On the fermionic T-duality of the $AdS_4 \times \mathbb{CP}^3$ sigma-model

Ido Adam,$^a$ Amit Dekel$^b$ and Yaron Oz$^b$

$^a$Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut),
Am Mühlenberg 1, D-14476 Golm, Germany
$^b$Raymond and Beverly Sackler School of Physics and Astronomy, Tel-Aviv University,
Ramat-Aviv 69978, Israel

E-mail: idoadam@aei.mpg.de, amitde@post.tau.ac.il, yaronoz@post.tau.ac.il

ABSTRACT: In this note we consider a fermionic T-duality of the coset realization of the type IIA sigma-model on $AdS_4 \times \mathbb{CP}^3$ with respect to the three flat directions in $AdS_4$, six of the fermionic coordinates and three of the $\mathbb{CP}^3$ directions. We show that the Buscher procedure fails as it leads to a singular transformation and discuss the result and its implications.

KEYWORDS: Duality in Gauge Field Theories, String Duality

ArXiv ePrint: 1008.0649
1 Introduction and summary

Since the $\mathcal{N} = 6$ superconformal Chern-Simons theory with matter was proposed by ABJM [1] as a dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$, which reduces in a certain limit to the type IIA superstring on $AdS_4 \times \mathbb{C}P^3$, much work has been devoted to understanding the properties of the ABJM field theory.

Several tree-level scattering amplitudes of the ABJM theory were computed [2] and were shown to possess a Yangian symmetry, which includes the non-local charges and the dual superconformal symmetry [3]. Some light-like polygonal Wilson loops in the ABJM theory were computed in [4] and hinted that the ABJM theory may have a scattering amplitudes/Wilson loop duality, which would further support the case in favor of the existence of dual superconformal symmetry. Additionally, a contour integral reproducing the known tree-level amplitudes has been recently proposed and was shown to have a Yangian symmetry [5]. Furthermore, a differential representation of a dual superconformal symmetry at tree-level has been constructed [6]. This representation involves variables dual to the ones parameterizing part of the R-symmetry in addition to the ones dual to the bosonic and fermionic momenta.

The corresponding findings in $\mathcal{N} = 4$ SYM in four dimensions were explained from the point of view of string theory on $AdS_5 \times S^5$ by a combination of bosonic and fermionic T-dualities, which is exact at the string tree-level [7, 8] (see [9] for a short review). Hence, it is interesting to see whether that is also the case for type IIA strings on $AdS_4 \times \mathbb{C}P^3$. Previously, it was found that the sigma-model for $AdS_4 \times \mathbb{C}P^3$, realized as the coset $\text{OSp}(6|4)/(\text{SO}(2,1) \times \text{U}(3))$ constructed in [10, 11], was not self-dual under T-duality involving both three directions in $AdS_4$ and six fermionic coordinates [12, 13]. In fact, one could not perform a fermionic T-duality in six fermionic isometries which together with the dualized bosonic ones form an Abelian subgroup of the whole isometry group.

In this note, in light of a suggestion that T-dualizing three isometries of $\mathbb{C}P^3$ is also required [3] and the new evidence [5, 6] from the field theory, we consider the fermionic T-duality along the three flat $AdS_4$ coordinates, three complex Killing vectors in $\mathbb{C}P^3$ (each one of real dimension one) as well as six of the fermionic coordinates, whose corresponding
tangent-space vectors generate an Abelian subgroup of the isometry group. We show that as in the case of dualizing just in $AdS_4$ and the fermions, the Buscher procedure fails as it leads to a singular transformation [12].

The outline of this note is as follows: in section 2 we apply the Buscher procedure for $T$-duality to the $OSp(6|4)/(SO(2,1) \times U(3))$ Green-Schwarz sigma-model describing type IIA strings on $AdS_4 \times \mathbb{CP}^3$ in a certain partial gauge-fixing and show that it fails. In section 3 we discuss the implications of the result. The $osp(6|4)$ algebra is given in appendix A.

