QED$_2$ Light-Front Hamiltonian reproducing all orders of covariant chiral perturbation theory

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Abstract

Light-Front (LF) Hamiltonian for QED in (1+1)-dimensions is constructed using the boson form of this model with additional Pauli-Villars type ultraviolet regularization. Perturbation theory, generated by this LF Hamiltonian, is proved to be equivalent to usual covariant chiral perturbation theory. The obtained LF Hamiltonian depends explicitly on chiral condensate parameters which enter in a form of some coupling constants.

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\section{Introduction}

Hamiltonian approach to Quantum Field Theory in Light-Front (LF) coordinates \cite{1}, $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$, $x^\perp = \{x^1, x^2\}$, with $x^+$ playing the role of time, is one of nonperturbative approaches which can be used in attempts to solve strong coupling problems \cite{2}. It has the advantage of having simple vacuum state description, because the physical vacuum is described on the LF as the lowest eigenstate of LF momentum operator $P^- \geq 0$, and this vacuum coincides with bare perturbative one on the LF.

However the specific LF singularities at zero LF momenta ($k^- \to 0$), being regularized via cutoff $|k^-| \geq \varepsilon > 0$ (which breaks Lorentz and gauge symmetries), can be the cause of noncomplete equivalence between LF theory and its original formulation in Lorentz coordinates. The nonequivalence can be found even when one compares LF perturbation theory with usual covariant one \cite{3, 4}. One needs to add new counterterms to canonical Hamiltonian to restore the equivalence \cite{3, 4}. A general method to find these counterterms to all orders in perturbation theory was described in \cite{4}. For gauge theories and, in particular, for QCD, in axial LF gauge, $A^- = 0$, this method gave an infinite number of possible new counterterms \cite{4}. To avoid this difficulty one needs to use other, more complicated regularization scheme \cite{5}.

However in (1+1)-dimensional space-time one can use the bosonization method \cite{6, 7, 8, 9} to treat the theory analogously to scalar field theories. These bosonic theories have essentially nonpolynomial form of the interaction Hamiltonian. We show in the present paper how the method of paper \cite{4} can be applied to these theories. For simplicity we use the example of $QED_2$. In this case the bosonization method results in field theory like Sine-Gordon model. As it is known, the Sine-Gordon model Lagrangian contains the interaction term in a form of $\cos(\beta u)$, and there is special case $\beta = \sqrt{4\pi}$, at which ultraviolet (UV) behaviour of the theory becomes worse. Exactly this case arises in boson form of $QED_2$, and it is considered in this paper. For a case $\beta < \sqrt{4\pi}$ similar problem was investigated in the paper \cite{10}. We construct LF Hamiltonian for $QED_2$ model (in boson form) using chiral perturbation theory to all orders. This Hamiltonian depends on fermionic condensate pa-
parameters which enter the Hamiltonian like coupling constants. These parameters depend essentially on UV regularization parameter (which is introduced via Pauli-Villars type regularization scheme) and can become infinite in the limit of removed regularization cutoff. The appearance of this divergency and the necessity of UV regularization in the model under consideration correspond to special case $\beta = \sqrt{4\pi}$ in Sine-Gordon type models.

The obtained LF Hamiltonian can be applied to the calculation of mass spectrum using chiral perturbation theory. It can be checked that results coincide with already known ones in the 2nd order [11]. Moreover, one can apply this LF Hamiltonian in nonperturbative calculations using the DLCQ method [12, 13].

2. The analysis of ultraviolet divergences

Let us start with the boson form of $QED_2$ [6, 7, 8, 9, 14] in Lorentz coordinates $x^\mu = \{x^0, x^1\}$. It can be described by Lagrangian density

$$L = \frac{1}{8\pi} \left( \partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2 \right) - \gamma (\cos \theta - : \cos(\varphi + \theta) :) ,$$

(1)

where $\varphi(x)$ is scalar field originated from UV-renormalized fermion currents; the $\theta$ – parameter characterizes "instanton" vacuum [8, 11]; $m = e/\sqrt{\pi}$ is Schwinger boson mass, $e$ is the coupling constant; $\gamma = M m e^c/(2\pi)$, $M$ is fermion mass and $c = 0.577216$ is Euler constant; normal ordering symbol, $::$, corresponds to the decomposition of the quantized field in creation and annihilation operators in the interaction picture, the zero mode of the field is excluded, $\int dx^1 \varphi(x) = 0$ (that’s why the action doesn’t include the term linear in $\varphi$).

