Chaotic star formation and the alignment of stellar rotation with disc and planetary orbital axes

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ABSTRACT
We investigate the evolution of the relative angle between the stellar rotation axis and the circumstellar disc axis of a star that forms in a stellar cluster from the collapse of a turbulent molecular cloud. This is an inherently chaotic environment with variable accretion, both in terms of rate and the angular momentum of the material, and dynamical interactions between stars. We find that the final stellar rotation axis and disc spin axis can be strongly misaligned, but this occurs primarily when the disc is truncated by a dynamical encounter so that the final disc rotation axis depends simply on what fell in last. This may lead to planetary systems with orbits that are misaligned with the stellar rotation axis, but only if the final disc contains enough mass to form planets. We also investigate the time variability of the inner disc spin axis, which is likely to determine the direction of a protostellar jet. We find that the jet direction varies more strongly for lighter discs, such as those that have been truncated by dynamical interactions or have suffered a period of rapid accretion. Finally, we note that variability of the angular momentum of the material accreting by a star implies that the internal velocity field of such stars may be more complicated than that of aligned differential rotation.

Key words: accretion, accretion discs – stars: formation – stars: interiors – stars: rotation – planetary systems: formation – planetary systems: protoplanetary discs

1 INTRODUCTION
Over the last 20 years or so our picture of the environment in which stars form and the way in which they do so has changed considerably. With a few notable exceptions (for example, Larson 1978), prior to 1990 the process of star formation was treated by forming one star at a time (see, for example the reviews by Shu, Adams & Lizano, 1987; McKee & Ostriker, 2007). The formation of binary stars was also considered, but mainly through numerical simulation of the dynamics of rotating rings or bars of material (see reviews by Bodenheimer & Burkert, 2001; Bonnell, 2001). In all these scenarios, the symmetries are such that the rotation axes of any stars formed tend to remain aligned with the rotation axis of any surviving circumstellar material or disc. Thus these simple models all predict that protostellar discs, and therefore any planetary system formed within them, will be closely aligned with the rotation axis of the central star. This expectation is reinforced by observation of the one system for which we have good data – the Solar System (Tremaine, 1991). We need to be aware, however, that while the solar system may be a good guide to the formation of planetary systems in general, because of our special place within it, it may also be atypical in some ways (e.g. Beer et al., 2004). In this paper we consider how an examination of the degree to which stellar rotation axes are aligned with their protostellar discs and/or planetary systems can give us information about the way in which stars form. This is becoming a relevant consideration, given a recent amount of growing evidence of star-disc misalignment from observations of spin-orbit misalignment of transiting exoplanets (Hébrard et al., 2008; Winn et al. 2009a, 2009b; Pont et al., 2009a, 2009b; Gillon 2009; Johnson et al., 2009; Narita et al. 2009).

The fact that many, if not most, stars are members of binary or multiple systems has led to the argument that the star formation process must be much more dynamical and interactive than the simple pictures described above (e.g. Pringle, 1989, 1991; Clarke & Pringle, 1991). Early multiplicity surveys of pre-main-sequence stars in nearby star-forming regions confirmed that most low-mass stars in the solar neighbourhood form in multiple systems (e.g. Leinert et al., 1993; Ghez, Neugebauer & Matthews 1993; Simon et
Indeed most stars within the Galaxy appear to form in clustered environments (see the reviews by Lada & Lada, 1995, Clarke, Bonnell & Hillenbrand, 2000) and there is evidence that disc sizes and presumably disc lifetimes are reduced in binary as compared to single stars (Cieza et al., 2009). The theoretical expectation is that close encounters in the early stages of formation when stars have substantial discs lead to substantial disc truncation and moreover, that the tidal effects of such encounters can also lead to the outer parts of the disc becoming strongly misaligned or warped (Heller, 1995; Hall, Clarke & Pringle, 1996; Larwood, 1997; Pfalzner, 2004; Moeckel & Bally, 2006). Such interactions imply that there is a strong possibility that the stellar rotation axis (determined by the angular momentum of material accreted early which forms the stars) is not aligned with the disc rotation axis (which might correspond to the angular momentum of material accreted later which forms the disc, and eventually planets).

Recent numerical simulations of the star formation process (Bate, Bonnell & Bromm, 2002a,b, 2003; Bate & Bonnell 2005; Bate 2005, 2009a,b) have considered the formation of stars within clusters, starting with initial turbulent conditions for the gas which are designed to mimic those seen within star forming molecular clouds. The picture which results involves turbulent and chaotic motions of both gas and stars, with disc fragmentation, competitive accretion and close dynamical interactions all playing a role. Close star-disc encounters occur throughout the star formation process at a variety of orientations. From this perspective it might seem reasonable to conclude the star-disc misalignment would be the norm rather than the exception. Here we investigate this possibility.

Because of the large range of scales involved it is not feasible to follow the evolution of gas starting from the turbulent molecular cloud (tens of parsecs) down to the sizes of protostellar discs (tens of au) and further to the accretion of matter onto the protostars themselves (a few solar radii). Numerical simulations of the star formation process begin with a turbulent molecular cloud but truncate the computation typically at a resolution scale of several au by means of sink particles. In this paper we are interested in what happens to the gas when it falls within the sink particles. We are therefore constrained to carry out a highly idealised computation using a strongly simplified, and not necessarily self-consistent, set of assumptions. We consider the evolution gas falling into a sink particle taken from a particular simulation. In Section 2 we describe the simulation. In Section 3 we describe the simplifications we have made in order to model the evolution of the gas when it has fallen into the sink particle from the formation of the sink particle (≈ 10^-3 M⊙) until after accretion onto the sink particle has finished and the 'star' is more or less fully formed with mass ≈ 0.2 M⊙. In Section 4 we present the results of our calculations. We discuss the implications in Section 5.

