Fitting Blazhko light curves

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ABSTRACT

The correct amplitude and phase modulation formalism of the Blazhko modulation is given. The harmonic order dependent amplitude and phase modulation form is equivalent to the Fourier decomposition of multiplets. The amplitude and phase modulation formalism used in electronic transmission technique as introduced by Benkó, Szabó & Paparó for Blazhko stars oversimplifies the amplitude and phase modulation functions; thus, it does not describe the light variation in full detail.

The results of the different formalisms are compared and documented by fitting the light curve of a real Blazhko star, CM UMa.

Key words: methods: data analysis – techniques: photometric – stars: horizontal branch – stars: individual: CM UMa – stars: oscillations – stars: variables: RR Lyrae.

1 INTRODUCTION

The periodic modulation of the light curve of a large percentage of RR Lyrae stars, the so-called Blazhko effect, is a hundred-year old enigma. The height and time of maximum light of these stars oscillate with the same period of several days, weeks, months or even years. Blazhko (1907) was the first who noted that no constant period could satisfy the observed times of maximum light of RW Dra, an RR Lyrae type star,1 and an oscillation in the fundamental period with a cycle length of 41.6 d had to be postulated. The striking changes in the height of the light maximum and in the shape of the light curve of RR Lyrae, which turned out to be periodic with 40 d, were discovered by Shapley (1916). Following these discoveries, the Blazhko effect has been detected and studied in a number of RR Lyrae stars.

Large-scale surveys such as Massive Compact Halo Objects project (MACHO) and Optical Gravitational Lensing Experiment (OGLE) (Alcock et al. 2003; Soszyński et al. 2003, 2011) detected 10–30 per cent incidence rate of the modulation among RRab stars in the Magellanic Clouds and the Galactic bulge, while a recent ground-based multicolour photometric survey (Jurcsik et al. 2009) and the highly accurate observations of the CoRoT and Kepler space missions (Chadid et al. 2009; Kolenberg et al. 2010) revealed that a significantly larger fraction of the fundamental-mode RR Lyrae stars (about 50 per cent) shows the effect.

The high occurrence rate of modulated RR Lyrae stars makes the Blazhko effect an ever more intriguing problem of the pulsation theory. No wonder that it has captivated the researchers’ interest again in recent years. In spite of the fact that the highly accurate space data and the ground-based multicolour photometries have given new insights into the Blazhko phenomenon (for a recent review, see Kolenberg 2011), no generally accepted theory that is able to explain the observed features of the effect exists (Kovács 2009).

The connection between the shock waves in the atmosphere and light-curve modulation is particularly interesting. The shock waves play a significant role in the appearance of the bump/hump features on the light curve of RR Lyrae stars. Preston, Smak & Paczyński (1965), investigating the spectral-feature variations during the 41-d modulation cycle of RR Lyrae, suggested a model in which ‘a critical level of shock wave formation moves up and down’ in the star’s atmosphere. Later, Chadid & Gillet (1997) have found that the amplitude of shock waves is greatly correlated with the Blazhko modulation.

To describe the light variations of Blazhko stars, mathematical models have been propounded. A model that aimed to describe the full Fourier spectrum of the modulation was suggested by Breger & Kolenberg (2006). The triplet was interpreted as the non-linear coupling of the main pulsation mode and a non-radial mode close to it, and many of their combination terms. However, no detailed confrontation of the possible predictions of this model with the observations (e.g. on the amplitudes of the components of the multiplets) has been performed. In other approaches (Szeidl & Jurcsik 2009; Benkó, Szabó & Paparó 2011, hereafter BSP11), the multiplet structure is the simple result of the modulation of purely radial pulsation.

In their comprehensive study, BSP11 presented an analytical formalism (applied in electronic signal transmission) for the description of the light curves of Blazhko RR Lyrae stars. Their model shows several light-curve characteristics similar to those observed in real Blazhko stars. It was also claimed that the new light-curve...
solution drastically reduced the number of necessary parameters compared to the traditional methods. However, in BSP11, the suggested method was not tested on real observational data.

