In this study, two-dimensional circular thickness effects on time period of skew plate made with nonhomogeneous material properties are computed in variable temperature environment. Clamped (C), simply supported (S), and free-edge (F) conditions are the various combinations considered in this work. One-dimensional circular density and two-dimensional parabolic temperature variation on the plate are considered due to the nonhomogeneous nature of the material and variable temperature environment. Rayleigh–Ritz method is used to compute the time period for the first two modes of vibration. A convergence study of frequency modes of skew plate on various edge conditions has also been performed. The present work has been compared with the frequency modes of parallelogram and rectangle plate on various edge conditions. The objective of the study is to present a numerical data of time period as well as to show how one can control the frequencies of plate by taking an appropriate variation in plate parameters.

1. Introduction

A large number of structures/equipments are made up of plates, and this fact has prompted the researchers to do a detailed study of vibrational frequencies of plates. Almost, all structures while working undergo some vibrations. A large vibration is a kind of waste that can reduce the performance of structures/equipments. In order to increase the performance and reliability of structures/equipment, the vibration of the structure needs to be decreased. The various parameters such as variation in shape, size, tapering, non-homogeneity, and temperature of the plate play a crucial role in minimizing the frequencies. Studies regarding plate’s vibration have been well documented in the literature. In the available literature, for the thickness of the plate, researchers focused on linear, parabolic variation (one dimensional, two dimensional), parabolic variation (one dimensional, two dimensional), and circular (only in one dimensional) variation. To the best of the knowledge of the authors, the effect of two-dimensional circular variation in thickness on time period of frequency modes has not been studied yet. This prompted the authors to carry out the research to study the effect of two-dimensional circular thickness impact on time period of parallelogram plate.

Natural frequency of rectangular plates has been computed in [1] on various combinations of clamped, simply supported, and free-edge condition by applying Galerkin’s averaging technique. The first three modes of frequencies comprise the effect of foundation, density, thickness, and aspect ratio of nonhomogeneous orthotropic rectangle plate which have been computed in [2] using the Chebyshev collocation technique. The semianalytical method has been used in [3] to study the effect of skew angle, aspect ratio, orthotropy, and boundary conditions on the vibration of orthotropic skew plates. Rayleigh–Ritz and kinematic methods have been used in [4] to analyze the effects of material, number of layers, and fiber orientation on frequencies and mode shapes of anisotropic, simply supported
plates of square planform. Natural frequencies and mode shapes of sector plate on simply supported edges have been computed in [5] using an analytical solution. Multiple scales method has been applied as in [6] to study the vibration of rectangle plate (cracked, fluid medium), and the effect of crack length, fluid level, fluid density, and immersed depth of the plate have been presented. High accurate solution (using accurate series) for rectangle plate with all possible combinations on edge conditions has been presented in [7]. A detailed comprehensive study of vibration of plates, beams, and shell has been studied in [8] using Rayleigh–Ritz method. Natural vibration of nonhomogeneous rectangle plate has been studied in [9] using Rayleigh–Ritz method, and effect of various plate parameters on time period of parallelogram plate. Effect of circular simply supported, and free edges and presented the various nonhomogeneous, nonuniform circular plates on clamped, boundary conditions under variable temperature field. The authors also computed one-dimensional circular density and two-dimensional parabolic thickness impact on time period. All the numerical data are presented in the form of tables. In order to authenticate the findings and to present the objectives, a comparative analysis of frequency modes of parallelogram and rectangle plate with the available published results is also presented in tabular form.

2. Analysis

A skew (parallelogram) plate made with nonhomogeneous material property with variable thickness $l$ having skew angle $\theta$, length $a$, breadth $b$, density $\rho$, and Poisson’s ratio $\nu$ is considered referred to skew coordinates $\zeta = x - y \tan \theta, \psi = y \sec \theta$ (refer Figure 1).

