Basic Analytical Study of CFRP Grid-Sprayed Mortar Behavior in Shear Strengthening of a RC Beam

by

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The shear failure of reinforced concrete (RC) beams is extremely dangerous as it can occur suddenly and it is difficult to predict. Therefore, strengthening methods to prevent this failure in a RC beam are especially important. Recent years, CFRP material is promising alternatives for rehabilitation of deteriorated concrete members. In this study, it is assumed that, shear resistance of a RC beam decreases because of corroded stirrups, thus it need to be strengthened by using carbon fiber-reinforced polymer (CFRP) grid and sprayed mortar. Subsequently, the part where potentially appears diagonal crack in this beam would be taken out to analyze. The work of CFRP grid inside the mortar layer is modelled as a frame put on an elastic-brittle springs foundation. Simultaneously, bond behavior between mortar layer and substrate concrete is also modelled as the same type springs. A system of CFRP frame and springs is analyzed to investigate about the failure process and the influence of the strengthening layer. Lastly, case studies are presented by FEM analysis to compare with results of some previous tests. There is an appropriate good agreement between FEM and experimental approaches. It is suggested that the damage zone starts at the bottom of the specimen and spreads to the top during loading. In addition, the effect of CFRP and sprayed mortar on the resistance capacity of the structure was also clarified.

Key words:
Shear failure, RC beam, CFRP grid, Sprayed mortar, Bond behavior, Elastic-brittle spring, Finite-Element Method (FEM)

1 Introduction

Presently, many concrete structures show signs of severe damage after years of long service, which affects their usability and safety. There are many studies on how to repair such aged structures. One of the most common methods is to remove the deteriorated surface layer, clean the corroded reinforcement, and then cover the structure with appropriate materials. Currently, CFRP materials are widely used as reinforcement owing to high strength to weight ratio, durability in adverse environments and high fatigue strength 1). The majority of the studies have focused on the use of CFRP sheets to strengthen beams or slabs under bending load, whereas the use of CFRP grids to improve shear failure resistance is not adequately studied 2). Moreover, in term of constructability and maintenance of repairing an existing construction, using CFRP grid has some significant advantages in comparison with CFRP sheet, for example: reduce work for preparing substrate surface, can construct in wet surface condition or can determine damage at the surface after strengthening, etc.

Reinforced concrete beams fail via brittle fracturing in shear mode, which is more dangerous than flexural tension failure. Furthermore, the shear failure of an RC beam is not well understood 3). If the beam design does not call for adequate shear reinforcement, e.g., stirrups and diagonal reinforcements, then overloading will induce shear collapse. Especially, in Japan, which is an earthquake-prone country with a long coastline, stirrups and other oblique reinforcements of RC beams can be corroded easily due to water ingress through small cracks. Until now, steel mesh occasionally has been applied on the repair work though having many anxieties about corrosion phenomenon. A superior alternative protection method is to combine CFRP materials with cement layers 4).

In this study, basing on the assumption that the shear failure resistance of a reinforced concrete beam decreases because of corroded stirrups. Therefore, this beam should be strengthened by using a CFRP grid and repair mortar (Fig. 1). To evaluate...
behavior of the repair layer when the beam is loading, the part in which potentially appears diagonal crack in this beam would be taken out to analyze. This element also includes 3 components: substrate concrete, mortar and CFRP grid.

In fact, diagonal cracks in shear failure of a RC beam causes by diagonal tension forces. As its role like steel rebar, the CFRP grid is embedded entirely inside the sprayed mortar layer and resists almost tensile force in the analyzed elements. Therefore, external load can transferred via the CFRP grid to the mortar, and then come to the interface between the mortar and concrete substrate. (Fig. 2). Consequently, the resistance of the specimens depend on 3 factors: strength of the CFRP, anchorage of the CFRP inside the mortar layer, bond between the concrete substrate and the mortar. Especially, the bond behavior of the interface is the most important because it is the last destination of external force before break and very difficult to estimate. In this study, the links between CFRP-mortar-mortar-concrete are idealized by the system of elastic-brittle spring (Fig. 3). The analysis intends to make clear about the influence of each component and their relationship to the failure process as well as the real experiment’s results. From that, it can be evaluated the effectiveness this repair method.

