Projectively-Compact Spinor Vertices and Space-Time Spin-Locality in Higher-Spin Theory

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To the memory of Mikhail Soloviev

Abstract

The concepts of compact and projectively-compact spin-local spinor vertices are introduced. Vertices of this type are shown to be space-time spin-local, i.e., their restriction to any finite subset of fields is space-time local. The known spinor spin-local cubic vertices with the minimal number of space-time derivatives are verified to be projectively-compact. This has the important consequence that spinor spin-locality of the respective quartic vertices would imply their space-time spin-locality. More generally, it is argued that the proper class of solutions of the non-linear higher-spin equations that leads to the minimally non-local (presumably space-time spin-local) vertices is represented by the projectively-compact vertices. The related aspects of the higher-spin holographic correspondence are briefly discussed.
1 Introduction

Higher-spin (HS) gauge theory has dramatic history. It results from a simply looking idea of promoting HS gauge symmetries associated with free Fronsdal theory \[1\] to the non-linear level. In old days HS gauge theory was argued not to exist due to the so-called no-go statements forbidding HS symmetries \[2, 3, 4\] (see \[5\] for a review). Later on it was shown that the problem is avoided by going from the flat background to AdS \[6\].

Fundamental properties of HS gauge theories are that they are consistently formulated in AdS background \[6\], contain higher derivatives in interactions of degrees increasing with spins \[7, 8, 9\], and involve infinite towers of fields of unlimited spins \[8, 9\]. Altogether, these properties imply that HS gauge theory is not a usual local theory, exhibiting certain degree of non-locality. The level of non-locality of HS gauge theory is still debatable in the literature within various formalisms. In this paper we elaborate a tool reducing the analysis of spin-locality of the HS theory in space-time to the much simpler one in the auxiliary spinor space.

In eighties, the role of AdS background was not appreciated so that its relevance was considered just as a peculiarity of HS theory. However, after AdS/CFT came into the game \[10, 11, 12\] Klebanov and Polyakov proposed a remarkable conjecture that HS theory is holographically dual to a 3d vector boundary sigma model \[13\] see also \[14, 15, 16, 17, 18\] for related precursor work. Further generalizations were worked out to supersymmetric \[19, 20\] and Chern-Simons extensions \[21, 22\].

Though Klebanov-Polyakov conjecture obeyed all kinematic constraints of the linearized holography expressed by the Flato-Fronsdal theorem \[23\] establishing the relation between currents (tensor products) build from free 3d conformal fields and free massless fields in AdS\(_4\), attempts to reconstruct HS interactions in the bulk led the authors of \[24, 25\] (see also \[26\]) to the conclusion that HS gauge theory must be essentially non-local beyond the leading order. (See, however, \[27\]). This point is wide spread these days even though, as discussed below, it has no solid grounds whatsoever\(^1\).

Let us now summarise what is really known on the status of the HS gauge theory. There are two main ways for its study. The most popular and seemingly simpler one is to analyse the properties of HS gauge theory based on the holographic duality principle

\[\text{Holography} \quad KP \text{ conjecture} \quad \Rightarrow \quad \text{HS gauge theory}. \quad (1.1)\]

This was used for obtaining both positive results in \[28, 29\] and negative ones in \[24\].

Alternatively, HS gauge theory can be studied directly in the bulk. This is hard at higher orders because of the presence of infinite towers of dynamical fields and the absence of the low-energy expansion parameter, that demands all higher derivatives be accounted on the same footing in the AdS background. Apart from standard Noether procedure typically

\(^1\)A couple of years ago one very good physicist and friend of mine told me at some conference: “You know, someone whose name I even do not remember, told me that HS gauge theory has been shown to be very poorly defined as the interacting theory being too non-local”.

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efficient at the lower-orders \[8\] (for incomplete list of references see also \[30, 31, 32, 33, 34, 35, 36, 37, 38, 39\]) but not beyond, there are two main tools to simplify the problem.

One is the light-cone formalism of \[7\] further developed in the series of papers of A. Bengtsson (see \[40\] and references therein) and Metsaev \[41, 42, 43, 44, 45, 46\], who in particular was able to find some quartic HS vertices in Minkowski space \[41\] that correspond to the self-dual HS theory, and others (see e.g. \[47\]).

An efficient covariant approach was suggested in \[48\] where a system of equations was formulated allowing to reconstruct on-shell HS vertices order by order. This way, using shifted homotopy approach, in \[49, 50, 51, 52, 53, 54, 55, 56\] some higher-order vertices in HS theory were reconstructed that all have been shown to be spin-local which roughly speaking means their locality for any finite subset of fields in the system. (For more detail see \[54\] and below.) However, to compare conclusions of the bulk analysis of the equations of \[48\] with the holographic reconstruction conclusion of \[24\] one has to find full $\phi^4$ vertex with $\phi$ being a scalar field component contained in the zero-form sector of HS fields $C$ in HS theory. This problem has been only partially solved so far.

The scheme of \[48\] is complete in the sense that it represents any solution to the problem including all possible field redefinitions. This has both advantages and disadvantages. The disadvantage is that equations of \[48\] do not directly lead to a local or minimally non-local form of the equations. In other words, to proceed one has to work out appropriate additional conditions that single out a proper form of HS field equations. This is somewhat analogous to the Schroedinger equation in quantum mechanics: most of its solutions have no relation to physics. One has to, first, impose an additional condition that the wave-function must belong to $L^2$ and, second, to find appropriate solutions. Analogously, in the analysis of HS gauge theory based on the equations of \[48\] a list of conditions has been identified in \[51\] that reduce the degree of non-locality of the resulting vertices. As a result, it was shown that all lowest-order vertices are properly reproduced by the equations of \[48\]. These results were then extended \[54\] to a class of non-linear vertices of the types discussed in \[24\]. Though so far not all of the HS vertices to be compared with \[24\] have been obtained from \[48\] the already obtained results indicate that the analysis of \[48\] along the lines of \[54\] leads to the conclusions essentially different from those predicted in \[24\].

It should be noted that the results of this paper further restrict a class of solutions of the HS equations of \[48\] leading to space-time spin-local HS dynamics. We conjecture that the projectively-compact spin-local vertices identified below just form the appropriate class of solutions of the non-linear HS equations of \[48\] analogous to $L^2$ in QM.

