An Improved Algorithm of D-S Evidence Fusion

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Abstract. In the theory of evidence fusion, it is a very important task to deal with the conflicts between evidences correctly. Based on the credibility degree of evidence proposed in [10], in order to increase the credibility degree of each group of evidence, the concept of falsity is introduced, which is another concept to represent the conflict between two groups of evidence, this treatment makes up for the deficiency of normalized evidence weight coefficient when one group of evidence credibility degree is zero. Because we consider not only the credibility degree of evidence, but also the falsity of evidence, the weight coefficient of unreliable information is effectively reduced. The numerical simulation results show that the algorithm has the advantages of simple idea, fast convergence speed and strong focusing.

1. Introduction

D-S evidence theory is a theory of dealing with imprecise reasoning proposed by Dempster[1] and Shafer[2], it has a strong ability to deal with uncertain information, and has a great deal of advantages: the required conditions are weaker than Bayesian probability theory; there is the ability to express “unknown” and “uncertain” directly [1]. Therefore, it is widely used in information fusion and uncertain reasoning of artificial intelligence [3-5].

In fact, in the process of information fusion, due to unreliability, uncertainty and conflict of sensors, the efficient fusion of multi-source uncertain information has become a difficult problem. Shafer [2] initially used normalization factor $k$ to express the conflict measurement between two evidences, however, a subsequent study shows that $k$ can't effectively measure the degree of conflict between evidences[4,5]. Many researchers have proposed many ways to express conflicts, such as Liu [6] proposed to measure the Pignistic probability distance of two evidences, and combined with the conflict coefficient to determine whether the evidence is in conflict. Jousselme et al. [7] proposed evidence distance to indicate the difference between evidences, which is often used to indicate the conflict between evidences. Jiang et al. [4] used $k$ and Jousselme's evidence distance to jointly represent evidence conflict.

In terms of different conflict measures, many scholars have proposed new evidence fusion methods, Murphy[8] proposed a weighted fusion algorithm of average combination of evidences; Deng et al. [9] introduced a distance function of evidence body to calculate the degree of support of each evidence in the system by other evidence, normalized the support degree to obtain the credibility of each evidence as the weight of evidence, and used Murphy method to realize information fusion; Wang et al. [5] proposed a method of evidence combination based on credibility degree and false degree (EC-CF for short). In [10], we proposed an information fusion algorithm based on closeness degree, which used the closeness degree to measure the evidence credibility, and then calculated the final fusion result by...
Murphy’s method. But when the closeness degree between one group and another group evidence is zero, the credibility degree of this group evidence is zero, a small positive number epsilon is proposed to replace the credibility degree of this group evidence, which brings inconvenience to the subsequent calculation. Therefore, this paper proposes an improved algorithm combining closeness degree and falsity, which has strong focus and target discrimination.

2. Preliminaries

Let $\Omega$ is a set of all possible values of variable $x$, and the elements in $\Omega$ are mutually exclusive, then it is called the discernment frame of $x$, propositions in the field are represented by subsets of $\Omega$.

**Definition 1.**[1] Let $m:2^{\Omega} \rightarrow [0,1]$ be a function, and it be a number $m \in [0,1]$, which satisfies the following:

$$m(\emptyset) = 0 \text{ and } \sum_{A \in \Omega} m(A) = 1$$

Then $m$ is called the basic probability assignment (BPA), $m(A)$ is called the basic probability number of proposition $A$, and which is regarded as the reliability accurately assigned to $A$. When $m(A) > 0$, $A$ is called a focal element of $\Omega$.

**Definition 2.**[1] (Dempster combination rule) Assume that $m_1$ and $m_2$ are two BPAs, the corresponding focal elements are $A_1, A_2, \cdots, A_n$ and $B_1, B_2, \cdots, B_n$, $m$ is a new BPA by combining $m_1$ and $m_2$.

