Exponential stretching in filaments as fast dynamos in Euclidean and curved Riemannian 3D spaces

by

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Abstract

A new antidynamo theorem for non-stretched twisted magnetic flux tube dynamo is obtained. Though Riemannian curvature cannot be neglected since one considers curved magnetic flux tube axis, the stretch can be neglect since one only considers the limit of thin magnetic flux tubes. The theorem states that marginal or slow dynamos along curved (folded), torsioned (twisted) and non-stretched flux tubes endowed with diffusionless plasma flows, if a constraint is imposed on the relation between poloidal and toroidal magnetic fields in the helical dynamo case. A formula for the stretch of flux tubes is derived. From this formula one shows that the Riemann flux tube is stretched by an interaction between the plasma flow vorticity and torsion, in accordance with our physical intuition. Marginal diffusionless dynamos are shown to exist obtained in the case of flux tube dynamos exponential stretching. Thus slow dynamos can be obtaining on the flux tube under stretching. Filamentary dynamos anti-dynamos are also considered. As flux tubes possess a magnetic axis torsioned filament, it can also be considered as the germ of a fast dynamo in flux tubes Riemannian curved space. It is shown that for non-stretched filaments only untwist and unfold filaments can provide dynamo action in diffusive case. A condition for exponential stretching and fast dynamos in filaments is given. These results are actually in agreement with Vishik argument that fast dynamo cannot be obtained in non-stretched flows. Actually the flux tube result is the converse of Vishik’s lemma. PACS numbers: 02.40.Hw:differential geometries. 91.25.Cw-dynamo theories.
I Introduction

Earlier a theorem stated and proved by Zeldovich (anti-dynamo theorem) [1] showed that planar flows cannot support dynamo action. Along with Cowling's theorem [2] which has been recently tested for black holes by Brandenburg [3], these are the two main and more traditional and well-tested antidynamo theorems ever. Actually in dynamo action one of the main ingredients is exponential stretching, so-well investigated by Friedlander and Vishik [4]. In this same paper they argued that there are several topological obstructions to the existence of Anosov flows [5] in Riemannian 3-D space. Anosov flows which are Riemannian spaces of constant curvature endowed with geodesic flows, have been shown recently by Chicone and Latushkin [6] to be simple fast dynamos in compact Riemannian manifolds. Other types of fast dynamo mechanisms with stretching flux tubes in Riemannian conformal manifolds have been obtained by Garcia de Andrade [7]. Yet much earlier M. Vishik [8] has argued that only slow dynamos can be obtained from non-stretching dynamo flows and no fast dynamos so well-known to be obtained from the Vainshtein and Zeldovich [9] work on stretch-twist and fold (STF) [10] magnetic dynamo generation mechanism could be obtained in this way. This sort of "anti-fast-dynamo" theorem on flows can here be generalize to flux tube dynamos and filaments. Actually in this paper we show that the Vishik argument can be extended as to provide a anti-fast-dynamo theorem for filaments and tubes in some particular cases and subject to bounds in poloidal and toroidal magnetic fields and the twist (torsion) of the magnetic flux tube axis. Exponential stretching of tube condition is derived and from it one is able to derive our results. Non-Stretching in diffusionless media is used for flux tube dynamos while diffusive filaments are used in the second case. Marginal dynamos are obtained in the case of steady dynamos, while constraints are placed on the magnetic fields for dynamo action to be effective. Diffusion processes have been previously also investigated in the context of Riemannian geometry by S. Molchanov [11]. Such slow dynamos which have previously obtained by Soward [12] which also argued that fast dynamos actions would be possible in regions where no non-stretching flows would be presented, such in some curved surfaces. These surfaces could be of course Riemannian. In the case of filaments it is shown that dynamo action for non-stretched filaments would be obtained if they were unfolded and untwisted.
as well. When torsion vanishes filaments are planar and by Zeldovich anti-dynamo theorem [1] cannot support dynamo action, actually it is shown that it generates a static magnetic initial field and a steady perturbation which may be a marginal dynamo at maximum. Thus we may conclude that in the same way fast dynamo are generated by stretch, folding and twisting of the loops or filaments it seems that non-stretched, fold and twisted filaments leads to slow dynamos filaments. The paper is organized as follows: In section 2 a brief review on dynamics of holonomic Frenet frame is presented with the discussion of vortex-filament exponential stretching. In section 3 the self-induction equation is solved in the framework of the Riccas [13] twisted magnetic flux tube in Riemannian 3D manifold and anti-dynamo theorem presented. In section 4 a twisted (torsioned) and curved filament is shown to be a fast dynamo in Euclidean space. Section 5 contains discussions and future prospects.
II Exponential stretching in dynamo flux tube

