Vector-Tensor and Vector-Vector Decay Amplitude Analysis of $B^0 \rightarrow \varphi K^0$
We perform an amplitude analysis of the decays $B^{0} \rightarrow \phi K^{*0}(1430)^{0}$, $\phi K^{*}(892)^{0}$, and $\phi(K\pi)^{0}_{S-wave}$ with a sample of about $384 \times 10^{6}$ $B\bar{B}$ pairs recorded with the BABAR detector. The fractions of longitudinal polarization $f_{L}$ of the vector-tensor and vector-vector decay modes are measured to be $0.853^{+0.066}_{-0.036}$ and $0.506 \pm 0.040 \pm 0.015$, respectively. Overall, twelve parameters are measured for the vector-vector decay and seven parameters for the vector-tensor decay, including the branching parameters and parameters sensitive to CP violation.

The interest in the polarization and CP-asymmetry measurements in $B \rightarrow \phi K^{*}$ decays is motivated by their potential sensitivity to physics beyond the standard model in the $b \rightarrow s$ transition, shown in Fig. 1(a) [1]. The polarization measurements of $B$ meson decays reveal both strong and weak interaction dynamics and are discussed in a recent review [2,3]. The large fraction of transverse polarization in the $B \rightarrow \phi K^{*}(892)$ decay measured by BABAR [4] and by Belle [5] indicates a significant departure from the naive expectation of predominant longitudinal polarization. This suggests other contributions to the decay amplitude, previously neglected, either within or beyond the standard model [6].

We now extend our investigation of the polarization puzzle with an amplitude analysis of the vector-tensor $B^{0} \rightarrow \phi K^{*0}(1430)^{0}$ decay. We also measure vector-vector $B^{0} \rightarrow \phi K^{*0}(892)^{0}$ and vector-scalar $B^{0} \rightarrow \phi(K\pi)^{0}_{0}$ decay amplitudes, where $(K\pi)^{0}_{0}$ is the $J^{P} = 0^{+} K\pi$ component. We use the dependence on the $K\pi$ invariant mass of the interference between the $J^{P} = 0^{+}$ and $1^{-}$ or $2^{+}$ components [7,8] to resolve the discrete ambiguity in the determination of the strong and weak phases otherwise present in the $B^{0} \rightarrow \phi K^{*0}(892)^{0}$ analysis [2,4,5] and to provide new measurements of the strong and weak phases relative to the vector-scalar decay amplitude.

The angular distribution of the $B \rightarrow \phi K^{*}$ decay can be expressed as a function of $\mathcal{H}_{i} = \cos\theta_{i}$, and $\Phi$ shown in Fig. 1(b). Here $\theta_{i}$ is the angle between the direction of the $K$ meson from the $K^{*} \rightarrow K\pi(\theta_{1})$ or $\phi \rightarrow KK(\theta_{2})$ and the direction opposite the $B$ in the $K^{*}$ or $\phi$ rest frame, and $\Phi$ is the angle between the decay planes of the two systems. The differential decay width has seven complex amplitudes $A_{J\lambda}$ corresponding to the spin of the $K\pi$ system $J$ and the helicity $\lambda = 0$ or ±1:

$$
\frac{d\Gamma}{d\mathcal{H}_{1}d\mathcal{H}_{2}d\Phi} \propto \left| \sum A_{J\lambda} Y_{J}^{\lambda}(\mathcal{H}_{1}, \Phi) Y_{1-\lambda}(-\mathcal{H}_{2}, 0) \right|^{2},
$$

where $Y_{J}^{\lambda}$ are the spherical harmonics with $J = 2$ for $K^{*0}(1430)$, $J = 1$ for $K^{*0}(892)$, and $J = 0$ for $(K\pi)_{0}^{0}$. We can reparameterize the amplitudes with the index $J$ suppressed as $A_{0}$ and $A_{\pm 1} = (A_{\parallel} \pm A_{\perp})/\sqrt{2}$.

We analyze $B^{0} \rightarrow \phi K^{*0} \rightarrow (K^{+}K^{-})(K^{\pm}\pi^{\mp})$ candidates using data collected with the BABAR detector [9] at the PEP-II $e^{+}e^{-}$ collider. A sample of $383.6 \pm 4.2$ million $Y(4S) \rightarrow B\bar{B}$ events was recorded at the center-of-mass energy $\sqrt{s} = 10.58$ GeV. Charged-particle momenta are measured in a tracking system consisting of a silicon vertex tracker with five double-sided layers and a 40-layer drift chamber, both within the 1.5-T magnetic field of a solenoid. Charged-particle identification is provided by measurements of the energy loss in the tracking devices and by a ring-imaging Cherenkov detector.

