DEVELOPING ONE-SIDED SPECIFICATION SIX-SIGMA FUZZY QUALITY INDEX AND TESTING MODEL TO MEASURE THE PROCESS PERFORMANCE OF FUZZY INFORMATION

K.-S. Chen1, C.-H. Wang1,2, K. Hua Tan1, S.-F. Chiu3
1National Chin-Yi University of Technology, Taiwan
2Nottingham Business School, University of Nottingham, United Kingdom
3De La Salle University, Philippines

Abstract

Depending on the quality characteristic, a process capability index (PCI) can be used for one-sided specifications or for bilateral specifications. They are widely employed in the evaluation and improvement of process performance and are also good tools for communication between sales departments and vendors. The earliest PCIs for one-sided specifications were proposed by Kane (1986) and have one-to-one relationships with yield. A number of researchers have investigated the statistical properties of one-sided specification indices and proposed methods for applications. The later introduction of the Six Sigma approach also assisted many firms in effectively enhancing their production capacities, reducing waste, and increasing effectiveness. Like the PCIs, the Six Sigma quality levels have also become good quality control methods and communication tools in the industry. As a result, many researchers have examined the relationships between PCIs and the Six Sigma and their applications so that both approaches can be easily used to solve engineering problems in practice. In view of this, Chen et al. (2017a) modified the PCI for one-sided specifications and proposed the Six Sigma Quality Index (SSQI), which coincidently equals the quality level and has a one-to-one relationship with yield. However, uncertainty in quality characteristic measurements is common in practice, which can lead to judgment errors in conventional process capability assessment methods. In these cases, fuzzy calculation methods can determine the actual process capability more precisely. This study therefore developed an SSQI for one-sided specifications based on the fuzzy testing method created by Buckley (2005) and developed a Six Sigma fuzzy evaluation index and testing model. In addition to having a simpler calculation procedure, the model takes the process capability and Six Sigma quality level into consideration and can process the uncertainties in the data to make it more convenient for the industry to solve engineering issues. Finally, we presented a practical example to demonstrate the application. The model proposed in this study can provide the industry with a practical approach to assess process quality in a fuzzy environment.

Keywords:
Six Sigma Quality Index, uncertain data, one-sided specification, fuzzy test.

1 INTRODUCTION

Process capability indices (PCIs) are effective tools used to determine whether the process capability and performance of products reach requirements. Depending on the quality characteristic, a PCI can be either for one-sided specifications or for bilateral specifications. With PCIs, users can analyze and improve process and product quality, ensure that the process quality of the product remains over a certain level, and also prevent the production of defective products. The earliest PCIs used to evaluate quality characteristics with one-sided specifications, Cpu and Cpl, were proposed by Kane (1986). These two indices have one-to-one relationships with yield and can therefore fully reflect process yield: \( Yield\% = \Phi(3C_{pu}) \) and \( Yield\% = \Phi(3C_{pl}) \), where \( C_{pu} = (USL - \mu)/3\sigma \) is a smaller-the-better (STB) index, \( C_{pl} = (\mu - LSL)/3\sigma \) is a larger-the-better (LTB) index, \( \mu \) and \( \sigma \) denote the process mean and standard deviation, respectively, and USL and LSL represent the upper and lower specification limits. A number of researchers have investigated the statistical properties of one-sided specification indices and proposed methods for applications, including Prasad and Bramorski (1998), Chou and Owen (1989), Chen et al. (2002), Pearn and Chen (2002), Chen et al. (2007), Hsu et al. (2016) and Chen et al. (2017b). These two indices are currently the PCIs most commonly used to evaluate quality characteristics with one-sided specifications and can be employed to assess the process capability performance of one-sided specifications in various industries.

The later introduction of the Six Sigma approach also assisted many firms in effectively enhancing their production capacities, reducing waste, and increasing effectiveness (Huang et al. (2010), Chen and Chen (2016), Chen et al. (2017a)). Like the PCIs, the Six Sigma quality levels have also become good quality control methods and communication tools in the industry. As a result, many researchers, such as Chen et al. (2009), Huang et al. (2010), Wang et al. (2011), Chen et al. (2012), and Chen et al. (2017a), have examined the relationships between PCIs and the Six Sigma and their applications so that both approaches can be easily used to solve engineering problems in practice. Chen et al. (2017a) indicated that conversions are needed for PCIs and Six Sigma quality levels, which makes them less convenient in practical application. In view of this, they proposed the Six Sigma Quality Index (SSQI), which can directly reflect the quality level and yield of processes. This index coincidently equals the quality level and has a one-to-one relationship with yield. For common differences with one-sided specifications, Chen et al. (2017a) proposed two SSQIs as follows:

\[
Q_u = \frac{\mu - LSL}{\sigma_x} \quad \text{(larger the better)} \quad (1)
\]

\[
Q_d = \frac{USL - \mu_x}{\sigma_x} + 1.5 \quad \text{(smaller the better)} \quad (2)
\]

where \( USL \) and \( LSL \) are respectively the upper specification limit and the lower specification limit, \( \mu_x \) is the process mean and \( \sigma_x \) is the process standard

\[
549
\]
development. The index $Q_{pu}$ is suitable for processes of the smaller-the-better (STB) type, whereas $Q_{pl}$ is suitable for processes of the larger-the-better (LTB) type.

In the Six Sigma approach, a deviation of $1.5\sigma$ from the target value is allowed in the process mean when the process quality level reaches $k\sigma$. This was designed for nominal-the-best quality characteristics, as a smaller deviation from the target value in the process mean results in less expected loss. However, for LTB quality characteristics, $T - \mu \leq 1.5\sigma$. As the target value of LTB quality characteristics is infinity ($\infty$), $T - \mu \leq 1.5\sigma$ is impossible in practice. Similarly, for STB quality characteristics, a deviation of $1.5\sigma$ from the target value is allowed in the process mean, which means that $\mu - T \leq 1.5\sigma$; zero can be regarded as the target value $T$ of a STB quality characteristic ($T = 0$). However, getting the process mean $\mu$ closer to 0 requires greater process technologies and costs, so it is clear that allowing a deviation of $1.5\sigma$ from the target value in the process mean is not suitable for quality characteristics with one-sided specifications. Thus, for LTB quality characteristics, we deduce that the rule of the process reaching the $k\sigma$ quality level should be $\mu - LSL \geq (k - 1.5)\sigma$. In other words, if $\mu - (k - 1.5)\sigma \geq LSL$ and the process reaches the $k\sigma$ quality level, then $Q_{pu} = k$ and the process yield is $\Phi(k - 1.5)$. For STB quality characteristics, if $\mu + (k - 1.5)\sigma \leq USL$ and the process reaches the $k\sigma$ quality level, then $Q_{pl} = k$ and the process yield is $\Phi(k - 1.5)$.

Past studies that performed process capability analysis generally indicate that the measurement data is assumed to be precise data. However, according to Wu (2009), Wu et al. (2013), Liao et al. (2014), Wu and Liao (2014), and Wu et al. (2016), measurement errors may prevent characteristic values from being clearly quantified. Another possibility is that the product has some unclear circumstances, such as unspecific quality characteristics, which can lead to uncertainty and judgment errors in conventional process capability assessment methods. For this reason, fuzzy calculation methods can determine the actual process capability more precisely. In view of this, Buckley (2005) proposed the use of the triangular fuzzy numbers generated by sets of confidence intervals of parameters to construct the fuzzy estimates $\tilde{\mu}$ and $\tilde{\sigma}$ of $\mu$ and $\sigma^2$. Wu and Kuo (2011), Wu et al. (2014), and Wu et al. (2016) applied this approach to develop fuzzy process capability evaluation models with various PCIs.

However, although the approach proposed by Buckley (2005) uses fuzzy calculations to assess process quality, the fuzzy calculations employ integration to calculate the area encompassed by the confidence intervals. From a practical perspective, these calculations are difficult, and the results obtained are only approximate values. We therefore developed an SSQI for one-sided specifications based on the concepts established by Chen et al. (2017a). We referred to and modified the fuzzy testing method created by Buckley (2005) and developed a Six Sigma fuzzy evaluation index and testing model. In addition to having a simpler calculation procedure, the model takes the process capability and Six Sigma quality level into consideration and can process the uncertainties in the data to make it more convenient for the industry to solve engineering issues. Chapter 2 of this study presents the development of a Six Sigma fuzzy quality index for one-sided specification processes. Chapter 3 shows the development of the hypothesis testing model for the SSQI. Chapter 4 presents a practical example to demonstrate the applications. Finally, Chapter 5 contains the conclusions and suggestions of this study. The model proposed in this study provides the industry with a practical approach to the assessment of process quality in a fuzzy environment.