2 SIMULATION DATA

We are interested here in illustrating the growth of a particular protostellar core. The data we use comes from a computation by Bate, Bonnell & Bromm (2003) which describes the formation of a cluster of stars from a turbulent self-gravitating parcel of dense (molecular) gas. The details of the numerical procedures are described fully in that paper, and here we draw attention to those details of particular relevance to the calculations below.

The calculation was performed using a 3D smoothed particle hydrodynamics (SPH) code. The smoothing lengths are variable in time and space, subject to the constraint that the number of neighbours for each particle must remain approximately constant at N_{neigh} = 50. The initial cloud is spherical and uniform in density with a mass of 50 M⊙ and diameter of 0.375 pc (77,400 au). At the initial temperature of 10 K, the thermal Jeans mass is 1 M⊙. The free fall timescale of the cloud is t_{ff} = 1.90 × 10^5 yr. An initial supersonic turbulent velocity field is imposed on the cloud with rms Mach number M = 6.4. The field is divergence-free and random Gaussian with a power spectrum P(k) ∝ k^{-4}, where k is the wavenumber. The local Jeans mass is resolved throughout the calculation. The number of particles used is 3.5 × 10^6, which means that each particle has a mass of 1.43 × 10^{-5} M⊙.

The calculation uses a barotropic equation of state that is isothermal up to densities of 10^{-13} g cm^{-3}, but with the temperature of the gas increasing at higher densities. In this way, the calculations mimic the opacity limit for fragmentation (Low & Lynden-Bell 1976; Rees 1976). Despite this, because of computational constraints, it is necessary to excise the high density protostellar cores from the bulk of the computation. This is done by inserting a sink particle (Bate et al., 1995) whenever the central density of a pressure-supported fragment exceeds a certain density, ρ_{crit} = 10^{-11} g cm^{-3}. The sink particle is formed by replacing the SPH gas particles contained within a radius R_{sink} = 5 au of the densest gas particle by a point mass with the same mass and momentum. Any gas that later falls within this radius is accreted by the point mass if both (a) it is bound and (b) its specific angular momentum is less than that required to form a circular orbit at radius R_{sink} from the sink particle. Sink particles interact with the gas only via gravity and by accretion. Sink particles interact with each other only by gravity. The gravitational acceleration between two sink particles is softened within a distance of 4 au. Thus sink particles cannot merge with each other. The typical initial mass of a sink particle in this calculation is ≈ 10^{-3} M⊙.

3 EVOLUTION EQUATIONS

As we have seen, because of resolution constraints, in the numerical simulations of chaotic star formation the 'stars' formed are represented by sink particles. Because of this, from the simulations we only have very basic information about material being accreted onto the protostellar cores. We have information about the rates of accretion of both mass and angular momentum onto the sink particles, but note that the accretion is discretized into that of individual SPH particle masses of ≈ 10^{-5} M⊙. The sink particle radius is set at 5 au., which is slightly less than the peak in the distribution of binary star separations for low mass stars. We also have information about the distance to the nearest neighbouring sink particle as a function of time. It is usually the case that for some of the time this distance is less than the radius of the sink particles.
Alignment of disc and stellar rotation axes

Since the information about the accretion process onto sink particles is so limited, we are strongly constrained in how we are able to model what might happen to the gas when it is accreted within the sink particle radius of $5 \text{ au}$. We proceed by making the simplest assumptions we can about the gas flow within the sink particle, noting that some of the implications of these are of necessity at times not self-consistent.

Within the sink particle, we model the gas flow as a twisted accretion disc around a central mass. The evolution of such a disc is computed in the manner described by Pringle (1992). The disc surface density is given as a function of time and radius by $\Sigma(r, t)$. Each disc annulus, at radius $R$, is assumed to rotate with angular velocity $\Omega(r, t)$ around the central mass, and to have a spin in the direction of the unit vector $\mathbf{l}(R, t)$. Thus the disc locally has an angular momentum density given by $\mathbf{L}(R, t) = \Sigma R^2 \Omega$. We shall also assume (cf. Lin & Pringle 1990) that any matter being added to the disc is added at the radius corresponding to its specific angular momentum, so that it arrives already on a circular orbit and in centrifugal balance.

Then (Pringle 1992) the evolution of the angular momentum density is given by

$$\frac{\partial \mathbf{L}}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left\{ \frac{1}{2} \Omega \Sigma R^2 \left[ \frac{\partial}{\partial R}(\Omega R^2 \Sigma) \right] \right\}$$

$$+ \frac{1}{R} \frac{\partial}{\partial R} \left\{ \frac{1}{2} \nu_2 R^2 \frac{\partial \mathbf{L}}{\partial R} \right\}$$

$$+ \frac{1}{R} \frac{\partial}{\partial R} \left\{ \left[ \frac{(1/2) \nu_2 R^2 \Omega \partial \mathbf{L}}{\partial R} \right] \right\}$$

$$+ \nu_1 \left( \frac{\partial \mathbf{L}}{\partial R} \right) \mathbf{L}$$

$$+ \mathbf{S}(R, t). \tag{1}$$

Here $\Omega' = d\Omega/dR$, $\nu_1$ is the shear viscosity normally associated with accretion discs and $\nu_2$ is associated with straightening out the orbit-of-planer motions (i.e. the warp or twist). The source term $\mathbf{S}(R, t)$ allows for the addition of material.

