Electron cyclotron drift instability and anomalous transport: two-fluid moment theory and modeling

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Abstract

In the presence of a strong electric field perpendicular to the magnetic field, the electron cross-field ($E \times B$) flow relative to the unmagnetized ions can cause the so-called electron cyclotron drift instability (ECDI) due to resonances of the ion acoustic mode and the electron cyclotron harmonics. This occurs in, for example, collisionless shock ramps in space, and in $E \times B$ discharge devices such as Hall thrusters. A prominent feature of ECDI is its capability to induce an electron flow parallel to the background $E$ field at a speed greatly exceeding predictions by classical collision theory. Such anomalous transport is important due to its role in particle thermalization at space shocks, and in causing plasma flows towards the walls of $E \times B$ devices, leading to unfavorable erosion and performance degradation, etc. The development of ECDI and anomalous transport is often considered requiring a fully kinetic treatment. In this work, however, we demonstrate that a reduced variant of this instability, and more importantly, the associated anomalous transport, can be treated self-consistently in a collisionless two-fluid framework without any adjustable collision parameter. By treating both electron and ion species on an equal footing, the free energy due to the inter-species velocity shear allows the growth of an anomalous electron flow parallel to the background $E$ field. We will first present linear analyses of the instability in the two-fluid five- and ten-moment models, and compare them against the fully-kinetic theory. At low temperatures, the two-fluid models predict the fastest-growing mode in good agreement with the kinetic result. Also, by including more ($\geq 10$) moments, secondary (and possibly higher) unstable branches can be recovered. The dependence of the instability on ion-to-electron mass ratio, plasma temperature, and background $B$ field strength is also thoroughly explored. We then carry out direct numerical simulations of the cross-field setup using the five-moment model. The development of the instability, as well as the anomalous transport, is confirmed and in excellent agreement with theoretical predictions. The force balance properties are also studied using the five-moment simulation data. This work casts new insights into the nature of ECDI and the associated anomalous transport and demonstrates the potential of the two-fluid moment model in efficient modeling of $E \times B$ plasmas.

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1. Introduction

In this work, we present two-fluid (electron and ion) investigations of an electrostatic instability due to the electron E × B drift relative to unmagnetized ions initially at rest, with the wavevector perpendicular to the uniform background magnetic field. In the fully kinetic description, this instability is often called the electron cyclotron drift instability (ECDI) due to the coupling between the ion acoustic wave and Doppler-shifted discrete electron cyclotron harmonics.

The research interest of ECDI dates back to the 1970s when references [16–20, 33, 34, 55] presented rather thorough kinetic analyses of this instability, motivated primarily by laboratory observations of enhanced fluctuations in low-β, collisionless plasma shocks perpendicular to a background magnetic field. At the shock ramp, a fraction of the incoming ions are reflected by the shock potential, picking up a fast drift relative to the incoming electrons, exciting ECDI and other microinstabilities. More recently, ECDI received revived interest in the Earth’s bow shocks due to the common observations of electron Bernstein waves in association with ion acoustic waves [7, 14, 22, 37–39, 53, 54], and was suggested to be a potentially important mechanism to allow efficient electron bulk thermalization [11, 54].

ECDI attracted significantly more attention in the Hall effect thruster (HET) research community, stimulated by the continuing efforts to develop electrically powered spacecraft propulsion [6, 13, 21]. In the HET design, a strong electric potential is applied between the anode at the closed end of an annular ceramic channel, and a cathode external to the open end of the channel. Propellant injected at the anode end are ionized by electrons streaming from cathode and accelerated by the applied electric field to produce thrust. To reduce the electron’s axial mobility towards the anode and prolong their residency time in the working channel, a radial magnetic field is applied to magnetize the electrons and confine them through the drift in the E × B azimuthal direction. In numerous studies, however, enhanced electron axial (that is, parallel to the applied electric field) mobility is observed that cannot be explained by classical diffusion due to electron–neutral or electron–ion collisions. A number of explanations have since been proposed to understand this anomalous electron transport [6, 47], with the most promising one being an azimuthal instability, which is the topic of this manuscript, the ECDI.

The role of ECDI in the HET context has been actively studied through laboratory experiments and fully kinetic particle-in-cell modeling, which could be computationally challenging for full-device studies. Existing fluid and hybrid (fluid-electrons-kinetic-ions/neutrals) modeling efforts have also been successful in producing useful HET operation results, but primarily rely on adjustable parameters, for instance, an adjustable, anomalous collision frequency, to reproduce the observed ECDI characteristics and anomalous electron transport. The conjecture that the enhanced mobility does not manifest self-consistently in a fluid or hybrid framework, is often implied. Thorough reviews on the HETs and the numerical efforts to model their physics to different levels of complexity, including ECDI and anomalous drift, can be found in [6, 21, 26, 47].

