Nematicity as a route to a magnetic-field–induced spin density wave order: Application to the high-temperature cuprates

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Abstract – The electronic nematic order characterized by broken rotational symmetry has been suggested to play an important role in the phase diagram of the high-temperature cuprates. We study the interplay between the electronic nematic order and a spin density wave order in the presence of a magnetic field. We show that a cooperation of the nematicity and the magnetic field induces a finite coupling between the spin density wave and spin-triplet staggered flux orders. As a consequence of such a coupling, the magnon gap decreases as the magnetic field increases, and it eventually condenses beyond a critical magnetic field leading to a field-induced spin density wave order. Both commensurate and incommensurate orders are studied, and the experimental implications of our findings are discussed.

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Introduction. – Typically, conduction electrons in strongly correlated materials are either localized due to interactions between them, or conduct with a uniform and isotropic distribution. Can electrons in metals assemble themselves and exhibit novel patterns seen in liquid crystals? If so, what is the mechanism behind such a self-organizing pattern, and what are the effects of such a metal on nearby phases? The electronic nematic phase characterized by an anisotropic conduction has been proposed as one such state. It was suggested that it plays a relevant role in determining the superconducting transition temperature in the cuprates [1], which motivated several studies on the interplay between the nematic and d-wave superconducting (dSC) states [2,3]. An evidence of its existence in the cuprates was found by neutron scattering on YBa2Cu3O6.45 (YBCO) [4]. On the other hand, the interplay between the nematic and the antiferromagnet, another nearby phase of the cuprates, has not been addressed.

In this paper, we study the relationship between the nematic and spin density wave (SDW) orders. It is trivial to see that a direct coupling term between the two is not allowed in the free energy, since they break distinctly different symmetries — the nematic order breaks the rotational symmetry between x- and y-directions, while the AF order breaks the translational, spin-rotational, and time reversal symmetries. Thus, a naive conclusion is that there is no strong influence between the two. We show that the situation can be dramatically different when the time reversal symmetry is broken by an external perturbation such as a magnetic field. We find that there is a direct coupling between the SDW and spin-triplet staggered flux (tSF) orders inside the nematic phase when the magnetic field is applied. The tSF phase, like the (spin-singlet) staggered flux (or sometimes called d-density wave) breaks the translational and rotational symmetries due to circulating currents with an alternating pattern. However, unlike the d-density wave state, it does not break the time reversal symmetry, because up- and down-spin circulations have opposite directions, which leads to the name of spin-triplet staggered flux. A consequence of such a coupling is that the SDW order can be induced by the magnetic field via the nematicity. This result applies to both commensurate and incommensurate orders — a finite coupling between incommensurate spin density wave (ISDW) and incommensurate triplet staggered flux (ItSF) orders occurs in the presence of a magnetic field leading to ISDW order beyond a critical magnetic field. We will discuss the experimental implications of our result in the context of the high-temperature cuprates.

Nematicity and commensurate SDW and tSF orders. – The nematic order, which breaks the 90 rotational symmetry between x- and y-directions, is
where the coupling \( \gamma_{ij} \) is a function of \( N_0 \) and the magnetic field \( B \) to respect the discrete symmetries of the system. First, the tSF order breaks the \( x-y \) symmetry, while the AF does not, thus the coupling between them vanishes, except inside the nematic phase. Therefore, \( \gamma(N_0 = 0, B) = 0 \). Second, the AF order breaks the time reversal symmetry, while the tSF order does not, which implies \( \gamma(N_0, B = 0) = 0 \). Therefore, the coupling between the tSF and AF orders is finite only in the presence of both a magnetic field and the nematic order, \( \gamma(N_0 \neq 0, B \neq 0) \neq 0 \). To linear order in \( N_0 \) and \( B \), the form of coupling is found to be \( F \propto N_0 B \cdot (S_Q \times T_Q) \), i.e. the index dependence of \( \gamma_{ij} \) is given by \( \gamma_{ij} \propto \epsilon_{ijk} B^k \). It is straightforward to check that such a term is allowed in the free energy based on the symmetry consideration discussed above. To compute \( \gamma_{ij} \) in eq. (5), it is useful to introduce \( \psi_k^\dagger = (\epsilon_k^\perp, \epsilon_k^\parallel + B \cdot Q) \). In the basis of \( \psi \), the Hamiltonian is written as

