ARTICLE

New Paradigm in Mapping: A Critique on Cartography and GIS

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ABSTRACT

As noted in the epigraph, a map was long ago seen as the map of the map, the map of the map, of the map, and so on endlessly. This recursive perspective on maps, however, has received little attention in cartography. Cartography, as a scientific discipline, is essentially founded on Euclidean geometry and Gaussian statistics, which deal respectively with regular shapes and more or less similar things. It is commonly accepted that geographic features are not regular and that the Earth’s surface is full of fractal or scaling or living phenomena: far more small things than large ones are found at different scales. This article argues for a new paradigm in mapping, based on fractal or living geometry and Paretian statistics, and – more critically – on the new conception of space, conceived and developed by Christopher Alexander, as neither lifeless nor neutral, but a living structure capable of being more living or less living. The fractal geometry is not limited to Benoit Mandelbrot’s framework, but tends towards Christopher Alexander’s living geometry and is based upon the third definition of fractal: A set or pattern is fractal if the scaling of far more small things than large ones recurs multiple times. Paretian statistics deals with far more small things than large ones, so it differs fundamentally from Gaussian statistics, which deals with more or less similar things. Under the new paradigm, I make several claims about maps and mapping: (1) the topology of geometrically coherent things – in addition to that of geometric primitives – enables us to see a scaling or fractal or living structure; (2) under the third definition, all geographic features are fractal or living, given the right perspective and scope; (3) exactitude is not truth – to paraphrase Henri Matisse – but the living structure is; and (4) Töpfer’s law is not universal, but the scaling law is. All these assertions are supported by evidence, drawn from a series of previous studies. This article demands a monumental shift in perspective and thinking from what we are used to in the legacy of cartography and GIS.

Keywords: third definition of fractal, fractal or living geometry, wholeness, head/tail breaks (ht-index), scaling law

RÉSUMÉ

Comme l’indique l’épigraphe, il fut un temps où la carte était considérée comme étant la carte de la carte, la carte de la carte et ainsi de suite à l’infini. Cette façon récursive d’envisager les cartes a toutefois suscité peu d’intérêt chez les cartographes. La cartographie, comme discipline scientifique, repose essentiellement sur la géométrie euclidienne et les statistiques gaussiennes, qui portent respectivement sur les formes régulières et les éléments plus ou moins similaires. Il est généralement convenu que les caractéristiques géographiques ne sont pas régulières et que la surface de la Terre regorge de phénomènes fractals ou d’échelle ou vivants : un nombre beaucoup plus important de petits éléments que de grands apparaissent à différentes échelles. L’auteur plaide pour un nouveau paradigme en cartographie, basé sur la géométrie fractale ou vivante, les statistiques parétiennes et, plus crucial encore, sur la nouvelle conception de l’espace imaginée et élaborée par Christopher Alexander selon laquelle l’espace n’est ni inerte ni neutre, mais constitue une structure vivante, pouvant l’être plus ou moins. La géométrie fractale ne se limite pas au cadre de référence de Mandelbrot, mais elle tend vers la géométrie vivante de Christopher Alexander et se fonde sur la troisième définition de la représentation fractale : un ensemble de motifs est fractal si l’échelle d’un nombre beaucoup plus important de petits éléments que de grands se répète à de multiples reprises. Les statistiques parétiennes portent sur le nombre beaucoup plus important de petits éléments que de grands, de sorte qu’elles diffèrent fondamentalement des statistiques gaussiennes, qui portent sur les éléments plus ou moins similaires. À partir du nouveau paradigme, l’auteur formule plusieurs affirmations au sujet des cartes et de la cartographie : 1) la topologie des éléments géométriquement cohérents – outre celle des primitives géographiques – nous permet de visualiser une structure d’échelle ou fractale ou vivante ; 2) selon la troisième définition, toutes les caractéristiques géographiques sont fractales ou vivantes, moyennant la perspective et l’étendue appropriées ; 3) l’exactitude n’est pas la vérité – pour paraphraser Henri Matisse –, mais la structure vivante l’est ; et 4) la loi de Töpfer n’est pas universelle, mais la loi d’échelle l’est. Toutes ces assertions sont étayées par des faits, tirés d’une série d’études antérieures. L’auteur
revendique un énorme virage dans la façon d'aborder et d'analyser le sujet par rapport à l'héritage familier de la cartographie et des SIG.

