An integral sliding mode control for the magnetic levitation system based on backstepping approach

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Abstract. Due to their reduced maintenance costs, increased power efficiency and reduced power consumption, the Magnetic Levitation (Maglev) system make a significant contribution to the industrial application. Maglev's production of electricity (e.g. wind turbines), maglev trains and medical devices (e.g. artificial heart pump magnetically suspended) are typical applications. This paper suggests designing a nonlinear control for the Maglev system model which represented by a third-order model consists of the mechanical (ball position and velocity) and electrical (the current) subsystems. The controller is designed utilizing the Integral Sliding Mode Control (ISMC) and based on the Backstepping approach. The tracking accuracy of the ball position to the desired reference is determined by computing the ultimate boundedness as a function to the controller parameters and that using the Lyapunov function. The numerical simulation results showed the robustness and the efficiency of the proposed controller where the tracking error limited by the computed bound.

Keywords: Maglev, Uncertain model, Ultimate boundedness, Lyapunov function, Backstepping approach.

1. Introduction
Because of the use of force exertion, noncontact (i.e., low friction) and effectively reduce mechanical vibrations, with an ever-increasing demand for precise and reliable manufacturing and research environments, the Maglev systems find increased utilization in applications such as machine tools. The Maglev systems, however, are subject to various control complexities like open-loop insecurity, inherent non-linearities within the model system, continuous biasing and unidirectional force input [1]. Because in Maglev systems there is no mechanical contact, friction or noise they can be used for accurate positioning. Consequently, they have many relevant applications, such as Maglev trains, magnetic rollers, wind tunnels and transmission systems, in addition to, Maglev systems are becoming popular in many applications where they can be used for precise positioning such as frictionless bearings and conveyor systems[2]. So, in many applications of engineering [3]-[14], Maglev is becoming a technology with the benefit of such flow costs and less environmental impacts and high-speed performance. Maglev system dynamics are usually unstable and complex. Thus, a good controller is required to position a ferromagnetic ball very accurately [11]. Moreover, the Maglev is a non-linear method, so it has been suggested some adaptive control algorithms [15]-[17]. But it is difficult to achieve the exact dynamic model of a Maglev.

However, the Maglev suffers from many problems, which complicate the design task like the nonlinear and open loop instability nature, the uncertainty in system model and the external
disturbance. These challenges have led to a significant need for developing control technologies for Maglev control systems [18]. Levitation technology can be divided between the following categories [19]-[21]: first, the technology of Electro Dynamic Suspension (EDS) and, second, the technology of Electro Magnetic Suspension (EMS). The levitation is carried out in the first group, based on a repulsive force. This is intrinsically stable system. Therefore, it is very suitable for high-speed operations [22]-[24]. In the second category for levitation, the magnetic attraction force is utilized. This approach can be applied more quickly and can be levitated at low or zero speed, so, it is more widely applied [25], [26]. Preliminary attempts are also made to address unknown system parameters and load disturbances [27],[28].

Historically, for a number of hundred years Maglev of contactless objects is a fantasy. The use of magnetic forces in many situations seems to be the ideal solution for such an objective. The magnetism laws restrict the use of magnetic forces to support an object without mechanical contact. This unstable behaviour, highly nonlinear differential equations can be presented [12]. Various techniques for controlling non-linear dynamics have been proposed: Benomair and others [12] used the exact linearization algorithm of feedback from the input state instead of the linear dynamic approximation. Recently, a diversity of control approaches has been used to design nonlinear control to the Maglev. The feedback linearization was used, as a first step, by many authors in order to transform the Maglev system model to a linear model like [11], [13], [14], and [29]. The performance of the control is affected, especially when the system model is uncertain, and the external disturbance varies as shown by [30]. A disturbance observer-based control design methodology was introduced by [14]. Moreover, [31] showed that the chattering problem cannot be avoided unless by using some approximation for the discontinuous term. This step will degrade the Sliding Mode Controller (SMC) performance.

Furthermore, Zhang et al. [32] designed the input control so that the Maglev's output state could follow the desired trajectory exponentially. Over that, Maglev's system of a train comprises a guide track with magnets was explained by Rehman et al. [33]. Sun et al. [34] modelled a system Maglev firstly and then an integral sliding mode controller with an integral switching gain is suggested. A Backstepping using nonlinear damping approach for the Maglev model was used by [29]. In [35] Delavari and Heydarinejad proposed an adaptive fractional order Backstepping sliding mode control schemes for a Maglev system. Backstepping algorithm was based on the Lyapunov theory. In addition to, Singh and Kumar [36] had implemented a Backstepping controller for the stabilization of the Maglev system. Nonlinear adaptive controller based on backstepping was proposed by Zhou and Liu [37] for the design of the Maglev system with parameter uncertainty.