In this work, we show how the E × B electron drift induce an azimuthal instability in a warm two-fluid (electron–ion) high-moment description without any adjustable parameters, and further, leads to anomalous axial transport of the electrons. This model treats all species, critically, the electrons, in the same manner by evolving their velocity moments, namely number density, velocity, and pressure (and possibly more). The development of an azimuthal instability in this framework is not surprising, due to the free energy available from the velocity difference between electrons and ions. In the cold plasma limit, it reduces to the magnetized Buneman instability [8, 28, 45], therefore the instability itself is not a new finding of this manuscript. The generation of axial transport due to this instability is less evident, though, but can be shown to be a natural result of the Lorentz force applied on the electrons, and, as we will show, is implied by the eigenstructure of the E × B instability. We shall also see that, somewhat similar to the fully kinetic description, this instability is due to the coupling between the ion acoustic mode and the Doppler-shifted hybrid wave where the electron cyclotron dynamics play a critical role. However, with only the lower-order velocity moments taken into account, effects due to higher cyclotron harmonics are lost, leading to less or no quantization and consequently greater deviation from a fully kinetic description as the plasma temperature increases.

In this manuscript, we do not intend to suggest the five-moment two-fluid model as a replacement for the fully-kinetic model, nor to report the discovery of a new fluid instability. Instead, the goals of this paper include (1) to suggest the growth of anomalous transport in a purely fluid description without any collision, which was previously thought to mandate kinetic treatment or anomalous transport; (2) to explore the nature and scaling of the electron drift instability in a finite-temperature two-fluid five-moment (scalar pressure) plasma; (3) to explore regimes when the fluid and kinetic prediction make a meaningful (not perfect), order-of-magnitude agreement; (4) to show that with more velocity moments included in this two-fluid, high-moment framework, the model may capture higher electron cyclotron harmonics and unstable branches, which helps the model to achieve better agreement with the fully-kinetic treatment at higher temperatures.
This manuscript is outlined as follows. In section 2, we present the two-fluid high-moment model framework and the linear theory of the ECDI in the five-moment model, which assumes adiabatic Maxwellian plasma species, as well as some results using the ten-moment model which captures more kinetic physics through higher-order moments. In section 3, we perform two-fluid five-moment simulations of the $E \times B$ configuration and demonstrate the development of ECDI and anomalous electron transport. In section 4, we compare five-moment and fully-kinetic Vlasov–Poisson simulations using experimental parameters to demonstrate the capabilities as well as limitation of the former in capturing ECDI and anomalous current. We conclude in section 5 by summarizing the results and providing future motivation to use high-moment models for cross-field instability studies.

2. Fluid linear theory

A unique and interesting feature of the ECDI in the fully-kinetic description is the discrete growth rates near wave numbers $k_m = m c_s E/B$, where $c_s$ and $v_{E,B}$ are the electron cyclotron frequency and drift velocity, and $m$ is an integer mode numbers. This is evident in the example in figure 1(c), where the green curves represent a typical dispersion relation of ECDI in the fully-kinetic (Vlasov) description. As shown in figure 1(c), the kinetic growth rate of ECDI peaks near integer multiples of $c_s/v_{E,B}$ due to the coupling of the ion acoustic wave and Doppler-shifted Bernstein harmonics at integral multiples of $c_s$. At lower temperatures, however, the quantized unstable branches expand and eventually form a single unstable interval, as indicated by the growth rates shown in figures 1(a) and (b). In this section, we will present linear analysis of the ECDI in the two-fluid five-moment and ten-moment theories to demonstrate their similarities and differences from a fully-kinetic description in describing the ECDI.

2.1. The two-fluid five-moment and ten-moment models

A key concept in the multifluid high-moment model framework is the equal treatment of all populations in the plasma, critically, the electrons. In other words, the electron flows, inertial, and thermal effects are self-consistently evolved instead of inferred from assumptions like quasi-neutrality, etc. In this work, we focus on the five-moment model in this framework, which assumes the electron and ion pressures to be isotropic, i.e., scalars [23],

\[
\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s v_s) = 0,
\]

\[
\frac{\partial (\rho_s v_s)}{\partial t} + \nabla p + \nabla \cdot (\rho_s v_s v_i) = n_s q_s (E + v_s \times B),
\]

\[
\frac{\partial \varepsilon}{\partial t} + \nabla \cdot [v_s (\rho_s + \varepsilon_s)] = n_s q_s v_s \cdot E.
\]

Here, $\rho_s$, $v_s$, $p_s$, and $\varepsilon_s = p_s/(\gamma_{ps} - 1) + \frac{3}{2} p_s c_s^2$ are the mass density, velocity, thermal pressure, and total energy of the plasma population $s$. For electrostatic problems (like ours), the magnetic field is supplied as a background, while the electric field is solved with the Poisson’s equation, coupling all plasma populations through their charge densities [23, 52].

