Bunch Expansion as a Cause for Pulsar Radio Emissions

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Received 2021 June 25; revised 2021 October 1; accepted 2021 October 1; published 2021 December 14

Abstract

Electromagnetic waves due to electron–positron clouds (bunches), created by cascading processes in pulsar magnetospheres, have been proposed to explain the pulsar radio emission. In order to verify this hypothesis, we utilized for the first time Particle-in-Cell (PIC) code simulations to study the nonlinear evolution of electron–positron bunches dependant on the initial relative drift speeds of electrons and positrons, plasma temperature, and distance between the bunches. For this sake, we utilized the PIC code ACRONYM with a high-order field solver and particle weighting factor, appropriate to describe relativistic pair plasmas. We found that the bunch expansion is mainly determined by the relative electron–positron drift speed. Finite drift speeds were found to cause the generation of strong electrostatic suprathermal waves at the bunch density gradients that reach up to \( E \sim 7.5 \times 10^7 \text{V cm}^{-1} \) and strong plasma heating. As a result, up to 15% of the initial kinetic energy is transformed into the electric field energy. Assuming the same electron and positron distributions, we found that the fastest (in the bunch reference frame) particles of consecutively emitted bunches eventually overlap in momentum (velocity) space. This overlap causes two-stream instabilities that generate electrostatic subluminal waves with electric field amplitudes reaching up to \( E \sim 1.9 \times 10^7 \text{V cm}^{-1} \) (\( E/(m_e \omega_p e^{-1}) \sim 0.11 \)). We found that in all simulations the evolution of electron–positron bunches may lead to the generation of electrostatic suprathermal or subluminal waves, which, in principle, can be behind the observed electromagnetic emissions of pulsars in the radio wave range.

Unified Astronomy Thesaurus concepts: Radio pulsars (1353); Plasma astrophysics (1261); High energy astrophysics (739); Pulsars (1306)

1. Introduction

Pulsars are strongly magnetized (10^8–10^13 G), fast rotating neutron stars that emit radio waves into narrow cones. Their emission is considered to be generated in the relativistically hot electron–positron magnetospheric plasma, studied for more than fifty years now (see, e.g., Kramer et al. 2002; Beskin 2018). A number of models have been proposed to explain the observed radio waves and their properties (Sturrock 1971; Ruderman & Sutherland 1975; Usov 1987; Petrova 2009; Philippov et al. 2020). Nevertheless, no consensus has been reached about the proper emission mechanism, the efficiency of the energy conversion, and its relation to the observed pulsar radio signals (Melrose & Rafat 2017; Melrose et al. 2021a).

The current “standard” model of the pulsar magnetospheric plasma is based on the formation of Goldreich–Julian currents (Goldreich & Julian 1969; Cheng & Ruderman 1977a) that screen the convective electric field in closed magnetic field lines in a way such that \( E \cdot B = 0 \) holds. By contrast, over the polar caps with open magnetic field lines, where particles can escape the pulsar magnetospheres, electric field components parallel to the magnetic field can be formed. In these regions known as “gaps,” strong parallel electric field components (\( E \cdot B = 0 \)) with \( E \approx 10^{12} \text{V m}^{-1} \) can exist.

If the plasma density is small, particles in the gap region are accelerated to ultrarelativistic velocities with Lorentz factors \( \gamma \sim 10^6–10^8 \), forming the so-called “primary beam” that emits \( \gamma \)-ray photons. The \( \gamma \)-ray photons propagate into arbitrary directions, eventually interacting with the magnetic field. As a result, they decay into electron–positron pairs: \( \gamma \rightarrow \text{photon} + B \rightarrow e^- + e^+ + B \) (Buschauer & Benford 1977; Cheng & Ruderman 1977a). The newly formed “secondary beam” of electrons and positrons is denser than the primary beam by a factor of \( 10^2–10^4 \) and their Lorentz factor is rather \( \gamma \sim 10^2–10^3 \) (Arendt & Eilek 2002). Initially, their velocity has components parallel and perpendicular to the magnetic field. The perpendicular velocity component quickly vanishes, however, by emitting synchrotron radiation within short times (\( t \leq \omega_p^{-1} \), where \( \omega_p \) is the plasma frequency). Hence, only their magnetic field aligned velocity component remains. The release of secondary particles is not a continuous process. As the secondary beam particles escape their generation region at relativistic speeds, the corresponding currents do not screen the electric fields anymore. Therefore, their sparking repeats again and again at timescales of \( \sim 500 \text{ns} \) (Ruderman & Sutherland 1975; Cheng & Ruderman 1977b).

The interaction of the bunches of electrons and positrons (sometimes called clouds) can generate plasma waves due to streaming instabilities that might emit radio waves (Eilek & Hankins 2016; Melrose 2017; Mitra 2017; Melrose et al. 2021a). Three basic scenarios (C1–C3) of the interaction between the particle bunches of primary and secondary beams have been suggested so far:

1. Relative streaming of the primary and secondary beam particles. A reactive instability with large growth rates could develop in the lower pulsar magnetosphere within a distance (10–100)\( R_n \), where \( R_n \) is the neutron star radius (Cheng & Ruderman 1977b; Buschauer & Benford 1977). However, this instability has been found to be inefficient due to its small growth rates (Benford & Buschauer 1977; Arons 1981) due to relativistic effects.

2. Acceleration of the secondary particles due to the net current. Electrons and positrons are accelerated in opposite directions, thus possibly causing a two-stream instability (Cheng & Ruderman 1977a). Thermal effects were taken into account by Weatherall (1994) and...
Asseo & Melikidze (1998). The instability calculations assumed an infinitely long interaction region with a constant particle density. When taking the spatial size of the bunch into account, the drift velocity between species separates the location of oppositely charged particles and creates electric discharges in the course of their evolution. Thus the constant-density approximation can be only sufficient for large bunches, or very short times \( \lessapprox \omega_p^{-1} \).

3. Relative streaming of the bunch particles. The sparking processes can create “trains” of hot bunches that expand in the course of their propagation along the pulsar magnetic fields. As a result, a streaming instability can occur when the fastest particles of a trailing bunch catch up with the particles of the leading bunch (Ruderman & Sutherland 1975; Usov 1987; Usov & Usov 1988; Asseo & Melikidze 1998). Consecutively emitted bunches are modulated by the gap with size \( H \sim 50 \) m, estimated by the time \( \tau_0 \) needed for the decay of a \( \gamma \)-ray photon to an electron–positron pair. Individual sparks that form the bunches need a time \( \simeq (30-40) \times \tau_0 \approx (10-\mu s) \) to grow and saturate, which gives an estimate of the bunch size \( H \sim (30-40) \times H \) (Ruderman & Sutherland 1975). Note that this model does not predict a specific spatial density profile. Usov (2002) estimated the bunch size as \( 30H \approx 3 \) km and the distance between bunches (gap size) as 100 m. Melrose et al. (2021a, Appendix A) pointed out that, although the time needed for catching up to slow particles of the leading bunch is sufficiently short, the time for catching up with particles moving at the mean velocity is too long for a viable pulsar emission mechanism in the lower pulsar magnetosphere.

