Numerical approximation of modified non-linear SIR model of computer viruses

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Abstract

In this paper, the non-linear modified epidemiological model of computer viruses is illustrated. For this aim, two semi-analytical methods, the differential transform method (DTM) and the Laplace-Adomian decomposition method (LADM) are applied. The numerical results are estimated for different values of iterations and compared to the results of the LADM and the homotopy analysis transform method (HATM). Also, graphs of residual errors and phase portraits of approximate solutions for \( n = 5, 10, 15 \) are demonstrated. The numerical approximations show the performance of the LADM in comparison to the LADM and the HATM.

keywords: Non-linear Susceptible-Infected-Recovered model, Differential transform method, Laplace transformations, Adomian decomposition method.

1 Introduction

The computer viruses are malware programs that have been able to infect thousands of computers and have hurt billions dollar in computers around the world. The virus should never be considered to be harmless and remain in the system. There are several types of viruses that can be categorized according to their source, technique, file type that infects, where they are hiding, the type of damage they enter, the type of operating system, or the design on which they are attacking. We can introduce some of famous and malicious viruses such as ILOVEYOU, Melissa, My Doom, Code Red, Sasser, Stuxnet and so on. Therefore, it is important that we study the methods to analyze, track, model, and protect against viruses. In recent years, many scientists have been illustrated the epidemiological models of computer viruses [2, 14, 17, 21, 32, 33, 36, 37, 45, 46, 47]. These models have been estimated by many mathematical methods such as collocation method [30], homotopy analysis method [5, 27], variational iteration method [23] and others [33, 41].

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One of applicable and important models is the classical Susceptible-Infected-Recovered (SIR) computer virus propagation model [21, 22, 33] which is presented in the following form:

\[
\frac{dS(t)}{dt} = f_1 - \lambda S(t)I(t) - dS(t),
\]
\[
\frac{dI(t)}{dt} = f_2 + \lambda S(t)I(t) - \varepsilon I(t) - dR(t),
\]
\[
\frac{dR(t)}{dt} = f_3 + \varepsilon I(t) - dR(t),
\]

where

\[ S(0) = S_0(t), I(0) = I_0(t), R(0) = R_0(t), \]

are the initial conditions of non-linear system of Eqs. (1). Functions and initial values of system (1) are given in Table 1.

| Parameters & Functions | Meaning | Values |
|------------------------|---------|--------|
| S(t)                   | Susceptible computers at time t | S(0) = 20 |
| I(t)                   | Infected computers at time t | I(0) = 15 |
| R(t)                   | Recovered computers at time t | R(0) = 10 |
| f_1, f_2, f_3         | Rate of external computers connected to the network | f_1 = f_2 = f_3 = 0 |
| \lambda                | Rate of infecting for susceptible computer | \lambda = 0.001 |
| \varepsilon            | Rate of recovery for infected computers | \varepsilon = 0.1 |
| d                      | Rate of removing from the network | d = 0.1 |

Recently, several numerical and semi-analytical methods are introduced to solve the mathematical and engineering problems [9, 10, 11, 24, 25, 26, 48] that we can apply them to solve the non-linear model (1). The DTM and the LADM are two important and efficient tools to solve the linear and non-linear problems arising in the mathematics, physics and engineering [6, 7, 29, 34, 35, 40, 43]. Specially, the LADM [12, 15, 31] obtained by combining the Adomian decomposition method [1, 3, 4, 16, 39, 44] and the Laplace transformations [18] similar to the HATM [8, 24, 27, 28], Laplace homotopy perturbation method [13, 38, 42] and so on.

The aim of this paper is to apply the DTM and the LADM to find the approximate solution of non-linear epidemiological system of Eqs. (1). The numerical results are compared with the HATM [8, 24, 27, 28] by plotting the residual errors function for different iterations. Also, the phase portraits of approximate solutions for n = 10 and different functions of S(t), I(t) and R(t) are presented. The numerical results show the abilities and capabilities of the LADM in comparison to the DTM and the HATM.
2 Differential transform method

Transformation of the $k$-th derivative of a function in one variable is as follows

$$F(k) = \frac{1}{k!} \left[ \frac{d^k f(t)}{dt^k} \right]_{t=t_0},$$

and the inverse transformation is defined by

$$f(t) = \sum_{k=0}^{\infty} F(k)(t - t_0)^k$$

where $F(k)$ is the differential transform of $f(t)$. In actual applications, the function $f(t)$ is expressed by a finite series and Eq. (4) can be rewritten as follows:

$$f(t) = \sum_{k=0}^{N} F(k)(t - t_0)^k$$

where $N$ is decided by the convergence of natural frequency. The fundamental operations of DTM have been given in Table 3.

