On high frequency Cherenkov-type radiation in pulsar magnetospheric electron-positron plasma

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ABSTRACT

Emission process of a charged particle propagating in a medium with a curved magnetic field is considered. This mechanism combines features of conventional Cherenkov and curvature emission. Thus, presence of a medium with the index of refraction larger than the unity is essential for the emission. In the present paper the generation of high frequency radiation by the mentioned mechanism is considered. The generated waves are vacuum-like electromagnetic waves and may leave the medium directly. Consequently, this emission mechanism may be important for the problem of pulsar X-ray and gamma-ray emission generation.

Subject headings:
pulsars: general — radiation processes: nonthermal

1. Introduction

In the electron-positron ($e^-e^+$) plasma filling the magnetosphere of pulsars can generate waves in the different frequency ranges. It is well known, that relativistic charged particle moving along the curved magnetic field lines emit radiation. In this context was considered the analogy of curvature radiation in the pulsar magnetosphere, for the generation of radio waves. It was found that for the beam of particles the wave interference processes can suppress the generated emission [Blandford 1975; Melrose 1980]. Later on the similar processes were reconsidered, taking into account the inhomogeneity, particularly the curvature of the field lines, which causes the drift motion of the particles [Kazbegi et al. 1991; Lyutikov et al. 1999]. It appeared that the $e^-e^+$ plasma can generate radio emission in this case, through the modified Cherenkov-curvature resonance

$$\omega - k_\parallel v_\parallel - k_\perp u_\perp = 0,$$

where, signs $\perp$ and $\parallel$ denote components across and along the pulsar magnetic field $B_\parallel$, $u_\perp = v_\parallel^2\gamma_r / R_\omega B$ is the drift velocity of the particles due to curvature of the magnetic field lines ($R_\omega$ is the curvature of the field lines, $\omega_B = eB/mc$, $B$ is the pulsar magnetic field, $e$ is the electrons charge and $m$ is its rest mass, $c$ is the speed of light). The curved field lines lie on a plane across which the particles perform the drift motion. This particularly violates the axial symmetry of the mentioned problem, leading to coupling of the parallel and perpendicular components of electric field of waves, provoking appearance of additional terms in the dispersion relation, which maintain the resonance condition (1). The issue of generation of electromagnetic radio waves in approximation of infinitely large magnetic field $B \to \infty$ was thoroughly investigated in the follow-\-ing works (Gedalin et al. 2002a,b). In these papers the medium inhomogeneity is neglected and one considers the oblique propagation of waves, which results the violation of the axial symmetry, as well. Even choosing the wave vector in the following way $k = (k_\perp, 0, k_\parallel)$ does not help to separate the parallel and perpendicular components of electric field of waves $E_\parallel$ and $E_\perp$, in the dispersion relation. Which by itself leads to terms containing the Cherenkov resonance in the dispersion relation

$$\omega - k_\parallel v_\parallel = 0.\hspace{1cm}(2)$$

The fact that this is a Cherenkov-type resonance immediately implies that the presence of a sublu-
minous wave with the phase velocity smaller than the speed of light is essential. It is well known, that if the speed of charged particle is greater than the speed of light in the medium the Cherenkov radiation is generated (Cerenkov 1937).

We suppose that the Cherenkov-curvature resonance can provide generation of high frequency (X-ray and gamma-ray) radiation, as well. This mechanism has been so far considered only for explanation of pulsar radio emission generation in the pulsars’ magnetosphere. Differently from radio emission, the high frequency radiation does not 'feel’ the medium and emerges freely from the region of its generation, as in the pulsar magnetosphere for X-ray and higher energy emission the condition \( \lambda < n^{-1/3} \) is satisfied (here \( n \) is the density of plasma particles, and \( \lambda \) is the wavelength of high frequency waves). Consequently, the emission of single particles can be summed up without taking into account the interference of waves. Generally accepted generation mechanisms of the high energy emission in pulsar magnetospheres are the synchrotron-curveature radiation and the inverse-Compton scattering. The Cherenkov type radiation has not been so far considered, as possible mechanism for generation of high energy emission in pulsar magnetospheres. In the present paper we investigate generation of high frequency (\( \omega \gg \omega_B/\gamma_b \)) waves in the relativistic \( e^-e^+ \) plasma filling the pulsar magnetosphere via the Cherenkov-curvature resonance.

