Economic Dispatch With Distributed Energy Resources: Co-Optimization of Transmission and Distribution Systems

Xinyang Zhou, Chin-Yao Chang, Andrey Bernstein, Changhong Zhao, and Lijun Chen

Abstract—The increasing penetration of distributed energy resources (DERs) in the distribution networks has turned the conventionally passive load buses into active buses that can provide grid services for the transmission system. To take advantage of the DERs in the distribution networks, this letter formulates a transmission-and-distribution (T&D) systems co-optimization problem that achieves economic dispatch in the transmission level and optimal voltage regulation in the distribution level by leveraging large generators and DERs. A primal-dual gradient algorithm is proposed to solve this optimization problem jointly for T&D systems, and a distributed market-based equivalent of the gradient algorithm is used for practical implementation. The results are corroborated by numerical examples with IEEE 39-Bus system connected with 7 different distribution networks.

I. INTRODUCTION

The rising electricity demand and the shortage of power supply have caused surging electricity price and even blackouts in peak hours; a few unfortunate events have occurred in recent years with or without market manipulation [1]. Meanwhile, the penetration of distributed energy resources (DERs) has been deepening in distribution systems, with residential photovoltaic (PV) devices, energy storage devices, and electric vehicles (EVs) becoming increasingly popular. Such DERs can potentially meet (part of) the demand from the distribution networks, as well as provide grid services like voltage regulation. Involving residential DERs for energy supply without disturbing distribution system operation becomes operationally desired and economically sensible for the overall transmission-and-distribution (T&D) systems.

In the literature, joint generator-side and load-side control has been proposed to assist power balancing and frequency regulation in the transmission systems [2]–[6]. These works usually focus on dynamics in the transmission system by treating load buses as controllable without detailing distribution system structure at the load buses.

Optimizing DERs in distribution systems has been extensively studied in the past decade. Various problem formulations and solution methods have been proposed to optimally coordinate DERs for the purpose of voltage regulation, loss minimization, dispatching signal tracking [7]–[9]. Works like [10], [11] propose a concept of virtual power plant that enables the distribution network to provide a certain amount of aggregate power output by coordinating the DERs within the network. Most of these works usually focus on distribution system analysis and do not model any transmission structure. There are also a few works on T&D co-optimization. In [12], concrete models for the transmission network, the distribution networks and DERs are formulated, and a multi-level solution method to solve the subproblems for each layer in sequence are proposed. In [13], a T&D coordination scheme is proposed by solving respective subproblems for the two levels. However, solving respective optimization problems for transmission and distribution networks independently may not achieve the same optimal points of solving the T&D system jointly, which has been addressed by few works.

In this letter, we first formulate a convex optimization problem featuring economic dispatch at the transmission level while ensuring optimal voltage regulation at the distribution level with linearized power flow equations. The outputs of the large generators in the transmission network and those of the DERs in the distribution networks are jointly optimized. Next we propose a primal-dual gradient algorithm to solve the optimization problem with provable convergence. A market-based distributed implementation of the gradient algorithm is then designed to provide practical application. Finally, we illustrate the performance of the proposed scheme on the IEEE 39-bus system connected with 7 different distribution feeders.

The rest of this letter is structured as follows. Section II models the transmission and distribution networks. Section III formulates the T&D system co-optimization problem and proposes a gradient algorithm for solving it. Section IV designs a market-based distributed implementation of the gradient algorithm. Section V presents numerical results and Section VI this letter.

Notation

In this paper, we use bold upper-case letters to represent matrices, e.g., $\mathbf{A}$, italic bold letters to represent vectors, e.g., $\mathbf{a}$, and non-bold letters to represent scalars, e.g., $A$ and $a$. Superscript $\top$ performs vector or matrix transpose. $|\cdot|$ denotes the cardinality of a set. $\lfloor \cdot \rfloor$ makes projection upon set $\Omega$. Operator $\times$ represents the Cartesian product of sets. $0_m$ is a $m \times 1$ vector of all zeros.
II. SYSTEM MODEL

In this section, we provide the model for both transmission and distribution systems, where the term “bus or control area” is used for the former and “node” is used for the latter.

