Ab — \textit{initio} No-Core Shell Model With Many-Body Forces

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March 30, 2022

Abstract

Working in the framework of the \textit{ab--initio} no-core shell model, we derive two-body effective interactions microscopically for specific harmonic-oscillator basis spaces from the realistic Argonne \textit{V8} nucleon-nucleon potential. However, our two-body effective interaction is only the dominant part of the \(A\)-body \((A \geq 3)\) effective interaction, which also contains three-, four- and, in general, up to \(A\)-body components. We model these \textit{multi--body} components as a simple two-parameter, basis-space-dependent interaction. We then investigate the coupling needed to provide accurate descriptions of selected nuclear observables. We successfully model the multi-body interaction as a zero-range two-body operator times the number operator \(\hat{N}_{\text{spa}}\), that measures the harmonic-oscillator quanta of the spectator nucleons.

1 Introduction

Despite the tremendous progress realized in the study of the bare nucleon-nucleon \((N,N)\) interaction as well as equally impressive advances in the theory of the effective \(NN\) interaction in the shell model, it has not been possible to systematically account for the properties of finite nuclei solely on the basis of two-body interactions. Studies of the roles played by many-body forces \((e.g. \, 3\text{-body and 4-body})\) suggest that they provide small yet important contributions
However, more careful analysis of nuclear many-body systems is needed to fully identify these roles. Adding to this complexity, the \textit{ab-initio} No Core Shell Model (NCSM) generates effective many-body forces which have been shown to decrease in strength with increasing basis space size. Overall, the nature of the "real" and the "effective" many-body forces remains quite complicated and elusive.

In this work, we present a phenomenological model of three-body and higher multi-body interactions as a simple, two-parameter and basis-space-dependent interaction. This multi-body interaction is added to our theoretically-derived two-body effective interaction, and its two parameters are adjusted in each basis space so as to reproduce results in agreement with experimental data. Thus, ours is a \textit{residual} multi-body interaction obtained in a phenomenological way. Our chosen functional form is a delta function reminiscent of the recent attention of Effective Field Theory (EFT) to expansions of nuclear interactions beginning with zero-range terms. In EFT there is an appropriate cutoff in momentum transfers, corresponding to the poorly-known short-range part of the nuclear potential. In a similar spirit, we model the poorly known many-body forces as harmonic-oscillator-cutoff-dependent zero-range interactions.

More specifically, we model the residual multi-body interaction by a one-parameter two-body delta interaction and a one-parameter multi-body interaction. The multi-body interaction consists of a two-body delta times a number operator, \( \hat{N}_{sps} \), that measures the excitations in oscillator quanta of the spectators. We expect and find that the contribution of the two-body part of the interaction weakens as the basis space increases and as our theoretically-derived effective two-body interaction approaches the exact two-body interaction. On the other hand, we find as expected that the multi-body part of our residual interaction remains relatively strong, which provides an indication of the role played by \( n \)-body forces (\( n > 2 \)) in nuclei.

2 The Shell-Model Hamiltonian and the Effective Interaction

The \textit{ab-initio} approach in shell-model studies of the nuclear many-body problem begins with the one- and two-body Hamiltonian for the \( A \)-nucleon system, i.e.,

\[
H = \sum_{i=1}^{A} \frac{\vec{p}_i^2}{2m} + \sum_{i<j}^{A} V_{ij}
\]

with \( m \) the nucleon mass and \( V_{ij} \) the \( NN \) interaction. We then add the center-of-mass harmonic-oscillator (HO) potential \( \frac{1}{2}m\Omega^2 \bar{R}^2 \), with \( \bar{R} = (1/A) \sum_{i=1}^{A} \vec{r}_i \). The total center-of-mass harmonic-oscillator hamiltonian,
\( H_{c.m.} = \vec{P}_{c.m.}^2 / 2Am + \frac{1}{2} Am\Omega^2 \vec{R}^2 \), \( \vec{P}_{c.m.} = \sum_{i=1}^{A} \vec{p}_i \), is later subtracted to leave a purely intrinsic Hamiltonian. At this stage we have a convenient mean field acting on each nucleon that helps our overall convergence when working in a HO set of basis states. We emphasize that we adopt proper measures to ensure that the intrinsic properties of the many-body system are not affected by the center-of-mass term and we discuss our specific procedure for this in more detail below. The modified Hamiltonian, thus, acquires a dependence on the HO frequency \( \Omega \), and can then be written as

\[
H_{c.m.} = \sum_{i=1}^{A} \left[ \frac{\vec{p}_i^2}{2m} + \frac{1}{2} m\Omega^2 r_i^2 \right] + \sum_{i<j}^{A} \left[ V_{ij} - \frac{m\Omega^2}{2A} (r_i - r_j)^2 \right]
\]

