Falsifying ΛCDM: Model-independent tests of the concordance model with eBOSS DR14Q and Pantheon

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ABSTRACT

We combine model-independent reconstructions of the expansion history from the latest Pantheon supernovae distance modulus compilation and measurements from baryon acoustic oscillation to test some important aspects of the concordance model of cosmology namely the FLRW metric and flatness of spatial curvature. We then use the reconstructed expansion histories to fit growth measurement from redshift-space distortion and obtain strong constraints on (Ω_m, γ, σ_8) in a model independent manner. Our results show consistency with a spatially flat FLRW Universe with general relativity to govern the perturbation in the structure formation and the cosmological constant as dark energy. However, we can also see some hints of tension among different observations within the context of the concordance model related to high redshift observations (z > 1) of the expansion history. This supports earlier findings of Sahni et al. (2014) & Zhao et al. (2017) and highlights the importance of precise measurement of expansion history and growth of structure at high redshifts.

Key words: methods: numerical – methods: statistical – large-scale structure of Universe – cosmology: theory – cosmological parameters – dark energy –

1 INTRODUCTION

The concordance model of cosmology is based on Einstein’s general theory of relativity (GR), which enabled us to come up with a theory of the Universe that is testable and can be falsified. The concordance flat ΛCDM model, which is based on GR and the assumptions of isotropy and homogeneity of the Universe, has been very successful at explaining various astronomical observations. This predictive model explains the dynamics of the Universe with only 6 free parameters. Ω_b and Ω_d (baryonic and dark matter densities) are the matter parameters. Assuming a flat universe and cosmological constant being responsible for late time acceleration of the Universe, we can derive Ω_Λ = 1 − (Ω_0 + Ω_d). r representing the epoch of reionization, H_0 the Hubble parameter, n_s the spectral index of the primordial spectrum and A_s the overall amplitude of the primordial spectrum are the other 4 parameters of this model. Out of these parameters, the first four dictate the dynamic of the Universe and the other two represent the initial condition through the primordial fluctuations given by P_R(k) = A_s \left( \frac{k}{r} \right)^{n_s - 1}, where k_s is the pivot point. Having the form of the primordial fluctuations and the expansion history of the Universe one can determine the growth of structure for this model on linear scales following the linearised perturbation equation and also run N-body simulations to study the small scales and non-linear regime. Despite the simplicity of the model, most astronomical observations are in great agreement with the concordance model and so far there has not been any strong observational evidence against it (e.g., Planck Collaboration XIII 2016; Alam et al. 2017; Scolnic et al. 2017). In this paper we test some important aspects of the concordance model of cosmology in light of the most recent cosmological observations in a model-independent manner. We test dark energy as the cosmological constant Λ, the FLRW metric and the flatness of the Universe, and we derive the H_0 parameters. We then use model independent reconstruction of the expansion history from supernovae data to fit growth of structure data and put model independent constrains on key cosmological parameters of (Ω_m, γ and σ_8). In § 2 we describe the background expansion and our tests on Λ dark energy, FLRW metric and flatness of the spatial curvature. Analysis on the growth of structure and testing general theory of relativity are presented in § 3, and our conclusions are drawn in § 4.

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2 BACKGROUND EXPANSION: TESTING Λ THE FLRW METRIC, AND THE CURVATURE

At the background level, it is possible to test dark energy as Λ, the FLRW metric, and the curvature of the Universe. In a FLRW universe with a dark energy component of equation of state w(z), the luminosity distance can be written for any curvature Ωk

\[ d_L(z) = \frac{c}{H_0} (1 + z) D(z), \]  

(1)

where

\[ D(z) = \frac{1}{\sqrt{\Omega_k}} \sin \left( \sqrt{-\Omega_k} \int_{0}^{z} \frac{dz'}{H(z')} \right) \]  

(2)

is the dimensionless comoving distance, and

\[ h^2(z) = \left( \frac{H(z)}{H_0} \right)^2 = \Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + (1 - \Omega_m - \Omega_k) \exp \left( \int_{0}^{z} \frac{1 + w(x)}{1 + x} dx \right) \]  

(3)

is the expansion history. Having different observables of the cosmic distances and expansion history one can then introduce novel approaches to examine the FLRW metric, flatness of the Universe and Λ dark energy in a model-dependent (e.g., Farooq & Ratra 2013) or independent way (Clarkson et al. 2008; Sahni et al. 2008; Sapone et al. 2014; L’Huillier & Shafieloo 2017; Zhao et al. 2017; Marra & Sapone 2017). Note that one can also test the metric and the curvature using gravitational lensing (e.g. Räsänen et al. 2015; Denis et al. 2018).