This section contains a very brief review of the Serret-Frenet holonomic frame [14] equations that are specially useful in the investigation of STF Riemannian flux tubes in magnetohydrodynamics (MHD) with magnetic diffusion. Here the Frenet frame is attached along the magnetic flux tube axis which possesses Frenet torsion and curvature, which completely determine topologically the filaments, one needs some dynamical relations from vector analysis and differential geometry of curves such as the Frenet frame \((t, n, b)\) equations

\[
\begin{align*}
\dot{t'} &= \kappa n & (\text{II.1}) \\
\dot{n'} &= -\kappa t + \tau b & (\text{II.2}) \\
\dot{b'} &= -\tau n & (\text{II.3})
\end{align*}
\]

The holonomic dynamical relations from vector analysis and differential geometry of curves by \((t, n, b)\) equations in terms of time

\[
\begin{align*}
\dot{t} &= [\kappa' b - \kappa \tau n] & (\text{II.4}) \\
\dot{n} &= \kappa \tau t & (\text{II.5}) \\
\dot{b} &= -\kappa' t & (\text{II.6})
\end{align*}
\]

along with the flow derivative

\[
\dot{t} = \partial_t t + (\vec{v}, \nabla) t & (\text{II.7})
\]

From these equations and the generic flow [13]

\[
\dot{X} = v_s t + v_n n + v_b b & (\text{II.8})
\]

one obtains

\[
\frac{\partial l}{\partial t} = (-\kappa v_n + v_s') l & (\text{II.9})
\]

where \(l\) is given by

\[
\begin{align*}
l &:= (\dot{X'}, \dot{X'})^{\frac{1}{2}} & (\text{II.10}) \\
l &= l_0 e^{\int (-\kappa v_n + v_s') dt} & (\text{II.11})
\end{align*}
\]
which shows that stretching which shows that if \( v_s \) is constant, which fulfills the solenoidal incompressible flow

\[
\nabla \cdot \mathbf{v} = 0 \tag{II.12}
\]

and \( v_n \) vanishes, one should have an non-stretched twisted flux tube. This is exactly the choice \( \mathbf{v} = v_0 \mathbf{t} \), where \( v_0 = \text{constant} \) is the steady flow one uses here. This definition of magnetic filaments is shows from the solenoidal character of the magnetic field

\[
\nabla \cdot \mathbf{B} = 0 \tag{II.13}
\]

where \( B_s \) is the toroidal component of the magnetic field. In the next section one shall solve the diffusion equation in the steady case in the non-holonomic Frenet frame as

\[
\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \tag{II.14}
\]

where \( \eta \) is the magnetic diffusion. Since in astrophysical scales, \( \eta \nabla^2 \approx \eta L^{-2} \approx \eta \times 10^{-20} \text{ cm}^{-2} \) for a solar loop scale length of \( 10^{10} \text{ cm} \) [6] one notes that the diffusion effects are not highly appreciated in astrophysical dynamos, though they are not neglected here. Let us now consider the magnetic field definition in terms of the magnetic vector potential \( \mathbf{A} \) as

\[
\mathbf{B} = \nabla \times \mathbf{A} \tag{II.15}
\]

the gradient operator is

\[
\nabla = \mathbf{t} \partial_s + \frac{1}{r} \mathbf{e}_\theta + \mathbf{e}_r \partial_r \tag{II.16}
\]
Let us now consider the Riemann metric of a flux tube in curvilinear coordinates \((r, \theta, s)\). This metric is encoded into the Riemann line element

\[
 ds^2 = dr^2 + r^2 d\theta^2 + K^2 ds^2
\]  

