Zero-mode contribution to the light-front Hamiltonian of Yukawa type models

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Abstract

Light-front Hamiltonian for Yukawa type models is determined without the framework of the canonical light-front formalism. Special attention is given to the contribution of zero modes.

1 Introduction

During the last years, quantization on the light front has been one of the intensively developing topics in field theory (see review [1] and the references therein). The most attractive feature of the light front formalism is the simplicity of definition of physical vacuum state. It is this feature which instils hope that light-front Hamiltonian approach might give nonperturbative solution of relativistic bound state problems in strong interaction. However for a complete attack on QCD to be feasible many technical obstacles remain still to be overcome. One of such obstacles is known as zero-mode problem [1, 2, 3] and will be the main topic of the paper.

In the canonical light-front formalism zero modes, i.e. fields that are constant along $x^- = \frac{1}{\sqrt{2}}(x^0 - x^3)$ ($x^+ = \frac{1}{\sqrt{2}}(x^0 + x^3)$ – plays the role of time), are not, generally speaking, dynamical variables. They have to be determined from the solution of nonlinear operator constraint equations. Only an approximate solution of such equations is possible and even in the case of two dimensional scalar field theories it causes considerable difficulties [4]. On the other hand zero modes are needed for computation of the Poincare generators. As was noticed in [5] the neglecting of zero modes contribution leads, for example, to breakdown of rotational invariance in the theory with fermions. As the result the operator constraint equations in canonical light-front formalism must be solved before the correct light-front Hamiltonian $P_+ = \frac{1}{\sqrt{2}}(P_0 + P_3)$ can even be written down. For gauge theories the zero-mode problem becomes much more complicated.

Various attempts were undertaken to avoid the solving of constraint equations and to obtain an effective light-front theory [6]. Nevertheless, many aspects of the zero-mode problem are not yet completely under control and there is still some necessity in developing other ways to construct the light-front Hamiltonians.

It is our intention to discuss here a method of construction of light-front Hamiltonians without the framework of canonical light-front formalism. The method was proposed in [7] and applied there to the scalar field theories. We investigate here the case of theories with fermions. We determine the matrix elements of the Hamiltonian $P_+$ in the same Fock space that is used in the canonical light-front formalism but via Green function equations and special analysis (without exact calculation) of Feynman diagrams for Green functions to all orders in perturbation theory. Such
analysis allows to pick out the contribution of zero modes to the light-front Hamiltonian without direct solving the constraint equations. We find that zero modes lead to additional contributions in comparison with naive canonical light-front Hamiltonian. The structure of such terms is discussed in detail.

In section 2 we put forward the method to construct the light-front Hamiltonian. We apply it to the case of Yukawa model and discuss contribution of zero modes to the Hamiltonian. Section 3 contains general consideration of Feynman diagrams. In this respect, a convenient technique is proposed to select role of zero modes. We conclude in section 4 with a brief summary of the obtained results.

2 Hamiltonian

We consider a system characterized by the Lagrangian

\[
L = \bar{\psi}(\gamma^\mu i\partial_\mu - M)\psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 + g \bar{\psi}\psi \phi \tag{2.1}
\]

The theory (2.1) admits canonical quantization on the \(x^0 = 0\) surface which we will assume to be fulfilled, that is

\[
[\phi(y), \partial_0 \phi(x)] \delta(y^0 - x^0) = \delta^{(4)}(y - x),
\]

\[
\{\psi_\alpha(y), \bar{\psi}_\beta(x)\} \delta(y^0 - x^0) = \delta_{\alpha\beta} \delta^{(4)}(y - x) \tag{2.2}
\]

and all the other (anti-) commutators are zero.

The fields \(\phi(x)\) and \(\psi(x)\) satisfy the equations

\[
(i\gamma^\mu \partial_\mu - M + g\phi)\psi = 0, \\
(\Box + m^2)\phi - g\bar{\psi}\psi = 0. \tag{2.3}
\]

These equations of motion (2.3) and the equal-time commutators then yield usual equations for the quantities \(T(\psi_{\alpha_1}(x_1) \ldots \bar{\psi}_{\beta_1}(y_1) \ldots \phi(z_1) \ldots)\), where \(T\) stands for the \(x^0\) – chronological ordering operation. We will use them below.

Define the operators \(a(\vec{k}), b(\vec{k}\lambda)d(\vec{k}\lambda)\) via fields on the light front

\[
a(\vec{k}) = \int d^3\vec{x}2 k_- e^{ikx} \phi(x) \mid_{x^0=0}, \quad k_- > 0,
\]

\[
b(\vec{k}\lambda) = \int d^3\vec{x}e^{ikx} \bar{u}(k\lambda) \gamma^+ \psi(x) \mid_{x^0=0}, \quad k_- > 0,
\]

\[
d(\vec{k}\lambda) = \int d^3\vec{x}\bar{\psi}(x)\gamma^+ v(k\lambda) e^{ikx} \mid_{x^0=0}, \quad k_- > 0. \tag{2.4}
\]

Here and throughout the paper the following notations are accepted: \(\vec{x} \equiv (x^- , x^+), \vec{k} \equiv (k_-, k_\perp), x^\pm \equiv (x^1, x^2), k_\perp \equiv (k_1, k_2), k_\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3), kx = k_+ x^+ + k_- x^- + k_1 x^1 + k_2 x^2\), \(u(k\lambda)\) and \(v(k\lambda)\) are free Dirac spinors with usual normalization conditions: \(\bar{u}(k\lambda)u(k\lambda) = 2M, \quad \bar{v}(k\lambda)v(k\lambda) = -2M\).
In the canonical light-front formalism the operators $a(\vec{k}), b(\vec{k}\lambda), d(\vec{k}\lambda)$ and their conjugate play the role of annihilation and creation operators and satisfy the following commutation relations

$$[a(\vec{k}), a^\dagger(\vec{p})] = (2\pi)^3 2\delta^{(3)}(\vec{k} - \vec{p}),$$
$$\{b(\vec{k}\lambda), b^\dagger(\vec{p}\mu)\} = (2\pi)^3 2\delta^{(3)}(\vec{k} - \vec{p})\delta_{\lambda\mu},$$
$$\{d(\vec{k}\lambda), d^\dagger(\vec{p}\mu)\} = (2\pi)^3 2\delta^{(3)}(\vec{k} - \vec{p})\delta_{\lambda\mu}. \tag{2.5}$$

the other (anti-)commutators are zero. In the canonical light-front formalism these commutation relations are postulated. In our case they should, in principle, be proved. It can be done at least in framework of perturbation theory but here we assume the relations (2.3) without any proof.

