Optimal planning of Renewable energy generators in modern power grid for enhanced system inertia

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Abstract
In recent times renewable energy sources have become an integral part of the modern power grid. As a result, the overall system inertia of the grid has been reduced, thus leading to frequency instability issues such as fast rate of change of frequency. Thus, to compensate for the declining inertia, it is important to carefully select renewable energy generators (REGs) and energy storage systems (ESS) in order to ensure the stability of the power grid, while also controlling greenhouse gases emissions in line with environmental standards. Therefore, this paper proposes an optimal planning model of REGs and ESS, considering the inertia requirement of the grid. The objective function is formulated to minimize the cost of operation, emissions, and investment in new REGs and energy storage units while maximizing the system inertia. The model was developed as a mixed integer linear programming problem and solved using CPLEX solver in GAMS. Finally, the model was validated using a modified IEEE 9-bus system and compared under three scenarios. The results show that in scenario 3 where system inertia is considered in the presence of REGs and ESS, higher system inertia of 8.776 s was achieved at minimal emission and cost, which justifies the aim of the study.

Keywords Inertia · Virtual inertia · GAMS · Generation expansion planning · Model

Nomenclature
Indices and sets
\( g \) · Thermal generating units
\( p \) · Pumped hydro storage (PHS) units
\( v \) · Battery energy storage (BES) units
\( w \) · Wind farm units
\( s \) · Solar PV plant
\( l \) · Set of transmission lines
\( rl \) · Transmission lines at the receiving bus
\( sl \) · Transmission lines at the sending bus

Parameters
\( ag, bg, cg \) · Fuel cost coefficients of thermal generators
\( Di \) · Load power demand at bus \( i \)
\( fl \) · Flexibility factor used to set different RES penetration level
\( \eta_v^{dis}, \eta_v^{cha} \) · Discharging and charging efficiency of BES, respectively
\( e_g \) · Emission coefficient of thermal generators
\( E_{\text{lim}} \) · Maximum CO\textsubscript{2} emissions permissible for all thermal generators
\( \overline{P}_g \) · Maximum active power generation from thermal generators

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### Variables

- $P_{g}$: Minimum active power generation from thermal generators
- $P_{\text{max}}^{ij}$: Maximum transmission line limit (MVA)
- $B_{ij}$: Susceptance of transmission line connecting bus $i$ to $j$
- $S_{\text{base}}$: Base apparent power of system (MVA)
- $S_{g}, S_{w}, S_{p}, S_{s}, S_{v}$: Apparent power of thermal generators, BES, PHS, wind farm, and solar PV plants, respectively
- $CO_{\text{inv}}^{v}, CO_{\text{inv}}^{p}, CO_{\text{inv}}^{w}, CO_{\text{inv}}^{s}$: Investment cost on BES, PHS, wind farm, and solar PV plants, respectively
- $CO_{\text{cul}}^{v}$: Cost of curtailment of the wind turbine (M$/MWh)
- $RPU_{g}$: Ramp up limit of thermal unit
- $RPD_{g}$: Ramp down limit of thermal unit
- $\lambda_{w,t}, \lambda_{s,t}$: Variability of wind turbines, and solar PV plants per time, respectively (p.u)
- $\overline{\lambda_{\text{dis}} v}, \overline{\lambda_{\text{dis}} s}$: Maximum and minimum discharging power of BES, respectively
- $\overline{\lambda_{\text{cha}} v}, \overline{\lambda_{\text{cha}} s}$: Maximum and minimum charging power of BES, respectively
- $SC_{\text{max}}, SC_{\text{min}}$: Maximum and minimum state of charge of BES, respectively
- $\overline{P_{p}}, \overline{P_{r}}$: Maximum and minimum power delivery by PHS, respectively
- $\overline{P_{\text{cap}}^{w}}, \overline{P_{\text{cap}}^{s}}$: Maximum power capacity of wind turbine, and solar PV plants connected to bus $i$, respectively (MW)
- $H_{g}, H_{w}, H_{p}, H_{s}$: Inertia constant of thermal generators, BES, PHS, wind farm, and solar PV plants, respectively (in seconds)
- $H_{\text{max}}, H_{\text{min}}$: Maximum and minimum overall system inertia constant set, respectively
- $\lambda_{e}$: Cost of emission of thermal generators ($/\text{tonsCO}_2$)
- $P_{g,t}, P_{g,t-1}$: Active output power generated by thermal generators (MW)
- $P_{g}^{+}, P_{g}^{-}$: Power flows into transmission line $l$
- $P_{g}^{+}, P_{g}^{-}$: Power flows out of transmission line $l$
- $H_{eq}$: Charging and discharging power of BES, respectively
- $\psi_{i}$: Overall system inertia constant of the power system
Introduction

The increasingly negative effects of climate change are partly attributed to emissions from fossil-fueled generators. Therefore, environmental control policies have been introduced to reduce and control these emissions. Also, power system engineers are replacing existing thermal generators with renewable energy generators (REGs) and energy storage systems (ESS) [1].

As more non-inertia renewable energy (RE) generators are being integrated into the power grid, the overall system inertia of the power grid is reduced, which makes the new grid less resistant to system contingencies such as sudden loss of load [2]. This makes the modern grid prone to frequency instabilities such as fast rate of change of frequency (RoCoF) and large frequency nadir. The use of ESS was investigated in [3] to mitigate grid frequency instability due to increasing RES penetration into the grid. The optimal operation and placement of ESS in the modern grid was also investigated in [4] using genetic algorithm, and in [5] using an improved binary particle swarm optimization algorithm.

