Parameters of the Link between the Optical and Radio Frames from Gaia DR2 Data and VLBI Measurements

V.V. Bobylev

Pulkovo Astronomical Observatory, Russian Academy of Sciences,
Pulkovskoe sh. 65, St. Petersburg, 196140 Russia

Abstract—Based on published data, we have assembled a sample of 88 radio stars for which there are both trigonometric parallax and proper motion measurements in the Gaia DR2 catalogue and VLBI measurements. A new estimate of the systematic offset between the optical and radio frames has been obtained by analyzing the GaiaDR2–VLBI trigonometric parallax differences: $\Delta \pi = -0.038 \pm 0.046$ mas (with a dispersion of 0.156 mas). This means that the Gaia DR2 parallaxes should be increased by this correction. The parallax scale factor is shown to be always very close to unity within $\sim 3$ kpc of the Sun: $b = 1.002 \pm 0.007$.

Our analysis of the proper motion differences for the radio stars based on the model of solid-body mutual rotation has revealed no rotation components differing significantly from zero: $(\omega_x, \omega_y, \omega_z) = (-0.14, 0.03, -0.33) \pm (0.15, 0.22, 0.16)$ mas yr$^{-1}$.

INTRODUCTION

Highly accurate stellar parallaxes are required to solve many stellar-astronomy problems. The trigonometric parallaxes are among the most reliable ones. However, it is necessary to check and eliminate the possible systematic offsets before using even the most reliable data.

The first data release of the Gaia space experiment was published in September 2016 (Prusti et al. 2016; Brown et al. 2016). The second data release of this experiment, Gaia DR2, appeared in April 2018 (Brown et al. 2018). This catalogue contains the trigonometric parallaxes and proper motions of $\sim 1.7$ billion stars. The derivation of their values is based on the orbital observations performed over 22 months. The mean errors of the trigonometric parallax and both stellar proper motion components in this catalogue depend on the magnitude. For example, for bright stars ($G < 15^m$) the parallax errors lie within the range 0.04–0.02 milliarcseconds (mas), while for faint stars ($G = 20^m$) they are about 0.7 mas.

Lindegren et al. (2018) pointed out the presence of a possible systematic offset $\Delta \pi = -0.029$ mas in the Gaia DR2 parallaxes with respect to the inertial reference frame. At present, there are several reliable distance scales a comparison with which allows, in the opinion of their authors, the systematics of the Gaia trigonometric parallaxes to be checked.

Stassun and Torres (2016) found quite a significant mean offset $\Delta \pi = -0.25 \pm 0.05$ mas of the Gaia DR1 trigonometric parallaxes with respect to the parallaxes of a calibration sample of eclipsing binaries. This result was soon confirmed by other authors based on an analysis of classical Cepheids close to the Sun (Casertano et al. 2017), the ground-based parallaxes of the nearest M dwarfs (Jao et al. 2016), and asteroseismology (Huber et al. 2017).
Having compared the parallaxes of 89 stars from the Gaia DR2 catalogue and calibration eclipsing binary stars, Stassun and Torres (2018) found a slight offset between the frames $\Delta \pi = -0.082 \pm 0.033$ mas. This value is also confirmed by other authors, in particular, when analyzing Cepheids (Riess et al. 2018) and asteroseismology (Zinn et al. 2018).

Therefore, the distances to radio stars determined by very long baseline interferometry (VLBI) are of interest. Here we have in mind the absolute parallaxes that are absolutized during the observations using quasars. At present, the VLBI observations aimed at determining highly accurate trigonometric parallaxes and proper motions of radio sources, in particular, galactic masers, are being performed by several research teams.

The accuracy of astrometric VLBI measurements depends on many factors. For example, estimates of the contributions from the position of a calibration source, the Earth’s orientation, the antenna position, and the tropospheric delay for radio sources located at different declinations can be found in Pradel et al. (2006). Furthermore, the mean VLBI parallax error depends on the observation frequency: the higher the frequency, the smaller this error. As a result, the mean VLBI parallax error in observations at 22.2 GHz is $\sim 0.01$ mas.

The Gaia DR2 catalogue contains the astrometric parameters for more than half a million quasars. This has allowed the optical, kinematically nonrotating Gaia DR2-Celestial Reference Frame (Gaia-CRF2) to be realized. Some of the quasars have accurate VLBI positions, which allows (Mignard et al. 2018) the axes of this frame to be aligned with the International Celestial Reference Frame (ICRF) specified by a set of radio sources, for example, ICRF2 or ICRF3 (which is being developed at present). As Mignard et al. (2018) showed, the coordinate axes of the Gaia DR2 catalogue and the ICRF3 prototype are aligned with errors of 20–30 mas, but more accurate values of these errors will be presented after a more detailed study of various errors. Therefore, determining the mutual rotation parameters between the two (optical and radio) frames using the proper motions of radio stars is of interest.

The goal of this paper is to produce a collection of VLBI observations of the absolute parallaxes and proper motions for radio stars common to the Gaia DR2 catalogue based on published data and to use this sample as a calibration one to check the Gaia DR2 distance scale.

**DATA**

In this paper we collected the VLBI observations of stellar trigonometric parallaxes and proper motions performed and published by various research teams. For example, these include the Japanese VERA (VLBI Exploration of Radio Astrometry) project devoted to the observations of H$_2$O masers at 22.2 GHz (Hirota et al. 2008) and a number of SiO masers at 43 GHz (Kim et al. 2008). Methanol (CH$_3$OH) and H$_2$O masers are observed in the USA on the VLBA (Reid et al. 2009). Similar observations are also performed within the framework of the European VLBI network (Rygl et al. 2010). The VLBI observations of radio stars in continuum at 8.4 GHz are also carried out with the same goals (Torres et al. 2012).

