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Formation of Deformation Bands and Work Hardening of FCC Crystals*

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I. Introduction

The deformation behaviour of crystals is comprehensively represented by the stress-strain curve, which is generally divided into three stages I, II and III as shown in Fig. 1. This characteristic is widely recognized not only in fcc crystals, which are emphasized in this lecture, but also in bcc and hcp crystals as well as crystals of diamond structure and alkali halides(1). As evident from the figure, the work hardening rate is largest in stage II, and it may be said that the main mechanism of work hardening is operating in this stage.

The work-hardening phenomenon of crystals has been studied by many investigators for many years since the work of Taylor(2) whose need for the clarification of the phenomenon was one of the motives for the birth of the dislocation theory. The direct observation of dislocation structures by transmission electron microscopy, a new wave in the 1960s, produced many notable achievements and based on these results, many theories have been proposed concerning work hardening(1).

Despite all these developments, “The phenomena of work hardening are still not well understood” as reminisced by Hirsch (1975)(1), who played a central role in the progress of research on work hardening. It seems that conventional theories were preoccupied with discussions on the mechanism whereby the flow stress is governed, and lacked in essential clues for the more difficult problem of how dislocations are accumulated and distributed as the strain increases. In fact, how stage II that mainly supports work hardening begins, or in other words, the necessity of the transition from stage I to II is not incorporated by any of existing theories of work hardening. Also, the sudden activation of secondary slip systems and a sharp reduction in the slip line length, which are distinct characteristics in the process of transition from stage I to II, have been taken into account a priori in the con-

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vential theories but no corroborating evidence is presented about the causes of such structural changes.

In his Honda Memorial Lecture (1973)\(^3\), the author critically reviewed the situation then surrounding the issue of work hardening as "We cannot see the wood for the trees" and pointed out that, preoccupied with the dislocation structures, researchers ignored the inhomogeneous nature of deformation and in particular the role of deformation bands. The author was strongly interested in the formation of deformation bands and their relation to work hardening for about ten years after the war, but was unable to clarify the problem. Fortunately, about 20 years later, the author was given another opportunity to consider the issue involved, on the occasion of the Honda Memorial Lecture. For about ten years since then, the author and members in the plasticity research group of his laboratory had tackled the problem, and confirmed the possibility of explaining in a unified manner the mechanism of work hardening within the framework of the deformation-band formation. This historical theme 'work hardening' that is not fashionable these days and is left unsolved, is chosen as the subject of this honorable lecture. It is a report on the homework the author has imposed on himself for many years.

II. Inhomogeneity of Deformation

The shear stress-shear strain curve under uniaxial deformation is usually drawn by assuming the homogeneous deformation of crystals for the sake of convenience. In reality, the deformation of crystals is essentially very inhomogeneous: Microscopically, the slip process of a crystal due to the dislocation motion is inhomogeneous, and macroscopically, inhomogeneities arise in the form of deformation bands as shown in Fig. 2. This indicates two types of deformation bands that developed when aluminium single crystals with different crystal orientations were deformed to stage III. Figure 2(a) presents the first type of deformation band or kink band, while Fig. 2(b) depicts the second type of deformation band or band of secondary slip. Fine slip lines are virtually invisible and the direction of primary main slip lines in each micrograph is as indicated by the arrow.

Studies on deformation bands in fcc crystals reported in the literature were made independently by workers such as Cahn\(^4\), Chen and Mathewson\(^5\), Honeycombe\(^6\), and Nishimura and Takamura\(^7\)(\(^8\)) in the early 1950s, and discussions on the subject were contributed about that time by Mott\(^9\) and Kuhlmann\(^10\). For several years thereafter, this subject was investigated in depth, also by Karashima\(^11\) and Kitajima\(^12\) in the western part of Japan. Among the crystallographic characteristics of deformation bands are:

(a) The first type of deformation band or kink band is a wall perpendicular to the direction of operative primary slip and composed of edge dislocations. The crystal is sharply bent with the \langle112\rangle direction as the axis of rotation.

(b) The second type of deformation band or band of secondary slip is a region of almost constant width and parallel to the primary slip plane. In this zone, the operation of primary slip is weak while the secondary slip system actively operates.

The formation of these deformation bands is governed by the deformation temperature, crystal orientation, alloying element, specimen
cross-section geometry, and many other factors. Why are the deformation bands governed by these factors or under what conditions are the deformation bands formed? If our present viewpoint is accepted that the extent of stage I, or the transition from stage I to stage II, depends on the ease with which the deformation bands are formed and that the essential nature of stage II does not appear before the formation of deformation bands, search of the causes for the formation of deformation bands is the key to solving the problem of work hardening.

However, the causes for the formation of deformation bands were little discussed, and even the presence of deformation bands themselves went out of the memory of most of the people investigating work hardening. In our early paper (1951)(7), the author pointed out that "the kink band wall is important for blocking the dislocation motion, and the problem of work hardening is difficult to solve unless the kink-band formation is taken into account". This thought was based on the observation(8) that "the work hardening rate sharply increases once the deformation bands are built up", and has occupied the author's mind to date.

