BeyondPlanck XIV. Polarized foreground emission between 30 and 70 GHz

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ABSTRACT

We constrain polarized foreground emission between 30 and 70 GHz with the Planck Low Frequency Instrument (LFI) and WMAP data within the framework of BeyondPlanck global Bayesian analysis. We combine for the first time full-resolution Planck LFI time-ordered data with low-resolution WMAP sky maps at 33, 40 and 61 GHz. Spectral parameters are fit with a likelihood defined at the native resolution of each frequency channel. This analysis represents the first implementation of true multi-resolution component separation applied to CMB observations for both amplitude and spectral energy distribution (SED) parameters. For synchrotron emission, we approximate the SED as a power-law in frequency and find that the low signal-to-noise ratio of the data set strongly constrains the number of free parameters that may be robustly constrained. We partition the sky into four large disjoint regions (High Latitude; Galactic Spur; Galactic Plane; and Galactic Center), each associated with its own power-law index. We find that the High Latitude region is prior-dominated, while the Galactic Center region is contaminated by residual instrumental systematics. The two remaining regions appear to be both signal-dominated and clean of systematics, and for these we derive spectral indices of $\beta_{\text{Spur}} = -3.15 \pm 0.07$ and $\beta_{\text{Planck}} = -3.12 \pm 0.06$. This agrees qualitatively with the WMAP-only polarization constraints presented by Dunkley et al. (2009), but contrasts with several temperature-based analyses. For thermal dust emission we assume a modified blackbody with a mean temperature of $T_{\text{CMB}} = 2.7255 \text{K}$ (Fixsen 2009). As such, its peak intensity occurs at 161 GHz, and the primary frequency range considered by most CMB experiments is therefore around 30 to 300 GHz. In addition to the CMB, a diverse range of astrophysical emission mechanisms contribute to the observed signal at these frequencies, both of Galactic and extragalactic origin. For intensity, the main contributors are synchrotron, free-free, CO, thermal dust, and anomalous microwave emission, while for polarization, synchrotron and thermal dust emission (e.g., Bennett et al. 2013; Planck Collaboration IV 2020, and references therein).

Key words. ISM: general – Cosmology: observations, polarization, cosmic microwave background, diffuse radiation – Galaxy: general

1. Introduction

One of the most important sources of information about the early universe is the cosmic microwave background (CMB; Penzias & Wilson 1965). By mapping and characterizing the statistical properties of this signal, cosmologists have during the last few decades constrained both the composition and evolution of the universe to percent accuracy (e.g., Bennett et al. 2013; Planck Collaboration I 2020).

As shown by the COBE-FIRAS experiment (Mather et al. 1994), the frequency spectrum of the CMB may to a very high precision be described in terms of a blackbody with a mean temperature of $T_{\text{CMB}} = 2.7255 \text{K}$ (Fixsen 2009). As such, its peak intensity occurs at 161 GHz, and the primary frequency range considered by most CMB experiments is therefore around 30 to 300 GHz. In addition to the CMB, a diverse range of astrophysical emission mechanisms contribute to the observed signal at these frequencies, both of Galactic and extragalactic origin. For intensity, the main contributors are synchrotron, free-free, CO, thermal dust, and anomalous microwave emission, while for polarization, synchrotron and thermal dust emission (e.g., Bennett et al. 2013; Planck Collaboration IV 2020, and references therein).
At the foreground minimum, around 80 GHz, the CMB anisotropies dominate over the combined foreground amplitude over most of the sky (Planck Collaboration Int. XLVI 2016), and CMB temperature extraction is therefore relatively straightforward. The same does not hold true for polarization, even at the foreground minimum, the sum of synchrotron and thermal dust emission is greater than the polarized CMB by almost a full order of magnitude on large angular scales. Polarized foreground estimation therefore plays a critically important role in contemporary cosmology, as such observations may contain signatures of primordial gravitational waves created during the Big Bang (e.g., Kamionkowski & Kovetz 2016, and references therein), and thereby provide a unique observational window on inflation and physics at the Planck energy scale.

However, the amplitude of the primordial gravitational wave signal is expected to be smaller than 10–100 nK on large angular scales (Tristram et al. 2020). This, combined with a plethora of confounding instrumental effects, such as temperature-to-polarization leakage (Svalheim et al. 2020), correlated noise (Ilie et al. 2020) and calibration uncertainties (Gjerløw et al. 2020), makes high-precision CMB polarization science a particularly difficult challenge. Furthermore, as summarized by the Planck team in a document called “Lessons learned from Planck”, the quality of current state-of-the-art CMB observations is limited by the interplay between instrumental and foreground effects. This insight formed the basis for the BeyondPlanck project (BeyondPlanck Collaboration 2020), which aims to implement an end-to-end Bayesian CMB analysis framework that jointly accounts for both systematic effects and astrophysical foregrounds, starting from raw time-ordered data. This framework employs an explicit parametric model that accounts jointly for cosmological, astrophysical, and instrumental parameters. These parameters are sampled with Markov Chain Monte Carlo, such as Gibbs sampling, as implemented in the Commander software. (Eriksen et al. 2004a, 2008; Galloway et al. 2020).

The BeyondPlanck results are described in a suite of 18 companion papers (see BeyondPlanck Collaboration 2020, and references therein), each focusing on a particular aspect of the analysis. The current paper focuses on polarized foreground characterization, both in terms of algorithms and results, with special attention paid to the spectral properties of polarized synchrotron and thermal dust emission. The current BeyondPlanck analysis considers only the Planck LFI observations in terms of time-ordered data, although selected preprocessed external data sets are also included to break critical degeneracies, specifically WMAP measurements between 33 and 61 GHz (Bennett et al. 2013). Planck HFI measurements at 353 and 857 GHz (Planck Collaboration I 2020; Planck Collaboration Int. LVII 2020), and the Haslam 408 MHz measurements (Haslam et al. 1982). Overall, the main emphasis of the present analysis lies on frequencies below the foreground minimum, between 30 and 70 GHz, and in particular on the spectral behavior of polarized synchrotron and thermal dust emission within this critically important frequency range. The present analysis is the first to combine high-resolution Planck measurements with low-resolution WMAP observations into a single coherent model, when estimating both foreground amplitudes and spectral parameters.

The rest of this paper is structured as follows: In Sect. 2, we briefly review the BeyondPlanck data model, focusing on the aspects that are relevant for polarized foreground analysis. In Sect. 3, we review the data sets that are included in the current analysis. In Sect. 4 we describe the basic algorithms, and connect these to the larger Gibbs sampling framework outlined in BeyondPlanck Collaboration (2020) and Andersen et al. (2020). Results are presented in Sect. 5, before we conclude in Sect. 6.

2. The BeyondPlanck data model

As described by BeyondPlanck Collaboration (2020), the main goal of the BeyondPlanck project is to perform end-to-end Bayesian CMB analysis, building on well-established statistical methods. The first step in any such Bayesian analysis is to write down an explicit parametric data model which then will be fit to observations through posterior mapping techniques. For BeyondPlanck, we adopt the following model for this purpose,

$$d_{ij} = g_{ij}P_{pp,j} \left[ B_{pp,j}^{\text{symm}} \sum_c M_c(\beta_{pp,j}, \Delta\nu_{pp,j})w_{pp,j} + B_{j}^{\text{asym}} (s^{\text{orb}} + s^{\text{fsl}}) \right] + n^{\text{corr}}_{ij} + n^{i}_{ij},$$

where \(j\) represents a radiometer (or detector) label, \(t\) indicates a single time sample, \(p\) denotes a single pixel on the sky, and \(c\) represents one single astrophysical signal component. Further, \(d_{ij}\) denotes the measured time-ordered data; \(g_{ij}\) denotes the instrumental gain; \(P_{pp,j}\) is a pointing matrix; \(B_{pp,j}\) denotes beam convolution; \(M_c(\beta_{pp,j}, \Delta\nu_{pp,j})\) denotes a foreground mixing matrix that depends on some set of spectral parameters, \(\beta\), and instrumental bandpass specification, \(\Delta\nu\); \(s^{\text{orb}}\) is the amplitude of astrophysical component \(c\) in pixel \(p\), measured at the same reference frequency as the mixing matrix \(M\), and expressed in brightness temperature units; \(s^{\text{fsl}}\) is the orbital CMB dipole signal; \(n^{\text{corr}}_{ij}\) denotes the contribution from far side-lobes; \(n^{i}_{ij}\) denotes correlated instrumental noise; and \(n^{i}_{ij}\) is uncorrelated instrumental noise with (diagonal) time-domain covariance matrix \(N^{i}_{ij}\). For further details regarding any of these objects, we refer the interested reader to BeyondPlanck Collaboration (2020).

