Abstract: In this paper a distributed Received Signal Strength (RSS) minimization algorithm is proposed that guarantees strong connectivity of the network topology while minimizing the RSS of the network received at a given eavesdropper’s location. The proposed algorithm is composed of multiple rounds of maximum consensus network communications implementing a distributed greedy solution of the problem. The proposed RSS minimization algorithm is distributed in the sense that nodes do not assume, estimate or communicate any network connectivity knowledge such as a routing table, the Laplacian matrix, neighbour lists or the total number of nodes. The proposed algorithm assumes that the initial network topology is strongly connected and that each agent knows its own location and that of the eavesdropper. We provide an extension of the proposed algorithm for dealing with multiple and moving eavesdropper. In this case, we also propose a heuristic for increasing nodal transmit power to effectively reshape the network topology according to the closer eavesdropper. Performance of the proposed algorithm is demonstrated in simulations.

Keywords: Multi-agent systems; Distributed optimisation for large-scale systems; Sensor networks

1. INTRODUCTION

A fundamental requirement that underpins the success in many applications where distributed autonomous systems (termed nodes throughout this paper) are employed, is the connectivity of the underlying communication network (Santilli et al., 2019; Ju et al., 2019; Zavlanos and Pappas, 2008; De Gennaro and Jadbabaie, 2006). In this paper we consider a variant of this problem with an added requirement: the nodes need to choose their transmission powers so that the amplitude of the signal received at a given position is minimised and the network remains connected. The desire for minimising the magnitude of the received signal at certain locations comes from the fact that often we would want to minimise the negative impact of the deployed system with the existing communication infrastructure (Lou et al., 2011). Alternatively, one might want to minimise the received signal strength at a location that corresponds to a known eavesdropper. In this paper, we adopt the latter terminology.

The majority of existing solutions to the problem of connectivity maintenance requires the nodes to freely exchange information about their positions with their neighbours. For example, similar to Sims et al. (2019) one can employ a methodology based on the seminal work of Zavlanos and Pappas (2008) in which the nodes estimate the topology and the location of other agents distributively in order to come up with a set of desired transmission powers for each of nodes. However, this might not be desirable in many scenarios where the success of the mission relies on keeping nodes’ positions in the environment (or other strategic knowledge) secret.

In this paper, first, a centralised greedy algorithm for minimising the received signal strength (RSS) at a given location is provided. The proposed greedy algorithm relies on the existence of a network connectivity oracle that determines if a network remains connected by removing an edge. We demonstrate that such an oracle is amenable to a distributed implementation by modifying a standard leader election algorithm. This in turn, leads to a distributed implementation of the proposed greedy algorithm for minimising the RSS at a given location. While, at the moment, no theoretical results regarding the (sub-)optimality of the proposed solution is available, we have never observed a scenario where the solution obtained by the proposed algorithm and the optimal solution obtained by enumerating all the possible solutions do not coincide.

Outline All the necessary background and the problem of interest are presented in the next section. A greedy solution to the problem of interest is proposed in Section 3. Furthermore, a distributed implementation of the solution is presented in the same section. Numerical examples are provided in Section 4. Concluding remarks and possible future research directions are discussed in the end.

2. PROBLEM FORMULATION

In this section, first we introduce the notation used to describe the network. Later, we provide a formal defini-
A directed network graph is denoted by $G = (V, E)$, where $V = \{1, 2, \ldots, N\}$ is the set of network nodes and $E$ is set of network edges $\{(i, j)\}$. An edge $(i, j) \in E$ if node $j \in V$ is able to receive messages from node $i \in V$.

**Definition 1.** A directed graph $G = (V, E)$ is strongly connected if there exists a path that follows the direction of the edges in $E$ from any node $i \in V$ to any other distinct node $j \in V$.

