Condition identification of bolted connections using a virtual viscous damper

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Abstract
Vibration-based condition identification of bolted connections can benefit the effective maintenance and operation of steel structures. Existing studies show that modal parameters are not sensitive to such damage as loss of preload. In contrast, structural responses in the time domain contain all the information regarding a structural system. Therefore, this study aims to exploit time-domain data directly for condition identification of bolted connection. Finite element model updating is carried out based on the vibration test data of a steel frame, with various combinations of bolts with loss of preload, representing different damage scenarios. It is shown that the match between the numerically simulated and measured acceleration responses of the steel frame cannot be achieved. The reason is that time-dependent nonlinearity is generated in bolted connections during dynamic excitation of the steel frame. To capture the nonlinearity, a virtual viscous damper is proposed. By using the proposed damper alongside the updated system matrices of the finite element model, the time-domain acceleration responses are estimated with great consistency with the measured responses. The results demonstrate that the proposed virtual damper is not only effective in estimating the time-domain acceleration responses in each damage case, but also has the potential for condition identification of bolted connections with such small damage as just one bolt with loss of preload. It can also be applied to other challenging scenarios of condition identification, where modal parameters are not sensitive to the damage.

Keywords
Time-domain condition identification, finite element model updating, nonlinear damping, bolt pretension loss, nonlinear constrained optimisation

Introduction
Bolted steel connections are commonly used in steel construction. The bolts are usually pretensioned to increase the structural capacity of bolted connection. Loss of pretension force in the bolts occurs over time due to viscoelastic creep, environmental effects and possible deterioration. Loss of pretension also occurs due to redistribution of stresses in the bolts due to weathering of materials in the surrounding steel members. It thus necessitates the use of condition monitoring of structures. By assessing the current condition, the current capacity of the structure can be evaluated. If the estimated capacity of the structure is found to be less than a threshold capacity, an indicator can be set for sanctioning repair or renovation of the structure.

Non-destructive testing tools have been extensively used in the existing literature\(^1\)\(^–\)\(^3\) for identifying existing pretension force in the bolts. However, non-destructive testing techniques become very expensive for structures with many bolts. Finite element (FE) model updating\(^4\) is commonly used for identifying condition of a structure by using global measurements taken along the length and breadth of the structure. FE model updating technique can be implemented on frequency-domain or time-domain measurements of the structural responses. For example, a virtual distortion method–based model updating technique is proposed in Lin et al.\(^5\) for detecting damage in cable structures of a bridge. Further

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work is needed to implement the virtual distortion method for identifying rotational spring constants at the connections of a steel frame. Compared with the frequency domain data, structural vibration responses in time domain are more sensitive to damage\textsuperscript{6,7} and are thus attracting increasing research interests. The accurate simulation of time-domain structural vibrations not only provides the opportunity for FE model updating using time-domain data, but also can be a reliable approach to constructing data dictionary for machine-learning-based method. FE model updating is carried out for identifying the stiffness of bolted connections from the measured strain time histories.\textsuperscript{8} Bilinear springs along with a Rayleigh damping matrix are used for the dynamic simulation of a pipe with bolted connection.\textsuperscript{9} However, the results\textsuperscript{9} show that the dynamic simulation can still be improved so that it matches closely with the measured responses.

Discontinuous interfaces at the bolted connections region introduce structural damping and nonlinear contact.\textsuperscript{10} Micro- and meso-scale parameters like geometry, roughness and bolt preload govern the contact nonlinearity,\textsuperscript{11} and it is computationally very expensive to include those effects in a macro-scale model. The study by Hammami et al.\textsuperscript{12} concluded that the level of junction coupling has sufficient influence on the dynamic system to induce damping. It thus necessitates integrating the damping effects introduced at a bolted connection in the dynamic analysis of the structure.

A nonlinear system identification technique is employed for modelling and analysing the nonlinear damping effects induced at a bolted connection.\textsuperscript{13} A nonlinear damping model is proposed by combining a viscous damping model with a quadratic polynomial damping force.\textsuperscript{14} The nonlinear damping effects modelled in Eriten et al.\textsuperscript{13} and Franchetti et al.\textsuperscript{14} are applicable for a rigid mass system. A polynomial type nonlinear damping model is proposed for a rigid mass system,\textsuperscript{15} where the inherent nonlinearity in the damping is estimated using wavelet transform. Matrix-based and modal parameter–based damping identification methods\textsuperscript{16,17} are also reported. Damping identification based on matrix method requires the full frequency response function matrix to be measured. Modal parameter–based damping identification requires prior knowledge about the damping matrix, which is again the sole objective of the method. A probability distribution of decay rate\textsuperscript{18} is proposed for developing a relation between time-domain damping to damage in a steel structure with bolted connections. However, it cannot quantify the effect of bolt loosening on the system matrices in the FE model of the structure.

