NEW SUPERSYMMETRIC TWO-HIGGS-DOUBLET 
STRUCTURE AT THE ELECTROWEAK ENERGY SCALE

ERNEST MA

Department of Physics, University of California,
Riverside, California 92521, USA

ABSTRACT

Contrary to common belief, the requirement that supersymmetry exists and that 
there are two Higgs doublets and no singlet at the electroweak energy scale does 
not necessarily result in the minimal supersymmetric standard model (MSSM). An 
interesting alternative is presented.

1. Introduction

It is generally believed that given the gauge group SU(2) × U(1) and the require-
ment of supersymmetry, the quartic scalar couplings of the Higgs potential (consisting 
of two doublets and no singlet) are completely determined in terms of the two gauge 
couplings. This is actually not the case because the SU(2) × U(1) gauge symmetry 
may be a remnant of a larger symmetry which is broken at a higher mass scale 
together with the supersymmetry. The structure of the Higgs potential is then de-
termined by the scalar particle content needed to precipitate the proper spontaneous 
symmetry breaking and to render massive the assumed fermionic content of the larger 
theory. Furthermore, the quartic scalar couplings are related to the gauge couplings 
of the larger theory as well as other couplings appearing in its superpotential. At the 
electroweak energy scale, the reduced Higgs potential may contain only two scalar 
doublets, but their quartic couplings may not be those of the minimal supersymmet-
ric standard model (MSSM). The work that I will describe in this contribution to 
Kamesh Wali’s Festschrift is an explicit first example that the MSSM structure is not 
unique. It is based on my very recent work with Daniel Ng of TRIUMF.

2. The Two-Doublet Higgs Potential

Consider two Higgs doublets \( \Phi_{1,2} = (\phi_{\pm,1,2}^0, \phi_{\mp,1,2}^0) \) and the Higgs potential

\[
V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)
\]
\[ + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) \\
+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \frac{1}{2} \lambda_6 (\Phi_2^\dagger \Phi_1)^2. \]  

In the MSSM, there are the well-known constraints

\[ \lambda_1 = \lambda_2 = \frac{1}{4} (g_2^2 + g_2^2), \quad \lambda_3 = -\frac{1}{4} g_1^2 + \frac{1}{4} g_2^2, \quad \lambda_4 = -\frac{1}{2} g_2^2, \quad \lambda_5 = 0, \]  

where \( g_1 \) and \( g_2 \) are the U(1) and SU(2) gauge couplings of the standard model respectively. Note that only the gauge couplings contribute to the \( \lambda \)'s. This is because that with only two SU(2) \( \times \) U(1) Higgs superfields, there is no cubic invariant in the superpotential and thus no additional coupling.

3. The \( E_6 \)-Inspired Left-Right Model

Consider now the gauge group SU(2)_L \( \times \) SU(2)_R \( \times \) U(1) but with an unconventional assignment of fermions.[3] An exotic quark \( h \) of electric charge \(-1/3\) is added so that \((u, d)_L\) transforms as \((2,1,1/6)\), \((u, h)_R\) as \((1,2,1/6)\), whereas both \(d_R\) and \(h_L\) are singlets \((1,1,−1/3)\). There are two scalar doublets \(\Phi_1\) and \(\chi\), as well as a bidoublet \(\eta = (\phi_0^2 \eta^\dagger \chi + \overline{\phi_0^2 \eta}^\dagger \eta^\dagger \chi)\), transforming as \((2,1,1/2)\), \((1,2,1/2)\), and \((2,2,0)\) respectively. Note that \(\Phi_1^\dagger \tilde{\eta} \chi\) is then an allowed term in the superpotential, where \(\tilde{\eta} \equiv \sigma_2 \eta^* \sigma_2\), so that its coupling \(f\) also contributes to the quartic scalar couplings of this model’s Higgs potential.

Let \(G_1\) be the U(1) gauge coupling and \(G_2\) the coupling of both SU(2)'s. Then

\[ V = V_{soft} + \frac{1}{8}(G_1^2 + G_2^2)\left[ (\Phi_1^\dagger \Phi_1)^2 + (\chi^\dagger \chi)^2 \right] \]
\[ + \frac{1}{4} G_2^2(Tr \eta^\dagger \eta)^2 - (Tr \eta^\dagger \tilde{\eta})(Tr \tilde{\eta}^\dagger \eta) + (f^2 - \frac{1}{4} G_2^2)(\Phi_1^\dagger \Phi_1 + \chi^\dagger \chi)Tr \eta^\dagger \eta \]
\[ - (f^2 - \frac{1}{2} G_2^2)(\Phi_1^\dagger \eta \eta^\dagger \Phi_1 + \chi^\dagger \eta \eta^\dagger \chi) + (f^2 - \frac{1}{4} G_1^2)(\Phi_1^\dagger \Phi_1)(\chi^\dagger \chi), \]  

where \(V_{soft}\) contains terms of dimensions 2 and 3, and breaks the supersymmetry. Let \(\chi^0\) acquire a vacuum expectation value \(u \neq 0\). Then SU(2)_L \( \times \) SU(2)_R \( \times \) U(1) breaks down to the standard SU(2)_L \( \times \) U(1)_Y with \(m^2(\sqrt{2} Re \chi^0) = (G_1^2 + G_2^2)u^2/2\) and \(m^2(\eta^+, \eta^0) = G_2^2u^2/2\). These heavy particles can be integrated out at the electroweak energy scale where only \(\Phi_{1,2}\) are left.

