1. INTRODUCTION

Gravitational wave astronomy is a rapidly developing field, with many ground-based interferometric detectors (e.g., the Laser Interferometer Gravitational-Wave Observatory [LIGO], Variability of Irradiance and Gravity Oscillations [VIRO], GEO, TAM) in place and taking data. These detectors are sensitive to (among other things) the waves emitted from the late stages of inspiral and merger of compact binaries with neutron stars (NSs) and black holes (BHs). Under certain conditions (as discussed in Apostolatos et al. 1994; Grandclément et al. 2003, 2004; Pan et al. 2004 and references therein) the BH spin in high–mass ratio binaries such as BH-NS binaries can significantly modulate the emitted inspiral waveform. While the more complex resulting waveforms can pose challenges for detection strategies, they also enable us to empirically determine the spin of rapidly rotating black holes, if they exist. We undertake the present study of the expected BH spin distribution for two reasons: (1) to examine whether special search methods for spinning compact objects for optimal detection efficiency are necessary from an astrophysical point of view, and (2) to provide the first steps for a theoretical understanding of the origin and magnitude of BH spins in BH-NS binaries that will be useful when such BH spins are empirically constrained in the future.

In §2 we briefly describe the population synthesis calculations we use to explore a broad range of possible scenarios that lead to the formation of BH-NS binaries that can merge within a Hubble time (hereafter referred to as “merging”). We summarize and explain the common formation channels for these binaries and describe the mass accretion history associated with them. In §3 we review the observational evidence for BH birth spin drawn from observations of isolated NSs and of X-ray binaries. On the basis of this evidence, we employ two fiducial choices for BH birth spin parameter \( a = J/M^2 \), i.e., \( a = 0 \) (nonspinning) or \( a = 0.5 \) (moderate spin). Given an understanding of the mass accretion history and a choice for the initial BH spin, we could in principle determine the final black hole spin if we knew the specific angular momentum of the material accreted by the BH in these systems. This factor remains substantially uncertain. Thus, in §4 we use a conservative model for the specific angular momentum, combined with the distribution of accretion histories and birth spins, to limit the rate at which high-spin black holes occur in merging BH-NS binaries.

2. ACCRETION HISTORY OF BLACK HOLES IN BH-NS BINARIES

To generate and evolve stellar populations until double compact object formation occurs, we use the StarTrack code first developed by Belczynski et al. (2002, hereafter BKB02) and recently significantly updated and tested as described in detail in K. Belczynski et al. (2005, in preparation). As with all binary population synthesis codes, StarTrack encapsulates the residual physical uncertainty in evolutionary processes of stars in a number of parameters, several of which can significantly affect the statistics of BH-NS mergers. In order to have a sample of merging BH-NS binaries drawn from a large archive of astrophysically a priori plausible models, we generated an archive of accumulated results from a broad swath of plausible models for population synthesis. As described in O’Shaughnessy et al. (2005a, hereafter OKB05), the archive was generated using flat probability distributions for seven of the most significant parameters: three parameters describing the supernova kick magnitude distribution (assumed to consist of two Maxwellian distributions with dispersions \( \sigma_k \in [0, 200] \) km s\(^{-1}\) and \( \sigma_k \in [200, 1000] \) km s\(^{-1}\) and the relative weight of small to large kicks, \( s \in [0, 1] \) ), the effective

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**ABSTRACT**

As a first step toward understanding the angular momentum evolution history of black holes in merging black hole/neutron star binaries, we perform population synthesis calculations to track the distribution of accretion histories of compact objects in such binaries. We find that there are three distinct processes that can possibly contribute to the black hole spin magnitude: a birth spin for the black hole, imparted at either (1) the collapse of a massive progenitor star to a black hole or (2) the accretion-induced collapse of a neutron star to a black hole, and (3) an accretion spin-up when the already formed black hole (via processes 1 or 2) goes through an accretion episode (through an accretion disk or a common-envelope phase). Our results show that, with regard to accretion-induced spin-up in merging BH-NS binaries (method 3 above), only accretion episodes associated with common-envelope phases and hypercritical accretion rates occur in the formation history of merging black hole/neutron star binaries. Lacking unambiguous experimental information about BH birth spins (i.e., regarding the results of processes 1 and 2), we choose two fiducial values for the BH birth angular momentum parameter \( a = J/M^2 \), consistent with observations of (1) NS birth spins (\( a \approx 0 \)) and (2) X-ray binaries (\( a = 0.5 \)). Using these two fiducial values and a conservative upper bound on the specific angular momentum of accreted matter, we discuss the expected range of black hole spins in the binaries of interest. We conclude with comments on the significance of these results for ground-based gravitational wave searches of inspiral signals from black hole binaries.

