Multiplicative structure on the Hochschild cohomology of crossed product algebras

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Abstract

Consider a smooth affine algebraic variety $X$ over an algebraically closed field $k$, and let a finite group $G$ act on $X$. We assume that $\text{char } k$ is greater than $\dim X$ and $|G|$. An explicit formula for multiplication on the Hochschild cohomology of a crossed product $HH^*(k[G] \ltimes k[X])$ is given in terms of multivector fields on $X$ and $g$-invariant subvarieties of $X$ for $g \in G$.

1 Introduction

Let $X$ be a smooth algebraic variety. It is well-known ([1], [4], [6]) that the groups $Ext^*_X(O_\Delta, O_\Delta)$ where $\Delta$ denotes the diagonal (these groups will be further referred to as Hochschild cohomology $HH^*(X)$) may be interpreted in terms of multivector fields on $X$:

$$HH^i(X) = \bigoplus_{p+q=i} H^p(X, \Lambda^q TX)$$

as vector spaces ([3], Thm. 8.4, [1], Corr. 4.2). For affine $X$ this becomes an algebra isomorphism

$$HH^*(X) \cong \bigoplus_i \Gamma(X, \Lambda^i TX)$$

where the multiplication is given by the cup product in the left hand side, and the wedge product in the right hand side.

Our goal is to extend this result to the situation of a smooth affine algebraic $X$ with an action of a finite group $G$.

The author would like to thank Pavel Etingof, Vasiliy Dolgushev, Boris Shoikhet and Sergei Fironov for useful discussions. The author would also like to express her special gratitude to Xiang Tang for pointing out an essential gap (which led to a significant shortening of the paper).
2 Main results

Let $X$ be a smooth affine algebraic variety over an algebraically closed field $k$ of characteristic greater than $\dim X$ and $|G|$. Denote by $A = \Gamma(X, \mathcal{O}_X)$ the algebra of regular functions on $X$, let $B = k[G] \ltimes A$ be crossed product algebra with multiplication defined by $ga = a^g g$, where $a^g$ denotes the result of $g$-action on $a$. Denote by $\pi^g$ the operator of symmetrization by $g$: if $g$ has order $k$, then $\pi^g = \frac{1}{k} \sum_{i=1}^{k} g_i$. Let it act on the multivectors of degree $l$ as $\pi^g \wedge \ldots \wedge \pi^g$ ($l$ components). Let $X^g_m$ for $g \in G$ denote the $m$-th connected component of the subvariety of $g$-invariants in $X$, let $d_{g,m} = \text{codim} \ X^g_m$.

**Theorem 1** (cf. [7], Theorem 3.11)

$$ HH^i(B) \cong \left( \bigoplus_{g \in G,m} \Gamma(X^g_m, (\Lambda^{i-\text{codim} X^g_m} TX^g_m \otimes \Lambda^{\text{top}} (N_X X^g_m))) \right)^G $$

as vector spaces.

A similar result was proven in [3], [5]:

**Theorem 2** ([3], [5]) For a complex symplectic vector space $V$ with a symplectic linear action of a finite group $G$ one has

$$ HH^i(C[G] \ltimes C[V]) \cong \left( \bigoplus_{g \in G} \Omega^{i-\text{codim} V^g}(V^g) \right)^G $$

as graded vector spaces.

Here $\Omega^k(Y)$ denotes the space of differential $k$-forms on $Y$. One can easily see that in presence of a symplectic form there is an isomorphism $\Omega^i(X^g_m) \cong \Lambda^{i-\text{codim} X^g_m} TX^g_m \otimes \Lambda^{\text{top}} (N_X X^g_m)$, so Theorem 2 is a special case of Theorem 1.