Consider a power transmission network, denoted by a graph \((\mathcal{K}, \mathcal{E})\) where \(\mathcal{K} = \{1, \ldots, K\}\) is a set of buses or control areas, and the set \(\mathcal{E} \subset \mathcal{K} \times \mathcal{K}\) collects undirected transmission lines connecting the buses.

Without loss of generality, for each bus \(k \in \mathcal{K}\), we assume there to be a dispatchable generator \(k\) with mechanical power input \(P^M_k\) and a distribution feeder indexed with \(k\) with a total real power load \(P^L_k\) injected at its substation. Denote by \(P^M = [P^M_k]_{k \in \mathcal{K}}\) and \(P^L = [P^L_k]_{k \in \mathcal{K}}\). Denote by \(P_0^0\) the remaining uncontrollable power injection at bus \(k\). We assume that the transmission system is lossless to have the following power balance equation:

\[
\sum_{k \in \mathcal{K}} \left( P^M_k - P^L_k(p_k, q_k) + P_0^0 \right) = 0. \tag{1}
\]

Distribution feeder \(k\) has a radial topology \((\mathcal{N}_k, \mathcal{E}_k)\) with a set \(\mathcal{N}_k\) collecting all its \(N_k\) nodes and a set \(\mathcal{E}_k\) collecting their connecting distribution lines. Let \(v_k := [v_{k,1}, \ldots, v_{k,N_k}]^T \in \mathbb{R}^{N_k}\) denote the voltage magnitudes vector in the distribution system, and \(p_k := [p_{k,1}, \ldots, p_{k,N_k}]^T \in \mathbb{R}^{N_k}\) is the real and reactive power injections from all its nodes. For the purpose of algorithms design, we leverage a linear power flow model \(^1\)

\[
v_k = A_k p_k + B_k q_k + c_k, \tag{2}
\]

\[
P^L_k = M_k p_k + N_k q_k + d_k, \tag{3}
\]

where \(A_k, B_k \in \mathbb{R}^{N_k \times N_k}\), \(c_k, M_k, N_k \in \mathbb{R}^{N_k}\), and \(d_k \in \mathbb{R}\) are system parameters that can be computed using methods such as [14], [15]. In the following, we use \(v_k(p_k, q_k)\) and \(P^L_k(p_k, q_k)\) to represent (2) and (3), respectively.

III. PROBLEM FORMULATION AND GRADIENT ALGORITHM

A. Controllable Devices

We assume that generator \(k\) has a cost function \(C^M_k(P^M_k)\) and a convex feasible set featuring its operational limits \(\Omega^M_k\). Meanwhile, distribution feeder \(k\) controls its total injected power \(P^L_k\) indirectly through \(p_k\) and \(q_k\) from DERs while ensuring its voltage constraints modeled as \(g_k(v_k(p_k, q_k)) \leq 0_{m_k}\). Similar to the generators, each DER \(i\) has a cost function denoted by \(C_{k,i}(p_{k,i}, q_{k,i})\) and a convex feasible set of \(\Omega_{k,i}\). Denote by \(\Omega^M = \times_{k \in \mathcal{K}} \Omega^M_k\) and \(\Omega = \times_{k \in \mathcal{K}} \times_{i \in \mathcal{N}_k} \Omega_{k,i}\). We have the following assumption on the cost and constraints functions.

**Assumption 1** Functions \(C_{k,i}(p_{k,i}, q_{k,i})\) for all \(i \in \mathcal{N}_k, k \in \mathcal{K}\) and \(C^M_k(P^M_k)\) for all \(k \in \mathcal{K}\) are continuously differentiable and strongly convex in \((p_{k,i}, q_{k,i})\) and \(P^M_k\), respectively, with bounded first-order derivatives. Functions \(g_k(v_k)\) are convex and differentiable functions of \(v_k\) for all \(k \in \mathcal{K}\).

