Prescaling and far-from-equilibrium hydrodynamics in the quark-gluon plasma

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Prescaling is a far-from-equilibrium phenomenon which describes the rapid establishment of a universal scaling form of distributions much before the universal values of their scaling exponents are realized. We consider the example of the spatio-temporal evolution of the quark-gluon plasma explored in heavy-ion collisions at sufficiently high energies. Solving QCD kinetic theory with elastic and inelastic processes, we demonstrate that the gluon and quark distributions very quickly adapt a self-similar scaling form, which is independent of initial condition details and system parameters. The dynamics in the prescaling regime is then fully encoded in a few time-dependent scaling exponents, whose slow evolution gives rise to far-from-equilibrium hydrodynamic behavior.

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Introduction. Universal scaling phenomena play an important role in our understanding of the thermalization process in quantum many-body systems. Topical applications range from heavy-ion collisions [1–3] to quenches in ultracold quantum gases [4–7]. Starting far-from-equilibrium, these systems exhibit transients a nonthermal fixed point regime where the time evolution of characteristic quantities becomes self-similar. As a consequence, details about initial conditions and underlying system parameters become irrelevant in this regime, and the nonequilibrium dynamics is encoded in universal scaling exponents and functions [2, 4, 8–12].

In heavy-ion collisions at sufficiently high energies, where the gauge coupling is small due to asymptotic freedom [13, 14], the time evolution of gluons (g) and quarks (q) is described by distribution functions $f_{g,q}(p\perp,p_z,\tau)$. Since the system is longitudinally expanding, the distributions depend on transverse ($p\perp$) and longitudinal momenta ($p_z$), and on proper time ($\tau$) [15, 16]. In the scaling regime the gluon distribution obeys

$$f_g(p\perp,p_z,\tau)\overset{\text{scaling}}{=}\tau^\gamma f_S(\tau^\beta p\perp,\tau^\gamma p_z),$$

with dimensionless $\tau \to \tau/\tau_{ref}$ and $p\perp,z \to p\perp,z/Q_s$, in terms of some (arbitrary) time $\tau_{ref}$ and characteristic momentum scale $Q_s$. The exponents $\alpha, \beta, \gamma$ are universal and the nonthermal fixed-point distribution $f_S$ is universal up to normalizations [2], which has been established numerically using classical-statistical lattice simulations [10]. The exponents are expected to be $\alpha_{BMSS} = -2/3$, $\beta_{BMSS} = 0$ and $\gamma_{BMSS} = 1/3$ according to the first scenario of ‘bottom up’ thermalization [1] based on number conserving and small-angle scatterings and neglecting inelastic processes at the same order in the coupling, or $\alpha_{BD} = -3/4$, $\beta_{BD} = 0$ and $\gamma_{BD} = 1/4$ in a variant of ‘bottom-up’ including the effects of plasma instabilities [17].

In this work we compute the evolution of the quark-gluon plasma approaching the nonthermal fixed-point using leading-order QCD kinetic theory [18]. Since this state-of-the-art description involves elastic and inelastic processes, conservation of particle number is not build in, and no small-angle approximation is assumed. In its range of validity it goes beyond classical-statistical approximations by including quantum effects that capture the late-time approach to thermal equilibrium [3, 19, 20].

We establish that the far-from-equilibrium dynamics according to leading-order QCD kinetic theory exhibits self-similar scaling. Comparing to the dynamics with elastic scatterings only, the softer momentum regions are efficiently populated by collinear radiation processes, which is seen to improve the universal scaling behavior of the distributions.

Most remarkably, we find that much before the scaling (1) with universal exponents is established, the evolution is already governed by the fixed-point distribution $f_S$ as

$$f_g(p\perp,p_z,\tau)\overset{\text{prescaling}}{=}\tau^{\alpha(\tau)} f_S(\tau^{\beta(\tau)} p\perp,\tau^{\gamma(\tau)} p_z),$$

with non-universal time-dependent exponents $\alpha(\tau)$, $\beta(\tau)$ and $\gamma(\tau)$. This represents a dramatic reduction in complexity already at this early stage: The entire evolution in this prescaling regime is encoded in the time dependence of a few slowly evolving exponents, and we point out the relation to hydrodynamic behavior far from equilibrium.

