Gamma Cherenkov-transition radiation produced by charged particles at an interface and in a stack of plates

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Abstract. The spectral and angular distributions as well as the total number of photons of gamma-ray Cherenkov-transition radiation (GCTR) in the photon energy region (1÷10) MeV produced by charged particles passing through an interface between two media and a stack of plates are calculated using the formulae of X-ray transition radiation. The concept of formation length for GCTR is discussed.

1. Introduction
It is well known [1] that one can present the dielectric constant and the refraction index of materials in the form $\varepsilon(\omega) = n^2(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega) = 1 + \chi'(\omega) + i\chi''(\omega)$ and $n(\omega) = 1 + \delta(\omega) + i\beta(\omega)$ where $\chi'$ and $\chi''$ are the real and imaginary parts of the susceptibility. In this expressions $\chi'(\omega)$ and $\delta(\omega)$ are connected with the refraction, while $\chi''(\omega)$ and $\Delta(\omega)$ describe the absorption of photons with absorption length $L_{abs} = 1/\mu(\omega) = c/\omega\varepsilon''(\omega)$ where $\mu$ is the linear absorption coefficient. On the other hand, it is known that when a charged particle with energy exceeding some threshold value passes through a medium Cherenkov radiation is produced in the optical region (see [2]) as well as in X-ray region (see [3, 4]) if $n(\omega) > 1$.

Recently in a difficult experiment it has been shown [5] that the refractive index of silicon in the gamma ray region $\sim (1–2)$ MeV is also greater than 1. Using QED calculations on Delbruck scattering the authors of [5] explained this fact and showed that for some other materials also $n(\omega) > 1$ in the larger photon energy interval from 1 MeV up to $\sim$10 MeV. This fact opens new applications in gamma optics, nuclear physics, etc. Taking that these results are true it has been shown [6, 7] that Cherenkov radiation will exist also in the gamma ray region. It is more reasonable to call this last radiation as gamma ray Cherenkov-transition radiation, GCTR, since more correct results on GCTR are obtained using the theory of transition radiation [6, 7]. As it is shown in [6, 7] GCTR also can find wide applications, in particular, GCTR can be used in a much easier way for finding new materials with $n(\omega) > 1$ in gamma region.

Let us note that for the first time the energy losses, in particular, the so called parametric Cherenkov radiation of the particles passing through stratified medium has been considered in the work [8]. Further the parametric Cherenkov radiation has been studied in the long wave region in the works [9] and [10]. Then it has been shown theoretically (see [3, 11, 12]) that the transition radiation (TR) formulae include also the CR, and in certain conditions it is difficult to separate CR from TR. It
has been shown that the radiation produced by relativistic particles at an interface between two media and in stratified media is emitted in X-ray region and is called X-ray transition radiation (XTR).

This work is devoted to the study of GCTR produced in a stack of plates. However, before developing the theory of the GCTR produced by particles passing stratified media it is necessary to consider GCTR produced at an interface of two media and the concept of formation length, \( L_{\text{form}} \), for GCTR, which were not studied in [6, 7].

### 2. Formation lengths of GCTR

Let us first consider the formation length of GCTR in medium. According to [12] the formation length of XTR is a complex magnitude and for relativistic energies is given by the expression

\[
Z_{\text{Med}}(\theta) = \frac{2\pi V}{\omega(\gamma^{-2} + \theta^2 - \chi^2)},
\]

in which \( \theta \) is the radiation angle, \( V \) and \( \gamma = E/mc^2 = 1/\sqrt{1-(V/c)^2} \) are the velocity and relativistic factor of the particle. Taking the module of (1) one obtains for the formation length of GCTR in medium, \( L_{\text{Med}}^{\text{Form}} \), the length at which the GCTR photon is separated from the charged particle

\[
L_{\text{Med}}^{\text{Form}} = |Z_{\text{Med}}| = \frac{2\pi V}{\omega} \frac{1}{\sqrt{(\gamma^{-2} + \theta^2 - \chi^2) + \chi'^2}}.
\]

Therefore, the formation length in vacuum \((\chi' = \chi'' = 0)\) is equal to

\[
L_{\text{Vac}}^{\text{Form}} = \frac{2\pi V}{\omega(\gamma^{-2} + \theta^2)},
\]

which has a maximum at \( \theta = 0 \) equal to \( L_{\text{Vac}}^{\text{Form}} = \lambda\gamma \) (\( \lambda = 2\pi c/\omega \) is the wavelength of the radiation photon) just as the formation length in vacuum of other types of radiation.

