Intrinsic vanishing of energy and momenta in a universe

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Abstract We present a new approach to the question of properly defining energy and momenta for non asymptotically Minkowskian spaces in General Relativity, in the case where these energy and momenta are conserved. In order to do this, we first prove that there always exist some special Gauss coordinates for which the conserved linear and angular 3-momenta intrinsically vanish. This allows us to consider the case of creatable universes (the universes whose proper 4-momenta vanish) in a consistent way, which is the main interest of the paper. When applied to the Friedmann-Lemaître-Robertson-Walker case, perturbed or not, our formalism leads to previous results, according to most literature on the subject. Some future work that should be done is mentioned.

Keywords Energy and momenta of the Universe · Non asymptotic flatness · Intrinsic vanishing of momenta

1 Introduction

1.1 General considerations

In General Relativity, the problem of associating linear and angular 4-momenta to a finite space-time region and the related idea to define (for a general space-time) such global quantities have been approached from different, but not necessarily...
contradictory, perspectives. See [1] for an extensive and critical review on the current status of the problem.

Diverse mathematical objects (energy-momentum pseudotensors and superpotentials, flat or curved background metrics, Killing vectors or other fields generating generalized symmetries, etc.) and several geometrical techniques (3+1 or 2+2 spacetime splittings, initial data constraints and boundary conditions, etc.) seem appropriate to deal with this issue. See, for example, [2–6] for some detailed explanations and general comments on these subjects.

However, nowadays, no consensus on a preferred approach nor any complete or definitive answer to the problem of how to associate linear and angular 4-momenta to a general space-time seem to have been reached by the relativistic community. Of course, the existing points of view don’t exclude each other and seem to point towards the correct understanding of the problem, while the possibility of new approaches remains still open.

Nevertheless, the reader should be warned about the presence of a lot of criticisms in the current literature to the pseudotensor approach to define the 4-momenta of a physical space-time, which is the approach adopted in the present paper. These criticisms stress that the approach is by no means a covariant one (see for example [7,8]), or argue against any definition of energy in General Relativity referring to 3-surface integrals, instead of being quasi-local (referring to 2-surface integrals) from the very beginning [9], or even accept the approach for asymptotically flat space-times but express some doubts for the non asymptotically flat ones [10].

Although the covariant approach to the definition of quasi-local conserved quantities in General Relativity followed in [7–9] and the results obtained seem very interesting, we cannot fully share those criticisms.

Before giving our personal opinion about it, let us begin remarking that in [11] a particular “covariant Hamiltonian approach to quasi-local energy” is presented. In this approach, each pseudotensor corresponds to a Hamiltonian boundary term, which brings the authors to the conclusion that “Hamiltonian approach to quasi-local energy-momentum rehabilitates the pseudotensors”. Quoting this conclusion, Vargas [12] used again the pseudotensorial method to calculate the energy of the universe in teleparallel gravity. The conclusion was quoted in [13] too.

Regarding the objection prescribing a quasi-local definition of energy in General Relativity [9], we recall that our pseudotensorial method yields 4-momenta that are quasi-local quantities, in the sense that they can be expressed subsequently as 2-surface integrals, even if their original definitions were through 3-surface integrals.

As far as the remarked [7,8] non covariance of the pseudotensorial method is concerned, we recall that there is nothing invalidating in the fact that the energy of a physical system can depend on the reference frame used and that, at the same time, we look for some natural special frame in order to define some “proper” energy and momenta: at least, nothing, apparently, that from the very beginning prevents us from approaching this problem. The same frame dependence is present in classical mechanics, as it is mentioned in [14] in relation to the general problem of defining energy in a covariant way. But, for a particle, for example, we can select a “natural” special frame (the one where the 3-momentum vanishes) to define the “proper” energy of the particle. In a similar way, but pointing to General Relativity, we must select a “natural”