In order to integrate the nonlinear damping effects in the FE modelling of structures with bolted connections, a virtual damper is proposed and introduced to the FE model of the steel frame. It is used to capture nonlinear and time-dependent damping effects generated by the system which comprises the structure and the impact device. In section ‘Methodology’, the methodology is described in detail, including the implementation of the virtual damper in the general dynamic equation of a structural system and the FE model updating technique. In section ‘Experimental test setup and modal parameter extraction’, details of a steel frame with bolted connections designed and assembled in the laboratory is presented. Numerical modelling of the steel frame and the model updating results are given in section ‘Dynamic response in the presence of virtual damper’. The numerical simulation results with and without the virtual damper have been compared with the measured acceleration responses for various damage cases, which demonstrate the effectiveness of the proposed method.

**Methodology**

*Introduction of a virtual viscous damper to the dynamical system*

The equation of motion of damped forced vibration of a structural system is given by

\[ M\ddot{X} + C\dot{X} + KX = F + W_0 \]  \hspace{1cm} (1)

where \( M, C \) and \( K \) are structural mass, damping and stiffness matrices of the system, \( X \) is the system displacements and \( F \) is the applied force. \( W_0 \) is a Brownian motion\textsuperscript{19} representing process noise that quantifies the uncertainties coming from modelling system dynamics. Without loss of generality, let the contribution of structural steel connection towards nonlinearity in the dynamic system response be expressed through a nonlinear damping matrix \( C^{NL} \), such that \( C^{NL} = C - C^L \).

Here, \( C \) is the damping matrix of the structural system, taking into account the linear as well as nonlinear component of damping. The dynamic system can now be written as

\[ M\ddot{X} + (C^L + C^{NL})\dot{X} + KX = F + W_0 \]  \hspace{1cm} (2)

where

\[ C^L = \alpha M + \beta K \quad \text{and} \quad C^{NL} = g_c(X, F) \]  \hspace{1cm} (3)

The nonlinear component of damping \( C^{NL} \) is taken as a time-varying function \( g_c \) of system states \( X \) and input force \( F \). Moving the nonlinear component of damping \( C^{NL} \) to the right-hand side, equation (2) can be written as

\[ M\ddot{X} + C^L\dot{X} + KX = F - C^{NL}\dot{X} + W_0 \]  \hspace{1cm} (4)
The Rayleigh damping factors are then evaluated using the measured modal damping factors and natural frequencies. In Step 4, the beam column joint of the steel frame is modelled using linear spring elements. A differential evolution (DE) algorithm is then implemented to update the initial FE model by estimating the stiffness of all the springs using the measured natural frequencies of the steel frame. In Step 5, Rayleigh damping matrix is constructed using the system mass and stiffness matrices extracted from the updated FE model and the Rayleigh damping factors evaluated in Step 3. The measured impulse force is then applied to the system, and the dynamic analysis of the system is carried out. The simulated acceleration responses of the system are then compared with the measured acceleration responses. The above procedure is then validated with 10 different damage scenarios.

**FE model updating for estimating spring stiffnesses given the measured natural frequencies**

FE model updating technique\(^4\) employed in Step 4.a (first stage of model updating) is an inverse problem that estimates the unknown spring stiffnesses in the FE model by minimising the cumulative percentile error between the measured and simulated frequencies. Let the vector of updating parameters \(\theta\) in the FE model updating be the multipliers to the initial spring stiffness, and are defined as \(\theta = \{Kms\}_i=1,2,\ldots,6\). The \(i\)th element of \(\theta\) is defined as \(Kms_i = Kf_i/k_i\), where \(k_i\) denotes the stiffness of \(i\)th spring before model updating and \(Kf_i\) denotes the stiffness of \(i\)th spring after model updating. An objective function \(g(\theta)\) is formed that minimises the sum of square of error between the measured and simulated frequencies. If \(f_m\) is the vector of measured frequencies and \(f_s(\theta)\) is the vector of simulated frequencies for a given set of parameters of interest \(\theta\), the objective function \(g(\theta)\) is defined as

\[
g(\theta) = Err^T \times Err, \quad \text{where} \quad Err = f_m - f_s(\theta) \tag{6}
\]

For a given candidate solution \(\hat{\theta}\) of the unknown parameter \(\theta\), multipliers \(Kms\) to the initial stiffness \(Kf\) are obtained as \(Kms = \hat{\theta}\). Stiffness of \(i\)th spring in the FE model is then calculated as \(k_i = Kms_i \times Kf_i\), where \(k_i\) denotes the stiffness of \(i\)th spring before model updating and \(Kf_i\) denotes the stiffness of \(i\)th spring after model updating. A frequency analysis is then carried out in the FE software OpenSEES\(^{21,22}\) to estimate the natural frequencies of the system \(f_s\) using the updates spring stiffnesses.
unknown parameter algorithm 23 was used to find the optimal values of the objective function (equation (6)). In this article, a DE population-based metaheuristic search algorithms. 24 The DE algorithm has shown robustness to multi-modal parameters (frequencies $j_i = 1, 2, ..., 6$ for a given candidate solution $\hat{\theta}$ of the unknown parameter $\theta$.

