Threshold corrections to the MSSM finite-temperature Higgs potential

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Abstract

In the minimal supersymmetric standard model (MSSM) we calculate the one-loop finite-temperature corrections from the squarks-Higgs bosons sector, reconstruct the effective two-Higgs-doublet potential and study possibilities of the electroweak phase transition in the full MSSM ($m_{H^\pm}$, $\tan \beta$, $A_{t,b}$, $\mu$, $m_Q$, $m_U$, $m_D$) parameter space. At large values of $A_{t,b}$ and $\mu$ of around 1 TeV, favored indirectly by LEP2 and Tevatron data, the threshold finite-temperature corrections from triangle and box diagrams with intermediate third generation squarks are very substantial. High sensitivity of the low-temperature evolution to the effective two-doublet and the MSSM squark sector parameters is observed, but rather extensive regions of the full MSSM parameter space allow the first-order electroweak phase transition respecting the phenomenological constraints at zero temperature.

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1 Introduction

The absence of antimatter in the Universe (the baryon asymmetry), a small ratio of the observed number of barions to the observed number of photons \( n_B/n_\gamma \sim 6 \times 10^{-10} \) and the absence of light \( (m_H \sim 100 \text{ GeV}) \) CP-even Higgs boson signal at LEP2 and Tevatron energies lay a specific claim to models of particle physics. The baryon asymmetry and an extremely small \( n_B/n_\gamma \) could be understood on the basis of Sakharov conditions, which are respected at the electroweak phase transition, expected to take place at the temperature of the order of \( 10^2 \) GeV [1]. The generation of nonzero vacuum expectation value \( v \) of the scalar field breaks the elctroneutral symmetry \( SU(2)_f \times U(1)_Y \) to the electromagnetic symmetry \( U(1)_{em} \). It is well-known [2] that in the simple isoscalar model with the standard-like Higgs potential \( U(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4 \), describing a thermodynamically equilibrium system of the scalar particles at the temperature \( T \), the equation for the vacuum expectation value \( v(T) \) has two solutions: \( v(0) = 0 \) and \( v^2(T) = \frac{\mu^2}{\lambda} - \frac{T^2}{4} \), demonstrating the second order phase transition at the critical temperature \( T_c = 2\mu/\sqrt{\lambda} = 2v(0) \), see Fig.1a. The thermal Higgs boson mass \( m_h^2 = -\mu^2 + \lambda T^2/4 \) vanishes at the critical temperature \( T_c \) thus restoring the spontaneously broken symmetry.

![Figure 1: Contours on the \((v, T)\) plane for (a) the second order phase transition (fracture point) and (b) the first order phase transition at the critical temperature \( T_c \) (dashed vertical line).](image)

However, in the cosmological evolution the stages with thermodynamically non equilibrium plasma and the first order phase transitions (see a typical \( v(T) \) contour in Fig.1b) are very important, so such simple picture in combination with the standard model CP-violation by means of the CKM mixing matrix turns out to be not sufficient to justify the observed ratio of baryon number to entropy. The situation becomes better in the minimal supersymmetric model (MSSM) where sparticles, extended two-doublet Higgs sector with the two background fields and nonstandard sources of CP-violation provide a number of new possibilities. In a number of approaches [3] the electroweak phase transition is defined by evolution of the finite temperature effective Higgs potential involving the cubic term in the background scalar fields \( v_1, v_2 \). The larger this term is, the stronger pronounced turns out to be the first order phase transition, which is essential for consistency with the Higgs boson mass beyond the LEP2 exclusion \( m_H < 115 \) GeV. Enhancement of the cubic term in the MSSM at the one-loop level is substantial in the class of MSSM scenarios with a light right stop [4]. Temperature loop corrections from the stop and other additional scalar states could be large and lead to the first order phase transition, the intensity of the latter depends on \( \xi = v(T_c)/T_c \), where \( v(T_c) = \sqrt{v_1^2(T_c) + v_2^2(T_c)} \) is the
vacuum expectation value at the critical temperature $T_c$. The electroweak baryogenesis could be explained if $v(T_c)/T_c > 1$ [5], the case of strong first order phase transition.

In a number of analyses the MSSM finite-temperature effective potential is taken in the representation

$$V_{\text{eff}}(v, T) = V_0(v_1, v_2, 0) + V_1(m(v), 0) + V_1(T) + V_{\text{ring}}(T),$$

(1)

where $V_0$ is the tree-level MSSM two-doublet potential at the SUSY scale, $V_1$ is the (non-temperature) one-loop resummed Coleman-Weinberg term, dominated by stop and sbottom contributions, $V_1(T)$ is the one-loop temperature term and $V_{\text{ring}}$ is the correction of re-summed leading infrared contribution from multi-loop ring (or daisy) diagrams. The MSSM relations between the $SU(2)_L \times U(1)_Y$ gauge couplings $g_2$ and $g_1$, and the quartic parameters $\lambda_{1,2,3,4}$ of the potential $V_0(v_1, v_2, 0)$ are very restrictive. Only two additional parameters $t g \beta = v_2/v_1$ and $m_{H^\pm}$ (charged scalar mass) determine the zero-temperature two-doublet Higgs sector at tree-level. The one-loop radiative corrections, both logarithmic and non-logarithmic generated at the threshold $M_{\text{SUSY}}$, can change strongly the tree-level picture. They depend on the parameters $(A_t, b, \mu, m_Q, m_U, m_D)$ of the scalar quarks-Higgs bosons interaction sector. In most cases for the analysis in the representation (1) numerical methods are used to find the critical temperature $T_c$, for example, by solving the equation for the determinant of second derivatives of the potential (1) at $v_{1,2} = 0$ [6]. Then the two background fields $v_{1,2}(T_c)$ are found at the minimum using the minimization conditions (i.e. the absence of linear terms of the effective potential representation in the "shifted" fields). The first order phase transition strength is dependent on the cubic term $ETv^3$ which appears from the infrared region.

Numerical high-precision Monte Carlo simulations on the lattice [7] have been developed and applied to MSSM in connection with the infrared problem [8] inherent to all analyses based on the effective potentials. Infrared divergences appear in the integration over bosonic static $(\omega_0 = 0)$ Matsubara modes, which in the loop expansion for the three-dimensional momentum space correspond to the intermediate massless bosons. The non-perturbative investigations of the problem have been performed in the framework of high-temperature dimensional reduction [9, 10], when an effective three-dimensional MSSM with the same Green’s functions as in the four-dimensional MSSM for the light bosons is constructed [11, 12, 13] by integrating out perturbatively the non-static modes. The corrections from squarks and gauge bosons are introduced after the reduction to the three-dimensional model.

In order to cover the temperature range from very low temperatures to the temperatures of the order of critical, in our analysis we are using different approach developed in [14, 15] for the general (non-temperature) two-Higgs doublet potential with complex-valued parameters $\mu_{12}^2$, $\lambda_5$, $\lambda_6$ and $\lambda_7$, which violates the CP-invariance explicitly. However, in this publication we consider a simplified situation of the Higgs potential in the CP conserving limit (the imaginary parts of the effective parameters $\lambda_5, \lambda_6, \lambda_7$ and $\mu_{12}^2$ are taken to be zero). The full MSSM effective potential in the generic $\Phi_1, \Phi_2$ basis has the form

$$U_{\text{eff}}(\Phi_1, \Phi_2) = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \mu_{12}^2(\Phi_1^\dagger \Phi_2) - \mu_{12}^2(\Phi_2^\dagger \Phi_1) + \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 +$$

$$+ \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \frac{\lambda_6}{2}(\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) +$$

$$+ \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_6(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + \lambda_7(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_7(\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) +$$

(2)

where the background fields (vev’s) are $\langle \Phi_1 \rangle = (0, v_1)/\sqrt{2}$ and $\langle \Phi_2 \rangle = (0, v_2)/\sqrt{2}$. The temperature corrections from squarks, both logarithmic and non-logarithmic (at the SUSY threshold)
are incorporated to $\lambda_1, ... \lambda_7$. In [14, 15] (see also [16]) a nonlinear transformation for masses and mixing angles $\lambda_i = \lambda_i(\alpha, \beta, m_h, m_H, m_A, m_{H^\pm} \lambda_6, \lambda_7)$, $i = 1, ... 5$ to the Higgs bosons mass basis can be found for a general case ($h, H$ and $A$ are the neutral and $H^+, H^-$ are the charged Higgs bosons, $\alpha$ is the $h-H$ mixing angle, $t\tan \beta = v_2/v_1$)

$$U_{eff}(\Phi_1, \Phi_2) \Rightarrow \frac{m_H^2}{2} (hh) + \frac{m_H^2}{2} (HH) + \frac{m_A^2}{2} (AA) + m_{H^\pm}^2 (H^+ H^-) + h, H, A, H^\pm \text{ interaction terms}$$

which allows to work with symbolic expressions for the temperature-dependent Higgs boson mass eigenstates.

