Self-consistent Stellar Radial Velocities from LAMOST Medium-resolution Survey DR7

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Abstract

Radial velocity (RV) is among the most fundamental physical quantities obtainable from stellar spectra and is rather important in the analysis of time-domain phenomena. LAMOST Medium-resolution Survey (MRS) DR7 contains five million single-exposure stellar spectra with spectral resolution $R \sim 7500$. However, the temporal variation of the RV zero-points (RVZPs) of the MRS, which makes the RVs from multiple epochs inconsistent, has not been addressed. In this paper, we measure the RVs of 3.8 million single-exposure spectra (for 0.6 million stars) with signal-to-noise ratios ($S/N$) higher than 5 based on the cross-correlation function method, and propose a robust method to self-consistently determine the RVZPs exposure by exposure for each spectrograph with the help of Gaia DR2 RVs. Such RVZPs are estimated for 3.6 million RVs and can reach a mean precision of $\sim 0.38$ km s$^{-1}$. The result of the temporal variation of RVZPs indicates that our algorithm is efficient and necessary before we use the absolute RVs to perform time-domain analyses. Validating the results with APOGEE DR16 shows that our absolute RVs can reach an overall precision of $0.84/0.80$ km s$^{-1}$ in the blue/red arm at $50 < S/N < 100$ and of $1.26/1.99$ km s$^{-1}$ at $5 < S/N < 10$. The cumulative distribution function of the standard deviations of multiple RVs ($N_{\text{obs}} \geq 8$) for 678 standard stars reaches 0.45/0.54, 1.07/1.39, and 1.45/1.86 km s$^{-1}$ in the blue/red arm at the 50%, 90%, and 95% levels, respectively. Catalogs of the RVs, RVZPs, and selected candidate RV standard stars are available at https://github.com/hypergravity/paperdata.

Unified Astronomy Thesaurus concepts: Radial velocity (1332); Surveys (1671); Astronomy data analysis (1858); Astronomy data reduction (1861); Spectroscopic binary stars (1557); Radio spectroscopy (1359); Spectroscopy (1558); Catalogs (205); Sky surveys (1464); Astrostatistics (1882); Robust regression (1949)

Supporting material: FITS files

1. Introduction

Large spectroscopic surveys, for example, RAVE (Steinmetz et al. 2006, 2020a), the Sloan Digital Sky Survey/SEGUE (Yanny et al. 2009), the Large Sky Area Multi-object Fiber Spectroscopic Telescope (LAMOST; Cui et al. 2012; Deng et al. 2012; Zhao et al. 2012; Luo et al. 2015), APOGEE (Majewski et al. 2017), GALAH (De Silva et al. 2015), Gaia-ESO (Gilmore et al. 2012), and the Gaia Radial Velocity Spectrometer (Gaia-RVS; Katz et al. 2004; Cropper et al. 2018), have obtained tens of millions of stellar spectra over the last two decades, aiming at understanding the formation and evolution of the Galaxy. Among the most fundamental physical quantities derived from stellar spectra is radial velocity (RV), which forms the basis of many studies, such as on stellar multiplicity (e.g., Gao et al. 2014, 2017; El-Badry et al. 2018; Yang et al. 2020), stellar kinematics (e.g., Tian et al. 2020; Bird et al. 2020), and Galactic substructures (e.g., Yang et al. 2019; Xu et al. 2020).

LAMOST, after a five-year low-resolution spectroscopic survey (LRS; $R \sim 1800$, 3800 Å $< \lambda < 9000$ Å), started a five-year medium-resolution spectroscopic survey (MRS; $R \sim 7500$, 4950 Å $< \lambda < 5350$ Å and 6300 Å $< \lambda < 6800$ Å; Liu et al. 2020) in 2017 September. DR6,11 the first data release of the MRS, contains the data obtained from 2017 September to 2018 June and is already available to the international astronomical community. DR7,12 including the data from 2017 September to 2019 June (about five million spectra for over 800,000 stars), is currently open to the Chinese astronomical community only.

In the beginning, an Sc lamp was used to calibrate the wavelength of LAMOST MRS spectra; later, it was switched to a ThAr lamp (in 2018). Due to the short wavelength coverage, sky lines are not used in wavelength calibration as in the LRS

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11 http://dr6.lamost.org/
12 http://dr7.lamost.org/
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(2010). The released spectra have undergone barycentric correction and the wavelength uses the vacuum standard. Both the LAMOST pipeline and Wang et al. (2019) have measured RVs from DR7 spectra and estimated a static RV zero-point (RVZP) by comparing the measured RVs to those in the literature of a sample of RV standard stars (Huang et al. 2018) for each spectrograph. However, the temporal variation of the RVZPs (between exposures) is not clear so far despite a few trials. For example, Liu et al. (2019b) and Zong et al. (2020) show the temporal variation of RVZPs based on data from the Kepler field using a sample of roughly selected RV-invariant stars, and Ren et al. (2021) show that the RVZPs of the red arm vary between exposures and the difference can reach 4 km s\(^{-1}\) in the MRS-N fields (nebula survey: Wu et al. 2021).

Physically, the variation of the LAMOST MRS RVZPs can be explained by several factors.

1. LAMOST has a long optical path and a large focal plane (1.75 m diameter) on which 4000 fibers are installed (Cui et al. 2012); temperature variation is unavoidable.
2. Currently, arc lamp exposures are taken about every 2 hr (typical observing time of a plate), which is not frequent enough. The instrument might change its state between lamp exposures.
3. Since the LAMOST MRS needs ~20 ThAr lamps to illuminate the big focal plane simultaneously, the lamps are easily damaged and frequently replaced, and the lamp exposure time needs frequent adjustment. These factors affect the signal-to-noise ratio (S/N) of the lamp spectra and thus the wavelength calibration consistency over time.
4. The MRS spectrographs are mounted and dismounted monthly because the MRS is scheduled on the 14 bright/gray nights, while the other nights are for the LRS. Therefore, the focuses are adjusted monthly.

All these factors and other potential defects in the data reduction pipeline are finally reflected in the RVZPs of the spectra. Therefore, it is insufficient to use the RVs from either the LAMOST pipeline or Wang et al. (2019) at different epochs and perform a time-domain analysis, such as in studies of pulsating stars (the LAMOST-Kepler Project; Zong et al. 2020; Fu et al. 2020) and spectroscopic binaries (Gao et al. 2014, 2017; Liu 2019) and even in searches for black holes (Liu et al. 2019a; Gu et al. 2019), which are important scientific goals of the MRS (Liu et al. 2020).

In this paper, aiming at subsequent time-domain analysis, we measure the RVs for the 3.8 million single-exposure spectra with S/N > 5 in LAMOST MRS DR7 v1.1,\(^\dagger\) and propose a robust method to self-consistently determine the absolute RVZPs with the help of Gaia DR2 data. In Section 2, we briefly describe the observational rules of the LAMOST MRS and the instrumental parameters. In Section 3, we describe how we measure the RVs. In Section 4, we show the algorithm that determines the RVZPs self-consistently. In Section 5, we present our RV and RVZP measurements and assess the precision and self-consistency, and select a sample of candidate RV standard stars based on our results. Information on and instructions regarding our data products are described in Section 6, and a summary of this work is given in Section 7.

\(^\dagger\) http://dr7.lamost.org/v1.1

### 2. The LAMOST MRS

#### 2.1. Targeting and Observational Rules

The scientific goals of the LAMOST MRS mainly include stellar multiplicity, stellar pulsation, star formation, emission nebulae, Galactic archeology, host stars of exoplanets, and open clusters. The details of the scientific plan and the survey strategy are described in Liu et al. (2020). In this section, we summarize them briefly.

Each scientific goal has a principal investigator who is responsible for its targeting. A planid is assigned to each MRS plate (pointing), which has the form [TD/NT]hmmsS[N/S]ddmmssXnn. The first two characters denote time-domain (TD, will be repeatedly observed during the five-year survey) or non-time-domain (NT, just observed on a single night), hhmmss[N/S]ddmmss represents the equatorial coordinates of the plate center, [N/S] means north/south, the digit X is used to denote the scientific goal (see Table 1), and the last two digits represent a serial number. For example, the planid TD062610N184524B01 means the plate is a time-domain plate dedicated to binarity/multiplicity research. There also exist some irregular planid that are testing fields, such as HIP8426401 and NGC216801. HIP8426401 means the central star of the plate is HIP 84264, and NGC216801 means a plate toward the star cluster NGC 2168.

The MRS uses the local modified Julian minute (LMJM), an 8 bit integer defined as 1440 × the local modified Julian date at the beginning of each exposure, as the stamp of each exposure. The typical exposure time is set to 1200 s, while 900 s and 600 s exposures also exist, depending on the brightness of the targets. Each NT plate is observed with three consecutive exposures while each TD plate is observed until it is observable (usually five to six 1200 s exposures are allowed). An arc lamp exposure is taken at the beginning of each plate, and at the end of an observing night (every ~2 hr). The targeting is mostly based on the Gaia DR2 source catalog (Gaia Collaboration et al. 2018). For a 1200 s exposure, the magnitudes corresponding to S/N = 5 in blue- and red-arm spectra are G ~ 14.5 and 15.2 mag, respectively (see Figure 1 in Liu et al. 2020). Magnitudes brighter than these cover the majority of the objects observed by the LAMOST MRS.

