A Directed Network of Greek and Roman Mythology

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We study the Greek and Roman mythology using the network theory. We construct a directed network by using a dictionary of Greek and Roman mythology in which the nodes represent the entries listed in the dictionary and make directional links from an entry to other entries that appear in its explanatory part. We find that this network is clearly not a random network but a directed scale-free network. Also measuring the various quantities which characterize the mythology network, we analyze the Greek and Roman mythology and compare it to other real networks.

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Current intense researches are focused on the widespread complex systems such as biological, social, technological, economic, and communications systems 1, 2, 3, 4, 5, 6. Such complex systems can be represented by the complex network which is composed of nodes which represent the diverse elements in the system and links by which the elements are connected if there are various interactions between them. For example, in a social acquaintance network nodes are individuals, links are connections between friends. In the World-Wide-Web (WWW) network nodes are web pages, links are hyper-links between them. Especially, in the WWW network, hyper-links have directions so it is called a directed network.

The complex networks are characterized by some topological and geometrical properties such as small-world, high degree of clustering, and scale-free topology. The small-world property denotes that the average path length \( L \) which is the average shortest path length between vertex pairs in a network, is small. It grows logarithmically with the network size \( N \). The clustering structure in a network is measured by the clustering coefficient \( C \) which is defined as the fraction of pairs between the neighbors of a vertex that are the neighbors of each other. The high degree of clustering indicates that if vertices A and B are linked to vertex C then A and B are also likely to be linked to each other. The scale-free (SF) topology reflects that the degree distribution \( P(k) \) follows a power law, \( P(k) \sim k^{-\gamma} \), where degree \( k \) is the number of edges attached to a vertex and \( \gamma \) is the degree exponent. Such network is called the SF network in which there are vertices of high degree which produces strong effects. Also recent attention has been focused on the hierarchical structure and the cyclic topology. A hierarchical structure appears in some real networks and it has been clarified by a power-law behavior of the clustering coefficient \( C(k) \) as a function of the degree \( k \) 6. This indicates that the networks are fundamentally modular.

It is an origin of the high degree of clustering of the networks. The cyclic topology is determined by loops with various sizes which can affect the delivery of information, transport process, and epidemic spreading behavior 8. The cyclic coefficient \( R \) which considers the loops of all sizes from three up to infinity is defined as the average of the local cyclic coefficient \( r_i \) over all the vertices 9. A local cyclic coefficient \( r_i \) for a vertex \( i \) is defined as the average of the inverse size of the smallest loop that connects the vertex \( i \) and its two neighbor vertices, i.e., \( r_i = \frac{2}{k_i(k_i-1)} \sum_{lm} \frac{1}{S_{lm}^i} \) where \( <lm> \) is for all the pairs of the neighbors of the vertex \( i \) and \( S_{lm}^i \) is the smallest size of the closed path that passes through the vertex \( i \) and its two neighbor vertices \( l \) and \( m \). The cyclic coefficient has a value between zero and 1/3. \( R=0 \) means that the network has a perfect tree-like structure without having any loops. Meanwhile if all the neighbor pairs of the vertices have direct links to each other, then the cyclic coefficient becomes \( R=1/3 \). The larger the cyclic coefficient \( R \) is, the more cyclic the network is.

Applications of the network analysis to the real systems have gradually broaden from the systems whose network structures are obviously exposed, for examples, trains 10, subways 11, airports 12, and internet communities 13, to the systems whose network structures are relatively hidden, for examples, language 14, seismology 15, jazz 16, tango 17, comics 18, dolphins 19. The complex network analysis provides a new successful standpoint to a wide range of real-life systems. Reversely, the studies of real-life networks are provided as basic data to construct the general theory of complex networks. Thus the studies for a variety of real-world systems using network analysis is very important.

In this paper, we study the Greek and Roman mythology (GRM) which is a reflection of human-life using the network analysis. By using a GRM dictionary we construct a directed mythology network in which the nodes correspond to myth characters and two nodes are linked when the corresponding characters have a relationship in the myth. We analyze the GRM by surveying the various properties for the GRM network and compare the myth-world network to other real-world networks.

