Power Flow and Efficiency Analysis of a Ravigneaux Hybrid Transmission

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Abstract. In urban mobility, special attention must be paid to the energy requirements in vehicles, air quality and noise pollution. This can be achieved in two ways: the first is the use of low-emission hybrid vehicles, the second is the use of gear transmission system with improved efficiency. Both will reduce fuel consumption and provide more environmental protection. In the current work, a low-emission Ravigneaux hybrid transmission with thirteen operation modes is shown to be feasible. A fuel-efficient strategy is developed to control the fuel consumption of hybrid modes and reduce emissions. An efficiency analysis for all driving modes of the Ravigneaux hybrid transmission is conducted. The concept of potential power is used to determine the power ratios of any three links in the fixed reference frame (FRF) when the input link in the moving reference frame (MRF) is predictable. A single equation is developed that can be manipulated to determine all the powers flowing through the train. A numerical example is provided to give a complete view of the expected efficiency values. The proposed Ravigneaux hybrid transmission is found to be suitable for urban rather than external routes. The results can provide a theoretical bases for design works of hybrid transmissions with complex planetary gears.

1. Introduction

Planetary Gears have many usage, in hybrid electric vehicles [1,2], wind power generators [3], hybrid scooter transmissions [4-6], and actuators [7,8]. Most hybrid transmissions are designed based on single-planet planetary gear trains [9-11]. Few designs are developed based on previous automatic transmissions such as Simpson, Ravigneaux and Lepelletier gear trains [12,13]. The Ravigneaux gear train is a double-planet gear train entity that is still widely used in automatic transmissions of vehicles and trucks. The seven-Link and eight-Link Ravigneaux planetary gear trains to obtain hybrid transmissions is a topic with a significant number of contributions along the years [14-16]. Hsieh and Tsai [17] developed a method for calculating the efficiency and ordering the most effective clutching sequences associated with the Simpson, Ravigneaux, and Type-6206 automatic transmissions. Hoang and Yan [16,18] synthesized hybrid transmission configurations with eight-link gear trains. Esmail proposed [13] motor integrated hybrid transmissions based on traditional automatic transmissions; Figure 1 shows the general configuration of the proposed hybrid transmissions. The present study extends the previous work through...
the addition of the efficiency analysis of the operation modes of the hybrid transmission to specify the efficient control strategy to be followed.

Figure 1. Configuration of the Hybrid Transmission [14].

The efficiency of gear transmission systems is critical to reducing fuel consumption and protecting the environment. Latterly, the concept of "potential power" is used to determine the power flow through PGTs. This concept basically defines the power ratios of any three links in the FRF when the input link in the MRF is predictable. In the current work, an extended analysis is performed to include single power input cases, two power outputs and two power inputs with and without a reaction link, giving full-range efficiency formulas to the Ravigneaux hybrid transmission. A simplified expression for calculating the loss factor ($\lambda$) for a pair of meshing gears can be written as [19]:

$$\lambda = \frac{1}{6} \left( \frac{1}{Z_q} + \frac{1}{Z_p} \right)$$

Where $Z_q$ and $Z_p$ are the gear teeth for the two gear $q$ and $p$; the minus (plus) sign refers to an internal (external) gear pair.

Taking into account elasto-hydrodynamic lubrication, velocity-dependent and torque-dependent losses, more sophisticated models of gear friction losses are suggested [20-24]. However, the loss of power resulting from different operating modes for hybrid transmission is still not fully examined. Tuplin model of power loss is utilized to determine the power loss characteristics of the proposed Ravigneaux hybrid transmission.

In order to select a most efficient control strategy of a hybrid transmission, an efficiency analysis is necessary. Due to the fact that the efficiency is a function of the local efficiencies, speed ratios and applied powers, it is essential to identify the relations among the angular velocities, torque distribution and the power flow in the hybrid transmission. It can be shown that power distribution in the hybrid transmission is related to its speed ratios and local efficiencies. Therefore, it is possible to express the efficiency of the hybrid transmission in terms of speed ratios, local efficiencies and applied powers.

This work aims to

1. Develop a simple method to estimate the efficiency of the hybrid transmission in its different operating conditions. As will be seen through this work, simplifying the method is not trivial.
2. Develop a single mathematical formula that can be addressed to identify all the powers flowing through the train from which efficiency can be easily calculated.
3. Provide conditions for smooth shifting: the variations in the input, output and reaction (stationary) members must be determined to provide conditions for smooth shifting.

4. Increase the efficiency of hybrid transmissions which is critical to improving performance, reducing emissions and reducing fuel consumption. There are two important ways to improve efficiency. The first is the rational use of engine power and the second is the adoption of a control strategy based on most efficient operating modes.

5. Develop an effective way to use the power of the engine to charge the battery when there is too much power, and discharge it if needed.

6. Identify the efficiency characteristics of the hybrid transmission and propose the most efficient control strategy.

2. **Operation Modes of the Ravigneaux Hybrid Transmission**

One of the possible alternatives to hybrid transmission is based on the conventionally available Ravigneaux automatic transmission. The motor integrated hybrid transmission shown in Figure 2 consists of two power sources, an engine and a battery, combined with a Ravigneaux gear train. Depending on the battery charging status, the engine and/or battery can power the transmission or the engine alone can provide power to the wheels and to charge the battery.

![Figure 2. The proposed transmission.](image)

The engine crankshaft can be coupled to sun gear by a rotating clutch (EC). The planet carrier can be fixed to the housing by a one way clutch (OWC) or by the reverse clutch (Cₚ) for rear movement.