2.1 Model-independent reconstruction of the expansion history from the Pantheon compilation

In order to reconstruct the expansion history h(z) at any redshift z, we apply the iterative smoothing method (Shafieloo et al. 2006; Shafieloo 2007; L’Huillier & Shafieloo 2017) to the latest compilation of supernovae distance modulus (Pantheon, Scollnic et al. 2017). In order to take into account the non-diagonal terms of the covariance matrix, we modified the method in the following way. Starting with some initial guess ˆµn, we iteratively calculate the reconstructed ˆµn+1 at iteration n + 1:

\[ ˆµ_{n+1}(z) = ˆµ_n(z) + N(z) \delta \mu_n^T \cdot C_{SN}^{-1} \cdot W(z), \]  

(4)

where

\[ W(z) = \exp \left( -\frac{\ln^2 \left( \frac{1 + z}{1 + z_\text{drag}} \right)}{2 \Delta^2} \right) \]  

(5)

is a vector of weights,

\[ N^{-1}(z) = I^T \cdot C_{SN}^{-1} \cdot W(z) \]  

(6)

is a normalization factor

\[ I = (1, \ldots, 1)^T \]  

(7)

is a column vector,

\[ \delta \mu_n = \mu_i - ˆ\mu_n(z_i) \]  

(8)

is the vector of residuals, and ˆC_{SN} is the covariance matrix of the Pantheon data. In case of uncorrelated data (C_{ij} = δ_{ij} σ_{ij}^2), we recover the formula used in Shafieloo (2007) and L’Huillier & Shafieloo (2017).

The χ^2 of the reconstruction ˆµ_n(z) is then defined as

\[ \chi_n^2 = \delta \mu_n^T \cdot C_{SN}^{-1} \cdot \delta \mu_n, \]  

(9)

and in this work we only consider reconstructions with χ^2 < χ^2_{ΛCDM} best-fit.

The result of the smoothing procedure is thus H_0 d_L(z) = 10^{h_\text{e}}. Under the assumption of a flat Universe, we can obtain h_\text{e}(z) = 1/d_L(z)/dz. Therefore, we obtain a non-exhaustive sample of plausible expansion histories, directly reconstructed by supernova data, and with no model assumption, which all give a better χ^2 to the Pantheon data than the best-fit ΛCDM model. This enables us to explore regions of the physical space of the expansion history beyond the flexibility of the concordance model that can fit the data reasonably well.

2.2 BAO measurements of cosmic distances and expansion history

The radial mode of the BAO measures H(z)r_d, while the transverse mode provides d_A(z)/r_d, where

\[ r_d = \frac{c}{\sqrt{3} a} \int_0^{1/(1+z_{\text{drag}})} \frac{da}{a^2 H(a) \sqrt{1 + \frac{3aH}{H_0}}} \]  

(10)

is the sound horizon at the drag epoch z_{drag}. We combined the Baryon Oscillation Spectroscopic Survey (BOSS) DR12 consensus values (Alam et al. 2017) and the extended-BOSS (eBOSS) DR14Q measurements (Zhao et al. 2018). We note that both BOSS DR12 and eBOSS DR14Q provide H(z)r_d/r_{d, fid} and d_A(z)/r_{d, fid}/r_d with r_{d, fid} = 147.78 Mpc. We also include the Dark Energy Survey DR1 (DES DR1) measurement of d_A/r_d at z = 0.81 (The Dark Energy Survey Collaboration et al. 2017). We use these BAO data along with our reconstructions of the expansion history from supernova data as two independent sets of observations to test some key aspects of the concordance model.

