Evaluation of Audio-Visual Alignments in Visually Grounded Speech Models

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Abstract

Systems that can find correspondences between multiple modalities, such as between speech and images, have great potential to solve different recognition and data analysis tasks in an unsupervised manner. This work studies multimodal learning in the context of visually grounded speech (VGS) models, and focuses on their recently demonstrated capability to extract spatio-temporal alignments between spoken words and the corresponding visual objects without ever been explicitly trained for object localization or word recognition. As the main contributions, we formalize the alignment problem in terms of an audiovisual alignment tensor that is based on earlier VGS work, introduce systematic metrics for evaluating model performance in aligning visual objects and spoken words, and propose a new VGS model variant for the alignment task utilizing cross-modal attention layer. We test our model and a previously proposed model in the alignment task using SPEECH-COCO captions coupled with MSCOCO images. We compare the alignment performance using our proposed evaluation metrics to the semantic retrieval task commonly used to evaluate VGS models. We show that cross-modal attention layer not only helps the model to achieve higher semantic cross-modal retrieval performance, but also leads to substantial improvements in the alignment performance between image object and spoken words.

Index Terms: cross-modal learning, audio-visual alignment, visual object localization, word segmentation

1. Introduction

Utilization of statistical dependencies between different modalities has the potential to replace or supplement supervised learning in many tasks, as the data streams can potentially be used as weak supervision for each other. As an example of such multimodal systems, so-called visually grounded speech (VGS) models have been recently proposed [1–6]. They are unsupervised audiovisual algorithms that can learn shared semantic concepts between visual and speech data, given a series of unlabeled images and utterances describing them (e.g., [3, 4]). As a result, the models can be used to search for semantically similar content in audio and images. VGS models are also interesting for modeling of infant language learning, which is essentially an unsupervised multimodal learning process with learners having access to both auditory and visual input (see, e.g., [7, 8]).

Although VGS models typically operate based on high-dimensional semantic embeddings from which spatial and temporal characteristics of the data have been abstracted away, the architecture of these models can also be modified to estimate alignments between visual objects and the corresponding spoken words in the input data. For instance, [3] introduced a latent audiovisual tensor that assigns explicit alignment scores on each spatial position and time-step for input pairs of images and utterances. One could also attempt to extract the object-word-alignments from other latent (e.g., audiovisual attention) layers of the models. This makes VGS models promising for unsupervised decoding (segmentation) of image and audio data into their constituent units in an unsupervised manner, and opens up a largely unexplored strategy to learn structural properties of initially unorganized and unlabeled data (e.g., visual object categorization or unsupervised language learning).

Localization of objects within images using text-based queries has been investigated in different lines of research, such as in object detection, given a category label [9] and in natural language object retrieval [10–13]. Given a query word, these models either directly try to predict the locations of object bounding boxes in images [12, 13], or by ranking alternative image regions in terms of the potential presence of the relevant object [15]. Even though the majority of existing multi-modal works have focused on phrase-based object detection, the multimodal learning and alignment task can also be viewed as symmetric with respect to both input modalities, i.e., learned objects in the visual input could also be used to segment correct words in the other modality, and vice versa. Moreover, object localization has been mainly evaluated using intersection over union (IoU) between the predicted and true bounding boxes of visual objects. IoU uses hard decisions on what pixels are detected, potentially followed by another thresholding to determine if a specific object is detected or not (see, e.g., [15]).

In [5], the proposed VGS model was evaluated using a number of heuristic approaches, including measuring the capability of the audiovisual tensor to conduct speech-based object localization, to form meaningful audio-visual pattern clusters, and in building an object-word concept dictionary. Although functional, these evaluation strategies are variable, dependent on the characteristics of the selected small test set, and require manual analysis of the results. Hence, in order to enable systematic comparison of alternative models and facilitate further unsupervised multimodal alignment system development, standardized definition of the alignment problem and associated evaluation metrics would be desirable.

Given this background, our present goal is to formally define the problem of multimodal alignment between speech and images, here studied in the context of VGS models, and to define evaluation metrics accord with the problem definition. We start from audiovisual alignment tensors (derived from [5]), and define two metrics that capture complementary aspects of the alignment performance in terms of finding visual objects for spoken words (or phrases), and finding words in speech, given visual objects. We also propose a new VGS model variant for unsupervised audiovisual alignment using audiovisual attention, and show that it outperforms the models from [5] in both alignment and semantic retrieval tasks. Although we place our work in the context of VGS models, the proposed methodology is applicable to both explicit and implicit alignment models, and should generalize beyond image and speech modalities.
The basic goal of the alignment process is to identify the link between spoken words or phrases and the corresponding visual objects or concepts in images. Formally, we define this link using an audiovisual alignment tensor $\mathbf{T}[x, y, t] \in \mathbb{R}_{\geq 0}^{N_T \times N_x \times N_y}$, where $x \in \{1, \ldots, N_x\}$, $y \in \{1, \ldots, N_y\}$, and $t \in \{1, \ldots, T\}$, which defines the association strength between each pixel $x, y$ in the image space with each time step $t$ in the speech signal (see Fig. 1 for illustration). $N_x \times N_y$ is the size of the image in pixels, and $T$ is the duration of the utterance in terms of 10-ms signal frames. In $\mathbf{T}[x, y, t]$, zeros stand for no association and the larger the (positive) value, the stronger the model’s confidence that the pixels and timestamps are related to each other. The goal of an alignment model is to derive this tensor, given an input image and an utterance. In practice, the tensor may be a final output of a model (e.g., using supervised training) or extracted as a latent representation of a VGS model (as in [3] and all models in our present experiments).

