Correlation-hole induced paired quantum Hall states in lowest Landau level

Yuan-Ming Lu,1,2 Yue Yu,1 and Ziqiang Wang2,1

1Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China
2Department of Physics, Boston College, Chestnut Hill, MA 02467

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A theory is developed for the paired even-denominator fractional quantum Hall states in the lowest Landau level. We show that electrons bind to quantized vortices to form composite fermions, interacting through an exact instantaneous interaction that favors chiral p-wave pairing. There are two canonically dual pairing gap functions related by the bosonic Laughlin wavefunction (Jastrow factor) due to the correlation holes. We find that the ground state is the Moore-Read pfaffian in the long wavelength limit for weak Coulomb interactions, a new pfaffian with an oscillatory pairing function for intermediate interactions, and a Read-Rezayi composite Fermi liquid beyond a critical interaction strength. Our findings are consistent with recent experimental observations of the 1/2 and 1/4 fractional quantum Hall effects in asymmetric wide quantum wells.

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The fractional quantum Hall effect (FQHE) observed at Landau level filling fraction $\nu = 5/2$ [1] signifies a new state of correlated electrons. This state is believed to be described by the Moore-Read pfaffian (MRP) [2] and supports fractionalized quasiparticle excitations [3] that obey nonabelian statistics relevant for topological quantum computing [4]. An outstanding question has been whether such nonabelian topological phases exist in the lowest Landau level (LLL). Several recent experiments [5,7] indeed observed FQHE at $\nu = 1/2$ and 1/4, suggesting that these, too, may be in the MRP phase. Although the abelian two-component Halperin (331) and (553) states [8] can be strong contenders for these FQHE states [9], fresh experiments and numerical studies found strong evidence for the one-component FQHE at $\nu = 1/2$ and 1/4 in asymmetric wide quantum wells [7,10]. Whether the observed FQHE can be understood as pfaffians in the LLL is the focus of this work.

The MRP is a chiral p-wave paired quantum Hall state [11]. In principle, it can emerge as a p-wave pairing instability of the composite Fermi liquid (CFL), a gapless state of electrons attached to flux tubes [13]. The leading-order statistical interaction mediated by the Chern-Simons (CS) gauge field fluctuations can produce a p-wave pairing potential for the composite fermion (CF). However, since the coupling between the CF and the CS gauge field is not small, diagrammatic perturbation theory is not controllable. Within the random-phase approximation, the gauge fluctuations are in fact singular and pair-breaking [15]. Therefore, the ground states at filling fractions 1/2 and 1/4 remained enigmatic [16].

The key to solve this problem is to properly account for the effects of the correlation hole, i.e. the local charge depletion caused by attaching flux to an electron. A CF feels the correlation hole of the other CFs, which is captured by the Jastrow factor in the Laughlin wavefunction. In the unitary CF theory [12,13], only an infinitely thin flux tube associated with a U(1) phase is attached to each electron without accounting for the Jastrow factor. Read improved the concept of CF by attaching finite size vortices to electrons [17,18] such that the Jastrow factor naturally appears in the ground state wavefunction. Binding zeros of the wavefunction to electrons keeps them apart and lowers the Coulomb energy. The vortex attachment can be achieved through a non-unitary but nevertheless canonical transformation on the electron operators [19]. The saddle point solutions of such a non-unitary CF (NUCF) field theory recover the Laughlin state for the odd denominator FQHE [19] and the Rezayi-Read CFL at $\nu = 1/2$ [20]. The effective interaction induced by the vortex attachment has also been studied at $\nu = 1/2$ [21].

