Z′ physics with early LHC data

Elena Accomando ∗,1,2, Alexander Belyaev †,1,2, Luca Fedeli ‡,3, Stephen F. King §,1 and Claire Shepherd-Themistocleous ¶2

1School of Physics & Astronomy, University of Southampton, Highfield, Southampton SO17 1BJ, UK
2Particle Physics Department, Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, UK
3INFN, 50019 Sesto F., Firenze, Italy and Department of Physics and Astrophysics, University of Florence, 50019 Sesto F., Firenze, Italy

We discuss the prospects for setting limits on or discovering spin-1 Z′ bosons using early LHC data at 7 TeV. Our results are based on the narrow width approximation in which the leptonic Drell-Yan Z′ boson production cross-section only depends on the Z′ boson mass together with two parameters \( c_u \) and \( c_d \). We carefully discuss the experimental cuts that should be applied and tabulate the theoretical next-to-next-to-leading order corrections which must be included. Using these results the approach then provides a safe, convenient and unbiased way of comparing experiment to theoretical models which avoids any built-in model dependent assumptions. We apply the method to three classes of perturbative Z′ boson benchmark models: \( E_6 \) models, left-right symmetric models and sequential standard models. We generalise each class of model in terms of mixing angles which continuously parametrize linear combinations of pairs of generators and lead to distinctive orbits in the \( c_u - c_d \) plane. We also apply this method to the strongly coupled four-site benchmark model in which two Z′ bosons are predicted. By comparing the experimental limits or discovery bands to the theoretical predictions on the \( c_u - c_d \) plane, we show that the LHC at 7 TeV with integrated luminosity of 500 pb\(^{-1}\) will greatly improve on current Tevatron mass limits for the benchmark models. If a Z′ is discovered our results show that measurement of the mass and cross-section will provide a powerful discriminator between the benchmark models using this approach.

I. INTRODUCTION

The end of the first decade of the millenium is an exciting time in particle physics, with the CERN LHC enjoying an extended run at 7 TeV, and the Fermilab Tevatron collecting unprecedented levels of integrated luminosity, eventually up to perhaps 10 fb\(^{-1}\), in the race to discover the first signs of new physics Beyond the Standard Model (BSM). Since spin-1 Z′ bosons are predicted by dozens of such models, and are very easy to discover in the leptonic Drell-Yan mode, this makes them good candidates for an early discovery at the LHC. For a review see Refs. [1–3] and references therein. Furthermore high mass Z′ bosons are more likely to be discovered at the LHC than the Tevatron [4], since energy is more important than luminosity for the discovery of high mass states. This makes the study of Z′ bosons both timely and promising and has led to widespread recent interest in this subject (see for example [5–21, 21]).

Since one of the purposes of this paper is to facilitate the connection between experiment and theory, it is worth being clear at the outset precisely what we shall mean by a Z′ boson. To an experimentalist a Z′ is a resonance “bump” more massive than the Z of the Standard Model (SM) which can be observed in Drell-Yan production followed by its decay into lepton-antilepton pairs. To a phenomenologist a Z′ boson is a new massive electrically neutral, colourless boson (equal to its own antiparticle) which couples to SM matter. To a theorist it is useful to classify the Z′ according to its spin, even though actually measuring its spin will require high statistics. For example a spin-0 particle could correspond to a sneutrino in R-parity violating supersymmetric (SUSY) models. A spin-2 resonance could be identified as a Kaluza-Klein (KK) excited graviton in Randall-Sundrum models. However a spin-1

∗ E-mail: e.accomando@soton.ac.uk
† E-mail: a.belyaev@soton.ac.uk
‡ E-mail: fedeli@fi.infn.it
§ E-mail: king@soton.ac.uk
¶ E-mail: C.H.Shepherd-Themistocleous@rl.ac.uk
\( Z' \) is by far the most common possibility usually considered, and this is what we shall mean by a \( Z' \) boson in this paper.

In this paper, then, we shall discuss electrically neutral colourless spin-1 \( Z' \) bosons, which are produced by the Drell-Yan mechanism and decay into lepton-antilepton pairs, yielding a resonance bump more massive than the \( Z \). We shall be particularly interested in the prospects for discovering or setting limits on such \( Z' \) bosons using early LHC data. By early LHC data we mean the present 2010/11 run at the LHC at 7 TeV, which is anticipated to yield an integrated luminosity approximately of 1 fb\(^{-1}\). Since the present LHC schedule involves a shut-down during 2012, followed by a restart in 2013, the early LHC data will provide the best information possible about \( Z' \) bosons over the next three years, so in this paper we shall focus exclusively on what can be achieved using these data, comparing the results with current Tevatron limits. In order to enable contact to be made between early LHC experimental data and theoretical models, we advocate the narrow width approximation, in which the leptonic Drell-Yan \( Z' \) boson production cross-section only depends on the \( Z' \) boson mass together with two parameters \( c_u \) and \( c_d \).

Properly defined experimental information on the \( Z' \) boson cross-section may then be recast as limit or discovery contours in the \( c_u-c_d \) plane, with a unique contour for each value of \( Z' \) boson mass. In order to illustrate how this formalism enables contact to be made with theoretical models we study three classes of \( Z' \) boson benchmark models: \( E_6 \) models, left-right (LR) symmetric models and sequential standard models (SSM). We also apply this method to the strongly coupled four-site benchmark model in which two \( Z' \) bosons are predicted [13]. Each benchmark model may be expressed in the \( c_u-c_d \) plane which enables contact to be made with the experimental limit or discovery contours.

Working to next-to-next-leading order (NNLO) we show that the LHC at 7 TeV with as little data as 500 pb\(^{-1}\) can either greatly improve on current Tevatron mass limits, or discover a \( Z' \), with a measurement of the mass and cross-section providing powerful resolving power between the benchmark models using this approach. We also briefly discuss the impact of the \( Z' \) boson width on search strategies. Although the width \( \Gamma_{Z'} \) into standard model particles may in principle be predicted as a function of \( M_{Z'} \), in practice there may be considerable uncertainty concerning \( \Gamma_{Z'} \) due for example to the possible decay into other non-standard particles, including supersymmetric (SUSY) partners and exotic states, for example.

At the outset we would like to highlight some of the strengths and limitations of our approach to the benchmark models. One of the strengths of our approach is that the considered benchmark models encompass two quite different types of \( Z' \) models: perturbative gauge models and strongly coupled models, where the perturbative models generally involve relatively narrow widths (which however can get larger if SUSY and exotics are included in the decays in addition to SM particles), while the strongly coupled models involve multiple \( Z' \) bosons with rather broad widths.

The perturbative benchmark gauge models are defined in terms of continuous mixing angles, in analogy to the \( E_6 \) class of models expressed through the linear combinations of \( \chi \) and \( \psi \) generators. This approach is generalised to the case of LR models involving linear combinations of the generators \( R \) and \( B-L \), and similarly we define a new class of SM-like \( Z' \) models involving linear combinations of \( L \) and \( Q \) generators, with the precise linear combination in each case parameterised by a separate angle. The strength of this approach is that it enables a finite number of classes, each containing an infinite number of benchmark models, to be defined, rather than just a finite number of models. For each class of models, the respective angle serves to parametrize the specific orbit which represents that class in the \( c_u-c_d \) plane. The different orbits turn out to be non-overlapping for the abovementioned three classes of models which in the following we label as: \( E_6(\theta) \), \( GLR(\phi) \) and \( GSM(\alpha) \) respectively. A limitation of the approach is that it ignores the effect of the SUSY and exotics on the width \( \Gamma_{Z'} \). In addition it also ignores the effects of \( Z-Z' \) mixing since this is model dependent. However any such mixing must be small due to the constraints from electroweak precision measurements, and we refer to such constraints on the mixing angle where possible. As regards the strongly coupled models, in principle \( Z' \) bosons could emerge from techni-rho bound states in Walking Technicolour models, or a series of strongly interacting resonances such KK excitations of the \( Z \) boson. A limitation of our approach here is that we only consider the four-site Higgsless model which contains just two excitations of the \( Z \) (and W) bosons, as a simple approximation to both the Walking technicolour models and the KK excitations of the \( Z \). Nevertheless the four-site model is representative of the physics of a typical strongly interacting \( Z' \) model and by representing it for the first time in the \( c_u-c_d \) plane, it is clearly seen that the associated \( Z' \) bosons may easily be distinguished from those of the perturbative gauge models.

The layout of the rest of the paper is as follows. In section [11] we describe our model independent approach based on the narrow width approximation and the variables \( M_{Z'}, c_u \) and \( c_d \). Higher order corrections to the cross-section are tabulated in the narrow width approximation and new K factors are defined which enable the \( c_u \) and \( c_d \) approach to

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1 We remark that in [14] the authors focussed on models obtained by taking a linear combination of \( Y \) and \( B-L \) which is related to the generalized LR models that we also consider, however the choice of generator basis and parametrisation is different and they did not use the mixing angle parametrisation that we propose here.
be reliably extended to NNLO. In this section we consider finite width effects and discuss the choice of invariant mass window around \( M_{Z'} \) which matches the narrow width approximation, showing that in the case of the LHC this choice is only weakly constrained. We also comment on the effects of interference and show that they may become important for invariant mass cuts close to 100 GeV but are negligible for the a suitable invariant mass cut around \( M_{Z'} \), which we therefore advocate. In section \( \text{III} \) we define our benchmark models based on the current Tevatron limits using the latest \( D0 \) results and the latest CMS potential based on the projected CMS limits on the \( Z' \) boson cross section normalized to the SM \( Z \)-boson one for an integrated luminosity \( L = 500pb^{-1} \). We use the CMS potential since they are more closely related to the narrow width approximation that we advocate, and we use CMS rather than ATLAS since the projected limits are publicly available. In both cases we express the experimental cross-section limits in the \( c_u-c_d \) plane and compare these limits to the benchmark models also displayed in the \( c_u-c_d \) plane. In the case of CMS we also show the discovery limits. We tabulate some of the results for some special choices of the mixing angle variable, including the width, the leptonic branching ratio, and we advocate, and we use CMS rather than ATLAS since the projected limits are publicly available. In both cases we express the experimental cross-section limits in the \( c_u-c_d \) plane and compare these limits to the benchmark models also displayed in the \( c_u-c_d \) plane. In the case of CMS we also show the discovery limits. We tabulate some of the results for some special choices of the mixing angle variable, including the width, the leptonic branching ratio, and similarly for the \( g_A \) couplings, which means that there are eight model dependent couplings to SM fermions \( g_{VA}^f \), with \( f = u, d, e, \nu \). These eight couplings are not all independent since they are related to the left (L) and right (R) couplings as follows:

\[
\mathcal{L}_{NC} = \frac{g'}{2} Z'_\mu \bar{f} \gamma^\mu (g^f_{V} - g^f_{A} \gamma^5)f.
\]

The values of \( g^f_{V}, g^f_{A} \) depend on the particular choice of \( U'(1) \) and on the particular fermion \( f \). We assume universality amongst the three families, \( g^u_{V} = g^d_{V} = g^e_{V} \), and \( g^{\nu}_{V} = g^{\tau}_{V} \), as well as \( g^{u}_{V} = g^{d}_{V} = g^{e}_{V} \), and similarly for the \( g^f_{A} \) couplings, which means that there are eight model dependent couplings to SM fermions \( g_{VA}^f \), with \( f = u, d, e, \nu \). These eight couplings are not all independent since they are related to the left (L) and right (R) couplings as follows:

\[
\mathcal{L}_{NC} = \frac{g'}{2} Z'_\mu \bar{f} \gamma^\mu (g^f_{V} - g^f_{A} \gamma^5)f = g' Z'_\mu \bar{f} \gamma^\mu (\epsilon^f_L P_L + \epsilon^f_R P_R)f.
\]

where \( P_{L,R} = (1 \mp \gamma_5)/2 \) and \( g_{VA,A}^L = \epsilon^f_L \pm \epsilon^f_R \). The couplings are not all independent since the left-handed fermions are in doublets with the same charges \( \epsilon^f_L = \epsilon^f_R \) and \( \epsilon^f_L = \epsilon^f_R \). Excluding the right-handed neutrinos (which we assume to be heavier than the \( Z' \)) there are really five independent couplings \( \epsilon^u_L, \epsilon^d_L, \epsilon^e_L, \epsilon^\nu_L, \epsilon^\tau_L, \epsilon^d_R, \epsilon^e_R \). However we prefer to work with the eight vector and axial couplings \( g_{VA}^f \). In addition, the strength of the gauge coupling \( g' \) is model dependent. Throughout, we follow the conventions of [1].

