Decoherence in Two Bose-Einstein Condensates

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In this paper, decoherence in a system consisting of two Bose-Einstein condensates is investigated analytically. It is indicated that decoherence can be controlled through manipulating the interaction between the system and environment. The influence of the decoherence on quantum coherent atomic tunneling (AT) between two condensates with arbitrary initial states is studied in detail. Analytic expressions of the population difference (PD) and the AT current between two condensates are found. It is shown that the decoherence leads to the decay of the PD and the suppression of the AT current.

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I. INTRODUCTION

Recently, much attention has been paid to experimental investigations [1-7] and theoretical studies [8-15] for systems consisting of two and multi Bose condensates since such systems give rise to a fascinating possibility of observing a rich set of new macroscopic quantum phenomena [16-20] which do not exist in a single condensate. Among important macroscopic quantum effects is the quantum coherent atomic tunneling (AT) between two trapped Bose condensates [15-18]. Several authors [19,20] showed that the AT can support macroscopic quantum self-trapping (MQST) due to the nonlinearity of atom-atom interactions in condensates. As is well known, no system can be completely isolated from its environment. In fact, in current experiments on trapped Bose condensates of dilute alkali atomic gases condensed atoms continuously interact with non-condensate atoms (environment). Interactions between a quantum system and environment cause two types of irreversible effects: dissipation and decoherence [21,22]. Mathematically, the dissipation and decoherence can be understood in the following way. Let $\hat{H}_S$ and $\hat{H}_B$ be Hamiltonian of the system and environment (bath), respectively, and $\hat{H}_I$ be interacting Hamiltonian between the system and environment. When $[\hat{H}_I, \hat{H}_S] \neq 0$, which implies that the energy of the system is not conservative, the interaction $\hat{H}_I$ describes the dissipation. When $[\hat{H}_I, \hat{H}_S] = 0$, the energy of the system is conservative, so the interaction $\hat{H}_I$ describes the decoherence. The dissipation effect, which dissipates the energy of the quantum system into the environment, is characterized by the relaxation time scale $\tau_d$. In contrast, the decoherence effect, which can be regarded as a mechanism for enforcing classical behaviors in the macroscopic realm, is much more insidious because the coherence information leaks out into the environment in another time scale $\tau_d$, which is much shorter than $\tau_d$, as the quantum system evolves with time. Since macroscopic quantum phenomena in Bose condensates mainly depend on $\tau_d$ rather than $\tau_r$, the discussions in present paper only focus on the decoherence problem rather than the dissipation effect. To study the decoherence in Bose condensates is not only of theoretical interest but also importance from a practical point of view, since decohering would always be present in any Bose condensate experiments of trapped atoms.

On the aspect of modeling dissipation and decoherence in trapped Bose condensates, some progress [9,23-26] has been made. In particular, Anglin [23] derived a master equation for a trapped Bose condensate by considering a special model of a condensate confined in a deep but narrow spherical square-well potential. In his model the reservoir of non-condensate atoms consists of a continuum of unbound modes obtained by the scattering solutions of the potential well. Making use of the Anglin’s master equation, Ruostekoski and Walls [24] numerically simulated dissipative dynamics of a Bose condensate in a double-well potential when the condensate is in the atomic coherent states, and shown that the interactions between condensate and non-condensate atoms make the MQST decay away. For a system consisting of two trapped weakly connected Bose condensates, there is quantum coherent AT between two condensates. Questions that naturally arise are, what is the effect of decoherence on the quantum coherent AT? Does decoherence increase or decrease the AT current between them? In this paper, we analytically study the decoherence problem in two Bose condensate, and investigate the influence of the decoherence on the quantum coherent AT between two trapped Bose condensates in terms of an exactly solvable Hamiltonian. We will present analytic expressions of the population difference (PD) and the AT current between two Bose condensates, and show that the decoherence leads to the PD decay and the suppression of the AT current.

This paper is organized as follows. In Sec.II, we present an approximate analytic solution of the system consisting of two Bose condensates with a tunneling coupling with-
out decoherence. In Sec. III, we introduce a decoherence model and apply it to the two- condensate system. We discuss the influence of decoherence on the atomic tunneling. Concluding remarks are provided in the last section.