### 2 FUZZY ESTIMATION OF ONE-SIDED SPECIFICATIONS SIX SIGMA QUALITY INDICES

As noted by Chang et al. (2014), if quality characteristic $X$ is the STB type, then $0 < X \leq USL$. Through the variable transformation method, $Y$ is set as $Y = X / USL$. The STB specification can then be converted to $0 < Y \leq 1$. Similarly, if quality characteristic $X$ is the LTB type, then $LSL < X < \infty$. As $Y = X / LSL$, the LTB specification can then be converted to $1 \leq Y < \infty$. Following this line of reasoning, the common variable $Y$ can be expressed as follows:

$$Y = \begin{cases} \frac{X}{USL}, & 0 < Y \leq 1 \text{ for STB} \\ \frac{X}{LSL}, & 1 \leq Y < \infty \text{ for LTB} \end{cases}$$

Thus, according to variable transformation, Eq. (3) can be rewritten as

$$(\mu_{x}, \sigma_{x}) = \begin{cases} \left[ \frac{\mu_{x} - \sigma_{x}}{USL-USL}, \frac{\mu_{x} + \sigma_{x}}{USL-USL} \right] & \text{for STB} \\ \left[ \frac{\mu_{x} - \sigma_{x}}{LSL-LSL}, \frac{\mu_{x} + \sigma_{x}}{LSL-LSL} \right] & \text{for LTB} \end{cases}$$

$Q_{pl} = \frac{\mu_{x} - \sigma_{x}}{\sigma_{x}} + 1.5$ and $Q_{pu} = \frac{1 - \mu_{x}}{\sigma_{x}} + 1.5$.

Next, we compiled the two indices into a general formula:

$Q_{x} (i) = \frac{\mu_{x} - \sigma_{x}}{\sigma_{x}} + 1.5$

where $\mu_{x} = (-1)^{i} (1 - \mu_{x})$. Then

$Q_{pl} = \frac{\mu_{x} - \sigma_{x}}{\sigma_{x}} + 1.5, i = 1$

$Q_{pu} = \frac{1 - \mu_{x}}{\sigma_{x}} + 1.5, i = 2$

Consider $X$ a random variable with normal distribution $N(\mu, \sigma^2)$, then random variable $Y$ is distributed as $N(\mu_{x}, \sigma_{x}^2)$. Let $Y_{1}, Y_{2}, ..., Y_{n}$ be a random sample and let $\mu_{x}$ and $\sigma_{x}^2$ denote the sample mean and standard deviation, respectively, as follows:

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_{i}$$

$$\sigma_{x} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (Y_{i} - \bar{Y})^2}$$

$$\mu_{x} = (-1)^{i} (1 - \mu_{x})$$

Then

$$\mu_{x} = \bar{Y}$$

$$\sigma_{x} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (Y_{i} - \bar{Y})^2}$$

$\mu_{x} = (-1)^{i} (1 - \mu_{x})$$

$1 - \mu_{x}, i = 2$
\[ Q'_p(i) = \frac{\mu^*_i}{\sigma_i} + 1.5 \]

\[ Q'_p(i) = \left\{ \begin{array}{ll}
\frac{\mu^*_i}{\sigma_i} + 1.5 = \frac{\mu_i}{\sigma_i} + 1.5, & i = 1 \\
\frac{\mu^*_i}{\sigma_i} + 1.5 = \frac{\mu_i - 1}{\sigma_i}, & i = 2
\end{array} \right. \]

Let 
\[ t_i = \left( \frac{-1}{\sqrt{n}}(\mu_i - \mu^*_i) \frac{\sigma_i}{\sqrt{n}}, \frac{\mu^*_i - \mu_i}{\sigma_i} \right), \quad i = 1, 2 \]
\[ t_i \sim t(n-1). \]

Thus, the \((1-\alpha)\times 100\%\) confidence interval of \(\mu^*_i\) is

\[ \left( \frac{\mu_i - t_{n/2,1} \frac{\sigma_i}{\sqrt{n}}}{\sigma_i}, \frac{\mu_i + t_{n/2,1} \frac{\sigma_i}{\sqrt{n}}}{\sigma_i} \right) \]

where \(t_{n/2,1}\) is the upper \(\alpha/2\) quantile of the \(t\) distribution with \(n-1\) degrees of freedom. Based on Buckley (2005), the triangular shaped fuzzy number \(\mu^*_i\) is

\[ \mu^*_i[\alpha] = \left[ \mu_i^1(\alpha), \mu_i^2(\alpha) \right], \quad 0.01 \leq \alpha \leq 1 \]

\[ \mu_i^1(0.01), \mu_i^2(0.01) \]

where

\[ \mu^*_i[\alpha] = \mu_i^1 + t_{n/2,1} \frac{\sigma_i}{\sqrt{n}} \]