2 T-dualizing $AdS_4 \times \mathbb{CP}^3$

We attempt to T-dualize $AdS_4 \times \mathbb{CP}^3$ along the directions corresponding to $P_a, Q_{\ell \alpha}, R_{kl}$, which form an Abelian subalgebra of the isometry group.

We assume that $\kappa$-symmetry can be partially gauge-fixed to set the six coordinates corresponding to $S^i_\alpha$ to zero and choose the coset representative
\begin{align}
g &= e^{x^a P_a + \theta^\alpha Q_{\ell \alpha} + y^{kl} R_{kl}} e^B, \\
e^B &= e^{\theta^\alpha \tilde{Q}_\alpha + \xi_\alpha S_{\ell \alpha} y^D \tilde{y}_{\ell}} e^{\tilde{R}_{kl}},
\end{align}
where the indices $a = 0,1,2$ run over the flat directions of $AdS_4$, $\alpha = 1,2$ are $AdS_4$ spinor indices and $\ell = 1,2,3$ are $U(3)$ fundamental representation indices (see appendix A for further details). The Maurer-Cartan one-form is
\begin{align}
K = J + j, \\
J = e^{-B} (dx^a P_a + d\theta^\alpha Q_{\ell \alpha} + dy^{kl} R_{kl}) e^B, \\
j = e^{-B} de^B.
\end{align}

Examining the algebra, one finds that the current $J$ takes values in the space spanned by $\{P_a, Q_{\ell \alpha}, R_{kl}, \tilde{Q}_\alpha^l, \lambda_{\ell}^l, \tilde{R}^{kl}\}$, while $j$ is valued in span $\{\tilde{Q}_\alpha^l, S_{\ell \alpha}, \tilde{S}_\alpha^l, D, M_{\alpha \beta}, \lambda_{\ell}^l, \tilde{R}^{kl}\}$.

Denoting the decomposition of $K$ into the $\mathbb{Z}_4$-invariant subspaces by $K_i \in \mathcal{H}_i$, the Green-Schwarz action takes the form
\begin{align}
S = \frac{R^2}{4\pi \alpha'} \int d^2 z \left\{ -\frac{1}{2} g_{ab} J_{P_a} \tilde{J}_{P_b} - J_{\tilde{D}} \tilde{J}_{\tilde{D}} - 2 J_{R_{kl}} (\tilde{J}_{\tilde{R}^{kl}} + \tilde{J}_{\tilde{R}^{kl}}) - 2 \tilde{J}_{\tilde{R}^{kl}} (J_{\tilde{R}^{kl}} + \tilde{J}_{\tilde{R}^{kl}}) - \frac{i}{2} C_{\alpha \beta} \left[ J_{Q_{\ell \alpha}} (\tilde{J}_{\tilde{Q}_{\ell}^a} + \tilde{J}_{\tilde{Q}_{\ell}^a}) - (J_{\tilde{Q}_{\ell}^a} + J_{\tilde{Q}_{\ell}^a}) \tilde{J}_{\tilde{Q}_{\ell}^a} - J_{S_{\ell \alpha}} \tilde{S}_{\ell}^a + J_{S_{\ell \alpha}} \tilde{S}_{\ell}^a \right] \right\}.
\end{align}

We attempt to T-dualize the action by using the Buscher procedure [14, 15] by introducing the new fields $A^a, A^{\alpha \beta}, A^{kl}, \bar{A}^a, \bar{A}^{\alpha \beta}$ and $\bar{A}^{kl}$ such that the current now reads
\begin{align}
J = e^{-B} (A^a P_a + A^{\alpha \beta} Q_{\ell \alpha} + A^{kl} R_{kl}) e^B,
\end{align}
while $j$, which does not contain $x^a$, $\theta^\alpha$ and $y^{kl}$, remains unmodified. In addition, the following Lagrange multiplier terms are added to the action:
\begin{align}
S_L = \frac{R^2}{4\pi \alpha'} \int d^2 z \left[ \tilde{x}_a (\partial A^a - \partial \bar{A}^a) + \tilde{\theta}_{\ell \alpha} (\partial A^{\alpha \beta} - \partial \bar{A}^{\alpha \beta}) + \tilde{y}_{kl} (\partial A^{kl} - \partial \bar{A}^{kl}) \right],
\end{align}
where $\tilde{x}_a, \tilde{\theta}_{\ell \alpha}$ and $\tilde{y}_{kl}$ are Lagrange multipliers.
The T-duality is performed by integrating out the gauge fields, whose equations of motion are