In this paper network nodes are assumed to establish wireless communication with each other via omni-directional radio frequency transceivers based on the following simplified free space propagation model which is based on Friis (1946).

$$R_i^j(T_i(t)) = \frac{T_i(t)}{||l_i - l_j||^2}. \quad (1)$$

Here, $R_i^j(T_i(t)) \in \mathbb{R}$ denotes the RSS at node $j$ which is proportional to node $i$’s transmitted power $T_i(t) \in \mathbb{R}$ and inversely proportional to the squared Euclidean distance between the two nodes. The vectors $l_i, l_j \in \mathbb{R}^n$ with $n \in \{2, 3\}$ denote the coordinates of the two nodes (assumed to be time-invariant relative to run-time of network control protocols) and $||l_i - l_j||$ denotes the Euclidean distance. The scalar $\gamma_i > 0$, $i \in V$, captures the unmodeled aspects of the model in Friis (1946), here assumed to be constant for all nodes. Let $R_{th} > 0$ be a transceiver-dependent RSS threshold that results in robust communications between two nodes. Moreover, the background noise in the environment is assumed to be constant and independent of nodes locations. This correlates to a signal to a noise ratio at a receiving node which is directly proportional to $R_i^j(t)$, allowing RSS to be used to determine connectivity.

The set of neighbours of node $i$ is defined as

$$N_i(T_i(t)) = \{j \mid R_i^j(T_i(t)) \geq R_e\}. \quad (2)$$

We assume all nodes have homogeneous transceiver devices and $R_{th}$ is identical for all nodes. Let $T_G(t) := [T_1(t), T_2(t), \ldots, T_N(t)]^T$ denote the vectorized transmission power of the network nodes. Then, the set of instantaneous edges of the graph is given as

$$E(T_G(t)) = \{(i, j) \mid i \in V, j \in N_i(T_i(t))\}. \quad (3)$$

Note further that this induces our graph as

$$G(T_G(t)) := (V, E(T_G(t))). \quad (4)$$

Node $i$ can broadcast messages $M_i(t) \in \mathbb{R}$ to nodes $j$ over this network, the message received at nodes $j$, $M_j^i(t)$, is according to the following definition.

$$M_j^i(t) = \begin{cases} M_i(t) & j \in N_i(T_i(t)) \\ \text{absent otherwise} \end{cases} \quad (5)$$

Here, $M_j^i(t) = \text{absent}$ implies that the transmitting power of agent $i$ and the distance between the two agents have not resulted in a strong enough signal at node $j$ to deliver the message broadcast of node $i$.

Let $l_e \in \mathbb{R}^n$ denote the coordinates of an eavesdropper agent $e$ in which the network’s effective RSS is to be minimized. The effective RSS signature at $l_e$ is defined as

$$R_e(t) = \max_{i \in V} \frac{T_i(t)}{||l_e - l_i||^2}. \quad (6)$$

This definition is based on the widely used Time-Division Multiplexing (TDM) protocol Flood (1997) in wireless communication networks in which agents alternate in broadcasting their signal on the channel.

**Remark 1.** If TDM is not utilised, and if signals can appear simultaneously on the channel, one can define the effective RSS signature of the network as the summation of the RSS of the individual nodes rather than the maximum RSS considered in (6).

Let us now formally define the problem considered in this paper.

**Problem 1.** (Distributed RSS Minimisation). Consider a set of nodes $i \in V$ with identical transceiver devices, propagation model (1) and RSS threshold $R_{th}$. Assume that the agents’ transmission power at time $t = 0$, $T_G(0)$, and their coordinates $l_i$ are such that they form a strongly connected directed graph $G(T_G(0))$. Let each agent $i$ have knowledge of its own coordinates $l_i$, the coordinates of the eavesdropper $l_e$ and an upper bound integer $N$ for the total number of nodes in $G(t)$, i.e. $N \leq N$. It is desired to design a distributed algorithm that solves the following optimisation problem

$$\min_{T_i(t), \forall \in V} R_e(t) \quad \text{subject to} \quad G(T_G(t)) \text{is strongly connected,}$$

where $R_e(t)$ is defined in (6).

**Remark 2.** Informally, Problem 1 entails pruning a strongly connected network such that the effective RSS is minimized while strong connectivity property is retained at all times. Furthermore, network nodes are to achieve this distribution without having knowledge of the total number and the coordinates of their peers in the network.

**Remark 3.** A candidate solution to Problem 1 is one in which agents estimate, in a distributed fashion, the topology and the location of other agents in order to coordinate their actions for accomplishing the pruning. This strategy was pursued in Simos et al. (2019) while it was originally introduced for a different problem setup in Zavlanos and Pappas (2008). In this work we aim to avoid communicating strategic knowledge such as coordinates or topology information due to the security nature of this problem.