The current paper focuses primarily on diffuse astrophysical foregrounds, which for practical purposes are stationary in time, and we are therefore not interested in the time-domain aspects of the model. For this reason, we rewrite Eq. (1) into the following compact form,

$$m_{v,p} = \left[ B_{pp}^{\text{symm}} \sum_c M_c(\beta_{pp}, \Delta\nu_{pp})w_{pp} \right] + n_{pp}^v,$$

where \(m_{v,p}\) is a binned sky map derived by co-adding all radiometer data within a single frequency channel, and we are for the moment conditioning on all time-domain parameters,

$$\left( \sum_{j,v} P_j(N_{ij}^v)^{-1} P_j \right) m_v = \sum_j P_j(N_{ij}^v)^{-1} r_j.$$

Here, \(r_j\) is the cleaned and calibrated time-ordered data for detector \(j\), as defined by Eq. (76) in BeyondPlanck Collaboration (2020), and full marginalization over time-domain parameters is done iteratively through Gibbs sampling; see Sect. 4.1 for further details.

In this paper, we are particularly interested in the total sky signal, which may be written as follows,

$$s = Ma \equiv \sum_{c=1}^{N_{\text{comp}}} a_c \int f_c(\nu; \beta) \tau(\nu) \, d\nu.$$

1 https://www.cosmos.esa.int/web/planck/lessons-learned
Here, $c$ again represents the astrophysical signal components, each of which is associated with an overall amplitude parameter $a$; a unit conversion factor $U$ for going from either thermodynamic or intensity units to brightness temperature; a spectral energy density, $f_{c}$, describing the intensity of the component relative to some reference frequency, $v_{0}$; and some set of spectral parameters, $\beta$, which characterize the frequency dependence of the various emission mechanisms. Finally, $r$ represents an instrumental bandpass response function that describes the detector sensitivity as a function of frequency. Again, we refer the interested reader to BeyondPlanck Collaboration (2020) for further details.

2.1. Polarized sky model

We assume that the polarized microwave sky may be well approximated by three physically distinct components; synchrotron emission, thermal dust emission, and CMB. A spinning dust, or anomalous microwave emission (AME) component is also commonly included in similar studies, however with an upper limit of its polarization fraction at <1% (Génova-Santos et al. 2016; Macellari et al. 2011), we regard it as negligible for this study. We do, however, direct the interested reader to Herman et al. (2020) for a study of polarized AME in a BeyondPlanck context.

Each component exhibits a distinctly different behavior both in terms of spatial structure and frequency dependence, and must be described by some unique set of spectral parameters, $\beta$. The choice of spectral parameters is both important and nontrivial. On the one hand, it is important that the adopted parametric model for each component is able to provide an acceptable goodness-of-fit, typically as measured by some $\chi^2$ statistic. On the other hand, it is also important that the model is not too flexible, as degeneracies between components may increase the effective noise level to arbitrary high levels. Generally speaking, degeneracies also tend to exacerbate instrumental systematic errors, by attempting to accommodate small signal discrepancies into the signal model. Choosing an appropriate parametric model is thus a delicate trade-off between allowing sufficient flexibility to model the real sky, while not introducing too many parameters, which otherwise may increase both systematic and statistical uncertainties to untenable levels.

For BeyondPlanck, we adopt the following parametric model for the polarized microwave sky,

$$s_{\nu} = a_{\text{CMB}} \frac{e^{x_{\nu}}}{(e^{x_{\nu}} - 1)^2} \frac{(e^{x_{\nu}} - 1)^2}{x_{\nu}^2}$$

$$+ a_{s} \left( \frac{v}{v_{0,s}} \right)^{\beta_s}$$

$$+ a_{d} \left( \frac{v}{v_{0,d}} \right)^{\beta_d + 1} \frac{e^{h\nu/kT_d}}{e^{h\nu/kT_d} - 1} - 1,$$

where all vectors are expressed in brightness (Rayleigh-Jeans) temperature units; $x = h\nu/kT_0$ (where $h$ and $k$ are Planck’s and Boltzmann’s constants respectively and $T_0$ is the CMB monopole of 2.7255 K; Fixsen 2009); and $v_{0,c}$ is the reference frequency for component $c$, $a$ is its amplitude, $\beta_s$ is the synchrotron power-law index, and $\beta_d$ and $T_d$ are the thermal dust power-law index and temperature, respectively. Even this minimal model, which adopts a power-law SED for synchrotron and modified blackbody SED for thermal dust, contains far too many parameters to be fit per pixel with the BeyondPlanck data set, and several informative priors will be imposed to regularize the system, as discussed in detail below.

2.1.1. Synchrotron emission

The first foreground component, and the brightest Galactic emission mechanism between 30 and 70 GHz, is synchrotron radiation. This emission is generated when relativistic cosmic-ray electrons ejected from supernovae gyrate in the Galactic magnetic field. From both temperature and polarization observations,
the synchrotron SED is known to be well approximated by a power-law over several decades in frequency (e.g., Lawson et al. 1987; Reich & Reich 1988; Platania, P. et al. 2003; Davies et al. 2006; Gold et al. 2009), at least from 1 GHz to 100 GHz, although some analyses also claim evidence for a slight spectral steepening towards higher frequencies (Kogut 2012; Jew et al. 2019). The GaLprop model (Orlando et al. 2018) is a physically motivated model that takes into account quantities such as the electron temperature ($T_e$) and large-scale magnetic field distributions, and this physical model also predicts SED flattening below 1 GHz depending on the model parameters. However, because the flattening occurs at frequencies well below 10 GHz, this is not relevant for the current analysis, which only considers frequencies above 30 GHz, and we therefore assume a straight power-law SED model for synchrotron emission in the following.

Next, several analyses have reported significant spatial variations in $\beta_s$ (e.g., Fuskeland et al. 2014; Krachmalnicoff et al. 2018; Fuskeland et al. 2019). In general, these analyses report flatter spectral indices (around $\beta_s = -2.8$) at low Galactic latitudes, and steeper at high Galactic latitudes (around $\beta_s = -3.1$). This picture appears roughly consistent with similar results derived from intensity observations (e.g., Vidal et al. 2014; Platania et al. 1998; Lawson et al. 1987; Reich & Reich 1988). On the other hand, when analyzing low-resolution WMAP polarization data with full correlated noise propagation, Dunkley et al. (2009) found only minor differences between low and high Galactic latitudes.

As shown in the following, the BEYONDPLANCK data combination has in general low statistical power to probe spatial variations. We therefore partition the sky into four large disjoint regions, as shown in the top panel of Fig. 1, and fit one spectral index for each region. For reference, the middle panel in this figure shows the Planck 2018 synchrotron amplitude map (Planck Collaboration IV 2020) with region outlines and a corresponding processing mask.

Since the spectral index is fit uniformly over large sky regions, it is important to impose a processing mask during the posterior evaluation to avoid pixels with poor goodness-of-fit from contaminating surrounding pixels. To this end, we construct individual spectral index processing masks for synchrotron by first fitting the component amplitudes given our chosen prior values for each spectral parameter. Using the resulting synchrotron amplitude map smoothed to 4$^\circ$ FWHM, we construct a mask by thresholding on the strongest 5% of the signal to remove the brightest part of the galactic plane. Next, we combine the mask with a $\chi^2$ map thresholded at the strongest 10% of the signal to remove the strongest point sources such as Tau A and a larger area around them. Finally, we use an un-smoothed $\chi^2$ map in order to remove particularly bright compact sources. All of these are visualized along with the region outlines applied to the Planck 2018 polarized synchrotron amplitude in the middle panel of Fig. 1.

