A braid model for the particle X(3872)

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Abstract

The Model of Quark Exchange (MQE) describes the particle X(3872) as a meson molecule. We asked whether braids influence the meson potential in the MQE. We used the Burau representation that parameterized braids with a variable $t$. The present result shows that $t$ rescales the coupling of the meson potential determining if it is attractive or repulsive. As a consequence, a capture diagram favored the molecular state for $t = 0.85$, it breaks for other values. For the future, braids may help to study others exotic states in geometrical terms.

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I. INTRODUCTION

In 2003 the Belle Collaboration discovered the particle X(3872) [1] which other experiments confirmed later [2–5]. More recently, the detector LHCb measured its quantum numbers [2]. These findings suggested that X(3872) is composed of something more than two heavy charm quarks ($\bar{c}c$). Nowadays, its composition includes one of the alternatives (e.g. [6]): a molecule made of two heavy mesons [7, 8]; a tetra quark [9] or a hybrid state.

The Model of Quark Exchange (MQE) describes the particle X(3872) as a molecule composed of mesons $D^0$ and $\bar{D}^{0*}$ (D-mesons) [4]. The model request a potential energy characterized by a coupling $\lambda_{ex}$ and a width $\gamma$ that were fitted with the mass of the X(3872). Moreover, the nature of these parameters was well established in terms of diagrams called capture and transfer [10, 11]. Although, the diagrams posses a vertex signalizing the exchange of two quarks, it is unclear how the vertex contributes to the potential energy. We answered this question assuming the capture and transfer diagrams content braids. Braids form groups of great interest in physics as they explain the origin of fermion statistics associated with exchange of particles [5]. We proved that braids modified the phase-shift calculated previously in ref. [4] because the coupling $\lambda_{ex}$ rescales by a factor originated from braids.

The section II describes briefly the method based on the quark exchange model for the particle X(3872). In the section III we included the generators of the braid group $B_3$ in the quark exchange model.

II. A BRAID MODEL FOR THE X(3872)

A. The phase-shift

The phase-shift ($\delta$) describes the state X(3872) when solving the Lippmann-Schwinger equation for the scattering process $D^0 + \bar{D}^{0*} \rightarrow J/\psi + \rho$ [12]. The authors in ref. [12] implemented a Lorentzian potential for mesons characterized by two parameters: Its strength $\lambda = 20.3$ GeV$^{-2}$ and a cutoff $\gamma = 0.8$. For these parameters the state X(3872) materializes as a molecule of D-mesons with mass of 3.872 GeV. The energy variable $z$ enters in the
scattering phase-shift $\delta(z)$ defined as

$$
\delta(z) = \arctan\left( \frac{\text{Im}[t(z)]}{\text{Re}[t(z)]} \right),
$$

where $t(z)$ relates to the T-matrix that solves the Lippmann-Schwinger equation (see eq. 10 of ref. [12]).

**B. The quark exchange mechanism**

The MQE proposed that a meson potential originates when the two D-mesons exchange two quarks [11, 13–15]. This description supposes that all heavy mesons interact with the potential

$$
U(p,p') = -C_{SFC} \, I(p,p')
$$

with the factor $C_{SFC} = \frac{1}{6}$ and $I(p,p')$ as the invariant matrix element of the scattering $D^0 + \bar{D}^{0*} \rightarrow J/\psi + \rho$. This matrix element was calculated using four contributions named capture and transfer diagrams [11]. The first capture diagram ($D1$) is shown in Fig.2. The

(A) $p = p_1 + p_2$

(B) $-p = p_3 + p_4$

\[ U \]

\[ \begin{align*}
\rho_1' + \rho_2' &= p' \\
\rho_3' + \rho_4' &= -p'
\end{align*} \] (C) (D)

FIG. 1: Up panel: A box diagram describes the meson potential. D-mesons are composed of the quarks up (u) and charm (c). Down panel: The meson potential approximates to the capture diagram $D1$ in the first ladder approximation. The factor eight derives from the product of: the two conjugate channels $D^0$, $\bar{D}^{0*}$ and $\bar{D}^0$, $D^{0*}$ [11]; the prior and post interaction [11, 15]; the initial and final momenta of each heavy quark also contributes with a factor two.
result for the capture diagram $D1$ was

$$I(p, p') = \langle \phi_p \phi_{-p'} | - D_1 | \phi_{p'} \phi_{-p} \rangle$$

$$= \sum_{p_1 \ldots p_4} \phi^*_p(p_1, p_2) \phi^*_B(p_3, p_4) \times$$

$$\mathbf{V}(p_1, p_4) \delta_{p_2, p_4} \delta_{p_3, p'_4}$$

$$\times \phi_{p'}(p'_1, p'_2) \phi_{-p'}(p'_3, p'_4) .$$

(3)

within the quark-quark interaction

$$\mathbf{V}(p_1, p_4) = -V e^{-2(p_1 - p_1')^2} \delta_{p_4, p_4'} \delta_{p_3, p_3'}$$

