Possible scenarios for soft and semi-hard components structure in central hadron-hadron collisions in the TeV region

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**Abstract**

Possible scenarios in $hh$ collisions in the TeV region are discussed in full phase space. It is shown that at such high energies one should expect strong KNO scaling violation and a $\ln(\sqrt{s})$ increase of the average charged multiplicity of the semi-hard component, resulting in a huge mini-jet production.

**Dedication**

This paper is dedicated to Peter Carruthers.

One of the authors (A.G.) had a long correspondence with him on the name of the distribution to the success of which in multi-particle dynamics Peter contributed so much, and which is at the basis of the present work. It can be found indeed in the literature under different names: in particular, the name “negative binomial” — currently used in our field — seemed too anonymous. The agreement with Peter was to start to call it “Pascal distribution”, in honour of Blaise Pascal, one of the scientists who used it first. The passing of Peter did not allow to solidify this proposal, which is now brought forward in his memory.
1 Introduction

The study of final particles multiplicity distributions (MDs) and related correlations structure in the new foreseen energy domain in the TeV region in hadron-hadron ($hh$) collisions is a challenging problem for multiparticle dynamics. Here the production of events with a huge number of final particles is indeed the most spectacular and fascinating although not yet fully understood phenomenon. The new fact is the occurrence of high parton density islands in regions where QCD parton evolution equations cannot be applied, and long range correlations among produced particles are expected to be quite large.

A sound theory of strong interactions cannot avoid to describe such complex high energy many-body system; at the same time to approach this problem is ancillary to the understanding of even more complex strongly interacting systems like proton-nucleus and heavy ions collisions. It should be reminded and stressed again and again that it is still a problem for QCD (in $hh$ collisions more than in others collisions due to the mentioned complexity of the reaction) how to extend the theory from the perturbative to the non-perturbative sector. Hadronization mechanism and more specifically how to calculate from first QCD principles multiplicity distributions and correlation structure of final particles states (the true observables) are here unanswered questions. In this region where standard QCD has shy predictions for a complex system like hadron-hadron scattering at very high c.m. energies one can rely only on models based on empirical observations on multiplicity distributions behaviour and related normalized factorial and cumulant moments both in full phase space, and in limited sectors of rapidity and transverse variables. Thus the first step of our programme is to examine critically what one learns on $hh$ collisions in full phase space from previous experimental and theoretical work starting from accelerator region results (in a subsequent paper we will extend our study to rapidity intervals).

Important hints are expected also to come from other reactions like $e^+e^-$ annihilation and deep inelastic scattering, where the simplicity of the projectile and/or of the reaction itself allowed already to isolate very interesting properties of the most elementary substructures (jets of given flavor [1]) at work in the interaction region.

The phenomenological work done in this area is impressive.

Many regularities (like the Pascal, or negative binomial, multiplicity distribution, Pa(NB)MD,\footnote{From now on the abbreviation 'Pa(NB)MD' will be used to mean: the Pascal (also known as negative binomial) multiplicity distribution.} behaviour and KNO scaling) have been analyzed in all classes of collisions since 1972: this effort reminds us of the precious and humble job done by the spectroscopists generation on atomic spectra in the pre-quantum mechanics era. However the general agreement is only on few statements. Nobody is questioning that particles are produced in jets, the most important QCD prediction in the field. In addition, observed events are considered either “soft” (events without mini-jets) or “semi-hard” (events with mini-jets), and the relevance of the latter is expected to increase with c.m. energy.\footnote{Here we rest with the standard definition of mini-jets as proposed by the UA1 Collaboration: groups of particles having total transverse energy larger than 5 GeV.}

Limiting to central collisions only one can say that particles are produced not independently one from the other (Feynman scaling violation). Two-particle correlations are probably dominant at lower c.m. energies; this fact favors here hierarchical correlation structure and supports the ”old” Pa(NB)MD regularity: a ”single” Pa(NB)MD provides indeed a satisfactory description of final particles MDs in the accelerator region up to
ISR center of mass energies but it doesn’t work above such energy due to the occurrence of a shoulder structure in the tail of the distribution, whose relevance is growing with energy. This fact can be interpreted here as the onset of semi-hard events. This remark follows from the excellent fits shown by the UA5 Collaboration \[2\] on final particle MDs in non-single-diffractive events in $hh$ collisions above 200 GeV in terms of the weighted superposition of soft and semi-hard events, the weight being given by the fraction of soft events and the MD of each component being of Pascal (negative binomial) type. The lesson one learns is that "old" Pa(NB)MD regularity is violated at c.m. energies higher than ISR when applied to the full sample of events but it continues to be valid when applied separately at the two individual components (soft and semi-hard) level. This result should be compared with the observation that, in $e^+e^-$ annihilation, the Pa(NB)MD regularity is also violated in the full sample of events (again due to shoulder effect), but it is restored at the two- and multi-jet component level \[3\]; in addition, residuals analysis on the two-jet sample of events indicates that the regularity in $e^+e^-$ should work even better at single quark-jet level \[4\] (the building block of multiparticle dynamics in the reaction): accordingly, the observed final particle multiplicity distribution and the related correlation structure are expected to be simply the result of the weighted superposition of the mentioned elementary substructures at different levels of investigation.