When the sink particle forms it has a mass, typically around $10^{-3} \, M_\odot$, which is much less than the eventual mass of the star. Thus during the accretion process we expect the overall mass of the star+disc system to grow by a factor of around $10^2 - 10^3$. To calculate the angular velocity $\Omega(R)$ we need to take account of the central time-varying force contributions from both the central star, mass $M_*(t)$, and the disc itself. Although an exact determination of $\Omega(R)$ could be made (e.g. Bertin & Lodato 1999), in general we shall find that the disc mass is much less than that of the central star. Thus for computational convenience (cf. Lin & Pringle 1990) we adopt the approximation

$$\Omega(R, t) = \left( \frac{GM_*(t)}{R^3} \right)^{1/2}, \tag{2}$$

where

$$M(R, t) = M_*(t) + \int_0^R \Sigma(R, t) 2\pi R dR \tag{3}.$$

In order to estimate the magnitudes of the viscosity terms it is strictly necessary to compute the disc structure at each radius. For protostellar discs this is not a straightforward matter and the physical processes relevant at various radii are still a matter for debate (see for example Lodato & Rice 2004, 2005; Kratter, Matzner & Krumholz, 2008; Terquem 2008). What would be required first is a detailed time-dependent model of the evolution of the disc surface density distribution, similar to that carried out by Lin & Pringle (1990), but taking more recent concepts and understanding into account (see, for example, Armitage, Livio & Pringle 2001; Rice & Armitage 2009; Zhu, Hartmann & Gammie 2009). In addition, a secondary, but nevertheless important, consideration would be to take account of how such discs might respond to misalignments – for example, the low viscosity regions (the dead zones) might transfer warp in a wave-like, rather than in a diffusive, manner, and gravitational bending torques might play a role in disc regions where self-gravity is important (Papaloizou & Lin 1995).

With these issues in mind, and in order to simplify matters in this initial investigation, we make the following assumptions. We assume that for each viscosity, i.e. for $i = 1, 2$, $\nu_i = \alpha_i \left( \frac{H}{R} \right)^2 R^2 \Omega.$ \tag{4}

Here $H$ is the disc semi-thickness, and we shall assume for convenience that $H/R = 0.1$, independent of radius (cf. Bell et al., 1997; Terquem, 2008). Thus $\alpha_1$ is the usual Shakura & Sunyaev (1973) viscosity parameter and we take typically (cf. King, Pringle & Livio, 2007)

$$\alpha_1 = 0.02, \tag{5}$$

although we also consider one case with $\alpha_1 = 0.2$. Typically $\alpha_2 \gg \alpha_1$ (Papaloizou & Pringle 1983) and we shall take (Lodato & Pringle 2007)

$$\alpha_2 = 2. \tag{6}$$

As far as the evolution of the system is concerned, what matters most are the relative sizes of the various physical timescales. Thus to zeroth order, the details of the viscous processes are less important than the magnitude. This gives rise to the viscous timescale for mass flow through the disc which is then given by

$$t_{\nu_1} \approx 9000 \left( \frac{\alpha_1}{0.02} \right)^{-1} \left( \frac{H}{0.1 R} \right)^{-2} \left( \frac{M}{M_\odot} \right)^{-1/2} \left( \frac{R}{5 \text{ au}} \right)^{3/2} \text{ yr}. \tag{7}$$

The timescale on which warp is propagated through the disc is less than this and is

$$t_{\nu_2} \approx 90 \left( \frac{\alpha_2}{2} \right)^{-1} \left( \frac{H}{0.1 R} \right)^{-2} \left( \frac{M}{M_\odot} \right)^{-1/2} \left( \frac{R}{5 \text{ au}} \right)^{3/2} \text{ yr}. \tag{8}$$

3.1 Numerical details

We take a fixed radial grid with inner radius $R_{\text{in}} = 10R_\odot$. The choice of an outer radius of the grid presents us with a consistency problem. Some of the sink particles in the calculation have resolved discs outside of the sink particle radius. The interactions between these discs and the discs which we model within the sink particles is not accounted for. This is problematic because the timescale governing the propagation of warps is much shorter than the viscous evolution timescale. Therefore, the direction of the spin axis in the disc inner parts is affected by that of the spin axis in the disc outer parts, which may contain most of the mass. However, even if the simulation of Bate et al. were to be performed...
again, there is no way to provide a continuous and consistent link between what is inside and what is outside the sink particle. When sink particles were originally invented, even the construction of boundary conditions that tried to minimise the unphysical effects on pressure and viscosity that were introduced outside the sink particle accretion radius proved extremely difficult to implement (Bate 1995; Bate, Bonnell & Price 1995). There is certainly no way to link the propagation of warps across the accretion radius. Instead, for the calculations presented here, we have chosen one of the many sink particles that does not have a resolved disc outside of the sink particle radius.