\(^1\)The linear model is used only to formulate the optimization problem and devise an efficient solution algorithm. The simulation experiments in Section V are performed using the exact (AC) power flow model.

B. T&D Co-Optimization Problem

In this part, we formulate a T&D co-optimization problem that achieves economic dispatch over all generators and DERs while maintaining voltage constraints by DERs in the distribution feeders.

Let \(N = \sum_{k \in \mathcal{K}} N_k\) be the total number of nodes in all distribution networks. Denote by \(p = [p_1^T, \ldots, p_K^T]^T\), \(q = [q_1^T, \ldots, q_K^T]^T \in \mathbb{R}^N\). Consider the following optimization problem subject to power flow and operational constraints:

\[
\begin{align}
& \min \sum_{k \in \mathcal{K}} \left( \sum_{i \in \mathcal{N}_k} C_{k,i}(p_{k,i}, q_{k,i}) + C^M_k(P^M_k) \right), \tag{4a} \\
& \text{over } (p, q) \in \Omega, P^M \in \Omega^M \text{ s.t.} \sum_{k \in \mathcal{K}} \left( P^M_k - P^L_k(p_k, q_k) + P_0^0 \right) = 0, \tag{4b} \\
& \quad \forall k \in \mathcal{K}, \tag{4c}
\end{align}
\]

where the cost function \(4a\) adds up the generation costs of all large generators and small DERs in the distribution feeders, the equality constraint \(4b\) ensures that power demand and supply are balanced, and inequality constraint \(4c\) confines voltage magnitudes to within acceptable ranges.

Let \(m = \sum_{k \in \mathcal{K}} m_k\) be the dimension of the transmission network. Introduce dual variables \(\lambda \in \mathbb{R}\) for the equality constraint \(4b\) and nonnegative vector \(\mu = [\mu_k]_{k \in \mathcal{K}} \in \mathbb{R}^m_+\) for the inequality constraints \(4c\) to have the following regularized Lagrangian of \(\mathcal{L}\) with a small constant \(\eta > 0\):

\[
\mathcal{L}(p, q, P^M; \lambda, \mu) = \sum_{k \in \mathcal{K}} \left( \sum_{i \in \mathcal{N}_k} C_{k,i}(p_{k,i}, q_{k,i}) + C^M_k(P^M_k) + \mu_k^T g_k(v_k(p_k, q_k)) \right) + \lambda \sum_{k \in \mathcal{K}} \left( P^M_k - P^L_k(p_k, q_k) + P_0^0 \right) - \eta(\lambda^2 + \|\mu\|_2^2) / 2. \tag{5}
\]

The introduction of the regularization term \(-\eta(\lambda^2 + \|\mu\|_2^2) / 2\) ensures the strong concavity of \(\mathcal{L}(p, q, P^M; \lambda, \mu)\) with respect to the dual variables, as well as provable convergence of gradient-based algorithms with a constant step size. However, a discrepancy that is proportional to \(\eta\) is also brought in, which can be negligible if \(\eta\) is small. We refer to Proposition 3.1 of [16] for detailed analytical characterization of the discrepancy.

C. Gradient-Based Algorithm Design

We next design a primal-dual gradient algorithm to solve for the unique saddle point of (5). To this end, we write down the partial gradient of \(\mathcal{L}\) with respect to decision variables as follows:

\[
\frac{\partial \mathcal{L}}{\partial p_{k,i}} = \nabla_{p_{k,i}} \left( \sum_{i \in \mathcal{N}_k} C_{k,i}(p_{k,i}, q_{k,i}) \right) - \lambda M_k + A_k^T \nabla_{v_k} g_k(v_k)^T \mu_k, \tag{6a}
\]

\[
\frac{\partial \mathcal{L}}{\partial q_{k,i}} = \nabla_{q_{k,i}} \left( \sum_{i \in \mathcal{N}_k} C_{k,i}(p_{k,i}, q_{k,i}) \right) - \lambda N_k + B_k^T \nabla_{v_k} g_k(v_k)^T \mu_k, \tag{6b}
\]

