Vacuum decay and internal symmetries

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ABSTRACT

We study the effects of internal symmetries on the decay by bubble nucleation of a metastable false vacuum. The zero modes about the bounce solution that are associated with the breaking of continuous internal symmetries result in an enhancement of the tunneling rate into vacua in which some of the symmetries of the initial state are spontaneously broken. We develop a general formalism for evaluating the effects of these zero modes on the bubble nucleation rate in both flat and curved space-times.
1 Introduction

During its early history the universe may have undergone a number of phase transitions. One or more of these may have been first-order transitions in which the universe was for a time trapped in a metastable “false vacuum” state, with the transition proceeding by the nucleation and expansion of bubble of the stable “true vacuum”. A crucial quantity for the development of such a transition is the bubble nucleation rate per unit volume $\Gamma$. A semiclassical procedure, based on a Euclidean “bounce” solution, has been developed for the calculation of $\Gamma$ at zero temperature [1], and extended to the case of bubble nucleation at nonzero temperature [2].

Recently [3] it was noted that there can be an enhancement of $\Gamma$ if a continuous internal symmetry of the false vacuum is completely or partially broken by the true vacuum. This enhancement can be understood as a consequence of the fact that, instead of a single possible final state, there is a continuous family of degenerate true vacuum states into which the decay can occur. More formally, the effect arises from the existence of additional zero eigenvalues in the spectrum of small fluctuations about the bounce solution.

The primary focus of Ref. [3] was on the case of a broken $U(1)$ symmetry (see also [4]). While a similar enhancement is expected for larger symmetry groups, the treatment of the zero modes becomes somewhat more complicated for the case of a non-Abelian symmetry. In this note we develop the formalism needed to deal with the case of an arbitrary symmetry. We also discuss some further implications of these results, including the extension of these results to bubble nucleation in curved space-time.

The remainder of this paper is organized as follows. In Sec. 2 we develop the general formalism and estimate the magnitude of the enhancement that can be achieved. As a concrete example, we apply this formalism to the case of $SU(2)$ symmetry in Sec. 3. In Sec. 4 we discuss the extension to this work to curved space-time, using the formalism of Coleman and De Luccia [5]. We show that although the zero mode contribution to the curved space-time nucleation rate appears at first sight to be rather different from its flat space-time counterpart, it does in fact give the expected result in the limit where gravitational effects are negligible. Section 5 contains some brief concluding remarks.

2 General Formalism

We consider a field theory whose fields we assemble into a column vector $\phi(x)$ with purely real components. The standard method [1] for the calculation of the quantum mechanical (i.e., zero temperature) bubble nucleation rate per unit volume, $\Gamma$, is based on the existence of a “bounce” solution $\phi_b(x)$ of the Euclidean field equations that tends to the false vacuum value $\phi_f$ at spatial infinity and approaches (although not necessarily reaches) the true vacuum value $\phi_t$ near a point that may be taken to be the origin. The result may be written in the form

$$\Gamma = \frac{1}{\Omega} \frac{I_b}{I_f} \tag{2.1}$$

Here $I_f$ and $I_b$ are the contributions to the Euclidean path integral of $e^{-S_E}$ (where $S_E$ is the Euclidean action) from the homogeneous false vacuum and bounce configurations,
respectively, while the division by $\Omega$, the volume of the four-dimensional Euclidean space, arises in order to obtain a rate per unit volume. The contribution to the path integral from a stationary point $\bar{\phi}(x)$ can be evaluated by expanding the field as

$$\phi(x) = \bar{\phi}(x) + \sum_j c_j \psi_j(x)$$  \hspace{1cm} (2.2)$$

where the $\psi_j(x)$ form a complete set of orthonormal eigenfunctions of the second variation operator $S''_E(\bar{\phi}) \equiv \delta^2 S_E/\delta \phi(x)\delta \phi(y)$. For $\bar{\phi}(x) = \phi_f$, this gives, in leading approximation, a product of Gaussian integrals over the real variables $c_j$ that results in

$$I_f = e^{-S_E(\phi_f)} \{\det[S''(\phi_f)]\}^{-1/2} \equiv e^{-S_E(\phi_f)} D_f$$  \hspace{1cm} (2.3)$$