The FE model updating technique implemented in this study involves finding the optimal solution of the objective function (equation (6)). In this article, a DE algorithm 23 was used to find the optimal values of the parameters ($\theta$) of the FE model given the measured frequencies ($f_m$) of the structural system. The DE algorithm is one of the most stable and versatile population-based metaheuristic search algorithms. 24 The DE algorithm has shown robustness to multi-modal nonlinear optimisation problems by iteratively improving a candidate solution based on an evolutionary process. The details of the DE algorithm are given in the paper by Storn and Price. 23

**Metropolis Hastings algorithm for generating samples from $p(F_h(Z_m)$**

Step 6.c in Figure 2 (second stage of model updating) is an inverse problem process of generating samples from the target probability distribution $p(F_h(Z_m)$ given the

![Figure 2](image-url)
output measurement data $Z_m$. While $p(F^d_h)$ is the prior density function of the uncertain parameter $F^d_h$, and $p(Z_m|F^d_h)$ is the likelihood of measurement with respect to the given parameter $F^d_h$, the steps involved in implementing the Metropolis-Hastings algorithm are

1. Start with an initial parameter $F^d_h,0$.
2. Generate a new parameter $F^d_h,t$ from proposal density function $q(F^d_h|F^d_h,n)$.
3. Accept $F^d_h,t$ with probability $\alpha^p$, defined as

$$
\alpha^p(F^d_h,t|F^d_h,n) = \min \left\{ 1, \frac{p(Z_m|F^d_h,t)p(F^d_h,t|F^d_h,n)q(F^d_h,n|F^d_h,t)}{p(Z_m|F^d_h,n)p(F^d_h,n|F^d_h,n)q(F^d_h,t|F^d_h,n)} \right\}
$$

4. Repeat Steps 2 to 5 till convergence or the maximum number of samples covered.

Convergence of the process is achieved when a Markov chain reaches a stage such that the posterior density function remains unchanged (i.e., variance of the last three generated sample is less than 0.01) for all subsequent stages. If the maximum number of samples is covered before convergence is achieved, the maximum number of samples is increased and the algorithm is run again. In this article, the likelihood is taken as a multivariate normal distribution. Each element of the measurement vector is taken as an independent and normally distributed random variable with mean as the measured value and standard deviation as 2% of mean.

**Experimental test set-up and modal parameter extraction**

**Test set-up**

In this study, a single-bay single-storey steel portal frame with bolted connections is designed and manufactured in the laboratory. The beam is connected to the columns with the help of gusset angles and pretensioned bolts. Bright zinc-plated high-tensile 10-mm bolts of grade 8.8 are used. The maximum recommended tightening torque for 10 mm bolts of grade 8.8 is 55 N m. A pretension torque of 55 N m is applied to each bolt using a torque wrench. The columns are welded to the base plates which are then bolted to the strong floor using 16 mm bolts. All the bolts used in this study are high-tensile bolts of grade 8.8. The geometric details of the frame and bolted connection details with specified bolt numbers are shown in Figure 3. In this study, 10 damage cases are considered. Damage is introduced in the portal frame by loosening pretensioned bolts. Care was taken to ensure that the bolts are only loosened and not removed from the connections. Details of the 10 damage cases are given in Table 1. The objective of this study is to identify the condition of the structural connections, given the occurrence of a damage scenario out of the 10 cases mentioned in Table 1. More details about the experiment and its results can be found in Zhang et al.7

**Natural frequencies**

The natural frequencies of the steel frame were extracted from the measured acceleration time signals...
using peak-picking technique. The extracted natural frequencies are given in Table 2.

### Modal damping factor estimation

Output-only modal identification techniques\(^\text{26,27}\) have shown great potential in identifying modal parameters from measured time-domain responses of a structural system. Modal damping factors are estimated from the measured time signals using the time-domain decomposition (TDD) technique.\(^\text{26}\) The TDD technique is implemented in this study, because of (1) its simplicity and efficiency and (2) possibility of extracting modal damping factors given any arbitrary input. The TDD technique for modal damping factor extraction can be described in four steps: (1) the prior information of the ranges of the frequencies are used along with a digital band-pass filter to isolate filtered time signals \(Y_i\) corresponding to each mode, where \(i\) is the mode number; (2) the energy correlation \(E_i\) of the \(i\)th mode is then calculated as \(E_i = Y_i^T \times Y_i\); (3) singular value decomposition of energy correlation matrix, defined as \(E_i = U \times \Omega \times U^T\), gives the singular vector matrix \(U\), the first column vector of which is designated as the mode shape of that isolated mode; and (4) half-power band method is then used to extract damping factors from the auto power spectrum of the isolated time signals corresponding to each mode.

