The CKM unitarity problem: A trace of new physics at the TeV scale?

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After the recent high precision determinations of $V_{us}$ and $V_{ub}$, the first row of the CKM matrix shows more than $4\sigma$ deviation from unitarity. Two possible scenarios beyond the Standard Model can be investigated in order to fill the gap. If a 4th quark $b'$ participates in the mixing, with $|V_{ub'}| \sim 0.04$, then its mass should be no more than 6 TeV or so. A different solution can come from the introduction of the gauge horizontal family symmetry acting between the lepton families and spontaneously broken at the scale of about 6 TeV. Since the gauge bosons of this symmetry contribute to muon decay in interference with Standard Model, the Fermi constant is slightly smaller than the muon decay constant so that unitarity is recovered. Also the neutron lifetime problem, that is about $4\sigma$ discrepancy between the neutron lifetimes measured in beam and trap experiments, is discussed in the light of the these determinations of the CKM matrix elements.

1. The Standard Model (SM) contains three fermion families in the identical representations of the gauge symmetry $SU(3) \times SU(2) \times U(1)$ of strong and electroweak interactions. One of its fundamental predictions is the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix of quark mixing in charged current

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (1)$$

Deviation from the CKM unitarity can be a signal of new physics beyond the Standard Model (BSM). The experimental precision and control of theoretical uncertainties in the determination of the elements in the first row of $V_{\text{CKM}}$ are becoming sufficient for testing the condition

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \quad (2)$$

Since $|V_{ub}| \approx 0.004$ is very small, its contribution is negligible and (2) reduces essentially to the check of the Cabibbo mixing: $|V_{us}| = \sin \theta_C$, $|V_{ud}| = \cos \theta_C$ and $|V_{us}/V_{ud}| = \tan \theta_C$. In essence, this is the universality test for the W-boson coupling $(g/\sqrt{2}) W^\mu_\gamma J^\mu_L + \text{h.c.}$ to the relevant part of the charged left-handed current

$$J^\mu_L = V_{ud} \bar{\nu}_L \gamma^\mu d_L + V_{us} \bar{\nu}_L \gamma^\mu s_L + \bar{\nu}_L \gamma^\mu e_L + \bar{\nu}_L \gamma^\mu \mu_L \quad (3)$$

For energies smaller than $W$-boson mass this coupling gives rise to the effective current $\times$ current interactions

$$-\frac{4G_F}{\sqrt{2}} \bar{\nu}_L (V_{ud} \gamma_\mu d_L + V_{us} \gamma_\mu s_L) (\bar{\nu}_L \gamma^\mu e_L + \bar{\nu}_L \gamma^\mu \mu_L) \quad (4)$$

which are responsible for lepton decays of the neutron, pions, kaons etc., as well as to the interaction

$$-\frac{4G_F}{\sqrt{2}} (\bar{\nu}_L \gamma_\mu e_L) (\bar{\nu}_L \gamma^\mu \mu_L) \quad (5)$$

2. The most precise determination of $|V_{ub}|$ is obtained from superallowed $0^- \rightarrow 0^+$ nuclear $\beta$-decays which are pure Fermi transitions sensitive only to the vector coupling constant $G_V = G_F |V_{ub}|$ [1]:

$$|V_{ub}|^2 = \frac{K}{2G^2_F \text{Fr} (1 + \Delta_R^V)} = 0.97147(20) \quad (6)$$

where $K = 2\pi^3 \ln 2/m^5 = 8120.2776(9) \times 10^{-10}$ s/GeV$^4$ and $\text{Fr}$ is the nucleus independent value obtained from

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the individual $f_\ell$-values of different $0^+ - 0^+$ nuclear transitions by absorbing in the latter all nucleus-dependent corrections, while $\Delta^V_R$ accounts for short-distance (transition independent) radiative corrections. For the second step, we take $F_1 = 3072.07(72) s$ obtained by averaging the individual $F_1$-values for fourteen superallowed $0^+ - 0^+$ transitions determined with the best experimental accuracy, and plug in the Fermi constant as $G_F = G_\mu = 1.1663787(6) \times 10^{-5} GeV^{-2}$ determined from the muon decay [3]. The major uncertainty is related to the so called inner radiative correction $\Delta^V_R$.

The element $|V_{us}|$ can be determined from analysis of semileptonic $K^3$ decays ($K_{L\mu3}$, $K_{L\epsilon3}$, $K^{+}e^-\epsilon$, etc.) [4]:

$$f_+(0)|V_{us}| = 0.21654 \pm 0.00041$$

where $f_+(0)$ is the $K \to \pi\nu$ vector form-factor at zero momentum transfer. On the other hand, by comparing the kaon and pion inclusive radiative decay rates $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$, one obtains [5]:

$$|V_{us}/V_{ud}| \times (f_K/f_\pi) = 0.27599 \pm 0.00038.$$  

Hence, the values $|V_{us}|$ and $|V_{us}/V_{ud}|$ can be independently determined using the lattice QCD results for the form-factor $f_+(0)$ and the decay constant ratio $f_K/f_\pi$.

3. Let us first consider the values of the CKM matrix elements $|V_{us}|$, $|V_{ud}|$ and their ratio $|V_{us}/V_{ud}|$ as quoted by Particle Data Group (PDG) review 2018 [5]:

$$|V_{us}| = 0.2238(8)$$

$$|V_{us}/V_{ud}| = 0.2315(10)$$

$$|V_{ud}| = 0.97420(21)$$

Here $|V_{us}|$ and $|V_{us}/V_{ud}|$ are obtained respectively from Eqs. (7) and (8) using the FLAG 2017 averages of 3–flavor lattice QCD simulations $f_+(0) = 0.9677(27)$ and $f_K/f_\pi = 1.192(5)$ [6]. $|V_{ud}|$ is obtained from Eq. (6) by taking $\Delta^V_R = 0.02361(38)$ as calculated in Ref. [7].

By imposing the CKM unitarity [2], the three data [9] reduce to three independent determinations of $|V_{us}|$. These determinations shown as A, B, C in upper panel of Fig. 1 (see also Table I for numerical values) are compatible within their error-bars [1]. Namely, B and C are almost equal while there is a modest tension (1.4σ) between A and B. Their average $A+B = 0.2245(6)$, practically coincides with the PDG 2018 average $|V_{us}|$ [5]. By averaging all three values we get $A+B+C = 0.2248(5)$ with $\chi^2_{\text{dof}} = 1.7$. Pulls of A, B and C relative to this average (given in Fig. 1) are compatible with a standard deviation. Summarizing, the dataset [9] adopted from PDG 2018 [5] is consistent with the CKM unitarity [2].