Subject headings: binaries: close — black hole physics — stars: neutron — stars: rotation
common-envelope efficiency, \( \alpha \in [0, 1] \), the stellar wind strength, \( w \in [0, 1] \), the power-law index controlling the distribution of the mass of the companion relative to the mass of the primary, \( r \in [-3, 0] \), and the fraction of mass accreted (as opposed to lost from the binary) during nonconservative mass transfer episodes, \( f_a \in [0, 1] \). Unfortunately, as is discussed in more detail in O’Shaughnessy et al. (2005b), we cannot succinctly describe the effect of these seven parameters on the formation channels and rates of BH-NS systems; physical predictions depend in a highly correlated way on all seven parameters. As in OKB05, we fixed the remaining population synthesis model parameters to physically reasonable values, such as setting the maximum neutron star mass to \( M_{\text{NS, max}} = 2 M_\odot \) and the metallicity to solar metallicity \( Z = 0.02 \) (see the parameters of model A in BKB02 for more details). From this archive we identified each merging BH-NS binary along with the details of its evolutionary history. Our results are summarized in Table 1.

Compact binaries tight enough to merge through the emission of gravitational waves within the Hubble time have progenitors that usually interacted strongly (e.g., through mass transfer) earlier in their evolution. For example, a common mode of interaction is conventional mass transfer through Roche lobe overflow and disk accretion. However, for merging BH-NS binaries, we find in our simulations that almost all of them have experienced a more dramatic phase of dynamically unstable mass transfer and common envelope (CE) evolution (see Table 1). During such phases it is expected that hypercritical accretion becomes possible. In this form of accretion (hypercritical common envelope; HCE), discussed in more detail in Brown (1995), Brown et al. (2000), and Fryer et al. (1996), a compact object spirals in through the envelope of its companion, rapidly accreting matter at highly super-Eddington (for photons), neutrino-cooled rates. As a result of this process, most of the companion’s envelope is ejected, bringing the post-CE binary very close together. While the detailed accretion onto the compact object is complex and ill understood, since we know that the whole envelope is lost during this phase, a straightforward application of the conservation of energy and mass during the quasi-circular spiral-in process can determine the final orbit, the final binary masses, and the mass of the ejected material; see Appendix A of BKB02 for details. Nevertheless, we stress that our treatment of HCE is essentially a simple, semi-analytical model, which is by no means guaranteed to accurately describe the quantitative effect of the HCE phase.

More specifically, as described in Table 1, the vast majority (all but 582 out of 5435) of the BH-NS binaries seen in our simulations form through an evolutionary channel that involves an HCE phase. In these channels, after an optional mass transfer phase, one star explodes, the compact remnant spirals through andstrips the envelope of its companion, and then the companion explodes. Black holes form either immediately after the first core-collapse event or through the accretion-induced collapse of a NS that forms in the first supernova event. Regardless of the nature of the first compact object (BH or NS), it most typically experiences HCE evolution and accretes a large amount of matter (potentially equal to its birth mass) during the brief HCE phase. Figure 1 shows the fraction of the initial compact object’s mass \( M_{\text{init}} \) that is accreted during the HCE phase. On the basis of the sample we have, we estimate an upper bound on the fractional mass accreted as a function of the initial mass to be

\[
\left( \frac{\Delta M}{M} \right)_{\text{max}} \approx 0.25 + \frac{(2 M_\odot)}{M_{\text{init}}}. \tag{1}
\]

This bound is valid for all channels.3 For systems that form their BH immediately after the first supernova (i.e., no accretion-induced collapse, so \( M_{\text{a, init}} > 2 M_\odot \)) we find an empirical lower bound of

\[
\left( \frac{\Delta M}{M} \right)_{\text{max}} \approx 0.05 + \frac{(0.6 M_\odot)}{M_{\text{init}}}. \tag{2}
\]

This latter bound may be understood as a requirement that the BH-NS system merge within the simulation time. Given that the second supernova rarely further tightens the orbit, if the binary is to merge within 10 Gyr, then the orbit must be smaller than roughly \( (3.4 R_\odot)/(M_\odot/M_\odot)(M_\odot/M_\odot)(M_\odot/M_\odot)]^{1/4} \) after the common envelope phase. Using the common-envelope evolution equations presented in Appendix A of BKB02 (with any choice for \( \alpha_{\text{CE}} \)), we find that this requirement approximately translates into the lower bound presented above.

3 We expect that systems that transfer more mass than this limit are brought so close that they merge during their common-envelope phase.

