Reconnection–less reconfigurable low–pass filtering topology suitable for higher–order fractional–order design

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Abstract

The paper discusses a new design of a current–mode reconnection–less reconfigurable fractional–order (FO) low–pass filter of various orders. The filtering structure is based on a 4th–order leap–frog topology using operational transconductance amplifiers as basic building blocks. The resulting order of the filter is given by the setting of current gains (allowing the reconnection–less reconfiguration) alongside with the values of the fractional–order capacitors realized by the RC ladder networks. For this purpose, RC ladder networks of orders 0.3, 0.4, 0.5, 0.6 and 0.7 have been designed. The fractional–order form of the filter contains from one up to four FO capacitors (remaining capacitors (if there are any) are of integer–order) allowing to obtain low–pass functions of order of $3 + \alpha_1, 2 + x, 1 + x, \Re \alpha + \Re \beta, \Re \alpha + \Re \beta + \Re \gamma, \Re \alpha + \Re \beta + \Re \gamma + \Re \delta$. The proposed filter offers a wide variety of possible order combinations with an increasing degree of freedom as the number of fractional–order capacitors within the structure increases. The proposal is supported by the PSpice simulations of magnitude and phase characteristics, pole frequency adjustment and stability analysis. Moreover, the experimental measurements of the implemented filter were carried out and compared with the simulation results. The possibility of the electronic control of the fractional order is also discussed and presented.

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Introduction

In recent years, the matter of the fractional–order (FO) calculus received an increased attention of many scientists due to its possible utilization in various spectrums of industry branches including medicine [1–3], agriculture [1,4] and, of course, electrical engineering [5–33] giving many new potential applications a chance to arise. In comparison to the integer–order circuits, FO circuits provide an increased degree of freedom, due to the presence of the non–integer–order parameter ($\alpha$). This can be quite beneficial when it comes to more accurate generation and measurement of biomedical signals, impedance of various organic objects, etc.
The electrical engineering field, in particular, covers areas of FO frequency filters [5–23], oscillators [24–26], controllers [27,28] and other circuits [29–33].

When discussing the FO filters, the slope of the transition between the band-pass and band-stop area is characterized by the relation $20\log(n + \alpha)$, where $n$ is a non-zero integer number and $\alpha$ provides the fractional–order parameter. It is possible to come over two basic design approaches applied to filtering structures in order to obtain fractional–order characteristics. The first approach supposes the usage of a FO capacitor (or inductor) more generally referred to as FOE (Fractional–Order Element). Fractional–order capacitor is a special element with its impedance given as $Z_C = 1/s^\alpha C$, where $s$ is the Laplacian operator (complex number), symbol $\alpha$ represents the fractional order in range of $0 < \alpha < 1$ and $C$ is a pseudo capacitance with the unit of F s$^{\alpha-1}$. Generally speaking, the order of a fractional–order capacitor has its value in between the behavior of a traditional resistor and capacitor [2]. There have been some reports of a fabrication of fractional–order capacitors [34–36] nonetheless, these elements are expensive and difficult to implement and they are not commercially available. Therefore, they are usually substituted by passive elements (RC ladder networks) [5–13] or electronic emulators [29–31] approximating the FO capacitor (or inductor). The other approach involves the approximation of Laplacian operator of fractional–order $s^\alpha$ by an integer–order function [13–23]. The approximation is applied on the filtering structure as a whole leading to the change of values of standard elements/parameters within the structure (capacitors, resistors, transconductances $g_m$, voltage or current gains $A, B$).

A FO filter of a low–pass (LP) $1 + \alpha$ function is used most commonly [11,14–17,21–23]. Other quite common FO function is a high–pass (HP) $1 + \alpha$ filter [7,16,18] and [21–23]. There are also reports of FO band–pass (BP) [5,7,10,17,20,21–23], band–stop (BS) [7,21–23] and all–pass (AP) [8,19] filtering structures. Some structures suppose the replacement of both capacitors (considering the second–order structure having 2 integer–order capacitors turned into a FO filter) by FO capacitors providing an additional degree of freedom of the order control. Let us mention LP $\alpha + \beta$ filters [6,9,13], LP $\alpha + \beta$ filter assuming two FO capacitors of identical orders [10] and even LP $\alpha + \beta + \gamma$ supposing a FO filter originated from a second–order filter consisting of 3 capacitors [12]. Furthermore, Refs. [15] and [17] report FO filters of higher–orders (LP $5 + \alpha$) where the higher–order is obtained by a cascade combination of a FO filter of order 1 $+ \alpha$ and integer–order filter of order 4 (in this case). Several filters [7,10,16,17,21–23] are able to offer more than one type of a FO transfer function. For this purpose, a structure modification is required in case of filter from [16]. Structures in [7] and [10] are of either single–input multiple–output (SIMO) or multiple–input single–output (MISO) type requiring switching between different nodes of the structure. Only the filtering structures in [21–23] offer the ability of the electronic reconnection–less reconfiguration between available functions without any structure modification or switching of nodes needed. Nonetheless, the reconnection–less reconfiguration in case of [22] and [23] is solved by the array of electronic switches controlled through a 4–to–16 decoder. Such solution can have rather complex control logic and more importantly does not offer a feature of a fine tuning of the transfer function as the switches are either on or off in comparison to a continuous control when employing a suitable amplifier similarly to [21] and this paper. The feature of the reconnection-less reconfiguration is a beneficial ability of so–called reconnection–less configurable filters [21,37–40]. These filters can change the resulting output response by adjusting electronically controllable parameters of various active elements such as transconductance $g_m$ in case of an operational transconductance amplifier (OTA) or voltage or current gain $A, B$ in case of a variable gain amplifier (VGA) and adjustable current amplifier (ACA), etc. Thus, no manual modification or switching of inputs/outputs, which are not available for on chip implementation, is required. Table 1 introduces a comparative overview of the above–mentioned FO filters and collates them with the filter proposed in this paper.

The first thing, which is necessary to understand from Table 1, is the fact that the proposed filter is based on a different premise than other mentioned filters. Some filters [8,11,14–17,19,21–23] work with one fractional–order parameter while filters in [5–7,9,10,13] consider two fractional–order parameters. Based on this fact these filters have either one or two (fractional) orders which can be controlled theoretically. These structures, however, do not suggest any other possible modification of the order unless it comes to the design of higher order filters, where a suitable integer–order filter has to be added to the fractional–order filter to provide the desired higher–order function. The proposed filter can offer higher degree of freedom as it operates with a structure of the 4th–order (and can use up to 4 FC as evident from Table 1) and it is up to the user what the resulting order is going to be used by addition or removal of another block by electronic means. In short, the presented design can offer a higher degree of freedom while choosing a desired order without any needs of the structure modification (physical addition of a next stage in a cascade). Most importantly, with an implementation of a suitable electronically controllable FC emulator, we can choose whether we want the resulting order to be of the integer–order, combination of the integer and fractional order, or a combination of multiple fractional orders (either resulting in integer or fractional order). Based on the above–mentioned fact, the proposed filter can offer nine combinations of functions (and 4 additional functions if considering integer–order functions as well) in comparison to only one function in case of [5,6,8,9,11–15,18–20]. This useful feature is not available in case of other discussed solutions. Furthermore, the intended function of the proposed filter is supported by experimental measurements, not available in case of [6–11,13,19,21–23].

The filtering structure introduced in this paper is designed by the Signal–Flow Graph method (SFG) using a 4th–order leap–frog topology as its foundation. The filter operates in the current mode (CM) and provides LP functions of the first, second, third and fourth order. The 4th–order leap–frog topology is based on OTA elements. The addition of a current amplifier with the ability to control the gain of each output separately to the proposed structure offers the ability of the reconnection–less reconfiguration of the resulting filtering response. The FO characteristics are obtained by the gradual replacement of integer–order capacitors within the structure by their FO counterparts (RC ladder networks). Such modified structure can offer FO functions of LP $3 + \alpha$, LP $2 + \alpha$, LP $1 + \alpha$, LP $2 + \alpha + \beta$, LP $1 + \alpha + \beta + \gamma$, LP $\alpha + \beta + \gamma$ and LP $\alpha + \beta + \gamma + \delta$. The design is supported by PSpice simulations and experimental measurements.

