is reached at $m_\sigma \simeq 1500$ GeV. Interestingly, according to the scan performed in [8], $\eta$ mass can vary between 100 and 700 GeV for $f = 800 – 1100$ GeV. Notice that $f = 1000$ GeV returns to be a viable option if a fraction of the DM relic density is accounted for by a different particle, as for instance an axion.

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Appendix A: Details of the Models

1. Fermionic Sector

The Lagrangian of the fermionic sector, including composite resonances, is given by

$$\mathcal{L}_f = \bar{q}_L i \partial \!\!\!\!/ q_L + \bar{t}_R i \partial \!\!\!\!/ t_R$$

(A1)
The explicit form of the form factors in terms of the parameters in (A1) is given in [8].

In addition, there can be interactions between composite resonances [8, 23]:

\[
\mathcal{L}_{int} = \sum_{\eta=L,R} (k_{ij}^\eta \hat{F}_i \gamma^\mu (g_\mu p_\mu - E_\mu) P_\eta F_j) + \sum_{\eta=L,R} (S_i \gamma^\mu (k_{ij}^\eta a_\mu + i\eta_{ij}^\eta d_\mu) P_\eta F_j + h.c).
\]

where \(\rho_\mu, a_\mu\) are massive vector resonances of the strong sector. Notice that these interactions do not enter the scalar couplings to \(gg\) and \(\gamma\gamma\) at one loop because they mix different species of composite fermions [61].

In order to compute the low energy effective theory of SM fermions, we need to integrate out the composite resonances. The result, in momentum basis, up to quadratic order in the fermions, is written as

\[
\mathcal{L}_{eff} = \Pi_{\eta} \hat{I}_L \hat{P}_L + \Pi_{\eta} \hat{I}_R \hat{P}_R - (\Pi_{\eta} \hat{t}_L \hat{t}_R + h.c).
\]  

(A2)

The form factors are written as

\[
\Pi_{\eta} = i\Pi_F + \frac{h^2}{f^2} \Pi_{1F}, \quad \Pi_{\eta} = iS + \frac{1 - h^2}{f^2} - \frac{\eta^2}{f^2} \Pi_{1S},
\]

\[
\Pi_{\eta} = i\Pi_F + \frac{h^2}{f^2} \Pi_{1F}, \quad \Pi_{\eta} = iS + \frac{1 - h^2}{f^2} - \frac{\eta^2}{f^2} \Pi_{1S},
\]

(A3)

The explicit form of the form factors in terms of the parameters in (A1) is given in [8].

2. Vector Resonances

The Lagrangian for vector resonances is given by

\[
\mathcal{L} = -\frac{1}{4} Tr(p_\mu^2) + \frac{f^2}{2} Tr(g_\mu p_\mu - E_\mu)^2
\]

\[
-\frac{1}{4} Tr(a_{\mu}^2) + \frac{f^2}{2\Delta^2} Tr(g_\mu a_\mu - \Delta d_\mu)^2.
\]  

(A4)

General cases of vector resonances are examined in [23] and mixing between \(\rho\) and \(E\) is described in [8, 23]. Similarly to the fermion case, integrating out heavy vector fields we obtain an effective Lagrangian for SM vector bosons given by, in momentum space,

\[
\mathcal{L} = \frac{F_{\mu\nu}}{2} (\Pi_0 (q^2) Tr(A_\mu A_\nu) + \Pi_1 (q^2) \Sigma^\mu_0 A_\mu A_\nu \Sigma
\]

\[
+ \Pi_0 (q^2) X_\mu X_\nu)
\]

(A5)

where \(A_\mu\) is a spurion obtained formally gauging all the SO(5) generators. In the physical configuration where only \(A_\mu^0 = W_\mu, A_\mu^3 = c_X B, X = s_X B\) are different from zero, with \(c_X = g_Y / g_L\) and \(s_X^2 = 1 - c_X^2\), the former expression reduces to

\[
\mathcal{L} = \frac{F_{\mu\nu}}{2} (\Pi_0 W_\mu W_\nu + \Pi_1 \frac{h^2}{f^2} (W_\mu W_\nu + W_\mu W_\nu) + \Pi_{BB} B_\mu B_\nu + \Pi_1 (\eta^2 - \lambda^2) Z_\mu Z_\nu)
\]

(A6)

where \(\Pi_B = s_X^2 \Pi_0^2 + c_X^2 \Pi_0^2\) and \(Z = \cos \theta_\omega W - \sin \theta_\omega B, \cos \theta_\omega = g_L / \sqrt{g_L^2 + g_Y^2}\). It is also customary to define

\[
\Pi_{WW} = \Pi_0 + \frac{h^2}{4f^2} \Pi_1, \quad \Pi_{BB} = \Pi_B + c_X^2 \frac{h^2}{4f^2} \Pi_1,
\]

\[
\Pi_{WB} = - c_X \frac{h^2}{4f^2} \Pi_1.
\]  

