Some thoughts on constructing a microscopic theory with holographic degrees of freedom

Yong Xiao

College of Physical Science and Technology, Hebei University, Baoding 071002, China

Holographic principle states that the maximum entropy of a system is its boundary area in Planck units. However, such a holographic entropy cannot be realized by the conventional quantum field theory. We need a new microscopic theory which naturally possesses all the holographic degrees of freedom. In this paper, we provide some preliminary thoughts on how to construct a theory with holographic degrees of freedom. It may shed light on the understanding of quantum properties of gravity and the early stage of the universe.

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I. INTRODUCTION

Holographic principle states that the maximum entropy contained in a system is its boundary area in Planck units [1–3]. However, the conventional quantum field theory (QFT) cannot provide enough degrees of freedom (DoFs) to account for the holographic entropy. As analyzed in [4], the energy and entropy of an ordinary QFT system can be expressed as \( E = L^3 \Lambda^4 \) and \( S = L^3 \Lambda^3 \) where \( L \) is the size of system and \( \Lambda \) is the effective ultraviolet cutoff of the system, which can also be easily obtained by dimensional analysis. Imposing a requirement that the maximum energy of the system does not exceed the energy of a black hole of the same size, i.e., \( E = L^3 \Lambda^4 \leq L/G \), it immediately leads to \( \Lambda \leq (\sqrt{G}L)^{-1/2} \) which is called the ultraviolet-infrared (UV-IR) relation in [4]. Substituting it into the entropy formula, one finds that the maximally realizable entropy of an ordinary QFT system is \( S \leq (A/G)^{3/4} \). In a word, though QFT is usually viewed as theory with infinite DoFs (attached with an unlimited value of \( \Lambda \)), after imposing the energy limitation of general relativity, the maximum realizable entropy becomes \( (A/G)^{3/4} \). This \( (A/G)^{3/4} \) entropy bound for conventional QFT has been obtained and verified in various contexts [1, 4–8]. Throughout the paper, we reserve the gravitational constant \( G \) to highlight the influence of gravity, while the other fundamental constants \( \hbar, c, K_B \) and those unimportant numerical factors are omitted in most expressions.

Cohen et al. also introduced another UV-IR relationship \( \Lambda \sim (GL)^{-1/3} \) in [4] with the purpose of saturating the holographic entropy bound, i.e., making \( L^3 \Lambda^3 \sim A/G \). But they soon excluded the UV-IR relation because \( E = L^3 \Lambda^4 \gg L/G \) in this case. Interestingly, such a kind of \( (GL)^{-1/3} \) behavior or its variants can always be found in the literature [8, 12]. A recent example is that the specific volume of the constituents of black holes is identified to be \( v = G r_h \) by comparing with the properties and equations of van der Waals fluids [13]. This can be easily translated to a length size \( (Gr_h)^{1/3} \) or \( (Gr_h)^{-1/3} \) in momentum space. Thus we should accept \( \Lambda = (GL)^{-1/3} \) as a useful UV-IR relation, which should be applied to a holographic system rather than a conventional QFT system. Meanwhile, the energy formulæ \( E = L^3 \Lambda^4 \) cannot be applicable for the case with \( \Lambda = (GL)^{-1/3} \). To make it clear, let us examine the gravitational correction to the energy of an ordinary QFT system. The self-gravitational potential energy can be estimated as \( G(L^3 \Lambda^3)/(L^2 \Lambda^2) \) and it is negligible compared to the energy \( E = L^3 \Lambda^4 \) for \( \Lambda \ll (\sqrt{G}L)^{-1/2} \). But with \( \Lambda > (\sqrt{G}L)^{-1/2} \), the gravitational correction is too large and make the ordinary QFT description invalid.