The Dempster combination rules are as follows:

$$m(\emptyset) = 0$$

$$m(A) = \frac{1}{1-k} \sum_{k \in A \cap B} m_1(A) m_2(B)$$

Where $k = \sum_{A \cap B = \emptyset} m_1(A) m_2(B)$ is called the conflict coefficient. Combination rule can’t be used when $k$ is 1.

Let $m_1, m_2, \cdots, m_n$ be $n$ groups BPA, according to formula (2), the new evidence $m$ after their combination can be expressed as

$$m(\emptyset) = 0$$

$$m(A) = \frac{1}{1-k} \prod_{A \cap B = \emptyset} m(A)$$

Where $k = \prod_{1 \leq i < j \leq n} m(A)$ is the conflict coefficient.

3. Closeness degree and falsity of evidence

**Definition 3.** Let $\Omega = \{\theta_1, \theta_2, \cdots, \theta_n\}$ be a discernment frame, $m_1$ and $m_2$ are two BPAs, the closeness degree of $m_1$ and $m_2$ is defined as:

$$C(m_1, m_2) = \frac{\sum_{A \in \Omega} (m_1(A) \land m_2(A))}{\sum_{A \in \Omega} (m_1(A) \lor m_2(A))}, A \in 2^{\Omega}$$

Evidently, the following are established:

1. $C(m_1, m_2) = C(m_2, m_1)$;
2. $0 \leq C(m_1, m_2) \leq 1$;
3. $m_1 = m_2 \iff C(m_1, m_2) = 1$;
4. $C(m_1, m_2) = 0 \iff (\cup A_1) \cap (\cup A_2) = \emptyset$, $A_1, A_2$ are focal elements of $m_1, m_2$, respectively.

**Example 1.** Suppose the discernment frame is $\Omega = \{\theta_1, \theta_2, \theta_3\}$, there are two BPAs which are defined as follows:
\[ m_1(\{\theta_1\}) = 0.85, m_1(\{\theta_1, \theta_2\}) = 0.1, m_1(\Omega) = 0.05 \]
\[ m_2(\{\theta_1\}) = 0.85, m_2(\{\theta_1\}) = 0.05, m_2(\{\theta_1, \theta_2\}) = 0.05, m_2(\Omega) = 0.05 \]
then
\[ C(m_1, m_2) = 0.026 \]

In the above example, the closeness degree of \( m_1 \) and \( m_2 \) is 0.026, it shows that there is a great conflict between \( m_1 \) and \( m_2 \). At this time, the conflict coefficient \( k \) is 0.8925 by the formula (2), it shows that the two evidences are highly conflicting.

**Definition 4.** (closeness matrix) Let \( \Omega \) be the discernment frame, \( m_i (i = 1, \ldots, n) \) be \( n \) groups BPA, then the closeness matrix among \( n \) groups evidences based on closeness degree is defined as

\[
CM = \begin{bmatrix}
1 & C(m_1, m_2) & \cdots & C(m_1, m_n) \\
C(m_2, m_1) & 1 & \cdots & C(m_2, m_n) \\
\vdots & \vdots & \ddots & \vdots \\
C(m_n, m_1) & C(m_n, m_2) & \cdots & 1
\end{bmatrix}
\]

(5)

The closer the two groups evidence are, the more they support each other, so, the credibility degree \( \text{Crd}_i (i = 1, \ldots, n) \) of \( m_i (i = 1, \ldots, n) \) can be measured by its closeness to other groups BPA. \( \text{Crd}_j (j = 1, \ldots, n) \) can be expressed as

\[ \text{Crd}_i = \frac{1}{n-1} \sum_{j \neq i} C(m_i, m_j), i = 1, \ldots, n \]

(6)

The degree of falsity was proposed by Schubert [11] based on the conflict coefficient \( k \), which is also a measure of evidence conflict.