### Rayleigh damping coefficients estimation

The estimated Rayleigh damping coefficients using the modal damping factors extracted in Appendix B are given in Table 3.

### Dynamic response in the presence of virtual damper

**Numerical modelling of connection stiffnesses**

OpenSEES\(^\text{21,22}\) is used in this study for FE modelling of the steel frame. A total of 33 nodes are assigned to the FE model as shown in Figure 4. Eleven equally spaced nodes are present in the beam and columns.
‘elasticBeamColumn’ elements in OpenSEES are used to model the beam and columns. Each beam column joint is modelled by three springs, two translational springs and one rotational spring (Figure 4). Let $k_1$, $k_2$ and $k_3$ be the spring stiffness in the left side joint, and $k_4$, $k_5$ and $k_6$ be the spring stiffness in the right side joint. As shown in Figure 4, $k_1$ and $k_4$ act along the length of beam, $k_2$ and $k_5$ act transverse to the length of beam, and $k_3$ and $k_6$ act along the rotational degrees of freedom between the beam and column. In OpenSEES, ‘zeroLength’ elements are used to model the springs. According to Eurocode 3 (BS EN 1993-1-8:2005),28 such joints can be treated as semi-rigid joints, which allow a certain amount of rotation. For a beam column joint, where the elastic rigidities of the beam and column are of the same order, the relationship between rotational spring stiffness $S$ and the fixity factor $\alpha_s$ can be written29 as

$$\alpha_s = \frac{1}{1 + \frac{3EI}{L}} \quad \text{or} \quad S = \frac{3EI}{L} \times \frac{\alpha_s}{1 - \alpha_s}$$  \hspace{1cm} (7)$$

where $E$ is the elastic modulus of beam, $I$ is second moment of inertia of beam and $L$ is length of beam. The fixity factor $\alpha_s$ depends on the type of joints,8,28 and for a semi-rigid joint $\alpha_s$ varies between 0.143 and 0.891. Assuming a fixity factor of 0.6, the rotational spring stiffnesses $k_3$ and $k_6$ are each calculated as 1,136,498 N m/rad. According to Eurocode 3 (BS EN 1993-1-8:2005),28 the translational stiffness of joints with more than two columns of preloaded bolts can be taken as an infinitely large number. In the FE model, the stiffness of each spring $k_1$, $k_2$, $k_4$ and $k_5$ is assumed to be $1 \times 10^{10}$ N/m.

Six accelerometers are used in this study to measure the dynamic behaviour of the steel frame when the steel frame is excited with an impulse hammer. The optimal location of accelerometers and position of impulse excitation are evaluated based on the optimal sensor positioning algorithm given in Biswal and Wang.30 The first three natural frequencies corresponding to the bending modes simulated for the damage case D0 are 85.23, 273.61 and 566.27 Hz, respectively. The cumulative percentile error between the measured (Table 2) and simulated frequencies is 9.7%.

DE algorithm23 is implemented in Python to find the unknown parameters (multipliers to the initial spring stiffness) as the roots of the objective function given in equation (6). The estimated multipliers to the initial stiffness of the springs for the damaged case D0 are $K_{ms1} = 0.0026$, $K_{ms2} = 8.9204$, $K_{ms3} = 0.2958$, $K_{ms4} = 7.3604$, $K_{ms5} = 8.9314$ and $K_{ms6} = 1.5821$. By multiplying the initial spring stiffness to the corresponding stiffness multipliers, the final values of the spring stiffness for damage case D0 are given in

$$\begin{align*}
K_{f1} &= 2.59 \times 10^7 \\
K_{f2} &= 8.92 \times 10^{10} \\
K_{f3} &= 3.36 \times 10^5 \\
K_{f4} &= 7.36 \times 10^{10} \\
K_{f5} &= 8.93 \times 10^{10} \\
K_{f6} &= 1.79 \times 10^6
\end{align*}$$

Table 4. Updated spring stiffness for damage case D0.

| Damage scenario | Spring stiffnesses |
|-----------------|--------------------|
|                 | $K_{f1}$, N/m      | $K_{f2}$, N/m      | $K_{f3}$, N m/rad | $K_{f4}$, N/m | $K_{f5}$, N/m | $K_{f6}$, N m/rad |
| 0               | $2.59 \times 10^7$ | $8.92 \times 10^{10}$ | $3.36 \times 10^5$ | $7.36 \times 10^{10}$ | $8.93 \times 10^{10}$ | $1.79 \times 10^6$ |
Table 4. The estimated vertical translation and rotation spring stiffness at the two connections were observed to have different values, even though for damage case D0 they are normally considered identical. The difference in the vertical translation and rotation spring stiffness at the two connections can be attributed to the possible imperfections in the manufacturing and assemblage of the steel portal frame. It is clear that stiffness of springs at left side connection is less than the respective stiffness at the right side connection. To verify the cause for this difference, additional accelerometers were installed on both columns of the portal frame. The mode shapes of the frame for the first three modes are given in Appendix C. The asymmetry behaviour of the frame is evident in Figures 13 to 15. It can be seen that the beam-to-column interaction is markedly different at the two ends of the beam. Due to the observed asymmetry in the connections, the sway displacement at top of the left column is larger than that of its counterpart on the right. This is reflected in the updated stiffness parameters at the two ends, as can be seen in Table 4 in the article.