### Table 1: Statistics for Evolutionary Channels

| Channel | \( n \) | AIC |
|---------|------|-----|
| (MT+)SNa HCE(b→a) | 4310 | 3656 |
| (MT+)SNa HCE(a→b) | 543 | 543 |
| (MT+)SNx MT+SNx | 319 | 0 |
| (MT+)SNx SNy | 263 | 0 |
| Total | 5435 | |

Notes: This table summarizes all evolutionary channels followed by systems that formed merging BH-NS binaries in our simulations. The first three columns describe the evolutionary channel (where SNx indicates a supernova of either the primary [a] or the secondary [b], HCE indicates a hypercritical common envelope phase, and MT indicates a stable Roche lobe overflow mass transfer; parentheses indicate an optional feature of the channel), the fourth column provides the number of merging BH-NS binaries that passed through this channel, and the fifth column lists the number that undergo accretion-induced collapse (i.e., the BH forms during the common-envelope phase from an accreting neutron star). Most systems formed via a HCE phase onto a compact object; this compact object is occasionally already a BH, but usually is a NS that usually interacted strongly (e.g., through mass transfer) earlier in their evolution. In other words, they show the relative increase in mass of the compact object \( \Delta M_{\text{HCE}}/M \) due to hypercritical common-envelope accretion. The upper solid line shows the empirical upper bound given in eq. (1). Our calculations assume lower and upper limits on NS mass of \( M = 1.3 \) and \( 2 M_\odot \), respectively.

![Figure 1](image-url)
Merging BH-NS binaries can form through other channels, as shown in Table 1. However, these channels involve substantially less mass transfer onto the BH: for the HCE-related channels, the smallest mass increase was 0.22 $M_\odot$, to be contrasted with the largest disk-mediated mass transfer, 0.007 $M_\odot$ (i.e., from the channel SNx+MT+SNy). As described in §4, this incredibly small disk-mediated mass transfer cannot significantly modify the birth spin of the BH. Therefore, we neglect these channels in subsequent discussion.

Unfortunately, our branching ratios for various evolutionary channels are not significant. When exploring different population synthesis parameter combinations, we normalized so that each population synthesis run contributes a roughly fixed number of merging NS-NS binaries to the archive. As a result, models associated with low NS-NS merger rates but high BH-NS merger rates should contribute disproportionately more merging BH-NS to our archive and vice versa. Moreover, even if each model contributed equally to the total number of BH-NS systems, our net numbers would then reflect the mean branching ratios over all population synthesis models considered rather than the branching ratio of, e.g., the most common model. Finally, as discussed in O’Shaughnessy et al. (2005b), not all population synthesis models we considered are equally compatible with observations. Nonetheless, given the current statistics, we fully expect that most BH-NS systems form via a hypercritical common envelope phase.

3. BIRTH SPINS OF BLACK HOLES IN BH-NS BINARIES

No direct measurements of BH birth spins (or even BH spins) exist yet. A priori, the BH birth spin cannot be constrained beyond the most fundamental level (i.e., to avoid naked singularities, we must have low angular momentum $J$: specifically, $a \equiv J/M^2 \leq 1$). For example, the collapse of a slightly hypermassive NS rotating at breakup will, on dimensional grounds alone, produce a BH of spin on the order of half that of maximal ($a \sim 0.5$) should it collapse; more realistic computations that allow for potential differential rotation support could push this value even higher (see, e.g., Duez et al. 2004; Shapiro & Shibata 2002; Baiotti et al. 2005).

Lacking unambiguous theoretical guidance for BH birth spins in merging BH-NS systems, and lacking direct observations of BH spin, we employ evidence for birth spin in similar systems to guide our choices for BH birth spin. On the one hand, on the basis of our understanding of NS birth spins and spin-up in NS-NS binaries, we expect low BH birth spins ($a = 0$). However, on the basis of suggestive observations of X-ray binaries, we suspect that moderate BH birth spins could occur ($a = 0.5$).

3.1. BH Birth Spin Estimates from NS Observations

The same processes that produce BH birth spins (i.e., core collapse and accretion-induced spin-up) also determine the spins of the well-observed NS population. Observations of the NS population can thus potentially provide us with some constraints on BH birth spins.

