LEVERAGING MATERIALS IN TOPOLOGY OPTIMIZATION

Projection-Based Implicit Modeling Method (PIMM) for Functionally Graded Lattice Optimization

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We propose a projection-based implicit modeling method (PIMM) for functionally graded lattice optimization, which does not require any homogenization techniques. In this method, a parametric projection function is proposed to link the implicit function of functionally graded lattice with the finite element background mesh. To reduce the number of design variables, the radial basis function is utilized to interpolate the implicit design field. The triply periodic minimal surface lattice is employed to demonstrate the proposed method. Several two- and three-dimensional lattice design examples are presented to solve the compliance problems. The proposed PIMM method is flexible and can potentially be extended to design graded irregular porous scaffold and non-periodic lattice infill designs.

INTRODUCTION

Lightweight designs are desirable in many industrial applications, and structural optimization is an effective way to generate lightweight structures. Topology optimization (TO) is an important tool to investigate the optimal design of engineering structures. Because of its importance in engineering designs, this subject has drawn much attention by academia for more than 20 years, and remarkable progress has been made since the pioneering work of Bendsøe and Kikuchi. Although the continuum topology optimization method has already achieved remarkable progress in recent years, there still exist several challenges for conventional density-based methods, such as incorporation of manufacturability constraints. In recent years, several parametric design methods have been reported. To control design complexity in an explicit geometric way, a moving morphable component (MMC) approach has been proposed by Guo et al. Meanwhile, another method, called the moving morphable voids (MMVs) solution framework was also proposed in recent years to achieve explicit topology optimization and geometry control. Based on the MMC or MMV approaches, Guo and co-workers further extended these methods to three-dimensional (3D) problems and further improved this method to solve more complex physical problems such as stress constraint and multi-material problems. Meanwhile, the MMC-based method has also been extended to solve geometric nonlinear problems. Recently, Tortorelli and co-workers proposed a method for continuum-based topology optimization made of discrete elements. This method lies in the framework of density-based topology optimization, and hence standard finite element progress and nonlinear programming algorithm can be applied. Furthermore, Narato et al. developed this method to resolve stress constraint problems, in which the optimized designs are created by assembling discrete geometric components like bars or plates. Lately, Tortorelli extended geometric projection work to three dimensions and designed unit cells for lattice materials based on inverse homogenization, where a negative Poisson’s ratio lattice material is achieved. Recently, Daniel et al. proposed a novel method to represent the density field with a truncated Fourier representation, where the number of design variables are significantly reduced. Du et al. proposed a novel efficient topology optimization based on image compression techniques. In fact, the above methods can be classified as different parametric design methods. A parametric design method is equivalent to finding an appropriate
density field representation methodology to replace the density field in real space in conventional density-based methods.

Cellular materials or lattice structures have been utilized in numerous applications and are common in nature, such as bone, wood, sponge, etc. Recently, these porous materials have been designed to achieve multi-functional materials for weight reduction, energy absorption, or heat transfer.\(^\text{10}\) Lattice structures made of metal are widely applied to product design in the field of orthopedic regenerative medicine.\(^\text{11}\) These include, for example, design of bone scaffolds and implants to replicate the biomechanical properties of host bones. A porous metallic structure is an ideal candidate for repairing or replacing damaged bone because of its tunable mechanical properties. More importantly, porous metals can be designed to be open-celled to promote in-growth of bone tissue, which accelerates the osseo-integration process. Conventional processes can be difficult or impossible for the fabrication of porous media due to the intricate internal architecture. Recent advancements in additive manufacturing (AM) has enabled the fabrication of lattice structures which has significantly increased the demands for implants with a customized mechanical performance.\(^\text{11}\) In fact, open-celled lattice structures are preferred for AM for many reasons, including\(^\text{12}\) (1) inherent porosity can minimize residual stress to reducing printed part distortion, (2) reduce support materials due to self-supporting unit cells, and (3) no enclosed voids so that the powder can be easily removed. Several approaches have been proposed in recent years to generate lattice structures, such as generic ground truss structure approaches, etc.\(^\text{13}\) Triply periodic minimum surfaces (TPMS) are becoming a promising microstructure for designing scaffolds due to their extraordinary mechanical performance. TPMS is composed of 3D continuous smooth surfaces, for which the average curvatures at each surface point is zero. These structures are called biomimetic structures, and are widely found in biological systems in nature, such as butterfly wings. Due to their particular geometric properties, these structures have the advantages of light weight, high strength, and high specific surface area. Another merit of TPMS is that these structures can be defined using implicit equations. In addition, TPMS has already been proven to be a versatile source for biomorphic scaffold designs, as well as providing a viable and stable environment to replace damaged bones because of the smooth bending properties and optimized fluid permeability.\(^\text{14}\) In this paper, we focus on the design of functionally graded lattice structure with TPMS unit cells due to their extraordinary mechanical properties.

