Black holes and neutron stars in the generalized tensor-vector-scalar theory

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(Received; Published)

Bekenstein’s Tensor-Vector-Scalar (TeVeS) theory has had considerable success as a relativistic theory of Modified Newtonian Dynamics (MoND). However, recent work suggests that the dynamics of the theory are fundamentally flawed and numerous authors have subsequently begun to consider a generalization of TeVeS where the vector field is given by an Einstein-Æther action. Herein, I develop strong-field solutions of the generalized TeVeS theory, in particular exploring neutron stars as well as neutral and charged black holes. I find that the solutions are identical to the neutron star and black hole solutions of the original TeVeS theory, given a mapping between the parameters of the two theories, and hence provide constraints on these values of the coupling constants. I discuss the consequences of these results in detail including the stability of such spacetimes as well as generalizations to more complicated geometries.

I. INTRODUCTION

Modified Newtonian Dynamics (MoND) has enjoyed significant success as a phenomenological model of gravity without the requirement for dark matter (for a review see [2]). However, the discovery of a complete relativistic theory generalizing MoND has proven extremely difficult. Recently, Bekenstein’s Tensor-Vector-Scalar (TeVeS) theory [3] has received considerable attention; in the weak acceleration limit TeVeS has been shown to reproduce MoND, and in the Newtonian limit the parametrized post-Newtonian coefficients, \( \beta \) and \( \gamma \), are consistent within limits imposed by solar system experiments (for a comprehensive review of TeVeS see [3]).

Despite its success, there exists mounting evidence that TeVeS suffers from dynamical problems. Firstly, Seifert [4] showed that the Schwarzschild-TeVeS solution [4] is unstable to linear perturbations for experimentally and phenomenologically valid values of the various coupling parameters. By analysing a broad variety of dynamical situations, Contaldi et al. [7] then showed that the vector field is prone to the formation of caustics analogously to Einstein-Æther theories where the vector field is given by a Maxwellian action [8]. Finally, Sagi [9] has shown that the vector field is constrained by the cosmological value of the scalar field in such a way that it prevents the scalar field from evolving and also forces it to be negative, thereby inducing superluminal propagation of scalar waves. Contaldi et al. [7] and Skordis [10] have provided a generalization of TeVeS, whereby the vector field action is not given by the Maxwellian form of TeVeS, but a more general Einstein-Æther form. In this way the vector field action is the “...most general diffeomorphism invariant action, which is quadratic in derivatives and consistent with the \( A^2 = -1 \) constraint” [7], whereas the tensor and scalar field actions are identical to those of TeVeS. This type of generalization to the vector field action is motivated from Einstein-Æther theories, where it was shown to stabilize dynamical problems. Indeed, this generalization of TeVeS has been shown to resolve the caustic formation problem present in the original TeVeS theory [7].

Calculations of various quantities in generalized TeVeS are still in their infancy. Skordis [10] presented the cosmological equations for the theory and showed that they are identical with the TeVeS equations up to a rescaling of Hubble’s constant. Sagi [9] has looked at the PPN parameters within this theory and found them to not conflict with solar system experiments for suitable values of the various parameters of the theory. I continue investigations by exploring strong-field solutions, finding the spherically symmetric, static solutions of the field equations for neutron stars, neutral and charged black holes (sections III and IV). Interestingly, I find all these solutions to be the same as for the original TeVeS theory given a simple substitution of the vector field coupling parameters. This implies that the work of Giannios [6] for neutral black holes in TeVeS, Sagi and Bekenstein [11] for charged black holes and Lasky et al. [12] for neutron stars is also correct for the generalized TeVeS theory, given the aforementioned substitution of parameters. Spherically symmetric neutron stars and black holes are therefore observationally indistinguishable in TeVeS and generalized TeVeS. In section V I discuss in detail further implications of this work, including generalizations to more complex spacetimes and stability arguments.