Production of secondary electron and positron beams is necessary for scenarios C2 and C3 to work. It has been studied by several authors. Arendt & Eilek (2002), e.g., derived the properties of the produced secondary pairs. According to them, the beam Lorentz factors range from \( 10^2 \) to \( 10^3 \) and the plasma inverse temperature is \( \rho = m_e c^2/k_BT = 1.0 - 1.5 \), depending on the photon energy of pulsars, assuming a magnetic field of the neutron star equal to \( B_\star = 10^{12} \) G. Though the estimated inverse temperature is in a quite narrow range, e.g., Weatherall (1994) and Rafat et al. (2019a) consider lower pulsar inverse temperatures, reaching up to \( \rho \approx 0.01 \). Philippov et al. (2020) numerically simulated the formation of electron–positron pair bunches by a cascading process. The cascading can occur along the open magnetic field at heights where the magnetic field is small enough to allow a velocity component perpendicular to the magnetic field direction, which is not immediately radiated. Cruz et al. (2021) investigated the development of the pair cascade analytically. They confirmed their analytical results utilizing one-dimensional PIC-code simulations taking quantum electrodynamic effects into account. Their studies revealed the formation of particle clouds associated with large-amplitude electrostatic waves.

Rafat et al. (2019a, 2019b, 2019c) studied the dispersion properties of a relativistically hot pair plasma and the consequent streaming instabilities in a pulsar magnetosphere. They found that the waves in a relativistic hot pulsar magnetosphere manifest quite different dispersion properties than their nonrelativistic counterparts. In particular, the generated (electrostatic) L-mode waves have superluminal and subluminal dispersion branches. These waves are the solution to the relevant dispersion relation for parallel propagation in the strong magnetic field, low-frequency, and non-gyrotropic approximations of the dispersion tensor \( \Lambda_{ij} \) (Rafat et al. 2019b, Appendix A),

\[
\text{det} \Lambda_{ij} = 0. \tag{1}
\]

Solutions of \( \Lambda_{13} = 0 \) are often denoted as L-mode waves or electrostatic waves. They represent the relativistic generalization of electrostatic Langmuir and beam waves. Most unstable is the subluminal L-mode wave close to the light line. These authors calculated the corresponding growth rates of the different modes associated with the streaming instabilities and concluded that they are too small to generate sufficiently strong emission in the lower pulsar magnetosphere.

Rahaman et al. (2020) compared the growth rates of the linear instabilities obtained for scenarios C2 and C3. They found that the growth rates of C2 can largely exceed those obtained of C3 for certain multipolar magnetic field configurations at the height of a few \( R_\star \) above the pulsar surface. For C3, however, the growth rates can be sufficiently large only for \( n_{GJ} \gtrsim 10^4 \), where \( n_{GJ} \) is the multiplicity factor.

Mantei et al. (2021) obtained the properties of the relativistic streaming instabilities for the scenario C3 by solving the linear dispersion relations and verifying them by PIC-code simulations. They could confirm their linear dispersion theory predictions by their numerical plasma simulations during the initial phase of the instability development until nonlinear effects prevail. They also showed that sufficiently large growth rates can be obtained for certain pulsar plasma parameters. In particular they showed that an instability can develop at timescales \( <10 \mu s \) for \( 13 < \gamma < 300, \rho_0, \rho_1 \gtrsim 1, \) and \( n_1/(\gamma_\rho n_0) \gtrsim 10^{-3} \), where \( \rho_0, \rho_1 \) are the inverse temperatures and \( n_1, n_0 \) are the densities of the background and the beam, respectively. The \( \gamma_\rho \) is the beam Lorentz factor.

Benáček et al. (2021) studied the nonlinear stage of the relativistic pair plasma streaming instability by means of PIC-code simulations. They found that nonlinear superluminal electrostatic L-mode waves can be formed in sufficiently cold plasmas \( \rho \gtrsim 1.66, \) or for sufficiently large beam velocities \( \gamma_\rho \gtrsim 40 \). These waves are generated by a nonlinear interaction between the initially generated subluminal L-mode waves and an accompanying density wave. These waves stay stable for sufficiently long times so that they can cause coherent electromagnetic (radio) wave emissions. These authors estimated the electrostatic energy density, which can be reached this way, to be \( 1.2 \times 10^4 \text{ erg cm}^{-3} \) for subluminal waves and up to \( 1.1 \times 10^5 \text{ erg cm}^{-3} \) for superluminal waves.

As far as the mechanisms of radio wave emission are concerned, three suggestions were made in the framework of scenarios C2 and C3 (Melrose 1995; Eilek & Hankins 2016; Melrose & Rafat 2017; Melrose et al. 2021a):

1. Relativistic plasma emissions can be caused by a modulational instability of longitudinal electrostatic L-mode waves, which propagate parallel to the magnetic field generating escaping electromagnetic waves via a wave–wave interaction \( T, L + M \rightarrow T \) (Weatherall 1997, 1998). The modulational wave \( M \) has a polarization vector component perpendicular to the magnetic field; the O-mode waves can escape the emission region.

2. Linear acceleration and free electron maser/laser emission mechanisms based on charges that undergo acceleration in the parallel or arbitrary direction to the magnetic field lines, respectively. A simplified model assumes particles oscillating
with frequency $\omega_0$ and an electromagnetic emission at the frequency $\gamma^2 \omega_0$, where $\gamma$ is the particle Lorentz factor in the pulsar reference frame (Cocke 1973; Melrose 1978; Kroll & McMullin 1979; Fung & Kuijpers 2004; Melrose & Luo 2009; Melrose et al. 2009; Reville & Kirk 2010; Timokhin & Arons 2013; Lyutikov 2021). This frequency characterizes the linear acceleration emission. It determines the modulational frequency for the free electron maser emission.

3. An anomalous Doppler emission takes place when the resonance condition $\omega(1 - \frac{\omega}{\gamma} \cos \theta) + \omega_{ce}/\gamma = 0$ is fulfilled, where $\gamma = 1$ is the resonance number for the cyclotron frequency $\omega_{ce}$, $n$ is the refractive index, and $\theta$ is the propagation angle with respect to the magnetic field. This condition for the emission of X- or O-mode waves requires $n > 0$ and $\gamma > v_A/c$, where $v_A$ the Alfvén speed, and $c$ is the speed of light. The emission frequency is $\omega = 2(v_A/c)^2 \omega_{ce}/\gamma$, which is in the radio band only for small values of $\omega_{ce}/\gamma$ (Machabeli & Usov 1979; Kazbegi et al. 1991; Lyutikov et al. 1999a, 1999b).