The organization of the paper is as follows: Section ‘Design of an integer–order reconnection–less reconfigurable filter’ introduces the design of the 4th–order leap–frog topology constructed as a reconnection–less reconfigurable LP filter of the first, second, third and fourth order. The simulation and experimental results of this filter are provided in Section ‘Results of the proposed integer–order reconnection–less reconfigurable filter’. The following section describes the modification of the integer–order LP filter from Section ‘Design of an integer–order reconnection–less reconfigurable filter’ into a FO reconnection–less reconfigurable filter. Section ‘Results of the proposed fractional–order
**Comparative sheet of relevant fractional–order filtering structures.**

| Reference number | Type or realization | Type (number) of active element(s) | Type (number) of passive elements | Provided functions | Type of structure | Results | Notes |
|------------------|---------------------|-----------------------------------|-----------------------------------|-------------------|------------------|---------|-------|
| [5]              | RC                  | Opam(3)                           | R(6), FC(2)                       | BP $\alpha \times \beta$ | Sim/ Meas        | Sim/ - | -     |
| [6]              | RC                  | DVC(1)                            | R(1), FC(2)                       | LP $\alpha \times \beta$ | Thomas Meas      | Sim/ - | -     |
| [7]              | RC                  | OTA(6)                            | FC(2)                             | HP $\alpha \times \beta$ | Unspecified IFLF | Sim 1  | -     |
| [8]              | RC                  | OPA(2)                            | FC(1)                             | AP $\alpha$        | Unspecified Sim/ | Sim/ - | -     |
| [9]              | RC                  | DDCC(1)                           | R(2), FC(2)                       | LP $\alpha \times \beta$ | Sim/ Unspecified| Sim/ - | -     |
| [10]             | RC                  | OTA(3)                            | FC(2)                             | LP $\alpha \times \beta$ | Unspecified Sim/ | Sim/ - | -     |
| [11]             | RC                  | Opam(3)                           | R(6), FC(1), FC(1)                | LP $1 \times \alpha$ | Thomas Sim/      | Sim/ - | -     |
| [12]             | RC                  | Opam(2)                           | R(6), FC(3)                       | LP $\alpha \times \beta \times \gamma$ | Unspecified Sim/ Meas/ - | -     |
| [13]             | RC                  | Opam(4)                           | R(18), C(4)                       | LP $\alpha \times \beta$ | Unspecified Sim/ | Sim/ 2 | -     |
| [14]             | Approx              | OTA(3)                            | C(3)                              | LP $1 \times \alpha$ | FLF Meas/ Sim/  | Sim/ - | -     |
| [15]             | Approx              | DDCC(3)                           | R(8), C(4)                        | LP $1 \times \alpha$ | IFLF Sim/ Meas/  | Sim/ - | -     |
| [16]             | Approx              | DFA(4)                            | R(8), C(4)                        | LP $1 \times \alpha$ | FLF Sim/ Meas/  | Sim/ - | -     |
| [17]             | Approx              | DFA(10)                           | R(10), C(3)                       | LP $5 \times \alpha$ | Cann. Sim/ Meas/ | Sim/ - | -     |
| [18]             | Approx              | OTA(3), ACA(3)                    | LC(3)                             | LP $1 \times \alpha$ | FLF Sim/ Meas/  | Sim/ - | -     |
| [19]             | Approx              | OTA(2)                            | R(6), FC(2)                       | LP $1 \times \alpha$ | Unspecified Sim/ | Sim/ - | -     |
| [20]             | Approx              | OTA(9)                            | C(4), trans(11)                   | LP $1 \times \alpha$ | IFLF Sim/ Meas/ | Sim/ - | -     |
| [21]             | Approx              | OA(3), ACA(4), CF(2)              | C(6)                              | LP $1 \times \alpha$ | IFLF Sim/ Meas/ | Sim/ - | -     |
| [22]             | Approx              | DFA(8)                            | R(16), C(3)                       | LP $1 \times \alpha$ | Sim/ Meas/       | Sim/ - | -     |
| [23]             | Approx              | N/A                               | N/A                               | LP $1 \times \alpha$ | Sim/ Meas/       | Sim/ - | -     |
| [24]             | Approx              | N/A                               | N/A                               | LP $1 \times \alpha$ | Sim/ Meas/       | Sim/ - | -     |
| Fig. 2           | RC                  | IOGC-CA(1), OTA(4)                 | C(0–4), FC(0–4)                   | LP $3 \times \alpha$, LP $2 \times \alpha$, LP $1 \times \alpha$, LP $2 \times \alpha \times \beta$, LP $1 \times \alpha \times \beta$, LP $1 \times \alpha \times \beta \times \gamma$, LP $\alpha \times \beta \times \gamma$, LP $\alpha \times \beta \times \gamma \times \delta$. | Leap-frog Sim/ Meas/ | Sim/ - | -     |

List of previously unexplained abbreviations used in this table:
- RC – RC ladder network, Opam – operational amplifier, R – resistor, FC – fractional–order capacitor, Sim – simulations, Meas – measurements, DVC – differential voltage current converter, IFLF – inverse follow–the–leader feedback topology, C – (standard integer–order) capacitor, approx. – approximation of Laplacian operator of fractional–order, CFOA – current feedback operational amplifiers, FLF – follow–the–leader feedback topology, DDCC – differential difference current conveyor, CF – current follower, IOGC–CA – individual output gain controlled current amplifier

Notes:
1. Reference presents two topologies (SIMO and MISO).
2. Reference introduces two structures (one based on the approximation, the other one on usage of the RC structure).
3. Paper also presents LP $5 \times \alpha$ created by a cascade combination of $1 \times \alpha$ filter and filter of the 4th–order constructed by DDCC(4), R(4), C(4).
4. Two different topologies (for LP and HP function).
5. LP and HP $5 \times \alpha$ filters are also presented created by a cascade combination with a 4th–order filter constructed by CFOA(4), R(9), C(4) for the LP $5 \times \alpha$ and CFOA(4), R(6), C(7) for HP $5 \times \alpha$.
6. The number of active/passive elements is specified for LP $5 \times \alpha$ filter (Opam(2), R(10), C(3) for HP $1 \times \alpha$).
7. The FO capacitor is approximated by a second–order IFLF topology (OTA(8), C(2)).
8. The presented topology is fully differential.
9. The structure is all transistor based.
10. The structure contains four integer–order capacitors which are gradually replaced by FO capacitors depending on specific FO function.

The ‘reconnection–less filter’ shows the results of the proposed FO filter and offers its stability analysis. Finally, the paper is concluded by Section ‘Conclusion’.

**Design of an integer–order reconnection–less reconfigurable filter**

The proposed structure was firstly designed in its integer–order form (described in this chapter). Its FO counterpart is described in Section ‘Modification of the proposed integer–order filter in order to obtain a fractional–order reconnection–less reconfigurable filter’.

**General structure design**

The initial filter design is made through the usage of Signal–Flow Graph method [41] and a 4th–order leap–frog topology. The filter is working in the current mode (has current input and current...
output). The input current is applied to different circuit nodes via electronically controllable current gains to ensure the presence of the reconnection–less reconfiguration of the transfer function. The proposed (simplified) signal–flow graph of the filtering structure is depicted in Fig. 1.

The transfer function of a signal–flow graph is determined by so–called Mason’s gain formula expressed as:

$$K = \frac{1}{\Delta} \sum_i P_i \Delta_i,$$  \hspace{1cm} (1)

where $P_i$ stands for a gain of $i$–th forward path and $\Delta$ is the determinant of the given graph.