So we need a new theory from \( (\sqrt{G}L)^{-1/2} \) to \( (GL)^{-1/3} \). Many new DoFs should emerge in this range and finally overcome the entropy gap from \( (A/G)^{3/4} \) to \( A/G \). The situation is visualized as follows.

\[
\Lambda : 0 \quad \frac{\text{Ordinary QFT}}{s \ll (A/G)^{3/4}} \quad \frac{\text{New theory}}{s \ll A/G} \quad (GL)^{-1/3}.
\]

In this paper, we aim to provide some thoughts on understanding the physics in the range from \( (\sqrt{G}L)^{-1/2} \) to \( (GL)^{-1/3} \), which has never be carefully studied before as far as we know. It is natural to expect the behavior in this range should be closely related to the quantum properties of gravity. But due to our lack of the knowledge of a complete theory of quantum gravity, we mainly rely on the holographic principle to guide us. We concentrate on answering such a question: how to construct a microscopic theory with holographic DoFs and how the theory is distinct from our familiar QFT. We shall show that such a holographic theory can be successfully constructed. And from this theory it is easy to derive the thermodynamical behaviors \( E \sim L/G \), \( S \sim A/G \) and \( T \sim L^{-1} \) that are typical for black holes.

II. LESSONS FROM THE DEBYE THEORY FOR SOLIDS

Holographic principle imposes a maximum DoFs for any quantum-gravitational system. And in solid physics,
Debye theory also has a limitation to the maximum DoFs of the corresponding system. So here is a suitable place for us to learn some lessons from the Debye theory. The Debye model considers a solid as N non-interacting quantum harmonic oscillators. So the total DoFs for the system is 3N. To insure this limitation, the concept of Debye frequency \( w_D \) is introduced and its value can be calculated from the requirement

\[
L^3 \int_0^{w_D} w^2 dw = 3N. \tag{1}
\]

As always, to avoid unnecessary complications, we omit an analysis of the differences between the longitudinal and transverse DoFs in the solid. From Eq. (1) we get \( w_D \sim N^{1/3}/L \). Knowing the Debye frequency, the energy of the system is expressed as

\[
E = L^3 \int_0^{w_D} \frac{w}{e^{w/T} - 1} w^2 dw. \tag{2}
\]

For a temperature far below \( w_D \), the upper limit of integration can be approximately extended to \( \infty \). Thus the energy is calculated as

\[
E \approx L^3 \int_0^{\infty} \frac{w}{e^{w/T} - 1} w^2 dw \sim L^3 T^4. \tag{3}
\]

For a temperature with \( T \gg w_D \), there is \( e^{w/T} - 1 \approx w/T \). Then the energy is expressed as

\[
E \approx L^3 \int_0^{w_D} T w^2 dw \sim NT. \tag{4}
\]

The physical picture of Debye model is clear. At low energy scale, \( T \ll w_D \), the system is described by phonon gas which exhibits the thermodynamical behaviors

\[
E = L^3 T^4, \quad S = L^3 T^3 \ll 3N. \tag{5}
\]

It gives an effective description of the situation where the DoFs of the system are far from being totally excited. In contrast, at the energy scale larger than \( w_D \), the behaviors of the system become

\[
E \sim NT, \quad S = 3N. \tag{6}
\]

In this situation, all the DoFs are excited and the thermodynamics of phonon gas is no longer a reasonable description of the system.

Now we have learned important lessons from the Debye theory: a system exhibits very different behaviors at different energy scale. Back to the problem we concerned, the ordinary QFT and its thermodynamical behaviors \( E = L^3 A^4 \) and \( S = L^3 A^3 \) are very similar to the low energy scale behaviors of the Debye model. The common characteristic is that they are only applicable to the cases where the DoFs are not fully excited. Then, after some scale, the physical behavior will be dramatically changed. The maximum DoFs that can be excited is \( A/G \) for a quantum-gravitational system and 3N for a Debye solid, save that the related physical mechanisms are different in the two kinds of systems.