It is important that HS holography is weak-weak \[13, 29\], i.e., it can be tested on the both sides in the weak coupling regime. In this situation disagreement of the bulk and boundary analysis implies that something goes wrong in the holographic correspondence. Since HS gauge theory is definitely one of the fundamental theories of nature, this has to be taken seriously because its analysis can affect the standard paradigm of the holographic correspondence in presence of HS gauge fields.

\[2\]Note that the results of \[54, 55, 56\] suggested that the shifted homotopy approach can be related to the star-product deformation as was explicitly demonstrated in \[55\].
In this paper we focus on the interplay between the notions of spin-locality in space-time and in the twistor-like spinor space. (Both are explained in the main text.) In practice, it is easier to analyse spinor spin-locality controlled by a number of theorems of \[51, 54\]. However, beyond the lowest order, spinor spin-locality does not necessarily imply spin-locality in space-time that makes it difficult to compare conclusions of the analysis of the HS theory in the spinor space against the space-time locality analysis. In this paper a simple sufficient criterion is presented guaranteeing that spinor spin-locality implies space-time spin-locality. It is also shown that this criterion is fulfilled by the local HS vertices obtained in \[49, 50\] hence implying that these vertices and their higher-order extensions are spin-local in the usual space-time sense as well.

Yet hypothetical spin-locality of the bulk HS theory suggests the need of a modification of the Klebanov-Polyakov conjecture. This can be achieved by replacing the boundary sigma-model respecting global HS symmetries by its gauged version of the sigma-model interacting with the conformal HS gauge theory at the 3d boundary. This idea was put forward in \[60\] being motivated by the direct holographic correspondence resulting from the so-called unfolded formulation originally introduced in \[61\]. The advantage from the HS theory perspective was the manifest control of HS gauge symmetries on the both sides of the holographic correspondence. We believe that this key feature of the HS gauge theory has to be respected by any scheme.

The proposal of \[60\] is non-standard in several respects. As usual, this type of holographic correspondence relates AdS bulk symmetry with the boundary conformal symmetry. However, the boundary conformal theory is not a standard CFT since conformal gravity and its HS extensions possess no gauge invariant stress tensor. As a result correlators in this theory should not necessarily respect conformal bootstrap which assumption was instrumental for the arguments of \[24\]. The prise is however that the boundary theory is much more involved than usual CFT. (3d theories of this class were considered in \[62, 63\].) Another distinction was that the proposal of \[60\] seemingly leaves unclear how to identify the sources for conformal fields once the HS gauge fields are engaged to be dynamical (rather than sources as in the conventional approach). This point will be briefly discussed in Section 6.

The rest of the paper is organized as follows. In Section 2 we discuss the peculiarities of the notion of locality in the models with infinite towers of fields like in HS theory. The notion of space-time spin-locality is introduced and the role of compact spin-local field redefinitions is emphasized. In Section 3 the unfolded formulation of free massless fields in AdS\(_4\) is recalled. The notion of spinor spin-locality is introduced in Section 4 while the class of projectively-compact spin-local vertices is introduced in Section 5 where it is shown that such vertices imply space-time spin-locality. The related aspects of the HS holographic correspondence are briefly discussed in Section 6. Conclusions are summarized in Section 7.

2 Locality, spin-locality and non-locality

Let us now discuss peculiarities of the notions of locality and non-locality in field theories like HS gauge theory. Recall that HS gauge theory contains interaction vertices with higher
derivatives both in Minkowski \([4, 5]\) and AdS \([3]\) backgrounds (for a broader class of HS models see also \([13]\)). The same time, HS gauge theories in \(d \geq 4\) were known to describe infinite towers of fields of different spins \([8]\) because HS symmetries \([9]\) are infinite-dimensional. Since the order of maximal derivatives \(\text{ord}(V)\) in a HS vertex \(V(s_1, s_2, s_3)\) for three fields with spins \(s_1, s_2, s_3\) increases with involved spins the theory contains an infinite number of derivatives once all spins are involved, thus being non-local in the standard sense. However, in such theories there are more options to be distinguished.

### 2.1 Interactions

Let some system describe fields \(\phi^A_s\) characterized by quantum numbers called spin \(s\) and some Lorentz indices \(A\) like tensor, spinor, etc. Consider field equations of the form

\[
E_{A_0, s_0}(\partial, \phi) = 0, \quad E_{A_0, s_0}(\partial, \phi) = \sum_{k=0, l=1}^{\infty} a^{n_1...n_k}_{A_0...A_l}(s_0, s_1, s_2, \ldots, s_l)\partial_{n_1} \ldots \partial_{n_k}\phi^{A_1}_{s_1} \ldots \phi^{A_l}_{s_l}.
\]

Here derivatives \(\partial_n := \frac{\partial}{\partial x^n}\) may hit any of the fields \(\phi^{A_k}_{s_k}\) with \(s_0\) being the spin of the field on which the linearized equation is imposed. Locality of the equations will be treated perturbatively, \(i.e.,\) independently at every order \(l\). In usual perturbatively local field theory the total number of derivatives is limited at any order \(l\) by some \(k_{\text{max}}(l)\):

\[
a^{n_1...n_k}_{A_0...A_l}(s_0, s_1, s_2, \ldots, s_l) = 0 \quad \text{at} \quad k > k_{\text{max}}(l).
\]

This condition can be relaxed to \(\text{space-time spin-locality condition}

\[
a^{n_1...n_k}_{A_0...A_l}(s_0, s_1, s_2, \ldots, s_l) = 0 \quad \text{at} \quad k > k_{\text{max}}(s_0, s_1, s_2, \ldots, s_l)
\]

with some \(k_{\text{max}}(s_0, s_1, s_2, \ldots, s_l)\) depending on the spins in the vertex. In the theories with the finite number of fields where \(s\) takes at most a finite number of values, the conditions (2.1) and (2.2) are equivalent. However in the HS-like models, where spin \(s\) can take an infinite number of values, the locality and spin-locality restrictions differ. Both types of theories have to be distinguished from the genuinely non-local ones in which there exists such a subset of spins \(s_0, s_1, s_2, \ldots, s_l\) that (2.2) is not true, \(i.e.,\) no finite \(k_{\text{max}}(s_0, s_1, s_2, \ldots, s_l)\) exists at all.

The relaxation of the class of local field theories with the finite number of fields to the spin-local class is the simplest appropriate for the models involving infinite towers of fields. However, it makes sense to further specify the concept of spin-local vertices as follows.