Similarly, let \(K = \left( \frac{n-1}{\sqrt{n}} \right)^2\), then \(K \sim \chi^2_{n-1}\) and the \((1-\alpha)\times 100\%\) confidence interval of \(\sigma_i\) is

\[ \left( \frac{n-1}{\chi^2_{n-2,1}} \sigma_i, \frac{n-1}{\chi^2_{n-2,1}} \sigma_i \right) \]

where \(\chi^2_{n-2,1}\) is the lower \(\alpha/2\) quantile of the chi-square distribution with \(n-1\) degrees of freedom. The \(\alpha\)-cuts of triangular shaped fuzzy number \(\sigma_i\) is

\[ \sigma^*_i[\alpha] = \left[ \sigma^*_i(\alpha), \sigma^*_i(\alpha), \sigma^*_i(\alpha) \right], \quad 0.01 \leq \alpha \leq 1 \]

\[ \sigma^*_i(0.01), \sigma^*_i(0.01) \]

where

\[ \sigma^*_i(\alpha) = \frac{n-1}{\chi^2_{n-2,1}} \sigma_i \]

As noted by Buckley (2005), calculations will be performed by \(\alpha\)-cuts. Choose \(a \in \mu^*_i[\alpha]\) and \(b \in \sigma^*_i[\alpha]\) and compute

\[ Q'_p(i)(a) = \frac{a}{b} + 1.5 \]

As \(a\) and \(b\) range through their intervals, we get an interval which comprises the \(\alpha\)-cuts of triangular shaped fuzzy number \(Q'_p(i)\) as follows:

\[ Q'_p(i)[\alpha] = \left[ Q'_p(i)(\alpha), Q'_p(i)(\alpha) \right], \quad 0.01 \leq \alpha \leq 1 \]

\[ Q'_p(i)(0.01), Q'_p(i)(0.01) \]

where

\[ Q'_p(i) = \frac{\hat{\mu}_i - \frac{\hat{\sigma}_i}{\sqrt{n}}}{\chi^2_{n-2,1}} \]

\[ Q'_p(i) = \frac{\hat{\mu}_i + \frac{\hat{\sigma}_i}{\sqrt{n}}}{\chi^2_{n-2,1}} \]

When \(\alpha = 1\), then

\[ Q'_p(i) = Q'_p(i) \]

Thus, the triangular shaped fuzzy number of \(Q'_p(i)\) is

\[ \Delta Q'_p(i) = (Q_L, Q_M, Q_R) \]

where

\[ Q_L = Q_p'(0.01) \]

\[ Q_L = Q_p'(i) - 1.5 \]

\[ Q_R = Q_p'(0.01) + 1.5 \]

\[ Q_R = Q_p'(i) + 1.5 \]

Let \(\eta_p'(i)\), then the fuzzy membership function of \(\psi'\) is as follows:

\[ \begin{cases} 
0 & \text{if } x \leq Q_L \\
y' & \text{if } Q_L < x < Q_M \\
y' & \text{if } Q_M < x < Q_R \\
0 & \text{if } Q_R \leq x 
\end{cases} \]

where \(y', y^*\) are determined by following two equations respectively.

\[ (Q_p'(i) - 1.5) - \frac{t_{\alpha,2,1}}{\sqrt{n}} \chi^2_{\alpha,2,1} = x \]

\[ (Q_p'(i) - 1.5) + \frac{t_{\alpha,2,1}}{\sqrt{n}} \chi^2_{\alpha,2,1} = x \]

Next, the arc triangular fuzzy diagram of \(Q'_p(i)\) is as displayed below (Fig 1):
3 FUZZY HYPOTHESIS TESTING

As mentioned previously, when \( Q_i(i) = k \), then the process reaches the \( k \sigma \) quality level. Thus, in order to do the hypothesis test \( H_0: Q_i(i) \geq k \) versus \( H_1: Q_i(i) < k \), from the random sample we compute the test statistically.

\[
Q^*_p(i) = \frac{Q(i) - \mu^*}{\sigma^*} + 1.5
\]

(21)

This will be a real number and not a fuzzy set. We then determine the statistic \( t' = \sqrt{n} \left( Q^*_p(i) - 1.5 \right) - t'(n-1, \delta^*) \), where \( \delta^* = \sqrt{n} \left( Q(i) - 1.5 \right) \).