3. **Analysis of modes by nomograph technique**

The nomograph technique is used to visualize the angular velocity, torque and power at each active link of a planetary gear train [14, 15]. In the present work, the nomograph is drawn to determine the Input/Output and clutch/brake positions to ensure that the transmission system is capable of providing all required operating modes. Depending on the nomographs shown in Figure 3, there are four cluthing conditions from which five main operating modes and thirteen sub-modes originate. Figure 4 shows the power flow of these modes, while Table 1 shows the clutches to be activated with each mode.
Figure 3. Nomographs for the operation modes of the Ravigneaux hybrid transmission
(a) MF (b) MR (c) E1 (d) P1 (e) E/C1 (f) E/C2 (g) E2 (h) P2 (i) R
Figure 4. Power flow through the Ravigneaux hybrid transmission for different operation modes (a) MF+MR (b) E1 (c) P1 (d) E/C1 (e) E/C2 (f) E2 (g) P2 (h) R.
Table 1. Active clutches and operating modes.

| Operating Modes | Engine Clutch (EC) | Forward One-Way Clutch (0WC) | Reverse/Regenerative Clutch (RC) | Motor Operating Condition |
|-----------------|--------------------|-----------------------------|-------------------------------|---------------------------|
| Electric Motor  | MF Forward direction | C ** | M*** |
| MR Backward direction | | C | M |
| Engine | E1 First engine mode | C | F |
| E2 Second engine mode | C | M |
| Engine/Charge | E/C1 First engine/charge mode | C | G |
| E/C2* Second engine/charge mode | C | G |
| Power | P1 First power mode | C | M |
| P2* Second power mode | C | M |
| Regenerative Braking | MF | R0 | C | G |
| MR | RR | C | G |
| E1+ E/C1+ P1 | R1 | C | G |
| E2+ E/C2 | R2 | C | G |
| P2 | R3 | C | G |

*Note that E/C2 and P2 provide CVT capability.
** "C" means that the corresponding clutch is activated.
***"M"=Motor; "F"=Freewheeling; "G"=Generator.

4. Kinematic Analysis
For any PGT, the sequence of angular velocities can be estimated by expressing planet gear ratios (PGRs) in terms of angular velocities. The "planet gear ratio" $N_{p,q}$ can be expressed as specified in reference [25] as:

$$N_{p,q} = \frac{Z_p}{Z_q} = \frac{\omega_q - \omega_e}{\omega_p - \omega_e}$$

(2)

Where (p) refers to the planet gear meshing with gear (q), the + (−) sign is for internal (external) meshing gears.

The velocity ratio $R_{x,y}^z$ among any three links x, y, and z is related to the gear ratios by the following well-known equation [25]:

$$R_{x,y}^z = \frac{N_{p,x} - N_{p,z}}{N_{p,y} - N_{p,z}} = \frac{\omega_x - \omega_z}{\omega_y - \omega_z}$$

(3)

The PGRs for the PGT shown in Figure 1 can be written as:
The second rule (Rule 2, i.e. therefore) can arrange the angular velocities in the following descending order:

\[ \frac{\omega_2 - \omega_3}{\omega_6 - \omega_3} > 0 \]

and

\[ \frac{\omega_4 - \omega_3}{\omega_6 - \omega_3} < 0 \]

Since the number of teeth \( Z_4 \) is always positive, we can conclude from equations (4), (5) and (6) that \( N_{6,1} < 0, 0 < N_{6,4} < 1, \) and \( 0 < N_{6,2} \). Note also that \( Z_6 \) is always smaller than \( Z_4 \).

The signs of \( A \) and \( B \) where \( A \) and \( B \) are the numerator and denominator of a rational number \( A/B \), depend on the sign of the rational number \( A/B \). To solve an inequality of the form \( A/B>0 \) (or \( A/B<0 \)), we first make a sign analysis to determine the signs of the numerator and denominator that satisfy the inequality. When \( A/B>0 \), then the signs of \( A \) and \( B \) are identical while they are different when \( A/B<0 \).

For the inequality \( \frac{\omega_1 - \omega_3}{\omega_6 - \omega_3} < 0 \), there are two cases:

The first case will be detailed only. Since \( \omega_1 - \omega_3 \) is positive, therefore \( \omega_1 > \omega_3 \). From \( \frac{\omega_2 - \omega_3}{\omega_6 - \omega_3} > 0 \) and since \( \omega_6 - \omega_3 < 0 \), therefore \( \omega_2 - \omega_3 < 0 \) or \( \omega_3 > \omega_2 \). From \( 1 > \frac{\omega_4 - \omega_3}{\omega_6 - \omega_3} > 0 \) and since \( \omega_6 - \omega_3 < 0 \), therefore \( \omega_3 > \omega_4 < 0 \) and \( \omega_3 > \omega_4 \). But \( Z_4 > Z_2 \) and \( \frac{1}{Z_4} \leq \frac{1}{Z_2} \) or \( \frac{Z_4}{Z_2} > 0 \) and we can write the lat gear rations in terms of the angular velocities as \( \frac{\omega_2 - \omega_3}{\omega_6 - \omega_3} > \frac{\omega_4 - \omega_3}{\omega_6 - \omega_3} \). By multiplying this inequality with \( (\omega_6 - \omega_3) \) which is a negative quantity, we obtain : \( \omega_4 - \omega_3 > \omega_2 - \omega_3 \) or \( \omega_4 > \omega_2 \). From the above results, we can arrange the angular velocities in the following ascending order : \( \omega_1 > \omega_3 > \omega_4 > \omega_2 \). Similarly for the second case, we can show that angular velocities are arranged in the following ascending order: \( \omega_1 < \omega_3 < \omega_4 < \omega_2 \). In general, that angular velocities are arranged in the following descending (ascending) order:

\[ \omega_1; \omega_3; \omega_4; \omega_2 \]

5. Torque and power flow analysis

To find the driving, and driven links in the FRF and MRF, we must use the following rules [26, 27]:

**Rule 1**: For any three links \( x, y, \) and \( z \) of a gear train entity (GTE) with their velocities arranged in the following ascending or descending order: \( \omega_x; \omega_y; \omega_z \), the torques \( T_x \) and \( T_z \) must have the same direction while \( T_y \) must have opposite directions [26].

