Flux Stabilization of D-branes in NSNS Melvin Background

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Abstract

In this paper we reexamine the D-brane spectrum in the Melvin background with non-constant NSNS B-field from the viewpoint of its world-volume and string world-sheet theory. We find that the stable D2-D0 bound state exists even though it does not wrap any nontrivial cycles. We show that this system is stabilized by the presence of the NSNS B-field and the magnetic flux $F$. Moreover from the non-abelian world-volume theory of D0-branes the bound state is regarded as a system of D0-branes expanding into a fuzzy torus.
1 Introduction

Recently many aspects of Melvin backgrounds [1] in string theory have been studied intensively. They give us interesting models of string theory in flux backgrounds. In the case of RR fluxes they are often called fluxbranes [2, 3, 4, 5, 6, 7, 8, 9] and one can realize them as classical solutions in supergravity theory. On the other hand, if we consider NSNS fluxes, then one can have exactly solvable string sigma models [10, 11, 12, 13, 14].

The latter is regarded as a $S^1$ fibration over $R^2$. This non-trivial fibration is due to two Kaluza-Klein (K.K.) gauge fields $A_\varphi$ and $B_\varphi$, which originate from K.K. reduction of metric $G_{\varphi y}$ and B-field $B_{\varphi y}$, respectively. The metric of this background $M_3$ is given by

$$ds^2 = d\rho^2 + \frac{\rho^2}{(1 + \beta^2 \rho^2)(1 + q^2 \rho^2)} d\varphi^2 + \frac{1 + q^2 \rho^2}{1 + \beta^2 \rho^2} (dy + A_\varphi d\varphi)^2,$$

$$A_\varphi = \frac{q \rho^2}{1 + q^2 \rho^2}, \quad B_{\varphi y} = B_\varphi = -\frac{\beta \rho^2}{1 + \beta^2 \rho^2}, \quad e^{2(\phi - \phi_0)} = \frac{1}{1 + \beta^2 \rho^2},$$

where $q, \beta$ are the (magnetic) parameters which are proportional to the strength of two gauge fields and $\phi_0$ is the constant value of the dilaton $\phi$ at $\rho = 0$. As a string model we can consider the ten dimensional background $R^{1,6} \times M_3$.

Since the presence of fluxes generically breaks all supersymmetries, the theory becomes unstable and often includes closed string tachyons (for the recent analysis of tachyon condensation in NSNS Melvin backgrounds see [15, 13, 14, 16, 17, 18]).

Another interesting phenomenon caused by the NSNS fluxes which we will discuss is the existence of expanded D2-branes (see Fig.1) whose world-volume has the form of a torus. This is accompanied with a quantized magnetic flux $F$ on it and thus is more properly said as a D2-D0 bound state. This kind of D2-branes has already been expected by the heuristic arguments on T-duality transformations in the paper [19]. However, we have been left with the important question why such a topologically trivial torus D2-brane is stable. In this short letter we answer this by investigating the D-brane from the viewpoint of the D-brane world-volume theory and the would-sheet theory. We conclude that such a D2-brane is stabilized by the combined effect of both the NSNS flux $B$ and the quantized magnetic flux $F$ on it.
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Let us consider what kind of D-branes can exist in the Melvin background (1.1). In the most part of this paper we assume that the values of the magnetic parameters are rational such that

\[ \frac{\beta \alpha'}{R} = \frac{k}{N}, \quad qR = \frac{l}{M}, \]  

(2.1)

where \((k, N)\) and \((l, M)\) are pairs of coprime integers. In particular we are interested in such D-branes that are localized in the \(\rho\) direction. Thus we assume that D-branes obey the Dirichlet boundary condition along \(\rho\). The other D-branes can also be investigated in the same way as the arguments below and we summarize the results in Table 1.

First let us discuss a D0-brane. If we put it in the Melvin background, then we can see that it can exist only at the origin \(\rho = 0\) because of the non-trivial \(\rho\) dependence of the dilaton \(\phi\) in (1.1). This is easily understood if we remember the value of D0-brane mass \(M_{D0} = e^{-\phi(\alpha')^{-\frac{1}{2}}}\), which takes its minimum value at \(\rho = 0\).