II. GENERALIZED TEVES FIELD EQUATIONS

Generalized TeVeS, as with the original TeVeS theory [3], is built upon three gravitational fields; an Einstein tensor field, \( g_{\mu\nu} \), a normalized timelike vector field, \( A^\mu \), and a dynamical scalar field, \( \varphi \). These three objects
combine in the following way to give the physical metric;
\[ \tilde{g}_{\mu \nu} = e^{-2\varphi} (g_{\mu \nu} + A_\mu A_\nu) - e^{2\varphi} A_\mu A_\nu, \] (1)
which is the metric measured by clocks and rods. Throughout this article, quantities with and without tildes are measured in the physical and Einstein frames respectively. The original and generalized versions of the theory differ in the vector field action. Bekenstein’s version had a Maxwellian action, and subsequently one vector field coupling constant, $K$. The generalized version has an Einstein-Æther vector field action with four associated coupling constants, $K$, $K_+$, $K_2$ and $K_4^1$. Bekenstein’s original TeVeS theory is recovered by letting $K_+ = K_2 = K_4 = 0$ and $K = K$.

The modified Einstein field equations can be expressed analogously to TeVeS as
\[ G_{\mu \nu} = 8\pi G \left( \tilde{T}_{\mu \nu} + (1 - e^{-4\varphi}) A^\alpha \tilde{T}_{\alpha (\mu} A_{\nu)} + \tau_{\mu \nu} \right) + \Theta_{\mu \nu}. \] (2)
Here, $G_{\mu \nu}$ is the Einstein tensor associated with the Einstein frame and $\tilde{T}_{\mu \nu}$ is the physical stress-energy tensor. The tensor, $\tau_{\mu \nu}$, has the same definition as that in the original theory
\[ \tau_{\mu \nu} := \frac{\mu}{kG} \left[ \nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{2} g^{\alpha \beta} \nabla_\alpha \varphi \nabla_\beta \varphi g_{\mu \nu} - A^\alpha \nabla_\alpha \varphi \right. 
\left. \times \left( A_{(\mu} \nabla_\nu \varphi \right) - \frac{1}{2} A^\beta \nabla_\beta \varphi g_{\mu \nu} \right] - \frac{F(\mu)}{2k^2 G} g_{\mu \nu}, \] (3)
where $k$ is the scalar field coupling constant, $\ell$ is a positive scalar of dimension length, $\mu$ is a function associated with the MoND acceleration scale, and $F(\mu)$ is a function that is not predicted by the theory. Only the strong-field limit of the theory is considered in this article, whereby an excellent approximation is $\mu = 1$ (for more details see [3, 8, 7, 11]). Moreover, in this case one finds that $F$ diverges logarithmically, but is exactly cancelled out in the field equations implying one is free to ignore this function [7]. It is worth noting that $F$ only diverges in the strong-field limit considered herein, implying this divergence is not a general concern for the theory.

