Velocity shift of surface acoustic waves due to interaction with composite fermions in a modulated structure

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We study the effect of a periodic density modulation on surface acoustic wave (SAW) propagation along a 2D electron gas near Landau level filling \( \nu = 1/2 \). Within the composite fermion theory, the problem is described in terms of fermions subject to a spatially modulated magnetic field and scattered by a random magnetic field. We find that a few percent modulation induces a large peak in the SAW velocity shift, as has been observed recently by Willett et al. As further support of this theory we find the dc resistivity to be in good agreement with recent data of Smet et al.

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The interaction of surface acoustic waves (SAW) with a 2D electron gas (2DEG) in a strong magnetic field was intensively studied during the last few years. It was found that the velocity shift \( \Delta v/v \) and the absorption coefficient \( \Gamma \) of the SAW show a rich structure reflecting the magnetooscillations and Hall quantization of the transport in a 2DEG. The SAW measurements provide a very efficient method of studying the (frequency- and momentum-dependent) conductivity of the 2DEG. The power of this method was demonstrated in a study near half-filling (\( \nu = 1/2 \)) of the lowest Landau level. It was found that the SAW velocity exhibits a pronounced minimum near \( \nu = 1/2 \), with a resonance structure appearing at very high SAW frequencies. These results are in good quantitative agreement with theoretical calculations based on the

Recently, new experimental results on SAW interaction with a 2DEG in the presence of a periodic array of parallel gates (grating) were obtained. It was found that a weak modulation potential, with a wave vector parallel to that of the SAW, changes the behavior of \( \Delta v/v \) near \( \nu = 1/2 \) drastically, inducing a large peak similar to those observed at quantum Hall states (\( \nu = 1, 1/3 \)).

The aim of this paper is to show that a large maximum in \( \Delta v/v \) is precisely what the CF theory predicts in the presence of a periodically modulated electric potential. In analogy to the Weiss oscillation phenomenon in low magnetic fields, even a weak modulation produces a large correction to the effective conductivity, which is observed in the SAW measurements.

We consider the SAW (frequency \( \omega \), wave vector \( \mathbf{q} \), velocity \( v_\omega = \omega/\omega = 2.8 \times 10^5 \text{ cm/s in GaAs} \) interacting with a 2DEG of density \( n \) in the presence of a periodic potential (and, consequently, density) modulation with a wave vector \( \mathbf{p} \). Near half-filling, the system is described in terms of the CF’s with Fermi wave vector \( k_F = (4\pi n)^{1/2} \) moving in the reduced effective magnetic field \( B_{\text{eff}} = B - 2hc/e \). At low temperatures, the main source of CF scattering is the random magnetic field due to the electron (or, equivalently, CF) density inhomogeneity produced by the impurity random potential. Similarly, the main role of the grating will be in creating a magnetic field modulation \( \Delta B(\mathbf{r}) \) related to the density modulation as \( \Delta B(\mathbf{r}) = -(2hc/e)\Delta n(\mathbf{r}) \). The direct effect of the scalar potential modulation being smaller by a factor \( (p/k_F)^2 \ll 1 \). A general formalism for calculating the velocity shift and the absorption coefficient in the presence of periodic modulation has been recently developed. The results are expressed in the form

\[
\Delta v/v = (\alpha^2/2)\Re(1 + \delta\sigma_{\text{eff}}(\mathbf{q}, \omega)/\sigma_m)^{-1};
\]

\[
\Gamma = -q(\alpha^2/2)\Im(1 + \delta\sigma_{\text{eff}}(\mathbf{q}, \omega)/\sigma_m)^{-1},
\]

