New Einstein-Hilbert-type Action and Superon-Graviton Model (SGM) of Nature

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Abstract

A nonlinear supersymmetric (NLSUSY) Einstein-Hilbert (EH)-type new action for unity of nature is obtained by performing the Einstein gravity analogue geometrical arguments in high symmetry spacetime inspired by NLSUSY. The new action is unstable and breaks down spontaneously into E-H action with matter in ordinary Riemann spacetime. All elementary particles except graviton are composed of the fundamental fermion “superon” of Nambu-Goldstone (NG) fermion of NLSUSY and regarded as the eigenstates of SO(10) super-Poincaré (SP) algebra, called superon-graviton model (SGM) of nature. Some phenomenological implications for the low energy particle physics and the cosmology are discussed. The linearization of NLSUSY including N=1 SGM action is attempted explicitly to obtain the linear SUSY local field theory, which is equivalent and renormalizable.

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1 Introduction

The standard model (SM) is established as a unified model for the strong-electroweak interactions. Nevertheless, there still remains many unsolved problems, e.g. it can not explain the particle quantum numbers \((Q, I, Y, color)\), i.e. \(1 \times 2 \times 3\) gauge structure, the three-generations structure of quarks and leptons and contains more than 28 arbitrary parameters (in the case of neutrino oscillations) even disregarding the mass generation mechanism for neutrino. The simple and beautiful extension to SU(5) GUT has serious difficulties, e.g. the life time of proton, \(\cdots\), etc and is excluded so far. The SM and GUT equipped naively with SUSY\([1]\)[2][3] have improved the situations, e.g. the unification of the gauge couplings at about \(10^{17}\), relatively stable proton (now threatened by experiments), etc., but they possess more arbitrary parameters which diminish the naturalness of the unification. Also SUSY model usually requires two times more number of elementary particles than non-SUSY model, e.g., at least about 60 for SUSY SM and 160 for SUSY GUT in curved spacetime.

SUSY is an essential notion to unify various topological and non-topological charges and gives a natural framework to unify spacetime and matter leading to the birth of supergravity theory (SUGRA)\([4]\)[5]. Unfortunately the maximally extended SO(8) SUGRA\([6]\) is too small to accommodate all observed particles\([7]\). The straightforward extension to SO(N) SUGRA with \(N > 9\) has a difficulty in writing down the action due to so called the no-go theorem on the gravitational interaction of massless elementary high spin \((> 2)\) (gauge) field\([8]\)[9]. (The massive high-spin field is another.)

We think that from the viewpoints of simplicity and beauty of nature it is natural to attempt the accommodation of all observed particles in a single irreducible representation of a certain algebra (group) especially in the case of spacetime having a certain boundary, i.e. a boundary condition. And the dynamics is to be described by the spontaneous breakdown of the high symmetry of spacetime by itself, which is encoded in (the nonliner realization of) the geometrical arguments of spacetime. Facing so many fundamental elementary particles and so many arbitrary parameters, we are tempted to imagine that they are the composite objects and/or that they should be attributed to the particular topological and geometrical structure of spacetime. Also the no-go theorem does not exclude the possibility that the fundamental action, if it exists, possesses the high-spin degrees of freedom (d.o.f.) not as the elementary massless fields but as certain composite eigenstates of a certain symmetry (algebra) of the fundamental action recasted by the asymptotic local fields. Note that the relativistic hydrogen atom is solved by O(4) symmetry and that the particular structure (and the boundary) of the bulk materials induces the high-\(T_c\) superconductivity containing s- and d-wave pairs. In this talk we would like to present a model along this scenario according to the chronological order of the studies.
The structure of this paper is as follows. In section 2, we show by the group theoretical arguments that three generations of quarks and leptons can be accommodated in the single irreducible representation of SO(10) SP algebra and propose SGM for a composite model of observed particles except graviton. In section 3, the fundamental SGM action of the vacuum EH-type is obtained. In section 4, the linearization of NLSUSY is discussed to obtain the the equivalent (broken) LSUSY theory, which is renormalizable. In section 5, SGM with spin 3/2 NG fermion is presented. In section 6, the cosmological meaning of SGM is discussed qualitatively. In section 7, as a summary some characteristic properties of SGM including many open questions are discussed briefly.

2 SGM and SO(10) SP Algebra

In this section we study a single irreducible representation which contains all observed elementary particles. By considering the structure of the helicity states of the representation of SP algebra it is natural to consider SO(N) extension of SUSY. We have shown that among all single irreducible representations of all SO(N) extended SP symmetries, the massless irreducible representations of SO(10) SP algebra(SPA) is the only one that accommodates minimally all observed particles including the graviton\cite{10,11}. That is, SO(10) SP is unique among all SO(N) SP extension of the framework of SGM discussed below\cite{10}.

SO(10) SP contains 10 generators $Q^N (N = 1, 2, .., 10)$, which are the ten dimensional fundamental representations of SO(10) internal symmetry. We have decomposed 10 generators $Q^N (N = 1, 2, .., 10)$ into $10 = \bar{5} + \bar{5}^*$ with respect to SU(5) following $SO(10) \supset SU(5) \times U(1)$. For the massless case the little algebra of SO(10) SPA for the supercharges in the light-cone frame $P_\mu = \epsilon (1, 0, 0, 1)$ becomes after a suitable rescaling

$$\{Q^M_\alpha, Q^N_\beta\} = \{\bar{Q}^M_\dot{\alpha}, \bar{Q}^N_\dot{\beta}\} = 0, \quad \{Q^M_\alpha, \bar{Q}^N_\dot{\beta}\} = \delta_{\alpha\dot{\beta}} \delta^{MN},$$

(1)

where $\alpha, \beta = 1, 2$ and $M, N = 1, 2, .., 5$\cite{12,13,14}. By identifying the graviton with the Clifford vacuum $|\Omega\rangle$ (SO(10) singlet) satisfying $Q^M_\alpha |\Omega\rangle = 0$ and performing the ordinary procedures\cite{15,16} we obtain 2·210 dimensional irreducible representation of the little algebra $\{\}$ of SO(10) SPA as follows\cite{14}:

- $[4]^+(+2), [10]^+(+\frac{3}{2}), [45]^+(+1), [120]^+(+\frac{5}{2}), [210]^0, [252]^-(\frac{1}{2}), [210]^-(1), [120]^-(\frac{3}{2}), [45]^-(2), [10]^-(\frac{5}{2}), [1]^-(3)] + [CPT-conjugate],$

where $d(\lambda)$ represents SO(10) dimension $d$ and the helicity $\lambda$. By noting that the helicities of these states are automatically determined by SO(10) SPA in the lightcone and that $Q^M_\alpha$ and $\bar{Q}^M_\dot{\alpha}$ satisfy the algebra of the annihilation and the creation operators for the massless spin $\frac{1}{2}$ particle, we speculate boldly that these massless states spanned upon the Clifford vacuum $|\Omega(\pm 2)\rangle$ are the massless (gravitational) eigenstates of spacetime and matter with SO(10) SP symmetric structure, which
are composed of the fundamental massless object $Q^N$, \textit{superon} with spin $\frac{1}{2}$ \cite{11} \cite{17}. Because they correspond merely to all possible nontrivial combinations of the multiplications of the spinor charges (i.e. \textit{creation} or \textit{annihilation operators}) of SO(10) SP algebra. Simultaneously we can escape from the no-go theorem in a sense that we can write down the fundamental action with $N > 9$. Therefore we regard $\tilde{5} + \tilde{5}^*$ as a \textit{superon-quintet} and an \textit{antisuperon-quintet}. And we call the model containing the above towers of the helicity states are superon-graviton model (SGM). The justification of this bold assumption is given later. Interestingly the composite model of matter based upon VA model \cite{2} was attempted long time ago \cite{18}. To survey the physical implications of superon model for matter we assign the following SM quantum numbers to superons and adopt the following symbols.

\[
\begin{align*}
\begin{bmatrix} 10 \end{bmatrix} &= \begin{bmatrix} 5 & 5^* \end{bmatrix} \\
 &= \left[ Q_a (a = 1, 2, 3), Q_m (m = 4, 5) \right] + \left[ Q^*_a (a = 1, 2, 3), Q^*_m (m = 4, 5) \right], \\
&= \left( [3, 1; -1, -1, -1], [1, 2; 1, 0] \right) + \left( [3^*, 1; 1, 1, 1], [1, 2^*; -1, 0] \right),
\end{align*}
\]  

where we have specified $(SU(3), SU(2); \text{electric charges})$ and $a = 1, 2, 3$ and $m = 4, 5$ represent the color and electroweak components of superons respectively. Our model needs only five superons as the fundamental elementary objects, which have surprisingly the same quantum numbers as the fundamental matter multiple $\tilde{5}$ of SU(5) GUT \cite{19} and satisfy the Gell-Mann–Nishijima relation.

\[ Q_e = I_z + \frac{1}{2} (B - L). \]  

Consequently all $2 \cdot 2^{10}$ states are specified uniquely with respect to $(SU(3), SU(2); \text{electric charges})$ \cite{11}. Here we suppose drastically that the fundamental action of SGM exists and that all such composite states are represented by local fields in a certain energy scale with $SU(3) \times SU(2) \times U(1)$ invariance, where by absorbing the lower helicity states through the superHiggs mechanism and through the diagonalizations of the mass terms the high-spin fields become massive through the spontaneous symmetry breaking $[SO(10) \rightarrow SU(3) \times SU(2) \times U(1)]$. We have carried out the recombinations of the helicity states and found surprisingly that besides the gauge bosons for $3 \times 2 \times 1$ all the massless states necessary for the SM with three generations of quarks and lepton (therefore, no sterile neutrinos) \cite{10}. As for the assignments of observed particles, we take for simplicity the following left-right symmetric assignment for quarks and leptons by using the conjugate representations naively, i.e. $\nu \bar{l} = \bar{\nu} l^+$, etc \cite{11}. Furthermore as for the identification of the generation of quarks and leptons we assume simply that the states with more (color-) superons turn to acquiring larger masses in the low energy
and no a priori mixings among generations. The surviving massless states identified with SM(GUT) are as follows.

(In the paper [11] the right-handed neutrinos are denoted as a new particle, for the right-handed neutrinos were not observed at that time.)

For three generations of leptons \([\nu_e, e], (\nu_\mu, \mu), (\nu_\tau, \tau)\] we take

\[
\left[ (Q_m \varepsilon_{ln} Q_l^* Q_m), (Q_m \varepsilon_{ln} Q_l^* Q_a Q_a^*), (Q_a Q_a Q_b Q_b Q_m^*) \right]
\]

and the conjugate states respectively. SGM contains initially four lepton generations and one of them (in the present case, \(Q_m Q_a Q_a^*\)) disappears by the superHiggs mechanism.

For three generations of quarks \([u, d], (c, s), (t, b)\] we have

\[
\left[ (\varepsilon_{abc} Q_b^* Q_c^* Q_m^*), (\varepsilon_{abc} Q_b^* Q_c^* Q_l \varepsilon_{mn} Q_m^* Q_n^*), (\varepsilon_{abc} Q_b^* Q_c^* Q_d Q_d^* \right]
\]

and conjugate states respectively.

For \(SU(2) \times U(1)\) gauge bosons \([W^+, Z, \gamma, W^-]\), \(SU(3)\) gluons \([G^a (a = 1, 2, \ldots, 8)]\), \([SU(2)\) Higgs Boson], \([X, Y]\) leptoquark bosons in GUTs, and a color- and \(SU(2)\)-singlet neutral gauge boson from \(3 \times 3^*\) (which we call simply \(S\) boson to represent the singlet) we have

\[
\left[ Q_1 Q_5^*, \frac{1}{\sqrt{2}} (Q_1 Q_1^* \pm Q_5 Q_5^*), Q_5 Q_4^* \right],
\]

\[
[Q_1 Q_3^*, Q_2 Q_3^*, -Q_1 Q_2^*, \frac{1}{\sqrt{2}} (Q_1 Q_1^* - Q_2 Q_2^*), Q_2 Q_1^*, \frac{1}{\sqrt{6}} (2Q_3 Q_3^* - Q_2 Q_2^* - Q_1 Q_1^*),
-\sqrt{Q_2 Q_2^*}, Q_3 Q_1^*], \left[ \varepsilon_{abc} Q_a Q_b Q_c Q_m \right], [Q_a Q_m^* \text{ and } Q_a Q_a^*, (and their conjugates)] \text{ respectively.}
\]

Now in order to test the superon picture and to see the potential of superon-quintet model(SQM) of matter in the low energy we try to interpret the Feynman diagrams of SM(GUT) in terms of the superon pictures of all particles in SM(GUT). We replace a single line of the propagator of a particle in the Feynman diagrams of SM(GUT) by the multiple lines of superons constituting each particle under the following two assumptoions at the vertex;

(i) the analogue of the OZI-rule of the quark model and
(ii) the superon number conservation.

The assumption (i) is natural, for all particles are assigned to each state of a single irreducible representation of \(SO(10)\) SP algebra. Fig.1 shows the corresponding forbidden superon-line Feynmann diagram showing the transition between the different eigenstates \(a\) and \(b\) without the interaction. Fig.2 is the allowed diagram showing a interacting(decay) vertex \(a \to b + c\). (ii) is a superon number conservation and gives naturally a selection rule at the vertex as read in Fig.2.
We have studied whether Feynmann diagrams of SM and (SUSY)GUT are reproduced (i.e., SGM allowed) or not (i.e., SGM forbidden) by the superon pictures under the asumptions (i) and (ii).

