Quantum information scrambling after a quantum quench

Vincenzo Alba\textsuperscript{1} and Pasquale Calabrese\textsuperscript{2,3}

\textsuperscript{1}Institute for Theoretical Physics, Universiteit van Amsterdam, Science Park 904, Postbus 94485, 1098 XH Amsterdam, The Netherlands
\textsuperscript{2}SISSA and INFN, via Bonomea 265, 34136 Trieste, Italy
\textsuperscript{3}International Centre for Theoretical Physics (ICTP), Strada Costiera 11, 34151 Trieste, Italy

How quantum information is scrambled in the global degrees of freedom of non-equilibrium many-body systems is a key question to understand local thermalization. Here we propose that the scaling of the mutual information between two intervals of fixed length as a function of their distance is a diagnostic tool for scrambling after a quantum quench. We consider both integrable and non-integrable one dimensional systems. In integrable systems, the mutual information exhibits an algebraic decay with the distance between the intervals, signalling weak scrambling. This behavior may be qualitatively understood within the quasiparticle picture for the entanglement spreading, predicting, in the scaling limit of large intervals and times, a decay exponent equal to 1/2. Away from the scaling limit, the power-law behavior persists, but with a larger (and model-dependent) exponent. For non-integrable models, a much faster decay is observed, which can be attributed to the finite life time of the quasiparticles: unsurprisingly, non-integrable models are better scramblers.

Introduction. The spreading of quantum information is a central process for our understanding of non equilibrium many-body systems. A fundamental question, key to this work, is how the quantum information encoded in the initial state gets dispersed globally during the dynamics following a quantum quench. A prominent idea, originally formulated in the context of the information paradox in black holes, is that in generic quantum many-body systems the initial information gets dispersed in the many-body global entanglement and correlation [1–3] (scrambling scenario). Unfortunately, very few explicit results are available for realistic systems, although calculations in conformal field theories (CFTs) with large central charge [4–7], holographic setups [8], and mean-field-like models [9] provide useful insights. Several tools have been proposed to diagnose scrambling, such as the tripartite information [10–13], out-of-time-order correlators [3, 14–18], and entanglement of operators [19–34].

Scrambling is unanimously expected to have a different nature in integrable and generic, i.e., non-integrable, systems, although a quantitative assessment has not been made and it is one of the main goals of this work. Indeed, integrable systems possess well-defined quasiparticles, which have a local-in-space nature, much alike to classical solitons. As these quasiparticles move ballistically, they spread the initial (EPR-like) correlations [35]. Crucially, initial correlations get “dressed” by non-trivial thermodynamic and many-body effects. For instance, the late-time properties of the quasiparticles after a quantum quench are described by an emergent thermodynamic macrostate (typically a Generalized Gibbs Ensemble (GGE) [36–40], in contrast to the thermal ensemble for non-integrable models [41–46]). Hence, the entanglement entropy carried by the quasiparticles becomes the thermodynamic entropy of the stationary ensemble [46–51]. In the appropriate space-time scaling limit, the entanglement evolution is quantitatively describable by a simple hydrodynamic framework (quasiparticle picture [35]), for both non-interacting and interacting systems [51–58]. Consequently, the scrambling of information takes place only through entanglement spreading and cannot be very effective. Conversely, non-integrable systems are expected to be better scramblers: they do not have stable quasiparticles and so they should loose memory of initial local correlations faster as compared to integrable ones. The question that we address here is: Is it possible to go beyond this qualitative scenario, characterizing quantitatively quantum information scrambling in integrable and non-integrable systems? Here we positively answer this question by studying the mutual information between two distant intervals $A_1$ and $A_2$ [59–61]

$$I_{A_1:A_2} = S_{A_1} + S_{A_2} - S_{A_1 \cup A_2}. \quad (1)$$

(Here $S_X \equiv -\text{Tr} \rho_X \ln \rho_X$ is entanglement entropy of the subsystem $X$ in terms of its reduced density matrix $\rho_X$.)

We consider two intervals of equal length $\ell$ at a distance $d$. The mutual information always exhibits a well defined peak at intermediate times, but its features depend on whether the system is integrable or not, reflecting the different degrees of scrambling. Indeed, for large distance $d$ and at fixed $\ell$, the peak amplitude decays algebraically in $d$ in the integrable case and much faster (likely exponentially) for non-integrable systems.

Scrambling and quasiparticles. We start discussing the mutual information scrambling within the quasiparticle picture, cf. Fig. 1. In generic integrable models there are, in principle, infinite species of quasiparticles [62] labelled by an integer $n$. Quasiparticles of the same species are identified by a real parameter $\lambda$, called rapidity, that for non-interacting particles is the momentum. Within each species, quasiparticles exhibit a non-trivial dispersion, i.e. a $\lambda$ dependent velocity $v(\lambda)$. We consider the situation in which only entangled pairs of quasiparticles with opposite rapidity $\lambda, -\lambda$ and opposite...
velocities \( v_n(\lambda) = -v_n(-\lambda) \) are created after the quench (more complicated situations, for instance with entangled triplets, can be also treated [63–65]). According to the standard picture [35], the entanglement entropy is proportional to the number of quasiparticles shared between the subsystems of interest.