III. Bend Gliding

As is well known, grip constraints under uniaxial tensile deformation of a crystal cause plastic bending of the slip plane. Cottrell(13) pointed out that a geometrical distribution of edge dislocations can be formed on the slip plane by this plastic bending. The author considered that the phenomenon of plastic bending is important in connection with the formation of deformation bands(6), and termed it "bend-gliding" and indicated that the excess dislocations distributed along the axis of plastic bending consist of not only edge components but also screw components(14).

According to this concept, the character of the excess dislocations comprising the bend gliding depends on the angle \( \omega \) between the slip direction and the axis of bending on the slip plane (Fig. 3). The bending axis (BA) is defined by the intersection of the neutral plane of bending, produced by the grip constraint, with the primary slip plane. The direction of the bending axis, or angle \( \omega \) depends on not only the crystal orientation but also the geometry of the specimen cross-section relative to the primary slip direction when the specimen is a plate. For a cylindrical crystal, the angle \( \omega \) is uniquely defined by the orientation of the specimen axis. If the screw-to-edge component ratio \( (b_s/b_e) \) of the mixed excess dislocations is denoted by cot \( \omega \), the following equation is derived(14):

\[
\cot \omega = \sin^2 \theta / (\cos^2 \theta \cot \eta + \tan \eta) \tag{1}
\]

where \( \theta \) is the angle between the specimen axis and the normal to the primary slip plane, and \( \eta \) is the angle between the primary slip direction and the direction of the maximum inclination of the primary slip plane.

The effect of grip constraints on the formation of bend gliding has been examined by stress analyses(15). According to our analyses for a rod crystal by using the infinitesimal bending theory and the deformation gradient (strain tensor), the constraint force due to uniaxial deformation consists of bending stress \( \sigma_{zz} \) and shear stress acting uniformly along the specimen length, \( \sigma_{xy} \), where axes \( z \) and \( y \) are the specimen axis and the projection of slip direction to the cross section, respectively. The former stress is important in the formation of bend-gliding and the latter is important as the additional stress to the secondary slip system.

In this way, the bend-gliding region is produced near the grip end by the bending stress generated under uniaxial deformation. The quite similar effect is expected to occur in any local region throughout the specimen(16)(17) where the operation of primary main slip is inhomogeneous for some reason, as understood
from the finite-element stress analysis shown in Fig. 4(18). Figure 4(a) shows the graphical plot of the mesh in which it is assumed that the primary slip direction is in the plane of drawing, that the Schmid factor is 0.5 and that the plastic strain in the shaded regions is only half of that in the other regions. Figure 4(b) shows the bend-gliding regions (shaded regions) of alternating signs, which form on the assumption that the specimen is plastically relaxed through the introduction of excess dislocations whose equilibrium distribution was calculated from the distribution of shear stress on the primary main slip system when an average tensile stress of 1 MPa is applied to the model crystal. From this figure it can be understood that the accumulated region of excess dislocations is formed in the shaded region perpendicular to the primary slip direction. This is the 'local' bend-gliding region observed in positions other than near the grip end.

IV. Local Bend-Gliding Region and Its Transmutation into Kink Band

1. Nature of local bend-gliding region

Figure 5 shows the change in the deformation structure in stage I of a nearly perfect Cu–1 at%Ge alloy crystal oriented near the [321] direction, as revealed by the etch-pit method(18). The observation was made on one of the side surfaces of the specimen with a hexagonal cross section, which is parallel to the cross slip plane containing the Burgers vector of the primary main slip. The direction of primary main slip is indicated by the arrow. The etch-pit structure of Fig. 5(a) was observed right after yielding (shear strain $\gamma = 0.2\%$) and exhibits the onset of propagation of slip bands. Figure 5(b) shows the state in which the specimen surface is entirely covered by slip bands ($\gamma = 5.5\%$), whereas Fig. 5(c) depicts the dislocation structure half way through stage I ($\gamma = 21.1\%$), where dislocations are densely accumulated in many striated regions almost perpendicular to the primary slip direction. These are the local bend-gliding regions.

For the sake of convenience, the notation of each slip system is listed in Table 1(3) together with the Thompson notation(19) shown in Fig. 6. For instance, $S_{11}$ indicates the slip system due to dislocations having the Burgers vector of the largest Schmid factor on the primary slip plane $S_{1}$, while $S_{12}$ and $S_{13}$ denote coplanar slip at $60^\circ$ to the main slip direction $S_{11}$ on the same slip plane $S_{1}$. On other slip planes also main slip and coplanar slip are distinguished.
Figure 7 shows the result of X-ray analyses of the specimen axis rotation in nine adjacent local regions 1.2 mm apart each in the central part of a Cu-1 at%Ge alloy crystal deformed in tension at room temperature, together with the stress-strain curve up to the beginning of stage I(16). The X-ray measurements were made at each strain indicated by small arrows on the stress-strain curve, but the results at strains Nos. 1, 3 and 6 alone are given in the right-hand part of Fig. 7. At the beginning of stage I (No. 1) no particular scattering of the crystal orientation is seen, whereas in the middle of stage I (No. 3) a wide range of scattering of the local specimen axes is observed along the dotted line that is the direction of the ideal crystal rotation due to primary main slip. At strain No. 6 at the beginning of stage II, there is observed a marked transverse deviation of local specimen axes from the direction of the ideal rotation.