2.3 Testing Λ Dark Energy

The solid black lines in Fig. 1 show the different reconstructed D(z), h(z) = 1/D′(z) and Om(z) from Pantheon supernovae compilation where Om(z) is defined as (Sahni et al. 2008):

\[ Om(z) = \frac{H^2(z) - 1}{(1 + z)^3} \]  

(11)

We also show in Fig. 1 the BAO data points for these quantities. Since the BAO measure H(z)r_d and d_A(z)/r_d, to have a good sense of comparison within the context of the concordance model, we normalize them by H_0r_d from Planck 2015 (TTTTEEE+LowP+Lensing) best fit ΛCDM model, and show on the top panel D(z) = (1 + z)H_0p_dA(z)/(cr_d), in the middle panel h(z) = H(z)r_d/H_0r_d and the corresponding
\[
\frac{\sigma_{\Delta(z)}}{H(z)} = \frac{\sigma_{\Delta d_{\Lambda}}(z)}{d_{\Lambda}(z)} = \frac{\sigma_{\Delta d_{\Lambda}}(z)}{d_{\Lambda}(z) r_{d}}
\]

where, assuming a flat-FLRW universe, \( h(z) = 1/D(z) \).

For Fig. 2, the error-bars are shown as a solid line. For the first method, we use the transverse BAO mode, which has smaller error-bars, coupled with direct reconstructions \( D(z) \) which do not use derivative.

The second method involves the use of the line-of-sight mode of the BAO, together with \( h(z) \) from supernovae data which is a derivative. Since the Pantheon data become scarce at \( z \geq 1 \), the estimation of \( h(z) \) becomes less precise at this range having large error-bars. Combination of these two results to large uncertainties for \( H_0r_d \) from the second method. On the other hand, it can be seen from Fig. 1 that while \( h(z) \) from SNIa are lower than the best-fit Planck \( \Lambda \)CDM model, \( h(z) \) from the BAO (scaled with best-fit Planck \( \Lambda \)CDM Planck cosmology. Combining these results of the comoving distances and expansion histories may show some inconsistency with flatness as we will see later in this work.

2.4 Estimating \( H_0r_d \)

L'Huillier & Shaﬁeeloo (2017) estimated \( H_0r_d \) in a model-independent way by combining BAO measurements and reconstructions of the expansion history from supernovae. For each reconstruction \( n \), we can calculate \( H_0r_d \) in two different ways

\[
H_0r_d|d_{\Lambda,n} = \frac{c}{1+z} \frac{D_n(z)}{d_{\Lambda}(z)} r_d
\]

(12a)

\[
H_0r_d|H,n = \frac{H(z)r_d}{h_n(z)}
\]

(12b)

and their associated errors

\[
\sigma_{H_0r_d}|d_{\Lambda,n} = \frac{c}{1+z} \frac{\sigma_{D_n}}{d_{\Lambda}(z)} \frac{d_{\Lambda}(z)}{r_d}
\]

(13a)

\[
\sigma_{H_0r_d}|H,n = \frac{\sigma_{H_0}}{h_n(z)}
\]

(13b)

For each reconstruction \( n \) and method \( X \), we have an error \( \sigma_{H_0r_d,X,n} \). They are of the same order for each reconstruction, so we define the final BAO error as the maximum value over all reconstructions. For each reconstruction \( n \) and method \( X \), we have an error \( \sigma_{H_0r_d,X,n} \). They are of the same order for each reconstruction, so we define the final BAO error as the maximum value over all reconstructions. This error-bar is shown as a solid error-bar in Fig. 2.

For the first method (in orange), the measurements of \( H_0r_d \) from combination of supernovae and SDSS BAO data are fully consistent with Planck. The DES data point, also using the transverse BAO mode, is an independent confirmation at intermediate redshift. However, for the second method, while at low redshift, the measurements are consistent with Planck, the eBOSS data points are systematically lower than the Planck best-fit at \( z \geq 1.2 \) while the errorbars become very large at this range. This can be understood by the following remarks.

The first method yields very consistent results thanks to the use of the transverse BAO mode, which has smaller error-bars, coupled with direct reconstructions \( D(z) \) which do not use derivative.