Mots clés : géométrie fractale ou vivante, loi d'échelle, ruptures tête/queue (indice ht), totalité, troisième définition de la représentation fractale

Two important characteristics of maps should be noticed. A map is not the territory it represents, but, if correct, it has a similar structure to the territory, which accounts for its usefulness. If the map could be ideally correct, it would include, in a reduced scale, the map of the map: the map of the map, of the map; and so on, endlessly, a fact first noticed by Royce.

– Alfred Korzybski (1994 [1933])

Introduction

As noted in the epigraph, a map was long ago seen as the map of the map, the map of the map, of the map, and so on endlessly. This recursive perspective on maps, however, has received little attention in cartography. Euclidean geometry has served as the foundation of cartography ever since human beings began to measure the magnitude of the Earth, if not even before that (Anson and Ormeling 2013; Robinson et al. 1995; Slocum et al. 2008). We cartographers tend to see geographic features individually rather than holistically, non-recursively rather than recursively; we tend to focus on individual scales rather than on all scales or the underlying scaling hierarchy ranging from the smallest to the largest (Jiang and Brandt 2016); we tend to believe – consciously or subconsciously – in more or less similar things, as reflected in Tobler's law (Tobler 1970), rather than far more small things than large ones, which is formulated as a scaling law (Jiang 2015a). This Euclidean geometric perspective is so stubborn that it makes some maps or mapping – for example automatic map generalization – difficult or virtually impossible. A cartographic curve is traditionally viewed as a collection of more or less similar line segments – a non-recursive perspective. From a recursive perspective, a cartographic curve consists of far more small bends than large ones, and small bends are embedded in large ones (Jiang and Brandt 2016). Inspired by the living geometry of Christopher Alexander (2002–05), a cartographic curve can be seen as a coherent whole, in which nested bends constitute coherent sub-wholes at different scales.

In general terms, geographic features look regular only at a local scale, but they are essentially irregular; geographic features look more or less similar only at one scale (note: “scale” means size rather than map scale), but there are essentially far more small geographic features than large ones. This notion of far more smalls than larges recurs multiple times, indicating a scaling hierarchy of numerous smallest, a very few largest, and some in between the smallest and the largest. However, the scaling hierarchy is quite well hidden in various representations of geographic information systems (GIS), such as raster and vector (Bian 2007; Goodchild 2018). These geographic representations, based on mechanically imposed geometric primitives of pixels, points, lines, and polygons, are unable to reveal the true scaling property of geographic features; see more discussion in Topology Matters for Seeing a Scaling or Fractal or Living Structure. This mechanistic thinking is limited, because the mechanically imposed geometric primitives do not correspond to what we perceive about geographic features (cf. Figure 5 in Topology Matters for Seeing a Scaling or Fractal or Living Structure). Thus with mechanistic thinking, we cannot effectively see the fractal or living nature of geographic features. Instead we only see fragmented geometric primitives as equivalent to geographic features. I am, therefore, advocating a paradigm shift in cartography and GIS.

This article intends to discuss with cartographers, both older and young, fractal geometric and Paretian statistical thinking and – more fundamentally – the new organic view of space: that space is neither lifeless nor neutral, but a living structure capable of being more living or less living. For this purpose, I have attempted to write in an accessible manner so that both academics and practicing cartographers can understand my arguments for the new paradigm in mapping. I am calling for a paradigm shift from Euclidean to fractal geometry, from Gaussian to Paretian statistics, and – more importantly – from the mechanistic thinking of Descartes (1954 [1637]) to the organic thinking of Alexander (2002–05). To see scaling or fractal or living structure clearly, we must shift our mentality from geometric details of locations, sizes, and directions to overall character through topology – the topology of coherent geometric entities such as rivers, cities, streets, buildings, and even tiny ornaments. The overall character refers to the underlying scaling or fractal or living structure of far more smalls than larges.

I argue that all geographic features are fractal or scaling, given the right perspective and scope. A tree is surely fractal, but we hardly see the fractal nature if we concentrate only on the scale of individual leaves (note: I am not referring to subscales of the leaves, which are likely to be fractal), which tend to be more or less similar in terms of size and shape. Similarly, with a non-recursive perspective, one can only see fragmented pieces rather than an interconnected whole. The Sierpinski (1915) carpet is fractal when seen as a whole, but we hardly see the fractal nature if we view it fragmented, as disconnected squares. The same applies for traditional and vernacular building façades: they are definitely fractal, because there are far more small things than large
The first definition is the strictest among the three and it dates back to the nineteenth century, when there were such fractals as Cantor (1883) dust, Koch (1904) curve, and the Sierpinski (1915) carpet (Table 1). Let us use the Koch curve – named after its inventor, the Swedish mathematician Helge von Koch (1870–1924) – as a working example to illustrate the first definition. It requires a power law relationship or a constant ratio between two parameters, $x$ and $y$, on their logarithmic scales, $y = x^{1.26}$, or equivalently $\ln(y) = -1.26 \ln(x)$, where $x$ and $y$ indicate the scale and the number of segments, respectively. A segment of one unit is divided into three equal thirds, and the middle one is replaced by the two sides of an equilateral triangle (see Figure 1 for Iteration 1 or the generator). This process of division and replacement is iterative, which means that scale decreases exponentially by a factor of one-third – $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{27}$ – and the number of segments increases exponentially by a factor of four – $1$, $4$, $16$, and $64$.