As discussed above, the deformation of a crystal in stage I proceeds as a whole in the direction of ideal rotation due to primary main slip but the rotation of the specimen axis advances and delays in the respective local regions. Such large scattering of local specimen axes, as seen in the middle of stage I, strongly suggests that the neighbouring local regions are connected by the bend-gliding region composed of the excess dislocations of such nature as predicted from the bend-gliding concept(14)(16)(18). It also indicates that in stage II where deformation bands develop and the secondary slip system starts to operate, the relationship characteristic of bend gliding collapses and a new rotational relationship takes place as seen in the case of No. 6 in Fig. 7.

2. Bend gliding and work hardening in stage I

The density of excess dislocations comprising the local bend-gliding region increases with the progress of deformation. This was concluded from the X-ray measurement of crystal rotation at various strains in stage I(16). Let $\Delta \xi$ be the parameter which is related to the curvature of the slip plane and is obtained by X-ray analysis from the angle of scattering between two adjacent local regions. Then, the density of excess dislocations, $\rho_{ex}$, comprising the local bend-gliding region can be given by(17)

$$\rho_{ex} = g \Delta \xi / b \sin \omega,$$

Fig. 6 Thompson's tetrahedron
where $b$ is the Burgers vector, and $g$ is a geometrical factor for the crystal orientation as given by the following equation

$$g = \frac{1 - \sin^2 \theta \cos^2 \eta}{\cos^2 \theta \sin \theta \cos \eta \sin \omega}.$$  

(3)

Here, angles $\theta$, $\eta$ and $\omega$ are the same as those in eq. (1). By measuring the value of $\Delta \tau$ with the progress of deformation, in a Cu-1 at% Ge alloy crystal, the relation between the flow stress in stage I, $\tau_1$, and the density of excess primary dislocations, $\rho_{ex}$, was found as $\tau_1 = \tau_0 + \mu \rho_{ex}$ \sqrt{\rho_{ex}}. 

(4)

where $\tau_0$ is the yield stress, $\mu$ the shear modulus, and $\alpha \approx 0.5$.

This relation implies that the stage-I work hardening is governed by the internal stress of the excess dislocations comprising the local bend-gliding region, and provides an important suggestion about the hardening mechanism of stage I that is still not well understood. Seeger et al. attributed the stage-I hardening to the stress required for the primary slip dislocations to pass each other, Hirsch et al. to the density of excess dislocations contained in multipole clusters of primary slip dislocations, and Basinski et al. to the dislocation density of secondary slip systems. Our results given by eq. (4) strongly support the idea of Hirsch et al., for we consider that the dipole and multipole clusters observed by electron microscopy are none other than the clusters of dislocations constituting the local bend-gliding region.

3. Formation of kink band

As discussed above, local bend-gliding regions are initiated by the inhomogeneous operation of primary slip under the constraint of uniaxial deformation. As the deformation proceeds in stage I, the density of excess dislocations accumulated in the bend-gliding region increases with increasing strain. Therefore the excess (mixed) dislocations tend to align on walls perpendicular to the primary slip plane, in the form of polygonization, so as to reduce the long-range stress field. The next and last stable configuration is a kink band wall into which the local bend-gliding region transmutes. In the course of building up the kink band wall, the screw component of excess dislocations in the bend-gliding region must be removed, because the kink wall consists of only edge dislocations. Thus the stabilization process of excess dislocations in the local bend-gliding region can be described by the progression as

Random Accumulation \downarrow

Polyagonization \downarrow

Kink Band Wall.

(5)

Figure 8 shows the etch pit structure in early stage II of a nearly perfect copper crystal deformed to $\gamma = 12.5\%$\textsuperscript{(23)}. The arrow at upper left indicates the slip direction $S_{11}$. From this figure are observed various features of the dislocation structure, e.g. not only the development of local bend-gliding regions but also the in-situ transmutation of local bend-gliding regions into micro kink bands.