(II.17)

where, accordingly to our hypothesis, \(K^2 = (1 - \kappa rcos\theta)^2\) which contributes to the Riemann curvature. These can be easily computed with the computer tensor package as the Riemann curvature components

\[
 R_{1313} = R_{rsrs} = -\frac{1}{4K^2}[2K^2 \partial_r A(r, s) - A^2] = -\frac{1}{2} K^4 \frac{1}{r^2} = -\frac{1}{2} r^2 \kappa^4 \cos^2 \theta
\]

(II.18)

\[
 R_{2323} = R_{\theta s\theta s} = -\frac{r}{2} A(r, s) = -K^2
\]

(II.19)

where \(A := \partial_r K^2\). In the case of thin tubes addressed here, where \(K^2(r, s) \approx 1\), these Riemann curvature reduces to

\[
 R_{1313} = R_{rsrs} = -\frac{1}{r^2}
\]

(II.20)

This shows that Riemann curvature of the tube is particularly strong when one approaches the tube. In the next section one applies some of the mathematical machinery derived in this section to the formulation of a new anti-dynamo theorem. Let us now consider the generic flow in the case of flux tube in Riemannian space. As in Ricca’s [12] one considers that no radial components of either flows or magnetic fields are present during the computations, thus

\[
 \dot{X} = v_t t + v_\theta e_\theta
\]

(II.21)

By considering the relation between the two bases \((e_r, e_\theta, t)\) and Frenet frame as

\[
 e_r = \cos \theta n + \sin \theta b
\]

(II.22)

and

\[
 e_\theta = -\sin \theta n + \cos \theta b
\]

(II.23)

one may express formula (II.21) as

\[
 \dot{X} = v_t t - v_\theta \sin \theta n + v_\theta \cos \theta b
\]

(II.24)

Comparison between (II.24) and (II.21) yields

\[
 v_n = -v_\theta \sin \theta
\]

(II.25)
and
\[ v_b = v_\theta \cos \theta \] (II.26)
which by a simple comparison with expression (II.22) yields
\[ \frac{\partial l}{\partial t} = (kv_\theta \sin \theta + v_s')l \] (II.27)
Thus one finally finds out an expression between the exponential stretching \( l \) as
\[ l = l_0e^{\int (kv_\theta \sin \theta + v_s')dt} \] (II.28)
From the solenoidal incompressible flow equation
\[ \nabla \cdot \mathbf{v} = 0 \] (II.29)
one obtains
\[ \partial_s v_\theta = r\kappa \tau \sin \theta v_\theta \] (II.30)
where one has taken the operator \( \partial_\theta = -\tau^{-1}\partial_s \) and use flux tube definition twist angle \( \theta = \theta_0 - \int \tau ds \). Note that substitution of this result into expression (II.28) and taking into account that in Ricca’s tube \( v'_s \) vanishes, upon integration leads to
\[ l = l_0e^{(r^2_0r\int \sin \theta dt)} \] (II.31)
where one has consider to simplify computations that the dynamos are helical which means that the torsion and Frenet curvature are taken as equal and constants. By the mathematical operator \( dt = -\omega_0^{-1}d\theta \) and substitution into the formula (II.31) yields upon integration that
\[ l = l_0e^{\left( \frac{r^2_0r}{2\omega_0}\int \sin \theta d\theta \right)} \] (II.32)
and
\[ l = l_0e^{\left( \frac{4\omega_0}{2\omega_0}\cos \theta \right)} \] (II.33)
where one has taken the following expression into account
\[ \omega_0r = v_\theta \] (II.34)
where \( \omega_0 \) is the constant rotation of the plasma. This shows that there is a stretching in the tube if \( (\omega_0 > 0) \) and a compression or non-stretching if \( (\omega_0 < 0) \). The plasma rotation inside the flux tube can be obtained from the vorticity equation
\[ \nabla \cdot \mathbf{\omega} = \nabla \cdot \nabla \times \mathbf{v} = 0 \] (II.35)
which yields the following PDEs

\[
\begin{align*}
\omega_r &= -\partial_s v_\theta \\
\omega_\theta &= \omega_0 = -\partial_r v_s \\
\omega_s &= -[\partial_r v_\theta - \frac{\cos \theta}{r} \tau_0 v_\theta] 
\end{align*}
\]  

