Temperature-induced stresses in reinforced concrete structures

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Abstract. The author estimates stress state in steel-and-concrete power conduit arranged at the concrete downstream face with regard to temperature and combination of loads. It is shown that the maximal tensile stresses are formed under minimum temperature in winter at the conduit points remote from the downstream face.

In case of the heat exchange in rock mass under conditions of high-amplitude seasonal variations in air temperatures, in deep level mines in the North, during operation of large concrete structures and in other similar cases, it is necessary to take into account temperature effects on stress–strain state of geomaterials.

Regarding waterworks, the most critical structures of their design are steel-and-concrete pressure conduits and spiral chambers of hydropower units. Reliable and safe operation of these critical structures is one of the top-priority objectives both at the stage of design and during service period. An obligatory element of serviceability and safety check in high-pressure waterworks and their structural elements is the stress–strain state modeling for the whole structure and its most critical elements.

This paper reports the 3D numerical modeling of stress state of the steel-and-concrete power conduit of Sayano-Shushenskaya hydropower plant subjected to thermal forces. The computation used bundled software for solving problems on thermal elasticity of block structured rock masses [1]. In the stress state calculation for a steel-and-concrete power conduit, the blocks are assumed the conduit layers.

A steel-and-concrete conduit of circular section was modeled by a multi-layer ring (Figure 1) composed of an inner steel shell structure and an external reinforced concrete shell structure with two layers of reinforcing steel (equivalent to two rows of hoop reinforcement). The thickness of each of the reinforcing rings was accepted based on the condition that cross section area of a ring was equal to cross section area of rebars in the corresponding reinforcement row.

The consequence of thermal effect on a block structured rock mass is the redistribution of stresses inside and in-between the blocks. The dynamics of temperature field in a block is taken into account with the help of the equation of transient heat conduction. The system of differential equations of the unbound quasistatic thermoelastic problem without the sources of heat and bulk forces is given by [1]:

\[
\mu^{[n]} \Delta u^{[n]} + (\lambda^{[n]} + \mu^{[n]}) \text{grad} \text{div} u^{[n]} = \gamma^{[n]} \text{grad} \theta^{[n]},
\]
\[ \Delta \theta^{[n]} = \beta \frac{\partial \theta^{[n]}}{\partial t}, \]  

where \( u^{[n]}(X, t) \)—vector of displacements at point \( X (x, y) \) at time \( t \); \( \theta^{[n]}(X, t) = T(X, t) - T_0 \)—temperature counted from the initial state; \( \lambda^{[n]}, \mu^{[n]} \)—Lamé constants; \( \gamma^{[n]} = \alpha^{[n]}(3\lambda^{[n]} + 2\mu^{[n]}) \); \( \alpha^{[n]} \)—coefficient of linear thermal expansion; \( \Delta \)—Laplace operator; \( \beta = 1/\alpha \), where \( \alpha \)—thermal diffusivity.

**Figure 1.** Computational scheme: (a)—bottom of the conduit; (b)—cross section I–I.

It is assumed that the temperature difference is such that the properties of rocks remain unaltered and, consequently, the stresses and strains are related by the Duhamel–Najman relation [1].

The temperature field is determined regardless of the field of displacement by solving Eq. (2) with the boundary and initial conditions:

\[ \theta^{[n]}(X, 0) = 0, \quad X \in \Gamma^{[n]} \cup \Gamma^{[n]}, \]  

\[ \frac{\partial \theta^{[n]}(Y)}{\partial n(Y)} + h^{[n]}(Y, t) \theta^{[n]}(Y, t) = h^{[n]}(Y, t) \bar{\theta}^{[n]}(Y, t), \quad Y \in \Gamma^{[n]}, \quad t > 0, \]  

\[ p_i(Y, t) = f_i(Y, t), \quad Y \in \Gamma^{[n]}, \quad t > 0, \quad i = 1, 2, \]  

\( h^{[n]}(Y, t) \)—heat exchange coefficient; \( \bar{\theta}^{[n]}(Y, t) \)—ambient temperature or the neighbor block temperature; \( \frac{\partial \theta^{[n]}}{\partial n(Y)} \)—derivative with respect to the outside normal to a block. The obtained value of the temperature is used to solve Eq. (1) where the time \( t \) is considered as a parameter.