Note that scenarios C2 and C3 have been so far studied only by means of oversimplifying approximations. The properties of the resulting electromagnetic emissions are not well understood yet. In particular, the consequences of density gradient effects are not taken into account, yet, in theoretical models trying to theoretically elaborate scenarios C2 and C3 (Weatherall 1994; Rafat et al. 2019e; 2019a; Benáček et al. 2021; Manthei et al. 2021). Also, the bunch expansion, their interaction with the background plasma, and the nonlinear evolution after the overlap have not been considered yet (Usov 2002; Usor & Usov 1988; Melikidze et al. 2000; Rahaman et al. 2020; Melrose et al. 2021b).

In this paper, we model the evolution of the bunches as particle clouds in scenarios C2 and C3; i.e., an enhanced central density with a smooth decrease of the density from the maxima to the level of the surrounding plasma. The plasma between bunches is also taken into account, with a low but finite density. We study the expansion of such bunch, the propagation through the low-density background, and in some cases, the eventual overlap of particles of adjacent bunches in phase space, the consequent growth of instabilities, and the formation and evolution of waves. We utilized one-dimensional (1D) PIC-code simulations, where the velocity distribution of only one finite component directed parallel to the magnetic field lines is taken into account. This approximation is valid for the lower pulsar magnetosphere, where the magnetic field is so strong that the energy of any perpendicular velocity component of the particles is immediately radiated away. In particular, we investigate the evolution of the bunches and waves for three sets of characteristic initial parameters: the plasma temperature, the distance between bunches, and the relative drift speed of electrons and positrons.

The paper is structured as follows. In Section 2 we describe the initial setup and numerical algorithms suitable for the kinetic study of relativistic pair plasmas utilizing the appropriate PIC code ACRONYM. In Section 3 we present the resulting evolution of the bunches. First, we discuss the main results and properties of all simulations. We then, in more detail, analyze the results of the two most representative simulations. An overall discussion of our results is contained in Section 4, and we state our conclusions in Section 5.
Maxwell–Jüttner velocity distribution function

\[ g(u) = \frac{1}{\gamma_d} \frac{n}{2K_1(\rho)} e^{-\rho u c (1 - \beta_d)}, \]  

(3)

where

\[ \gamma = \sqrt{1 + \frac{u^2}{c^2}}, \quad \beta = \frac{v}{c} = \frac{u}{c \gamma}, \quad \beta_d = \frac{\beta u}{c} = \frac{\beta u_d}{c \gamma_d}, \]  

(4)

where \( \rho = m_e c^2/k_B T \) is the inverse temperature, \( k_B \) is the Boltzmann constant, \( \sigma = 1 \) for electrons, \( \sigma = -1 \) for positrons, and \( K_1 \) is the MacDonald function of the first order (a modified Bessel function of the second kind). Particles are generated using the rejection method that selects particles from a velocity interval much wider than the expected distribution width. The produced distributions converge to the analytical prescription as the number of particles increases.

The periodic boundary conditions of our simulations allow one to choose an arbitrary bunch position. Because we want to study the dynamics that occur between the bunches, we select the middle point between them as the center of the simulation at \( x = 0 \), and, thus, we get half of each bunch on either side. This may also be interpreted as half of the leading and half of the trailing bunch. Moreover, Usov’s model assumes a “waterbag” (step) density profile of the bunch and vacuum between bunches. Nevertheless, for the simulations, we choose to represent the density more smoothly by a \( n \sim e^{-ax^2} \) profile, where \( a \) is an arbitrary constant and \( x_c \) is the distance from the bunch center. This profile is smoother than the discontinuous waterbag function and, therefore, it can avoid instabilities that may result from the evolution of such a discontinuous function. In addition, the plasma properties simulated by PIC codes are statistically better resolved for a significant number of particles per cell. That is why we choose a finite but low density between bunches as \( 0.1 n_0 \), where \( n_0 \) is the density in the bunch center. This density is high enough for a statistically significant representation of the plasma properties but low enough to not significantly influence the bunch expansion. Then, the initial particle density \( n(x) \) may be described as

\[ n(x) = \begin{cases} 
0.1 n_0, & |x| \leq \frac{L}{2}, \\
0.1 n_0 \exp \left\{ -\left( \frac{L - |x|}{x_0} \right)^6 \right\}, & |x| > \frac{L}{2},
\end{cases} \]  

(5)

\[ x_0 = |\ln(0.1)|^{-\frac{1}{6}} \frac{L - \frac{L}{2}}{\ln(0.1)} \approx 0.870 \cdot \frac{L - \frac{L}{2}}{2} \]  

(6)

where \( x_0 \) is chosen such that \( n(x) \) is a smooth function in \( x = \pm L/2 \). We assume that the initial plasma density is an even function. The simulation part where \( x < -L/2 \) covers half of the trailing bunch, while \( x > L/2 \) covers half of the leading bunch. We initially have the same number of electrons and positrons in each grid cell. The net current is subtracted in each time step. In all simulations we represent the density \( n_0 \) by \( 10^4 \) macroparticles per cell. The Debye length for a relativistically hot plasma with a 1D velocity function is given by

\[ \lambda_D = \frac{c'}{\omega_p} = \frac{c}{\omega_p} \left[ \frac{1}{G_{1D}} \frac{dG_{1D}}{d\rho} \left( \frac{dG_{1D}}{d\rho} + \frac{1}{\rho} \right)^{-\frac{1}{2}} \right] \]  

(7)

\[ G_{1D}(\rho, 0) = \frac{K_2(\rho \gamma)}{K_0(\rho \gamma)} \]  

(8)

where \( c' \) is the relativistic “sound” speed. In such a plasma, particles have two degrees of freedom (one for the electron and one for the positron), and the adiabatic index is 4/3. For \( \rho \to 0 \), the relativistic sound speed approaches the speed of light. For \( \rho = 1/0.01 \), the ratio of the Debye length with respect to the grid size ranges from \( \lambda_D / \Delta x = 18.2–20.0 \) at the bunch center to \( \lambda_D / \Delta x = 57–63 \) between the bunches. Thus, the number of particles in a Debye length is \( (1.8–6.3) \times 10^7 \).

