Evolution of Néel order and localized spin moment in the doped two-dimensional Hubbard model

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We investigate effects of doped holes’ hopping on Néel order in the two-dimensional Hubbard model. Semiclassical staggered moments are computed by solving saddle point equations derived from a path-integral formalism. Effects of quantum fluctuations are taken into account by the Schwinger boson mean field theory. We argue that hopping of doped holes is ineffective in suppressing Néel order compared to rapid suppresstion of Néel order in high-temperature superconductors. After destruction of Néel order, the quantum disordered phase sets in. Taking the strong coupling limit in the quantum disordered phase leads to a model of spinless fermions and bosons but no gauge field interaction.

I. INTRODUCTION

In high-temperature superconductors, one remarkable feature is rapid destruction of Néel order by hole doping. In fact, only 2% doping hole concentration is enough to suppress Néel order. While critical disorder is 50% for the bond percolation threshold and 41% for the site percolation threshold. How do doped holes suppress Néel order in such an effective way?

Naive expectation is that the hopping process of doped holes suppresses Néel order. In this paper, we examine the effect of disorder brought by hopping of doped holes. As a model, we take the Hubbard model because at half-filling it is reduced to the $S = 1/2$ antiferromagnetic Heisenberg model that describes the undoped parent compound of high-temperature superconductors, and perhaps it is the simplest model to see disorder effect by doped holes’ hopping on Néel order. We use path-integral formalism developed by Schulz. [1,2] We solve saddle point equations to compute the magnitude of the staggered moment. Since saddle point equations are semiclassical equations, solutions do not contain effects of quantum fluctuations. We take into account quantum fluctuation effects in terms of Schwinger boson mean field theory. [3]

The remainder of this paper is organized as follows: In sec.II, we rewrite the Hubbard model following Schulz. [1,2] In sec.III, we derive saddle point equations. Solving these equations, we compute doping dependence of the semiclassical staggered moment. We examine effects of quantum fluctuations on Néel order by the Schwinger boson mean field theory. We show that there is the quantum disordered regime in which quantum fluctuations suppress Néel order. In sec.IV, we discuss the effective action in the quantum disordered phase. Sec. V is devoted to summary and discussion.

II. PATH INTEGRAL FORMULATION OF THE HUBBARD MODEL

In order to investigate doped holes’ hopping effects on Néel order, we first rewrite the model in a convenient form following Schulz. [1,2] In the coherent state path-integral formulation, the partition function of the model is given by $\mathcal{Z} = \int D\pi Dc \exp(-S)$ with $S = S_0 + S_U$, where

$$S_0 = \int_0^\beta d\tau \left[ \sum_j \tau_j (\partial_\tau - \mu) c_j - i \sum_{(i,j)} (\tau_i c_j + \tau_j c_i) \right],$$

(1)

$$S_U = \int_0^\beta d\tau \sum_j U n_{j\uparrow} n_{j\downarrow}.$$ 

(2)

Hereafter $\tau$ dependence of fields is implicit. The summation $\sum_{(i,j)}$ is taken over the nearest neighbor sites. Carrier fields are represented in a spinor: $c_i = \tau (c_{i\uparrow}, c_{i\downarrow})$ and $\tau_i = (\tau_{i\uparrow}, \tau_{i\downarrow})$. Using the identity, [4] the on-site Coulomb interaction term can be rewritten as, $U \sum_j n_{j\uparrow} n_{j\downarrow} = -(U/4) \sum_j \left[ (n_{j\uparrow} + n_{j\downarrow})^2 + (\tau_{j\uparrow} \sigma c_j)^2 + (n_{j\uparrow} + n_{j\downarrow}) \right]$, where the components of the vector $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. Introducing Hubbard-Stratonovich fields for the charge and spin fluctuations, [1,2] we obtain $\mathcal{Z} = \int D\pi Dc D\tilde{S} D\phi \exp(-S_0 - S_U)$, where,

$$S_U = \int_0^\beta d\tau \left[ U \sum_j \tilde{S}_j^2 - U \sum_j \tilde{S}_j \cdot \tau_j \sigma c_j + \frac{U}{4} \sum_j \phi_{c,j}^2 - \frac{U}{2} \sum_j \phi_{c,j} \tau_j c_j \right].$$

