PRE-SALE ORDERING STRATEGY BASED ON THE NEW RETAIL CONTEXT CONSIDERING BOUNDED CONSUMER RATIONALITY

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Abstract. The purpose of this paper is to study the impact of bounded consumer rationality on the order quantity and profitability of the seller in the advance period and the spot period in the context of the combination of new retail and pre-sale. In this paper, we develop a seller order model in the context of the combination of new retail and pre-sale, with and without reference price dependence. Besides, the model considers the order cancellation and delayed purchase behavior of consumers. We then discuss the optimal profit and optimal order quantity under different conditions and the effect of different reference price dependence and value-added offline service on them. Our research shows that: First, the seller tends to set the deposit too low in pre-sales. Second, reference price dependence has different effects on order quantities in different periods. The seller should pay more attention to the impact of reference price dependence. Third, on the whole, consumer rationality benefits the seller. The seller, or the public policymaker, can benefit new retail businesses by increasing consumer rationality. Last, in the new retail context, an increase in offline service value-added, even if it increases total order quantity, is not always beneficial to the seller and may reduce profits. Therefore, the seller should weigh all factors to determine the optimal value-added offline services.

1. Introduction. Over the past few years, the physical retail outlook has been so gloomy that it’s been described as the “end of retail” [15]. To solve the bottlenecks in the development of physical retail and find new development points for e-commerce, Alibaba proposed the concept of “New Retail” for the first time in October 2016. At present, there are many definitions of new retail, among which the description from China’s Ministry of Commerce is the most representative: new retail is a commodity trading method that puts consumers as the core, aims to improve efficiency and reduce costs. New retail is driven by technological innovation, and is a comprehensive evolution of elements. Traditional retail, catalog, and online businesses are single-channel models for selling products or services [37]. Multichannel retail is a set of activities that involves selling goods or services to...
consumers through multiple channels [24]. Omnichannel retail is the process by which customers can access information, make purchases, and ultimately acquire goods seamlessly between any channel [6]. Compared with the above three types of retail, new retail is similar to omnichannel retail. Still, the biggest difference is that omnichannel retail focuses on the comprehensiveness of sales channels. In contrast, new retail not only includes omnichannel retail, but also uses big data, artificial intelligence and other emerging technologies to transform and upgrade the production and sales process to provide customers with the most extreme consumption experience.

After several years of development and practice, new retail has gradually entered people’s lives. For example, Amazon opened its first unmanned retail store in Seattle, using artificial intelligence technology to share online member information without waiting in line at checkout. Walmart opens online “Sam’s Club Store” on Jingdong platform, sharing Jingdong’s integrated warehouse and distribution service. In the new retail environment, consumers gradually show behavioral characteristics such as two-tier purchasing, personalized demand, scene diversification, and value participation. It becomes challenging for businesses to forecast demand. Ordering and inventory management is an important part of business operations management and is the core issue of cost control. Uncertainty in demand significantly influences on increasing average inventory holding levels and reduces inventory management efficiency in retail businesses [22].

In reality, a pre-sales strategy can improve a retailer’s understanding of a product’s market potential and reduce demand uncertainty [8]. Pre-sales refers to a marketing practice in which the seller induces the buyer to promise to buy a product before it is consumed [55]. Pre-sales allow that demand uncertainty during the advance period to be completely eliminated (since reservations are pre-committed). That demand during the spot period can be more accurately predicted through reservation quantities [42]. The pre-sale strategy also has many benefits for retailers. For example, it can increase sales by taking advantage of consumer uncertainty in valuation during the advance period [56], allow the retailer to receive all or part of the payment [54], and also reduce the risk of inventory backlogs and stock-outs [42, 43]. Pre-sale has been widely used in reality as a cost-effective marketing strategy. For instance, in May 2019, the Volkswagen I.D.3 ST opened for pre-sale in Europe with a deposit of €1,000, and it took only 24 hours to reach an order book of more than 10,000. In October 2019, US skincare brand Estee Lauder made nearly 500 million yuan ($70.65 million) in sales in just 25 minutes after pre-sales began on Tmall’s Double Eleven, surpassing the company’s sales for the entire day on November 11 last year.

In addition, advances in information technology are not only driving the development of new retail but also enabling strategic consumer behavior [50]. Strategic consumer behavior refers to the intertemporal shifting of consumer purchase decisions. That is, consumers strategically choose the best time to buy to maximize their utility [58], which can be followed by situations such as delayed purchases.

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and reference price dependence [21, 16, 46, 59, 10]. Simultaneously, ShopSavvy, Shopbrain, and other shopping price comparison software\(^9\), so that bounded consumer rationality reflected more prominent. If sellers ignore bounded consumer rationality, this can cause significant deviations in order levels and reduce corporate revenues [50].

Based on the above background, we propose a combined new retail and pre-sales model. Online retailers explore offline channels to sell their products and use pre-sales strategies to reduce demand uncertainty while considering consumer experience and bounded consumer rationality. This paper’s main objective is to delve into the impact of consumer experience and bounded consumer rationality on retailers’ operational strategies in the case of new retail combined with pre-sales. Because new retail can take many forms (e.g., an online retailer tapping into offline channels, an offline retailer tapping into online channels, or a traditional retailer tapping into omnichannel sales), this model is not all-inclusive. Still, it can provide retailers and public policymakers with corresponding decision-making references. The framework of this study is arranged as follows: Section 2 introduces the references relevant to this paper. Section 3 introduces the research questions and develops the research model. Section 4 examines the ordering strategy of retailers in new retail context without considering reference price dependence. Section 5 examines the ordering strategy of retailers in new retail context with reference price dependence. Section 6 analyses and proves the research model and gets some propositions. Section 7 is a numerical simulation of the model and draws some conclusions. The paper will conclude in Section 8.

2. Literature review. (1) The first part is about new retail. The concept of new retail has been proposed in recent years and has become a hotspot for scholarly research. Zhao and Xu (2017) [60] summarized the meaning, model, and development path of new retail and suggested that its core is to improve user experience. Briel (2018) [44] conducted a four-phase Delphi study that concluded that the future of competition in the retail industry will be based on the overall customer experience and that omnichannel retail will improve operational productivity. Hu et al. (2020) [18] summarized the concept of new retail. They found that its core is consumer experience, analyzed the operation and management practices of new retail models of leading companies such as Jingdong and Alibaba, and looked forward to future research trends of new retail models.

Many other scholars have studied business operations management in the context of new retail, primarily focusing on omnichannel supply chain issues. Wang and Ng (2018) [47] developed a dual oligopoly model consisting of new retail model firms and online firms. The results showed that new retail model firms have higher cost inputs but can make more profits when consumer perception is difficult. Pereira and Frazzon (2020) [30] proposed a data-driven approach that combines machine learning demand forecasting with simulation-based operational planning optimization to accommodate demand and supply synchronization in the omnichannel retail supply chain. Nageswaran et al. (2020) [27] modeled cross-channel pricing in omnichannel retail. They allowed cross-channel returns, internalized customers’ purchase, and return decisions. They found that companies with more brick-and-mortar customers should offer full refunds, while companies with more online store customers should charge for online returns to induce customers to return in-store.

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\(^9\)https://www.pcmag.com/news/the-best-shopping-apps-to-compare-prices
However, there are two limitations in the above literature. On the one hand, some of the literature only studies the concept of new retail and does not study the model in depth for the new retail context. On the other hand, even though some literature studies the supply chain model in the new retail context, it does not highlight the core of new retail, i.e., consumer experience.

(2) The second part is about the pre-sale. Şeref et al. (2016) [38] studied retailer’s inventory and pricing decision problems in pre-sale situations with strategic consumer involvement to obtain optimal inventory management and pricing policies. Gupta and Chutani (2020) [12] considered a two-period pricing model in which sellers offer freebies when they sell their products in advance and production is limited by capacity. The study obtained the optimal price that maximizes the seller’s expected profit, the quality level of the freebies, and the production quantity decision.

Some scholars use pre-sale information to predict what will happen later in the sale to reduce demand uncertainty risk. Tang et al. (2004) [42] studied the pre-sale strategy for perishable goods with uncertain demand, using the order volume in the advance period to predict the order volume in the later period and giving the optimal discounted price that maximizes the retailer’s expected profit. Boyacı and Özer (2010) [8] studies found that manufacturers can significantly reduce order quantity bias and improve manufacturers’ profits by predicting spot sales through pre-sale information. Kuthambalayan et al. (2015) [23] used pre-sale orders to update demand for the spot period and analyzed the impact of discounted pre-sale versus maximizing the company’s expected profits.

Consumers may change their valuation during the advance period and cancel their orders, and many scholars have studied pre-sale strategies in the case of order cancellation. He et al. (2019) [14] studied an all-pay pre-sale mechanism for a new product that allowed cancellation of a reservation before delivery and a full refund. They found that the oversubscription strategy was beneficial to manufacturers, but that higher capacity did not necessarily lead to higher profits. Cheng et al. (2020) [9] developed an inventory model of spoiled goods that allow for order cancellations for retailers that offer two-stage pre-sale to customers and gave an easy-to-use method to determine the optimal selling price and the optimal sales cycle to maximize the retailer’s total profit.

Overall, most of the previous studies on pre-sales are based on traditional or online retail contexts, such as real estate pre-sale [51, 20], fresh produce pre-sale [13, 53], or online e-commerce pre-sale [41]. However, research on pre-sale based on the new retail context is relatively vacant. In fact, the successful combination of new retail and pre-sale has long been applied in real life. For example, in 2017, the Tmall Double Eleven shopping festival was a massive success with total sales of 168.2 billion yuan, combining new retail and pre-sales. Besides, previous studies related to pre-sale have rarely considered the impact of order cancellations on order volume during the advance period. A few scholars have considered the impact of order cancellations. Still all assume that the amount of order cancellations is inversely proportional to time or a fixed value, without starting from the nature of consumer cancellations, i.e., consumers cancel orders because their valuation of the product drops to an unacceptable threshold.

