Optimization and Simulation of DMC Algorithm with Terminal Weight Based on LQR Controller

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Abstract. Based on the multivariable dynamic matrix control algorithm, there are many similarities between the dynamic matrix control algorithm and LQR in the form of state equation, state variable and objective function. We can use the objective function of LQR to improve the objective function of DMC and improve the control effect of the system. Using the given mathematical model, the terminal weighting matrix of LQR is applied to DMC algorithm, and the optimized multivariable dynamic matrix control algorithm is simulated. The results show that the algorithm can effectively reduce the response time of the system, make the system sink faster and reach the given value faster.

1. Introduction
In the current industrial control engineering, predictive control algorithm is the most used and the best advanced control algorithm. The dynamic matrix control algorithm is one of the predictive control algorithms. Dynamic matrix control (DMC) algorithm was proposed by Cutler in 1980[1]. It is an optimal control algorithm based on the prediction model of step response, rolling implementation and feedback correction. In recent decades, in order to deal with different situations and needs in industry, algorithms have made great progress in theory and practical engineering, especially in the field of process control. The main development directions are as follows: (1) the combination of DMC and other algorithms, such as the combination of the classical proportional integral differential control algorithm and DMC control, this improved algorithm not only has the strong adaptability and flexibility of PID algorithm, but also has the strong robustness and rolling optimization of DMC algorithm Strong coupling and large absolute error integral of DMC. (2) The algorithm optimizes its own formula. For example, a new optimization is made on the calculation of the characteristic polynomial of DMC, which further simplifies the DMC algorithm which is not complicated. Firstly, this paper introduces the DMC control briefly, and expounds its principle; Then, the feedback correction part is optimized to improve the performance of the system; Finally, taking the distillation column as the object, experimental simulation is carried out to verify the feasibility and superiority of the optimized algorithm[2].

2. Multivariable dynamic matrix control algorithm
For MIMO systems, in general, due to the strong coupling of the system, each output of the system is determined by all the inputs. Therefore, it is not necessary to consider the control problem of model pairing. Assuming that the system has m inputs and N outputs, the step response \( \alpha_j(t) \) of each output \( y_j \) to each input \( u_j \) can be obtained according to the nonparametric model:
\[ \alpha_y = [\alpha_y(1), \ldots, \alpha_y(N)] \]

Where \( n \) is called the model length.

### 2.1. Prediction model

Although the step response is a nonparametric model, because the linear system has the properties of scale and superposition, according to the output model \( \alpha_y \), the output value of the system can be obtained by single variable prediction and accumulation[3]. Under the action of input \( u_j \), the predicted value of output \( y_i \) is derived. Assuming that the system is single input single output, there is a single increment at \( k \) when the control action is constant \( \Delta U(k) \), the initial prediction value of the output at the next \( N \) times is as follows:

\[ y_{i,N1}(k) = y_{i,N0}(k) + \alpha_y \Delta u(k) \]  

(2)

Among,

\[
y_{i,N1}(k) = \begin{bmatrix} \hat{y}_{i,1}(k + |1| k) \\ \vdots \\ \hat{y}_{i,N}(k + |N| k) \end{bmatrix}, \\ y_{i,N0}(k) = \begin{bmatrix} \hat{y}_{i,0}(k + |1| k) \\ \vdots \\ \hat{y}_{i,0}(k + |N| k) \end{bmatrix}.
\]

Each component in \( y_{i,N0}(k) \) represents the predicted value of zero time in the future \( n \) times when all control variables \( u_1, \ldots, u_m \) at \( k \) time remain unchanged.

