Accretion in a Dynamical Spacetime and the Spinning Up of the Black Hole in the Gamma-Ray Burst Central Engine

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Abstract

We compute the evolution of a quasi-spherical, slowly rotating accretion flow around a black hole, whose mass and spin evolve adequately to transfer of mass and energy through the horizon. Our model is relevant for a central engine driving a long gamma-ray burst (GRB) that originates from the collapse of a massive star. The computations of a GRB engine in a dynamically evolving spacetime metric are important specifically due to the transient nature of the event, in which a huge amount of mass is accreted and changes the fundamental black hole parameters—its mass and spin—during the process. We discuss the results in the context of the angular momentum magnitude of the collapsing star. We also study the possible formation and evolution of shocks in the envelope, which may temporarily affect accretion. Our results are important for the limitations on the mass and spin range of black holes detected independently by electromagnetic observations of GRBs and gravitational waves. We speculate on the possible constraints for the final masses and spins of these astrophysical black holes. It is shown that the most massive black holes are not formed in a powerful GRB explosion if the cores of their progenitors were only weakly rotating.

Key words: accretion, accretion disks – black hole physics – gamma rays: general – hydrodynamics

1. Introduction

The collapsar model was originally proposed to describe long-duration gamma-ray bursts (GRBs; Woosley 1993; Paczynski 1998). In this scenario, the total energetics of an explosion is consistent with the amount of total binding energy of the progenitor star, while the duration of the event, time variability of the prompt phase, details of the afterglow emission, observations of host galaxies, and statistical studies in connection with supernovae may give further insight to the physics of collapsing massive stars (Podsiadlowski et al. 2004; Crowther 2007).

In general, all GRBs are presumably powered by the accretion of a rotationally supported torus onto a newly born black hole. The black hole itself should also rotate fast, in order to efficiently transport the power to remote jets. Both the Blandford–Znajek process and neutrino annihilation may act as the source of this power, and for these two mechanisms, their efficiencies are similarly scaled with the black hole spin (McKinney & Gammie 2004; Liu et al. 2015; Janiuk 2017). The process is further mitigated by the magnetic fields. For long GRBs, the rotation of the progenitor star is a key property supporting the accretion over relatively long activity periods and also sustaining the rotation of the black hole (Barkov & Komissarov 2008; Janiuk et al. 2008; Janiuk & Proga 2008).

In 2015, LIGO discovered gravitational waves from a binary black hole merger for the first time, marking the beginning of a new era in astrophysics (Abbott et al. 2016). This revolutionary discovery challenged our understanding of compact objects, including the formation of very heavy black holes from the direct collapse of massive stars (Spera et al. 2015). The process of collapse inevitably involves feeding the new black hole with mass and angular momentum. Because rotation of the black hole is essential for the occurrence of GRBs, an open question remains about how fast the collapsed black hole of a given final mass can rotate (see, e.g., Lopez-Camara et al. 2010; Battar & Lee 2016).

Or, in other words, whether black holes as heavy as those found to date by LIGO (GW150914, GW151226, LVT151012) can also posses a large spin, and hence could they have been in the past the progenitors of extremely energetic or ultra-long-GRB explosions. That could in principle be possible if the black hole had been spun up by its companion in a close binary system (Barkov & Komissarov 2010; Janiuk et al. 2013). The black hole spin constraints are, however, not directly available from the gravitational waveform analysis, which only gives the mass-weighted projection of their values, i.e., the so-called “effective spin” of the binary black hole merger (Cutler & Flanagan 1994).

Our study of star collapse accounts for the simultaneous numerical constraint for the black hole mass and spin. It can help obtain a more consistent picture of stellar-mass black hole formation. This is particularly important in light of currently available and future multimessenger observations, and the ongoing debate on the relevance of electromagnetic observations for the measurement of intrinsic black hole parameters (Pankov et al. 2017). In this work, we perform self-consistently a numerical simulation of the collapse process and black hole growth, considering the full general relativistic framework. We account for the change in black hole mass and spin during the accretion of mass–energy through the black hole horizon, and we evolve the spacetime as a sequence of Kerr solutions. The stellar structure is described using a slowly rotating, quasi-spherical flow, with the relativistic solution for the Bondi–Michel radial dependence of density and the specific angular momentum concentrated at the equator (Janiuk & Proga 2008; see also, e.g., Mach et al. 2018). Our simulations utilize the changing Kerr metric coefficients and black hole growth, which we have implemented within the HARM code (High Accuracy Relativistic Magnetohydrodynamics; Gammie et al. 2003).

Our study aims to probe the influence of the amount of angular momentum in the quasi-spherical flow during the star’s collapse on the final black hole properties. We follow the
assumption that only fast spinning black holes, accompanied by the action of a rotationally supported “mini disk” embedded in the collapsar envelope, may drive the long-GRB emission (Janiuk et al. 2008). In addition, the possible existence and evolution of shock fronts, which are common features in the low-angular momentum black hole accretion flows (Chakrabarti & Das 2001; Das 2002; Sukova & Janiuk 2015; Sukova et al. 2017), are studied. The occurrence of these shocks may affect accretion and lead to a transient variability and other potentially observable effects in long GRBs. Finally, we speculate that our computations may help solve the puzzle of the formation of the most massive stellar black holes as observed by LIGO, and maybe put an independent constraint on their possible maximum spin values.

2. Evolution of the Black Hole Mass and Spin

The model computations are based on the axisymmetric, general relativistic MHD code HARM, described by Gammie et al. (2003) and Noble et al. (2006). We investigate the evolution of an axisymmetric, slowly rotating, non-magnetized plasma accreting onto a black hole. Its key parameters (mass and spin) evolve with time as for a GRB central engine, where the transient accretion episode results in quickly changing physical conditions, leading to black hole growth. We describe the collapsar evolution, taking into account the changing Kerr metric due to the evolving mass and spin of black holes (see also Hamersky & Karas 2013, who studied the runaway instability of relativistic tori around accreting black holes). In this sense, we significantly expand our previous calculations of the GRB central engine, which were conducted in a fixed background metric as suited for short GRBs (Janiuk et al. 2013; Janiuk 2017).

