A Novel Treatment of the Josephson Effect

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A new picture of the Josephson effect is devised. The radio-frequency (RF) signal, observed in a Josephson junction, is shown to stem from bound electrons, tunneling periodically through the insulating film. This holds also for the microwave mediated tunneling. The Josephson effect is found to be conditioned by the same prerequisite worked out previously for persistent currents, thermal equilibrium and occurrence of superconductivity. The observed negative resistance behaviour is shown to originate from the interplay between the normal and superconducting currents.

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I. INTRODUCTION

The Josephson effect was initially observed1–3 in the kind of circuit sketched in Fig.1 and has kept arousing an unabated interest, in particular because of its relevance to electronic devices4,5 and quantum computation6–8. For simplicity, both superconducting leads A,B are assumed here to be made out of the same material. They are separated by a thin (<10−8m) insulating film, enabling electrons to tunnel through it. If A,B were made of a normal metal, a constant current I = U / (R + Rτ) would flow through the circuit. Nevertheless, this simple setup has attracted considerable attention because of Josephson’s predictions9:

1. there should be ⟨I⟩ ≠ 0 for ⟨U⟩ = 0, which entails d⟨I⟩ / d⟨U⟩ (⟨U⟩ = 0) → ∞ (⟨I⟩, ⟨U⟩) refer to time t averaged values of I(t),U(t);
2. I(t),U(t) should oscillate at frequency ω = 2e⟨U⟩ / ℏ with e being the electron charge.

However the characteristic I(U), reproduced in Fig.2, indicates rather ⟨I⟩ (⟨U⟩ = 0) = 0 with finite d⟨I⟩ / d⟨U⟩ (⟨U⟩ = 0) ≈ 0.02Ω−1. Likewise, since the origin ⟨I⟩ = ⟨U⟩ = 0 is not indicated in Fig.3, the accuracy is poor and d⟨I⟩ / d⟨U⟩ (⟨U⟩ = 0) ≈ 0.06Ω−1 is also finite, claim 1 cannot be validated on the basis of the experimental data. Besides a periodic signal was indeed observed2,10, but in the RF range, i.e. ω < 100MHz, rather than in the microwave one, i.e. ω > 1GHz, as inferred from Josephson’s formula, given the measured ⟨U⟩ values. Consequently, the numerous experimental data, documenting the electrodynamical behaviour of the Josephson junction4,5, have been interpreted so far by resorting10 to a formula, relating I(t),U(t) to Ginzburg and Landau’s phase11 ΦGL. However the time behaviour of ΦGL(t) has been derived with help of a perturbation calculation9,12–14, which is well-suited to describe the random tunneling of a single particle, either electron or Bogolyubov-Valatin excitation14, but cannot account for the coherent tunneling of bound electrons, such as those making up the superconducting state15–18, for some reason to be given below. Therefore, this work is rather intended at presenting an alternative explanation of the Josephson effect, unrelated to ΦGL, by studying the time-periodic tunneling motion19 of bound electron pairs15–18 through the insulating barrier.

The outline is as follows: the expression of the tunneling current, conveyed by independent electrons, is recalled in section 2, whereas that of bound electron current is worked out in section 3; this enables us to solve, in section 4, the electrodynamical equation of motion of the circuit, depicted in Fig.1; sections 5,6 deal respectively with the microwave mediated Josephson effect2,3 and the negative resistance induced signal10. The results are summarised in the conclusion.
II. RANDOM TUNNELING

As in our previous work\textsuperscript{15–18,20–22}, the present analysis will proceed within the framework of the two-fluid model, for which the conduction electrons comprise superconducting and independent electrons, in respective concentration $c_s, c_n$. The superconducting and independent electrons are organized, respectively, as a many-body system, including the leads $A, B$, and the conduction electrons comprise su-

\[ j_n = \frac{e^2 \rho(E_F) v_F T}{2} U \Rightarrow R_t \propto \frac{1}{\rho(E_F)} , \]  

with $\rho(E_F), v_F, T$ standing for the one-electron density of states at the Fermi level, the Fermi velocity and the one-electron transmission coefficient through the insulating barrier ($\Rightarrow 0 < T < 1$). Several remarks are in order, regarding Eq.(1)

- Eq.(1) is seen to agree with the corresponding formula, available in textbooks\textsuperscript{14};

- the independent electrons contribute thence the current $I_n(t) = U(t)/R_t$ to the total current $I(t)$. However, despite $I_n$ obeying Ohm’s law, the tunneling electrons suffer no energy loss inside the insulating barrier;

- because $c_n$ is expected to grow\textsuperscript{16} at the expense of $c_s$ with growing $|I|$, this implies that $\rho(E_F)$ and $R_t$ will, respectively, increase and decrease with increasing $|I|$. The negative resistance effect, addressed in section 6, stems from this property.