Table 1 gives the proper motion and trigonometric parallax differences for 88 stars. The stars have a different evolutionary status. Some of them are very young stars with maser emission (H$_2$O and CH$_3$OH masers). Asymptotic giant branch stars observed as OH, H$_2$O, and SiO masers constitute the other part of the sample. Quite a few sources were observed in continuum. This is true for such objects as pulsars, Wolf–Rayet stars, systems with black holes, and a number of T Tauri stars.
Table 1: Gaia DR2–VLBI stellar proper motion and parallax differences

| Star              | Type or spectrum | $\Delta \mu_\alpha \cos \delta$ | $\sigma \Delta \mu_\alpha \cos \delta$, mas yr$^{-1}$ | $\Delta \mu_\delta$, mas yr$^{-1}$ | $\sigma \Delta \mu_\delta$, mas yr$^{-1}$ | $\Delta \pi$, mas | $\sigma \Delta \pi$, mas | Ref |
|-------------------|------------------|----------------------------------|------------------------------------------|----------------------------------|------------------------------------------|----------------|--------------------------|-----|
| SY Scl            | Mira             | .541                             | .328                                     | −.155                            | .314                                     | −.075         | .229                     | (1) |
| S Per             | RSG              | .480                             | .458                                     | −1.380                           | .451                                     | −.191         | .123                     | (2) |
| HII 174           | RS CVn           | .020                             | .122                                     | −.105                            | .172                                     | −.111         | .057                     | (3) |
| HII 625           | BY Dra           | .409                             | .134                                     | −.121                            | .275                                     | −.008         | .070                     | (3) |
| HII 1136          | RS CVn           | −.800                            | .098                                     | .346                             | .246                                     | −.161         | .057                     | (3) |
| HII 2147          | RS CVn           | −2.579                           | .112                                     | .879                             | .171                                     | −.119         | .062                     | (3) |
| V773 Tau          | T Tau            | −1.321                           | .929                                     | −3.935                           | .391                                     | .113          | .164                     | (4) |
| HIP 20097         | T Tau            | −.020                            | .129                                     | −.115                            | .064                                     | −.084         | .059                     | (5) |
| HDE 283572        | T Tau            | .158                             | .150                                     | .106                             | .134                                     | −.107         | .065                     | (5) |
| T Tau N           | T Tau            | −.994                            | .128                                     | −2.037                           | .112                                     | .109          | .066                     | (6) |
| V1201 Tau         | T Tau            | −.370                            | .115                                     | −1.309                           | .096                                     | −.197         | .083                     | (5) |
| V807 Tau          | T Tau            | .986                             | 1.237                                    | 8.544                            | 1.009                                    | .935          | .667                     | (5) |
| V1110 Tau         | RS CVn           | .438                             | .112                                     | −.021                            | .096                                     | −.281         | .154                     | (5) |
| HIP 26233         | B2/3V            | −1.421                           | .404                                     | 1.311                            | .409                                     | −2.196        | .187                     | (8) |
| LSI +61 303       | BH               | −.146                            | .041                                     | .185                             | .067                                     | —             | —                       | (7) |
| DG Tau            | T Tau            | −.644                            | .876                                     | −.197                            | .932                                     | —             | —                       | (9) |
| HD 118216         | F2+K2            | −.013                            | .202                                     | −.031                            | .164                                     | —             | —                       | (10)|
| WR 112            | WR               | .625                             | 1.164                                    | 1.596                            | 1.436                                    | —             | —                       | (11)|
| WR 125            | WR               | −.964                            | .503                                     | .606                             | .604                                     | —             | —                       | (11)|
| WR 140            | WR               | .377                             | .206                                     | −.847                            | .115                                     | —             | —                       | (11)|
| WR 146            | WR               | 2.284                            | .696                                     | −1.375                           | 2.242                                    | —             | —                       | (11)|
| WR 147            | WR               | −1.097                           | .803                                     | −1.100                           | 1.198                                    | —             | —                       | (11)|
| PSR J0437–47      | Pulsar           | 1.185                            | 1.198                                    | .654                             | 1.672                                    | 1.929         | .679                     | (12)|
| V999 Tau          | T Tau            | −4.040                           | .677                                     | −3.306                           | .433                                     | 1.166         | .438                     | (5) |
| HD 282630         | T Tau            | .410                             | .191                                     | .078                             | .152                                     | −.798         | .148                     | (5) |
| T Lep             | Mira             | −4.887                           | .555                                     | 1.108                            | .739                                     | −.101         | .193                     | (13)|
| V1699 Ori         | YSO              | −.209                            | .428                                     | −.144                            | .399                                     | .062          | .267                     | (8) |
| GMR G             | YSO              | −.048                            | .139                                     | .745                             | .188                                     | −.242         | .067                     | (8) |
| GMR F             | YSO              | −.250                            | .128                                     | .231                             | .164                                     | −.048         | .074                     | (8) |
| Parenago 1469     | YSO              | .026                             | .107                                     | −.047                            | .120                                     | .013          | .049                     | (8) |
| Parenago 1540     | PMS              | .162                             | .128                                     | .212                             | .109                                     | −.096         | .063                     | (8) |
| Parenago 1724     | YSO              | .199                             | .209                                     | .153                             | .170                                     | −.078         | .057                     | (8) |
| Parenago 1778     | YSO              | .332                             | .499                                     | .284                             | .728                                     | −.116         | .312                     | (8) |
| Parenago 1955     | YSO              | −2.249                           | .694                                     | −4.038                           | 1.053                                    | −.594         | .215                     | (8) |
| Parenago 2148     | YSO              | 2.276                            | .347                                     | .909                             | .530                                     | .606          | .429                     | (8) |
| V621 Ori          | YSO              | .475                             | .463                                     | −.335                            | .293                                     | .269          | .115                     | (8) |
| HIP 26220         | HAe/Be           | −3.274                           | .187                                     | 2.653                            | .184                                     | −.253         | .145                     | (8) |
| HIP 26314         | B3III            | .392                             | .142                                     | .366                             | .160                                     | .170          | .076                     | (8) |
| RW Lep            | Mira             | 1.139                            | .634                                     | −2.792                           | .724                                     | .735          | .209                     | (14)|
| HD 294300         | T Tau            | 7.695                            | .682                                     | −7.858                           | 1.376                                    | −.514         | .373                     | (8) |
Table 1. Contd.

