On the Capacity of Pairwise Collaborative Networks

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Abstract—We derive expressions for the achievable rate region of a collaborative coding scheme in a two-transmitter, two-receiver Pairwise Collaborative Network (PCN) where one transmitter and receiver pair, namely relay pair, assists the other pair, namely the source pair, by partially decoding and forwarding the transmitted message to the intended receiver. The relay pair provides such assistance while handling a private message. We assume that users can use the past channel outputs and can transmit and receive at the same time and in the same frequency band. In this collaborative scheme, the transmitter of the source pair splits its information into two independent parts. Interestingly, the relay pair employs the decode and forward coding to assist the source pair in delivering a part of its message and re-encodes the decoded message along with private message, which is intended to the receiver of the relay pair, and broadcasts the results. The receiver of the relay pair decodes both messages, retrieves the private message, re-encodes and transmits the decoded message to the intended destination. We also characterize the achievable rate region for Gaussian PCN. Finally, we provide numerical results to study the rate trade off for the involved pairs. Numerical result shows that the collaboration offers gain when the channel gain between the users of the relay pair are strong. It also shows that if the channel conditions between transmitters or between the receivers of the relay and source pairs are poor, such a collaboration is not beneficial.

Index Terms—Pairwise collaborative network, rate splitting, decode and forward.

I. INTRODUCTION

In a multi user network users may collaborate to jointly convey the information. Van der Meulen [1] introduced the relay channel where a relay forwards the data from a source to the destination. Cover and ElGamal [2] proved some capacity theorems for a single relay channel. In a collaborative network, users may collaborate to transmit message of other users while handling their own private messages; this can be regarded as a generalization of the traditional relay channel. We present a pairwise relaying collaboration model where a pair of transmitter and receiver collaborates with the source pair in delivering the message of the source pair along with its own private message. Our proposed model differs from previous research in that we consider collaboration schemes that the transmitter and receiver of the relay pair handles a private message, which, to the best of our knowledge, no previous work has considered in this setting. Figure 1 represents such a network where the 1st user, intends to send a message to the 4th user, and the 2nd user to the 3rd user. We propose two collaboration schemes where in the first scheme, the transmitter of the relay pair, the 1st user, splits its message into two independent parts. The relay pair collaborates with the source pair via decode and forward coding to transmit a part of the message of the source pair and the private message of the relay pair. In the second scheme the relay pair partially cancels the interference of other users and sends the compressed observed signal to the intended receiver of source pair, the 4th user.

Collaboration between wireless users has been investigated recently by several authors. Liang and Veeravalli [3] studied a cooperative relay broadcast channel with three users where relay links are incorporated into standard two-user broadcast channels to support user cooperation. Liang and Kramer [4] have found improved bounds for the relay broadcast channel. Tannious and Nosratinia in [5] developed decode and forward and compress and forward strategies for a network of one relay channel with private messages where in addition to the traditional communication from source to destination (assisted by relay), the source has a private message for the relay, and the relay has a private message for the destination (see [6] for a survey on decode and forward and compress and forward strategies). Akhavan and Gazor [7] investigated multi-hopping strategies and resource allocation in such networks. Reznik, Kulkarni and Verdu in [8] further studied the relay broadcast collaborative model for the case of more than two destinations. Sendonaris, Erkip and Aazhang in [9], [10] showed that collaboration enlarges the achievable rate region in a channel with two collaborative transmitters and a single receiver. Laneman, Tse and Wornell considered a fading channel with two cooperative transmitters and two non-cooperative receivers [11]. Host-Madsen in [12], [13] presented the achievable rate regions for channels with transmitter and/or receiver collabo-
ration. Ng, Jindal, Goldsmith and Mitra [14] investigated capacity improvement from transmitter and receiver cooperation in a two-transmitter, two-receiver network.

In this paper, we extend the results of [5], [7] and study the achievable rate region of the decode and forward coding schemes in the PCN. We present in Section II the network model. In Section III we develop the rate splitting in conjunction with decode and forward coding scheme for the PCN and determine its capacity. We investigate the additive white Gaussian noise PCN rate region in Section IV. Finally in Section V we give the concluding remarks.

II. System Model

The PCN consists of inputs $x_i$ where $i \in \{1, 2, 3\}$, outputs $y_j$ where $j \in \{2, 3, 4\}$ and the transition probability $p(y_2, y_3, y_4 | x_1, x_2, x_3)$ (see Figure I). The 1st user wishes to send the message $w_1$ to the 4th user, while the 2nd user wishes to send the message $w_2$ to the 3rd user.