2 Analytical investigation

2.1 Outline

Generally, in the concrete specimens repaired by the above-described method, the interface between old and new materials is the weakest part. Therefore, damages often start to appear at the interface. However, with the development and use of adhesive materials, such as epoxy primers, the cohesive strength of the interface significantly increased, therefore damage can first occur at the interface, inside the repair mortar, or even in the substrate concrete. In this study, it was assumed that the specimens were damaged because of the stress that develops at the interface and inside the mortar layer. Indeed, if considering that the CFRP frame is entirely inside the mortar layer, the resistance capacity of the element depends on the interface’s bond behavior and the mortar properties.

2.2 Work of elastic-brittle spring

The link between the CFRP-mortar and mortar-substrate concrete is modeled by a system of nonlinear springs. However, because of elastic–brittle characteristics of these materials, performance of the springs needs to be simulated equivalently. The springs should behave as nonlinear elastic springs (Fig. 4). The relation between spring’s reaction force and displacement of the spring is explained in the following manner. Before reaching the critical displacement $\delta_c$, the spring works like an elastic material. Therefore, relationship between the reaction force $R$ and displacement $\delta$ is linear. At the critical point of maximum stress, the spring switches to softening behavior. In this stage, the vertical deformation of the spring continues increasing until reaching $\delta_{max}$ while the reaction force is decreasing. At $\delta_{max}$, the reaction force of the spring becomes zero, which means the bond between the materials is broken and the resistance region moves to the next springs of this location. Consequently, the structure is damaged completely when all springs lose their capacities.

The spring’s coefficients can be estimated in the following method. The critical displacement $\delta_c$ and maximum displacement $\delta_{max}$ are same as in the original concrete material. The maximum capacity force of the spring depends on the shear strength of the interface, which is obtained from direct shear tests, and the tensile strength of the mortar. Each spring is represented for a part of the interface area and a part of the mortar’s cross section. Therefore, based on the distance between each spring, the grid spacing and the thickness of the mortar, the maximum force that each spring can bear can be estimated.

As can be seen in Fig. 5, the reaction force of each spring includes the shear force ($R_s$) on one unit area of the interface ($h \times s \text{ mm}^2$) and the tensile force ($R_t$) on one unit cross-sectional area in the mortar ($t \times s \text{ mm}^2$). The maximum reaction force ($R_{max}$) is attained, if these areas reach maximum capacity, and is given by the equation below

$$R_{max} = R_s + R_t = (h \times s \times \sigma_g + t \times s \times \sigma_t) \times \beta \quad (1)$$

In where $\beta$ is an adjustable coefficient, $s$ is the spring spacing, $t$ is the thickness of the mortar, $h$ is the height of the resistance area, $\sigma_g$ is the shear strength of the interface, and $\sigma_t$ is the tensile strength of the mortar.
2.3 Influence of CFRP type

In this model, the CFRP grid works like a frame structure is supported on a spring foundation. In the CFRP structure, the vertical CFRP bar is considered as a truss element that only carries the axial force, whereas the horizontal bars act like a flexural beam.

To evaluate the effect of CFRP, a simple model is considered to assess the role of each component. As it is shown in Fig. 6, this simple structure includes a truss, a beam, and a spring that carries the concentrated point load. It is noticed that when subjected to a vertical load, the two types of spring work in parallel; thus it can be replaced by an equivalent one. According to the principle of conservation of energy, the work done by external forces on a structure is converted to internal work or internal stored energy.

Assuming that the external load \( P \) gradually increases to the maximum displacement \( \Delta \), the external work \( W \) is

\[
W = \frac{1}{2} P \cdot \Delta
\]

(2)

The internal work of this structure comprises three components; the strain energies of the CFRP beam, the truss, and the spring.  

\[
U = U_{spring} + U_{truss} + U_{beam}
\]

(3)

From the strain energy of the components, the internal work:

\[
U = \frac{1}{2} k \Delta^2 + \frac{1}{2} E \epsilon^2 \Delta^2 + 2 \frac{3Eh^3}{L^3} \Delta^2
\]

(4)

In which \( k \) is the spring stiffness, \( n \) is the displacement, \( F \) is the cross-sectional area of the CFRP bar, \( I \) is moment of inertia, \( E \) is the Young’s modulus, and \( L \) is the length of the truss or beam. Since \( W = U \), therefore:

\[
\Delta = \frac{P}{k + \frac{6EI}{L^2} + \frac{6EI}{L^2}}
\]

(5)

In Eq. (5), the displacement of point \( A \) depends on the magnitude of the external load \( P \), the properties of the CFRP structure \( (E, I, F, \text{and} L) \), and the properties of the spring \( k \). More specifically, the deflection \( \Delta \) is inversely proportional to the elastic modulus of CFRP, the cross-sectional area of the CFRP bar, and the stiffness of the spring, but it is proportional to the load and the grid spacing of the CFRP frame.

