Flux surface shaping effects on tokamak edge turbulence and flows

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(Dated: November 20, 2018)

Shaping of magnetic flux surfaces is found to have a strong impact on turbulence and transport in tokamak edge plasmas. A series of axisymmetric equilibria with varying elongation and triangularity, and a divertor configuration are implemented into a computational gyrofluid turbulence model. The mechanisms of shaping effects on turbulence and flows are identified. Transport is mainly reduced by local magnetic shearing and an enhancement of zonal shear flows induced by elongation and X-point shaping.

PACS numbers: 52.25, 52.35Ra

I. INTRODUCTION

It can be regarded as one of the most intriguing results of half a century in physics of hot magnetized plasmas, that in macroscopically stable equilibria the confinement is nevertheless determined by micro-scale instabilities and fluctuations, with transport of energy and particles across magnetic flux surfaces being dominated by ubiquitous turbulence on small (gyro radius) scales and low (drift) frequencies [1, 2, 3, 4, 5, 6, 7]. This turbulence is regulated by the formation of mesoscopic zonal structures out of the turbulent flows [8, 9, 10, 11, 12, 13].

Criteria for design of new magnetized plasma experiments for fusion research like ITER are primarily based on empirical scaling laws for the energy confinement time . First-principle based transport models still require reference to experimental scalings, and are validated mainly only for core plasmas excepting the pedestal region [5]. An improved understanding of the underlying turbulent dynamics and of its relation to design parameters, like the shape of the confining magnetic field, will facilitate the development of advanced magnetic confinement experiments.

The plasma shape of a tokamak enters into confinement and transport modelling through parameters specifying a vertical elongation \( \kappa \geq 1 \) and an outboard side triangularity \( \delta \geq 0 \) that describe the deviation from a simple circular torus. The design criteria of the ITER plasma shape with \( \kappa = 1.7 \) and \( \delta = 0.33 \) (at 95% flux surface) are based on a conservative regime that is well established in present experiments [11]. More extreme variations of flux surface shaping were investigated with the experiment TCV (Tokamak à Configuration Variable) which is able to achieve configurations up to \( \kappa = 3 \) and \( \delta = \pm 0.5 \) [12]. Experimental evidence suggests that large elongation is always beneficial for local and global energy confinement, whereas the role of triangularity depends also on the value of the plasma pressure gradient.

An important task for computational plasma physics is to determine the effects of flux surface shaping on turbulence and transport by first-principle direct numerical simulation. Shaping effects are complicated even for apparently simple circular equilibria [16], and efforts to perform modelling for edge plasmas most often incorporate technically complicated prescriptions in an inflexible way [17]. An advantageous way to explore shaping is through the use of a covariant metric in a model with Cartesian topology [18, 19], but even then it was found that the use of experimental geometry, while most realistic for modelling purposes, may not be helpful to theoretical understanding, and hence standard “kappa-delta” models were used [20]. In attempt to understand the effects of stellarator geometry, a set of analytical forms were used for the metric, varying each in turn to provide a small set of steps from tokamak to stellarator geometry [21]. Forms
like this were also used to characterise the X-point effects, again making simple control tests tractable.  

In the following, we report on the results of a more systematic series of “kappa-delta” tokamak configurations with varying elongation and triangularity, and an actual ASDEX Upgrade divertor configuration with the equilibrium code HELENA. These are implemented into the gyrofluid code GEM, which is updated from Ref. 21 for computations of edge turbulence and flows. Specific effects of flux surface shaping on drift-Alfvén (DALF) and ion temperature gradient (ITG) turbulence in the tokamak edge plasma are analysed and compared, and the mechanisms of the coupling between geometry and turbulence are identified. The emphasis is on physical mechanism, which is complimentary to and should also be helpful to the understanding of the more experimentally oriented modelling efforts.

It is found that turbulent transport fluxes are reduced by increasing flux surface elongation $\kappa$. The influence of triangularity $\delta$ is much weaker and generally also depends on elongation and on a pressure shift. The geodesic acoustic oscillation modes (GAM) and the overall turbulence frequency spectra depend on flux surface geometry. An enhancement of zonal flow amplitude in the spectra by elongation and X-point shaping is found to be weak in the DALF case and more pronounced in the ITG case. The reduction of transport observed in our simulations in shaped tokamak geometry is mainly a result of local and global magnetic shear.