To confirm crystallographically the in-situ transmutation of the local bend-gliding region into the kink band, the change in the bending axis of the primary slip plane was measured by X-ray as shown in Fig. 9\textsuperscript{(23)}. To minimize the ambiguity of the analysis, a huge bend-gliding region near the grip end was selected as the area of measurement, in a Cu-0.5 at% Ge alloy crystal oriented near the [321] orientation. The huge bend-gliding region at $\gamma = 13.6\%$ is
shown in Fig. 9(a), and at a higher strain ($\gamma=19.8\%$) the very sharp and large kink band was formed from that bend-gliding region as shown in Fig. 9(b). These micrographs are the etch-pit structure on the cross plane, and clearly indicate the same place as is seen from the persisting subboundaries marked by the white dots. The results of X-ray analysis are given in Fig. 9(c). The full circle labelled BA (bending axis) marks the theoretical position of the rotation axis in the bend-gliding region and is determined as illustrated in the left-hand diagram. The theoretical bending axis of the kink band is signified by the symbol KA (kink axis), i.e., the $[12\overline{1}]$ direction. The open circle near BA marks the measured position for Fig. 9(a) while the open circle close to KA corresponds to the condition of Fig. 9(b). Although the experimental accuracy is not so high, it is clear that the bending axis was altered from BA to KA on transmutation of the bend-gliding (polygonized) region into the kink band.

As the dislocation density in the bend-gliding region increases, the excess dislocations are rearranged in the form of polygonization to reduce their internal stresses. The stress field set up by the edge component of the excess dislocations in the polygonization process disappear exponentially with distance so as to reduce the strain energy and to stabilize the dislocation structure. However, the long-range stress field by the screw component of the excess dislocations does not disappear exponentially. If the coordinates are such that plane $xz$ is the slip plane and plane $yz$ is the polygonization wall and the dislocation spacing is
denoted by $h$, the stress field due to the screw component for $r > h/2\pi$ is given by

$$\sigma_{xy} = \mu b_s / 2h,$$

where $\mu$ is the shear modulus and $b_s$ is the screw component of the Burgers vector. The presence of such long-range stress field due to the screw component has an effect of preventing the excess dislocations from collapsing into a sharp wall.

As the deformation proceeds, the density of excess dislocations in the bend-gliding region eventually becomes so high that tilt walls of low energy, i.e. kink band walls are formed. This occurs through reorienting the excess dislocations from the mixed character into the pure edge type, resulting in the corresponding movement of the bending axis of the primary slip plane from BA to KA as illustrated in Fig. 9(c). Accordingly, the screw component which has been eliminated through the kink-band wall formation must be compensated by some additional stresses. The newly generated stresses are important in relation to the operation of secondary slip systems accompanying the formation of kink bands, and will be discussed later separately.

4. Media for formation of local bend-gliding region

As described above, the local bend-gliding region is formed from the positional inhomogeneity of the operation of primary main slip, under the constraint of uniaxial deformation. Two problems are involved here: One is the cause of the inhomogeneous operation of primary main slip and the other is the anchoring mechanism of dislocations composing the local bend-gliding region. The former will be discussed in detail in the next section in connection with the formation of coplanar slip zones and the latter is considered here.

Local bend-gliding regions may not act as efficient barriers against dislocation motion, because excess dislocations in the regions are merely statistically accumulated according to the curvature of the primary slip plane. The excess dislocations can easily move under external forces, and as can be understood from the dislocation arrangement in Fig. 4(b), many of the positive and negative dislocations may mutually annihilate when unloaded. The structure, which should be unstable from the standpoint of mechanics, is observed in reality to be stable as revealed by the etch-pit technique. This suggests that the dislocations in the excess dislocation clusters comprising the bend-gliding region are trapped by the interaction with some obstacles. Such interaction was actually observed\(^{(18)(23)}\): For less perfect crystals with many subboundaries, huge local bend gliding regions are formed because of the frequent interaction of the excess dislocations with subboundaries, and in nearly perfect crystals, small bend-gliding regions are formed due to the interaction of the excess dislocations with the minority secondary slip that operated in the pre-yield stage\(^{(24)(25)}\). In this connection, it should be noted that the extent of stage I is much reduced by the presence of grown-in subboundaries\(^{(16)(23)}\).

V. Coplanar Slip Zone and Band of Secondary Slip

1. Formation of primary coplanar slip zone

In the preceding section, the formation of kink bands or the first type of deformation bands by the in-situ transmutation of local bend-gliding regions and its leading role in the transition from stage I to II have been discussed. Regarding the second type of deformation bands or bands of secondary slip (BSS) that appear in stage II (see Fig. 2(b)), little is known about their nature and origin except for the characteristics revealed by optical microscopy that is a wide region almost parallel to the primary slip plane where the secondary slip actively operates while the operation of the primary slip is less active.

If we take a view of the origin of deformation bands ‘BSS’ that the presence of BSS is recognized for the first time after the operation of the secondary slip system, some presage of BSS should be apparent in stage I. This idea is the same as the concept that the local bend-gliding region was already present in stage I as the precursory phenomenon of the kink band.
In fact, the precursory phenomenon of BSS was discovered, and that is the primary coplanar slip zone (23).