Evaluation of the alignment is based on ground-truth knowledge on visual objects that are related to specific acoustic words or phrases, basically word/phrase timestamps together with object pixel masks. Given a concept (“class”) $c$, we define $\{x, y\} \in S_c$ as the set of pixels corresponding to visual object $o_c$ and $\{\text{font}, \ldots, \text{object}\} \in T_c$ as the set of time-frames corresponding to the related word $w_c$. Together they define a subset of $\mathbf{T}[x, y, t]$ corresponding to the ground-truth alignment.

We propose two new primary metrics to evaluate how well $\mathbf{T}$ relates to the ground-truth: alignment score (AS) and glancing score (GS), both measured separately in time and space.

2.1. Alignment score

AS measures how well the model attends to the correct visual object(s) throughout a spoken word, or how well the attention across all the pixels of an object focuses on the time-instances of the corresponding word. In the case of object alignment given a spoken utterance, $\text{AS}_{\text{object}}(c) \in [0, 1]$ is calculated as follows.

First, each frame in $\mathbf{T}$ is normalized to sum up to one $\tilde{\mathbf{T}}'[x, y, t] = \mathbf{T}[x, y, t] / \sum_{x,y,t} \mathbf{T}[x, y, t]$ to measure the distribution of “attention” across the image at each time step. Then the proportion of attention on the target object is obtained by

$$\text{AS}_{\text{object}}(c) = \frac{1}{|T_c|} \sum_{x,y,t \in S_c} \tilde{\mathbf{T}}'[x, y, t]$$

(1)

where $|T_c|$ is the number of frames in $w_c$. As a result, $\text{AS}_{\text{object}}(c)$ obtains a value of 0 if none of the attention is on the target object $o_c$ during the word $w_c$, and 1 if all attention is located on the object $o_c$ during the entire duration of $w_c$.

Word alignment score $\text{AS}_{\text{word}}(c) \in [0, 1]$ is calculated in an analogous manner, now first normalizing each pixel of $\mathbf{T}$ to sum up to one across the utterance: $\tilde{\mathbf{T}}''[x, y, t] = \mathbf{T}[x, y, t] / \sum_{t} \mathbf{T}[x, y, t]$, and then measuring the average temporal overlap of the resulting scores and $w_c$ across the $o_c$ pixels:

$$\text{AS}_{\text{word}}(c) = \frac{1}{|S_{t_c}|} \sum_{x,y,t \in S_{t_c}} \tilde{\mathbf{T}}''[x, y, t]$$

(2)

If $\text{AS}_{\text{word}}(c)$ is 0, the temporal attention at all the pixels of $o_c$ have zero value during $w_c$, whereas score of 1 means that attention at all of the $o_c$ pixels is always completely within the bounds of $w_c$ and zero during other time steps.

2.2. Glancing score

The alignment score enforces each word time-step or object pixel to have consistent attention on the corresponding target in the other signal modality. However, in some use cases we may only be interested whether the model “takes a look” at the correct object once it recognizes a word, or tags the correct word in time given a partial observation of an object. As an example, a temporally causal model simulating human eye-gaze during speech comprehension could “glance” at the correct object once it recognizes the corresponding word [16]. However, such a model would not be able to maintain sustained attention on the target object before identifying the word as it unfolds in time. To evaluate such a behavior, we introduce glancing score GS.

