Light adjoint scalars and unification at the string scale.

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Abstract

Following the suggestion of Bachas, Fabre and Yanagida (BFY), we analyze the gauge coupling unification at the two-loop order, in a supersymmetric scenario where scalars belonging to the adjoint representations contribute to the evolution of the couplings from intermediate scales onward, and the unification scale is pushed towards the string scale. Thereafter, we compare the masses of these adjoint scalars to the scale at which the hidden sector gauge coupling reaches the non-perturbative limit at the two-loop order for various possible hidden sector gauge groups motivated by the the conjecture of BFY that the masses of these adjoint scalars are related to gaugino condensation. We also compute the predictions for the top and bottom quark masses in this scenario and compare them with those of MSSM. The predicted bottom mass improves in the BFY scenario for smaller values of $\alpha_s$. 

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The gauge couplings unify at \( M_{\text{string}} = 2 \times 10^{16} \text{GeV} \) when Minimal Supersymmetric Standard Model (MSSM) degrees of freedom contributes to the evolution of the gauge couplings. However, if the field theory of MSSM is a low energy remnant of the string theory, there exists a noteworthy discrepancy. This field theoretic unification scale is different from the scale of the string theory, \( M_{\text{String}} \), which is fixed by the intrinsic scale of the string theory, namely \( M_{\text{Plank}} \) by the relation,

\[
M_{\text{string}} = g_{\text{string}} M_{\text{Plank}}.
\]  

One loop string effects could lower this tree level value of the string scale somewhat, and one calculates \[1, 2\] that the string unification scale is modified to,

\[
M_{\text{string}} = g_{\text{string}} \times 5.27 \times 10^{17} \text{ GeV} \simeq 5.27 \times 10^{17} \text{ GeV},
\]  

where we have assumed \( g_{\text{string}} = O(1) \). Consequently, the scale \( M_{\text{string}} \) is higher than the scale \( M_{\text{GUT}} \) by a factor of 20 approximately.

In the literature one finds several approaches to rectify the aforementioned difference between the string scale and the unification scale. There can be intermediate symmetry groups \[3, 4\], which alter the running of the gauge couplings and likewise the unification occurs at the scale of \( M_{\text{string}} \). Non-standard hypercharge normalizations, which arise in various string models, can also accommodate the apparent mismatch between the string scale and the unification scale \[5\]. The heavy string threshold corrections from the string states \[6\] at the Plank scale or below have been used to reconcile the mismatch between the string scale and the unification scale. Non-standard exotic matter \[7\] has also been shown to serve the purpose.

The string models having a \( G \times G \) structure, when broken to the diagonal subgroup, naturally contains adjoint scalars with zero hypercharge. Bachas, Fabre and Yanagida (BFY) have pointed out \[9\] that, if the mass of these zero-hypercharge adjoint scalars lie in the well motivated intermediate scale \( M_I \sim M^{2/3} m_{\text{SUSY}}^{1/3} \sim 10^{13} \text{ GeV} \), the unification scale is pushed up to the string scale at the one-loop level. In this paper we present a two-loop analysis of this scenario and find ranges of the masses of the \( SU(2) \) and \( SU(3) \) adjoint scalars, namely, \( M_2 \) and \( M_3 \) for the allowed
range of $\alpha_s(m_Z)$ which gives rise to the gauge coupling unification at the scale $M_{\text{string}}$. At the two-loop level there is also a mild variation of the unification scale to the ratio $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$, because at the two-loop level, the running of the gauge couplings are affected by the simultaneous running of the Yukawa couplings. The adjoint scalars have no Yukawa couplings with the ordinary Fermions and so we consider the effect of only the ordinary Yukawa couplings on the evolution of gauge couplings.