We begin in Section II with a brief review and discussion of the role of sheared flows in tokamak turbulence in context of the toroidal flux surface geometry. In section III the gyrofluid electromagnetic model GEM for edge turbulence computations is presented, and the flux tube representation of tokamak geometry within a GEM turbulence code is discussed in Section IV. Transport scalings by elongation and triangularity, that are obtained with this model, are presented in Section V. Resulting frequency and wave number spectra in tokamak divertor geometry are discussed in Section VI. Conclusions and implications of the results are presented in Section VII.

II. THE ROLE OF SHEARED FLOWS IN TOKAMAK TURBULENCE

One of the main requirements for successful performance of fusion experiments is the prospect of operation in a high confinement H-mode. The formation of a characteristic transport barrier in magnetically confined toroidal H-mode plasmas is closely related to the presence of a radially varying electric field near the last closed flux surface. A radical electric field, $E_r$, gives rise to a perpendicular E-cross-B (ExB) flow through the relation $v_E = (c/B^2)E \times B$. If $E_r$ varies with $r$, then this flow is sheared, and this flow shear is what is invoked to explain the transport barrier. The flow shear (vorticity) is expected to suppress turbulent transport either by shearing the eddies apart or by simply tilting them so that their Reynolds-stress interaction with the vorticity layer tends towards energy transfer out of the eddies and into the flow. Neo-classical equilibrium effects can also produce these shear layers. As opposed to these mean ExB flows, the fluctuating zonal flows in direct interaction with the turbulence have also been observed in tokamak edge regions.

The transition between states of low (L) plasma confinement to the high (H) confinement regime featuring such an edge transport barrier is up to now not satisfactorily described by any of the existing first-principle theories. The notion that generation of zonal flows by direct drive of the turbulence can act to self-regulate by suppressing the driving turbulent vortices has lead to the development of predator-prey type bifurcation models that are able to describe specific characteristics of the L-H transition (see e.g. [37]). Early computations based on the resistive-g and collisional drift wave models found self-generated sheared flows which could be argued as an L-H transition trigger. However, more recent models in more comprehensive treatment of the toroidal geometry and also the full range of scales of motion do not find this. The phenomenology of these self-consistent zonal flows receives wide interest, but they serve as a moderator of the turbulence rather than a suppression mechanism, as the energetic interactions with not only the pressure but also magnetic parts of the equilibrium tend to place each part of the overall dynamical system, which naturally includes the flows themselves.

After the initial discovery of the H-mode in the divertor experiment ASDEX, it has been found in numerous large tokamaks around the world that the presence of a divertor (originally designed to improve the impurity exhaust) significantly enhances the prospect of reaching this state. However, up to now both the transition models as well as the numerical simulations have relied mainly on simplified representations of the plasma geometry. In our present work we investigate the effects of realistic flux surface shaping on tokamak edge turbulence using a gyrofluid model recently corrected for energy conservation properties, thereby updating previous gyrofluid results. This provides a counterweight to existing efforts based on collisional fluid equations and associated numerical schemes which cannot treat the ion gyroradius scales and therefore overestimate linear phenomena based in the larger scales. The ion gyroradius necessarily enters because the dynamics is sensitive to phenomena in the range of the spectrum with perpendicular wavenumbers only slightly less than the inverse of the drift scale ($\rho_s$, also called “ion sound gyroradius”), and with comparable ion and electron temperatures $\rho_s$ is commensurate with the ion gyroradius $\rho_i$. A schematic view of a shaped plasma cross section including a flux-tube section from turbulence computations is shown in Fig. 4.
Transport in a fusion edge plasma is dominated by turbulent low-frequency drift wave motion that causes a fluid-like convection through ExB vortices in a plane perpendicular to the magnetic field direction, acting in concert with parallel coupling dynamics tending towards dissipation. Toroidal compressibility of zonal (flux surface averaged) ExB flows opens channels in the nonlinear dynamics of the vortex-flow interaction that strongly affect the flow energetics. The origin of the compression is the geodesic curvature of the magnetic field lines. The flow is thereby coupled with poloidally asymmetric pressure sidebands which are consumed by the turbulence and global Alfvén or parallel flow dynamics. This energetics was given in Ref. and the sensitivity of the turbulence to the geodesic energy transfer effect was explored using an additional scaling parameter for the strength of its coupling.

This geodesic transfer mechanism represents a restoring loss channel for the zonal flows, ultimately placing them in statistical equilibrium with the turbulence, with the Reynolds stress (spin-up) mechanism continuing to operate. Since the latter acts on all frequencies, the geodesic acoustic oscillation (GAM) itself need not be present for the geodesic transfer mechanism to operate. The net result of this is that turbulent transport in 3D toroidal edge computations is found to be reduced but not completely suppressed by the self-generated zonal flows. This geodesic transfer effect was first studied for the case of a simple circular toroidal magnetic field. In the following we give both numerical evidence and a physical picture of how plasma shaping and the presence of a divertor via its modification of plasma geometry can influence the outcome of the gyrofluid turbulence/flow interaction.

