High success standard quantum teleportation using entangled coherent state and two-level atoms in cavities

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Abstract

We propose here a new idea for quantum teleportation of superposed coherent state which circumvents the problem of performing non-unitary $Z_c = |\alpha\rangle\langle\alpha| - | -\alpha\rangle\langle-\alpha|$ operation on coherent state qubit. The receiver only requires to perform either no unitary operation or a $\pi$ phase shift in order to recreate the information state. We use entangled resource $\sim |\alpha, \alpha\rangle_{\sqrt{2}} - | -\alpha, -\alpha\rangle_{\sqrt{2}}$ in contrast with the usual $\sim |\alpha, \alpha\rangle - | -\alpha, -\alpha\rangle$ (both states unnormalized). Bob receives state which is then superposition of the states $|\pm\alpha\rangle_{\sqrt{2}}$. Bob mixes these with even or odd coherent states involving superposition of states $|\pm\alpha\rangle_{\sqrt{2}}$ to obtain a two-mode state which is one of $\sim |I, 0\rangle \pm |0, I\rangle$, $|I\rangle$ being the information state. Bob then obtains the teleported information by using the interaction of one of these modes in two cavities with resonant two-level atoms. This scheme results in average fidelity of $\simeq 0.95$ for $|\alpha|^2 \simeq 10$, which increases with $|\alpha|^2$ and tends to 1 asymptotically, varying as $1 - \frac{\pi^2}{16|\alpha|^2} + \frac{\pi^2(\pi^2+8)}{256|\alpha|^4}$ for large values of $|\alpha|^2$.

Keywords Quantum teleportation · Coherent state · Entangled Coherent state · Two-level atom

We dedicate this paper in the memory of Prof. Ranjana Prakash and Prof. Hari Prakash. May their soul rest in peace

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1 Introduction

Quantum teleportation is a faithful transmission of quantum information between two parties by sharing an appropriate entangled resource and a classical communication channel. Since its first theoretical proposal by Bennet et al. [1] for teleporting the quantum state of a two-level system (spin state of spin-1/2 particles or polarization state of a photon), it has been realized experimentally in a variety of physical systems including, optical modes, photonic qubits, trapped ion, etc. [2]. Quantum communication relies on the ability to perform efficient quantum teleportation over arbitrarily large distances. However, the pre-requisite demands to establish entanglement between the sender and receiver, which is limited by various noise factors. Entangled state passing through a quantum channel degrades naturally into a mixed one, thereby reducing the fidelity of teleported state. To overcome losses, schemes such as quantum repeaters and relays can be employed for entanglement purification [3–5]. Furthermore, performing Bell state measurement, deterministically, is important to achieve good success probability. In the context of two two-level bosonic states, it had been shown by Lütkenhaus et al. [6] that a never-failing Bell state measurement is not possible, and in fact, using only linear optical devices such as phase shifters, beam splitters, and photo-detectors only two out of four Bell states can be discriminated.

Recently, quantum teleportation using hybrid quantum channel has been reported in which encoding of information can be changed from discrete variable to continuous variable or vice versa [7,8]. Such a scheme has the advantage that whereas discrete systems, like trapped atoms, have longer information holding capacity, continuous variable shows greater robustness against environmental de-coherence and thus suitable for transmitting information over noisy channel [9,10]. For all the progress that has been made in the past three decades, the motivation to go further remains unaltered, finding physical systems which are more efficient and, at the same time, experimentally feasible.

Coherent states of radiation field [11], which are eigenstates of photon annihilation operator and are closest to classical noiseless field, form a valuable tool in performing various quantum information processing tasks. The coherent states are in general non-orthogonal, and in fact, for two coherent state $|\alpha\rangle$ and $|\beta\rangle$ (where $\alpha$ and $\beta$ are, in general, complex numbers) their overlap is given by,

$$|\langle \alpha | \beta \rangle|^2 = \exp(-|\alpha - \beta|^2).$$

(1)

For $\beta = -\alpha$, $|\langle \alpha | - \alpha \rangle|^2 = \exp(-2|\alpha|^2) < 10^{-3}$ for $|\alpha|^2 \geq 3$. Therefore, for moderately large coherent amplitudes, we can have a close correspondence between logical qubits $|0\rangle$, $|1\rangle$ and phase opposite coherent states $|\alpha\rangle$, $|-\alpha\rangle$, enabling us to encode and manipulate information in their superposition [12]. Various schemes were proposed for the generation of such superposition states and also their entangled counterparts using nonlinear interactions, photon subtraction from squeezed vacuum states, mixing of squeezed vacuum with a coherent light, etc., with appreciable large size [13–18]. Proposals for generating freely travelling multi-partite resources such as GHZ, W and cluster entangled coherent state (ECS) were also proposed [19,20]. Realizing the advantages that superposed coherent state (SCS) can offer, with regard
to its robustness over noisy channel and the fastest mode of transmitting information, the field is since then ever growing [21–24].

Jeong et al. studied quantum information processing using mixed-entangled coherent state, further extending it for efficient quantum computing [21,22]. Van Enk et al. [25] presented a scheme of achieving quantum teleportation of a single qubit information, encoded in phase opposite coherent state of the form,

\[
|I\rangle = \varepsilon_+|\alpha\rangle + \varepsilon_-|-\alpha\rangle
\]

using ECS as quantum channel, similar to Standard Quantum Teleportation scheme of Bennet et al. [1] for the teleportation of single qubit state living in 2-d Hilbert space. The success probability of such a scheme was, however, shown to be only 1/2. This has been generalized for teleportation of ECS by Wang [26], with success probability of 1/2. The cause of failure is the inability of the receiver to find a valid unitary transformation that may change \(\varepsilon_+|\alpha\rangle - \varepsilon_-|-\alpha\rangle\) to \(\varepsilon_+|\alpha\rangle + \varepsilon_-|-\alpha\rangle\) which correspond to making a \(Z_c = |\alpha\rangle\langle\alpha| - | -\alpha\rangle\langle -\alpha|\) operation on SCS. Z_c is a non-unitary operation due to non-orthogonality of coherent states, and it may become unitary in the limit when \(|\alpha|^2 \rightarrow \infty\). Jeong et al. [22] showed that an arbitrary rotation about z-axis can be obtained by displacement operator, however, such a displacement is rather hard to implement in practice requiring the state to be mixed with intense coherent beam (\(|\alpha|^2 \rightarrow \infty\)) using highly reflecting beam splitter [27]. Ralph et al. [23] discussed the implementation of such single flip or \(Z_c\) gate operation using quantum teleportation and making \(X\) gate error correction. Although, \(X\) operation for SCS can be implemented by simply passing the mode through a \(\pi\) phase shifter, this scheme fails half of the time. Cheong et al. [28] presented a teleportation scheme for SCS in which the required \(Z_c\) operation is approximately obtained by interacting the mode with two-level atom in a cavity. N Ba An [29] discussed a scheme of quantum teleportation of SCS within a network, also with success probability of 1/2. Prakash et al. [30] presented a modified scheme for the teleportation of SCS, where it was shown that by adopting a photon counting strategy that can discriminate between a zero, nonzero even and an odd photon count, almost perfect teleportation for appreciable coherent state amplitude can be achieved. This scheme circumvents the problem of performing non-unitary \(Z_c\) transformation by performing a unitary \(Z\) operation on even (\(|EVEN, \alpha\rangle\)) and odd (\(|ODD, \alpha\rangle\)) coherent states,

