A Synthesis Approach to Output Feedback MPC for LPV Model With Bounded Disturbance

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ABSTRACT The model with polytopic parametric uncertainty and bounded disturbance is controlled by the approach named dynamic OFRMPC (Output Feedback Robust Model Predictive Control). A key knob for control performance and region of attraction for this approach is the selection of Lyapunov matrix. A Lyapunov matrix, which does not have structural restriction, is proposed. In the ICCA (Iterative Cone Complementary Approach), which is invoked in optimizing the control law parameters, the starting up steps are designed as a variant CCA (Cone Complementary Approach). ICCA designs an outer loop, over CCA, for searching the minimum cost bound, while the variant CCA omits the outer loop by adding the cost bound in CCA objective function. This starting up can reduce the computational burden. The suboptimal dynamic OFRMPC (where CCA is avoided) is discussed, and a previous approach is re-formulated. A numerical example is given to show the advantages of the proposed approach.

INDEX TERMS Output feedback, model predictive control, polytopic uncertainty, bounded disturbance.

I. INTRODUCTION

In the industrial circle, MPC (Model Predictive Control) has been proven as the most successful among advanced control techniques (see e.g., [1]–[6]). Other advanced control techniques such as sliding mode control (see [7]) and adaptive control (see e.g., [8]–[10]) have also attracted lots of attentions. Some works have combined adaptive control with MPC and applied the approach to highly nonlinear systems such as Brain-Machine Interface (BMI) system (see e.g., [11], [12]). Based on the mathematical model of the system, MPC solves an optimization problem at each control interval. By carefully designing a penalty and a constraint on the terminal state, stability of the closed-loop system has been ensured [13], [14]. LPV (Linear Parameter Varying) model, where a set of linear sub-models are utilized to cover a sufficiently large region around equilibrium, has been widely utilized in MPC literature (see e.g., [15]–[19] theoretically and [20], [21] practically). In this paper, as usual, LPV model refers to the model with polytopic parametric uncertainty, which takes a linear form but can represent a wide variety of nonlinear and/or uncertain systems. The work in [22] has provided an LMI (Linear Matrix Inequality) based min-max MPC approach to handle system with model parametric uncertainty, which is extended in several works with improvements on computational efficiency (see e.g., [21]) or control performance (see e.g., [23]). With an addition of unknown disturbance on linear model, a variety of MPC works are available, where QB (Quadratic Boundedness, see [24], [25]) technique and the constraint tightening approach (see e.g., [26]–[29]) seem to be mostly promising.

When the system state is not measured, the output feedback control may utilize the measured output of the system to estimate the state (see e.g., [9], [10], [30], [31]), and this scheme applies to MPC (see e.g., [32]–[35]). The existence of state estimation complicates the design in several aspects, e.g., increase of the computational burden (see e.g., [33]), ruin of the recursive feasibility (see e.g., [32]). Regarding the output feedback MPC for LPV model with bounded disturbance, there are a few works available (see e.g., [25], [36]); the notion of QB, which is concise for specifying the invariance and stability properties for the system with additive bounded disturbance, is introduced into dynamic OFRMPC. In [25], a dynamic OFRMPC approach is proposed with guaranteed recursive feasibility and closed-loop stability, and an ICCA (Iterative Cone Complementary Approach) is proposed to find the controller solution. The work [36] improves [25] in several aspects, including the utilization of full
Lyapunov matrix, the treatment of physical constraints with norm-bounding technique, and the introduction of multi-step approach.

In [25], a non-diagonal block of the Lyapunov matrix is a fixed transformation of the diagonal block. In [36], it requires a non-diagonal block of the Lyapunov matrix to be invertible. In order to remove these limitations and improve the control performance, in the present paper we develop a dynamic OFRMPC approach without restrictions on the structure of Lyapunov matrix, for LPV model with bounded disturbance. The proposed approach fuses the merits in [36] and previous works, with further improvements. It is shown that the newly proposed optimization problem can cover the one in [36] via appropriate congruence transformations. Then, for handling the nontrivial optimization systematically with less computational burden, we propose an ICCA which includes the previous one and has a potential to reduce the computational burden without sacrifice of the control performance. Finally, by an appropriate congruence transformation and the dilation technique, the suboptimal solution to dynamic OFRMPC is given, which can have considerably lighter computational burden. It is shown that the previous approach in [37] can be re-formulated according to this suboptimal solution.

The main contribution of the paper can be further detailed as follows.

- In [36], the dimension of the controller state \( x_c \) required to be equal to the dimension of the true state \( x \). This limitation will be removed by utilizing the general structured Lyapunov matrix and applying some proper matrix transformations. Moreover, this paper will also include the approach in [25].
- The work [36] uses CCA (Cone Complementarity Approach) to handle the mutual inverse positive-definite matrices in the optimization problem, and invoke an outer loop over CCA in order to decrease the cost bound. We call this CCA with outer loop as ICCA. In the starting up steps of ICCA, this paper will use a variant CCA. This variant CCA modifies the objective function of CCA, i.e., adds the cost bound to the objective function of CCA. The completion of variant CCA usually yields solution close to ICCA. Then, the major role of ICCA is to guarantee the recursive feasibility. Therefore, introducing the variant CCA will reduce the computational burden.
- By utilizing a dilation technique, the optimization problem will be solved in a suboptimal manner. This mainly concerns pre-specifying (once off-line, or inherited from the previous control interval) a part of the control law parameters. The approach in [37] is re-formulated with relaxation.

The structure of this paper is organized as follows. In Section II, the problem of dynamic OFMPC with general structured Lyapunov matrix is formulated. In Section III, both near-optimal and sub-optimal dynamic OFRMPC approaches are proposed with guaranteed recursive feasibility and closed-loop stability. In Section IV, the proposed approaches are applied to a fuel cell model. Section V gives the conclusion.