Let us consider the structure of Feynman diagrams of the perturbation theory with respect to (w. r. t.) $\gamma$. The vertices with $j$ entering lines give the factors

$$i^{j+1} \gamma c_j(\theta), \quad j \geq 2,$$

(2)

where

$$c_j(\theta) = \begin{cases} 
\cos \theta, & \text{for even } j \\
 i \sin \theta, & \text{for odd } j 
\end{cases} .$$

(3)
Propagators can be written as

\[ \Delta(k) = \frac{i}{\pi} \frac{1}{k^2 - m^2 + i0}, \quad \Delta(x) = \int dk \, e^{ikx} \Delta(k), \quad (4) \]

where \( dk = dk_0dk_1, \ kx = k_0x^0 + k_1x^1. \)

First of all we investigate whether the perturbation theory in \( \gamma \) is UV-finite. The interaction Hamiltonian is nonpolynomial in the field \( \varphi \), and one has infinite number of diagrams at any finite order in \( \gamma \). One can see that all separate diagrams are finite (possible logarithmically divergent diagrams, shown in fig. 1, are excluded owing to normal ordering).

Fig. 1. Logarithmically divergent diagram excluded by normal ordering.

Nevertheless infinite sum of these diagrams in a given order in \( \gamma \) can diverge, and this divergence is UV, because it can be removed by UV-regularization (as it is shown below). Let us consider, for example, the sum \( s_{ln} \)

\[ s_{ln} = \sum_{m=2}^{\infty} D_{lnm}, \quad s_{ln} = s_{nl} \quad (5) \]

of diagrams \( D_{lnm} \) shown in fig. 2.

Fig. 2. The diagram \( D_{lnm} \) with \( l + n \) external and \( m \) internal lines; \( l \) and \( n \) lines are connected to the left and right vertex, accordingly; \( p \) is the total momentum going through the diagram.
This sum is

\[
s_{ln} = \frac{\gamma^2}{l!n!(1 + \delta_{ln})} \sum_{m=2}^{\infty} \frac{1}{m!} c_{l+m}(\theta) c_{n+m}(\theta) i^{l+m+1} \times \\
\times \int \prod_{j=1}^{m} dk_j (2\pi)^2 \delta(\sum_{j=1}^{m} k_j - p) \prod_{j=1}^{m} \Delta(k_j),
\]

where \( p \) is total momentum going through the diagram, \( \delta_{ln} \) is Kronecker symbol. "Symmetry" coefficients of the diagrams are taken so that the sum of expressions \( s_{ln} \) with all transpositions of external momenta gave the contribution to Green function. We obtain

\[
s_{ln} = -\frac{\gamma^2}{l!n!(1 + \delta_{ln})} \sum_{m=2}^{\infty} \frac{1}{m!} c_{l+m}(\theta) c_{n+m}(\theta) i^{l+n+2m+2} \int dx e^{-ipx} \Delta(x)^m = \\
-\frac{\gamma^2}{l!n!(1 + \delta_{ln})} \int dx e^{-ipx} \times \\
\times (c_{l+1}(\theta)c_{n+1}(\theta) (\Delta(x) - \text{sh}(\Delta(x))) + c_l(\theta)c_n(\theta) (\text{ch}(\Delta(x)) - 1)),
\]

where the eq. (3) was taken into account. At \( x \to 0 \)

\[
\Delta(x) \sim \ln \frac{1}{x^2},
\]

so that we get the integral over \( x \) logarithmically divergent.

