Defect-induced electronic smectic state at the surface of nematic materials

Aritra Lahiri,1,* Avraham Klein,2 and Rafael M. Fernandes1,†

1 School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA
2 Physics Department, Ariel University, Ariel 40700

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Due to the intertwining between electronic nematic and elastic degrees of freedom, lattice defects and structural inhomogeneities commonly found in crystals can have a significant impact on the electronic properties of nematic materials. Here, we show that defects commonly present at the surface of crystals generally shift the wave-vector of the nematic instability to a non-zero value, resulting in an incommensurate electronic smectic phase. Such a smectic state onsets above the bulk nematic transition temperature and is localized near the surface of the sample. We argue that this effect may explain not only recent observations of a modulated nematic phase in iron-based superconductors, but also several previous puzzling experiments that reported signatures consistent with nematic order before the onset of a bulk structural distortion.

Electronic nematicity has been observed in a wide range of systems, including high-$T_c$ superconductors [1–3], heavy-fermion materials [4–6], topological superconductors [7, 8], cold atoms [9], and twisted moiré devices [10, 11]. Among those, iron-based superconductors (FeSC) have provided unique insight into this quantum electronic state due to the nearly-universal and unambiguous presence of nematic order and nematic fluctuations in their phase diagrams [3, 12–16]. Despite significant progress, essential questions remain unresolved, related not only to the microscopic mechanisms of nematicity, but also to its general phenomenology [17]. For instance, since early studies of FeSC, various probes in nominally unstrained samples have reported signatures consistent with nematicity above the nematic transition temperature $T_{nem}$ established by thermodynamic probes [18–29]. More recently, experiments have found evidence for a spatially-modulated nematic phase – i.e. an electronic smectic phase [30–33].

The probes used in many of these experiments are particularly sensitive to the surface, e.g. angle-resolved photo-emission spectroscopy (ARPES) [25–27], scanning tunneling microscopy (STM) [21, 30–32], spatially resolved photomodulation [19], and photo-emission electron microscopy (PEEM) [33]. Moreover, the onset of these interesting phenomena does not usually show typical phase-transition signatures in thermodynamic quantities, such as specific heat [34] and elasto-resistance [3].

This suggests that both effects – nematic manifestations above $T_{nem}$ and modulated nematic order – may signal a surface nematic transition at higher temperatures than the bulk one [19], reminiscent of the so-called extraordinary transition [35]. The key question is whether a surface nematic transition is particular to some FeSC compounds or a more general phenomenological property of nematic compounds.

While a purely electronic mechanism was previously invoked to explain surface nematicity [30], in this Letter we focus on the role of the elastic degrees of freedom. The nemato-elastic coupling $g$ is known to significantly impact the nematic state, particularly in FeSC [15, 16, 37–42]. For instance, coupling to elastic fluctuations (acoustic phonons) renders the nematic transition mean-field like [41, 43–46], whereas intrinsic random strain fosters behaviors associated with the random-field Ising-model [47–49]. Here, we show that defects commonly found in the surfaces of crystals, such as steps separating terrace domains, promote an electronic smectic state localized near the surface and that onsets at a temperature $T_{smc} > T_{nem}$ (see Fig. 1). The smectic state survives down to a temperature $T_{smc-nem}$, which decreases as the sample thickness is reduced, at which point a homogeneous nematic phase takes over. Our results establish a hitherto unexplored facet of electronic nematic phases in elastic media, which we argue can explain the intriguing observation of Ref. [33] of a mesoscopic nematic wave in FeSC.

To understand why defects induce a surface transition, note that elastic fluctuations increase the nematic transition temperature $T_{nem}$ from its bare purely-electronic value $T_{nem}^{(0)}$. In a clean system, some of the elastic modes are expected to be frozen near the surface, resulting in $T_{nem}^{(surface)} < T_{nem}$ [50]. However, the fact that the exposed surface is more disordered than the bulk changes this picture dramatically. To see this, consider a random distribution of defects, such as vacancies and dislocations, on the surface of a crystal whose bulk is clean. Defects locally induce large strains that decay slowly with distance [51]. Since they are concentrated at the surface, they rapidly screen each other as one moves deeper into the bulk. However, near the surface, they do not screen efficiently, causing not only an enhancement of $T_{nem}$ at the surface, but also creating a “speckle” pattern in the nematic fluctuation spectrum, with typical spot size set by the algebraic strain correlations rather than by the defect density. This disorder-induced pattern imposes a preferred wavelength for the condensation of the nematic order parameter, driving the formation of an electronic smectic state.

To derive these results, we solve a Ginzburg-Landau...
model of a generic nematic order parameter coupled to elastic strain induced by simple types of surface quenched disorder, such as steps and anisotropic point defects. We find that the defect distribution induces a non-local effective potential for the nematic order parameter. After averaging over defect realizations, the minimum of the resulting nematic free energy appears at a higher temperature \( T_{\text{smc}} = T_{\text{nem}} + \Delta T_{\text{smc}} \) (with \( \Delta T_{\text{smc}} > 0 \)) and at a non-zero wave-vector \( q_{\text{smc}} \), resulting in an electronic smectic phase. In terms of the disorder strength \( \sigma^2 \), we find

\[
q_{\text{smc}} \propto g^2 \sigma^2, \quad \Delta T_{\text{smc}} \propto q_{\text{smc}}^2. \tag{1}
\]

The smectic order parameter is inhomogeneous and localized at the surface, decaying exponentially into the bulk with a penetration depth \( \propto 1/q_{\text{smc}} \). Eventually, below \( T_{\text{smc-nem}} \), which is lower than the bulk nematic transition temperature \( T_{\text{nem}} \), the smectic solution becomes unfavorable and the uniform \( q = 0 \) nematic state is established throughout the sample.

Surface step disorder and induced strain. – To elucidate our results, we consider an Ising-nematic order parameter \( \eta \) that breaks the equivalence between the \( x \) and \( y \) directions of a crystal (i.e. it transforms as the \( B_{1g} \) irreducible representation of the tetragonal group). In the presence of strain, the nematic action is given by:

\[
S = \int \left[ \left( \frac{T - T_{\text{nem}}}{2T_{\text{nem}}} \right) \eta_x \right] \eta^2 + \int \left[ \frac{b_y}{2} \left( \partial_x \eta_x \right)^2 - g \varepsilon \eta_x \eta_y + \frac{u_{xy}}{4} \eta_y^4 \right] \tag{2}
\]

where repeated indices are implicitly summed; \( b_y = b_y = b_y = b_y \) and \( b_y \) are the nematic stiffness coefficients; \( \eta \) is the quartic coefficient; \( r_0 \) is of the order of the Fermi energy (action has dimensions of energy); \( \varepsilon \) is the \( B_{1g} \) shear strain, which acts as a conjugate field to the nematic order parameter; and \( T_{\text{nem}} \) are the nematic transition temperatures without and with the enhancement from elastic fluctuations. For a clean unstrained crystal, \( \varepsilon \) is only present as a fluctuating field whose properties are determined by the crystal’s elastic constants. However, for a crystal with quenched disorder, a static slow-decaying strain \( \varepsilon_{\text{st}} \) is generated by the various types of defects. In both cases, an effective nematic potential emerges in the action due to either thermal fluctuations or average over disorder configurations. While the former scenario has been widely studied [43–46], the latter has received much less attention [52, 53].

