Search for a Stable Six-Quark State at BaBar

J. P. Lees,1 V. Poireau,1 V. Tisserand,1 E. Grauges,2 A. Palano,3 G. Eisen,4 D. N. Brown,5 Yu. G. Kolomensky,5 M. Fritsch,6 H. Koch,6 T. Schroeder,6 C. Heartyab,7 T. S. Mattisonb,7 J. A. McKennew,7 R. Y. Soe,7 V. E. Blinovabc,8 A. R. Buzykaev,8 V. P. Druzhininab,8 V. B. Golubevabc,8 E. A. Kozyrevaabc,8 E. A. Kravchenkoab,8 A. P. Omuchinab,8 S. I. Serednyakovab,8 Yu. I. Skopyenab,8 E. P. Solodovab,8 K. Yu. Todorchevab,8 A. J. Lankford,9 J. W. Gary,10 O. Long,10 A. M. Eisner,11 W. S. Lockman,11 W. Panduro Vazequez,11 D. S. Chao,12 C. H. Cheng,12 B. Echenard,12 K. T. Flood,12 D. G. Hitlin,12 J. Y. Li,12 T. S. Miyashita,12 P. Ongmongkolk12,12 F. C. Porter,12 M. Rörken,12 Z. Huard,13 B. T. Meadows,13 B. G. Pushpawela,13 M. D. Sokoloff,13 L. Sun,13 J. G. Smith,14 S. R. Wagner,14 D. Bernard,15 M. Verderi,15 D. Bettoni16,16 C. Bozzi16,16 R. Calabreseab,16 G. Cibinettoab,16 E. Fioravantiab,16 I. Garziaab,16 E. Luppi16,16 V. Santoroa,16 A. Calcaterra,17 R. de Sangro,17 G. Finocchiaro,17 S. Martellotti,17 P. Patteri,17 I. M. Peruzzi,17 M. Piccolo,17 M. Rotondo,17 A. Zallo,17 P. Passaggio,18 C. Patrignani,18 H. M. Lacker,19 B. Bhuyan,20 U. Malikc,21 C. Chen,22 J. Cochran,22 S. Prell,22 A. V. Gritsan,23 N. Arnaud,24 M. Davier,24 F. Le Diberder,24 A. M. Lutz,24 G. Wormser,24 D. J. Lange,25 D. M. Wright,25 J. P. Coleman,26 E. Gabathuler,26 D. E. Hutechcroft,26 D. J. Payne,26 C. Touramanis,26 A. J. Bevan,27 F. Di Lodovico,27 R. Sacco,27 G. Cowan,28 Sw. Banerjee,29 D. N. Brown,29 C. L. Davis,29 A. G. Denig,30 W. Gradl,30 K. Griessinger,30 A. Hafner,30 K. R. Schubert,30 R. J. Barlow,31,31 G. D. Lafferty,31 R. Cenci,32 A. Jawahery,32 D. A. Roberts,32 R. Cowan,33 S. H. Robertsonab,34 R. M. Seddon,34 B. Deyab,35 N. Neria35 F. Palomboa,35 R. Cheaib,36 L. Cremaldi,36 R. Godang,36 D. J. Summers,36 P. Taras,37 G. De Nardo,38 C. Sciacci,38 G. Raven,39 C. P. Jessop,40 J. M. LoSecco,40 K. Hanscheid,41 R. Kass,41 A. Gaze,42 M. Margoniab,42 M. Possocca,42 G. Simibia,42 F. Simonettob,42 R. Stroiliab,42 S. Akar,43 E. Ben-Haim,43 M. Bomben,43 R. G. Bonneaua,43 G. Calderinib,43 J. Chauveauc,43 G. Marchioria,43 J. Ocariza,43 M. Biasiaab,44 E. Manonib,44 A. Rossiab,44 G. Batignanib,45 S. Bettarinbic,45 M. Carpinellia,45 G. Casorosac,45 M. Chrzaszcza,45 F. Fortib,45 M. A. Giorgib,45 A. Lusianbac,45 B. Oberholfb,45 E. Paoloniab,45 M. Rama,45 G. Rizzoab,45 J. J. Walshab,45 L. Zanib,45 A. J. Smith,46 F. Anullia,47 R. Facchinib,47 F. Ferrartott,47 F. Ferronib,47 A. Pillonib,47 G. Piredda,47 C. Bünger,48 S. Dittrich,48 O. Grünberg,48 M. Heß,48 T. Leddig,48 C. Voß,48 R. Waldib,48 T. Adye,49 F. F. Wilson,49 S. Emery,50 G. Vasseur,50 D. Aston,51 C. Cartaro,51 M. R. Convery,51 J. Dorfan,51 W. Dunwoodie,51 M. Ebert,51 R. C. Field,51 B. G. Fulsom,51 M. T. Graham,51 C. Hast,51 W. R. Innes,51 P. Kim,51 D. W. G. Seith,51 S. Luizi,51 B. D. MacFarlane,51 D. R. Muller,51 H. Neal,51 B. N. Ratcliff,51 A. Roozman,51 M. K. Sullivan,51 J. Va’vra,51 W. J. Wismieski,51 V. M. Purohit,52 J. R. Wilson,52 A. Randle-Conde,53 S. J. Sekula,53 H. Ahmed,54 M. Bellis,55 P. R. Burchat,55 E. M. T. Puccio,55 M. S. Alam,56 J. A. Ernst,56 R. Gorodeisky,57 N. Guttman,57 D. R. Peimer,57 A. Soffer,57 S. M. Spanier,58 J. L. Ritchie,59 R. F. Schawers,59 J. M. Izen,60 X. C. Lou,60 F. Bianchiab,61 F. De Morab,61 A. Filippib,61 D. Gambaab,61 L. Lanceri,61 L. Vitale,62 F. Martinez-Vidal,63 A. Oyanguren,63 J. Albertb,64 A. Beaulieu,64 F. U. Bernlochnerb,64 G. J. King,64 R. Kowalewski,64 T. Lueck,64 I. M. Nugentb,64 J. M. Roneyab,64 R. J. Sobieab,64 N. Tasneemb,64 T. J. Gershon,65 P. F. Harrison,65 T. E. Latham,65 R. Prepost,66 and S. L. Wu66