Let us show that, in spite of this, the sum of all diagrams \( s_{ln} \), contributing to Green functions, is finite. Indeed the divergent part of (7) is proportional to

\[
s_{ln} = -\frac{\gamma^2}{2l!n!(1 + \delta_{ln})} \left( c_l(\theta)c_n(\theta) - c_{l+1}(\theta)c_{n+1}(\theta) \right).
\]

As it follows from definitions (3), the quantity (9) is

\[
s_{ln} = -\frac{\gamma^2}{4l!n!(1 + \delta_{ln})} \left( (-1)^l + (-1)^n \right).
\]

The Green function includes the sum

\[
\sum_{0 \leq l \leq j/2} s_{l,j-l},
\]
the divergent part of it is
\[
\sum_{0 \leq l \leq j/2} s_{l,j-l}^\infty = \frac{1}{2} \sum_{l=0}^{j} s_{l,j-l}(1 + \delta_{l,j-l}) =
\]
\[
= -\frac{\gamma^2 i^j}{8 j!} \left( \sum_{l=0}^{j} \frac{j!}{l!(j-l)!} (-1)^l + \sum_{l=0}^{j} \frac{j!}{l!(j-l)!} (-1)^{j-l} \right) =
\]
\[
= -\frac{\gamma^2 i^j}{8 j!} ((-1 + 1)^j + (1 - 1)^j) = 0,
\]
as it is stated above.

We shall suppose that UV-finiteness of Green functions holds to any order in \( \gamma \) in this model. But we must consider always only sums like (11), but not separate \( s_{ln} \). As it will be explained in Sect. 3, this fact makes impossible the perturbative construction of LF Hamiltonian, by the method of [4], if we don’t introduce proper UV regularization.

3. The perturbative construction of the Hamiltonian

Let us try to construct a LF Hamiltonian regularized by the cutoff in LF momenta, \( |k_-| \geq \varepsilon > 0 \), (i.e. there are no modes with \( |k_-| < \varepsilon \) in the Fourier decomposition of the field \( \varphi \)), and to generate a perturbation theory, equivalent to the usual covariant one in the limit \( \varepsilon \to 0 \). Let us start from canonical LF Hamiltonian which follows from the Lagrangian (1). The interaction part of this Hamiltonian has the form:
\[
H_I^{can} = \gamma (\cos \theta - : \cos(\varphi + \theta) : ) = \]
\[
= \gamma \cos \theta (1 - : \cos \varphi : ) + \gamma \sin \theta : \sin \varphi : .
\]

It can be shown [15, 16] that noncovariant perturbation theory, obtained with LF Hamiltonian, can be transformed into equivalent covariant perturbation theory via resummation of diagrams, but in this theory the integrations over \( k_- \) are limited by cutoff \( |k_-| \geq \varepsilon \), and one should also integrate firstly over \( k_+ \), then over \( k_- \). The \( \varepsilon \to 0 \) limit of such LF calculation of the diagram we call LF diagram. To construct LF Hamiltonian one has to find all necessary counterterms to canonical LF Hamiltonian which compensate the differences between LF and covariant diagrams.
The method of estimating these differences that allows to find all diagrams, giving nonzero difference, is described in [4]. For its application the requirement \( \omega_+ < 0 \) is necessary, where \( \omega_+ \) is the index of UV-divergence of the diagram in \( k_+ \). This requirement means that Feynman integrals are to be UV-convergent in \( k_+ \). It is easy to see, that for the considered theory this requirement is fulfilled.

Let us describe briefly the method of [4]. Consider, for example, an arbitrary 1-loop Feynman diagram with external entering momenta \( p_{i \mu}^\perp, \ i = 1, 2, \ldots \). The loop momentum \( k_- \) is bounded by cutoff conditions \( |k_- - \sum p_{i \perp}| \geq \varepsilon \), stemming from restriction on the propagator. On the other side, analogous covariant diagram contains the integration over all \( k \). Therefore the difference between these diagrams can be found as the sum of integrals over the bands \( |k_- - \sum \varepsilon p_{i \perp}| < \varepsilon \).