A slightly more complicated setup is needed to describe the four-site model which, in this paper, has been chosen to represent Higgsless multiple \( Z' \)-boson theories. The corresponding framework will be given in Sec. \( \text{IIIB} \). Throughout the paper we shall ignore the couplings of the \( Z' \) to beyond SM particles such as right-handed neutrinos, SUSY partners and any exotic fermions in the theory, which all together may increase the width of the \( Z' \) by up to about a factor of five [22] and hence lower the branching ratio into leptons by the same factor.
B. \(Z'\) production and decay in the narrow width approximation

The \(Z'\) contribution to the Drell-Yan production cross-section of fermion-antifermion pairs in a symmetric mass window around the \(Z'\) mass (\(|M - M_{Z'}| \leq \Delta\)) may be written as:

\[
\sigma_{f\bar{f}} = \int_{(M_{Z'} - \Delta)^2}^{(M_{Z'} + \Delta)^2} \frac{d\sigma}{dM^2}(pp \rightarrow Z' \rightarrow f\bar{f}X) dM^2.
\] (II.2)

In the narrow width approximation (NWA), it becomes

\[
\sigma_{f\bar{f}} \approx \left( \frac{1}{3} \sum_{q=u,d} \left( \frac{dLq\bar{q}}{dM_{Z'}^2} \right) \hat{\sigma}(q\bar{q} \rightarrow Z') \right) \times Br(Z' \rightarrow f\bar{f}) \tag{II.3}
\]

where the parton luminosities are written as \(\left( \frac{dLq\bar{q}}{dM_{Z'}^2} \right)\) and \(\hat{\sigma}(q\bar{q} \rightarrow Z')\) is the peak cross-section given by:

\[
\hat{\sigma}(q\bar{q} \rightarrow Z') = \frac{\pi}{12} g^2 [(g_u^q)^2 + (g_d^q)^2].
\] (II.4)

The branching ratio of the \(Z'\) boson into fermion-antifermion pairs is

\[
Br(f\bar{f}) \equiv Br(Z' \rightarrow f\bar{f}) = \frac{\Gamma(Z' \rightarrow f\bar{f})}{\Gamma_{Z'\rightarrow Z'\rightarrow f\bar{f}}},
\] (II.5)

where \(\Gamma_{Z'}\) is the total \(Z'\) width and the partial widths into a particular fermion-antifermion pair of \(N_c\) colours is given by

\[
\Gamma(Z' \rightarrow f\bar{f}) = N_c g'^2_{Z'} M_{Z'} [(g_u^f)^2 + (g_d^f)^2].
\] (II.6)

Assuming only SM fermions in the final state and neglecting in first approximation their mass, one finds the total width:

\[
\Gamma_{Z'} = \frac{g'^2_{Z'}}{48\pi} M_{Z'} \left[ 9(g_v^u)^2 + (g_A^u)^2 + 9(g_v^d)^2 + (g_A^d)^2 + 3(g_v^\nu)^2 + (g_A^\nu)^2 + 3(g_v^\tau)^2 + (g_A^\tau)^2 \right].
\] (II.7)

Specializing to the charged lepton pair production cross-section relevant for the first runs at the LHC, Eq.\(\text{[II.3]}\) may be written at the leading order (LO) as \(\text{[3]}\):

\[
\sigma_{e^+e^-}^{\text{LO}} = \frac{\pi}{48\pi} \left[ c_u w_u(s, M_{Z'}^2) + c_d w_d(s, M_{Z'}^2) \right]
\] (II.8)

where the coefficients \(c_u\) and \(c_d\) are given by:

\[
c_u = \frac{g^2}{2} (g_v^u)^2 Br(\ell^+\ell^-), \quad c_d = \frac{g^2}{2} (g_v^d)^2 Br(\ell^+\ell^-),
\] (II.9)

and \(w_u(s, M_{Z'}^2)\) and \(w_d(s, M_{Z'}^2)\) are related to the parton luminosities \(\left( \frac{dLq\bar{q}}{dM_{Z'}^2} \right)\) and \(\left( \frac{dL\bar{q}q}{dM_{Z'}^2} \right)\) and therefore only depend on the collider energy and the \(Z'\) mass. All the model dependence of the cross-section is therefore contained in the two coefficients, \(c_u\) and \(c_d\). These parameters can be calculated from \(g_{V}^1, g_{A}^1\) and \(g_{V}'\), assuming only SM decays of the \(Z'\) boson. The corresponding values for all models which predict a single \(Z'\) boson purely decaying into SM fermions are given in Table \(\text{[I]}\)

A slight complication arises in Higgsless theories, which in the present paper are represented by the four-site model. Here in fact the two neutral extra gauge bosons, \(Z_{1,2}\), decay preferably into di-boson intermediate states. Their total width gets therefore two contributions:

\[
\Gamma_{Z_i} = \Gamma_{Z_i}^{f\bar{f}} + \Gamma_{Z_i}^{VV} \quad (i = 1, 2)
\] (II.10)

where the two terms on the right-hand side represent the fermionic and bosonic decay, respectively. More in detail,

\[
\Gamma_{Z_i}^{f\bar{f}} = \frac{1}{48\pi} M_{Z_i} \left[ 9(g_v^u)^2 + (g_A^u)^2 + 9(g_v^d)^2 + (g_A^d)^2 + 3(g_v^\nu)^2 + (g_A^\nu)^2 + 3(g_v^\tau)^2 + (g_A^\tau)^2 \right]
\] (II.11)
\[ \Gamma_{Z_1}^{WW} = \frac{1}{3\pi} \left( \frac{1}{16} \right)^2 \frac{M_{Z_1}^3}{M_W^2} (1-z^4)(1+z^2) \]  
(II.12)

\[ \Gamma_{Z_2}^{WW} = \frac{1}{3\pi} \left( \frac{1}{16} \right)^2 \frac{M_{Z_2}^3}{M_W^2} z^4(1-z^2)^3[1+10z^2+z^4] \]  
(II.13)

with \( i=1,2 \), where in this case we have included the \( g' \) coupling in the definition of \( g_{1,2}^t \) and \( g_{1,2}^l \). In the above formulas \( M_{Z_1} \) and \( M_{Z_2} \) are the masses of the two extra gauge bosons, \( Z_{1,2} \), while \( z \) is their ratio, i.e. \( z = M_{Z_1}/M_{Z_2} \). The direct consequence of this peculiarity is that the \( Z_{1,2} \) leptonic branching ratio acquires a not trivial dependence on the \( Z_{1,2} \) boson mass which reflects in an intrinsic mass dependence of the \( c_u \) and \( c_d \) coefficients. In addition, there is an external source of variation with mass. As all vector and axial couplings in the four-site model can be expressed in terms of the three independent free parameters \( (g_{2V}^t, M_{Z_{2}}, z) \), \( c_u \) and \( c_d \) are completely specified by these quantities as well: \( c_{u,d} = c_{u,d}(g_{2V}^t, M_{Z_{2}}, z) \). This means that at fixed masses, \( M_{Z_{2}} \) and \( z = M_{Z_1}/M_{Z_2} \), these coefficients get constrained by the EWPT bounds acting on \( g_{2V}^t \). As these limits vary with mass (see Fig. 6), \( c_u \) and \( c_d \) acquire this extra \( M_{Z_{2}} \) dependence. The net result opens up a parameter space in the \( c_d - c_u \) plane which will be displayed at due time.

As emphasized in [4], the \( c_d - c_u \) plane parametrization is a model-independent way to create a direct correspondence between the experimental bounds on \( pp(\bar{p}) \rightarrow Z' \rightarrow \ell^+\ell^- \) cross sections and the parameters of the Lagrangian. An experimental limit on \( \sigma(pp(\bar{p}) \rightarrow Z' \rightarrow \ell^+\ell^-) \) for a given \( Z' \) mass gives in fact a linear relation between \( c_u \) and \( c_d \),

\[ c_u = a - bc_d \]  
(II.14)

where \( a, b \) can be regarded as known numbers given by:

\[ a = \frac{488}{\pi} \frac{\sigma_{\ell^+\ell^-}^{exp}}{w_u}, \quad b = \frac{w_d}{w_u}. \]  
(II.15)

where \( \sigma_{\ell^+\ell^-}^{exp} \) represents the 95\% C.L. upper bound on the experimental Drell-Yan cross section which can be derived from observed data.

In practice, it is more convenient to use a log-log scale resulting in the limits appearing as contours for a fixed \( Z' \) mass in the \( c_d - c_u \) plane. We use this representation in the next subsections.

### C. Higher-order corrections

At higher-orders, the expression for \( Z' \) production given by Eq. (II.8) strictly speaking is no longer valid. However, as it was shown in Ref. [4], the additional terms which are not proportional to \( c_u \) and \( c_d \) in Eq. (II.8) can be neglected at NNLO. Therefore, Eq. (II.8) gives a quite accurate description of the approach we are discussing here even at NNLO.

In the following, we take into account QCD NNLO effects as implemented in the WZPROD program [29] as a correction to the total \( Z' \) production cross section in the NWA.\(^\text{2}\) We have adopted this package for simulating the \( Z' \) production, and have linked it to an updated set of Parton Density Functions (PDF). This set includes in particular the most recent versions of CTEQ6.6 [29, 31] and MSTW08 [31] PDF’s, which we use in our analysis. We can provide the complete code upon request.

The QCD NNLO \( Z' \) production cross sections obtained using CTEQ6.6 and MSTW08 PDFs are in a good agreement. Their difference is in fact at the 2-3\% level over a wide \( Z' \) mass spectrum as shown in Fig. 1 where we plot the total \( pp(\bar{p}) \rightarrow Z' \) cross section at the Tevatron and LHC@7 TeV versus the \( Z' \) mass. Here, we have taken as factorization scale the value \( Q = M_{Z'}. \) The further detailed analysis of the cross section variation with the scale is outside of the scope of the current paper.

It is also convenient to define customary NLO and NNLO \( K \)-factors which can be useful for experimentalists in establishing \( Z' \) exclusion limits:

\[ K_i = \frac{\sigma(pp(\bar{p}) \rightarrow Z'_i)}{\sigma(pp(\bar{p}) \rightarrow Z'_0)}. \]  
(II.16)

\(^{2}\) We would like to note, that study of NLO effects for kinematical distributions involving leptons from \( Z' \) decay [29, 30] is beyond the scope of this paper.
FIG. 1: $\sigma(pp(p) \rightarrow ZZ')_{NLO}$ for Standard Model-like $Z'$ production at the Tevatron (left panel) and the LHC@7TeV (right panel) for CTEQ6.6 and MSTW08 PDF’s.

where the index $i = 1, 2$ corresponds to NLO and NNLO $K$-factors, respectively. As an example, in Fig. 2 we present the values of these $K_i$-factors for Standard Model-like $Z'$ production at the Tevatron (left panel) and the LHC@7TeV (right panel) for CTEQ6.6 and MSTW08 PDF’s.