As an example of the allowed diagram, $\beta$ decay (Fig.3) is drawn in Fig.4 in terms of superons.
We find many remarkable results and show below only some of them. In the SM (the low energy); the naturalness of the mixing of $K^0$, $D^0$, and $B^0$, (the difference of the mass eigenstates and the electroweak eigenstates), no CKM-like (but the different origins beyond SM) mixings among the lepton generations, $\pi^0 \rightarrow 2\gamma$ as an ordinary tree diagram of the dominating decay mode, no $\mu \rightarrow e + \gamma$ despite compositeness, \cdots, etc.

Beyond the SM; $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$ transitions, the origins of the observed (strong) CP-violation and their qualitative differences among $K^0$, $D^0$, and $B^0$, the tiny values of the SM Yukawa couplings constants as effective couplings, \cdots, etc.

In (SUSY)GUT; no (Fig.6) dangerous tree diagrams inducing (Fig.5) proton decay (without introducing R-parity by hand for SUSY GUT), \cdots etc.

Fig.5 shows as an example the dangerous Feynmann diagram of the main decay mode of proton $p \rightarrow \pi^0 + e$ in GUT. Fig.6 shows that the gauge coupling vertex (the dotted circle of Fig.5) can not be reproduced in terms of superons, which indicates proton does not decay $p \rightarrow \pi^0 + e$ at the tree level. Also proton is stable against the decay $p \rightarrow K^+ + \bar{\nu}$ in SUSY GUT, for the dangerous box-type Feynmann diagram of the decay $p \rightarrow K^+ + \bar{\nu}$ in SUSY GUT can not be reproduced in the superon picture.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4}
\caption{Fig.4}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig5}
\caption{Fig.5}
\end{figure}
Among predicted new particles in the low energy (i.e., the states escape from being absorbed)\[11\]: one lepton-type electroweak-doublet ($\nu_T, \Gamma^-$) with spin $\frac{3}{2}$, with the mass of the electroweak scale ($\leq Tev$) and an electroweak-singlet and double-charge Dirac particle $E^{++}$ with spin $\frac{1}{2}$ which should have large mass($> Tev$) are color-singlet states for matter and can be produced directly.

And the effects of S gauge boson may be observed in the coming (high energy) experiments, particularly in $B^0$ (and $D^0$) decay and in the various mixings of the electroweak eigenstates.

Considering SUSY SM is usually equipped with $SU(3) \times SU(2) \times U(1) \times U(1)$ superon-quintet model(SQM)\[11\] of matter may be the most economic gauge model containing the three generations of quarks and leptons.

Finally we just remark about the (superHiggs) mass generation mechanism assumed boldly above. This may be probable if the symmetry between the states containing up to 5 superons and above 5 superons is broken spontaneously. And/or it is probable if the symmetry between the massless states with spin $J$ and $J-1$ ($J \leq 3$) is broken. One half of the helicity states (e.g. the superpartners) become massive and decouple, provided the mass is huge.
3 Fundamental Action of SGM

In this section we justify the superon hypothesis. The supercharges $Q$ of VA action of NLSUSY\(^{[2]}\) is computed by the supercurrents\(^{[22]}\)

$$ J^\mu(x) = \frac{1}{i} \sigma^\mu \psi(x) - \kappa \{ \text{the higher order terms of } \kappa, \psi(x) \text{ and } \partial \psi(x) \} \quad (6) $$

(6) means the field-current identity between the elementary NG spinor field $\psi(x)$ and the supercurrent, which justifies our bold assumption that the generator(supercharge) $Q^N \sim \int J^0 dx^3$ (N=1,2,..10) of SO(10) SPA in the light-cone frame represents the fundamental massless particle, superon with spin $\frac{1}{2}$. Therefore superon is the NG fermion of NLSUSY and the fundamental theory of SGM for spacetime and matter at(above) the Planck scale is SO(10) NLSUSY in the curved spacetime(corresponding to the Clifford vacuum $| \Omega(\pm 2) \rangle$).

It is well known that it is impossible to write down the action of SUGRA with $N > 8$ due to so called the no-go theorem on massless high spin($> 2$) field based upon the S-matrix arguments.

However we show that disregarding a priori S-matrix constraints at the begining and giving weight to the geometrical arguments we can construct N=10 extended SUSY theory containing the gravitational interaction.

Extending the geometrical arguments of Einstein general relativity theory\(^{[2]}\) on Riemann spacetime to new (SGM) spacetime where besides the Minkowski coordinate $x^a$ the coset space coordinates $\psi$ for SL(2C) of $^{\text{super}}GL(4,R)$/$GL(4,R)$ turning to the NG fermion degrees of freedom(d.o.f.) are attached at every spacetime point, we obtain the following N=10 SGM action\(^{[17]}\):

$$ L_{SGM} = -\frac{c^3}{16\pi G} |w|(\Omega + \Lambda), \quad (7) $$

$$ |w| = det w^a_\mu = det(e^a_\mu + t^a_\mu), \quad t^a_\mu = \frac{i\kappa^4}{2} (\bar{\psi}^j \gamma^a \partial_\mu \psi^j - \partial_\mu \bar{\psi}^j \gamma^a \psi^j), \quad (8) $$

where $w^a_\mu$ and $e^a_\mu$ are the vierbeins of unified SGM spacetime and Riemann spacetime of EGRT respectively, $\psi^j$($j = 1,2,..,10,)$ is NG fermion(superon) originating from the coset space coordinates of $\frac{N=10 \; ^{\text{super}}GL(4,R)}{GL(4,R)}$, G is the gravitational constant, $\kappa^4 = \left( \frac{c^3 \Lambda}{16\pi G} \right)^{-1}$ is a fundamental volume of four dimensional spacetime of VA model\(^{[2]}\), and $\Lambda$ is a small cosmological constant related to the strength of the superon-vacuum coupling constant. Therefore SGM contains two mass scales, $\frac{1}{\sqrt{G}}$(Planck scale) in the first term describing the curvature energy and $\kappa \sim O(1)$ in the second term describing the vacuum energy of SGM. $\Omega$ is a new scalar curvature analogous to the Ricci scalar curvature $R$ of EGRT, whose explicit expression
is obtained by just replacing $e^a_\mu(x)$ by $w^a_\mu(x)$ in Ricci scalar $R$. These results can be understood intuitively by observing that

$$w^a_\mu(x) = e^a_\mu(x) + t^a_\mu(x),$$

(9)

inspired by

$$\omega^a = dx^a + \frac{i\kappa^4}{2}(\bar{\psi}^j\gamma^a\psi^j - d\bar{\psi}^j\gamma^a\psi^j) \sim w^a_\mu dx^\mu,$$

(10)

where $\omega^a$ is the NLSUSY invariant differential forms of VA, is invertible, i.e.,

$$w^\mu_a = e^\mu_a - t^\mu_a - t^\mu_{\rho\sigma}t^\rho_{\sigma}t^\mu a + \cdots,$$

(11)

which terminates with $(t)^{10}$ and $s_{\mu\nu} \equiv w^a_\mu\eta_{ab}w^b_\nu$ and $s^{\mu\nu}(x) \equiv w^a_\mu(x)w^a_\nu(x)$ are a unified vierbein and a unified metric tensor in SGM spacetime. It is straightforward to show $w^a_\mu w_{\mu b} = \eta_{ab}$, $s_{\mu\nu} w^a_\mu w^b_\nu = \eta_{ab}$, etc. [As shown in (8), throughout the paper the first and the second indices of $t$ represent those of the $\gamma$-matrix and the derivative, respectively.] It seems natural that the ordinary vierbein and the stress-energy-momentum tensor of superon contribute equally to the vierbein of the unified (SGM) spacetime.

The SGM action (7) is invariant at least under the following symmetries; global unified (SGM) spacetime.

$$\delta^{NL}_L \psi^i(x) = \frac{1}{\kappa^4} L^i + i\kappa^4 (\bar{\psi}^j\gamma^i\psi^j(x))\partial_\rho \psi^i(x), \quad \delta^{NL}_L e^a_\mu(x) = i\kappa^4 (\bar{\psi}^j\gamma^a\psi^j(x))\partial_\rho e^a_\mu(x),$$

(12)

where $L^i, (i = 1, \ldots, 10)$ is a constant spinor and $\partial_\rho e^a_\mu(x) = \partial_\rho e^a_\mu - \partial_\mu e^a_\rho$, the following GL(4R) transformations due to (12)

$$\delta_L w^a_\mu = \xi^\nu\partial_\nu w^a_\mu + \partial_\mu \xi^\nu w^a_\nu, \quad \delta_L s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\kappa\mu},$$

(13)

where $\xi^\rho = i\kappa^4 (\bar{\psi}^j\gamma^\rho\psi^j(x))$, and the following local Lorentz transformation on $w^a_\mu$

$$\delta_L w^a_\mu = e^a_\beta w^\beta_\mu,$$

(14)

with the local parameter $\epsilon_{ab} = (1/2)\epsilon|_{ab}(x)$ or equivalently on $\psi$ and $e^a_\mu$

$$\delta_L \psi(x) = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi, \quad \delta_L e^a_\mu(x) = e^a_\epsilon e^\beta_\mu + \frac{\kappa^4}{4}\epsilon^{abcd}\bar{\psi}^j_i\gamma^i_d\psi^j (\partial_\mu \epsilon_{bc}),$$

(15)

The local Lorentz transformation forms a closed algebra, for example, on $e^a_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}] e^a_\mu = \beta^a_\beta e^\beta_\mu + \frac{\kappa^4}{4}\epsilon^{abcd}\bar{\psi}^j_i\gamma^i_d\psi^j (\partial_\mu \beta_{bc}),$$

(16)
where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ae} \epsilon_1 c_b - \epsilon_{2be} \epsilon_1 c_a$. The commutators of two new NLSUSY transformations (12) on $\psi(x)$ and $e^a_{\mu}(x)$ are GL(4R), i.e. new NLSUSY (12) is the square-root of GL(4R);

$$[\delta_1, \delta_2] \psi = \Xi_\mu \partial_\mu \psi, \quad [\delta_1, \delta_2] e^a_{\mu} = \Xi^\rho \partial_\rho e^a_{\mu} + e^a_{\rho} \partial_\mu \Xi^\rho,$$

(17)

where $\Xi^\mu = 2i\kappa (\bar{\zeta}_2 \gamma^\mu \zeta_1) - \zeta_1^\rho \zeta_2^\sigma e^a_{\mu}(\partial_{\rho} e^a_{\sigma})$. They show the closure of the algebra. (The ordinary GL(4R) invariance is trivial by the construction.) SGM action (7) is invariant at least under

$$[\text{global new NLSUSY}] \otimes [\text{local GL}(4, \mathbb{R})] \otimes [\text{local Lorentz}] \otimes [\text{global SO}(10)],$$

(18)

which is isomorphic to SO(10)SP whose single irreducible representation gives the group theoretical description of SGM.

Note that the no-go theorem is overcome in a sense that the massless N-extended theory with $N > 8$ has been written down explicitly. Here we just mention that the superon d.o.f. can be gauged away neither by the ordinary GL(4R) transformations of $e^a_{\mu}(x)$ connecting $x^b$ and $x^a$ nor by the local spinor translation, e.g. $\delta \psi(x) = \zeta(x)$, $\delta e^a_{\mu}(x) = -i\kappa (\bar{\zeta}_2 \gamma^\mu \psi(x) + \bar{\psi}(x) \gamma^\mu \zeta(x))$ which is nothing but a translation(redefinition) of the spinor coordinate in SGM spacetime. Therefore the action is a nontrivial generalization of the EH action. Further details are read in Sec.7.

Also it should be noticed that SGM action poses two types of flat space which are not equivalent, i.e. SGM-flat($e^a_{\mu}(x) \to \delta^a_{\mu}$) and Riemann flat($e^a_{\mu}(x) \to \delta^a_{\mu}$). This structure plays important roles in the cosmology of SGM discussed in Sec 6. The linearization of such a theory with a high nonlinearity is an interesting and inevitable to obtain an equivalent local field theory which is renormalizable and describes the observed low energy (SM) physics. We discuss these problems in the next section.

4 Toward Linearization of SGM

4.1 Linearization of N=1 NLSUSY in flat spacetime

In advance of the linearization of the SGM we investigate the linearization of VA model in detail. The linearization of VA model has been investigated by many authors and proved that N=1 VA model of NLSUSY is equivalent to N=1 scalar supermultiplet action of LSUSY which is renormalizable. The general arguments on the constraints which gives the relations between the linear and the nonlinear realizations of global SUSY have been established. Following the general arguments we have shown explicitly that the nonrenormalizable N=1 VA
model is equivalent to a renormalizable action of a U(1) gauge supermultiplet of the linear SUSY [1] with the Fayet-Iliopoulos(FI) D term [2] indicating a spontaneous SUSY breaking [28]. Remarkably we find that the magnitude of FI term (vacuum value) is determined uniquely to reproduce the correct sign of VA action and that a U(1) gauge field constructed explicitly in terms of NG fermion fields is an axial vector field for N=1.