The region set was defined as follows: We started from the 24-region partitioning defined by Fuskeland et al. (2014), which itself was based on the nine year WMAP polarization analysis mask (Bennett et al. 2013). We then ran preliminary analyses to determine the effective posterior width resulting from each region. If this was found to be strongly prior-dominated, we merged neighboring regions. This process resulted in four main regions, which we will refer to as 1) high-latitudes; 2) the Galactic Spur or simply Spur; 3) the Galactic Center; and 4) the Galactic plane. As we will show later, even after this process the high-latitude region does not have sufficient signal-to-noise ratio to significantly constrain $\beta_s$, as shown in the middle panel of Fig. 1; the synchrotron emission in this region is generally very faint, and there is thus little leverage to estimate spectral variations as a function of frequency.

We also note that synchrotron emission is sensitive to Faraday rotation (e.g., Beck et al. 2013) in areas with high electron densities and magnetic fields, which is the case near the Galactic center. This effect is caused by circular birefringence, where left- and right-handed circularly polarized emission traverse the magnetic field of an ionized medium at different velocities, generating a net linear polarization signal with a polarization angle proportional to the field strength. However, since this effect is proportional to the squared wavelength of the radiation, it is most prominent at frequencies below 5 GHz, and it is negligible for most of the sky above 30 GHz. In this paper, we do not make any corrections for Faraday rotation to any frequency channel. However, as shown in Sect. 5, our synchrotron constraints for the Galactic center region are contaminated by systematic effects, and these are most likely due to a combination of instrumental and astrophysical mis-modeling effects.

### 2.1.2. Thermal dust emission

The second significant polarized foreground component between 30 and 70 GHz is thermal dust emission generated by interstellar dust grains that collectively account for $\approx 1\%$ of the mass of the interstellar medium. See, e.g., Hensley & Draine (2020) for a recent review of relevant physics and observations constraints. The size of these grains typically varies from a few Angstroms to a few tenths of a $\mu$m, and they are heated up by the interstellar radiation field to a temperature of about 20 K. This heat is then re-emitted thermally with a peak frequency between 1000 and 2000 GHz. Furthermore, due to paramagnetic dissipation resulting from interactions between rotating grains and the local magnetic field, the dust grains tend to align with their short axes parallel to the local magnetic field. In turn, this alignment can induce a polarization signal with a polarization fraction of 20% or more, as reported by Planck Collaboration XI (2020).

The most common thermal dust SED model is that of a modified blackbody function, as defined in Eq. (7). This SED has three free parameters; 1) the amplitude $a_d$, which traces the surface density of dust particles along each line-of-sight; 2) a spectral index, $\beta_d$, that quantifies the low-frequency slope of the SED, and depends on the physical composition of the dust grains; and 3) the dust temperature, $T_d$, which affects the peak location of the modified blackbody function, and depends on the local radiation field strength at any given position. More complex, and perhaps more realistic, representations do exist, including multi-component modified blackbody (e.g., Finkbeiner et al. 1999; Meisner & Finkbeiner 2015) or physical dust grain models (Guillet et al. 2018).

The particular data combination used in the BEYONDPLANCK analysis has a very low signal-to-noise ratio for constraining thermal dust SED variations, whether of frequency or spatial origin, as we are working at synchrotron dominated frequencies and in this paper we only consider one single free parameter for the thermal dust SED, a single spatially constant value of $\beta_d$. The bottom panel in Fig. 1 shows the Planck thermal dust amplitude map (Planck Collaboration IV 2020) with its corresponding processing mask used in the sampling procedure, which was gener-

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2 Note that the BEYONDPLANCK data combination does not include the WMAP K-band sky, as this channel would otherwise dominate over Planck LFI 30 GHz in terms of total signal-to-noise ratio.
Table 1. Overview of all included data bands in the BeyondPlanck polarization analysis. The columns represent the following: 1) Science experiment for that band, 2) frequency band name, 3) data processing pipeline 4) HEALPix resolution in $N_{\text{side}}$, 5) highest included multipole $\ell_{\text{max}}$, 6) beam resolution in arcminutes, 7) bandpass center frequency in GHz, 8) bandpass width in GHz, 9) noise rms given in $\mu K$ scaled by pixel size, 10) noise format.

| Experiment | Band | Processing | $N_{\text{side}}$ | $\ell_{\text{max}}$ | $v_c$ | Bandwidth | $\sigma$ | $\sigma$ | Noise Format |
|------------|------|------------|-------------------|---------------------|------|-----------|---------|---------|---------------|
| Planck LFI | 30   | BEYONDPLANCK | 512               | 1500                | 32/4 | 28.4      | 5.7     | 260     | TOD+White noise |
| Planck LFI | 44   | BEYONDPLANCK | 512               | 2000                | 27/1 | 44.1      | 8.2     | 370     | TOD+White noise |
| Planck LFI | 70   | BEYONDPLANCK | 1024              | 2500                | 13/3 | 70.1      | 14.0    | 270     | TOD+White noise |
| Planck HFI | 353  | NPIPE      | 1024              | 3000                | 4’/4 | 353       | 88.2    | 380     | White noise |
| WMAP       | Ka   | WMAP       | 16               | 64                  | 40’  | 33        | 7.0     | 410     | Full covariance |
| WMAP       | Q    | WMAP       | 16               | 64                  | 31’  | 41        | 8.3     | 380     | Full covariance |
| WMAP       | V    | WMAP       | 16               | 64                  | 21’  | 61        | 14.0    | 460     | Full covariance |

The same argument applies to the WMAP $K$-band channel (Bennett et al. 2013). While this channel provides strong constraints on both AME and polarized synchrotron emission, its statistical sensitivity is so high that it would dominate over the Planck LFI 30 GHz channel if included in the BeyondPlanck analysis, which again would undermine the main purpose of the current work. For this reason, we include only the WMAP $K_a$, $Q$, and $V$-band channels in the following, centered on 33, 41 and 61 GHz, respectively. (The $W$-band channel is omitted because it is known to be contaminated by systematic residuals; Bennett et al. 2013.)

In addition, we include the Haslam 408 MHz survey to constrain synchrotron emission in temperature, see Andersen et al. (2020). This results in a total of eight frequency channels in intensity, and seven frequency channels in polarization, which in principle should be sufficient to constrain a model with five intensity components and three polarization components.

One of the important novel algorithmic developments that will be described in section 4 is true multi-resolution component separation for both linear and non-linear foreground parameters. Specifically, we consider in the following only the low-resolution WMAP polarization data at a HEALPix resolution of $N_{\text{side}} = 16$, for which dense pixel-pixel noise covariance matrices are available (Bennett et al. 2013). This ensures that we have at least nominally a complete noise description for all frequency channels relevant for CMB extraction, and also that small-scale polarization results are entirely dominated by Planck LFI.