Let \( m_1, m_2, \ldots, m_n \) be \( n \) groups BPA, according to formula (3), the global conflict between them is defined as

\[ k_0 = \sum_{C \neq A, A = A \cap k, k \neq 1} \left( \prod_{i=1}^{n} m_i(A) \right) \]

(7)

If evidence \( m_j \) is deleted from \( n \) groups evidence, the conflict coefficient between the remaining evidences is

\[ k_j = \sum_{C \neq A, A = A \cap k, k \neq 1} \left( \prod_{i=1}^{n} m_i(A) \right) \]

(8)

Obviously, \( 0 \leq k_j \leq k_0 \leq 1 \), then the false degree of evidence \( m_j \) is defined as

\[ F(m_j) = \frac{k_0 - k_j}{1 - k_j}, j = 1, 2, \ldots, n \]

(9)

When there is only two evidences, \( F(m_1) = F(m_2) = k_0 \).

**4. Evidence combination algorithm based on closeness degree and falsity**

Murphy’s method is simple in calculation, but it does not consider the difference of weight of each evidence. However, in the actual fusion system, the contribution of the information provided by each evidence to the system is not the same, the weight of each evidence is need to be considered.

The greater the falsity of an evidence, the greater the conflict between the evidence and other evidence. Therefore, the weight coefficient can be determined by combining the closeness and falsity of evidence. We propose a new method to construct weights, which are defined as follows:

\[ \alpha_i = \text{Crd}_i + 1 - F(m_i), i = 1, 2, \ldots, n \]

(10)

Evidently, \( \alpha_i > 0 \). Normalized \( \alpha_i \), we can get

\[ \omega_i = \frac{\alpha_i}{\sum_{i=1}^{n} \alpha_i}, i = 1, 2, \ldots, n \]

(11)

Therefore, the weight coefficient vector of \( n \) groups BPA is obtained
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\[ \omega = (\omega_1, \omega_2, \ldots, \omega_n) \]  

(12)

To sum up, the improved algorithm of evidence fusion is given as follows:

1. Input \( n \) groups BPA \( m_1, m_2, \ldots, m_n \);
2. Compute the closeness degree \( C(m_i, m_j), i, j = 1, 2, \ldots, n \) from formula (4);
3. Compute the closeness matrix \( CM \) from formula (5);
4. Compute the credibility degree \( Crd_i, i = 1, 2, \ldots, n \) from formula (6);
5. Compute the falsity \( F(m_i), j = 1, 2, \ldots, n \) from formula (7), (8) and (9);
6. Compute the weight coefficient vector \( \omega \) from formula (10) and (11);
7. Obtain \( m \) by weighting average of \( n \) groups BPA according to \( \omega \) in formula (12);
8. Obtain final BPA \( m \) by combining \( m \) for \( n-1 \) times according to the formula (2) or (3).

5. Example simulation and analysis

In this paper, an example in reference [5] is used to simulate the experiment, the fusion results of this method and other methods are compared and analyzed.

**Example 2.** Suppose the discernment frame is \( \Omega = \{\theta_1, \theta_2, \theta_3\} \), there are five groups BPA which are defined as follows:

- \( m_1(\theta_1) = 0.5, m_1(\theta_2) = 0.2, m_1(\theta_3) = 0.3, m_1(\theta_4) = 0, m_1(\theta_5) = 0.15 \)
- \( m_2(\theta_1) = 0.6, m_2(\theta_2) = 0.1, m_2(\theta_3) = 0.3, m_2(\theta_4) = 0.25, m_2(\theta_5) = 0.1 \)
- \( m_3(\theta_1) = 0.55, m_3(\theta_2) = 0.1, m_3(\theta_3) = 0.35 \)

We obtain the closeness matrix from formula (4) and (5) as follows:

\[
CM = \begin{bmatrix}
1 & 0.212 & 0.818 & 0.667 & 0.818 \\
0.212 & 1 & 0.143 & 0.212 & 0.143 \\
0.818 & 0.143 & 1 & 0.667 & 0.905 \\
0.667 & 0.212 & 0.667 & 1 & 0.6 \\
0.818 & 0.143 & 0.905 & 0.6 & 1
\end{bmatrix}
\]