Mathematically, if one variable decreases exponentially and another increases exponentially, these two variables constitute a power law relationship, $y = x^{1.26}$. In the power law plot, points $(1, 1)$, $(1/3, 4)$, $(1/9, 16)$, and $(1/27, 64)$ are exactly on the trend line. This is where the problem arises. When scale is decreased to infinite shortness, the curve becomes infinitely long. This is the so-called conundrum of length (Percival 1966 [1958]; Richardson 1961), which puzzled scientists for over 100 years, until the French mathematician Benoit Mandelbrot (1967, 1983) established fractal geometry. In the framework of Euclidean geometry, anything should be measurable, no matter how big or small it is. In fact, this is a limitation of Euclidean geometry. The second definition is less strict or more relaxed than the first one. Mandelbrot (1967) noticed that the first definition of fractal is too rigorous for the Koch curve to be a meaningful model of the real world. As a matter of fact, there is no need for scale to decrease to exactly one-third or to have the number of segments increase exactly four times to retain the power law relationship. In other words, with a decrease of scale to approximately one-third and an increase in the number of segments of approximately four times, the power law relationship still holds, not exactly but approximately or statistically (Figure 1). On the power law plot, a set of points, such as $(1 \pm e_1, 1 \pm d_1)$, $(1/3 \pm e_2, 4 \pm d_2)$, $(1/9 \pm e_3, 16 \pm d_3)$, and $(1/27 \pm e_4, 64 \pm d_4)$, are.
In contrast to the first two definitions, the third definition is not constrained by the power law relationship. Instead, it examines, given a set or pattern, whether the scaling of far more small things than large ones recurs multiple times (Jiang and Yin 2014). These multiple times or the ht-index would answer the "how complex" question about the set or pattern. The new paradigm is actually to confront or to address the issue of "how complex"; more specifically, maps and mapping must reflect the underlying complex, scaling, fractal, or living structure of the Earth's surface.

To further illustrate the third definition, let us examine the 100 numbers that exactly and strictly follow Zipf's law (1949): 1, 1/2, 1/3, . . ., and 1/100. The average of the 100 numbers is 0.05, which partitions these numbers into two parts: the first 19 numbers (about 20 percent, all greater than the average, called the head) and the last 81 numbers (about 80 percent, all less than the average, called the tail). The average of the first 19 numbers is 0.19, which again partitions the 19 numbers into two parts: the first five (25 percent, all greater than the second average, called the head); and the remaining 14 (75 percent, all less than the second average, called the tail). This recursive

where $e_i$ and $d_i$ indicate very small epsilons or deviations, are around the trend line rather than on the trend line, as in the first definition (Ma and Jiang 2018). The resulting curves look very natural, such as clouds, city skylines, and coastlines, dramatically different from the rigorous Koch curve. This shift from the first to the second definition of "fractal" is probably another example supporting the statement that exactitude is not truth in science (see further details in Exactitude Is Not Truth), because the second is more relaxed or less rigorous than the first.

The third definition is still less strict or more relaxed than the first two. Neither Koch curves nor coastlines are measurable, and their lengths depend on the measuring scale; the shorter the measuring scale, the longer the curves. For complex curves such as coastlines, what matters is not how long they are, but how complex they are. "How long" is an unanswerable question, one that concerns Euclidean geometry or simple science in general, whereas "how complex" is an answerable question that concerns fractal geometry or complexity science in general (Jiang and Ma 2018). The first two definitions are top-down in nature; for example, given a line segment of one unit, and the generator, a fractal curve is generated iteratively. In other words, the generator is applied again and again at increasingly fine scales. Eventually, very convoluted and very complex curves are generated. In contrast to the first two definitions, the third definition is not constrained by the power law relationship. Instead, it examines, given a set or pattern, whether the scaling of far more small things than large ones recurs multiple times (Jiang and Yin 2014). These multiple times or the ht-index would answer the "how complex" question about the set or pattern. The new paradigm is actually to confront or to address the issue of "how complex"; more specifically, maps and mapping must reflect the underlying complex, scaling, fractal, or living structure of the Earth's surface.