Finally, KNO scaling is probably a correct asymptotic (how far?) prediction and its early occurrence at lower energies should be considered—as suggested by Léon Van Hove—an inessential transient regime or, as we prefer, limited in $hh$ collisions to the soft component only. This still open question notwithstanding, we believe KNO scaling to be a useful tool in order to describe extreme scenarios of the production process; in addition it should be remembered that Pa(NB)MD for average charged multiplicity $\bar{n}$ larger than $k$ (with $k > 1$) can be written as a gamma distribution in the scaled variable $z = n/\bar{n}$:

$$\bar{n} P_n^{\text{Pa(NB)}} = \frac{k^k}{\Gamma(k)} z^{k-1} e^{-kz} \tag{1}$$

i.e., it has KNO form. KNO scaling behaviour occurs when $k$ becomes an energy independent parameter. Remember that the parameter $k$ is linked to the dispersion $D$ by

$$k^{-1} = \frac{D^2 - \bar{n}}{\bar{n}^2} \tag{2}$$

(We notice in passing that, when the distribution is not a Pa(NB)MD, eq. (2) can be taken as a definition for the parameter $k$ in terms of dispersion and average multiplicity).

The correlation structure of multiparticle production in $hh$ collisions can be investigated also by means of other important observables like the factorial moments of order $n$ of the multiplicity distributions, $F_n$, and the corresponding cumulant moments, $K_n$, which are particularly sensitive to the tail of the distributions and currently used in order to study intermittent behaviour in very small rapidity intervals. Of interest is also the ratio $H_n$ of the above mentioned observables, i.e. $K_n/F_n$, when plotted as a function of its order, integer $n$. It shows indeed a characteristic oscillatory behaviour already seen in $e^+e^-$ annihilation and there explained \[4\] as due to the same cause which originated the shoulder effect in the multiplicity distributions in that reaction, i.e., the weighted superposition of events of different topology. It should be pointed out that observed oscillatory behaviour of $H_n$ versus $n$ in $hh$ collisions can also be explained in terms of the
same cause which allowed to understand the shoulder effect in final particle MDs, i.e.,
the superposition of soft and semi-hard events\footnote{5}. These facts, if confirmed, would point
in the direction of what is the real goal of multiparticle dynamics, an integrated description
of correlation structure and multiplicity distributions in collisions belonging to the
same class, whereas the superposition mechanism of elementary subprocesses seems to be
common to all classes of collisions. It is also striking that most elementary substructures
in \(hh\) collisions as well as in \(e^+e^-\) annihilation, i.e., soft and semi-hard components and
two- and multi-jet contributions respectively, are well described by Pa(NB)MDs. In this
sense Pascal (negative binomial) regularity is still valid and should be considered even
more fundamental than originally thought in the field.

The aim of this paper is to explore charged particle multiplicity distributions and
Corresponding correlation structure in hadron-hadron collisions in the TeV region in full
phase space within the just sketched general phenomenological framework by using the
above mentioned collective variables. Accordingly, we propose to describe MDs in full
phase space in the new horizon in terms of the weighted superposition of the MDs of soft
and semi-hard events, with each component assumed to be of Pa(NB)MD type. With
these two simplifications the full problem is reduced to determine the energy dependence
of the Pa(NB)MD parameters, i.e., of the average charged multiplicity, \(\bar{n}\), and of parameter
\(k\) for the soft and semi-hard components.

It appears that in this essential framework at least three scenarios are possible:

1. KNO scaling limit is achieved in the TeV region for both components.

2. KNO scaling is valid for the soft component, but it is heavily violated for the semi-
   hard component in the TeV region.

3. A QCD-inspired scenario can be obtained by assuming that the form of perturbative
   QCD predictions for the width of the MD can be used also in the non-perturbative
   sector.

The first two scenarios should be considered quite extreme possibilities. They deter-
mine in a certain sense reasonable bounds to the variation of the Pa(NB)MD parameters.
The third one turns out to give Pa(NB)MD parameters behaviour intermediate between
the first two.

We are aware of the fact that the assumptions of the proposed description are not
unique. They are dictated to us by our experience in the field and by our personal taste.
This consideration notwithstanding we believe that this research line should be pursued
in order to give some hints to future experimental and theoretical work, and to explore
whether new phenomena are predicted by the present approach at higher energies. Our
findings should be confronted with results of other possible phenomenological approaches
(which we are demanding) leaving to experiments, when available, to decide among dif-
ferent realistic alternatives.

