Secure Two-Party Feature Selection

Vanishree Rao¹, Yunhui Long², Hoda Eldardiry², Shantanu Rane², Ryan A. Rossi³, and Frank Torres²

¹ Intertrust Technologies Corporation, Sunnyvale, CA 94085, USA; contribution to this work was done while this author was at PARC
² Palo Alto Research Center (PARC), Palo Alto CA 94304, USA
³ Adobe Research, San Jose CA 95110, USA

Abstract. In this work, we study how to securely evaluate the value of trading data without requiring a trusted third party. We focus on the important machine learning task of classification. This leads us to propose a provably secure four-round protocol that computes the value of the data to be traded without revealing the data to the potential acquirer. The theoretical results demonstrate a number of important properties of the proposed protocol. In particular, we prove the security of the proposed protocol in the honest-but-curious adversary model.

Keywords: Secure two party feature selection · Feature selection · Classification · Privacy preserving data mining · Homomorphic encryption.

1 Introduction

According to the report “Data Never Sleeps 6.0” published recently by Domo Inc., an estimated 1.7 MB of data will be created every second for each person on earth by 2020. The owners of this staggering amount of data sometimes provide it readily to others, but often hold back despite the value that data trading could provide. Both privacy concerns and the desire to monetize data at a fair market value are barriers, as both could be compromised if data are revealed before terms have been negotiated. A method to assess the value of a data trade without first revealing the data would help make data trading a more efficient transaction, whether the aim is to trade at a fair market price, apply some type of differential privacy, or both.

Finding business value in ‘distributed’ data: When data on different aspects of a system are captured by different stakeholders, trading the data can provide a more complete perspective of the system. For instance, in an Internet-of-Things (IoT) ecosystem, IoT devices owned by different parties (manufacturers, service providers, consumers, etc.) often collect data that reveal only a partial understanding of behaviors and events. Creating a marketplace for trading the data would enable a party to get a more complete understanding when required, without spending extra time and money deploying additional IoT devices to collect data that another party already has. As long as stakeholders can establish a fair
price for the data, inefficient duplication of efforts can be avoided, benefiting both parties of a transaction. However, identifying trade partners and tagging a cash value to the data can be a tricky challenge, particularly because the value depends on the quality and content of the data held by both partners.

Maximizing data utility while protecting individual privacy: When considering how to share sensitive datasets, potential collaborators may seek to analyze how different statistical privacy options affect the utility of data. The party applying statistical privacy to their data before sharing may like to work with a potential collaborator to experiment with different choices of statistical privacy methods and parameters, in order to deliver desensitized data of the highest possible utility. Applications include both business-to-business transactions and business-to-government transactions.

Data trading scenarios: An owner of a dataset may want to release only subsets of their data to control proliferation, but they need a way to determine utility of subsets in order to choose the right one for each potential collaborator. An owner may also want to limit the number of times data are shared, either to mitigate security and privacy risks or to maintain a desired monetary price for access to the data. Choosing customers that have the highest utility for the data will help maximize monetary return, as those customers will in principle pay a higher price. An owner may want to sell access to data at a full value-based price, but rational purchasers may insist on a discounted price to compensate for any risk associated with uncertain utility. Thus, answering the following question is important:

How can one securely measure utility of data and the impact of applying statistical privacy enhancement techniques, without access to the actual data?

1.1 This Work

In this work, we try to answer the above question for a specific potential acquirer’s task, where the parties freely share data dictionaries. Specifically, we provide a protocol with which a potential provider and a potential acquirer can determine the value of the data with respect to the latter’s task at hand, without the latter learning anything more about the data, other than its specification in the data dictionary. The specific sub-case we consider is the provider having a binary feature vector and the acquirer having a binary class vector. The acquirer would like to learn if the provider’s feature vector can improve the correctness of the acquirer’s classification. Thus, the utility we consider is whether the data shared by the provider is expected to improve the classification of the acquirer’s existing dataset. To quantify utility, we use the $\chi^2$-statistic studied by Yang and Pederson (1997) for the related problem of feature selection. We employ Pallier homomorphic encryption for the required privacy-preserving computations.
1.2 Roadmap

The protocols in this paper assume parties share primary keys for their data, in order for data elements to be aligned. In future work, we will integrate private set intersection protocols, such as the Practical Private Set Intersection Protocols published by De Cristofaro and Tsudik \cite{7}, in order to relax this assumption. We also plan to study extensions of the work to more sophisticated feature selection, based on combining multiple columns in the provider’s dataset to generate more complex feature candidates.

2 Background

In this work, we consider a structured dataset, and we are interested in classification based on all the features available. Specifically, we consider two parties, Carol and Felix. Carol has a dataset consisting of certain feature columns and a class vector generated from her available features. Felix possesses an additional feature column \( f \) that might be useful for Carol in improving the classification of her dataset.