However, recent progress in the determination of the CKM elements allows to test the unitarity with improved precision. Significant redetermination of $|V_{ud}|$ is related to new calculation of inner radiative corrections with reduced hadronic uncertainties, $\Delta^V_R = 0.02467(22)$ [8]. Employing also the recent result $f_+(0) = 0.9696(18)$ from new 4–flavor ($N_f = 2+1+1$) lattice QCD simulations [9] and the FLAG 2019 four-flavor average $f_K/f_\pi = 1.1932(19)$ [10], one arrives to the following set [2]:

$$|V_{us}| = 0.22333(60)$$

$$|V_{us}/V_{ud}| = 0.23130(50)$$

$$|V_{ud}| = 0.97370(14)$$

This dataset, again by imposing the CKM unitarity, reduces to independent $|V_{us}|$ values A, B, C shown in lower panel of Fig. 1 (numerical values are given in Table I).

Now we see that the values A, B, C are in tensions among each other. Namely, there is a 5.3σ discrepancy between A and C, and 3.2σ between B and C. The tension between the determinations A and B, both from kaon physics, is 2.7σ. More conservatively, one can take their average $A+B$. The discrepancy of the latter with C is 4.5σ. Fitting these values, we get $A+B+C = 0.22546(31)$ but the fit is bad, $\chi^2_{\text{dof}} = 13.9$. C, A and A+B have large pulls, 3.9σ, −3.6σ and −2.3σ.

This tension can be manifested also by analyzing the data [10] in a different way. Without imposing the unitarity condition [2], we perform a two parameter fit of

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1 Throughout this paper A is the direct determination of $|V_{us}|$ obtained from Eq. [7]. B and C are the values of $|V_{us}|$ obtained respectively from $|V_{us}/V_{ud}|$ and $|V_{ud}|$ by assuming unitarity.

2 Alternatively, one could use the FLAG 2019 average $f_+(0) = 0.9706(27)$ [10] (not including result of Ref. [7]) leading to a minor change of $|V_{us}|$ in [10] from 0.22333(60) to 0.22310(75).
the three independent values $\{10\}$. In Fig. 2 we show the gaussian hill of the probability distribution with the confidence level (C.L.) contours around the best fit point ($|V_{us}| = 0.22449$, $|V_{ud}| = 0.97369$), with $\chi^2_{\text{min}} = 6.1$. (This $\chi^2$-value seems large for a two parameter fit, but it is dominated by the tension between the determinations A and B of $|V_{us}|$ from kaon data and perhaps this tension will disappear with more accurate lattice simulations.) The red solid line corresponding to the three family unitarity condition (1) and the dashed red line corresponds to the “extended” unitarity (11) with $|V_{ub'}| = 0.04$.

4. “If the Hill will not come to the CKM, the CKM will go to the Hill.” The unitarity line can be moved down towards the probability distribution hill in Fig. 2 if the unitarity condition is extended to more families. One can introduce, besides the three down quarks $d, s, b$, a 4th state $b'$ which is also involved in quark mixing. Then the first row unitarity condition will be modified to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 + |V_{ub'}|^2 = 1.$$  \hspace{0.5cm} (11)

In particular, the red dashed line in Fig. 2 passing through the best fit point on the top of the probability hill corresponds to $|V_{ub'}| = 0.04$ (at 95% C.L. this additional mixing is limited as $|V_{ub'}| = 0.04 \pm 0.01$). Plugging this value in Eq. (11), the dataset \(10\) gives the modified determinations of $|V_{us}|$ for the three cases named above as A, B and C (for numerical values see in 3rd column of Table \[\]). Clearly, the case A in this list remains the same as in 2nd column but B and especially C are shifted down. Fig. 3 shows that consistency between these values is significantly improved compared to lower panel of Fig. 1. The fit for $A + B + C$ is acceptable, $\chi^2_{\text{dof}} = 3$. Pulls of C and A+B are practically vanishing. There remains a tension between A and B but it is softened to 2.4$\sigma$ from 2.7$\sigma$ of Fig. 1.

Let us discuss now in which conditions one could obtain so large mixing with the 4th species, $|V_{ub'}| \approx 0.04$. In the SM the three families ($i = 1, 2, 3$ is the family index) of left-handed (LH) quarks $Q_i = (u_i, d_i)_L$ and leptons $\ell_L = (\nu_i, e_i)_L$ transform as weak isodoublets of $SU(2) \times U(1)$ and the right-handed (RH) quarks $u_R, d_R$ and leptons $e_R$ are the isosinglets. The existence of a fourth sequential family is excluded by the SM precision tests in combination with the direct limits from the LHC, but one can introduce additional vector-like fermions. Let us briefly sketch a vanilla picture of this type, adding just a vector-like couple of isosinglet down-type quarks $b'_L, b'_R$ having a large Dirac mass $M b' b'$, $b' = b'_L + b'_R$. In this way, we obtain the modified $3 \times 4$ matrix of the quark mixing in left-handed charged current:

$$\tilde{V}_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \end{pmatrix}.$$  \hspace{0.5cm} (12)

The condition (11) regards the first row of this matrix.\(^3\)

Without losing generality, the Yukawa terms can be divided in two parts. The first part

$$Y_{ui} \tilde{\phi} Q_{Li} u_R j + Y_{d'i} \tilde{\phi} Q_{Li} d_R j + Y_{e'i} \tilde{\phi} \ell_{Li} e_R j + \text{h.c.}$$  \hspace{0.5cm} (13)

comprises the SM Yukawa terms of three standard families with the Higgs doublet $\phi$, $Y_{u,d,e}$ being the Yukawa constant matrices and $\tilde{\phi} = i\tau_2\phi^*$.

The second part

$$h_i \tilde{\phi} Q_{Li} b'_R + M b'_L b'_R + \text{h.c.}$$  \hspace{0.5cm} (14)

involves the extra state $b'$. Fermion masses emerge from the vacuum expectation value (VEV) of the Higgs, $\langle \phi^0 \rangle = v_w = 174$ GeV (for a convenience, we use this

\(^3\) One can introduce also a fourth upper quark $t'$, so that two singles $b', t'$ would form a family in some sense, and the mixing matrix (13) would become a $4 \times 4$ matrix. However, this modification of the minimal picture is irrelevant since $t'$ will have no impact on the first row unitarity. In addition, it can be easily shown that introduction of fourth vector-like isodoublet family $Q'_{L,R} = (t', b')_{L,R}$ cannot generate large enough mixing $V_{ub'}$.\)
normalization of the Higgs VEV instead of “standard” normalization $\langle \phi \rangle = v/\sqrt{2}$, i.e. $v = \sqrt{2v_\mu}$.