III. COHERENT TUNNELING

Unlike the random diffusion of independent electrons across the insulating barrier, the tunneling motion of bound electrons takes place as a time-periodic oscillation to be analysed below. Their energy per unit volume $\mathcal{E}$ depends on $c_s$ only and is related to their chemical potential $\mu$ by $\mu = \frac{\partial \mathcal{E}}{\partial c_e}$. Before any electron crosses the barrier, the total energy of the whole bound electron system, including the leads $A, B$, reads

\[ \mathcal{E}_i = 2\mathcal{E}(c_e) + cc_e U , \]  

with $c_e$ referring to the bound electron concentration at thermal equilibrium. Let $n >> 1$ of bound electrons cross the barrier from $A$ toward $B$. The total energy becomes

\[ \mathcal{E}_f = \mathcal{E}(c_e + \frac{n}{V}) + \mathcal{E}(c_e - \frac{n}{V}) + e(c_e - \frac{n}{V})U , \]  

with $V$ being the volume, taken to be equal for both leads $A, B$. Energy conservation requires $\mathcal{E}_i = \mathcal{E}_f$, which leads finally to

\[ n = \frac{eV}{\frac{\partial \mathcal{E}}{\partial c_e}} U . \]  

The wave-functions $\varphi_i, \varphi_f$, associated with the twofold degenerate eigenvalue $\mathcal{E}_i = \mathcal{E}_f$, read

\[ \varphi_i = \varphi_A(c_e) \otimes \varphi_B(c_e) \]  

\[ \varphi_f = \varphi_A(c_e - \frac{n}{V}) \otimes \varphi_B(c_e + \frac{n}{V}) \]  

with $\varphi(c_e)$ being the MBE, $c_e$ dependent eigenfunction\textsuperscript{16,23}. The coherent tunneling motion of $n$ electrons across the barrier is thence described by the wave-function $\psi(t)$, solution of the Schrödinger equation

\[ H = \omega_1 \sigma_x \]  

\[ \omega_1 = \langle \varphi_i | V_b | \varphi_f \rangle . \]
The Hamiltonian $H$ and the potential barrier $V_0$, hindering
the electron motion through the Josephson junction
and including the applied voltage $U$, are expressed in
frequency unit, $\frac{V_0}{\hbar}$, is taken as the origin of energy, whereas
$\psi$ and the Pauli matrix $\sigma_x$ have been projected onto
the basis $\{\varphi_i, \varphi_f\}$. The tunneling frequency $\omega_t$, taken to lie
in the RF range, i.e. $\omega_t < 100MHz$, as reported by
Shapiro$^2$, is realized to describe the tunneling motion of
bound electrons in a similar way as the matrix element
$T_{kq}$ does for the random tunneling of a single electron
in the mainstream view$^{14}$. Finally Eq.(6) is solved$^{25}$ to yield
\begin{equation}
\psi(t) = \cos \left( \frac{\omega t}{2} \right) \varphi_i - i \sin \left( \frac{\omega t}{2} \right) \varphi_f ,
\end{equation}
whence the charge $Q_s, -Q_s$, piling up in $A,B$ respectively,
is inferred, thanks to Eq.(4), to read
\begin{equation}
Q_s(t) = -ne|\langle \psi(t)|\varphi_f \rangle|^2 = C_eU \sin^2 \left( \frac{\omega t}{2} \right) ,
\end{equation}
with the effective capacitance $C_e = -\frac{e^2}{\hbar} \frac{\partial\varphi}{\partial x}$. Since
$\frac{\partial \varphi}{\partial x} > 0$ has been shown to be a prerequisite for the exis-
tence of persistent currents$^{15}$, thermal equilibrium$^{16}$ and
occurrence of superconductivity$^{18}$, it implies that $C_e > 0$. In
addition, given the estimate$^{16}$ of $\frac{\partial \varphi}{\partial x}$, it may take a
very large value up to $C_e \approx 1F$. At last, by contrast
with $I_n$ being incoherent, the bound electrons contribute
an oscillating current $I_s(t) = Q_s = \frac{\partial \varphi}{\partial x}$ to $I(t)$.