(3) The last part is about bounded consumer rationality. Kahneman and Tversky (2013) [21] first introduced the concept of reference dependence by stating that
when the consumer buys at a price lower than the reference price, then he will perceive a gain, and vice versa perceive a loss, which is greater than the perceived gain for the same price difference. Hsieh and Dye (2017) [16] incorporated reference price dependence into a worsening inventory problem. They explored the relationship between the optimal pricing decision and the initial reference price based on moderate assumptions. Crettez et al. (2020) [10] discussed the existence of an optimal dynamic pricing strategy when demand depends on the reference price. The existence of a unique pricing strategy parameterized in the reference price’s initial value is also demonstrated.

In reality, consumers choose to delay the purchase of goods to gain maximum utility, which suggests bounded consumer rationality. Su (2009) [40] developed a model of inertial consumer decision making, where consumer inertia influences consumers’ choice to buy immediately or to delay their purchase and found that consumer inertia has both positive and negative effects on profits: it reduces demand in the first stage, it increases competition among consumers in the second stage. Mishra and Venkataraman (2020) [26] considered a monopoly retailer of a seasonal product, faced with random demand from strategic customers who have delayed purchases and determined the optimal order quantity to maximize the retailer’s expected profits in a monopoly market.

Many previous studies on operations management have not considered the bounded consumer rationality, when consumer reference price dependence and delayed purchase behavior can lead to demand uncertainty and thus to retailer ordering bias, which can be potentially costly to ignore.

In summary, this paper utilizes model building, analytical proof, and numerical simulation based on the theories related to new retail, pre-sale, and bounded consumer rationality. This type of research methodology has been applied in many literature [19, 26, 27, 39]. We study the impact of consumer experience and bounded consumer rationality on retailers’ operating strategies in the combination of new retail and pre-sale.

In addition, this paper contributes the following:

a. This paper summarizes the new retail concept and compares and analyses it with other retail forms. The paper delves into supply chain models in the context of new retail. It highlights the consumer experience, adding to the literature gap on new retail supply chain models.

b. This paper introduces pre-sale into the new retail context to study the impact of pre-sale on new retail firms’ operational decisions. We use pre-sale to reduce the demand uncertainty of new retail firms and consider the effect of pre-sale order cancellations on firms’ decisions, making them more rational and fills a gap in the literature.

c. This paper considers the impact of pre-order cancellations on order volume, starting from the nature of consumer cancellations and assuming that consumers cancel their orders because their valuation of the product is reduced to an unacceptable threshold.

d. Our model takes into account bounded consumer rationality to make it more realistic. In-depth study of the impact of bounded consumer rationality (especially reference price dependence) on the pre-sale strategy with new retail firms, and give corresponding decision recommendations for new retail firms.
3. Description of model. In the model, we assume that a consumer purchases at most one product. In this paper, price is an exogenous variable. In the spot period, a uniform selling price is used for both online and offline channels. Sellers use a discounted presale strategy where products are sold at a lower price in the advance period than in the spot period. To ensure the profitability of the seller, the advance price of the product is greater than the wholesale price, and the wholesale price is greater than the out-of-stock loss and salvage value, i.e., \( p > \lambda p > w > \max(s, g) \).

If a consumer who placed an order during the advance period cancels the order, then the seller does not refund the deposit. Consumers are strategic, and valuations are denoted by \( v \). The density function of \( v \) is \( h(v) = \gamma e^{-\gamma v} \) and the distribution function is \( H(v) \).

We assume that consumers fall into two categories in the pre-sale model based on the new retail context. One category is those who pay attention to pre-sale information and reach the market during the advance period, \( X \sim \mathcal{N}(\mu_x, \delta_x^2) \). The other category is those who do not pay attention to pre-sale information and reach the market during the spot period, and the new retail influences them, \( \tau Y \sim \mathcal{N}(\tau \mu_y, (\tau \delta_y)^2) \). In fact, this assumption is similar to other literature on pre-sales that divides consumers into informed and uninformed consumers [33]. The impact of the new retail on the second category of consumers is similar to the omnichannel showroom effect and will increase consumers across all channels [7].

The sales period is divided into two phases: the advance period and the spot period. During the advance period, consumers pay a deposit \( \alpha \) to confirm their order upfront and decide whether to pay the final payment \( (\lambda p - \alpha) \) at the end of the advance period. At the end of the advance period, the seller will open an offline channel, which, together with the online channel, will form a dual-channel structure for spot sales. Consumers can place orders at any time during the advance period, and at the moment \( t_0 \), the seller needs to send total order requests to the supplier to meet market demand for the entire sales period. At this point, the seller can update the demand in the online and offline channels for periods \( t_0 \sim T_1 \) (i.e., late advance period) and \( T_1 \sim T_2 \) (i.e., the spot period) based on the demand \( x_1 \) that occurred during period \( 0 \sim t_0 \) (i.e., early advance period), indicates the lead time for the production of a product. The seller obtains the goods at moment \( (t_0 + L) \) to ensure that they can be shipped on order in time for the beginning of the spot period.

The seller opens the offline channel based on customer segments, which helps meet the customer base’s needs that prefer to shop in offline stores and enhance the user experience, thereby increasing market demand [16]. Therefore, the total market demand under the dual-channel model will be higher than that under the single online marketing model. The increased demand stemming from the promotional effect of the offline stores during the spot period. Assuming that \( \tau \in (1, +\infty) \), then the number of customers reaching the market in the spot period is \( \tau Y \). It is also similar to the showroom effect in omnichannel. Showrooms in offline channels increase demand in both overall and online channels, creating operational spill-overs to other channels by attracting customers with a higher average cost of service [7].

The difference between online and offline channels is that online shopping is more convenient and saves consumers time and energy. In contrast, the offline channel provides consumers with personalized services such as the physical experience of products. We assume that the offline channel adds value to the consumer’s service relative to the convenience of the online channel is \( S_{off} \). When \( S_{off} \geq 0 \), consumers
are more satisfied with the offline channel’s service, and conversely, when \( S_{off} < 0 \), consumers perceive the convenience of the online channel to be more valuable than the service provided by the offline channel. Given this, the seller can influence the consumer’s shopping decision by controlling the value added by the offline channel service. Assume that the cost of providing services through the offline channel is \((S_{off} - l)^2c_0/2\) and \( S_{off} \in (l, +\infty) \). When \( S_{off} = l \), consumers have the lowest valuation of the offline channel’s services, at which point the merchant’s cost of value-added services is zero. The sequence of events is shown in Figure 1. The meaning of the symbols in this paper is shown in table 1.

Figure 1. The pre-sale model based on the new retail context

4. Ordering model without reference price dependence.

4.1. Ordering strategies for the advance period for the seller. The seller publishes advance and spot prices at the beginning of the advance period. The consumer’s purchase utility during the advance period is affected by the advance price, deposit loss aversion, and waiting time. The utility of consumer purchases during the presale period is expressed as \( U^{(1)}_a = v - \lambda p - (1 - \xi) r_0 \alpha - \eta t \). In the spot period, the utility of the product purchased by consumers in the online and offline channels is expressed as \( U^{(1)}_{on} = v - p \), \( U^{(1)}_{off} = v - p + s_{off} \), respectively.

For consumers arriving in the market during the advance period, consider placing an order during the advance period when \( U^{(1)}_a \geq 0 \). At this point, the expected market size is \( \overline{H}(\lambda p + (1 - \xi) r_0 \alpha + \eta t)X \). Given consumers’ strategic nature, it is when \( U^{(1)}_a \geq \max(U^{(1)}_{on}, U^{(1)}_{off}) \) that they will decide to place an order during the advance period, otherwise they will consider buying during the spot period. When \( U^{(1)}_a = \max(U^{(1)}_{on}, U^{(1)}_{off}) \), the consumer has the same utility in the advance period as in the spot period. At this point, his sensitivity to waiting time is \( \eta_1 = \frac{(1 - \lambda)p - (1 - \xi)r_0 \alpha - \max(s_{off}, 0)}{(1 - \lambda)p - (1 - \xi)r_0 \alpha - \max(s_{off}, 0)} \). That these heterogeneous consumers have different willingness to pay for advance period purchases due to different wait time sensitivities in the marketplace. By making \( \eta_1 = \min\{\eta | U^{(1)}_a = \max(U^{(1)}_{on}, U^{(1)}_{off}), 1\} \), we can obtain the consumer market segmentation as shown in Figure 2.

When \( \eta \in [0, \eta_1] \), consumers are less sensitive to waiting time and are willing to pay a lower price in exchange for a longer waiting time. When \( \eta \in (\eta_1, 1] \), consumers are more sensitive to waiting time and are willing to pay more to get the product and use it as soon as possible. In particular, when \( \eta = 0 \), consumers focus only on the product’s price, on the satisfaction of a low price, and are insensitive to long waiting times. When \( \eta = 1 \), consumers are unwilling to make any wait for a
Table 1. The meaning of the symbols

