Approximating the probability distribution laws for critical frequency deviations of the ionospheric F2 layer by fourth-degree polynomials

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Abstract. One of important tasks in studying the ionosphere is to determine its perturbation level. Determining and predicting the ionosphere perturbation degree is necessary for solving practical problems, as well as for a theoretical study of processes occurring in it. There is also an urgent problem of creating a new daily-seasonal ionospheric perturbation scale. One of approaches to solve these problems can be to determine the distribution laws for deviations in the critical frequencies of the F2 layer from their median values calculated for previous 27 days. The optimal method for calculating polynomial dependencies for determining the probability distribution laws for deviations in the critical frequency of the ionospheric F2 layer for the longitude chain of observation stations in cases when they are different from the normal one is suggested in this article. The optimal degree of the approximating polynomial is determined. Polynomial coefficients are calculated. The percentage of data from the present period that exceeds the limits of the obtained dependencies in each case is calculated. The analysis of the obtained results is carried out, as a result of which the possibility of describing the behavior of the ionosphere by the obtained dependencies is determined.

1. Introduction
One of the important tasks in studying the ionosphere is to determine its perturbation level both in real time and in the predicted future. Determining and predicting the disturbance degree of the ionosphere is necessary to solve practical problems (determining the possibility of propagation of short and very-high-frequency radio waves in a certain area, conditions for their propagation, predicting extreme natural phenomena, preventing problems in ultra-long radio communication systems, etc.), as well as for a theoretical study of processes taking place in it. Research in this area has been conducted for many years. Short-term ionospheric forecasting remains a particularly difficult issue. One of the first works on this problem is [1], which considers the possibility of conducting a temporal extrapolation of the critical frequency values of the ionosphere F2 layer to supplement the existing short-term forecasts of critical frequency variations. Every year, due to the development of methods and means of studying the ionosphere, the amount of statistically accumulated data increases, the number of errors in obtaining them decreases, which makes it possible to obtain correlation ratios with greater accuracy. In a number of works, such as [2-5], new data are used to model the behavior of certain ionosphere areas, as well as to correct existing models. As can be seen from these studies, an important problem in predicting critical frequency variations is establishing relationships between ionosphere behavior and external factors. Thus, the paper [6] studies variations in the correlation level of long-term series of full electron content and plasmasphere content with solar and geomagnetic activity indices. The papers [7, 8] analyze the daily and seasonal deviations in the critical frequency values of the ionospheric F2
layer from their median values, as well as the influence of external factors, such as solar and geomagnetic activity and the geographical location of the ionospheric station, on its behavior. The paper [9] explores the relationship of short-period ionospheric perturbation with solar activity and variations in the winter polar stratosphere. The establishment of dependencies between changes in the critical frequency of the ionospheric F2 layer and various external factors allows defining optimal entry conditions for finding laws, which can be used for describing the ionosphere behavior in the future. Despite the ongoing research, a universal approach that can be used to study the ionosphere as a whole, and not specific local areas of it at a certain period, has not been found at present, and the task remains relevant.

In addition, the task of developing an ionospheric perturbation scale should be distinguished as a separate direction. For example, in [10], ionospheric perturbation is classified based on data from the Moscow station. However, the article does not consider the full cycle of solar activity and does not analyze critical frequencies from other ionospheric stations. For the analysis, data only from 1975 for 1986 were used. This interval is small in comparison with the full period of studying the critical frequencies of the ionospheric F2 layer and also does not contain newer values of critical frequency. Thus, the problem of creating a new daily-seasonal scale of ionospheric disturbances is also relevant, since the existing point scale does not take into account the daily, seasonal, and latitudinal features of the ionosphere and is not universal. The development of a new daily-seasonal scale will allow operational control and prediction of the ionosphere state in changing heliogeophysical conditions.

One approach to solving these problems may be to determine the laws of distributing deviations of critical frequencies of the F2 layer from their median values, calculated for the previous 27 days. In the future, this will allow determining the possible maximum ionospheric variations under specific geomagnetic conditions in order to determine the level of ionospheric disturbance determined by a complex of geophysical conditions of various nature. This article is a continuation of the study [11], where it was shown that the normal distribution was not valid for all cases. In addition, [11] tested statistical hypotheses about the proposed laws of distribution, which showed that none of the known distribution laws was the solution to the task.

