Generation of high quality electron beams from a quasi-phase-stable cascaded laser wakefield accelerator with density-tailored plasma segments

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Abstract
By controlling electron injection into the second period of the laser-driven wakefield in a downward density ramp, a high-quality low-energy electron beam can be accelerated in a short segment of high-density plasma. After a second downward density ramp followed by a low-density plasma plateau, the pre-accelerated electron beam can be seeded into the first period of the laser-driven wakefield for cascaded acceleration at an optimized phase. A monoenergetic electron beam with peak energy of ∼1.2 GeV can be generated from plasma with a length of 12 mm and density of $9 \times 10^{17} \text{ cm}^{-3}$, driven by a laser pulse with peak power of 77 TW. By modifying the acceleration stage comprising several density-ascending plasma segments, the peak energy of the quasi-monoenergetic electron beam can be efficiently increased by about 50% via a quasi-phase-stable multiple-cascade acceleration scheme.

1. Introduction
Laser plasma acceleration has attracted great interest since Tajima and Dawson proposed the idea of the laser wakefield accelerator (LWFA) in 1979 [1]. Major breakthroughs and significant progress have been made over the past decade thanks to the rapid development of high-power laser technology [2–7]. A laser-driven plasma wakefield capable of sustaining strong electric fields on the order of 100 GV m$^{-1}$ can allow the development of electron accelerators having a scale notably smaller than that of radiofrequency accelerators. Electron beams with energies up to multi-GeV levels and energy spreads of a few percent have been achieved by several groups [8–10], and some theoretical works have been done for 100 GeV scale LWFA design [11]. Employing these electron beams, a number of potential applications, such as table-top x- and γ-ray radiation sources using Thomson/Compton scattering [12, 13] and x-ray free-electron lasers [14–17], will be developed. However, it may not be an efficient way for single-stage accelerators to generate controllable low-energy-spread and high-energy electron beams beyond 1 GeV. This is because electron injection and acceleration, having different requirements for wakefield and plasma parameters, cannot be controlled independently. The LWFA’s cascading injection and acceleration with two-segment plasma provides a promising route to the generation of high-quality GeV e-beams [18].

In the so-called ‘blow out’ or ‘bubble’ regime [19–22], electrons are self-injected into bubbles by the nonlinear evolution of an intense femtosecond pulse in plasma. Quasi-monoenergetic electron beams produced in this scheme have already reached 4.2 GeV with 6% root-mean-square (rms) energy spread [9]. Several methods, such as ionization-induced injection [23–29], pulse colliding injection [30–32], and density transition injection [33–41], have been proposed to control the electron injection process so as to optimize the quality of the produced electron beam and improve the stability of experiments. In the density transition injection scheme, which was initially proposed by Bulanov [41] for plasma wakefield accelerators, the plasma density exhibits
controlled energy spread. Unlike the self-injection scheme acceleration separately in a cascaded LWFA. At the densities and two downward density ramps are designed for controlling electron injection, seeding and further density ramp is operated to control the seeding of the accelerated electron beam into the could be avoided by optimizing the parameters of the laser pulse and plasma density. The second downward period of the laser-driven wake field of 2 mm, using laser power of 50 TW.

multi-GeV electron acceleration straightforwardly. higher energy via a quasi-phase-stable multiple-cascade acceleration scheme. This scheme can be scaled up to segments of plasmas, the peak energy of the monoenergetic electron beam can be efﬁciently boosted to much higher energy via a quasi-monoenergetic electron beams beyond 0.5 GeV over an acceleration distance of 2 mm, using laser power of 50 TW.