(4)

and $V = 113.39 \text{ GeV}^{-2}$ as a parameter used to fit the meson spectrum $[11]$. In this paper, the volume in momentum space is one and the spin-spin interaction is neglected since it is hundred times smaller than $V$. The wave function for a meson $A$ is represented by $\phi_p(p_1, p_2)$ with amplitude $\phi_A$, similarly for the other mesons. They define the product $\phi_A \phi_B \phi_C \phi_D (2\pi)^6 = 6.312 \text{ GeV}^{-6}$. In the continuum limit for the invariant matrix, the sum (3) becomes an integration yielding the factor $0.12 \text{ GeV}^6$. Therefore the capture diagram $D1$ contributes with the coupling value $[11, 16]

$$\lambda_{ex} = -8 C_{SFC} \frac{\phi_A \phi_B \phi_C \phi_D}{(2\pi)^6} V \left(0.12 \text{ GeV}^6\right) ,$$

$$= -114.35 \text{ GeV}^{-2} .$$

(5)

Thus the strength $|\lambda_{ex}|$ is around six times bigger than the coupling of the meson potential $\lambda$. Besides the first capture diagram $D1$ supplies a repulsive interaction. However, the four capture and transfer diagrams previously introduced in ref. $[11]$ reduces the strength until half of $\lambda$ $[16]$. Probably higher orders in the ladder approximation may help to reduce this disagreement. However, We solved this problem using braids inside the capture diagram $D1$.

The phase-shift is calculated using the method previously described in ref. $[12]$ with a capture diagram (Fig. 1) modified by braids (Fig. 4).

C. Definition of braids

K. Murasugi introduced braids in three dimensions as the pictures in Fig. 2 $[17, 18]$. Braids may form groups $[19, 20]$. The full braid group $B_n$ consists of the $n - 1$ generators $\sigma_1,$
\(\sigma_2, \ldots, \sigma_{n-1}\) \[19, 20\]. For instance, the group \(B_3\) is composed of two generators as in Fig. 2\(b\) while the groups with one generator are \(B_1\) in Fig. 2\(a\) (the trivial group) and \(B_2\) in Fig. 2\(b\) (the cyclic group) \[20\]. The group \(B_n\) (also called the Artin group) is non-abelian for \(n \geq 3\). Besides it emerges as the fundamental group of \(\mathbb{R}^2\) \[21, 22\].

\[
\varphi_n(\sigma_i) = \begin{pmatrix}
I_{i-1} & & & \\
& 1-t & t & \\
& 1 & 0 & \\
& & 1_{n-i-1} & \\
\end{pmatrix}
\] \quad (6)

with \(i = 1, 2, \ldots, n-1\). The empty spaces in (6) consist of zeros; the identity \(m \times m\) matrix named \(I_m\) disappears from \(\psi_n(\sigma_1)\) as \(i = 1\) and from \(\psi_n(\sigma_{n-1})\) as \(i = n-1\). Thus the braid group \(B_2\) is generated by

\[
\varphi_2(\sigma_1) = \begin{pmatrix}
1-t & t \\
1 & 0
\end{pmatrix}
\] \quad (7)

and the braid group \(B_3\) by

\[
\varphi_3(\sigma_1) = \begin{pmatrix}
1-t & t & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad \varphi_3(\sigma_2) = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1-t & t \\
0 & 1 & 0
\end{pmatrix}
\] \quad (8)

These matrices (8) depend uniquely on \(t\) in comparison with other representations that use more than one variable \[20, 24, 26\].
III. THE RESULTS

A. A braid structure in the quark exchange diagrams

The quark exchange potential (4) determines the capture diagram in Fig. 1 when the quarks \( \bar{u} \) and \( \bar{c} \) exchange because of the constraint given by

\[
\delta_{p_4,p_2} + (p_1 - p_1') \delta_{p_2,p_4} \delta_{p_3,p_4'} ,
\]

this constraint permutes the momenta subscripts \((2,3,4)\) if the symbol prime is ignored. The permutation of three numbers is the element \( \Pi_2 \) of the permutation group \( S_3 \) ([27], p. 301)

\[
\Pi_2 = \begin{pmatrix} 2 & 3 & 4 \\ 4 & 3 & 2 \end{pmatrix} .
\]

(10)

Since \( \Pi_2 \) represents also the \( 3 \times 3 \) matrix ([27], p. 301-304)

\[
D(\Pi_2) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} ,
\]

(11)

the trace of the permutation matrix \( \Pi_2 \) contributes to the sum (3). This procedure reaches only until fifty percent of the coupling \( \lambda \) when including the whole capture and transfer diagrams together [16]. Besides, the permutation matrix \( \Pi_2 \) allows arbitrarily all type of quark exchange contributions. We limited the contributions by considering the exchange diagrams as braids. Each braid has two alternatives for crossings that distinguishes the type of quark exchange. We proved this statement by tracing the product of the permutation matrix \( \Pi_2 \) with \( \varphi(\sigma_1^{-1}) \) and \( \varphi(\sigma_1) \) (8). Hence the traces

\[
Tr \left[ \varphi_3(\sigma_1)D(\Pi_2) \right] = 0 ,
\]