\[
|EVEN, \alpha\rangle = \left[\sqrt{2(1 + x^2)}\right]^{-1} (|\alpha\rangle + |-\alpha\rangle)
\]

\[
|ODD, \alpha\rangle = \left[\sqrt{2(1 - x^2)}\right]^{-1} (|\alpha\rangle - |-\alpha\rangle),
\]

which forms an orthogonal basis [31], where \(x = e^{-|\alpha|^2}\). This photon counting strategy has a great theoretical advantage over all the previously existing schemes that involve coherent states and it has been widely used for obtaining almost perfect success for many other quantum information processing task using ECS as a resource [32–40]. However, the experimental feasibility of the required \(Z\) unitary operations demands a change between \(|EVEN, \alpha\rangle \leftrightarrow |ODD, \alpha\rangle\) which seems difficult to achieve and
so far no one has shown how this would be done experimentally. It is, therefore, important to devise a scheme which leads to teleportation of SCS with good success rate and fidelity, and at the same time permits its experimental feasibility using optimal resources.

We, in our work, show that an almost perfect teleportation of SCS is obtainable without having to perform non-unitary $Z_c$ transformation. We shall show that by making or not making a phase-shift of $\pi$ in the state Bob receives and then adding extra step of mixing this received state with a preferred cat state (odd or even coherent state, conditioned to the classical information of sender) gives an almost perfect teleportation of SCS. We also discuss the effect of decoherence on the quality of teleportation.

2 Proposed scheme for quantum teleportation of SCS: getting two mode state $\sim |I, 0\rangle \pm |0, I\rangle$ at Bob’s station

Let Alice possess a single qubit information state encoded in phase opposite coherent states given by Eq. 2, existing in mode 0. We can equivalently write the information state in orthogonal Cat state basis by using Eq. 3 in the form,

$$|I\rangle = A_+ |EVEN, \alpha\rangle + A_- |ODD, \alpha\rangle,$$

with $A_\pm = \sqrt{(1 \pm x^2)/2(\epsilon_+ + \epsilon_-)}$. While preserving the normalization, $|A_+|^2 + |A_-|^2 = 1$, one can write $A_+/A_- = e^{i\phi} \tan(\theta/2)$, using Bloch representation of a

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Fig. 2 Variation of concurrence of the entangled channel, $|E\rangle_{1,2}$, used in our scheme with respect to mean photons in the coherent state, $|\alpha|^2$. The concurrence becomes almost unity for $|\alpha|^2 \geq 2$.

Fig. 3 Variation of probability of occurrence for case (i) with respect to information parameter $\theta$ and average photon number in the coherent state $|\alpha|^2$. The probability decreases rapidly to become vanishingly small for $|\alpha|^2 \geq 3$.

qubit. We can also expand information in terms of Fock basis $|n\rangle$ as,

$$|I\rangle_0 = \sum_{n=0}^{\infty} p_{In} |n\rangle_0$$  \hspace{1cm} (5)

where, $p_{In} = \sqrt{x} [\epsilon_+ + (-1)^n \epsilon_-] \alpha^n / \sqrt{n!}$ is the expansion coefficient. Normalization demands,
Fig. 4  a and b shows the variation of probability of occurrence for cases (i) (same for case (ii)) and case (iii) (same for case (iv)) respectively, with respect to information parameter $\theta$ and average photon number in the coherent state $|\alpha|^2$. The probabilities converges to a constant value of 0.25 when there is appreciable mean photons in the coherent state $|\langle I|I\rangle|^2 = \sum_{n=0}^{\infty} P_{In} = |\epsilon_+|^2 + |\epsilon_-|^2 + x(\epsilon_+^* \epsilon_- + \epsilon_+ \epsilon_-^*) = 1$ (6)

where, $P_{In} = |p_{In}|^2$ is the probability of counting $n$ photons in $|n\rangle$. Alice and Bob shares mode 1 and 2 of the entangled state, $|E\rangle_{1,2} = \frac{1}{\sqrt{2(1-x^3)}} \left( |\alpha, \alpha \sqrt{2}\rangle_{1,2} - |\alpha, -\alpha \sqrt{2}\rangle_{1,2} \right)$ (7) respectively. The reason for selecting such an entangled channel shall become clear in a while. We may note that $|E\rangle_{1,2}$ can be generated using Schrodinger’s cat state $|O D D, \sqrt{3}/2\alpha\rangle$ incident on one of the input port of a lossless beam splitter with reflectivity, $R = 1/3$, and vacuum on the other input port. Assuming the beam splitter do not impart any phase shift to the input modes, the output ports will be in required entangled state, $|E\rangle_{1,2}$.

In any quantum teleportation protocol, the degree of entanglement in the quantum resource, plays a vital role in determining the success probability as well as fidelity of teleportation. For a two-qubit pure state, $|\phi\rangle$, a good entanglement measure is concurrence [41], defined as

$$ C = |\langle \phi | \sigma_y \otimes \sigma_y | \phi^* \rangle|, $$

where, $|\phi^*\rangle$ is the complex conjugate of $|\phi\rangle$, and $\sigma_y$ is the Pauli-matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. In order to find the concurrence ($C$) of $|E\rangle_{1,2}$, we write it in orthonormal basis,
\(|EVEN, \alpha/\sqrt{2}\), |ODD, \alpha/\sqrt{2}\rangle\) using Eq. 3 and use Eq. 8 to obtain,

\[
C = \frac{(1 + x)(1 + x^2)^{1/2}}{1 + x + x^2}.
\]  

(9)

It becomes evident that \(|E\rangle_{1,2}\) is not a maximally entangled state in terms of its concurrence due to difference in coherent amplitudes of the two modes. However, the value of \(C\) is nearly 0.943 for \(\alpha \rightarrow 0\), increasing monotonically to become nearly unity for \(|\alpha|^2 \geq 3\) as shown in Fig. 2. This is in contrast to the ECS used usually, which has equal amplitude [21,25,30] and exactly 1-e bit entanglement for all \(\alpha \neq 0\) [9]. We shall be working for coherent amplitudes appreciable enough so that the entanglement is nearly unity.