Our aim is to illustrate the Pa(NB)MD composition technique and to apply it to
a simple problem. Of course one relevant fraction of events is expected to come also
from diffraction which affects both the soft and semi-hard components. Accordingly,
one can study in this context substructures generated by diffraction in the two previous
components by using again the composition in terms of weighted Pa(NB)MDs of diffractive
and non-diffractive events. The use of a Pa(NB)MD for describing diffractive events leads
indeed to results not far from those of a modified gamma distribution \[6\]. We do not intend to explore for the moment this new perspective, which we postpone to future work, but which we are ready to use when a correct residual analysis on MDs will show indications of new substructures. What is studied in this paper is indeed the structure of non-single-diffractive events only.

2 \(P_n\) vs \(n\) behaviour and \(H_q\) vs \(q\) oscillations in full phase space in the GeV and in the TeV energy domains

It has been shown in the accelerator region that final \(n\) charged multiplicity distributions in full phase space, \(P_n\), are initially narrower than a Poisson distribution \((1/k_{\text{Pa(NB)MD}}\) parameter is negative\) and the distribution is indeed a binomial distribution, produced particles are very few and like to stay far apart one from the other, particles anti-correlations seem here to be favored. Then at approximately 30 GeV c.m. energy the observed MD becomes Poissonian: \(1/k_{\text{Pa(NB)MD}}\) parameter is zero in this region, particles are produced independently one from the other (as predicted by the naive multiperipheral model). Above 30 GeV c.m. energies up to ISR energies the distribution is a true \(\text{Pa(NB)MD}\): \(1/k\) parameter becomes positive, the number of produced particles is larger, two particle correlation are dominant as requested by hierarchical correlations structure. A single \(\text{Pa(NB)MD}\) due to the flexibility of its \(1/k\) parameter is describing quite well all the above mentioned experimental facts, which are apparently dominated by soft events. A shoulder structure in the tail starts then to appear at higher energies as shown by the UA5 Collaboration \[7\]: a single \(\text{Pa(NB)MD}\) cannot describe the new effect (one talks of Pascal (negative binomial) regularity violation), which can be interpreted as the onset of semi-hard events (events with mini-jets). Notice that Pythia Monte Carlo calculations give at present unsatisfactory results in this area \[8\], although a careful optimization of the parameters can of course improve this trend.

Accordingly, as reminded in the introduction, it has been proposed to describe the observed shoulder structure as the weighted superposition of soft events (events without mini-jets) and semi-hard events (events with mini-jets), the weight being the fraction of soft events and the MD of each component being of \(\text{Pa(NB)MD}\) type. The resulting master equation for \(P_n\) turns out to be the following

\[
P_n(\alpha_{\text{soft}}; \bar{n}_{\text{soft}}, k_{\text{soft}}; \bar{n}_{\text{semi-hard}}, k_{\text{semi-hard}}) =
\alpha_{\text{soft}}(\sqrt{s})P_n^{\text{Pa(NB)}}(\bar{n}_{\text{soft}}(\sqrt{s}), k_{\text{soft}}(\sqrt{s})) +
(1 - \alpha_{\text{soft}}(\sqrt{s}))P_n^{\text{Pa(NB)}}(\bar{n}_{\text{semi-hard}}(\sqrt{s}), k_{\text{semi-hard}}(\sqrt{s}))
\]

(3)

whose physical content is self-explanatory. Notice that we do not consider interference terms because the classification of events as soft or semi-hard is based on the final hadronic state, not on the underlying partonic event.

Excellent fits have been obtained for \(P_n\) \[3\], an example of which is shown in Figure \[3a\]: the plot of residuals (\(b\) and \(c\) in the same figure) shows how the fit gets better when using two \(\text{Pa(NB)MDs}\) (although, it should be mentioned, the fit is not completely satisfactory, which can indicate the presence of further substructures. The use of residuals should also be taken \textit{cum grano salis} in this case, because a minimum chi-square test has been used to find the \(\text{Pa(NB)MDs}\) parameters: although the run test might still be asymptotically
Figure 1: a) Multiplicity distribution at c.m. energy 546 GeV (UA5 data), with the two components of eq. (3), corresponding residual analysis of b) a fit with one single Pa(NB)MD and of c) the fit with eq. (3); d) ratio of moments $H_q$, calculated from eq. (3) after truncation.

-independent of the chi-square test, which does not use information on the sign and sequence of the deviations, its distribution is in fact not known, thus the pattern of the residual is only indicative.\footnote{We thank S. Krasznovszky for discussion on this point.}

In the fits, the soft component fraction decreases from 93 percent at 200 GeV to 72 percent at 900 GeV, $k_{\text{soft}}$ is taken constant as the energy increases (as requested by an early KNO scaling behaviour) and its best fit numerical value is 7, whereas $k_{\text{semi-hard}}$ decreases from 79 at 200 GeV (it describes a nearly Poissonian behaviour) to 13 at 900 GeV, indicating strong KNO scaling violation. The average charged particle multiplicity is approximately two times larger for the semi-hard component than for the soft component (as observed by UA1 Collaboration \footnote{We thank S. Krasznovszky for discussion on this point.}). It is interesting (and remarkable) that $H_q$ vs. $q$ obtained by data on multiplicity distributions oscillates in this region and that the oscillations are quite well described by the $K_q$ over $F_q$ ratio calculated by using equation (3), as shown in Figure 1d.