A mythology comprises a collection of stories belonging
to a single culture group which frequently feature both anthropomorphic or theriomorphic divine figures, heros, or animals. The GRM becomes the patterns upon which Freudian psychiatrists base their interpretation of human behavior: painters, composers, sculptors and writers, deliberately or unconsciously, imitate the mythical patterns of the past. Myth furnishes us more than a repertoire of literary plots and themes. The GRM has been recounted by hundreds of writers throughout the world over the course of nearly three thousand years. It incorporates vast myth-tale motifs in which so many myth characters are connected each other. The mythic world constitute a mythic-social network composed of myth characters and connections between them.

The dictionary of classical mythology by M. Grant and J. Hazel is a biographical dictionary including the extensive Greek and Roman mythological characters, from major deities to the lesser-known nymphs. It includes, as 1647 entries, gods, heros, monsters, mortals, fairies. Each entry has an biographical explanatory part in which the dictionary are made. The entry Acacallis, as an example, has the explanatory part as "Daughter of Minos and Pasiphae. She bore Apollo a son, Amphithemis, and perhaps Miletos also." Acacallis has directed links to the five entries, Minos, Pasiphae, Apollo, Amphithemis, and Miletos. Also, if the linked entries refer to Acacallis in their explanatory part opposite-directed links are formed between them. Figure 1 shows the connection of Acacallis to its five neighbors. There are bi-directional links between Acacallis and Minos, Miletos, and Amphithemis, while links between Acacallis and Pasiphae and Apollo are mono-directional. It results from that the explanatory parts of Minos, Miletos, and Amphithemis refer to Acacallis, while those of Pasiphae and Apollo do not refer to Acacallis. In this way, the node Acacallis has in-degree \( k_{in} = 3 \) and out-degree \( k_{out} = 5 \) which are the number of links incoming upon it and outgoing from it, respectively. Thus we construct the directed mythology network where the number of nodes, total in-links (out-links), and total undirected links are 1637, 6687 and 8938 respectively. Here total undirected links are counted in the undirected network where we neglect the direction of links so two entries are connected if they refer to or refer by each other. The difference between the numbers of total undirected links and total directional links is same as the number of unilateral references in the dictionary.

We measure the degree distribution of the GRM network by using the cumulative degree distribution function. The cumulative degree distribution function \( P_c(k) \) is the probability that a randomly selected node has over \( k \) links, i.e., \( P_c(k) = \sum_{k' \geq k} P(k') \). The cumulative distribution also follows power-law, but with exponent \( \gamma + 1 \) rather than exponent \( \gamma \), so that \( P_c(k) \sim k^{-\gamma+1} \). We measured the out-degree, the in-degree, and the total-degree distribution which shows the number of common links between the dictionary's entries. We measure the cumulative degree distribution function \( P_c(k) \) and plot it using a log-log scale, which shows a linear relationship with a slope of \( \gamma = 2.61 \) for the out-degree distribution, \( \gamma = 2.93 \) for the in-degree distribution, and \( \gamma = 2.49 \) for the total-degree distribution. These results suggest that the GRM network follows a power-law degree distribution, indicating a scale-free network structure.

The cumulative out-degree distribution and the cumulative in-degree distribution are shown in Figure 2, which displays the distribution of the number of links outgoing from and incoming to each node, respectively. The slope of the solid line in Figure 2(a) is 1.93, indicating a power-law distribution for the out-degree with an exponent of \( \gamma_{out} = 2.93 \). Similarly, Figure 2(b) shows the cumulative in-degree distribution with a slope of 1.49, indicating a power-law distribution for the in-degree with an exponent of \( \gamma_{in} = 2.49 \).

The cumulative degree distribution of the GRM network ignoring the directions of the edges is shown in Figure 3. The slope of the solid line is 1.61, which shows that the network's degree distribution follows a power-law with an exponent of \( \gamma = 2.61 \). This indicates that the GRM network is a scale-free network, where a few nodes have a disproportionately large number of links, while the majority of nodes have a small number of links.
Table I: Seven most connected entries with the corresponding out-degrees and in-degrees and degrees.