Oppositely to what happens for the aforementioned exact NNLO $Z'$ production cross section, where the agreement between CTEQ6.6 and MSTW08 PDF predictions is optimal, in this case there is a noticeable difference in the behavior of $K_{NLO}$ and $K_{NNLO}$ factors as a function of the $Z'$ mass when convoluting the $Z'$ production cross section with CTEQ6.6 or MSTW08 PDF’s. This difference is related to the way of fitting the LO PDF’s of CTEQ and MSTW collaborations (see e.g. [31, 32]). Furthermore, both $K_{NLO,NNLO}$ factors display a strong dependence on the $Z'$ mass. As an example, $K_{CTEQ6}^{NNLO}$ varies between 10-40% at the Tevatron and 10-30% at the LHC@7TeV for potentially accessible $Z'$ masses.

Applying a universal $K$-factor can be highly misleading. As shown above, the $K_{NLO,NNLO}$-factor has indeed a two-fold source of dependence: PDF set and energy scale (i.e. $M_{Z'}$). A uniform setup must be fixed when comparing
experimental limits on different models.

Since Eq. (II.8) gives an accurate description even at NNLO [4], and noting that QCD NNLO corrections are universal for up- and down-quarks, one can effectively apply the same $K_{NNLO}$-factor derived for SM-like $Z'$ to generic $Z'$ models without losing of generality. Owing to the remarkable $Z'$ mass dependence of the $K_{NNLO}$-factor, we first convolute the LO $Z'$ production cross section with the respective LO PDF’s and then we multiply it by $K_{NNLO}(M_{Z'})$.

For convenience and clarity, we provide in Tables III and IV shown in Appendix A the values of $K_{NNLO}$-factors and cross sections for the SM-like $Z'$-boson production process at the Tevatron and the LHC@7TeV: $p\bar{p}(p) \rightarrow Z' + X$. The first table contains the results obtained with MSTW08 PDF, the latter with CTEQ6.6 PDF. The quoted numbers correspond to the curves visualised in Figs. II2

In narrow width approximation, the two-fermion cross section is the product of the production cross section and the respective branching ratio. When considering the complete $Z'$-boson production and decay in the Drell-Yan channel, one has to keep in mind that QCD NNLO corrections also affect the $Z'$ branching ratio even for purely leptonic decays, $Br(Z' \rightarrow ℓ^+ ℓ^-)$, since the $Z'$ total decay width will be corrected at NNLO. This reflects into an higher order correction to the $c_u$ and $c_d$ coefficients, through $Br(Z' \rightarrow ℓ^+ ℓ^-)$ which explicitly enters the expression for $c_u$ and $c_d$ given in Eq. (II.13). The NNLO Drell-Yan cross section can be thus written as:

$$\sigma_{NNLO}^{ℓ+ℓ-} = \frac{π}{488} \left[ c_u^{NNLO} w_u(s, M_{Z'}^2)^{NNLO} + c_d^{NNLO} w_d(s, M_{Z'}^2)^{NNLO} \right]$$

$$= K_{NNLO}^{PDF} K_{NNLO}^{BR} \frac{π}{488} [c_u w_u(s, M_{Z'}^2) + c_d w_d(s, M_{Z'}^2)] = K_{NNLO}^{PDF} K_{NNLO}^{BR} \sigma_{NNLO}^{LO}$$ (II.17)

The leading NLO QCD correction to the total $Z'$ width is known to be $α_s/π$ [33, 34]. This gives an enhancement of the order of 2-3% to the $Z'$ width for $M_{Z'}$ in the range 500-2000 GeV. The $Br(Z' \rightarrow ℓ^+ ℓ^-)$ will thus decrease accordingly by $(2 - 3\%) \times Br(Z' \rightarrow hadrons)$. The net result corresponds to a $1 - 2\%$ depletion of the leptonic branching ratio within the SM-like $Z'$ model. An effect of the same order is expected for the other classes of $Z'$ models under consideration. In the current study, we neglect this effect and use the following formula for establishing limits on $Z'$ models:

$$\sigma_{NNLO}^{ℓ+ℓ-} \approx K_{NNLO}^{PDF} \sigma_{NNLO}^{LO}$$ (II.18)

### D. Finite width effects

So far we have have discussed the $Z'$ boson production using narrow width approximation. However, the experimental search for an extra $Z'$ boson and the descrimination of the SM backgrounds could strongly depend on the realistic $Z'$ width. Moreover the theoretical prediction of the $Z'$ production cross section also depends on its width as we discuss below.

We start this discussion with Fig. 3 where we present the di-lepton invariant mass distribution for the $Z'$ boson production within various models at the Tevatron (left panel) and the LHC@7TeV (right panel). We consider three representative models: the SM-like $Z'$ model (black line), the N-type $E_6$ model defined in Table I (red line), and the weakly coupled SM-like $Z'$ model where the $Z'$ boson gauge coupling to SM fermions is reduced by a factor 10 (blue line). From top to bottom, the last two distributions are normalized to the integral under the first one. We first consider the SM-like $Z'$ model distribution at the Tevatron. It is important to stress that the total cross section of $p\bar{p} \rightarrow Z' \rightarrow ℓ^+ ℓ^-$ process integrated over the entire $M_{ℓ^+ ℓ^-}$ range is actually almost as twice as large as the SM-like $Z'$ in the narrow width approximation. The main reason for this effect is the specific shape of the $M_{ℓ^+ ℓ^-}$ distribution in the region of small $M_{ℓ^+ ℓ^-}$ far away from $M_{Z'}$. This region is exhibited by a non-negligible tail due to the steeply rising PDF in the region of low $M_{ℓ^+ ℓ^-}$ even though the $Z'$ boson is extremely far off mass-shell in this region. The integral over this region can even double the cross section evaluated in the NWA in the case of $Z'$ production at the Tevatron.

This effect, which is related to the off-shellness of the extra gauge boson, varies according to the total $Z'$ width. In the $Z_N$ model, it brings an additional 20% contribution to the narrow width approximation cross section at the Tevatron. In the weakly coupled SM-like $Z'$ model, the far off-shellness effects are effectively negligible (below 1%).

We can see that in general experimental limits would and should strongly depend on the particular $Z'$ model predicting a specific $Z'$ width. On the other hand, if one requires a di-lepton mass window cut around the $Z'$ mass, one can establish a quasi model-independent experimental upper limit on $σ(p\bar{p} → Z' × Br(Z' → ℓ^+ ℓ^-))$ versus $M_{Z'}$ and apply this limit to constraint different classes of models.

In Fig. 4 we present the effect of a symmetric mass window cut around $M_{Z'}$ for the SM-like $Z'$ model and two other representative models (see Table I) at the Tevatron and the LHC@7TeV. We fix the $Z'$ mass to be $M_{Z'} = 1$ TeV.
We plot the relative difference between the full cross section for \( pp(\bar{p}) \to Z' \to \ell^+\ell^- \) evaluated taking into account the finite \( Z' \) width (\( \sigma \)) and the cross section computed in narrow width approximation (\( \sigma^{NWA} \)). The relative difference is presented as a function if the \( \Delta M/M \) symmetric mass window cut \( |M_{\ell^+\ell^-} - M_{Z'}| < \Delta M \) applied to the full cross section (\( \sigma \)). Three different representative models are considered for \( M_{Z'} = 1 \) TeV (see Table I).

FIG. 3: Di-lepton invariant mass distribution for the \( Z' \) boson production in various models at the Tevatron (left panel) and LHC@7TeV (right panel).

FIG. 4: Relative difference between the full cross section for \( pp(\bar{p}) \to Z' \to \ell^+\ell^- \) evaluated taking into account the finite \( Z' \) width (\( \sigma \)) and the cross section computed in narrow width approximation (\( \sigma^{NWA} \)). The relative difference is presented as a function if the \( \Delta M/M \) symmetric mass window cut \( |M_{\ell^+\ell^-} - M_{Z'}| < \Delta M \) applied to the full cross section (\( \sigma \)). Three different representative models are considered for \( M_{Z'} = 1 \) TeV (see Table I).
be an optimal mass window cut consistent with all models. The choice of the mass window cut to gain agreement with the narrow width approximation also depends on $M_{Z'}$. This dependence is defined by proton parton densities and is therefore model-independent. The net effect is again to make all the lines cross the abscissa at about the same value of $\Delta M/M_{Z'}$, where this point depends on $M_{Z'}$. Therefore for every given mass one can work out a quasi model-independent mass window cut where the full cross section matches the narrow width approximation. The additional advantage of this choice is that in the selected mass window around the $Z'$ mass the model-dependent interference effect between $Z'$ signal and SM background is highly suppressed.

The experimental limits would be quasi model-independent if one would apply this cut on the $M_{t+\bar{t}}$ around the $M_{Z'}$: it brings in agreement the cross section calculated in the narrow width approximation and in the finite width approximation as well as removes model-dependent shape of the $M_{t+\bar{t}}$ distributions in the region of low $M_{t+\bar{t}}$—especially for the case of large $Z'$ width effects as, for example, take place for SM-like $Z'$. Moreover, the cut on $M_{t+\bar{t}}$ around the $M_{Z'}$ plays an important role in reducing an effect of $Z'$ interference with $Z/\gamma$ down to a few% level, which again, allows to establish and use experimental limits in model-independent way.

For example, in case of SM-like $Z'$ production at the Tevatron, the relative interference, which is defined as $R_i = |\sigma(p\bar{p} \to Z'/Z/\gamma \to \ell^+\ell^-) - \sigma(p\bar{p} \to Z' \to \ell^+\ell^-) - \sigma(p\bar{p} \to Z/\gamma \to \ell^+\ell^-)|/\sigma(p\bar{p} \to Z' \to \ell^+\ell^-)$ is as large as about $-19$ (meaning $-1900\%$ of interference(!)) for $M_{t+\bar{t}} > 100$ GeV cut but it drops down to $-6\%$ for $|M_{t+\bar{t}} - M_{Z'}| < 0.15M_{Z'}$: cut which matches NWA and finite width cross sections. The effect of the mass window cuts is also quite large for the case of SM-like $Z'$ production at the LHC, where interference is about $-300\%$ for $M_{t+\bar{t}} > 100$ GeV cut and only about $-2\%$ for $|M_{t+\bar{t}} - M_{Z'}| < 0.15M_{Z'}$: cut.

We can see, that there is a strong motivation to use an invariant mass window cut for conducting a model-independent analysis. The size of this cut, if one aims to match the NWA and finite width cross sections, is collider dependent: it is about $15\%$ of $M_{Z'}$ for the Tevatron and about $40\%$ of $M_{Z'}$ for the LHC7+TeV.

In this paper we are using results of experimental analysis which are based on $M_{t+\bar{t}}$ mass window cut similar to what we are advocating. This would allow us to use precise NNLO model predictions and perform a respective model-independent interpretation of the experimental limits.

### III. Benchmark Models

In this section, we extend and classify the benchmark $Z'$ models present in the literature. We divide such classes into two main types: perturbative and strongly coupled gauge theories.