(II.36)  

(II.37)  

(II.38)

From vorticity expression (II.37) one obtains

\[
\omega_0 r = -v_s 
\]

(II.39)

Substitution of the expression for \( \partial_s v_\theta \) above into expression (II.36) now yields

\[
\omega_r = -\tau_0^2 r \sin \theta v_\theta 
\]

(II.40)

for the thin flux tube. For steady dynamos one obtains [18]

\[
(v \cdot \nabla) B = (B \cdot \nabla) v 
\]

(II.41)

\[
\frac{B_\theta}{B_s} = \frac{v_\theta}{v_s} 
\]

(II.42)

By making use of the vorticity equations one obtains

\[
\frac{B_\theta}{B_s} \approx \frac{1}{\tau_0 r \cos \theta} 
\]

(II.43)

Note then that near to the flux tube axis \((r \approx 0)\) one obtains that the toroidal magnetic field decreases from toroidal field. A dynamo test can be obtained by computing the integral of Zeldovich

\[
\frac{4\pi d\epsilon_M}{dt} = \int B \cdot (B \cdot \nabla) v dV 
\]

(II.44)

where another term of diffusion has been dropped since here diffusion vanishes. After some algebra one obtains for the flux tube the following equation

\[
\frac{4\pi d\epsilon_M}{dt} = \int \left[ B_\theta \tau_0^2 \sin \theta [v_\theta - \tau_0^{-1} v_s] + B_s v_s \right] \left( B_s - \frac{B_\theta \tau_0^{-1}}{r} \right) dV 
\]

(II.45)

which shows that

\[
B_s = \frac{B_\theta \tau_0^{-1}}{r} 
\]

(II.46)
implies the existence of a marginal dynamo, where $\frac{4\pi dM}{dt}$ vanishes. Note that this result does not depend directly on the stretching but on the torsion which indirectly is responsible for stretching, therefore exponential stretching may exists even for marginal dynamos, which would represent a converse result of a Vishik’s antidynamo theorem for flux tubes. Non-stretching flows implies necessarily slow dynamos, but slow dynamos do not imply necessarily non-stretching. The weak torsion approximation used here, is in agreement with the torsion (twist) in the solar twisted coronal loop torsion ($\tau_0 \approx 10^{-10} cm^{-1}$) [15].
III Anti-dynamo theorem in non-stretching filaments

In this section one considers the anti-dynamo formulation of non-stretching dynamo flows in twisted filaments. This can be done by simply considering the gradient along the filament as \( \nabla = t \partial_s \) and computing the total magnetic energy on a diffusive medium as

\[
\frac{4 \pi d \epsilon_M}{dt} = \int B_s^2 v_s \mathbf{t} \cdot \mathbf{n} dV := 0 \quad (III.47)
\]

since \( \mathbf{n} \cdot \mathbf{t} = 0 \). Here one has considered that \( \mathbf{B} := B_s \mathbf{t} \). Thus one must say that in a diffusive media since \( v'_s = 0 \) and \( v_n \) also vanishes by assumption that the following lemma has been proved.

**lemma:**

Non-stretching vortex filaments in a diffusionless media gives rise to a marginal or slow dynamo. No fast dynamo being possible.

This can be considered as a sort of anti-fast dynamo theorem from Vishik's idea. Now we shall relax the idea of diffusionless media and introduce diffusion into the problem. As note previously by Zeldovich [1] this leads us to the dynamo action and possibly to fast dynamos.