All data except the NPIPE 353 GHz channel are analyzed as provided by the respective team without any preprocessing or smoothing. The 353 GHz channel, however, is smoothed to an angular resolution of 10’ and re-pixelized at a resolution of $N_{\text{side}} = 1024$, corresponding to a pixel size of 3/4. This is done both in order to speed up the analysis process (BeyondPlanck Collaboration 2020; Galloway et al. 2020), and to suppress small-scale correlated noise which would otherwise lead to a significant $\chi^2$ excess. Table 1 provides an overview over all data sets included in the current analysis.
4. Methods

As outlined above and by BeyondPlanck Collaboration (2020), the main goal of BeyondPlanck is to perform end-to-end Bayesian analysis of CMB measurements, starting from the data model defined in Eq. (1). We define \( \omega = \{ \xi, n_{\text{corr}}, \Delta_{\text{bp}}, \alpha, \beta, \ldots \} \) to be the set of all free parameters in this model, of both instrumental and astrophysical origin. Our main goal is then to map out the corresponding joint posterior distribution, which is given by Bayes’ theorem,

\[
P(\omega \mid d) = \frac{P(d \mid \omega)P(\omega)}{P(d)} \propto L(\omega)P(\omega). \tag{8}
\]

In this expression, \( P(\omega \mid d) \) is the posterior distribution, and describes our knowledge about \( \omega \) after performing the experiment in question: \( L(\omega) \equiv P(d \mid \omega) \) is called the likelihood, and quantifies the information content in \( d \) regarding \( \omega \) and \( P(\omega) \) is called the prior, which quantifies our beliefs regarding \( \omega \) before doing the experiment. The prior can also be used actively to regularize specific degeneracies in the model.

This posterior distribution is large and complex with billions of free parameters. Attempting to directly map out the full posterior distribution by brute-force is infeasible. Instead, we resort to Markov Chain Monte Carlo methods, and draw samples from the posterior distribution. In particular, we find that the Gibbs sampling algorithm (Geman & Geman 1984) is particularly well suited to handle the complexity of this model. We therefore adopt the Commander CMB Gibbs sampling framework as the starting point for our analysis. This software was first introduced by Eriksen et al. (2004b) for optimal CMB power spectrum estimation applications, and later generalized to also account for joint CMB and component separation by Eriksen et al. (2008) and Seljebotn et al. (2019). In this operational mode, Commander played a central role in the official Planck analysis, as summarized by Planck Collaboration I (2014, 2016, 2020) and references therein. BeyondPlanck has now generalized this framework further to also account for low-level TOD processing and mapping, effectively turning the entire CMB analysis challenge into one global problem.

4.1. Gibbs sampling

Gibbs sampling formalizes the idea of iterative analysis within a rigorous statistical language. In short, the theory of Gibbs sampling states that samples from a complex joint distribution may be drawn by iteratively sampling from each (typically simpler) conditional distribution. The full BeyondPlanck Gibbs chain (BeyondPlanck Collaboration 2020) illustrates how this is done in practice, and can be written as

\[
g \leftarrow P(g \mid d, \xi, \Delta_{\text{bp}}, \alpha, \beta, C_l) \tag{9}
\]

\[
n_{\text{corr}} \leftarrow P(n_{\text{corr}} \mid d, g, \xi, \Delta_{\text{bp}}, \alpha, \beta, C_l) \tag{10}
\]

\[
\xi \leftarrow P(\xi) \mid d, g, n_{\text{corr}}, \Delta_{\text{bp}}, \alpha, \beta, C_l \tag{11}
\]

\[
\Delta_{\text{bp}} \leftarrow P(\Delta_{\text{bp}} \mid d, g, n_{\text{corr}}, \xi, \alpha, \beta, C_l) \tag{12}
\]

\[
a \leftarrow P(a \mid d, g, n_{\text{corr}}, \xi, \Delta_{\text{bp}}, \beta, C_l) \tag{13}
\]

\[
\beta \leftarrow P(\beta \mid d, g, n_{\text{corr}}, \xi, \Delta_{\text{bp}}, a, C_l) \tag{14}
\]

\[
C_l \leftarrow P(C_l \mid d, g, n_{\text{corr}}, \xi, \Delta_{\text{bp}}, a, \beta). \tag{15}
\]

Here, the symbol \( \leftarrow \) means setting the variable on the left-hand side equal to a sample from the distribution on the right-hand side. For convenience, we also define the notation \( \omega \setminus \xi \) to imply the set of parameters in \( \omega \) except \( \xi \).

The parameters not already introduced are noise power spectrum parameters \( (\xi_i) \), bandpass corrections \( (\Delta_{\text{bp}}) \), and the CMB power spectrum \( (C_l) \). Since the current paper focuses on polarized foregrounds, we are here primarily interested in the amplitude parameters, \( a \), sampled in Eq. (13), and the spectral parameters, \( \beta \), sampled in Eq. (14). We will therefore describe these two sampling steps in detail in the following, and refer the interested reader to BeyondPlanck Collaboration (2020) and references therein for details regarding the remaining steps.

4.2. Signal amplitude sampling

We first consider the signal amplitude conditional distribution, \( P(a \mid d, \omega \setminus a) \), which has already been a primary focus of interest for a long line of studies, including Jewell et al. (2004), Erken et al. (2004a, 2008), and Seljebotn et al. (2019). In the following, we give a brief summary of these developments, and also highlight two important novel features introduced in the current work.

The appropriate starting point for this conditional distribution is the data model described in Eq. (2). We first note that \( a \) contains all signal amplitude maps, using some appropriate linear basis, stacked column-wise, such that \( a = [a_{\text{SB}, m}, a_{\text{SF}, m}, a_{\text{FG}, m} \ldots] \), where the \( t \) superscript denotes the transpose. Further, we adopt a spherical harmonics basis to describe each diffuse component, such that \( a_{i} = [a_{i, \ell_{\text{int}}, m_{\text{int}}} a_{i, \ell_{\text{int}}, m_{\text{int}}} a_{i, \ell_{\text{int}}, m_{\text{int}}}] \) contains the various temperature and polarization spherical harmonics coefficients (Zaldarriaga & Seljak 1997) for component \( i \). Second, for notational convenience we combine the beam and mixing matrix operators in Eq. (2) into one joint linear operator, \( A \equiv B m, \) such that this equation may be written succinctly as

\[
m_{\nu} = A_{\nu} a + n_{\nu}. \tag{16}
\]

Noting that the noise, \( n_{\nu} = m_{\nu} - A_{\nu} a \), is assumed to be Gaussian distributed with vanishing mean and a known covariance matrix, \( N_{\nu} \), Bayes’ theorem then allows us to write the conditional distribution of interest in the following form,

\[
P(a \mid d, \omega \setminus a) \propto P(d \mid a, \omega \setminus a) P(a) \tag{17}
\]

\[
\propto P(m \mid a) P(a) \tag{18}
\]

\[
\propto \prod_{\nu} \exp \left( -\frac{1}{2} (m_{\nu} - A_{\nu} a) N_{\nu}^{-1} (m_{\nu} - A_{\nu} a) \right) \cdot \exp \left( -\frac{1}{2} (a - \bar{a}) S_{\nu}^{-1} (a - \bar{a}) \right), \tag{19}
\]

Here, the first line holds because TOD binning is a deterministic operation when conditioning on \( \omega \) (BeyondPlanck Collaboration 2020), and the set of binned sky maps, \( m, \) is a sufficient statistic for \( a \); no additional information in the TOD will provide more knowledge about the foreground amplitude maps beyond that stored in \( m \). In the last line adopt a Gaussian signal prior with mean \( \bar{a} \) and covariance matrix \( \Sigma \), discussed further below. In this paper, we set \( \bar{a} = 0 \), and only use \( \Sigma \) for smoothing purposes. Since the product of two Gaussian distributions is another Gaussian, \( P(a \mid d, \omega \setminus a) \) is also Gaussian, and the appropriate sampling algorithm is given by a standard multivariate Gaussian, which is discussed in detail in Appendix A in BeyondPlanck Collaboration (2020). In particular, a proper sample may be drawn by solving the following linear equation for \( a \),

\[
\left( S_{\nu}^{-1} + \sum_{\nu} A_{\nu} N_{\nu}^{-1} A_{\nu} \right) a = \sum_{\nu} A_{\nu} N_{\nu}^{-1} m_{\nu} + \sum_{\nu} A_{\nu} N_{\nu}^{-1/2} \eta_{\nu} + S_{\nu}^{-1/2} \eta_{\nu},
\]
where \( \eta_r \) and \( \eta_0 \) are random Gaussian vectors of independent \( \mathcal{N}(0, 1) \) variates. Because of its large dimensionality, this equation must in practice be solved using iterative linear algebra techniques, and we use a preconditioned conjugate gradient solver for this purpose (e.g., Shewchuk 1994).