**Theorem 3** The multiplication on $HH^i(B)$ is given by

$$( \sum_{g \in G,m} \alpha_{g,m} \otimes \beta_{g,m} ) \cdot ( \sum_{h \in G,n} \gamma_{h,n} \otimes \delta_{h,n} ) = $$

$$= (-1)^{d_{g,m}(j-d_{h,n})} \sum_{u \in G,k} \sum_{gh=u, X^g_m \cap X^h_n \ni X^u_k} (\pi^u \alpha_{g,m} |X^u_k \wedge \pi^u \gamma_{h,n} |X^u_k) \otimes (\beta_{g,m} |X^u_k \wedge \delta_{h,n} |X^u_k) $$

where $\alpha_{g,m}, \gamma_{g,m} \in \Gamma(X^g_m, \Lambda^{i-d_{g,m}} TX^g_m)$ and $\beta_{g,m}, \delta_{g,m} \in \Gamma(X^g_m, \Lambda^{d_{g,m}} N_X X^g_m)$. The sum in the RHS is taken over all $g$ and $h$ such that $X^g_k$ is a component of $X^m \cap X^n$ for some $k,m,n$. 

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Another description of the multiplication in $HH^*(B)$ was given in [3]. When $X$ is a symplectic variety, the formula simplifies in the following way:

**Theorem 4** Let $X$ be a symplectic variety with a symplectic form $\omega$, and let the action of $G$ be symplectic. Then

$$HH^i(B) \cong (\bigoplus_{g \in G} \Gamma(X_m^g, \Lambda^{i-d_{g,m}}TX_m^g))^G$$

as vector spaces. The multiplication is given by

$$\left( \sum_{g \in G, m} \alpha_{g,m} \right) \cdot \left( \sum_{h \in G, n} \gamma_{h,n} \right) = (-1)^{d_{g,m}(j-d_{h,n})} \sum_{u \in G,k} \sum_{gh=u, X_m^g \cap X_n^h \supset X_k^u} (\pi^u \alpha_{g,m}|X_k^u \wedge \pi^u \gamma_{h,n}|X_k^u).$$

### 3 Preliminaries

Define an $A$-bimodule $Ag$ as a submodule $Ag = \{ag \mid a \in A\}$ in $B$. The following proposition was proved in [2]:

**Proposition 1** ([2], Prop. 3)

$$HH^i(B) \cong (\bigoplus_{g \in G} H^i(A,Ag))^G$$

as vector spaces.

In fact, the complexes $C^*(B,B)$ and $(\bigoplus_{g \in G} C^*(A,Ag))^G \cong (C^*(A,B))^G$ are quasiisomorphic. The multiplication is defined on the Hochschild cochains, hence on $(C^*(A,B))^G$. To write it down in a convenient basis one should extend it to $C^*(A,B)$ (possibly losing cohomological properties beyond $G$-invariants). We have a following lemma-definition:

**Lemma 1** The multiplication on the Hochschild cochains $C^*(A,B)$ given by the map

$$\mu : C^n(A,B) \otimes C^j(A,B) \to C^{n+j}(A,B)$$

$$\mu(\phi_1 \otimes \psi)_i(a_1 \otimes \ldots \otimes a_i) = \phi_i(a_1 \otimes \ldots \otimes a_i) \cdot \psi_j(a_{i+1} \otimes \ldots \otimes a_{i+j}) gh,$$

where $\phi \in \text{Hom}(A^{\otimes i}, A)$, $\psi \in \text{Hom}(A^{\otimes j}, A)$, on the $G$-invariant cohomology $(H^*(A,B))^G$ coincides with the multiplication that comes from the isomorphism with $HH^*(B)$.

The proof is a direct computation.
4 The local case

In this section let \( X \) be a linear space \( V \cong k^n \), so \( k[V] = k[x_1, \ldots, x_n] \) is the polynomial algebra. Let the action of the group \( G \) on \( X \) be linear. For \( g \in G \) denote by \( V^g \) the space of \( g \)-invariant vectors in \( V \), and by \((V^g)^\vee\) the subspace of \( V \) generated by eigenvectors of \( g \) with eigenvalues different from one. The matrix of \( g \) is diagonalizable, so \( V = V^g \oplus (V^g)^\vee \). Note that \( k[V^g]g \cong k[V^g] \) as an \( A \)-bimodule (compare with Lemma 3).