(The BaBar Collaboration)

1Laboratoire d’Annecy-le-Vieux de Physique des Particules (LAPP), Université d'Annecy, CNRS/IN2P3, F-74941 Annecy-le-Vieux, France
2Universitat de Barcelona, Facultat de Fisica, Departament ECM, E-08028 Barcelona, Spain
3INFN Sezione di Bari and Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy
4University of Bergen, Institute of Physics, N-5007 Bergen, Norway
5Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA
6Ruhr Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany
7Institute of Physics”; University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1
8Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Novosibirsk State University, Novosibirsk 630090, Russia
9University of California at Irvine, Irvine, California 92697, USA
10University of California at Riverside, Riverside, California 92521, USA
Recent investigations have suggested that the six-quark combination $uuddss$ could be a deeply bound state ($S$) that has eluded detection so far, and a potential dark matter candidate. We report
A new stable state of matter may still be undiscovered. While the vast majority of known hadrons can be described as either quark-antiquark or three-quark combinations, other multi-quark possibilities are allowed by quantum chromodynamics (QCD). Among those, the six-quark configuration $uuddss$ is of particular interest, as it may be a deeply bound, stable system as proposed by Farrar [1]. This state, tentatively named $S$ [2], is a spin 0, flavor singlet boson with quantum numbers $Q = 0$, $B = 2$, and $S = -2$. Unlike any other six-quark configuration, its spatial wave function is completely symmetric; generic arguments imply that it should be the most tightly bound state of its class (see e.g. Ref. [3]). This property was noticed by Jaffe 40 years ago [4]. He predicted the existence of a loosely bound $uuddss$ state with a mass close to 2150 MeV [5], dubbed the H-dibaryon. As its mass is above the $m_p + m_e + m_A = 2055$ MeV threshold, the H-dibaryon would have a typical weak interaction lifetime. Numerous negative experimental results were taken as evidence against such a particle, including observations of doubly strange hypernuclei decays [6, 7], searches for narrow $Λpπ^-$ resonances in $Υ$ decays [8] and direct searches for new neutral particles (see e.g. Ref. [9–13]).