#### A. Perturbative gauge theories

1. $E_6$ Models

In these models one envisages that at the GUT scale the gauge group is $E_6$. The gauge group $E_6$ is broken at the GUT scale to $SO(10)$ and a $U(1)_\psi$ gauge group,

$$E_6 \rightarrow SO(10) \times U(1)_\psi.$$  \hspace{1cm} (III.1)

The $SO(10)$ is further broken at the GUT scale to $SU(5)$ and a $U(1)_\chi$ gauge group,

$$SO(10) \rightarrow SU(5) \times U(1)_\chi.$$  \hspace{1cm} (III.2)

Finally the $SU(5)$ is broken at the GUT scale to the Standard Model (SM) gauge group,

$$SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y.$$  \hspace{1cm} (III.3)

All these breakings may occur at roughly the GUT scale. The question which concerns us here is what happens to the two Abelian gauge groups $U(1)_\psi$ and $U(1)_\chi$ with corresponding generators $T_\psi$ and $T_\chi$. Do they both get broken also at the GUT scale, or may one or other of them survive down to the TeV scale? In general it is possible for some linear combination of the two to survive down to the TeV scale,

$$U(1)' = \cos \theta \ U(1)_\chi + \sin \theta \ U(1)_\psi,$$  \hspace{1cm} (III.4)

where $-\pi/2 < \theta \leq \pi/2$. More correctly the surviving $E_6$ generator $Q_{E_6}$ should be written as,

$$Q_{E_6} = \cos \theta \ T_\chi + \sin \theta \ T_\psi.$$  \hspace{1cm} (III.5)
Some popular examples of such $U(1)'$ are shown in Table III.

The resulting heavy $Z'$ couples as $g' Q_{E_R} Z'$. Note that in $E_6$ models it is reasonable to assume that the $Z'$ gauge coupling $g'$ is equal to the GUT normalized $U(1)_Y$ gauge coupling of the SM, $g_1(M_Z) = (e/c_W)\sqrt{5/3} \approx 0.462$ where $e = 0.3122(2)$ and $c_W = \sqrt{1 - s_W^2}$ where the $\overline{MS}$ value is $s_W^0 = 0.2312$. Thus we take $g' \approx 0.46$. GUT normalization also implies that the $T_\psi$ charges of the fermions in the $SO(10)$ 16 representation for the $\psi$ case are all equal to $1/\sqrt{24}$, while for the $\chi$ case the $T_\chi$ charges of the $SU(5)$ representations $(10, 5, 1)$ are $(-1/\sqrt{40}, 3/\sqrt{40}, -5/\sqrt{40})$. Recalling that $g'_{V,A} = e_L \pm e_R$, and $10 \to Q, u^c, e^c$ and $\overline{5} \to L, d^c$, and that $u^c, d^c, e^c$ have the opposite charges to $u_R, d_R, e_R$, this results in the values of the $g'_{V,A}$ charges for the $U(1)_\psi$ and $U(1)_\chi$ cases as shown in Table III. The general charges as a function of $\theta$ are then simply given as,

$$g_{V,A}^f(\theta) = \cos \theta \ g_{V,A}^f(\chi) + \sin \theta \ g_{V,A}^f(\psi),$$

where the numerical charges for the popular models quoted in the literature are listed in Table III.

2. Generalised Left-Right Symmetric Models (GLRs)

These models are motivated by the left-right (LR) extensions of the SM gauge group with the symmetry breaking,

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to SU(2)_L \times U(1)_Y$$

which, from the point of view of $Z'$ models essentially involves the symmetry breaking,

$$U(1)_R \times U(1)_{B-L} \to U(1)_Y$$

where $U(1)_R$ involves the generator $T_{3R}$ corresponding to the third component of $SU(2)_R$, while $U(1)_{B-L}$ involves the generator $T_{B-L} = (B - L)/2$. The hypercharge generator is then just given by $Y = T_{3R} + T_{B-L}$. Assuming a left-right symmetry, the gauge couplings of $SU(2)_{L,R}$ are then equal, $g_L = g_R$ and the resulting heavy $Z'_{LR}$ then couples as $g_1 Q_{LR} Z'$ where

$$Q_{LR} = \sqrt{\frac{3 \alpha T_{3R} - \frac{1}{\alpha} T_{B-L}}{5}}$$

with $\alpha = \sqrt{\cot^2 \theta_W - 1} \approx 1.53$ and $g_1 \approx 0.462$ as before.

The left-right symmetric models therefore motivate a $U(1)_{LR}$ which is a particular linear combination of $U(1)_R$ and $U(1)_{B-L}$ with a specific gauge coupling. From this perspective the special cases where the $Z'$ corresponds to a pure $U(1)_R$ or a pure $U(1)_{B-L}$ are not well motivated. Nevertheless these types of $Z'$ have been well studied in the literature and so it is useful to propose a generalization of the LR models which includes these special cases. To this end we propose a generalized left-right (GLR) symmetric model in which the $Z'$ corresponds to a general linear combination of the generators of $U(1)_R$ and $U(1)_{B-L},$

$$Q_{GLR} = \cos \phi \ T_{3R} + \sin \phi \ T_{B-L},$$

where $-\pi/2 < \phi \leq \pi/2$. The gauge coupling $g'$ is fixed so that for a particular value of $\phi$ the $Z'$ of the GLR may be identified with the $Z'$ of the LR symmetric model above. To be precise, we identify, for a particular value of $\phi$:

$$g_1 Q_{LR} = g' Q_{GLR}$$

which implies $\tan \phi = -1/\alpha^2$ which corresponds to $\phi = -0.128\pi$ for $\alpha \approx 1.53$ and we find $g' = 0.595$. Keeping $g' = 0.595$ fixed, we are then free to vary $\phi$ over its range where $\phi = -0.128\pi$ gives the LR model, but other values of $\phi$ define new models.

Clearly $\phi = 0$ gives a $U(1)_R$ model while $\phi = \pi/2$ gives a $U(1)_{B-L}$ model. In the GLR model the value of $\phi = \pi/4$ also defines a $Z'$ which couples to hypercharge $Y = T_{3R} + T_{B-L}$ (not to be confused with the sequential SM $Z'$ which couples like the $Z$). The couplings of the $Z'$ for the special cases of the GLR models are give in Table III. The general charges as a function of $\phi$ are then simply given as,

$$g_{V,A}^f(\phi) = \cos \phi \ g_{V,A}^f(R) + \sin \phi \ g_{V,A}^f(B - L),$$

where the numerical charges for particular models are shown in Table III.
During the early stage of the LHC. Sizeable couplings to SM matter. Hence, they could be tested in the favoured Drell-Yan channel at the Tevatron and Minimal Walking Technicolour [35, 36] and the four site Higgsless model [13, 37, 38] predict extra new resonances must be fermiophobic in order to evade EWPT constraints. However, in recent years, new models have been proposed that are able to satisfy the EWPT bounds without imposing such a strong condition. Both the

| Parameter | \( g_\psi \) | \( g_\phi^\prime \) | \( g_\phi^\prime \) | \( g_\phi^\prime \) | \( g_\phi^\prime \) | \( g_\phi^\prime \) | \( g_\phi^\prime \) |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( E_6' \) (g' = 0.462) | \( \theta \) | \( \alpha \) | \( \beta \) | \( \gamma \) | \( \delta \) | \( \epsilon \) | \( \zeta \) |
| \( U(1)_X \) | 0 | 0 | -0.316 | -0.632 | 0.316 | 0.316 | -0.474 | 0.474 |
| \( U(1)_Y \) | 0.5 | 0 | -0.408 | -0.408 | 0 | -0.204 | 0.204 | 0.204 |
| \( U(1)_T \) | -0.29 | 0 | -0.316 | -0.387 | -0.129 | 0.387 | -0.129 | 0.129 |
| \( U(1)_S \) | 0.129 | 0 | -0.129 | -0.581 | 0.452 | 0.581 | 0.452 | 0.516 |
| \( U(1)_I \) | 0.21 | 0 | 0 | 0.5 | -0.5 | -0.5 | -0.5 | -0.5 |
| \( U(1)_N \) | 0.42 | 0 | 0.316 | -0.158 | 0.474 | 0.158 | 0.474 | 0.316 |
| GLR (g' = 0.595) | \( \phi \) | \( \omega \) | \( \eta \) | \( \zeta \) | \( \xi \) | \( \chi \) | \( \psi \) |
| \( U(1)_R \) | 0 | 0.5 | -0.5 | -0.5 | 0.5 | -0.5 | 0 | 0 |
| \( U(1)_{B-L} \) | 0.5 | 0.333 | 0 | 0.333 | 0 | -1 | 0 | -0.5 |
| \( U(1)_L \) | -0.128 | 0.392 | -0.46 | -0.591 | 0.46 | 0.46 | 0.196 | 0.196 |
| \( U(1)_Y \) | 0.25 | 0.833 | -0.5 | -0.167 | 0.5 | -1 | 0.5 | -0.5 |
| GSM (g' = 0.760) | \( \alpha \) | \( \beta \) | \( \gamma \) | \( \delta \) | \( \epsilon \) | \( \zeta \) | \( \xi \) |
| \( U(1)_{SM} \) | -0.072 | 0.193 | 0.5 | -0.347 | -0.5 | -0.0387 | -0.5 | 0.5 |
| \( U(1)_{TL} \) | 0 | 0.5 | 0.5 | -0.5 | -0.5 | -0.5 | 0.5 | 0.5 |
| \( U(1)_Q \) | 0.5 | 1.333 | 0 | -0.666 | 0 | -2.0 | 0 | 0 |

TABLE I: Benchmark model parameters and couplings. The angles \( \theta, \phi, \alpha \) are defined in the text.

3. Generalised Sequential Models (GSMs)

No study is complete without including the sequential standard model (SSM) \( Z'_{SSM} \) which is defined to have identical couplings as for the usual \( Z \), namely given by \( \frac{g_\phi}{c_W} Q Z_{SSM} \) and \( Q_Z = T_Z - s_W^2 Q \) where \( s_W^2 = 0.2312 \) and \( \alpha_2(M_Z) = g_Z^2/(4\pi) \approx 0.0338 \) imply that \( \frac{g_\phi}{c_W} \approx 0.74 \). Similar to the GLR models, it is useful to define a generalised version of the SSM called GSM where the heavy gauge boson \( Z'_{GSM} \) then couples as \( g' Q_{GSM} Z'_{GSM} \) where \( Q_{GSM} \) corresponds to a general linear combination of the generators of \( U(1)_{TL} \) and \( U(1)_Q \),

\[
Q_{GSM} = \cos \alpha T_{3L} + \sin \alpha Q,
\]

and where \(-\pi/2 < \alpha \leq \pi/2\). The gauge coupling \( g' \) is fixed so that for a particular value of \( \alpha \) the \( Z'_{GSM} \) of the GSM may be identified with the \( Z'_{SSM} \) of the SSM above. To be precise, we identify, for a particular value of \( \alpha \):

\[
\frac{g_\phi}{c_W} Q_Z = g' Q_{GSM}.
\]

This implies that the GSM reduces to the SSM case for \( g' = \frac{g_\phi}{c_W} \sqrt{1 + s_W^2} \approx 0.76 \) and tan \( \alpha = -0.23 \) which corresponds to \( \alpha = -0.072 \pi \). Keeping \( g' = 0.76 \) fixed, they are then free to vary \( \alpha \) over its range where \( \phi = -0.072 \pi \) gives the usual SSM, but other values of \( \alpha \) define new models. Clearly \( \alpha = 0 \) gives a \( U(1)_{TL} \) model while \( \alpha = \pi/2 \) gives a \( U(1)_Q \) model.