4. Reduced Higgs Potential of the Left-Right Model

The quartic scalar couplings of the reduced Higgs potential at the electroweak
energy scale are now given by

\[
\lambda_1 = \frac{1}{4}(G_1^2 + G_2^2) - \frac{(4f^2 - G_1^2)^2}{4(G_1^2 + G_2^2)},
\]

(5)

\[
\lambda_2 = \frac{1}{2}G_2^2 - \frac{(4f^2 - G_2^2)^2}{4(G_1^2 + G_2^2)},
\]

(6)

\[
\lambda_3 = \frac{1}{4}G_2^2 - \frac{(4f^2 - G_1^2)(4f^2 - G_2^2)}{4(G_1^2 + G_2^2)},
\]

(7)

\[
\lambda_4 = f^2 - \frac{1}{2}G_2^2, \quad \lambda_5 = 0,
\]

(8)

where the second terms on the right-hand sides of the equations for \(\lambda_{1,2,3}\) come from the cubic interactions of \(\sqrt{2}Re\chi^0\). In the limit \(f = 0\) and using the tree-level boundary conditions \(G_2 = g_2\) and \(G_1^{-2} + G_2^{-2} = g_1^{-2}\), it can easily be shown from the above that the MSSM is recovered. However, \(f\) is in general nonzero, although it does have an upper bound because \(V\) must be bounded from below. Hence

\[
0 \leq f^2 \leq \frac{1}{4}(g_1^2 + g_2^2) \left(1 - \frac{g_1^2}{g_2^2}\right)^{-1},
\]

(9)

where the maximum value is obtained if \(V_{soft}\) is also left-right symmetric.

5. Phenomenological Consequences

For illustration, let \(f = f_{max}\) and \(x \equiv \sin^2 \theta_W\), then

\[
\lambda_1 = 0, \quad \lambda_2 = \frac{e^2}{2x} \left[1 - \frac{2x^2}{(1 - x)(1 - 2x)}\right] + \frac{g_2^2 \epsilon}{4M_W^2 \sin^4 \beta},
\]

(10)

\[
\lambda_3 = \frac{e^2}{4x} \left[1 - \frac{2x}{1 - 2x}\right] = -\lambda_4, \quad \lambda_5 = 0,
\]

(11)

where

\[
\epsilon = \frac{3g_2^2 m_t^4}{8\pi^2 M_W^2} \ln \left(1 + \frac{\tilde{m}^2}{m_t^2}\right)
\]

(12)

is an extra term coming from radiative corrections and \(\tan \beta \equiv \langle \phi_2^0 \rangle / \langle \phi_1^0 \rangle\). Comparing against the MSSM, the lighter of the two neutral scalar bosons is now constrained by

\[
m_h^2 < m_A^2 \sin^2 \beta, \quad m_h^2 < 2M_W^2 \left[1 - \frac{2x^2}{(1 - x)(1 - 2x)}\right] \sin^4 \beta + \epsilon,
\]

(13)

instead of \(m_A^2 \cos^2 2\beta + \epsilon / \tan^2 \beta\) and \(M_Z^2 \cos^2 2\beta + \epsilon\), where \(m_A\) is the mass of the pseudoscalar boson. Assuming \(m_t = 150\) GeV and \(\tilde{m} = 1\) TeV, this means that

\[
m_h < 120\text{ GeV}
\]

(14)
in this model, whereas \( m_h < 115 \) GeV in the MSSM. There is also the sum rule

\[
m^2_{H^\pm} = m_A^2 + \frac{1}{2} M_W^2 \left( 1 - \frac{2x}{1 - 2x} \right)
\]

(15)

instead of the corresponding \( m^2_{H^\pm} = m_A^2 + M_W^2 \) in the MSSM. In the limit of large \( m_A \), both models reduce to the standard model with \( h \) as its one Higgs boson while keeping their respective mass upper limits.

6. Outlook

If supersymmetry exists and future experiments discover two and only two Higgs doublets at the electroweak energy scale, it does not mean necessarily that the MSSM will be confirmed. If this model with \( f \neq 0 \) or some other is found, then it will point to a larger theory such as \( SU(2)_L \times SU(2)_R \times U(1) \) or some other at a higher energy scale.

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8. References

1. E. Ma and D. Ng, Univ. of Calif., Riverside Report No. UCRHEP-T103 (1993).
2. E. Ma and D. Ng, Univ. of Calif., Riverside Report No. UCRHEP-T107 (1993).
3. E. Ma, \textit{Phys. Rev.} \textbf{D36}, 274 (1987); K. S. Babu, X.-G. He, and E. Ma, \textit{ibid.} \textbf{36}, 878 (1987).