#### 2.2. LAMOST MRS Spectra

The whole LAMOST MRS DR7 catalog (R ~ 7500) contains over five million single-exposure spectra of over 800,000 stars obtained from 2017 September to 2019 June. In this work, 3,753,659 spectra with S/N > 5 either in the blue arm or in the red arm for 600,771 stars are selected. Their distributions on the number of exposures and time span are

| Table 1 |
|---|---|---|
| Abbreviations of Scientific Goals (X) Used in planid |
| X | NT/TD | Science |
| --- | --- | --- |
| K | TD | Kepler fields |
| H | TD | high-frequency Kepler fields |
| B | TD | binarity/multiplicity |
| T | TD | TESS fields |
| M | NT | Milky Way |
| S | NT | star formation |
| C | NT | star cluster |
| N | NT | nebula |
presented in Figure 1. The spectra are oversampled. The typical wavelength steps are 0.11 and 0.14 Å at the blue and red arms, respectively. The sampling rate (\(\lambda/\Delta \lambda \sim 45,000\)) is as high as six times the spectral resolution (\(R \sim 7500\)). In Figure 2, we show a spectrum of a K-type giant star as a demo of the LAMOST MRS. The blue arm and red arm are designed mainly for the Mg I triplet at around 5175 Å and for H\(\alpha\) at around 6564 Å. The released spectra are not corrected with response curves.

3. Measurement of RVs

3.1. Preparation for RV Measurements

We notice that cosmic rays frequently pollute MRS single-exposure spectra and are neither identified nor removed by the LAMOST pipeline. Therefore, in our method, the first step is to carefully remove cosmic rays in spectra. We smooth spectra with a 21-pixel median filter followed by a 9-pixel Gaussian filter, and remove the original pixels that deviate from the smoothed spectrum by four and eight times the local standard deviation in the upper and lower directions. The parameters of the filters are set empirically so that the absorption lines in the spectra of A-, F-, G-, and K-type stars are not affected. The removed pixels are replaced by linearly interpolated values using neighboring pixels. We also note that the two ends of both the blue and red arms are sometimes tilted and show unreasonable flux values probably due to extrapolation of the sky flux modeling, so we trim 50 pixels at both edges of each arm.

The second step is to normalize spectra to a pseudocontinuum to place the spectral features (usually absorption lines) on the same scale. We iteratively fit the spectrum with a smoothing spline function and clip the pixels away from the median values by three times the standard deviation in each 100 Å window. The number of iterations is set to 3. As shown in Figure 2, for a typical K-type star with a medium S/N, the normalization is quite adequate for the subsequent RV measurements.

3.2. Spectral Templates

We adopt the synthetic grid published by Allende Prieto et al. (2018) based on the ATLAS9 stellar atmosphere model (Kurucz 1979; Mészáros et al. 2012) as our spectral library. We degrade the spectral resolution from 10,000 to 7500 to fit the MRS configuration and convert the air wavelength to vacuum wavelength using the formula proposed by Morton (2000). To limit the computational cost of the subsequent RV measurements to a reasonable amount, we interpolate the synthetic library to generate 100 spectral templates with stellar parameters randomly drawn from a uniform distribution in the ranges of 3500 < \(T_{\text{eff}}/K\) < 15,000, 0 < \(\log g/\text{dex}\) < 5, \(-2 < [\text{Fe}/\text{H}]/\text{dex} < 0.5\), and \(-0.5 < [\alpha/\text{Fe}]/\text{dex} < 0.7\). Extremely metal-poor and extremely hot templates are not considered because the spectral features are not significant. Tests on high-S/N MRS spectra show that the sparsity of our templates induces statistical errors \(\sim 0.10\) and 0.20 km s\(^{-1}\) in the blue and red arms, respectively, which are negligible as compared to other sources of uncertainties. Figure 3 shows the distribution of parameters of the 100 spectral templates and a series of PARSEC isochrones (Bressan et al. 2012) with solar metallicity and logarithmic age \(\log t = 7, 8, 9,\) and 10. These spectral templates are normalized to a pseudocontinuum in the same way as in Section 3.1.

3.3. RV Estimates

The cross-correlation function (CCF; Tonry & Davis 1979) method is widely used in spectroscopic surveys to measure stellar RVs (e.g., Nidever et al. 2015; Steinmetz et al. 2020b). One important advantage is that the CCF can be accelerated using fast Fourier transformation (FFT) once a spectrum is continuum-subtracted and resampled to a logarithmic wavelength grid. However, the drawback of such a scheme is that the sampling of the resulting CCF is generally very sparse. To evaluate the CCF at a smaller RV step, we do not follow the FFT way. In our implementation, the CCF at an RV of \(v\) is evaluated as

\[
\text{CCF}(v|F, G) = \frac{\text{Cov}(F, G(v))}{\sqrt{\text{Var}(F)\text{Var}(G(v))}},
\]

where \(F\) is the vector of the normalized observed spectrum, \(G(v)\) is the vector of the normalized synthetic spectrum shifted by an RV \(v\) and resampled to the wavelength grid of \(F\), Var represents the variance operator, and Cov represents the covariance operator (see Appendix A for more details).

Deriving the final RV consists of three steps:

1. The initial estimates are made from an RV grid of \(-1500\) to \(+1500\) km s\(^{-1}\), with a step of 10 km s\(^{-1}\). The template with the maximum CCF value is selected as the best-match template, and the corresponding RV is adopted as the initial guess of the final RV of the observed star. The parameters of the template \((T_{\text{eff}}, \log g, [\text{Fe}/\text{H}],\) and \([\alpha/\text{Fe}])\) are recorded, which make a good prior for some following analyses, such as for stellar atmospheric parameter determination.
2. With the best-match template, we maximize the CCF to determine the final RV ($v_{\text{obs}}$) using the optimization routine `scipy.optimize.minimize` with the Nelder–Mead algorithm (Nelder & Mead 1965). The corresponding CCF value is recorded as CCFMAX to assess the likelihood between the best-match template and the observed spectrum. The $S/N$–CCFMAX relations are shown in Figure 4.

3. To obtain the measurement error $\sigma_{v,\text{obs}}$, we use a Monte Carlo method—namely, we repeat this process 100 times and each time we add Gaussian random noise to the spectrum according to the flux error. The measurement error is computed using the 16th and 84th percentiles, i.e., $\sigma_{v,\text{obs}} = (v_{94} - v_{16})/2$. The $S/N$–$\sigma_{v,\text{obs}}$ relations are shown in Figure 5.

We avoid Gaussian fitting to the CCF, which is widely used in literature (e.g., Nidever et al. 2015; Wang et al. 2019). Our reasons for doing this include the fact that the CCF peak is obviously non-Gaussian and that at low $S/N$ the fitting process is fragile. The final error of the RV includes the measurement error and a noise floor term that is due to systematics (e.g., background, detector imperfections, temperature changes, or focusing issues) and will be assessed below.

In this work, one RV is estimated for the blue arm ($v_B$) and two for the red arm ($v_R$ and $v_{Rm}$). The $v_R$ is measured using the H$\alpha$-masked red-arm spectrum. As shown in Figures 4 and 5, these RV measurements deteriorate rapidly at $S/N < 20$. For cool stars (e.g., FGK types), at a given $S/N$, $v_B$ is more precise than $v_R$ because of rich spectral features in the blue arm (for a detailed discussion of spectral information content in MRS spectra, see Zhang et al. 2020b). However, for hot stars, $v_R$ is more reliable than $v_B$ because of the H$\alpha$ feature and $v_{Rm}$ is significantly less precise than $v_R$ due to the absence of H$\alpha$. Therefore, we recommend that readers only consider $v_{Rm}$ when the targets have (possible) H$\alpha$ emissions.

4. RVZPs

In essence, the RVZP is the bias of the wavelength solution of a specific spectrum as compared to its true wavelength solution in terms of RV. It is affected by many factors, e.g., the condition of the instrument, the quality of the arc lamp exposure, the reduction algorithm, and the nonsimultaneous nature of the arc lamp exposures and the object exposures. In this paper, we define the RVZP correction value $\Delta v$ by

$$v_{\text{obs}} = v_{\text{abs}} + \Delta v,$$

where $v_{\text{abs}}$ is the absolute RV and $v_{\text{obs}}$ the RV directly measured from the spectrum.
4.1. The Scheme

LAMOST has 16 spectrographs of which each has 250 fibers (4000 fibers in total). Excluding a few tens of sky fibers and a few problematic fibers, each spectrograph typically produces \( \lesssim 200 \) spectra in an exposure, depending on the targeting, the condition of the instrument, the data quality, and the reduction.

Figure 4. The left/middle/right panel shows the S/N–CCFMAX relation for the B (blue arm) / R (red arm) / Rm (red arm with H\( \alpha \) masked) CCF results of 5000 randomly selected spectra, respectively. The color denotes the \( T_{\text{eff}} \) of the best-match template.

