Research Article

New Restricted Liu Estimator in a Partially Linear Model

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In this paper, we introduce a new restricted Liu estimator in a partially linear model when addition linear constraints are assumed to hold. We also consider the asymptotic normality of the new estimator. Finally, a numerical example and a simulation study are listed to illustrate the performance of the new estimator.

1. Introduction

Consider the following partially linear model:

\[ y_i = x_i' \beta + f(t_i) + \varepsilon_i, \quad i = 1, \ldots, n, \]  

where \( y_i \) shows a scalar dependent variable, \( x_i = (x_{i1}, \ldots, x_{ip})' \), \( t_i \) denotes explanatory variables, and \( \beta = (\beta_1, \ldots, \beta_p)' \) denotes a vector of unknown parameters, and in this paper, we suppose that \( f(\cdot) \) is an unknown smooth function, \( t_i's \) show values of an extra univariate variable such as the time at which the observation is made, and \( \varepsilon_i \) be an independent random error with \( E(\varepsilon_i) = 0 \) and \( \text{Var}(\varepsilon_i) = \sigma^2 < \infty \).

The statisticians have mainly discussed how to estimate the parametric component of \( \beta \), and many methods have been proposed to estimate \( \beta \) such as methods by Akdeniz and Duran [1], Akdeniz et al. [2], Duran and Akdeniz [3], Akdeniz et al. [4], Akdeniz et al. [5], Heckman [6], Liang [7], Liu et al. [8], Speckman [9], Wu [10, 11], Wu and Asar [12], Yatchew [13], Yang et al. [14], and Yang and Li [15].

As we all know, in regression analysis, the presence of multicollinearity among regressor variables can lead to highly unstable least squares estimates of the regression parameters. To deal with this problem, some biased estimating methods have been proposed, such as Hoerl and Kennard [16], Liu [17], and Xu and Yang [18, 19].

The theory and approach on biased estimation were mainly with the linear regression model. How to obtain biased estimator in partially linear models is also important and interesting. In literature studies, some biased estimators have been proposed to estimate \( \beta \) in partially linear models. Hu [20] introduced a ridge estimator by the parametric component \( \beta \). Liu et al. [8] introduced a PCR estimator in partially linear models. For more references, one can refer to Roozbeh [21], Roozbeh et al. [22], Akdeniz and Roozbeh [23], Roozbeh et al. [24], Roozbeh and Hanzah [25], and Wei and Wang [26].

In practice, we may find that the unknown parameter \( \beta \) may have some restrictions, such as linear restrictions, stochastic linear restrictions, and other restrictions. Many authors have studied these linear restrictions; namely, Akdeniz and Tabakan [27] introduced the restricted ridge estimator for the parametric component \( \beta \) and Akdeniz and Duran [1] proposed a restricted Liu estimator in a partially linear model.

As we all know, there are few papers discussing the asymptotic normality of the estimator of \( \beta \). In this paper, we continue this work in this aspect. First, we will introduce a new restricted Liu estimator for the parametric component \( \beta \) when some addition linear restrictions are assumed to hold, and this estimator is different form the Liu-type estimator which Akdeniz and Duran [1] proposed. Second, we will study the asymptotic normality of the new estimator.

The rest of the paper is organized as follows. In Section 2, we propose a new restricted Liu estimator for the linear parametric component to deal with multicollinearity, and we...
will discuss the asymptotic normality of the new estimator in Section 3. A numerical example is given to show the performance of the proposed estimator in Section 4. A simulation study is given to show the performance of the proposed estimator in Section 5, and some conclusion remarks are listed in Section 6.

2. New Restricted Liu Estimator

In this section, based on the profile least squares estimator, we will present a new restricted Liu estimator for the unknown parameter $\beta$.

To present the new estimator, we first give some assumptions (Gao [28]).