As mentioned, the extra degrees of freedom attributed to the generalized version of TeVeS appear in the vector-field action, which manifests itself partly in the $\Theta_{\mu \nu}$ term of equation (2) (for details of these derivations see [7, 10]). For clarity in the expressions, $\Theta_{\mu \nu}$ can be expressed as a linear sum of terms associated with each coupling parameter, plus one term associated with the Lagrange multiplier, $\lambda$:
\[ \Theta_{\mu \nu} := \Theta^K_{\mu \nu} + \Theta^K_{\mu \nu} + \Theta^K_{2 \mu \nu} + \Theta^K_{4 \mu \nu} + \Theta^\lambda_{\mu \nu}, \] (4)
where
\[ \Theta^K_{\mu \nu} := K \left( F_{\alpha \mu} F^\alpha_{\nu} - \frac{4}{g_{\mu \nu}} F_{\alpha \beta} F^{\alpha \beta} \right), \] (5)
\[ \Theta^K_{\mu \nu} := K_+ \left[ S_{\mu \alpha} S^\alpha_{\nu} - \frac{4}{g_{\mu \nu}} S_{\alpha \beta} S^{\alpha \beta} \right. 
\left. + \nabla_\alpha \left( A^\alpha S_{\mu \nu} - S^\alpha_{(\mu} A_{\nu)} \right) \right], \] (6)
\[ \Theta^K_{2 \mu \nu} := K_2 \left[ g_{\mu \nu} \nabla_\alpha \left( A^\alpha \nabla_\beta A^\beta \right) - A_{\mu (\nabla_\nu} (\nabla_\alpha A^\alpha) \right. 
\left. - \frac{1}{2} g_{\mu \nu} \nabla_\alpha A^\alpha \nabla_\beta A^\beta \right], \] (7)
\[ \Theta^K_{4 \mu \nu} := K_4 \left[ \dot{A}_\mu \dot{A}_\nu + \dot{A}_\alpha A_{\mu (\nabla_\nu} A^\alpha \right. 
\left. - \nabla_\alpha \left( \dot{A}^\alpha A_\mu A_\nu \right) - \frac{1}{2} g_{\mu \nu} \dot{A}_\alpha A^\alpha \right], \] (8)
\[ \Theta^\lambda_{\mu \nu} := - \lambda A_\mu A_\nu. \] (9)
Here, $F_{\mu \nu} := \nabla_{[\mu} A_{\nu]}$, $S := \nabla (\nabla A_{\mu})$ and $\dot{A}_\mu := A^\alpha \nabla_\alpha A_{\mu}$.
Additional to the modified Einstein equations, one has the vector field equation given by
\[ K \nabla_\alpha F^{\alpha \mu} + K_+ \nabla_\alpha S^\alpha_{\mu} + K_2 \nabla_\mu (\nabla_\alpha A^\alpha) + \lambda A^\mu \]
\[ + K_4 \left[ \nabla_\alpha \left( \dot{A}^\mu A^\alpha \right) - A^\alpha \nabla_\mu A_\alpha \right] + \frac{8\pi \mu}{k} A^\alpha \nabla_\alpha g^{\mu \beta} \nabla_\beta \varphi \]
\[ = 8\pi G (1 - e^{-4\varphi}) g^{\mu \alpha} \tilde{T}_{\alpha \beta} A^\beta, \] (10)
which again reduces to the original TeVeS vector field equation when $K_+ = K_2 = K_4 = 0$ and $K = K$. Finally, the scalar field equation is
\[ \nabla_\beta \left[ \mu \left( g^{\alpha \beta} - A^\alpha A^\beta \right) \nabla_\alpha \varphi \right. \]
\[ = kG \left( g^{\alpha \beta} + (1 - e^{-4\varphi}) A^\alpha A^\beta \right) \tilde{T}_{\alpha \beta}, \] (11)
which is equivalent to that of the original TeVeS theory.

III. NEUTRON STARS AND NEUTRAL BLACK HOLES

Giannios [6] first considered static, spherically symmetric vacuum solutions of the original TeVeS field equations. He found two branches of solutions; one where the vector field has only a non-zero timelike component, and one where the vector field has both non-zero radial and temporal components. Lasky et al. [12] then considered spherically symmetric neutron star solutions of the original TeVeS field equations where the vector field only has a non-zero temporal component. The black hole solution of Giannios [6] can be recovered from the neutron star solutions of Lasky et al. [12] by simply letting the matter variables vanish throughout the spacetime. I therefore

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1 I follow the notation of Sagi [3], which is different to that of Skordis [10] who denotes the four parameters by $K_B$, $K_+$, $K_0$ and $K_A$. These are related as follows; $K \equiv K_B \equiv (c_1 - c_3)/2$, $K_+ \equiv K_+ \equiv (c_1 + c_3)/2$, $K_2 \equiv K_0 \equiv c_2$ and $K_4 \equiv K_A \equiv -c_4$, where the $c_i$ are those of the Einstein-Æther theory (see Jacobson [11]).
begin here by deriving the neutron star solutions and sub-
sequently the black hole solutions. Moreover, throughout
this section I consider a vector field which points in the
temporal direction, for which Bekenstein [34] has shown
support for the general case of static spacetimes (for a
discussion of generalizing this work to a more general
vector field ansatz see section 3).

