Quantum gravitational processes in a hot ultrarelativistic gas and their effect on the isotropic Universe evolution.

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Abstract

The variant of quasiclassical (half–quantum) theory of gravity in strong gravitational field is presented. The exact solution of the problem of the renormalized energy-momentum tensor calculation is performed in terms of non-local operator-signed function. The procedure of quasilocalization is proposed, which leads to the equations of non-equilibrium thermodynamics for temperature and curvature. The effects of induced particle creation and media polarization are taking into account and used to solve the problem of non-Einstein’s branches damping. The problem of Universe creation from “nothing” is also discussed.

1 Introduction

It is well known that in a strong gravitational field $R/m^2 \gg 1$ the effects of 1) vacuum polarization, 2) real particle creation, and 3) interaction between real particles and self-consistent gravitational field take place [1], [2], [3]. We would like to emphasize that in a gravitating medium no only vacuum polarization, but also the polarization of medium occurs, and the particle creation may be no only spontaneous, but the induced one.

The purposes of our research are both to conduct the quantitative analysis of these phenomena and to discuss their influence on the cosmological evolution of spacetime geometry and of hot ultrarelativistic medium. We show, that, being considered together, the polarization of medium and the induced particle creation solve the problem of non-Einstein’s branches damping. Such branches, as it is shown in [4], [5], are the solutions of Einstein’s equations with radiative corrections including the higher derivatives of metric.
2 Method of calculation of quantum gravitational corrections to the Einstein’s equations in a strong gravitational field.

The subjects of our calculations are the Green functions (GF) of quantum fields in the coinciding space-time points. For example, in the case of scalar field $\varphi(x, t)$, this function renormalized by the Pauli-Willars method is defined as

$$D_{\text{ren}}(x, t) = \langle \varphi(x, t) \varphi(x, t) \rangle_{\text{ren}} - \sum_a c_a \langle 0 | \Phi_a(x, t) \Phi_a(x, t) | 0 \rangle. \quad (1)$$

Here $a$ run over a number of Pauli-Willars’s fields, $c_a$ are the renormalized coefficients; $\langle 0 |$ is Heisenberg’s vacuum state vector of auxiliary Pauli-Willars’s fields; $\langle \ldots \rangle_T$ is the density matrix averaging in Heisenberg’s representation. Both vector $\langle 0 |$ and matrix $\langle \ldots \rangle_T$ are defined in some initial time moment.

In the isotropic Universe

$$ds^2 = a^2(\eta) (d\eta^2 - dl^2) \quad (2)$$

one can use the Fourier image of Heisenberg’s quantum fields $\varphi^p(\eta)$ and to introduce the Fourier image of GF:

$$D_p(\eta) = \langle \varphi^+_{\!p}(\eta) \varphi_p(\eta) \rangle_T. \quad (3)$$

Then GF (1) is transformed to

$$D_{\text{ren}}(\eta) = \sum_p D_p(\eta) - \sum_a c_a \sum_p D_{pa}(\eta) \quad (4)$$

and the task of radiative correction calculation is divided into several steps:

(i) to obtain the solution for the Fourier images $D_p(\eta)$;

(ii) to integrate $D_p$ over momentums according to (4); this gives the corrections to GF;

(iii) to calculate the energy-momentum tensor (EMT) as a functional of the renormalized GF.

We have calculate the exact solutions for GF and EMT in the form of the operator-signed function, which depend from operator $\hat{J}$:

$$\hat{J} f = \frac{1}{4} \int_{\tau_0}^{\tau} \frac{1}{r^2(\tau_1)} d^3 \tau_1 \left( \frac{1}{r^2(\tau_1)} f \right) d\tau_1,$$

where $f$ is an arbitrary function, time $\tau$ is defined as $r^3 d\tau = r d\eta = dt$. The analytic functions of operator $\hat{J}$ are defined as the Teylor series; but the calculation shows that both GF and EMT are non-analytic functions. This fact necessarily shows the presence of non-local effect, which will be discussed below.