**Notations.** Let \( c = (c_1, c_2, \ldots, c_n) \) be the class label vector with Carol, and \( f = (f_1, f_2, \ldots, f_n) \) be the feature vector with Felix. We assume both the class labels and the features are binary attributes, leaving generalization to multinomial classifiers for a future paper. That is, for all \( 1 \leq i \leq n \), \( c_i \in \{0, 1\} \) and \( f_i \in \{0, 1\} \).

Let \( c_i \) denote the class variable of the \( i \)-th record in Carol’s dataset. Let \( f_i \) be the feature value, in Felix’s feature vector, corresponding to the \( i \)-th record in Carol’s dataset.

2.1 \( \chi^2 \) Feature Selection

Feature selection is the process of removing non-informative features and selecting a subset of features that are useful to build a good predictor \cite{14}. The criteria for feature selection vary among applications. For example, Pearson correlation coefficients are often used to detect dependencies in linear regressions, and mutual information and \( \chi^2 \) statistics are commonly used to rank discrete or nominal features \cite{14,23}.

In this paper, we focus on determining utility of binary features. We choose \( \chi^2 \) statistics as a measure of utility, due to its wide applicability and its amenability towards cryptographic tools. More specifically, unlike mutual information which involves logarithmic computations, the calculation of \( \chi^2 \) statistics only involves additions and multiplications.

For the class label vector \( c \) and the corresponding feature vector \( f \), \( A \) is defined to be the number of rows with \( f_i = 0 \) and \( c_i = 0 \). \( B \) is defined to be the number of rows with \( f_i = 0 \) and \( c_i = 1 \). \( C \) is defined to be the number of rows with \( f_i = 1 \) and \( c_i = 0 \). \( D \) is defined to be the number of rows with \( f_i = 1 \) and \( c_i = 1 \). Table 1 shows the two-way contingency table for \( f \) and \( c \). The \( \chi^2 \) statistic of \( f \) and \( c \) is
defined \( \chi^2(f, c) \) to be:

\[
\chi^2(f, c) = \frac{n(AD - BC)^2}{(A + C)(A + B)(C + D)(B + D)}.
\]

**Table 1.** Two-Way Contingency Table of \( f \) and \( c \)

| \( f \) | \( c \) | 0   | 1   |
|-------|-------|-----|-----|
| 0     |       | A   | B   |
| 1     |       | C   | D   |

\( \chi^2(f, c) \) is used to test the independence of \( f \) and \( c \). Table 2 shows the confidence of rejecting the independence hypothesis under different \( \chi^2 \) values. For example, when \( \chi^2(f, c) \) is larger than 10.83, the independence hypothesis can be rejected with more than 99.9% confidence, indicating that the feature vector \( f \) is very likely to be correlated with the class label vector \( c \).

**Table 2.** Confidence of Rejecting the Hypothesis of Independence under Different \( \chi^2 \) Values

| \( \chi^2(f, c) \) | Confidence |
|---------------------|------------|
| 10.83               | 99.9%      |
| 7.88                | 99.5%      |
| 6.63                | 99%        |
| 3.84                | 95%        |
| 2.71                | 90%        |

### 2.2 Cryptographic Tools

**PKE scheme and CPA security.** We recall the standard definitions of public-key encryption (PKE) schemes and chosen plaintext attack (CPA) security, which are used in this paper.

**PKE schemes.** A scheme PKE with message space \( \mathcal{M} \) consists of three probabilistically-polynomial-time (PPT) algorithms Gen, Enc, Dec. Key generation algorithm Gen(1\(^k\)) outputs a public key \( pk \) and a secret key \( sk \). Encryption algorithm Enc\((pk, m)\) takes \( pk \) and a message \( m \in \mathcal{M} \), and outputs a ciphertext \( c \). Decryption algorithm Dec\((sk, c)\) takes \( sk \) and a ciphertext \( c \), and outputs a message \( m \). For correctness, we require that \( \text{Dec}(sk, c) = m \) for all \( m \in \mathcal{M} \), all \( (pk, sk) \leftarrow \text{Gen}(1^k) \), and all \( c \leftarrow \text{Enc}(pk, m) \).
Negligible Function. A function $f : \mathbb{N} \rightarrow \mathbb{R}$ is negligible if for every possible integer $c$, there exists an integer $N$ such that for all $x > N$, $|f(x)| \leq \frac{1}{2^c}$. We denote negligible functions as $\text{negl}(\cdot)$.

The CPA Experiment. We now describe the chosen-plaintext attack (CPA) game with an adversary $A$ against a PKE scheme $PKE$.