That’s in summary all we know in the GeV region on our variables behaviour. Coming to the TeV region which we want to explore we will proceed by extrapolating Equation (3) in the new energy domain. Although highly simplified our approach still requires to determine the energy dependence of Pa(NB)MD parameters $\bar{n}$ and $k$ for the soft and semi-hard components, as well as of soft component fraction $\alpha_{\text{soft}}$, which we propose to do in the following.

2.1 $\bar{n}_{\text{soft}}$ and $\bar{n}_{\text{semi-hard}}$ energy dependence.

For $\bar{n}_{\text{soft}}$ it is assumed that its fitted values in the multiplicity distributions in the GeV region can be extrapolated to higher energy domains, i.e.,

$$
\bar{n}_{\text{soft}}(\sqrt{s}) = -5.54 + 4.72 \ln(\sqrt{s})
$$

(dashed line in Figure 3), with $\sqrt{s}$ in GeV.

\footnote{We thank S. Krasznovszky for discussion on this point.}
Figure 2: Average multiplicity $\bar{n}$ vs. c.m. energy. The figures shows experimental data (filled triangles) from ISR and SPS colliders, the UA5 analysis with two Pa(NB)MDs of SPS data (circles: soft component; squares: semi-hard component), together with our extrapolations (lines: dotted: total distribution; dashed: soft component; short-dashed: semi-hard component, eq. (5.A); dot-dashed: semi-hard component, eq. (5.B)).

Figure 3: Energy dependence of the superposition parameter $\alpha_{\text{soft}}$ (fraction of soft events) in the two cases of a linear (solid line, eq. (5.A)) and quadratic (dashed line, eq. (5.B)) dependence of the average multiplicity of the semi-hard component on c.m. energy. The triangles are the result of the UA5 analysis [2].
Assuming UA1 analysis on mini-jets to be approximately valid also at higher energies one has for $\bar{n}_{\text{semi-hard}}$ (short-dashed line in Figure 2):

$$\bar{n}_{\text{semi-hard}}(\sqrt{s}) \approx 2\bar{n}_{\text{soft}}(\sqrt{s}) \quad (5.A)$$

Alternatively one can postulate that $\bar{n}_{\text{semi-hard}}(\sqrt{s})$ is increasing more rapidly with energy and correct Eq. (5.A) by adding on its right side a $\ln^2(\sqrt{s})$ term, accordingly one obtains

$$\bar{n}_{\text{semi-hard}}(\sqrt{s}) \approx 2\bar{n}_{\text{soft}}(\sqrt{s}) + c' \ln^2(\sqrt{s}) \quad (5.B)$$

(dash-dotted line in Figure 2). This simple correction might take into account observed deviations from Eq. (5.A) behaviour at 900 GeV and estimate from the fit at the same time parameter $c'$ in Eq. (5.B), which turns out to be $\approx 0.1$.

Finally $\bar{n}_{\text{total}}$ of the resulting multiplicity distribution in agreement with the common wisdom is given by a quadratic fit (dotted line in Figure 2):

$$\bar{n}_{\text{total}} = 3.01 - 0.474 \ln(\sqrt{s}) + 0.754 \ln^2(\sqrt{s}) \quad (6)$$

Being now in this approach

$$\bar{n}_{\text{total}} = \alpha_{\text{soft}} \bar{n}_{\text{soft}} + (1 - \alpha_{\text{soft}}) \bar{n}_{\text{semi-hard}} \quad (7)$$

the energy dependence of $\alpha_{\text{soft}}$ can easily be determined. It turns out to be in the two cases described by Eq.s. (5.A) and (5.B)

$$\alpha_{\text{soft}} = 2 - \bar{n}_{\text{total}}/\bar{n}_{\text{soft}} \quad (8.A)$$

and

$$\alpha_{\text{soft}} = 1 + [\bar{n}_{\text{soft}} - \bar{n}_{\text{total}}]/[\bar{n}_{\text{soft}} + c' \ln^2(\sqrt{s})] \quad (8.B)$$

respectively.

In Figure 3 the soft events fraction is shown to be quickly decreasing with energy; the general trend is to invert the situation observed in the GeV region where soft events fraction was dominant: semi-hard events fraction is here increasing from 11 (0.2 TeV) to 75 (20 TeV) percent of the reaction. Significant changes are introduced by the presence of the $\ln^2(\sqrt{s})$ term in eq. (5.B) at higher energies: the fraction of semi-hard events is in this case 10 percent at 0.2 TeV but increases only to 62 percent at 20 TeV. In the previous case, the semi-hard component becomes larger than the soft one at 3–4 TeV, but now the $\ln^2(\sqrt{s})$ term induces this change at approximately 7 TeV. This difference will be visible especially in scenarios 1 and 3.