| Original functions | Transformed functions |
|--------------------|-----------------------|
| $f(t) = u(t) \pm v(t)$ | $F(k) = U(k) \pm V(k)$ |
| $f(t) = \beta u(t)$ | $F(k) = \beta U(k)$ |
| $f(t) = u(t)v(t)$ | $F(k) = \sum_{i=0}^{k} U(k)V_{k-i}(k)$ |
| $f(t) = \frac{du(t)}{dt}$ | $F(k) = (k+1)U(k+1)$ |
| $f(t) = \frac{d^m u(t)}{dt^m}$ | $F(k) = (k+1)(k+2) \cdots (k+m)U(k+m)$ |
| $f(t) = \int_{t_0}^{t} u(\xi)d\xi$ | $F(k) = \frac{U(k-1)}{k}, k \geq 1$ |
| $f(t) = t^m$ | $F(k) = \delta(k - m)$ |
| $f(t) = \exp(\lambda t)$ | $F(k) = \lambda^k$ |
| $f(t) = \sin(\omega t + \alpha)$ | $F(k) = \frac{\omega}{\pi} \sin\left(\frac{k\pi}{2} + \alpha\right)$ |
| $f(t) = \cos(\omega t + \alpha)$ | $F(k) = \frac{\omega}{\pi} \cos\left(\frac{k\pi}{2} + \alpha\right)$ |
By applying the presented method to system of Eqs. (1), we get

\[ S_{k+1} = \frac{1}{k+1} \left[ f_1 - \lambda \sum_{i=0}^{k} S_i I_{k-i} - dS_k \right], \]

\[ I_{k+1} = \frac{1}{k+1} \left[ f_2 - \lambda \sum_{i=0}^{k} S_i I_{k-i} - \varepsilon I_k - dR_k \right], \quad (6) \]

\[ R_{k+1} = \frac{1}{k+1} \left[ f_3 - \varepsilon I_k - dR_k \right]. \]

The differential transform method series solution for system (1) can be obtained as

\[ S(t) = \sum_{j=0}^{n} S_j t^j, \]

\[ I(t) = \sum_{j=0}^{n} I_j t^j, \quad (7) \]

\[ R(t) = \sum_{j=0}^{n} R_j t^j, \]

3 Laplace-Adomian decomposition method

We apply the Laplace transformation \( \mathcal{L} \) as

\[ \mathcal{L}[S(t)] = \frac{S(0)}{z} + \frac{\mathcal{L}[f_1]}{z} - \frac{\lambda}{z} \mathcal{L}[S(t)I(t)] - \frac{d}{z} \mathcal{L}[S(t)], \]

\[ \mathcal{L}[I(t)] = \frac{I(0)}{z} + \frac{\mathcal{L}[f_2]}{z} + \frac{\lambda}{z} \mathcal{L}[S(t)I(t)] - \frac{\varepsilon}{z} \mathcal{L}[I(t)] - \frac{d}{z} \mathcal{L}[R(t)], \quad (8) \]

\[ \mathcal{L}[R(t)] = \frac{R(0)}{z} + \frac{\mathcal{L}[f_3]}{z} + \frac{\varepsilon}{z} \mathcal{L}[I(t)] - \frac{d}{z} \mathcal{L}[R(t)]. \]

By putting the initial conditions we have

\[ \mathcal{L}[S(t)] = \frac{S(0)}{z} + \frac{f_1}{z^2} - \frac{\lambda}{z} \mathcal{L}[A] - \frac{d}{z} \mathcal{L}[S(t)], \]

\[ \mathcal{L}[I(t)] = \frac{I(0)}{z} + \frac{f_2}{z^2} + \frac{\lambda}{z} \mathcal{L}[A] - \frac{\varepsilon}{z} \mathcal{L}[I(t)] - \frac{d}{z} \mathcal{L}[R(t)], \quad (9) \]

\[ \mathcal{L}[R(t)] = \frac{R(0)}{z} + \frac{f_3}{z^2} + \frac{\varepsilon}{z} \mathcal{L}[I(t)] - \frac{d}{z} \mathcal{L}[R(t)], \]
where \( A = SI \) and

\[
S = \sum_{j=0}^{\infty} S_j, \quad I = \sum_{j=0}^{\infty} I_j, \quad R = \sum_{j=0}^{\infty} R_j.
\]

(10)

Also, the non-linear operator \( A \) is called the Adomian polynomials and it is presented as

\[
A = \sum_{j=0}^{\infty} A_j,
\]

(11)

where

\[
A_0 = S_0 I_0,
\]

\[
A_1 = S_0 I_1 + S_1 I_0,
\]

\[
A_2 = S_0 I_2 + S_1 I_1 + S_2 I_0,
\]

\[
A_3 = S_0 I_3 + S_1 I_2 + S_2 I_1 + S_3 I_0,
\]

(12)

\[
A_4 = S_0 I_4 + S_1 I_3 + S_2 I_2 + S_3 I_1 + S_4 I_0,
\]

\[
\vdots
\]

