Quarks, monopoles and dyons at large N

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\textbf{Abstract}

We study a system of external particles of various charges in $\mathcal{N} = 4$ super Yang-Mills in the large $N$ limit at finite temperature. We demonstrate that at high enough temperature partial or complete screening of the particles can occur. At zero temperature the total electric or magnetic charge cannot be screened, while higher multipole moments of these charges can be screened. The specific case of a quark, a monopole and a dyon is worked out and the above properties are verified. We also discuss the free energy of isolated particles and show that their entropy is independent of the temperature.

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1 Introduction

The recently proposed connection between anti-de Sitter (AdS) supergravity and conformal field theory at its boundary \[1, 2, 3\] (see also \[4\]) permitted the calculation in the large-N $\mathcal{N} = 4$ super Yang-Mills theories of quantities that were previously inaccessible. In \[5, 6\] the temporal Wilson loop for a quark anti-quark pair at zero temperature was calculated and their interaction energy thus deduced. This was later extended to finite temperature in \[7, 8\] and to a system of a quark and a monopole at zero temperature in \[9\]. Furthermore, a demonstration of the existence of a mass gap in this theory was given in \[10\]. See \[11\] for other recent results.

What makes the physics nontrivial is that for any nonzero temperature the system is actually in a ‘high-temperature’ phase \[10\]. This phase persists at zero temperature as one of two possible phases, and is the one examined here and in the literature. (The ‘low-temperature’ confining phase would be obtained by taking the temperature to zero before taking the volume to infinity and is, arguably, physically less interesting \[10\].) As a result, isolated quarks can exist even at zero temperature, signaling the breakdown of $Z_N$ symmetry and the presence of a conformally invariant screening in the gluon vacuum.

In this paper we will investigate the general properties of a system of external particles in this theory, concentrating on issues of screening and clustering. As a representative example we will study and explicitly work out the properties of a system consisting of three particles: a quark, a monopole and a dyon at nonzero temperature. The charges are chosen such that the particles can form a singlet bound state, so that questions of clustering versus screening as the system is heated up can be addressed. At sufficiently high temperature, one of the particles is screened from the other two and at even higher temperature we get complete screening of all three. A similar analysis holds for more general collections of particles and the features that generalize are pointed out.

The entropy of isolated particles is also calculated and shown to be independent of the temperature. This determines the effective internal states of external particles in this theory and raises the interesting issue of a microscopic derivation of these states.

2 The setup

Consider the specific case of a quark, a monopole and a dyon. For simplicity we will assume that the three particles are situated along a line. As an example, we choose to put the dyon in the middle.

The euclidean metric of the non-extremal D3-brane is given by

$$ds^2 = \alpha' \left( \frac{U^2}{R^2} \left\{ (1 - U_T^4/U^4) d\tau^2 + dx_i^2 \right\} + R^2 (1 - U_T^4/U^4)^{-1} \frac{dU^2}{U^2} \right)$$  \(1\)
Figure 1: A configuration connecting a quark, a dyon and a monopole.

The horizon is at $U_T$ and has become the origin of the euclidean coordinates. Space is restricted to the $U \geq U_T$ domain. The periodicity of euclidean time $\tau$, ensuring the absence of conical singularities at $U = U_T$, is $\beta = 1/T = \pi R^2/U_T$, while the coupling constant $g_{YM}$ of the corresponding large-$N$ theory is given as $g_{YM}^2 N = R^4$, (we have $g = g_{YM}^2/4\pi$). We note that $R$ is dimensionless while $U_T$ has dimensions of energy.