Without loss of generality, the matrix $Y_d$ can be chosen diagonal, $Y_d = Y_d^{\text{diag}} = \text{diag}(y_d, y_s, y_b)$. The Yukawa terms in (14) induce the mixing of three known quarks $d, s, b$ to the 4th quark $b'$. Thus, $4 \times 4$ mass matrix of all down-type quarks has a form:

$$
\begin{pmatrix}
Y_{d_{1}w} & 0 & 0 & h_{d_{2}w} \\
0 & Y_{s_{1}w} & 0 & h_{s_{2}w} \\
0 & 0 & Y_{b_{1}w} & h_{b_{2}w} \\
0 & 0 & 0 & 0
\end{pmatrix}
$$

(15)

In this basis, the up quark Yukawa matrix is non-diagonal, $Y_u = V_{uL} Y_{u}^\text{diag} V_{uL}^\dagger$, $Y_{u}^{\text{diag}} = \text{diag}(y_u, y_c, y_t)$, where $3 \times 3$ unitary matrix $V_{uL}$ in fact determines the ordinary three-family part (1) of the quark mixing, i.e. $V_{uL}^2 = V_{\text{CKM}}$. Then $3 \times 4$ extended mixing matrix $V_{\text{CKM}}$ (2) is completed by diagonalization of the matrix (15). Namely, the off-diagonal terms in (15) determine the mixing of $d, s, b$ with $b'$, $V_{u1b'} \simeq h_{d_{2}w}/M$, $V_{u2b'} \simeq h_{s_{2}w}/M$ and $V_{u3b'} \simeq h_{b_{2}w}/M$ which values are generically complex. This mixing practically does not affect the diagonal elements in (15). Hence, $m_{d,s,b} = Y_{d,s,b}^\text{diag}$ and $m_{b'} = M$.

The LHC limit on extra $b'$ mass $M > 880$ GeV [4] implies that $|V_{ub'}| \simeq 0.04$ can be obtained if $h_d > 0.2$, much larger than the Yukawa constant $y_t$. In turn, by taking $|V_{ub'}| > 0.03$ in $M = \sqrt{h_d |V_{ub'}|}$, and assuming (for the perturbativity) $h_d \lesssim Y_t \lesssim 1$, we get an upper limit on the extra quark mass, $M < 6$ TeV or so.

The extension of the SM by adding an extra isosinglet quark $b'$ generates significant contributions in flavor-changing processes. E.g., $Wb'$ box diagram shown in Fig. 4 induces $D^0 - \bar{D}^0$ mixing. For $|V_{ub'}| \simeq 0.04$, its contribution would exceed the experimental value of their mass splitting, $\Delta M \simeq 10^{10}$ s$^{-1}$, unless $|V_{cb'}/V_{ub'}| \times (M/1$ TeV $) < 1/3$ or so. In addition, as one can see from (14), the 4th quark has tree level flavor-changing couplings with the Higgs bosons $H$ and also with $Z$-boson:

$$
\frac{M}{\sqrt{2} v_w} H (V_{ub'} d_L + V_{cb'} s_L + V_{tb'} b_L) b_R' + \text{h.c.}
$$

$$
\frac{g}{2c_W} Z (V_{ub'} d_L + V_{cb'} s_L + V_{tb'} b_L) \gamma^\mu b_L' + \text{h.c.}
$$

(16)

Then the $Zb'$ box diagram shown in Fig. 4 contributes to $K^0 - \bar{K}^0$ mixing. Interestingly, for $|V_{ub'}| \simeq 0.04$ this new contribution in CP-violating $\epsilon_K$ parameter would be larger than the SM one unless $\arg(V_{cb'}/V_{ub'}) \times |V_{ub'}/V_{ub}| \times (M/1$ TeV $) < 1/10$ or so. For $|V_{ub'}| \sim |V_{ub}| = 0.04$, the analogous $Zb'$ box diagram with external $b$ quark would give a contribution to $B_d - \bar{B}_d$ meson mixing comparable to the SM contribution.

These flavor-changing and CP-violating effects can be suppressed if $V_{cb'}$ and $V_{ub'}$ are much less than $V_{ub}$, or at least have rather small complex parts. (Accidentally, $|V_{ub'}| \simeq 0.04$ is comparable to $|V_{cb'}|$ and ten times larger than $|V_{ub}|$.) The picture with the 4th state $b'$ having a larger mixing with the first family than with (heavier) 2nd and 3rd families looks somewhat ad hoc, but it is not excluded by the present experimental limits. The implications of a TeV scale extra quark $b'$ with significant $|V_{ub'}|$ mixing deserve careful analysis.

5. “But what if the Hill comes to the CKM?” Here we discuss just the opposite possibility: instead of moving the unitarity line to the probability distribution Hill in Fig. 2, we move the Hill towards the unitarity line.

Namely, we consider that the Fermi constant $G_F$ in the effective interaction (4) which is responsible for leptonic decays of hadrons can be different from the effective constant $G_{\mu}$ determined from the muon lifetime. We assume that besides the SM interaction (5) mediated by charged $W$-boson, there is also a new operator:

$$
- \frac{4G_F}{\sqrt{2}} (\overline{\nu_L} \gamma_\mu \mu_L)(\overline{\nu_R} \gamma^\mu \nu_e)
$$

(17)

mediated by a hypothetical lepton flavor changing neutral gauge boson $F$. The respective diagrams, shown in Fig. 5, have positive interference for the muon decay. Namely, by Fierz transformation this new operator can be brought to the form (5), so that the sum of these two diagrams effectively gives the operator

$$
- \frac{4G_{\mu}}{\sqrt{2}} (\overline{\nu_L} \gamma_\mu \mu_L)(\overline{\nu_R} \gamma^\mu \nu_e),
$$

(18)

the same as (5) but with the coupling constant

$$
G_{\mu} = G_F + G_{F} = G_F (1 + \delta_{\mu}), \quad \frac{G_F}{G_{\mu}} \equiv \delta_{\mu} > 0.
$$

(19)

Constant $G_{\mu} = 1.1663787(6) \times 10^{-5}$ GeV$^{-2}$ is determined with great precision from the muon decay [3]. Now Eqs.
6 and 4, instead of $|V_{ud}|$ and $|V_{us}|$, are determined respectively the values $|V_{ud}| \times G_F/G_H$ and $|V_{us}| \times G_F/G_H$. Instead the value of $|V_{us}/V_{ad}|$ determined from (8) remains unchanged since the Fermi constant cancels out.

Thus, under our hypothesis, the dataset (10) should be modified to the following:

$$
|V_{us}| = 0.22333(60) \times (1 + \delta_\mu)
$$

$$
|V_{us}/V_{ad}| = 0.23130(50)
$$

(20)

$$
|V_{ad}| = 0.97370(14) \times (1 + \delta_\mu)
$$

Now, involving the extra parameter $\delta_\mu$ but assuming the 3-family unitarity [2], the fit of the above dataset has acceptable quality, $\chi^2 = 6.1$, and the best fit point corresponds to $\delta_\mu = 0.00076$. This situation is shown in Fig. 6 in which the values of $|V_{us}|$ and $|V_{us}/V_{ad}|$ are determined by taking $\delta_\mu = 0.00076$. By this choice of the extra parameter the fit becomes perfectly compatible with the unitarity [2]. The probability distribution Hill is moved up so that its top now lies on the unitarity line.

By imposing the unitarity condition $|V_{ud}|^2 + |V_{us}|^2 = 1 - |V_{ad}|^2$, the list of dependent determinations A, B, C of $|V_{us}|$. Fig. 6 shows these determinations for $\delta_\mu = 0.00076$. Taking into account that $G_F/\sqrt{2} = g^2/8M_W^2 = 1/4v_w^2$, where $v_w = 174$ GeV is the weak scale, and parametrizing similarly $G_F/\sqrt{2} = 1/4v_F^2$, we see that $\delta_\mu = G_F/G_F = 0.00076$ corresponds to $v_F/v_w = 36.3$, or to the flavor symmetry breaking scale $v_F = 6.3$ TeV. More widely, the range of $\delta_\mu$ consistent with unitarity at the 68% C.L. is $\delta_\mu = (7.6 \pm 1.6) \times 10^{-4}$ which corresponds to the new scale in the interval $v_F = [5.7 \div 7.1]$ TeV.