(12)

\[
Tr \left[ \varphi_3(\sigma_1^{-1})D(\Pi_2) \right] = 1 - \frac{1}{t} .
\]

(13)

affect the value of \( \lambda_{ex} \) such that the potential scales with the new strength

\[
\lambda = \lambda_{ex} \left( 1 - \frac{1}{t} \right) ,
\]

(14)

As a result the quarks exchange as the braid \( \sigma_1^{-1} \) (Fig. 3) instead of \( \sigma_1 \). We calculated the
phase-shift (4) as a function of the energy \((z)\) and the variable \(t\). Our method is the same that in ref. [12] but with a change induced by the coupling (14). The Lorentzian form factors \(L(p)\) and \(R(p)\) have cutoff \(\gamma = 0.8\). We observed that \(t = 0.85\) makes a molecule of \(D^0\) and \(\bar{D}^0\). Moreover, a sharp behavior of the phase-shift around 3.872 GeV remains. We obtained the same result than in ref. [12] only when \(t \to \infty\).

![FIG. 3: The braids \(\sigma_1\) or \(\sigma_1^{-1}\) represents the exchange of quarks \(\bar{c}\) and \(\bar{u}\) as well as in Fig. 1.](image1)

![FIG. 4: Phase-shift as a function of energy for different values for \(t\). The phase-shift jumps in \(\pi\) at the energy of 3.872 GeV (\(\lambda = 20.3\) GeV\(^{-2}\), \(\gamma = 0.8\)).](image2)
IV. DISCUSSION

By including braids in the MQE we found that they regulate the repulsive or attractive character of the meson potential (2). Besides the braid model explains the origin of the coupling $\lambda$ at the quark level. Therefore, the braids in the Fig. 3 makes the phase-shift (Fig. 4) to jump in $\pi$. It confirms that the pair of mesons $D^0, \bar{D}^{0*}$ forms a molecule with mass of 3.872 GeV for $t = 0.85$, other values destroy it. Although, for the molecule, we obtained zero for the binding energy it may differ if all capture and transfer diagrams are included [16].

This braid model requires two quarks exchange (one heavy $\bar{c}$ and one light $\bar{u}$) but other models proposed a meson exchange ($\rho$ or $\omega$) containing only light quarks [7, 8, 28]. Nevertheless, whether the $X(3872)$ prefers the exchange with two quarks rather than a meson is unclear.

We observed within one capture diagram that $t = 1$ destroys the molecule because the meson potential vanishes (see the phase-shift in Fig. 4). Moreover, for $t > 1$ the potential gets repulsive (disfavors the molecule) while for $t < 1$ attractive (favors the molecule). An attractive potential rises also for $t > 1$ when considering all the capture and transfer diagrams leading a small chance for the braids effects to disappear. Although other braids belongs to the braid group $B_3$ we have chosen those as in Fig. 3 with one crossing so the potential attenuates only with the polynomial $1 - \frac{1}{t}$. We believe the physical meaning of $t$ would emerge if a relation between the geometrical and the matrix representation of braids is established. Unfortunately such relation is absence in literature.

The robust calculation of the phase-shift in this molecular model includes the quark structure of mesons something common in all exotic states. The meson potential solves analytically the Lippmann-Schwinger equation since it separates in two form factors ($L(p), R(p)$). Either Lorentzian or Gaussian form factors yield almost the same phase-shift with small adjustments for the cutoff $\gamma$. We neglected the width of the $\rho$ meson in the scattering process $D^0 + \bar{D}^{0*} \rightarrow J/\psi + \rho$ causing the phase-shift to jump sharply at 3.872 GeV. A similar behavior is well known for the Deuteron system [29].

The charged mesons $D^+$ and $\bar{D}^{*+}$ may affect the phase-shift for energies above 3.872 GeV as previously was studied in [16]. We plan to test this braid model for other exotic states since current descriptions do not apply well for all [6]. The braid model fits well with the
predictions for a hot and dense medium created in heavy ion collisions where the X(3872) may form \([30, 31]\). The capture diagram used in Fig. 1 corresponds to the first order in the ladder approximation. Therefore these findings must be interpreted with caution for higher order.

To summarize, braids may help as a mechanism for production of exotic states. The braid \(\sigma_1^{-1}\) modifies the meson potential because the coupling \(\lambda_{ex}\) rescales by a factor \(1 - \frac{1}{t}\) such that a \(X(3872)\) forms as a molecule of \(D^0\) and \(\bar{D}^{0*}\) for \(t = 0.85\). The value of \(t\) may change significantly when adding more capture and transfer diagrams. Nevertheless, it informs if the meson potential is attractive or repulsive. Future studies testing this braid model for other exotic states and predictions for heavy ion collisions are needed.

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