**Notations:** For any vector \( x \) and matrix \( W \), \( \|x\|^2_W := x^T W x \). \( x[i](k) \) is the value of \( x \) at time \( k + 1 \). \( I \) is the identity matrix with appropriate dimension. \( r_M := \{ \xi \mid |\xi^T M | \leq 1 \} \) denotes the ellipsoid associated with the positive-definite matrix \( M \). An element belonging to a polytope CoS is convex combination of the elements in set \( S \), with all scalar nonnegative combing coefficients summing as 1. The symbol \( \ast \) induces a symmetric structure in a square matrix. A value with superscript \( * \) means that it is the solution of the optimization problem. The time-dependence of the MPC decision variables is often omitted for brevity.

## II. PROBLEM STATEMENT

Consider the following LPV model:

\[
\begin{align}
x(k+1) &= A(k)x(k) + B(k)u(k) + D(k)w(k), \\
y(k) &= C(k)x(k) + E(k)w(k),
\end{align}
\]

where \( u \in \mathbb{R}^{n_u}, x \in \mathbb{R}^{n_x}, y \in \mathbb{R}^{n_y} \) and \( w \in \mathbb{R}^{n_w} \) are the control input, system state, output and unknown disturbance, respectively. The physical constraints are

\[
|u(k)| \leq \bar{u}, |z(k)| \leq \bar{z}, \quad k \geq 0,
\]

where \( z(k) = C(k)x(k) + E(k)w(k) \in \mathbb{R}^{n_y} \) is the constrained signal (e.g., \( z = x, z = y \)); \( \bar{u} = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_{n_u}]^T \); \( \bar{z} = [\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_{n_y}]^T \); \( \bar{u}_j > 0, j = 1, \ldots, n_u; \bar{z}_j > 0, j = 1, \ldots, n_z \).

The disturbance and model parametric matrices satisfy the following assumptions.

**Assumption 1:** \( \|w(k)\| \leq 1 \) for all \( k \geq 0 \).

**Assumption 2:** (polytope) \( A |B| C |D| E |C| E |k \) \( \in \Omega := \text{Co}([A_1 |B_1| C_1 |D_1| E_1 |C_1| E_1 |k] \mid |i| = 1, \ldots, L] \), i.e., there exist unknown nonnegative coefficients \( \lambda_i(k) \), \( i = 1, \ldots, L \) such that \( \sum_{i=1}^L \lambda_i(k) = 1 \) and \( |A|B|C|D|E|C|E|k \) \( = \sum_{i=1}^L \lambda_i(k) [A_i |B_i| C_i |D_i| E_i |C_i| E_i |k] \).

As in [36], invoke the following dynamic output feedback control law:

\[
\begin{align}
x_c(i+1) &= A_c(k)x_c(i)(k) + B_c(k)y(i)(k), \\
u(i)(k) &= C_c(k)x_c(i)(k) + D_c(k)w(i)(k), \quad i \geq 0,
\end{align}
\]

where \( x_c \in \mathbb{R}^{n_x} \) is the controller state; \( \{A_c, B_c, C_c, D_c\} \) are the control law parameters. The closed-loop model is obtained based on (1) and (3), i.e.,

\[
\begin{align}
\tilde{x}(i+1) &= \Phi(i, k)\tilde{x}(i)(k) + \Gamma(i, k)w(i)(k), \quad i \geq 0, \\
\tilde{x}(0)(k) &= \tilde{x}(k),
\end{align}
\]

where

\[
[\Phi(i, k), \Gamma(i, k)] = \sum_{l=1}^L \lambda_l(i)(k) \sum_{j=1}^L \lambda_j(i)(k) [\Phi_j(k), \Gamma_j(k)],
\]
\[
\Phi_i(k) := \begin{bmatrix} A_i + B_iD_i(k)C_j & B_1C_i(k) \\ B_i(k)C_j & A_i(k) \end{bmatrix},
\]
\[
\Gamma_i(k) := \begin{bmatrix} D_i + B_iD_i(k)E_j \\ B_i(k)E_j \end{bmatrix}.
\]

The stability/convergence property of (4) can be specified by the notion of QB, extending the application range of [24].

**Definition 1:** In the sense for all allowable \( \lambda_i(i,k), \lambda_j(i,k) \) and \( w(i,k) \), the system (4) is quadratically bounded with a common Lyapunov matrix \( Q^{-1} \), if
\[
\tilde{x}(i)TQ^{-1}\tilde{x}(i) \geq 1
\Rightarrow \tilde{x}(i + 1)TQ^{-1}\tilde{x}(i + 1) \leq \tilde{x}(i)TQ^{-1}\tilde{x}(i).
\] (5)

**Lemma 1:** Consider the system (4). In the sense for all allowable \( \lambda_i(i,k), \lambda_j(i,k) \) and \( w(i,k) \), the following facts are equivalent:

(a) (4) is quadratically bounded with a common Lyapunov matrix \( Q^{-1} \);

(b) the ellipsoid \( \varepsilon_{Q^{-1}} \) is a positively invariant set for (4), i.e., \( \tilde{x}(k) \in \varepsilon_{Q^{-1}} \) leads to \( \tilde{x}(k) \in \varepsilon_{Q^{-1}} \) for all \( i \geq 1 \).

Define the following disturbance-free model:
\[
\tilde{x}_u(i + 1) = \Phi(i,k)\tilde{x}_u(i,k), \quad i \geq 0,
\tilde{x}_u(0) = \tilde{x}(k).
\]
Correspondingly, \( y_u(i,k) = C(i,k)\tilde{x}_u(i,k) \) and \( u_0(i,k) = C_e(k)\tilde{x}_u(i,k) + D_e(k)y_u(i,k) \) are the disturbance-free output and input, respectively.

Define \( Q := \begin{bmatrix} Q_1 \\ Q_2 Q_3 \end{bmatrix} \) and \( M := \begin{bmatrix} M_1 & M_2 & M_3 \end{bmatrix} = Q^{-1} \).