In this paper, we show that paired quantum Hall states emerge in the LLL using the NUCF field theory where a vortex with vorticity $\tilde{\phi}$ (with $\tilde{\phi}$ even integer) is attached to an electron at filling fraction $\nu = 1/\tilde{\phi}$. An important feature of attaching vortices to electrons is that the diamagnetic coupling, quadratic in the gauge field, to the CF density is canceled by its dual contribution from the correlation hole associated with the vortex. As a result, we show that the gauge fluctuations can be integrated out exactly, leading to an instantaneous statistical interaction between the NUCFs which is attractive for chiral $p_x - ip_y$ pairing [21] and scales linearly with the vorticity $\tilde{\phi}$. We construct the variational ground state wavefunction for such a correlation-hole induced paired quantum Hall state, introducing two canonically dual gap functions related by the radial distribution function of the corresponding bosonic Laughlin wavefunction. Variational calculations are carried out in the presence of the pair-breaking Coulomb interaction $V_c(r) = e^2/4\pi\epsilon_0 r, \epsilon_0 = \lambda^2 V_c(\ell_B) h\omega_c$, where $\ell_B$ is the magnetic length and $\lambda$ is a dimensionless coupling constant. We find that for weak Coulomb interaction $\lambda < \lambda_c$, the ground state is a MRP in the long wavelength limit. However, the pairing function deviates significantly from
that of the MRP at shorter distances, consistent with recent numeric studies in the projected LLL [22]. Remarkably, we find that for intermediate Coulomb interactions, $\lambda_{c1} < \lambda < \lambda_{c2}$, the paired state is different from the MRP even in the asymptotic long wavelength limit. The pairing function in this new phase is oscillatory with its amplitude decaying as the inverse square root of the distance. For sufficiently strong Coulomb interactions $\lambda > \lambda_{c2}$, the paired state becomes unstable and the ground state undergoes a transition to the Rezayi-Read CFL [18] phase. The topological properties and the effect of a short-ranged interaction are also studied.

We consider 2D spin-polarized interacting electrons described by the field operator $\psi_e$ in a uniform perpendicular magnetic field $B$. The electron density is $n_e$ and the density operator $\rho = \psi_e^\dagger \psi_e$. The vortex attachment is through the following non-unitary transformation [19]:

$$\Phi(r) = e^{-iJ_\phi(r)}\psi_e(r), \quad \Pi(r) = \psi_e^\dagger(r)e^{iJ_\phi(r)}, \quad \Psi(r) = \sqrt{\frac{\nu e}{\nu \phi e B}} \rho(r),$$

$$J_\phi(r) = I_\phi(r) - \frac{|z|^2}{4e}, \quad e_i = \sqrt{\frac{\nu e}{\nu \phi e B}}, \text{ and}$$

$$I_\phi(r) = \frac{\nu c}{\nu \phi e B} \int d^2r' \rho(r') \log(z - z'), \quad z = x + iy.$$  

We set $\hbar = |e|/c = 1$ hereafter. The imaginary part of $I_\phi$ coincides with the unitary CS transformation, while its real part describes the finite vortex core (correlation hole) accompanying the flux attachment. Note that the fields $\Phi$ and $\Pi$ are not hermitian conjugate, $\Phi = \Phi e^{iJ_\phi}$. However, they form a pair of canonical fields obeying fermion anti-commutation relations; the operator $\Pi$ creates a NUCF while $\Phi$ annihilates one and $\rho = \Phi\Pi$. The transformed Hamiltonian reads

$$H^{CF} = \frac{1}{2m_e} \int d^2r \Pi(r) [-i \nabla - (A - v_\phi)]^2 \Phi(r)$$

$$+ \frac{1}{2} \int d^2r d^2r' \delta \rho(r) \delta \rho(r') V_\phi(r - r') \delta \rho(r')$$

$$\text{where } V_\phi(r) = -i \nabla J_\phi = a(\bar{z}) + i\bar{n} \times a(r) - i B \bar{n} \text{ with } \bar{n} \text{ normal to the 2D plane and the CS gauge field is}$$

$$a(r) = \frac{e}{\nu c} \int d^2r' \rho(r') \text{Im} \log(z - z').$$

The statistical magnetic field $b = \nabla \times a = 2\pi e \phi_B$. One of the physical justifications to introduce such a NUCF field theory lies in the fact that the resulting mean-field states give rise to numerically well-tested wavefunctions [19, 20]. At the mean field level, one takes gauge field $a$ to be a classical one determined by [33] with a uniform density $\rho(r) = n_e$, and $\bar{a}(r) = -\nu e B \bar{n} \times r$. Thus, the mean-field theory describes free NUCFs in an effective magnetic field $\Delta B = B - \nu \phi B = \nabla \times \Delta A$ with $\Delta A = A - \bar{a}$. At even-denominator filling fractions $\nu = 1/\phi$, $\ell_0 = \ell_B$ and the effective $\Delta B = 0$, the mean-field ground state is the Rezayi-Read CFL [18].