An N = 1 U(1) gauge supermultiplet is given by a real superfield [13] [14]

\[ V(x, \theta, \bar{\theta}) = C + i\theta \chi - i\bar{\theta} \tilde{\chi} + \frac{1}{2} i\theta^2 (M + iN) - \frac{1}{2} i\bar{\theta}^2 (M - iN) - \theta \sigma^m \bar{\theta} v_m \]

\[ + i\theta^2 \bar{\theta} \left( \lambda + \frac{1}{2} i \sigma^m \partial_m \chi \right) - i\bar{\theta}^2 \theta \left( \tilde{\lambda} + \frac{1}{2} i \sigma^m \partial_m \tilde{\chi} \right) + \frac{1}{2} \theta^2 \bar{\theta}^2 \left( D + \frac{1}{2} \Box \zeta \right) \]

where \( C(x), \ M(x), \ N(x), \ D(x) \) are real scalar fields, \( \chi_a(x), \ \lambda_a(x) \) and \( \tilde{\chi}_a(x), \ \tilde{\lambda}_a(x) \) are Weyl spinors and their complex conjugates, and \( v_m(x) \) is a real vector field. We adopt the notations in ref. [12]. Following refs. [25], we define the superfield \( \tilde{V}(x, \theta, \bar{\theta}) \) by

\[ \tilde{V}(x, \theta, \bar{\theta}) = V(x', \theta', \bar{\theta}'), \quad x'^m = x^m + i\kappa \left( \zeta(x) \sigma^m \bar{\theta} - \theta \sigma^m \bar{\zeta}(x) \right), \quad \theta' = \theta - \kappa \zeta(x), \quad \bar{\theta}' = \bar{\theta} - \kappa \bar{\zeta}(x). \]

\[ \tilde{V} \]

may be expanded as [19] in component fields \( \{ \tilde{\phi}_i(x) \} = \{ \tilde{C}(x), \tilde{\chi}(x), \tilde{\lambda}(x), \ldots \} \), which can be expressed by \( C, \chi, \tilde{\chi}, \ldots \) and \( \zeta, \bar{\zeta} \) by using the relation (20). \( \kappa \) is now defined with the dimension (length)². They have the supertransformations of the form

\[ \delta \tilde{\phi}_i = -i \kappa \left( \zeta \sigma^m \bar{\epsilon} - \epsilon \sigma^m \bar{\zeta} \right) \partial_m \tilde{\phi}_i. \]

Therefore, a condition \( \tilde{\phi}_i(x) = \) constant is invariant under supertransformations. As we are only interested in the sector which only depends on the NG fields, we eliminate other degrees of freedom than the NG fields by imposing SUSY invariant constraints

\[ \tilde{C} = \tilde{\chi} = \tilde{M} = \tilde{N} = \tilde{v}_m = \tilde{\lambda} = 0, \quad \tilde{D} = \frac{1}{\kappa}. \]

Solving these constraints we find that the original component fields \( C, \chi, \tilde{\chi}, \ldots \) can be expressed by the NG fields \( \zeta, \bar{\zeta} \). Among them, the leading terms in the expansion of the fields \( v_m, \lambda, \tilde{\lambda} \) and \( D \), which contain gauge invariant degrees of freedom, in \( \kappa \) are

\[ v_m = \kappa \zeta \sigma^m \bar{\zeta} + \ldots, \quad \lambda = i \zeta - \frac{1}{2} \kappa^2 \zeta \left( \zeta \sigma^m \partial_m \zeta - \partial_m \zeta \sigma^m \zeta \right) + \ldots, \quad D = \frac{1}{\kappa} + i \kappa \left( \zeta \sigma^m \partial_m \zeta - \partial_m \zeta \sigma^m \zeta \right) + \ldots, \]

where \( \ldots \) are higher order terms in \( \kappa \). Our discussion so far does not depend on a particular form of the action. We now consider a free action of a U(1) gauge
supermultiplet of LSUSY with a FI \( D \) term. In component fields we have

\[
S = \int d^4x \left[ -\frac{1}{4} v_{mn} v^{mn} - i\lambda^m \partial_m \bar{\lambda} + \frac{1}{2} D^2 - \frac{1}{\kappa} D \right].
\]

(25)

The last term proportional to \( \kappa^{-1} \) is the FI \( D \) term. The field equation for \( D \) gives \( D = \frac{1}{\kappa} \neq 0 \) in accordance with eq. (23), which indicates the spontaneous breakdown of supersymmetry. We substitute eq. (24) into the action (25) and obtain an action for the NG fields \( \zeta, \bar{\zeta} \) which is exactly N=1 VA action.

\[
S = -\frac{1}{2\kappa^2} \int d^4x \det \left[ \delta_m^n + i\kappa^2 \left( \zeta^m \partial_n \bar{\zeta} - \partial_m \zeta^n \bar{\zeta} \right) \right].
\]

(26)

For N=1, U(1) gauge field becomes \( v_m \sim \kappa \bar{\zeta} \gamma_m \gamma_5 \zeta + \cdots \) in the four-component spinor notation, which is unfortunately an axial vector and cannot be identified with the observed vector gauge boson of SM. However these are very suggestive and favourable to SGM and as shown in the next section the vector gauge field and \( SU(2) \) gauge structure appear simultaneously in N=2.

### 4.2 Linearization of N=2 NLSUSY in flat spacetime

Next we focus our attention to the \( N = 2 \) SUSY and discuss a connection between the VA model and an \( N = 2 \) vector supermultiplet [31] of the linear SUSY in four-dimensional spacetime. In particular, we show that for the \( N = 2 \) theory a SUSY invariant relation between component fields of the vector supermultiplet and the NG fermion fields can be constructed by means of the method used in Ref. [21] starting from an ansatz given below (Eq. [30]). We also briefly discuss a relation of the actions for the two models.

Let us denote the component fields of an \( N = 2 \) U(1) gauge supermultiplet [31], which belong to representations of a rigid SU(2) [38], as follows; namely, \( \phi \) for a physical complex scalar field, \( \lambda^i_R \) \((i = 1, 2)\) for two right-handed Weyl spinor fields and \( A_a \) for a U(1) gauge field in addition to \( D^I \) \((I = 1, 2, 3)\) for three auxiliary real scalar fields required from the mismatch of the on-shell degrees of freedom between bosonic and fermionic physical fields. \( \lambda^i_R \) and \( D^I \) belong to representations 2 and 3 of SU(2) respectively while other fields are singlets. By the charge conjugation we define left-handed spinor fields as \( \lambda^i_L = C \lambda^i_R \). We use the antisymmetric symbols \( \epsilon^{ij} \) and \( \epsilon_{ij} \) \((\epsilon^{12} = \epsilon_{21} = +1)\) to raise and lower SU(2) indices as \( \psi^i = \epsilon^{ij} \psi_j, \psi_i = \epsilon_{ij} \psi^j \).

The \( N = 2 \) LSUSY transformations of these component fields generated by constant spinor parameters \( \zeta^i_L \) are

\[
\delta_Q \phi = -\sqrt{2} \zeta^i_R \lambda^i_L,
\]
\[
\delta Q \lambda_{Li} = -\frac{1}{2} F_{ab} \gamma^{ab} \zeta_{Li} - \sqrt{2} i \gamma^{a} \partial_{a} \phi \zeta_{Ri} + i (\zeta_{L} \sigma^{I})_{i} D^{I}, \\
\delta Q \lambda_{a} = -i \bar{\zeta}_{L} \gamma_{a} \lambda_{L} - i \bar{\zeta}_{R} \gamma_{a} \lambda_{R}, \\
\delta Q D^{I} = \bar{\zeta}_{L} \sigma^{I} \gamma^{a} \partial_{a} \lambda_{L} + \bar{\zeta}_{R} \sigma^{I} \gamma^{a} \partial_{a} \lambda_{R},
\]

(27)

where \( \zeta_{Ri} = C \bar{\zeta}_{Li} \), \( F_{ab} = \partial_{a} A_{b} - \partial_{b} A_{a} \), and \( \sigma^{I} \) are the Pauli matrices. The contractions of SU(2) indices are defined as \( \bar{\zeta}_{Ri} \gamma_{a} \lambda_{L} = \bar{\zeta}_{Ri} \lambda_{L}^{i} \), \( \bar{\zeta}_{Ri} \sigma^{I} \lambda_{L} = \bar{\zeta}_{Ri} (\sigma^{I})^{j} \lambda_{L}^{j} \), etc. These supertransformations satisfy a closed off-shell commutator algebra

\[
[\delta Q (\zeta_{1}), \delta Q (\zeta_{2})] = \delta P (v) + \delta g (\theta),
\]

(28)

where \( \delta P (v) \) and \( \delta g (\theta) \) are a translation and a U(1) gauge transformation with parameters

\[
v^{a} = 2i (\bar{\zeta}_{1L} \gamma^{a} \zeta_{2L} - \bar{\zeta}_{1R} \gamma^{a} \zeta_{2R}), \theta = -v^{a} A_{a} + 2\sqrt{2} \bar{\zeta}_{1L} \zeta_{2R} \phi - 2\sqrt{2} \bar{\zeta}_{1R} \zeta_{2L} \phi^{*}.
\]

(29)

Only the gauge field \( A_{a} \) transforms under the U(1) gauge transformation

\[
\delta g (\theta) A_{a} = \partial_{a} \theta.
\]

(30)

Although our discussion on the relation between the linear and NLSUSY transformations does not depend on a form of the action, it is instructive to consider a free action which is invariant under Eq. (27)

\[
S_{\text{lin}} = \int d^{4} x \left[ \partial_{a} \phi \partial^{a} \phi^{*} - \frac{1}{4} F_{ab}^{2} + i \bar{\lambda}_{R} \partial_{a} \lambda_{R} + \frac{1}{2} (D^{I})^{2} - \frac{1}{\kappa} \xi^{I} D^{I} \right],
\]

(31)

where \( \kappa \) is a constant whose dimension is (mass)\(^{-2} \) and \( \xi^{I} \) are three arbitrary real parameters satisfying \( (\xi^{I})^{2} = 1 \). The last term proportional to \( \kappa^{-1} \) is an analog of the FI \( D \) term in the \( N = 1 \) theories [29]. The field equations for the auxiliary fields give \( D^{I} = \xi^{I} / \kappa \) indicating a spontaneous SUSY breaking.

On the other hand, in the \( N = 2 \) VA model [18] we have a NLSUSY transformation law of the NG fermion fields \( \psi_{i}^{L} \)

\[
\delta Q \psi_{i}^{L} = \frac{1}{\kappa} \bar{\zeta}_{L}^{i} - i \kappa \left( \bar{\zeta}_{L} \gamma^{a} \psi_{L} - \bar{\zeta}_{R} \gamma^{a} \psi_{R} \right) \partial_{a} \psi_{i}^{L},
\]

(32)

where \( \psi_{Ri} = C \bar{\psi}_{Li}^{T} \). This transformation satisfies off-shell the commutator algebra (28) without the U(1) gauge transformation on the right-hand side. The VA action invariant under Eq. (32) reads

\[
S_{VA} = -\frac{1}{2 \kappa^{2}} \int d^{4} x \ \det w,
\]

(33)

14
where the $4 \times 4$ matrix $w$ is defined by

$$w^a_b = \delta^a_b + \kappa^2 t^a_b, \quad t^a_b = -i\bar{\psi}_L \gamma^a \partial_b \psi_L + i\bar{\psi}_R \gamma^a \partial_b \psi_R.$$ (34)

The VA action (33) is expanded in $\kappa$ as

$$S_{VA} = -\frac{1}{2\kappa^2} \int d^4x \left[ 1 + \kappa^2 t^a_a + \frac{1}{2} \kappa^4 (t^a_a t^b_b - t^a_b t^b_a) 
- \frac{1}{6} \kappa^6 \epsilon_{abcd} \epsilon_{efgh} t^a_e t^b_f t^c_g - \frac{1}{4!} \kappa^8 \epsilon_{abcd} \epsilon_{efgh} t^a_e t^b_f t^c_g t^d_h \right].$$ (35)

We would like to obtain a SUSY invariant relation between the component fields of the $N = 2$ vector supermultiplet and the NG fermion fields $\psi^i$ at the leading orders of $\kappa$. It is useful to imagine a situation in which the linear SUSY is broken with the auxiliary fields having expectation values $D^I = \xi^I / \kappa$ as in the free theory (31). Then, we expect from the experience in the $N = 1$ cases (23) (24) (25) (26) and the transformation law of the spinor fields in Eq. (27) that the relation should have a form

$$\lambda_{Li} = i\xi^I (\psi_L \sigma^I) + O(\kappa^2), \quad D^I = \frac{1}{\kappa} \xi^I + O(\kappa), \quad (\text{other fields}) = O(\kappa).$$ (36)

Higher order terms are obtained such that the LSUSY transformations (27) are reproduced by the NLSUSY transformation of the NG fermion fields (32).