\[
\frac{\partial \mathcal{L}}{\partial P^M_k} = \sum_{i \in \mathcal{N}_k} dC^M_k(P^M_k) / dP^M_k + \lambda, \tag{6c}
\]

\[
\frac{\partial \mathcal{L}}{\partial \mu_k} = g_k(v_k(p_k, q_k)) - \eta \mu_k, \tag{6d}
\]
\[
\frac{\partial C}{\partial x} = \sum_{k \in K} (P_k^M - P_k^L(p_k, q_k) + P_k^B) - \eta \lambda,
\]
where \( \nabla_v g_k(v_k) \) is the Jacobian matrix of \( g_k \) and are for all \( k \in K \).

For notational simplicity, we further let \( x = [p^T, q^T, (P^M)^{\top}]^T \) collect all the primal variables and \( y = [\lambda, \mu^T]^T \) collect all the dual variables. Then, the iterative primal-dual gradient algorithm for finding the unique saddle point of the regularized Lagrangian (5) is given by:

\[
x(t+1) = x(t) - \epsilon \frac{\partial \mathcal{L}(x(t), y(t))}{\partial x} \bigg|_{x=x(t), \Omega=\Omega(t)} \quad (7a)
\]
\[
y(t+1) = y(t) + \epsilon \frac{\partial \mathcal{L}(x(t), y(t))}{\partial y} \bigg|_{x=x(t), \Omega=\Omega(t)} \quad (7b)
\]
where the partial gradient is calculated by (6), \( \epsilon \) is a constant stepsize, and \( t \) is the iteration index.

**D. Convergence Analysis**

Define gradient operator \( T(x; y) = \left[ \frac{\partial \mathcal{L}(x; y)}{\partial x}, -\frac{\partial \mathcal{L}(x; y)}{\partial y} \right] \).

Based on Assumption 1 and the regularization terms we have added for the dual variables to the Lagrangian, the next lemma follows by definition.

**Lemma 1** Based on Assumption 1, \( T(x; y) \) is an s-strongly monotone operator with some \( s > 0 \) and is \( l \)-Lipschitz continuous with some \( l > 0 \), for any feasible \( x \in \Omega \times \Omega_M \) and \( y \in \mathbb{R} \times \mathbb{R}^M \).

**Theorem 1** Based on Assumption 1, given a constant stepsize \( \epsilon \) such that \( 0 < \epsilon \leq \bar{\epsilon} < 2/s^2 \), the primal-dual gradient dynamics (7) converges to the unique saddle point of the regularized Lagrangian (5).

We omit the detailed proof of Theorem 1 here due to space limit. It can be found in numerous literature, e.g., [17], [18].

**IV. DISTRIBUTED MARKET-BASED IMPLEMENTATION**

**A. Economic Model**

Unlike most utility-owned power plants, the user-owned DERs in the distribution feeders are usually not obliged to follow the gradient steps specified in (6a)-(6d). On the contrary, users are naturally driven to minimize their overall cost (or maximize their overall utility) featuring a trade-off between their DERs generation cost and electricity bills, which is formulated as follows:

\[
\min_{p_{k,i}, q_{k,i}} f_{k,i}(p_{k,i}, q_{k,i}) = C_{k,i}(p_{k,i}, q_{k,i}) + \alpha_{k,i} p_{k,i} + \beta_{k,i} q_{k,i},
\]

Here, \( \alpha_{k,i}, \beta_{k,i} \in \mathbb{R} \) are the incentive signals/electricity price for real and reactive power, respectively, set by the network operator for user \( i \) in distribution feeder \( k \). When \( \alpha_{k,i} \) and \( \beta_{k,i} \) are positive (resp. negative), users are incentivized to reduce (resp. increase) the values of their \( p_{k,i} \) and \( q_{k,i} \). Moreover, once the problem formulation—specifically, \( C_{k,i} \) and \( \Omega_{k,i} \)—is revealed, \( \alpha_{k,i} \) and \( \beta_{k,i} \) can be used to induce certain values of \( p_{k,i} \) and \( q_{k,i} \) by solving (8).