In the present paper, we propose a new scheme by controlling the electron injection process in the second period of the laser-driven wakefield during a downward density ramp. Three segments of plasmas with different densities and two downward density ramps are designed for controlling electron injection, seeding and further acceleration separately in a cascaded LWFA. At the first downward density ramp, the injection process is confined to a limited time scale of the order of a few femtoseconds to obtain a low-energy electron beam with a small energy spread. Unlike the self-injection scheme [18], the injection process in this scheme may be controlled ﬂexibly by adjusting the position, length, and slope of the downward density ramp. Moreover, the effect of the slippage electrons, which are from the tail of the ﬁrst wakeﬁeld period, on the injected electrons could be avoided by optimizing the parameters of the laser pulse and plasma density. The second downward density ramp is operated to control the seeding of the accelerated electron beam into the ﬁrst wake period of the third segment of plasma. By modifying the third segment of plasma as several step-by-step density-ascending segments of plasmas, the peak energy of the monoenergetic electron beam can be efﬁciently boosted to much higher energy via a quasi-phase-stable multiple-cascade acceleration scheme. This scheme can be scaled up to multi-GeV electron acceleration straightforwardly.

2. Scheme of a cascaded LWFA

2.1. Electron injection in the second wakeﬁeld period

Figure 1 sketches the scheme of a cascaded LWFA. Three segments of plasmas with different densities and two downward density ramps are designed to control electron injection, seeding and further acceleration separately in the cascaded LWFA. The ﬁrst segment with a higher density has a downward density ramp over a length of $\Delta l_1$, where the density gradient injection into the second wakeﬁeld period is controlled by adjusting the position, length and slope of the ﬁrst downward density ramp. The second segment with a density has a downward density ramp over a length of $\Delta l_2$, where the injected electrons are seeded into the ﬁrst wakeﬁeld period in the third segment for acceleration.

As mentioned by Fubiani et al [38], the injection of electrons in a downward density ramp is attributed to two effects: (1) a decrease in plasma density along the laser propagation enlarges the wavelength of the wakeﬁeld so as to change the phase position of trapped electrons relative to the wakeﬁeld, and (2) a decrease in plasma density reduces the phase velocity of the plasma wave and also affects the group velocity of the laser pulse. These two effects depend nonlinearly on normalized laser amplitude $a_0$ and its evolution.

To distinguish the difference in injection conditions between the ﬁrst and second wake periods, a sharp density down-ramp with density changing from $n_1$ to $n_3$ over a length of $l$ is considered. The transition length $l$ is so short that the laser parameters can be treated as constants. The change in phase of trapped electrons after density transition is then $\Delta \psi = \psi_2 - \psi_1 = \psi_1[(n_3/n_1)^{3/2} - 1]$, where $\psi_1$ and $\psi_2$ are the phases of the electrons.
before and after transition. For instance, the phase variation is $\Delta \psi_1 = 2\pi [1 - (n_2/n_1)^{1/2}]$ for electrons at the base of the first wake period and $\Delta \psi_2 = 2\Delta \psi_1$ for electrons at the base of the second wake period. The electrons at the base of the second period are thus easier to trap at the same downward density transition because a lower forward momentum threshold for trapping is required in the case of these electrons, which are shifted to a further forward position with respect to the wakefield.

The phase velocity $v_p$ of the plasma wave in the one-dimensional model can be expressed as

$$v_p = c\left(1 + \frac{x}{k_p \frac{dp}{dz}}\right)^{-1},$$

(approximately under the assumption $v_g = c$, where $v_g$ is the group velocity of the driving laser pulse, $c$ is the velocity of light in a vacuum, $\xi$ is the relative distance to the laser pulse ($\xi < 0$ behind the pulse) and $k_p$ is the wavenumber of the plasma wave. Since $\frac{dp}{dz} < 0$ for a downward density ramp, the phase velocity of the plasma wave behind the driving pulse decreases as the wave propagates at the density transition ramp. The phase velocity of the base of the first wakefield period can be written as $v_{p1} = c/(1 + d\lambda_p/\xi)$ for $\xi_1 = -\lambda_p$. Similarly, the phase velocity of the base of the second period is $v_{p2} = c/(1 + 2d\lambda_p/\xi)$, and therefore, $v_{p1} < v_{p2}$ since $d\lambda_p/\xi < 0$ for a downward density ramp. Additionally, the condition of electron trapping in the single-particle dynamics model has been deduced as [44]