TO for designing functionally graded lattice has been a hot topic in recent years. Designs with varying microstructures have attracted great attention and become the active field at the moment. The earliest work for designing functionally graded lattices for AM was described by Brackett et al.\(^\text{15}\) Several efficient topology optimization methods have been proposed based on homogenization theory by assuming that the two scales are separated,\(^\text{16}\) and some algorithms have been developed based on a reduced order modeling technique.\(^\text{17}\) In recent years, some new hierarchical lattice structure design methods have also been proposed. Liu et al.\(^\text{18}\) proposed an optimal structural design method considering symmetry and pattern repetition constraints by combining a parameterized level-set method with a radial basis function (RBF) interpolation strategy, which allows for decoupling RBF knots from the finite element (FE) analysis mesh. A generative design and optimization method for TPMS-based functionally graded cellular structures with homogenization technology have been developed by Li et al.\(^\text{19}\) This work also uses TPMS functions to generate periodic basic lattice structures. Liu et al.\(^\text{20,21}\) proposed an AM-oriented graded lattice structures design method by combining the MMC/MMV explicit topology optimization methods with coordinate perturbation functions, which can generate more general graded lattice structures and has fewer design variables. Xia\(^\text{22}\) has reviewed recent advances in designing multiscale structural modeling and design of nonlinear structures. With recent advances in scale-related periodic design, perfect connectivity between different optimized unit cells is guaranteed.\(^\text{23,24}\) Zong et al.\(^\text{25}\) proposed a variable cutting (VCUT) level set-based topology optimization method to design functionally graded cellular structures, in which a variable and continuous cutting function is described to generate functionally graded material. The VCUT method has been further developed by Liu et al.\(^\text{26}\) and Xia et al.\(^\text{27}\) The VCUT method shares some similarity with the proposed method in this paper, where a cutting plane and an implicit (level-set) representation of the infill are applied. However, we need to mention that the VCUT method is in the framework of level set methods, while the proposed method in this paper is in the framework of density-based methods. In addition, the sensitivity computation and description of the cutting plane are different. Some other advanced multiscale design methods for simultaneous achieving macro- and microscale topology optimization, such as decoupling multiscale analysis, can be found in Refs.\(^\text{28,29}\).

The conventional computer-aided design (CAD) technique creates geometric objects using surfaces, which is an ideal solution for visualization and conventional subtractive manufacturing processes, such as computer numerical controlled machine tools. However, this surface-based shape representation method is not ideal for designs for AM. The AM technology builds an object in a layer-by-layer way, which can be applied to print extremely complex designs such as porous scaffolds or lattice structures. In general, geometric models for AM can
be implemented using voxels, tetrahedra, parametric solids, or implicit field functions\textsuperscript{30} defined in 3D space. However, using voxel points or a set of tetrahedra can be expensive in terms of storage space. Moreover, representation with voxels or tetrahedra provide only an approximation of the real object. For parametric representation, it is extremely tedious and difficult to design lattice structures using constructive solid geometry.\textsuperscript{31} Compared to the above methods, a ready-to-print geometric object is to describe a geometry as a 3D function $F(x, y, z)$, which directly informs the AM machine to determine whether the point $P(x, y, z)$ should be printed. Recent research has shown that geometry modeling using implicit functions is particularly suitable for modeling lattice and porous media. This AM-friendly modeling method has attracted great attention from academia and industry for AM design, and has already been used to model any complex geometry in general. More importantly, this implicit field modeling method for AM has already been commercialized and has achieved great success by a software company called nTopology. The main advantages of implicit modeling are (1) an implicit geometry is directly defined in the physical space, which can directly provide precise information of objects to a 3D printer, (2) implicit modeling is a lightweight geometric modeling technique without requirement of massive storage space, and (3) recent research has shown that implicit functions are particularly suitable for modeling microporous structures.\textsuperscript{30,32} Therefore, using implicit functions to model porous media or lattice scaffolds is a more advanced and feasible approach for future AM-oriented design.