At the two-loop level the evolution of the gauge couplings are governed by the equation,

$$\frac{d\alpha_i}{dt} = b_i \frac{1}{2\pi} \alpha_i^2 + \sum_j b_{ij} \frac{1}{8\pi^2} \alpha_i \alpha_j - \sum_k a_{ik} \frac{1}{8\pi^2} \alpha_i \alpha_k^2 Y_k,$$

(3)

where,

$$b_i = \begin{pmatrix} 2n_f + \frac{3}{5}n_d \\ -6 + 2n_f + n_d + 2 \Theta(\mu - M_2) \\ -9 + 2n_f + 3 \Theta(\mu - M_3) \end{pmatrix} \quad a_{ik} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix},$$

(4)

and,

$$b_{ij} = \begin{pmatrix} \frac{38}{15}n_f + \frac{2}{5}n_d \\ \frac{2}{3}n_f + \frac{3}{5}n_d \\ \frac{11}{5}n_f \end{pmatrix} \begin{pmatrix} -24 + 14n_f + 7n_d + 24 \Theta(\mu - M_2) \\ 3n_f \end{pmatrix} \begin{pmatrix} \frac{8}{15}n_f \\ 8n_f \end{pmatrix} - 54 + \frac{68}{3}n_f + 54 \Theta(\mu - M_3),$$

(5)

where, i,j=1,3; k=t,b,\tau; n_d are the number of Higgs doublets, $n_f$ is the number of Fermion generations, $t = \ln \mu$ and $\Theta(x) = 1$, whenever $x \geq 0$. Fixing the scale $M_X = M_{\text{string}}$ we can find three quantities, from a system of three evolution equations, using as inputs the quantities $\alpha_1(m_Z)$, $\alpha_2(m_Z)$ and $\alpha_s(m_Z)$. The evaluated unknowns are $\alpha_G$, $M_2$ and $M_3$. The calculated values of $M_2$ and $M_3$ are plotted in Figure (1.b). The magnitude of $M_3$ has an appreciable sensitivity to the input value of $\alpha_s(m_Z)$. This is also displayed in the Figures (1.a) and (1.b).

It is noticeable from Figure (1.b) that the adjoint scalars having the mass near $10^{12} - 10^{14}$ GeV. Numerically, this scale is roughly near $M_{\text{Plank}}^{2/3}m_{\text{susy}}^{1/3}$. This is a striking coincidence, as noted by BFK. Now to check further, we will assume the unification of the gauge couplings of the hidden and observable sectors at the scale $M_{\text{string}}$. Using the initial value of the observable sector coupling $\alpha_G(M_{\text{string}})$ we evolve the hidden gauge coupling backwards and search for the scale $M_C$.
Figure 1: The values of \( M_2 \) and \( M_3 \) for various values of the unification scale are in Figure (b). The shaded region refers to the variation of \( \tan \beta \). In Figure (a) we have compared the values of \( M_2 \) and \( M_3 \) with the condensation scale \( M_C \) when \( M_{\text{string}} \) is \( 5.27 \times 10^{17} \text{ GeV} \) and \( G_H \) is taken as \( SU(3) \), \( SU(5) \) and \( SO(10) \).

at which the hidden sector gauge coupling becomes non-perturbative. We have, for the simplicity, considered only matter-free gauge groups which are the subgroups of \( E_8 \), eg, \( SU(2) \), \( SU(3) \), \( SU(5) \) or \( SO(10) \). In Figure (1.a) we have compared the scale \( M_C \) with \( M_2 \) and \( M_3 \) when \( G_H \) is \( SU(3) \), \( SU(5) \) and \( SO(10) \) respectively. The condensation scale, when \( G_H \) is \( SO(10) \) is large compared to the scales \( M_2 \) and \( M_3 \). The choice \( SU(3) \) as the hidden group returns a closer value of \( M_C \) to the scales \( M_3 \) and \( M_2 \) than if \( G_H \) is chosen as \( SU(5) \). A comment about the results in two-loop calculation is in order. Consider the case when \( G_H \) is \( SU(5) \) and \( M_{\text{string}} = 2 \times 10^{17} \text{ GeV} \). In the one loop case the scale \( M_C \) is \( 10^{13.5} \text{ GeV} \) but at the two-loop case this scale becomes \( 10^{15.2} \text{ GeV} \). Hence, the two-loop calculation shows a departure from the one-loop expectation and brings \( M_C \) closer to the intermediate scales \( M_2 \) and \( M_3 \) when the hidden group is \( SU(3) \). This can be seen in Figure (2.a), where the running of the couplings are plotted. The running of the couplings in the MSSM are plotted in Figure (2.b) for comparison\(^1\). We have taken \( SU(5) \) as the unification

\(^1\)One of the authors B.B acknowledges discussions with Q. Shafi on the running of the hidden sector gauge coupling.
group from $M_X \simeq 2 \times 10^{16}$ GeV up to $M_{\text{string}}$.