However, such private information of users is usually inaccessible to the network operator. We next present an iterative method to find the optimal signals to incentivize the users to react in a certain way that concurrently solves the T&D optimization problem (4) [19]. As we will see, the resultant design can be seen as a market-based equivalent implementation of the primal-dual gradient algorithm (7).

**B. Market-Based Distributed Implementation**

To incentivize the users to act according to (7) so that problem (4) can be solved, network operator needs to carefully design the incentive signals. Note that when user \( k,i \) solves its cost minimization problem (9), the gradient he/she takes is in the form of

\[
\frac{\partial f_{k,i}}{\partial p_{k,i}} = \partial C_{k,i} + \alpha_{k,i}, \quad (9a)
\]
\[
\frac{\partial f_{k,i}}{\partial q_{k,i}} = \partial C_{k,i} + \beta_{k,i}, \quad (9b)
\]

Denote the signals vector of distribution feeder \( k \) as \( \alpha_k = [\alpha_{k,1}, ..., \alpha_{k,N_k}]^T \) and \( \beta_k = [\beta_{k,1}, ..., \beta_{k,N_k}]^T \). By comparing (9) with (6a)-(6b), we design the incentive signals as follows:

\[
\alpha_k = -\lambda M_k + A_k^T \nabla_v g_k(v_k)^T \mu_k, \quad (10a)
\]
\[
\beta_k = -\lambda N_k + B_k^T \nabla_v g_k(v_k)^T \mu_k, \quad (10b)
\]

which relies on network information without any private information from the users. Using (10), we propose a market-based iterative distributed algorithm presented as Algorithm 1. We also illustrate the algorithm in Fig. 1. By design, we have the following formal statement.

**Proposition 1** Algorithm 1 is equivalent to the primal-dual gradient algorithm for solving the saddle point of (5).

Since Algorithm 1 and the primal-dual gradient dynamics (7) are equivalent, they share the same convergence properties.

**C. Nonlinear (AC) Power Flow Feedback**

In the primal-dual gradient update (7), as well as in Algorithm 1 while the Jacobian matrices \( \nabla \mathcal{L}(x; y)/\partial y \) and
Algorithm 1 Distributed Market-Based T&D Optimization

while stopping criterion not met do
[S1] Given incentive signals $\alpha_{k,i}(t)$ and $\beta_{k,i}(t)$, user $i \in \mathcal{N}_k$, $k \in \mathcal{K}$ takes gradient step towards solving his/her own optimization problem (8) by:
\[
p_{k,i}(t+1) = \left[ p_{k,i}(t) - \epsilon \left( \frac{\partial C_{k,i}(p_{k,i}(t), q_{k,i}(t))}{\partial p_{k,i}} \right)\right]_{\Omega_k},
\]
\[
q_{k,i}(t+1) = \left[ q_{k,i}(t) - \epsilon \left( \frac{\partial C_{k,i}(p_{k,i}(t), q_{k,i}(t))}{\partial q_{k,i}} \right)\right]_{\Omega_k}.
\]
[S2] Dispatchable generator $k \in \mathcal{K}$ updates its power setpoints by:
\[
P_k^M(t+1) = \left[ P_k^M(t) - \epsilon \left( \frac{dC_k^M(P_k^M(t))}{dP_k^M} + \lambda(t) \right) \right]_{\Omega_k^M}.
\]
[S3] Network operator updates the power flow by:
\[
P_k(t+1) = M_k^p p_k(t+1) + N_k^q q_k(t+1) + d_k,
\]
\[
v_k(t+1) = A_k p_k(t+1) + B_k q_k(t+1) + c_k.
\]
[S4] Network operator updates the dual variables as
\[
\lambda(t+1) = \lambda(t) + \epsilon \left( \sum_{k \in K} (P_k^M(t+1) - sP_k^L(t+1) + P_k^0) \right)
\]
\[-\eta \lambda(t)\),
\[
\mu_k(t+1) = [\mu_k(t) + \epsilon (g_k(v_k(t+1)) - \eta \mu_k(t))]_{\mathbb{R}^{N_k}},
\]
and the incentive signals as
\[
\alpha_k(t+1) = -\lambda(t+1)M_k + A_k^T \nabla g_k(v_k(t+1))^\top \mu_k(t+1),
\]
\[
\beta_k(t+1) = -\lambda(t+1)N_k + B_k^T \nabla g_k(v_k(t+1))^\top \mu_k(t+1),
\]
and sends the updated signals to the users.
end while