The purpose of this study is to approximate the probability distribution laws for deviations in the critical frequency of the ionospheric F2 layer at time T from the sliding median of these values Me (for F2 (T)) calculated for the previous 27 days for cases when the law is different from normal. T in this case is 03, 09, 15 and 21 UT.

2. Approximating distribution laws by polynomials

The first part of the study used foF2 observation data from the Moscow, Rostov, and Leningrad ionospheric stations for the observation period from 1957 to 2017 with low solar (w < 75) and geomagnetic activity (0 to 10).

The optimal polynomial degree was determined and polynomial coefficients were calculated. For this, the criteria for choosing an approximation method were defined. The envelope of all distribution histograms has a parabolic central region and edge regions with values close to zero. Thus, the main criterion for choosing the approximation method was the proximity of the polynomial to the central part of the distribution histogram envelope. An additional criterion was the minimum possible degree of the polynomial at which the desired shape of the central portion could be obtained.

Consider the approximation polynomials of the third, fourth, and fifth degrees. One of the histograms was used to analyze the polynomial data (figure 1).
Figure 1. Example of approximating a distribution histogram by third-, fourth-, and fifth-degree polynomials

It can be seen from the drawing that the third-degree polynomial does not produce a satisfactory result; therefore, it does not fit the first criterion. The difference between the fourth- and fifth-degree polynomials is little, hence by both criteria, the fourth-degree polynomial best describes the considered histograms. Thus, a nonlinear approximation by fourth-order polynomials was used to solve the set problems. The least-squares method found the polynomials for the corresponding distribution histograms. Table 1 shows, for example, the polynomials for the Moscow station at 03 UT.

Table 1. Polynomial dependencies to describe histograms of deviation distribution.

| Months | Polynomials                                      |
|--------|--------------------------------------------------|
| 1      | \( y = 6.0293x^4 - 2.186x^3 - 5.6561x^2 + 0.8917x + 1.321 \) |
| 2      | \( y = 0.4707x^4 - 1.352x^3 - 2.2456x^2 + 0.6921x + 1.0617 \) |
| 3      | \( y = 2.788x^4 + 2.1608x^3 - 3.2507x^2 - 1.3914x + 1.1208 \) |
| 4      | \( y = -0.0223x^4 - 0.345x^3 - 0.7921x^2 + 0.0222x + 0.7514 \) |
| 5      | \( y = 0.2793x^4 + 0.22x^3 - 0.9136x^2 - 0.3472x + 0.7411 \) |
| 6      | \( y = -0.0639x^4 - 0.5505x^3 - 1.2241x^2 - 0.3122x + 0.8587 \) |

Polynomials of a similar kind also describe the rest of the distribution histograms.

Given that polynomials were chosen taking into account the shape of only the central part of the distribution law envelope, the law of distribution of the edge parts of a function, starting from the value at which the function crosses the abscissa axis, was considered zero.

3. Testing the hypothesis of using polynomials to describe the ionosphere

The second part of the study was to determine the possibility of describing the ionosphere behavior by the resulting polynomials. It was carried out using the obtained polynomials, as well as hourly observation data for FoF2 obtained by the Moscow, Rostov, and Leningrad ionospheric stations for the period of observation from 2017 to 2019 with the low solar and geomagnetic activity.

The quantiles of the levels 0.05, 0.03 at negative deviations from the median, and the levels 0.95, 0.97 at positive deviations from the median were calculated from the previously obtained functions. After finding the values of quantiles according to new data (for the period from 2017 to 2019) from the same ionospheric stations, with the same levels of solar and geomagnetic activity, the amount of data that goes beyond quantiles was calculated. It was impractical to use the polynomial to study the
ionosphere behavior, if more than three percent of the data went beyond the quantile of the 0.03 or 0.97 level, and more than five percent of the data went beyond the quantile of the 0.05 or 0.95 level.

The study confirmed the possibility of describing the ionosphere behavior by the obtained polynomials for all histograms except the stations:

- Leningrad 10 out of 44, especially at 15 UT in winter and autumn;
- Moscow 5 out of 44;
- Rostov 4 out of 44.

In other cases, more than three percent of the data went beyond the level of 0.03 and 0.97 quantiles. However, in several cases (Rostov, August, 09 UT; Moscow, October, 15 UT; Moscow, April, 09 UT; Leningrad, July, 03 UT; Leningrad, November, 15 UT; Moscow, November, 15 UT), the data percent outside the quantiles of the 0.05 and 0.95 levels was less than five (figure 2).