The formally exact expressions for GF and EMT are
\[ \pi^2 D_{ren}(\tau) = \frac{1}{8r^2} \hat{J} \ln \hat{J} - \frac{5}{24} \frac{r^2}{r^8} + \frac{5}{48} \frac{r''}{r^7} - \frac{1}{8} \ln(\mu^2 r^2) \left( \frac{r^2}{r^8} - \frac{1}{2} \frac{r''}{r^7} \right) + \frac{1}{8\pi^2} \int_0^\infty \frac{p^3 dp}{p^2 + J e^{p/\theta} - 1} \]  

\[ \pi^2 \langle T^0_0 \rangle_{ren}(\tau) = \frac{1}{32r^6} \left( \frac{1}{r^2} \hat{J} \ln \hat{J} \right)'' - \frac{1}{8r^4} \hat{J}^2 \ln \hat{J} + \left( -\frac{3}{8} \frac{r^4}{r^{16}} + \frac{1}{4} \frac{r^2 r''}{r^{15}} + \frac{1}{64} \frac{r^2}{r^{14}} \right) - \frac{1}{32} \frac{r^4}{r^{15}} \ln(\mu^2 r^2) + \frac{121}{960} \frac{r^4}{r^{16}} \]  

\[ + \frac{1}{32\pi^2 r^6} \left[ r^4 \int_0^\infty \frac{p^3 dp}{p^2 + J e^{p/\theta} - 1} \right]'' + \frac{1}{8\pi^2 r^2} \int_0^\infty \frac{p^5 dp}{p^2 + J e^{p/\theta} - 1} \]  

Here prime denotes \( ' = d/d\tau \); \( \theta = Ta \) is a conformal temperature of a medium; \( \mu \) is renormalization scale.

The terms in (5), (6), which do not contain the Bose-Einstein function, describe the vacuum effects; the terms, which contain this function, describe the quantum gravitational effects in the real particle sector. Two types of nonanalyticities are present in equations (5), (6): those in vacuum sector are logarithms from operator \( \hat{J} \); those in real particle sector are integrals containing operator \( \hat{J} \) and Bose–Einstein’s distribution function. We consider vacuum and ”matter” terms together and made the asymptotic expansion of the integrals on parameter \( \left( \hat{J}/T^2 \right)^{1/2} \), then the nonanalyticities of \( \ln \hat{J} \) type are canceled. All terms \( F_n = \left( \hat{J}/T^2 \right)^{n/2} \), \( n = 0, 1, 2, 3, \ldots \) in asymptotic expansion are divided into two type. For even \( n \) all \( F_n \) are local functions of metric, for odd \( n \) they are principally non-local. For example, the linear on temperature term in EMT is

\[ \langle T^0_0 \rangle_\theta = \theta \left[ -\frac{1}{16\pi^2 r^2} \left( \frac{1}{r^2} \hat{J}^{1/2} \right)'' + \frac{1}{4r^4} \hat{J}^{3/2} \right] \]  

This term and other odd on temperature terms, being non-local ones, describe the effect of the induced creation of real particles. In the next sections we work in the framework of nonequilibrium thermodynamics and so we develop a special method to make a quasilocalization of term (7) and similar ones.

3 The problem of quasilocalization of nonanalytic operator-signed functions.

Notice, the above described calculations are made when conformal temperature \( \theta \) is supposed to be constant. The procedure of quasilocalization means the nonlocal
dependence would be transferred from operator $J$ degrees to the conformal temperature, which became irreversibly dependent from time $\tau$. At the first step we find approximations

$$\frac{1}{4\pi^2} \left( \frac{1}{r^2} J^{1/2} \right)'' \approx \left( \frac{1}{r^2} \right)' \left( \frac{1}{r^2} \right)' + \frac{1}{8} \left( \frac{1}{r^2} \right)' \left( \frac{1}{r^2} \right)'' - \frac{1}{27} \left( \frac{1}{r^2} \right)'^3 \cdot r^2$$

$$\frac{4}{\pi^2} \left( \frac{1}{r^2} \right)^{3/2} \approx \frac{1}{3} \left( \frac{1}{r^2} \right)'^3 + \frac{1}{2} \left( \frac{1}{r^2} \right)''' + \frac{1}{2} \left( \frac{1}{r^2} \right)' \left( \frac{1}{r^2} \right)' \left( \frac{1}{r^2} \right)'' + \frac{1}{18} \int_{\tau_0}^{\tau} \sqrt{-g} R^2 d\tau$$