The unbound quasistatic thermoelastic problem is reduced to sequential solution of the system of integral equations for finding the densities \( \phi_i, i = 1, 2, 3 \). The values of the stresses and strains are determined by integrating over the boundary (for the densities \( \phi_1, \phi_2 \) obtained from the system of integral equations of isothermic elasticity) or over the boundary and time (for \( \phi_3 \) from the integral equation of thermal potential of simple fiber [1, 2]).

At the interfaces of adjacent media (solids, e.g. neighbor blocks \( G^{[n]} \) and \( G^{[m]} \)) the conjugation conditions are set for heat flows and temperatures [4]: \( q_{S_{n,m}} = q_{S_{m,n}} \); \( t_{S_{n,m}} = t_{S_{m,n}} \), where \( q_{S_{n,m}}, t_{S_{n,m}} \)—respectively, heat flow and temperature of a boundary element \( S_{n,m}^+ \) at its contact with the block \( G^{[n]} \), and
$q_{S_{m,n}}$ and $t_{S_{m,n}}$ — heat flow and temperature of a boundary element $S_{m,n}$ at its contact with the neighbor block $G^{(m)}$.

The conditions of conjugation of heat flows at the interface of the immobile medium (solid) and a fluid (water, air) are set in terms of the coefficients of heat transfer $\alpha$ W/(m$^2$·deg) and heat exchange $h = \alpha/k$ (1/m), where $k$ — heat conduction coefficient, W/(m·deg).

The problem is implemented using the method of regularization by Tikhonov. For practical problems taking much time to be solved, algorithms are based on the concept of parallel computing [1, 2].

![Figure 2. Annual cycle of water temperature at the water intake and air temperature in the area of Sayano-Shushenskaya HPP.](image)

The stresses and strains were analyzed in structural elements in the middle of the bottom section of the steel-and-concrete conduit (Figure 1) under seasonal variation in the water temperature at the water intake and in the air temperature (Figure 2). The steel-and-concrete conduit of circular section was modeled by a multi-layer ring composed of an inner steel shell structure and an external reinforced concrete shell structure with two layers of reinforcing steel (equivalent to two rows of hoop reinforcement). The thickness of each of the reinforcing rings was accepted based on the condition that cross section area of a ring was equal to cross section area of rebars in the corresponding reinforcement row.

It is assumed that steel, concrete and material the reinforcing rings are made of are the linearly deformable materials. Table 1 presents the source data for the stress calculation of the steel-and-concrete power conduit. Given the conjugation of heat flows at the solid and fluid (water, air) interface [3], the heat transfer coefficients are $\beta_1 = 6$ W/(m$^2$·°C) for air and $\beta_2 = 600$ W/(m$^2$·°C) for water; the heat exchange coefficient are $h_1 = \beta_1/\lambda_b = 3.45$ 1/m for air–concrete system and $h_2 = \beta_2/\lambda_b = 17.14$ 1/m for water–steel system.

The calculations accounted for fracture formation of concrete [1, 3, 6].

| Physical property             | Steel shell | Concrete | Reinforcing ring material |
|-------------------------------|-------------|----------|---------------------------|
| Elasticity modulus $E$, MPa   | $2.1\times10^5$ | $2.9\times10^4$ | $2.1\times10^5$ |
| Density $\rho$, kg/m$^3$      | 7794        | 2400     | 7784                      |
| Poisson’s ratio $\nu$         | 0.28        | 0.17     | 0.25                      |
| Linear thermal expansion $\alpha$, 1/deg | $1.2\times10^{-5}$ | $0.95\times10^{-5}$ | $1.1\times10^{-5}$ |
| Thermal conduction $k$, W/(m·deg) | 35         | 1        | 30                        |
| Thermal diffusivity $\alpha$, m$^2$/s | $7.32\times10^{-6}$ | $0.54\times10^{-6}$ | $0.84\times10^{-6}$ |
| Heat transfer $\alpha_t$, W/(m$^2$·deg) | 3.51        | 0.87     | 2.1                       |