The situation is markedly different if the potential is deeply attractive. Below $m_S = 2055$ MeV, the $uuddss$ configuration acquires a cosmological lifetime, as its decay would have to proceed via doubly-weak interactions, and it is absolutely stable if $m_S < 2(m_p + m_e) = 1878$ MeV. Intriguingly, recent lattice QCD investigations suggest the possibility of a strongly bound $B = 2, S = -2$ state [14], though the calculations need further improvements to provide a definitive answer. Neither hypernuclei decays nor direct searches for long-lived neutral states have excluded such a possibility so far (the latter were limited to masses above ~2 GeV due to the large neutron background [15]).

Although not all authors agree (see e.g. Ref. [15]), a stable six-quark state might also have cosmological implications. If dark matter is composed of nearly equal numbers of $u, d, s$ quarks, its formation rate is driven by the quark-gluon plasma transition to the hadronic phase and the quark and anti-quark abundances. As the same source is also responsible for determining the residual amount of ordinary matter in the universe, this framework would explain both the dark matter density and the baryon asymmetry, two seemingly unrelated quantities. A specific realization of this scenario, six-quark dark matter with $m_S ∼ 1860 – 1880$ MeV, can reproduce the observed ratio of dark matter to ordinary matter densities within ~15% [16].

Being a flavor singlet, the $S$ particle does not couple to pions or other mesons. The $S$-nucleon interaction cross-section is expected to be suppressed compared to that of nucleon-nucleon interactions, and its production rate is several orders of magnitude below that for neutrons. Given that a low-mass $S$ is difficult to kinematically distinguish from a neutron, these attributes might explain why this state has escaped detection so far. Despite these difficulties, several search strategies have been proposed. Among them, the exclusive decay $Υ^+ → SΛ$ [12] stands out for its simplicity and robustness. The short-distance nature of the gluonic source increases the overlap with the compact $S$ wave function, enhancing its production rate compared to other mechanisms involving baryons. Heuristic arguments suggest an inclusive six-quark production rate in $Υ(1S, 2S, 3S)$ decays at the level of $10^{-7}$, albeit with significant uncertainties. No specific prediction for the exclusive $SΛ$ final state has been made so far, though this channel could conceivably account for a large fraction of the total production rate.

We report herein the first search for a stable, doubly strange six-quark configuration produced in $Υ(2S, 3S)$ decays [18]. For completeness, we probe the entire mass range compatible with a stable state: $0$ GeV < $m_S$ < 2.05 GeV. The analysis is based on a sample containing $90 × 10^6 Υ(2S)$ and $110 × 10^6 Υ(3S)$ decays collected with the Babar detector at the PEP-II asymmetric-energy $e^+e^-$ collider operated at the SLAC National Accelerator Laboratory. The integrated luminosity of the $Υ(2S)$ and $Υ(3S)$ samples are 14 fb$^{-1}$ and 28 fb$^{-1}$, respectively [19]. Additional samples of 428 fb$^{-1}$ collected at the $Υ(4S)$ peak, as well as in the vicinity of the $Υ(2S, 3S)$ resonances, are used to estimate the background. The Babar detector is described in detail elsewhere [20, 21]. To avoid experimental bias, we examine the data signal region only after finalizing the analysis strategy.

Simulated events are used to optimize the selection procedure and assess the signal efficiency. Signal events are generated for $0$ GeV < $m_S$ < 2.2 GeV in steps of 0.2 GeV. The $S$ angular distribution is simulated using an effective Lagrangian based on a constant matrix element for the different arrangements of angular momentum between the final state particles, assuming that angular momentum suppression effects are small [22] (see appendix for a detailed description). A second model based on a
phase space distribution is used to assess systematic uncertainties. The interaction between six-quark states and matter is expected to be similar to that of neutrons, albeit with reduced cross-sections. For the purpose of simulating the signal, we model these interactions similarly to those of neutrons. As an extreme alternative, we simulate six-quark states as non-interacting particles, and we assign the difference between these two models as a systematic uncertainty. To study the background, we generate generic \( \Upsilon(2S, 3S, 4S) \) decays with EvtGen \[23\], while the continuum \( e^+e^- \rightarrow q\bar{q} \ (q = u, d, s, c) \) background is estimated using a data-driven approach described below. The detector acceptance and reconstruction efficiencies are determined using a Monte Carlo (MC) simulation based on GEANT4 \[24\]. Time-dependent detector inefficiencies and background conditions, as monitored during data-taking periods, are included in the simulation.