We call a spin-local vertex \(\text{compact}\) if \(a^{n_1...n_k}_{A_0...A_l}(s_0, s_1, s_2, \ldots, s_k+t_k, \ldots, s_l) = 0\) at \(t_k > t_k^0\) with some \(t_k^0\) for any \(0 \leq k \leq l\) and \(\text{non-compact}\) otherwise. (Note that here the compactness is in the space of spins, not in space-time.) In HS theory both types of vertices are present. For instance, the cubic HS vertices \(\omega * \omega\) constructed in \([3]\), that are built from the HS gauge connections \(\omega\), are spin-local-compact since they are non-zero iff spins \(s_0, s_1, s_2\) obey the triangle inequalities \(s_0 \leq s_1 + s_2\) etc. On the other hand, vertices associated with the conserved currents built from gauge invariant field strengths like those considered in \([7]\) are spin-local non-compact. Indeed, these include in particular vertices \(a^{n_1...n_k}(s_0, 0, 0)\) that describe conserved currents of any integer spin \(s_0\) built from two spin-zero fields.
2.2 Field redefinitions

A class of local field theories with finite sets of fields is invariant under perturbatively local field redefinitions

\[ \phi^B_{s_0} \to \phi^B_{s_0} + \delta \phi^B_{s_0}, \quad \delta \phi^B_{s_0} = \sum_{k=0, l=1}^{\infty} b^{Bn_1 \ldots n_k}_{A_1 \ldots A_l}(s_0, s_1, \ldots, s_l) \partial_{n_1} \ldots \partial_{n_k} \phi^{A_1}_{s_1} \ldots \phi^{A_l}_{s_l} \]  

(2.3)

with at most a finite number of non-zero coefficients \( b^{Bn_1 \ldots n_k}_{A_1 \ldots A_l}(s_0, s_1, \ldots, s_l) \) at any given order. It should be stressed that application of a non-local perturbative field redefinition to a local field theory makes it seemingly non-local. Other way around, to answer the question if one or another model is perturbatively local or not it is not enough to check given vertices. Instead one has to analyse a less trivial problem whether there exists a non-local field redefinition transforming a seemingly non-local model to the manifestly local form. Once such a field redefinition is found, the model is shown to be local and should be analysed in this local frame (field variables). As long as such a field redefinition is unknown it is an open question whether the model is local or not. It should be stressed that in practice this means that it may be hard to prove that one or another model is essentially non-local.

If the (spin-)local frame of a model is known, the next question is what is the proper class of field redefinitions that leave the form of vertices perturbatively local or spin-local? In field theories with a finite number of fields the answer is that these are perturbatively local field redefinitions involving a finite number of derivatives at every order.

In the theories with an infinite number of fields the situation is more subtle. Naively one might think that appropriate field redefinitions in spin-local theories are also spin-local. This is not necessarily true, however, because the modified vertex

\[ \delta E_{A_0, s_0}(\partial, \phi) = \sum_{s_p=0}^{\infty} \sum_{k'=0, l'=1}^{\infty} a^{A_1 \ldots A_l}_{A_0 A_1 \ldots A_l}(s_0, s_1, s_2, \ldots, s_p, \ldots, s_l) \]  

(2.4)

\[ \partial_{n_1} \ldots \partial_{n_k} \phi^{A_1}_{s_1} \ldots \phi^{A_{p-1}}_{s_{p-1}} \phi^{A_p}_{s_{p+1}} \ldots \phi^{A_l}_{s_l} B_{B_1 \ldots B_{l'}}(s_p, s_{l+1}, \ldots, s_{l+l'}) \partial_{m_1} \ldots \partial_{m_{k'}} \phi^{B_1}_{s_{l+1}} \ldots \phi^{B_{l'}}_{s_{l'+l'}} \]

may contain an infinite summation over the spin \( s_p \) of the redefined field. If the vertex and field redefinition were spin-local the result of such a field redefinition can still be non-local and even ill-defined because an infinite number of terms with the same field pattern and any number of derivatives may result from the terms with different \( s_p \).

This difficulty is avoided if the field redefinition (2.3) is spin-local-compact in which case the summation over \( s_p \) is always finite and the modified vertex is both well-defined and spin-local. Thus, in the spin-local theories with infinite sets of fields a proper counterpart of the local field redefinitions in usual theories with finite numbers of fields is represented by spin-local-compact field redefinitions. One of the important consequences of this analysis is that non-compact spin-local field redefinitions at the lower order may produce non-localities.

3Note that we only consider field redefinitions expandable into power series of derivatives. The ill-defined non-localities like \( \Box^{-1} \) are not allowed. Allowing the latter, any theory can be reduced to a free theory (see, e.g., [64]) which option is not interesting.
at higher orders. This implies that the choice of field variables in the theories with infinite towers of fields is a delicate issue from the very first step. Once a spin-local frame of a theory is found, a very restricted class of spin-local-compact field redefinitions is compatible with spin-locality.

One of the central problems in HS gauge theory is to find its spin-local frame if exists. In [56] (and references therein) the local frame was found for a number of vertices including some up to the fifth order. This was achieved in the spinor formalism which we sketch now.

3 Free fields unfolded

For simplicity, in this paper we focus on the most elaborated example of $4d$ HS theory. The idea of our consideration applies to other HS theories as well, including those in $3d$ [65] and any $d$ [56, 57].

3.1 Free equations in $AdS$

For the vacuum one-form connection $W_0$ of $sp(4) \sim o(3, 2)$, that describes $AdS_4$,

$$W_0 = \frac{1}{2} w^{AB}(x) Y_A Y_B, \quad dw^{AB} + w^{AC} C_{CD} w^{DB} = 0 \quad (3.1)$$

with the $sp(4)$ invariant form $C_{AB}$ ($A, B \ldots = 1, \ldots, 4$), the unfolded system for free massless fields described by the one-forms $\omega(y, \bar{y}; K | x)$ and zero-forms $C(y, \bar{y}; K | x)$ reads as [11]

$$R_1(y, \bar{y}; K | x) = \frac{i}{4} \left( \eta H^{\hat{k} \hat{\beta}} \partial_\alpha \bar{\partial}_\beta C(0, \bar{y}; K | x) k + \bar{\eta} H^\alpha_{\hat{\beta}} \partial_\alpha \partial_\beta C(y, 0; K | x) \bar{k} \right), (3.2)$$

$$\tilde{D}_0(C(y, \bar{y}; K | x) k) = 0, \quad (3.3)$$

where $w^{AB} = (\omega^{\alpha \beta}, \omega^{\hat{\alpha} \hat{\beta}}, e^{\alpha \dot{\alpha}})$ describes Lorentz connection, $\omega^{\alpha \beta}, \omega^{\hat{\alpha} \hat{\beta}}$, and vierbein, $e^{\alpha \dot{\alpha}}$, with two-component spinor indices $\alpha = 1, 2, \hat{\alpha} = 1, 2$ (at the convention $A^\alpha = e^{\alpha \beta} A_\beta, A_\alpha = e_{\alpha \dot{\alpha}} A^{\dot{\alpha}}$)

$$\partial_\alpha := \frac{\partial}{\partial y^\alpha}, \quad \bar{\partial}_\hat{\alpha} := \frac{\partial}{\partial \bar{y}^{\hat{\alpha}}}, \quad (3.4)$$