\[
H_{nem}^0 = \sum_k \psi_k^\dagger (\epsilon_k + B) \gamma_3 - \mu_k I \psi_k ,
\]

where \( \sigma \) is the Pauli matrix, \( i = x, y, z \), and \( Q = (\pi, \pi) \). It is straightforward to check that there is no direct coupling between the nematic and AF orders based on the distinctly different quantum numbers associated with their order parameters.

However, when we consider the following tSF order parameter, the situation changes:

\[
T_Q^i = i \sum_k d(k) c_{k\alpha}^\dagger \sigma_{\alpha\beta} c_{k+Q\beta}.
\]

The tSF order is characterized by circulating currents with an alternating pattern like the \( d \)-density wave state, but up- and down-spins circulate in opposite directions. Thus, there is no net charge current, but a finite spin current. It breaks the translational, \( x-y \) rotational, and spin-rotational symmetries, but preserves the time reversal symmetry. This state was identified as a component of the 6-dimensional superspin which can be rotated under the 15 generators forming the \( SO(6) \) group [7] which includes as a subset the \( SO(5) \) group suggested for a unified theory of the high-\( T_c \) cuprates [8]. It was also discussed as one of non-zero angular-momentum condensate states in ref. [9]. In a dSC state, both the tSF and AF modes are gapped while the AF gap, i.e. the magnon gap, is smaller than the tSF gap as shown in the box (a) in fig. 1, due to the proximity of the AF phase in the phase diagram.

**Effects of a magnetic field in the nematic phase.**

- Since both \( T_Q \) and \( S_Q \) carry charge 0, spin 1, and momentum \( Q \) one may expect that there is a direct coupling between the two order parameters such as

\[
F = \gamma_{ij} (N_0, B) S_Q^i T_Q^j,
\]

while the coupling \( \gamma_{ij} \) is a function of \( N_0 \) and the magnetic field \( B \) to respect the discrete symmetries of the system. First, the tSF order breaks the \( x-y \) symmetry, while the AF does not, thus the coupling between them vanishes, except inside the nematic phase. Therefore, \( \gamma(N_0 = 0, B) = 0 \). Second, the AF order breaks the time reversal symmetry, while the tSF order does not, which implies \( \gamma(N_0, B = 0) = 0 \). Therefore, the coupling between the tSF and AF orders is finite only in the presence of both a magnetic field and the nematic order, \( \gamma(N_0 \neq 0, B \neq 0) \neq 0 \). To linear order in \( N_0 \) and \( B \), the form of coupling is found to be \( F \propto N_0 B \cdot (S_Q \times T_Q) \), i.e. the index dependence of \( \gamma_{ij} \) is given by \( \gamma_{ij} \propto \epsilon_{ijk} B^k \). It is straightforward to check that such a term is allowed in the free energy based on the symmetry consideration discussed above. To compute \( \gamma_{ij} \) in eq. (6), it is useful to introduce \( \psi_k^\dagger = (\epsilon_k^\perp, \epsilon_k^\parallel + B \cdot Q) \). In the basis of \( \psi \), the Hamiltonian is written as

\[
H_{nem}^0 = \sum_k \psi_k^\dagger (\epsilon_k + B) \gamma_3 - \mu_k I \psi_k ,
\]

where \( \mu_k = \mu - \frac{\epsilon_k^\perp + \epsilon_k^\parallel + Q}{2} = \mu_k + Q \), \( \epsilon_k^\perp = \epsilon_k^\perp - \frac{\epsilon_k^\perp + \epsilon_k^\parallel + Q}{2} = -\epsilon_k^\perp + Q \), and \( B = B \cdot Q \). Note that the AF and tSF fluctuations couple