**Rule 2**: In the FRF: When \( T_i \omega_i>0 \), then link \( i \) is a driving link in the FRF and will be given the symbol \( (x) \). When \( T_i \omega_i<0 \), then link \( i \) is a driven link in the FRF and will be given the symbol \( (y) \). For example, when the hybrid transmission is in the first engine charge mode listed in **Table 1**, the driving and driven links can be estimated as follows:

It is shown that : \( \omega_1 > \omega_3 > \omega_4 > \omega_2 \). Since link 3 is stationary, then \( \omega_3 = 0 \) and \( \omega_2 \) will be negative, i.e. \( \omega_1 > 0 > \omega_4 > \omega_2 \). For link 2 to be input link, \( T_2 \omega_2 \) must be larger than zero. Since \( \omega_2 < 0 \), then \( T_2 \) must be negative for link 2 to be input. For links 3, 4, and 2, since \( T_2 < 0 \), then according to Rule 1, \( T_3 <
and T₃ > 0. Since ω₄ < 0 and T₄ > 0, then T₄ω₄ < 0 and ring 4 is a driven link. Similarly, For links 1, 3, and 2, since T₂ < 0, then according to Rule 1, T₁ < 0 and T₃ > 0. Since ω₁ > 0 and T₃ < 0, then T₁ω₁ < 0 and sun gear 1 is a driven link too. In a similar way, thirteen modes of operation are obtained according to their input, output and fixed links as shown in Table 2.

| Operating Modes | Link Condition | Velocity range |
|-----------------|----------------|----------------|
| Electric Motor  | MF Input fixed output | T₃ > 0 |
| MR Input fixed output |  |
| Engine          | E1 fixed output Input |  |
| E2 fixed output Input |  |
| Engine/Charge   | E/C1 output fixed output Input |  |
| E/C2 or CVT1 output output Input |  |
| Power           | P1 Input fixed output Input |  |
| P2 or CVT2 Input output Input |  |
| Regenerative    | R0 output fixed Input |  |
| RR output fixed Input |  |
| R1 output fixed Input |  |
| R2 output fixed Input |  |
| R3 output fixed Input |  |

From Table 2, we observe that MF, MR, E1, E2, E/C1, P1, R0, RR, R1, R2, and R3 modes are operating with a reaction link, while E/C2 and P2 modes work without a reaction link. In the latter case, the power supplied to the wheels is regulated either by loading the generator or by providing additional power through the motor.

**Rule 3**: In the MRF where the planet carrier is relatively stationary: When \( T₁ \times (ω₁ - ω₃) > 0 \), then link i is a driving link in the MRF and will be given the symbol (x). When \( T₁ \times (ω₁ - ω₃) < 0 \), then link i is a driven link in the MRF and will be given the symbol (y). When the carrier is stationary, the FRF and MFR are the same.

For example, when the hybrid transmission is in the E/C2 mode listed in Table 2, the driver and driven links in the MRF can be estimated as follows:

It is known that: \( ω₁ > ω₃ > ω₄ > ω₂ \). Since link 3 is relatively fixed, then \( (ω₁ - ω₃) > 0 \) \( (ω₄ - ω₃) > (ω₂ - ω₃) \) and \( (ω₂ - ω₃) \) will be negative. Since \( T₂ < 0 \), then \( T₂ \times (ω₂ - ω₃) \) is larger than zero and link 2 is input link in the MRF also. For links 1, 4, and 2, since \( T₂ < 0 \), then according to Rule 1, \( T₁ < 0 \) and \( T₄ > 0 \). Since \( (ω₄ - ω₃) < 0 \) and \( T₄ > 0 \), then \( T₄ \times (ω₄ - ω₃) < 0 \) and ring gear 4 is a driven link in the MRF. Similarly, Since \( (ω₁ - ω₃) > 0 \) and \( T₃ < 0 \), then \( T₃ \times (ω₁ - ω₃) < 0 \) and sun gear 1 is a driven link in the MRF.

### Table 3: Input and output links for E2, E/C2 and P2 modes in the MRF.

| Operating Modes | Link Condition | Velocity range |
|-----------------|----------------|----------------|
| Engine/Charge   | E/C2 or CVT1 output Relatively fixed output Input | ω₁ > ω₃ > ω₄ > ω₂ |
| Engine          | E2 output Relatively fixed output Input | (ω₁ = 0) > ω₃ > ω₄ > ω₂ |
Rule 4: According to the power balance and relations of angular speeds, the powers flowing through any GTE are related as in the following equation [27]:

\[
\frac{P_x}{R_{y,x}^f} = -\frac{P_y}{\eta_{c(x-y)}R_{y,x}^f} = \frac{P_c}{(\eta_{c(x-y)}-R_{y,x}^f)R_{c,x}^f} \tag{8}
\]

Where $\eta_{c(x-y)}$ is the efficiency related to a GTE in which $x$, and $y$ are the driving and driven links in the carrier-MRF. If one of the three links of the GTE is stationary, only the mathematic expressions that do not contain the subscript denoting the stationary link are used.

