DISSIPATION AND DECOHERENCE
IN PHOTON INTERFEROMETRY

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Abstract

The propagation of polarized photons in optical media can be effectively modeled by means of quantum dynamical semigroups. These generalized time evolutions consistently describe phenomena leading to loss of phase coherence and dissipation originating from the interaction with a large, external environment. High sensitive experiments in the laboratory can provide stringent bounds on the fundamental energy scale that characterizes these non-standard effects.
1. INTRODUCTION

Quantum systems are usually treated as isolated: their time evolution is unitary, driven by the appropriate hamiltonian operator. In general however, this should be regarded as an approximation: any external environment $\mathcal{E}$ unavoidably interacts with the system $\mathcal{S}$ under study, making the resulting dynamics rather involved.[1-3]

The global system $\mathcal{S} + \mathcal{E}$ is closed, and its time evolution is determined by the operator $e^{-iH_{\text{tot}}t}$, involving the total hamiltonian, that can be always decomposed as:

$$H_{\text{tot}} = H + H_\mathcal{E} + H', \quad (1.1)$$

where $H$ drives the system $\mathcal{S}$ in absence of $\mathcal{E}$, $H_\mathcal{E}$ describes the internal environment dynamics, while $H'$ takes into account the interaction between $\mathcal{S}$ and $\mathcal{E}$. Nevertheless, being interested only in the evolution of the subsystem $\mathcal{S}$ and not in the details of the dynamics of $\mathcal{E}$, one finally integrates over the environment degrees of freedom. Describing the states of subsystem and environment by appropriate density matrices, the evolution in time of $\mathcal{S}$ will then be given by the transformation:

$$\rho(0) \mapsto \rho(t) = \text{Tr}_\mathcal{E} \left[ e^{-iH_{\text{tot}}t} \rho(0) \otimes \rho_\mathcal{E} e^{iH_{\text{tot}}t} \right], \quad (1.2)$$

where $\rho_\mathcal{E}$ describes the state of the environment at $t = 0$ (for simplicity, we assume $\mathcal{S}$ and $\mathcal{E}$ to be initially uncorrelated; see [4] for a generalization).

The resulting map $\rho(0) \mapsto \rho(t)$ is rather complex, involving in general non-linear and memory effects; it consistently describes decoherence effects, leading to irreversibility and dissipation. An explicit and mathematically precise description in terms of quantum dynamical semigroups is however possible when the interaction between the subsystem $\mathcal{S}$ and the environment is weak. These generalized time evolutions are represented by linear transformations, mapping density matrices into density matrices, while preserving very basic physical properties, like forward in time composition law (semigroup property), entropy increase (irreversibility) and complete positivity (that guarantees the correct physical interpretation of the dynamics in all situations).[1-3]

Thanks to its generality and physical self-consistency, the description of open systems in terms of quantum dynamical semigroups can be applied to model the dynamics of any system in weak interaction with a large environment.[1-7] In particular, it has recently been applied to describe possible effects of irreversibility and dissipation induced by the evolution of strings and branes. Indeed, quite in general the fundamental dynamics of these extended objects gives rise at low energies to a weakly coupled environment, and as a consequence to decoherence phenomena.[8, 9]

From a more phenomenological point of view, similar effects have also been described in the framework of quantum gravity: the quantum fluctuation of the gravitational field and the appearance of virtual black holes make space-time look “foamy” at distances comparable to Planck’s length, inducing non-standard phenomena leading to possible loss of quantum coherence.[10-16] Dissipation and decoherence are also the general result of the dynamics in theories with large extra dimensions;[17] indeed, the possible energy leakage into the bulk of space-time due to gravity effects would inevitably inject noise into the boundary, thus inducing irreversibility and dissipation at low energy in our brane-world.
Our present knowledge of string theory does not allow precise estimates of the magnitude of these non-standard effects. Using a rough dimensional analysis, one can nevertheless conclude that they should be rather small, being suppressed by at least one inverse power of a large, fundamental mass scale (most likely the Planck mass). Despite this, they can be studied using interferometric phenomena. Indeed, detailed investigations involving various elementary particle systems (neutral mesons [18-22], neutrons [23], neutrinos [24, 25]) have shown that present and future experiments might soon reach the required sensitivity in order to detect the new, non-standard phenomena.

This possibility looks particularly promising for photon interferometry and more in general optical physics.[5-7] The sophistication of present laboratory experiments in quantum optics is so high that decoherence effects induced by a fundamental, “stringy” dynamics might be studied using available setups.