(3)
up to constant. Here the vector $\tilde{S}_j$ represents the localized spin moment. The scalar $\phi_{cj}$ is associated with charge fluctuations. For the charge degrees of freedom, we take the uniform value at the saddle point: $\phi_{cj} = (\tau_j \sigma_0 c_j) = 1 - \delta$, with $\delta$ the doped hole concentration.

Thus, the approximate action is given by

$$S = S_0 + \int_0^\beta d\tau \left[ -U \sum_j \tilde{S}_j \cdot \tau_j \sigma c_j + U \sum_j \tilde{S}_j^2 \right]. \quad (4)$$

Note that the first term in the square brackets has the form of Hund coupling between the localized spin moment and the carrier’s spin.

### III. Evolution of Néel Order by Hole Doping

Now we consider doped holes’ hopping effects on Néel order. Our strategy is the following: First, we estimate semiclassical staggered magnetic moments by solving saddle point equations. Secondly, we examine stability of Néel order against quantum fluctuations. For analysis of quantum fluctuations, we use Schwinger boson mean field theory. [3] In the Schwinger boson mean field theory, the localized spin moment $\tilde{S}_j$ is given by

$$\tilde{S}_j = (-1)^j \tilde{S} \hat{e}_z. \quad (5)$$

Substituting this into Eq.4, and then performing Fourier transforms, we obtain

$$S = \sum_k' \sum_{i \omega_n} \left\{ \tau_k(i \omega_n) \tau_{k+Q}(i \omega_n) \right\} \times \left( \begin{array}{cc} -i \omega_n + \epsilon_k - \mu & -U \tilde{S}_z \\ -U \tilde{S}_z & -i \omega_n + \epsilon_k - \mu \end{array} \right) \left( \begin{array}{cc} \epsilon_k(i \omega_n) \\ \epsilon_{k+Q}(i \omega_n) \end{array} \right) + \beta NU \tilde{S}^2, \quad (6)$$

where $\epsilon_k = -2t(\cos k_x + \cos k_y)$ and the summation in $\bf{k}$-space is taken over half of the first Brillouin-Zone. The energy dispersion of the carriers is given by $\pm E_k$ with $E_k = \sqrt{\epsilon_k^2 + (US)^2}$. After integrating out fermions, we obtain

$$S = -2 \sum_k \left\{ \ln \left[ 1 + e^{-\beta(E_k - \mu)} \right] + \ln \left[ 1 + e^{-\beta(-E_k - \mu)} \right] \right\} + \beta NU \tilde{S}^2 \quad (7)$$

At zero temperature, variation of the action with respect to the chemical potential $\mu$ yields

$$\frac{1}{N} \sum_k [\theta(E_k + \mu) + \theta(-E_k + \mu)] = 1 - \delta \quad (8)$$

and variation with respect to $\tilde{S}$ yields

$$\frac{1}{2N} \sum_k \frac{U}{E_k} [\theta(E_k + \mu) - \theta(-E_k + \mu)] = 1, \quad (9)$$

where $\theta(x) = 1$ for $x > 0$ and zero otherwise.

