A biased firm in a market with complementary products. A note on the welfare effects

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Abstract
We analyse a duopoly setting with complementary products, in which a firm has a bias about its absolute advantage. We show that the bias can internalize parts of the negative externality that the complementarity of goods creates implying a higher producer’s surplus. Moreover, we analyse additional conditions, which lead to an increase in the consumer’s surplus. Counterintuitively, we show that the presence of a bias can lead to a positive welfare effect.

KEYWORDS
Bias, complementary products, oligopoly

JEL CLASSIFICATION
D21; D62; L13

1 | INTRODUCTION

We analyse a duopoly with differentiated products, which are complements. One of the two firms in this market is biased in its perception of the demand. This means that one firm can either overestimate or underestimate its own absolute advantage. At first glance, intuition suggests that market forces should decrease the viability of the biased firm. However, we show that under certain conditions, producers’ surplus increases compared to the unbiased case. We show that the bias—if not too large—can internalize parts of the negative externality that the complementarity of goods creates. Moreover, we are able to prove that also the consumers’ surplus increases, therefore implying an increase in the total welfare.

Recently, the study of different kinds of biases on the part of economic actors has received increasing attention. On the one hand, this comes from a need to accommodate growing evidence from psychology. For example, Weinstein (1980) and more recently Sharot (2011) point to the fact that people not only tend to be unrealistically...
optimistic about the future but also when updating beliefs put more weight on positive rather than negative outcomes, which then leads to a persistence of optimism. Related to this, Brunnermeier and Parker (2005) find that individuals' utility can be affected by their own assessment of probabilities of future success. On the other hand, Heifetz, Shannon, and Spiegel (2007) show that, contrary to the traditional view that biases should be driven out of the market (Alchian, 1950; Friedman, 1953), biases can persist despite the pressure of evolutionary processes.

Examples of managerial biases are well documented, for example, the impact of biased CEOs on investment decisions (Malmendier & Tate, 2005), on acquisition (Malmendier & Tate, 2008) or on earnings smoothing (Bouwman, 2014).

Roughly two strands of theoretical literature on the effects of biases on firm performance can be distinguished. The first encompasses models that aim at capturing biases of self perception, for example, overconfidence about one’s own capabilities. The second strand focuses on biases about pertinent features of the environment, such as overoptimistic views of market characteristics. Examples of the first strand are organizational economics models, in which actors have biases about their own characteristics. (Gervais & Goldstein, 2007; de la Rosa, 2011; Santos-Pinto, 2008). The second, and more closely related to our model, studies firm performance and behaviour if managers have misperceptions about some fundamentals of the market. This approach is complementary to the one on strategic delegation starting with Vickers (1985) and Fershtman and Judd (1987). There, firm owners can incentivize managers strategically by appropriately designing incentive schemes, thereby signalling a commitment to competitors. Building on this, Englmaier (2010) and Englmaier and Reisinger (2014) develop this literature along two dimensions. First, it is not necessarily always possible to use delegating contracts which are conditioned on more than profits. Second, rather than assuming that the characteristics of incentive contracts are observable to all players, they allow for different types of agents, who differ according to their optimism about the market. Arguably, the assumption that manager types are public knowledge rather than the contracts, appears less restrictive in many cases. Others add to this literature, such as Nakamura (2016), who studies what effects networks have on the outcomes.

Our approach complements both strands of literature in the following ways. On the one hand, we use the study of the effects of the bias within organizations as starting point for having biased firms in market; If agents within an organization can be biased it follows that the observable behaviour of the organization can also be biased. On the other hand, the literature on the strategic use of biased managers is exclusively focussed on substitute products. Even if in our model the bias is not used strategically, it remains relevant to this literature, because we make the goods complementary and study the effects on firms' profits and welfare.

2 | Model

Consider two firms (denoted by \( i \in \{1, 2\} \)), which sell complementary products with quantity \( q_i \) at a price \( p_i \). The demands, as in Dixit (1979), come from a representative consumer with preferences described by the quadratic subutility function:

\[
U(q_1, q_2) = a_1 q_1 + a_2 q_2 - \frac{1}{2} \left( \beta_1 q_1^2 + 2 \gamma q_1 q_2 + \beta_2 q_2^2 \right)
\]

Since we focus on the case of complementary goods, we assume that \( \gamma < 0 \). All other parameters are positive. Additionally, \( \gamma^2 < \beta_1 \beta_2 \) must be satisfied to ensure strict concavity of the utility. The consumer maximizes \( q_0 + U(q_1, q_2) \) subject to the budget constraint \( p_1 q_1 + p_2 q_2 + q_0 \leq m \) with \( q_0 \) being the numéraire commodity and \( m \) the income. The solution to the consumer's problem implies that the firms face the linear inverse demand functions:

\[
p_1 = a_1 - \beta_1 q_1 - \gamma q_2
\]

\[
p_2 = a_2 - \beta_2 q_2 - \gamma q_1
\]
The parameter $\alpha_i$, with nonidentical products, can be interpreted as the absolute advantage in the demand for firm $i$.\(^1\)

We assume w.l.o.g. that production is performed at zero marginal cost and therefore firms’ profits are simply $\pi_i = p_i q_i$.