The denominator of the proposed filter, which corresponds with the determinant $\Delta$ of the graph in Fig. 1, is calculated as:

$$D(s) = 1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_4 + L_2 L_4 + L_1 L_3,$$  \hspace{1cm} (2)

denote then:

$$D(s) = 1 - \left( -\frac{L_1}{g_{m1}} - \frac{L_2}{g_{m2}} - \frac{L_3}{g_{m3}} - \frac{L_4}{g_{m4}} \right) +$$

$$+ \left[-\frac{L_1}{g_{m1}} \left( -\frac{L_2 / g_{m2}}{\Delta_2} \right) \right] + \left[-\frac{L_3 / g_{m3}}{\Delta_3} \left( -\frac{L_4 / g_{m4}}{\Delta_4} \right) \right] +$$

$$+ \left[\frac{L_1 / g_{m1}}{\Delta_2} \left( -\frac{L_2 / g_{m2}}{\Delta_2} \right) \right] + \left[\frac{L_3 / g_{m3}}{\Delta_3} \left( -\frac{L_4 / g_{m4}}{\Delta_4} \right) \right] +$$

$$+ \left[\frac{L_1 / g_{m1}}{\Delta_2} \left( -\frac{L_2 / g_{m2}}{\Delta_2} \right) \right] + \left[\frac{L_3 / g_{m3}}{\Delta_3} \left( -\frac{L_4 / g_{m4}}{\Delta_4} \right) \right] +$$

$$+ \left[\frac{L_1 / g_{m1}}{\Delta_2} \left( -\frac{L_2 / g_{m2}}{\Delta_2} \right) \right] + \left[\frac{L_3 / g_{m3}}{\Delta_3} \left( -\frac{L_4 / g_{m4}}{\Delta_4} \right) \right] +$$

The numerator of the transfer function is given as:

$$N(s) = s^3 C_1 C_2 C_3 g_{m3} B_4 + s^2 C_1 C_2 C_3 g_{m3} B_3 + s^2 C_1 C_3 g_{m2} B_4 +$$

$$+ s C_1 g_{m1} B_4 + + s C_3 g_{m1} B_4 + s C_4 g_{m1} B_4 +$$

$$+ s C_2 g_{m1} B_4 + + s C_3 g_{m2} B_4 + s C_3 g_{m3} B_4 +$$

$$+ s C_3 g_{m4} B_4 + + s C_4 g_{m2} B_4 + s C_4 g_{m3} B_4 +$$

$$+ s C_4 g_{m4} B_4 + + s C_4 g_{m3} B_4 +$$

$$+ s C_4 g_{m4} B_4 + + s C_4 g_{m4} B_4 +$$

(3)

$$N(s) = s^3 C_1 C_2 C_3 g_{m3} B_4 + s^2 C_1 C_2 C_3 g_{m3} B_3 + s^2 C_1 C_3 g_{m2} B_4 +$$

$$+ s C_1 g_{m1} B_4 + + s C_3 g_{m1} B_4 + s C_4 g_{m1} B_4 +$$

$$+ s C_2 g_{m1} B_4 + + s C_3 g_{m2} B_4 + s C_3 g_{m3} B_4 +$$

$$+ s C_3 g_{m4} B_4 + + s C_4 g_{m2} B_4 + s C_4 g_{m3} B_4 +$$

$$+ s C_4 g_{m4} B_4 + + s C_4 g_{m4} B_4 +$$

(4)

From (4), it can be seen that the resulting output response of the filter is controlled by the current gains $B_1$ to $B_4$ and depends on their adjustment. For the 4th–order LP function, current gain $B_1$ is set to 1 and remaining gains are zero, for example. Based on this fact, the numerator turns into $g_{m3} B_3 g_{m2} B_2 g_{m1} B_1$, other terms of the numerator are canceled. In case of the 1st–order function, current gain $B_4$ is set to 1 and other gains are zero thus, the numerator takes form of $s^3 C_1 C_2 C_3 g_{m3} B_4 + s^2 C_1 C_2 C_3 g_{m3} B_3 + s C_1 C_3 g_{m2} B_4 + s C_4 g_{m2} B_4$. Similarly, for the 2nd and 3rd–order function, where the current gain $B_2$ or $B_3$ is set and the remaining gains are zero, the numerator will change accordingly. In order to obtain a specific function, the current gains $B$ are either set to 1 or 0. It is also possible (depending on particular implementation of current amplifiers and their abilities of the control) to set the value higher or lower than 1 although that will shift the function in the matter of gain (the pass band will not have the unity gain). Nonetheless, this feature can be beneficial in case of the fine tuning of the pass–band area of available LP functions if the pass–band is not exactly 0 dB due to inaccuracy of filter parameters in real case or if the function does not have its pass band at 0 dB to start with originating from the design itself (which is common for some functions of higher–order filters). The other possibility is to adjust the stop–band area by partial addition of another term (0 < $B$ < 1) as presented in [39], for instance. The resulting functions, based on the setting of current gains $B$ in ideal case, are stated in Table 2.

The graph in Fig. 1 can be turned into a circuit scheme (see Fig. 2) considering active elements (described in the following subsection) suitable for the required function fulfillment.

### Implemented active elements

The active elements used in the proposed are four OTAs and one current amplifier with independent gain control of each output separately proposed in [42] (labeled as individual output gain controlled current amplifier (IOGC–CA) throughout the text). The structure also contains four grounded capacitors. The OTA element is represented by the schematic symbol depicted in Fig. 3 a). The relation between the terminal of the OTA is expressed as $N_{OUT} = N_{IN} - V_{SET_B}$, where $V_{SET}$ is the transconductance of this element. Fig. 3 b) shows a possible (used in the case of the measurement results) implementation of this element by a universal current conveyor (UCC) [43,44] and one resistor $R$ connected to the X terminal. The value of the resistor determines the value of the resulting transconductance by the relation $g_{m} = 1/R$. This solution does not directly offer the electronic control of the transconductance nonetheless, the OTA elements in the structure (in order to provide the electronic control of transconductances) could be realized by a combination of LT1228 device [45] and EL4083 device [46] as suggested in Fig. 4 (both devices are commercially available), for example. LT1228 device has its transconductance controlled electronically by a DC bias current but it only offers one output. We can obtain two outputs of both polarities by adding EL4083 device at the output of LT1228 device.

The addition of the IOGC–CA to the proposed structure allows the possibility of the reconnection–less reconfiguration of the transfer function. The schematic symbol of the IOGC–CA element can be seen in Fig. 5(a). The original structure of the IOGC–CA from [42] has been modified since the UCC offers two non–inverting and two inverting outputs and thus, the IOGC–CA itself offers the same polarities of its outputs. To ensure that all available functions of the proposed filter have the same polarity (all are non–inverting), all the outputs of the IOGC–CA must have the same polarity. It can be easily solved in case of the (transistor–level) simulations and on–chip implementation of whole circuitry from Fig. 2 when directly designed with this requirement in mind. For the specific implementation using the UCC and EL2082 [47] devices (current gain $B$ is controlled by the DC bias voltage $V_{SET, B}$), there are two additional OPAB660 devices [48] added in series with EL2082 in paths from $V_C$ outputs of the UCC (B1 and B3 paths as apparent in Fig. 5(b)). OPAB60 devices are used to invert the signal polarity of these outputs so all transfer functions have the same polarity. This could be also solved by using a different type of a current

![Fig. 1. Simplified signal-flow graph of the proposed 4th-order leap-frog topology suitable for the design of a reconnection-less reconfigurable low-pass fractional-order filter.](image-url)
amplifier with an opposite output polarity than the EL2082 provides nevertheless, it is quite possible such active element would have a different driving force (current instead of voltage) or even if using voltage as its driving force, the dependence of the current gain on the driving force could be dissimilar. Therefore, the solution utilizing OPA860 devices has been used to make the reconfiguration easier and more transparent. The behavior of the IOGC–CA is given as $I_{\text{OUT}} = \frac{1}{C_0} B_i(I_{\text{IN}})$, where $i = \{1, 2, 3, 4\}$ depending on the individual output.

