Mathematical modeling of heavy particle concentration in a jig bed

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Abstract. The mathematical model for motion of heavy particles surrounded with grains in a jig is proposed in the present paper. The grains are modeled by balls of a certain radius. The statistical approach to the process description is considered. The Brownian particle motion model is used, where the researchers consider and take into account the kinetic energy of grains surrounding the bead of a heavy particle moving under the influence and power of the vibrating jig, rather than the kinetic energy of the random thermal motion of the molecules bombarding the Brownian particle. This force depends on the amplitude and frequency, being, the parameters of the jigging cycles. The mathematical modeling allowed derivation of the Fokker-Planck-type equation for fractions distributed by density in the jig chamber. The dynamic curves for distribution of heavy grains along the height of a jig bed are obtained.

1. Introduction
The gravity jigging of mineral materials gives a rise of typical problems on determination of the heavy fraction concentration in a jig bed in terms of jig specifications. The research subject of the present paper is the mathematical modeling of the jigging process.

The statistical approach with consideration of stochastic processes along with determinate mechanical ones is applied to describe the process. The known publications [1, 2, 3, 4] do not propose proper demonstration on what pre-requisites and allowances the Fokker–Plank equation rests on and how it is solved. The paper proposes the physical jigging model serving the basis for the relationship of Fokker–Planck-type equation. Brownian particle motion model in the Earth gravity field is used to solve this equation [5]. The obtained solution can be expanded to the case when there is a majority of non-interacting particles. Equation has unknown parameters which can be determined based on the physical model and ascertained by means of physical modeling.

2. Mathematical model and numerical test results
Consider the following problem. The jig bed was stuffed with \( N \) of identical balls of density \( \rho_1 \) and radius \( r_0 \). Balls are in a certain medium of density \( \rho_1 \), and viscosity \( \eta_1 \). At a certain height \( h \) there is a ball of the same size, but higher density \( \rho_0 > \rho_1 \).
The jig bed (a physical vessel as supposed) is set in oscillation motion of a certain frequency $\omega_0$ and amplitude $a_0$. Thereto, a test particle should diffuse downward under gravity force. It is required to find a probability of particle location at a random time moment.

The problem is stated in 1D variant.

First under consideration is the system in which at the initial moment a heavy particle is in the rest state and surrounded with light particles. Under assumption that gravity forces and boundaries are absent, in other words, there is an unlimited system, and oscillation motion is rendered to the system the particle would move according to Markovian processes. In this case it is a Brownian particle which moves chaotically under thermal motion molecules of the surrounding medium. Therewith, molecules which bombarding the particle have certain root-mean-square velocities with regard to temperature. In our case the motion of the heavy particle in the jig bed is affected by surrounding light particles forced to move under external periodical forces. The light particles of the bed (surrounding medium) will have kinetic energy corresponding to oscillation motion of the medium, namely

$$\frac{m(v)^2}{2} = \frac{m a_0^2 \omega_0^2}{2},$$

where $a_0$ and $\omega_0$ are amplitude and frequency of oscillating motions of the medium. The particle is affected by two type of forces: gradient (as every time moment is considered as a statistical assembly) $F_g$ and resistance forces. Resistance forces are conditioned by co-collision of the particle under consideration and balls surrounding it $F_1$ and viscosity $F_2$, provided that the process runs in water, for example. Thereto, resistance force is $F_s = F_1 + F_2$.

Resistance force arising owing to collisions can be determined as follows. Averaged retarding force equals to an impulse loss. Under every collision it completely loses the directed velocity. This force approximately equals to:

$$F_i = -\frac{dp}{dt} = -m \frac{dv}{dt} \approx -m \bar{v} \frac{d\bar{v}}{\tau}, \quad (1)$$

where $\bar{v}$—average velocity of particles, $\tau$—average time of running between collisions.

Average value of the free run time can be found from oscillation characteristics of the system. The root-mean-square velocity of particles under oscillation motions of the jig machine is:

$$\langle v \rangle = a_0 \cdot \omega_0. \quad (2)$$

Average run length is equal to oscillation amplitude, then the average run time is equal to oscillation period:

$$\tau = \frac{2\pi}{\omega_0}. \quad (3)$$

Thus:

$$\bar{v} \approx \frac{m \omega_0}{2\pi}. \quad (4)$$

The medium resistance force can be assumed equal to Stokes force:

$$F_2 = -6\pi \eta \bar{v}, \quad (5)$$

where $\eta$—medium viscosity factor.
With the purpose to determine the gradient force let consider 1D heterogeneous system, viz., 1D problem where \( n(x) \) depends only on single variable \( x \), a vessel with inhomogeneous distribution of particles (Figure 1). Assume that the concentration of particles in the left side nearby a certain boundary is higher than that in the right side \( n_1 > n_2 \).