Let us first consider the case of a single quasiparticle with fixed velocity \( v(\lambda) = v, \forall \lambda \), as it happens in CFTs [35]. Given two intervals of equal length \( \ell \) at distance \( d \), the mutual information at time \( t \) is the width of the shaded area in Fig. 1 [35, 66], i.e. \( I_{A_1:A_2} = \max(d/2, vt) + \max((d + 2\ell)/2, vt) - 2\max((d + \ell)/2, vt) \).

Here \( s_n(\lambda) \) is the quasiparticle contribution to the GGE entropy of the steady state [51] and \( v_n(\lambda) \) the quasiparticle velocity [69]. Both \( s_n \) and \( v_n \) can be calculated using thermodynamic Bethe ansatz [52]. Eq. (2) captures the correct mutual information dynamics in the scaling limit when both \( d \) and \( \ell \) are of order of \( v_{\text{max}}t \) (with \( v_{\text{max}} \) the maximum velocity of all quasiparticles). According to Eq. (2), \( I_{A_1:A_2} \) is zero for \( t < d/(2v_{\text{max}}) \), then it increases linearly up to \( t \leq (d + \ell)/(2v_{\text{max}}) \). At later times it exhibits a short and slower increase until it reaches a maximum and finally it slowly decays at long times. Both the growth after \( t > (d + \ell)/(2v_{\text{max}}) \) and the asymptotic slow decay are due to the presence of slow quasiparticles.

An intriguing feature of \( I_{A_1:A_2} \) is that it collapses on a scaling function of \( t/\ell \) (or \( t/d \)); hence the peak close to \( t \approx (d + \ell)/(2v_{\text{max}}) \) does not depend on \( d \) and there is no actual sign of scrambling, as for the case with fixed velocity, as indeed tested in few instances [51, 66]. However, here we change the perspective and consider intervals of fixed length with increasing distance, i.e. \( \ell \ll d \). In principle this limit is not captured by the quasiparticle picture because \( \ell \) should be proportional to \( d \). Yet, we can think to the situation in which \( \ell = ad \) with \( a \ll 1 \). Thus, the quasiparticle picture might describe an intermediate regime between very small \( \ell \) and the scaling regime with \( \ell \) of the same order of \( d \). In fact, it is really illuminating to look at this regime within the quasiparticle picture: in Eq. (2) only the quasiparticles within a shell of width proportional to \( \ell/d \) close to \( v \approx v_{\text{max}} \) may contribute to the height of the peak of \( I_{A_1:A_2} \).

As a consequence, the peak of the mutual information vanishes as \( d \to \infty \). Thus, even for integrable models there is a sort of weak scrambling, related to the non-trivial dispersion of the quasiparticles.

The above reasoning can be made quantitative by expanding up to second order \( v_n(\lambda) \) and \( s_n(\lambda) \) around the rapidity \( \lambda_{\text{max}} \) of the fastest quasiparticle. We focus on the maximum \( I_{\text{max}}^{(n)} \) of the mutual information associated with quasiparticle species \( n \). At the leading order in \( \ell/d \), Eq. (2) provides

\[
I_{\text{max}}^{(n)} = \frac{4}{3} \left( \frac{2v_{\text{max}}\ell}{v_{\text{max}}d} \right)^{2/3} s_{n,\text{max}} \ell, \quad d \gg \ell. \tag{3}
\]

In (3), \( s_{n,\text{max}} \) is the thermodynamic entropy of the fastest quasiparticle of the species \( n \) with rapidity \( \lambda_{\text{max}} \), and \( v_{n,\text{max}}^{\prime} \equiv d^2v_n(\lambda)/d\lambda^2|_{\lambda_{\text{max}}} \). We also assume that \( v_{n,\text{max}}^{\prime} \neq 0 \), that \( v_n(\lambda) \) has only one maximum, and that \( s_{n,\text{max}} \neq 0 \). Hence, each quasiparticle species is responsible for a peak \( I_{\text{max}}^{(n)} \), in the mutual information decaying for large distance as \( d^{-1/2} \). As we anticipated, Eq. (3) may describe an intermediate regime of \( \ell \) such that \( 1 \ll \ell \ll d \).