Next we consider a D2-brane whose world-volume is the torus \(0 \leq \varphi < 2\pi, \quad 0 \leq y < 2\pi R, \quad \rho = \text{constant}\). However, it is easy to see that the mass of it is proportional to \(\rho\) (set \(F\) to zero in (2.2)) as in the flat space. Thus it should squash and cannot exist.

In this way we have observed that any D2-branes and D0-branes cannot exist at \(\rho \neq 0\).

Then what happens if we consider D2-D0 bound states? We start with a D2-D0 bound state which is made of \(p\) D2-branes and \(q\) D0-branes \((p\) and \(q\) are coprime) and assume that its world-volume is the same torus. The mass\(^3\) of this object is given by

\[ M_{p,q} = \frac{e^{-\phi}}{4\pi^2(\alpha')^2} \int dy \, d\varphi \text{Tr} \sqrt{\det(G + B + F)} \]

\[ = \frac{e^{-\phi_0}pR}{(\alpha')^2} \sqrt{(F\beta - 1)^2\rho^2 + F^2}, \]  

(2.2)

where \(F\) is the constant flux which generates \(q\) D0-branes and is quantized as usual\(^4\)

\[ F = \alpha' R^{-1} \frac{q}{p}. \]  

(2.3)

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\(^3\)Here we ignore the quantum corrections. On the other hand we will not have such a problem for the supersymmetric example discussed at the next section.

\(^4\)Note that this value \(F\) is determined by the quantization law \(\frac{1}{2\pi \alpha'} \int \text{Tr} \, F = q \in \mathbb{Z}\).
Table 1: D-brane spectrum in the Melvin background with rational values of parameters $\beta \alpha'/R = k/N, qR = l/M$. We show how the D-branes defined in the free field representation $(Y', \rho, \varphi'')$ correspond to those in the original Melvin background. In the above table the Neumann and Dirichlet boundary condition are denoted by $N$ and $D$. The tension $T_p$ represents that of the standard $Dp$-brane. The D-branes marked by * are regarded as fractional D-branes and even for irrational case they have finite tensions, while others have infinite tensions for irrational case.

In order for this D-brane to exist at $\rho \neq 0$, the $\rho$ dependence of the energy should disappear and we have the constraint $F = 1/\beta$. Since we assume the rational cases (2.1), this can be satisfied for $p = k, q = N$. Furthermore the mass of the object for this particular value of flux is given by

$$M_{k,N} = N T_0 \quad (T_0 = e^{-\phi_0 (\alpha')^{-1/2}}),$$

(2.4)

where $T_0$ is the mass of a D0-brane at $\rho = 0$. This result (2.4) tells us an interesting fact that the D2-brane part of the mass $M_{k,N}$ is effectively zero (so called tensionless brane). This is the reason why such a expanded D-brane is allowed which does not wrap any nontrivial cycles. Moreover from the RR-coupling and eq.(2.3) we can obtain the correct RR-charges of $k$ D2-branes and $N$ D0-branes. This mechanism may be regarded as a non-compact CFT version of the stabilization of spherical D2-branes in SU(2) WZW-model (NS5-brane background) [21, 22, 23, 24, 25, 26].

It is also interesting to examine the limit $\rho \rightarrow 0$. Since the net D2-brane charge is zero for this torus configuration, we have only $N$ D0-branes localized at $\rho = 0$. This is

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| Free field | $(Y', \rho, \varphi'')$ | Melvin $(\rho, \varphi, Y)$ | Tension |
|------------|----------------|----------------|---------|
| D0*        | DDD            | D0 fixed at $\rho = 0$ | $T_0$   |
| D0         | DDD            | D2-D0 bound state $(\rho, Y = \text{fixed})$ | $N T_0$ |
| D1         | DND            | D3-D1 bound state | $N T_1$ |
| D2*        | DNN            | D2 $(Y = \text{fixed})$ | $T_2$   |
| D1*        | NDD            | D1 fixed at $\rho = 0$ | $T_1$   |
| D1         | NDD            | Spiral D1 $(\rho, \varphi + qY = \text{fixed})$ | $M T_1$ |
| D2         | NND            | Spiral D2 $(\varphi + qY = \text{fixed})$ | $M T_2$ |
| D3*        | NNN            | D3                 | $T_3$   |
similar to the decay of a $D2 - \overline{D2}$ system due to tachyon condensation [27]. However, note that our process $\rho \to 0$ is an exactly marginal deformation of boundary conformal field theory as we will see. If we say these results in the opposite way, $N$ D0-branes with the torus $D2$-brane can leave from the origin $\rho = 0$. This behavior is very similar to that of fractional D-branes in $\mathbb{Z}_N$ orbifolds [28, 29]. Indeed as has been already pointed out in our previous paper [19] by heuristic arguments of T-duality transformations, we can identify the $D2$-$D0$ bound state in the original coordinate system $(\rho, \varphi, Y)$ as the system of $N$ different fractional D-branes in the other coordinate system $(\rho, \varphi'', Y')$ (see Fig.1). Let us see this correspondence more explicitly.