6. The non-abelian gauge horizontal flavor symmetry $G_H$ between the fermion families can be the key for understanding the quark and lepton mass and mixing pattern [11][12]. Namely, the form of the Yukawa matrices $Y_{u,d,e}$ in [13] can be determined by the $G_H$ symmetry breaking pattern, i.e. by the VEV structure of the horizontal scalar fields (flavons) responsible for this breaking.

Then the fermion mass hierarchy is related to the hierarchy between these VEVs. In Refs. [11] this conjecture was coined as hypothesis of horizontal hierarchies. In this picture the fermion masses emerge from the higher order operators involving, besides the Higgs doublet $\phi$, also flavon scalars which transfer their VEV structure to the Yukawa matrices $Y_{u,d,e}$. These so called “projective” operators in the UV-complete renormalizable theory can be obtained via integrating out some extra heavy fields, scalars [12] or vector-like fermions [11]. In particular, this concept implies that the fermion masses cannot emerge if $G_H$ symmetry is unbroken. Thus, $G_H$ cannot be a vector-like symmetry but it should have a chiral character transforming the LH and RH particle species in different representations. In particular, in Refs. [11][14] the horizontal symmetry $G_H$ was considered as $SU(3)_H$ with the LH fermions of the three families transforming as triplets and the RH ones as anti-triplets, as it is motivated by the grand unification.

However, in the Standard Model framework one has more possibilities. Namely, in the limit of vanishing Yukawa couplings $Y_{u,d,e} \to 0$ in [13], the SM Lagrangian acquires a maximal global chiral symmetry $U(3)_Q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$, under which fermion species $Q$, $u$ etc. transform as triplets of independent $U(3)$ groups. It is tempting to consider that the non-abelian $SU(3)$ factors of this maximal flavor symmetry are related to gauge symmetries [5].

Let us concentrate on the lepton sector and discuss the gauge symmetry $SU(3)_\ell \times SU(3)_e$ [10] under which the LH and RH lepton fields transform as

$$
\ell_{L\alpha} = \left( \frac{\nu_\alpha}{e_\alpha} \right)_L \sim (3_\ell, 1), \quad e_{R\gamma} \sim (1, 3_e)
$$

(21)

where $\alpha = 1, 2, 3$ and $\gamma = 1, 2, 3$ are the indices of $SU(3)_\ell$ and $SU(3)_e$ respectively. This set of fermions is not
anomaly free. The ways of the anomaly cancellation were discussed in Ref. [10] and in this letter we shall not concentrate on this issue.

For breaking $SU(3)_c \times SU(3)_c$ we introduce flavon fields, three triplets $\eta_{\alpha}$ of $SU(3)_c$ and three triplets $\xi_{\gamma}$ of $SU(3)_c$, $i = 1, 2, 3$. Then the charged lepton masses emerge from the gauge invariant dimension-6 operator

$$\frac{y_{ij}}{M^2} \eta_{i\alpha} \bar{\eta}_{j\beta} \phi \ell_{\alpha \gamma} \epsilon_{\beta \gamma} + \text{h.c.} \tag{22}$$

where $y_{ij}$ are order one constants, $\phi$ is the Higgs doublet and $M$ is a cutoff scale. In an UV-complete theory such operators can be induced via see-saw-like mechanism by integrating out some heavy scalar or fermion states [11, 12]. However, concrete model building is not the scope of this paper, and for our demonstration effective operator analysis is sufficient. As for the neutrinos, their Majorana masses are induced by the higher order operator

$$\frac{h_{ij}}{M^2} \eta_{i\alpha} \bar{\eta}_{j\beta} \phi \ell_{\alpha \beta} C \ell_{\beta} + \text{h.c.} \tag{23}$$

where $h_{ij} = h_{ji}$. The cutoff scale $M_\nu$ of this operator is not necessarily the same as the scale $M$ of operator (22).

In order to generate non-zero masses of all three leptons $e, \mu, \tau$, all three $SU(3)_c$ flavons $\eta_i$ as well as $SU(3)_c$ $\xi_i$ should have non-zero VEVs with disoriented directions. This means that the VEVs $\langle \eta_{\alpha} \rangle$ should form a rank-3 matrix. Without losing generality, the flavon basis can be chosen so that the matrix $\langle \eta_{\alpha} \rangle$ is diagonal: $\langle \eta_{\alpha} \rangle = w_i \delta_{i\alpha}$, i.e. the flavon VEVs are orthogonal:

$$\langle \eta_1 \rangle = \begin{pmatrix} w_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \eta_2 \rangle = \begin{pmatrix} 0 \\ w_2 \\ 0 \end{pmatrix}, \quad \langle \eta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ w_3 \end{pmatrix} \tag{24}$$

Analogously, for $\xi$-flavons we take $\langle \xi_{\gamma} \rangle = v_\gamma \delta_{\gamma \gamma}$. After plugging these VEVs into (22) we obtain the leptonic Yukawa matrices in the SM Lagrangian [13] as

$$Y_{c}^{ij} = \frac{y_{ij}}{M^2} w_i v_j$$

Since the couplings (22) should give the lepton mass hierarchy, we consider that the latter emerges due to the VEV hierarchy $v_3 \gg v_2 \gg v_1$ in $SU(3)_c$ symmetry breaking, i.e. $v_3 : v_2 : v_1 \sim m_\tau : m_\mu : m_e$ as it is described in Ref. [10]. On the other hand, operator (23) should give the observed neutrino mass pattern, $m_\nu^2 = h_{ij} w_i w_j v_i^2 / M^2$, and in particular the large neutrino mixing. This implies that $SU(3)_c$ breaking flavons $\eta_i$ should have comparable VEVs, $v_3 \sim v_2 \sim v_1$.

Gauge bosons $F_a$ of $SU(3)_c$, associated to the Gell-Mann matrices $\lambda_\alpha$, $\alpha = 1, 2, \ldots, 8$, interact as $g F_a^\mu J_{a\mu}^\nu$, with the respective currents $J_{a\mu} = J_{a\mu}^{(\nu)} + J_{a\mu}^{(\rho)} = \frac{1}{2} \epsilon_{\nu \rho \lambda} \lambda_a e_L + \frac{i}{2} \epsilon^{\nu \rho \lambda} \lambda_a \gamma_{\mu} \nu_L$, where $g$ is the gauge coupling constant, $e_L = (e_1, e_2, e_3)_L^T$ and $e_L = (v_1, v_2, v_3)_L^T$ respectively denote the family triplets of the LH charged leptons and neutrinos.

