Vibration analysis of milling process with helix-angle cutter based on Euler method

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Abstract. The chatter vibration is the major limiting factor in increasing the metal removal rates of the machine tools. In this paper, a program for dynamic analysis of thin-walled milling process with helical milling cutter is developed, based on Euler iteration. The program can take into account the characteristics of thin-walled workpiece, the contact separation between workpiece and cutter, the helical angle of cutter teeth and other actual cutting process characteristics. Based on the stability lobe diagram obtained by semi-discrete method, the stability simulation in different cutting parameters around the stable island was carried out. The unstable form of the cutting process was determined by spectrum analysis and Poincare mapping method. The results show that the method is in good agreement with the results of stability Lobe diagram and can capture the stability island in small radial cutting process. This method can well grasp the cutting force and vibration evolution process in cutting process, and provide an effective means for stability analysis of cutting process.

1. Introduction
Chatter in milling results in low machining quality, poor accuracy and surface finish, unpleasant noise and sound, as well as accelerated tool wear, aside from damaging the cutting spindle and machined part. Chatter has generally been analyzed and modeled in the frequency [1-2] or time domain [3-6]. Such as, the zeroth-order approximation method developed Altintas and Budak [1], the multi-frequency solution suggested by Merdol et al. [2], the semi-discretization method (SDM) proposed by Insperger and Stepan [3], and the temporal finite element analysis provided by Bayly et al. [4]. More recently, the full-discretization method [5-7] was used to efficiently obtaining stability diagrams.

In this paper, Euler method is used to research the milling stability with helix angle cutter. The work of this paper is organized as follows. Firstly, one milling dynamic model considering an integral model of cutting force is introduced. Then, Euler method is used to obtain the vibration message in the milling process and is utilized to analyze the stability island caused by helix angle in small radial depth of cut. The final section describes the conclusions from this work.

2. Cutting force modeling
Compared with solid parts, the stiffness thin-walled parts is relatively small in the direction of workpiece thickness. For this direction, its dynamic equation can be expressed as

\[ m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t) \]
where \( m \) is modal mass, \( c \) is modal damping, \( k \) is modal stiffness. These parameters can be obtained by hammering experiment. \( F(t) \) is the cutting force. If the effect of sectional characteristics (see figure 1) of milling with helical cutters on cutting force\[6,7\], \( F(t) \) can be obtained by

\[
F(t) = -\sum_{j=1}^{N} G(\phi_j(t, z)) \frac{R}{\tan \beta} \int_{\phi_j(0)}^{\phi_j(t)} (K_1(t)(y(t) - y(t - \tau)) + K_2(t)) d\phi
\]

(2)

where \( K_1(t) = k_c s^2 - k_c s^2 \), \( K_2(t) = k_s f_s c - k_s f_s c - k_c c \) \( (c = \cos \phi_j, s = \sin \phi_j) \)

\[ \phi_0(t) = \max(\phi_j(t, a_z), \phi_a), \phi_j(t) = \min(\phi_j(t, 0), \phi_a) \]

Figure 1. Schematic diagram of milling process.

3. Method based on Euler’s method

3.1 Simulation method

Euler method is one of the numerical solutions of ordinary differential equations. In this paper, the cutting force signals are calculated at several times in each cutting cycle, and then solved according to the milling dynamics formula. In this way, the milling vibration signals are obtained by iterating all the times one by one. The specific model is shown in figure 2.

Its process is as follows:

1. Cyclic variables are set to 1, while acceleration, velocity and displacement are all set to 0.
2. \( R, Kt, Kr, m, c, K \) and other basic parameters are inputted.
3. \( F_j \) is calculated.
4. Acceleration is calculated based on \( \ddot{y}_j = (F_j - c\dot{y}_{j-1} - ky_{j-1}) / m \), speed is calculated based on \( \dot{y}_j = \dot{y}_{j-1} + \ddot{y}_j \Delta t \), while displacement is calculated based on \( y_j = y_{j-1} + \dot{y}_j \Delta t \).
5. Judge whether \( j \geq H \) is valid. If it is valid, the program ends. If not, cycle number plus 1 and iterate continues.
3.2 Method validation

Figures 3 shows the stability lobe diagram used the SDM [3] in the cutting process under the condition: tool radius 9.525mm, the number of teeth 3, cutter helix angle 30°, radial depth ratio 5.25 %, feed per tooth $1.78 \times 10^{-4}$ m/s. Because the calculation result in this graph has been proved by experiments in the literature [6], it can be taken as a reference to examine the accuracy of the calculation results of the Euler method proposed in this paper. Eight points are selected around the stability island near the third lobe (see figures 3) and their cutting parameters are shown in Table 1. The corresponding results are shown in figures 4-9 as well as their spectrum diagrams and Poincare maps.

![Figure 3 stability lobe diagram and the selected cutting points](image)

| Points | Radial depth of cut ratio $a_D$ | Axial depth of cut $a_p$ /mm | Speed $n$/(r/min) |
|--------|-------------------------------|-------------------------------|------------------|
| A      | 0.0525                        | 9.1                           | 2200             |
| B      | 0.0525                        | 8.0                           | 2170             |
| C      | 0.0525                        | 8.0                           | 2200             |
Consequently, the stability states corresponding to above eight cases are as following (it should be noted that the results corresponding to cases E, G and H are not given due to space limitation):

1) Case A and B: two stable processes since the 1/tooth passage displacement samples approached a single fixed point value (see figures 4b and 5b) and there are only tooth passing frequency and its harmonics (see figures 4c and 5c).

2) Cases C and F: two unstable processes. They are characterized by two points in the Poincare section (see Figures 6b and 8b), i.e., so-called the period-two while their vibration histories are divergent (see Figures 6a and 8a). Meanwhile, it can be seen from the figures 6c and 8c that the large amplitude vibrations occur at the chatter frequency of 165Hz.

3) Cases D: one stable process, obviously.
In this paper, a program for dynamic analysis of thin-walled milling process with helical milling cutter is developed, based on Euler iteration. The program can take into account the characteristics of thin-walled workpiece, the contact separation between workpiece and cutter, the helical angle of cutter teeth and other actual cutting process characteristics. The stability simulation in different cutting parameters around the stable island by was semi-discrete method carried out. The unstable form of the cutting process was determined by spectrum analysis and Poincare mapping method. The comparing results show that the proposed method is in good agreement with the results by theoretical calculation. So, it can capture the stability island in small radial cutting process. Considered its good ability in grasping the cutting force and vibration evolution process in cutting process, it can provide one effective means for stability analysis of cutting process.

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**Reference**
[1] Altintas Y, Engin S and Budak E 1999 *J. Manuf. Sci. Eng* **121** 173-178.
[2] Merdol SD and Altintas Y J 2004 *J. Manuf. Sci. Eng* **126** 459–466.
[3] Budak E 2003 *J. Manuf. Sci. Eng* **125** 29-34.
[4] Bayly PV, Halley JE and Mann BP 2003 *J. Manuf. Sci. Eng* **125** 220–225.
[5] Ding Y, Zhu L, Zhang X and Ding H 2010 *Int. J. Mach. Tools. Manuf* **50** 502 – 509.
[6] Jin G, Qi HJ and Cai YJ 2016 *Math. Method. Appl. Sci* **39**(4) 949–958.
[7] Jin G, Qi HJ, Li ZJ and Han JX 2018 *Commun. Nonlinear. Sci* **63** 38-56.