However, even for those sink particles not surrounded by resolved discs, much of the material accreted onto the sink particle has an angular momentum which would put it on a circular orbit at just inside the sink particle radius of 5 au. Thus we expect the disc evolution to lead to disc expansion outside 5 au. This is of course not permitted in the original calculation. However, it would be unrealistic to choose the outer disc radius at 5 au and so to simply remove all disc mass which expands beyond 5 au as that leads to a significant fraction of the material accreted onto the sink particle not being accreted by the central star. Instead it would be lost back to the computational domain. Thus whatever choice we make leads to an inconsistency. As a compromise we choose the outer disc radius to be \( R_{\text{out}} = 50 \) au and note that we therefore allow the disc to expand outside the radius of the sink particle \( R_{\text{sink}} = 5 \) au.

We employ an explicit first-order finite difference scheme, as detailed in Pringle (1992), to solve the time-evolution of equation [1]. We take 120 logarithmically spaced grid points. Thus the ratio of radii of neighbouring grid points is 1.06 so that \( dR/R = 0.06 \). The timestep is adjusted to satisfy the usual numerical stability criteria. For most of the time it is set by the diffusion time-scale across the innermost radial zone.

At each time step the following procedure is carried out.

1. Mass is added to the disc at the appropriate radius. This is the radius at which the specific angular momentum of the added matter equals that of the local disc material. The largest radius at which matter can be added corresponds to the the radius of the sink particle \( R_{\text{sink}} = 5 \) au which is a factor of 10 smaller than the outer disc radius at all times. If the added material has sufficiently small angular momentum that it lies within the inner disc radius \( R_{\text{in}} = 10R_{\odot} \) then the mass and angular momentum is added to the central star. Because the mass of individual SPH particles are quite massive compared to the mass of the disc we spread the accretion of mass onto the disc in time by assuming that at each time \( t \) the accretion rate is a constant given by the mass of an SPH particle divided by the time between its arrival and that of its predecessor.

2. At this stage because mass has been added, and because disc evolution has taken place (see step 3), the angular velocity profile of the disc must be adjusted. First the angular velocity profile is updated using equation [2] Then the amount of material in each grid zone is adjusted to restore centrifugal balance. To achieve this we move material inwards in the manner described by Bath & Pringle (1981; see also Lin & Pringle, 1990). This process conserves angular momentum to machine accuracy, but conserves mass only to the order of the numerical scheme.

3. The evolution of the disc angular momentum distribution is computed from equation [1]. The numerical scheme is written in such a manner that it conserves angular momentum to machine accuracy, but conserves mass only to the order of the scheme. At the inner edge of the grid we apply a zero-torque boundary condition. Mass and angular momentum are freely accreted onto the star, and the stellar mass \( M_*(t) \) and angular momentum \( L_*(t) \) are appropriately updated. At the outer edge of the grid we allow angular momentum (and mass) to flow freely outwards.

3.2 Mass conservation

We have noted that the although the various steps in the numerical scheme conserve angular momentum to machine accuracy (except for the small loss at the outer disc edge), they do not conserve mass. We show in the Appendix that for Step 2 this leads at each timestep to a systematic overestimate of the mass added by a factor of order \( (dR/R)^2 \). Similar considerations apply to Step 3.

3.3 Interaction with other sink particles

At each time we have the distance to the nearest sink particle \( R_{\text{next}}(t) \) and have noted that it is often the case the \( R_{\text{next}} < R_{\text{sink}} \). We are only able to take the interaction with the neighbouring sink particle into account in an idealised fashion. To do so we take note of the finding (e.g. Hall et al, 1996) that the main effect of the interaction between a disc and the fly-by of a point mass is to unbind the outer disc regions and so to truncate the disc. Here we shall mimic this interaction by removing all disc material which is at radii \( R < 0.5R_{\text{next}} \). We also note that an encounter is likely to perturb the angular momentum of the disc, particularly near the truncation radius. We are forced to neglect any such effect, but we note that a typical dynamical encounter is also associated with the accretion of new material into the sink particle and that the addition of this new material, whose angular momentum is likely to be uncorrelated with that of the existing star and disc, is likely to be more important than the tidal effect of the encounter on the pre-existing disc. Indeed, as will be seen in the calculations presented below, when truncation of the disc through interactions with other sink particles is included, the stellar and disc axes ending up strongly misaligned because the angular momentum of the new material is unrelated to the star’s angular momentum. If anything, including the perturbation suffered by the pre-existing disc during an encounter would typically serve only to increase the degree of misalignment, not decrease it, and therefore would not alter the fundamental result.

3.4 Spin directions

Since we know the total angular momentum accreted onto the central star we can compute the direction of that vector in space. We use an arbitrary fixed coordinate system and denote the space direction of the stellar spin vector in spherical polar coordinates as \( (\theta_*(t), \phi_*(t)) \). If and when the star is able to smooth out any internal differential rotation this would represent the stellar rotation axis.

We also measure the spin direction of the disc. We take
this to be represented by the spin direction \((\theta_d(t), \phi_d(t))\) of the annulus close to the inner disc edge at \(\approx 3.2R_{\text{in}} = 32R_\odot\).