A crystal with an exposed surface can be modeled by an isotropic elastic half-space \( (z \geq 0) \) with Young’s modulus \( E \) and Poisson ratio \( \nu \). Each type of surface defect generates a characteristic dipolar local force, which in turn can be used to calculate \( \varepsilon_{\text{st}} \) via standard methods [54–60]. Here, we consider idealized infinite step defects parallel to the \( y \)-axis, as shown in Fig. 1 (we consider point defects in the Supplementary material (SM)). A single step at \( x = x' \) is parametrized by the force density \( f_{x'} = h_{x'}[\partial_x \delta(x - x')](z) \), where \( \delta(z) \) is the Dirac delta function, the force \( h_{x'} \) characterizes the strength of the defect, and \( x = x', z \). For simplicity, we consider steps that create forces along the \( z \)-axis only, i.e. \( h_x = 0 \) and \( h_z = 0 \). The lattice displacement created by a single step is given by \( \eta_{x'} = h_{x'} \partial_x G_{x'z}(x - x', z) \), where \( G_{x'z} \) is the Green’s function for an infinite line-force along the \( y \)-axis in half-space [51]. The \( B_{1g} \) strain \( \varepsilon_{\text{st}} \), generated by a single defect is [51]

\[
\varepsilon_{\text{st}} = -\frac{4(1 + \nu) h_{x'}}{\sqrt{2\pi E}} \left[ \frac{(\nu - 1) \delta x^3 z + (\nu + 1) \delta x z^3}{(\delta x^2 + z^2)^3} \right], \tag{3}
\]

where \( \delta x = x - x' \). A distribution of such steps at random positions \( x = x_j \) and with random strength \( h_{x,j} \), results in the net \( B_{1g} \) strain \( \varepsilon^{B_{1g}}_{\text{st}} = \sum_j h_{x,j} \delta_{x} G_{x,z}(x - x', z - 0) = \sum_j h_{x,j} \varepsilon_{\text{st}}(x, z) \). The nematic action (2) for the finite crystal with dimensions \( L_x = L_y = L \parallel \) and \( L_z = L \parallel \) becomes:

\[
S = \int \left[ \left( \frac{T - T_{\text{nem}}}{2T_{\text{nem}}} \right) \eta_x \right] \eta^2 + \int \left[ \frac{b_y}{2} \left( \partial_x \eta_x \right)^2 - g \varepsilon_{\text{st}} \eta_x \eta_y + \frac{u_{xy}}{4} \eta_y^4 \right] \tag{4}
\]

where we defined \( \rho_x = \sum_j h_{x,j} \delta(x - x_j) \).
Effective nematic potential and smectic state.— For a random distribution of steps, \( \langle h_i h_j \rangle = \sigma^2 \delta_{i,j} \), the step density \( \rho_s \) follows a Gaussian distribution with variance \( \sigma^2 (N_{\text{step}}/L||)(a||/L_\xi) \), where \( N_{\text{step}} \) is the number of steps, \( a|| \) is the in-plane lattice constant, and \( L_\xi \) is a length scale larger than \( a|| \) but smaller than the nematic correlation length. Integrating out the step density in Eq. (4) (equivalent to the standard procedure of averaging over quenched disorder [61–67]) generates a new quadratic term in the nematic action:

\[
S_d = \int_0^L dz \sum_{q_s} V_{q_s,z,z'} q_{q_s,z} q_{q_s,z'}
\]

with an effective potential experienced by the nematic order parameter

\[
V_{q_s,z,z'} = -\frac{(g\sigma)^2}{2} e^{-|q_s||z|+|z'|} \times \frac{q_s^2}{4}(q_s||z|+2\nu-1)||q_s||z'\rangle + 2\nu-1).
\]

Here, \( \beta = [(1 + \nu)/\sqrt{2}E]^2N_{\text{step}}(L_\xi/a||) \) and \( \eta_{q_s,z} = (1/L_||) \int \eta_{q_s,z} e^{-|q_s|\cdot x}. \) The potential \( V_{q_s,z,z'} \) is non-local, depending on both \( z \) and \( z' \). Moreover, it vanishes quadratically as \( q_s \to 0 \) and exponentially as \( z, z' \to \infty \) or \( q_s \to \infty \). Thus, the potential has a negative-valued minimum at a non-zero \( q_s \) and is significant only near the surface. These features are a consequence of the algebraic decay of the strain fields generated by defects, rather than the type of defects (see SM).

While the defect-generated potential in Eq. (6) is minimized by \( q_s \neq 0 \), the nematic stiffness term \( b||q_s^2 \) in Eq. (4) favors a uniform \( q_s = 0 \) state. This competition causes the nematic instability to take place at a nonzero wave-vector \( q_s \), resulting in an electronic smectic state. This effect is restricted to the vicinity of the surface due to the exponential suppression of \( V_{q_s,z,z} \) with \( |z| \). This can be more clearly seen by an approximate analytical solution of the problem. Re-expressing \( V_{q_s,z,z'} \) in terms of \( z = (z+z')/2 \) and \( \delta z = z - z' \), \( V_{q_s,z}\delta z \) is peaked at \( \delta z \sim 1/q_s \) and \( \delta z = 0 \). Assuming that \( \eta_{q_s,z} \) varies slowly near the surface over a depth \( L_s \sim 1/q_s \), before eventually decaying exponentially away from the surface, the action (5) becomes:

\[
S_d = \int_0^L dz \sum_{q_s} V_{q_s,z,z'} \eta_{q_s,z} \eta_{q_s,z'}^* \approx -L_s^2 \sum_{q_s} \frac{(g\sigma)^2}{2} \frac{\beta}{2} \langle (|q_s|) \eta_{q_s,0} |^2 \rangle.
\]

In the regime of vanishing \( z \)-component stiffness \( b \to 0 \), the quadratic part of the action (4), \( S(2) \), is given by:

\[
S(2) \approx L_s^2 \sum_{q_s} \left[ \left( \frac{r_0}{2T_{\text{nem}}} \right) + \frac{b||q_s^2}{2} \right] \eta_{q_s,0}^2.
\]