The novelty of the present work is a new framework proposed to combine implicit geometry modeling with a projection-based method to achieve functionally graded lattice designs. The method has more flexibility to design irregular and complex scaffolds compared with a homogenization-based framework, since material periodicity is not required. The paper is organized as follows. In second section, we describe a projection-based implicit modeling method (PIMM) for functionally graded lattice design in detail by using the TPMS structure as an example. Third section presents the formulation of minimum compliance optimization problems, and a sensitivity analysis of PIMM is derived based on the chain rule. Fourth section presents the optimized designs based on the PIMM method. The paper ends with conclusions in fifth section.

**IMPLICIT MODELING FOR FUNCTIONALLY GRADED LATTICE**

**Generation of Functionally Graded Lattices Based on Implicit Modeling**

In general, lattice unit cells can be constructed using a surface-based representation method. Using the TPMS as an example, the structure can be defined by an implicit function (i.e., $f(x, y, z) = t$), where $t$ is the parameter that governs the offset from the level sets, and $t$ can vary in the design domain. There are several different types of TPMS, as described in Ref. 12, and the four typical structures of TPMS\textsuperscript{33} are shown in Fig. 1. The TPMS can be utilized to create lattice structures with unique mechanical characteristics. Furthermore, the lattice structures generated by TPMS have a higher surface-to-volume ratio compared with traditional strut-based lattice structures.\textsuperscript{34} The gyroid is one of the most popular TPMS with a robust mechanical performance. In this paper, we focus on the lattice design based on the gyroid minimum surface.

In general, TPMS can be constructed using inequality conditions expressed as\textsuperscript{12}:

$$f(x, y, z)^2 \leq t(t > 0)$$  \hfill (1)

The control equation for the gyroid surface is

$$f_G(x, y, z) = \sin(\lambda_x x) \cdot \cos(\lambda_y y) + \sin(\lambda_y y) \cdot \cos(\lambda_z z) + \sin(\lambda_z z) \cdot \cos(\lambda_x x)$$  \hfill (2)

where $\lambda_i(i = x, y, z)$ is the function periodicity, expressed as:

$$\lambda_i = \frac{2\pi}{L_i} \quad \text{(with } i = x, y, z)$$  \hfill (3)

In Eqs. 2 and 3, $L_i$ is the absolute dimension which defines the length of a unit cell. To design functionally graded lattice structures, the material grading in 3D space can be realized by operating through 4D representation $(x, y, z, t)$, where $t$ is an iso-value matrix in the $(x, y, z)$ space. Therefore, a functionally graded lattice can be represented in an implicit way as follows:

$$f(x, y, z)^2 \leq t(x, y, z)$$  \hfill (4)

In Eq. 4, $(x, y, z)$ controls the spatial variation of unit cell volume fraction in 3D Cartesian space. Therefore, designing a functionally graded TPMS lattice is equal to varying the variable $t(x, y, z)$ in the design domain. The design domain for $t(x, y, z)$ in Cartesian space can be discretized by voxels. The continuous function $t(x, y, z)$ in space can be reconstructed through field values at every voxel. In fact, the periodicity, $\lambda_i$, can also be varied in 3D Cartesian space. However, the periodicity is fixed in the design domain for the present work.