We choose this radius as it gives a reasonable estimate of the direction in which any jet might be launched, since the velocity of such jets indicates that they must be launched from close to the central star (e.g. Pringle, 1993; Livio, 1997; Price, Pringle & King 2003). In addition, by the end of the calculation at a time of \(5 \times 10^4\) yr, taken to be well after the last accretion event, the disc is essentially coplanar, and so this direction represents the plane of the protostellar disc and any subsequent planetary system.

4 RESULTS

In this section we consider the accretion history of one particular sink particle taken from the 50 which formed during the calculation. Because of the inherent uncertainties and inconsistencies of dealing with closely interacting sink particles we choose to follow a sink particle which ends up as a single star. The total mass accreted onto the sink particle is shown as a function of time from the formation of the sink particle in Figure 1 (black solid line). It can be seen that accretion continues until a time of \(8 \times 10^{11}\) s = \(2.5 \times 10^4\) yr. The total mass accreted is \(4 \times 10^{32}\) g \(\approx 0.2\) \(M_\odot\). Also in Figure 1 we show the distance to the nearest sink particle \(R_{\text{next}}\) as a function of time (red short-dashed line). The sink particle is the second sink particle that formed in the original hydrodynamical calculation and starts its evolution as a companion to the first sink particle formed in the calculation. As can be seen from Figure 1 for some \(2 \times 10^4\) yr these two sink particles form a binary with separation of around 20 au. After that, the binary encounters a tighter binary consisting of the 3rd and 10th objects to form in the calculation. After a chaotic interplay lasting about 3000 years, during which the distance of closest approach occasionally become as small as 1 au, the sink particle we are considering is ejected from the system (at the time of \(2.5 \times 10^4\) yr in Figure 1) leaving objects 3 and 10 as a very tight 1 au binary (that survives until the end of the calculation) with the first sink particle as a wide companion to the tight binary with a separation of about 60 AU. This wider companion does not survive until the end of the calculation – it is unbound during another dynamical encounter 5000 years later. Accretion onto the sink particle that we consider in this paper comes to an abrupt end when it is ejected from the multiple system and is left as a single star. The computation continues until a time \(5 \times 10^4\) yr has elapsed.

As can be seen from Figure 1 the accretion rate is not a constant. Moreover, the accretion onto the sink particle occurs from a variety of directions. We have seen that it is not possible to make a set of fully consistent assumptions about how to treat the material that falls into the sink particle. For this reason we shall consider two extreme possibilities. First, we follow the evolution of the disc ignoring any interaction with neighbouring sink particles. In this way we model a star forming in the centre of a protostellar disc which is being fed material from a variety of directions. Second, we take account of the interaction of neighbouring sink particles which pass through the disc by simply truncating the disc in the manner described in Section 3.3 and removing the truncated matter. This is, of course, not fully realistic.
Figure 3. Same as Figure 2 except that the disc viscosity is an order of magnitude larger at \( \alpha = 0.2 \). Note that the mass passes through the disc more quickly than in the low viscosity case and so the central star grows in mass more quickly (a significant reservoir of material). The disc does not maintain a significant reservoir of material.

Figure 4. Same as Figure 2 except that the disc viscosity is an order of magnitude larger at \( \alpha = 0.2 \). Note that the mass passes through the disc more quickly than in the low viscosity case and so the central star grows in mass more quickly and because the disc drains quickly, accretion onto the central star stops soon after the accretion into the sink particle ends. The disc does not maintain a significant reservoir of material.

as in reality the material should be put back into the SPH computational domain. There it would interact with other circumstellar material and would stand a chance of being re-accreted.

4.1 Without sink particle interaction

We first consider the evolution of the system without taking account of any interaction with the neighbouring sink particle. As we mentioned above, although not strictly self-consistent, this serves to illustrate the general behaviour of a star-disc system which is accreting material with a wide range of angular momenta.

We consider the case when the disc viscosity is given by \( \alpha = 0.02 \). In Figure 2 we show the accretion rate onto the central star as a function of time (solid black line) and the time-evolution of the directions of the stellar rotation axis (solid black line) and of the rotation axis of the inner disc (long-dashed lines). At the end of the computation the stellar mass is \( M_\ast = 0.113 \, M_\odot \) and the disc mass is \( M_d = 3.4 \times 10^{-2} \, M_\odot = 0.3 \, M_\ast \) (self-gravity is likely to play a role in providing the effective viscosity of the disc in this particular case; Lodato & Rice 2004, 2005). Thus about a quarter of the mass accreted onto the sink particle has been lost at the outer disc edge. This is to be expected since much of the matter is accreted with an angular momentum corresponding to a centrifugal radius \( \approx R_{\text{sink}} \) and angular momentum conservation leads to the disc expanding beyond this radius. The fraction of matter required to carry away the angular momentum to radius \( R_{\text{out}} \) is approximately \( \sqrt{R_{\text{sink}}/R_{\text{out}}} \approx 0.32 \).

For \( \alpha = 0.02 \) the accretion timescale (Equation 7) from where most of the matter is accreted into the disc (at \( R < R_{\text{sink}} = 5 \, \text{au} \)) is comparable to the typical timescale for the growth of stellar mass (\( \approx 2 \times 10^4 \, \text{yr} \)). Thus, there is still a non-negligible amount of mass in the disc at the end of the computation. At that time the accretion rate onto the star is \( M \approx 3.8 \times 10^{-7} \, M_\odot \, \text{yr}^{-1} \). Comparing these values with the conventional picture of the evolution of low-mass stars, our modelling corresponds to the Class 0/I phases of protostellar evolution and ends at a stage comparable to the beginning of the Class II or T-Tauri phase, both in terms of disc mass and stellar accretion rates. During the evolution, the stellar rotation axis and the disc spin axis both converge quite quickly to their final values. Because the disc evolution timescales are comparable to the stellar accretion timescale, although its spin direction is somewhat influenced by late accretion of material (especially when that material is accreted at radii close to the inner disc radius where the disc spin is measured) it does not differ substantially from that of the star. At the end of the computation the angular distance between these two axes is \( 4.2^\circ \).