We construct the credibility degree vector according to formula (6) as follows:

\[
\mathbf{CRD} = (Crd_1, Crd_2, Crd_3, Crd_4, Crd_5) = (0.629, 0.178, 0.633, 0.537, 0.617)
\]

From formula (7), the global conflict coefficient of five groups BPA is

\[
k_i = \sum_{A \in \mathcal{A}} \left( \prod_{i=1}^{5} m_i(A) \right) = 0.9991
\]

According to formula (8), the conflict coefficient between partial evidence is

\[
k_1 = 0.9963, k_2 = 0.8891, k_3 = 0.9942, k_4 = 0.9936, k_5 = 0.9944
\]

The vector of the falsity of \( m_1, m_2, m_3, m_4, m_5 \) by formula (9) is

\[
F = (0.757, 0.992, 0.845, 0.859, 0.839)
\]

Normalized weight coefficient vector according to (10) and (11) is

\[
\omega = (0.264, 0.056, 0.239, 0.205, 0.236)
\]

Obtain \( \bar{m} \) by weighting average of five groups BPA according to \( \omega \):

\[
\bar{m}(\theta_1) = 0.5384, \bar{m}(\theta_2) = 0.1992, \bar{m}(\theta_3) = 0.2624
\]

Obtain final BPA \( m \) by combining \( \bar{m} \) for four times according to the formula (3):

\[
m(\theta_1) = 0.966, m(\theta_2) = 0.007, m(\theta_3) = 0.027
\]

In the following, through the above example, comparing the methods of Dempster combination[1], Murphy’s method[8], Deng’s method[9], method of EC-CF[5], and the method of this paper, we can fuse the BPAs, as shown in Table 1.

| Various methods | \( \oplus_{i=1}^{3} m_i \) | \( \oplus_{i=1}^{4} m_i \) | \( \oplus_{i=1}^{5} m_i \) |
|-----------------|-----------------|-----------------|-----------------|
| Dempster’s method | \( m(\theta_1) = 0 \) | \( m(\theta_2) = 0 \) | \( m(\theta_3) = 0 \) |
It can be seen from the above table that Dempster’s method can’t recognize $\theta_1$, this is due to $m_*(\{\theta_1\}) = 0$, the conflict between the second BPA and the other four groups BPA is too great, which results in “one vote veto”. As can be seen from the five groups BPA, except for the second evidence, the other four groups evidence greatly support $\theta_1$, from the comparison results, Murphy’s method needs to collect four groups evidence to identify target $\theta_1$ ($m(\{\theta_1\}) = 0.683$). However, Deng Yong’s method, EC-CF method and our method only need to collect three groups evidence to completely identify the target $\theta_1$. With more and more evidences, the support of target $\theta_1$ increases rapidly, and the discriminative power between targets increases further, which is consistent with the results of intuitive analysis, simulation results show that these methods are beneficial to the final decision.

When the system collects the fifth groups evidence, the credibility degree of the target $\theta_1$ is $m(\{\theta_1\}) = 0.966$ in our method, this results are larger than Deng Yong’s method and EC-CF method, this is because our method not only considers the credibility of evidence, but also considers the falsity of evidence, which effectively reduces the weight coefficient of unreliable information.

6. Conclusions
In the process of evidence fusion, it is very common for evidence to conflict with each other. Therefore, it is a very important subject to deal with the conflict between evidences. In this paper, we use the concept of falsity, which represents the conflict between two groups evidence, to increase the credibility degree of each group evidence, the weight coefficient of each group evidence not only considers the credibility degree of evidence, but also considers the falsity of evidence, which effectively reduces the weight coefficient of unreliable information.

The method of this paper effectively solves the problem of “one vote veto” and high conflict integration. The numerical simulation results show that the algorithm has the advantages of simple idea, fast convergence speed and strong focusing performance. How to effectively deal with the conflict between evidences is always an open topic. How to construct a reasonable and effective method is our next research content.

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