By substituting series (10) and (11) into (9) we get

\[
\mathcal{L} \left[ \sum_{j=0}^{\infty} S_j \right] = \frac{S(0)}{z} + \frac{f_1}{z^2} + \frac{\lambda}{z} \mathcal{L} \left[ \sum_{j=0}^{\infty} A_j \right] - \frac{d}{z} \mathcal{L} \left[ \sum_{j=0}^{\infty} S_j \right],
\]

\[
\mathcal{L} \left[ \sum_{j=0}^{\infty} I_j \right] = \frac{I(0)}{z} + \frac{f_2}{z^2} + \frac{\lambda}{z} \mathcal{L} \left[ \sum_{j=0}^{\infty} A_j \right] - \frac{\varepsilon}{z} \mathcal{L} \left[ \sum_{j=0}^{\infty} I_j \right] - \frac{d}{z} \mathcal{L} \left[ \sum_{j=0}^{\infty} R_j \right],
\]

(13)

\[
\mathcal{L} \left[ \sum_{j=0}^{\infty} R_j \right] = \frac{R(0)}{z} + \frac{f_3}{z^2} + \frac{\varepsilon}{z} \mathcal{L} \left[ \sum_{j=0}^{\infty} I_j \right] - \frac{d}{z} \mathcal{L} \left[ \sum_{j=0}^{\infty} R_j \right].
\]
Now, the following relations can be obtained:

\[ \mathcal{L}[S_0] = \frac{S(0)}{z} + \frac{f_1}{z^2}, \]
\[ \mathcal{L}[I_0] = \frac{I(0)}{z} + \frac{f_2}{z^2}, \]
\[ \mathcal{L}[R_0] = \frac{R(0)}{z} + \frac{f_3}{z^2}, \]

(14)

\[ \mathcal{L}[S_1] = \frac{\lambda}{z} \mathcal{L}[A_0] - \frac{d}{z} \mathcal{L}[S_0], \]
\[ \mathcal{L}[I_1] = \frac{\lambda}{z} \mathcal{L}[A_0] - \frac{\varepsilon}{z} \mathcal{L}[I_0] - \frac{d}{z} \mathcal{L}[R_0], \]
\[ \mathcal{L}[R_1] = \frac{\varepsilon}{z} \mathcal{L}[I_0] - \frac{d}{z} \mathcal{L}[R_0], \]

and for term \( j \) we have

\[ \mathcal{L}[S_j] = \frac{\lambda}{z} \mathcal{L}[A_{j-1}] - \frac{d}{z} \mathcal{L}[S_{j-1}], \]
\[ \mathcal{L}[I_j] = \frac{\lambda}{z} \mathcal{L}[A_{j-1}] - \frac{\varepsilon}{z} \mathcal{L}[I_{j-1}] - \frac{d}{z} \mathcal{L}[R_{j-1}], \]
\[ \mathcal{L}[R_j] = \frac{\varepsilon}{z} \mathcal{L}[I_{j-1}] - \frac{d}{z} \mathcal{L}[R_{j-1}], \]

(15)

Applying the inverse Laplace transformation \( \mathcal{L}^{-1} \) for first equations of (14) as follows

\[ S_0 = S(0) + f_1 t, \]
\[ I_0 = I(0) + f_2 t, \]
\[ R_0 = R(0) + f_3 t, \]

(16)
By putting \( S_0, I_0, R_0 \) in second equations of (14) and using the Laplace transformations we have

\[
\mathcal{L}[S_1] = \frac{\lambda}{z} \left( \frac{S(0)I(0)}{z} + \frac{S(0)f_2}{z^2} + \frac{I(0)f_1}{z^2} + \frac{2f_1f_2}{z^3} \right) - \frac{d}{z} \left( \frac{S(0)}{z} + \frac{f_1}{z^2} \right),
\]

\[
\mathcal{L}[I_1] = \frac{\lambda}{z} \left( \frac{S(0)I(0)}{z} + \frac{S(0)f_2}{z^2} + \frac{I(0)f_1}{z^2} + \frac{2f_1f_2}{z^3} \right) - \frac{\varepsilon}{z} \left( \frac{I(0)}{z} + \frac{f_2}{z^2} \right) - \frac{d}{z} \left( \frac{R(0)}{z} + \frac{f_3}{z^2} \right),
\]

\[
\mathcal{L}[R_1] = \frac{\varepsilon}{z} \left( \frac{I(0)}{z} + \frac{f_2}{z^2} \right) - \frac{d}{z} \left( \frac{R(0)}{z} + \frac{f_3}{z^2} \right),
\]

(17)

and by applying the inverse Laplace transform \( \mathcal{L}^{-1} \) we can find \( S_1, I_1 \) and \( R_1 \). By repeating above process, the other terms \( S_2, \cdots, S_j, I_2, \cdots, I_j, R_2, \cdots, R_j \) can be obtained. By using the relations

\[
S_n = \sum_{j=0}^{n} S_j, \quad I_n = \sum_{j=0}^{n} I_j, \quad R_n = \sum_{j=0}^{n} R_j,
\]

(18)

the \( n \)-th order approximate solutions can be estimated.