The couplings of the \( Z' \) for the special cases of the GLR models are give in Table I. The general charges as a function of \( \alpha \) are then simply given as,

\[
g_{V,A}(\alpha) = \cos \alpha g_{V,A}(L) + \sin \alpha g_{V,A}(Q),
\]

where the numerical charges for particular models are shown Table II.

B. Strongly coupled gauge theories

Strongly interacting gauge theories provide an alternative mechanism for the electroweak symmetry breaking (EWSB). The EWSB is not driven by a light Higgs boson anymore, but it happens in a dynamical way. Such theories date back to decades. However, even if they predict the existence of new gauge bosons in order to delay at high energy the perturbative unitarity violation in vector boson scattering amplitudes, they are not considered when performing searches of \( Z' \) bosons in the dilepton Drell-Yan channel. The reason is that historically the predicted new resonances must be fermiophobic in order to evade EWPT constraints. However, in recent years, new models have been proposed that are able to satisfy the EWPT bounds without imposing such a strong condition. Both the Minimal Walking Technicolour [33, 39] and the four site Higgsless model [13, 40, 38] predict extra \( Z' \) bosons with sizeable couplings to SM matter. Hence, they could be tested in the favoured Drell-Yan channel at the Tevatron and during the early stage of the LHC.
Higgsless models emerge naturally from local gauge theories in five dimensions. Their major outcome is delaying the unitarity violation of vector-boson scattering (VBS) amplitudes to higher energies, compared to the SM without a light Higgs, by the exchange of Kaluza-Klein excitations [39]. Their common drawback is to reconcile unitarity with the ElectroWeak Precision Test (EWPT) bounds. Within this framework, and in the attempt to solve this dichotomy, many models have been proposed [40–48].

In this paper, we consider the four site Higgsless model [49] as representative of strongly coupled theories. This model belongs to the class of the so called deconstructed theories [50–58] which come out from the discretization of the fifth dimension on a lattice, and are described by chiral lagrangians with a number of gauge-group replicas equal to the number of lattice sites. The simplest version of this class of models is related to the old BESS model [53, 60], a lattice with only three sites and $SU(2)_L \times SU(2)_1 \times SU(2)_2 \times U(1)_Y$ gauge symmetry (for it, sometimes called three-site Higgsless model). In order to reconcile unitarity and EWPT-bounds, this minimal version predicts indeed the new triplet of vector bosons to be almost fermiophobic. Hence, only di-boson production, vector boson fusion and triple gauge boson production processes can be used to test these models. All these channels require high energy and luminosity and will be proper for a future upgrade of the LHC [61–63].

In the strongly coupled scenario, the four site Higgsless model represents a novelty in this respect [13, 37, 38]. Its phenomenological consequences are quite similar to those of the Minimal Walking Technicolour [35, 36]. The four site model, based on the

$$SU(2)_L \times SU(2)_1 \times SU(2)_2 \times U(1)_Y$$

(III.16)
gauge symmetry, predicts two neutral and four charged extra gauge bosons, $Z_{1,2}$ and $W_{1,2}^{\pm}$, and is capable to satisfy EWPT constraints without necessarily having fermiophobic resonances. Within this framework, the more promising Drell-Yan processes become particularly relevant for the extra gauge boson search at the LHC.

The four site Higgsless model is described by four free parameters: $g_{1V}^e, g_{2V}^e, M_{Z1}, M_{Z2}$ that is the two vector couplings between $Z_{1,2}$-bosons and SM electrons and the two $Z_{1,2}$ masses (charged and neutral gauge bosons are degenerate).
FIG. 6: Parameter space in the plane \((g^e_{2V}, M_{Z2})\) where \(g^e_{2V}\) is the \(Z_2\)-boson vector coupling to SM electrons, and \(M_{Z2}\) is the \(Z_2\)-boson mass. The red solid lines restrict the area allowed by EWPT and unitarity. One sample case has been considered: \(z=0.8\).

In terms of the mass eigenstates, the Lagrangian describing the neutral current interaction is given by\(^3\)

\[
\mathcal{L}_{NC} = \frac{1}{2} \bar{f} \gamma^{\mu} \left[ (g^f_{1V} - g^f_{1A} \gamma_5) Z^1_1 + (g^f_{2V} - g^f_{2A} \gamma_5) Z^1_2 \right] f
\]

where \(g^f_{1,2V}, g^f_{1,2A}\) are the vector and axial couplings of the extra \(Z_{1,2}\) gauge bosons to ordinary matter. In the above formula, we have included the \(g'\) coupling in the definition of \(g^f_{1,2V}\) and \(g^f_{1,2A}\).

The energy range, where the perturbative regime is still valid is plotted in the left panel of Fig.5 for different values of the ratio \(z = M_{Z1}/M_{Z2}\). Owing to the exchange of the extra gauge bosons, the perturbative unitarity violation can be delayed up to an energy scale of about \(\sqrt{s} \simeq 3\) TeV. Hence, the mass spectrum of the new particles is constrained to be within a few TeV.

In the past, the only way to combine the need of relatively low mass extra gauge bosons with EWPT was to impose the new particles to be fermiophobic. In the four-site Higgsless model, this strong assumption is not necessary anymore. In the right panel of Fig.5 we plot the bounds on the vector couplings of the \(Z_{1,2}\)-bosons to SM electrons coming from the EWPT expressed in terms of the \(\epsilon_{1,3}\) parameters [64] (\(\epsilon_2\) is ineffective due to SU(2)-custodial symmetry). The outcome is that \(\epsilon_3\) constrains the relation between the two couplings, while \(\epsilon_1\) limits their magnitude.

allows to reconcile unitarity and EWPT bounds, leaving a calculable and not fine-tuned parameter space, where the new gauge bosons are not fermiophobic.

Using the linear relation shown in the right panel of Fig.5 we can express the \(Z_1\)-boson vector coupling to SM electrons as a function of the \(Z_2\)-boson vector coupling to SM electrons (by computing back the bare-parameters of the Lagrangian, all other \(Z_{1,2}\)-boson-fermion couplings can be simultaneously derived). In this way, the number of independent free parameters describing the four-site Higgsless model gets reduced to three. We choose the following physical observables: \(M_{Z2}, g^e_{2V}\), and \(z\). In terms of these new variables, the parameter space allowed by EWPT and perturbative unitarity is shown in Fig.6 for one representative \(z\)-value: \(z=0.8\). The outcome is that one can reconcile unitarity and EWPT bounds, leaving a calculable and not fine-tuned parameter space, where the new gauge bosons are not fermiophobic.

Compared to the popular extra \(U'(1)\) theories summarized in Table I, the four-site Higgsless model does not predict fixed values for the couplings of the extra gauge bosons to ordinary matter. One has indeed a parameter space

\[^3\text{For details see }[13, 37].\]
on the $Z$ of $M$ every contour corresponding to a well defined properties and limits. We use this framework in the next sections, when discussing four-site model

second remark concerns the experimental sensitivity to the $Z$ excluded at the Tevatron up to masses bounded but still large enough to accommodate rather sizeable $Z_2$-boson couplings to SM fermions. They can range from zero to the order of SM couplings. We use this framework in the next sections, when discussing four-site model properties and limits.

IV. APPLICATION OF MODEL INDEPENDENT APPROACH TO THE BENCHMARK MODELS

A. Current limits from Tevatron

As discussed above, collider limits on the $Z'$-boson mass can be presented via contours in the $c_u - c_d$ plane, with every contour corresponding to a well defined $M_{Z'}$. Simultaneously, in the same plane, one can also show the values of $c_{d,u}$ couplings allowed by a specific $Z'$ model. As a result, one can immediately visualize and derive a mass bound on the $Z'$-boson predicted by that particular model.

We start presenting our results at the Tevatron which is running at $\sqrt{s} = 1.96$ TeV. We use the most recent 95% C.L. upper bound on $\sigma(pp \rightarrow Z' \rightarrow \ell^+\ell^-)$ reported at the ICHEP 2010 conference by the D0 collaboration for the di-electron channel where the $\Delta M/M_{Z'} \sim 15\%$ cut was used in the analysis. This limit is shown in Fig.7(left panel), together with its ‘translation’ into the $c_u - c_d$ plane for different $M_{Z'}$ masses as shown in Fig.8(right panel).

In the following, we use Fig.7(right panel) to interpret current limits from Tevatron, and to derive mass bounds on the $Z'$ boson predicted in the classes of models described in the previous section. The results are shown in Fig.8. The top-left panel displays the contour representing $E_6$ Models in the $c_u - c_d$ plane, the top-right panel shows the Generalised Left-Right Models (GLR), the bottom-left panel contains the Generalised Sequential Models (GSM), and finally the bottom-right one gives the Four Site Higgsless Model (4S). In the first three mentioned panels, the colour code corresponds to four equidistant intervals for the mixing angle in the $[-\pi/2, \pi/2]$ range for $E_6$, GLR, GSM models, represented by continuous and closed contours. The black dots on these contours denote the popular benchmark models quoted in Table I. In the bottom-right panel, which shows the parameter space of the Four Site Model, the colour indicates different mass values for $M_{Z_1}$ and $M_{Z_2}$. The line style distinguishes the $Z_1$ mass (solid line) from the $Z_2$ mass (dashed line). For the $Z_1$ boson, the following mass values have been chosen: $M_{Z_1} = 480$ (red), $800$ (green) and $1600$ (blue) GeV. For the chosen sample of free parameters, $z = 0.8$, the corresponding values for the $Z_2$ bosons are: $M_{Z_2} = 600, 1000$ and $2000$ GeV shown with the same colour coding.

Several comments are in order. The first remarkable fact is that there is almost no overlap between contours for $E_6$, GLR, GSM models. This means that, if a $Z'$ boson will be discovered and its cross section will be measured with a reasonable accuracy, these classes of $Z'$ models can be well distinguished using just this basic information. The second remark concerns the experimental sensitivity to the $Z'$ production within different models. Comparing the four plots, it is clear that the highest sensitivity is to the GSM class of models. In particular, the $Q$-model can be excluded at the Tevatron up to masses $M_{Z'} \geq 1260$ GeV. Among the GLR models, which are second in terms of the

| $U(1)'$ | $\text{Br}(e^+ e^-)$ | $c_u$ | $c_d$ | $c_{u/d}$ | $M_{Z'}(\text{GeV})$ | $M_{Z''}(\text{GeV})$ | $\theta_{ZZ'}$ |
|--------|-------------------|-------|------|----------|-------------------|-------------------|----------|
| $U(1)_X$ | 0.0606 | $6.46 \times 10^{-4}$ | $3.23 \times 10^{-3}$ | 0.2 | 0.0117 | 915 | $1141^{+}_{-3}$ | $1.6 \times 10^{-3}$ |
| $U(1)_\Psi$ | 0.0444 | $7.90 \times 10^{-4}$ | $7.90 \times 10^{-4}$ | 1 | 0.0053 | 915 | $481^{e}_{c}$ | $1.8 \times 10^{-3}$ |
| $U(1)_\eta$ | 0.0371 | $1.05 \times 10^{-3}$ | $6.59 \times 10^{-4}$ | 1.6 | 0.00636 | 940 | $434^{e}_{c}$ | $4.7 \times 10^{-3}$ |
| $U(1)_S$ | 0.0656 | $1.18 \times 10^{-4}$ | $3.79 \times 10^{-3}$ | 0.31 | 0.0117 | 847 | $1257^{e}_{c}$ | $1.3 \times 10^{-3}$ |
| $U(1)_T$ | 0.0667 | 0 | $3.55 \times 10^{-3}$ | 0 | 0.0106 | 795 | $1204^{e}_{c}$ | $1.2 \times 10^{-3}$ |
| $U(1)_N$ | 0.0555 | $5.94 \times 10^{-4}$ | $1.48 \times 10^{-3}$ | 0.40 | 0.00635 | 892 | $623^{e}_{c}$ | $1.5 \times 10^{-3}$ |

TABLE II: Model predictions and current constraints. The direct limits above on the $Z'$ mass, $M_{Z'}$, are the result of the analysis performed in this paper while the indirect limits, $M_{Z''}$, come from either electroweak (e) fits or contact (c) interactions at LEP2.
experimental sensitivity to a $Z'$ boson, the Y-model can be already excluded up to $M_{Z'} \geq 1125$ GeV with the current Tevatron data. Interestingly, the lowest experimental sensitivity is to $E_6$ models, that is one of the most popular class of $Z'$ models. Within this class, the strongest limit can be derived for $\theta \in [-\pi/4, 0]$ providing the mass bound $M_{Z'} \geq 955$, as one can read from the red-coloured part of the $E_6$ contour.