Algorithm 1 The $\text{PubK}^{\text{CPA}}_{A,PKE}$ Experiment

**Input:** Security parameter $k$

1: $(pk, sk) \leftarrow \text{Gen}(1^k)$
2: The adversary $A$ is given $1^k$, $pk$, and oracle access to $\text{Enc}_{pk}(\cdot)$. $A$ outputs a pair of messages $(m_0, m_1)$ of the same length
3: A uniform bit $b \in \{0, 1\}$ is chosen, and $c \leftarrow \text{Enc}_{pk}(m_b)$ is given to $A$
4: $A$ continues to have access to $\text{Enc}_{pk}(\cdot)$, and outputs a bit $b'$

**Output:** 1 if $b' = b$, and 0 otherwise

CPA Security \[16\]. A PKE scheme $PKE = (\text{Gen}, \text{Enc}, \text{Dec})$ has indistinguishable encryptions under a chosen-plaintext attack, or is CPA-secure, if for all probabilistic polynomial-time adversaries $A$ there is a negligible function $\text{negl}$ such that

$$\Pr\left[\text{PubK}^{\text{CPA}}_{A,PKE}(k) = 1\right] \leq \frac{1}{2} + \text{negl}(k),$$

where the experiment $\text{PubK}^{\text{CPA}}_{A,PKE}$ is defined in Algorithm 1 and the probability is taken over the randomness of $A$ and of the experiment.

Paillier Encryption. We use Paillier encryption to maintain privacy in our two-party feature selection algorithm, and employ the additive homomorphic property of Paillier encryption to calculate the $\chi^2$ statistics that quantify feature utility. We recall the Paillier encryption scheme in Figure 1 \[16\].

Note that while we use Paillier homomorphic encryption, the proposed protocols can accommodate any semantically secure additively homomorphic encryption scheme.

Paillier encryption supports additive and scalar multiplication homomorphism. We briefly recall the definitions of additive homomorphism and scalar multiplication homomorphism \[16\].

Additive Homomorphism. A PKE scheme $PKE = (\text{Gen}, \text{Enc}, \text{Dec})$ is said to be additively homomorphic, if there exists a binary operation $\oplus$, such that the following holds for all $k \in \mathbb{N}$, and for all $m_1, m_2 \in \mathcal{M}$,

$$\Pr\left[m^* = m_1 + m_2 \mid (pk, sk) \leftarrow \text{Gen}(1^k)\right. = 1 - \text{negl}(k).$$

$$c_1 \leftarrow \text{Enc}_{pk}(m_1), c_2 \leftarrow \text{Enc}_{pk}(m_2)$$
$$e^* \leftarrow c_1 \oplus c_2$$
$$m^* \leftarrow \text{Dec}_{sk}(e^*)$$
Paillier Encryption Scheme

Let \text{GenModulus} be a polynomial-time algorithm that, on input \(1^k\), outputs \((N, p, q)\) where \(N = pq\) and \(p\) and \(q\) are \(k\)-bit primes (except \(p\) or \(q\) is not prime with probability negligible in \(k\)). Define the following encryption scheme:

- \text{Gen}: on input \(1^k\) run \text{GenModulus} \((1^k)\) to obtain \((N, p, q)\). The public key is \(pk = N\), and the private key is \(sk = \langle N, \phi(N) \rangle\), where \(\phi(N) = (p - 1)(q - 1)\).

- \text{Enc}: on input of a public key \(N\) and a message \(m \in \mathbb{Z}_N\), choose a uniformly random \(r \leftarrow \mathbb{Z}_N^*\) and output the ciphertext \(c := [(1 + N)m \cdot r^N \mod N^2]\).

- \text{Dec}: on input of a private key \(\langle N, \phi(N) \rangle\) and a ciphertext \(c\), compute \(m := \left[\frac{c^{\phi(N)} \mod N^2 - 1}{N} \cdot \phi(N)^{-1} \mod N\right]\).

Fig. 1. Paillier Encryption Scheme.

Scalar Multiplication Homomorphism. A PKE scheme \(\text{PKE} = (\text{Gen}, \text{Enc}, \text{Dec})\) is said to be \textit{scalar multiplication homomorphic}, if there exists a binary operation \(\otimes\), such that the following holds for all \(k \in \mathbb{N}\), and for all \(m_1, m_2 \in \mathcal{M}\),

\[
\Pr \left[ m^* = m_1 m_2 \left| \begin{array}{c}
(pk, sk) \leftarrow \text{Gen}(1^k) \\
c \leftarrow \text{Enc}_{pk}(m_2) \\
c^* \leftarrow m_1 \otimes c \\
m^* \leftarrow \text{Dec}_{sk}(c^*)
\end{array} \right. \right] = 1 - \text{negl}(k).
\]

3 Proof of Privacy

We first present the high-level argument for how our protocols will protect each party’s data. We have one of the parties (Carol) choose the encryption key, and encrypt her data using this key before sending it to the other party (Felix). Thus, Carol’s privacy will be guaranteed by the semantic security assumption of the encryption scheme. Meanwhile, Felix will also encrypt his data using Carol’s key, but he will blind all of the outputs he sends to Carol with randomness of his choosing, ensuring that Carol can learn nothing about his data. We now make these notions precise by first providing a formal definition of privacy protection in the honest-but-curious adversary model, and a formal proof of privacy for the protocol that attempts to protect privacy in the above described manner.