We use two kinematic variables: $\Delta E = (E_{i}E_{B} - p_{B} \cdot p_{i} - s/2)/(\sqrt{s}$ and $m_{ES} = ((s/2 + p_{i} \cdot p_{B})^{2}/E_{i}^{2} - p_{B}^{2})^{1/2}$, where $(E_{i}, p_{i})$ is the $e^{+}e^{-}$ beam four-momentum, and $(E_{B}, p_{B})$ is the four-momentum of the $B$ candidate. We require $|\Delta E| < 0.1$ GeV and $m_{ES} > 5.25$ GeV. The requirements on the invariant masses are $0.99 < m_{K\bar{K}} < 1.05$ GeV and $0.75 < m_{K\pi} < 1.05$ GeV (lower $m_{K\pi}$ range) or $1.13 < m_{K\pi} < 1.53$ GeV (higher $m_{K\pi}$ range).

To reject the dominant $e^{+}e^{-}$ → quark-antiquark background, we use variables calculated in the center-of-mass frame. We require $|\cos\theta_{T}| < 0.8$, where $\theta_{T}$ is the angle between the $B$-candidate thrust axis and that of the rest of the event. We construct a Fisher discriminant, $f$, that combines the polar angles of the $B$-momentum vector and the $B$-candidate thrust axis with respect to the beam axis, and the two Legendre moments $L_{0}$ and $L_{2}$ of the energy flow around the $B$-candidate thrust axis [10].

We remove signal candidates that have decay products with invariant mass within 12 MeV of the nominal mass values for $D_{s}^{+}$ or $D_{s}^{-} \rightarrow \phi \pi^{*}$. In about 5% of events, more than one candidate is reconstructed, and we select the one whose four-track vertex has the lowest $\chi^{2}$. We define the flavor sign $Q$ to be the charge of the pion.

We use an unbinned, extended maximum-likelihood fit [4] to extract the event yields $n_{j}$, flavor asymmetries $A_{j\lambda}$, and the probability density function (PDF) parameters,
denoted by \( \xi \) for the polarization parameters and \( \xi \) for the remaining parameters. The data model has five event categories: \( B \to \phi(K\pi)_{J=0,1/2}, B \to f_0(980)K^* \), and combinatorial background. The combinatorial background PDF is found to account well for both the dominant quark-antiquark background and the random tracks from the \( B \) decays. The likelihood \( \mathcal{L}_i \) for each candidate \( i \) is defined as \( \mathcal{L}_i = \sum_{k} \mathcal{P}_j(x_i; \xi, \xi) \), where each of the \( \mathcal{P}_j \) is the PDF for variables \( x_i = \{ \mathcal{H}_1, \mathcal{H}_2, \Phi, m_{K\pi}, m_{ES}, \Delta E, m_{ES}, \mathcal{F}, \mathcal{Q} \} \). The flavor index \( k \) corresponds to the value of \( Q \), that is \( \mathcal{P}^k \equiv \mathcal{P}_j \times \delta_{Q^k} \).

We define \( n_j = n_j^+ + n_j^- \) and \( \mathcal{A}_j = (n_j^+ - n_j^-)/(n_j^+ + n_j^-) \). The polarization parameters, with the index \( J \) suppressed, are defined as \( f_L = |A_0|^2/\Sigma|A_\lambda|^2 \), \( f_\perp = |A_\perp|^2/\Sigma|A_\lambda|^2 \), \( \phi_0 = \arg(A_0)/A_0 \), and \( \phi_\perp = \arg(A_\perp)/A_0 \). We allow for \( CP \)-violating differences between the \( B^0 \) \((Q = +1)\) and \( B^0 \) \((Q = -1)\) decay amplitudes (\( A \) and \( A \)) and incorporate them via the replacements \( f_L \to f_L \times (1 + \mathcal{A}_0 \times Q) \), \( f_\perp \to f_\perp \times (1 + \mathcal{A}_\perp \times Q) \), \( \phi_0 \to \phi_0 + \Delta \phi_0 \times Q \), and \( \phi_\perp \to \phi_\perp + \pi/2 + (\Delta \phi_\perp + \pi/2) \times Q \) [2].