**Proposition 2**

1. \( H^i(k[V], k[V]g) \cong \Lambda^{i-d_g} V^g \otimes \Lambda^{d_g} (V^g)^\vee \otimes k[V^g] \).

2. \( H^i(k[V], k[V^g]) \cong \Lambda^i V \otimes k[V^g] \).

3. The cohomological map \( H^i(k[V], k[V]g) \to H^i(k[V], k[V^g]) \) which arises from the morphism \( k[V]g \to k[V^g]g \cong k[V^g] \) is the natural inclusion \( \Lambda^{i-d_g} V^g \otimes \Lambda^{d_g} (V^g)^\vee \otimes k[V^g] \to \Lambda^i V \otimes k[V^g] \).

**Proof.** Choose a basis \( \{v_1, \ldots, v_n\} \) in \( V \) so that \( v_1, \ldots, v_{n-d_g} \) are \( g \)-invariant and span \( V^g \), and \( v_{n-d_g+1}, \ldots, v_n \) are eigenvectors of \( g \) with eigenvalues \( \lambda_i \neq 1 \), hence span \((V^g)^\vee\). Let \( x_i \) be the dual basis of \( V^* \subset A \).

The algebra \( k[V] \) has a Koszul resolution, so the groups \( H^i(k[V], k[V]g) \) can be computed as the cohomology of the complex

\[
0 \leftarrow \Lambda^n V \otimes k[V]g \leftarrow \Lambda^{n-1} V \otimes k[V]g \leftarrow \cdots \leftarrow V \otimes k[V]g \leftarrow k[V]g \leftarrow 0
\]

with the differential

\[
d(\xi \otimes ag) = \sum_{i=1}^n v_i \wedge \xi \otimes (x_i - x_i^g)ag = \sum_{i=n-d_g+1}^n v_i \wedge \xi \otimes (1 - \lambda_i)x_iag
\]

and the groups \( H^i(k[V], k[V^g]) \) as the cohomology of

\[
0 \leftarrow \Lambda^n V \otimes k[V^g] \leftarrow \Lambda^{n-1} V \otimes k[V^g] \leftarrow \cdots \leftarrow V \otimes k[V^g] \leftarrow k[V^g] \leftarrow 0
\]

with zero differential. Then the proposition is straightforward. □.

The situation of a vector space with a linear action of a finite group has a nice property: the Koszul complex \( \Lambda^* V \otimes k[V]g \) is a direct summand in the Hochschild complex \( \text{Hom}(k[V]^{\otimes*}, k[V]g) \), and the complex formed by the cohomology (with zero differential) \( \Lambda^{i-d_g} V^g \otimes \Lambda^{d_g} (V^g)^\vee \otimes k[V^g] \) is in turn a direct summand in the Koszul complex (here we use the fact that \( k[V^g] \) is a submodule of \( k[V]g \)), hence \( H^i(k[V], k[V]g) \) is a direct summand in \( C^i(k[V], k[V]g) \). Now the map \( \mu \) from Lemma 4 combined with the projection, gives
a map $H^i(k[V], k[V]) \otimes H^j(k[V], k[V]) \to H^{i+j}(k[V], k[V])$ that coincides with the multiplication in the $G$-invariant part. In the setting of Proposition 2 this map may be computed directly. To do this in a convenient way, we need following lemma:

**Lemma 2** If $(V^g) \cap (V^h) = \{0\}$, then $V^{gh} = V^g \cap V^h$.

**Proof.** Obviously $V^g \cap V^h \subset V^{gh}$. The symmetrization by $g$ (denoted by $\pi^g$) projects $V$ onto $V^g$. Let $(V^g) \cap (V^h) = \{0\}$. The kernel of $\pi^g$ is $(V^g)$. If we take any $v \in V^{gh}$, then $v = gh \cdot v$ and $0 = \pi^g v - \pi^gh \cdot v = \pi^g(v - h \cdot v)$, hence $(Id - h)v \in (V^g)$. But the operator $Id - h$ is the projection onto $(V^h)\cap$, hence $(Id - h)v = 0$ and $v \in V^h$. By the same argument $v \in V^g$, and since $v$ was an arbitrary element of $V^{gh}$, we have $V^{gh} \subset V^g \cap V^h$. $\Box$

**Proposition 3** The map

$$\bar{\mu} : \Lambda^{i-d_g} V^g \otimes \Lambda^{d_g}(V^g) \otimes \Lambda^{i-d_h} V^h \otimes \Lambda^{d_h}(V^h) \to \Lambda^{i+j-d_{gh}} V^{gh} \otimes \Lambda^{d_{gh}}(V^{gh})$$

which is zero if $V^{gh} \neq V^g \cap V^h$ and sends $(\xi^1 \otimes \xi^2 \otimes f) \otimes (\nu^1 \otimes \nu^2 \otimes e)$ to $(-1)^{d_g(j-d_h)} \pi^g \xi^1 \cap \pi^h \nu^1 \otimes \xi^2 \otimes (fe)|_{V^{gh}}$ otherwise, induces the multiplication on the $G$-invariant part, which coincides with the cup product.

**Proof.** The maps

$$\psi_{i,g} : Hom(k[V]^\otimes i, k[V]_g) \to \Lambda^{i-d_g} V^g \otimes \Lambda^{d_g}(V^g) \otimes k[V]$$

may be written down as follows:

$$\psi_{i,g}(\alpha g) = \sum_{1 \leq m_1 < \ldots < n - d_g} \alpha(x_{m_1} \otimes \ldots x_{m_{1-d_g}} \otimes x_{n-d_g+1} \otimes \ldots x_n) v_{m_1} \wedge \ldots v_{m_{1-d_g}} \otimes v_{n-d_g+1} \wedge \ldots v_n f|_{V^g}.$$ 

As for the maps in the inverse direction, they are quite complicated, but since $\psi_{i,g}$ are computed only on linear functions, we can only consider the linear parts of cochain maps:

$$\phi_{i,g} : \Lambda^i V \otimes k[V]_g \to Hom((V^*)^\otimes i, k[V]_g)$$

which are of the form

$$\phi_{i,g}(\xi_1 \wedge \ldots \wedge \xi_i \otimes f)(y_1 \otimes \ldots \otimes y_i) = \frac{1}{i!} \sum_{\sigma \in S_i} (-1)^\sigma \langle \xi_1, y_{\sigma(1)} \rangle \cdot \ldots \cdot \langle \xi_i, y_{\sigma(i)} \rangle f g.$$
Hence the product $\psi_{i+j,gh}\mu(\phi_{i,g} \otimes \phi_{j,h})$ of two vector fields $\xi = \xi_1 \wedge \ldots \wedge \xi_r \otimes f \in \Lambda^1 V \otimes k[V^g]$ and $\nu = \nu_1 \wedge \ldots \wedge \nu_s \otimes e \in \Lambda^1 V \otimes k[V^h]$ is the $\Lambda^{i+j-d_{gh}} V^{gh} \otimes \Lambda^{d_{gh}} (V^{gh})^\vee$-component of $\xi_1 \wedge \ldots \wedge \xi_r \otimes \nu_1 \wedge \ldots \wedge \nu_s \otimes f \otimes e \in \Lambda^1 V \otimes k[V^{(g, h)}]$. Then if $\xi$ and $\nu$ are taken in the cohomology, $\xi$ contains $\Lambda^{top} (V^g)^\vee$, and $\nu$ contains $\Lambda^{top} (V^h)^\vee$. If $(V^g)^\vee \cap (V^h)^\vee \neq \{0\}$, then $\Lambda^{top} (V^g)^\vee \cap (V^h)^\vee = \{0\}$, and the product is automatically zero; if $(V^g)^\vee \cap (V^h)^\vee = \{0\}$, then by Lemma 4 $V^{gh} = V^g \cap V^h$ and $(\Lambda^{gh}) = (V^g)^\vee \oplus (V^h)^\vee = (V^g)^\vee \oplus ((V^h)^\vee)^\vee$. Note that in this case $(fe)^\vee V^{gh} = (fe)^\vee$. Now its time to write $\xi = \xi_1 \otimes \xi_2 \in \Lambda^{i-d_{gh}} V^g \otimes \Lambda^{d_{gh}} (V^g)^\vee$, $\nu = \nu_1 \otimes \nu_2 \in \Lambda^{i-d_{gh}} V^h \otimes \Lambda^{d_{gh}} (V^h)^\vee$. Then the product may be rewritten as $(-1)^{d_{gh}(j-d_{gh})} \xi_1 \otimes (\nu_1)^g \otimes \xi_2 \otimes (\nu_2)^h \otimes (fe)|_{V^{gh}}$. Note that for any vector $w \in V$ we have $w - w^g \in (V^g)^\vee$, so after taking the wedge product with $\xi_2 \in \Lambda^{top} (V^g)^\vee$ we have $w \wedge \xi^2 = w^g \wedge \xi^2$, hence the product simply equals $(-1)^{d_{gh}(j-d_{gh})} \xi_1 \wedge \nu_1 \wedge \xi_2 \wedge \nu_2 \otimes (fe)|_{V^{gh}}$ which in turn equals $\pi^{gh} \xi_1 \wedge \pi^{gh} \nu_1 \otimes \xi_2 \wedge \nu_2 \otimes f |_{V^{gh}}$ and we are done. \[\square\]

If $V$ carries a symplectic form $\omega$, and the action of $G$ is symplectic, then for all $g$ the forms $\omega|_{(V^g)^\vee}$ are nondegenerate, and we can construct nonzero elements $s_g \in \Lambda^{d_{gh}} (V^g)^\vee$ such that if $V^{gh} = V^g \cap V^h$, then $s_{gh} = s_g \wedge s_h$; namely, take $s_g$ dual to $(\omega|_{(V^g)^\vee})^{\vee}$.

Using sections $s_g$, another canonical isomorphism can be established:

**Proposition 4** If $V$ is symplectic, then $H^i(k[V], k[V]^g) \cong \Lambda^{i-d_{gh}} V^g \otimes k[V^g]$. The multiplication is given by the wedge product: the product of two vector fields $\xi_g \in \Lambda^{i-d_{gh}} V^g \otimes k[V^g]$ and $\xi_h \in \Lambda^{i-d_{gh}} V_h \otimes k[V^h]$ is zero if $V^{gh} \neq V^g \cap V^h$ and $(-1)^{d_{gh}(j-d_{gh})} \xi_g \wedge \xi_h$ otherwise.

## 5 The affine case

Now we can return to the case of smooth affine algebraic $X$ and $A = k[X]$ its algebra of regular functions. The group $G$ acts algebraically on $X$; for $g \in G$ denote by $X^g$ the $m$-th connected component of the algebraic subvariety of $g$-invariant points. By finiteness of $G$ the variety $X^g$ is smooth. Denote by $A_{g,m} = k[X^g]$ its algebra of regular functions.

**Lemma 3** $A_{g,m} \simeq A_{g,m}^g$ as $A$-bimodules.

Note that the normal bundle $N_X X^g$ is naturally embedded into the restriction $TX|_{X^g}$ as a subbundle generated by $g$-semiinvariant vectors.