Alice mixes modes 0 and 1 in her possession to modes 3 and 4 using a symmetric lossless beam splitter, \(B_1\) fitted with two phase shifters at its second input and second output port which changes state \(|\alpha\rangle\) to \(|-i\alpha\rangle\). For coherent states, \(|\alpha\rangle\) and \(|\beta\rangle\) at the input ports \(x\) and \(y\), this combination transforms it to modes \(u\) and \(v\) as, \(|\alpha, \beta\rangle_{x,y} \rightarrow |^{\alpha+\beta}_{\sqrt{2}} \cdot ^{\alpha-\beta}_{\sqrt{2}}\rangle_{u,v}\). Following [30] we write,

\[
|x\rangle = \frac{1 - x^2}{\sqrt{2}} |NZE, \sqrt{2}\alpha\rangle + \sqrt{\frac{1 - x^4}{2}} |ODD, \sqrt{2}\alpha\rangle,
\]

where, \(|NZE, \sqrt{2}\alpha\rangle = [\sqrt{2}(1 - x^2)]^{-1} (|\sqrt{2}\alpha\rangle + |\sqrt{2}\alpha\rangle - 2x|0\rangle)\) is the normalized state containing nonzero even Fock states. We expand modes 3 and 4 of \(|\psi\rangle_{3,4,2}\) in the orthogonal basis, \(|0\rangle, |NZE, \sqrt{2}\alpha\rangle, |ODD, \sqrt{2}\alpha\rangle\). State of the system consisting of modes 3,4 and 2 can then be written as,

\[
|\psi\rangle_{3,4,2} = \frac{1}{\sqrt{2}(1 - x^3)} \left\{ x|0\rangle_3|0\rangle_4 (\epsilon_+ + \epsilon_-) \left( \left| \frac{\alpha}{\sqrt{2}} \right|_2 + \left| -\frac{\alpha}{\sqrt{2}} \right|_2 \right) \\
+ \frac{1 - x^2}{\sqrt{2}} \left[ |NZE, \sqrt{2}\alpha\rangle_3|0\rangle_4 (\epsilon_+ \left| \frac{\alpha}{\sqrt{2}} \right|_2 - \epsilon_- \left| -\frac{\alpha}{\sqrt{2}} \right|_2 \right) \\
+ |0\rangle_3|NZE, \sqrt{2}\alpha\rangle_4 \left( \epsilon_+ \left| \frac{-\alpha}{\sqrt{2}} \right|_2 + \epsilon_- \left| \frac{\alpha}{\sqrt{2}} \right|_2 \right) \right\}.
\]

(10)

She then counts photons in modes 3 and 4, for which the detectors outcomes \((D_3, D_4)\) can be one of the following, \((0,0), (NZE,0), (0,NZE), (\text{odd},0)\) or \((0,\text{odd})\) (cases (i) to (v)) and conveys this information to Bob. For the case (i), when both detectors do not click, the collapsed state with Bob is \(|ODD, \frac{\alpha}{\sqrt{2}}\rangle\) and it is not possi-
Table 1 shows the result of photon counting done by Alice in modes 3 and 4, with corresponding probability of occurrence. The teleported state with Bob in mode 2, the conditional unitary operation applied by him, the mixing mode and the entangled output state of $B_2$ is also given. Here, $P_+$ and $P_-$ are given by Eqs. 13 and 14, respectively, while $I$ is the identity operation and $P(\pi)$ is phase shift by $\pi$. The BS-II outputs $|\psi\rangle_{7,8}$ and $|\phi\rangle_{7,8}$ are given by Eq. 16 and 17.

| Cases  | Alice’s PC result | Prob. | Bob’s State in mode 2 (un-normalized) | U (Bob) | Mixing mode | BS-II output |
|--------|-------------------|-------|--------------------------------------|---------|-------------|--------------|
| (i)    | 0, 0              | $P_i$ | $|\alpha\rangle_{\sqrt{2}} - |\alpha\rangle_{\sqrt{2}}$ | -       | -           | -            |
| (ii)   | NZE, 0            | $P_+$ | $\epsilon_+ |\alpha\rangle_{\sqrt{2}} - \epsilon_- |\alpha\rangle_{\sqrt{2}}$ | $I$     | $ODD, \alpha\rangle_{\sqrt{2}}$ | $|\psi^+\rangle_{7,8}$ |
| (iii)  | 0, NZE            | $P_+$ | $\epsilon_+ |\alpha\rangle_{\sqrt{2}} - \epsilon_- |\alpha\rangle_{\sqrt{2}}$ | $P(\pi)$ | $ODD, \alpha\rangle_{\sqrt{2}}$ | $|\psi^-\rangle_{7,8}$ |
| (iv)   | ODD, 0            | $P_-$ | $\epsilon_+ |\alpha\rangle_{\sqrt{2}} + \epsilon_- |\alpha\rangle_{\sqrt{2}}$ | $I$     | $EVEN, \alpha\rangle_{\sqrt{2}}$ | $|\phi^+\rangle_{7,8}$ |
| (v)    | 0, ODD            | $P_-$ | $\epsilon_+ |\alpha\rangle_{\sqrt{2}} - \epsilon_- |\alpha\rangle_{\sqrt{2}}$ | $P(\pi)$ | $EVEN, \alpha\rangle_{\sqrt{2}}$ | $|\phi^-\rangle_{7,8}$ |
ble to transform this to information state using any means. However, the probability of occurrence is,

\[ P_i = \frac{x P_{I0}}{1 + x + x^2} \]  \hspace{1cm} (11)

which becomes almost zero for appreciable coherent amplitude \( |\alpha|^2 \geq 3 \) and for all \( \theta \) (Fig. 3). For information state \( |\psi^I\rangle \) and teleported state \( |\psi^T\rangle \), the teleportation fidelity is defined as,

\[ F = |\langle \psi^I | \psi^T \rangle|^2. \]  \hspace{1cm} (12)

For this case of photon counts \((0,0)\) in the modes 3 and 4, the fidelity becomes,

\[ F_i = \frac{x^3/2-\sqrt{2}(1-x\sqrt{2})^2|\epsilon_+ - \epsilon_-|^2}{2(1-x)}. \]  \hspace{1cm} (13)

Denoting by \( P_+ \) and \( P_- \) the probability of occurrence for cases (ii) (same for (iii)), and, case (iv) (same for (v)), respectively, we have,

\[ P_+ = \frac{(1+x^2)(1-P_{I0})}{4(1+x+x^2)} \]  \hspace{1cm} (14)

\[ P_- = \frac{(1+x^2)(1+P_{I0})}{4(1+x+x^2)}. \]  \hspace{1cm} (15)

\( P_+ \) and \( P_- \) are plotted in Fig. 4 where both converges to a constant value of \( 1/4 \) for appreciable coherent amplitudes, imitating the common SQT where all the four Bell states are equally probable.

It is at this step where our scheme differs from the previous ones (see Fig. 1). Alice communicates to Bob her measurement outcome using a classical channel. Bob on receiving the classical information, passes mode 2 through a conditional phase shifter (CPS), which transform it to mode 5. The CPS introduces a phase shift of \( \pi \) for cases (iii) and (v), and no phase shift for cases (i), (ii) and (iv). For cases (ii) and (iii), Bob mixes mode 5 with an odd coherent state \( |ODD, \alpha \sqrt{2} \rangle_6 = 2(1-x)|\alpha \sqrt{2} \rangle_6 - |\alpha \sqrt{2} \rangle_6 \) leading to state in modes 7 and 8,

\[ |\psi^\pm\rangle_{7,8} = \pm [2(1-P_{I0})]^{-1/2} (|I, 0\rangle_{7,8} - |0, I\rangle_{7,8}), \]  \hspace{1cm} (16)

respectively. On the other hand, for cases (iv) and (v), Bob mixes mode 5 with an even coherent state \( |EVEN, \alpha \sqrt{2} \rangle_6 = 2(1+x)|\alpha \sqrt{2} \rangle_6 + (|\alpha \sqrt{2} \rangle_6 \) leading to state in modes 7 and 8,

\[ |\phi^\pm\rangle_{7,8} = \pm [2(1+P_{I0})]^{-1/2} (|I, 0\rangle_{7,8} + |0, I\rangle_{7,8}), \]  \hspace{1cm} (17)
respectively. The states in Eqs. 16 and 17 are an entangled mixture of vacuum and the information. Table 1 shows the possible measurement outcomes of photon counting by Alice in modes 3 and 4, corresponding probability of occurrence, the state in mode 5, unitary transformation to be performed by Bob and the state with which Bob mixes mode 5 using $B_2$ and the final state in modes 7 and 8.