At low energies these couplings induce four-fermion (current × current) interactions:

$$L_{\text{eff}} = -\frac{g^2}{2} J_{a\mu}^\nu (M^2)^{-1} J_{a\rho}^\nu$$

where $M_a^2$ is the squared mass matrix of gauge bosons $F_a$ which in the flavon VEV basis (24) is essentially diagonal apart of a non-diagonal $2 \times 2$ block related to $F_\mu^\nu - F_\mu^\nu$ mixing. Namely, the masses of $F_{\mu,
u}^1, F_{\mu,
u}^4, 5$ and $F_{\mu,
u}^6, 7$ are

$$M_{1,2}^2 = \frac{g^2}{2} (w_2^2 + w_1^2) \approx \frac{g^2}{2} v_2$$

$$M_{3,5}^2 = \frac{g^2}{2} (w_3^2 + w_1^2), \quad M_{6,7}^2 = \frac{g^2}{2} (w_3^2 + w_2^2).$$

As for $F_\mu^3$ and $F_\mu^8$ they have a mass mixing and their mass matrix reads

$$M_{3,8}^2 = \frac{g^2}{2} \left( \frac{w_2^2 + w_1^2}{\sqrt{2}} - \frac{\sqrt{3}}{4} (w_3^2 + w_1^2 + w_2^2) \right)$$

Notice that if $w_1 = w_2 = v_\nu / \sqrt{2}$, this matrix becomes diagonal. In the following, for the simplicity of our demonstration, we analyze this case. Then for the gauge boson masses we have $M_a^2 = (g^2/2)(x_a v_a)$, where

$$x_{1,2,3}^2 = 1, \quad x_{4,5,6,7}^2 = \frac{r + 1}{2}, \quad x_8^2 = \frac{2r + 1}{3} \tag{29}$$

and $r = 2w_3^2 / v_2^2$. Then operators (26) can be rewritten as $L_{\text{eff}} = L_{\text{eff}}^{\text{ee}} + L_{\text{eff}}^{\text{ee}} + L_{\text{eff}}^{\text{ee}}$ where

$$L_{\text{eff}}^{\text{ee}} = -\frac{2G_F}{\sqrt{2}} \sum_{a=1}^8 \left( \epsilon_{L}^a \gamma_\mu \lambda_a x_a \right) \left( \epsilon_{R}^a \gamma_\mu \lambda_a \nu_L \right)$$

$$L_{\text{eff}}^{\text{ee}} = -\frac{G_F}{\sqrt{2}} \sum_{a=1}^8 \left( \epsilon_{L}^a \gamma_\mu \lambda_a x_a \right)^2 \tag{30}$$

$$L_{\text{eff}}^{\text{ee}} = -\frac{G_F}{\sqrt{2}} \sum_{a=1}^8 \left( \epsilon_{R}^a \gamma_\mu \lambda_a \nu_L \right)^2$$

where $4G_F / \sqrt{2} = 1 / v_\nu$. Obviously, the factor $g^2 / 2$ in operators cancels out and the strength of these operators is determined solely by the VEVs (24).

The first term $L_{\text{eff}}^{\text{ee}}$ contains operator (17) which contributes to the muon decay $\mu \rightarrow e \nu_\mu \nu_\tau$ as $G_{\mu} = G_F G_F$. It is induced by exchange of gauge bosons $F_a^\mu$ and $F_b^\mu$, or more precisely by the combination $\left( F_a^\mu \pm iF_b^\mu \right) / \sqrt{2}$, as in second diagram of Fig. [5]. As it was pointed out in previous section, for restoring the CKM unitarity one needs $\delta_\mu = G_F / G_F = (v_\mu / v_F)^2$ to be around $7 \times 10^{-4}$ which corresponds to the flavor scale $v_F \approx 6 \div 7$ TeV.

5 Similar analysis can be done also for a general case $w_1 \neq w_2$, along the lines of Ref. [10] where such analysis was done for the RH gauge sector $SU(3)_c$. 

6
The similar operators in $\mathcal{L}_{\text{eff}}^{\text{SM}}$ mediated by the gauge bosons $F_{\mu}^a$ and $W_{\mu}^a$ contribute to the tau lepton decays $\tau \rightarrow \nu_\tau \bar{\nu}_e$ and $\tau \rightarrow \nu_\tau \bar{\nu}_\mu$. Then, in the case $w_1,2,3 \sim v_F$ but $w_1 \neq w_2$, the branching ratio $\Gamma(\tau \rightarrow \nu_\tau \bar{\nu}_e)/\Gamma(\tau \rightarrow e\nu_\tau \bar{\nu}_\mu)$ can have up to $O(10^{-3})$ deviation from the SM prediction 0.9726 which can be experimentally testable. (For a comparison, the present experimental value of this ratio is 0.9752(28)\cite{5}.) In addition, in $\mathcal{L}_{\text{eff}}^{\text{SM}}$, the terms with the “diagonal” generators $\lambda_3$ and $\lambda_8$ give rise also the non-standard neutrino interactions with leptons with coupling constants $\sim G_F = \delta_\mu G_F$, well below the experimental constraints.

The last term $\mathcal{L}_{\text{eff}}^{\text{SM}}$ in (30) contains the non-standard interactions between neutrinos, but present experimental limits on such interactions are rather weak. On the other hand, the second term in (30) containing charged leptons in principle is testable for the scale $v_F$ of several TeV.

Interestingly, if the flavor eigenstates $e_1, e_2, e_3$ are the mass eigenstates $e, \mu, \tau$, the terms (30) do not contain any LFV operators inducing processes like $\mu \rightarrow 3e$, $\tau \rightarrow 3\mu$ etc. However, the lepton flavor-conserving contact operators $\frac{2}{\lambda^3} (\tilde{e}L_i \gamma_{\mu} \mu L_j) \tilde{\nu}_L \mu \nu_L \lambda \nu_L \mu \nu_L$, etc. are restricted by the “compositeness” limits $\lambda_{\Lambda}(eeee) > 10.3$ TeV and $\lambda_{\Lambda}(ee\mu\mu) > 9.5$ TeV. Comparing these operators with the corresponding terms in (30) and taking into account the relations (29), the ‘compositeness’ scales can be expressed in terms of the scale $v_F$. Hence, we obtain the limit

$$v_F > \left(\frac{r + 1}{r + 0.5}\right)^{1/2} \times 2.1 \text{ TeV}.$$  \hspace{1cm} (31)

Here the $r$-dependent pre-factor approaches 1 when $r \gg 1$ and it becomes $\sqrt{2}$ in the opposite limit $r \ll 1$. Thus, the strongest limit emerges in the latter case, $v_F > 3$ TeV or so, which is anyway fulfilled for our benchmark range $v_F \simeq (6 \div 7)$ TeV.