As it follows from (2) when GCTR takes place in medium and the condition for CR production \( \gamma^{-2} + \theta^2 - \chi' = 0 \) is satisfied, the formation length of GCTR in medium \( L_{\text{Med}}^{\text{Form}} = 2\pi L_{\text{abs}} \). For GCTR predicted and considered in [6, 7] when the GCTR photons have \( \lambda \approx 0.5 \times 10^{-11} \) cm and \( \gamma \approx 4 \times 10^4 \), \( L_{\text{Med}}^{\text{Form}} \approx 0.08 \) cm, which is less than the absorption length of GCTR in Si equal to a few cm [6, 7], i.e. \( L_{\text{Med}}^{\text{Form}} > L_{\text{abs}} \gg L_{\text{Vac}}^{\text{Form}} \). On the other hand all the total thickness of any GCTR radiator must be less than the radiation length \( X_0 \), i.e. \( L_{\text{rad}} \ll X_0 \). Otherwise, the electron energy will be degraded due to radiation losses. For Si \( X_0 \approx 9.36 \) cm. Therefore, it is reasonable to take the thickness of the single radiator and the total thickness of the stack of plate equal to, say, \( L_{\text{rad}} \approx 0.1 X_0 \approx 1 \) cm. Therefore, for the above parameters for production of GCTR one has \( L_{\text{rad}} < L_{\text{abs}} < L_{\text{Med}}^{\text{Form}} \). These facts result in the following two interesting consequences:

a) It is well known that the formula for XTR from one interface produced between a semi-infinite medium and vacuum it is not obligatory to have a medium with infinite thickness: it is enough to have a medium thicker than a few formation length in the medium, and the validity of the XTR formula for an interface is justified because in contrast to GCTR for XTR \( L_{\text{abs}}^{\text{Form}} \gg L_{\text{Med}}^{\text{Form}} \). For \( \hbar \omega >> (1-10) \) keV and \( \gamma \approx 4 \times 10^4 \) \( L_{\text{abs}}^{\text{Form}} \) is less then a few tens and \( L_{\text{Med}}^{\text{Form}} \) is about a few microns, see, for instance [12]). On the other hand, for the validity of GCTR formula for an interface (see below) requiring to have instead of semi-infinite medium
a radiator thicker than a few formation length one has to take radiator thicker than $X_0$ in
which the velocity and energy will be changed essentially due to radiation losses. Therefore,
the validity of below formula for GCTR from one interface is under question.

b) It is well known that the formula for XTR (or resonance transition radiation, RTR, see, for
instance [12]) produced in a stack of $M$ plates with plate thickness $l_1^{\text{XTR}}$ greater than the
formation length of XTR in medium, $L_{\text{XTR,Med}}^{\text{Form}}$, and distances $l_2^{\text{XTR}}$ between the plates greater
than the XTR formation length $L_{\text{XTR,Med}}^{\text{Form}}$ gives RTR yield equal to the sum of XTR from M
plates. Now using the above said one can show that for a stack of plates the maximal GCTR
yield can be equal to the sum of the GCTR yield from one plate, if the total thickness of the
stack of plate is equal to the thickness of the single plate and if the distance between the
plates of the stack $l_1 > L_{\text{Form}}^{\text{Vac}}$ when there is no interference between GCTR from various
plates of the stack. Indeed, as it has been shown in [7] for GCTR from one layer for the same
case when $\lambda \leq 5 \cdot 10^{-13}$ cm and $\gamma \approx 4 \cdot 10^4$ (see figure 5 of [7]) the dependence of the GCTR
yield upon the radiator thickness is linear around $L_{\text{rad}} \approx 1$ cm, while for $L_{\text{rad}} << 1$ cm this
dependence is quadratic. As a result the GCTR yield from $M$ independent plates with
thickness $l_1$ is equal to that from a single plate with thickness $ML_1$ only if $l_1$ is not much less
than 1 cm, and the GCTR yield from $M$ independent plates will be less less than that from a
single plate with thickness $ML_1$ if $l_1$ is much less than 1 cm. Such a property of GCTR
produced in a stack of plates is connected with the fact that in the case of GCTR the
absorption length of the produced MeV photons equal to a few cm is much greater than the
absorption length of XCR and XTR photons with energies $\sim (1–10)$ keV. This means that in
contrast to XTR and XCR, in the case of GCTR it is not reasonable to take stack of plates
instead of one plate with the same total thickness. This expectation will be confirmed below
by numerical calculations.

3. GCTR produced at an interface between two media
Now let us derive the expression for spectral-angular distribution of the number of photons of GCTR
produced at an interface between medium and vacuum substituting the values of $\chi'$ and $\chi''$ into the
formulae (1.59) of the work [12] for XTR in small $\theta$ and $\gamma >> 1$ approximation. One obtains

$$
\frac{d^2N_{\text{GCTR}}}{d(\omega)d\theta} = \frac{2\alpha\theta^3}{\pi\hbar\omega} \left(\frac{(\chi'^2 + \chi''^2)}{(\theta^2 + \gamma^{-2})^2 - (\theta^2 + \gamma^{-2})^2 + \chi'^2} \right). 
$$

Let us note that the formula (4) can be derived from the formula for GCTR produced in a single plate
[6, 7] taking $L_{\text{abs}} >> L_{\text{rad}}$, i.e. when the radiator is much thinner than the absorption length. Let us also
note that as it has been shown above the formula (4) has no practical value.