where \( \alpha^2/2 \) is the piezoelectric coupling constant, \( \sigma_m = \varepsilon v_m/2\pi \), and \( \varepsilon \) is an effective dielectric constant of the background. To present the expression for the effective conductivity \( \sigma_{\text{eff}} \), we introduce tensorial conductivities \( \delta_{\mu\nu} \), with an index \( s \) referring to the spatial Fourier component with the wave vector \( \mathbf{q}_s = \mathbf{q} + s\mathbf{p} \) (\( s = 0, \pm 1, \pm 2, \ldots \)), and the longitudinal conductivity \( \sigma_{x'x''} = (\mathbf{q}_s/g_s)\delta_{ss'}(q_{s''}/g_{s''}) \). The superscript \( e \) serves to distinguish the electronic conductivities from those of the CF’s, and the hat denotes the matrix structure in the \( xy \) plane. We will assume the density modulation to be of the single-harmonic form, \( \Delta n(\mathbf{r}) = \eta n \cos \mathbf{p} \mathbf{r} \), where \( \eta \ll 1 \), and with a wave length much shorter than that of the SAW, \( p \gg q \). Then \( \sigma_{\text{eff}}(\mathbf{q}, \omega) = \sigma^e(\mathbf{q}, \omega) + \delta\sigma_{\text{eff}}(\mathbf{q}, \omega) \),
with
\[
\delta\sigma_{\text{eff}}(\mathbf{q},\omega) = \sigma_{0,0}^{(2)} - \frac{\sigma_{0,1}^{(1)}\sigma_{1,0}^{(1)} + \sigma_{0,-1,0}^{(1)}}{\sigma_0(p,\omega) - i(q/p)\sigma_m} .
\]

Here the upper index \((i)\) refers to the \(i\)-th order of the expansion in \(\eta\); \(\sigma^{(i)}(q,\omega) = \sigma_{0,0}^{(i)}\) and \(\sigma^{(i)}(p,\omega) = \sigma_{1,1}^{(i)}\) being the longitudinal conductivities at \(\eta = 0\). In the sequel, we will drop the superscript \((0)\), keeping \((1)\) and \((2)\) only.

In the random phase approximation, the resistivity tensor of the electrons is related to that of the CF’s via
\[
\rho_{s,s'} = (2h/e^2)\epsilon\delta_{s,s'} + \rho_{ss'},
\]
where \(\epsilon\) is the antisymmetric tensor with \(\epsilon_{xy} = -\epsilon_{yx} = 1\). This relation yields
\[
\begin{align*}
\hat{\rho}_{00}^{(2)} &= -\hat{\rho}_{00}\hat{\rho}_{00}^{(2)}\hat{\rho}_{00} + 2\hat{\rho}_{00}\hat{\rho}_{01}^{(1)}\hat{\rho}_{10}^{(1)}\hat{\rho}_{00}, \\
\hat{\rho}_{01}^{(1)} &= -\hat{\rho}_{00}\hat{\rho}_{01}^{(1)}\hat{\rho}_{11}, \quad \hat{\rho}_{10}^{(1)} = -\hat{\rho}_{11}\hat{\rho}_{10}^{(1)}\hat{\rho}_{00},
\end{align*}
\]
where the grating-induced corrections to the resistivity tensor \(\hat{\sigma}_{ss'}^{(i)}\) by
\[
\hat{\rho}_{00}^{(2)} = -\hat{\rho}_{00}\hat{\rho}_{00}^{(2)}\hat{\rho}_{00} + 2\hat{\rho}_{00}\hat{\rho}_{01}^{(1)}\hat{\rho}_{10}^{(1)}\hat{\rho}_{00}, \quad \hat{\rho}_{01}^{(1)} = -\hat{\rho}_{00}\hat{\rho}_{01}^{(1)}\hat{\rho}_{11}, \quad \hat{\rho}_{10}^{(1)} = -\hat{\rho}_{11}\hat{\rho}_{10}^{(1)}\hat{\rho}_{00}.
\]