We compute $\tilde{S}$ by solving saddle point equations (8) and (9). Figure 1 shows $U/t$ dependence of $\tilde{S}$ at half-filling. The same result was obtained in Ref. [5] based on the Hartree-Fock approximation. Figure 2 shows doping dependence of $\tilde{S}$ at $U/t = 6, 8, 12$. Figure 3 shows effective exchange interaction $J(x)/J \equiv (2\tilde{S})^2$. Temperature dependence of the antiferromagnetic correlation length is given by $\xi_{AF} = 0.26 \exp(1.38J(x)/T)$ according to the renormalization group analysis of non-linear sigma model and numerical simulations. [6]

Note that non-zero values of $\tilde{S}$ do not necessarily imply that there is Néel order. Because solutions of saddle point equations neglect quantum fluctuation effect, we need to examine stability of Néel order against quantum fluctuations. For this purpose, we apply the Schrödinger-Peierls mean field theory. [3] In the Schrödinger-Peierls mean field theory, the localized spin moment $\tilde{S}_j$ is represented by

$$\tilde{S}_j = \frac{1}{2} \tilde{\tau}_j \sigma \tilde{z}_j, \quad (10)$$

with the constraint $\sum_{\sigma=\uparrow,\downarrow} \tilde{z}_{j\sigma} \tilde{z}_{j\sigma} = 2 \tilde{S}$. We derive the effective action of the localized spin system by integrating out the carrier fields. Detail of calculations is given in Appendix A. The result is,

$$S_{\text{spin}} = \int_0^\beta d\tau \left[ \sum_{j\sigma} \tilde{\tau}_j \sigma \partial_\tau \tilde{z}_{j\sigma} - \frac{J}{2} \sum_{(i,j)} \tilde{A}_{ij} \tilde{A}_{ij} \right], \quad (11)$$

where $J = 4t^2/U$ and $\tilde{A}_{ij} = \tilde{\tau}_i \tilde{z}_{j\uparrow} - \tilde{z}_{i\uparrow} \tilde{z}_{j\uparrow}$ and $A_{ij} = z_{i\uparrow} z_{j\downarrow} - z_{i\downarrow} z_{j\uparrow}$. (In terms of the fields $\tilde{S}_j$, $S_{\text{spin}}$ is

$$S_{\text{spin}} = \sum_j S_{\text{spin}}^{\text{Berry}} + \int_0^\beta d\tau J \sum_{(i,j)} \tilde{S}_i \cdot \tilde{S}_j, \quad (12)$$

where $S_{\text{spin}}^{\text{Berry}}$ denotes the Berry phase term for the localized spin moment $\tilde{S}_j$. This is nothing but the spin $\tilde{S}$ antiferromagnetic Heisenberg model.)

In the Schrödinger-Peierls theory, Néel order is stabilized if $\tilde{S}$ is larger than $S_c = 0.19660$ as shown in Ref. [3]. (This result is briefly summarized in Appendix B.) The parameter regions of $\tilde{S} > S_c$ and $0 < \tilde{S} < S_c$ are shown in Fig. 4. In the $\tilde{S} > S_c$ regime, Néel order is stabilized. Whereas in the $0 < \tilde{S} < S_c$ regime, Néel order is suppressed by quantum fluctuations. The point $\tilde{S} = S_c$ can be taken as a quantum critical point. [7] The $0 < \tilde{S} < S_c$ regime is called the quantum disordered regime. [8] Note that this quantum disordered regime is identical to that in Ref. [8]. The condition of $\tilde{S} > S_c$ for the stability of Néel order turns out to be $g < g_c$ in Ref. [8].

In Fig. 4, the $\tilde{S} = 0$ regime is also shown. Contrary to the other regimes, there is no antiferromagnetic Heisenberg type correlations in this regime. Therefore, spin
fluctuations in this regime are disconnected to the original spin correlations at half-filling.