This paper falls within the context of the EMS system. The problem of levitation control in a nonlinear Maglev system is considered. There is instability in the system that can be caused by unknown parameters and external load disturbances. The uncertainty may vary quickly. No other information is known except its possible bound. A robust controller based on Backstepping approach is proposed in this paper which utilizes the integral sliding mode. The integral sliding mode will robustly eliminate the perturbation term and enforcing the induced current to make the ball level to follow the desired reference. The organization of this paper is as follows: the mathematical model of the Maglev system is given in section two. In section three, controller design and stability analysis are derived. The simulation results and discussion, in section four and conclusion in section five.

2. Dynamics of the Maglev System: Mathematical Model
Maglev system dynamics are generally unstable and complicated. To achieve high accuracy of a ferromagnetic ball positioning, a good controller is therefore necessary. The Maglev system keeps a small steel ball in a steady position at a stable levitation. An electromagnet produces forces to support the weight of the ball (see Figure 1). The electromagnetic forces are connected with the electric current that passes via the electromagnetic coil. The Apling Kirchhoff’s Voltage Law on electrical system loop can calculate the electromagnetic force generated by current [12].
The Maglev systems generally comprise an electrical loop and an electromechanical loop. An analysis of electromagnetic and mechanical subsystems will form the dynamic attitude of the Maglev system [38].

Firstly, let us analyze the Electromagnetic Dynamic Modeling where the current flowing through the coil using the Kirchhoff voltage law produces the electromagnetic force;

\[ V_{in} = V_R + V_L = iR + \frac{di}{dt}L(x) \]  

(1)

Where \( V_{in} \) is the voltage applied, \( i \) is the current in Electromagnet coil, \( R \) is the coil resistance and \( L \) is the coil inductance.

Secondly, a free body concept for a ferromagnetic ball that is unsettled by equating the electromagnetic force \( F_{em}(x, i) \) and the gravitational force \( F_g \) is modeled as follows. When the ball is in equilibrium situation.

\[ F_g = F_{em} \]  

(2)

To apply the Newton’s 3rd law of motion, friction and the air force resistance etc. are disregarded. \( F_{net} \) that affects ball is the net force.

\[ F_{net} = F_g - F_{em} \]  

(3)

Where \( m \) is the ball mass, \( x \) is the ball position, \( g \) is the gravitational constant and \( C \) is the magnetic force constant.

Finally, for the Maglev, the non-linear model is derived as follows: the model can be depicted in differential equations terms as the following basis of an electro-mechanical non-linear Maglev system model:

\[ V_{in} = iR + L \frac{di}{dt}L(x) \]  

(4)

**Figure 1. Active Maglev ball system**
\[
m\ddot{x} = mg - C\left(\frac{i}{x}\right)^2
\]

For inductance limitation in the Maglev method, several different approximations were used. Taking the approximation that inductance varies depending on the inverse ball position, namely:
\[
L(x) = L_C + \frac{L_0x_0}{x} + \ldots
\]

Eq. 6 indicates that \( L(x) \) is a nonlinear ball position function \( x \) where \( L_C \) is the coil inductance constant in the absence of a ball, \( L_0 \) is the additional inductance of the ball and \( x_0 \) is the balance. The consequence of (6) substitution with (4) is:
\[
\begin{align*}
V_{in} &= Ri + \frac{d}{dt}\left(\frac{2C}{x}\right)i + L(x)\frac{di}{dt} \\
&= Ri + L\frac{di}{dt} + 2\left(\frac{d}{dt}\left(\frac{C}{x}\right)\right)i \\
&= iR + L\frac{di}{dt} - 2C\left(\frac{i}{x^2}\right)\frac{dx}{dt}
\end{align*}
\]

Now, let the input control signals and state to be selected as:
\[
x_1 = x, x_2 = v, x_3 = i, u = V_{in}, \text{ and } \delta = [x_1 \quad x_2 \quad x_3]^T \text{ is the state vector.}
\]

Therefore, as a state space type, the model of the Maglev system is as follows:
\[
\begin{align*}
\frac{dx_1}{dt} &= x_2 \\
\frac{dx_2}{dt} &= g - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 \\
\frac{dx_3}{dt} &= -\frac{R}{L}x_3 + \frac{2C}{L}\frac{x_2x_3}{x_1^2} + \frac{1}{L}u
\end{align*}
\]