The equation for kinetic energy $T_s$ and strain energy $V_s$ for natural transverse vibration of the nonuniform parallelogram is given by the expression as [16]

$$T_s = \frac{1}{2} \omega^2 \cos \theta \int \rho \Phi^2 d\zeta \, d\psi,$$

$$V_s = \frac{1}{2} \cos^2 \theta \int D_1 \left[ \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 - 4 \sin \theta \left( \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \right) \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right) \right. + 2 \left( \sin^2 \theta + \nu \cos^2 \theta \right) \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right) \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right) + 2 \left( 1 + \sin^2 \theta - \nu \cos^2 \theta \right) \left( \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \right)^2 \left. \right] d\zeta \, d\psi,$$

where $D_1 = (E t^3 / 12)(1 - \nu^2)$ is the flexural rigidity. Here, $\nu$, $E$, and $\Phi$ are Poisson’s ratio, Young’s modulus, and deflection function, respectively.

Rayleigh–Ritz method requires that maximum strain energy must be equal to maximum kinetic energy; that is,
\[ J = \delta(V_s - T_s) = 0. \] (3)

On substituting (1) and (2) in (3), we get

\[ J = \frac{1}{2} \cos^3 \theta \int_{D_1} \left[ \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 - 4 \sin \theta \left( \frac{\partial^3 \Phi}{\partial \zeta^2 \partial \psi} \right) \right. \]

\[ + 2 \left( \sin^2 \theta + \nu \cos^2 \theta \right) \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \frac{\partial \Phi}{\partial \psi} \right) \]

\[ \left. + 2 \left( 1 + \sin^2 \theta - \nu \cos^2 \theta \right) \left( \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \right)^2 \right] \]

\[ d\psi \times \frac{1}{2} \omega^2 \cos \theta \int \rho \Phi^2 d\zeta \ d\psi = 0. \] (4)

\[ \eta = \eta_0 \left( 1 - \frac{\zeta^2}{a^2} \right) \left( 1 - \frac{\psi^2}{b^2} \right), \] (6)

where \( \tau \) and \( \tau_0 \) denote the temperature excess above the reference temperature on the plate at any point and the origin, respectively. The temperature dependence modulus of elasticity for engineering structures is given by

\[ E = E_0 \left( 1 - \gamma \eta \right), \] (7)

where \( E_0 \) is the Young's modulus at mentioned temperature (i.e., \( \eta = 0 \)), and \( \gamma \) is the slope of variation.

Substituting (6) in (7), we get the following expression:

\[ E = E_0 \left[ 1 - \kappa \left( 1 - \frac{\zeta^2}{a^2} \right) \left( 1 - \frac{\psi^2}{b^2} \right) \right], \] (8)

where \( \kappa = \gamma \eta_0, \) (0 \( \leq \kappa < 1 \)) is called temperature gradient.

Using (5) and (8), (4) becomes
\[
\begin{align*}
J &= \frac{D_0}{2} \int_0^a \int_0^b \left[ 
\begin{array}{c}
1 - \kappa \left( 1 - \frac{\zeta^2}{a^2} \right) \left( 1 - \frac{\psi^2}{b^2} \right) \\
\left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 - 4Y \sin \theta \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right) \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right) \\
+ 2Y^2 \left( \sin^2 \theta + \nu \cos^2 \theta \right) \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right) \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right) \\
+ 2Y^2 \left( 1 + \sin^2 \theta - \nu \cos^2 \theta \right) \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 \\
- 4Y^2 \sin \theta \left( \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \right) \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right) \\
+ Y^4 \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right)^2 \\
\end{array} \right] \\
&\quad \times d\psi \ d\zeta - \lambda^2 \cos^4 \theta \int_0^a \int_0^b [1 - \varphi \Lambda_1],
\end{align*}
\]

(9)

The two-term deflection function that satisfies all the edge conditions can be taken as

\[
\Phi (\zeta, \psi) = \left[ \begin{array}{c} \zeta^4 \frac{e}{a} \binom{\psi}{b} \left( 1 - \frac{\zeta}{a} \right)^{\beta} \left( 1 - \frac{\psi}{b} \right)^{\eta} \\
\sum_{i=0}^{N} \Omega_i \left[ \begin{array}{c} \zeta^4 \frac{e}{a} \binom{\psi}{b} \left( 1 - \frac{\zeta}{a} \right)^{\beta} \left( 1 - \frac{\psi}{b} \right)^{\eta} \\
\end{array} \right] \end{array} \right],
\]