The initial electric field is zero. The magnetic field is not included in the simulation, as the particle distribution is only one dimensional, and all quantities vary only in the \( x \)-direction and do not interact with the magnetic field. The currents and evolution of the electric field can be only along this direction. Then, the curl operator acting on electric and magnetic fields is always zero in the Maxwell equations; the electric and magnetic fields do not influence each other. The parameters of the different simulation runs are summarized in Table 1. Because the parameters of plasma when it leaves the gap regions are not known, we have decided to use the known properties from the pair creation process by Arendt & Eilek (2002). They found the plasma temperature \( \gamma \sim 1 \), drift Lorentz factor \( \gamma_d = 80–400 \) (for \( B = 10^{12} \) G), and from the above-mentioned Usov’s model the implicit condition that electrons and positrons have initially the same distributions, i.e., they are treated as a charge neutral fluid with zero drift speed (\( \gamma_d = 1 \)).

We vary these parameters to better comprehend their influence on the bunch evolution. The initial values range as follows: inverse temperature values \( \rho = (1, 0.1, 0.01) \), distance between bunches \( l/L = (1/30, 1/3) \), and drift (momentum) velocity range \( u_d/c = (0, 0.1, 1, 10) \) corresponding to Lorentz factors \( \gamma_d \approx (1, 1.005, 1.414, 10.05) \).

### 3. Results

We denote the bunch in \( x < 0 \) as the “left bunch,” the bunch in \( x > 0 \) as the “right bunch,” and the region around \( x \approx 0 \) as the “center.” All processes are described and presented in the simulation reference frame that is moving with respect to the pulsar reference frame with arbitrary Lorentz factor \( \gamma_c \).
of electromagnetic vector fields in the simulations, the generation of only electric fields and L-mode electrostatic longitudinal waves are allowed due to the 1D approximation (for more details, see the text above). We denote these waves as “electrostatic waves” to describe both subluminal and superluminal waves. When we specify one mode, we use the notation “superluminal electrostatic waves” or “subluminal electrostatic waves.” We note that our description is not limited to the case of a Maxwell–Jüttner distribution, but Equation (1) is valid for arbitrary velocity distributions.

### 3.1. Evolution of Simulations

#### 3.1.1. Energy Evolution

Figure 1 shows the evolution of the ratio of the electric to the initial total kinetic energy for all simulations. The evolution of Simulation 1 is plotted as a blue line in all subfigures as the reference. The initial inverse plasma temperature, distance between bunches, and drift between species are varied between the different simulations.

In Simulations 1–4 with zero initial drift between species (Figure 1(a),(b)), particles from both bunches begin to expand after the simulations start. The expansion is not suppressed by ambipolar diffusion as in the case of an electron–ion plasma. Both particle species have the same initial velocity distribution, and both expand in the same way due to the same inertia of electrons and positrons. The fastest particles from the left bunch reach the fastest particles from the right bunch in the simulation center, and they cause a two-stream instability. The start of the mutual particle interaction is denoted by a black arrow for Simulations 1–4. By using the electric field evolution in the whole simulation domain, we fit the function \( E_E(t, \Gamma) \sim e^{\frac{\Gamma}{\omega_p}t} \), where \( \Gamma \) is the growth rate and \( t \) is time. The growth rates \( \Gamma/\omega_p \) are \((3.48 \pm 0.05) \times 10^{-3}\) for Simulation 1 in the time interval \( \omega_p t = [2082, 3390] \), \((3.35 \pm 0.04) \times 10^{-3}\) for Simulation 2 in the time interval \( \omega_p t = [2060, 2880] \), and \((5.8 \pm 0.4) \times 10^{-4}\) for Simulation 3 in the time interval \( \omega_p t = [2610, 3650] \). These growth rates can be understood as the effective instability growth for a given initial simulation parameter set. They are expected to significantly change along the simulation domain and in time, dependent on the separation of the velocity distributions, densities of particles from both bunches, and their density ratios. With increasing plasma temperature, the instability growth rates decrease, and the amount of converted kinetic energy into electrostatic energy also decreases. In the hottest case, the energy ratio increases only slightly above the initial energy level. The evolution slightly differs in Simulation 4, where the instability produced by the interaction of expanding particles with the background plasma has enough time to develop. The growth rates of this instability is \( \Gamma/\omega_p = (7.8 \pm 0.5) \times 10^{-5}\) in the time interval \( \omega_p t = [2920, 10, 150] \). Moreover, particles from both bunches need more time to approach each other during the bunch expansion. That occurs due to the larger bunch separation and the slowdown of particles by the developed streaming instability. The streaming instability in the center has a lower growth rate \( \Gamma/\omega_p = (2.69 \pm 0.02) \times 10^{-3}\) in the time interval \( \omega_p t = [10, 100, 10, 980] \).

In Simulations 5–7 with nonzero initial drift velocity (Figure 1(c)), the simulation evolution is different. Although the initial distributions of species undergo a two-stream instability, the electrostatic energy by expansion effects rises with the growth rate \( \Gamma/\omega_p \sim 1 \) as a strong instability due to expansion effects at density gradients. Particles quickly form local regions with nonzero charge densities and cause ambipolar diffusion that generates stronger electric fields than the two-stream instability. Even a small increase in drift Lorentz factor from \( \gamma_d = 1 \) to \( \gamma_d = 1.005 \) increases the amount of the released energy by \( \sim 3 \) orders of magnitude in Simulation 5. As the drift velocity increases in Simulations 6 and 7, the amount of converted energy approaches \( < 0.15\% \) of the total initial kinetic energy. In all three Simulations 5–7, the electrostatic energy gradually decreases after a short time, and then the system evolves with a constant energy ratio.

#### 3.1.2. Spatial Evolution along Simulation Domain

In Figures 2–4, we present the evolution of the electrostatic energy density and electron density, which is normalized to the initial density at the bunch center \( n_0 \), along the simulation box. Figure 2 shows the evolution for Simulations 1–3. As particles from the bunches expand, the electrostatic energy forms a visible “web” structure of waves in the background electric field (blue regions). When particles begin to interact in the simulation center at \( \omega_p t \sim 3000 \), electrostatic waves are formed at time \( \omega_p t \sim 4000 \), which are stable for long time. Electrostatic waves generated at \( x < 0 \) start propagating to the left, while those at \( x > 0 \) move to the right, respectively. The distance between generated waves increases with time. New waves are formed at the edges of expanding bunches. With increasing temperature, the distance
between them increases while the intensity of their electric fields decreases. The total particle density (Figure 2(d)–(f)) decreases in the centers of the bunches as they expand, and the low density increases in the center.

Figure 3 shows the effect of increasing distance between bunches in Simulation 4. Waves generated by the streaming instability have higher energies and smaller separations between them. Furthermore, the density fluctuations are smaller than in Simulation 1.