Similarly, when \( u_j \) has \( M \) incremental changes \( \Delta u_j(k), \ldots, \Delta u_j(k + M - 1) \) in order, \( M \) is called control time domain[4]. Under its action, the predicted value of \( y_i \) in \( P \) time is shown in equation (3) (\( P \) is prediction time domain):

\[ \hat{y}_{i,PM}(k) = \hat{y}_{i,P0}(k) + A_y \Delta u_{j,M}(k) \]

(3)

Among,

\[
\hat{y}_{i,PM}(k) = \begin{bmatrix} \hat{y}_{i,M}(k + |1| k) \\ \vdots \\ \hat{y}_{i,M}(k + |P| k) \end{bmatrix}, \\ \hat{y}_{i,P0}(k) = \begin{bmatrix} \hat{y}_{i,0}(k + |1| k) \\ \vdots \\ \hat{y}_{i,0}(k + |P| k) \end{bmatrix}, \\ A_y = \begin{bmatrix} \alpha_y(1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \alpha_y(P) & \cdots & \alpha_y(P - M + 1) \end{bmatrix}, \\ \Delta u_{j,M}(k) = \begin{bmatrix} \Delta u_j(k) \\ \vdots \\ \Delta u_j(k + M - 1) \end{bmatrix}.
\]

When \( u_1, \ldots, u_m \) acts together, the general multivariable system prediction model can be obtained by adding \( y_i \) under different \( u_j \) actions:

\[ \hat{y}_{N1}(k) = \hat{y}_{N0}(k) + A \Delta u(k) \]  

(4)

\[ \hat{y}_{PM}(k) = \hat{y}_{P0}(k) + A \Delta u(k) \]  

(5)
Among,

\[
\hat{y}_{n1}(k) = \begin{bmatrix} \hat{y}_{1,n1}(k) \\ \vdots \\ \hat{y}_{p,n1}(k) \end{bmatrix}, \quad \hat{y}_{n0}(k) = \begin{bmatrix} \hat{y}_{1,n0}(k) \\ \vdots \\ \hat{y}_{p,n0}(k) \end{bmatrix},
\]

\[
\hat{y}_{pm}(k) = \begin{bmatrix} \hat{y}_{1,pm}(k) \\ \vdots \\ \hat{y}_{n,pm}(k) \end{bmatrix}, \quad \hat{y}_{p0}(k) = \begin{bmatrix} \hat{y}_{1,p0}(k) \\ \vdots \\ \hat{y}_{n,p0}(k) \end{bmatrix},
\]

\[
\bar{A} = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1m} \\ \vdots & \ddots & \vdots \\ \alpha_{n1} & \cdots & \alpha_{nm} \end{bmatrix}, \quad A = \begin{bmatrix} A_{11} & \cdots & A_{1m} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nm} \end{bmatrix},
\]

\[
\Delta u(k) = \begin{bmatrix} \Delta u_1(k) \\ \vdots \\ \Delta u_m(k) \end{bmatrix}, \quad \Delta u_M(k) = \begin{bmatrix} \Delta u_{1,M}(k) \\ \vdots \\ \Delta u_{m,M}(k) \end{bmatrix}.
\]

2.2. Rolling optimization

In multivariable DMC control, the system output \( y_i \) must be infinitely close to its corresponding expected value \( \omega_i \) in the predictive time domain \( P \), in order to achieve the ideal control effect, and in the control time domain \( m \), the control increment can be changed to affect the dynamic response of the system, that is the control increment is constrained. Therefore, the performance index can be described as follows:

\[
\min J(k) = \|W(k) - \hat{y}_{pm}(k)\|_Q^2 + \|\Delta u_M(k)\|_R^2
\]

Among,

\[
W(k) = \begin{bmatrix} \omega_1(k) \\ \vdots \\ \omega_p(k) \end{bmatrix}, \quad \omega_i = \begin{bmatrix} \omega_i(k+1) \\ \vdots \\ \omega_i(k+P) \end{bmatrix}, i = 1, \cdots, p
\]