The calculation of the bounce contribution is complicated by the presence of zero eigenvalues in the spectrum of $S''_E(\phi_b)$. Four of these correspond to translation of the bounce in the four-dimensional Euclidean space. We will assume that the remainder are all associated with internal symmetries of the false vacuum that are not symmetries of the bounce. These zero modes are handled by eliminating the corresponding normal mode coefficients $c_i$ in favor of an equal number of collective coordinates $z_i$. The zero modes about a bounce configuration $\phi_b(x, z)$ can then be written in the form

$$\psi_i(x, z) = N_{ij}(z) \frac{\partial \phi_b(x, z)}{\partial z_j}$$  \hspace{1cm} (2.4)$$

where the $N_{ij}$ satisfy the equation

$$[(N^\dagger N)^{-1}]_{kl} = \int d^4x \frac{\partial \phi_b^\dagger(x; z)}{\partial z_l} \frac{\partial \phi_b(x; z)}{\partial z_k} \equiv 2\pi (M_b)_{kl}$$  \hspace{1cm} (2.5)$$

that follows from the orthonormality of the $\psi_i$. (The factor of $2\pi$ is for later convenience.)

The bounce contribution can then be written as a product of two factors. The first

$$I_b^{(1)} = e^{-S_E(\phi_b)} \frac{1}{2} |\det[S''(\phi_b)]|^{-1/2} \equiv e^{-S_E(\phi_b)} D_b$$  \hspace{1cm} (2.6)$$

arises from the integration over the modes with nonzero eigenvalues. The prime indicates that the functional determinant is to be taken in the subspace orthogonal to the zero modes, while the factor of $1/2$ arises from the integration over the single negative eigenvalue mode. The second factor, from integrating over the remaining $n$ variables, is

$$I_b^{(2)} = (2\pi)^{-n/2} \int d^n z \det \left[ \frac{\partial c_i}{\partial z_j} \right]$$  \hspace{1cm} (2.7)$$

where the factors of $2\pi$ compensate for the absence of $n$ Gaussian integrations. To calculate the Jacobian determinant, we first equate the change in the field resulting from an infinitesimal change in the $z_i$ with that corresponding to a shift of the $c_i$, to obtain

$$\psi_i(x, z) \, dc_i = \frac{\partial \phi_b(x; z)}{\partial z_j} \, dz_j$$  \hspace{1cm} (2.8)$$
Using the orthonormality of the $\psi_j$, we then find that

$$(2\pi)^{-n/2} \det \left[ \frac{\partial c_i}{\partial z_j} \right] = (2\pi)^{n/2} \det [M_b(z) N^\dagger(z)] = [\det M_b(z)]^{1/2}$$

so that

$$I_b^{(2)} = \int d^n z [\det M_b(z)]^{1/2}$$

The fact that the zero modes all arise from symmetries of the theory might lead one to expect that the integrand in this equation would be independent of the $z_j$. Actually, this is true only if the measure $d^n z$ is invariant under the symmetry transformations. If it is not, let $\mu(z)$ be such that $\mu(z)^{1/2} d^n z$ is an invariant measure and write

$$I_b^{(2)} = \int d^n z \mu(z)^{1/2} [\mu(z)^{-1} \det M_b(z)]^{1/2}$$

The quantity in brackets is now $z$-independent and can be taken outside the integral.

One can always choose coordinates so that this expression can be written as a product of a contribution from the translational zero modes and a contribution from the internal symmetry zero modes. For the former, the natural choice of collective coordinates are the spatial coordinates $z^\mu$ of the center of the bounce. The derivative of the field with respect to these is, up to a sign, the same as the spatial derivative of the field. Furthermore, with these coordinates $\mu(z) = 1$, so the integration over the $z^\mu$ simply gives a factor of $\Omega$. Hence, the contribution of these modes to $I_b^{(2)}$ is $\Omega J_{\text{trans}}^{b}$, where\[J_{\text{trans}}^{b} = (2\pi)^{2} \left[ \prod_{\mu=1}^{4} \int d^4 x (\partial_\mu \phi)^2 \right]^{1/2}$