III. THE GYROFLUID ELECTROMAGNETIC MODEL GEM

The dynamical character of cross-field ExB drift wave turbulence in the edge region of a tokamak plasma is governed by electromagnetic and dissipative effects in the parallel response. The most basic drift–Alfvén (DALF) model to capture the drift wave dynamics includes nonlinear evolution equations of three fluctuating fields: the electrostatic potential, electromagnetic potential, and density. The tokamak edge usually features a more or less pronounced density pedestal, and the dominant contribution to the free energy drive to the turbulence by the inhomogeneous pressure background is thus due to the density gradient. On the other hand, a steep enough ion temperature gradient (ITG) in the edge does not only change the turbulent transport quantitatively, but adds new interchange physics into the dynamics. In addition, more field quantities have to be treated: parallel and perpendicular temperatures and the associated parallel heat fluxes, for a total of six moment variables for each species. Finite Larmor radius effects introduced by warm ions require a gyrofluid description of the turbulence equations. Both the resistive DALF and the ITG models can be covered by using the six-moment electromagnetic gyrofluid model GEM, but we will refer for the DALF model to its more economical two-moment version.

The gyrofluid model is based upon a moment approximation of the underlying gyrokinetic equation. The first complete six-moment formulation was given for slab geometry, and later extended to incorporate toroidal effects using a ballooning-based form of flux surface geometry. Electromagnetic induction and electron collisionality were then included to form a more general gyrofluid for edge turbulence, with the geometry correspondingly replaced by the version from the edge turbulence work, which does not make ballooning assumptions and in particular represents slab and toroidal eigenmode types equally well and does not require radial periodicity. Energy conservation considerations were solidified first for the two-moment version, and most recently for the six-moment version. This latter reference currently defines the GEM model, energetics, and also the numerical techniques. The present paper uses the six-moment equation set for each species (Eqs. 99-104), induction and polariisation equations (Eqs. 89,92), the zero gyroradius limit for electrons, the local model with separate linear drive terms and Dirichlet boundary conditions (Sec. IV A), Padé forms for gyroaveraging operators (Sec. IV B), and the dissipation model and numerical scheme (Secs. IV C-E), all with equation and Section numbers from Ref. This is the standard local form for the GEM code.

The flux surface geometry of a tokamak enters into the gyrofluid equations via the curvilinear generalisation of differentiation operators and via inhomogeneity of the magnetic field strength \( B \). The different scales of equilibrium and fluctuations parallel and perpendicular to the magnetic field motivate the use of field aligned flux coordinates. The differential operators in the field aligned frame are the parallel gradient

\[
\nabla_\| = (1/B)(\mathbf{B} + \mathbf{B}_\perp) \cdot \nabla, 
\]

with magnetic field disturbances \( \mathbf{B}_\perp = (-1/B)\mathbf{B} \times \nabla \mathbf{A}_\| \) as additional nonlinearities, the perpendicular Laplacian

\[
\nabla_\perp^2 = \nabla \cdot [(-1/B^2)\mathbf{B} \times (\mathbf{B} \times \nabla)],
\]

and the curvature operator

\[
\mathcal{K} = \nabla \cdot [(c/B^2)\mathbf{B} \times \nabla].
\]

The dynamical character of the system is further determined by a set of parameters characterising the relative role of dissipative, inertial and electromagnetic effects in addition to the driving by gradients of density.
and temperature. In particular, we specify collisional

\( C = 0.51(\nu e_cR_{Lz}/L_{\perp})(m_e/M_i) \), magnetic induction

\( \beta = 4(\pi p_e/B^2) \), electron inertia \( \hat{\mu} = \hat{\mu}(m_e/M_i) \) and ion

inertia \( \hat{\epsilon} = (qR/L_{\perp})^2 \), where \( L_{\perp} \) is the background gra-
dient scale length, and \( \epsilon_s = \sqrt{T_e/M_i} \) is the sound speed.

We use a standard set of edge parameters given as \( C = 5, \beta = 1, \hat{\mu} = 5 \) and \( \hat{\epsilon} = 18350 \).