To evaluate the CF conductivities, we use the Boltzmann equation for the distribution function \(F(\mathbf{r},\mathbf{n},t)\) in the presence of an electric field \(\mathbf{E}(\mathbf{r}) = \mathbf{E}_c\exp(i\mathbf{q}\mathbf{r})\),
\[
\left\{-i\omega + v_F n \nabla + [\omega_c + \Delta\omega_c(\mathbf{r})] \frac{\partial}{\partial \phi} - C\right\} F = v_F e n E,
\]
where \(n = (\cos \phi, \sin \phi)\) determines the direction of the momentum, \(\omega_c = eB_{eff}/mc\) is the cyclotron frequency, \(\Delta\omega_c(\mathbf{r}) = e\Delta B(\mathbf{r})/mc\), and \(C\) is the collision integral describing scattering by the random magnetic field with a transport time \(\tau\). To simplify the calculation, we assume the low-momentum SAW field condition, \((q/p)^2/(2\omega\tau) \ll 1\), where \(l\) is the CF mean free path (which is marginally valid for \(\omega = 2\pi \cdot 300\) MHz, the lowest frequency tested in the experiment [6]). This will be sufficient to explain this experiment, where no essential dependence on \(q\) was observed anyway. Under the condition assumed, we can set \(q = 0\) when calculating the conductivities entering the r.h.s. of Eqs. (6), (7). In particular, \(\hat{\sigma}_{00}\) is then approximated by the Drude form,
\[
\hat{\sigma}_{00} = \left(\rho_{00}\right)^{-1} = \frac{\sigma_0(\tilde{\tau}/\tau)}{1 + \tilde{S}^2} \begin{pmatrix} 1 & -\tilde{S} \\ \tilde{S} & 1 \end{pmatrix} ;
\]
\[
\sigma_0 = (e^2/2h)k_F l; \quad \tilde{S} = \omega_c/\tilde{\tau}; \quad \tilde{\tau}^{-1} = \tau^{-1} - i\omega .
\]
Furthermore, the grating-induced contributions \(\hat{\rho}_{00}^{(2)}\), \(\hat{\rho}_{01}^{(1)}\) can be found via the method developed in [6,10] as
\[
\hat{\rho}_{00}\hat{\rho}_{00}^{(2)}\hat{\rho}_{00} = \frac{\eta^2}{2} \left(\frac{2h}{e^2}\right)^2 \hat{\epsilon}\sigma(\mathbf{p},\omega)\hat{\epsilon} ;
\]
\[
\hat{\rho}_{00}\hat{\rho}_{01}^{(1)}\hat{\rho}_{00} = -\frac{h}{e^2}\hat{\epsilon}\sigma(\mathbf{p},\omega) ; \quad \hat{\rho}_{10}^{(1)}\hat{\rho}_{00} = -\frac{h}{e^2}\hat{\epsilon}\sigma(\mathbf{p},\omega)\hat{\epsilon} .
\]

We choose \(\mathbf{p} \parallel \mathbf{e}_x\) and assume first that the SAW wave vector (and the SAW electric field) is orthogonal to \(\mathbf{p}\), i.e. \(\mathbf{q} \parallel \mathbf{e}_y\). Using Eq. (10), we find that the two terms in the expression for \(\rho_{00}^{(2)}\), Eq. (11), cancel each other, while \(\hat{\rho}_{01}^{(1)} = \hat{\rho}_{10}^{(1)} = \hat{\eta}(h/e^2)\epsilon\). Substituting this into Eqs. (6)–(10) and using \(\omega_c, \tau, \omega \tau \ll k_F l\), we get
\[
\delta\sigma_{\text{eff}} = \frac{\eta^2}{2} \left[ \frac{\sigma_{11,yy}^{(1)} - \sigma_{11,yy}^{(2)}}{\sigma_{11,xx}^{(1)} - i(q/p)\sigma_m} \right] = \frac{\eta^2}{2} \rho_{11,yy} - i(q/p)(2h/e^2)^2\sigma_m
\]
(10)
(we used also \((q/p)\sigma_m\rho_{11,xx} \ll 1\) in the second line). This finally yields \(\sigma_{\text{eff}}\) in terms of the transverse resistivity of the CF’s in the absence of the grating,
\[
\sigma_{\text{eff}}(\mathbf{q},\omega) = (e^2/2h)\rho_{xx}(\mathbf{q},\omega)
\]
\[
\quad + \frac{\eta^2}{2} \rho_{yy}(\mathbf{p},\omega) - i(q/p)(2h/e^2)^2\sigma_m .
\]
(11)