From Table 1, it can be easily seen that apart from overall negative sign, that does not affect the foregoing analysis, cases (ii) and (iii), and cases (iv) and (v) give similar entangled mixtures of information with vacuum in modes 7 and 8. In order to get perfect success, Bob needs to discriminate which of the mode contains the information state, and that too non-destructively.

Measurement in quantum mechanics brings an irreversible change in the state of the measured system due to the collapse of wavefunction. Quantum non-demolition measurement is an interesting measurement scheme where the state of a system can be revealed without altering its state. Measuring certain properties of optical field, for example, photon number and optical quadratures, non-destructively, has been discussed in the past with many applications in quantum optics and quantum information science [42,43]. However, the mode in which we are interested in making the measurement can be vacuum or the superposition of coherent state for which the trivial schemes of non-destructive measurement cannot be appropriate. Moreover, the information state $|I\rangle$ itself contains the vacuum, thus, this cannot be done with unit success. In the next section, we discuss a scheme of discriminating between vacuum state and the information, using the interaction of the mode of the light with a two-level atom (TLA). As we shall see, by entangling the mode with a TLA in a cavity and using atom as a probe, the desired task can be obtained with fidelity approaching unity with increasing $|\alpha|^2$.

### 3 Getting replica of information by using cavity-QED two exactly two-level atoms

Before we proceed to discuss our scheme to obtain a replica of information, let us first briefly discuss the dynamics of a single TLA (in lower state $|l\rangle$ or in excited state $|u\rangle$) interacting with a cavity mode in Fock state $|n\rangle$. The quantum mechanical interaction of cavity mode with TLA is best given by the Jaynes–Cumming model for which the interaction Hamiltonian can be written as,

$$H_{int.} = \hbar g (a^\dagger \sigma_- + a \sigma_+),$$

(18)

where $a^\dagger$ ($a$) is the creation (annihilation) operator of the field mode, $g$ is the atom field coupling constant and $\sigma_{\pm}$ are the atomic raising and lowering operators [44]. Under such an interaction, the combined state of single cavity mode having ’$n$’ photons and a TLA initially in its ground state $|l\rangle$ or in upper state $|u\rangle$, evolves as,

$$|n, l\rangle \longrightarrow \cos \phi_n |n, l\rangle - i \sin \phi_n |n - 1, u\rangle,$$

(19)
or

\[ |n, u\rangle \rightarrow \cos \phi_{n+1} |n, u\rangle - i \sin \phi_{n+1} |n + 1, l\rangle \] (20)

where, \( \phi_n = g \sqrt{nt} \). We use these to find the replica of information.

Let Bob pass mode 8 to interact in a cavity \( C \) with a resonant TLA in its ground state \( |l\rangle \). If the changed state of radiation is denoted by mode 9, the state of modes 7 and 9 and the TLA is

\[ |\psi^\mp\rangle_{7,9,C} = [2(1 \mp P_{I0})]^{-1/2}[|I\rangle_7 \mp p_{I0}|0\rangle_7]|0\rangle_C |l\rangle_C \]

\[ \mp \sum_{n=1}^{\infty} p_{I_n} |\gamma\rangle (\cos \phi_n |n\rangle_9 |l\rangle_C - i \sin \phi_n |n - 1\rangle_9 |u\rangle_C) ] \] (21)

The upper and lower signs in \( \mp \) correspond to cases (ii) or (iii), and for cases (iv) or (v), respectively, here and later. The interaction time is so chosen that,

\[ t = t_0, g|\alpha|t_0 = \pi/2. \] (22)

A Von Neumann measurement (VNM) performed on TLA shall give us results \( |l\rangle \) or \( |u\rangle \) with conditional probabilities (Fig. 5),

\[ P(\mp |l\rangle) = [2(1 \mp P_{I0})]^{-1} \sum_{n=0}^{\infty} p_{I_n} \cos^2 \phi_n + 1 \mp 2 P_{I0} \] (23)

\[ P(\mp |u\rangle) = [2(1 \mp P_{I0})]^{-1} \sum_{n=0}^{\infty} p_{I_n} \sin^2 \phi_n, \] (24)

Upon performing VNM, Bob makes a photon counting (PC) on one of the modes 7 or 9. This may result in situations for each of cases I to V:

- **A\( ^\mp \)**- TLA in \( |l\rangle_C \), PC on 9 gives \( n = 0 \)
- **B\( ^\mp \)**- TLA in \( |l\rangle_C \), PC on 9 gives \( n \neq 0 \)
- **C\( ^\mp \)**- TLA in \( |u\rangle_C \), \( n=0 \) in mode 7 is indicated

We shall now discuss these situations separately.

### 3.1 Situations **A\( ^\mp \)**

The term corresponding to this case in \( |\psi^{\pm}\rangle_{7,9,C} \) is given by,

\[ [2(1 \mp P_{I0})]^{-1/2}(|I\rangle_7 \mp p_{I0}|0\rangle_7)|0\rangle_9 |l\rangle_C \]. (25)

Bob accepts state in mode 7 as the teleported state, which is given by

\[ |T^{(\mp,A)}\rangle = (1 \mp P_{I0})^{-1/2}(|I\rangle_7 \mp p_{I0}|0\rangle_7). \] (26)
Fig. 5  a and b show the variation of probability for TLA in the cavity $C$ to be found in excited state and ground state for case (ii) (same for case (iii)) respectively, while c and d for case (iii) (same for case (iv)), with respect to information parameter $\theta$ and average photon number in the coherent state $|\alpha|^2$.

Fig. 6  a and b show the variation of fidelity for cases $A^-$ and $A^+$ with respect to $\theta$ and $|\alpha|^2$ respectively.
The conditional probabilities of occurrence and the fidelities for such cases are given by,

\[ P(-|A) = \frac{1}{2}, \quad P(+|A) = \frac{1 + 3P_{I0}}{2(1 + P_{I0})} \]  
\[ F^{(-,A)} = 1 - P_{I0}, \]  
\[ F^{(+,A)} = \frac{1 + 2P_{I0} + P_{I0}^2}{1 + 3P_{I0}}. \]  

\( F^{(\mp, A)} \) becomes almost unity for \(|\alpha|^2 \geq 3\) (Fig. 6), thus, we can say that for these cases, an almost perfect teleportation is established between Alice and Bob.