In the literature, several studies have been carried out on the optimal scheduling and planning of new power system generators into the power grid using heuristic algorithms, metaheuristic algorithms, and mathematical methods. The authors in [6] developed a dynamic multi-objective model for power system planning to minimize system cost and emissions. The model was solved using binary particle swarm optimization algorithm. The authors however did not consider the need for RE generators, and energy storage units (ESUs) in their model formulation. Furthermore, the authors in [7] developed an economical-environmental-technical dispatch model to minimize the total system cost, system emission, voltage deviation, and power loss. The model was solved using three meta-heuristic optimization algorithms, coronavirus herd immunity optimizer, salp swarm algorithm, and ant lion optimizer. Key findings of the study reveal that coronavirus herd immunity optimizer technique outperformed the other optimization approaches. However, the authors did not consider the inertia requirement of the grid in the model formulation. Similarly, authors in [8] developed an economic-environmental dispatch model to minimize the total system cost and emissions. The model was implemented on a 30-bus IEEE test system and solved using Harris hawks optimization algorithm and flower pollination algorithm. Key findings of the study reveal that Harris hawks optimization technique outperforms flower pollination technique in achieving the model objectives. However, the model also did not consider the inertia requirement of the grid as well as the need for ESUs in their model formulation. A multi-objective optimal power flow model was developed in [9] to minimize cost, emissions, and power loss using a decomposition-based multi-objective particle swarm optimization algorithm. The model was implemented on a modified IEEE 30-bus and IEEE 57-bus test power systems. Findings of the study reveal that the developed model algorithm performed better other meta-heuristic optimization algorithms. However, the model also did not consider the inertia requirement of the grid as well as the need for ESUs to combat frequency instabilities, in their model formulation framework. The authors in [10] developed an optimal power flow optimization model to minimize cost and emissions. The model was then solved using a hybrid optimization algorithm comprising of salp swarm optimization algorithm and...
particle swarm optimization algorithm. Key findings of their study reveal that the hybrid optimization model performed better than other metaheuristic optimization algorithms used in the analysis. However, the model also did not consider the inertia requirement of the grid in their model formulation as well as the need for energy storage units.

On the other hand, limited studies have considered the inertia requirement of the grid in operation and expansion planning model. The authors in Ref. [11] explained the importance of virtual inertia as a solution to the declining inertia of the power grid, while the authors in [12] explained the importance of adequate inertia in an AC power system network in order to ensure frequency stability. The authors further proposed a minimum effective inertia constant of 3.6 s required in the conventional grid to ensure frequency stability. Authors in reference [13] highlighted the various types of energy storage systems that can be used to provide virtual inertia to the grid. The various characteristics of the energy storage systems were also explained. Vetoshkin et al. in [14] carried out a comparative analysis of the stability of the grid using different virtual inertia control techniques. Findings of their work reveal that virtual inertia control techniques can be used to improve the stability of the modern power grid, however, generation expansion planning in the presence of ESUs was not considered in their model analysis. The authors in [15] investigated the effect of decreasing system inertia on the frequency stability of the modern grid. Key findings of their study reveal that low inertia grid will be associated with frequency instabilities, however, these instabilities can be mitigated using fast frequency responding devices. Mudaheranwa et al. in [16] investigated the system inertia of Rwanda’s power system under varying RES penetration. The research findings indicate that the overall system inertia in Rwanda’s power system decreases with an increase in RES penetration, thus causing the system frequency to deviate above acceptable limit during system contingency. However, the impact of the study on other system parameters such as cost and emissions was not investigated. In [17], the impact of increasing RE penetration on the rotational inertia of the European power system was investigated. Findings of their study reveal that setting a minimum inertia limit in the grid may truncate the net zero carbon goals by 2050. However, this study did not consider the need for energy storage units in their model formulation.

In expansion planning models, Wogrin et al. in [18] highlighted the importance of inertia consideration in generation expansion planning (GEP). Authors in reference [19] presented an analytical framework for frequency constraint formulation in an economic dispatch problem. More so, the authors in [20] developed a mixed integer linear programming (MILP) model using simplified system frequency constraints to plan for expansion in the power system. Their results show the importance of enhancing system inertia in order to combat frequency instability in the expanding grid. However, the expansion model was developed for an integrated power and natural gas system, while the need to minimize the resulting emissions from the model was not considered. The authors in [21], carried out curtailment analysis of the Nordic power system by setting a minimum inertia limit of 113GWs in the grid. Findings of their study reveal that considering the inertia requirement of the grid in planning will not lead to significant energy curtailment in the grid. However, the study did not consider the need to minimize emissions from thermal generators in their model formulation. A generation expansion planning model was developed in [22] to accommodate new RE generators in the grid. The model was then solved using a differential evolution algorithm to minimize system cost and emissions. Findings of their research reveal that as RES penetration to the grid increases the overall system cost increases while the system efficiency decreases. However, the model did not consider the inertia requirement of the grid, and the need for ESS in their model analysis. Similarly, the authors in [23] developed a generation expansion planning model to minimize cost, and CO₂ emissions. The model also considered investment in RE generators. Key findings of their study reveal that investment in wind turbines reduces the total CO₂ emission in the system, however, the total system cost increases. The study also did not consider system inertia and the need for energy storage units in their model analysis. The authors in [24] incorporated battery energy storage into their generation expansion planning model. The model was then developed as a MILP model and solved using an agglomerative hierarchical clustering decomposition algorithm. However, the influence of system inertia was not considered in the model formulation and analysis.