The flavor eigenstates $e_1, e_2, e_3$ coincide with the mass eigenstates $e, \mu, \tau$, if the Yukawa matrix $Y_i^\mu$ in (25) is diagonal. This can be achieved by imposing some additional discrete symmetries between the flavons $y_i$ and $\xi_i$ of $SU(3)_L$ and $SU(3)_e$ sectors which would forbid the non-diagonal terms $y_{ij}$ in operator (22). However, in general case the initial flavor basis of the LH leptons is related to the mass basis by the unitary transformation

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}_L = U_L \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}, \quad U_L = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} \\ U_{2e} & U_{2\mu} & U_{2\tau} \\ U_{3e} & U_{3\mu} & U_{3\tau} \end{pmatrix}.$$  \hspace{1cm} (32)

Then, in the basis of mass eigenstates, the operators $\mathcal{L}_{\text{eff}}^{\text{SM}}$ read as in (30) but with the substitution $\lambda_{3a}/x_a \rightarrow U^\dagger (\lambda_{3a}/x_a) U$. Interestingly, in the limit $r = 1$, i.e. when the VEVs $w_1,2,3$ are equal and so $x_a = 1$, all flavor bosons $F_{a}^\mu$ have equal masses, and the substitution $\lambda_{3a} \rightarrow U^\dagger \lambda_{3a} U$ is simply a basis redefinition of the Gell-Mann matrices. Therefore, no LFV effects will emerge in this case since the global $SO(8)_e$ symmetry acts as a custodial symmetry. Namely, by Fierz transformations, using also the Fierz identities for the Gell-Mann matrices, we obtain

$$- \frac{G_F}{\sqrt{2}} \sum_{a=1}^{8} (\bar{e}_L \gamma_\mu \lambda_a e_L)^2 = - 4 \frac{G_F}{3 \sqrt{2}} (\bar{e}_L \gamma_\mu e_L)^2$$ \hspace{1cm} (33)

Obviously, the latter expression is invariant under the unitary transformation (32).

In general case $r \neq 1$, the mixing (32) gives rise to the LFV operators as e.g. the one inducing $\mu \rightarrow 3e$ decay:

$$- \frac{4G_{\mu e e c}}{\sqrt{2}} (\bar{e}_L \gamma_\mu \mu L)(\bar{\tau} \gamma_\mu e_L) + \text{h.c.},$$

$$\frac{4G_{\mu e e c}}{\sqrt{2}} = C(r) \left[ 1 + \frac{1 - r}{r} |U_{3e}|^2 \right] U_{3e}^* T_{3\mu} ,$$  \hspace{1cm} (34)

where the function $C(r) = (r - 1)\p((r + 1)(r + 0.5))^{-1}$ is limited as $|C(r)| < 1$, reaching the maximal value at $r > 1$, and it vanishes at $r = 1$. Then, taking $|U_{3e}| < 1$, we obtain for the branching ratio of $\mu \rightarrow 3e$ decay

$$\Gamma(\mu \rightarrow e e e) = \frac{1}{2} \left| \frac{\mu_{\text{eff}}}{G_F} \right|^2 = \frac{1}{8} (\delta_\mu C(r) |U_{3e}^* T_{3\mu}|)^2$$ \hspace{1cm} (35)

The experimental upper bound on this branching ratio is $10^{-12}$\cite{5}. Taking $\delta_\mu = (v_\mu/v_F)^2 = 7 \times 10^{-4}$, the limit $\delta_\mu |CU_{3e}^* T_{3\mu}|/\sqrt{8} < 10^{-6}$ translates into $|CU_{3e}^* T_{3\mu}| < 0.4 \times 10^{-2}$ which is nicely satisfied if the lepton mixing angles in (32) are comparable with the CKM mixing angles in (1) or even larger. E.g. if the VEF ratio is in between $r = 0.5 \div 1.5$, then $|C(r)| < 1/7$ so that $|U_{3e}^* T_{3\mu}| < (1/6)^2$ or so would suffice for properly suppressing the $\mu \rightarrow 3e$ decay rate. This means that in this case the matrix elements $|U_{3e}|$ and $|U_{3e}|$ can be almost as large as the Cabibbo angle $\sin \theta_W = V_{us}$. The experimental limits on other LFV effects as e.g. $\tau \rightarrow 3\mu$ are weaker, and following the lines of Ref.\cite{6}, one can show that in our model with $v_F \approx 6$ TeV or so, they are fulfilled even for whatever large mixings in (32). Once again, for $r = 1$ all LFV effects are vanishing owing to custodial symmetry, see Eq. (33).

7. Let us discuss briefly how the hypothesis $G_\mu \neq G_F$ could affect the SM precision tests. In the SM, at tree level, the weak gauge boson masses are $M_W = g v_w/\sqrt{2} = e v_w/\sqrt{2} \sin \theta_W$ and $M_Z = M_W/\cos \theta_W$ where $\theta_W$ is the weak angle. For precision tests the radiative corrections are important which depend also on the top quark and Higgs mass.

The world averages of experimentally measured masses of $Z$ and $W$ reported by PDG 2018 are \cite{5}:

$$M_{\text{exp}}^{Z} = 91.1876(21) \text{ GeV},$$

$$M_{\text{exp}}^{W} = 80.379(12) \text{ GeV},$$  \hspace{1cm} (36)

while the SM global fit yields to the following values:

$$M_{\text{SM}}^{Z} = 91.1884(20) \text{ GeV},$$

$$M_{\text{SM}}^{W} = 80.358(4) \text{ GeV}.$$  \hspace{1cm} (37)
Hence, the theoretical and experimental values of $Z$-mass are in perfect agreement while for $W$-boson the two values have about 1.6σ discrepancy:

$$M_W^{\text{exp}} - M_W^{\text{SM}} = (21 \pm 13) \text{ MeV} \quad (38)$$

In the SM the mass of $W$-boson, including radiative corrections, is determined as

$$M_W = \frac{A_0}{s_Z(1 - \Delta t_W)^{1/2}} \quad (39)$$

where $A_0 = (\pi \alpha/\sqrt{2} G_F)^{1/2} = 37.28039(1) \text{ GeV}$ taking $G_F = G_\mu$, the factor $1 - \Delta t_W = 0.93084(8)$ includes the main radiative corrections and $s_Z^2 = 1.0348(2)s_W^2$ is the corrected value of $\sin^2 \theta_W(M_Z)$ by including the top and Higgs mass dependent corrections. The theoretical mass $M_W = 80.358(4) \text{ GeV}$ \cite{37} is then obtained by substituting in $A_0$ of the value $s_Z^2 = 0.23122(3)$ obtained from the SM global fit \cite{3}. In our scenario, however, $G_F \neq G_\mu$. Should we set $A_0$ instead of $G_F = G_\mu$ the “corrected” value $G_F = (1 + \delta_\mu)^{-1} G_\mu$, then $A_0$ should be rescaled by a factor $(1 + \delta_\mu)^{1/2}$, and correspondingly the “theoretical” value of $M_W$ \cite{39} too. In particular, for $\delta_\mu = 7 \times 10^{-4}$ we would get $M_W = 80.386 \text{ GeV}$, right in the ball-park of the experimental values \cite{37}. However, this is not the right thing to do.