### III. A THEORY WITH HOLOGRAPHIC DOFS

#### A. Thermodynamical analysis

A system can exhibit very different behaviors in different energy scale. We therefor hope to conceive a flexible thermodynamical formulae applicable to various situations. We suggest that the thermodynamics of a system can be generally put into the form

\[
S = L^3 \Lambda^3, \tag{7}
\]

\[
E = L^3 \Lambda^3 T. \tag{8}
\]

Here the crucial setting is that we treat \( \Lambda \) and \( T \) differently, after all they are respectively attached to momentum and energy. The parameter \( \Lambda \) is understood as the effective moment cutoff. By intuition every Dofs is located in a size of \( \Lambda^{-1} \), so the number of independent elements is \( L^3/\Lambda^{-3} \) and it is consistent with the entropy formula. In contrast, the temperature \( T \) is understood as the average energy distributed to every DoF. We did not introduce the mass parameter \( m \), because a system consisting of relativistic massless particles always has more entropy and thus is more appropriate for the analysis of entropy bounds.

Now look at the ordinary QFT case with \( E = L^3 A^4 \) and \( S = L^3 A^3 \). Compared with Eqs. (7) and (8), we find \( T \sim \Lambda \). It reflects the fact that in conventional QFT the energy and momentum are treated on the same foot. Concretely speaking, for massless particles of the QFT we always have the energy-moment relation

\[
\varepsilon = c p, \tag{9}
\]

where \( p = |\vec{p}| \).

Then we want to describe a system with all the holographic DoFs being excited. To fit with \( S = A/G \) and \( E = L/G \), we must have \( T = L^{-1} \) and \( \Lambda = (GL)^{-1/3} \) in Eqs. (7) and (8). Considering that \( T \) and \( \Lambda \) are respectively related to energy and momentum, there should be

\[
\varepsilon = G p^3, \tag{10}
\]

rather than \( \varepsilon = cp \) as in the ordinary QFT situation. We call this new type of particles as “holographic particles”. The relation looks very weird, but it is comprehensible because we should expect the gravitational constant \( G \) plays a central role in the holographic case. Whatever, it is worthy to mention that we don’t expect a racial change of the energy-moment relation. The parameter \( \varepsilon \) should always be understood as the energy distributed to a DoF.

We will come back to its explanation later in the paper.

#### B. The validity of \( \varepsilon = G p^3 \)

We have found a relation \( \varepsilon = G p^3 \) for holographic particles. Surely we are eager to find something unfamiliar
to avoid going round in the circle of ordinary QFT and finally to capture the holographic DoFs. But we still need some confidence of the relation before making further explanation. Next we shall illustrate the validity of the relation $\varepsilon = Gp^3$. Using $\varepsilon = Gp^3$ as our starting point, we show that the holographic thermodynamics $E \sim L/G$, $S \sim A/G$ and $T \sim L^{-1}$ can be derived consistently.

For photon fields confined in a box of size $L$, the ensemble approach will lead us to the thermodynamical behaviors

$$E = \frac{\pi^2}{15} L^3 T^4, \quad S = \frac{4\pi^2}{45} L^3 T^3,$$

which are the standard QFT behaviors.

Now keeping $\varepsilon = Gp^3$ in mind, we have a new microscopic theory for holographic particles. All the analysis for photon system can be translated to the new system, expect that the relation $\varepsilon = cp$ has to be replaced by $\varepsilon = Gp^3$. The logarithm of the partition function now is

$$\ln \Xi = - \sum_i \ln(1 - e^{-\beta \varepsilon})$$
$$= -\frac{g L^3}{2\pi^2} \int_0^\infty \ln \left(1 - e^{-\beta Gp^3}\right) p^2 dp = \frac{g L^3}{12 G \beta},$$

where $\beta \equiv 1/T$ and $g$ represents other DoFs such as polarizations. Then we get the expressions for the energy and entropy of the system as