2. Linear Waves in Pair \( (e^-e^+) \) Plasma

Let us consider the wave propagation in \( e^-e^+ \) plasma of pulsar magnetosphere with small inclination angles to the magnetic field \( B_0 \), when the waves electric field components \( E_\parallel \) and \( E_\perp \) are bounded with each other. In this case for the electromagnetic waves, the Maxwell’s equations mutually with the Vlasov equation for the pair plasma in the linear approximation can be written as (Volokitin 1985):

\[
\left(k^2 c^2 - \omega^2 \epsilon_{xx}\right) \left(k^2 c^2 - \omega^2 \epsilon_{zz}\right) = \left(k_x k_y c^2 - \omega^2 \epsilon_{xy}\right)^2. \tag{3}
\]

Here \( \omega \) is the wave frequency, \( \epsilon_{ij} \) is the dielectric tensor for plasma \( (i, j = x, y, z) \) and \( B_0 \parallel z \). The components of dielectric tensor written for the weakly curved magnetic field lines \( (u_\perp/c \ll 1) \) have the following form (Kazbegi et al. 1991; Lyutikov et al. 1999):

\[
\epsilon_{xx} = 1 - \frac{1}{2} \Sigma_a \frac{\omega_{pa}^2}{\omega^2} \int \frac{dp_\parallel}{\gamma} (\omega - k_\parallel v_\parallel) A^+_a f_a - \Sigma_a \omega_{pa}^2 \int \frac{dp_\parallel}{\gamma} \frac{f_a}{\Omega_a^2} \frac{k^2_\parallel u_\parallel^2}{\omega^2} \left(1 - \frac{\omega v_\parallel}{k_\parallel c^2}\right),
\]

\[
\epsilon_{yy} = 1 - \frac{1}{2} \Sigma_a \frac{\omega_{pa}^2}{\omega^2} \int \frac{dp_\parallel}{\gamma} (\omega - k_\parallel v_\parallel - k_x u_a) A^+_a f_a,
\]

\[
\epsilon_{zz} = 1 - \Sigma_a \omega_{pa}^2 \int \frac{dp_\parallel}{\gamma} \frac{f_a}{\Omega_a^2} \left[1 - \frac{k_x u_a}{\omega}\right] \times \left[1 - \frac{k_x u_a}{\omega} - \frac{\gamma_0^2}{\gamma^2}ight],
\]

\[
\epsilon_{xy} = - \epsilon_{yx} = - \frac{1}{2} \Sigma_a \omega_{pa}^2 \int \frac{dp_\parallel}{\gamma} k_x v_\parallel - \Sigma_a \omega_{pa}^2 \int \frac{dp_\parallel}{\gamma} \frac{f_a}{\Omega_a^2} \frac{k_x u_a}{\omega}, \tag{4}
\]

where

\[
A^+_a = \left(\frac{1}{\Omega_a^+} + \frac{1}{\Omega_a}\right), \tag{5}
\]

\[
\Omega_a^\pm = (\omega - k_\parallel v_\parallel - k_x u_a \pm \omega_B/\gamma), \quad \Omega_a^0 = (\omega - k_\parallel v_\parallel - k_x u_a). \tag{6}
\]

Here \( \omega_{pa}^2 = 4\pi e^2 n_{pa}/m \), \( n_{pa} \) is the concentration of a-type particles, \( f_a \) is the one-dimensional distribution function of a components of \( e^-e^+ \) plasma. Due to specific mechanism of filling

\[
\begin{align*}
\end{align*}
\]
the pulsar magnetosphere with charged particles, the distribution function of pair $e^-e^+$ plasma consists of three components: the bulk of plasma with the Lorentz-factor $\gamma_p \approx (3 - 10)$ and the particle concentration at the star surface $n_p \approx 10^{20}\text{cm}^{-3}$, the long extended in one direction tail on the distribution function with $\gamma_t \approx (10^4 - 10^5)$, $n_t \approx (10^{16} - 10^{17})\text{cm}^{-3}$ and the most energetic primary beam with $\gamma_b \approx (10^6 - 10^7)$ (Osmanov & Rieger 2009), $n_b \approx (10^{13} - 10^{14})\text{cm}^{-3}$ (Goldreich & Julian 1969; Sturrock 1971; Tademaru 1973; Ruderman & Sutherland 1975) (see Fig. 1). In the expression for the dielectric tensor $\varepsilon_{ij}$, when summing up by different types of particles one should only take into account the contribution of bulk plasma. The situation changes, when the resonance conditions are satisfied for the beam particles (Exp. (1)). As in this case the beam particles appear to have the equal contribution in expression (4), as the bulk particles. Consequently, in the hydrodynamical approximation for the particle distribution function we apply summing for beam and bulk plasma particles $f_a = \delta(v - v_a)$, $a = p, b$.