Firm 2 has a bias on its absolute advantage $\alpha_2$, which means it believes it to be $\hat{\alpha}_2 = \alpha_2 + b$. Here, $b$ can be any number as it describes a positive or negative bias. In this context, we assume, in the sense of Morris (1995), that both firms agree to disagree on the bias $b$. In this case, the best response functions for the firms are:

$$BR_1(q_2) = \frac{\alpha_1 - \gamma q_2}{2\beta_1} \tag{4}$$

$$BR_2(q_1) = \frac{\hat{\alpha}_2 - \gamma q_1}{2\beta_2} \tag{5}$$

**Lemma 1** In equilibrium, firms produce

$$q_1^* = \frac{2\alpha_1\beta_2 - \gamma (\alpha_2 + b)}{4\beta_1\beta_2 - \gamma^2} \tag{6}$$

$$q_2^* = \frac{2(\alpha_2 + b)\beta_2 - \gamma \alpha_2}{4\beta_1\beta_2 - \gamma^2} \tag{7}$$

**Proof.** It is straightforward to see that (6) and (7) are the coordinates of the intersection of the best responses (4) and (5).

Prices and firms’ profits are obtained after substitution of the previous equations. In general, linear demands can lead to negative quantities and/or prices. We proceed with the analysis only for ranges of the parameters that ensure positive outcomes.

The effect of an increase in the bias $b$, in equilibrium, is synthesized in the following corollary to Lemma 1:

**Corollary 1**

$$\frac{\partial q_1^*}{\partial b} > 0 \quad \frac{\partial q_2^*}{\partial b} > 0 \quad \frac{\partial p_1(q_1^*, q_2^*)}{\partial b} > 0 \quad \frac{\partial p_2(q_1^*, q_2^*)}{\partial b} < 0$$

**Proof.** Differentiating (6) and (7), the effects on the quantities are immediate. For the prices, using (2) and (3):

$$\frac{\partial p_1(q_1^*, q_2^*)}{\partial b} = \beta_1 \frac{\partial q_1^*}{\partial b} - \gamma \frac{\partial q_2^*}{\partial b} = -\frac{\beta_1 \gamma}{4\beta_1\beta_2 - \gamma^2} > 0$$

$$\frac{\partial p_2(q_1^*, q_2^*)}{\partial b} = \beta_2 \frac{\partial q_2^*}{\partial b} - \gamma \frac{\partial q_1^*}{\partial b} = -\frac{2\beta_1 \beta_2 - \gamma^2}{4\beta_1\beta_2 - \gamma^2} < 0$$

where the inequalities follow from the assumption $\gamma^2 < \beta_1\beta_2$.

\(^1\)The observation is from Dixit (1979, p. 26). A more detailed interpretation of all the parameters can be found in Vives (2001, p. 145).

\(^2\)Alternatively, one could model a bias on the slope of the inverse demand, $\beta_2$. Nothing would substantially change in the analysis.
If a positive bias increases the biased firm will produce more, because it erroneously thinks its absolute advantage is larger than it is. Accordingly, an increase in the severity of a negative bias (b becoming more negative) will make the biased firm produce less. For the unbiased firm 1 a positive bias of firm 2 will increase production, because of the increased production of 2. Similar reasoning holds for the effect on the prices. Note that for firm 1 there are two opposite effects of an increase in the bias on the price. On the one hand, a higher production of firm 2 shifts firm 1’s demand upwards, which ceteris paribus increases the price $p_1$. On the other hand, also firm 1 produces more, which decreases the price. The corollary says that the former effect dominates the latter. The effects on the profits of both firms are the following.

**Proposition 1**

(i) $\pi_1^*$ is an increasing function in $b$;

(ii) $\pi_2^*$ is an inverted-U-shaped function in $b$.

*Proof.*

(i) For the profit of firm 1:

$$\frac{\partial \pi_1(q_1^*, q_2^*)}{\partial b} = \frac{\partial p_1(q_1^*, q_2^*)}{\partial b} q_1^* + p_1(q_1^*, q_2^*) \frac{\partial q_1^*}{\partial b}$$

The conclusion follows from Corollary 1.

(ii) For the profit of firm 2:

$$\pi_2(q_1^*, q_2^*) = p_2(q_1^*, q_2^*) q_2^* - 2\beta_1(2\beta_1\beta_2 - \gamma^2)b^2 + \gamma^2(2\beta_1\alpha_2 - \gamma\alpha_1)b + \beta_2(4\beta_2^2\alpha_2^2 - 4\alpha_4 \alpha_2 \beta_1 \gamma + \alpha_1^2 \gamma^2)$$

$$= \frac{(4\beta_1\beta_2 - \gamma^2)^2}{(4\beta_1\beta_2 - \gamma^2)^2}.$$  

Inspection of the numerator suggests that $\pi_2(q_1^*, q_2^*)$ is a concave parabola in $b$.