The conductivity tensor \(\sigma_{\mu\nu}(\mathbf{p},\omega)\) was calculated in Ref. [6], leading to the following result for the transverse resistivity in terms of Bessel functions:
\[
\rho_{yy}(\mathbf{p},\omega) = (iQS/4\sigma_0)[J_{-1} - J_{+1}] + J_{+1},
\]
\[
J_{-1} = \frac{J_{+1}}{1 + 2\beta},
\]
\[
J_{+1} = \frac{J_{+1}}{1 + 2\beta},
\]
where \(Q = qR_c\) with \(R_c = v_F/\omega_c\) being the cyclotron radius, \(S = \omega_c\tau\), and \(T = \omega/\omega_c\). Eq. (12) is valid for random magnetic field scattering (\(\beta = 1\)), as well as for isotropic potential scattering (\(\beta = 0\)).

Using Eqs. (10), (11), (12), and (13), one can evaluate the SAW velocity shift and the absorption coefficient as functions of the effective magnetic field. The velocity shift is plotted in Fig. 1 for the experimentally relevant values of the density \(n\), SAW frequency \(\omega\), and the grating period \(d = 2\pi/p\). We also used the typical experimental values of the CF effective mass \(m = 0.88m_e\) \((m_e\) being the free electron mass), the CF transport relaxation time \(\tau\) \(= 40\) ps, and the conductivity parameter \(\sigma_m = 0.6 \times 10^5\) cm/s

As is seen from Fig. 1, a weak (3-5%) modulation produces a large peak in \(\Delta v/v\), with an amplitude of the order of \(1 \times 10^{-4}\) (the scale is set by \(\alpha^2/2 = 3.2 \times 10^{-4}\) in GaAs), i.e. approximately of the same magnitude as the maxima observed at the quantum Hall fractions \((\nu = 1, 1/3\) in agreement with experiment [10,22]. The reason for a weak modulation to be sufficient to produce such a drastic effect is as follows. The resistivities entering Eq. (10) are of order of \(\sigma_0^{-1}\), so that the second term becomes comparable to the first one at \((\eta k_F l)^2/2 \sim 1\),
which with $k_{\text{F}}l \sim 50$ yields $\eta \sim 0.03$. Such an enhancement is familiar from the Weiss oscillations effect in low magnetic fields. For $\mathbf{q} \parallel \mathbf{p} \parallel \mathbf{e}_x$, all the indices $y$ are replaced by $x$ in the first line of (11), yielding a negligibly small correction $\delta\sigma_{\text{eff}}$, in agreement with experiment.

We comment now on the role played by the scattering mechanism. In Fig. 3 the SAW velocity shift is plotted for scattering by a random magnetic field and by a random potential, respectively, using the same value of the transport time $\tau$ and keeping all other parameters fixed. While the amplitude of the peak is approximately the same in both cases, the shape is very different. For random potential scattering, the peak is considerably sharper, with an oscillatory structure, not observed in Fig. 1. In contrast, for random magnetic field scattering, we find a broad and smooth peak, the shape and width of which are in agreement with the experiment [6].

There are some differences between the experimental and theoretical results, for which the present theory does not seem to account. The theoretical value of $\Delta v/v$ in the center of the peak decreases with $\eta$, while it increased in the experiment. Also, the theory predicts that the peak width increases with $p$ (at $p \gg q$), while in the experiment the width was weakly dependent on $p$ and $q$. We think that the latter feature does not have a deep meaning and may only be valid in a restricted range of the parameters.