Let us consider the possibility of generation of high frequency waves, which satisfy the following conditions $\omega_B^2/\gamma_b^2 \ll 1 \ll \omega_p^2/\gamma_p^2$. The frequency of the excited waves can be written as $\omega = kc(1 - \Delta)$, where $\Delta \ll 1$ and $k = k_\parallel (1 + \tan^2 \theta)^{1/2}$. Here $\tan \theta = k_\perp / k_\parallel$ and as we are considering the small angles of propagation ($\theta \ll 1$), one can write $k \approx k_\parallel (1 + \theta^2/2)$. We assume that $k_\parallel = 0$ and taking into account the following conditions $u_d/c \ll 1$, $\omega \gg \omega_p \gamma_p$, $\omega \gg \omega_B / \gamma_b$ and $1/\gamma_b^2 \ll \Delta \ll 1/\gamma_p^2$. The components of dielectric tensor calculated from Eq. (4) take the form:

$$
\varepsilon_{xx} = \varepsilon_{yy} = 1,
\varepsilon_{zz} = 1 + \frac{\omega_p^2}{\gamma_b^2 \gamma_p k_\parallel^2 c^2 \Delta^2 \theta^2 / c^2},
\varepsilon_{xz} = \frac{\omega_B \theta}{\omega^2 \gamma_b \Delta^2} \pm \frac{\omega_B}{k_\parallel c^2 \gamma_b^2}.
$$

(7)

It should be mentioned that the above expressions were taken, assuming that $\theta/2 = u/c$. Using this expressions for $\varepsilon_{ij}$, in place of Eq. (3) we will get:

$$
\Delta^2 - \frac{\omega_B^2}{2k_\parallel c^2 \gamma_b} \Delta - \frac{\omega_B^2}{k_\parallel c^2 \gamma_b^2} = 0.
$$

(8)

From where:

$$
\Delta = \frac{\omega_B}{k_\parallel c \gamma_b} \pm \left[ \left( \frac{\omega_B}{4k_\parallel c^2 \gamma_b} \right)^2 + \frac{\omega_B^2}{4k_\parallel c^2 \gamma_b^2} \right]^{1/2}.
$$

(9)

Taking into account the estimations done for the parameters, this expression can be reduced to the following form:

$$
\Delta = \frac{\omega_B}{k_\parallel c \gamma_b}.
$$

(10)

From the general equations for fields (Volokitin 1985; Arons & Barnard 1986), its easy to define the polarization of this waves when $\theta \neq 0$, $E_x/E_z \gg 1$ and the wave is practically linearly polarized, its electric field vector lies in the $k, B_0$ plane. The phase velocity of these waves can be defined as

$$
v_{ph} = \frac{\omega}{k} = c(1 - \Delta).
$$

(11)

The value of $\Delta$ defines how the phase velocity of the wave in the $e^-e^+$ differs from the speed of light in the vacuum. The condition

$$
v_b - v_{ph} > 0,
$$

(12)
defines the generation of Cherenkov emission. We write the relativistic velocity of the particles in the following way \( v_b \approx c(1 - 1/(2\gamma_b)) \) and in this case the condition rewrites as

\[
\Delta > \frac{1}{2\gamma_b^2}.
\]

(13)

The condition (13) and expression (10) defines the conditions for generation of high energy Cherenkov emission in the pulsar magnetosphere

\[
k||c < 2\gamma_b\omega_{B_0}\left(\frac{r_0}{r}\right)^3,
\]

(14)

where \( \omega_0 = eB_0/mc \) is the cyclotron frequency at the surface and \( r_0 \) is the star radius. Let us assume that the distance changes from star surface \( r = r_0 \) to the light cylinder radius \( r = r_{LC} \), which can be taken as the limit distance for the generation region of pulsed emission in pulsars. Then one can find from Eq. (14) the maximum photon energy that can be radiated through the Cherenkov mechanism at the certain distance

\[
\epsilon_{ph} \approx 8.3 \cdot 10^{-15}\gamma_b\omega_{B_0}\left(\frac{r_0}{r}\right)^3.
\]

(15)