| Symbol | Description |
|--------|-------------|
| $\xi$  | Probability of consumers not canceling their orders during the advance period. It’s the consumer’s prediction of their behavior. $(1 - \xi)$ is the probability that the consumer will cancel the order. $\xi \in [0,1]$ |
| $\beta$ | Consumer preference for online channels, $(1 - \beta)$ is consumers’ preference for offline channels. |
| $c_i, g_0, c_1$ | Price of the unit of product in the spot period/Wholesale cost per unit of product per unit of time/Cost of value-added services per unit of product in the offline channel/Order costs for the seller. |
| $p, S, w$ | The discount rate of a product’s advance price compared to its spot price/Consumer valuation of products. |
| $\lambda, v$ | Advance deposits for unit products/Out-of-stock losses per unit of product. |
| $\eta$ | Consumer sensitivity to waiting time for the seller shipments. |
| $t_0$ | Point in time when the seller sends an order request. |
| $T_1, T_2$ | The end of the advance period/The end of the spot period. |
| $L$ | The lead time, i.e. from the time an order is placed by the seller to the time the goods are delivered to the warehouse. |
| $\tau$ | Consumer aversion coefficient for “deposit loss”/Consumer’s reference dependence coefficient on “acquisition”/The end of the advance period. $0 < r_1 < r_2 \leq 1$. |
| $X$ | Consumers who are concerned about pre-sales and they will enter the market during the pre-sales period. $X \sim N(\mu_x, \delta_x^2)$. |
| $Y$ | Consumers who are concerned about pre-sales and they will enter the market during the pre-sales period. $Y \sim N(\mu_y, \delta_y^2)$. |
| $\tau_Y$ | $\tau \in (1, +\infty)$, $\tau_Y$ represents the number of consumers reaching the market in the new retail model in the spot period. |
| $S_{off}$ | Value-added services for consumers in offline channels, $S_{off} \in (l, +\infty)$. |
| $\rho$ | Indicates the correlation coefficient between market size in period $t_0 \sim T_1$ and in period $\rho_x$ represents the correlation coefficient between the market size in the advance period and in period $0 \sim t_0$. $\rho_y$ represents the correlation coefficient between the market size in the spot period and in period $0 \sim t_0$. |
| $U^{(i)}_A(j)$ | In case (i), the utility of the consumer’s purchase during the advance period. $i = 1, 2, 1$ and 2 denote the new retail marketing model when reference price dependence is not considered and considered, respectively. |
| $U^{(i)}_S(j)$ | In case (i), the utility of the consumer’s purchase in the spot period. $s = on$, $off$, on and off denote online and offline channels respectively. |
| $\Pi^{(i)}_j$ | In case (i), the seller’s profit during stage $j$, $j = 1, 2, 3, 1, 2, 3$ denote the period $0 \sim t_0, t_0 \sim T_1, T_1 \sim T_2$, respectively. |
| $D^{(i)}_j, D^{(i)}_j'$ | In case (i), the market demand in period $j, D^{(i)}_j \sim N(\mu^{(i)}_j, (\delta^{(i)}_j)^2)$. In case (i), the updated market demand in period $j, D^{(i)}_j' \sim N(\mu^{(i)}_j, (\delta^{(i)}_j)^2)$. |
| $C^{(i)}$ | In case (i), the seller’s total inventory cost. |
| $Q^{(i)}_j, x^{(i)}_j$ | In case (i), the order quantity of the seller to meet the demand in period $j$. In case (i), the time-sensitive thresholds for when a consumer’s purchase utility in the advance period equals the purchase utility in the spot period. |
| $\eta_i$ | In case (i), the threshold at which a consumer’s purchase utility equals zero during the presale period. |
low price. So, when the seller pre-sells the product since consumers have different sensitivities about waiting time, the probability that consumers will choose to spend their money during the advance period is $P(\eta < \eta_1)$ that they are willing to take the time to wait. The probability of consumers choosing to buy in the spot period is $P(\eta_1 \geq \eta)$ and they prefer to pay a higher price for less waiting time [39]. $\eta$ obeys a uniform distribution of $[0, 1]$.

The probability that a consumer will place an order in the advance period is $P(\eta < \eta_1) = \frac{(1-\lambda)p-(1-\xi)r_0\alpha}{f_{\eta}(\eta_1)}$, and the probability that a consumer will buy in the spot period is $P(\eta_1 \geq \eta) = 1 - \frac{(1-\lambda)p-(1-\xi)r_0\alpha}{f_{\eta}(\eta_1)}$. For the sake of a brief-expression, make $\theta_1 = \lambda p + (1-\xi)r_0\alpha + \eta t$, then $P(\eta < \eta_1)\overline{H}(\theta_1)X$ denotes the number of consumers who place orders during the advance period. For consumers who arrive in the market during the spot period, they will purchase the goods when $\min(U_{on}, U_{off}) \geq 0$.

During the advance period of a new product, consumers will always delay their purchases in expectation of maximum utility, resulting in fewer orders in the early advance period and a significant increase in orders in the late advance period [23]. Given this phenomenon of delayed purchases, we assume that the volume of presale orders grows exponentially over time, i.e., the growth factor is $e^t$.

According to Tang et al. (2004) [42], the seller updates $D_2^{(1)}$ using the actual quantity of orders $x_1^{(1)}$ that occur in period $0 \sim t_0$. The updated market demand is $D_2^{(1)'}$. It has a density function of $g_2(x)$ and a distribution function of $G_2(x)$, $D_2^{(1)'} \sim N(\mu_2^{(1)'}, (\delta_2^{(1)'})^2)$. The details are as follows.

$$
\mu_2^{(1)'} = \mu_2^{(1)} + \text{corr}(D_2^{(1)}, D_1^{(1)})(x_1^{(1)} - \mu_1^{(1)}) \left(1 - \frac{e^{\lambda_0}}{\epsilon T_t}\right) P(\eta < \eta_1)\overline{H}(\theta_1)\delta_x
= \mu_2^{(1)} + \rho(x_1^{(1)} - \mu_1^{(1)}) e^{T_1} - e^{\lambda_0},
$$

$$
\delta_2^{(1)'} = \delta_2^{(1)} \sqrt{1 - \text{corr}(D_2^{(1)}, D_1^{(1)})^2} = \delta_2^{(1)} \sqrt{1 - \rho^2},
$$

$$
D_2^{(1)'} \tilde{N}\left((1 - \frac{e^{\lambda_0}}{\epsilon T_t})P(\eta < \eta_1)\overline{H}(\theta_1)\mu_x + \rho(x_1^{(1)} - \mu_1^{(1)}) e^{T_1} - e^{\lambda_0},
(1 - \rho^2)(1 - \frac{e^{\lambda_0}}{\epsilon T_t})P(\eta < \eta_1)\overline{H}(\theta_1)\delta_x^2 \right).
$$

In period $t_0 \sim T_1$, the seller’s optimal order quantity for the late advance period is:
Q(1)_2^* = G_2^{-1}(\frac{\lambda p + g - w - (T_1 - t_0 - L)c}{\lambda p + g - s}), \hspace{1cm} (4)

At this point, the seller’s optimal profit is:

\Pi(1)_2^* = (\lambda p + g - w - (T_1 - t_0 - L)c)Q(1)_2^* - g\mu(1)_2^* - \int_{v_0(1)}^{x_1(1)} h(v)dv. \hspace{1cm} (5)

Considering order cancellation, when \( v_0(1) < \lambda p - \alpha \), the sum of the consumer’s valuation and the utility of the “acquisition” is lower than the final payment, the consumer forgoes the final payment and cancels the order. The critical valuation of the consumer abandonment orders is \( v_0(1) = \lambda p - \alpha \). When \( v < v_0(1) \), the consumer will forgo paying the final payment and when \( v \geq v_0(1) \), he will choose to pay the final payment.

The expected net income of consumers who placed orders during the advance period is:

\[ E(1)(U) = \int_{0}^{v_0(1)} (-\alpha)h(v)dv + \int_{v_0(1)}^{\infty} (v - \lambda p - \eta)t)h(v)dv. \] \hspace{1cm} (6)

In this expression, the first part represents the expected income for consumers who have not paid the final payment. The second part represents the expected income for consumers who have paid the final payment. If \( E(1)(U) \geq 0 \), the simplification results in \( \alpha \leq \frac{\int_{x_1(1)}^{\infty} (v - \lambda p - \eta)t)h(v)dv}{H(v_0(1))} = E(v, v_0(1)). \) Since the deposit cannot be greater than the presale price, i.e., \( \alpha \leq \lambda p, \alpha(1)_0 = \min(E(v, v_0(1)), \lambda p). \)

In period 0 \( \sim T_1 \) (i.e., the advance period), the number of people placing orders is \( x_1(1) + Q(1)_2^* - \int_{0}^{Q(1)_2^*} G_2(x)dx \), and therefore the number of order cancellations is:

\[ q_0(1) = (x_1(1) + Q(1)_2^* - \int_{0}^{Q(1)_2^*} G_2(x)dx) \int_{v_0(1)}^{x_1(1)} h(v)dv \]
\[ = H(v_0(1))(x_1(1) + Q(1)_2^* - \int_{0}^{Q(1)_2^*} G_2(x)dx), \] \hspace{1cm} (7)

To meet the demand for the period 0 \( \sim T_1 \) (i.e., the presale period), the seller’s optimal order quantity is:

\[ Q(1)_a = x_1(1) + Q(1)_2^* - q_0(1) \]
\[ = x_1(1) + Q(1)_2^* - H(v_0(1))(x_1(1) + Q(1)_2^* - \int_{0}^{Q(1)_2^*} G_2(x)dx), \] \hspace{1cm} (8)

The seller’s total profit for the advance period is:

\[ \Pi(1)_a = (\lambda p - w - (T_1 - t_0 - L)c)x_1(1) + \Pi(1)_2^* - (\lambda p - \alpha - w)q_0(1). \] \hspace{1cm} (9)
4.2. Ordering strategies for the spot period for the seller. Next, we update the spot period demand $D_{on}^{(1)}, D_{off}^{(1)}$ using the actual order quantity $x_1^{(1)}$ that has occurred over period $0 \sim t_0$.