![Figure 2.](image)

**Figure 2.** Daily dependence of quantiles of the calculated polynomial for the Leningrad station, July. The figure shows: solid line quantiles of the calculated dependence for the 0.03 and 0.97 levels; bar-dashed line quantiles of the calculated dependence for the 0.05 and 0.95 levels. Individual points suspected data errors. The dotted line is the boundaries of the corresponding data samples for 2017-2019.

In the first four cases, there are positive deviations, in the fifth and sixth negative ones.

The cases listed have single emissions significantly removed from the rest of the sample data and do not have any certain temporal and seasonal dependencies. In this regard, it could be assumed that the calculated dependencies correctly describe the histograms studied, and the presence of data falling outside the quantiles is related to the operator error.

The volumes of the obtained data samples in several cases (Leningrad 03 UT 1-3, 9-12 and 21 UT 1-3, 11-12) were not sufficiently significant, for them, it is impossible to check by this method.

Figure 3 shows an example graph for the daily dependence of the quantiles of distributions 0.03 and 0.97 and the boundaries of the corresponding data samples for 2017-2019 for the Moscow station in February (figure 3).
Figure 3. Daily dependence of quantiles of the polynomials for the Moscow station, February. The figure indicates: the solid line quantiles of the calculated dependence for the 0.03 and 0.97 levels. The dotted line is the boundaries of the corresponding data samples for 2017–2019.

In this case, the polynomial for 03 UT requires correction, the rest polynomials correctly model the behavior of the ionosphere at the appropriate points in time, since the percentage of data beyond the boundaries of the quantiles is less than 3. In most cases where the polynomials do not describe the histograms under investigation, correction of the polynomial is required in terms of the definition of edge areas by another function (figure 4).

Figure 4. Histograms of deviation distribution for the Moscow station, at 03, 09, 15 and 21 UT in February with graphs of obtained dependencies applied on them. In the figure, blue vertical lines indicate quantiles of the 0.03 and 0.97 levels; red vertical lines the value, which exceeds their limits.

Polynomials, which do not fully describe the histograms, were calculated by data from the stations:
- Leningrad, 21 UT, April; Leningrad, 21 UT, June; Leningrad, 21 UT, August (positive deviations) Leningrad 09, UT May (negative deviations);
- Moscow, 03 UT, February; Moscow, 03 UT, September;
- Rostov, 21 UT, February and November (both positive and negative deviations), August (positive deviations).

In several cases, it is necessary to find a completely different polynomial. This may indicate a change in the ionosphere behavior at a given geographical site during the relevant period (Figure 5). These cases are observed at the Leningrad station, at 15 UT, in January, February, March, September, October, and at 09 UT in April. Besides, in all these cases, negative deviations are observed.
4. Conclusions
The following conclusions can be drawn from the study.
1. The distribution histograms obtained according to the data from the Rostov station, to a greater extent at 03, 09, and 15 UT, as well as histograms according to the data from the Moscow station at 21 UT mainly have the normal law of distribution.
2. The approximation polynomials calculated in this article fully determine the ionosphere behavior at the stations:
   - Moscow in almost all cases except for those described by the normal distribution law and two cases at 03 UT in various seasons;
   - Rostov in all cases except for those described by the normal distribution law and three cases at 21 UT, two of which in winter have both positive and negative deviations beyond the calculated quantiles;
   - Leningrad in 26 cases out of 44, mainly at 03 and 09 UT and in summer, and can be used to describe the ionosphere behavior in appropriate periods.
3. For nine histograms, fourth-degree polynomials are not enough to describe the law of critical frequency variation distribution. They require the correction of the polynomials in terms of setting the edge sections of the function. In Rostov, these histograms are only at 21 UT, in Leningrad mainly at 21 UT. These cases have no seasonal dependence. Since in Moscow at 21 UT, virtually all cases can be described by the normal law, it can be assumed that histograms at 21 UT have a form closer to the normal law than the rest.
4. The ionosphere behavior in some cases cannot be determined by the resulting dependencies. These cases are observed at the Leningrad station, in the autumn-winter period at 15 UT, as well as at 09 UT in April. They have clear temporal, seasonal, and geographic dependencies, suggesting changes in the ionosphere behavior during the specified periods.
5. The approximation polynomials calculated in this article can be further used to determine the possible maximum ionospheric variations under various geomagnetic conditions. This will help solve the problem of determining the level of ionospheric disturbance, as well as create a new daily-seasonal scale for it.

Further studies will be presented in subsequent publications.

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