The initial stiffness for damage cases D1 to D10 are taken same as the final stiffness for damage case D0. The estimated multipliers to the initial stiffness for damage cases D1 to D10 are given in Table 5 for damage cases 1–10. The comparison between simulated frequencies using updated stiffness of springs in the FE model with the measured frequencies is shown in Table 6. From the results shown in Table 6, the maximum cumulative percentile error for the three simulated frequencies is 1.364%. It is possible to quantify damage at the connections using the estimated spring stiffness corresponding to different damage cases. A damage indicator can be proposed to quantify damage at the connections based on the changes in the estimated spring stiffness values. The possible damage indicators will be explored in a future study. The FE model with the updated stiffnesses of springs is used to extract the structural system matrices. In OpenSEES, a ‘Static’ analysis with ‘FullGeneral’ system is performed to extract the full stiffness matrix, and a ‘Transient’ analysis with ‘FullGeneral’ system is performed to extract the full mass matrix. The damping matrix is estimated in the

Table 5. Estimated multipliers to initial spring stiffness in the finite element model.

| Damage scenario | Multipliers to spring stiffnesses |  |  |
|-----------------|-----------------------------------|---|---|
|                 | $K_{ms_1}$ | $K_{ms_2}$ | $K_{ms_3}$ | $K_{ms_4}$ | $K_{ms_5}$ | $K_{ms_6}$ |
| 1               | 1.0000     | 1.0000     | 1.0000     | 0.9457     | 1.0000     | 1.0000     |
| 2               | 1.0000     | 1.0000     | 1.0000     | 0.9505     | 1.0000     | 1.0000     |
| 3               | 0.9984     | 0.9998     | 1.0000     | 0.9504     | 0.9988     | 0.9997     |
| 4               | 0.5320     | 0.8675     | 0.9046     | 0.9177     | 0.7649     | 0.2803     |
| 5               | 0.9965     | 0.9964     | 1.0000     | 0.9504     | 0.8995     | 0.9999     |
| 6               | 0.9922     | 0.9995     | 1.0000     | 0.9457     | 0.9973     | 0.9999     |
| 7               | 0.5078     | 0.8809     | 0.9052     | 0.9177     | 0.7244     | 0.2023     |
| 8               | 0.9831     | 0.9973     | 1.0000     | 0.9563     | 0.5000     | 0.9995     |
| 9               | 0.3692     | 0.7815     | 0.8586     | 0.9103     | 0.5811     | 0.1674     |
| 10              | 0.2499     | 0.4402     | 0.6202     | 0.8506     | 0.2427     | 0.1155     |

Table 6. Comparison of estimated to measured natural frequencies.

| Damage scenario | Measured natural frequencies in Hz | Simulated natural frequencies in Hz | Cumulative error in % |
|-----------------|-----------------------------------|-----------------------------------|----------------------|
|                 | 1st | 2nd | 3rd | 1st | 2nd | 3rd |                  |
| 0               | 82.430 | 563.200 | 258.800 | 82.430 | 563.198 | 258.800 | 3.55e–4 |
| 1               | 83.340 | 563.300 | 254.300 | 82.430 | 563.052 | 254.299 | 1.136 |
| 2               | 83.340 | 564.600 | 254.700 | 82.430 | 563.065 | 254.699 | 1.364 |
| 3               | 82.920 | 563.300 | 254.700 | 82.430 | 563.064 | 254.696 | 0.634 |
| 4               | 81.630 | 560.700 | 251.700 | 81.630 | 560.700 | 251.700 | 0.000 |
| 5               | 82.920 | 562.500 | 254.700 | 82.427 | 562.943 | 254.698 | 0.516 |
| 6               | 83.340 | 563.700 | 254.300 | 82.429 | 563.045 | 254.298 | 1.210 |
| 7               | 81.630 | 560.700 | 251.700 | 81.631 | 560.731 | 251.713 | 0.011 |
| 8               | 82.920 | 561.600 | 255.200 | 82.409 | 562.047 | 255.195 | 0.538 |
| 9               | 81.210 | 559.000 | 250.900 | 81.214 | 559.120 | 250.937 | 0.041 |
| 10              | 78.640 | 550.100 | 245.300 | 78.640 | 550.099 | 245.300 | 1.817e–4 |
next section, which is then used for the dynamical analysis of the steel frame with bolted connection.