| Star          | Type or spectrum | $\Delta \mu_\alpha \cos \delta$, mas yr$^{-1}$ | $\sigma_{\Delta \mu_\alpha \cos \delta}$, mas yr$^{-1}$ | $\Delta \mu_\delta$, mas yr$^{-1}$ | $\sigma_{\Delta \mu_\delta}$, mas yr$^{-1}$ | $\Delta \pi$, mas | $\sigma_{\Delta \pi}$, mas | Ref |
|---------------|------------------|-----------------------------------------------|-------------------------------------------------|-----------------------------------|-----------------------------------|----------------|----------------|-----|
| TYC 5346-538-1 | B8.1             | .147                                          | .171                                            | .188                              | .290                              | .045           | .091           | (8) |
| HD 290862     | B3/5             | −.607                                         | .291                                            | −1.482                            | .836                              | −.020          | .549           | (8) |
| U Lyn         | Mira             | −2.257                                        | .607                                            | −.297                             | .602                              | −.690          | .232           | (15) |
| R UMa         | Mira             | 1.436                                         | .551                                            | .691                              | .517                              | .075           | .208           | (16) |
| RT Vir        | M8III            | —                                             | —                                               | —                                 | —                                 | −2.367         | .320           | (17) |
| FV Boo        | Mira             | —                                             | —                                               | —                                 | —                                 | −.397          | .191           | (18) |
| S Crt         | M6III            | −.869                                         | .327                                            | .460                              | .268                              | .316           | .195           | (19) |
| R Cas         | Mira             | 1.400                                         | 2.384                                           | 1.660                             | 1.786                             | −.328          | 1.965          | (20) |
| RX Boo        | M7.5/8           | −3.572                                        | 1.178                                           | 1.809                             | 2.432                             | .519           | .583           | (21) |
| S CrB         | Mira             | −1.671                                        | .526                                            | 1.172                             | .467                              | −.038          | .366           | (22) |
| U Her         | Mira             | −.261                                        | .360                                            | −.911                             | .392                              | −1.991         | .628           | (22) |
| WLY 2–11      | T Tau            | 2.725                                         | .361                                            | −.388                             | .301                              | .231           | .181           | (23) |
| YLW 24        | T Tau            | −.083                                        | .213                                            | .268                              | .148                              | −.143          | .166           | (23) |
| DoAr21        | T Tau            | −.554                                        | .269                                            | .155                              | .176                              | .061           | .243           | (23) |
| rho Oph S1    | T Tau            | −.120                                        | .254                                            | 3.163                             | .167                              | .917           | .145           | (23) |
| VSSG11        | T Tau            | .739                                         | 1.118                                           | 14.217                            | .776                              | −.523          | .521           | (23) |
| DROXO 71      | PMS              | −.799                                        | .640                                            | 1.376                             | .525                              | −.812          | .312           | (23) |
| SFAM 87       | T Tau            | 1.143                                        | .142                                            | −2.653                            | .111                              | .345           | .115           | (23) |
| GWAYL 5       | T Tau            | −1.188                                       | .568                                            | .532                              | .428                              | −.669          | .342           | (23) |
| DoAr51        | T Tau            | −.396                                        | 1.071                                           | 1.572                             | .726                              | .265           | .387           | (23) |
| VX Sgr        | RSG              | 2.091                                        | .883                                            | 3.691                             | .875                              | .147           | .232           | (24) |
| [GFM2007] 11  | YSO              | −.654                                        | .139                                            | .862                              | .170                              | −.072          | .109           | (25) |
| [GFM2007] 65  | YSO              | 3.397                                        | 1.873                                           | 2.574                             | 2.086                             | −.780          | .852           | (25) |
| W 40 IRS 5    | B1               | .360                                         | .404                                            | −.487                             | .360                              | −.249          | .221           | (25) |
| W 40 IRS 1c   | YSO              | −3.102                                       | .892                                            | 2.848                             | .752                              | .840           | .476           | (25) |
| [KGP2010] 133 | YSO              | −1.177                                       | .472                                            | −.881                             | .520                              | −.379          | .245           | (25) |
| PN K 3–35     | PN               | .545                                         | .157                                            | 2.459                             | .194                              | .123           | .131           | (26) |
| RR Aql        | Mira             | 3.713                                        | .883                                            | 1.077                             | .614                              | 1.566          | .499           | (22) |
| Cyg X–1       | BH               | −.102                                        | .077                                            | .229                              | .132                              | −.117          | .046           | (27) |
| IRAS 20126+4104| YSO             | −1.853                                       | .790                                            | −5.558                            | .861                              | .275           | .306           | (28) |
| IRAS 20143+3634| YSO             | −.123                                        | .193                                            | 1.447                             | .454                              | −.047          | .080           | (29) |
| V404 Cyg      | BH               | −.729                                        | .176                                            | −.205                             | .176                              | .021           | .103           | (30) |
| HIP 101341    | O6.5+            | −1.443                                       | .985                                            | 3.075                             | 1.282                             | .028           | .227           | (31) |
| NML Cyg       | RSG              | 1.282                                        | 1.260                                           | 3.727                             | 1.310                             | .906           | .570           | (32) |
| UX Cyg        | Mira             | 3.381                                        | .810                                            | .254                              | 1.621                             | −.364          | .178           | (33) |
| SS Cyg        | Df Nova          | −.047                                        | .133                                            | .209                              | .117                              | −.076          | .130           | (34) |
| IRAS 22480+6002| RSG             | −.075                                        | .354                                            | −.250                             | .212                              | .079           | .082           | (35) |
| IM Peg        | RS CVn           | .111                                         | .164                                            | .419                              | .159                              | −.320          | .114           | (36) |
| R Aqr         | M6.5e            | −9.800                                       | .632                                            | −1.239                            | .593                              | −1.578         | .847           | (37) |
| PZ Cas        | RSG              | .590                                         | .232                                            | .192                              | .320                              | .064           | .085           | (38) |
Table 1. end.