(11)

where

\[
D_0 = \left( E_0 \rho_0^2 / 12(1 - \nu^2) \right), \quad \Lambda_1 = \left( 1 - \sqrt{1 - \frac{\zeta^2}{a^2}} \right),
\]

\[
\Lambda_2 = \left( 1 - \sqrt{1 - \frac{\psi^2}{b^2}} \right), \quad \lambda^2 = \rho_0 \omega^2 L_0 a^4 / D_0 \text{ and } Y = (a/b).
\]

(10)
which is the product of two functions. Here, the first function represents the boundary conditions depending on the value of \( e, f, g, \) and \( h \), which can take different values depending upon the support edge condition. Values 0, 1, 2 are assigned for free edge, simply supported, and clamped edge, respectively. The second function represents the number of modes of frequencies, and \( \Omega_i, i = 0, 1, 2, \ldots, N \) represents arbitrary constants.

In order to minimize the function given in (9), we require the following condition:

\[
\frac{\partial J}{\partial \Omega_i} = 0, \quad i = 0, 1, 2, 3 \ldots N.
\]  

(12)

After simplifying (12), we get a homogeneous system of equations in \( \Omega \), whose nonzero solution gives the equation of frequency as

\[
|P - \lambda^2 Q| = 0,
\]

(13)

where \( P = [p_{ij}]_{N \times 1} \) and \( Q = [q_{ij}]_{N \times 1} \) are the square matrix of order \((n + 1), i = 0, 1, 2 \ldots N, \) and \( j = 0, 1, 2 \ldots N \).

The following expression is used for calculating the time period:

\[
K = \frac{2\pi}{\lambda},
\]

(14)

where \( \lambda \) is a frequency obtained from (13).

3. Numerical Results and Discussion

Time period \( K \) of parallelogram plate on CCCC, CCCS, CCSS, CSSS, SSSS, CSCS, and SCSC edge conditions (refer Figure 3) is computed and shown the effect of various plate parameters (especially two-dimensional circular thickness impact) on the behavior of time period of frequency modes with the help of tables. The value of aspect ratio \((a/b) = 1.5, \) skew angle \( \theta = 30^\circ, \) and Poisson’s ratio \( \nu = 0.345 \) is taken throughout the calculation.

Table 1 summarizes the time period \( K \) of frequency modes of parallelogram plate on CCCC, CCCS, CCSS, CSSS, SSSS, CSCS, and SCSC edge conditions corresponding to tapering parameters \( \omega_1, \omega_2 \) for a fixed value of thermal gradient \( \kappa = 0.2 \) and nonhomogeneity \( \varphi = 0.4. \) From Table 1, the following conclusions can be made:

(i) Time period \( K \) decreases for both the increasing value of tapering parameters \( \omega_1, \omega_2 \) on each edge condition.

(ii) The rate of decrement in time period \( K \) in case of tapering \( \omega_2 \) is more in comparison to the rate of decrement in time period \( K \) in case of tapering \( \omega_1 \) on all edge conditions.

(iii) The time period \( K \) is higher on SCSC edge condition and lower on CCSS edge condition. But the rate of decrement in time period \( K \) is less in CCSS edge condition and higher on CSSS edge condition.

(iv) The order of increase in time period \( K \) for different edge conditions is CCSS < CSSS < CSCS < SSSS < CCSC < SCSC, corresponding to tapering parameters \( \omega_1, \omega_2. \)

Table 2 represents the time period \( K \) of frequency modes of parallelogram plate on CCCC, CCCS, CCSS, CSSS, SSSS, CSCS, and SCSC edge conditions corresponding to thermal gradient \( \kappa \) for a fixed value of nonhomogeneity \( \varphi = 0.4 \) and variable values of tapering parameters \( \omega_1, \omega_2 \) from 0.0 to 1.0. From Table 2, the following facts can be interpreted:

(i) Increase in the thermal gradient \( \kappa \) results in an increase in the time period \( K, \) but the increasing value of both tapering parameters \( \omega_1, \omega_2 \) results in the decrease in time period \( K, \) on all edge conditions.

(ii) The rate of increment in time period \( K \) in case of thermal gradient \( \kappa \) is less than that in case of tapering parameters \( \omega_1, \omega_2, \) on all edge conditions.

(iii) Here, also the time period \( K \) is higher on SCSC edge condition and lower on CCSS edge condition as shown in Table 1. But the rate of increment in time period \( K \) is less on CSSS edge condition and higher on SCSC edge condition.