The summarized review of related literature is presented in Table 1, from which the novelty of the proposed study can be seen in relation to related studies. To the best of the authors’ knowledge as seen in the reviewed literature and in Table 1, no study was found that carried out generation expansion planning considering the inertia requirement of the grid in the presence of energy storage systems and renewable energy generators. To this end, this research aims to develop a new deterministic optimization model that maximizes the overall system inertia in the grid, while minimizing the cost of operation, emissions from thermal generators, and investment on new RE generators and energy storage systems. The model is developed as a mixed integer linear programming optimization problem and solved using CPLEX solver in GAMS. The resulting model is then validated using the modified IEEE 9-bus system to test the robustness of the system, while the optimal scheduling of the generators was obtained. The main contributions of the proposed model are outlined as follows:

- Formulation of a new deterministic generation expansion planning model of new renewable energy generators and energy storage systems for enhanced system inertia.
• Carry out performance evaluation of the developed model in terms of cost, and system inertia.
• Carry out sensitivity analysis of the developed model under different RES penetration levels.

The remaining part of this paper is structured as follows. Section 2 explains the concept of system inertia and relays the inter-relationship between system inertia and frequency stability. Section 3 gives the mathematical formulation of the new deterministic optimization model and its implementation on a modified IEEE 9-bus system. Section 4 presents the model simulation results and discussion, while Sect. 5 concludes the study.

### Background Concept of Inertia in Modern Power Grid

Inertia can be defined as the amount of rotating energy stored in the rotor of synchronous generators (SGs) which tends to resist changes in grid frequency particularly during times of contingency [30]. Synchronous inertia is inherent in synchronous machines, because of their rotating mass, hence the moment of inertia of a synchronous generator (SG) can be defined as in Eq. (1).

\[
E = \frac{1}{2} J \omega^2 = S \cdot H
\]

where \(E\) defines the kinetic energy of the SG, \(J\) defines the moment of inertia in (Kg/m²), \(\omega\) defines the angular frequency in (rad/s), \(S\) defines the base apparent power in VA, and \(H\) is the inertia constant in seconds.

Also, from Eq. (1), \(H\) can be expressed as in Eq. (2);

\[
H = \frac{E}{S} = \frac{1}{2} \frac{J \omega^2}{S} = \frac{J \omega^2}{2S}
\]  (2)

Furthermore, from the swing equation, the power imbalance can be represented as in Eq. (3);

\[
P_m - P_e = P_a = \frac{dE}{dt} = J \frac{d\omega}{dt}
\]  (3)

Equation (3) can then be expressed in terms of torque as in Eq. (4);

\[
\tau_m - \tau_e = \tau_a = J \frac{d\omega}{dt} = J \frac{d^2 \theta}{dt^2}
\]  (4)

where \(P_m\) defines the mechanical power developed in p.u, \(\tau_m\) defines the mechanical torque developed, \(\tau_e\) defines the electrical torque, \(P_e\) defines the electrical output power in p.u, \(P_a\) defines the acceleration power in p.u, \(\theta\) defines the angular displacement of the rotor in rad, \(t\) is the time in seconds, \(f\) defines the supply frequency, and \(df/dt\) is the RoCoF of the system.

From Eqs. (2) and (3), we can obtain Eq. (5);

\[
\frac{2H}{\omega} = \frac{J \omega}{S} = \frac{dt}{d\omega} \left[ \frac{P_m - P_e}{1} \right]
\]  (5)

Rearranging Eq. (5) gives Eq. (6);

\[
\frac{2H}{\omega} \frac{d\omega}{dt} = \left[ \frac{P_m - P_e}{S} \right]
\]  (6)

Replacing the angular frequency \(\omega\) with the supply frequency \(f\) and making RoCoF the subject, we have Eq. (7);
Equation (7), shows the dependence of RoCoF (df/dt) on the power change $P_a$, and the inertia constant ($H$). It further reveals that the inertia constant is inversely proportional to RoCoF such that the higher the inertia constant the smaller the RoCoF and vice versa. Therefore, a reliable and resilient power system can be achieved by increasing the overall system inertia constant. Figure 1, shows the variations of system frequency with system inertia, it is seen that the higher the system inertia the more stable the power system becomes. In light of this, frequency instability in the modern power grid can primarily be mitigated by providing adequate inertia to the grid.

In modern power system comprising conventional thermal generators, solar plants, and wind farms, the effective inertia constant $H_{eq}$ of the grid can be estimated as the sum of the individual inertia of all connected generators to the grid, aggregated as a single generator model. This can be expressed in Eq. (8) as defined by reference [31].

$$H_{eq} = \frac{\sum_{i=1}^{n} H_i S_i + \sum_{p=1}^{np} H_{p\cdot S_{pv}} + \sum_{w=1}^{nw} H_w S_w}{S_B}$$

where $H_i$ is the inertia constant of the thermal generator, $S_i$ is the rated apparent power of the thermal generator, $n$ is the total number of the connected SGs in the power system, $S_B$ is the base apparent power, $S_{pv}$ is the apparent power of solar plant, $np$ is the total number of connected solar plant, $nw$ is the total number of connected wind farm, $H_{p\cdot}$ is the virtual inertia constant of the solar plant, and $H_w$ is the virtual inertia constant of the wind farm in seconds.