**Definition 1 (Honest-but-curious security of two-party protocol).** We begin with the following notation:

- Let \(g_c\) and \(g_f\) be probabilistic polynomial-time functionalities and let \(\Pi\) be a two-party protocol for computing \(g = (g_c, g_f)\). Let the parties be Carol, Felix, with inputs \(c, f\) respectively.
– The view of the party \( A \in \{ Carol, Felix \} \) during an execution of \( \Pi \) on \((c, f)\) and security parameter \( k \) is denoted by \( \text{view}_A^\Pi(c, f, k) \) and equals \( (w, r^A, m_1^A, \ldots, m_t^A) \), where \( w \in \{c, f\} \) (w’s value depending on the value of \( A \)), \( r^A \) equals the contents of the party \( A \)'s internal random tape, and \( m_j^A \) represents the \( j \)-th message that it received.

– The output of the party \( A \) during an execution of \( \Pi \) on \((c, f)\) and security parameter \( k \) is denoted by \( \text{output}_A^\Pi(c, f, k) \) and can be computed from its own view of the execution.

Let \( g = (g_c, g_f) \) be a functionality. We say that \( \Pi \) securely computes \( g \) in the presence of semi-honest adversaries if there exist probabilistic polynomial-time algorithms \( S_c \) and \( S_f \) such that

\[
\begin{align*}
\{ (\text{view}_{\text{Carol}}^\Pi(c, f, k), \text{output}_{\text{Carol}}^\Pi(c, f, k)) \}_{c, f, k} &\cong \{ (\text{view}_A^\Pi(c, f, k), \text{output}_A^\Pi(c, f, k)) \}_{c, f, k} \\
\{ (\text{view}_{\text{Felix}}^\Pi(c, f, k), \text{output}_{\text{Felix}}^\Pi(c, f, k)) \}_{c, f, k} &\cong \{ (\text{view}_A^\Pi(c, f, k), \text{output}_A^\Pi(c, f, k)) \}_{c, f, k}
\end{align*}
\]

\( c, f \in \{0, 1\}^* \) such that \(|c| = |f|\), and \( k \in \mathbb{N} \).

4 Protocol

In this section, we describe a four-round protocol for \( \chi^2 \) statistic calculation under a two-party setting. For convenience, we continue to refer to the parties as \( Carol \), who has the class vector \( c \), and \( Felix \), who has the feature vector \( f \). Carol’s objective is to learn \( \chi^2(f, c) \) and Felix’s objective is to not reveal any further information about \( f \) while Carol computes the utility of Felix’s data for her classifier. In this section, Felix uses multiplicative binding to keep the detailed mathematics a little simpler, but an alternative protocol that uses additive blinding is provided in Section 6 for situations where the security of multiplicative blinding is a concern.

As before, \( A \) is the number of rows with \( f_i = 0 \) and \( c_i = 0 \). \( B \) is the number of rows with \( f_i = 0 \) and \( c_i = 1 \). \( C \) is the number of rows with \( f_i = 1 \) and \( c_i = 0 \). \( D \) is the number of rows with \( f_i = 1 \) and \( c_i = 1 \).

Round 1.

Carol performs the following operations:

1. Generate a Paillier key pair \((pk, sk) = \text{Gen}(1^k)\).
2. Encrypt all class labels with \( pk \): \( \text{Enc}_{pk}(c_1), \text{Enc}_{pk}(c_2), \ldots, \text{Enc}_{pk}(c_n) \).
3. Compute \( \frac{B + D}{A + C} \). Note that Carol can obtain this value by computing \( \frac{\sum_{i=1}^n c_i}{n - (\sum_{i=1}^n c_i)} \), since \( B + D = \sum_{i=1}^n c_i \) and \( A + C = n - (B + D) \), based on the contingency table.
4. Encrypt \( \frac{B + D}{A + C} \) with \( pk \): \( \text{Enc}_{pk}(\frac{B + D}{A + C}) \).
5. Send the following values to Felix:

\[
\left( pk, \text{Enc}_{pk}(c_1), \text{Enc}_{pk}(c_2), \ldots, \text{Enc}_{pk}(c_n), \text{Enc}_{pk}(\frac{B + D}{A + C}) \right).
\]
Round 2.
Felix performs the following operations:
1. Compute $\mathcal{E}_{pk}(D)$. Note that Felix can obtain this value by computing
   \[ \bigoplus_{i=1}^n (f_i \otimes \mathcal{E}_{pk}(c_i)) = \bigoplus_{i=1}^n \mathcal{E}_{pk}(f_i c_i), \]
   since $\sum_{i=1}^n f_i c_i = D$.
2. Sample $r \leftarrow \mathbb{Z}_N$, and compute $r \otimes \mathcal{E}_{pk}(D) = \mathcal{E}_{pk}(rD)$.
3. Send the following value to Carol:
   \[ \mathcal{E}_{pk}(rD) \]

