The influence of feedback channel delay of servo motor control system on current loop bandwidth and dynamic response

Chaoran Wang¹, Jian Luo¹
¹School of Mechatronics Engineering and Automation, Shanghai University, China
Merkaba@sina.com

Abstract. In the servo control system of permanent magnet synchronous motor, the current loop, as the innermost loop, has the greatest impact on the performance of the whole system. For many reasons, the delay time exists in the current loop. This paper focuses on the analysis of the less analyzed feedback channel delay in the previous papers, obtains the relationship between the feedback channel delay time and the current loop bandwidth, and compares the relationship between the forward channel delay time and the current loop bandwidth, and analyzes the relationship between the feedback channel delay and the current loop dynamic response, which provides a theoretical basis for the actual project to improve the current loop bandwidth.

1. Introduction
Permanent magnet synchronous motor servo control system usually adopts three closed-loop structure, from the inside to the outside are current loop, speed loop and position loop. Because the current loop is the innermost loop, the control effect of the current loop will directly affect the operation performance of the whole system. The higher the current loop bandwidth, the better the current loop control effect and the better the performance of the whole system. By means of internal model control, the bandwidth of current loop can be infinite in theory. [1]

But in the actual system, because of the control method, power switch, sampling and other reasons, the forward channel and feedback channel of the current loop have delays, which will restrict the improvement of the current loop bandwidth. [2]

This paper explains the reason why the feedback channel delay has little effect on the current loop bandwidth compared with the forward channel delay through the analytical solution of the current loop bandwidth under different conditions, and analyzes its dynamic response ability, which provides a reference for the actual engineering current loop bandwidth expansion. It provides support for engineering practice.

2. The influence of feedback channel delay on bandwidth
2.1. Current loop model
In the servo motor control system, the current loop model is shown in Figure 1.
In the figure, the PI regulator is used as the controller, and its transfer function is \( k_p \left( 1 + \frac{k_i}{s} \right) \), where, \( k_p \) is the proportionality coefficient, \( k_i \) is the integral coefficient. The transfer function of delays in forward channel is \( \frac{1}{T_d s + 1} \), and the transfer function of delays in feedback channel is \( \frac{1}{T_f s + 1} \). The controlled object is a permanent magnet synchronous motor (PMSM), its transfer function is \( \frac{1}{L s + R} \).

The mathematical model of PMSM needs to assume that it is in ideal state. [3]
1. The magnetic saturation, hysteresis loss and eddy current loss are neglected.
2. It is considered that the parameters of resistance and inductance of motor do not vary with temperature and speed.
3. Neglecting the influence of back EMF on motor.
4. The three-phase electricity supplied to the motor is balanced.

In the analysis of current loop, ideal state analysis is often used, that is, there is no forward channel delay and feedback channel delay in current loop. For PI regulator, PI parameters are set by internal model control method. [4] \( k_p = 2\pi f L \), \( k_i = \frac{R}{L} \). Where \( f \) is the ideal bandwidth setting value, when there is no delay in the system.

2.2. Current Loop Model with Only Feedback Channel Delay
A current loop model with only feedback channels delay is shown in Figure 2.

The feedback channel delay is mainly caused by the A/D sample conversion. In systems where there is only feedback channel delay, the transfer function of forward channel is \( G(s) = \frac{k_p(s+k_i)}{s(Ls+R)} \), the transfer function of feedback channel is \( H(s) = \frac{1}{T_f s + 1} \). The PI regulator is tuned according to ideal conditions, that is, the method of internal model control is used for setting, \( k_p = 2\pi f L \), \( k_i = \frac{R}{L} \). Then, the closed-loop transfer function of the whole system is
According to equation (3), when only the feedback channel has delay, the system is a second order system with one zero point.

Build the following equation.

\[
G_c(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{2\pi f (T_f s + 1)}{s^2 + \frac{1}{T_f} s + \frac{2\pi f}{T_f}}
\]  

(1)

In equation (2),

\[
\omega_n = \sqrt{\frac{2\pi f}{T_f}}
\]

\[
\xi = \frac{1}{2\sqrt{2\pi f T_f}}
\]

\[
\tau = T_f
\]

The definitions of \(\xi\) and \(\omega_n\) are the same as in the previous case, except that the delay time is different.