BH birth spin from core collapse.—We expect that we can estimate the birth spins $a = J/M^2$ of BHs, and particularly low-mass BHs, through the estimated birth angular momenta $J$ of young NSs. For BHs and NSs of comparable mass, the collapse process should be nearly identical; therefore, the collapse product should have very similar values of $J$ and $M$. Assuming that no other process intervenes (for example, as discussed in Lindblom & Owen [2002], $r$-mode damping is not expected to significantly change the NS angular momentum of young NSs), NSs will spin down electromagnetically, and thus NS spins at birth can be estimated from the observed pulsar NS sample. In the past few years a number of studies of observed radio pulsars have estimated the NS spin periods at birth (see Lorimer et al. 2005; Kramer et al. 2003; Migliazzò et al. 2002 and references therein) in the range of 10–140 ms. These values are significantly slower than breakup spins (for typical NS equations of state) and correspond to $a \sim 0.005–0.02$ (assuming a NS radius of 10 km and rigid rotation).

BH birth spin from accretion-induced collapse.—Our simulations show that a significant fraction (i.e., 4199 out of 5435 binaries) of the BHs in merging BH-NS systems were originally NSs that experienced HCE and collapsed into BHs. Therefore, birth spins for this BH class are related to the spin-up of NSs during a CE phase. Our current understanding of this spin-up process is quite limited. However, we can obtain guidance from the mildly recycled pulsars in known double neutron star binaries. These pulsars are believed to have been spun up during a CE phase (where collapse to a BH was avoided, however), and it is evident that they are not spinning at close to breakup speeds: the fastest known pulsar in a double NS binary is PSR J0737–3039A, spinning at $\sim$20 ms or else having $a = 0.01$ (Burgay et al. 2003). Therefore, post-CE NS spins appear to correspond to values of $a$ no larger than $a = 0.01$.

On the basis of the observational considerations discussed above, we would expect the birth BH spin to be negligible (i.e., $a \leq 0.03$). However, a number of uncertainties in our modeling suggest that very different values of $a$ could be equally plausible.

Uncertainties in collapse model.—For black holes that form from the supernova of a massive star, we assume that the birth spins of these BHs are comparable to the birth spins of comparable-mass NSs, and we then assume that those NS birth spins may be estimated from electromagnetic (pulsar) spin-down from present-day pulsars. However, most of these low-mass black holes form through fallback of post-supernova ejecta onto the hole; if the post-supernova fallback material carries significant angular momentum, it can spin up the nascent BH to high values of $a$, even if the proto–neutron star is not spinning with a period shorter than $\sim$10 ms. Furthermore, if young proto–neutron stars can spin down rapidly through other mechanisms, such as $r$-modes (Lindblom & Owen 2002), straightforward electromagnetic-based extrapolations may significantly underestimate the proto–neutron star spin. If either of these mechanisms took place, then the birth BH spin could be significantly larger than the naive estimate we outlined above.

Uncertainties in HCE model.—The known pulsars in double NS binaries have avoided accretion-induced collapse into BHs. Those that do not avoid this collapse must accrete a higher amount of mass, and therefore it is still possible that they get spun up to shorter spin periods and therefore lead to BHs formed with higher $a$-values than expected.

Given these uncertainties, we cannot be sure whether the relevant BH birth spins are negligible (i.e., $a = 0$) or large, comparable to the breakup spin of a NS (i.e., $a \sim 0.5$).

3.2. Black Hole Spins in X-Ray Binaries

Black hole X-ray binaries (XRBs) offer the possibility of directly measuring the spin of a compact object, since they involve a highly relativistic accretion flow in the strong field of a BH. Two techniques are prominent in the literature: interpreting quasi-periodic oscillations and fitting iron-line profiles.

Quasi-periodic oscillations (QPOs) are believed to be modes of the inner, highly relativistic regions of the BH accretion disk. As such, their frequency is expected to be related to the Kepler frequency of the inner edge of the disk, which in turn is intimately
connected with the properties of the black hole. Measurements of high-frequency QPOs that are inconsistent with the innermost stable circular orbit of a nonspinning black hole, as by Strohmayer (2001) and Remillard et al. (2002), have been interpreted as evidence for BH spin. Moreover, evidence suggests that the spin suggested by these measurements is not merely a relic of past accretion. Unfortunately, QPO frequencies cannot be unambiguously translated to black hole angular momenta; their interpretation depends strongly on disk mode modeling (see, e.g., Török et al. 2005; Rezzolla et al. 2003), with results that vary from \( a \approx 0.1 \) to \( a \approx 1 \) depending on the assumptions used. However, even if the QPO interpretation is correct, the associated BH spin estimates need not reflect BH birth spins; such spins are most probably reached via long-term disk accretion, a process that is not relevant to BHs in BH-NS merging binaries.