We select events containing at most 5 tracks and two \( \Lambda \) candidates with the same strangeness, reconstructed in the \( \Lambda\Lambda \rightarrow p\pi^-\pi^- \) final state with \( 1.10 \text{ GeV} < m_{p\pi} < 1.14 \text{ GeV} \). One additional track not associated with a \( \Lambda \) candidate with a distance of closest approach from the primary interaction point (DOCA) larger than 5 cm is allowed to account for particles produced from secondary interactions with the detector material. The (anti)protons must be selected by particle identification (PID) algorithms. This requirement, which is approximately 95% efficient for identifying both protons and antiprotons, removes a large amount of background from four-pion final states. To further improve the signal purity, the \( \Lambda \) flight vector is measured as the distance between the primary interaction point and the \( \Lambda \) decay vertex. The flight significance of each \( \Lambda \) candidate, defined as the length of this vector dived by its uncertainty, must be larger than 5. The cosine of the angle between the \( \Lambda \) momentum and the flight vector must also be greater than 0.9. In addition, the total energy of clusters in the electromagnetic calorimeter not associated with charged particles, \( E_{\text{extra}} \), must be less than 0.5 GeV. To account for possible interactions between the \( S \) candidate and the calorimeter, the sum excludes clusters that are closer than an angle of 0.5 rad to the inferred \( S \) direction. Moreover, the distance between the cluster and the proton is required to be greater than 40 cm to reduce the contribution of cluster fragments. The \( E_{\text{extra}} \) distribution after applying all other selection criteria is shown in Fig. 1. The selection procedure is tuned to maximize the signal sensitivity, taking into account the systematic uncertainties related to \( S \) production and interaction with detector material in the calculation. The \( p\pi^- \) mass distribution obtained after applying these criteria is shown in Fig. 2. A total of 8 \( \Upsilon \rightarrow S\Lambda\bar{\Lambda} \) candidates are selected.

The events are then fit, imposing a mass constraint to each \( \Lambda \) candidate and requiring a common origin, compatible with the beam interaction point within its uncertainty. We select combinations with \( \chi^2 < 25 \) (for 8 d.o.f.), retaining half of the previously selected candidates. The signal is identified as a peak in the recoil mass squared against the \( \Lambda\Lambda \) system, \( m_{\text{rec}}^2 \), in the region \( 0 \text{ GeV}^2 \lesssim m_{\text{rec}}^2 \lesssim 5 \text{ GeV}^2 \). The recoil mass squared allows for negative values arising from the limited resolution on the reconstructed \( \Lambda \) candidates, providing a better estimator of the efficiency near \( m_S \sim 0 \) GeV than the recoil mass. The \( m_{\text{rec}}^2 \) distribution is shown in Fig. 3a, together with various background predictions and a simulated signal assuming \( m_S = 1.6 \text{ GeV} \). No events are observed in the signal region.

The continuum \( e^+e^- \rightarrow q\bar{q} \ (q = u, d, s, c) \) background is estimated from the data collected at the \( \Upsilon(4S) \) peak. This data sample contains contributions from both continuum and \( \Upsilon(4S) \) events. The latter is evaluated from the generic \( \Upsilon(4S) \) MC sample and found to be negligible, as those decays tend to have higher multiplicity and are much more suppressed than continuum production by our selection. The data collected at the \( \Upsilon(4S) \) resonance are therefore a good representation of the continuum background.

The \( \Upsilon(2S, 3S) \) background components are estimated from the corresponding MC simulations. The contributions are normalized using sideband data obtained by applying all the selection criteria previously described but requiring \( E_{\text{extra}} \) to be greater than 0.5 GeV instead of below that threshold. The \( \Upsilon(2S, 3S) \) MC components are found to underestimate the observed \( p\pi \) yield, and we adjust their overall normalizations to improve the agree-
FIG. 2: The distribution of the $p\pi$ invariant mass, $m(p\pi)$, before performing the kinematic fit for the combined $\Upsilon(2S)$ and $\Upsilon(3S)$ data sets, together with various background estimates. All other selection criteria are applied. Continuum background is scaled from $\Upsilon(4S)$ data using integrated luminosity; the $\Upsilon(2S,3S)$ MC are normalized using $E_{\text{extra}}$ sideband data. Two entries per event are plotted.

