LATEST QCD RESULTS FROM LEP

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We summarise the latest experimental QCD studies based on data from LEP. Measurements of the quark and gluon jet fragmentation functions are discussed, including a new algorithm to infer the properties of unbiased gluon jets. We describe a new test for destructive interference in the radiation of soft gluons from a three-parton system. Finally, we report the latest combined value of the strong coupling, measured using event shape observables.

1 Introduction

The LEP collider at CERN was used to study $e^+e^-$ annihilation at centre-of-mass energies in the range $\sqrt{s} = 91–209$ GeV, during the years 1989–2000. Four multi-purpose detectors (ALEPH, DELPHI, L3 and OPAL) were positioned at 90$^\circ$ intervals around the circular accelerator. Events of the type $e^+e^- \rightarrow Z^0/\gamma \rightarrow$ hadrons were used extensively to test QCD predictions, and to measure the colour factors and strong coupling. In this article, we outline four of the most recent studies performed by the LEP collaborations.

The text is organised as follows: in Sections 2 and 3, we report measurements of quark and gluon jet properties, published by the OPAL Collaboration. Section 2 deals with the scaling of fragmentation functions over a range of energies, for jet samples with a variety of flavour compositions, while Section 3 focuses on a new study of ‘unbiased’ gluon jets. In Section 4, we discuss an innovative test for the presence of destructive interference in the radiation of particles from a three-jet system, by the DELPHI Collaboration. Finally, in Section 5, we report the progress of a combined measurement of the strong coupling derived from event shape observables, performed by the LEP QCD Working Group.

We emphasise that the results discussed here do not constitute an exhaustive list of the recent and ongoing QCD studies by the LEP collaborations. In particular, the results of pentaquark searches, colour reconnection studies, and $\gamma\gamma$ physics will be omitted due to time constraints.
Scaling violations of quark and gluon jet fragmentation functions

The fragmentation function, \( D_a^h(x, Q^2) \), is defined as the probability that a parton \( a \), which is produced at a short distance of order \( 1/Q \), fragments into a hadron \( h \) carrying a fraction \( x \) of the momentum of \( a \). QCD predicts that the multiplicity of soft gluon emission from a gluon source should be higher than that from a quark source, due to the inequality of the colour factors \( C_A \) and \( C_F \). We therefore expect softer fragmentation functions for gluon jets. Furthermore, one can predict the dependences of the fragmentation functions on the scale \( Q^2 \), by means of the splitting functions \( P_{q\rightarrow qg} \sim C_F \) and \( P_{g\rightarrow gg} \sim C_A \): the scaling violations for gluon jets are expected to be larger than those for quark jets.

In a recent study by the OPAL Collaboration,\(^1\) the fragmentation functions have been measured for a variety of quark and gluon jet samples in \( e^+e^- \rightarrow (Z^0/\gamma) \rightarrow q\bar{q}(g) \) interactions at \( \sqrt{s} = 91.2 \) and 183–209 GeV. There are two experimental approaches to the identification of jets. Firstly, one can use a jet-finding algorithm (the Durham algorithm in this case): jets obtained by this method are biased, because their properties depend on the choice of jet finder, and on its associated parameters. Alternatively, jets may be defined as inclusive hemispheres of particles in a back-to-back \( q\bar{q} \) or \( gg \) system: these unbiased jets correspond to the definitions commonly used in theoretical calculations. The new OPAL measurements include seven types of fragmentation functions: the udsc, b, gluon and flavour-inclusive fragmentation functions for biased jets, and the udsc, b and flavour-inclusive quark fragmentation functions for unbiased jets. A previous study of unbiased gluon jets has been performed with rare OPAL events of the type \( e^+e^- \rightarrow q\bar{g}_{\text{incl}}, \) in which the gluon \( 'g_{\text{incl}}' \) is identified as the hemisphere opposite to a hemisphere containing two tagged b quark jets which are almost collinear.\(^2\) The new results presented here for biased gluon jets will be compared with the older results for unbiased jets. An alternative approach to unbiased gluon jets will be discussed in the next section.