Results of the proposed integer–order reconnection–less filter

The appropriate operation of the proposed filter is supported by PSpice simulations alongside with experimental measurements.

Simulation and measurement setup

The simulation results have been carried out by means of transistor–level simulation models based on CMOS 0.18 μm TSMC process. The IOGC–CA elements have been constructed by a current follower and current amplifiers. The transistor–level models of these elements were adopted from [49,50], respectively. The current gain $B$ of the used current amplifier model is controlled by a DC bias current. The OTA element is based on a model available in [51]. The transconductance of this particular model is set via a DC bias current. The supply voltage for all the used simulation models is ±1 V. A practical implementation of the used active elements is done with help of a UCC and EL2082s as described in the previous section. The UCC (created in cooperation of Brno University of Technology and ON Semiconductor design center) is implemented in CMOS 0.35 μm IT3T technology. The chip is labeled UCC–N1B_0520 where each chip contains one UCC and a second–generation current conveyor with two outputs (CCII±). The supply voltage of the UCC is ±1.65 V and 5 V for the EL2082. The measurement is performed by a network analyzer Agilent 4395A and voltage–to–current, current–to–voltage converters constructed around OPA860 [48] and OPA861 [52] devices. The block diagram of the measurement setup can be seen in Fig. 6 (a). Implemented V–I and I–V converters are based on a simple principle of a suitable connection of a CCII. In case of the V–I (Fig. 6 (b)), the input voltage is connected to the voltage input node of the CCII and converted to the output current by means of a resistor R connected to the current input node. The function of the I–V converter (Fig. 6 (c)) is as follows, the current from the proposed filter is connected to the current input of the CCII (working as a current follower) and mirrored to the current output node of the CCII. The output current is then transferred to voltage through the resistor R. A voltage buffer (present in the package of OPA860 together with the CCII) connected to the output of the converter provides the impedance separation. In order to eliminate the impact of these converters on obtained measurement results, a careful calibration of the signal path was performed. The actual measuring workspace including the supply voltage and control sources is shown in Fig. 7 (currently measuring order of 3 + 0.5).

Numerical design and results

The numerical design (calculation of individual coefficients of the transfer function in regard to the filter order and chosen approximation) has been made with help of NAF software [53]. Used coefficients stated in (5) are given for the following specifications: Butterworth approximation, the operational angular frequency $\omega_0 = 300,000$ rad/s ($f_0 = 47$ kHz), transfer in pass–band $K_P = -3$ dB, the stop–band frequency $f_s = 470$ kHz, transfer in stop–band $K_S = -80$ dB.

\[
\begin{align*}
    b_4 &= 1 \\
    b_3 &= 7.8440 \cdot 10^5 \\
    b_2 &= 3.0764 \cdot 10^{11} \\
    b_1 &= 7.0680 \cdot 10^{16} \\
    b_0 &= 8.1193 \cdot 10^{21}
\end{align*}
\]
These coefficients have been applied onto the denominator (expressed in (3)) of the transfer function. The transconductances $g_{m1}$ to $g_{m4}$ can be calculated as:

$$g_{m1} = b_1 C_1,$$

$$g_{m2} = \frac{b_1 C_1 C_3 C_4 - C_1 C_2 g_{m3} g_{m4}}{C_1 C_3 g_{m3} + C_1 C_4 g_{m1}},$$

$$g_{m3} = -\frac{C_1 C_3 C_4 b_2 g_{m1}}{C_1 b_3}.$$

For selected values of capacitors ($C_1 = C_2 = C_3 = C_4 = 1 \text{ nF}$), the resulting values of the transconductances (in accordance with Fig. 5. Individual output gain controlled current amplifier (IOGC–CA): (a) schematic symbol, (b) its implementation by one UCC and four EL2082 devices (and two OPA860).

Fig. 6. Measurement setup: (a) block diagram of the measurement, (b) basic principle of the used V–I converter, (c) basic principle of the used I–V converter.
coefficients in (5) are $g_{m1} = 784.4 \, \mu S$, $g_{m2} = 277.3 \, \mu S$, $g_{m3} = 190.3 \, \mu S$ and $g_{m4} = 196.1 \, \mu S$.

The list of available transfer functions for the filter from Fig. 2 is given in Table 3. The setting is now stated for control voltages in comparison to Table II, where $V_{SET\,B} = 1 \, V$ corresponds to current gain $B$ equal to 1.

The theoretical (indicated by black dashed lines) and simulation (represented by colored lines) results of the proposed integer–order filter are compared in Fig. 8. Magnitude characteristics of the first, second, third and fourth LP function are depicted in Fig. 8(a) while their phase characteristics are given in Fig. 8(b). Obtained simulation (black dashed lines) and experimental (colored lines) results of the same circuit are shown in Fig. 9. It can be seen that the theoretical and simulation results as well as the simulation and experimental results are in good accordance with each other. Any possible differences are mainly a result of the effect of parasitic characteristics of used simulation models and chips used for the implementation. As we are talking about the influence of the parasitic characteristics which affect the transfer function at higher frequencies, this is mainly caused by the impedance characteristics of the current inputs of used active elements. In the ideal case, the impedance of the current inputs is zero. The actual impedance of the X terminal of the UCC is 0.7 $\Omega$ at 10 kHz and it is increasing to 8.7 $\Omega$ at 1 MHz and it is already 75 $\Omega$ at 10 MHz. The value of the impedance of the current input of the EL2082 device is given (by the datasheet) to be 95 $\Omega$. These values will influence the attenuation in stop–band area at higher frequencies (the function will not continue with given slope of attenuation but rather stay at specific value of attenuation). Around 10 MHz, a slight increase of the transfer function (especially evident in case of the 1st–order function) is a typical parasitic feature of the UCC. The measured results are also affected by the features of used converters and the recognition abilities of the used Analyzer (around –80 dB). The above–mentioned effect of parasitics applies to all presented results. The functions are available based on the setting of current gains $B_1$ to $B_4$ as suggested in Table 2 and the results confirm the intended reconnection–less reconfiguration of the transfer function.

| $V_{SET\,B1}$ | $V_{SET\,B2}$ | $V_{SET\,B3}$ | $V_{SET\,B4}$ | function |
|--------------|--------------|--------------|--------------|-----------|
| 1 V          | 0 V          | 0 V          | 0 V          | 4th–order LP |
| 0 V          | 1 V          | 0 V          | 0 V          | 3rd–order LP |
| 0 V          | 0 V          | 1 V          | 0 V          | 2nd–order LP |
| 0 V          | 0 V          | 0 V          | 1 V          | 1st–order LP |

Modification of the proposed integer–order filter in order to obtain a fractional–order reconnection–less reconfigurable filter