Recently, Guggenberger et al. (2012) made an attempt at applying the proposed new analytic modulation formalism to the Kepler data of the complex Blazhko star KIC 6186029 = V445 Lyr. Although the model light curve showed the global properties of the observed one, the fitted light curve deviated from the observed data significantly in certain phases of the pulsation and the modulation (see fig. 16 in Guggenberger et al. 2012). The surprisingly high variance of the residuals was explained by method- and object-specific reasons. The method did not describe the migration of the bumps and humps, and as any ‘stationary’ model, it was unable to follow an irregular, time-dependent phenomenon (Guggenberger et al. 2012).

In this paper, we look into the possible shortcomings of the formalism introduced in BSP11. The mathematical formalisms are given in Section 2, and the results are documented by the different fits of the light curve of a Blazhko star, CM Uma, in Section 3. As the modulation of CM Uma is quite simple and regular, no secondary modulation is detected, and no modulation components of order higher than quintuplet are present in the Fourier spectrum; therefore, no bias of the results arises from any time-dependent irregularity of the modulation, which was the case in V445 Lyr.

2 MATHEMATICAL CONSIDERATIONS

The Fourier decomposition of the light curve of Blazhko RR Lyrae stars is a common technique to analyse their modulation properties. If \( f_0 \) and \( f_m \) indicate the fundamental and modulation frequencies, in the typical Fourier representation of the light curve, the harmonics (\( jf_0 \)) and the multiplets (triplets, quintuplets, septuplets, etc.: \( jf_0 \pm 2jf_m \)), as well as the harmonics of the Blazhko frequency (\( kf_m \)), appear in the spectrum.

Let \( \omega = 2\pi f_0 \) and \( \Omega = 2\pi f_m \). Then the Fourier representation of the modulated light curve is

\[
m(t) = m_0 + \sum_{k=1}^{i} b_k \sin (k\Omega t + \phi_{bk}) \\
+ \sum_{l=1}^{n} \left[ a_{l} \sin (i\omega t + \phi_{il}) + \sum_{j=1}^{l^+} a_{ij} \sin \left( i\omega t + j\Omega t + \phi_{ij} \right) \right] \\
+ \sum_{j'=1}^{l'} a_{j'} \sin \left( i\omega t - j'\Omega t + \phi_{j'} \right),
\]

(1)

where the amplitudes \( b, a \) and angles \( \phi_{bk}, \phi_{il} \) are constants (latter ones are epoch dependent). In the expression of \( m(t) \), the first sum \( \sum_{k=1}^{i} b_k \sin (k\Omega t + \phi_{bk}) \) corresponds to the mean light-curve variation during the Blazhko cycle.

The application of formula (A2) (see Appendix A) to equation (1) leads to the following form:

\[
m(t) = m_0 + \sum_{k=1}^{i} b_k \sin (k\Omega t + \phi_{bk}) \\
+ \sum_{l=1}^{n} \left[ a_{l} + f_{\lambda}(t) \right] \sin (i\omega t + \phi_{il} + f_{\psi}(t)) ,
\]

(2)

where \( a, b, \psi \) and \( \phi \) are amplitude and phase constants, while \( f_{\lambda}(t) \) and \( f_{\psi}(t) \) are the amplitude and angle (phase or frequency, see Appendix B) modulation functions of the \( l \)th harmonics of the pulsation. They depend only on \( \Omega(t) = 2\pi f_m t \) and the constant parameters of the Fourier series (equation 1). Therefore, they are periodic functions according to \( f_m^{-1} (= 2\pi \Omega^{-1}) \); consequently, they can be approximated by the Fourier series:

\[
f_{\lambda}(t) = \sum_{j=1}^{l^+} a_{ij} \sin \left( j\Omega t + \phi_{ij} \right)
\]

(3)

and

\[
f_{\psi}(t) = \sum_{j'=1}^{l'} a_{j'} \sin \left( j'\Omega t + \phi_{j'} \right),
\]

(4)

where \( a^\lambda, d^\lambda, \phi^\lambda \) and \( \psi^\lambda \) are amplitude and phase constants.

In equations (1)–(4), the harmonic orders of the pulsation and the different modulation series \( n, l, l^+, l', l'' \) are limited by the accuracy of the observations.