In addition to the five-moment model, in this work, we will also present some results from the ten-moment model where plasma pressures are treated as full tensors with potentially unequal diagonal and non-vanishing off-diagonal elements [24]:

\[
\frac{\partial \rho_s}{\partial t} + \frac{\partial}{\partial x_i} (\rho_s v_s) = 0,
\]

\[
m \frac{\partial (\rho_s v_s)}{\partial t} + \frac{\partial \varepsilon_s}{\partial x_i} = n_s q_s (E_i + \varepsilon_s u_i B_k),
\]

\[
\frac{\partial P_{ij}}{\partial t} = n_s q_s E_j + \frac{q}{m} \varepsilon_{ijkl} P_{kli} B_l.
\]

Here, we have neglected subscripts $s$ for simplicity, $P_{ij} = \int d\mathbf{v} n_{ij} f(v)$ is the stress tensor in the rest frame, the square brackets around indices represent the minimal sum over permutations of free indices needed to yield completely symmetric tensors (for example, $u_i E_j = u_i E_j + u_j E_i$). The ten-moment model retains more kinetic effects resulting in one more electron harmonic which is particularly insightful for the ECDI physics. The ten-moment results will be used to better demonstrate the capability of high-moment models in capturing ECDI, though this paper focuses on the five-moment results primarily.

2.2. Dispersion relations in the five-moment, ten-moment, and fully-kinetic Vlasov models

Consider 1D electrostatic modes in a 1D two-fluid plasma with fully magnetized electrons and unmagnetized ions. The background magnetic field $B_0 = B_0 \hat{e}_z$ is along $z$ and the wavevector $k = k \hat{e}_x$ is along $x$, perpendicular to $B_0$. There exists a background electric field $E = E_0 \hat{e}_z$ and the fully magnetized electrons flow at $E \times B$ drift velocity $v_{E,B} = E \times B/B^2 = v_0 \hat{e}_z$, along $x$.

The dispersion relation in the five-moment regime can be written as

\[
1 = \frac{\omega_{pi}^2}{\omega^2 - k^2 c_{ti}^2} + \frac{\omega_{pe}^2}{(\omega - k v_{E,B})^2 - k^2 c_{te}^2 - \Omega_{ce}^2},
\]

where $c_{ti} = \sqrt{\gamma_{pi}/\rho_i}$, $\omega_{pi}$, and $\Omega_{ce}$ are the sound speed, plasma and cyclotron frequencies for the species $i$, respectively. Compared to the well-known cold-plasma Buneman instability, the formal difference here lies in the presence of the electron cyclotron term, i.e., the role of the first electron–electron resonance. Therefore, the name ‘ECDI’ is still proper for this fluid instability, though the possible quantization due to higher electron cyclotron resonances is missing and leads to greater deviation compared to the fully kinetic counterpart when the plasma temperature goes up. It is worth noting that the existence of drift instabilities in a magnetized two-fluid regime is not surprising, as indicated by Janhunen et al [28] in the cold plasma limit. However, as we will see, this fluid instability has an important and surprising implication for anomalous transport, an important phenomenon in $E \times B$ devices. This is what motivates our in-depth investigation of
Figure 1. Dispersion relations of the ECDI in fluid moment and fully-kinetic models at different electron/ion temperatures. The parameters are \( E_0 = 20 \text{kV/m, } B_0 = 0.005 \text{T, } n = 5 \times 10^{16} \text{m}^{-3}, \ T_i = 0.1 \text{eV, } T_e = 0.5 \text{eV} \), and \( T_e = 5 \text{eV, } T_e = 10 \text{eV, } T_e = 20 \text{eV} \). The five-moment, ten-moment, and Vlasov dispersion relations are in blue, orange, and green, respectively. Bottom panel (e) shows the growth rates of the fastest-growing mode (FGM) predicted by the five-moment and the Vlasov models as a function of the ratio \( c_{se}/v_{E \times B} \). The bottom panel (f) shows the relative error between the five-moment and Vlasov predictions of the FGM growth rates.

The origin and manifestation of the ECDI in the two-fluid model.

As mentioned earlier, the ten-moment model retains the full plasma thermal pressure and one more electron resonance. Its dispersion relation is more complex compared to the five-moment counterpart and is not given here. Instead, we numerically solve both the five- and ten-moment dispersion relations using a matrix-based algorithm [24, 51, 56] to find the real and imaginary frequencies for any wavenumber and a set of background physical parameters.

As an example, figure 1 show the growth rates from electrostatic five-moment, ten-moment, and Vlasov dispersion relations for parameters relevant to those used in the various benchmarks: axial electric field \( E_0 = 20 \text{kV/m, } \) radial magnetic field \( B_0 = 0.005 \text{T, \ number density } n = 5 \times 10^{16} \text{ m}^{-3}, \) ion temperature \( T_i = 0.5 \text{ eV, } \) and varying electron temperatures \( T_e = 2.5 \text{ eV, } 5 \text{ eV, } 10 \text{ eV, } 20 \text{ eV} \). Though the five-moment and ten-moment results cannot obtain all the quantized unstable branches that are seen in the kinetic dispersion, the moment models do provide reasonable estimation of fastest growth rate at lower \( T_e \). It is interesting to note, however, that the ten-moment is able to capture a secondary unstable branch near \( k = \Omega_{ce}/v_{E \times B} \) due to its inclusion of the full pressure tensor. In the future work, one may include even higher velocity moments recover higher electron cyclotron branches, giving better agreement with the Vlasov prediction.