An important, non-perturbative feature of this NUCF theory is that the usual diamagnetic fermion density-gauge field coupling of the form $\delta \mathbf{a} \cdot \mathbf{A}$, where $\delta \mathbf{a} = \mathbf{a} - \bar{\mathbf{a}}$, is canceled in Eq. (2) by the contribution from the correlation holes since $(\delta \mathbf{a} \pm i \bar{\mathbf{n}} \times \delta \mathbf{a})^2 = 0$. As a result, the gauge fluctuations in the CS action can be exactly integrated out to obtain a closed-form effective Hamiltonian:

$$H^{CF} = \left(\xi_k + \frac{\omega_e}{2}\right) \Pi_k \Phi_k$$

$$+ \frac{\pi \phi}{m_b} \sum_{q \neq 0} \frac{k + p}{q} \Pi_{q-k} \Pi_{q-p} \Phi_{-p} \Phi_k + \sum_{q \neq 0} \frac{e^2}{4} \Pi_{q-k} \Phi_k \Pi_{q-p} \Phi_{-p} \Phi_k.$$  

Here $\xi_k = \frac{\hbar^2 k^2}{2m_b}$ and $\omega_e = |B|/m_b$ is the cyclotron frequency. The second term is the induced instantaneous statistical interaction written in terms of the complex momenta $k = k_x + ik_y$ (similarly for $p$, $q$). For $k = p$, it reduces to a singular pairing interaction that scatters a pair of NUCFs with zero center-of-mass momentum from $(k, -k)$ to $(k', -k')$ with momentum transfer $q = k - k'$. Expanding in the angular-momentum channels ($l$):

$$\frac{1}{2} \left| \frac{k + p}{k - k'} \right| = 1 + \sum_{l \neq 0} \text{sign}(l) \left| \frac{k'}{k} \right| e^{i l \theta_{k'k}} \theta(1 - \left| \frac{k'}{k} \right|^{|l|},$$

where $\theta_{k'k} = \arg(k') - \arg(k)$, we see that the pairing potential is attractive for $l < 0$ with dominant $p_x - ip_y$ scattering. The Coulomb interaction in Eq. (4), where $\nu = e^2/2\epsilon_0 |q|$, is purely repulsive in all channels. In the absence of Coulomb interaction, it can be shown that the MRP, being an antiholomorphic function, is an exact ground state of the NUCF Hamiltonian.

To study the variational ground state of Hamiltonian [43], we generalize the BCS theory to the non-unitary case. Introduce the Dirac ket and bra

$$|G^{(l)}\rangle \equiv e^{\frac{1}{2} \sum_k |g_k^{(l)}|^2 \Pi_k \Pi_{-k}} |0\rangle, \quad \langle G^{(l)}| \equiv \langle 0| e^{\frac{1}{2} \sum_k |g_k^{(l)}|^2 \Phi_{-k}}, 

The corresponding electron wavefunction is $\Psi_e (\{r_i\}) = \text{Pf} \left[ g^{(l)}(r_i, r_j) \right] \Pi_{l \neq 0} (z_i - z_j)^l e^{-\sum_l |z_i|^2 l^2 / 4}$. Here the pairing function $g^{(l)}$ is an eigenfunction of the angular momentum $L_z = 1 \hbar (k_x \delta_{k_z} - k_x \delta_{k_z}) = -1 \hbar \delta_{k_z}$, i.e. $g^{(l)} = e^{i l \theta_k} R_k$ with $R_k = R(|k|)$ a real function of $|k|$. The parity of the pairing function must be odd for spin-polarized fermions, i.e. $g^{(l)} = -g^{(l)}$. Thus $l$ must be an odd integer. The expectation value of an operator $\hat{O}$ is given by $\langle \hat{O} \rangle = \langle G^{(l)}| \hat{O} |G^{(l)}\rangle / \langle G^{(l)}|G^{(l)}\rangle$.