The 4S class of $Z'$ models must be considered separately. First of all, it predicts two $Z'$ bosons with two different masses. Secondly, the parameter space of the 4S model is described by more than just one parameter, so the model would be represented by an area in the $c_u - c_d$ plane rather than by a contour. In order to interpret Fig. 8 (bottom-right panel) and following analogous figures correctly, a clarification is needed. In the 4S model, the two extra gauge bosons can decay into both SM fermion pairs and boson pairs. While the contribution to the total width coming from the decay into fermion pairs is linear in the extra gauge boson mass, the contribution from the diboson decay grows with the third power of the extra gauge boson mass (see Eq. 2). As a consequence, and oppositely to the other perturbative gauge models, the $Z_{1,2}$ boson branching ratio into lepton pairs acquires a mass dependence (see Sec. II B for details). This reflects into a mass dependence of the $c_u$ and $c_d$ parameters which parametrize the 4S model. So, Fig. 8 (bottom-right) should be interpreted as the full parameter space of the four site model projected into the $c_u - c_d$ plane. To simplify the visualisation of this area, we have varied the vector coupling between the $Z_2$ boson and SM electrons, $g_{2Y}$, within the allowed region of Fig. 7 for the sample scenarios: $z=0.8$ and $M_{Z_2} = 600$, 1000 and 2000 GeV. This setup should give a full representation of the parameter space, $M_{Z_2} = 600$ GeV being the minimum allowed mass and $M_{Z_2} = 2000$ GeV being close to the maximum value of the mass permitted by unitarity. The parameter space for these values of $M_{Z_2}$ and the respective $M_{Z_1} = 0.8 \times M_{Z_2}$ is presented in Fig. 8 (bottom-right) by coloured lines (see caption). Whenever the coloured line describing a given $M_{Z1,2}$ value for the 4S model crosses the black contour corresponding to the same mass value, that crossing point would give the experimental sensitivity to a $Z_{1,2}$ boson with that mass. The portion of the coloured line above this crossing point would represent the excluded region for a $Z_{1,2}$ boson branching ratio into lepton pairs. When the colour line comes from the cross point between the dashed green line and the black contour labelled by 1000 GeV. Since, the two extra gauge bosons would be simultaneously produced, from the discussed green lines one should deduce that the most restrictive bound on the $c_{u,d}$ couplings comes from the $Z_1$ boson.
In the 4S model, the coupling between the mass. Since the linearly with the physics discovery, the bound on the mass can be translated into limits on the \( c_u, c_d \) coefficients. From there, one can then trace back exactly the lagrangian parameters of the 4S model which are in turn excluded. In case of new physics discovery, the \( c_{u,d} \) approach allows one to uniquely determine the \( c_{u,d} \) values corresponding to that observed mass. Since the \( c_{u,d} \) coefficients are strictly related to the new gauge boson couplings, this in turn enables one to extract informations on their size. In the 4S model, the coupling between the \( Z_2 \) boson and SM electrons, \( g_{2V}^\alpha \), grows linearly with \( c_u \) and \( c_d \) as shown in Fig. 8. Any mass measurement therefore translates into a coupling determination.
Moreover, $g_{2V}^e$ is one of the three free parameters of the model. The measurement of the mass of the new gauge bosons would therefore allow one to derive direct informations on the bare lagrangian parameters.

![Image](image1.png)

**FIG. 9:** $g_{2V}^e$ fermion-boson coupling versus $c_u$(left) or $c_d$(right) in four-site model

![Image](image2.png)

**FIG. 10:** Left: CMS limit on the $Z'$ boson production cross section in the di-electron channel, normalized to the SM $Z$ boson cross section, as a function of the $Z'$ mass. The limit is projected at 500pb$^{-1}$ [69]. Right: Limits in the $c_u - c_d$ plane, based on the projected LHC 500pb$^{-1}$ limit shown in the right panel. In the $c_u - c_d$ representation, the limits appear as contour lines corresponding to different $M_{Z'}$ values.

**B. Expected LHC potential at 7 TeV to probe $Z'$ models**

We now explore the LHC@7TeV potential to test the classes of $Z'$ models under consideration. We use the projected limits from LHC. In particular, we rely on the limits given by the CMS Exotica group for 500pb$^{-1}$ of integrated luminosity which hopefully will be available in about one year from now. This limit is shown in Fig.[10] (left panel), which is taken from the public web page of the CMS Exotica group [69]. The projected 500pb$^{-1}$ limit from CMS is given as a ratio $\sigma_{Z'}/\sigma_Z$, where $\sigma_Z$ is the Z-boson production cross section in the 60 < $m_{ee} <$ 120 GeV window and we have converted this limit into the limit on the NNLO production cross section for the $Z'$ boson shown in terms of $c_{u,d}$ coefficients in $c_u - c_d$ plane for different $Z'$ masses given in the right panel of Fig.[10]. This representation is
the analog of what done before at the Tevatron. Comparing Fig. and Fig., one can observe the strong gain in sensitivity one gets at the LHC@7TeV with 500pb$^{-1}$, compared to the Tevatron with 5.4fb$^{-1}$, at fixed $c_{u,d}$ value.

Now we can estimate the LHC@7TeV potential for deriving bounds on $Z'$ models at 500pb$^{-1}$. The results are shown in Fig. where the legend scheme is the same as in Fig. From Fig. and Fig., one can see that for the models with small-intermediate values of $Z'$ boson couplings to SM fermions (that is $E_6$ models, GLR-models partly, and some GSM models), the LHC@7TeV can extend the limits on $M_{Z'}$ by about 500 GeV when compared to the Tevatron. For example, the limit on the SM-like $Z'$ boson could be extended from 1020 GeV to about 1520 GeV. On the other hand, the limits for larger $c_{u,d}$ coefficients and respectively larger masses could be extended in the near future up to a 2 TeV scale, which is unreachable at the Tevatron. For the Q-model, belonging to the GSM class, the mass bound could be improved from 1210 GeV (current Tevatron) to 2250 GeV. Regarding the 4S model, one can see that the scenario characterized by $M_{Z_1} = 800$ GeV and $M_{Z_2} = 1000$ GeV could be totally excluded in the $c_u - c_d$ plane shown in Fig. The solid green line, representing the $Z_1$ boson parameter space, lies in fact beyond the 800 GeV black contour line in the displayed plane. No improvement, compared to the current Tevatron, would instead be possible for the scenario: $M_{Z_1} = 1600$ GeV and $M_{Z_2} = 2000$. The LHC@7TeV and 500 pb$^{-1}$ will not have sensitivity enough, as one can deduce from observing that the blue lines never cross the black contour lines labelled by their respective mass values. Exploring this region of the 4S parameter space would require higher integrated luminosity.
and preferably higher collider energy.

\[ \begin{align*}
\alpha &: (\frac{-\pi}{2}, -\frac{\pi}{4}) \\
\phi &: (0, \frac{\pi}{2})
\end{align*} \]

We also use Fig. 10 to estimate the LHC@7TeV discovery limits for 500pb\(^{-1}\). In this analysis, we assume that the significance grows as \( \sqrt{L} \), where \( L \) is the total integrated luminosity and that the signal over background ratio is constant for the same selection cuts. The last assumption is motivated by the fact that for a chosen invariant di-lepton mass window the \( q\bar{q} \) parton densities are very similar for signal and background processes and defined mainly by the \( \sqrt{s} \) value.

The LHC discovery potential for various \( Z' \) models is shown in Fig. 12 in the \( c_u - c_d \) plane. The legend scheme is the same as in Fig. 8. The upper(dashed) and lower(dot-dashed) contours correspond to the uncertainty in Fig. 10(left) reflected in the width of the band. One can see that discovery limits are typically 150-200 GeV lower than exclusion ones.
V. IMPACT OF Z' WIDTH ON SEARCH STRATEGIES

Invariant mass distributions may be examined in a number of ways for evidence of resonant structures. The sensitivity of any particular approach has a dependence on the intrinsic width of any possible resonance. The simplest approach is to bin the invariant mass distribution and determine the compatibility of the number of events in any bin with the Standard Model prediction. A p-value may be used to quantify this compatibility. In this approach the width of the bins for optimal sensitivity depends on the intrinsic width of possible resonances and the detector resolution. In the case where the width of the resonance is much smaller than the detector resolution this parameter defines the optimal bin size. For intrinsic widths comparable to the detector resolution then the optimal width depends on both of these parameters.

An alternative is to use a parameterization of the expected distribution in the two alternative hypotheses of, a distribution resulting only from Standard Model Physics and one resulting from the addition of a new physics process. A comparison of some measure of the quality of the fit in both cases allows a determination of the probability of the presence of New Physics. In this case the functional form of the resonance structure is typically taken to be some variant on a convolution of a Gaussian and a Breit-Wigner. For a resonance where the width of the Breit-Wigner is small in comparison to the width of the Gaussian the sensitivity of the search is insensitive to the width of the Breit-Wigner. Such circumstances result in the greatest possible experimental sensitivity. For resonances with large widths compared to the experimental resolution then the Breit-Wigner width must be included as a further parameter in any fits. In cases where the interference effect is large this must be included in the functional form used to fit to the invariant mass distribution. We show here that in the mass regions which will provide the greatest sensitivity for low integrated luminosities at the LHC the interference effect is negligible and may be ignored without compromising the search sensitivity.

The above descriptions of possible methods of searching for a resonance in an invariant mass distribution illustrate that a knowledge of the width of the resonances being searched for has an impact on the search procedure used. In order to obtain the best sensitivity, it is thus important to have a knowledge of the magnitude of the widths from New Physics models. The results of such searches depend on the assumptions made in the analysis and can’t be easily interpreted in other circumstances.

Besides that, in the case of a discovery the Z' boson mass and decay width will be determined from fits to the reconstructed invariant mass distribution of di-lepton candidate events. In this section, we thus focus on the prediction and the possible measurement of the total decay width, comparing the various classes of models under consideration.