In the present work, we shall discuss in detail how these non-standard, dissipative phenomena can affect the propagation of polarized photons immersed in optically active media. A preliminary discussion has been presented in [26]. There, it has been shown that the dissipative phenomena manifest themselves via depolarizing effects, that accumulate with time. Limits on the magnitude of the parameters describing the new phenomena can therefore be obtained from astrophysical and cosmological observations.[27]

In the following, different aspects of the quantum dynamical semigroup description of photon propagation will be analyzed, focusing on the discussion of possible laboratory tests. As we shall see, the possibility of actually detecting the new, dissipative effects are greatly enhanced by making them interfere with those induced by time-dependent optical media. For slowly varying media, the use of the adiabatic approximation is justified. In this case, explicit expressions for relevant physical observables will be given and discussed; the formulas can be used to fit actual experimental data. The outcome of our investigation is that, at least in principle, bounds on some of the parameters describing dissipation and decoherence can be obtained using existing laboratory setups.

2. QUANTUM DYNAMICAL SEMIGROUPS

In describing the evolution of polarized photons we shall adopt the standard effective description in terms of a two-dimensional Hilbert space, the space of helicity states.[28-31] A convenient basis in this space is given by the circularly polarized states $|R\rangle, |L\rangle$. With respect to this basis, any partially polarized photon state can be represented by a $2 \times 2$ density matrix $\rho$, i.e. by an hermitian operator, with positive eigenvalues and constant trace:

$$\rho = \begin{bmatrix} \rho_1 & \rho_3 \\ \rho_4 & \rho_2 \end{bmatrix}, \quad \rho_4 = \rho_3^* . \quad (2.1)$$

As explained in the Introduction, the evolution in time of $\rho$ will be described by mean of a quantum dynamical semigroup, i.e. by a linear transformation generated by an equation of the following form:[1-3, 32-34]

$$\frac{\partial \rho(t)}{\partial t} = -i[H(t), \rho(t)] + L[\rho(t)] . \quad (2.2)$$
The first term in the r.h.s. is of hamiltonian form, while the piece \( L[\rho] \) takes into account the interaction with the external environment and leads to irreversibility and dissipation.\(^\dagger\)

As mentioned in the introductory remarks, it is convenient to make the photons cross an additional, non-dissipative, time-dependent optical medium, whose properties can be suitably controlled. This will in general induce extra birefringence effects on the polarized photons, and these can be conveniently described in terms of a time-dependent, effective hamiltonian \( H(t) \). We shall assume a simple harmonic dependence on time:

\[
H(t) = \begin{bmatrix} \omega_0 + \mu & \nu e^{-i\lambda t} \\ \nu e^{i\lambda t} & \omega_0 - \mu \end{bmatrix}; \quad (2.3)
\]

this form is of sufficient generality for the considerations that follow. In (2.3), the parameter \( \omega_0 \) represents the average photon energy, while the real constants \( \mu \) and \( \nu \) induce the level-splitting \( \omega = (\mu^2 + \nu^2)^{1/2} \) among the two instantaneous eigenstates. As compared with the effects of this splitting, the dependence on time of \( H(t) \), characterized by the real frequency \( \lambda \), will be assumed to be slow: \( \lambda \ll \omega \); this is the situation that is most likely to be reproduced by actual laboratory setups.

The additional piece \( L[\rho] \) in the evolution equation (2.2) is not of hamiltonian form, and induces a mixing-enhancing mechanism leading in general to irreversibility and loss of quantum coherence. In order to write it down explicitly, it is useful to adopt a vector-like notation and collect the entries \( \rho_1, \rho_2, \rho_3, \rho_4 \) of the density matrix (2.1) as the components of the four-dimensional abstract vector \( |\rho\rangle \). The evolution equation (2.2) can then be rewritten as a Schrödinger (or diffusion) equation:

\[
\frac{\partial}{\partial t} |\rho(t)\rangle = \left[ H(t) + L \right] |\rho(t)\rangle, \quad (2.4)
\]

where the \( 4 \times 4 \) matrix \( H \) takes into account the hamiltonian contributions,

\[
H(t) = i \begin{bmatrix} 0 & 0 & \nu e^{i\lambda t} & -\nu e^{-i\lambda t} \\ 0 & 0 & -\nu e^{i\lambda t} & \nu e^{-i\lambda t} \\ \nu e^{-i\lambda t} & -\nu e^{-i\lambda t} & -2\mu & 0 \\ -\nu e^{i\lambda t} & \nu e^{i\lambda t} & 0 & 2\mu \end{bmatrix}, \quad (2.5)
\]

while the dissipative part \( L \) can be fully parametrized in terms of six real constants \( a, b, c, \alpha, \beta, \) and \( \gamma \), as follows:[1, 18]

\[
L = \begin{bmatrix} -D & D & -C & -C^* \\ D & -D & C & C^* \\ -C & C^* & -A & B \\ -C & C & B^* & -A \end{bmatrix}, \quad (2.6)
\]