The second method however, uses the line-of-sight mode of the BAO, together with \( h(z) \) from supernovae data which is a derivative. Since the Pantheon data become scarce at \( z \geq 1 \), the estimation of \( h(z) \) becomes less precise at this range having large error-bars. Combination of these two results to large uncertainties for \( H_0r_d \) from the second method. On the other hand, it can be seen from Fig. 1 that while \( h(z) \) from SNIa are lower than the best-fit Planck \( \Lambda \)CDM model, \( h(z) \) from the BAO (scaled with best-fit Planck \( \Lambda \)CDM model. 0149.1-149.13 (2018)

\[
\frac{\sigma_{\Delta(z)}}{H(z)} = \frac{\sigma_{\Delta d_{\Lambda}}(z)}{d_{\Lambda}(z)} = \frac{\sigma_{\Delta d_{\Lambda}}(z)}{d_{\Lambda}(z) r_{d}}
\]

This tension is also visible clearly looking at the \( Om \) diagnostic in bottom plot of Fig. 1, which is also consistent with the finding of (Sahni et al. 2014). If dark energy is a cosmological constant, the \( Om \) diagnostic should be constant in redshift. Therefore, having different values from different observations suggests some tension among the data within the framework of the concordance model.

Meanwhile, the comoving distances \( D(z) \) from BAO and SNIa are fully consistent together and with the best-fit Planck (2015) best fit \( \Lambda \)CDM model. The black, solid lines are the reconstructed expansion histories from the Pantheon data which are fully model independent, and the purple line is the prediction from (Planck Collaboration XIII 2016) for the best-fit \( \Lambda \)CDM model.
2.5 Test of the FLRW metric and the curvature

L’Huillier & Shafieloo (2017) reformulated the Clarkson et al. (2008) $O_k$ diagnostic by introducing the $\Theta$ diagnostic so that it now only depends on the BAO and supernovae observations:

\[
O_k(z) = \frac{\Theta(z) - 1}{D^2(z)}
\]

\[
\Theta(z) = \frac{1 + z}{c} H(z)r_d \frac{d_k(z)}{r_d} \frac{D'(z)}{D(z)}
\]

For a FLRW Universe, $O_k(z) \equiv \Omega_k$, and in case of flatness, $O_k(z) \equiv 0$ and $\Theta(z) \equiv 1$. We can then calculate for each reconstruction the associated $O_k, r_d$, and $\Theta_n(z)$. Similarly to § 2.4, we calculated the median of $O_k$ and $\Theta$ over all reconstructions, and defined the SN error as the minimal and maximal values, and the BAO error as the maximal error over all reconstructions. Fig. 3 shows $\Theta(z)$ (top) and $O_k(z)$ (bottom). Both are consistent with a flat FLRW metric up to $z \approx 1.2$.

However, at high redshift, some deviation from flatness can be seen. Again, this can be explained by the previous remarks. In addition to the scarcity of the SN data at $z \gtrsim 1.5$, which results in into poor constraints on $h(z)$, the BAO seem to show some internal tensions. While $d_A(z)/r_d$ are consistent with the Planck best-fit, $H(z)r_d$ are lower than expected. However, the $\Theta$ and $O_k$ statistics assume a FLRW metric, where $d_A$ and $H$ are related to each other. Thus, discrepancy between $d_A$ and $H$ combined with the higher $h$ values at high-redshift ($z \gtrsim 1$) yields lower values for $\Theta$ and $O_k$.

3 GROWTH OF STRUCTURE VERSUS EXPANSION: TESTING GR

At the perturbation level, the cosmological growth of structure can also serve as a test of gravity (Nesseris & Perivolaropoulos 2008; Song & Percival 2009; Basilakos 2012; Shafieloo et al. 2013; Pavlov et al. 2014; Gómez-Valent et al. 2015; Ruiz & Huterer 2015; Mueller et al. 2016; Nesseris et al. 2017; Marra & Sapone 2017; Solá et al. 2017; Kazantzidis & Perivolaropoulos 2018). In the linear regime,
the growth of structure follows
\[
\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\delta = 0,
\]  
(17)
where \(\delta = \rho/\bar{\rho} - 1\) is the density contrast with respect to the mean density of the Universe \(\bar{\rho}\). The growth rate
\[
f(a) = \frac{\ln \delta}{\ln a},
\]  
(18)
can be approximated for a wide range of cosmologies by (Lahav et al. 1991; Wang & Steinhardt 1998; Linder 2005)
\[f(z) = \Omega_m^8(z),\]
(19)
where
\[\Omega_m(z) = \frac{\Omega_m(1 + z)^3}{h^2(z)}.\]
(20)
In general relativity (GR), \(\gamma \approx 0.55\). \(f\sigma_8\) is thus a powerful probe of gravity. Observationally, redshift-space distortion enables to measure the combination (e.g. Song & Percival 2009)
\[f\sigma_8(z) \approx \sigma_8 \Omega_m^8(z) \exp \left( -\int_0^z \Omega_m^8(x) \frac{dx}{1 + x}\right),\]
(21)
where \(\sigma_8 = \sigma_8(z = 0)\) is the rms fluctuation in \(8\ h^{-1}\)Mpc spheres. Following this formalism, having model independent reconstructions of the expansion history and \(f\sigma_8(z)\) data, one can obtain constraints on \(\Omega_m, \gamma, \) and \(\sigma_8\) (L’Huillier et al. 2018).