Simple kinematical arguments \[8, 7\] show that the operators (2.4) annihilate the physical vacuum $|0\rangle$. This feature and the commutation relations (2.3) permit us to construct a light-front Fock space above the physical vacuum from the basis vectors like

$$|k_1\lambda_1 \ldots q_1\mu_1 \ldots t_1 \ldots\rangle = b^\dagger(\vec{k}_1\lambda_1) \ldots d^\dagger(\vec{q}_1\mu_1) \ldots a^\dagger(\vec{t}_1) \ldots |0\rangle \tag{2.6}$$

Consider a set of wave functions $\langle k_1\lambda_1 \ldots q_1\mu_1 \ldots t_1 \ldots |P\rangle$, where $|P\rangle$ is any eigenstate of the operator $P_\mu$, i.e. $P_\mu|P\rangle = p_\mu|P\rangle$. Determination of matrix elements of $P_\mu$ in the basis (2.4) is equivalent to finding Schrödinger equation for the wave functions. To obtain these equations we rewrite the wave function in the form of integral of Bethe-Salpeter (BS) amplitude $\langle 0\rangle[T(\psi(x) \ldots \tilde{\psi}(y) \ldots \phi(z) \ldots)|P\rangle$

$$\langle k_1\lambda_1 \ldots q_1\mu_1 \ldots t_1 \ldots |P\rangle =$$
$$= \int \prod_i \left( d^4x_i e^{ik\cdot x_i} \delta(x_i^+) \bar{u}_{\alpha'/(k_i\lambda_i)}(k_i\lambda_i) \gamma^+_{\alpha'\alpha_i} \right) \prod_j \left( d^4y_j e^{iq\cdot y_j} \delta(y_j^+) \gamma^+_{\beta_j\beta'_j} y_{\beta_j}(q_j\mu_j) \right) \times$$
$$\times \prod_i \left( d^4z_i 2t_i - e^{it_i z_i} \delta(z_i^+) \right) \langle 0\rangle[T(\psi_{\alpha_1}(x_1) \ldots \tilde{\psi}_{\beta_1}(y_1) \ldots \phi(z_1) \ldots)|P\rangle \tag{2.7}$$

Here the symbol $T$ means the chronological ordering operation along $x^0$. Without $T$-ordering the right-hand side of relation (2.7) is just substitution of definitions (2.4). The representation (2.7) is possible due to the fact that difference between product of fields and $T$-product of fields is translationally invariant quantity but the Fourier transform of such quantity is proportional to $\delta(\sum k_i - \sum q_j - \sum t_i) = 0$ as longitudinal momenta $k_{i-}, q_{j-}, t_{i-}$ are positive.

Let $\langle 0\rangle[T(\psi_{\alpha_1}(k_1) \ldots \tilde{\psi}_{\beta_1}(q_1) \ldots \phi(t_1) \ldots)|P\rangle$ denote the Fourier transform of the BS amplitude $\langle 0\rangle[T(\psi_{\alpha_1}(x_1) \ldots \tilde{\psi}_{\beta_1}(y_1) \ldots \phi(z_1) \ldots)|P\rangle$. Then one has

$$\langle k_1\lambda_1 \ldots q_1\mu_1 \ldots t_1 \ldots |P\rangle =$$
$$= \int \prod_i \left( \frac{dk_i}{2\pi} \bar{u}_{\alpha'/(k_i\lambda_i)}(k_i\lambda_i) \gamma^+_{\alpha'\alpha_i} \right) \prod_j \left( \frac{dq_j}{2\pi} \gamma^+_{\beta_j\beta'_j} y_{\beta_j}(q_j\mu_j) \right) \prod_l \left( \frac{dt_l}{2\pi} 2t_l \right) \times$$
$$\times \langle 0\rangle[T(\psi_{\alpha_1}(k_1) \ldots \tilde{\psi}_{\beta_1}(q_1) \ldots \phi(t_1) \ldots)|P\rangle \equiv$$
$$\equiv \prod_l 2t_l \prod_i \bar{u}_{\alpha'/i}(k_i\lambda_i) \gamma^+_{\alpha'\alpha_i} \prod_j \gamma^+_{\beta_j\beta'_j} y_{\beta_j}(q_j\beta'_j) \times$$
$$\times \langle 0\rangle[T(\psi_{\alpha_1}(k_1) \ldots \tilde{\psi}_{\beta_1}(q_1) \ldots \phi(t_1) \ldots)|P\rangle, \tag{2.8}$$
where we have introduced short notation for the integral over plus-component of momenta (the overline).

The equations to be found will be obtained from the relations

\[
\langle k_1 \lambda_1 \ldots q_1 \mu_1 \ldots t_1 \ldots | P_+ | P \rangle = p_+ \langle k_1 \lambda_1 \ldots q_1 \mu_1 \ldots t_1 \ldots | P \rangle = \prod_i 2t_i \prod_i \bar{u}_{i\alpha}(k_i \lambda_i) \gamma^+_{\alpha\alpha_i} \prod_j \gamma^+_{\beta_j \beta_j'} v_{\beta_j \beta_j'}(q_j \beta_j') \times \\
(\sum k_i - \sum q_j + \sum t_l) \langle 0 | T(\psi_\lambda(k_1) \ldots \tilde{\psi}_\beta(q_1) \ldots \phi(t_1) \ldots | P \rangle 
\]

(2.9)

Let us consider only one of the items of the sum in the right hand side of equation (2.9). The other items are considered analogically. From the equations for \(T\)-products of fields we get

\[
\gamma^+ k_{1+} \langle 0 | T(\psi(k_1) \ldots | P \rangle \frac{M^2 + k_{1+}^2}{2k_{1-}} \gamma^+ \langle 0 | T(\psi(k_1) \ldots | P \rangle - \\
g \left( \frac{M - \gamma^+ k_{1+}}{2k_{1-}} \gamma^+ + \frac{1}{\sqrt{2}} \gamma \gamma^- \right) \int \frac{d^3 \bar{l}_1}{(2\pi)^3} \frac{d^3 \bar{l}_2}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\bar{l}_1 - \bar{l}_1 - \bar{l}_2) \times \\
\langle 0 | T(\psi(l_1) \phi(l_2) \ldots | P \rangle, 
\]

(2.10)

where dots denote all the other fields that enter in BS amplitude and are the same in the left- and right-hand sides of equation (2.10). A term with \(\delta\)-function which usually has place in equations for Green functions disappear due to positivity of \(k_{1-}, q_{j-}\).

The first term in equation (2.10) having substituted in (2.9) gives \((M^2 + k_{1+}^2)/(2k_{1-})\) \(\langle k_1 \lambda_1 \ldots | P \rangle \) but the second term in equation (2.10) must be transformed further to be presented in the form of linear combination of wave functions. Indeed, it includes \(\gamma^- \langle 0 | T(\psi \ldots | P \rangle \) instead of \(\gamma^+ \langle 0 | T(\psi \ldots | P \rangle \) that enters in the wave function representation (2.8). Besides the region of integration in (2.10) over \(l_{1-}, l_{2-}\) is spread from \(-\infty\) to \(+\infty\) and includes the points \(l_{1-} = 0\). If the function \(\langle 0 | T(\psi(l_1) \phi(l_2) \ldots | P \rangle \) has a behavior like \(\delta(l_{1-})\) it gives a singular contribution of zero modes. Revealing such contribution is the main aim of our consideration.