**Proposition 5**

1. $H^i(A, A_g) \simeq \bigoplus_m \Gamma(X^g, \Lambda^{i-d_{a,m}} TX^g \otimes \Lambda^{d_{a,m}} N_X X^g)$;

2. $H^i(A, A_{g,m}) \simeq \Gamma(X^g, \Lambda^i TX|_{X^g})$;
3. The cohomological map $H^i(A, Ag) \to \bigoplus_m H^i(A, A_{g,m})$ induced by the map $Ag \to \bigoplus_m A_{g,m}g \simeq \bigoplus_m A_{g,m}$ comes from the natural inclusions $\Lambda^{i-d_{g,m}}TX^g_m \otimes \Lambda^{d_{g,m}}N_XX^g_m \to \Lambda^iTX|_{X^n}^g$.

Proof. It is convenient to switch to the language of coherent sheaves. By definition $H^i(A, \cdot) = Ext^i_{A \otimes A^{op}}(A, \cdot)$; the algebra $A$ is commutative, hence $A^{op} \simeq A$, and $A \otimes A = \Gamma(X \times X, \mathcal{O}_{X \times X})$. Let $\Delta \subset X \times X$ be the diagonal, $Y \subset X \times X$ be the graph of $g : X \to X$. Then $H^i(A, Ag) = Ext^i_{X \times X}(\mathcal{O}_\Delta, \mathcal{O}_Y)$, and $H^i(A, A_{g,m}) = Ext^i_{X \times X}(\mathcal{O}_\Delta, \mathcal{O}_{\Delta^m_n})$.

Our main tool in the proof of the proposition will be the following lemma:

**Lemma 4** Let $F, G$ be coherent sheaves on an algebraic variety $Y$, let $f : F \to G$ a morphism of coherent sheaves. Then $f$ is an isomorphism (resp. injective) iff for any point $y \in Y$ in a formal neighborhood of $y$ it becomes an isomorphism (resp. injective).

Then the proof goes as follows: we construct the maps of sheaves on $X \times X$:

$$Ext^i_{X \times X}(\mathcal{O}_\Delta, \mathcal{O}_Y) \xrightarrow{\phi'} \bigoplus_m Ext^i_{X \times X}(\mathcal{O}_\Delta, \mathcal{O}_{\Delta^m_n})$$

$$\bigoplus_m Ext^i_{X \times X}(\mathcal{O}_\Delta, \mathcal{O}_{\Delta^m_n}) \xleftarrow{\phi} \bigoplus_m \Lambda^i\Delta|_{\Delta^m_n}$$

$$\bigoplus_m \Lambda^i\Delta|_{\Delta^m_n} \xrightarrow{\phi''} \bigoplus_m \Lambda^{i-d_{g,m}}T\Delta^g_m \otimes \Lambda^{d_{g,m}}N_\Delta \Delta^g_m$$

(1)

then apply Lemma 4 to prove that $\phi$ is an isomorphism and $\phi'$ is an inclusion; then we can take the composition map $\psi = \phi'' \circ \phi'^{-1} \circ \phi'$ and after applying Lemma 4 once more, see that $\psi$ is an isomorphism.

Let us define the maps: $\phi'$ is the functorial map arising from $\mathcal{O}_Y \to \bigoplus \mathcal{O}_{\Delta^m_n}$ (note that $\bigcup \Delta^m_n = \Delta \cap Y$); the last one, $\phi''$, is the projection (recall that $T\Delta|_{\Delta^m_n} = T\Delta^g_m \otimes N_\Delta \Delta^g_m$). To define $\phi$, take a resolution of $\mathcal{O}_\Delta$ by free $\mathcal{O}_{X \times X}$-sheaves $\Gamma(X \times X, \mathcal{O}_\Delta) \otimes_n \mathcal{O}_{X \times X}$ (in fact, the bar resolution) and construct the map $\phi_i : \Lambda^i\Delta|_{\Delta^m_n} \to Hom_k(\Gamma(X \times X, \mathcal{O}_\Delta) \otimes^i, \Gamma(X \times X, \mathcal{O}_{\Delta^m_n}))$ directly:

$$(\phi_i(\xi_1 \wedge \ldots \wedge \xi_i))(a_1 \otimes \ldots \otimes a_i) = \frac{1}{i!} \sum_{\sigma \in S_i} (-1)^{\sigma} \partial_{\xi_1} a_{\sigma(1)} \cdots \partial_{\xi_i} a_{\sigma(i)},$$

(2)

where RHS is a well-defined function on $\Delta^m_n$. By an easy calculation, the image of $\phi$ lies in the kernel of the differential in the bar resolution, so it induces a map $\phi : \Lambda^i\Delta|_{\Delta^m_n} \to Ext_{X \times X}(\mathcal{O}_\Delta, \mathcal{O}_{\Delta^m_n})$. 

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When $g = 1$ this map is the famous Hochschild-Kostant-Rosenberg isomorphism [4], which appears in this exact form in [6], 4.6.1.1.

To proceed, we need one more lemma:

**Lemma 5** If the order $|G|$ of a finite group $G$ is prime to the characteristic of the ground field $k$, then any action of $G$ on a formal polydisc over $k$ is equivalent to a linear action.

Now for $x \in X \times X - \Delta^g$ all the considered sheaves are zero in the formal neighborhood of $x$, and for $x \in \Delta^g_{\mathfrak{m}}$, which corresponds to a point $x \in X^g_{\mathfrak{m}}$, the action of $G$ on the formal neighborhood of $x \in X$ is linear by Lemma 3 and we are in the situation of Proposition 2 with $V = T_xX$. All previous constructions of cohomology groups and their maps commute with the transition to the formal neighborhood, and in the formal neighborhood the statements follow from Proposition 2. Then we can apply Lemma 4 to finish the proof. □

Propositions 1 and 5 together prove Theorem 1.

Note that (2) actually defines a map of $\Gamma(X^g_{\mathfrak{m}}, \Lambda^i T^n X|_{X^g_{\mathfrak{m}}})$ into Hochschild cochains $C^i(A, A_{g,m})$, so we can introduce a multiplication: $H^i(A, A_g) \otimes H^j(A, A_h) \rightarrow C^{i+j}(A, A_{gh})$ The equalities from Theorems 3 and 4 hold locally by Propositions 3 and 4 hence they hold globally.

**References**

[1] A. Caldararu, *The Mukai pairing, II: the Hochschild-Kostant-Rosenberg isomorphism*, math.AG/0308080.

[2] V. Dolgushev, P. Etingof, *Hochschild cohomology of quantized symplectic orbifolds and the Chen-Ruan cohomology*, Int. Math. Res. Not. 2005, no. 27, 1657-1688, also math.QA/0410562.

[3] V. Ginzburg, D. Kaledin, *Poisson deformations of symplectic quotient singularities*, Adv. Math. 186, 1 (2004) 1-57, also math.AG/0212279.

[4] G. Hochschild, B. Kostant, A. Rosenberg, *Differential forms on regular affine algebras*, Trans. AMS 102 (1962), 383-408.

[5] D. Kaledin, *Multiplicative McKay correspondence in the symplectic case*, math.AG/0311409.

[6] M. Kontsevich, *Deformation quantization of Poisson manifolds, I*, Lett. Math. Phys. 66 157-216 (2003), also q-alg/9709040.
[7] N. Neumaier, M. J. Pflaum, H. B. Posthuma, X. Tang, *Homology of formal deformations of proper etale Lie groupoids*, math.KT/0412462.

[8] S. J. Witherspoon, *Products in Hochschild cohomology and Grothendieck rings of group crossed products*, math.RA/0212003.

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