QCD motivated three-body force and light nuclei

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Abstract

The need for the three-body forces (3BF) for reproduction of physical properties of light nuclei has been evident for some time. These 3BF are doing well for light nuclei, but there seems to be scope for other 3BF which may arise from other physical considerations not included so far. Here we discuss a QCD motivated new 3BF which should be included in studies of nuclei, in particular the light ones. We perform variational Monte Carlo calculations including this new 3BF and discuss improvements brought in thereby for light nuclei.

The study of light nuclei is of special significance, firstly as being composed of a few nucleons one hopes that their structure should be simple and secondly the understanding gained therefrom should be of basic importance for the study of heavier nuclei. It has therefore been somewhat of a surprise that these light nuclei themselves present several fundamental problems. It is however true that at present a rather good understanding has been achieved. However, there are still some open issues: (i) Is our variational wave function, having no explicit dependence on quantum chrodynamics (QCD) effects, complete?, (ii) Are ground-state energies of nuclei sensitive to QCD effects? And if so in what manner? The answer to these questions, shall be the focus of our attention here.

For since over a decade, it has been known that the best microscopic calculations using successful two-body forces (2BF) for light nuclei $A = 3, 4$ give underbinding and too large radii [1]. This hinted at the requirement of other forces like the three-body

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forces (3BF). Once included these do improve the fits to the binding energies and the radii of these nuclei.

However, one may ask as to all the 3BF included in calculations so far [1, 2] is it all that one needs? Quite clearly this is not correct. There may be other 3BF which have not been included so far. Here we shall discuss inclusion of a new 3BF which comes from QCD and quark model considerations [3, 4]. Based on basic considerations as to confinement in QCD, this new 3BF would be essential in the studies of nuclei and in particular the light ones.

We perform a variational Monte Carlo calculation [2, 5] where we take the Hamiltonian to be

\[ H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}. \]  

Here \( v_{ij} \) and \( V_{ijk} \) are NN and NNN potentials. For NN potential we use Argonne \( v_{18} \) (AV18) potential [6] which is written as a sum of 18 terms,

\[ v_{ij} = \sum_{p=1,18} v_p(r_{ij}) O_{ij}^p, \]

where the first 14 terms are charge independent,

\[ O_{ij}^{p=1,14} = \left[ 1, \sigma_i \cdot \sigma_j, S_{ij}, L \cdot S, L^2, L^2 \sigma_i \cdot \sigma_j, (L \cdot S)^2 \right] \otimes \left[ 1, \tau_i \cdot \tau_j \right] \]

and the rest,

\[ O_{ij}^{p=15,18} = \left[ 1, \sigma_i \cdot \sigma_j, S_{ij} \right] \otimes T_{ij}, (\tau_z i + \tau_z j), \]

are charge independence breaking terms.

For the NNN potential, we have

\[ V_{TNj}^{ijk} = V_{ij}^{2\pi} + V_{ijk}^{R}. \]

We take Urbana-IX potential model of the Urbana series potentials[7, 8, 9]. The first term is the two- pion exchange attractive component and the second is the short but still finite range phenomenological repulsive component. Here, we introduce a new term \( V_{ij}^{Q} \) (more about it a little later) as

\[ V_{ij} = V_{ij}^{TNj} + V_{ij}^{Q}. \]
which arises from QCD and Quark Model (QM) considerations [3, 4]. The variational wave function [2] for nuclei may be generalized to write the wave function after properly incorporating the \( V_{ijk} \) term,

\[
| \Psi_v \rangle = \left[ 1 + \sum_{i<j<k}^A (U_{ijk} + U_{ijk}^{TNI}) + \sum_{i<j}^A U_{ij}^{LS} \right] \left[ S \prod_{i<j}^A (1 + U_{ij}) \right] | \Psi_J \rangle. \tag{7}
\]

The \( | \Psi_J \rangle \) is the antisymmetric Jastrow wave function

\[
| \Psi_J \rangle = \left[ \prod_{i<j<k}^A f_{ij}^{Q} \right] \left[ \prod_{i<j<k}^A f_{ij}^{C} \right] \left[ A \prod_{i<j}^A f_{ij}^{C}(r_{ij}) \right] A | \phi_A(\text{JMT T}_3) \rangle. \tag{8}
\]