In section 2 we calculate various one-loop temperature corrections to the potential. Section 3 contains some examples of the electroweak phase transition for the finite-temperature effective potential reconstructed in the full MSSM parameter space. The potential of scalar quarks - Higgs bosons interaction and some technical details of evaluation can be found in the Appendix.

## 2 Finite temperature corrections of squarks

In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies $\omega_n = 2\pi n T$ ($n = 0, \pm 1, \pm 2, ...$), lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n = -\infty}^{\infty} \int \frac{dk}{(2\pi)^3} \prod_{i=1}^{b} \frac{(-1)^b}{(k^2 + \omega^2_n + m_j^2)},$$

(4)

Here $k$ is the three-dimensional momentum in a system with the temperature $T$. In the following calculations first we perform integration with respect to $k$ and then take the sum, using the reduction to three-dimensional theory in the high-temperature limit for zero frequencies. At $n \neq 0$ the result is [17, 18]

$$I[m_1, m_2, ..., m_b] = 2T (2\pi T)^{3-2b} \frac{(-1)^b \pi^{3/2}}{(2\pi)^3} \frac{\Gamma(b - 3/2)}{\Gamma(b)} S(M, b - 3/2),$$

(5)

where

$$S(M, b - 3/2) = \int \{dx\} \sum_{n=1}^{\infty} \frac{1}{(n^2 + M^2)^{b-3/2}}, \quad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$  

(6)

For $b > 1$ the parameter $m^2$ is a linear function dependent on $m^2$ and the variables $\{dx\}$ of Feynman parametrization, which are the integration variables in (6). At the integer values of $b$ the integrand in (3) is a generalized Hurwitz zeta-function [19]. Note that for the leading threshold corrections to effective parameters of the two-doublet potential $b > 2$, so the wavefunction renormalization appears in connection with the divergence at $b = 2$ (which is suppressed by vertex factors, see [14]).

A number of integrals can be easily calculated. We calculate the integral

$$J_0[a_1, a_2] = \int \frac{dk}{(2\pi)^3} \frac{1}{(k^2 + a_1^2)(k^2 + a_2^2)} = \frac{1}{4\pi(a_1 + a_2)};$$

(7)

taking a residue in the spherical coordinate system. Here $a_{1,2}^2$ are the sums of squared frequency and squared mass, see (4). Derivatives of $J_0$ with respect to $a_1$ and $a_2$ can be used for calculation of integrals

$$J_1[a_1, a_2] = \int \frac{dk}{(2\pi)^3} \frac{1}{(k^2 + a_1^2)^2(k^2 + a_2^2)} = -\frac{1}{2a_1} \frac{\partial J_0}{\partial a_1} = \frac{1}{8\pi a_1(a_1 + a_2)^2},$$

(8)
Note that the series (12) are divergent, but the derivatives (13) and (14) are convergent for all $n$ and divergent series for (7) to (9), where

$$I_0[m_1, m_2] = \sum_{n=-\infty, n\neq 0}^{\infty} J_0^n[m_1, m_2] = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{4\pi(\sqrt{4\pi^2n^2T^2 + m_1^2} + \sqrt{4\pi^2n^2T^2 + m_2^2})}. \quad (12)$$

or, after redefinition of mass parameters $M_{1,2} = m_{1,2}/2\pi T$ the temperature corrections to effective potential are expressed by summed integrals

$$I_1[M_1, M_2] = -\frac{1}{64\pi^4T^2} \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{\sqrt{M_1^2 + n^2(\sqrt{M_1^2 + n^2 + \sqrt{M_2^2 + n^2})}}}, \quad (13)$$

$$I_2[M_1, M_2] = \frac{1}{256\pi^5T^4} \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{\sqrt{M_1^2 + n^2\sqrt{M_2^2 + n^2(\sqrt{M_1^2 + n^2 + \sqrt{M_2^2 + n^2})}}}}. \quad (14)$$

Note that the series (12) are divergent, but the derivatives (13) and (14) are convergent for all $M_{1,2}$. In the following it will be convenient to keep separately the terms for zero and nonzero modes in the sum. Both terms will be temperature-dependent since the zero-mode integrals coincide with (7)-(9), where $a_i^2 = m_i^2$ and the factor $T$ should be accounted for. Numerical check of the zero temperature limiting case $T \to 0$ demonstrates that the non-temperature field theory results are successfully reproduced. In the high-temperature limit the zero mode gives dominant contribution in agreement with a known suppression of quantum effects at increasing temperatures.

The sum of integrals (13) and (14) can be expressed by means of the generalized zeta-function. Such forms can be derived if we introduce Feynman parameters in the integrand of (7)

$$\frac{1}{[k^2 + m_a^2][k^2 + m_b^2]} = \int_0^1 \frac{dx}{(k^2 + m_a^2)x + (k^2 + m_b^2)(1-x)^2}, \quad (15)$$

and redefine $k \rightarrow p = k/2\pi T$, $M_a^2(M_a, M_b, x) = (M_a^2 - M_b^2)x + M_b^2$. Then we get

$$\frac{1}{|k^2 + m_a^2|^2} = \frac{1}{(2\pi T)^4} \int_0^1 \frac{dx}{(p^2 + n^2 + M^2)^2}. \quad (16)$$

and divergent series for (7) ($dk = (2\pi T)^3 dp$)

$$I_0[M_a, M_b] = \frac{1}{2\pi T} \int_0^1 dx \sum_{n=-\infty, n\neq 0}^{\infty} \int \frac{dp}{(2\pi T)^3} \frac{1}{(p^2 + n^2 + M^2)^2}. \quad (17)$$

$^1$The same results for $J_3$ and $J_4$ can be found in [11] and [12], where they appear in the context of high temperature dimensional reduction.
With the help of dimensional regularization or differentiating the integral
\[
\int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{(\mathbf{p}^2 + M^2)} = -\frac{M}{4\pi} + O\left(\frac{M^2}{T^2}\right)
\] (18)
over the parameter \(M\), the equation (17) can be reduced to

\[
I_0[M_a, M_b] = \frac{1}{16\pi^2 T} \int_0^1 dx \, \zeta(2, \frac{1}{2}, M^2),
\] (19)

where \(\zeta(u, s, t)\) is the generalized Hurwitz zeta-function [19]:

\[
\zeta(u, s, t) = \sum_{n=1}^{\infty} \frac{1}{(n^u + t)^s}.
\] (20)

So in the case under consideration the sums of integrals (13) and (14) can be calculated by differentiation of (19) with respect to mass parameters participating in \(M = M(M_a, M_b, x)\). Differentiation increases the power \(s\) in the denominator of (19) giving convergent integrals

\[
I_1[M_a, M_b] = \frac{T}{2M_a} \frac{\partial}{\partial M_a} I_0 = -\frac{1}{64\pi^4 T^2} \int_0^1 dx \, x \, \zeta(2, \frac{3}{2}, M^2),
\] (21)

\[
I_2[M_a, M_b] = -\frac{1}{2M_b} \frac{\partial}{\partial M_b} (-I_1) = \frac{3}{256\pi^6 T^4} \int_0^1 dx \, (1 - x) \, \zeta(2, \frac{5}{2}, M^2).
\] (22)

The integrals (21) and (22) are equal to the series (13) and (14), respectively.

![Diagram](image)

Figure 2: Threshold corrections (left and central diagram) and diagram contributing to the wavefunction renormalization (right).

