Chiral transition and deconfinement in $N_f = 2$ QCD

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The transition is studied by means of a disorder parameter detecting condensation of magnetic monopoles in the vacuum. The deconfining transition is found to coincide with the chiral transition and the susceptibility $\rho$, related to the disorder parameter, is consistent with a first order phase transition.

1. Introduction

Understanding confinement and deconfinement in the presence of dynamical fermions is still an open issue in strong interaction physics. There is general agreement on the order-disorder nature of the deconfining transition in the quenched case. The popular order parameter is the Polyakov line $\langle L \rangle$; the symmetry involved is $Z_N$. Alternatively the dual (‘t Hooft) line $\langle \tilde{L} \rangle$ [1] can be used as a disorder parameter [2,3], (order parameter of the disordered phase) corresponding to the dual $\tilde{Z}_N$ symmetry.

In full QCD, i.e. in the presence of dynamical quarks, the situation is less clear: $Z_N$ and $\tilde{Z}_N$ symmetries are explicitly broken. At zero quark mass there is a phase transition at some $T_c$ where chiral symmetry is restored, the chiral condensate being the order parameter: at the same $T_c$ also the susceptibility of the Polyakov loop shows a maximum which can be related to deconfinement.

However it is not clear theoretically what the chiral transition has to do with the deconfinement transition. Moreover, in the presence of physical quark masses also the chiral symmetry is explicitly broken, so that neither the Polyakov loop nor the chiral condensate are true order parameters.

In a series of papers [4,5,6] we have tested and verified in the quenched case the mechanism of confinement based on dual superconductivity of the QCD vacuum proposed by ’t Hooft [7]. More precisely we have constructed an operator $\mu$ which creates a magnetic monopole in a $U(1)$ subgroup of the color gauge group selected by abelian projection. If the magnetic symmetry is realized à la Wigner and if $\mu$ carries non zero net magnetic charge, then $\langle \mu \rangle = 0$. Therefore $\langle \mu \rangle \neq 0$ implies Higgs breaking of the magnetic $U(1)$ symmetry and thus dual superconductivity. It has indeed been shown that the quenched QCD vacuum is a dual superconductor ($\langle \mu \rangle \neq 0$) in the confined phase and goes to normal ($\langle \mu \rangle = 0$) at the deconfinement transition. It has also been shown that $\langle \mu \rangle$ being different or equal to zero is independent of the particular abelian projection chosen [6,8].

Both the construction of the disorder parameter $\langle \mu \rangle$ and the related magnetic symmetry stay unaltered if dynamical fermions are introduced: $\langle \mu \rangle$ is therefore a good candidate disorder parameter also in full QCD, especially if one expects, in the spirit of the $N_c \rightarrow \infty$ limit, that the mechanism which drives confinement and the deconfinement phase transition be the same with and without dynamical quarks.

In Ref. [9] we have indeed demonstrated that also in full QCD with two light flavors, $\langle \mu \rangle$ shows a transition from a low temperature phase, where it is different from zero thus signaling dual superconductivity, to a high temperature phase where it is exactly zero, thus signaling the disappearance of dual superconductivity, or deconfinement. We have also demonstrated that the transition for $\langle \mu \rangle$ coincides with the chiral phase transition. Analogous results have been obtained in Ref. [10].
measuring a parameter related to the monopole free energy.

The aim of the present work is to extend the analysis and to show that \( \langle \mu \rangle \) scales with the correct critical indices at the phase transition, thus demonstrating that it is indeed a valid disorder parameter. We will discuss the results of a finite size scaling analysis of \( \langle \mu \rangle \) around the phase transition, showing that it gives critical indices which are consistent with those found through finite size scaling of the specific heat (and of the chiral susceptibility), as presented in [11].