We define an additive white Gaussian noise (AWGN) PCN with the input output relation: $Y_i = Z_i + \sum_{j \neq i} \sqrt{h_{ij}} X_j$ where $X_i, Y_i$ and $Z_i$ denote input, output and channel noise with normal distributions, i.e. $Z_i \sim \mathcal{N}(0, N_i)$, respectively. Let denote the power gain of the communication channel between the $i$th and $j$th user by $h_{ij}$. We impose the power constraints $E(X_i^2) \leq P_i$ for all channel inputs. We assume that the users can transmit and receive at the same time and in the same frequency band.

In this paper, let $X$, $x$ and $\bar{x}$ denote a random variable, a scalar and a vector, respectively. We define $x = 1 - x$ and $\mathcal{C}(x) = \frac{1}{2} \log(1 + x)$.

III. Collaboration via Partial Decode-and-Forward

In this section we consider a collaborative scheme, partial decode and forward, which includes rate splitting technique at the source pair transmitter and decode and forward relaying at the relay pair transmitter and receiver. The source pair transmitter splits the message $w_1$ into two independent parts, $w_{11}$ and $w_{12}$. The source pair transmitter, 1st user, encodes $w_{11}$ and $w_{12}$ to the codeword $x_1$. The relay pair transmitter, 2nd user, decodes $w_{12}$ and re-encodes both its message, i.e. $w_2$, and $w_{12}$, to $x_2$. The relay pair receiver, 3rd user, retrieves $w_{12}$ and $w_2$ from $y_3$ and re-encodes $w_{12}$ to $x_3$. Finally, the source pair receiver, 4th user, by using $y_4$ estimates its intended message $w_{12}$ and $w_{11}$.

In the following we prove that the rates $(R_1, R_2)$, given by (I) shown at the top of the next page, are achievable for the PCN for some joint distribution

$$p(x_3) p(w_1 | x_3) p(w_2 | x_1, x_3) p(x_2 | x_1, u_1 x_3).$$

For this achievable region, we apply Fourier-Motzkin elimination to eliminate $R_11$ and $R_21$ from the bounds and then obtain the region (I) which provides a simpler form.

We use the coding strategies developed in [2], [5], [6], [15] for relay and multiple access channels (MACs). The 1st user uses a three-level superposition block Markov encoding, while the 2nd user uses a two-level and the 3rd user a single-level superposition coding. Furthermore, we use the regular encoding/backward decoding techniques. We divide the messages $w_1$ and $w_2$ into $B$ blocks for $b = 1, 2, ..., B$ and send these message blocks in $B + 2$ transmission blocks. In the following we construct the codebooks and discuss the decoding in each block.

Random Codebook Construction:

1) We generate $2^n R_1$ i.i.d. $x_3 = (x_{31}, x_{32}, ..., x_{3n})$ sequences, each with distribution $p(x_3) = \prod_{i=1}^{n} p(x_{3i})$ and label them $x_3(w'_1)$.  
2) For each $x_3(w'_1)$, we generate $2^n R_2$ i.i.d. $x_2$ sequences, each with distribution $p(x_2) = \prod_{i=1}^{n} p(x_{2i}|x_{3i}, u_{1i})$ and label them $x_2(w_2, w'_3)$.  
3) For each pair $x_3(w'_1, x_2)$ and $x_3(w'_1)$, we generate $2^n R_2$ i.i.d. $x_2$ sequences, each with distribution $p(x_2) = \prod_{i=1}^{n} p(x_{2i}|x_{3i}, u_{1i})$ and label them $x_2(w_2, w'_3)$.  
4) For each pair $x_3(w'_1, x_2)$ and $x_3(w'_1)$, we generate $2^n R_2$ i.i.d. $x_2$ sequences, each with distribution $p(x_2) = \prod_{i=1}^{n} p(x_{2i}|x_{3i}, u_{1i})$ and label them $x_2(w_2, w'_3)$.  
5) For each triplet $w_1(w'_1, w'_2, w'_3)$, we generate $2^n R_1$ i.i.d. $x_3$ sequences, each with distribution $p(x_3) = \prod_{i=1}^{n} p(x_{3i}|x_{3i}, u_{1i}, u_{2i})$ and label them $x_3(w_1, w'_2, w'_3)$.  

Encoding:

For each time $b = 1, 2, ..., B + 2$ the users send the following sequences:

1) $x_3(w_{11:b}, w_{12:b}, 1, 1), x_2(w_{2:b}, 1, 1), x_3(1) b = 1$  
2) $x_3(w_{11:b}, w_{12:b}, w_{12:b-1}, 1), x_2(w_{2:b}, w_{12:b-1}, 1), x_3(1) b = 2$  
3) $x_3(w_{11:b}, w_{12:b}, w_{12:b-1}, w_{12:b-2}), x_2(w_{2:b}, w_{12:b-1}, w_{12:b-2}), x_3(w_{12:b-2}) b = 3, ..., B$  
4) $x_3(1, 1, w_{12:b-1}, w_{12:b-2}), x_2(1, w_{12:b-1}, w_{12:b-2}), x_3(w_{12:b-2}) b = B + 1$  
5) $x_3(1, 1, w_{12:b-2}), x_2(1, w_{12:b-2}), x_3(w_{12:b-2}) b = B + 2$  