In Figures 3 and 4 we consider an identical case, except that the disc viscosity is increased by an order of magnitude to \( \alpha = 0.2 \). In this case the accretion timescale is much less than the timescale for the growth of the stellar mass. Thus, the gas passes through the disc quickly, the central star grows more quickly in mass (Figure 3), the disc mass is lower at any given time, and significant accretion from the disc onto the central star ceases essentially at the same time as the accretion into the sink particle ends (Figure 4). Because the disc mass is lower, the spin of the inner
Alignment of disc and stellar rotation axes

disc is perturbed more strongly by incoming material and so the stellar and disc spins are generally less aligned with each other than in the low-viscosity case. At the end of the computation, the stellar mass is $M_\star = 0.122 \, M_\odot$, the disc mass is only $M_d = 3.0 \times 10^{-5} \, M_\odot = 2.5 \times 10^{-3} \, M_\star$, the accretion rate onto the star is only $\dot{M} \approx 2.3 \times 10^{-3} \, M_\odot$ yr$^{-1}$, and the angle between the stellar and disc rotation axes is 21.3$^\circ$.

4.2 With sink particle interaction

We have seen in Figure 1 that throughout the time accretion is taking place there is a nearby sink particle in close attendance. For the first $2 \times 10^4$ yr or so the sink particle is sufficiently far away ($R_{\text{next}} \approx 20$ au) that it does not interfere strongly with the evolution of the disc, although its occasional sorts to within 10 au, imply (under our assumptions of disc truncation) that loss of angular momentum from the outer disc regions can be significant. For the final $\approx 5000$ yr of accretion, the neighbouring sink particle regularly approaches within 4 au, before becoming unbound. Thus during late accretion the disc is severely truncated. This implies that the material which is in the disc at the end of the computation will have arrived only shortly before the neighbour sink particle vanished and accretion terminated. For comparison with the above we take $\alpha_\odot = 0.02$. In Figure 5 we show the amount of material which has accreted onto the central star as a function of time (long-dashed blue line). At the end of the computation the stellar mass is $M_\star = 1.1 \times 10^{-2} \, M_\odot$ and the disc mass is $M_d = 2.2 \times 10^{-4} \, M_\odot = 2.1 \times 10^{-3} \, M_\star$. The accretion rate is $\dot{M} \approx 10^{-10} \, M_\odot$ yr$^{-1}$. Thus the effect of a nearby companion has been to severely decrease the amount of mass accreted onto the central star. This is, of course, a direct result of our somewhat extreme assumptions about truncation.

In Figure 6 we show the angular coordinates of the stellar angular momentum vector $(\theta_\star, \phi_\star)$ and of the inner disc spin vector $(\theta_d, \phi_d)$ as functions of time. In this case it is apparent that while the companion sink particle keeps its distance, the spin axes evolve more or less in tandem as before, except that since the disc is now less massive it is more easily buffeted by incoming material. Thus we see that the disc spin axis can be quite variable throughout the accretion process (The points here are separated by 10 year intervals.). Moreover, during the brief final episode of chaotic close encounters disc spin axis becomes erratic as the disc is stripped. The upshot of this is that the final disc spin axis is unrelated to that of the star, with the angular distance between them being 122$^\circ$.

5 DISCUSSION

We have considered in an approximate and idealised manner what happens to gas when it falls into a sink particle in a numerical simulation of a cluster of stars. Material is accreted onto the sink particles at a variable rate and from a variety of directions. The aim of our calculation has been to try to understand how the direction of the stellar rotation axis might evolve, how the direction of the spin axis of the inner disc (presumably related to the direction of any accretion-induced jet) might evolve, and to what extent, if
any, the final stellar spin direction and the final disc plane (and hence any resulting planetary orbits) might be related.

We have limited ourselves to considering the evolution of a single sink particle from a particular simulation. In addition because of the limitations of the data available, and the nature of assumed accretion by sink particles, the nature of the computations we are able to carry out are of necessity not fully self-consistent and contain various idealisations. Thus our efforts here should be regarded as an illustrative exploration of the kind of things that might happen rather than definitive predictions. Nevertheless we feel that are able to draw some general conclusions which indicate that further consideration of this problem will be worthwhile.