In the case of diffusive filaments one notes that the magnetic induction equation

\[
\frac{d}{dt} \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{B} + \eta \nabla^2 \mathbf{B} \quad (III.48)
\]

where \( \eta \) is the magnetic resistivity or diffusion, which here one considers as constant, reduces to three scalar equations

\[
\frac{dB_s}{dt} = -\eta \kappa B_s \quad (III.49)
\]

\[
\kappa' = \eta \kappa \tau \quad (III.50)
\]

\[
-\kappa \tau = \eta \kappa' + v_s \quad (III.51)
\]

Solution of these equations can be easily obtained as

\[
B_s = \exp[-\eta \int \kappa^2 ds] B_0 \quad (III.52)
\]

where \( \int \kappa^2 ds \) is the total Frenet curvature energy integral. The remaining solutions are

\[
\kappa = \exp[\eta \int \tau ds] \kappa_0 \quad (III.53)
\]
where $\int \tau ds$ is the total torsion, and in the case of helical filaments where torsion equals curvature and are constants,

$$-\tau_0^2 = v_s \quad (\text{III.54})$$

Note that the decaying or not of the magnetic field depends on the sign of the integral but since the average value of this integral is positive the magnetic field decays and no dynamo action is possible, even slow dynamos. Thus in the case of non-stretching filaments, the presence of diffusion actually enhances the non-dynamo character in Vishik’s lemma.

### IV Filamentary fast dynamos in Euclidean space

Earlier Arnold has argued [16] that no fast dynamo are usually found in Euclidean 3D spaces. Actually Arnold himself found a stretching and compressed fast dynamo in curved Riemannian space. Though this served as motivation for the study of flux tube dynamos in 3D curved Riemannian space, in this section one shall address the problem of finding a filamentary fast dynamo in Euclidean space in 3D. The stretching followed by squeezing is a path to finding a growing magnetic field. Recently Nuñez [17] has considered a similar problem, also making use of Frenet frame as here, by investigating eigenvalues in plasma flows. In this section one shows that the stretching condition in filaments is fundamentally connected to the incompressibility of the flow. This is simply understood if one considers the exponent in exponential stretching in expression

$$\gamma := -\kappa v_n + v_s' \quad (\text{IV.55})$$

which shows that stretching factor gamma is a fundamental quantity to be examined when one wants to find out a fast dynamo action. Note that when $\gamma \geq 0$ or $\gamma < 0$, one would have respectively either a fast or slow or marginal dynamo and a decaying magnetic field as found before. Let us now drop the constraint that $v_s$ and substitute the flow

$$\mathbf{v} = v_n \mathbf{n} + v_s \mathbf{t} \quad (\text{IV.56})$$

into the solenoidal incompressible flow

$$\nabla \cdot \mathbf{v} = 0 \quad (\text{IV.57})$$
and \( v_n \) vanishes, one should have a non-stretched twisted flux tube. This is exactly the choice \( \mathbf{v} = v_0 \mathbf{t} \), where \( v_0 = \text{constant} \) is the steady flow one uses here. This definition of magnetic filaments is shows from the solenoidal carachter of the magnetic field

\[
\nabla \cdot \mathbf{v} = -\kappa v_n + v_s' = 0 \quad (\text{IV.58})
\]

This result is exactly \( \gamma = 0 \) which implies no stretching at all! This lead us to note that if a fast filamentary dynamo action be possible, a modification of the flow has to be performed. To investigate this possibility one considers the following form of the dynamo flow

\[
\mathbf{v} = v_n \mathbf{n} + v_s \mathbf{t} + v_0 \mathbf{b} \quad (\text{IV.59})
\]

which minimally generalizes (IV.56). In this case expression for \( \gamma \) does not vanish and is equal to

\[
\gamma = -\kappa v_n + v_s' = \tau_0 v_0 \quad (\text{IV.60})
\]

Note that again for \( \gamma > 0 \) one obtains a fast dynamo since \( \eta = 0 \) already and stretching is possible if \( \tau_0 \) and \( v_0 \) possesses the same sign. Actually this flow leads to the following three scalar dynamo equations

\[
\frac{d}{dt} B_s = \gamma B_s - \tau_0^2 B_n \quad (\text{IV.61})
\]

\[
\frac{d}{dt} B_n = \tau_0 (v_s - \tau_0 - \frac{1}{\tau_0} \partial_s v_n) B_s \quad (\text{IV.62})
\]

\[
B_s \tau_0 v_n = 0 \quad (\text{IV.63})
\]

