Modeling for Lane Changing Trajectory with Odd Order Polynomial

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Abstract. In this paper, we study the virtual trajectory planning method for lane changing of vehicle in automated highway system, derive the trajectory model for lane changing on circular road by using odd-order polynomial constraints. Based on the trajectory model for lane changing on straight road, the motion for lane changing of vehicle on circular road can be decomposed into the linear centripetal motion and the circular motion around the instantaneous centre of the circle road. The simulation results verify the feasibility of trajectory planning method for lane changing proposed in this paper on circular road.

Introduction
At present, the research result for lane changing on curve section of road is very little. In reference [2], Hatipoglu et al researched the control method for lane changing based on the yaw rate tracking, by using the curvature of the circular road, designed the mathematical model of desired yaw angle and yaw rate for lane changing, but it ignored the curvature difference between the inside lane and the outside lane of the road, so the vehicle position can’t be guaranteed at the centerline of the target lane at the end of the lane changing process, namely lane changing trajectory exists a deviation. In reference [3], authors considered the impact of lateral movement of vehicle on curvature of lane changing trajectory, without assuming that the starting lane and the target lane have the same curvature, the trajectory model planed for lane changing did not exist deviation. In above references [2] and [3], the length constraint of the road section corresponds to the lane changing process had not been considered, but in fact lane changing behavior usually occurs in the case when there is an obstacle in front of the current vehicle position or the spacing between vehicles is limited, it is needed to consider the driving distance during the process of lane changing, therefore, the road section length constraints for lane changing behavior should be considered when planning the lane changing trajectory, such as in reference [4], authors established a minimum safety distance model for lane changing by applying elliptic model, on which the changing lane model considering spatial constraints was designed based, but it did not consider the curvature of road.

Based on references [2] and [3], using odd-order polynomial constraints as reference [5], this paper continues to research trajectory planning method for lane changing on circular road, not only consider the lateral distance between lanes of circular road, but the longitudinal length constraints for road section is also included, make a further promotion for existing research results.

Modeling for Lane Changing Trajectory
In this paper, assume that the displacement of the vehicle centroid moved along the longitudinal direction is \( X_d(t) \), along the lateral direction is \( Y_d(t) \), where \( t \in [t_{on}, t_{off}] \), and \( t_{on}, t_{off} \) represent the start time and the end time of the lane changing process, respectively.

Constraint Conditions
In fact, road sections are usually not linear, it is necessary to study how to change lanes on the curved road. In the following, we should extend the trajectory model for lane changing on straight road to
circular road based on polynomial constraint. It is different from straight road, the centerlines $l_0$, $l_1$ and $l_2$ of road and its two lanes are changed from straight line into an arc line, as shown in Figure 1. Assuming $l_0$, $l_1$ and $l_2$ have the same center of curvature $O_r$, namely the instantaneous center of the road, let the radius of curvature of $l_0$ as $R$, the difference between the radius of curvature of $l_1$ and $l_2$ is lane spacing $d$, so the radius of curvature of $l_1$ and $l_2$ are $R+d/2$ and $R-d/2$, respectively. Let $S'$ and $S''$ as the intersection points of $l_0$, $l_2$ with the connect line between start point $S$ and the instantaneous center of road $O_r$, $F'$ and $F''$ as the points of intersection between $l_1$, $l_0$ and the extension of line connecting $O_r$ and the end point $F$. Assuming the arc length corresponding a lane changing process, namely the length of road centerline between the point $S'$ and point $F'$ is $L$, then the corresponding circumference angle $\alpha = L/R$, and arc length between points $S''$ and $F$ is $(R-d/2)\alpha$, arc length between points $S$ and $F''$ is $(R+d/2)\alpha$. When a vehicle changes lanes on a circular road, the motion of vehicle centroid $M$ can be decomposed into the linear centripetal motion from point $S$ to instantaneous center $O_r$ and the circular motion around $O_r$.