(iv) The time period \( K \) for different edge conditions corresponding to thermal gradient \( \kappa \) is in the following ascending order: CCSS < CSSS < CSCS < SSSS < CSSS < CCSC < SCSC.

The time period \( K \) of frequency modes of parallelogram plate on CCCC, CCCS, CCSS, CSSS, SSSS, CSCS, and SCSC edge conditions corresponding to non-homogeneity \( \varphi \) for a fixed value of thermal gradient \( \kappa = 0.4 \) and variable values of tapering parameters \( \omega_1, \omega_2 \) from 0.0 to 1.0 is summarized in Table 3. From Table 3, it can be concluded that

(i) Time period \( K \) decreases with increasing value of nonhomogeneity \( \varphi \) as well as for the increasing
value of both tapering parameters $\omega_1$, $\omega_2$, for each edge conditions.

(ii) The rate of decrement in time period $K$ in case of both the tapering parameters $\omega_1$, $\omega_2$ is higher when compared with a rate of decrement in time period $K$ for nonhomogeneity $\varphi$, on all edge conditions.

(iii) Here, also the time period $K$ is higher on SCSC edge condition and lower on CCCS edge condition as shown in Tables 1 and 2. But the rate of decrement in time period $K$ is lower on CCCS edge condition and higher on CSSS edge condition. The time period $K$ for different edge conditions increases in the following sequence: CSSS < CCSS < CSCS < SSSS < CCCS < CSSS < SCSS, corresponding to nonhomogeneity $\varphi$.

4. Convergence of Results

This section reports convergence study done on frequency modes of parallelogram plate on CCCS, CCSS, CSSS, CSCS, SSSS, CSSS, and SCSS edge condition, for the plate parameters in range specified, that is, $\omega_1 = \omega_2 = \varphi = \kappa = 0.2$, $\varphi = 0.4$.

| $\omega_1$ | $\omega_1 = 0$ | $\omega_1 = 0.2$ | $\omega_1 = 0.4$ | $\omega_1 = 0.6$ | $\omega_1 = 0.8$ | $\omega_1 = 1.0$ |
|---|---|---|---|---|---|---|
| $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 0.0 | 0.0215 | 0.0862 | 0.0208 | 0.0819 | 0.0190 | 0.0764 | 0.0177 | 0.0718 | 0.0164 | 0.0679 | 0.0157 | 0.0639 |
| 0.0 | 0.0209 | 0.0834 | 0.0197 | 0.0785 | 0.0185 | 0.0740 | 0.0172 | 0.0693 | 0.0161 | 0.0657 | 0.0153 | 0.0619 |
| 0.4 | 0.0203 | 0.0806 | 0.0190 | 0.0768 | 0.0179 | 0.0718 | 0.0167 | 0.0673 | 0.0158 | 0.0637 | 0.0147 | 0.0601 |
| 0.6 | 0.0196 | 0.0779 | 0.0185 | 0.0738 | 0.0177 | 0.0693 | 0.0165 | 0.0650 | 0.0155 | 0.0615 | 0.0145 | 0.0580 |
| 0.8 | 0.0190 | 0.0753 | 0.0179 | 0.0717 | 0.0164 | 0.0674 | 0.0157 | 0.0633 | 0.0149 | 0.0600 | 0.0140 | 0.0568 |
| 1.0 | 0.0185 | 0.0728 | 0.0173 | 0.0684 | 0.0161 | 0.0649 | 0.0152 | 0.0618 | 0.0143 | 0.0588 | 0.0135 | 0.0559 |

Table 1: Time period of parallelogram plate on various edge conditions vs tapering parameters $\omega_1$, $\omega_2$ for a fixed value of aspect ratio $(a/b) = 1.5$. 