2.4 FEM model

The most simple finite-element model is represented by a frame that consists of trusses, beams, and springs (Fig. 7). In this model, the horizontal bars are simulated by pure bending beams (elements 2 and 4) with only bending moment. The vertical bars (element 6) are truss elements. End of the beam and truss elements belong to the same nodes that act as the rigid link between two components. The beam and truss element have the same properties as the actual CFRP materials.

Springs (elements 1, 3, 5, and 7) are of the elastic-brittle type, with one end connected with the CFRP element and the other linked to a fixed foundation. Because two springs are parallel, the equivalent spring stiffness \( k \) can be calculated as the total stiffness of each spring. The spring breaks when a given threshold (maximum displacement) is exceeded. At that moment, all these spring strain energy is released and the spring is not considered in the calculations in the subsequent loading steps. Therefore, the spring’s stiffness \( k \) is not constant but is a function of the spring deformation. In the computations, an iterative approach is used to find the solution to this problem.

After considering the boundary conditions, the unknown displacements \( (u, \text{translation} \text{and} \theta, \text{rotation}) \) of each node are also shown in Fig. 7. The stiffness matrix of the bending beam elements (elements 2 and 4) is

\[
[K_{2,4}] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}
\]

(6)

The stiffness matrix of a truss element (element 6) is

\[
[K_6] = \frac{E}{I} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}
\]

(7)

The stiffness matrices of the spring elements (elements 1, 3, 5, and 7) are

\[
[K_{1,3,5}] = \begin{bmatrix} k & -k & 0 \\ -k & k & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

(8)

\[
[K_7] = \begin{bmatrix} k' & -k' & 0 \\ -k' & k' & 0 \end{bmatrix}
\]

(9)

Equation

\[
[K][q] = [P]
\]

(10)

is transformed to Eq. (11). Solving it can give the unknowns.
3 Case study and comparison with previous tests

3.1 Outline

In this section, a specific case study is presented to evaluate more detail about behavior of the proposal model and to compare with results of previous actual element tests. Especially, it is focused on comparing maximum external load, failure process, failure pattern and influence of CFRP types in the two approaches.

The tests were carried out in laboratory, in which the specimens also comprised 3 components: concrete, mortar and CFRP grid (Fig. 8). There are 4 types of CFRP, which differs in thickness (CR6, CR8) and grid spacing (100 mm and 50 mm), were used. The mechanical properties of CFRP is obtained from its product catalog as shown in Table 1.

Due to machinery condition, only one CFRP bar in the middle of each specimen was extended and applied a pull force. Firstly, results of experiments has confirmed the role of CFRP to the failure of these specimen. As can be seen in Table 2, the specimen has a larger cross section of CFRP bar can resist higher maximum load. Furthermore, the smaller the grid steps are, the better the bearing capacity could be achieved.

The influence of CFRP type also presents through the failure patterns that shown in Fig. 9. The specimens used CR6 type were damaged causes by pulling out of CFRP bar. There are not many cracks appear on the surface of mortar. On the other hand, CR8 bar has not been pulled out in the tests. The specimens with CR8 and 50 mm grid spacing was peeled. Before the ultimate load, a vertical crack along the center CFRP bar appeared. Type of failure in the specimen has CR8-100 mm grid spacing is fracturing on the mortar’s surface and peeling at the interface. The last row of CFRP was totally broken down and led to peel in all of the cross section. Especially, from experimental investigation, the failure is suggested that damage increases steps by step from the bottom of mortar to the top and failures always start from the grid point and propagate in horizontal direction until all of the cross section areas are covered.

The analytical models were simulated similar to the experiment specimens. The model is computed by using the FEM Marc-Mentat software as shown in Fig.10. At each node of the CFRP element, there are two springs (below and above) that

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### Table 1: Type of CFRP

| CFRP Type | CR6 | CR8 |
|-----------|-----|-----|
| Cross-section area (mm²) | 17.5 | 26.4 |
| Tensile strength (N/mm²) | 1400 | |
| Young's modulus (kN/mm²) | 100 | |
| Grid step (mm) | 50, 100 | 50, 100 |