$$\gamma_{\text{min}} = \gamma_p\left(1 + \gamma_p\Delta \phi\right) - \gamma_p\beta_p\left[\left(1 + \gamma_p\Delta \phi\right)^2 - 1\right]^{1/2},$$

where $\beta_p = v_p/c$, $\gamma_p = \left(1 - \beta_p^2\right)^{-1/2}$ is the Lorentz factor of the plasma wave, and $\Delta \phi = \phi_{\text{max}} - \phi_{\text{min}}$ for the normalized electrostatic potential $\phi = e\Phi/m_ec^2$, which is assumed to oscillate in the range $\phi_{\text{min}} < \phi < \phi_{\text{max}}$ periodically within a plasma wavelength. In the limit of $\gamma_p\Delta \phi \gg 1$, the minimum energy necessary for trapping electrons is approximately given by $\gamma_{\text{min}} \approx \beta_p/(2\Delta \phi) + \Delta \phi/(1 + \beta_p)$. Thus, the lower phase velocity of the second wakefield period results in a lower electron injection threshold as compared with the first wakefield period.

Considering the reasons mentioned above, more electron injections occur in the second wakefield period than the first wakefield period, similar result was demonstrated in the paper by Brantov et al [45]. This property may reduce the normalized laser amplitude $a_0$ of the driving pulse needed for realizing proper electron injection in plasma. Moreover, the injected electron beam parameters, such as the beam charge and energy spread, are tunable because $\Delta \psi_1$ and $v_p$ for the electron injection process can be manipulated by adjusting the position, length, and density variation of the first downward ramp. Additionally, the longitudinal injection process [46], which is supposed for generating lower-emittance electron beams as compared with a transverse injection process, possibly occurs in the second wake period rather than the first. This is attributed to the fact that the second period is filled with slippage electrons from the tail of the first period, and some of these electrons might be trapped when their phases in the wakefield shift through a density downward ramp. For the first period, however, the background electrons are mostly blown out by the intense driving pulse, and the phase variations of the remaining electrons are not large enough for trapping. Thus, the longitudinal injection process would not occur in the first wake period.

2.2. Dephasing and phase-stable effect

Before seeding the injected beam directly into the acceleration stage of a cascaded LWFA [25], the injected beam can be accelerated in the second wake period until dephasing in the second-segment plasma. Thus, the energy of the injected beam will be increased over the dephasing length [47], given by

$$L_{\text{de}} \approx \frac{2}{3} \frac{\omega_{p2}^2}{\omega_{b}} R_2,$$

where $\omega_b$ is the laser frequency, $\omega_{p2}$ the plasma frequency of the second-segment plasma, and $R_2$ the corresponding bubble radius. Moreover, the evolution of the laser intensity affects the bubble length and the acceleration process. If the focused radius of the driving pulse well matches the bubble radius, there is a relationship $R_b \approx 2\sqrt{a}/k_p$, and the bubble length $L_b$ can then be calculated as $L_b \approx 2R_b \approx 4\sqrt{a}/k_p$ [47], where $a$ is the normalized vector potential, which is a variant owing to the focusing and defocusing of the laser pulse, and $k_p$ is a constant for a uniform plasma. Since the phase velocity $v_p$ of the bubble base would increase as $L_b$ decreases, the velocity variation of the first bubble caused by the pulse intensity evolution can be expressed as $-dL_b/dt \approx -2/(k_p\sqrt{a}) \times da/dt$. As the phase velocity variation of the base of the second bubble is doubled, we can get an explicit expression for the phase velocity of the base of the second bubble as

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\[ v_{ib} \approx \frac{2dL_b}{dt} + v_o \approx \frac{4}{k_L \sqrt{a}} \frac{da}{dt} + v_o, \]

where \( v_p \approx c \left[ 1 - 3\omega_p^2/(2\omega_0^2) \right] \) is the phase velocity of the bubble [47] without accounting for the effect of the pulse intensity evolution. For efficient acceleration and high energy gain, the phase-locking [48] or phase-stable effect can be obtained when \( v_{ib} = c \) is met as \( a_0 \) decreases. Hence, the first downward density ramp is placed near the position where \( a \) starts to decrease.