Before we conclude, we give some results of a two-loop Yukawa coupling analysis of this model. The relevant beta function coefficients are summarized in Ref [4]. The Top quark mass is determined from the quasi-infrared fixed point of the top quark Yukawa coupling [11]. The presence of the adjoints alter the running of the gauge couplings above the intermediate scale which has secondary effects on the running of the Yukawa couplings. Even though we do not expect a large change in the individual Yukawa couplings at low energy, the ratios of the Yukawa couplings are sensitive to the presence of these extra matter. To check these issues, we have plotted the prediction of the top quark and the bottom quark masses in the Figures (3.a) and (3.b). The bottom quark mass can be calculated under the well-known assumption of b-τ unification. The prediction of the bottom quark mass, having been calculated from the ratio of the bottom and τ Yukawa coupling, differs notably when the extra adjoint matter is present. To see this we have

\footnote{b-τ unification is natural in GUT models like $SU(5)$ or $SO(10)$. Here, we consider a similar scenario in string unification.}
Table 1: The masses of the adjoint scalars of $SU(2)$ and $SU(3)$ that leads to gauge coupling unification at the scale $M_{\text{string}}$ is in the two bottom lines of the table. The values quoted are for $\alpha_s = 0.117$. It can be compared to the scale $M_C$ at which the hidden sector coupling becomes of order one for various gauge groups. For comparison the corresponding one-loop values are shown inside the brackets. The gauge groups have been designated by their respective quadratic casimirs.

| $G_H$ | $C_2(G)$ | $M_{\text{string}} = 2 \times 10^{17}$ | $M_{\text{string}} = 5.27 \times 10^{17}$ | $M_{\text{string}} = 7 \times 10^{17}$ |
|-------|----------|-------------------------------------|-------------------------------------|-------------------------------------|
| $SU(2)$ | 2        | $M_C = 10^{10.1} (10^{0.8})$        | $M_C = 10^{10.9} (10^{8.7})$        | $M_C = 10^{11.1} (10^{9.0})$        |
| $SU(3)$ | 3        | $M_C = 10^{13.0} (10^{11.0})$        | $M_C = 10^{13.7} (10^{11.7})$        | $M_C = 10^{13.8} (10^{11.9})$        |
| $SU(5)$ | 5        | $M_C = 10^{15.2} (10^{13.5})$        | $M_C = 10^{15.7} (10^{14.1})$        | $M_C = 10^{15.9} (10^{14.3})$        |
| $SO(10)$ | 8        | $M_C = 10^{16.3} (10^{14.9})$        | $M_C = 10^{16.7} (10^{15.5})$        | $M_C = 10^{16.9} (10^{15.6})$        |

$\alpha_s = 0.117$ $SU(2)$ adjoint $M_2 = 10^{14.8}$ $M_2 = 10^{14.1}$ $M_2 = 10^{14.9}$
$\alpha_s = 0.117$ $SU(3)$ adjoint $M_3 = 10^{14.2}$ $M_3 = 10^{13.4}$ $M_3 = 10^{13.1}$

compared the present case with that of MSSM in Figure (3.a) and (3.b) in dashed lines. The bands in the figures (3.a) and (3.b) are obtained by varying $\tan \beta$ in the range 5-60 and $M_{\text{string}}$ in the range $2 - 7 \times 10^{17}$ GeV. In MSSM the unification occurs for only a narrow region in the $\alpha_s$ space, whereas, the presence of adjoint moduli opens up the parameter space further and as an welcome result unification occurs for a larger range in $\alpha_s$. This fact is reflected in Figure (3) where the prediction of the top and bottom quark masses are displayed as narrow lines, whereas in the present case with adjoint moduli the predictions are wide bands. The bottom quark mass can be within the experimental range for lower values of $\alpha_s$, for which the prediction in the MSSM case is unavailable due to the absence of gauge coupling unification and hence $b-\tau$ unification. In Table 2 we have compared the predictions of $m_{t_{\text{pole}}}$, $m_{b_{\text{pole}}}$ and $\tan \beta$ in the MSSM with those obtained in the presence of adjoints for the required value of $\alpha_s(m_Z)$ which gives rise to unification in MSSM [Note that this specific value of $\alpha_s$ depends on $Y_b$ or equivalently on $\tan \beta$]. Typically, the value of $m_{t_{\text{pole}}}$ diminishes while $m_{b_{\text{pole}}}$ increases slightly in the BFY scenario.

