Gauged Noncommutative Wess-Zumino-Witten Models

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Abstract

We investigate the Kac-Moody algebra of noncommutative Wess-Zumino-Witten model and find its structure to be the same as the commutative case. Various kinds of gauged noncommutative WZW models are constructed. In particular, noncommutative $U(2)/U(1)$ WZW model is studied and by integrating out the gauge fields, we obtain a noncommutative non-linear $\sigma$-model.

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1 Introduction

Noncommutative field theory has emerged from string theory in certain backgrounds [1, 2, 3, 4]. The noncommutativity of space is defined by the relation,

\[ [x^\mu , x^\nu ] = i\theta ^{\mu \nu } \]  

where \( \theta ^{\mu \nu } \) is a second rank antisymmetric real constant tensor. The function algebra in the noncommutative space is defined by the noncommutative and associative Moyal \( \ast \)-product,

\[ (f \ast g)(x) = e^{\frac{i}{2} \theta ^{\mu \nu } \frac{\partial }{\partial \xi ^{\mu }} \frac{\partial }{\partial \eta ^{\nu }}} f(\xi )g(\eta )|_{\xi = \eta = x}. \]  

A noncommutative field theory is simply obtained by replacing ordinary multiplication of functions by the Moyal \( \ast \)-product. An interesting field theory whose noncommutative version would be of interest is the WZW model. In [5], a noncommutative non-linear \( \sigma \)-model has been studied and an infinite dimensional symmetry is found. They also derived some properties of noncommutative WZW model. In [6], the \( \beta \)-function of the \( U(N) \) noncommutative WZW model was calculated and found to be the same as that of ordinary commutative WZW model. Hence, the conformal symmetry in certain fixed points is recovered. In [7] and [8], the derivation of noncommutative WZW action from a gauge theory was carried out. The connection between noncommutative two-dimensional fermion models and noncommutative WZW models was studied in [9, 10].

In this letter, we study the noncommutative two-dimensional field theory of WZW model and its gauged versions. In section 2, we calculate the current Kac-Moody algebra of the noncommutative WZW model and find that the structure and central charge of the algebra are the same as commutative WZW model. In section 3, we construct different versions of gauged noncommutative WZW models. As an example, we consider the axial gauged \( U(2) \) WZW model by its diagonal \( U(1) \) subgroup. The obtained gauged action contains infinite derivatives in its \( \ast \) structure and hence is a nonlocal field theory. Integration over the gauge fields requires solving an integral equation which we solve by perturbative expansion in \( \theta \). The result is a noncommutative non-linear \( \sigma \)-model, which may contains singular structures or a black hole.
2 The Current Algebra of Noncommutative WZW Model

The action of the noncommutative WZW model is [7]:

\[
S(g) = \frac{k}{4\pi} \int_{\Sigma} d^2 z \text{Tr}(g^{-1} \star \partial g \star g^{-1} \star \bar{\partial} g) - \frac{k}{12\pi} \int_{M} \text{Tr}(g^{-1} \star dg)^3,
\]

(2.1)

where \( M \) is a three-dimensional manifold whose boundary is \( \Sigma \), and \( g \) is a map from \( \Sigma \) (or from its extension \( M \)) to the group \( G \). We assume that the coordinates \((z, \bar{z})\) on the worldsheet \( \Sigma \) are noncommutative but the extended coordinate \( t \) on the manifold \( M \) commutes with others:

\[
[z, \bar{z}] = \theta, \quad [t, z] = [t, \bar{z}] = 0.
\]

(2.2)

We define the group-valued field \( g \) by,

\[
g = e^{i\pi^a T_a} = 1 + i\pi^a T_a + \frac{1}{2}(i\pi^a T_a)^2 + \cdots,
\]

(2.3)

where the \( T_a \)'s are the generators of group \( G \).

Inserting the \( \star \)-product of two group elements in the eq. (2.1), we find the noncommutative Polyakov-Wiegmann identity,

\[
S(g \star h) = S(g) + S(h) + \frac{1}{16\pi} \int d^2 z \text{Tr}(g^{-1} \star \bar{\partial} g \star \partial h \star h^{-1}),
\]

(2.4)

which is the same as ordinary commutative identity with products replaced by \( \star \)-products.