| Star      | Type or spectrum | $\Delta \mu_\alpha \cos \delta$, mas yr\(^{-1}\) | $\sigma_{\Delta \mu_\alpha \cos \delta}$, mas yr\(^{-1}\) | $\Delta \mu_\delta$, mas yr\(^{-1}\) | $\sigma_{\Delta \mu_\delta}$, mas yr\(^{-1}\) | $\Delta \pi$, mas | $\sigma_{\Delta \pi}$, mas | Ref |
|-----------|------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|------------------|-----------------|-----|
| UX Ari    | RS CVn           | 5.089                                        | .525                                          | 2.111                                        | .411                                          | .443             | .452            | (39) |
| HR 1099   | RS CVn           | −1.304                                       | .355                                          | −.082                                        | .332                                          | −.127            | .478            | (39) |
| HIP 79607 | RS CVn           | −1.275                                       | .104                                          | −.265                                        | .154                                          | .205             | .119            | (39) |
| HD 199178 | G5III            | −.277                                        | .415                                          | .498                                         | .435                                          | .312             | .332            | (39) |
| AR Lac    | RS CVn           | −.110                                        | .137                                          | .160                                         | .195                                          | −.537            | .371            | (39) |
| AM Her    | polar            | .063                                         | .223                                          | −.784                                        | .183                                          | .105             | .082            | (40) |
| W Hya     | Mira             | −7.533                                       | 2.418                                         | −4.408                                       | 3.234                                         | −4.089           | 2.497           | (20) |
| VY CMa    | RSG              | 3.726                                        | 1.865                                         | −9.074                                       | 1.847                                         | −6.772           | .827            | (41) |

Mira—Mira Ceti variable; RSG—red supergiant; RS CVn—Canes Venatici variable; BY Dra—BY Draconis variable; T Tau—T Tauri variable; PMS—pre-main-sequence star; HAe/Be—Herbig Ae/Be star; YSO—young stellar object; PN—planetary nebula; Df Nova—dwarf nova; BH—one of the binary components is a black hole; WR—Wolf-Rayet star.

(1) Nyu et al. (2011); (2) Asaki et al. (2010); (3) Melis et al. (2014); (4) Torres et al. (2012); (5) Galli et al. (2018); (6) Loinard et al. (2007); (7) Dhawan et al. (2006); (8) Koukel et al. (2017); (9) Rivera et al. (2015); (10) Abbuhl et al. (2015); (11) Dzib and Rodriguez (2009); (12) Deller et al. (2008); (13) Nakagawa et al. (2014); (14) Kamezaki et al. (2014); (15) Kamezaki et al. (2016a); (16) Nakagawa et al. (2016); (17) Zhang et al. (2017); (18) Kamezaki et al. (2016b); (19) Nakagawa et al. (2008); (20) Vlemmings et al. (2003); (21) Kamezaki et al. (2012); (22) Vlemmings, and van Langevelde (2007); (23) Ortiz-Leon et al. (2017a); (24) Xu et al. (2018); (25) Ortiz-Leon et al. (2017b); (26) Tafoya et al. (2011); (27) Reid et al. (2011); (28) Xu et al. (2013); (29) Burns et al. (2014); (30) Miller-Jones et al. (2009); (31) Dzib et al. (2013); (32) Zhang et al. (2012a); (33) Kurayama et al. (2005); (34) Miller-Jones et al. (2013); (35) Imai et al. (2012); (36) Ratner et al. (2012); (37) Min et al. (2014); (38) Kusuno et al. (2013); (39) Lestrade et al. (1999); (40) Gawroński et al. (2018); (41) Zhang et al. (2012b).
In our previous paper (Bobylev 2010) we used 23 radio stars from this list to study the tie-in of the Hipparcos catalogue (1997) to the inertial reference frame. In this paper the list was expanded significantly both through an increase in the number of VLBI observations and owing to a great density of the Gaia DR2 catalogue.

The first column in the table gives the names of the radio stars using which they are easily found in the SIMBAD electronic search system. The second column lists the types of the stars or their spectral types. The next columns provide the proper motion and trigonometric parallax differences. The dispersions of the differences are given for each type of differences. For example, for the proper motions the formula to calculate the dispersions of the differences is as follows:

$$\sigma_{\Delta \mu} = \sqrt{\sigma_{\mu_{\text{Gaia}}}^2 + \sigma_{\mu_{\text{VLBI}}}^2},$$

for the dispersions of the parallax differences the expression is similar in form after an appropriate substitution.

There are no parallax differences for eight stars in Table 1. This is either due to negative parallaxes in the Gaia DR2 catalogue or the absence (for example, for Wolf-Rayet stars) of VLBI measurements. At the same time, we used these eight stars to analyze the proper motion differences. For two stars, RT Vir and FV Boo, there are data only on their VLBI parallax measurements.

It can be seen from Table 1 that several stars have differences that differ significantly from the expected zero. For example, these include the stars R Aqr with $\Delta \mu_\alpha \cos \delta = -9.800 \pm 0.632$ mas yr$^{-1}$, VSSG11 with $\Delta \mu_\delta = 14.217 \pm 0.776$ mas yr$^{-1}$, or VY CMa with $\Delta \pi = -6.772 \pm 0.827$ mas. Note that the presence of a long tail in the distribution of radio source position differences was established by Petrov and Kovalev (2017) when analyzing a large sample of quasars from the Gaia catalogue with VLBI measurements.

When the optical and radio images of stars are compared, the size and pattern of the radio-emitting region can play an important role. The supergiant S Per can serve as an example of a “good”, symmetric radio image. As can be seen from Fig. 5 in Asaki et al. (2010), more than 40 maser spots are distributed quite uniformly in a region with a radius of about 50 mas, while, according to Fig. 10 in the cited paper, the residual velocity vectors excellently pinpoint the position of the image center. As can be seen from our table, all differences for the star S Per are close to zero.

On the other hand, the radio emission can be associated with the jets or vast disk structures surrounding the radio star. In that case, the probability of the appearance of a significant offset when comparing the optical and radio images of a star is great.

Finally, the optical image of a radio star can also be asymmetric. The well-known star VY CMa can serve as such an example. This is a red supergiant; the star has a record size. It is actually a presupernova and is surrounded by a nebula with a highly asymmetric shape.

All of this necessitates using constraints on the differences being investigated when solving our problems. Such constraints were selected through several iterations to eliminate the largest discrepancies.
RESULTS

Comparison of the Proper Motions

We use the following coupling equations to determine the three angular velocities of mutual rotation of the two frames around the equatorial coordinate axes $\omega_x, \omega_y, \omega_z$:

$$
\Delta \mu_\alpha \cos \delta = -\omega_x \cos \alpha \sin \delta - \omega_y \sin \alpha \sin \delta + \omega_z \cos \delta,
\Delta \mu_\delta = \omega_x \sin \alpha - \omega_y \cos \alpha,
$$

where the Gaia–VLBI differences are on the left-hand sides of the equations. We use the stellar proper motion differences whose absolute values do not exceed 6 mas yr$^{-1}$. There are a total of 81 such differences; their distribution is given in Fig. 1.