During operation of conduits, their stress state is conditioned by: stress state of the bottom edge of the dam, process stresses generated during construction, cyclical hydrostatic loads,
seasonal variation in the temperature of ambient air and water in the conduit. Since the bottom edged of the dame is compressed in the longitudinal direction (along the conduit), it is assumable that the steel shell structure and the reinforcement of the conduit are also axially compressed and inactive under the action of hydrostatic pressure in the conduit [2–5]. The circumferencial stresses in the steel shell and reinforcement are generated under the hydrostatic load and seasonal temperature variation in air and water.

The tensile (circumferencial) stresses have the determining influence on the strength of the power conduit constructs. Table 2 and Figure 3 give the maximum values of \( \sigma_1 \) for the main load combination at the level of head water (LHW) of 539 m at the minimal and maximal temperatures of air and water in the conduit.

The circumferencial stresses are nonuniformly spread along the conduit perimeter: maximal tensile stresses are formed in the steel shell structure at the most remote points from the bottom edge of the dam while minimal tensile stresses concentrate nearby the bottom edge (see Figure 3). In summer, \( \sigma_1 \) in the upper part of the shell lowers by 10% (Table 2).

### Table 2. Maximal principal tensile stresses \( \sigma_1 \) (MPa) in structural elements of the power conduit under the main combination of loads.

| Structural element          | \( T_{\text{air}}^{\text{min}} = -17.3{}^\circ\text{C} \); \( t_w = 2{}^\circ\text{C} \) | \( T_{\text{air}}^{\text{max}} = +17.9{}^\circ\text{C} \); \( t_w = 12.5{}^\circ\text{C} \) |
|-----------------------------|-------------------------------------------------|-------------------------------------------------|
| Steel shell                 | 132.1                                           | 119.5                                           |
| Inner reinforcing ring      | 80.0                                            | 57.1                                            |
| External reinforcing ring   | 98.2                                            | 71.4                                            |

### Figure 3. Epure of principal stress \( \sigma_1 \) (MPa) in the steel shell structure and inner reinforcing ring in the straight section of the conduit (Figure 1b) at LHW = 539 m, \( T_{\text{air}} = -17.3{}^\circ\text{C} \), \( t_w = 2{}^\circ\text{C} \).

The same distribution of \( \sigma_1 \) is observed in the inner reinforcing ring: the tensile stresses in the sides are not higher than 88% of the maximal values (\( \sigma_1 \) reaches 80 MPa in the top portion of the reinforcing ring); lowest values of the tensile stresses are recorded at the conduit attachment with the bottom edge of the dam. The tensile stresses \( \sigma_1 \) in the external reinforcing ring are higher than in the inner ring by 15–20%.

In cold season when the temperature of ambient air and water lowers (see Figure 2), the tensile stresses in concrete and in reinforcement nearby the outside boundary of the conduit grow. In the steel shell structure, the tensile stresses increase insignificantly. In summer, due to heating of the external concrete ring and the external shell structure, the tensile stresses reduce in the external reinforcement:
σ₁ nearby the outside boundary of the conduit decrease significantly, from 98.2 MPa at $T_{\min}^{\text{air}} = -17.3^\circ\text{C}$ to 71.4 MPa at $T_{\max}^{\text{air}} = +17.9^\circ\text{C}$. The rise in the water temperature in the conduit results in the reduction in the tensile stresses in the steel shell structure.

Sum up, the efficiency of the solution of the quasistatic thermoelastic problems by the method of singular integral equations for piecewise-homogenous media has been illustrated in terms of the stress state calculation for a multi-layered steel-and-concrete power conduit of the real waterworks. The results of the numerical calculations prove their applicability in the analysis of thermal stress state in a composite material with regard to the temperature variation in adjacent media (rock mass, ambient air and water in the conduit). The calculated stresses in the structural elements of the hydropower plant conduit under thermal effects are quantitatively and qualitatively similar to the in situ stress measurements [4, 5].

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