\[ (1 + \beta^2 \rho^2) \partial \varphi'' = \partial(\varphi + qY) + \beta \partial Y, \]
\[ (1 + \beta^2 \rho^2) \bar{\partial} \varphi'' = \bar{\partial}(\varphi + qY) - \beta \bar{\partial} Y, \]
\[ (1 + \beta^2 \rho^2) \partial Y' = \partial Y - \beta \rho^2 \partial(\varphi + qY), \]
\[ (1 + \beta^2 \rho^2) \bar{\partial} Y' = \bar{\partial} Y + \beta \rho^2 \bar{\partial}(\varphi + qY). \]

Figure 1: The equivalence between $N$ D0-branes in the free field theory and a $D2$-$D0$ bound state in the original sigma model of the Melvin background.

As shown in [11, 12], the sigma model of the NSNS Melvin background (1.1) can be solvable. Indeed we can show that if we perform T-duality twice, then the sigma model is equivalent to that of the flat background with nontrivial boundary conditions (for more details see the review part in [19]). The world-sheet fields for the trivial background, which define a free field theory, are denoted by $(X' = \rho e^{i\varphi''}, X' = \rho e^{-i\varphi''}, Y')$. The relation between these free fields and world-sheet fields in the original NSNS Melvin background is given as follows

\[ (1 + \beta^2 \rho^2) \partial \varphi'' = \partial(\varphi + qY) + \beta \partial Y, \]
\[ (1 + \beta^2 \rho^2) \bar{\partial} \varphi'' = \bar{\partial}(\varphi + qY) - \beta \bar{\partial} Y, \]
\[ (1 + \beta^2 \rho^2) \partial Y' = \partial Y - \beta \rho^2 \partial(\varphi + qY), \]
\[ (1 + \beta^2 \rho^2) \bar{\partial} Y' = \bar{\partial} Y + \beta \rho^2 \bar{\partial}(\varphi + qY). \]
The Dirichlet boundary conditions of D0-branes are
\[ \partial_2 \varphi'' = 0, \quad \partial_2 Y' = 0, \quad (2.6) \]
at \( \sigma_1 = 0, \pi \) (from now on we will define the boundary conditions in the open string picture). If we rewrite the above equations from the viewpoint of the original Melvin sigma model by using (2.5), they become
\[ i\partial_2 (\varphi + qY) - \beta \partial_1 Y = 0, \]
\[ i\partial_2 Y + \beta \rho^2 \partial_1 (\varphi + qY) = 0. \quad (2.7) \]
Thus we have obtained mixed Neumann-Dirichlet boundary conditions. By comparing this result (2.7) with the general formula of the boundary condition
\[ G_{\mu\nu} \partial_1 X^\nu + i (B_{\mu\nu} + F_{\mu\nu}) \partial_2 X^\nu = 0, \quad (2.8) \]
where \( X^\mu \) denotes the world-sheet field, we obtain the non-trivial value of the flux
\[ F = F_{\varphi Y} = \frac{1}{\beta} = \alpha' R^{-1} \frac{N}{k}. \quad (2.9) \]
This value does match with the previous value (2.3) if we set \( p = k, \quad q = N \). Therefore we can conclude that \( N \) D0-branes at \( \rho \neq 0 \) in the free field picture in \((\rho, \varphi'', Y')\) is T-dual equivalent to a bound state of \( N \) D0-branes and \( k \) D2-branes wrapping around the torus \((\varphi, Y)\) with \( \rho \neq 0 \) in the original coordinate picture. This shows that expanding the D2-brane corresponds to moving the fractional D0-branes and thus this is an exactly marginal deformation of boundary conformal field theory. It would be also interesting that the quantization of flux \( F \) requires the rational values of \( \frac{\beta \alpha'}{R} \). In the irrational cases we will have to require \( N \to \infty \) in order to move D0-branes, and the bound state becomes infinitely massive.