(2)

Actual shell-model calculations can only be carried out in a finite basis space defined by a projection operator \( P \), with the complementary space to the finite basis space (i.e. the excluded space) defined by the projection operator \( Q = 1 - P \). Furthermore, due to its strong short-range part, the realistic nuclear interaction in Eqs. (1) and (2) will yield pathological results unless we derive a basis-space dependent effective Hamiltonian:

\[
H_{\Omega}^{\text{eff}} = P \sum_{i=1}^{A} \left[ \frac{\vec{p}_i^2}{2m} + \frac{1}{2} m\Omega^2 r_i^2 \right] P + P \sum_{i<j}^{A} \left[ V_{ij} - \frac{m\Omega^2}{2A} (r_i - r_j)^2 \right]_{\text{eff}} P
\]

(3)

The second term in Eq. (3) is an effective interaction that is, in general, an A-body interaction. If the effective interaction is determined without any approximations, the basis-space Hamiltonian provides an identical description of a subset of states as the exact original Hamiltonian \( H \). From among the eigenstates of the Hamiltonian (3), it is necessary to choose only those that correspond to the same center-of-mass energy. This can be achieved first by working in a complete \( N\hbar\Omega \) basis space, and then by projecting the center-of-mass eigenstates with energies greater than \( \frac{3}{2}\hbar\Omega \) (representing spurious center-of-mass motion) upwards in the energy spectrum. In our case, we do this by adding \( \beta PH_{c.m.}^0 P \) to and subtracting \( \beta \frac{3}{2}\hbar\Omega P \) from equation (3) above. One unit of \( H_{c.m.}^\Omega \) has already been incorporated, as mentioned above, and it is also subtracted at this stage. The resulting shell-model Hamiltonian takes the form

\[
H_{\Omega}^{\text{eff}} = \sum_{i<j}^{A} \left[ \frac{(\vec{p}_i - \vec{p}_j)^2}{2Am} + \frac{m\Omega^2}{2A} (r_i - r_j)^2 \right] P
\]

(4)
where $\beta$ is a sufficiently large positive parameter.

When applied in a suitably chosen finite $HO$ basis space, this procedure removes the spurious center-of-mass motion, and has no effect on the intrinsic spectrum of states \cite{5}. Our choice of the $P$-space is fixed by $N\hbar\Omega$ that signifies a many-body basis space of all $HO$ Slater determinants having total $HO$ quanta less than or equal to the cutoff given by $N$. We refer to this as a complete $N\hbar\Omega$ basis space. It is complete in the sense it allows the precise removal of spurious center-of-mass motion effects.

In principle, the effective interaction introduced in Eqs. (3) and (4) above should reproduce exactly the full-space results in the finite basis space for some subset of physical states. Furthermore, an $A-$body effective interaction is required for an $A-$nucleon system. In practice, however, the $A-$body effective interaction cannot be calculated exactly, and it is approximated by a two-body effective interaction determined for a two-nucleon sub-system or two-nucleon cluster.

In this work, we follow the procedure described in Refs. \cite{5,6} in order to construct the two-body effective interaction. The procedure employs the Lee-Suzuki \cite{11} similarity transformation method, which yields an interaction in the form

$$P_2V_{eff}P_2 = P_2V_P + P_2V_Q\omega P_2,$$

with $\omega$ the transformation operator satisfying $\omega = Q_2\omega P_2$, and $P_2, Q_2 = 1 - P_2$ operators that project on the two-nucleon finite basis and complementary spaces, respectively. Note that we distinguish between the two-nucleon system projection operators $P_2, Q_2$ and the $A-$nucleon system projection operators $P, Q$. Note also that the size of the two-nucleon basis is dictated by the choice of $N\hbar\Omega$ for the $A$-body problem.