Let us estimate one of these "\( \varepsilon \)"-band integrals. We shift the variable \( k_- \) in this integral so that \( |k_-| < \varepsilon \). Then we change the scale:

\[
k_- \longrightarrow \varepsilon k_-, \quad k_+ \longrightarrow \frac{1}{\varepsilon} k_+.
\]

that makes the integration interval independent of \( \varepsilon \), while keeps Lorentz invariant products like \( k_+ k_- \) or \( dk_+ dk_- \). Propagators, corresponding to internal lines whose momenta are outside of the \( \varepsilon \)-band (owing to external momentum \( p_- \), going through the line) change as follows:

\[
i \frac{1}{\pi} \frac{1}{2(k_+ + p_+)(k_- + p_-) - m^2 + i0} \rightarrow i \frac{1}{\pi} \frac{1}{2(\frac{1}{\varepsilon} k_+ + p_+)(\varepsilon k_- + p_-) - m^2 + i0} \approx \frac{i}{\pi} \frac{\varepsilon}{2k_+ p_-}.
\]

In the paper [4] we used denotations for the lines with momenta outside and inside of the \( \varepsilon \)-band. First one was called \( \Pi \)-line and the last one, \( \varepsilon \)-line. It follows from eq. (15) that every \( \Pi \)-line gives a factor of order \( O(\varepsilon) \) while every \( \varepsilon \)-line gives a factor of order \( O(1) \). Therefore the integral over the band is zero in \( \varepsilon \rightarrow 0 \) limit if at least one of \( \Pi \)-lines is present in the diagram.

Similar analysis can be made for an arbitrary many-loop Feynman diagram. The difference between LF and covariant calculation of this
diagram can be estimated again by considering all possible configurations of Π- and ε- lines in the diagram [4]. It was shown in the paper [4] (for a wide class of field theories) that each of these configurations can be estimated in ε as having the order \(O(\varepsilon^\sigma)(1 + O(\log \varepsilon))\) with

\[
\sigma = \min(\tau, \omega_- - \omega_+ - \mu - \eta),
\]

where the minimum is to be taken w.r.t. all subdiagrams of the diagram at some configuration of Π- and ε- lines in it; \(\omega_±\) are indices of UV-divergency in \(k_±\) of a given subdiagram; \(\mu\) is the index of total UV-divergency in \(k_-\) of all Π-lines in the subdiagram; \(\tau\) is total power of the \(\varepsilon\) that arises, after the change \(k_- \rightarrow \varepsilon k_-\) of loop variables \(k_-\), from numerators of all propagators of the diagram and from all volume elements in the integrals over \(k_-\); \(\eta\) is the part of the \(\tau\) related with only those numerators and volume elements (used in the definition of \(\tau\)) that are not present in the considered subdiagram. Let us apply this general result to our scalar field theory. All propagators have simple structure. Only possible contribution to the \(\tau\) comes from the volume elements \(dk_-\). Because we are interested only in the difference of LF and covariant diagrams, any configuration should contain at least one integration over \(k_-\) in the \(\varepsilon\)-band. Therefore, \(\tau > 0\) (and \(\eta \geq 0\)). Due to Lorentz-invariant form of diagrams in \(k_+, k_-\) we have \(\omega_+ - \omega_- = 0\). It follows from the expression (15) for a Π-line propagator that the \(\mu\) can be counted as the number of Π-lines in the subdiagram taken with the minus sign. Therefore, one has \(-\mu + \eta > 0\) (and, hence, \(\sigma > 0\)) if at least one of Π-lines is present. Thus, only configurations without Π-lines, i.e. at \(\mu = \eta = \sigma = 0\), can contribute to the difference between LF and covariant diagrams. All these configurations are connected with diagrams, shown in Fig. 3, which are equal to zero in LF perturbation theory. Therefore, all difference between LF and covariant perturbation theory is related with the sum of these covariant diagrams.

However the conclusion made before was based on the estimation of the difference between LF and covariant calculation for each separate diagram. To estimate this difference for infinite sums of diagrams which are present in our model at any given order of perturbation theory, one must be sure that infinite sums of separate estimations converge uniformly
in \( \varepsilon \). But it is not so in our scheme, due to divergency of some partial sums of diagrams (like \( s_{ln} \) that we considered above). That is why we use in the following some additional regularization that makes these partial sums finite.