The electron beam will be seeded into the first wake period in the third-segment plasma at the second downward density ramp for further acceleration before dephasing in the second wake period. The dephasing length for this acceleration process is

\[ L_{d2} \approx \frac{2}{3} \frac{\omega_p^2}{\omega_{p3}^2} R_3, \]

where \( \omega_{p3} \) is the plasma frequency of the third-segment plasma and \( R_3 \) is the corresponding bubble radius. Therefore, the total dephasing length in the cascaded LWFA is

\[ L_d = L_{d1} + L_{d2} \approx \frac{2}{3} \left( \frac{R_2}{\omega_{p2}^2} + \frac{R_3}{\omega_{p3}^2} \right), \]

which becomes longer than that in a single-stage LWFA.

3. Particle-in-cell simulations

To demonstrate the scenarios of electron injection, seeding and acceleration in the cascaded LWFA, two-dimensional (2D) relativistic particle-in-cell (PIC) simulations were carried out. A linearly polarized laser pulse with a wavelength of \( \lambda_L = 0.8 \) \( \mu \)m enters the left boundary of the simulation box. The initial normalized amplitude of the laser pulse was \( a_0 = 3 \). The laser pulse profile was chosen to be Gaussian transversely with a spot radius of 16 \( \mu \)m at 1/e² of the peak intensity and sine-squared longitudinally with a full width at half maximum (FWHM) duration of 40 fs. The simulation window had dimensions of 102 \( \times \) 102 \( \mu \)m² and moved at the speed of light. The size of the simulation grid was \( k_{0x} = 0.18 \) in the laser propagation direction \( x \) and \( k_{0y} = 0.37 \) in the transverse direction. Four particles per cell were used and the ions were assumed to be stationary.

Simulations were performed by setting different parameters for the first downward density ramp, e.g. the plasma density in the first-segment plasma is set as \( n_{n1} = 5.0 \times 10^{18} \text{ cm}^{-3} \) and the first downward ramp has a density variation \( \Delta n_1 = -0.5 \times 10^{18} \text{ cm}^{-3} \) with an adjustable transition length from \( \Delta l_1 = 4.8 \) \( \mu \)m to \( \Delta l_1 = 12.0 \) \( \mu \)m. As shown in figures 2(a) and (b), electrons are injected into the second wake period under both conditions of \( \Delta l_1 = 4.8 \) and 12.0 \( \mu \)m. However, electrons can also be injected into the first wake period for \( \Delta l_1 = 4.8 \) \( \mu \)m but not for \( \Delta l_1 = 12.0 \) \( \mu \)m. This results from suppression of electron injection into the first wake period due to the lower density gradient for the longer transition length. Two electron beams are injected separately in space into the second wake period after the first down-ramp, as shown in figures 2(a) and (c). Beam 1, which has a smaller transverse size than beam 2, comes from longitudinal injection as we explained in section 2. As \( \Delta l_1 \) increases to 12.0 \( \mu \)m as shown in figures 2(b) and (d), beam 1 is suppressed. Thus, the injected electron beams in the second-stage period are controllable by adjusting the transition length of the first downward density ramp.