\(^\dagger\) It should be noticed that in general the interaction with the environment can also produce hamiltonian pieces in (2.2);[1-3, 9, 25] however, in the present case these contributions can not be distinguished from those originating from other birefringence phenomena.
where for later convenience the combinations:

\[ A = \alpha + a , \quad B = \alpha - a + 2ib , \quad C = c + i\beta , \quad D = \gamma , \]  

\[ (2.7) \]

have been introduced. The six parameters are not all independent: they need to satisfy the following inequalities:[1-3, 18, 35]

\[ 2R \equiv \alpha + \gamma - a \geq 0 , \quad RS - b^2 \geq 0 , \quad a \geq 0 , \]
\[ 2S \equiv a + \gamma - \alpha \geq 0 , \quad RT - c^2 \geq 0 , \quad \alpha \geq 0 , \]
\[ 2T \equiv a + \alpha - \gamma \geq 0 , \quad ST - \beta^2 \geq 0 , \quad \gamma \geq 0 , \]
\[ RST - 2bc\beta - R\beta^2 - Sc^2 - Tb^2 \geq 0 . \]  

\[ (2.8) \]

These relations are the consequence of the property of complete positivity that assures the correct physical interpretation of the time evolution \(|\rho(0)\rangle \rightarrow |\rho(t)\rangle\) generated by (2.4) in all situations; without this condition, serious inconsistencies in general arise (for more details, see [35]).

The effective environment generated by the fundamental “stringy” dynamics can be considered to be in thermal equilibrium:[9] the decoherence effects induced on the photons are therefore stationary, so that the six parameters \(a, b, c, \alpha, \beta, \gamma\) in (2.6), (2.7) can be taken to be time-independent. Nevertheless, let us mention that the evolution equation (2.2) can be generalized to take into account non-stationary dissipative contributions: these would typically arise for environments that are out of equilibrium, giving rise in general to time-dependent interactions with the photons.[32-34] On the other hand, a physically consistent, general formulation of non-linear dissipative dynamics is not yet available.

Once the evolution equation (2.4) is solved, one can easily compute any physical property involving polarized photons. Indeed, in the formalism of density matrices, any observable \(O\) is represented by an hermitian matrix, that can be decomposed as in (2.1). The evolution in time of its mean value is then obtained by taking its trace with the density operator \(\rho(t)\):

\[ \langle O(t) \rangle \equiv \text{Tr} \left[ O \rho(t) \right] = O_1 \rho_1(t) + O_2 \rho_2(t) + O_3 \rho_3(t) + O_4 \rho_4(t) \equiv \langle O | \rho(t) \rangle . \]  

\[ (2.9) \]

In the case of photons, of particular interest is the observable that correspond to a fully polarized state, identified by the two angles \(\theta\) and \(\varphi\); it is explicitly given by the following projector operator

\[ O_{\theta,\varphi} = \frac{1}{2} \begin{bmatrix} 1 + \sin \varphi \sin 2\theta & \cos 2\theta - i \cos \varphi \sin 2\theta \\ \cos 2\theta + i \cos \varphi \sin 2\theta & 1 - \sin \varphi \sin 2\theta \end{bmatrix} . \]  

\[ (2.10) \]

Its mean value gives the probability \(P_{\theta,\varphi}(t)\) that the evolved state \(|\rho(t)\rangle\) be found at time \(t\) in the polarization state determined by \(\theta\) and \(\varphi\); it is proportional to the intensity curve that can be detected at an appropriate interferometric apparatus.
3. TRANSITION PROBABILITIES

In order to find explicit solutions of the evolution equation (2.4), it is convenient to perform a time-dependent unitary transformation and study it in the basis of instantaneous eigenvectors \( |v(\pm)(t)\rangle \) of the hamiltonian (2.3), \( H(t) |v(\pm)(t)\rangle = (\omega_0 \pm \omega) |v(\pm)(t)\rangle \); using the four-vector notation, one then writes

\[
|\tilde{\rho}(t)\rangle = U(t) |\rho(t)\rangle ,
\]

where

\[
U(t) = \frac{1}{2\omega} \begin{bmatrix}
\omega + \mu & \omega - \mu & \nu e^{i\lambda t} & \nu e^{-i\lambda t} \\
\omega - \mu & \omega + \mu & -\nu e^{i\lambda t} & -\nu e^{-i\lambda t} \\
-\nu e^{-i\lambda t} & \nu e^{i\lambda t} & \omega + \mu & -(\omega - \mu)e^{-2i\lambda t} \\
-\nu e^{i\lambda t} & \nu e^{-i\lambda t} & -(\omega - \mu)e^{2i\lambda t} & \omega + \mu
\end{bmatrix} .
\]