The agreement between five-moment and Vlasov predictions can be better understood through the scaling laws in panels (e) and (f) of figure 1 using the parameters above but with varying \( T_e \). For these parameters, the relative error is below 25% when the electron sound speed, \( c_{se} \), is less than about 0.5 times the drift speed \( v_{E \times B} \). Similar estimations hold for typical Hall thruster parameters. At higher temperatures, higher moments would be required for the moment fluid code to achieve good agreement with the fully kinetic model. This will be investigated in future studies. As a first step, this paper will focus on the analysis of the five-moment model, which will serve as the foundation for future research using higher-moment models.
Figure 2. Predicting the location of the FGM using the crossing of the ion-acoustic wave (3) and the Doppler-shifted hybrid wave (4). The left and right panels are the $k$-$\omega$-$\mathbf{R}$ and $k$-$\gamma$ dispersion relation plots, respectively. For better visualization, simplified parameters are employed here: $m_i/m_e = 25$, $\omega_{pe}/\Omega_{ce} = 4$, $c_e/v_{E,B} = 0.2$, $c_i/v_{E,B} = 0.1$. The solid curves are the actual dispersion relation: the red curves mark the branch with appreciable growth, while the black curves are the rest of the full dispersion relation. The dotted and dashed lines are asymptotic solutions at the large $k$ limit: the blue lines are the ion-acoustic solution (3), the orange lines are the Doppler-shifted hybrid solution (4). The $\times$ symbol marks the actual location of the FGM in the $k > 0$, $\omega > 0$ quadrant. The thin, black vertical lines mark the crossing of the dotted blue and orange asymptotic lines, i.e., the $k_{\text{FGM}}$ given by equation (5) as a prediction of the location of the FGM.

Figure 3. The dependence of the growth rate on the ion-to-electron mass ratio, $m_i/m_e$. Different curves represent dispersion relations due to a vast range of $m_i/m_e$ values. All cases have identical values for the following parameters: $\omega_{pe}/\Omega_{ce} = 10$, $\sqrt{\gamma p_e/m_e}/v_{E,B} = 0.2$, $\sqrt{\gamma p_i/m_i}/v_{E,B} = 0.02$.

2.3. Location of the fastest-growing mode

In the long wavelength limit, the five-moment ECDI dispersion relation has two asymptotic solutions: an ion-acoustic-like wave,

\begin{equation}
1 \approx \frac{\omega_{\text{pi}}^2}{\omega^2 - k^2c_s^2} \Rightarrow \omega^2 - k^2c_s^2 \approx \omega_{\text{pi}}^2,
\end{equation}

and a Doppler-shifted 'hybrid' wave, given by

\begin{equation}
1 \approx \frac{\omega_{pe}^2}{(\omega - kv_{E,B})^2 - k^2c_s^2 - \Omega_{ce}^2},
\end{equation}

or

\begin{equation}
(\omega - kv_{E,B})^2 - k^2c_s^2 \approx \Omega_{ce}^2 + \omega_{pe}^2.
\end{equation}

The unstable region of the five-moment ECDI on the $k$-$\omega$ graph is near the crossings of the two waves. Neglecting the $\omega_{\text{pi}}$ terms for simplicity and equating the two asymptotic dispersion relations, we find the wavenumber of the fastest-growing mode (FGM) in the $\omega > 0$ and $k > 0$ quadrant,

\begin{equation}
k_{\text{FGM}} \approx \sqrt{\frac{\Omega_{ce}^2 + \omega_{pe}^2}{(kv_{E,B} - c_e)^2 - c_s^2}},
\end{equation}

and the associated real-frequency

\begin{equation}\label{eq:omega_FGM}
\omega_{\text{FGM}} \approx \sqrt{k_{\text{FGM}}^2c_s^2 + \omega_{pe}^2}.
\end{equation}

The agreement between the numerically found (black crosses) and predicted (thin vertical line) locations of the FGM is shown in figure 2 for a set of artificial parameters listed in the figure caption. For a wide range of parameters, $k_{\text{FGM}}$ and $\omega_{\text{FGM}}$ give good prediction of the FGM’s wavenumber and real-frequency, respectively.