If \(l_{1-} \neq 0\) one rewrites term with \(\gamma^-\) in the form of terms with \(\gamma^+\) using again the equations for \(T\)-products

\[
\frac{1}{\sqrt{2}} \gamma^0 \gamma^- \langle 0 | T(\psi(l_1) \ldots | P \rangle = \frac{M + \gamma^+ l_{1+}}{2l_{1-}} \gamma^+ \langle 0 | T(\psi(l_1) \ldots | P \rangle - \\
g \left( \frac{M - \gamma^+ l_{1+}}{2l_{1-}} \gamma^+ + \frac{1}{\sqrt{2}} \gamma \gamma^- \right) \int \frac{d^3 \bar{l}_1}{(2\pi)^3} \frac{d^3 \bar{l}_2}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\bar{l}_1 - \bar{l}_1 - \bar{l}_2) \gamma^+ \langle 0 | T(\psi(l_1) \phi(l_2) \ldots | P \rangle 
\]

(2.11)

We have not written the terms with \(\delta\)-functions because fermion fields marked by dots enter in equation (2.9) in the form \(\psi \gamma^+\) but \(\{\gamma^0 \gamma^- \psi(x), \psi(y) \gamma^+\} \delta(x^0 - y^0) = 0\). As a result we obtain

\[
\gamma^+ k_{1+} \langle 0 | T(\psi(k_1) \ldots | P \rangle = \frac{M^2 + k_{1+}^2}{2k_{1-}} \gamma^+ \langle 0 | T(\psi(k_1) \ldots | P \rangle - 
\]
second term of equation (2.12) functions. For example, we can symbolically write for the BS amplitude in the expanded into a sum of connected BS amplitudes multiplied by connected Green function (2.9) excluded). Let us discuss the contribution to the Schrödinger equation from other terms of equation (2.12). It proves convenient to define at this stage the connected Bethe-Salpeter amplitude the region of integration over longitudinal momenta in equation (2.12) is, it has no singular contribution of zero modes. Therefore, for connected BS amplitude due to positivity of corresponding \( k \). In respect to \( l_1 \) the integrals in equation (2.12) are taken in the sense of principal value that is marked by \( \text{PV} \) (i.e. \( l_1 = 0 \) is excluded).

The first term in (2.12) gives the following contribution to the Schrödinger equation (2.13).

\[
- g \text{PV} \int \frac{d^3 l_1}{(2\pi)^3} \frac{d^3 l_2}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\vec{k}_1 - \vec{l}_1 - \vec{l}_2) (M - \gamma^+ l_{1\perp} + M + \gamma^+ l_{1\perp}) \times \gamma^+ (0|T(\psi(l_1)\phi(l_2)\ldots)|P) + \\
+ g^2 \text{PV} \int \frac{d^3 l_{11}}{(2\pi)^3} \frac{d^3 l_{12}}{(2\pi)^3} \frac{d^3 l_2}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\vec{k}_1 - \vec{l}_1 - \vec{l}_2) \gamma^+ \times \\
\times (0|T(\psi(l_{11})\phi(l_{12})\phi(l_2)\ldots)|P) + \ldots \\
(2.12)
\]

where dots mean possible singular contribution of the mode \( l_{1\perp} = 0 \) that comes from the \( (0|T(\psi(l_1)\phi(l_2)\ldots)|P) \). In respect to \( l_{1\perp} \) the integrals in equation (2.12) are taken in the sense of principal value that is marked by \( \text{PV} \) (i.e. \( l_{1\perp} = 0 \) is excluded).

The first term in (2.12) gives the following contribution to the Schrödinger equation (2.13).
\[
p_+(k_1 \lambda_1 \ldots \ldots |P) = \frac{M^2 + k_{1\perp}^2}{2k_{1\perp}} (k_1 \lambda_1 \ldots \ldots |P) + \ldots \\
(2.13)
\]

Let us discuss the contribution to the Schrödinger equation from other terms of equation (2.12). It proves convenient to define at this stage the connected Bethe-Salpeter amplitude \( (0|T(\psi \ldots \phi \ldots)|P) \) as an amplitude which cannot be presented in the form like \( (0|T(\ldots)|0)(0|T(\ldots)|P) \), where dots denote fields belonging to subsets into which the set \( \psi, \ldots, \bar{\psi}, \ldots, \phi, \ldots \) is divided. Arbitrary BS amplitude can be expanded into a sum of connected BS amplitudes multiplied by connected Green functions. For example, we can symbolically write for the BS amplitude in the second term of equation (2.12).

\[
\langle 0|T(\psi(l_1)\phi(l_2)\ldots)|P\rangle = \langle 0|T(\psi(l_1)\phi(l_2)\ldots)|P\rangle_c + \\
+ \sum \langle 0|T(\psi(l_1)\ldots)|0\rangle_c \langle 0|T(\phi(l_2)\ldots)|P\rangle_c + \\
+ \sum \langle 0|T(\phi(l_2)\ldots)|0\rangle_c \langle 0|T(\psi(l_1)\ldots)|P\rangle_c \\
(2.14)
\]

In this decomposition we take into account that \( l_{1\perp} + l_{2\perp} = k_{1\perp} > 0 \) and all the other longitudinal (minus-component) momenta are positive. The sum in (2.14) is spread over all possible unordered partitions of the set of fields \( \psi(k_2), \ldots, \bar{\psi}(q_1), \ldots, \phi(t_1), \ldots \) into subsets and the dots denote fields belonging to these subsets.

Introduction of connected BS amplitude is justified by the fact that the wavefunctions \( \langle k_1 \lambda_1 \ldots q_1 \mu_1 \ldots t_1 \ldots |P\rangle \) have as the integrand in equation (2.8) only connected BS amplitude due to positivity of corresponding \( k_-, q_-, t_- \). Having substituted the decompositions like (2.14) into equation (2.12) and then into equation (2.8) one transforms the right-hand side of (2.8) to desirable form except for the region of values for variables \( l_{1\perp} \). In wave functions all of the longitudinal momenta must be positive.