### 3. MAIN RESULT

In this section, we present a solution to Problem 1 and provide some discussions around the properties of this solution. Later, we consider variations to the proposed solution to deal with the case where there are multiple and non-stationary target points. Solving Problem 1 exactly requires exploring all the possible networks with underlying graphs whose edge sets are subsets of $E(T_G(0))$. This becomes prohibitively expensive as the number of agents and network edges increase. To this end, in this section we explore a greedy algorithm for solving and approximate solution to this problem. However, before we present a distributed solution to Problem 1, we outline a greedy algorithm for a modified version of Problem 1 and then discuss how this greedy algorithm can be implemented in a distributed way. To this aim we introduce the following problem.
Problem 2. Consider a set of nodes $i \in V$ with homogeneous transceiver devices, propagation model (1) and RSS threshold $R_{th}$. Assume that the agents’ transmission power at time $t = 0$, $T_G(0)$, and their coordinates $l_i$ are such that they form a strongly connected directed graph $G(T_G(0))$. The problem is how to monotonically 1 reduce the transmission powers $T_G(t)$ and hence the effective RSS (6) at $l_i$ such that $G(T_G(t))$ remains strongly connected. ■

As it will be discussed below, Problem 2 can be solved in a distributed manner, and the solution to this problem is a sub-optimal solution to Problem 1.

Before proposing a solution to this problem we need to introduce a connectivity oracle as defined below.

Definition 2. (Connectivity Oracle). Consider the mapping $\Omega$ from the set of directed graphs to $\{0, 1\}$. The mapping is termed the connectivity oracle if it returns 0 for the case where its argument is not strongly connected and 1 otherwise.

A possible solution to Problem 2 is presented in Algorithm 1. Briefly, Algorithm 1 works in the following manner. While possible a node $i$ with maximum contribution to the RSS at $l_e$ is dropped by the amount that will result in disconnection of $i$ from its farthest neighbour $j$. The connectivity oracle is invoked to check whether this change will maintain the strong connectivity of the network. If this is not the case the change is reverted back and the algorithm repeats until no further change is possible.

Algorithm 1: Centralised Greedy RSS Minimization with Guaranteed Strong Connectivity of the Network

1. **Requires** $G(T_G(0))$: strongly connected.
2. **Inputs:** $T_G(0)$
3. $V \leftarrow V$
4. $k \leftarrow 0$
5. while $V \neq \emptyset$ do
6.     $i \leftarrow \arg \max_{e \in V} T_G^e$
7.     $T_G(k+1) \leftarrow T_G(k)$
8.     $T_G(k+1)[i] \leftarrow \min_{T} T$
9.     s.t. $N_i(T) = N_i(T_G(k)[i]) \setminus \{(i, j)\}$
10. $j = \arg \max_{e \in \mathcal{N}_i(T_G(k)[i])} R^e_i(T_G(k)[i])$
11. if $\Omega(G(T_G(k+1))) = 0$ then
12.     $T_G(k+1)[i] \leftarrow T_i(k)$
13.     $V \leftarrow V \setminus \{i\}$
14. end
15. $k \leftarrow k + 1$
16. end

The solution obtained from Algorithm 1 is not guaranteed to solve Problem 2 exactly. In other words, it does not always return a network with the smallest RSS at a location of interest. However, it is guaranteed to result in a strongly connected network with the RSS at a location of interest smaller than the original RSS.

1 Defined with respect to element-wise inequality, i.e. $T_G(t_1) \leq T_G(t_2)$ for all positive scalars $t_1$ and $t_2$ such that $t_1 \leq t_2$.

There are two challenges in adapting Algorithm 1 to obtain in a distributed greedy solution to Problem 1. First, $\Omega$ needs to be implemented in a distributed way. Second, each node $i$ is not explicitly aware of its neighbour set or their positions. It is worthwhile to remember that the nodes in $\mathcal{N}_i$ can receive information from $i$ and not vice versa. Thus, it cannot be assumed that $i$ knows where they are located and which node belongs to this set. Therefore, step 8 of Algorithm 1 needs to be implemented via an alternative distributed approach as well. In the first instance, we introduce a methodology for implementing $\Omega$ in a distributed manner. This implementation in turn relies on the well-known Distributed Maximum-Consensus (DMC) algorithm Lynch (1996). Conceptually, this algorithm is defined as follows. Given an initial scalar value $R_i$ per node $i \in V$ and a strongly connected network $G$ (over which the agents communicate), the DMC algorithm yields

$$\text{DMC}(R_i, T_i) = \begin{cases} 1 & \text{if } R_i = \max_{j \in V} R_j, \\ 0 & \text{otherwise.} \end{cases}$$

Note that the DMC algorithm converges in less than $N - 1$ steps, the convergence time of the worst case scenario associated with a directed ring formation. Since no information other than an upper bound $N \leq N$ on the network size is assumed, the algorithm is set to run for $N - 1$ to ensure convergence.