Our calculations start with exact solutions of the two-body Hamiltonian

$$H_2^{\Omega} = H_2^{\Omega} + V_2^{\Omega} \quad (5)$$

$$= \left[\frac{\vec{p}_1^2 + \vec{p}_2^2}{2m} + \frac{1}{2}m\Omega^2(r_1^2 + r_2^2)\right] + \left[V_{12}(\vec{r}_1 - \vec{r}_2) - \frac{m\Omega^2}{2A}(\vec{r}_1^2 - \vec{r}_2^2)\right],$$

which follows from reducing the shell-model Hamiltonian (2) to the two-nucleon case. The exact solutions are obtained by assuming a $0\hbar\Omega$ motion of the center-of-mass of nucleons 1 and 2. Let $|\alpha_P\rangle$ denote the two-nucleon $HO$ states which form the basis space signified by $P_2$, and let $|\alpha_Q\rangle$ denote those that form the $Q_2-$space. Dropping the subscript “2” for convenience, one can then express the $Q-$space components of an eigenvector $|k\rangle$ of the Hamiltonian (5) as a combination of the $P-$space components with the help of the operator $\omega$:

$$\langle\alpha_Q|k\rangle = \sum_{\alpha_P} \langle\alpha_Q|\omega|\alpha_P\rangle \langle\alpha_P|k\rangle \quad (6)$$
If the dimension of the basis space is \( d_p \), we may then choose a set \( K \) of \( d_p \) eigenvectors for which relation (6) will be exactly satisfied [12]. Typically, these \( d_p \) states will be the lowest states obtained in a given channel. Under the condition that the \( d_p \times d_p \) matrix \( \langle \alpha_P | k \rangle \) for \( |k \rangle \in K \) is invertible, the operator \( \omega \) can be determined from Eq. (7), and the effective Hamiltonian can then be constructed as follows:

\[
\langle \gamma_P | H_{2eff} | \alpha_P \rangle = \sum_{k \in K} \left( \langle \gamma_P | k \rangle E_k \langle k | \alpha_P \rangle + \sum_{\alpha_Q} \langle \gamma_P | k \rangle E_k \langle k | \alpha_Q \rangle \langle \alpha_Q | \omega | \alpha_P \rangle \right)
\]

(7)

This Hamiltonian, when diagonalized in a finite basis space, reproduces exactly the set \( K \) of \( d_p \) eigenvalues \( E_k \) of the defined 2-body problem. Note that the effective Hamiltonian (7) is not Hermitian, and that it can be transformed into a Hermitian form \( \tilde{H}_{2eff} \) by applying a similarity transformation determined from the metric operator \( P_2 (1 + \omega \dagger \omega) P_2 \) [13]:

\[
\tilde{H}_{2eff} = \left[ P_2 (1 + \omega \dagger \omega) P_2 \right]^{1/2} H_{2eff} \left[ P_2 (1 + \omega \dagger \omega) P_2 \right]^{-1/2}
\]

The two-body effective interaction actually used in the present calculations is determined from this two-nucleon effective Hamiltonian as \( V_{2eff} = \tilde{H}_{2eff} - \tilde{H}_0^\Omega \). The resulting two-body effective interaction \( V_{2eff} \) depends on \( A \), on the \( HO \) frequency \( \Omega \), and on \( N_{max} \), the maximum many-body \( HO \) excitation energy (above the lowest configuration) defining the \( P \)-space. Furthermore, when used in the shell-model Hamiltonian (4), it results in the factorization of our many-body wave function into a product of a center-of-mass \( \frac{\hbar}{\omega} \Omega \) component times an intrinsic component, which allows exact correction of any observable for spurious center-of-mass effects, thus preserving translational invariance. This feature distinguishes our approach from most phenomenological shell-model studies that involve multiple \( HO \) shells.

So far, the most important approximation used in our approach is the neglect of contributions coming from higher than two-body clusters to our effective Hamiltonian. Although our method can be readily generalized to include the effects of three-body clusters, leading to the derivation of a three-body effective interaction, so far this has only been done in \( A = 3 \) and \( A = 4 \) systems [14, 15], and it is not yet clear how difficult it will be to extend it to the \( p \)-shell nuclei with \( A \geq 5 \) and with the same range of \( N \hbar \Omega \) values accessible at the 2-body cluster level.

In order to approximately account for the many-body effects neglected when using only a two-body effective interaction, we introduce a simple residual interaction that depends on all particles. More specifically, our residual many-body interaction contains a delta function times the sum of the oscillator quanta for the \( A - 2 \) spectators denoted by \( N_{sp} \), and defined by
\[ N_{sps} \equiv N_{sum} - N_\alpha = N'_{sum} - N_\gamma \]  

(8)

where \( N_{sum} \) and \( N'_{sum} \) are the total oscillator quanta in the initial and final many-body states, respectively. The quantities \( N_\alpha \) and \( N_\gamma \) are the total oscillator quanta in the initial and final two-nucleon states \( |\alpha\rangle \) and \( |\gamma\rangle \), respectively, interacting through the delta function.

