General RGEs for dimensionful couplings in the MS scheme

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Recently, Poole & Thomsen have presented the Renormalization Group Equations (RGEs) in the MS scheme for dimensionless parameters of a general renormalizable gauge theory in a new formalism based on the Local Renormalization Group. In this Letter, we apply the dummy field method to the expressions in this formalism in order to derive the RGEs for the dimensionful couplings which so far have been missing in this approach. The complete set of RGEs has been implemented in a new version of the public code PyR@TE dedicated to the automatic computation of the RGEs for any general renormalizable (non-supersymmetric) gauge theory.

Introduction. — The knowledge of Renormalization Group Equations (RGEs) is crucial in the context of studies of gauge theories since they provide the necessary link between the couplings of the theory at different energy scales. The expressions for the dimensionless couplings RGEs of a general gauge theory were first presented almost forty years ago [1-6]. These results were later extended to the case of dimensionful parameters [7] using the so-called dummy field method [7,8], which relates the $\beta$-functions of dimensionful parameters to those of dimensionless ones. A thorough presentation of the dummy field method is given in [9] along with a proper derivation of the general two-loop RGEs for dimensionful couplings in the MS scheme, correcting some of the results presented in [7]. Using these general expressions by hand can be quite involved and error-prone. For this reason, they have been implemented in the Mathematica package SARAH [10] and the Python package PyR@TE [11,12], which are both publicly available.

Recently, a new formalism was proposed [13] for the RGEs of general gauge theories that possesses many interesting features compared to the old Machacek & Vaughn formalism, both on the conceptual and practical sides. Within the framework of the Local Renormalization Group [13,16], relations can be derived that involve the coefficients appearing in the $\beta$-functions of gauge, Yukawa and quartic couplings at different loop orders: these are the so-called Weyl Consistency Conditions. Based on these relations and the existing results in the literature at two-loop, the authors of [13] were able to present for the first time the general expressions of the gauge coupling $\beta$-functions at the 3- and 4-loop order in Yukawa and gauge $\beta$-functions, respectively.

Currently, there is however one missing piece compared to the previous formalism, namely the expressions of the RGEs for all (renormalizable) dimensionful couplings. The aim of the present Letter is to bridge this gap providing general expressions for the $\beta$-functions of dimensionful couplings, obtained by properly applying the dummy field method to the formalism of Poole & Thomsen.

Description of the formalism. — We consider a general gauge theory containing an arbitrary set of real scalars $\phi_a$, and Weyl fermions $\psi_i$. Following [13], we define the Majorana spinor

$$\Psi_i \equiv \left(\begin{array}{c} \psi_i \\ \psi_i^\dagger \end{array}\right).$$

With this definition, the most general Lagrangian density may be written (see [13] for details):

$$\mathcal{L} = -\frac{1}{4} G_{AB}^2 F_{\mu \nu}^A F^{B \mu \nu} + \frac{1}{2} (D_\mu \phi_a (D^\mu \phi)_a - \frac{i}{2} \psi_i \gamma^\mu \sigma^{\mu \nu} \bar{\psi}^j \Psi_j \phi_a - \frac{1}{2} \frac{1}{m_{ij}} \lambda_{ij} \psi_i \psi_j$$

where $G_{AB}$ is a matrix containing the gauge couplings associated to the (semi-)simple gauge group of the theory,
which can be decomposed as:
\[ G = \bigotimes_{u=1}^{N} G_u. \] (3)

Letting \( d_u \) be the dimension of the gauge factor \( G_u \), the covariant derivatives for fermions and scalars are respectively defined as:
\[ D_\mu \Psi_i = \partial_\mu \Psi_i - i \sum_{u=1}^{N} d_u V^A_u (T^A)_{ij} \Psi_j, \] (4)
\[ D_\mu \phi_a = \partial_\mu \phi_a - i \sum_{u=1}^{N} d_a V^A_u (T^A)_{ab} \phi_b. \] (5)