6. Power Flow Relations

There are many methods that can be used to calculate the power flowing through different elements of a hybrid transmission. Under steady state operation conditions, the power flowing through any three links of the system can be calculated from equation (8). To facilitate the power flow analysis, the planet gears carrier (link 3) has been used as the relatively fixed link in the MRF (it is already the fixed link in the FRF for this hybrid transmission), therefore the efficiency terms in eq. (8) are reduced to the efficiency of a conventional gear train.

But this requires writing equation (8) twice for two-input (two-output) modes: once per input (or output). For modes without reaction link, the net power passing through the planet carrier (link 3) is zero. For modes with link 3 as reaction link, $P_3 = 0$.

For one-input (one-output) modes, and since link 3 is the stationary link, only the mathematic expressions that do not contain the subscript denoting the stationary link are used.

\[
\frac{P_x}{R_{y,x}^f} = -\frac{P_y}{\eta_{3(x-y)}R_{y,x}^f} \tag{9}
\]

7. Efficiency Formulas for Operating Modes

7.1. Electric Motor Mode

When the motor starts rotating, the OWC becomes active and the motor alone drives the car forward. By putting $x = 1, y = 4, z = f = 3$ into Eq. (9), the following equation is obtained:

\[
P_1 = -\frac{P_x}{\eta_{3(1-4)}} \tag{10}
\]

The efficiency of the motor mode in the reverse and forward directions are the same and can be written from Eq. (10) as:

\[
\eta_{MF} = \eta_{MR} = -\frac{P_x}{P_1} = \eta_{3(1-4)} = \eta_1\eta_4 \tag{11}
\]

7.2. First Engine Mode

When the engine starts operating, the OWC becomes active and the engine alone drives the car forward. By putting $x = 2, y = 4, z = f = 3$ into Eq. (9), we get:

\[
\eta_{E1} = \eta_{3(2-4)} = -\frac{P_x}{P_2} = \eta_2\eta_3\eta_4 \tag{12}
\]
7.3. First Power Mode
When the maximum speed is needed, the electric motor and the engine provide the vehicle with power at the same time. Equation (9) is required to be written twice; once for each input. By putting \( x = 1, y = 4, z = f = 3 \) into Eq. (9), we get:

\[
P_1 = -\frac{P_1^1}{\eta_1 \eta_4} \quad (13)
\]

By putting \( x = 2, y = 4, z = f = 3 \) into Eq. (9), we get:

\[
P_2 = -\frac{P_2^2}{\eta_2 \eta_3 \eta_4} \quad (14)
\]

The output power is given by:

\[
P_4 = P_4^1 + P_4^2 \quad (15)
\]

\[
P_4 = -\eta_1 \eta_4 P_{\text{motor}} - \eta_2 \eta_3 \eta_4 P_{\text{engine}} \quad (16)
\]

The efficiency of the first power mode is:

\[
\eta_{P1} = -\frac{P_4}{P_1 + P_2} = \frac{\eta_1 \eta_4 P_{\text{motor}} + \eta_2 \eta_3 \eta_4 P_{\text{engine}}}{P_{\text{motor}} + P_{\text{engine}}} \quad (17)
\]

7.4. First Engine/Charge Mode
In this mode, the OWC prevents the carrier from rotation and the engine is used to operate the wheels and generator simultaneously. By putting \( x = 2, y = 4, z = f = 3 \) into Eq. (9), we get:

\[
P_{2^1} = -\frac{P_4}{\eta_2 \eta_3 \eta_4} \quad (18)
\]

By putting \( x = 2, y = 1, z = f = 3 \) into Eq. (9), we get:

\[
P_{2^2} = -\frac{P_1}{\eta_1 \eta_2 \eta_3} \quad (19)
\]

The input power is given by:

\[
P_2 = P_{2^1} + P_{2^2} \quad (20)
\]

\[
P_{\text{engine}} = \frac{P_{\text{wheel}}}{\eta_1 \eta_2 \eta_3} - \frac{P_{\text{generator}}}{\eta_1 \eta_2 \eta_3} \quad (21)
\]

The efficiency of the first engine/charge mode is given by:

\[
\eta_{E/C1} = -\frac{P_{\text{wheel}} + P_{\text{generator}}}{P_{\text{engine}}} = \frac{P_{\text{wheel}} + P_{\text{generator}}}{P_{\text{engine}}} \quad (22)
\]

7.5. Second Engine/Charge or CVT1 Mode
In this mode, link 3 is freewheeling and the engine is used to operate the wheels and generator simultaneously. By putting \( x = 2, y = 4, z = 3 \) into Eq. (8), we get:
By assigning \( x = 2, y = 1, z = 3 \) into Eq. (8), we get:

\[
\frac{P_{31}}{R_{4,2}^2} = \frac{-P_4}{\eta_{3(2-4)}R_{4,2}^4} = \frac{P_{32}}{(\eta_{3(2-4)}R_{4,2}^2)R_{4,2}^2}
\]  

(23)

The input power is given by:

\[
P_2 = P_{21} + P_{22}
\]  

(25)

\[
P_2 = \frac{R_{4,2}^3 P_4}{\eta_{3(2-4)}R_{4,2}^4} - \frac{R_{4,2}^3 P_1}{\eta_{3(2-1)}R_{4,2}^4} P_3
\]  

(26)

\[
P_{\text{engine}} = -\frac{R_{4,2}^3 P_{\text{wheel}}}{\eta_{3(2-4)}R_{4,2}^4} - \frac{R_{4,2}^3 P_{\text{generator}}}{\eta_{3(2-1)}R_{4,2}^4}
\]  

(27)

Since link 3 is freewheeling, the net power passing through it is zero.

\[
P_{31} + P_{32} = 0
\]  

(28)