Figure 10 is the etch-pit structure of a Cu-1 at% Ge alloy crystal oriented near [321], which was deformed to the transition stage from stage I to II ($\gamma = 36.8\%$) at room temperature. The observation surface is the cross plane containing the primary main slip direction $S_{11}$. Broad and light bands parallel to the direction $S_{11}$ indicated by the arrow are the region in question. In the figure, the transmutation of the local bend-gliding region into the kink band is also observed. The region in question appears bright because the dislocation density of $S_{11}$ is lower than that in the surrounding dark region.

The formation of this region is affected by the crystal orientation and the crystal perfection. The common feature is such that, besides primary main slip $S_{11}$, primary coplanar slips $S_{12}$ and $S_{13}$ operate in approximately the same amount in this region (23). For this reason, we named the region as the coplanar slip zone. This is a typical manifestation of the inhomogeneous operation of primary main slip and is a precursor of BSS in stage I.

2. Characteristics of coplanar slip zone

To ascertain that the primary coplanar slip is operative in this region, the etch-pit observation was made on the {111} plane on which the Burgers vector of gliding dislocations can be easily identified. In Fig. 10, for example, the cross plane was chosen to investigate the operation of $S_{11}$. To find the operation of $S_{13}$ with the second largest Schmid factor in a crystal oriented near [321], the etch-pit observation was made on the conjugate plane on which the Burgers vector of $S_{13}$ is lying (see Table 1 and Fig. 6). The result is shown in Fig. 11, which exhibits the dislocation structure of a Cu-1 at% Ge alloy crystal deformed to the latter half of stage I ($\gamma = 30\%$). Although the specimens are different, the dark band in Fig. 11 corresponds to the light band in Fig. 10. Namely, it can be understood that $S_{13}$ was operative in the region where the operation of $S_{11}$ was weak. Figure 12, which is the enlargement of a part of Fig. 11, shows that the etch pits of $S_{13}$ are large and deep because its Burgers vector is lying on the observation surface, whereas the etch pits of $S_{11}$ whose Burgers vector is inclined to the surface are small and shallow. It is partly for this reason that dark and light regions parallel to the primary slip plane are observed. It is clear that the dislocations of main slip $S_{11}$ also exist in a large number in the coplanar slip zone.
If only $S_{11}$ and $S_{13}$ are operative in the coplanar slip zone, the direction of rotation of the specimen axis in the local region should deviate from the direction of ideal rotation due to the primary main slip $S_{11}$ alone. Yet, such a deviation has never been observed in stage I, as shown in Fig. 7. This strongly suggests that not only the primary coplanar slip $S_{13}$ but also the other coplanar slip $S_{12}$ must be operating in almost equal amounts in the same coplanar slip zone developed in stage I, in spite of the fact that their Schmid factors are largely different.

Figure 13 provides the direct evidence for this\(^{23}\). This figure demonstrates excellent matching of the coplanar slip zones as revealed by the etch-pit technique on the conjugate and critical planes of a Cu–1 at%Ge alloy crystal whose initial specimen axis lies near [941], at which the Schmid factor of $S_{11}$ is nearly 0.5. This specimen was deformed to later stage I ($\gamma = 44\%$) at 77 K. The simultaneous operation of $S_{13}$ and $S_{12}$ is clearly understood from the corresponding etch-pit zones observed on both sectional planes. Since the primary main slip $S_{11}$ was also operating in every part of the specimen in early stage I (see Fig. 5(b)), it is obvious that dislocations of all primary slip systems, i.e. $S_{11}$, $S_{12}$ and $S_{13}$ coexist in the same coplanar slip zone.

Thus, the primary coplanar slip zone is a characteristic region where coplanar slips $S_{13}$ and $S_{12}$ operate in almost the same amount in addition to main slip $S_{11}$, and is formed in stage I. No operation of the secondary slip system is observed in the zone until the onset of stage II; when the deformation proceeds from stage I to II, the secondary slip is induced for the first time in the zone and the coplanar slip zone directly transmutes into BSS, i.e. the band of secondary slip.

Figure 14 is a scanning electron micrograph that depicts an example of a band of secondary slip. Fine conjugate slip lines and coarse cross slip lines are observed within this deformation band. The arrow indicates the slip direction $S_{11}$ and the specimen is a Cu–1 at%Ge alloy crystal deformed to stage II ($\gamma = 93\%$) at 77 K.

### 3. Origin of coplanar slip zone

A typical manifestation of the inhomogeneous operation of primary main slip is the development of coplanar slip zone as noted above. There are at least three factors responsible for the formation of the coplanar slip zone. One factor is the local decline in the
S11 operation; primary coplanar slip is triggered in regions where the continuous operation of S11 is impeded by the interaction with grown-in subboundaries and/or with the minority secondary slip that operated in the pre-yield stage in nearly perfect crystals. Since the operation of the secondary slip system on pre-yielding is affected by the crystal orientation, a noticeable orientation dependence is expected to occur in the development of the coplanar slip zone.