In [36], by defining \( N_1 := M_1^{-1}, P_1 := Q_1^{-1} \) and \( U := -M_1^{-1}M_2^T \), the following formulations of Lyapunov matrices are utilized:
\[
Q = \begin{bmatrix} U^{-1}(Q_1 - N_1) & U^{-1}(Q_1 - N_1) U^{-T} \end{bmatrix},
M = \begin{bmatrix} M_1 & * & * \\ M_2 & M_2(M_1 - P_1)^{-1}M_2^T \end{bmatrix},
\] (6)

which naturally satisfy \( M = Q^{-1} \).

Taking the block-matrix inverse of \( Q \), it is easy to show that
\[
M = \begin{bmatrix} M_1 \\ -Q_3^{-1}Q_2M_1 \\ Q_3^{-1} + Q_3^{-1}Q_2M_1Q_2^T \end{bmatrix}.
\]
By applying \( MQ = I \), it is easy to show that \( Q_1 = N_1 + Q_3^TQ_2 \). It is clear that \( U \) also satisfies \( U = Q_1^{-1} \). Further define \( P_3 := Q_3^{-1} \). In this paper, without loss of generality, we set
\[
Q = \begin{bmatrix} N_1 + UQ_3U^T & * \\ Q_3U^T & Q_3 \end{bmatrix},
M = \begin{bmatrix} M_1 \\ -U^TM_1 \\ P_3 & U^TM_1U \end{bmatrix},
\] (7)

which naturally satisfy \( M = Q^{-1} \). Define \( x^0 = Ux_e \). For all \( k > 0 \), it will find \( x^0(k) \) and \( M_e(k) \), such that \( x(k) = x^0(k) \in \varepsilon_{M_e(k)} \).

**Remark 1:** In (6), the parameter \( U \) has to be nonsingular. This restriction has been removed in (7).

**Theorem 1:** Consider the system (1) satisfying Assumptions 1–2. The dynamic output feedback control law (3), which drives the closed-loop system (4) quadratically bounded, minimizes the worst-case upper bound \( \gamma(k) \) of performance cost \( \sum_{t=j_0}^{\infty} [\|y_u(i,k)\|^2 + \|u_0(i,k)\|^2] \), and satisfies \( |u_k| \leq \bar{u} \) and \( |\gamma(k) + 1| \leq \bar{y} \) for all \( i \geq 0 \), can be approximated by solving (8)-(14), as shown at the bottom of the next page, where
\[
\begin{cases}
\Phi_{ij} := \begin{bmatrix} 0 & \alpha_i j \gamma \end{cases}, \\
\Gamma_{ij} := \begin{bmatrix} M & * & * \\ Q & \Phi_{ij} & \Gamma_{ij} \end{bmatrix}.
\end{cases}
\]

**Proof:** Refer to [38]. Note that \( (Q_1 - E_0^TQ_3E_0)^{-1} \leq \varepsilon_{M_e(k)} \) in [38] is replaced by \( M_1 \leq \varepsilon_{M_e(k)} \). Since \( Q_1 - UQ_3U^T = M_1^{-1} \), the main difference between (8)-(14) and the approach in [38] is the replacement of \( E_0^T \) with \( U \).

The conditions (9)-(14) are, respectively, on
current estimation error set, current controller state;
QB of the closed-loop model;
optimality of the disturbance-free model;
control input from \( k \) onwards;
constrained signal for all future time;
mutable inverse for Lyapunov matrices.

**Remark 2:** In this paper, \( x_e \) is not required to have the same dimension with \( x \), which means that the controller state can have lower dimension than the true state. This property can be invoked to reduce the computational burden. Moreover, note that, in [38] the general form (7) is not given.

Utilize “Proposition 2” of [39] to handle the double convex combinations in \{10, 11, 13\}. For example, (10) is guaranteed by either Set 1 or Set 2 in the following:

Set 1: \( n = 2 \)
(i) \( \gamma_{ij} \geq 0, l \in \{1, \ldots, L\} \),
(ii) \( \gamma_{ij} + \gamma_{ij} \geq 0, j > l, l, j \in \{1, \ldots, L\} \);

Set 2: \( n = 3 \)
(i) \( \gamma_{ij} \geq 0, l \in \{1, \ldots, L\} \),
(ii) \( \gamma_{ij} + \gamma_{ij} + \gamma_{ij} \geq 0, j > l, l, j \in \{1, \ldots, L\} \),
(iii) \( \gamma_{ij} + \gamma_{ij} + \gamma_{ij} + \gamma_{ij} + \gamma_{ij} + \gamma_{ij} \geq 0, l > j > l, l, j, t \in \{1, \ldots, L\} \).

Here, \( n \) is the complexity parameter. By increasing \( n \), less conservative and asymptotically necessary conditions can be obtained.

**Proposition 1:** For the special case when (6) replaces (7), the problem (8)-(14) can be equivalently transformed into (15)-(21), as shown at the page 5.
The controller parameters \( \{A_c, B_c, C_c\} \) are obtained through \( A_c = -U^{-1}\tilde{A}_c(M_1 - P_1)^{-1}M_2^T, B_c = -U^{-1}\tilde{B}_c, \) and \( C_c = \tilde{C}_c(M_1 - P_1)^{-1}M_2^T. \)

**Proof:** The details can be found in [36].

Both optimization problems (8)-(14) and (15)-(21) are nontrivial (unable to be solved via single convex optimization). Reference [36] has utilized CCA to achieve (21). In order to reduce \( \gamma \), an iterative procedure is invoked in [36] as an outer loop of CCA. Recently in [40], it shows that the outer loop can be reduced without much influence on the control performance. A linearization method, instead of CCA, is adopted in [40] to handle \( Q = M^{-1} \). In this paper, we propose a new iterative method based on CCA, which is a close improvement on [36].

Remark 3: Note that other techniques such as the nonlinear decoupling method (see e.g., [41], [42]) can be applied to handle the nonlinear constraints in (8)-(14) and (15)-(21). However, the result can be conservative.