\[
0 = -\frac{1}{2} h_{bc} [e^{-B} P_a e^B] P_b J_{c} + \frac{i}{2} C_{\alpha\beta} \left[ [e^{-B} P_a e^B] Q_{\alpha} (J_{\beta} + \delta J_{\beta}) - [e^{-B} P_a e^B] Q_{\beta} (J_{\alpha} + \delta J_{\alpha}) - 2 [e^{-B} P_a e^B] R_{\alpha} (J_{\beta} + \delta J_{\beta}) - 2 [e^{-B} P_a e^B] R_{\beta} (J_{\alpha} + \delta J_{\alpha}) \right] - \partial \bar{\theta}_{\alpha} ,
\]

\[
0 = -\frac{1}{2} h_{bc} [e^{-B} Q_{\alpha e^B} R_{\beta} J_{c} + \frac{i}{2} C_{\alpha\beta} \left[ [e^{-B} Q_{\alpha e^B} R_{\beta}] Q_{\alpha} (J_{\beta} + \delta J_{\beta}) - [e^{-B} Q_{\alpha e^B} R_{\beta}] Q_{\beta} (J_{\alpha} + \delta J_{\alpha}) - 2 [e^{-B} Q_{\alpha e^B} R_{\beta}] R_{\alpha} (J_{\beta} + \delta J_{\beta}) - 2 [e^{-B} Q_{\alpha e^B} R_{\beta}] R_{\beta} (J_{\alpha} + \delta J_{\alpha}) \right] - \partial \bar{\theta}_{\alpha} ,
\]

\[
0 = -\frac{1}{2} h_{bc} [e^{-B} R_{k l e^B} P_{b} J_{c} + \frac{i}{2} C_{\alpha\beta} \left[ [e^{-B} R_{k l e^B} P_{b}] Q_{\alpha} (J_{\beta} + \delta J_{\beta}) - [e^{-B} R_{k l e^B} P_{b}] Q_{\beta} (J_{\alpha} + \delta J_{\alpha}) - 2 [e^{-B} R_{k l e^B} P_{b}] R_{\alpha} (J_{\beta} + \delta J_{\beta}) - 2 [e^{-B} R_{k l e^B} P_{b}] R_{\beta} (J_{\alpha} + \delta J_{\alpha}) \right] + \partial \bar{\theta}_{\alpha} ,
\]

for the holomorphic fields and

\[
0 = -\frac{1}{2} h_{bc} [e^{-B} P_a e^B] P_b J_{c} - \frac{i}{2} C_{\alpha\beta} \left[ [e^{-B} P_a e^B] Q_{\alpha} (J_{\beta} + \delta J_{\beta}) - [e^{-B} P_a e^B] Q_{\beta} (J_{\alpha} + \delta J_{\alpha}) - 2 [e^{-B} P_a e^B] R_{\alpha} (J_{\beta} + \delta J_{\beta}) - 2 [e^{-B} P_a e^B] R_{\beta} (J_{\alpha} + \delta J_{\alpha}) \right] - \partial \bar{\theta}_{\alpha} ,
\]

\[
0 = -\frac{1}{2} h_{bc} [e^{-B} Q_{\alpha e^B} R_{\beta} J_{c} - \frac{i}{2} C_{\alpha\beta} \left[ [e^{-B} Q_{\alpha e^B} R_{\beta}] Q_{\alpha} (J_{\beta} + \delta J_{\beta}) - [e^{-B} Q_{\alpha e^B} R_{\beta}] Q_{\beta} (J_{\alpha} + \delta J_{\alpha}) - 2 [e^{-B} Q_{\alpha e^B} R_{\beta}] R_{\alpha} (J_{\beta} + \delta J_{\beta}) - 2 [e^{-B} Q_{\alpha e^B} R_{\beta}] R_{\beta} (J_{\alpha} + \delta J_{\alpha}) \right] - \partial \bar{\theta}_{\alpha} ,
\]