2. Disorder parameter

The operator \( \mu \) is defined in full QCD exactly in the same way as in the quenched theory [11, 12, 13, 14, 15, 16].

\[
\langle \mu \rangle = \frac{\tilde{Z}}{Z},
\]

\[
Z = \int \langle DU \rangle e^{-\beta S},
\]

\[
\tilde{Z} = \int \langle DU \rangle e^{-\beta \tilde{S}}.
\]

(1)

\( \tilde{Z} \) is obtained from \( Z \) by changing the action in the time slice \( x_0, S \to \tilde{S} = S + \Delta S \). In the Abelian projected gauge the plaquettes

\[
\Pi_{i0}(\tilde{x}, x_0) =
\]

\[
U_i(\tilde{x}, x_0)U_0(x + i, x_0)U_1^\dagger(\tilde{x}, x_0 + 0)U_0^\dagger(\tilde{x}, x_0)
\]

(2)

are changed by substituting

\[
U_i(\tilde{x}, x_0) \to \tilde{U}_i(\tilde{x}, x_0) = U_i(\tilde{x}, x_0)e^{iTb_0(\tilde{x} - \tilde{y})}
\]

(3)

where \( \tilde{b}(\tilde{x} - \tilde{y}) \) is the vector potential of a monopole configuration centered at \( \tilde{y} \) in the gauge \( \nabla \tilde{b} = 0 \), and \( T \) is the diagonal gauge group generator corresponding to the monopole species chosen. It can be shown that, as in the quenched case, \( \mu \) adds to any configuration the monopole configuration \( \tilde{b}(\tilde{x} - \tilde{y}) \).

Instead of \( \langle \mu \rangle \) we measure the quantity

\[
\rho = \frac{d}{d\beta} \ln(\langle \mu \rangle).
\]

(4)

It follows from Eq. (1) that

\[
\rho = \langle S \rangle_S - \langle \tilde{S} \rangle_{\tilde{S}}
\]

(5)

the subscript meaning the action by which the average is performed. In terms of \( \rho \)

\[
\langle \mu \rangle = \exp \left( \int_0^\beta \rho(\beta') d\beta' \right).
\]

(6)

A drop of \( \langle \mu \rangle \) at the phase transition corresponds to a strong negative peak of \( \rho \).

3. Numerical Results

We have made simulations with two degenerate flavors of Kogut-Susskind quarks, using the standard gauge and fermion actions. Configuration updating was performed using the standard Hybrid R algorithm. The lattice temporal size was fixed at \( N_t = 4 \). Different spatial sizes \((L = 12, 16, 20, 24, 32)\) and values of the quark mass were used. For a more detailed account on simulation parameters we refer to [11].

We can assume the following general scaling form for \( \langle \mu \rangle \) around the phase transition:

\[
\langle \mu \rangle = L^k \Phi(\tau L^{1/\nu}, m L^{\eta_m})
\]

(7)

where \( \tau \) is the reduced temperature and \( m \) the quark mass. Analyticity arguments [11, 12] suggest that in the infinite volume limit the mass dependence in the scaling function factorizes, so that \( \rho = \frac{d}{d\beta} \ln(\langle \mu \rangle) \) does not depend on the mass. We then obtain the following scaling law:

\[
\rho = L^{1/\nu} \phi(\tau L^{1/\nu}).
\]

In Figure 1 we show the quality of scaling assuming \( \nu = 1/3 \), i.e. a first order phase transition: a good agreement is clearly visible. The deviation from scaling in the deconfined region is well understood [13] and related to the disorder parameter \( \langle \mu \rangle \) being exactly zero on that side.

For comparison we show in Figure 2 the quality of scaling assuming the \( O(4) \) critical index \( \nu = 0.75 \).

The \( O(4) \) universality class is clearly excluded, while there is good agreement with a first order phase transition, confirming results obtained through an analysis of the specific heat and of the chiral susceptibility [11].
4. Conclusions

We have investigated the scaling properties of the parameter $\langle \mu \rangle$, detecting dual superconductivity of the vacuum, around the phase transition of full QCD with 2 flavors of staggered fermions, in order to check if $\langle \mu \rangle$ behaves as a good disorder parameter for the confinement - deconfinement phase transition, as it does in quenched QCD.

We have shown that a finite size scaling analysis of $\langle \mu \rangle$ indicates a first order phase transition and excludes $O(4)$ critical behaviour, in agreement with previous hints [9,10] and with a detailed analysis of the specific heat and the chiral susceptibility [11]. This confirms that also in full QCD $\langle \mu \rangle$ can be considered as a valid disorder parameter.

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