4 Numerical Illustration

In this section, the numerical results of the DTM and the LADM for solving the system of Eqs. (11) are presented. The approximate solutions for \( n = 5 \) by using the DTM are obtained in the following form

\[
S_5(t) = 20 - 2.3t + 0.15425t^2 - 0.00790458t^3 + 0.000309711t^4,
\]

\[
I_5(t) = 15 - 2.2t + 0.04575t^2 + 0.00573792t^3 - 0.000406169t^4,
\]

\[
R_5(t) = 10 + 0.5t - 0.135t^2 + 0.006025t^3 - 7.17708 \times 10^{-6}t^4,
\]

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and for $n = 10$ we have

$$S_{10}(t) = 20 - 2.3t + 0.15425t^2 - 0.00790458t^3 + 0.000309711t^4$$
$$- 7.74864 \times 10^{-6}t^5 - 1.35996 \times 10^{-8}t^6 + 1.41005 \times 10^{-8}t^7$$
$$- 8.93373 \times 10^{-10}t^8 + 3.32927 \times 10^{-11}t^9,$$

$$I_{10}(t) = 15 - 2.2t + 0.04575t^2 + 0.00573792t^3 - 0.000406169t^4$$
$$+ 9.82135 \times 10^{-6}t^5 + 1.12052 \times 10^{-7}t^6 - 1.97453 \times 10^{-8}t^7$$
$$+ 9.96904 \times 10^{-10}t^8 - 3.2067 \times 10^{-11}t^9,$$

$$R_{10}(t) = 10 + 0.5t - 0.135t^2 + 0.006025t^3 - 7.17708 \times 10^{-6}t^4$$
$$- 7.97984 \times 10^{-6}t^5 + 2.96686 \times 10^{-7}t^6 - 2.63764 \times 10^{-8}t^7$$
$$- 2.13846 \times 10^{-10}t^8 + 1.34528 \times 10^{-11}t^9,$$

and finally for $n = 15$ the approximate solutions are obtained as

$$S_{15}(t) = 20 - 2.3t + 0.15425t^2 - 0.00790458t^3 + 0.000309711t^4$$
$$- 7.74864 \times 10^{-6}t^5 - 1.35996 \times 10^{-8}t^6 + 1.41005 \times 10^{-8}t^7$$
$$- 8.93373 \times 10^{-10}t^8 + 3.32927 \times 10^{-11}t^9 - 5.68453 \times 10^{-13}t^{10}$$
$$- 2.34166 \times 10^{-14}t^{11} + 2.59234 \times 10^{-15}t^{12} - 1.28786 \times 10^{-16}t^{13} + 3.78868 \times 10^{-18}t^{14},$$

$$I_{15}(t) = 15 - 2.2t + 0.04575t^2 + 0.00573792t^3 - 0.000406169t^4$$
$$+ 9.82135 \times 10^{-6}t^5 + 1.12052 \times 10^{-7}t^6 - 1.97453 \times 10^{-8}t^7$$
$$+ 9.96904 \times 10^{-10}t^8 - 3.2067 \times 10^{-11}t^9 + 4.21668 \times 10^{-13}t^{10}$$
$$+ 2.88892 \times 10^{-14}t^{11} - 2.70438 \times 10^{-15}t^{12} + 1.28307 \times 10^{-16}t^{13} - 3.62709 \times 10^{-18}t^{14},$$

$$R_{15}(t) = 10 + 0.5t - 0.135t^2 + 0.006025t^3 - 7.17708 \times 10^{-6}t^4$$
$$- 7.97984 \times 10^{-6}t^5 + 2.96686 \times 10^{-7}t^6 - 2.63764 \times 10^{-8}t^7$$
$$- 2.13846 \times 10^{-10}t^8 + 1.34528 \times 10^{-11}t^9 - 4.55198 \times 10^{-13}t^{10}$$
$$+ 7.97151 \times 10^{-15}t^{11} + 1.74314 \times 10^{-16}t^{12} - 2.21438 \times 10^{-17}t^{13} + 1.07465 \times 10^{-18}t^{14}.$
Now, by applying the LADM we get

\[ S_0(t) = 20, \quad I_0(t) = 15, \quad R_0(t) = 10, \]
\[ S_1(t) = -2.3t, \quad I_1(t) = -2.2t, \quad R_1(t) = 0.5t, \]
\[ S_2(t) = 0.15425t^2, \quad I_2(t) = 0.04575t^2, \quad R_2(t) = -0.135t^2, \]
\[ \vdots \quad \vdots \quad \vdots \]
\[ S_{10}(t) = -5.68453 \times 10^{-13}t^{10}, \quad I_{10}(t) = 4.21668 \times 10^{-13}t^{10}, \quad R_{10}(t) = -4.55198 \times 10^{-13}t^{10}, \]
\[ \vdots \quad \vdots \quad \vdots \]