The functions \( f_c(r_{ij}) \) and \( u_p(r_{ij}) \) are the same as given in Ref.[5]. The \( f_{ij}^{C} \) is a three-nucleon correlation induced by \( v_{ij} \) which is discussed in Ref.[2] and in references therein. Its explicit mathematical expression is

\[
f_{ij}^{C} = 1 + q_{1}^{c}(r_{ij} \cdot r_{ik})(r_{ji} \cdot r_{jk})(r_{ki} \cdot r_{kj}) \exp(-q_{2}^{c}R_{ijk}). \tag{9}\]

Here \( R_{ijk} = r_{ij} + r_{jk} + r_{ki} \). An optimal variational wavefunction \( (\Psi_v) \) is used to calculate the ground-state energy,

\[
E_v = \frac{\langle \Psi_v | H | \Psi_v \rangle}{\langle \Psi_v | \Psi_v \rangle} \geq E_0, \tag{10}
\]

for a wide spectrum of nuclei, nuclear and neutron matter[10].

Keeping QCD considerations in mind, the \( V_{ijk}^Q \) (more about it below) gives an infinite repulsion if an accepted set of spatial configurations passes the following conditions operative simultaneously

\[
r_{ij} \leq \lambda_1, r_{jk} \leq \lambda_1, r_{ki} \leq \lambda_1 \tag{11}
\]

and

\[
R_{ijk} \leq \lambda_2. \tag{12}
\]

This imposes a corresponding correlation condition on the wave function that may be made effective by setting \( f_{ij}^{Q} \) to zero, thereby setting \( \Psi_J \) to zero in such case and hence the variational wave function \( \Psi_v \) as given in Eqs. [7,8]. The second condition Eq. [12] is imposed to get a reasonable triangular shape of triplets. Herein, the \( \lambda_1 \) and \( \lambda_2 \) are
treated as variational parameters in the wave function for each nucleus. The $V_{ijk}^Q$ is completely absorbed in the wave function. Its existence in Eq.[6], therefore, will be decided by the variational principle, which is a sacred law in all variational calculations.

Now we address as to how $V_{ijk}^Q$ arises. As we shall be discussing structure of nuclei in the ground state, we have to ensure that it is the ground state structure of nucleons itself that we take into account. Hence we take nucleon as consisting of three constituent quarks in the s-state. Note that though the rms radius of nucleon is taken as 0.8 fm, it is a very diffuse system with matter distribution given by $\rho(r)=\rho_0 e^{-\nu r}$.

As the u- and d- quarks are almost degenerate in masses, we take the flavour symmetry $SU(2)_F$ as a good symmetry, which should be relevant to the study of ground state properties of nucleus. Hence for our purpose here significant group would be $SU(12) \supset SU(4)_{SF} \otimes SU(3)_c$. Here $SU(3)_c$ is the QCD group and $SU(4)_{SF} \supset SU(2)_F \times SU(2)_S$, where S denotes spin. Upto 12 quarks can sit in the s-state for the group SU(12). Colour confinement is a property which should follow from the symmetry and dynamics of QCD. Therefore such multiquarks built up of 3-, 6-, 9- and 12- quarks should be colour singlet objects. The 3- q is our familiar baryon. What role do the 6-, 9- and 12- quarks play in nuclei? In as much as we can successfully study nuclear physics with nucleonic degrees of freedom the higher multi-quark configuration should play no role. But it has been found that in physical situations where to understand the reality, if the relative distances between nucleons have to be less than 1 fm, then one can not escape considering 6- q, 9- q and 12- q configurations [3, 4, 11].