Figure 5. The left/middle/right panel shows the S/N–\( \sigma_{\text{obs}} \) (measurement error) relation for the B (blue arm) / R (red arm) / Rm (red arm with H\( \alpha \) masked) CCF results of 5000 randomly selected spectra, respectively. The color denotes the \( T_{\text{eff}} \) of the best-match template.

Figure 6. Grouped pointings of the LAMOST MRS DR7 observations using a friends-of-friends method with a linking length of 5°. The field of view of LAMOST is a circle with a 2.5° radius if all spectrographs are in operation. The figure uses the equatorial coordinate system with Mollweide projection, and the pointings in a particular group are shown with the same color.
algorithm. Let $i$ denote the exposure epoch or LMJM, $j$ the spectrograph ID, and $k$ the fiber ID; ideally, we seek the solution of the RVZP for each fiber, for each spectrograph, and exposure by exposure, namely, $\Delta v_{i,j,k}$. This scheme is infeasible because the true/reference RVs $v_{i,j,k,\text{abs}}$ of the targets are not always known.

In this work, assuming that the fibers in a spectrograph in one exposure (hereafter, we refer to it as a spectrograph exposure unit, or an SEU) share similar RVZPs, the systematic RVZP $\Delta v_{i,j}$ can be determined as long as a homogeneous reference set of RVs can be found for a fraction of fibers in that SEU. The assumption is quite reasonable given the fact that the wavelength calibration of a multifiber spectrograph is done by fitting a 2D grating equation. And as we will see in Section 4.2, the Gaia DR2 RVs (Katz et al. 2019) meet our needs for the reference set.

As a contrast, both the LAMOST pipeline and Wang et al. (2019) calculate $\Delta v_j$ assuming the RVZPs for a specific spectrograph do not vary with time, and, therefore, get around the temporal variation of RVZPs. However, we notice that there exist some weird absolute RVs (as we will see in the results in Section 5).

### 4.2. Gaia DR2 RVs as the Reference Set

Thanks to the European Space Agency’s Gaia mission (Gaia Collaboration et al. 2016), we now have access to the largest RV data set, which matches the LAMOST MRS in terms of
velocity precision and magnitude limit. The spectral resolution of Gaia-RVS \((R \sim 11,500; \text{ Cropper et al. 2018})\) is slightly higher than that of the MRS \((R \sim 7500)\). The magnitude limit of the Gaia DR2 RV catalog \((\text{ Katz et al. 2019})\) is at \(G_{\text{RVS}} = 12 \text{ mag} \) or \(G \sim 14 \text{ mag}\) depending on spectral type and line-of-sight interstellar extinction, which is slightly brighter than the LAMOST MRS magnitude limit. Gaia DR2 contains qualified median RVs for 7,224,631 stars derived from the Gaia-RVS spectra, with \(T_{\text{eff}}\) in the range \([3550, 6900]\) K, excluding large RV-variant stars (see \text{ Katz et al. 2019} for details). At the faint end, \(G_{\text{RVS}} = 11.75 \text{ mag}\), the precisions for \(T_{\text{eff}} = 5000 \text{ and } 6500 \text{ K}\) are 1.4 and 3.7 \(\text{ km s}^{-1}\), respectively.

Aiming at studying time-domain astrophysical phenomena, e.g., spectroscopic binaries, we proceed to carry out the second-best scheme—\(\Delta v_{ij}\). Cross-matching the LAMOST MRS DR7 catalog with Gaia DR2, we find 1,582,948 out of the 3,753,659 single-exposure spectra (42.1\%) have Gaia RVs, and the common objects usually have good S/N in the MRS catalog because they are relatively bright in that survey. The number of objects in Gaia DR2 is \(\sim 1000\) times larger than that in catalogs of RV standard stars, such as that of \text{Huang et al. (2018)}. The challenge arises because not all of the seven million objects in the Gaia-RVS catalog are RV-invariant, i.e., quite a number of them are pulsating stars or binary/multiple systems that have periodic/nonperiodic RV variations. Below, we discuss a robust method that can determine the RVZP self-consistently for each spectrograph in each exposure (\(\Delta v_{ij}\)) by comparing the observed RVs to the Gaia DR2 RVs without identifying RV standard stars.

4.3. Self-consistent RVZPs

Assuming that the RV variables vary with random periods at random phases or nonperiodically, and are not the majority of the observed stars, we can regard them as outliers and use a robust method, e.g., least absolute residual (LAR) regression (or least absolute deviation regression; see \text{Press et al. 2007}), to estimate \(\Delta v_{ij}\) (the common RV bias shared by the objects in an SEU). From a Bayesian perspective, LAR regression originates from an exponential likelihood while least squares (LSQ) regression comes from a Gaussian likelihood. Utilizing the LAR technique, extreme values have less influence on the fit compared to those in LSQ regression. Besides, since we aim at time-domain analysis, as long as our RVZPs are temporally self-consistent, the absolute scales are not very important.

Using the indices proposed in Section 4.1, for each group of pointings, we construct a global cost function \(f\) as follows:

\[
f(\Delta v) = \Lambda_1 \sum_{i,j,k} \frac{|v_{i,j,k,\text{obs}} + \Delta v_{ij} - v_{i,j,k,\text{GAIA}}|}{\sqrt{\sigma_{\text{min}}^2 + \sigma_{i,j,k,\text{obs}}^2 + \sigma_{i,j,k,\text{GAIA}}^2}} + \Lambda_2 \sum_{i,j,k} \frac{|v_{i,j,k,\text{obs}} + \Delta v_{ij} - \tau_{\text{obs}}|}{\sqrt{\sigma_{\text{min}}^2 + \sigma_{i,j,k,\text{obs}}^2 + \sigma_{\text{obs}}^2}},
\]

where \(\Delta v\) is the vector of \(\{\Delta v_{ij}\}\) for all relevant SEUs in the group of pointings; \(v_{i,j,k,\text{obs}}\) and \(\sigma_{i,j,k,\text{obs}}\) are the RV and associated measurement error of the \(k\)th star in the SEU \{\(i, j\); \(v_{i,j,k,\text{GAIA}}\) and \(\sigma_{i,j,k,\text{GAIA}}\) are the Gaia RV and associated uncertainty of the \(k\)th star in SEU \{\(i, j\); \(\sigma_{\text{min}}\) is the noise floor of the measured RV, which indicates the stability of the wavelength calibration, i.e., the dispersion of \(v_{i,j,k,\text{GAIA}}\) in SEU \{\(i, j\); \(\Delta v_{ij}\) is the RVZP correction value of SEU \{\(i, j\), which is a free variable to be solved; \(\tau_{\text{obs}}\) and \(\sigma_{\text{obs}}\) are the median and scatter of the (RVZP-corrected) measured RVs of the star \{\(i, j, k\)\} in other SEUs; and \(\Lambda_1\) and \(\Lambda_2\) are the regularization parameters of the two terms. In this scheme, the first term guarantees that the absolute scale of our RVZP-corrected RVs is close to that of Gaia DR2, while the second term makes use of multiple exposures and guarantees that the relative RVZPs are self-consistent. We set \(\Lambda_1 = \Lambda_2 = 1\) so that the final correction of each SEU is determined as an average of the two effects. Then the vector \(\Delta v\) is determined by minimizing the cost function \(f\), i.e., \(\Delta v = \arg \min f\). This algorithm can be implemented by minimizing

\[
f_{ij}(\Delta v_{ij}) = \Lambda_1 \sum_k \frac{|v_{i,j,k,\text{obs}} + \Delta v_{ij} - v_{i,j,k,\text{GAIA}}|}{\sqrt{\sigma_{\text{min}}^2 + \sigma_{i,j,k,\text{obs}}^2 + \sigma_{i,j,k,\text{GAIA}}^2}} + \Lambda_2 \sum_k \frac{|v_{i,j,k,\text{obs}} + \Delta v_{ij} - \tau_{\text{obs}}|}{\sqrt{\sigma_{\text{min}}^2 + \sigma_{i,j,k,\text{obs}}^2 + \sigma_{\text{obs}}^2}},
\]

for each SEU \{\(i, j\)\} sequentially and iteratively, where \(\Delta v_{ij}\) is the RVZP correction value for SEU \{\(i, j\)\}. Therefore, the

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**Figure 8.** Histograms of \(\Delta v_B\), \(\Delta v_R\), and \(\Delta v_{Rm}\). The Sc- and ThAr-lamp-calibrated data are shown in gray and cyan, respectively. The \(\mu\) and \(\sigma\) are calculated using the median and \((q_{84} - q_{16})/2\), respectively.
problem is to solve the vector $\Delta v$ that has $N_{SEU}$ elements, where $N_{SEU}$ is the number of related SEUs. We claim that $\Delta v$ is determined when the $L_\infty$ norm of the difference between the solutions in the $l$th and $(l+1)$th iterations is less than a specified value, i.e., $\max(|\Delta v_l - \Delta v_{l+1}|) < \epsilon$, where $\epsilon$ is the tolerance and is set to 0.075 km s$^{-1}$.