The \( \beta \)-functions of the dimensionless couplings of the model – namely gauge, Yukawa and quartic couplings – are defined as follows [13]:
\[ \beta_{\Delta} \equiv \frac{d\Delta}{dt} = \frac{1}{2} \sum_{\text{perm}} \sum_{\ell} \frac{1}{(4\pi)^{2\ell}} G^2_{\Delta C} \beta^{(\ell)}_{C D} G^2_{D \Delta}, \] (6)
\[ \beta_{\Delta i j} \equiv \frac{d\Delta_{i j}}{dt} = \frac{1}{2} \sum_{\text{perm}} \sum_{\ell} \frac{1}{(4\pi)^{2\ell}} \beta^{(\ell)}_{i j}, \] (7)
\[ \beta_{\Delta a b c d} \equiv \frac{d\Delta_{a b c d}}{dt} = \frac{1}{4!} \sum_{\text{perm}} \sum_{\ell} \frac{1}{(4\pi)^{2\ell}} \beta^{(\ell)}_{a b c d}, \] (8)

with \( \ell \) denoting the perturbative loop-order. We generalize this definition to the dimensionfull couplings of the model (fermion mass, scalar trilinear and scalar mass couplings respectively):
\[ \beta_{i j} \equiv \frac{d\Delta_{i j}}{dt} = \frac{1}{2} \sum_{\text{perm}} \sum_{\ell} \frac{1}{(4\pi)^{2\ell}} \beta^{(\ell)}_{i j}, \] (9)

In [13], the expressions of the dimensionless \( \beta^{(\ell)} \) are given as a sum of individual contributions, each one associated with a particular diagram and weighted by a renormalization scheme-dependent coefficient.

The dummy field method is particularly well-suited for such a diagram-based approach. Starting from the diagrams contributing to the Yukawa and quartic \( \beta \)-functions, the procedure consists in amputating one or two (in the case of scalar mass couplings) scalar legs in order to obtain diagrams respectively contributing to fermion mass, trilinear or scalar mass \( \beta \)-functions.

Results. — We now turn to the presentation of the results. The various quantities appearing in the expressions of the \( \beta \)-functions are directly taken from [13]. Therefore, we invite the reader to refer to the definitions presented therein. In addition, the founding principles of the dummy field method and its application are outlined in [9], allowing us to skip the related technical details and to summarize our results at once. We show below the whole set of RGEs obtained in the \( \overline{\text{MS}} \) scheme for the fermion mass, trilinear and scalar mass couplings. The following equations complete the list of RGEs presented in [13] for the dimensionless couplings. We use a notation where the fermion indices are made implicit. In this context, \( m, y_a \), and any other tensor carrying two fermion indices may be seen as matrices in the space of the fermions of the theory.

**Fermion mass \( \beta \)-functions**

At 1-loop:
\[ \beta^{(1)} = \xi_1^{(1)} m C_2(F) + \xi_2^{(1)} y_s y_t m + \xi_3^{(1)} m \tilde{Y}_2(F). \] (12)

At 2-loop:
\[ \beta^{(2)} = \xi_1^{(2)} \tilde{C}_2(F) m C_2(F) + \xi_2^{(2)} m C_2(F) C_2(F) + \xi_3^{(2)} m C_2(F, G) + \xi_4^{(2)} m C_2(F, S) + \xi_5^{(2)} m C_2(F, F) + \xi_6^{(2)} y_s T^A y_t m T^B G_{A B} + \xi_7^{(2)} Y_2(F) T^A m T^B G_{A B} + \xi_8^{(2)} y_s y_t m C_2(S) |_{bc} + \xi_9^{(2)} Y_2(F, C_S) m + \xi_{10}^{(2)} y_s \tilde{y}_t y_c |_{b c d} + \xi_{11}^{(2)} y_s \tilde{y}_t m \tilde{y}_c y_b + \xi_{12}^{(2)} Y_2(F, C_F) m + \xi_{13}^{(2)} y_s \tilde{y}_t m \tilde{y}_c y_e + \xi_{14}^{(2)} y_s \tilde{y}_t m \tilde{y}_c y_b + \xi_{15}^{(2)} Y_4(F) m + \xi_{16}^{(2)} y_s \tilde{y}_t m \tilde{y}_c Y_2(S) |_{b c} + \xi_{17}^{(2)} Y_2(F, Y_F) m. \] (13)
Trilinear couplings $\beta$-functions