![Figure 1. Scheme of 1D inhomogeneously distributed system.](image)

The force acting on the left boundary of \( dx \) domain is proportional to the concentration and equals to \( kn_1 \Delta S \), that on the right side is \( kn_2 \Delta S \), \( k = \frac{m (v)^2}{3} = \frac{m a \omega_0^2}{3} \). The force acting on a single particle inside of the domain is:

\[
F_{gr} = \frac{k(n_2 - n_1)}{n(t, x) \cdot \Delta V} \cdot \Delta S = \frac{k(n(t, x) - n(t, x + dx)) \cdot \Delta S}{n(t, x) \cdot \Delta S \cdot dx} = \frac{k}{n(t, x)} \cdot \frac{dn(t, x)}{dx}.
\]

or in 3D form:

\[
\vec{F}_{gr} = -\frac{k}{n(t, \vec{r})} \nabla n(t, \vec{r}).
\]

Mathematical expression of the law of the matter conservation is the equation of continuity:

\[
\frac{\partial n(t, \vec{r})}{\partial t} + \nabla \cdot (n(t, \vec{r}) \cdot \vec{v}) = 0.
\]

The balance of forces acting on the given particle

\[
\sum_i F_i = F_{gr} + \vec{F}_1 + \vec{F}_2 = 0
\]

gives ratio:

\[
- \frac{k}{n(t, \vec{r})} \nabla n(t, \vec{r}) - \frac{m a \omega_0}{2 \pi} \vec{v} - 6 \pi \eta \rho_0 \vec{v} = - \frac{k}{n(t, \vec{r})} \nabla n(t, \vec{r}) - \alpha \vec{v} = 0,
\]

where \( \alpha = \frac{m a \omega_0}{2 \pi} + 6 \pi \eta \rho_0 \).
From (8) and (10) we have:

\[
\frac{\partial n(t, \vec{r})}{\partial t} = \frac{k}{\alpha} \Delta n(t, \vec{r}) = D \Delta n(t, \vec{r}).
\]

(11)

where \( D = \frac{k}{\alpha} \) is macro-diffusion coefficient.

The obtained expression is Fokker-Plank equation for a free particle. Solutions of this classical equation are known, in particular, the fundamental solution of this equation for 1D case is:

\[
n(t, x) = \frac{1}{\sqrt{4\pi D t}} \exp \left( -\frac{x^2}{4Dt} \right).
\]

(12)

Let consider motion of a particle in Earth gravity field. A particle is in a vessel limited with nonpermeable wall from below and is surrounded with other particles of less density. Under oscillating motions the particle diffuses downwards. The gravity force acting on the particle is:

\[
F_3 = mg,
\]

(13)

where \( m = \frac{4}{3} \pi \rho_0 r_0^3 \) is mass of the study particle.

Then Fokker-Plank equation is:

\[
\frac{\partial n(t, x)}{\partial t} - \frac{mg \, \partial n(t, x)}{\alpha \, \partial x} - D \Delta n(t, x) = 0
\]

(14)

with initial and boundary conditions:

\[
n(0, x) = \delta(x - h),
\]

(15)

\[
\int_0^\infty n(t, x) dx = 1,
\]

(16)

\[
n(t, x)|_{x \to \pm \infty} = 0,
\]

(17)

\[
\frac{\partial n(t, x)}{\partial x} |_{x \to \pm \infty} = 0,
\]

(18)

\[
\left( \frac{mg}{k} n(t, x) + \frac{\partial n(t, x)}{\partial x} \right)_{x=0} = 0.
\]

(19)

The last relationship (19) is the condition for absence of a flow through the lower surface.

To solve this problem we use the solution of a similar problem for Brownian particle [5] and obtain the following analytical expression:
\[ n(t,x) = \frac{1}{\sqrt{4\pi D t}} \left( \exp \left( -\frac{(x-h)^2}{4Dt} \right) + \exp \left( -\frac{(x+h)^2}{4Dt} \right) \right) \exp \left( -\frac{(mg)^2 t}{4k\alpha} - \frac{mg(x-h)}{2k} \right) + \\
\quad + \frac{mg}{k} \exp \left( -\frac{mgx}{k} \right) \frac{1}{\sqrt{4\pi D t}} \int_{-\infty}^{\frac{mg}{x-h}} \exp \left( -\frac{\eta^2}{4Dt} \right) d\eta. \] (20)

Figure 2 demonstrates the curves of relationships according to (20) at different time moments; the height of a vessel with balls is directed along horizontal axis.

As it is obvious in Figure 2, distribution of probabilities for location of the particle first spreads about, moves downwards and finally transforms to Boltzmann distribution.

The above problem is equivalent to the problem given that majority \( N \) of particles of \( \rho_0 \) density are distributed at the same plane at height \( h \). Then distribution of a number of particles will be described by curve (20), therewith a number of particles is proportional to area of the figure limited inside of the domain adjoining with horizontal axis and curve in Figure 2.

Factually, the heavy mineral particles, for example, grains of diamond crystals in the oversize product of jigging are randomly present in a volume of the jig bed. It is possible to use the random number generator to model this distribution. In the case of greater number of the study material grains the initial distribution is approximately uniform. In view of this let consider a uniformly distributed system (Figure 3).
Figure 3. Uniformly distributed system.

For such system the cumulative distribution is equal to:

\[ n(t,x) = \frac{1}{N} \sum_{i=1}^{N} n_i(t,x). \]  

(21)

In Figure 4 the graphs of relationships of summary distribution are shown for the case when the initial state has the uniform distribution of the study grains at different time moments.

Figure 4. Distribution of probabilities of the study particles location in the case of the initial uniform distribution.

3. Conclusions
At the preset values of the initial fractions (for example, in percents of the total volume) characterized with the initial uniform distribution the obtained distribution allows calculation of a probable time during which a certain preset material bed is formed at the bottom of the jig bed vessel with the target distribution of heavy (valuable) fraction. The results obtained with the help of the model are important for description of the oversize variant of the diamond jigging operation owning to the parameters: time
of complete heavy-fraction deposition from the mineral-bearing stream and the height of the concentration zone (volume of recovered oversize concentrate).

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