Three methods have been used to distinguish between quark and gluon jets, and between different flavours of quark jets. In the b-tag method, a neural network based algorithm is used to select b quark jets in three-jet events. The untagged jet in an event containing two b-tagged jets (or the lowest-energy untagged jet in an event with one b-tag) is selected as a biased gluon jet. In events with no b-tagged jets, all three jets are selected as biased udsc quark jets; a correction is applied to remove gluon jets from the sample. In the energy-ordering method, the jets 1, 2 and 3 of a three-jet event are ordered such at \( E_1 > E_2 > E_3 \). Jet number 2 is assigned to the biased flavour-inclusive quark jet sample, while jet number 3 is assigned to the biased gluon jet sample. Finally, in the hemisphere method, a b-tagging algorithm is applied to an inclusive sample of hadronic events. Each event contains two hemispheres, which are regarded as unbiased udsc or b quark jets. In all three methods, an unfolding procedure is employed to correct for impurities in the selection.

For experimental purposes, we define the scale \( Q \) for a biased jet of energy \( E_{\text{jet}} \) to be \( Q_{\text{jet}} = E_{\text{jet}} \sin(\theta/2) \), where \( \theta \) is the angle to the closest jet. In the case of unbiased jets, we take \( Q = \sqrt{s}/2 \). The momentum fraction \( x \) is replaced by the quantity \( x_E = E_h/E_{\text{jet}} \), where \( E_h \) is the energy of the hadron and \( E_{\text{jet}} \) is the energy of the jet to which it is assigned.

Next-to-leading order QCD predictions for the fragmentation functions have been calculated by three groups: Kniehl, Kramer and Pötter (KKP),\(^3\) Kretzer (Kr),\(^4\) and Bourhis, Fontannaz, Guillet and Werlen (BFGW).\(^5\) The predictions correspond to unbiased jets, and are derived using the DGLAP evolution equations, from a set of measured fragmentation functions at a fixed input scale.

The measured scale dependences for udsc, b and gluon fragmentation functions are shown in Figures 1 and 2, for different ranges of \( x_E \). For the udsc quark fragmentation functions, good agreement is found with the NLO predictions, except in the lowest and highest regions of \( x_E \). The agreement is poorer in the case of b quark and gluon jets; however, the gluon jet
Figure 1: Scale dependence of the udsc and b jet fragmentation functions in different bins of $x_E$. The ‘scale’ denotes $Q_{jet}$ for the biased jets and $\sqrt{s}/2$ for the unbiased jets. The inner and outer error bars indicate statistical and total uncertainties respectively. The data are compared to NLO predictions by KKP, Kr, and BFGW.
fragmentation functions do exhibit stronger scaling violations than the quark jets, as expected. Good agreement is found between the biased and unbiased jet samples, suggesting that $Q_{\text{jet}}$ is an appropriate scale in events with a three-jet topology. The results obtained using the b-tag and energy-ordering methods are also consistent with one another, and with previous results from DELPHI and OPAL.

3 Studies of unbiased gluon jets using the jet boost algorithm

As we have mentioned in Section 2, a distinction exists between the biased jets obtained from experimental data using a jet-finding algorithm, and the unbiased jets used in theoretical calculations. An unbiased gluon jet would correspond to one hemisphere of a back-to-back $gg$ system, which is not seen in $e^+e^-$ annihilation. Instead, rare events of the type $e^+e^- \rightarrow q\bar{q}g_{\text{incl}}$ have been used, in which the $q$ and $\bar{q}$ jets are approximately collinear, leaving an unbiased ‘$g_{\text{incl}}$’ jet in the opposite hemisphere. Another OPAL study has used a more indirect method, whereby the results obtained from two-jet $q\bar{q}$ events are subtracted from those in $q\bar{q}g$ events. Unbiased gluon jets have also been obtained from radiative $\Upsilon \rightarrow \gamma gg$ decays at CLEO. Recently, however, a new approach known as the jet boost algorithm has been considered in $e^+e^-$ annihilation. In this method, a $q\bar{q}g$ system is decomposed into two independent $qg$ and $\bar{q}g$ dipoles. The dipoles are boosted into a symmetric frame, such that the angle $2\alpha$ between the $q$ and $g$ is the same as that between the $\bar{q}$ and $g$, as shown in Figure 3(a). Further Lorentz boosts $\beta = \cos \alpha$ are then applied independently to the two dipoles, such that they are each back-to-back, as in Fig-
Figure 3: The jet boost algorithm

Figure 4: The mean charged particle multiplicity of unbiased gluon jets, \( \langle n_{\text{gluon}}^{\text{ch}} \rangle \), as a function of the jet energy \( E_g^* \).

ure 3(b). Finally the two dipoles in their different frames can be recombined, yielding an event with the colour-structure of a gg system in a colour singlet state, as shown in Figure 3(c). The hemisphere containing the gluon in this event corresponds to the theoretical definition of an unbiased gluon jet. Unlike the unbiased quark jets discussed in Section 2, the energies \( E_g^* \) of these gluon jets are not fixed by the \( e^+e^- \) centre-of-mass energy.