Figure 4 illustrates the effects of the initial drift velocity in Simulations 5–7. Please note that these figures have different color schemes than Figures 2–3. The mean intensity of electrostatic waves is much higher in these three cases. The most intense electrostatic waves are created in regions with the largest density gradients. Until \( \omega pt \sim 1000 \), the group velocity of these waves is close to zero. Then, it increases toward the simulation center, where they eventually interact and may be absorbed. In these three simulations, the density evolution does not fully correspond to the evolution of the electric field. While for \( \gamma_d = 1.005 \), the density gradient evolves more or less smoothly, stronger density waves are formed with increasing drift velocity. Moreover, the propagation direction of the particle density waves is not only toward the simulation center, but also outwards.

Table 2 summarizes the properties of the electric field intensity in all simulations. We select the final time 15,000 \( \omega pt \) to analyze the simulations with \( \gamma_d = 1 \) and time 1000 \( \omega pt \) for simulations with \( \gamma_d > 1 \). The spatial region is selected to cover the region of generated waves in all cases. Maximum, as well as mean values, are calculated in the selected regions. Note that \( E_\rho(x, t) \) is electrostatic energy density varying with position and time, while \( E_{\gamma}(t = 0) \) (Figure 1) is the mean kinetic energy density at the simulation start.

**Figure 2.** Evolution of the electrostatic energy density (a–c) and particle density (d–f) for inverse temperatures \( \rho = [1, 0.01] \) along the x-direction. Simulation 1 (a, d), Simulation 2 (b, e), and Simulation 3 (c, f). In all cases \( \gamma_d = 1 \), \( l/L = 1/30 \).

**Figure 3.** Evolution of the electrostatic energy density (a) and particle density (b) for distance between bunches \( l/L = 1/3 \), \( \rho = 1 \), \( \gamma_d = 1 \) along the x-direction in Simulation 4. Compare with Figure 2.
When particles from both bunches. The wavenumber of both medium- and high-intensity waves decreases in time. The most intense waves are created during the interaction of particles from both bunches. The wavenumber of both medium- and high-intensity waves decreases in time. In addition, the wavenumber of these waves decreases with increasing plasma temperature. When particles from the bunches begin to interact at ωpt ∼ 3000, only electrostatic waves are created (green-yellow lines). The number of generated waves and their wavelengths decrease with increasing plasma temperature. In addition, the wavenumber of these waves decreases in time and additional waves are formed at larger wavelengths.

When bunches are separated by a larger distance in Simulation 4 (Figure 6), the wavenumbers of electrostatic waves manifest a slightly different behavior. The initial (ωpt < 3000) low-amplitude background waves are not created in pairs. Instead, only waves with constant wavenumber in time are present. As the expanding particles from the bunches interact with the background (3000 < ωpt < 10,500), they form beam waves. Finally (ωpt > 10,500), the most intense waves are created during the interaction of particles from both bunches. The wavenumber of both medium- and high-intensity waves decreases in time.

Figure 7(a)–(c); (for Simulations 5–7) show the evolution of the wavenumber for increasing initial drift velocity. The waves are created at kc/ωpt ∼ 0 right after the simulation start; their wavenumbers increase in time.

### 3.1.4. Dispersion Properties

Figures 8–10 present dispersion diagrams of the electrostatic and density waves. In order to build the dispersion diagrams, the region x/d_e = [0, 6000] and the whole simulation time every 120th time step are considered.
The dispersion diagrams of the electrostatic waves (Figure 8(a)) and the subluminal waves (Figure 8(b)) show the generation of two types of waves, the superluminal electrostatic wave and the diagonal mode with subluminal electrostatic waves. The frequencies of both superluminal and subluminal waves decrease with increasing temperature due to dispersion effects in the relativistically hot plasma rest frame (see, e.g., Rafat et al. 2019b, Figure 7) as $\omega_p / \omega_p \sim [0, 0.3]$ and $k c / \omega_p \sim [-0.5, 0]$. Subluminal waves with $\omega_p / \omega_p = [0, 0.1]$ and $k c / \omega_p = [-0.1, 0]$ are enhanced, and the subluminal waves with positive $k$ do not form contiguous modes.

The dispersions of electrostatic waves (Figure 10a–c) mostly manifest superluminal electrostatic waves with negative wavenumbers. The intensity of waves increases with increasing drift velocity. Low-intensity subluminal electrostatic waves are enhanced with $v_p \approx c$ and positive wavenumber. The density dispersion (Figure 10d–f) shows subluminal waves with negative wavenumber and increasing intensity of waves $v_p \approx c$ with increasing temperature.

3.2. Effects of the Initial Drift Velocity

In this section, we present detailed results of Simulations 1 and 6. In Simulation 1, the initial drift is zero, in Simulation 6, the initial drift between electrons and positrons equals the thermal velocity.

3.2.1. Evolution of the Electron Phase Space

Figure 11 shows the electron phase spaces of the whole domain for three selected time steps. As particles overlap in phase space in Simulation 1, they begin to form an instability and waves. The electrostatic waves influence the slower particles with $|u|/c < 2$ and form phase-space holes. In Simulation 6, the shown range of velocities is larger. Plasma heats up in the regions with the strongest electrostatic waves (Figure 11(c),(f)). The tail of the distribution can exceed $|u|/c \sim 50$. For $x > 0$, particles have mostly positive velocities, while there are mostly negative ones for $x < 0$. Their distributions are not symmetric with respect to the axis $u = 0$, and generally, they are stable with respect to the streaming instability formed in Simulation 1.

3.2.2. Evolution of Electrostatic Energy Density

The evolution of the mean electrostatic energy density in selected regions along $x$ is shown in Figure 12. In Simulation 1, the initial energy density evolution is not the same for all positions. It increases from the simulation center to the edge as the particle density increases. As the instability starts at the center, the energy density starts to grow there first. Then, further regions manifest exponential energy increases as they are further away from the center. The estimated growth rates $\Gamma / \omega_p$ of the exponentially
Growing parts are: 

\((4.43 \pm 0.05) \times 10^{-3}\) for \(x/d_e = [-250, 250]\); 
\((2.28 \pm 0.03) \times 10^{-3}\) for \(x/d_e = [1750, 2250]\); 
\((1.24 \pm 0.02) \times 10^{-3}\) for \(x/d_e = [3750, 4250]\); 
\((1.61 \pm 0.03) \times 10^{-3}\) for \(x/d_e = [5750, 6250]\); and 
\((1.37 \pm 0.02) \times 10^{-3}\) for \(x/d_e = [7750, 8250]\). 

The growth rate decreases along \(x\) because the particle velocity distributions broaden, and the two-stream instability weakens. However, the saturation energy density is almost the same in all regions, reaching the mean energy density of \(\sim 2 \text{ erg cm}^{-3}\). After the start of Simulation 6, most of the electrostatic energy density is generated in \(x/d_e = [1750, 2250]\), where their mean energy density reaches \(\sim 3.5 \times 10^4 \text{ erg cm}^{-3}\). Although the energy density is not constant along \(x\) in the selected boxes, the mean value along a sufficiently large region can be a good representative. The time evolution of the energy densities has a decreasing trend with the exception of the simulation center \(x/d_e = [-250, 250]\), where the energy density varies in time and remains \(>10^3 \text{ erg cm}^{-3}\) until the simulation end.