FIG. 3: The distribution of the recoil mass squared against the $\Lambda\Lambda$ system, $m_{\text{rec}}^2$, after performing the kinematic fit for the combined $\Upsilon(2S)$ and $\Upsilon(3S)$ data sets, together with various background estimates for (a) the $E_{\text{extra}} < 0.5$ GeV signal region and (b) the $E_{\text{extra}} > 0.5$ GeV sideband data sample. A signal spectrum assuming $m_S = 1.6$ GeV and a branching fraction $B(\Upsilon \to \Lambda\Lambda) = 1 \times 10^{-7}$ is shown as an example. Continuum background is scaled from $\Upsilon(4S)$ data using integrated luminosity; the $\Upsilon(2S,3S)$ MC are normalized using $E_{\text{extra}}$ sideband data.

The efficiency as a function of the $S$ mass is derived from the corresponding MC sample. For each mass hypothesis, we define a signal region in the $m_{\text{rec}}^2$ distribution with a profile likelihood method with the data. The resulting correction factors are then used throughout the analysis.

A data-driven estimate of the background is also derived from the sideband data. Similarly to signal events, this sample contains predominantly two real $\bar{\Lambda}$ particles with additional (undetected) particles. Since the difference in $E_{\text{extra}}$ is essentially due to the interaction of those particles with the calorimeter, sideband data provide a good approximation of the expected background in the signal region. The corresponding recoil mass distribution is displayed in Fig. [3b]. Both MC and data-driven methods predict a negligible background in the signal region.

The efficiency as a function of the $S$ mass is derived from the corresponding MC sample. For each mass hypothesis, we define a signal region in the $m_{\text{rec}}^2$ distribution as the symmetric interval around the nominal $S$ mass containing 99% of the reconstructed $S$ candidates. Its typical size is of the order of 2.5 GeV$^2$. The efficiency rises from 7.2% near threshold to 8.2% near $m_S = 2$ GeV, and is well approximated by a second order polynomial. The efficiency is mainly driven by the detector acceptance and the $\Lambda \to p\pi$ branching fraction.

The main uncertainties on the efficiency arise from the modeling of the $\Upsilon \to S\bar{\Lambda}\Lambda$ angular distribution and the limited knowledge of the $S$-matter interactions. The former varies between 4% to 15%, assessed by comparing the predictions based on the simplified Lagrangian to those obtained using a phase space distribution for $\Upsilon$ decays. The latter is estimated by using simulations modeling the $S$ as a neutron or a non-interacting particle. The corresponding uncertainties ranges from 8% to 10%. A systematic uncertainty of 8% is included to account for the difference in $\Lambda$ reconstruction efficiencies between data and MC, determined from control samples in data [29]. Both the uncertainty on the $\Lambda \to p\pi$ branching fraction (1.6% [20]) and the limited MC statistics ($\sim 1.5\%$) are also propagated.

In the absence of a significant signal, we derive 90% confidence level (CL) upper limits on the $\Upsilon(2S,3S) \to S\bar{\Lambda}\Lambda$ branching fractions, scanning $S$ masses in the range $0 \text{ GeV} < m_S < 2.05 \text{ GeV}$ in steps of 50 MeV (approximately half the signal resolution). For each mass hypothesis, we evaluate the upper bound on the number of signal events from the $m_{\text{rec}}$ distribution with a profile likelihood
method [27]. This approach treats the background as a Poisson process whose unknown mean is estimated from the number of observed background events, set to zero in this instance. Systematic uncertainties are included by modeling the signal efficiency as a Gaussian distribution with the appropriate variance. In addition to the contributions previously described, the limits include an additional uncertainty of 0.6% associated with the uncertainty on the number of $\Upsilon(2S)$ and $\Upsilon(3S)$ decays. The results are shown in Fig. 4 for the $\Upsilon(2S)$ and $\Upsilon(3S)$ data sets, as well as the combined sample assuming the same partial width.

In conclusion, we performed the first search for a stable $uuddss$ configuration in $\Upsilon$ decays. No signal is observed, and 90% CL limits on the combined $\Upsilon(2S,3S) \rightarrow S\bar{A}A$ branching fraction of $(1.2 - 1.4) \times 10^{-7}$ are derived for $m_S < 2.05$ GeV. These results set stringent bounds on the existence of a stable, doubly strange six-quark state.