Using the Polyakov-Wiegmann identity, we can show that the action (2.1) is invariant under the following transformations:

\[
g(z, \bar{z}) \rightarrow h(z) \star g(z, \bar{z}) \star \bar{h}(\bar{z}).
\]

(2.5)

The corresponding conserved currents are,

\[
J(z) = \frac{k}{2\pi} \bar{\partial} g \star g^{-1},
\]

\[
\bar{J}(\bar{z}) = \frac{k}{2\pi} g^{-1} \star \partial g.
\]

(2.6)

By use of the equations of motion, we can show that these currents are indeed conserved,

\[
\bar{\partial} J(z) = \partial \bar{J}(\bar{z}) = 0.
\]

(2.7)
We quantized this model by using Poisson brackets. To illustrate this method, consider the following arbitrary action which is first order in time derivative,

$$ S = \int dt A_i(\phi) \star \frac{d\phi^i}{dt}, \quad (2.8) $$

Under the infinitesimal change of fields $\phi^i \to \phi^i + \delta\phi^i$, one can calculate the variation of the action \[11\],

$$ \delta S = \int dt F_{ij} \star \delta \phi^i \star \frac{d\phi^j}{dt}, \quad (2.9) $$

in which $F_{ij} = \partial_i A_j - \partial_j A_i$. The Poisson bracket of any two dynamical variables $X$ and $Y$ in the phase space can be written as,

$$ [X, Y]_{PB} = \sum_{i,j} F^{ij} \frac{\partial X}{\partial \phi^i} \frac{\partial Y}{\partial \phi^j}, \quad (2.10) $$

where $F^{ij}$ is the inverse of $F_{ij}$. The advantage of the above method is that no explicit introduction of coordinates and momentum is necessary. Now, consider the case for the action \[2.1\], whose variation under $g \to g + \delta g$ leads to,

$$ \delta S = \frac{k}{2\pi} \int d\sigma d\tau Tr(g^{-1} \star \delta g \star \frac{d}{d\sigma} (g^{-1} \star \frac{dg}{d\tau})). \quad (2.11) $$

Let $(g^{-1} \star \delta g) = (g^{-1} \star \delta g)^a T_a$ and $(g^{-1} \star \frac{dg}{d\sigma}) = (g^{-1} \star \frac{dg}{d\sigma})^b T_b$, where $T_a$ and $T_b$ are the group generators. So one can introduce $(g^{-1} \star \delta g)^a$ and $(g^{-1} \star \frac{dg}{d\sigma})^b$ as $\delta\phi^i$ and $\frac{d\phi^i}{d\sigma}$ in \[2.9\], respectively. Therefore the $F_{ab}$ can be read as,

$$ F_{ab} = \delta_{ab} \frac{k}{2\pi} \frac{d}{d\sigma} \quad (2.12) $$

and its inverse is,

$$ F^{ab} = \delta^{ab} \frac{2\pi}{k} \frac{d}{d\sigma}^{-1} \quad (2.13) $$

Now in \[2.10\] we take $X = \frac{2\pi}{k} J_a = Tr(T_a \frac{dg}{d\sigma} \star g^{-1})$ and $Y = \frac{2\pi}{k} J_b = Tr(T_b \frac{dg}{d\sigma} \star g^{-1})$. To find the $\frac{\partial X}{\partial \phi^i} \frac{\partial Y}{\partial \phi^j}$ term, we can evaluate the variation of $X$ and $Y$,

$$ \delta X \delta Y = \delta\phi^i \delta\phi^j \frac{\partial X}{\partial \phi^i} \frac{\partial Y}{\partial \phi^j}, \quad (2.14) $$

then drop the $\delta\phi^i \delta\phi^j$. After some algebra one can find,

$$ [X, Y] = \frac{2\pi}{k} i\delta(\sigma - \sigma') Tr[T_a, T_b] \frac{dg}{d\sigma} \star g^{-1} + \frac{2\pi}{k} i\delta'(\sigma - \sigma') Tr(T_a T_b), \quad (2.15) $$
or,

\[ [J_a(\sigma), J_b(\sigma')] = \delta(\sigma - \sigma')if_{ab}^c J_c(\sigma) + \frac{k}{2\pi}i\delta'(\sigma - \sigma')\delta_{ab}, \]  

and a similar relation holds for commutation of \( \bar{J}'s \).