Why do we introduce such a dependence of the many-body forces? We are motivated by the experience with the “multi-G” method of addressing effective many-body forces [4], which is based on the role of the spectators in a many-body matrix element. As the spectators are excited, the effective \( P_2 \) space for the interacting pair decreases and this increases the attraction of the \( G \)-matrix. The spectator dependence in the multi-\( G \) approach to the effective many-body forces predominantly lowers the excited multi-\( h\Omega \) states.

In this connection, one might imagine, as we do, the spectator dependence to be of a form where the strength is weakest when the spectators have 0\( h\Omega \) excitation (above their minimum value) and strongest when they have the maximum excitation. These two situations correspond, respectively, to when the 2-particles roam over the entire \( N h\Omega \) and over only the lowest \( h\Omega \) range. Thus, we expect this \( N_{sps} \)-dependent term to be attractive and account for the leading effects observed in the multi-\( G \) investigations.

We also allow for a spectator-independent term in the many-body force and we will take it to be a delta function for simplicity. We expect this term will approximate the many-body effective force components that are largely independent of the spectator configurations. We expect this contribution to be repulsive in order to correct for the overbinding found in smaller model spaces when \( H_{eff} \) is approximated with the 2-body cluster form [4]. We will see below that the fits to nuclear spectra verify these expectations.

Our total effective many-body interaction is then expressed in the form of a zero-range two-body interaction, and two parameters, one of which multiplies the many-body variable \( N_{sps} \) introduced above:

\[ V_{res}(N_{sps}) \equiv (a + bN_{sps}) \times G\delta(\vec{r}_1 - \vec{r}_2) \]  

(9)

where \( G \) is a fixed strength parameter taken to be 1 MeV fm\(^3\) in the present work. The parameters \( a \) and \( b \) are basis-space dependent dimensionless parameters to be determined by fits to selected many-body observables.

We will then use this schematic residual interaction to study the role played by the many-body forces that were neglected in our derivation of the effective two-body interaction. We shall derive the latter from the realistic Argonne \( V8' \) \( NN \) potential [1], and perform calculations of some basic observables in the \( A = 4 \) system with and without the addition of the residual many-body interaction \( V_{res}(N_{sps}) \), and in various basis spaces. We solve for the properties of \( A = 4 \) nuclei using the \( m \)--scheme Many-Fermion Dynamics code [17] that was developed for No-Core Shell-Model (NCSM) calculations.
3 Application to $A = 4$ Nuclei

In this section we apply the methods outlined in Sec. II to derive the effective two-body interaction, based on the Argonne $V8'$ NN potential, which we then use in the remainder of our work. However, it is now well-known that two-body interactions alone are not sufficient, and in order to better account for such experimental facts as the binding energy of nuclei, one is led to introduce three-body and higher multi-body forces. In the following discussion, we first consider calculations done with only the Argonne $V8'$ two-body interaction in order to see how such two-body calculations compare with experiment. We then ask the following question: what couplings $(a, b)$ for the residual many-body interaction are needed in order to achieve as close an agreement with experiment as possible?

We use a complete $N\hbar\Omega$ basis space with, e.g., $N = 8$ for the positive-parity states of $^4He$. This means that nine major HO shells are employed. The two-nucleon basis space is defined in our calculations by $N_{\text{max}}$, e.g., for an $8\hbar\Omega$ calculation for $^4He$ we have $N_{\text{max}}=8$. The restriction of the HO shell occupation is given by $(N_1 + N_2) \leq N_{\text{max}}$ where the single-particle HO quanta are given by $N_i = 2n_i + l_i$. The same conditions hold for the relative $2n + l$, center-of-mass $2N + L$, and $2n + l + 2N + L = N_1 + N_2$ quantum numbers, respectively.

In the case of the $A = 4$ system, we seek to reproduce the following observables as accurately as possible: the binding energy of the ground state (GS) as well as the energies of the first three excited states. We are also interested in reproducing the root-mean-square point-charge radius $R$.

3.1 The Experimental Situation

In Table 1 we present the experimental energies of the low-lying states that are relevant to our present study. The binding energies of the ground states are given first, followed by the excitation energies of the first three excited states (all energies are in $MeV$) [18, 19]. Furthermore, the root-mean-square point-charge radius of the ground state of $^4He$ is known experimentally to be 1.46 fm, which we also take to be the point mass radius.