After some calculations we obtain the relation between the fields in the linear theory and the NG fermion fields as

$$\phi(\psi) = \frac{1}{\sqrt{2}} i\kappa \xi^I \bar{\psi}_L \sigma^I \psi_L - \sqrt{2} \kappa^3 \xi^I \bar{\psi}_L \gamma^a \psi_L \bar{\psi}_R \sigma^I \partial_a \psi_L 
- \frac{\sqrt{2}}{3} \kappa^3 \xi^I \bar{\psi}_R \sigma^I \psi_L \bar{\psi}_R \sigma^J \psi^I \psi^J + O(\kappa^5),$$

$$\lambda_{Li}(\psi) = i\xi^I (\psi_L \sigma^I)_i + \kappa^2 \xi^I \gamma^a \psi_R \bar{\psi}_R \sigma^I \partial_a \psi_L + \frac{1}{2} \kappa^2 \xi^I \gamma^{ab} \psi_L \sigma^I \partial_a \psi_L 
+ \frac{1}{2} \kappa^2 \xi^I (\psi_L \sigma^I \psi^J \bar{\psi}_R \sigma^J \psi^I \psi^J) + O(\kappa^4),$$

$$A_a(\psi) = -\frac{1}{2} \kappa \xi^I \left( \bar{\psi}_L \sigma^I \gamma^a \psi_L - \bar{\psi}_R \sigma^I \gamma^a \psi_R \right) 
+ \frac{1}{4} i\kappa^3 \xi^I \left[ \bar{\psi}_L \sigma^I \psi_R \bar{\psi}_R \left( 2\delta^I_j \delta^j_a - \sigma^I_j \sigma^J_j \gamma_a^b \right) \partial_b \psi_L \right. 
- \frac{1}{4} \bar{\psi}_L \gamma^{cd} \psi_R \bar{\psi}_R \sigma^I \left( 2\gamma_{a}^{cd} \gamma^b - \gamma^b \gamma_{cd} \gamma\right) \partial_b \psi_L + \left( L \leftrightarrow R \right) \left. + O(\kappa^5) \right]$$

$$D^I(\psi) = \frac{1}{\kappa} \xi^I - i\kappa \xi^I \left( \bar{\psi}_L \sigma^I \psi^J \psi^J - \bar{\psi}_R \sigma^I \psi^J \psi^J \psi^J \right)$$
\[ + \kappa^3 \xi^J \left[ \bar{\psi}_L \sigma^I \psi_R \partial_a \bar{\psi}_R \sigma^J \partial^a \psi_L - \bar{\psi}_L \sigma^K \gamma^I \psi_L \right] \{ i \epsilon^{JK} \partial_c \bar{\psi}_L \partial \psi_L 
 \]
\[ - \frac{1}{2} \partial_a \bar{\psi}_L \sigma^I \sigma^K \gamma_c \partial^a \psi_L + \frac{1}{4} \partial_a \bar{\psi}_L \sigma^I \sigma^J \sigma^K \gamma_c \partial \psi_L \}
\[ - \frac{1}{4} \bar{\psi}_L \sigma^K \psi_R \left\{ \partial_a \bar{\psi}_R \sigma^J \sigma^I \sigma^K \gamma^b \gamma^a \partial_b \psi_L - \bar{\psi}_R \left( 2 \delta^{IK} + \sigma^I \sigma^K \right) \sigma^J \square \psi_L \right\} 
\]
\[ + \frac{1}{16} \bar{\psi}_L \gamma^{cd} \psi_R \left\{ \partial_a \bar{\psi}_R \sigma^J \sigma^K \gamma^b \gamma^a \gamma^c \partial_b \psi_L + \bar{\psi}_R \sigma^J \sigma^K \gamma^b \gamma^a \gamma^c \partial_a \partial_b \psi_L \right\} 
\[ + (L \leftrightarrow R) \right] + \mathcal{O}(\kappa^5). \] (37)

The transformation of the NG fermion fields \([32]\) reproduces the transformation of the linear theory \([27]\) except that the transformation of the gauge field \(A_a(\psi)\) contains an extra U(1) gauge transformation

\[ \delta_Q A_a(\psi) = -i \bar{\zeta}_L \gamma_\alpha \lambda_L (\psi) - i \bar{\zeta}_R \gamma_\alpha \lambda_R (\psi) + \partial_a X, \] (38)

where

\[ X = \frac{1}{2} i \kappa^2 \xi^I \bar{\zeta}_L \left( 2 \delta^{IJ} - \sigma^{IJ} \right) \psi_R \bar{\psi}_R \sigma^J \psi_L + (L \leftrightarrow R). \] (39)

The U(1) gauge transformation parameter \(X\) satisfies

\[ \delta_Q (\zeta_1) X (\zeta_2) - \delta_Q (\zeta_2) X (\zeta_1) = -\theta, \] (40)

where \(\theta\) is defined in Eq. \((29)\). Due to this extra term the commutator of two supertransformations on \(A_a(\psi)\) does not contain the U(1) gauge transformation term in Eq. \((28)\). This should be the case since the commutator on \(\psi\) does not contain the U(1) gauge transformation term. For gauge invariant quantities like \(F_{ab}\) the transformations exactly coincide with those of the LSUSY. In principle we can continue to obtain higher order terms in the relation \((37)\) following this approach. However, it will be more useful to use the \(N=2\) superfield formalism \([34]\) as was done in Refs. \([23, 26, 27, 28]\) for the \(N=1\) theories.

We note that the leading terms of \(A_a\) in Eq. \((37)\) can be written as

\[ A_a = -\kappa \xi^1 \chi \gamma_5 \gamma_a \varphi + i \kappa \xi^2 \bar{\chi} \gamma_\alpha \varphi - \frac{1}{2} \kappa \xi^3 (\bar{\chi} \gamma_5 \gamma_\alpha \chi - \bar{\varphi} \gamma_5 \gamma_\alpha \varphi) + \mathcal{O}(\kappa^3), \] (41)

where we have defined Majorana spinor fields

\[ \chi = \psi^1_L + \psi_{R1}, \quad \varphi = \psi^2_L + \psi_{R2}. \] (42)

When \(\xi^1 = \xi^3 = 0\), this shows the vector nature of the U(1) gauge field \([33]\) as we expected.
The relation (37) reduces to that of the $N = 1$ SUSY by imposing, e.g. $\psi^2_L = 0$. When $\xi^1 = 1$, $\xi^2 = \xi^3 = 0$, we find $\lambda_{L2} = 0$, $A_a = 0$, $D^3 = 0$ and that the relation between $(\phi, \lambda_{L1}, D^1, D^2)$ and $\psi^1_L$ becomes that of the $N = 1$ scalar supermultiplet obtained in Ref. [28]. When $\xi^1 = \xi^2 = 0$, $\xi^3 = 1$, on the other hand, we find $\lambda_{L1} = 0$, $\phi = 0$, $D^1 = D^2 = 0$ and that the relation between $(\lambda_{L2}, A_a, D^3)$ and $\psi^1_L$ becomes that of the $N = 1$ (axial) vector supermultiplet obtained in Refs. [25, 28].

Our result (37) does not depend on a form of the action for the linear SUSY theory. We discuss here the relation between the free linear SUSY action $S_{\text{lin}}$ in Eq. (31) and the VA action $S_{\text{VA}}$ in Eq. (33). It is expected that they coincide when Eq. (37) is substituted into the linear action (31) as in the $N = 1$ case [25, 26, 27, 28].

We have explicitly shown that $S_{\text{lin}}$ indeed coincides with the VA action $S_{\text{VA}}$ up to and including $O(\kappa^0)$ in Eq. (33)[33].

Now we summarize our results in this section.

We have constructed the SUSY invariant relation between the component fields of the $N = 2$ vector supermultiplet and the NG fermion fields $\psi^I_L$ at the leading orders of $\kappa$. We have explicitly shown that the $U(1)$ gauge field $A_a$ has the vector nature in terms of the two NG fermion fields in contrast to the models with the $N = 1$ SUSY [33]. The vector state with two NG fermion fields belongs to the adjoint representation of SGM scenario as expected. The relation (37) contains three arbitrary real parameters $\xi^I/\kappa$, which can be regarded as the vacuum expectation values of the auxiliary fields $D^I$. When we put $\psi^2_L = 0$, the relation reduces to that of the $N = 1$ scalar supermultiplet or that of the $N = 1$ vector supermultiplet depending on the choice of the parameters $\xi^I$. We have also shown that the free action $S_{\text{lin}}$ in Eq. (31) with the FI $D$ term reduces to the VA action $S_{\text{VA}}$ in Eq. (33) at least up to and including $O(\kappa^0)$. From these results we anticipate the equivalence of the action of $N$-extended standard supermultiplets of LSUSY to the $N$-extended VA action of NLSUSY, which is favorable for the SGM scenario.

It is interesting that SU(2) gauge structure of the SM is explained naturally if the gauge bosons are the SUSY-composites of SGM-type.

The derivation of the equivalent interacting LSYSY theory is yet to be studied.

### 4.3 Linearizing SGM

In this section we would like to attempt [35, 36, 37] the linearization of the new EH type action ($N=1$ SGM action) to obtain the equivalent LSUSY theory in the low energy.

Considering a phenomenological potential of SGM, though qualitative and group theoretical, discussed in [10, 11] based upon the composite picture of LSUSY representation and the recent interest in NLSUSY in superstring(membrane) world, the
linearization of NLSUSY in curved spacetime may be of some general interest.

The linearization of SGM is physically interesting in general, even if it produced an existing SUGRA-like theory, for the consequent broken LSUSY theory is shown to be equivalent and gives a new insight into the fundamental structure of nature behind the low energy effective theory.

For convenience we review N=1 SGM action briefly. SGM action is given by [17];

$$L_{SGM} = -\frac{c^3}{16\pi G} |w|(\Omega + \Lambda),$$  \hspace{1cm} (43)

$$|w| = \text{det} w^\alpha_\mu = \text{det}(e^\alpha_\mu + t^\alpha_\mu), \quad t^\alpha_\mu = \frac{i\kappa^4}{2} (\bar{\psi}\gamma^\alpha \partial_\mu \psi - \partial_\mu \bar{\psi}\gamma^\alpha \psi),$$  \hspace{1cm} (44)

which is invariant at least under the following symmetry [24]: ordinary GL(4R), the linearization of NLSUSY in curved spacetime may be of some general interest.

The local Lorentz transformation forms a closed algebra, for example, on $\psi$ and $e^\alpha_\mu$;

$$\delta_L w^a_\mu = \epsilon^a_b w^b_\mu,$$  \hspace{1cm} (47)

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$ or equivalently on $\psi$ and $e^\alpha_\mu$;

$$\delta_L \psi(x) = -\frac{i}{2} \epsilon_{ab}\sigma^{ab} \psi, \quad \delta_L e^\alpha_\mu(x) = \epsilon^a_b e^b_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}\gamma^a_5 \gamma^d \psi(\partial_\mu \epsilon_{bc}).$$ \hspace{1cm} (48)

The local Lorentz transformation forms a closed algebra, for example, on $e^\alpha_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}] e^\alpha_\mu = \beta^a_\mu e^b_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}\gamma^a_5 \gamma^d \psi(\partial_\mu \epsilon_{bc}),$$ \hspace{1cm} (49)

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ac}\epsilon^c_1 - \epsilon_{2bc}\epsilon^c_1$. The commutators of two new NL SUSY transformations [13] on $\psi(x)$ and $e^\alpha_\mu(x)$ are GL(4R), i.e. new NL SUSY [13] is the square-root of GL(4R);

$$[\delta_{\xi_1}, \delta_{\xi_2}] \psi = \Xi^\mu \partial_\mu \psi, \quad [\delta_{\xi_1}, \delta_{\xi_2}] e^\alpha_\mu = \Xi^\rho \partial_\mu e^\alpha_\mu + e^\alpha_\mu \partial_\mu \Xi^\rho, \hspace{1cm} \Xi^\mu = 2i(\bar{\xi}_1 \gamma^a \xi_1^a - \xi_1^a \bar{\xi}_2 \gamma^a_5 \epsilon_{a\mu}(\partial_\mu \epsilon^a_\sigma)).$$ \hspace{1cm} (50)

where $\Xi^\mu = 2i(\bar{\xi}_1 \gamma^a \xi_1^a - \xi_1^a \bar{\xi}_2 \gamma^a_5 \epsilon_{a\mu}(\partial_\mu \epsilon^a_\sigma))$. They show the closure of the algebra. SGM action [13] is invariant at least under [24]

$$[\text{global new NLSUSY}] \otimes [\text{local GL}(4, R)] \otimes [\text{local Lorentz}],$$ \hspace{1cm} (51)
which is isomorphic to SP whose single irreducible representation gives the group theoretical description of SGM.\[11\] In the preceding section the linearization has been carried out by using the superfield formalism and/or by the heuristic and intuitive arguments on the relations between the component fields of LSUSY and NLSUSY. In either case it is crucial to discover the SUSY invariant relations which connect the supermultiplets of L and NL theories and reproduce the LSUSY transformations. In abovementioned cases of the global SUSY in flat spacetime the SUSY invariant relations are obtained straightforwardly, for L and NL supermultiplets are well understood and the algebraic structures are the same SP. (However as demonstrated the naive application of the superfield technique have produced the free theory of the linear supermultiplet.)

The situation is rather different in SGM, for (i) the supermultiplet structure of the linearized theory of SGM is unknown except it is expected to be a broken LSUSY SUGRA-like theory containing graviton and a (massive) spin 3/2 field as dynamical d.o.f. and (ii) the algebraic structure (51) is changed into SP. And we should seek the linearization which produces the interacting theory of the linearized supermultiplet and unifies all charges.

Therefore by the heuristic arguments and referring to SUGRA we discuss for the moment the linearization of N=1 SGM.

At first, we assume faithfully to SGM scenario that;
(i) the linearized theory should contain the spontaneously broken global (at least) LSUSY
(ii) graviton is an elementary field (not composite of superons corrsponding to the vacuum of the Clifford algebra) in both L and NL theories
(iii) the NLSUSY supermultiplet of SGM \( (e^a_\mu(x), \psi(x)) \) should be connected to the composite supermultiplet \( (\tilde{e}^a_\mu(e(x), \psi(x)), \tilde{\lambda}_\mu(e(x), \psi(x))) \) for elementary graviton field and a composite (massive) spin 3/2 field of the SUGRA-like linearized theory.

From these assumptions and following the arguments performed in the flat space cases we require that the SUGRA gauge transformation with the global spinor parameter \( \zeta \) should hold for the supermultiplet \( (\tilde{e}^a_\mu(e, \psi), \tilde{\lambda}_\mu(e, \psi)) \) of the (SUGRA-like) linearized theory, i.e.,

\[
\begin{align*}
\delta \tilde{e}^a_\mu(e, \psi) &= i\kappa \bar{\zeta} \gamma^a \tilde{\lambda}_\mu(e, \psi), \\
\delta \tilde{\lambda}_\mu(e, \psi) &= 2 \frac{D_\mu \zeta}{\kappa} = -i \bar{\omega}_\mu \epsilon^{ab} \sigma_{ab} \zeta,
\end{align*}
\]

where \( \sigma^{ab} = \frac{i}{4} [\gamma^a, \gamma^b] \), \( D_\mu = \partial_\mu - \frac{i}{4} \bar{\omega}_\mu (e, \psi) \sigma_{ab} \), \( \zeta \) is a global spinor parameter and the variations in the left-hand side are induced by NLSUSY (51).
We put the following SUSY invariant relations which connect \( e^a_{\mu} \) to \( \tilde{e}^a_{\mu}(e, \psi) \):

\[
\tilde{e}^a_{\mu}(e, \psi) = e^a_{\mu}(x). \tag{54}
\]

This relation (54) is the assumption (ii) and holds simply the metric conditions. Consequently the following covariant relation is obtained by substituting (54) into (52) and computing the variations under (45);

\[
\tilde{\lambda}_{\mu}(e, \psi) = \kappa \gamma_a \gamma^\rho \psi(x) \partial_{[\rho} e^a_{\mu]}(x). \tag{55}
\]

(As discussed later these should may be considered as the leading order of the expansions in \( \kappa \) of SUSY invariant relations. The expansions terminate with \( \psi^4 \).) Now we see LSUSY transformation induced by (45) on the (composite) supermultiplet \( (\tilde{e}^a_{\mu}(e, \psi), \tilde{\lambda}_{\mu}(e, \psi)) \).