II. TWO BOSE-EINSTEIN CONDENSATES WITH TUNNELING COUPLING

Let us consider a system of two Bose condensates with weak nonlinear interatomic interactions and the Josephson-like coupling. Such a condensate system, in principle, can be produced in a double trap with two condensates coupled by quantum tunneling and ground collisions, or in a system with two different magnetic sublevels of an atom, in which case the two species condensates correspond to two electronic states involved. In the formalism of the second quantization, Hamiltonian of such a system can be written as

\[ \hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_{\text{int}} + \hat{H}_{\text{Jos}}, \]  
\[ \hat{H}_i = \int dx \psi_i^\dagger(x) \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_i(x) \right] \psi_i(x), \quad (i = 1, 2), \]
\[ \hat{H}_{\text{int}} = U_{12} \int dx \psi_1^\dagger(x) \psi_2^\dagger(x) \psi_1(x) \psi_2(x), \]
\[ \hat{H}_{\text{Jos}} = \Lambda \int dx [\psi_1^\dagger(x) \psi_2(x) + \psi_1(x) \psi_2^\dagger(x)]. \]

Here \( i = 1, 2 \), \( \psi_i(x) \) and \( \psi_i^\dagger(x) \) are the atomic field operators which annihilate and create atoms at position \( x \), respectively. They satisfy the commutation relation \( [\psi_i(x), \psi_j^\dagger(x')] = \delta_{ij} \delta(x - x') \). \( \hat{H}_1 \) and \( \hat{H}_2 \) describe the evolution of each species in the absence of interspecies interactions. \( \hat{H}_{\text{int}} \) describes interspecies collisions. \( \hat{H}_{\text{Jos}} \) is the Josephson-like tunneling coupling term. Atoms are confined in harmonic potentials \( V_i(x) \) \((i = 1, 2)\). Interactions between atoms are described by a nonlinear self-interaction term \( U_i = 4\pi \hbar^2 a_i^c / m \) and a term that corresponds to the nonlinear interaction between different species \( U_{12} = 4\pi \hbar^2 a_{12}^c / m \), where \( a_i^c \) is s-wave scattering lengths of species \( i \) and \( a_{12}^c \) that between species 1 and 2. For simplicity, throughout this paper we set \( \hbar = 1 \), and assume that \( a_1^c = a_2^c = a^c \), \( V_1(x) = V_2(x) \).

It has been well known that the Hamiltonian (1) can reduce to a two-mode Hamiltonian [27] by the use of the approximation of the atomic field operators: \( \psi_i(x) = \hat{a}_i \phi_i(x) \), where \( \hat{a}_i = \int dx \phi_i(x) \psi_i(x) \) are correspondent mode annihilation operators with real distribution functions \( \phi_i(x) \) and \( \hat{a}_i, \hat{a}_i^\dagger = 1 \). Then the Hamiltonian (1) can be reduced to the two-mode Hamiltonian

\[ \hat{H} = \omega_0 (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) + q (\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_2 \hat{a}_1) + g (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + 2\chi \hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2, \]  
\[ \hat{H}_1 = \omega_1 (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) + g (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + 2\chi \hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2. \]

where \( q, \chi \) and \( g \) are coupling constants which characterize the strength of interatomic interaction in each condensate, interspecies interaction, and the Josephson-like coupling, respectively.

The valid conditions of the two-mode approximation were demonstrated in Ref. [19] which indicate that the two-mode approximation is valid for weak many-body interactions, i.e., for small number of condensed atoms. As shown in Ref. [19], the two-mode approximation should be acceptable for the number of atoms \( N \leq 2000 \) if the scattering length typically taken as \( a = 5 \text{nm} \) for a large trap with the size \( L = 10 \mu m \).

We note that the two-mode approximate Hamiltonian has the same form of that of a two-mode nonlinear optical directional coupler [28]. It can not be exactly solved, but a closed analytical solution can be obtained under the rotating wave approximation suggested by Alodjanc et al. [29]. The approximate analytic solution is valid for the weak interactions between atoms, but it sheds considerable light on the ATU under our consideration.