\[
Q = \text{block diag} \left( Q_1, \cdots, Q_n \right); \quad Q = \text{diag} \left[ q(1), \cdots, q(p) \right], i = 1, \cdots, n; \quad R = \text{block diag} \left( R_1, \cdots, R_m \right); \quad R_j = \text{diag} \left[ r_j(1), \cdots, r_j(M) \right], j = 1, \cdots, m. \text{ Each element in Q and R has a clear physical meaning, in which Q is the error weight matrix, which is used to feedback the error of the system at different times and directly determine the steady-state error of the system [9]. } Q_1, \cdots, Q_n \text{ correspond to n outputs of the system, while } q_i(1), \cdots, q_i(p) \text{ represent tracking errors at different times[5]. Similarly, } R_1, \cdots, R_m \text{ correspond to m control inputs of the system, which affect the regulation speed of the system, that is, the response time of the system, while } r_j(1), \cdots, r_j(M) \text{ correspond to the inhibition of } u_j \text{ increment at different times. Without considering constraints, the optimal control increment of performance index can be obtained according to the prediction model shown in equation}(5), \text{ according to the formula}(6), \text{ and make this } \frac{\partial J(k)}{\partial u(k)} = 0, \text{ we can find out:}
\]

\[
\Delta u_{ij}(k) = \left( A^TQA + R \right)^{-1} A^TQ \left[ W(k) - \hat{y}_{p0}(k) \right]
\]
According to equation (7), it can be deduced that the control increment at the current time is:

$$\Delta u(k) = D \left[ W(k) - \hat{y}_{p0}(k) \right]$$

(8)

Among,

$$D = L \left( A^T QA + R \right)^{-1} A^T Q$$

$$D = \begin{bmatrix} a_{11}^T & \cdots & a_{1n}^T \\ \vdots & \ddots & \vdots \\ a_{m1}^T & \cdots & a_{mn}^T \end{bmatrix}$$

Notice that

$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

is an $m \times mM$ matrix, in which the first row, the first column and the $m$-th row, multiplication of $(m-1)$ and the $m$-th column elements results is 1 and all other elements are 0. $d_j^T (i = 1, \cdots, n; j = 1, \cdots, m)$ is a $p$-dimensional vector.

2.3. Feedback correction

According to the prediction model shown in equation (1), at $k$ time, after the control is implemented, the predicted output values of the system at $P$ time in the future can be calculated, including the predicted value $\hat{y}_{i,1}(k+1 | k), i = 1, \cdots, n$ of each output at $K+1$ time[6]. In addition, the actual output value $y_{i,1}(k+1)$ of the system at $t + 1$ can be measured. By comparing the two, the error vector can be obtained:

$$e(k+1) = \begin{bmatrix} e_1(k+1) \\ \vdots \\ e_n(k+1) \end{bmatrix} = \begin{bmatrix} y_{1,1}(k+1) - \hat{y}_{1,1}(k+1 | k) \\ \vdots \\ y_{n,1}(k+1) - \hat{y}_{n,1}(k+1 | k) \end{bmatrix}$$

(9)

According to the error between the actual output value and the predicted output value, the system is compensated and corrected by weighting. The corrected prediction vector can be obtained by equation (9):

$$\hat{y}_{cor}(k+1) = \hat{y}_{N,1}(k) + He(k+1)$$

(10)

Among,

$$H = \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nn} \end{bmatrix}, h_{ij} = \begin{bmatrix} h_{ij}(1) \\ \vdots \\ h_{ij}(N) \end{bmatrix}, i, j = 1, \cdots, n$$

$H$ is the error correction matrix, which is used to compensate the error vector. It consists of a series of correction vectors $h_{ij}$. For multi input multi output system, the original error of the system is unknown, and the prediction error of each output has different cross influence on the prediction value, that is, the coupling degree of the system is different. Therefore, the selection of error correction vector $h_{ij}$ is random. For the convenience of calculation, only the main diagonal block in the main diagonal of $H$ is usually reserved, that is, only its own error is used to modify the predicted value output by weighting. The corrected prediction vector $\hat{y}_{cor}(k+1)$ moves the time base point from time $k$ to time $K + 1$ through the shift matrix $s$, and the initial prediction value at time $K + 1$ can be obtained:

$$y_{N,0}(k+1) = S_0 \hat{y}_{cor}(k+1)$$

(11)

Among,
The control process of multivariable dynamic matrix control is repeated according to this process.

### 3. LQR algorithm

In modern control theory, the linear system represented by state space equation is the main research object of linear quadratic regulator (LQR). The quadratic function of control input or the state of control object is designed as the objective function.

Consider a system:

\[ x(t) = f[x(t), u(t)] \]  
(12)

Where \( x(t) \) is the state variable and \( u(t) \) is the control quantity.