and finally the approximate solution of epidemiological model of computer viruses for \( n = 10 \) is in the following form

\[ S_{10}(t) = \sum_{j=0}^{10} S_j(t) = 20 - 2.3t + 0.15425t^2 - 0.00790458t^3 + 0.000309711t^4 \]
\[ - 7.74864 \times 10^{-6}t^5 - 1.35996 \times 10^{-8}t^6 + 1.41005 \times 10^{-8}t^7 \]
\[ - 8.93373 \times 10^{-10}t^8 + 3.32927 \times 10^{-11}t^9 - 5.68453 \times 10^{-13}t^{10}, \]
\[ I_{10}(t) = \sum_{j=0}^{10} I_j(t) = 15 - 2.2t + 0.04575t^2 + 0.00573792t^3 - 0.000406169t^4 \]
\[ + 9.82135 \times 10^{-6}t^5 + 1.12052 \times 10^{-7}t^6 - 1.97453 \times 10^{-8}t^7 \]
\[ + 9.96904 \times 10^{-10}t^8 - 3.2067 \times 10^{-11}t^9 + 4.21668 \times 10^{-13}t^{10}, \]
\[ R_{10}(t) = \sum_{j=0}^{10} R_j(t) = 10 + 0.5t - 0.135t^2 + 0.006025t^3 - 7.17708 \times 10^{-6}t^4 \]
\[ - 7.97984 \times 10^{-6}t^5 + 2.96686 \times 10^{-7}t^6 - 2.63764 \times 10^{-9}t^7 \]
\[ - 2.13846 \times 10^{-10}t^8 + 1.34528 \times 10^{-11}t^9 - 4.55198 \times 10^{-13}t^{10}. \]

In order to show the accuracy of the presented methods, following residual errors are presented. Also, the numerical results are compared to the obtained results of the HATM for
\( n = 5, 10 \). The results are presented in Tables 3, 4 and 5.

\[
E_{n,S}(t) = S_n'(t) - f_1 + \lambda S_n(t)I_n(t) + dS_n(t),
\]

\[
E_{n,I}(t) = I_n'(t) - f_2 - \lambda S_n(t)I_n(t) + \varepsilon I_n(t) + dR_n(t),
\]

\[
E_{n,R}(t) = R_n'(t) - f_3 - \varepsilon I_n(t) + dR_n(t).
\]

The comparative graphs between the residual errors of the LADM, the DTM and the HATM for \( n = 5, 10, 15 \) are demonstrated in Figs. 11–22 and 23. Also, phase portraits of \( S-I, S-R, I-R \) and \( S-I-R \) which are obtained by 10-th order approximation of the DTM and the LADM are presented in Figs. 3 and 5. According to the generated results, the LADM has suitable scheme than the DTM and the HATM.

Table 3: Numerical comparison of residual error \( E_{n,S}(t) \) between LADM, DTM and HATM for \( n = 5, 10 \).

| t   | \( E_{n,S}(t) \)-LADM | \( E_{n,S}(t) \)-DTM | \( E_{n,S}(t) \)-HATM | \( E_{n,I}(t) \)-LADM | \( E_{n,I}(t) \)-DTM | \( E_{n,I}(t) \)-HATM |
|-----|------------------------|----------------------|----------------------|------------------------|----------------------|----------------------|
| 0.0 | 0                      | 0                    | 0                    | 0                      | 0                    | 0                    |
| 0.2 | 1.98295 \times 10^{-11} | 6.22316 \times 10^{-8} | 0.0000116327         | 4.4409 \times 10^{-16} | 4.4409 \times 10^{-16} | 4.22713 \times 10^{-11} |
| 0.4 | 4.38413 \times 10^{-10} | 9.99998 \times 10^{-17} | 0.0000188901         | 1.77636 \times 10^{-15} | 8.71449 \times 10^{-10} |
| 0.6 | 3.87637 \times 10^{-9}  | 5.07724 \times 10^{-16} | 0.0000214386         | 1.77636 \times 10^{-15} | 5.9508 \times 10^{-14} | 4.95175 \times 10^{-9} |
| 0.8 | 3.9464 \times 10^{-8}   | 0.0000161631         | 0.000015492          | 2.57572 \times 10^{-14} | 5.94032 \times 10^{-15} | 1.57635 \times 10^{-8} |
| 1.0 | 1.18760 \times 10^{-7}  | 0.0000094305         | 0.000029508          | 2.30028 \times 10^{-13} | 5.97122 \times 10^{-12} | 4.47742 \times 10^{-8} |

Table 4: Numerical comparison of residual error \( E_{n,I}(t) \) between LADM, DTM and HATM for \( n = 5, 10 \).

