Monte Carlo cluster simulations to determine the rate of compact star inspiraling to a central galactic black hole

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Abstract. The capture and gradual inspiral of stellar mass objects by a massive black hole at the centre of a galaxy has been proposed as one of the most promising source of gravitational radiation to be detected by LISA. Unfortunately rate estimates for this process suffer from many uncertainties. Here we report on the use of our newly developed Monte Carlo stellar dynamics code to tackle this problem. We present results from simple galactic nuclei models that demonstrate the high potential of our approach and point out the aspects of the problem where an improved treatment seems desirable.

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1. Introduction

In the past decade, the harvest of observational evidences for the presence of massive dark objects in the centre of most bright galaxies has become very impressive. It is generally accepted that these mass concentrations are most likely massive black holes (MBHs) with detected masses ranging between $10^6$ and $10^{10} M_\odot$. For all but two galaxies, however, today’s observations in the electro-magnetic spectrum lack resolution to rule out concurrent interpretations concerning the nature of the dark object [1, 2].

The motion of a “test particle” (with mass $\ll M_{\text{MBH}}$) on a relativistic orbit, monitored by the emitted gravitational waves (GW), is an ideal way to get much more precise information from the immediate vicinity of the putative black hole. By detecting such signals and comparing them with general relativity predictions, we could, at the same time, test Einstein’s theory in the strong field regime, confirm the existence of MBHs (in a statistical sense, if not in particular galaxies) and get precise determinations of their mass and spin [3].

However, two main classes of theoretical difficulties have to be worked around. First, the computation of the trajectory of a particle around a Kerr BH without resorting to strong simplifying assumptions, is a formidable, still unsolved, problem. The second difficulty is the prediction of the number of potential sources, i.e. the rate with which stars get so deeply bound to a central MBH that their subsequent orbital evolution is driven by GW-emission.
The range of MBH masses that can be probed by LISA is $3 \times 10^5 - 3 \times 10^7 M_\odot$. Only for the highest $M_{\text{MBH}}$ values can MS stars be captured on relativistic orbits without being teared off by tidal forces so we expect most capture events to feature a compact remnant. It can be brought onto relativistic orbits through 2-body relaxation or non-disruptive collisions with other stars if the velocity dispersion exceeds $V_*$, an unlikely situation for compact stars. Repeated impacts on an accretion disk could also channel stars onto relativistic orbits \[4\], but this process is probably ineffective for compact remnants as they would not sweep much disk’s material at each crossing.

The rate of captures on orbits with strong emission of gravitational radiation (hereafter “GR-captures”) has been estimated in three previous works \[5, 6, 7\]. Unfortunately, these authors use different galactic nuclei models, assumptions and computation methods... and quite expectedly get discordant capture rates, ranging from $2 \times 10^{-8}$ to $10^{-4}$ yr$^{-1}$. Besides the determination of such a rate, important issues raised by these works are the following. To what extent is the mass-segregation process efficient at concentrating stellar remnants towards the central regions? Is the GR-capture rate sustainable for many Gyrs? How does the stellar nucleus evolve on the long run in response to this and other stellar dynamical processes (relaxation, tidal disruptions, collisions, stellar evolution)?

2. The Monte Carlo cluster evolution code

2.1. Overview of the code

We recently developed a new computer code to follow the evolution of a central star cluster surrounding a MBH over time scales as long as $10^{10}$ years. This tool, which we describe in \[8\], is the backbone of our work aimed at a better modelization and understanding of the influence of the MBH on the stellar system.

The numerical scheme we chose is a Monte Carlo (MC) method based on the pioneering work of Hénon \[9\]. This class of programs proved very successful in the realm of globular clusters studies \[10, 11, 12\]. In its basic form, the method is more or less equivalent to resolving the Fokker-Planck (FP) equation, but with no implicit integration of the diffusion coefficients. The relaxation-driven secular evolution of a stellar system is simulated without wasting CPU time at integrating orbits. Compared to direct FP resolutions, the main advantages of the MC method are (1) its simplicity, (2) the fact that any stellar mass function and any velocity distribution (both variable in time and position) are naturally coped with, as is the cluster’s self-gravity, and (3) the ease with which extra physical processes can be included, even those that are not continuous, like collisions. Although they are also particle-based, MC simulations are enormously faster than relaxational $N$-body integrations whose computing time increases like $N_{\text{par}}^{2-3}$ while the scaling is almost linear for our MC code. Furthermore, as each particle represents a spherical shell of stars with same orbital and stellar properties, the number of simulated stars can be made arbitrarily high. Of course, MC methods
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also suffer from a few drawbacks. On the one hand, they produce much noisier results than FP methods. On the other hand, they rely on more simplifying assumptions than N-body schemes; the strongest of them are spherical symmetry, constant dynamical equilibrium and treatment of relaxation as the integrated effect of distant, uncorrelated, 2-body encounters.