In the large-$N$, large-$g_{YM}^2 N$ domain the expectation value of the Wilson-’t Hooft loops corresponding to the insertion of external particles can be approximated by the classical minimum of the action of the corresponding AdS string configuration. Figure 1 shows the configuration that we have in mind for the three particles. Each is represented by a string, a $(1,0)$ string with tension $1/2\pi\alpha'$ for the quark, a $(0,1)$ string with tension $1/2\pi\alpha' g$ for the monopole and a $(-1,-1)$ string with tension $\sqrt{1 + 1/g^2}/2\pi\alpha'$ for the dyon. They connect in a vertex at $U = U_0$. This kind of three string junctions has been extensively discussed in the literature, [13].

The general shape of the strings is determined by locally minimizing the world-sheet surface area, i.e.,

$$S = \frac{T}{2\pi} \int dx \sqrt{(\partial_x U)^2 + (U^4 - U_T^4)/R^4}. \quad (2)$$

It is found that

$$\frac{U_i^4 - U_T^4}{\sqrt{(\partial_x U)^2 + (U^4 - U_T^4)/R^4}} = R^2 \sqrt{U_i^4 - U_T^4}, \quad (3)$$

where $U_i$ are the minima of $U$ for the three strings respectively. (In case of the dyon string it is the minimum of the extension of the string past the vertex.)

At the vertex, the forces exerted by the three strings due to the tension must sum to zero. By using the above expressions for the shapes of the strings we find
the following conditions. In the horizontal direction we find

\[ y_1 = \sqrt{1 + 1/g^2 y_2 + \frac{1}{g y_3}}, \]

while in the vertical direction we find

\[ \sqrt{1 - y_1^2} + \frac{1}{g} \sqrt{1 - y_3^2} = \sqrt{1 + 1/g^2 \sqrt{1 - y_2^2}}. \]

For convenience we have defined

\[ y_i^2 = \frac{U_i^4 - U_T^4}{U_0^4 - U_T^4}. \]

It will also be convenient to define

\[ x_i = \frac{U_i^2}{U_0^2} = \sqrt{y_i^2 + \frac{U_i^4}{U_0^4}(1 - y_i^2)}. \]

The conditions (4) and (5) are solved by

\[ y_2 = \frac{g y_1 - \sqrt{1 - y_1^2}}{\sqrt{1 + g^2}} \]

and

\[ y_3 = \sqrt{1 - y_1^2}. \]

We will use \( y_1 \) as a parameter. Together with \( U_0 \) it parametrizes all the configurations of the system (only the two relative distances between the particles are relevant).

The free energy \( F \) of the configuration is obtained by dividing the total worldsheet action of the above configuration by the inverse temperature. This is the relevant quantity when considering equilibrium conditions and transitions from metastable states. The mean energy \( E \) of the configuration (in excess of the thermal vacuum), on the other hand, is found by differentiating the total action with respect to the inverse temperature, and would be relevant to local nonequilibrium processes. We will mostly consider the free energy in this work.

The worldsheet area is infinite, due to the branches extending to \( U = \infty \), and needs to be regularized. The origin of this is the infinite mass of the heavy particles inserted \( [5] \), which should be subtracted. This corresponds to the free energy of three isolated particles at zero temperature, calculated by evaluating the worldsheet action of three free strings reaching down from infinity to \( U = 0 \) in the extremal geometry (\( U_T = 0 \)). We can perform this subtraction by introducing a cutoff \( \Lambda \) for \( U \) in both the extremal and the near-extremal case, keeping \( R \) the same and introducing the same periodicity \( \beta \) to ensure the same asymptotic geometry at \( U = \Lambda \). (A subtlety involving a slightly different periodicity in the two cases, which appears when calculating the total world volume action \( [14, [10] \), is actually irrelevant here.)
This should be contrasted to the procedure used previously in [7, 8] where the subtraction was with a free string down to \( U = U_T \) in the near-extremal space, reducing the free energy of all isolated particles to zero at any temperature. We obtain for the free energy, mean energy and entropy of an isolated particle

\[
F = -Q \sqrt{\pi g N T}, \quad E = 0, \quad S = Q \sqrt{\pi g N}
\]

where \( Q = \sqrt{p^2 + q^2/g^2} \) is the BPS mass of the particle in quark mass units. The result \( E = 0 \) signifies that thermal vacuum polarization effects lead to no additional accumulation of energy around the particle. Interestingly, we obtain a constant value for the entropy of a particle, a result also valid at zero temperature. The above would imply that the presence of the particle, apart from a shift of the energy by a constant (which applies towards renormalizing the mass of the particle), introduces a constant degeneracy factor \( e^S \) corresponding, presumably, to the effective internal states of the particle in this phase.