The model. To benchmark our results, we consider the spin-1/2 chain described by the Hamiltonian

\[
H = \sum_{i=1}^{L} \left[ \frac{1}{2} (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) + \Delta S_i^z S_{i+1}^z \right] + J_{nn} \sum_{\alpha=x,y,z} \sum_{i=1}^{L-2} S_i^\alpha S_{i+2}^\alpha + h_s \sum_{i=1}^{L} S_i^z. \tag{4}
\]
Here $S_{x-x'}^z$ are spin-1/2 operators. $J_{nn}$ is the strength of the next-nearest-neighbor interaction, $h_x$ a longitudinal magnetic field, and $\Delta$ an anisotropy parameter. For $J_{nn} = h_x = 0$ the model is the XXZ chain, which is integrable by Bethe ansatz. For $J_{nn} = 0$ and $h_x = 1$, $H$ is integrable only at $\Delta = 1$. For $J_{nn} \neq 0$ the model is not integrable. For $J_{nn} = h_x = 0$, $H$ defines the XX chain, which is mappable onto the free-fermion tight binding model $H = \sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i)$, with $c_i$ standard fermionic operators. Here we only consider the quench from the Néel state ($|N\rangle \equiv |\uparrow\downarrow\cdots\rangle$).

**Free-fermion scramblers.** Let us first focus on the tight binding model. The fermionic mutual information dynamics is calculable from the time-dependent two-point correlation function $C_{x,x'}$ restricted to $A_1 \cup A_2$ [70] (fermionic and spin mutual informations are different [78, 79]). In free-fermion language the Néel state is $|N\rangle = \prod_{i=1}^d c_{2i}^\dagger |0\rangle$. A straightforward application of Wick theorem yields the time-dependent correlation function in the thermodynamic limit which reads

$$
\langle c_{x}^\dagger(t) c_{x'}^\dagger(t) \rangle = \frac{\delta_{x,x'}^2}{2} \left(1 + \frac{(-1)^x}{2} \right) \int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{ik(x-x')} e^{4it \cos(k)} = \frac{\delta_{x,x'}}{2} \left(1 + \frac{(-1)^x}{2i(x-x')} \right) J_{x-x'}(4t),
$$

with $J_\nu(x)$ the Bessel function of the first kind. Denoting with $\lambda_i$ the eigenvalues of the correlation matrix restricted to a subsystem $A$, the entanglement entropy is $S_A = -\sum_i (\lambda_i \ln \lambda_i + (1 - \lambda_i) \ln (1 - \lambda_i))$ [70], and the mutual information follows from the definition (1).

It is instructive to consider first the mutual information between two fermions at distance $d$. The 2-by-2 correlation matrix $C_{x,x'}$ is just (5) with eigenvalues $\lambda_{\pm}$

$$
\lambda_{\pm} = \frac{1}{2} \pm \frac{d^{-\frac{1}{2}}}{6\pi^2 \Gamma\left(\frac{3}{4}\right)}.
$$

and so $I_{A_1:A_2} = 2^{-3/2}/(3^{3/4} \Gamma^2(2/3))/d^{3/2}$. For fermions, the peak of the mutual information for $\ell = 1$ decays as $d^{-2/3}$. We are going to show that this behavior persists for larger, but finite, $\ell$.

For finite $\ell$, the mutual information peak is at $t = (d + \ell)/4$, since $v_{\text{max}} = 2$. The asymptotic behavior of $I_{A_1:A_2}$ for $d \to \infty$ is obtained from $C_{x,x'}$. The two points $x, x'$ are either in the same interval ($A_1$ or $A_2$) or in different ones. In the former case, the contribution of $k(x - x')$ in (5) is negligible for finite $\ell$. Hence the integral (5) for large $d$ is given by the stationary points at $k = 0, \pm \pi$, i.e.

$$
\int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{\frac{ik}{2} \cos(k)} \to \sqrt{\frac{2}{\pi d}} \cos\left(\frac{\pi}{4} - d\right).
$$

If $x$ and $x'$ are in different intervals $A_1$, then $x - x' \propto d$, and the integral is dominated by the stationary point at $k = -\pi/2$. Crucially, the saddle point contribution vanishes at $k = \pi/2$, and one has to consider the order $k^3$, which gives

$$
\int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{\frac{ik}{2} \cos(k)} \to \frac{1}{d^{\frac{3}{2}}} \int_{-\infty}^{\infty} \frac{e^\frac{ik^3}{2\pi}}{2\pi} = \frac{-6^2}{d^{3/2} \Gamma\left(\frac{3}{4}\right)}.
$$

The exponent $1/3$ appearing in (8) is ubiquitous in free-fermion models and it is related to the Airy processes [71–77]. At this point, the mutual information is dominated by the elements of $C_{x,x'}$ coming from $x, x'$ in different intervals and, since $\ell$ is finite, the same $d^{-2/3}$ behavior holds in general. This is explicitly tested in the inset of Fig. 2 for $\ell = 5$ where the decay $d^{-2/3}$ is observed neatly.