The marked difference between the random diffusion current $I_n(t)$ and the time-periodic one $I_s(t)$ ensues from the
property that energy must be conserved during tun-
neling. This is automatically ensured$^{14}$ for an independ-
ent particle because its eigenenergy is defined uniquely
all over the electrodes $A,B$ and the insulating barrier,
whereas special care, as expressed in Eq.(4), must be
taken to enforce energy conservation for bound electrons
tunneling through a barrier. Unfortunately this cru-
cial constraint has been overlooked in the mainstream
analysis$^{9,12-14}$.

IV. ELECTRODYNAMICAL BEHAVIOUR

The total current $I(t)$ comprises 3 contributions, namely
$I_n = \frac{U}{R_t}, I_s = \dot{Q}_s$ and a component $C\dot{U}$, loading
the Josephson capacitor ($C$ refers to its capacitance),
so that the electrodynamical equation of motion reads
\begin{equation}
U_s = U + RI , \quad I = \frac{U}{R_t} + \dot{Q}_s + C\dot{U} ,
\end{equation}
which is finally recast into
\begin{equation}
\dot{U} = \frac{U_s - U \left( 1 + \frac{R}{R_t} + \frac{RC\omega t}{2} \sin (\omega t) \right)}{R \left( C + C_e \sin^2 \left( \frac{\omega t}{2} \right) \right)} . \quad (8)
\end{equation}

It is worth noticing that, due to $\frac{C_0}{C_e} \gg 1$, the denominator in the right-hand side of Eq.(8) would vanish for
$C_e < 0$, at some $t$ value, so that Eq.(8) cannot be solved
unless $C_e > 0 \Rightarrow \frac{\partial \varphi}{\partial x} < 0$, which confirms a previous$^{15-18}$
conclusion, derived independently.

$\frac{\partial \varphi}{\partial x}$ is expected$^{16}$ to increase with increasing $|I|$ and
is no longer defined for $|I| > I_M$, the maximum value of the bound electron current, because the sample goes
thereby normal. Consequently for practical purposes,
Eq.(8) has been solved by assuming $R_t (|I| \leq I_M) = \frac{R_0 g \left( \left| \frac{U}{R_M} \right| \right)}{R_n}$, $R_t (|I| > I_M) = R_n$ with $\frac{R_0}{R_n} \gg 1$,
$C_e (|I| \leq I_M) = C_0 g \left( \left| \frac{U}{R_M} \right| \right)$, $C_e (|I| > I_M) = 0$ with
$\frac{C_0}{C_e} \gg 1$, and $g(x) = 1 - x^2$. Regardless of the ini-
tial condition $U(0)$, the solution $U(t)$ of Eq.(8) becomes
time-periodic, i.e. $U(t) = U \left( t + \frac{2\pi}{\omega} \right)$, $\forall t$, after a short
transient regime.

Eq.(8) has been solved with the assignments $C = 1pF, C_0 = 1mF, R = 10\Omega, R_n = 100\Omega, R_0 = 10K\Omega$, and
the corresponding $U(t)$ have been plotted in Fig.4. The large slope $|\frac{dU}{dt}(0)| >> 1$ stems from $\frac{C\phi}{e} >> 1$. Since no experimental data of $U(t)$ have been reported in the literature to the best of our knowledge, no comparison between observed and calculated results can be done. Nevertheless, the large $u_M >> 1$ values, seen in Fig.4, have been indeed observed\textsuperscript{2}. Likewise, the calculated $u_M$ values have been found to increase very steeply with $U_s$ decreasing toward 0. Hence the thermal noise, generated by the $U_s$ source, will suffice even at $U_s = 0$ to give rise to sizeable $u_M$, which is likely to be responsible for the misconception\textsuperscript{0}, conveyed by claim 1 above. As a matter of fact, the noisy behaviour of the circuit sketched in Fig.1 has been reported\textsuperscript{2}.