3.1.1 Calculations Using Only Two-Body Effective Interactions

Before we embark upon the investigation of the many-body force, let us first examine the theoretical predictions of the $^4He$ spectrum using only the Argonne $V8'$ plus the Coulomb interaction. In Table 2, we present results of calculations of the first two positive-parity and the first two negative-parity states, using the interaction in increasingly larger basis spaces (indicated by $N_{\text{max}}$, as defined above). The results depend somewhat on the choice of $\hbar\Omega$, the HO energy parameter, but we shall adopt the value $\hbar\Omega=19 \ MeV$, which is suggested by the phenomenological expression [20].
\[ h \Omega = \frac{45}{A^{1/3}} - \frac{25}{A^{2/3}}. \]

In the large \( N_{\text{max}} \) limit our results become independent of the choice of \( h \Omega \) \cite{22}.

As shown in Table 2, it is clear that the Argonne \( V/8' \) interaction overbinds the ground state for \( N_{\text{max}} < 8 \) and underbinds it for \( N_{\text{max}} = 8 \) and 10, with the overbinding decreasing (and the underbinding increasing) as \( N_{\text{max}} \) increases. On the other hand, the excitation energy of the \( 0^+_2 \) state steadily decreases with \( N_{\text{max}} \). If a two-body force is alone sufficient, and if the Argonne \( V/8' \) plus Coulomb interaction represents a sufficiently accurate form of this two-body force, we would expect our calculations to converge to the experimental values. By examining Table 2, it becomes clear that while the first positive-parity excited state (\( 0^+_1 \)) and the negative-parity states (\( 0^- \) and \( 2^- \)) are becoming more bound with increasing \( N_{\text{max}} \), the ground state itself (\( \text{GS} \)) is becoming less bound.

Furthermore, while the discrepancy between the calculated and the experimental ground-state energy is reduced with increasing \( N_{\text{max}} \) (from 12\% with \( N_{\text{max}} = 2 \) to less than 4\% with \( N_{\text{max}} = 10 \)), the improvement is less favorable for the energies of the excited states (e.g., from 51\% with \( N_{\text{max}} = 2 \) to 23\% with \( N_{\text{max}} = 10 \) for the \( 0^+_2 \) state). The percentages used here represent the percent error (in absolute value) of the calculated energy relative to the experimental value. For the ground state, this is based on the total binding energy, whereas for the excited states it is based on the excitation energy.

We are, thus, led to conclude that the two-body force used is somewhat deficient and we need either a better two-body force, or three-body and/or higher many-body forces, in general. This is exactly the situation that we wish to investigate with our zero-range, spectator-dependent, residual delta interaction.

### 3.1.2 The effect of adding a residual many-body force

Next, we consider the effect of adding our residual delta interaction to the derived effective 2-body interaction in order to obtain a \( 2 + \) many-body potential:

\[
V = V_{\text{eff ArgonneV/8'}} + V_{\text{Coul}} + [a + b N_{\text{sps}}] \times G \delta(\vec{r}_1 - \vec{r}_2).
\]

By adding the spectator-dependent phenomenological interaction introduced in Eq. (9), we are simply modelling the contribution of all residual many-body forces, omitted by our derived two-body effective interaction. We then vary the coefficients \( a \) and \( b \) independently for each basis space, in order to achieve good agreement with the experimental ground-state binding energy and excitation energy of the first excited \( 0^+ \) state of \( ^4\text{He} \). The results are shown in Table 3 and illustrated in Fig. 1. We note that \( a \) is positive, while \( b \) is negative, and that both are decreasing functions of \( N_{\text{max}} \), with \( a \) decreasing somewhat faster than
b. As the ground state was found to be overbound, whereas the first excited state was found to be underbound by the pure two-body interaction (see Table 2), it should not be surprising that $a$ turns out to be positive, while $b$ is negative. It is also reassuring to see both $a$ and $b$ decrease steadily (in absolute value) with $N_{\text{max}}$, indicating that the need for the residual many-body interaction $V_{\text{res}}$ decreases and that the adequacy of the two-body interaction increases as the basis space becomes larger. More precisely, the result that $a \to 0$ as $N_{\text{max}} \to \infty$, is not surprising, as it indicates that there is little or no correction needed at the pure two-body level to our Hamiltonian. On the other hand, the $b$-term, which carries all the information about the "true" many-body interaction, is not converging to zero. We, therefore, conclude that it is possible to accommodate the contribution of the missing many-body interaction and to represent it in the simple form of $b N_{\text{sp}} \times G \delta(\vec{r}_1 - \vec{r}_2)$ for these observables.