For multiquark system (with quark number greater than 3) the concept of hidden colour plays a basic role. Hidden colour represents that part of the multiquark wavefunction which as per the confinement idea of QCD can not be separated out in terms of physical hadrons and so manifests itself only inside multiquark systems. If relevant to nuclear physics, this could represent unique QCD based quark aspects of nuclear physics. There have been some claims that hidden colour may not be a useful or unique concept for nuclear physics as these may be rearranged in terms of asymptotic colour singlet states [12]. But as discussed in Ref. [12] the hidden colour concept is not unique only when the two clusters do not overlap strongly and can be separated out asymptotically. However it has been demonstrated convincingly that when the clusters of 3-q overlap strongly so that the relative distance between them goes to zero, the hidden colour concept becomes relevant and unique [3, 4, 12]. It is only in these situations
Table 1: Variational parameters $\lambda_1$ and $\lambda_2$ in units of fm.

|       | $^4\text{He}(0^+)$ | $^3\text{He}(1^+_\frac{1}{2})$ | $^3\text{H}(1^+_\frac{1}{2})$ |
|-------|---------------------|-------------------------------|-------------------------------|
| $\lambda_1$ | 0.55  | 0.63  | 0.62  |
| $\lambda_2$ | 1.2   | 1.5   | 1.5   |

that we shall make use of the concept of hidden colour in our calculation. Ref.[12] also discusses how antisymmetrization and short range repulsion are related as per the hidden colour idea at short range. What is important [[12]] is that how near $r = 0$ range hidden colour representation is unique.

Group theoretically hidden colour [13] component of 6- q system was found to be 80%. Next it was found that the 9- q and 12- q components of hidden colour are predominant i.e. 97.6% and 99.8%, respectively [3, 4]. The $A = 3, 4$ nuclei $^3\text{H}$, $^3\text{He}$ and $^4\text{He}$ have experimental point-proton radii of 1.60 fm, 1.77 fm and 1.47 fm respectively. These radii are obtained by subtracting a proton mean-square radius of 0.743 fm$^2$ and $N/Z$ times a neutron mean square radius of -0.116 fm$^2$ from the squares of the measured charge radii. Given the fact that each nucleon is itself a rather diffuse object, quite clearly in a size $\leq 1$ fm at the centre of these nuclei, the three and four nucleons would overlap strongly. There would be an effective repulsion at the center keeping the three and four nucleons away from the centre, as the corresponding 9- and 12- q have predominantly hidden colour components. The nucleonic system resists going into this configuration. This conceptual framework has been found to be useful in making further predictions for normal and neutron rich nuclei [11].

Let us here concentrate upon the 3BF part. Quite clearly as discussed above, arguments based on QCD and quark model, indicate that there should be a unique short range repulsive force in $A = 3, 4$ nuclei. This is the new $V_{ij}^Q$ that we have proposed above. The $V_{ij}^Q$ is clearly three body analogue of the short range hard core repulsion in the 2BF [3, 4, 11]. Just as for 2BF one has hard core of infinite repulsion and size $\sim 0.5$ fm so here too we implement the $V_{ij}^Q$ as infinitely repulsive hard core. This hard core is implemented in terms of two parameters $\lambda_1$ and $\lambda_2$ (as above) in a variational Monte Carlo calculations to fit to the physical quantities of relevance for $A = 3, 4$ nuclei $^3\text{H}$ and $^3\text{He}$ and $^4\text{He}$. 
In Table 1, we present best values of variational parameters $\lambda_1$ and $\lambda_2$ tuned to obtain ground-state energies for the nuclei under consideration. For the mirror nuclei: $^3\text{He}$ and $^3\text{H}$ whose rms radii are very close to each other, $\lambda_1$ is found to be 0.63 fm and 0.62 fm respectively, and $\lambda_2$ is found to be about 1.5 fm. For the compact nucleus, $^4\text{He}$, the values of $\lambda_1$ and $\lambda_2$ are smaller which are found to be 0.55 fm and 1.2 fm. They are down almost in the same proportion. Thus, $\lambda_1$ and $\lambda_2$ seem to be correlated with the compactness of the nucleus. Our calculations reproduce the experimental rms radii which are shown in Table 2. In Table 3, the variational energy breakdown for kinetic energy ($T$), two-body potential energy ($v_{ij}$), total two-body energy ($T + v_{ij}$), three-body potential energy ($V_{ijk}$) and total energy ($E = T + v_{ij} + V_{ijk}$) with $V_{ijk}^Q$ along with experimental ground state energy are presented. We also report the same without $V_{ijk}^Q$ in Table 4. We note an improvement in the upperbound ground-state energy of the nuclei considered herein, by about -0.04 MeV, -0.03 MeV and -0.01 MeV in case of $^4\text{He}$, $^3\text{He}$ and $^3\text{H}$, which may be small but can not be ignored in fine calculations while obtaining nuclear spectra through nuclear forces\[14\]. Though the Urbana-IX 3BF is already having a strong phenomenological short range repulsion, the idea of infinite repulsion still improves variational results. One may also implement this idea at the