Allowing
\[
x^T = [x_1 \ x_2 \ x_3] = [h \ h \ i], \quad a_1 = \frac{C}{m}, a_2 = \frac{R}{L}, a_3 = \frac{2C}{L} \text{ and } b = \frac{1}{L}, \text{ equations (8) can be rewritten as follows:}
\]
\[
\dot{x} = \begin{bmatrix}
x_2 \\
g - a_1x_3^2x_1^{-2} \\
-a_2x_3 + a_3x_2x_3x_1^{-2}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
u
\end{bmatrix}
\]

Considering the system uncertainties due to deviations of the system parameters and loading mass, equation (9) is rewritten in the form of equation (10)
\[
\dot{x} = \begin{bmatrix}
x_2 \\
g - a_{1n}x_3^2x_1^{-2} \\
-a_{2n}x_3 + a_{3n}x_2x_3x_1^{-2}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
u \\
d_1 \\
d_2
\end{bmatrix}
\]

Where \( d_1 = \Delta a_1x_3^2x_1^{-2} \) and \( d_2 = \Delta a_2x_3 + \Delta a_3x_2x_3x_1^{-2} + \Delta bu, \) in which \( \Delta a_1, \Delta a_2, \Delta a_3 \) and \( \Delta b \) denote the deviations of \( a_1, \ a_2, \ a_3 \) and \( b \) respectively.

The Maglev Model can also be written in a more suitable form to control design by firstly defining the following error function states: \( e_1 = x_1 - x_d \) and \( e_2 = x_2 - \dot{x}_d \) where \( x_d \) and \( \dot{x}_d \) are the desired ball position and velocity respectively. Equation (10) according to this definition becomes;
Secondly, by defining the following state and control transformations,

\[ z_1 = e_1, \quad z_2 = e_2 \quad \text{and} \quad z_3 = g - a_{1n}x_3^2x_1^{-2} - \ddot{x}_d \]

Eq. 11 transformed to the following form:

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3
\end{bmatrix} =
\begin{bmatrix}
f(x) \\
g(x)
\end{bmatrix} u +
\begin{bmatrix}
d_1 \\
d_2
\end{bmatrix}
\]  

(12)

Where

\[
f(x) = 2a_{1n}\{a_{2n}x_3^2x_1^{-2} + x_3^2x_2x_1^{-3}(1 - a_{3n}x_1^{-1})\} - \ddot{x}_d,\]

\[
g(x) = -2a_{1n}b_n x_3 x_1^{-2} \quad \text{and} \quad d_2 = -2a_{1n}x_3x_1^{-2}d_2
\]

Finally, by defining the following transformation in control input

\[ u = \frac{1}{g(x)} (-f(x) + u_1) \]  

(13)

the Maglev system dynamics is given by

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} 
\begin{bmatrix}
z_1 \\
z_2 \\
z_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u +
\begin{bmatrix}
d_1 \\
d_2
\end{bmatrix}
\]  

(14)

Or

\[ \dot{z} = Az + Bu_1 + D \]  

(15)

Where

\[
z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 0 \\ d_1 \\ d_2 \end{bmatrix}
\]

When the Maglev model is certain \((D = 0)\), Eq. 15 represents the system model in a phase variable canonical. Therefore, Eq. 15 gives a phase variable canonical Maglev model under a non-vanishing perturbation \(D\) when the uncertainty is considered. Additionally, the matching condition will not be satisfied here and that because \(D\) does not lie within the span of \(B\), which is because of the presence of \(d_4\) in the second line in Eq. 14.

3. Controller Design and Stability analysis

The model of the Maglev system, which has been subject to uncertainty in system parameters, is also unstable open-loop and tightly nonlinear. Nonlinear and unstable nature of the system makes it a real challenge to control the system. These systems have unstable open loop response, to make the response of the system stable optimal control design based on sliding mode for a Maglev system was used.
Based on the mathematical model in Eq. (14), the proposed controller here is based on Backstepping methodology. Two steps are performed to design the proposed controller. In the first step, a virtual controller is designed for the mechanical subsystem that represented by the following two equations;

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\
z_2
\end{bmatrix} + \begin{bmatrix} 0 \\
1
\end{bmatrix} z_3 + \begin{bmatrix} 0 \\
d_1
\end{bmatrix}
\]

(16)

In these equations, the mechanical subsystem is affected by a non-vanishing perturbation term \(d_1\), and by the variable \(z_3\). \(z_3\) will be taken as the virtual controller \(z_{3v}\). In this work, the virtual controller \(z_{3v}\) is selected as;

\[z_{3v} = -k_1 z_1 - k_2 z_2\]