### 3.2 Situations \( B^\mp_n \)

Due to nonzero contribution of vacuum in the information, it may happen that TLA is not at all excited while interacting with the information state. It is this case for which we get nonzero PC result in mode 9 and TLA in its ground state. The corresponding term for these cases in \(|\psi^\pm\rangle\)\( \gamma, 9, C \) are,

\[ [2(1 \mp P_{I0})]^{-1/2}[\mp P_{I0} |0\rangle_7 (\cos \phi_n |n\rangle_9 |l\rangle_C - i \sin \phi_n |n - 1\rangle_9 |u\rangle_C)], \]  

for \( n \neq 0 \). Since the teleported state with Bob is \(|0\rangle_7\), the fidelity of teleportation is,

\[ F^{(\mp B_n)} = P_{I0}. \]  

The conditional probability of occurrence for these cases becomes,

\[ P(\mp|B_n) = [2(1 \mp P_{I0})]^{-1} \sum_{n=1}^{\infty} P_{I0} \cos^2 \phi_n. \]  

We note that \( P(\mp|B_n) \leq 10^{-3} \) for \(|\alpha|^2 \geq 5\), therefore, such a case has negligibly small probability of occurrence for moderately large coherent amplitudes.

### 3.3 Situations \( C^\pm \)

If VNM on TLA gives result \(|u\rangle\), then it must be something (at least one photon) that caused the excitation, ensuring that the mode 9 comes out of \(|I\rangle\) while 7 is vacuum. The term corresponding to this case in \(|\psi^\pm\rangle\)\( \gamma, 9, C \) is given by,

\[ -i[2(1 \mp P_{I0})]^{-1/2} \mp \sum_{n=1}^{\infty} P_{I0} |0\rangle_7 (\sin \phi_n |n - 1\rangle_9 |u\rangle_C)]. \]  

However, due to absorption of photon from mode 8 in exciting the atom, the state (information) is changed. To reverse this effect (atleast approximately), we pass mode
9 to another cavity \((C')\) where TLA is initially in \(|u\rangle\), and let it interact for another time \(t_0\). Here, we are taking advantage of the phenomenon of collapses and revivals of atomic population when a TLA is allowed to interact with a cavity mode which is the superposition of Fock states (for our case, the information state). The interaction of mode 9 with second cavity \(C'\) transforms as

\[
|n-1\rangle_9|u\rangle_{C'} \rightarrow \cos \phi_n |n-1\rangle_{10}|u\rangle_{C'} - i \sin \phi_n |n\rangle_{10}|l\rangle_{C'}. \tag{33}
\]

A VNM is again performed on \(C'\), resulting in following two cases. We note that, except for the expression of conditional probabilities, fidelity as well as teleported state shall remain same for cases (ii) to (v).

### 3.3.1 When VNM in \(C'\) gives \(|l\rangle\)

Using Eqs. 32 and 33, the term corresponding to these cases are,

\[
- [2(1\mp P_{I0})]^{-1/2} \sum_{n=1}^{\infty} P_{In} \sin^2 \phi_n |n\rangle_{10}|l\rangle_{C'}. \tag{34}
\]

with respective conditional probability of occurrence given by,

\[
P(\mp|C_l) = [2(1\mp P_{I0})]^{-1} \sum_{n=1}^{\infty} P_{In} \sin^4 \phi_n. \tag{35}
\]

The teleported state in mode 10 is,

\[
|T^{(C_l)}\rangle = \frac{\sum_{n=1}^{\infty} P_{In} \sin^2 \phi_n |n\rangle}{\sqrt{\sum_{n=1}^{\infty} P_{In} \sin^4 \phi_n}}, \tag{36}
\]

therefore, the fidelity of teleportation becomes,

\[
F^{(C_l)} = \frac{[\sum_{n=1}^{\infty} P_{In} \sin^2 \phi_n]^2}{\sum_{n=1}^{\infty} P_{In} \sin^4 \phi_n}. \tag{37}
\]

We have numerically plotted \(F^{(C_l)}\) in Fig. 7 with respect to \(|\alpha|^2\), and information parameters \(\theta\) and \(\phi\). The fidelity becomes almost unity for \(|\alpha|^2 \geq 3.

### 3.3.2 When VNM in \(C'\) gives \(|u\rangle\)

There is still a nonzero probability that the TLA remains excited despite of the interaction with mode 9, containing the vacuum state. The term corresponding to
Fig. 7 Variation of fidelity $F^{(C_L)}$ with respect to $\theta$ and $|\alpha|^2$. The fidelity becomes nearly unity as mean photons in the coherent state increases and for all $\theta$ these cases are,

$$- [2(1 \mp P_{10})]^{-1/2} \sum_{n=1}^{\infty} P_{1n} \sin \phi_n \cos \phi_n |n - 1 \rangle_{10} |u \rangle_{C'} ,$$

with respective conditional probabilities,

$$P(\mp|C_u) = [2(1 \mp P_{10})]^{-1} \sum_{n=1}^{\infty} P_{1n} \sin^2 \phi_n \cos^2 \phi_n .$$

The teleported state with Bob in mode 10 is,

$$|T^{(C_u)} \rangle = \frac{\sum_{n=1}^{\infty} P_{1n} \sin \phi_n \cos \phi_n |n - 1 \rangle}{\sqrt{\sum_{n=1}^{\infty} P_{1n} \sin^2 \phi_n \cos^2 \phi_n} \phi_n} .$$
Fig. 8  Variation of average fidelity of teleportation for the proposed scheme with respect to \( \theta \) and \(|\alpha|^2\). 
\( F_{\text{avg.}} \) increases monotonically with \(|\alpha|^2\), and, is above 0.9 for \(|\alpha|^2 \geq 6 \) and for all \( \theta \).

This gives the fidelity,

\[
F(C_u) = \frac{| \sum_{n=1}^{\infty} P_{I_n}^* P_{I_n} \sin \phi_n \cos \phi_n |^2}{\sum_{n=1}^{\infty} P_{I_n} \sin^2 \phi_n \cos^2 \phi_n \phi_n}.
\] (41)

4 Overall quality of teleportation

To estimate overall quality of teleportation, we calculate the average fidelity of our teleportation scheme. Following Prakash et al. [45], we define the average fidelity of teleportation as the sum of products of probability of occurrence with corresponding fidelity for each possible outcomes. Therefore, we can write average fidelity for our teleportation scheme as,
By substituting the values of various terms, and on simplifying, we get,

\[
F_{\text{avg.}} = \frac{1}{2(1-x^2)} \left[ x^{2-3/2-\sqrt{2}}(1-x^{\sqrt{2}})^2 \left| e_+^2 - e_-^2 \right|^2 - x(1-x) P_{l0} \right] 
+ \frac{1}{2} \left[ 1 + P_{l0}^2 + P_{l0} S_1 + |S_2|^2 + |S_3|^2 \right].
\]  

where,

\[
S_1 = \sum_{n=1}^{\infty} P_{ln} \cos^2 \phi_n,
\]

\[
S_2 = \sum_{n=1}^{\infty} P_{ln} \sin^2 \phi_n,
\]

\[
S_3 = \sum_{n=1}^{\infty} P_{ln}^2 \sin \phi_n \cos \phi_n.
\]

The average fidelity is a function of mean coherent amplitude $|\alpha|^2$ as well as information parameters, $\theta$ which is plotted numerically in Fig. 8. We need to find analytically the dependence of $F_{\text{avg.}}$ over these independent parameters in order to analyse such a variation more closely.