**Threshold corrections from the triangle and box diagrams**, shown in Fig. 2, are denoted by \(\Delta\lambda_i^{th}\), \(i=1, \ldots, 7\). They contribute additively to the parameters \(\lambda_i = \lambda_i^{SUSY} - \Delta\lambda_i^{th}\). In the following we are using the normalization conventions from [14]. Calculation of the finite-temperature diagrams for the general case of complex-valued \(\mu\) and \(A_{t,b}\) gives the result (see details in the Appendix)

\[
\Delta\lambda_i^{thr} = 3h_i^4|\mu|^4 I_2[m_Q, m_U] + 3h_i^4|A|^4 I_2[m_Q, m_D] + \]

\[+ h_i^2|\mu|^2(-\frac{g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U] + 2g_1^2 I_1[m_U, m_Q])
\] (23)

\[\text{Note that (non-generalized) Hurwitz zeta-function is defined by } \zeta(s, t) = \sum_{n=0}^{\infty} \frac{1}{(n + t)^s}.\]
\[ + h_b^2 |A_b|^2 \left( \frac{12 h_t^2 - g_1^2 - 3 g_2^2}{2} I_1[m_Q, m_D] + (6 h_t^2 - g_1^2) I_1[m_D, m_Q] \right) \]

\[ \Delta \lambda_2^{thr} = 3 h_t^4 |A_t|^4 I_2[m_Q, m_U] + 3 h_b^4 |\mu|^4 I_2[m_Q, m_D] + \]

\[ + h_t^2 |A_t|^2 \left( \frac{g_1^2 + 3 g_2^2}{2} I_1[m_Q, m_D] + g_1^2 I_1[m_D, m_Q] \right) + \]

\[ + h_b^2 |A_b|^2 \left( \frac{12 h_t^2 + g_1^2 - 3 g_2^2}{2} I_1[m_Q, m_U] + (6 h_t^2 - 2 g_1^2) I_1[m_U, m_Q] \right) \]

\[ \Delta \lambda_3^{thr} = h_t^2 \left( (|\mu|^2 \frac{3 g_2^2 + g_1^2}{12} + |A_t|^2 \frac{12 h_t^2 - g_1^2 - 3 g_2^2}{12} ) I_1[m_Q, m_U] + \right) \]

\[ + (|\mu|^2 \frac{3 h_t^2 - g_1^2}{3} + |A_t|^2 \frac{g_1^2}{3} ) I_1[m_U, m_Q] \right) + \]

\[ + (h_b^2 (|\mu|^2 \frac{3 g_2^2 - g_1^2}{12} + |A_b|^2 \frac{12 h_t^2 - g_1^2 - 3 g_2^2}{4} ) I_1[m_Q, m_D] + \]

\[ + (|\mu|^2 \frac{6 h_t^2 - g_1^2}{6} + |A_b|^2 \frac{g_1^2}{6} ) I_1[m_D, m_Q] \right) + \]

\[ + h_t^2 |\mu|^2 |A_t|^2 I_2[m_Q, m_U] + h_b^2 |\mu|^2 |A_b|^2 I_2[m_Q, m_D] + \]

\[ + h_t^2 h_b^2 (2(A_t A_b - |\mu|^2) I_3[m_Q, m_U, m_D] + (|\mu|^4 + |A_t|^2 |A_b|^2 - 2 A_t A_b |\mu|^2) I_4[m_Q, m_U, m_D] \]

\[ \Delta \lambda_4^{thr} = 6 h_t^4 |\mu|^2 |A_t|^2 I_2[m_Q, m_U] + 6 h_b^4 |\mu|^2 |A_b|^2 I_2[m_Q, m_D] + \]

\[ + h_t^2 \left( (|\mu|^2 \frac{12 h_t^2 + g_1^2 - 3 g_2^2}{4} - |A_t|^2 \frac{g_1^2 - 3 g_2^2}{4} ) I_1[m_Q, m_U] + \right) \]

\[ + (|A_t|^2 g_1^2 - |\mu|^2 (g_1^2 - 3 h_t^2) ) I_1[m_U, m_Q] \right) + \]

\[ + h_b^2 \left( (|\mu|^2 \frac{-12 h_t^2 + g_1^2 + 3 g_2^2}{4} - |A_b|^2 \frac{g_1^2 + 3 g_2^2}{4} ) I_1[m_Q, m_D] + \right) \]

\[ + \frac{1}{2} (|A_b|^2 g_1^2 - |\mu|^2 (g_1^2 - 6 h_b^2) ) I_1[m_D, m_Q] \right) - \Delta \lambda_3^{th} \]

\[ \Delta \lambda_5^{thr} = 3 h_t^4 |A_t|^2 A_t^2 I_2[m_Q, m_U] + 3 h_b^4 |\mu|^2 A_b^2 I_2[m_Q, m_D] \]

\[ \Delta \lambda_6^{thr} = -3 h_t^4 |A_t|^2 I_2[m_Q, m_U] - 3 h_b^4 |\mu|^2 A_b I_2[m_Q, m_D] + \]

\[ + h_t^2 |A_t| (\frac{g_1^2 + 3 g_2^2}{2} I_1[m_Q, m_U] - g_1^2 I_1[m_U, m_Q] \right) + \]

\[ + h_b^2 |A_b| (\frac{-12 h_t^2 + g_1^2 + 3 g_2^2}{4} I_1[m_Q, m_D] - \frac{6 h_t^2 - g_1^2}{2} I_1[m_D, m_Q] \right) \]

\[ \Delta \lambda_7^{thr} = -3 h_t^4 |A_t|^2 I_2[m_Q, m_U] - 3 h_b^4 |\mu|^2 I_2[m_Q, m_D] + \]

\[ + h_t^2 |A_t| (\frac{-g_1^2 + 3 g_2^2}{2} I_1[m_Q, m_D] - \frac{g_1^2}{2} I_1[m_D, m_Q] \right) + \]

\[ + h_b^2 |A_b| (\frac{12 h_t^2 + g_1^2 - 3 g_2^2}{4} I_1[m_Q, m_U] - (3 h_t^2 - g_1^2) I_1[m_U, m_Q] \right) \]

where \( g_1, g_2 \) are U(1) and SU(2) gauge couplings, \( \mu \) is the Higgs superfield mass parameter, \( A_t, A_b \) are the trilinear squarks-Higgs bosons parameters, \( h_t, h_b \) are the Yukawa couplings and
$m_Q, m_U, m_D$ denote the scalar quark mass parameters, in terms of which the physical masses are expressed.

** Corrections of ”fish” diagrams, see Fig.3, give the following contributions to the effective parameters 

$$-\Delta \lambda_1^f = \left[ h_0^2 - \frac{g_1^2}{6} \right]^2 (I(m_Q) + I(m_D)) + \frac{g_1^4}{9} I(m_U),$$

$$-\Delta \lambda_2^f = \left[ h_1^2 + \frac{g_1^2}{6} \right]^2 I(m_Q) + [h_1^2 - \frac{g_1^2}{6}]^2 I(m_U) + \frac{g_1^4}{36} I(m_D),$$

$$-(\Delta \lambda_3 + \Delta \lambda_4)^f = \frac{1}{t^2} \left(-g_1^4 + 6(h_0^2 - h_1^2)g_1^2 + 9(g_1^2 - 2(h_0^2 + h_1^2)) \right) I(m_Q) +$$

$$+ \frac{g_1^2}{3} (h_1^2 - \frac{g_1^2}{3}) I(m_U) + \frac{g_1^2}{6} (h_0^2 - \frac{g_1^2}{6}) I(m_D),$$

$$-\Delta \lambda_3^f = \frac{1}{t^2} \left(-g_1^4 + 6(h_0^2 - h_1^2)g_1^2 + 9(g_1^2 - 2(h_0^2 + h_1^2)) \right) I(m_Q) +$$

$$+ \frac{g_1^2}{3} (h_1^2 - \frac{g_1^2}{3}) I(m_U) + \frac{g_1^2}{6} (h_0^2 - \frac{g_1^2}{6}) I(m_D) + h_1^2 h_0^2 I(m_U, m_D).$$

The three-dimensional integrals in (30)-(34) are

$$J(m_I) = \frac{1}{8\pi m_I}, \quad J(m_U, m_D) = \frac{1}{4\pi (m_U + m_D)}. \quad (35)$$

see (7), leading to series analogously to (12) and (17).