The evolution of the direction of the stellar spin axis is probably reasonably well modelled. It illustrates how the spin axis might evolve given that the angular momentum of the material from which the star is forming is significantly variable in direction. The total stellar spin direction converges quite quickly to its final value. This is because the inner disc direction is controlled quite closely by the plane of the outer disc (equation 8) and thus the average spin direction of material arriving in the disc is communicated efficiently to the star. Thus to a first approximation the stellar spin direction just reflects the sum of everything that arrives and once most of the mass has been accreted it is essentially fixed. What we do not know, however, is how the direction of the surface spin of the star relates to its total angular momentum. Since these stars are fully convective for much of the time it may be that the surface follows the total quite well. But we have here a significant difference from the usual considerations in that the accretion does not all occur axi-symmetrically and thus the internal mixing of angular momentum in the star is much more complicated than what is usually considered. This has implications for dynamo driving – the driving is likely to be quite severe as the star sorts out the fact that different shells started rotating about different axes. Thus the usual relationships between surface rotation and magnetic activity might need to be modified. Thus for very young stars we might expect that the dynamo activity depends as much on accretion history as on its surface rotation rate. Further, as stars with masses above 0.4 M⊙ approach the main sequence, their cores become radiative and so able to decouple from the convective outer layers. It might be that these central radiative regions can retain some memory of the variable spin direction of the formation process.

As a measure of the direction of disc spin we have taken the direction of the angular momentum vector of the innermost disc radii. We do this for two reasons. First, while accretion is active, this is likely the direction of any accretion-driven jet. And, second, because the warp smoothing timescale (equation 8) is relatively short, once accretion onto the disc has ceased this is a good measure of the spin of the final disc as a whole. Given the limitations of the analysis discussed above, we limit ourselves to general comments about our expectations for the evolution of the direction of disc spin. We take as two possibly extreme examples of the nature of the accretion process the case when the disc retains a substantial fraction of the total mass (Figures 2 and 3, 4) and the case when the disc contains little mass, either because it is severely truncated by interactions with a neighbouring star (Figures 5, 6) or has undergone rapid accretion (Figures 8, 9).

Since the accretion process is variable both with regard to mass flux and with regard to vectorial angular momentum direction, the disc spin can in both cases be variable during the process of star formation. While the angular momentum of the accreted material is relatively high, so that the mass is added at relatively large radii, the accretion process is smoothed by the diffusive effects of the disc and the direction changes fairly smoothly. However, as is particularly evident in the truncated case for which the disc mass is low, there are times when the angular momentum of the material being accreted is sufficiently low that material is added at small radii and the disc direction changes rapidly. Thus during the period of close interaction when the disc is continuously truncated by repeated close encounters, the disc mass is low and so the smoothing effect of the disc is reduced. At these times the disc spin direction can vary quite strongly and quite rapidly. This also occurs without truncation if the disc mass is low because the viscosity is high. It has long been recognised that the outflows from young stars – the Herbig-Haro flows and jets – can provide probes of early stellar evolution (Reipurth & Bally, 2001). It was argued early on (Stahler 1994) that the broad molecular flows seen around young objects are most likely driven by entrainment by high speed jets emanating close to the central star and it was also clear that a jet with variable direction is more able to entrain material efficiently (Stone & Norman, 1994; Smith et al., 1997). In addition there is considerable evidence that jets and collimated outflows vary both in outflow rate and direction (see for example Cunningham, Moeckel & Bally, 2009; and the reviews by Bally, Reipurth & Davis, 2007; Bally, 2007). In addition we have seen that accretion of mass and angular momentum becomes particularly erratic during close interactions with companions, in line with the ideas of Reipurth (2000).

We have seen that once accretion onto the disc has ceased, the disc settles down to its final planar configuration well before the disc is able to drain onto the star. This occurs because typically ν2 ≫ ν1. The correspondence between the spin axis of the final disc and the spin axis of the star depends on the extent to which the final disc has memory of the direction vector of the total angular momentum accreted onto the disc, and thence communicated to the star. Thus if the disc remains essentially undisturbed throughout accretion process, the stellar and disc rotation axes are reasonably well aligned. However, if the outer parts of the disc (where most of the angular momentum resides) are lost, then the final disc direction corresponds to the spin axis of the most recently accreted material. This axis might or might not be related to the spin of the star. Similarly, the action of removing the disc itself (i.e. a dynamical encounter with another object) may misalign the remnant disc from the stellar spin axis.

It is interesting to speculate about the possible dependence of misalignment on stellar mass. For example, in the hydrodynamical calculation presented by Bate et al. (2002, 2003) and subsequent calculations brown dwarfs are produced when protostars that have recently begun to form suffer dynamical encounters and are kicked out of the cloud terminating their accretion before they have been able to accrete to stellar masses. In this case, the brown dwarf may...
be left with a misaligned disc if either the disc and stellar rotation axes had not had time to converge (because the object had recently formed) or if the dynamical encounter itself changed the disc’s rotation axis. Because the object has been ejected from the cloud it is unlikely to subsequently accrete material with different angular momentum. Conversely, intermediate and massive stars in such hydrodynamical calculations typically suffer many dynamical encounters with other objects (e.g. Bate et al. 2003; Bate & Bonnell 2005; Bonnell & Bate 2005). These encounters often leave the star embedded in the dense gas and, thus, repeatedly destroy their discs but also allow them to reform their discs from freshly accreted material. Again, this may lead to discs being misaligned with the stellar rotation axis if the majority of the stellar mass was accreted before the final encounter and growth of a new disc.