Minimizing the full action \( S_d + S(2) \) with respect to \( q_s \) gives a finite smectic wave-vector \( q_{\text{smc}} = (g\sigma)^2 \beta [(\nu - \frac{1}{2})^2 + \nu^2] / 2b|| \) and an enhanced smectic transition temperature \( T_{\text{smc}} = T_{\text{nem}} + (T_{\text{nem}}/r_0) b||q_{\text{smc}}^2 \), consistent with Eq. (1). The actual spatial profile of \( \eta_{x,z} \) and the precise \( q_{\text{smc}} \) are obtained by solving the saddle-point equation in real space:

\[
\frac{T - T_{\text{nem}}}{T_{\text{nem}}} - b||q_s^2 \eta_{x,z}^2 + \frac{1}{L_\xi} \int_0^L dz' \sum_{q_s} \frac{q_s^2}{2} \langle \eta_{x,z}, \eta_{x,z'} \rangle = 0
\]

where \( \eta_{x,z}, \eta_{x,z'} \) is the inverse Fourier transform of \( V_{q_s,z,z'} \) (see SM whose asymptotic behavior is:

\[
\eta_{x,z} \sim \begin{cases} -(z+z')^{-3} & (\delta x) \ll z, z' \\ + (z+z')(\delta x)^{-4} & (\delta x) \gg z, z' \\ \end{cases}
\]

Therefore, as a function of \( \delta x/(z+z') \), \( \eta_{x,z} \) has a negative central trough at \( \delta x = 0 \), crosses zero at \( \delta x \sim z+z' \), and then remains positive as it decays algebraically. The sign change in real-space means that the effective potential favors an oscillatory \( \eta_{x} \) solution.

The numerical solution of Eq. (9), shown in Fig. 2(a), confirms the main results of our analytical approximation. The quartic term of the nematic action (4) stabilizes a single smectic wave-vector over the entire temperature range \( T_{\text{smc-nem}} < T < T_{\text{smc}} \), as it acts as a repulsive biquadratic interaction \( u_2|\eta_{q_s}|^2|\eta_{q_s'}|^2 \) between states with different wave-vectors. Consequently, only the smectic wave-vector corresponding to the highest critical temperature develops. For the same reason, in a fully 3D crystal with \( L \gg 1/q_{\text{smc}} \), the uniform bulk nematic phase is preferred for \( T < T_{\text{nem}} \), as its free-energy gain scales extensively with the system size. However, for smaller values of \( L \) comparable to \( 1/q_{\text{smc}} \), the smectic free-energy can compete with the bulk nematic one. Consequently, the smectic-nematic transition is pushed to a lower temperature \( T_{\text{smc-nem}} < T_{\text{nem}} \), which decreases with decreasing sample thickness. Figs. 2(b)-(c) show the corresponding profile of \( \eta_{q_s,z} \) in momentum space, highlighting the change in wave-vector above and below \( T_{\text{smc-nem}} \).

The temperature dependence of the uniform nematic and smectic order parameters is shown in Fig. 3(a). The continuous onset of surface smectic order is evident, eventually dropping discontinuously to zero, concomitant to the onset of uniform nematic order. Fig. 3(b) shows the numerically obtained phase diagram as a function of increasing defect disorder strength \( \sigma^2 \).

Discussion.— The mechanism unveiled in this work for the emergence of a surface electronic smectic state above the onset of bulk electronic nematicity is rather general, as it relies solely on the existence of defects commonly observed at crystal surfaces. While here we focused on
Figure 2. (color online) (a) Spatial profile of the nematic order parameter $\eta_{x,z}$ for three representative solution of the saddle-point equation (9). For $T > T_{\text{smc}}$, the nematic order parameter is effectively zero everywhere. As temperature is lowered towards $T_{\text{smc-nem}} < T < T_{\text{smc}}$, $\eta_{x,z}$ displays a sinusoidal $x$-dependence characterized by a single smectic wave-vector $q_{\text{smc}}$ [panel (b)]. Below the bulk nematic transition $T < T_{\text{smc-nem}}$, a uniform nematic state emerges with zero wave-vector [panel (c)]. The enhancement of $\eta_{x,z}$ at the corners is an artifact of the boundary conditions. The profile of the nematic order parameter $\eta_{x,z}$ in Fourier space is shown in panels (b) (for $T_{\text{smc-nem}} < T < T_{\text{smc}}$) and (c) (for $T < T_{\text{smc-nem}}$). The parameters used are (in arbitrary units): $r_0 = 1$, $b = 0.5$, $\beta = 0.25$, $\nu = 0.495$, $L_0 = 44$, $L = 9$, $(q\sigma)^2/2 = 1$, and $u_q = 5$. In panels (b) and (c), the nematic fields were normalized.

steps, other defects with nonzero dipolar elastic moments are expected to promote a similar behavior, since they also generate algebraically-decaying strain fields that are poorly screened at the surface (see SM) [68, 69]. Our result uncovers yet another aspect of the rich phenomenology of electronic nematicity caused by the coupling to the elastic degrees of freedom.

The impact of the effect we found on a given nematic system depends on the disorder strength $\sigma$ and on the nemato-elastic coupling $g$, as shown in the phase diagram of Fig. 3(b). FeSC stand out as compounds with strongly coupled nematic and elastic degrees of freedom, as manifested by, e.g., the large orthorhombic distortion seen in the nematic phase [70]. In contrast, in other tetragonal correlated systems that display nematic tendencies, such as Hg-based cuprates [71] and heavy-fermion systems [5, 6], a lattice distortion is difficult to be resolved experimentally. The potentially large period $2\pi/q_{\text{smc}}$ of the smectic state may explain why certain surface-sensitive probes, such as ARPES and STM, observe signatures consistent with nematic order above the temperature where a bulk orthorhombic distortion onset sets. Among the various experimental findings that have indicated the existence of a smectic phase in FeSC [30–33], the PEEM data reported in Ref. [33] provides the most straightforward platform to perform comparisons with our theory and extract relevant physical estimates. That work found a sinusoidal modulation of the nematic order parameter with a long and material-dependent period. Moreover, when Fourier-transformed to momentum space (see the SM), the PEEM data, available in [72], displays a distinctive speckle pattern corresponding to a spot of size $q_{\text{smc}}$, reminiscent of our theoretically calculated nematic potential $V_{q_{x,z}}$. As shown in detail in the SM, combining the experimental results of Ref. [33] with our theoretical model, we find two interesting results: (i) The size of a typical region of parallel stripes, observed in that work, is small enough that a homogeneous nematic phase may not be stabilized at $T_{\text{nem}}$. (ii) The characteristic energy scale per defect is of the order $E_d \sim 100\mu\text{eV}$. This scale is much smaller than both the Fermi energy and the bulk nematic orbital order energy splitting observed in FeSe. These results highlight that
the smectic order is not the result of a particular defect distribution with fine-tuned disorder strength, but of the subtle effects of the long-range strain generated by the defect distribution.