** The logarithmic corrections** for non-degenerate squark masses can be defined following [20] and [21]. Schematically, in the results of [15]) we replace $\ln \left( \frac{M_{susy}}{m_t^2} \right)$ by $\ln \left( \frac{m_{QWU}}{m_t^2} \right)$:

$$\Delta \lambda_1^{log} = -\frac{1}{384\pi^2} \left( 11g_1^4 - 36h_0^2 g_1^2 + 9 \left( g_2^4 - 4h_0^2 g_2^2 + 16h_1^4 \right) \right) \ln \left( \frac{m_{QWU}}{m_t^2} \right), \quad (36)$$

$$\Delta \lambda_2^{log} = -\frac{1}{1536\pi^2} \left( 44g_1^4 - 144h_0^2 g_1^2 + 36g_2^4 + 576h_0^4 - 144g_2^4 h_1^2 \right) \ln \left( \frac{m_{QWU}}{m_t^2} \right), \quad (37)$$

Figure 3: ”Fish” diagrams
Large logarithms not connected with the renormalization group appear also in the wave-function renormalization yield, see below.

It is known that in order to renormalize the the $\lambda \varphi^4$ theory, one needs to renormalize the self-coupling and the mass of the scalar field. If the $\lambda \varphi^4$ theory is supplemented by fermions with interactions defined by the Yukawa term, an additional wave-function renormalization is necessary. Similar situation takes place in the two-doublet model. Expanding the self-energy diagram (see the insertion to the leg in Fig.2, right) calculated with non-degenerate masses at finite temperature, we get at $p^2=0$ the wave-function renormalization (w.f.r.) correction, which is defined by a factor in front of $p^2$. At zero temperature two ways of w.f.r. calculation can be used [23]. Our calculation is based on the integration of convergent w.f.r. contribution over the momentum squared, previously which has been used in differentiation. The standard subtraction scheme at zero momentum (BPSZ-scheme) in the divergent expression for the self-energy contribution, when the divergent pole part is subtracted, turns out to be not convenient at finite temperatures, because in summation over Matsubara frequencies not divergent integrals, but divergent series must be subtracted. Following [14] we can write

$$\Delta \lambda_3^{\text{wfr}} = \frac{1}{384\pi^2} \left(-11g_1^4 + 18\left(h_b^2 + h_t^2\right)g_1^2 + 9\left(g_2^4 - 2\left(h_b^2 + h_t^2\right)g_2^2 + 16h_b^2h_t^2\right)\ln\left(\frac{m_\nu m_U}{m_t^2}\right)\right),$$

$$\Delta \lambda_4^{\text{wfr}} = \frac{3}{64\pi^2} \left(g_2^4 - 2\left(h_b^2 + h_t^2\right)g_2^2 + 8h_b^2h_t^2\right)\ln\left(\frac{m_\nu m_U}{m_t^2}\right).$$

The sum of all w.f.r. corrections to $\lambda_{5,6,7}$ vanishes.

It is useful to check that the finite temperature corrections are reduced to the structures of zero-temperature MSSM, which play a role of boundary condition at $T=0$. Indeed in the

$$A_{ij} = \begin{pmatrix} 2 \cdot 3h_i^2 & F(m_{Q_1}^2, m_U^2, T) \left[ |\mu|^2 & -\mu^* A_U \right] & \left( U \rightarrow D, A \leftarrow \mu \right) \end{pmatrix} \left( 1 - \frac{1}{2} l \right),$$

include the series (compare with Eq.(113) in [9], taking into account differentiation to get the finite w.f.r. yield)

$$F(m_1^2, m_2^2, T) = T \sum_{n=-\infty}^{+\infty} \frac{1}{\sqrt{m_1^2 + (2\pi n T)^2} + \sqrt{m_2^2 + (2\pi n T)^2}} = \frac{T}{(m_1^2 + m_2^2)^3} + 2T \sum_{n=1}^{+\infty} \frac{1}{\sqrt{m_1^2 + (2\pi n T)^2} + \sqrt{m_2^2 + (2\pi n T)^2}}.$$
limiting case of \( T=0 \) and degenerate squark mass parameters all equal to \( M_{SUSY} \) the threshold corrections given by Eq.(23)-(29) are reduced to previous zero-temperature results [14, 22]. For example, let us take \( \Delta \lambda_1 \) for \( m_a = m_b = M_{SUSY} \)

\[
\Delta \lambda_1 = 3h_i^2|\mu|^4I_2[M_{SUSY}] + 3h_i^4|A|^4I_2[M_{SUSY}] + \]

\[
+h_i^2|\mu|^2\frac{(g_1^2 - 3g_2^2)}{2}I_1[M_{SUSY}] + 2g_i^2I_1[M_{SUSY}] +
\]

\[
+h_i^2|A|^2\frac{(12h_i^2 - g_1^2 - 3g_2^2)}{2}I_1[M_{SUSY}] + (6h_i^2 - g_1^2)I_1[M_{SUSY})],
\]

where the integrals are

\[
I_1[M_{SUSY}] \equiv -\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + M_{SUSY}^2)^3} = -\frac{1}{16\pi^2} \frac{1}{2M_{SUSY}^2},
\]

\[
I_2[M_{SUSY}] \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + M_{SUSY}^2)^4} = \frac{1}{16\pi^2} \frac{1}{6M_{SUSY}^4} - \frac{2}{\pi^2} \frac{1}{M_{SUSY}^2} + \frac{2}{\pi^2} \frac{1}{M_{SUSY}^4}.
\]

Transformation to Minkowski space leads to the change of sign in (44). The equality of the temperature series for \( I_{1,2} \) to the symbolic expressions for the integrals can be numerically verified.

In the limiting case of \( T=0 \) and different squark mass parameters the reduction of (13) and (14) to the four-dimensional \( I_1 \) and \( I_2 \) can be achieved using

\[
\frac{1}{[p^2 + m_a^2]_2[p^2 + m_b^2]} = -\frac{1}{2m_a} \frac{\partial}{\partial m_a} \frac{1}{[p^2 + m_a^2][p^2 + m_b^2]},
\]

Differentiating (46) with respect to \( m_b \)

\[
\frac{1}{[p^2 + m_a^2]_2[p^2 + m_b^2]} = -\frac{1}{2m_b} \frac{\partial}{\partial m_b} \frac{1}{[p^2 + m_a^2][p^2 + m_b^2]},
\]

then using Feynman parametrization (15), differentiation in the same way as in (46) and (47), and dimensional regularization to integrate over the four-momentum \( p \) with the following integration over the Feynman parameter, we arrive at

\[
I_1 = -\int \frac{d^4p}{(2\pi)^4} \frac{1}{[p^2 + m_a^2]_2[p^2 + m_b^2]} = -\frac{m_a^2 - m_b^2(1 + 2\ln \frac{m_a}{m_b})}{16\pi^2(m_a^2 - m_b^2)^2},
\]

\[
I_2 = \int \frac{d^4p}{(2\pi)^4} \frac{1}{[p^2 + m_a^2]_2[p^2 + m_b^2]} = \frac{m_a^2 - m_b^2 - (m_a^2 + m_b^2)\ln \frac{m_a}{m_b}}{8\pi^2(m_a^2 - m_b^2)^3}.
\]

In the limit \( m_a = m_b \) these formulas coincide with the expressions for degenerate squark masses (44) and (45).