The second factor is the stress required for the passage of primary coplanar dislocations S12 or S13 through a local region where main dislocations S11 are accumulated by some interaction stated above. This stress is smaller for S12 or S13 than the stress for the passage of S11 dislocations, because the interaction force f_{CM} between coplanar and main dislocations is given by

$$f_{CM} \approx \frac{1}{2} f_{MM},$$

where f_{MM} is the interaction between S" dislocations. Thus the coplanar slip plays an important role in the relaxation of internal stresses in local regions where main dislocations S11 tend to accumulate.

The third factor is the equivalent operation of three kinds of slip S11, S12 and S13 in the same coplanar slip zone, in spite of the fact that their Schmid factors are very different. This may be understood in terms of dislocation configuration with a low self-energy; the coplanar slip zone tends to have a structure similar to a kind of twist boundary so as to reduce the long-range stress field. In this connection, we invoke the analysis of small-angle dislocation boundaries in terms of Frank's formula given by

$$B = \theta (\mathbf{u} \times \mathbf{v}).$$

Here, \(\mathbf{v}\) is any arbitrary unit vector lying in the plane of the boundary, \(\mathbf{u}\) is a unit vector parallel to the axis of relative rotation, \(\theta\) is the rotation angle, and \(\mathbf{B}\) is the vector sum of the Burgers vectors of all dislocations intersected by \(\mathbf{v}\). The small-angle dislocation boundary which satisfies this formula has no long-range stress field and thus has a low self-energy.

The actual coplanar slip zone may not satisfy eq. (8) in a strict sense. However, it should be kept in mind that twist boundaries are often observed in stage II by transmission electron microscopy in the so-called "carpet" structure, which can be formed by the coplanar slip zone and its transmutative band of secondary slip as will be discussed later. Thus it seems that the coplanar slip zone approximates the twist boundary and has the dislocation configuration with a low energy.

VI. Unsolved Problems in the Stage-II Hardening

At the transition from stage I to II, two noticeable structure changes on a macroscopic scale have been commonly accepted, i.e. (a) the onset of activity on secondary slip systems and (b) a marked reduction in the slip line length. Another important phenomenon in stage II is, (c) the development of dislocation cell structures. The origin of these three structural changes, however, has not been identified with any certainty. The following will clarify these phenomena.

1. Operation of secondary slip systems

The operation of secondary slip systems is not observed at all in stage I except the pre-yield stage, and becomes pronounced for the first time in stage II. On one hand, as described in Section IV, the transition from stage I to II is triggered by the formation of kink bands. Therefore, a close causal relation is naturally expected between the formation of kink bands and the sudden operation of secondary slip.

\[\text{Recently, Nabarro has suggested the rigid glide of the twist boundary, based on the characteristics of the coplanar slip zone we found. According to his theorem, if the material on the upper side of a glide plane is rotated about the normal to the plane with respect to that on the lower side by angle } \theta, \text{ and this rotation is accommodated by a dislocation network, then when this network glides rigidly with velocity } u, \text{ the material on the upper side of the glide plane glides relative to that on the lower side with velocity } v= u \times \theta. \text{ The glide behaviour of the coplanar slip zone described in the text can be understood by this rigid glide of the twist boundary.}\]
Figure 15 provides direct evidence for this. The specimen was a pure copper crystal oriented near [321], and deformed by different amounts of strain up to early stage II as \( \gamma = 2.2 \), 5.0 and 8.3\% in Figs. 15(a), (b) and (c), respectively. After the deformation, the specimen was chemically sectioned and forest dislocations were observed on the primary slip plane. It is to be noted that, on transition from stage I to II at \( \gamma = 5.0\% \) in Fig. 15(b), sharp arrays of forest dislocations appeared in the [12\( \bar{1} \)] direction along the wall of kink bands. In early stage II at \( \gamma = 8.3\% \) in Fig. 15(c), a rapid increase in the forest dislocation density was observed not only in the neighbourhood of kink bands but also in the matrix region between kink bands.

A detailed analysis of Fig. 15(c) indicates that the dislocations induced at the kink band are surprisingly the cross slip dislocations \( S_{42} \) whose Schmid factor is zero\( ^{(23)} \). This strongly suggests the presence of the stress concentration due to the \( S_{11} \) dislocation clusters piled-up against the wall of kink band. When calculated by a simple pile-up model, the secondary slip systems such as \( S_{31}, S_{21}, S_{33} \) and \( S_{42} \) are subjected to fairly large internal stresses due to the stress concentration in the respective order. In fact, the operation of the critical and conjugate slip systems was confirmed in crystals of this orientation. Thus, the formation of the kink band that governs the transition from stage I to II brings about the pile up of \( S_{11} \) dislocations and triggers the operation of secondary slip systems.

