Error-Tracking Learning Control Method Based on Iterative Extended State Observer

To cite this article: Xiao Hong Hao et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 382 052003

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Error-Tracking Learning Control Method Based on Iterative Extended State Observer

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Abstract: we propose an error-tracking learning control method based on iterative extended state observer for a class of nonparametric uncertain systems, aiming at relaxation of initial positioning conditions for conventional iterative learning control method so as to allow any position of the initial value at each iteration and effectively process the effect of uncertain system items on tracking accuracy. It firstly constructs an appropriate expected error trajectory, followed by the linear iterative extended state observer proposed for uncertain system items which evaluates uncertain system items with LIESO, and then presents a learning control algorithm based on this method, which allows convergence of actual tracking error to the default error trajectory to realize that the system state accurately tracks reference signals within the work interval. Simulation results indicate the effectiveness of the method.

1. Introduction

In 1980s, Japanese scholar Arimoto firstly proposed the idea of iterative learning control\[1\]. The initial condition of the iterative learning control is the restriction of system with the repetitive positioning operation of the initial point, in order to guarantee the convergence of the control system\[2\]. In recent years, in the field of ILC research, with the Adaptive Iterative Learning Control method\[3,4\] has proposed and has developed, parametric uncertainty has been effectively processed. However, the research results about the non parametric uncertain relatively are few. Therefore, it is very important that the practical application of the iterative learning control need to study the initial value problem about the non parametric uncertain system.

Reference [5] puts forward to an adaptive iterative learning control method based on initial correction of filter error for a class of nonlinear systems with parametric and nonparametric uncertainties. Reference [6] puts forward to a learning control scheme on initial correction of filter error improved on the basis of Reference [5] for a class of nonparametric uncertain systems with dead-zone nonlinearity and nonzero initial error. Reference [7] discusses the tracking problem on a class of uncertain motion systems repeating operation in a limited space which proposes an adaptive iterative learning control algorithm on spatial period by introducing space state differential operator and space composite energy function, but it also has certain requirements on initial error. Aiming at a class of nonparametric uncertain systems, the paper puts forward to an error-tracking learning control method based on iterative extended state observer. The method is designed to relax the initial positioning condition for conventional iterative learning control method which is to allow the initial value to be positioned at any position during each iteration and effectively reduce the effect of nonparametric items of system on tracking accuracy so as to improve iterative efficiency and extend the application scope of iterative learning control.
2. Problem Description
Consider the nonlinear time varying systems of the 2-D form as shown below:

\[
\begin{align*}
\dot{x}_1(t,k) &= x_2(t,k), \\
\vdots \\
\dot{x}_{n-1}(t,k) &= x_n(t,k), \\
\dot{x}_n(t,k) &= f(x(t,k),t) + u(t,k).
\end{align*}
\]

(1)

\(x_i\) is the measurable system state, \(u\) is the control input of the system, \(t \in [0,T]\) is the time, \(k\) is the number of iterations, \(n\) is the system order, \(f\) is the unknown term of system, \(r_i(t,k)\) is the desired trajectory.

Wrote down: \(x^T = [x_1, x_2, \cdots, x_n] \), \(r^T = [r_1, r_2, \cdots, r_n] \). The initial value of the system is different from the reference signal in the starting time, that is \(x_i(0,k) \neq r_i(0,k)\), which is not satisfied with the initial condition requirement of the conventional iterative learning control algorithm.

Hypothesis 1: expected trajectory \(r(t)\) is bounded, and there is \(N\) derivative \(t \in [0,T]\) in the interval.

Hypothesis 2: \(f\) is a bounded local Lipschitz continuous unknown nonlinear time-varying function, and \(\frac{\partial f}{\partial t}\) bounded. Set \(x^1\) and \(x^2\) are any two state, there are

\[
\left| f(t,x^1) - f(t,x^2) \right| \leq L_f \left\| x^1 - x^2 \right\|, \\
\left| f(t,x) \right| \leq B_f.
\]

(2)

We can get the error dynamic as following forms.

\[
\begin{align*}
\dot{e}_1(t,k) &= e_2(t,k), \\
\vdots \\
\dot{e}_{n-1}(t,k) &= e_n(t,k), \\
\dot{e}_n(t,k) &= f(t,x) + u(t,k) - r_n.
\end{align*}
\]

(4)

Among them \(t \in [0,T]\).