With the above regularization, the free energy of our configuration is found by adding a piece for each of the string segments, subtracting the contribution for three free strings reaching down from infinity to the horizon and adding the free energy of each isolated particle. The result is:

\[
F_{QDM} = \frac{U_1}{2\pi} \int_1^\infty \left( \frac{\sqrt{z^4 - U_1^4}}{\sqrt{z^4 - 1}} - 1 \right) - \frac{U_1}{2\pi} \int_1^{1/\sqrt{x_1}} \frac{\sqrt{z^4 - U_1^4}}{\sqrt{z^4 - 1}}
\]

\[
+ \frac{1}{g} \left( \frac{U_3}{2\pi} \int_1^\infty \left( \frac{\sqrt{z^4 - U_3^4}}{\sqrt{z^4 - 1}} - 1 \right) - \frac{U_3}{2\pi} \int_1^{1/\sqrt{x_3}} \frac{\sqrt{z^4 - U_3^4}}{\sqrt{z^4 - 1}} \right)
\]

\[
+ \sqrt{1 + 1/g^2} \left( \frac{U_2}{2\pi} \int_{1/\sqrt{x_2}}^\infty \left( \frac{\sqrt{z^4 - U_2^4}}{\sqrt{z^4 - 1}} - 1 \right) - \frac{U_0}{2\pi} \right) \quad \text{(11)}
\]

We also need the distances between the particles as indicated in the figure. We find

\[
L_1 = \frac{R^2}{U_1} \sqrt{1 - \frac{U_1^4}{U_1^4} \left( \int_1^\infty \frac{1}{\sqrt{z^4 - U_1^4}} \sqrt{z^4 - 1} \right) + \int_1^{1/\sqrt{x_1}} \frac{1}{\sqrt{z^4 - U_1^4}} \sqrt{z^4 - 1} \right) \quad \text{(12)}
\]

\[
L_2 = \text{sign}(y_2) \frac{R^2}{U_2} \sqrt{1 - \frac{U_2^4}{U_2^4} \left( \int_1^{1/\sqrt{x_2}} \frac{1}{\sqrt{z^4 - U_2^4}} \sqrt{z^4 - 1} \right) \quad \text{(13)}
\]

and

\[
L_3 = \frac{R^2}{U_3} \sqrt{1 - \frac{U_3^4}{U_3^4} \left( \int_1^{1/\sqrt{x_3}} \frac{1}{\sqrt{z^4 - U_3^4}} \sqrt{z^4 - 1} \right) \quad \text{(14)}
\]
Note that we allow for the dyon to be to the right or to the left of the vertex by being careful with the sign of $y_2$ or equivalently $L_2$. In the above expressions $U_i$ should be expressed as a function of $y_i$ using the vertex conditions.

The above configuration will compete with three other possibilities depicted in figure 2. The first connects the quark and the monopole as in [J], with a $(1,1)$ string reaching down from $U_0$ to the horizon. The vertex condition is

$$y_1 = \frac{1}{g} y_3$$

and

$$\sqrt{1 - y_1^2} + \frac{1}{g} \sqrt{1 - y_3^2} = \sqrt{1 + 1/g^2}. \quad (16)$$

