Profiling procedure for disk cutter to generate the male rotor,
screw compressors component, using the “Substitute Family Circle” - graphic method in AUTOCAD environment

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Abstract. This paper proposes a profiling method for the tool which generates the helical groove of male rotor, screw compressor component. The method is based on a complementary theorem of surfaces enveloping - "Substitute Family Circles Method". The specific theorem of family circles of substitution has been applied using AUTOCAD graphics design environment facility. The frontal view of the male rotor, screw compressor component, has been determine knowing the transverse profile of female rotor, and using this theorem of "Substitute Family Circle". The three-dimensional model of the rotor makes possible to apply the same theorem, leading to the surface of revolution enveloping the helical surface.
An application will be also presented to determine the axial profile of the disk cutter, numeric and graphics, following the proposed algorithm.

1. Introduction

The helical surface generating, as constituent surfaces of machine bodies (gears, active elements of helical pumps, screw compressors), is done using tools bounded with revolution surfaces, disk cutter type tool. A distinctive problem in these types of tools is to determine the shape of revolution surface that must be conjugated to the helical surface. The mathematical solution to this problem is known as Theorem I Olivier, or Gohman Theorem [1].

Lyukshin [2] has also developed a vectorial solution to the problem, based on decomposition of helical motion in several rotation motions - Nikolaev Theorem. This solution represents the most often form used for solving analytically the problem of profiling the primary surface of the disk cutter, due the ease expressing of the enveloping condition - the condition of coplanarity of three vectors: helix axis, tool axis disk and a vector that connects two points of those axes.

Also have been elaborated specific complementary theorems as: "Substitute Family Circles" and "Minimum Distance Method" as analytical theorems for approaching this issue [3].

Graphic design software products in AutoCAD or CATIA environments allowed approaching the problem graphically [4,5,6]. The graphic solving is very rigorous and intuitive.

2. Screw Compressor Rotors Front View Profile [7]

In the following we are proposing a graphical method developed in AutoCAD, based on the theorem of "Substitute Family Circles", for profiling the disk cutter which generates the male rotor, screw compressor component, first of all for a particular case, but with prospect to generalizing the method to the other helical surfaces (Figure 1).

The mobiles systems X₂Y₂Z₂ and X₁Y₁Z₁ will be defined, being attached to, respectively, the male rotor and female rotor, in their transverse section, while they will rotate with the global systems x₀y₀z₀ and x₁y₁z₁.

The female rotor profile is made up of a series of arcs: A₁B₁, arc, radius r₁.
\[
A_1B_1 = \begin{cases} 
X_1 = r_2 - r \cos \theta_1; \\
y_1 = -r \sin \theta_1; \\
Z_1 = 0;
\end{cases} \quad \theta_{1_{\text{min}}} = 0; \quad \theta_{1_{\text{max}}} = \left| \arccos \frac{r + r_0}{2r_2} \right|;
\]

\[
B_2C_2 \text{ arc, radius } r_0;
\]

\[
B_2C_2 = \begin{cases} 
X_2 = r_2 \cos \tau + r_0 \cos(\theta_{\text{max}} - \theta_2); \\
y_2 = -r_2 \sin \tau - r_0 \sin(\theta_{\text{max}} - \theta_2); \\
Z_2 = 0;
\end{cases} \quad \theta_{2_{\text{min}}} = 0; \quad \theta_{2_{\text{max}}} = (\pi - \theta_{1_{\text{max}}});
\]

\[
C_2D_2 \text{ arc, radius } r_2 + r_0;
\]

Knowing the absolute movements:

\[
\begin{align*}
x_2 &= \omega^T(\varphi_2) \cdot X_2; \\
x_1 &= \omega^T(-\varphi_1) \cdot X_1;
\end{align*}
\]

and the relative position of global systems,

\[
x_1 = x_2 + \begin{bmatrix} A_{12} \bigg| 0 \end{bmatrix},
\]

it will be determine the relative motion

\[
X_1 = \omega_1(-\varphi_1) \begin{bmatrix} \omega^T \cdot X_2 \bigg| A_{12} \bigg| 0 \end{bmatrix}.
\]
This motion will determine the envelopes of profiles made up of arcs \( A_3B_2, B_2C_2, C_2D_2 \), representing the female rotor profile. One circle belongs to the “Substitute Circles Family” (Figure 2) is described by equations as follow:

\[
\begin{align*}
C_i: & \quad X = -R_i \sin \varphi_i + r_i \sin \beta_i, \\
& \quad Y = R_i \cos \varphi_i - r_i \cos \beta_i,
\end{align*}
\]

\( r_i, \beta_i, \varphi_i \) variable.