(8)

$$\langle T_0^0 \rangle \approx \frac{\pi^3}{32} \theta \left[ - \left( \frac{1}{r^2} \right)^3 \left( \frac{1}{r^2} \right)' \left( \frac{1}{r^2} \right)' \left( \frac{1}{r^2} \right)'' + \frac{1}{6} \left( \frac{1}{r^2} \right)^2 \int_{\tau_0}^{\tau} \sqrt{-g} R^2 d\tau \right]$$

(9)

In equations (8), (9) $\theta = \theta(\eta_0) = \text{const}$ is the initial conformal temperature, but the integral term describe the particle creation. At the second step the non-local time dependence transforms to the conformal temperature irreversible time dependence $\theta = \theta(\eta)$. In order to vanish integral from EMT the law of entropy increase should be as follows:

$$\dot{\theta}^2 = \frac{1}{\pi^2 k_1 r^2}$$

(10)

where the dot denotes $\frac{d}{d\eta}$, the coefficients $k_1$, $k_3$ see below.

Notice, the proposed way of localization of operator-signed functions is the simplest one, but other ways are not excluded also. This question is the discussible one.

As a result Einstein’s equation with the renormalized EMT

$$R_0^0 - \frac{1}{2} R = \omega \langle T_0^0 \rangle_{\text{ren}}$$

in the isotropic space–time with metric (2) has a form

$$\frac{1}{l_{Pl}^2} r^2 = \beta \frac{\dot{r}^4}{r^4} + k_1 \theta^4 + k_2 \frac{\dot{r}^2}{r^2} + k_3 \theta \left( \frac{\dot{r}^3}{r^3} - \frac{\dot{r} \ddot{r}}{r^2} \right) - k_4 \left( \frac{2 \dot{r} \dddot{r}}{r^2} - \frac{\dot{r}^2}{r^2} - 4 \frac{\dot{r}^2 \ddot{r}}{r^3} \right) \ln \frac{\mu^2 r^2}{\theta^2}.$$  

(11)

For one scalar field the coefficients are

$$\beta = \frac{1}{1440\pi^2}, \quad k_1 = \frac{\pi^4}{30}, \quad k_2 = \frac{\pi^2}{24}, \quad k_3 = \frac{\pi^3}{32}, \quad k_4 = \frac{9}{8\pi^4}.$$  

Equations (10), (11) form the closed self-consistent system for the cosmological variables $r(\eta)$ and $\theta(\eta)$.

4 Damping of non-Einstein branches of cosmological solutions.

The system (10), (11) of differential equations is solved, utilizing the theory of continuous symmetry groups [4], [5]. At the first step one should reduce the order of this
system; at the second step the WKB-solution on small parameter — dimensionless Hubble constant

\[ h = H l_{Pl} = \frac{\dot{r}}{r^2} l_{Pl} \ll 1 \]  

is found:

\[ \ln \frac{r}{r_0} = -\frac{1}{2} \ln h - A \left( \frac{k_4}{h} \right)^{1/4} \exp \left( -\frac{1}{4} \frac{k_3}{k_4 \sqrt{k_1}} \phi \right) \cdot \cos(\phi - \phi_0); \]  

\[ \frac{1}{l_{Pl}} (t - t_0) = \frac{1}{2} h - Ah \left( \frac{k_4}{h} \right)^{1/4} \exp \left( -\frac{1}{4} \frac{k_3}{k_4 \sqrt{k_1}} \phi \right) \cdot \cos(\phi - \phi_0); \]  