The LSUSY transformation on \( \tilde{e}^a_{\mu} \) becomes as follows. The left-hand side of (52) gives

\[
\delta \tilde{e}^a_{\mu}(e, \psi) = \delta^{NL} e^a_{\mu}(x) = i\kappa^2 (\bar{\zeta} \gamma^\rho \psi(x)) \partial_{[\rho} e^a_{\mu]}(x). \tag{56}
\]

While substituting (54) into the righ-hand side of (52) we obtain

\[
i\kappa^2(\bar{\zeta} \gamma^\rho \psi(x)) \partial_{[\rho} e^a_{\mu]}(x) + \cdots \text{(extra terms)}. \tag{57}
\]

These results show that (54) and (55) are not SUSY invariant relations and reproduce (52) with unwanted extra terms which should be identified with the auxiliary fields. The commutator of the two LSUSY transformations induces GL(4R) with the field dependent parameters as follows;

\[
[\delta_{\zeta_1}, \delta_{\zeta_2}] \tilde{e}^a_{\mu}(e, \psi) = \Xi^p \partial_{\rho} e^a_{\mu}(e, \psi) + \tilde{e}^a_{\rho}(e, \psi) \partial_{\mu} \Xi^p, \tag{58}
\]

where \( \Xi^p = 2i(\bar{\zeta}_2 \gamma^\mu \zeta_1) - \xi_1^i \xi_2^j e^a_{\mu}(\partial_{[\rho} e^a_{\sigma]}). \)

On \( \tilde{\lambda}_{\mu}(e, \psi) \), the left-hand side of (53) becomes apparently rather complicated;

\[
\delta \tilde{\lambda}_{\mu}(e, \psi) = \kappa \delta (\gamma_a \gamma^\rho \psi(x) \partial_{[\rho} e^a_{\mu]})
\]

\[
= \kappa \gamma_a [\delta^{NL} \gamma^\rho \psi(x) \partial_{[\rho} e^a_{\mu]} + \gamma^\rho \delta^{NL} \psi(x) \partial_{[\rho} e^a_{\mu]} + \gamma^\rho \psi(x) \partial_{[\rho} \delta^{NL} e^a_{\mu]}]. \tag{59}
\]

However the commutator of the two LSUSY transformations induces the similar GL(4R);

\[
[\delta_{\zeta_1}, \delta_{\zeta_2}] \tilde{\lambda}_{\mu}(e, \psi) = \Xi^p \partial_{\rho} \tilde{\lambda}_{\mu}(e, \psi) + \tilde{\lambda}_{\rho}(e, \psi) \partial_{\mu} \Xi^p. \tag{60}
\]

These results indicate that it is necessary to generalize (52), (53) and (55) for obtaining SUSY invariant relations and for the closure of the algebra. Furthermore due to the complicated expression of LSUSY (54) which makes the physical and mathematical structures are obscure, we can hardly guess a linearized invariant action which is equivalent to SGM.
Now we generalize the linearization by considering the auxirialy fields such that LSUSY transformation on the linearized fields induces SP transformation.

By comparing (53) with (59) we understand that the local Lorentz transformation plays a crucial role. As for the local Lorentz transformation on the linearized asymptotic fields corresponding to the observed particles (in the low energy), it is natural to take (irrespective of (48)) the following forms

\[\delta_L \tilde{\lambda}_\mu(x) = -\frac{i}{2} \epsilon_{ab} \sigma^{ab} \tilde{\lambda}_\mu(x), \quad \delta_L \tilde{e}^a_{\mu}(x) = \epsilon^a_b \tilde{e}^b_{\mu}, \quad (61)\]

where \(\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)\) is a local parameter. In SGM the local Lorentz transformations \((47)\) and \((48)\), i.e. the local Lorentz invariant gravitational interaction of superon, are introduced by the geometrical arguments in SGM spacetime[24] following EGRT. While in SUGRA theory the local Lorentz transformation invariance \((61)\) is realized as usual by introducing the Lorentz spin connection \(\omega^{ab}_{\mu}\). And the LSUSY transformation is defined successfully by the (Lorentz) covariant derivative containing the spin connection \(\tilde{\omega}^{ab}_{\mu}(e, \psi)\) as seen in (53), which causes the super-Poincaré algebra on the commutator of SUSY and is convenient for constructing the invariant action. Therefore in the linearized (SUGRA-like) theory the local Lorentz transformation invariance is expected to be realized as usual by defining (61) and introducing the Lorentz spin connection \(\omega^{ab}_{\mu}\). We investigate how the spin connection \(\tilde{\omega}^{ab}_{\mu}(e, \psi)\) appears in the linearized (SUGRA-like) theory through the linearization process. This is also crucial for constructing a nontrivial (interacting) linearized action which has manifest invariances.

We discuss the Lorentz covariance of the transformation by comparing (53) with the right-hand side of (59). The direct computation of (53) by using SUSY invariant relations \((54)\) and \((55)\) under \((45)\) produces complicated redundant terms as read off from (59). The local Lorentz invariance of the linearized theory may become ambiguous and lose the manifest invariance.

For a simple restoration of the manifest local Lorentz invariance we survey the possibility that such redundant terms may be adjusted by the d.o.f of the auxiliary fields in the linearized supermultiplet. As for the auxiliary fields it is necessary for the closure of the off-shell superalgebra to include the equal number of the fermionic and the bosonic d.o.f. in the linearized supermultiplet. As new NL SUSY is a global symmetry, \(\tilde{\lambda}_\mu\) has 16 fermionic d.o.f.. Therefore at least 4 bosonic d.o.f. must be added to the off-shell SUGRA supermultiplet with 12 d.o.f.\([38]\, [39]\) and a vector field may be a simple candidate.

However, counting the bosonic d.o.f. present in the redundant terms corresponding to \(\tilde{\omega}^{ab}_{\mu}(e, \psi)\), we may need a bigger supermultiplet e.g. \(16 + 4 \cdot 16 = 80\) d.o.f., to carry out the linearization, in which case a rank-3 tensor \(\tilde{\phi}_{\mu\nu\rho}\) and a rank-2 tensor-spinor \(\lambda_{\mu\nu}\) may be candidates for the auxiliary fields.
Now we consider the simple modification of SUGRA transformations (algebra) by adjusting the (composite) structure of the (auxiliary) fields. We take, in stead of (52) and (53),
\[\delta \tilde{e}_a^\mu(x) = i\kappa \bar{\zeta} \gamma^a \tilde{\lambda}_\mu(x) + \bar{\zeta} \tilde{\Lambda}^a_\mu,\]
(62)
\[\delta \tilde{\lambda}_\mu(x) = \frac{2}{\kappa} D_\mu \zeta + \tilde{\Phi}_\mu \zeta = -\frac{i}{\kappa} \bar{\omega}^{ab}_\mu \sigma_{ab} \zeta + \tilde{\Phi}_\mu \zeta,\]
(63)
where \(\tilde{\Lambda}^a_\mu\) and \(\tilde{\Phi}_\mu\) represent auxiliary fields which are functionals of \(e^a_\mu\) and \(\psi\). We need \(\tilde{\Lambda}^a_\mu\) term in (62) to alter (56), (58), (59) and (60) toward that of super-Poincaré algebra of SUGRA. We attempt the restoration of the manifest local Lorentz invariance order by order by adjusting \(\tilde{\Lambda}^a_\mu\) and \(\tilde{\Phi}_\mu\). In fact, the Lorentz spin connection \(\bar{\omega}^{ab}_\mu(e,\psi)\) (i.e. the leading order terms of \(\bar{\omega}^{ab}_\mu(e,\psi)\)) of (63) is reproduced by taking the following one
\[\tilde{\Lambda}^a_\mu = \kappa \frac{3}{4} \left[ i e_b^\rho [\partial_\rho e^a_\mu] - \gamma^a \tilde{\lambda}^\mu_\mu - \gamma^a \tilde{\lambda}_\mu \right],\]
(64)
where (58) holds. Accordingly \(\tilde{\lambda}_\mu(e,\psi)\) is determined up to the first order in \(\psi\) as follows;
\[\tilde{\lambda}_\mu(e,\psi) = \frac{1}{2\kappa} (i\kappa \gamma^a \gamma^\rho \psi(x) \partial_\rho e^a_\mu - \gamma^a \tilde{\lambda}_\mu) = -\frac{i\kappa}{2} \omega^{ab}_\mu(e) \sigma_{ab} \psi,\]
(65)
which indicates the minimal Lorentz covariant gravitational interaction of superon. Substituting (65) into (63) we obtain the following new LSUSY transformation of \(\tilde{\lambda}_\mu\) (after Fiertz transformations)
\[\delta \tilde{\lambda}_\mu(e,\psi) = -\frac{i\kappa}{2} \left\{ \delta^{NL} \omega^{ab}_\mu(e) \sigma_{ab} \psi + \omega^{ab}_\mu(e) \sigma_{ab} \delta^{NL} \psi \right\} \]
\[= -\frac{i}{2\kappa} \omega^{ab}_\mu(e) \sigma_{ab} \zeta + \frac{i\kappa}{2} \left\{ \bar{\epsilon}^{ab}(e,\psi) \sigma_{ab} \cdot \omega^{ab}_\mu(e) \sigma_{cd} \psi + \cdots \right\}.\]
(66)
Remarkably the local Lorentz transformations of \(\tilde{\lambda}_\mu(e,\psi)\) (i.e. the second term) with the field dependent antisymmetric parameters \(\bar{\epsilon}^{ab}(e,\psi)\) is induced in addition to the intended ordinary global SUSY transformation. This shows that (65) is the SUSY invariant relations for \(\tilde{\lambda}_\mu(e,\psi)\), for the SUSY transformation of (65) gives the right hand side of (63) with the extra terms. Interestingly the commutator of the two LSUSY transformations on (65) induces GL(4R);
\[[\delta_{\xi_1}, \delta_{\xi_2}] \tilde{\lambda}_\mu(e,\psi) = \Xi^\rho \partial_\rho \tilde{\lambda}_\mu(e,\psi) + \partial_\mu \Xi^\rho \tilde{\lambda}_\rho(e,\psi),\]
(67)
where \(\Xi^\rho\) is the same field dependent parameter as given in (58). (58) and (67) show the closure of the algebra on SP algebra provided that the SUSY invariant relations (54) and (55) are adopted. These phenomena coincide with SGM scenario [10][11]
from the algebraic point of view, i.e. they are the superon-graviton composite (eigenstates) corresponding to the linear representations of SP algebra. As for the redundant higher order terms in (63) we can adjust them by considering the modified spin connection \( \tilde{\omega}_{ab\mu}(e, \psi) \) particularly with the contorsion terms and by recasting them in terms of (the auxiliary field d.o.f.) \( \tilde{\Phi}_\mu(e, \psi) \). In fact, we found that the following supermultiplet containing 160 (= 80 bosonic + 80 fermionic) d.o.f. may be the supermultiplet of the SUGRA-like LSUSY theory which is equivalent to SGM, i.e., for 80 bosonic d.o.f.

\[
[ \tilde{e}^a_\mu(e, \psi), a_\mu(e, \psi), b_\mu(e, \psi), M(e, \psi), N(e, \psi), \\
A_\mu(e, \psi), B_\mu(e, \psi), A^a_\mu(e, \psi), B^a_\mu(e, \psi), A^{[ab]}_\mu(e, \psi) ]
\]

(68)

and for 80 fermionic d.o.f.

\[
[ \tilde{\lambda}_{\mu a}(e, \psi), \tilde{\Lambda}^{a}_{\mu a}(e, \psi) ],
\]

(69)

where \( a = 1, 2, 3, 4 \) are indices for Majorana spinor. The gauge d.o.f. of the local GL(4R) and the local Lorentz of the vierbein are subtracted. Note that the second line of (68) and the second term of (69) are equivalent to the off-shell (auxiliary) field with spin 3 and spin 5/2, respectively.

The \emph{a priori} gauge invariance for \( \tilde{\lambda}_{\mu a}(e, \psi) \) is not necessary for massive case, which can be anticipated by the spontaneous SUSY breaking. For it is natural to suppose that the equivalent linear theory may be a coupled system of graviton and massive spin 3/2 with the spontaneous global SUSY breaking, which may be an analogue obtained by the super-Higgs mechanism in the spontaneous local SUSY breaking of \( N=1 \) SUGRA\cite{21}. Although at the moment the arguments are independent of the form of the action.