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Figure 1. The first two definitions of "fractal"

Note: The first definition is too rigorous, requiring a very strict power-law relationship between scale and length, while the second definition is less rigorous and more statistical. In other words, the scale decreases not strictly by 1/3, but by approximately 1/3 with a small epsilon; the number of segments or length increases not exactly four times, but approximately four times with a small deviation. The two portraits from left to right are Helge von Koch (1870–1924) and Benoît Mandelbrot (1924–2010).

Sources: Von Koch from "Portrait of Nils Fabian Helge von Koch" (n.d.) and Mandelbrot from at IBM (n.d.).
process is called head/tail breaks (Jiang 2013a, 2015b). The head/tail breaks process continues recursively or iteratively three times and the scaling of far more small numbers than large ones recurs three times, implying four hierarchical levels for these 100 numbers; that is, \([0.01, 0.05], (0.05, 0.19], (0.19, 0.46], (0.46, 1)\]. The number of recurrent times is called the ht-index – an alternative to fractal dimension for characterizing the complexity of fractals or geographic features in particular (Jiang and Yin 2014). The notion of far more small things than large ones is also well reflected in the four hierarchical levels. There are 81, 14, 3, and 2 numbers at the four levels from the lowest to the highest, respectively. The ratio of upper class to lower class is always a minority to a majority, that is, \(14/81, 3/14, 2/3\), which reflects the underlying scaling hierarchy of far more smalls than larges.

The notion of far more smalls than larges must be comprehended not only statistically, but also in terms of the underlying geometry (or spatial configuration to be more precise). Assuming the 100 numbers are 100 city sizes, their distribution over a region of space follows the scaling hierarchy, characterized by central place theory (CPT; Christaller 1966 [1933]; Chen and Zhou 2006). The CPT model implies that large cities are surrounded or supported by medium-sized cities, which are further surrounded or supported recursively by small cities, forming a scaling hierarchy. This geometric aspect indicates that cities are adapted to each other or that nearby cities are more or less similar. This adaptation can also be seen from Tobler’s law (1970), which states that nearby things (or cities in particular) tend to be more or less similar. Therefore, the third definition of “fractal” involves both statistical and geometric aspects. This definition implies that not only coastlines but also highways are fractal (Figure 2). As mentioned at the start of this article, a cartographic curve should be more correctly viewed as a collection of recursively defined bends and a recurrent scaling of far more small bends than large ones. However, highways are not fractal under the first two definitions, since they tend to be smooth or regular. The third definition of fractal, which is closer to living geometry (Alexander 2002–05), provides a theoretical basis to support a new paradigm in cartography.

The New Paradigm in Cartography

A new paradigm occurs in science when the basic concepts and experimental practices of a scientific discipline undergo drastic revision. More than just replacing techniques, a paradigm shift means an entirely new way of looking at the real world (Kuhn 1970). The new paradigm in cartography is essentially built on the new cosmology – the conception of physical reality – conceived by Christopher Alexander through his life’s work: The Nature of Order: An Essay on the Art of Building and the Nature of the Universe (Alexander 2002–05), a four-volume opus on art, science, nature, and beauty (cf. Alexander 2003 for a short summary of the masterful work for a scientific audience). In the new cosmology, space is unlike what we were told under the mechanistic framework (Descartes 1954 [1637]), not being lifeless or neutral, but a structure capable of being more or less living. For example, the Koch curves are living structure; those in the high iterations are more living than those in the low iterations; or equivalently, those in the low iterations are less living than those in the high iterations. This new organic worldview or new cosmology is built directly on the wholeness, which is defined as follows:

I propose a view of physical reality which is dominated by the existence of this one particular structure, \(W\), the wholeness. In any given region of space, some subregions have higher intensity as centers, others have less. Many subregions have weak intensity or none at all. The overall configuration of the nested centers, together with their relative intensities, comprise a single structure. I define this structure as “the” wholeness of that region. (Alexander 2002–05, Book 1, 96)

The wholeness is a recursive structure that is defined mathematically and exists physically in nature and in what we build and make (Alexander 2002–05; Jiang 2015d, 2016, 2019). The recursive structure recurs at different levels of scale in the deep; it is so deep that “each time it occurred, it took a different form, and was yet, nevertheless always the same” (Alexander 2006). In the new cosmology or world picture, the Earth’s surface is considered to be an unbroken whole. A map eventually reflects the truth of the wholeness of the Earth’s surface or part of it. Thus, the truth, or capturing the truth, should be the essence of all mapping activities. Given the circumstance, quality of maps is a matter of fact rather than personal preferences or opinions, as commonly conceived.