According to the definition of bandwidth, the corresponding frequency when the magnitude-frequency characteristic decreases by 3dB is the bandwidth. The bandwidth can be calculated as

\[
20 \log |G_c(j\omega)| = 20 \log \frac{1}{\sqrt{2}} \approx -3
\]

(3)

And because

\[
|G_c(s)| = \sqrt{\text{Re}[G_c(j\omega)] + \text{Im}[G_c(j\omega)]} = \sqrt{\frac{\tau^2 \omega^2 + 1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2 \frac{\omega^2}{\omega_n^2}}}
\]

It can be solved that the bandwidth \(\omega\) of the current loop under the magnitude-frequency characteristic is

\[
\omega = \omega_n \sqrt{(1 + \tau^2 \omega_n^2 - 2\xi^2) + \sqrt{(1 + \tau^2 \omega_n^2 - 2\xi^2)^2 + 1}}
\]

(4)

It can be seen from the reference [5] and reference [6] that the bandwidth equation with delay only in the forward channel is

\[
\omega = \omega_n \sqrt{1 - 2\xi^2} + \sqrt{(1 - 2\xi^2)^2 + 1}
\]

(5)
Comparing equation (4) and equation (5), it can be observed that when there is only feedback channel delay in the system, there's an extra part in the bandwidth equation as \( \tau \omega_n^2 \).

Because \( \tau^2 \omega_n^2 \) is greater than zero, \( 1 + \tau^2 \omega^2 - 2\xi^2 \) is greater than \( 1 - 2\xi^2 \). When the delay time is same, the bandwidth of the system with only feedback channel delay is higher than the system with only forward delays. Therefore, compared with the forward channel delay, the feedback channel delay has a weak ability to suppress the bandwidth of the current loop.

2.3. Dynamic response only when the feedback channel has delay

In order to meet the dynamic requirements of the system, the overshoot of the current loop of the system is also needed. According to industrial requirements, set the PI regulator if the overshoot does not exceed 5%.

In the case of unit step signal, \( R(s) = \frac{1}{s} \), the response of Equation (2) is

\[
C(s) = G_i(s)R(s) = \frac{\omega_n^2 (\tau s + 1)}{s^2 + 2\zeta \omega_n s + \omega_n^2} \cdot \frac{1}{s} \quad \text{or}
\]

\[
= \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \cdot \frac{1}{s} + \frac{\omega_n^2 \tau}{s^2 + 2\zeta \omega_n s + \omega_n^2} = C_1(s) + C_2(s)
\]

(6)

In equation (6),

\[
C_2(s) = \frac{\omega_n^2 \tau}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

\[
= C_1(s) \cdot \tau s
\]

\[
c_2(t) = \tau \frac{dc_1(t)}{dt}
\]

Because the closed loop pole is \( s_{1,2} = -\zeta \omega_n \pm j\omega_n \sqrt{1 - \zeta^2} = -\zeta \omega_n \pm j\omega_d \). So

\[
C_1(s) = \frac{1}{s} - \frac{s + \zeta \omega_n}{(s + \zeta \omega_n + j\omega_d)(s + \zeta \omega_n - j\omega_d)}
\]

\[
= \frac{1}{s} - \frac{s + \zeta \omega_n}{s + \zeta \omega_n} \cdot \frac{\zeta \omega_n}{\omega_d} \cdot \frac{\omega_d}{(s + \zeta \omega_n)^2 + \omega_d^2}
\]

(7)

The Laplace transform of equation (7) gives the dynamic equation in the time domain,

\[
c_1(t) = L^{-1}[C_1(s)]
\]

\[
= 1 - e^{-\zeta \omega_d t} \cos \omega_d t - \frac{\zeta \omega_n}{\omega_d} e^{-\zeta \omega_d t} \sin \omega_d t
\]

\[
= 1 - \frac{e^{-\zeta \omega_d t}}{\sqrt{1 - \zeta^2}} \left( \sqrt{1 - \zeta^2} \cos \omega_d t + \zeta \sin \omega_d t \right)
\]

\[
= 1 - \frac{e^{-\zeta \omega_d t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t + \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta}), t \geq 0
\]

