Fast control of the reflection of a ferroelectric by means of an extremely short pulse

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Abstract
We propose a new type of optical switch based on a ferroelectric. It is based on the gap which exists for waves propagating from a dielectric to a ferroelectric material. This gap depends on the polarization of the ferroelectric. We show that it can be shifted by means of a control electromagnetic pulse in such a way that the material becomes transparent. This device would produce a shift in a time much shorter than the relaxation time of the ferroelectric (1 ns). Estimates are given for a real material.

Keywords: ferroelectric materials, switching in ferroelectrics, optical switches

(Some figures may appear in colour only in the online journal)

1. Introduction

The rapid control of light is an area that has been studied for a long time, in particular because of the applications. Ferroelectric materials are good candidates for providing this control because they can be activated using an electric field. The principle is the following. The reflection and transmission properties of a ferroelectric material depend on its state of polarization, and in particular its spontaneous polarization which exists in the absence of an electric field. The controlling electric field will shift this polarization in such a way that one can then control the reflection and transmission of any electromagnetic wave. This phenomenon is the basis of a switching device. The materials used can be solid state ferroelectrics [1–4], as bulk or thin films. Recently liquid crystals have also been used [5–10]. These are usually in the form of thin films. Two main configurations are used: a waveguide [6] and a directional coupler [9]. More details can be found in the reviews [11, 12].

To control the polarization, a first idea is to use a slowly varying electric field, for example [2]. Then one needs to account for the relaxation of the ferroelectric which is about 1 ns and this limits considerably the possibilities for applications. Another idea is to use a fast electromagnetic pulse so that the relaxation of the ferroelectric can be neglected. Here the signals are of two types: there is the high frequency signal that we want to control, and the other signal, of lower frequency, is the control signal. To fix ideas we choose the high frequency signal to be about a femtosecond in period and the control signal to be ten times slower. The time scales are such that we can neglect the relaxation of the polarization. In this way we achieve light control with light, using an ultra-short pulse. In this paper, we show that the reflection coefficient is equal to 1 in a frequency region: a
2. The basic equations

To describe the propagation of a pulse of any duration we use the total macroscopic Maxwell equations. Recall that the macroscopic equations are derived by averaging the microscopic Maxwell equations over a small volume. This causes a limitation on the duration of the pulse described by the macroscopic electrodynamics. However the propagation of femtosecond pulses may be investigated using these macroscopic Maxwell equations [13].

The Maxwell equations in a dielectric medium are taken in the following form:

\[ \nabla \times \mathbf{E} = -\mathbf{B}, \quad \nabla \cdot \mathbf{E} = -\varepsilon_0^{-1} \nabla \cdot \mathbf{P}, \]
\[ c^2 \nabla \times \mathbf{B} = \mathbf{E}_f + \varepsilon_0 \varepsilon_0^{-1} \mathbf{P}_t, \quad \nabla \cdot \mathbf{B} = 0, \]

where the subscripts indicate the time derivative. From these equations the wave equation results as

\[ c^2 \nabla^2 \mathbf{E} - \mathbf{E}_{\cdot \cdot t} = \varepsilon_0^{-1} [\mathbf{P}_{\cdot t} - \nabla (\nabla \cdot \mathbf{P})]. \]

The geometry is shown in figure 1; it is a dielectric layer for \( z < 0 \) and a ferroelectric layer for \( z \geq 0 \). We assume the electric field to be polarized along \( x \). The spontaneous polarization of the ferroelectric is also supposed to be along \( x \) and to depend only on \( z \). Because of this the wave equation above reduces to

\[ c^2 E_{zz} - E_{\cdot \cdot t} = \frac{P_{\cdot t}}{\varepsilon_0}. \]  

In particular the term \( \nabla (\nabla \cdot \mathbf{P}) \) is zero because the polarization does not have a longitudinal \( (z) \) component. The linear dielectric material \((z < 0)\) will be described by the Drude–Lorentz model [14], so the polarization \( \mathbf{P} \) is as follows:

\[ P_{\cdot t} + \alpha_0^2 P = \varepsilon_0 \omega_0^2 \delta E, \]  

where \( \omega_0 \) is the polarization frequency and \( \omega_0 \) the plasma frequency. The polarization in the ferroelectric is given by the Landau–Khalatnikov equation [15]

\[ \tau^2 P_{\cdot t} - AP + BP^3 = \varepsilon_0 E, \]  

where \( A = \alpha (T_c - T) > 0 \) and \( B > 0 \) are the Landau–Ginzburg coefficients [17–19] and where \( \tau \) is the characteristic time of the polarization. The three equations (1)–(3) describe completely the field and polarization of the media.