We are now in a position to examine the predictive power of our extended effective Hamiltonian. We begin by asking how well such a residual many-body interaction (determined in the case of $^4\text{He}$) can help reconcile the calculated spectra of $^4\text{H}$ and $^4\text{Li}$ with the experimental situation? In Tables 4 and 5 we show the results obtained with and without the residual many-body interaction and compare them to experiment. We are now addressing ($T = 1$) negative-parity states, so we perform our calculations in an $N_{\text{max}} = 9$ basis space, and we use the values $a = 0.63$ and $b = -2.43$, which were determined in the earlier fit to $^4\text{He}$ (see Table 3).

It is clear from the results shown in Tables 4 and 5 that a much better agreement with experiment is achieved for both $^4\text{H}$ and $^4\text{Li}$ when our residual many-body interaction is added to the pure two-body interaction. Aside from the improved agreement for the ground states and first excited states, we observe that it is only by including the residual many-body interaction that the calculated $0^-$ and $1^-_2$ states in $^4\text{Li}$ become bound, in accord with experiment.

Based on the overall results for the $A = 4$ isotopes, presented above, we can identify a procedure for improving the many-body force. In particular, the $^4\text{He}$ fit can be interpreted as determining the spin-isospin averaged values of the couplings for our many-body force. By allowing spin-isospin dependence in the couplings, we could fine tune the coupling strengths, while preserving their average values, so as to obtain an improved description of the other isotopes. Thus, we can introduce an isospin dependence in our residual many-body force as follows:

$$V_{\text{res}} = [(a + a' \vec{\tau}_1 \cdot \vec{\tau}_2) + (b + b' \vec{\tau}_1 \cdot \vec{\tau}_2) N_{\text{sp}}] \times G \delta(\vec{r}_1 - \vec{r}_2).$$

or, equivalently, by assigning different values for $a$ and $b$ in the $T = 0$ and $T = 1$ channels, denoted by $a_0$, $a_1$ and $b_0$, $b_1$. We do this while keeping the isospin average of $a_0$ and $a_1$ to be equal to $a$, and likewise for $b_0$ and $b_1$. Applying a minimum root-mean-square convergence criterion on the calculated energies of the first two states in each of the $A = 4$ nuclei considered relative to their
experimental values, we find that it is possible to improve the calculations when using an isospin-dependent version of the residual many-body interaction (Table 6).

It would be interesting to study whether our model of the residual many-body interaction can be generalized to the study of $A > 4$ systems. For example, the test of the spin-isospin dependent many-body force can then be carried out in the $A = 5$ systems. This effort is underway and will be reported in another paper.

To summarize, we have presented a simple model for treating multi-body interactions in nuclear-structure calculations in terms of a two-parameter, basis-space-dependent delta interaction. We determine the two-parameters as a function of basis-space size by fits to the lowest two $0^+$ states of $^4\text{He}$. The parameter values decrease with basis-space size, as anticipated. The obtained multi-body interaction also yields improved agreement with experiment for negative parity states in $^4\text{He}$ and for the $T = 1$ states of $^4\text{H}$ and $^4\text{Li}$, which were not included in the empirical fits. The advantages of our model for the multi-body interaction in nuclei are two-fold: (1) the model provides a simple method for including multi-body interactions into nuclear-structure calculations, thereby yielding improved agreement with experiment, and (2) the model will give us insight into the nature of such multi-body interactions, so that we will be able to model them more realistically in the future.

4 Acknowledgements

M.S. Fayache and B.R. Barrett acknowledge partial support from NSF Grant No. PHY0070858. J.P. Vary acknowledges support from USDOE Grant No. DE-FG02-87ER-40371. This work was partly performed under the auspices of the U. S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48 (P. Navrátil).
References

[1] S.A. Coon, M.D. Scadron, P.C. McNamee, B.R. Barrett, D.W.E. Blatt, and B.H.J. McKellar, Nucl. Phys. A 317, 242 (1979).

[2] S.A. Coon and H.K. Han, Few-Body Systems Supplement 30, 131 (2000).

[3] B.S. Pudliner, V.R. Pandharipande, J. Carlson, and R.B. Wiringa, Phys. Rev. Lett. 74, 4396 (1995); S. C. Pieper, V.R. Pandharipande, R.B. Wiringa, and J. Carlson, Phys. Rev. C 64, 014001 (2001) and references therein.

[4] D.C. Zheng, B.R. Barrett, L. Jaqua, J.P. Vary, and R.J. McCarthy, Phys. Rev. C 48, 1083 (1993); D.C. Zheng, J.P. Vary, and B.R. Barrett, Phys. Rev. C 50, 2841(1994); D.C. Zheng, Barrett, J.P. Vary, W.C. Haxton and C.L. Song, Phys. Rev. C 52, 2488 (1995).