All extra U(1) theories summarized in Table II make the assumption, for sake of simplicity, that the Z'-boson decays purely into SM fermions. Under this approximation, the total decay width is given by Eq.[11] and its value never exceeds a few percent of the corresponding mass \( \Gamma_{Z'} / M_{Z'} \leq 3\% \), as shown in Table II. This property has a direct implication on the possibility to measure the Z' decay width at the LHC, being correlated to the experimental di-lepton mass resolution. If indeed \( \Gamma_{Z'} > R \), being \( R \) the mass resolution, one can have direct access to the decay width of the observed spin-1 particle. During the early stage of the 7 TeV LHC, the expected di-electron mass resolution is about \( R_{LHC} \approx 2\% M \). As a consequence, within the majority of models summarized in Table II the total Z'-boson width is hardly measurable. This is visualized in Fig. 13 where the ratio between Z' width and mass is plotted as a function of the mixing angle parametrizing the three classes of models listed in Table II. E6, GLR and GSM. All E6 inspired models predict a quite narrow Z' boson. For some benchmark model within the generalized Left-Right class (GLR), the ratio becomes slightly bigger than the early LHC resolution. The scenario changes when we consider generalized sequential models (GSM). Here, the width over mass ratio is well above the early 2% resolution of the LHC. Another example of measurable decay width is given by the four-site Higgsless model. Here, in fact, in most part of the parameter space \( \Gamma_{Z_{1,2}} / M_{Z_{1,2}} \geq 2\% \). This property is shown in Fig. 14 for different values of the free \( z \) parameter. In this case, the distinctive behaviour is due to the fact that the \( Z_{1,2} \)-bosons predicted by the four-site model decay preferably into the diboson channel: \( Z_1 \rightarrow WW \) and \( Z_2 \rightarrow W_1 W_2 \). Thus, their width grows with the third power of the corresponding mass, as shown in Eqs.[11][12][13][14] making it wider compared to the other models. This feature is common to all Higgsless and Technicolor models.

This discussion is appropriate when considering the Z' boson production in the di-electron DY channel. For di-muon final states, the analysis would change drastically. The estimated di-muon mass resolution during the early stage of the 7 TeV LHC is in fact around \( R \approx 8\% \). Hence, only in a very restricted range of the GSM mixing angle the Z' width could be measurable. An exception is given by the four-site Higgsless model, where the Z2 boson width could be determined in a large portion of the parameter space.

The discussed differences between classes of models should be taken into account for improving search strategies and possibly measurements.
\[\chi \psi \eta S I N \theta (\text{Rad}) \]

\[\Gamma / M(\%)\]

\[\theta (\text{Rad})\]

\[\phi (\text{Rad})\]

\[\alpha (\text{Rad})\]

\[\Gamma_1 / M(Z_1) (\%)\]

\[\Gamma_2 / M(Z_2) (\%)\]

\[M_1 (\text{GeV})\]

\[M_2 (\text{GeV})\]

FIG. 13: \(Z'\) boson width over its mass as a function of the mixing angle parametrizing the \(E_6, \text{GLR}\) and \(\text{GSM}\) class of models given in Table [I]. The colour code corresponds to four equidistant intervals in the \([-\pi/2, \pi/2]\) region. The black dots on the contours correspond to the benchmark models listed in Table [I].

FIG. 14: Left: \(Z_1\)-boson width over its mass as a function of \(M_{Z_1}\) for different values of the free \(z\)-parameter. Right: same for the \(Z_2\)-boson. We assume the maximal \(Z_2\)-boson coupling to SM fermions.

VI. SUMMARY AND CONCLUSIONS

In this paper we have discussed the prospects for setting limits on or discovering spin-1 \(Z'\) bosons using early LHC data at 7 TeV. In order to facilitate the connection between experimental data and theoretical models, we have advocated the narrow width approximation, in which the leptonic Drell-Yan \(Z'\) boson production cross-section only depends on the \(Z'\) boson mass together with two parameters \(c_u\) and \(c_d\). These variables provide a convenient way of expressing the experimental limits or discovery information about the mass and cross-section which enables a direct comparison to be made with the predictions arising from theoretical models. The experimental limits on the \(Z'\) boson cross-section may be expressed as contours in the \(c_u - c_d\) plane, with a unique contour for each value of \(Z'\) boson mass. If a discovery is made then the measurement of the mass and cross-section corresponds to some unique contour, or in practice a unique band including the error. Such contours may be compared to the theoretical prediction of \(c_u\) and \(c_d\) arising from particular models, enabling limits to be set on models or a discovery to discriminate between particular models. However the application of this strategy requires the experimental cross-sections to be properly defined and the theoretical cross-sections to be accurately calculated as we now discuss.

On the experimental side, we have seen that the use of the narrow width approximation requires an appropriate di-lepton invariant mass window cut around the mass of the \(Z'\) boson. The effect of the cuts is rather subtle since it depends on both the width of the \(Z'\) boson and the energy of the collider, with higher widths and lower collider energies leading to more prominent signal tails at low invariant masses. Fortunately we have seen that at LHC energies the suitable experimental cut is rather insensitive, especially for models with lower \(Z'\) boson widths, and furthermore...
may be optimised at a unique value suitable for all models, although there is some unavoidable dependence on the $Z'$ mass. One important conclusion is that whatever cut is chosen should involve invariant masses well above 100 GeV otherwise interference effects will be dominant. In summary, we have demonstrated that this cut plays a crucial role: it diminishes a possibly huge model-dependent interference effect, removes model-dependent shape of the $M_{\ell^+\ell^-}$ distributions in the region of low $M_{\ell^+\ell^-}$ especially for the case of large $Z'$ width and brings into agreement NWA and FWA cross sections. On the theoretical side, we have evaluated cross-sections at NNLO using updated ZWPROD package. One should stress that $K_{NNLO}$ factors are depend on the $Z'$ mass and we have tabulated them for convenience for both the Tevatron and LHC. Moreover the $K_{NNLO}$ are very PDF dependent and one should specify which PDF is being used to apply a respective $K_{NNLO}$.

We have applied the approach above to two quite different types of $Z'$ models: perturbative gauge models and strongly coupled models. Among the perturbative gauge models we have studied three classes: $E_6$ models, left-right symmetric models and sequential standard models. Each class of model is defined in terms of a continuous mixing angle variable. This enabled infinite classes of benchmark models to be defined, rather than just a finite number of models, where for each class of model, the respective angles serve to parametrize different orbits in the $c_u$-$c_d$ plane. These orbits turn to be non-overlapping for these three classes of models. A limitation of this approach is that it ignores the effect of the SUSY and exotics (and right-handed neutrinos) on the width $\Gamma_{Z'}$. Assuming only SM particles in the final state we have calculated the widths of the benchmark classes of models and seen that the perturbative models generally involve relatively narrow widths (which however can be increased if SUSY and exotics are included in the decays), while the strongly coupled models involve multiple $Z'$ bosons with rather broad widths. We have also commented on the significance of the width on search strategies which if measured would a complementary for a discrimination of the underlying $Z'$ model as well as it would allow to test non-SM $Z'$ decays menationed above. Another limitation of our approach is that it ignores the effects of $Z-Z'$ mixing which is quite model dependent. However such effects must be small due to the constraints from electroweak precision measurements, so such effects will not have a major effect on direct collider searches considered here, although of course they will affect the precise vector and axial couplings (see for example [6–8] where the $U(1)_N$ vector and axial vector couplings are calculated including the mixing effects). As regards the strongly coupled models, we only considered the four-site Higgsless model which contains just two excitations of the $Z$ (and $W$) bosons. However it is representative of models of walking technicolour models and the KK excitations of the $Z$(and $W$) which is considered for the first time in the $c_u$-$c_d$ plane. It is clearly seen that the associated $Z'$ bosons may easily be distinguished from those of the perturbative gauge models.

In conclusion, our results support the use of the narrow width approximation in which the leptonic Drell-Yan $Z'$ boson production cross-section only depends on the $Z'$ boson mass together with two parameters $c_u$ and $c_d$. However, as discussed in this paper, care must be taken concerning the experimental cuts and the theoretical $K_{NNLO}$ factors tabulated here must be included correctly. Providing the experimental cross-section is appropriately defined, according to the recipe we provide in Fig.4 and the theoretical cross-sections are properly calculated at NNLO, we have shown that such a strategy is safe, convenient and provides the most unbiased way of comparing experiment to theoretical models which avoids any built-in model dependent assumptions. The experimental limits or discovery bands may then be reliably confronted with the theoretical predictions on the $c_u-c_d$ plane as shown in Fig.5 which represent the main results of our study, leading to the new limits which we derive here for the Tevatron and to the projected limits for LHC. The results show that the LHC at 7 TeV with as little data as 500 pb$^{-1}$ can either greatly improve on current Tevatron mass limits, or discover a $Z'$, with a measurement of the mass and cross-section providing a powerful discriminator between the benchmark models using this approach.

Acknowledgments

We would like to thank Pavel Nadolsky, Alexander Pukhov, Douglas Ross and Ian Tomalin for stimulating discussions. A.B. and S.K. would like to thank Chriss Hays and Sam Harper for help with understanding details of the experimental analysis at CDF and CMS. A.B. would like to acknowledge a support of International Joint Project Royal Society Grant #JP090598. L.F. would like to thank the School of Physics & Astronomy of the University of Southampton for hospitality.

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Appendix A: $K_{NNLO}$-factors and cross sections for the $pp(p) \to Z' + X$ production process at the Tevatron and the LHC@7TeV