For zero electric field, the spontaneous polarization of the ferroelectric is given by

\[ \text{Dielectric} \quad \text{Ferroelectric} \]

Figure 1. Top panel: schematic drawing of the dielectric–ferroelectric interface. The electric field \( E \) is incident normally to the interface. It is polarized along \( x \) and the polarization is also along \( x \). We show the control pulse \( E_0 \) and the controlled small amplitude wave \( \delta E \) of frequency \( \omega_0 \), the carrier frequency (see the text for details). The bottom panel indicates the Fourier spectra of these waves.

\[ -AP_0 + BP^3_0 = 0, \quad \rightarrow P_0 = \pm \sqrt{\frac{A}{B}}. \]

Here the + (resp. -) sign is for a polarization in the direction of +\( x \) (resp. -\( x \)). Now assume a field \( E_0 \) constant over a time interval \( t_p \). The polarization will then shift and satisfy

\[ -AP_0 + BP^3_0 = \varepsilon_0 E_0. \]

For small \( E_0 \) we can estimate \( P_0 \) using \( E_0 \) as a perturbation. We get

\[ P_0 = \pm \frac{A}{B} + \frac{\varepsilon_0 E_0}{2A} + O(E_0^2). \]

Note that we did not consider the polarization close to 0 because it is unstable. During the time interval \( t_p \) we can send a small electromagnetic wave \( \delta E \). This will shift the polarization by \( \delta P \). Assuming that \( E = E_0 + \delta E, \quad P = P_0 + \delta P \) where \( |\delta E| \ll E_0 |\delta P| \ll P_0 \) in equations (1)–(3) yields the following linear system for in the ferroelectric:

\[ c^2 \delta E_{zz} - \delta E_{\cdot \cdot t} = \varepsilon_0^{-1} \delta P_{\cdot t}, \]
\[ \tau^2 \delta P_{\cdot t} - \Lambda \delta P + 3BP_0^2 \delta P = \varepsilon_0 \delta E. \]

In the dielectric layer, \( P_0 = 0 \), so the second equation should be replaced by

\[ \delta P_{\cdot t} + \alpha_0^2 \delta P = \varepsilon_0 \omega_0^2 \delta E. \]

The three linear equations (7)–(9) represent the small oscillations \( (\delta E, \delta P) \) around the function point \((E_0, P_0)\) which exists for the time \( t_p \).
3. Solutions of the wave equations in the frequency domain

The system of linear equations above can now be solved completely using Fourier transforms in $t$ and matching $\delta E$ and its derivative at the interface $z = 0$. Taking the Fourier transform in $t$ we get for $z < 0$

$$\tilde{\delta}E_{zz} + k_0^2 \tilde{\delta}E = -\frac{q_0^2 k_0^{-1} \tilde{\delta}P}{k_0^2 \omega^2},$$  \hspace{1cm} (10)

$$\tilde{\delta}P = \varepsilon_0 \tilde{E} - \frac{\omega_p^2}{\omega_0^2 - \omega^2}.$$  \hspace{1cm} (11)

Plugging the second equation into the first one, we obtain

$$\tilde{\delta}E_{zz} + k_0^2 \left(1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}\right) \tilde{\delta}E = 0.$$  \hspace{1cm} (12)

For in the ferroelectric medium, for $z > 0$, starting from equation (7) and following a procedure similar to that for $z < 0$ we get

$$\tilde{\delta}E_{zz} + k_0^2 \left(1 + \frac{1}{-\tau^2 \omega^2 - A + 3Bp^2}\right) \tilde{\delta}E = 0.$$  \hspace{1cm} (13)

Substituting in the expression (6) for the spontaneous polarization $P_0$, we finally get

$$\tilde{\delta}E_{zz} + k_0^2 \left(1 + \frac{1}{2A + 3\varepsilon_0 \varepsilon_E \sqrt{B/A} - \tau^2 \omega^2}\right) \tilde{\delta}E = 0.$$  \hspace{1cm} (14)

Thus we have a piecewise wave equation for $z < 0$ (12) and $z > 0$ (14). In such linear media we can introduce the dielectric permittivity to describe wave propagation. We get

$$\varepsilon_{\text{dil}}(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}$$

in the dielectric layer and

$$\varepsilon_{\text{ferr}}(\omega) = 1 + \frac{1}{2A + 3\varepsilon_0 \varepsilon_E \sqrt{B/A} - \tau^2 \omega^2}$$

in the ferroelectric layer. The solution in the two different regions is a linear combination of harmonic waves with wavenumber $k_1$ for $z < 0$ and wavenumber $k_2$ for $z > 0$, where

$$k_1 = k_0 \left(1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}\right)^{1/2}.$$  \hspace{1cm} (15)

$$k_2 = k_0 \left(1 + \frac{1}{2A + 3\varepsilon_0 \varepsilon_E \sqrt{B/A} - \tau^2 \omega^2}\right)^{1/2}.$$  \hspace{1cm} (16)

These formulas show us that the external electromagnetic pulse can control the dielectric properties of the ferroelectric material.