$D_{on}^{(1)} \sim N(\mu_{on}^{(1)}, (\delta_{on}^{(1)})^2)$. It has a density function of $g_{on}(y_1)$ and a distribution function of $G_{on}(y_1)$. The details are as follows.

$$
\mu_{on}^{(1)} = \beta(P(\eta > \eta_1)\mu_x^{(1)} + \tau \mu_y^{(1)})H(v_2),
$$  

(10)

$$(\delta_{on}^{(1)})^2 = (\beta P(\eta > \eta_1)H(v_2)\delta_x^{(1)})^2 + (\beta H(v_2)\tau \delta_y^{(1)})^2 + 2\rho_y \beta^2 P(\eta > \eta_1)H(v_2)^2\tau \delta_y^{(1)}.
$$  

(11)

$D_{off}^{(1)} \sim N(\mu_{off}^{(1)}, (\delta_{off}^{(1)})^2)$. It has a density function of $g_{off}(y_2)$ and a distribution function of $G_{off}(y_2)$. The details are as follows.

$$
\mu_{off}^{(1)} = (P(\eta > \eta_1)\mu_x^{(1)} + \tau \mu_y^{(1)})(H(v_2) - H(v_1) + (1 - \beta)H(v_2)),
$$  

(12)

$$(\delta_{off}^{(1)})^2 = (P(\eta > \eta_1)(H(v_2) - H(v_1) + (1 - \beta)H(v_2))\delta_x^{(1)})^2 + ((H(v_2) - H(v_1) + (1 - \beta)H(v_2))\tau \delta_y^{(1)})^2 + 2\rho_y \beta^2 P(\eta > \eta_1)(H(v_2) - H(v_1) + (1 - \beta)H(v_2))\tau \delta_y^{(1)}.
$$  

(13)

In period $T_1 \sim T_2$ (i.e., the spot period), the optimal order quantity for the seller’s online channel is:

$$
\overline{Q}_{on}^{(1)*} = G_{on}^{-1}(\frac{p + g - w - (T_2 + T_1 - 2t_0 - 2L)c}{p + g - s + \frac{(T_2 - T_1)c}{2}}).
$$  

(14)

In period $T_1 \sim T_2$ (i.e., the spot period), the optimal profit for the seller’s online channel is:

$$
\Pi_{on}^{(1)} = -(p + g - s + \frac{(T_2 - T_1)c}{2}) \int_{0}^{\overline{Q}_{on}^{(1)*}} G_{on}(y_1)dy_1 + (p + g - w - \frac{(T_2 + T_1 - 2t_0 - 2L)c}{2})\overline{Q}_{on}^{(1)*} - g\mu_{on}^{(1)}.
$$  

(15)

In period $T_1 \sim T_2$ (i.e., the spot period), the optimal order quantity for the seller’s offline channel is:

$$
\overline{Q}_{off}^{(1)*} = G_{off}^{-1}(\frac{p + g - w - (T_2 + T_1 - 2t_0 - 2L)c}{p + g - s + \frac{(T_2 - T_1)c}{2}}),
$$  

(16)

In period $T_1 \sim T_2$ (i.e., the spot period), the optimal profit for the seller’s offline channel is:

$$
\Pi_{off}^{(1)} = -(p + g - s + \frac{(T_2 - T_1)c}{2}) \int_{0}^{\overline{Q}_{off}^{(1)*}} G_{off}(y_2)dy_2 + (p + g - w - \frac{(T_2 + T_1 - 2t_0 - 2L)c}{2})\overline{Q}_{off}^{(1)*} - g\mu_{off}^{(1)} - \frac{(s_{off} - l)^2}{2}c_0.
$$  

(17)
5. Ordering model with reference price dependence.

5.1. Ordering strategies for the advance period for the seller. Given the consumer’s reference price dependence, and assuming that represent the consumer’s reference dependence coefficients on “acquisition” and “loss”, respectively, the consumer’s purchase utility in advance period is \( U_{on}^{(2)} = v - \lambda p + \xi r_1(p - \lambda p) - (1 - \xi) r_0 \alpha - \eta t \). In the spot period, the consumer’s purchase utility in the online channel is \( U_{on}^{(2)} = v - p - r_2(p - \lambda p) \), and that in the offline channel is \( U_{off}^{(2)} = v - p - r_2(p - \lambda p) + s_{off} \).

For consumers who arrive at the market during the advance period, they will consider placing an order during the advance period when \( U_{on}^{(2)} \geq 0 \). At this point the expected market size is \( \overline{H}(\lambda p - \xi r_1(p - \lambda p) + (1 - \xi) r_0 \alpha + \eta t)X \). Given the strategic nature of the consumer, when \( U_{on}^{(2)} \geq \max(U_{on}^{(2)}, U_{off}^{(2)}) \), they will decide to place an order during the advance period. Otherwise, they will only consider whether to buy during the spot period. When \( U_{on}^{(2)} = \max(U_{on}^{(2)}, U_{off}^{(2)}) \), the consumer’s utility is the same in the advance and spot periods. At this point, his wait time sensitivity is \( \eta_2 = \frac{1 - \lambda \rho + \xi r_1(p - \lambda p) + r_2(p - \lambda p) - (1 - \xi) r_0 \alpha - \max(s_{off}, 0)}{\xi} \). The probability that a consumer will place an order during the advance period is \( P(\eta < \eta_2) = \frac{(1 - \lambda \rho + \xi r_1(p - \lambda p) + r_2(p - \lambda p) - (1 - \xi) r_0 \alpha - \max(s_{off}, 0)}{\xi} \). The probability of a consumer buying in spot period is \( P(\eta \geq \eta_2) = 1 - \frac{(1 - \lambda \rho + \xi r_1(p - \lambda p) + r_2(p - \lambda p) - (1 - \xi) r_0 \alpha - \max(s_{off}, 0)}{\xi} \).

For simplicity, make \( \theta_2 = \lambda p - \xi r_1(p - \lambda p) + (1 - \xi) r_0 \alpha + \eta t \), then \( P(\eta < \eta_2) \overline{H}(\theta_2)X \) denotes the number of consumers who placed orders during the presale period. For consumers who arrive in the market during the spot period, consumers will purchase the product when \( \min(U_{on}^{(2)}, U_{off}^{(2)}) \geq 0 \).

Same as above, the seller updates \( D_{2}^{(2)} \) using the actual quantity of orders \( x_1^{(2)} \) that occur in period \( 0 \sim t_0 \). The updated market demand is \( D_{2}^{(2)} \). It has a density function of \( z_2(x) \) and a distribution function of \( Z_2(x) \). \( D_{2}^{(2)} \) has a distribution function of \( N(\mu_2^{(2)}', \delta_2^{(2)}') \) for simplicity. The details are as follows.

\[
\mu_2^{(2)'} = \mu_2^{(2)} + \text{corr}(D_{2}^{(2)}, D_{1}^{(2)})(x_1^{(2)} - \mu_1^{(2)}) \left(1 - \frac{\epsilon_{10}}{\epsilon_{T1}}\right)P(\eta < \eta_2) \overline{H}(\theta_2)\delta_x
\]
\[
= \mu_2^{(2)} + \rho(x_1^{(2)} - \mu_1^{(2)}) \left(1 - \frac{\epsilon_{10}}{\epsilon_{T1}}\right)P(\eta < \eta_2) \overline{H}(\theta_2)\delta_x
\]
\[
\delta_2^{(2)} = \delta_2 \sqrt{1 - \left[\text{corr}(D_{2}^{(2)}, D_{1}^{(2)})\right]^2} = \delta_2 \sqrt{1 - \rho_1^2},
\]
\[
D_{2}^{(2)} \sim N\left(\left(1 - \frac{\epsilon_{10}}{\epsilon_{T1}}\right)P(\eta < \eta_2) \overline{H}(\theta_2)\mu_x + \rho(x_1^{(2)} - \mu_1^{(2)}) \left(1 - \frac{\epsilon_{10}}{\epsilon_{T1}}\right)P(\eta < \eta_2) \overline{H}(\theta_2)\delta_x \right),
\]
\[
(1 - \rho_1^2)\left(1 - \frac{\epsilon_{10}}{\epsilon_{T1}}\right)P(\eta < \eta_2) \overline{H}(\theta_2)\delta_x \right).
\]

In period \( t_0 \sim T_1 \), the seller’s optimal order quantity for the late advance period is:
\[
Q_2^{(2)*} = Z_2^{-1} \left(\frac{\lambda p + g - w - (T_1 - t_0 - L)e}{\lambda p + g - s}\right),
\]
At this point, the seller’s optimal profit is:
\[ \Pi_2^{(2)*} = -\lambda p + g - s \int_0^{Q_2^{(2)*}} Z_2(x) dx + (\lambda p + g - w - (T_1 - t_0 - L)c) Q_2^{(2)*} - g \mu_2^{(2)*}. \]  

(22)

Same as above, when \( v_0^{(2)} + r_1(p - \lambda p) < \lambda p - \alpha \), the sum of the consumer’s valuation and the utility of the “acquisition” is lower than the final payment, the consumer forgoes the final payment and cancels the order. The critical valuation of the consumer abandonment orders is \( v_0^{(2)} = \lambda p - \alpha - r_1(p - \lambda p) \). When \( v < v_0^{(2)} \), the consumer will forgo paying the final payment and when \( v \geq v_0^{(2)} \), he will choose to pay the final payment.

The expected net income of consumers who placed orders during the advance period is:

\[ E^{(2)}(U) = \int_0^{v_0^{(2)}} (-\alpha)h(v) dv + \int_{v_0^{(2)}}^{\infty} (v - \lambda p + r_1(p - \lambda p) - \eta t) h(v) dv. \]  

(23)

In this expression, the first part represents the expected income for consumers who have not paid the final payment. The second part represents the expected income for consumers who have paid the final payment. If \( E^{(2)}(U) \geq 0 \), the simplification results in \( \alpha \leq \frac{\int_{v_0^{(2)}}^{\infty} (v - \lambda p + r_1(p - \lambda p) - \eta t) h(v) dv}{H(v_0^{(2)})} = E(v, v_0^{(2)}). \) Since the deposit cannot be greater than the presale price, i.e., \( \alpha \leq \lambda p, \alpha^{(2)*} = \min(E(v, v_0^{(2)}), \lambda p) \).