involutive Klein elements $K = (k, \bar{k})$ defined to obey

$$\{k, y_\alpha\} = 0, \quad [k, \bar{y}_{\dot{\alpha}}] = 0, \quad k^2 = 1, \quad [\bar{k}, y_\alpha] = 0, \quad \{\bar{k}, \bar{y}_{\dot{\alpha}}\} = 0, \quad \bar{k}^2 = 1, \quad [k, \bar{k}] = 0, \quad (3.5)$$

$$H_{\alpha \beta} := e_{\alpha \dot{\alpha}} e_{\beta \dot{\beta}}^\dot{\alpha}, \quad \overline{H}^{\alpha \beta} := e_{\dot{\alpha} \alpha} e^{\beta \dot{\beta}}_{\dot{\beta}},$$

$$R_1(y, \bar{y}; K | x) := D_{\alpha \beta}^a \omega(y, \bar{y}; K | x) \quad D_{\alpha \beta}^{ad} = D^L - e^{\alpha \beta} \left( y_\alpha \bar{\partial}_\beta + \partial_\alpha \bar{y}_\beta \right), \quad (3.6)$$

$$D^L = d_x - \left( \omega^{\alpha \beta} y_\alpha \partial_\beta + \omega^{\hat{\alpha} \hat{\beta}} \bar{y}_{\dot{\alpha}} \bar{\partial}_\hat{\beta} \right),$$

$$\tilde{D}_0 = D^L + e^{\alpha \beta} \left( y_\alpha \bar{y}_\beta + \partial_\alpha \bar{y}_\beta \right) \quad (3.7)$$

The massless fields obey

$$\omega(y, \bar{y}; -k, -\bar{k} | x) = \omega(y, \bar{y}; k, \bar{k} | x), \quad C(y, \bar{y}; -k, -\bar{k} | x) = -C(y, \bar{y}; k, \bar{k} | x). \quad (3.8)$$
3.2 Zero-form sector

The linearized HS equations decompose into independent subsystems associated with different spins. We start with the simpler equations (3.3) on the gauge invariant zero-forms $C$

$$C(Y; K|x) = \sum_{A=0}^{1} \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} C_{\alpha_{1}\alpha_{n} \hat{\alpha}_{1} \ldots \hat{\alpha}_{m}}^{\hat{A}1-A}(x) y^{\alpha_{1}} \ldots y^{\alpha_{n}} \bar{y}^{\hat{\alpha}_{1}} \ldots \bar{y}^{\hat{\alpha}_{m}} k^{A} \bar{k}^{1-A}.$$  

Spin-s zero-forms are $C_{\alpha_{1}\alpha_{n} \hat{\alpha}_{1} \ldots \hat{\alpha}_{m}}^{\hat{A}1-A}(x)$ with

$$n - m = \pm 2s.$$  

Eq. (3.7) rewritten in the form

$$D^{L} C^{A1-A} = e^{\alpha \hat{\beta}} \frac{\partial^{2}}{\partial y^{\alpha} \partial \bar{y}^{\hat{\beta}}} C^{A1-A} + \text{lower-derivative and nonlinear terms}$$  

implies that higher-order terms in $y$ and $\bar{y}$ in the zero-forms $C(y, \bar{y}|x)$ (discarding from now on indices $A$) describe higher-derivative descendants of the primary components $C(y, 0|x)$ and $C(0, \bar{y}|x)$. Generally, $C_{\alpha_{1}\alpha_{n} \hat{\alpha}_{1} \ldots \hat{\alpha}_{m}}(x)$ contain $\frac{n + m}{2} - \{s\}$ space-time derivatives of the spin-$s$ dynamical fields. In particular, self-dual and anti-self-dual components of the generalized Weyl tensor $C_{\alpha_{1}\alpha_{2}}(x)$ and $C_{\hat{\alpha}_{1}\hat{\alpha}_{2}}(x)$ originally introduced by Weinberg [4] (see also [5]) contain $[s]$ derivatives of the dynamical (Fronsdal) massless field, including spin-zero and spin-1/2 matter fields. As a result, the presence of zero-forms $C$ in the HS vertices may induce infinite towers of derivatives and, hence, non-locality.

Using the frame one-form $e^{\alpha \hat{\beta}}$, whatever the $r.h.s.$ of equations (3.10) is it can be represented in the form

$$D^{L} C(y, \bar{y}|x) = e^{\alpha \hat{\beta}} \left( \partial_{\alpha} \bar{\partial}_{\hat{\beta}} F^{++}(y, \bar{y}|x) + y_{\alpha} \bar{y}^{\hat{\beta}} F^{-+}(y, \bar{y}|x) + \partial_{\alpha} \bar{y}^{\hat{\beta}} F^{+-}(y, \bar{y}|x) + y_{\alpha} \bar{y}^{\hat{\beta}} F^{--}(y, \bar{y}|x) \right).$$  

(3.11)

Note that this form of the dynamical equations is not unfolded since their $r.h.s.$ may contain via $F^{ab}$ components of the dynamical fields not necessarily in the form of wedge products. For instance, a one-form $E(y, \bar{y}) = dx^{\alpha} E_{\alpha}(y, \bar{y})$ is represented by $E^{\pm\pm}(y, \bar{y})$ with

$$N_{y} N_{\bar{y}} E^{++}(y, \bar{y}) := y^{\alpha} \bar{y}^{\hat{\beta}} e^{\alpha \hat{\beta}} e_{\alpha \hat{\beta}} \bar{E}_{\alpha}(y, \bar{y}), \quad (N_{y}+2) N_{\bar{y}} E^{-+}(y, \bar{y}) := \partial_{\alpha} \bar{y}^{\hat{\beta}} e^{\alpha \hat{\beta}} \bar{E}_{\alpha}(y, \bar{y}),$$  

(3.12)

$$N_{y} (N_{y}+2) E^{+-}(y, \bar{y}) := y^{\alpha} \partial_{\alpha} e^{\alpha \hat{\beta}} \bar{E}_{\hat{\beta}}(y, \bar{y}), \quad (N_{y}+2)(N_{\bar{y}}+2) E^{--}(y, \bar{y}) := \partial_{\alpha} \bar{\partial}_{\hat{\beta}} e^{\alpha \hat{\beta}} \bar{E}_{\alpha}(y, \bar{y}).$$  

(3.13)

$$N_{y} := y^{\alpha} \partial_{\alpha}, \quad N_{\bar{y}} := \bar{y}^{\hat{\beta}} \bar{\partial}_{\hat{\beta}},$$  