The equality of equations (1) and (2) demands that the time history of a modulated star must be equally well described by either of them, with about the same variance of the residual, no matter which of the two formalisms is chosen. Then, the question is raised: why does the formalism of BSP11 give an inferior fit to the Fourier decomposition? If equation (2) is compared to equation (49) of BSP11, the difference is at once conspicuous. In equation (2), the amplitude and angle modulation functions (equations 3 and 4) depend on the harmonic order \( i \) of the Fourier sequence of the unmodulated light curve (carrier wave), whereas BSP11 assume that the modulation functions depend on the harmonic order in a very restricted way. To be more specific, in their approach, each harmonic term of the Fourier series of the unmodulated light curve is modulated in amplitude and angle, respectively, by the formulæ

\[
g_{\lambda}(t) = a_{i}^\lambda \sum_{p'=1}^{q} a_{p'}^\lambda \sin \left( 2\pi p' f_m t + \phi_{p'} \right)
\]

(5)

and

\[
g_{\psi}(t) = \sum_{p=1}^{q} a_{p}^\psi \sin \left( 2\pi p f_m t + \phi_{p} \right).
\]

(6)

We note here that in BSP11 the constant term of the amplitude modulation (AM) function \( g_{\lambda}(t) \) is identified with the difference between the magnitude and intensity mean values, and the constant term of the phase modulation function \( g_{\psi}(t) \) is contracted into the \( \psi \) phase constant of the pulsation.

In equations (5) and (6), the BSP11 notation holds, with the exception of denoting the harmonic order of the pulsation with \( i \), instead of \( j \). Here \( g_{\lambda} \) does not depend on the harmonic order of the unmodulated light curve in the least, which is inconsistent with equation (3), while in \( g_{\psi} \), the harmonic order is not taken into account correctly with the sole multiplication of the modulation function by \( i \) (cf. equation 4).

2.1 Consequences

Due to the adoption of simplified formulæ for the amplitude and angle modulations, BSP11 drastically reduce the number of parameters in their formalism. However, the outcome is an inferior fit compared to the Fourier fit (see details in Section 3.1). The modulation assumed by BSP11 can be effectuated in electronic signal transmission, but, a priori, it is not known how the star modulates its pulsation light curve (the ‘signal’).

Another serious problem is the change of the mean brightness during the Blazhko cycle. Obviously, the angle modulation cannot
contribute to it. BSP11 state that ‘the formalism naturally explains the mean brightness variations’; however, in their description it is directly proportional to the AM. Like their AM formalism describes only an averaged harmonic order dependence, it can happen that the mean varies in a more complex way than the BSP11 formalism demands. In real stars, it is actually observed: e.g. neither the magnitude nor the intensity-averaged variation of the mean V brightness of MW Lyr is linearly proportional to the amplitude variation of the star during the Blazhko cycle (see fig. 14 in Jurcsik et al. 2008b). Moreover, their difference is Blazhko phase dependent, and thus it cannot be taken into account as a sole constant, as in BSP11.

In principle, the summation of \( j \) and \( j' \) in equations (3) and (4) can start from zero; the \( j = j' = 0 \) terms modify the amplitude and phase of the unmodulated light curve in each harmonic order. These constant terms are included in the \( a_i \) and \( \psi_i \) parameters of equation (2). Therefore, the choice of the starting index of the summation from 1 does not have any consequence on the results. These extra terms express the differences between the mean light curve of a Blazhko star and the light curve of an unmodulated RRab star with the same physical properties.

The difference between the phase and AM functions of BSP11 and the present paper (modulation functions deduced from the Fourier series) has another consequence, too. The observations prove that the amplitude versus phase diagrams for the different harmonic orders of the light curve during the Blazhko cycle show intrinsically different forms (see details in Section 3.2). These diagrams, determined according to the formalism of BSP11, are morphologically exactly the same for each harmonic order. This fact also refers to the alteration of the real phase and amplitude relations of the observations due to the reduction of the degree of freedom (decreasing the number of free parameters) by BSP11.

A further important question, which is generally regarded as trivial, is the definition/determination of the phase difference between the angle and the AMs. In equations (3) and (4), the angles depend on the starting epoch, but their difference \( \Phi_{i,j} = \psi_{i,j} - \psi_{j,i} \) is epoch independent. Since the \( \Phi_{ij} \) values can be different for the possible combinations of \( i \) and \( j \), the definition of the phase difference between the angle and the AMs is not at all obvious.