In order for the proposed integer–order filter to provide FO characteristics, we suppose the filtering structure from Fig. 2 is being understood (and worked with) as two second–order filter sections connected in a cascade [54] which are then considered as integer–order (2nd–order) or fractional–order (either $1 + \alpha$ or $\alpha + \beta$) depending on a specific function. This fact is established due to that a FO filter can be stable only when the order of the filter is less than 2 as stated in [17]. The integer–order capacitors in the filtering structure are gradually replaced by their FO counterparts implemented by RC ladder networks. This gradual exchange can offer functions of LP \(3 + \alpha\), LP \(2 + \alpha\), LP \(1 + \alpha\), LP \(2 + \alpha + \beta\), LP \(1 + \alpha + \beta\), LP \(1 + \alpha + \beta + \gamma\), LP \(\alpha + \beta + \gamma\) and LP \(\alpha + \beta + \gamma + \delta\) based on the function which best suits our needs. The presence of multiple FO orders provides an additional degree of freedom since their summation can provide more possible combinations (more obtainable orders from which some can better meet our requirements for the specified slope of transition between the pass–band and stop–band area or specific phase shift). The FOE elements are substituted by the 5th–order RC structures of the Foster I type [55] depicted in Fig. 10. The purposes of the proposed filter, four printed circuit boards (presented in Fig. 11) have been constructed to replace integer–order capacitors. Each board contains five RC structures of orders 0.3, 0.4, 0.5, 0.6 and 0.7. The values of the components within the RC structures for the implemented orders are calculated with help of the Oustaloup approximation [56] (using a Matlab script). All the values (calculated for the operational $f_0 = 50 \, kHz$) are stated in Table 4. This does not directly offer the reconnection–less reconfigurable control of the fractional orders (the integer order can be added to the fractional order by electronic means by current gains $B$), nevertheless, used RC structures could be replaced by an electronically adjustable emulator introduced in [57] or some other FO emulators in order to ensure the electronic control of the order. This fact will be shown later in the paper in Section ‘Results of the proposed fractional–order reconnection–less filter’. Since the values for the RC structure are not standard series values, they are made by a parallel combination of E24 resistors (with tolerance of 5%) and a parallel combination of E12 capacitors (with tolerance of 5–20%).

The denominator of one section of the filter is expressed by (10) if one of the integer–order capacitors is replaced by a FO capacitor and (11) if both capacitors are replaced by their FO counterparts.

\[
D(s) = s^{a + \alpha} + s^{a + \gamma} + \frac{g_{m1} + g_{m2}}{C_1} s^{a + \beta} + \frac{g_{m1} + g_{m2}}{C_2} s^{a + \delta} \tag{10}
\]

\[
D(s) = s^{a + \beta} + s^{a + \gamma} + \frac{g_{m1} + g_{m2}}{C_1} s^{a + \alpha} + \frac{g_{m1} + g_{m2}}{C_2} s^{a + \delta} \tag{11}
\]

All FO filters of higher–order were constructed by a cascade combination of a 1 + $\alpha$ filter with an integer–order structure so far. Our solution is working with whole topology (4th–order in this case) and you can simply decide what is the resulting order of the filter by selecting the setting of current gains $B_1$ to $B_4$.

Results of the proposed fractional–order reconnection–less filter

The simulations and measurements of the FO filter are using the same setup as specified for the simulations and measurement of the integer–order filter (simulations done with CMOS 0.18 $\mu m$ TSMC models and the experimental measurement with help of the UCCs, EL2082s, network analyzer and converters). All FO trans-
Filter functions are designed to have their $f_0$ being equal approximately to 50 kHz. All simulation results are indicated by black dash lines while colored lines stand for the experimental results except for Figs. 12, 14 and 16 which compare the theoretical (black dash lines) and simulation (colored lines) results. Orders of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$

Figs. 12–17 show the results (magnitude and phase characteristics) of the proposed filter when capacitor $C_4$ is replaced by one of the implemented RC structure board. The denominator of the filter with $C_4$ being of fractional order is expressed as:

$$D(s) = s^{3 + \alpha}C_1C_2C_3C_4 + s^{2 + \alpha}C_1C_2C_3C_4g_{m_1} + s^{1 + \alpha}C_3C_4g_{m_1}g_{m_2} +$$
$$s^{1 + \alpha}C_1C_2C_4g_{m_2}g_{m_3} + s^2C_1C_2g_{m_2}g_{m_3}g_{m_4} + s^1C_2g_{m_3}g_{m_4} +$$
$$s^2C_4g_{m_3}g_{m_2}g_{m_3}g_{m_4} + g_{m_1}g_{m_2}g_{m_3}g_{m_4}$$

(12)

Such modification offers FO low-pass transfer functions of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$ in correlation with the setting of current gains. The
Table 4
Specification of the part values of the RC structures.

| $\alpha$ [-] | component | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  |
|-------------|-----------|------|------|------|------|------|
| $R_0$ [Ω]  |           | 149.4| 57.0 | 50.6 | 29.5 | 17.2 |
| $R_1$ [Ω]  |           | 118.7| 96.3 | 71.6 | 49.3 | 31.1 |
| $R_2$ [Ω]  |           | 241.3| 246.1| 229.5| 197.8| 155.9|
| $R_3$ [Ω]  |           | 465.5| 590.6| 685.1| 734.7| 720.4|
| $R_4$ [Ω]  |           | 902.6| 1431.1| 2077.6| 2793.9| 3444.0|
| $R_5$ [Ω]  |           | 1960.1| 2461.7| 2298.3| 15643.7| 29039.1|
| $C_0$ [nF] |           | 0.547| 0.752| 1.13 | 1.82 | 3.22 |
| $C_1$ [nF] |           | 2.34 | 2.56 | 3.06 | 3.96 | 5.59 |
| $C_2$ [nF] |           | 10.58| 9.29 | 8.92 | 9.27 | 10.54|
| $C_3$ [nF] |           | 47.49| 33.37| 25.62| 21.23| 19.19|
| $C_4$ [nF] |           | 190.36| 100.41| 56.44| 33.0 | 19.81|

Fig. 12. Theoretical and simulation results of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$ orders for $\alpha = 0.3$ (a) magnitude characteristics, (b) phase characteristics.

Fig. 13. Simulation and experimental results of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$ orders for $\alpha = 0.3$ (a) magnitude characteristics, (b) phase characteristics.

Fig. 14. Theoretical and simulation results of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$ orders for $\alpha = 0.5$ (a) magnitude characteristics, (b) phase characteristics.
Fig. 15. Simulation and experimental results of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$ orders for $\alpha = 0.5$ (a) magnitude characteristics, (b) phase characteristics.

Fig. 16. Theoretical and simulation results of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$ orders for $\alpha = 0.7$ (a) magnitude characteristics, (b) phase characteristics.

Fig. 17. Simulation and experimental results of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$ orders for $\alpha = 0.7$ (a) magnitude characteristics, (b) phase characteristics.

| $\alpha$ | $g_{m3}$ [mS] | $g_{m4}$ [mS] |
|----------|----------------|----------------|
| 0.3      | 0.610          | 3.060          |
| 0.4      | 0.757          | 2.530          |
| 0.5      | 0.933          | 2.103          |
| 0.6      | 1.139          | 1.764          |
| 0.7      | 1.375          | 1.496          |

Table 5: Values of transconductances $g_{m3}$, $g_{m4}$ in relation to the values of alpha.

| $\alpha$ | $g_{m3}$ [mS] | $g_{m4}$ [mS] |
|----------|----------------|----------------|
| 0.3      | 1.46           | 3.47           |
| 0.4      | 0.4            | 2.57           |
| 0.5      | 0.5            | 2.03           |
| 0.6      | 0.6            | 1.67           |
| 0.7      | 0.7            | 1.41           |