For conventional density-based topology optimization, the design domain is discretized by a finite element (FE) mesh, in which each element works as a design variable. To connect the implicit field with the density-based method, a Heaviside function-based projection method is implemented to map the implicit field to the background FE mesh,
which enables a topology optimization algorithm to be performed on a fixed grid. An approximate Heaviside function can be defined as:

$$ H_{a,c}(x) = \frac{1}{1 + e^{-a(x-c)}} $$  \hspace{1cm} (5)

where $a$ and $c$ are two parameters that can control the shape of the Heaviside function. A projection from the parametric design space $t(x,y,z)$ to the density field $\rho$ can be expressed as:

$$ \rho(x,y,z) = H_{a_{0},c_{0}}(t(x,y,z)) \cdot H_{a_{1},c_{1}}(t(x,y,z) - f(x,y,z)^{2}) $$  \hspace{1cm} (6)

where $H_{a_{0},c_{0}}$ and $H_{a_{1},c_{1}}$ are Heaviside functions defined in Eq. 5 with different control parameters. Note that the $t(x,y,z)$ is closely related to the volume fraction of the unit cell. Thus, the first term is used to control the volume fraction of the gyroid lattice. If the volume fraction at point $(x,y,z)$ is a small value, the first term will tend to zero so that the material at this point can be removed. The second term works as a projection to map the gyroid lattice to the density field. Hence, Eq. 6 is capable of mapping the design space $t(x,y,z)$ to the density field $\rho(x,y,z)$. In practice, the design space $t(x,y,z)$ is a continuous differentiable function in the design domain. To effectively construct the implicit field $t(x,y,z)$ in the entire design domain with a single globally continuous and differentiable function, the RBFs are introduced here to model the implicit field $t(x,y,z)$. RBFs are able to interpolate scattered data to generate a smooth surface, and are an effective way to approximate complex functions. They are radially symmetric functions centered at a specific point, called an RBF knot, which can be expressed as follows:

$$ \phi_{i}(x) = \varphi(\|x-x_{i}\|) $$  \hspace{1cm} (7)

where $\|\|$ denotes the Euclidean norm and $x_{i}$ is the position of the knot. There are several possible RBFs, including thin-plate spline, Gaussian, etc.

In this paper, The Gaussian function has been chosen to work as the RBF kernel, where the explicit form of the Gaussian function is expressed as follows:

$$ \varphi(\|x-x_{i}\|) = e^{-\left(\frac{\|x-x_{i}\|}{\epsilon}\right)^{2}} $$  \hspace{1cm} (8)

where $\epsilon$ is a parameter to control the shape of the Gaussian function. The implicit function $t(x)$ in the design domain can be interpolated via the RBF functions as:

$$ t(x) = \sum_{i=1}^{N} z_{i}\phi_{i}(x) $$  \hspace{1cm} (9)

where $z_{i}$ is the expansion coefficient of the RBF positioned at the $i$th knot. The above equations can be rewritten as

$$ t(x) = \phi^{T}(x)z $$  \hspace{1cm} (10)

where

$$ \phi(x) = [\phi_{1}(x), \phi_{2}(x), \ldots, \phi_{N}(x)]^{T}, z = [z_{1}, z_{2}, \ldots, z_{N}]^{T} $$  \hspace{1cm} (11)

Using the RBFs to model the implicit function $t(x,y,z)$, we have

$$ t(x,y,z) = \phi^{T}(x,y,z)z_{t} $$  \hspace{1cm} (12)

where $z_{t}$ is the design variable, which directly determines the implicit function $t(x,y,z)$. The relationship between the RBF knot and the density is illustrated in Fig. 2. While each density point is located at the center of each element of the FE mesh, the locations of the RBF knots and density points can be independently chosen and do not need to coincide with each other. In our proposed scheme, the element densities are obtained through the projection function defined in Eq. 6, where the implicit field $t(x,y,z)$ is constructed using the RBF
knot through Eq. 12. Therefore, the density field
\( \rho(x, y, z) \) can be constructed via the following expression:

\[
\rho(x, y, z) = H_{a_0, e_0}(\phi^T(x, y, z)\mathbf{x}_t) \\
\cdot H_{a_1, e_1}(\phi^T(x, y, z)\mathbf{x}_t - f(x, y, z)^2)
\]

(13)

Note that the coordinate of \((x, y, z)\) should be normalized accordingly to the range of \([0,1]^3\) for optimization purposes.