2.4. Dependence of the fastest growth rate on characteristic parameters

The five-moment ECDI dispersion relation (2) can be written in the dimensionless form,

\begin{equation}
\frac{1}{r} = \frac{1}{m} \frac{\tilde{\omega}^2 - k^2\tilde{c}_s^2}{\tilde{\omega}^2 - k^2\tilde{c}_i^2} + \frac{1}{\left(\tilde{\omega} - \tilde{k}\right)^2 - k^2\tilde{c}_s^2 - 1},
\end{equation}

where $\tilde{\omega} \equiv \omega/\Omega_{ce}$, $\tilde{k} \equiv kv_{E,B}/\Omega_{ce}$, $m \equiv m_i/m_e$, $r = \omega_{pe}/\Omega_{ce}$, $c_e = c_e/v_{E,B}$, $c_i = c_i/v_{E,B}$. In other words, the five-moment ECDI dispersion relation is characterized by the four parameters: $m$, $r$, $c_e$, $c_i$. It is thus useful to further understand how the FGM scales with these parameters. To this end, we start from the baseline parameters $m = 400$, $c_e = 0.2$, and $c_i = 0.02$, $r = 10$, and then vary them in isolation to understand how the ECDI mode growth scales. Note that these parameters are chosen not to match experiments but to amplify the effects of each parameter and make the scaling studies below more clear. Note that it is possible to further develop the analytical form.
cases have identical values for √kce. Another notable observation is that the unstable range has a mass ratio increases, stacking the dispersion relation curves. of unstable wavenumbers also shrinks as the ion-to-electron drops substantially as the mass ratio increases. The range of the dispersion relation in various asymptotic limits but those will be left for future work.

Figure 3 shows the dependence of the growth rate on the ion-to-electron mass ratio, mi/me. It is clear to see that the location of the FGM does not change significantly across a vast range of mi/me ratio. The maximum growth rate, however, drops substantially as the mass ratio increases. The range of unstable wavenumbers also shrinks as the ion-to-electron mass ratio increases, stacking the dispersion relation curves. Another notable observation is that the unstable range has a lower limit at k ∼ Ωce/vpe. The strong dependence of ECDI on ion-to-electron mass ratio indicates a critical difference in the role of this instability in the space plasma, dominated by light ions like hydrogen and oxygen, versus HET plasmas, dominated by heavy ions like xenon and sometimes krypton.

Figure 4 demonstrates how the growth rate depends on electron and ion sound speeds, and thus indirectly, the species temperatures. From the left two panels, higher plasma temperature moves the interval with nonzero growth towards greater wavenumbers and small growth rate. A more subtle observation is that the growth rate drops slower with the electron temperature than with the ion temperature. This is evident in the right panel of figure 4, which shows the maximum growth rate over a matrix of cE and cI values. In this figure, the gradient along the (vertical) ion axis is much greater than that along the (horizontal) electron axis.

Figure 5 shows the dependence of the growth rate on the background magnetic field. Different curves represent different ωpe/Ωce values. All cases have identical values for the following parameters: mi/me = 400, √γpe/me/vpe = 0.2, √γpe/me/vpe = 0.02. The growth rates (vertical coordinates) are normalized by ωpe. of the dispersion relation in various asymptotic limits but those will be left for future work.

Table 1. Normalized eigenvector for the FGM for parameters used by the simulation in section 3.

| nE   | vsel | vsel | vsel |
|------|------|------|------|
| nE   | vsel | vsel | vsel |
| 0.987 | 0.0181i | 0.0181i | 0 |
| nE   | vsel | vsel | vsel |
| 0.0636 + 0.109i | −0.000331 + 0.000666i | 0 |

2.5. Eigenvector and its indication of anomalous electron transport

The matrix-based dispersion relation solver provides the eigenvectors associated with the eigenfrequencies. Here, we consider the parameters m = mi/me = 1836, r = ωpe/Ωce = 5.07, cE = ce/vpe = 0.3, cI = vpe/vpe = 0.0022. Its FGM occurs at k ≈ 5.44 322 Ωce/vpe with a growth rate γ = 0.26892cE−1. Its normalized eigenvector is listed in table 1. Particularly, the value for vsel is non-trivial. In linear analysis, this term and the vsel perturbation stem from the Lorentz force.
due to the background magnetic field. A direct consequence of the non-trivial $v_{cei}$ value is the development of appreciable anomalous axial (i.e., along the applied electric field) transport of the electrons. Direct numerical simulation of anomalous electron transport is discussed in the next section.

3. Numerical simulations scanning simplified parameters

In this section, we perform direct numerical simulations of the ECDI by integrating the five-moment equations (1), coupled with the Poisson’s equation for the electric field. The simulations are performed using the multi-moment solvers in the Princeton code, Gkeyll [23, 24, 52], that has been verified extensively for a number of plasma physics problems [10, 15, 40–44, 46, 48–50]. Similar models have also been implemented by other groups for various applications [1–3, 29, 32, 35, 36].

3.1. Simulation setup

The simulation uses the 1D cross-field configuration, where the simulation domain is along $x$, the initial electric and magnetic fields are along $y$ and $z$, so that the initial electron drift is along $x$. The wavenumber of the FGM is $k_{FGM} = 5.44 \Omega_{ce}/v_{EB}$ with a growth rate $\gamma_{FGM} = 0.27\Omega_{ce}^{-1}$. Its normalized eigenvector is given in section 2.5. The periodic simulation domain length is $L = 10\lambda_{FGM} = 10 \cdot 2\pi/k_{FGM} = 11.54v_{EB}/\Omega_{ce}$ and discretized with $N_x = 1280$ cells. The CFL number is 0.95. The gas gamma $\gamma_{gas}$ is set to 3.