These conditions are solved by

$$y_1 = \frac{1}{\sqrt{1 + g^2}} \quad (17)$$

and

$$y_3 = \frac{g}{\sqrt{1 + g^2}}. \quad (18)$$

The second connects the quark and the dyon giving:

$$y_1 = \sqrt{1 + 1/g^2} y_2$$

and

$$\sqrt{1 - y_1^2} + \frac{1}{g} \sqrt{1 - y_2^2} = \sqrt{1 + 1/g^2} \sqrt{1 - y_2^2} \quad (20)$$

These conditions are solved by

$$y_1 = 1 \quad (21)$$

and

$$y_2 = \frac{g}{\sqrt{1 + g^2}} \quad (22)$$

Note that the quark string is always at its minimum at $U_0$. (This is due to the fact that the $(1,0)$ string and the vertical $(0,-1)$ string must join at right angles.) Finally we have the dyon and the monopole with

$$\sqrt{1 + 1/g^2} y_2 = \frac{1}{g} y_3 \quad (23)$$

and

$$\sqrt{1 + 1/g^2} \sqrt{1 - y_2^2} = \frac{1}{g} \sqrt{1 - y_3^2} + 1. \quad (24)$$

These conditions are solved by

$$y_2 = \frac{1}{\sqrt{1 + g^2}} \quad (25)$$
Now it is the monopole string that always is at its minimum at $U_0$.

We need the free energies for these configurations as well. At zero temperature these can be obtained as functions of the distances between the particles. The strategy is then to pick a value for $y_1$, calculate the free energy for the QDM configuration, read off the corresponding distances, plug them into the expressions for the paired configurations and compare. At non zero temperature we can not do that. Instead we have to tune $U_0$, for the paired configurations, to match the distance and then read off the free energy.

In the case of the QD configuration, which is the one we will study in detail, the free energy is given by

$$F_{QD} = \frac{U_0}{2\pi} \int_1^{\infty} \left( \frac{\sqrt{z^4 - U_T^4}}{U_0^4} - 1 \right) - \frac{U_0}{2\pi} + \frac{1}{g} \frac{U_0 - 2U_T}{2\pi}$$

and the distance between the quark and the dyon is

$$L_{QD} = \frac{R^2}{U_0} \sqrt{1 - \frac{U_T^4}{U_0^4}} \int_1^{\infty} \frac{1}{\sqrt{z^4 - U_T^4\sqrt{z^4 - 1}}}$$

$$+ \frac{R^2}{U_0} \sqrt{1 - \frac{U_T^4}{U_0^4}} \int_{1/\sqrt{z^2}}^{\infty} \frac{1}{\sqrt{z^4 - U_T^4\sqrt{z^4 - 1}}}.$$  

We have included the free energy also for the isolated monopole.
3 Discussion

We will now compare the free energy for the various configurations as the temperature is increased.

At zero temperature explicit numerical integration shows that the QDM configuration always has the lowest free energy. The physical interpretation is that there is no finite-range screening. The three particles always feel a Coulomb-like force from the others. Even if one of the particles is very far from the other two it is nevertheless advantageous to have a string connecting to the other particles rather than dropping it straight down to the horizon. The last case would have implied screening. The very long connecting string does not cost very much since it runs close and parallel to the horizon which is a null surface.

It should be clear that the above no-screening result generalizes to any configuration. The exact statement is that, at zero temperature, there can be only one type of strings reaching down to \( U = 0 \), all of the same irreducible \((p, q)\) type \((p, q\) relatively prime). Indeed, if there were more than one type of strings, we could always decrease the action by recombining two strings of different type some distance \( \Delta U \) from \( U = 0 \) into a unique string. (The gain in the \( U \)-direction is of order \( \Delta U \) while the loss in the \( x_i \)-direction is of order \( \Delta U^2 \).) Reducible strings of type \((np, nq)\) \((n > 0)\) can be considered as degenerate cases of \( n \) overlapping \((p, q)\) strings. In general, such strings will want to split in order to globally minimize the action, although we can set up situations where they do not (e.g., when they all end up on the same \((np, nq)\) particle). For a singlet configuration, the above means that no string can end on \( U = 0 \). Thus, if there are no singlet subclusters of particles, all particles are connected to each other through a string network and are, therefore, interacting.