This exact equation was discussed in detail by Seljebotn et al. (2019), who also introduced two novel and efficient preconditioners for partial-sky observations based on pseudo-inverse and multi-grid techniques. However, in the present work, for which no analysis mask is imposed during the main Gibbs sampling analysis, we adopt the simpler diagonal pre-conditioner described by Eriksen et al. (2008), which both converges slightly faster with full-sky data, and has a lower computational cost per iteration.

The two main novel features regarding Eq. (20) introduced by BeyondPlanck (as described by Andersen et al. (2020) for temperature and by this paper for polarization) concern the data and model representation and active use of spatial priors.

### 4.2.1. Vector representation and object-oriented programming

Equation (20) is written in a general vector form, without reference to any specific vector representation or basis, or to any specific form of either the effective mixing matrix, \( A_r \), or noise covariance matrix, \( N_r \). All of these may, in principle, be chosen per component and frequency channel. As detailed by Galloway et al. (2020), we have implemented such flexibility in the most recent version of Commander adopting an object-oriented programming style, where separate classes are defined for each object. For instance, the current implementation contains specific classes for three general types of components, namely diffuse components (for which \( a \) is represented in terms of spherical harmonic coefficients, \( a_m \)), point source objects (for which \( a \) is represented in terms of a single flux density per compact source), and fixed spatial templates (for which \( a \) is represented in a single multiplicative amplitude per template). Any combination of such objects may all be fit simultaneously and jointly through Eq. (20). It is also relatively straightforward to add new types of objects as the need may arise. Two examples of classes that might be important for future applications include pixel- or needlet-based components, which could be useful for modeling partial sky experiments.

Each of the instrumental objects are also implemented in terms of individual classes. This is particularly relevant for the noise covariance matrix, \( N_r \), which may have very different representations for different experiments. For example, in the current analysis, we model the Planck LFI noise as a sum of correlated and white noise, where the correlated noise is treated as a stochastic variable in the Gibbs chain, and sampled over directly, while the white noise uncertainty is propagated analytically through a diagonal \( N_r \). This approach supports, for the first time, propagation of both correlated and white noise at all angular resolutions. For WMAP, however, we do not yet have access to time-ordered data within our framework, so \( N_r \) is defined in terms of the precomputed dense low-resolution noise covariance matrices provided by the WMAP team (Bennett et al. 2013). For this reason, WMAP contributes only to large angular scales in the current analysis. Finally, for the Planck 353 GHz measurements, which are essential for modeling polarized thermal dust emission at full angular resolution, we are for now only able to propagate white noise uncertainties with a diagonal \( N_r \) as given by Planck Collaboration Int. LVII (2020). Of course, this will necessarily lead to an underestimation of dust-related uncertainties; hence modeling Planck HFI observations in time-domain is clearly a high-priority issue for future work, but outside the scope of the current project.

In summary, the first critically important novel feature provided by the current Commander implementation is the ability to operate with fundamentally different types of data sets within one analysis and thereby exploit complementary features from each data set to break degeneracies within the full model. At the time of publication, there is direct support for only two types of noise covariance matrices, namely diagonal and WMAP-style dense Stokes \( QU \) matrices. However, using the new infrastructure described here and by Galloway et al. (2020), it is straightforward to add support for other types of matrices. Possibly useful examples include block-diagonal or banded covariance matrices, or noise covariance matrices with specific subspaces projected out through the Sherman-Morrison-Woodbury formula (Sherman & Morrison 1950; Woodbury 1950). The latter could be particularly useful for ground-based experiments that tend to have limited statistical sensitivity on large angular scales, but high systematic uncertainties. We hope that such features can be implemented, and made publicly available, by third-party authors as Open Source contributions (Gerakakis et al. 2020).

### 4.2.2. Gaussian spatial priors

The second novel feature supported by the latest Commander implementation is the use of active spatial priors for the diffuse components. For practical purposes, we currently support only Gaussian priors, as described by Eq. (19), as non-Gaussian priors would lead to a prohibitively high computational cost associated with the current conjugate gradient-based approach. In practice, imposing a spatial prior is therefore equivalent to specifying a prior mean, \( \bar{a} \), and covariance matrix, \( \mathbf{S} \), for each component. In the current polarization-oriented analysis, however, we do not wish to enforce any informative priors on any of the three free components (polarized CMB, synchrotron or thermal dust emission), so we set \( \bar{a} = 0 \) for all three. Instead, we use \( \mathbf{S} \) to adjust the allowed level of fluctuations around zero for each component as a function of angular scale; this is numerically equivalent to choosing an appropriate effective smoothing scale for each component, and might for this paper be considered more of a technical issue than a prior in the normal sense. For an example of application of active priors, however, see Andersen et al. (2020), in which proper informative spatial priors are imposed on both free-free and anomalous microwave emission.

The remaining question is how to choose a specific form of \( \mathbf{S} \) for each component. There are two main requirements for this choice. First, \( \mathbf{S} \) is the covariance of \( a \) and therefore dictates the overall fluctuation level of the fitted components. A large value of \( \mathbf{S} \) implies a weak prior, and the fitted component will then be dominated by the data-driven likelihood term in Eq. (19), while a low value of \( \mathbf{S} \) implies a strong prior, resulting in values close to the prior mean. In practice, we want the prior to play a limited role where the data have a large signal-to-noise ratio, but a stronger role where the data are noise dominated. It is therefore in general desirable to choose scale-dependent priors, in which the smoothing becomes gradually stronger; the prime example of this is a standard Gaussian smoothing kernel.
As already mentioned, we adopt spherical harmonics as our basis set for $a$, writing each polarized signal component as

$$a_X(\hat{n}) = \sum_{\ell m} a_{\ell m}^X Y_{\ell m}(\hat{n}).$$  \hspace{1cm} (21)

For each component we therefore also define an angular power spectrum prior of the form

$$\hat{D}_X^\ell = \langle |w_{\ell m}^X|^2 \rangle \ell (\ell + 1)/2\pi,$$  \hspace{1cm} (22)

where $X = \{E, B\}$. Note that this power spectrum prior representation is conceptually similar to the assumptions made by SMICA (Cardoso et al. 2008), one of the four CMB extraction codes used by Planck.

To be specific, we adopt the following priors for each of the three astrophysical components,

$$\hat{D}_{\text{CMB}}^\ell = 1000 \left(\frac{\ell}{20}\right)^{-0.5} \mu K^2$$  \hspace{1cm} (23)

$$\hat{D}_{\nu}^\ell = 200 e^{-\ell/10^2(30')^2} \mu K^2$$  \hspace{1cm} (24)

$$\hat{D}_{\theta}^\ell = 500 e^{-\ell/10^2(40')^2} \mu K^2,$$  \hspace{1cm} (25)

where $\sigma^2(\ell, \theta_{\text{FWHM}}) \equiv (8/\pi)\pi/(180 \cdot 60)\theta_{\text{FWHM}}^2$ is the standard deviation of a Gaussian distribution expressed in terms of a full width at half maximum, $\theta_{\text{FWHM}}$, in arcminutes. Because a typical $\Lambda$CDM model predicts a CMB EE power spectrum amplitude of less than $20 \mu K^2$ at $\ell \leq 500$, and Planck LFI has very low signal-to-noise ratio above $\ell \approx 10$, the $\hat{D}_{\text{CMB}}(\ell)$ prior has negligible effect on any scientific CMB constraint—but it does prevent degeneracies due to unconstrained CMB modes from contaminating the foreground model at small angular scales.\footnote{We do note, however, that this specific CMB prior does become comparable to the $\Lambda$CDM predictions around $\ell \approx 1000$, which is unfortunate for aesthetic reasons, even if it may be inconsequential for actual results. In the next version of the BEYONDPLANEK analysis, the amplitude of the CMB prior will be increased by an order of magnitude to eliminate this issue.}

For the polarized synchrotron and thermal dust amplitudes, the priors correspond to Gaussian smoothing of 30’ and 10’ FWHM, respectively. The overall scaling factors are chosen to be 200 and 500 $\mu K^2$, respectively, which correspond to the observed angular power spectrum of each component on large angular scales. In these cases, the priors are used to apodize the resulting foreground amplitude maps with Gaussian smoothing kernels, to avoid ringing around bright sources and the Galactic plane and unphysical degeneracies at high multipoles.