As we will see in next section the function \( \langle 0|T((\gamma^+\psi)\ldots(\psi\gamma^+)\ldots\phi\ldots)|P\rangle_c \) is zero if at least one of the longitudinal momenta of the fields is negative and it has no singular contribution of zero modes. Therefore, for connected BS amplitude the region of integration over longitudinal momenta in equation (2.12) is, in fact, restricted to \( l_{1\perp} > 0 \). That also means that all zero mode contributions to Schrödinger equation and Hamiltonian can come only from the factors
In the decomposition into connected components and from the function \( \langle 0|T((\gamma^-\psi(l_1))(\gamma^+\psi(k_2)) \ldots (\psi^+ \ldots \phi \ldots)|P\rangle_c \) (see equation (2.10) and below)

In next section we will analyse these questions via consideration of Feynman diagrams for connected Green functions. Let us briefly summarize here some results of this consideration. First of all, the function \( \langle 0|T(\ldots)|0\rangle_c \), where dots mean fields \( \gamma^+ \psi, \bar{\psi}, \gamma^+ \phi \), with the number of fields more than two is proportional to \( \prod_i \delta(k_{i-}) \), where \( k_{i-} \) are longitudinal momenta of the fields. Secondly, two-point functions have a term proportional to the \( \prod_i \delta(k_{i-}) \) and an additional term which is

\[
\gamma^+ \langle 0|T(\psi(l)\psi(k))|0\rangle_c \gamma^+ = (2\pi)^3 \delta^{(3)}(\vec{l} + \vec{k}) \theta(0) \gamma^+, \quad k_- > 0
\] (2.15)

for fermion fields and

\[
\langle 0|T(\phi(l)\phi(k))|0\rangle_c = (2\pi)^3 \delta^{(3)}(\vec{l} + \vec{k}) \frac{\theta(0)}{2k_-}, \quad k_- > 0
\] (2.16)

for boson fields. \( \theta(x) \) is the step function.

These additional terms reflect the fact that nonzero modes of the fields on the light front satisfy commutation relations (2.25).

Thirdly, \( \langle 0|T((\gamma^-\psi(l_1))(\gamma^+\psi(k_2)) \ldots (\psi^+ \ldots \phi \ldots)|P\rangle_c \) has a singular part (due to zero mode \( l_{1-} = 0 \)) which is

\[
\langle 0|T(\gamma^-\psi(l_1)\gamma^+\psi(k_2) \ldots \psi(q)\gamma^+ \ldots \phi(t) \ldots)|P\rangle_c \mid \text{zero mode } l_{1-} =~

\begin{align*}
&= g A(l_1) \left\{ \sum_{i=1}^{n_f} \frac{1}{2t_{i-}} \langle 0|T(\gamma^-\psi(l_1) \psi(l_1 + t_i)\gamma^+\psi(k_2) \ldots \psi(q)\gamma^+ \ldots \phi(t) \ldots)|P\rangle_c + \\
&+ \sum_{i=1}^{n_f} (-1)^{n_f+i} \langle 0|T(\gamma^-\psi(l_1) \phi(l_1 + q_i)\gamma^+\psi(k_2) \ldots \psi(q_i)\gamma^+ \ldots \phi(t) \ldots)|P\rangle_c \right\}
\end{align*}

(2.17)

where \( A(l) \sim \delta(l_-) \) and \( A(l) = \frac{1}{2} \gamma^- \gamma^+ \int \frac{dl}{2\pi} D(l) S^{-1}(l) G(l) \gamma^- \gamma^+ \) and \( D(l) = i/(l^2 - M^2 - i\epsilon) \), \( S(l) = i/(l - M + i\epsilon) \), \( G(l) \) is the Fourier transform of \( \langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle \) (\( G(l) = \int d^4x \exp(il(x-y)) \langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle \)). Underbrace under some term means that this term is dropped.

Taking into account enumerated results the decomposition (2.14), for example, becomes

\[
\langle 0|T(\gamma^+\psi(l_1)\phi(l_2) \ldots)|P\rangle = \langle 0|T(\ldots)|P\rangle_c + \\
+ \sum_{i=1}^{n_f} (-1)^{n_f+i} \langle 0|T(\gamma^+\psi(l_1) \psi(q_i)\gamma^+)|0\rangle_c \langle 0|T(\gamma^+\psi(l_1) \phi(l_2) \ldots (\psi(q_i)\gamma^+ \ldots)|P\rangle_c + \\
+ \sum_{i=1}^{n_f} \langle 0|T(\phi(l_2)\phi(t_i))|0\rangle_c \langle 0|T(\gamma^+\psi(l_1) \phi(l_2) \ldots \phi(t_i) \ldots)|P\rangle_c + \\
+ \langle 0|\phi(l_2)|0\rangle \langle 0|T(\gamma^+\psi(l_1) \phi(l_2) \ldots)|P\rangle_c
\] (2.18)
We took into account that fermion fields enter in (2.13) in one of forms \(\gamma^+\psi, \bar{\psi}\gamma^+\).

Having substituted (2.18) into (2.12) and then into (2.9) we obtain the contribution of the second term from (2.12) to Schrödinger equation

\[
p_+\langle k_1\lambda_1 \ldots |P \rangle = \ldots - g\sum_{l_1} \int [dl_1][dl_2](2\pi)^3\delta^{(3)}(\vec{k}_1 - \vec{l}_1 - \vec{l}_2) \times
\]

\[
\times \bar{u}(k_1\lambda_1)\bigg(\frac{M - \gamma^+ k_{1\perp}}{2k_{1\perp}}\gamma^+ + \gamma^+ \frac{M - \gamma^+ l_{1\perp}}{2l_{1\perp}}\bigg)u(l_1\sigma_1)\langle k_1\lambda_1 l_1\sigma_1 l_2 \ldots |P \rangle -
\]

\[
- g\sum_{i=1}^{n_f}(-1)^{n_f+i} \bar{u}(k_1\lambda_1)\bigg(\frac{M - \gamma^+ k_{1\perp}}{2k_{1\perp}}\gamma^+ - \gamma^+ \frac{M + \gamma^+ q_{1\perp}}{2q_{1\perp}}\bigg)v(q_i\mu_i) \times
\]

\[
\times \langle k_1\lambda_1(k_1 + q_i) \ldots q_{i\mu_i} \ldots |P \rangle -
\]

\[
- \frac{g}{2k_{1\perp}}\sum_{\lambda} \bar{u}(k_1\lambda_1) \bigg(\frac{M - \gamma^+ (k_{1\perp} + t_i\perp)}{2(k_{1\perp} + t_i\perp)}\bigg)u((k_1 + t_i)\sigma) \times
\]

\[
\times \langle k_1\lambda_1(k_1 + t_i)\sigma \ldots t_i \ldots |P \rangle -
\]

\[
- \frac{2g\langle \phi \rangle M}{2k_{1\perp}}\langle k_1\lambda_1 \ldots |P \rangle
\]

To obtain (2.19) we used the relation

\[
\gamma^+ = \frac{1}{2l_-}\gamma^+ \sum_{\sigma} u(l\sigma)\bar{u}(l\sigma)\gamma^+
\]

Notation \([dl] \equiv \frac{dl_1 dl_2}{(2\pi)^2 l_-}, \ l_+ > 0\) was also introduced.