The summation terms, $S_1$, $S_2$ and $S_3$ follows from Appendix. Using Eqs. 65, 66 and 70, we can write the average fidelity,

\[
F_{\text{avg.}} = 1 + \frac{1}{2(1-x^2)} \left[ x^{2-3/2-\sqrt{2}}(1-x^{\sqrt{2}})^2 \left| e_+^2 - e_-^2 \right|^2 - x(1-x) P_{l0} \right] 
+ \frac{1}{2} \left\{ P_{l0}^2 + \frac{P_{l0} \pi^2}{32} \left[ \frac{2}{|\alpha|^2} - \frac{1}{|\alpha|^4} + \left( 8 - \frac{5}{|\alpha|^2} + \frac{1}{|\alpha|^4} \right) X \right] - P_{l0} \right\} 
+ \frac{\pi^2 (\pi^2 + 8)}{128|\alpha|^4} - \frac{\pi^2}{2} \left[ 1 - \frac{(10 + \pi^2)}{16|\alpha|^2} - \frac{(16 + 9\pi^2)}{16|\alpha|^4} \right] X + \pi^2 \left[ 2 - \frac{1}{4|\alpha|^2} \right] 
+ \frac{41}{32|\alpha|^4} X^2 \right] 
+ \frac{x^2 \pi^2 |k_1|^2}{256|\alpha|^4} - \frac{x^2 \pi^2 |k_2|^2}{144|\alpha|^2} \left[ (\pi^2 + 3)^2 - \frac{2(\pi^4 + 10\pi^2 + 21)}{|\alpha|^2} \right]
+ \frac{(4\pi^4 + 9\pi^2 + 18)}{4|\alpha|^4} + \frac{x^2 \pi^2 (k_1^2 + k_2^2 + k_1 k_2^*)}{192|\alpha|^2} \left[ (\pi^2 + 3)^2 - \frac{(4\pi^4 + 15\pi^2 + 60)}{6|\alpha|^2} \right],
\]

(45)
At small coherent amplitude $|\alpha|^2 \to 0$, considerable variation of $F_{\text{avg}}$, with respect to $\theta$ is observed, maximizing for $\theta = \pi$. The reason for this is that the terms having $\theta$ dependence (terms containing $\epsilon_{\pm}$) are multiplied by either $x$ or its square, which contributes considerably to such limits. However, $x$ decreases very rapidly to become equal to zero for $|\alpha|^2 \geq 3$ beyond which the variation ceases to exist. Also, the lower value of $F_{\text{avg}}$ may be attributed to the lower values of concurrence of $|E'\rangle_{1,2}$ and much higher probability for the case (i) to occur, compared to other cases.

For a large enough value of coherent amplitude, say $|\alpha|^2 \geq 5$, we can afford to discard terms proportional to $x$ and $X$. We can then write the average fidelity in this limit as,

$$F_{\text{avg}} \simeq 1 - \frac{\pi^2}{16|\alpha|^2} + \frac{\pi^2(\pi^2 + 8)}{256|\alpha|^4},$$

(47)

giving $F_{\text{avg}} = \{0.947, 0.971, 0.980\}$ at $|\alpha|^2 = \{10, 20, 30\}$ respectively, and increasing monotonically to become unity. Therefore, we can say that our scheme gives an almost perfect quantum teleportation of SCS and it becomes more and more effective as we increase the coherent amplitude.

## 5 Effect of decoherence on the quality of teleportation

In any quantum teleportation protocol, the entangled state has to be previously distributed between the sender, Alice, and the receiver, Bob. In a realistic scenario, the light mode passing through the quantum channel gets attenuated as it interacts with the environment. In our present scheme, we account for the effect of losses by assuming that mode 1 and 2 of the entangled channel follows transformation [25],

$$|\alpha\rangle|0\rangle_R \rightarrow |\sqrt{\eta}\alpha\rangle|\sqrt{1-\eta}\alpha\rangle_R$$

(48)

where the mode is interacting with the reservoir mode $|0\rangle_R$ and $\eta$ is the noise parameter. We assume that the modes 1 and 2 suffer equal loss. We then write the decohered entangled state as,

$$|\tilde{E}\rangle_{1,2} = \frac{1}{\sqrt{2(1-x^3)}} \left( |\tilde{\alpha}, \frac{\tilde{\alpha}}{\sqrt{2}}\rangle_{1,2} |\Lambda\rangle_{R12} - |\tilde{\alpha}, \frac{-\tilde{\alpha}}{\sqrt{2}}\rangle_{1,2} |-\Lambda\rangle_{12R} \right)$$

(49)
where $\tilde{\alpha} = \sqrt{\eta} \alpha$, $|\Lambda\rangle_{R12} = |\lambda\rangle_{R1}|\lambda/\sqrt{2}\rangle_{R2}$ and $\lambda = \sqrt{1-\eta} \alpha$. Once the entangled resource is distributed, Alice prepares the information state in mode 0 which takes the form below as it gets attenuated [25]

$$|\tilde{I}\rangle_0 = |\tilde{N}|^{-1/2}\epsilon_+|\tilde{\alpha}\rangle_0 + \epsilon_-|\tilde{-\alpha}\rangle_0,$$

(50)

where $\tilde{N} = |\epsilon_+|^2 + |\epsilon_-|^2 + 2\tilde{x}Re(\epsilon_+\epsilon_-)$ and $\tilde{x} = \exp(2\eta |\alpha|^2)$. Following the techniques as described in Sect. 2, i.e., Alice mixes modes 0 and 1 in her possession using symmetric beam splitter, $B_1$, and performs PC at the output ports. Depending upon the PC outcome as conveyed to Bob through some classical channel, Bob performs CPS on the received state and mixes it either with the decohered even or odd coherent state, $|ODD, \alpha/\sqrt{2}\rangle_6 = [2(1-x)]^{-1/2}(|\tilde{\alpha}/\sqrt{2}\rangle_6 - |\tilde{-\alpha}/\sqrt{2}\rangle_6)$, or $|EVEN, \alpha/\sqrt{2}\rangle_6 = [2(1+x)]^{-1/2}(|\tilde{\alpha}/\sqrt{2}\rangle_6 + |\tilde{-\alpha}/\sqrt{2}\rangle_6)$ using $B_2$. The resulting entangled state (un-normalized) with Bob at the $B_2$ becomes, $|\tilde{\psi}^\pm\rangle_{7,8} \sim |\tilde{I}, 0\rangle_{7,8} - |0, \tilde{I}\rangle_{7,8}$, for cases (ii) and (iii), and $|\tilde{\phi}^\pm\rangle_{7,8} \sim |\tilde{I}, 0\rangle_{7,8} + |0, \tilde{I}\rangle_{7,8}$ for cases (iv) and (v). Here, $|\tilde{I}\rangle_7 = \epsilon_+|\tilde{\alpha}\rangle |\Lambda\rangle_{R12} + \epsilon_-|\tilde{-\alpha}\rangle |\Lambda\rangle_{R12}$.

Bob now, does cavity-QED measurement in order to find out which mode (7 or 8) contains the information exactly in a way discussed in Sect. 3. Our primary concern is the effect of photon loss on the fidelity of teleported state. At moderately large coherent amplitudes, the fidelity is near unity for situations $A^\mp$ and $C^\mp$ (when VNM in $C'$ gives $|l\rangle$). For remaining situations, the fidelity and the corresponding probability of occurrence is almost zero for $|\alpha|^2 \geq 10$.