As can be seen from the table, the data are unequally accurate. Therefore, we solve the system of conditional equations (2) both with unit weights ($p = 1$) and with weights inversely proportional to the measurement errors

$$
p = 1/\sqrt{\sigma^2_{\mu Gaia} + \sigma^2_{\mu VLBI}},
$$

where the dispersions $\sigma_\Delta$ listed in the corresponding columns of the table are in the denominator (see Eq. (1)).

Having solved the system of 162 conditional equations (2) by the least-squares method with unit weights, we obtained the rotation components

$$
\omega_x = -0.44 \pm 0.20 \text{ mas yr}^{-1},
\omega_y = -0.05 \pm 0.29 \text{ mas yr}^{-1},
\omega_z = -0.27 \pm 0.21 \text{ mas yr}^{-1}.
$$

At the same time, with weights (3) we obtained the rotation components

$$
\omega_x = -0.14 \pm 0.15 \text{ mas yr}^{-1},
\omega_y = +0.03 \pm 0.22 \text{ mas yr}^{-1},
\omega_z = -0.33 \pm 0.16 \text{ mas yr}^{-1},
$$
Figure 2: The histogram of Gaia–VLBI parallax differences constructed from all differences: a Gaussian with an expectation value of $-0.30$ mas and a dispersion of $0.40$ mas (a) and a Gaussian with the constraint on the difference $\sigma_{\Delta \pi} < 0.25$ mas (here it has an expectation value of $-0.35$ mas and a dispersion of $0.18$ mas) (b) are shown.

where $\omega_x$ decreased greatly compared to the solution (4); the errors in the parameters being determined also decreased.

**Comparison of the Parallaxes**

To compare the parallaxes, we use 75 stars selected in such a way that the relative parallax errors from the Gaia DR2 catalogue and the VLBI parallax errors do not exceed 50%.

First, we found the mean $\Delta \pi = -0.030 \pm 0.073$ (0.404) mas from the Gaia-VLBI parallax differences. The mean was calculated with unit weights, the error of the mean calculated from the formula $\sqrt{\sum (x - \bar{x})^2/n(n-1)}$, is given, and the dispersion $\sigma = \sum (x - \bar{x})^2/n$ (here the square of the rms deviation) is given in parentheses. Then, we calculated the weighted mean with weights (3)

$$\Delta \pi = -0.038 \pm 0.046$ (0.156) mas,$$

where the error of the weighted mean is given and the corresponding dispersion is given in parentheses. We see that the errors and dispersions differ greatly. This effect is explained by the fact that we used significantly inhomogeneous data. Very broad distribution wings can be seen already from the distribution of stellar proper motion differences (Fig. 1), namely (a) a central clump that can be described by a Gaussian with a small dispersion and (b) broad wings that can be described by a Gaussian with a considerably larger dispersion.

The effect is more pronounced in the distribution of stellar parallax differences. The histogram of differences for 75 stars is presented in Fig. 2a. This figure shows a Gaussian with an expectation value of $-0.30$ mas and a dispersion of $0.40$ mas that poorly describes the distribution. Two Gaussians with significantly differing dispersions would be better suited for the description of this distribution. However, we did otherwise. To construct the histogram in Fig. 2b, we used 49 stars that were selected under constraints on the error in the differences (see (1) and the table): $\sigma_{\Delta \pi} < 0.25$ mas. The parameters of the Gaussian found (an expectation value of $-0.35$ mas and a dispersion of $0.18$ mas) are now in excellent agreement with the result (6). On this basis we conclude that the application of weights
(3) gives a result consistent with the available data; this approach allows the entire set of available data to be used. To determine the scale factor $b$, we set up a system of conditional linear equations

$$\pi_{\text{Gaia}} = a + b \cdot \pi_{\text{VLBI}}, \quad (7)$$

from the solution of which we can estimate two parameters, $a$ and $b$. As above, we use 75 stars with relative parallax errors less than 50%. Solving the system of conditional equations (7) by the least-squares method with weights (3) yields the following result:

$$a = -0.048 \pm 0.059 \text{ mas},$$
$$b = +1.002 \pm 0.007. \quad (8)$$

In Fig. 3 the parallaxes of the radio stars from the Gaia DR2 catalogue are plotted against their VLBI parallaxes. The scales are clearly seen to be virtually identical within about 3 kpc of the Sun, and only at greater distances does the Gaia DR2 parallax scale become longer than the VLBI parallax one.

**DISCUSSION**

Liu et al. (2017) studied the frame of the Gaia DR1 catalogue (Brown et al. 2016). In particular, the TGAS (Tycho-Gaia Astrometric Solution) version was compared with the Tycho2 catalogue (Høg et al. 2000) and the version of the Hipparcos catalogue (1997) improved by van Leeuwen (2007) using the model of solid-body rotation (2). These authors found the rotation vector components $(\omega_x, \omega_y, \omega_z) = (0.008, 0.010, -0.014) \pm (0.007, 0.007, 0.009) \text{ mas yr}^{-1}$ from the Hipparcos-TGAS proper motion differences for $\sim87000$ stars and $(\omega_x, \omega_y, \omega_z) = (0.011, 0.013, 0.024) \pm (0.004, 0.004, 0.005) \text{ mas yr}^{-1}$ from the Tycho2-TGAS
proper motion differences for \( \sim 2 \) million stars. Thus, Liu et al. (2017) revealed no significant mutual rotations between these frames.

However, based on the Ogorodnikov-Milne model, Liu et al. (2017) performed a kinematic analysis of \( \sim 23,000 \) K–M giants from the TGAS catalogue and found nonzero components pointing to a possible residual rotation in the Gaia DR1 frame or the presence of problems in the kinematic model. The rotation components were found to be \( \omega_{YG} = -0.38 \pm 0.15 \) mas yr\(^{-1}\) and \( \omega_{YG}' = -0.29 \pm 0.19 \) mas yr\(^{-1}\), which are interpreted as an additional rotation around the Galactic \( Y \) axis.