$K$ is lower on CCCS edge condition and higher on SSSS edge condition. $K$ is higher on CCSS and lower on SCSS edge condition.

\[ \kappa = \frac{1}{a} \]
Table 2: Time period of parallelogram plate on various edge conditions vs thermal gradient $\kappa$ for a fixed value of aspect ratio $(a/b) = 1.5$.

| $\kappa$ | $\omega_1 = \omega_2 = 0.0$ | $\omega_1 = \omega_2 = 0.2$ | $\omega_1 = \omega_2 = 0.4$ | $\omega_1 = \omega_2 = 0.6$ | $\omega_1 = \omega_2 = 0.8$ | $\omega_1 = \omega_2 = 1.0$ |
|----------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
|          | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 0.0      | 0.02057 | 0.08228 | 0.01886 | 0.07530 | 0.01723 | 0.06885 | 0.01572 | 0.06298 | 0.01434 | 0.05768 | 0.01308 | 0.05294 | 0.01428 | 0.05768 | 0.01308 | 0.05294 | 0.01428 | 0.05768 |
| 0.2      | 0.02156 | 0.08629 | 0.01967 | 0.07865 | 0.01970 | 0.07168 | 0.01627 | 0.06540 | 0.01479 | 0.05978 | 0.01345 | 0.05477 | 0.01428 | 0.05768 | 0.01308 | 0.05294 | 0.01428 | 0.05768 |
| CCCC     | 0.4   | 0.02271 | 0.09096 | 0.02060 | 0.08253 | 0.01865 | 0.07493 | 0.01688 | 0.06816 | 0.01528 | 0.06215 | 0.01386 | 0.05683 | 0.01428 | 0.05768 | 0.01308 | 0.05294 | 0.01428 | 0.05768 |
| 0.6      | 0.02406 | 0.09651 | 0.02168 | 0.08708 | 0.01971 | 0.07872 | 0.01767 | 0.07135 | 0.01583 | 0.06848 | 0.01430 | 0.05919 | 0.01428 | 0.05768 | 0.01308 | 0.05294 | 0.01428 | 0.05768 |
| 0.8      | 0.02569 | 0.10326 | 0.02294 | 0.09254 | 0.02049 | 0.08321 | 0.01833 | 0.07512 | 0.01644 | 0.06808 | 0.01478 | 0.06194 | 0.01428 | 0.05768 | 0.01308 | 0.05294 | 0.01428 | 0.05768 |

Table 2 presents the comparison of frequency modes of parallelogram plate on SSSS, CCCC, SCSC and CSCS edge conditions. The results are shown in Table 3. Table 3 also concludes that the frequency modes of parallelogram plate converge up to five decimal places in the fifth approximation.

5. Results Comparison

In order to validate the findings of the present study as well as to present the objective of the study, a comparison of frequency modes obtained in the present study was done with the following:

(i) Frequency modes of parallelogram plate obtained in [11] on CCCC and CSCS edge condition corresponding to tapering parameters $\varphi_1$, $\varphi_2$, and aspect ratio $(a/b)$.

(ii) Frequency modes of rectangle plate obtained in [17, 18] on CCCC, SSSS, and CSCS edge condition corresponding to thermal gradient $\kappa$. The results are reported in tabular form (refer Tables 6 and 7 for parallelogram plate and refer Tables 8 and 9 for rectangle plate). For comparison of frequency modes of rectangle plate and obtained in [17, 18], the authors exclude the nonhomogeneity $\varphi$ in the present study as it was not considered in [17, 18].

Table 6 presents the comparison of frequency modes of parallelogram plate and the frequency obtained in [11] on CCCC and CSCS edge condition corresponding to tapering parameters $\varphi_1$, $\varphi_2$, and aspect ratio $(a/b) = 1.5$. Table 6 clearly shows that the frequency modes of parallelogram plate is less in the fifth approximation.

The convergence of frequency modes of parallelogram plate on SSSS, CCCC, and CSCS edge conditions is presented in Table 7. Table 7 also concludes that the frequency modes of parallelogram plate converge up to five decimal places in the fifth approximation.

The frequency modes of parallelogram plate converge up to five decimal places in the fifth approximation. The convergence of frequency modes of parallelogram plate on SSSS, CCCC, and CSCS edge conditions is presented in Table 7. Table 7 also concludes that the frequency modes of parallelogram plate converge up to five decimal places in the fifth approximation.

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when compared with the frequency modes obtained in [11] with the increasing value of tapering parameters $\bar{\omega}_1, \bar{\omega}_2$ on both CCCC and CSCS edge conditions.