In the new basis, the hamiltonian contribution in (2.4) becomes diagonal:

\[
\tilde{H} = U(t) H(t) U^\dagger(t) = \text{diag}[0, 0, -2i\omega, 2i\omega] ;
\]

the four entries coincide with the eigenvalues of the operator \(-i[H(t), \cdot]\), and therefore are given by the differences of the eigenvalues \(\omega_0 \pm \omega\) of \(H(t)\). However, since \(U(t)\) is time dependent, the evolution equation for the transformed vector \(|\tilde{\rho}(t)\rangle\) involves an effective hamiltonian:

\[
\frac{\partial}{\partial t} |\tilde{\rho}(t)\rangle = \left[ H_{\text{eff}}(t) + \tilde{\mathcal{L}}(t) \right] |\tilde{\rho}(t)\rangle ,
\]

with

\[
H_{\text{eff}}(t) = \tilde{H} + \dot{U}(t) U^\dagger(t) ;
\]

further, the dissipative contribution becomes time-dependent:

\[
\tilde{\mathcal{L}}(t) = U(t) \mathcal{L} U^\dagger(t) .
\]

One can check that its explicit form is as in (2.6), with the new parameters \(\bar{A}, \bar{B}, \bar{C}, \bar{D}\) linear combinations of the old ones \(A, B, C, D\):†

\[
\bar{A} = A + \frac{\nu^2}{2\omega^2} \left[ 2D - A + \Re \left( Be^{2i\lambda t} \right) \right] - \frac{2\mu \nu}{\omega^2} \Re \left( Ce^{-i\lambda t} \right) ,
\]

\[
\bar{B} = e^{-2i\lambda t} \left\{ \left( 1 - \frac{\nu^2}{2\omega^2} \right) \Re \left( Be^{2i\lambda t} \right) + \frac{i\mu}{\omega} \Im \left( Be^{2i\lambda t} \right) + \frac{2\mu \nu}{\omega^2} \Re \left( Ce^{-i\lambda t} \right) \\
- \frac{2i\nu}{\omega} \Im \left( Ce^{-i\lambda t} \right) - \frac{\nu^2}{2\omega^2} \left( 2D - A \right) \right\} ,
\]

\[
\bar{C} = e^{i\lambda t} \left\{ \left( 1 - \frac{\nu^2}{2\omega^2} \right) \Re \left( Ce^{-i\lambda t} \right) + \frac{i\mu}{\omega} \Im \left( Ce^{-i\lambda t} \right) - \frac{\mu \nu}{2\omega^2} \left[ 2D - A + \Re \left( Be^{2i\lambda t} \right) \right] \\
+ \frac{i\nu}{2\omega} \Im \left( Be^{2i\lambda t} \right) \right\} ,
\]

\[
\bar{D} = D - \frac{\nu^2}{2\omega^2} \left[ 2D - A + \Re \left( Be^{2i\lambda t} \right) \right] + \frac{2\mu \nu}{\omega^2} \Re \left( Ce^{-i\lambda t} \right) .
\]

† This is a general property of any quantum dynamical semigroup, whose explicit form is in fact basis-independent.[1-3]
When the system hamiltonian $H(t)$ is slowly varying, the explicit dependence on time of $H_{\text{eff}}(t)$ is very mild, so that the adiabatic approximation can be used in studying (3.4). In general, this is justified when the transitions induced by the explicit time dependence of the hamiltonian are suppressed with respect to its natural level splitting.[36] In the present case, this condition is guaranteed by the starting assumption: $\lambda \ll \omega$. Within this approximation, one can neglect the off-diagonal terms in the contribution $\hat{U}(t)\hat{U}^\dagger(t)$, so that $H_{\text{eff}}$ becomes diagonal:

$$H_{\text{eff}} = \text{diag}[0, 0, -2i(\omega + \lambda_B), 2i(\omega + \lambda_B)].$$

(3.8)

(As explained in the Appendix, in the case of the hamiltonian (2.3) this result can be directly checked.)