SEUs with RVZP correction values $|\Delta v_{ij}| > 50$ km s$^{-1}$ or associated uncertainties $\sigma_{\Delta v_{ij}} > 10$ km s$^{-1}$ (see Section 4.5) are excluded from the iterations. These results are generally due to (1) the number of Gaia DR2 objects being $\leq 10$, (2) the spectral S/N being too low, or (3) bad spectra due to saturation or instrumental problems. We do not think that for these SEUs our scheme and assumptions are valid, so we only keep their initial guesses of $\Delta v_{ij}$ (see Section 4.4) and their uncertainties (see Section 4.5) in our catalog (see Section 6).

### 4.4. Tricks to Accelerate the Algorithm

Several tricks are used to accelerate the algorithm. The first trick is to get a good initial estimation of $\Delta v$. A good approximation can be made by ignoring the second term in Equation (4), so that with Gaia DR2 RVs we can roughly estimate $\Delta v_{ij}$ by minimizing

$$f_i,\text{init}(\Delta v_{ij}) = \sum_k \frac{|v_i,j,k,\text{obs} + \Delta v_{ij} - v_i,j,k,\text{GAIA}|}{\sqrt{\sigma_{\text{min}}^2 + \sigma_{i,j,k,\text{obs}}^2 + \sigma_{i,j,k,\text{GAIA}}^2}}.$$ (5)

We note that if an SEU has only a few objects in common with the Gaia catalog, the estimation is risky. Therefore, we require that an SEU at least have 10 objects in common with Gaia DR2 to proceed; otherwise we calculate the initial guess $\Delta v_{ij,\text{init}}$ but exclude this SEU in the iteration process.

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**Figure 9.** The temporal variance of $\Delta v_B$ before/after 2019 May 1 for the blue arm. The gray filled areas show the 16th and 84th percentiles of $\Delta v_B$ in the two time intervals, and the black solid lines show the medians. The colors denote the square root of the peak flux of the Th5231 line, which can be used as an indicator of S/N.
The second trick is to cut down $N_{\text{SEU}}$ by separating physically detached SEUs, which hastens the index evaluation in each iteration. In Equations (3) and (4), the second terms contain cross-terms, meaning that when solving the $N_{\text{SEU}}$ elements, an iteration process is needed to guarantee that the solution of $\Delta v$ is stable. However, the evaluation of indices is computationally expensive when $N_{\text{SEU}}$ grows. Therefore, before the optimization process, physically detached sky areas can be separated, so that the many index evaluation processes can be accelerated by cutting down the array sizes. We group the pointings of LAMOST MRS DR7 using a friends-of-friends algorithm with a 5° linking length, which is double the radius of the field of view of LAMOST in case of any possible common stars between them. Eventually, 137 groups are obtained, as shown in Figure 6. Initial guesses of $\Delta v$ for each SEU are made by optimizing the cost function of Equation (3) for each group of plates.

In practice, we find that if $\epsilon$ is too small, there is the possibility that $\Delta v$ jumps back and forth between two solutions and does not converge. Further analysis shows that this is an optimization-method-related problem (we use the Nelder–Mead solver; changing it to the Powell solver does not solve the problem but the two solutions are different). We guess that this might be due to the numerical problem of the optimization routine. To avoid such a situation, we then add random processes into the algorithm, i.e., if in the $l$th iteration the solution is $\Delta v_l$ and after looping over all related SEUs the solution is $\Delta v_{l,\text{opt}}$, we evaluate the $(l+1)$th solution by $\Delta v_{l+1} = \eta(\Delta v_{l,\text{opt}} - \Delta v_l) + \Delta v_l$, where $\eta$ is the learning rate randomly generated between $\eta_0$ and $\eta_1$. We set $\eta_0 = 0.5$, $\eta_1 = 1.0$, and $\epsilon = 0.075$; considering the expectation for $\eta$ is 0.75, the effective tolerance of our solution of $\Delta v$ is 0.10 km s$^{-1}$, which is acceptable when compared to the typical precision of measured RVs (e.g., $\sim 1.5$ km s$^{-1}$ as reported by Wang et al. 2019).

Finally, the RVZP corrections for all the 137 groups of pointings converge after several tens of iterations with a Dell Precision R740 workstation with two Intel Xeon Platinum 8260 CPUs (2.40 GHz), among which the longest solution takes $\sim$10 hr. Compared to the computation of RV measurements...
| Ref. Data Set | S/N | This Work | LAMOST | Wang et al. (2019) | Gaia DR2 RVS |
|--------------|-----|----------|---------|-------------------|---------------|
|              |     | Blue Arm | Red Arm | Blue Arm | Red Arm |
| 5 < S/N < 10 | N   | 166,218  | 89,032  | 89,009  | 166,218  |
|              | μ   | -0.02    | -0.05   | -0.07   | -0.56    |
|              | σ   | 1.68     | 2.38    | 2.47    | 3.78     |
| 10 < S/N < 20| N   | 301,963  | 164,068 | 164,010 | 301,963  |
|              | μ   | -0.04    | -0.10   | -0.10   | -0.53    |
|              | σ   | 1.42     | 1.74    | 1.81    | 2.52     |
| 20 < S/N < 50| N   | 572,355  | 531,044 | 530,913 | 572,355  |
|              | μ   | -0.03    | -0.07   | -0.08   | -0.44    |
|              | σ   | 1.22     | 1.39    | 1.44    | 1.88     |
| 50 < S/N < 100| N  | 270,880  | 497,328 | 497,126 | 270,880  |
|              | μ   | 0.00     | -0.00   | -0.01   | -0.33    |
|              | σ   | 1.00     | 1.10    | 1.14    | 1.45     |
| 5 < S/N < 10 | N   | 35,352   | 23,362  | 23,354  | 35,352   |
|              | μ   | 0.22     | 0.08    | 0.06    | -0.39    |
|              | σ   | 1.26     | 1.99    | 2.17    | 3.72     |
| 10 < S/N < 20| N   | 49,482   | 38,971  | 38,987  | 49,482   |
|              | μ   | 0.13     | 0.08    | 0.02    | -0.41    |
|              | σ   | 0.95     | 1.27    | 1.38    | 2.24     |
| 20 < S/N < 50| N   | 80,556   | 87,168  | 87,159  | 80,556   |
|              | μ   | 0.06     | 0.06    | 0.03    | -0.41    |
|              | σ   | 0.82     | 0.92    | 0.99    | 1.51     |
| 50 < S/N < 100| N | 41,479   | 73,363  | 73,350  | 41,479   |
|              | μ   | -0.04    | 0.04    | 0.04    | -0.41    |
|              | σ   | 0.84     | 0.80    | 0.85    | 1.22     |
| 5 < S/N < 10 | N   | 2753     | 1659    | 1661    | 2753     |
|              | μ   | 0.44     | 0.33    | 0.36    | 0.28     |
|              | σ   | 1.11     | 1.53    | 1.69    | 3.34     |
| 10 < S/N < 20| N   | 3404     | 2890    | 2890    | 3404     |
|              | μ   | 0.43     | 0.36    | 0.36    | 0.01     |
|              | σ   | 0.84     | 1.13    | 1.18    | 2.02     |
| 20 < S/N < 50| N   | 5050     | 5923    | 5920    | 5050     |
|              | μ   | 0.42     | 0.39    | 0.40    | 0.01     |
|              | σ   | 0.76     | 0.81    | 0.87    | 1.31     |
| 50 < S/N < 100| N | 2507     | 4917    | 4916    | 2507     |
|              | μ   | 0.38     | 0.44    | 0.47    | 0.06     |
|              | σ   | 0.69     | 0.74    | 0.79    | 0.98     |
| 5 < S/N < 10 | N   | 30,272   | 17,317  | 17,271  | 30,272   |
|              | μ   | 0.32     | 0.26    | 0.22    | -0.25    |
|              | σ   | 1.37     | 2.18    | 2.37    | 4.06     |
| 10 < S/N < 20| N   | 42,242   | 32,885  | 32,768  | 42,242   |
|              | μ   | 0.27     | 0.29    | 0.23    | -0.20    |
|              | σ   | 1.00     | 1.40    | 1.54    | 2.48     |
| 20 < S/N < 50| N   | 52,015   | 63,642  | 63,593  | 52,015   |
|              | μ   | 0.26     | 0.27    | 0.22    | -0.10    |
|              | σ   | 0.83     | 0.96    | 1.01    | 1.66     |
| 50 < S/N < 100| N | 18,796   | 37,239  | 37,239  | 18,796   |
|              | μ   | 0.24     | 0.29    | 0.27    | -0.07    |
|              | σ   | 0.72     | 0.78    | 0.81    | 1.16     |
| 5 < S/N < 10 | N   | 593      | 414     | 414     | 593      |
|              | μ   | 0.53     | 0.93    | 0.77    | 0.13     |
|              | σ   | 2.17     | 2.99    | 3.08    | 4.63     |
using the CCF for the 3.8 million spectra, including the blue and red arms (~1 week with the same machine), computing the RVZPs is quite fast.