The PDF \( \mathcal{P}(x_i; \xi, \xi) \) for a given candidate \( i \) is a joint PDF for the helicity angles, resonance masses, and \( Q \), and the product of the PDFs for each of the remaining variables. The helicity part of the exclusive \( B \) decay PDF is the ideal angular distribution from Eq. (1), where the amplitudes \( A_\lambda \) are expressed in terms of the polarization parameters \( \xi \), multiplied by an empirically-determined acceptance function \( G(\mathcal{H}_1, \mathcal{H}_2, \Phi) = G_1(\mathcal{H}_1) \times G_2(\mathcal{H}_2) \). A relativistic \( J \)-spin Breit-Wigner amplitude parameterization is used for the resonance mass [3,11], except for the \( (K\pi)^0 m_{K^*} \) amplitude parameterized with the LASS function [7]. The latter includes the \( K_0^0(1430)^0 \) resonance together with a nonresonant component.

The interference between the \( J = 1 \) or \( 2 \) and the \( S \)-wave \((K\pi) \) contributions is modeled with the three terms \( 2R \mathcal{A}_0 \mathcal{A}_0^\ast \) in Eq. (1) with the four-dimensional angular and \( m_{K\pi} \) dependence. It has been shown in the decays \( B^0 \to J/\psi(K\pi)^0 \) and \( B^+ \to \pi^+(K\pi)^0 \) [8] that the amplitude behavior is consistent with that observed by LASS except for a constant phase shift. We allow an unconstrained overall shift, again with the index \( J \) suppressed, \((\delta_0 + \Delta \delta_0 \times Q)\) between the LASS amplitude phase and either the vector (\( J = 1 \)) or the tensor (\( J = 2 \)) resonance amplitude phase.

The parameters \( \xi \) describe the background or the remaining signal PDFs. They are left free to vary in the fit for the combinatorial background or are fixed to the values extracted from Monte Carlo (MC) simulation [12] and calibration \( B \)-decay channels for the exclusive \( B \) decays. We use a sum of Gaussian functions for the parameterization of the signal PDFs for \( \Delta E, m_{ES}, m_{ES}, \mathcal{F} \). For the combinatorial background, we use polynomials, except for \( m_{ES} \) and \( \mathcal{F} \) distributions which are parameterized by an empirical phase-space function and by Gaussian func-

\[ \text{FIG. 2. Projections onto the variables } m_{K\pi} \text{ (a), } m_{K^*} \text{ (b), } \Delta E \text{ (c), and } m_{ES} \text{ (d) for the signal } B^0 \to \phi(K\pi) \text{ candidates. Data distributions are shown with a requirement on the signal-to-background probability ratio calculated with the plotted variable excluded. The solid (dashed) lines show the signal-plus-background (background) PDF projections.} \]

\[ \text{FIG. 3. Projections onto the variables } \mathcal{H}_1 \text{ (a), } \mathcal{H}_2 \text{ (b), } \Phi \text{ (c), and the differences between the } \Phi \text{ projections for events with } \mathcal{H}_1 \mathcal{H}_2 > 0 \text{ and with } \mathcal{H}_1 \mathcal{H}_2 < 0 \text{ (d) for the signal } B^0 \to \phi K^* \text{ candidates following the solid (dashed) line definitions in Fig. 2. The } D_{(i)} \text{-meson veto causes the sharp acceptance dips seen in (a).} \]

\[ 051801-5 \]
The nonresonant $K^+ K^-$ contribution under the $\phi$ is accounted for with the $B^0 \to f_0^0 K^{*0}$ category. Its yield is consistent with zero in the higher $m_{K\pi}$ range and is 89 ± 18 events in the lower $m_{K\pi}$ range. The uncertainties due to $m_{K\bar{K}}$ interference are estimated with the samples generated according to the observed $K^+ K^-$ intensity and with various interference phases analogous to $\delta_0$ in $K\pi$. These are the dominant systematic errors for the $\xi$ parameters of the $B^0 \to \phi K^{*}(892)^0$ decay.

We vary those parameters in $\xi$ not used to model combinatoric background within their uncertainties and derive the associated systematic errors. We allow for the flavor-dependent acceptance function and the reconstruction efficiency in the study of asymmetries. The biases from the finite resolution of the angle measurement, the dilution due to the presence of fake combinations, or other imperfections in the signal PDF model are estimated with MC simulation. Additional systematic uncertainty originates from $B$ background, where we estimate that only a few events can fake the signal. The systematic errors in efficiencies are dominated by those in particle identification and track finding. Other systematic effects arise from event-selection criteria, $\phi$ and $K^{*0}$ branching fractions, and a number of $B$ mesons.