We conclude that the total electric and magnetic charge cannot be screened at zero temperature, since the strings from non-singlet clusters cannot drop to \( U = 0 \) and will always connect to strings from other non-singlet clusters at an arbitrary distance away. The only exception is when all clusters are of the same \((p, q)\) type, in which case they form a BPS-saturated state and supersymmetry ensures that there is no net force between these clusters. Higher multipole moments, on the other hand, can in principle be screened since there is nothing that prevents the strings within a (sufficiently isolated) singlet cluster to totally close among themselves and prevent the cluster from interacting with the rest of the system.

Coming back to the QDM system, as the temperature is increased, a critical point is eventually reached where the free energy for the QDM configuration coincides with the free energy for one of the paired configurations. When the critical point is passed the system is only at a local minimum of its free energy. The system is in a metastable state, and there is a thermal ‘tunneling’ probability for a transition to the stable state of lower energy. An estimate of this transition rate can be found by evaluating the minimal action of a string configuration interpolating between the two local minima.
In the particular case described below we have positioned the monopole some distance away from the other two. We find that the QD configuration wins at a sufficiently high temperature. The monopole is screened from the rest by the thermal bath. This happens at free energy less than the one for isolated particles. If the system is heated further, the free energy of the quark-dyon pair approaches the isolated value from below. When the two are equal, there is a transition to a free system with complete screening.

All of these features are illustrated in figure 3 which shows $F_{QDM}$ and $F_{QD}$ as functions of $U_T$. We have chosen $g = 1$ and have subtracted off the free energy of the three isolated particles. In the figure $F_{QDM}$ has been drawn for $y_1 = 0.9$ and $U_0 = 1$ while $F_{QD}$ is drawn for $U_0 = 0.9$. These values correspond to $L_{QD}/R^2$ ranging from 0.9772 to 0.8896 for the QDM system and from 0.9655 to 0.8904 for the QD system as $U_T$ goes from 0 to 0.7. For the QDM system we have $L_{QM}/L_{QD}$ going from 2.6 to 2.1. A careful readjustment of the parameters to keep $L_{QM}/L_{QD}$ constant would not change the conclusions. For the particular configuration that we are considering, the free energies for the QM and DM configurations are always higher and for high enough temperature the configurations do not exist.

The limit in which some of the particles become much heavier than the rest is particularly nice. Consider, for instance, the limit where $g$ becomes very small and thus the monopole and the dyon acquire a much bigger mass than the quark. Then the AdS solution for the monopole-dyon system is the same as if the quark
did not exist, since the pull from the quark string is negligible. This MD solution is essentially the same as for a quark-antiquark system since the two masses are almost equal and the contribution of the ‘leftover’ $(-1,0)$ string is negligible. The quark string, on the other hand, has the option to either go straight to the horizon or end on the string joining the monopole and the dyon at an almost right angle at the joint. (The preferred configuration will be the one with least action.) In effect, the light quark is moving in the background field of the heavy monopole-dyon system.

To make the situation even simpler, consider that the monopole-dyon distance is bigger than the critical one for screening and therefore the AdS solution for them corresponds to straight strings down to the horizon. Their free energy is the same as if they were isolated. The quark string has the option of either dropping to the horizon or joining at a right angle either of these vertical strings (which act, now, essentially like horizons). In the latter case, the action of the quark string, by symmetry, will be half the action of a string connecting the quark and an antiquark positioned at the mirror image of the quark with respect to the heavy particle. So the free energy of that configuration will be half the free energy of a quark-antiquark pair at double the distance of the quark-heavy particle pair.