The additional phase contribution $\lambda_B$ to the finite-time evolution operator $e^{H_{\text{eff}}t}$ has a precise physical meaning: it gives the Berry phase that in general accumulates with time:[37, 38] indeed, one easily checks that:

$$\lambda_B = \frac{\lambda}{2} \left(1 - \frac{\mu}{\omega}\right) \equiv \mp i \langle v^{(\pm)}(t) | \frac{\partial}{\partial t} | v^{(\pm)}(t) \rangle.

(3.9)

Being encoded in the diagonal part of $\hat{U}(t)\hat{U}^\dagger(t)$, Berry’s contribution is directly connected to the characteristic properties of the starting hamiltonian $H(t)$, and not to the use of the adiabatic approximation.

In absence of the dissipative piece, $\tilde{\mathcal{L}} = 0$, the evolution in time of any given initial state $|\rho(0)\rangle$ can then be written as:

$$|\rho(t)\rangle = \hat{U}^\dagger(t) \cdot \mathcal{M}_0(t) \cdot \hat{U}(0) |\rho(0)\rangle, \quad \mathcal{M}_0(t) = e^{H_{\text{eff}}t}.$$  

(3.10)

Using this expression, one can compute the evolution of physically relevant observables, and in particular transition probabilities. An experimentally relevant example is given by the probability $\mathcal{P}_\theta(t)$ of finding an initially left-polarized photon in a state with linear polarization along the direction $\theta$ at time $t$. Using the general definition (2.9) and the expression in (2.10) with $\varphi = 0$, from the evolution map (3.10) one explicitly finds:

$$\mathcal{P}_\theta(t) = \frac{1}{2} \left\{ 1 + \frac{\mu \nu}{\omega^2} \cos(2\theta - \lambda t) \left[ \cos[2(\omega + \lambda_B)t - \lambda t] - 1 \right] \right. $$

$$+ \left. \frac{\nu}{\omega} \sin(2\theta - \lambda t) \sin[2(\omega + \lambda_B)t - \lambda t] \right\}. 

(3.11)

This expression further simplifies for a vanishingly small $\mu$; in this case, it can be conveniently rewritten as:

$$\mathcal{P}_\theta(t) = \frac{1}{2} \left\{ 1 + \frac{1}{2} \left[ \cos(2\omega t + \lambda t - 2\theta) + \cos(2\omega t - \lambda t + 2\theta + \pi) \right] \right\}. 

(3.12)

\footnote{Indeed, the evolution generated by the hamiltonian $H(t)$ in (2.3) can be written in closed form. In general however, this is no longer possible when the dissipative contribution in (2.2) is non-vanishing.}
This intensity pattern can be studied by means of an interferometric setup: one can then extract amplitudes and phases of the various Fourier components that characterize the probability (3.12). In particular, the presence of the modulation $e^{-i\lambda t}$ in the Hamiltonian (2.3) describing in the optical medium crossed by the photon beam leads to a symmetric shift of the fundamental birefringence frequency $2\omega$ by the small amount $\lambda$. Notice that this result is a consequence of the presence of Berry's phase contribution, that now takes the simplified expression $\lambda_B = \lambda/2$; indeed, neglecting this contribution would have produced an asymmetric split of the fundamental frequency. An experimental analysis of the intensity pattern (3.12) can then allow a direct identification of Berry’s phase.

To see how this description is modified by the presence of dissipative phenomena, one needs to study the evolution equation (3.4) with a non-vanishing $\tilde{\mathcal{L}}$. Although in general the effects induced by the interaction with the environment are parametrized by the six real constants $a$, $b$, $c$, $\alpha$, $\beta$, and $\gamma$, there are physically motivated instances for which only one of them is actually non-zero. For example, this happens when $\gamma$ is vanishingly small; in this case, the inequalities (2.8) further imply $a = \alpha$ and $b = c = \beta = 0$.\(^\dagger\) In this case, the entries of the matrix $\tilde{\mathcal{L}}$ are all proportional to $\alpha$, and assuming as before $\mu = 0$, from (3.7) one explicitly obtains:

$$\tilde{A} = \tilde{D} = \alpha , \quad \tilde{B} = \alpha e^{-2i\lambda t} , \quad \tilde{C} = 0 .$$