At 1-loop:

$$\beta^{(1)}_{abc} = \tau_1^{(1)} [C_2(S)]_{ae} t_{ebc} + \tau_2^{(1)} \lambda_{abef} t_{efc} + \tau_3^{(1)} [Y_2(S)]_{ae} t_{ebc} + \tau_4^{(1)} [m_{\bar{y}a} y_b \bar{y}_b].$$  \hspace{1cm} (14)

At 2-loop:

$$\beta^{(2)}_{abc} = \tau_1^{(2)} (T^A T^C)_{ae} G^2_B G^2_D (T^{B T^D})_{bf} t_{efc} + \tau_2^{(2)} (T^A T^C)_{ab} G^2_B G^2_C D (T^B T^D)_{ef} t_{efc} + \tau_3^{(2)} [C_2(S)]_{ae} [C_2(S)]_{ef} t_{ebc} + \tau_4^{(2)} [C_2(S)]_{ae} t_{ebc} + \tau_5^{(2)} [C_2(S, G)]_{ae} t_{ebc} + \tau_6^{(2)} [C_2(S, S)]_{ae} t_{ebc} + \tau_7^{(2)} [C_2(S, F)]_{ae} t_{ebc} + \tau_8^{(2)} (T^A_{ae} (T^B_{ef}) b f G^2_B \lambda_{efgh} t_{egh} + \tau_9^{(2)} \lambda_{abef} [C_2(S)]_{fg} t_{egc} + \tau_{10}^{(2)} [C_2(S)]_{ae} t_{efg} \lambda_{fgc} + \tau_{11}^{(2)} [C_2(S)]_{ae} \lambda_{ebfg} t_{fgc} + \tau_{12}^{(2)} \lambda_{acdg} \lambda_{cfgh} t_{ehbc} + \tau_{13}^{(2)} t_{aebc} \lambda_{egbh} \lambda_{fgc} + \tau_{14}^{(2)} \lambda_{abef} \lambda_{egbc} t_{ghc} + \tau_{15}^{(2)} \lambda_{abef} \lambda_{fgbc} t_{ghc} + \tau_{16}^{(2)} (T^A T^C)_{ab} G^2_B G^2_C D T^{B T D} [m_{\bar{y}a} y_b \bar{y}_b] + \tau_{17}^{(2)} [Y_2(S, C)]_{ae} t_{ebc} + \tau_{18}^{(2)} [Y_2(S, C)]_{ae} t_{ebc}.$$  \hspace{1cm} (15)

Scalar mass $\beta$-functions

At 1-loop:

$$\beta^{(1)}_{ab} = \sigma_1^{(1)} [C_2(S)]_{ae} \mu_{eb} + \sigma_2^{(1)} \lambda_{abef} \mu_{ef} + \sigma_3^{(1)} \lambda_{abef} \mu_{ef} + \sigma_4^{(1)} [Y_2(S)]_{ae} \mu_{eb} + \sigma_5^{(1)} [Y_2(S)]_{ae} \mu_{eb} + \sigma_6^{(1)} [Y_2(S)]_{ae} \mu_{eb} + \sigma_7^{(1)} [Y_2(S)]_{ae} \mu_{eb} + \sigma_8^{(1)} [Y_2(S)]_{ae} \mu_{eb} + \sigma_9^{(1)} [Y_2(S)]_{ae} \mu_{eb} + \sigma_10^{(1)} [Y_2(S)]_{ae} \mu_{eb} + \sigma_11^{(1)} [Y_2(S)]_{ae} \mu_{eb} + \sigma_12^{(1)} [Y_2(S)]_{ae} \mu_{eb}.$$  \hspace{1cm} (16)

