SPIN MATRIX ELEMENTS FOR THE ELASTIC PROTON-PROTON AND PROTON-ANTIPROTON COLLISIONS

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Abstract
Motivated by the present extensive RHIC (BNL) spin program and further PAX project at FAIR, we study the possibility of explicit reconstruction of full set of helicity amplitudes by joint consideration of elastic proton-proton and proton-antiproton scattering. Procedure is based on the derivative relations for the helicity amplitudes, explicit parametrization of the leading spin non-flip amplitudes and crossing-symmetry relations. Some preliminary results are presented for the PAX energies to show the expected magnitudes of the spin dependent observables as functions of invariant Mandelstam’s variables $s$ and $t$.

1 Introduction
The quantitative description of high-energy soft processes is not yet achieved in frame of the QCD. Elastic scattering of hadrons has always been a crucial tool for study the dynamics of strong interaction. Despite of important results at the interface between soft and hard physics and recent progress in nonperturbative QCD, elastic scattering cannot be described in a pure QCD framework because of this reaction is soft one essentially. In such a situation the direct reconstruction of the scattering matrix from the complete set of the experimental data will be the appropriate method. One may hope that such spin programs will be realized at RHIC, FAIR (PAX project) and other facilities in near future.

In order to make the predictions for the observables in those facilities one needs to have a well justified method, contains a small number of free parameters and applicable at wide range of the kinematics variables. Below we propose such a technique. Its correctness may be tested by the joint consideration of proton-proton and proton-antiproton elastic scattering data. Thus the further study of the $pp$ and $p\bar{p}$ elastic scattering in wide range of momentum transfer and initial energies is extremely important task.

2 Preliminary results
Two methods have been suggested for reconstruction the full set of helicity amplitudes for elastic $p\bar{p}$-collisions in \textsuperscript{1,2}. We describe in details the method for deducing the helicity amplitudes based on crossing-symmetry relation and the derivative relations. As known
the amplitudes for the binary reaction $A + B \to C + D$ in $s$-, $t$-, and $u$- channels are described by just one set of analytic functions. The following preliminary expression for full set of helicity amplitudes of $p\bar{p}$ elastic scattering via set of helicity amplitudes for $pp$ elastic reaction has been derived:

\[
\begin{align*}
\Phi_1^{pp} &= 1/2 \bigg[ \sin^2 \psi (\Phi_1^{pp} + \Phi_2^{pp} + \Phi_3^{pp}) + (1 + \cos^2 \psi) \Phi_4^{pp} \bigg] \\
\Phi_2^{pp} &= 1/2 \bigg[ \sin^2 \psi (\Phi_1^{pp} + \Phi_3^{pp} - \Phi_4^{pp}) - (1 + \cos^2 \psi) \Phi_2^{pp} \bigg] \\
\Phi_3^{pp} &= 1/2 \bigg[ \sin^2 \psi (\Phi_1^{pp} + \Phi_2^{pp} - \Phi_4^{pp}) - (1 + \cos^2 \psi) \Phi_3^{pp} \bigg] \\
\Phi_4^{pp} &= 1/2 \bigg[ (1 + \cos^2 \psi) \Phi_4^{pp} - \sin^2 \psi (\Phi_3^{pp} + \Phi_2^{pp} - \Phi_4^{pp}) \bigg] + 2\Phi_5^{pp} \sin \psi \\
\Phi_5^{pp} &= 1/2 \cos \psi [\sin \psi (\Phi_1^{pp} + \Phi_2^{pp} + \Phi_3^{pp} - \Phi_4^{pp}) + 2\Phi_5^{pp}] \\
\end{align*}
\]

(1)

where

\[
\cos \psi = \frac{st}{(s-4m_p^2)(t-4m_p^2)}; \quad \sin \psi = 2m_p \sqrt{\frac{4m_p^2 - s - t}{(s-4m_p^2)(t-4m_p^2)}},
\]

$\psi$ - the $t \to s$ crossing angle [3], $m_p$ - proton mass. The system (1) shows apparent analytical forms for the full set of amplitudes of elastic $p\bar{p}$ scattering $\{\Phi_j^{pp}\}_{j=1-5}$ via helicity amplitudes $\{\Phi_j^{pp}\}_{j=1-5}$ for $pp$ channel. It should be noted that the relation $\Phi_5^{pp} = -\Phi_6^{pp}$ has been taken into account in the (1) already.