Round 3.
Carol performs the following operations:
1. Decrypt $\mathcal{E}_{pk}(rD)$ using $sk$.
2. Compute $r^2 D^2$ and $rD$, and encrypt them.
3. Send the following values to Felix:
   \[ \left( \mathcal{E}_{pk} \left( \frac{r^2 D^2}{(B + D)(A + C)} \right), \mathcal{E}_{pk} \left( \frac{rD}{A + C} \right) \right). \]

Round 4.
Felix performs the following operations:
1. Cancel $r$ by computing
   \[ r^{-2} \otimes \mathcal{E}_{pk} \left( \frac{r^2 D^2}{(B + D)(A + C)} \right) = \mathcal{E}_{pk} \left( \frac{D^2}{(B + D)(A + C)} \right) \]
   and
   \[ r^{-1} \otimes \mathcal{E}_{pk} \left( \frac{rD}{A + C} \right) = \mathcal{E}_{pk} \left( \frac{D}{A + C} \right). \]
2. Compute an encryption of $\chi^2(f, c)$ by computing:
   \[ \left( \frac{n^3}{(A + B)(C + D)} \otimes \mathcal{E}_{pk} \left( \frac{D^2}{(B + D)(A + C)} \right) \right) \]
   \[ \oplus \left( \frac{n(C + D)}{A + B} \otimes \mathcal{E}_{pk} \left( \frac{B + D}{A + C} \right) \right) \oplus \left( \frac{-2n^2}{A + B} \otimes \mathcal{E}_{pk} \left( \frac{D}{A + C} \right) \right), \]
   where $C + D$ and $A + B$ are computed as
   \[ C + D = \sum_{i=1}^n f_i, \]
   and
   \[ A + B = n - (C + D). \]
We see below that the above computation gives $\mathcal{E}_{pk}(\chi^2(f, c))$. Since $AD - BC = (A + B + C + D)D - (B + D)(C + D)$, $\chi^2(f, c)$ can be decomposed as follows:
\[
\chi^2(f, c) = \frac{n(AD - BC)^2}{(A + C)(A + B)(C + D)(B + D)}
\]
\[
= \frac{n^3}{(A + B)(C + D)(B + D)(A + C)} + \frac{n(C + D)(B + D)}{(A + B)(A + C)} + \frac{2n^2}{(A + B)(A + C)}.
\]
3. Send the following value to Carol:

\[ \text{Enc}_{pk}(\chi^2(f, c)) \].

Local computation.
Carol decrypts \( \text{Enc}_{pk}(\chi^2(f, c)) \) to obtain \( \chi^2(f, c) \).

Remark 1. We note that only Carol receives the value \( \chi^2(f, c) \). Depending on the application, if Felix also needs to know the value of \( \chi^2(f, c) \), Carol can simply then send it to Felix after running the protocol.

Remark 2. If Felix needs to know the value of \( \chi^2(f, c) \) but does not trust Carol to send the true value, then the parties can use a two-stream protocol wherein both parties compute and send encrypted values in round one and both parties send encrypted values of \( \chi^2(f, c) \) in round four. Since the computation for \( \chi^2(f, c) \) is symmetric with respect to \( c \) and \( f \), both parties should end up with the same value of \( \chi^2(f, c) \), assuming they used the same data in both streams (i.e., did not cheat). To verify that the parties did not cheat, they can re-encrypt their \( \chi^2(f, c) \) values with a new, single-use key, send their re-encrypted \( \chi^2(f, c) \) to the other party, and then send the one-use key after receiving the re-encrypted \( \chi^2(f, c) \) message from the other party. If cheating occurred, the decrypted value of the other party’s \( \chi^2(f, c) \) will not match their own.

5 Proof of Security

With respect to the notion of security specified in Definition 1, we first prove the following key lemma that will allow us to argue that our two-party protocol is secure against an honest-but-curious adversary. Specifically, the lemma captures the crux of proof, and its extension to the main theorem is straightforward.

Lemma 1. Suppose that in a two-party protocol \( \Pi' \), Carol runs the key generation algorithm of a CPA-secure homomorphic public-key encryption scheme and gives the public key to Felix. Also, suppose that all messages sent from Carol to Felix are encrypted with the generated public key, and all messages sent from Felix to Carol are either encryptions of elements randomly distributed in the plaintext space and independent of Felix’s inputs, or encryptions of the final output. Then, the protocol \( \Pi \) is secure in the honest-but-curious adversary model.