In recent simulations of binary neutron star mergers, the evolution of spacetime is fully coupled with the evolution of matter until the stars merge and a compact remnant is formed. The massive compact remnant then dominates the spacetime evolution in the system, while the contribution of the surrounding disk is not very large due to its small mass (see, for instance, Kastaun et al. 2017). Recent collapsar simulations, on the other hand, involve the conservation equation for the stress–energy tensor, including the fluid and radiation fields, and the metric evolution is followed through the standard BSSN method. The black hole growth is, however, not followed further after core collapse (Ott et al. 2018), or the black hole is found and diagnosed by means of the baryon mass enclosed inside a certain (i.e., Schwarzschild) radius. Its mass is then fixed and the black hole is not rotating, so the metric is frozen (Kuroda et al. 2018).

Solving the full set of Einstein equations that describe the collapsar evolution in a dynamical spacetime for the entire duration of the event is a very complex task. In our paper, we use a simple physical picture of the collapsing massive star, and we start our simulation after the black hole has already formed. We assume that its gravitational field determines the subsequent spacetime evolution. However, instead of using a stationary metric with the black hole having constant mass and spin and simply allowing the matter to accrete onto it, we follow the sequence of stationary solutions with black hole mass and spin parameters updated by a very small value in each time step. This approach assumes that the effect of the extended matter is still not as dynamically important as the effect of the black hole on the spacetime evolution (see, e.g., Semerak & Sukova 2010 and

\[
J = \int d\theta d\phi \sqrt{-g} \ T_{\phi\phi},
\]

\[
M = \dot{E} = \int d\theta d\phi \sqrt{-g} \ T_{t\phi},
\]

where the components of the stress–energy tensor of the accreting matter, \( T_{\mu\nu} \), are integrated over the black hole horizon.

The changing black hole spin and mass subsequently affect the spacetime metric. The mass growth is discretized and updated in every time step according to Janiuk et al. (2008):

\[
\Delta M = M_{BH}^{\text{curr}} - M_{BH}^{\text{old}},
\]

where \( M_{BH}^{\text{old}} \) is the initial mass of the black hole, and the current mass is given by integrating the rest-mass flux over the horizon at every time step:

\[
M_{\text{BH}}^{\text{curr}} = M_{\text{BH}}^{\text{old}} + \int_{r=r_{in}} dM_{in} 2\pi d\theta \sqrt{-g} \Delta t,
\]

where

\[
dM_{in} = -\rho u^r.
\]

We subsequently update the six relevant coefficients of the \( g_{\mu\nu} \) metric in the Kerr–Schild form, which are dependent on the central mass and are also sensitive to the spin change, namely

\[
g_{tt} = -1 + 2(1 + \Delta M)\frac{r}{r^2 + a^2 \cos^2 \theta},
\]

\[
g_{rr} = 2(1 + \Delta M)\frac{r}{r^2 + a^2 \cos^2 \theta},
\]

\[
g_{\theta\theta} = -2(1 + \Delta M)ar\frac{\sin^2 \theta}{r^2 + a^2 \cos^2 \theta},
\]

\[
g_{rr} = 1 + 2(1 + \Delta M)\frac{r}{r^2 + a^2 \cos^2 \theta},
\]

\[
g_{\phi\phi} = -a \sin^2 \theta (1 + 2(1 + \Delta M)\frac{r}{r^2 + a^2 \cos^2 \theta})
\]

\[
+ a^2 \sin^2 \theta \left(1 + 2(1 + \Delta M)\frac{r}{r^2 + a^2 \cos^2 \theta} \right).
\]

The change of the spin parameter of the black hole is computed as

\[
a' = a^{-1} + \left( \frac{J}{M_{BH}^{\text{curr}}} - \frac{a^{-1}}{M_{BH}^{\text{curr}}} \dot{E} \right) \Delta t.
\]
Note that the original HARM code works in dimensionless units, \( G = c = M_{\text{BH}} = 1 \), so they do not appear in the metric coefficients explicitly. The current mass of the black hole is given by \( M_{\text{BH}}^{\text{cur}} / M_{\text{BH}}^{\text{0}} = 1 + \Delta M \).

The spin parameter \( a \) here is not dimensionless, but it has units of \( M_{\text{BH}} \). The spatial and time coordinates are also measured in units of the black hole mass \( M_{\text{BH}} \). The dimensionless black hole spin, defined as \( s = a / M = J / M^2 \), is the one for which Equation (1) holds. Hence, by determining the derivative of \( a, da/dt = (ds/dt) M + s (dM/dt) \), we get relation (7) for the black hole spin evolution.

In the following sections, we consider a specific astrophysical scenario for the collapsing star and GRB progenitor, so that physical units will be chosen. We scale our calculations in a such way that the mass of the star is in the range of a few tens of solar mass. In order to compute the mass increase properly, we evolve the ratio \( \Delta M \). We consider an initial black hole mass of a certain value, \( M_{\text{BH}}^{\text{0}} = 3M_{\odot} \), which is on the order of an initial iron-core mass in the collapsing star.

### 3. Initial Conditions and Dynamical Model

Our initial conditions consider the case relevant to long gamma-ray bursts. The initial condition is prescribed as a slowly rotating spherical cloud of matter, given by the Bondi solution in the Kerr metric, and supplied with a small angular momentum. The Bondi flow extends from the outer boundary of the grid, typically located at a thousand gravitational radii, to the black hole horizon, and is parameterized by the location of the sonic radius. We integrate the initial density and radial velocity profiles upwards and downwards from the sonic radius.

The angular momentum is prescribed as a fraction of the critical angular momentum, which is the value at the circularization radius for a test particle orbiting a black hole. Here, the energy and angular momentum are given by

\[
\varepsilon = \frac{1 - 2/r + a/r^{3/2}}{\sqrt{1 - 3/r + 2a/r^{3/2}}}, \quad (8)
\]

\[
l = \frac{r^{1/2} - 2a/r + a^2/r^{3/2}}{\sqrt{1 - 3/r + 2a/r^{3/2}}}. \quad (9)
\]

We start all simulations in this work with non-spinning black holes, and in most cases we take the value of the angular momentum at the last stable orbit for a non-spinning black hole (i.e., at \( r = 6 r_{\text{s}} \)), which is then mapped onto the entire radial grid. The angular velocity in the Boyer–Lindquist coordinates is then given by

\[
u^\phi = g^\psi (-\varepsilon) + g^\phi l, \quad (10)
\]

where \( g^\psi = -2ar/(\Sigma \Delta) \) and \( g^\phi = (\Delta - a^2 \sin^2 \theta)/(\Sigma \Delta \sin^2 \theta) \), with \( \Sigma = r^2 + a^2 \cos^2 \theta \) and \( \Delta = r^2 - 2r + a^2 \). In addition, we want to scale the rotation of the cloud (collapsing envelope) to have a maximum at the equatorial plane and to tend to zero at the poles; hence, we introduce an additional factor of \( \sin^2 \theta \). Finally, we scale our models with a parameter defining the ratio between our collapsar’s angular momentum and the above (critical) angular momentum value at the circular orbit. Therefore, we use

\[
l_{\text{spec}} = S \frac{\langle u^\phi \rangle \sin^2 \theta}{u_t}, \quad (11)
\]

with \( u^\phi \) defined in Equation (10) and \( S = l / l_{\text{crit}} \) being a model parameter, in principle larger, or smaller, than unity.