Assumption 1. There exist bounded functions $h_j(\cdot)$ over $[0,1]$, $j = 1, \ldots, p$, so that

\[ x_{ij} = h_j(t_i) + u_{ij}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, p, \tag{2} \]

where $u_i = (u_{i1}, \ldots, u_{ip})$ denotes real vectors which fulfill

\[ \lim_{n \to \infty} \frac{\sum_{i=1}^n u_{ij}^2}{n} = \Sigma. \tag{3} \]

Assumption 2. Let the functions $f(\cdot)$ and $h_j(\cdot)$ have the Lipschitz condition of order 1 on $[0,1], s = 1, \ldots, p$. The Lipschitz condition is that with any function $f(x)$, there exists a constant $k$, with any $x, y$ in $[0,1]$, so that $|f(x) - f(y)| \leq k|x - y|$.

Assumption 3. The positive weight function $W_{mn}(\cdot)$ fulfills the following:

(i) $\max_{1 \leq s \leq n} \sum_{j=1}^p W_{mn}(t_j) = O(1)$
(ii) $\max_{1 \leq s \leq n} \sum_{j=1}^p W_{mn}(t_j)\Pi(t_{ij} - t_j) = O(n^{-2/3})$
(iii) $\max_{1 \leq s \leq n} \sum_{j=1}^p W_{mn}(t_j)\Pi(t_{ij} - t_j)\sigma_n = O(d_n)$

Here, $I$ denotes the indicator function, $d_n$ appasses $\limsup_{n \to \infty} nd_n^3 < \infty$, and $c_n$ appasses $\limsup_{n \to \infty} nc_n^3 < \infty$.

2.1. Profile Least Squares Estimator. In this paper, we let $(x_{i1}, t_i, y_i, \quad i = 1, \ldots, n)$ satisfy partially linear model (1). When we obtain an estimator of $\beta$, then we can obtain an estimator of $f(\cdot)$, which is given as follows:

\[ \hat{f}(t, \beta) = \sum_{i=1}^n W_{mn}(t_i)(y_i - x_i'\beta). \tag{4} \]

And we also suppose that the positive weight functions $W_{mn}(\cdot)$ satisfy Assumption 3.

By (1) and (4), we obtain the following:

\[ \bar{y} = \bar{X}'\beta + \varepsilon, \tag{5} \]

where $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_n)'$, $\bar{X} = (\bar{x}_1, \ldots, \bar{x}_n)'$, $y_i = y_i - \sum_{j=1}^p W_{nj}(t_i)y_j$, and $x_i = x_i - \sum_{j=1}^p W_{nj}(t_i)x_j$ with $i = 1, \ldots, n$.

To estimate $\beta$, one can usually use the profile least squares method to get the profile least squares (PLS) estimator:

\[ \hat{\beta} = \left( \bar{X}'\bar{X} \right)^{-1} \bar{X}'\bar{y}. \tag{6} \]

2.2. New Restricted Liu Estimator. In order to deal with multicollinearity, Akdeniz and Duran [1] proposed the Liu estimator (LE), which is denoted as follows:

\[ \hat{\beta}(d) = \left( \bar{X}'\bar{X} + dI_p \right)^{-1} \left( \bar{X}'\bar{y} + d\hat{\beta} \right), \quad 0 < d < 1. \tag{7} \]

Now, we consider the following linear restrictions:

\[ R\beta = r, \tag{8} \]

where $R$ denotes a known $m \times p$ matrix with rank $m < p$ and $r$ shows a known $m \times 1$ vector. For models (1) and (8), restricted profile least squares (RPLS) estimator [1] is denoted by

\[ \hat{\beta}_r = \hat{\beta} - \left( \bar{X}'\bar{X} \right)^{-1} R' \left( R \left( \bar{X}'\bar{X} \right)^{-1} R' \right)^{-1} (R\hat{\beta} - r). \tag{9} \]

We are now ready to introduce a new restricted Liu estimator, which is obtained by combining the Liu estimator proposed by Akdeniz and Duran [1] and the restricted profile least squares (RPLS) estimator as follows:

\[ \hat{\beta}_r(d) = \hat{\beta}(d) - \left( \bar{X}'\bar{X} \right)^{-1} R' \left( R \left( \bar{X}'\bar{X} \right)^{-1} R' \right)^{-1} (R\hat{\beta}(d) - r), \tag{10} \]

where $\hat{\beta}(d) = \left( \bar{X}'\bar{X} + dI_p \right)^{-1} \left( \bar{X}'\bar{y} + d\hat{\beta} \right)$ is the Liu estimator.