[5] P. Navrátil and B.R. Barrett, Phys. Rev. C 54, 2986 (1996).

[6] P. Navrátil, J.P. Vary and B.R. Barrett, Phys. Rev. Lett. 84, 5728 (2000); Phys. Rev. C 62, 054311 (2000).

[7] C. Ordonez and U. van Kolck, Phys. Lett. B 291, 459 (1992); C. Ordonez, L. Ray and U. van Kolck, Phys. Rev. Lett. 72, 1982 (1994); Phys. Rev. C 53, 2086 (1996); U. van Kolck, Phys. Rev. C 49, 2932 (1994).

[8] Nuclear Physics with Effective Field Theory II, Proceedings from the Institute for Nuclear Theory, Vol. 9, P. F. Bedaque, M. J. Savage, R. Seki, and U. van Kolck (Eds), (World Scientific, Singapore, 2000).

[9] L. Jaqua, D.C. Zheng, B.R. Barrett, and J.P. Vary, Phys. Rev. C 48, 1765 (1993).

[10] D.H. Gloeckner and R.D. Lawson, Phys. Lett. 53 B, 313 (1974).

[11] K. Suzuki and S.Y. Lee, Progr. Theor. Phys. 64, 2091 (1980).

[12] For a discussion of mathematical flexibility, see C.P. Viazminsky and J.P. Vary, J. Math. Phys., 92, 2055 (2001).

[13] K. Suzuki, Progr. Theor. Phys. 68, 246 (1982); K. Suzuki and R. Okamoto, ibid. 70, 439 (1983).

[14] P. Navrátil and B.R. Barrett, Phys. Rev. C 57, 562 (1998); 59, 3119 (1998).

[15] P. Navrátil, G.P. Kamuntavicius, and B.R. Barrett, Phys. Rev. C 61, 044001 (2000).
[16] B.S. Pudliner, V.R. Pandharipande, J. Carlson, S.C. Pieper and R.B. Wiringa, Phys. Rev. C 56, 1720 (1997); R.B. Wiringa, Nucl. Phys. A 631, 70c (1998).

[17] J.P. Vary, “The Many-Fermion-Dynamics Shell-Model Code”, Iowa State University, (1992) (unpublished); J.P. Vary and D.C. Zheng, ibid. (1994).

[18] G. Audi and A. H. Wapstra, Nucl. Phys. A 595, 409 (1995).

[19] Nuclear Data Retrieval [http://www.nndc.bnl.gov].

[20] G. F. Bertsch, The Practitioner’s Shell Model, (North Holland, Amsterdam, 1972).

[21] B. R. Barrett, P. Navrátíl, W. E. Ormand and J. P. Vary, Proceedings of the Polish Summer School of Physics, 2001 (in press).
Figure Captions

Fig. 1: The strengths $a$ (solid curve) and $b$ (dashed curve) of the residual delta interaction needed to achieve agreement with experiment in the case of $^4He$ and using $\hbar \Omega = 19 \, MeV$. 
Table 1: Experimental binding energies of the ground states (GS) and excitation energies of the first three excited states (in MeV) in the $A = 4$ system.

|          | $^4He$ (T = 0) | $^4Li$ (T = 1) | $^4H$ (T = 1) |
|----------|---------------|---------------|---------------|
| $0^+_1$ (GS) | -28.295674 5 | 2.578058      | 2.578058      |
| $0^+_2$      | 20.210        | 1.030         | 1.030         |
| $0^-_1$      | 21.010        | 2.080         | 2.080         |
| $2^-_1$      | 21.840        | 2.850         | 2.830         |

Table 2: Ground-State energy and excitation energies (in MeV) of the first two $0^+$ states and the first two negative-parity states of $^4He$ using only the Argonne V8' and the Coulomb potentials (i.e., $a = 0$ and $b = 0$), as calculated in shell-model spaces characterized by $N_{max}h\Omega$ excitations and with $h\Omega = 19$ MeV.