The initial black hole mass is assumed fixed and equal to \( M_{\text{BH}} = 3M_{\odot} \). The total mass of the surrounding gas is computed taking into account the density scaling unit, which gives a cloud of mass \( M_{\text{cloud}} \approx 25 M_{\odot} \) contained within a Bondi sphere of size \( R_{\text{out}} = 1000 r_{\odot} \). This reflects the core-collapse scenario for the central part of a massive evolved star, which is contained within some \( 5 \times 10^8 \) cm, while the outer layers that contain mostly the light hydrogen envelope are ignored in our simulations, for practical reasons (cf. Janiuk & Proga 2008).

The sonic point is chosen to be a parameter of our initial configuration and determines the density and velocity profile according to the transonic Bondi solution. In our models, the sonic radius is initially located at \( r_s = 80 r_{\odot} \), well inside the computational domain. The models assume a weak rotation profile as described above, but ignore the magnetic fields. The equation of state adopted in our simulations is given by \( P = (\gamma - 1)u \), where \( P \) is the pressure and \( u \) the internal energy of the gas, and we use the adiabatic index of \( \gamma = 4/3 \).

To follow the evolution of the gas dynamics near a black hole, we use the numerical MHD code HARM-2D (Gammie et al. 2003; Noble et al. 2006). The original code was designed to solve magnetohydrodynamic equations in the stationary metric around a black hole. The code is written in a conservative, shock-capturing scheme, and has low numerical viscosity.

In this work, we modify the original code to account for the change of the background metric, according to the amount of mass sinking under the horizon, and supplementing the black hole with mass and energy–momentum. The metric is updated every dynamical time step during the simulation. To speed up the computations and obtain better efficiency, we use our own parallelization scheme, based on Open-MP and optimized through the distribution of the processes among the physical system directions.\(^8\)

The grid of all models presented here have a 256 \times 256 point numerical resolution in the \( r \) and \( \theta \) directions. The grid is logarithmic in radius and condensed in the polar direction toward the equatorial plane, as in Gammie et al. (2003).

### 4. Results

The full list of models and their parameters is given in Table 1. Note that the CL and SL, as well as the CT and ST runs, have the same values of the physical parameters. They differ from each other with respect to the final time of the evolution. Still, information about the collapsar properties at the earlier end time of the simulation is important, because for some particular values of the parameters, the longer simulation runs were leading to almost complete evacuation of the accreting cloud (at the level of the numerical density “floor”).

#### 4.1. Evolution of the Flow for Different Rotation Magnitudes

The mass and spin of the black hole grew in time during the dynamical simulations, according to Equations (1) and (2). As expected, the fastest growth of the black hole mass occurs in the case of almost non-rotating flow, when the matter inside the cloud flows in supersonically through the horizon from all directions. The rotation halts the gas particles that are closer to

\(^8\) Provided by the function MPI_Dims_create(nprocs, 2, dims) using an appropriate divisibility algorithm.
the equatorial plane and form a kind of “mini disk” there. In models with higher $S$, the black hole mass grows only due to the accretion of gas from the polar direction.

Figure 1 presents the results of both mass and dimensionless spin evolution for the three models, with subcritical, $S = 0.4$; critical, $S = 1.0$; and supercritical, $S = 1.4$, rotations. The critical value of $S$ means that the condition for the formation of a long-living mini disk is satisfied. Still, only in the model with $S = 1.4$ does the black hole spin increase, up to the value of $s = a/M \approx 0.99$, and it remains high until the end of our simulation run. In the subcritical rotation case, the larger the rotation parameter, the larger the black hole spin value obtained at its maximum. For $S = 1.0$, the black hole temporarily achieves a spin as high as 0.99. However, the initial fast rotating bubble is accreted within 1 second. Then, the spin of the black hole decreases again and saturates at $s = a/M \approx 0.65$. For the slowest rotation model, $S = 0.4$, the black hole spin at peak reaches the value of 0.35, and then decreases to about 0.2.

We also checked if the maximum value of the spin is sensitive to the chosen circularization radius (cf. runs RL in Table 1). In most of our models, $r_c = 6$ was chosen. We