Note that in the above commutation relation, \( \theta \) does not appear, and it is just as commutative ordinary affine algebra with the same central charge. The absence of \( \theta \) has been expected, since the currents are holomorphic by equations of motion and hence commutative in the sense of \( \ast \)-product.

Constructing the energy momentum tensor is also straightforward,

\[ T(z) = \frac{1}{k + N} : J_i(z)J_i(z) : + \frac{1}{k} : J_0(z)J_0(z) :, \]  

where \( J_i's \) are \( SU(N) \) currents and \( J_0 \) is the \( U(1) \) current corresponding to the subgroups of \( U(N) = U(1) \times SU(N) \). Again the products in (2.17) are commutative because of holomorphicity of the currents. So the Virasoro algebra is also the same as usual standard form and its central charge is unchanged,

\[ c = \frac{kN^2 + N}{k + N}. \]  

(2.18)

3 Gauged Noncommutative WZW Models

In this section, we want to gauge the chiral symmetry (2.3) as,

\[ g(z, \bar{z}) \rightarrow h_L(z, \bar{z}) \ast g(z, \bar{z}) \ast h_R(z, \bar{z}). \]  

(3.1)

where \( h_L \) and \( h_R \) belong to \( H \) some subgroup of \( G \). For finding the invariant action under the above transformation we need to add gauge fields terms to the action (2.1) as follows,

\[ S(g, A, \bar{A}) = S(g) + S_A + S_{\bar{A}} + S_2 + S_4, \]  

(3.2)

where, \( S(g) \) is the action (2.1) and,

\[
\begin{align*}
S_A &= \frac{k}{4\pi} \int d^2z \text{Tr}(A_L \ast \partial_\ast g \ast g^{-1}) \\
S_{\bar{A}} &= \frac{k}{4\pi} \int d^2z \text{Tr}(\bar{A}_R \ast g^{-1} \ast \partial_\ast g) \\
S_2 &= \frac{k}{4\pi} \int d^2z \text{Tr}(\bar{A}_R \ast A_L) \\
S_4 &= \frac{k}{4\pi} \int d^2z \text{Tr}(\bar{A}_R \ast g^{-1} \ast A_L \ast g).
\end{align*}
\]  

(3.3)
Gauge transformations for the gauge fields are,

\[ A_L \rightarrow h_L \star (A_L + d) \star h_L^{-1}, \]
\[ A_R \rightarrow h_R^{-1} \star (A_R + d) \star h_R. \] (3.4)

Using the Polyakov-Wiegmann identity (2.4), one can find,

\[ S(g') = S(h_L \star g \star h_R) = S(h_L) + S(g) + S(h_R) + \frac{k}{4\pi} \int d^2z Tr\{g^{-1}h_L^{-1}\partial h_L g\bar{\partial}h_R h_R^{-1}
+ g^{-1}\partial g\bar{\partial}h_R h_R^{-1} + h_L^{-1}\partial h_L \bar{\partial}g^{-1}\}, \] (3.5)

where \{ \} \ast means all products inside brackets are \ast-products. The gauge fields terms (3.3), under (3.1) and (3.4) transform as,

\[ S_A \rightarrow S'_A = S_A + \frac{k}{4\pi} \int d^2z Tr\{A_L(h_L^{-1}\partial h_L + g\bar{\partial}h_R h_R^{-1} g^{-1})
- \partial h_L h_L^{-1}\partial h_L h_L^{-1} - h_L^{-1}\partial h_L \bar{\partial}g^{-1} - h_L^{-1}\partial h_L g\bar{\partial}h_R h_R^{-1} g^{-1}\}, \]  