In calculations of the temperature dependent parameters \( \lambda_i(T) \) of the effective MSSM potential at moderate temperatures we used truncated series with fifty terms (50 Matsubara frequencies). Relative contributions of the remaining terms are less than \( 10^{-2} \) percent at \( T=50 \) GeV, decreasing with an increasing \( T \). At small temperatures of the order of a few GeV an acceptable accuracy is achieved with 1000 terms. The effective parameters \( \lambda_i(T) \) are less than one, justifying the perturbative approach, as a rule, at the squark mass parameters around several hundred GeV. However, strong parametric dependence is observed here, for example, at the squark mass parameters 200, 500 and 800 GeV the criteria \( \lambda_i(T) < 1 \) is valid up to \( T \sim 860 \) GeV, while taking degenerate squark masses at 600 GeV we found that at \( T > 600 \) GeV the perturbative regime cannot be used.
3 Thermal evolution and the critical temperature

In view of the effective two-doublet potential structure defined by (2) one could assume that broken symmetry of \( U_{\text{eff}}(v_1, v_2) \) with a local minima at \( T = 0, v_{1,2} \neq 0 \) appears in the sum of the potential terms with \( \mu_1^2, \mu_2^2 \) and \( \mu_{12}^2 \) of dimension 2 in the fields, which form a 'saddle' (a hyperboloid in the \((v_1, v_2)\) space), and of the dimension 4 terms \( \lambda_{1,\ldots,7} \) which are increasing quartically, being unbounded from above. However, the situation is more involved because \( \mu_{1,2,12}^2 \) and \( \lambda_i \) respect a number of constraints. In this section we are going to describe roughly some possible scenarios of temperature evolution in the effective two-doublet MSSM Higgs sector with threshold, logarithmic and wave-function renormalization one-loop corrections. The squark mass parameters in the following numerical calculations are \( m_Q = 500 \) GeV, \( m_U = 200 \) GeV, \( m_D = 800 \) GeV, giving substantial mass splitting. With these parameters the third generation squark eigenstates \( m_{t_{1,2}} \) and \( m_{b_{1,2}} \) have masses in the range from 400 GeV \( (m_{b_1}) \) to 900 GeV \( (m_{t_2}) \) at the values of \( A_{t,b} \) and \( \mu \) of the order of 1 TeV (only an extremely high-tangent \( \beta \sim 50 \) region is sometimes not suitable). Large difference of the stop masses is necessary to respect constraints following from the LEP2 experimental limit \( m_h > 115 \) GeV. The values of \( \tan \beta \) above 5 and large soft supersymmetry breaking parameters \( A_{t,b} \) and \( \mu \) of the order of \( m_Q \) also lead to an acceptable Higgs boson mass \( m_h \) (but weaken the strength of the electroweak phase transition if taken too large). At the same time substantial threshold corrections appear in the MSSM scenarios with large \( A_{t,b} \) and \( \mu \), like the BGX scenario [24] and the CPX scenario [25], or the regions of MSSM parameter space close to BGX and CPX.

The potential (2) includes temperature-dependent parameters \( \lambda_i(T), i=1,\ldots,7 \), and \( v_{1,2}(T) \), see Eq.(23)-(29), which define the thermal evolution from some high temperature \( T \) of the order of several hundred GeV down to zero. At the SUSY scale, where \( \lambda_1 = \lambda_2 = (g_1^2 + g_2^2)/8 \), \( \lambda_3 = (g_2^2 - g_1^2)/4, \lambda_4 = -g_2^2/2 \) and \( \lambda_5 = \lambda_6 = \lambda_7 = 0 \), the zero-temperature potential \( U_0(v_1, v_2) = -(g_1^2 + g_2^2)(v_1^2 - v_2^2)^2/32 \) is unbounded from below and has two 'flat directions' \( v_1 = \pm v_2 \), see Fig.4. Threshold corrections at zero temperature can be found in [22]. As a rule they transform the decreasing function in Fig.4 to a saddle configuration, slowly increasing along one of the 'flat directions' and more rapidly decreasing along the other. Necessary ingredients of the two-doublet Higgs system are the minimization conditions which set to zero the linear terms in the
Figure 5: Development of the saddle configuration in the potential $U_{\text{eff}}(v_1, v_2)$, see (2), at the critical temperature $T_c = 120$ GeV. The squark sector parameter values are $m_Q = 500$ GeV, $m_U = 200$ GeV, $m_D = 800$ GeV, $A_t = A_b = 1200$ GeV, $\mu = 500$ GeV, the charged Higgs boson mass $m_{H^\pm} = 150$ GeV. Horizontal plane corresponds to $U_{\text{eff}} = 0$.

physical fields $h$, $H$ and $A$ and ensure a local minimum at any point of the surface $U_{\text{eff}}(v_1, v_2)$ in the background fields space (see e.g. [14])

$$\mu_1^2 = \lambda_1 v_1^2 + (\lambda_3 + \lambda_4 + \text{Re}\lambda_5)\frac{v_1^2}{2} - \text{Re}\mu_1^2 v_1 \tan \beta + \frac{v_1^2 s_\beta^2}{2} (3 \text{Re}\lambda_6 \text{ctg} \beta + \text{Re}\lambda_7 \text{tg} \beta), \quad (50)$$

$$\mu_2^2 = \lambda_2 v_2^2 + (\lambda_3 + \lambda_4 + \text{Re}\lambda_5)\frac{v_2^2}{2} - \text{Re}\mu_2^2 v_2 \tan \beta + \frac{v_2^2 c_\beta^2}{2} (3 \text{Re}\lambda_6 \text{ctg} \beta + \text{Re}\lambda_7 \text{tg} \beta), \quad (51)$$

where ($m_A$ is the CP-odd scalar mass)

$$\text{Re}\mu_1^2 = \sin \beta \cos \beta [m_A^2 + \frac{v_1^2}{2} (2 \text{Re}\lambda_5 + \text{Re}\lambda_6 \text{ctg} \beta + \text{Re}\lambda_7 \text{tg} \beta)].$$

Important input parameters of the two-doublet potential are $\tan \beta = v_2/v_1$ and the charged Higgs boson mass

$$m_{H^\pm}^2 = m_W^2 + m_A^2 - \frac{v_1^2}{2} (\text{Re}\Delta\lambda_5 - \Delta\lambda_4) \quad (52)$$

where the effective temperature-dependent mass of the longitudinal $W$-boson is $m_{W_L}(v, T) = m_W^2(v) + \Pi_{W_L}(T), \Pi_{W_L}(T) = 5 g_2^2 T^2/2$ (with the one-loop Standard Model and third-generation squarks contributions included in the polarization operator; $m_W^2 = v^2 g_2^2/2$). We denote $\lambda_{345} = \lambda_3 + \lambda_4 + \text{Re}\lambda_5$. If in the process of thermal evolution, when the system moves along some trajectory in the $v_1(T), v_2(T)$ plane, we require the minimization of $U$ with respect to the scalar fields oscillation in the extremum $v_1(T), v_2(T)$ and continuously admit the interpretation of the system in terms of scalar states $h, H$ and $A$, then $\mu_1^2, \mu_2^2$ and $\mu_4^2$ can be expressed by means of the effective parameters $\lambda_{1,\ldots,7}$ [14].

$^4$Although only the CP-conserving limit is considered, we keep the notation of real parts for the variables where a phase factor could appear in the general case.

$^5$The normalization of $\lambda_{1,2}$ in [14] is different from [15] by a factor of 2.
First we consider the simplified case $\lambda_6 = \lambda_7 = 0$. The two-doublet Higgs potential without $\lambda_6$ and $\lambda_7$ terms has been considered in the context of discrete Peccei-Quinn symmetry [26]. In this case the effective potential $U_{\text{eff}}(v_1, v_2) = -(\lambda_1 v_1^4 + \lambda_2 v_2^4 + \lambda_{345} v_1^2 v_2^2)/4$ is positively defined and unbounded from above if the Sylvester's criteria for the quadratic form $U_{\text{eff}}(v_1^2, v_2^2)$ is respected

$$\lambda_1 < 0, \quad \lambda_2 < 0, \quad \lambda_1 \lambda_2 - \frac{\lambda_{345}^2}{4} < 0$$

(53)

At the critical temperature defined by the equation $\lambda_1 \lambda_2 - \frac{\lambda_{345}^2}{4} = 0$ the positively defined potential starts to develop the saddle configuration which is unbounded from below, see Fig.5. The regions of positively and negatively defined $\lambda_1$ and $\lambda_2$ and the contour for Sylvester’s criteria (53) are shown in Figs. 6 and 7 at the temperature $T = 150$ GeV in the $(A = A_t = A_b, \mu)$ plane.