\[
0 = -\frac{1}{2} h_{bc} [e^{-B} R_{k l e^B} P_{b} J_{c} - \frac{i}{2} C_{\alpha\beta} \left[ [e^{-B} R_{k l e^B} P_{b}] Q_{\alpha} (J_{\beta} + \delta J_{\beta}) - [e^{-B} R_{k l e^B} P_{b}] Q_{\beta} (J_{\alpha} + \delta J_{\alpha}) - 2 [e^{-B} R_{k l e^B} P_{b}] R_{\alpha} (J_{\beta} + \delta J_{\beta}) - 2 [e^{-B} R_{k l e^B} P_{b}] R_{\beta} (J_{\alpha} + \delta J_{\alpha}) \right] + \partial \bar{\theta}_{\alpha} ,
\]

for the anti-holomorphic ones. (The complexity of the equations arises from the fact that, unlike in the $AdS_5 \times S^5$ case, $J$ is valued in a space larger than the one that is actually dualized.)

For the purpose of solving these equations, the properties of the field-dependent group-theoretic factors must be understood. In particular, it should be checked whether the coefficients of the gauge fields have non-trivial kernels.

In order to do so, we resort to explicitly expressing the currents in terms of the coordinates. We denote $C = \theta^a Q_{\alpha} + \xi^a S_{\alpha}$ and examine the commutators

\[
[P_a, C] = -\frac{i}{\sqrt{2}} \gamma_{\alpha\beta} \xi^a Q_{\beta} \equiv \Xi_{\alpha} P_{\beta} Q_{\gamma} ,
\]

\[
[Q_{\alpha\beta}, C] = \frac{1}{\sqrt{2}} (\gamma^a C)_{\beta a} \theta^a P_a + \frac{1}{\sqrt{2}} C_{\beta a} \xi^a R_{\alpha} \equiv \Theta^{Q}_{\alpha\beta} P_a + \Xi^Q R_{\alpha} \equiv M_{\alpha\beta} ,
\]

\[
[R_{kl}, C] = -\frac{i}{\sqrt{2}} (\tilde{\theta}^a \delta^a_{\beta} - \tilde{\theta}^a \delta^a_{\gamma}) Q_{\alpha\beta} \equiv \Theta^{R}_{\alpha\beta} Q_{\gamma} .
\]
We further define
\[ N_{i_\alpha}^{\kappa\beta} = \Theta_{i_\alpha}^{\kappa a} \Xi_{a}^{\mu \kappa \beta} + \Xi_{\alpha}^{\kappa} \Theta_{\mu}^{\kappa \beta} \]  
(2.9)
and note that \([M_{i_\alpha}, C] = N_{i_\alpha}^{\kappa \beta} Q_{k_\beta}\) and \([Q_{i_\alpha}, C] = M_{i_\alpha}\). Using the formula \(e^{-B} A e^B = A + [A, B] + \frac{1}{2}[A, [A, B], B] + \ldots\), we get
\[ e^{-C}(dx^a P_a + dy^k R_k) e^C = dx^a P_a + dy^k R_k + \]
\[ + \left( dx^a \Xi_a^{\mu \kappa \beta} + dy^k \Theta_{k_\beta} \right) \left[ \left( \frac{\cosh \sqrt{N} - 1}{\sqrt{N}} \right) \right]^{k \beta}_{\mu \kappa \beta} M_{k_\beta} + \left( \frac{\sinh \sqrt{N}}{\sqrt{N}} \right)_{\mu \kappa \beta} Q_{k_\beta} + \]
\[ + d\theta^{\kappa \beta} \left[ \left( \frac{\sinh \sqrt{N}}{\sqrt{N}} \right)_{\mu \kappa \beta} M_{k_\beta} + \left( \cosh \sqrt{N} \right)_{\mu \kappa \beta} Q_{k_\beta} \right]. \]
(2.10)