Substituting the values of \( P_{31} \) and \( P_{32} \) from equations (23) and (24) into eq. (28) and arranging, we get:

\[
\frac{P_{\text{wheel}}}{P_{\text{generator}}} = -\frac{\eta_4(\eta_1 \eta_2 \eta_3 - R_{4,2}^2)}{\eta_1(\eta_2 \eta_3 \eta_4 - R_{4,2}^2)} R_{4,1}^4
\]  

(29)

The efficiency of the second engine/charge mode is given by:

\[
\eta_{E/C2} = \frac{1 + \frac{P_{\text{wheel}}}{P_{\text{generator}}}}{\frac{R_{4,2}^2}{\eta_1 \eta_2 \eta_3 R_{4,2}^4} + \frac{R_{4,2}^2}{\eta_2 \eta_3 \eta_4 R_{4,2}^4} P_{\text{generator}}}
\]  

(30)

Substituting the value of \( \frac{P_{\text{wheel}}}{P_{\text{generator}}} \) from equation (29) into eq. (30), we get:

\[
\eta_{E/C2} = \frac{1 - \frac{\eta_4(\eta_1 \eta_2 \eta_3 - R_{4,2}^2)}{\eta_1(\eta_2 \eta_3 \eta_4 - R_{4,2}^2)}}{\frac{R_{4,2}^2}{\eta_1 \eta_2 \eta_3 R_{4,2}^4} + \frac{(\eta_1 \eta_2 \eta_3 - R_{4,2}^2)R_{4,2}^2}{\eta_1 \eta_2 \eta_3 \eta_4 R_{4,2}^4} R_{4,2}^4}
\]  

(31)

7.6. Second Engine Mode

As a complement to the E/C1 or P2 mode, when \( k (= \omega_1 / \omega_2) \) reaches the mechanical point, i.e. the generator/motor speed is zero, the motor behaves as a stationary link and the engine is used to drive the vehicle in a second engine mode. By assigning \( x = 2, y = 4, z = 3 \) and \( f = 1 \) into Eq. (8), we get:

\[
\frac{P_{31}}{R_{4,2}^2} = \frac{-P_4}{\eta_{3(2-4)}R_{4,2}^4} = \frac{P_{32}}{(\eta_{3(2-4)}R_{4,2}^2)R_{4,2}^2}
\]  

(32)
By assigning $x = 2, y = 1, z = 3$ and $f = 1$ into Eq. (8), we get:

$$
\frac{P_{2^2}}{R_{1,2}^4} = \frac{P_{3^2}}{(\eta_{(2-1)} - R_{1,2}^4)R_{1,2}^4}
$$

(33)

Since link 3 is freewheeling, the net power passing through it is zero, i.e. $P_{3^1} = -P_{2^2}$, and Eq. (33) becomes after arranging:

$$
P_{2^2} = -\frac{R_{1,2}^4P_{3^1}}{(\eta_{(2-1)} - R_{1,2}^4)R_{1,2}^4}
$$

(34)

But, from Eq. (32), we can write $P_{3^1}$ as:

$$
P_{3^1} = -\frac{(\eta_{(3-2)} - R_{3,2}^4)R_{1,2}^4p_4}{\eta_{(3-2)}R_{1,2}^4}
$$

(35)

Substituting the value of $P_{2^2}$ from equation (35) into eq. (34), we get:

$$
P_{2^2} = \frac{(\eta_{(3-2)} - R_{3,2}^4)R_{1,2}^4}{\eta_{(3-2)}(\eta_{(2-1)} - R_{1,2}^4)R_{4,2}^4}p_4
$$

(36)

from Eq. (32), we can write $P_{3^1}$ as:

$$
P_{2^1} = -\frac{R_{3,2}^4}{\eta_{(3-2)}R_{4,2}^4}p_4
$$

(37)

The input power is given by:

$$
P_2 = P_{2^1} + P_{2^2}
$$

(38)

Substituting the values of $P_{2^2}$ and $P_{2^1}$ from Eqs. (36) and (37) into eq. (38), we get:

$$
P_2 = \left[\frac{(\eta_{(3-2)} - R_{3,2}^4)R_{1,2}^4}{\eta_{(3-2)}(\eta_{(2-1)} - R_{1,2}^4)R_{4,2}^4} - \frac{R_{3,2}^4}{\eta_{(3-2)}R_{4,2}^4}\right]p_4
$$

(39)

The efficiency of the second engine mode is given by:

$$
\eta_{E_2} = \frac{P_4}{P_2}
$$

(40)

From Eq. (39), we can write:

$$
\eta_{E_2} = -\frac{1}{\frac{(\eta_{(3-2)} - R_{3,2}^4)R_{1,2}^4}{\eta_{(3-2)}(\eta_{(2-1)} - R_{1,2}^4)R_{4,2}^4} + \frac{R_{3,2}^4}{\eta_{(3-2)}R_{4,2}^4}}
$$

(40)

After some simplification and arrangement, we get:

$$
\eta_{E_2} = \frac{\eta_2\eta_3\eta_4}{R_{3,2}^4 - \frac{R_{2,2}^4}{\eta_2\eta_3\eta_4 - R_{3,2}^4}}
$$

(41)
7.7. Second Power Mode

7.7.1. Under drive

In this mode, link 3 is freewheeling and the engine and motor simultaneously are used to operate the wheels. The motor speed is smaller than the engine speed. By assigning \( x = 2, y = 4, z = 3 \) into Eq. (8), we get:

\[
\frac{P_{31}}{R_{4,2}} = - \frac{P_4}{\eta_{3(2-4)}R_{4,2}} = \frac{P_{31}}{(\eta_{3(2-4)}-R_{4,2})R_{3,2}} \tag{42}
\]