At 2-loop:

$$\beta^{(2)}_{ab} = \sigma_1^{(2)} (T^A T^C)_{ae} G^2_B G^2_D (T^{B T D})_{bf} \mu_{ef} + \sigma_2^{(2)} (T^A T^C)_{ab} G^2_B G^2_C D (T^B T^D)_{ef} \mu_{ef} + \sigma_3^{(2)} [C_2(S)]_{ae} [C_2(S)]_{ef} \mu_{eb} + \sigma_4^{(2)} [C_2(S)]_{ae} [C_2(S)]_{ef} \mu_{eb} + \sigma_5^{(2)} [C_2(S, G)]_{ae} \mu_{eb} + \sigma_6^{(2)} [C_2(S, S)]_{ae} \mu_{eb} + \sigma_7^{(2)} [C_2(S, F)]_{ae} \mu_{eb} + \sigma_8^{(2)} [C_2(S)]_{ae} \lambda_{fgc} \mu_{fgc} + \sigma_9^{(2)} [C_2(S)]_{ae} \lambda_{egbc} t_{egb} + \sigma_{10}^{(2)} [C_2(S)]_{ae} \lambda_{egbc} t_{egb} + \sigma_{11}^{(2)} [C_2(S)]_{ae} \lambda_{egbc} t_{egb} + \sigma_{12}^{(2)} [C_2(S)]_{ae} \lambda_{egbc} t_{egb}.$$  \hspace{1cm} (17)
\begin{align*}
+\sigma^{(2)}_{13} \lambda_{afg} \phi_{fgh} \\
+\sigma^{(2)}_{15} \lambda_{abef} t_{egh} t_{fg} \\
+\sigma^{(2)}_{17} \lambda_{abef} \phi_{fgh} \\
+\sigma^{(2)}_{19} (T^A T^C)_{ab} G^2_{CB} G^2_{CD} \text{Tr} [T^D T^B \tilde{m} m] \\
+\sigma^{(2)}_{21} [C_2(S)]_{ae} [Y_2(S)]_{ef} \mu_{fb} \\
+\sigma^{(2)}_{23} t_{ac} f_{eg} t_{egh} \\
+\sigma^{(2)}_{25} G^2_{AB} \text{Tr} [y a T^A \tilde{y} m T^B \tilde{m}] \\
+\sigma^{(2)}_{27} [C_2(S)]_{ae} \text{Tr} [y a \tilde{m} m] \\
+\sigma^{(2)}_{29} \text{Tr} \left[ y a \tilde{m} m \tilde{C}_2 \right] \\
+\sigma^{(2)}_{31} \text{Tr} \left[ y a \tilde{m} m \tilde{C}_2 \right] \\
+\sigma^{(2)}_{33} \text{Tr} \left[ y a \tilde{y} a \tilde{y} a \tilde{y}_c \right] \mu_{ef} \\
+\sigma^{(2)}_{35} \text{Tr} \left[ m \tilde{y} e m \tilde{y}_c \right] \lambda_{efab} \\
+\sigma^{(2)}_{37} \text{Tr} \left[ y a \tilde{m} m \tilde{y} e \tilde{y}_c \right] \mu_{ef} \\
+\sigma^{(2)}_{39} [Y_4(S)]_{ae} \mu_{eb} \\
+\sigma^{(2)}_{41} \text{Tr} \left[ m \tilde{y} e m \tilde{y} e \tilde{y}_c \right] \\
+\sigma^{(2)}_{43} \text{Tr} \left[ y a \tilde{m} m \tilde{y} e \tilde{y}_c \right] \
+\sigma^{(2)}_{45} \text{Tr} \left[ y a \tilde{y} e \tilde{m} m \tilde{y}_c \right] \\
+\sigma^{(2)}_{47} \text{Tr} \left[ y a \tilde{m} y e \tilde{m} m \tilde{y}_c \right] \\
+\sigma^{(2)}_{49} \text{Tr} \left[ y a \tilde{m} y e \tilde{y} e \tilde{y}_c \right] \\
+\sigma^{(2)}_{51} \text{Tr} \left[ m \tilde{y} a \tilde{m} m \tilde{C}_2 \right].
\end{align*}