The jet boost algorithm has recently been studied by the OPAL Collaboration, using \( e^+e^- \rightarrow q\bar{q}g \) events at the \( Z^0 \) resonance. Gluon jets are selected using a combination of b-tagging and energy-ordering; cuts then are imposed on the softness and collinearity of the quark and gluon jets. Using the HERWIG Monte Carlo event generator, it has been established that the unbiased gluon jets obtained using the jet boost algorithm should have properties consistent with a true back-to-back gg pair. It has also been shown that the properties are essentially independent of the jet-finder used to identify the q, \( \bar{q} \) and g jets, as expected for an unbiased jet.

The mean charged multiplicity of the unbiased gluon jets, \( \langle n_{\text{gluon}}^{\text{ch}} \rangle \), is shown in Figure 4, as a function of the jet energy \( E_g^* \). The results are found to be consistent with previous OPAL measurements based on the ‘g\_incl’ and ‘subtraction’ methods, and with the HERWIG prediction for genuine gg pairs. QCD evolution fits are also performed for two calculations: one based on next-to-next-to-next-to-leading order (3NLO) perturbation theory for a scale-dependent \( \alpha_s \), and the other based on an exact solution for a fixed \( \alpha_s \). Two free parameters are used in each case, and no hadronisation corrections are applied. The QCD scale parameter \( \Lambda \) is hence found to be \( \Lambda = 0.296 \pm 0.037 \) (total) GeV, compared to the corresponding quark jet result \( \Lambda = 0.190 \pm 0.032 \) (stat.) GeV. Similar analyses have been performed for the first two non-trivial factorial moments of the charged particle multiplicity distribution, \( F_{2,\text{gluon}} \) and \( F_{3,\text{gluon}} \).

Fragmentation functions have also been obtained for the unbiased gluon jets, at energies...
$E_g^* = 14.24$ and 17.72 GeV. Since the OPAL Collaboration has previously measured the fragmentation functions for unbiased gluon jets at 40.1 GeV$^2$ and for unbiased quark jets at 45.6 GeV$^2$ the DGLAP evolution equations can be used to construct a QCD prediction for the new measurements at lower energies. Such predictions are dependent on the strong coupling $\alpha_s$, which can therefore be extracted from a one-parameter fit. The fits are in good agreement with the measured fragmentation functions, yielding $\alpha_s(M_Z) = 0.128 \pm 0.008$ (stat.) $\pm 0.015$ (syst.). Although not competitive with other measurements of the strong coupling, this result is compatible with the world average, and has provided a unique consistency test of QCD.

4 Coherent soft particle production in $Z^0$ decays into three jets

Interference effects are fundamental to all quantum mechanical theories, including the gauge theories of the Standard Model. Evidence for coherence effects in QCD comes, for example, from the “hump-backed plateau” in the logarithmic scaled momentum spectrum of hadrons, due to suppression of low energy particle production, and from the string effect in three-jet $e^+e^-$ annihilation events. However, there are arguments against the conclusiveness of this evidence.

In a new study by the DELPHI Collaboration, a direct test is made for the presence of destructive interference in the radiation of soft gluons from a three-jet system. When a quark emits a hard gluon at a small opening angle, the quark-gluon system may behave on large distance scales as a single entity. A soft gluon radiated perpendicular to the $q\bar{q}$ pair will not be able to resolve the individual colour charges of the quark and gluon; it should therefore be regarded a coherent emission from the parton ensemble. The leading order cross section, $d\sigma_3$, has been calculated for coherent soft gluon emissions perpendicular to the plane of a $q\bar{q}g$ system: 

$$d\sigma_3 = d\sigma_2 \cdot \frac{C_A}{4C_F} \left[ \frac{1}{N_c^2} \hat{q}g + \frac{1}{N_c^2} \hat{q}q \right].$$

Here $d\sigma_2$ represents the corresponding cross section for soft gluon emissions perpendicular to the axis of a two-jet ($q\bar{q}$) event, and the antenna function $\hat{i}j$ for a pair of partons $(i,j)$ with an opening angle $\theta_{ij}$ is defined by $\hat{i}j = 1 - \cos \theta_{ij}$. The last term of Equation (1), inversely proportional to the square of the number of colours $N_c = 3$, is due to destructive interference. By measuring the ratio $d\sigma_3/d\sigma_2$ in samples of two- and three-jet events, one should be able to verify this $(1/N_c^2)\hat{q}q$ interference term, and to determine the colour factor ratio $C_A/C_F$ based on the leading order expression given above.