Figure 6: Contours of negatively defined $\lambda_1$ (left, dark grey area) and $\lambda_2$ (right, dark grey area) in the $(A_t = A_b, \mu)$ plane at the temperature 150 GeV. The squark sector parameter values $m_Q = 500$ GeV, $m_U = 200$ GeV, $m_D = 800$ GeV.

Figure 7: Contour of negatively defined determinant $\lambda_1 \lambda_2 - \frac{\lambda_{345}^2}{4}$ (dark grey area) in the $(A_t = A_b, \mu)$ plane at the temperature 150 GeV. The squark sector parameter values $m_Q = 500$ GeV, $m_U = 200$ GeV, $m_D = 800$ GeV.
plane. The squark mass parameters $m_Q$, $m_U$ and $m_D$ are fixed as mentioned in the beginning of the section, the $(A, \mu)$ parameters are chosen in the vicinity of the contours which separate positively and negatively defined $\lambda$-parameters in (53). The critical temperature in this case is slightly above 120 GeV, insignificantly dependent on the values of $(A_t, b, \mu)$ if they are changing along the right segment of the contour in Fig.7, separating the light grey and the dark grey areas. For the general case of nonzero $\lambda_6$ and $\lambda_7$ defined by Eqs.(28) and (29) the effective potential $U_{\text{eff}}(v_1, v_2) = - (\lambda_1 v_1^4 + \lambda_2 v_2^4 + \lambda_3 v_1^2 v_2^2 + 2\lambda_6 v_1^2 v_2 + 2\lambda_7 v_1 v_2^3)/4$ always demonstrates a saddle configuration which slopes become steeper with an increase of the temperature. Typical shape of $U_{\text{eff}}(v_1, v_2)$ is shown in Fig.8.

Figure 8: The potential $U_{\text{eff}}(v_1, v_2)$, see (2), with nonzero $\lambda_6$ and $\lambda_7$ at the temperature $T = 120$ GeV. The squark sector parameter values $m_Q = 500$ GeV, $m_U = 200$ GeV, $m_D = 800$ GeV, $A_t = A_b = 1500$ GeV, $\mu = 1000$ GeV, charged Higgs boson mass $m_{H^\pm} = 150$ GeV. Horizontal plane corresponds to $U_{\text{eff}} = 0$.

So far we assumed that we are working in the framework of effective field theory at the $m_{\text{top}}$ energy scale, when the contributions from squarks decouple or a contribution of the potential terms with squarks (see the Appendix) is practically constant. However, if it is not the case and the Higgs bosons-squarks quartic term is positive definite with the global minimum at the origin $v_1 = v_2 = 0$, the phase transition may occur due to the development of a saddle configuration by the $\mu_1^2 \mu_2^2$ and $\mu_{12}^2$ terms. Such situation may take place when the vacuum expectation values of charged and colored superpartners participate in the full MSSM scalar potential, possibly giving charge and color breaking minima [27]. It is convenient to use the polar coordinates $v_1(T) = v(T) \cos \beta(T), \quad v_2(T) = v(T) \sin \beta(T)$ for the vacuum expectation values. The mass term of the two-doublet potential has the form

$$U_{\text{mass}}(v, \bar{\beta}) = -\frac{v^2}{2} (\mu_1^2 \cos^2 \bar{\beta} + \mu_2^2 \sin^2 \bar{\beta}) - \frac{v^2}{2} \mu_{12}^2 \sin 2\bar{\beta}$$

By definition at the critical temperature the gradient of $U_{\text{mass}}(v, \bar{\beta})$ is zero along some direction in the $(v_1, v_2)$ plane, then $\partial U_{\text{mass}}/\partial v = 0$ and $1/v \partial U_{\text{mass}}/\partial \bar{\beta} = 0$; it follows from these two equations

$$\tan 2\bar{\beta} = \frac{2\mu_{12}^2}{\mu_1^2 - \mu_2^2}, \quad (\mu_1^2 \mu_2^2 - \mu_{12}^4)[(\mu_1^2 - \mu_2^2)^2 + 4\mu_{12}^4] = 0$$
The phase transition is characterized by the critical angle $\bar{\beta}(T)$ which defines the flat direction for the mass term at the temperature $T_c$ in the background fields plane $(v_1, v_2)$, and at the real-valued $\mu_1$, $\mu_2$ and $\mu_{12}$ the critical temperature is defined by the equation

$$\mu_1^2 \mu_2^2 = \mu_{12}^4$$

(56)

For a fixed set of the squark sector parameters the thermodynamical evolution of the effective potential is described by a $\text{grad} U_{\text{eff}} = 0$ trajectory in the three-dimensional $(v, T, \tan \beta)$ space, which is defined by the intersection of the two surfaces, corresponding to the equation (55) for the critical angle ("$\beta$-surface"), and the equation (56) for the $\mu_1$, $\mu_2$ and $\mu_{12}$ ("$\mu$-surface"). We show the cross sections of $\mu$-surface (calculated without any approximations numerically) by the plane at fixed $T=0$, giving the $(v, \tan \beta)$ contour, and the cross section at $\tan \beta = 1$ giving the $(v, T)$ contour in Fig.9. The presence of nonzero effective parameters $\lambda_6$ and $\lambda_7$ is essential to

get the critical temperature of the order of 100 GeV. Useful analytical approximation can be obtained using the minimization conditions (50)-(51), then the critical angle $\bar{\beta}(T)$ defined by (55) can be expressed as

$$\tan 2\bar{\beta} = \tan 2\beta \left( \frac{v^2}{2m_A^2} - a_1 \right) - \lambda_{345} + \frac{2m_A^2}{\alpha_1} + \alpha_2$$

(57)

Figure 9: Cross sections of the $\mu$-surface, see (55), at the temperature close to zero (right panel) and $\tan \beta = 1$ (left panel, see also Fig.1). White area in the right plot corresponds to the parameter values when the effective mass term (54) has a saddle configuration, $A_{t,b} = 1800$ GeV, $\mu = 2000$ GeV. The $\beta$-surface is very close to the $\mu$-surface. The squark sector parameter values are $m_Q = 500$ GeV, $m_U = 200$ GeV, $m_D = 800$ GeV, the charged Higgs boson mass $m_{H^\pm} = 150$ GeV.

\footnote{Flat directions may exist also in the quartic term separately taken, see e.g. [28].}

\footnote{Mathematica package [29] with encoded representations of $\lambda_i(T)$ by means of series with $n = 50$ was used to scan the MSSM parameter space. At low temperatures the convergence of Matsubara series becomes worse, so the number of terms up to $n = 1000$ is needed to reach an acceptable accuracy.}

\footnote{In different context analogous relation between $\tan \beta$ and $\tan \bar{\beta}$ can be found in [6], where the mass term of the form $v^2 f(\beta, T)$ with $f(\beta, T) = a(\beta) T^2 - b(\beta)$ was analyzed for a special case of degenerate squark masses, $A = \mu = 0$ and with the high-temperature expansion. Our quartic potential is very different from the tree-level $(g_1^2 + g_2^2)/8 \lambda_T v^4 \cos^2 2\beta$ plus a logarithmic term [6], so the approximate expression $3E/\lambda_T > 1$ of Bochkarev-Shaposhnikov criteria $v(T_c)/T_c > 1$ for the absence of sphaleron in the broken phase is not suitable with nonzero threshold corrections.}
where
\[ \alpha_1 = \lambda_5^2 + \frac{1}{4} (\lambda_6 \cot \theta + \lambda_7 \tan \theta), \quad \alpha_2 = \lambda_6 (\tan 2\theta - \cot \theta) - \lambda_7 (\tan \theta + \tan 2\theta). \] (58)

If we assume that \( \bar{\beta}(T) = \beta(0) \), i.e. only the modulo of \( v(T) \) but not the direction in \( (v_1, v_2) \) plane are changed in the process of thermal evolution, then for the critical angle (57) we get
\[ -\frac{m_A^2}{v^2}(2\lambda_5 + \lambda_6 \cot \theta + \lambda_7 \tan \theta) + \frac{v^2}{m_A^2} \left[ \left( \frac{2\lambda_1 - 2\lambda_2 \tan 2\theta + \lambda_6 (\tan \theta - \cot \theta) + \lambda_7 (\tan \theta - 3\tan 2\theta)}{1 - \tan^2 \theta} \right) - \lambda_3 \frac{\gamma}{345} \right] = 0. \] (59)