The critical region \( C = \{ Q^*_p(i) \leq C_0 \} \) and where \( C_0 \) is determined by \( P \left( Q^*_p(i) \leq C_0, Q^*_p(i) = k \right) = \beta \), where \( \beta \) is the significance level of the test and usually \( \beta \) are 0.01, 0.05, 0.10. Thus, we have

\[
C_0 = \frac{\mu^*}{\sigma^*} + 1.5
\]

(22)

where \( \delta^* = \sqrt{n} (k - 1.5) \) and \( \mu^* = x_{\beta-1, \alpha} \) is the lower \( q \) quantile of the non-central \( t \) distribution with \( n-1 \) degrees of freedom and non-central parameter \( \delta^* \). Now when the null hypothesis is true, the decision rule is as follows:

1. Reject \( H_0 \) if \( Q^*_p(i) \leq C_0 \).
2. Do not reject \( H_0 \) if \( Q^*_p(i) > C_0 \).

Similarly to \( Q^*_p(i) \), the new transformation of \( \alpha \)-cuts of triangular shaped fuzzy critical value number \( C_0 \) are

\[
C_0^* = \left[ \frac{C_{01}(\alpha), C_{02}(\alpha)}{C_{01}(0.01), C_{02}(0.01)} \right], \quad 0.01 \leq \alpha \leq 0.05
\]

(23)

where

\[
C_{01}(\alpha) = \left( C_0 - 1.5 \right) + \frac{\beta_{\alpha/2, n-1}}{\sqrt{n}} \sqrt{\frac{\beta_{\alpha/2, n-1}^2}{n-1} + 1.5}
\]

(24)

\[
C_{02}(\alpha) = \left( C_0 - 1.5 \right) + \frac{\beta_{\alpha/2, n-1}}{\sqrt{n}} \sqrt{\frac{\beta_{\alpha/2, n-1}^2}{n-1} + 1.5}
\]

(25)

The triangular shaped fuzzy number of \( C_0 \) is \( \Delta C_0 = \{ C_L, C_M, C_U \} \), where

\[
C_L = C_{01}(0.01) = \left( C_0 - 1.5 \right) + \frac{\beta_{0.005, n-1}}{\sqrt{n}} \sqrt{\frac{\beta_{0.005, n-1}^2}{n-1} + 1.5}
\]

(26)

\[
C_U = \left( C_0 - 1.5 \right) + \frac{\beta_{0.995, n-1}}{\sqrt{n}} \sqrt{\frac{\beta_{0.995, n-1}^2}{n-1} + 1.5}
\]

\[
C_M = \left( C_0 - 1.5 \right) + \frac{\beta_{0.5, n-1}}{\sqrt{n}} \sqrt{\frac{\beta_{0.5, n-1}^2}{n-1} + 1.5}
\]

(27)

\[
C_k = C_{01}(0.01) = \left( C_0 - 1.5 \right) + \frac{\beta_{0.005, n-1}}{\sqrt{n}} \sqrt{\frac{\beta_{0.005, n-1}^2}{n-1} + 1.5}
\]

(28)

Obviously, \( Q^*_p(i) \leq C_0 \) if and only if \( Q_M \leq C_M \).

Next, the arc triangular fuzzy diagram of \( \tilde{C}_0 \) and \( \tilde{Q}^*_p(i) \) is as shown below (Fig2):

\[
\alpha = 1
\]

\[
\alpha = 0.01
\]

(29)

With the shaded area \( \{ A_y \} \) on the right side of \( x = C_M \) as the numerator and the total area \( \{ A_y \} \) of the arc triangular fuzzy number \( \tilde{Q}(i) \) as the denominator, Buckley (2005) used \( A_y / A_r \) to perform fuzzy tests. If \( d_r \) is the length of the bottom of \( A_y \), then \( d_r = C_M - Q_M \). Clearly, \( A_y \) increases with \( d_r \). Furthermore, if \( d_r \) is the length of the bottom of \( A_r \), then \( d_r = d_r^+ + d_r^- \), where \( d_r^+ = Q_M - Q_u \) and \( d_r^- = Q_u - Q_l \). For the sake of convenience, let

\[
d = Q_M - C_M \quad \text{and} \quad d_r = d_r^- - Q_l \quad \text{and}
\]

\[
d_r/d_r = \begin{cases} 0 & \text{if } C_M \leq Q_u \\ 1 & \text{if } Q_u \leq C_M \leq Q_x \\ \frac{1}{2}d_r^-/d_r & \text{if } Q_x < C_M < Q_u \end{cases}
\]

(29)

Obviously, \( 0 \leq d_r/d_r \leq 1 \) and when the value of \( d_r/d_r \) is bigger, then the value of \( A_y / A_r \) is bigger. In fact, when \( Q_M = C_M \) \( Q^*_p(i) = C_0 \), then \( d_r/d_r = A_y / A_r = 1/2 \) and \( C_M \leq Q_M \) \( Q^*_p(i) \leq C_0 \), then \( d_r/d_r = A_y / A_r = 0 \).