Note, however, that one should keep in mind that Eq. (19) is an approximate fit only. In particular, \(\gamma\) may not be exactly constant for quintessence (dark energy modelled by a scalar field with some potential minimally coupled to gravity, Polarski et al. 2016). Still both for \(\Lambda\)CDM and for quintessence-CDM this fit is good since \(\sigma_8^2\) is small as far as \(\Omega_m\) is not too small (see also Polarski & Gannouji 2008). For modified gravity theories like the (R) gravity, the situation can be different (Gannouji et al. 2009; Motohashi et al. 2010).

### 3.1 Cosmological constraints on \(\Omega_m, \gamma, \sigma_8\)

Following L’Huillier et al. (2018), we combined the Pantheon compilation with the latest measurements of \(f\sigma_8\): 2dFGRS (Song & Percival 2009), WiggleZ (Blake et al. 2011), 6dFGRS (Beutler et al. 2011), VIPERS (de la Torre & Peacock 2013), the SDSS Main galaxy sample (Howlett et al. 2015), 2MTF (Howlett et al. 2017), BOSS DR12 (Gil-Marín et al. 2017), FastSound (Okumura et al. 2016), and eBOSS DR14Q (Zhao et al. 2018). In this section, we assume a flat Universe, therefore
\[h(z) = \frac{1}{D(z)}.\]
(22)
For each reconstructed expansion history \(h_n(z)\), we vary \((\Omega_m, \gamma, \sigma_8)\) and calculate \(f\sigma_8(z|\Omega_m, \gamma, \sigma_8, h_n)\) via eq. (21). We can then fit the RSD data and since the RSD and SN data are independent, we then calculate the total \(\chi^2\) by summing
\[\chi^2_{n,tot} = \chi^2_{n,f\sigma_8} + \chi^2_{n,SN},\]
(23)
where
\[\chi^2_{n,f\sigma_8} = \delta f\sigma_8^T C^{-1}_{f\sigma_8} \cdot \delta f\sigma_8,\]
(24)
and where the \(i\)th component of the residual vector \(\delta f\sigma_{8n}\) is
\[\delta f\sigma_{8ni} = f\sigma_8(z_i|\Omega_m, \gamma, \sigma_8, h_n) - f\sigma_8(z_i|\Omega_m, \gamma, \sigma_8, h_0).\]
(25)
The \(f\sigma_8\) data used in this work are shown in Fig. 4. The black lines are computed from eq. (21) using a randomly selected combination of expansion histories from supernovae data, \(\Omega_m, \gamma, \) and \(\sigma_8\). All lines have a better \(\chi^2\) to the SN+growth data than the best-fit \(\Lambda\)CDM model.

The red contours in the \((\sigma_8, \Omega_m)\) plane in Fig. 5a show the 1\(\sigma\) and 2\(\sigma\) regions of the parameter space in the flat \(\Lambda\)CDM case, that is, flat-\(\Lambda\)CDM expansion history and \(\gamma = 0.55\). The blue contours in Fig. 5a show the allowed parameter space in the model-independent case. Namely, for any point in the blue contours, one can find at least one reconstruction \(h(z)\) which, combined to the corresponding \((\Omega_m, \gamma, \sigma_8)\), gives a better fit to the data than the best-fit \(\Lambda\)CDM. In the \((\sigma_8, \Omega_m)\) plane, the model-independent case is fully consistent with the \(\Lambda\)CDM case. Moreover, the flexibility of the model-independent approach allows a larger area of the parameter space to be consistent to the data. For instance, for larger values of \(\sigma_8\) and lower values of \(\Omega_m\), one can find reconstructed expansion histories that give a better total fit to the data (SN1a+growth) with respect to the best fit \(\Lambda\)CDM model. For the model-independent case, \(\gamma\) is fully consistent with 0.55, as expected from GR. Moreover, lower value of \(\gamma\), combined with lower value of \(\Omega_m\) and larger \(\sigma_8\), can also provide good fit to the data.