The internal symmetry zero modes arise from the action of the gauge group $G$ on the bounce solution. Because the bounce tends asymptotically toward the false vacuum $\phi_f$, normalizable modes are obtained only from the unbroken symmetry group $H_f \subset G$ of the false vacuum. Furthermore, there are no such modes from the subgroup $K_b \subset H_f$ that leaves the bounce solution invariant. Hence, the corresponding collective coordinates span the coset space $H_f/K_b$. The contribution from these to $I_b^{(2)}$ is then

$$J_{b}^{H_f/K_b} \mathcal{V}(H_f/K_b) = [\mu(g_0)^{-1} \det M_b^{H_f/K_b}(g_0)]^{1/2} \mathcal{V}(H_f/K_b)$$

where $\mathcal{V}(H_f/K_b)$ is the volume of the coset space, $M_b^{H_f/K_b}$ is the submatrix corresponding to the internal symmetry zero modes, and $g_0$ is an arbitrary point of $H_f/K_b$. To evaluate $J_{b}^{H_f/K_b}$ it is convenient to take $g_0$ to correspond to the identity element of $H_f$. Writing the group elements near the identity in the form $e^{i\alpha_j T_j}$, we may take the collective coordinates to be the parameters that multiply the $T_j$ that that span the coset $H_f/K_b$. Evaluated at the identity element, the function $\mu(g)$ is then equal to unity, while the derivatives with respect

\[J_{b}^{\text{trans}} = (2\pi)^{2} \left[ \prod_{\mu=1}^{4} \int d^4 x (\partial_\mu \phi)^2 \right]^{1/2}$

For the case of a spherically symmetric bounce in a scalar field theory with a standard kinetic energy term, $J_{b}^{\text{trans}}$ can be expressed in terms of the bounce action, with $J_{b}^{\text{trans}} = [S_E(\phi_b) - S_E(\phi_f)]^2 / 4\pi^2$.\[\]
to the collective coordinates are given simply by the action of the generators on the bounce solution. Hence
\[ J_{b}^{H_{f}/K_{b}} = \left\{ \det \left[ (2\pi)^{-1} \int d^{4}x \phi_{b}^{\dagger}(x) T_{i}^{\dagger} T_{i} \phi_{b}(x) \right] \right\}^{1/2} \] (2.14)
with the \( T_{i} \) being the generators of \( H_{f}/K_{b} \). Gathering our results together, we obtain
\[ \Gamma = e^{-B} \frac{D_{b}^{\text{trans}} J_{b}^{H_{f}/K_{b}}}{D_{f}} V(H_{f}/K_{b}) \] (2.15)
where \( B \equiv S_{E}(\phi_{b}) - S_{E}(\phi_{f}) \).

It is important to note that \( K_{b} \) is determined by the symmetry of the bounce, and not by that of the true vacuum; it is conceivable (although we believe it unlikely) that the latter is invariant under a larger subgroup \( K_{t} \subset H_{f} \) than the former. Even if \( K_{t} \) is identical to \( K_{b} \), it is not in general the same as the unbroken symmetry group \( H_{t} \subset G \) of the true vacuum. For example, if \( G \) is unbroken in the true vacuum and completely broken in the false vacuum, \( H_{f} \), and hence \( K_{t} \), are trivial even though \( H_{t} = G \). In addition, the subgroup \( K_{b} \) depends not only on the symmetries of the true and false vacua, but also on their relative orientation.