(A7)

3. Dilaton Potential

Unlike other Goldstone bosons, a non derivative self-interaction term for the dilaton is allowed and indeed it is expected at tree level:

\[
V_{tree}(\chi) = \frac{\kappa}{4!} \chi^4.
\]  

(A8)

Corrections are generated by loops of self interactions and loops of heavy resonances. The first gives

\[
V_{eff} = V_{tree} + \frac{3\kappa^2}{32\pi^2} \frac{\chi^4}{f^4} \left( \log \frac{\kappa^2}{2 \mu^2} - \frac{1}{2} \right)
\]

\[
= \frac{1}{32\pi^2} \frac{\chi^4}{f^4} \left( \hat{k}_0 \log \frac{\chi^2}{f^2} + \hat{k}_1 \right)
\]

(A9)

where

\[
\hat{k}_0 = 3\kappa^2 f^4,
\]

\[
\hat{k}_1 = \frac{32\pi^2 \kappa^4}{4!} + 3\kappa^2 f^4 \left( \log \frac{\kappa^2}{2 \mu^2} - \frac{1}{2} \right).
\]  

(A10)

Gauge and fermion contributions to the potential are obtained from the form factors at \(h = \eta = 0:\)

\[
V(\chi) = \int \frac{d^4 p_E}{(2\pi)^4} \left( \frac{3}{2} \log |\Pi_0 \Pi_B| - 6 \log |\Pi_0 \Pi_1 (\Pi_1 + \Pi_0)| \right).
\]  

(A11)
Recalling the general formula

$$\int \frac{d^4p_E}{(2\pi)^4} \log[p_E^2 + U^2] = \frac{1}{32\pi^2} U^4(\log U^2 \mp \frac{1}{2})$$  \hspace{1cm} (A12)$$

the result can be expressed as

$$V(\chi) \simeq \frac{1}{32\pi^2} f^4 (\kappa_0 \log \frac{\chi^2}{f^2} + \kappa_1)$$  \hspace{1cm} (A13)$$

where trivially

$$\kappa_0 = \frac{2\pi^2 f^4}{3} \frac{\chi}{f^2} \frac{\partial^2 V}{\partial \chi^4}, \quad \kappa_1 = \frac{4\pi^2 f^4}{3} \frac{\partial^4 V}{\partial \chi^4} \bigg|_{\chi \to f} - \kappa_0.$$  \hspace{1cm} (A14)$$

We now move to study the Vacuum Expectation Value (VEV) and the mass of the dilaton. We start with the potential

$$V_{\text{eff}}(h, \eta, \chi) = \frac{\chi^4}{f^4} (V(h, \eta) - V(0, \chi)) + \frac{\chi^4}{f^4} (\kappa_0 \log \frac{\chi^2}{f^2} + \kappa_1)$$  \hspace{1cm} (A15)$$

where $V(h, \eta)$ is the sum of the gauge and fermion contributions. Imposing the condition $\langle \chi \rangle = f$ we obtain

$$\kappa_1 = -32\pi^2 V(v, 0) - \frac{\kappa_0}{2},$$  \hspace{1cm} (A16)$$

and then

$$V_{\text{eff}}(h, \eta, \chi) = \frac{\chi^4}{f^4} (V(h, \eta) - V(0, \chi)) + \chi \frac{\chi}{f^2} \frac{\partial^2 V}{\partial \chi^2} \bigg|_{\chi \to f} - \frac{\kappa_0}{4}.$$  \hspace{1cm} (A17)$$

Therefore the mass of the dilaton is given by

$$m^2_{\sigma} = -\frac{\kappa_0}{4\pi^2 f^2}$$  \hspace{1cm} (A18)$$

and the effective potential (A17) can be rewritten as

$$V_{\text{eff}}(h, \eta, \chi) = \frac{\chi^4}{f^4} (V(h, \eta) - V(0, \chi)) + \frac{m^2_{\sigma}}{4} \frac{\chi^4}{f^2} (\log \frac{\chi}{f} - \frac{1}{4}).$$  \hspace{1cm} (A19)$$

We assumed $\kappa_0 > 0$ in order to have a potential bounded from below. Finally we notice that because of the tree level term the dilaton mass model dependent and therefore in our phenomenological analysis we treat it as a free parameter.