Another factor for the operation of secondary slip system in stage II is the generation of the twisting moment, expected to occur when the bend-gliding region transmute into the kink band. The twisting moment is produced when the screw component of the excess dislocations that constitute the bend-gliding region is eliminated with the formation of the kink band. Figure 16 illustrates an example of stress analysis by the finite element method, where the distribution of twisting moment around the axis normal to the primary slip plane is plotted along the specimen axis (horizontal axis). The magnitude of this mo-

![Fig. 15 Initiation of secondary slip at kink bands](image1)

![Fig. 16 Twisting moment induced by formation of kink band](image2)
ment depends on the crystal orientation as can be understood from eq. (1), and increases with increasing the amount of the screw component in the excess dislocations composing the bend-gliding region. This generation of the twisting moment contributes to the induction of secondary slip systems, and plays a particularly important role in the transmutation of the coplanar slip zone into the band of secondary slip.

The twisting moment newly generated with the formation of kink band has a large resolved shear stress component on the coplanar slip system $S_{12}$ or $S_{13}$. As a result, unequal but extensive operation of $S_{12}$ and/or $S_{13}$ is expected to occur in stage II in the coplanar slip zone, where, in stage I, $S_{12}$ and $S_{13}$ operated in almost same amount in addition to $S_{11}$. Actually, such unequal operation of coplanar slip has been confirmed in stage II by scanning electron microscopy and optical interferometry, particularly in the neighbourhood of the boundaries of bands of secondary slip. The unequal operation of coplanar slip systems gives rise to long-range stress fields which would induce the activation of secondary slip in the coplanar slip zone.

Thus the coplanar slip zone changes into the band of secondary slip at the time when the kink band wall is built up from the bend-gliding region, i.e. at the transition from stage I to II, and also in the course of stage II during which new kink bands are continuously formed.

In the above, the activation of secondary slip at the kink band and in the band of secondary slip has been discussed. The remaining problem is the multiplication of secondary slip dislocations in the matrix region between the kink bands as shown in Fig. 15(c). The random distribution of these dislocations seems to validate the mechanism suggested by Kuhlmann-Wilsdorf and Comins, who assume a Taylor-type lattice composed of positive and negative edge dislocations in the matrix region between kink bands, and propose that secondary slip is induced by shear stresses generated in the lattice. This is the third factor for the operation of secondary slip in stage II.

2. Change in the length of primary slip lines

On transition from stage I to II, a marked decrease in the length of primary slip lines is observed. As pointed out by Hirsch, no satisfactory explanation has been provided yet for this phenomenon. Considering that stage I terminates with the formation of kink bands and also that primary main slip is constrained by the kink band walls, it is clear that the sharp reduction in the slip line length in stage II is closely related to the formation of kink bands.

The etch-pit observation shows that the spacing between kink bands is reduced with increasing strain, because of the development of new kink bands in between previously formed kink bands which remain unchanged. Figure 17 exhibits the relationship between the mean kink-band spacing ($L$) and the shear strain ($\gamma$) in early stage II of a nearly perfect copper crystal oriented near $[321]$. The following equation is derived from this figure:

$$L = \lambda/(\gamma - \gamma_{II}),$$

where $\lambda$ is a constant that is approximately $1.1 \times 10^{-5} \text{m}$, and $\gamma_{II}$ is the extent of stage I. This equation is of the very same form as the relationship for slip line lengths by Mader.
and Seeger(33), and even the value of $A$ is quite close to their value of $0.8 \times 10^{-5}$ m.

The above discussion strongly suggests that the decrease in the slip line length in stage II is caused by the decrease in the spacing between kink band walls which act as barriers for primary slip. In other words, the number of kink bands increases linearly with strain in stage II through the transmutation from local bend-gliding regions newly formed in stage II. An equation similar to eq. (9) has also been obtained in the case of bands of secondary slip(23).

3. Dislocation cell structures

As described in the previous section, the number of kink bands and bands of secondary slip increases with the progress of deformation in stage II. This leads to the development of the well-defined etch pit structure as shown in Fig. 18, which consists of micro kink bands perpendicular to the primary slip direction and bands of secondary slip approximately parallel to the primary plane. The specimen is a nearly perfect copper crystal oriented near [321], which was deformed to early stage II ($\gamma = 9.1\%$).

In order to obtain a more clearly defined structure by eliminating redundant dislocations, specimens deformed up to early stage II were annealed at a suitable temperature in vacuum under constant load. Figure 19 shows a structure stabilized thus in a Cu–1 at% Ge alloy crystal oriented near [321] that was deformed to early stage II ($\gamma = 40\%$). This structure is composed of etch pit arrays elongated along two directions; one is perpendicular to the direction of primary main slip and the other is parallel to the primary plane.

Fig. 18 Dislocation cell structure of as-deformed specimen observed by etch-pit technique

Fig. 19 Dislocation cell structure of specimen subjected to annealing after deformation.

The arrow in each figure indicates the direction of $S_{11}$.

As has been clarified by many investigations using transmission electron microscopy, the most characteristic dislocation structure in stage II is the cell structure(34). Many investigators have shown that the dislocation structure characteristic for later stage II consists of carpets or mats of dislocations approximately parallel to the primary plane. The presence of multipole walls perpendicular to the primary slip direction was also pointed out by a careful observation(29).