**Fig. 1 Effect of system inertia on frequency deviation**

![Graph showing the effect of system inertia on frequency deviation](image)

Effects of declining inertia on the power grid: A report from various countries

Several countries such as New Zealand, United States, United Kingdom, Cyprus, and Ireland with high penetration of renewable energy sources have now been affected by the declining inertia of the power grid. These countries have in recent times already reported cases of power disturbances due to large frequency nadir (up to 47.5 Hz) and fast rate of change of frequency (up to 0.73 Hz/s) which led to power interruption to a large number of customers, tripping of frequency relays, and unplanned load shedding [32]. Figure 2 shows the frequency nadir after a power loss of 2750 MW at the electric reliability council of Texas (ERCOT) power system due to increased RES penetration into the grid.
Concept of Virtual Inertia Control Strategy

Renewable energy sources such as photovoltaic (PV) spower output of each generating unity systems which lack rotating mass can be made to provide inertia virtually using an energy storage system such as battery energy storage (BES) or pump hydro storage (PHS) etc. with a suitable converter control strategy. These virtual inertia control strategies simply mimic the characteristics of SGs in providing inertia virtually to the grid.

Several virtual inertia (VI) control strategies have been adopted in the literature such as synchronverter, virtual synchronous machine (VSM), synchronous Power Controller (SPC), virtual synchronous generator (VSG), virtual oscillator control (VOC), virtual synchronous control (VSYNC), and Inducerverter. These control strategies vary in terms of their characteristics. Further description of the various types of control strategies can be found in [10]. Figure 3, shows the basic configuration of virtual inertia control strategy.

Model formulation

The description of the proposed generation expansion planning model formulated to maximize the overall system inertia while minimizing the total cost and CO₂ emissions of the system is given in this section.

![Fig. 3 Basic configuration of virtual inertia control system](image-url)
Objective function formulation

This model seeks the optimal planning of new energy storage systems and renewable energy generators in such a way as to minimize the total cost and CO₂ emissions while maximizing the system inertia. The total cost comprises of operational cost of thermal generators, and investment cost of new RE generators and ESS. In order to minimize the total cost and maximize system inertia simultaneously, a multi-objective optimization model is formulated. PV systems and wind turbines are considered as candidate REGs, while BES and PHS are considered as candidate energy storage units. The multi-objective optimization problem is then converted into a single objective function in order to obtain a Pareto optimal solution using the weighted sum approach as presented in Eq. (9). The model assumes that the inertia from the synchronous generators and the virtual inertia from renewable energy generators and energy storage system can be aggregated.

\[
\begin{align*}
\text{min} & \quad \alpha \left[ \sum_{n} \left( \sum_{k} a_n(P_{g,k})^2 + b_n(P_{g,k}) + c_n \right) \right] \quad \beta \left[ \sum_{n} \left( \sum_{k} \Delta_{\text{mg}}(P_{g,k}) + \Delta_{\text{mg}}(P_{g,k}) \right) \right] \\
& \quad \alpha + \beta = 1
\end{align*}
\]

(9)

The non-negative weighting factors in Eq. (10) are used to transform the economic and technical objective into a single objective function.

\[
\sum_{g} a_g(P_{g})^2 + b_g(P_{g}) + c_g = S(P_{g})
\]

(11)

\[
S(P_{g}) = a\left(p_{\text{min,}g}\right)^2 + b\left(p_{\text{min,}g}\right) + c_{\text{g,u,r}} + \sum_{k} S_k^g p_k^g
\]

(12)

where,

\[
S_g^k = \frac{C_{g,\text{fin}}^k - C_{g,\text{ini}}^k}{\Delta p_g^k}
\]

(13)

\[
c_{g,\text{ini}}^k = a\left(P_{g,\text{ini}}^k\right)^2 + b\left(P_{g,\text{ini}}^k\right) + c
\]

(14)

\[
c_{g,\text{fin}}^k = a\left(P_{g,\text{fin}}^k\right)^2 + b\left(P_{g,\text{fin}}^k\right) + c
\]

(15)

The total operating cost is given as the sum of the quadratic fuel cost of the thermal generators, curtailment cost of wind turbines, and the cost of emissions from thermal generators. The quadratic fuel cost in Eq. (9) is then linearized by approximating it by a set of piecewise segments \( k \) as presented in Fig. 4 [4]. The resulting linear equation can then be solved easily and faster using a linear programming solver.

The linear version of the fuel cost calculation is expressed as in Eq. (12), while the mathematical formulation used for the linearization is given in Eqs. (11)-(17).

\[
\sum_{g} a_g(P_{g})^2 + b_g(P_{g}) + c_g = S(P_{g})
\]

\[
S(P_{g}) = a\left(p_{\text{min,}g}\right)^2 + b\left(p_{\text{min,}g}\right) + c_{\text{g,u,r}} + \sum_{k} S_k^g p_k^g
\]

\[
S^k = \frac{C_{g,\text{fin}}^k - C_{g,\text{ini}}^k}{\Delta p_g^k}
\]

\[
c_{g,\text{ini}}^k = a\left(P_{g,\text{ini}}^k\right)^2 + b\left(P_{g,\text{ini}}^k\right) + c
\]

\[
c_{g,\text{fin}}^k = a\left(P_{g,\text{fin}}^k\right)^2 + b\left(P_{g,\text{fin}}^k\right) + c
\]

Fig. 4 Piecewise linear cost function
Model constraint

The proposed MILP model is subjected to investment, power generation, power balance, inertia, DC optimal power flow, ramp rate, emission, budgetary, energy storage, and renewable energy flexibility constraints.