**Appendix B: Decoupling of Heavy Composite Fermions**

We discuss here the effect of heavy fermionic resonances on the couplings of the dilaton $\sigma$ to $\gamma\gamma$ and $gg$. They contribute entering the beta function coefficients $b_{1R}^f$ and also circulating in triangular loops. In the limit of mass much larger than $m_{\sigma}/2$ the two effects cancel and in the following we review this property. Indeed in extra dimensional construction heavy KK modes of bulk fermions do not generate corrections for radion couplings, as shown in [25]. We obtain the same result in a four dimensional language.

We consider $N_F$ and $N_S$ heavy Dirac fermions with quantum numbers under the SM gauge group $SU(N_c) \times SU(2)_L \times U(1)_Y$

$$F = (N_c, 2)_{1/2} \oplus (N_c, 2)_{1/3}, \quad S = (N_c, 1)_{2/3}.$$  \hspace{1cm} (B1)$$

They enter the Lagrangian

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} (b_{1R,F}^f + b_{1R,S}^f) \sigma \frac{G_{\mu\nu} G^{\mu\nu}}{f} + \frac{\alpha_{em}}{8\pi} (b_{1R,F}^f + b_{1R,S}^f) \sigma F_{\mu\nu} F^{\mu\nu}$$  \hspace{1cm} (B2)$$

contributing with

$$b_{1R,F}^f = -\frac{10}{3} N_F, \quad b_{1R,S}^f = \frac{N_c}{27} 152 N_F,$$

$$b_{1R,S}^f = -\frac{2}{3} N_S, \quad b_{1R,S}^f = \frac{N_c}{27} 16 N_S.$$  \hspace{1cm} (B3)$$

The second contribution comes from loop diagrams. For $\sigma gg$ it has the form

$$\mathcal{L}_{\text{eff}} \supset \frac{\alpha_s}{8\pi} (\frac{5 N_F}{2} F_{1/2}(x_F^2) + \frac{N_S}{2} F_{1/2}(x_S^2)) \sigma \frac{G_{\mu\nu} G^{\mu\nu}}{f} \bigg|_{x = f \leftarrow x}$$  \hspace{1cm} (B4)$$

where $x_F, S = 2 m_{F,S}/m_\sigma$. Note that $F_{1/2}(x)$ quickly saturates to $4/3$ for $x > 1$. Since typical masses of heavy composite fermions are larger than $m_\sigma/2$ the limit is justified and we have a perfect cancellation in the infinite limit mass. Similarly for $\sigma \gamma\gamma$

$$\mathcal{L}_{\text{eff}} \supset \frac{\alpha_{em}}{8\pi} N_c (\frac{38 N_F}{9} F_{1/2}(x_F^2) + \frac{4 N_S}{3} F_{1/2}(x_S^2)) \sigma \frac{F_{\mu\nu} F^{\mu\nu}}{f} \bigg|_{x = f \leftarrow x}$$  \hspace{1cm} (B5)$$

and the same cancellation is in place. Therefore we verify, at one loop, the decoupling of heavy fermions states, confirming the expectation from extra dimensional models.

**Appendix C: Dilaton Decay Widths**

$$\Gamma_{\sigma \rightarrow \psi \psi} = \frac{3 m_\psi^2 (m_\sigma^2 - 4 m_\psi^2)^{3/2}}{8 \pi f^4 m_\sigma^2},$$

$$\Gamma_{\sigma \rightarrow hh} = \frac{\sqrt{m_\sigma^2 - 4 m_h^2 (m_h^2 + 2 m_\sigma^2)^2}}{32 \pi f^2 m_\sigma^2},$$

$$\Gamma_{\sigma \rightarrow WW} = \frac{\sqrt{m_\sigma^2 - 4 m_W^2 (m_W^2 - 4 m_\sigma^2 m_W^2 + 12 m_h^4)}}{16 \pi f^2 m_\sigma^2},$$

$$\Gamma_{\sigma \rightarrow gg} = \frac{\alpha_s^2}{32 \pi^2 f} (b_{1R}^f - b_{1U}^f + \frac{1}{2} F_{1/2}(x_t)^2 m_\sigma^2),$$
\[
\Gamma_{\sigma \rightarrow \gamma \gamma} = \frac{\alpha^2}{256\pi^2} (b_{mW}^2 - b_{\ell W}^2 + \frac{4}{3} F_{1/2}(x_1) - F_1(x_W))^2 m_\sigma^3 \frac{r^2}{f^2},
\]
\[
\Gamma_{\sigma \rightarrow \eta \eta} = \frac{\sqrt{m_\sigma^2 - 4m_\eta^2 (m_\sigma^2 + 2m_\eta^2)^2}}{32\pi f^2 m_\eta^2}. \quad (C1)
\]