This last point can be illustrated using a theory with global \( SU(5) \) symmetry. Let us assume that there is a single scalar field \( \phi \), in the adjoint representation, with the potential such that the false vacuum has unbroken \( SU(4) \times U(1) \) symmetry and the unbroken symmetry of the true vacuum is \( SU(3) \times SU(2) \times U(1) \). Without loss of generality we may choose the false vacuum configuration to be of the form
\[ \phi_{f} = \text{diag} \left( a, a, a, a, -4a \right) \] (2.16)
The \( SU(5) \) orientation of this field of this configuration influences that of the true vacuum bubbles that nucleate within it. Thus, decays to the true vacua with
\[ \phi_{t}^{(1)} = \text{diag} \left( b, b, b, -\frac{3}{2}b, -\frac{3}{2}b \right) \] (2.17)
and
\[ \phi_{t}^{(2)} = \text{diag} \left( -\frac{3}{2}b, -\frac{3}{2}b, b, b, b \right) \] (2.18)
are governed by inequivalent bounce solutions and proceed at different rates [6]. If the bounces have the maximum possible symmetry, then in the former case \( K_{b} = SU(3) \times U(1) \times U(1) \) and there are six internal symmetry zero modes, while in the latter \( K_{b} = SU(2) \times SU(2) \times U(1) \times U(1) \) and there are eight such modes. Of course, bubble nucleation could with equal probability lead to any configuration obtained by applying an \( SU(4) \times U(1) \) transformation to \( \phi_{t}^{(1)} \) or \( \phi_{t}^{(2)} \); this is taken into account by the integration over the coset spaces \( (SU(4) \times U(1))/(SU(3) \times U(1) \times U(1)) \) and \( (SU(4) \times U(1))/(SU(2) \times SU(2) \times U(1) \times U(1)) \). However, there are true vacuum configurations that cannot be obtained by such transformations. In general, there are no bounce solutions corresponding to these, reflecting the fact that if a bubble of such a vacuum were to form, the external false vacuum would exert forces that would realign the field in the bubble interior.

Finally, let us estimate the magnitude of the zero mode corrections that we have found. For definiteness, we will consider the case of a scalar field theory whose potential can be
written as $V(\phi) = \lambda F(\phi)$ with $F(\phi)$ containing no small dimensionless parameters. Standard scaling arguments using the fact that the bounce is a stationary point of the action show that the bounce has a radius $\sim 1/m$ (where $m$ is a characteristic scalar mass) and an action (relative to that of the false vacuum) of order $1/\lambda$. The typical magnitude of the bounce field is $\phi_b(x) \sim m/\sqrt{\lambda}$, while the $T_4$ are all of order unity, so $J_b^{H_f/K_b} \sim (\lambda m^2)^{-N/2}$, where $N$ is the number of internal symmetry zero modes. The ratio $D_b/D_f$ is of order unity in $\lambda$, but is proportional to a dimensionful parameter $\sim m^{N+4}$ arising from the fact that the contribution of the zero eigenvalue modes has been deleted from $D_f$. Finally, the coset volume is of order unity. Overall, then, we have

$$\Gamma = c_1 \lambda^{-(N+4)/2} m^4 e^{-c_2/\lambda} \quad (2.19)$$

where $c_1$ and $c_2$ are of order unity; the effect of the internal symmetry zero modes has been to enhance the nucleation rate by a factor of order $\lambda^{-N/2}$. Phrased somewhat differently, the enhancement is roughly by a factor of $B^{N/2}$.

3 SU(2) Symmetry

As a concrete example, let us consider the case where the symmetry group of the false vacuum is $H_f = SU(2)$ but the bounce solutions break this symmetry. A natural set of collective coordinates is given by the Euler angles. Thus, given one bounce solution $\phi_b^0(x)$, we can define a three-parameter family of solutions by

$$\phi_b(x; \varphi, \theta, \psi) = e^{i\varphi T_3} e^{i\theta T_2} e^{i\psi T_3} \phi_b^0(x) \equiv U(\varphi, \theta, \psi) \phi_b^0(x) \quad (3.1)$$

where the $T_j$ are the appropriate (possibly reducible) representation of the generators of $SU(2)$. Differentiation of this expression gives

$$\partial_j \phi_b(x; \varphi, \theta, \psi) = iU(\varphi, \theta, \psi) \tilde{T}_j \phi_b^0(x) \quad (3.2)$$

where

$$\tilde{T}_3 = e^{-i\varphi T_3} e^{-i\theta T_2} T_3 e^{i\varphi T_3} e^{i\psi T_3}$$

$$= \cos \psi \sin \theta \ T_1 + \sin \psi \sin \theta \ T_2 + \cos \theta \ T_3$$