Investment constraints

\[ \sum v \mu_{1v} \leq \bar{v} \quad \forall v = 1, 2, \ldots, nv \]  
(18)

\[ \sum p \mu_{2p} \leq \bar{p} \quad \forall p = 1, 2, \ldots, np \]  
(19)

\[ \sum w \mu_{3w} \leq \bar{w} \quad \forall w = 1, 2, \ldots, nw \]  
(20)

\[ \sum s \mu_{4s} \leq \bar{s} \quad \forall s = 1, 2, \ldots, ns \]  
(21)

\[ \mu_{1v}, \mu_{2p}, \mu_{3w}, \mu_{4s} \in \{0, 1\} \]  
(22)

Equations (18)-(21) set an upper bound on number of investment on BES, PHS, wind farm, and solar PV technology, respectively. Equation (22) defines the binary investment variables used for selecting investment in BES, PHS, wind turbine, and solar PV systems, respectively. The variable is equal to 1 if a particular technology is to be built, and 0 otherwise.

Power generation constraints

The power output of each generating unit is limited by the following constraint.

\[ 0 \leq P_{g,t}^k - \Delta P_{g}^k \quad \forall k = 1 : n \]  
(16)

\[ \Delta P_{g}^k = \frac{p_{g}^{\text{max}} - p_{g}^{\text{min}}}{n} \]  
(17)

Equation (23) sets the minimum and maximum operating limit for the thermal generators. Equations (24) and (25) set the operating limit of the solar PV system and wind farm, respectively. Equation (26) specifies the power curtailment for the wind farm. Equation (27) specifies the wind farm available power per time.

Power Balance constraint

\[ \sum g \sum v P_{g,vt} + \sum v \sum p P_{p,vt} + \sum p \sum s P_{s,vt} + \sum s \sum w P_{w,vt} + \sum w \sum s P_{s,vt} - \sum s \sum l P_{l,vt} \geq x \cdot \sum D_{t} \]  
(28)

Equation (28) states that the sum of power from all online thermal units, wind farms, solar PV systems, pumped hydro storage, discharge power from BES, charging power from BES, and the power flowing in and out of the transmission lines should be greater than or equal to the total load demand. A power demand safe margin \( x \) is introduced to cater for an unanticipated increase in demand.

DC Optimal Power flow constraints

\[ -\frac{\pi}{2} \leq \psi_{i} \leq \frac{\pi}{2} \]  
(29)

\[ P_{ij} = B_{ij}(\psi_{i} - \psi_{j}) \]  
(30)

\[ -p_{ij}^{\text{max}} \leq P_{ij} \leq p_{ij}^{\text{max}} \]  
(31)

Equation (29) sets the voltage angle at bus within a specified limit to ensure that the power flow in the transmission line is below the static stability limit. Equation (30) states that the transmission line limit is a function of the voltage angle at bus and the line susceptance. Equation (31) specifies the transmission line flow limit.

Emission constraint

\[ \sum g s_{g}(P_{g}) \leq E_{\text{lim}} \]  
(32)

Equation (32) limits the total emissions from thermal generators to a maximum permissible value.

Inertia constraints

\[ H_{\text{min}} \leq H_{eq} \leq H_{\text{max}} \]  
(33)

Equation (33) sets the permissive overall inertia constant in the power system model within a specified range.
Ramp Rate constraint

\[ P_{g,t-1} - P_{g,t} \leq RPD_g \]  
(34)

\[ P_{g,t} - P_{g,t-1} \leq RPU_g \]  
(35)

Equations (34) and (35) constraint the active power of the thermal power plants by its ramp rate limits. Equation (34) gives the ramp down limit, while Eq. (35) gives the ramp up limit.

Energy storage constraints

Energy storage systems are used to increase the flexibility of power system operation as they discharge their stored energy in times of energy deficiency and charge during times of excessive energy [9, 33]. In addition, they are used to provide virtual inertia to the grid through virtual inertia control schemes. Investment decisions on energy storage systems should be based on the inertia requirement of the grid as well as on the individual characteristics of the energy storage systems.

\[ SC_{v,t} = SC_{v,t-1} + \left( P_{cha,v} \cdot \eta_{cha}^{v} - P_{dis,v} \right) \]  
(36)

\[ SC_{v,t}^{\min} \cdot u_{1,v} \leq SC_{v,t} \leq SC_{v,t}^{\max} \cdot u_{1,v} \]  
(37)

\[ \mu S_{p,v}^{cha} \cdot P_{cha,v} \leq P_{cha,v} \leq \mu S_{p,v}^{cha} \cdot \frac{P_{cha,v}}{\eta_{cha}} \]  
(38)

\[ \mu S_{p,v}^{dis} \cdot P_{dis,v} \leq P_{dis,v} \leq \mu S_{p,v}^{dis} \cdot \frac{P_{dis,v}}{\eta_{dis}} \]  
(39)

\[ \mu S_{v}^{cha,v} \cdot \mu S_{v}^{dis,v} \in \{0, 1\} \]  
(40)

\[ \mu S_{v}^{cha,v} + \mu S_{v}^{dis,v} = 1 \]  
(41)

\[ \sum_v \mu_{1,v} + \sum_p u_{2,p} \leq \bar{e} \]  
(42)

Equation (36) describes the state of charge of the BES. Equation (37) gives the state of charge capacity limit of prospective BES. Equations (38) and (39) give the charging and discharging power limit of BES, respectively. Equations (40) and (41) describe the binary variable used to prevent simultaneously charging and discharging of the BES at the same time. Equation (42) sets the maximum number of ESS comprising BES and PHS that can be installed during the planning horizon.