A comparison of frequency modes of the present study (parallelogram plate) and the frequency modes obtained in [11] is presented in Table 7 on CCCC and CSCS edge conditions.
Table 6: Comparison of frequency modes of the present study (parallelogram plate) and obtained in [11] corresponding to tapering parameters $\omega_1, \omega_2$ for a fixed value of thermal gradient $\kappa = 0.0$, nonhomogeneity $\varphi = 0.0$, skew angle $\theta = 30^\circ$, and aspect ratio $(a/b) = 1.5$.

| $\omega_1 \lambda_1$ | $\omega_2 = 0.0 \lambda_2$ | $\omega_2 = 0.2 \lambda_2$ | $\omega_2 = 0.4 \lambda_2$ | $\omega_2 = 0.6 \lambda_2$ | $\omega_2 = 0.8 \lambda_2$ | $\omega_2 = 1.0 \lambda_2$ |
|----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.0                  | 294.60 74.00   | 311.99 78.29   | 331.52 82.97   | 352.97 87.97   | 376.12 93.22   | 400.76 98.69   |
| 0.1                  | 303.38 76.45   | 321.15 80.84   | 341.12 85.63   | 363.06 90.74   | 386.75 96.12   | 411.98 101.71  |
| 0.2                  | 324.47 81.52   | 358.75 90.10   | 396.58 99.50   | 437.05 109.47  | 479.49 119.85  | 523.43 130.52  |
| 0.4                  | 312.54 79.01   | 330.70 83.50   | 351.12 88.39   | 373.58 93.62   | 397.84 99.12   | 423.68 104.84  |
| 0.6                  | 355.02 89.24   | 392.23 98.58   | 433.33 108.80  | 477.33 119.65  | 523.49 130.95  | 571.30 142.57  |
| 0.8                  | 322.04 81.68   | 340.61 86.26   | 361.51 91.26   | 384.50 96.61   | 409.34 102.42  | 435.83 108.09  |
| CCCC                 | 386.09 97.12   | 426.28 107.22  | 470.72 118.28  | 518.31 130.03  | 568.27 142.26  | 620.02 154.85  |
| 0.6                  | 331.87 84.43   | 350.86 89.11   | 372.25 94.23   | 395.79 99.70   | 421.24 105.45  | 448.39 111.44  |
| 0.8                  | 417.56 105.12  | 460.79 116.00  | 508.61 127.91  | 559.84 140.57  | 613.64 153.75  | 669.38 167.32  |
| 1.0                  | 342.00 87.27   | 361.43 90.25   | 383.32 97.28   | 407.43 102.87  | 433.31 108.75  | 461.33 114.88  |
|                      | 449.35 113.21  | 495.64 124.88  | 546.88 137.65  | 601.80 151.23  | 659.48 165.37  | 719.27 179.93  |

Bold font values are obtained in [11].

Table 7: Comparison of frequency modes of the present study (parallelogram plate) and obtained in [11] corresponding to aspect ratio $(a/b)$ for variable values of tapering parameters $\omega_1, \omega_2$, a fixed value of thermal gradient $\kappa = 0.0$, nonhomogeneity $\varphi = 0.0$, and skew angle $\theta = 30^\circ$.

| $\omega_1 \lambda_1$ | $\omega_2 = 0.0 \lambda_2$ | $\omega_2 = 0.2 \lambda_2$ | $\omega_2 = 0.4 \lambda_2$ | $\omega_2 = 0.6 \lambda_2$ | $\omega_2 = 0.8 \lambda_2$ | $\omega_2 = 1.0 \lambda_2$ |
|----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.25                 | 1960.78 478.17  | 2141.81 522.49 | 2348.03 571.79 | 2579.23 625.74 | 2835.11 684.09 | 3115.34 746.68 |
|                      | 498.67 122.88  | 544.32 134.25 | 596.11 146.85 | 653.99 160.59 | 717.91 175.42 | 787.78 191.28 |
| 0.50                 | 498.67 122.88  | 607.62 149.66 | 734.95 180.83 | 880.55 216.28 | 1044.30 255.95 | 1226.09 299.75 |
|                      | 245.09 62.31   | 266.95 68.07  | 291.55 74.42  | 318.88 81.32  | 348.91 88.77  | 381.63 96.61  |
| CCCC                 | 194.81 50.56   | 236.95 65.41  | 285.59 75.47  | 340.62 88.66  | 402.07 104.72 | 469.92 122.44 |
| 1.0                  | 194.81 50.56   | 236.95 65.41  | 285.59 75.47  | 340.62 88.66  | 402.07 104.72 | 469.92 122.44 |
|                      | 225.15 57.63   | 245.11 62.96  | 267.52 68.83  | 292.38 75.20  | 319.68 82.03  | 349.40 89.31  |
| 1.25                 | 225.15 57.63   | 274.03 70.16  | 330.56 84.68  | 394.75 101.14 | 466.57 119.53 | 546.01 139.80 |
| 1.50                 | 294.60 74.00   | 321.15 80.84  | 351.12 88.39  | 384.50 96.61  | 421.24 105.45 | 461.33 114.88 |
|                      | 294.60 74.00   | 358.75 90.10  | 433.33 108.80 | 518.31 130.03 | 613.64 153.75 | 719.27 179.93 |