| t   | \( E_{n,I}(t) \)-LADM | \( E_{n,I}(t) \)-DTM | \( E_{n,I}(t) \)-HATM | \( E_{n,S}(t) \)-LADM | \( E_{n,S}(t) \)-DTM | \( E_{n,S}(t) \)-HATM |
|-----|------------------------|----------------------|----------------------|------------------------|----------------------|----------------------|
| 0.0 | 0                      | 0                    | 0                    | 0                      | 0                    | 0                    |
| 0.2 | 2.06857 \times 10^{-10} | 7.88132 \times 10^{-8} | 0.0000116327         | 4.4409 \times 10^{-16} | 4.4409 \times 10^{-16} | 4.22713 \times 10^{-11} |
| 0.4 | 6.48712 \times 10^{-9}  | 1.26470 \times 10^{-16} | 0.000029508          | 2.57572 \times 10^{-14} | 5.94032 \times 10^{-15} | 1.57635 \times 10^{-8} |
| 0.6 | 4.78160 \times 10^{-8}  | 6.42035 \times 10^{-16} | 0.0000397647         | 1.77636 \times 10^{-15} | 5.94032 \times 10^{-15} | 4.47742 \times 10^{-8} |
| 0.8 | 1.95560 \times 10^{-7}  | 0.0000203449         | 8.51494 \times 10^{-16} | 3.15303 \times 10^{-14} | 5.96412 \times 10^{-15} | 1.65033 \times 10^{-8} |
| 1.0 | 5.78838 \times 10^{-7}  | 0.000009794           | 0.000149914          | 2.89548 \times 10^{-15} | 4.50395 \times 10^{-15} | 3.7476 \times 10^{-8} |

Table 5: Numerical comparison of residual error \( E_{n,R}(t) \) between LADM, DTM and HATM for \( n = 5, 10 \).

| t   | \( E_{n,R}(t) \)-LADM | \( E_{n,R}(t) \)-DTM | \( E_{n,R}(t) \)-HATM | \( E_{n,S}(t) \)-LADM | \( E_{n,S}(t) \)-DTM | \( E_{n,S}(t) \)-HATM |
|-----|------------------------|----------------------|----------------------|------------------------|----------------------|----------------------|
| 0.0 | 0                      | 0                    | 0                    | 0                      | 0                    | 0                    |
| 0.2 | 5.69638 \times 10^{-8}  | 6.38387 \times 10^{-8} | 1.95392 \times 10^{-6} | 1.11022 \times 10^{-16} | 3.30067 \times 10^{-16} | 1.04154 \times 10^{-12} |
| 0.4 | 1.82284 \times 10^{-8}  | 1.02142 \times 10^{-16} | 0.0000285538         | 3.30067 \times 10^{-16} | 1.22125 \times 10^{-16} | 8.17403 \times 10^{-12} |
| 0.6 | 3.8422 \times 10^{-7}   | 5.17994 \times 10^{-16} | 0.0000112891         | 7.21645 \times 10^{-16} | 4.56875 \times 10^{-16} | 1.57476 \times 10^{-10} |
| 0.8 | 5.8339 \times 10^{-7}   | 0.0000063427         | 0.0000299756         | 9.43689 \times 10^{-17} | 6.10734 \times 10^{-17} | 1.85486 \times 10^{-9} |
| 1.0 | 1.70812 \times 10^{-6}  | 0.0000098892         | 0.0000601621         | 8.77631 \times 10^{-14} | 4.55164 \times 10^{-14} | 8.65334 \times 10^{-9} |
Figure 1: Comparison between error functions of LADM, DTM and HATM for $S_5(t)$, $I_5(t)$, $R_5(t)$. 
Figure 2: Comparison between error functions of LADM, DTM and HATM for $S_{10}(t), I_{10}(t), R_{10}(t)$.
Figure 3: Comparison between error functions of LADM, DTM and HATM for $S_{15}(t), I_{15}(t), R_{15}(t)$. 
Figure 4: Phase portraits of $S_{10}(t), I_{10}(t), R_{10}(t)$ by using the LADM.
Figure 5: Phase portraits of $S_{10}(t), I_{10}(t), R_{10}(t)$ by using the DTM.
5 Conclusion

In this study, two robust and applicable methods, the DTM and the LADM were applied to solve the non-linear epidemiological model of computer viruses. In order to show the efficiency and accuracy of presented method, the residual errors for different iterations were presented based on the LADM, DTM and HATM. Also, the graphs of residual error were demonstrated to show the abilities of the LADM than the other methods.

References

[1] S. Abbasbandy, Extended Newton’s method for a system of nonlinear equations by modified Adomian decomposition method, Applied Mathematics and Computation, 170, 648-656 (2005).

[2] F. Cohen, Computer viruses: theory and experiments, Comput. Secur., 6, 22-35 (1987).