Iron-line profiles potentially offer in principle a more direct probe into the inner disk rotation profile and thus the BH spin. The iron line can show strong Doppler shifts (due to rotation) and redshifts (due to strong gravity); in particular, very rapidly spinning BHs should show asymmetric red "tails." While pioneered for use with active galactic nuclei (AGNs), this same technique has been applied to BH XRBs (see, e.g., Miller et al. 2002, 2004a, 2004b). We treat the results, several claims of extreme spins \( a > 0.8 \), with great caution, however. On the one hand, these systems are strongly model dependent, and many physically relevant details (e.g., proper accounting of light bending near the hole; see Beckwith & Done 2004) have yet to be included in the models. On the other hand, just like in the case of QPO interpretations, the estimated spins may be entirely due to the disk accretion in the XRB and therefore unrelated to the birth BH spin.

3.3. Birth Black Hole Spins Adopted
To summarize, on the basis of observations of isolated NSs, we believe the birth spins of NSs, and thus of low-mass BHs, to be small. On the other hand, observations of X-ray binaries (QPOs and iron lines) suggest that higher mass BHs (\( M > 5 M_\odot \)) could be born with moderate spin. To allow for this substantial uncertainty, in what follows we consider the implications of two models for the birth BH spin: \( a = 0 \) and 0.5.

4. MASS ACCRETION AND SPIN-UP
Figure 1 demonstrates that the black hole accretes a significant fraction of its mass during the HCE phase. Since that matter probably carries some amount of angular momentum, the black hole could conceivably spin up significantly during the HCE phase. Unfortunately, the details of HCE accretion, particularly at the fine level of detail needed to resolve the amount of angular momentum advected onto the compact object, are not understood.

Lacking a quantitatively sound choice for the specific angular momentum, we argue instead that the specific angular momentum accreted should be at most the angular momentum of the marginally stable equatorial particle orbit. This model, equivalent to assuming that the accretion proceeds through a thin disk, implies that the final black hole spin depends only on the total amount of matter accreted and the initial black hole spin \( J \) (or, as commonly denoted, \( a = J / M^2 \)); as derived initially by Bardeen (1970; see also Thorne 1974), for a black hole that is initially nonspinning with mass \( M \), the BH spin parameter \( a \) is given by the following expression
\[
a_g(M, M_t) = \left( \frac{2}{3} \right)^{1/2} \left( \frac{M_t}{M} \right) \left[ 4 - \left( \frac{18M_t^3}{M^2} - 2 \right)^{1/2} \right].
\]

We consider this estimate to represent an upper limit to the BH value of \( a \), since we do not expect that accretion in an HCE event occurs through a thin disk. Instead, we expect that on almost all scales the accretion flow is nearly radial.

4.1. Accretion onto Nonspinning Holes
We use our results on compact object masses at the onset and at the end of HCE to estimate the values of \( a \) under the thin disk assumption. We consider two cases: (1) all mass accreted during the CE phases contributes to spin-up, and (2) if accretion-induced collapse (AIC) occurs, only the fraction of the mass accreted after AIC contributes to spin-up.

All mass contributes to spin-up.—As demonstrated in Figure 2 (top), because of the substantial amount of mass accreted during HCE phases, nearly all BHs could conceivably spin up to very large values of \( a \); a histogram of the values of \( a \) we observe would be highly concentrated near \( a \sim 1 \). In particular, many low-mass BHs (i.e., those that formed via AIC during an HCE phase) have accreted on the order of half their final mass and can potentially spin up to be on the order of \( a \sim 1 \). Furthermore, every system that undergoes HCE evolution accretes enough mass to spin up significantly (\( a > 0.2 \)). In this case, the BH-NS binaries with the largest spins (i.e., those that have \( a \approx 1 \)) have mass ratios between 2:1 and 3.5:1. However, for NSs that undergo AIC, this approach strongly overestimates the maximum attainable spin, since it assumes disk accretion at the Schwarzschild radius, even before the BH forms.

Only post-AIC mass contributes to spin-up.—Since current astrophysical evidence suggests that NSs do not get strongly
spin up during CE events, we also consider the possibility that only the mass accreted after the BH is formed through AIC contributes to its spin-up. (Those BHs that form through direct collapse are treated as in the previous case.) In this case, summarized in Figure 2 (bottom), we find a much broader distribution of possible a-values (i.e., a histogram of possible a-value outcomes would be more nearly flat from a = 0 to 1). In this case, the systems with the largest spins have a mass ratio near 3.5:1.