This approximation may be too rough at small \( m_H \), as pointed out in [6]. The saddle configuration changes not only the shape, but also the horizontal orientation in the process of thermal evolution. For the case \( \lambda_5 = \lambda_6 = \lambda_7 = 0 \) (59) is reduced to
\[ \tan^2 \theta = \frac{2\lambda_1 - \lambda_3 \frac{\gamma}{345}}{2\lambda_2 - \lambda_3 \frac{\gamma}{345}} \] (60)

Combining (51) and (59), where we keep only the leading power terms in \( \lambda_i \) and omit \( \lambda_5 \ll m_A^2/v^2 \), the equation (56) can be written in the form
\[ \lambda_1 (2\lambda_2 - \lambda_3 \frac{\gamma}{345})^2 + 2\lambda_1 (2\lambda_1 - \lambda_3 \frac{\gamma}{345})^2 + \lambda_3 \frac{\gamma}{345} (2\lambda_1 - \lambda_3 \frac{\gamma}{345}) (2\lambda_2 - \lambda_3 \frac{\gamma}{345}) = 0 \] (61)

The vacuum expectation value \( v \) and mass \( m_A \) do not explicitly participate in this equation, only the dimensionless effective parameters \( \lambda_i \) of the quartic potential terms. The left-hand side of (61) approaches zero from below as \( v \) increases, demonstrating however no solution for the saddle configuration. This can be understood qualitatively if we rewrite (61) in the form
\[ \lambda_1 \cot 2\theta + \lambda_2 \tan 2\theta + 2\lambda_3 = 0 \] where the numerical values in the \( \lambda \)-pattern, see Fig.16, calculated in the BGX scenario \( \lambda_1 < 0, \lambda_2 > 0 \) and \( \lambda_3 < 0 \), so in (60) \( \tan 2\theta < 1 \).

Turning back to the case of effective field theory when the squarks decouple at the \( m_{top} \) energy scale, the evaluation of thermal masses of Higgs bosons, mixing angles and couplings can be done using results of [14]. For example, the thermal evolution of the CP-even Higgs bosons \( h \) and \( H \) is expressed by
\[ m_h^2 = c_{\alpha-\beta}^2 m_A^2 + v^2 (2\lambda_1 s_{\alpha}^2 c_{\beta}^2 + 2\lambda_2 c_{\alpha}^2 s_{\beta}^2 - 2(\lambda_3 + \lambda_4)c_{\alpha} s_{\alpha} s_{\beta} + \Re \lambda_5 (s_{\alpha}^2 c_{\beta}^2 + c_{\alpha}^2 s_{\beta}^2)) - 2c_{\alpha-\beta} (\Re \lambda_6 s_{\alpha} c_{\beta} - \Re \lambda_7 c_{\alpha} s_{\beta})), \] (62)
\[ m_H^2 = s_{\alpha-\beta}^2 m_A^2 + v^2 (2\lambda_1 c_{\alpha}^2 c_{\beta}^2 + 2\lambda_2 s_{\alpha}^2 s_{\beta}^2 + 2(\lambda_3 + \lambda_4)c_{\alpha} s_{\alpha} s_{\beta} + \Re \lambda_5 (s_{\alpha}^2 c_{\beta}^2 + c_{\alpha}^2 s_{\beta}^2)) + 2s_{\alpha-\beta} (\Re \lambda_6 c_{\alpha} c_{\beta} + \Re \lambda_7 s_{\alpha} s_{\beta})), \] (63)

where the mixing angle \( \alpha \) of the CP-even states \( h \) and \( H \) is
\[ \tan^2 \alpha = \frac{s_{2\beta} m_A^2 - v^2 ((\lambda_3 + \lambda_4) s_{2\beta} + 2c_{\beta}^2 \Re \lambda_6 + 2s_{\beta}^2 \Re \lambda_7)}{c_{2\beta} m_A^2 - v^2 (2\lambda_1 c_{\beta}^2 - 2\lambda_2 s_{\beta}^2 - \Re \lambda_5 c_{2\beta} + (\Re \lambda_6 - \Re \lambda_7) s_{2\beta})}. \] (64)

We show the regions of the \((v, T)\) plane where the CP-even Higgs boson masses \( m_h \) and \( m_H \) are positively defined in Fig.10 - 12 (shaded areas). "Tachyonic" areas (shown in white colour) correspond to the negative squared masses of \( m_h \) or \( m_H \), see (3), when the fluctuations of physical fields \( h \) and \( H \) near the unstable local extremum \((v_1(T), v_2(T))\) grow exponentially with time. Configuration of the effective potential \( U_{eff}(h, H, A, T) \) expressed in the physical fields \( h, H \) in these areas is a saddle or a function with negative or indefinite sign values unbounded from below. The 'saddle' temperature \( T_c = 120 \) GeV (see Fig.5) is close to the temperature, when the thermal mass \( m_h(T) \) vanishes, only at the low \( \tan \beta \) values. The heavy scalar mass \( m_H(T) \) vanishes at the temperatures substantially higher than \( T_c \). In the scenario under consideration at higher \( \tan \beta \) one should increase the charged scalar mass to respect the zero-temperature condition \( v(0) = 246 \) GeV.
Figure 10: In the shaded areas $m_h$ (left panel) and $m_H$ (right panel) are positively defined at the parameter values $\tan \beta = 5$, $m_{H^\pm} = 180 \text{ GeV}$, $A_t, A_b = 1200 \text{ GeV}$, $\mu = 500 \text{ GeV}$. Isocontours of constant $m_h$ and $m_H$ masses are indicated. The squark sector parameter values are $m_Q = 500 \text{ GeV}$, $m_U = 200 \text{ GeV}$, $m_D = 800 \text{ GeV}$.

Figure 11: The same contours as in Fig.10 at $\tan \beta = 15$, $m_{H^\pm} = 230 \text{ GeV}$

## 4 Summary

Our analysis of the effective MSSM finite-temperature potential is based on a calculation of various one-loop temperature corrections from the squark-Higgs sector for the case of nonzero trilinear parameters $A_t, A_b$ and Higgs superfield parameter $\mu$. Quantum corrections are incorporated in the parameters $\lambda_1, \ldots, \lambda_7$ of the effective two-doublet potential (2), which is then explicitly rewritten in terms of the Higgs boson mass eigenstates, using the approach developed in [14, 15]. The effective parameters $\lambda_1(T), \ldots, \lambda_7(T)$ include the threshold corrections from triangle, box and "fish" diagrams together with the logarithmic and the wave-function renormalization terms. The dominant contribution comes from the triangle and box graphs (Fig.2, left and central) and can be written in a compact form by means of the generalized Hurwitz zeta-function.

Temperature evolution of the potential $U_{\text{eff}}(v_1, v_2)$ expressed in terms of the background
Figure 12: The same contours as in Fig.10 at $\tan\beta = 40$, $m_{H^\pm} = 260$ GeV