4.5. Uncertainty Estimation

Rigorous uncertainties are very difficult to obtain for our RVZP corrections. The uncertainties of the RVZP correction values consist of two parts, namely the tolerance in the iteration process and the formal error. The tolerance is $\epsilon = 0.1 \text{ km s}^{-1}$ as mentioned above. For the latter part, based on the discussion presented in Appendix B, we use the 16th and 84th percentiles to construct a fiducial error of our RVZP correction values $\Delta v_{ij}$ divided by an empirical correction $\xi$ to construct the formal error. Hence, the total uncertainties of the RVZP correction values are evaluated via

$$\sigma^2_{\Delta v_{ij}} = \left(\frac{q_{84,ij} - q_{16,ij}}{2\xi/\sqrt{N_{ij}}}\right)^2 + \epsilon^2,$$

where $i$ and $j$ index the SEUs, $q_{16}$ and $q_{84}$ denote the 16th and 84th percentiles of the residuals of the Gaia DR2 RVs and the RVZP-corrected LAMOST MRS RVs, $N_{ij}$ is the number of Gaia DR2 objects with RVs, and $\xi$ is the empirical correction factor for small-number statistics.

5. Results and Validation

In total, we have measured RVs from 3,181,157/3,723,934 single-exposure blue/red-arm spectra in LAMOST MRS DR7 with S/N higher than 5. For 36,301/37,624 37,624 B/Rm SEUs, we have successfully derived the initial values of the RVZPs. After eliminating bad SEUs with the criteria described at the end of Section 4.3, we estimate the final RVZPs for 33,073/35,207/35,199 (B/R/Rm) SEUs, which cover 2,985,015/3,631,023/3,629,895 B/R/Rm RVs. Roughly, the percentages of coverage are 87.9%/93.6%/93.6% for B/R/Rm in terms of SEUs and 93.8%/97.5%/97.5% for B/R/Rm in terms of RVs.

5.1. The Temporal Variation of RVZPs

In Figure 7, we present the $\Delta v_{ij}$ for each SEU for Sc and ThAr arc lamps versus the date. The Sc lamp was in use until 2018 October, after which it was replaced by the ThAr lamp. The mean uncertainties of $\Delta v_{B}$, $\Delta v_{R}$, and $\Delta v_{Rm}$ are all ~0.38 km s$^{-1}$, which is quite good. The median uncertainties are even 20% smaller. The $\Delta v_{R}$ and $\Delta v_{Rm}$ have different patterns while $\Delta v_{B}$ and $\Delta v_{Rm}$ are very similar. In Figure 8 we show the distribution of $\Delta v_{B}$, $\Delta v_{R}$, and $\Delta v_{Rm}$ of the SEUs solved. The $\mu$ here is estimated using the median, and $\sigma$ is estimated using $(q_{84} - q_{16})/2$. The $\mu$ are 0.49 and 6.47 km s$^{-1}$ for the ThAr and Sc lamps in the blue arm, while in the red arm.

### Table 2 (Continued)

| Ref. Data Set | S/N | This Work | LAMOST | Wang et al. (2019) | Gaia DR2 |
|---------------|-----|----------|--------|-------------------|----------|
|               |     | Blue Arm | Red Arm | Blue Arm | Red Arm | RVS     |
| RAVE DR6      |     |          |        |          |         |         |
| $10 < S/N < 20$ |    | $N = 1273$ | $N = 719$ | $N = 719$ | $N = 1273$ | $N = 862$ | $N = 146$ |
|               |     | $\mu = 0.47$ | $\mu = 0.50$ | $\mu = 0.37$ | $\mu = -0.29$ | $\mu = -0.19$ | $\mu = 0.14$ |
|               |     | $\sigma = 1.94$ | $\sigma = 2.08$ | $\sigma = 2.03$ | $\sigma = 3.10$ | $\sigma = 2.55$ | $\sigma = 3.02$ |
| $20 < S/N < 50$ |   | $N = 3343$ | $N = 2273$ | $N = 2273$ | $N = 3343$ | $N = 2539$ | $N = 1487$ |
|               |     | $\mu = 0.39$ | $\mu = 0.62$ | $\mu = 0.55$ | $\mu = -0.09$ | $\mu = -0.16$ | $\mu = 0.15$ |
|               |     | $\sigma = 1.81$ | $\sigma = 2.15$ | $\sigma = 2.17$ | $\sigma = 2.34$ | $\sigma = 2.14$ | $\sigma = 2.57$ |
| $50 < S/N < 100$ |   | $N = 2332$ | $N = 3209$ | $N = 3209$ | $N = 2332$ | $N = 1798$ | $N = 2368$ |
|               |     | $\mu = 0.35$ | $\mu = 0.41$ | $\mu = 0.35$ | $\mu = 0.08$ | $\mu = 0.05$ | $\mu = -0.25$ |
|               |     | $\sigma = 1.77$ | $\sigma = 1.88$ | $\sigma = 1.86$ | $\sigma = 2.11$ | $\sigma = 1.93$ | $\sigma = 2.10$ |
they are 0.26 and 4.91 km s⁻¹, respectively. The σ are 1.07 and 1.06 km s⁻¹ for the ThAr and Sc lamps in the blue arm, and in the red arm they are 0.85 and 0.68 km s⁻¹. Generally, the different systematics for the Sc- and ThAr-lamp-calibrated data are consistent with Wang et al. (2019). Despite the large systematics, we find that the precision of the Sc-lamp-calibrated data is no worse than that of the data calibrated by the ThAr lamp. The Rm results are very similar to those of R, except their μ have a 0.3 km s⁻¹ difference. We note that this is reasonable considering that the systematics vary with wavelength as shown in Ren et al. (2021). In addition, ~4000 single-exposure spectra calibrated using an Ne lamp are also found in DR7 v1.1. We confirm that the Ne lamp was used to calibrate the LRS spectra and these mistakenly calibrated data will be removed from the internationally available version of DR7, so we exclude these data in the following analysis.

It is clear that the RVZPs are reasonably stable except after around 2019 May 1, which seems to be correlated with the arc

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**Figure 12.** A comparison of the RVs of standard stars observed in LAMOST MRS DR7 (planid = HIP8426401) determined in this work, in LAMOST, and in Wang et al. (2019). Each color represents a unique RV standard star. The thick ticks show the RVs from Huang et al. (2018), the dashed lines show the RVs before RVZP corrections, and the circles connected by solid lines show the RVZP-corrected RVs. The first exposure is in 2018 April and is calibrated with an Sc lamp, so the correction value is quite different from those of other exposures.
lamp exposure flux.\textsuperscript{14} We plot the $\Delta v$ for each spectrograph for the time interval with available arc lamp intensity in Figures 9 and 10. Since our mean uncertainty of $\Delta v_{i,j}$ is $0.38 \text{ km s}^{-1}$, we regard $\Delta v_{i,j}$ larger than $1 \text{ km s}^{-1}$ as significant values, including the 7th, 12th, 13th, and 15th spectrographs of the blue arm and the 7th, 9th, and 15th spectrographs of the red arm. It is currently not known whether these shifts are due to the fact that the new ThAr lamps were brought into use at around 2019 May 1 or to some other issues induced in the maintenance. Probably in DR8, with the RVZP data in a longer time baseline, we can address the problem.

\subsection*{5.2. Validating with Other Data Sets}

We also validate our RVs with stars common to other data sets, namely, Gaia DR2 (Katz et al. 2019), APOGEE DR16 (Column $\text{VHELIO\_AVG}$; Jönsson et al. 2020), the RV standard stars from Huang et al. (2018), GALAH DR3 (Column