### 3 A REAL CASE STUDY: CM UMA

In this section, we compare the results of fitting the light curve of a real Blazhko star with

(i) the Fourier series of Blazhko-frequency-spaced multiplets around each pulsation frequency component (case A, equation 1),

(ii) the amplitude- and phase-modulated sum of the pulsation light curve (case B, equation 2),

(iii) the formalism introduced by BSP11 (equation 49 therein) (case C, equation 7).

In order to index and parametrize the equations in a homogeneous manner, equation (49) of BSP11 is rewritten in the following form:

\[
m(t) = m_0 + \left[ a_0 + \sum_{j=1}^4 a_j \sin (j \Omega t + \psi_j) \right]
\]

\[
\left\{ a_0 + \sum_{i=1}^n a_i \sin \left[ i \omega t + \frac{\theta}{2} \sum_{j=m}^{i-1} a_j \sin (j \Omega t + \psi_j) + \phi_i \right] \right\}.
\]

The only difference between equation (49) of BSP11 and equation (7) is an additional term: \( m_0' \). Without adding this extra constant to the equation, a real light curve cannot be fitted. We also note that the magnitude zero-point of equation (7) is not identical with the zero-points of equations (1) and (2): \( m_0' = m_0 - a_0' \times a_0 \).

CM UMa, one of the Blazhko stars, which was extensively observed in the course of the Konkoly Blazhko Survey II (Södor 2012) has been chosen for the case study. CM UMa was observed with the 60-cm automatic telescope of the Konkoly Observatory (Budapest, Svábhegy) in the 2009 and 2010 observing seasons. The detailed analysis of the multicolour light curve of CM UMa will be published elsewhere.

The light curve of CM UMa can be described simply with the combination terms of one pulsation \( (f_0) \) and one modulation \( (f_m) \) frequency components. No secondary modulation or additional frequency has been detected in the Fourier spectrum, which makes CM UMa an ideal target for such an investigation. Its light-curve solution consists of the Fourier sum of quintuplets around the pulsation frequency and its harmonics, and the \( f_m \) and \( 2f_m \) modulation frequencies.

The \( V \) light curve of CM UMa (shown in Fig. 1) is fitted with different order series of the pulsation \( (n) \) and the modulation \( (l, l_1, l_2, l_3, l_4, l_5, l_6, l_7, l_8) \) components according to equations (1), (2) and (7). In each solution, the harmonic orders of the ‘+’ and ‘−’ side multiplets or the amplitude and phase modulations are taken to be identical and the same for each harmonic of the pulsation, i.e. \( N = l_1 = l_2 = l_3 = l_4, i = 1, \ldots, n \) (equations 1, 3 and 4). When applying equation (7), the orders of the amplitude- and phase-modulation series are also taken to be identical: \( N = l \). The zero-point of the magnitudes \( (m_0, m_0') \) are also fitted in all the solutions. A low-order \( (l = 1, 2) \) sum of the modulation frequency in case A and case B has been taken into account, too. The fits thus comprise \( p = m(4N + 2) + 2l + 1 \) parameters when using equations (1) and (2), while \( p = 2n + 4N + 3 \) for equation (7).

The pulsation and modulation frequencies are not fitted in any of the procedures; \( f_0 = 1.697434 \) and \( f_m = 0.036066 \), determined from the Fourier analysis of the light curve, are accepted and taken as constants.
Table 1. Comparison of the rms of the residuals when fitting the light curve of CM UMa according to equations (1), (2) and (7).

| Case A (equation 1) | Case B (equation 2) | Case C (equation 7) |
|--------------------|--------------------|--------------------|
| \( p(n, N, l) \)   | \( p(n, N, l) \)   | \( p(n, N) \)      |
| rms (mag)          | rms (mag)          | rms (mag)          |
| 185(18,2,2) 0.00911 | 185(18,2,2) 0.00928 |                   |
| 135(13,2,2) 0.00945 | 135(13,2,2) 0.00976 |                   |
| 83(13,1,2) 0.00987 | 83(13,1,2) 0.01033 | 63(20,5) 0.01924  |
| 57(9,1,1) 0.01056  | 57(9,1,1) 0.01100  | 55(18,4) 0.01930  |
| 39(6,1,1) 0.01213  | 39(6,1,1) 0.01254  | 43(16,2) 0.01946  |