Future analyses may wish to use $S$ to directly estimate the angular power spectrum of each foreground component, fully analogous to the CMB case (Colombo et al. 2020; Paradisio et al. 2020). This will then both alleviate the need of specifying the prior parameters by hand before executing the analysis, and it will ensure optimal smoothing properties for each component, resulting in minimal high-$\ell$ degeneracies between the various components.

Finally, we conclude this section by emphasizing that the above priors are in fact only priors, and not deterministic operators. Thus, the resulting synchrotron and thermal dust component maps will be determined by the properties of the observed data wherever the data are stronger than the prior. This is important to bear in mind for instance when trying to estimate the angular power spectrum of the resulting maps; the component maps $a$ that result from solving Eq. (20) correspond to a model of the sky without instrumental beam convolution, but with spatially varying noise properties, depending on the local signal-to-noise ratio of the data. The angular resolutions of the synchrotron and thermal dust maps are not given precisely by a Gaussian beam of 30’ and 10’, respectively, but will generally be higher where the data are sufficiently strong. As such, the behavior of these foreground maps is conceptually similar to the GNILC algorithm (Remazeilles et al. 2011), in which a spatially varying angular resolution also results from signal-to-noise ratio variations. Likewise, it is also important to note that the noise properties of these maps are highly non-trivial, and the only statistically robust way of assessing and propagating their uncertainties is through the ensemble of sky map samples produced by the algorithm itself.

### 4.3. Spectral parameter sampling

The second of the two main conditional distributions discussed in this paper is $P(\beta | d, \omega \setminus \beta)$, which describes the foreground SED parameters. In general, $a$ and $\beta$ are strongly correlated for a given component, especially for high signal-to-noise data. For temperature-oriented foreground analysis, the BEYONDPLANEK Gibbs sampler therefore implements a special-purpose sampling step for these parameters, by exploiting the definition of a conditional distribution, $P(a, \beta | d) = P(a | d, \beta)P(\beta | d)$ (Stivoli et al. 2010; BEYONDPLANEK Collaboration 2020; Andersen et al. 2020). That is, we first sample spectral parameters from the marginal distribution with respect to $a$, and then sample $a$ conditionally with respect to $\beta$. The resulting algorithm is thus effectively an independence sampler in $(a, \beta)$ (a sampler where the new proposal does not depend on the previous) with an internally vanishing Markov chain correlation length. In this case, any long-term Markov chain correlations come from degeneracies with other parameters.

However, as described by Stivoli et al. (2010), this algorithm is only computationally practical for observations with the same angular resolution, as the computational expense for evaluating the marginal distribution $P(\beta | d)$ otherwise becomes prohibitively high. In practice, this means that all data maps must be smoothed to a common angular resolution. For temperature data, this is not a major problem, but for polarization analysis it is non-trivial, since the smoothing operation correlates the instrumental noise. In addition, the data combination used in the current paper involves low-resolution WMAP data with dense noise covariance matrices, and smoothing all data to this resolution is impractical.

Since we focus on polarization in this work, we instead adopt a standard Gibbs sampler for $\beta$. In this case, the appropriate conditional posterior distribution may be derived from Eq. (16), assuming the amplitude to be known, and that mixing matrix in $M$ in $A$ is given by the spectral parameters $\beta$ by noting that the instrumental noise is assumed to be Gaussian distributed with covariance matrix $N_{\nu}$, and therefore

$$P(\beta | d, a) \propto P(d | a, \beta)P(\beta)$$  \hspace{1cm} (26)

$$\propto \left[ \prod_{\nu} \exp \left( -\frac{1}{2} (d_{\nu} - A(\beta)a)^T N_{\nu}^{-1} (d_{\nu} - A(\beta)a) \right) \right] P(\beta),$$

where $d = \{d_{\nu}\}$ is the set of all available frequency maps (which within the larger BEYONDPLANEK framework may be a specific set of frequency map sky samples), and $P(\beta)$ is a user-defined prior, typically a Gaussian distribution with physically motivated mean and standard deviation.

Note that Eq. (26) is also written with a general vector notation, and makes no reference to a specific vector basis for either
a or β; it therefore allows the various data sets to be defined in different basis sets. The only requirement is that there must exist a well-defined mapping between β and the effective mixing matrix at each frequency, A_β(β). This generality is precisely what is needed to analyze multi-resolution observations jointly, for instance high-resolution LFI data together with low-resolution WMAP data.

As noted in Sect. 2.1.1, the signal-to-noise ratios of the Planck and WMAP polarization data are modest, so we have defined broad regions for β_c as shown in Fig. 1. To sample these values, we employ a standard Metropolis algorithm, as outlined in Algorithm 1, using Eq. (26) as target distribution and a standard symmetric Gaussian proposal distribution, T(β_c^n | β_c^{n-1}) ~ N(β_c^{n-1}, C_β), where C_β is a tunable proposal matrix. When evaluating Eq. (26), we first propose β_c^n independent for each region i, and then smooth the resulting map with a 10° FWHM Gaussian beam to suppress edge effects. This smoothed map is then used to evaluate the mixing matrix A_β at the appropriate resolution for each frequency channel.

To assess the goodness-of-fit locally across the sky, we compute a χ^2 map per pixel. Since WMAP is pixelized with N_side = 16, the χ^2 map is too. The sum of this map, after application of an optional analysis mask, is used to evaluate the exponent in Eq. (26) and the corresponding Metropolis acceptance rate.

As summarized in Algorithm 1, this method has two free tunable parameters, the proposal matrix C_β, and the number of Metropolis steps per main Gibbs iteration, n. Before starting a full Gibbs analysis, we perform a tuning run using a diagonal C_β and adjust the diagonal elements until the resulting accept rate is between 0.2 and 0.8. Once that happens, we run a longer chain with fixed C_β and replace this matrix with the covariance matrix from the resulting samples. Finally, we run a third chain, computing the Markov autocorrelation length from the resulting chain, and setting n such that the empirical correlation between samples β_c^n and β_c^(n+1) is less than 0.1. These tuning steps are only performed once for each analysis setup, and files are stored on disk for subsequent runs.

The only missing part of the algorithm is now a specification of the priors. For the synchrotron and thermal dust spectral indices, we adopt for the main analysis Gaussian priors of β_s = −3.1 ± 0.1 and β_T = 1.56 ± 0.10, respectively, motivated by the official Planck results (Planck Collaboration Int. XLVI 2016; Planck Collaboration IV 2020). Different prior values are studied in Sect. 4.2.2. However, the high-Galactic and Galactic Center regions are treated separately, because of strong degeneracies seen in these two regions. To prevent potential residual systematic errors from contaminating other parameters within these two regions, we choose to include only the prior when evaluating the Metropolis accept rate. That, we do marginalize over the prior for these two regions, but we do not let the data themselves constrain β_s and we therefore only gain new information in two of the four regions. While clearly non-ideal, we consider this approach to be preferable to simply fixing the synchrotron index at some given value, as was done for most previous Planck polarization analyses, e.g., Planck Collaboration: Diffuse component separation (2020).