In the global fit of SM $M_Z$ is one of the input parameters with smallest experimental errors, along with the fine structure constant $\alpha$ and the “muon” Fermi constant $G_\mu$. Essentially, this is the main reason of the good coincidence between $M_Z^{\text{exp}}$ and $M_Z^{\text{SM}}$. In fact, the SM implies the relation

$$M_Z = \frac{M_W}{\hat{c}_Z^2 \hat{\rho}_Z^{1/2}} = \frac{A_0}{s_Z^2(1 - \Delta t_W)^{1/2} \hat{\rho}_Z^{1/2}} \quad (40)$$

where $\hat{\rho} = 1 + \rho_t + \delta_\rho = 1.01013(5)$ includes the weak isospin breaking effects, dominantly from the quadratic $m_t$ dependent corrections $\rho_t = 3G_F m_t^2 / 8\sqrt{2}\pi^2$. Therefore, taking the experimental value of $Z$-mass \cite{36}, Eq. \eqref{40} can be used for determination of $s_Z^2$ parameter, $s_Z^2 = 0.23123(3)$. This, in turn, from $M_W = M_{Z\hat{\rho}}^{1/2} \hat{c}_Z$ gives $M_W = 80.357(4)_{\text{SM}} \text{ GeV}$, i.e. practically the same as the global fit result \cite{37}. This is because the determination of the parameter $s_Z^2$ in the SM global fit is dominated by the results of $Z$-pole measurements.

However, in our scenario rescaling $A_0 \to A_0(1 + \delta_\rho)^{1/2}$ changes the value of $s_Z^2$. In particular, taking $\delta_\rho = (7.6 \pm 1.6) \times 10^{-4}$, we get $s_Z^2 = 0.23148(3)_{\text{SM}} s_\mu$. Then, again from $M_W = M_{Z\hat{\rho}}^{1/2} \hat{c}_Z$, we get $M_W = 80.344(4)_{\text{SM}} s_\mu \text{ GeV}$. Thus, unfortunately, while the effect is there, in reality it goes right to the opposite direction. So, our determination of $M_W$ differs from $M_W^{\text{SM}}$, $M_W^{\text{SM}} - M_W^{\text{exp}} = (13 \pm 3) \text{ MeV}$, but, with $M_W^{\text{SM}}$ already being in tension with the experimental value \cite{36}, our result has more tension: $M_W^{\text{exp}} - M_W^{\text{SM}} = (35 \pm 13) \text{ MeV}$ \cite{270}. However, let us remark that the tension with the latest results of ATLAS $M_W^{\text{ATL}} = 80.370(19)$ is less, $M_W^{\text{ATL}} - M_W^{\text{SM}} = (26 \pm 20) \text{ MeV} \quad (1.3\sigma)$. If the tension will increase with future precision, this would mean that one has to admit at least some minimal step beyond the SM.

The relation between $W$ and $Z$ masses can be improved by increasing of $\rho$-parameter via e.g. the VEV of a scalar triplet of the electroweak $SU(2) \times U(1)$, or by diminishing $Z$ mass by few MeV e.g. via its mixing with some extra gauge bosons like $Z'$ or perhaps also with the flavor gauge bosons considered in the previous section.

8. The value $|V_{ud}|$ can be extracted also from free neutron decay by combining the results on the measurements of the neutron lifetime $\tau_n$ with those of the axial current coupling constant $g_A = G_A / G_V$. The master formula reads (see e.g. in a recent review \cite{17}):

$$|V_{ud}|^2 = \frac{K / \ln 2}{G_F^2 F_n \tau_n (1 + 3g_A^2) (1 + \Delta_Y^V)}$$

$$= \frac{5024.46(30) \text{ s}}{\tau_n (1 + 3g_A^2) (1 + \Delta_Y^V)} \quad (41)$$

where $F_n = f_n(1 + \delta_f^\prime)$ is the neutron $f$-value $f_n = 1.6887(1)$ corrected by the long-distance QED correction $\delta_f^\prime = 0.01402(2) \quad (18)$. This equation, taking the values $\tau_n = 880.2 \pm 1.0 \text{ s}$ and $g_A = 1.2724 \pm 0.0023$ adopted in PDG 2018 \cite{15}, would give the value $|V_{ud}| = 0.97577(5)_{\text{PDG}}(146)_{\text{PDG}}(18)_{\Delta_Y^V} = 0.97577(157)$. It is compatible with $|V_{ud}| = 0.97370(10)_{\text{PDG}}(10)_{\Delta_Y^V} = 0.97370(14)$ obtained from \cite{6} and used in \cite{3}, but has an order of magnitude larger error.

However, rather than for determination of $|V_{ud}|$, Eq. \eqref{41} can be used for a consistency check. Namely, by comparing it with Eq. \eqref{6} we get a relation between $\tau_n$ and $g_A$ \cite{19}:

$$\tau_n = \frac{2F_l}{\ln 2 F_n (1 + 3g_A^2)} = \frac{5172.0(1.1) \text{ s}}{1 + 3g_A^2} \quad (42)$$

In Fig. 8 this relation is shown by the red band. This formula is very accurate since the common factors in Eqs. \eqref{6} and \eqref{41} cancel out, including the Fermi constant and radiative corrections $\Delta_Y^V$.

For the axial current coupling $g_A$, the PDG 2018 quotes a value $g_A = 1.2724 \pm 0.0023$. However, the results of the latest and most recent experiments \cite{20, 22} which measured $\beta$-asymmetry parameter using different techniques (the cold neutrons in PERKEO II and PERKEO III experiments \cite{20, 22} and ultra-cold neutrons in the UCNA experiment \cite{21}), are in perfect agreement among each other, and their average determines the axial current coupling $g_A$ with impressive (better than one per mille) precision:

$$g_A = 1.27625 \pm 0.00050. \quad (43)$$

Fig. 8 shows the results of Refs. \cite{20, 22} and their average (vertical grey band). For $g_A$ in this range Eq. \eqref{12} gives the Standard Model prediction for the neutron lifetime $\tau_n^{\text{SM}} = 878.7 \pm 0.6 \text{ s} \quad (44)$.
From the experimental side, the neutron lifetime is measured in two types of experiments. The trap experiments measure the disappearance rate of the ultra-cold neutrons (UCN) by counting the survived neutrons after storing them for different times in the UCN traps and determine the neutron decay width $\Gamma_n = \tau_n^{-1}$. The beam experiments are the appearance experiments, measuring the width of $\beta$-decay $n \to pe^+\nu_e$, $\Gamma_\beta = \tau_\beta^{-1}$, by counting the produced protons in the monitored beam of cold neutrons. In the Standard Model the neutron decay should always produce a proton, and so both methods should measure the same value $\Gamma_n = \Gamma_\beta$.

However, there is tension between the results obtained using different methods, as it was pointed out in Refs.~[23]. Fig.~8 clearly demonstrates the discrepancy. Namely, by averaging the presently available results of eight trap experiments [24–31] one obtains:

$$\tau_{\text{trap}} = 879.4 \pm 0.6 \text{ s}, \quad (45)$$

which is compatible with the SM prediction (44). On the other hand, the beam experiments [32, 33] yield

$$\tau_{\text{beam}} = 888.0 \pm 2.0 \text{ s}, \quad (46)$$

which is about 4.4σ away from the SM predicted value (44).  