$$E = -\frac{\partial}{\partial \beta} \ln \Xi = \frac{g}{12} L^3 T^2 G,$$

and

$$S = \ln \Xi + E/T = \frac{g L^3 T}{6 G}.$$

Substituting into $T = GA^3$, these formulae can also be reexpressed as

$$E \sim G L^3 A^6, \quad S \sim L^3 A^3.$$

Obviously the system has distinct thermodynamical behaviors from those of conventional QFT. For example, the system has lower temperature and larger entropy density compared to a conventional QFT system with the same energy. When we require the system has an energy $E \sim L/G$, it immediately follows that $T \sim L^{-1}$ and $S \sim A/G$. In addition, the Komar mass corresponds to $(\rho + 3p)V$, so we get $M = 4E = \frac{4 L^3 T^2}{G}$ (the relation $\rho = p$ is derived right away). Comparing with Eq. (14), we observe a relation $M = 2TS$, which is the same as the relation for a Schwarzschild black hole.

Thus, using $\varepsilon = Gp^3$ as the starting point, the typical holographic thermodynamics $E \sim L/G$, $T \sim L^{-1}$ and $S \sim A/G$ can be obtained. Note that though the temperature $T$ is of the order of $L^{-1}$, the corresponding momentum $\Lambda$ is of the order of $(GL)^{-1/3}$, which was inconspicuous in previous knowledge of black hole thermodynamics.

C. The validity of $\varepsilon = Gp^3$ II

We can also calculate the pressure of the system as

$$p = -\frac{\partial F}{\partial V} = T \frac{\partial \ln \Xi}{\partial V} = \frac{g T^2}{12 G}.$$

Comparing with $\rho = U/V = \frac{g T^2}{12 G}$, we get $\rho = p$. Fundamentally this behavior comes from the fact that $\varepsilon_i = G p^3_i = \frac{4}{3} \sqrt{n_x^2 + n_y^2 + n_z^2} \sim \frac{4}{3} \rho$, where the momenta are quantized as $\vec{p} = \frac{2}{3} (n_x, n_y, n_z)$. Thus there is naturally

$$P = -\sum_i a_i \frac{\partial \varepsilon_i}{\partial V} = \sum_i a_i \varepsilon_i = \frac{U}{V} = \rho.$$

Interestingly, Fischler and Susskind [16] applied the holographic principle to cosmology and found that in the flat FRW universe case the holographic entropy bound can only be saturated with the equation of state $p = \rho$.

Banks and Fischler [17] contributed many efforts on the holographic cosmology with $p = \rho$ which can dismiss the Big Band singularity, and they further proposed a holographic eternal inflation model by noting that a black hole with dS interior can be embedded in a $p = \rho$ background. Moreover, the $p = \rho$ fluid has also been used to construct stable dense stars as the endpoints of gravitational collapse, which have no event horizons and no singularities [21]. We think the natural derivation of $\rho = p$ in our model as an evidence of the validity of $\varepsilon = Gp^3$.

IV. THE FIELD-THEORETICAL VIEWPOINT

Now we have a microscopic theory from which the complete holographic thermodynamics can be derived, and its only difference with the familiar photon gas is that we require $\varepsilon = Gp^3$ other than $\varepsilon = p$. Certainly we cannot expect such a simple theory to describe all the profound phenomena of quantum gravity. Maybe the constitutes obeying $\varepsilon = Gp^3$ should only be viewed as the collective excitations or quasi-particles for the corresponding systems. In the following, we try to explain the meaning of the relation $\varepsilon = Gp^3$ from the field-theoretical viewpoint. Under some assumptions, we shall see that $\varepsilon$ should be understood as the energy distributed to a DoF. As in the above calculation for thermodynamics, the gravitational effects are only reflected by the gravitational constant $G$ and we don’t intend to be involved in a complicated analysis of curved space-time.