Table 6: Values of transconductances $g_{m3}$, $g_{m4}$ in relation to the values of alpha and beta.
first section of the filter (involving OTA1, OTA2 and capacitors C1 and C2) is expressed as a standard 2nd–order filter with its 
\[ f_0 = \frac{1}{2\pi} \frac{g_{m1}g_{m2}}{C_1 C_2} \]  
For \( f_0 = 50 \text{ kHz} \) and selected values of capacitors 
\( C_1 = C_2 = 4.7 \text{ nF} \), the values of transconductances \( g_{m1} \) and \( g_{m2} \) were calculated to be \( g_{m1} = g_{m2} = 1.477 \text{ mS} \). The second section

is using \( C_3 = 4.7 \text{ nF} \) and the values of transconductances \( g_{m3} \) and \( g_{m4} \) (stated in Table 5) are calculated in relation to the value of the FO capacitor which is depending on FO parameter \( a \). Figures present the comparison of the theoretical and simulation results and then simulation and experimental results of functions of
\[ 3 + \alpha, 2 + \alpha \text{ and } 1 + \alpha \] when \( \alpha \) is equal to 0.3 in case of Figs. 12 and 13, it is \( \alpha = 0.5 \) for Figs. 14 and 15 and \( \alpha = 0.7 \) for Figs. 16 and 17. The results show a good correlation with each other. The resulting orders are usually slightly lower than the expected values (about 0.1 less than expected). These differences from the theory are mainly caused by the rounding or calculated values, error caused by the resulting values of parallel combination of resistors/capacitors which do not exactly match the desired values and the tolerance of used parts. Note that this error will be added to each other as more integer–order capacitors are replaced by the implemented RC structures. Nevertheless, this could be easily solved by using higher values of \( \alpha \) to better match the required order, if keeping this fact in mind. Other possible solution is the implementation of more accurate RC structures.
The proposed filter can offer FO low–pass functions of $2 + a + b$, $1 + a + b$ and $a + b$ by replacing two capacitors ($C_3$ and $C_4$) in the topology by FO capacitors. The denominator for this case is given as:

$$D(s) = s^{2 + a + b}C_1C_2C_3C_4 + s^{1 + a + b}C_2C_3C_4C_5 + s^{a + b}C_3C_4C_5C_6 + s^{a + b}C_2C_3C_4C_5C_6C_7 + s^{a + b}C_1C_2C_3C_4C_5C_6C_7C_8 + 1.$$  

(13)

Table 7

| $f_0$/transconductance | 25 kHz | 50 kHz | 100 kHz |
|------------------------|--------|--------|--------|
| $g_{m1}$ [mS]          | 0.47   | 0.93   | 1.87   |
| $g_{m2}$ [mS]          | 1.05   | 2.10   | 4.21   |
| $g_{m3}$ [mS]          | 0.73   | 1.46   | 2.91   |
| $g_{m4}$ [mS]          | 1.01   | 2.03   | 4.06   |

Orders of $2 + a + b$, $1 + a + b$ and $a + b$

The proposed filter can offer FO low–pass functions of $2 + a + b$, $1 + a + b$ and $a + b$ by replacing two capacitors ($C_1$ and $C_2$) in the topology by FO capacitors. The denominator for this case is given as:

$$D(s) = s^{2 + a + b}C_1C_2C_3C_4 + s^{1 + a + b}C_2C_3C_4C_5 + s^{a + b}C_3C_4C_5C_6 + s^{a + b}C_2C_3C_4C_5C_6C_7 + s^{a + b}C_1C_2C_3C_4C_5C_6C_7C_8 + 1.$$  

(13)
The first section of the filter is set as in the previous case (\(C_1 = C_2 = 4.7 \, \text{nF}, \, g_{m1} = g_{m2} = 1.477 \, \text{mS}\)). The second section is involving two FO capacitors. Used values of \(g_{m3}\) and \(g_{m4}\) are summarized in Table 6. The value of \(g_{m3}\) is depending on parameter \(a\) but the change is minimal (going from 1.45 mS to 1.46 mS for \(a\) changing from 0.3 to 0.7). Therefore, transconductance \(g_{m3}\) has been set to 1.46 mS for all used \(a\). Transconductance \(g_{m4}\) is depending on the value of \(b\). Fig. 18 introduces the results of \(2 + a + b\) function for four different combinations of \(a\) and \(b\) values. The following setting has been used: \(a = 0.3, \, b = 0.4, \, a = 0.5, \, b = 0.3, \, a = 0.5, \, b = 0.6\) and \(a = 0.7, \, b = 0.7\) resulting in orders 2.7, 2.9, 3.1 and 3.3. The characteristics of \(1 + a + b\) functions are provided in Fig. 19. The \(a\) and \(b\) value combination for this case is \(a = 0.4, \, b = 0.3, \, a = 0.6, \, b = 0.3, \, a = 0.4, \, b = 0.7\) and \(a = 0.7, \, b = 0.6\) (orders 1.7, 1.9, 2.1 and 2.3).

Orders of \(1 + \alpha + \beta + \gamma\) and \(\alpha + \beta + \gamma\)

Fractional-order low-pass functions of \(1 + \alpha + \beta + \gamma\) and \(\alpha + \beta + \gamma\) are available if three capacitors (\(C_2, C_3, C_4\)) are replaced by the designed RC structures. The denominator takes form of:

\[
D(s) = s^{1+\alpha+\beta+\gamma}C_1C_2C_3C_4 + s^{\alpha+\beta+\gamma}C_2C_3C_4g_{m1} + s^{\beta+\gamma}C_3C_4g_{m2} + s^{1+\alpha+\beta+\gamma}C_1C_2C_3C_4g_{m1} + s^{\alpha+\beta+\gamma}C_2C_3C_4g_{m2}g_{m4} + s^{\beta+\gamma}C_3C_4g_{m3} + s^{1+\alpha+\beta+\gamma}C_1C_2C_3C_4g_{m1}g_{m4} + s^{\alpha+\beta+\gamma}C_2C_3C_4g_{m2}g_{m3} + s^{\beta+\gamma}C_3C_4g_{m1}g_{m4} + s^{1+\alpha+\beta+\gamma}C_1C_2C_3C_4g_{m1}g_{m3}g_{m4} + s^{\alpha+\beta+\gamma}C_2C_3C_4g_{m2}g_{m3}g_{m4} + s^{\beta+\gamma}C_3C_4g_{m1}g_{m2}g_{m4} + s^{1+\alpha+\beta+\gamma}C_1C_2C_3C_4g_{m1}g_{m2}g_{m3}g_{m4}.
\]

(14)

The first section of the filter is handled as a \(1 + \alpha\) filter the same way as the second section in case of \(3 + a\), \(2 + a\) and \(1 + \alpha\) functions. The values of transconductances \(g_{m1}\) and \(g_{m2}\) are then same as the values of \(g_{m3}\) and \(g_{m4}\) in Table 5. The second section is identical to the previous case (\(g_{m3}\) and \(g_{m4}\) are set according to Table 6). The
The results of the $1 + \alpha + \beta + \gamma$ functions are shown in Fig. 20 for combinations $\alpha = 0.3$, $\beta = 0.3$, $\gamma = 0.5$, $\beta = 0.4$, $\gamma = 0.4$, $\alpha = 0.5$, $\beta = 0.6$, $\gamma = 0.6$ and $\alpha = 0.7$, $\beta = 0.7$, $\gamma = 0.7$ (orders 1.9, 2.3, 2.7 and 3.1). Fig. 21 introduces results of $\alpha + \beta + \gamma$ function for $\alpha = 0.5$, $\beta = 0.3$, $\gamma = 0.3$, $\alpha = 0.3$, $\beta = 0.5$, $\gamma = 0.7$ and $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 0.7$.

Orders of $\alpha + \beta + \gamma + \delta$

The last possibility with all four FO capacitors suggests both sections of the filter being set according to Table 6 depending on the values of FO orders. The denominator is:

$$D(s) = s^{\alpha + \beta + \gamma + \delta}C_1C_2C_3C_4 + s^{\alpha + \beta + \gamma + \delta}C_2C_3C_4G_{m1} +$$ $+ s^{\alpha + \beta + \gamma + \delta}C_1C_4G_{m1}G_{m2} + s^{\alpha + \beta + \gamma + \delta}C_1C_3G_{m2}G_{m3} +$$ $+ s^{\alpha + \beta + \gamma + \delta}C_1C_2G_{m3}G_{m4} + s^{\alpha + \beta + \gamma + \delta}C_4G_{m1}G_{m2}G_{m3}G_{m4} +$$ $+ s^{\alpha + \beta + \gamma + \delta}C_4G_{m1}G_{m2}G_{m3}G_{m4} + s^{\alpha + \beta + \gamma + \delta}C_4G_{m1}G_{m2}G_{m3}G_{m4}.$

(15)

Characteristics of $\alpha + \beta + \gamma + \delta$ function (for $\alpha = 0.3$, $\beta = 0.3$, $\gamma = 0.5$, $\delta = 0.3$, $\alpha = 0.3$, $\beta = 0.6$, $\gamma = 0.5$, $\delta = 0.4$, $\alpha = 0.7$, $\beta = 0.5$, $\gamma = 0.6$, $\delta = 0.4$, $\alpha = 0.7$, $\beta = 0.7$, $\gamma = 0.6$, $\delta = 0.6$) are available in Fig. 22.