IV. QUANTUM DISORDERED PHASE IN THE STRONG COUPLING LIMIT

In this section, we discuss the effective action for the quantum disordered regime in the strong coupling limit. The effective action in the quantum disordered regime is given by

\[
S = \int_0^\beta d\tau \left[ \sum_j \bar{c}_j (\partial_\tau - \mu) c_j - t \sum_{\langle i,j \rangle} (\bar{c}_i c_j + \bar{c}_j c_i) - \frac{U}{2} \sum_j (\bar{c}_j \sigma c_j) (\bar{c}_j \sigma z_j) + \sum_{j\sigma} \bar{c}_{j\sigma} \partial_\tau z_{j\sigma} + \sum_{j\sigma} \lambda_j \left( \bar{c}_{j\sigma} z_{j\sigma} - \bar{S} \right) - \frac{J}{2} \sum_{\langle i,j \rangle} (\bar{c}_{i\uparrow} \bar{c}_{i\downarrow} - \bar{c}_{j\uparrow} \bar{c}_{j\downarrow}) (z_{i\uparrow} z_{j\downarrow} - z_{i\downarrow} z_{j\uparrow}) \right].
\]  

(12)

This action consists of the free fermion part and the Schwinger boson part that describes the spin \( \bar{S} \) antiferromagnetic Heisenberg model. The interaction between them is of the form of Hund coupling. The action (12) is so-called the spin-fermion model.

Now let us consider the strong coupling limit, \( U/t \to \infty \). Since the Hund coupling term is dominant in this regime, we move to a frame in which the Hund coupling term is diagonalized. In such a frame, the doped hole's spin at the \( j \)-site is in the direction of the localized spin moment \( \bar{S}_j \). The transformation to this frame is given by

\[
c_j = U_{ij} f_j,
\]

(13)

where the matrix \( U_{ij} \) is given by

\[
U_{ij} = \begin{pmatrix} z_{ij} & \bar{z}_{ij} \\ \bar{z}_{ij} & \bar{z}_{ij} \end{pmatrix}.
\]

(14)

After this transformation, the action reads

\[
S = \int_0^\beta d\tau \left[ \sum_j \bar{f}_j (\partial_\tau - \mu + \bar{U}_{ij} \partial_\tau U_{ij}) f_j - t \sum_{\langle i,j \rangle} (\bar{f}_i U_{ij} f_j + \bar{f}_j U_{ij} f_i) - \frac{U}{2} \sum_j (-1)^J \bar{f}_j \sigma f_j + \sum_{j\sigma} \bar{z}_{j\sigma} \partial_\tau z_{j\sigma} + \sum_{j\sigma} \lambda_j \left( \bar{z}_{j\sigma} z_{j\sigma} - \bar{S} \right) - \frac{J}{2} \sum_{\langle i,j \rangle} (\bar{f}_{i\uparrow} \bar{f}_{i\downarrow} - \bar{f}_{j\uparrow} \bar{f}_{j\downarrow}) (z_{i\uparrow} z_{j\downarrow} - z_{i\downarrow} z_{j\uparrow}) \right].
\]

(15)

Note that the hopping for the fermions from \( j \)-site to \( i \)-site contains a matrix,

\[
\bar{U}_{ij} U_{ij} = \begin{pmatrix} F_{ij} & -A_{ij} \\ A_{ij} & F_{ij} \end{pmatrix},
\]

(16)

with \( F_{ij} = \bar{z}_{i\uparrow} z_{j\downarrow} + \bar{z}_{i\downarrow} z_{j\uparrow} \) and \( A_{ij} = z_{i\uparrow} z_{j\downarrow} - z_{i\downarrow} z_{j\uparrow} \). Note that \( F_{ij} \) describes ferromagnetic correlations and \( A_{ij} \) describes antiferromagnetic correlations in the localized spin system. In fact, \( \langle F_{ij} \rangle \) is taken for the mean field in the ferromagnetic spin system and \( \langle A_{ij} \rangle \) is taken for the mean field in the antiferromagnetic spin system in the Schwinger boson mean field theory. [3]

A similar action can be derived in the slave-fermion mean field theory of the t-J model. However, there is a crucial difference. If we take the strong coupling limit of \( U/t \to \infty \), then one finds that the fermion hopping only couple to ferromagnetic correlations in the localized spin system. Therefore, coupling between fermions and the gauge field that describes antiferromagnetic fluctuations is absent.