A distributed implementation of the connectivity oracle $\Omega$, is presented in Algorithm 2. The proposed denote this distributed connectivity oracle by $\Omega^d$.

Algorithm 2: Distributed Connectivity Oracle $\Omega^d$

1. **Inputs:** $T_i, \bar{T}_i, R^e_i, R_i = \text{DMC}^m(R^e_i, T_i)$.
2. **Outputs:** $\Omega^d(T_i, \bar{T}_i, R_i) = \begin{cases} 1 & \text{if } \bar{G}(T_i) \text{ is strongly connected} \\ 0 & \text{otherwise} \end{cases}$
3. $\Omega^d \leftarrow 1$
4. $R_i \leftarrow \text{DMC}^m(R^e_i, \bar{T}_i)$
5. $f_i \leftarrow \begin{cases} 1 & \text{if } R_i \neq \bar{R}_i \\ 0 & \text{otherwise} \end{cases}$
6. $\bar{M}_i \leftarrow \text{DMC}^m(f_i, T_i)$
7. if $\bar{M}_i = 1$ then
8.     $\Omega^d \leftarrow 0$
9. end

Algorithm 2 works as follows. The algorithm takes as input the original transmission power $T_i$ that resulted in the strongly connected graph $G(T_i)$, a modified version, $\bar{T}_i$, that results in an alternative $\bar{G}(T_i)$, the original RSS of agent $i$ at eavesdropper agent $e$ and the maximum RSS of the network $G(T_i)$. The last input would have been obtained using the DMC algorithm (7). The algorithm proceeds to initialising the output to 1. A round of DMC algorithm will run effectively on the alternative network topology $\bar{G}(T_i)$. If the maximum consensus value achieved at any node $j$ is different to the original maximum RSS of the original graph $G$, $\bar{R}_j$, then that node will select $f_j = 1$ as a flag to indicate that it has lost the path to the
maximum contributing RSS node of the original network, in the new network topology. Another round of DMC algorithm now initiated with this flag values will run on the original graph. If the maximum consensus on the flagged value is equal to 1, the nodes (and most importantly the node proposing the network change) will know that the new topology is not strongly connected.

Now we proceed to provide a distributed greedy solution for implementing step 8 of Algorithm 1. Recall that, in a distributed setup nodes are not aware of their neighbours as defined in (2). Therefore, dropping a particular edge, as is required in step 8 of Algorithm 1 is not straightforward. To address this problem, we propose that the node with maximum RSS contribution, to the eavesdropper, to reduce its transmission power to a fraction of its original level. Let \( 0 < \alpha < 1 \), then the following algorithm implements Algorithm 1 in a distributed fashion.

**Algorithm 3:** Distributed Greedy Effective RSS Minimization at each \( i \)

1. **Requires** \( G(T_G(0)) \): strongly connected.
2. **Input:** \( T_i(0) \)
3. **Output:** \( T_G \)
4. \( k \leftarrow 0 \)
5. \( o_i \leftarrow 0 \)
6. **while no termination condition is satisfied do**
    7. **for** \( i \in V \) **do**
        8. \( R_i \leftarrow (1 - \alpha) R_i^c(T_i) \)
        9. \( \ell_i \leftarrow \text{DMC}^i(R_i, T_i) \)
        10. \( \hat{R}_i \leftarrow \text{DMC}^n(R_i, T_i) \)
        11. \( \hat{T}_i \leftarrow \begin{cases} \alpha T_i & \text{if } \ell_i = 1 \\ T_i & \text{otherwise} \end{cases} \)
        12. **if** \( \ell_i = 1 \) **and** \( \Omega^d(T_i, \hat{T}_i, G, \hat{R}_i) = 0 \)** **then**
            13. \( T_i(k + 1) \leftarrow T_i(k) \)
            14. \( o_i \leftarrow 1 \)
        15. **end**
    16. **end**
    17. \( k \leftarrow k + 1 \)
18. **end**
19. \( T_G \leftarrow T_G(k) \)