### III. NEW SYNTHESIS APPROACHES OF DYNAMIC OFRMPNC

When \( \{A_c, B_c, C_c, D_c\} \) are simultaneously optimized, it is referred to as the near-optimal dynamic controller. In the sequel, we first present a near-optimal dynamic OFRMPNC, then give the suboptimal dynamic OFRMPNC by taking appropriate transformations.

#### A. NEAR-OPTIMAL DYNAMIC OFRMPNC

When all the four parameters \( \{A_c, B_c, C_c, D_c\} \) are optimized simultaneously, one cannot find the solution via single convex optimization; \( Q = M^{-1} \) has to be satisfied, where both \( Q \) and \( M \) are the decision variables of optimization problem. CCA has been proven as an appropriate approach for handling \( Q = M^{-1} \). When \( Q \) and \( M \) are pre-specified, it is easy to solve (8)-(14).

**Algorithm 1:** (Near-optimal solution to (8)-(14))

Add
\[
\begin{bmatrix}
M_1 & I & N_1 \\
I & N_3
\end{bmatrix} \succeq 0, \quad \begin{bmatrix}
Q_3 & I \\
I & P_3
\end{bmatrix} \succeq 0. \tag{22}
\]

- **Off-line stage:** Select the convergence rates \( \kappa_i \ (i = 1, \ldots, n_c) \), sufficiently large integers \( n^0 > 2 \). Pre-specify \( \{a_{ij}, l, j = 1, \ldots, L; \eta_{1s}, s = 1, \ldots, n_s; \eta_{2s}, \eta_{3s}, s = 1, \ldots, q\} \) (see [36]) and \( \sigma^0 = 2(n_c+n_d) \). Choose \( \tilde{\gamma} \) mostly likely to be the upper bound of the performance cost.

- **On-line stage:**
  a. (Fast starting up—variant CCA)

\[
\begin{aligned}
\min_{\gamma, \varphi, \alpha, \eta_{1s}, \eta_{2s}, \eta_{3s}, \kappa_1, \kappa_2, \kappa_3, \Lambda_c, \kappa_c, \Delta_c, D_c, M_1, N_1, Q_3, P_3} \gamma,
\end{aligned}
\]

\[
\begin{aligned}
s.t. \quad M_1 \leq Q M_c(k), \quad \begin{bmatrix}
1 - \varphi & * \\
* & Q_3
\end{bmatrix} \succeq 0, \quad \gamma \end{aligned}
\]

\[
\begin{aligned}
\sum_{i=1}^{L} \lambda_i(i) \sum_{j=1}^{L} \lambda_j(i) \gamma_{lj}^{lb} \succeq 0, \quad \gamma \end{aligned}
\]

\[
\begin{aligned}
\sum_{i=1}^{L} \lambda_i(i) \sum_{j=1}^{L} \lambda_j(i) \gamma_{lj}^{opt} \succeq 0, \quad \gamma \end{aligned}
\]

\[
\begin{bmatrix}
M & * & * \\
0 & I & * \\
\end{bmatrix} \succeq 0, \quad \gamma
\]

\[
\begin{bmatrix}
\frac{1}{\sqrt{1-\eta_{1s}}} \xi_s C_c & \frac{1}{\sqrt{1-\eta_{1s}}} \xi_s D_c E_j & \xi_j^2 \hat{u}_s^2 \\
\end{bmatrix} \succeq 0, \quad \gamma
\]

\[
\sum_{i=1}^{L} \lambda_i(i) \sum_{j=1}^{L} \lambda_j(i) \begin{bmatrix}
M & * & * \\
0 & I & * \\
\end{bmatrix} \succeq 0, \quad \gamma
\]

\[
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 \\
\end{bmatrix} \succeq 0, \quad \gamma
\]

\[
\begin{aligned}
\kappa_1 = \frac{1}{\sqrt{(1-\eta_{3s})(1-\eta_{1s})}} \psi_s C_h \left[ A_h + B_h D_c C_j \right], \\
\kappa_2 = \frac{1}{\sqrt{(1-\eta_{2s})\eta_{3s}}} \psi_s C_h (D_l + B_l D_c E_j), \\
\kappa_3 = \bar{\gamma}_s \xi_s^2 \psi_s C_h \xi_s^T \psi_s^T, \\
M_1 = N_1^{-1}, \quad Q_3 = P_3^{-1}.
\end{aligned}
\]

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\begin{align}
\min_{\gamma, \vartheta, \nu_1, \nu_2, \nu_3, E_1, E_2, C_1, D_1, Q_1, P_1, M_1, N_1} & \quad \gamma, \\
\text{s.t.} & \quad M_1 \leq \varrho M_c(k), \\
& \sum_{l=1}^{L} \lambda_l(i|k) \sum_{j=1}^{L} \lambda_j(i|k) \left[\begin{array}{cc}
1 - \vartheta & Q_1 - N_1 \\
0 & 0
\end{array}\right] \geq 0, \\
& \sum_{l=1}^{L} \lambda_l(i|k) \sum_{j=1}^{L} \lambda_j(i|k) \left[\begin{array}{ccc}
M_1 & 0 & 0 \\
0 & M_1 & 0 \\
0 & 0 & M_1
\end{array}\right] \geq 0, \\
& \sum_{l=1}^{L} \lambda_l(i|k) \sum_{j=1}^{L} \lambda_j(i|k) \left[\begin{array}{ccc}
A_j + B_j D_c C_j & B_j \hat{C}_c & 0 \\
B_j C_j & \hat{A}_c & 0 \\
0 & 0 & \hat{R}_c^{1/2} [C_j 0]
\end{array}\right] \geq 0, \\
& \sum_{l=1}^{L} \lambda_l(i|k) \sum_{j=1}^{L} \lambda_j(i|k) \left[\begin{array}{ccc}
M_1 & 0 & 0 \\
0 & M_1 & 0 \\
0 & 0 & M_1
\end{array}\right] \geq 0, \\
& \sum_{l=1}^{L} \lambda_l(i|k) \sum_{j=1}^{L} \lambda_j(i|k) \left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \geq 0.
\end{align}

1) **(Initialization)** Set $t = 0$, iflg = 0. Select an initial $n (2 \leq n \leq n^0)$. Solve

\begin{align}
\min_{\gamma, \varrho, A_c, B_c, C_c, D_c, M_1, N_1, Q_3, P_3} & \quad \gamma, \\
\text{s.t.} & \quad (9) - (13) \text{ and } (22).
\end{align}

If (23) is infeasible, then go to step g; else, denote \([M_1, N_1, Q_3, P_3]^0 = [M_1, N_1, Q_3, P_3]^*\).