The perpendicular scale length for the DALF model is

set as \( L_{\perp} = L_n \) to the density gradient length, and for

the ITG model as \( L_{\perp} = L_{T_i} = 0.5L_n = 0.5L_{T_e} \) to the ion

temperature gradient length, so that \( \eta_i = L_n/L_{T_i} = 2 \). The ITG model has otherwise identical parameters. Accu-

rate experimental measurement of ion temperature

profiles in the tokamak pedestal region are only recently

being advanced and have previously been plagued by

large uncertainties. The present experimental knowledge

of edge parameters can thus be seen commensurate with

both the DALF and ITG model cases used here.

The computational domain is set to \( 64 \times 256 \) nodes in

units of the drift scale \( \rho_e = (c/eB)\sqrt{T_e/M_i} \) for \( (x,y) \)

and 16 nodes in one field line connection length \( (2\pi qR) \)
in \( -\pi < z < \pi \).

The dimensions are chosen to appropriately account

for statistical isotropy in small scales in both perpendicular

directions with satisfactory spectral overlap, and for

an extended box size in the electron drift direction \( y \) to

achieve convergence. A box resolution down to \( \rho_e \) guar-

antees inclusion of all scales necessary for the nonlinear

dynamics that are essential for the drift wave turbulence

characteristics.

![FIG. 2: Metric element \( g^{xx}, g^{xy} \) (unshifted), normal curvature \( K^y \), geodesic curvature \( K^x \), magnetic field strength \( B \) and local magnetic shear \( S \) in a tokamak with elongation \( \kappa = 1 \) and triangularity \( \delta = 0 \) (dashed lines), compared to a configuration with \( \kappa = 2 \) and \( \delta = 0.4 \) (thin solid lines) and to an actual ASDEX Upgrade configuration (bold solid lines).](image)

### IV. FLUX TUBE REPRESENTATION OF TOKAMAK GEOMETRY

Tokamak equilibria are computed by solving the Grad-

Shafranov / Lüst-Schlüter equation with the code HE-

LENA [24], implemented as described in Ref. [21]. A set

of nested flux surfaces in straight field line Hamada coor-

dinates \( (V, \theta, \zeta) \) is obtained by specification of given radi-

al profiles of pressure and rotational transform, and of

the shape of the bounding last closed flux surface. These

Hamada coordinates are then transformed into a field

aligned system and re-scaled into local flux tube coor-

dinates \( (x,y,z) \). Global consistency in the parallel

boundary condition for one connection length is maintained

[53]. A transformation of the coordinate \( y \), which signifies

the electron diamagnetic drift direction perpendicu-

lar to the magnetic field within flux surfaces, is ap-

plied in order to avoid grid deformation by local magnetic

shear [19]. Otherwise, grid cells are sheared strongly in \( y \)
direction particularly near the X-point region, with ma-

ligence consequences on nonlinear dynamics, especially in

the vorticity, that lead to a violation of the basic drift

wave character and overestimate linear MHD-like dyna-

mics. The differential operators are then expressed in terms

of the flux tube coordinates: The curvature operator be-

comes

\[
K = K^x(z)\nabla_x + K^y(z)\nabla_y, \tag{4}
\]

the perpendicular Laplacian in flute mode ordering is

\[
\nabla_{\perp}^2 = g^{xx}(z)\partial_{xx} + 2g^{xy}(z)\partial_{xy} + g^{yy}(z)\partial_{yy}, \tag{5}
\]

and the parallel derivative is

\[
\nabla_{\parallel} = b^x(z)\partial_z, \tag{6}
\]

noting that the factor of \( B^2 \) in \( \rho_e^2 \) also depends on \( z \).

Some metric coefficients \( g^{ij} \) that were obtained for elonga-

tion \( \kappa = 1 \) and \( \kappa = 2 \) with triangularity \( \delta = 0 \) and \( \delta = 0.4 \)

are shown in Fig. 2. Increasing elongation \( \kappa \) specifi-

cally rises the local magnetic shear \( S = \nabla_{\parallel}(g^{xy}/g^{xx}) \) and

reduces \( K^x \) both in the upper and lower regions of the

torus that correspond to flux tube coordinates \( z = \pm \pi/2 \).

In the following, the configuration with \( \kappa = 1 \) and \( \delta = 0 \)

will be referred to as a “simple circular torus” (SCT).