### 4. The construction of LF Hamiltonian with Pauli-Villars type regularization

We need a Lorentz invariant regularization that can be used in covariant and LF calculations simultaneously. We use Pauli-Villars type one, modifying the Lagrangian (4) in the following way:

\[
L = \frac{1}{8\pi} \sum_{l=0,1} (-1)^l \left( \partial_\mu \varphi_l \partial^\mu \varphi_l - m_l^2 \varphi_l^2 \right) - \gamma (\cos \theta - : \cos (\varphi + \theta : ) , \quad (17)
\]

where \( \varphi = \varphi_0 + \varphi_1 \), \( \varphi_0 \) being the original field of the mass \( m_0 = m \), and \( \varphi_1 \) being the additional (ghost) field with a large mass \( m_1 \) playing the role of UV cutoff. Only the sum of propagators of \( \varphi_0 \) and \( \varphi_1 \) fields enter the Feynman diagrams. This sum defines regularized propagator in the form

\[
\Delta(k) = \frac{i}{\pi} \frac{1}{k^2 - m_0^2 + i0} - \frac{i}{\pi} \frac{1}{k^2 - m_1^2 + i0} = \frac{i}{\pi} \frac{m_0^2 - m_1^2}{(k^2 - m_0^2 + i0)(k^2 - m_1^2 + i0)}. \quad (18)
\]
At finite $m_1$ the corresponding propagator $\Delta(x)$ is finite at $x = 0$, and one avoids the divergency for sums like $s_{ln}$. One can repeat now all arguments of Sect. 3 and conclude that the difference between LF and covariant perturbation theories exists only for diagrams shown in Fig. 3. And the difference between these covariant and LF diagrams coincides with covariant diagrams, because LF diagrams are equal to zero. Therefore this difference does not depend on $\varepsilon$.

The counterterms which must be added to canonical LF Hamiltonian should generate all diagrams shown in Fig. 3. Let us denote such arbitrary diagram of order $n$, with $l$ external and $m$ internal lines entering the same vertex, by $R_{lm}^{n(i)}$, where the index $i$ numerates different these diagrams at fixed $n, l, m$. One can express all these diagrams in terms of only $R_{0m}^{n(i)}$ taking into account vertex factors (2) and symmetry coefficients of diagrams:

$$R_{lm}^{n(i)} = \begin{cases} R_{0m}^{n(i)} r_m^{n(i)} (-1)^{l/2}, & \text{for even } l \\ R_{0m}^{n(i)} r_m^{n(i)} (-1)^{(l-1)/2} (-\tan \theta), & \text{for odd } l \text{ and even } m \\ R_{0m}^{n(i)} r_m^{n(i)} (-1)^{(l+1)/2} (-\cot \theta), & \text{for odd } l \text{ and } m \end{cases}$$

where $r_m^{n(i)}$ is a number of vertices in the diagram $R_{0m}^{n(i)}$ which are equivalent to the vertex, to which the external momenta are joined in the diagram $R_{lm}^{n(i)}$ (including this vertex).

The counterterm generating the sum of diagrams $R_{lm}^{n(i)}$ with all even $l$ is

$$\sum_{k=1}^{\infty} B_{2k,m}^{n(i)} \frac{i}{(2k)!} : \varphi^{2k} : = \sum_{k=1}^{\infty} B_{0m}^{n(i)} r_m^{n(i)} (-1)^k \frac{i}{(2k)!} : \varphi^{2k} := -ir_m^{n(i)} R_{0m}^{n(i)} (1 - \cos \varphi).$$