2) **(Feasibility judgement)** Increase $t$ by 1. Solve

\begin{align}
\min_{\gamma, \varrho, A_c, B_c, C_c, D_c} & \quad \gamma, \\
\text{s.t.} & \quad (9) - (13), \quad M_1 = M_1^{t-1}, \\
& \quad N_1 = (M_1^{t-1})^{-1}, \quad P_3 = P_3^{t-1}, \\
& \quad Q_3 = (P_3^{t-1})^{-1}.
\end{align}

If (24) is feasible, then denote \([M_1, P_3]^* = [M_1, P_3]^{t-1}, \gamma^* = \gamma^0, \{N_1, Q_3\}^* = (M_1^{t-1})^{-1}, (P_3^{t-1})^{-1}\), set iflg = 1 and go to step b. If (24) is infeasible at $t = 1$, denote $\gamma^0 = \hat{\gamma}$. If (24) is infeasible and $t = t^0$, then go to step 5).

3) **(Equalization between CCA and $\gamma$)** Solve

\begin{align}
\min_{\gamma, \varrho, A_c, B_c, C_c, D_c, M_1, N_1, Q_3, P_3} & \quad \sigma^0 \gamma + \gamma^0 \sigma, \\
\text{s.t.} & \quad (9) - (13) \text{ and } (22),
\end{align}

where $\sigma = \text{trace}(M_1^{t-1} N_1 + M_1 N_1^{t-1} + Q_3^{t-1} P_3 + Q_3 P_3^{t-1})$. If (25) is infeasible (this can only happen at $t = 1$), then go to step 5); else, denote \([M_1, P_3]^* = [M_1, P_3]^*\).

4) **(Iteration)** If $t < t^0$, then go to step 2).
5) (Complexity increment) If iflg = 0 and n < n^0, then increase n by 1, set \( \{M_1, P_3\}^0 = \{M_1, P_3\}^{t-1} \), set t = 0 and go to step 2.

b) (Transfer to ICCA)

6) (Re-initialization) Set t = 0, i = 1, \( \kappa = \kappa_1, \{M_1, P_3\}^0 = \{M_1, P_3\}^* \).

7) (Restoring recursive feasibility) If \( k > 0 \) and \( \gamma^o > \gamma(k-1) \), then solve

\[
\min_{\gamma, \phi, A, B, C, D, E} \{M_1, P_3\} = \{M_1, P_3\}^{t-1}, \quad \text{s.t} \quad (9)-(13), \quad M_1 = M_1(k-1), \quad N_1 = N_1(k-1), \quad P_3 = P_3(k-1), \quad Q_3 = Q_3(k-1), \quad \gamma \leq \gamma(k-1),
\]

and if (26) is feasible, then \( \gamma^o = \gamma^* \) and \( \{M_1, P_3\}^0 = \{M_1, P_3\}(k-1) \).

c) (Judging feasibility for CCA) Increase t by 1. Solve

\[
\min_{\gamma, \phi, A, B, C, D, E} \{M_1, P_3\} = \{M_1, P_3\}^{t-1}, \quad \text{s.t} \quad (9)-(13), (22), \quad \text{and} \quad \gamma \leq \kappa^0 \gamma^o
\]

(28)

Then the problem (8)-(14) can be recursively feasible.

d) (Minimizing trace for CCA) Solve

\[
\min_{\gamma, \phi, A, B, C, D} \{M_1, P_3\} = \{M_1, P_3\}^{t-1}, \quad \text{s.t} \quad (9)-(13), (22), \quad \text{and} \quad \gamma \leq \kappa^0 \gamma^o
\]

Then the problem (8)-(14) can be recursively feasible.

e) (Reducing \( \gamma \)) Set \( \{M_1, P_3\}^0 = \{M_1, P_3\}^{t-1}, \) set \( t = 0 \) and go to step c).

f) (Speeding up) If \( i < n_o \), then increase i by 1, set \( \kappa = \kappa_i \) and \( \{M_1, P_3\}^0 = \{M_1, P_3\}^{t-1}, \) set \( t = 0 \) and go to step c).

g) Terminate.

**Remark 4:** The following points are useful for understanding Algorithm 1.

- The problems (23)-(28) are LMI optimizations. Increasing \( n \) makes it more likely to yield the feasible solutions, and tends to improve the control performance. However, the larger \( n \), the larger the number of LMI, and the heavier the computational burden. We restrict \( n \leq n^0 \) for computational reasons.
- If the algorithm terminates with \( \text{iflg} = 1 \), then the feasible solution to (8)-(14) is obtained.
- The maximum iteration times \( t^0 \) is a regular parameter in CCA.
- Step a) is a variant CCA. The objective function of CCA (e.g. in (28)) is \( \sigma \). In the variant CCA (see (25)), the objective function equally include the cost bound \( \gamma \). Steps c)-e) composed of ICCA. As compared with ICCA, in the variant CCA there is no gradual reduction of \( \gamma \) (by adding gradual constraint \( \gamma \leq \kappa^0 \gamma^o \)). Since the variant CCA does not include an outer iteration, it has much lighter computational burden than ICCA. For ICCA, the optimization problem with complexity \( O(9^3 \cdot 2) \), where \( 9^3 \) is the number of scalar LMI variables and \( 2 \) is the number of scalar LMI rows (see [43]), will be solved for several to thousands times more than variant CCA.
- If the algorithm only takes step a) (i.e., omitting b)-f)), then the whole algorithm becomes variant CCA. However, in order to guarantee the recursive feasibility, we add step b) which can adjust the solution of the variant CCA. Usually, the solution of variant CCA in step a) is close to ICCA. ICCA not only withholds the recursive feasibility, but also may further reduce \( \gamma(k) \).