With the aim of comparison with canonical light-front Hamiltonian it is convenient to extract from (2.19) the expression for the Hamiltonian \(P_+\) in the operator form. For example, the first term of (2.19) gives the following operator expression

\[
- g\sum_{\lambda\sigma} \int [dp][dq][dk](2\pi)^3\delta^{(3)}(p - k - q) \times
\]

\[
\times \bar{u}(p\lambda)\bigg(\frac{M - \gamma^+ p_{\perp}}{2p_{\perp}}\gamma^+ + \gamma^+ \frac{M - \gamma^+ q_{\perp}}{2q_{\perp}}\bigg)u(q\sigma) b^\dagger(\bar{p}\lambda)b(\bar{q}\sigma)a(\vec{k})
\]

(2.21)

It is exactly the same term that appears in naive canonical light-front formalism without zero modes. Analogically, one can determine operator expressions for other terms of (2.19) and establish the correspondence with naive canonical expressions for all terms except the last one which represents effective contribution of zero modes and is equal to

\[
\sum_{\lambda} \int [dk](-1)^{2g\langle \phi \rangle M\frac{b^\dagger(\vec{k}\lambda)b(\vec{k}\lambda)}{2k_-}}
\]

(2.22)

Similar consideration of the third term in (2.12) leads to operator terms some of which coincide with canonical ones, and we do not discuss them. We concentrate only on the difference between our and naive canonical Hamiltonian. This difference is connected with zero modes. In the decomposition of the function
\[ \langle 0 | T(\psi(l_{11}) \phi(l_{12}) \phi(l_{2}) \ldots | P) \] into connected components we have the following singular zero mode terms

\[ \langle 0 | T(\phi(l_{12}) \phi(l_{2})) | 0 \rangle_c, \quad \langle 0 | T(\psi(l_{11}) \phi(l_{12}) \phi(l_{2}) \ldots | P) | 0 \rangle_c, \]

\[ \langle 0 | T(\psi(l_{11}) \phi(l_{12}) \phi(l_{2}) \ldots | P) | 0 \rangle_c, \]

\[ \langle 0 | \phi(l_{12}) | 0 \rangle \quad \langle 0 | \phi(l_{2}) | 0 \rangle \quad \langle 0 | T(\psi(l_{11}) \phi(l_{12}) \phi(l_{2}) \ldots | P) | 0 \rangle_c, \]

\[ \langle 0 | \phi(l_{2}) | 0 \rangle \quad \langle 0 | T(\psi(l_{11}) \phi(l_{12}) \phi(l_{2}) \ldots | P) | 0 \rangle_c \]

They generate the addition to the fermion mass term

\[ g^2 \sum_{\sigma} \int [dk] b^\dagger(\bar{k} \sigma)b(\bar{k} \sigma) \frac{1}{2k_-} \left( \langle \phi \rangle^2 + \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{2(k_- + l_-)} \langle 0 | T(\phi(l_1) \phi(l_2)) | 0 \rangle_c \right) \]

and the following interaction that must be added to (2.24)

\[ g^2 \langle \phi \rangle \sum_{\lambda \sigma} \int [dp][dq][dk](2\pi)^3 \delta^{(3)}(\bar{p} - \bar{q} - \bar{k}) b^\dagger(\bar{p} \lambda)b(\bar{q} \sigma) a(\bar{k}) u(p \lambda)(\frac{\gamma^+}{2p_-} + \frac{\gamma^+}{2q_-})u(q \sigma) \]

Note that the last term in (2.27) can be presented as a sum of two parts: first one is

\[ \int \frac{d^3 l}{(2\pi)^3} \frac{2k_-}{2l_-} \]

and it is called in the literature the self-induced inertia. In canonical light-front formalism it arises from normal ordering of the seagull terms. The second part actually presents zero mode contribution and is absent in naive canonical expression. It is the \( \langle 0 | (\phi - \langle \phi \rangle)^2 | 0 \rangle \) (subscript 0 indicates that we take into account only zero mode).

Now let us discuss zero mode contribution caused by (2.17). Substituting (2.17) into (2.1) and then into (2.9) we obtain the following contribution to Schrödinger equation

\[ p_+ \langle k_1 \lambda_1 \ldots | P \rangle = \ldots - \frac{g^2}{2k_1} \bar{u}(k_1 \lambda_1) \frac{1}{\sqrt{2}} \gamma^0 \int \frac{d^3 \bar{l}}{(2\pi)^3} A(\bar{l}) u(k_1 \lambda_1) \frac{1}{2k_1} \langle k_1 \lambda_1 \ldots | P \rangle - \]

\[ -g^2 \sum_{i=1}^{n_b} \int \frac{d^3 \bar{l}_i}{(2\pi)^3} \left( (l_1 + t_i) \sigma l_2 k_1 \lambda_1 k_2 \lambda_2 \ldots t_i \ldots | P \right) - \]

\[ - \sum_{i=1}^{n_f} (-1)^{n_f+i} \int \frac{d^3 \bar{l}_i}{(2\pi)^3} \frac{g^2}{2(l_1 + q_i^-)} \int [dl_2](2\pi)^3 \delta^{(3)}(\bar{k}_1 - \bar{l}_1 - \bar{l}_2) \bar{u}(k_1 \lambda_1) \frac{1}{\sqrt{2}} \gamma^0 A(\bar{l}_i) \sum_{\sigma} u(l_i + t_i \sigma) \times \]

\[ \times \frac{1}{2(t_i + l_-)} \langle l_i + t_i \sigma l_2 k_1 \lambda_1 k_2 \lambda_2 \ldots t_i \ldots | P \rangle - \]

\[ A(\bar{l}_i) v(q_i \mu_i) \langle l_i + q_i l_2 k_1 \lambda_1 k_2 \lambda_2 \ldots q_i \mu_i \ldots | P \rangle \] (2.29)
The first term in (2.29) gives an addition to the fermion mass term of the light-front Hamiltonian and agrees with results found by Burkardt [3, 8].

\[
\sum_{\lambda} \int [dk] b^\dagger(\hat{k}\lambda) b(\hat{k}\lambda) \frac{-g^2}{2k_+} \left( \bar{u}(k\lambda) \frac{1}{\sqrt{2}} \gamma^0 \int \frac{d^3 l}{(2\pi)^3} A(l) u(k\lambda) \right. \left. \frac{1}{2k_+} \right) \tag{2.30}
\]

The contribution of other terms of (2.29) to the Hamiltonian has the following operator form

\[
-g^2 \sum_{\lambda\sigma} \int [dk_1][dk_2][dk_3][dk_4](2\pi)^3\delta^{(3)}(k_1 + k_2 - k_3 - k_4) \times
\left( \bar{u}(k_1\lambda) \frac{1}{\sqrt{2}} \gamma^0 \right. \left. A(\bar{k}_3 - \bar{k}_2) u(k_3\sigma) b^\dagger(\bar{k}_3\lambda) a^\dagger(\bar{k}_4) b(\bar{k}_3\sigma) a(\bar{k}_4) + \left. \bar{u}(k_1\lambda) \frac{1}{\sqrt{2}} \gamma^0 A(\bar{k}_3 - \bar{k}_2) \nu(k_2\sigma) b^\dagger(\bar{k}_3\lambda) d^\dagger(\bar{k}_2\sigma) a(\bar{k}_4) a(\bar{k}_3) \right) \tag{2.31}
\]

However, since these operator terms contain two creation operators (fermion and boson or antifermion) and, as consequence, they act on states with at least two corresponding particles we, to obtain correct expression for the Hamiltonian, have to consider the other items in (2.9) which correspond to boson and antifermions. Analysis analogical to the above-mentioned leads for these cases to the expressions that together with (2.31) cancel each other. So the full contribution of such zero mode terms to the Hamiltonian is zero. For the same reason the zero mode contribution of fermion and antifermion fields to the boson mass term of the Hamiltonian is equal zero.