Additionally, the injected beam charge is tunable by adjusting transition length \( \Delta l_1 \) and density variation \( \Delta n_1 \). As shown in figure 3, the injected beam charge decreases as \( \Delta l_1 \) increases for the cases \( \Delta n_1 = -0.3 \times 10^{18} \text{ cm}^{-3} \). Similar results have been reported by several groups [15, 37] for density transition injection in the first wake period. We explain this as two electron beams transversely and longitudinally injected into the second wakefield period in cases with shorter transition lengths, such as \( \Delta l_1 = 4.8 \) and 8.0 \( \mu \)m. Thus, the charge in these cases would be larger than that in the case of \( \Delta l_1 = 12.0 \mu m \), where only one transversely injected electron beam is obtained. The charge in the case of \( \Delta l_1 = 4.8 \) \( \mu \)m is larger than that in the case of \( \Delta l_1 = 8.0 \) \( \mu \)m, as shown in figure 3. We explain this as the charge of the longitudinally injected beam decreasing as \( \Delta l_1 \) increases since fewer electrons are trapped for a longer \( \Delta l_1 \) owing to a higher injection threshold. Additionally, the charge of the transversely injected beam is nearly invariable because the electrons shifted from the base of the bubble were mostly trapped in these cases. On the other hand, the beam charge can increase from several picocoulombs to tens of picocoulombs as the density variation \( \Delta n_1 \) changes from \( -0.3 \times 10^{18} \text{ cm}^{-3} \) to \( -0.5 \times 10^{18} \text{ cm}^{-3} \).

The following acceleration process that the injected beams undergo in the second-segment plasma is shown in figure 4(a). In the case of \( \Delta n_1 = -0.3 \times 10^{18} \text{ cm}^{-3} \) and \( \Delta l_1 = 4.8 \mu m \), two electron beams are injected into
the second bubble after the driving pulse propagates about 1.2 mm. The transversely injected beam behind the longitudinally injected beam then interacts with the electron sheath at the base of the second wake period, as a result of the contraction of the bubble length due to the decrease in $a_0$. By contrast, the longitudinally injected electron beam undergoes phase-stable acceleration from $z = 1.1$ to $1.7$ mm.

Also shown in figure 4 are the evolutions of the peak energy (b), energy spread (c) and normalized transverse emittance (d) of electron beams injected into the second-segment plasma as functions of $z$ for different downward-ramp parameters. In the three cases of $\Delta n_1 = -0.3 \times 10^{18}$ cm$^{-3}$ with $\Delta l_1 = 4.8$ $\mu$m, $\Delta n_1 = -0.3 \times 10^{18}$ cm$^{-3}$ with $\Delta l_1 = 8.0$ $\mu$m, and $\Delta n_1 = -0.5 \times 10^{18}$ cm$^{-3}$ with $\Delta l_1 = 4.8$ $\mu$m, there are two electron beams injected into the second wake period and only the longitudinally injected beam is taken into account because the electrons in the transversely injected beam spread out when interacting with the electron sheath at the base of the second wake period. By controlling the injection process in a limited time...
(Δl = 4.8 μm) and reducing the density variation (Δn₀ = −0.3 × 10¹⁸ cm⁻³), a high-quality electron beam with relative rms energy spread below 1%, normalized transverse emittance less than 0.36 mm·mrad, and peak energy up to 300 MeV is generated after pre-acceleration in the second-segment plasma. The simulation results indicate that a high-quality electron beam can be injected into the second bubble for acceleration if the transition length is short enough (i.e. from 4.8 to 8.0 μm), as shown in figures 3 and 4. Additionally, the electrons would be injected longitudinally for a wide range of density variations Δn₁ because Δn₁ mainly affects the transversely injected electrons and not the longitudinally injected electrons. In the experiment, such a sharp density transition can be produced by inserting a razor blade into a supersonic gas flow from a de Laval nozzle [42], or by employing a second laser to induce a transit density ramp via plasma expansion [33, 39, 49–52].

Before dephasing, the pre-accelerated electron beam in the second wakefield period will be seeded into the first period of the third-segment low-density plasma for further acceleration owing to the increase in the bubble size there. A preformed plasma channel was designed for the third-segment plasma to ensure the guiding of the driving pulse and thus an efficient acceleration can be achieved. The preformed density distribution in the y-direction of the third-segment plasma with the parabolic profile \( n_{y1}(10^{18} \text{ cm}^{-3}) = 1.9(y/\text{μm})^2 - 75 \)/ 5625 + 0.9 was chosen in our simulations, as shown in figure 5(a). This transverse parabolic plasma channel can be produced by employing an ablative or gas-filled discharge capillary [53]. As a result, the normalized amplitude \( a₀ \) remains in the range from 3 to 4, as shown in figure 5(b). Figure 5(c) shows the evolution of the transverse beam profile along the laser propagation direction z.