| rank | entries            | $k_{out}$ | $k_{in}$ | $k$  |
|------|--------------------|-----------|----------|------|
| 1    | Heracles           | 140       | 223      | 243  |
| 2    | Poseidon           | 92        | 187      | 230  |
| 3    | Odysseus           | 83        | 144      | 177  |
| 4    | Zeus               | 79        | 140      | 156  |
| 5    | Argonaut           | 77        | 137      | 149  |
| 6    | Theseus            | 69        | 97       | 122  |
| 7    | Dionysus           | 55        | 87       | 102  |

degree distributions. The plots of the cumulative out-degree and in-degree distribution functions as a function of the out-degree and the in-degree are shown in Fig.2(a) and (b), respectively. The slopes of the straight guide lines are 1.93 and 1.49, which represents that the out-degree and the in-degree exponents are $\gamma_{out} \approx 2.93$ and $\gamma_{in} \approx 2.49$. Thus we found that the GRM network is a directed scale-free network, which means that all characters do not play an equal role and some characters play an more central role than other characters in the Greek and Roman mythic world. Also we obtained the values of degree exponent $\gamma \approx 2.61$ in the undirected network as shown in Fig. 3.

Table II shows seven most connected entries with the corresponding numbers of out-degrees, in-degrees, and undirected-degrees in order of ranks. We found that Heracles ranks the first with 140 outgoing links for out-degrees, while Zeus ranks the first with 223 incoming links and 243 undirected links for in-degrees and undirected-degrees. In the GRM network, the fact that a nodes has more links than other nodes means that its corresponding myth character appears more frequently in myth tales. Heracles has the most outgoing links, which means that he appears as a leading character in many different myth tales. On the one hand Zeus who has most incoming links most frequently appears as a supporting character in different myth tales. Also we notice that the characters well known to the mass of the people hold high ranks on the whole.

For the undirected GRM network, we measure various quantities which characterize the network. First, we obtained the average path length $L = 3.47$ and the clustering coefficient $C = 0.41$. Comparing these values with the random network of the same number of nodes and links for which the values of the average shortest path length and the clustering coefficient are 6.86 and 0.0034, we find that the GRM network has the small-world property and high degree of clustering as the other various complex networks. The small-world effect also underlies some well-known parlar games, particularly the calculation of Erdös numbers \cite{21}. Similarly, we measured Heracles(Zeus) number of a node is defined as the shortest path length between the node and Heracles(Zeus). The Heracles(Zeus) numbers range from 0(himself) to 7(6) and the average Heracles(Zeus) number of the network is 2.18(2.11).

Figure 4 shows the log-log plot of the clustering coefficient $C(k)$ versus the degree $k$. The straight guide line represents that $C(k)$ follows a power-law, $C(k) \sim k^{-\beta}$ with $\beta \approx 0.63$. It means that the GRM network forms a hierarchical structure. We also measured the cyclic coefficient $R$ and obtained $R \approx 0.23$. Figure 5 shows the plot of the distribution of local cyclic coefficient. There are the first and second peaks at $r = 0$ and $r = 1/3$ and two peaks have almost equivalent values. This result represents that there are many tree-like and triangular patterns in the GRM networks. It is different from the network structures of the other real-networks where the only one between tree-like and triangular pattern is certainly dominant \cite{22}. While, except for $r = 0$, it is similar to that of the movie actor network \cite{22}, in which nodes are actors and two nodes are linked if the corresponding actors have acted in the same movie together. It reflects the biographical nature of the mythology dictionary: two entries have high possibilities of appearance in the explanatory parts of each other when the corresponding characters have jointly appeared in the same myth story as two actors have costarred in a same film. That is, although the GRM network is constructed by using a biographical dictionary, the GRM network can be regarded as a kind of a social network.

In summary, we studied the relationship among characters appeared in the GRM with the help of the latest complex network theory. By using the biographical dictionary of GRM, we constructed the directed GRM network in which the nodes correspond to the entries(mythology characters) and a directional link was made from an entry A to an entry B when the entry B was appeared in the explanatory part of the entry A. It was founded that the GRM network is a scale-free network and has prop-
properties such as the small-world and high degree of clustering. Also by measuring the clustering coefficient $C(k)$, we found that the GRM network forms a hierarchical structure. The distribution of local cyclic coefficient tell us that the GRM network is a social-like network such as the movie actor network as well as a dictionary network.

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