Proof. To prove the security of the protocol, we need to consider two cases – one, where Carol is corrupted, and the other, where Felix is corrupted. In each case, we will prove that the corrupted party will not learn anything more about the other party’s output than the protocol output. Specifically, we show that there exist PPT algorithms \( S_c \) and \( S_f \), that simulate the non-corrupted party’s messages without knowing the non-corrupted party’s inputs but only knowing the output, in cases where Carol and Felix are corrupted, respectively. This corresponds to establishing equations (1) and (2) in Definition 1.
Case 1: When Felix is corrupted by an adversary. We show how to simulate Carol’s messages sent to Felix, by describing the simulator $S_f$. For every ciphertext to be sent from Carol to Felix, $S_f$ chooses a random plaintext in the message space and sends an encryption of it. If Felix can tell apart the views of communicating with Carol and with the simulator, then there exists an adversary that can break CPA security of the underlying encryption scheme. Since, by assumption, no such PPT adversary exists, we have that Equation (2) holds.

Case 2: When Carol is corrupted by an adversary. We show how to simulate Felix’s messages sent to Carol, by describing the simulator $S_c$. For every ciphertext that encrypts a randomly distributed plaintext, sent by Felix to Carol, $S_c$ samples a uniform random element in the plaintext space, encrypts it with Carol’s public key, and sends the resulting ciphertext to Carol. For the ciphertext encrypting the final output, note that $S_c$ gets the final output as an input. Using this, the simulator can compute its encryption, and send the resulting ciphertext to Carol. Since the messages sent by $S_c$ to Carol are distributed identically to Felix’s messages to Carol, we have that Equation (1) holds.

We will now simply extend the core lemma into the main theorem.

Theorem 1. The two-party protocol $\Pi$ described in Section 4 is secure in the honest-but-curious adversarial model.

Proof. We note the following aspects in the protocol $\Pi$. All the messages sent from Carol to Felix are encrypted using Carol’s public key under Pailler encryption scheme. The messages sent from Felix to Carol are either encryptions of elements randomly distributed in $\mathbb{Z}_N$, the plaintext space, or encryption of the final output, $\chi^2(f, c)$. Since these aspects conform to the conditions in Lemma 1 based on the lemma, we have that the protocol $\Pi$ is secure in the honest-but-curious adversary model.

6 Alternative Protocol

In this section, we describe an alternative protocol for $\chi^2$ statistic calculation under a two-party setting, wherein Felix uses additive blinding rather than multiplicative blinding to introduce the random number $r$. In theory, taking advantage of additive rather than multiplicative homomorphism provides stronger security [5], albeit at a cost in computational efficiency and complexity. For this alternative protocol, round one is unchanged:

Round 1.
Carol performs the same operations as in Round 1 of section 4, including sending the following values to Felix:

$$\left( pk, \text{Enc}_{pk}(c_1), \text{Enc}_{pk}(c_2), \ldots, \text{Enc}_{pk}(c_n), \text{Enc}_{pk}\left( \frac{B + D}{A + C} \right) \right).$$

Round 2.
Felix performs the following operations:
1. Compute $\text{Enc}_{pk}(D)$. Note that Felix can obtain this value by computing
$$\bigoplus_{i=1}^n (f_i \otimes \text{Enc}_{pk}(c_i)) = \bigoplus_{i=1}^n \text{Enc}_{pk}(f_i c_i) = \text{Enc}_{pk}(\sum_{i=1}^n f_i c_i),$$
since $\sum_{i=1}^n f_i c_i = D$.

2. Sample $r \leftarrow \mathbb{Z}_N$, and compute $r \oplus \text{Enc}_{pk}(D) = \text{Enc}_{pk}(r + D)$.

3. Send the following value to Carol:
$$\text{Enc}_{pk}(r + D)$$

**Round 3.**
Carol performs the following computations.
1. Decrypt $\text{Enc}_{pk}(r + D)$ using $sk$.
2. Compute five values:
$$\frac{(r+D)^2}{(B+D)(A+C)}, \quad \frac{(r+D)}{(B+D)(A+C)}, \quad \frac{(r+D)}{(A+C)} \quad \text{and} \quad \frac{1}{(B+D)(A+C)},$$
and encrypt them, obtaining:
$$\text{Enc}_{pk}\left(\frac{(r+D)^2}{(B+D)(A+C)}\right), \quad \text{Enc}_{pk}\left(\frac{(r+D)}{(B+D)(A+C)}\right), \quad \text{Enc}_{pk}\left(\frac{(r+D)}{(A+C)}\right), \quad \text{Enc}_{pk}\left(\frac{1}{(B+D)(A+C)}\right) \quad \text{and} \quad \text{Enc}_{pk}\left(\frac{1}{A+C}\right).$$