The problem is the correspondence between the cell structure observed with transmission electron microscopy and the etch pit structure as shown in Fig. 18. The observation made by Wilkens(34) for copper crystals with the X-ray topography has a decisive meaning in this respect. Besides the layer-like structure roughly parallel to the primary slip plane as reported by many workers, he found dislocation walls with lateral dimensions of a few hundred micrometers perpendicular to the primary slip direction. He also found that these walls are accompanied on either side by an extended region with an excess density of primary edge dislocations of the same sign. These results are in excellent agreement with our observations on micro kink bands.

Regarding the carpet structure parallel to the primary slip plane, Steeds(28) found in copper crystals that some misoriented regions of the carpet structure are bounded by twist boundary carpets composed of dislocations with all the Burgers vectors belonging to the primary slip system. Karashima et al.(29) also reported the presence of twist boundaries parallel to the primary slip plane, in the substructure devel-
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oped during creep deformation of copper crystals. These observations clearly show that the carpet structure corresponds to the coplanar slip zone and/or the band of secondary slip where $S_{11}$ coexists with $S_{12}$ and $S_{13}$.

It may be concluded from the above discussion that the framework of the dislocation cell structure whose origin has not been clarified yet, is composed of kink bands and bands of secondary slip.

**VII. Conclusions**

The author has discussed above that the work hardening mechanism of fcc crystals can be understood in a unified manner, from the standpoint that the accumulation of dislocations governing the internal stress forms two types of deformation bands; the kink band that is a wall perpendicular to the primary slip direction and the band of secondary slip that is almost parallel to the primary slip plane. Particular attention has been focused on the process of transition from stage I to II where the make-up of the framework of the dislocation structure characteristic of stage II is most clearly revealed. It has been clarified that the transition from stage I to II is governed by the transmutation of the local bend-gliding region into the kink band.

The extent of stage I is affected by such factors as deformation temperature, stacking fault energy of the solute, solute concentration, crystal perfection, crystal orientation, specimen size and the cross-sectional shape. As was already pointed out, the effect of these factors can be explained without contradiction, by considering the ease with which the local bend-gliding region transmute into the kink band. From all the results we obtained, the formation process of deformation bands and the related phenomena such as the dislocation structure may be correlated as illustrated in Fig. 20. The outline of this diagram is explained as in the following.

With the progress of deformation in stage I, the activity of primary main slip $S_{11}$ becomes diminished in some local regions of specimen as the $S_{11}$ dislocations interact with subboundaries in imperfect crystals, or with secondary slip dislocations operated at pre-yielding in nearly perfect crystals. The deformation process in these local regions is complemented by the equal operation of the primary coplanar slip $S_{12}$ and $S_{13}$, leading to the formation of the primary coplanar slip zone that consists of low-energy dislocation configurations similar to twist boundaries. This coplanar slip zone is a typical representation of the inhomogeneity of the primary slip operation.

Then, the local bend-gliding region is formed as a result of the inhomogeneous operation of $S_{11}$ under the constraint of uniaxial deformation. The excess dislocations that constitute the bend-gliding region consist of not only edge but also screw components. The dislocation density in the local bend-gliding region increases with the progress of deformation, and, to relax the long-range stress field of randomly accumulated dislocations, the polygonization occurs and eventually the kink band is formed. Once the kink band is built up, a rapid pile-up of primary dislocations against the kink band wall is brought about, accompanying the sudden operation of secondary slip systems induced by the stress concentration due to the pile-up. Thus the transition from stage I to II occurs. It is to be noted that grown-in subboundaries and the minority secondary slip dislocations operated in the pre-yield stage contribute to the anchoring of dislocations constituting the local bend-gliding

Fig. 20 Formation process of deformation bands and related phenomena.
region that is otherwise unstable mechanically.

The kink band is the wall composed of pure edge dislocations, so that the screw component must be eliminated when the local bend gliding region which contains screw dislocations transmutes into the kink band. This action produces the twisting moment, activates the unequal operation of primary coplanar slip $S_{12}$ and $S_{13}$ in the coplanar slip zone, induces the secondary slip by its long-range stress field, and eventually leads to the in-situ transmutation of the coplanar slip zone into the band of secondary slip.

From the point of view on the basis of the deformation-band formation, the unsolved problems in stage II, i.e., (1) sudden operation of secondary slip systems, (2) marked reduction in the slip line length, and (3) formation of dislocation cell structures, can be comprehended totally.

The development of deformation bands is generally observed when there are inhomogeneities of slip operation under the constraint of deformation, so that this mechanism is also expected to operate for any type of crystals as well as fcc crystals. The deformation bands should also form under the constraint of deformation as in rolling and are considered to play an important role in controlling not only deformation textures but also recrystallization textures which develop through the nucleation at the region of deformation band.

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