Finally, conjugating with \(y^D e^{y_{\hat{r}_k} R_k}\) yields the current
\[ J = \frac{dx^a}{y} P_a + dy^k (R_k + 2i \sqrt{2} y_k \lambda^q + 2 \hat{y}_k \hat{y}_l \hat{R}_{pq}) + \]
\[ + \left( dx^a \Xi_a^{\mu \kappa \beta} + dy^k \Theta_{k_\beta} \right) \left[ \left( \frac{\cosh \sqrt{N} - 1}{\sqrt{N}} \right) \right]^{k \beta}_{\mu \kappa \beta} M_{k_\beta} + \left( \frac{\sinh \sqrt{N}}{\sqrt{N}} \right)_{\mu \kappa \beta} Q_{k_\beta} + \]
\[ + \frac{1}{y^{1/2}} \left[ \left( \frac{\sinh \sqrt{N}}{\sqrt{N}} \right)_{\mu \kappa \beta} M_{k_\beta} + \left( \cosh \sqrt{N} \right)_{\mu \kappa \beta} Q_{k_\beta} \right] \times \]
\[ \times (Q_{k_\beta} + i \sqrt{2} y_k \hat{Q}^\rho_{k_\beta}), \]
(2.11)
where \(\tilde{M}_{k_\beta} \equiv y^{-D} M_{k_\beta} y^D = \frac{1}{y} \Theta_{i_\alpha}^{\kappa a} P_a + \Xi_{\alpha}^{\kappa} R_{k_\beta}\).

Unfortunately, \(j\) is even more complicated. However, before plunging into its computation in a closed form it is worthwhile to examine it to the lowest order in \(\hat{\theta}^{\kappa \beta}\) and \(\xi^{\kappa \beta}\). Doing so yields,
\[ j = \frac{d\hat{\theta}^{\kappa \beta}}{y^{1/2}} \hat{Q}_{\alpha} + y^{1/2} d\xi^{\kappa \beta} S_{\alpha} - i \sqrt{2} y^{1/2} \hat{y}_{kl} d\xi^{\kappa \beta} S_{\alpha} + \frac{dy}{y} D + dy^p \hat{R}_{pq} + O(\hat{\theta}^{\kappa \beta}, \xi^{\kappa \beta}). \]

Having the currents, we can take a look at the action to lowest order in \(\hat{\theta}^{\kappa \beta}\) and \(\xi^{\kappa \beta}\):
\[ S = \frac{R^2}{4 \pi \alpha'} \int d^2 z \left\{ - \frac{1}{2} \eta_{ab} \partial x^a \partial x^b - \partial y \partial \hat{y} - 2 \partial y (2 \hat{y}_{kl} \hat{y}_{pq} + \hat{y}_{kl}) - \frac{1}{2} y C_{\alpha \beta} \left[ \partial \theta^{\kappa \beta} (i \sqrt{2} \hat{y}_{kl} \partial \theta^{\kappa \beta} + \partial \hat{\theta}^{\kappa \beta}) - \right] - (i \sqrt{2} \hat{y}_{kl} \partial \xi^{\kappa \beta} + \partial \hat{\theta}^{\kappa \beta}) \partial \theta^{\kappa \beta} + \frac{1}{2} y C_{\alpha \beta} \right\}. \]

The term quadratic in the \(\theta^{\kappa \beta}\) derivatives is multiplied by a three-dimensional antisymmetric matrix, whose rank is two, and the higher order terms in \(\hat{\theta}^{\kappa \beta}\) and \(\xi^{\kappa \beta}\) cannot make
The matrix's kernel trivial. Thus the term quadratic in the fermionic gauge fields in the
dualized action will be multiplied by a singular matrix and the fermionic gauge fields will
be multiplied by a singular matrix in the equations of motion — one cannot T-dualize all
the six fermionic coordinates.

Since the obstruction to T-dualizing the fermionic coordinates is at the zeroth order
in the spectator fermions, it appears that modifying the $\kappa$-symmetry gauge-fixing of these
fermionic degrees of freedom would not change the above conclusion.