A spherically symmetric, static spacetime can be ex-
pressed in the Einstein frame as
\[ g_{\alpha\beta}dx^\alpha dx^\beta = -e^{\nu(r)}dt^2 + e^{\xi(r)}dr^2 + r^2d\Omega^2, \]

where \( d\Omega^2 := d\theta^2 + \sin^2\theta d\phi \). As mentioned, the vector
field is chosen such that is has only a non-zero temporal
component. Normalization of this implies
\[ A^\mu = \delta^\mu_\nu e^{-\nu/2}, \]

further implying that the physical line element is
\[ \tilde{g}_{\alpha\beta}dx^\alpha dx^\beta = -e^{\nu' + 2\nu}dt^2 + e^{-2\nu} \left( e^{\xi}dr^2 + r^2d\Omega^2 \right), \]

where symmetries also imply \( \varphi = \varphi(r) \).

Stress-energy tensors in TeVeS and its generalization
are defined in terms of the physical frame. Therefore, a
perfect fluid takes the form
\[ T_{\mu\nu} = \left( \hat{\rho} + \hat{P} \right) \tilde{u}_\mu \tilde{u}_\nu + \hat{P} \tilde{g}_{\mu\nu}, \]

where \( \tilde{u}_\mu \), \( \hat{\rho} \) and \( \hat{P} \) are respectively the four-velocity,
energy-density and pressure of the fluid as measured in
the physical frame. Conservation of the stress-energy
tensor is given by \( \nabla_\alpha T^\alpha_\mu = 0 \), where \( \nabla_\mu \)
is the unique metric connection associated with the physical metric.
The radial component of this equation implies the pressu-
re gradient is expressed as
\[ -\hat{P}' = \left( \frac{\nu}{2} + \varphi' \right) \left( \hat{\rho} + \hat{P} \right), \]

where a prime denotes differentiation with respect to the
radial coordinate.

The scalar field equation (11) can be expressed in terms
of the metric coefficients, and once integrated to give
\[ \varphi' = \frac{kGM_\varphi}{4\pi r^2} e^{(\xi - \nu)/2}, \]

as \( \varphi \) is the unique component provides an equation for the
Lagrange multiplier, \( \lambda \). Therefore, this is essentially a periphery equation as the
Lagrange multiplier is used in the modified field equa-
tions in the \( \Theta^\lambda_\nu \) term of equation (9). The only remain-
ing equations are the modified Einstein field equations
(20). After much work, the \( tt, rr \) and \( \theta\theta \) components can
be respectively shown to be
\[ r\varphi' + e^{\xi} - 1 = 8\pi G\rho e^{-\varphi} - \frac{\nu'}{2} \left( K + K_+ - K_4 \right) \]
\[ \times \left( \nu'^2 - \nu' \varphi' + 2\nu'' + \frac{4\nu'}{r} \right) + \frac{4\pi r^2}{k} \varphi'^2, \]

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where \( \varphi' \) also implies \( \xi = \xi(r) \).

A striking similarity between this system of equations
and the equivalent system in the original TeVeS theory
presented in Lasky et al. [12], their equations (21-23), is
now observed. In fact, the system of equations presented
for the original TeVeS theory is exactly the same, where
\[ K + K_+ - K_4 = \mathcal{K}, \]

in the above equations. That is, given the neutron star
solution of the original TeVeS theory, one derives the
solution of generalized TeVeS by making the substitu-
tion (22). The spacetime of neutron stars in both theo-
ries is therefore identical. Moreover, allowing the matter
terms to vanish in equations (16-21), and therefore re-
covering the equations governing the generalized TeVeS
Schwarzschild spacetime, one can show that the structure of
the spacetime is exactly equivalent to that presented in
Giannios [4] for the original TeVeS theory. To show this,
one must first perform a coordinate transformation given by
\( r = r(R) \) such that \( r = e^{(\xi - \nu)/2} \) to put the system
in the same set of coordinates used by Giannios. Then,
allowing the matter terms to vanish, the system of equa-
tions becomes exactly equivalent to equations (28-30) of
Giannios [4], given the expression in (22). That is, the
structure of black hole spacetimes, under the symmetry
assumptions posed hitherto, is exactly the same as that
of the original TeVeS theory.