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1. While the electronic nematic order refers to a spontaneous broken symmetry, our current results can be also applied for a nematicity arising from an external perturbation such as orthohombicity.
Fig. 2: Diagram to compute the coupling constant $\gamma_{xy}$ in eq. (5).

to fermions as

$$H' = g_1 \sum_k S^x_{Q_k} \frac{\hat{c}_k}{\hat{c}_k + B} + g_2 \sum_k d(k) T^y_{Q_k} \frac{\hat{c}_k + Q_\perp}{\hat{c}_k + Q_\perp + \text{h.c.}}$$

$$= \sum_k \psi_k^\dagger (g_1 S^x_{Q_k} \tau_1 + g_2 d(k) T^y_{Q_k} \tau_1) \psi_k,$$

(7)

where $g_1$ and $g_2$ are the interaction strengths in the AF and tSF channels, respectively. We have chosen x- and y-component of the AF and tSF fluctuations assuming that there is a small anisotropy such as Dzyaloshinskii-Moriya interaction aligning the staggered moment in the $(x, y)$-plane.

Then $\gamma_{xy}$ in the nematic state and in the presence of a magnetic field is obtained by computing the Feynman diagram shown in fig. 2:

$$\gamma_{xy} = g_1 g_2 \sum_k \frac{d(k)}{\hat{c}_k + B} [n_F(\hat{c}_k + B - \mu_k)$$

$$- n_F(-\hat{c}_k - B - \mu_k)],$$

(8)

which is proportional to $N_0 B$. For example, for $t = 1$, $t' = -0.3$, and $\mu = -0.867$, $\gamma_{xy} = 2.36 N_0 B g_1 g_2$.

### Field-induced antiferromagnetism, collective modes, and neutron scattering.

- The free energy deep in the nematic state (static $N_0$), expressed in terms of the AF and tSF fluctuations, is given by

$$\mathcal{F} = \frac{1}{2 m_s} \left[ (1 + N_0)(\partial_x S_Q)^2 + (1 - N_0)(\partial_y S_Q)^2 \right]$$

$$+ \frac{1}{2 m_t} \left[ (1 + N_0)(\partial_x T_Q)^2 + (1 - N_0)(\partial_y T_Q)^2 \right]$$

$$+ \frac{1}{2} \Delta_s S_Q + \frac{1}{2} \Delta_t T_Q - \gamma S_Q \cdot T_Q,$$

(9)

where $\Delta_s$ and $\Delta_t$ represent the gap of AF and tSF, respectively, when a magnetic field is absent.

The above free energy becomes unstable to the formation of antiferromagnetism when the following condition is satisfied:

$$|\gamma| = \sqrt{\Delta_s \Delta_t}.$$ 

(10)

Strictly speaking, the condensate mode is not a pure AF, but a mixture of AF and tSF. However, if $\Delta_t \gg \Delta_s$, the mode is dominated by AF. Assuming that $\Delta_t \propto (x - x_c)$, the doping dependence of the critical field for the onset of the field-induced AF state is given by

$$B_c \propto \frac{\sqrt{(x - x_c)} \Delta_t}{N_0},$$

(11)

which determines the phase boundary between the pure nematic phase and the field-induced AF phase as shown in the dashed line in fig. 1.

The collective mode dispersions are shown by different lines in the boxes in fig. 1. The gapped magnon denoted by the thick line in the nematic state, as shown in fig. 1(a), can be detected by neutron scattering, and its intensity should be anisotropic in momentum at a given frequency. The anisotropic intensity of magnetic excitations was reported in YBCO, and interpreted as evidence of the nematic state [4,10]. While the magnon can be detected by the neutron scattering, the tSF mode does not couple to the magnetic field directly. Therefore, the tSF mode shown in the dotted line in fig. 1(a) cannot be detected by the neutron scattering technique. However, in the presence of a field, the two modes couple as we have shown above. The magnon gap is pushed down, while the tSF gap is pushed up, and the coupling also generates a finite but small intensity of the tSF mode shown by the dashed line in fig. 1(b), so the two modes should be in principle detectable by neutron scattering in a field. A further magnetic field decreases the magnon gap as shown in fig. 1(c) leading a field-induced AF order, while the tSF gap becomes larger.