Let us comment the world-volume theory on a D2-D0 bound state. Because of the presence of B-flux it becomes noncommutative theory \([30, 31]\). Following the prescription \([31]\), it is easy to see that the noncommutativity \( \theta \) of noncommutative torus \( A_{\theta} \) is exactly given for any value of \( \rho \) as follows
\[ \theta = \frac{\beta \alpha'}{R} = \frac{k}{N} \in \mathbb{Q}. \quad (2.10) \]
This shows that it is identified with the fuzzy torus which allows finite dimensional representations.

Such a noncommutativity can be seen more explicitly from the analysis of the non-abelian Dirac-Born-Infeld (DBI) action of \( N \) D0-branes. The non-abelian DBI action is
already proposed in [32, 33] where the author determines its form from the T-duality covariance of DBI action. Especially the action of $N$ D0-branes is written by

$$S_{DBI} = -T_0 \int dt \text{STr} \left[ e^{-\phi} \sqrt{-P\{E_{00} + E_{0i}(Q^{-1}_{ij} - \delta^i_j)E_{jk}E_{k0}\} \det(Q^i_j)} \right], \quad (2.11)$$

where we defined $E_{\mu\nu}$ and $Q^i_j$ as $E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}$ ($\mu, \nu = 0, \cdots, 9$), $Q^i_j = \delta^i_j + i[\Phi^i, \Phi^k]E_{kj}$ ($i, j, k = 1, \cdots, 9$), and STr and $P$ denote the symmetrized trace and the pull back onto the D0-brane world-volume, respectively. Here we do not consider the time dependence of fields, thus we can set $\frac{d\Phi^i}{dt}$ to zero. Moreover since the transverse fluctuations for $R^{1,6}$ directions are irrelevant in this situation, we can also set such fields to zero. After all, the potential part of the action becomes the following form

$$V = T_0 e^{-\phi_0} \text{STr} \left[ \sqrt{1 + \rho^2 \{i[\Phi^\varphi, \Phi^Y] + \beta\}^2} \right]. \quad (2.12)$$

Here we set $\Phi^\rho$ to $\rho$ because we want to consider the expanding of D0-branes into the torus form of D2-branes with constant $\rho \neq 0$. From the above equation we can see that the potential is always greater than $NT_0 e^{-\phi_0}$, which is the mass of $N$ D0-branes. To realize such the lowest limit of the potential the condition $[\Phi^\varphi, \Phi^Y] = i\beta$ should be needed. If we normalize coordinates by replacing $\Phi^\varphi$ with $\frac{\Phi^\varphi R_{2\pi\alpha'}}{2\pi\alpha'}$, then we can get the following relation

$$[\Phi^\varphi, \Phi^Y] = 2\pi i\theta. \quad (2.13)$$

This is exactly the algebra of noncommutative torus. However this relation holds only for the infinite dimensional representation of $\Phi^\varphi$ and $\Phi^Y$, while here we consider the finite dimensional $(N \times N)$ representation. We can approximate by using $N$ dimensional fuzzy torus algebra generated by $e^{i\Phi^\varphi} = U$ and $e^{i\Phi^Y} = V$ with the relation $UV = e^{-2\pi i\frac{k}{N}}VU$. Then the potential is not exactly equal to the mass of $N$ D0-branes. Such a difference comes from the $\frac{1}{N}$ order correction which can be seen in other non-abelian world-volume analyses [33, 34, 26]. Any way the noncommutative algebra of the torus (2.13) is a good approximation for large $N$ and we have seen the explicit noncommutativity on the world-volume of the D2-brane.