Figure 5(d) shows the evolutions of the peak energy and relative rms energy spread of the accelerated electron beam as functions of z in the range of 0–12 mm for the first downward density ramp of Δl₁ = 4.8 μm and \( Δn₁ = −0.3 \times 10¹⁸ \text{ cm}^{-³} \). The whole dephasing length can be calculated from equation (6) to be about 20 mm, which is longer than the pump depletion length of 12 mm in the simulation. The peak energy of the electron beam can be boosted to 1147 MeV at a final position of 12 mm after being seeded into the third-segment plasma. The rate of increase in energy of the electron beam slows gradually as a result of the weakening of the acceleration electric field due to the phase shift and pump depletion of the laser pulse. The relative rms energy spread of the accelerated beam remains at 0.71% to 0.83% before the peak energy increases to 612 MeV at 4.4 mm. The relative energy spread then begins to increase because the longitudinal energy distribution of the accelerated electron beam evolves to be a positive chirped profile, when the electrons in the front of the accelerated beam...
have lower energies than those at the back where the acceleration field is stronger, resulting in the greater energy spread.

The normalized transverse emittance, expressed as $\epsilon_{y} = \frac{\langle y^2 \rangle \langle p_y^2 \rangle}{\langle y^2 \rangle} / (m_0 c)$ ($m_0$ is the rest mass of an electron), is estimated particle by particle statistically. The space charge force is large enough to affect the evolution of emittance because the generated electron beam has a current of several kilo-ampere. The normalized transverse emittance of the accelerated beam in plasma evolves linearly from 0.06 to 3.6 mm·mrad over a range of 1.4–9.2 mm and changes little after 9.6 mm, as shown in figure 5(e). In contrast, the normalized transverse emittance increases rapidly if the beam propagates after 2 mm in a vacuum. Therefore, the transverse focusing force in the wakefield effectively neutralizes the space charge force of the beam and suppresses transverse expansion.

Figures 5(f)–(h) show the energy–space distribution of the accelerated electron beam when the driving laser pulse propagates at $z = 4$ mm in the third-segment plasma. The peak energy is 559 MeV with an rms energy spread of 0.71%, as shown in figure 5(f). Figures 5(g) and (h) show the 2D and on-axis ($y = 0$) distributions of electron density, respectively. The normalized transverse emittance is evaluated to be $1.2 \text{ mm·mrad}$, the beam charge is estimated as $4.3 \text{ pC}$, and the FWHM bunch duration is about 2 fs according to the beam longitudinal density distribution as shown in the inset of figure 5(h).
4. Quasi-phase-stable multiple-cascade acceleration

In the acceleration stage, the electron beam moves faster than the laser pulse in plasma. The electron beam will gradually move toward the center of the wakefield, where the longitudinal acceleration electric field is weaker, and experience the dephasing process shortly thereafter. Therefore, the strong electric field at the base of the wakefield is not fully utilized in a typical LWFA with uniform plasma density owing to the dephasing process. Two methods have been proposed for keeping the accelerated electron beam locked at a phase for efficient acceleration. One employs a tailored plasma density profile [36], while the other relies on the decay of the driving pulse intensity [48] to ensure the reduction in the bubble size so that the electron beam can be kept near the base of the bubble throughout the acceleration stage. However, given that both these methods rely sensitively on the nonlinear evolution of the laser pulse, it may not always be easy to control them. Here, we modify the third segment of plasma as several step-by-step density-ascending segments, as shown in figure 6(a). Three-step segments of plasmas form a quasi-phase-stable multiple-cascade acceleration scheme. The length and plasma density of each segment should be designed so as to ensure that the accelerated electron beam in the former segment can be seeded into the latter segment at a proper phase before dephasing.