\[ S_{\bar{A}} \rightarrow S'_{\bar{A}} = S_{\bar{A}} + \frac{k}{4\pi} \int d^2z Tr\{\bar{A}_R(\partial h_R h_R^{-1} + g^{-1}h_L^{-1}\partial h_L D)
+ \partial h_R h_R^{-1} g^{-1}h_L^{-1}\partial h_L D + \partial h_R h_R^{-1} g^{-1}D + h_R^{-1}\partial h_R h_R^{-1}\partial h_R\}, \]  

\[ S_2 \rightarrow S'_2 = \frac{k}{4\pi} \int d^2z Tr\{h_R^{-1}\bar{A}_R h_R h_L A_L h_L^{-1} + h_R^{-1}\bar{A}_R h_R h_L \partial h_L
+ h_R^{-1}\partial h_R h_L A_L h_L^{-1} + h_R^{-1}\partial h_R h_L \partial h_L\}, \]  

\[ S_4 \rightarrow S'_4 = S_4 + \frac{k}{4\pi} \int d^2z Tr\{-h_R^{-1} g^{-1}h_L^{-1}\partial h_L g\bar{\partial}h_R
- g^{-1}h_L^{-1}\partial h_L g\bar{\partial}h_R + h_R^{-1} g^{-1}A_L g\bar{\partial}h_R\}. \] (3.6)

To find an invariant action \( S(g, A, \bar{A}) \), we have to choose constraints on the subgroup elements \( h_L \) and \( h_R \). The first consistent choice is,

\[ h_R = h_L^{-1} \equiv h, \] (3.7)

and yields to following transformations,

\[ g \rightarrow g' = h^{-1} \ast g \ast h \]
\[ A \rightarrow A' = h^{-1} \ast (A \ast h + \bar{\partial}h) \]
\[ \bar{A} \rightarrow \bar{A}' = h^{-1} \ast (\bar{A} \ast h + \bar{\partial}h). \] (3.8)

The corresponding invariant action, called vector gauged WZW action, is,

\[ S_{V}(g, A, \bar{A}) = S(g) + S_A - S_{\bar{A}} + S_2 - S_4 \]
\[ S(g) + \frac{k}{2\pi} \int d^2z \text{Tr} \{ A \partial g g^{-1} - \bar{A} g^{-1} \partial g + A \bar{A} - g^{-1} A g \} \star. \]  
(3.9)

The second choice is to take \( h_L = h_R = h \) with \( h \) belonging to an Abelian subgroup of \( G \). In this case we find the following gauge transformations,

\[
\begin{align*}
g & \rightarrow \ g' = h \ast g \ast h \\
A & \rightarrow \ A' = h \ast (A \ast h^{-1} + \partial h^{-1}) \\
\bar{A} & \rightarrow \ \bar{A}' = h^{-1} \ast (\bar{A} \ast h - \partial h),
\end{align*}
\]
(3.10)

with the so called axial gauged WZW action,

\[
S_A(g, A, \bar{A}) = S(g) + S_A + S_{\bar{A}} + S_2 + S_4
\]

(3.11)

There are some other choices for the gauged transformations, which can be constructed obviously as the commutative case [12], however they are less common and we will not discuss them here.

By integrating out the \( A \) and \( \bar{A} \) from the actions (3.9) and (3.11), in principle, we find the effective actions as noncommutative non-linear \( \sigma \)-models. We take here the noncommutative gauged axial \( U(2) \) WZW action (3.11), gauged by the subgroup \( U(1) \) diagonally embedded in \( U(2) \). The group element of \( U(2) \) is,

\[
g = \begin{pmatrix}
a_1 & a_2 \\
a_3 & a_4
\end{pmatrix},
\]
(3.12)

with the following constraints,

\[
\begin{align*}
a_1 \ast a_1^\dagger + a_2 \ast a_2^\dagger &= 1, \\
a_3 \ast a_3^\dagger + a_4 \ast a_4^\dagger &= 1, \\
a_1 \ast a_3^\dagger + a_2 \ast a_4^\dagger &= 0.
\end{align*}
\]
(3.13)