2.2 $k_{\text{soft}}$ and $k_{\text{semi-hard}}$ energy dependence

$k_{\text{soft}}$ was found to be constant in the GeV region by the UA5 collaboration; since in addition $\bar{n}_{\text{soft}}$ is growing with energy, see Eq. (4), to assume $k_{\text{soft}}$ constant in the new energy domain implies

$$D^2_{\text{soft}}/\bar{n}_{\text{soft}}^2 \approx \text{constant} \approx 0.143 \quad (9)$$

which corresponds to say that KNO scaling behaviour is valid for the soft component in the TeV region. We stay with this assumption on $k_{\text{soft}}$. It should be pointed out that $k_{\text{soft}}$ is not affected by the introduction of the $\ln^2\sqrt{s}$ term in eq. (5.B).

As anticipated in the introduction, the discussion on the behaviour of $k_{\text{semi-hard}}$ opens at least three possible scenarios which are discussed in the following.
3 The three scenarios

3.1 Scenario 1

KNO scaling holds in the TeV region also for the semi-hard component, i.e., we assume that $k_{\text{semi-hard}}$ is decreasing until 900 GeV (its value is 13 at this c.m. energy) and then it remains constant in the new region. Being $\bar{n}_{\text{semi-hard}}$ even larger than $\bar{n}_{\text{soft}}$, $D_{\text{semi-hard}}^2/\bar{n}_{\text{semi-hard}}^2 \approx 0.09$ throughout all the explored energy range. See Figure 4a,b.

The effect of a quadratic growth of $\bar{n}_{\text{semi-hard}}$ with energy, eq. (5.B), in this scenario is to increase the value of $1/k_{\text{total}}$ (curves B in Figure 4a,b), via the change in $\alpha_{\text{soft}}$, eq. (8.B). This fact is consistent with our assumption of the superposition mechanism.

Expected multiplicity distributions at 1.8 TeV and 14 TeV c.m. energies and corresponding $H_q$ vs. $q$ oscillations fitted by using the composition of the soft and semi-hard substructures are shown in figure 5. The energies of 1.8 and 14 TeV have been chosen because they are respectively the c.m. energy of Tevatron and the expected c.m. energy of LHC. It is interesting to remark that the shapes of the multiplicity distributions of the two components are similar at all energies, but the heights of the corresponding maxima are reversed in going from the lowest to the top energy. In addition oscillations seem to be stretched in shape as the energy increases, indicating their tendency to be highly reduced at higher energies (notice that we show here $H_q$ moments calculated without truncating the MD, because the truncation depends solely on the size of the experimental sample; thus we only show the moments computed for the total MD, as the two components are each of Pa(NB)MD type and therefore, individually considered, show no oscillations).

A rather large change in the shape of the MD is introduced when we consider the quadratic term for $\bar{n}_{\text{semi-hard}}$, eq. (5.B): even at 20 TeV the maximum of the semi-hard component has not yet become larger than that of the soft component. On the other hand, the shoulder has become more evident, due to the higher average multiplicity that was introduced for the semi-hard component and the smaller resulting value of $\alpha_{\text{soft}}$; at a c.m. energy of 14 TeV the area of the maximum of the total MD is rather flat and shows a small dip (see fig. 5a). Correspondingly the oscillations of the $H_q$ moments become approximately 4 times larger in amplitude, but don’t vary much in period, and the first minimum is not shifted.

3.2 Scenario 2.

The main assumption is that the general trend observed in the GeV region for the total distribution continues to be valid in the TeV region, i.e., $D_{\text{total}}^2/\bar{n}_{\text{total}}^2$ is logarithmically growing, suggesting strong KNO scaling behaviour violation:

$$k_{\text{total}}^{-1} = -0.082 + 0.0512 \ln \sqrt{s}$$

See Figure 6c,d.

The effect of a quadratic growth of $\bar{n}_{\text{semi-hard}}$, eq. (5.B), in this scenario is to decrease the value of $1/k_{\text{semi-hard}}$ (curves labelled B in the figure). This fact is again consistent with $1/k_{\text{total}}$ growing with energy.

In Figure 6 are shown the multiplicity distributions and the corresponding $H_q$ vs. $q$ oscillations of this scenario.

The first remark is that the shapes of the two components are totally different, a wide queue in the semi-hard component is visible in the high multiplicity channels. The
Figure 4: The parameter $1/k$ (figures a,c,e) and the KNO scaling parameter $D^2/\bar{n}^2$ (figures b,d,f) are plotted for the scenarios described in the text (from top to bottom: a,b: scenario 1; c,d: scenario 2; e,f: scenario 3). The figures shows experimental data (filled triangles) from ISR and SPS colliders, the UA5 analysis with two Pa(NB)MDs of SPS data (circles: soft component; squares: semi-hard component), together with our extrapolations (lines: dotted: total distribution; dashed: soft component; short-dashed: semi-hard component).
heights at the maxima favor initially the soft component and then the heights of the two components are almost equal. It is striking that $H_q$ vs. $q$ oscillations disappear as the c.m. energy increases, indicating that single Pa(NB)MD behaviour is the dominant feature in this energy domain.