As far as we know, our code is the first implementation of the Hénon’s scheme devoted to MBH-hosting dense galactic nuclei. We spent much time to include stellar collisions (between MS stars, for the time being) as realistically as possible $^{[13, 14]}$. From more than 12 000 MS-MS collision simulations carried out with a Smoothed Particle Hydrodynamics code (SPH, $^{[13]}$), we built a grid of collisional outcomes from which the result of any collision is interpolated when it occurs during a cluster evolution simulation. We also included “loss-cone” effects $^{[16]}$ to account for the tidal disruptions of stars in the Roche zone around the MBH. Finally, for this LISA symposium, we adapted this loss-cone routine in order to detect GR-capture events.

2.2. Computing capture rates with the Monte Carlo code

Following Sigurdsson & Rees $^{[3]}$, we consider that a star is “captured” by the MBH if it gets on an orbit whose shrinkage time by GW-emission, $T_{GW}$, is shorter than the time 2-body relaxation would take to modify it substantially, $T_{mod}$. The most likely way for a star to get sufficiently close to the MBH to be captured is to experience 2-body encounters that make its orbit nearly radial. For such a highly eccentric orbit, $T_{mod}$ is the time taken by relaxation to introduce a relative change in the pericentre distance of order 1. Then, $T_{mod} \simeq (1 - e)T_{relax} \ll T_{relax}$ where $e$ is the eccentricity. Typical values of $1 - e$ for captured particles are $10^{-7} - 10^{-3}$.

Using time steps $\delta t$ as small as a fraction of $T_{mod}$ would completely jeopardize the efficiency of the MC scheme for which $\delta t \simeq 0.001 - 0.01 T_{relax}$ is otherwise sufficiently small. For a star with fixed $|\vec{V}|$ at a given distance from the centre, the capture orbits form a thin conical bundle with aperture angle $\theta_{GRC} \ll \pi$. If we impose too large a $\delta t$, $\vec{V}$ jumps over this tiny “loss cone” and most captures are missed. This problem is closely reminiscent to the detection of tidal disruptions. Thus we solved it the same way. We keep time steps that are a (small) fraction of $T_{relax}$ (and/or $T_{coll}$) but we “over-sample” each of them by simulating the random walk of the tip of $\vec{V}$ on a sphere during $\delta t$. We simply piggy-backed the procedure for GR-captures detections on the routine for tidal disruptions. Hence, as a star would be tidally destroyed only if it stays on a loss cone orbit until next pericentre passage, the time resolution of the random walk is the orbital period $T_{orb}$. This is not well adapted to GR-captures for which $T_{orb}$ has not special meaning. However, entries in the GR-capture loss cone are correctly simulated as a diffusion process as long as individual steps in the random walk are smaller than $\theta_{GRC}$. But this is always true because, by definition, the R.M.S. diffusion angle during $T_{GW}$ is $\theta_{GRC}$ and $T_{GW}$ is much longer than $T_{orb}$.

We think that the main intrinsic limitation in our approach is to treat relaxation
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with small angle approximation. Indeed, close gravitational encounters with scattering angles of order $\pi$, although they only contribute a fraction $\ln(\Lambda)^{-1} < 0.1$ to the overall relaxation, can dominate the rate of captures $[\mathbb{L}]$. Another limitation is of statistical nature. To ensure energy conservation, each particle stands for a fixed number of stars. Consequently there are only 1300 SBH particles out of $2 \times 10^6$.