By (10), we can see that the new restricted Liu estimator is different from the restricted Liu estimator $\hat{\beta}(d) = \hat{\beta}(d) - \left( \bar{X}'\bar{X} + I_p \right)^{-1} R' \left( R \left( \bar{X}'\bar{X} + I_p \right)^{-1} R' \right)^{-1} (R\hat{\beta}(d) - r)$ proposed by Akdeniz and Duran [1], and so this Liu estimator is a new restricted estimator.

From the definition of $\hat{\beta}_r(d)$, it is easy to see that $\hat{\beta}_r(1) = \hat{\beta}$.

In this paper, we mainly discuss how to estimate $\beta$. For the estimator of $f$, we will discuss it in the next study.

3. Properties of the New Estimator

To obtain the properties of the new estimator, we first list some lemmas.

Lemma 1. If Assumptions 1–3 hold, we have

\[ \lim_{n \to \infty} n^{-1}\bar{X}'\bar{X} = \Sigma. \tag{11} \]

Proof. See Gao et al.’s study [28].\hfill \square

Lemma 2. Under Assumptions 1–3, PLS estimator $\hat{\beta}$ is an asymptotic normality of $\hat{\beta}$, i.e.,
\[
\sqrt{n}(\hat{\beta} - \beta) \longrightarrow ^{l} N(0, \sigma^2 \Sigma^{-1}),
\]
where \(\longrightarrow ^{l}\) denotes convergence in distribution.

Proof. See Gao et al.’s study [28]. \(\square\)

**Theorem 1.** Under Assumptions 1–3, Liu estimator \(\tilde{\beta}(d)\) is an asymptotic normality of \(\beta\), i.e.,
\[
\sqrt{n}(\bar{\beta}(d) - \beta) \longrightarrow ^{l} N(0, \sigma^2 \Sigma^{-1})
\]

Proof. By (7), we have
\[
\bar{\beta}(d) = (X'X + dIp)X'Y + d\bar{\beta}) - \beta
= (X'X + dIp)^{-1}(X'X + dIp)(\tilde{\beta} - \beta)
+ (d - 1)(X'X + dIp)^{-1}\beta.
\]
Then,
\[
\sqrt{n}(\tilde{\beta}(d) - \beta) = \sqrt{n}\left[(X'X + dIp)^{-1}(X'Y + d\bar{\beta}) - \beta\right]
= (X'X + dIp)^{-1}(X'X + dIp)(\tilde{\beta} - \beta)
+ (d - 1)\sqrt{n}(\tilde{\beta}(d) - \beta).
\]

By Lemma 1, it is easy to derive that
\[
\frac{1}{n}(X'X + dI_p) \longrightarrow ^p \Sigma,
\]
\[
\frac{1}{n}(X'X + dI_p) \longrightarrow ^p \Sigma,
\]
\[
\tilde{\beta}(d) - \beta = \bar{\beta}(d) - (X'X)^{-1}R^t\left[R(X'X)^{-1}R\right]^{-1}(\tilde{\beta}(d) - \beta) = (\tilde{\beta}(d) - \beta) - (X'X)^{-1}R^t\left[R(X'X)^{-1}R\right]^{-1}(\tilde{\beta}(d) - \beta) = (\tilde{\beta}(d) - \beta) - (X'X)^{-1}R^t\left[R(X'X)^{-1}R\right]^{-1}(\tilde{\beta}(d) - \beta).
\]