The quantity $R_{0m}^{n(i)}$ is one of vacuum diagrams contributing to vacuum energy density. Summing expressions (20) for all different $R_{lm}^{n(i)}$ with the same $R_{0m}^{n(i)}$, and then summing over $i$ and $m$, we get

$$H_1^n = -in (1 - \cos \varphi) \sum_{i,m} R_{0m}^{n(i)},$$

because the external lines can be joined to representative of each group of equivalent vertices (to one representative for each group), and the number

$$\sum_{i,m} R_{0m}^{n(i)} = \sum_{i,m} R_{lm}^{n(i)} = \sum_{i,m} R_{0m}^{n(i)}.$$
of vertices in such group is equal \( r^n_m(i) \). Summing also over all \( n \) we obtain the following form of the counterterm generating all diagrams of Fig. 3 with even number of external lines:

\[
H_1 = \sum_n H_1^n = \tilde{C}_1 \left( 1 - \cos \varphi : \right), \quad \tilde{C}_1 = -i \sum_{i,m,n} n R^n_m(i).
\] (22)

The coefficient \( \tilde{C}_1 \) depends on parameters \( \gamma, \theta, m_0, m_1 \) and can become infinite in \( m_1 \to \infty \) limit.

For the odd \( l \) we can find, that

\[
\sum_{k=0}^{\infty} R^n_{2k+1,m} \frac{i}{(2k+1)!} : \varphi^{2k+1} :=
\]

\[
= \sum_{k=0}^{\infty} R^n_m(i) (-1)^k (-\tan \theta) \frac{i}{(2k+1)!} : \varphi^{2k+1} := -i \tan \theta r^n_m(i) R^n_m : \sin \varphi :
\] (23)

for even \( m \), and

\[
\sum_{k=0}^{\infty} R^n_{2k+1,m} \frac{i}{(2k+1)!} : \varphi^{2k+1} :=
\]

\[
= \sum_{k=0}^{\infty} R^n_m(i) (-1)^{k+1} (-\cot \theta) \frac{i}{(2k+1)!} : \varphi^{2k+1} := i \cot \theta r^n_m(i) R^n_m : \sin \varphi :
\] (24)

for odd \( m \). Every vertex in the diagram \( R^n_0m \) has the factor \( \cos \theta \) if the number of it’s legs is even, and the factor \( \sin \theta \) if this number is odd. The factors \( -\tan \theta \) in (23) for even \( m \), or \( \cot \theta \) for odd \( m \) in (24), can be produced by the action of the derivative \( \frac{\partial}{\partial \theta} \) on vertex factors \( \cos \theta \), or \( \sin \theta \), accordingly. Summing at fixed \( m \) expressions (23) and (24) for all different \( R^n_{l,m} \) with the same \( R^n_0m \) and taking into account the factors \( r^n_m(i) \), we get the sum of \( n \) terms, in each of which the derivative \( \frac{\partial}{\partial \theta} \) acts on its own factor, so that after summing over \( i \) and \( m \) we have

\[
H^n_2 = i : \sin \varphi : \frac{\partial}{\partial \theta} \sum_{i,m} R^n_0m(i).
\] (25)

Summing over all \( n \) we obtain the counterterm generating all diagrams of Fig. 3 with odd number of external lines:

\[
H_2 = \sum_n H^n_2 = \tilde{C}_2 : \sin \varphi :, \quad \tilde{C}_2 = i \frac{\partial}{\partial \theta} \sum_{i,m,n} R^n_0m(i).
\] (26)
The coefficient $\tilde{C}_2$ depends on the same parameters as $\tilde{C}_1$.

Now we can write the corrected LF interaction Hamiltonian as follows:

$$ H_I = H^\text{can}_I + H_1 + H_2 = C_1 (1 - : \cos \varphi :) + C_2 : \sin \varphi : $$

$$ C_1 = \gamma \cos \theta + \tilde{C}_1, \quad C_2 = \gamma \sin \theta + \tilde{C}_2. $$

(27)

The corrected LF Hamiltonian generates the theory, which is perturbatively equivalent to covariant theory, if the coefficients $C_1$ and $C_2$ depend properly on the parameters $\gamma$ and $\theta$ of the covariant theory.