Note the paper by Fedorov et al. (2017), where it was found from a comparison of the stellar proper motions from the Gaia DR1 catalogue with a number of ground-based catalogues based on the model (2) that the component \( \omega_y \) changes dramatically from \(+0.5\) to \(-1.5\) mas yr\(^{-1}\) with magnitude. In our case (5) this component is small, \( \omega_y = 0.03 \pm 0.22 \) mas yr\(^{-1}\).

It has been shown by Lindegren et al. (2018) that the optical reference frame defined by Gaia DR2 is aligned with ICRS and is non-rotating with respect to the quasars to within 0.15 mas yr\(^{-1}\). Since a large number of stars were used, the random errors of rotational parameters are small, less than 10\%. The dependence of \( \omega_x, \omega_y, \omega_z \) on magnitude is clearly seen from Fig. 4 of cited publication. For example, for \( G \approx 10^m \), which is typical for the sample of stars considered in this paper, we will have \( (\omega_x, \omega_y, \omega_z) \approx (0.1, -0.1, -0.15) \) mas yr\(^{-1}\). We see good agreement of these values with our estimates (5).

As has already been noted in the Introduction, from a comparison with the Gaia DR2 data for 89 detached eclipsing binaries Stassun and Torres (2018) found a correction \( \Delta \pi = -0.082 \pm 0.033 \) mas. Here the dispersion of the Gaussian 0.033 mas should be compared with our value of 0.156 mas in the solution (6). These stars are interesting in that they were selected from published data using very rigorous criteria imposed on the photometric characteristics. As a result, the relative errors in the stellar radii, effective temperatures, and bolometric luminosities (from which the distances are estimated) do not exceed 3\%. The spectral types of the stars in this sample lie in a wide range, from late O to M; most of the stars belong to the main sequence and there are also a few giants. According to Stassun and Torres (2016), the relative parallax errors for eclipsing binaries, on average, do not exceed 5\% and do not depend on the distance.

Riess et al. (2018) estimated \( \Delta \pi = -0.046 \pm 0.013 \) mas based on a sample of 50 long-period Cepheids by comparing their parallaxes with those from the Gaia DR2 catalogue. They used the photometric characteristics of these Cepheids measured onboard the Hubble Space Telescope. Interestingly, relative to the highly accurate calibration scale of Riess et al. (2016), in which the relative Cepheid distance errors are 1–2\%, these authors determined the scale factor \( b = 1.006 \pm 0.033 \) that differs little from that found by us in the solution (8).

One might expect that the stellar parallaxes from the Gaia DR1 and DR2 catalogues do not greatly differ systematically. For example, based on a kinematic analysis of stars from the Gaia TGAS catalogue, Bobylev and Bajkova (2018) concluded that the distances to them calculated from their trigonometric parallaxes do not require using any additional correction factor. This conclusion is also confirmed by our study with regard to the stellar parallaxes from the Gaia DR2 catalogue.

Zinn et al. (2018) found \( \Delta \pi = -0.083 \pm 0.002 \) mas by comparing the distances of \( \sim 3000 \) giants from the APOKAS-2 catalogue (Pinsonneault et al. 2018) with the Gaia DR2 data. The distances to these stars belonging to the red giant clump were calculated from asteroseismology. According to these authors, here the parallax errors are approximately
equal to the estimation errors of the stellar radius and are, on average, 1.5%. Such small errors in combination with a huge number of stars allowed $\Delta \pi$ to be determined with a high accuracy.

Young stars from the Gould Belt, the distances to which have been measured by VLBI, constitute a significant fraction of our sample. Using data on 55 such stars (they are all presented in our table as PMS, YSO, and T Tau), Kounkel et al. (2018) found the following parameters based on relation (7): $a = -0.073 \pm 0.034$ mas and $b = +0.9947 \pm 0.0066$. The value of these parameters are in excellent agreement with our estimates (8).

CONCLUSIONS

Based on published data, we produced a sample of 88 radio stars for which there are both trigonometric parallax measurements in the Gaia DR2 catalogue and VLBI measurements.

A new estimate of the systematic offset between the optical and radio frames of the parallaxes, $\Delta \pi = -0.038 \pm 0.046$ (0.156) mas, was obtained by analyzing the Gaia–VLBI trigonometric parallax differences for the radio stars. If the VLBI parallaxes are assumed to be more accurate, then the correction found should be added to the parallaxes from the Gaia DR2 catalogue. In this case, the distances to the stars calculated from the corrected Gaia DR2 parallaxes slightly decrease, i.e., the stars will become closer to the Sun.

The scale factor $b$, whose value differs from 1 by no more than 1%, is determined with confidence. Such a situation is observed within 3 kpc of the Sun, and only at greater distances is the Gaia DR2 parallax scale slightly extended compared to the VLBI parallax one.

Based on the model of solid-body mutual rotation, we determined the rotation vector components in equatorial coordinates from the Gaia-VLBI proper motion differences for radio stars, $(\omega_x, \omega_y, \omega_z) = (-0.14, 0.03, -0.33) \pm (0.15, 0.22, 0.16)$ mas yr$^{-1}$.

ACKNOWLEDGMENTS

I am grateful to the referee for the useful remarks that contributed to an improvement of the paper. This work was supported by Basic Research Program P–28 of the Presidium of the Russian Academy of Sciences, the subprogram “Cosmos: Studies of Fundamental Processes and their Interrelations”.