$$\tilde{T}_2 = e^{-i\varphi T_3} T_2 e^{i\psi T_3}$$

$$= -\sin \psi \ T_1 + \cos \psi \ T_2 \quad (3.3)$$

Thus, if $z_j = (\varphi, \theta, \psi)$,

$$\partial_j \phi_b(x; \varphi, \theta, \psi) = iK_{jk} U(\varphi, \theta, \psi) T_k \phi_b^0(x) \quad (3.4)$$

where

$$K = \begin{pmatrix}
\cos \psi \sin \theta & \sin \psi \sin \theta & \cos \psi \cos \theta \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{pmatrix} \quad (3.5)$$
Substitution of this into Eq. (2.5) yields

\[ (M_{SU(2)}/K_b)_{ij} = (2\pi)^{-1} K_{ij} \int d^4 x \phi_b^0(x) T_j^i T_k^k \phi_b^0(x) \]  

(3.6)

Now recall that an invariant measure on SU(2) is given by \( \sin^2 \theta d\phi d\theta d\psi \), so we may take

\[ \mu(\phi, \theta, \psi) = \sin^2 \theta = (\det K)^2. \]

Hence,

\[ \mu^{-1} \det M_{SU(2)/K_b} = \det \left[ (2\pi)^{-1} \int d^4 x \phi_b^0(x) T_j^i T_k^k \phi_b^0(x) \right] \]

(3.7)

This is independent of the collective coordinates, as promised, and is in agreement with Eq. (2.14).

Three specific cases may serve to illustrate some of the possible behaviors:

a) One SU(2) doublet: If the bounce involves an SU(2) doublet, then the bounce completely breaks the SU(2) symmetry. The coset volume factor is

\[ V(H_f/K_b) = V(SU(2)) = 16\pi^2 \]  

(3.8)

while \( M_{SU(2)/K_b} = M_{SU(2)} \) is proportional to the unit matrix, with

\[ \mu^{-1} \left[ M_{SU(2)} \right]_{ij} = \delta_{ij} (2\pi)^{-1} \int d^4 x \phi_b^1(x) \phi_b(x) \]  

(3.9)

(Because our formulas have been derived using real fields, one must use a four-dimensional real representation, rather than a two-dimensional complex representation, for the \( T_j \) when obtaining this result from Eq. (3.7).)

b) One SU(2) triplet: If the bounce is constructed from a single real triplet whose direction is independent of \( x \) [i.e., such that \( \phi_b(x) \) can be written as \( (0, 0, f(x)) \)], then \( K_b = U(1) \). There are only two zero modes and \( H_f/K_b \) is the two-sphere spanned by \( \theta \) and \( \varphi \) with

\[ V(H_f/K_b) = V(SU(2)/U(1)) = 4\pi \]

(3.10)

\( M_{SU(2)/K_b} \) is now a \( 2 \times 2 \) matrix, with

\[ \mu^{-1} \left[ M_{SU(2)/U(1)} \right]_{ij} = \delta_{ij} (2\pi)^{-1} \int d^4 x \phi_b^2(x) \]  

(3.11)

\[ \phi_b^2(x) \]

is a \( 2 \times 2 \) matrix, with

\[ \mu^{-1} \left[ M_{SU(2)/U(1)} \right]_{ij} = \delta_{ij} (2\pi)^{-1} \int d^4 x \phi_b^2(x) \]  

(3.11)

The matrix \( \mu^{-1} M_{SO(3)} \) has three unequal eigenvalues.

4 Bubble Nucleation in Curved Space-Time

Coleman and De Luccia [5] showed that the bounce formalism could be extended to include the effects of gravity by requiring that both the bounce and the homogeneous false vacuum
configurations be solutions of the coupled Euclidean matter and Einstein equations. For a scalar field theory with $V(\phi) \geq 0$, as we henceforth assume, the false vacuum solution consists of a uniform scalar field $\phi_f$ on a four-sphere of radius

$$\tilde{H}_f^{-1} = \sqrt{\frac{3M_{Pl}^2}{8\pi V(\phi_f)}} \quad (4.1)$$

with total Euclidean action (including gravitational contributions)

$$S_E(\phi_f) = -\frac{3M_{Pl}^4}{8\pi V(\phi_f)} \quad (4.2)$$

The bounce solution has the same topology, with regions of approximate true vacuum and false vacuum separated by a wall region. If the matter mass scale $\mathcal{M}$ is much less than the Planck mass $M_{Pl}$, then both the radius $R_b$ of the true vacuum region and the difference between the bounce action and the false vacuum action differ from the corresponding flat space quantities by terms of order $(\mathcal{M}/M_{Pl})^2$.