(3.14)

where $e^{\alpha \hat{\beta}}$ is the inverse vierbein. The leading term in (3.10), that determines the higher components in $C(y, \bar{y}|x)$ via space-time derivatives of the lower ones with smaller $n + m$, just has the form of $F^{++}(y, \bar{y}|x)$ (3.11). Clearly, the projector $\Pi^{des}$ to the part $F^{++}$ that contains descendants in (3.10) is

$$\Pi^{des} := N_{y}^{-1} \bar{N}_{\bar{y}}^{-1} y^{\alpha} \bar{y}^{\hat{\beta}} \frac{\partial}{\partial e^{\alpha \hat{\beta}}}.\quad (3.15)$$
Suppose now that all non-linear corrections to the r.h.s. of the field equations (3.10) contain dependence on $y^a$ or $\dot{y}^\dot{a}$ in the combinations $e^{\alpha\dot{a}} y_\alpha \tilde{\phi}_\dot{a}(y,\dot{y})$ or $e^{\alpha\dot{a}} \ddot{y}_\alpha \phi_{\dot{a}}(y,\dot{y})$. In that case the expressions for the components $C_{\alpha_1...\alpha_n,\dot{\alpha}_1...\dot{\alpha}_m}(x)y^{\alpha_1}...y^{\alpha_n}\ddot{y}^{\dot{\alpha}_1}...\ddot{y}^{\dot{\alpha}_m}$ with higher $n + m$ (descendants) via space-time derivatives of those with the lower ones will preserve the same form as in the free theory being insensitive to the non-linear corrections to (3.10). In that case the non-linear corrections to the unfolded HS equations will contribute to the dynamical equations (the r.h.s. of the Fronsdal equations and Bianchi identities associated with them) not affecting the expressions for descendants in $C(Y|x)$ via space-time derivatives of the primaries. This simple observation will allow us in Section 5 to formulate the equivalence condition for the concepts of space-time and spinor spin-locality.

### 3.3 One-form sector

In the sector of one-forms $\omega(Y; K|x)$ spin-$s$ fields are described by the degree $s - 1$ homogeneous monomials in $Y$: $\omega(\mu Y; K|x) = \mu^{2(s-1)} \omega(Y; K|x)$, i.e., the spin-$s$ gauge fields in the generating function

$$\omega(Y; K|x) = \sum_{A=0}^{1} \sum_{n,m=0}^{\infty} \frac{1}{2n! m!} \omega^A_{\alpha_1...\alpha_n,\dot{\alpha}_1...\dot{\alpha}_m}(x)(kk)^A y^{\alpha_1}...y^{\alpha_n}\ddot{y}^{\dot{\alpha}_1}...\ddot{y}^{\dot{\alpha}_m}$$

are $\omega^A_{\alpha_1...\alpha_n,\dot{\alpha}_1...\dot{\alpha}_m}(x)$ with $n + m = 2(s - 1)$. Dynamical HS fields, that contain Fronsdal fields, are those with $n = m$ for bosons and $|n - m| = 1$ for fermions. Other components contain derivatives of them. In other words, dynamical fields belong to the bisectrix on the plane $n$, $m$ for bosons or to its nearest neighbours for fermions. For given $s$ the number of derivatives in the field $\omega^A_{\alpha_1...\alpha_n,\dot{\alpha}_1...\dot{\alpha}_m}$ (discarding from now on the dependence on $K$ and $x$) equals to the half a distance from the nearest line of the dynamical fields on the $n, m$ plane. This has the important consequence that the spin-$s$ components of $\omega(Y)$ contain at most $s - 1$ derivatives of the spin-$s$ Fronsdal field.

The interpretation of the components $\omega^A_{\alpha_1...\alpha_n,\dot{\alpha}_1...\dot{\alpha}_m}$ depends on whether $n > m$ or $n < m$. From (3.16) it follows that at $n > m$ every next spin-$s$ component with $n > m$ is expressed by (3.3) via the space-time derivatives of the previous one

$$D^L \omega(y, \dot{y}) - e^{\alpha\dot{\beta}} \partial_{\alpha} \tilde{y}^\dot{\beta} \omega(y, \dot{y}) + ... = 0, \quad n \geq m,$$  \hspace{1cm} (3.17)

where ellipsis denotes the lower-derivative terms as well as the l.h.s. of the Fronsdal equations or Bianchi identities. Analogously, at $m \geq n$

$$D^L \omega(y, \dot{y}) - e^{\alpha\dot{\beta}} y_\alpha \ddot{y}^{\dot{\beta}} \omega(y, \dot{y}) + ... = 0, \quad m \geq n.$$  \hspace{1cm} (3.18)

Introducing (non-orthogonal) projectors $P_\pm$:

$$P_+ \omega(y, \dot{y}) = \omega(y, \dot{y}) \quad n \geq m, \quad P_+ \omega(y, \dot{y}) = 0 \quad n < m,$$  \hspace{1cm} (3.19)
\[ P_\omega(y, \bar{y}) = \omega(y, \bar{y}) \quad m \geq n, \quad P_\omega(y, \bar{y}) = 0 \quad m < n, \] (3.20)
equations (3.17), (3.18) can be put into the form
\[ D^L \omega(y, \bar{y}) - e^{\alpha\beta} \left( P_+ \partial_\alpha \bar{y}_\beta + P_- y_\alpha \bar{\partial}_\beta \right) \omega(y, \bar{y}) + \ldots = 0. \] (3.21)

Any one-form \( \omega(y, \bar{y}) \) can be represented in the form
\[ \omega(y, \bar{y}) = e^{\alpha\dot{\alpha}} \left( \partial_\alpha \bar{\partial}_\dot{\alpha} \Omega^{++}(y, \bar{y}) + y_\alpha \bar{\partial}_\dot{\alpha} \Omega^{-+}(y, \bar{y}) + \partial_\alpha \bar{y}_\dot{\alpha} \Omega^{-+}(y, \bar{y}) + y_\alpha \bar{y}_\dot{\alpha} \Omega^{--}(y, \bar{y}) \right) \] (3.22)
with zero-forms \( \Omega^{\mu\nu}, \nu, \mu = + \) or \(-\). It is not hard to see that
\[ e^{\alpha\beta} \bar{\partial}_\beta y_\alpha \omega(y, \bar{y}) = \frac{1}{2} \left( (N_\dot{y} + 2) H^{\alpha\beta} (y_\alpha \partial_\beta \Omega^{++}(y, \bar{y}) + y_\alpha \bar{\partial}_\dot{\beta} \Omega^{-+}(y, \bar{y}) \right) - N_\dot{y} H^{\alpha\beta} \partial_\alpha \bar{\partial}_\beta \Omega^{++}(y, \bar{y}) \right), \] (3.23)
\[ e^{\alpha\beta} \partial_\alpha \bar{y}_\beta \omega(y, \bar{y}) = \frac{1}{2} \left( (N_\dot{y} + 2) H^{\alpha\beta} (y_\alpha \bar{\partial}_\beta \Omega^{-+}(y, \bar{y}) + \bar{y}_\alpha \partial_\dot{\beta} \Omega^{--}(y, \bar{y}) \right) - N_\dot{y} H^{\alpha\beta} \partial_\alpha \bar{\partial}_\beta \Omega^{++}(y, \bar{y}) \right). \] (3.24)

Note that the r.h.s. of equation (3.2) also has this form with \( C(y, 0; K|x) \) and \( C(0, \bar{y}; K|x) \) in place of the appropriate components of \( \Omega^{++} \).