Algorithm 3 conceptually works as follows. In the loop, each node \( i \) initialises \( R_i \) with their RSS at \( T_i \). If \( R_i \) is less than the RSS at \( T_i \), then the node will pick a secondary transmission power \( \hat{T}_i \) as fraction of its original value and rest of the nodes will pick their original transmission power value. Agents then run a synchronised distributed connectivity oracle. This will indicate to the node with \( \ell_i = 1 \) whether the secondary transmission power will maintain the strong connectivity of the network. If this is not the case, the original transmission power value is kept and that node is excluded from reducing it transmission power in the following rounds of the algorithm. The variable \( k \) is the iterator for the inner loop of Algorithm 3 and is not the same as the time iterator \( t \). It should be noted that each DMC algorithm will run for \( N - 1 \) time steps. Therefore, \( k = 3(N - 1) \).

**Remark 4.** (Time Synchronicity). In this paper we have utilised a time synchronised version of the DCM algorithm which renders (7) and Algorithm 3 time synchronised as well. This is because this choice is conceptually more straightforward for the presentation. However, we note that there are asynchronised versions of this algorithm that will be more suitable for some applications. For instance, a token approach was introduced in Zavlanos and Pappas (2008) that can make our algorithms asynchronised. An asynchronised approach can potentially shorten the time frame of each DCM round as they are currently based on the worst case convergence time of an upper estimate of the network size.

The following result is immediate.

**Proposition 1.** Let \( G(T_G) \) be the underlying graph of the network obtained from the application of Algorithm 3. Then \( G(T_G(\tau)) \) is strongly connected.

Regarding the optimality of the solutions obtained from Algorithm 1. After, many numerical examples, we have yet to observe a case where the proposed greedy solution fails to return an optimal solution to the original problem. Thus, we hazard stating the following conjecture.

**Conjecture 1.** Algorithm 1 returns an optimal solution to Problems 1 and 2.

### 3.1 Multiple Non-Stationary Eavesdropping Agents

In a scenario where there are more than one eavesdroppers \( e = 0, 1, 2, \cdots, X \), there are a number of ways to give an alternative definition of the effective RSS of the network. Two obvious choices are the summation over the effective RSS of the network over each one of the eavesdroppers or the maximum effective RSS over these agents. In this section we will consider the latter option. In this section we will also discuss non-stationary eavesdroppers. In this case, we still assume that their locations are known at all times.

Algorithm 3 still applies in these variants of Problem 1, albeit with the following minimal modifications.

**Re-growing the Network:** Let \( \beta, 1 < \beta < 1 + \alpha \), be a small growth factor fixed for all nodes. Adding the following lines after line 13 of Algorithm 3 will be considered.

\[
\begin{align*}
\text{if } & \ell_i = 0 \ \& \ o_i = 0 \ \text{then } T_i(k + 1) \leftarrow \beta \times T_i(k) \\
\text{if } & \ell_i = 0 \ \text{then } o_i \leftarrow 0
\end{align*}
\]

Choosing a small \( \beta \) will result in a small growth in the transmission power of the nodes that identified as not having the maximum RSS contribution at that time. This in conjunction with the power reduction of nodes identified each round as having the maximum RSS will result in a dynamic reshaping of the network towards a maximally connected network. However, due to the mobility of the eavesdroppers, we set \( o_i = 0 \) so that the excluded nodes...
Fig. 1. Initial and final network topologies in Scenario 1 have the chance to participate further in power reduction/increase after two exclusion rounds.

4. ILLUSTRATIVE EXAMPLE

In this section, we illustrate the performance of the proposed Algorithm 3 in two scenarios involving $N = 7$ nodes in a V-formation.