The spectra of the small fluctuations about these solutions again contain one zero mode for each symmetry of the Lagrangian that is broken by the solution. However, because the Euclidean solutions are on closed manifolds with finite volumes, the modes due to symmetries broken by the false vacuum are normalizable, in contrast with the flat space case. Hence, we would expect the flat space factors given in Eq. (2.13) to be replaced by

$$J_{G/K_b} J_{G/H_f} = J_{G/H_f} [1 + \mathcal{O}(\mathcal{M}/M_{Pl})^8] \quad (4.3)$$

Although the volume factors give the same result as in flat space, the Jacobean factors appear quite different. Yet, for $\mathcal{M} \ll M_{Pl}$, where gravitational corrections should be small, this should approach the flat space result.

To see how this comes about, let us denote by $t_j$ the generators of $H_f/K_b$ and by $s_j$ those of $G/H_f$. The Jacobean determinant in the numerator of Eq (4.3) has contributions from matrix elements containing both types of generators, whereas the determinant in the denominator only involves $s_i s_j$ matrix elements. Because the $t_j$ annihilate the false vacuum, the matrix elements involving these have nonzero contributions only from the region, of volume $\sim R_b^4$, where the bounce solution differs from the false vacuum and hence are suppressed by a factor of order $(\tilde{H}_f R_b)^4 \sim (\mathcal{M}/M_{Pl})^4$ relative to the $s_i s_j$ matrix elements. (We are assuming that $R_b \sim \mathcal{M}^{-1}$; this will be the case for generic values of the parameters.) This implies that, up to corrections of order $(\mathcal{M}/M_{Pl})^8$, the determinant can be written as a product of a determinant involving only the $t_i$ and one involving only the $s_i$; i.e.,

$$J_{G/K_b} J_{G/H_f} = J_{G/H_f} J_{G/H_f} [1 + \mathcal{O}(\mathcal{M}/M_{Pl})^8] \quad (4.4)$$

The first factor on the right hand side differs from unity only by an amount proportional to the fraction $\sim (\mathcal{M}/M_{Pl})^4$ of the Euclidean space where the bounce differs from the false vacuum. The second factor differs from the corresponding flat-space term only by the
replacement of the matter fields of the flat space bounce by those of the curved space bounce, and so clearly reduces to the flat space result as $\mathcal{M}/M_{\text{Pl}} \to 0$.

The fact that the bounce solution is a closed manifold, with the true and false vacuum regions both finite, suggests that it can contribute not only to the nucleation of a true vacuum bubble within a false vacuum background, but also to the nucleation of a false vacuum bubble within a true vacuum background, with the rate for the latter process obtained from that of the former by making the substitution $\phi_f \to \phi_t$ \([7]\). To leading order, the ratio of these two rates is

$$\frac{\Gamma_{t \to f}}{\Gamma_{f \to t}} = e^{S_E(\phi_t) - S_E(\phi_f)} = \exp \left[ -\frac{3M_{\text{Pl}}^4}{8} \left( \frac{1}{V(\phi_t)} - \frac{1}{V(\phi_f)} \right) \right]$$

(4.5)

The continued nucleation and expansion of bubbles of one vacuum within the other will result in a spacetime that is a rather inhomogeneous mixture of the two vacua. There is an intriguing thermal interpretation of this mixture if $V(\phi_f) - V(\phi_t) \ll (V(\phi_f) + V(\phi_t))/2 \equiv \bar{V}$, so that the geometry of space is approximately the same in the regions of either vacua, with a Hubble parameter

$$\bar{H} \approx \sqrt{\frac{8\pi \bar{V}}{3M_{\text{Pl}}^2}}$$

(4.6)