(17)

To analyze the stability of the mechanical subsystem with the selected virtual controller, let us consider \(z_3 = z_{3v}\), accordingly, the mechanical subsystem dynamics in Eq. 16 becomes

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \begin{bmatrix} z_1 \\
z_2
\end{bmatrix} + \begin{bmatrix} 0 \\
1
\end{bmatrix} d_1
\]

Or

\[
\dot{z} = \hat{A} \dot{z} + Gd_1
\]

(18)

Where

\[
\dot{z} = \begin{bmatrix} z_1 \\
z_2
\end{bmatrix}, \quad \hat{A} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \text{ and } G = \begin{bmatrix} 0 \\
1
\end{bmatrix}
\]

By selecting \(k_1 \& k_2 > 0\) and with the assumption of a bounded norm for the perturbation \(d_1\), the mechanical subsystem is stable. Since \(d_1\) is not a vanishing perturbation, we will only determine the ultimate boundedness [30] in terms of the perturbation norm. To estimate the ultimate boundedness, the following candidate Lyapunov function is utilized;

\[
V = \dot{z}^T P \dot{z}
\]

(19)

Where, \(P\) is a positive definite matrix, which it given by:

\[
P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix}
\]

Using Eq. (18), the time derivative of \(V\) is obtained as:

\[
\dot{V} = \dot{z}^T P \dot{z} + \dot{\dot{z}}^T P \dot{z} = -\dot{z}^T Q \dot{z} + 2 \dot{z}^T P G d_1
\]

(20)

Where, \(Q\) is a positive definite matrix, and \(P\) and \(Q\) are related by the following Lyapunov equation [30].

\[
P \hat{A} + \hat{A}^T P = -Q
\]

In terms of the disturbance norm for \(d_1\), \(\dot{V}\) satisfies the following inequality

\[
\dot{V} \leq -\dot{z}^T Q \dot{z} + 2|\dot{z}^T P G d_1|
\]
where $|d_1| < \delta, \forall t$. As a result, the ultimate boundedness for the mechanical subsystem is given by:

$$\|\dot{x}\| \leq \frac{2(p_2^2 + p_3^2)}{\lambda_{\text{min}}(Q)} \delta$$

Consequently, the maximum error between the ball position and the desired position is given by:

$$|e_1| = |x_1 - x_d| = \frac{2(p_2^2 + p_3^2)}{\lambda_{\text{min}}(Q)} \delta$$

To attenuate the effect of the perturbation $d_1$, suitable virtual control parameters $k_1 & k_2 > 0$ should be selected such that the term $\frac{2(p_2^2 + p_3^2)}{\lambda_{\text{min}}(Q)} \delta$ is minimized as will be done in section 4. The term $\frac{2(p_2^2 + p_3^2)}{\lambda_{\text{min}}(Q)} \delta$ is named here as the attenuation factor.

In the second design step, the state $z_3$ is enforced to follow the virtual control $\dot{z}_{3v}$ as in the following; define the new error variable $\eta = z_3 - z_{3v}$, Eq. 14 can be rewritten including the $\eta$ error dynamics as follows;

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} -k_1 & -k_2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ d_1 \\ \dot{d}_2 - \dot{z}_{3v} \end{bmatrix}$$

The component $\dot{d}_2 - \dot{z}_{3v}$ is unknown, because, as defined above, $\dot{d}_2$ is an unknown perturbation term, also, the derivative of $z_{3v}$ will contain unknown terms like $d_1$. In this work, we propose the use of integral sliding mode controller (ISMC) for the $\eta$ dynamics in order to cancel the unknown term ($\dot{d}_2 - \dot{z}_{3v}$) in the third equation in (23) and then to regulate $\eta$ to the origin via a linear control term with a desired characteristic. As a result, $z_1$ will ultimately bounded by inequality (22).

The ISMC for the $\eta$ dynamics, is given by

$$\begin{cases} s = \eta + y \\ \dot{y} = k_3 \eta \\ u_1 = -k_3 \eta - k_4 \text{sign}(s) \end{cases}$$

where $s$ is the sliding variable and $y \in R$ is a new variable with initial condition $y(0) = -\eta(0)$ which will ensure that the $\eta$ dynamics will initiate and maintain at the sliding manifold $s = 0$ [31]. To examine the dynamic stability of the Maglev system according to the proposed controller, an equivalent dynamics model for the Maglev system that given in Eq. 23 is derived here according to the equivalent SMC theory. The equivalent control $u_{eq}$ is derived by applying the equivalent operator to the time derivative of $s$ as follows;

$$\dot{s} = \dot{\eta} + \dot{y} = u_1 + (\dot{d}_2 - \dot{z}_{3v}) + k_3 \eta$$
Then $[s]_{eq}$ is given by

$$[s]_{eq} = [u_1 + (d_2 - z_{3v}) + k_3 \eta]_{eq} = [u_1]_{eq} + (d_2 - z_{3v}) + k_3 \eta$$