The characteristics $I(U)$, plotted in Fig.5, have been reckoned as

$$\langle f \rangle = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} f(u)du ,$$

with $f = U, I$. In all cases, there is $\langle I \rangle (0) = 0$ with finite $\frac{d\langle I \rangle}{dU}(0)$ in agreement with the experimental data in Fig.2. However the slope $\frac{d\langle I \rangle}{dU}(0)$, calculated for $\omega_t = 100MHz$, is much larger than the one at $\omega_t = 1MHz$. Accordingly, the characteristics, reproduced in Figs.2,3, differ markedly by their slope at the origin, which might then hint at different tunneling frequencies.

Noteworthy is that there are no observed $\langle I \rangle$ data in Fig.3 over a broad $\langle U \rangle$ range, starting from $\langle U \rangle \approx 0$ up to a value big enough for the sample to go into the normal state, characterised by constant $I = I_n > I_M$. This feature might result\textsuperscript{2} from $U_s \propto \sin(\omega_p t)$ with $\omega_p = 60Hz$. Thus since the tunneling frequency $\omega_t$ is expected to decrease exponentially\textsuperscript{14,19} with increasing $n$ and hence $U$, this entails that the signal could indeed no longer be observed for $\omega_t < \omega_p$. Likewise, the observed frequency modulation\textsuperscript{3,14}, i.e. $\omega_t(n)$ is time-periodic, ensues from $n(t) \propto U(t)$ being time-periodic too (see Eq.(4)).

V. MICROWAVE MEDIATED TUNNELING

By irradiating the Josephson junction, depicted in Fig.1, with an electromagnetic microwave of frequency $\omega$, Shapiro observed\textsuperscript{2} the step-like characteristic $I(U)$, recalled in Fig.6. The discontinuities of $\frac{d\langle I \rangle}{dU}$, showing up at $\langle U \rangle = \frac{m\omega}{2\pi}$ with $m > 0$ being an integer, brought forward a cogent proof that the MBE state comprises an even number of electrons. In order to explain this experimental result, let us begin with studying the microwave induced tunneling of one bound electron pair across the $U_m = \frac{m\omega}{2\pi}$ biased barrier. The corresponding Hilbert space, describing the system before and after crossing, is subtended by the basis $\{ \varphi_{\pm} = \varphi_A(c_e) \otimes \varphi_B(c_e), \varphi_1 = \varphi_A(c_e + \frac{\pi}{4}) \otimes \varphi_B(c_e - \frac{\pi}{4}) \}$ of respective energies $V\xi_{\pm}, V\xi_{1} + m\omega$. The tunneling motion of one electron pair is then described by $\psi_0(t)$, solution of the Schrödinger equation

$$i\frac{\partial \psi_0}{\partial t} = H_0(t)\psi,$$

$$H_0 = m\omega_\sigma + 2 (\omega_t + \omega_s \sin(\omega t)) \sigma_x . \hspace{0.5cm} (9)$$

The Hamiltonian $H_0$ is expressed in frequency unit, $\frac{V\xi_0}{\hbar} + \frac{m\omega}{2\pi}$ is taken as the origin of energy, $\omega_r$ stands for the dipolar, off-diagonal matrix element\textsuperscript{28} (the microwave power is $\propto \omega^2$), and $\sigma_z, \sigma_x$ are Pauli’s matrices\textsuperscript{25}, projected onto $\{ \varphi_1, \varphi_{\pm} \}$. It is worth pointing out that Eq.(9) could be readily solved like Eq.(6), if $H_0$ were $t$ independent. Accordingly, in order to get rid of the $t$ dependence of $H_0$, we shall take advantage of a procedure devised for nonlinear optics\textsuperscript{29,30}.

To that end, $H_0$ is first recast into

$$H_0 = P_0 + f(t)\sigma_z , \hspace{0.5cm} (10)$$

for which $P_0 = m\omega_\sigma + 2\omega_1\omega_2$ is a Hermitian, $2 \times 2$, $t$ independent matrix, such that $(P_0)_{1,1} + (P_0)_{2,2} = 0$, $(P_0)_{2,2} - (P_0)_{1,1} = m\omega$, and $f(t) = \omega_t \sin(\omega t)$ is a real function of period $= \frac{2\pi}{\omega}$, having the dimension of a fre-
quency, such that \( \langle f \rangle = \int_0^{2\pi} f(t)dt = 0 \). Then \( H_0 \) is projected onto \( \{ \psi_-, \psi_+ \} \), the eigenbasis of \( P_0 \)

\[
G = TH_0T^{-1} = e\sigma_z + d(t)\sigma_x + g(t)\sigma_y \quad .
\]