Finally let us turn to a D1-brane in the Melvin background [35, 19]. As shown in [19], this is equivalent to the previous D2-D0 bound state by the T-duality along $Y'$, which interchanges $q$ and $\beta$ [12, 13]. The boundary condition of a D1-brane in the free field theory can be rewritten in terms of the fields $(\rho, \varphi, Y)$ and the result is

$$\partial_2(\varphi + qY) = \partial_1 Y = 0. \quad (2.14)$$

\footnote{Of course this relation exactly holds for the infinite number of D0-branes, however in that case the potential value becomes infinite and this configuration may be singular. This consideration may be related to the D-brane picture in the Melvin background with irrational magnetic parameters [19].}
This exactly represents a D1-brane wrapping the ‘geodesic line for D-branes’ \( \varphi + qY = \text{constant} \), which is defined\(^7\) for the ‘D-brane metric’ \( e^{-2\phi}(ds)^2 \).

As pointed out in the calculation of the boundary state \([19]\), for rational values of \( qR = \frac{l}{M} \) the mass of the D1-brane is finite and it winds \( M \) times along \( Y \) and \( l \) times along \( \varphi \), while for irrational values it becomes infinite because the D1-brane should wind infinitely many times. These facts are all consistent with the T-dual equivalence to the previous D2-D0 bound state.

### 3 D-branes in Higher Dimensional Melvin Background

In previous section we considered the D2-D0 bound state in the Melvin background. However, in the presence of the flux this background breaks the target space supersymmetry completely, and the D2-D0 bound system is not a BPS state.

On the other hand, as shown in several papers \([13, 14]\), we can extend the exactly solvable NSNS Melvin background \( R^{1,6} \times M_3 \) to more higher dimensional ones \( R^{1,8-2n} \times M_{2n+1} \). The most important nature of higher dimensional backgrounds is that the partial supersymmetry can be preserved even though these backgrounds are curved. Thus, we can consider BPS D-branes in these backgrounds. They are stable, and the classical analysis is reliable. Therefore it is interesting to consider D-branes in these higher dimensional backgrounds.

Here we consider one example of supersymmetric higher dimensional models; the product space of the trivial five dimensional Minkowski Space \( R^{1,4} \) and the nontrivial curved space \( M_5 \) which is topologically equivalent to \( S^1 \) fiberation over \( R^2 \times R^2 \). In the following we parameterize \( S^1 \) and \( R^2 \times R^2 \) with \( Y \) and \((\rho, \varphi), (r, \theta)\), respectively. Then, its world-sheet action is given as follows \([19]\)

\[
S = \frac{1}{\pi \alpha'} \int d^2 \sigma \left[ \bar{\partial} \rho \partial \rho + \bar{\partial} r \partial r + \rho^2 \bar{\partial} \bar{\varphi} \partial \varphi + r^2 \bar{\partial} \bar{\theta} \partial \theta \right]
+ (1 + \beta_1^2 \rho^2 + \beta_2^2 r^2)^{-1} (\bar{\partial} Y + \beta_1 \rho^2 \bar{\partial} \bar{\varphi} + \beta_2 r^2 \bar{\partial} \bar{\theta}) (\partial Y - \beta_1 \rho^2 \partial \bar{\varphi} - \beta_2 r^2 \partial \bar{\theta}), \tag{3.1}
\]

where we have defined \( \bar{\varphi} = \varphi + q_1 Y, \bar{\theta} = \theta + q_2 Y \). This model includes four flux parameters \( q^1, q^2, \beta^1 \) and \( \beta^2 \), and as you can see from the Killing spinor analysis \([13, 19]\), this model keeps half of the maximal supersymmetry if \( q_1 = q_2, \beta_1 = \beta_2 \) or \( q_1 = -q_2, \beta_1 = -\beta_2 \).

Here we consider a D2-D0 bound state in the higher dimensional background with rational parameters \( \beta_i = \frac{k_i}{N} \) \((i = 1, 2)\), where \( N, k_1 \) and \( k_2 \) are coprime integers. For

\(^7\) Note that the usual geodesic line for the metric \((ds)^2\) is given by \( \varphi + (q \pm \beta)Y = \text{constant} \).
simplicity we set $q_i$ ($i = 1, 2$) to zero. This system becomes a BPS state \cite{19} if the background keeps supersymmetry. The main motivation to analyze this is the same as that in the previous section: the stabilization mechanism which comes from NSNS B-field effect. The analysis is almost the same as that in the previous section, while there appears one nontrivial constraints which we will see.