**Comparison with Homogenization-Based Lattice Design**

Homogenization-based functionally graded lattice design has become a popular method in recent years.\(^{37,38}\) The general procedure of this method is as follows: (a) compute the effective mechanical properties of the unit cell by the asymptotic homogenization (AH) method;\(^{39}\) (b) use the conventional density-based TO method with effective material properties computed based on AH, and (3) complete the lattice reconstruction based on density optimized results, where the volume fraction of each unit cell is directly determined by material density distribution from the density-based TO method.

Compared with homogenization-based topology optimization, the pros and cons of the proposed method are as follows:

1. The AH method is effective when the unit cell is sufficiently small in size compared to the macrostructure, while the proposed method is not limited by unit cell size.
2. Several advanced homogenization-based lattice design methods have been proposed in recent years,\(^{46}\) where the configuration of the single unit cell is allowed to be optimized and varied in space. However, the microscale topology is fixed in these methods.
3. The homogenization-based design method is not feasible for irregular porous scaffold designs,\(^{41}\) but is widely used and preferred for tissue engineering. For example, the Voronoi foam design,\(^{42}\) which does not possess any periodicity in space, cannot be designed using homogenization-based methods. However, because the irregular porous scaffold can be described by the implicit field,\(^ {42}\) the projection-based method proposed in this work can be readily employed in such situations.
4. The computational cost of the proposed method is much higher than homogenization-based methods, especially when the length scale of unit cell is small. The proposed method becomes practical for lattice design where the unit cell size is on the same order with the macroscopic structural feature size, while homogenization-based methods are preferred in situations where the length scale of unit cell is relatively small.
5. Employing the RBF mapping method to describe the design domain increases the computational cost and implementation complexity compared with the standard homogenization-based method.

The PIMM method can be classified as a non-homogenization method for lattice design based on implicit modeling.

**TOPOLOGY OPTIMIZATION FORMULATION BASED ON IMPLICIT MODELING**

**Minimum Compliance**

In this section, the implicit modeling method described in the previous section is utilized to develop the TO formulation of compliance minimization.\(^ {43}\) The density field is controlled by RBF knots in the design domain. Hence, the TO will iteratively optimize a functionally graded lattice through updating the RBF knots in the design domain until the design achieves the optimal stiffness. Here, the RBF knots are defined as the design variables for evolving the true density field in the design domain during the optimization. Thus, the optimization problem can be expressed as:

\[
\begin{align*}
\text{Find : } & \mathbf{z}_t \\
\text{Min: } & C(\mathbf{u}, \mathbf{z}_t) = \frac{1}{2} \int_{\Omega} \varepsilon(\mathbf{u})^T \mathbf{D}(\rho(\mathbf{z}_t)) \varepsilon(\mathbf{u}) \, d\Omega \\
\text{s.t. : } & \frac{1}{V_{\text{prescribe}}} \int_{\Omega} \rho(\mathbf{z}_t) \, d\Omega - V_{\text{prescribe}} \leq 0
\end{align*}
\]

(14)

where \( C \) is the objective function defined by the structural compliance, \( \mathbf{z}_t \) is the weight vector of the RBF knots in the design space, \( \rho \) is the density distribution in the design domain, \( \Omega \), and \( V_{\text{prescribe}} \) is the prescribed volume fraction. In the finite element model, \( \mathbf{u} \) is the unknown displacement field, \( \varepsilon \) is the strain, and \( \mathbf{D} \) is the elastic tensor matrix.
Design Sensitivity Analysis Based on the Chain Rule