Pole frequency control

Other beneficial ability of the designed filter is the control of the pole frequency which can be adjusted by the change of transconductances $G_{m1}$ to $G_{m4}$ as long as the ratio between transconductances given by the FO approximation remains unchanged. This ability has been tested for three different pole frequencies (25 kHz, 50 kHz and 100 kHz) in case of $\alpha + \beta + \gamma$ function where $\alpha = \beta = \gamma = 0.5$. The values of transconductances are specified in Table 7. As it can be seen from the graph, the possibility of the pole
frequency control while maintaining the desired order of the filter is working as presumed. The limitation of the tunable range of $f_0$ is the range in which used FO approximation stays valid (up to four decades for used 5th-order RC structures). This range can be extended by increasing the order of used RC structures (for the price of increasing circuit complexity).

Stability analysis

The analysis of the stability of analog circuits, frequency filters, in particular, is an important part of the circuit design. Therefore, it is very important to determine whether the filter will be stable or not. A general approach to stability analysis of fractional-order circuits by the transformation of the $s$-plane into a general plane that includes the fractional orders so called $W$-plane was introduced in [58]. $W$-plane is defined as $s^{1/m}$, where $m$ is a positive integer [58]. The unstable and stable regions of the $s$-plane transform into $|\omega W| > \pi/m$ and into $\pi/2m < |\omega W| < \pi/m$ as suggested in Fig. 24. The region where $|\omega W| > \pi/m$ is non-physical. Thus, the system will be stable if all roots in the $W$-plane lie in the region $|\omega W| > \pi/2m$ and unstable if at least one root is in the region $|\omega W| < \pi/2m$. In order to investigate and prove the stability of the proposed fractional-order filter, positions of poles projected into the $W$-plane of various situations were carried out using a Matlab script based on the mathematical procedure introduced in [59]. Figs. 25–27 depict $W$-planes for the $3 + \alpha$ filter, where Fig. 25 shows a situation for $\alpha = 0.3$, Fig. 26 for $\alpha = 0.5$ and Fig. 27 for $\alpha = 0.7$. $W$-planes in case of the $2 + \alpha + \beta$ filter are presented in Fig. 28, Fig. 29 and Fig. 30, respectively for situations $\alpha = \beta = 0.3$, $\alpha = \beta = 0.5$ and $\alpha = \beta = 0.7$. The last three pictures (Fig. 31, Fig. 32 and Fig. 33) offer the $W$-planes of the $1 + \alpha + \beta + \gamma$ filter for combinations of different fractional-orders (the first combination is $\alpha = 0.3$, $\beta = 0.5$, $\gamma = 0.7$, the second one is $\alpha = 0.5$, $\beta = 0.7$, $\gamma = 0.3$ and the third one is $\alpha = 0.7$, $\beta = 0.5$, $\gamma = 0.3$). It is evident that the filter is stable for all tested cases.

Electronic control of the fractional order

As mentioned before, the proposed filter was implemented with RC networks to substitute the function of fractional-order capacitors due to rather high complexity of the constructed structure, unless it is all designed on one chip, and the complexity of the resulting control necessary for proper function. The electronic control of the fractional order and thus, the reconfiguration of both the integer and fractional order can be achieved by the replacement of the RC ladder structures by electronically adjustable emulators of a fractional-order capacitor/inductor introduced in [29–31,57], for example. The other possible

Table 8

| Parameter | 0.3 | 0.5 | 0.7 |
|-----------|-----|-----|-----|
| $g_m1$ [mS] | 772.3 | 590.6 | 451.7 |
| $g_m2$ [mS] | 449.4 | 295.3 | 164.1 |
| $B_1$ [-] | 0.4 | 0.2 | 0.09 |
| $B_2$ [-] | 1 | 1 | 1 |
| $B_3$ [-] | 2.5 | 5 | 11.8 |

Fig. 33. Roots in the $W$-plane for $1 + \alpha + \beta + \gamma$ filter where $\alpha = 0.7$, $\beta = 0.5$, $\gamma = 0.3$.

Fig. 34. Current-mode integrator based on FLF topology used for the aproximation of a fractional-order integrator.

Fig. 35. Simulation results of $3 + \alpha$, $2 + \alpha$ and $1 + \alpha$ orders for the solution with the RC structure and FO integrator when $\alpha = 0.3$ (a) magnitude characteristics, (b) phase characteristics.
solution is to replace the building blocks (OTA + C) of the designed structure i.e. the integrator in this particular case by its fractional–order counterpart. Possible solutions of fractional–order integrators can be found in [60–62], for instance. For the demonstration of the electronic control of a fractional order, we have decided to adopt the approach described in [60]. The introduced structure is an electronically tunable integrator based on the FLF topology and approximated to function as a fractional–order integrator with order of \( \alpha \). Since the circuit in [60] operates in the voltage mode and the proposed filter requires an current–mode integrator (the input and output of each stage is current), the structure has been redesigned to operate in the current mode. The resulting current–mode integrator used for the aproximation on a fractional–order current–mode integrator is presented in Fig. 34.

The circuit in Fig. 34 is described by the equation:

\[
K(s) = \frac{s^2 C_1 C_2 + s C_2 \mu_m B_2 + \mu_m \mu_{in} B_3}{s^2 C_1 C_2 + s C_2 B_{in} + \mu_m \mu_{in} B_{in2}}.
\]  

(16)

The general transfer function of a fractional–order (lossless) integrator is given as \( H(s) = 1/(\tau s)^{\alpha} \). The second–order approximation of \( s^2 \) is described as:

\[
H(s) = \frac{(s^2 - 3\alpha + 2)s^2 + (8 - 2\alpha^2)s + (\alpha^2 + 3\alpha + 2)}{(s^2 + 3\alpha + 2)s^2 + (8 - 2\alpha^2)s + (\alpha^2 - 3\alpha + 2)}.
\]  

(17)

By the substitution of the general transfer function of a fractional–order integrator and the second–order approximation of \( s^2 \), the transfer function takes form of:

\[
H(s) = \frac{(s^2 + 2\alpha^2 - \alpha + 2)s^2 + (\alpha^2 + 2\alpha^2 - \alpha - 2)s + (\alpha^2 + 2\alpha - \alpha^2)}{(s^2 + \alpha^2 + 2\alpha - \alpha^2)s^2 + (\alpha^2 - 2\alpha^2 + \alpha + 2)s + (2\alpha - 2\alpha^2 - \alpha^2)}.
\]  

(18)

Combining Eqs. (16) and (18), the parameters of (16) for desired fractional order can be easily calculated. The values of transconductances and current gains of the FLF topology in Fig. 34, in correspondence with the desired fractional orders (0.3, 0.5 and 0.7) calculated for the center frequency being 100 kHz and \( C_1 = 470 \text{ pF} \), \( C_2 = 4.7 \text{ nF} \), are stated in Table 8.