Hadronic $Z^0$ decays were recorded by the DELPHI detector in $e^+e^-$ annihilation at $\sqrt{s} = 91$ GeV. The angular ordered Durham algorithm was used, with a fixed resolution parameter $y_{cut} = 0.015$, to determine the number of jets in each event. For each three-jet event, the charged particle multiplicity $N_3$ was measured in a $30^\circ$ cone oriented perpendicular to both sides of the event plane. The corresponding multiplicity $N_2$ was also measured in two-jet events, for a cone perpendicular to the event axis; the azimuthal angle of the cone was chosen randomly in this case. In Figure 5, we show the three-jet cone multiplicity $N_3$ as a function of the inter-jet angles $\theta_2$ and $\theta_3$, where $\theta_i$ is the angle between the two jets opposite to jet $i$, and the jets are energy-ordered such that $E_1 < E_2 < E_3$. From the measured value of $N_2$, a prediction can be calculated for $N_3$ using Equation (1): these predictions are shown in Figure 5, with and without the destructive interference term. The expression without the $(1/N_c^2)\hat{q}q$ term is incompatible with the data, while the fully coherent prediction is in good agreement. If the ratio of colour factors, $C_A/C_F$, is fitted to the data, one obtains the value $2.211 \pm 0.014$ (stat.) $\pm 0.053$ (syst.) with $\chi^2/ndf = 1.3$. Although this result is valid only at leading order, and does not include an estimate of the theoretical uncertainty, it is in astonishingly good agreement with the expectation $C_A/C_F = 2.25$. The leading order QCD prediction for the soft gluon multiplicity, including destructive interference, has therefore been verified convincingly by the data.
5 A combined measurement of $\alpha_s$ using event shape observables

Event shape observables have been used extensively to test QCD predictions, and to measure the strong coupling $\alpha_s$, in both $e^+e^-$ annihilation and deep inelastic scattering.\textsuperscript{16} The LEP collaborations have published measurements of $\alpha_s$ for events in the energy range $\sqrt{s} = 91$–209 GeV,\textsuperscript{17} using the following six event shape observables: thrust ($T$), heavy jet mass ($M_H$), $C$-parameter, total jet broadening ($B_T$), wide jet broadening ($B_W$), and the Durham two-to-three jet resolution parameter ($y_3$). For each distribution, an $O(\alpha_s^2)$ perturbative prediction is matched with an NLLA resummation and corrected for hadronisation effects using the PYTHIA Monte Carlo event generator. The QCD predictions are then fitted to the LEP data, with $\alpha_s$ as a free parameter. The LEP QCD Working Group has combined these into a single preliminary\textsuperscript{6} measurement of $\alpha_s$ at the $Z^0$ mass scale, with particular attention to estimation of theoretical uncertainties\textsuperscript{18} and to the treatment of correlations.\textsuperscript{17} The result is

$$\alpha_s(M_Z) = 0.1202 \pm 0.0003 \text{ (stat.)} \pm 0.0009 \text{ (expt.)} \pm 0.0013 \text{ (hadr.)} \pm 0.0047 \text{ (theo.)},$$

where the three systematic uncertainties are due to experimental effects (‘expt.’), Monte Carlo hadronisation corrections (‘hadr.’) and higher-order terms in the perturbative QCD predictions (‘theo.’). Our measurement is in good agreement with the current world average.\textsuperscript{19}

\textsuperscript{6}A publication is expected later in 2004, when measurements provided by the collaborations have become final.
Figure 6: *Left figure:* Measurements of the strong coupling \(\alpha_s(Q)\) using event shape observables at LEP. Each point is a combination of all available measurements at a single energy; the inner error bar represents the combined statistical uncertainty, while the outer bar is the total uncertainty. The curve indicates the running of \(\alpha_s(Q)\) predicted by QCD at three-loop order, based on our combined measurement of \(\alpha_s\) at the \(Z^0\) mass scale, \(\alpha_s(M_Z)\). The statistical and total uncertainties in \(\alpha_s(M_Z)\) are indicated by the dotted curve and grey band respectively.

*Right figure:* Combined measurements of \(\alpha_s(M_Z)\), using individual event shape observables.

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