For situations $A^\mp$, i.e. when VNM on $C$ gives result $|l\rangle$ and PC on mode 9 gives $n = 0$, the teleported state with Bob, in mode 7, is entangled with the two reservoir

\[ \text{Fig. 9} \quad \text{Variation of fidelity } F(A, -) \text{ for information state, } \theta = \pi/3 \text{ and } \phi = 0, \text{ with respect to mean coherent amplitude, } |\alpha|^2, \text{ for different values of noise parameter, } \eta \]
modes, $R_1$ and $R_2$. We can find the reduced state with the Bob, by tracing out the environment degree of freedom as following,

$$\rho^{(-,A)} = [Tr(\rho^{(-,A)})]^{-1} \sum_{n,m=1}^{\infty} q_n q_m^* (|\epsilon_+|^2 + (-1)^{n+m} |\epsilon_-|^2$$

$$+ e^{-3|\alpha|^2(1-\eta)} \{(-1)^m \epsilon_+ \epsilon_-^* + (-1)^n \epsilon_+^* \epsilon_- \}|n\rangle \langle m|\] (51)

and

$$\rho^{(+,A)} = [Tr(\rho^{(+,A)})]^{-1} \sum_{n,m=0}^{\infty} (\delta_{0,n} \delta_{0,m} + \delta_{0,n} + \delta_{0,m} + 1) q_n q_m^* (|\epsilon_+|^2$$

$$+ (-1)^{n+m} |\epsilon_-|^2 + e^{-3|\alpha|^2(1-\eta)} \{(-1)^m \epsilon_+ \epsilon_-^* + (-1)^n \epsilon_+^* \epsilon_- \}|n\rangle \langle m|\] (52)

where, $\delta_{l,k}$ is the usual Kronecker delta function which gives 1 for $l = k$ and zero otherwise, and $q_n = (\tilde{\alpha^n}) / \sqrt{n!}$. The normalization constraints for reduced density operator $\rho^{(-,A)}$ and $\rho^{(+,A)}$ are given by $Tr(\rho^{(-,A)}) = \sum_{n=0}^{\infty} |q_n|^2 (|\epsilon_+|^2 + |\epsilon_-|^2 + 2e^{-3|\alpha|^2(1-\eta)} (-1)^n Re[\epsilon_+^* \epsilon_-])$ and $Tr(\rho^{(+,A)}) = \sum_{n=0}^{\infty} (1 + 3\delta_{0,n}) |q_n|^2 (|\epsilon_+|^2 + |\epsilon_-|^2 + 2e^{-3|\alpha|^2(1-\eta)} (-1)^n Re[\epsilon_+^* \epsilon_-])$ respectively. Similarly, when VNM in $C'$ gives

Fig. 10 Variation of fidelity $\tilde{F}(C_i)$ for information state, $\theta = \pi/3$ and $\phi = 0$, with respect to mean coherent amplitude, $|\alpha|^2$, for different values of noise parameter, $\eta$
result $|l\rangle$, the reduced density operator for teleported state becomes,

$$
\rho^{(C_l)} = [Tr(\rho^{(C_l)})]^{-1} \sum_{n,m=1}^{\infty} q_n q_m^* (|\epsilon_+\rangle^2 + (-1)^{n+m}|\epsilon_-\rangle^2 + e^{-3|\alpha|^2(1-\eta)} \langle -1\rangle^m \epsilon_+ \epsilon_-^* + (-1)^n \epsilon_+^* \epsilon_-^* | \sin^2\phi_n \sin^2\phi_m |n\rangle \langle m|)
$$

(53)

where, $Tr(\rho^{(C_l)}) = \sum_{n=1}^{\infty} |q_n|^2 [|\epsilon_+|^2 + |\epsilon_-|^2 + 2e^{-3|\alpha|^2(1-\eta)}(-1)^n \Re\langle\epsilon_+\epsilon_-\rangle] \sin^4\phi_n$.

For information state, $\rho_I$, and teleported state, $\rho_T$, the fidelity of teleportation is given by, $F = Tr[\rho_T \rho_T^*]$. For situations $A^\pm$ the fidelity becomes,

$$
\tilde{F}^{(-,A)} = [Tr(\rho^{(-,A)})]^{-1} \sum_{n,m=1}^{\infty} p_n^* p_m q_n^* q_m^* (|\epsilon_+\rangle^2 + (-1)^{n+m}|\epsilon_-\rangle^2 + e^{-3|\alpha|^2(1-\eta)}\langle -1\rangle^m \epsilon_+ \epsilon_-^* + (-1)^n \epsilon_+^* \epsilon_-^*)
$$

(54)

and,

$$
\tilde{F}^{(+,A)} = [Tr(\rho^{(+,A)})]^{-1} \sum_{n,m=0}^{\infty} (\delta_{0,n}\delta_{0,m} + \delta_{0,n} + \delta_{0,m} + 1)p_n^* p_m q_n^* q_m^* (|\epsilon_+\rangle^2 + (-1)^{n+m}|\epsilon_-\rangle^2 + e^{-3|\alpha|^2(1-\eta)}\langle -1\rangle^m \epsilon_+ \epsilon_-^* + (-1)^n \epsilon_+^* \epsilon_-^*) ,
$$

(55)

while for case $C_l$ the fidelity is given by,

$$
\tilde{F}^{(C_l)} = [Tr(\rho^{(C_l)})]^{-1} \sum_{n,m=1}^{\infty} p_n^* p_m q_n^* q_m^* (|\epsilon_+\rangle^2 + (-1)^{n+m}|\epsilon_-\rangle^2 + e^{-3|\alpha|^2(1-\eta)} \langle -1\rangle^m \epsilon_+ \epsilon_-^* + (-1)^n \epsilon_+^* \epsilon_-^*) \sin^2\phi_n \sin^2\phi_m .
$$

(56)

We note that the expressions for fidelity $\tilde{F}^{(+,A)}$ and $\tilde{F}^{(C_l)}$ reduce to $F^{(+,A)}$ and $F^{(C_l)}$ as given by Eq. 28 and Eq. 37 in the absence of noise ($\eta = 1$). It is also evident that the decohered fidelity is now a function of $\eta$ along with information parameters, $\theta$ and $\phi$, and mean coherent amplitude. In order get an idea about the variation of decohered fidelity with respect to $|\alpha|^2$, we numerically plot $\tilde{F}^{(-,A)}$ and $\tilde{F}^{(C_l)}$ for information parameters, $\theta = \pi/3$ and $\phi = 0$, in Figs. 9 and 10 respectively, for different values of $\eta = \{0.9, 0.7, 0.5, 0.3, 0.1\}$. In contrast to noiseless case, where $\tilde{F}^{(-,A)}$ and $\tilde{F}^{(C_l)}$ increase monotonically to become almost unity with increasing $|\alpha|^2$, in the presence of noise, the fidelities first increase, reach a maximum value, and then start to decrease. Also, with increasing $\eta$ the obtained maxima shifts towards the lower values of $|\alpha|^2$ and the decrement becomes sharper.
6 Discussion and conclusion

