Mathematical model of viral marketing

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Abstract. The article presents the results of studies of mathematical aspects of the viral marketing. A mathematical model of the spread of a marketing virus as a prefractal graph was developed. An algorithm for the dissemination of viral information was developed.

1. Introduction
In epidemiology, an epidemic is defined as the spread of an infectious disease in time and space. The term is interpreted as a process of spreading anything among people (e.g., epidemy of tourism). Viral marketing is an impact on the audience, promotion of goods and services at the expense and by the means of the target audience consciously or unconsciously participating in the distribution of information about a product.

Open advertising is perceived as negative, intrusive and useless information from which people try to protect themselves. The nature of viral marketing is that original information, the idea as a ready-made marketing message is virally transmitted as information recommended for viewing. Thus, information is associated with advantages rather than an imposed advertising appeal. One of the most effective ways to disseminate information on the Internet is social networks. Social networks are a powerful tool for influencing public opinion. They influence business, politics and society. Thanks to the free interaction of a large number of participants, dissemination of information in social networks has characteristics similar to those of the epidemic. However, classical models of the spread of epidemics do not take into account peculiarities of viral marketing and social networks. The article analyzes a mathematical model describing the digital marketing using an epidemiological approach that takes into account principles of interaction in social networks.

The purpose: construction of a mathematical apparatus that describes the viral marketing and identifies its characteristics affecting the development of networks.

2. Materials and research methods
Agent-based modeling, analysis, forecasting, simulation modeling.

3. Results
The multi-agent mathematical model reveals vulnerable vertices and edges of the graph, calculates an involvement threshold, predicts the development of networks with certain characteristics.

Studies on the phenomenon of information dissemination in social networks is not an easy task, since connections are often random in nature, the networks are dynamic; it is also difficult to predict the interest of new users in information dissemination. A prefractal graph serves as a simulation multi-agent model. The use of a simulation multi-agent approach in modeling epidemics is due to the fact that the
approach allows it to take into account various factors influencing the epidemic process and makes it possible to carry out numerical experiments. A social network is a structure consisting of agents and relationships (a set of relationships between agents – acquaintance, friendship, cooperation). A social network is a graph \( G = (V, E) \) where \( V \) is the set of vertices (agents) and \( E \) is the set of edges describing the relationship of agents. In the viral marketing in social networks, an agent is a research participant. Agents are people with individual parameters.

The agent is active, in a certain state which may change under the influence of certain factors. Properties of the agent include characteristics which form the “level of involvement” \( u \in [0,1] \) of agent \( p \): numerical - age (\( v \)), gender (\( s \)), income (\( d \)); categorical - marital status (\( q \)), education (\( e \)), social and professional criteria (\( g \)), reaction speed (\( b \)). The “level of engagement” \( v \in [0,1] \) is a parameter indicating how many people are resistant to advertising. It should be emphasized that one agent may have different levels of involvement regarding different marketing products; for example, one person may be interested in cosmetics rather than in car accessories. [1]

Marketing network participants can have two \( p \) states:
- **active** agents \( p = 1 \), if the agent accepts an invitation and distributes a marketing virus;
- **passive** \( p = 0 \), if the agent accepts an invitation, but does not distribute a marketing virus.

The position will depend on the level of involvement, since each network participant has an individual set of characteristics. The model is an artificial city of agents represented as a social network with the same set of properties different in values, i.e. some agent database. Let us imagine that you have created a new company and are planning to attract customers through the viral growth. The first customers, can be friends, etc. The model will have the following inputs:

| Variable | Description | Example |
|----------|-------------|---------|
| \( V_0 \) | Initial set of clients | 8 |
| \( n \) | Number of customers to whom an invitation is sent from one customer | 10 |
| \( v_k \) | The share of invitations that become customers (it depends on individual parameters of customers who received invitations) | 25% |

Each network agent may successfully involve a certain number of clients. It is a viral coefficient \( K=n\cdot v_k \) [6].

\( v_0 \) – the vertice of the graph which will correspond to the owner of the company (zero network member). \( v_{i\theta}, i = 1, n \) – vertices representing individual customers who received an invitation from the customer \( v_0 \). A set of first-level active and passive agents is formed. The process of sending invitations will be denoted by a directed edge on the graph. Let us connect the vertex \( v_0 \) with vertex \( v_{i\theta}, i = 1, n \) with directed edges coming from \( v_0 \). In terms of the graph theory, the vertex denoting the primary source of infection is a root of the marketing virus. Let us combine the resulting set of vertices and edges into graph \( H = (W, Q) \):

![Figure 1. Graph \( H = (W, Q) \).](image-url)
Active agents of the first level send invitations to potential members of the network of the second level. Each vertex of the first level of graph \( H = (W, Q) \), is connected with \( n \) vertices of the second level \( v_{2i}, i = 1, n^2 \). Each first-level agent sends invitations to potential second-level agents [2].