|          | (a) Positive Parity | (b) Negative Parity |
|----------|---------------------|---------------------|
| $N_{max}$ | $E(GS)$ | $E_x(0^+_2)$ | $N_{max}$ | $E_x(0^-)$ | $E_x(2^-)$ |
| 2        | -31.853 | 30.577   | 3        | 26.560 | 28.692   |
| 4        | -30.758 | 29.825   | 5        | 25.694 | 27.600   |
| 6        | -29.328 | 27.019   | 7        | 23.884 | 25.644   |
| 8        | -28.149 | 25.213   | 9        | 22.607 | 24.211   |
| 10       | -27.247 | 24.902   |          |        |          |
Table 3: Values of the parameters $a$ and $b$ in the interaction $V_{\text{ArgonneV8'}}^{\text{eff}} + V_{\text{Coul}} + |a + b*N_{\text{sps}}| \times G_\delta (\vec{r}_1 - \vec{r}_2)$ and corresponding energies as calculated in various basis spaces characterized by $N_{\text{max}} \hbar \Omega$ excitations, with $\hbar \Omega = 19 \text{ MeV}$ for $^4\text{He}$.

| $a$ | $b$ | (a) Positive Parity | (b) Negative Parity |
|-----|-----|---------------------|---------------------|
|     |     | $N_{\text{max}}$ | $E(GS)$ | $R_{GS}$ | $E_x(0^+_1)$ | $N_{\text{max}}$ | $E_x(0^-)$ | $E_x(2^-)$ |
| 2.22 | -6.86 | 2 | -28.305 | 1.390 | 20.223 | 3 | 23.811 | 26.017 |
| 2.09 | -5.80 | 4 | -28.291 | 1.341 | 20.221 | 5 | 23.638 | 25.353 |
| 1.51 | -3.12 | 6 | -28.286 | 1.354 | 20.212 | 7 | 22.446 | 23.977 |
| 0.63 | -2.43 | 8 | -28.301 | 1.383 | 20.238 | 9 | 21.541 | 22.903 |
| 0.09 | -1.78 | 10 | -28.295 | 1.409 | 20.209 |   |       |       |

Table 4: Experimental and calculated energies (in MeV) of the first few states in $^4\text{He}$, with and without the residual many-body interaction ($a = 0.63, b = -2.43$) in an $N_{\text{max}} = 9$ basis space and using $\hbar \Omega = 19 \text{ MeV}$.

| State | Experimental Energy | Energy calculated without residual many-body interaction | Energy calculated with residual many-body interaction |
|-------|---------------------|--------------------------------------------------------|----------------------------------------------------|
| $2^-_1$ (GS) | -5.575 | -2.994 | -4.922 |
| $1^-_1$ | -5.265 | -2.452 | -4.792 |
| $0^-_1$ | -3.495 | -1.403 | -3.861 |
| $1^-_2$ | -2.745 | -0.935 | -3.864 |

Table 5: Experimental and calculated energies (in MeV) of the first few states in $^4\text{Li}$, with and without the residual many-body interaction ($a = 0.63, b = -2.43$) in an $N_{\text{max}} = 9$ basis space, and using $\hbar \Omega = 19 \text{ MeV}$.

| State | Experimental Energy | Energy calculated without residual many-body interaction | Energy calculated with residual many-body interaction |
|-------|---------------------|--------------------------------------------------------|----------------------------------------------------|
| $2^-_1$ (GS) | -4.618 | -1.409 | -3.433 |
| $1^-_1$ | -4.298 | -0.933 | -3.343 |
| $0^-_1$ | -2.538 | 0.097 | -2.422 |
| $1^-_2$ | -1.768 | 0.532 | -2.444 |
Table 6: Experimental and calculated energies (in $MeV$) of the first two states in the $A = 4$ system, with the isospin-independent and the isospin-dependent residual many-body interaction in $N_{max} = 8, 9$ basis spaces, and using $\hbar\Omega = 19\,MeV$.

| Nucleus (State) | Experimental Energy | Using $a_0 = a_1 = 0.63$ and $b_0 = b_1 = -2.43$ | Using $a_0 = 0.52, a_1 = 0.67$ and $b_0 = -2.88, b_1 = -2.28$ |
|-----------------|---------------------|-----------------------------------------------|--------------------------------------------------|
| $^4He(0^+_1\,(GS))$ | -28.297            | -28.301                                        | -28.627                                          |
| $^4He(0^+_2)$    | -8.086             | -8.063                                         | -8.697                                           |
| $^4H(2^-_1\,(GS))$ | -5.575            | -4.922                                         | -5.255                                           |
| $^4H(1^-_1)$     | -5.265             | -4.792                                         | -5.157                                           |
| $^4Li(2^-_1\,(GS))$ | -4.618            | -3.433                                         | -3.775                                           |
| $^4Li(1^-_1)$    | -4.298             | -3.343                                         | -3.713                                           |
