Theoretical Derivation and Method Implementation of the Pseudo-Static Method in Seismic Analysis of Underground Structures

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Abstract. Based on the principles of the substructure method, a quasi-static method was derived to solve the seismic response of underground structures. On the basis of this theoretical derivation, the pseudo-static calculation model was proposed, and its calculating parameters were analyzed and solution implemented. Finally, the proposed method in this paper was compared with the dynamic FEM method. The results show that the parameters of quasi-static method have clear physical meaning and convenient solution. Compared with dynamic FEM method, the calculation parameters of quasi-static method are simpler, and the maximum seismic response of the structure can be obtained by static calculation, avoiding the complicated soil-structure dynamic FEM calculation.

1. Introduction

Underground structures are commonly employed in urban transportation projects for mass transit or underground roadways. In addition to static loads, in seismically active areas, these structures have to be designed to withstand significant seismic forces. Domestic and foreign researchers have carried out a lot of research on seismic analysis of underground structures and proposed a variety of seismic design analysis methods for underground structures since the Hanshin earthquake in Japan in 1995 [1]. Penzien et al. [2] and Huo et al. [3] equated the seismic load with the static load at infinity and proposed an analytical method for solving the internal force and the deformation of the underground structure. Considering of the vibration characteristics of underground structures, Tateishi A et al [4], Rushan Liu et al [5] and Jingbo Liu et al [6,7] proposed the reaction displacement method, reaction stress method, reaction acceleration method, integral reaction displacement method, etc. Guoxing Chen et al [8], Liping Jing [9] and Lianjin Tao, et al. [10] used large-scale finite element or finite difference software to analyse the seismic response of underground structures.

The quasi-static method is simple in form, clear in physical meaning, and can comprehensively consider the characteristics of seismic response of underground structures. At present, the most popular quasi-static methods are reactive displacement methods, which can better reflect the force and deformation of underground structures under earthquake action and have been applied to Japanese and Chinese specifications [11].
In this paper, based on the substructure method, the theory of the quasi-static method of underground structure is deduced. Then, based on this theoretical derivation, the meaning of various expressions in the derivation is described. Finally, the quasi-static method and the dynamic FEM method are compared and analysed by numerical examples.

2. Theoretical derivation

For the soil-structure interaction problem, the dynamic system can be decomposed into two substructures, as shown in the Figure 1.

![Figure 1. Substructure decomposition of the soil-structure system](image)

Based on the above decomposition, the basic motion equation under the soil-structure interaction can be obtained by the substructure method as shown in equation (1) [12].

\[
\begin{bmatrix}
[S_s] & [S_{sb}] \\
[S_{ss}'] & [S_{sb}']
\end{bmatrix}\begin{bmatrix}
[u_s(t)] \\
[u_{sb}(t)]
\end{bmatrix} = \begin{bmatrix}
{0} \\
{0}
\end{bmatrix}
\]

\(1\)

In the formula, \([S]\), \(\{u(t)\}\) represents the dynamic stiffness matrix and the displacement vector respectively; the superscript character \(s, b\) represents the structure and the soil-junction contact surface respectively; the superscript character \(s, g\) represents the structure and the soil after excavation, respectively.

Assume that the excavated soil is a structure, as shown in Figure 2, there are:

\[
[S_{eo}'] = [S_{eo}] \\
[S_{eo}] = [S_{eo}^e] + [S_{eo}^f]
\]

\(2\)

The superscript character \(e, f\) in the formula is represents the excavated soil and the free field, respectively.

![Figure 2. Substructure decomposition of the soil-excavated system](image)

Bring equation (2) into equation (1), where \([S_s] = 0\), \(\{u_s(t)\} = \{u_e^e(t)\}\), then:

\[
[S_{eo}]\{u_e^e(t)\} = [S_{eo}']\{u_e^e(t)\}
\]

\(3\)

Bringing equation (3) into equation (1), there are:

\[
\begin{bmatrix}
[S_s] & [S_{ss}] \\
[S_{ss}'] & [S_{ss}']
\end{bmatrix}\begin{bmatrix}
[u_s(t)] \\
[u_{ss}^e(t)]
\end{bmatrix} = \begin{bmatrix}
{0} \\
{0}
\end{bmatrix}
\]

\(4\)

The dynamic stiffness matrix \([S]\) can be replaced by equation (5) [4, 12].
\[
[S] = [K] + i\omega [C] - \omega^2 [M]
\] (5)

Where, \([K], [M], [C]\) represent the stiffness matrix, the mass matrix, and the damping matrix, respectively; \(\omega\) represent the frequency.