It would be instructive to compare the average fidelity of teleportation obtained from our scheme with some of the previous schemes [22,30]. In [22], authors discussed efficient quantum computation and quantum teleportation using optical coherent states. The authors propose to use second order nonlinear interaction and a high transmission ($T \rightarrow 1$) beam splitters for making quasi-Bell state measurement. The required unitary operation $X_c$ is obtained by a $\pi$ phase shifter while $Z_c$ is approximately obtained by mixing the signal with intense coherent beam ($|\alpha|^2 \rightarrow \infty$) using the high transmission beam splitter. Their scheme gives an average fidelity of $F_{avg}^j \approx 0.970$ at $|\alpha|^2 = 10$, increasing monotonically to become unity with $|\alpha|^2$. We then have the scheme suggested by Prakash et al. [30] that works in orthogonal basis $\{|\text{EVEN}, \alpha\rangle, |\text{ODD}, \alpha\rangle\}$ to obtain minimum average fidelity, $F_{avg}^p = 1 - 0.98 \times 10^{-4}$ at $|\alpha|^2 = 2$, while increasing sharply to become unity. However, the scheme requires conversion between even and odd coherent states which is not trivial in practice, and so far, no one has suggested a way of doing this.

We presented a scheme for quantum teleportation of SCS using linear optics and photon detectors. The receiver only requires to perform either no unitary operation or a $\pi$ phase shift which is conditioned to the PC result of the sender. Furthermore, mixing of the unitary transformed state with either even or odd coherent state using a symmetric lossless beam splitter gives directly the entangled mixture of information with vacuum as the output. This entangled mixture ensures that only one of the output mode contains the information and other is a vacuum. Our scheme relies on using two exactly resonant TLA in cavities to obtain the replica of information. It is, however, demanding to find out more reliable method for exact extraction of $|I\rangle$, which is still open. The robustness of ECS against noise makes it an important choice for establishing long-distance quantum teleportation as well as for other quantum information processing tasks for which our scheme can be found useful.

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A Appendix: Approximating $S_1$, $S_2$ and $S_3$

We need to find the summation of terms, $\sum_{n=1}^{\infty} p_{In} \cos^2 \phi_n$, $\sum_{n=1}^{\infty} p_{In} \sin^2 \phi_n$, $\sum_{n=1}^{\infty} p_{In}^* p_{In} \sin \phi_n \cos \phi_n$, which we abbreviated by $S_1$, $S_2$ and $S_3$. We expand $\phi_n$ about $n = |\alpha|^2$ as,

$$\phi_n = \frac{\pi}{2} \sqrt{1 + y} \simeq \frac{\pi}{2} \left( 1 + \frac{y}{2} - \frac{y^2}{8} \right) \; (57)$$

where, $y = \frac{\delta n}{|\alpha|^2}$ and $\delta n = n - |\alpha|^2$. As $y << 1$, we shall only keep terms upto cubic order in $|\alpha|$. We shall further expand the trigonometric function to obtain,
\[
\sin \phi_n = 1 - \frac{\pi^2 y^2}{32} + \frac{\pi^2 y^3}{64}
\]
\[
\cos \phi_n = -\frac{\pi y}{4} + \frac{\pi y^2}{16} + \frac{\pi^3 y^3}{384}
\] (58)

which can be used further to obtain,
\[
\sin^2 \phi_n = 1 - \frac{\pi^2 y^2}{16} + \frac{\pi^3 y^3}{32},
\] (59)
\[
\cos^2 \phi_n = -\frac{\pi y^2}{16} - \frac{\pi^3 y^3}{32},
\] (60)

and
\[
\sin \phi_n \cos \phi_n = -\frac{\pi y}{4} + \frac{\pi y^2}{16} + \frac{\pi^3 y^3}{96}.
\] (61)

We note that \(P_{In}\) is sharply peaked at \(n = |\alpha|^2\), while the trigonometric functions are flat and hence they can be replaced by their approximate values near \(n = |\alpha|^2\). Therefore, finding the sum is reduced into calculating terms, \(\sum_{n=0}^{\infty} y^m P_{nI}\), for \(m = 1, 2\) and 3. We use the fact that,
\[
\sum_{n=0}^{\infty} f(n) P_{nI} = \langle I | f(N) | I \rangle,
\] (62)

where \(N = a^\dagger a\) is the number operator , together with,
\[
\langle I | a^{\dagger 2m} a^{2m} | I \rangle = |\alpha|^{4m}
\]
\[
\langle I | a^{\dagger 2m+1} a^{2m+1} | I \rangle = |\alpha|^{4m} (1 - X),
\] (63)

where, \(X = \frac{2x^2 (|\epsilon_+ + \epsilon_-|^2 - 1)}{1 - x^2}\), to obtain,
\[
\sum_{n=0}^{\infty} y P_{nI} = X,
\]
\[
\sum_{n=0}^{\infty} y^2 P_{nI} = \frac{1}{|\alpha|^2} + \left( 2 - \frac{1}{|\alpha|^2} \right) X,
\]
\[
\sum_{n=0}^{\infty} y^3 P_{nI} = \frac{1}{|\alpha|^4} + \left( -4 + \frac{3}{|\alpha|^2} - \frac{1}{|\alpha|^4} \right) X.
\] (64)

We thus have,
\[
S_1 = \frac{\pi^2}{32} \left[ \frac{2}{|\alpha|^2} - \frac{1}{|\alpha|^4} + \left( 8 - \frac{5}{|\alpha|^2} + \frac{1}{|\alpha|^4} \right) X \right] - P_{0I}.
\] (65)
and,

\[ S_2 = 1 - (S_1 + P_{0I}). \]  \hspace{1cm} (66)  

Little more effort is required to find \( S_3 \). We note that,

\[ S_3 = \frac{x}{\alpha^*} \sum_{n=1}^{\infty} Q(n) \sqrt{n} \left[ |\epsilon_+|^2 - |\epsilon_-|^2 + (-1)^n (\epsilon_+^* \epsilon_- - \epsilon_+ \epsilon_-^*) \right] \cos \phi_n \sin \phi_n \]  \hspace{1cm} (67)  

where, \( Q(n) = x |\alpha|^2 / n! \). We can write, \( \sqrt{n} = |\alpha| (1 + y/2 - y^2/8) \) upto second order in \( y \). We use the fact that,

\[ \left\langle \alpha | f(N) | \alpha \right\rangle = \sum_{n=0}^{\infty} Q(n) f(n) \]  \hspace{1cm} (68)  

which on putting in Eq. 67 and using

\[ \left\langle -\alpha | a^{\dagger 2m} a^{2m} | \alpha \right\rangle = x^2 |\alpha|^{4m} \]  \hspace{1cm} (69)  

we obtain,

\[ S_3 = \frac{\alpha}{|\alpha|} \left\{ \frac{\pi x^2 (|\epsilon_+|^2 - |\epsilon_-|^2)}{16|\alpha|^2} \left( -1 + \frac{\pi^2 + 6}{6|\alpha|^2} \right) - \frac{\pi x^2 (\epsilon_+^* \epsilon_- - \epsilon_+ \epsilon_-^*)}{12} \right\} \]  \hspace{1cm} (70)  

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