In period 0 \( \sim T_1 \) (i.e., the advance period), the number of people placing orders is \( x_1^{(2)} + Q_2^{(2)*} - \int_0^{Q_2^{(2)*}} Z_2(x) dx \), and therefore the number of order cancellations is:

\[ q_0^{(2)} = (x_1^{(2)} + Q_2^{(2)*} - \int_0^{Q_2^{(2)*}} Z_2(x) dx) \int_0^{v_0^{(2)}} h(v) dv = H(v_0^{(2)}) (x_1^{(2)} + Q_2^{(2)*} - \int_0^{Q_2^{(2)*}} Z_2(x) dx), \]  

(24)

To meet the demand for the period 0 \( \sim T_1 \) (i.e., the presale period), the seller’s optimal order quantity is:

\[ Q_2^{(2)*} = x_1^{(2)} + Q_2^{(2)*} - q_0^{(2)} = x_1^{(2)} + Q_2^{(2)*} - H(v_0^{(2)}) (x_1^{(2)} + Q_2^{(2)*} - \int_0^{Q_2^{(2)*}} Z_2(x) dx), \]  

(25)

The seller’s total profit for the advance period is:

\[ \Pi_a^{(2)} = (\lambda p - w - (T_1 - t_0 - L)c)x_1^{(2)} + \Pi_2^{(2)*} - (\lambda p - \alpha - w) q_0^{(2)}. \]  

(26)

5.2. Ordering strategies for the spot period for the seller. Same as above, we update the spot period demand \( D_{on}^{(2)}, D_{off}^{(2)} \) using the actual order quantity \( x_1^{(2)} \) that has occurred over period 0 \( \sim t_0 \).

\( D_{on}^{(2)} \tilde{N}(\mu_{on}^{(2)}, \tau_{on}^{(2)}c) \), It has a density function of \( z_{on}(y_1) \) and a distribution function of \( Z_{on}(y_1) \). The details are as follows.

\[ \mu_{on}^{(2)} = \beta(P(\eta > \eta_2) \mu_x^{(2)} + \tau \mu_y^{(2)}), \]  

(27)
Proposition 1. Analyze and explain.

In the following, we give and prove some propositions based on the above model. The proof process will be presented in the appendix.

Proposition 1. \( \alpha^{(2)*} \geq \alpha^{(1)*} \), this suggests that the seller should set higher deposits in the advance period due to the reference dependence of consumer behavior.

Because of the reference price dependence, consumers are willing to bear a higher risk of deposit loss to achieve a higher purchase utility.
Proposition 2. (1) $Q_{2}^{(2)*} > Q_{1}^{(1)*}$, this suggests that sellers should increase their late advance period orders due to the reference dependence of consumer behavior.
(2) $\partial Q_{2}^{(2)*} / \partial r_{1} > 0$, $\partial Q_{2}^{(2)*} / \partial r_{2} > 0$. It shows that considering the reference price dependence, the seller’s order quantity in the late advance period increases with the increase of the “acquisition” and “loss” coefficients.

(3) When $Q_{2}^{(2)*} - Q_{1}^{(1)*} \geq \frac{\phi(\mu_{2} - \mu_{1}) + (\lambda_{e} + \sigma - \eta)(\int Q_{2}^{(1)*} F_{2}(x) dx - \int Q_{2}^{(1)*} G_{2}(x) dx)}{(\lambda_{e} + \sigma - (1 - \mu_{2} - \lambda_{e} \mu_{1}) \sigma)}$, the reference price dependence has a positive effect on the seller's profit in the late advance period.

Because consumers can purchase at lower prices during the pre-sale period, generating an acquisition utility that increases the total purchase utility of consumers during the pre-sale period, more consumers are willing to wait longer to obtain lower-priced products. The order quantity at the late advance period increases with the increase of “acquisition” and “loss” coefficient. The seller considers the reference price dependence in the late advance period, the optimal profit is greater than the optimal profit without considering the reference price dependence, so the seller should consider the reference price factor in the actual pre-sale, and increase the order quantity at the late advance period.

Proposition 3. (1) $Q_{a}^{(2)*} \geq Q_{a}^{(1)*}$,
(2) $Q_{on}^{(2)*} < Q_{on}^{(1)*}$, $Q_{off}^{(2)*} < Q_{off}^{(1)*}$, (1) and (2) show that the reference price dependence has a positive effect on demand in the advance period but negatively affects demand in the spot period.

(3) $\partial Q_{on}^{(i)*} / \partial r_{1} < 0$, $\partial Q_{off}^{(i)*} / \partial r_{2} < 0, i = \{1, 2\}$, this indicates that the quantity of dual-channel orders in the spot period decreases with the increase of the “acquisition” and “loss” coefficients.

When considering the reference price dependence, on the one hand, more consumers arriving in the market during the advance period choose to place their orders in this period, while fewer customers are willing to wait until the spot period to consume. On the other hand, the spot period consumers’ purchase utility is lower, resulting in a lower amount of the spot period consumers. And the ordering quantity in the late advance period increases with the increase of the “acquisition” and “loss” coefficients, but the ordering quantity in the spot period for the dual-channel decreases with the increase of the “acquisition” and “loss” coefficients. Therefore, considering the reference price dependence, the seller should increase the advance period order quantity and decrease the spot period order quantity.

Proposition 4. (1) $\begin{cases} \partial Q_{on}^{(i)*} / \partial s_{off} \geq 0, s_{off} \geq 0, \\ \partial Q_{off}^{(i)*} / \partial s_{off} < 0, s_{off} < 0, \end{cases} i = \{1, 2\}.$

(2) $\partial Q_{off}^{(i)*} / \partial s_{off} \geq 0, s_{off} \in R, i = \{1, 2\}$, (1) and (2) show that in the spot period, for the online channel, an increase in offline value-added service has a variable impact on the order quantity, specifically related to the positive and negative impact of offline value-added service. For the offline channel, an increase in offline value-added service positively impacts the order quantity.

This is because when value-added service is less than zero, i.e. when consumers perceive the level of offline service as higher than the convenience of the online channel, it does not affect the total demand in the spot period. Stills, more consumers are shifting to offline channels, and online channels are becoming less in demand.
And when the value-added service is greater than zero and gradually increasing, consumers in the advance period begin to shift to the spot period, expanding the market demand in the spot period and increasing demand in both online and offline channels, thus increasing the quantity of orders. Therefore, to expand market demand and gain more profits, the seller should appropriately increase the service level in the offline of the spot period.

7. Numerical simulations. In this section, we use numerical simulations to test the propositions and conclusions above and go further to investigate seller ordering strategies. Based on past market information, we assume that the spot price of the product is \( p = 70 \). The cost of storage per unit of product per unit of time is \( c = 1 \). The out-of-stock loss per unit of product is \( g = 30 \). The wholesale price per unit of product is \( w = 40 \). The pre-sale deposit for a unit of product is \( \alpha = 30 \). The ordering point is \( t_0 = 24 \). The length of the advance period is \( T_1 = 30 \) and the end of the spot period is \( T_2 = 50 \). Consumers estimate the likelihood that they will pay the final payment as \( \xi = 0.8 \). The discount rate of a product’s advance price compared to its spot price is \( \lambda = 0.8 \). Consumer sensitivity to acquisition is \( r_1 = 0.3 \), sensitivity to loss is \( r_2 = 0.4 \), and aversion to loss of deposit is \( r_0 = 0.5 \). The correlation coefficient of demand between the early advance period and late advance period is \( \rho = 1 \). The correlation coefficient of demand between early advance period and the advance period is \( \rho_x = 1 \). The correlation coefficient of demand between early advance period and the spot period is \( \rho_y = 0.7 \). The difference between the actual early advance period orders and the mean demand is \( x_1 - u_1 = 1000 \). The production lead time for the product is \( L = 4 \). The size of the number of people reaching the market during the advance period is \( X \sim N(10000, 1000^2) \). The size of the number of people reaching the market during the spot period is \( Y \sim N(8000, 1000^2) \). Delivery time for pre-sale products is \( t = [1, 2, 3, ..., 28, 29, 30] \). We also assume that the value-added of offline experience services is \( s_{off} = 5 \), and the increase in demand in the new retail model in the spot period is \( \tau = 1.25 \). Consumers’ preference for online channels is \( \beta = 0.6 \). The cost of value-added services per unit of product in the offline channel is \( c_0 = 150 \), and the minimum service value is \( l = -5 \).

From Figures 3 and 4: (1) Whether or not the reference price dependence is taken into account, the longer the waiting time for shipment, the lower the demand for the advance period and the lower the order quantity in this period. However, demand for the spot period increases as the waiting time for shipment becomes longer, and the order quantity in this period increases as well. When the shipment wait time is short, the order quantity remains the same for both the advance and spot periods. (2) The reference price dependence has a positive effect on order intake in the advance period and negatively affects order intake in the spot period.

Considering the reference price dependence, the seller should increase the order quantity during the advance period. Because consumers can buy at a lower price during the advance period and are willing to wait longer for a lower-priced product. The seller should reduce the order quantity during the spot period because consumers in the post period know that the product was sold at a lower price during the advance period, which creates a loss mentality that reduces the consumer’s purchasing utility, lowers market demand, and ultimately leads to fewer orders. In summary, when considering the effect of reference price dependence, the seller should increase the order quantity in the advance period, but should not be blind to the fact that the order quantity in the spot period will also increase as
a result of the increase in the advance period order quantity, but instead should appropriately reduce the order quantity in the spot period. This is similar to previous studies where price reference dependence can significantly bias sellers’ order quantities [52, 4]. Still, in these studies, reference price dependence has only one
effect on order quantities in some cases. Still, we find that under a two-phase pre-
sale strategy, reference price dependence has different effects on order quantities in
different periods. The seller should pay more attention to the impact of reference
price dependence.

Figure 5. Changes in the seller’s profit over the advance period
with acquisition and loss coefficients.

Figure 6. Changes in the seller’s profit over the spot period with
acquisition and loss coefficients.

From Figures 5 and 6: (1) As consumers’ sensitivity to acquisition and loss
increases, i.e., the less rational consumers are, there is a positive effect on the
seller’s advance period profit, and a negative effect on the spot period profit. (2)
When the acquisition and loss coefficients are both zero, i.e., when the consumer
is perfectly rational, the seller’s total profit is greater than when the consumer is finitely rational gain.