Finally, the grating-induced correction to the $dc$ conductivity is given by the first term in Eq. (3), $\sigma_{\text{eff}}^{(2)}$ (in the limit $q \to 0$ and then $\omega \to 0$), which yields, according to the above calculation,

$$\sigma_{\text{eff}}^{(2)} = (\eta^2/2)\hat{\sigma}(\mathbf{p}',0). \tag{13}$$

Since $\sigma_{\text{yy}}(\mathbf{p},0)$ is the only non-zero component of $\hat{\sigma}(\mathbf{p},0)$, Eq. (13) implies a correction to the $xx$-component of the macroscopic resistivity tensor $\rho_{\text{dc}} = (\hat{\sigma}_{\text{xx}})^{-1}$ measured in $dc$ experiments [13]

$$\rho_{\text{dc},xx} = \sigma_0^{-1} + 2\eta^2(h/e^2)^2\sigma_{\text{yy}}(\mathbf{p},0) \tag{14}$$
An experiment on \( dc \) transport near \( \nu = 1/2 \) in a modulated structure was performed recently by Smet et al.\(^4\). In Fig. 4 we present the theoretical result, Eq. (14), for the parameters of the sample A (Fig. 1 of \([4]\)). The modulation amplitude is estimated from the comparison with experiment as \( \eta = 0.026 \), which is in good agreement with \( \eta = 0.032 \) found from the fit of the low-field Weiss oscillations (also presented in Fig. 1 of \([4]\)) to the theoretical formula \([4]\) taking into account the scattering by the smooth random potential. As is seen from Fig. 4, the theoretical results reproduce well the width of the grating induced minimum in \( \rho_{dc,xx} \) around \( \nu = 1/2 \). The commensurability oscillations are again (almost) washed out due to the scattering by the random magnetic field. Only the first minimum is weakly developed at \( B_{eff} \approx 0.4T \), possibly corresponding to the shoulders observed experimentally. The difference between the theoretical and the experimental curve at small \( B_{eff} \) (V-shaped minimum vs. plateau) is probably due to open CF orbits \([4]\), which are not taken into account by our perturbative-in-\( \eta \) calculation and should produce an additional positive magneto-resistance in a range \( |B_{eff}| < B_c = \eta(2h\nu/e) \approx 0.4T \).

In conclusion, we have studied the propagation of SAW interacting with a 2DEG near \( \nu = 1/2 \) in the presence of a weak density modulation. Within the CF theory, the problem is described in terms of fermions subject to a modulated magnetic field and scattered by a random magnetic field. Using the Boltzmann equation approach, we have calculated the SAW velocity shift and found that it exhibits, at modulation strength \( \sim 3\% \), a pronounced maximum, with amplitude of order of the piezoelectric coupling constant, in agreement with the experiment \([3]\). The calculated correction to the \( dc \) resistivity describes reasonably well the magnitude and the width of the minimum of \( \rho_{xx} \) near \( \nu = 1/2 \) observed in \([4]\).

While working on this project, we became aware of the preprint \([7]\), where the same problem was addressed. The authors of \([7]\) arrived at a formula similar to our Eq. \((11)\), and then proceeded via numerical solution of the Boltzmann equation for isotropic scattering. Our result for the \( dc \) case, Eq. \((14)\), is however different from that obtained in \([7]\).

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11. Theoretically, the value of \( \sigma_m \) changes between \( 0.35 \times 10^6 \text{cm/s} \) for \( qD \ll 1 \) and \( 0.65 \times 10^6 \text{cm/s} \) for \( qD \gg 1 \), where \( D \) is the distance between the 2D gas layer and the sample surface; see S. H. Simon, Phys. Rev. B 54, 13878 (1996). We expect that some additional increase in \( \sigma_m \) may appear due to the screening by the gates producing the modulation. We have checked that results very similar to those shown in Figs. 1–3 are obtained with \( \sigma_m = 0.35 \times 10^6 \text{cm/s} \) and \( \sigma_m = 1.00 \times 10^6 \text{cm/s} \).
12. Note that for the relatively low frequency assumed in Fig. 1 the minimum in \( \Delta \nu/v \) at \( \eta = 0 \) is very weak.
13. We emphasize that the transport coefficient \( \sigma_{\nu y}(p,0) \) in \([4]\) describes the response to a pure electric field and does not include the diamagnetic contribution \( \propto p^2/i\omega \) (which would diverge for \( \omega \to 0 \)).
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