$$M_{\text{in}} = 3M_\odot$$

**Table 1**

Summary of the Models

| Model | $r_c$ | $S$ | $M_{\text{end}}^\text{cloud}$ | $\rho_{\text{end}}$ | $a_{\text{end}}$ | $s_{\text{end}}$ | Metric Change | $M_{\text{end}}^\text{cloud}$ | $M_\text{in}$ | $M_\text{out}$ |
|-------|------|-----|------------------------------|-----------------|--------------|-------------|---------------|-----------------|-------------|--------------|
| CL-04 | 6    | $0.4$ | 13.53 | 0.8 | 0.89 | 0.19 | yes | 2.69 | 10.53 | $10^{-19}$ |
| CL-10 | 6    | $1.0$ | 9.49  | 0.8 | 2.22 | 0.70 | yes | 7.39 | 6.5 | $6 \times 10^{-4}$ |
| CL-14 | 6    | $1.4$ | 4.52  | 0.8 | 1.21 | 0.80 | yes | 20.51 | 1.52 | 1.11 |
| SL-04 | 6    | $0.4$ | 14.51 | 3.0 | 0.98 | 0.20 | yes | 0.0007 | 11.51 | 0.0004 |
| SL-10 | 6    | $1.0$ | 11.88 | 3.0 | 2.53 | 0.64 | yes | 0.014 | 8.88 | 0.002 |
| SL-14 | 6    | $1.4$ | 6.27  | 3.0 | 2.09 | 0.99 | yes | 9.98 | 3.27 | 1.82 |
| CT-04 | 6    | $0.4$ | 3.0   | 0.8 | 0 | 0 | no | 16.61 | 5.09 | $10^{-16}$ |
| CT-10 | 6    | $1.0$ | 3.0   | 0.8 | 0 | 0 | no | 19.77 | 2.26 | 0.52 |
| CT-14 | 6    | $1.4$ | 3.0   | 0.8 | 0 | 0 | no | 20.97 | 1.31 | 1.54 |
| ST-04 | 6    | $0.4$ | 3.0   | 3.0 | 0 | 0 | no | 2.34 | 13.72 | $10^{-16}$ |
| ST-10 | 6    | $1.0$ | 3.0   | 3.0 | 0 | 0 | no | 8.05 | 7.73 | 0.51 |
| ST-14 | 6    | $1.4$ | 3.0   | 3.0 | 0 | 0 | no | 13.82 | 4.50 | 2.36 |
| RL-04 | 10   | $0.4$ | 14.52 | 3.0 | 1.07 | 0.22 | yes | 0.0007 | 11.52 | 0.005 |
| RL-10 | 10   | $1.0$ | 6.38  | 3.0 | 2.12 | 0.99 | yes | 0.49 | 3.38 | 1.57 |
| RL-14 | 10   | $1.4$ | 6.82  | 3.0 | 2.00 | 0.88 | yes | 7.81 | 3.82 | 4.27 |

Note. Mass is given in units of $M_\odot$, radius in units of $GM/c^2$. The end time of the simulations is given in seconds for a black hole mass of $3M_\odot$. Mass lost through the inner boundary is integrated over the time of the simulation and given in units of solar mass. $M_{\text{end}}^\text{cloud}$ is equal to $3M_\odot$ in all runs, and $M_{\text{cloud}}^0 = 25M_\odot$ and $a^2 = 0$ initially.
checked that when the value of $r_c$ is larger, then the mini-disk size is larger, if the rotation is equal to or larger than the critical value.

The spin-up of the black hole occurs more effectively only in the case of critical rotation, and the final spin at the end of the run RL-10 is at the maximum Kerr limit. For supercritical rotation, the mass lost through the outer boundary due to the centrifugal force was in fact larger than the mass accreted onto the black hole. And since the rotationally supported mini disk kept the material in orbit and the increase in the BH spin per unit mass was lower, the final value of the dimensionless spin was even smaller in this run (RL-14) than for the generic model (SL-14).

Figure 2 shows the accompanying change in the accreting cloud total mass, as integrated over the simulation volume. The parameters of our model were chosen such that the initial mass was always equal to $25 M_\odot$. The actual mass of the cloud decreases then due to accretion (inflow through the horizon), but also due to the outflow through the outer boundary. The final mass depends on how much matter is still halted by the mini disk or has not been accreted by the end of the simulation. For subcritical rotation, the cloud mass will asymptotically drop to zero; however, the long simulation runs show that a small blob remains even at late times in the equatorial plane, when subsonic material arrives at the innermost regions and is decelerated by the rotation.

4.2. Comparison with the Static Metric Models

The mass and spin of the black hole, by definition, do not change in the static metric models. This setup, which is used for comparison and code testing, assumes that the mass is lost from the simulation volume through the inner (and also outer) boundaries; however, it neither contributes further to black hole growth nor to metric change.

The bottom panel of Figure 2 shows for comparison the evolution of the cloud mass in the stationary metric, when the black hole mass and spin are constant (cf. runs ST in Table 1). As shown in Figure 2, the static metric models result in a much shallower decrease of the accreting cloud mass. The mass lost through $r_{in}$ was of moderate amount. The mass of the cloud was also lost through the outer boundary, due to the free outflow boundary condition.

The simulations with metric evolution in general show that a lot more mass is accreted onto the black hole through the inner boundary, while the mass outflow at the outer boundary is rather negligible (see Table 1).

We also stress that the quantity conserved in the code is the comoving density, $D = \rho u^t$. This is why the numbers in columns 4, 9, 10, and 11 do not add up to the value of the initial cloud mass ($25 M_\odot$). We verified that the conservation scheme works to within a good accuracy in both setups. The volume-integrated $D$ is conserved, taking into account its outflow through the outer and inner boundaries, over the simulation time, with accuracies of 0.2% and 0.3% for the changing and static metric runs, respectively.

In general, for the static metric runs, one can notice a smoother decrease of the mass and slower accretion in the case of subcritically rotating flows. For the supercritical rotation, the mass decrease is similar in both cases, i.e., it does not drop to zero in either changing or static metric simulations. The final mass of the cloud at time $t = 3$ s is $M_c \approx 10 M_\odot$ and $M_c \approx 14 M_\odot$ for runs SL-14 and ST-14, respectively. We also note that a small jump occurs in these runs at $t = 0.25$ s; see Figure 2. This is attributed to the shock accretion and formation of the mini disk (see next section), but the magnitude of this jump is not affected by the metric evolution.

4.3. Mass Accretion Rate and “Mini-disk” Formation

In Figure 3, we show the time evolution of the mass accretion rate onto the black hole. The temporary accretion rate is large, due to the fast supersonic inflow, and for our parameters and normalization of the physical units, it is on the
order of a few tens of $M_\odot$ s$^{-1}$. The accretion rate for the subcritical rotation model is highest at peak and steeply grows with time at the beginning of the simulation. The subsequent decrease of the accretion rate occurs when there is almost no matter left, and the cloud is emptied out. The flow here is highly supersonic, and this is why the velocity of the inflow results in the rapid evacuation of cloud material and growth of the black hole. In the case of critical rotation, an additional effect is the temporary flickering of the accretion rate around $t \sim 0.37$ s (25,000 M),\footnote{Due to the choice $M_{\text{BH}}^0 = 3 M_\odot$, the conversion from geometrical units to physical units is such that 10,000 M corresponds to $\sim 0.148$ s.} which is connected to the vertical oscillations of the shock bubble. For supercritical rotation, the accretion rate is quite stable and constant with time during the oscillations of the shock bubble. For supercritical rotation, the flow is rather turbulent inside the disk, and gas with different amounts of angular momenta is mixed within the disk body. This manifests in the variability of the accretion rate at later times.