According to our previous studies (see [25], [36]), at each time \( k > 0 \), if we choose \( \{U, x^0, M_2\}(k) \) as

\[
U(k) = U(k-1), \quad x^0(k) = U(k)x_c(k), \quad Q(k) = 1 - x_c(k)^T Q_3(k-1)^{-1} x_c(k), \quad M_2(k) = Q(k) M^*_2(k-1), \quad M_2(k) \geq M_o(k).
\]

then the problem (8)-(14) can be recursively feasible.

On the other hand, a tighter bound of \( x(k) \), specified by \( \{M'_t, x^0\}(k) \), may be found by looking at the following conditions:

\[
\{x(k-1) - x^0(k-1) \in \mathcal{E}_{M_o(k-1)}, \quad \|w(k-1)\| \leq 1 \} \Rightarrow x(k) - x^0(k) \in \mathcal{E}_{M'_t(k)}, \quad \|w(k)\| \leq 1 \}
\]

(33)

\[
\Rightarrow M'_t(k) \geq M_o(k).
\]

(34)

For all \( \{x, w\}(k-1) \), the condition (33), satisfying (34), holds if

\[
\sum_{i=1}^{L} \sum_{j=1}^{L} \lambda_{ij} (k-1) \lambda_{ij}(k-1) \geq 0,
\]

(36)

for some positive scalars \( \{\phi_1, \phi_2\} \), where \( Q_o(k) = M'_t(k-1) \).

The proof of (36) is referred to as in [36]. Moreover, the
condition (35) is equivalent to
\[ Q_t(k) \leq M_t(k)^{-1}. \] (37)
Therefore, \{\(M_t^*, x^0\)(k)} satisfying (33)-(35) are obtained by solving
\[
\min_{\phi_1, \phi_2, \tilde{Q}(k), U(k)} \text{trace}(Q_t(k)), \quad \text{s.t.} \ (36) \text{ and } (37),
\] (38)
and calculating \(M_t^* = Q_t(k)^{-1}\). The double convex combination in (36) can be handled similar to (10), (11), (13). In order to ensure the good numerical property of \(U(k)\), add
\[
\begin{bmatrix}
\delta I & U^*(k) - U(k - 1) \\
U^*(k)^T - U(k - 1)^T & I
\end{bmatrix} \geq 0,
\] (39)
and further solve
\[
\min_{\phi_1, \phi_2, U(k)} \delta, \quad \text{s.t.} \ (36), \ (37) \text{ and } (39),
\] (40)
where \(Q_t(k)\) is fixed as the optimal solution of (38).

**Algorithm 2:** (Near-optimal Dynamic OFRMPC)
Off-line, take off-line stage of Algorithm 1.
On-line, at each \(k \geq 0\),
a) for \(k = 0\), take \(U(0) = I\);
b) for \(k > 0\), calculate \(x_c(k) = A_c(k - 1)x_c(k - 1) + B_c(k - 1)\gamma(k - 1)\), and refresh \(\{U, x^0, M_t\}(k)\) as in (29)-(32); solve (38) and, if (38) is feasible, then
- change \(M_t(k) = Q_t^*(k)^{-1}\),
- solve (40), and change \(U(k) = U^*(k)\) and \(x^0(k) = U^*(k)x_c(k)\);
c) solve (8)-(14) to find \(\{A_c, B_c, C_c, D_c, M_1, N_1, Q_3, P_3\}(k)\) (see on-line stage of Algorithm 1);
d) implement \(u(k) = C_c(k)x_c(k) + D_c(k)\gamma(k)\).

**Theorem 2:** For the system (1) with Assumptions 1–3, Algorithm 2 is adopted. Suppose (8)-(14) is feasible (if \(lg = 1\)) at time \(k = 0\). Then
i) (8)-(14) is feasible (if \(lg = 1\)) at each \(k > 0\);
ii) \{\(\tilde{x}, y, u\)\} converge to a neighborhood of \(\{0, 0, 0\}\), and (2) holds for all \(k \geq 0\).

**Proof:** i) Suppose (8)-(14) is feasible at \(k - 1 \geq 0\). At time \(k\), let us choose \(\{\gamma, \tilde{\gamma}\}, \{\tilde{A}_c, \tilde{B}_c, \tilde{C}_c, \tilde{D}_c, \tilde{M}_1, \tilde{N}_1, \tilde{Q}_3, \tilde{P}_3\}(k)\) \(= \{\gamma, \tilde{A}_c, \tilde{B}_c, \tilde{C}_c, \tilde{D}_c, \tilde{M}_1, \tilde{N}_1, \tilde{Q}_3, \tilde{P}_3\}^*(k - 1)\). Then, the conditions \{(10), (11), (12), (13)\} are satisfied. Consider the following two cases:
i.1) (38) is infeasible. The condition (9) holds according to (31)-(32).
i.2) (38) is feasible. The condition (9) holds according to (35), (31) and (32).
Hence, at each \(k \geq 1\), a feasible solution to (8)-(14) can be constructed by that at time \(k - 1\).

ii) This is similar to [25] and [36], so the details are omitted.

**B. SUBOPTIMAL DYNAMIC OFRMPC**
Let us take a dilation matrix \(\tilde{T} = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_3 \end{bmatrix}\), and define \(\tilde{A}_c = A_cT_3\) and \(\tilde{C}_c = C_cT_3\).