(11)

\( T \) is the unitary transfer matrix from \( \{ \varphi_i, \varphi_1 \} \) to \( \{ \psi_-, \psi_+ \} \) and \( \sigma_x, \sigma_y \) have been projected onto \( \{ \psi_-, \psi_+ \} \). The corresponding eigenvalues are \( \mp \frac{\epsilon}{2} \) with \( \epsilon = \sqrt{(m\omega)^2 + \omega_i^2} \equiv m\omega \) because of \( \omega_l < \omega \), while the real functions \( d(t), g(t) \) have the same properties as \( f(t) \) in Eq.(10). Let us now introduce\(^{29,30} \) the unitary transformation \( R_1(t) \), operating in the Hilbert space, subtended by \( \{ \psi_-, \psi_+ \} \)

\[
R_1(t) = e^{i\Phi(t)} |\psi_- \rangle \langle \psi_-| + e^{-i\Phi(t)} |\psi_+ \rangle \langle \psi_+| \quad ,
\]

(12)

with the dimensionless \( \Phi(t) = \frac{m\omega}{2} - \int_0^t d(u)du \). We then look for \( \psi_1 = R_1^{-1}\psi_0 \), solution of the Schrödinger equation

\[
i\frac{\partial \psi_1}{\partial t} = H_1 \psi_1 \quad ,
\]

\[
H_1 = P_1 + R(z_1(t))\sigma_x + \Im(z_1(t))\sigma_y \quad ,
\]

(13)

for which the Hermitian \( 2 \times 2 \) matrix \( P_1 \) has the same properties as \( P_0 \) in Eq.(10), except for \( (P_1)_{2,1} \approx (m - 1)\omega, (P_1)_{2,2} = \omega_l = \omega_l/2 \) instead of \( (P_0)_{2,2} - (P_1)_{2,1} = m\omega, (P_0)_{2,2} = \omega_l \), the Pauli matrices \( \sigma_x, \sigma_y \) have been projected onto \( \{ \psi_-, \psi_+ \} \), and \( \Re(z_1(t)), \Im(z_1(t)) \) which are the real and imaginary parts of the complex function \( z_1(t) \), have the same properties as \( f(t) \) in Eq.(10). Consequently, iterating this procedure \( m \) of times yields finally

\[
i\frac{\partial \psi_m}{\partial t} = H_m \psi_m \quad ,
\]

\[
H_m = P_m + R(z_m(t))\sigma_x + \Im(z_m(t))\sigma_y \quad ,
\]

(14)

\[
P_m = \eta\sigma_x + 2(\Re(\omega_m))\sigma_x + \Im(\omega_m)\sigma_y
\]

for which the Pauli matrices \( \sigma_x, \sigma_y \) have been projected onto the eigenbasis of \( P_m \), \( \{ \psi_-, \psi_+ \} \), and \( \eta = 0 \), \( |\omega_m| < \omega \). The Fourier series \( \Re(z_m(t)), \Im(z_m(t)) \) of fundamental frequency \( \omega \) play no role, because the resonance condition\(^{25} \) \( |(P_m)_{1,1} - (P_m)_{2,2}| = \eta \) is not fulfilled due to \( |(P_m)_{1,1} - (P_m)_{2,2}| = |\eta| < \omega \), so that Eq.(14) is finally solved, similarly to Eq.(6), to give

\[
\psi_m = \cos \left( \frac{\omega_m t}{2} \right) \psi_- - i \sin \left( \frac{\omega_m t}{2} \right) \psi_+ \quad .
\]

The solution of Eq.(9) is thereby inferred to read

\[
\psi_0(t) = \left( \prod_{i=1,m} R_i(t) \right) \psi_m(t) \quad .
\]

(9)