Decoding:

1) The 2nd user decodes $w_{12:b}$ by looking at $w_{12:b}$ such that $x_2(w_{2:b}, w_{12:b-1}, w_{12:b-2}), x_3(w_{12:b-2}), x_3(w_{12:b-1}, w_{12:b-2})$ and $x_2(w_{2:b}, w_{12:b}, w_{12:b-1}, w_{12:b-2})$ are jointly typical. The decoding is reliable if $R_2 < I(Y_2; U_2 | X_2, X_3)$.  
2) The 3rd user decodes $w_{12:b}, w_{12:b-1}, w_{11:b}$ and $w_{12:b}$ by looking at $w_{12:b}, w_{12:b-1} - w_{11:b}$ and $w_{12:b}$ such that $x_2(w_{2:b}, w_{12:b-1}, w_{12:b-2}), x_2(w_{2:b}, w_{12:b}, w_{12:b-1}, w_{12:b-2}), x_2(w_{2:b}, w_{12:b-1}, w_{12:b-2}), x_3(w_{12:b-2})$ and $x_2(w_{2:b}, w_{12:b}, w_{12:b-1}, w_{12:b-2})$ are jointly typical. Here, the 1st and 2nd users attempt to transmit a common message $w_{12}$ along with their private messages, i.e. $w_{11}$ and $w_2$, respectively. It is shown in [15], [16] that this step can be made reliably...
The rate region in (1) follows from combining the achievable
in the scenario where pairs do not collaborate. In the absence
\[ R_{11} < \min \{ I(Y_4; X_1 | U_1, U_2, X_2, X_3), I(Y_3; X_1 | X_2, X_3, U_1, U_2) \} \]
\[ R_{12} < I(Y_2; U_2 | U_1, X_2, X_3) \]
\[ R_2 < \min \{ I(Y_3; X_2 | X_1, X_3, U_1, U_2), I(Y_4; X_2 | U_1, U_2, X_1, X_3) \} \]
\[ R_2 + R_{11} < \min \{ I(Y_3; X_1, X_2 | X_3, U_1, U_2), I(Y_4; X_1, X_2 | U_1, U_2, X_1, X_3) \} \]
\[ R_2 + R_{11} + R_{12} < \min \{ I(Y_3; X_1, X_2, U_1, U_2 | X_3), I(Y_4; U_1, U_2, X_1, X_2, X_3) \} \]  
\[ R_1 = R_{11} + R_{12} \]

Now, we concentrate on the AWGN PCN. Employing partial
decide and forward scheme, the rates \((R_1, R_2)\), are achievable
for the AWGN PCN:
\[ R_1 < C \left( \frac{\alpha \beta h_{12} P_1}{\delta h_{12} P_1 + N_2} \right) + \phi_1 \]
\[ R_2 < \min \left\{ C \left( \frac{\beta h_{23} P_2}{N_3} \right), \frac{\delta h_{24} P_2}{N_4} \right\} \]
\[ R_1 + R_2 < \min \{ \phi_2, C \left( \frac{\alpha \beta h_{12} P_1}{\alpha \beta h_{12} P_1 + N_2} \right) + \phi_3 \} \]  
where \(\phi_1, \phi_2\) and \(\phi_3\) are given by (3) shown at the top of the
following page. We use the following independent normal
distributions to find the rate region for the partial decode
and forward coding scheme (2): \(A \sim \mathcal{N}(0, \alpha P_1), B \sim \mathcal{N}(0, \alpha \beta P_1), C \sim \mathcal{N}(0, \alpha \beta \gamma P_1), D \sim \mathcal{N}(0, \alpha \beta \gamma P_1), E \sim \mathcal{N}(0, \delta P_3), \) \(\alpha, \beta, \gamma, \delta \in [0, 1]. \) Furthermore, we let \(X_1 = A + B + C + D, X_2 = E + C + D, X_3 = D, U_1 = C + D\) and \(U_2 = B + C + D. \)

Here, we move on to study the condition under which collaboration
improves the achievable rate of pairs. Numerical result shows that the collaboration offers capacity gain when
the channel gain between the source pair is week and the corre-
sponding channel between relay users is strong. It also shows
that if the channels condition between the transmitters, \(h_{12}\) and between the receivers, \(h_{34},\) are poor, such a collaboration
is not beneficial.

We consider the AWGN PCN with \(P_1 = P_2 = P_3 = 1\)
and \(N_2 = N_3 = N_4 = 1\) and we examine the proposed
collaboration scheme under different channel conditions.
We compare the achievable rate region of the proposed scheme
with the scheme where pairs do not collaborate, i.e. interference
channel. We also consider the scenario where both pairs
acquire the same rate as \(R_1 = R_2\) and study the capacity gain
of the schemes.