By continuing the heuristic arguments order by order referring to the familiar SUGRA supermultiplet we find the following SUSY invariant relations up to \( O(\psi^2) \):

\[
\tilde{e}^a_\mu(e, \psi) = e^a_\mu, \\
\tilde{\lambda}_\mu(e, \psi) = -i\kappa(\sigma_{ab}\psi)\omega^{ab}_\mu, \\
\tilde{\Lambda}^{a}_{\mu a}(e, \psi) = \frac{\kappa^2}{2}e^{abcd}(\gamma_5\gamma_d\psi)\omega_{bc\mu}, \\
A_\mu(e, \psi) = \frac{i\kappa^2}{4}[(\bar{\psi}\gamma^\rho\partial_\rho\tilde{\lambda}_\mu) - (\bar{\psi}\gamma^\rho\tilde{\lambda}_a)\partial_\mu e^a_\rho - (\tilde{\lambda}_\rho\gamma^\rho\partial_\mu\psi)] \\
+ \frac{\kappa^3}{4}[(\bar{\psi}\sigma^{a\rho}\gamma^b\partial_\rho\psi)(\omega_{ab\mu} + \omega_{a\mu b}) + (\bar{\psi}\sigma^{a\rho}\gamma^c\partial_\mu\psi)\omega_{cab}] \\
+ \frac{\kappa^2}{8}(\bar{\psi}\lambda_{\mu a}\gamma^a\psi)\omega^{ab}_\mu, \\
B_\mu(e, \psi) = \frac{i\kappa^2}{4}[-(\bar{\psi}\gamma_5\gamma^\rho\partial_\rho\tilde{\lambda}_\mu) + (\bar{\psi}\gamma_5\gamma^a\lambda_a)\partial_\mu e^a_\rho - (\tilde{\lambda}_\rho\gamma_5\gamma^\rho\partial_\mu\psi)]
\]

(70)  (71)  (72)  (73)
\[
\delta \tilde{e}^\mu_a = i \kappa \tilde{\zeta} \gamma^a \tilde{\lambda}_\mu - e^a_{\mu b} \tilde{e}^b + \tilde{\zeta} \Lambda^a_{\mu},
\]
\[
\delta \tilde{\lambda}_\mu = -\frac{i}{\kappa} (\sigma_{ab} \zeta) \omega_{ab \mu} + \frac{i}{2} \epsilon^{ab}(\sigma_{ab} \tilde{\lambda}_\mu)
= A_{\mu} \zeta + B_{\mu}(\gamma_5 \zeta) + A^a_{\mu}(\gamma_a \zeta) + B^a_{\mu}(\gamma_5 \gamma_a \zeta) + A^{ab}_{\mu}(\sigma_{ab} \zeta),
\]
\[
\delta \tilde{\Lambda}^a_{\mu} = \frac{1}{2} \epsilon^{abcd}(\gamma_5 \gamma_d \zeta) \omega_{bc \mu},
\]
\[
\delta A_{\mu} = -\frac{1}{8} \left[ i(\tilde{\zeta} \gamma^\rho D_\rho \tilde{\lambda}_{a}) e^a_{\mu} + 3i(\tilde{\zeta} \gamma^a D_\mu \tilde{\lambda}_{a}) + 2(\tilde{\zeta} \sigma^{\rho \gamma} \gamma_\mu D_\rho \tilde{\lambda}_{a}) \right]
- \frac{1}{4\kappa} \left[ 3(\tilde{\zeta} D_\mu \tilde{\lambda}^a_{\mu}) + i(\tilde{\sigma}^{ab} D_\mu \tilde{\lambda}_{ab}) + i(\tilde{\sigma}^{\rho \gamma} D_\mu \tilde{\lambda}_{(ab)} e^b) \right]
+ \frac{1}{16} \left[ 4i(\tilde{\zeta} \gamma^\rho \tilde{\lambda}_{a}) \omega_{\rho a \mu} + 4(\tilde{\zeta} \sigma^{bc} \gamma^a \tilde{\lambda}_{a}) \omega_{bc \mu} - 4(\tilde{\zeta} \sigma^{\rho \gamma} \gamma_\mu \tilde{\lambda}_{a}) \omega_{|ab|\mu} \right]
+ 4(\tilde{\sigma}^{ab} \gamma^a \tilde{\lambda}_{a}) \omega_{|ab|\mu} - 3(\tilde{\zeta} \sigma^{bc} \gamma^a \tilde{\lambda}_{a}) \omega_{|bc|\mu} + 2i(\tilde{\zeta} \sigma^{ab} \gamma_\mu \sigma^{cd} \tilde{\lambda}_{a}) \omega_{cd \mu},
\]
\[
A^a_{\mu}(e, \psi) = \frac{i \kappa^2}{4} \left[ (\gamma^\rho \gamma^a \gamma^\rho \gamma_\mu \tilde{\lambda}_b - \gamma^\rho \gamma^a \gamma^\rho \gamma_\mu \tilde{\lambda}_b) \tilde{\epsilon}_b + (\tilde{\lambda}_\rho \gamma^a \gamma^\rho \gamma_\mu \tilde{\lambda}_b) \tilde{\epsilon}_b \right]
+ \frac{\kappa^3}{4} \left[ -\tilde{\psi} \sigma^b \gamma^a \gamma^\rho \gamma_\mu \tilde{\epsilon}_b (\omega_{\mu cb} + \omega_{bc \mu}) - (\gamma^{bc} \gamma^\rho \gamma_\mu \tilde{\lambda}_b) \tilde{\epsilon}_b \right]
- \frac{\kappa^2}{8} (\tilde{\lambda}_\mu \gamma_5 \epsilon^{ab \rho}) \omega_{ab \rho},
\]
\[
B^a_{\mu}(e, \psi) = \frac{i \kappa^2}{4} \left[ (\tilde{\psi} \gamma_5 \gamma^n \gamma^a \gamma^\rho \gamma_\mu \tilde{\lambda}_b - \gamma_5 \gamma^n \gamma^a \gamma^\rho \gamma_\mu \tilde{\lambda}_b) \tilde{\epsilon}_b + (\tilde{\lambda}_\rho \gamma_5 \gamma^n \gamma^a \gamma^\rho \gamma_\mu \tilde{\lambda}_b) \tilde{\epsilon}_b \right]
+ \frac{\kappa^3}{8} \left[ -\tilde{\psi} \sigma^b \gamma^a \gamma^\rho \gamma_\mu \tilde{\epsilon}_b (\omega_{\mu cb} + \omega_{bc \mu}) - (\gamma^{bc} \gamma^\rho \gamma_\mu \tilde{\lambda}_b) \tilde{\epsilon}_b \right]
- \frac{\kappa^2}{8} (\tilde{\lambda}_\mu \gamma_5 \epsilon^{ab \rho}) \omega_{ab \rho},
\]
\[
\delta A^a_\mu = \frac{1}{8} \left[ -4i(\bar{\zeta} D_\mu \bar{\lambda}^a) + i(\bar{\zeta} \gamma^\nu \gamma^\rho D_{[\mu} \bar{\lambda}_{\nu]} \rho) + 2(\bar{\zeta} \sigma^{\nu \rho} \gamma^\alpha \gamma^\rho D_{\nu} \bar{\lambda}_{\rho}) \right] \\
+ \frac{1}{4\kappa} \left[ -i(\bar{\zeta} \sigma^{b \rho} \gamma^\alpha D_{[\mu} \bar{\lambda}_{b]}_{[\rho]} - i(\bar{\zeta} \sigma^{\nu \rho} \gamma^\alpha D_{\nu} \bar{\lambda}_{b \rho}) \right] \tilde{c}^b_\mu + (\bar{\zeta} \gamma^c \gamma^a \gamma^b D_{\mu} \bar{\lambda}_{bc} \) \\
+ \frac{1}{16} \left[ -4i(\bar{\zeta} \gamma^\rho \gamma^a \bar{\lambda}_{b \rho}) \omega^b_{\rho \mu} - 2(\bar{\zeta} \gamma^\nu \gamma^a \sigma^{b c} \bar{\lambda}_{[\mu]} \omega_{[b c] [\nu]} + 2(\bar{\zeta} \sigma^{b c} \gamma^a \gamma^b \bar{\lambda}_{bc}) \omega_{\nu \mu} c d \right] \\
+ 2(\bar{\zeta} \sigma^{c d} \gamma^a \gamma^b \bar{\lambda}_{b c}) \omega_{c d \mu} + 4(\bar{\zeta} \sigma^{b c} \gamma^a \gamma^c \bar{\lambda}_{[\mu]} \omega_{[b c] [\nu]} - 4(\bar{\zeta} \sigma^{b c} \gamma^a \gamma^c \bar{\lambda}_{b c}) \omega_{\nu \mu} d c \\
- (\bar{\zeta} \sigma^{a \rho} \gamma^c \sigma^{de} \bar{\lambda}_{bc}) \omega_{de \mu} + 2(\bar{\zeta} \sigma^{a \rho} \gamma^a \sigma^{de} \bar{\lambda}_{bc}) \omega_{de \mu} \\
+ \frac{1}{8\kappa} \left[ (\bar{\zeta} \sigma^{b \rho} \gamma^a \sigma^{de} \bar{\lambda}_{bc}) \omega_{de \mu} + i(\bar{\zeta} \sigma^{de} \gamma^c \gamma^a \bar{\lambda}_{bc}) \omega_{de \mu} + i(\bar{\zeta} \sigma^{de} \gamma^c \gamma^a \bar{\lambda}_{bc}) \omega_{de \mu} \right] \\
+ \frac{\kappa}{2}(\bar{\zeta} D_{\mu} \Lambda^{\mu}) - \frac{\kappa}{4}(\bar{\zeta} \gamma^c \gamma^a \Lambda^{ab}) \omega_{bc \mu}, \tag{82}
\]
\[
\delta A^{[ab]}_\mu = \frac{1}{4} \left[ -2i(\bar{\zeta} \gamma^\rho \sigma^{ab} D_{\rho} \bar{\lambda}_c) \tilde{c}^c_\mu + i(\bar{\zeta} \sigma^{ab} \gamma^\rho D_{\rho} \bar{\lambda}_c) \tilde{c}^c_\mu + i(\bar{\zeta} \sigma^{ab} \gamma^c D_{\mu} \bar{\lambda}_c) - 2(\bar{\zeta} \sigma^{ab} \gamma^a \gamma^b D_{\nu} \bar{\lambda}_{\rho}) \right] \\
+ \frac{1}{2\kappa} \left[ -i(\bar{\zeta} \sigma^{ab} D_{\mu} \Lambda^c) + i(\bar{\zeta} \sigma^{cd} \sigma^{ab} D_{\mu} \bar{\lambda}_{cd}) + i(\bar{\zeta} e^b \sigma^{ab} D_{\mu} \bar{\lambda}_{cd}) \right] \tilde{c}^c_\mu \\
+ \frac{1}{8} \left[ 4i(\bar{\zeta} \gamma^\rho \sigma^{ab} \bar{\lambda}_c) \omega^c_{\rho \mu} + 4(\bar{\zeta} \sigma^{ab} \gamma^c \sigma^{de} \bar{\lambda}_{[cd]} \omega_{[cd] [\nu]} - 4(\bar{\zeta} \sigma^{ab} \gamma^c \sigma^{de} \bar{\lambda}_c) \omega_{\nu \mu} d e \right] \\
- (\bar{\zeta} \sigma^{ab} \gamma^a \sigma^{de} \bar{\lambda}_{bc}) \omega_{de \mu} - 2i(\bar{\zeta} \sigma^{ab} \gamma^a \sigma^{de} \bar{\lambda}_{bc}) \omega_{de \mu} \\
- 4i(\bar{\zeta} \sigma^{ab} \gamma^a \sigma^{de} \bar{\lambda}_{bc}) \omega_{de \mu} + 2(\bar{\zeta} \sigma^{ab} \gamma^a \sigma^{de} \bar{\lambda}_{bc}) \omega_{de \mu} \\
+ \frac{1}{4\kappa} \left[ -4(\bar{\zeta} \sigma^{[ab]} \Lambda^d) \omega^d_{c \mu} + i(\bar{\zeta} \sigma^{ab} \sigma^{cd} \bar{\lambda}_c) \omega_{c \mu d} - (\bar{\zeta} \sigma^{ab} \sigma^{cd} \bar{\lambda}_c) \omega_{c \mu d} - (\bar{\zeta} \sigma^{ab} \sigma^{cd} \bar{\lambda}_c) \omega_{c \mu d} \right] \\
- (\bar{\zeta} \sigma^{ab} \sigma^{cd} \bar{\lambda}_c) \omega_{de \mu} - 2(\bar{\zeta} \sigma^{ab} \sigma^{cd} \bar{\lambda}_c) \omega_{de \mu} \right], \tag{83}
\]

where \(\epsilon^{ab}\) is the Lorentz parameter and we put \(\epsilon^{ab} = \xi^b \omega_{ab}\). \(\delta B_\mu\) and \(\delta B^a_\mu\) are similar to \(\delta A_\mu\) and \(\delta A^a_\mu\) respectively and omitted for simplicity. In the right-hand side of (82) and \(\delta B^a_\mu\), the last terms contain \(\Lambda^a_\mu\) which is defined by \(\Lambda^a_\mu = -\epsilon^{abcd} \gamma^c \omega_{bc \mu}\). Note that \(\Lambda^a_\mu\) is not the functional of the supermultiplet \(\{3\}\), so we may have to treat \(\Lambda^a_\mu\) as a new auxiliary field. However, if we put \(\epsilon^{ab} = \epsilon^{ab} \tilde{\lambda}^a_\mu, \tilde{\Lambda}^a_\mu\), e.g. \(\epsilon^{ab} = \tilde{\zeta} \gamma^a \tilde{\lambda}^b_\mu, \tilde{\Lambda}^a_\mu\) does not appear in the right-hand side of (82) and \(\delta B^a_\mu\). As a result, the LSUSY transformation on the supermultiplet \(\{3\}\) and (83) are written by using the supermultiplet itself at least at the leading order of supercon \(\psi\). The higher order terms remain to be studied. However we believe that we can obtain the complete linearized off-shell supermultiplets of the SP algebra by repeating the similar procedures (on the auxiliary fields) order by order which terminates with \(\psi^4\). It may be favorable that 10 bosonic auxiliary fields, for example \(a_\mu(e, \psi), b_\mu(e, \psi), M(e, \psi), N(e, \psi)\) are arbitrary up now and available for the closure of the off-shell SP algebra in higher order terms.