As it is fairly difficult to calculate \( A_y / A_r \), Buckley (2005) could only use approximation to calculate it. We therefore suggest replacing \( A_y / A_r \) with \( d_r/d_r \) in the fuzzy test.

Let \( 0 < < \phi_k < 0.5 \), then the fuzzy test rule is as follows:

1. If \( \phi_k < d_r/d_r < \phi_k^* \), then reject \( H_0 \) and conclude that \( Q(i) > k \).
2. If \( \phi_k^* < d_r/d_r < \phi_k^* \), then continue evaluation,
(3) If \( \delta_i \leq d_R/d_T = 1 \), then do not reject \( H_0 \) and conclude that \( Q_p(i) \leq k \).

4 EMPIRICAL ANALYSIS AND DISCUSSION

Shafts in the mechanical industry are key components in many products, such as machine tools, conveyors, power tools, vehicles, deceleration mechanisms, and household appliances. Their primary function is to coordinate with rotation and enable machine equipment to start up and operate smoothly with the least noise. Precision, strength, wear, and noise are the core elements of a shaft. In another aspect, modern people are placing an increasing amount of importance on nutrition and food safety, and home cooking (DIY foods) are growing in popularity in Europe and the US, thereby leading to healthy sales in related equipment. Electric mixers produced by a renowned American household appliance manufacturer can be used to make bread at home and are thus popular with consumers in Europe and the US. To improve their core technologies, professional production, and service, the business model of this manufacturer is to obtain appropriate resources from outside their company to participate in production and service. They therefore endeavored to commission precision machinery manufacturer A in central Taiwan to manufacture the key component in their electric mixer: the shaft.

The quality of shaft manufacturing has direct impact on the precision, strength, wear, and noise of the electric mixer. The process that is crucial to the quality of the outer diameter of shafts during the manufacturing process is the grinding process. Poor roundness during the grinding process will cause the accompanying journal bearing to become loose, which in turn affects the operations and damages the entire machine. The case company found that the process capability with regard to the outer diameter of the shaft was not very stable; a small number of products exceeded the quality control limits, causing an increase in defected products and customer complaints. To cope with market competition and maintain a long-term relationship with the American household appliance manufacturer, the case company must increase their yield and attain the specification of roundness smaller than 0.002 mm that the customer demands. Vibrations during the grinding process affect the precision of roundness measurements and add uncertainty to the data. We therefore applied the proposed Six Sigma fuzzy quality index and testing model to assess the performance of the shaft outer diameter process.

As the roundness of the shaft outer diameter process must be smaller than 0.002 mm (i.e. \( \text{USL} = 0.002 \)), the decision makers in the case company hoped to aim for the goal of 6sigma in the future to maintain their long-term relationship with the American household appliance manufacturer but first wanted to determine whether their process capability can attain the 6sigma level required by the customer (thus, the tested hypothesis is \( H_O : Q_P(i) \geq 5 \) versus \( H_O : Q_P(i) < 5 \)). After stabilizing the process and confirming that its results follow a normal distribution, we took random samples of output data (\( n = 64 \)) and calculated the mean and standard deviation of the roundness of the samples from the observed values: \( \bar{x} = 0.0016 \) and \( s = 0.00012 \).

Substituting the known USL and the \( \bar{x} \) and \( s \) we obtained into Eq. (6), we can derive the index estimate for the Six Sigma quality level: \( Q_p(i) = 4.58 \). Next, under the condition of the probability of type I error being \( \beta = 0.05 \), we first calculated \( \delta_i = \sqrt{n(k-1.5)} = \sqrt{64(5-1.5)} = 28 \) and \( t_{0.05,63}(28) = 24.1541 \), and then obtained the critical value \( C_0 = 4.52 \) using Eq. (22). We then substituted the critical value \( C_0 \) into Eqs. (26)-(28) to derive \( \Delta C_0 = (C_1, C_{\alpha}, C_1) \), the fuzzy triangular membership function of critical value \( C_0 \), as follows:

\[
C_1 = C_{tr}(0.01) = \left( C_0 - 1.5 \right) - \frac{t_{0.05,63}(28)}{\sqrt{n}} \left( \frac{\delta_i}{n-1} \right) + 1.5 = 4.26
\]

\[
C_M = C_{tr}(0.15) = \left( C_0 - 1.5 \right) - \frac{t_{0.05,63}(28)}{\sqrt{n}} \left( \frac{\delta_i}{n-1} \right) + 1.5 = 4.50
\]

\[
C_b = C_{tr}(0.01) = \left( C_0 - 1.5 \right) - \frac{t_{0.05,63}(28)}{\sqrt{n}} \left( \frac{\delta_i}{n-1} \right) + 1.5 = 5.63
\]