$\partial L(x; y)/\partial x$ are calculated based on the linearized relationships (4)–(5) to simplify the computation, they lose accuracy. To improve this, a feedback mechanism can be applied to reduce such modelling discrepancy. Specifically, when performing Algorithm 1 instead of using linearized power flow model in [S3], nonlinear power flow is used to calculate $v_k$ and $P_k^L$ which are further fed into [S4] to update dual variables. Stability analysis with the feedback mechanism can be found in [19]. We also use nonlinear (AC) power flow for numerical results in Section V next.

V. NUMERICAL RESULTS

A. System Setup

1) Transmission and Distribution Networks: We use New England IEEE 39-Bus system as the transmission network with 9 controllable generators located at buses 30–38, and bus 39 (the slack bus) connected to the rest of US/Canada grid. We connect 7 different distribution networks—case18, case22, case33bw, case69, case85, case141 [20], and SCE 42-bus system [21]—to load buses indexed 3, 7, 12, 18, 26, 28, and 31, respectively. The total power injected into the distribution feeders will be used as the demand of the corresponding load bus of the transmission system. See Fig. 2 for the locations of generators and distribution feeders.

2) Parameters: We assign quadratic cost functions $c_k P_k^M^2$, with $c_k$ set to 1, 1.5, 1.3, 1.7, 1.8, 1.2, 0.8, and 1.2, respectively, for the generators 1–9. We assign homogeneous cost functions $p_{k,i}^2 + 0.1 q_{k,i}^2$ for DERs in distribution feeders. The inequality constraints $g_k(v_k) \leq 0$ are set to 0.95 p.u. $\leq v_k \leq 1.05$ p.u. for all $k$. Linearization parameters are based on LinDistFlow model to generate parameters $A_k, B_k, c_k$ and $M_k, N_k, d_k$ in Eqs. (2)–(3) for the distribution networks. Nonlinear power flow is solved to update both the transmission and the distribution systems every iteration with MATPOWER 7.0 [20].

3) Simulation Process: The system is initialized at a non-optimal point with voltage constraints violated in some distribution network. The system then approaches the optimal with primal-dual gradient dynamics before generator 7 is set to go down at iteration 10,000. At this point, we enlarge the feasible set of DERs by two times to allow them to contribute more to the grids without loss of generality. The system will then approach a new optimal point with all constraints satisfied.

4) Handling the Slack Bus: In MATPOWER and other power system analysis tools, slack buses are essential to compensate loss and balance power. However, this automatic balancing functionality causes problems when we intend to manually balance Eq. (6b) with generators and DERs from load buses. Specifically, in MATPOWER, $\sum_{k \in K} (P_k^M - P_k^L(p_k, q_k) + P_k^0)$ is always zero thanks to the slack bus, making updating $\lambda$ with gradient (6c) impossible. To address this issue, we apply the following techniques to fix the output of the slack bus. We record the initial real power of the slack bus as $P_{\text{slack}}^0$. Afterwards, the output of the slack bus changes according to all the other generation and load buses, denoted by $P_{\text{slack}}(P^M, P^L(p, q))$. Then, instead

\[\text{Fig. 2: We use IEEE 39-Bus system as the transmission network with 7 different distribution networks connected to the load buses marked by red boxes and 9 controllable generators connected to Buses 30–38. Bus 39 is the slack bus.}\]
of enforcing the constraint Eq. (4b), we use the constraint $P_{\text{slack}}^0 = P_{\text{slack}}(P^M, P^L(p,q))$ to achieve power balance using the generation and load buses while prohibiting the slack bus from participation.