Sinusoidal perturbations are applied to the electrical field $E_x$ so that the spectral energy $|E_x(k)|^2$ is evenly distributed across $kl/2\pi = 0, 1, \ldots, 32$. These modes then compete with each other and, over time, the FGM, presumably the $k = k_{FGM}$ one, dominates. The electron number density $n_e$ is also perturbed to satisfy Gauss’s law initially. Finally, the perturbation magnitudes are controlled by a parameter $\delta = 10^{-5}$ so that the mode $E_x(k_0)$ leads to density fluctuation $\delta \cdot n_e/k_0L$ (and the same $|E_x(k_0)|$).

3.2. Linear development

We first examine the development of electron density fluctuation and azimuthal electric field shown in figure 6. Both show clear dominance of a mode with a wavelength $\lambda = L/10$, or $k = k_{FGM}$ as predicted. Near the end of the simulation at $t = 40\Omega_{ce}^{-1}$, the linear development is saturated and the simulation enters a nonlinear stage. In this paper, we focus on the linear stage only.

An interesting observation is that the $n_e$ and $E_x$ are not entirely out-of-phase; in other words, a nonzero average $\langle n_e E_x \rangle$ develops during the simulation. Consistent with reference [31], this indicates a nonvanishing cross-field electron mobility in the collisionless limit: $\mu_{ce} = -\langle n_e E_{azimuthal} \rangle/n_e E_{axial} \theta_{radial}$ and enhances the electron anomalous transport. In the subsequent subsections, we will study the anomalous transport in more detail.

Next, we compare the simulation results with the linear theory prediction. The left panel of figure 7 shows the snapshots of the electron and ion density fluctuations at the early linear stage, late linear stage, and early nonlinear stage. In the linear stage, $\delta n_i$ is only a small fraction of $\delta n_e$, consistent with the eigenvector prediction given in table 1. Entering the nonlinear stage, the electron density profile becomes highly spiky as the waves start to break. For a more quantitative comparison to theory, the right panel of figure 7 shows the time evolution of different components of the eigenvectors using electron-to-ion ratios in blue curves, along with their predicted values from the FGM as horizontal dashed lines. It is clear that beginning from about $t = 16\Omega_{ce}^{-1}$, the simulated ratios approaches the predicted values. Again, as predicted by theory, a nonvanishing axial electron transport $\int v_{ce} dx$ develops and leads to anomalous transport. Near the end of the simulation, where the evolution is nonlinear, the ratios begin to show deviations from the linear prediction. These results provide excellent verification of the linear development in our simulation.

3.3. Anomalous electron transport

As mentioned earlier, the correlated fluctuations in $n_e$ and $E_x$ indicate the existence of anomalous electron transport. The left panel of figure 8 shows the temporal-spatial profile of the anomalous electron current. A positive net current develops in the linear stage at the predicted wavelength. The right panel shows the growth of the integrated anomalous current. In the early stage of the simulation, the competition between modes of different wavelengths causes a wide spectrum of fluctuations. Beginning from about $t = 16\Omega_{ce}^{-1}$, one dominant mode arises and its linear growth lasts about $20\Omega_{ce}^{-1}$, which is determined by the initial perturbation level. Fitting the data between the primary region of linear growth, at approximately $20 < \Omega_{ce} < 37.5$, we find a growth rate $\gamma = 0.268 \pm 0.008 \Omega_{ce}$, which is in excellent agreement with the theoretical prediction $0.268 \pm 0.008 \Omega_{ce}$.

So far, we have shown the development of nonzero $\langle n_e E_x \rangle$ and the electron anomalous current. It is useful to further examine how they are related. We start by examining the electron momentum equation along the azimuthal direction $x$,

\[
\frac{\partial (\rho_e v_{ce})}{\partial t} + \frac{\partial \rho_e v_{ce}^x}{\partial x} = n_e q_e (E_x + v_{by} B_z),
\]  

(8)

to understand the role of the azimuthal electric force term $n_e q_e E_x$. The left panel of figure 9 shows the decomposition of this equation in the middle of the linear stage at $t = 25\Omega_{ce}^{-1}$. In this snapshot, the pressure gradient force (red) is smaller in magnitude and the $V \times B$ force (green) term is negligible. The $n_e q_e E_x$ term (orange) and the flow divergence term $\partial_i (\rho_e v_{ce}^2)$ (blue) are much larger in magnitude but appear to largely cancel each other. The net acceleration, i.e., the time derivative term (magenta), is a fraction of the $n_e q_e E_x$ and $\partial_i (\rho_e v_{ce}^2)$ terms, indicating the importance of both terms.

Next, we divide this momentum equation by $B_z$ and integrate each of the components of equation (8) along the $x$-coordinate at every time step. This way, we obtain the time evolution of the net and decomposed currents along the $y$-direction.
Figure 6. Time evolution of density fluctuation $\delta n_e \equiv n_e - n_{e0}$ and azimuthal electric field $E_z$ in the numerical simulation.