The above definition of wholeness can be rephrased as the scaling hierarchy of far more smalls than larges. Space, geographic space in particular, is neither lifeless nor neutral,
but a living structure. When I say it is neither lifeless nor neutral, I am not saying that geographic space is not dynamic along the time dimension. Instead, I am saying that at any instant in time, geographic space is not neutral and it has the capacity to be more living or less living. We need to adopt a holistic view in order to see the capacity of geographic space. The scaling hierarchy cannot be effectively characterized by Euclidean geometry, but it can be by fractal geometry (Mandelbrot 1983) – or living geometry, particularly under the third definition: A set or pattern is fractal if the scaling of far more small things than large ones recurs multiple times with the h-index being at least three (Jiang and Yin 2014). It is important to realize the paradigm shift from Descartes’ mechanistic world picture to Alexander’s organic conception of the physical world (Figure 3).

The living geometry of Alexander (2002–05) is more profound than the fractal geometry of Mandelbrot (1983) as a way to characterize the Earth’s surface. On the one hand, the Earth’s surface is a whole and is part of a larger whole, and so on endlessly towards the entire universe. On the other hand, the Earth’s surface contains countries, which further contain cities, streets, and buildings, down to an architectural scale of millimetres (e.g., to see the living structure of an ornament). The wholeness of the Earth’s surface is what the new paradigm is largely based on, and it is what maps attempt to depict. To this point, I wish to correct a statement I made in the early 1990s: visualization as the core of cartography. Visualization cannot be the core of cartography, and it is just appearance. The core of cartography is the deep structure of the wholeness, or the fractal or scaling or living structure of geographic space. Maps and mapping, such as visualization, symbolization, map generalization, and even cognitive mapping, should reflect the wholeness or the scaling hierarchy of far more smalls than larges.

In addition to the new cosmology, emerging geospatial big data add another incentive for the new paradigm in cartography. Big data differ fundamentally from small data in terms of three data characteristics (Mayer-Schonberger and Cukier 2013; Jiang and Thill 2015). First, big data are considered to be all rather than samples. Second, big data are accurately measured at a very high resolution, while small data are at a low resolution or roughly estimated. Third, big data are defined at the individual scale rather than aggregated as small data. These three characteristics imply that big data are better than small data in reflecting the wholeness of the Earth’s surface, which tends to be very heterogeneous and diverse. The heterogeneity and diversity cannot be well seen in raster and vector representations of GIS, since they are based on geometric primitives of pixels, points, lines, and polygons (Bian 2007; Goodchild 2018), which tend to be more or less similar rather than far more smalls than larges; see for an illustration, see Figure 5 in Topology Matters for Seeing a Scaling or Fractal or Living Structure. Instead, we should take spatially or geometrically coherent entities as basic units, such as named or natural streets, and assess how they constitute a coherent whole, from which sub-wholes can be identified. A coherent whole emerges from a holistic perspective, or more truly from its spatial configuration point of view.

This new paradigm requires shifting our ways of thinking, not only geometrically (Figure 3) but also statistically (Figure 4). The third definition of “fractal” is based on the notion of far more smalls than larges, actually indicating a Paretian distribution. It is not the bell curve shown in the histogram, but the long tail in the rank–size plot (Zipf 1949). This long-tailed distribution can be shown to have hierarchical levels through head/tail breaks (Jiang 2013a, 2015b). Under the new paradigm, different types of mapping can be considered as head/tail breaks processes for thematic mapping, for map generalization, for cognitive mapping, and even for perception of beauty (Jiang 2013a, 2013b, 2015c; Jiang et al. 2013; Jiang and Sui 2014). This
beauty is a new kind of beauty that exists in deep structure – structural beauty – out of the deep structure of wholeness (Jiang and Sui 2014). The new paradigm implies that cartography should go beyond conventional GIS representations towards topological representations that enable us to see the underlying scaling or fractal or living structure of the wholeness of the Earth’s surface. I will further discuss this implication and others in the next section.