In this case the effect of a quadratic growth of $\bar{n}_{semi-hard}$ is much less noticeable than in scenario 1 in the shape of the MD: the shoulder is only slightly more visible and the amplitude of the $H_q$ oscillations is moderately larger than in the case of linear $\bar{n}_{semi-hard}$.

### 3.3 Scenario 3.

This is the QCD inspired scenario. QCD predicts, at the leading order\[^4\] for the parameter $k$ of the multipicity distribution \[^11\]

\[ k^{-1} = a + b\sqrt{\alpha_{\text{strong}}} \tag{11} \]

where

\[ \alpha_{\text{strong}} \approx 1/\ln(Q/Q_0) \tag{12} \]

and $Q$, $Q_0$ are the initial virtuality and the cut-off of the parton shower. QCD predicts (for $e^+e^-$ annihilation) $a \approx 0.33$ and $b \approx -0.9$, but in order to apply the above equation to our problem we leave the $a$, $b$ and $Q_0$ parameters free.

Since the constants can be determined by a least square fit to the values found for $k$ at c.m. energies 200 GeV, 500 GeV and 900 GeV one obtains, assuming Eq. (11) to control $k_{semi-hard}$ component behaviour,

\[ k_{semi-hard}^{-1} = 0.38 - 0.42/\sqrt{\ln(\sqrt{s}/10)} \tag{13} \]

The result is presented in Figure 4\[^e,f\]. Notice how $1/k_{total}$ is indeed lower than in scenario 2 but higher that in scenario 1.

The effect of a quadratic growth of $\bar{n}_{semi-hard}$, eq. (5.B), in this scenario is once again to increase the value of $1/k_{total}$ similarly to what happens in scenario 1.

The new situation for $P_n$ vs $n$ and $H_q$ vs $q$ is summarized in Fig. 7. It is interesting to remark that this scenario gives predictions on both variables which are intermediate between the two previous extreme ones of scenarios 1 and 2. In Figure 7\[^a\] one sees in fact that the tail of $P_n$ vs $n$ is increasing with c.m. energy but high multiplicity channels are not populated as in Figure 6, although they are larger than those in Figure 5. Mini-jets production is intermediate between the two scenarios. Accordingly, in Figure 7\[^b\] $H_q$ vs $q$ oscillations are decreasing with c.m. energies but not as much as in scenario 2, indicating that the absence of oscillations is here an asymptotic prediction and coincides with expectations of $H_q$ oscillations description in terms of a single Pa(NB)MD. It is in fact quite clear that in the limit $\alpha_{soft} \to 0$ mini-jets production is dominant with respect to soft events and the corresponding multiplicity distribution is described almost fully by a single Pa(NB)MD.

The effect of a quadratic growth of $\bar{n}_{semi-hard}$ is quite noticeable here, as the shoulder structure, almost disappeared above 10 TeV with linear $\bar{n}_{semi-hard}$ is now well visible up

\[^4\text{in view of the approximations involved, more sophisticated calculations \[^10\] are not useful in this framework.}\]
Figure 5: a) Multiplicity distributions for scenario 1 at c.m. energies of 1.8 (TEVATRON energy) and 14 (LHC expected energy) TeV; the first row refers to solution A for the average multiplicity in semi-hard events, the second row to solution B (solid line: total distribution; dashed line: soft component; short-dashed line: semi-hard component).

b) filled circles: $H_q$ vs $q$ oscillations (without truncation of the MD) corresponding to the total distribution of part a; the line is drawn to guide the eye; again the first row refers to solution A for the average multiplicity in semi-hard events, the second row to solution B.
Figure 6: Same content as Figure 5 but for scenario 2.
Figure 7: Same content as Figure 6 but for scenario 3.
to 20 TeV, because the semi-hard component has not yet become dominant. Correspondingly, $H_q$ oscillations are approximately 3 times as large as in the linear case.

It appears that probably it is a too “black and white” attitude to assume for $k_{\text{semi-hard}}$ strong KNO scaling behaviour (scenario 1) or strong KNO scaling violation (scenario 2). These too extreme choices were done of course on purpose in order to fix the boundary conditions to our exploration: the real world might very well be (as illustrated by scenario 3) between the above two.

In conclusion the reaction is controlled by the ratio of soft to semi-hard events. This ratio favors soft events production up to ISR energies and a single Pa(NB)MD describes here quite well all experimental facts, above such energies semi-hard events start to play a more important role and are revealed by two effects (the onset of shoulder structure in the multiplicity distributions and $H_q$ vs $q$ oscillations in related correlations); both effects can be cured by using the weighted composition of two Pa(NB)MDs, one for the soft part of the reaction, and the second for its semi-hard part. In the TeV region semi-hard events become dominant, they obscure soft events production and following our assumptions asymptotically ($\alpha_{\text{soft}} \to 0$) a single Pa(NB)MD is describing again quite well multiplicity distributions and corresponding correlation structure.