(8)
In the same way, 
\[ c_2(t) = \tau \frac{dc_1(t)}{dt} \]
\[ = \tau \left[ 1 - \frac{e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t + \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta}) \right]' \]
\[ = -\frac{\tau}{\sqrt{1 - \zeta^2}} \left[ e^{-\zeta^2t} \sin(\omega_d t + \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta}) \right]' \]
\[ = \frac{\tau \omega_d e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \sin(\arctan \frac{\sqrt{1 - \zeta^2}}{\zeta} - \omega_d t - \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta}) \]
\[ = \frac{\tau \omega_d e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t) \]

The unit step response of the current loop model when only the feedback channel has delay can be obtained by combining equation (8) and (9),
\[ c(t) = c_1(t) + c_2(t) \]
\[ = 1 - \frac{e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t + \beta) + \frac{\tau \omega_d e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t) \]
\[ = 1 - \frac{e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t) - e^{-\zeta^2t} \cos(\omega_d t) + \frac{\tau \omega_d e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t) \]
\[ = 1 - \frac{e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \left[ (\zeta - \tau \omega_n) \sin(\omega_d t) + \sqrt{1 - \zeta^2} \cos(\omega_d t) \right] \]
\[ = 1 - \frac{e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \sqrt{(1 - \zeta^2)^2 + (\zeta - \tau \omega_n)^2} \]
\[ \sin(\omega_d t) \cos \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta - \tau \omega_n} + \cos(\omega_d t) \sin \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta - \tau \omega_n} \]
\[ = 1 - \frac{e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \left[ \frac{1}{\tau} - 2 \zeta \omega_n \frac{1}{\tau} + \omega_n^2 \right] \]
\[ \sin(\omega_d t + \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta} + \arctan \frac{\omega_n \sqrt{1 - \zeta^2}}{1 - \zeta^2 \omega_n}) \]
\[ = 1 - \frac{e^{-\zeta^2t}}{\sqrt{1 - \zeta^2}} \frac{1}{\z} \sin(\omega_d t + \beta + \phi) \]

In equation (10),
Overshoot occurs at peak time $T_p$. From the definition of peak time, it can be known that when the system reaches peak time, the derivative of current loop response is 0,

$$\frac{dc(t)}{dt} = [1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1 - \zeta^2}} \frac{l}{z} \sin(\omega_d t + \beta + \phi)]'$$

$$= -\frac{1}{\sqrt{1 - \zeta^2}} \frac{l}{z} \sin(\omega_d t + \beta + \phi)[']$$

$$= -\frac{1}{\sqrt{1 - \zeta^2}} \frac{l}{z} [-\zeta \omega_n e^{-\zeta \omega_n t} \sin(\omega_d t + \beta + \phi) + \omega_d \cos(\omega_d t + \beta + \phi)]$$

$$= \frac{\omega_d l}{z\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} [\sin(\omega_d t + \beta + \phi) \cos \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta} -$$

$$\cos(\omega_d t + \beta + \phi) \sin \arctan \frac{\sqrt{1 - \zeta^2}}{\zeta}]$$

$$= \frac{\omega_d l}{z\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \beta + \phi - \beta)$$

$$= \frac{\omega_d l}{z\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)$$

(11)

According to Equation (11), when equation (11) is equal to 0, $t_p = \frac{\pi - \phi}{\omega_d}$.

It can be seen from the definition of overshoot $\sigma\%$ that overshoot is defined as the percentage of the ratio between the difference between the maximum deviation and the final value (i.e., the stable value) and the final value,

$$\sigma\% = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

(12)

It can be obtained from equation (12),

$$\sigma\% = e^{-\frac{\zeta}{\sqrt{1 - \zeta^2}}(\pi - \phi) \sqrt{\frac{z^2 - 2\zeta \omega_n z + \omega_n^2}{z}}}$$

(13)
According to the requirements of industrial design, the overshoot cannot exceed 5%, and the overshoot is

\[ e^{\frac{\zeta}{\sqrt{1-\zeta^2}} \sqrt{\frac{1}{\zeta^2-2\zeta\omega_n z + \omega_n^2}} z} \leq 0.05 \]  

(14)

Substitute \( z = \frac{1}{T_f} \), \( \zeta = \frac{1}{2\sqrt{2\pi fT_f}} \), \( \omega_n = \frac{2\pi f}{\sqrt{T_f}} \) into Equation (14),

\[ \sqrt{2\pi fT_f} e^{-\frac{1}{8\pi fT_f} (\pi - \text{arctan}(\frac{8\pi fT_f}{1}))} \leq 0.05 \]  

(15)

It can be solved by equation (15),

\[ \omega \cdot T_f = 2\pi f \cdot T_f \leq 0.461 \]  

(16)

3. Simulation and verification

Different current loop models are simulated by MATLAB.