\textsuperscript{14} We have also gained access to the peak flux of Th5231 and Ar6752 of each ThAr lamp exposure since 2018 November, which are not included in the formal data release.
The mean $\mu$ and scatter $\sigma$ derived from Gaussian fitting to the residuals are tabulated in Table 2 and shown as functions of $S/N$ in Figure 11. Also shown are the results of the LAMOST pipeline and Wang et al. (2019). Note that, in DR7 v1.1, the LAMOST pipeline only provides two RVs measured from the blue arm using ELODIE empirical templates (Moultaka et al. 2004) and ATLAS9 synthetic templates (Castelli & Kurucz 2003), respectively. In future MRS data releases, the RVs using ELODIE templates will be removed, and the RVs based on ATLAS9 will be provided for both blue and red arms for better performance. Therefore, we use calibrated RVs of blue arms based on ATLAS9 templates in our comparison (Column rv_kul). The Wang et al. (2019) catalog is a subset of ours (including data taken during the first year and a half with an $S/N$ cut at 10). Spectra without our measurements (i.e., either $S/N < 5$ or $\Delta v$ is invalid) are excluded from this comparison, so that it is fair to the other two RV sources. At the high-$S/N$ end (50–100), we find the standard deviations derived from Gaussian fitting for our results can reach 1.00/1.10, 0.84/0.80, 0.69/0.74, 0.72/0.78, and 1.77/1.88 km s$^{-1}$ with respect to Gaia DR2, APOGEE DR16, Huang et al. (2018), GALAH DR3, and RAVE DR6 in the blue/red arm. The common stars among LAMOST MRS DR7, Gaia DR2, and the other four reference sets are used to calculate the fiducial accuracy of Gaia DR2. At the high-$S/N$ end ($50 < S/N < 100$), the LAMOST MRS gets close to the performance of Gaia DR2 (see Figure 11). Note that, in our algorithm, the Gaia DR2 RVs are used as a reference set, so the comparison with Gaia DR2 is not an independent validation. These results of the comparisons are quite fascinating. At the high-$S/N$ end, we outperform Wang et al. (2019) and the LAMOST pipeline by $\sim$20% and 30%, respectively, according to the blue-arm results compared to APOGEE DR16. At the low-$S/N$ end (10–20), the advantages increase to 58% and 47%, indicating that our algorithm of RV measurements and RVZP determinations is quite efficient. Since the Huang et al. (2018) sample is from APOGEE DR14, the comparison to Huang et al. (2018) has a 0.4 km s$^{-1}$ systematic bias, which does not exist in our comparison to APOGEE DR16. This may be due to the update of the APOGEE data release. GALAH DR3 has a 0.23 km s$^{-1}$ systematic bias as compared to Gaia DR2. The comparison with RAVE DR6, whose spectral resolution is the same as that of

Table 3

| Percentile | This Work | LAMOST | Wang et al. (2019) |
|------------|-----------|--------|-------------------|
|            | Blue Arm  | Red Arm | H$\alpha$ Masked  |
| $q = 50\%$ | 0.45 (16.5%) | 0.54 | 0.59 | 0.53 | 0.54 (−0.7%) | 0.50 |
| $q = 90\%$ | 1.07 (35.5%) | 1.39 | 1.61 | 1.66 | 1.76 (−5.8%) | 1.72 |
| $q = 95\%$ | 1.45 (37.5%) | 1.86 | 2.10 | 2.32 | 2.29 (1.5%) | 2.08 |

Note. Each column shows the RV standard deviation for 678 standard stars at corresponding levels of the CDF. In the parentheses we show the advantages over the LAMOST blue-arm results.
of the LAMOST MRS but is lower than that of Gaia-RVS, APOGEE, and GALAH, shows a large scatter but it is still reasonable.

### 5.3. Self-consistency

Besides the precision, a check on self-consistency is necessary and important before using RVs in time-domain research. As a demonstration, in Figures 12 and 13 we validate the temporal RV variations of the standard stars (whose RVs are assumed invariant) from Huang et al. (2018) in two pointings, namely, planid = HIP8426401 and HIP4312101, which have 10 and 17 exposures and contain 27 and 41 RV standard stars, respectively. The RVs from the LAMOST pipeline show large fluctuations, which are as expected from the comparison in Section 5.2. It turns out that for many stars, the RVs of multiple exposures from the Wang et al. (2019) catalog have exactly the same values. This is due to the failure of their Gaussian fitting process and then they fall back on the best estimation from the 1 km s⁻¹ RV grid, which is a defect in the algorithm. By comparing the measured RVs and the RVZP-corrected RVs, we find that the RVZP-corrected RVs are much cleaner, indicating that the RVs do benefit from our algorithm for Δv₁₂ determination. The LAMOST pipeline and Wang et al. (2019) assume that the RVZP for a spectrograph is static so that the RVZP corrections are basically a shift of the RVs.

To quantify the performance in self-consistency, we select RV standard stars from Huang et al. (2018) with at least eight exposures having valid RVZP-corrected RVs in our catalog and calculate their standard deviation, empirically corrected for the small-number statistic effect, which is discussed in Appendix B. The cumulative distribution function (CDF) of the standard deviations is shown in Figure 14. The absolute RVs from the blue arm (B) show the best self-consistency, followed by those from the red arm (R) and the Hα-masked red arm (Rm). By calculating the 50th, 90th, and 95th percentiles of the CDFs for the blue-arm results, we find our absolute RVs have significant advantages over those of the LAMOST pipeline, namely 16.5%, 35.5%, and 37.5% better (see Table 3), while Wang et al. (2019) are at nearly the same level as the LAMOST pipeline. This reveals the excellent self-consistency of our absolute RVs, which will be used in the time-domain analysis. Note that these estimations of advantages are very conservative due to the fact that the Wang et al. (2019) data set cuts the S/N at 10 but LAMOST and our sample cut at 5, for example. Besides, we do not exclude any of the “exactly the same” RVs from Wang et al. (2019) as readers may find that the CDFs for LAMOST and Wang et al. (2019) jump at around the 0 km s⁻¹ position.

With this excellent self-consistency, we select 10,320 candidate RV standard stars by requiring that in the blue arm (B) and red arm (R)

1. their numbers of exposures are at least 8,
2. their absolute RVs have standard deviations (corrected for small-number statistics) of less than 1.45 and 1.85 km s⁻¹ (corresponding to the 95% level of the CDF, or to 95% completeness with respect to Huang et al. 2018), and
3. their time baselines are longer than at least 180 days.

These stars can be useful for the RV calibration of low-resolution surveys, such as the LAMOST LRS (R ~ 1800).

### Table 4

The 3.8 Million RVs (vobs) Obtained from LAMOST MRS DR7 (v1.1)

| Index | Label (FITS) | Format | Units | Description |
|-------|--------------|--------|-------|-------------|
| 1     | obsid        | Integer| ...   | LAMOST observational ID (unique for each .fits file) |
| 2     | lmjm         | Integer| min   | LMJM² |
| 3     | bjdmid       | Double | ...   | Barycentric Julian date of the middle of exposure |
| 4     | planid       | String | ...   | Plan ID |
| 5     | spid         | Short  | ...   | Spectrograph ID |
| 6     | fiberid      | Short  | ...   | Fiber ID |
| 7     | ra           | Double | deg   | R.A. (J2000) |
| 8     | dec          | Double | deg   | Decl. (J2000) |
| 9     | snr_B        | Float  | ...   | S/N of blue arm |
| 10    | snr_R        | Float  | ...   | S/N of red arm |
| 11    | lamp_B       | String | ...   | Lamp used to calibrate blue arm |
| 12    | lamp_R       | String | ...   | Lamp used to calibrate red arm |
| 13    | rv_B         | Float  | km s⁻¹ | RV (vobs) |
| 14    | rv_err_B     | Float  | km s⁻¹ | RV measurement error (σvobs) |
| 15    | rv_teff_B    | Float  | K     | Teff of the best template |
| 16    | ccfmax_B     | Float  | ...   | CCF max value |
| 17    | rv_R         | Float  | km s⁻¹ | RV (vobs) |
| 18    | rv_err_R     | Float  | km s⁻¹ | RV measurement error (σvobs) |
| 19    | rv_teff_R    | Float  | K     | Teff of the best template |
| 20    | ccfmax_R     | Float  | ...   | CCF max value |
| 21    | rv_Rm        | Float  | km s⁻¹ | RV (vobs) |
| 22    | rv_err_Rm    | Float  | km s⁻¹ | RV measurement error (σvobs) |
| 23    | rv_teff_Rm   | Float  | K     | Teff of the best template |
| 24    | ccfmax_Rm    | Float  | ...   | CCF max value |

Notes. The suffixes _B, _R, and _Rm represent results for the blue arm, red arm, and red arm without Hα, respectively. Table 4 is published in its entirety in machine-readable (FITS) format. A portion is shown here for guidance regarding its form and content.

² The LMJM is 1440× the local modified Julian date of the beginning of exposure, which is an 8 bit integer assigned to each exposure.

(This table is available in its entirety in FITS format.)
The data products of this work include

1. a catalog of ∼3.8 million measured RVs (but 5 million rows for completeness), associated errors, and information on observations for ∼0.8 million stars (Table 4),
2. a catalog of RVZP corrections ($\Delta v_{\text{L},i,j}$) for B, R, and Rm and their uncertainties for all SEUs (Table 5), and
3. a catalog of 10,320 candidate RV standard stars with at least eight exposures and standard deviation less than 1.45/1.86 km s$^{-1}$ in the blue/red arm over a time baseline longer than 180 days (Table 6).

The RV and RVZP catalogs can be cross-matched using the columns spid and lmjm. All catalogs will be available online in FITS format and also on GitHub.\(^{15}\)

A few tips: Users who want to correct Doppler effects of their spectra (e.g., Zhang et al. 2020a) should use RVs without RVZP corrections, while those who want to use absolute RVs can obtain them from our catalogs via Equation (2). The uncertainties of the absolute RVs can be evaluated via

$$\sigma_{\text{abs}}^2 = \sigma_{v,\text{obs}}^2 + \sigma_{\text{min}}^2 + \sigma_{\Delta v}^2 + \sigma_{\text{mod}}^2,$$

where $\sigma_{v,\text{obs}}$ is the measurement error; $\sigma_{\text{min}}$ is the wavelength calibration error floor, which we can infer from the comparison to APOGEE DR16 to be approximately 0.85 km s$^{-1}$ or conservatively 1 km s$^{-1}$; $\sigma_{\Delta v}$ is the uncertainty of the RVZP; $\sigma_{\text{mod}}$ is the uncertainty of the model.