First we examine the boundary condition of this D2-D0 bound state. In this case we can transform the original coordinates $(Y, \rho, \varphi, r, \theta)$ in (3.1) into the free fields $(Y', \rho, \varphi', r, \theta')$ by using T-duality two times and several field redefinitions. To analyze this system quantitatively we transform the following boundary conditions of D0-branes

$$\partial_2 \varphi'' = \partial_2 \theta'' = \partial_2 Y'' = 0,$$

into those which are represented by the original coordinate $(\varphi, \theta, Y)$. The relation between these coordinates are obtained in the same way as (2.5). Then the result becomes

$$\beta_2 \partial_2 \varphi - \beta_1 \partial_2 \theta = 0,$$

$$i \partial_2 \varphi - \beta_1 \partial_1 Y = 0,$$

$$i \partial_2 Y + \beta_1 \rho^2 \partial_1 \varphi + \beta_2 r^2 \partial_1 \theta = 0. \quad (3.3)$$

In these equations the first equation indicates that the following condition should be satisfied on the world-volume of the D2-brane:

$$\beta_2 \varphi = \beta_1 \theta + \text{constant}. \quad (3.4)$$

Then by comparing the result (3.3) to the general formula of the boundary condition (2.8) with an additional constraint (3.4), we can see the following flux on the world-volume of D2-branes

$$\beta_1 F_{\varphi Y} + \beta_2 F_{\theta Y} = 1. \quad (3.5)$$

Then this flux is properly quantized on the world-volume of the D2-brane

$$\frac{1}{4\pi^2 \alpha'} \int F = \frac{1}{4\pi^2 \alpha'} \int d\xi_1 d\xi_2 F_{\xi_1 \xi_2} = N, \quad (3.6)$$

where $\xi_1, \xi_2$ parameterize the world-volume of the D2-brane: $\{(\varphi, \theta, Y) \mid \beta_2 \varphi = \beta_1 \theta + \text{constant}, \ 0 \leq \varphi < 2\pi k_1, \ 0 \leq \theta < 2\pi k_2\}$\footnote{This periodicity of $\varphi$ and $\theta$ is effective one which is only available for the world-volume of a D2-brane.}. From this we can see that this system represents a bound state of $N$ D0-branes and one D2-brane.
Moreover we can see the stabilization mechanism of the D2-brane by the analysis of
the Dirac-Born-Infeld theory in the same way as (2.2). The total mass turns to be equal
to that of $N$ D0-branes. Namely, the D2-brane part again becomes tensionless by the
total effect of the NSNS B-field and the magnetic flux $F$ (3.3). The analysis from the
world-volume theory of D0-branes is the same as before. We can see the structure of the
fuzzy torus with the noncommutativity $\theta = \frac{1}{N}$.

4 Conclusions

In this paper we have investigated the D-brane spectrum in the two parameter NSNS
Melvin background from the viewpoint of both D-brane world-volume theory and string
world-sheet theory. In particular we have found that for non-zero rational values of
the B-field parameter $\beta \alpha'/R = k/N$, neither pure D0-branes nor D2-branes can exist
except at the origin $\rho = 0$. Instead we can put D2-D0 bound states for any $\rho \neq 0$
(see Fig.1). Interestingly, this object, which wraps a topologically trivial torus in the
Melvin background, is indeed stabilized by the presence of $B$ and $F$ flux. The existence
of it is also completely consistent with the analysis of boundary states [13] and indeed
the bound state just corresponds to a system of $N$ fractional D0-branes in the free field
representation. Furthermore, we have argued that this torus brane can be constructed
from $N$ D0-branes by using the proposed non-abelian DBI-action of D0-branes. This
result is consistent with the fact that the effective world-volume theory on the D2-D0
bound state is identified with noncommutative torus (fuzzy torus) $\mathcal{A}_{\theta}$ ($\theta = k/N$).

In the same way we can also determine in the original Melvin background what kind of
objects correspond to the other types of D-branes in the free field representation ($Y', \rho, \varphi''$)
(see Table.1). Furthermore we have shown that the similar kind of D2-D0 bound states
exists for the higher dimensional Melvin background [13, 14]. For the special values of
the parameters we obtain BPS D2-D0 bound states.

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