\begin{align*}
\text{Fermion mass coefficients} \\
\xi_1^{(1)} = -6, \quad \xi_2^{(1)} = 2, \quad \xi_3^{(1)} = 1.
\end{align*}

At 1-loop:
\begin{align*}
\xi_1^{(2)} &= 0, \quad \xi_2^{(2)} = -3, \quad \xi_3^{(2)} = -\frac{97}{3}, \quad \xi_4^{(2)} = \frac{11}{6}, \quad \xi_5^{(2)} = \frac{5}{3}, \quad \xi_6^{(2)} = 12, \\
\xi_7^{(2)} &= 0, \quad \xi_8^{(2)} = 6, \quad \xi_9^{(2)} = 10, \quad \xi_{10}^{(2)} = 6, \quad \xi_{11}^{(2)} = 9, \quad \xi_{12}^{(2)} = -\frac{1}{2}, \\
\xi_{13}^{(2)} &= -\frac{7}{2}, \quad \xi_{14}^{(2)} = -2, \quad \xi_{15}^{(2)} = 2, \quad \xi_{16}^{(2)} = 0, \quad \xi_{17}^{(2)} = -2, \quad \xi_{18}^{(2)} = 0, \\
\xi_{19}^{(2)} &= -2, \quad \xi_{20}^{(2)} = -\frac{1}{4}, \quad \xi_{21}^{(2)} = -1, \quad \xi_{22}^{(2)} = -\frac{3}{4}.
\end{align*}

At 2-loop:
\begin{align*}
\tau_1^{(1)} &= -9, \quad \tau_2^{(1)} = 3, \quad \tau_3^{(1)} = \frac{3}{2}, \quad \tau_4^{(1)} = -12.
\end{align*}
At 2-loop:

\[
\begin{align*}
\tau_1^{(2)} &= 6, & \tau_2^{(2)} &= 30, & \tau_3^{(2)} &= 0, & \tau_4^{(2)} &= \frac{9}{2}, & \tau_5^{(2)} &= -\frac{143}{4}, & \tau_6^{(2)} &= \frac{11}{4} \\
\tau_7^{(2)} &= \frac{10}{4}, & \tau_8^{(2)} &= -9, & \tau_9^{(2)} &= 24, & \tau_{10}^{(2)} &= -\frac{9}{2}, & \tau_{11}^{(2)} &= -9, & \tau_{12}^{(2)} &= \frac{1}{4} \\
\tau_{13}^{(2)} &= -3, & \tau_{14}^{(2)} &= -3, & \tau_{15}^{(2)} &= 0, & \tau_{16}^{(2)} &= -36, & \tau_{17}^{(2)} &= -36, & \tau_{18}^{(2)} &= \frac{15}{2} \\
\tau_{19}^{(2)} &= 0, & \tau_{20}^{(2)} &= -3, & \tau_{21}^{(2)} &= 0, & \tau_{22}^{(2)} &= 0, & \tau_{23}^{(2)} &= 12, & \tau_{24}^{(2)} &= 6, \\
\tau_{25}^{(2)} &= -24, & \tau_{26}^{(2)} &= -24, & \tau_{27}^{(2)} &= 6, & \tau_{28}^{(2)} &= 6, & \tau_{29}^{(2)} &= 0, & \tau_{30}^{(2)} &= 0, \\
\tau_{31}^{(2)} &= -\frac{3}{2}, & \tau_{32}^{(2)} &= -\frac{9}{4}, & \tau_{33}^{(2)} &= 24, & \tau_{34}^{(2)} &= 12, & \tau_{35}^{(2)} &= 12, & \tau_{36}^{(2)} &= 24, \\
\tau_{37}^{(2)} &= 12, & \tau_{38}^{(2)} &= 12. 
\end{align*}
\]  