Local and global magnetic shear are in general known
to have a damping influence on tokamak edge turbu-

lence [22], whereas geodesic curvature acting through \( K^x \)

upon the axisymmetric component \( (k_y = 0 \) components, or “modes”) maintains the coupling for a loss channel from zonal flow energy eventually to turbulent vortices

[17]. Both mechanisms help to reduce turbulent trans-

port. Normal curvature \( K^y \) on the other hand strengthens

primarily the interchange forcing of the turbulence

\( (k_y \neq 0) \).
V. TRANSPORT SCALING BY ELONGATION AND TRIANGULARITY

A series of tokamak equilibria is constructed for elongation $1 \leq \kappa \leq 2$ and triangularity $0 \leq \delta \leq 0.4$ with equal profiles of pressure and rotational transform. The flux tube is chosen at a radial position $r = \sqrt{V/V_0} = 0.95$, where $V_0$ is the volume enclosed by the last closed flux surface. The background density profile and rotational transform for the GEM turbulence computations are linearised within the bounds of the radial computational domain. We restrict our simulations for now to the closed field line region lying a few tens of ion gyroradii inside from the separatrix, thus avoiding complications that occur by a divergent metric and associated grid deformation when the last closed flux surface is approached in the vicinity of an X-point. Ultimately, the goal of edge turbulence simulations will be to combine sufficiently well resolved nonlocal drift wave computations with a representation of the realistic field line geometry crossing the separatrix to the bounded scrape-off layer region.

When the plasma shape is thus varied, we have a particular interest in the effects on fluctuation time and spatial scales, on the radial variation of flux surface averaged (zonal) flows, and on turbulent transport. We first apply the ensemble of shaped tokamak equilibria to the DALF model. The cross-field turbulent electron transport is determined by the flux surface average of the radial $E \times B$ convection of the fluctuating density by $F_e = \langle \dot{\bar{n}}_e \bar{n}_e \rangle_{yz}$, given in standard gyro-Bohm normalisation to $n_e c_s (\rho_s/L_L)^2$. The transport flux $F_e(t)$ is fluctuating in time, and for specifying a quantitative value to it the time average over a sufficiently long window, that covers all relevant frequency scales, is taken during the final phase of simulations after all initial linear transients and spin-up of flows and oscillations have reached saturation.

As shown in Fig. 3 we find that $F_e$ is reduced to around a third when elongation is increased from a circular cross section to $\kappa = 2$. Scaling $F_e$ against triangularity $\delta$, we find in Fig. 4 that for $\kappa = 1$ the transport flux is independent from $\delta$ within the error bars given by the deviation of the fluctuating data from its time average. For higher elongation $\kappa = 2$ the transport $F_e$ is increased from the normalised value of 0.20 to 0.28 by increasing $\delta$ from 0 to 0.4. A good fit to the transport data in the DALF model within fluctuation error bars is obtained by

$$F_e^{\text{DALF}}(\kappa, \delta) \approx F_{1.0} \cdot \kappa^{-2.15+1.46\delta} \quad (7)$$

where $F_{1.0} = F(\kappa=1, \delta=0)$. This reduction of transport by elongation can be related to three different causes:

1. Increase of local and global magnetic shear (magnetic shear damping),
2. Reduction of curvature drive (interchange coupling), and
3. Enhancement of sheared zonal flows (geodesic transfer).

In Section VI we will present results showing that the effect of flow shear enhancement by a weakening of the geodesic transfer mechanism is insignificant for DALF parameters. Also for these resitive electromagnetic edge parameters, the interchange dynamics in the normal curvature forcing is acting more as a catalyst than as a drive mechanism for the turbulence, with only weak ballooning of the fluctuating quantities observed. On the other hand, it has been established that local magnetic shear by flux surface deformation has a strong damping influence on tokamak edge turbulence: comparing the case of a simply circular torus (with global shear only) to a case where the local magnetic structure is typical of a shaped tokamak (but the curvature properties are those of a circular torus), a reduction of transport to around a half was found. Moreover, the effective topological global shear (defined, e.g., in Eq. 29 of Ref. [54]) is also increased by finite aspect ratio and elongation, but is not significantly affected by moderate triangularity ($\delta < 0.6$ or so). It can thus be argued, that the major mechanism
for reduction of transport by elongation in the DALF case is magnetic shear damping, with an additional (slightly lesser) influence by interchange drive reduction, and an only insignificant zonal flow enhancement.