We then fix \(\gamma = 0.55\), as we did for the \(\Lambda\)CDM case, and show in Fig. 5b the corresponding confidence contours. This effectively allows for a non-\(\Lambda\)CDM background expansion, with gravity as GR. This time, since we do not allow \(\gamma\) to vary, the region with low \(\Omega_m\) and high \(\sigma_8\) is now forbidden.

Finally, following L’Huillier et al. (2018), we focus on combinations of \(h(z)\) and \(\Omega_m\) that respect the positive dark energy condition
\[\Omega_{\text{de}}(z) = h^2(z) - \Omega_m(1 + z)^3 \geq 0 \quad \forall z.\]
(26)

We show this region in dark-blue in Figs. 5a and 5b. Imposing equation (26) effectively forbids large values of...
\( \Omega_m \), and dramatically reduces the allowed parameter space of the model-independent case. The allowed region of the parameter space is then fully consistent with the model-dependent case, as in L’Huillier et al. (2018). This is a strong support from the data for combination of \( \Lambda \) model-independent case. The allowed region of the parameter space is then consistent with the model-independent constraints on \( \Omega_m \) and \( \sigma_8 \) and a lower bound limit on \( \sigma_8 > 0.70 \). These are in fact model independent constrains on these key cosmological parameters.

4 SUMMARY AND CONCLUSIONS

We used the Pantheon supernovae compilation to reconstruct the expansion history in a model-independent way, using an improved version of the iterative smoothing method (Shafieloo et al. 2006; Shafieloo 2007; L’Huillier & Shafieloo 2017), which we modified to take into account the non-diagonal terms of the full covariance matrix. We then combined the reconstructed expansion histories to measure \( H(z) \) and \( d_A(z)/r_d \) from BOSS DR12 and eBOSS DR14Q to model-independently measure \( H_0 \) and test the FLRW metric. Our measurements of \( H_0 \) are consistent with the Planck 2015 values, where the metric test is consistent with a flat-FLRW metric. However, for the eBOSS DR14Q data points, while \( d_A(z)/r_d \) is consistent with the prediction from the Planck best-fit \( \Lambda \)CDM cosmology, the \( H(z)/r_d \) measurements are slightly but systematically lower. This yields some hints for a departure from flat-FLRW (Fig. 3) and supports previous findings of Sahni et al. (2014) & Zhao et al. (2017).

We then fit the growth data from redshift space distortion, mainly from SDSS survey using the model-independent reconstructions of the expansion history, and put model-independent constraints on \( \Omega_m \) and \( \sigma_8 \) from growth and expansion data. In plot (b) we have fixed \( \gamma = 0.55 \) (assuming GR). The red contours are the 1\( \sigma \) and 2\( \sigma \) confidence levels for the \( \Lambda \)CDM case. The blue contours are associated to the combination of the parameters and reconstructions of the expansion history that yield a better \( \chi^2 \) with respect to the best-fit \( \Lambda \)CDM model. The dark-blue region satisfy positive dark energy density condition as expressed in equation (26).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Model independent cosmological constrains on \( (\Omega_m, \gamma, \sigma_8) \) from growth and expansion data. In plot (b) we have fixed \( \gamma = 0.55 \) (assuming GR). The red contours are the 1\( \sigma \) and 2\( \sigma \) confidence levels for the \( \Lambda \)CDM case. The blue contours are associated to the combination of the parameters and reconstructions of the expansion history that yield a better \( \chi^2 \) with respect to the best-fit \( \Lambda \)CDM model. The dark-blue region satisfy positive dark energy density condition as expressed in equation (26).}
\end{figure}

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