Is the quasiparticle prediction (3) recovered for larger $\ell$? This question is investigated in Fig. 2 by diagonalizing the correlation matrix numerically. We report the height of the mutual information peak after the Néel quench for different $\ell$. It is evident that the curves for $\ell = 25, 50, 100$ show a crossover from the $d^{-1/2}$ scaling for not so large $d$, to a truly asymptotic $d^{-2/3}$ decay for very large $d$. The quasiparticle prediction (3) well describes the data in a preasymptotic regime, as expected.

**Interacting integrable and non-integrable scramblers.** We finally consider the effect of interactions and of integrability-breaking perturbations. The quench from the Néel state with several parameters of the Hamiltonian (4) is studied by means of tDMRG [80] simulations. It is however computationally very demanding to calculate the entanglement entropy of two disjoint intervals. The computational cost is $\propto \chi^3$, with $\chi$ the bond dimension which must grow exponentially with $\ell$. Here we use $\chi \lesssim 1500$. Thus, we restrict ourselves to simulate small intervals with $\ell = 2$ placed at the ends of an open chain and so we can only explore the regime $\ell \sim O(1)$, without accessing the possible quasiparticle regime. All our data are reported in Fig. 3. In panels (a), (b), and (d) we focus on the integrable case. Quite generically, $I_{A_1:A_2}$ exhibits a clear peak at intermediate times. The peak decreases as a function of $d$ and it remains visible even at large $d$. (Interestingly, at $\Delta = 1/2$ for $d = 22$ a second peak appears, reflecting the presence of two species...
Mutual information $I_{\text{mutual information}}$ of quasiparticles. However, $d = 22$ is not large enough to resolve the two peaks neatly.

The picture changes dramatically upon breaking integrability. In Fig. 3 (c) we break integrability by switching on a longitudinal magnetic field $h_x = 1$ (cf. (4)). Now a peak is visible only for small $d$, whereas it decays rapidly at large $d$. This suggests a much faster decay of the mutual information in non-integrable models. We now move to a different integrability-breaking perturbation by setting $J_{nn} = 1$ in (4). Fig. 3 (e-f) shows results for $J_{nn} = 1$ and $\Delta = 1/2, 1$. Surprisingly, a peak is only visible for $d = 6, 8$, and it rapidly melts into a broad plateau at larger $d$. The height of the plateaux decreases quickly with $d$.

In Fig. 3 (g) and (h) we analyze the decay of the mutual information peak/plateaux as a function of $d$. The integrable cases are summarized in (g). For $\Delta = 0$, i.e., for the XX chain, the expected decay $d^{-2/3}$ is visible already for relatively small $d$. For other values of $\Delta$ a clear power-law behavior is found. For instance, at $\Delta = 1/2$ the data follow a $1/d$ behavior, while for $\Delta = 1, 2$ they suggest a faster algebraic decay. However, larger values of $d$ would be needed to extract the correct power-law reliably for all $\Delta$. For the non-integrable case in Fig. 3 (h) a faster decay is observed as compared with the integrable case in Fig. 3 (g). The data are compatible with an exponential decay, in spite of the fact that we have only one decade of data for $d$. Thus our data are compatible with a strong-scrambling scenario for non-integrable models. Note that in the kicked Ising chain, which is regarded as a maximally chaotic model, the mutual information peak is completely absent [81]. In chaotic random circuits, the mutual information is exponentially small for $\ell < d$ [49].

Conclusions. We investigated the information scrambling after a quantum quench in integrable and non-integrable systems, focusing on the mutual information between far apart intervals of fixed lengths. While the standard configuration in the quasiparticle picture with $\ell \propto d$ has an enormous computational cost, intervals of fixed lengths can be simulated easily, as noticed long ago [82]. We found that for integrable systems the mutual information decays as a power-law of the distance between the intervals. For non-integrable models a faster decay, compatible with an exponential, is observed.

Our work motivates further studies in many new directions. First, it is important to corroborate the correctness of our conclusions for other systems and different quenches, both analytically and numerically. Then, it is natural to wonder whether the model dependent exponent for the decay of the mutual information in integrable systems can be obtained analytically. An intriguing possibility would be to understand whether this exponent can distinguish between interacting and free integrable systems [83]. Our results may suggest that in interacting theories the exponent is larger than in free ones. Comitantly, further checks of the $d^{-2/3}$ decay in other non-interacting systems are necessary to assess its universality. A natural question is also whether a crossover between algebraic and exponential decay can take place in models with unstable but long lived quasiparticles, as for instance in prethermalization scenario [84–87] or in confining models [88–92]. Finally, it would be interesting to investigate the mutual information scrambling in those models without a maximum quasiparticle velocity, such as integrable non-relativistic quantum field theories.

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