fields ($v_1, v_2$) is very sensitive to the MSSM scenario under consideration. We are using the scenarios with large $A_{t,b}$ and $\mu$ (about of the order of 1 TeV), favored by available experimental data. When the non-temperature threshold corrections [22] to the tree-level zero temperature MSSM Higgs potential, see Fig.4, are included at some values of $m_Q, m_U, m_d, A_{t,b}$ and $\mu$, it changes the shape, becoming sign indefinite and unbounded from above, developing a saddle configuration. The evolution of a saddle with the increase of temperature is strongly dependent on the availability of $\lambda_6$ and $\lambda_7$ terms and the magnitude of possible contributions from the MSSM squarks-Higgs bosons sector. If the latter are small (or practically constant in the background field space), heavy superpartners of quarks decouple at the top mass scale and $U_{\text{eff}}(v_1, v_2)$ is adequate to the temperature range of the order of $10^2$ GeV, then the phase transition at some critical temperature $T_c$ of the order of 100 GeV takes place in an extensive regions of the MSSM parameter space and is described by a development of the saddle structure which is defined by the quartic dimensionless parameters $\lambda_i(T)$, $i = 1, \ldots, 5$, see Fig.5. The potential $U_{\text{eff}}(h, H, A)$ written in terms of physical Higgs fields (i.e. Higgs mass eigenstates $h$, $H$ and $A$) demonstrates the spectrum of scalars with positively defined masses which are reaching zero at different temperatures, see Figs.10 - 12. These temperatures are, as a rule, not close to the 'critical' temperature $T_c$, when the potential $U_{\text{eff}}(v_1, v_2)$, $\lambda_6 = \lambda_7 = 0$, forms a horizontal 'narrow gully', so not only the first, but also the second derivatives are zero in some direction. It is interesting that the transformation of the potential configuration at zero temperature unbounded from below, see Fig.4, to the saddle configuration unbounded from above, see 5, in the framework of large $A_{t,b}$ and $\mu$ MSSM scenarios is rapid if thermal masses of scalars do not change substantially. The isocontours for CP-even scalar masses $m_h$ and $m_H$ fall down in an extremely narrow temperature region near $T = 0$, see Figs.10 - 12, so during the 'overturn' of the potential a nearly step-like decrease of $v(T)$ must happen to keep constant masses. If the contribution from the squarks-Higgs bosons sector is large and the decoupling approximation for squarks is not valid, the phase transition is generated by a saddle configuration of the mass term with $\mu_2^2$, $\mu_3^2$ and $\mu_{12}^2$, dimension 2 with respect to the fields. Although our evaluation is performed at the one-loop only and not all possible corrections are considered, usually it is possible to adjust $\mu$, $A_{t,b}$ (or $X_{t,b}$), $m_Q$, $m_U$ and $m_D$ of the MSSM parameter space in such a way that the boundary condition for zero temperature $v(0) = 246$ GeV is respected, the
critical temperature is of the order of 100 GeV or higher, and the phase transition is of the first order. Independently on the temperature evolution scenarios, one should not forget that the value of $\tan \beta$ at zero temperature must be consistent with the range from 5 to 50-60, provided by phenomenological restrictions from LEP2 and Tevatron data for the reactions $e^+e^- \rightarrow hZ$, $h \rightarrow b\bar{b}$, and $pp \rightarrow t\bar{t}$, $t \rightarrow H\pm b$. In the nearest future useful information about the allowed regions of the MSSM parameter space could be provided by the LHC Higgs physics program [30]. Availability of the criteria $v(T_c)/T_c \sim 1$ for the absence of sphaleron in the broken phase deserves a more careful study with the evaluation of radiative corrections from other sources, especially the infrared ones.

We considered only the case of real-valued MSSM parameters $A_{t,b}$ and $\mu$. Generalization to the complex-valued parameters (the case of explicit CP violation in the squark-Higgs and the two-doublet Higgs sectors) is straightforward with radiation corrections defined by Eqs.(27)-(29), where phases of $A_{t,b}$ and $\mu$ can be introduced. Complex $\lambda_{5,6,7}$ lead to the mixing of CP-even $h,H$ and CP-odd $A$ scalars resulting in the Higgs bosons without a definite CP-parity $h_1,h_2$ and $h_3$ with specific properties, modifying the qualitative picture described above.

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5 Appendix

Inputs and some details of various quantum corrections calculation, see Eq.(23) and the following
group of formulas, are given below. Supersymmetric potential of the Higgs bosons - third generation of
couplings for the third generation squarks interaction has the form [20]

\[ V^0 = V_M + V_T + V_\Lambda + V_{\tilde{Q}}, \]

where

\[ V_M = -\mu_d^2 \Phi_1^\dagger \Phi_2 + M_Q^2 (\tilde{Q}^\dagger \tilde{Q}) + M_U^2 \tilde{U}^* \tilde{U} + M_D^2 \tilde{D}^* \tilde{D}, \]
\[ V_T = \Gamma^P_i \left( \Phi_1^\dagger \tilde{Q} \right) \tilde{D} + \Gamma_i^U \left( i \Phi_i^T \sigma_2 \tilde{Q} \right) \tilde{U} + \Gamma_i^D \left( \tilde{Q}^\dagger \Phi_i \right) \tilde{D}^* - \Gamma_i^U \left( i \tilde{Q}^\dagger \sigma_2 \Phi_i^* \right) \tilde{U}^*, \]
\[ V_\Lambda = \Lambda_i g_i \left( i \Phi_i^T \sigma_2 \Phi_i \right) \tilde{D}^* \tilde{U} + \text{h.c.}, \]

\[ V_{\tilde{Q}} \]

denotes the four scalar quarks interaction terms, \( \sigma_2 \equiv \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \), and \( \Lambda^I \) are defined by

the tree-level equalities

\[ \Lambda^Q = \text{diag}\left\{ \frac{1}{4}(g_2^2 - g_1^2 Y_Q), \quad h_U^2 - \frac{1}{4}(g_2^2 - g_1^2 Y_Q) \right\}, \]
\[ \Lambda^U = \text{diag}\left\{ \frac{1}{4} g_1^2 Y_U, h_U^2 + \frac{1}{4} g_1^2 Y_U \right\}, \]
\[ \Lambda^D = \text{diag}\left\{ h_D^2 - \frac{1}{4} g_2^2 Y_D, \frac{1}{4} g_2^2 Y_D \right\}, \quad \Lambda = -h_U h_D. \]

Here the squark hypercharges are \( Y_{Q_i} = 1/3(-1), \) \( Y_{D_i} = 2/3(2), \) \( Y_{U_i} = -4/3, \) and the Yukawa

couplings for the third generation squarks \( h_t = \sqrt{\frac{m_t}{v \sin^2 \beta}}, \) \( h_b = \sqrt{\frac{m_b}{v \cos^2 \beta}}. \) In the general case of

complex-valued parameters

\[ \Gamma_{\{1;2\}}^{U} = h_U \{-\mu^*; A_U\}, \quad \Gamma_{\{1;2\}}^{D} = h_D \{A_D; -\mu^*\}, \]

In order to calculate, e.g., the one-loop threshold corrections (23)-(29) first we extract from the

potential (65) the triple and quartic interactions presented in Fig.13 and Fig.14. The triangle and box diagrams which contribute, for example, to the threshold corrections included in \( \lambda_1 \)

are shown in Fig.15 together with symbolic expressions for the temperature one-loop integrals. Their sum multiplied by the color factor 3 gives \( \lambda_1^{\text{thr}} \), see (23).
Figure 13: The triple interactions extracted from the squark-Higgs sector, see (65).
Figure 14: The quartic interactions extracted from the squark-Higgs sector, see (65).
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Figure 16: Histograms for temperature-dependent $\lambda_i$ ($i=1,...,7$) with various quantum corrections in the framework of the CPX-like scenario [25], $A_t = A_b = 1000 \text{ GeV}, \mu = 2000 \text{ GeV}$, in the cases of (a) degenerate squark masses $m_Q = m_t = m_b = M_{SUSY} = 500 \text{ GeV}$, zero temperature, (b) degenerate squark masses $m_Q = m_t = m_b = M_{SUSY} = 500 \text{ GeV}$, $T = 200 \text{ GeV}$, (c) different squark masses $m_Q = 500 \text{ GeV}, m_t = 800 \text{ GeV}, m_b = 200 \text{ GeV}$ at zero temperature, and (d) different squark masses $m_Q = 500 \text{ GeV}, m_t = 800 \text{ GeV}, m_b = 200 \text{ GeV}$ at $T = 200 \text{ GeV}$. 
Figure 17: Effective temperature-dependent parameters $\lambda_i$ ($i=1,...,7$) with the one-loop threshold and logarithmic corrections at $m_Z = 91.19$ GeV, $m_b = 3$ GeV, $m_t = 175$ GeV, $m_W = 79.96$ GeV, $g_2 = 0.6517$, $g_1 = 0.3573$, $G_F = 1.174 \cdot 10^{-5}$ GeV$^{-2}$, $M_{SUSY} = 500$ GeV, $m_Q = 500$ GeV, $m_t = 800$ GeV, $m_b = 200$ GeV, $\mu = 2000$ GeV, $A = X_t + \mu/\tan \beta$, $X_t = 700$ GeV, $\tan \beta = 5$, $h_t = 1$, $h_b = 0.1$. 