Here we just mention the systematic linearization by using the superfield for-
malism applied to study the coupled system of VA action with SUGRA. We can define on such a coupled system a local spinor gauge symmetry which induces a super-Higgs mechanism converting VA field to the longitudinal component of massive spin 3/2 field. The consequent Lagrangian may be an analogue that we have anticipated in the composite picture but with the elementary spin 3/2 field. Developing the superfield formalism on SGM spacetime may be crucial for carrying out the linearization along the SGM composite scenario, especially for \( N > 1 \).

Finally, we discuss the commutators for more general cases. Here we consider a functional of \((e^a_{\mu}, \psi)\) and their derivatives as

\[
\begin{align*}
  f_A(\psi, \bar{\psi}, e^a_\rho; \psi_\rho, \bar{\psi}_\rho, e^a_{\rho\sigma}), \quad (A = \mu, \mu \nu, \ldots \text{etc.})
\end{align*}
\]

with \( \psi_\rho = \partial_\rho \psi, \ldots \text{etc.} \), and we suppose that \( f_A \) is the functional of \( O(\psi^2) \) for simplicity. Then we have the variation of \( f_A \),

\[
\begin{align*}
  \delta f_A &= \frac{\partial f_A}{\partial \psi} \delta \psi + \delta \bar{\psi} \frac{\partial f_A}{\partial \bar{\psi}} + \frac{\partial f_A}{\partial e^a_\rho} \delta e^a_\rho + \frac{\partial f_A}{\partial \psi_\rho} (\delta \psi)_\rho + (\delta \bar{\psi})_\rho \frac{\partial f_A}{\partial \bar{\psi}_\rho} + \frac{\partial f_A}{\partial e^a_{\rho\sigma}} (\delta e^a_\rho, \sigma) \quad \text{(85)}
\end{align*}
\]

and the commutator for \( f_A \) becomes

\[
\begin{align*}
  [\delta_1, \delta_2] f_A &= \frac{\partial f_A}{\partial \psi} [\delta_1, \delta_2] \psi + [\delta_1, \delta_2] \bar{\psi} \frac{\partial f_A}{\partial \bar{\psi}} + \frac{\partial f_A}{\partial e^a_\rho} [\delta_1, \delta_2] e^a_\rho \\
  &+ \frac{\partial f_A}{\partial \psi_\rho} ([\delta_1, \delta_2] \psi)_\rho + ([\delta_1, \delta_2] \bar{\psi})_\rho \frac{\partial f_A}{\partial \bar{\psi}_\rho} + \frac{\partial f_A}{\partial e^a_{\rho\sigma}} ([\delta_1, \delta_2] e^a_\rho, \sigma) \quad \text{(86)}
\end{align*}
\]

If we substitute the commutators for \((e^a_{\mu}, \psi)\) of Eq.(8) into Eq.(86), we obtain

\[
\begin{align*}
  [\delta_1, \delta_2] f_A &= \Xi^\lambda \partial_\lambda f_A + G_A, \quad \text{(87)}
\end{align*}
\]

where \( G_A \) is defined by

\[
\begin{align*}
  G_A &= \partial_\rho \Xi^\lambda \left( \frac{\partial f_A}{\partial e^a_\rho} e^a_\lambda + \frac{\partial f_A}{\partial \psi_\rho} \partial_\lambda \psi + \partial_\lambda \bar{\psi} \frac{\partial f_A}{\partial \psi_\rho} + \frac{\partial f_A}{\partial e^a_{\sigma\rho}} \partial_\sigma e^a_\lambda + \frac{\partial f_A}{\partial e^a_{\rho\sigma}} \partial_\sigma e^a_\lambda \right) \\
  &+ \partial_\rho \partial_\sigma \Xi^\lambda \frac{\partial f_A}{\partial e^a_{\rho\sigma}} e^a_\lambda. \quad \text{(88)}
\end{align*}
\]

The first term in r.h.s. of Eq.(87) means the translation of \( f_A \). Therefore Eq.(87) shows that the closure of the commutator algebra on \( \text{GL}(4, \mathbb{R}) \) for the various functionals \( f_A \) in the previous argument depends on \( G_A \) of Eq.(88), and these argument reproduces all the previous commutators respectively.

The linearization of SGM action with the extra dimensions, which gives another unification framework describing the observed particles as elementary fields,
is open. And the linearization of SGM action for spin 3/2 NG fermion field [44] discussed in the next section (with extra dimensions) to be discussed in the next section may be in the same scope.

Now we summarize the results as follows: (i) Referring to SUGRA transformations we have obtained explicitly the SUSY invariant relations up to $O(\psi)^2$ and the corresponding new LSUSY transformations among 80+80 off-shell supermultiplet of LSUSY. (ii) The new LSUSY transformations on 80+80 linearized supermultiplet are different apparently from SUGRA transformations but close on super-Poincaré. (iii) It is interesting that the simple relation $\lambda_\mu = e^{a_\mu} \gamma_a \psi + \cdots$, which is suggested by the flat spacetime linearization, seems disfavor with the SGM linearization in our present method, so far. From the physical viewpoint what LSUSY SP may be to SGM in quantum field theory, what $O(4)$ symmetry is to the relativistic hydrogen model in quantum mechanics. The complete linearization to all orders up to $O(\psi)^4$, which can be anticipated by the systematics emerging in the present study, needs specifications of the auxiliary fields and remains to be studied. The details will appear separately [42].

5 SGM with spin 3/2 Superon

In this section we extend the SGM [17] to a higher spin NG fermion. Following the arguments of VA, the action of NG fermion $\psi_\alpha^\mu(x)$ with spin 3/2 is already written down by Baaklini as a nonlinear realization of a new superalgebra containing a vector-spinor generator $Q_\alpha^\mu$ [43]. We study in detail the gravitational interaction of Baaklini model [43]. We will see that the similar arguments to SGM can be performed and produce a gauge invariant action, which is the straightforward generalization of SGM action. The phenomenological implications of spin 3/2 fundamental constituents are discussed briefly.

In ref. [43], a new SUSY algebra containing a spinor-vector generator $Q_\alpha^\mu$ is introduced as follows:

$$\{Q_\alpha^\mu, Q_\beta^\nu\} = \varepsilon^{\mu\nu\lambda\rho} P_\lambda (\gamma_\rho \gamma_5 C)_{\alpha\beta},$$  \hfill (89)

$$[Q_\alpha^\mu, P^\nu] = 0,$$  \hfill (90)

$$[Q_\alpha^\mu, J^{\lambda\rho}] = \frac{1}{2} (\sigma^{\lambda\rho} Q_\alpha^\mu) + i \eta^{\lambda\mu} Q_\alpha^\rho - i \eta^{\rho\mu} Q_\alpha^\lambda,$$  \hfill (91)

where $Q_\alpha^\mu$ are vector-spinor generators satisfying Majorana condition $Q_\alpha^\mu = C_{\alpha\beta} \overline{Q}_\beta^\mu$, $C$ is a charge conjugation matrix and $\frac{1}{2} \{\gamma^\mu, \gamma^\nu\} = \eta^{\mu\nu} = (+, -, -, -)$. By extending the arguments of VA model [2] of NLSUSY, they obtain the following action as the
nonlinear representation of the new SUSY algebra.

\[ S = \frac{1}{\kappa} \int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \frac{1}{\kappa} \int \det w_{ab} d^4 x, \quad (92) \]

\[ w_{ab} = \delta_{ab} + t_{ab}, \quad t_{ab} = i \kappa \varepsilon_{acde} \bar{\psi}^c \gamma^d \gamma_5 \partial_b \psi^e, \quad (93) \]

where \( \kappa \) is up to now arbitrary constant with the dimension of the fourth power of length (i.e., a fundamental volume of spacetime) and \( \omega_a \) is the following differential forms

\[ \omega_a = dx_a + i \kappa \varepsilon_{abcd} \bar{\psi}^b \gamma^c \gamma_5 d\psi^d, \quad (94) \]

which is invariant under the following (super)translations

\[ \psi^a_{\alpha} \rightarrow \psi^a_{\alpha} + \zeta^a_{\alpha}, \quad (95) \]

\[ x_a \rightarrow x_a + i \kappa \varepsilon_{abcd} \bar{\psi}^b \gamma^c \gamma_5 \zeta^d, \quad (96) \]

where \( \zeta^a_{\alpha} \) is a constant Majorana tensor-spinor parameter.

Now we consider the gravitational interaction of Baaklini model (89). We show that the arguments performed in SGM of spin 1/2 NG field \( \psi_{\alpha}(x) \) can be extended straightforwardly to spin 3/2 Majorana NG field \( \psi_a^{\alpha} \). In the present case, as seen in (94), (95) and (96) NLSUSY SL(2C) degrees of freedom (i.e. the coset space coordinates \( \psi^a_{\alpha} \) representing NG fermions) in addition to Lorentz SO(3,1) coordinates are embedded at every curved spacetime point with GL(4R) invariance. Following the arguments of SGM[17], it is natural to introduce formally a new vierbein field \( w^a_{\mu}(x) \) through the NLSUSY invariant differential forms \( \omega_a \) in (94) as follows:

\[ \omega^a = w^a_{\mu} dx^\mu, \quad (97) \]

\[ w^a_{\mu}(x) = e^a_{\mu}(x) + t^a_{\mu}(x), \quad t^a_{\mu}(x) = i \kappa \varepsilon_{abcd} \bar{\psi}^b \gamma^c \gamma_5 \partial_\mu \psi^d, \quad (98) \]

where \( e^a_{\mu}(x) \) is the vierbein of Einstein General Relativity Theory (EGRT) and Latin \((a,b,..)\) and Greek \((\mu, \nu, ..)\) are the indices for local Lorentz and general coordinates, respectively. By noting \( (\psi^\mu_{\alpha}(x))^2 = 0 \), we can easily obtain the inverse of the new vierbein, \( w^a_{\mu}(x) \), in the power series of \( t^a_{\mu} \) which terminates with \( (t^a_{\mu})^4 \):

\[ w^a_{\mu} = e^a_{\mu} - t^a_{\mu} + \bar{t}_{\alpha a} t^\alpha_{\rho} \cdots. \quad (99) \]

Note that the first and the second indices of \( t^a_{\mu} \) represent those of \( \gamma \)-matrix and the derivative, respectively. Similarly we introduce formally a new metric tensor \( s^{\mu \nu}(x) \) in the abovementioned curved spacetime as follows:

\[ s^{\mu \nu}(x) \equiv w^a_{\mu}(x) w^a_{\nu}(x). \quad (100) \]

It is easy to show \( w^a_{\mu} w_{b\mu} = \eta_{ab} \), \( s_{\mu \nu} w^a_{\mu} w^b_{\nu} = \eta_{ab} \), etc. In order to obtain simply the action in the abovementioned curved spacetime, which is invariant at least under
GL(4R), NLSUSY and local Lorentz transformations, we follow formally EGRT as performed in SGM. That is, we require that the new unified vierbein $w^a_\mu(x)$ and the metric $s^{\mu\nu}(x)$ should have formally a general coordinate transformations under the supertranslations:

$$\delta x_\mu = -\xi_\mu, \quad \delta \psi^a = \zeta^a,$$  

(101)

where $\xi^\mu = i\kappa \varepsilon^{\mu
u\rho\sigma} \bar{\psi}_\nu \gamma_\rho \gamma_5 \zeta_\sigma$.

Remarkably we find that the following global new NLSUSY transformations

$$\delta \psi^a(x) = \zeta^a - i\kappa (\varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\nu \gamma_\rho \gamma_5 \zeta_\sigma) \partial_\mu \psi^a,$$

(102)

$$\delta e^a_\mu(x) = i\kappa (\varepsilon^{\rho\sigma\lambda\mu} \bar{\psi}_\nu \gamma_\rho \gamma_5 \zeta_\lambda) \partial_\rho e^a_\mu,$$

(103)

induce the desirable transformations on $w^a_\mu(x)$ and $s^{\mu\nu}(x)$ as follows:

$$\delta \zeta^1 w^a_\mu = \xi^\nu \partial_\nu w^a_\mu + \partial_\mu \xi^\nu w^a_\nu,$$

(104)

$$\delta \zeta^1 s^{\mu\nu} = \xi^\kappa \partial_\kappa s^{\mu\nu} + \partial_\mu \xi^\kappa s^{\kappa\nu} + \partial_\nu \xi^\kappa s^{\mu\kappa},$$

(105)

where $\xi^a_1 = i\kappa \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\nu \gamma_\rho \gamma_5 \zeta_\sigma$. These show that $w^a_\mu(x)$ and $s^{\mu\nu}(x)$ have general coordinate transformations under the new NLSUSY transformations (102) and (103). Therefore replacing $e^a_\mu(x)$ in EH Lagrangian of general relativity by the new vierbein $w^a_\mu(x)$ we obtain the following Lagrangian which is invariant under (102) and (103):

$$L = -\frac{c^3}{16\pi G} |w|(\Omega + \Lambda),$$

(106)

$$|w| = \text{det} w^a_\mu = \text{det}(e^a_\mu + t^a_\mu),$$

(107)

where the overall factor is now fixed uniquely to $\frac{c^3}{16\pi G}$, $e^a_\mu(x)$ is the vierbein of EGRT and $\Lambda$ is a probable cosmological constant. $\Omega$ is a (mimic) new unified scalar curvature analogous to the Ricci scalar curvature $R$ of EGRT. The explicit expression of $\Omega$ is obtained by just replacing $e^a_\mu(x)$ in Ricci scalar $R$ of EGRT by $w^a_\mu(x) = e^a_\mu + t^a_\mu$, which gives the gravitational interaction of $\psi^a_\alpha(x)$. The lowest order term of $\kappa$ in the action (106) gives the EH action of general relativity. And in flat spacetime, i.e. $e^a_\mu(x) \rightarrow \delta^a_\mu$, the action (106) reduces to VA model with $\kappa^{-1} = \frac{c^3}{16\pi G} \Lambda$. Therefore our model predicts a non-zero (small) cosmological constant.