Budgetary constraints on investment

Equations (43)- (47) define budgetary allocation limits on candidate REGs and ESSUs.

\[ \sum_v CO^{inv}_v \cdot \mu_{1,v} \leq \overline{BU}_v \]  
(43)

\[ \sum_p CO^{inv}_p \cdot \mu_{2,p} \leq \overline{BU}_p \]  
(44)

\[ \sum_w CO^{inv}_w \cdot \mu_{3,w} \leq \overline{BU}_w \]  
(45)

\[ \sum_{i} CO^{inv}_i \cdot \mu_{4,i} \leq \overline{BU}_i \]  
(46)

\[ \sum_{i} CO^{inv}_i \cdot \mu_{4,i} + \sum_p CO^{inv}_p \cdot \mu_{2,p} + \sum_v CO^{inv}_v \cdot \mu_{3,v} + \sum_{i} CO^{inv}_i \cdot \mu_{4,i} \leq \overline{BU}_i \]  
(47)

Equations (43–46) give the budgetary constraint that also guides investment in BES, PHS, wind farm, and solar PV technology, respectively. It subjects the total investment per technology to be less than or equal to its budgetary allocation. Equation (47) constrains the total investment on candidate REGs and ESS to a maximum monetary budget.

Renewable energy flexibility constraint

Equations (48) - (50) constrain RES penetration into the grid within a specified limit in order to carry out sensitivity analysis of the model. RES penetration target levels are set to 33%, 66%, and 100%, using a flexible parameter \( f_l \). Equation (48) constrains the total power generation from RES using the flexible parameter \( f_l \), which is given as a fraction of the total load demand. Equation (49) limits the total power generation from thermal generators to at most \((1 - f_l)\) percent of total load demand [34]. Equation (50) defines the range of values of the flexible parameter, \( f_l \), which is used to confine the RES penetration level to be between 33 and 100% of the total load demand.

\[ \left[ \sum_{i=1}^{ns} P_x \cdot \mu_{4,x} + \sum_{i=1}^{nw} P_w \cdot \mu_{3,w} \right] \leq f_l \cdot \left[ \sum_i D_i \right] \]  
(48)

\[ \left[ \sum_{i=1}^{n} P_x \right] \leq (1 - f_l) \cdot \left[ \sum_i D_i \right] \]  
(49)

\[ f_l \in [0.33, 0.66, 1.0] \]  
(50)
Model Design

The proposed MILP model is analyzed for a time horizon of one year. Tables 2 and 3 gives the data for the thermal generators and candidate RE generators and ESUs used in the model formulation. Transmission line parameters and load demand data at buses can be obtained from reference [35].

In order to make provision for expansion in the power network, the load demand at buses is assumed increased by 50%. Candidate RE generating units are then placed at buses with the least load demand (bus 5,6,8,9) because of the intermittency of REGs, while ESUs are placed at buses with the highest load demand (bus 1,4,5,6). Figure 5 gives the flowchart of the model design.

Furthermore, Table 4 gives the system parameters used for modelling, while the time variability data of the wind and solar resources is given in Table 5.

Study case and Model Validation

The proposed MILP model is validated using an IEEE 9-bus test system with prospective investment on new RE generators and energy storage units to reduce emissions and enhance system inertia. The configuration of the IEEE 9-bus test system is shown in Fig. 6. The model comprises 13 existing transmission lines, 8 candidate ESS (4 BES, and 4 PHS), 8 candidates RE generating units (4 wind farms, 4 solar PV systems), 9 load centers, 5 thermal generators, and an average load of 3414 MW. The developed model was solved using the CPLEX solver in GAMS with an Intel(R) core i3(TM), CPU 2.53 GHz personal computer.

Results and Discussion

The developed model was evaluated under three scenarios.

Scenarios 1. Model without renewable energy generators and energy storage systems.
Scenario 2. Inertia is not considered in the GEP model with available renewable energy generators and energy storage systems.
Scenario 3. Inertia is considered in the GEP model with available renewable energy generators and energy storage systems.

Model Results

The comparison of results obtained under all three scenarios is presented in Table 6. The results will be discussed in terms of the economic (cost) and technical (system inertia) implications of the design.
Scenarios 1: In this case, there is no provision for renewable energy generators and energy storage systems, hence the overall system inertia is provided only by thermal unit. It can be observed that in this scenario the system inertia is high compared to scenario 2, however, the emissions from thermal generators is the highest among the three scenarios. Furthermore, the fuel cost, operational cost, and the total cost is highest in this scenario compared to scenarios 2 and 3.