4. Scattering of linear waves off the interface $z = 0$

The reflection and transmission coefficients of harmonic waves can be computed as a function of the control field $E_0$. Note that the two orientations of polarization will give the same reflection coefficient for $E_0 = 0$. Only on adding the control $E_0$ is one able to distinguish the two states of polarization. We set up the scattering formalism assuming a wave incident from the left, a reflected wave and a transmitted wave:

$$\tilde{E}(z, \omega) = E_{\text{in}}(\omega) e^{ik_1 z} + E_{\text{r}}(\omega) e^{-ik_1 z}$$

in the dielectric and

$$\tilde{E}(z, \omega) = E_{\text{t}}(\omega) e^{ik_2 z}$$

in the ferroelectric medium. In the absence of the surface charges and currents, the jump conditions on the interface read [16]

$$E(0-, \omega) = \tilde{E}(0+, \omega), \quad E_{\text{r}}(0-, \omega) = \tilde{E}_{\text{r}}(0+, \omega).$$

Using these conditions and the solution of the wave equation one can find the Fresnel relations connecting the amplitudes of the incident wave $E_{\text{in}}(\omega)$, reflected wave $E_{\text{r}}(\omega)$ and transmitted wave $E_{\text{t}}(\omega)$:

$$E_{\text{t}}(\omega) = \frac{k_1 - k_2}{k_1 + k_2} E_{\text{in}}(\omega).$$  \hspace{1cm} (17)

$$E_{\text{r}}(\omega) = \frac{2k_1}{k_1 + k_2} E_{\text{in}}(\omega).$$  \hspace{1cm} (18)

These relations are correct for any low amplitude waves: both for solitary waves and for harmonic waves. The reflection and transmission coefficients are then, respectively

$$R = \frac{E_{\text{r}}}{E_{\text{in}}} = \frac{k_1 - k_2}{k_1 + k_2}, \quad T = \frac{E_{\text{t}}}{E_{\text{in}}} = \frac{2k_1}{k_1 + k_2}.$$  \hspace{1cm} (19)

5. Discussion

Figure 2 shows the modulus of the reflection coefficient $|R|^2$ as a function of the reduced frequency $\tau^2 \omega^2$ for three different values of the control $E_0 = 0$ (continuous line, red online), $E_0 = 0.3$ (long dashes, red online) and $E_0 = -0.3$ (short dashes, blue online). As can be seen, the boundary of the gap for which there is total reflection of the wave
Figure 3. Square of the modulus of the reflection coefficient $|R|^2$ for a fixed reduced frequency $\tau^2 \omega^2 = 0.8$ as a function of the normalized control $3\epsilon_0 E_0/P_4$.

is shifted to higher frequencies (resp. lower frequencies) for $E_0 > 0$ (resp. $E_0 < 0$). We have assumed the plus sign in the expression for $k_2$ (15). To illustrate how the field $E_0$ can be used to block a wave, we have plotted in figure 3 $|R|^2$ as a function of $E_0$ for $\tau^2 \omega^2 = 0.8$. For $E_0 > 0.4$ the ferroelectric is transparent. As $E_0$ is decreased, $|R|^2$ increases sharply and reaches 1 for $E_0 = 0.2$. Below that value, the ferroelectric is opaque to this particular frequency.