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* Present address: Theoretical Physics 4, University of Würzburg, 97074 Würzburg, Germany
† rfernandez@umn.edu

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Supplementary Material: Defect-induced electronic smectic state at the surface of nematic materials

DEFECT-GENERATED NEMATIC POTENTIAL

In this section, we discuss the properties of the effective nematic potential as a function of $z$ and $z'$. In momentum space (i.e. $q_x$ space), the potential is given by (see Eq. (6) of the main text):

$$V_{q_x,z,z'} = -\frac{(g\sigma)^2\beta}{2} q_x^2 |q_x| |z + 2\nu - 1||z' + 2\nu - 1|e^{-|q_x|(z+z')}.$$  \hfill (S1)

For a fixed $q_x$, this function, shown in Fig. S1 for $\nu = 0.4$ and $q_x = 1$, is characterized by a well-defined negative minimum centered around $z,z' \sim 1/q_x$ (corresponding to a positive peak of $-V_{q_x,z,z'}$). The width of this minimum is approximately the same along the directions $z$, $z'$ and $z - z'$. As a result, when the defect-induced potential is rewritten in terms of the quantities $\bar{z} = (z+z')/2$ and $\delta z = z - z'$, as shown in Fig. S1(b), the minimum has approximately the same width along both $\bar{z}$ and $\delta z$ coordinate directions. In terms of these coordinates, the minimum is centered at $\delta z = 0$ and $\bar{z} \sim 1/q_x$.

It is also convenient to study the potential in the real $x$-space. Performing a Fourier transform of Eq. (S1), we obtain:

$$V_{\delta x,z,z'} = -\frac{(g\sigma)^2\beta}{2} L_{\parallel} (2\pi) \left[ \frac{48zz'}{(z+z')^5} \frac{1-10\xi^2+5\xi^4}{(\xi^2+1)^5} + \frac{12(2\nu-1)}{(z+z')^3} \frac{1-6\xi^2+\xi^4}{(\xi^2+1)^4} + \frac{4(2\nu-1)}{(z+z')^3} \frac{1-3\xi^2}{(\xi^2+1)^3} \right].$$ \hfill (S2)

Consistent with the analysis in momentum $q_x$-space, the relevant range of $\delta x$ is of the order of $(z+z')$, corresponding to the momentum scale $q_x \sim 1/(z+z')$. This is illustrated by the behavior of the auxiliary function $\psi$ shown in Fig. S2. In panel (a), we note that the peak-to-trough distance increases with increasing mean depth $(z+z')/2$. More importantly, the change in sign of $\psi$ over this distance makes the values of the nematic order parameter at points separated by this distance to also have opposite signs, thereby generating a modulation. Panel (b) demonstrates that two $\psi$ curves with the same mean depth $(z+z')/2$ but different $z - z'$ have a very similar shape.

Figure S1. The negative of the defect-induced nematic potential, $-V_{q_x,z,z}$, in units of $(g\sigma)^2\beta/2$, and as a function of (a) $z$ and $z'$, and (b) $\bar{z} = (z+z')/2$ and $\delta z = z - z'$. In both panels, $q_x = 1$ and $\nu = 0.4$. Note that $-V_{q_x,z,z}$ is peaked at $z = z' \sim 1/q_x$, corresponding to $\bar{z} \sim 1/q_x$ and $\delta z = 0.$
Here we derive the smectic critical temperature $T_{\text{smc}}$ by minimizing the linearized disorder-averaged action. Using Eq. (5) of the main text, the disorder-averaged nematic action is given by,

$$S = L^2 \sum_{q_x} \int_{z=0}^{L} \left[ \left( r_0 \frac{T - T_{\text{nem}}}{T_{\text{nem}}^{(0)}} \right) |\eta_{q_x,z}|^2 + \frac{b_0}{2} |q_x|^2 |\eta_{q_x,z}|^2 + \frac{b_0}{2} |\partial_z \eta_{q_x,z}|^2 - \int_{z'=0}^{L} \frac{(g\sigma)^2}{2} q_x^2 [|q_x z + 2\nu - 1||q_x z' + 2\nu - 1] \right] \times e^{-|q_x z + z'| \eta_{q_x,z}^* \eta_{q_x,z}^*},$$

(S3)

Defining the constant parameter $R = \int_{z'} |q_x||q_x z' + 2\nu - 1|e^{-|q_x z'| \eta_{q_x,z}^* \eta_{q_x,z}^*}$, the linearized saddle-point equation is given by,

$$\partial^2_{z} \eta_{q_x,z} - \left( \frac{r_0}{T_{\text{nem}}^{(0)}} - \frac{b_0 q_x^2}{b} \right) \eta_{q_x,z} + (g\sigma)^2 \beta R |q_x||q_x z + 2\nu - 1|e^{-|q_x z|} = 0.$$

(S4)

For a sample occupying the half-space $z \geq 0$, its solution is readily obtained as,

$$\eta_{q_x,z}^{(\text{sp})} = (g\sigma)^2 \beta e^{-\frac{\sqrt{\left(\frac{t+q_x^2}{b} \right) \left[ 2 \left( \frac{t+q_x^2}{b} \right) (1-\nu) + 2 \nu q_x^2 \right]}}{b}} R + \frac{(g\sigma)^2 \beta}{b} \frac{e^{-|q_x z| \left( \frac{t+q_x^2}{b} - q_x^2 \right)}}{\left( \frac{t+q_x^2}{b} - q_x^2 \right)^2} R,$$

(S5)

In this expression, we defined $t = r_0(T - T_{\text{nem}})/T_{\text{nem}}^{(0)}$ for brevity and used the von Neumann boundary condition $\partial_z \eta(z \to 0) = 0$ since no nematic surface terms are present. The smectic critical temperature for a given wave-vector $q_x$ is obtained from the self-consistency condition, $R = \int_{z'=0}^{L} |q_x||q_x z' + 2\nu - 1|e^{-|q_x z'| \eta_{q_x,z}^{(\text{sp})} \eta_{q_x,z}^{(\text{sp})}^*}$, as shown in Fig. S3(a). As expected, the reduced critical temperature vanishes for $q_x = 0$. Upon increasing $q_x$, it rises due to the defect contribution to the potential, and is eventually peaked at a finite $q_x$, followed by a suppression caused by the nematic stiffness contribution to the potential. The actual smectic critical temperature, obtained from the peak values in Fig. S3(a), is shown in Fig. S3(b). It is found to increase quadratically with the effective defect strength $(g\sigma)^2 \beta/2$. The observed smectic wave-vector, corresponding to the peak positions in Fig. S3(a), is shown in Fig. S3, and varies approximately linearly with the defect strength. (c).