Figure 7. (Left) Electron and ion number density fluctuation in, from top to bottom, the earlier linear stage, later linear stage, and early nonlinear stage. The competing of various modes, the dominance of the FGM, and the steepening of the waves, are evident in these three stages. (Right) Time evolution of ratios of spatially integrated fluctuations as they approach values from linear-theory predictions of the FGM. The integrated quantities, from top to bottom, are $\int |n_e - n_0| \, dx / \int |n_i - n_0| \, dx$, $\int |v_x e - v_x e_0| \, dx / \int |v_x i| \, dx$, and $\int |v_x e - v_x e_0| \, dx / \int |v_x i| \, dx$. Their expected values due to the FGM from the linear theory are marked by horizontal dashed lines, which are taken from table 1: $|n_{e1}| / |n_i| \approx 7.8373$, $|v_{xe1}| / |v_{xi}| \approx 5.27388$, $|v_{xe1}| / |v_{xi}| \approx 128.562$.

Figure 8. Time evolution of anomalous current in the numerical simulation. (Left) Anomalous axial current $J_y$ as a function of $(t, x)$. The horizontal and vertical axes are time and $x$-coordinates, respectively. Note that in this simulation $J_y$ is due to the anomalous transport of electrons only since ions are unmagnetized and do not contribute to $J_y$. (Right) Time evolution of the anomalous current, $\int_0^L J_y \, dx$, integrated over the entire domain (blue curve) along with a linear fit (orange dashed line). The vertical dashed lines mark the range where the fit is made.

\[
\int dx \left[ n_e q_e v_y + \frac{n_e q_e E_x}{B_z} - \frac{1}{B_z} \frac{\partial (\rho_p v_x e)}{\partial t} - \frac{1}{B_z} \frac{\partial (\rho_p v_x e_0)}{\partial x} \right].
\]

The results are shown in the right panel of figure 9. Here, the dashed curve is the total current $\int n_e q_e v_y \, dx$. The current due to the flow divergence term (green) and the pressure gradient (magenta) are negligible. The main contribution comes from the $E_x \times B_z$ current $\int dx n_e q_e E_z / B_z$ (red), with small cancellation due to time-derivative inertial term, $- \int dx \partial (\rho_p v_x e_0) / B_z$ (orange). The sum of the two agrees with the net anomalous
Figure 9. (Left) Components of the electron momentum equation along the $x$ (azimuthal) direction in the mid-linear stage at $t = 25\Omega_{ce}^{-1}$. The magenta term (the last term in the figure legend) is the net acceleration and are the summation of the remaining terms. (Right) Temporal development of the spatially-integrated axial electron current $\int_{-L}^{L} J_y e dx$ (dotted blue curve) and its decomposition (various solid curves). The terms corresponding equation (9) are $J_y e = n_e q_e v_y e$, $J_y e E \times B = n_e q_e E_x / B_z$, $J_y e, inert = -\partial_t (\rho_e v_{xe}) / B_z$, $J_y e, div = -\partial_x (\rho_e v_{xe}^2) / B_z$, $J_y e, diamag = -\partial_x p_e / B_z$.

Figure 10. Comparison of five-moment and Vlasov simulations in section 4 using parameters $E_0 = 20 \text{kV m}^{-1}$, $B_0 = 0.005 \text{T}$, $m_i / m_e = 241 074$, $n_0 = 5 \times 10^{16} \text{ m}^3$, $T_e = 5 \text{ eV}$, and $T_i = 0.1 \text{ eV}$. All five-moment diagnostics are in blue, and all Vlasov diagnostics are in green. (Upper left panel (a)) Dispersion relations of the ECDI for parameters used in the comparative five-moment vs Vlasov simulations. This panel is a zoomed-in version of figure 1(b) with additional vertical dashed lines marking wavenumbers of initial Fourier modes $k_m = 2\pi m / L_x$, where mode numbers $m = 1, 2, \ldots, 30$. Particularly, the red, purple, and brown vertical lines denote mode numbers $m = 21, 22, \text{and } 23$, which are within the range of peak growths for either model. (Lower left panel (b)) Time evaluation of the integrated electron anomalous current power in the five-moment (blue curve) and Vlasov (green curve) simulations. The dashed straight lines represent the linear-growth fits for the two runs. (Right panel (c)) The growth of different Fourier modes in the five-moment (blue curves) and Vlasov (green curves) simulations, and linear-growth fits (thick, translucent straight lines).