The same reasoning applies when considering a quark-monopole system at a separation $L$ with the mass of the monopole going to infinity. We get for the effective potential $V_{QM}(L,T) = F_{QM}(L,T) - F_Q(T) - F_M(T)$:

$$V_{QM}(L,T) = \frac{1}{2} V_{Q\bar{Q}}(2L,T)$$

(29)

at any temperature. This, in particular, explains Minahan’s result that at zero temperature the quark-monopole potential is $1/4$ times the quark-antiquark potential \[4\], since at zero temperature the potential scales like $1/L$. In the opposite limit of the monopole becoming much lighter than the quark we similarly get $1/4$ times the monopole-antimonopole potential.

We conclude by mentioning that the above results respect duality and scale invariance. This can be shown by rescaling $x_i$ and $\tau$ in the metric (1) and appropriately redefining $U$. Specifically, the free energy $F(Q,L;T,g)$ for any configuration of a number of static particles $Q$ at mutual distances $L$ as a function of the temperature $T$ and the coupling constant $g = g_{YM}^2/4\pi$ obeys

$$F(Q,L;T,g) = \lambda F(Q,\lambda L;T/\lambda,g) = F(\hat{Q},L;T,1/g)$$

(30)

where $\lambda$ is any (positive) constant and $\hat{Q}$ are the dual particles. In particular, if the particles involved are all of type $(np,nq)$ for fixed $p,q$ the free energy becomes

$$F(L,T;p,q,g) = \sqrt{gp^2 + q^2/g} F(L;T)$$

(31)

which is consistent with duality. (The above is not a BPS-saturated state since different $n$ can be both positive and negative.) One can also verify that in this case the total potential is a sum of two-body terms.
References

[1] J. Maldacena, “The large N limit of superconformal field theories and supergravity,” hep-th/9711002.

[2] S. Gubser, I. Klebanov and A. Polyakov, “Gauge theory correlators from non-critical string theory,” hep-th/9802109.

[3] E. Witten, “Anti de Sitter space and holography,” hep-th/9802150.

[4] A. Polyakov, “String theory and quark confinement”, hep-th/9711002.

[5] J. Maldacena, “Wilson loops in large N field theories,” hep-th/9803002.

[6] S.J. Rey, J. Yee, “Macroscopic strings as heavy quarks of large N gauge theory and Anti-de Sitter Supergravity”, hep-th/9803001.

[7] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, “Wilson loops in the large N limit at finite Temperature,” hep-th/9803137.

[8] S.-J. Rey, S. Theisen and J.-T. Yee, “Wilson-Polyakov loop at finite temperature in large N Gauge Theory and Anti-de Sitter Supergravity,” hep-th/9803135.

[9] J. Minahan, “Quark-Monopole potentials in large N Super Yang Mills,” hep-th/9803111.

[10] E. Witten, “Anti de Sitter space, thermal phase transition, and confinement in gauge theories,” hep-th/9803131.