\( U_m \) can be fitted to get \( \eta = 0 \). Thus, for the sake of illustration, calculated \( |\omega_m| \) and \( \delta_m = 1 - \frac{2U_m}{m\omega_m} \) are indicated in table I. As expected, \( |\omega_m| \) decreases steeply

\[
\begin{array}{c|c|c|c|c|c}
\hline
\omega_l & 100MHz & \omega_l & 1MHz \\
\hline
m & |\omega_m| & \delta_m & |\omega_m| & \delta_m \\
\hline
1 & 0.5 & 2 \times 10^{-4} & 0.5 & 2 \times 10^{-5} \\
2 & 5 \times 10^{-5} & 8 \times 10^{-5} & 5 \times 10^{-7} & 3 \times 10^{-5} \\
3 & 3 \times 10^{-6} & 3 \times 10^{-5} & 3 \times 10^{-6} & 8 \times 10^{-6} \\
4 & 10^{-10} & 2 \times 10^{-5} & 6 \times 10^{-13} & 4 \times 10^{-6} \\
5 & 5 \times 10^{-12} & 10^{-5} & 5 \times 10^{-12} & 3 \times 10^{-6} \\
6 & 5 \times 10^{-13} & 7 \times 10^{-6} & 5 \times 10^{-13} & 2 \times 10^{-6} \\
7 & 5 \times 10^{-13} & 5 \times 10^{-6} & 5 \times 10^{-13} & 10^{-6} \\
8 & 7 \times 10^{-13} & 4 \times 10^{-6} & 7 \times 10^{-13} & 9 \times 10^{-7} \\
9 & 10^{-12} & 3 \times 10^{-6} & 10^{-12} & 7 \times 10^{-7} \\
10 & 3 \times 10^{-12} & 3 \times 10^{-6} & 3 \times 10^{-12} & 6 \times 10^{-7} \\
11 & 4 \times 10^{-12} & 2 \times 10^{-6} & 4 \times 10^{-12} & 5 \times 10^{-7} \\
\hline
\end{array}
\]

with increasing \( m \) but, remarkably enough, \( |\omega_{2m+1}| \) decreases more slowly than \( |\omega_{2m}| \), all the more so since \( \omega_l \) is weaker. This property ensues\(^{25,28} \) from \( \omega_{2m} = 0 \), \( \forall m \) for \( \omega_l = 0 \).

Let us neglect \( 2U_m < 10^{-20} \), so that the energy of \( \psi_0 \) is taken to be constant and equal to \( V_E \). The coherent tunneling of \( \eta \gg 2 \) of bound electrons will thence be described by Eq.(7), except for \( \{ \psi_0, \varphi_f \}, \langle U \rangle - U_m, \langle I_m \rangle \), showing up instead of \( \{ \varphi_i, \varphi_j \}, \langle U \rangle, \langle I \rangle \), respectively, which entails that \( \langle I_m \rangle (U) - U_m = \langle I \rangle (U) \), as illustrated by Fig.5. Likewise, the contributions \( \langle I_{m=1,2,3,...} \rangle \) will add up together to give the step-like characteristic \( I(U) \), recalled in Fig.6. At last, Shapiro noticed\(^2 \) that

\[
\begin{array}{c|c|c|c|c|c}
\hline
\omega_l & 100MHz & \omega_l & 1MHz \\
\hline
m & |\omega_m| & \delta_m & |\omega_m| & \delta_m \\
\hline
1 & 0.5 & 2 \times 10^{-4} & 0.5 & 2 \times 10^{-5} \\
2 & 5 \times 10^{-5} & 8 \times 10^{-5} & 5 \times 10^{-7} & 3 \times 10^{-5} \\
3 & 3 \times 10^{-6} & 3 \times 10^{-5} & 3 \times 10^{-6} & 8 \times 10^{-6} \\
4 & 10^{-10} & 2 \times 10^{-5} & 6 \times 10^{-13} & 4 \times 10^{-6} \\
5 & 5 \times 10^{-12} & 10^{-5} & 5 \times 10^{-12} & 3 \times 10^{-6} \\
6 & 5 \times 10^{-13} & 7 \times 10^{-6} & 5 \times 10^{-13} & 2 \times 10^{-6} \\
7 & 5 \times 10^{-13} & 5 \times 10^{-6} & 5 \times 10^{-13} & 10^{-6} \\
8 & 7 \times 10^{-13} & 4 \times 10^{-6} & 7 \times 10^{-13} & 9 \times 10^{-7} \\
9 & 10^{-12} & 3 \times 10^{-6} & 10^{-12} & 7 \times 10^{-7} \\
10 & 3 \times 10^{-12} & 3 \times 10^{-6} & 3 \times 10^{-12} & 6 \times 10^{-7} \\
11 & 4 \times 10^{-12} & 2 \times 10^{-6} & 4 \times 10^{-12} & 5 \times 10^{-7} \\
\hline
\end{array}
\]