### Table 2: Results of experiments.

| Specimen | CFRP | Grid spacing (mm) | No. of CFRP row | Maximum load (kN) |
|----------|------|------------------|-----------------|-------------------|
| CR6-100  | CR6  | 100              | 3               | 8.11              |
| CR6-50   | CR6  | 50               | 5               | 14.92             |
| CR8-100  | CR8  | 100              | 3               | 12.23             |
| CR8-50   | CR8  | 50               | 5               | 17.42             |

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### Table 3: Results of maximum load in analysis (kN).

| Specimen | No. of CFRP row N |
|----------|-------------------|
|          | N=2 | N=3 | N=4 | N=5 | N=6 |
| CR6-100  | 7.01 | 9.54 | 12.04 | 14.04 | 14.5 |
| CR8-100  | 9.07 | 12.71 | 15.34 | 17.5  | 19.28 |
| CR6-50   | 8.54 | 11.75 | 17.84 | 20.61 | 23.24 |
| CR8-50   | 11.11 | 15.76 | 25.07 | 31.12 | 32.61 |
The maximum loads are obtained in the models CR8-50 and the values in the actual tests in Fig. 8. It is noticed that, the height of the supporting area under the first row of the CFRP is two or four times smaller than in the grid 100 (or 3 to 11 in grid 50) because it can represent the height to 1m of a general RC beam in a concrete bridge.

Stiffness of the springs is calculated by Eqs. (1) based on the actual value tensile strength of mortar and shear strength of the interface. The shear strength was obtained from a direct shear test in laboratory. Therefore, the parameters for the elastic–brittle springs are $\delta_s = 0.1$ mm; $\delta_{max} = 0.3$ mm; the distance between springs is $s = 2.5$ mm; the mortar tensile strength and shear strength of the interface are $\sigma_t = 2.02$ N/mm$^2$ and $\sigma_s = 4.69$ N/mm$^2$, respectively. Based on the assumption of decreasing distributed stress to the vicinity area around the location of spring, only a half of the corresponding area near each spring in the both direction resist the external load. Consequently, the adjustable coefficient $\beta$ is supposed as 0.5. Other dimensions is used same as values in the actual tests in Fig. 8.

Using these parameters to apply for Eq. (1) and round up these values, the maximum reaction force of one spring is assumed as $R_{max1} = 250$ N for the lowest row and $R_{max2} = 700$ N (in grid 100 mm) or $R_{max2} = 400$ N (in grid 50 mm) for the higher rows. It is noticed that, the height of the supporting area under the first row of the CFRP is two or four times smaller than in the higher ones (25 mm compared with 50 or 100 mm). Therefore, the capacity of the springs in the lowest row is the smallest.

3.3 Results and discussions

In total, 20 models were analyzed (Table 3). The influence of CFRP to the load capacity is good agreement with the results of FEM analysis. As can be seen in Fig. 11, a thicker CFRP bar and CFRP with small grid spacing can withstand higher loads. The maximum loads are obtained in the models CR8-50 and the model of CR6-100 represent the worst capacity. It also found that, if considering the ratio of maximum tensile capacity of CFRP bar (CR6 is 24.5 kN; CR8 is 36.96 kN) to the maximum load, CFRP grid 100mm in both types increase from over 20% ($N=2$) to more than 60% ($N=6$). On the other hand, the corresponding ratio of 50mm grid type are 30 % and 90%, respectively.

In comparison with testing results, the maximum loads in the FEM analysis are 4% to 27% differs with those in the experiments (average value 14%). The different of analytical and experimental results would be mainly due to the bond between CFRP and mortar is not considered, especially the vertical bars.
The capacity of the springs in the lowest row is the smallest. Higher ones (25 mm compared with 50 or 100 mm). Therefore, actual value tensile strength of mortar and shear strength of the interface are assumed as values in the actual tests in Fig. 8. It is noticed that, the height of the supporting area under the first row of the CFRP is two or four times smaller than in the both rows. It is supposed as 0.5. Other dimensions is used same as values in the both rows.

Next, it is found that the shear resistance area moved stepwise from the bottom to the top of all specimens. Damage always begins at the grid points and propagates in the horizontal direction until it covers all the cross-sectional area (Fig. 15 a,b,c). Thus, the ratio of tension capacity of CFRP bar to the maximum load, which reflect the effective of CFRP material, rises when the number of CFRP row increases and the grid spacing is smaller.

In summary, the work of CFRP grid can open a possibility to replace steel stirrup in recovery of the RC beam, which has been reduced shear capacity.

This research was supported by Asian Human Resources Fund from Tokyo Metropolitan Government.

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