An analytical approximation may be obtained in the limit $b \to 0$. This is a reasonable approximation for layered materials, such as the iron-based superconductors. In this limit, the saddle-point solution is given by,

$$\eta_{q_x,z}^{(\text{sp})} = (g\sigma)^2 \beta \frac{|q_x| |q_x z + 2\nu - 1|e^{-|q_x z|} R}{\left( \frac{r_0}{T_{\text{nem}}^{(0)}} + \frac{b_0 q_x^2}{b} \right)},$$

(S6)
from which the critical temperature is obtained by imposing the same self-consistency condition as before,

\[ r_0 \frac{T_{\text{nem}}(q_x) - T_{\text{nem}}}{T_{\text{nem}}^{(0)}} = (g\sigma)^2 \beta \left( \nu - \frac{1}{2} \right)^2 + \nu^2 \]  \quad \text{[S7]}

It is clear that the maximum transition temperature happens at a non-zero \( q_x \), leading to the wave-vector:

\[ |q_{\text{smc}}| = \frac{(g\sigma)^2 \beta}{2b_\parallel} \left[ (\nu - \frac{1}{2})^2 + \nu^2 \right] \quad \text{[S8]}

The smectic critical temperature is obtained by substituting \( T_{\text{smc}} = \max_{q_x} T_{\text{nem}}(q_x) \),

\[ r_0 \frac{T_{\text{smc}} - T_{\text{nem}}}{T_{\text{nem}}^{(0)}} = \frac{(g\sigma)^4 \beta^2}{4b_\parallel} \left[ (\nu - \frac{1}{2})^2 + \nu^2 \right] \]  \quad \text{[S9]}

Furthermore, from the spatial profile of the nematic order parameter, Eq. S6, we note that \( \eta_{x,z} \) is exponentially localized on the surface and peaked at \( z \sim 1/|q_{\text{smc}}| \).

### SMECTIC ORDER IN THE CASE OF POINT DEFECTS

In this section, we study the case where electronic smecticity is induced not by step-like defects, but by point-like anisotropic defects illustrated in Fig. S4. We follow the same procedure as in the case of infinite steps and start by obtaining the strain field for a point-like anisotropic defect. We subsequently obtain the defect-induced nematic potential by averaging over a distribution of such defects.

The strain created by a defect in equilibrium is modeled by a localized force density \( f_{\mu} \) with \( \mu = x,y \) along the crystallographic axes, this leads to two possibilities for the defect force densities, namely, \( f_{\mu}^{(1)} = h_{\mu}^{(1)} \zeta \delta(x-x') \delta(y-y') \delta(z) \) and \( f_{\mu}^{(2)} = h_{\mu}^{(2)} \zeta \delta(x-x') \partial_y \delta(y-y') \delta(z) \), along with superpositions of these two forces. Here \( h_{\mu}^{(1,2)} \) denote the corresponding forces, and \( \zeta \approx a_\parallel \) is a microscopic length scale of the order of the lattice constant. The first case, \( f_{\mu}^{(1,1)} \), is depicted in Fig. S4.

Now, we derive the \( B_{1g} \) strain field generated by the defect force \( f_{\mu}^{(1)} \). The strain field corresponding to \( f_{\mu}^{(2)} \) is obtained by interchanging \( x \leftrightarrow y \) and \( y \leftrightarrow -x \). As in the main text, for simplicity, we restrict ourselves to the case where only the \( z \)-component of the force is present, i.e. \( f_{x,y}^{(1)} = 0 \), but \( f_{z}^{(1)} = f^{(1)} \neq 0 \). The Green’s function for a
unit point force density \( \delta(x)\delta(y)\delta(z) \) applied normally to the surface of a semi-infinite elastic half-space \((z \geq 0)\) at the origin is given by [51]

\[
G_{xz}(x, y, z) = \frac{(1 + \nu)}{2\pi E} \left[ \frac{zx}{\varepsilon^3} + \left( 3 - 4\nu \right) \frac{\delta_{j,3}}{\varepsilon} - \frac{(1 - 2\nu)}{\varepsilon} \left( \delta_{j,3} + \frac{x}{\varepsilon} \right) \right],
\]

where \( \varepsilon = \sqrt{x^2 + y^2 + z^2} \). Hence, the deformation profile created by this single defect at the origin is given by,

\[
u_{x}^{(1)} = h^{(1)} \zeta \partial_{x} G_{xz}(x, z) = \left( 1 + \nu \right) \zeta h^{(1)} \frac{1}{2\pi E} \partial_{x} \left( \frac{zx}{\varepsilon^3} - \frac{(1 - 2\nu)}{\varepsilon} \frac{x}{\varepsilon^2} \right),
\]

\[
u_{y}^{(1)} = h^{(1)} \zeta \partial_{y} G_{yz}(x, z) = \left( 1 + \nu \right) \zeta h^{(1)} \frac{1}{2\pi E} \partial_{y} \left( \frac{zy}{\varepsilon^3} - \frac{(1 - 2\nu)}{\varepsilon} \frac{y}{\varepsilon^2} \right).
\]

Here, we have approximated the denominator \( \varepsilon + z \approx \varepsilon \) to obtain an analytical closed-form expression in Fourier domain. This is valid over long distances \( \varepsilon \), corresponding to \( q_{\|} \to 0 \). Defining \( r = \sqrt{x^2 + y^2} \), \( x = r \cos \theta \), \( y = r \sin \theta \), and \( q_{\|} = (q_{x}, q_{y}) = q_{\|} (\cos \phi, \sin \phi) \), along with \( \omega = q_{\|} r \) and \( \gamma = q_{\|} z \), we have the following Fourier transformed deformations,

\[
u_{x}^{(1)} \approx \frac{1}{2\pi E} \left[ \gamma e^{-\gamma} - (1 - 2\nu) \gamma K_{1,\gamma} \right] \cos^2(\phi),
\]

\[
u_{y}^{(1)} \approx \frac{1}{2\pi E} \left[ \gamma e^{-\gamma} - (1 - 2\nu) \gamma K_{1,\gamma} \right] \cos(\phi) \sin(\phi),
\]

where \( K_{n,\gamma} \) is the modified Bessel function of the second kind. Therefore, the \( B1g \) strain for a defect with unit force \( h^{(1)} = 1 \), \( \varepsilon_{B1g} = (\partial_{x} u_{x} - \partial_{y} u_{y})/\sqrt{2} \), \( h_{x}^{(1)} = 1 \), is given by

\[
\varepsilon_{B1g}^{1} \approx \frac{1}{2\sqrt{2\pi E} L_{\xi}} \left[ \gamma e^{-\gamma} - (1 - 2\nu) \gamma K_{1,\gamma} \right] \cos^3(\phi) - \cos(\phi) \sin^2(\phi).
\]