The controlled object adopts permanent magnet synchronous motor. The rotor permanent magnet is a surface-mount structure, and the quadrature-axis inductance and the direct-axis inductance are equal. The specific motor parameters are shown in Table 1.

| Table 1. Motor parameters |
|---------------------------|
| Parameter                | Value       |
|---------------------------|
| Inductance of Quadrature-axis and Direct-axis (H) | 0.000492 |
| Permanent Magnet Flux Linkage (Wb) | 0.0272 |
| Stator Resistance (Ω) | 0.2 |

For the PI regulator, PI parameters are set by means of internal model control, that is \( k_p = 2\pi fL \) and \( k_i = \frac{R}{\pi f} \) is the bandwidth setting value of internal mode control, \( L \) is the inductance parameter of the motor, and \( R \) is the resistance parameter of the motor. Set the ideal current loop bandwidth as 3000Hz under internal mode control.

In general, the switching frequency of power components in the servo control system is 10kHz. For the forward channel, the time between AD sampling and PWM cycle update is the most important delay. For the feedback channel, AD sampling is converted to a major delay.

In engineering practice, overshoot generally do not exceed 5%. The calculation formula of bandwidth and overshoot in reference [5] and [6] can be used to calculate the bandwidth of a current loop with only forward channel delay when the dynamic response is limited. The results are shown in Figure 3.
Figure 3. The available range of current loop bandwidth with only forward channel delay under overshoot constraints

From Equations 4 and 16, the bandwidth of the current loop with only feedback channel delay under the condition of limited dynamic response can be obtained. The results are shown in Figure 4.

Figure 4. The available range of current loop bandwidth with only feedback channel delay under overshoot constraints

4. Conclusion

It can be seen from Figure 3 and Figure 4, no matter the forward channel delay or the feedback channel delay, the transfer function of the system is second-order with resonance, so the current loop bandwidth will increase first and then decrease after reaching the extreme value. In the case that the PI regulator adopts the same parameter setting, the ideal bandwidth setting value is the same, and there is the same 5% overshoot limit, compared with Figure 1 and Figure 2, it can be found that the current loop can get a higher bandwidth when there is only feedback channel delay.

Moreover, the current loop bandwidth extreme value is greater when only the feedback channel delay exists, and the falling speed after reaching the extreme value is lower. It can be inferred that the feedback channel delay has less influence on the current loop bandwidth than the forward channel delay.

Firstly, this paper focuses on the analysis of the feedback channel delay which is less analyzed in the past, the current loop model with only feedback channel delay is obtained. The transfer function of current loop with delay only in feedback channel is a second order equation with zero point, this paper gives the analytical solution of bandwidth and dynamic response parameters in this case.

This article provides theoretical assistance for engineering practice. According to the analysis in this paper, the feedback channel delay has little effect on the current loop bandwidth, so for systems with
high sampling requirements, the feedback channel delay can be appropriately increased to improve the sampling accuracy.

**References**

[1] Li Niu. Research on current loop bandwidth expansion technology of AC permanent magnet synchronous servo system [D]. Harbin University of technology, 2010

[2] Hongjia Wang, Ming Yang, Dianguo Xu, et al. Current loop bandwidth expansion in permanent magnet ac servo system. Chinese journal of electrical engineering, 2010, 30(12).

[3] Guoquan Mei. Research and design of PMSM vector control system [D]. Nanjing University of technology, 2013

[4] Lei Yuan, Bingxin Hu, keyin Wei, et al. Control principle and MATLAB simulation of modern permanent magnet synchronous motor (Beihang University Press, Beijing, 2016), pp. 73-74.

[5] Bao Song, Xiaoqi Tang, Jiankun Wu Parameter setting of servo current regulator based on second-order system. Machinery and electronics, 2004(9).

[6] Yi Ruan, Boshi Chen. Automatic control system for electric power drive: motion control system (China Machine Press, Beijing, 2009), pp. 77-78.