Quantum walk of a trapped ion in phase space

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We implement the proof of principle for the quantum walk of one ion in a linear ion trap. With a single-step fidelity exceeding 0.99, we perform three steps of an asymmetric walk on the line. We clearly reveal the differences to its classical counterpart if we allow the walker/ion to take all classical paths simultaneously. Quantum interferences enforce asymmetric, non-classical distributions in the highly entangled degrees of freedom (of coin and position states). We theoretically study and experimentally observe the limitation in the number of steps of our approach, that is imposed by motional squeezing. We propose an altered protocol based on methods of impulsive steps to overcome these restrictions, in principal allowing to scale the quantum walk to several hundreds of steps.

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A quantum walk is the deterministic quantum mechanical extension of a classical random walk. A simple classical version requires two basic operations: Tossing the coin (coin-operation), allowing for two possible and random outcomes. Dependent on this outcome, the walker performs a step to the right or left (step-operation). In the quantum mechanical extension the operations allow for coherent superpositions of entangled coin and position states. After several iterations the probability to be in a certain position is determined by quantum mechanical interference of the walker state that leads to fundamentally different characteristics of the walk.

The motivation for studying quantum walks is twofold. On the one hand, many classical algorithms include random walks. Examples can be found in biology, psychology, economics and physics, for example Einstein’s simple model for Brownian motion. The extension of the walk to quantum mechanics might allow for substantial speed-up of related quantum versions, as in prominent algorithms suggested by Shor and Grover due to other quantum-subroutines. On the other hand, the quantum walk could lead to new insights into entanglement and decoherence in mesoscopic systems. These topics might be explored by increasing the amount of walkers - even before any algorithm might benefit from the quantum random walk.

Quantum walks have been thoroughly investigated theoretically and first attempts at implementation have been performed with a very limited amount of steps due to a lack of operation fidelity or fundamental restrictions within the protocol. Some aspects have been realized on the longitudinal modes of a linear optical resonator and in a nuclear magnetic resonance experiment. An implementation based on neutral atoms in a spin-dependent optical lattice has resulted in an experiment recently. Other proposals considered an array of microtraps illuminated by a set of microlenses and Bose-Einstein condensates. Travaglione and Milburn proposed a scheme for trapped ions to transfer the high operational fidelities obtained in quantum information processing (QIP) into the field of quantum walks. We implement the proof of principle for a discrete, asymmetric quantum walk of one trapped ion along a line in phase space. After three steps, each performed with a fidelity exceeding 0.99, we reveal the differences to a classical random walk. The limit of coherent displacements to states inside the Lamb-Dicke regime was foreseen by, experimentally observed in a different context, and is confirmed by us.

In the experimental realization, we confine one $^{25}\text{Mg}^+$ ion of mass $m$ in a linear multi-zone Paul trap. Motion in the axial ($z$-) direction is harmonic with an os-
ciliation frequency $\omega_z = 2\pi \cdot 21$ MHz and energy eigenstates $|n\rangle$. The ion’s/walker’s state $\Psi_N$ after $N$ steps can be described by the state of the coin, $|T\rangle$ and $|H\rangle$, and positions $|\alpha_i\rangle$ (with $i$ integer) on the line, as depicted in Fig. 1. The coin states are composed of the $^2S_{1/2}$-ground-state hyperfine levels of the ion, $|F = 3, M_F = 3\rangle$ and $|F = 2, M_F = 2\rangle$, labeled $|T\rangle$ and $|H\rangle$, respectively (splitting $\omega_0 \approx 2\pi \cdot 1.8$ GHz, a magnetic field of $6 \cdot 10^{-4}$ Tesla lifts the degeneracy of states of identical $F$). Ideally, the position of the walker is encoded into the coherent state

$$|\alpha_i\rangle = e^{-|\alpha_i|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_i^n}{\sqrt{n!}} |n\rangle,$$

whith $i$ running from $-N$ to $N$.