The gauge parts of the action (3.11) is,

\[
S_{\text{gauge}} = \frac{k}{2\pi} \int d^2z \{ A \sum_i \partial a_i \ast a_i^\dagger + \sum_i a_i^\dagger \ast \partial a_i \bar{A} + 2A \bar{A} + \sum_i A \ast a_i \ast \bar{A} \ast a_i^\dagger \},
\]
(3.14)

To illustrate the integrating over the gauge fields \( A \) and \( \bar{A} \), we consider abbreviated notations as follows,

\[
\int DAD\bar{A} e^{-S_{\text{gauge}}} = \int DAD\bar{A} e^{-\int d^2z (A \ast b \ast \bar{A} \ast \bar{b} + A \ast b)}
\]

\[
= \int DAD\bar{A} e^{-\int d^2z ((A \ast b') \ast (\bar{A} \ast \bar{b}') - b' \ast \bar{b} \ast \bar{A} \ast \bar{b})},
\]
(3.15)
where

\[ b' \ast \mathcal{O} = b, \]  
(3.16)  

\[ \mathcal{O} \ast \bar{b}' = \bar{b}. \]  
(3.17)  

The result of integration would be,

\[ e^{-S_{eff}} = (\det \mathcal{O})^{-1/2} e^{\int d^2z b' \ast \bar{b}} e^{-S(g)}. \]  
(3.18)  

By comparing eq. (3.15) with (3.14), \( b \) and \( \bar{b} \) could be read as follows,

\[ b = \frac{k}{2\pi} \sum_i a_i^\dagger \ast \partial a_i \]

\[ \bar{b} = \frac{k}{2\pi} \sum_i \bar{\partial} a_i \ast a_i^\dagger, \]  
(3.19)

and \( \mathcal{O} \) can be read from quadratic terms of gauge fields \( A \) and \( \bar{A} \) in (3.14). In fact by using the Fourier transformation of Moyal \( \ast \)-products of functions,

\[ f_1(x) \ast f_2(x) \ast \cdots \ast f_n(x) = \int f_1(p_1) f_2(p_2) \cdots f_n(p_n) e^{i \sum_{i<j} p_i \wedge p_j} e^{i \sum_i p_i x \partial p_i \cdots \partial p_n}, \]  
(3.20)

the explicit Fourier transform of \( \mathcal{O} \) is as follows,

\[ \mathcal{O}(p_1, p_2) = 2 e^{i(p_1 \wedge p_2)} \delta(p_1 + p_2) + \int a_i(p_3) a_i^\dagger(p_4) e^{i(p_1 \wedge p_2 + p_1 \wedge p_3 + p_1 \wedge p_4 + p_2 \wedge p_3 + p_3 \wedge p_4 + p_4 \wedge p_2)} \]

\[ \times \delta(p_1 + p_2 + p_3 + p_4) dp_3 dp_4. \]  
(3.21)  

To find \( b' \), we need to inverse the \( \mathcal{O} \) operator in (3.16), and this is equivalent to solving Fourier transform of (3.16) which is an integral equation as follows,

\[ 2b'(p) + \int b'(p - p_1 - p_2) a_i(p_1) a_i^\dagger(p_2) e^{-i(p_1 \wedge p_2 + p_1 \wedge p_3 + p_1 \wedge p_4 + p_2 \wedge p_3 + p_3 \wedge p_4 + p_4 \wedge p_1)} dp_1 dp_2 = b(p), \]  
(3.22)

where

\[ b(p) = i \int a_i^\dagger(p_1) a_i(p_1) p_1 e^{i p_1 \wedge p_1} dp_1. \]  
(3.23)  