We now include ion temperature gradient (ITG) driven dynamics by allowing for a gradient ratio \( \eta_i = 2 \) and using the full six-moment version of GEM. In the ITG model, the flux-surface averaged turbulent transport can be characterised by particle transport \( F_n = \langle \dot{n} \hat{v}_x \rangle \) and by heat transport \( Q_i = Q_i^{cv} + Q_i^{cd} \) with convective component \( Q_i^{cv} = \frac{3}{2} \tau_i (\langle \hat{n} \hat{v}_x \rangle + \langle T_{||} \hat{w}_x \rangle) \) and conductive component \( Q_i^{cd} = \tau_i (\langle \frac{1}{2} T_{||} \hat{v}_x \rangle + \langle T_{\perp} \hat{w}_x \rangle + \langle \hat{n} + 2T_{\perp} \rangle \hat{w}_x \rangle \), where \( \hat{w}_x \) is the gyro-averaged ExB velocity and \( \hat{w}_x \) is its FLR correction. Normalisation is to standard gyro-Bohm units with the gradient scale length set to unity, and \( \tau_i = T_i / T_e \) is the temperature ratio.

When displaying heat and particle transport in relation to elongation and triangularity now for the ITG model, we find in Fig. 5 a scaling of \( Q_i \sim \kappa^{-2.6} \) and \( F_n \sim \kappa^{-2.3} \). The reduction by elongation is thus stronger than for the DALF model. One evident mechanism for this increased reduction in the ITG case is the weakening of the interchange drive of the ITG instability acting via the normal curvature \( K^x \), which is slightly reduced by elongation in the ballooning region of the flux tube (compare Fig. 2). In contrast to DALF dynamics, the ITG turbulence is strongly driven by “unfavourable” normal curvature, leading to a pronounced ballooning character of the fluctuations, as shown in Fig. 6.

We find as an additional mechanism in the ITG case a significant weakening of the geodesic transfer mechanism mediated by \( K^y \), which is discussed in the following section.

Similar as in the DALF case, both heat and particle transport are slightly increased with higher triangularity for an elliptical cross section \( (\kappa = 2) \) in Fig. 5 while for small \( \kappa \) no influence from triangularity is observed within the fluctuation error bars.

Summarising the results of this section on the influence of flux surface shaping on turbulent transport for different edge parameters, we find that the main contribution to transport reduction in both the DALF and ITG regimes is magnetic shear damping by elongation.

Shear flow enhancement by the reduced geodesic transfer mechanisms is found to be relatively weak in the DALF case, but is becoming relevant in the ITG model, as can be seen from analysing the wave number and frequency spectra discussed in the next section.

For fixed gradients, the local diffusivity is proportional to the transport and thus to the roughly inverse quadratic scaling with \( \kappa \) given above. Global transport across an entire magnetic flux surface can in principle be obtained by integration of the local transport coefficients, that takes the variation of surface area with elongation into account.

FIG. 5: ITG: Dependence of turbulent particle transport \( F_n \) and heat transport \( Q_i \) (in gyro-Bohm units) on elongation \( \kappa \) for triangularity \( \delta = 0 \).

FIG. 6: ITG: Dependence of turbulent particle transport \( F_n \) and heat transport \( Q_i \) (in gyro-Bohm units) on triangularity \( \delta \) for elongation \( \kappa = 1 \) and 2.

FIG. 7: ITG: In the shaped configuration with \( \kappa = 2 \) the coupling quantity \( h_\phi (z) = \bar{h}_\phi (z) - \hat{\phi} (z) \) has a reduced squared amplitude compared to \( \hat{\phi} (z) \), indicating a weaker interchange ballooning character of the turbulence than in a circular torus.
VI. FREQUENCY AND WAVENUMBER
SPECTRA OF FLOWS AND FIELDS

Time traces are resolved down to 1/20th of the drift frequency $\omega_{DW} = c_s/L_{\perp}$, and long computational runs are required to account for zonal flows with $\omega \approx 0$. A typical number of time steps is $10^9$.

Geodesic acoustic oscillations (GAM) may be distinctly detectable for some parameters and are expected for circular geometry around $\omega/\omega_{DW} = (2L_{\perp}/R)\sqrt{(1 + \tau_i)/2}$. In the DALF case for $\tau_i = 0$ we have $\omega_{GAM} = 0.035\omega_{DW}$, and in the ITG case for $\tau_i = 1$ we have $\omega_{GAM} = 0.05\omega_{DW}$.

In general, the GAM frequency is determined by the geodesic curvature term $K^x$ and is influenced by flux surface deformation. The geodesic curvature effect is the action of the curvature operator $K$ upon the axisymmetric component ($k_y = 0$). In local flux tube coordinates we find $K^x \sim (1/\kappa)$, and the factor $1/\kappa$ in the geodesic curvature term does also accordingly scale the geodesic acoustic frequency: stronger elongation shifts the GAM resonance frequency in the spectrum closer to the zero-frequency zonal flow. The same argument, now applied to the normal curvature term $K^y$, also accounts for a $\kappa^{-1}$ scaling of the (interchange) drift wave frequency.