By assigning \( x = 2, y = 1, z = 3 \) into Eq. (8), we get:

\[
\frac{P_{32}}{R_{1,2}} = - \frac{P_4}{\eta_{3(2-1)}R_{1,2}} = \frac{P_{32}}{(\eta_{3(2-1)}-R_{1,2})R_{3,2}} \tag{43}
\]

The input power is given by these two parts:

\[
P_{21} = - \frac{R_{3,2}}{\eta_{3(2-4)}R_{4,2}} P_4 \tag{44}
\]

and

\[
P_{22} = - \frac{R_{3,2}}{\eta_{3(2-1)}R_{1,2}} P_1 \tag{45}
\]

The input power is given by:

\[
P_2 = P_{21} + P_{22} \tag{46}
\]

Substituting the values of \( P_{21} \) and \( P_{22} \) from Eqs. (44) and (45) into eq. (46) and renaming, we get:

\[
\frac{P_{\text{engine}}}{P_{\text{motor}}} = - \frac{R_{3,2}}{\eta_{3(2-4)}R_{4,2}} \frac{P_{\text{wheel}}}{P_{\text{motor}}} = \frac{R_{3,2}}{\eta_{3(2-1)}R_{1,2}} \tag{47}
\]

From Eq.(42), we can write:

\[
P_{31} = - \frac{(\eta_{3(2-4)}-R_{4,2})R_{3,2}^f}{\eta_{3(2-4)}R_{4,2}^f} P_4 \tag{48}
\]

From Eq.(43), we can write:

\[
P_{32} = - \frac{(\eta_{3(2-1)}-R_{1,2})R_{3,2}^f}{\eta_{3(2-1)}R_{1,2}^f} P_1 \tag{49}
\]

Since link 3 is freewheeling, the net power passing through it is zero.

\[
P_{31} + P_{32} = 0 \tag{50}
\]

Substituting the values of \( P_{31} \) and \( P_{32} \) from equations (48) and (49) into eq. (50) and arranging we get:

\[
\frac{P_{\text{wheel}}}{P_{\text{motor}}} = \frac{\eta_{4}y_{2}y_{3}-R_{4,2}^f}{\eta_{4}y_{2}y_{3}+R_{4,2}^f} R_{4,1}^f \tag{51}
\]

The efficiency of the second engine/charge mode is given by:
\[ \eta_{P_2} = - \frac{P_{\text{wheel}}}{P_{\text{motor}} + P_{\text{engine}}} = - \frac{P_{\text{wheel}}}{P_{\text{motor}}} \frac{P_{\text{engine}}}{1 + \frac{P_{\text{engine}}}{P_{\text{motor}}}} \]  

(52)

Substituting the values of \( \frac{P_{\text{engine}}}{P_{\text{motor}}} \) and \( \frac{P_{\text{wheel}}}{P_{\text{motor}}} \) from equations (47) and (51) into eq. (52), we get:

\[ \eta_{P_2, \text{under\text{-}drive}} = \frac{\eta_4 \left( \eta_2 \eta_3 - R_{1,2}^f \right) R_{4,1}^f}{\eta_1 \left( \eta_2 \eta_4 - R_{1,2}^f \right) R_{4,1}^f} \]  

(53)

7.7.2. Direct Drive

With both the engine and motor rotate at the same speed, the gear set locked up as a unit, and power is provided to the vehicle without power loss.

\[ \eta_{P_2, \text{direct\text{-}drive}} = 1 \]  

(53)

7.7.3. Over Drive

When the motor moves faster than the engine, the vehicle operates in the over drive area. By putting \( x = 1, y = 2, z = 3 \) into Eq. (8), we get:

\[ \frac{P_1}{R_{2,1}^f} = - \frac{P_{21}}{\eta_1 (1-2) R_{2,1}^f} = \frac{P_{31}}{(\eta_1 (1-2) - R_{2,1}^f) R_{3,1}^f} \]  

(54)

By putting \( x = 4, y = 2, z = 3 \) into Eq. (8), we get:

\[ \frac{P_4}{R_{3,4}^f} = - \frac{P_{22}}{\eta_3 (4-2) R_{3,4}^f} = \frac{P_{32}}{(\eta_3 (4-2) - R_{3,4}^f) R_{3,4}^f} \]  

(55)

The input power is given by these two parts:

\[ P_{2^1} = - \frac{\eta_1 (1-2) R_{2,1}^f}{R_{2,1}^f} P_1 \]  

(56)

and

\[ P_{2^2} = - \frac{\eta_3 (4-2) R_{3,4}^f}{R_{3,4}^f} P_4 \]  

(57)

The input power is given by:

\[ P_2 = P_{2^1} + P_{2^2} \]  

(58)

Substituting the values of \( P_{2^1} \) and \( P_{2^2} \) from Eqs. (56) and (57) into eq. (58), we get:

\[ P_{\text{engine}} = - \eta_1 (1-2) R_{2,1}^f P_{\text{motor}} - \eta_3 (4-2) R_{3,4}^f P_{\text{wheel}} \]  

(59)

\[ \frac{P_{\text{engine}}}{P_{\text{motor}}} = - \frac{\eta_1 (1-2) R_{2,1}^f}{R_{2,1}^f} \frac{P_{\text{wheel}}}{P_{\text{motor}}} \]  

(59)

From Eq.(54), we can write:

\[ P_3^1 = \frac{(\eta_3 (1-2) R_{3,4}^f) R_{3,4}^f}{R_{3,4}^f} P_1 \]  

(60)
From Eq.(55), we can write:
\[ P_{3} = \frac{(\eta_{3}(4-2)-R_{4}^{3})R_{1,4}^{1,4}}{R_{2,4}^{2}}p_{4} \]  \hfill (61)