5. Results

5.1. Goodness-of-fit

Before turning our attention to the final astrophysical products, we assess the goodness-of-fit of the fitted sky model. Our first statistic is the total χ^2 per pixel, summed over frequencies and Stokes Q and U parameters and averaged over Gibbs samples, as shown in Fig. 2. To aid visual interpretation, this map is plotted in the form (χ^2 - n_{dof})/√2n_{dof}, where n_{dof} = 20 390 is an estimate of the total number of degrees of freedom (i.e., number of full-frequency data pixels summed over frequencies, minus the number of fitted parameters) within each pixel. If the model performs as expected, this quantity should have zero mean and unit standard deviation. Overall, we see that our model appears to perform well at high Galactic latitudes, while the Galactic plane shows clear excess χ^2. This behavior is not surprising, as the Galactic Center is far more complex and difficult to model, both in terms of astrophysics and instrumental effects. We do note that the morphological structure of the excess χ^2 appears to be correlated with Galactic emission, rather than instrumental effects.

Another useful statistic for evaluating the model goodness-of-fit is data-minus-model residual maps per frequency, r_c = d_c - s_c. These residual maps provide a visual summary of remaining systematics on a band-to-band basis, which is useful when physically interpreting specific artifacts in the χ^2 map. Figure 3 shows such residual maps for the Planck frequency bands, smoothed to 2° FWHM to suppress white noise. Generally, these maps indicate excellent model performance, with amplitudes of ≤3 μK. High Galactic latitudes appear consistent with noise, while small deviations are seen around the Galactic Center, with a morphology that may suggest residual bandpass-induced temperature-to-polarization leakage (Svalheim et al. 2020). The 44 GHz channel exhibits the largest relative variations. This is expected from algorithmic arguments, by noting that this channel does not dominate the determination of any single fit.
ted astrophysical component. In contrast, the 30 GHz channel dominates synchrotron determination; the 70 GHz channel dominates CMB determination; and the 353 GHz channel dominates thermal dust determination. As such, any excess fluctuations in these channels will rather be interpreted as signal belonging to whichever foreground component that uses this as a reference band. For component separation purposes, it is clearly advantageous to have multiple frequency maps with comparable signal-to-noise ratio per astrophysical component, as instrumental artifacts are then much easier to identify. Employing the full set of Planck frequency maps in a future analysis will improve these results, and provide many more internal cross-checks. However, we once again stress that the main aim of the current study is not to present a new state-of-the-art sky model, but rather to demonstrate the BeyondPlanck algorithm.

Figure 4 shows the remaining residual maps, i.e., WMAP Ka, Q, and V-band. In all three cases, we see coherent large-scale structures that are clearly morphologically inconsistent with instrumental noise. These residual maps match very closely a set of correction templates presented by Jarosik et al. (2007) that account for known transmission imbalance between the A- and B-sides of the instrument,\(^5\) as shown in the second and fourth column of Fig. 4. In our study, we do not apply any explicit corrections for these templates; hence, they appear in the frequency residual maps. However, they are accounted for in the WMAP covariance matrices, and the corresponding modes are therefore appropriately down-weighted when fitting the astrophysical parameters with Eqs. (20) and (26). Transmission imbalance effects will therefore not bias any astrophysical results, but only result in larger uncertainties. At the same time, these residual maps clearly suggest how a future joint time-domain processing of WMAP and Planck will be able to constrain the WMAP transmission imbalance parameters with high precision. Once that happens, the corresponding spatial modes will no longer need to be algebraically projected out, as is effectively done now, but may rather be used for scientific inference, on the same footing as any other mode.

\(^5\) Note that Jarosik et al. 2007 provide two correction templates per WMAP differencing assembly, and both Q- and V-bands are therefore associated with four templates each. Only one of these are shown in Fig. 4 for intuition purposes; the other templates look qualitatively similar.
5.2. Amplitude results

We now turn our attention to the astrophysical results, and start with the posterior amplitude maps, $a$. First, Fig. 5 shows the thermal dust amplitude, plotted at an angular resolution of 10' FWHM at $N_{side} = 1024$ for the polarization amplitude $P = \sqrt{Q^2 + U^2}$, as well as $Q$ and $U$ averaged over an ensemble of Gibbs samples. The right column shows the corresponding posterior distribution standard deviation per pixel, with values peaking around $3 \mu K_{RJ}$. As mentioned in the previous subsection, the thermal dust amplitude is for all practical purposes determined by the pre-computed HFI 353 GHz band in the BeyondPlanck processing, and the LFI bands have little influence. As a result, the thermal dust standard deviation maps shown in Fig. 5 are essentially given deterministically by the input 353 GHz standard deviation map. These uncertainties are underestimated in the Galactic plane, where systematic effects must be significant. Overall, the polarized thermal dust amplitude map is in good agreement with previous results.

Figure 6 shows corresponding results for the polarized synchrotron amplitude map. At high Galactic latitudes, we see that the standard deviation of this component traces the LFI 30 GHz scanning strategy, and is correspondingly largely determined by instrumental white noise. However, in this case the Galactic plane is in fact dominated by temperature-to-polarization leakage due to bandpass and gain uncertainties, resulting in a morphology that matches the 30 GHz intensity map.

In Fig. 7, we show Stokes parameter maps of the difference between the BeyondPlanck map of polarized synchrotron amplitude and two independent synchrotron tracers. The top panel shows differences with respect to the full WMAP $K$-band frequency map at 23 GHz. To account for its different effective frequency, we scale the $K$-band map according to a power-law model with $\beta_s = -3.1$, or, explicitly, by a factor of 0.38. The
Fig. 6. Same as Fig. 5, but for polarized synchrotron emission evaluated at 30 GHz. In this case, the effective angular resolution is 1° FWHM.

differences are quite substantial between the two results, around \(5 \mu K_{RJ}\) even at high Galactic latitudes. Considering that the effective frequency difference between \(K\)-band and 30 GHz is only 7 GHz, spatial variations in the synchrotron spectral index are unlikely to be relevant, as a difference of \(\Delta \beta_s = 0.02\) only translates into approximately \(0.5 \mu K\) in the relevant regions. Rather, noting that the dominant feature appears to be dipolar in nature, it is natural to suspect calibration-related issues playing a role. This hypothesis is more likely when considering the bright foreground levels at 23 and 30 GHz, which makes accurate calibration of these channels particularly challenging. At low Galactic latitudes, the differences are dominated by bandpass-induced temperature-to-polarization leakage uncertainties (Svalheim et al. 2020).

The bottom panel of Fig. 7 shows similar difference maps with respect to the polarized synchrotron amplitude map derived from NPIPE (Planck Collaboration Int. LVII 2020). In this case, the high-latitude residuals are dominated by the Planck scanning strategy, with an overall morphology that closely matches the LFI gain residual template produced by Planck Collaboration II (2020) and discussed by Gjerløw et al. (2020). At the same time, no striking correlations are seen between the \(K\)-band and NPIPE residual maps, and the overall levels of variation in these two difference maps are comparable. A future joint time-domain analysis that includes both LFI and WMAP will clearly be beneficial for both datasets, and this work is already on-going (Watts et al. 2020).

5.3. Spectral index results

Finally, we consider the spectral parameters for synchrotron and thermal dust, \(\beta_s\) and \(\beta_d\).

5.3.1. Synchrotron spectral index

Before presenting the synchrotron spectral index posterior distribution, we motivate our final choice of sampling regions and priors by considering the preliminary analysis configuration sum-
Fig. 7. (Top panel:) Difference between the BeyondPlanck polarized synchrotron amplitude and the raw WMAP K-band map (Bennett et al. 2013), the latter being scaled to 30 GHz assuming a spectral index of $\beta_s = -3.1$. (Bottom panel:) Similar difference between the BeyondPlanck and NPIPE (Planck Collaboration Int. LVII 2020) synchrotron amplitude maps. Left and right columns show Stokes $Q$ and $U$ parameters respectively, and all maps are smoothed to a common angular resolution of $1^\circ$ FWHM.

...

Next, we see that the Gum Nebula region also converges to a notably low value, and this is most likely related to the same systematic errors and/or prior volume effects (e.g., Dunkley et al. 2009). Indeed, when sampling jointly over the full range of time-ordered systematic corrections, we find that these regions converge to $\beta_s \approx -2.5$. As in the case of the Galactic Center, we do not take this as evidence for a truly flatter spectral index in these regions, but rather as an indication that there are low-level residual systematics present in either BeyondPlanck, WMAP, or NPIPE 353 GHz (or, possibly, all of them) at a level that is sufficient to bias the spectral index in the faintest regions.