Both models have the same behavior for systems with large final BH masses (M_{bh,f} > 4 M_\odot). Good empirical lower and upper bounds for values of a for M_{bh,f} > 4 M_\odot follow directly from the mass-accretion bounds (eqs. [1] and [2]):

\[
a < a_B \left( M_{bh,f}, \frac{4M_{bh,f}}{5} - \frac{8 M_\odot}{5} \right),
\]

(4)

\[
a > a_B \left( M_{bh,f}, 0.95M_{bh,f} - 0.57 M_\odot \right),
\]

(5)

where M_{bh,f} and M_{bh,i} are the BH final and birth masses, respectively.

### 4.2. Accretion onto Spinning Holes

Black holes that are initially spinning present a more complex accretion challenge in principle: the BH spin need not be aligned with the disk angular momentum axis, so the accreted material could just as well spin down as spin up the hole. However, by following the same procedure we used to circumvent the (substantial) uncertainties we addressed earlier, we limit attention to bounding the BH spin. The most conservative bound is obtained by assuming a corotating equatorial disk.

Bardeen’s formula (eq. [3]) applies equally well to BHs that are initially spinning, if M_i is chosen so the true initial black hole mass M_{bh,i} and spin a_i satisfy a_B(M_{bh,i}, M_i) = a_i. To be concrete, if the birth BH spin is a_i = 0.5, then choosing

\[
M_i = 0.84M_{bh,i}
\]

yields a_B(M_{bh,i}, M_i) = 0.5. When we repeat the analysis of \S 4.1, we find the results that are summarized in Figure 3. Since Bardeen’s spin-up relation is monotonic, the empirical upper limit presented in equation (4) (for a_i = 0) translates directly into a corresponding spin limit when a_i = 0.5:

\[
a < a_B \left[ M_{bh,f}, 0.84 \left( \frac{4M_{bh,f}}{5} - \frac{8 M_\odot}{5} \right) \right],
\]

(7)

\[
a > a_B \left[ M_{bh,f}, 0.84 \left( 0.95M_{bh,f} - 0.57 M_\odot \right) \right],
\]

(8)

Briefly, the results are qualitatively similar to the nonspinning case but compressed in scale such that the minimum possible spin is a = 0.5 instead of a = 0. Since the construction of spin as a function of BH mass is similar, the spin upper bound also follows equation (4), with the Bardeen formula calculated choosing M_i so the initial black hole has spin a = 0.5.

### 4.3. General Conclusions

Our simulations have shown that almost all merging BH-NS binaries form through a hypercritical common envelope (HCE) phase, where they accrete a substantial fraction of their final mass. Lacking a quantitative model for the specific angular momentum accreted during HCE, we use the most pessimistic model: we assume the BH spins up through a thin equatorial disk according to the Bardeen formula (eq. [3]). Even setting aside those systems with even greater physical uncertainties—that systems with final BH mass M < 4 M_\odot, which form through accretion-induced collapse of the NS and whose final spin depends additionally on the details of the collapse and angular momentum transport onto the NS during that collapse—and even ignoring the possibility of BH birth spin, we find that large spins (a \sim 1) could easily be attained in principle from the HCE phase alone: with such a substantial fraction of its final mass transferred, the BH could easily spin up. However, for BHs with M > 4 M_\odot, significant spin-up after birth becomes increasingly difficult, as larger black holes require ever larger (and more unlikely) mass transfers in order to spin up to large values of a. For example, if we assume nonrotating initial BHs, the only BHs we found with spin a > 0.8 are those with final masses between 3 and 10 M_\odot; similarly, for BHs with a = 0.5 at birth, the only BHs we found with spin a > 0.95 are those with final masses in this same range.

### 5. BLACK HOLE SPIN AND DETECTION OF BH-NS INSPIRAL

In BH-NS binaries, the spin of the BH can, if large, have a significant effect on the late stages of inspiral and in particular the emitted gravitational waveforms (see Apostolatos et al. 1994; Apostolatos 1996; Grandclément et al. 2004; Pan et al. 2004; Buonanno et al. 2004 and references therein). As a result, in BH-NS binaries measurements of gravitational waves offer the potential to extract information about the spin. Furthermore, because of precession, such binaries will be much more reliably detectable, suffering less frequently from the effects of poor source orientation.

But these advantages come at a price. The more complex waveforms of spinning binaries are vastly more complicated to model. At present only ad hoc models are considered computationally feasible (Pan et al. 2004), and even these methods...
involve substantial computational challenges, particularly at more extreme mass ratios.

Our calculations provide extremely conservative upper bounds on the BH spin in BH-NS binaries. These bounds tell us that employing spin need not pose such a dramatic dilemma on astrophysical grounds: if we employ spin, we could require fewer computational resources than we might expect, and if we neglect it, we will not lose a dramatic fraction of potentially observable events.