To solve eq. (3.22), we expand the \( b' \) and exponential factors in terms of \( \theta \),

\[ b'(p) = b'_0(p) + \theta b'_1(p) + \theta^2 b'_2(p) + \cdots. \]  
(3.24)
In zero-th order of θ, one finds,

\[ b'_0(z, \bar{z}) = \frac{1}{4} a_i^\dagger \partial a_i, \quad (3.25) \]

and in first order of θ,

\[ b'_1(z, \bar{z}) = \frac{1}{4} (+4 \partial \partial a_i \bar{a}_i^\dagger - 4 \bar{\partial} \partial a_i a_i^\dagger). \quad (3.26) \]

In obtaining the above expressions, we have used the unitarity conditions \((3.13)\) and the equations of motion \((2.7)\). The effective action arising from the \(\int d^2 z b' \star \bar{b'}\) term in eq. \(3.18\) could be found as a power series in θ,

\[ S_{\text{eff}} = S(g) + \frac{1}{2} Tr \ln(O) + S_{\text{eff}}^{(0)} + \theta S_{\text{eff}}^{(1)} + \cdots, \quad (3.27) \]

in which

\[ S_{\text{eff}}^{(0)} = -\frac{k}{8\pi} \int d^2 z a_i^\dagger \partial a_i \bar{a}_j a_j^\dagger, \]

\[ S_{\text{eff}}^{(1)} = -\frac{k}{8\pi} \int d^2 z ((\partial a_i^\dagger \partial a_i + a_i^\dagger \partial \partial a_i)(\bar{\partial} \partial a_j a_j^\dagger + \bar{\partial} a_j \bar{a}_j^\dagger) + a_i^\dagger \partial a_i(\partial \partial a_j \bar{a}_j^\dagger - \bar{\partial} \partial a_j \bar{a}_j^\dagger)) \]

\[ - (\partial \partial a_i a_i^\dagger - \partial \partial a_i a_i^\dagger) \partial a_k a_k^\dagger). \quad (3.28) \]

By looking at equations \((3.25), (3.26)\) and \((3.28)\), we suggest the following exact forms for \(b'(z, \bar{z})\) and \(S_{\text{eff}}\),

\[ b'(z, \bar{z}) = \frac{1}{4} \partial a_i \star a_i^\dagger \]

\[ S_{\text{eff}} = S(g) + \frac{1}{2} Tr \ln(O) - \frac{k}{8\pi} \int d^2 z \partial a_i \star a_i^\dagger \star \partial a_j \star a_j^\dagger, \quad (3.29) \]

We have to fix the gauge freedom by a gauge fixing condition on \(a_i\)’s. Under infinitesimal axial gauge transformations \((3.10)\), we find,

\[ a'_i = a_i + a_i \epsilon + \epsilon \star a_i, \quad (3.30) \]

in which \(\epsilon\) is the infinitesimal parameter of the gauge transformation. We can find (at least perturbatively in \(\theta\)) some \(\epsilon\) such that \(\Re(a'_i) = 0\) and one may take this relation as the gauge fixing condition. It is worth mentioned that the \(U(2)/U(1)\) model after applying all conditions gives us a three dimensional non-linear noncommutative \(\sigma\)-model.

\(^3\)The term \(\frac{1}{2} Tr \ln(O)\) gives the noncommutative effective action for dilaton field.
4 Conclusions

We found current algebra and energy-momentum tensor of noncommutative WZW models are the same as corresponding quantities in commutative WZW models. By gauging the chiral symmetry of noncommutative WZW model, we showed that different gauge actions could be constructed. After integrating out the gauge fields from the axial gauged noncommutative WZW model, we observed that the final result is a perturbative noncommutative non-linear $\sigma$-model. In general, integrating over the quadratic action demands inverting the quadratic operator $O$, but the inversion of operators in noncommutative space is not obvious and it is natural to find an infinite series (in powers of $\theta$) for the $\sigma$-model action, however, we suggested an exact form for the effective action containing $\star$-products. Applying the gauge condition will reduce this effective action to a $\sigma$-model on a three dimensional noncommutative target space. The geometrical study of this target space which may contain singular structure will be interesting.

Acknowledgement

We would like to thank F. Ardalan and V. Karimipour for useful discussions.

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