In the lower $m_{K\pi}$ range, the yield of the $\phi (K\pi)^{00}$ contribution is 60 ± 15 events with the statistical significance of 7.9$\sigma$, including the interference term. The dependence of the interference on the $K\pi$ invariant mass [7,8] allows us to reject the other solution near $(2\pi - \phi_{||}, \pi - \phi_{\perp})$ relative to that in Table II for the $B^0 \to \phi K^{*}(892)^0$ decay with significance of 5.4$\sigma$, including systematic uncertainties.

We also resolve this ambiguity with statistical significance of more than 4$\sigma$ with the $B^0$ or $B^0$ decays independently. Because of the low significance of our measured $f_0^0 = (1 - f_L - f_{\perp}) (2.9\sigma)$ and $f_{\perp} (1.6\sigma)$ in the $B^0 \to \phi K^*_\pi(1430)^0$ decay, we have insufficient information to constrain $\phi_{||}$ and $\phi_{\perp}$ at higher significance and to measure five asymmetries, and so we fix these asymmetry parameters to zero in the fit in the higher $m_{K\pi}$ range.

The $(V - A)$ structure of the weak interactions and the $s$-quark spin flip suppression in the diagram in Fig. 1(a) suggest $|A_0| \gg |A_{-1}| \gg |A_{+1}| [1,6]$. This expectation is consistent with our measurements in the vector-tensor $B^0 \to \phi K^*_\pi(1430)^0$ decay, but disagrees with our observed vector-vector polarization. In the $B^0 \to \phi K^*(892)^0$ decay, we obtain the solution $\phi_{||} \approx \phi_{\perp}$ without discrete ambiguities. Combined with the approximate solution $f_L \approx 1/2$ and $f_{\perp} \approx (1 - f_L)/2$, this results in the approximate decay amplitude hierarchy $|A_0| \approx |A_{+1}| \approx |A_{\perp}|$ (and $|A_0| \approx |A_{-1}|$).

We find more than 5$\sigma$ (4$\sigma$) deviation, including systematic uncertainties, of $\phi_{||}(\phi_{\perp})$ from either $\pi$ or zero in the $B^0 \to \phi K^*(892)^0$ decay, indicating the presence of final-state interactions (FSI) not accounted for in naive factorization. The effect of FSI is evident in the phase shift of the cosine distribution in Fig. 3(d).

Our measurements of eight $CP$-violating parameters rule out a significant part of the physical region and are consistent with no $CP$-violation in this decay. Significant nonzero $CP$-violating parameters would indicate the presence of new amplitudes with different weak phases. The $\Delta \phi_{\perp}$ and $\Delta \phi_{||}$ are particularly interesting due to sensitivity to the weak phases of the amplitudes without hadronic uncertainties [2], such as the relative weak phases of $A_{+1}$ and $A_0$, while the $CP$-violating $\Delta \delta_0$ parameter represents potential differences of weak phases among decay modes.

In summary, we have performed an amplitude analysis and searched for $CP$-violation in the angular distribution with the $B^0 \to \phi K^{*0}$ decays with the tensor, vector, and scalar $K^{*0}$. Our results are summarized in Tables I and II and supersede corresponding measurements in Ref. [4]. Our vector-tensor results are in agreement with quark spin flip suppression and $A_0$ amplitude dominance.
TABLE II. Summary of polarization results. The dominant fit correlation coefficients ($C$) are presented for the $K^*(892)^0$ mode where we show correlations of $\delta_0$ with $\phi_\parallel/\phi_\perp$ and of $\Delta \delta_0$ with $\Delta \phi_\parallel/\Delta \phi_\perp$. For the $K^*_2(1430)^0$ mode, the dominant values of $C$ are 32% for ($\delta_0, \phi_\parallel$) and 26% for ($\phi_\parallel, \phi_\perp$).