Assuming the typical parameters for radio pulsars, \( B_0 \approx 10^{12}\text{G}, \gamma_b \approx 10^6 \) and the neutron star radius \( r_0 \approx 10^6\text{cm} \), we find that the most energetic photons can be emitted near the star surface (see Fig. [2]). In particular, at the star surface the beam electrons emitting through the Cherenkov mechanism can generate high energy gamma-ray photons (\( \epsilon_{ph} \approx 24\text{GeV} \)). On the other hand emitting through the same mechanism at the light cylinder distances (\( r_{LC} \approx 10^8\text{cm} \)) the radiation comes in X-ray domain (\( \epsilon_{ph} \approx 24\text{KeV} \)). As we are searching for the generation of high frequency emission by Cherenkov mechanism, more interesting is to consider the process near the star surface, where the upper limit on generated radiation comes in gamma-ray domain. It is important to find also the lower limit of Cherenkov emission generated near the star surface. For this purposes, we take into account the conditions for the wave frequency that were assumed during the calculations. In particular, the highest lower limit for the wave frequency can be obtained from the following condition

\[
\Delta \ll \frac{1}{\gamma_p^2}.
\]

Taking into account the Eq. (10) for the \( \Delta \), one obtains

\[
k||c \gg \frac{\omega_{B_0}\gamma_b^2}{\gamma_p}\left(\frac{r_0}{r}\right)^3.
\]

(17)

Using expression (14) and (17) we define the energy domain for the Cherenkov radiation generated near the star surface, which covers X-ray up to gamma-ray domains (see Fig. [3]).

It is also interesting to estimate the luminosity of the high frequency radiation generated through the Cherenkov-curvature mechanism in the pulsar magnetosphere. The luminosity can be calculated by the following expression

\[
L = 4\pi r^2 F_{\Omega_1},
\]

(18)

where \( r \) shows the location of emission generation region, \( F \) is the emission flux and \( F_{\Omega_1} \) is the beaming fraction. For typical radio pulsars \( 0.4 < F_{\Omega_1} < 1 \) (Narayan & Vivekanand 1983). For estimations we set the value of beaming fraction to 1 and consider the generation of the emission near the star surface (\( r = r_0 \)). The emission is generated due to the kinetic energy of the resonant particles. Consequently, for the emission flux one can write

\[
F = mc^3\gamma_b n_b,
\]

(19)

here \( n_b \) is the density of emitting particles. Using the values for typical radio pulsars, we find that the luminosity of the high frequency radiation generated near the star surface \( L \approx 10^{35} - 10^{36}\text{erg/s} \).

3. Conclusion

In this paper we considered a Cherenkov-curvature emission mechanism which combines features of conventional Cherenkov and curvature radiation. It is essential that even a weak inhomogeneity of the magnetic fields results in a drift motion of the particle perpendicular to the local plane of the magnetic field line, which is weakly relativistic when the motion of the particles along the magnetic field is ultrarelativistic. This causes generation of vacuum-like waves, propagating freely in the pulsar magnetosphere that can reach a observer as pulsar radiation. Physical origin of the emission in the case of Cherenkov-type and synchrotron-type processes is quite different. Cherenkov-type process the emission may be attributed to the electromagnetic polarization shock.
front that develops in a dielectric medium due to the passage of a charged particle with speed larger than phase speed of waves in a medium. It is virtually a collective emission process. Čerenkov-type emission is impossible in vacuum and in a medium with the refractive index smaller than unity. The conventional Čerenkov and curvature emission mechanisms may be viewed as corresponding limits of the Čerenkov-curvature mechanism in the cases of homogeneous magnetic field (in a medium), medium without magnetic field, and inhomogeneous magnetic field without a medium. The Čerenkov-curvature instability develops on the rising part of the beam distribution function (see Fig. 1). The free energy for the growth of the instability comes from the non-equilibrium, anisotropic distribution of the fast particles. For the development of the instability it’s essential that the medium supports subluminous waves, i.e. its index of refraction is larger than unity.

In our case, we have studied the possibility of generation of high frequency radiation through the mentioned Čerenkov-curvature mechanism. It revealed that this mechanism can provide excitation of emission from the broad energy domain. In particular near the star surface generation of X-rays up to high energy gamma-rays is possible, if the resonant particles are the most energetic primary beam electrons. This emission can freely emerge in the pulsar magnetosphere and reach an observer. The interesting feature of this type of radiation should be its angular distribution. The calculations showed that the emission is generated in the cone centered at the angle $\theta/2 = u/c$. Consequently, one can find the angular distribution of the generated radiation ($\theta \approx 2c^2\gamma_b/R_c\omega_B$) for the chosen location of the generation region. Taking into account that the beam has a very narrow distribution, one should expect also a small opening angle of the radiation cone, that inevitably will cause detection of the narrow pulses.

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Fig. 3.— The energy range of photons radiated through the Cherenkov mechanism in the pulsar magnetosphere near the star surface.