[11] K. Sfetsos and K. Skenderis, “Microscopic derivation of the Bekenstein-Hawking entropy formula for non-extremal black holes”, hep-th/9711138, H. J. Boonstra, B. Peeters and K. Skenderis, “Branes and anti-de Sitter spacetimes,” hep-th/9801206. P. Claus, R. Kallosh, and A. van Proeyen, “M Five-brane and Superconformal (0, 2) tensor multiplet in six-dimensions”, hep-th/9711161. P. Claus, R. Kallosh, J. Kumar, P. Townsend, and A. van Proeyen, “Conformal field theory of M2, D3, M5, and D1-branes + D5-branes”, hep-th/9801206. R. Kallosh, J. Kumar and A. Rajaraman, “Special conformal symmetry of worldvolume actions”, hep-th/9712073. S. Ferrara and C. Fronsdal, “Conformal Maxwell theory as A singleton field theory on AdS(5), IIB three-branes and duality”, hep-th/9712239. N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, “Supergravity and the large N limit of theories M. Gunaydin and D. Minic, “Singletonons, Doubletons, and M Theory”, hep-th/9802047. G. T. Horowitz and H. Ooguri, “Spectrum of large N gauge theory from supergravity,” hep-th/9802116. S. Ferrara and C. Fronsdal, “Gauge fields as composite boundary excitations”, hep-th/9802126. S. Kachru and E. Silverstein, “4d Conformal field theories and strings on orbifolds,” hep-th/9802183.
M. Berkooz, “A supergravity dual of a (1, 0) field theory in six dimensions”, hep-th/9802195. V. Balasumramanian and F. Larsen, “Near horizon geometry and black holes in four dimensions”, hep-th/9802198. M. Flato and C. Fronsdal, “Interacting singletons”, hep-th/9803013. A. Lawrence, N. Nekrasov and C. Vafa, “On conformal theories in four dimensions”, hep-th/9803015. M. Bershadsky, Z. Kakushadze, and C. Vafa, “String expansion as large N expansion of gauge theories”, hep-th/9803076. S. S. Gubser, A. Hashimoto, I. R. Klebanov, and M. Krasnitz, “Scalar absorption and the breaking of the world volume conformal invariance”, hep-th/9803023. I. Ya. Aref’eva and I. V. Volovich, “On Large N Conformal theories, field theories on Anti-de Sitter space, and singletons”, hep-th/9803025. L. Castellani, A. Ceresole, R. D’Auria, S. Ferrara, P. Fré and M. Trigiante, “G/H M-branes and AdS_{p+2} geometries”, hep-th/9803039. S. Ferrara, C. Fronsdal and A. Zaffaroni, “On N = 8 supergravity on AdS_5 and N = 4 superconformal Yang-Mills theory”, hep-th/9802203. O. Aharony, Y. Oz and Z. Yin, “M theory on AdS_{p+1}S^{11−p} and superconformal field theories”, hep-th/9803051. S. Minwalla, “Particles on AdS_{4/7} and primary operators on M_{2/5} brane worldvolumes”, hep-th/9803053. R. G. Leigh and M. Rozali, “The large N limit of the (2, 0) superconformal field theory”, hep-th/9803068. M. Bershadsky, Z. Kakushadze and C. Vafa, “String expansion as large N expansion of gauge theories”, hep-th/9803076. A. Rajaraman, “Two-form fields and the gauge theory description of black holes”, hep-th/9803082. S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, “Membranes and fivebranes with lower supersymmetry and their AdS supergravity duals”, hep-th/9803109.

[12] J. Gomis, “Anti de Sitter geometry and strongly coupled gauge theories”, hep-th/9803119. G.T. Horowitz and S.F. Ross, “Possible resolution of black hole singularities from large N gauge theory”, hep-th/9803085.

[13] O. Aharony, J. Sonnenschein and S. Yankielowicz, Nucl. Phys. B474 (1996) 309, hep-th/9603009. J.H. Schwarz, Nucl. Phys. Proc. Suppl. 55B (1997) 1, hep-th/9607201. K. Dasgupta and S. Mukhi, “BPS nature of 3-string junctions,” hep-th/9711094. A. Sen, “String Network,” hep-th/9711130. S-J. Rey and J-T. Yee, “BPS dynamics of triple (p,q) string junction,” hep-th/9711202. M. Krogh and S. Lee, “String network from M-theory,” hep-th/9712050. Y. Matsuo and K. Okuyama, “BPS Condition of string junction from M theory,” hep-th/9712070. O. Bergman, “Three-pronged strings and 1/4BPS states in $N = 4$ Super-Yang-Mills,” hep-th/9712211. M.R. Gaberdiel, T. Hauer and B. Zwiebach, “Open string - string junction transitions,” hep-th/9801205. C.G. Callan, L. Thorlacius, “Worldsheet dynamics of string junctions,” hep-th/9803097.

[14] S.W. Hawking and D. Page, Commun. Math. Phys. 87 (1983) 577.