There is no need to further derive solutions of these sys-
tems of equations for both neutron stars and black holes,
as the above result implies the results of Lasky et al. [12]
and Giannios [4] hold for the generalized TeVeS theory,
providing equation (22) is utilized. One observes from
equations (16-21) that the individual coupling constants
do not appear independently of one another, but rather
neutron stars and black holes can only be used to con-
strain the combination of parameters, \( K + K_+ - K_4 \). As
with Lasky et al. [12], we can use observations of neutron star masses to constrain these coupling constants. In that paper, we gave a conservative estimate of $K \lesssim 1$ based on the existence of neutron stars with masses of at least $\sim 1.5M_\odot$. Therefore, using the same conservative estimate, the combination of the vector coupling constants is constrained to $K + K_+ - K_4 \lesssim 1$.

In some ways the relationship between the two TeVeS theories given in (22) is to be expected. As was discussed in section II, one recovers the original TeVeS theory by setting $K_+ = K_2 = K_4 = 0$ and $K = K$, which is an obvious subset of solutions offered by the above equation. However, (22) does not imply that this has to be the case, but rather any combination of $K, K_+ and K_4$ may hold such that (22) is true. Moreover, these spacetimes are completely independent of the parameter $K_2$. I discuss the consequences of these result in considerably more detail in section IV.

### IV. CHARGED BLACK HOLES

Sagi and Bekenstein [11] considered spherically symmetric, charged black holes in the original TeVeS theory, where the vector field is again aligned with the temporal direction. In this section I reproduce those calculations in the context of the generalized TeVeS theory.

The ansatz and the symmetries applied herein imply the metric and vector field are given by equations (12) and (13) respectively. The stress-energy tensor is now taken to be that of an Einstein-Maxwell field as measured in the physical frame;

$$\tilde{T}_{\mu\nu} = \frac{1}{4\pi} \left( \tilde{F}_{\alpha\mu} \tilde{F}_{\alpha\nu} - \frac{1}{4} g_{\mu\nu} \tilde{F}_{\alpha\beta} \tilde{F}^{\alpha\beta} \right),$$  \hspace{1cm} (23)

where $\tilde{F}_{\mu\nu}$ is the electromagnetic Faraday tensor in the physical frame. Conservation of stress-energy then leads to the Maxwell equations, $\nabla_\alpha F^{\mu\alpha} = 0$. The symmetries of the problem imply the only non-zero contribution to the Faraday tensor is the $\tilde{F}_{rt} = -\tilde{F}_{tr}$ component [11]. Moreover, only the temporal component of Maxwell’s equations is non-zero, which is once integrated to give

$$\tilde{F}_{rt} = \frac{Q}{r^2} e^{2\varphi + (\nu + \zeta)/2}.$$ \hspace{1cm} (24)

Here, $\tilde{Q}$ is a constant of integration which is the charge of the black hole. As with section II, the only non-zero contribution from the vector field equation (11) is from the temporal component, which gives the Lagrange multiplier expressed as a function of the metric coefficient. Evaluating the modified Einstein equations [3] and substituting the Lagrange multiplier back through gives the $tt, rr$ and $\theta\theta$ components respectively as

$$r\zeta' + e^\zeta - 1 = \frac{G\tilde{Q}^2}{k r^2} e^{\zeta + 2\varphi} + \frac{r^2}{4} (K + K_+ - K_4) \times \left( \frac{\nu'' - \nu'\nu'}{2} + 2
u' + 4
u
nu' k \right) + \frac{4\pi r^2}{k} \varphi'^2,$$ \hspace{1cm} (25)

$$rv' + e^\zeta + 1 = -\frac{G\tilde{Q}^2}{r^2} e^{\zeta + 2\varphi} - (K + K_+ - K_4) \frac{r^2}{8} \nu'^2$$

$$+ \frac{4\pi r^2}{k} \varphi'^2.$$ \hspace{1cm} (26)