| $B^0 \rightarrow K^*_2(1430)^0$ | $B^0 \rightarrow K^*(892)^0$ | $C$ |
|---------------------------------|---------------------------------|-----|
| $f_L$                            | $0.853^{+0.061}_{-0.069} \pm 0.036$ | $0.506 \pm 0.040 \pm 0.015$ | $-53\%$ |
| $f_\perp$                         | $0.945^{+0.049}_{-0.046} \pm 0.013$ | $0.227 \pm 0.038 \pm 0.013$ | $61\%$ |
| $\phi_\parallel$                 | $2.90 \pm 0.39 \pm 0.06$ | $2.31 \pm 0.14 \pm 0.08$ | $37\%$ |
| $\phi_\perp$                      | $5.72^{+0.55}_{-0.57} \pm 0.11$ | $2.24 \pm 0.15 \pm 0.09$ | $22\%$ |
| $\delta_0$                        | $3.54^{+0.12}_{-0.14} \pm 0.06$ | $2.78 \pm 0.17 \pm 0.09$ | $37\%$ |
| $\delta_0^{+}$                   | $-0.03 \pm 0.08 \pm 0.02$ | $-0.03 \pm 0.16 \pm 0.05$ | $-51\%$ |
| $\delta_0^{-}$                   | $+0.24 \pm 0.14 \pm 0.08$ | $+0.19 \pm 0.15 \pm 0.08$ | $61\%$ |
| $\delta_0^0$                     | $+0.21 \pm 0.17 \pm 0.08$ | $37\%$ |

whereas the vector-vector mode contains substantial $A_{+1}$ amplitude from a presently unknown source either within or beyond the standard model [6].

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), MEC (Spain), and PPARC (United Kingdom). Individuals have received support from the Marie Curie EIF (European Union) and the A. P. Sloan Foundation.

*Also with Università di Perugia, Dipartimento di Fisica, Perugia, Italy
† Also with Università della Basilicata, Potenza, Italy
‡ Also with IPPP, Physics Department, Durham University, Durham DH1 3LE, United Kingdom

[1] A. Ali et al., Z. Phys. C 1, 269 (1979); G. Valencia, Phys. Rev. D 39, 3339 (1989); G. Kramer and W. F. Palmer, Phys. Rev. D 45, 193 (1992); H.-Y. Cheng and K.-C. Yang, Phys. Lett. B 511, 40 (2001); C.-H. Chen et al., Phys. Rev. D 66, 054013 (2002); M. Suzuki, Phys. Rev. D 66, 054018 (2002); A. Datta and D. London, Int. J. Mod. Phys. A 19, 2505 (2004).
[2] A. V. Gritsan and J. G. Smith, “Polarization in B Decays” review in [3], J. Phys. G 33, 833 (2006).
[3] W.-M. Yao et al. (Particle Data Group), J. Phys. G 33, 1 (2006).
[4] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 91, 171802 (2003); Phys. Rev. Lett. 93, 231804 (2004).
[5] K.-F. Chen et al. (Belle Collaboration), Phys. Rev. Lett. 91, 201801 (2003); Phys. Rev. Lett. 94, 221804 (2005).
[6] A. L. Kagan, Phys. Lett. B 601, 151 (2004); Y. Grossman, Int. J. Mod. Phys. A 19, 907 (2004); C. W. Bauer et al., Phys. Rev. D 70, 054015 (2004); P. Colangelo et al., Phys. Lett. B 597, 291 (2004); M. Ladisa et al., Phys. Rev. D 70, 114025 (2004); E. Alvarez et al., Phys. Rev. D 70, 115014 (2004); H. Y. Cheng et al., Phys. Rev. D 71, 014030 (2005); H. N. Li and S. Mishima, Phys. Rev. D 71, 054025 (2005); P. K. Das and K. C. Yang, Phys. Rev. D 71, 094002 (2005); C. H. Chen and C. Q. Geng, Phys. Rev. D 71, 115004 (2005); Y. D. Yang et al., Phys. Rev. D 72, 034009 (2005); C. S. Huang et al., Phys. Rev. D 73, 034026 (2006); M. Beneke et al., Phys. Rev. Lett. 96, 141801 (2006); C. H. Chen and H. Hatanaka, Phys. Rev. D 73, 075003 (2006).
[7] D. Aston et al. (LASS Collaboration), Nucl. Phys. B 296, 493 (1988); W. M. Dunwoodie (private communications).
[8] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 71, 032005 (2005); Phys. Rev. D 72, 072003 (2005).
[9] B. Aubert et al. (BABAR Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 479, 1 (2002).
[10] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 70, 032006 (2004).
[11] E. M. Aitala et al. (E791 Collaboration), Phys. Rev. Lett. 86, 765 (2001).
[12] S. Agostinelli et al., Nucl. Instrum. Methods Phys. Res., Sect. A 506, 250 (2003).