Bold font values are obtained in [11].
conditions corresponding to aspect ratio $a/b$ and tapering parameters $\omega_1, \omega_2$ for a fixed value of thermal gradient $\kappa = 0.0$ and nonhomogeneity $\varphi = 0.0$ and skew angle $\theta = 30^\circ$. Here also, the frequency modes obtained in the present study (parallelogram plate) are less when compared with the frequency modes obtained in [11] with the increasing value of aspect ratio $(a/b)$ and tapering parameters $\omega_1, \omega_2$ on both CCC and CSCS edge conditions.

Table 8 shows the comparison of frequency modes of rectangle plate (by taking skew angle $\theta = 0^\circ$ in the present study) and obtained in [17] on CCC, SSS, and CSCS edge conditions corresponding to tapering parameters $\omega_1, \omega_2$ for a fixed value of $\kappa = 0.0$ and aspect ratio $(a/b) = 1.5$.

Table 8: Comparison of frequency modes of the present study (rectangle plate) and obtained in [17] corresponding to tapering parameters $\omega_1, \omega_2$ for a fixed value of thermal gradient $\kappa = 0.0$ and aspect ratio $(a/b) = 1.5$.

| $\omega_2$ | $\omega_1 = 0.0$ | $\omega_1 = 0.2$ | $\omega_1 = 0.4$ | $\omega_1 = 0.6$ | $\omega_1 = 0.8$ |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|            | $\lambda_1$    | $\lambda_2$    | $\lambda_1$    | $\lambda_2$    | $\lambda_1$    | $\lambda_2$    |
| 0.0        | 217.54          | 54.15           | 223.98          | 55.93           | 230.70          | 57.782          | 237.66          | 59.701          | 244.86          | 61.68           |
|            | 240.47          | 60.85           | 271.43          | 68.92           | 287.81          | 73.31           | 305.20          | 77.97           | 323.46          | 82.85           |
| 0.2        | 230.55          | 57.31           | 237.37          | 59.18           | 244.48          | 61.12           | 251.84          | 63.14           | 259.46          | 65.22           |
|            | 277.75          | 70.26           | 294.19          | 74.65           | 311.85          | 79.36           | 330.58          | 84.35           | 350.23          | 89.56           |
| CCC        | 0.4            | 245.16          | 60.76           | 252.41          | 62.73           | 259.96          | 64.77           | 267.78          | 66.90           | 275.87          | 69.08           |
|            | 302.35          | 76.32           | 320.15          | 81.04           | 339.26          | 86.10           | 359.50          | 91.44           | 380.72          | 97.02           |
| 0.6        | 261.22          | 64.44           | 268.93          | 66.52           | 276.97          | 68.68           | 285.29          | 70.91           | 293.90          | 73.21           |
|            | 329.48          | 82.85           | 348.78          | 87.94           | 369.47          | 93.37           | 391.37          | 99.09           | 414.32          | 105.07          |
| 0.8        | 278.55          | 68.31           | 286.77          | 70.50           | 295.32          | 72.78           | 304.19          | 75.13           | 313.35          | 77.55           |
|            | 358.70          | 89.75           | 379.61          | 95.21           | 402.00          | 101.04          | 425.68          | 107.17          | 450.47          | 113.38          |

Table 9: Comparison of frequency modes of the present study (rectangle plate) and obtained in [18] corresponding to thermal gradient $\kappa$ for a fixed value of tapering parameters $\omega_1 = \omega_2 = 0.0$ and aspect ratio $(a/b) = 1.5$.