**B. Convergence**

As shown in Fig. 3 and Fig. 4, the power outputs of generators and distribution substations gradually stabilize before Generator 7 goes down at iteration 10,000, disturbing the imminent convergence. After Generator 7 goes down, the value of $\lambda$ increases to reflect a higher price for balancing the power demand and supply, as Fig. 7 shows. The larger $\lambda$ then leads to more power outputs from the remaining generators and DERs in the distribution network; see Fig. 6 for increasing generator outputs and Fig. 3 for reducing load bus demand. As a result, we see from Fig. 5 that the total output from distribution feeders compensate for the lost generator, with the slack bus maintaining constant output as expected.

Fig. 4 plots the voltage convergence at sampled nodes in all 7 distribution feeders. It is worth noticing that most voltage magnitudes are pushed towards the 1.05 p.u. upper limits because DERs are incentivized to generate more real power to support transmission system, increasing the voltage levels. Here, negative reactive power is used to allow for more real power generation without violating the voltage bounds.

In Fig. 7 we record and plot the convergence of signal $\lambda$, along with signals $\alpha$ and $\beta$ of sampled nodes from the 7 distribution feeders. Note that the value of $\lambda$ increases after Generator 7 goes down to incentivize more real power input from other generators and DERs, $\alpha$ is smaller than $\lambda$ because $B_k \nabla \nabla_k g_k(v_k)^T \mu_k$ in Eq. (10a) is negative due to over-voltages; $\beta$ is even smaller because $A_N \mu_k = 0$ here and $B_k > A_k$ element-wise for distribution networks in Eq. (10b), driving more negative reactive power to lower the voltages.

**C. Cost Analysis**

We plot the convergence of the value of the total cost function in Fig. 8. For comparison, we conduct another set of simulation without involving distribution system in the economic dispatch, i.e., set $\lambda = 0$ in the signals sent to DERs in (10). A higher total cost is recorded without DERs’ participation. Such results are expected because even though DERs and generators share similar cost functions, the marginal costs are significantly different: to provide the same amount of power, it would be more economic and more optimal to replace some of the high-marginal-cost generator outputs with low-marginal-cost DERs outputs.

**VI. Conclusion**

This letter formulated a co-optimization problem for economic dispatch in the transmission system and voltage regulation in distribution networks that are connected to the load buses of the transmission system. Large generators in the transmission system and DERs in distributed systems are jointly leveraged to achieve the optimal points of the T&D
system. We proposed a primal-dual gradient algorithm, as well as its distributed market-based equivalence, to solve this problem. Numerical results validated that DERs can provide economic grid services to the transmission system while helping maintain operational constraints in the distribution feeders.