4. Five-Moment fluid vs fully-kinetic simulations using experimental parameters

In this section, we compare 1D five-moment and fully-kinetic Vlasov–Poisson simulations of the instability using parameters relevant to $E \times B$ devices. The goal is to demonstrate the capability and limitations of the model for realistic, experimental parameters, particularly in the linear growth and saturation of the anomalous electron current. We chose the second current very well. Therefore, in this simulation, the anomalous electron current is supported by the $E \times B$ flux, consistent with some previous works [31] but differs from the conclusions of [27] (the latter observed $E \times B$ fluxes that is not large enough to fully account for the anomalous transport). A more comprehensive understanding of this issue would require exhaustive numerical experiments in various parameter regimes and could be the topic of future work.
Figure 11. More comparison of five-moment and Vlasov simulations in section 4. (Left four panels) For a linear stage frame at $t = 800 \Omega_{ce0}^{-1}$, the spatial profile of the anomalous current in the five-moment (upper left, blue line) and Vlasov (upper right, green line) simulations, the power of the first Fourier modes (lower left, blue line with dots) and Vlasov (lower right, green line with dots) simulations. Consistent with figure 10(a), the red, purple, and brown vertical dashed lines denote mode numbers $m = 21, 22, 23$. (Right four panels) The same diagnostics in the saturation stage at $t = 160 \Omega_{ce0}^{-1}$.

The five-moment simulation gives a qualitatively reasonable prediction of the saturation level for the total anomalous current in terms of order of magnitude. The agreement would vary with the parameters like the electron temperature. As implied by the dispersion relations in figure 10(a), the kinetic simulation would allow two comparable modes to develop at $m = 22$ and 23, while the five-moment fluid simulation only has one dominating mode at $m = 22$. The simulations confirmed this difference. Figure 10(c) shows the power of different Fourier components of the anomalous current $J_{ye}$ (for simplicity, only the dominating modes between $21 \leq m \leq 23$ are shown). The simulations clearly captured the expected linear growth rates, marked by thick, translucent straight lines, for either model.

Figure 11 shows the configuration and wavenumber space profiles of the anomalous current during typical linear (left, at $t = 800 \Omega_{ce0}^{-1}$) and saturation (left, at $t = 160 \Omega_{ce0}^{-1}$) stages. In the linear stage, both the spatial profile and Fourier component powers clearly show the dominance of the $m = 22$ mode in the five-moment simulation, while the kinetic simulation shows the overlapping of and competence between the $m = 22$ and $m = 23$ modes. In the saturation stage, the five-moment simulation patterns become very 'spiky' and remain dominated by the single $m = 22$ mode, while the kinetic simulation develops a broader range of wave modes, notably at larger wavelengths.

In summary, using realistic experimental parameters and at relatively low electron temperature, the five-moment fluid model is capable of capturing the growth of anomalous current roughly in the correct regime. The five-moment model itself clearly lacks the broad electron harmonics excited in the fully-kinetic simulation, but the total saturated anomalous current seems to be a good indicator of the kinetic values for lower temperatures. Again, such agreement relies on the parameter regime and becomes less satisfactory as the temperature rises and higher cyclotron harmonics become important. On the other hand, the multifluid high-moment model may indeed capture more cyclotron harmonics by including higher velocity moments in future studies.
5. Discussions and conclusions

The ECDI due to the electron $E \times B$ drift in a cross-field setup is an important research topic actively studied in the HET community and has been drawing increased attention from the space physics community as well. Traditional ECDI studies often rely on fully kinetic models. Models based on the fluid or hybrid description are often thought to require additional collision models and adjustable parameters to correctly capture produce anomalous transport. In this paper, we show detailed theoretical proof and numerical evidence how this instability develops in a collisionless two-fluid plasma, and leads to enhanced axial electron anomalous transport.

In the five-moment model, only the lowest-order electron resonance is captured, and the coupling between a Doppler shifted hybrid wave associated with this resonance, and an ion-acoustic-like wave, leads to the development of ECDI. Compared to the fully kinetic theory featured by a highly quantized nature of the unstable modes due to higher-order electron resonances, the five-moment gives reasonable prediction of the FGM in terms of both wavelength and growth rate, when the plasma temperature is low. The prediction gets worse in comparison to fully kinetic descriptions as the plasma temperature increases.

As indicated by the secondary unstable branch when using the ten-moment model, we may capture a more accurate dispersion relation, including the discrete patterns noted in the kinetic description, by including higher fluid moments. Due to the very low cost of these fluid moment models, this provides a promising new approach for future modeling of HETs and other space physics phenomena where cross-field instabilities are important.

We presented preliminary comparison against fully-kinetic simulations using realistic, experimental parameters, which confirmed the model’s ability to predict the growth and saturation of anomalous current roughly in the correct regime at relatively low electron temperatures. The comparison also shows the model’s inherent limitation of not being able to excite higher harmonics and broader wave modes, which may be partially overcome by including higher velocity moments (like the heat-flux tensor) in the fluid equations. Finally, it should be noted that, the focus of this work is on the fundamental properties and scaling of the dispersion relation and the development of anomalous transport without additional collisions. The work performed here may be extended in the future to use the ten-moment model, and possibly even higher-order moment fluid models, with improved plasma closure relations based on physical constraints [2, 4, 5, 25, 42, 44, 58] or data-driven approaches [12, 59, 60].

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Data availability statement

The Gkeyll software can be acquired from https://gkeyll.readthedocs.io/. The data that support the findings of this study are available upon reasonable request from the authors.

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