By Lemma 1 and (22), we obtain
\[
(X'X)^{-1}R^t\left[R(X'X)^{-1}R\right]^{-1}R \longrightarrow ^{p} \Sigma^{-1}R^t\left[R\Sigma^{-1}R\right]^{-1}R.
\]
Thus by (23), we have
\[
\left[I - \Sigma^{-1}R^t\left[R\Sigma^{-1}R\right]^{-1}R\right]\Sigma^{-1}\left[I - \Sigma^{-1}R^t\left[R\Sigma^{-1}R\right]^{-1}R\right]' = \Gamma,
\]
where \(\Gamma = \Sigma^{-1} - \Sigma^{-1}R^t\left[R\Sigma^{-1}R\right]^{-1}R\Sigma^{-1}\). Thus, by (22)–(24) and the Slutsky theorem, we derive
\[
(1 - d)\sqrt{n}(X'X + dIp)\beta = O_p(n^{-1/2}),
\]
where \(\longrightarrow ^p\) shows convergence in probability and \(O_p(\cdot)\) denotes the infinitesimal of higher order. Thus, by Lemma 2 and (15)–(18), we obtain
\[
(X'X + dIp)^{-1}(X'X + dIp)X'X + dIp = \Sigma^{-1}.
\]
Then, by the Slutsky theorem and (15) and (19), we obtain
\[
\sqrt{n}(\bar{\beta}(d) - \beta) \longrightarrow ^{l} N(0, \sigma^2 \Sigma^{-1}).
\]

Now, we present the asymptotic normality of the new restricted Liu estimator. \(\square\)

**Theorem 2.** Under Assumptions 1–3, the new restricted Liu estimator \(\tilde{\beta}(d)\) is an asymptotic normality of \(\beta\), i.e.,
\[
\sqrt{n}(\tilde{\beta}(d) - \beta) \longrightarrow ^{l} N(0, \sigma^2 \Gamma),
\]
where \(\Gamma = \Sigma^{-1} - \Sigma^{-1}R^t\left[R\Sigma^{-1}R\right]^{-1}R\Sigma^{-1}\).

Proof. By (10), we obtain
\[
\tilde{\beta}(d) - \beta = \tilde{\beta}(d) - (X'X)^{-1}R^t\left[R(X'X)^{-1}R\right]^{-1}(\tilde{\beta}(d) - \beta) = (\tilde{\beta}(d) - \beta) - (X'X)^{-1}R^t\left[R(X'X)^{-1}R\right]^{-1}(\tilde{\beta}(d) - \beta) = (\tilde{\beta}(d) - \beta) - (X'X)^{-1}R^t\left[R(X'X)^{-1}R\right]^{-1}(\tilde{\beta}(d) - \beta).
\]

\[
\sqrt{n}(\tilde{\beta}(d) - \beta) \longrightarrow ^{l} N(0, \sigma^2 \Gamma).
\]

Remark 1. By the asymptotic covariance matrices of \(\tilde{\beta}(d)\) and \(\bar{\beta}(d)\), we note that \(\Sigma^{-1} - \Gamma = \Sigma^{-1}R^t\left[R\Sigma^{-1}R\right]^{-1}R\Sigma^{-1}\) is a positive definite matrix. That is, to mean if the linear restrictions (8) are assumed to hold, the new restricted Liu estimator \(\tilde{\beta}(d)\) is more efficient than the Liu estimator \(\bar{\beta}(d)\), and this agrees with the practice; thus, the new restricted Liu estimator contains more information for \(\beta\) than the Liu estimator.
4. Numerical Example

In this section, we will use a numerical example to illustrate the performance of the new estimator. We consider the hedonic prices of housing attributes. The partially linear model was estimated by Ho [29] using semiparametric least squares. The data consist of 92 detached homes sold during 1987 in the Ottawa area [30].