To show how this scheme can work in reality, we examine parameters for a real material. Consider the study by Noguchi et al [20] on the Bi$_4$Ti$_3$O$_{12}$SrBi$_4$Ti$_4$O$_{15}$ intergrowth ceramics. Such material was shown to have a large spontaneous polarization. In addition, its Curie temperature is high, so it is stable. To estimate $A$ and $B$ from the measurements of [20] we recall that

$$\frac{A}{B} = P_c^2, \quad \frac{2}{3} A \sqrt{\frac{A}{3B}} = \epsilon_0 E_c,$$

where $P_c$ and $E_c$ are respectively the coercive polarization and coercive field. Using the values from [20] we get

$$A = 2 \times 10^{-3}, \quad B = 10^{-1} \text{ m}^4 \text{ C}^{-2}.$$  

This value of $A$ defines a resonant frequency $\omega_r$ such that

$$\tau^2 \omega_r^2 = 2A.$$  

We get

$$\tau \omega_r = 4.5 \times 10^{-2}.$$  

The value of $\tau$ given by estimates of the inertia of molecular assemblies is about $\tau = 10^{-10}$ [21]. This gives

$$\omega_r = 4.5 \times 10^8 \text{ Hz}.$$  

An important point is that at resonance the two terms $2A$ and $\tau^2 \omega^2$ are almost equal, so their difference is very small. Then a small shift due to the term $3\epsilon_0 E_0 \sqrt{B/A}$ will displace the resonance. Let us estimate the field $E_0$ needed to shift the resonance from $\omega_r$ to $\omega_r/2$. We have

$$3\epsilon_0 E_0 \sqrt{\frac{B}{A}} = \tau^2 \omega_r^2 \frac{\omega_r}{4}.$$  

This gives

$$E_0 \approx 10^6 \text{ V} \text{ m}^{-1}.$$  

This value of the electric field can be achieved using a laser.

6. Conclusion

The reflection of the electromagnetic wave at the interface between a linear dielectric medium and a ferroelectric was considered. We assume that the electromagnetic wave is a superposition of a high frequency wave and a spike-like electromagnetic signal. The spectrum of the spike is located near the zero of frequency and can be considered as low frequency. We showed that the spike-like signal induced an extra contribution to the total ferroelectric polarization. This causes a fast change of the reflection coefficient in the high frequency domain. In the time duration of the spike-like signal the relaxation processes can be neglected. Thus one can achieve rapid control of light with light, using an extremely short, spike pulse.

References

[1] Sherman V, Astafiev K, Setter N, Tagantsev A, Vendik O, Vendik I, Hoffmann-Effert S and Waser R 2001 IEEE Microw. Wirel. Compon. Lett. 11 407–9
[2] Mishina E D, Shershnyuk N E, Stadnichuk V I, Sigov A S, Mukhorotov V M, Golovko Yu I, van Etterge A and Rasing Th 2003 Appl. Phys. Lett. 83 2402–4
[3] Vizdrik G, Ducharme S, Fridkin V M and Yudin S G 2003 Phys. Rev. B 68 094113
[4] Kedzierski D, Kirichenko E V and Stephanovich V A 2011 Phys. Lett. A 375 685–8
[5] Meadows M R, Handschy M A and Clark N A 1989 Appl. Phys. Lett. 54 1394
[6] Gros E and Dupont L 2001 IEEE Photon. Technol. Lett. 13 115–7
[7] Ntogari G, Tsipouridou D and Kriezis E E 2005 J. Opt. A: Pure Appl. Opt. 7 S2
[8] Rodriguez B J, Jesse S, Baddorf A P, Kim S-H and Kalinin S V 2007 Phys. Rev. Lett. 98 247603
[9] Lee W Y, Lin J S, Lee K Y and Chuang W C 1995 J. Lightwave Technol. 13 2236–43
[10] Liu S W and Xiao M 2006 Appl. Phys. Lett. 88 143512
[11] Sirleto L, Hermann D S, Scalia G, Komitov L, De Marco F, Abbate G, Lindgren M, Mornmle P and Richini G C 2002 Fiber Integr. Opt. 21 277–93
[12] Sitter N et al 2006 J. Appl. Phys. 100 051606
[13] Brasbee T and Kraus F 2000 Rev. Mod. Phys. 72 545–91
[14] Rosenfeld L 1965 Theory of Electrons (New York: Dover) p 68
[15] Landau L D, Pitaevskii L P and Lifshitz E M 1984 Electrodynamics of Continuous Media 2nd edn (Oxford: Butterworth–Heinemann)
[16] Born M and Wolf E 1999 Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light 7th edn (Cambridge: Cambridge University Press)
[17] Ginzburg V L 1945 JETP 15 739 (in Russian)
[18] Devonshire A F 1949 Phil. Mag. 40 1040
[19] Ginzburg V L 2001 Phys.—Usp. 44 1037
[20] Noguchi Y, Miyayama M and Kudo T 2000 Appl. Phys. Lett. 77 3639–41
[21] Caputo J-G, Mainistov A I, Mishina E D, Kazantseva E V and Mukhertov V M 2010 Phys. Rev. B 82 094113