FIG. 6. Characteristics \( I(U) \), recorded by Shapiro\(^2 \) at 9.3GHz for A (vertical scale is 58.8 pV/cm, horizontal scale is 67 nA/cm) and 24.85GHz for B (vertical scale is 50 pV/cm, horizontal scale is 50 pA/cm).
some contributions $\langle I_m \rangle$ were missing in Fig.6. As explained above in section 4, this might result from the corresponding $|\omega_m| < \omega_p$ and thence would confirm $\omega_t << \omega$.

VI. NEGATIVE RESISTANCE

Signals $U(t), I(t) \propto \sin(\omega t)$, with the RF frequency $\omega$ defined by the resonance condition $LC\omega^2 = 1$, have been observed \cite{10} in the kind of setup, sketched in Fig.7. Due to $\omega \neq \omega_1$, the bound electron tunneling plays no role and the oscillation rather stems from $R(t)I(t)$ decreasing \cite{16} down to $R_n$ with $|I|$ increasing up to $I_M$, as indicated in section 4. Accordingly, since the voltage drop across the coil is equal to $LI$, the electrodynamical equation of motion reads

$$I = \frac{U}{R_t} + CU \Rightarrow \ddot{U} = \omega^2(U_s - U) - \frac{\dot{U}}{R_tC} \quad \text{.} \tag{15}$$

Linearising Eq.(15) around the fixed point $U_0 = U_s \Rightarrow I_0 = \frac{U_s}{R_t(I_0)}$, yields the differential equation

$$\ddot{U} = -\omega^2U - \frac{\dot{U}}{R_tC} \quad , \tag{16}$$

with the effective resistance $R_e$, defined by $R_e = R_t(I_0) + I_0\frac{dR}{dI}(I_0)$. Due to $\frac{dR}{dI} < 0$, the fixed point may be unstable in case of negative resistance $R_n < 0$, which will give rise to an oscillating solution of Eq.(15), $U(t) \propto \sin(\omega t)$. As a matter of fact, integrating Eq.(15) leads to the sine-wave, depicted in Fig.8. Note that, unlike $U(t)$ in Fig.4, every harmonic $\propto \sin(m\omega t)$ with $m > 1$ is efficiently smothered by the resonating $L, C$ circuit due to $LC(m\omega)^2 \neq 1$ for $m > 1$. At last, we have checked that Eq.(15) has no sine-wave solution for $\frac{R_n}{R_m} < 50$ or $U_s > R_nI_M$, because those inequalities entail that $R_n > 0$, which corresponds to a stable fixed point of Eq.(16).

VII. CONCLUSION

All experimental results \cite{1,2}, illustrating the Josephson effect, have been accounted for on the basis of bound electrons tunneling periodically across the insulating barrier. Likewise, the very existence of the Josephson effect has been shown to be conditioned by $\frac{\partial U}{\partial t} < 0$, which had previously been recognized as a prerequisite for persistent currents \cite{15}, thermal equilibrium \cite{16} and a stable superconducting phase \cite{18} too. The negative resistance feature \cite{10} has been ascribed to the tunneling resistance of independent electrons decreasing with increasing current, flowing through the superconducting electrodes, which confirms the validity of an analysis of the superconducting-normal transition \cite{16}.

By contrast with this work, $I_n(t), I_s(t)$ are dealt with on the same footing in the mainstream view \cite{9,12,14}, both resulting from the tunneling of independent particles, obeying Fermi-Dirac statistics. The only difference appears to be the one-particle density of states, namely either that associated with normal electrons for $I_n$ or Bogoliubov-Valatin \cite{26,27} excitations for $I_s$.

The coherent tunneling of bound electrons is thus concluded to be the very signature of the Josephson effect. Furthermore it has two noticeable properties:

- since coherent tunneling has been ascribed in section 3 to the properties of a MBE state, the time-periodic tunneling of bound electrons through a thin insulating barrier might be observed on a Josephson capacitor, for which the electrodes $A, B$ are made of magnetic metals \cite{31};

- the coherent tunneling motion seems to have no counterpart in the microscopic realm. For instance, the electrons, involved in a covalent bond, cannot tunnel between the two bound atoms because of their thermal decay toward the bonding ground-state. As for the Josephson effect, the bonding eigenfunction and its associated energy would read

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FIG. 7. Sketch of the electrical setup, displaying the negative resistance behaviour. $L$ refers to the self-inductance of the coil.