One must consider $pp$ elastic scattering in order to obtain some additional relations for set of helicity amplitudes $\{\Phi_j^{pp}\}_{j=1-5}$. According to the model independent analysis of unpolarized differential cross section [4] the following parametrization is suggested for spin non-flip amplitude $\Phi_1^{pp}$:

\[
\Phi_1^{pp} = i\sqrt{A} \exp \left[ \left( \frac{B}{2} + \alpha' \ln \left( \frac{s}{s_0} \right) - i\alpha' \frac{\pi}{2} \right) t + \left( \sqrt{C_0} - i\sqrt{C_\infty} \right) \exp(Dt/2) \right], \quad (2)
\]

where $s_0 = 1$GeV$^2$ and $\sqrt{A}$, $B$, $\alpha'$, $\sqrt{C_0}$, $\sqrt{C_\infty}$, $D$ are free parameters obtained from experimental data for elastic $pp$ reaction in the interval $0.15 \leq |t| \leq 5.0$ GeV$^2$ [4].

The two different assumptions were made in calculations: $1 - \Phi_2^{pp} = 0$ and $\Phi_1^{pp} = \Phi_3^{pp}$; $2 - \Phi_2^{pp} = -\Phi_4^{pp}$ and $\Phi_1^{pp} = \Phi_3^{pp}$. Similar assumptions were used in the analysis of $pp$ elastic collisions at RHIC [5] and for recent study of $p\bar{p}$ elastic interactions in PAX [1] by another method of reconstruction the full set of helicity amplitudes for nucleon-nucleon elastic scattering.

The derivative relations allow to express the spin single-flip amplitude $\Phi_5^{pp}$ and the spin double-flip amplitude $\Phi_4^{pp}$ via $\Phi_1^{pp}$ [6]:

\[
\Phi_5^{pp}(s,t) = C_1^{pp}(s) \frac{\partial}{\partial(\sqrt{-t})} \Phi_1^{pp}(s,t); \quad \Phi_4^{pp}(s,t) = C_2^{pp}(s) \frac{\partial^2}{\partial(\sqrt{-t})^2} \Phi_1^{pp}(s,t), \quad (3)
\]

where $C_k^{pp}(s) = C_k^{pp}(s) + iC_k^{pp}(s), \ k = 1,2$ - complex parameters in general.

The Odderon hypothesis is important for definition of unknown parameters $C_k(s)^{pp}$ in the derivative relations [3]. We use the asymptotic total cross section, differential cross...
section, $\rho$ and $B$ parameters both in $pp$ and $p\bar{p}$ elastic reactions in order to obtain the complex unknown parameters. The suggested reconstruction procedure is described in details for asymptotic energies in [2]. According to [3] one can assume that $C_k(s)$ are real constants at low and medium energies for $pp$ scattering. Numerical values of these parameters have been found in crude approach from comparison with early results for $pp$ differential cross section [7, 8] at qualitatively level: $C_1 = -0.05$ GeV and $C_2 = 0.02$ GeV. But further study is planed for this hypothesis.

We present below preliminary results for elastic $p\bar{p}$ collisions at energy range of PAX experiment obtained by discussed above reconstruction procedure. We have calculated $|t|$-dependence of differential cross-section, polarization, some elements of the second-rank spin tensors, namely, spin-correlation ($C_{ik}$) and depolarization ($D_{ik}$) tensors and Wolfenstein parameters. These parameters have been calculated according to the general analytical formulas in Regge-pole theory [9] for 1-st and 2-d versions of assumptions for the $pp$ helicity amplitudes described above.

The preliminary $t$-dependence is shown for each of observable in fig.1 and in fig.2 for 1-st and 2-d version of assumptions for a range of $0.15 \leq |t| \leq 5.0$ GeV$^2$. The validity region for [3] is $-t \ll s$ [3] but the derivative relations are used at all values of $t$-invariant allowed by parameterization [2] even at lowest energy $\sqrt{s} = 3$ GeV. One can see that all dependences are smooth at this energy and no any dramatic features at large $|t|$ are seen. But the applicability of derivative relations at large $|t|$ requires additional study.

One can see that differential cross section is almost the same for two versions in the domain of $|t| \geq 2.0$ GeV$^2$ but there is difference between $d\sigma/dt$ values at smaller $|t|$ for various versions (fig.1a, 2a) and the value of this difference increases with energy increasing. The maximum value of this difference is at $|t| \simeq 1.0 - 1.5$ GeV$^2$ and removes to larger $|t|$ at energy increasing. There is a qualitative agreement between our predictions and the experimental data [7, 8, 10]. Polarization (fig.1b, 2b) shows good agreement with early Regge model predictions and three-pole fits [8] at lower energies but our calculation predicts opposite sign and different magnitude of polarization for $|t| \leq 1.0$ GeV$^2$ at highest energy $\sqrt{s} = 14.7$ GeV. One needs more precise experimental data for separating unambiguously between two versions of our calculations (fig.1b, 2b). Thus we have obtained correct energy dependence of polarization at qualitative level. It is worthwhile to note, that the sign on polarization for $p\bar{p}$ scattering is opposite to the sign of $P$ for elastic $pp$ scattering independent of versions of our calculations.