Scenarios 2: In this case, there is provision for renewable energy generators and energy storage systems, but no consideration is made for system inertia in the GEP model design, hence the overall system inertia is lowest among all three scenarios because of inappropriate investment decisions. Furthermore, the emissions from thermal generators is lower than in scenario 1 because of the presence of REGs and ESS. Also, the cost of investment on new RE generators and energy storage systems is lower than in scenarios 3. Sub-optimal investment
decisions were made in this case, which favour the installation of solar PV systems S1 and S2, wind farms W3 and W4, and pumped hydro storage P1, P2, P3, and P4.

Scenarios 3: This case is the most preferred among all three scenarios, as the inertia requirement of the grid is considered in the GEP model in the presence of REGs and ESS. This scenario achieved the highest overall system inertia among all three scenarios because appropriate investment decisions on candidate RE generators and energy storage systems were made. In addition, the emissions from thermal generators is lower in scenario 3 than in scenario 1, with just a slight increase in the cost of investment on RE generators and ESS compared to scenario 2. Overall, this scenario achieves the highest system inertia at the lowest possible cost, thus validating the importance of this research aimed at enhancing the system inertia of the modern power grid comprising of RE generators and energy storage systems. The investment decisions in this scenario favour the installation of solar PV systems S3 and S4, wind farm W1 and W2, and battery energy storage B1, B2, B3, and B4 with virtual inertia capabilities.

It can also be observed that this scenario prefers investment on battery storage system compared to pumped hydro storage unit because battery storage system offers higher virtual inertia compared to the pumped hydro storage. Furthermore, the result obtained in this scenario reveals that by considering the inertia requirement of the grid in planning, higher system inertia can be achieved in the grid compared to scenarios 1 and 2 where inertia is not considered in the planning model.

In addition, a slight increase in investment cost on candidate RE generators and storage systems is observed as seen in Table 6, because of the extra cost incurred to equip the REGs and BES with virtual inertia capabilities. Thus, with higher system inertia, the new model is more stable than the conventional model without inertia consideration as already established by Eq. (7).

Table 7 gives the operational power dispatch of the thermal generators in all scenarios, while Fig. 7 gives the generating capacity of candidate RE generators in scenarios 2 and 3. It can be observed from Table 7 that the generating capacity of the thermal generators is highest in scenarios 1 and lowest in scenarios 2 and 3. This is a result of the new REGs and ESS introduced into the model.

Furthermore, the optimal power flow across the transmission line in the network is given in Table 8. It can be observed from Table 8 that the power flow across the transmission lines is well below its maximum power flow limit with no congested line. Also, Fig. 8 gives the voltage angle across the buses in the network at various times. It can also be observed that the bus voltage angle is maintained within standard limit and ranges between 0.245 pu and 0.007pu.

**Sensitivity analysis of system inertia with increasing RE penetration**

In this sub-section, the effect of increasing RE generation on the overall system inertia of the developed model is evaluated.
The sensitivity analysis is carried out under different levels of RE penetration outlined as follows; 33% RES penetration, 66% RES penetration, and 100% RES penetration. REGs and ESS selected for the analysis are assumed to be equipped with virtual inertia capability. Table 9 and Fig. 9 give the variation of system inertia under the different RES penetration levels.

![IEEE 9-bus network with existing thermal and candidate generating and storage units (dash lines are related to candidate generating and energy storage units)](image)

Table 6  Result of generation expansion planning model under three scenarios

| Scenarios | Inertia constant (seconds) | Emissions (tonnes) | Total cost ($10^6) | Operational cost ($10^6) | Cost of emission (10^6) | Cost of investment on ESS and RE generators ($10^6) | Cost of wind curtailment ($10^6) | Fuel cost ($10^6) |
|-----------|---------------------------|--------------------|--------------------|--------------------------|------------------------|-----------------------------------------------|---------------------------------|------------------|
| Scenario1 | 7.204                     | 414.562            | 1,808.432          | 1,808.432                | 3.616                  | -                                             | No curtailment                 | 1804.8           |
| Scenario 2 | 5.067                     | 389.532            | 1,795.57           | 1,795.22                 | 3.4123                 | 0.38422                                       | 7.0080                         | 1784.8           |
| Scenario 3 | 8.776                     | 389.532            | 1,806.125          | 1,805.7                  | 3.4123                 | 0.39293                                       | 17.520                          | 1784.8           |

The sensitivity analysis is carried out under different levels of RE penetration outlined as follows; 33% RES penetration, 66% RES penetration, and 100% RES penetration. REGs and ESS selected for the analysis are assumed to be equipped with virtual inertia capability. Table 9 and Fig. 9 give the variation of system inertia under the different RES penetration levels. It can be seen from Table 9 that the overall system inertia

![Operational dispatch of thermal generators in all scenarios](image)

Table 7  Operational dispatch of thermal generators in all scenarios

| Generators | Installed capacity (MW) | Operational dispatch (MW) Scenarios 1 | Operational dispatch (MW) Scenarios 2 | Operational dispatch (MW) Scenarios 3(New model) |
|------------|-------------------------|----------------------------------------|----------------------------------------|-----------------------------------------------|
| G1         | 250                     | 203.1                                  | 81.2                                   | 81.2                                          |
| G2         | 240                     | 194.4                                  | 77.8                                   | 77.8                                          |
| G3         | 625                     | 500                                    | 100.0                                  | 100.0                                         |
| G4         | 500                     | 400                                    | 40.0                                   | 40.0                                          |
| G5         | 550                     | 441.8                                  | 44.2                                   | 44.2                                          |

![Generating capacity of RE generators in scenarios 2 and 3](image)
constant decreased at a rate of 4.4% with increase in RES penetration. It can also be observed that the total synchronous inertia energy decreased as RES penetration increased. It is worthy of note that at 100% RES penetration level, the total synchronous inertia energy in the power system is zero, even though the overall system inertia constant is still above the permissible range, because of the provision of system inertia by the virtual inertia providing REGs and ESS. However, the stability of the grid at 100% RES penetration level will be a study in the future because of the obvious decline in the amount of synchronous inertia available in the model.