Next, we substituted the critical value \( Q_p(i) = 4.58 \) into Eqs. (15)-(17) to derive \( Q_p(i) = (Q, Q, Q) \), the fuzzy triangular membership function of critical value \( Q_p(i) = 4.58 \), as follows:

\[
Q_1 = Q_{tr}(0.01) = \left( Q_p(i) - 1.5 \right) - \frac{t_{0.05,63}(28)}{\sqrt{n}} \left( \frac{\delta_i}{n-1} \right) + 1.5 = 3.63
\]

\[
Q_M = Q_{tr}(0.15) = \left( Q_p(i) - 1.5 \right) - \frac{t_{0.05,63}(28)}{\sqrt{n}} \left( \frac{\delta_i}{n-1} \right) + 1.5 = 4.56
\]

\[
Q_b = Q_{tr}(0.01) = \left( Q_p(i) - 1.5 \right) - \frac{t_{0.05,63}(28)}{\sqrt{n}} \left( \frac{\delta_i}{n-1} \right) + 1.5 = 5.70
\]

Based on the concept proposed by Buckley (2005), we let \( d_R \) be the length of the bottom of \( A_R \), so \( d_R = C_M - Q_1 = 0.87 \). Next, we let \( d_T \) be the length of the bottom of \( A_T \), so \( d_T = d_T^+ + d_T^- = 0.3 \), where \( d_T^+ = Q_1 - Q_M = 1.14 \), \( d_T^- = Q_M - Q_1 = 0.93 \), and \( d = Q_M - C_M = 0.04 \), as shown in Figure 3. In this case example, we let the two values in the test model be \( \phi_1 = 0.25 \) and \( \phi_2 = 0.4 \) and set \( 0 < \phi_1 < \phi_2 < 0.5 \). Due to \( Q_1 < C_M < Q_M \), the calculation results of Eq. (29) show that \( d_R/d_T = (1-d/d_T) / 2 = 0.478 \), which means that \( \phi_2 \leq d_R/d_T \leq 0.478 \leq 1 \). Thus, \( H_0 \) is rejected, which means that the manufacturing process do not reaches the quality level required by the customer.

![Figure 3. Arc triangular fuzzy diagram of \( \hat{C}_b \) and of shaft roundness process.](image-url)
reach the 5 sigma quality level and the measured data are uncertain or cannot be explicitly quantified, applying the traditional testing method of process capability, p-value or the critical value used as criteria judgment is unable to identify that the process capability does not meet the 5 sigma quality level. If Six Sigma quality index $Q_p(i)$ and the fuzzy testing model proposed by this study are employed to evaluate the process performance of uncertain data, they are able to tell that the real process does not meet the requirement of the quality level and able to avoid misjudging the sampled data containing uncertainty.

5 CONCLUSIONS
This study developed Six Sigma Index $Q_p(i)$ and the fuzzy testing model for unilateral specifications to evaluate the process performance with uncertain data. First, based on Six Sigma Index $Q_p(i)$, this study employed the fuzzy estimate $Q_p'(i)$ of the triangular fuzzy number construction index $Q_p(i)$ resulted from the sets of the confidence intervals of parameters, and established the fuzzy triangular membership function $\Delta Q_p'(i) = (\hat{Q}_L, \hat{Q}_M, \hat{Q}_R)$ of the estimate $Q_p(i)$. Besides, this study referred to the fuzzy statistic testing method proposed by Buckley (2005) and derived $\Delta C_p' = (C_{Lp}, C_{Mp}, C_{Rp})$, the fuzzy triangular membership function of critical value $C_p'$. Next, according to the total area, $A_T$, surrounded by the membership function $Q_p'(i)$, and the area surrounded by vertical lines of critical value $C_p'$, we let $d_L$ be the length of the bottom of $A_L$ and $d_R$ be the length of the bottom of $A_R$, developed a fuzzy testing method of distance ratio $d_L/d_R$, and built up a decision making procedure for the fuzzy process yield performance. To explain the application of this method, this study employed the real case. When the real process capability of the product was unable to reach the 5 sigma quality level and the measured data were unable to be explicitly quantified, applying the traditional testing method of process capability was unable to identify that the process capability did not meet the 5 sigma quality level. If the fuzzy testing model proposed by this study is employed, it is able to tell that the real process does not meet the requirement of the quality level and able to avoid misjudging the sampled data containing uncertainty. Therefore, the calculating process of this method is more simplified and able to simultaneously consider process capability as well as Six Sigma quality level of the product and solve the problem of the inexplicit quantification of the quality characteristics under the fuzzy environment of the industry. It is a practical method which can help the industry conduct process performance evaluations under the fuzzy environment.