It seems plausible that the fraction of space contained in each of the vacua might tend to a constant, with the nucleation of true vacuum bubbles in false vacuum regions being just balanced by the nucleation of false vacuum bubbles in true vacuum regions. For such an equilibrium to hold, the volumes $\Omega_f$ and $\Omega_t$ of false and true vacuum must satisfy

$$\frac{\Omega_f}{\Omega_t} = \frac{\Gamma_{t \to f}}{\Gamma_{f \to t}} \approx e^{-\Omega_{\text{hor}}[V(\phi_f) - V(\phi_t)]/T_H}$$

(4.7)

where the horizon volume $\Omega_{\text{hor}} = (4\pi/3)\bar{H}^{-3}$ and the Hawking temperature $T_H = \bar{H}/2\pi$. If we view the de Sitter space as being somewhat analogous to an ensemble of quasi-independent horizon volumes in a thermal bath, then this leading contribution to the volume ratio is essentially a Boltzmann factor.

The zero mode corrections to the nucleation rate are consistent with this thermodynamic picture. Their effect is to multiply the ratio in Eq. (4.7) by

$$\frac{\mathcal{V}(G/H_f)}{\mathcal{V}(G/H_t)} \frac{J_{f}^{G/H_f}}{J_{t}^{G/H_t}} = \left( \frac{\bar{H}}{\sqrt{3}\pi} \right)^{N_t - N_f} \frac{\mathcal{V}(G/H_f)}{\mathcal{V}(G/H_t)} \left[ \frac{\det \left[ (\Omega_{\text{hor}}T_H/2\pi) \left( \phi_j^i T_i T_j \phi_f \right) \right]}{\det \left[ (\Omega_{\text{hor}}T_H/2\pi) \left( \phi_j^i T_i T_j \phi_t \right) \right]} \right]^{1/2}$$

(4.8)

where $N_f$ and $N_t$ are the number of internal symmetry zero modes in the false and true vacua, respectively. We recognize the dimensionless ratio on the right hand side as the ratio of two classical partition functions of the form

$$\int d^N z d^N p \frac{e^{-Nz/T_H}}{(2\pi)^N} = \int d^N z \left[ (\Omega_{\text{hor}}T_H/2\pi) \left( \phi_j^i T_i T_j \phi \right) \right]^{1/2}$$

(4.9)

that follow from the effective Lagrangian

$$L_z = \frac{1}{2} \Omega_{\text{hor}} \left( \phi_j^i T_i T_j \phi \right) \dot{z}_i \dot{z}_j$$

(4.10)
that describes the collective coordinates dynamics for a horizon volume with spatially uniform scalar field $\phi$.

The presence of a dimensionful prefactor in Eq. (4.8) is required by the differing numbers of zero modes about the true and false vacua, which implies a dimensional mismatch between the functional determinants over the nonzero eigenvalue modes. This suggests that the factor of $(\bar{H}/\sqrt{3}\pi)^{N_t-N_f}$ should, like the functional determinants themselves, be understood as related to the first quantum corrections to the vacuum energies.

5 Concluding Remarks

We have seen that when a metastable false vacuum decays to a true vacuum that breaks some of the internal symmetries of the false vacuum, the presence of $N > 0$ zero modes about the bounce solution can lead to an enhancement of the bubble nucleation rate. In a theory characterized by an overall scalar coupling $\lambda$, the zero-temperature, quantum mechanical, tunneling rate is increased by a factor of order $\lambda^{-N/2} \sim B^{N/2}$. A straightforward extension of our methods to finite temperature thermal tunneling shows that, although the $\lambda$-dependence is changed, the enhancement is still of order $B^{N/2}$. Since the nucleation rate falls exponentially with $B$, we therefore have the curious situation that the enhancement is greatest when the overall rate is smallest.

These results may be of particular interest for the symmetry-breaking phase transitions that arise in the context of a grand unified theory. For any given nucleation rate, the numerical effect of the zero mode corrections will, of course, almost always be negligible compared to the uncertainties due to the undetermined couplings in the scalar field potential. The zero mode effects could, however, be significant when there are competing decays to vacua with different degrees of symmetry breaking, such as are encountered in many supersymmetric models.

Acknowledgments

A.K and E.W. would like to thank the Aspen Center for Physics, where part of this work was done. This work was supported in part by the U.S. Department of Energy. K.L. is supported in part by the NSF Presidential Young Investigator program.

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