REFERENCES

1. E. Abbulh, R. L. Mutel, C. Lynch, and M. Guedel, Astrophys. J. 811, 33 (2015).
2. Y. Asaki, S. Deguchi, H. Imai, K. Hachisuka, M. Miyoshi, and M. Honma, Astrophys. J. 721, 267 (2010).
3. V. V. Bobylev, Astron. Lett. 41, 156 (2015).
4. V. V. Bobylev and A. T. Bajkova, Astron. Lett. 44, 184 (2018).
5. A. G. A. Brown, A. Vallenari, T. Prusti, J. de Bruijne, F. Mignard, R. Drimmel, C. Babusiaux, C. A. L. Bailer-Jones, et al. (Gaia Collab.), Astron. Astrophys. 595, A2 (2016).
6. A. G. A. Brown, A. Vallenari, T. Prusti, J. de Bruijne, C. Babusiaux, C. A. L. Bailer-Jones, M. Biermann, D. W. Evans, et al. (Gaia Collab.), Astron. Astrophys. 616, 1 (2018).
7. R. A. Burns, Y. Yamaguchi, T. Handa, T. Omodaka, T. Nagayama, A. Nakagawa, M. Hayashi, T. Kamezaki, et al., Publ. Astron. Soc. Jpn. 66, 102 (2014).
8. S. Casertano, A. G. Riess, B. Bucciarelli, and M. G. Lattanzi, Astron. Astrophys. 599, 67 (2017).
9. A. T. Deller, J. P. W. Verbiest, S. J. Tingay, and M. Bailes, Astrophys. J. 685, L67 (2008).
10. V. Dhawan, A. Mioduszewski, and M. Rupen, in Proceedings of the 6th Microquasar Workshop: Microquasars and Beyond, September 18–22, 2006, Como, Italy, (2006), p.52.1.
11. S. A. Dzib and L. F. Rodriguez, Rev. Mex. Astron. Astrofis. 45, 3 (2009).
12. S. A. Dzib, L. F. Rodriguez, L. Loinard, A. J. Mioduszewski, G. N. Ortiz-León, and A. T. Araudo, Astrophys. J. 763, 139 (2013).
13. S. Dzib, L. Loinard, L. F. Rodriguez, A. J. Mioduszewski, G. N. Ortiz-León, M. A. Kounkel, G. Pech, J. L. Rivera, et al., Astrophys. J. 801, 91 (2015).
14. P. N. Fedorov, V. S. Akhmetov, and A. B. Velitchko, Mon. Not. R. Astron. Soc. 476, 2743 (2017).
15. P. A. B. Galli, L. Loinard, G. N. Ortiz-León, M. Kounkel, S. A. Dzib, A. J. Mioduszewski, L. F. Rodriguez, L. Hartmann, et al., Astrophys. J. 859, 33 (2018).
16. M. P. Gawroński, K. Goździewski, K. Katarzyński, and G. Rycyk, Mon. Not. R. Astron. Soc. 475, 1399 (2018).
17. The HIPPARCOS and Tycho Catalogues, ESA SP–1200 (1997).
18. T. Hirota, T. Bushimata, Y. K. Choi, M. Honma, H. Imai, I. Hiroshi, K. Iwadate, T. Jike, et al., Publ. Astron. Soc. Jpn. 60, 37 (2008).
19. E. Høg C. Fabricius, V. V. Makarov, S. Urban, T. Corbin, G. Wycoff, U. Bastian, P. Schwerkendiek, and A. Wiecèe, Astron. Astrophys. 355, L27 (2000).
20. D. Huber, J. Zinn, M. Bojsen-Hansen, M. Pinsonneault, C. Sahlholdt, A. Serenelli, V. S. Aguirre, K. Stassun, et al., Astrophys. J. 844, 102 (2017).
21. H. Imai, N. Sakai, H. Nakanishi, H. Sakano, M. Honma, and T. Miyaj, Publ. Astron. Soc. Jpn. 64, 142 (2012).
22. W.-C. Jao, T. J. Henry, A. R. Riedel, J. G. Winters, K. J. Slatten, and D. R. Gies, Astrophys. J. Lett. 832, L18 (2016).
23. T. Kamezaki, A. Nakagawa, T. Omodaka, T. Kurayama, H. Imai, D. Tafoya, M. Matsui, and Y. Nishida, Publ. Astron. Soc. Jpn. 64, 7 (2012).
24. T. Kamezaki, T. Kurayama, A. Nakagawa, T. Handa, T. Omodaka, T. Nagayama, H. Kobayashi, and M. Shizugami, Publ. Astron. Soc. Jpn. 66, 107 (2014).
25. T. Kamezaki, A. Nakagawa, T. Omodaka, T. Handa, K.-I. Inoue, T. Kurayama, H. Kobayashi, T. Nagayama, et al., Publ. Astron. Soc. Jpn. 68, 71 (2016a).
26. T. Kamezaki, A. Nakagawa, T. Omodaka, K.-I. Inoue, J. O. Chibueze, T. Nagayama, Y. Ueno, and N. Matsunaga, Publ. Astron. Soc. Jpn. 68, 75 (2016b).
27. M. K. Kim, T. Hirota, M. Honma, H. Kobayashi, T. Bushimata, Y. K. Choi, H. Imai, K. Iwadate, et al., Publ. Astron. Soc. Jpn. 60, 991 (2008).
28. M. Kounkel, L. Hartmann, L. Loinard, G. N. Ortiz-León, A. J. Mioduszewski, L. F. Rodriguez, R. M. Torres, G. Pech, et al., Astrophys. J. 834, 142 (2017).
29. M. Kounkel, K. Covey, G. Suarez, C. Román-Zuniga, J. Hernandez, K. Stassun, K. O. Jaehnig, E. Feigelson, et al., Astron. J. 156, 84 (2018).
30. T. Kurayama, T. Sasa, and H. Kobayashi, Astrophys. J. 627, L49 (2005).
31. K. Kusuno, Y. Asaki, H. Imai, and T. Oyama, Astrophys. J. 774, 107 (2013).
32. F. van Leeuwen, Astron. Astrophys. 474, 653 (2007).
33. J.-F. Lestrade, R. A. Preston, D. L. Jones, R. B. Phillips, A. E. E. Rogers, M. A. Titus, M. J. Rioja, and D. C. Gabuzda, Astron. Astrophys. 344, 1014 (1999).
34. L. Lindegren, J. Hernandez, A. Bombrun, S. Klioner, U. Bastian, M. Ramos-Lerate, A. de Torres, H. Steidelmuller, et al. (Gaia Collab.), Astron. Astrophys. 616, 2 (2018).
35. N. Liu, Z. Zhu, J.-C. Liu, and C.-Y. Ding, Astron. Astrophys. 599, 140 (2017).