Implications of the New Paradigm in Cartography

The new paradigm has deep implications for cartography and GIS and for mapping and geospatial analysis in particular. Under the new paradigm, a map would become the truth of the wholeness of the Earth’s surface, and mapping processes of various kinds should be largely guided by the scaling law (Jiang 2013b, 2015a, 2015c). In general terms, cartography is a science – one based on complexity, fractals, scaling hierarchy, and living structure, just to mention a few examples. Under the new paradigm, conventional mathematics such as Euclidean geometry and Gaussian statistics remains valid for measuring and analyzing geographic objects with respect to Tobler’s law or to the notion of more or less similar things, but is unlikely to be of much use for developing new insights with respect to spatial heterogeneity or the scaling law.

TOPOLOGY MATTERS FOR SEEING A SCALING OR FRACtal OR LIVING STRUCTURE

The foundation of the new paradigm is the organic world picture, in which geographic space – or space in general – is viewed as a scaling or fractal or living structure. To see this living structure clearly, we must adopt a topological perspective – the topological relationship among geometrically coherent entities such as rivers, lakes, streets, and buildings. Conventional GIS is essentially based on a geometric perspective, focusing on geometric details of locations, sizes, and directions, and based on geometric primitives of pixels, points, lines, and polygons (Bian 2007; Goodchild 2018). A street network is a very good example. A street network is conventionally seen as a graph of street nodes or street segments (Figure 5a). Structurally speaking, the graph to the left is very homogeneous with characteristic scales, since each node or street segment has more or less similar number of connections. Three or four can be said to be a characteristic scale of the node’s connectivity; all segments have more or less similar length, which can be said to be another characteristic scale. Traditional mathematical description and quantitative analyses are essentially based on characteristic scales.

In fact, the street network can be more truly seen as a graph of individual streets, defined, for example, by unique names. This is the topological perspective, in which we can see far more short streets than long ones geometrically, or far more less-connected streets than well-connected ones topologically. Thus, streets constitute a fractal or living structure. It should be noted that not only streets but also street blocks – the spaces between streets – are fractal, since they involve far more small blocks than large ones (Jiang and Liu 2012). This is in line with the notion that if a pattern or set is fractal, its complement set tends to be fractal also (Chen 2017). The transformation from the geometric representation to the topological representation ignores the geometric details. This is because an entire street has been abstracted as one node and, more importantly, this node has no geometric information at all except its topological information, such as degrees of connectivity. Many researchers (e.g., Ratti 2004) mistakenly argued that topological representation suffers from the loss of geometric information, so
it is of less use than geometric representation. This is an extremely biased, prejudiced, and blinkered view. In fact, it is exactly through ignorance (rather than loss) of geometric information that the topological representation gains penetrating insights into the underlying scaling structure of far more less-connected streets than well-connected ones. Geometrically a street network is not fractal, but topologically it is. The topological representation is considered to be the first and foremost, while the geometric one is secondary. In other words, we do not give up the geometric representation entirely, and we only give up our devotion to it, since there is something more important – the topology – than the geometry.

ALL GEOGRAPHIC FEATURES ARE FRACTAL OR LIVING

Under the third definition, all geographic features are fractal or living, given the right perspective and scope. We have already seen that topology among meaningful geographic features is the right perspective for seeing the fractal or living structure of a street network. As for the scope, it is usually the case that bigger is better for seeing fractal or living structure. For example, a country is better than a city for seeing fractal structure, a city is better than a building, a building is better than a façade, and a façade is better than an ornament. However, as a matter of fact, fractal or living structure can be seen at different levels of scale. These examples can be extended to biology. A human body is better than an organ for seeing fractal structure, an organ is better than a tissue, and a tissue is better than a cell. In summary, the larger the scope, the more heterogeneous or more diverse the things are.

Given that all geographic features are fractal, we must adopt the head/tail breaks (Jiang 2013a, 2015c; Lin 2013) for classification and visualization rather than the commonly used natural breaks or k-means or other classifications (Jenks 1967). For example, any digital elevation model (DEM) involves far more low elevations than high one. Current colour rendering for DEM unconsciously exaggerates high elevations (Figure 6a), so it distorts – rather than reflects – the underlying fractal or living structure. Using head/tail breaks, the DEM should be rendered as in Figure 6b. Figures 6a and 6b look very different, but Figure 6b reflects well the underlying scaling or fractal or living structure of far more low elevations than high ones. This difference can be seen clearly in the two corresponding histograms in the figure, with one showing a Gaussian-like distribution (Figure 6c), and the other a long tail distribution (Figure 6d).