3.2.3. Wave Profiles

Figure 13 shows two example profiles of the electron and positron density and the intensity of the electric field. We select typical regions where one of the largest electrostatic waves appears. For Simulation 1, the wave electric intensity reaches \(\sim 15 \text{ kV cm}^{-1}\). At the same position, the electrons and positrons form a density fluctuation. The particle densities to the left from the wave are almost constant; only small fluctuations are visible. On the right from the wave center, the particle density decreases with respect to the left side. In Simulation 6, the density and the electric intensity vary more strongly along \(x\). The maximal intensity of the electric field reaches \(\sim -200 \text{ kV cm}^{-1}\). The density varies in the
The velocity $\gamma$ (regions along the phase-space holes are formed, and the electric potential energy potential wall is formed with an electric potential energy height of $0.3\,\rho_{\text{e}}$, in Simulation 1 for $x/d_e = [-7750, 8250]$ from the center, only one peak at negative velocities and a plateau for positive velocities are present. By contrast, at the center of Simulation 6, the plasma is quickly heated after the simulation start. Electrons also arrive into the simulation center during the evolution. Though we select exactly the simulation center, at $\omega_d t = 1000$ the tail of the distribution contains significantly more electrons with positive velocities. Despite the limited number of macroparticles in the simulation, the distribution tail is covered well. Moreover, $\sim 0.1\%$ of electrons exceed the velocity $u/c = 150$. The plasma is slightly cooled down at later simulation times, and the distribution becomes more thermalized with a mean value of $\sim 0.6c$, but it is still significantly hotter than the initial plasma. Moreover, the distribution function also significantly varies with the distance from the simulation center. While the plasma is hot in the simulation center, it becomes cooler with increasing distance from the simulation center.

3.2.6. Electrostatic Wave Dispersion along Simulation Domain

Figure 16 shows dispersion diagrams for selected regions along the $x$-axis. As these regions are smaller than in Figure 5–7, they do not cover such a high range of plasma densities, and the broadening of the superluminal electrostatic mode is also smaller. In Simulation 1 for $x/d_e = [-250, 250]$, the enhanced subluminal electrostatic waves show a group velocity $u/c \sim 0$. Its velocity increases with increasing distance from the simulation center and approaches the speed of light. Also, the superluminal electrostatic waves show up the relativistic shift as the plasma drifts toward the simulation center with relativistic velocities. Figures (a–c) are overlaid with analytical dispersion solutions of superluminal and subluminal electrostatic waves for a Maxwell–Jüttner distribution with $\rho = 1$. Relativistic corrections with $v/c = 0, 0.3,$ and 0.5 are applied to superluminal electrostatic waves. The subluminal waves are overlaid with beam-mode waves with relativistic corrections $u/c = 0, 0.55,$ and 0.75. In Simulation 6, the frequency of the superluminal electrostatic waves increases with increasing distance from the simulation center.

4. Discussion

A number of proposed pulsar radio emission models are based on the consideration of plasma instabilities due to interacting streaming bunches of positrons and electrons in the lower parts of pulsar magnetospheres. In order to verify this conjecture, we conducted a series of PIC-code simulations to comprehend the nonlinear evolution of the instabilities of a corresponding relativistically hot pair plasma for parameters appropriate to describe this situation: for a range of initial particle thermal velocities, of distances between the bunches, and the drift velocities between electrons and positrons. We
Figure 10. Dispersion diagrams of the electrostatic waves (a–c) and the particle density waves (d–f) for initial drift Lorentz factors $\gamma_d = [1.005, 10.05]$ along the $x$-direction. All quantities selected in $x/d_e = [0, 6000]$ and the whole simulation time. Simulation 5 (a, d), Simulation 6 (b, e), and Simulation 7 (c, f). In all cases $\rho = 1$, $l/L = 1/30$. Compare with Figures 8 and 9.

Figure 11. Electron phase-space evolution of Simulation 1 (a–c) and Simulation 6 (d–e). Each case for three selected time moments. Note that the shown range of velocities is larger in Simulation 6.
found that the initial drift velocity between the particle species is the main parameter governing the bunch evolution. We found that if there is no initial relative drift, the bunches just expand until the fastest particles of two bunches reach each other to cause, eventually, a streaming instability. However, if there is a finite initial drift between the species, the expansion becomes suppressed since strong electrostatic fields are generated. As a result, the plasma becomes strongly heated, the particles become confined, and no streaming instability takes place when the bunches overlap.

Already previous studies revealed an important constraint for the bunch interaction mechanism to work: a sufficient long time interval is needed to allow bunch expansion and interaction. More details about this constraint in typical pulsar magnetospheres are discussed in Melrose et al. (2021a). We discuss more details about expansion properties and additional conditions below.

### 4.1. Effects of Finite Drift Speed

For a finite mutual drift speed of the electrons and positrons (Simulations 5–7), two types of instabilities are generated. The two-stream instability between electrons and positrons along the initially generated bunch is the first one. Its growth rates are not known from the literature for our parameter set. Notwithstanding, in Simulations 5 and 6, the electron and positron distributions are not separated enough, and the instability growth rates approach zero. A significant positive growth rate only appears in Simulation 7. In the literature, we found the closest value for an integrated growth rate $\Gamma/\omega_p \approx 1.413 \times 10^{-3}$ (Manthei et al. 2021, Figure 7(b)) in the reference frame of one of the beams. In this reference frame, one beam is taken as a background plasma with density $n_0$, while the second beam with density $n_1$ has a Lorentz factor $\gamma'_b = 200$. We also considered the values $\rho_0 = \rho_1 = 1$ and the density ratio $r_e = n_1/\gamma'_b n_0 = 1$. The second instability is the ambipolar diffusion effect between species that occurs when they expand asymmetrically at the density gradients of the bunches. That generates strong electrostatic waves on timescales of order $\omega_p^{-1}$. Because these waves are significantly more intense (approximately by two to four orders of magnitude for Simulations 5–7) and grow in a significantly shorter time than the two-stream instability, they dominate the contribution to the produced electrostatic waves, bunch dynamics, and possible radio emission.