A. The explanation of $\varepsilon = Gp^3$

We begin with a review of the simplest theory for massless scalar fields. The action is $S = \int d^4x \partial^\mu \varphi \partial_\mu \varphi$. The Hamiltonian is $H = \int d^3x [\varphi^2 + (\nabla \varphi)^2]$. When confined
to a box of volume $V$, the scalar field can be decomposed as

$$
\varphi = \sum_k \frac{1}{\sqrt{2w_kV}} \left[ a_k e^{i(\vec{k} \cdot \vec{r} - w_k t)} + a_k^\dagger e^{-i(\vec{k} \cdot \vec{r} - w_k t)} \right],
$$

where $w_k = |\vec{k}|$. For a particle explanation, we should require

$$
[a_k, a_l^\dagger] = \delta_{kl}, \quad [a_k, a_l] = [a_k^\dagger, a_l^\dagger] = 0.
$$

With Eqs. (18) and (19), the Hamiltonian can be simplified and expressed as

$$
H = \sum_k w_k a_k^\dagger a_k.
$$

As known, the theory describes infinite quantum harmonic oscillators with each particle excitation has energy $w = cp$.

Now we assume a general system can be effectively described by a field denoted as $\Phi$. Next we require $\Phi$ to be a dimensionless field, then the action naturally takes the form $S = \frac{1}{\sqrt{G}} \int d^3x \partial_\mu \phi \partial^\mu \phi$ by dimensional analysis. The corresponding Hamiltonian is

$$
H = \frac{1}{\sqrt{G}} \int d^3x [\dot{\Phi}^2 + (\nabla \Phi)^2].
$$

The factor $1/\sqrt{V}$ is in order to cancel out the integral of coordinates in the Hamiltonian for a while. The factor $G^{3/2}p^{3-\frac{3}{2}}$ is a general form insuring $\Phi$ to be a dimensionless quantity. The value of $\alpha$ is arbitrary here but it will be discussed later. The commutation relation (19) should still hold to insure the physical picture of particle excitation. Substituting Eqs. (19) and (21) into the Hamiltonian, we find

$$
H = \sum_k G^{2a-1}w_k^{4a-1}a_k^\dagger a_k.
$$

When $\alpha = 1/2$ we recover Eq. (20), so the energy distributed to a DoF is $\varepsilon = w$. It is easy to understand because $\Phi/\sqrt{G}$ has dimension 1 and using it as the fundamental field the theory returns to the standard scalar theory. In contrast, when $\alpha = 1$ we get the expected

$$
H = \sum_k Gw_k^3 a_k^\dagger a_k.
$$

The theory is different from conventional QFT in various aspects. First, the energy distributed to a DoF is $\varepsilon = Gw^3$ other than $w$. This gives a field-theoretical explanation of $\varepsilon = Gp^3$. Note that the energy-momentum relation $w = p$ still holds since $e^{-i(\vec{k} \cdot \vec{r} - w_k t)}$ is the planar wave solution of $\partial^\mu \partial_\mu \Phi = 0$. There are two different energies at hand now. It reminds that in a gravitational system the energy measured by a local observer is red-shifted to a distant observer and this effect can remarkably influence the thermodynamics of a system. So it is possible that $w$ could be explained as the intrinsic energy of the oscillator and $Gw^3$ as the energy measured by an exterior observer. Second, the theory with $\varepsilon = Gw^3$ possesses the entire holographic DoFs as we analyzed. Obviously the DoFs increase from $(A/G)^{3/4}$ to $A/G$ as the parameter $\alpha$ changes from $1/2$ to 1. Third, the standard canonical commutation relation $[\Phi(\vec{x}), \pi(\vec{x})] = i(\text{together with } \varepsilon = w)$ is only applicable to the conventional QFT case with $\alpha = 1/2$. For the holographic case with $\alpha = 1$, a detailed analysis will lead to $[q, p] = i(1 + Gp^2)$ where the operator $q$ is constructed from $a_k$ and $a_k^\dagger$, and the conjugate $p$ is extracted from the corresponding Lagrangian. Notably, it has the same form as the second term of a generalized commutation relation $[q, p] = i(1 + Gp^2)$ which corresponds to the generalized uncertainty principle (GUP). The proposal of GUP $\Delta q \Delta p \geq \frac{1}{2} \left(1 + G(\Delta p)^2\right)$ is mainly to incorporate with the minimum length $l_p = \sqrt{G}$ of quantum gravity. The momentum $p$ can also be taken as $p_o + Gp^3$ with $[x, p_o] = i$ to realize the GUP. Again the second term has the same form as our holographic formula $\varepsilon = Gp^3$, though the exact physical connection between them is not clear for now.