Let us show that the coefficients $C_1$, $C_2$ can be written in the form of some condensate parameters of the field quantized at $x^0 = 0$ in Lorentz coordinates. Firstly, let us write the eq. (22) in the form

$$ \tilde{C}_1 = -i \gamma \frac{\partial}{\partial \gamma} \sum_{i,m,n} R^{n(i)}_{i,m} = -i \gamma \frac{\partial}{\partial \gamma} \tilde{G}_0, $$

(28)

where $\tilde{G}_0$ is the density of connected vacuum Green function, i. e.

$$ \tilde{G}_0 = \frac{1}{V} \ln G_0, $$

(29)

where the $V$ is the space-time volume, and the $G_0$ is vacuum Green function:

$$ G_0 = \langle 0 | T \exp (i S_I) | 0 \rangle, $$

(30)

with the interaction part of the action defined in the interaction picture by the expression

$$ S_I = \int dx \gamma (: \cos (\varphi + \theta) : - \cos \theta) $$

(31)

in Lorentz coordinates. Analogously, the eq. (26) can be written in the form

$$ \tilde{C}_2 = i \frac{\partial}{\partial \theta} \tilde{G}_0. $$

(32)

Hence we get

$$ C_1 = \gamma \cos \theta - i \frac{\gamma}{V G_0} \langle 0 | T \int dx \: : \cos (\varphi + \theta) : - \cos \theta \rangle e^{i S_I} | 0 \rangle = $$

$$ = \gamma \cos \theta + \frac{\gamma}{\langle 0 | T e^{i S_I} | 0 \rangle} \langle 0 | T \exp (i S_I) | 0 \rangle = $$

$$ = \gamma \cos \theta + \gamma \langle \Omega | ( : \cos (\varphi + \theta) : - \cos \theta) | \Omega \rangle = \gamma \langle \Omega | : \cos (\varphi + \theta) : | \Omega \rangle, $$

(33)
and
\[ C_2 = \gamma \sin \theta + i \frac{1}{V G_0} \langle 0 | T \int dx i \gamma (- : \sin(\varphi + \theta) : + \sin \theta) e^{i S_I} | 0 \rangle = \]
\[ = \gamma \sin \theta + \gamma \frac{\langle 0 | T ( : \sin(\varphi + \theta) : - \sin \theta) e^{i S_I} | 0 \rangle}{\langle 0 | T e^{i S_I} | 0 \rangle} = \]
\[ = \gamma \sin \theta + \gamma \langle \Omega | ( : \sin(\varphi + \theta) : - \sin \theta) | \omega \rangle = \gamma \langle \Omega | : \sin(\varphi + \theta) : | \Omega \rangle, \] (34)

where the \(| \Omega \rangle\) is physical vacuum state. Now the interaction Hamiltonian (27) can be rewritten in the form
\[ H_I = \gamma (1 - : \cos \varphi : ) \langle \Omega | : \cos(\varphi + \theta) : | \Omega \rangle + \]
\[ + \gamma : \sin \varphi : \langle \Omega | : \sin(\varphi + \theta) : | \Omega \rangle. \] (35)

Notice that the normal ordering inside of vacuum matrix elements corresponds to bare (perturbative) vacuum of usual formulation in Lorentz coordinates. It follows from the bosonization procedure that these vacuum condensate parameters are equal (up to a constant, included in the \(\gamma\)) to fermionic condensate parameters \(\langle \bar{\Psi} \Psi \rangle\) and \(\langle \bar{\Psi} \gamma^5 \Psi \rangle\), which require UV-regularization [11], achieved now by introducing the cutoff mass \(m_1\).

Let us notice also that the formula (35) for the LF Hamiltonian can be directly obtained via the method of the limiting transition to LF Hamiltonians starting from usual Hamiltonian formulation [17]. The approximation used in [17] becomes true owing to the analysis of the difference between LF and covariant perturbation theories diagrams, carried out above, so that this method also can be applied.

One can use the obtained LF Hamiltonian in nonperturbative calculations of the mass spectrum, that can be done on the LF numerically by DLCQ method [12, 13], and compare the results with those obtained in usual formulations of \(QED_2\). In these calculations the cutoff in momenta \(k_-\) must be removed before the removing UV cutoff.

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