V. SUMMARY AND DISCUSSION

In this paper, we investigate effects of doped holes’ hopping on Néel order. What we have found is that disorder effects induced by holes’ hopping on Néel order is rather small. In fact, the critical doping concentration is \( \delta_c \approx 0.40 \) at \( U/t = 10 \). This value is substantially larger than that in high-temperature superconductors. Therefore, hopping processes of doped holes are not so effective in suppressing Néel order. For destruction of Néel order in high-temperature superconductors, holes must behave like an excitation which suppresses Néel order more effectively. Such an excitation would be intimately connected with properties of the localized spin system.

After destruction of Néel order, the quantum disordered phase appears. (This quantum disordered phase is special to the antiferromagnetic correlations. In fact, there is no such phase in case of the ferromagnetic correlations because there is no quantum fluctuations that suppress the semiclassical long-range order as in the antiferromagnetic case.) If we take the strong coupling limit \( U/t \to \infty \), then the coupling between the doped holes and the antiferromagnetic fluctuations are lost. Here we first derive the spin-fermion model starting from the Hubbard model. After representing the localized spin moments by the Schwinger bosons, we take the strong coupling limit. There is another way of taking this strong coupling limit. If we take the strong coupling limit first at the spin-fermion model, it is believed that the model is reduced to the t-J model. Applying the slave-boson theory, we obtain a system of spinless fermions and bosons with a gauge field interaction. In the derivation of the
slave-boson representation of the t-J model, one big assumption is that there is a deconfinement phase of a U(1) gauge field theory. Whether there is a deconfinement phase or not is still an unsolved issue. By contrast, there is no need to assume a deconfinement phase when we introduce boson fields to describe the localized spin moments at the spin-fermion model. After taking the strong coupling limit, we obtain a system of spinless fermions and bosons but there is no gauge field interaction. Our analysis suggests that taking the strong coupling limit is not justified or indirectly suggests that there is no deconfinement phase. A situation such that taking the strong coupling limit is not allowed occurs when there is a term of spin-orbit coupling like \( H_{so} = i \sum_{i,j} c_i^\dagger \mathbf{A}_{ij} \cdot \sigma c_j + \text{h.c.} \). In the presence of such a term, doped holes rotate their spin at every hopping process. [9] Therefore, we cannot take the strong coupling limit.

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APPENDIX A: DERIVATION OF THE ANTIFERROMAGNETIC HEISENBERG MODEL

In this appendix, we derive the antiferromagnetic Heisenberg model from \( S_0 + S_U \), where \( S_0 \) is defined by Eq. (1) and \( S_U \) is defined by Eq. (4), by applying second order perturbation theory with respect to \( t \). In order to describe the localized spin moments \( S_j \), we introduce the Schwinger bosons through Eq. (10), with the constraint

\[
\sum_{\sigma = \uparrow, \downarrow} z_{j \sigma}^\dagger z_{j \sigma} = 2 S_j.
\]

We integrate out \( \mathbf{f}_j \) and \( f_j \):

\[
S_{\text{eff}} = -\text{Tr} \ln \left( \left( \partial_\tau - \mu + \mathbf{U}_j \partial_{\mathbf{r}_j} U_j - \frac{U}{2} \sigma_z \right) \delta_{ij} - t_{ij} \mathbf{U}_j U_j \right),
\]

where \( t_{ij} = t \) for the nearest neighbor sites and \( t_{ij} = 0 \) otherwise.