Since link 3 is freewheeling, the net power passing through it is zero.
\[ P_{3} = -P_{3} \]  \hfill (62)

Substituting the values of \( P_{3} \) and \( P_{3} \) from equations (60) and (61) into eq. (62) and arranging we get:
\[ \frac{P_{\text{wheel}}}{P_{\text{motor}}} = \frac{(\eta_{2} \eta_{4} - R_{4,1}^{3})R_{4,1}^{1,4}R_{1,4}^{1,4}}{(\eta_{2} \eta_{4} - R_{4,1}^{3})R_{1,4}^{1,4}R_{1,4}^{1,4}} \]  \hfill (63)

The efficiency of the second engine/charge mode is given by:
\[ \eta_{P_{2}} = -\frac{P_{\text{wheel}}}{P_{\text{motor}} + P_{\text{engine}}} = -\frac{\frac{P_{\text{wheel}}}{P_{\text{motor}}}}{1 + \frac{P_{\text{engine}}}{P_{\text{motor}}}} \]  \hfill (64)

Substituting the values of \( \frac{P_{\text{engine}}}{P_{\text{motor}}} \) and \( \frac{P_{\text{wheel}}}{P_{\text{motor}}} \) from equations (59) and (63) into eq. (64), we get:
\[ \eta_{P_{2,\text{over--drive}}} = \frac{(\eta_{2} \eta_{4} - R_{4,1}^{3})R_{4,1}^{1,4}R_{1,4}^{1,4}}{(\eta_{2} \eta_{4} - R_{4,1}^{3})R_{1,4}^{1,4}R_{1,4}^{1,4}} \]  \hfill (65)

Table 4. shows the efficiency formulas for all driving modes.

| Operating Modes | Efficiency |
|-----------------|------------|
| Electric Motor  | \( \eta_{\text{MF}} = \eta_{1} \eta_{4} \) |
| MR             | \( \eta_{\text{MR}} = \eta_{1} \eta_{4} \) |
| Engine          | \( \eta_{E_{1}} = \frac{\eta_{2} \eta_{3} \eta_{4}}{\eta_{2} \eta_{3} \eta_{4}} \) |
| E2             | \( \eta_{E_{2}} = \frac{R_{4,1}^{3}}{R_{4,1}^{1,4}} \cdots \) |
| Engine/Charge   | \( \eta_{E/C_{1}} = \frac{P_{\text{wheel}} + P_{\text{generator}}}{\eta_{1} \eta_{2} \eta_{3} R_{4,1}^{1,4} \cdots} \) |
| E/C2 or CVT1    | \( \eta_{E/C_{2}} = \frac{1 - \frac{R_{1,2}^{3} R_{1,2}^{1,4}}{R_{1,2}^{3} R_{1,2}^{1,4}} \cdots \frac{\eta_{1} \eta_{2} \eta_{3} \eta_{4} - R_{4,2}^{3}}{\eta_{1} \eta_{2} \eta_{3} \eta_{4} - R_{4,2}^{3}} \cdots} \) |
| Power           | \( \eta_{P_{1}} = \frac{\eta_{1} \eta_{4} \frac{P_{\text{motor}}}{P_{\text{motor}} + P_{\text{engine}}} + \eta_{2} \eta_{3} \eta_{4} \frac{P_{\text{engine}}}{P_{\text{motor}} + P_{\text{engine}}} \cdots} \) |
8. Control Strategy

To develop a control strategy, we first have to answer the question: how to move from one mode to the next? Returning to Table 1, we observe the following:

a. The transition from MF and MR to R0 and RR is accomplished by changing the motor status from M to G and vice versa.

b. The shift between E/C1, E1, P1 and R1 is accomplished by changing the motor status from M or F to G and deactivation the EC.

c. Similarly, switching between E/C2, E2, P2 and R2; and between P2 and R3 where only the (Rc) is activated is accomplished by changing the motor status from M or F to G.

The operating conditions given in Table 1 can be re-categorized as shown in Table 2. Each of the first and second drives consists of a power mode, an engine mode, an engine/charge mode and a regenerative mode. Each of the under drive, direct drive, and over drive consists of a P and R modes.

| Operating Modes | Clutching Condition | Motor Operating Condition |
|-----------------|---------------------|---------------------------|
| Electric Motor - Forward | MF | Engine Clutch (EC) | Forward One-way Clutch (OWC) | Reverse/Regenerative Clutch (RC) | M |
| | R0 | C | | | G |
| Electric Motor - Reverse | MR | E/C1 | C | C | G |
| | RR | E1 | C | C | F |
| | | P1 | C | C | M |
| First drive | | R1 | C | C | G |

Regenerative

\[ \eta_{P2_{\text{under-drive}}} = \frac{\eta_4(n_1n_2n_3 - R_{1,2}^3)}{\eta_1(n_2n_3n_4 - R_{4,2}^3)} R_{4,1}^f \\
1 + \frac{(n_1n_2n_3 - R_{1,2}^3)R_{4,1}^3}{\eta_1n_2n_3(n_2n_3n_4 - R_{4,2}^3)R_{1,2}^f} - \frac{R_{1,2}^3}{n_1n_2n_3R_{1,2}^f} \]

\[ \eta_{P2_{\text{direct-drive}}} = 1 \]

\[ \eta_{P2_{\text{over-drive}}} = \frac{(n_1n_2n_3 - R_{2,1}^3)R_{2,1}^3 R_{4,1}^f}{(n_2n_3n_4 - R_{4,2}^3)R_{4,1}^3} + \frac{n_1n_2n_3(n_1n_2n_3 - R_{2,1}^3)R_{4,1}^f}{(n_2n_3n_4 - R_{4,2}^3)R_{4,1}^3} \]

| \[ \eta_R = \eta_1 \eta_4 \] |
Second drive

| E/C2 or CVT1 | C | G |
|--------------|---|---|
| E2          | C | M |
| P2          | C | M |
| R2          |   | G |

Direct Drive, and
Over Drive

| P2 or CVT2 | C | M |
|------------|---|---|
| R3         | C | G |

Figure 5 illustrates the paths adopted in developing a control strategy.