Suppose that non-linear corrections to the r.h.s. of the field equations (3.12) do not contribute to the projected terms of (3.23) and (3.24) \( e^{\alpha\beta} \left( P_+ \partial_\alpha \bar{y}_\beta + P_- y_\alpha \bar{\partial}_\beta \right) \omega(y, \bar{y}) \). Then expressions for the components of \( \omega(y, \bar{y}) \) associated with higher space-time derivatives of the Fronsdal fields keep the same form as in the free theory being insensitive to the non-linear corrections to (3.2). Hence, the latter will only contribute to the r.h.s. of the Fronsdal equations and associated Bianchi identities not affecting the expressions for higher components of \( \omega(y, \bar{y}|x) \) via space-time derivatives of the lower ones. This fact underlies the analysis of the equivalence of space-time and spinor spin-locality in Section 3.

## 4 Spinor spin-locality

Unfolded HS equations have the following form originally proposed in [61]
\[ d_\omega \omega + \omega \ast \omega = \Upsilon(\omega, \omega, C) + \Upsilon(\omega, \omega, C, C) + \ldots, \] (4.1)
\[ d_\omega C + \omega \ast C - C \ast \omega = \Upsilon(\omega, C, C) + \Upsilon(\omega, C, C, C) + \ldots. \] (4.2)

As in [61], the perturbative expansion is in powers of the zero-forms \( C \) with one-forms \( \omega \) treated as having order zero. Only wedge products of differential forms are used with the wedge symbol implicit.

Note that from the above results it is not hard to check that the vertices (4.1) and (4.2) that are at most linear in the zero-forms \( C \) are spin-local and, moreover, spin-local-compact. On the other hand, the vertices bilinear in \( C \) are \textit{a priori} not spin-local and may contain infinite towers of derivatives for any given set of spins. Nevertheless, in the papers [51, 54, 56] a field frame was found in which these vertices are manifestly spin-local though non-compact.
As explained in Section 3, equations (3.10) tell us that, in the lowest order, \( \frac{\partial}{\partial x} \) is equivalent to \( \frac{\partial^2}{\partial y \partial \bar{y}} \). However, at higher orders relation (3.10) gets corrected due to the contribution of the non-linear vertices to the unfolded HS equations at lower orders.

The customary field-theoretic setup is that of space-time derivatives. From the perspective of non-linear HS equations the analysis of locality in terms of spinor variables \( y \) and \( \bar{y} \) is most natural, being technically far simpler and allowing simple criteria of spinor spin-locality elaborated in [51, 54]. Since the two approaches are different we will call them differently using the name space-time spin-locality for the former and spinor spin-locality for the latter.

Let us now explain how the spinor spin-locality works [51]. The vertices can be put into the form

\[
\Upsilon(C, C, \ldots) = F(y, t^i, p^l, \bar{y}; \bar{p}^k)\omega(Y_1) \ldots \omega(Y_k)C(Y_{k+1}) \ldots C(Y_n)\bigg|_{Y_i=0},
\]

where

\[
t^i_\alpha := \frac{\partial}{\partial y^\alpha_i}, \quad \bar{t}^i_\dot{\alpha} := \frac{\partial}{\partial \bar{y}^\dot{\alpha}_i}
\]

act on the argument of the \( i^{th} \) factor of \( \omega \) and

\[
p^i_\alpha := \frac{\partial}{\partial y^\alpha_i}, \quad \bar{p}^i_\dot{\alpha} := \frac{\partial}{\partial \bar{y}^\dot{\alpha}_i}
\]

act on the argument of the \( i^{th} \) factor of \( C \). The function \( F(y, t^i, p^l, \bar{y}; \bar{p}^k) \) depends on various Lorentz-covariant contractions of \( y_\alpha, p^i_\alpha \) and \( t^i_\alpha \) and their conjugates like

\[
P^{ij} := p^i_\alpha p^{j \alpha}, \quad \bar{P}^{ij} := \bar{p}^i_\dot{\alpha} \bar{p}^{j \dot{\alpha}}, \quad T^i := y^\alpha t^i_\alpha, \quad \bar{T}^i := \bar{y}^{\dot{\alpha}} \bar{t}^i_\dot{\alpha}.
\]

Generally, as a consequence of non-locality of the (Moyal) star product underlying the full nonlinear system [18], once no special care of the perturbative scheme is taken, the functions \( F(P^{ij}, \bar{P}^{kl}) \) are non-polynomial. Since an infinite number of derivatives \( p^i_\alpha \) implies by (3.10) an infinite number of space-time derivatives, such a general vertex is non-local.

Note that there was a number of studies of the star-product induced non-locality in the literature (see e.g. [69, 70, 71, 72]). However, the HS setup of [48] provides special tools to control locality in HS theories as explained in [73, 51, 54] and in this paper.

Since a spin-\( s \) one-form connection \( \omega \) contains at most a finite number of derivatives dominated by \( s \), from the spinor spin-locality perspective the potential non-local contribution to the vertices is due to the zero-forms \( C \). If, however, there exists such a perturbative scheme that \( F(P^{ij}, \bar{P}^{kl}) \) is polynomial in either \( P^{ij} \) or \( \bar{P}^{ij} \) for any pair of \( i, j \), the vertex is called spinor spin-local. In the lowest order this implies space-time spin-locality. Indeed, since the projection on the fixed spins relates degrees in \( P^{ij} \) and \( P^{ij} \) to each other via (3.9), if the function was polynomial in \( P^{ij} \), its projection to fixed spins is simultaneously polynomial in \( P^{ij} \) and vice versa because the degrees in \( y \) and \( \bar{y} \) differ by \( 2s \).

\[\text{4Note that earlier the term spin-locality was sometimes used for slightly different concepts [51].}\]
Remarkably the perturbative scheme elaborated in \[52, 53, 54, 55, 56\] reproduces spinor spin-local vertices of this type not only in the $C^2$ sector but also in the holomorphic $\Upsilon^{\eta\eta}(\omega, C, C, C)$ and antiholomorphic $\Upsilon^{\bar{\eta}\bar{\eta}}(\omega, C, C, C)$ parts of the vertex $\Upsilon(\omega, C, C, C)$ proportional to $\eta^2$ and $\bar{\eta}^2$, respectively where $\eta$ is the coupling constant in the HS equations of \[48\] (for review see \[74\]. Once these results are extended to the mixed vertex $\Upsilon^{\eta\bar{\eta}}(\omega, C, C, C)$ this proves spin-locality of the $C^3$ ($C^4$ at the action level) that was argued to be non-local in \[24\]. At the moment this is still work in progress.