In order to obtain an approximate analytic solution of the Hamiltonian (5), we introduce a new pair of bosonic operators:

\[ \hat{A}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2)e^{-igt}, \hat{A}_2 = \frac{1}{\sqrt{2}}(\hat{a}_1 - \hat{a}_2)e^{igt}, \]

which satisfy the usual bosonic commutation relation: \( [\hat{A}_1, \hat{A}_2^\dagger] = \delta_{ij} \). Then the Hamiltonian (5) reduces to the following form

\[ \hat{H} = \Omega \hat{N} + g(\hat{A}_1^\dagger \hat{A}_1 - \hat{A}_2^\dagger \hat{A}_2) + \frac{1}{4} q(3\hat{N}^2 - 2\hat{N}) - (\hat{A}_1^\dagger \hat{A}_1 - \hat{A}_2^\dagger \hat{A}_2)^2 \]
\[ + \frac{1}{2} \chi \hat{N}^2 - \chi \hat{A}_1^\dagger \hat{A}_1 \hat{A}_2^\dagger \hat{A}_2 + \hat{H}', \]

where \( \Omega = (\omega_0 - \omega) \), the total number operator \( \hat{N} = \hat{A}_1^\dagger \hat{A}_1 + \hat{A}_2^\dagger \hat{A}_2 = \hat{A}_1^\dagger \hat{A}_1 + \hat{A}_2^\dagger \hat{A}_2 \) is a conservative constant, and \( \hat{H}' \) is a nonresonant term which oscillates at the frequency \( 4g \) in the sense of the Alodjanc et al.’s proposal [29]. The account of the fast oscillating terms results only in some additional oscillations which play no essential role in the evolution of the measurable quantities specifying the macroscopic quantum phenomena of the two- condensate system, so that the nonresonant term is fully negligible. This is the rotating wave approximation (RWA) in the sense of Ref. [29]. After neglecting the nonresonant term \( \hat{H}' \), we get the following approximate Hamiltonian:

\[ \hat{H}_A = \Omega \hat{N} + g(\hat{A}_1^\dagger \hat{A}_1 - \hat{A}_2^\dagger \hat{A}_2) + \frac{1}{4} q(3\hat{N}^2 - 2\hat{N}) - (\hat{A}_1^\dagger \hat{A}_1 - \hat{A}_2^\dagger \hat{A}_2)^2 \]
\[ + \frac{1}{2} \chi \hat{N}^2 - \chi \hat{A}_1^\dagger \hat{A}_1 \hat{A}_2^\dagger \hat{A}_2. \]
It is worthwhile noting that the dynamics of the non-RWA Hamiltonian (5) is often chaotic. A detailed investigation on chaotic behaviors in the two-condensate system is beyond the scope of present paper, and will be given elsewhere. Nevertheless, the RWA Hamiltonian (8) is an integrable Hamiltonian whose dynamics is regular, does not exhibit chaos. Hence, the terms neglected in the RWA lead to chaos when they kept in the two-condensate system. This is very analogous to the case of the Jaynes-Cummings Model (JCM) [30] which describes the interaction of a two-level atom with a single-mode electromagnetic field in quantum optics. It was well known that the RWA JCM is exactly solvable, but the non-RWA JCM [31] exhibits chaos. As shown in Ref.[31], the chaos was a consequence of inclusion of terms normally neglected in the RWA.

Obviously, the Hamiltonian $\hat{H}_A$ is diagonal in the Fock space of the ($A_1$, $A_2$) representation defined by

$$|n, m\rangle = \frac{1}{\sqrt{n!m!}}A_1^nA_2^m|0, 0\rangle,$$

where $n$ and $m$ take nonnegative integers. And we have $\hat{H}_A|n, m\rangle = E(n, m)|n, m\rangle$ with the eigenvalues

$$E(n, m) = (\Omega - g/2)(n + m) + g(n - m) + \frac{1}{4}(3g + 2\chi)\times(n + m)^2 - \frac{g}{4}(n - m)^2 - \chi nm.$$  \hspace{1cm} (9)

For simplicity, all calculations below shall be carried out in the ($\hat{a}_1$, $\hat{a}_2$) representation with the basis: $\{|n, m\rangle\}$, which is related to the ($\hat{a}_1$, $\hat{a}_2$) representation with a set of basis: $\{|n, m\rangle = \hat{a}_1^\dagger\hat{a}_2^m/\sqrt{n!m!}|0, 0\rangle, n, m = 0, 1, 2, ...\}$ through the following relation

$$|n, m\rangle = \sum_{r=0}^{n} \sum_{s=0}^{m} \frac{n!m!(n - r + s)!m!(s + r)!}{2^{(n+m)/2}(n-r)!(m-s)!}\times e^{-i(2r-3m)\pi/2}|n - r + s, m - s + r\rangle.$$  \hspace{1cm} (10)