3. Send the five encrypted values to Felix.

**Round 4.**
Felix performs the following computations.
1. Eliminate $r$ from the first and third encrypted values by computing
$$\text{Enc}_{pk}\left(\frac{(r+D)^2}{(B+D)(A+C)}\right) \oplus \left(\frac{r^2}{(B+D)(A+C)} \otimes \text{Enc}_{pk}\left(\frac{1}{(B+D)(A+C)}\right)\right)$$
$$\oplus \left(-2r \otimes \text{Enc}_{pk}\left(\frac{(r+D)}{(B+D)(A+C)}\right)\right) = \text{Enc}_{pk}\left(\frac{D^2}{(B+D)(A+C)}\right)$$
and
$$\text{Enc}_{pk}\left(\frac{(r+D)}{(A+C)}\right) \oplus \left(-r \otimes \text{Enc}_{pk}\left(\frac{1}{(A+C)}\right)\right) = \text{Enc}_{pk}\left(\frac{D}{(A+C)}\right).$$

2. Compute an encryption of $\chi^2(f, c)$ by computing:
$$\left(\frac{n^3}{(A+B)(C+D)} \otimes \text{Enc}_{pk}\left(\frac{D^2}{(B+D)(A+C)}\right)\right)$$
$$\oplus \left(\frac{n(C+D)}{A+B} \otimes \text{Enc}_{pk}\left(\frac{B+D}{A+C}\right)\right) \oplus \left(-2n^2 \otimes \text{Enc}_{pk}\left(\frac{D}{A+C}\right)\right),$$
where $C+D$ and $A+B$ are computed as
$$C+D = \sum_{i=1}^n f_i,$$
and
\[ A + B = n - (C + D). \]

We see below that the above computation gives \( \text{Enc}_{pk}(\chi^2(f, c)) \). Since \( AD - BC = (A + B + C + D)D - (B + D)(C + D) \), \( \chi^2(f, c) \) can be decomposed as follows:

\[
\chi^2(f, c) = \frac{n(AD - BC)^2}{(A + C)(A + B)(C + D)(B + D)} - \frac{n^3 D^2}{(A + B)(C + D)(B + D)(A + C)} + \frac{n(C + D)(B + D)}{(A + B)(A + C)} - \frac{2n^2 D}{(A + B)(A + C)}.
\]

3. Send the following value to Carol:

\[ \text{Enc}_{pk}(\chi^2(f, c)). \]

Local computation.
Carol decrypts \( \text{Enc}_{pk}(\chi^2(f, c)) \) to obtain \( \chi^2(f, c) \).

7 Related Work

There has been extensive research on privacy-preserving data mining (PPDM), which aims at completing data mining tasks on a union of several private datasets, each owned by a different party. The goal of PPDM can be achieved by either adding noise and perturbations \[2,9\] or using cryptographic tools. This paper falls into the latter category.

General SMPC \[24,13,12,15,18,4\] can be used to calculate any functions between multiple parties without revealing the input of each party. However, currently-known general SMPC protocols are computationally inefficient. Therefore, it is impractical to do large-scale multi-party feature selection using these protocols. Compared to general SMPC protocols, the protocol proposed in this paper is more efficient in handling feature selection.

Recent studies have proposed several efficient SMPC protocols to accomplish different data mining tasks such as statistics computations \[8,6\], set intersections \[10,11\], classification \[21,17,20,22\], clustering \[19\], and regression \[11\]. However, to the best of our knowledge, not much research has been done in secure multi-party feature selection. As a commonly-used pre-processing technique, feature selection can be used in conjunction with many of the previously mentioned SMPC data mining protocols or as a metric to estimate data quality for classification tasks.

There are many feature selection methods. \[8\] proposes an algorithm for privacy-preserving calculation of Pearson correlation coefficients among distributed parties. However, different from our approach, they use perturbation techniques to achieve privacy protection. \[3\] proposes a secure multi-party feature selection protocol using virtual dimensionality reduction, but their protocol requires users to exchange unencrypted intermediate results such as the dot product of two attribute vectors. Our protocol achieves a stronger privacy protection: each participating party only learns the \( \chi^2 \) coefficient between the two attributes, and no intermediate results are leaked.
8 Conclusion

Data trading will become more and more important as devices generate more and more data. In this work, we initiate a study on how to securely evaluate the value of trading data without requiring a trusted third party, by considering the specific case of data classification tasks. We present a secure four-round protocol that computes the value of the data to be traded without revealing the data to the potential acquirer.

We employed additive homomorphic encryption as a core building block to compute the $\chi^2$-statistic in a privacy-preserving manner.

9 Acknowledgement

The authors acknowledge and express appreciation for partial funding for this work from the U.S. Department of Transportation Federal Highway Administration Exploratory Advanced Research Program, grant ID DTFH6115H00006, and the support of FHWA Program Manager Dr. Ana Maria Eigen.