\[
\sigma_{\eta \eta \rightarrow WW} = \frac{m_W^4 \sqrt{m_W^2 - m_\eta^2}}{32\pi^2 f^4} \left( 2 + \left( \frac{2m_\eta^2 - m_W^2}{m_W^2} \right)^2 \right) \left[ \frac{144m_\eta^4}{|m_W^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2} + 48m_\eta^2 m_W^2 (-4m_\eta^2 + f^2 \lambda_\eta(1 - \xi))(16m_\eta^4 - 4m_\eta^2 (m_\sigma^2 + m_\eta^2) + m_\sigma^2 m_\eta^2 + \Re(\Pi_\sigma(4m_\eta^2))\Re(\Pi_h(4m_\eta^2))) \right.
\]
\[
\left. \frac{|m_W^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2 |m_W^2 - 4m_\eta^2 + i\Re(\Pi_h(4m_\eta^2))|^2}{|m_W^2 - 4m_\eta^2 + i\Re(\Pi_h(4m_\eta^2))|^2} \right] \right], \quad (D1)
\]

\[
\sigma_{\eta \eta \rightarrow ss} = \frac{m_W^4 \sqrt{m_W^2 - m_\eta^2}}{32\pi^2 f^4} \left( 2 + \left( \frac{2m_\eta^2 - m_W^2}{m_W^2} \right)^2 \right) \left[ \frac{16(4m_\eta^4 - f^2 \lambda_\eta(1 - \xi)^{3/2}(1 - \xi))}{|m_W^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2} \right. 
\]
\[
\left. + \left( \frac{m_W^2 - 4m_\eta^2 + i\Re(\Pi_h(4m_\eta^2))}{|m_W^2 - 4m_\eta^2 + i\Re(\Pi_h(4m_\eta^2))|^2} \right) \right], \quad (D2)
\]

\[
\sigma_{\eta \eta \rightarrow \psi \psi} = \frac{3m_W^4}{8\pi^2 m_\eta^2} \left( \frac{36m_\eta^4}{f^4|m_\eta^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2} \right. \left. \right] \left[ \frac{16(4m_\eta^4 - f^2 \lambda_\eta(1 - \xi)^{3/2}(1 - \xi))}{|m_W^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2} \right. 
\]
\[
\left. + \frac{8(4m_\eta^4 - m_\eta^2)(4m_\eta^2 - f^2 \lambda_\eta(1 - \xi)^{3/2}(1 - \xi))}{v^2 f^2(1 - \xi)^{3/2} |m_W^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2} \right. 
\]
\[
\left. + \frac{12m_\eta^2 (m_\eta^2 - 4m_\eta^2)}{v^2 f^2(1 - \xi)|m_W^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2} + \frac{|m_\eta^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2}{v^2 (1 - \xi)^{3/2}} \right] \left. \right], \quad (D3)
\]

\[
\sigma_{\eta \eta \rightarrow \sigma \sigma} = \frac{m_W^4}{4\pi^2 f^4 (m_\sigma^2 - 2m_\eta^2)^2} |m_W^2 - 4m_\eta^2 + i\Re(\Pi_\sigma(4m_\eta^2))|^2, \quad (D4)
\]