We expand the logarithm in Eq. (A4) with respect to \( t_{ij} \). The second order term is

\[
S_{\text{eff}}^{(2)} = \frac{1}{2} \text{Tr} \left[ \frac{1}{\partial_\tau - \mu + \mathbf{U}_j \partial_{\mathbf{r}_j} U_j - \frac{U}{2} \sigma_z} \frac{1}{t_{ij} \mathbf{U}_j U_j} \right].
\]

Applying the derivative expansion technique, we obtain

\[
S_{\text{eff}}^{(2)} = \frac{\beta^2}{\omega_n} \sum_{(i,j)} \sum_{\sigma, \sigma'} \frac{1}{\omega_n + \mu + \frac{U}{2} \sigma} \frac{1}{\omega_n + \mu + \frac{U}{2} \sigma'} \times \int_0^\beta d\tau \langle \mathbf{U}_j U_j(\tau) | \sigma' \rangle \langle \sigma' | \mathbf{U}_j U_j(\tau) | \sigma \rangle + \text{(higher derivatives)}
\]

After the summation over the fermion Matsubara frequencies, we take \( \beta U \to \infty \) limit. Thus, we obtain

\[
S_{\text{eff}}^{(2)} = -\frac{J}{2} \int_0^\beta d\tau \sum_{(i,j)} \mathbf{A}_{ij} A_{ij},
\]

where \( J = 4t^2 / U \) and \( A_{ij} = z_{i \uparrow} z_{j \downarrow} - z_{i \downarrow} z_{j \uparrow} \) and \( \mathbf{A}_{ij} = \mathbf{A}_{i \uparrow} \mathbf{A}_{j \downarrow} - \mathbf{A}_{i \downarrow} \mathbf{A}_{j \uparrow} \).

On the other hand, the expansion of the logarithm in Eq. (A4) with respect to \( \mathbf{U}_j \partial_{\mathbf{r}_j} U_j \) gives the Berry phase term for the localized spin moments as follows. The term with the first order of \( \mathbf{U}_j \partial_{\mathbf{r}_j} U_j \) is

\[
S_{\text{Berry}} = -\text{Tr} \left[ \frac{1}{\partial_\tau - \mu + \frac{U}{2} \sigma_z} \mathbf{U}_j \partial_{\mathbf{r}_j} U_j \right].
\]

Applying the derivative expansion technique, we obtain

\[
S_{\text{Berry}}^{(2)} = \frac{1}{\omega_n} \sum_{\sigma} \frac{1}{\omega_n + \mu + \frac{U}{2} \sigma} \int_0^\beta d\tau \langle \mathbf{U}_j \partial_{\mathbf{r}_j} U_j | \sigma \rangle.
\]

After the summation over the fermion Matsubara frequency, we take \( \beta U \to \infty \) limit. Thus, we obtain

\[
S_{\text{Berry}}^{(2)} = \int_0^\beta d\tau \sum_{j \sigma} \mathbf{z}_{j \sigma} \partial_{\mathbf{r}_j} \mathbf{z}_{j \sigma}.
\]

This is nothing but the Schwinger boson representation of the spin's Berry phase. In terms of the original spin moment fields \( S_j \), the action \( S_{\text{spin}} = S_{\text{Berry}}^{(2)} + S_{\text{eff}}^{(2)} \) turns out to be the antiferromagnetic Heisenberg model.
APPENDIX B: THE COMPUTATION OF $S_c$

Here we briefly summarize the result of Ref. [3] for $S_c = 0.19660$.

In terms of the Schwinger boson fields, which is defined in Eq. (10), the antiferromagnetic Heisenberg Hamiltonian $H = J \sum_{\langle i, j \rangle} \vec{S}_i \cdot \vec{S}_j$, reads

$$H = - J 2 \sum_{\langle i, j \rangle} \left( z_{i\uparrow}^i \zeta_{j\downarrow}^j \zeta_{j\uparrow}^j \zeta_{i\downarrow}^i \right) + 2 J \tilde{S}_N^2.$$  

\hspace{1cm} (B1)

We introduce mean fields $A_{ij} = \langle z_{i\uparrow}^i z_{j\downarrow}^j \rangle$ and $A^{*}_{ij} = \langle \zeta_{i\uparrow}^i \zeta_{j\downarrow}^j \rangle$ and assume the uniform value $A_{ij} = A^{*}_{ij} = A =$ const. Then, the free energy of the system is given by

$$\mathcal{F} = \frac{2}{\beta N} \sum k \ln \left[ 2 \sinh \left( \frac{\beta \omega_k}{2} \right) \right] + JA^2 - \lambda (2 \tilde{S} + 1),$$  