To obtain the sensitivity of objective function with respect to weights of RBF knots, the chain rule is employed. The adjoint method\(^44\) is applied to obtain the sensitivity with respect to the density field \(\rho\):

\[
\frac{\partial C}{\partial \rho} = \gamma^T \frac{\partial K}{\partial \rho} u
\]

where \(\gamma\) is the adjoint vector computed from the adjoint equation \(K\gamma = -f\), and \(K\) is the assembled stiffness matrix (see Ref. 43). According to the chain rule, the sensitivity of objective \(C\) with respect to design variables, \(\mathbf{x}_i\), can be expressed as:

\[
\frac{\partial C}{\partial \mathbf{x}_i} = \sum_{i=1}^{\text{nele}} \frac{\partial C}{\partial \rho_i} \frac{\partial \rho_i}{\partial \mathbf{x}_i}
\]

where nele is the total number of elements, and \(\rho_i\) denotes the density of the \(i\)th element. We simply describe the derivation of density, \(\rho\), with respect to RBF knots, \(\mathbf{x}_i\), as follows:

\[
\begin{align*}
\frac{\partial \rho}{\partial \mathbf{x}_i} &= H'_{a_0,c_0}(\phi^T(x,y,z)\mathbf{x}_i) \cdot \phi(x,y,z) \\
&\quad \cdot H_{a_1,c_1}(\phi^T(x,y,z)\mathbf{x}_i - f(x,y,z)^2) \\
&\quad + H'_{a_2,c_2}(\phi^T(x,y,z)\mathbf{x}_i) \\
&\quad \cdot H_{a_1,c_1}(\phi^T(x,y,z)\mathbf{x}_i - f(x,y,z)^2) \cdot \phi(x,y,z)
\end{align*}
\]

where \(H'_{a_0,c_0}()\) and \(H'_{a_1,c_1}()\) denote the first derivative of \(H_{a_0,c_0}\) and \(H_{a_1,c_1}\). The explicit formulation of \(H'_{a_0,c_0}()\) and \(H'_{a_1,c_1}()\) can be readily obtained using a symbolic differentiation system, which is available in a build-in module in MATLAB\(^45\).

NUMERICAL EXAMPLES AND DISCUSSION

This section, several 2D and 3D numerical examples demonstrated in detail for designing functionally graded gyroid lattice structures. The classic MBB beam in two dimensions is first investigated to demonstrate the effectiveness of the proposed implicit modeling method for lattice design. The parameters for all the numerical examples are chosen as: \(a_0 = 50, c_0 = 0.2, a_1 = 500, c_1 = 0\), and \(\epsilon = 0.1\). The method of moving asymptotes (MMA)\(^46\) is applied to solve the optimization problem. The number of RBF knots in each direction are chosen based on our experience, with fewer RBF knots resulting in a simpler topology shape, while more RBF knots will inevitable increase the computational cost. In this paper, ten RBF knots along each direction are implemented in all the numerical examples. Note that, for 2D problems, the implicit function for describing the lattice structure \(f_{2D}(x,y,z)\) is chosen as follows:

\[
f_{2D}(x,y) = \sin(\lambda x) \cdot \cos(\lambda y)
\]

Compliance Optimization for MBB Design

The MBB beam\(^47\) is a popular test and benchmark problem in topology optimization. The symmetry is used for the design, and the right half of the beam is modeled. The design of the MBB beam with the loading and boundary conditions is illustrated in Fig. 3a. The design domain is uniformly meshed by \(200 \times 200\) elements with unit length. The prescribed volume fraction is set as 30%. The elastic constants are chosen as follows: elastic modulus \(E = 1\) and Poisson’s ratio \(\mu = 0.3\). The \(10 \times 10\) uniformly distributed RBF knots are generated along two directions. The initial weights of the RBF knots are chosen as 0.1. The periodicity parameters are selected as: \(L_x = 1\) and \(L_y = 0.1\). The optimized design is plotted in Fig. 3b. As shown in the optimal results, the method proposed in this paper is able to generate functionally graded lattice infill structures.