ACKNOWLEDGMENTS

We thank G. Farrar for providing us with the theoretical model to simulate the signal and useful discussions. We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BaBar. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Education and Science of the Russian Federation, Ministerio de Economía y Competitividad (Spain), the Science and Technology Facilities Council (United Kingdom), and the Binational Science Foundation (U.S.-Israel). Individuals have received support from the Marie-Curie IEF program (European Union) and the A. P. Sloan Foundation (USA).

* Now at: Wuhan University, Wuhan 430072, China
† Now at: Università di Bologna and INFN Sezione di Bologna, I-47921 Rimini, Italy
‡ Deceased
§ Now at: University of Huddersfield, Huddersfield HD1 3DH, UK
¶ Now at: University of South Alabama, Mobile, Alabama 36688, USA
** Also at: Università di Sassari, I-07100 Sassari, Italy
†† Also at: Università di Sassari, I-07100 Sassari, Italy
‡‡ Also at: Università di Sassari, I-07100 Sassari, Italy
§§ Also at: Wuhan University, Wuhan 430072, China

\[ h = c = 1 \] are used throughout this paper.

\[ \text{[Phys. Rev. Lett. 38, 617 (1977)]} \]

\[ \text{[Phys. Rev. Lett. 87, 1164 (2001).]} \]

\[ \text{[Phys. Rev. Lett. 84, 2593 (2000).]} \]

\[ \text{[Phys. Rev. Lett. 84, 2593 (2000).]} \]

\[ \text{[Phys. Rev. Lett. 37, 474 (1976).]} \]

\[ \text{[Phys. Rev. Lett. 37, 474 (1976).]} \]

\[ \text{[Phys. Rev. Lett. 87, 132504 (2001).]} \]

\[ \text{[Phys. Rev. Lett. 87, 212502 (2001).]} \]

\[ \text{[Phys. Rev. Lett. 110, 222002 (2013).]} \]

\[ \text{[Phys. Rev. Lett. 87, 132504 (2001).]} \]

\[ \text{[Phys. Rev. Lett. 87, 1164 (2001).]} \]

\[ \text{[Phys. Rev. Lett. 37, 474 (1976).]} \]

\[ \text{[Phys. Rev. Lett. 37, 474 (1976).]} \]

\[ \text{[Phys. Rev. Lett. 37, 474 (1976).]} \]

\[ \text{[Phys. Rev. Lett. 37, 474 (1976).]} \]

\[ \text{[Phys. Rev. Lett. 37, 474 (1976).]} \]

\[ \text{[Phys. Rev. Lett. 37, 474 (1976).]} \]
APPENDIX

The S angular distribution is simulated using the following amplitude:

\[
|A|^2 = \frac{2mM(m^2 - \alpha) - M^2\beta + 2(mM + \alpha - \beta)(\beta - \gamma - mM) - mM(\gamma - m^2) + m^2(\alpha - \beta + \gamma - M^2_Y)}{(m^2 - M^2 - 2\alpha + M^2_Y)(m^2 - M^2 - 2\gamma + M^2_Y)} \\
+ \frac{2m^2(M^2 + m^2 - 2\alpha + M^2_Y) - 2mM(m^2 - \alpha - \beta + \gamma) + 2(m^2 - \alpha)(\beta - \gamma) - \beta(M^2_Y + m^2 - M^2 - 2\alpha)}{(m^2 - M^2 - 2\alpha + M^2_Y)^2} \\
+ \frac{2m^2(M^2 - m^2 - 2\gamma + M^2_Y) + 2mM(\alpha - \beta + \gamma - m^2) - 2(m^2 - \gamma)(\alpha - \beta) - \beta(M^2_Y - m^2 - M^2 - 2\gamma)}{(m^2 - M^2 - 2\gamma + M^2_Y)^2}
\]

where \(\alpha = p \cdot q\), \(\beta = p \cdot p'\), \(\gamma = p' \cdot q\), \(q\) is the 4-momentum of the \(T(2S, 3S)\), \(p\) (\(p'\)) is the 4-momentum of the first (second) \(\Lambda\), \(m\) is the \(\Lambda\) mass and \(M\) is an effective mass, taken to be \(m_A\).