Figure 5. The flowchart that illustrates the paths adopted in developing a control strategy.

A practical schedule is the flow on any path on the bold line of Figure 5. To further reduce emissions, a fuel-efficient strategy is developed to control the fuel consumption of hybrid modes. An efficient schedule is MF → E/C1 → E/C2 → P2.

Example. To demonstrate the feasibility of the proposed design, a numerical example is presented to calculate the efficiency of the operation modes of the Ravigneaux hybrid transmission. Assuming that $Z_1 = 30, Z_2 = 26, Z_4 = 74, Z_5 = 18$ and $Z_6 = 22$, respectively. Using Eqs. (1), four gear ratios are calculated. Using Eqs. (2), the efficiency of the gear pairs are calculated. Table 6 summarize the results.

| $GP_1$ | q | p | $\eta_{GP_1}$ |
|-------|---|---|--------------|
| $GP_1$ | 1 | 6 | -22/30 | 0.9842 |
| $GP_2$ | 2 | 5 | -18/26 | 0.9812 |
| $GP_3$ | 5 | 6 | -22/18 | 0.9798 |
| $GP_4$ | 4 | 6 | 22/74 | 0.9936 |

Eight speed ratios of the Ravigneaux hybrid transmission are calculated from Eq. (3) as shown in Table 7.

| $R_{1_2}^1$ | 0.3514 |
| $R_{1_4}^1$ | $\frac{1}{R_{1_2}^1}$ | 2.8462 |
| $R_{1_2}^2$ | -0.8667 |
| $R_{1_4}^2$ | $\frac{1}{R_{1_2}^2}$ | -1.1538 |

Table 6. Efficiency of the gear pairs of the Ravigneaux hybrid transmission.

Table 7. Eight speed ratios of the Ravigneaux hybrid transmission.
\[ R_{12}^f = k^* \quad R_{21}^f = \frac{1}{R_{12}^f} = \frac{1}{k} \]

\[ R_{12}^1 = 0.6274 \quad R_{21}^f = \frac{0.04685 - 0.02495k}{0.07179k} \]

\[ k = \omega_1/\omega_2 \]

### Table 8. Efficiency for each driving mode

| Operating Modes | Efficiency | k = \omega_1/\omega_2 |
|-----------------|------------|------------------|
| Electric Motor  |            |                  |
| MF              | \eta_{ME} = 0.9779 | k = -0.8667 |
| MR              | \eta_{ME} = 0.9779 | k = -0.8667 |
| Engine          |            |                  |
| E1              | \eta_{E1} = 0.9673 | k = -0.8667 |
| E2              | \eta_{E2} = 0.9397 | k = 0 |
| Engine/Charge   |            |                  |
| E/C1            | For \(0 \leq \frac{P_{\text{generator}}}{P_{\text{wheel}}} \leq 0.6 \) or \(0 \leq \frac{P_{\text{wheel}}}{P_{\text{generator}}} \leq \frac{1}{0.6} \), \(0.9462 \leq \eta_{E/C1} \leq 0.9552 \) | k = -0.8667 |
| E/C2 or CVT1    | \eta_{E/C2} = \frac{(1.9393 + 0.0328k)}{2.0091} | \(-0.8667 < k < 0\) |
| Power           |            |                  |
| P1 Under drive  | For \(0 \leq \frac{P_{\text{motor}}}{P_{\text{engine}}} \leq 0.6 \) or \(0 \leq \frac{P_{\text{engine}}}{P_{\text{motor}}} \leq \frac{1}{0.6} \), \(0.9552 \leq \eta_{P1} \leq 0.9780 \) | k = -0.8667 |
| P2 Under drive  | \eta_{P2\text{under-drive}} = \frac{1.9393 + 1.0328k}{k + 2.0091} | \(0 < k < 1\) |
| P2 Direct drive | \eta_{P2\text{direct-drive}} = 1 | k = 1 |
| P2 Over drive   | \eta_{P2\text{over-drive}} = \frac{(1.7993 + 0.8584k)}{(1.4316 + 1.3257k)} | \(0 < \frac{1}{k} < 1\) |
| Regenerative    | \eta_R = 0.9779 | k = -0.8667 |

From Table 8, note that the efficiency is high in all modes except the second power mode when the vehicle is over-driven.

### 9. Conclusions

The hybrid transmission presented in this paper uses the commercially available Ravigneau gear train, which features the quality of an experienced automatic transmission. The Ravigneau Hybrid Transmission can be implemented in cars that operate most of the time within urban areas. It is highly efficient at low and medium speeds and less at very high speeds. The Ravigneau hybrid transmission features operational flexibility, compactness and simplicity. Only one electric motor integrated with one set of planetary gears and three clutches perform thirteen operating modes. In the event of a malfunction, the vehicle can be driven by the engine or motor alone.
Nomographs are used to visualize the operating modes of the hybrid transmission. A method is proposed to calculate the efficiency of the hybrid transmission based on the actual powers flowing through it. For any three links of a planetary gear train, a single equation is developed that can be manipulated to determine all the powers flowing through the train. The use of a single power equation in multi-mode hybrid transmission analysis can make this work different and novel. The results indicate that it is very important to consider the efficiency of hybrid modes in determining the appropriate control strategy.

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