In general, iterative learning control is a kind of data driven control method, an expected control will be determined by repeated iteration, and the uncertain term \(f(t,x)\) is compensated gradually of the iterative process by the controlled quantity. If the input and output data can seek out explicitly \(f(t,x)\), which will significantly improve the efficiency of the iteration. The extended state observer can estimate the \(f(t,x)\) primarily for the shape of system(1). If we decide to choose the linear extended state observer\(^8\), the time domain linear extended state observer of the system (1) is expressed as
The Proposed Method

Firstly, we will construct the suitable expectation error trajectory, then lead to the LIESO of iterative domain according to the LESO of time domain. Finally, we will give and prove the related algorithms and theorem.

3.1 Construction of the Desired Error Trajectory

Constructing a suitable expected error trajectory $e^*_i(t,k)$ is the base of the design error tracking controller. The expected error trajectory of the proposed structure $e^*_i(t,k)$ must be satisfied

$$e^*_i(0,k) = e_i(0,k), e^*_i(t,k) = 0, t \in [T_i, T].$$

The construction scheme of the desired error trajectory is given in the following

i. When $T_i \leq t \leq T$,

$$e^*_i(t,k) = e^*_1(t,k) = \cdots = e^*_n(t,k) = 0. \tag{7}$$

ii. When $0 \leq t < T_i$, we will choose

$$e^*_i(t,k) = a_{i,k} + a_{i+1,k}t + \cdots + a_{2n+1,k}t^{2n+1}, \tag{8}$$

$$e^*_i(t,k) = e^*_{i+1}(t), i = 2, 3, \ldots, n. \tag{9}$$

Its coefficient is selected as

$$a_{0,k} = e_{1,k}(0),$$

$$a_{j,k} = \frac{1}{j!}e_{j+1,k}(0), 1 \leq j \leq n. \tag{10}$$

3.2 Construction of Linear Iterative Extended State Observer

Transforming the coordinate of the LESO’s system (5) according to the following coordinate

$$\left\{\begin{array}{l}
z_1(t,k) = \hat{x}_1(t,k); \\
z_i(t,k) = \hat{x}_i - \sum_{j=1}^{i-1} \frac{a_{i-j}}{e^i_j}(z_{i-j} - x_{i-j}), \\
i = 2, 3, \ldots, n+1.
\end{array}\right. \tag{11}$$

$$\left\{\begin{array}{l}
z_0(t,k) = \int_0^t z_1(r,k) \, dr, \\
x_0(t,k) = \int_0^t x_1(r,k) \, dr.
\end{array}\right. \tag{12}$$

The LESO’s system (6) can be changed to the following form:
\[
\begin{align*}
\dot{z}_1(t, k) &= z_2, \\
\vdots \\
\dot{z}_{n-1}(t, k) &= z_n, \\
\dot{z}_n(t, k) &= -\sum_{j=0}^{n} \alpha_j \delta_j(t, k) + u.
\end{align*}
\] (13)

As a result, \(z_i\) and \(\hat{x}_j\) are the approximation of \(x_j\). The formula (14) can be written as the error equation:

\[
\begin{align*}
\dot{\delta}_1(t, k) &= \delta_2(t, k), \\
\vdots \\
\dot{\delta}_{n-1}(t, k) &= \delta_n(t, k), \\
\dot{\delta}_n(t, k) &= \hat{f}(t, k) + u(t, k) - \dot{x}_n.
\end{align*}
\] (14)

\[
\hat{f}(t, k) = -\sum_{j=0}^{n} \alpha_j \delta_j.
\] (15)

\[
\begin{align*}
w_r(t, 0) &= 0 \\
w_r(t, k) &= \\
w_r(t, k-1) + \sum_{j=0}^{n} \alpha_j e_j(t, k), & k \in \mathbb{Z}^+.
\end{align*}
\] (16)

\[
\begin{align*}
e_0(t, k) &= \int_0^t e_1(\tau, k) d\tau. 
\end{align*}
\] (17)

So \(w_r(t, k)\) can be used as an estimate of \(f(t, r)\), the modeling error \(\sigma(t, k)\) is estimated in real time at each iteration, and is compensated in the next time, we can get the accurate estimation of \(f(t, r)\) gradually. Formula (17) can be used as LIESO of the system (1).