To obtain the defect-induced nematic potential, we follow the procedure presented in the main text for the case of step defects. Each defect, randomly distributed and indexed by \( j \), is located at the sample surface \((z = 0)\) with the location specified by \( r_{\|,j} = (x_{j}, y_{j}) \) and aligned along one of the crystallographic axes, \( (m) = \{ (1), (2) \} \). The net strain created by the random distribution of defects can be written as \( \varepsilon_{r}^{B1g} = \sum_{j} \left[ h_{j}^{(1)} \varepsilon_{1, r_{j}}^{B1g} + h_{j}^{(2)} \varepsilon_{2, r_{j}}^{B1g} \right] \). It is convenient to define the defect force density

\[
\rho_{r}^{(m)} = \sum_{j} h_{j}^{(m)} \delta(x - x_{j})\delta(y - y_{j}),
\]

Similar to the step defects considered in the main text, the variance of this continuous defect distribution is \( \sigma_{m}^2 \).
bare nematic correlation length (as introduced in the main text). We thus obtain the action,

\[ S = \int_{r_{i|z}} \eta r_{i|z} \frac{r_0 \frac{\tau_{\text{num}}}{\tau_{\text{num}}}}{2} - \eta r_{i|z} - g \sum_{m=1,2} \int_{r_{i|z}} \rho_{m}^{(m)} \eta_{m, r-r_{i|z}} \eta r_{i|z}, \tag{S17} \]

After averaging over the defect distributions, we find

\[ S = \int_{r_{i|z}} \eta r_{i|z} \frac{r_0 \frac{\tau_{\text{num}}}{\tau_{\text{num}}}}{2} - \eta r_{i|z} + \frac{m_{\text{def}}}{2} \int_{r_{i|z}} \int_{r_{i|z}} \sum_{m=1,2} \left( \frac{g \sigma m}{2} \right) \eta_{m, r-r_{i|z}} \right] \int_{q_{i|z}} \left( \frac{L{\text{nem}}}{2} L{\text{nem}} \right) \int_{q_{i|z}} \int_{q_{i|z}} \sum_{m=1,2} \eta \left( m, q_{1|z} \right) \left( m, q_{1|z} \right) \eta \left( m, q_{1|z} \right) \right], \tag{S18} \]

where, from Eq. (S15), \( F_{m, q_{1|z}} = \frac{L{\text{nem}}}{2} \left( \frac{1+\nu}{2} \right) \gamma K_{1,\gamma} \left[ \cos^2(\phi m) - \cos^2(\phi m) \sin^2(\phi m) \right] \) with \( = \phi + (m-1) \frac{\pi}{2} \). The last expression defines the analogue of \( V_{q_{1|z}} \) defined by Eq. (6) in the main text, but for the point defects being considered here. The effective coupling in Fourier space is then given by \( (g \sigma m)^2 \beta/2 \), with \( \beta = \left( N_{\text{def}} \left( L{\text{nem}}^2 / a_{\text{def}}^2 \right) \right) \left( 1 + \nu \right) \left( 2 \sqrt{2} \pi E \right)^2 \). As a result, the defect-induced potential is explicitly given by

\[ V_{q_{1|z}} \sim \sum_{m=1,2} \frac{(g \sigma m)^2 \beta}{2} q_{1|z}^2 \left[ q_{1|z} \cos(q_{1|z}) - (1 - 2
\nu) q_{1|z} K_{1,\gamma} \right] ^2 \cos^2(\phi m) - \cos^2(\phi m) \sin^2(\phi m) \right] \right]. \tag{S19} \]

**Figure S5.** The analytical approximation to the defect-generated potential \( V_{q_{1|z}} \) from Eq. (S19), for (a) \( m = 1 \) and (b) \( m = 2 \), considering \( z = z' \). In (a), troughs appear at \( q_x \neq 0, q_y = 0 \) while in (b), troughs are found at \( q_y \neq 0, q_x = 0 \).

This potential, plotted in Fig. S5, has a trough at finite \( q_{1|z} \) and, hence, it favors a modulated nematic order. Note that, depending on the character of the defect distribution, \( \sigma_1 \) and \( \sigma_2 \) could be different. In this case, the nematic potential would have deeper troughs along one direction than the other, leading to stripe smectic patterns.

To estimate the smectic wave-vector, we write the nematic potential in Eq. (S19) terms of the variables \( \bar{z} = (z + z')/2 \) and \( \delta z = z - z' \). Similarly to the case of step defects, the (negative) potential is peaked at \( \bar{z} \sim 1/q_{1|z} \) and \( \delta z = 0 \), leading to a nematic order parameter \( \eta_{1|z} \) that is peaked near the surface (\( z = 0 \)). Assuming that \( \eta_{1|z} \) varies slowly near the surface over a depth \( L_z \sim 1/q_{1|z} \), and eventually decays exponentially away from the surface, the integral in
Eq. (5) yields,
\[ S_d = L_s^2 \int_{z=-L}^{0} \int_{\delta z=-L_s/2}^{L_s/2} \sum_{q_i} V_{i,q,i,\delta z} |\eta_{q_i,0}|^2 \]
\[ \approx L_s^2 L_s^2 \sum_{q_i} \sum_{m=1,2} \frac{-\langle g(\sigma m)^2 \beta \rangle}{2} q_i |\eta_{q_i,0}|^2 \int_{\omega=0}^{|\omega'|=\infty} d\omega \left[ \omega e^{-\omega} - (1-2\nu)\omega K_{1,\omega} \right]^2 \left[ \cos^2(\phi_m) - \cos(\phi_m) \sin^2(\phi_m) \right]^2, \]
\[ = L_s^2 L_s^2 \sum_{q_i} \sum_{m=1,2} \frac{-\langle g(\sigma m)^2 \beta \rangle}{2} q_i d(\nu, \phi, m)|\eta_{q_i,0}|^2, \]
where,
\[ d(\nu, \phi, m) = \left\{ \begin{array}{l} 1 \quad \text{if } (1-2\nu)^2 \frac{3\pi^2}{32} - \frac{4(1-2\nu)}{5} \left[ \cos^2(\phi_m) - \cos(\phi_m) \sin^2(\phi_m) \right]^2 < 0 \end{array} \right. \]  
(S20)

Recall that \( \phi_m = \phi + (m-1)\pi/2 \). In the limit \( b \to 0 \), the minimization of \( S \approx L_s^2 L_v [(T - T_{\text{nem}})/2T_{\text{nem}} + \beta d(|q_{\text{smc}}|^2)/2|\eta_{q_{\text{smc}},0}|^2 + S_d \) takes place for a non-zero \( q_{\text{smc}} \), with \( \phi = n\pi/2 \) and \( n \in \mathbb{Z} \), and at the temperature \( T_{\text{smc}} = T_{\text{nem}}(1 + \beta q_{\text{smc}}^2) \).