As for the Lorentz invariance we again require that the new vierbein $w^a_\mu(x)$ should have formally a local Lorentz transformation as for SGM with spin 1/2 NG fermion. Then we find that the following (generalized) local Lorentz transformations

$$\delta_L \psi^a(x) = e^a_b \psi^b - \frac{i}{2} \varepsilon_{bde} \gamma^e \psi^a,$$

(108)

$$\delta_L e^a_\mu(x) = e^a_b e^b_\mu - i\kappa \varepsilon^{abcd} \{ \bar{\psi}_b \gamma_c \gamma_5 \psi_d (\partial_\mu \epsilon^e) - \frac{i}{4} \varepsilon^{efg} \bar{\psi}_b \gamma_g \psi_d (\partial_\mu \epsilon^e) \}$$

(109)
induce the desirable transformation. The equation (109) also reduces to the familiar form of the Lorentz transformations if the global transformations are considered (for $g^{\mu\nu}$).

Therefore, as in spin 1/2 SGM case, replacing $e^a_\mu(x)$ in EH Lagrangian of GR by the new vierbein $u^a_\mu(x)$ defined by (98), we obtain the Lagrangian (106) of the same form as (7), which is invariant under (102), (103), (108) and (109).

The commutators of two new supersymmetry transformations (102) and (103) on $\psi^a(x)$ and $e^a_\mu(x)$ are now calculated as

\[ [\delta_{\zeta_1}, \delta_{\zeta_2}] \psi^a = 2i\kappa (\varepsilon^{\mu\nu\rho\sigma} \zeta_{2\rho} \gamma_5 \zeta_{1\sigma} - \xi_{1}^{\rho} \xi_{2,\rho} (\partial_{\rho} e^a_\sigma)) \partial_\mu \psi^a, \]

\[ [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a_\mu = 2i\kappa (\varepsilon^{\rho\nu\sigma\tau} \zeta_{2\rho} \gamma_5 \zeta_{1\sigma} - \xi_{1}^{\rho} \xi_{2,\rho} (\partial_{\rho} e^a_\sigma)) \partial_{\mu} (\partial_{\rho} e^a_\sigma). \]

These can be rewritten as GL(4R);

\[ [\delta_{\zeta_1}, \delta_{\zeta_2}] \psi^a = \Xi^\mu \partial_\mu \psi^a, \]

\[ [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a_\mu = \Xi^\rho \partial_\rho e^a_\mu + e^a_\rho \partial_\rho \Xi^\rho, \]

where $\Xi^\mu$ is now a generalized gauge parameter defined by

\[ \Xi^\mu = 2i\kappa (\varepsilon^{\mu\nu\rho\sigma} \zeta_{2\rho} \gamma_5 \zeta_{1\sigma} - \xi_{1}^{\rho} \xi_{2,\rho} (\partial_{\rho} e^a_\sigma)). \]

Also, the commutator of the local Lorentz transformation on $e^a_\mu(x)$ of Eq.(109) is calculated as

\[ [\delta_{L_1}, \delta_{L_2}] e^a_\mu = \beta^a_\mu e^b_\mu - i\kappa \varepsilon^{\nu\rho\sigma\tau} \psi_\rho \gamma_5 \psi_\sigma (\partial_\mu \beta_\nu e^c_\tau - \frac{i}{4} \varepsilon^{c\rho g} \bar{\psi}_b \gamma_5 \psi_\rho (\partial_\mu \beta_\nu)). \]

where $\beta_{ab}$ is the same as SGM with spin 1/2. The equations (109) and (113) explicitly reveal a generalized local Lorentz transformation with the parameters $\epsilon_{ab}$ and $\beta_{ab}$, which forms a closed algebra.

Therefore our action (106) is invariant at least under

\[ [\text{global new NLSUSY}] \otimes [\text{local GL(4,R)}] \otimes [\text{local Lorentz}] \otimes [\text{global SO(N)}], \]

when it is extended to global SO(N). It is interesting that the spin 3/2 massless field can couple consistently with graviton besides SUGRA. SGM formalism [17] can be generalized to the spacetime with extra dimensions for the inclusion of the non-abelian internal symmetries. It may give a potential new framework for the simple unification of spacetime and matter.

Finally we just mention the phenomenological implications of our model. As read off from the above discussions it is easy to introduce (global) SO(N) internal symmetry in our model by replacing $\psi^a_\alpha(x) \rightarrow \psi^{ia}_\alpha(x), (i = 1, 2, \ldots, N)$, which may enable
us to consider SGM[5] with spin 3/2 superon. However the fundamental internal symmetry for superons may be rather different from SGM, for the generator of a new algebra shifts spin by 3/2 and one-superon states correspond to spin 1/2 states but the adjoint representation is a vector as well. Also it is worthwhile to consider SGM with extra dimensions. We think that the above result is useful when we consider the gravitational interaction of the massless field with higher half-integer spin (> 5/2), though the algebra itself contains the negative norm states[15].

6 Cosmology of New EH-type Action

There remain many unsolved interesting problems, even qualitatively, in the physics of the universe, e.g. the birth of the universe which is expanding, the origins of the inflation and the big bang, the tiny value of the cosmological constant, the critical value(∼ 1) of the energy density, the dark energy, the baryon number genesis, ... etc. These problems should be understood in terms of the knowledges of the unified local field theory of particle physics. We discuss briefly and qualitatively the potential of SGM for these unsolved problems.

We regard that the ultimate entity of nature is high symmetric SGM spacetime inspired by NLSUSY, where the coset space coordinates ψ of $\frac{\text{superGL}(4,R)}{\text{GL}(4,R)}$ turning to the NG fermion d.o.f. in addition to the ordinaly Minkowski coordinate $x^a$, i.e. $\text{local SL}(2C) \times \text{local SO}(3,1)$ d.o.f., are attached at every spacetime point. The geometry of new spacetime is described by SGM action (7) of vacuum EH-type and gives the unified description of nature. The fundamental action (7) on new (SGM) spacetime is unstable against the new global NLSUSY transformation and induces the self-contained spontaneous (symmetry) breakdown into ordinary observed Riemann spacetime and the massless superon-quintet matter which expands rapidly, for the curvature-energy of SGM spacetime is converted into those of Riemann spacetime and energy-momentum of the superon(matter). This may be regarded as the gapless phase transition of spacetime from SGM to Riemann. Also this may be the birth of the present expanding universe, i.e. the big bang and the consequent rapid expansion (inflation) of spacetime in the quark-lepton SM era. And we think that the birth of the universe by the spontaneous breakdown of self-contained SGM spacetime of vacuum action of EH-type (7) may explain qualitatively the observed critical value(∼ 1) of the energy density of the universe.

Note that SGM action posees two inequivalent flat spaces, one is SGM-flat($w^a_\mu(x) \rightarrow \delta^a_\mu$) which allows nontrivial configurations of $e^a_\mu$ and $t^a_\mu$ and the other is Riemann flat($e^a_\mu(x) \rightarrow \delta^a_\mu$), which are crucial for the spontaneous breakdown from SGM to Riemann spacetime. As proved for EH action of GR [16], the energy of SGM action of EH-type is expected to be positive (for positive $\Lambda$).
Remarkably the observed Riemann spacetime of EGRT and matter(superon) appear simultaneously from (the vacuum) SGM action by the spontaneous decay of SGM spacetime, i.e. by the gapless phase transition of spacetime. The catastrophe problem of the gravitational collapse should be reconsidered in SGM due to the massless NG mode at the Planck scale, i.e. the phase transition to and subsequently from the unstable SGM spacetime. It is interesting if SGM could give new insights into these unsolved problems.

7 Discussions

A new EH-type action (called tentatively SGM from the composite viewpoints) in NLSUSY inspired (SGM) spacetime is obtained by the geometrical arguments similar to Einstein general relativity theory in Riemann spacetime. Despite the simple expressions of the unified vierbein defined on N=1 SGM spacetime

\[ w^a \mu = e^a \mu + t^a \mu, \]

\[ w^\mu_a = e^\mu_a - t^\mu_a + t^\rho_a t^\mu \rho t^\sigma \rho + t^\rho_a t^\mu \rho t^\nu \rho t^\mu \nu, \]

(Note that the second index of \( t \) represents the derivative.) and the consequent metric \( s_{\mu
u}, \cdots \), etc, SGM is a non-trivial generalization of EH action. In fact, as for the bosonic gauge transformation we can show explicitly that by the redefinitions (variations under GL(4R) with the field dependent parameters) only on the vierbein, e.g. \( e^a \mu \rightarrow e^a \mu - t^a \mu \) and the consequent variations on \( e^\mu_a \) it is impossible to gauge away \( \psi \) in compatible with new NLSUSY, for the new NLSUSY induces the square-root of GL(4R) on \( w \) (and \( s \)) and defined on the multiplet \( (e^a \mu, \psi) \).

Next we discuss on the confusive local spinor transformation which leaves SGM action invariant. SGM action (7) is invariant under the following local spinor translation with a local parameter \( \epsilon(x) \), \( \delta \psi = \epsilon \) and \( \delta e^a \mu = -i\kappa^2(\bar{\psi}\gamma^a \partial_\mu \psi + \bar{\psi}\gamma^a \partial_\mu \epsilon) \) which give \( \delta w^a \mu = \delta w^\mu_a = 0 \). It should be noticed that the local fermionic d.o.f. \( \psi \) would not be transformed (gauged) away. At a glance, the choice \( \delta \psi = \epsilon = -\psi \) seemingly gauges away \( \psi \). However, such an effect is canceled precisely by the simultaneous gauge transformation \( \delta e^a \mu \) with \( \epsilon = -\psi \), i.e. \( w(\epsilon, \psi) = w(e+\delta e, \psi+\delta \psi) = w(e+t, 0) \) as indicated by \( \delta w = 0 \), which reproduces precisely SGM action describing NLSUSY invariant gravitational interaction of superon. The commutators of these local spinor transformations on \( e^a \mu \) and \( \psi \) vanish identically. Therefore the local spinor translation mentioned above is a fake (gauge) transformation in a sense that, in contrast with the local Lorentz transformation invariance in EGRT, it can not eliminate gauge d.o.f. with spin 1/2, for the unified vierbein \( w = e + t \) is the only gauge field on SGM spacetime that contains only integer spin. (Note that the puzzling (space-time origin) local spinor symmetry plays an essential role in the linearization, i.e. in the superHiggs phenomena as demonstrated in the coupled system of VA action of NLSUSY and SUGRA of LSUSY equipped with a mass term and a cosmological constant.) This confusive situation comes from the funny geometrical formulation.
of SGM on unfamiliar SGM spacetime where, besides the Minkowski coordinates $x^a$, $\psi$ is a Grassmann coordinate (i.e., the another fundamental d.o.f.) defining the tangential spacetime with $SO(3, 1) \times SL(2C)$ d.o.f. inspired by NLSUSY. (Note that $SO(3, 1)$ is the twice covering group of $SL(2C)$.) And $\delta \psi = \epsilon(x)$ is just a coordinate translation(redefinition) on SGM flat spacetime. These situations can be understood easily by observing that the unified vierbein gauge field $w^a_{\mu}(x) = e^a_{\mu}(x) + t^a_{\mu}(x)$ is defined by $\omega^a = dx^a + \frac{i}{2} (\bar{\psi}^j \gamma^a d\psi^j - d\bar{\psi}^j \gamma^a \psi^j) \sim w^a_{\mu} dx^\mu$, where $\omega^a$ is the NLSUSY invariant differential form of VA and that $x^a$ and $\psi$ are coordinates of flat spacetime inspired by NLSUSY and SGM are encoded as a spacetime symmetry. From these geometrical viewpoints (in SGM spacetime) we can understand that $\psi$ is a coordinate and would be neither transformed away nor gauge-fixed away and the structure of SGM (flat) spacetime is preserved. Note that putting $\psi = 0$ by formal arguments concerning the local spinor symmetry makes SGM based upon NLSUSY vacuum(VA flat spacetime action) reduce to EH action based upon different vacuum(Minkowski flat spacetime), which is another theory based upon another vacuum. SGM (7) is a nontrivial generalization of EH action and Born-Infeld action\cite{47}. SGM possesses some hidden global symmetries originating from the fact that the graviton and (the energy-momentum of) the superon contribute equally to the unified vierbein $w_{\mu}$\cite{23}.

In this talk we have presented an attempt to describe the unity of nature as a geometry of new spacetime manifold with high symmetry and rich structures, which is called tentatively SGM spacetime from the viewpoints of the compositeness of matter. New (SGM) spacetime is the ultimate physical entity described by EH-type vacuum action \cite{7} and induces spontaneously the phase transition to observed Riemann spacetime and matter. We have depicted the potential of new EH-type (SGM) action. The study of the vacuum structure of SGM action in the broken phase (i.e. SGM action in Riemann spacetime) is important and challenging.

SGM with the extra dimensions to be compactified is open, which may allow the unification by means of the elementary fields. In this case the mechanism of the conversion of the spacetime d.o.f. into the dynamical d.o.f. is duplicate, i.e. by the compactification of Kaluza-Klein type and by the new mechanism adopted in SGM. SGM with spin 3/2 NG fermion may be in the same scope but remains to be studied.
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