Thus the full contribution of zero modes to the light-front Hamiltonian of Yukawa model consists of additions to the fermion mass term (2.22), (2.27) and (2.30) and additional three-particle interaction (2.28) and analogical terms with antifermion operators and terms that are hermitean conjugate to them.

### 3 Feynman diagram analysis

To prove statements used in section 2 let us consider an arbitrary Feynman diagram that represents connected \((n+m)\)-point connected Green function \(W_{n|m}(k_1, \ldots, k_n|−p_1, \ldots, −p_m)\). Momenta corresponding to amputated lines are \(p_1, \ldots, p_m\), they are ingoing and \(p_{i_−} > 0\). Momenta \(k_1, \ldots, k_n\) are outgoing. There are no restriction on the sign of \(k_{i−}\). (For Fourier transform we use the same definition as in section 2)

The expression for the integrand of the diagram is

\[
I(W) = C(W) \prod_{i=1}^{L} P(k_i) \frac{i}{k_i^2 - m_i^2 + i\epsilon} \prod_{v=1}^{V} (2\pi)^4\delta^{(4)}(P_v - \sum_n \varepsilon_{vn} k_n) \tag{3.1}
\]

Here \(C(W)\) contains all factors belonging to the vertices. \(k_i\) are momenta of both external nonamputated lines \((l = 1, \ldots, n)\) and internal lines \((l = n + 1, \ldots, L)\), \(V\) – is number of vertices. We assume at the moment that \(V \geq 1\). Special case
with \( V = 0 \) corresponds to free two-point functions. Polinom \( P(k_l) \) is 1 for scalar lines, \( P(k_l) = k_l + m_l \) for fermion lines with one exception that we indicate below. \( P_v = \sum p_k \) is the sum of momenta \( p_k \) going into the vertex \( v \). \( \varepsilon_{vl} \) is the vertex-edge incidence matrix of the diagram \([10]\). Direction of fermion lines is chosen along the charge spreading. If the direction of external fermion line and corresponding momentum are opposite then for this line \( P(k_l) = - k_l + m_l \). The orientation of scalar internal lines is chosen arbitrary.

It proves convenient to use the following integral representation of free scalar propagator and \( \delta \)-function

\[
\frac{i}{k_l^2 - m_l^2 + i\epsilon} = \int d\alpha_l \theta(\alpha_l) e^{i\alpha_l (k_l^2 - m_l^2 + i\epsilon)}, \tag{3.2}
\]

\[
2\pi \delta(P_{v+} - \sum \varepsilon_{vl} k_{l+}) = \int dy^+ \theta(y^+) e^{-i(P_{v+} - \sum \varepsilon_{vl} k_{l+})y^+}, \tag{3.3}
\]

and to change \( P(k_l) \to P(i\frac{\partial}{\partial \xi_l})e^{-ik_l \xi_l} |_{\xi_l=0} \).

Integration of \( I(W) \) over \( k_{l+}, (l = 1, \ldots, L) \) then yields

\[
I = \int \prod_{l=1}^{L} \frac{dk_{l+}}{2\pi} I(W) = C(W) \prod_{v=1}^{V} (2\pi)^3 \delta(2) (P_{v+} - \sum l \varepsilon_{vl} k_{l+}) \times \]

\[
\times \int \prod_{v=1}^{V} \left( dy^+ \delta(P_{v-} - \sum \varepsilon_{vl} k_{l-})e^{-iy^+_v P_{v+}} \right) \int \prod_{l=1}^{L} \left( d\alpha_l \theta(\alpha_l) P(i\frac{\partial}{\partial \xi_l}) \times \right) \]

\[
\times \delta(2\alpha_l k_{l-} + \sum_v y^+_v \varepsilon_{vl} - \xi_l^+) e^{-i\alpha_l (k_{l+}^2 - m_l^2 - i\epsilon) - i\varepsilon_{vl} k_{l-}} \bigg|_{\xi_l=0} \tag{3.4}
\]

Note that we integrate not only over internal lines but also over external lines (see \([2.8]\)).

Consider at first conditions that are determined by \( \delta \)-functions

\[
\prod_{l=1}^{L} \delta(2\alpha_l k_{l-} + \sum_v y^+_v \varepsilon_{vl} - \xi_l^+) \prod_{v=1}^{V} \delta(P_{v-} - \sum \varepsilon_{vl} k_{l-}) \tag{3.5}
\]

Let us resolve them in respect to \( k_{l-} \) and \( y^+_v \). Introducing a matrix \( C_{vv'} = \sum_i \varepsilon_{vi} \frac{1}{2\alpha_i} \varepsilon_{vi} \)

we get

\[
k_{l-} = \sum_{vv'} \frac{\varepsilon_{vl}}{2\alpha_i} C_{vv'}^{-1}(P_{v'} - \sum \varepsilon_{v'l} \xi^+_l) + \xi^+_l \tag{3.6}
\]

\[
y^+_v = -C_{vv'}^{-1}(P_{v'} - \sum \varepsilon_{v'l} \xi^+_l) \tag{3.7}
\]

Let us imagine that all external nonamputated lines have common additional vertex ( \( (V + 1) \)-th vertex) and denote such new diagram as \( G' \). Then the following representation will take place \([11]\)

\[
\sum_{vv'} C_{vv'}^{-1} a_v b_{v'} = \frac{1}{D(\alpha)} \frac{T_2}{\prod_{l \in T_2} (2\alpha_l)} \left( \sum_{halftT_2} a_v \right) \left( \sum_{halftT_2} b_v \right) \tag{3.8}
\]
for vectors $\vec{a}$ and $\vec{b}$ having $\sum_{v=1}^{V+1} a_v = \sum_{v=1}^{V+1} b_v = 0$ and
\[
D(\alpha) = \sum_{T_1} (\prod_{l \notin T_1} 2\alpha_l)
\]