**Dynamic response without virtual damper**

Using the structural stiffness matrix \( K \) and mass matrix \( (M) \) of the steel frame extracted from OpenSEES in section ‘Numerical modelling of connection stiffnesses’, and the Rayleigh damping coefficients \( \alpha \) and \( \beta \) extracted in section ‘Rayleigh damping coefficients estimation’, the linear damping matrix \( (C^L) \) is calculated as shown in equation (3). Let \( F \) be the vector of forces applied to the steel frame. All the elements of vector \( F \) are zero except at the degree of freedom where the impulse is applied. A measured force from impulse hammer test is shown in Figure 5. Let \( Z_m \) be the vector of measured accelerations. Six accelerometers are used in this study, so the size of \( Z_m \) is \( 6 \times 1 \). Since the FE model (Figure 4) has 33 nodes, two nodes are fixed, and every node has 3 degrees of freedom (two translation and one rotation), and the size of stiffness matrix \( K \) is \( 93 \times 93 \). The size of \( Y^{n+1} \) in equation (14) is \( 186 \times 1 \). If \( Z_e \) be the vector of estimated acceleration responses at the same degrees of freedom as the measured accelerations \( Z_m \), at time step \( n+1 \), \( Z_e \) is calculated as

\[
Z_e^{n+1} = H \times Y^{n+1}
\]

where \( H \) is a rectangular sparse matrix with the number of rows equal to the number of measurements, and the number of columns is equal to the number of degrees of freedom of the structural system (e.g. length of stiffness matrix). Let \( nm \) be the number of measurements and \( nd \) is the number of degrees of freedom of the system. Then, the size of the matrix \( H \) is \( nm \times nd \), and elements of \( H \) are defined as

\[
H_{ij} = \begin{cases} 
1 & \text{for measurement number } i \text{ and the corresponding degree of freedom } j \\
0 & \text{otherwise}
\end{cases}
\]

If the first measurement is taken along the \( nf \)th degree of freedom, then \( H_{1,nf} = 1 \), and all the elements in the first row of matrix \( H \) are zeros. Here, the size of matrix \( H \) is \( 6 \times 186 \). The estimated acceleration responses \( Z_e \) are plotted against the measured responses \( Z_m \) in Figure 6.

From Figure 6, it is clear that apart from the acceleration at location 4, the estimated accelerations at all other locations do not match closely to the measured accelerations at those locations. Even though the updated FE model has been used along with Rayleigh damping coefficients generated from the measured modal damping factors, a big difference between the estimated and the measured responses can be seen in Figure 6. As reported in Ertiten et al.,13 Franchetti et al.14 and Chandra and Sekhar,15 the nonlinear damping effects generated from the structural connections must be taken into account in the dynamic system equation for analysing the dynamic behaviour of a structure.

**Dynamic analysis of steel frame in the presence of the virtual damper**

The external virtual damping force \( F_d^v \) is estimated following the procedure described in section ‘Methodology’. The estimated damping force for damage case 0 is shown in Figure 7. In equation (5), at degree of freedom where the impulse is applied, the element of \( F_d^v \) is replaced with the estimated damping force \( F_d^e \). By keeping all other elements of \( F_d^v \) as zero, equation (5) is then solved as in equation (14). The estimated acceleration responses are then compared with the measured accelerations at the respective locations and are shown in Figure 8. To quantify the difference between the measured and simulated accelerations, the average power is calculated between measured accelerations and simulated accelerations before and after damping update. If \( \ddot{x}(t) \) be a time signal, the average power \( P_i(t) \) between time \( t_1 \) and \( t_2 \) is given by

\[
P_i(t) = \int_{t_1}^{t_2} |\ddot{x}(t)|^2 \, dt \quad (10)
\]

The percentile absolute change in average power between measured and simulated accelerations is given in Table 7. From the results shown in Table 7, it is clear that the damping update substantially reduces the difference in average power between the measured and simulated accelerations. The mean of difference in average power corresponding to the six locations of accelerometers is 874.98 in the absence of virtual damper, whereas in the presence of virtual damper the mean of difference in average power reduces to 8.25.

The frequency spectrum of measured accelerations at location 1 was compared to the frequency spectrum of simulated accelerations with and without virtual damper and is shown in Figure 9. It is clear from Figure 9 that the proposed technique not only
estimates accelerations that compare well in the time domain, the estimated accelerations also compare well to those of measured accelerations in the frequency domain. It is also clear that there is a large deviation between the frequency spectrum of measured acceleration and the frequency spectrum of simulated acceleration without the virtual damper, thus highlighting the importance of the introduction of the proposed virtual damper to the dynamical system.