In conclusion, we have computed the two-loop running of the gauge couplings in a supersymmetric scenario with extra adjoint matter from intermediate scales onwards. Setting the unification scale at the string scale, the masses of these adjoints and the unification coupling have been calcu-
Figure 3: The range of the bottom and the top quark mass plotted against $\alpha_s$. This range is obtained by varying $\tan \beta$ in the range approximately 5-60 and $M_{\text{string}}$ in the range $2 - 7 \times 10^{17}$ GeV. Dashed lines are the prediction for the MSSM varying $\tan \beta$ in the same range.

The masses of the adjoints turn out to be of the same order as that of the expected gaugino condensation scale $M_P^{2/3} m_{\text{susy}}^{1/3}$ in some cases as displayed in Figure (1) and Table 1. Taking the gauge coupling $\alpha_H$ for the hidden sector group $G_H$, to be equal to the unification coupling at the scale $M_{\text{string}}$, we have evolved back this value to the scale $M_C$, where $\alpha_H$ becomes non-perturbative at the two-loop order, for different choices of the hidden (matter free) group $G_H$. The two-loop running of the hidden sector is called for, because the scale $M_C$ differs from the one-loop expectation (shown inside brackets in Table-1) by more than one order of magnitude. When $G_H$ is $SU(3)$ or $SU(5)$ the scale $M_C$ works out to be of the order of the masses of $M_2$ and $M_3$. In comparison to MSSM, unification occurs for wider range of $\alpha_s$. The bottom quark pole mass prediction from $b-\tau$ unification improves compared to that of MSSM for smaller values of $\alpha_s$ like 0.115. The top quark mass is evaluated assuming that the top quark Yukawa coupling at $M_{\text{string}}$ is within domain of the quasi-infrared fixed point at the scale $m_{\text{top}}$. The range for the prediction of the top quark mass is consistent with experimental numbers in the moduli scenario.
Table 2: The predictions of $m_t^\text{pole}$ and $m_b^\text{pole}$ in MSSM have been compared with those of BFY scenario for $\alpha_s(m_Z)$ which gives rise to unification in the MSSM case. For the BFY scenario $M_{\text{string}} = 5.27 \times 10^{17}$, $Y_b = h_b^2/4\pi$, where $h_b$ is the bottom quark Yukawa coupling. The low energy value of $\tan \beta$ has been calculated using $Y_\tau(m_\tau)$ from the RGE and $m_\tau = 1.777$ GeV; and this value of $\tan \beta$ has been used to estimate the top and the bottom masses.

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| $Y_t(M_{\text{string}}), Y_b(M_{\text{string}})$ | $\alpha_s(m_Z)$ | $m_t^\text{pole}$ | $m_b^\text{pole}$ | $\tan \beta$ |
|----------------------------------|-----------------|------------------|------------------|--------------|
|                                  |                 | MSSM Moduli | MSSM Moduli | MSSM Moduli |
| 1, 1                             | 0.120           | 189  187  | 3.84  3.94   | 63.2  61.0   |
| $1, 10^{-1}$                      | 0.122           | 193  190  | 3.91  4.00   | 59.1  57.7   |
| $1, 10^{-2}$                      | 0.123           | 199  197  | 4.06  4.18   | 39.9  40.6   |
| $1, 10^{-3}$                      | 0.124           | 202  200  | 4.16  4.31   | 15.8  16.5   |
| $1, 10^{-4}$                      | 0.124           | 199  197  | 4.18  4.33   | 5.1   5.3    |
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