Bringing equation (5) into equation (4):

\[
\begin{bmatrix}
[M_a] \\
[M_a]
\end{bmatrix} \begin{bmatrix} 0 \\
0
\end{bmatrix} + \begin{bmatrix}
[K_a] \\
[K_a]
\end{bmatrix} \begin{bmatrix} u(t) \\
u(t)
\end{bmatrix} = \begin{bmatrix}
[K_a] \\
[K_a]
\end{bmatrix} \begin{bmatrix} \dot{u}(t) \\
\dot{u}(t)
\end{bmatrix} + \begin{bmatrix}
[K_{a0}] \\
K_{a0} + [K_{a0}]
\end{bmatrix} \begin{bmatrix} \ddot{u}(t) \dot{u}(t) \\
\ddot{u}(t) \dot{u}(t)
\end{bmatrix} \begin{bmatrix} \ddot{u}(t) \\
\ddot{u}(t)
\end{bmatrix}
\] (6)

According to the research in the literature [13], due to the restraining effect of the surrounding soil, the vibration characteristics of the underground structure itself is not obvious, and the basic vibration mode is the first mode. Thus, the internal forces of the underground structure will reach a maximum at its maximum relative displacement, and we assume that this moment is \(t_0\). At the \(t_0\) time, the speed of the underground structure and the soil are relatively low, then the damping force is relatively small and can be ignored. Thus, the \(t_0\) state of the underground structure is written as:

\[
\begin{bmatrix}
K_a \\
K_a
\end{bmatrix} \begin{bmatrix} u(t) \\
u(t)
\end{bmatrix} = \begin{bmatrix}
K_a \\
K_a + [K_{a0}]
\end{bmatrix} \begin{bmatrix} \dot{u}(t) \\
\dot{u}(t)
\end{bmatrix} \begin{bmatrix} \ddot{u}(t) \\
\ddot{u}(t)
\end{bmatrix} \begin{bmatrix} \ddot{u}(t) \\
\ddot{u}(t)
\end{bmatrix} + \begin{bmatrix}
[K_{a0}]
\end{bmatrix} \begin{bmatrix} \dot{u}(t) \\
\dot{u}(t)
\end{bmatrix} \begin{bmatrix} \ddot{u}(t) \\
\ddot{u}(t)
\end{bmatrix} \begin{bmatrix} \ddot{u}(t) \\
\ddot{u}(t)
\end{bmatrix}
\] (7)

3. Quasi-static method implementation

According to formula (7), to characterize the state of force of the underground structure at the \(t_0\) moment, simply solve the two terms on the right side of the equation (7). The specific steps are as follows:

The first step is to obtain the displacement \(u(t_0)\) and acceleration \(\ddot{u}(t_0)\) of the soil at the location of the underground structure from the free-field seismic response.

The second step, the boundary load is obtained from the free-field finite element model shown in Figure 3. The free field displacement \(u(t_0)\) and acceleration \(\ddot{u}(t_0)\) are applied at the location of the structure in the free field, and the boundary load is obtained by the node reaction force of the structure in contact with the soil.

\[
F(z) = [K_{a0}] [u(t_0)] + [K_{a0}] [u(t_0)] + [M*] [\dot{u}(t_0)] = ([K_{a0}] + [K_{a0}]) [u(t_0)] + [M*] [\dot{u}(t_0)]
\] (8)

In the formula, \(F(z)\) represent the node reaction force at the \(z\) depth.

In the third step, the quasi-static method is used to calculate the internal force and deformation of the underground structure. The calculation model is shown in Figure 4. A finite element model of the soil-structure interaction is established, a boundary load \(F(z)\) is applied to the interface between the structure and the soil, and an inertial force is applied to the structure to calculate the structural response.
4. Case analysis

4.1. Calculation object
In order to verify the validity of the pseudo-static calculation method proposed in this paper, 0.1g El Centro wave is used as the input seismic wave, as shown in Figure 5. The numerical calculation is carried out by changing the section form of the structure, the stiffness of the soil layer and the buried depth of the structure. The calculation conditions are shown in Table 1. The cross-sectional dimensions of the two selected underground structures are shown in Figure 6 (Figure 6-(a) shows 4m × 8m, Figure 6-(b) shows 8m × 4m).

![Figure 4. Calculation model of quasi-static method](image)

![Figure 5. The acceleration time history of El Centro wave](image)

![Figure 6. Sketch of the cross-section of the underground structures (unit: m)](image)

| Influencing factor | Working condition | structure size(b×h) | Soil stiffness | Structure depth(m) |
|-------------------|-------------------|---------------------|---------------|-------------------|
| Structure size    | 1                 | 4m×8m               | 1             | 4.0               |
|                   | 2                 | 8m×4m               | 1             | 4.0               |
|                   | 3                 | 4m×8m               | 0.5           | 4.0               |
|                   | 4                 | 4m×8m               | 1             | 4.0               |
|                   | 5                 | 4m×8m               | 2             | 4.0               |
| Soil stiffness    | 6                 | 4m×8m               | 1             | 2.0               |
| Structure depth   | 7                 | 4m×8m               | 1             | 4.0               |
|                   | 8                 | 4m×8m               | 1             | 10.0              |
4.2. Analysis of calculation results

4.2.1. Different cross-section dimensions

Figure 7 and Figure 8 show the internal force diagrams of the two cross-sectional shapes. The numbers on the horizontal axis indicate different positions of the structure. The numerical meaning is numbered 1 from the upper left corner of the structure, and the clockwise winding structure. One circle is numbered sequentially and is divided into 24 units, and the vertical axis represents the internal force value.