**B. Holographic state space and holographic entropy bound**

In Eq. (8) the dimension of the Hilbert space for the conventional QFT is found to be $e^{(A/G)^{3/4}}$. The method can be utilized to analyze the holographic state space.

When confined to a region of size $L$, the particle’s momentum $\vec{p}$ is quantized as $\frac{1}{L} (n_x, n_y, n_z)$. Introducing an effective ultraviolet momentum cutoff $\Lambda$, the total number of the quantized modes is $N = \sum_k 1 \sim L^3 \Lambda^3$. Then the state space of the system can be constructed by acting $a_k^\dagger$ in sequence on the vacuum state $|\Omega\rangle$, that is

$$
(a_{k_1}^\dagger)^{n_1} (a_{k_2}^\dagger)^{n_2} \cdots (a_{k_N}^\dagger)^{n_N} |\Omega\rangle.
$$

Different sets of the occupation number $\{n_i\}$ corresponds to independent quantum states. Now we consider the states satisfying the gravitational stable requirement $E = n_1\varepsilon_{i_1} + n_2\varepsilon_{i_2} + \cdots + n_N\varepsilon_{i_N} \leq E_{bh}$ as the physically permitted state, where $\varepsilon = Gp^3$ should be applied for the holographic case and $E_{bh} = L/G$ is the black hole energy of the same size. To get the dimension of the physically permitted Hilbert space, we need to count the total number of these states.

We start from the simplest states $|a_{k_1}^\dagger|^{n_1} |a_{k_2}^\dagger|^{n_2} |\Omega\rangle (i_1 \neq i_2)$ with only two modes being excited. The number of states satisfying $n_{i_1}\varepsilon_{i_1} + n_{i_2}\varepsilon_{i_2} \leq E_{bh}$ can be evaluated as
\[ \frac{1}{m!} S_2 = \frac{1}{m!} \sum_{i_1 < i_2}^{N} \frac{E_{bh}}{\varepsilon_{i_1}} \frac{E_{bh}}{\varepsilon_{i_2}}. \] The calculation is easy to be generalized to the counting of the states with \( m \) modes simultaneous excited, which is

\[ \frac{1}{m!} S_m = \frac{1}{m!} \sum_{i_1 < i_2 \ldots < i_m}^{N} \frac{E_{bh}}{\varepsilon_{i_1}} \frac{E_{bh}}{\varepsilon_{i_2}} \ldots \frac{E_{bh}}{\varepsilon_{i_m}} \leq \frac{1}{(m!)^2} \left( L^3 \int_0^\Lambda \sqrt{E_{bh}/Gp^3} dp \right)^{2m} \]
\[ = \frac{1}{(m!)^2} \left( \frac{E_{bh} L^6 \Lambda^3}{G} \right)^m. \]