5 Space-time : spinor spin-locality equivalence for projectively-compact vertices

Though spinor and space-time spin-locality are equivalent in the lowest order this is not necessarily true at higher orders. The reason is that non-linear contributions to the field equations (3.10) may affect the expressions for higher components of the zero-forms $C$ via space-time derivatives of the lower ones adding some higher-derivative non-linear terms to the former. In that case, spinor spin-locality will not imply the space-time one. There exist, however, a special class of projectively-compact vertices for which this does not happen and spinor spin-locality implies the space-time one. Remarkably, as discussed below, such vertices have been already identified in HS theory as carrying the minimal number of derivatives.

5.1 Spin-local-compact vertices

If an order-$n$ vertex is spinor spin-local-compact it will only produce space-time spin-local vertices. This is simply because for a given set of spins at most a finite number of higher-derivative terms will contribute to the next-order space-time vertex in full analogy with the spin-local-compact field redefinitions (2.3). For instance, the redefinition of the descendant HS fields due to the presence of the spin-local-compact vertices $\omega \ast \omega$ and $\omega \omega C$ do not spoil space-time spin-locality at the next order. Though the situation with the non-compact spinor spin-local vertices $V$ containing several factors of the zero-forms $C$ is more subtle it can be controlled in a similar way in case its projection $\Pi^{\text{des}} V$ to the sector determining the descendant fields in terms of the derivatives of the primary ones (cf. (3.10), (3.15)) is spin-local-compact. We will call such vertices projectively-compact spin-local. In particular, this is true if $\Pi^{\text{des}} V = 0$.

5Recently, a closely related attempt of the analysis of holomorphic vertices pretending to be unrelated to equations of \[18\] and the approach of \[55\] (though using the star-product of \[55\]) was suggested in \[75\] where however neither its formal consistency nor potential divergencies were considered.
5.2 Projectively-compact spin-local vertices in $d_x C$

Let us start with the simpler case of equation (4.2) on the zero-form $C$. Suppose that the vertex obeying the spinor spin-locality conditions of Section 4 has the form (4.3) with

$$F(y, t^i, p^l, \bar{y}, \bar{p}^k) = \tilde{T} F'(P^{ij}, \bar{P}^{kl}, \ldots) \quad (5.1)$$

with $\tilde{T} = T$ or $\tilde{T} (4.4)$. Then non-linear corrections resulting from this vertex do not affect the space-time spin-locality at the next order. To see this one has to take into account that in the lowest order the one-form $\omega$ contributes via its zero-order background part associated with the vierbein. The presence of the factor of $\tilde{T}$ implies that each vierbein comes via $e_\alpha^\dot{\alpha}$ or $\bar{e}_\alpha^{\dot{\alpha}}$. However, as explained in Section 3, such terms do not contribute to $F^{++}$ in (3.11) that determines the expressions for higher components in spinor variables in $C(Y|x)$ via space-time derivatives of the lower ones. So, in this case, spinor spin-locality with $\Pi^{des}$ (3.15) is equivalent to the space-time spin-locality at least up to the higher-order contributions.

The vertex found in [49]

$$\Upsilon = \Upsilon_\eta(e, C) + \Upsilon_{\bar{\eta}}(e, C) \quad (5.2)$$

has the following projectively-compact spin-local form presented in [50]

$$\Upsilon_\eta(e, C) = \frac{1}{2} \eta \exp (i P^{1,2}) \int_0^1 d\tau e(y, (1-\tau)\bar{p}_1 - \tau\bar{p}_2) C(\tau y, \bar{y}; K) C(-(1-\tau)y, \bar{y}; K) * k, \quad (5.3)$$

$$\Upsilon_{\bar{\eta}}(e, C) = \frac{1}{2} \bar{\eta} \exp i(P^{1,2}) \int_0^1 d\tau e((1-\tau)p_1 - \tau p_2, \bar{y}) C(y, \tau \bar{y}; K) C(y, -(1-\tau)\bar{y}; K) * \bar{k}, \quad (5.4)$$

where $e^{\alpha\dot{\alpha}}$ is the $AdS_4$ vierbein, $\eta$ is a free complex parameter of the HS theory, and

$$e(a, \bar{a}) := e^{\alpha\dot{\alpha}} a_\alpha \bar{a}_{\dot{\alpha}}. \quad (5.5)$$

It is spinor spin-local since either $P^{12}$ or $\bar{P}^{1,2}$ enter non-polynomially but not both. Since the frame field $e^{\alpha\dot{\alpha}}$ is contracted either with $y$ or to $\bar{y}$ being of the form (5.1) it is also projectively-compact not affecting the interplay between spinor and space-time spin-locality at the next order. This implies in particular that the spinor spin-local $C^3$ vertices of [53, 55, 56] are space-time spin-local provided that one starts with $C^2$ vertices (5.3), (5.4).

The following comment is now in order. A special property of the vertex (5.1) is that it has a lowest number of space-time derivatives compared to other vertices containing factors like $t_\alpha^i p^{j\alpha}$ in place of $t_\alpha^i y^\alpha$. This is because the replacement of $y$ by $p$ increases the degree of the respective component of the zero-form $C$ which by virtue of (3.10) implies increase of the number of space-time derivatives carried by $C$. The lesson is that, to reach equivalence between space-time and spinor spin-locality, one has to choose the vertices with the minimal number of derivatives among various local ones. Note that the vertex of [49] was successfully tested in [78, 79] to show that it properly reproduces the holographic predictions.

\textsuperscript{6}Note that, since the cubic spin-zero vertex is zero in the HS theory [20, 76], the relation between descendants and primary fields is not affected by the interactions in this sector. This observation implied equivalence of the space-time and spinor spin-locality for the quartic spin-zero vertex analysed in [24].
5.3 Projectively-compact spin-local vertices in $d_x \omega$

As explained in the end of Section 3.3, the terms in the vertex that have the form

$$P_+ \left( (\bar{N} + 2) H^{\alpha \beta} (y_\alpha \bar{\partial}_\beta \Omega^+(y, \bar{y}) + y_\alpha y_\beta \Omega^-(y, \bar{y})) - N \bar{H}^{\dot{\alpha} \dot{\beta}} \bar{\partial}_{\dot{\alpha}} \bar{\partial}_{\dot{\beta}} \Omega^{++}(y, \bar{y}) \right) \tag{5.6}$$

or

$$P_- \left( (N + 2) \bar{H}^{\dot{\alpha} \dot{\beta}} (\bar{y}_\dot{\alpha} \bar{\partial}_{\dot{\beta}} \Omega^{++}(y, \bar{y}) + \bar{y}_\dot{\alpha} \bar{y}_\dot{\beta} \Omega^{--}(y, \bar{y})) - \bar{N} H^{\alpha \beta} \partial_\alpha \partial_\beta \Omega^{--}(y, \bar{y}) \right) \tag{5.7}$$

with $P_\pm$ (3.19) do not affect the expressions for the one-form descendant fields via derivatives of the primary ones hence being projectively-compact. In that case, spinor spin-locality of the next-order vertex implies its space-time spin-locality.