**Discussion of results**

Based on the obtained simulation results from all three scenarios. It can be seen that considering system inertia in generation expansion planning model (scenario 3) leads to a significant increase in the overall system inertia, and decrease in total system cost and CO₂ emissions.

It is observed that in scenario 3 where system inertia is considered in the planning model in the presence of wind turbines and solar PV plant, the overall system inertia constant is increased from 7.204 s in scenario 1 to 8.776 s in scenario 3 (21.8% increase), and from 5.067 s in scenario 2 to 8.776 s in scenario 3 (73.2% increase). This shows that considering system inertia in GEP model helps in improving the overall system inertia constant through making appropriate investment decisions.

In addition, the introduction of REGs and ESS into the model reduces the total CO₂ emissions in the model. This can be observed as the total CO₂ emissions reduced from 414.562 tons CO₂ in scenario 1 to 389.532 tons CO₂ in scenario 3 (6% decrease).

Also, the overall system cost was reduced due to the incorporation REGs and ESS in the model which resulted in reduced emissions and fuel cost. The total cost reduced from 1,808.432 million USD in scenario 1 to 1,806.125 million USD in scenario 3 (0.13% decrease). These results reveal the importance of the proposed model which incorporates system inertia into generation expansion planning model in the presence of REGs and ESS.

Finally, the sensitivity analysis of system inertia with increasing RE generation shows that as RE penetration into the grid increased from 33% to 100 RES penetration, the total system inertia of the grid decreases from 6.45267 s to 6.167873 s. It is also noteworthy that the synchronous inertia energy also decreases as the RE penetration into the grid increases. The synchronous inertia of the grid at 100% RES penetration is seen to be reduced to zero, thus the stability of the grid at 100% RES penetration level will be investigated in future studies.

### Conclusion

Generation expansion planning model based on the system inertia requirement of the grid is proposed in this paper. The proposed model is designed to guide investment on renewable energy generators and energy storage systems in such a way as to increase the overall system inertia of the grid and by implication improve the resilience of the grid, while

### Table 8 Optimal power flow on transmission line

| Transmission line connecting bus | Power flow (MW) | Transmission line connecting bus | Power flow (MW) |
|---------------------------------|----------------|---------------------------------|----------------|
| 1–6                             | 150.7          | 4–9                             | 156.8          |
| 2–6                             | 87.2           | 5–4                             | 294.1          |
| 3–2                             | 131.8          | 6–7                             | 186.3          |
| 3–6                             | 66.9           | 6–8                             | 1.543          |
| 4–1                             | 99.6           | 8–7                             | 0.867          |
| 4–3                             | 39.7           | 9–8                             | 0.914          |
| 4–6                             | 75.0           |                                 |                |

![Fig. 8 Voltage angle at bus at various times](image)

### Table 9 Capacity mix for different RES penetration levels

| Generator installed capacity per technology(MW) | RES Penetration level (%) |
|------------------------------------------------|---------------------------|
|                                                 | 33%           | 66%           | 100%          |
| Thermal                                        | 2165          | 1365          | -             |
| Wind                                           | 800           | 1600          | 1600          |
| Solar                                          | 200           | 400           | 1600          |
| BES                                            | 200           | 200           | 100           |

System inertia constant[s]: 6.45267 6.265985 6.167873
Synchronous inertia energy: 7565 5190 0
minimizing the system cost and CO₂ emissions. The model was developed as a mixed integer linear programming problem and solved using CPLEX solver in GAMS. The model was then validated using a modified IEEE 9-bus test system.

Key findings of the research reveal the following: (1) The developed model which considers system inertia in the model formulation (scenarios 3) achieved higher system inertia of 8.776 s compared to the conventional model in scenarios 1 (7.204 s) and 2 (5.067 s). (2) Sub-optimal investment decisions could be prevented by considering the inertia requirement of the grid in generation expansion planning. (3) Sensitivity analysis of the model reveals that the overall system inertia constant decreases at a rate of 4.4% with increase in RE penetration into the grid. (4) Investment on virtual inertia-equipped renewable energy generators and energy storage systems helped in enhancing the overall inertia constant of the grid as in scenario 3 where system inertia was considered in the planning model. (5) The developed GEP in scenario 3 achieved 0.13% decrease in total cost and 6% decrease in CO₂ emissions due to the incorporation of REGs and ESS into the model design.

Finally, it should be noted that investment decisions taken in this model were based on the IEEE 9-bus test system used for validating the developed model, therefore in the future, the model will be extended to a larger test system, while transmission expansion planning will also be incorporated.

**Data Availability** Authors can confirm that all relevant data are included in the article.

**Declarations**

**Conflict of Interest** The authors declare no conflict of interest in this research.

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