3 Discussion

We showed that the application of the Buscher T-duality procedure to the coset
OSp(6|4)/(SO(2, 1) × U(3)) fails when dualizing along the AdS$_4$ flat directions, three of the
(real) CP$^3$ directions and six fermionic directions. There are several ways to explain this
apparent tension between the field theory tree-level evidence and the sigma-model analysis.

The simplest and most obvious explanation is that the dual superconformal symmetry
exists only in the weakly-coupled field theory description and breaks down at the
strong-coupling regime, which is described by the string theory dual. A second possibility
is that in this case the dual superconformal symmetry is not related to the ordinary
superconformal symmetry by a T-duality transformation but in a more intricate way.

A third possibility is that the coset formulation does not capture the entire superstring
description. The coset is obtained by a partial gauge-fixing of the $\kappa$-symmetry of the
full AdS$_4$ × CP$^3$ sigma-model [16] by setting the fermionic coordinates corresponding to
the eight broken supersymmetries to zero. However, as noted in [16], this gauge-fixing
is not compatible with all the possible string configurations. Thus, it does not have a
representation for certain field theory operators, which might amount to a (possibly
inconsistent) truncation of the field theory that does not preserve the dual superconformal
symmetry. A way to resolve this issue could be to use a better gauge-fixing of the $\kappa$-symmetry as proposed in [13, 16].

Acknowledgments

We would like to thank Y.-t. Huang and A. E. Lipstein for sharing a draft of their paper [6]
with us before its publication. I.A. is supported in part by the German-Israeli Project
cooperation (DIP H.52) and the German-Israeli Fund (GIF).

A The osp(6|4) superalgebra

The osp(6|4) algebra’s commutation relations in the $\text{so}(1, 2) \oplus \text{u}(3)$ basis are given by