Although the resulting expression for $\tilde{\mathcal{L}}$ is still explicitly time-dependent, the evolution equation (3.4) can be exactly integrated; one finds:

$$|\tilde{\rho}(t)\rangle = \mathcal{M}(t) |\tilde{\rho}(0)\rangle , \quad \mathcal{M}(t) = e^{-\alpha t} \begin{bmatrix} \Theta(t) & 0 \\ 0 & \Xi(t) \end{bmatrix} ,$$

where the $2 \times 2$ matrices $\Theta(t)$ and $\Xi(t)$ can be expressed in terms of the Pauli matrices $\sigma_1$, $\sigma_3$ and the identity $\sigma_0$:

$$\Theta(t) = e^{\alpha t \sigma_1} , \quad \Xi(t) = e^{-i\lambda t \sigma_3} \left[ \cos 2\Omega t \sigma_0 - \frac{i\omega}{\Omega} \sin 2\Omega t \sigma_3 + \frac{\alpha}{2\Omega} \sin 2\Omega t \sigma_1 \right] ,$$

and

$$\Omega = \sqrt{\omega^2 - \alpha^2/4} .$$

Using the expression of the evolution matrix $\mathcal{M}(t)$ above in place of $\mathcal{M}_0(t)$ in (3.10), one finally obtains the dynamical map $|\rho(0)\rangle \rightarrow |\rho(t)\rangle$ in presence of dissipative effects. Accordingly, the expressions of physically interesting observables change. In particular, the

\(^\dagger\) There are essentially two known ways of implementing the condition of weak interaction between subsystem and environment:[1-3] the singular coupling limit (in which the time-correlations in the environment are assumed to be much smaller than the typical time scale of the subsystem) and the weak coupling limit (in which it is the subsystem characteristic time scale that becomes large). One can check that the second situation leads precisely to the condition $\gamma = 0$.\([9, 25]\)
transition probability $P_\theta(t)$ of finding an initial circularly polarized photon in a linearly polarized state at time $t$ becomes:

$$P_\theta(t) = \frac{1}{2} \left\{ 1 + \frac{\omega}{2\Omega} e^{-\alpha t} \left[ \cos \left( 2\Omega t + \lambda t - 2\theta \right) + \cos \left( 2\Omega t - \lambda t + 2\theta + \pi \right) \right] \right\} . \quad (3.17)$$

The presence of dissipation affects the expression of $P_\theta(t)$ through the introduction of the exponential damping term together with the amplitude rescaling by the factor $\omega/\Omega$, and the change in the birefringence frequency from $\omega$ to $\Omega$. On the other hand, note that the symmetric shift in frequency by the amount $\lambda$ induced by Berry’s phase contribution remains unchanged. This is not surprising: the geometrical mechanism leading to the presence of Berry’s phase is completely different from the physical phenomena leading to irreversibility and dissipation, and this fact is clearly reflected in the expression of the transition probability (3.17). As a result, the dissipative contributions and those originating from Berry’s phase can be independently probed.

Similarly to the expression in (3.12), also the intensity pattern described by (3.17) can be, at least in principle, experimentally studied using Fourier analysis. Notice however that the oscillatory behaviour in (3.17) critically depends on the magnitude of the non-standard effects induced by the presence of the environment; indeed, for sufficiently large $\alpha$, the frequency $\Omega$ becomes purely imaginary, so that the only remaining harmonic dependence in $P_\theta(t)$ is driven by the small frequency $\lambda$:

$$P_\theta(t) = \frac{1}{2} \left\{ 1 + \frac{\omega}{\Omega} e^{-\alpha t} \sinh(\Omega t) \sin(2\theta - \lambda t) \right\} . \quad (3.18)$$

In any case, independently from the relative magnitude of $\alpha$ and $\omega$, the damping effects always prevail for large times: in this limit, the transition probability $P_\theta(t)$ takes the constant value $1/2$. One can show that this result is independent from the approximation used to derive (3.17). Actually, in presence of dissipative phenomena all transition probabilities asymptotically tend to constant values, corresponding to the transition to a totally depolarized state.[39, 26]

A different treatment is possible when the non-standard parameters $a, b, c, \alpha, \beta,$ and $\gamma$ can be considered to be small in comparison with the characteristic system energy $\omega$. This is likely to be the case in most standard laboratory situations: indeed, the main source of birefringence effects is usually the propagation in laboratory controlled optical media, and not the weak interaction with an external environment. In this case, the additional piece $\tilde{L}$ in (3.4) can be treated as a perturbation, and the evolution matrix $M(t)$ in (3.14) can thus be expressed as the following series expansion:

$$M(t) = e^{\mathcal{H}_{\text{eff}}t} \left\{ 1 + \int_0^t dt_1 e^{-\mathcal{H}_{\text{eff}}t_1} \tilde{L}(t_1) e^{\mathcal{H}_{\text{eff}}t_1} \\
+ \int_0^t dt_1 \int_0^{t_1} dt_2 e^{-\mathcal{H}_{\text{eff}}t_1} \tilde{L}(t_1) e^{\mathcal{H}_{\text{eff}}(t_1-t_2)} \tilde{L}(t_2) e^{\mathcal{H}_{\text{eff}}t_2} + \ldots \right\} . \quad (3.19)$$