For DALF parameters used in our computations, the GAM peak does not protrude very distinctly but can still be identified on the flat top of the spectra. In Fig. 8 the discrete Fourier transform spectrum $\tilde{\phi}(\omega)$ for a measurement of $\hat{\phi}$ at a local point in the centre of the $xyz$-domain is shown: the spectrum below the GAM resonance is mostly flat except for the distinct zonal flow peak at $\omega = 0$. The higher frequency part shows a typical $\omega^{-\alpha}$ cascade structure, where the exponent $\alpha$ changes around the drift frequency $\hat{\omega} \approx 1$, which for our choice of edge parameters coincides also with the Alfvén frequency.

One may identify three ranges in the spectrum: A flat top region between the zonal flows and GAM frequencies, one intermediate range between the GAM and drift wave frequencies with $\alpha \approx 1 - 2$, and a cascade range between the drift wave and the dissipation times scale with $\alpha \approx 3 - 5$. As expected, both GAM and drift wave frequencies are reduced by increasing elongation to lower values $\omega \rightarrow \omega/\kappa$. The whole spectrum is thus shifted to the left for $\kappa = 2$ in Fig. 8.

Using ITG parameters, the GAM peak is more clearly visible for the circular torus case but vanishes for elongation $\kappa = 2$, as can be seen in Fig. 9. The whole spectrum experiences again a shift to lower frequencies for the elongated case. The $\omega = 0$ zonal flow component in the ITG spectrum exhibits significantly higher amplitude for elongation $\kappa = 2$ than for the circular torus.

In addition to computing the flux surface shapes with varying elongation and triangularity discussed above, HELENA is also used to obtain equilibria with an outer boundary defined by an experimentally reconstructed separatrix position. In this way we obtain an equilibrium in the typical shape of an ASDEX Upgrade (AUG) lower single null divertor configuration (as shown in Fig. 1). The additional asymmetric shaping by the presence of an X-point in the lower part of the torus leads to an increase of local magnetic shear near $z = -\pi/2$ and an associated local reduction of geodesic curvature. The elongation and triangularity of this configuration are otherwise comparable to the kappa-delta model with $\kappa \approx 1.6$ and $\delta = 0.3$. This AUG configuration was first used in edge turbulence computations in Ref. [13], where the dependence of transport on $\hat{\beta}$ was established: it was found that the onset of MHD ballooning mode turbulence is prevented for typical tokamak edge parameters due to the shaping effects in realistic geometry, and the nature of transport is still basically of the drift-Alfvén wave character.

![Fig. 8: DALF: Local frequency spectra $\tilde{\phi}(\omega)$ for $\kappa = 1$ (thin line) and $\kappa = 2$ (bold line), both for $\delta = 0$. Arrows indicate the drift wave and GAM resonances for $\kappa = 1$.](image1)

![Fig. 9: ITG: Local frequency spectra $\tilde{\phi}(\omega)$ for $\kappa = 1$ (thin line) and $\kappa = 2$ (bold line), both for $\delta = 0$. Arrows indicate the drift wave and GAM resonances for $\kappa = 1$.](image2)
The AUG model has, for the present flux tube position, properties relevant to the turbulence dynamics that are in some aspects in between those of up-down symmetric configurations with \( \kappa = 1.6 \) and 2.0. Transport in the AUG model is for DALF parameters found to be reduced to \( F_e = 0.5 \) compared to \( F_e = 0.9 \) for the circular torus. The nonlinear DALF growth rate \( \Gamma_N = F_e/2E_{tot} \) is only slightly lower for the AUG model with \( \Gamma_N = 0.007 \pm 0.001 \) than for the circular torus with \( \Gamma_N = 0.0085 \pm 0.001 \).

For ITG parameters, the particle transport is nearly identical in the AUG and SCT cases, both at \( F_e = 1.0 \). The nonlinear ITG growth rate is also similar, \( \Gamma_N = 0.0080 \pm 0.0020 \) for AUG and \( \Gamma_N = 0.0088 \pm 0.0024 \) for the circular torus.

A reduction of geodesic curvature by elongation and X-point shaping might be expected to lead to a weakening of the geodesic transfer coupling mechanism for energy from zonal flows to GAMs. This can be analysed by comparing the flux surface averaged vorticity \( \langle \Omega_E(x) \rangle = \partial_x \langle u^x_E(x) \rangle \) for the configurations of the circular torus with the AUG model. The zonal vorticity \( \langle \Omega \rangle / \Gamma_N \), set in relation to the nonlinear drive rate \( \Gamma_N \), thus represents the radial shearing of zonal flows, which is considered responsible for the turbulent shear flow decorrelation and energetic damping of vortices.