Here $T_1$ is a 1-tree of the graph $G'$, i.e. a connected subgraph containing all vertices of $G'$ and not having cycles. $T_2$ is a 2-tree of the graph $G'$, i.e. a subgraph of $G'$ containing all lines, not having cycles and consisting of exactly two connected components. Applying (3.3) to (3.6) we obtain
\[
D(\alpha) k_{l-} = \sum_{T_{1l}} (\prod_{j \notin T_{1l}} 2\alpha_j)( \sum_{\text{half}T_{1l}} P_{v-}) - \sum_{l' = 1}^{L} \frac{1}{D(\alpha)} \sum_{T_{1l'}} (\prod_{j \notin T_{1l'}} 2\alpha_j) \sum_{T_{2l'}} (\prod_{j \notin T_{2l'}} 2\alpha_j) \xi_{l'}^+(-1)^{a_{v'}}(\sum_{T_{1l'}} \sum_{j \notin T_{1l'}} 2\alpha_j)) =
\]

\[
= \prod_{l = 1}^{L} \delta(2\alpha_l k_{l-} + \sum_{v} y_{v}^+ \varepsilon_{v l} - \xi_{l}^+) \prod_{v = 1}^{V} \delta(P_{v-} - \sum_{l} \varepsilon_{v l} k_{l-}) =
\]

\[
= \prod_{l = 1}^{L} \delta(D(\alpha) k_{l-} - \sum_{T_{1l}} (\prod_{j \notin T_{1l}} 2\alpha_j)( \sum_{\text{half}T_{1l}} P_{v-}) + \sum_{l' = 1}^{L} \frac{1}{D(\alpha)} \sum_{T_{1l'}} (\prod_{j \notin T_{1l'}} 2\alpha_j) \sum_{T_{2l'}} (\prod_{j \notin T_{2l'}} 2\alpha_j) \xi_{l'}^+(-1)^{a_{v'}}(\sum_{T_{1l'}} \sum_{j \notin T_{1l'}} 2\alpha_j)) \times
\]

\[
= \prod_{v = 1}^{V} \delta(y_{v}^+ + \frac{1}{D(\alpha)} \sum_{T_{2l}} (\prod_{j \notin T_{2l}} 2\alpha_j)( \sum_{\text{half}T_{2l}} P_{v-}) - \frac{1}{D(\alpha)} \sum_{l' = 1}^{L} \frac{1}{D(\alpha)} \sum_{T_{1l'}} (\prod_{j \notin T_{1l'}} 2\alpha_j) \xi_{l'}^+(-1)^{a_{v'}}(\sum_{T_{1l'}} \sum_{j \notin T_{1l'}} 2\alpha_j))\varepsilon_{T_{2v l'}})
\]

(3.10)

It follows from (3.9) that for external lines $k_{l-}$ can be only nonnegative when $\alpha_i \geq 0$ and $\xi_i^+ = 0$; indeed, $P_{v-} (v = 1, \ldots, V)$ are positive and momenta of external lines are outgoing. As a consequence of (3.10) we obtain $I = 0$ if at least one of external momenta is negative.

We will have a singular contribution of zero mode, i.e. $\delta(k_{l-})$, if $\sum_{T_{1l}} (\prod_{j \notin T_{1l}} 2\alpha_j) (\sum_{\text{half}T_{1l}} P_{v-}) = 0$ exactly for all values of $\alpha$. It has place for diagrams without amputated lines (all $P_{v-} = 0$). As we saw in previous section such cases correspond to some vacuum expectation values in the light-front Hamiltonian. For theories with only scalar fields it exhausts all possible contributions of zero modes. For theories with fermion fields there is also another possibility. If some of $\alpha_i$, th are exactly zero, i.e. there are some $\delta(\alpha_i)$, then we also can get $\delta(k_{l-})$. It is simply to understand that fact from the following analogy with electrical circuits. If we identify $2\alpha_i$ as a resistance of the link $i$, $k_{i-}$ as a current in the link $i$ and $(-g_{v}^+)$ as a potential of the junction $v$ then the equations that are determined by $\delta$-functions (3.5) are nothing but Ohm’s law for the links of the circuit that is presented by the diagram $G'$. The
joint \((V + 1)\)-th vertex of external lines has zero potential by definition. Now if 
\(\alpha_i = 0\) then both ends of the link have the same potential and we can reduce this 
link to the one point without changing currents in the other links. If as a result of 
such reduction we obtain a diagram \(G''\) that consists of two parts connected with 
each other only through this point and if one part of two has no external amputated 
lines, i.e. no external current goes in it, then currents in all links of this part will be 
exactly zero. We will call such part of diagram \(G'\) generalized tadpole and denote 
it \(G'_i\). The reason of appearence \(\delta(\alpha)\) in (3.4) is the following. In expression (3.4) 
for fermion fields there is \(P(i\frac{\partial}{\partial \xi^+})\) which contains a term with \(\gamma^+ i\frac{\partial}{\partial \xi^+}\), and we have 

derivative of \(\delta\)-function \(i\frac{\partial}{\partial \xi^+}\delta(2\alpha_i k_{l-} + \sum_v y_v^+ \varepsilon_{vl} - \xi^+_l)\) that leads to \(\delta(\alpha)\). To see 
that let us transform (3.4) in a following way. We introduce 1 in (3.4) in the form 
\[1 = f(\prod_{v=1}^V d\lambda_v \delta(\lambda_v - \sum_l \varepsilon_{vl} \alpha_l))\]. Then we resolve 
\[\prod_{l=1}^L \delta(2\alpha_i k_{l-} + \sum_v y_v^+ \varepsilon_{vl} - \xi^+_l) \prod_{v=1}^V \delta(\lambda_v - \sum_l \varepsilon_{vl} \alpha_l)\]
in respect to \(\alpha_l\) and \(y_v^+\) (as we did above in respect to \(k_l\) and \(y_v^+\)) and get \(\alpha_l\) and \(y_v^+\) 
as functions of \(k_{l-}, \xi^+_l\) which have the same form as (3.6 – 3.9) with rechange \(\alpha_l \leftrightarrow k_{l-}, \lambda_v \leftrightarrow P_{v-}\). The result of differentiation over \(\xi^+_j\) can be rewritten in the form 
\[\left\{\prod_l \delta(2\alpha_l k_{l-} + \sum_v y_v^+ \varepsilon_{vl} - \xi^+_l) \prod_{v=1}^V \delta(P_{v-} - \sum_l \varepsilon_{vl} k_{l-})\right\} \times \]
\[\times \left[\sum_{l=1}^L \frac{\partial \alpha_l}{\partial \xi^+_j} i \frac{\partial}{\partial \alpha_l} + \sum_{v=1}^V \frac{\partial y_v^+}{\partial \xi^+_j} i \frac{\partial}{\partial y_v^+}\right] \left\{\prod_l (\theta(\alpha_l)e^{-i\alpha_l(k_{l-} + m^2 - i\varepsilon)} \prod_{v=1}^V e^{-iy_v^+ P_{v-}}\right\}\]
(3.11) 

where 
\[\frac{\partial \alpha_l}{\partial \xi^+_j} = (-1)^{s_{li}} \frac{1}{D(k)} \sum_{T_{li}} \left( \prod_{j \not\in T_{li}} 2k_{j-} \right),\]
(3.12) 
\[\frac{\partial y_v^+}{\partial \xi^+_j} = \frac{1}{D(k)} \sum_{T_{li}} \sum_{j \not\in T_{li}} \varepsilon(T_{li}) \left( \prod_{j \not\in T_{li}} 2k_{j-} \right)\]
(3.13) 