Here \( \gamma = (v_s' - \tau_0 v_n) \) since we are considering helical dynamo filaments. From equation (IV.63) one obtains \( v_n = 0 \) which simplifies much the other equations. Since in astrophysical scales, torsion is very weak as happens in \( \tau_0^2 \approx \eta \times 10^{-20} \text{cm}^{-2} \) for a solar coronal loop scale, the terms proportional to torsion squared may be dropped. In this approximation a fast dynamo solution is found as

\[
B_s = B_0 e^{\gamma t} \quad (\text{IV.64})
\]

and

\[
\frac{d}{dt} B_n = \tau_0 (v_s - \tau_0) B_s \quad (\text{IV.65})
\]

which yields

\[
B_n = \frac{1}{v_0} (v_s - \tau_0) e^{\gamma t} \quad (\text{IV.66})
\]
where to obtain $B_n(t, s)$ one made use of the stationary property of the flow $\partial_t v_s = 0$, in order to being able to integrate (IV.65). Thus a fast dynamo action for filamentary flows in Euclidean 3D space was obtained. Note that this action is natural and somewhat expected since when we change the dynamo flow we obtain a three-dimensional $(v_s, v_n, v_b)$ from a two dimensional flow $(v_s, v_n)$, which is strictly forbidden from Zeldovich anti-dynamo theorem. Besides if one integrates the flow equation (IV.60) yields

$$v_s = \tau_0 v_0 s + c_1$$  \hspace{1cm} (IV.67)

where $c_1$ is an integration constant. Expression (IV.67) indicates that the flow along the filament undergoes a uniform strain [18] due to the stretching of the magnetic filament. Now if one uses these relations into the total magnetic energy integral, yields

$$\epsilon_M = \frac{a^2}{8} [B_0^2 s + \tau_0 (v_s - v_0)^3 v_0] e^{\tau_0 v_0 t}$$  \hspace{1cm} (IV.68)

where one has put $c_1 := 0$ for convenience. This magnetic energy indicates that the stretching of the magnetic field accumulates energy and gives rise to a fast dynamo as long as $v_s \geq v_0$ is bounded from below. Physically this means that the flow along the magnetic filament is stronger than any magnetic orthonormal perturbation and the system might be stable.

V Conclusions

Vishik’s idea that the non-stretched dynamos cannot be fast is tested once more here, by showing that the fluctuations modes of the solar dynamos are strongly suppressed when the long wavelength dynamo modes, or small dynamo wave numbers are effective. Stretching are therefore fundamental in the effective dynamo convective region of the Sun [6]. The twist or Frenet torsion is also small, and slow dynamo are shown to be present in this astrophysical loops. By following the analogy proposed by Friedlander and Vishik in dynamo theory between vorticity equations and dynamo equations and considering exponential stretching one shows that flux tube dynamos are in complete agreement with Ricca’s original Riemannian magnetic flux flux tube model. STF Zeldovich-Vainshtein fast dynamo generation method, is not the only Riemannian method that can be applied as in Arnold’s cat map but other conformal
fast kinematic dynamo models as the conformal Riemannian one has been recently obtained. Small scale dynamos in Riemannian spaces can therefore be very useful for our understanding of more large scale astrophysical dynamos. Other applications of plasma filaments such as stretch-twist and fold fractal dynamo mechanism which are approximated Riemannian metrics have been recently put forward by Vainshtein et al [14]. Finally one has shown that the Vishik's result is strongly enhanced and no fast dynamo or even slow dynamo is found and decay of the magnetic field in non-stretched filaments in presence of magnetic diffusion and the Frenet torsion is found. As considered by Arnold [16] and Zeldovich et al [18] no fast dynamos exists in 3D Euclidean space and a fast dynamo was obtained in 3D curved Riemannian space by Arnold [16] where stretching in some directions is compensated by compression in other. In this paper a fast filamentary dynamo was obtained in Euclidean 3D space. Note that when torsion vanishes in the last section a marginal dynamo is obtained, this result, a sort of filamentary antidynamo theorem, has been recently obtained [19] in the non-holonomic frame.

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