\hspace{1cm} (B2)

where $\lambda$ is a Lagrange multiplier to impose the constraint $\sum_{\sigma} z_{i\sigma}^i z_{j\sigma}^j = 2 \tilde{S}$, and $\omega_k = \sqrt{\lambda^2 - 4 A^2 J^2 \alpha_k^2}$ with $\alpha_k = (\sin k_x + \sin k_y)/2$.

The variation with respect to $\lambda$ and $A$ yields

$$\frac{1}{N} \sum_k \frac{J \alpha_k^2}{\omega_k} \coth \left( \frac{\beta \omega_k}{2} \right) = \frac{1}{2},$$  

\hspace{1cm} (B3)

$$\frac{1}{N} \sum_k \frac{1}{\omega_k} \coth \left( \frac{\beta \omega_k}{2} \right) = \frac{2 \tilde{S} + 1}{\lambda}.$$  

\hspace{1cm} (B4)

At zero temperature, Eq. (B4) has the following form:

$$\tilde{S} = \frac{1}{2} \left( 4 \pi^2 \int_0^1 d \gamma K(\sqrt{1 - \gamma^2}) \sqrt{1 - p^2 \gamma^2} - 1 \right),$$  

\hspace{1cm} (B5)

with $K$ the complete elliptic function and $p = 2AJ/\lambda$. The right hand side is a monotonically increasing function with respect to $p$ and it takes the maximum value $S_c \equiv 0.19660$ at $p = 1$. Therefore, Eq. (B5) has a solution of $p < 1$ for $\tilde{S} < S_c$. Solutions of $p < 1$ imply that spin wave excitation have gap, and there is no Bose-Einstein condensation of Schwinger bosons. [11]. Therefore, there is no Néel order for $\tilde{S} < S_c$.

[1] H. J. Schulz, Phys. Rev. Lett. 65, 2462 (1990).
[2] See, also, Z. Y. Weng, C. S. Ting, and T. K. Lee, Phys. Rev. B 43, 3790 (1991).

\hspace{1cm} [3] D. P. Arovas and A. Auerbach, Phys. Rev. B 38, 316 (1988).
[4] See, for example, T. Moriya, Spin Fluctuations in Itinerant Electron Magnetism, Springer-Verlag, Berlin (1985).
[5] A. Singh and Z. Tesanović, Phys. Rev. B 41, 614 (1990).
[6] E. Manousakis, Rev. Mod. Phys. 63, 1 (1991).
[7] J. A. Hertz, Phys. Rev. B 14, 1165 (1976); A. J. Millis, Phys. Rev. B 48, 7183 (1993).
[8] S. Chakravarty, B. I. Halperin, and D. R. Nelson, Phys. Rev. Lett. 60, 1057 (1988).
[9] T. Morinari, Phys. Rev. B 65, 064513 (2002).
[10] P. Lacour-Gayet and M. Cyrot, J. Phys. C 7, 400 (1974).
[11] D. Yoshioka, J. Phys. Soc. Jpn. 58, 3733 (1989); S. Sarker et al., Phys. Rev. B 40, 5028 (1989).
FIG. 3. The doping dependence of the effective exchange interaction $J(\delta)/J \equiv (2\tilde{S})^2$ for $U/t = 6, 8, 12$.

FIG. 4. The Néel order regime $\tilde{S} > S_c$ (Néel) and the quantum disordered regime $0 < \tilde{S} < S_c$ (QD) on the $U/t$-$\delta$ plane. In the $\tilde{S} = 0$ regime, there is no antiferromagnetic correlation that is associated with the Heisenberg antiferromagnetic type correlation.