The initial state of each walk - $\Psi_0 = |T\rangle \otimes |0\rangle$ - is prepared by laser cooling to $\Gamma (N = 0) < 0.03$, close to the ground state of motion $|0\rangle$ and optically pumping into the electronic state $|T\rangle$ $17$. To implement the coin toss, we couple state $|T\rangle$ and $|H\rangle$ with a resonant radio-frequency (rf) pulse. The pulse implements a rotation of the coin-state on the Bloch sphere $18$

$$\hat{R}(\Theta, \Phi) = \begin{pmatrix} \cos(\frac{\omega}{2}) & -ie^{-i\Phi} \sin(\frac{\omega}{2}) \\ -ie^{i\Phi} \sin(\frac{\omega}{2}) & \cos(\frac{\omega}{2}) \end{pmatrix},$$

where $|T\rangle = (0, 1)^T$ and $|H\rangle = (1, 0)^T$. The angle $\Theta$ is proportional to the duration of the rf-pulse and $\Phi$ is its phase. For the coin-operation we chose $\hat{R}(\pi/2, -\pi/2)$ with the rotation axis perpendicular to the cross-section of the Bloch sphere depicted in Fig. 1a.

The conditional unitary step-operator

$$\hat{S} = \sum_i |\langle T| \otimes |\alpha_{i+1}\rangle \langle \alpha_i | + |H\rangle \langle H| \otimes |\alpha_{i-1}\rangle \langle \alpha_i | \rangle,$$

is realized by a state dependent optical dipole force $0, 20, 21$. For this, two Raman laser beams at frequencies $\omega_1$ and $\omega_2$, both detuned ($2\pi \cdot 80$ GHz) blue from the $^2S_{1/2} \rightarrow ^2P_{3/2}$ electric dipole transition ($\lambda \approx 280$ nm) induce an AC-Stark shift of the electronic states. Their linear polarization and directions are chosen to provide forces ($F_T = -\frac{1}{2} F_H$) on the walker/ions that are conditioned on the coin state, as depicted in Fig. 1b.

If $\omega_2 - \omega_1 = \omega_z$, the forces would resonantly drive the ion, leading to coherent displacements $|\alpha_i\rangle$ along a line in phase space (co-rotating at $\omega_z$ within this letter). In our implementation, we choose a detuning $\delta = \omega_2 - \omega_1 - \omega_z \approx 2\pi \cdot 100$ kHz that the forces initially dephase and then rephase after a duration $t_d = 2\pi / \delta \approx 10 \mu$s. We stop the coherent drive after $t_d/2$, then do a $\hat{R}(\pi, 0)-$pulse to exchange $|T\rangle$ and $|H\rangle$ and then drive for $t_d/2$ again and finish the step sequence with another $\hat{R}(\pi, 0)-$pulse. The $\hat{R}(\pi, 0)-$pulses also serve as spin-echos $20$ that make our implementation less susceptible to coin state dephasing, e.g. due to magnetic field fluctuations. Ideally, the walk of $N$-steps could be performed by iterating coin- and step-operations $N$-times, with the ideal evolution depicted in Fig. 2.
as $\Omega_{n,n+1} = \sqrt{n + 1}/\Omega$, where $\Omega$ is the Rabi frequency of the carrier transition $|T, 0\rangle \rightarrow |H, 0\rangle$. Typical values in our implementation are $\Omega \approx 2\pi \cdot 500 \text{kHz}$, $z_0 \approx 10 \text{nm}$ and $\eta = 0.31$. This experiment is performed $> 10^5$ times for each set of parameters to measure the average probability $P_T^e(N)$ of occupation in state $|T\rangle$ and repeated for step wise increased pulse duration $t$ on the blue-sideband [22]. A discrete Fourier analysis of the recorded fluorescence data allows to deduce the relative contributions of specific frequency components $\Omega_{n,n+1}$ and therefore provides $P_{\alpha_i}^e(N)$. Since the frequency separation and therefore resolution is maximal between states $|n = 0\rangle$ and $|n = 1\rangle$, we can increase our experimental sensitivity by coherently displacing the motional state to be measured, $|\alpha_i\rangle$, $t$-times back on the $|\alpha_0\rangle$ state.