Because as consumers’ sensitivity to “acquisition” increases, the utility of purchasing during the presale period increases, and more consumers are willing to place orders in this period. In addition, when consumers’ loss aversion sensitivity increases, they are more willing to wait longer for a lower purchase price during the advance period. On the whole, consumer rationality is beneficial to the seller. This finding does not conflict with previous researches. This is because rational consumers enable the seller to predict consumer behavior better and develop operational strategies. In contrast, consumers with limited rationality can make it more difficult for sellers to operate [34]. Certain strategies that the sellers, or the public policymaker, can use to improve consumer rationality that is beneficial to the seller, and these strategies will be considered in future research.

![Figure 7. The impact of value-added offline service on dual-channel order quantities.](image)

From Figure 7, we can see the effect of value-added offline service on demand in the spot period: an increase in value-added offline services increases both online and offline demand, thus increasing the total order quantity. This is shown by the fact that when the value-added offline service is less than zero and gradually increases, the order quantity of online channels gradually decreases, but the order quantity of offline channels gradually increases.

This is because even if the value-added offline service is less than zero, the opening of offline channels will attract some consumers from online channels to offline channels, resulting in lower demand from online channels if the total demand in the spot period remains unchanged. When the value-added offline service is greater than zero and increases, consumers in the advance period gradually shift to the spot period, increasing the market demand for the spot period.
Figure 8. The impact of value-added offline services on dual-channel profits

From Figure 8, we can see the impact of value-added offline service on the profit of the spot period. Due to the influence of consumer channel preference and service cost factors, the offline channel profit does not increase steadily with the increase of value-added offline service, but instead, there is a spill-over effect to benefit the online channel. This is shown by the fact that as the value-added offline service increases, the profit in the online channel first decreases and then increases. When the value-added offline service is less than zero, some of the demand in the online channel shifts to the offline channel, and when the value-added offline service is greater than zero, demand in the advance period shifts to the spot period, and demand in the online channel increases. As value-added offline service increases, offline profits are volatile, as they are also influenced by channel preference and service cost factors. This is because increasing value-added offline service not only increases demand in the offline channel, but also raises the marginal cost per unit of service. As a result, the seller should take a comprehensive measure to determine the optimal value-added offline service.

These findings are similar to previous researches, where an increase in value-added offline services can increase demand and profitability [45, 25]. However, in the new retail context, an increase in offline service value-added, even if it increases total order quantity, is not always beneficial to the seller and may reduce profits. Therefore, the seller should weigh all factors to determine the optimal value-added offline services. The value-added offline services should not be increased indefinitely.

8. Conclusions. This paper proposes a combined new retail and pre-sales model in which online retailers explore offline channels to sell their products and use pre-sales strategies to reduce demand uncertainty while considering consumer experience and bounded consumer rationality. We study the ordering decisions of the seller in both the advance and spot periods, with and without reference price dependence.
Our use of modeling analysis and numerical analysis yields some valuable management insights that differ from previous literature. For example, (1) The seller tends to set the deposit too low in pre-sales, but in practice, the seller should set a higher deposit in pre-sales due to the reference dependence of consumer behavior. (2) We find that under a two-phase pre-sale strategy, reference price dependence has different effects on order quantities in different periods. The seller should pay more attention to the impact of reference price dependence. (3) On the whole, consumer rationality benefits the seller. The seller, or the public policymaker, can benefit new retail businesses by increasing consumer rationality. (4) In the new retail context, we find that an increase in offline service value-added, even if it increases total order quantity, is not always beneficial to the seller and may reduce profits. Therefore, the seller should weigh all factors to determine the optimal value-added offline services.

However, there are some future extensions to our paper. Firstly, new retail and pre-sale can be used in many other areas, such as energy and information. For example, Fleet Cor offers customers networked fueling services in the form of fuel cards\footnote{https://www.fleetcor.com/solutions/fuel/}. Second, we will consider different forms of uncertainty assessment and the stability of the model in the future study. Due to various uncertainties in reality, we consider parameter uncertainties in our future research. We will refer to the research on robust optimization [29, 28, 5] to study the decision problem under uncertainty. We can also refer to the research on some continuous optimization [48, 3, 2, 11] and data mining [49] to apply in predicting the order cancellation behavior, return behavior and order quantity of consumers. Third, in the future, there are very many use cases for new retail and pre-sale, such as in the food retail industry and the auto parts industry. Our work can be used not only in the industries mentioned above.

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Appendix

Appendix 1. Steps to update the demand $D^{(1)}_2 \rightarrow D^{(2)}_2$ in the period $t_0 \sim T_1$ when reference effects are not taken into account. We assume that the demand in period $0 \sim t_0$ is $D^{(1)}_1$, and $D^{(1)}_1 \sim N(\mu^{(1)}_1, (\delta^{(1)}_1)^2)$. $\mu^{(1)}_1 = \frac{e_{t_0}}{e^{t_0}} P(\eta < \eta_1) \overline{H}(\theta_1) \mu_x, \delta^{(1)}_1 = \frac{e_{t_0}}{e^{t_0}} P(\eta < \eta_1) \overline{H}(\theta_1) \delta_x$, that is $D^{(1)}_1 \sim \frac{e_{t_0}}{e^{t_0}} P(\eta < \eta_1) \overline{H}(\theta_1) N(\mu_x, \delta^{(1)}_x)$. The demand in period $t_0 \sim T_1$ is $D^{(1)}_2$, and $D^{(1)}_2 \sim N(\mu^{(1)}_2, (\delta^{(1)}_2)^2)$. $\mu^{(1)}_2 = (1 - \frac{e_{t_0}}{e^{t_0}}) P(\eta < \eta_1) \overline{H}(\theta_1) \mu_x, \delta^{(1)}_2 = (1 - \frac{e_{t_0}}{e^{t_0}}) P(\eta < \eta_1) \overline{H}(\theta_1) \delta_x$, that is $D^{(1)}_2 \sim (1 - \frac{e_{t_0}}{e^{t_0}}) P(\eta < \eta_1) \overline{H}(\theta_1) N(\mu_x, \delta^{(1)}_x)$. 

\footnote{https://www.fleetcor.com/solutions/fuel/}
The seller predicts demand of late advance period at moment $t_0$. The order quantity that has actually occurred in period $0 \sim t_0$ is $x_1^{(1)}$. We update the demand $D_2^{(1)}$ using $x_1^{(1)}$ according to Tang et al. (2004). The updated market demand is $D^{(1)_2}$, with a density function of $g_2(x)$ and a distribution function of $G_2(x), D^{(1)_2} \sim N(\mu^{(1), 2}, (\delta^{(1), 2})^2)$. $\mu^{(1), 2} = \mu^{(1)} + \text{corr}(D^{(1)_1}, D^{(1)_2})(x_1^{(1)} - \mu^{(1)})(1 - \frac{e^{-\lambda_0}}{\sigma_0})P(\eta < \eta_1)\Pi_\theta_x = \mu^{(1)} + \rho(s_1^{(1)} - \mu^{(1)})\frac{e^{-\lambda_0} - e^{-\lambda_0}}{\sigma_0}\delta^{(1)} = \delta^{(1)} \sqrt{1 - \rho^2}, D^{(1)_2} \sim N((1 - \frac{e^{-\lambda_0}}{\sigma_0})P(\eta < \eta_1)\Pi_\theta_x + \rho(s_1^{(1)} - \mu^{(1)})\frac{e^{-\lambda_0} - e^{-\lambda_0}}{\sigma_0}\delta^{(1)}), (1 - \rho^2)((1 - \frac{e^{-\lambda_0}}{\sigma_0})P(\eta < \eta_1)\Pi_\theta_x)^2).

**Appendix 2.** Steps to find the seller’s expected profit, the optimal late advance period order quantity, and the optimal profit in period $t_0 \sim T_1$ when reference effects are not taken into account.

In the $t_0 \sim T_1$ period, the seller determines the optimal order quantity by constructing a newsvendor model that maximizes the sales profit during that period. Profit on sales = sales revenue - wholesale cost + salvage value - shortage loss - inventory cost. The mean market demand over the period was $u_2^{(1)}$. The seller's expected sales quantity is $S(Q_2^{(1)}(x)) = E(\min(Q_2^{(1)}, D_2^{(1)})) = Q_2^{(1)} - \int_0^{Q_2^{(1)}} G_2(x)dx$, the seller's expected residual is $I(Q_2^{(1)}) = E(Q_2^{(1)} - x_2^{(1)})^+ = Q_2^{(1)} - S(Q_2^{(1)}), the seller's expected stock-out is $L(Q_2^{(1)}) = E(x_2^{(1)} - Q_2^{(1)})^+ = \mu^{(1)} - S(Q_2^{(1)})$, $c$ represents the cost of storage per unit of product per unit of time, the profit for the seller is as follows. $\Pi_2^{(1)} = \lambda p \min(Q_2^{(1)}, D_2^{(1)}) - w Q_2^{(1)} + s(Q_2^{(1)} - D_2^{(1)})^+ - g(D_2^{(1)} - Q_2^{(1)})^+ - g_2^{(1)} + (Q_2^{(1)} - D_2^{(1)})^+ (T_1 - t_0 - L)c).

Evaluating the expected value of the above equation yields the seller’s expected profit in period $t_0 \sim T_1$, which is $\Pi_2^{(1)} = \lambda p S(Q_2^{(1)}) - w Q_2^{(1)} + s(Q_2^{(1)} - S(Q_2^{(1)})) - g(\mu^{(1)} - S(Q_2^{(1)}))^+ - Q_2^{(1)}(T_1 - t_0 - L)c = -(\lambda p + g - s) \int_{Q_2^{(1)}} G_2(x)dx + (\lambda p + g - w - (T_1 - t_0 - L)c) Q_2^{(1)} - \mu^{(1)}).