4. Theory of GCTR produced in stack of plates
Following the methods used in [3, 6, 7] we shall consider GCTR produced in a stack of plates with the
help of the formulae of the theory of XTR [11–13], substituting the values of $\chi'$, $\chi''$ for gamma ray
region measured and calculated in [5]. Using the formula (3.15) of the work [12] for the spectral
angular distribution of XTR, one obtains the following formulae for GCTR produced in a radiator
consisting of $M$ plates with thicknesses $l_1$ placed in vacuum at distances equal to $l_2$

$$
\frac{d^2N_{\text{GCTR}}}{d\omega d\theta} \bigg|_{\text{MPI}} = \frac{d^2N_{\text{GCTR}}}{d\omega d\theta} \bigg|_{\text{Int}} F_{\text{MPI}} F_{\text{MPI}}, 
$$

3
where \( \frac{d^2N_{\text{GCTR}}}{d\theta d\phi} \) is GCTR produced at one interface and given by (4) and the factors \( F_p \) and \( F_{mpl} \) which are due to interference in one and \( M \) plates are given by

\[
F_p = (1 - Q^{1/2})^2 + 4Q^{1/2} \sin^2 Y, \\
F_{mpl} = \frac{(1 - Q^{M/2})^2 + 4Q^{M/2} \sin^2 MX}{(1 - Q^{1/2})^2 + 4Q^{1/2} \sin^2 X},
\]

where

\[
Q = \exp(-\frac{\hbar \omega}{\hbar c} \chi'' l_1), \quad Y = \frac{\hbar c l_1}{4 \hbar c} (\theta^2 + \gamma^2 - \chi'), \\
X = \frac{\hbar \omega}{4 \hbar V} [l_1 (\gamma^2 - \theta^2 + \chi') + l_2 (\gamma^2 + \theta^2)].
\]

The purpose of this work is to calculate the 1) spectral, 2) angular distributions and 3) the total number of GCTR photons numerically integrating (5) over \( \theta, \hbar \omega \) and both \( \theta \) and \( \hbar \omega \), respectively.

5. Numerical results and discussion

As in [6, 7] the below results are obtained using the data on \( \chi', \chi'' \) for Si given in [5]. The electron energy is taken equal to \( E = 20 \) GeV. To compare with the results of [6,7] the calculations are carried out for three Si radiators with parameters: the total thickness of the three stratified radiators is taken equal to 1 cm. Taking the number of the Si layers in vacuum or air equal to \( M = 10 \), 100 and 1000, means that \( l_1 = 10^{-1}, 10^{-2} \) and \( 10^{-3} \) cm. Taking a \( l_2 = 1 \) mm for which the distances between the plates are greater than the GCTR formation length in vacuum, and the yields of GCTR are the sums from each plate in the stack, one obtains the radiators thicknesses equal to 1, 10 and 100 cm, respectively.

Figure 1 a), b), c) and d) show the spectral distributions of GCTR photon number produced per electron in a single plate \( (M = 1) \) with \( L_{\text{rad}} = 1 \) cm [7], and in above described three radiators with \( l_1 = 0.1 \) cm \( (M = 10), \), \( l_1 = 0.01 \) cm \( (M = 100), \) c) and \( l_1 = 0.001 \) cm \( (M = 1000), \) respectively.

As it was expected the spectrum (figure 1b) from the first stack of plates, consisting of 10 plates with \( l_1 = 0.1 \) cm, is very similar to that calculated for one thicker radiator with \( L_{\text{rad}} = 1 \) cm [7], because the taken \( l_2 = 1 \) mm is in the average larger than the formation length in vacuum with maximum \( L_{\text{Vac}}^{\text{Form}} = \lambda \gamma^2 \), and GCTR produced in various plates of the stack is summed. The intensities from the other radiators (figure 1c and d) are less, than that from one thicker radiator with thickness equal to the sum of the thicknesses of the plates of the second and third stack of plates. This fact is explained with the radiator thickness dependence of the GCTR given in figure 5 of [7] from which it follows that for their thicknesses of the plates \( (l_1 = 0.01 \) and \( 0.001 \) cm) the GCTR yield is less than 0.01 and 0.001 times.
Figure 1. The spectral distributions of GCTR produced in a single plate a) and the 3 radiators described in the text b), c) and d).

Figure 2. The angular distributions of GCTR produced in a single plate d) and the 3 radiators a), b) and c).
Figure 2 shows the angular distributions of GCTR produced per electron in these same radiators. As it is seen these angular distributions differ from each other and from the angular distribution of GCTR produced in 1 cm thick radiator (figure 3 of [7]) which is explained by the facts that the thicknesses of the plates in the cases a), b), c) and d) are different.

Finally, the total number of GCTR photons are equal to $N_{GCTR} = 1.6, 1.6, 0.8$ and $0.2$, for the radiator a), b), c) and d), respectively.

6. Conclusion
Thus, as it has been discussed above in contrast to XTR [12] and XCR [3], in the case of GCTR it is not reasonable to take stack of plates instead of one plate with the same total thickness. This conclusion is explained by the fact that in the case of GCTR the absorption length of the produced MeV photons equal to a few cm is much greater than the absorption length of XCR and XTR photons with energies ~ (1-10) keV. Of course, one can make the stack consisting of centimetre plates. However, in this case due to the energy losses and production of new electron positron pairs the energy of electrons can be less than the GCTR production threshold and accurate Monte Carlo simulations are necessary.

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