From the observations of transits of hot Jupiters, recent measurements of the angle between the planetary orbital plane and the stellar rotation direction show that in a substantial fraction of cases the orbital plane is misaligned with the stellar rotation axis (e.g. Hébrard et al., 2008; Winn et al. 2009a, 2009b; Pont et al., 2009a, 2009b; Gillon 2009; Johnson et al., 2009; Narita et al. 2009). Winn et al. (2009a) conclude that a model on misalignment angles which supposes two planetary populations – one perfectly aligned and one isotropically orientated – is consistent with the data. They interpret this as being due to two different causes of inward planetary migration – one via a gaseous disc and one via planet–planet scattering. We have found, however, that even gaseous discs can show substantial variation in misalignment angles depending on the history or accretion and more especially on the history of close stellar interactions while accretion is still taking place.

However, it also needs to be borne in mind that the accretion of the disc material remaining at the end of our calculations can still affect the stellar rotation. A number of authors (Cameron & Campbell, 1993; Yi, 1994; Ghosh, 1995; Armitage & Clarke, 1996) have argued that the rotation rates of young stars can be regulated by magnetic linkage between the star and a surrounding accretion disc. By the same argument, the misalignment of spins between disc and star could also be regulated by the same mechanism. For example the amount of mass $\Delta M$ needed to significantly change the stellar spin is of order $\Delta M/M_\ast \sim (k R_\ast / R_M)^2$ where $k R_\ast$ is the star’s radius of gyration and $R_M$ is the magnetospheric radius. For typical values of $k^2 \approx 0.2$ (Armitage & Clarke, 1996) and $R_M/R_\ast \approx 5$–10 (Gregory et al., 2008) we estimate $\Delta M/M_\ast \approx 10^{-2} – 10^{-3}$. We note further that formation of the solar planets requires a ‘minimum solar nebular’ mass of around $M_\ast/M_\odot \geq 0.02$. Determining how much of this mass is actually able to interact with the central star and so change its spin will depend on the details of the planet formation process, and in particular on the timing and extent of the formation of any inner gap which would decouple disc and star.

6 CONCLUSIONS

We have considered the evolution of the stellar and disc spin axes during the formation of a star which is accreting in a variable fashion from an inherently chaotic environment. To model this process we have modelled the evolution of a warped disc which has material added to it in a variable fashion. We take the input of mass and angular momentum to the disc to that acquired by the ‘sink particle’ in an SPH simulation of the formation of a cluster of stars in from self-gravitating turbulent gas. We have noted that many of the assumptions we have used cannot be made self-consistent. Thus we caution against drawing specific conclusions from our analysis. The calculations we show here should be regarded simply as illustrative. Nevertheless there are some general points which can be made.

First, the variability of the direction of the spin of material accreting onto the central protostar implies that the internal velocity field of such stars may be more complicated than the usual assumption of aligned differential rotation.

Second the lighter the disc (and the disc can lose mass either through rapid accretion or through interaction with a nearby protostar) the more able is the accreted material to cause the inner disc spin, which we identify with the direction of a jet, to vary.

Third, the final stellar rotation axis and the final disc spin axis can be strongly misaligned. However, this occurs most strongly when the disc is truncated so that rotation direction of the final disc material depends simply on what fell in last. In this case, although the disc might be misaligned with the star, it might not contain enough material to form planets. And we have noted that, depending on the details of the models, it may be that a misaligned disc which is massive enough to form planets, may also be massive enough to reduce the misalignment.

In conclusion, we have shown that it should be possible make some deductions about the accretion history of a young star by observations of jet direction variability, of star/planetary orbit misalignments, and perhaps even of its internal rotation structure. It is clear that further work is required to improve models of the star formation process with better account being taken of the nature of the accretion process within the final 100 au or so.

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APPENDIX: ADDITION AND SHIFTING OF MASS

We discuss here our algorithm for adding material to the disc.

Suppose that the disc grid cells are at radii \( r_i, i = 1, N \) corresponding to specific angular momenta \( h_i, i = 1, N \). In a timestep \( \Delta t \) we add a mass \( \Delta M \) with angular momentum \( \Delta J \). Thus the specific angular momentum of the added material is

\[
h = \frac{\Delta J}{\Delta M}
\]

where \( \Delta J = [\Delta J] \). Matter is than added to the cells \( k \) and \( k+1 \) where \( h_k < h < h_{k+1} \).

Then the angular momentum in cell \( k \) is incremented by an amount

\[
\Delta J_k = h_{k+1} - h_k \left( \frac{h_{k+1}}{h_k} - 1 \right) \Delta J
\]

and that in cell \( k+1 \) by an amount

\[
\Delta J_{k+1} = h_k - h_{k+1} \left( \frac{h_k}{h_{k+1}} - 1 \right) \Delta J
\]

In this manner angular momentum is exactly conserved.

However, the amount of mass actually added is now

\[
\Delta M' = \frac{\Delta J_{k+1}}{h_{k+1}} + \frac{\Delta J_k}{h_k}
\]

which needs to be compared with the actual mass to be added \( \Delta M = \Delta J/h \).

If we write \( h_{k+1} = h(1 + \epsilon_2) \) and \( h_k = h(1 - \epsilon_1) \), where
$\epsilon_2 > 0$ and $\epsilon_1 > 0$ then to first order in the small quantities $\epsilon$ it is simple to show that
\[
\frac{\Delta M'}{\Delta M} - 1 = \epsilon_2 \epsilon_1 > 0.
\] (13)

Thus the algorithm for the addition of angular momentum leads to a systematic over-estimate of the amount of mass added of an amount which is on average (i.e. when $\epsilon_1$ and $\epsilon_2$ are of comparable magnitude) second order in $dR/R$. 