3.1 Comparison of the accuracies of the fits

Table 1 compares the rms of the residuals of several light-curve solutions using equations (1), (2) and (7) with different parameter combinations. In case A and case B, the solutions that have the same number of parameters have similar accuracies; the rms of the Fourier solution (equation 1) is only marginally, 3–5 per cent, smaller than the rms of the amplitude and phase modulation formalism according to equation (2). This is not at all the case, however, when using equation (7). Even if similar number of parameters are fitted (57/55, 39/43), the residuals of equation (7) are 60–90 per cent larger than the residuals of the other solutions. This situation cannot be improved by increasing the number of parameters when fitting equation (7), as the rms of the fit decreases only marginally (by 0.5 per cent) when the orders of the pulsation and the modulation are increased from (16, 2) to (18, 4). If even higher orders (20, 5) of the pulsation and the modulation are fitted, the decrement of the residual is only 0.3 per cent. We note here, however, that fitting fifth-order amplitude and phase modulations is quite unrealistic, as even in the case of the very complex modulation of V445 Lyr only a third-order frequency modulation (FM) and a first-order AM were assumed (Guggenberger et al. 2012). Also as neither pulsation nor modulation components have been detected in the Fourier spectrum at harmonic orders higher than 18, the \( n = 20 \) solution has no relevance compared to the \( n = 18 \) one. As no secondary modulation of CM UMa has been detected, the parameter number when applying the formalism of equation (7) cannot be increased this way either. Therefore, the \( p = 55 \) \( (n = 18, N = 4) \) solution of the BSP11 formalism is accepted and used for the comparison.

The similar accuracy of the solutions of equations (1) and (2) confirms that these forms are, in fact, equivalent in practice. Of course, minor difference between these solutions may arise from the different role and significance of the parameters in the two formulae. Therefore, the solutions do not necessarily have to have exactly the same accuracies when the same number of parameters \((n, N, l)\) are fitted.

In the top panels of Fig. 2, the light curves in the lowest and highest amplitude phases of the modulation are shown and their fits using equation (1) \((p = 185)\), equation (2) \((p = 185)\) and equation (7) \((p = 55)\) are overplotted. [The \( p = 63 \) solution of equation (7) does not give any notable difference if compared to the results of the \( p = 55 \) one.] While the first two fits are indistinguishable from each other and they follow even the smallest features of the observed light variations accurately, this is not true for the fit obtained using equation (7). Large, 0.02–0.05 mag systematic differences, especially around light minima and maxima, appear between this fit and the observations. The bottom panels show the differences (fit – observation) for the three light-curve solutions. While the first two show nothing but noise around zero, in the third case the fit does not follow the light variation correctly in any phase of the pulsation.

The spectra of the three residuals shown in Fig. 3 also document that the differences between the results are substantial. While the mean levels of the residual spectra are around 0.0003 mag in the first two cases, it is as high as 0.0009 mag in the third one. Moreover, when using equation (7), 0.006–0.009 mag amplitude signals at around the positions of the pulsation frequency and its harmonics remain in the residual.

3.2 Comparison of the amplitude and phase relations of the fits

The maximum phase–maximum brightness diagrams of Blazhko RR Lyrae stars show a large variety. The direction of going around these loops and their shape are defined by the relative amplitudes and phases of the maximum brightness and maximum phase variations during the Blazhko cycle. The same is true for the shapes of the maximum phase–maximum brightness loops in the different harmonic orders of the pulsation. These curves can be determined from the amplitude and phase variations of the different harmonics of the pulsation light curve during the Blazhko cycle.

The amplitude and phase variations have been shown to have different shapes in the different harmonic orders even in a very regular Blazhko star, MW Lyr (see fig. 12 in Jurcsik et al. 2008a). Striking differences between the variations of the amplitudes of \( f_0 \)
Residual spectra of the light curve pre-whitened for the solutions of equation (1) ($p = 185$), equation (2) ($p = 185$) and equation (7) ($p = 55$) are shown. Note that the $y$ scales are the same in the three panels. The vertical grid indicates the positions of the pulsation frequency and its harmonics.

and $2f_0$ ($A_1$ and $R_{21}$) of RR Lyr have been also shown (see fig. 12 in Kolenberg et al. 2011).