EXACTITUDE IS NOT TRUTH

The title of this subsection is borrowed from the artist Henri Matisse, who made some very cogent statements about his art. Matisse (1978 [1947], 117) noted that the overall character of a human face does not depend on “the exact copying of natural forms, nor on the patient assembling of exact details, but on the profound feeling of the artist before the objects which he has chosen, on which his attention is focused and the spirit of which he has penetrated”. Figure 7 illustrates the scene while Matisse was drawing his four self-portraits, as seen in a mirror. These four portraits differ from each other in terms of local details of the nose, chin, and eyes, yet they all look unmistakably like the face and character of Henri Matisse. The artist argued that everything has an inherent truth that must be distinguished from its surface appearance, and this is the only truth that matters. He noticed that it is essentially the truth of an object that makes a drawing or painting
Figure 6. Renderings of the DEM based on (a) natural breaks and (b) on head/tail breaks, and (c) and (d) their corresponding histograms.

Figure 7. A photo of Henri Matisse and his four self-portraits.

Note: The local details in each portrait are different, but in each of them we see the unmistakable face and character of Henri Matisse (1869–1954) – the wholeness. The wholeness of the face can be summarized as such: the bald head, with the eyes spreading concentrating downward to the mouth, and with the low parts such as mustache and jaw spreading outward.

Source: Photo of Henri Matisse by Hélène Adant; four self-portraits from Matisse (1948), collection of Madame Marguerite Duthuit.
successful. Christopher Alexander (2002–05) claimed that the truth is what he termed the wholeness. The wholeness exists physically in space and matter at different levels of scale and is reflected in our minds and cognition psychologically. More importantly, the wholeness is essentially a recursive structure that can be defined mathematically (Alexander 2002–05; Jiang 2015d, 2016, 2019).

Contrary to the assertion, our desire for exactitude in GIS and cartography has become stronger and stronger. Cartographers and GIS experts in general are fond of high-resolution imagery and high-quality data in maps or GIS databases. This situation is understandable, given that cartography is essentially founded on Euclidean geometry, and its initial goal was to depict the underlying structures or patterns of geographic space through scientific abstraction. Such depiction requires high exactitude in terms of locations, sizes, and directions. In this regard, many different map projections have been developed for different purposes of measurement and navigation (Yang and others 1999). All of these achievements constitute the legacy of cartography, and have been well retained in GIS (Bian 2007; Goodchild 2018). However, cartography has been facing a critical change from data collection to knowledge discovery.

The past six decades of GIS history have experienced two major distinct phases of transformation: the transformation from data to information, and the transformation from information to knowledge. The former phase concerns data collection — transforming raw data into computerized information — whereas the latter is more interested in how to obtain useful information or knowledge for various forms of spatial planning and decision making. The Euclidean geometric paradigm works well in the first phase, but it has critical limitations in the second phase. Referring to Figure 7 again, the photo has the highest data quality — similar to the person in appearance — whereas the four portraits capture the most data or personal character — similar to the person in character. The difference between similarity to the person in appearance and in character is what underlies the notion that “exactitude is not truth.”

The issue of exactitude or overall data character has been discussed not only in art, but also in science. The Argentine writer Jorge Luis Borges (1998 [1946]) wrote a one-paragraph story entitled “On exactitude in science.” The story, styled as an extract from a historic travel book dating on 1658 by the fictitious author Suarez Miranda, praised the value of abstraction or reduced scales of maps instead of maps of 1:1 scale. Maps of 1:1 scale are useless due to their lack of abstraction or generalization. Privileging more data of less exactitude opens new paths for big-data analytics: “We don’t give up on exactitude entirely; we only give up our devotion to it. What we lose in accuracy at the micro level we gain in insight at the macro level” (Mayer-Schonberger and Cukier 2013, 13–14). The topological representation as discussed in Topology Matters for Seeing a Scaling or Fractal or Living Structure provides another good example for the fact that exactitude is not truth. Geometric details actually prevent us from seeing the truth — the underlying scaling or fractal or living structure.

TÖPFER’S LAW IS NOT UNIVERSAL, WHILE THE SCALING LAW IS

Töpfer’s law, also called the principle of section or radical law (Töpfer and Pillweizer 1966), provides a guideline for how many map objects should be selected or retained from the source map to the derived map. It is an empirical law, developed by counting the number of map objects in both the source and derived maps. This way of establishing the empirical law was justified in the paper map ages, when maps were mainly produced by human cartographers. However, the law was established through individual map sheets, which are artificially and mechanically determined. Each of these determined map sheets is not a whole or sub-whole. A whole is referred to something natural or organic rather than something mechanical or artificial. For example, the Earth’s surface is a whole, and a continent is a sub-whole; if a country is referred to as a whole, then its cities are sub-wholes; if a human body is a whole, then the heart or brain is a sub-whole.