\[ [\lambda^l_k, \lambda^m_n] = \frac{i}{\sqrt{2}} (\delta^l_m \lambda^k_n - \delta^k_m \lambda^l_n), \]  
\[ [\lambda^l_k, R_{mn}] = \frac{i}{\sqrt{2}} (\delta^l_m R_{kn} - \delta^l_n R_{km}), \quad [\lambda^l_k, \hat{R}^{pq}] = -\frac{i}{\sqrt{2}} (\delta^l_p \hat{R}^{kq} - \delta^l_q \hat{R}^{kp}) \]  
\[ [R_{mn}, R_{kl}] = 0, \quad [R_{mn}, \hat{R}^{kl}] = \frac{i}{\sqrt{2}} (\delta^k_m \lambda^l_n - \delta^l_m \lambda^k_n - \delta^k_n \lambda^l_m + \delta^l_n \lambda^k_m) \]
\[ [P_a, P_b] = 0, \quad [K_a, K_b] = 0, \quad [P_a, K_b] = \eta_{ab} D - M_{ab} \] 
\[ [M_{ab}, M_{cd}] = \eta_{ac} M_{bd} + \eta_{bd} M_{ac} - \eta_{ad} M_{bc} - \eta_{bc} M_{ad} \]
\[ [M_{ab}, P_c] = \eta_{ac} P_b - \eta_{bc} P_a \]
\[ [D, P_a] = P_a, \quad [D, K_a] = -K_a \]
\[ [D, Q_{la}] = \frac{1}{2} Q_{la}, \quad [D, S_{la}] = -\frac{1}{2} S_{la} \]
\[ [K_a, Q_{la}] = \frac{i}{\sqrt{2}} (\gamma_a)_{\alpha} Q_{l\alpha}, \quad [K_a, S_{la}] = \frac{i}{\sqrt{2}} (\gamma_a)_{\alpha} S_{l\alpha} \]
\[ [R_{kl}, \hat{Q}^p_{\alpha}] = \frac{i}{\sqrt{2}} (\delta^p Q_{ka} - \delta_k Q_{p\alpha}), \quad [R_{kl}, \hat{S}^p_{\alpha}] = -\frac{i}{\sqrt{2}} (\delta^p S_{ka} - \delta_k S_{p\alpha}) \]
\[ [\hat{R}^{kl}, Q_{p\alpha}] = \frac{i}{\sqrt{2}} (\delta^p Q_{k\alpha} - \delta_k Q_{p\alpha}), \quad [\hat{R}^{kl}, S_{p\alpha}] = \frac{i}{\sqrt{2}} (\delta^p S_{k\alpha} - \delta_k S_{p\alpha}) \]
\[ [\hat{R}^{kl}, Q_{p\alpha}] = \frac{i}{\sqrt{2}} (\delta^p Q_{k\alpha} - \delta_k Q_{p\alpha}) \]
\[ [\hat{R}^{kl}, S_{p\alpha}] = \frac{i}{\sqrt{2}} (\delta^p S_{k\alpha} - \delta_k S_{p\alpha}) \]
\[ [\lambda^l_{\alpha}, Q_{p\alpha}] = \frac{i}{\sqrt{2}} \delta^p Q_{k\alpha}, \quad [\lambda^l_{\alpha}, S_{p\alpha}] = \frac{i}{\sqrt{2}} \delta^p S_{k\alpha} \]
\[ [\lambda^l_{\alpha}, \hat{Q}^p_{\alpha}] = -\frac{i}{\sqrt{2}} \delta^p \hat{Q}^l_{\alpha}, \quad [\lambda^l_{\alpha}, \hat{S}^p_{\alpha}] = -\frac{i}{\sqrt{2}} \delta^p \hat{S}^l_{\alpha} \]
\[ \{Q_{la}, Q_{kb}\} = 0, \quad \{Q_{la}, S_{kb}\} = 0, \quad \{S_{la}, S_{kb}\} = 0 \]
\[ \{Q_{la}, S_{kb}\} = -\frac{1}{\sqrt{2}} C_{\alpha\beta} R_{lk}, \quad \{Q_{la}, \hat{S}^k_{\beta}\} = -\frac{1}{\sqrt{2}} C_{\alpha\beta} \hat{R}^{lk} \]
\[ \{Q_{la}, \hat{Q}^k_{\beta}\} = -\frac{i}{2} \delta^k \left( C_{\alpha\beta} D + i\frac{1}{2} (\gamma^{ab} C)_{\alpha\beta} M_{ab} \right) + \frac{1}{\sqrt{2}} C_{\alpha\beta} \lambda^l_k \]
\[ \{Q_{la}, \hat{S}^k_{\beta}\} = i\frac{1}{2} \delta^k \left( C_{\alpha\beta} D - i\frac{1}{2} (\gamma^{ab} C)_{\alpha\beta} M_{ab} \right) + \frac{1}{\sqrt{2}} C_{\alpha\beta} \lambda^l_k \]
The bilinear forms are given by

\[
\begin{align*}
\text{Str}(R_{kl}, \hat{R}^{pq}) &= \delta_k^q \delta_l^p - \delta_k^p \delta_l^q, \\
\text{Str}(\lambda^l_k, \lambda_o^p) &= -\delta_k^o \delta_l^p, \\
\text{Str}(Q_{\alpha k}, \hat{S}^k_{\beta}) &= i\delta_l^k C_{\alpha \beta}, \\
\text{Str}(S_{\alpha k}, \hat{Q}^k_{\beta}) &= -i\delta_l^k C_{\alpha \beta}, \\
\text{Str}(P_a, K_b) &= -\eta_{ab}, \\
\text{Str}(D, D) &= -1, \\
\text{Str}(M_{ab}, M_{cd}) &= \eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}. \tag{A.21}
\end{align*}
\]

The \(\mathbb{Z}_4\) subspaces with the invariant locus of \(U(3) \times SO(3,1)\) which gives the semi-symmetric space \(AdS_4 \times \mathbb{C}P^3\) are

\[
\begin{align*}
\mathcal{H}_0 &= \{P_a - K_a, M_{ab}, \lambda_k^l\}, \\
\mathcal{H}_1 &= \{Q_{\alpha k} - S_{\alpha k}, \hat{Q}_{\alpha}^l - \hat{S}_{\alpha}^l\}, \\
\mathcal{H}_2 &= \{P_a + K_a, D, R_{kl}, \hat{R}^{kl}\}, \\
\mathcal{H}_3 &= \{Q_{\alpha k} + S_{\alpha k}, \hat{Q}_{\alpha}^l + \hat{S}_{\alpha}^l\}. \tag{A.22}
\end{align*}
\]