### III. INFLUENCE OF DECOHERENCE ON ATOMIC TUNNELING

We now consider the effect of the decoherence. We use a reservoir consisting of an infinite set of harmonic oscillators to model environment of condensate atoms in a trap, and we assume the total Hamiltonian to be

$$\hat{H}_T = \hat{H}_A + \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k + F(\{\hat{S}\}) \sum_k c_k (\hat{b}_k^\dagger + \hat{b}_k)$$

$$+ F(\{\hat{S}\})^2 \sum_k \frac{c_k^2}{\omega_k},$$

where the second term is the Hamiltonian of the reservoir. The last term in Eq.(12) is a renormalization term [32]. The third term in Eq.(12) represents the interaction between the system and the reservoir with a coupling constant $c_k$, where $\{\hat{S}\}$ is a set of linear operators of the system or their linear combinations in the same picture as that of $\hat{H}_A, F(\{\hat{S}\})$ is an operator function of $\{\hat{S}\}$. In order to enable what the interaction between the system and environment describes is decoherence not dissipation, we require that the linear operator $\hat{S}$ commutes with the the Hamiltonian of the system $\hat{H}_A$. Then, the interaction term in Eq.(12) commutes with the Hamiltonian of the system. This implies that there is no energy transfer between the system and its environment. So that it does describe the decoherence. The concrete form of the function $F(\{\hat{S}\})$, which may be considered as an experimentally determined quantity, may be different for different environment. Therefore, the decohering interaction in (12) can not only describe decoherence caused by the effect of elastic collisions between condensate and non-condensate atoms for a Bose condensate system, but also simulate decoherence caused by other decohering sources through properly choosing the operator function of the system $F(\{\hat{S}\})$.

The Hamiltonian (12) can be exactly solved by making use of the following unitary transformation

$$\hat{U} = \exp[\hat{H}_A \sum_k \frac{c_k}{\omega_k} (\hat{b}_k^\dagger - \hat{b}_k)].$$

Corresponding to the Hamiltonian (12), the total density operator of the system plus reservoir can be expressed as

$$\hat{\rho}_T(t) = e^{-i\hat{H}_A t} \hat{U} e^{-it\sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k} \hat{\rho}_T(0) \hat{U}^{-1} e^{it\sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k} \hat{U} e^{i\hat{H}_A t}.$$  \hspace{1cm} (13)

In the derivation of the above solution, we have used $\hat{\rho}_T(t) = \hat{U}^{-1} \hat{\rho}_T(0) \hat{U}$, where $\hat{\rho}_T(0) = e^{-i\hat{H}_A t} \hat{\rho}_T(0) e^{i\hat{H}_A t}$ with $\hat{H}_T = \hat{U} \hat{H}_T \hat{U}^{-1}$ and $\hat{\rho}_T(0) = \hat{U} \hat{\rho}_T(0) \hat{U}^{-1}$, where $\hat{\rho}_T(0)$ the initial total density operator.

We assume that the system and reservoir are initially in thermal equilibrium and uncorrelated, so that $\hat{\rho}_T(0) = \hat{\rho}(0) \otimes \hat{\rho}_R$, where $\hat{\rho}(0)$ is the initial density operator of the system, and $\hat{\rho}_R$ the density operator of the reservoir, which can be written as $\hat{\rho}_R = \prod_k \hat{\rho}_k(0)$ with $\hat{\rho}_k(0)$ is the density operator of the $k$-th harmonic oscillator in thermal equilibrium. After taking the trace over the reservoir, from Eq.(14) we can get the reduced density operator of the system, denoted by $\hat{\rho}(t) = tr_R \hat{\rho}_T(t)$, its matrix elements in the ($\hat{A}_1$, $\hat{A}_2$) representation are explicitly written as

$$\rho(m', n')(m, n)(t) = \rho(m', n')(m, n)(0) R(m', n')(m, n)(t)$$

$$\times e^{-iF(S(m', n')) - F(S(m, n))} t,$$

$$\rho(m', n')(m, n)(t) = \rho(m', n')(m, n)(0) R(m', n')(m, n)(t)$$

$$\times e^{-iF(S(m', n')) - F(S(m, n))} t,$$

$$\rho(m', n')(m, n)(t) = \rho(m', n')(m, n)(0) R(m', n')(m, n)(t)$$

$$\times e^{-iF(S(m', n')) - F(S(m, n))} t,$$

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$$\times e^{-iF(S(m', n')) - F(S(m, n))} t,$$