### Table 5

| Index | Label (FITS) | Format | Units | Description |
|-------|--------------|--------|-------|-------------|
| 1     | planid       | String |       | Plan ID     |
| 2     | lmjm         | Integer|       | LMJM*      |
| 3     | spid         | Integer|       | Spectrograph ID |
| 4     | rv_corr0_B   | Float  | km s$^{-1}$ | Initial guess of RVZP correction |
| 5     | nStar_fnt_B  | Integer|       | Number of stars in this SEU with Gaia DR2 RVs |
| 6     | rv_corr2_B   | Float  | km s$^{-1}$ | Number of stars for first term of cost function |
| 7     | nF1_B        | Integer|       | Number of stars for second term of cost function |
| 8     | nF2_B        | Integer|       | Minimum number of exposures |
| 9     | nOther_med_B | Integer|       | Median number of exposures |
| 10    | nOther_max_B | Integer|       | Maximum number of exposures |
| 11    | nOther_min_B | Integer|       | Minimum number of exposures |
| 12    | rv_corr2_unc_B | Float | km s$^{-1}$ | Uncertainty of the final RVZP correction ($\sigma_{\Delta v}$) |
| 13    | nStar_fnt_R  | Integer|       | Number of stars in this SEU with Gaia DR2 RVs |
| 14    | rv_corr2_R   | Float  | km s$^{-1}$ | Final RVZP correction ($\Delta v$) |
| 15    | nF1_R        | Integer|       | Number of stars for first term of cost function |
| 16    | nF2_R        | Integer|       | Number of stars for second term of cost function |
| 17    | nOther_med_R | Integer|       | Median number of exposures |
| 18    | nOther_max_R | Integer|       | Maximum number of exposures |
| 19    | nOther_min_R | Integer|       | Minimum number of exposures |
| 20    | rv_corr2_Rm  | Float  | km s$^{-1}$ | Number of stars in this SEU with Gaia DR2 RVs |
| 21    | nStar_fnt_Rm | Integer|       | Final RVZP correction ($\Delta v$) |
| 22    | rv_corr2_Rm  | Float  | km s$^{-1}$ | Number of stars for first term of cost function |
| 23    | nF1_Rm       | Integer|       | Number of stars for second term of cost function |
| 24    | nF2_Rm       | Integer|       | Median number of exposures |
| 25    | nOther_med_Rm| Integer|       | Maximum number of exposures |
| 26    | nOther_max_Rm| Integer|       | Minimum number of exposures |
| 27    | nOther_min_Rm| Integer|       | Minimum number of exposures |
| 28    | rv_corr2_unc_Rm | Float | km s$^{-1}$ | Uncertainty of the final RVZP correction ($\sigma_{\Delta v}$) |

Notes. The suffixes _B, _R, and _Rm represent results for the blue arm, red arm, and red arm without H$\alpha$, respectively. Table 5 is published in its entirety in machine-readable (FITS) format. A portion is shown here for guidance regarding its form and content.

* The LMJM is 1440:14:19pp:00:00:00, which is also on GitHub.\(^{15}\)

(This table is available in its entirety in FITS format.)

### 6. Data Products

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### Table 6

| Index | Label (FITS) | Format | Units | Description |
|-------|--------------|--------|-------|-------------|
| 1     | ra           | Double | deg   | R.A. (J2000) |
| 2     | dec          | Double | deg   | Decl. (J2000) |
| 3     | rvmed_B      | Float  | km s$^{-1}$ | Median absolute RV |
| 4     | rvstd_B      | Float  | km s$^{-1}$ | Standard deviation of absolute RVs |
| 5     | Nexp_B       | Integer|       | Number of exposures |
| 6     | ts_B         | Double | days  | Time span |
| 7     | rvmed_R      | Float  | km s$^{-1}$ | Median absolute RV |
| 8     | rvstd_R      | Float  | km s$^{-1}$ | Standard deviation of absolute RVs |
| 9     | Nexp_R       | Integer|       | Number of exposures |
| 10    | ts_R         | Double | days  | Time span |

Note. The suffixes _B and _R represent results for the blue arm and red arm, respectively. Table 6 is published in its entirety in machine-readable (FITS) format. A portion is shown here for guidance regarding its form and content.

(This table is available in its entirety in FITS format.)

\(^{15}\)https://github.com/hypergravity/paperdata
and $\sigma_{\text{mod}}$ is the contribution from the sparsity of the spectral templates (0.10 km s$^{-1}$ for the blue arm and 0.20 km s$^{-1}$ for the red arm).

### 7. Summary

In this paper, we measure the RVs from LAMOST MRS DR7 stellar spectra and determine the RVZPs with the help of Gaia DR2 RVs, aiming at making the absolute RVs self-consistent and proper for time-domain analysis. More specifically,

1. we have measured the RVs of $\sim$3.8 million single-exposure spectra for more than 0.8 million stars obtained from LAMOST MRS DR7, including the blue arm and red arm (with and without Hα);
2. we determine the RVZPs exposure by exposure (for 3.6 million spectra) by comparing the measured RVs to those of Gaia DR2 and multiple MRS exposures using a robust method to a mean precision of 0.38 km s$^{-1}$;
3. we find the RVZPs vary significantly for some spectrographs before/after 2019 May 1, which confirms the utility of our algorithm for determining RVZPs;
4. we find good consistency in the comparisons of our absolute RVs with those of APOGEE DR16, RV standard stars (Huang et al. 2018), GALAH DR3, and RAVE DR6, and the precision at $5 < S/N < 100$ can reach 1.00/1.10, 0.84/0.80, 0.69/0.74, 0.72/0.78, and 1.77/1.88 km s$^{-1}$ in the blue/red arm, respectively;
5. we show that compared to those of the LAMOST pipeline and Wang et al. (2019), our absolute RVs have 16.5%, 35.5%, and 37.5% better self-consistency at the 50%, 90%, and 95% levels of the CDF of the standard deviations, respectively, which benefits the subsequent time-domain analysis; and
6. we select a set of 10,320 candidate RV standard stars whose standard deviations of RVs are less than 1.45 and 1.86 km s$^{-1}$ in the blue arm and red arm, respectively, over a time baseline of at least 180 days.

LAMOST MRS DR7 v1.2 and v1.3 have been released. We confirm that, in DR7 v1.2/1.3 the spectra are the same as those in v1.1 while the catalogs and parameters have some minor changes. Therefore, our results can be cross-matched with the v1.2/v1.3 catalogs directly. And we will release a new version of RVs on github once DR8 is released. On the other hand, since Gaia eDR3 is the same as DR2 but with moderate changes. Therefore, our results can be cross-matched with Gaia eDR3. In future LAMOST MRS data releases, we will update our RVs using the most recent Gaia RVs as a reference set.

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Facility: LAMOST.

Software: laspac (Zhang 2020a), regli (Zhang 2020b), berlin (Zhang 2020c), NumPy (van der Walt et al. 2011), SciPy (Virtanen et al. 2020), Astropy (Astropy Collaboration et al. 2013, 2018).

### Appendix A

#### CCF

In this section, we explain our definitions of mean, variance, covariance, and CCF. Let $X$ denote a vector containing $N$ elements $\{X_i\}$ (e.g., a continuum-normalized spectrum with $N$ pixels); the mean is defined as

$$\mathbf{X} = \frac{1}{N} \sum_i X_i,$$  \hspace{1cm} (A1)

the variance as

$$\text{Var}(X) = \frac{1}{N} \sum_i (X_i - \bar{X})^2,$$  \hspace{1cm} (A2)

and the covariance of two vectors $X$ and $Y$ as

$$\text{Cov}(X, Y) = \frac{1}{N} \sum_i (X_i - \bar{X})(Y_i - \bar{Y}).$$  \hspace{1cm} (A3)

A normalized CCF can be calculated with standardized $f$ and $g$, namely

$$\text{CCF}(f, g) = \frac{\text{Cov}(F, G)}{\sqrt{\text{Var}(F) \text{Var}(G)}},$$  \hspace{1cm} (A4)

where $F$ is one vector and $G$ is another vector but it is shifted by the RV $v$. When utilizing this CCF to estimate stellar RVs, $G$ is usually a spectral template whose $S/N$ is infinite and covers the wavelength range of $F$. Therefore, the shift could be implemented with interpolation. The CCF in this form is essentially the linear correlation coefficient and varies between $-1$ and $1$.

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16 http://dr7.lamost.org/v1.2/doc/dr7_update
17 http://dr7.lamost.org/v1.3/doc/dr7_update
Appendix B
Bias in Small-number Statistics

The estimators that characterize dispersion are often underestimated when the number of samples is small. For example, if we only have three or five measurements of a physical quantity, the standard deviation could be underestimated. In this section, we propose an empirical correction of this bias for the error of mean and standard deviation assuming a Gaussian distribution $P(x|\mu, \sigma)$, where $\mu$ is its position and $\sigma$ is its standard error.