![Figure 1. Model of a segment of circular road.](image1)

Take the start point $S$ as origin point to establish coordinate system, X-axis is along the tangential direction of arc $l_1$ at point S, and Y-axis towards the instantaneous centre of road, as shown in Figure 2, where $X_c(t)$ and $Y_c(t)$ denote the position coordinates of vehicle centroid $M$ at time $t$; $r(t)$ and $\theta(t)$ denote the instantaneous radius of changing lane trajectory and rotate angular displacement of vehicles around the instantaneous center, respectively.

![Figure 2. Schematic diagram of trajectory planning for lane changing on circular road.](image2)
Assuming that the lane spacing, time requirement, length of road section traveled during the lane changing process, and the constraints on vehicle states at the start point and end point are the same as above changing lane scene on a straight road, based on the coordinate system shown in Figure 2, the constraint conditions to be satisfied of the vehicle displacement, velocity and acceleration at the start time and end time for lane changing on circular road can be determined as

\[
\begin{align*}
\dot{X}_c(t_{on}) &= X_d(t_{on}) \\
\dot{Y}_c(t_{on}) &= Y_d(t_{on}) \\
\ddot{X}_c(t_{on}) &= \ddot{X}_d(t_{on}) \\
\ddot{Y}_c(t_{on}) &= \ddot{Y}_d(t_{on}) \\
\dot{X}_c(t_{off}) &= \dot{X}_d(t_{off}) \cos \alpha - \dot{Y}_d(t_{off}) \sin \alpha \\
\dot{Y}_c(t_{off}) &= \dot{X}_d(t_{off}) \sin \alpha + \dot{Y}_d(t_{off}) \cos \alpha
\end{align*}
\]  

(1)

(2)

(3)

In above equation (3), \(\dot{X}_d(t_{off})/(R + d/2)\) and \(\dot{X}_d(t_{off})/(R - d/2)\) denote the centripetal acceleration caused by desired velocity along the tangential direction of the lane changing trajectory at the start time and end time.

**Trajectory Model**

Instantaneous radius of lane changing trajectory \(r(t)\) should meet the boundary conditions \(r(t_{on}) = R + d/2\) and \(r(t_{off}) = R - d/2\). Using the trajectory planning results for lane changing of vehicle on straight road, during a lane changing process, the linear motion displacement of the vehicle centroid \(M\) to \(O_{r}\) is just as the same as \(Y_d(t)\) at the time \(t\), so it is reasonable to design \(r(t)\) as the following form.

\[
r(t) = R + d/2 - Y_d(t)
\]  

(4)

According to Figure 3, along the directions of coordinate axes, the vehicle displacement, velocity and acceleration model for the lane changing behavior on circular road are deduced as follows:

\[
\begin{align*}
\dot{X}_c(t) &= [R + d/2 \cdot Y_d(t)]\sin \theta \\
\dot{Y}_c(t) &= R + d/2 - [R + d/2 \cdot Y_d(t)]\cos \theta \\
\ddot{X}_c(t) &= [R + d/2 - Y_d(t)]\dot{\theta}(t) \cos \theta - \dot{Y}_d(t) \sin \theta \\
\ddot{Y}_c(t) &= [R + d/2 - Y_d(t)]\dot{\theta}(t) \sin \theta + \dot{Y}_d(t) \cos \theta
\end{align*}
\]  

(5)

(6)

(7)

Accordingly, the desired velocity along the tangential directions of lane changing trajectory can be calculated as the following form.
The desired yaw angle and yaw rate are deduced as

\[ \psi_c(t) = \arctan \frac{\dot{Y}_c(t)}{\dot{X}_c(t)} \]  

\[ \psi_c(t) = \frac{\ddot{Y}_c(t)\dot{X}_c(t) - \ddot{X}_c(t)\dot{Y}_c(t)}{\dot{X}_c(t)^2 + \dot{Y}_c(t)^2} \]  

Due to the rotate angular displacement \( \theta(t) \) desired of vehicles around the instantaneous center \( O_r \) should meet the conditions such as continuity, smoothness and monotonic, in this paper, we assume it meets the quintic polynomial constraint, namely:

\[ \theta(t) = c_0 t^5 + c_1 t^4 + c_2 t^3 + c_3 t^2 + c_4 t + c_5 \]  

Boundary conditions of \( \theta(t) \) and its rate of change met can be determined bases on the vehicle’s states at the start time and the end time of the lane changing process, based on the constraints conditions for lane changing behavior, such as lane spacing and location requirement of road section, the multinomial coefficient of \( \theta(t) \) can be determined from the boundary conditions.