The first part says that the unbiased firm 1 is better off for a positive bias of the competitor. The second part implies that if the bias becomes too large in either direction the biased firm will be worse off.

As mentioned in the introduction, our findings are related to recent literature. There the analysis is on substitute goods. For instance, Engmaier and Reisinger (2014) found that when a firm in a duopoly strategically delegates production to a positively biased manager, the quantity increases. The best response of the competitor is then to also hire the same type of manager, increasing production as well. These incentives lead to a prisoners’ dilemma resulting in higher quantities and lower profits. Even though in our model the choice of the biased firm is not strategic, the same result would apply if products were substitutes: in this case an increase in the quantity is detrimental for profit. Conversely, the bias through the complementarity of goods can be beneficial in terms of profit as will be shown in the next proposition. However, not only is it possible for firm 2 to be better off, but we can also give conditions for which there is an increase in the consumers’ surplus. This main result is summarized in the following proposition.

**Proposition 2** There exists a bias $\tilde{b} > 0$ such that both firms’ profits in $(0, \tilde{b})$ are higher than in the unbiased case.

Moreover, if $\gamma < 0$ is relatively small in absolute value also the consumer’ surplus is higher compared to the unbiased case.

*Proof.* Since the profit of firm 1 is always increasing in $b$ for $\gamma < 0$ and the profit of firm 2 is a concave parabola (Proposition 1), we only need to prove that the maximizer $\tilde{b}$ of the parabola of $\pi_2^*$ is positive. We have:

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2 Observe that for the unbiased firm 1 an increase in the quantity of the biased firm due to the increase in the bias is detrimental if products are substitutes, i.e. $\gamma > 0$. In fact, recall from the proof of Proposition 1 that $\frac{\partial \pi_1(q_1^*, q_2^*)}{\partial \gamma} = \frac{\partial p_1(q_1^*, q_2^*)}{\partial \gamma} q_1^* + p_1(q_1^*, q_2^*) \frac{\partial q_1^*}{\partial \gamma}$. From the proof of Corollary 1 we have that $\frac{\partial \pi_1(q_1^*, q_2^*)}{\partial \gamma} < 0$ for $\gamma > 0$ and from Equation (6) it is evident that $\frac{\partial p_1(q_1^*, q_2^*)}{\partial \gamma} < 0$ for $\gamma > 0$. Hence, it necessarily follows that $\frac{\partial \pi_2(q_1^*, q_2^*)}{\partial \gamma} < 0$ for $\gamma > 0$.
Then, $\hat{b}$ is positive if $\gamma < \frac{2 \beta_2}{\beta_1}$ which is satisfied because $\gamma < 0$.

It is always possible to find a range of the parameters that ensures non-negative prices and quantities. In fact, for $\gamma < 0$ and $b > 0$ the quantities are always positive. Price $p_1$ is an increasing function in $b$. The only possibility would be to always observe a $p_2 < 0$ because it is decreasing in $b$. However, the expression is:

$$ p_2(q_1^*, q_2^*) = \frac{2a_2\beta_1\beta_2 - 2b\beta_1\beta_2 - a_1\beta_2 \gamma + b\gamma^2}{4\beta_1\beta_2 - \gamma^2} $$

and given that the only negative term in the numerator is $-2b\beta_1\beta_2$, one can always have a $b$ such that also $p_2(q_1^*, q_2^*) > 0$.

To show that there is a possible range of $b$, in which the consumer is also better off, observe that the consumer’s surplus is:

$$ \text{CS}(\cdot, \cdot, b) = U(q_1^*, q_2^*) - p_1(q_1^*, q_2^*)q_1^* - p_2(q_1^*, q_2^*)q_1^* $$

Differentiating yields

$$ \frac{\partial \text{CS}(\cdot, \cdot, b)}{\partial b} = \frac{a_1\gamma^3 + \beta_1 (a_2 + b) (4\beta_1\beta_2 - 3\gamma^2)}{(4\beta_1\beta_2 - \gamma^2)^2} $$

which, for a given $b > 0$, is positive for a relatively small (in absolute value) $\gamma < 0$.

The consumer surplus is affected by the interplay between changing prices and quantities. If the two goods are not too strong complements, any positive bias is good for the consumer. However, recall that the unbiased firm increases both quantity and price for complementary goods. Since this effect is stronger the larger the degree of complementarity, at some point the effect of the price increase is stronger than the effect of the quantity increase.

3 | CONCLUSION

We analyse the effects of a biased firm in a duopoly. If there is no bias present, the strategic nature of the competition makes both firms underproduce. If the bias is positive but relatively small, and the goods are mildly complementary, this acts as an internalization of the externality that the two firms exert on one another. This is always an advantage for the unbiased firm. However, a too large bias will make the biased firm worse off, because then the detrimental effect of a decreased price outweighs the advantageous effect of a higher quantity.

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