The phenomenon of prescaling describes the rapid establishment of universal nonequilibrium results for certain quantities ($f_S$), though others still deviate from their universal values ($\alpha, \beta, \gamma$). This has to be distinguished from standard corrections due to finite size/time scaling behavior, from which asymptotic universal values are inferred without taking the infinite volume/time limit. Coined by Wetterich based on the notion of partial fixed points [21, 22], prescaling has recently been explored in the context of scaling violations in the short-distance behavior of correlation functions for Bose gases [23].

QCD kinetic theory. We employ the leading order QCD kinetic theory of Ref. [18] to evolve the gluon and quark distributions by

$$\partial_\tau f_g(p,\tau) - \frac{p_z}{\tau} \partial_{p_z} f_g(p,\tau) = -C_g^{2+2}[f] - C_g^{1+2}[f],$$

as
and equivalently for \( f_q \) with collision integrals \( C_{g,q}^{2+2} \) and \( C_{q,q}^{1+2} \), using the numerical setup of Refs. [19, 20, 24]. Here \( C_{g,q}^{2+2}[f] \) represent the collision integrals for leading-order elastic scatterings in the coupling \( \alpha_s \equiv g^2/(4\pi) \). This involves scatterings \( gg \leftrightarrow gg, qg \leftrightarrow qg, gg \leftrightarrow qq \) as well as particle conversion \( gg \leftrightarrow qq \) processes. To this order we also include number changing processes \( C_{g,q}^{1+2}[f] \) of medium induced collinear gluon radiation \( g \leftrightarrow gg, q \leftrightarrow qg \) and quark pair production \( g \leftrightarrow q\bar{q} \). The effective 1 \( \leftrightarrow \) 2 splitting rate is calculated by the resummation of multiple interactions with the medium and includes the Landau-Pomeranchuk-Migdal suppression of collinear radiation [25–28]. The soft momentum exchange is regulated by isotropic screening [29].

From free-streaming to universal scaling. We consider the initial distributions \( f_{g,q}^0(p) = A_{g,q} \exp\left(-\langle p^2 \rangle + \xi^2 p^2_\perp/Q_s^2 \right) \), where \( A_g = 0.5 \) for quarks and \( A_g = \sigma_0 g^2 \) for gluons. This reflects that for energetic collisions the bosonic gluons are expected to be highly occupied \( \sim g^2 \) with a characteristic momentum scale \( Q_s \), while fermion occupancies are bounded \( f_q \leq 1 \) by Fermi-Dirac statistics [30, 31]. Here \( \sigma_0 \) characterizes the initial gluon density, taken to be \( \sigma_0 = 0.1, 0.6 \), and \( g = 10^{-3} \) in view of the range of validity of kinetic theory. The initial anisotropy is controlled by \( \xi \), and we employ \( \xi = 2 \). Starting at \( \tau_0 Q_s = 70 \) and choosing \( \tau_\text{ref} Q_s = 7000 \), we solve the coupled set of kinetic equations (3) for the gluon and quark distributions numerically [32].