We conclude that dividing into nine sky regions is sub-optimal (not to mention 24, which was the starting point of the analysis; see Sect. 2.1.1), as most regions do not have sufficient signal-to-noise ratio to independently constrain $\beta_s$, so they are highly susceptible to residual systematic uncertainties. In particular, small changes in the absolute calibration can introduce...
Fig. 8. Synchrotron spectral index as a function of Gibbs iteration for an analysis that includes nine disjoint regions (see inset sky map). The prior is in this case $\beta_s = -2.8 \pm 0.1$, and all regions are constrained using the full posterior distribution.

Fig. 9. Synchrotron spectral index as a function of Gibbs iteration binned using the final BeyondPlanck analysis configuration with four disjoint regions and a prior of $\beta_s = -3.1 \pm 0.1$. Dotted lines indicate regions that are sampled exclusively from the prior distribution, while solid lines indicate regions that are sampled from the full posterior distribution.

confusing CMB dipole leakage at high latitudes, while bandpass variations can cause problems near the Galactic Center.

We therefore reduce the number of disjoint regions further, and combine the four high Galactic latitude regions into one, and we merge all regions along the galactic plane, except for the Galactic center. Even in this minimal configuration, the high Galactic latitude region is not well constrained, and we therefore sample this from the prior alone, as for the Galactic Center. This leaves only two regions (the Galactic Spur and the Galactic Plane) to be sampled properly with the full posterior distribution, and, fortunately, both of these appear to be both signal-dominated and stable with respect to instrumental parameter variations.
A part of the resulting Gibbs chain is shown in Fig. 9, and the full posterior mean and standard deviation synchrotron spectral index maps are shown in Fig. 10. Additionally, a histogram of the spur and plane regions can be seen in Fig. 11 along with the prior, where the two regions appear to be likelihood-dominated. For the two data-constrained regions, we find $\beta_{\text{Spur}} = -3.15 \pm 0.07$ and $\beta_{\text{Plane}} = -3.12 \pm 0.06$. First, both of these values are consistent with previous constraints, for instance as reported by Dunkley et al. (2009) and Planck Collaboration X (2016). At the same time, we find no compelling evidence for spectral index steepening toward the Galactic Spur, and our results are therefore qualitatively similar to those reported by Dunkley et al. (2009), who found a variation of only $\Delta \beta_s = 0.08$ from high to low Galactic latitudes.

Figure 12 compares histograms of $\beta_{\text{Spur}}$ as sampled with three different analysis configurations. The blue distributions are sampled with the default Gaussian spectral index prior of $\beta_s = -3.1$ (shown as a dashed blue curve), while the red distribution is sampled with a Gaussian prior of $\beta_s = -2.8$ (dashed red curve). Furthermore, the thick blue distribution is sampled with full marginalization over TOD parameters, while the thin blue distribution is sampled by fixing the LFI frequency maps at one arbitrary Gibbs sample. Thus, the former marginalizes over the full BEYONDPLANCK data model, while the latter only marginalizes over foreground parameters and white noise, as does the red distribution.

Several important conclusions can be drawn from this plot. First, we note that thick blue histogram is almost twice as wide as the thin blue histogram. This indicates that more than half of the full uncertainty of $\beta_s$ is actually due to low-level TOD parameter variations (i.e., correlated noise, gain and bandpasses), and marginalizing only with respect to foreground amplitudes and white noise will significantly underestimate its full error budget. This observation underscores the importance of global modeling. Second, we see that although the red histogram is pulled toward higher values, as expected from the shifted prior, it is centered on the low tail of the prior distribution. The data clearly prefer values $\beta_s \lesssim -3$. Overall, our literature-based default prior of $\beta_s = -3.10 \pm 0.10$ appears to be a good match to the current dataset.

As an additional validation of these results, we replace the three BEYONDPLANCK-processed LFI frequency map samples with
6. Summary and conclusions

The two main goals of this paper are to introduce a Bayesian sampling algorithm for polarized CMB foreground models as embedded within the end-to-end BeyondPlanck framework, and to present the first results from this pipeline as applied to the Planck LFI data set. This is the first time a joint global parametric model that accounts for both instrumental and astrophysical parameters has been fit to the Planck LFI data within a single joint posterior distribution, allowing for seamless end-to-end error propagation. This is also the first time a joint analysis of the Planck LFI and WMAP data has resulted in physically meaningful spectral indices for both polarized synchrotron and thermal dust emission. This analysis thus paves the way for future analyses that should integrate and analyze more data sets.

Indeed, we stress that the current analysis configuration has been specifically designed to demonstrate the properties and performance of the algorithm itself, not to derive a new best-fit sky model. Specifically, critically important data sets, such as WMAP K-band and Planck HFI, have been intentionally omitted, precisely because of their high signal-to-noise ratios; if they had been included, the results would have been dominated by WMAP and HFI methodology. Including these data sets, and other important ones like C-BASS (Jew et al. 2019) or QUIJOTE (Génova-Santos et al. 2015) will be done in future work, either by members within the current BeyondPlanck team or by external researchers, and either by starting from time-ordered or from pre-pixelized sky maps. In many respects, the current analysis configuration may very possibly represent one of the most difficult challenges that the BeyondPlanck pipeline will face, since it is the least constrained; future analyses will always have access to more data, and the resulting sky models will therefore be less degenerate.

With that important caveat in mind, particularly notable highlights from the current analysis include the following:

1. We constrain the spectral index of polarized synchrotron emission, $\beta_s$, in two large and disjoint regions of the sky, covering the Galactic Spur and the Galactic Plane, with best-fit values $\beta_{spur} = -3.15 \pm 0.06$ and $\beta_{plan} = -3.12 \pm 0.05$, respectively. These results are statistically consistent with previous WMAP-only results (Dunkley et al. 2009). The current analysis finds no compelling evidence for statistically significant spatial variations in $\beta_s$. At the same time, we note that the high Galactic latitude regions have a too low signal-to-noise ratio to support any robust conclusions regarding $\beta_s$, while the Galactic center region exhibits too strong residual systematic effects.

2. We constrain the spectral index of thermal dust emission between 30 and 70 GHz to $\beta_d = 1.62 \pm 0.04$, which is somewhat steeper than previously reported by Planck HFI (Planck Collaboration Int. LVII 2020). This means value is thus slightly steeper than expected based on HFI, with a statistical significance of about 2 $\sigma$. Furthermore, the uncertainty is significantly smaller than the prior width, which suggests that the result is indeed data-driven, even when marginalizing over the full BeyondPlanck instrument model.

3. Through joint analysis of the Planck and WMAP data, we have been able to isolate and highlight the effect of transmission imbalance in the WMAP observations. This strongly suggests that a future joint analysis of Planck and WMAP data in time-domain will be able to constrain the WMAP transmission imbalance parameters to high accuracy.

Figure 14 provides an overview of the main polarized microwave components in the frequency range from 10 to 1000 GHz, as described by the posterior BeyondPlanck results and our assumed sky model with the addition of spinning dust with a polarization fraction of 1% (Génova-Santos et al. 2016).
Here, each component is represented in terms of the standard deviation of the polarization amplitude evaluated over 88% (top edge of each band) and 27% (bottom edge of each band) of the sky, respectively. An artificial broadening has been applied to the low-amplitude tails of the foreground spectra to visually indicate uncertainties in low signal-to-noise regions for the relevant components. The EE CMB spectrum is generated from ideal CMB simulations based on the best-fit Planck ΛCDM model, while the BB limits are derived with tensor-to-scalar ratios of $r = 10^{-2}$ and $10^{-4}$, respectively. Additionally, we visualize an upper limit on the polarization fraction of 1% as has been reported in literature (Génova-Santos et al. 2016; Macelli et al. 2011).

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