| $M_{Z'}$ (GeV) | Tevatron | | | | LHC@7TeV | | | |
|---|---|---|---|---|---|---|---|---|
| | $\sigma_{LO}$ (pb) | $\sigma_{NNLO}$ (pb) | $K_{NNLO}$ | $\sigma_{LO}$ (pb) | $\sigma_{NNLO}$ (pb) | $K_{NNLO}$ | $\sigma_{LO}$ (pb) | $\sigma_{NNLO}$ (pb) | $K_{NNLO}$ |
| 100 | $3.96 \times 10^3$ | $5.57 \times 10^4$ | 1.41 | $1.63 \times 10^4$ | $2.12 \times 10^4$ | 1.29 | |
| 150 | $1.08 \times 10^3$ | $1.56 \times 10^3$ | 1.44 | $4.45 \times 10^3$ | $5.87 \times 10^3$ | 1.32 | |
| 200 | $4.15 \times 10^2$ | $6.09 \times 10^2$ | 1.47 | $1.70 \times 10^3$ | $2.28 \times 10^3$ | 1.34 | |
| 250 | $1.90 \times 10^2$ | $2.83 \times 10^2$ | 1.49 | $7.89 \times 10^2$ | $1.06 \times 10^3$ | 1.35 | |
| 300 | $9.67 \times 10^1$ | $1.45 \times 10^2$ | 1.50 | $4.14 \times 10^2$ | $5.60 \times 10^2$ | 1.35 | |
| 350 | $5.25 \times 10^1$ | $7.91 \times 10^1$ | 1.51 | $2.37 \times 10^2$ | $3.20 \times 10^2$ | 1.35 | |
| 400 | $2.96 \times 10^1$ | $4.49 \times 10^1$ | 1.51 | $1.44 \times 10^2$ | $1.95 \times 10^2$ | 1.35 | |
| 450 | $1.72 \times 10^1$ | $2.61 \times 10^1$ | 1.52 | $9.20 \times 10^1$ | $1.24 \times 10^2$ | 1.35 | |
| 500 | $1.02 \times 10^1$ | $1.54 \times 10^1$ | 1.52 | $6.10 \times 10^1$ | $8.22 \times 10^1$ | 1.35 | |
| 550 | $6.05 \times 10^0$ | $9.20 \times 10^0$ | 1.52 | $4.16 \times 10^1$ | $5.60 \times 10^1$ | 1.34 | |
| 600 | $3.62 \times 10^0$ | $5.51 \times 10^0$ | 1.52 | $2.92 \times 10^1$ | $3.91 \times 10^1$ | 1.34 | |
| 650 | $2.17 \times 10^0$ | $3.30 \times 10^0$ | 1.52 | $2.08 \times 10^1$ | $2.79 \times 10^1$ | 1.34 | |
| 700 | $1.29 \times 10^0$ | $1.97 \times 10^0$ | 1.52 | $1.52 \times 10^1$ | $2.02 \times 10^1$ | 1.33 | |
| 750 | $7.68 \times 10^{-1}$ | $1.16 \times 10^0$ | 1.52 | $1.12 \times 10^1$ | $1.49 \times 10^1$ | 1.33 | |
| 800 | $4.52 \times 10^{-1}$ | $6.83 \times 10^{-1}$ | 1.51 | $8.35 \times 10^0$ | $1.11 \times 10^1$ | 1.32 | |
| 850 | $2.63 \times 10^{-1}$ | $3.97 \times 10^{-1}$ | 1.51 | $6.30 \times 10^0$ | $8.32 \times 10^0$ | 1.32 | |
| 900 | $1.51 \times 10^{-1}$ | $2.28 \times 10^{-1}$ | 1.51 | $4.80 \times 10^0$ | $6.32 \times 10^0$ | 1.32 | |
| 950 | $8.52 \times 10^{-2}$ | $1.28 \times 10^{-1}$ | 1.50 | $3.69 \times 10^0$ | $4.84 \times 10^0$ | 1.31 | |
| 1000 | $4.72 \times 10^{-2}$ | $7.11 \times 10^{-2}$ | 1.51 | $2.86 \times 10^0$ | $3.73 \times 10^0$ | 1.31 | |
| 1050 | $2.56 \times 10^{-2}$ | $3.85 \times 10^{-2}$ | 1.50 | $2.23 \times 10^0$ | $2.90 \times 10^0$ | 1.30 | |
| 1100 | $1.35 \times 10^{-2}$ | $2.03 \times 10^{-2}$ | 1.50 | $1.74 \times 10^0$ | $2.26 \times 10^0$ | 1.30 | |
| 1150 | $6.95 \times 10^{-3}$ | $1.04 \times 10^{-2}$ | 1.50 | $1.37 \times 10^0$ | $1.78 \times 10^0$ | 1.29 | |
| 1200 | $3.45 \times 10^{-3}$ | $5.20 \times 10^{-3}$ | 1.51 | $1.09 \times 10^0$ | $1.40 \times 10^0$ | 1.29 | |
| 1250 | $1.65 \times 10^{-3}$ | $2.49 \times 10^{-3}$ | 1.51 | $8.63 \times 10^{-1}$ | $1.11 \times 10^0$ | 1.29 | |
| 1300 | $7.52 \times 10^{-4}$ | $1.14 \times 10^{-3}$ | 1.52 | $6.88 \times 10^{-1}$ | $8.82 \times 10^{-1}$ | 1.28 | |
| 1350 | $3.25 \times 10^{-4}$ | $4.95 \times 10^{-4}$ | 1.52 | $5.50 \times 10^{-1}$ | $7.03 \times 10^{-1}$ | 1.28 | |
| 1400 | $1.32 \times 10^{-4}$ | $2.02 \times 10^{-4}$ | 1.53 | $4.11 \times 10^{-1}$ | $5.63 \times 10^{-1}$ | 1.28 | |
| 1450 | $4.97 \times 10^{-5}$ | $7.66 \times 10^{-5}$ | 1.54 | $3.55 \times 10^{-1}$ | $4.51 \times 10^{-1}$ | 1.27 | |
| 1500 | $1.71 \times 10^{-5}$ | $2.65 \times 10^{-5}$ | 1.55 | $2.86 \times 10^{-1}$ | $3.63 \times 10^{-1}$ | 1.27 | |
| 1550 | | | | | | | | |
| 1600 | | | | | | | | |
| 1650 | | | | | | | | |
| 1700 | | | | | | | | |
| 1750 | | | | | | | | |
| 1800 | | | | | | | | |
| 1850 | | | | | | | | |
| 1900 | | | | | | | | |
| 1950 | | | | | | | | |
| 2000 | | | | | | | | |
| 2050 | | | | | | | | |
| 2100 | | | | | | | | |
| 2150 | | | | | | | | |
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| 2300 | | | | | | | | |
| 2350 | | | | | | | | |
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| 2450 | | | | | | | | |
| 2500 | | | | | | | | |

TABLE III: Values of $K_{NNLO}$-factors and cross sections for the $pp(p) \to Z' + X$ production process at the Tevatron and the LHC@7TeV, obtained with ZWPROD package for MSTW08 PDF.
| $M_{Z'}$ (GeV) | Tevatron $\sigma_{\text{LO}}$ (pb) | Tevatron $\sigma_{\text{NNLO}}$ (pb) | LHC@7TeV $\sigma_{\text{LO}}$ (pb) | LHC@7TeV $\sigma_{\text{NNLO}}$ (pb) |
|----------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 100            | $3.94 \times 10^{3}$            | $5.40 \times 10^{3}$            | $1.52 \times 10^{4}$            | $1.95 \times 10^{3}$            |
| 150            | $1.12 \times 10^{3}$            | $1.54 \times 10^{3}$            | $4.25 \times 10^{3}$            | $5.46 \times 10^{3}$            |
| 200            | $4.47 \times 10^{2}$            | $6.08 \times 10^{2}$            | $1.64 \times 10^{3}$            | $2.13 \times 10^{3}$            |
| 250            | $2.12 \times 10^{2}$            | $2.85 \times 10^{2}$            | $7.66 \times 10^{2}$            | $1.00 \times 10^{3}$            |
| 300            | $1.11 \times 10$               | $1.47 \times 10^{2}$            | $4.04 \times 10^{2}$            | $5.29 \times 10^{2}$            |
| 350            | $6.14 \times 10$               | $8.03 \times 10$               | $2.32 \times 10^{2}$            | $3.04 \times 10^{2}$            |
| 400            | $3.54 \times 10$               | $4.57 \times 10$               | $1.42 \times 10^{2}$            | $1.85 \times 10^{2}$            |
| 450            | $2.10 \times 10$               | $2.67 \times 10$               | $9.08 \times 10$               | $1.18 \times 10^{2}$            |
| 500            | $1.26 \times 10$               | $1.58 \times 10$               | $6.04 \times 10$               | $7.85 \times 10$               |
| 550            | $7.61$                         | $9.46$                         | $4.13 \times 10$               | $5.35 \times 10$               |
| 600            | $4.63$                         | $5.68$                         | $2.90 \times 10$               | $3.74 \times 10$               |
| 650            | $2.81$                         | $3.42$                         | $2.08 \times 10$               | $2.67 \times 10$               |
| 700            | $1.70$                         | $2.05$                         | $1.51 \times 10$               | $1.94 \times 10$               |
| 750            | $1.03 \times 10^{-1}$          | $1.22$                         | $1.12 \times 10$               | $1.42 \times 10$               |
| 800            | $6.12 \times 10^{-1}$          | $7.21 \times 10^{-1}$          | $8.38$                         | $1.06 \times 10$               |
| 850            | $3.61 \times 10^{-1}$          | $4.22 \times 10^{-1}$          | $6.35$                         | $7.98$                         |
| 900            | $2.11 \times 10^{-1}$          | $2.44 \times 10^{-1}$          | $4.85$                         | $6.06$                         |
| 950            | $1.21 \times 10^{-2}$          | $1.39 \times 10^{-1}$          | $3.74$                         | $4.64$                         |
| 1000           | $6.80 \times 10^{-2}$          | $7.77 \times 10^{-2}$          | $2.90$                         | $3.59$                         |
| 1050           | $3.75 \times 10^{-2}$          | $4.26 \times 10^{-2}$          | $2.27$                         | $2.78$                         |
| 1100           | $2.02 \times 10^{-2}$          | $2.28 \times 10^{-2}$          | $1.78$                         | $2.18$                         |
| 1150           | $1.06 \times 10^{-3}$          | $1.19 \times 10^{-2}$          | $1.41$                         | $1.71$                         |
| 1200           | $5.36 \times 10^{-3}$          | $6.04 \times 10^{-3}$          | $1.12$                         | $1.35$                         |
| 1250           | $2.62 \times 10^{-3}$          | $2.96 \times 10^{-3}$          | $8.88 \times 10^{-1}$          | $1.07$                         |
| 1300           | $1.23 \times 10^{-3}$          | $1.39 \times 10^{-3}$          | $7.10 \times 10^{-1}$          | $8.48 \times 10^{-1}$          |
| 1350           | $5.49 \times 10^{-4}$          | $6.24 \times 10^{-4}$          | $5.70 \times 10^{-1}$          | $6.77 \times 10^{-1}$          |
| 1400           | $2.31 \times 10^{-4}$          | $2.65 \times 10^{-4}$          | $4.58 \times 10^{-1}$          | $5.42 \times 10^{-1}$          |
| 1450           | $9.08 \times 10^{-5}$          | $1.05 \times 10^{-5}$          | $3.70 \times 10^{-1}$          | $4.35 \times 10^{-1}$          |
| 1500           | $3.28 \times 10^{-5}$          | $3.86 \times 10^{-5}$          | $2.99 \times 10^{-1}$          | $3.50 \times 10^{-1}$          |
| 1550           |                               |                                | $2.42 \times 10^{-1}$          | $2.82 \times 10^{-1}$          |
| 1600           |                               |                                | $1.97 \times 10^{-1}$          | $2.28 \times 10^{-1}$          |
| 1650           |                               |                                | $1.60 \times 10^{-1}$          | $1.85 \times 10^{-1}$          |
| 1700           |                               |                                | $1.31 \times 10^{-1}$          | $1.50 \times 10^{-1}$          |
| 1750           |                               |                                | $1.06 \times 10^{-1}$          | $1.22 \times 10^{-1}$          |
| 1800           |                               |                                | $8.70 \times 10^{-2}$          | $9.92 \times 10^{-2}$          |
| 1850           |                               |                                | $7.12 \times 10^{-2}$          | $8.09 \times 10^{-2}$          |
| 1900           |                               |                                | $5.83 \times 10^{-2}$          | $6.60 \times 10^{-2}$          |
| 1950           |                               |                                | $4.77 \times 10^{-2}$          | $5.40 \times 10^{-2}$          |
| 2000           |                               |                                | $3.92 \times 10^{-2}$          | $4.41 \times 10^{-2}$          |
| 2050           |                               |                                | $3.21 \times 10^{-2}$          | $3.61 \times 10^{-2}$          |
| 2100           |                               |                                | $2.64 \times 10^{-2}$          | $2.96 \times 10^{-2}$          |
| 2150           |                               |                                | $2.17 \times 10^{-2}$          | $2.43 \times 10^{-2}$          |
| 2200           |                               |                                | $1.78 \times 10^{-2}$          | $2.00 \times 10^{-2}$          |
| 2250           |                               |                                | $1.46 \times 10^{-2}$          | $1.64 \times 10^{-2}$          |
| 2300           |                               |                                | $1.20 \times 10^{-2}$          | $1.35 \times 10^{-2}$          |
| 2350           |                               |                                | $9.88 \times 10^{-3}$          | $1.11 \times 10^{-2}$          |
| 2400           |                               |                                | $8.12 \times 10^{-3}$          | $9.10 \times 10^{-3}$          |
| 2450           |                               |                                | $6.67 \times 10^{-3}$          | $7.48 \times 10^{-3}$          |
| 2500           |                               |                                | $5.48 \times 10^{-3}$          | $6.15 \times 10^{-3}$          |

TABLE IV: Values of $K_{\text{NNLO}}$-factors and cross sections for the $p\bar{p}(p) \to Z' + X$ production process at the Tevatron and the LHC@7TeV, obtained with ZWPROD package for CTEQ6.6 PDF.