5. Appearance of Shocks in the Flows

In Figure 7, we show the profiles of the radial Mach number in the three simulations with different specific angular momenta, at the time $t = 0.44$ s (50,000 M). The sonic point, initially located at $r_s = 80 r_g$ in all the models, moves outwards and reaches about 500 $r_g$ at this time in the simulation for the subcritical rotation models, CL-04 and CL-10. The sonic surface diffusion is slower in case of the critical model CL-10, and the sonic radius at this time in the simulation is close to 300 $r_g$. For model CL-14, the sonic surface does not have a spherical shape. Most of the flow in the equatorial regions is subsonic, and the Mach number in the “mini disk” is rather small.

In Figure 8, we plot the zoomed-in distributions of the radial Mach number and sonic surfaces, as taken for the critically rotating model, CL-10. The time snapshots were taken in the range of $0.14-0.37$ s (which corresponds to the geometric times 10,000, 15,000, 20,000, and 25,000 M). As seen in Figure 3, the accretion rate onto the black hole at this time varies in this model. We notice that this is the effect of the mini shock bubble, which appeared close to the inner radius, below $\sim 50 r_g$. The shock bubble was subsequently accreted onto the black hole after that time. The evolution of the shock front position is described in more detail below.

In Figure 9, we present the evolution of the shock and sonic point positions with time, for models SL-04 and SL-10. All of these results are plotted at the equatorial plane. We note that the supersonic parts of the flow for model SL-14 also exist, but they are present in the polar regions, above and below the equator (cf. Figure 7).

The plots in Figure 9 encompass the time of evolution for long runs, up to $t = 1.48$ s (100,000 M). The cloud was initially filled with Bondi flow, and during the simulation time, matter was supplied to the black hole from the outer regions. As indicated in Table 1, at the end of the long runs, there was almost no gas left in the domain, unless it was kept in orbit by the rotation (run SL-14).

The shock position is computed from the profile of the radial Mach number, when the local sound speed is estimated as

$$c_s = \sqrt{\frac{4}{3} \frac{P}{\rho + 4P}},$$

which holds for the $\gamma = 4/3$ adiabatic index.

The local radial velocity is computed as follows. Under a change of coordinates, the $x^\mu \rightarrow x^\hat{\mu}$ components transform as

$$g_{\hat{\mu}\hat{\nu}} = \frac{\partial x^\mu}{\partial x^\hat{\nu}} \frac{\partial x^\nu}{\partial x^\hat{\mu}} g_{\mu\nu}.$$  

Therefore,

$$\left( \frac{du}{d\hat{\nu}} \right) = \begin{pmatrix} \frac{\partial u}{\partial \theta} & \frac{\partial u}{\partial \phi} \\ \frac{\partial u}{\partial \theta} & \frac{\partial u}{\partial \phi} \end{pmatrix} \left( \frac{du}{d\hat{\nu}} \right).$$

We transform the velocity from the code coordinates $x^\nu(x^0, x^1, x^2, x^3)$ to the velocity in the Boyer–Lindquist coordinates, $(t, r, \theta, \phi)$. The radial velocity in the code coordinates is $V_r = \frac{dx^1}{dt}$, and the radial velocity in the
Figure 4. Structure of the slowly rotating accretion flow, at time $t = 50,000$ M. The model assumes transonic flow with $r_s = 80 \ r_g$ and almost no rotation ($l/l_{\text{crit}} = 0.4$). The maps show the (i) density, (ii) temperature of the plasma, and (iii) specific angular momentum. Parameters: initial black hole mass $M_{\text{BH}} = 3M_\odot$ and initial mass in the cloud $M_{\text{cloud}} = 25.0 M_\odot$. The maps show the inner 500 $r_g$.

Figure 5. Structure of the slowly rotating accretion flow, at time $t = 50,000$ M. The model assumes transonic flow with $r_s = 80 \ r_g$ and slow rotation ($l/l_{\text{crit}} = 1.4$). The maps show the (i) density, (ii) temperature of the plasma, and (iii) specific angular momentum. Parameters: initial black hole mass $M_{\text{BH}} = 3M_\odot$ and initial mass in the cloud $M_{\text{cloud}} = 25 M_\odot$. The maps show the inner 500 $r_g$. 
Figure 6. Velocity fields at \( t = 50,000 \, \text{M} \), for the almost non-rotating \( (l/l_{\text{crit}} = 0.4, \text{left}) \), critical \( (l/l_{\text{crit}} = 1.0, \text{middle}) \), and weakly rotating \( (l/l_{\text{crit}} = 1.4, \text{right}) \) models. The cloud mass at this time is between \( M_{\text{cloud}} \approx 2.5 \) and \( 20 \, M_{\odot} \), depending on the rotation.

Figure 7. Distribution of the radial Mach number \( (v_r/c_s) \), at \( t = 50,000 \, \text{M} \) for the almost non-rotating \( (l/l_{\text{crit}} = 0.4, \text{left}) \), critical \( (l/l_{\text{crit}} = 1.0, \text{middle}) \), and weakly rotating \( (l/l_{\text{crit}} = 1.4, \text{right}) \) models. The thick solid line marks the sonic surface, i.e., the \( M = 1 \) value. The cloud mass at this time is \( M_{\text{cloud}} \approx 2.7 \, M_{\odot} \), or \( 7.4 \, M_{\odot} \), or \( 20 \, M_{\odot} \), respectively.
Boyer–Lindquist coordinates is $U_{bl}^r = dr/dt$. Therefore, $U_{bl}^r = \frac{dr}{dt} = \frac{dr}{dt} = \frac{dr}{dt}$.

In each model, one or more shock fronts develop during the evolution, which affect the accretion rate and subsequently the growth of the black hole mass and spin. The presence of the subsonic bubble (“mini disk”) changes the accretion rate along the equatorial plane, while the supersonic material accretes through the polar funnels. Because the shock bubble can develop vertical oscillation (see the results for the shock evolution in the case where the black hole has constant mass and spin, summarized in Sukova et al. 2017), the presence of the shock front typically leads to the flickering of the accretion rate with peaks reaching one order of magnitude.