[3] J.S. Duan, R. Rach, A.M. Wazwaz, Higher order numeric solutions of the Lane-Emden-type equations derived from the multi-stage modified Adomian decomposition method, Int. J. Comput. Math., 94(1), 197-215 (2017).

[4] A. Ebaid, M. D. Aljoufi, A.M. Wazwaz, An advanced study on the solution of nanofluid flow problems via Adomian’s method, Appl. Math. Lett., 46, 117-122 (2015).

[5] A.A. Freihat, M. Zurigat, A. H. Handam, The multi-step homotopy analysis method for modified epidemiological model for computer viruses, Afrika Matematika, 26 (3-4), 585-596 (2015).

[6] M.A. Fariborzi Araghi, Sh. Behzadi, Solving nonlinear Volterra-Fredholm integro-differential equations using the modified Adomian decomposition method, Computational Methods In Applied Mathematics, 9 (4), 1-11 (2009).

[7] M.A. Fariborzi Araghi, A. Fallahzadeh, Dynamical Control of accuracy using the stochastic arithmetic to estimate the solution of the ordinary differential equations via Adomian decomposition method, Asian Journal of Mathematics and Computer Research, 8(2), 128-135 (2016).

[8] M.A. Fariborzi Araghi, S. Noeiaghdam, Homotopy analysis transform method for solving generalized Abel’s fuzzy integral equations of the first kind, IEEE (2016).

[9] M.A. Fariborzi Araghi, S. Noeiaghdam, Fibonacci-regularization method for solving Cauchy integral equations of the first kind, Ain Shams Eng J., (8), 363-369 (2017).

[10] M. Fariborzi Araghi, S. Noeiaghdam, A novel technique based on the homotopy analysis method to solve the first kind Cauchy integral equations arising in the theory of airfoils, Journal of Interpolation and Approximation in Scientific Computing, (1), 1-13 (2016).
[11] M.A. Fariborzi Araghi, S. Noeiaghdam, Homotopy regularization method to solve the singular Volterra integral equations of the first kind, Jordan Journal of Mathematics and Statistics, 10(4), (2017).

[12] H.E. Gadian, Solving coupled pseudo-parabolic equation using a modified double Laplace decomposition method, Acta Mathematica Scientia, 38 (1), 333-346 (2018).

[13] S. Gupta, D. Kumar, J. Singh, Analytical solutions of convection-diffusion problems by combining Laplace transform method and homotopy perturbation method, Alexandria Engineering Journal, 54 (3), 645-651 (2015).

[14] X. Han, Q. Tan, Dynamical behavior of computer virus on Internet, Appl. Math. Comput., 217, 2520-2526 (2010).

[15] F. Haq, K. Shah, G. ur Rahman, M. Shahzad, Numerical solution of fractional order smoking model via laplace Adomian decomposition method, Alexandria Engineering Journal, In press, 13, (2017).

[16] S. Gh. Hosseini, E. Babolian, S. Abbasbandy, A new algorithm for solving Van der Pol equation based on piecewise spectral Adomian decomposition method, Int. J. Industrial Math., 8(3), 177-184 (2016).

[17] J.O. Kephart, T. Hogg, B.A. Huberman, Dynamics of computational ecosystems, Phys. Rev. A, 40 (1), 404-421 (1998).

[18] S. Kumar, D. Kumar, S. Abbasbandy, M.M. Rashidi, Analytical solution of fractional Navier-Stokes equation by using modified Laplace decomposition method, Ain Shams Eng. J., 5, 569-574 (2014).

[19] S. Kumar, J. Singh, D. Kumar, S. Kapoor, New homotopy analysis transform algorithm to solve volterra integral equation, Ain Shams Eng J., 5, 243-246 (2014).

[20] D. Kumar, J. Singh, Sushila, Application of homotopy analysis transform method to fractional biological population model, Romanian Reports in Physics, 65, 63-75 (2013).

[21] B.K. Mishra, N. Jha, Fixed period of temporary immunity after run of anti-malicious software on computer nodes, Appl. Math. Comput., 190, 1207-1212 (2007).

[22] B.K. Mishra, S.K. Pandey, Fuzzy epidemic model for the transmission of worms in computer network, Nonlinear Anal.: Real World Appl., 11, 4335-4341 (2010).

[23] S. Noeiaghdam, A novel technique to solve the modified epidemiological model of computer viruses, SeMA Journal, (2018). https://doi.org/10.1007/s40324-018-0163-3.

[24] S. Noeiaghdam, Solving a non-linear model of HIV infection for CD4+ T cells by combining Laplace transformation and Homotopy analysis method, [arXiv:1809.06232].
[25] S. Noeiaghdam, M.A. Fariborzi Araghi, S. Abbasbandy, Finding optimal convergence control parameter in the homotopy analysis method to solve integral equations based on the stochastic arithmetic, Numer Algor, (2018). https://doi.org/10.1007/s11075-018-0546-7.