**SMECTIC-NEMATIC TRANSITION**

Here we study the smectic-nematic phase transition, focusing on the nature of the transition as well as on its dependence on the sample thickness. There are two competing factors affecting the transition. On the one hand, the defect-induced potential makes the smectic mass (i.e. the coefficient of the quadratic term in the action) more negative than the uniform nematic mass, which favors smectic order. On the other hand, the extensive character of the bulk-nematic free energy gain favors the bulk nematic order at low temperatures.

The quartic term \( (1/4)u_q|\eta_q|^4 \), when written in momentum space, is equivalent to a biquadratic “repulsion” \( \sum_{q,q'} (1/4)u_q|\eta_q|^2|\eta_{q'}|^2 \). Thus, for any temperature, it selects the order with the lowest mass. Consequently, for the transition to the bulk-nematic phase to occur, the extensive nematic free energy gain must compensate for the defect-induced enhancement of the non-extensive surface-smectic free energy. From Eq. (2) in the main text, the free energy can be written as
\[ F = F_{\text{nem}}^{(2)} + F_{\text{smc}}^{(2)} + F_{\text{smc-nem}}^{(4)}, \]  
(S21)
where we neglect all \( q \) wave-vectors except the homogeneous nematic wave-vector \( q = 0 \) and the preferred smectic wave-vector \( q_{\text{smc}} \). The quadratic part of the nematic free energy,
\[ F_{\text{nem}}^{(2)} = L_s^2 L_0 T_{\text{nem}} \frac{T - T_{\text{nem}}}{2T_{\text{nem}}} \eta_{q_s = 0}^2, \]  
(S22)
scales extensively with the system size. The surface smectic free energy,
\[ F_{\text{smc}}^{(2)} = L_s^2 L_s \left[ \frac{T - T_{\text{nem}}}{2T_{\text{nem}}} - \frac{b q_{\text{smc}}^2}{2} \right] |\eta_{q_{\text{smc}},0}|^2, \]  
(S23)
does not scale extensively along the \( z \)-direction, as the smectic layer is restricted to a depth \( L_s \sim 1/q_{\text{smc}} \), where \( q_{\text{smc}} = (g\sigma)^2 \beta \left[ (\nu - 1/2)^2 + \nu^2 \right]/2b_\parallel \) as derived in the main text. Lastly, the quartic contribution to the free energy is given by
\[ F_{\text{smc-nem}}^{(4)} = L_s^2 L_s \left[ \frac{12u_q}{4} |\eta_{q_{\text{smc}}}|^2 \eta_{q_s = 0}^2 + \frac{6u_q}{4} |\eta_{q_{\text{smc}}}|^4 \right] + L_0^2 \frac{u_q}{4} |\eta_{q_1}|^4. \]  
(S24)

The defect-induced strain \( \varepsilon_{1,1}^{B1} \) decays over a depth \( z \sim 1/q_1 \). In a sufficiently thick sample with \( L \gg 1/q_1 \), the description of the system in terms of an elastic half-space is valid. Then, the smectic-nematic transition temperature \( T_{\text{smc-nem}} \) can be obtained by equating the masses of the bulk nematic and of the surface smectic free energies, yielding,
\[ \frac{T_{\text{smc-nem}} - T_{\text{nem}}}{T_{\text{nem}}} = \frac{-b q_{\text{smc}}^2}{L_s} - 1 < 0, \]  
(S25)
Clearly, while the phase boundary approaches $T_{\text{smc-nem}} = T_{\text{nem}}$ for $L \to \infty$, as depicted in Fig.3, it moves below and farther away from $T_{\text{nem}}$ with decreasing $L$. Since the smectic wave-vector $q_{\text{smc}}$ is expected to remain largely unchanged with varying $L/L_{\text{crit}}$, the suppression of $T_{\text{smc-nem}}$ below $T_{\text{nem}}$ should persist for even thinner samples.

Note that, when the transition occurs at $T_{\text{smc-nem}} = T_{\text{nem}}$, which happens for $L \to \infty$, the nematic order parameter develops continuously from zero, whereas the smectic field discontinuously drops to zero, $\eta_{q=0}(T = T_{\text{smc-nem}}) = 0$. However, when $L$ becomes comparable to $1/q_{\text{smc}}$, such that $T_{\text{smc-nem}} < T_{\text{nem}}$, the nematic field discontinuously jumps to $\eta_{q=0}(T = T_{\text{smc-nem}}) = [(T_{\text{nem}} - T_{\text{smc-nem}})/u_qT_{\text{nem}}]^{1/2}$, in conjunction with the smectic field discontinuously dropping to zero.

### COMPARISON WITH THE EXPERIMENTAL PEEM DATA

In this section, we compare our results with the PEEM data published in Ref. [33]. We first address the puzzling result that the smectic state survives deep into the temperature range where a homogeneous nematic phase is expected. We also use the experimental results to estimate the typical energy scale associated with the defects, which we find to be reasonable. This energy scale could in principle be compared to microscopic calculations as a further verification, but that is beyond the scope of the current work.

We start by plotting the Fourier transform of the linear dichroism (LD) signal of the PEEM experiment [33] in Figs. S6, S7, S8 and S9. We note that the experimental data was extracted from Zenodo [72], where it is available, as stated in the manuscript where the data was originally published, Ref. [33]. Figs. S6 and S8 show the Fourier-transformed data for FeSe and doped Ba-122 respectively, while Figs. S7 and S9 show the same data with the intensity clipped for better visualization. In all cases, the momentum-space data displays a speckle pattern, from which we can infer the data for FeSe and doped Ba-122 respectively, while Figs. S7 and S9 show the same data with the intensity clipped for S6 and S8 is dominated by a pair of points. The radius of this spot corresponds to the better visualization. In all cases, the momentum-space data displays a speckle pattern, from which we can infer the data for FeSe and doped Ba-122 respectively, while Figs. S7 and S9 show the same data with the intensity clipped for S6 and S8 is dominated by a pair of points. The radius of this spot corresponds to the experimentally observed $q_{\text{smc}}$, and is present already at high temperatures. While the size of the spot changes only slightly as the temperature is lowered, the spectral weight is not only enhanced but also redistributed as $T$ decreases. In particular, at low enough temperatures, the spectral weight of the spot is concentrated at a sharply defined pair of spots along a specific direction, signaling the onset of a static smectic state.

This behavior is consistent with our theoretical description, from which we found that the defect-strains generate an effective smectic potential, $V_{q_{\text{smc}}}(\hat{r})$. As shown in Fig. S5, this potential displays sharp spots at the smectic wave-vector $q_{\text{smc}}$. This is consistent with the data in Figs. S6 and S8 at the lowest temperatures, where smectic order sets in. Above the smectic transition temperature, the spectral weight is distributed inside an ellipse with radius of the order of $q_{\text{smc}}$, signaling the build-up of smectic fluctuations.