According to formula (1) and (5), the angular displacement constraints that \( \theta(t) \) met at the start and end time of lane changing can be determined as

\[
\begin{align*}
\theta(t_{on}) &= 0 \\
\theta(t_{off}) &= \alpha
\end{align*}
\]

According to formula (2) and (6), the angular velocity constraints that \( \dot{\theta}(t) \) met at the start and end time of lane changing can be determined as

\[
\begin{align*}
\dot{\theta}(t_{on}) &= \ddot{X}_d(t_{on})/(R + d/2) \\
\dot{\theta}(t_{off}) &= \ddot{X}_d(t_{off})/(R - d/2)
\end{align*}
\]

Furthermore, according to formula (3) and (7), the angular acceleration constraints that \( \ddot{\theta}(t) \) met at the start and end time of lane changing can be determined as

\[
\begin{align*}
\ddot{\theta}(t_{on}) &= [(R + d/2)\ddot{X}_d(t_{on}) + 2\dot{X}_d(t_{on})\dot{Y}_d(t_{on})]/(R + d/2)^2 \\
\ddot{\theta}(t_{off}) &= [(R - d/2)\ddot{X}_d(t_{off}) + 2\dot{X}_d(t_{off})\dot{Y}_d(t_{off})]/(R - d/2)^2
\end{align*}
\]

Let vector \( C = [c_0, c_1, c_2, c_3, c_4, c_5]^T \), from the boundary constraints conditions that \( \theta(t) \), \( \dot{\theta}(t) \) and \( \ddot{\theta}(t) \) met, i.e., (12), (13)and(14), by solving the following equation

\[
[\theta(t_{on}), \dot{\theta}(t_{on}), \ddot{\theta}(t_{on}), \theta(t_{off}), \dot{\theta}(t_{off}), \ddot{\theta}(t_{off})]^T = TC
\]

where

\[
T = \begin{bmatrix}
\xi_{on} & \eta_{on} & \zeta_{on} & \eta_{on} & \zeta_{on} & 1 \\
5\xi_{on} & 4\eta_{on} & 3\zeta_{on} & 2\eta_{on} & 1 & 0 \\
20\xi_{on} & 12\eta_{on} & 6\zeta_{on} & 2 & 0 & 0 \\
\xi_{off} & \eta_{off} & \zeta_{off} & \eta_{off} & \zeta_{off} & 1 \\
5\xi_{off} & 4\eta_{off} & 3\zeta_{off} & 2\eta_{off} & 1 & 0 \\
20\xi_{off} & 12\eta_{off} & 6\zeta_{off} & 2 & 0 & 0
\end{bmatrix}
\]

the undetermined coefficient involved in the multinomial (11) can be acquired.

**Simulation Results**

Assume that curvature radius of the road \( R = 200m \), the distance between the centerlines of lanes \( d = 3.5m \), the length of the centerline of the road corresponded with the lane changing
process $L = 80m$; the start time for lane changing $t_{on} = 0$, the end time $t_{off} = 4s$. the states of vehicle at the start time are $X_d(0) = 15m/s$, $\dot{X}_d(0) = 5m/s^2$, $Y_d(0) = 0.5m/s$, $\dot{Y}_d(0) = 0.2m/s$; the states of vehicle at the end time are $X_d(4) = 25m/s$, $\dot{X}_d(4) = 0$, $\dot{Y}_d(4) = 0$, $\ddot{Y}_d(4) = 0$. Fig. 3 shows the trajectory of the position during changing lanes, in which the dotted lines represent centerlines of two lanes; Fig. 4 denote the desired yaw angle of vehicle changing lanes.

**Summary**

Both the lane spacing, and the length constraint of the road section corresponds to the lane changing process are considered in this paper, the lane changing trajectory model designed here is more general than the existed research results of lane changing on circular road.

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