The first-order derivative of the above equation is $\frac{\partial \Pi_2^{(1)}}{\partial Q_2^{(1)}} = -(\lambda p + g - s) G_2(Q_2^{(1)}) + (\lambda p + g - w - (T_1 - t_0 - L)c).

Since $\frac{\partial^2 \Pi_2^{(1)}}{\partial Q_2^{(1)}} = -(\lambda p + g - s) g_2(Q_2^{(1)}) < 0$, when $\frac{\partial \Pi_2^{(1)}}{\partial Q_2^{(1)}} = 0$, i.e., $G_2(Q_2^{(1)}) = \frac{\lambda p + g - w - (T_1 - t_0 - L)c}{\lambda p + g - s}$, there exists an optimal order quantity, $Q_2^{(1)}$, that makes the seller the most profitable in the period $t_0 \sim T_1$.

To meet market demand in the period $t_0 \sim T_1$, the optimal order quantity in the late advance period is $Q_2^{(1)*} = G_2^{-1}(\frac{\lambda p + g - w - (T_1 - t_0 - L)c}{\lambda p + g - s})$. At this point, the optimal profit for the seller is $\Pi_2^{(1)*} = -(\lambda p + g - s) \int_{Q_2^{(1)*}} G_2(x)dx + (\lambda p + g - w - (T_1 - t_0 - L)c) Q_2^{(1)*} - \mu^{(1)*}.

**Appendix 3.** Steps to update spot period demand $D_{on}^{(1)}, D_{off}^{(1)}$ using the actual order quantity $x_1^{(1)}$ that occurred in the period $0 \sim t_0$ when reference effects are not taken into account when $\max(U_{on}^{(1)}, U_{off}^{(1)})$, then $v_1 = p - \min(s_{off}, 0)$. When $\min(U_{on}^{(1)}, U_{off}^{(1)}) = 0$, then $v_2 = p - \min(s_{off}, 0)$. Consumers will buy goods in
the spot period only if \( v \geq v_1 \). The total demand in the spot period market is \( P(\eta > \eta_1)H(v_1)\mu_x + \overline{H}(v_1)\mu_y \).

Assume that \( s_{off} \geq 0 \). At this point, consumers whose valuation satisfies \( v_1 \leq v < v_2 \) will buy from offline channels, and the size of demand is \((P(\eta > \eta_1)X + \tau Y) \int_{v_2}^{\infty} h(v) dv\). For consumers whose valuation satisfies \( v \geq v_2 \), they are free to choose which channel to buy from, depending on their channel preference.

Suppose the consumer’s preference for the online channel is \( \beta \), then the preference for the offline channel is \((1 - \beta)\), and the size of demand for the offline channel is \( \beta(P(\eta > \eta_1)X + \tau Y) \int_{v_2}^{\infty} h(v) dv\), the size of demand for the online channel is \((1 - \beta)(P(\eta > \eta_1)X + \tau Y) \int_{v_2}^{\infty} h(v) dv\). The market segmentation of consumers in the spot period is shown in Figure 9.

![Figure 9. Market segmentation of consumers in the spot period.](image-url)

In summary, when \( s_{off} > 0 \), the total market demand for the offline channel is

\[
D_{off}^{(1)} = (P(\eta > \eta_1)X + \tau Y) \int_{v_2}^{\infty} h(v) dv + (1 - \beta) \int_{v_2}^{\infty} h(v) dv.
\]

The updated distribution of market demand is as follows.

\[
D_{off}^{(1)} = \beta(P(\eta > \eta_1)X + \tau Y) \int_{v_2}^{\infty} h(v) dv + (1 - \beta)(P(\eta > \eta_1)X + \tau Y) \int_{v_2}^{\infty} h(v) dv.
\]

The spot period is shown in Figure 9.
\[ D_{\text{off}}(\mu_{\text{off}}(1), \delta_{\text{off}}(1)^2) \] gives \( \Pi_{\text{off}}^2 \) and a distribution
function of \( G_{\text{off}}(y_2) \), and \( \mu_{\text{off}}(1) = (p(\eta > \eta_1)\mu_2 + \tau \mu_2)(H(v_2) - H(v_1) + (1 - \beta)\Pi(v_2)), \delta_{\text{off}}(1)^2 = (p(\eta > \eta_1)(H(v_2) - H(v_1) + (1 - \beta)\Pi(v_2))\delta_x(1)^2 + ((H(v_2) - H(v_1) + (1 - \beta)\Pi(v_2))\delta_{\text{off}}^2(1) + (1 - \beta)\Pi(v_2))\delta_{\text{off}}^2(1) - H(v_1) + (1 - \beta)\Pi(v_2))\delta_{\text{off}}^2(1). \]

### Appendix 4

Steps to find the seller’s expected profit, optimal order quantity, and optimal profit for the online channel in period \( T_1 \sim T_2 \) when reference effects are not taken into account.

In period \( T_1 \sim T_2 \) (i.e., the spot period), for the online channel, the mean market demand is \( \mu_{\text{on}}(1) \). Expected sale volume for the online channel is \( S(Q_{\text{on}}^{(1)}) = E(\min(Q_{\text{on}}^{(1)}, D_{\text{on}}^{(1)})) = Q_{\text{on}}^{(1)} - \int_{0}^{Q_{\text{on}}^{(1)}} G_{\text{on}}(y_1)dy_1 \). The seller’s expected residual is \( I(Q_{\text{on}}^{(1)}) = E(Q_{\text{on}}^{(1)} - x_{\text{on}}^{(1)})^+ = Q_{\text{on}}^{(1)} - S(Q_{\text{on}}^{(1)}) \). The seller’s expected stock-out is \( L(Q_{\text{on}}^{(1)}) = E(x_{\text{on}}^{(1)} - Q_{\text{on}}^{(1)})^+ = \mu_{\text{on}}(1) - S(Q_{\text{on}}^{(1)}) \).

Then the seller’s profit in the online channel is \( P_{\text{on}}^{(1)} = p \min(Q_{\text{on}}^{(1)}, D_{\text{on}}^{(1)}) - wQ_{\text{on}}^{(1)} + s(Q_{\text{on}}^{(1)} - D_{\text{on}}^{(1)})^+ - g(D_{\text{on}}^{(1)} - Q_{\text{on}}^{(1)})^+ \). The expected sale volume for the online channel in period when reference effects are not taken into account is \( \Pi_{\text{on}}^{(1)} = -(p + g - s + (T_2 - T_1)c)G_{\text{on}}(Q_{\text{on}}^{(1)}) + (p + g - w - (T_2 + T_1 - 2t_0 - 2L)c)Q_{\text{on}}^{(1)} - g\mu_{\text{on}}^{(1)} \).

The first-order derivative of the above equation yields \( \frac{\partial \Pi_{\text{on}}^{(1)}}{\partial Q_{\text{on}}^{(1)}} = -(p + g - s - (T_2 - T_1)c)g_{\text{on}}(Q_{\text{on}}^{(1)}) + (p + g - w - (T_2 + T_1 - 2t_0 - 2L)c) \), and the expected residual value is \( \frac{\partial^2 \Pi_{\text{on}}^{(1)}}{\partial Q_{\text{on}}^{(1)}} = -(p + g - s - (T_2 - T_1)c)g_{\text{on}}(Q_{\text{on}}^{(1)}) < 0 \), so \( \frac{\partial \Pi_{\text{on}}^{(1)}}{\partial Q_{\text{on}}^{(1)}} = 0 \), it follows that \( G_{\text{on}}(Q_{\text{on}}^{(1)}) = \frac{p + g - w - (T_2 + T_1 - 2t_0 - 2L)c}{p + g - s + (T_2 - T_1)c} \).

In order to meet the market demand in the online channel during the spot period, the optimal order quantity for the seller is \( \overline{Q}_{\text{on}}^{(1)} = G_{\text{on}}^{-1}(\frac{p + g - w - (T_2 + T_1 - 2t_0 - 2L)c}{p + g - s + (T_2 - T_1)c}) \).

### Appendix 5

Steps to find the seller’s expected profit, optimal order quantity, and optimal profit for the offline channel in period when reference effects are not taken into account.

In period \( T_1 \sim T_2 \) (i.e., the spot period), for the offline channel, the mean market demand is \( \mu_{\text{off}}(1) \). Expected sale volume for the offline channel is \( S(Q_{\text{off}}^{(1)}) = E(\min(Q_{\text{off}}^{(1)}, D_{\text{off}}^{(1)})) = Q_{\text{off}}^{(1)} - \int_{0}^{Q_{\text{off}}^{(1)}} G_{\text{off}}(y_2)dy_2 \). The seller’s expected residual is \( I(Q_{\text{off}}^{(1)}) = E(Q_{\text{off}}^{(1)} - x_{\text{off}}^{(1)})^+ = Q_{\text{off}}^{(1)} - S(Q_{\text{off}}^{(1)}) \).

The seller’s expected stock-out is \( L(Q_{\text{off}}^{(1)}) = E(x_{\text{off}}^{(1)} - Q_{\text{off}}^{(1)})^+ = \mu_{\text{off}}(1) - S(Q_{\text{off}}^{(1)}) \).

Then the seller’s profit in the offline channel is \( P_{\text{off}}^{(1)} = p \min(Q_{\text{off}}^{(1)}, D_{\text{off}}^{(1)}) - wQ_{\text{off}}^{(1)} + s(Q_{\text{off}}^{(1)} - D_{\text{off}}^{(1)})^+ - g(D_{\text{off}}^{(1)} - Q_{\text{off}}^{(1)})^+ \).