\(\varepsilon(T_{li}) = \pm 1\); it depends on whether the \(i\)-th line goes out or in the half of \(T_{li}\) which 
contains the vertex \(v\). The sum in (3.13) is over such \(T_{li}\) which have the vertexes \(v\) and \((V + 1)\) in different 
halves of \(T_{li}\). \(D(k) = \sum_{T_i} (\prod_{j \not\in T_i} 2k_{j-})\). 

As a result we obtain terms with 
\[\frac{\partial}{\partial \alpha_l} \theta(\alpha_l) = \delta(\alpha_l)\].

Note that we must here consider only derivatives \(\frac{\partial}{\partial \xi^+_j}\) for external lines because 
for internal lines we do, in fact, integration over internal longitudinal momenta.

Let \(\alpha_l = 0\) leads to generalized tadpole subgraph. Until \(\xi^+_l \neq 0\) longitudinal 
momenta in generalized tadpole subgraph are of the order \(\xi^+_l\) (see (3.9)) and, therefore, 
\[\sum_{T_i} (\prod_{j \not\in T_i} 2k_{j-}) = O((\xi^+_l)^{C-1}),\] where \(C\) is the maximal number of independent 
cycles made up of lines belonging to \(G'_i\) and the line \(l\). At the same time 
\[\sum_{T_{li}, j \not\in T_{li}} 2k_{j-} = \left\{\begin{array}{ll} O((\xi^+_l)^{C-1}) & \text{if } i \in G'_i \\ O((\xi^+_l)^C) & \text{if } i \not\in G'_i \end{array}\right.\]
So \( \frac{\partial \delta |_{\xi^+ = 0}}{\partial \xi^+_i} \) will be different from zero only if \( i \)-th line belongs to the subgraph \( G'_i \) and in this case for external line \( \frac{\partial \delta |_{\xi^+ = 0}}{\partial \xi^+_i} = (-1)^{\sigma_a} \frac{1}{2k_i} \) as it follows from (3.12).

The derivative \( \frac{\partial \delta}{\partial \xi^+_i} \) always appears in combination with \( \gamma^+ \). Therefore for functions \( \langle 0 | T(\gamma^- \psi(l_1) \gamma^+ \psi(k_2) \ldots \psi(q) \gamma^+ \ldots \phi(t) \ldots) | P \rangle_c \) derivative \( \gamma^+ \frac{\partial}{\partial \xi^+_i} \) can not appear. Thus for these functions there are not additional contributions of zero modes.

We want to find singular contribution of zero mode \( l_{1-} \) in \( \langle 0 | T(\gamma^- \psi(l_1) \gamma^+ \psi(k_2) \ldots \psi(q) \gamma^+ \ldots \phi(t) \ldots) | P \rangle_c \) which is contained in equation (2.10). As was showed above we must take into consideration only the derivative \( \gamma^+ \frac{\partial}{\partial \xi^+_i} \) and it must belong to the generalised tadpole subgraph. The tadpole fermion line containing \( l_{1-} \)-th line can lean on scalar or antifermion external lines for which the \( \delta(\alpha_i) \) appears. It is obviously that after taking off \( \delta(\alpha_i) \) in (3.4) the contribution of generalized tadpole subgraphs are factorized, and we can write

\[
\langle 0 | T(\gamma^- \psi(l_1) \gamma^+ \psi(k_2) \ldots \psi(q) \gamma^+ \ldots \phi(t) \ldots) | P \rangle_c \big|_{\text{zero mode } l_{1-}} = \\
= \sum_{i=1}^{n_f} \frac{g}{2t_{l_i}} A(\tilde{l}_i) \langle 0 | T(\gamma^- \psi(l_1) \psi(l_1 + t_i) \gamma^+ \psi(k_2) \ldots \psi(q) \gamma^+ \ldots \phi(t) \ldots) | P \rangle_c + \\
+ g \sum_{i=1}^{n_f} (-1)^{n_f+i} A(\tilde{l}_i) \langle 0 | T(\gamma^- \psi(l_1) \phi(l_1 + q_i) \gamma^+ \psi(k_2) \ldots \psi(q_i) \gamma^+ \ldots \phi(t) \ldots) | P \rangle_c
\]

(3.14)

where

\[
A(\tilde{l}) = \frac{1}{2} \gamma^- \gamma^+ \int \frac{4\pi}{2k_i} D(l) S^{-1}(l) G(l) \gamma^- \gamma^+
\]

and

\[
D(l) = i/(2 - M^2 + i\epsilon), \quad S(l) = i/(2 - M + i\epsilon), \quad G(l) = \text{the Fourier transform of } \langle 0 | T(\psi(x) \bar{\psi}(y)) | 0 \rangle \quad (G(l) = \int d^4 x \exp(i(x-y)l) \langle 0 | T(\psi(x) \bar{\psi}(y)) | 0 \rangle),
\]

As was shown above \( A(\tilde{l}) \sim \delta(l_{1-}) \).

4 Summary

We have proposed noncanonical approach to obtain Schrödinger equation and light-front Hamiltonian in the light-front Fock space. In this method we deal with BS amplitudes and light-front Hamiltonian is extracted from the equations for these amplitudes. To do this we have also carried out special analysis of Feynman diagrams. The advantage of the proposed method is that we obtain light-front Hamiltonian directly in normal form in respect to light-front annihilation and creation operators and quite simply get contribution of zero modes to this Hamiltonian. The terms caused by zero modes include as a factor initially unknown vacuum expectation values (VEV) such as \( \langle \phi \rangle \), \( \langle \phi^2 \rangle \) and some integrals of \( \langle 0 | T(\psi(x) \bar{\psi}(y)) | 0 \rangle \) and these factors are effective manifestation of zero modes. As discussed in [4, 8] if the eigenvalues and the eigenvectors of \( P_\mu \) are known then one can calculate these VEVs remaining in the light-front approach despite unknowledge of exact operator expressions for zero modes. Thus, solution of field theoretical models in the light-front Hamiltonian approach needs a self-consistent simultaneous determination both of the spectrum of \( P_\mu \) and these VEVs.
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