**Theorem 3:** When \(\{B_c, D_c\}\) are pre-specified, the optimization problem (8)-(14) can be transformed into (41)-(46), as shown at the bottom of the next page, where
\[
\begin{align*}
\odot = A_t + B_tD_cC_j, \\
S &= \begin{bmatrix} T_2^T + T_3 - Q_3U^T & * \\
T_1^T + T_1 - N_1 - UQ_jU^T & T_3^T + T_3 - Q_3
\end{bmatrix},
\end{align*}
\]

**Proof:** By taking congruence a transformation via \(\text{diag}(\tilde{T}, I)\), and applying the dilation technique \(\tilde{T}^T + \tilde{T} - M \leq \tilde{T}^TM^{-1}\tilde{T}\), it is shown that (10)-(13) are equivalent to (43)-(46). The condition (9) is transformed into (42) by applying the Schur complement.

On the feasibility aspect, (8)-(14) is equivalent to (41)-(46). However, for (41)-(46), it is inappropriate for applying CCA, so let us give its suboptimal solution.

**Algorithm 3:** (A suboptimal solution to (8)-(14))
- **Off-line stage:** Pre-specify \(\{\gamma_j, l, j = 1, \ldots, L; \eta_{1s}, s = 1, \ldots, n_u; \eta_{2s}, \eta_{3s}, s = 1, \ldots, q\}, \{U, \{B_c(0), D_c(0)\}\} (see [36]).
- **On-line stage:** At each \(k \geq 0\),
a) b) see a)-b) of Algorithm 2;
c) solve (41)-(46)
  c1) first for \(\{\gamma, \tilde{\gamma}, \tilde{A}_c, \tilde{C}_c, N_1, Q_3, T_1, T_2, T_3\}^*(k)\), by fixing \(\{B_c, D_c\}(k) = [B_c, D_c](k - 1);\)
c2) then for \(\{\gamma, \tilde{\gamma}, \tilde{A}_c, \tilde{C}_c, D_c, N_1, Q_3, T_3\}^*(k)\), by fixing \(\{T_1, T_2\}(k)\) at the above \(\{T_1, T_2\}^*(k);\)
d) implement \(u(k) = C_c(k)x_c(k) + D_c(k)\gamma(k)\).

Algorithm 3 finds a suboptimal solution which may be far from optimal, but it is computationally more efficient than Algorithm 2. On the other hand, Algorithm 3 solves (41)-(46) twice at each time \(k\), which may still be over-expensive. In order to further alleviate the computational burden, we can fix \(\{B_c, D_c\}(k) = [B_c, D_c](0);\) this simplifies Algorithm 3 as the following.

**Algorithm 4:** (Suboptimal Dynamic OFRMPC)
Same as Algorithm 3, except that
a) c) solve (41)-(46) for \(\{\gamma, \tilde{\gamma}, \tilde{A}_c, \tilde{C}_c, N_1, Q_3, T_1, T_2, T_3\}^*(k)\), by fixing \(\{B_c, D_c\}(k) = [B_c, D_c](0)\).

The stability proof of Algorithms 3–4 is similar to Theorem 2, so is omitted here for brevity. Let us utilize the notations in this paper to re-formulate the optimization problem in [37], i.e.,
\[
\min_{\gamma, \tilde{\gamma}, \tilde{A}_c, \tilde{C}_c, \tilde{U}} \gamma, \quad \text{s.t.} \ |u(i[k])| \leq \tilde{u}, \ |\Psi x(i + 1[k])| \leq \tilde{\psi}, \ |e(i + 1[k])| \leq \tilde{\varepsilon}, \ \tilde{x}(k) \in \mathbb{E}^(\tilde{\gamma}, \tilde{u}),
\] (47)
\[
\|\tilde{x}(i[k])\|_Q^{-1} \geq 1 \ \Rightarrow \\|\tilde{x}(i[k])\|_Q^{-i} - \|\tilde{x}(i + 1[k])\|_Q^{-i} \geq 0,
\] (48)
\[
\|\tilde{x}(i[k])\|_Q^{-i} - \|\tilde{x}(i + 1[k])\|_Q^{-i} \geq 0,
\] (49)
\[
1/\gamma \left[\|x_u(i[k])\|^2 + \|u_x(i[k])\|^2\right],
\] (50)
\[
\|\tilde{x}(i[k])\|_Q^{-i} - \|\tilde{x}(i + 1[k])\|_Q^{-i} \geq 0,
\] (51)
\[
\|\tilde{x}(i[k])\|^2 + \|u_x(i[k])\|^2 \leq \tilde{\psi},
\] (52)
where \( \hat{e} = [\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_n]^T \) is pre-specified such that \( e(0) \in \{e| - \hat{e} \leq e \leq \hat{e}\} = \text{Co}[\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_n]. \)

**Corollary 4:** The problem (53)-(58) can be approximated as (53)-(58), shown at the bottom of the next page.

**Proof:** This is analogous to Theorem 3.

**Proposition 2:** Adding \( U \) as a simultaneous decision variable, the problem (53)-(58) is less conservative than (26) in [37], i.e., the former includes the latter as a special case.

**Proof:** This is similar to “Proposition 1” of [44], only that in [44] \( \bar{Q} = \begin{bmatrix} Q_1 & * \\ Q_2 & Q_3 \end{bmatrix} \) and \( e = x - x_c \). Note that all the technical tools/knobs of [37] are improved and incorporated in this paper. Hence, it is deserved that the conclusion holds.

Therefore, adding \( U \) as a simultaneous decision variable, this paper improves the optimization problem in [37]. However, notice that [37] does not have \( U \), and not pre-specify any element in \( Q \). Compared with (41)-(46), the problem (53)-(58) have too weaknesses:

i) the recursive feasibility is not guaranteed;

ii) the optimization problem becomes conservative due to the existence of constraint on estimation error, (57).

Hence, this paper retrieves recursive feasibility of the optimization problem in [37].

**Remark 5:** The proposed approach provides a systematic way for controlling uncertain systems by utilizing MPC, and it can remove some weaknesses of the previous approaches. Moreover, a sub-optimal algorithm is provided in case the computational efficiency is a crucial issue in real applications. Even through the proposed approach is applicable to moderate dimensional systems, it may not be applicable for high dimensional systems. Other techniques for improving the computational efficiency can be studied and applied.