Solving for $[u_1]_{eq}$ by equating $[s]_{eq}$ to zero \cite{31}, yields

$$[u_1]_{eq} = -(d_2 - z_{3v}) - k_3 \eta \quad (25)$$

Consequently, by substituting $[u_1]_{eq}$ in the Maglev system model (Eq. (23)), the equivalent Maglev model, is given by

$$\frac{\dot{z}_1}{\dot{z}_2} = \begin{bmatrix} 0 & 1 & 0 \\ -k_1 & -k_2 & 1 \\ 0 & 0 & -k_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ u_1 + d_1 \\ 0 \end{bmatrix} \quad (26)$$

It is clear from Eq. (26) with $k_3 > 0$, that $\eta$ will be asymptotically stabilized to zero, after that $z_1$ will ultimately bounded by the inequality (22) as $t \rightarrow \infty$.

4. Simulation Results and Discussion

The simulation results are obtained by using the original Maglev model which is given in Eq. (8), with the control input as given in Eq. 13 and Eq. 24. The desired position of the ball will be taken in two cases; step and sinusoidal. The gains of the controller for the two cases shown in the following:

$$k_1 = 62500, k_2 = 500, k_3 = 5, k_4 = 3500$$

Where in words, $k_3$ and $k_4$ are the nominal and discontinuous control terms, which will enforce the state the selection of these values of $k_1$ and $k_2$. Accordingly, the attenuation factor is equal to

$$\frac{2\sqrt{p_z^2 + p_{zv}^2}}{\lambda_{min}(Q)} = 2 \sqrt{p_z^2 + p_{zv}^2} \approx 0.002$$

where $Q$ is taken as the identity matrix ($\lambda_{min}(Q) = 1$).

In eq. 24, the control matrix contains the signum function which is a discontinuous function of the sliding variable $s$. The discontinuous term will induce undesirable chattering behavior in system response. To greatly attenuate chattering, the signum function was replaced by the arctan function, which is a continuous function;

$$\text{sign}(s) \rightarrow \tan^{-1}(q * s), q > 1$$

where $q = 10$ in the present work.

Two cases of the reference ball position were considered in the present work; in the first case, a constant ball position reference was considered while a sinusoidal reference was taken in the second case.

For the case of constant reference $x_d = 0.03 \text{ m}$; Figure 2 shows the position tracking where the steady state error is less than $0.05 \text{ mm}$. The sliding variable dynamic is shown in Figure 3 where it stays from the first instant and for all future time at a small value near the zero level. The sliding variable is initiated and then stays near the zero for all future time. This behaviour is a feature of the Integral sliding mode but staying near the zero level and not identically zero is due to using the continuous control law. Additionally, a relatively small control input voltage was spent to enforce the ball to follow the desired position as shown in Figure 4.
Figure 2. Position of the Ball with Constant Reference

Figure 3. Sliding Variable with Constant Reference

Figure 4. Control Voltage with Constant Reference
In the second case, the reference ball position is a sinusoidal reference \( x_d = 0.015 + 0.01 \sin 2t \). The ball position tracking is clarified in Figure 5 which is almost identical. The sliding variable shows in Figure 6, where it can be seen that it is bounded by a small value not exceeding \( 11 \times 10^{-3} \). In addition, a fluctuated control effort is required for this case as shown in Figure 7.
5. Conclusion
The Maglev system is an open loop unstable and is in nature a dynamic nonlinear system. The suggested controller meets this challenge where an integral sliding mode controller was proposed based on the backstepping approach. The backstepping approach was used in order to overcome the mismatched problem, which appears due to the presence of a non-vanishing perturbation $d_j$ in the mechanical subsystem model as given in Eq. 16. This causes a steady state error in the ball position with respect to the reference position as can be deduced from the inequality (22), which was derived based on the Lyapunov function. For the complete Maglev control system, which was proposed here, the stability was successfully proved by utilizing the equivalent control theory.

The numerical simulation was performed for two ball position references; the constant reference and the sinusoidal reference. The results demonstrated the performance and the robustness of the proposed controller where the ball position follows the desired references with a very small error not exceeding 0.05 mm. In addition, the control task was performed using less than 7 volts as the control effort.

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