Moreover, the scalar field equation (11) can be shown to reduce to

$$\left[ \varphi' r^2 e^{(\nu - \zeta)/2} \right]' = \frac{kG\tilde{Q}^2}{4\pi r^2} e^{2\varphi + (\nu + \zeta)/2}. \hspace{1cm} (28)$$

Sagi and Bekenstein [11] derived the same set of equations in the original TeVeS theory, although they used isotropic coordinates as opposed to Schwarzschild coordinates. Performing the same coordinate transformation as section II, i.e. $r = r(R)$ such that $r = e^{\varphi(R)/2} R$ where the Einstein metric has component $g_{RR} = e^{\varphi(R)}$, and also using equation (22), one finds that equations (25-28) given above are identical to equations (49-52) of Sagi and Bekenstein [11]. That is, given the charged black hole solutions of Sagi and Bekenstein [11], one can derive the solution of generalized TeVeS simply making the substitution (22), implying the spacetimes in the two theories are identical.

### V. IMPLICATIONS AND GENERALIZATIONS

Herein I have shown that solutions of the field equations within Bekenstein’s original TeVeS framework for spherically symmetric neutron stars [13], neutral black holes [9] and charged black holes [11] are exactly the same solutions for the generalized TeVeS theory [2, 10], given the substitution (22). An immediate question one must ask is: how much can these results be generalized? That is, if more complex solutions of the original TeVeS field equations are found, will the same solution of the generalized TeVeS equations exist given the substitution (22)? The answer to this question is a simple “no”. In fact, a quick calculation reveals that this is not even the case in more complex spherically symmetric spacetimes. For example, consider the same metric ansatz utilized in this paper, and allow the vector field to have a non-zero radial and temporal component. One can then show that the divergence of the vector field is non-zero, i.e. $\nabla_\alpha A^\alpha \neq 0$, and consequently that the $K_2$ vector field
coupling now has a non-zero contribution to the modified Einstein equations. Therefore, the simple mapping given in (22) can no longer hold between the original TeVeS theory and the generalized version. Moreover, one can immediately see that when time dependence or rotation is included into the spacetime, the $K$ term will generally have non-trivial contributions due to the non-zero divergence of the vector field, and subsequently solutions of the original TeVeS theory will differ markedly from those of the generalized TeVeS theory.

Implications for observations of neutron stars and black holes given the above information is abundantly clear. Indeed, this article implies that spherically symmetric neutron stars and black holes in the generalized TeVeS theory, where the vector field is temporally aligned, are indistinguishable from those of the original TeVeS theory. Meanwhile, the above discussion suggests that observations of rotating black holes and neutron stars will differ between the theories, although when slow rotation approximations are valid, one would expect the observations to not differ significantly.

The relation provided in (22) is not unique to this article. In fact, Skordis [10] showed that the combination $K + K_+ - K_4$ plays the same role in cosmological perturbations of the generalized TeVeS theory as $K$ in the original theory. Moreover, he showed that to keep the energy density of cosmological perturbations positive, the constant must satisfy $0 < K + K_+ - K_4 < 2$, which is consistent with phenomenological constraints placed of $0 < K < 2$ in the original TeVeS theory [11, 12]. This is further consistent with Einstein-Æther theories, where this combination of constants (commonly denoted $c_{14} \equiv c_1 + c_4$) must also satisfy $c_{14} < 2$ such that Newton’s constant is positive.