**IV. THE FUEL CELL EXAMPLE**

Consider the nonlinear fuel cell model from [45], which has been handled in [46]. The model parameters are

\[
L = 8,
A_l = \begin{bmatrix}
0.9617 & 0 & 0 \\
0 & 0.9872 & 0 \\
0 & 0 & 0.6564
\end{bmatrix},
\]

(41)
\[ B_l = \begin{bmatrix} 45.4498 \\ 0 \\ 119.0970 \end{bmatrix}, \quad D_l = \begin{bmatrix} -0.0004525 \\ 0.0004525 \\ -0.0006790 \end{bmatrix}, \]
\[ C_l = 21.0606 C_l, \quad E_l = -0.63, \quad \xi_l = 0, \quad l = 1, \ldots, 8, \]
\[ C_l = \begin{bmatrix} 10.9952 & -0.6717 & 5.6465 \\ 10.9952 & -0.6717 & 2.4199 \\ 10.9952 & -0.2879 & 5.6465 \\ 10.9952 & -0.2879 & 2.4199 \\ 4.7122 & -0.6717 & 5.6465 \\ 4.7122 & -0.6717 & 2.4199 \\ 4.7122 & -0.2879 & 5.6465 \\ 4.7122 & -0.2879 & 2.4199 \end{bmatrix}, \]

The physical constraints are
\[
\begin{align*}
|u| & \leq 0.0008, \\
|x(k + 1)| & \leq 0.4 \times [0.1516, 2.4811, 0.1476]^T, \\
|z(k + 1)| & \leq 10.63, \quad k \geq 0.
\end{align*}
\]

In the simulation,
\[
y = 21.0606 \times \left[ \ln \left( \frac{(x_1 + 0.1516)(x_3 + 0.1476)^{1/2}}{x_2 + 2.4811} \right) - \ln \left( \frac{0.1516 \times (0.1476)^{1/2}}{2.4811} \right) \right].
\]

We compare Algorithm 2 (denoted as Alg2) and Algorithm 4 (denoted as Alg4) in this paper with the approach in [25] (denoted as CCC2011). We also compare with [40] (denoted as ASCC2019), where another method based on linearization is utilized. Choose \( \{\mathcal{D}, \mathcal{S}, \eta_1, \eta_2, \eta_3\} = \{1, 1, \frac{1}{2}, 0, \frac{1}{2}\} \) and \( \{0, n\} = \{25, 2\} \). Choose
\[
\begin{align*}
x_c(0) &= \frac{1}{\sqrt{3c(0)}}, \\
&\times [0.4 \times 0.1516, -0.2 \times 2.4811, 0.4 \times 0.1476]^T, \\
x_c(0) &= \frac{1}{\sqrt{3c(0)}}, \\
&\times [0.6 \times 0.1516, -0.4 \times 2.4811, 0.6 \times 0.1476]^T, \\
M_c(0) &= c(0)^2 \text{diag}(1088, 4.061, 1147.8).
\end{align*}
\]

\[
\begin{align*}
\min_{\gamma \in \mathbb{R}_+, \lambda_c, \hat{c}_c, \tau_1, \tau_2, \tau_3, \eta_1, \eta_3} \gamma', \\
\text{s.t.} \quad \left[ \begin{array}{c} \bar{c} \\ N_1 \end{array} \right] \geq 0, \quad \left[ \begin{array}{c} 1 - \bar{c} \\ x_c(k) \end{array} \right] \geq 0, \quad l \in \{1, \ldots, 2^n_c\}, \\
\sum_{i=1}^L \lambda_i(k) \sum_{j=1}^L \lambda_j(i[k]) \left[ \begin{array}{c} S \\ 0 \\ 0 \end{array} \right] \geq 0, \\
\sum_{i=1}^L \sum_{j=1}^L \lambda_i(k) \lambda_j(i[k]) \left[ \begin{array}{c} S \\ 0 \\ 0 \end{array} \right] \geq 0, \\
\sum_{i=1}^L \sum_{j=1}^L \lambda_i(k) \lambda_j(i[k]) \left[ \begin{array}{c} S \\ 0 \\ 0 \end{array} \right] \geq 0, \\
\sum_{i=1}^L \sum_{j=1}^L \lambda_i(k) \lambda_j(i[k]) \left[ \begin{array}{c} S \\ 0 \\ 0 \end{array} \right] \geq 0,
\end{align*}
\]
where any choice of $c(0)$ satisfies Assumption 3. Choose $c(0) = 2.0$. For the input/state/output responses, see Figures 1–5, which verify the closed-loop QB and constraint satisfactions. The total amounts of computational time for $k \leq 300$ for applying Alg2, Alg4, CCC2011 and ASCC2019 are 72.5, 0.81, 161.7 and 68.1 hours, respectively. This is not a real-time simulation.

It is observed that the proposed approach in this paper can have a much smaller cost bound and a more aggressive control input.

V. CONCLUSION

Researches on OFRMPMC for LPV model with bounded disturbance have been undergone for over 10 years. The dynamic output feedback, where the controller and estimator parameters are not separately designed, has been proven as successful. The controller has been improved over the past ten years, where one of the concentration is on the selection of appropriate Lyapunov matrix. In the previous studies, the Lyapunov matrix either has some fixed structure, or some of its non-diagonal blocks are required to be invertible. In this paper, these restrictions are removed. In this scenario, this paper has found appropriate transformations in order to transform OFRMPMC into a solvable form. Not only the general near-optimal solution, but also the suboptimal solution with much lighter computational burden, are proposed in this paper.

In the future work, we plan to introduce free control moves to OFRMPMC, which is a subject of great value so that the control performance can be improved and the region of attraction can be enlarged. However, due to the difficulty of guaranteeing the recursive feasibility, this subject needs further exploration.

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