It can be seen from Figure 7 that the calculation results of this method are closer to the dynamic FEM than the calculation results of the reaction displacement method. From the internal force distribution of each position of the structure, the method is closer to the dynamic FEM. And the shape of the internal force distribution is closer to the dynamic FEM.

It can be seen from Figure 8 that the calculation result of this method is close to the dynamic FEM; from Figure 8-(b), the shear force distribution obtained by the method almost the same as the dynamic FEM (the error is about 10%).

In general, from the comparison calculation results of the two underground structures, the calculation results of this method are close to the calculation results of the dynamic FEM.

4.2.2. Different soil stiffness

This paper lists the key parts of the structure: the internal force values of sections A, B, C, D, (see Figure 6) as shown in Table 2.

It can be seen from Table 2 that the calculation results of the critical section bending moment is consistent with the dynamic FEM. It can also be seen from Table 2 that as the stiffness of the soil increases, the internal force of the structure decreases. So, the seismic performance of the structure can be improved by increasing the stiffness of the soil around the underground structure.

| Section | Axial Force | Shear Force | Bending Moment |
|---------|-------------|-------------|----------------|
| A       | 100         | 80          | 200            |
| B       | 90          | 70          | 190            |
| C       | 80          | 60          | 180            |
| D       | 70          | 50          | 170            |
### 4.2.3. Different structure buried depth

It can be seen from the Table 3 that the method is in the shallow burial condition (condition 6), the calculation result of the method only differs by 10%. The calculation results of axial force and shear force of this method are close to the dynamic FEM.

| Working condition | Calculation method | Bending moment (kN·m) | Axial force (kN) | Shear force (kN) |
|-------------------|--------------------|-----------------------|-----------------|-----------------|
|                   |                    | A/D | B/C | A    | B   | C   | D   | A    | B   | C   | D   |
| 3                 | This paper         | 134 | 104 | -25  | 59  | 115 | 70  | 67   | 94  | 84  | 26  |
|                   | Dynamic FEM        | 127 | 98  | -30  | 55  | 100 | 74  | 69.7 | 99  | 83  | 41  |
| 4                 | This paper         | -134| 104 | -25  | 59  | 115 | 70  | 67.2 | 94  | 84  | 26  |
|                   | Dynamic FEM        | -127| 98  | -30  | 55  | 100 | 74  | 69.7 | 99  | 83  | 41  |
| 5                 | This paper         | -48 | 42  | -37  | 54  | 99  | 59  | 43.9 | 46  | 52  | 30  |
|                   | Dynamic FEM        | -44 | 6   | -40  | 63  | 95  | 72  | 37.7 | 56  | 59  | 35  |

#### Table 3. Calculation results of different buried depths

| Working condition | Calculation method | Bending moment (kN·m) | Axial force (kN) | Shear force (kN) |
|-------------------|--------------------|-----------------------|-----------------|-----------------|
|                   |                    | A/D | B/C | A    | B   | C   | D   | A    | B   | C   | D   |
| 6                 | This paper         | -80 | 42  | -21  | 47  | 69  | -43 | 48   | 74  | 66  | 34  |
|                   | Dynamic FEM        | -70 | 49  | -24  | 51  | 86  | -48 | 40   | 72  | 70  | 28  |
| 7                 | This paper         | -134| 104 | -25  | 59  | 115 | -70 | 67   | 94  | 84  | 26  |
|                   | Dynamic FEM        | -127| 98  | -30  | 55  | 100 | -74 | 69   | 99  | 83  | 41  |
| 8                 | This paper         | -194| 151 | -59  | 76  | 132 | -113| 105  | 131 | 106 | 51  |
|                   | Dynamic FEM        | -171| 92  | -64  | 85  | 149 | -134| 103  | 123 | 113 | 84  |

### 5. Conclusion

Based on the substructure method, the quasi-static method is deduced. Then, a quasi-static method for solving the seismic response of underground structures is proposed. Through the above theoretical derivation and comparative analysis of the examples, the following conclusions can be drawn:

1. The method of this paper is based on theoretical derivation, and the parameters of the method have clear physical meaning and it is convenient to achieve.
2. In the case of different cross-section dimensions, different soil stiffness and different structure buried depth, the overall distribution of the internal forces of the structure obtained by the method is close to the dynamic FEM method.
3. Compared with the dynamic FEM, the calculation parameters obtained by the method are based on the free field seismic response, and the internal force of the structure is obtained by static calculation, which avoids the complicated soil-structure dynamic FEM calculation, then, the calculation efficiency is higher.
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