So the dimension of the Hilbert space is

\[ W = \sum_{m=1}^{N} \frac{1}{m!} S_m < \sum_{m=1}^{N} \frac{1}{(m!)^2} z^m \sim \frac{1}{2 \sqrt{3} \pi z^{1/3}} e^{3z^{1/3}}, \]

where \( z \equiv \frac{E_{bh} L^6 \Lambda^3}{G} \). In the above summation, the state number \( \frac{1}{m!} S_m \) peaks at \( m_0 = (\frac{E_{bh} L^6 \Lambda^3}{G})^{1/3} \) and when \( m > m_0 \) the state number drops dramatically to 0. On the other hand, there is surely a physical limitation on the maximum value of excited modes \( m \). The lowest energy state with \( m \) modes excited is the state with one particle occupying one mode successively. The value of \( m \) could be so large that even the state with the lowest energy has \( E > E_{bh} \) and should not contribute to the counting of physically permitted states. The detailed physical analysis leads to \( m_0 = (\frac{E_{bh} L^6 \Lambda^3}{G})^{1/2} \). Then the consistency between the mathematical and physical scenarios requires \( (\frac{E_{bh} L^6 \Lambda^3}{G})^{1/3} = (\frac{E_{bh} L^6 \Lambda^3}{G})^{1/2} \), which immediately leads to \( \Lambda = (GL)^{-1/3} \) as we expect for a holographic theory. Substituting \( E_{bh} = L/G \) and \( \Lambda = (GL)^{-1/3} \) into Eq. \([25]\), we get the dimension of the Hilbert space \( W < e^{A/G} \) and the holographic entropy bound \( S = \ln W < A/G \).

V. CONCLUSION

The conventional QFT is only applicable to the scale \( \Lambda \leq (\sqrt{GL})^{-1/2} \). In this paper, we suggested a new theory exists at the scale from \( (\sqrt{GL})^{-1/2} \) to \( (GL)^{-1/3} \), where new DoFs should emerge and the entropy gap from \( (A/G)^{3/4} \) to \( A/G \) can be overcome. We provided some preliminary thoughts in this direction. By a thermodynamical analysis we proposed that for the holographic theory the energy distributed to a microscopic DoF should be \( \varepsilon = Gp^3 \). Using this relation rather than \( \varepsilon = p \) as the starting point, the standard statistical analysis verifies that it leads to the complete behaviors of holographic thermodynamics: \( E \sim L/G, T \sim 1/L \) and \( S \sim A/G \). It furthermore gives the equation of state \( \rho = p \) for the holographic constitutes, which happens to be consistent with the cosmological holographic entropy bound. Finally, we have tried to give a field-theoretical explanation of \( \varepsilon = Gp^3 \) and discussed several differences between the theory and the conventional QFT. Using the field theory viewpoint, we also constructed the state space of the holographic theory and derived the holographic bound.

Though these thoughts are quite rough, it may still shed some light on the understanding of holographic principle and quantum properties of gravity. Here we want to stress that it can also provide new ideas to the understanding of the early stage of the universe. When we gradually trace back to the early stage of the universe, we encounter higher and higher energy scale physics from atomic physics to nuclear physics and to grand unified physics. In this spirit, our work strongly suggests a holographic stage of the universe before the conventional quantum fields dominated stage. Even for the earlier inflation stage, the holographic eternal inflation model proposed in [20] serves as a good choice. So we expect the universe starts from a holographic stage. Afterwards the density of the holographic fluid with \( w = \rho/p = 1 \) will be diluted by conventional constitutes with \( w = 1/3 \) (radiation) and \( w = 0 \) (matter), since the cosmological evolution favors to lower the value of \( w \). We hope some kinds of remnant indications of this holographic stage could be detected in future experiment.

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We prove that $\rho = p$ can be derived from a holographic thermodynamics. But the converse statement that $\rho = p$ leads to a holographic theory is not always correct. To fit with $T dS = dE + pdv \sim L/G$, both the case $T = (\sqrt{GL})^{-1/2}$, $S = (A/G)^{3/4}$ and the case $T = 1/L$, $S = A/G$ can be applied nicely, depending on the concrete microscopic picture of the theory. Thus, as a conventional QFT, the scalar field theory with $\rho = p$ should still obey the $S = (A/G)^{3/4}$ entropy bound.