In Fig. 1, we show the evolution of the pressure anisotropy \( P_L/P_T \) (solid curve) as a function of dimensionless time \( \tau \rightarrow \tau/\tau_\text{ref} \) for \( \sigma_0 = 0.1 \). The longitudinal and transverse pressures are defined from the energy-momentum tensor

\[
T_{\mu\nu}(\tau) = \int \frac{d^3p}{(2\pi)^3} p_\mu p_\nu \left( \langle \nu_g f_g(p, \tau) + 2N_f \nu_q f_q(p, \tau) \rangle \right)
= \text{diag}(e, P_T, P_T, P_L),
\]

where \( \nu_g = 2(N_c^2 - 1) = 16 \) and \( \nu_q = 2N_c = 6 \) for \( N_c = 3 \) colors and \( N_f = 3 \) quark flavors. One observes how the pressure anisotropy starts to deviate from collisionless expansion (dashed line) relevant at earliest times, and bends over to a milder power-law dependence on time once interactions start to compete with expansion. This is in line with previous results using different approximations [10, 19, 33] and, for comparison, we also show the result by taking only elastic collisions into account (dotted curve). The difference to the evolution with full collision kernel is comparably small, indicating that the inelastic processes contribute mainly at low momenta, which are phase-space suppressed for bulk quantities such as pressure.

The emergence of scaling can be efficiently analyzed from moments of the distribution functions for gluons

\[
n_{m,n}(\tau) = \nu_g \int \frac{d^3p}{(2\pi)^3} p_\perp^m p_z^n f_g(p_\perp, p_z, \tau),
\]

and equivalently for quarks. Effectively, different moments probe different momenta and are thus sensitive to

\[
\begin{align*}
\alpha(\tau) &= \frac{\tau}{\tau_\text{ref}} \int \frac{d^3p}{(2\pi)^3} p_\perp^2 f_g(p_\perp, p_z, \tau), \\
\beta(\tau) &= \frac{\tau}{\tau_\text{ref}} \int \frac{d^3p}{(2\pi)^3} p_z^2 f_g(p_\perp, p_z, \tau), \\
\gamma(\tau) &= \frac{\tau}{\tau_\text{ref}} \int \frac{d^3p}{(2\pi)^3} p_\perp p_z f_g(p_\perp, p_z, \tau).
\end{align*}
\]

These are the fundamental scaling moments of QCD that can be related to pressure anisotropies and momentum flows while retaining information about momentum-space correlations.

In Fig. 2, we show the time-dependent scaling exponents (dotted curves) for different initial settings with \( \sigma_0 = 0.1 \). The results are in line with expectations from universal scaling [10, 19, 33]. For \( \sigma_0 = 0.1 \), the pressure anisotropy is dominated by the elastic scatterings (dashed line), while the result by taking only elastic collisions into account (dotted curve) is in line with previous results using different approximations [10, 19, 33]. The difference to the evolution with full collision kernel is comparably small, indicating that the inelastic processes contribute mainly at low momenta, which are phase-space suppressed for bulk quantities such as pressure.

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In Fig. 3, we show the time-dependent scaling exponents (dotted curves) for different initial settings with \( \sigma_0 = 0.6 \). The results are in line with expectations from universal scaling [10, 19, 33]. For \( \sigma_0 = 0.6 \), the pressure anisotropy is dominated by the elastic scatterings (dashed line), while the result by taking only elastic collisions into account (dotted curve) is in line with previous results using different approximations [10, 19, 33]. The difference to the evolution with full collision kernel is comparably small, indicating that the inelastic processes contribute mainly at low momenta, which are phase-space suppressed for bulk quantities such as pressure.
scaling in a particular momentum regime. If a distribution function shows prescaling (2), the precise values of $\alpha(\tau), \beta(\tau)$ and $\gamma(\tau)$ in general depend on the history of the evolution from the reference time 1 to $\tau$. Therefore, it can be useful to redefine the exponents in (2) to reflect the instantaneous scaling properties with

$$\tau^{\alpha(\tau)} \rightarrow \exp \left[ \int_1^{\tau} \frac{d\tau'}{\tau} \alpha(\tau) \right],$$

where for constant $\alpha$ reduces to the power law $\tau^\alpha$. Then the rate of change of a particular moment $n_{m,n}$ is given by a linear combination of scaling exponents

$$\frac{d \log n_{m,n}(\tau)}{d \log \tau} = \alpha(\tau) - (m+2) \beta(\tau) - (n+1) \gamma(\tau).$$

By looking at different moments $n,m = 0,1,\ldots$ one obtains a set of algebraic equations from which $\alpha(\tau), \beta(\tau)$ and $\gamma(\tau)$ can be determined. Since the choice of moments is not unique, one can probe different momentum regimes and test how well (pre-)scaling is realized.