Therefore, due to consistency with the SM prediction, it is more likely that the true value of the neutron lifetime is the one measured by trap experiments (45). About 1 per cent deficit of produced protons in the beam experiments [32, 33] might be due to some unfixed systematic errors. Alternatively, barring the possibility of uncontrolled systematics and considering the problem as real, a new physics must be invoked which could explain about one per cent deficit of protons produced in the beam experiments. One interesting possibility can be related to the neutron–mirror neutron $(n - n')$ oscillation [34], provided that ordinary and mirror neutrons have a tiny mass difference $100 \text{ neV}$ or so [35]. Then in large magnetic fields (5 Tesla or so) used in beam experiments $n - n'$ conversion probability can be resonantly enhanced to about $\sim 0.01$ and thus corresponding fraction of neutrons converted in mirror neutrons will decay in an invisible (mirror) channel without producing ordinary protons.

Concluding this section, let us remark that the present precision calculation of the short-range radiative corrections [$\Delta^{\mu}_F$][8] and respective redetermination of $V_{ud}$ has no influence on the determination of the neutron lifetime (44) obtained from Eq. (42) which in fact directly relates the value of $\tau_\nu$ to the value $\mathcal{F}t$ accurately measured in superallowed $0^+ - 0^+$ nuclear transitions and to the value $g_A = G_A/G_V$ obtained from accurate measurements of $\beta$-asymmetry. Notice that the relation (42) remains valid also in the presence of non-standard vector or axial interactions contributing to the neutron decay, since the value of $G_V$ (independently whether it is equal to $G_F|V_{ud}|$ or not) anyway cancels out [36] and only the ratio $g_A = G_A/G_V$ remains relevant which value is accurately determined from the measurements of $\beta$-asymmetry. In particular, Eq. (42) remains valid in our model with $G_F \neq G_\mu$ discussed in previous section, or more generally for any modification of the SM introducing new vector and axial couplings contributing in operator (4).

9. As concluding remarks, the present experimental and theoretical accuracy in independent determination of the first row elements of the CKM matrix indicates towards about 4.4σ deviation from the unitarity (4). This can be indication to the new physics at the scale of few TeV. We investigated two possible scenarios in order to fill the gap. The respective results are summarised it Table 1.

The first, rather straightforward possibility is related to the existence of “fourth family” in the form of a vector-like couple of isosinglet down-type quarks $b'_L$, $b'_R$, with the mass of few TeV, which has rather strongly mixed with the first family $V_{ud'} \simeq 0.04$. However, apart of the persistent question “who has ordered that?” it has some rather unnatural features. In particular, in order to avoid strong flavor changing effects in Kaon physics etc., the 4th quark $b'$ should have weaker mixings with 2nd and 3rd families than with the first one. Perhaps such a situation is possible by some conspiracies, however a priori it looks rather weird.
Alternatively, an additional effective operator contributing to muon decay in positive interference with the Standard Model contribution can restore unitarity. In this case the Fermi constant would be slightly different from muon decay constant, $G_F = G_{\mu}/(1 + \delta_{\mu})$, where $\delta_{\mu} \simeq 7 \times 10^{-4}$ would suffice for restoring the unitarity. Namely, the values of $V_{us}$ and $V_{ud}$ (which are normally extracted by assuming $G_F = G_{\mu}$) are shifted by a factor $1 + \delta_{\mu}$ while their ratio is not affected. The needed effective operator can be mediated by a flavor changing boson related to a gauge horizontal symmetry $SU(3)_c$ acting between the three lepton families, which symmetry is spontaneously broken at the scale of $6 - 7$ TeV.

Considering the gauge symmetry group $SU(3)_c \times SU(3)_c$, acting on left-handed and right-handed leptons respectively, one can get a natural understanding on the origin of the mass hierarchy among charged leptons and large mixing angles for neutrinos which is related to the pattern of spontaneous breaking of the symmetry. Interestingly, despite the fact that these gauge bosons are have flavor–changing couplings with the leptons, their exchanges do not induce dramatic LFV effects as decays $\mu \rightarrow 3e$, $\tau \rightarrow 3\mu$ etc., which can be kept under control thanks to approximate custodial symmetry.

Analogously, one can consider gauge symmetry $SU(3)_Q \times SU(3)_u \times SU(3)_d$ between the quark families. Its breaking pattern can be at the origin of the quark mass and mixing hierarchy, and the flavor-changing gauge bosons of $SU(3)_Q$ can contribute to hadronic decays of kaons, hyperons, etc. In supersymmetric extension of the SM, the chiral gauge symmetries $SU(3)_c \times SU(3)_c$ for leptons and $SU(3)_Q \times SU(3)_u \times SU(3)_d$ for quarks can be also motivated as a natural possibility of the realizing the minimal flavor violation scenario 37, 38.

One interesting possibility, discussed in Ref. 16, is that these flavor gauge symmetries are common symmetries between the ordinary and mirror particle sector, which is also motivated to the possibility of cancellation of triangle anomalies of gauge $SU(3)$ factors between the ordinary and mirror particles 37. Mirror matter is also a viable candidate for dark matter (see e.g. reviews 39). Since flavor gauge bosons are messengers between the two sectors, then they are a portal for direct detection of mirror dark matter 10 but also they mediate new flavor violating phenomena such as muonium–mirror muonium, kaon–mirror kaon oscillations 10.

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|                  | CKM [PDG] | CKM [post 2018] | CKM+$b'$ | CKM+$F$ |
|------------------|-----------|-----------------|---------|---------|
| C                | 0.2257(9) | 0.22780(60)     | 0.22443(61) | 0.22460(61) |
| B                | 0.2256(10)| 0.22535(45)     | 0.22518(45) | 0.22535(45) |
| A                | 0.2238(8) | 0.22333(60)     | 0.22333(60) | 0.22350(60) |
| $A+B$            | 0.2245(6) | 0.22463(36)     | 0.22452(36) | 0.22469(36) |
| $A+B+C$          | 0.2248(5)| 0.22546(31)     | 0.22449(31) | 0.22467(31) |
| $\chi^2$        | 3.4       | 27.7 †          | 6.1      | 6.1      |

TABLE I. The 1st column shows independent $|V_{us}|$ determinations A, B, C from the PDG dataset 9 by assuming 3-family CKM unitarity 2, their averages and total $\chi^2$ value. The last two rows show the conservative estimation of $|V_{us}|$ with error-bar rescaled by $\sqrt{\chi^2_{\text{dof}}}$ and the corresponding value of $|V_{ud}|$. Other columns show the same but obtained from after 2018 dataset 10 by assuming respectively 3-family CKM unitarity 2, unitarity extended to 4th quark $b'$ with $|V_{ub'}| = 0.04$, and 3-family CKM but taking $G_{\mu}/G_F = 1 + \delta_{\mu}$ with $\delta_{\mu} = 7.6 \times 10^{-4}$. Mark † in 2nd column indicates that for that large $\chi^2$ the error-rescaling by $\sqrt{\chi^2_{\text{dof}}} = 3.7$ does not make much sense since the data are incompatible.

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