In the resulting graph-model, the number of vertices and edges will increase, but the structure of the graph which consists of self-similar parts remains stationary. The parts are incentives, and the graph model is a prefractal graph. Each vertex of the last level is replaced with incentive \( H = (W, Q) \) – replacement of the vertex with an incentive (RVI). When repeating the process at \( l \rightarrow \infty \), the structure of relations will be a fractal graph \( G = (V, E) \) created by the incentive. There are no ideal cases in practice, therefore it is necessary to consider a case when the RVI is performed by various incentives \( H = \{H_1, H_2, ..., H_s\}, s \in Z \). [3]

The marketing model will be most appropriate if it takes into account a number of factors:

For social networks, a key indicator is a “viral threshold” (percolation threshold) \( \lambda_n \) which is critical level of involvement above which the marketing virus covers the entire network. If the social network is represented by a random prefractal graph, the virus with viral coefficient \( K \) exponentially multiplies above the viral threshold; the virus with viral coefficient \( K \) exponentially “dies out” below the percolation threshold. Let us assume that a marketing network agent with an involvement level \( u > \lambda_n \) received an invitation from a passive agent; if \( u \leq \lambda_n \) the agent will be passive. Let us assign certain weights to the graph vertices \( u(v_i), v_i \in V, i = 1, n, 0 \leq u(v_i) \leq 1 \); random numbers are randomly assigned to the graph vertices by the generator [4].

The percolation theory considers left-to-right or bottom-up processes; we define the concept “percolation” on the prefractal graph with weighted vertices which are a source and a drain. We assume that there is a flow between different fixed incentives of the same rank \( S_i = H_i = (W_i, Q_i) \) and \( S_2 = H_2 = (W_2, Q_2) \), which are a source and a drain if there is a route \( (u_i, v_j) \) from vertexes \( u(v_i) > \lambda \), connecting two vertexes where \( u_i \in W_i \) and \( v_j \in W_j, u_i, v_j \in 1 \cup R, i, j = 1, n \).

4. Algorithm for searching for a percolation threshold of a prefractal graph

1. \( \beta_i := 1; \)

2. The search for different routes connecting the source and the drain at \( \beta_i \); if there is such a route, let us return to step 1 at \( \beta_i := \beta_i/2 \); otherwise \( \beta_i := \beta_i \) and let us return to step 3;

3. \( \beta_i := \frac{\beta_i + \beta_i}{2} \). Если \( |\beta_2 - \beta_1| < \xi \), \( \beta := \beta_1 \) and the algorithm will stop working, otherwise let us return to step 2. [5]

Let us denote the value \( \beta \) as \( \lambda_n – \text{percolation threshold.} \)

To identify the relationships to be blocked to prevent further the development of the marketing virus, it is necessary to specify a rank of the incentive and indicate a share of active agent nodes at which quarantine measures are possible. When the share of active agents reaches the specified rank, all edges are blocked. These measures may become necessary if the invitation to join the network is harmful; it may be the dissemination of false information, illegal goods. On the other hand, these edges must be strengthened, if we need to develop a network.

Let us assume that the model is constructed in the form of a prefractal graph weighted by all vertices. Let us consider the dissemination of a marketing virus if the primary source of the virus is given. From a mathematical point of view at the stage \( 1 = 1 \), the structure of distribution of invitations is \( (n_i + 1) \)-vertex star \( H_1 = (W_i, Q_i) \). [1]
Putting the mathematical description into the verbal correspondence, we will proceed to the next level \( r + 1 \) of distribution by the following rules:

1. A lot of incentives \( H = \{H_1, \ldots, H_l, \ldots, H_T\} \), \( T \geq 2 \), build the graph which corresponds to the random number of infected individuals of the level \( r \);
2. If the vertex \( \nu \in V_r \) is not pendant, it is replaced;
3. The replaced vertex \( \nu \in V_r \) is selected from the subset of pendant vertexes, the edge is incident to the star center;
4. If any pendant vertex \( \nu \in V_r \) is not replaced, it is frozen.

Thus, applying the RVI \( H_2 = (W_2, Q_2) \) to pendant vertexes, we have graph \( G_2 = (V_2, E_2) \), which was the structure of virus dissemination at the next stage \( l=2 \). Any prefactal graph \( G_l = (V_l, E_l) \), \( l=1,2, \ldots \) is a dissemination structure at stage \( l \). Figure 2 shows prefactal graph \( G_l = (V_l, E_l) \), created by a set of different incentives-stars. Having performed \( L \) transitions, we have a root tree \( G_L = (V_L, E_L) \), which is a tree of distribution of the infectious disease \( D_L = (V_L, E_L) \).

5. Conclusion
A software simulation product was developed in the FreePascal environment. It is based on the agent approach that allows for simulation of development of marketing networks, calculation of a viral threshold, a percolation threshold, choosing measures for strengthening edges that affect the spread of a marketing virus.

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