The time-dependent exponents obtained from various combinations of moments $n_{n,m}$ with $n,m < 4$ for initial conditions with $\sigma_0 = 0.1$ are shown in Fig. 2, exhibiting a remarkable overlap of the results from different moment ratios [34]. In this case, one expects free-streaming scaling exponents $\alpha \rightarrow 0, \beta \rightarrow 0, \gamma \rightarrow 1$ at very early times. Accordingly, both $\alpha(\tau)$ and $\gamma(\tau)$ approach the nonthermal fixed-point limit from above for initial conditions with $\sigma_0 = 0.1$ [35]. At later times $\tau > 1$ we fit the power-laws with constant exponents and obtain $\alpha \approx -0.73$, $\beta \approx -0.01$ and $\gamma \approx 0.29$. These values are reasonably close to the analytic estimates given above and consistent with previous lattice results [10]. One clearly observes the prescaling regime, for which different moments can be described by the common set of time-dependent scaling exponents even before the asymptotic scaling is reached. To check the sensitivity of the prescaling regime to initial conditions, we varied the shape of initial gluon distribution, while keeping the initial gluon number and energy densities fixed. Only a moderate spread in exponent values was observed, comparable to the spread from the different sets of moments shown in Fig. 2. However, to emphasize that the time-dependence of the exponents is not universal, we show in Fig. 3 the results for larger initial gluon density $\sigma_0 = 0.6$ such that free streaming is suppressed. In this case we also see that at very early times $\tau < 0.03$ there is no unique notion of scaling exponents. But very quickly the results from different sets of moments collapse again to a single curve, much before the exponents attain their universal constant values.

Universal scaling form of the distributions. With the results for exponents, we can now extract the universal scaling form $f_S$. We first consider rescaling with the constant values of exponents obtained from the late time fit. The left panel of Fig. 4 shows the rescaled gluon distribution $\tau^{-\alpha}g^2f_g$ as a function of $p_\perp$ at different times $\tau$ for $p_z = 0$ (solid lines) for initial conditions with $\sigma_0 = 0.1$. After an initial period, all rescaled curves at different times collapse to a single scaling curve to very good accuracy.

We see that with full collision kernel, the low-momentum part of the distribution function develops a $\sim 1/p_T$ behavior. In contrast, only elastic processes are not efficient in developing these thermal-like features of a low-momentum bath (grey dashed curves) [36]. The softer momentum region is efficiently populated by the collinear radiation processes, and we observe excellent scaling properties also in that regime where particle number changing processes are essential.

Prescaling states that the very same distribution function $f_S$ can be extracted at much earlier times, before the scaling exponents take on their universal values. To verify this, we rescale the distribution function according to

![Graph](image)
\[ \tau^{-\alpha} g^2 f_{g}(p_{\perp} = 1, p_{z}, \tau) \]

\[ \tau^{-\gamma} p_{z} \]

\[ \tau^{\gamma(\tau)} p_{z} \]

\[ J^{\mu} = \nu_{g} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{p^{\mu}}{p^{0}} f_{p}, \]

\[ I^{\mu\nu\sigma} = \nu_{g} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{p^{\mu}p^{\nu}p^{\sigma}}{p^{0}} f_{p}, \]

where we focus on the gluonic part, i.e. \( \nu_{q} = 0 \). Integrating the kinetic equation (3) with the appropriate powers of \( p^{0} \) yields the equations of motion for these quantities.