The expected value of the above equation gives \( \Pi_{\text{off}}^{(1)} = pS(Q_{\text{off}}^{(1)}) - wQ_{\text{off}}^{(1)} + s(Q_{\text{off}}^{(1)} - S(Q_{\text{off}}^{(1)}) - \frac{Q_{\text{off}}^{(1)} + (Q_{\text{off}}^{(1)} - D_{\text{off}}^{(1)})^+}{2}(T_2 - T_1)c - \frac{(s_{\text{off}} - 1)^2}{2}c_0 \). The expected value of the above equation gives \( \Pi_{\text{off}}^{(1)} = pS(Q_{\text{off}}^{(1)}) - wQ_{\text{off}}^{(1)} + s(Q_{\text{off}}^{(1)} - S(Q_{\text{off}}^{(1)}) - \frac{Q_{\text{off}}^{(1)} + (Q_{\text{off}}^{(1)} - D_{\text{off}}^{(1)})^+}{2}(T_2 - T_1)c - \frac{(s_{\text{off}} - 1)^2}{2}c_0 \).
Proof of Proposition 2.

Let $g\mu^{(1)} - S(Q^{(1)}_{off})^+$

$- Q^{(1)}_{off}(T_1 - t_0 - L)c - \frac{Q^{(1)}_{off} + Q^{(1)}_{off} - S(Q^{(1)}_{off})}{\frac{g\mu^{(1)} - S(Q^{(1)}_{off})}{2}}(T_2 - T_1)c - \frac{\{s_{off} - l\}^2}{2}c_0$

$= -(p + g - s + \frac{(T_2 - T_1)c}{2}) \int_{Q^{(1)}_{off}} G_{off}(y_2)dy_2 + (p + g - w - \frac{(T_2 + T_1 - 2L)c}{2})Q^{(1)}_{off}$

$- g\mu^{(1)} - \frac{\{s_{off} - l\}^2}{2}c_0$

In the same vein as above, meeting the market demand in the offline channel at the current sale stage is $G_{off}(Q_{off}^{(1)*}) = \frac{p + g - w - \frac{(T_2 + T_1 - 2L)c}{2}}{p + g - s + \frac{(T_2 - T_1)c}{2}}$. At this point, the optimal order quantity for the seller is $Q_{off}^{(1)*} = \frac{G_{off}^{-1}(p + g - w - \frac{(T_2 + T_1 - 2L)c}{2})}{p + g - s + \frac{(T_2 - T_1)c}{2}}$

In period $T_1 \sim T_2$ (i.e., the spot period), the optimal profit for the seller in the offline channel is $\Pi_{off}^{(1)*} = -(p + g - s + \frac{(T_2 - T_1)c}{2}) \int_{Q_{off}^{(1)*}} G_{off}(y_2)dy_2 + (p + g - w - \frac{(T_2 + T_1 - 2L)c}{2})Q_{off}^{(1)*} - g\mu^{(1)} - \frac{\{s_{off} - l\}^2}{2}c_0$

Appendix 6. The situation when reference effects are taken into account. This section is omitted because it is similar to the process when reference effects are not considered.

Proof of Proposition 1.

Given that $v^{(1)}_0 = \lambda p - \alpha^{(1)}$ and $v^{(2)}_0 = \lambda p - \alpha^{(2)} - r_1(p - \lambda p)$. Assuming $\alpha^{(2)} > \alpha^{(1)}$, then $v^{(1)}_0 \geq v^{(2)}_0$, we can see that $E(v, v^{(2)}_0) > E(v, v^{(1)}_0)$ and $H(v^{(1)}_0) \geq H(v^{(2)}_0)$. At this point, if $E(v, v^{(2)}_0) \leq \lambda p$, then $\alpha^{(2)*} > \alpha^{(1)*}$, the assumption holds. If $E(v, v^{(1)}_0) > \lambda p$, then $\alpha^{(1)*} = \alpha^{(2)*} = \lambda p$, the assumption holds. Proof over.

Proof of Proposition 2.

(1) According to $\eta_1 > \eta_2$, $\theta_1 < \theta_2$, we can know that $p(\eta < \eta_1) < p(\eta < \eta_2)$, $H(\theta_2) > H(\theta_1)$. In addition, to achieve comparability, we assume that the difference between the expected demand and its actual occurrence in period $0 \sim t_0$ is equal, with or without reference price effects, that is $x^{(2)}_1 - \mu^{(2)}_1 = x^{(1)}_1 - \mu^{(1)}_1$, then $\mu^{(2)}_2 > \mu^{(1)}_2, \sigma^{(2)}_2 > \sigma^{(1)}_2$. Since $F_2(Q^{(2)*}) = G_2(Q^{(1)*})$, we know that $Q^{(2)*} > Q^{(1)*}$, Proof over.

(2) Since $\partial Q^{(2)}_2/\partial r_1 > 0$, $\partial Q^{(2)}_2/\partial r_2 > 0$, $\partial Q^{(2)}_2/\partial r_1 > 0$, $\partial Q^{(2)}_2/\partial r_2 > 0$, it follows that $\partial Q^{(2)}_2/\partial r_1 > 0$, $\partial Q^{(2)}_2/\partial r_2 > 0$. Proof over.

(3) Given that $\Pi^{(2)*}_2 - \Pi^{(1)*}_2 \geq 0, (\lambda p + g - s) (\int_0^{Q^{(1)*}_2} G_2(x)dx - \int_0^{Q^{(2)*}_2} G_2(x)dx) + (\lambda p + g - w - (T_1 - t_0 - L)c)$

\[(Q^{(2)*}_2 - Q^{(1)*}_2) - g(\mu^{(2)*}_2 - \mu^{(1)*}_2) \geq 0,
\]

\[Q^{(2)*}_2 - Q^{(1)*}_2 \geq \frac{g(\mu^{(2)*}_2 - \mu^{(1)*}_2) + (\lambda p + g - s)(\int_0^{Q^{(2)*}_2} G_2(2x)dx - \int_0^{Q^{(1)*}_2} G_2(x)dx)}{(\lambda p + g - w - (T_1 - t_0 - L)c)}.
\]

So when the reference price effect has a positive effect on the seller’s profit in the late advance period. Proof over.

Proof of Proposition 3.

(1) It follows from the above that $x^{(1)}_1 + Q^{(1)*}_2 < x^{(2)}_1 + Q^{(2)*}_2$, making $x^{(1)}_1 + Q^{(1)*}_2 + \Delta_1 = x^{(2)}_1 + Q^{(2)*}_2$, $\Delta_1 \geq 0$. Given that $v^{(2)}_0 < v^{(1)}_0$, then $H(v^{(2)}_0) < H(v^{(1)}_0)$.
Based on actual business operations, the order cancellation rate is bound to be less than 50% [57], so \( H(v_0^{(2)}) < H(v_0^{(1)}) < 0.5 \).

\[
Q_a^{(2)*} - Q_a^{(1)*} = (1 - H(v_0^{(2)}))(x_1^{(1)} + Q_2^{(1)*} + \Delta_1) + H(v_0^{(2)}) \int_0^{Q_2^{(2)*}} F_2(x)dx - (1 - H(v_0^{(1)}))(x_1^{(1)} + Q_2^{(1)*}) - H(v_0^{(1)}) \int_0^{Q_2^{(1)*}} G_2(x)dx
\]

\[
= (1 - H(v_0^{(2)}))\Delta_1 + (H(v_0^{(1)}) - H(v_0^{(2)})(x_1^{(1)} + Q_2^{(1)*})) - [H(v_0^{(1)}) - H(v_0^{(2)})] \int_0^{Q_2^{(2)*}} G_2(x)dx - H(v_0^{(2)}) \int_0^{Q_2^{(2)*}} F_2(x)dx + \nabla_1 \text{ when } \int_0^{Q_2^{(2)*}} F_2(x)dx > \int_0^{Q_2^{(1)*}} G_2(x)dx > 0, \text{ given that } (x_1^{(1)} + Q_2^{(1)*}) > \int_0^{Q_2^{(1)*}} G_2(x)dx, \text{ then } Q_a^{(2)*} - Q_a^{(1)*} \geq 0. \text{ When } 0 < \int_0^{Q_2^{(2)*}} F_2(x)dx < \int_0^{Q_2^{(1)*}} G_2(x)dx, \text{ making } \int_0^{Q_2^{(1)*}} G_2(x)dx = \int_0^{Q_2^{(2)*}} F_2(x)dx + \nabla_1, \text{ it can be seen that } \nabla_1 < \Delta_1, Q_a^{(2)*} - Q_a^{(1)*} = (1 - H(v_0^{(2)}))\Delta_1 + (H(v_0^{(1)}) - H(v_0^{(2)}))(x_1^{(1)} + Q_2^{(1)*}) - \int_0^{Q_2^{(2)*}} F_2(x)dx - H(v_0^{(1)})\nabla_1, \text{ so } Q_a^{(2)*} - Q_a^{(1)*} > 0, \text{ holds.}
\]

(2) From \( \eta_1 = (1 - \lambda)p - (1 - \xi)\rho_\alpha, \eta_2 = (1 - \lambda)\rho + \xi_1(p - \lambda\rho) + r_2(p - \lambda\rho) - (1 - \xi)\rho\alpha \), we have

\[
\eta_1 < \eta_2, P(\eta > \eta_1) > P(\eta > \eta_2), \overline{H}(p - \min(s_{off}, 0)) > \overline{H}(p + r_2(p - \lambda\rho) - \min(s_{off}, 0)).
\]

To achieve comparability, we assume that the difference between the expected demand and its actual occurrence in period 0 \( \sim t_0 \) is equal, with or without reference price effects, that is \( x_1^{(2)} - \mu_1^{(2)} = x_1^{(1)} - \mu_1^{(1)} \), then \( \mu_x^{(2)} < \mu_x^{(1)}, \mu_y^{(2)} < \mu_y^{(1)}, \delta_x^{(2)} = \delta_x^{(1)}, \delta_y^{(2)} = \delta_y^{(1)} \), \( \mu_{on} < \mu_{on}, \delta_{on} < \delta_{on} \), so \( \overline{Q}_{on}^{(1)*} > \overline{Q}_{on}^{(2)*} \) holds.

Similarly, \( \overline{Q}_{off}^{(1)*} > \overline{Q}_{off}^{(2)*} \) holds. Proof over.

**Proof of Proposition 4.**

This proposition is easy to derive, so omit it.

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