In this section, the amplitude and phase variations in the different harmonic orders of the light curve of CM UMa are shown, and compared to the same plots derived from synthetic data according to the three light-curve solutions introduced in the previous section.

Using the parameters of $p = 185$, 185 and 55 solutions of the three different formalisms, synthetic light curves of CM UMa have been generated. Dividing the synthetic data into small, homogeneous Blazhko phase bins, the variations of the pulsation light curves during the Blazhko cycle predicted by the different fits can be followed and compared to the observations. Both the observed and the fitted pulsation light curves in the different Blazhko phases are then fitted with a 15th-order Fourier sum of the pulsation frequency. The phases and amplitudes of the $f_0$, $2f_0$, ..., $if_0$ components characterize the modulations of the pulsation components in the different orders; they define the maximum phase–maximum brightness loops.

In Fig. 4, the amplitude versus phase variations of the first nine harmonics of the pulsation during the Blazhko cycle are plotted. The observed and fitted (via equations 1 and 2) loops are significantly different in the different orders; the most striking differences are in the phase ranges of the loops. The phase ranges, even if normalized by the division of the order number, vary between 0.1 and 0.6 rad. Large diversity in the morphology of the loops is also evident, e.g. it is ‘egg-shaped’ in the first order, while in the second and sixth orders the loops are degenerated, they are reduced to one-dimensional curves in these harmonic orders as the amplitude and phase variations (see Fig. 4) are quite symmetrical in these orders, and their phase differences are 180°.

The synthetic data corresponding to equation (7) do not follow the real variations of the phases and amplitudes of the maxima with the harmonic orders, due to the strong restrictions of the variations in the amplitude and phase modulations of the formalism of BSP11. The morphologies of these loops are identical to each order, as differences only in the relative scaling of the amplitudes are allowed.

The observed and fitted values are somewhat uncertain in these harmonic orders.

The observed and fitted light curves of the different harmonic orders of the pulsation light curve during the Blazhko cycle are shown. The phase values are divided and the amplitudes are multiplied by the number of the harmonic order ($i$) in each plot for a better visualization and comparison. Note that the same, 0.7-rad phase ranges are shown in each panel, while the amplitude ranges are different in the different rows of panels. The symbols denoting the observations and the three different synthetic fits are the same as in Fig. 2.

Fig. 5 documents the phase and amplitude variations of the pulsation light curve versus Blazhko phase in the different harmonic orders. The observations and the results of the three fits are plotted in separate columns here. Different amplitude and phase ranges are...
Figure 5. Variations of the Fourier amplitudes and phases of the harmonic orders of the pulsation light curves during the Blazhko cycle. From left to right, the panels show the results obtained directly from the observations and from the three different fits. Filled symbols denote the phase variations.

shown for the different harmonic orders but the same ranges are used for the observed and fitted values in each order (in each row in Fig. 5). Again, we can see that the first two fits give back the observed variations similarly well, with high precision. In case C, however, neither the mean values nor the amplitudes and shapes of the predicted variations match the observations correctly. The BSP11 formalism is not capable of fitting the observed differences of the phase relations between the amplitude and phase variations in the different orders.

4 CONCLUSIONS

The Fourier sequence describing the light curve of a Blazhko star (equation 1) has been transformed into the form of an amplitude- and angle-modulated signal (equation 2). The two descriptions are fully equivalent to each other. The correctly deduced amplitude and angle modulation functions, $f_A(t)$ and $f_{\phi}(t)$ depend upon the harmonics of the unmodulated light curve ($\ell$). Since these are periodic functions, they can be expressed as Fourier sums (equations 3 and 4). A priori, there is no knowledge about the parameters of these equations; they can be determined only through observations. The possible correlations among them can be revealed by observations, as well.

In their study, BSP11 employed an amplitude and angle modulation pattern used in electronic signal transmission. (In this technique, the coding of the modulation of a signal is known in advance and can be controlled.) They assume that the AM function does not depend upon the harmonic order of the unmodulated signal (light curve) and the difference between the angle modulation functions in the different orders is simply the multiplication by the harmonic-order number. This oversimplified procedure leads to the drastically reduced number of parameters, but on the cost of an unacceptably poor fit.