[26] S. Noeiaghdam, D. Sidorov, V. Sizikov, Control of accuracy on Taylor-collocation method to solve the weakly regular Volterra integral equations of the first kind by using the CESTAC method, arXiv:1811.09802v1.

[27] S. Noeiaghdam, M. Suleman, H. Budak, Solving a modified nonlinear epidemiological model of computer viruses by homotopy analysis method, Mathematical Sciences, 1-12 (2018).

[28] S. Noeiaghdam, E. Zarei, H. Barzegar Kelishami, Homotopy analysis transform method for solving Abel’s integral equations of the first kind, Ain Shams Eng. J., 7, 483-495 (2016).

[29] M. Nourifar, A. Aftabi Sani, A. Keyhani, Efficient multi-step differential transform method: Theory and its application to nonlinear oscillators, Communications in Nonlinear Science and Numerical Simulation, 53, 154-183 (2017).

[30] Y. Oztiirk, M. Gulsu, Numerical solution of a modified epidemiological model for computer viruses, Applied Mathematical Modelling, 39 (23-24), 7600-7610 (2015).

[31] S. Paul, S. Prasad Mondal, P. Bhattacharyya, Numerical solution of Lotka Volterra prey predator model by using Runge-Kutta-Fehlberg method and Laplace Adomian decomposition method, Alexandria Engineering Journal, 55 (1), 613-617 (2016).

[32] J.R.C. Piqueira, A.A. de Vasconcelos, C.E.C.J. Gabriel, V.O. Araujo, Dynamic models for computer viruses, Comput. Secur., 27, 355-359 (2008).

[33] J.R.C. Piqueira, V.O. Araujo, A modified epidemiological model for computer viruses, Appl. Math. Comput., 213, 355-360 (2009).

[34] M.M. Rashidi, E. Erfani, A new analytical study of MHD stagnation-point flow in porous media with heat transfer, Computers & Fluids, 40, 172-178 (2011).

[35] M.M. Rashidi, E. Erfani, New analytical method for solving Burger’s and nonlinear heat transfer equations and comparison with HAM, Computer Physics Communications, 180, 1539-1544 (2009).

[36] J. Ren, X. Yang, L. Yang, Y. Xu, F. Yang, A delayed computer virus propagation model and its dynamics, Chaos Solit Fractal, 45, 74-79 (2012).

[37] J. Ren, X. Yang, Q. Zhu, L. Yang, C. Zhang, A novel computer virus model and its dynamics, Nonlinear Anal.-Real., 3, 376-384 (2012).

[38] C. Rodrigue Bambe Moutsinga, E. Pindza, E. Maré, Homotopy perturbation transform method for pricing under pure diffusion models with affine coefficients, Journal of King Saud University-Science, 30 (1), 1-13 (2018).
[39] H. Roohani Ghehsareh, S. Abbasbandy, B. Soltanalizadeh, Analytical solutions of the slip Magnetohydrodynamik viscous flow over a stretching sheet by using the Laplace Adomian Decomposition Method, Zeitsehrift fur Naturorshung A , 67(a), 248-254 (2012).

[40] K. Shah, T. Singh, A. Kiliçman, Combination of integral and projected differential transform methods for time-fractional gas dynamics equations, Ain Shams Eng J., In press, (2017).

[41] J. Singh, D. Kumar, Z. Hammouch, A. Atangana, A fractional epidemiological model for computer viruses pertaining to a new fractional derivative, Applied Mathematics and Computation, 316 (1), 504-515 (2018).

[42] M.H. Tiwana, K. Maqbool, A.B. Mann, Homotopy perturbation Laplace transform solution of fractional non-linear reaction diffusion system of Lotka-Volterra type differential equation, Engineering Science and Technology, an International Journal, 20 (2), 672-678 (2017).

[43] A.M. Wazwaz, The combined Laplace transform-Adomian decomposition method for handling nonlinear Volterra integro-differential equations, Applied Mathematics and Computation, 216(4), 1304-1309 (2010).

[44] A.M. Wazwaz, R. Rach, J.S. Duan, Adomian decomposition method for solving the Volterra integral form of the Lane-Emden equations with initial values and boundary conditions, Applied Mathematics and Computation, 219(10), 5004-5019 (2013).

[45] J.C. Wierman, D.J. Marchette, Modeling computer virus prevalence with a susceptible-infected-susceptible model with reintroduction, Comput. Stat. Data An., 45, 3-23 (2004).

[46] H. Yuan, G. Chen, Network virus-epidemic model with the point-to-group information propagation, Appl. Math. Comput., 20, 357-367 (2008).

[47] Q. Zhu, X. Yang, J. Ren, Modeling and analysis of the spread of computer virus, Commun. Nonlinear Sci. Numer. Simul., 17 (12), 5117-5124 (2012).

[48] E. Zarei, S. Noeighdham, Solving generalized Abel’s integral equations of the first and second kinds via Taylor-collocation method, arXiv:1804.08571.