The columns of the following matrices indicate the initial coordinates (in meters) of the nodes and the eavesdroppers.

$$l_N = \begin{bmatrix} 2 & 2 & 8 & 8 & 5 & 5 & 11 \\ 8 & -8 & 2 & -2 & 5 & -5 & 0 \end{bmatrix}, \quad l_X = \begin{bmatrix} 2 & 2 \\ 16 & -16 \end{bmatrix}.$$ 

We consider a simulation time step of 0.1 seconds. The transmission power reduction and growth rates are set to $\alpha = 0.8$ and $\beta = 1.01$, respectively. The propagation constants $\gamma_i = 1$, for all $i \in V$, $R_t = 1$ are chosen in order to simplify the analysis. Initially, the network topology is densely connected and all nodes have transmission power given as

$${T_i}(0) = \text{Max}_j, \in V_j ||l_i(0) - l_j(0)||^2.$$ 

4.1 Scenario 1 - Stationary Setup with One Eavesdropper

The first scenario, considers only stationary nodes and a single stationary eavesdropper located at the first column of $l_X$. The scenario configuration is depicted in Fig. 1a. Our objective is to compare the performance of the proposed distributed Algorithm 3 against that of the greedy centralized Algorithm 1 and the optimal centralized algorithm. The optimal solution is obtained by considering all the possible networks with underlying graphs whose edge sets are subsets of $E(T_G(0))$ and picking one with the minimum effective RSS at the eavesdropper.

Fig. 2 shows the results. As can be seen in this particular example the greedy (Algorithm 1) and optimal solutions happen to coincide. The proposed distributed solution (Algorithm 3) convergences close to these solutions. The reason that it does not completely track these solutions can be traced back to the proposed power reduction methods in Line 9 of the proposed algorithm. There, we proposed a constant reduction factor $\alpha = 0.8$. One can increase $\alpha$ arbitrary close to 1 in order to get arbitrary close to the greedy solution. However, that would significantly increase the run time of the distributed algorithm. On the other hand, one can speed up the distributed algorithm by decreasing $\alpha$ which could lead to a larger asymptotic error with respect to the centralised greedy solution. Fig. 1b shows the pruned network configuration after 50 seconds. As can be seen the nodes closer to the eavesdropper are using shorter distance links to communicate while the farther away nodes are able to reach more nodes using higher transmission power for their transceivers.

4.2 Scenario 2 - Multiple Non-stationary Eavesdropping Agents

In the second scenario here we consider two eavesdroppers which rotate around the origin. It is assumed that the eavesdroppers remain stationary during any inner loop of Algorithm 3. We have implemented a modified version of Algorithm 3 according to the modifications discussed in Section 3.1 that allows the network to reintroduce edges if possible. It can be seen that the dynamic nature of the eavesdroppers have caused the network topology to be pruned to a sparsely connected but yet strongly connected network. The topology of the network for four different time instants are depicted in Fig. 3d. The transmission powers of each node for this scenario is presented in Fig. 4. The RSS levels at each of the eavesdroppers are illustrated in Fig. 5.

5. CONCLUSION

In this paper, we have proposed a methodology where the nodes in a network modify their transmission powers in order to minimise the overall received signal strength of the network at a given eavesdropper location. The proposed solution is a distributed greedy algorithm.

Proving the conjecture on the optimality the proposed solution is a future direction. Another possible future direction is analysing the case where the network is moving in the environment and there are multiple locations at which the RSS needs to be minimised.

REFERENCES

De Gennaro, M.C. and Jadabaie, A. (2006). Decentralized control of connectivity for multi-agent systems. In
Fig. 3. Network topology at different time-steps in Scenario 2.

Fig. 4. Nodes’ transmission powers in Scenario 2. The simulation is for 500 steps with each time-step represents 0.1s.

Lou, T., Tan, H., Wang, Y., and Lau, F.C. (2011). Minimizing average interference through topology control. In International Symposium on Algorithms and Experiments for Sensor Systems, Wireless Networks and Distributed Robotics, 115–129. Springer.

Lynch, N.A. (1996). Distributed algorithms. Elsevier.

Santilli, M., Mukherjee, P., Gasparri, A., and Williams, R.K. (2019). Distributed connectivity maintenance in multi-agent systems with field of view interactions. In 2019 American Control Conference (ACC), 766–771.

Sims, B., Zamani, M., and Hunjet, R. (2019). Distributed connectivity control in low probability of detection operations. In 2019 12th Asian Control Conference (ASCC), 1155–1160. IEEE.

Zavlanos, M.M. and Pappas, G.J. (2008). Distributed connectivity control of mobile networks. IEEE Transactions on Robotics, 24(6), 1416–1428.