$$\rho(m', n')(m, n)(t) = \rho(m', n')(m, n)(0) R(m', n')(m, n)(t)$$

$$\times e^{-iF(S(m', n')) - F(S(m, n))} t,$$

$$\rho(m', n')(m, n)(t) = \rho(m', n')(m, n)(0) R(m', n')(m, n)(t)$$

$$\times e^{-iF(S(m', n')) - F(S(m, n))} t,$$

$$\rho(m', n')(m, n)(t) = \rho(m', n')(m, n)(0) R(m', n')(m, n)(t)$$

$$\times e^{-iF(S(m', n')) - F(S(m, n))} t,$$
where $F\{S(m,n)\}$ is an eigenvalue of the operator function $F\{S\}$ in an eigenstate of $\hat{H}_A$. $R_{(m',n')(m,n)}(t)$ is a reservoir-dependent quantity given by
\begin{equation}
R_{(m',n')(m,n)}(t) = e^{-i\frac{1}{2}(F^2(S(m',n')) - F^2(S(m,n)))Q_1(t)} - e^{-i\frac{1}{2}F(S(m',n')) - F(S(m,n)))^2Q_2(t)},
\end{equation}
where the two reservoir-dependent functions are given by
\begin{equation}
Q_1(t) = \int_0^\infty d\omega J(\omega)\frac{2(\omega)}{\omega^2} \sin(\omega t),
\end{equation}
\begin{equation}
Q_2(t) = 2\int_0^\infty d\omega J(\omega)\frac{2(\omega)}{\omega^2} \sin^2(\frac{\omega t}{2}) \coth(\frac{\beta\omega}{2}),
\end{equation}
Here we have taken the continuum limit of the reservoir modes: $\sum_k \to \int_0^\infty d\omega J(\omega)$, where $J(\omega)$ is the spectral density of the reservoir, $c(\omega)$ is the corresponding continuum expression for $c_k$, and $\beta = 1/k_BT$ with $k_B$ and $T$ being the Boltzmann constant and temperature, respectively.

It is well known that decoherence corresponds to the decay of off-diagonal elements of the reduced density matrix of a quantum system. For the case under our consideration, the degree of decoherence is determined by the decaying factor in Eq.(16). It is interesting to note that if we choose a proper operator function $F\{S\}$ to make $F\{S(m',n')\} = F\{S(m,n)\}$ for $(m',n') \neq (m,n)$, then we find that
\begin{equation}
\rho_{(m',n')(m,n)}(t) = \rho_{(m',n')(m,n)}(0)
\end{equation}
which indicates that the quantum system maintains its initial quantum coherence, namely, the time evolution of decoherence-free of the quantum system is realized. Therefore, we conclude that one can control decoherence by manipulating interaction function $F\{S\}$.

Eqs.(15) and (16) indicate that the interaction between the system and its environment induces a phase shift and a decaying factor in the reduced density operator of the system. We now consider the PD between the two condensates in the presence of the decoherence, defined by $p(t) \equiv N_1(t) - N_2(t)$ with $N_i = \langle \hat{a}_1^\dagger \hat{a}_i \rangle$. We find that
\begin{equation}
p(t) = -2\sum_{r,s} \sqrt{s(r+1)}|\rho_{(r+1,s-1)(r,s)}(0)| \sin[\theta_{rs} - v_{rs}^- (t - v_{rs}^+ Q_1(t))] e^{-v_{rs}^- Q_2(t)},
\end{equation}
where we have introduced the symbols:
\begin{equation}
\rho_{(r+1,s-1)(r,s)}(0) = |\rho_{(r+1,s-1)(r,s)}(0)| e^{i\theta_{rs}},
\end{equation}
\begin{equation}
v_{rs}^+ = F\{S(r+1,s-1)\} \pm F\{S(r,s)\}.
\end{equation}

From Eq.(20) we see that if we do not take into account the influence of the decoherence, i.e., set $Q_1(t) = Q_2(t) = 0$, then we get a expression of the PD between two condensates
\begin{equation}
p(t) = -2\sum_{r,s} \sqrt{s(r+1)}|\rho_{(r+1,s-1)(r,s)}(0)| \sin[\theta_{rs} - v_{rs}^- (t - v_{rs}^+ Q_1(t)]
\times e^{-v_{rs}^- Q_2(t)},
\end{equation}
which implies that the time evolution of the PD is periodic. In particular, if we take $F\{S\} = \hat{H}_A$, we find that when $2\gamma/(q - \chi) = K$ (being an integer), we have a nonzero time-average value of the PD
\begin{equation}
\bar{p} = -2\sum_{r,s} \sqrt{(r-K+1)(r+1)}|\rho_{(r+1,r-K)(r,r-K+1)}(0)| \sin[\theta_{rs} - v_{rs}^- (t - v_{rs}^+ Q_1(t)]
\times e^{-v_{rs}^- Q_2(t)}.
\end{equation}

From Eqs.(18), (20) and (25) we can immediately draw one important qualitative conclusion: since $Q_2(t)$ is positive definite, the existence of the decoherence is always tend to suppress the PD and the AT current between the two condensates. This answers the question: “Does the decoherence increase or decrease the AT?”.