Remarkably, the cubic vertices found in [50] do indeed have such a form. Moreover, they only contain the $y, \bar{y}$-independent terms with nonzero $\Omega^{++}$. (For explicit expressions that indeed contain the projectors $P^\pm$ in the form of appropriate Cauchy integral see Eq. (3.7) of [50].) In fact, this again implies that the respective vertices carry the minimal number of derivatives.

6 Bulk to boundary correspondence

The idea of the bulk-to-boundary correspondence suggested by the unfolded dynamics analysis of [60] consists of the observation that, upon an appropriate change of the reality conditions on spinor variables, the unfolded equations (3.3) make sense independently of how many space-time coordinates are involved. In the case of $AdS_4$ with local coordinates $x^\alpha \dot{\beta}$ and $AdS_4$ connection $W_0$ these equations describe massless fields in $AdS_4$. The same equations with the 3d local coordinates $x^\alpha \beta = x^{\beta \alpha}$ and 3d flat $O(3, 2)$ connection describe 3d conformal conserved currents. This is a manifestation of the observation of [76] that the rank-two unfolded equations with the doubled spinor variables describe conformal conserved currents (which provides a field-theoretic realization of the Flato-Fronsdal theorem). The HS connections $\omega(Y; K | x^\alpha \dot{\beta})$ describe HS gauge fields in $AdS_4$ or 3d conformal HS gauge fields $\omega(Y; K | x^\alpha \beta)$ originally considered in [80] (in the absence of $K$, however).

Analogously, the 4d system of non-linear HS equations of [48] can be reinterpreted as a 3d non-linear system that describes interactions of 3d conformal currents with 3d conformal HS gauge fields of the Fradkin-Tseytlin type [1, 2], which however carry no local degrees of freedom analogously to 3d gravity [83] and supergravity [84]. That allowed us to conjecture in [60] that the HS gauge theory in $AdS_4$ is holographically dual to the 3d conformal HS theory that describes interactions of 3d conformal currents with the 3d conformal HS gauge fields. It was shown in [60] how this picture is reproduced in the boundary limit $z \to 0$ with $z$ being the Poincaré coordinate.

This conjecture can be considered as a HS gauged version of the original Klebanov-Polyakov conjecture [13]. In particular, the boundary conformal HS currents of [60] can be realized as bilinears $J(y_1, y_2 | x) = \sum_{i=1}^N \phi_i(y_1 | x) \phi_i(y_2 | x)$ that obey the same unfolded
equiv 4.1 equations hence describing the same representation of the conformal group. The $N \to \infty$ limit guarantees that the currents are not subjected to additional constraints.

The important point is that the HS gauge fields $\omega$ behave at the boundary as the shadow fields for the currents represented by the zero-forms $C$. Therefore, the holographic prescription is conjectured to be modified with the correlators reproduced by the variations

$$\langle J(x_1) \ldots J(x_n) \rangle = \frac{\delta^n S(\omega)}{\delta \omega(x_1) \ldots \delta \omega(x_n)}$$

(6.1)

at the boundary with local coordinates $x$ of some on-shell gauge invariant functional $S(\omega, C)$ of the type proposed in [85].

7 Conclusion

In this paper we introduce a class of projectively-compact spin-local vertices establishing equivalence between space-time and spinor spin-locality (i.e., locality of vertices with any finite set of spins) in HS gauge theory. Spinor spin-locality of the HS gauge theory is known to be much easier to analyse than the space-time one. However, generally, the procedure of the derivation of the space-time vertices from those in the spinor space is not only technically involved but also may induce higher-derivative corrections beyond the lowest order. Hence, spinor spin-locality does not automatically imply space-time spin-locality. In this paper the conditions are found guaranteeing equivalence of the spinor and space-time spin-locality.

Namely, it is shown that for so-called projectively-compact spin-local vertices spinor spin-locality implies the space-time one. It is also checked that the spin-local vertices of [49, 50] are projectively-compact that provides an important step towards verification of the space-time spin-locality of HS gauge theory via verification of its spinor spin-locality. We conjecture that the projectively-compact spin-local vertices form a proper functional class in which HS gauge theory has to be analysed. Though this is still work in progress our preliminary results suggest that HS gauge theory may be spin-local at the quartic order as well. Once this is shown to be really true, it would imply space-time spin-locality of the HS gauge theory in disagreement with the conclusions of [24] relying on the Klebanov-Polyakov conjecture [13]. Most likely, this will demand a modification of the latter along the lines of [10] sketched in Section 3. The most significant new point is that in that case the boundary dual theory is not just a CFT but rather a conformal HS gauge theory. (Note that as such it does not possess a gauge invariant stress tensor.) All this can significantly affect the paradigm of the holographic correspondence replacing gauge - gravity correspondence by the gravity - conformal gravity one. In particular, this kind of the correspondence spoils the assumptions of [87].

For simplicity, in this paper we considered the most elaborated example of 4d HS theory. The idea of our construction applies to other HS theories as well, including those in 3d [65].

\footnote{This is true for spins $s \geq 1$. For spins $s = 0$ or $1/2$ the prescription of [13] is unchanged with the sources and currents distinguished by the boundary behaviour of the respective components of the zero-forms $C$.}
and any \( d \). In the general case the term \textit{spinor spin-locality} has to be changed to the \textit{fiber spin-locality} since the fibers may have non-spinor local coordinates (e.g., vectors in the \( d \)-dimensional HS theory of \([17]\)). Still the rule is that a vertex is projectively-compact spin-local if its projection to the part of unfolded equations that determines descendant fields via derivatives of the primary ones is spin-local-compact.

In the end, let us make the following comment. Though the analysis of the effect of projectively-compact vertices is perturbative, it implies space-time spin-locality in higher orders as well since the corrections to descendants are projectively-compact spin-local. It suffices to prove projective spin-locality inductively at any order to prove equivalence of space-time and fiber spin-locality in all orders. It is also worth to mention that both the notion of space-time (and, hence, space-time locality) and spin-locality are intimately related via the underlying symmetry \( G \), which is \( Sp(4) \) in the \( AdS_4 \) case. Space-time is where \( G \) is geometrically realized while spin is associated with the appropriate \( G \)-modules. Space-time derivatives are identified with descendants of those modules while spin-locality demands a number of descendants in the vertex be limited for any finite subset of \( G \)-modules. Both of the concepts of background geometry and spin are perturbative.

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