(21)

Scalar mass coefficients

At 1-loop:

\[
\sigma_1^{(1)} = -6, \quad \sigma_2^{(1)} = 1, \quad \sigma_3^{(1)} = 1, \quad \sigma_4^{(1)} = 1, \quad \sigma_5^{(1)} = -4, \quad \sigma_6^{(1)} = -2. \quad (22)
\]

At 2-loop:

\[
\begin{align*}
\sigma_1^{(2)} &= 2, & \sigma_2^{(2)} &= 10, & \sigma_3^{(2)} &= 0, & \sigma_4^{(2)} &= 3, & \sigma_5^{(2)} &= -\frac{143}{6}, & \sigma_6^{(2)} &= \frac{11}{6}, \\
\sigma_7^{(2)} &= \frac{10}{6}, & \sigma_8^{(2)} &= -3, & \sigma_9^{(2)} &= 8, & \sigma_{10}^{(2)} &= 8, & \sigma_{11}^{(2)} &= -3, & \sigma_{12}^{(2)} &= -3, \\
\sigma_{13}^{(2)} &= \frac{1}{6}, & \sigma_{14}^{(2)} &= -1, & \sigma_{15}^{(2)} &= 1, & \sigma_{16}^{(2)} &= 2, & \sigma_{17}^{(2)} &= 0, & \sigma_{18}^{(2)} &= 0, \\
\sigma_{19}^{(2)} &= -12, & \sigma_{20}^{(2)} &= 5, & \sigma_{21}^{(2)} &= 0, & \sigma_{22}^{(2)} &= -1, & \sigma_{23}^{(2)} &= -1, & \sigma_{24}^{(2)} &= 0, \\
\sigma_{25}^{(2)} &= 0, & \sigma_{26}^{(2)} &= 0, & \sigma_{27}^{(2)} &= 2, & \sigma_{28}^{(2)} &= 4, & \sigma_{29}^{(2)} &= -8, & \sigma_{30}^{(2)} &= -8, \\
\sigma_{31}^{(2)} &= -4, & \sigma_{32}^{(2)} &= -4, & \sigma_{33}^{(2)} &= 2, & \sigma_{34}^{(2)} &= 4, & \sigma_{35}^{(2)} &= 1, & \sigma_{36}^{(2)} &= 0, \\
\sigma_{37}^{(2)} &= 0, & \sigma_{38}^{(2)} &= 0, & \sigma_{39}^{(2)} &= -1, & \sigma_{40}^{(2)} &= -\frac{3}{2}, & \sigma_{41}^{(2)} &= 4, & \sigma_{42}^{(2)} &= 8, \\
\sigma_{43}^{(2)} &= 8, & \sigma_{44}^{(2)} &= 4, & \sigma_{45}^{(2)} &= 4, & \sigma_{46}^{(2)} &= 4, & \sigma_{47}^{(2)} &= 4, & \sigma_{48}^{(2)} &= 4, \\
\sigma_{49}^{(2)} &= 4, & \sigma_{50}^{(2)} &= 2, & \sigma_{51}^{(2)} &= 2. 
\end{align*}
\]  

(23)

Conclusions. — We have presented in this Letter the general expressions of the two-loop $\overline{\text{MS}}$ $\beta$-functions for dimensionful parameters in the formalism of Poole & Thomsen. This completes the set of dimensionless RGEs given in [13] up to the respective loop orders 3-2-2 for gauge, Yukawa and quartic couplings. The full set of RGEs in this new formalism has been implemented in a new version of the PyR@TE software, PyR@TE 3 [21], which will be soon officially released [22].

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