Useful information on the presence of dissipative effects can already be obtained by considering only first order terms in the small parameters. Within this approximation, the
transition probability $\mathcal{P}_\theta(t)$ takes the following explicit form:

$$
\mathcal{P}_\theta(t) = \frac{1}{2} + \frac{e^{-(D+A/2)t}}{2} \left\{ -\Delta(t) \cos(2\theta - \lambda t) + \left[ \left( 1 + \frac{|B|}{2\lambda} \sin \lambda t \sin(\lambda t + \phi_B) \right) \sin 2\omega t - \Phi(t) \right] \sin(2\theta - \lambda t) \right\},
$$

(3.20)

where

$$
\Delta(t) = \frac{|C|}{2} \left[ \frac{2\lambda}{4\omega^2 - \lambda^2} \sin \phi_C - \frac{\sin(2\omega t + \lambda t - \phi_C)}{2\omega + \lambda} - \frac{\sin(2\omega t - \lambda t + \phi_C)}{2\omega - \lambda} \right] + \frac{|B|}{8} \left[ \frac{2\omega}{\omega^2 - \lambda^2} \sin \phi_B + \frac{\sin(2\omega t - 2\lambda t - \phi_B)}{\omega - \lambda} - \frac{\sin(2\omega t + 2\lambda t + \phi_B)}{\omega + \lambda} \right], \quad (3.21a)
$$

and

$$
\Phi(t) = \frac{|B|}{4} \sin(\lambda t + \phi_B) \left[ \frac{\sin(2\omega + \lambda)t}{2\omega + \lambda} - \frac{\sin(2\omega - \lambda)t}{2\omega - \lambda} \right] + 2|C| \sin(\lambda t/2 - \phi_C) \left[ \frac{\sin(2\omega - \lambda/2)t}{4\omega - \lambda} + \frac{\sin(2\omega + \lambda/2)t}{4\omega + \lambda} \right], \quad (3.21b)
$$

while $\phi_B$ and $\phi_C$ are the phases of $B$ and $C$, the combination of dissipative parameters introduced in (2.7). In writing (3.20) we have reconstructed the exponential damping factor by putting together terms linear in $t$: this is consistent at the used level of approximation; the large time asymptotic behaviour of $\mathcal{P}_\theta(t)$ mentioned before is thus reproduced. The expression in (3.20) is clearly much more involved than the ones presented before: it represents the most general form that the transition probability $\mathcal{P}_\theta(t)$ can take in presence of small dissipative effects.

4. DISCUSSION

The propagation of polarized photons in optical media can be consistently discussed within the formalism of open quantum systems, i.e. as a system in interaction with a large environment. This treatment can be physically justified in the framework of string and brane theory, whose dynamics can be effectively described at low energies as a weakly coupled environment, inducing non-standard phenomena leading in general to decoherence and dissipation. Quantum dynamical semigroups give a physically consistent and mathematically precise description of these non-standard effects; it turns out that they can be fully parametrized in terms of the phenomenological constants $a, b, c, \alpha, \beta, \gamma$ introduced in (2.6), (2.7).

As discussed in the previous section, the presence of these constants modify in a distinctive way the time evolution of physically interesting observables, that can be experimentally studied using interferometric setups. In particular, the new, dissipative phenomena manifest themselves through depolarizing effects, via the presence of exponential damping factors, and suitable shifts in the frequencies describing birefringence effects.
Although a detailed discussion on possible devices that can be used to measure such effects is surely beyond the scope of the present investigation, some general considerations can nevertheless be given.† Recalling for instance the expression (3.17) for the transition probability $P_\theta(t)$, one immediately realizes that the possibility of detecting the depolarizing effects induced by the non-standard, dissipative phenomena is connected with the ability of isolating and extracting the exponential factor $e^{-\alpha t}$ from the experimental data. The sensitivity of this measure clearly increases with $t$, so that large optical paths are in general required. This can be achieved by using high quality optical cavities. By adjusting in a controlled way the “finesse” and the optical properties of the cavity, one should be able to reconstruct from the measured signal the time (or path-length) dependence of the probability $P_\theta(t)$, and therefore extract information on the dissipative parameters both from the damping factors and the oscillating terms.