For the case of homogeneous boost-invariant expansion of a conformal system, (8) and (9) have only three independent components, namely the particle number density \( n = \eta^{0} \) and the tensors \( 2I^{0xx} = 2I^{0yy} \) and \( I^{0zz} \), which together with \( T^{00} \) evolve according to

\[ \partial_{\tau} n + \frac{n}{\tau} = -C_{J} \]

\[ \partial_{\tau} T^{00} + \frac{T^{00} + T^{zz}}{\tau} = 0 \]

\[ \partial_{\tau} I^{0xx} + \frac{I^{0xx}}{\tau} = -C_{J}^{xx} \]

\[ \partial_{\tau} I^{0zz} + \frac{3I^{0zz}}{\tau} = -C_{J}^{zz} \]

with the collision integrals \( C_{J} = \nu_{g} \int d^{3}p/((2\pi)^{3}p^{0})C[f] \) and \( C_{J}^{\sigma} = \nu_{g} \int d^{3}p/((2\pi)^{3}p^{0})p^{\nu}p^{\sigma}C[f] \). Noting that \( n = n_{0,0} \), \( 2I^{0xx} = n_{0,2} \) and \( I^{0zz} = n_{0,2} \), we see that the equations of motion for these moments can be mapped to the scaling exponents using (7). Specifying the scaling dependence of the collision kernels \( C_{J} \) and \( C_{J}^{xx} \) based on the rescaling property (2) of the distributions then closes the system of hydrodynamic equations of motions. This generalized hydrodynamic description around the nonthermal fixed-point distribution \( f_{S} \) resembles anisotropic hydrodynamic formulations, which
are based on expansion around the deformed equilibrium distribution [38–40]. A major difference is that in our formulation the anisotropic distribution is a scaling solution which is a result of the far-from-equilibrium dynamics. We leave the solution of the generalized hydrodynamic equations to a subsequent study.

**Conclusion.** The scaling regime of far-from-equilibrium QGP near the nonthermal fixed point reflects an enormous simplification of the plasma’s temporal evolution, which is governed only by the rescaling of the arguments and the overall normalization of the universal scaling distribution function $f_S$. We point out that this loss of detail does not rely on the scaling exponents being constant, but imposes relations between different moments of the distribution function. Using QCD kinetic theory we demonstrate that longitudinally expanding weakly coupled QGP evolves transiently towards a nonthermal fixed-point distribution with universal scaling exponents, which we confirm in the presence of both elastic and particle number changing processes. The approach to the asymptotics in the *prescaling* regime is characterised by slowly changing scaling exponents, which give rise to the hydrodynamic description of the far-from-equilibrium system, which is not based on an approach to local thermal equilibrium. The subsequent evolution of QGP towards thermal equilibrium with QCD kinetic theory is the subject of a separate work [20].

Though we have focussed on prescaling in the quark-gluon plasma, the generalized prescaling relation (2) should be relevant to other far-from-equilibrium systems exhibiting scaling. For instance, earlier theoretical [4, 5, 12] and very recently also experimental [6, 7] studies on quenches in nonequilibrium Bose gases may be interpreted along these lines, and it would be very interesting to revisit the results in view of our findings. In particular, our method of moments for the extraction of interesting to revisit the results in view of our findings. In particular, our method of moments for the extraction of salient scaling exponents, which we confirm in the presence of both elastic and particle number changing processes. The approach to the asymptotics in the *prescaling* regime is characterised by slowly changing scaling exponents, which give rise to the hydrodynamic description of the far-from-equilibrium system, which is not based on an approach to local thermal equilibrium. The subsequent evolution of QGP towards thermal equilibrium with QCD kinetic theory is the subject of a separate work [20].

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[36] This is in apparent contrast to elastic small-angle scattering approximations, where the $1/p_T$ behavior is also seen [33].

[37] For the relativistic/massless particles the tracelessness conditions read $I^{000} = I^{0xx} + I^{0yy} + I^{0zz}$ and $C^{00} = C^{xx} + C^{yy} + C^{zz}$ with $I^{0xx} = I^{0yy}$ and $C^{xx} = C^{yy}$.

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