The function of the designed fractional–order integrator has been tested in case of simulations for functions 3 + \( \alpha \), 2 + \( \alpha \) and 1 + \( \alpha \) (OTA4 and \( C_4 \) in Fig. 2 were replaced by the integrator from Fig. 34) for \( \alpha \) being equal to 0.3, 0.5 and 0.7 and the results are compared with simulation results of the corresponding solutions with the RC structures (Section ‘Orders of 3 + \( \alpha \), 2 + \( \alpha \) and 1 + \( \alpha \)'). The values of parameters of the filter (transconductances and current gains of the filter itself not the circuit from Fig. 34) are the same as in Section ‘Orders of 3 + \( \alpha \), 2 + \( \alpha \) and 1 + \( \alpha \)’. Figs. 35–37 compare the simulation results of the filter when using the RC structure (black dashed lines) with the simulation results of the filter when using the FO integrator (colored lines). The results yield a good agreement with each other and support the possible employment of the electronic control of the fractional order. There is a slight shift in the frequency for the results with the FO integrator increasing the operational frequency as \( \alpha \) is getting higher nonetheless, this undesired behavior can be compensated as demonstrated in subsection Electronic control of the fractional order.

Conclusion

The proposed structure of a FO reconnection–less reconfigurable filter can offer LP functions of 3 + \( \alpha \), 2 + \( \alpha \), 1 + \( \alpha \), 2 + \( \alpha + \beta \), 1 + \( \alpha + \beta \), 1 + \( \alpha + \beta + \gamma \), \( \alpha + \beta + \gamma \) and \( \alpha + \beta + \gamma + \delta \) (plus 4
additional functions if considering that the filter can provide integer–order functions as well as suggested in chapters describing the design and measurement of the proposed integer-order filter) in dependence on the setting of control gains $B_1$ to $B_3$ and replacement of standard integer–order capacitors by their FO counterparts offering an additional degree of freedom when it comes to the obtainable values of the filter order. The theoretical, simulation and experimental results, which are available for various combinations of orders (see Figs. 12–22), confirm the design intentions. The full electronic control of the order can be achieved if the used RC structures are replaced by the electronically controllable FO impedance emulators which is supported by simulation results in Figs. 35–37. The controllability of the pole frequency is supported by the results in Fig. 23 for three settings. The stability analysis of the proposed structure for various orders is shown in Figs. 24–33. All simulation and experimental results are in good correlation with each other. Considering the lowest and highest value of the RC networks being 0.1 and 0.9, for example, the proposed filter could offer a LP function of the theoretical order range from 0.4 up to 3.6. In other words, it can be fairly simple to choose an order which best fits the situation in relatively wide range of available orders.

Declaration of Competing Interest

The authors declared that there is no conflict of interest.

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References

1. Elwakil AS. Fractional-order circuits and systems: An emerging interdisciplinary research area. IEEE Circuits Syst Mag 2010;10(4):40–50.
2. Freeborn TJ. A survey of fractional-order circuit models for biology and biomedicine. IEEE J Emerging Sel Top Circuits Syst 2013;1(3):416–24.
3. Freeborn T, Maundy B, Elwakil AS. Extracting the parameters of the double-dispersion Cole bioimpedance model from magnitude response measurements. Med Biol Eng Comput 2014;52(9):749–58.
4. Freeborn T, Maundy M, Elwakil AS. Cole impedance extractions from the step-response of a current excited fractal. Comput Electron Agric 2013;98:100–8.
5. Kubanek D, Freeborn T, Koton J. Fractional-order band-pass filter design using fractional-characteristic specimen functions. Microelectron J 2019;86:77–86.
6. Mishra SK, Gupta M, Upadhyay DK. Active realization of fractional order Butterworth lowpass filter using DVCC. J King Saud Univ – Eng Sci 2020;32(2):158–65.
7. Kaskouta E, Kamilaris T, Sotner R, Jerabek J, Psychalinos C. Single-input multiple-output and multiple-input single-output fractional-order filter designs. In: Proc 41st international conference on telecommunications and signal processing (TSP), 2018, Athens, Greece. p. 356–9.
8. Kamath DV, Navya S, Soubhayargsaeneth N. Fractional order OTA-C current-mode all-pass filter. In: Proc. 2nd international conference on inventive communication and computational technologies (ICICCT 2018), 2018, Coimbatore, India. p. 383–7.
9. Mishra SK, Gupta M, Upadhyay DK. A low-power compact DDCC based fractional-order filter. In: Proc. 2nd international conference on power, energy and environment: towards smart technology (ICPEE), 2018, Shillong, India. p. 1–6.
10. Verma R, Pandey N, Pandey R. Electronically tunable fractional order filter. Arab J Sci Eng 2017;42:3409–22.
11. Soni A, Gupta M. Design of fractional order butterworth filter using genetic algorithm. In: Proc 2nd IEEE international conference on power electronics, intelligent control and energy systems (ICEPEICES-2018), 2018, Delhi, India. p. 1052–5.
12. Hassannein AM, Soltan A, Said LA, Madian AH, Radwan AG. Analysis and design of fractional-order low-pass filter with three elements of independent orders. In: Proc 2019 novel intelligent and leading emerging sciences conference (NILES), 2019, Giza, Egypt. p. 218–21.
international conference mixed design of integrated circuits and systems (MIXDES2019), 2019, Rzeszow, Poland. p. 1–5.

[40] Sotner R, Petrzela J, Jerabek J, Vrba K, Dostal T. Solutions of reconnection-less OTA-based biquads with electronical transfer response reconfiguration. In: Proc 25th int conf radioelektronika, 2015, Pardubice, Czech Republic. p. 40–5.

Nise NS. Signal-flow graphs. The control handbook. IEEE Press; 1996. 1566 pages.

[42] Langhammer L, Sotner R, Dvorak J. Modification of current follower/amplifier with controllable intrinsic resistance. In: Proc IEEE 2018 41st international conference on telecommunications and signal processing (TSP2018), 2018, Athens, Greece. p. 70–3.

[43] Sponar R, Vrba K. Measurements and behavioral modelling of modern conveyors. Int J Comput Sci Netw Security 2006;3A(6):57–63.

[44] UCC-N1B – Universal Current Conveyor (UCC) and Second-Generation Current Conveyor (CCII+) (datasheet). Czech Republic: Brno University of Technology; 2010 [accessible on http://www.utko.feec.vutbr.cz/~koton/soubory/UCC_N1B_Rev0.pdf].

[45] Texas Instruments – OPA860 – Wide Bandwidth Operational Transconductance Amplifier (datasheet) [accessible on http://www.ti.com/lit/ds/symlink/opa860.pdf].

[49] Jerabek J, Sotner R, Vrba K. Electronically adjustable triple-input single-output filter with voltage differencing transconductance amplifier. Revue Roumaine des Sciences Techniques – Serie ´Electrotechnique et ´Energ´ etique 2015;59(2):163–72.

[50] Sotner R, Herencsar N, Jerabek J, Prokop R, Kartci A, Dostal T, et al. Z-copy controlled-gain voltage differencing current conveyor: advanced possibilities in direct electronic control of first-order filter. Elektronika ir Elektrotechnika 2014;20(6):77–83.

[52] Texas Instruments – OPA861 – Wide Bandwidth Operational Transconductance Amplifier (datasheet) [accessible on http://www.ti.com/lit/ds/symlink/opa861.pdf].

[53] Hajek K, Sedlacek J. NAFeD program as powerful tool in filter education area. Proc of the conference CIBLIS'97, 1997, Leicester, United Kingdom. 1997. pp. PK-4 1–10.

[54] Deliyannis T, Sun Y, Fidler JK. Continuous-time active filter design. US: CRC Press; 1998.

[55] Tsirimokou G. A systematic procedure for deriving RC networks of fractional-order elements emulators using MATLAB. Int J Electron Commun (AEU) 2017;78:7–14.

[57] Tsirimokou G, Psychalinos C, Freeborn TJ, Elwakil AS. Emulation of current excited fractional-order capacitors and inductors using OTA topologies. Microelectron J 2016;55:70–81.