Concerning the geometric effect on this, we find different results for the DALF and ITG parameters.

In the DALF case, the cumulative amplitude of the components in the radial spectrum of flow shear \( \langle \Omega \rangle / \Gamma_N \) is the same within fluctuation error bars for AUG geometry as for the circular torus. The lowest \( k_x \) component in these spectra, which is found to be stronger in the AUG case, makes up for only approximately 8% of the total vorticity. The spectra are shown in Fig. [10]. Flow shear in the DALF case thus is not significantly enhanced by flux surface shaping. Also the amplitude of zonal flows in the DALF frequency spectra in Fig. [5] is found to be comparable for circular and shaped configurations. The reduction of turbulent edge transport is in this case therefore mainly a result of the local magnetic shear effect.

In the ITG case, the flow shear rate deduced from the \( k_x \) spectra of zonal vorticity is clearly higher in all components for the AUG tokamak than for the circular torus, as can be seen in Fig. [11]. The cumulative amplitude in the spectra is by 70% higher in AUG geometry than in simple torus geometry. Flux surface shaping introduced by elongation and X-point shaping clearly enhances flow shear significantly for ITG edge parameters. The zonal flow amplitude in Fig. [5] is also by a factor of three larger in the elongated configuration.

This shear flow enhancement contributes to the overall turbulent transport reduction by elongation, as has been discussed in section V, in addition to the effects of magnetic shear damping and the reduction of interchange drive. The detailed relative importance of these three mechanisms may however change with varying plasma parameters.

**VII. CONCLUSIONS AND OUTLOOK**

We have discussed the effects of flux surface shaping on tokamak edge turbulence and flows. Elongation strongly reduces the turbulent transport, whereas triangularity was found to have only weak (transport enhancing) influence. Two regimes of edge plasma parameters were studied: an electromagnetic resistive drift wave (DALF) case, and the (ITG) case with an additional ion temperature gradient of \( \eta_i = 2 \).

The scaling of local transport is roughly with the inverse square of elongation, showing a stronger reduction for ITG parameters than for DALF parameters. The ITG result is consistent with earlier findings with the previous version of the GEM model.

The mechanisms of flux surface shaping effects on turbulence have been discussed. The major cause for trans-
port reduction by elongation is damping of turbulence by stronger local magnetic shear at the upper and lower parts of the torus on top of increased global magnetic shear. The supervening reduction of the interchange drive by a lower normal curvature has an additional turbulence reduction effect in particular for ITG parameters, where the interchange drive is slightly weakened. In the ITG case a significant enhancement of zonal flow shear by elongation and X-point shaping further contributes to transport reduction. The shear flow effect is on the other hand found to be negligible for DALF parameters.

Both the DALF and ITG parameter regimes used in our computations seem consistent with the presently still poor experimental knowledge on edge ion temperature profiles, though this situation is rapidly improving. It can be assumed that additional core plasma heating, e.g. by neutral beam injection (NBI) used in particular for obtaining the transition to an H-mode, is both increasing the ion temperature at the pedestal top and the ion temperature gradient in the overall edge pedestal region. Based on our results, the shaping of tokamak flux surfaces by elongation and by the additional presence of an X-point like in AUG is expected to facilitate an enhancement of zonal flows when the role of the ion temperature gradient dynamics is pronounced by additional heating.

It can be expected that this enhancement of zonal flows found for ITG parameters is even more pronounced for more strongly shaped flux surfaces nearer to the separatrix. Preliminary results suggest that such initial radially local flows have the ability to spread over wider regions of the pedestal in simulations with global profile evolution.

Some L-H transition theories include zonal flows as a trigger mechanism to induce the transition to a sustained mean shear flow. The enhancement of zonal flows by flux surface and X-point shaping in the presence of ion temperature gradient steepening may offer an additional explanation for the observation that H-mode access in tokamak experiments is facilitated by the presence of a divertor.

The possibility to obtain a confinement transition within first principle computations of edge turbulence will thus have to be studied with a code that includes full temperature dynamics, realistic flux surface geometry, global profile evolution, and a coupling of edge and SOL regions including realistic sheath boundary conditions, while in addition it maintains sufficient grid resolution, grid deformation mitigation, and energy plus enstrophy conservation in the vortex/flow system.

Acknowledgement

This work was supported by the European Commission under contract FU06-CT-2003-00332.

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