4. Simulation Research

The second order nonlinear uncertain systems are considered:

\[
\begin{align*}
\dot{x}_1(t, k) &= x_2(t, k), \\
\dot{x}_2(t, k) &= f(t, x) + u(t, k).
\end{align*}
\] (19)

We can assume that \(f(t, x)\) satisfies the condition of Theorem 1, but the structure and the concrete parameters are unknown.

\[
f(t, x) = \frac{g \sin x_1 - ml(x_2)^2 \cos x_1 \cdot \sin x_1}{m_x + m}.
\] (20)

\[g = 9.8, m_x = 1, m = 0.1, l = 0.5.\] The desired trajectory is
\[
\begin{align*}
    r_1(t) &= \sin t + \sin 2t, \\
    r_2(t) &= \cos t + 2\cos 2t, \\
    r_3(t) &= \sin t - 4\sin 2t.
\end{align*}
\]  

(21)

We adopt the parameters: \(a_0 = 1, a_i = 3, \varepsilon=0.05\), and the feedback gain : \(\beta=5\).

We can construct the desired error trajectory according to the scheme proposed in this paper and, select \(T_1=0.4, T=2\). If \(T_1 \leq t \leq T\), we can get \(e_1^*(k,t) = 0, e_2^*(k,t) = 0\); If \(0 \leq t \leq T_1\), we can get

\[
e_1^*(k,t) = a_{0,k} + a_{1,k}t + a_{2,k}t^2 + a_{3,k}t^3 + a_{4,k}t^4 + a_{5,k}t^5, \\
e_2^*(k,t) = a_{1,k} + 2a_{2,k}t + 3a_{3,k}t^2 + 4a_{4,k}t^3 + 5a_{5,k}t^4,
\]

Iteration results are shown in figure 1-6.

![Figure 1 State \(x_1\) and its reference trajectory \(r_1\)](image)

![Figure 2 State \(x_2\) and its reference trajectory \(r_2\)](image)

![Figure 3 \(e_1\) and its expectation \(e_1^*\)](image)

![Figure 4 \(e_2\) and its expectation \(e_2^*\)](image)

![Figure 5 \(f(t,r)\) and its estimation \(w(t,k)\)](image)

![Figure 6 Performance evaluation index \(J_1(k)\) and \(J_2(k)\)](image)

5. Conclusion

In this paper, an error-tracking learning control method based on iterated extended state observer is proposed for a class of non parametric uncertain systems. Specifically, the observer achieves the state of the system track accurately the reference signal under the condition of arbitrary initial in the
operating range; and by using the extended state observer to deal with the uncertainty of the system, making the influence of the tracking accuracy caused by uncertainty is reduced effectively, which can improve the iterative efficiency and expand the scope of application of iterative learning control. The simulation results show that the proposed method is effective.

Acknowledgment
This research was supported by The natural science foundation of Gansu Province No. 1610RJZA024.

References
[1] Uchiyama M. Formation of high-speed motion pattern of mechanical arm by trial. Transactions of the Society of Instrumentation and Control Engineering[J], 1978, 14(6):706-712.
[2] Sun Ming-xuan, Huang Bao-jian. Iterative Learning Control[J]. Beijing: National Defense Industry Press,1999:265-271.
[3] Chi R H, Hou Z S, Xu J X. Adaptive ILC for a class of discrete-time systems with iteration-varying trajectory and random initial condition[J]. Automatica,2008,44(8):2207-2213.
[4] Sun M X, Ge S S. Adaptive repetitivie control for a class of nonlinearily parametrized systems[C]. IEEE Transactions on Automatic Control,2006,51(10):1684-1688.
[5] Yan Qiu, Sun Mingxuan, Li He. Iterative Learning Control for Nonlinear Uncertain Systems with Arbitrary Initial State[J]. Acta Automatica Sinica,2016,42:545-555.
[6] Yan Qiu, Sun Mingxuan, Cai Jianping. Filtering-error Rectified Iterative Learning Control for Systems with Input Dead-zone[J]. Control Theory & Applications. 2017.
[7] Liu Jiaolong, Dong Xinmin, Xue Jianping, Wang Haitao. Spatial Iterative Learning Control for A Class of Uncertain Motion Systems[J]. Control Theory & Applications. 2017, 32(2):197-204.
[8] Han J Q. Active Disturbance Rejection Control Technique. Beijing[J]: Press of National Defense Industry,2008:201-211.