First, we measure the average probabilities $P_T^e(N)$ and $P_H^e(N)$ of occupation in state $|T\rangle$ and $|H\rangle$, respectively. After step #1 and #2, these probabilities $P_T^e(1)$, $P_H^e(2)$ ($P_T^e(1)$, $P_H^e(2)$) to observe state $|H\rangle$ ($|T\rangle$) are 0.5±0.01, identical to the classical evolution. Tossing the coin for the third time reveals the coin asymmetry due to interference in the quantum walk. The ideal state after step #3 is $|\Psi_3\rangle = |H\rangle (|\alpha_{-3}\rangle + |\alpha_1\rangle) + |T\rangle ( - |\alpha_{-1}\rangle - 2|\alpha_1\rangle + |\alpha_3\rangle)$. We experimentally observe $P_T^e(3) = 0.741 \pm 0.003$ and $P_H^e(3) = 0.259 \pm 0.001$ (statistical errors only), clearly deviating from the classical expectation, in good agreement with the ideal ratio $P_T^e(3)/P_H^e(3) = 3.124$.

To investigate the motional coherence within the walk we can either undo a certain number of steps and measure the probability to return to the initial state or, more significant, we can slightly vary the duration of the walker pulses, thus perturbing the overlap of the final coherent states (positions). The results are depicted in Fig. 3. Deviations from the correct walker-pulse duration lead to distinguishable positions for all different paths of the walker. Therefore the coin-state interference is completely lost and $P_T^e(3) \approx P_H^e(3) \approx 0.5$ for pulse length deviations of ±2%, in good agreement with our theoretical simulations (solid line in Fig. 3). We further investigate the probabilities $P_{\alpha_i}^e(N) = |\langle \alpha_i | \Psi_N \rangle |^2$ for the positions $|\alpha_i\rangle$ of the walker/ion. After step #1 and #2, we determine probabilities $P_{\alpha_1}^e(1)$ and $P_{\alpha_2}^e(2)$ identical to the classical ones. The probabilities observed after step #3 are depicted in Fig. 4, with uncertainties due to statistical errors only. As in the classical counterpart, only one of eight paths reaches each of the outermost positions and therefore $P_{\alpha_{-3}}^e(3) = 1/8 = 0.125$. We find $P_{\alpha_{-1}}^e = 0.128 \pm 0.0012$ and $P_{\alpha_1}^e(3) = 0.115 \pm 0.002$. However, the remaining 6 paths destructively interfere to give $P_{\alpha_{-3}}^e = 0.121 \pm 0.007$ ($P_{\alpha_{-1}} = 1/8$) and constructively add up, leading to $P_{\alpha_1}^e = 0.633 \pm 0.0012$ ($P_{\alpha_1} = 5/8 = 0.625$). The small populations in motional states of even $i$ result from the residual overlap of the (non-orthogonal) approximately coherent position states. This is in agreement with our theoretical simula-
tions, based on the code of Ref. [15]. Thus, step #3 again reveals the quantum nature of the walk.

In summary, we have implemented the proof of principle for the quantum walk of one ion with a single step fidelity exceeding 0.99. We clearly reveal the differences to a classical counterpart with experimental uncertainties on the (sub-)percent level. The number of steps is limited by the present protocol. More steps lead to higher motional excitations beyond the Lamb-Dicke regime where higher order effects in the interaction modify the transition rates. For our actual experimental realization, transition rates start to decrease around \( n = 8 \) (in our present implementation: \( \pi(1) = 1.33; \pi(2) = 4.71; \pi(3) = 9.08 \)) leading to motional squeezing [13, 22] and state reflection \( (\Omega_{n,n+1} \approx 0) \) at \( n \approx 36 \) that cannot be overcome with the implemented walker steps. Reducing \( \eta \) by increasing the axial confinement and/or reducing the angle between the \( k \)-vectors of the Raman beams, and thus \( \Delta k \), should allow to extend this protocol to \( N \leq 20 \) at comparable fidelities. Walker states with much higher \( N \) can be achieved by switching to a method different to that in [6, 14]. Pairs of resonant, intense and short \( \hat{R}(\pi,0) \)-pulses will displace the state of the ion independent of its position in phase space. This would make it possible to walk far outside the Lamb-Dicke regime with almost arbitrarily many steps, ultimately limited by the anharmonicity of the trapping potential and/or its depth, which is of the order of several electron volts. However, read out of position states is not straightforward for large \( N \). Incorporating the availability of deterministic entanglement of the electronic states of more than one ion [16, 25], more sophisticated quantum walks become possible, including the “meeting problem”, where two or several entangled particles perform the walk on the basics of the simulation code of Ref. [15] and Ignacio Cirac and Gerhard Rempe for their great intellectual and financial support.

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