The depolarization tensor depends on $|t|$ in similar way for both versions at large values $|t| \geq 1.5$ GeV$^2$ (fig.1c, 2c). But values of $D$ are different for 1-st and 2-d version of assumption significantly at low $|t|$. The element $C_{NN}$ of spin-correlation tensor depends on initial energy at $|t| \leq 2.0$ GeV$^2$ (fig.1d) but $C_{NN}$ is almost independent of $s$ and $|t|$ at larger $|t|$ (fig.1d, 2d). Our preliminary results differ from early predictions [7] at all energies under study. The element of spin-correlation tensor $C_{kp}$ shows non trivial $|t|$-dependence (fig.1e, 2e).

The $|t|$-dependences for the first pair of Wolfenshtein parameters $R$ and $A$ are shown at fig.1f (fig.2f) and fig.1g (fig.2g) for 1-st (2-d) version assumptions for $pp$ helicity amplitudes respectively. The results for first ($A$ and $R$) pair of Wolfenstein parameters differ from predictions based on Regge-pole model significantly [7]. But it should be emphasized that the early predictions were made for more narrow range $|t| \leq 1.0$ GeV$^2$. The preliminary results for second pair of Wolfenshtein parameters $R'$ and $A'$ are shown in fig.1h (fig.2h).
and fig.1i (fig.2i) respectively. All Wolfenshtein parameters show the $|t|$-dependences which are similar each to other at qualitative level for all energies with the exception for the lowest one $\sqrt{S} = 3 \text{ GeV}$.

3 Conclusion

The main results of this paper are following. We propose the analytical method for calculation of the full set of helicity amplitudes for elastic $p\bar{p}$ reaction. This method is based on fundamental crossing-symmetry property and derivative relations for helicity amplitudes. Using this method the preliminary expressions for $p\bar{p}$ helicity amplitudes were derived. The known approximation of $pp$ helicity amplitudes (at least $\Phi_{pp}^{1}$) and some supplementary conditions are used in order to obtain the full set of helicity amplitudes for elastic $p\bar{p}$ scattering in wide range of $|t|$ parameter for the first time. The problem of definition of free parameters from the derivative relations is discussed. The preliminary numerical results are obtained for wide a set of observables in intermediate (PAX) energy domain. Our results agree with Regge-pole model predictions for some parameters qualitatively. The unified analysis of the $pp$ and $p\bar{p}$ data and new experimental results will allow to check in details the proposed method. It seems this analytical procedure might be useful for direct reconstruction of the spin matrix elements in elastic $pp$ and $p\bar{p}$ scattering at wide initial energy and square of momentum transfer domains.

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Figure 1: $t$-Dependence of some observable parameters for elastic $p\bar{p}$ scattering in intermediate (PAX) energy domain: 1 - $\sqrt{S} = 3$ GeV (black), 2 - $\sqrt{S} = 5$ GeV (red), 3 - $\sqrt{S} = 6.5$ GeV (green), 4 - $\sqrt{S} = 14.7$ GeV (blue). The first version of additional relations between $pp$ helicity amplitudes is under consideration: $a$ - differential cross section, $b$ - polarization, $c$ - depolarization parameter, $d$ - correlation of the normal components of polarization, $e$ - correlation of the transverse components of polarization, $f$ - transverse polarization rotation parameter, $g$ - longitudinal polarization rotation parameter, $h$ - correlation of transverse-longitudinal components of polarization, $i$ - correlation of longitudinal-longitudinal components of polarization.
Figure 2: $t$-Dependence of some observable parameters for elastic $p\bar{p}$ scattering in intermediate (PAX) energy domain: 1 - $\sqrt{S} = 3$ GeV (black), 2 - $\sqrt{S} = 5$ GeV (red), 3 - $\sqrt{S} = 6.5$ GeV (green), 4 - $\sqrt{S} = 14.7$ GeV (blue). The second version of additional relations between $pp$ helicity amplitudes is under consideration: $a$ - differential cross section, $b$ - polarization, $c$ - depolarization parameter, $d$ - correlation of the normal components of polarization, $e$ - correlation of the transverse components of polarization, $f$ - transverse polarization rotation parameter, $g$ - longitudinal polarization rotation parameter, $h$ - correlation of transverse-longitudinal components of polarization, $i$ - correlation of longitudinal-longitudinal components of polarization.