Many objects such as mountains, rivers, and streets may stretch across several map sheets, and are not constrained to any one of them. Therefore, one cannot effectively count the number of objects. Some objects, such as settlements or buildings, are countable, but belong to individual clusters, which cannot be effectively detected or counted in map sheets. In this regard, it would be reasonable to take a country as a whole, and its individual cities as sub-wholes, and so on. Or, if possible, take the entire world as a whole, and individual countries and cities as sub-wholes, and sub-wholes of the sub-wholes. All in all, the topological perspective rather than ordinary geometric perspective, as discussed in Topology Matters for Seeing a Scaling or Fractal or Living Structure, helps us to see whole or sub-wholes. The scaling law is essentially built on this holistic view of space, and the notion of far more smalls than larges recurs at different levels of scale. Therefore, the scaling law is universal, while Töpfer’s law is not.

There are two basic functions of maps: reading detailed individual information and illustrating overall scaling patterns. For the reading function, maps are presented conventionally with a detailed map legend, a map scale, and a compass. For the function of showing scaling patterns, map elements such as legend, scale, and compass are unnecessary (e.g., Figures 5b and 6b). The latter function is in line with fractal geometry focusing on patterns rather than individuals. Cartography is a true science. I therefore suggest change the wording in the definition of cartography from “the art, science, and technology of making maps” (Meynen 1973) to “the science, art, and technology of making maps.” First and foremost, cartography is a science,
and the art is for the sake of science, to paraphrase Mandelbrot (1989). The art or the artistic aspect arises from the underlying scaling or fractal or living structure rather than being subjective or idiosyncratic (Griffin 2017). The fractal or living structure can evoke a sense of beauty – structural beauty that can be measured quantitatively, as well as sensed by human beings (Jiang 2015d; Jiang and Sui 2014; Wu 2015; Roʃe 2016). A beautiful map must reflect the underlying living structure, which accounts for a majority of beauty, while aspects of surface beauty such as colour and design account for a minority.

Conclusion

Arguing for a new paradigm in mapping, this article provides a critical analysis of the state of the art of cartography and GIS: its stubborn Euclidean geometric and Gaussian statistical thinking, and – more fundamentally – its deadly mechanistic thinking, as reflected in many GIS representations such as raster and vector. This new paradigm is established on the new organic cosmology (Alexander 2002–05) in which the universe is a coherent unbroken whole, and space is neither lifeless nor neutral, but a living structure capable of being more or less living. According to modernism, postmodernism, and deconstructionism, so-called fashionable nonsense (Sokal and Bricmont 1998), a map is considered not to be the truth. Contrary to this claim, I argue that a map, if correct, is essentially about the truth of the wholeness of geographic space – the essence of the argument for the new paradigm – and quality of maps is a matter of fact rather than of opinion. I call for a paradigm shift, from Euclidean geometry to fractal geometry, from Gaussian statistics to Paretoian statistics, and – most importantly – from the mechanistic thinking of Descartes (1954 [1637]) to the organic thinking of Alexander (2002–05). I have presented three definitions of fractal and discussed how one definition gets relaxed from – actually beyond – another, yet opens new horizons to see our surroundings insightfully. The third definition is unique in the sense that it enables us to see things organically rather than mechanistically. The new paradigm may raise discomfort in the profession, but it nevertheless opens new ways of thinking that are highly challenging to the academic establishment of cartography and GIS.

The new paradigm has some profound implications for cartography and GIS, and for mapping practices and geospatial analysis in particular. It implies that mapping, including cognitive mapping, can be considered a head/tail breaks process (Jiang 2013b). It implies that the topological perspective rather than the perspective focusing on geometric details enables us to see the scaling or fractal or living structure of the Earth’s surface or its sub-wholes. It implies that all geographic features are fractal under the new, relaxed third definition of fractal. It implies that the wholeness of the Earth’s surface rests little on geometric details, but on the overall character – the very notion of far more smalls than larges. It implies that a map is the truth of the wholeness of the Earth’s surface, and cartography is a true science. I hope that this paper can help promote healthy debates in departments of cartography and GIS and in the cartographic community as a whole about the legacy and future of cartography and GIS.

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