### Incommensurate SDW and tSF orders.

Recently it was reported that the intensity of inelastic neutron scattering peaks at the incommensurate wave vectors, $Q = (\pi \pm \delta, \pi)$ in YBCO increases when the magnetic field is applied [11]. Here we show that the field induced SDW can be also applied to incommensurate orders for both collinear and spiral cases. We will discuss how to distinguish spiral and collinear orders at the end of the section.

Let us first consider a collinear spin density wave order:

$$S_Q^c = \frac{1}{2} \sum_k \left( c^\dagger_{k,\delta} c_{k,\delta} + c^\dagger_{k,\delta} c_{k,\delta} + c^\dagger_{k,\delta} c_{k,\delta} + c^\dagger_{k,\delta} c_{k,\delta} + \text{h.c.} \right),$$

(12)

where we choose the spin density wave as $S^c(r) \propto \cos (Q \cdot r)$. Taking into account broken symmetries as before, the following incommensurate tSF couples to ISDW linearly under the magnetic field:

$$T_Q^\sigma = \frac{1}{2} \sum_k d(k, Q) \left( c^\dagger_{k,\delta} c_{k,\delta} - c^\dagger_{k,\delta} c_{k,\delta} - c^\dagger_{k,\delta} c_{k,\delta} + \text{h.c.} \right),$$

(13)

where $d(k, Q) = 2(\sin k_x, \sin \frac{Q_x}{2}, - \sin k_y, \sin \frac{Q_y}{2})$. We used a symmetric form of the order parameter for convenience. Note that the form factor of $d(k, Q)$ has no longer a d-waveness when Q deviates from $(\pi, \pi)$ due to the constraint of the current conservation at each site [12].

Note that due to the incommensurability $\delta$, a deviation from $\pi$, there is a finite coupling between ISDW and ItSF.
under the field, even in the absence of the nematicity:

\[
\gamma_{xy} = \frac{g_1 g_2}{4} \sum_k \frac{\delta(k, Q)}{2(\epsilon_k - \frac{Q}{2} + B)} \left[ n_F \left( \epsilon_k - \frac{Q}{2} + B - \mu_k - \frac{Q}{2} \right) - n_F \left( -\epsilon_k + \frac{Q}{2} - B - \mu_k + \frac{Q}{2} \right) \right].
\]

However, the coupling \(\gamma_{xy}\) is proportional to \(\delta^2\) which makes \(\gamma_{xy}\) to be \(0.022B\) when \(\delta = 0.12\pi\). On the other hand, when the nematicity is finite, \(\gamma_{xy} = 2.4N_0B g_1 g_2\) similar to the AF case.

Now let us study a non-collinear or spiral spin density wave order. The spiral spin density wave can be written as

\[
S_Q = \sum_k \left( \epsilon^\dagger_{k-\frac{Q}{2}+i} c^\dagger_{k+\frac{Q}{2}+i} c^\dagger_{k-\frac{Q}{2}+i} c^\dagger_{k-\frac{Q}{2}+i} \right),
\]

where \(S_Q(r) = \cos Q \cdot r \hat{x} + \sin Q \cdot r \hat{y}\) with spins lying in the \((x, y)\)-plane. A similar spiral incommensurate staggered flux can be defined as

\[
T_Q = \sum_k \tilde{d}(k, Q) \left( \epsilon^\dagger_{k-\frac{Q}{2}+i} c^\dagger_{k+\frac{Q}{2}+i} + \epsilon^\dagger_{k-\frac{Q}{2}+i} c^\dagger_{k-\frac{Q}{2}+i} \right),
\]

Similar to the spiral spin density wave order, the spiral staggered flux can be viewed as a pattern of incommensurate staggering current where a spin quantization axis shifts from site to site determined by \(Q\). Note that the coupling between the spiral spin density wave and the spiral staggered flux in the presence of a magnetic field is the same as that for the collinear case.