| $\kappa$ | CCC | SSS | CSS | CSCS | CCS |
|----------|-----|-----|-----|------|-----|
|          | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ |
| 0.0      | 217.54 | 54.15 | 220.03 | 66.66 | 286.23 | 41.21 | 256.84 | 28.98 | 435.88 | 59.96 |
|          | 256.20 | 64.83 | 256.92 | 34.25 | 260.97 | 39.75 | 288.81 | 41.94 | 357.65 | 55.24 |
| 0.4      | 196.84 | 48.92 | 200.96 | 66.66 | 265.28 | 38.04 | 234.30 | 25.96 | 425.66 | 55.07 |
|          | 243.05 | 61.50 | 243.73 | 32.50 | 252.60 | 38.37 | 273.99 | 39.79 | 353.70 | 53.12 |
| 0.8      | 173.70 | 43.01 | 179.87 | 20.80 | 242.52 | 34.58 | 209.34 | 22.52 | 415.19 | 49.65 |
|          | 229.15 | 57.98 | 229.79 | 30.64 | 243.95 | 36.92 | 258.32 | 37.51 | 349.71 | 50.91 |

Bold font values are obtained in [17].

A comparison of frequency modes of rectangle plate (by taking skew angle $\theta = 0^\circ$ in the present study) and obtained in [18] on CCC, SSS, CSS, CSCS, and CCS edge conditions corresponding to thermal gradient $\kappa$ for fixed value of tapering parameters $\omega_1, \omega_2$.
tapering parameters $\varpi_1 = \varpi_2 = 0.0$ and aspect ratio $(a/b) = 1.5$ is shown in Table 9. Here also, one can easily see that the frequency modes of the rectangle plate are very less in comparison to the frequency modes obtained in [18] on all the above edge conditions with the increasing value of thermal gradient $\kappa$.

6. Conclusion

The effect of various plate parameters (especially the effect of two-dimensional tapering parameters $\varpi_1, \varpi_2$) on time period of the first two modes of frequency of parallelogram plate on various boundary conditions is computed. Based on the results obtained from numerical simulation and comparison done, the authors would like to conclude the following facts:

(a) Frequency modes of parallelogram plate in case of two-dimensional circular thickness in this study are less when compared with the frequency modes obtained in [11] in case of two-dimensional linear thickness on both CCCC and CSCS edge conditions. The frequency modes of the present study and obtained in [11] match at $\varpi_1 = \varpi_2 = 0.0$ on both CCCC and CSCS edge conditions (refer Table 6).

(b) Frequency modes of parallelogram plate in case of aspect ratio\'s impact on two-dimensional circular thickness in the current study provide less frequency when compared with frequency modes obtained in [11], in case of aspect ratio\'s impact on the two-dimensional linear thickness on CCCC and CSCS edge conditions. The frequency modes of the present study obtained in [11] coincide at $(a/b) = 0.25$ on CCCC and CSCS edge conditions (refer Table 7).

(c) Frequency modes of rectangle plate in case of two-dimensional circular thickness (present study) are very less in comparison to frequency modes obtained in [17] in case of two-dimensional linear thickness on CCCC, SSSS, and CSCS edge conditions (refer Table 8).

(d) In the current study, frequency modes of rectangle plate in case of two-dimensional parabolic temperature are very less as compared to frequency modes obtained in [18] in case of two-dimensional linear temperature on CCCC, SSSS, CSSS, CSCS, and CCSCS edge conditions (refer Table 9).

(e) Tapering parameter $\varpi_2$ dominates the time period (rate of decrement) more than the tapering parameter $\varpi_1$ (refer Table 1).

(f) Increase in both the tapering and nonhomogeneity on the plate results in the decrease in time period (refer Tables 1 and 3) while an increase in thermal gradient on plate results in an increase in the time period (refer Table 2).

(g) We can minimize the vibrational frequencies of the plate by choosing appropriate variation in plate parameters (refer points a to d). We can also control the variation (rate of increment/decrement) in time period of frequency modes by choosing appropriate variation in plate parameters (refer points e and f).

(h) A numerical data in the form of time period is calculated which shows the impact of various plate parameters.

Data Availability

The research data used to support the findings of this study are currently under embargo, while the research findings are commercialized. Requests for data, 6 months after the publication of this article, will be considered by the corresponding authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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