Assuming the initial state \( x(t_0) = x_0 \), the general form of quadratic evaluation function is as follows:

\[
J = \frac{1}{2} \int_{t_0}^{\infty} \left[ x^T(t)Q(t)x(t) + u^T(t)R(t)u(t) \right] dt
\]  
(13)

Where \( Q(t) \) is the semi positive definite terminal weighting matrix, \( Q(t) \) is the semi positive definite weighting matrix, and \( R(t) \) is the positive definite weighting matrix. \( Q(t), R(t) \) is usually a diagonal matrix.\(^7\)

By minimizing the objective evaluation function \( J \), the optimal control quantity \( u(t) \) of the system is obtained, and the system is transferred from the initial state to the final state, so as to achieve the control effect of keeping a small error with the minimum consumption and achieve the purpose of comprehensive optimization of energy and error. The feedback control law is designed:

\[
\dot{u} = -Kx = - \left[ \left( R + B_d^T P B_d \right)^T \right. \\
\left. B_d^T P A_d \right] x
\]  
(14)

Where \( p \) is the solution of Riccati equation

\[
P A + A^T P - R B_d P B_d^T + Q = 0.
\]

### 4. Improvement of multivariable dynamic control

The standard DMC cost function is similar to the cost function for an LQR controller with output weighting, as shown in the following equation:

\[
J(u) = \sum_{i=1}^{\infty} y(k+i)^T Q y(k+i) + u(k+i)^T R u(k+i)
\]  
(15)

The two cost functions are equivalent if the DMC cost function is:

\[
J(u) = \sum_{i=1}^{n-1} \left( y(k+i)^T Q y(k+i) + u(k+i-1)^T R u(k+i-1) \right) + x(k+p)^T Q_p x(k+p)
\]  
(16)

First, calculate the Riccati matrix \( Q \), \( L^T L = Q \), by lqry instruction in MATLAB. Next, define auxiliary unmeasured output variables \( y_c = L x \), such that \( y_c^T y_c = x^T Q_p x \). Augment the output vector of the plant such that it includes these auxiliary outputs, as shown in equation (17): \( C = \begin{bmatrix} C \\ L \end{bmatrix} \)

\[
C_d = \begin{bmatrix} C \\ L \end{bmatrix}
\]  
(17)

### 5. Simulation

Predictive control is the most widely used advanced control in the field of industrial control. The
simulation of the improved DMC algorithm is carried out. The results show that the improved DMC algorithm is better than the traditional DMC Algorithm in performance.

The given state space model is:

\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = \begin{bmatrix}
    1 & 0 \\
    0.1 & 1
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} + \begin{bmatrix}
    0.1 & 0.005
\end{bmatrix} \begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix}
\]

\[
y_1 = \begin{bmatrix}
    1 & 0
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
\]

\[
y_2 = \begin{bmatrix}
    0 & 1
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
\]

DMC parameters: sampling period TS = 0.1s, model length n = 10, control time domain M = 200s, optimization time domain P = 200s. LQR parameter value: \( q = [10, 0, 1], R=1 \).

The auxiliary output is weighted, and the simulation waveforms of the system with different weights are shown in Fig. 1 ~ Fig. 3.

Figure 1. Auxiliary output weighted values are 1 and 1

Figure 2. Auxiliary output weighted values are 3 and 3
Through the comparison of Figure 1 and Figure 2, it is found that when the weighted value is greater than 1, the optimized DMC control algorithm curve has a faster descent speed. Comparing Figure 1 and Figure 2 with figure 3, it is found that when the terminal weight is larger, the time to converge to a given value is shorter, but the corresponding output will have a larger overshoot.

6. Conclusions
Based on DMC algorithm, the terminal weight is added to optimize the algorithm. From the simulation results, the improved algorithm improves the performance and control effect of the system. The optimized algorithm has less response time, which makes the system sink faster and reaches the given value faster. In addition, it is found that the improved algorithm has a large overshoot when the weight is given. Therefore, the control algorithm needs to be further improved, but it still has some innovation and application prospects.

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