5.1. Mass Ratio Spin Constraints and Detection

When constructing template banks for BH-NS inspiral a priori, parameter ranges are typically chosen to cover a very conservative parameter range; for example, in one of the few papers that presented results for variable mass ratio, Pan et al. (2004) built a template bank based on all binaries with masses in the range $M_{\text{BH}} \in [7, 12] M_\odot$ and $M_{\text{NS}} \in [1, 3] M_\odot$ and BH spins in the range $a \in [0, 1]$. However, most of the templates in the template bank come from extreme mass ratio systems, involving a high-mass BH, which we have just demonstrated is difficult to spin up. Therefore, a different template bank that neglected astrophysically irrelevant systems could perform the same search much more rapidly. Detailed computations of the computational savings we could obtain by moderating our template bank are far beyond the scope of this paper and will depend sensitively on two poorly constrained parameters, the BH birth spin and the maximum NS mass (here assumed to be $M = 2 M_\odot$). However, in an optimistic case we expect that a more judicious choice of templates could reduce the template bank size by a significant factor (cf. § VI C of Pan et al. 2004).

5.2. Limits on the Decrease in Event Rate Due to Omitting BH Spin

For the purposes of detection, spinning templates are required only when nonspinning templates fail to have sufficient overlap with the physical signal to guarantee that most inspiral events are detected. Loosely speaking, a nonspinning template fails to mimic the effect of precession on the waveform. Because of spin-orbit coupling, angular momentum is exchanged, causing the orbital plane to precess around the total angular momentum. Therefore, spinning templates are needed to detect those systems where the effects of precession are strongest; namely, those with (1) strong BH spins $a \approx 1$ that are (2) strongly misaligned with the orbital angular momentum (see, e.g., Fig. 2 in Grandclément et al. 2004).

Grandclément et al. (2004) have quantitatively examined the expected overlap between nonspinning templates and the expected signal from binaries in which the BH spin magnitude and orientation are independently varied (see their Fig. 2), assuming BH-NS binaries with $M_{\text{BH}} = 10 M_\odot$ and $M_{\text{NS}} = 1.4 M_\odot$. They then further convolved this function with an expected spin orientation distribution to derive an estimate for the fraction of events that could be seen that will be seen as a function of spin magnitude of the BH (see their Fig. 3). They find that even if all BHs were spinning maximally, only about 30% of the potentially detectable BH-NS mergers would be missed.

The Grandclément et al. (2004) results, however, are not directly applicable in our circumstances. Spin-dependent modulations depend significantly on mass ratio, a parameter that is varied in our BH-NS binaries but fixed for those of Grandclément et al.

Nonetheless, if we employ their relation between (1) the probability of detecting a detectable source when ignoring spin and (2) the BH spin magnitude of that source, then we find that when we convolve with our Monte Carlo sample of BH spins, which, recall, was designed to produce the largest plausible spin values, given known mass transfer, we would lose at most 30% of the events in extreme cases (i.e., high BH birth spin or all HCE mass accreted onto NS) and slightly less (25%) in more conservative cases (i.e., mass before AIC does not contribute to spin). Briefly, searches that use nonspinning template banks will likely find most detectable inspiral signals. Practically speaking, given the considerable uncertainty in the underlying event rate itself, upper limits produced using precise templates will have only marginally more astrophysical impact than upper limits produced using nonspinning template banks. Of course, for parameter estimation, correct templates are essential in order to extract the BH spin and thus to constrain its accretion history.

6. SUMMARY AND FUTURE DIRECTIONS

In this paper we perform population synthesis calculations to track the history of systems that evolve into merging BH-NS binaries, demonstrating that these objects form through only a few channels, almost exclusively involving a hypercritical common envelope phase and often involving accretion-induced collapse of a NS into a BH. We show that a significant amount of mass is typically accreted during this HCE phase. Using an upper limit based on thin-disk accretion, we find that the BHs accrete enough matter to potentially spin up to large $a$-values, independent of the BH birth spin. Finally, we note that the observed sample of neutron stars suggests that the birth spins of the black holes in these binaries should be small; therefore, the BH spins in BH-NS binaries should arise almost entirely from the matter they accrete in an HCE phase. Unfortunately, the HCE phase is poorly understood. While our result is a first step of a program to better understand the expected BH spins for those systems that LIGO and other gravitational wave detectors could observe, more simulations of these processes are needed to obtain firmer conclusions.