From Eqs.(17),(18), (20) and (25) we see that all necessary information about the effects of the environment on the PD and the AT current is contained in the spectral density of the reservoir. To proceed further let us now specialize to the Ohmic case [33] with the spectral distribution $J(\omega) = \frac{\alpha}{\pi\omega} e^{-\omega/\omega_c}$, where $\omega_c$ is the high frequency cut-off, $\eta$ is a positive characteristic parameter of the reservoir. With this choice, at low temperature the functions $Q_1(t)$ and $Q_2(t)$ are given by the following expressions
\begin{equation}
Q_1(t) = \eta \tan^{-1}(\omega_c t),
\end{equation}
\begin{equation}
Q_2(t) = \eta\left\{\frac{1}{2} \ln[1 + (\omega_c t)^2] + \ln\left(\frac{\beta}{\pi t} \sinh\left(\frac{\pi t}{\beta}\right)\right)\right\}
\end{equation}
Recent experiments [1,7] on two condensates have established a typical time scale at which the two condensates preserve coherence. The value of the typical time scale is $t \approx 100 ms$. In the meaningful domain of time $\omega c t \gg 1$ which requires $\omega_c \gg 10Hz$ which can be easily
satisfied for an usual reservoir [34,32,33], at zero temperature, we have \( Q_1(t) = \eta/\omega_c t^2 \), and \( Q_2(t) = \eta \ln(\omega_c t) \), then we find

\[
p(t) = -2 \sum_r \sum_s \sqrt{s(r+1)}|\rho_{(r+1,s-1)(r,s)}(0)| \times \sin[\theta_{rs} - v_{rs}^{-}(t - v_{rs}^{+} Q_1(t))](\omega_c t)^{-\eta(v_{rs}^{-})^2},
\]

which indicate that the PD and the AT current decay away according to the “power law”, where we have noted which indicate that at finite temperature the PD decays exponential law, and the AT current and found that for the reservoir-spectral density of the Ohmic case, the PD and the AT current decay away by the “power law” at zero temperature; at finite temperature, the PD decays away by the “exponential law” while the decay of the AT current contains both “exponential-law” and “power-law” components. It is worthwhile to note that our results are obtained for arbitrary initial states of the two condensates, our entire analysis is carried out without invoking the assumption of Bose-broken symmetry which has recently been shown to be unnecessary for a Bose condensate of trapped atoms [35]. Also it should be pointed out that these results are obtained under the RWA in the sense of Alodjani et al.’s proposal, they are valid for interatomic weak non-linear interactions in two condensates. The RWA essentially changes the two-condensate system into an integrable system, hence it suppresses the chaotic behaviors of the two-condensate system.

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**IV. CONCLUDING REMARKS**

We have present a decoherence model which is exactly solvable, and applied it to study decoherence in two Bose condensates. We have indicated that one can control decoherence by manipulating interaction between a quantum system and environment. We have investigated the influence of the decoherence on quantum coherent AT between two trapped Bose condensates with arbitrary initial states, and shown that the decoherence suppresses the PD and the AT current between two condensates. We have obtained analytic expressions of the PD and the AT current and found that for the reservoir-spectral density of the Ohmic case, the PD and the AT current decay away by the “power law” at zero temperature; at finite temperature, the PD decays away by the “exponential law” while the decay of the AT current contains both “exponential-law” and “power-law” components. It is worthwhile to note that our results are obtained for arbitrary initial states of the two condensates, our entire analysis is carried out without invoking the assumption of Bose-broken symmetry which has recently been shown to be unnecessary for a Bose condensate of trapped atoms [35]. Also it should be pointed out that these results are obtained under the RWA in the sense of Alodjani et al.’s proposal, they are valid for interatomic weak non-linear interactions in two condensates. The RWA essentially changes the two-condensate system into an integrable system, hence it suppresses the chaotic behaviors of the two-condensate system.

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