For the subcritical case, SL-04, the shock develops only in the late stage of the simulation, at about $t = 1.5$ s ($100,000$ M). At that time, the remaining mass of the cloud is very low, several orders of magnitude lower than the initial mass, hence the accretion rate is also very low. Therefore, it does not influence the final mass and spin of the black hole.
In the critical case, SL-10, the rotation causes the very fast appearance of the inner shock bubble, which is growing in the equatorial plane. The subsonic region merges with the sonic point very quickly, at about $t = 0.03$ s (2000 M). After that time, accretion proceeds as subsonic flow. However, as the mass of the black holes grows, material becomes supersonic close to the black hole, especially along the axis, and eventually it encompasses the subsonic gas on the equator, where the rotation is the fastest, forming another shock bubble at about $t = 0.15$ s (11,000 M). This bubble halts part of the material before being accreted, so that the accretion rate slightly drops down temporarily at $t = 0.15$–0.22 s ($t \in (11,000–15,000)$ M).

The mass of the black hole is however still increasing, mainly due to the accretion through the polar regions, and after a while it reaches the regime where the shock bubble starts to oscillate. The vertical oscillations of the bubble again cause the flickering of the accretion rate; however, now the flickering appears during the time when the accretion rate is very high (see Figure 3 at $t \in (0.25; 0.35)$ s). During the next $\sim 0.15$ s (10,000 M), this shock bubble is accreted completely.

Between $t = 0.35$ s and $t = 1.5$ s (25,000–101,000 M), no shock is present in the flow, while the sonic surface is inflating up to about 700 $R_g$. At $t = 1.51$ s (102,000 M), yet another shock front appears, and this one is growing very fast, transferring the supersonic accretion in the subsonic one and causing more flickering. This happens, however, again in the very low accretion rate regime, and it is not substantial for black hole growth.

In the supercritical model, SL-14, due to the rotation, the shock forms and expands very quickly, so that it merges with the sonic surfaces, leaving almost the entire accretion flow subsonic. Only in the very vicinity of the black hole is there an inner sonic surface, which is no longer spherical. The locations of the sonic points depend on the altitude, and the characteristic shape of the sonic surface, close to the shape of “eight,” develops in the inner region of the shock bubble, as it is inflating. The flow inside the mini disk is fully subsonic, and the material becomes supersonic mainly along the black hole rotation axis.

In this case, the accretion rate is substantially lower than in the two previous cases, so that the remaining mass in the cloud shrinks more slowly. Hence, the whole process takes a longer time and is less energetic, leaving the black hole less massive, but spinning faster. The “eight” shape develops as the shock expands. As the rotating gas approaches the initial sonic surface, the rotation quickly modifies the surface shape. The cause of this shape is the slowdown of the gas in the equatorial region due to centrifugal force. In fact, the gas is pushed outward along the equator as the centrifugal force and gas pressure halt the inflow.

From comparison with Figure 1, one can notice that in the case of CL-10, there is a jump in spin at the time around $t = 0.3$ s. At that moment, the dimensionless spin is 0.99, and the spin decreases again to reach the stable value of about $s = 0.65$ (see Table 1). We also notice that the jump in the spin for the subcritical rotation model is not as high, and the stable value of the shock front position is achieved when the constant black hole spin saturates late in the simulation, at about $t = 1$ s (70,000 M).

The sonic surface for RL models moves outwards more slowly than that for generic SL runs. For instance, in the model with regularization radius $r_c = 10$ and critical rotation (i.e., run RL-10), the sonic radius at time $t = 50,000$ M (0.739 s) is located at about 200 $r_g$, while in the model $r_c = 6$ (run SL-10), the sonic radius at the same time is at about 400 $r_g$. The mass of the cloud was equal to about 15.5 $M_\odot$, and 7.3 $M_\odot$, respectively. In the case of the supercritical rotation models, as noted above, the sonic surface is not spherically symmetric; however, its maximum size is also much larger for run RL-14 at the end of the simulation than for run SL-14.

6. Summary

In this work, we calculated the general relativistic, hydrodynamical model of the collapse of the star’s central parts and the growth of the black hole due to accretion. We calculated the changing Kerr metric coefficients, due to the evolving mass and spin of the black hole, at every time step in the simulation. Such a scenario is relevant for new black hole formation and a long-GRB progenitor, provided that the resultant black hole spin is large enough to power the GRB jet via the Blandford–Znajek process. If not, the scenario is still applicable for the collapsar model without a powerful GRB emission. In this way, it is used to put a limit on the maximum spin of the black hole, whose mass has grown substantially during the collapse. Another constraint can be obtained for the maximum mass of the black hole, whose rotation is fast enough to power the GRB emission.

We also provided here the constraints for the formation of a rotationally supported mini disk in the center of the collapsar. In this context, an important issue is the radiative feedback that can occur during the collapse of a star into a black hole. It can easily deposit some additional energy in the stellar envelope and unbind the outer layers of the star (Batta et al. 2017). This will halt accretion, resulting in a less massive black hole that is even below the limit for the black hole mass obtained here (56% of the collapsing envelope mass, obtained for a supercritically rotating star). Our present model is still rather simplified in the sense that it ignores radiative processes, e.g., neutrino transport, and therefore the feedback mechanism is not accounted for. However, we can speculate that the most massive black holes detected by LIGO could have formed only if there was essentially no rotation in the progenitor star, hence all of the envelope falls into the black hole radially and no feedback occurs. Such a black hole would also rotate with a spin of $s < 0.2–0.3$ at maximum, and hence no powerful GRB would be associated with this event.

These limits are imposed by the amount of angular momentum in the star’s envelope. The masses of the merging black holes detected by the LIGO gravitational wave signals are larger than those typically found for stellar-mass black holes in X-ray binaries and have values between 10 and 30 $M_\odot$. The results of our computations suggest that their spins should be rather small. The details depend on the rotational properties of the evolved massive stars, which formed the LIGO black holes. As of today, no remnants of past GRB activities have been reported in the case of these black holes, so the picture presented in our work seems consistent with black holes being formed by the direct collapse of a very slowly rotating stellar core.
In our calculations, the central engine model of a GRB takes into account the non-stationary spacetime, due to the evolution of the black hole parameters. Previously, our computations were conducted in a fixed background metric in the context of a short-GRB central engine modeled with a high angular momentum torus accretion (see Janiuk et al. 2013; Janiuk 2017). The computations were based on the code that utilized the stationary geometry of spacetime, as is relevant for sub-Eddington accretion rates in active galactic nuclei, rather than GRBs. The present paper goes beyond such simplification. We have shown here that accounting for the metric change quantitatively affects the results of the speed of the accreting cloud evacuation. In particular, ignoring the black hole growth led to some overestimation of the mass loss from the system through the outer boundary, i.e., winds.