FIG. 8. Plots of the periodic solution $I(t), U(t)$ of Eq.(15), reckoned with $U_s = 10\mu V, I_M = 0.1mA, L = 1\mu H, C = 100\mu F, \omega = 100 MHz$. 

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\[ \varphi_b = \frac{\varphi_i + \varphi_f}{\sqrt{2}} \] and \[ V \mathcal{E}_i - \frac{\hbar \omega}{2}, \] respectively, but the relaxation from the tunneling state \[ \psi(t) \] in Eq.(6) toward \[ \varphi_b \] might occur only inside the insulating barrier, which is impossible because the valence band, being fully occupied, can thence accommodate no additional electron.

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1. P.W. Anderson and J.M. Rowell, Phys.Rev.Lett., 10, 230 (1963)
2. S. Shapiro, Phys.Rev.Lett., 11, 80 (1963)
3. S. Shapiro et al., Rev.Mod.Phys., 36, 223 (1964)
4. T. Nagatsuma et al., J.Appl.Phys., 54, 3302 (1983)
5. D. R. Gulevich et al., Prog.In.Electro.Res.Symp., IEEE, 3137 (2017)
6. B. Douçot and J. Vidal, Phys. Rev. Lett., 88, 227005 (2002)
7. M. H. Devoret and R. J. Schoelkopf, Science, 339, 1169 (2013)
8. A. Deville and Y. Deville, Quant.Inform.Process., 18, 320 (2019)
9. B.D. Josephson, Phys.Lett., 1, 251 (1962)
10. D.E. McCumber, J.Appl.Phys., 39, 3113 (1968)
11. V.L. Ginzburg and L.D. Landau, Zh.Eksperim.i.Teor.Fiz., 20, 1064 (1950)
12. N.R. Werthamer, Phys.Rev., 147, 255 (1966)
13. A. I. Larkin and Y. N. Ovshinnikov, Sov.Phys.JETP, 24, 1035 (1967)
14. A. Barone and G. Paterno, Physics and Application of the Josephson Effect, ed. John Wiley & Sons (1982)
15. J. Szeftel, N. Sandeau and M. Abou Ghandous, Eur.Phys.J.B, 92, 67 (2019)
16. J. Szeftel, N. Sandeau and M. Abou Ghandous, J.Supercond.Nov.Magn., 33, 1307 (2020)
17. J. Szeftel, N. Sandeau, M. Abou Ghandous and A. Khater, EPL, 131, 17003 (2020)
18. J. Szeftel, N. Sandeau, M. Abou Ghandous and M. El-Saba, J.Supercond.Nov.Magn., 34, 37 (2021)
19. L. Schiff, Quantum Mechanics, ed. McGraw-Hill (1969)
20. J. Szeftel, N. Sandeau and A. Khater, Phys.Lett.A, 381, 1525 (2017)
21. J. Szeftel, N. Sandeau and A. Khater, Prog.In.Electro.Res.M, 69, 69 (2018)
22. J. Szeftel, M. Abou Ghandous and N. Sandeau, Prog.In.Electro.Res.L, 81, 1 (2019)
23. J. Bardeen, L.N. Cooper and J.R. Schrieffer, Phys.Rev., 108, 1175 (1957)
24. N.W. Ashcroft and N. D. Mermin, Solid State Physics, ed. Saunders College (1976)
25. A. Abragam, Nuclear Magnetism, ed. Oxford Press (1961)
26. R.D. Parks, Superconductivity, ed. CRC Press (1969)
27. J.R. Schrieffer, Theory of Superconductivity, ed. Addison-Wesley (1993)
28. R. Boyd, Nonlinear Optics, ed. Academic Press USA (1992)
29. J. Szeftel, N. Sandeau and A. Khater, Opt.Comm., 282, 4602 (2009)
30. J. Szeftel et al., Opt.Comm., 305, 107 (2013)
31. P.L. Lévy, Magnetism and Superconductivity, ed. Springer (2000)