Thus, we have now found an additional necessary condition refining the bunch interaction mechanism: the velocity-space distributions of both electrons and positrons have to be similar all along the whole bunch. Specifically, if the accumulation of expanding particles creates a region with a locally enhanced charge density, electric fields are generated, creating a potential step for expanding charged particles. If the potential energy level is higher than the mean kinetic energy of particles, approximated as $\langle \gamma - 1 \rangle$ mev, where $\langle \rangle$ denotes the average value over the particles that try to pass over this potential step, the bulk of them is restrained. For Simulation 6, the electric potential height can reach $(13–50)$ mev (Figure 14(d–f)), while the typical expanding particle kinetic energy may be approximated as the sum of drift and thermal energies $\approx [(\gamma_b - 1) + 1/\rho] m_e c^2 \approx 1.414 m_e c^2$. Thus,
Figure 14. Electron phase spaces at selected distances from the simulation center overlaid with the electric potential (magenta line). (a) Simulation 1 at $\omega_{pt} = 15,000$. (b) Simulation 6 at $\omega_{pt} = 1000$. See Figure 15 for velocity profiles.

Figure 15. Electron velocity distributions as a function of time and distance from the simulation center: (a–b) Simulation 1, and (c–d) Simulation 6. (a,c) $x/d_e = [-250, 250]$ at selected time moments. (b) Selected regions at $\omega_{pt} = 15,000$. (d) Selected regions at $\omega_{pt} = 1000$. 
in all three analyzed regions in time $\omega_p t = 1000$, the bulk of particles is restrained from expanding. We found that even for a small drift Lorentz factor $\gamma_d/c = 1.005$, the created electric fields and potential levels at bunch density gradients are large enough to influence the expansion and locally heat the plasma significantly. Oppositely, the bunch can expand and eventually interact, if the diffusions of both species are similar and the generated potential steps do not confine the bulk of particles. For example, in Simulation 1, the electric potential height reaches $\approx (0.18 - 0.35)m_e c^2$. The typical particle kinetic energy is $\approx (\gamma_p - 1 + 1/\rho)m_e c^2 \approx m_e c^2$, so the bulk of particles can cross this potential barrier.

These conditions refine the opportunities of the interacting bunch mechanism to work because the pulsar magnetosphere is very dynamic, and the pair bunches are created in a cascading process. It may be statistically possible that particles are distributed in a way that allows the ambipolar effect to develop. However, whether the velocity distributions and density profiles, constraining or allowing bunch expansion, are systematically generated requires additional investigation of the plasma leaving the spark region in the context of the global pulsar magnetosphere properties.

Dependent on the nonzero initial drift velocity, the energy density of generated waves in Simulations 5–7 reached up to $E_{\gamma} \sim (0.017 - 2.5) \times 10^5$ erg cm$^{-3}$ ($E_{\gamma}/(n_e m_e c^2) \sim (0.066 - 9.6) \times 10^{-1}$) and the maximal electric field amplitude reached up to $E \sim (0.62 - 7.5) \times 10^5$ erg cm$^{-3}$ ($E/(m_e c \omega_p e^{-1}) \sim (0.37 - 4.4)$).

4.2. Effects of Zero Drift Speed

In the case of a vanishing initial drift, both species would expand symmetrically, and the fastest particles (in the bunch reference frame) would escape the bunch. They can interact with the background plasma, forming a weak streaming instability as they propagate. Later they “catch up” with fast particles of the adjacent bunch (expanding in the opposite direction in the bunch reference frame). They cause a two-stream instability that saturates at $\sim 1000 \omega_p t$ after their overlap and eventually form electrostatic waves that survive until the simulation end. Dependent on the initial plasma temperature, the electrostatic wave energy density is in the range $E_{\gamma} \approx (0.14 - 1.6) \times 10^5$ erg cm$^{-3}$ ($E_{\gamma}/(n_e m_e c^2) \approx (0.53 - 6.1) \times 10^{-1}$) and the electric field amplitude $E \approx (0.56 - 1.9) \times 10^4$ V cm$^{-1}$ ($E/(m_e c \omega_p e^{-1}) \approx (0.33 - 1.1) \times 10^{-1}$).

The growth rates of the streaming instability by particle phase-space overlap in Simulation 1 vary with distance from the simulation center as the density of one beam increases and
that of the second beam decreases. Together with the fact that the velocity distributions of both streams differ from their initial profiles, the growth rates are generally a function of time and position, as the velocity distribution function changes from an unstable double-peak to a stable peak-plateau distribution. If we treat the velocity distribution in Simulation 1 in the range \( x = [-250, 250] \) and time \( \omega_p = 5000 \) (Figure 15(a)) in the cold-plasma approximation, which neglects the background and beam thermal velocities, the growth rate can be estimated as (Hinata 1976; Shalaby et al. 2017)

\[
\Gamma = \frac{1}{\omega_p} = \frac{1}{2\sqrt{2\gamma_b^3}} \tag{9}
\]

Transforming the distribution maxima \( u_{max}/c \approx \pm 1.2 \) \( \gamma_{\text{max}} \approx 1.56 \); Figure 15) to the reference frame of one of the beams, we get a Lorentz factor of \( \gamma_b' \approx 3.9 \) in this frame. That gives a growth rate \( \Gamma/\omega_p \approx 2.3 \times 10^{-2} \), larger than the values estimated from our fits. However, assuming that the same velocity distribution may be approximated by two Maxwell–Jüttner distributions with temperature ratio \( \epsilon_p \) and density ratio \( r_n \), the maximal growth rate may be estimated by Rafat et al. (2019a) as

\[
\Gamma_{\text{max}} = \frac{\epsilon_p^{1/2} r_n^{3/2}}{2\gamma_\lambda^3} \tag{10}
\]

In this case we estimate \( (\gamma) \approx \gamma_b' \) and the density ratio decreases due to the relativistic transformation by a factor \( r_n \sim \gamma_{\text{max}}^{-1} \). We get a growth rate \( \Gamma/\omega_p \approx 4.3 \times 10^{-3} \). This value is in good agreement with the estimated value \( \Gamma/\omega_p \approx 4.3 \times 10^{-3} \) at the simulation center (Section 3.2.2). The instability in the simulation center cannot be approximated by a weak-beam approximation, where the density ratio is \( r_n \ll 1 \), but the growth rates are low enough for the condition of weak instability, i.e., \( \Gamma/\omega_p \ll 1 \). The beam velocities used for the calculation of the growth rates by Manthei et al. (2021) are too high to be directly compared with Simulations 1–3, because they assume more separated beam distributions, \( \gamma_b' \gtrsim 12 \). In addition, the growth rates decrease with increasing distance from the simulation center because the density ratio \( r_n \) and the velocity separation of both distributions decrease.

Taking several effects of the bunch expansion into account, we found that electrostatic waves are formed even in a highly relativistic plasma with \( \rho < 1 \) in Simulations 2 and 3. Their associated electrostatic wave energy is only slightly above the thermal noise levels (less than one order of magnitude in Simulation 3), but their noise levels are due to a relatively small amount of particles in the simulation box. In a real plasma, this enhanced thermal noise may be up to three orders of magnitude (for a plasma density six orders higher) because the number of physical particles is much larger than the number of numerical particles. This implies that in a real plasma, the instability increase may be more significant.