The specification of the partially linear model is

\[ y_i = \beta_0 + \beta_1 (frplc)_i + \beta_2 (grge)_i + \beta_3 (lux)_i + \beta_4 (avginc)_i + \beta_r (sds)_i + \beta_c (lotarea)_i + \epsilon_i. \]

(26)

The matrix \( X'X \) has eigenvalues 3.79, 4.58, 15.76, 18.59, 20.99, 90.73, and 236956.33 and the condition number of \( X \) is 250.069, which is large. The Gaussian kernel function

\[ K\left( \frac{t_i - t_j}{h} \right) = \frac{1}{h} \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{(t_i - t_j)^2}{2h^2} \right\} \]

(27)

is used when obtaining the matrices \( W_{t_i} \) and \( h \) is taken to be 0.5. We use the method presented by Wu and Asar [12] to estimate \( W_{\beta_r} \). In these data, we choose \( R\beta = 0 \), where

\[ R = \begin{bmatrix} 0.5 & 0.5 & 0 & -0.5 & -0.5 & -0.5 & 0 \end{bmatrix}. \]

(28)

And we use Liu [17] to estimate \( d \). Then, we obtain

\[
\begin{align*}
\text{MSE}(\hat{\beta}) &= 423.5498, \\
\text{MSE}(\hat{\beta}(d)) &= 392.1701, \\
\text{MSE}(\hat{\beta}_r) &= 415.6479, \\
\text{MSE}(\hat{\beta}_r(d)) &= 355.2262.
\end{align*}
\]

(29)

By the numerical example, we can see that the new estimator has smaller MSE than other estimators.

5. Monte Carlo Simulation Study

5.1. Design of the Simulation. In this section, we use a Monte Carlo simulation to discuss the performances of the estimators PLS, LE, and RPLS and the new estimator in the sense of MSE criterion. Although the purpose of this paper is to compare the estimators when there is multicollinearity, we use the following equation to obtain the explanatory variables having different degrees of multicollinearity:

\[ x_{ij} = (1 - \gamma^2)^{1/2} z_{ij} + \gamma z_{ip}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, p, \]

(30)

where \( z_{ij} \) show independent standard normal pseudo-random numbers. We form the data matrix as \( X = [x_{ij}] \). We discuss three different values of the degree of correlation \( \gamma \) corresponding to 0.9, 0.99, and 0.999.

In order to obtain the \( n \) observations of the dependent variable, we first use the following Doppler function:

\[ f(t_i) = \sqrt{t_i (1 - t_i)} \sin \frac{2.1\pi}{t_i + 0.05}, \]

(31)

where \( t_i = (i - 0.5)/n, \ i = 1, 2, \ldots, n \) and the sample size \( n \) varies between 30, 50, 100, 200, and 400 in this simulation study. After estimating the function \( f \), we use equation (1) to obtain the dependent variables. Finally, we estimate the function \( f \) again using equation (4).

The Gaussian kernel function

\[ K\left( \frac{t_i - t_j}{h} \right) = \frac{1}{h} \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{(t_i - t_j)^2}{2h^2} \right\} \]

(32)

is used when obtaining the matrices \( W_{t_i} \) and \( h \) is taken to be 0.05. In this paper, we study \( p = 4 \) and consider the following restriction:

\[ R = \begin{bmatrix} 1 & 5 & -3 & -1 \\ -1 & 2 & -1 & 0 \end{bmatrix}. \]

(33)

Moreover, we also consider \( \sigma^2 = 5 \).

5.2. Results of the Simulation. The results of the simulation are presented in Tables 1–5. From Tables 1–5, we see that an increase in the degree of correlation makes an increase in the simulated MSE values of the estimators in all cases.
newestimatorperformsbestcomparedtootherestimators
and the sample size makes a decrease in the MSE values. And the
estimators are satisfied in all of the cases, namely, an increase in the
MSE.

Moreover, the asymptotic behaviours of the estimators are satisfi ed in all of the cases, namely, an increase in the sample size makes a decrease in the MSE values. And the new estimator performs best compared to other estimators in all cases.

6. Conclusions

In this paper, we propose a new restricted Liu estimator when some additional linear restrictions are assumed to hold on the linear parametric component. And we study the properties of the proposed estimator. Finally, a data example and a simulation study are given to show the performance of these estimators.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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