B.1. The Deviation of the Mean

We can minimize an $L_1$-norm or $L_2$-norm cost function, namely, $\sum |x_i - \hat{\mu}|$ or $\sum (x_i - \hat{\mu})^2/2$, respectively, to get an estimate of the mean $\hat{\mu}$. The true deviation is by definition

$$\delta_{\text{true}} = |\hat{\mu} - \mu|. \quad (B1)$$

However, in practice we do not know $\mu$ when we tackle such a problem. A fiducial deviation associated with $\hat{\mu}$ can be constructed using the 84th and 16th percentiles or the interquantiles; see Lupton 1993; Ivezić et al. 2014, i.e.,

$$\delta_{\text{est}} = \frac{(x_{i_{84}} - x_{i_{16}})}{2\sqrt{N}}, \quad (B2)$$

where $N$ is the sample size. To obtain an empirical relation between $\delta_{\text{est}}$ and $\delta_{\text{true}}$, we assume the following form:

$$\delta_{\text{true}} = \delta_{\text{est}} / \xi(N), \quad (B3)$$

where $\xi(N)$ is the empirical correction factor and is a function of $N$. Then, we draw mock data from a standard Gaussian distribution using the numpy.random module. In each experiment, we draw $N$ samples and calculate $\delta_{\text{est}}$ and $\delta_{\text{true}}$, and derive $\xi$. We repeat this experiment 3000 times for each $N$.

**Figure B1.** The empirical correction factor for the error of mean ($\xi$) and for the standard error ($\zeta$) of the Gaussian distributions in small-number statistics.

| Cost Function | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ |
|---------------|-----------|-----------|-----------|-----------|-----------|-----------|
| $\sum |x_i - \hat{\mu}|$ | 0.07349721 | -0.60647022 | 1.97806105 | -3.22994084 | 2.72007585 | -0.92812989 |
| $\sum (x_i - \hat{\mu})^2/2$ | 0.08434813 | -0.69694429 | 2.28047526 | -3.73821453 | 3.15612997 | -0.99158461 |

Note. Our definition of a polynomial is $\text{poly}(x|\beta) = \sum \beta_i x^i$.
Appendix C

Several Related Python Packages

Three packages are developed in this work.

1. laspec (Zhang 2020a): A toolkit for LAMOST MRS/ LRS spectra, including modules for file IO, spectral convolution, continuum normalization, removal of cosmic rays, CCFs, and empirical correction evaluation (Appendix B).

2. regli (Zhang 2020b): The Regular Grid Linear Interpolator, a multidimensional linear interpolator based on gridded data. It is faster than scipy.interpolate.LinearNDInterpolator in the Python standard library in our performance test.

3. berliner (Zhang 2020c): A toolkit for manipulating MIST (Dotter 2016) and PARSEC (Bressan et al. 2012) stellar evolutionary tracks and isochrones, including a Python interface for downloading PARSEC isochrones from the CMD 3.4 website (http://stev.oapd.inaf.it/cgi-bin/cmd).

The source code and some tutorials of these packages can be found at https://github.com/hypergravity. Readers who are interested in LAMOST MRS spectra might find them useful for their research.

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References

Allende Prieto, C., Koesterke, L., Hubeny, I., et al. 2018, A&A, 616, A5
Astropy Collaboration, Price-Whelan, A. M., Sipőcz, B. M., et al. 2018, AJ, 156, 123
Astropy Collaboration, Robitaille, T. P., Tollerud, E. J., et al. 2013, A&A, 558, A33
Bird, S. A., Xue, X.-X., Liu, C., et al. 2020, arXiv:2005.05980
Bressan, A., Marigo, P., Girardi, L., et al. 2012, MNRAS, 427, 127
Buder, S., Sharma, S., Kos, J., et al. 2021, MNRAS, 506, 150
Castelli, F., & Kurucz, R. L. 2003, in IAU Symp. 210, Modelling of Stellar Atmospheres, ed. N. Piskunov, W. W. Weiss, & D. F. Gray (San Francisco, CA: ASP), A20
Cropper, M., Katz, D., Sartoretti, P., et al. 2018, A&A, 616, A5
Cui, X.-Q., Zhao, Y.-H., Chu, Y.-Q., et al. 2012, RAA, 12, 1197
De Silva, G. M., Freeman, K. C., Bland-Hawthorn, J., et al. 2015, MNRAS, 449, 2604
Deng, L.-C., Newberg, H. J., Liu, C., et al. 2012, RAA, 12, 735
Dotter, A. 2016, ApJS, 228, 8
El-Badry, K., Ting, Y.-S., Rix, H.-W., et al. 2018, MNRAS, 476, 528
Fu, J.-N., Cat, P. D., Zong, W., et al. 2020, RAA, 20, 167
Gaia Collaboration, Brown, A. G. A., Vallenari, A., et al. 2018, A&A, 616, A1
Gaia Collaboration, Prusti, T., de Bruijne, J. H. J., et al. 2016, A&A, 595, A1
Gao, S., Liu, C., Zhang, X., et al. 2014, ApJL, 788, L37
Gao, S., Zhao, H., Yang, H., & Gao, R. 2017, MNRAS, 469, L68
Gilmore, G., Randich, S., Asplund, M., et al. 2012, Msngr, 147, 25
Gu, W.-M., Mu, H.-F., Fu, J.-B., et al. 2019, ApJL, 872, L20
Huang, Y., Liu, X. W., Chai, Q., et al. 2018, AJ, 156, 90
Ivezić, Z., Connelly, A. J., VanderPlas, J. T., & Gray, A. 2014, in Statistics, Data Mining, and Machine Learning in Astronomy, ed. Z. Ivezić et al. (Princeton, NJ: Princeton Univ. Press)
Jönsson, H., Holtzman, J. A., Allende Prieto, C., et al. 2020, AJ, 160, 120
Katz, D., Munari, U., Cropper, M., et al. 2004, MNRAS, 354, 1223
Katz, D., Sartoretti, P., Cropper, M., et al. 2019, A&A, 622, A205
Kurucz, R. L. 1979, ApJS, 40, 1
Liu, C. 2019, MNRAS, 490, 550
Liu, C., Fu, J., Shi, J., et al. 2020, arXiv:2005.07210
Liu, J., Zhang, H., Howard, A. W., et al. 2019a, Natur, 575, 618
Liu, N., Fu, J.-N., Zong, W., et al. 2019b, RAA, 19, 078
Luo, A. L., Zhao, Y.-H., Zhao, G., et al. 2015, RAA, 15, 1095
Lupton, R. 1993, Statistics in Theory and Practice (Princeton, NJ: Princeton Univ. Press)
Majewski, S. R., Schiavon, R. P., Frinchaboy, P. M., & Soubiran, C. 2004, PASP, 116, 693
Nelder, J., & Mead, R. 1965, CompJ, 7, 308
Nidever, D. L., Holtzman, J. A., Allende Prieto, C., et al. 2015, AJ, 150, 173
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 2007, Numerical Recipes: The Art of Scientific Computing (Cambridge: Cambridge Univ. Press)
Ren, J.-I., Wu, H., Cai, C.-J., et al. 2021, RAA, 21, 051
Sinnefeld, M., Guiglion, G., McMillan, P. J., et al. 2020a, AJ, 160, 83
Sinnefeld, M., Matijević, G., Enke, H., et al. 2020b, AJ, 160, 82
Sinnefeld, M., Zwitter, T., Siebert, A., et al. 2006, AJ, 132, 1645
Tian, H., Liu, C., Wang, Y., et al. 2020, ApJ, 899, 110
Torry, J., & Davis, M. 1979, AJ, 84, 1511
van der Walt, S., Colbert, S. C., & Varoquaux, G. 2011, CSF, 13, 22
Virtanen, P., Gommers, R., Oliphant, T. E., et al. 2020, NatMe, 17, 261
Wang, R., Luo, A. L., Chen, J. J., et al. 2019, ApJS, 244, 27
Wang, S., Qin, H., & Ye, Z. 2010, ExA, 28, 195
Wu, C.-J., Wu, H., Zhang, W., et al. 2021, RAA, 21, 096
Xu, Y., Liu, C., Tian, H., et al. 2020, ApJ, 905, 6
Yang, C., Xue, X.-X., Li, J., et al. 2019, ApJL, 886, 154
Yang, F., Long, R. J., Shan, S.-S., et al. 2020, ApJS, 249, 31
Yanny, B., Rockosi, C., Newberg, H. J., et al. 2009, AJ, 137, 4377
Zhang, B. 2020a, hypergravity/laspec: A Toolkit for LAMOST Spectra, Zenodo, doi:10.5281/ZENODO.4381155
Zhang, B. 2020b, hypergravity/regli: REGular Grid Linear Interpolator, Zenodo, doi:10.5281/ZENODO.4381160
Zhang, B. 2020c, hypergravity/berliner: A Toolkit for Stellar Tracks and Isochrones, Zenodo, doi:10.5281/ZENODO.4381165