Our results in general support a magnetic-field-induced spin density wave including both collinear and spiral orders. Which one of these can be finally stabilized in spin density wave including both collinear and spiral orders. Which one of these can be finally stabilized in spin density wave including both collinear and spiral orders. Notethat the incommensurate SDW orders, where the electron pocket remains the same\(^2\), while the hole pocket is sensitive to the incommensurability.

Fig. 3: (Colour on-line) The topology of a Fermi surface in a field-induced AF via the nematic order. Note the anisotropy between the electron pockets near \((\pi, 0)\) and \((0, \pi)\) due to the nematic order. It is straightforward to generalize it for incommensurate SDW orders, where the electron pocket remains the same\(^2\), while the hole pocket is sensitive to the incommensurability.

The full effect of the direct interaction between magnetic field and AF order will be discussed later in the context of high-temperature cuprates, as it depends on circumstances. However, the effect of the Zeeman coupling on the quasi-particles is rather straightforward to compute. When the staggered moment lying in the \((x, y)\)-plane, and the magnetic field is along \(\hat{z}\), it changes the electronic dispersion as \(E_k = \frac{\epsilon_k + \epsilon_k + Q}{2} \pm \frac{1}{2} \sqrt{(\epsilon_k - \epsilon_k + Q + 2B)^2 + 4S_Q^2}\), where there is no linear \(B\) dependence. However, when the staggered moment and the magnetic field are parallel, say along \(\hat{z}\), then the dispersion is \(E_k = \frac{\epsilon_k + \epsilon_k + Q + 2B}{2} \pm \frac{1}{2} \sqrt{(\epsilon_k - \epsilon_k + Q)^2 + 4S_Q^2}\), with the linear \(B\) dependence similar to the effect of the magnetic field on spin singlet condensate states such as a charge density wave.

**Fermi surface and quantum oscillations.** The nematic phase in the absence of magnetic fields is metallic, and thus the field-induced SDW phase is expected to be a metal\(^2\). For example, as shown in fig. 3, in the field-induced AF coexisting with nematic order, there are elongated pockets—one electron-like and one hole-like. It is straightforward to generalize the Fermi surface for a spiral order, where the electron pocket is qualitatively the same.

\(^2\)When the induced AF gap is large, it is possible to open a gap on a whole Fermi surface. However, such a large gap would require an enormous magnetic field.

\(^3\)For the collinear case, a higher order mixing of bands is ignored, assuming that magnetic breakdown occurs for such a small band gap.
In a metal with a closed Fermi surface, magnetization $M$ and conductivity oscillate in $1/B$ as

$$M \propto \sum_n A_n \cos \left( \frac{n(A_g + \phi)}{B} \right),$$

where $A_n$ is the area of a closed Fermi surface at the chemical potential. Since there are two types of pocket, the primary periodicities ($n = 1$) are determined by the size of both the hole and electron pockets. Using the same parameter set of $t = 1$, $t' = -0.3$, $\mu = -0.867$ for $\gamma$, and setting $\langle S_0^Q \rangle = 0.07$ and $N_0 = 0.05$, we obtain the periodicity of $540T$ and $900T$ for the electron and hole-pocket, respectively, as a unit cell of square lattice $\sim 3.82 \times 3.89 A^2$. Due to the nematic order, the electron pockets around $(\pi, 0)$ and $(0, \pi)$ are elongated along different directions, but their area is the same yielding a single frequency of the quantum oscillations. As discussed above, in a Fermi liquid, the Zeeman effect generates a magnetic field dependence, implying a single phase of oscillation.

It is tempting to argue that our finding is relevant to the quantum oscillations observed in high-temperature cuprates, in particular in the context of understanding the phenomena observed in the high-temperature cuprates, in particular in the context of anisotropic magnetic excitations and quantum oscillations observed in YBCO materials. This work is supported by NSERC of Canada, Canadian Institute for Advanced Research, and Canada Research Chair.

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