The actual visibility of these parameters clearly depends on their magnitude. A precise a priori evaluation would require a detailed knowledge of string theory; nevertheless, an order of magnitude estimate can be obtained using the general theory of open systems. Indeed, quite in general the dissipative effects induced by the weak interaction with an external environment can be roughly evaluated to be at most proportional to the square of the typical energy scale of the system, while suppressed by an inverse power of the characteristic energy scale of the environment.

In the case of polarized photons, the system energy coincides with the average photon energy $\omega_0$, while the typical energy scale of the environment coincides with the mass $M_F$ that characterizes the fundamental, underlying dynamics. As a consequence, the values of the parameters $a, b, c, \alpha, \beta, \gamma$ can be predicted to be roughly of order $\omega_0^2/M_F$.

In the case of laboratory experiments using ordinary laser beams, the photon energy $\omega_0$ is fixed; therefore the expected magnitude of the new, non-standard effects is determined by the value of $M_F$. This fundamental scale can be as large as the Planck mass, but can also be considerably smaller in models of large extra dimensions. Fortunately, as stressed before, the description of decoherence phenomena by means of quantum dynamical semigroups is very general, and quite independent from the actual microscopic mechanism responsible for the appearance of the new effects. As a result, an experimental study of the transition probabilities discussed in the previous section can give model-independent indications of the presence of the non-standard, dissipative phenomena. In turn, this would allow the derivation of interesting bounds on the magnitude of the fundamental scale $M_F$, thus providing useful information on the underlying “stringy” dynamics.

The analysis of the previous sections have been limited to the study of dissipative evolutions for polarization states of a single photon. The whole treatment can be naively extended by linearity to include also the case of multi-photon states. Generalizing the one-photon dissipative dynamics generated by (2.2) to the case of multi-photon states is however not completely straightforward. Indeed, the photons obey the Bose statistics, and this property should be preserved by the time-evolution. It turns out that physically

† An additional discussion, although referred to the analysis of a specific experimental apparatus, can be found in Ref.[40].
acceptable multi-photon dissipative dynamics can not be simply expressed as the product of single-photon time-evolutions: a more refined treatment is necessary (see [41] for details). This fact might have interesting consequences in various aspects of quantum optics.

APPENDIX

As mentioned in the text, the evolution flow generated by the hamiltonian \( H(t) \) in (2.3) can be exactly integrated. The explicit expression for the corresponding unitary evolution operator \( U(t) \) is given by:

\[
U(t) = e^{-i\lambda t\sigma_3/2} \left[ \cos \Omega_0 t \sigma_0 - i \left( \frac{2\mu - \lambda}{2\Omega_0} \right) \sin \Omega_0 t \sigma_3 - \frac{i\nu}{\Omega_0} \sin \Omega_0 t \sigma_1 \right], \tag{A.1}
\]

where \( \Omega_0 = \left[ (\mu - \lambda/2)^2 + \nu^2 \right]^{1/2} \), while \( \sigma_1 \) and \( \sigma_3 \) are Pauli matrices and \( \sigma_0 \) the identity. Indeed, one easily verifies that:

\[
\dot{U}(t) = -iH(t)U(t), \quad U(0) = \sigma_0. \tag{A.2}
\]

Having the explicit solution of (A.2), one can now check directly the correctness of the adiabatic approximation used in Section 3. To this purpose, one needs to consider the appropriate evolution operator \( \tilde{U}(t) \) in the basis of the instantaneous eigenvalues of the hamiltonian \( H(t) \). The change of basis is provided by the transformation matrix

\[
T(t) = \frac{1}{\sqrt{2\omega(\mu + \omega)}} \begin{bmatrix} \mu + \omega & -\nu e^{-i\lambda t} \\ \nu e^{i\lambda t} & \mu + \omega \end{bmatrix}. \tag{A.3}
\]

In the limit of small \( \lambda \), one then easily verifies that the new evolution operator:

\[
\tilde{U}(t) = T^\dagger(t)U(t)T(0), \tag{A.4}
\]

indeed becomes diagonal:

\[
\tilde{U}(t) = \begin{bmatrix} e^{-i(\omega + \lambda_B) t} & 0 \\ 0 & e^{i(\omega + \lambda_B) t} \end{bmatrix}, \tag{A.5}
\]

where \( \lambda_B = \frac{\lambda}{2} \left( 1 - \frac{\mu}{\omega} \right) \) is precisely the Berry phase contribution.
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