References

[1] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, \(N=6\) superconformal Chern-Simons-matter theories, \(M2\)-branes and their gravity duals, \textit{JHEP} 10 (2008) 091 [arXiv:0806.1218] [SPIRES].

[2] A. Agarwal, N. Beisert and T. McLoughlin, Scattering in mass-deformed \(N \geq 4\) Chern-Simons models, \textit{JHEP} 06 (2009) 045 [arXiv:0812.3367] [SPIRES].

[3] T. Bargheer, F. Loebbert and C. Meneghelli, Symmetries of tree-level scattering amplitudes in \(N=6\) superconformal Chern-Simons theory, \textit{Phys. Rev. D} 82 (2010) 045016 [arXiv:1003.6120] [SPIRES].

[4] J.M. Henn, J. Plefka and K. Wiegandt, Light-like polygonal Wilson loops in 3D Chern-Simons and ABJM theory, \textit{JHEP} 08 (2010) 032 [arXiv:1004.0226] [SPIRES].

[5] S. Lee, Yangian invariant scattering amplitudes in supersymmetric Chern-Simons theory, \textit{arXiv:1007.4772} [SPIRES].

[6] Y.-T. Huang and A.E. Lipstein, Dual superconformal symmetry of \(N=6\) Chern-Simons theory, \textit{arXiv:1008.0041} [SPIRES].

[7] N. Berkovits and J. Maldacena, Fermionic T-duality, dual superconformal symmetry and the amplitude/Wilson loop connection, \textit{JHEP} 09 (2008) 062 [arXiv:0807.3196] [SPIRES].

[8] N. Beisert, R. Ricci, A.A. Tseytlin and M. Wolf, Dual superconformal symmetry from \(AdS_5 \times S^5\) superstring integrability, \textit{Phys. Rev. D} 78 (2008) 126004 [arXiv:0807.3228] [SPIRES].

[9] N. Beisert, T-duality, dual conformal symmetry and integrability for strings on \(AdS_5 \times S^5\), \textit{Fortsch. Phys.} 57 (2009) 329 [arXiv:0903.0609] [SPIRES].

[10] B. Stefanski jr., Green-Schwarz action for type IIA strings on \(AdS_4 \times \mathbb{C}P^3\), \textit{Nucl. Phys. B} 808 (2009) 80 [arXiv:0806.4948] [SPIRES].
[11] G. Arutyunov and S. Frolov, *Superstrings on AdS$_4 \times \mathbb{CP}^3$ as a coset $\sigma$-model*, *JHEP* 09 (2008) 129 [arXiv:0806.4940] [SPIRES].

[12] I. Adam, A. Dekel and Y. Oz, *On integrable backgrounds self-dual under fermionic T-duality*, *JHEP* 04 (2009) 120 [arXiv:0902.3805] [SPIRES].

[13] P.A. Grassi, D. Sorokin and L. Wulff, *Simplifying superstring and D-brane actions in AdS$_4 \times \mathbb{CP}^3$ superbackground*, *JHEP* 08 (2009) 060 [arXiv:0903.5407] [SPIRES].

[14] T.H. Buscher, *A symmetry of the string background field equations*, *Phys. Lett.* B 194 (1987) 59 [SPIRES].

[15] T.H. Buscher, *Path integral derivation of quantum duality in nonlinear $\sigma$-models*, *Phys. Lett.* B 201 (1988) 466 [SPIRES].

[16] J. Gomis, D. Sorokin and L. Wulff, *The complete AdS$_4 \times \mathbb{CP}^3$ superspace for the type IIA superstring and D-branes*, *JHEP* 03 (2009) 015 [arXiv:0811.1566] [SPIRES].