4 Clan analysis of the soft and semi-hard component substructures

The decision taken by UA5 Collaboration to describe soft and semi-hard events component substructures in full phase space in the GeV region in terms of Pa(NB)MDs and here extended to the TeV region allows to interpret all the above mentioned results in the framework of clan analysis [12, 13], that is we express and comment now our previous results in terms of the two parameters:

$$\bar{N} = k \ln (1 + \bar{n}/k)$$  \hspace{1cm} $$\bar{n}_c = \bar{n}/\bar{N}$$ \hspace{1cm} (14)

See Figure 8. The above definition is valid for a single Pa(NB)MD. Notice that for the total multiplicity distribution, clans cannot be defined: the fact, as shown in Figures 5 to 7, that the total MD presents oscillations in the ratio of moments, $H_q$, implies, via the theorems proven in [14], that the total MD is not an infinitely divisible distribution (IDD): indeed only for IDDs it is possible to generalize the definition of clans from that of eq. (14) to that of intermediate sources produced according to a Poisson distribution. This is the reason why we will discuss the behaviour of clan parameters only for each component separately.

The soft component substructure is the same in all three scenarios. The average number of clans is here a slowly increasing variable with c.m. energy and the average number of particles per clan goes from 2.6 at 1.8 TeV to 3.1 at 20 TeV, indicating that phase space is homogeneously filled by independent almost equal size clans as requested by a KNO scaling regime. One can talk in this context pictorially of independent equally populated sources whose number is a slowly increasing function of available c.m. energy.

In scenario 1, which assumes KNO scaling behaviour also for the semi-hard component, the average number of particles per clan is approximately the same as that seen in the soft component substructure, whereas the average number of clans is two times larger in the semi-hard component than in the soft one. This finding is consistent with the assumption of equation (1.4).
Figure 8: Clan parameters $\tilde{N}$ (figures a,c,e) and $\tilde{n}_c$ (figures b,d,f) are plotted for the scenarios described in the text (from top to bottom: a,b: scenario 1; c,d: scenario 2; e,f: scenario 3). The figures show experimental data (filled triangles) from ISR and SPS colliders, the UA5 analysis with two Pa(NB)MDs of SPS data (circles: soft component; squares: semi-hard component), together with our extrapolations (lines: dotted: total distribution; dashed: soft component; short-dashed: semi-hard component).
Scenario 2 shows a dramatic increasing with energy of the $D^2_{\text{semi-hard}}/\tilde{n}_{\text{semi-hard}}^2$ ratio as requested by strong KNO scaling violation. Accordingly the average number of clans is a quickly decreasing function of energy and then it becomes a very slowly decreasing quantity. This behaviour should be confronted with that of the average number of particles per clan which at 5 TeV is almost two times larger than at 900 GeV and becomes three times larger at 20 TeV. Notice that this huge cascading phenomenon is associated with a limited number of independent intermediate sources and this is striking. The two scenarios are indeed —as already mentioned— quite extreme. The highly ordered and homogeneous structure of phase space in scenario 1 becomes here highly inhomogeneous favoring huge branching production in each source to be compared with the fully independent production of the sources (clans). Available c.m. energy goes more in particle production within a clan than in clan production, contrary to what was found in scenario 1.

Finally scenario 3 which it should be remembered is a QCD inspired scenario. KNO scaling violation for the semi-hard component although effective is not as strong as in scenario 2. The average number of clans is 18.2 at 5 TeV (about the same size seen at 900 GeV) and 17.5 at 20 TeV, suggesting an almost energy independent production of the average number of clans over a large fraction of the TeV region; in addition the average number of particles per clan is growing with energy but not as much as in scenario 2, suggesting a moderate average branching consistent with particle production in mini-jets.

In Figure 8 $\bar{N}$ and $\bar{n}_c$ are plotted for the individual substructures of the collision in the three scenarios as functions of the c.m. energy.

It is interesting to remark here a linear growth of $\bar{N}$ with the maximum allowed rapidity for the soft component and for the semi-hard component of scenario 1 to be contrasted with an almost parabolic decrease of the average number of clans for the semi-hard component in scenarios 2 and 3 with respect to the same variable. It is also remarkable that the average number of clans is more rapidly decreasing in scenario 2 than in scenario 3, indicating the occurrence of more branching in each clan. In addition in scenario 3 the starting of a very slowly decreasing region for $\bar{N}$ is already clearly visible at 10 TeV and expected to develop over a large sector of the TeV region.