Figure 4 demonstrates the trade off between the achieved
rate region of the source and relay pairs. The achievable rate
region of the source pair expands as the transmitter increases
its transmit power. Similar to the source pair, increasing the
transmit power of the relay pair increases the achievable rate
of the relay pair.
First we investigate the case where the communication channel between the source pair is poor. The channel condition \(h_{12} = 1, h_{13} = 10, h_{14} = 1, h_{23} = 10; h_{24} = 10\) and \(h_{34} = 1\) exemplifies such a condition. Figure 2(a) shows the rate region of the involved pairs employing the proposed collaboration scheme in conjunction with rate region of interference channel. We observe that under this condition, collaboration offers small capacity gain. We also observe that the relay pair has incentive to collaborate and obtain more rate than interference channel, only if the source pair demands for more rate. We plotted the line \(X = Y\) to investigate the achievable rate on condition that both pairs demands equal rates, i.e. \(R_1 = R_2\). We observe that under this condition collaboration is not beneficial.

Exploiting strong communication links between the 1st and the 2nd users and between the 3rd and the 4th users, pairs obtain a considerable capacity gain which is shown in Figure 2(b) with \(h_{12} = 10, h_{13} = 10, h_{14} = 1, h_{23} = 10; h_{24} = 10\) and \(h_{34} = 10\). In this scenario, the source pair is suffering from poor direct channel gain, between the 1st and the 4th users, however, the communication channel between pair users to relay users is strong. In that case collaboration enhances the achievable rate of both pairs. It also offers dramatic gain if both pairs are interested in equal rates.

Collaboration only improves the achievable rate if the direct link between relay pair, i.e. the 2nd and the 3rd user, is strong. Otherwise as shown in Figure 2(c) with \(h_{12} = 10, h_{13} = 10, h_{14} = 10, h_{23} = 10; h_{24} = 10\) and \(h_{34} = 10\), collaboration does not enlarge the rate region. However, increasing the channel gain between the 3rd and 4th users and between the 1st and the 2nd, the pairs gain as much as the non collaborative scheme (see Figure 2(d)), with \(h_{12} = 1, h_{13} = 10, h_{14} = 10, h_{23} = 10; h_{24} = 10\) and \(h_{34} = 1\). This emphasizes that the efficiency of proposed scheme significantly depends on the channel gain between the relay users.

Lastly, in equal channel condition for both pairs, i.e. \(h_{12} = 10, h_{13} = 10, h_{14} = 10, h_{23} = 10; h_{24} = 10\) and \(h_{34} = 10\) (Figure 2(e)), the channel gain between the 1st and the 2nd users and between the 3rd and the 4th users, increases the capacity gain for involved pairs.

V. Conclusion

We have considered a network of collaborative transmitter-receiver pairs in which one pair (relay pair) acts as relay to assist the source pair in delivering the message of the source pair as well as its own private message. We have studied partial decode and forward collaborative schemes and established the capacity of this coding schemes for the PCN. In the proposed scheme we let the transmitter of the source pair to split its message into two independent parts. The relay pair decodes and forwards only one part of the message of the source pair and re-encodes and transmits the decoded message along with the relay pair private message. Having decoded both messages, the receiver of the relay pair decodes and transmits the message of the source pair to the intended destination. For AWGN PCNs, we have characterized the achievable rate regions. We have also provided numerical results and compared the proposed collaboration scheme with achievable rate of a non collaborative scheme, i.e. interference channel. We have examined the channel conditions under which such a collaboration is beneficial. We have shown that when the channel gain between the source pair is weak collaboration offers capacity gain to both pairs. However, if the channels condition between the involved pairs are poor such a collaboration is not beneficial.

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Fig. 2. The achievable rate region for collaborative, partial decode and forward (PDF) and non collaborative, interference channel (IFC) in a pairwise collaborative network with different scenarios for channel conditions: a) $h_{12} = 1, h_{13} = 10, h_{14} = 1, h_{23} = 10, h_{24} = 10$ and $h_{34} = 1$ b) $h_{12} = 10, h_{13} = 10, h_{14} = 1, h_{23} = 10, h_{24} = 10$ and $h_{34} = 10$ c) $h_{12} = 10, h_{13} = 10, h_{14} = 10, h_{23} = 1, h_{24} = 10$ and $h_{34} = 10$ d) $h_{12} = 1, h_{13} = 10, h_{14} = 10, h_{23} = 10, h_{24} = 10$ and $h_{34} = 1$ e) $h_{12} = 10, h_{13} = 10, h_{14} = 10, h_{23} = 10, h_{24} = 10$ and $h_{34} = 10$.

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