It is important to note that $g^{(l)}$ is not independent of $g^{(l)}$ in the physical Hilbert space. The hermiticity of electron pairing,

$$\langle \psi_e^\dagger (r + x) \psi_e^\dagger (x) \rangle = \langle \psi_e^\dagger (x) \psi_e (r + x) \rangle^*$$
implies, through relations (1), the following constraint:

\[ \langle \Pi(\mathbf{r} + \mathbf{x})\Pi(\mathbf{x}) \rangle \approx (g_\phi^2(r)\langle \Phi(\mathbf{x})\Phi(\mathbf{r} + \mathbf{x}) \rangle)^* \]  

(5)

where \( g_\phi^2(r) \) is, remarkably, the radial distribution function of the bosonic Laughlin wavefunction \( \Psi_{\Phi}(\{ \mathbf{r}_i \}) \equiv \prod_{i<j}(z_i - z_j)^{\phi}e^{-\sum |z_i|^2/4} \) which is shown in Fig. 1(a) for \( \phi = 2, 4, 6 \). This constraint describes mathematically how one NUCF feels the vortices (correlation holes) around the others. Consequently, the two gap functions \( \Delta_{k} = -\sum_{k'} V(k, k')\langle \Pi_{-k'}\Pi_{k'} \rangle \) and \( \Delta_{k'} = -\sum_{k} V(k', k)\langle \Phi_{-k}\Phi_{k} \rangle \) are related through the correlation holes. We find,

\[ \Delta_{k} = \Delta_{k'} + E_k \int_0^\infty \frac{\Delta_{k'}}{E_{k'}} \mathcal{H}_k(k', k')k' dk' \]  

(6)

where \( \mathcal{H}_k(k', k') \equiv \frac{1}{(2\pi)^2} \int_0^{2\pi} d\theta_{k'k'} e^{-i\theta_{k'k'}} h_{\phi}(|k - k'|) \) and \( h_{\phi}(|q|) \) is the Fourier transform of the pair correlation function defined as \( h_{\phi}(r) \equiv g_\phi^2(r) - 1 \).

Minimizing the ground state energy \( E^{(l)} = \langle \hat{H}_{\text{eff}}^{(l)} \rangle \) with respect to \( R_k \) and \( R_{k'} \), we obtain the self-consistent Bogoliubov-de Gennes (B-dG) equations,

\[ R_k = \frac{E_k - \xi_k}{\Delta_k} \quad \Delta_k = \frac{\Delta_{k'} - \xi_k'}{E_k + \xi_k} \quad R_{k'} = \frac{E_k - \xi_k}{\Delta_{k'}} \quad \Delta_{k'} = \frac{\Delta_k - \xi_k'}{E_k + \xi_k} \]  

(7)

to the Coulomb interaction in the particle-hole channel where \( \xi_k^{PH} = -\frac{2e^2}{m}\int_0^\infty n_k'k'dk'|M_k^{(0)}(|k'|) + M_{k'}^{(0)}(|k|) \) is given by \( |k - k'|^{-1} = \sum_{l} M_{k|k'}^{(l)}e^{i\theta_{l}} \). The dimensionless interacting strength \( \lambda \) measures the ratio of the Coulomb interaction strength to the Fermi energy: \( \lambda = \frac{V_c(R_k^{-1})/4\epsilon_F}{\pi e^2/\epsilon} \) with \( k_F = \sqrt{2}/\ell_B \) the Fermi wavevector. The momentum distribution is \( n_k = (E_k - \xi_k^2)/2E_k \).

It is important to stress that the relation (6) between the two canonical conjugate gap functions projects the non-unitary theory onto the physical Hilbert space. Assuming \( \Delta_k = \Delta_k^{\pm} \) would violate this constraint and fail to capture the correlation-hole effects. One can show that, if \( \Delta_k = \Delta_k^{\pm} \) is assumed, the solution of the BdG equation (7) is \( \Delta_k = \Delta_k^{\pm} = 0 \) for all \( k \).

We solved the BdG equations (7) under the constraint (6) for possible values of \( l \) and found that the leading pairing instability has indeed \( l = -1 \), i.e. \( p_x - ip_y \) wave symmetry, which will be our focus. For weak Coulomb interactions \( \lambda < \lambda_0 \), the two conjugate gap functions are in-phase, i.e. \( \Delta_k \Delta_{k'} > 0 \) and \( R_k \epsilon_k > 0 \) for all \( k \), as shown in Fig. 1(b). In this case, the asymptotic solutions can be obtained analytically in the long wavelength limit: \( \Delta_k, \Delta_{k'} \propto |k| \to 0 \). Thus, the pairing function \( g_k^{-1} \propto 1/k \) and the paired state is asymptotically a MRP. To study the paired state quantitatively for all \( k \), we numerically solve for the gap functions using an ultraviolet cutoff in momentum space, e.g., \( |k| \leq \Lambda = 1.4k_F \). It is clear from Fig. 1(b) that \( 1/R_k \) deviates significantly from the linear behavior such that the wavefunction of the paired state \( \Psi_{\Phi}(\{ \mathbf{r}_i \}) \) is different from a MRP at shorter distances \( |\mathbf{r}_i - \mathbf{r}_j| \leq \ell_B \).