REFERENCES

[1] P. L. Joskow, “California’s electricity crisis,” Oxford Review of Economic Policy, vol. 17, no. 3, pp. 365–388, 2001.
[2] E. Mallada, C. Zhao, and S. Low, “Optimal load-side control for frequency regulation in smart grids,” IEEE Trans. on Automatic Control, vol. 62, no. 12, pp. 6294–6309, 2017.
[3] C. Zhao, U. Topcu, N. Li, and S. Low, “Design and stability of load-side primary frequency control in power systems,” IEEE Trans. on Automatic Control, vol. 59, no. 5, pp. 1177–1189, 2014.
[4] X. Wu, J. He, Y. Xu, J. Lu, N. Lu, and X. Wang, “Hierarchical control of residential hvac units for primary frequency regulation,” IEEE Trans. on Smart Grid, vol. 9, no. 4, pp. 3844–3856, 2017.
[5] A. Kasis, E. Devane, C. Spanias, and I. Lestas, “Primary frequency regulation with load-side participation—part i: Stability and optimality,” IEEE Trans. on Power Systems, vol. 32, no. 5, pp. 3505–3518, 2016.
[6] L. Chen and S. You, “Reverse and forward engineering of frequency control in power networks,” IEEE Trans. on Automatic Control, vol. 62, no. 9, pp. 4631–4638, 2017.
[7] L. Gan, N. Li, U. Topcu, and S. H. Low, “Exact convex relaxation of optimal power flow in radial networks,” IEEE Trans. on Automatic Control, vol. 60, no. 1, pp. 72–87, 2014.
[8] P. Sule, S. Backhaus, and M. Chertkov, “Optimal distributed control of reactive power via the alternating direction method of multipliers,” IEEE Trans. on Energy Conversion, vol. 29, no. 4, pp. 968–977, 2014.
[9] D. K. Molzahn, F. Dörfler, H. Sandberg, S. H. Low, S. Chakrabarti, R. Baldick, and J. Lavaei, “A survey of distributed optimization and control algorithms for electric power systems,” IEEE Trans. on Smart Grid, vol. 8, no. 6, pp. 2941–2962, 2017.
[10] E. Dall’Anese, S. S. Guggilam, A. Simonetto, Y. C. Chen, and S. V. Dhillon, “Optimal regulation of virtual power plants,” IEEE Trans. on Power Systems, vol. 33, no. 2, pp. 1868–1881, 2018.
[11] A. Bernstein and E. Dall’Anese, “Real-time feedback-based optimization of distribution grids: A unified approach,” IEEE Transactions on Control of Network Systems, vol. 6, no. 3, pp. 1197–1209, 2019.
[12] M. Caramanis, E. Ntakou, W. W. Hogan, A. Chakrabortty, and R. Dhople, “Optimal regulation of virtual power plants,” IEEE Trans. on Power Systems, vol. 9, no. 4, pp. 3844–3856, 2017.
[13] A. Papavasiliou and I. Mezghani, “Coordination schemes for the integration of transmission and distribution system operations,” Proc. of Power Systems Computation Conference (PSCC), pp. 1–7, 2018.
[14] S. Bolognani and S. Zampieri, “On the existence and linear approximation of the power flow solution in power distribution networks,” IEEE Transactions on Power Systems, vol. 31, no. 1, pp. 163–172, 2016.
[15] A. Bernstein, C. Wang, E. Dall’Anese, J. Le Boudec, and C. Zhao, “Load flow in multiphase distribution networks: Existence, uniqueness, nonsingularity and linear models,” IEEE Trans. on Power Systems, vol. 33, no. 6, pp. 5832–5843, 2018.
[16] J. Koshal, A. Nedić, and U. V. Shanbhag, “Multiuser optimization: Distributed algorithms and error analysis,” SIAM Journal on Optimization, vol. 21, no. 3, pp. 1046–1081, 2011.
[17] D. P. Bertsekas and J. N. Tsitsiklis, Parallel and Distributed Computation: Numerical Methods. Prentice Hall Englewood Cliffs, NJ, 1989, vol. 23.
[18] S. Boyd, S. P. Boyd, and L. Vandenberghe, Convex Optimization. Cambridge University Press, 2004.
[19] X. Zhou, E. Dall’Anese, L. Chen, and A. Simonetto, “An incentive-based online optimization framework for distribution grids,” IEEE Trans. on Automatic Control, vol. 63, no. 7, pp. 2019–2031, 2018.
[20] R. D. Zimmerman and C. E. Murillo-Sánchez, “Matpower 6.0 user’s manual,” Power Systems Engineering Research Center, vol. 9, 2016.
[21] X. Zhou, M. Farivar, Z. Liu, L. Chen, and S. Low, “Reverse and forward engineering of local voltage control in distribution networks,” IEEE Trans. on Automatic Control, 2020.