Compliance Optimization for Three-Dimensional Cantilever Beam Design

In this section, a 3D cantilever beam example is presented for compliance optimization. The cantilever beam is modeled by a \(200 \times 100 \times 60\) hexahedral mesh, and the dimensions of the design are shown in Fig. 4a. A uniform line force, \(F = 1\), is applied on the bottom-right of the rectangle domain. The \(10 \times 10 \times 10\) uniformly distributed RBF knots in the design domain are as shown in Fig. 4b. Note that \(10 \times 10 \times 10\) uniformly distributed RBF knots applied here are only for implementation convenience, which will inevitably result in a difference in length scale in the optimization if a different distribution is used. To obtain a cubic unit cell, the number of knots in every direction should be proportional to the mesh number in each direction. However, it is worth mentioning that our method is applicable to a rectangle unit cell, which is generally not preferred for lattice optimization based on the homogenization model. Note that the left side of rectangle is fixed. The elastic constants are chosen as follows: elastic modulus, \(E = 1\), and Poisson’s ratio, \(\mu = 0.3\). The initial weights of the RBF knots are chosen as 0.1, and the periodicity parameters are selected as: \(L_x = 0.1, L_y = 0.1,\) and \(L_z = 0.1\). The optimization converges after 30 iterations as presented in Fig. 5b. The optimized lattice infill result is presented in Fig. 5.

Compliance Optimization for 3D Bracket Design

In this section, a 3D bracket design example is presented for compliance optimization. The four
corners are constrained by the planar joint with a point load, $F = 1$, at the center, as shown in Fig. 6a. The 3D bracket is modeled by a $160 \times 160 \times 80$ hexahedral mesh, and the dimensions of the design are shown in Fig. 6a. The $10 \times 10 \times 10$ uniformly distributed RBF knots in the design domain are as shown in Fig. 6b. The initial weights of the RBF knots are chosen as 0.1, and the periodicity parameters are selected as: $L_x = 0.1$, $L_y = 0.1$, and $L_z = 0.1$. The elastic constants are chosen as follows:

Fig 3. (a) MBB beam example, (b) optimal lattice infill design.

Fig 4. Three-dimensional cantilever beam.

Fig 5. Optimal lattice infill design and convergence history.
elastic modulus, $E = 1$, and Poisson’s ratio, $\mu = 0.3$. Actually, the method proposed in this work is able to produce shape-preserving results which are preferred for AM, because 0–1 topology optimization designs can sometimes not be manufactured, such as overhangs, as support structures are needed beneath them. Furthermore, removing support structures is time-consuming and requires additional post-processing. To produce a shape-preserving design, the value of parameter $c_0$ in Eq. 6 is set to be $c_0 = 0$. To make a comparison, two distinct optimization results with different values for parameter $c_0$ are demonstrated in Fig. 7, and the convergence history for the two different designs is presented in Fig. 8. The optimized compliance values are close for these two different designs. As shown in Fig. 7c and d, the optimized lattice structure is able to maintain the initial geometry configuration, where varied density lattice structures are generated, and materials tend to concentrate on the four corners and the loading point, and no block materials are removed. This shape-preserving result is preferred for the design of complex domains, where no overhang constraints are needed if the initial design domain is self-supporting.

**CONCLUSION**

A new projection-based algorithm based on an implicit field for gyroid lattice design has been proposed and demonstrated in detail. The PIMM algorithm is able to design a functionally graded lattice without the need for any homogenization. Thus, the lattice design based on this method is not
limited to periodic structures, and can be extended to irregular porous scaffold designs. This point will be demonstrated and verified in the future. The unit cell size for lattice design can be large and is not limited by size effects (a homogenization necessary condition\cite{49}), which is preferred for AM. Now that the geometry is defined by the implicit function, the geometry information is far less than in feature-based geometry modeling \cite{50}, which is sometimes extremely tedious for modeling porous media or lattice structures, and the data communication between the implicit field with AM systems is well-addressed by Ref. \cite{30}.

CONFLICT OF INTEREST

On behalf of all authors, the corresponding author states that there are no conflicts of interest.

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