In object glancing score $\text{GS}_{\text{object}} \in [0, 1]$, the cumulative attention map across the image during the entire word $w_c$ is first calculated as $a_c[x, y] = \sum_{t \in T_c} \tilde{\mathbf{T}}'[x, y, t]$. This is then normalized to a spatial distribution proper $a'_c[x, y] = a_c[x, y] / \sum_{x,y} a_c[x, y]$, and compared to ground-truth pixels to measure the proportion of attention on the target:

$$\text{GS}_{\text{object}}(c) = \sum_{(x,y) \in S_c} a'_c[x, y]$$

(3)

The corresponding word glancing score $\text{GS}_{\text{word}} \in [0, 1]$ is obtained by first measuring the total object attention as a function of time by $a[t] = \sum_{(x,y) \in S_{t_c}} \tilde{\mathbf{T}}'[x, y, t]$ and normalizing it to have a sum of 1 across the utterance with $a'_t[t] = a[t] / \sum_t a[t]$. The score is then obtained by

$$\text{GS}_{\text{word}}(c) = \sum_{t} a'_t[c]$$

(4)

In essence, $\text{GS}_{\text{object}}$ is the proportion of attention within the spatial extent of object $o_c$ during word $w_c$, whenever $T > 0$, but the attention does not have to be defined (positive-valued) throughout the duration of the word. In an analogous manner, $\text{GS}_{\text{word}}$ corresponds to the proportion of attention on the target word $w_c$ compared to other words, given the pixels of $o_c$, but not all pixels have to have positive attention values in $\mathbf{T}$. The more there is relative attention outside the correct targets, the lower the scores will be. If $a(x, y)$ or $a[t]$ fully zero before normalization, they are replaced by a uniform distribution across their elements.

Equations above describe scoring for individual word-object pairs $w_c-o_c$ in individual images-utterance samples. In practice, the score should be calculated for each ground-truth pair in each image-utterance-pair of the test set. Also note that confusion errors for both AS and GS can be derived from the equations simply by comparing $S_{t_c}$ and $T_{x,y}$, where $c_1 \neq c_2$.

3. Model Description

Our models follow the main structure of VGS models (see, e.g., [2,4]), where two branches of neural layers embed data from
speech and image domains, respectively. Then the modality-
specific vector embeddings are mapped into a shared semantic
space. Input to the system consists of images paired with spoken
descriptions of them. Output is a similarity score indicat-
ing the semantic relatedness of the input pairs. The network is
trained using a triplet loss [1] that tries to assign higher scores to
semantically related image-speech pairs compared to unrelated
pairs, and with the maximum separation limited by a margin \( M \).

However, unlike other common VGS models (e.g., [14]
[5]), encoder outputs of the so-called DAVEnet model [3] main-
tain modality-specific signal representations as a function of input
speech time (1-D) and image position (2-D). The embed-
dings vectors are then mapped (“aligned”) together through matrix
product, resulting a 3-D tensor \( T[w, h, n] \) spanning in both spatial location \( (w, h) \) and time frames \( (n) \). This audio-visual
alignment tensor \( T \) allows the model to co-localize patterns
within both modalities. The overall similarity score between input
speech and images can then be obtained by taking the sum
and/or maximum of \( T \) in time and/or spatial dimensions, result-
ing in three different model criteria for training: MISA (max over image, sum over audio), SIMA (sum over image, max over audio), SISA (sum over image, sum over audio) [3], whereas maxing over both dimensions appears to be difficult to train.

In our experiments, the first model variant is similar to the
DAVEnet (here: CNNi). In CNNi, the speech encoder is stack of five convolutional layers (with layer sizes of [128, 256, 256, 512, 512]) with gradual temporal downsampling with maxpooling
over time. VGG16 model [17], up to the last convolutional layer and pretrained on ImageNet data, is used as the image
encoder. This is followed by one trainable 2D-convolution layer with 512 filters. Both the speech and image encoder
branches are then fed to a dense linear layer, followed by L2
normalization, and then combined into \( T \) with matrix product.

Our CNN\textsubscript{ATT} (Fig. 2) is obtained from CNNi by adding a
cross-modal attention layer on top of the \( T[w, h, n] \) to help
the model to attend to specific image objects and spoken words
using the information from the another modality. This is in-
spired by the fact that the audiovisual tensor \( T \) introduced in [3]
is similar to scoring function applied in dot product attention mechanism [18][19]. Thus, we extended the model to have a complete
attentional module by applying softmax non-linearity separately in space (for a query in time; Fig. 2 [left branch]) and in time (for a query in space; Fig. 2 [right branch]) in order to produce a distribution of weights over image space and speech frames, respectively. In parallel to the softmax, we also apply a dense layer with a sigmoid activation function to produce an extra
attentional representation, as we found this to outperform the
use of softmax only in the retrieval task. These attention scores
are then used to produce corresponding spatially weighted rep-
resentations for audio (left) and time-weighted representations
for image (right), followed by average pooling to get rid of ab-
solute positional information. In the final stage, outputs of the
softmax and sigmoid layer attentions are concatenated with the
original image and speech representations to produce the final
speech and image embeddings. These embeddings are then L2
normalized and compared using a similarity score (dot product)
in triplet-loss training, and alignments can be extracted from \( T \)
or after taking the softmax in time or space.