In the simulation, the three-step accelerator stage is designed such that the first-step segment is from $z = 2$ to $4.72$ mm; the second-step segment is from $z = 4.72$ to $7.2$ mm, and the third-step segment is from $z = 4.72$ to $9.4$ mm. The preformed parabolic profiles of plasma density in the last two segments are chosen as $n_{p2}(10^{18} \text{ cm}^{-3}) = 2.0(y/\mu m) - 75y^2/5625 + 1.5$ and $n_{p3}(10^{18} \text{ cm}^{-3}) = 2.0(y/\mu m) - 75y^2/5625 + 2.5$. To guide the laser pulse, figures 6(b)–(e) show the 2D electron density distribution at different propagation distances $z$ of the driving pulse. The base of the first wake period moves forward with respect to the simulation box when entering the second-step segment, as shown in figures 6(b) and (c). The electron beam is then seeded into the wakefield in the second-step segment and accelerated under a stronger acceleration field in this segment, resulting in a higher energy gain. Additionally, the emittance of the beam is further lowered by the stronger focusing field, which is capable of confining the electron beam into a smaller transverse space. Before dephasing in the second-step segment, the electron beam will be seeded into the third-step segment for acceleration up to $z = 9.4$ mm, where the laser pulse is eventually pump-depleted. The pump-depletion length is shorter than that in the case of uniform density because of the higher average plasma density.

As shown in figure 7, the peak energy evolution has an approximately linear profile, in contrast to that shown in figure 5(d). This shows that quasi-phase-stable acceleration can be maintained by employing the step-by-step density-ascending plasma segments for the acceleration stage. The peak energy of the electron beam is boosted to

![Figure 6](https://example.com/figure6.png)

**Figure 6.** Three-step segments of plasmas are used to realize the quasi-phase-stable multiple-cascade acceleration scheme (a). The first-step segment is from $z = 2$ to $4.72$ mm, the second-step segment is from $z = 4.72$ to $7.2$ mm and the third-step segment follows from $z = 7.2$ mm. Parabolic profiles of plasma density of the second-step segment and the third-step segment in the $y$-direction are chosen to prevent the driving pulse from defocusing. The 2D electron density distributions at pulse positions of $4.72$ mm (b), $4.96$ mm (c), $7.20$ mm (d), and $7.28$ mm (e) are displayed.
1.8 GeV over a shorter acceleration distance of 9.4 mm as compared with the 1.15 GeV over a distance of 12 mm in the case of uniform density. Moreover, for the same peak energy, the normalized transverse emittance is lower than that in the uniform density case. It is inferred that this quasi-phase-stable multiple-cascade acceleration scheme can be employed and scaled up to generate multi-GeV electron beams in a more efficient way than employing the single-staged LWFA if a driving laser pulse with higher power is used.

5. Conclusion

We have proposed a new scheme for controlling the electron injection process in the second period of the laser-driven wakefield via a downward density ramp. Three segments of plasmas with different densities and two downward density ramps were designed to control electron injection, seeding and further acceleration separately in a cascaded LWFA. The injection and acceleration processes were analyzed. Two-dimensional numerical simulations showed that the injection threshold for the second wake period is lower than that for the first period, and longitudinally injected beams were obtained in the second wake period. Moreover, the injected beam charge is tunable by adjusting the parameters of the first downward density ramp. A long dephasing length was achieved when the beam was transferred to the first wake period at the second downward density ramp. A high-quality quasi-monoenergetic electron beam with peak energy of 559 MeV, relative rms energy spread of 0.71%, normalized transverse emittance of 1.2 mm mrad, charge of 4.3 pC, and FWHM duration of 2 fs was obtained at a propagation distance of 4 mm in plasma. These electron beam properties may be suitable for driving short-wavelength free-electron lasers [14].

Furthermore, by modifying the last accelerator stage to several step-by-step density-ascending plasma segments, a quasi-phase-stable multiple-cascade acceleration scheme was demonstrated, boosting the peak energy of the electron beam to 1.8 GeV over a short acceleration distance of 9.4 mm driven by a 77 TW laser pulse. This scheme may be scaled up to high-quality multi-GeV electron acceleration straightforwardly.

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