An interesting question regarding the stability of the generalized TeVeS theory now arises. Indeed, the first blow was dealt to the stability of the original TeVeS theory when Seifert [5] found that the Schwarzschild-TeVeS solution [6] is unstable. As the structure of the spacetime in the generalized theory is the same, does this necessarily imply that the black holes are also unstable? It is possible that this is not the case for the following reason; instabilities are a consequence of dynamical perturbations of the spacetime. Therefore, while the overall structure of the background spacetime is the same for the original and generalized TeVeS theories, the field equations differ due to the extra contributions of the vector field action, and therefore the dynamics of the perturbations will also differ. Whilst full calculations of this are still required, one can draw on experience from Einstein-Æther theory. There it has been shown that the vector field develops caustic singularities as it falls into the potential well of a gravitational field when the vector action is Maxwellian [8]. For this reason, generalizations of Maxwellian vector fields are generally considered in modern Einstein-Æther literature. Indeed this was the original motivation for generalizing TeVeS in such a way, although I emphasize the need for these arguments to be verified in the generalized TeVeS theory by way of rigorous calculations.

Whilst it is true that general perturbations will behave differently due to the different field equations of the theory, a certain subset of perturbations will behave in the same manner. For example, Sotani [14] studied perturbations of neutron star solutions in the original TeVeS theory by assuming a Cowling approximation. Here, the perturbation equations are simplified considerably by only dealing with perturbations of the fluid variables, and not the background spacetime (i.e. the metric, vector field and scalar field remain fixed). As the fluid equations in the physical frame are identical for the original and generalized TeVeS theories, the results of Sotani [14] also hold true in generalized TeVeS, where again the mapping between the parameters is given by (22).

In summary, in this article the equations governing spherically symmetric, static spacetimes representing neutron stars, neutral and charged black holes in the generalized TeVeS theory have been presented. These turn out to be equivalent to those of Bekenstein’s original TeVeS theory [3] given by Lasky et al. [12], Giannios [6] and Sagi and Bekenstein [11], where the coupling constants of the two theories are related by (22). Therefore, the solutions of those equations and hence the structure of the spacetimes presented by those authors is the same as for the generalized theory, and indeed these objects are observationally indistinguishable in this regime. I have used this to show that the combination of vector coupling constants can be constrained to $K + K_+ - K_4 \lesssim 1$ based on conservative neutron star observations. Moreover, whilst problems with the original TeVeS theory were initiated by studies of spherically symmetric black holes, it is not expected that this will be the case for the generalized theory, given that perturbations and dynamics of such spacetimes will behave differently.

I thank Costas Kokkotas, Dimitrios Giannios, Bronwen Thomas and Trixi Willburger for valuable discussions, and also the referee for valuable comments regarding the manuscript. This work was supported by the Alexander von Humboldt Foundation by way of a Postdoctoral Research Fellowship and the Transregio 7 “Gravitational Wave Astronomy,” financed by the Deutsche Forschungsgemeinschaft DFG (German Research Foundation).

[1] M. Milgrom, Astrophys. J. 270, 365 (1983).
[2] R. H. Sanders and S. S. McGaugh, Ann. Rev. Astron. Astrophys. 40, 263 (2002).
[3] J. D. Bekenstein, Phys. Rev. D 70, 083509 (2004).
[4] C. Skordis, Class. Q. Grav. 26, 143001 (2009).
[5] M. D. Seifert, Phys. Rev. D 76, 064002 (2007).
[6] D. Giannios, Phys. Rev. D 71, 103511 (2005).
[7] C. R. Contaldi, T. Wiseman, and B. Withers, Phys. Rev. D 78, 044034 (2008).
[8] T. Jacobson and D. Mattingly, Phys. Rev. D 64, 024028 (2001).
[9] E. Sagi, Phys. Rev. D 80, 044032 (2009).
[10] C. Skordis, Phys. Rev. D 77, 123502 (2008).
[11] E. Sagi and J. D. Bekenstein, Phys. Rev. D 77, 024010 (2008).
[12] P. D. Lasky, H. Sotani, and D. Giannios, Phys. Rev. D 78, 104019 (2008).
[13] T. Jacobson (2008), arXiv:0801.1547.
[14] H. Sotani, Phys. Rev. D 79, 064033 (2009).