Previously, in Sukova et al. (2017), we also showed different regimes of accretion of low-angular-momentum flows in the case of a constant background Kerr metric. The existence of multisonic solutions is determined by the interplay of the mass and spin of the black hole and the angular momentum of the gas. Here, we study how the feeding of the black hole by accretion changes this picture. Because the black hole parameters are changing while the angular momentum of the gas does not, the mutual relation of the parameters also changes. Thus, different regimes of accretion and shock behavior arise during the evolution of the system.

The presence of shock fronts in the gas, on the other hand, influences the growth of the black hole and its spin-up. If the combination of the three parameters mentioned above leads to the formation of the shock, less material accretes along the equator, and the accretion rate could be temporarily lowered. If the shock bubble oscillates, flickering of the accretion rate occurs. Consequently, the time evolution of the black hole parameters is nonlinear. Depending on the rotation profile of the star, even when the total mass of the cloud (collapsar) is the same, the final values of the black hole parameters are affected by the shock behavior.

Because the process of accretion is believed to power the outgoing jets from the collapsar, the flickering and other changes in the accretion rate translate to jet propagation, thus leading to observable consequences. If the Lorentz factor of the outgoing blobs in the jets depends on the time-varying accretion rate, then shells with different speeds can be produced. Their collisions far away from the central engine can produce highly energetic particles and radiation (Piran 2004). In this way, the shock presence can be important for the details of the light curve and energy spectrum of the measured GRBs. Most important is the influence of shock on the total duration of the event, and we question whether that shock is able to halt accretion. The supercritical regime of rotation shown in our work proves that a maximally spinning black hole can be supported by the rotation of the mini disk as long as there is enough material in the collapsar envelope to be accreted through it. The accretion rate at late times in this model is non-zero, but it is highly variable. Here, however, the feedback mechanism mentioned above may quantitatively change the picture. The study of this mechanism is the subject of our future work.

We note that in this work we explored only a limited parameter space and the presented plots cover only their fiducial values. Our goal was to introduce the method, which is explored here for the first time as applied to the collapsar model, and to briefly highlight one of its possible outcomes. The initial mass of the black hole of $3 M_\odot$ is, however, a quite representative value, and agrees well with the results of the core-collapse simulations (cf. $M_{\text{BH}} = 2.6 M_\odot$; see Kuroda et al. 2018). As for the mass of the progenitor star, the numbers used in various works range from above 70 solar masses (Woosley et al. 2002) and down to 15 solar masses (Lentz 2015). The 70 solar mass pre-collapse star of Takahashi et al. (2014) in fact had mass enclosed up to the helium layer equal to 31 solar masses. Our choice of 25 solar masses being enclosed in our computational domain of the size of 1000 gravitational radii meets the above constraints. For the initial spin of the core, we presented here the simplest case of zero spin. We also checked that other values (such as $a_0 = 0.3$ or $a_0 = 0.6$) lead to a very similar qualitative behavior in the simulation and that the supercritical rotation case is the only one that results in the maximum final spin of the black hole. In this case, the black hole mass growth is only modest. The subcritical rotation models, on the other hand, result in more massive black holes, which are not spinning fast. In fact, the dimensionless spin value at the end of the simulation being smaller than the initial one is also possible if only the envelope were not rotating sufficiently fast.

We conclude therefore that the qualitative behavior of the flow does not depend much on these parameters and is a generic feature resulting from our approach. The main uncertainty lies in the assumed rotation profile of the collapsar. This profile is, however, the main unknown, and has not been solved yet by either the stellar evolution models or the massive stars observations.

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Appendix

Analytical solutions for low-angular-momentum black hole accretion in the evolving spacetime metric do not exist. However, in order to verify the performance of our code and test the method on the simplest examples, we performed two types of additional computations, in which the numerical solutions can be compared with those for the stationary Bondi case. The first type of test assumes zero, or very low, angular momentum accretion onto the black hole, when the mass of the accreting cloud is negligibly small in comparison with the mass of the black hole. We call this computation here the “Light Cloud” test. The second test assumes a heavy cloud, whose mass ratio to the initial black hole mass is the same as in the main text, but now we ignore the angular momentum. This computation is called “Pure Spherical” test.

Appendix A.1

Light Cloud Test

The first test assumes that the mass of the cloud, which surrounds the $3 M_\odot$ mass black hole and is contained within a
volume of size $R_{\text{out}} = 1000$ gravitational radii, is equal to $2.5 \times 10^{-6} M_\odot$. As shown in Figure 10, the mass accretion rate is very small, so that the mass of the black hole does not grow during the simulation. Neither changes the black hole spin. We checked that it stays equal to $a = 0$ throughout the simulation, and as a consequence, the metric update terms are practically not affected. Therefore, the solutions do not depend on the metric update routine, even though this part of the code is activated.

In Figure 11, we show the radial profiles of density and velocity in the cloud for several time snapshots during the simulation. In the left panels, it can be seen that the non-rotating flow conserves our initial transonic Bondi solution for the radial velocity. The density profile also has the same slope for all snapshots; however, its normalization decreases as the cloud is very slowly emptied (cf. Figure 10). The sonic point was initially located at $80 \, r_g$, and it moved outwards only slightly (at the end of the simulation, $t = 2.95 \, \text{s}$, it shifted to $\sim 120 \, r_g$). This is because the systematic decrease in density was not completely compensated by the decrease in pressure, since there is no matter supplied from the outer boundary.

For comparison, we show here also the rotating “Light Cloud” simulation. This model produced a local excess in the density profile, above the Bondi solution, because the matter is slowed down by rotation and accumulates. This part of the flow extends only up to about $100 \, r_g$. The flow here becomes almost completely subsonic in the equatorial plane, and the Mach number is equal to 1 only very close to the black hole ($r_s \approx 3 \, r_g$).
Appendix A.2

Pure Spherical Test

In this test, we analyze the purely spherical simulation, when the mass of the black hole grows very fast, as imposed by the large ratio of the accreting cloud mass to the mass of the black hole. We verify that the solution in general keeps the properties of the Bondi flow, with regard to the profiles of the density and radial velocity, as before. We checked that also in this test the spin of the black hole is constant and equal to zero. However, the black hole mass grows significantly, while a fast decrease of the cloud’s mass occurs. This is illustrated in Figure 12.