\[
\sigma_{\eta \eta \rightarrow \sigma h} = \sqrt{m_\sigma^2 + (4m_\eta^2 + m_h^2 - 2m_\sigma^2(4m_\eta^2 + m_h^2)) \left[ \frac{4(m_\sigma^2 + 8m_\eta^2 - m_h^2)^2 (m_\sigma^2 - \xi v^2 \lambda_\eta(1 - \xi) + m_\eta^2)}{(m_\sigma^2 - 4m_\eta^2 + m_h^2)^2} \right. 
\]
\[
\left. \left( m_\sigma^2 - 4m_\eta^2 + m_h^2 \right) \right] \left[ \frac{-4(m_\sigma^2 + 8m_\eta^2 - m_h^2)(4m_\eta^2 + m_h^2 - m_\sigma^2 - 4m_\eta^2) (m_\sigma^2 - \xi v^2 \lambda_\eta(1 - \xi)}{(m_\sigma^2 - 4m_\eta^2 + m_h^2)^2} \right. 
\]
\[
\left. + \frac{(-4\lambda_\eta v^2 + (-m_\eta^2 + 4m_\eta^2 + m_h^2 + \lambda_\eta v^2)^2) + (m_\sigma^2 - 4m_\eta^2 + m_h^2)^2 (m_\sigma^2 - \xi v^2 \lambda_\eta(1 - \xi) + m_\eta^2)}{(m_\sigma^2 - 4m_\eta^2 + m_h^2)^2 \Re(\Pi_\sigma(4m_\eta^2))\Re(\Pi_h(4m_\eta^2))} \right. 
\]
\[
\left. + \frac{4(m_\sigma^2 - 4m_\eta^2)(m_\sigma^2 - 4m_\eta^2)(8m_\eta^2 + m_h^2 - m_\sigma^2)(-\lambda_\eta v^2 + (m_\sigma^2 + \lambda_\eta v^2))(-\lambda_\eta v^2 + (m_\sigma^2 + \lambda_\eta v^2))}{(m_\sigma^2 - 4m_\eta^2 + m_h^2)^2 \Re(\Pi_\sigma(4m_\eta^2))\Re(\Pi_h(4m_\eta^2))} \right. 
\]
\[
\left. + \frac{2(-4\lambda_\eta v^2 + \lambda_\eta v^2 \xi (m_\eta^2 + 20m_\eta^2 + m_h^2 + 8\lambda_\eta v^2)^2 + \xi^2 (4m_\eta^2 + \lambda_\eta v^2) (m_\eta^2 - 4m_\eta^2 + 4\lambda_\eta v^2)^2)}{(m_\sigma^2 - 4m_\eta^2 + m_h^2)^2 \Re(\Pi_\sigma(4m_\eta^2))\Re(\Pi_h(4m_\eta^2))} \right] \left. \right], \quad (D5)
\]
\[ \sigma_{\nu\eta \rightarrow hh} = \frac{m_h \sqrt{m^2 - m_h^2}}{4m_h^4 f^8} \left[ 64v^4(m_n^2 - f^2h_\eta(1 - \xi)^4) + \frac{(4m^2_n - f^2h_\eta(1 - \xi)^2(8(2m_n^2 + m_h^2)v^2 - 3f^2m_n^2(1 - \xi))^2}{(1 - \xi)^2(m_n^2 - 4m_n^2 + i\tilde{\Theta}(\Pi_\eta(4m_n^2))^2) - (2m_n^2 - m_h^2)(1 - \xi)^2(2m_n^2 - 4m_n^2 + i\tilde{\Theta}(\Pi_\eta(4m_n^2))^2) - 16(m^2_n - m_h^2)v^2(4m^2_n - f^2h_\eta(1 - \xi)(m_n^2 - f^2h_\eta(1 - \xi))^2(8(2m_n^2 + m_h^2)v^2 - 3f^2m_n^2(1 - \xi))}{(2m_n^2 - m_h^2)(1 - \xi)^2(2m_n^2 - 4m_n^2 + i\tilde{\Theta}(\Pi_\eta(4m_n^2))^2)} + 16(m^2_n - f^2h_\eta(1 - \xi))^2(8(m^2_n - 2m_h^2)v^4 + f^4( - h_\eta v^2 + (m_n^2 + 2h_\eta v^2 + \xi^2(m_n^2 - h_\eta v^2)))}{(2m_n^2 - m_h^2)(1 - \xi)^2} + \frac{2f^4(8(2m_n^2 + m_h^2)v^2 - 3f^2m_n^2(1 - \xi))(8(-2m_n^2 + m_h^2)v^2 - \xi(m_n^2 + 2h_\eta v^2) + \xi^2(m_n^2 - h_\eta v^2))}{v^2(4m^2_n - m_h^2)^{-1}(4m^2_n - f^2h_\eta(1 - \xi))^{-1}m^2_n - 4m_n^2 + i\tilde{\Theta}(\Pi_\eta(4m_n^2))^2(1 - \xi)^2} + \frac{8(2m_n^2 - m_h^2)v^4 + f^4( - h_\eta v^2 - (m_n^2 + 2h_\eta v^2))}{v^2(1 - \xi)^4} - \frac{48f^2v^2m_n^2(4m^2_n - m_h^2)(m_n^2 - f^2h_\eta(1 - \xi))^2(m_n^2 + 2m^2_n + m_h^2)}{(2m_n^2 - m_h^2)^2m^2_n - 4m_n^2 + i\tilde{\Theta}(\Pi_\eta(4m_n^2))^2(1 - \xi)} + \frac{9f^4m_n^2(2m^2_n + m_h^2)^2}{v^2(1 - \xi)^4}\right]. \]

(D6)
The couplings of $\eta$ to SM fermions other than top depend on the formal embedding of the SM quarks into SO(6) representations. We fix for convenience the same couplings for all the quarks. Small changes should not make a difference our analysis.