References

1. Agrawal, R., Evfimievski, A., Srikant, R.: Information sharing across private databases. In: Proceedings of the 2003 ACM SIGMOD international conference on Management of data. pp. 86–97. ACM (2003)
2. Agrawal, R., Srikant, R.: Privacy-preserving data mining. In: ACM Sigmod Record. vol. 29, pp. 439–450. ACM (2000)
3. Banerjee, M., Chakravarty, S.: Privacy preserving feature selection for distributed data using virtual dimension. In: Proceedings of the 20th ACM international conference on Information and knowledge management. pp. 2281–2284. ACM (2011)
4. Ben-David, A., Nisan, N., Pinkas, B.: Fairplaymp: a system for secure multi-party computation. In: Proceedings of the 15th ACM conference on Computer and communications security. pp. 257–266. ACM (2008)
5. Bianchi, T., Piva, A., Barni, M.: Analysis of the security of linear blinding techniques from an information theoretical point of view. In: International Conference on Acoustics, Speech and Signal Processing (ICASSP). pp. 5852–5855. IEEE (2011)
6. Canetti, R., Ishai, Y., Kumar, R., Reiter, M.K., Rubinfeld, R., Wright, R.N.: Selective private function evaluation with applications to private statistics. In: Proceedings of the twentieth annual ACM symposium on Principles of distributed computing. pp. 293–304. ACM (2001)
7. De Cristofaro, E., Tsudik, G.: Practical private set intersection protocols with linear complexity. In: International Conference on Financial Cryptography and Data Security. pp. 143–159. Springer (2010)
8. Du, W., Atallah, M.J.: Privacy-preserving cooperative statistical analysis. In: Computer Security Applications Conference, 2001. ACSAC 2001. Proceedings 17th Annual. pp. 102–110. IEEE (2001)
9. Dwork, C.: Differential privacy: A survey of results. In: International Conference on Theory and Applications of Models of Computation. pp. 1–19. Springer (2008)
10. Freedman, M.J., Nissim, K., Pinkas, B.: Efficient private matching and set intersection. In: International Conference on the Theory and Applications of Cryptographic Techniques. pp. 1–19. Springer (2004)
11. Gascon, A., Schoppmann, P., Balle, B., Raykova, M., Doerner, J., Zahur, S., Evans, D.: Secure linear regression on vertically partitioned datasets. In: 2016 IEEE Symposium on Security and Privacy (2016)
12. Goldreich, O.: Secure multi-party computation. Manuscript. Preliminary version pp. 86–97 (1998)
13. Goldreich, O., Micali, S., Wigderson, A.: How to play any mental game—a completeness theorem for protocol with honest majority. In: Proc. 19th ACM Symposium on the Theory of Computing. pp. 218–229 (1987)
14. Guyon, I., Elisseeff, A.: An introduction to variable and feature selection. Journal of machine learning research 3(Mar), 1157–1182 (2003)
15. Henecka, W., Sadeghi, A.R., Schneider, T., Wehrenberg, I., et al.: Tasty: tool for automating secure two-party computations. In: Proceedings of the 17th ACM conference on Computer and communications security. pp. 451–462. ACM (2010)
16. Katz, J., Lindell, Y.: Introduction to modern cryptography. CRC press (2014)
17. Kikuchi, H., Ito, K., Ushida, M., Tsuda, H., Yamaoka, Y.: Privacy-preserving distributed decision tree learning with boolean class attributes. In: Advanced Information Networking and Applications (AINA), 2013 IEEE 27th International Conference on. pp. 538–545. IEEE (2013)
18. Malkhi, D., Nisan, N., Pinkas, B., Sella, Y., et al.: Fairplay-secure two-party computation system. In: USENIX Security Symposium. vol. 4. San Diego, CA, USA (2004)
19. Vaidya, J., Clifton, C.: Privacy-preserving k-means clustering over vertically partitioned data. In: Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining. pp. 206–215. ACM (2003)
20. Vaidya, J., Clifton, C.: Privacy preserving naïve bayes classifier for vertically partitioned data. In: SDM. pp. 522–526. SIAM (2004)
21. Vaidya, J., Clifton, C.: Privacy-preserving decision trees over vertically partitioned data. In: IFIP Annual Conference on Data and Applications Security and Privacy. pp. 139–152. Springer (2005)
22. Wright, R., Yang, Z.: Privacy-preserving bayesian network structure computation on distributed heterogeneous data. In: Proceedings of the tenth ACM SIGKDD international conference on Knowledge discovery and data mining. pp. 713–718. ACM (2004)
23. Yang, Y., Pedersen, J.O.: A comparative study on feature selection in text categorization. In: ICML. vol. 97, pp. 412–420 (1997)
24. Yao, A.C.C.: How to generate and exchange secrets. In: Foundations of Computer Science, 1986., 27th Annual Symposium on. pp. 162–167. IEEE (1986)