(13)

here

\[ \phi = \frac{1}{4h} \cdot \frac{1}{\ln \frac{1}{h}} > 0, \quad A, \phi_0 — \text{integration constants} \]

It is easy to see that the first terms in (13) represent the Friedmann-Robertson-Walker (FRW) solution for radiative-dominant plasma \( r \sim t^{1/2} \). The second terms correspond to the non-Einstein branches. The problem is: for the existence of the observed FRW Universe non-Einstein branches must be damped. From our general solution (13) it is obvious that the key role is played by the parameters \( k_3, k_4 \). Being introduced in EMT (right side of Einstein’s equations (11)), they describe the effects of induced particle creation and matter polarization in the Universe.

Now it is clear why the problem of non-Einstein’s branches was unsolved until now. Usually in quasiclassical theory the key effects — induced particle creation and matter polarization were not taken into account (see for example [6], [7]). (Formally this case correspond to \( k_3 = 0 \).) In this case in (13) \( h \to 0 \) as \( t \to \infty \), the exponent is absent, the pre-exponential multiplier is leading term and non-Einstein’s branches grow up as \( \sim 1/h^{1/2} \); their oscillations predominate over FRW terms — this result was obtained numerically (see, for example, [3]). We show it is of great value that \( k_3 > 0 \), and non-Einstein’s branches are exponentially damped \( \sim 1/h^{1/2} \exp(-k_3/h) \).

Our consideration show that the problem of non-Einstein branching damping do not need to use any exotic ideas like n-dimension gravity. The problem can be solved by punctual account of all matter effects (particularly, both spontaneous and induced particle creation; and vacuum and medium polarization) in the framework if usual quasiclassical (half-quantum) theory of gravity.

5 The problem of Universe creation from “nothing”.

We assume that the obtained equations (10), (11) can be used for the formal description of early stages of Universe evolution. Particularly, this description performs to solve the problem of Universe creation from “nothing”, here “nothing” means the physical state with \( R = 0, T = 0 \).
We found the asymptotical at $R \ll R_{Pl}$, $T \ll T_{Pl}$ solution of equations (10), (11) in the form:

$$\frac{r}{r_0} \sim \exp \left[ \alpha \frac{(t - t_0)^3}{l_{Pl}^3} \right],$$

(14)

here $r_0$, $\alpha$, $t_0$ are arbitrary constants. This equation corresponds to the laws of curvature and temperature increasing

$$R_0^0 - \frac{1}{2} R \sim (t - t_0)^4,$$

$$T \sim t - t_0,$$

(15)

such increasing means the particle creation in early Universe. It’s obvious that curvature $R_0^0$ and temperature $T$ are equal to zero at the initial time moment:

$$(R_0^0 - \frac{1}{2} R)|_{t_0} = 0, \quad T|_{t_0} = 0$$

This solution at $t = t_0$ corresponds to the empty Minkowski space. In quasiclassical theory just this state can be interpreted as “nothing”. During the evolution this initial state decays to geometry and particles under the action of “initial push”, which in mathematical terms is described as nonzero fourth derivative:

$$(R_0^0 - \frac{1}{2} R)^{(4)}|_{t_0} \neq 0.$$  

(16)

The assumption about thermodynamic equilibrium at early stages of cosmological evolution used in (10), (11), physically corresponds to the hypothesis about quick thermalization of the created particles. This hypothesis is obviously rough because in the vicinity of a singularity near the Planck space-time scales the characteristic time of particle creation and the relaxation time of their gas are comparable. From this point of view the model is phenomenological, but nevertheless it involves all the main points of the discussed phenomena.

To clarify the whole phase picture of model (10), (11), and to count a whole number of the created particles $N|_{t=\infty}$ it’s necessary to found a whole set of analytical asymptotics and to make the numerical integration of system (10), (11). This work is in progress.

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