The comparison of estimated damping forces corresponding to various damage scenarios both in the time domain and frequency domain is given in Appendix D. As shown in Figures 16 and 17, variation of the estimated damping force among various damage scenarios is very small apart from the damage scenario 10, where all the four bolts at the right side connection were loosened. In damage scenario 10, this could be attributed

### Table 7. Percentage change in average power.

| Updating scenario     | Accelerometer locations |
|-----------------------|-------------------------|
|                       | 1          | 2          | 3               | 4          | 5          | 6          |
| Without virtual damper| 492.02     | 438.92     | 331.18         | 25.27      | 544.22     | 418.27     |
| With virtual damper   | 5.37       | 6.54       | 12.43          | 1.40       | 14.97      | 8.79       |

### Figure 6. Comparison of acceleration responses.

### Figure 7. Estimated damping force.
to the total loss of friction forces between the connection angles and the web of the beam by loosening all the four bolts at the connection. For completeness, the comparison of estimated acceleration responses for all considered damage scenarios is given in Appendix E.

In the paper by Cao et al.,\textsuperscript{31} it has been reported that nonlinear damping force generated due to structural connection is dependent on the amplitude of impulse excitation. To evaluate the effect of amplitude of impulse excitation on the estimated damping force,
the steel frame is excited with 10 excitations of varying amplitude, for each damage case. The amplitude of the 10 applied impulse excitations for damage case 1 are shown in Figure 10. The corresponding estimated damping forces are shown in Figure 11. It can be noted from Figure 11 that even though the amplitudes of estimated damping forces are different for different amplitude of applied impulse excitations, the estimated damping forces are shown to follow a similar pattern. This can be very beneficial in developing damage classification algorithms based on the measured time signals. A future study is focused on developing machine learning tools for identifying damages, where a set of neural networks are being trained using the estimated damping forces corresponding to various damage cases.

To get a more qualitative explanation and a better visual understanding of the dependence of amplitude of excitation on estimated nonlinear damping force, the measured accelerations are normalised with respect to the maximum amplitude of applied impulse excitations. All the measured accelerations are normalised with reference to a 100 N of maximum amplitude for the 10 applied impulse excitations. The estimated damping forces using the normalised measured accelerations for all the damage cases are shown in Figure 12. As shown in Figure 12, for a given damage case, the estimated damping forces not only follow the same pattern but also the amplitudes to a great extent. It thus can be stated that there is a linear relationship between the amplitude of applied impulse excitation and that of the estimated damping force.
Conclusion

Assessment of the current condition of a steel frame with different combinations of loosening of bolts in the structural connections has been carried out in this study. Numerical modelling of the steel frame is performed in OpenSEES. The stiffness of the FE model is updated using measured natural frequencies so that the cumulative percentile error in the simulated frequencies is less than 2%. The estimated system matrices predict the current condition of a structural connection as a single unit, represented by mechanical springs. A virtual damper to the steel frame is proposed for identifying the system in time domain. The virtual damper generates a time-varying force which when applied to the dynamical system gives very good agreement between the measured accelerations and accelerations simulated from the FE model of the steel frame in all damage cases and in both time and frequency domains.

From the results, it can be seen that the estimated damping forces have a linear relationship to the amplitude of applied impulse excitations. The observed linear relationship can be advantageous in condition identification and damage detection in structures using output only methods or techniques based on machine learning. The estimated damping forces in cases and in both time and frequency domains.

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### Appendix A: solution of stochastic differential equation (1)

The equation of motion of damped forced vibration (equation (1)) is given by

\[
M\ddot{X} + C\dot{X} + KX = F + W_0
\]  

(11)

Let, \( X_1 = X \), and \( X_2 = \dot{X} \) or \( dX_1 = X_2\,dt \)

Then, \( \ddot{X} = \dot{X} = M^{-1}F - M^{-1}CX - M^{-1}KX + W_1 \),

where \( W_1 = M^{-1}W_0 \)

or \( dX_2 = [M^{-1}F - M^{-1}CX - M^{-1}KX]\,dt + \sigma\,dW \)  

(12)

Putting in a vector form

\[
\begin{align*}
\{dX_1\} &= \left\{\begin{array}{c}
X_2 \\
M^{-1}F - M^{-1}CX - M^{-1}KX
\end{array}\right\}dt + \left\{\begin{array}{c}
0 \\
\sigma
\end{array}\right\}dW
\end{align*}
\]

or \( dX = \mu_X\,dt + \sigma_X\,dW \)  

(13)

The drift term \( \mu_X \) in the stochastic differential equation (equation (13)) is of order 2, and the diffusion term \( \sigma_X \) in equation (13) is of order 1.5. Order 1.5 strong Taylor expansion\(^{19}\) can be used to solve equation (13). An implicit scheme is required to avoid instability when a large step size is used for time integration. Using implicit order 1.5 strong Taylor scheme,\(^{19}\) the solution of above stochastic differential equation (equation (13)) can be written as

\[
Y_{n+1} = AY_n + BU_n + DR
\]  