### Table 2: rms point proton and neutron radii in units of fm.

| $^A_Z(J^\pi)$ | $p$    | $n$    | $p(Expt.)$ |
|---------------|--------|--------|------------|
| $^4\text{He}(0^+)$ | 1.466  | 1.466  | 1.47       |
| $^3\text{He}(\frac{1}{2}^+)$ | 1.766  | 1.592  | 1.77       |
| $^3\text{H}(\frac{1}{2}^+)$ | 1.613  | 1.713  | 1.60       |

### Table 3: Energy breakdown for $^4\text{He}$, $^3\text{He}$ and $^3\text{H}$ with $V_{ijk}^Q$. All quantities are in units of MeV.

| $^A_Z(J^\pi)$ | $T$         | $v_{ij}$   | $T + v_{ij}$ | $V_{ijk}^{TN}$ | $E$         | $E(Expt.)$ |
|---------------|-------------|------------|--------------|----------------|-------------|------------|
| $^4\text{He}(0^+)$ | 107.44(20)  | -129.93(19) | -22.48(2)    | -5.28(2)       | -27.77(1)   | -28.30     |
| $^3\text{He}(\frac{1}{2}^+)$ | 48.95(16)   | -55.45(16)  | -6.50(1)     | -1.04(1)       | -7.54(1)    | -7.72      |
| $^3\text{H}(\frac{1}{2}^+)$ | 49.39(23)   | -56.69(22)  | -7.30(2)     | -1.02(2)       | -8.32(1)    | -8.48      |
level of two-body force (2BF) as represented by AV18 in this calculation. Obviously, the effect of the infinite repulsion will be more evident with increasing number of nucleons coming together, Therefore, 3BF is a better candidate where QCD and QM considerations should be implemented first. We aim to implement the same at the level of 2BF later. The individual energy pieces $T$, $v_{ij}$ and $V_{ijk}$ are affected significantly due to the presence of $V_{ijk}^{Q}$. We infer that QCD effects are manifested through $V_{ijk}^{Q}$ in all the light nuclei considered herein. We recommend that this force be included in all energy calculations for both finite and infinite nuclear systems, specially for dense and compact systems.

It may be noted that the parameter $(\lambda_1)$ above is directly related to confinement size for a 9-quark system. One knows that so far it has not been possible to obtain the confinement size of proton/neutron from any fundamental considerations of QCD, hence it would be hopeless to expect to do the same for $\lambda_1$. Also as of now since there exists no definitive experimental candidates for the tri-baryonic systems, hence it would not be possible to lay hands on the $(\lambda_1)$ parameter from any phenomenological quark model perspective. Hence, our method above of treating it as a variational fitting parameter.

In Fig. ??, we plot density profiles for $^{4}$He, $^{3}$He and $^{3}$H in three different panels. The filled circles and open circles (which appeared to be strongly overlapping) represent density profiles with and without $V_{ijk}^{Q}$, respectively. The cross points represent the calculations after setting $f_{ijk}^{c}=1$ and $U_{ijk}^{R}=0$, thereby deleting the central correlation induced by $v_{ij}$ and short range-repulsive correlation from the variational wave function. The depression in densities at small $r$ is mainly due to $f_{ijk}^{c}$ and $U_{ijk}^{R}$ which is a correlation due to $V_{ijk}^{R}$ in the wave function.
In summary, we have found that the QCD motivated new 3BF $V_{ijk}^{Q}$ proposed herein is essential for the variational wave function and for the variational ground-state energies. Also in $^4\text{He}$ as per our model there should be an additional short range repulsive four-body force which we wish to include in future. Results here have been obtained within the framework of successful variational Monte Carlo calculations.

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