3. Simulations of simple models of galactic nuclei

3.1. Initial model & included physics

To explore the potential of our MC code in predicting GR-capture rates, we run a few simulations that are variations around our “standard” galactic nucleus model. The stellar cluster is set as a $W_0 = 8$ King model, with a core radius of 0.47 pc and $3.6 \times 10^8$ stars. As stellar evolution is not simulated, we have to include an evolved stellar population, containing compact remnants, from the beginning of the computations. It is prepared according to the prescription of Miralda-Escudé & Gould $[\mathbb{L}]$. The number fractions of white dwarfs (WDs), neutron stars (NSs) and SBHs are 0.056, $3.5 \times 10^{-3}$ and $6.5 \times 10^{-4}$, and their individual masses 0.6, 1.4 and 7.0 $M_\odot$, respectively. We do not include giant stars. An initial “seed” black hole ($M_{\text{BH}}/M_{\text{clust}} \approx 10^{-4}$) is set at the centre. It accretes all gas released by stellar collisions and tidal disruptions with no delay. GR-captured stars are also assumed to be immediately swallowed by the BH. Direct plunges through the horizon are detected. Collisions between MS stars are treated realistically thanks to our SPH grid but tidal disruptions are assumed to be always total.

This model is not meant to be highly realistic. It is probably too dense and massive to represent a real galactic nucleus. To bracket the density value at the Galactic centre $[\mathbb{L}]$, we also used another model with a number of stars reduced by a factor 10. A better representation for a galactic nucleus would require a power law density profile at large radii ($\rho \propto R^{-\alpha}$ with $\alpha \leq 2$ typically) instead of a steep cut-off. This point is problematical as it imposes to put large amounts of particles at large distances and consequently reduce the resolution near the MBH where all the action takes place.

3.2. Simulation results

Some aspects of the evolution of the “standard” model are shown in figure 1 while figure 2 illustrates the evolution of the lighter nucleus. Both simulations were realized with $2 \times 10^6$ particles.

We see that relaxational segregation of SBHs towards the centre occurs quickly. An early broad peak in the capture rate for this species ensues. $\dot{N}_{\text{SBH}}$ then steadily decreases due to exhaustion of SBHs in the central regions. At late times, the capture rate is dominated by WDs, NSs, and, for dense clusters, low-mass MS stars ($\langle M_* \rangle \approx 0.2 M_\odot$). However, we noted that for many of these MS stars, we have $T_{\text{GW}} \gg 10^9$ years even though the capture condition ($T_{\text{GW}} < T_{\text{mod}}$) is obeyed. Further investigations about these cases are called for. Typical remnant capture rates at $T = 1 - 2 \times 10^{10}$ years are
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Figure 1. Evolution of the “standard” model (see text). Panel (a) shows the rate of captures by emission of gravitational radiation for various stellar types. Panel (b) depicts the evolution of a few Lagrangian radii. Panel (c) shows the contribution of various processes to the growth of the central BH.

Figure 2. Same as figure 1 for a nucleus model which is 10 times less massive.

$\sim 10^{-5}$ yr$^{-1}$ and $\sim 5 \times 10^{-7}$ yr$^{-1}$ for the standard and “light” models. A nucleus with 10 times the mass of the standard model yields $\sim 10^{-4}$ yr$^{-1}$.

A massive BH ($M > 10^6 M_\odot$) can be grown from a seed if the initial stellar density is high enough. However, the stellar density after a Hubble time remains uncomfortably high ($\sim 10^7 M_\odot$ pc$^{-3}$ at 0.1 pc). Tidal disruptions are the major contributors to the BH’s mass with stellar collisions playing only a minor role. In the lighter model, GR-captures of SBHs dominates the early growth of the central BH with tidal disruptions catching up later on. This is probably connected to the stronger relaxation in this model.

3.3. Future improvements

There are many ways our simulations could be improved in the future. Let’s mention a few of them that are relevant for the problem of GR-captures.

- The lack of resolution for rare species, such as SBHs, and the problem of extending the initial model to larger radii could be solved if each particle could represent
a different number of stars. We presently do not know how to achieve this. Alternatively, a massive increase in the number of particles would solve these difficulties. A simulation with $10^7$ particles would require $\sim 1$ month of CPU time, but we run short of computer memory. To go beyond this number, a parallel version of the code should be developed.

- Stellar evolution should be included. In that case, we have to determine the fate of the gas lost by stars, and particularly the fraction accreted onto the central BH. For our IMF, more than 40% of the ZAMS mass is lost from the stars in the first $10^{10}$ years, which amounts to $5.9 \times 10^7 M_\odot$ for the standard model! Preliminary calculations including stellar evolution indicate that the GR-capture rate at late stages is not substantially altered when all the emitted gas is lost from the nucleus.

- A better treatment of GR-capture events should be attempted. It could go beyond the simple $T_{GW} < T_{\text{mod}}$ criterion and the assumption of immediate swallowing.

- The contribution of large angle scatterings to GR-captures must be clarified and included in our simulations if important.

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