Furthermore, on the basis of our rough understanding of the range of expected spin magnitudes, and using the expected effect of spin on gravitational wave detection drawn from Grandclément et al. (2004), we demonstrated that neglecting spin in gravitational wave searches should only moderately reduce the detection rate. However, our analysis was based on the work of Grandclément et al. (2004), which, like many papers in the gravitational wave literature, assumed that all BHs in BH-NS binaries had $M = 10 M_\odot$ (see Bulik et al. 2004 for the first discussion of the implications of realistic mass and mass-ratio distributions on gravitational wave detection). In fact, on the basis of our calculations we expect that (1) the mass ratio can vary and should be biased toward larger values (i.e., closer to 3 : 1 than 10 : 1.4) and (2) the sample of BH-NS binaries should show strong correlations between mass ratio and spin magnitude, because higher mass BHs are harder to spin up. We hope to undertake a more thorough Monte Carlo study of the effect of various expected distributions for BH spin magnitude, spin tilt, and BH-NS mass ratio in a future paper.

Finally, we note that in our simulations we consider only BH-NS binaries in the Galactic field; we do not account for any stellar interactions relevant to centers of globular clusters and similar dense environments. If BH-NS binaries form in significant numbers in cluster centers, then their spin properties could be entirely unrelated to the analysis presented in this study.

4 Specifically, they plot the average of the cube of the fitting factor vs. the BH spin $a$. 
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REFERENCES

Apostolatos, T. A. 1996, Phys. Rev. D, 54, 2438
Apostolatos, T. A., Cutler, C., Sussman, G. J., & Thorne, K. S. 1994, Phys. Rev. D, 49, 6274
Baaijens, L., Hawke, I., Montero, P. J., Löffler, F., Rezzolla, L., Stergioulas, N., Font, J. A., & Seidel, E. 2005, Phys. Rev. D, 71, 024035
Bardeen, J. M. 1970, Nature, 226, 64
Beckwith, K., & Done, C. 2004, MNRAS, 352, 353
Belczynski, K., Kalogera, V., & Bulik, T. 2002, ApJ, 572, 407 (BKB02)
Brown, G. E. 1995, ApJ, 440, 270
Brown, G. E., Lee, C.-H., & Bethe, H. A. 2000, ApJ, 541, 918
Bulik, T., Gondek-Rosinska, D., & Belczynski, K. 2004, MNRAS, 352, 1372
Buonanno, A., Chen, Y., Pan, Y., & Vallisneri, M. 2004, Phys. Rev. D, 70, 104003
Burgay, M., et al. 2003, Nature, 426, 531
Duez, M. D., Liu, Y. T., Shapiro, S. L., & Stephens, B. C. 2004, Phys. Rev. D, 69, 104030
Fryer, C. L., Benz, W., & Herant, M. 1996, ApJ, 460, 801
Grandclement, P., Ihm, M., Kalogera, V., & Belczynski, K. 2004, Phys. Rev. D, 69, 102002
Grandclement, P., Kalogera, V., & Vecchio, A. 2003, Phys. Rev. D, 67, 042003
Kramer, M., et al. 2003, MNRAS, 342, 1299
Lindblom, L., & Owen, B. 2002, Phys. Rev. D, 65, 063006
Lorimer, D. R., et al. 2005, in ASP Conf. Ser. 328, Binary Radio Pulsars, ed. F. A. Rasio & I. H. Stairs (San Francisco: ASP), 113
Migliazzo, J. M., Gaensler, B. M., Backer, D. C., Stappers, B. W., Strom, R. G., & van der Swaluw, E. 2002, in ASP Conf. Ser. 271, Neutron Stars in Supernova Remnants, ed. P. O. Slane & B. M. Gaensler (San Francisco: ASP), 57
Miller, J. M., et al. 2002, ApJ, 570, L69
———. 2004a, ApJ, 601, 450
———. 2004b, ApJ, 606, L131
O’Shaughnessy, R., Kalogera, V., & Belczynski, K. 2005a, ApJ, 620, 385 (OKB05)
O’Shaughnessy, R., Kim, C., Fragos, T., Kalogera, V., & Belczynski, K. 2005b, ApJ, in press
Pan, Y., Buonanno, A., Chen, Y., & Vallisneri, M. 2004, Phys. Rev. D, 69, 104017
Remillard, R. A., Muno, M. P., McClintock, J. E., & Orosz, J. A. 2002, ApJ, 580, 1030
Rezzolla, L., Yoshida, S., Maccarone, T. J., & Zanotti, O. 2003, MNRAS, 344, 137
Shapiro, S. L., & Shibata, M. 2002, ApJ, 577, 904
Strohmayer, T. E. 2001, ApJ, 552, L49
Thorne, K. S. 1974, ApJ, 191, 507
Török, G., Abramowicz, M. A., Kluzniak, W., & Stuchlík, Z. 2005, A&A, 436, 1