From the data in Figs. S6, we can roughly estimate from the distance between the sharp spots at low temperatures a smectic wave-vector of $q_{\text{smc}} \sim 1 \times 10^{-2}$ nm$^{-1}$, corresponding to a wavelength of the order of hundreds of nanometers. Note that, according to Ref. [33], the typical smectic domain size is of order of tens of microns.

We now perform a quantitative analysis to estimate the defect strength $\sigma$ required to obtain the experimentally determined value for $q_{\text{smc}}$. Our goal is to determine whether this analysis gives a reasonable value for the defect strength – alternatively, it can be compared to microscopic calculations, which are however beyond the scope of this work. We focus on FeSe, for which many of the material properties needed are known from other experiments.

First, we construct the energy scale (per defect) associated with the defect distribution, $E_d = (1/N_{\text{step}})(h/\sigma)^2$, where $h$ is the average force exerted by each step defect. Using Eq. S9 for the smectic critical temperature, $r_0(T_{\text{smc}} - T_{\text{nem}})/T_{\text{nem}} = b_{||}q_{\text{smc}}^2 = [(g\sigma)^4\beta^2/(4b_{||})][(\nu - 1/2)^2 + \nu^2]$, as well as the parameter $\beta$ defined in the main text, $\beta = [(1 + \nu)/(\sqrt{2}E)]^2N_{\text{step}}(L_0/a_{||})$, we can express $E_d$ as:

$$E_d = \left(\frac{g^4h^4[(\nu - 1/2)^2 + \nu^2]^2(1+\nu/E\sqrt{2})^4}{4b_{||}r_0(T_{\text{smc}} - T_{\text{nem}})/T_{\text{nem}}}\right)^{1/2}$$

where

$$= \frac{g^2h^2[(\nu - 1/2)^2 + \nu^2](1+\nu/E\sqrt{2})^2}{2b_{||}q_{\text{smc}}}.$$

Here, we approximated $L_0/a_{||} \approx 1$ for simplicity.

In order to obtain an order-of-magnitude estimate for $E_d$, we need to estimate the various parameters that appear in Eq. (S27): $g$, $b_{||}$, $h$, and the elastic parameters $E$ (the Young modulus) and $\nu$ (the Poisson ratio). We will use the experimentally-determined value for the smectic wave-vector, $q_{\text{smc}} \sim 1 \times 10^{-2}$ nm$^{-1}$. 


To obtain $g$ and $b_\parallel$, we need an estimate of $r_0$, which sets the energy scale of the inverse nematic susceptibility. Considering the nematic order’s electronic origin, a reasonable estimate is $r_0 \sim 1/N_F \sim E_F$, where $N_F$ is the density of states at the Fermi level and $E_F$ is the Fermi energy [73]. The latter can be approximated by the Fermi energy for the hole band of FeSe, $E_F \sim 20$meV [33, 42, 74].

As for the elastic parameters, firstly, we use the elastic constants reported in Refs. [39, 75] to estimate the $B_{1g}$ elastic constant $C_{B1g} = (C_{11} - C_{12})/\sqrt{2} \sim 20$GPa. Note that Ref. [39] considers the 2-Fe unit cell, (the puckered arrangement of the chalcogen/pnictogen atoms leads to two inequivalent Fe atoms in the Fe plane, as depicted in, say, Ref. [?]), where the $x, y$ coordinate axes are aligned along the line joining two equivalent Fe atoms. In this situation, the structural distortion associated with nematicity occurs in the $B_{2g}$ channel, corresponding to the softening elastic constant $C_{66}$. Theoretical models often consider a 1-Fe unit cell (see, for instance, Ref. [?]) for simplicity, with the $x, y$ coordinate axes aligned along the line joining two inequivalent Fe atoms, which is rotated relative to the one employed in the 2-Fe unit cell by 45°. In this case, the same nematic structural distortion happens in the $B_{1g}$ channel. Hence, for the present discussion based on the Ginzburg-Landau formalism, the shear modulus $C_{66}$ presented in Ref. [39] plays the role of our $C_{B1g}$. Secondly, the Young’s modulus and the Poisson’s ratio are estimated as $E \sim 65$GPa, and $\nu \sim 0.2$, respectively [75].

The nemato-elastic coupling $g$ can be obtained from the renormalization of the nematic critical temperature, for which we require not only the actual renormalized nematic critical temperature $T_{nem} \sim 90$K, but also the bare (i.e. not renormalized by the lattice) nematic transition $T_{nem}^{(0)}$. The latter has been obtained via Raman spectroscopy [16, 42], which found $T_{nem}^{(0)} \sim 40$K. Using the relationship $T_{nem}^{(0)} = T_{nem}^{(0)} - (g^2/C_{B1g})(T_{nem}^{(0)}/r_0)$, we obtain $g \sim 1 \times 10^{-5}$J/m$^3/2$.

The nematic stiffness $b_\parallel$ can be obtained from the nematic correlation length $\xi$ via the relation $\xi = \sqrt{b_\parallel T_{nem}^{(0)}/[r_0(T - T_{nem})]}$. The latter was obtained via inelastic X-ray scattering [41], which reported $\xi(T = 100$K) $\sim 70$Å, yielding $b_\parallel \sim 4 \times 10^{-38}$Jm$^2$.

Finally, while there is no data on $h$, the force exerted by each step, for iron-based superconductors, estimates for $h$ have indeed been obtained for simpler systems, yielding [77] $h \sim 10^{-10}$N. For instance, Stewart et al. [56] compare the displacement field measured by transmission electron microscopy to the corresponding analytical results on Si
surfaces, obtaining $h = 9.3 \times 10^{-10}$N. Similarly, Shilkrot and Srolovitz \cite{55, 76} obtain $h = 2.9 \times 10^{-10}$N on Ni and Au surfaces. We assume that the forces exerted by step defects in the systems considered in this study have similar magnitudes, and thus approximate $h \sim 10^{-10}$N.

Substituting all the estimated parameters in Eq. (S27), we find:

$$E_d \sim 100 \mu eV.$$  \hspace{1cm} (S27)

We emphasize that this value is just an order of magnitude estimate.
Figure S8. The LD PEEM data for BaFe$_2$(As$_{0.87}$P$_{0.13}$)$_2$ in momentum space. The dashed white line, which is a guide to the eyes, marks the “spot” of fixed size $\sim q_{smc}$. Such a spot is more clearly seen in Fig. S9. This plot was generated from the real-space experimental data of Ref. [33], which is available at Zenodo [72].

Figure S9. The same LD PEEM data for BaFe$_2$(As$_{0.87}$P$_{0.13}$)$_2$ plotted in Fig. S8, but with the magnitude clipped to highlight the “spot” (dashed white line).