36. L. Loinard, R. M. Torres, A. J. Mioduszewski, L. F. Rodriguez, R. A. Gonzalez-Lopezlira, R. Lachaume, V. Vazquez, and E. Gonzalez, Astrophys. J. 671, 546 (2007).
37. C. Melis, M. J. Reid, A. J. Mioduszewski, J. R. Stauffer, and G. C. Bower, Science (Washington, DC, U. S.) 345, 1029 (2014).
38. F. Mignard, S. A. Klioner, L. Lindegren, J. Hernández, U. Bastian, A. Bombrun, D. Hobbs, U. Lammers, et al. (Gaia Collab.), Astron. Astrophys. 616, 14 (2018).
39. J. C. A. Miller-Jones, P. G. Jonker, V. Dhawan, W. Brisken, M. P. Rupen, G. Nelemans, and E. Gallo, Astrophys. J. 706, 230 (2009).
40. J. C. A. Miller-Jones, G. R. Sivakoff, C. Knigge, E. G. Kording, M. Templeton, and E. O. Waagen, Science (Washington, DC, U. S.) 340, 950 (2013).
41. C. Min, N. Matsumoto, M. K. Kim, T. Hirota, K. M. Shibata, S.-H. Cho, M. Shizugami, and M. Honma, Publ. Astron. Soc. Jpn. 66, 38 (2014).
42. A. Nakagawa, M. Tsushima, K. Ando, T. Bushimata, Y. K. Choi, T. Hirota, M. Honma, H. Imai, et al., Publ. Astron. Soc. Jpn. 60, 1013 (2008).
43. A. Nakagawa, T. Omodaka, T. Handa, M. Honma, N. Kawaguchi, H. Kobayashi, T. Oyama, K. Sato, et al., Publ. Astron. Soc. Jpn. 66, 101 (2014).
44. A. Nakagawa, T. Kurayama, M. Matsui, T. Omodaka, M. Honma, K. M. Shibata, K. Sato, and T. Jike, Publ. Astron. Soc. Jpn. 66, 101 (2016).
45. D. Nyu, A. Nakagawa, M. Matsui, H. Imai, Y. Sofue, T. Omodaka, T. Kurayama, R. Kamohara, et al., Publ. Astron. Soc. Jpn. 63, 53 (2011).
46. G. N. Ortiz-León, L. Loinard, M. A. Kounkel, S. A. Dzib, A. J. Mioduszewski, L. F. Rodriguez, R. M. Torres, R. A. González-Lópezlira, et al., Astrophys. J. 834, 141 (2017a).
47. G. N. Ortiz-León, S. A. Dzib, M. A. Kounkel, L. Loinard, A. J. Mioduszewski, L. F. Rodriguez, R. M. Torres, G. Pech, et al., Astrophys. J. 834, 143 (2017b).
48. L. Petrov and Y. Y. Kovalev, Mon. Not. R. Astron. Soc. 467, 71 (2017).
49. M. H. Pinsonneault, Y. P. Elsworth, J. Tayar, A. Serenelli, D. Stello, J. Zinn, S. Mathur, R. Garcia, et al., arXiv: 1804.09983 (2018).
50. N. Pradel, P. Charlot, and J.-F. Lestrade, Astron. Astrophys. 452, 1099 (2006).
51. T. Prusti, J. H. J. de Bruijne, A. G. A. Brown, A. Vallenari, C. Babusiaux, C. A. L. Bailer-Jones, U. Bastian, M. Biermann, et al. (Gaia Collab.), Astron. Astrophys. 595, A1 (2016).
52. M. I. Ratner, N. Bartel, M. F. Bietenholz, D. E. Lebach, J.-F. Lestrade, R. R. Ransom, and I. I. Shapiro, Astrophys. J. Suppl. Ser. 201, 5 (2012).
53. M. J. Reid, K. M. Menten, X. W. Zheng, A. Brunthaler, L. Moscadelli, Y. Xu, B. Zhang, M. Sato, et al., Astrophys. J. 700, 137 (2009).
54. M. J. Reid, J. E. McClintock, R. Narayan, L. Gou, R. A. Remillard, and J. A. Orosz, Astrophys. J. 742, 83 (2011).
55. A. G. Riess, L. Macri, S. L. Hoffmann, D. Scolnic, S. Casertano, A. V. Filippenko, B. E. Tucker, M. J. Reid, et al., Astrophys. J. 826, 56 (2016).
56. A. G. Riess, S. Casertano, W. Yuan, L. Macri, B. Bucciarelli, M. G. Lattanzi, J. W. MacKenty, J. B. Bowers, et al., Astrophys. J. 861, 126 (2018).
57. J. L. Rivera, L. Loinard, S. A. Dzib, G. N. Ortiz-León, L. F. Rodriguez, and R. M. Torres, Astrophys. J. 807, 119 (2015).
58. K. L. J. Rylg, A. Brunthaler, M. J. Reid, K. M. Menten, H. J. van Langevelde, and Y. Xu, Astron. Astrophys. 511, 2 (2010).
59. K. G. Stassun and G. Torres, Astrophys. J. Lett. 831, L6 (2016).
60. K. G. Stassun and G. Torres, Astrophys. J. 862, 61 (2018).
61. D. Tafoya, H. Imai, Y. Gomez, J. M. Torrelles, N. A. Patel, G. Anglada, L. F. Miranda, M. Honma, et al., Publ. Astron. Soc. Jpn. 63, 71 (2011).
62. R. M. Torres, L. Loinard, A. J. Mioduszewski, A. F. Boden, R. Franco-Hernandez, W. H. T. Vlemmings, and L. F. Rodriguez, Astrophys. J. 747, 18 (2012).
63. W. H. T. Vlemmings, H. J. van Langevelde, P. J. Diamond, H. J. Habing, and R. T. Schilizzi, Astron. Astrophys. 407, 213 (2003).
64. W. H. T. Vlemmings and H. J. van Langevelde, Astron. Astrophys. 472, 547 (2007).
65. S. Xu, B. Zhang, M. J. Reid, K. M. Menten, X. Zheng, and G. Wang, Astrophys. J. 859, 14 (2018).
66. B. Zhang, M. J. Reid, K. M. Menten, X. W. Zheng, and A. Brunthaler, Astron. Astrophys. 544, 42 (2012a).
67. B. Zhang, M. J. Reid, K. M. Menten, and X. W. Zheng, Astrophys. J. 744, 23 (2012b).
68. B. Zhang, X. Zheng, M. J. Reid, M. Honma, K. M. Menten, A. Brunthaler, and J. Kim, Astrophys. J. 849, 99 (2017).
69. J. C. Zinn, M. H. Pinsonneault, D. Huber, and D. Stello, arXiv: 1805.02650 (2018).