When the expanding plasma bunches overlap in the simulation center, their distributions are well separated for velocities around \( u \approx 0 \) even for ultrarelativistically hot plasmas, while the shapes of the distribution tails approach the bunch initial distribution (Figure 15(a),(b)). This also answers the question raised in a review of pulsar emission processes by Melrose et al. (2021b): whether the velocity-space distributions of overlapping bunch particles can be sufficiently well separated to become unstable to generate waves. They can be if the particle overlap is allowed. Note that this finding differs from the temperature statement condition \( \rho \gtrsim 1.66 \) for wave formation in a homogeneous uniform streaming instability found by Benáček et al. (2021). The main difference is that we set up the Maxwell–Jüttner distribution with a specific temperature for the initial bunch, and the distribution is deformed during the bunch expansion due to spatial inhomogeneities. But Benáček et al. (2021) assumed that all species initially have the Maxwell–Jüttner distributions, with spatially uniform densities, causing the streaming instability; lower temperatures or higher beam velocities are needed for a sufficient instability development.

We also found that the most intense electrostatic waves differ between the cases without and with a finite initial drift velocity. For a zero initial drift velocity, the subluminal electrostatic waves with phase velocities \( v_p = (0–0.75)c \) are excited. Superluminal waves are fainter in a low-temperature plasma, \( \rho \sim 1 \), but their intensity increases with increasing plasma temperature. For a nonzero drift velocity, the superluminal electrostatic waves are the most intense waves, and the subluminal modes are weak.

4.3. Radio Emission Mechanisms

For the waves, we determined from our simulations (see, e.g., the dispersion diagrams in Figures 8–10) that several electromagnetic emission mechanisms could apply:

First, electromagnetic waves could be directly generated by a linear acceleration mechanism. This mechanism would be more efficient in the case of finite initial drifts between species, in which strong electrostatic fields are generated (Melrose & Rafat 2017; Melrose et al. 2021a). An oscillation frequency close to the plasma frequency was suggested to be the emission frequency (Eilek & Hankins 2016). We found, instead, that waves are generated over a broad frequency range. The local electrostatic superluminal wave frequency decreases as \( \sim (\gamma^{-5}) \) with temperature. The typically found frequencies are in the range \( \omega/\omega_p \sim 0.01-0.5 \), i.e., a factor 2–100 times smaller than the plasma frequency. The motion of individual particles in such a field must be taken into account to quantify this emission process properly. The prediction of emission properties is nontrivial as there can be several types of emitting particles, e.g., particles that can be captured in electric fields and particles with a kinetic energy large enough to pass through such electrostatic waves. The situation is more complicated due to the fact that the oscillation frequency can be smaller than the local plasma frequency, but the surroundings of the bunch can be sufficiently dilute to allow the propagation of electromagnetic waves. The relativistic frequency shift further increases the emission frequency.

A second plausible mechanism is a relativistic plasma emission. In this process an electrostatic wave interacts with other waves, e.g., another electrostatic wave or a density wave, to generate electromagnetic waves. One option is that the subluminal electrostatic wave propagates in a given direction and interacts with a counterpropagating wave. However, no sufficiently intense waves were generated in all our simulations propagating in the opposite direction at a sufficiently high frequency and intensity. Instead, the subluminal electrostatic waves mostly propagate along the same direction. Another possibility would be a modulation by an external wave with a component oscillating perpendicular to the magnetic field.
(Weatherall 1997). Such electromagnetic waves could escape the emission region.

Other plasma emission mechanisms—the free electron maser, curvature emission, the electron-cyclotron maser—do not specifically apply to instabilities caused by a bunch—bunch interaction. Curvature emission assumes independent beams, not necessarily an interaction among them. Free electron maser and electron-cyclotron maser mechanisms require a finite energy of perpendicular particle motion and specific types of velocity-space distribution functions that cannot be assumed to be formed in the lower pulsar magnetosphere.

5. Conclusions

We studied the nonlinear evolution and interaction of electron–positron bunches (clouds) in pulsar magnetospheres depending on the relative drift speed of electrons and positrons, the plasma temperature, and the distance between the bunches. The resulting plasma instabilities and particle oscillations are supposed to cause coherent radio emissions.

We found that already a low drift speed between electrons and positrons causes the generation of large-amplitude electrostatic waves. The wave energy reaches up 15% of the total kinetic particle energy. These waves heat the plasma, but no streaming instability between the bunches is triggered. For drift Lorentz factor $\gamma_d = 10.05$, the electrostatic energy density can locally reach up to $E_E \sim 2.5 \times 10^7 \text{ erg cm}^{-3} (E_E/(\bar{n}_e \bar{c}^2) \sim 0.96)$ and an electric field intensity can become as large as $E \sim 7.5 \times 10^5 \text{ V cm}^{-1} (E/(\bar{n}_e \bar{c} \omega_{pe} e^{-1}) \sim 4.4)$. For zero drift speeds between the species, the expanding particles from bunches overlap in phase space and cause streaming instabilities with density ratio $r_n \sim 1$ (in the bunch reference frame). The generated electrostatic waves are weaker compared to the preceding scenario. Their electrostatic energy density reach up to $E_E \sim 1.6 \times 10^7 \text{ erg cm}^{-3} (E_E/(\bar{n}_e \bar{c}^2) \sim 6.1 \times 10^{-5})$ and the electric field intensity $E \sim 1.9 \times 10^7 \text{ V cm}^{-1} (E/(\bar{n}_e \bar{c} \omega_{pe} e^{-1}) \sim 1.1 \times 10^{-4})$. Such a regime very much requires, however, drift speeds <0.1 c, which do not create local electric discharges, fields, and potential steps strong enough to constrain the relativistic particles—which further constrains the applicability conditions of the bunch overlap mechanism.

The authors acknowledge the support by the German Science Foundation (DFG) projects BU 777-17-1 and MU-4255/1-1. We acknowledge the developers of the ACRONYM code (Verein zur Förderung kinetischer Plasmasimulationen e. V.). The authors gratefully acknowledge the Gauss Centre for Supercomputing e.V. (www.Gauss-centre.eu) for partially funding this project by providing computing time on the GCS Supercomputer SuperMUC-NG at the Leibniz Supercomputing Centre (www.lrz.de), projects pr74vi and pn73ne. This work was supported by the Ministry of Education, Youth, and Sports of the Czech Republic through the e-INFRA CZ (ID:90140). Part of the simulations was carried out on the HPC-Cluster of the Institute for Mathematics of the TU Berlin. The authors also thankfully acknowledge their enlightening discussions with Axel Jessner of the Max-Planck-Institute for Radioastronomy in Bonn.

**Software**: PIC code ACRONYM, Python

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