(14)

where

\[
Y = \left\{\begin{array}{c}
X_{n+1} \\
X_{2,n+1}
\end{array}\right\}
\]

(15)

\[
\begin{align*}
I - \frac{1}{2}\Delta^2M^{-1}K &
-
\Delta M^{-1}K + \frac{1}{2}\Delta^2M^{-1}CM^{-1}K
\end{align*}
\]

(16)

\[
B = \left[\begin{array}{cc}
\frac{1}{2}\Delta^2M^{-1} & 0 \\
\Delta M^{-1} - \frac{1}{2}\Delta^2M^{-1}CM^{-1} & \frac{1}{2}\Delta^2M^{-1}
\end{array}\right]
\]

(17)

\[
U = \left\{\begin{array}{c}
F \\
\dot{F}
\end{array}\right\}
\]

(18)

\[
D = \left[\begin{array}{cc}
0 & \sigma \\
\sigma & -M^{-1}C\sigma
\end{array}\right]
\]

(19)

and

\[
R = \left\{\begin{array}{c}
\Delta W \\
\Delta Z
\end{array}\right\}
\]

(20)

In this study, \( \Delta W \) and \( \Delta Z \) are generated from a multivariate normal distribution with mean \( R_{\text{mean}} \) and covariance matrix \( R_{\text{cov}} \), where

\[
R_{\text{mean}} = \left[\begin{array}{cc}
0 & 0 \\
0 & 0
\end{array}\right] \quad \text{and} \quad R_{\text{cov}} = \left[\begin{array}{cc}
\Delta & \frac{1}{2}\Delta^2 \\
\frac{1}{2}\Delta^2 & \frac{1}{3}\Delta^3
\end{array}\right]
\]

(21)

### Appendix B: Rayleigh damping coefficients from measured modal damping ratios

Using Rayleigh’s proportional damping, the modal damping factors can be written as
\[ \zeta_i = \frac{1}{2\omega_i} \times \alpha + \frac{\omega_i}{2} \times \beta \]  

(22)  

The Rayleigh damping coefficients are then estimated using least square sense as

\[
\begin{bmatrix}
\zeta_1 \\
\zeta_2 \\
\vdots \\
\zeta_n
\end{bmatrix} = \begin{bmatrix}
\frac{1}{2\omega_1} & \frac{\omega_1}{2} \\
\frac{1}{2\omega_1} & \frac{\omega_1}{2} \\
\vdots & \vdots \\
\frac{1}{2\omega_1} & \frac{\omega_1}{2}
\end{bmatrix} \times \begin{bmatrix}
\alpha \\
\beta
\end{bmatrix}, \text{ or } \{\zeta\} = [A_w] \times \begin{bmatrix}
\alpha \\
\beta
\end{bmatrix},
\]

where \([A_w] = \begin{bmatrix}
\frac{1}{2\omega_1} & \frac{\omega_1}{2} \\
\frac{1}{2\omega_1} & \frac{\omega_1}{2} \\
\vdots & \vdots \\
\frac{1}{2\omega_1} & \frac{\omega_1}{2}
\end{bmatrix}\]  

(23)  

Then

\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = [A_w^T \times A_w]^{-1} \times [A_w^T] \times \zeta
\]

(24)  

\[\text{Appendix C: normalised mode shapes of the portal frame}\]

\[\text{Figure 13. Normalised mode shape corresponding to the first natural frequency.}\]

\[\text{Figure 14. Normalised mode shape corresponding to the second natural frequency.}\]

\[\text{Figure 15. Normalised mode shape corresponding to the third natural frequency.}\]

\[\text{Appendix D: comparison of virtual damping force for different damage scenarios}\]

The comparison of estimated damping forces corresponding to various damage scenarios both in the time domain and frequency domain is shown in Figures 16 to 19.
Figure 16. Estimated damping force for multiple damage scenarios.

Figure 17. Estimated damping force for multiple damage scenarios (zoomed).

Figure 18. Frequency spectrum of estimated damping force for multiple damage scenarios.
Appendix E: comparison of acceleration responses for damage cases 1 to 10

The estimated acceleration responses for damage scenarios 1 to 10 are plotted against the corresponding measured responses.
Figure 21. Comparison of acceleration responses for damage case 2 after damping update.

Figure 22. Comparison of acceleration responses for damage case 3 after damping update.
Figure 23. Comparison of acceleration responses for damage case 4 after damping update.

Figure 24. Comparison of acceleration responses for damage case 5 after damping update.
Figure 25. Comparison of acceleration responses for damage case 6 after damping update.

Figure 26. Comparison of acceleration responses for damage case 7 after damping update.
Figure 27. Comparison of acceleration responses for damage case 8 after damping update.

Figure 28. Comparison of acceleration responses for damage case 9 after damping update.
Figure 29. Comparison of acceleration responses for damage case 10 after damping update.