The effect of a quadratic growth of $\bar{n}_{\text{semi-hard}}$, eq. (5.B), is visible only in scenario 2, being negligible in scenarios 1 and 3; obviously the effect appears only in the semi-hard component, and not in the soft one. These facts are a consequence of how we constructed our scenarios: indeed in scenarios 1 and 3 the term $\ln^2(\sqrt{s})$ appears only in $\bar{n}$ inside the logarithm (see eq. (14)), whereas in scenario 2 it appears also in $k_{\text{semi-hard}}$. The average number of clans is increased by about 20 percent, while the average number of particles per clan is decreased by approximately 10 percent.

It should be pointed out that the mild energy dependence of the average number of clans in full phase space (f.p.s.) over a given energy interval for a (soft and/or semi-hard events) substructure of Pa(NB)MD type has important consequences:

1) the probability to produce empty events (events without charged particles) is also mildly energy dependent in view of the equation

$$P_0 = \exp(-\bar{N})$$ (15)

2) this fact establishes a upper bound to the production of clans at any energy $\bar{N}(\delta y) \leq \bar{N}_{\text{f.p.s.}}$ and a lower bound to the production of events without particles in any rapidity interval at a given c.m. energy $P_0(\text{f.p.s.}) \leq P_0(\delta y)$. 

5 New experimental data

After we finished our work, the paper by Matinyan and Walker \[15\] appeared, providing data from the E735 experiment at Fermilab. The Authors of that paper find that MD data in f.p.s. up to 1800 GeV c.m. energy can be described in terms of events belonging to two independent classes, only one of which obeys KNO scaling. The threshold for the appearance of the second class of events is estimated approximately between 100 and 200 GeV. These results agree with the starting point of the present analysis. The Authors of \[15\] interpret the two classes as events generated by one collision and by more than one collision, respectively. Alternatively, we prefer to interpret the two classes as soft and semi-hard events. It should be remarked that in the data presented by the E735 experiment the MD are systematically wider than the corresponding ones of the UA5 Collaboration. This consideration notwithstanding, we decided to compare the new data at 1.8 TeV c.m. energy to our predictions, presented in the previous sections. As can be seen in Fig. 9, just by inspection, scenario 2 with option A seems to be favored. We conclude that according to our description one should expect strong KNO scaling violation in the MD and logarithmic growth with energy of the average charged multiplicity of the semi-hard component. It is interesting to remark that what we considered an extreme situation turns out to be less extreme from an experimental point of view: it is clear indeed that a huge mini-jet production will be the main characteristic of the new energy domain. Assuming that observed deviations of E735 results from our predictions of scenario 2 in fig. 9 will be confirmed, they imply that our $k_{\text{semi-hard}}$ parameter decreases more rapidly than in our scenario. This fact has important consequences since the mentioned deviations occur mainly in the tail region of the distribution, where the multiplicity is higher. Accordingly, the integrated two-particle correlation, $\int C_{2,\text{semi-hard}}(\eta_1, \eta_2) d\eta_1 d\eta_2 = \bar{n}_{\text{semi-hard}}^2 / k_{\text{semi-hard}}$, are much larger. Therefore one should expect here a production of more densely populated mini-jets characterized by a higher internal two-particle correlation structure. Whether this is the onset of a new component to be added to the previous ones or not will be decided by future experiments at LHC.

6 Summary

Possible scenarios of multiparticle production in hadron-hadron collisions in the TeV region have been discussed. It has been shown that the most spectacular facts are here the occurrence of two classes of events and in this framework the dominance of semi-hard events with respect to the soft ones; this last result should be confronted with the behaviour of the two components in the GeV region where just the opposite occurs, i.e. the soft component is dominant. Assuming that soft component events multiplicity distributions behave according to KNO scaling expectations, two extreme situations for the semi-hard component structure of the multiplicity distributions, i.e. an effective KNO scaling regime and a strong KNO scaling violation regime, have been compared to a QCD inspired set of predictions. Essential ingredient of the analysis have been to think to the final charged particle multiplicity distributions at various energies in terms of the weighted superposition of the two above mentioned components, the weight being the fraction of soft events. In addition following the success of the fits in the GeV region each component

\[5\] A more detailed discussion on the experimental results of the E735 Collaboration and their relevance for the study of MDs in the TeV region is postponed to a forthcoming letter.
Figure 9: Comparison of our predictions on MDs at 1.8 TeV c.m. energy (solid lines) with the recent data published by the E735 Collaboration (triangles) for each scenario and each option.
has been assumed to be of Pa(NB)MD type. Scenario 1 is apparently excluded by E735 data at 1.8 TeV c.m. energy and scenario 2, when compared with the same set of data, turns out to be less extreme than previously thought suggesting that the main feature in the new region, in our framework, is a huge mini-jet production together with a possible production of a new species of mini-jets. The QCD inspired scenario amazingly gives predictions which are intermediate between the other two.

Clan structure analysis when applied to the identified substructures of the reaction in full phase space in the TeV region reveals unexpected and interesting properties which might be relevant for studying empty events production and bounds to rapidity gaps.

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