The figure shows for comparison the results for the code with the metric update routine deactivated. In this case, the mass accretion rate stays small at the beginning of the simulation and gradually decreases. It cannot be constant because of the lack of matter supplied from the outer boundary. In contrast, the update to the black hole mass and metric change results in a huge mass accretion rate at the beginning of the simulation. This is because the gravitational attraction of a growing black hole pulls the matter much faster.

Figure 11. Density and velocity as a function of time for a very light cloud accreting onto a 3 $M_\odot$ mass black hole. The label $S = 0.0$ denotes solutions for a non-rotating flow, while $S = 1.0$ involves slow rotation, imposed by the specific angular momentum as defined in Equation (11). The lines on each plot refer to several snapshots in time, starting from $t = 0$, until $t = 2.95$ s.

Figure 12. Density and velocity as a function of radius for a pure spherical test. The labels $S = 0.0$ and $S = 1.0$ correspond to non-rotating and slow-rotating flows, respectively. The lines on each plot show the profiles for different times $t = 0, 2.95$ s.

Figure 13. Profiles of density and velocity for several time snapshots, showing the effects of changing the metric. The left panel shows a constant metric, while the right panel shows a changing metric. The slopes of the density profiles are constant for the first $\sim 0.5$ s, and the flow remains transonic;
however, the black hole mass growth is very dynamic. Later on, the cloud is emptying and accretes very slowly onto the black hole, whose mass is already constant. The Bondi profile of the density is reproduced, but for purely supersonic flow. The position of the sonic point is not constant, because it depends on both the mass of the black hole and on the sound speed at infinity. After the first ∼0.15 s, the black hole mass grows by about 1.5 times, while the sonic point shifts from 80 $r_g$ to 140 $r_g$, i.e., by 1.75 times. However, at the end of the simulation, when the mass of the black hole increased by a factor of 5, the sonic point reached almost the outer boundary of the grid and was located at a radius more than 11 times larger than that initially. This fact can be explained as the response of the accreting matter to the growing mass of the black hole and the metric change. Both density and pressure have to adjust to the new conditions.

The profiles of the velocity conserve their constant slopes, the same as in the initial condition, while the magnitude of the velocity systematically increases. We note that the apparently superluminal velocities are reached below the current black hole horizon ($r_H = 2M$ for a Schwarzschild black hole). It shows, therefore, that the black hole mass growth by about five
times (from $3 M_\odot$ up to $\sim 15 M_\odot$) resulted in the proportional growth of the horizon size, as marked by the black points on the right panel of Figure 13. We conclude that this test confirms the consistency of our simulation, because the radius of the black hole horizon, in contrast to the sonic radius, should depend only on the black hole mass.

**References**

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016, PhRvL, 116, 061102
Barkov, M. V., & Komissarov, S. S. 2008, MNRAS, 385, L28
Barkov, M. V., & Komissarov, S. S. 2010, MNRAS, 401, 1644
Batta, A., & Lee, W. H. 2016, MNRAS, 459, 2140
Batta, A., Ramirez-Ruiz, E., & Fryer, C. 2017, MNRAS, 846, L15
Chakrabarti, S., & Das, S. 2001, MNRAS, 327, 808
Crowther, P. A. 2007, A&ARv, 15, 177
Cutler, C., & Flanagan, E. E. 1994, PhRvD, 49, 2658
Das, T. 2002, ApJ, 577, 880
Gammie, C. F., McKinney, J. C., & Shapiro 2004, ApJ, 602, 312
Gammie, C. F., McKinney, J. C., & Toth, G. 2003, ApJ, 589, 444
Hamersky, J., & Karas, V. 2013, A&A, 555, 32
Janiuk, A. 2017, ApJ, 837, 39
Janiuk, A., Bejger, M., Charynski, S., & Sukova, P. 2017, NewA, 51, 7
Janiuk, A., Charynski, S., & Bejger, M. 2013, A&A, 560, A25
Janiuk, A., Mioduszewski, P., & Moscibrodzka, M. 2013, ApJ, 776, 105
Janiuk, A., Moderski, R., & Proga, D. 2008, ApJ, 687, 433
Janiuk, A., & Proga, D. 2008, ApJ, 675, 519
Kastaun, W., Ciolfi, R., Endrizzi, A., & Giacomazzo, B. 2017, PhRvD, 96, 043019
Kuroda, T., Kotake, K., Takiwaki, T., & Thielemann, F.-K. 2018, MNRAS, 477, L80
Lentz, E. J. 2015, ApJL, 807, 31
Liu, T., Hou, S.-J., Xue, L., & Gu, W.-M. 2015, ApJS, 218, 12
Lopez-Camara, D., Lee, W. H., & Ramirez-Ruiz, E. 2010, ApJ, 716, 1308
Mach, P., Pirog, M., & Font, J. 2018, CQGra, 35, 095005
McKinney, J. C., & Gammie, C. F. 2004, ApJ, 611, 977
Noble, S. C., Gammie, C. F., McKinney, J. C., & Del Zanna, L. 2006, ApJ, 641, 626
Ott, C., Roberts, L. F., da Silva Schneider, A., et al. 2018, ApJL, 855, 3
Paczynski, B. 1998, ApJL, 494, 45
Pankov, C., Sampson, L., Perri, L., et al. 2017, ApJ, 834, 154
Piran, T. 2004, RvMP, 76, 1143
Podsiadlowski, P., Mazzali, P. A., Nomoto, K., et al. 2004, ApJ, 607, 17
Semenov, O., & Sukova, P. 2010, MNRAS, 404, 545
Spera, M., Mapelli, M., & Bressan, A. 2015, MNRAS, 451, 4086
Sukova, P., Charynski, S., & Janiuk, A. 2017, MNRAS, 472, 4327
Sukova, P., & Janiuk, A. 2015, MNRAS, 447, 1565
Takahashi, K., Umeda, H., & Yoshida, T. 2014, ApJ, 794, 40
Woosley, S. E. 1993, ApJ, 405, 273
Woosley, S. E., Heger, A., & Weaver, T. A. 2002, RvMP, 74, 1015