The analysis of the light curve of the Blazhko RR Lyrae star, CM UMa (Section 3), clearly shows that the BSP11 formalism gives a poor fit, especially around the minimum and maximum light and the ascending branch of the light curve. Even if the number of the parameters is increased, the fit does not improve. It should be emphasized that these parts of the light curve reflect the changing strength and occurrence of the shock waves during the Blazhko cycle (Preston et al. 1965; Chadid & Gillet 1997). We thus conclude that the BSP11 approach does not take the non-linear interactions, which determine the final form of the light variation, fully into account because of the strong restrictions of the formalism. Its capability to describe the changes of the pulsation light-curve shape is seriously limited, since it only shifts the pulsation light curve in phase and scales it in amplitude periodically during the modulation.

Plots that show the amplitude and phase variations of the different harmonic orders during the Blazhko cycle (Figs 4 and 5) reveal convincingly that the amplitude and angle modulation functions given in BSP11 are unsuited to describe the Blazhko modulation correctly.

Nevertheless, the Blazhko modulation can be correctly interpreted as amplitude- and angle-modulated signal (equations 2–4). This fact hints at the possibility that during the Blazhko cycle the radial pulsation of an RR Lyrae star is modulated in phase/frequency and amplitude, and the fundamental period is subject to real oscillation. Up to now, the only model that is in conformation with this scenario has been suggested by Stothers (2006), although the inside physics of this model has been strongly criticized by Smolec et al. (2011) and Molnár, Kolláth & Szabó (2012) recently.

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APPENDIX A: TRIGONOMETRIC IDENTITIES

From the well-known identities of trigonometry, we obtain

\[ A_1 \sin(\omega + \chi_1) + A_2 \sin(\omega + \chi_2) = A \sin(\omega + \chi), \tag{A1} \]

where

\[ A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\chi_2 - \chi_1)} \]

and

\[ \tan \chi = \frac{A_1 \sin \chi_1 + A_2 \sin \chi_2}{A_1 \cos \chi_1 + A_2 \cos \chi_2}. \]

By reiterating the formula (A1), we come to the relation

\[ \sum_{i=1}^{n} A_i \sin(\omega + \chi_i) = A \sin(\omega + \chi), \tag{A2} \]

where both \( A \) and \( \chi \) are bounded functions of \( (A_1, A_2, \ldots, A_n, \chi_1, \chi_2, \ldots, \chi_n) \). This relation holds even if \( A_i \) and \( \chi_i \) are time dependent.

APPENDIX B: FREQUENCY AND PHASE MODULATIONS

Two types of angle modulation exist, FM and phase modulation (PM).

In the case of FM, the frequency of the unmodulated wave (carrier signal), \( f_0 \), is modulated by the modulating signal, \( \Theta_p(t) \). The instantaneous frequency is

\[ F(t) = f_0 + k_p \Theta_p(t), \tag{B1} \]

where the constant \( k_p \) depends upon the modulating system. The instantaneous phase of the modulated wave is

\[ \Psi(t) = 2\pi f_0 t + 2\pi k_p \int_0^t \Theta_p(\tau) \, d\tau. \tag{B2} \]

(For simplicity, zero phase is assumed at \( t = 0 \).)

In the case of PM, the phase of the modulated wave is

\[ \Psi(t) = 2\pi f_0 t + k_p \Theta_p(t), \tag{B3} \]

where \( \Theta_p(t) \) and \( k_p \) are the modulating signal and a constant, respectively. The instantaneous frequency of a PM wave is

\[ F(t) = f_0 + k_p \frac{d\Theta_p(t)}{dt}. \tag{B4} \]

In both cases, the wave’s frequency and phase vary from moment to moment. From equations (B1) and (B4), it follows that the two modulations describe the same modulated signal, if the following relation holds:

\[ \Theta_p(t) = \frac{k_p}{k_F} \int_0^t \Theta_p(\tau) \, d\tau. \tag{B5} \]

The only difference between the two descriptions is that in the case of FM the frequency modulation of the carrier is given by the time derivative of the PM modulated signal. Unless some information is available about the modulation in advance (as it is the case in electronic signal transmission), it may not be obvious which one of the two types (FM or PM) is realized. If no information is available regarding the angle modulation (as in the case of Blazhko modulation), it is impossible to identify it as an FM or PM signal. This explains why the expression ‘angle modulation’ is used throughout this paper.

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