We tested three alternative variations for the CNN\textsubscript{ATT} archi-
tecture: CNN\textsubscript{ATT}0 with the same speech encoder as the CNNi,
CNN\textsubscript{ATT}1 with the first maxpool layer removed to obtain \( T \) at a higher temporal resolution of 128 frames (compared to 64 frames in other variants), and CNN\textsubscript{ATT}2 which was equal to
CNN\textsubscript{ATT}0 but with less filters [64, 128, 256, 256, 512].

4. Experimental setup

For our experiments, we used MSCOCO [29], which consists of images of everyday objects and their contexts, together with their speech-synthesized captions from SPEECH-COCO [21].

The dataset includes a total of 123,287 images with 91 common object categories (e.g., dog, pizza, chair) from 11 super-
categories (e.g., animal, food). In SPEECH-COCO, each image is paired by five synthetic speech captions describing the scene
using the object categories. In our experiments, we used the training set of MSCOCO images and their verbal descriptions for model training and validation. The original validation set of \( \sim 40k \) images was used as a held-out test set for evaluation, using one randomly chosen spoken caption per image.

SPEECH-COCO contains metadata on words and their
timestamps, while MSCOCO contains manually annotated
pixel masks for its 80 visual object categories. However, there
is no direct one-to-one mapping between image object labels
(e.g., [dog]) and spoken words (e.g., “puppy”). We derived
ground-truth pairings of objects and words automatically us-
ning semantic similarities from a pre-trained Word2vec model
[22]. We first extracted nouns of the caption transcripts using
NLTK-toolbox, and then used Word2vec cosine similarity be-
tween words and object labels as a means of identifying seman-
tically matching pairs. Words and objects with Word2vec simi-
larity above a threshold of 0.5 were considered as a ground-truth
word-object pair \( w_i \rightarrow o_j \) (a concept). Selection of this threshold
was based on manual observation of the similarity histograms
across the dataset, where 0.5 provided a reasonable cutoff point
in the somewhat bimodal (but noisy) similarity distribution.

Model training followed the same triplet-loss protocol as in
[1][8]. Adam optimizer was used for both models initial
learning rate of \( lr = 1 \times 10^{-4} \) and triplet loss margin \( M = 0.1 \).

In the speech processing channel, we applied ReLU activation
after each convolutional layer, followed by batch normalization.
We measured recall@10 audiovisual semantic retrieval score
[23] using the representation similarity scores to ensure that the
models have learned the semantic relationships between the two
modalities. For each variant, we saved the best model based on the
recall score of the validation set. In both CNNi and CNN\textsubscript{ATT}
models, recall scores were measured using speech and image
embedding layers before audiovisual \( T \) (I and A in Fig. 3).

AS and GS scores were then measured for each word-object

Table 2: Alignment and glancing scores for the model variants.

| Model     | $AS_{\text{obj}}$ | $AS_{\text{word}}$ | $GS_{\text{obj}}$ | $GS_{\text{word}}$ |
|-----------|-------------------|-------------------|-------------------|-------------------|
| baseline  | 0.158             | 0.011             | 0.158             | 0.073             |
| CNN$_0$   |                   |                   |                   |                   |
| SISA      | 0.200             | 0.016             | 0.200             | 0.102             |
| MISA      | 0.270             | 0.020             | 0.267             | 0.132             |
| SIMA      | 0.223             | 0.041             | 0.222             | 0.214             |
| CNN$_{\text{ATT}}$ |       |                   |                   |                   |
| v0        | 0.285             | 0.028             | 0.282             | 0.168             |
| v1        | 0.292             | 0.038             | 0.295             | 0.227             |
| v2        | 0.279             | 0.030             | 0.277             | 0.180             |
| v0 softmax| 0.504             | 0.052             | 0.504             | 0.317             |
| v1 softmax| 0.518             | 0.076             | 0.518             | 0.446             |
| v2 softmax| 0.501             | 0.056             | 0.501             | 0.327             |

Finally, Fig. 3 illustrates how the AS and GS depend on ground truth visual object size and word duration (for CNN$_{\text{ATT}}$v1 softmax). Both object detection (red) and word detection (blue) scores increase with increasing visual object size. They also increase more than what is expected by chance with larger targets. Fig. 3 also shows that AS and GS for word detection increase for longer words, whereas the object detection scores are not affected by the word duration. Similar patterns were observed across all the compared models.

6. Conclusions

In this paper, we proposed alignment and glancing scores as two novel metrics for evaluating performance of visually grounded speech (VGS) systems in an alignment task between visual objects and spoken words. We also introduced a VGS model variant based on cross-modal attention, and compared how different VGS models perform on the alignment task using our proposed metrics. The results show that the metrics can capture different aspects of cross-modal alignments, and that the attention-based VGS models outperform the earlier non-attentional alignment approach in both alignment and semantic retrieval tasks.

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