Remarkably, a completely new paired state emerges for intermediate Coulomb interactions. When \( \lambda > \lambda_0 \), the gap function \( \Delta_k \) changes sign at \( k_0 \) where \( R_k \) is singular as shown in Fig. 1(c) and Fig. 2(a,b). This singularity causes the pairing function \( g_k^{-1}(r) \) to oscillate in real space as shown in Fig. 1(d). We find that in the long-wavelength limit its amplitude decays as \( 1/\sqrt{r} \) according to \( g_k^{-1}(r) \sim (\sqrt{|z|/z}) \cos(k_0|z| - \pi/4) \). Despite the sign change in the gap function \( \Delta_k \), this oscillatory pfaffian state (OPS) remains fully gapped and topologically stable with quasiparticle dispersions shown in Figs. 2(a,b). The topological winding number associated with the mapping, via the pairing function \( g_k \), from a compactified complex plane \( k \equiv k_x + ik_y \) to the two-sphere \( \tilde{k} \equiv (2 R_{k_0}, 2 \text{Im} g_k, 1 - |g_k|^2)/(1 + |g_k|^2) \) is one, generic of a chiral \( p \)-wave paired state. This state can be detected by spectroscopy measurements since the singularity at \( k_0 \) produces a kink in the quasiparticle density of states that moves toward the Fermi level as \( k_0 \) approaches \( k_F \) with increasing \( \lambda \). For \( \lambda > \lambda_2 \), the paired state is destroyed and the Rezayi-Read CFL becomes the variational ground state. This quantum phase transition
is signaled by the closing of the quasiparticle excitation gap $\Delta_{\text{eff}} \equiv \min_k \{ E_k \}$ shown in Fig. 2(a,b) near the transition. We find that both $\lambda_1$ and $\lambda_2$ increase monotonically with $\tilde{\nu} = \nu^{-1}$, as shown in Fig. 2(d).

Since the interaction at short distances is reduced efficiently by the correlation-hole, the stability of the CFL against pairing must rely on the long-range part of the Coulomb interaction. As a result, the effects of screening become important. To illustrate this, we consider the 3D screened Coulomb interaction of the Yukawa form: $V_{sc}(r) = V_\kappa(r) e^{-\kappa r}$, where $\kappa$ is the inverse screening length. Fig. 2(c) shows that a CFL stabilized by a large enough $\lambda > \lambda_{c2}$ can be driven through a continuous transition into a paired state by increasing screening, i.e., reducing the screening length. Concomitantly, the logarithmic divergence of the effective mass in the CFL at $\kappa = 0$ is removed. We note that the MRP has been shown to be more stable when finite layer thickness is considered in the Coulomb interaction at $\nu = 5/2$.

To summarize, we have shown that the correlation hole, i.e., the binding of electrons to quantized finite-size vortices, is crucial for forming the paired quantum Hall states. The effective interaction is gauge-field free, instantaneous, and favors chiral $p$-wave pairing. The pairing potential is a direct consequence of the lowering of the Coulomb energy due to the correlation hole. We find that, with increasing Coulomb interaction strength, the ground state evolves from a $p - ip$ state asymptotically equivalent to the MRP, to a new oscillatory pfaffian state and finally to a CFL via a continuous phase transition as illustrated in Fig. 2(d).

Recently, FQHEs at $\nu = 1/2$ and $1/4$ have been observed in wide GaAs quantum wells with higher electron density than in previous experiments that reported no signs of FQHE. Indeed, there are direct evidences that the signatures of FQHE become stronger with increasing electron density. This is consistent with our theory since $\lambda$ is proportional to $k_F^{-1} \sim n_e^{-1/2}$, a smaller density would tend to destabilize the paired state. Hence, the paired quantum Hall states proposed in this work can be prospective candidates for the observed even-denominator FQHE in the LLL.

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