REFERENCES
1. Buckley J.J., Fuzzy statistics: hypothesis testing, Soft Computing, 2005, 9(7), 512-518.
2. Chang T.C., Wang K.J., Chen K.S., Sputtering process assessment of ITO film for multiple quality characteristics with one-sided and two-sided specifications, Journal of Testing and Evaluation, 2014, 42(1), 196-203.
3. Chen H.T., Chen K.S., A paired-test method to verify service speed improvement in the Six Sigma approach: a restaurant's case study, Total Quality Management and Business Excellence, 2016, 27(11-12), 1277-1297.
4. Chen K.S., Chen H.T., Chang T.C., The construction and application of Six Sigma quality indices, International Journal of Production Research, 2017, 55(8), 2365-2384.
5. Chen K.S., Chen H.T., Lin C.L., Applying a revised SQPM in the define step of six sigma and a case study, Total Quality Management and Business Excellence, 2012, 23(3-4), 309-324.
6. Chen K.S., Huang H.L., Huang C.T., Control charts for one-sided capability indices, Quality and Quantity, 2007, 41(3), 413-427.
7. Chen K.S., Li R.K., Liao S.J., Capability evaluation of a product family for processes of the larger-the-better type, International Journal of Advanced Manufacturing Technology, 2002, 20(11), 824-832.
8. Chen K.S., Ouyang L.Y., Hsu C.H., Wu C.C., 'The common bridge to Six Sigma and process capability indices, Quality and Quantity, 2009, 43(3), 463-469.
9. Chen K.S., Wang K.J., Chang T.C., A novel approach to deriving the lower confidence limit of indices $C_{pu}$, $C_{pk}$, and $C_{pk}$ in assessing process capability, International Journal of Production Research, 2017, 55(8), 2365-2384.
10. Chou Y.M., Owen D.B., On the distributions of the estimated process capability indices, Communications in Statistics – Theory and Methods, 1989, 18(2), 4549-4560.
11. Huang C.T., Chen, K.S., Chang, T.C., An Application of DMADV Methodology for Increasing the Yield Rate of Surveillance cameras, Microelectronics Reliability, 2010, 50(2), 266-272.
12. Hsu C.H., Chen K.S, Yang C.M., Construction of closed interval for process capability indices $C_{pu}$, $C_{pk}$, and $S_B$ based on Boole's inequality and de Morgan's laws, Journal of Statistical Computation and Simulation, 2016, 86(18), 3701-3714.
13. Kane V.E., Process capability indices, Journal of Quality Technology, 1986, 18(1), 41-52.
14. Liao M.Y., Wu C.W., Zhang F.Y., Fuzzy yield index for optimal tool replacement policy, Advanced Materials Research, 2012, 1028, 139-144.
15. Peam W.L, Chen K.S., One-sided capability indices $C_{pu}$ and $C_{pk}$: Decision making with sample information, International Journal of Quality & Reliability Management, 2002, 19(3), 221-245.
16. Prasad S., Bramorski T., Robust process capability indices, Omega, 1998, 26(3), 425-435.
17. Wang C.C., Chen K.S., Chang C.H., Chang, P.H., Application of 6-sigma design system to developing an improvement model for multi-process multi-characteristic product quality, Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 2011, 225(7), 1205-1216.
18. Wu C.W., Decision-making in testing process performance with fuzzy data, European Journal of Operational Research, 2009, 193(2), 499-509.
19. Wu C.W., Chang Y.C., Liao M.Y., Fuzzy estimation for process loss assessment, Journal of the Chinese Institute of Engineers, 2014, 37(1), 1-6.
20. Wu C.W., Kuo N.C., A Fuzzy Approach to Evaluate Process Performance Based on Imprecise Data, Journal of Quality Technology 2011, 18(6), 475-487.
21. Wu C.W., Liao M.Y., 2014, Fuzzy nonlinear programming approach for evaluating and ranking process yields with imprecise data, Fuzzy Sets and Systems, 246, 142-155.
22. Wu C.W., Liao M.Y., Lin C.Y., On ranking multiple touch-screen panel suppliers through the CTO: Applied fuzzy techniques for inspection with unavoidable measurement errors, Neural Computing and Applications, 2013, 25(2), 481-490.
23. Wu C.W., Liao M.Y., Lin C.W., Lin T.L., Testing and ranking multiple wafer-manufacturing processes with fuzzy-quality data, Journal of Testing and Evaluation, 2016, 44(5), 1970-1977.