The circle, center \( O_i \), fulfils the tangency conditions to \( \sum \) profile. The tangency condition between circle \( C_i \) and \( \sum \) profile means: conditions of contact points (Equation 8) and common tangent in contact point (Equation 9):

\[
\begin{align*}
X(u) &= R_i \sin \varphi_i - r_i \sin \beta_i, \\
Y(u) &= R_i \cos \varphi_i + r_i \cos \beta_i, \\
\dot{X}_u &= -r_i \cos \beta_i, \\
\dot{Y}_u &= -r_i \sin \beta_i,
\end{align*}
\]

The equations assembly (8) and (9) allows us to determinate the next condition, as follow:

\[
\dot{Y}_u \sin \varphi_i - \dot{x}(u) \cos \varphi_i = \frac{x(u) \dot{X}_u + y(u) \dot{Y}_u}{R_i},
\]

and

\[
tg \beta_i = \frac{Y_u}{X_u},
\]

meaning the enveloping condition.

The transposition in rolling motion of the substitute family circles leads to envelope shape:

\[
\begin{align*}
\xi &= -r_i \cos(\varphi_i + \varphi_2 + \beta_i) - R_i \cos \varphi_2, \\
\eta &= -r_i \sin(\varphi_i + \varphi_2 + \beta_i) + R_i \sin \varphi_2,
\end{align*}
\]

Figure 2. Circle of substitute circles family; generated profile \( \sum \); reference systems

\[\sum_{1}^{2} \]

\[\begin{array}{c}
C_1 \\
C_2
\end{array}\]
Equation 13 represents the rolling condition of the two centrodes; \( r, r_0, r, r_j, r_2 \) are known sizes.

It will propose, in the following, a graphical solution to the problem, using the “Substitute Family Circles” method.

3. The male rotor profiling

The substitute family circles method [3] assumes the next definition for the family circles: family circles belonging to a compound profile, linked of a couple of centrodes in rolling motion, is the family circles having centres on the associated centrode and in tangency with the compound profile, at the same time, as shown in Figure 3. \( A_2B_2C_2 \) is made up of some arcs with centres on centrode \( C_2 \) and the substitute family circles linked with centrode \( C_2 \) is reduced of two circles of radius \( r_1 \) and centre \( O_1 \), respectively, radius \( r_0 \) and centre \( O_0' \).

The substitute family circles for \( C_2D_2 \) is made up of circles of radius \( r_0 \) and centres on centrode \( C_2 \) - circles \( c_1, c_2 \).

In this particular case, the transposed of substitute family circles on centrode \( C_1 \), to whom the arcs radius \( r \), belong, is made up of arcs, having centres on centrode \( C_1 \), in \( O_1' \), for \( A_1B_1 \) and in \( O_2' \), for \( B_4C_1 \), as shown in Figure 4. \( O_2' \) will be determinate through condition that arc \( O_0'O_0' \) measured on centrode \( C_2 \) to be equal to arc \( O_1'O_1' \) measured on centrode \( C_1 \).

\( A_1B_1 \) belongs to circle radius „r” centre in \( O_1' \).

\( B_4C_1 \) belongs to circle radius “r_0” centre in \( O_2' \).

\( B_2 \), \( B \) belonging to female rotor profile, will be determinate using point B, which is initially superposed to point \( B_2 \), in the relative motion of the two centrodes.
The successive positions of the two centrodes, in the rolling process, and the position of point B, in relation to centrode $C_1$ attached to male rotor, as well, are represented in Figure 5.

Will be considered a large number of points, on arc $O_1^iB$ angularly equidistant, thus arc $O_1^i1$ belonging to centrode $C_1$, to be equal to arc $O_1^i1'$ belonging to centrode $C_2$, and so on.

The male rotor profile is described by the reference system, whose $X_1'$ axis represents the symmetry axis of the gap between the two successive lobes belonging to male rotor, Figure 6.

Coordinate transformation between the two systems is described by Equation 14:

$$
\begin{bmatrix}
X_1' \\
Y_1'
\end{bmatrix} = \begin{bmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
X_1 \\
Y_1
\end{bmatrix}
$$

(14)
4. The disk cutter for male rotor

Will be make the solid model of male rotor, Figure 7, reported to the system $X_1'Y_1'Z_1'$, for a rotor left screw, pitch 188.67mm.

The substitution family circles method stipulates that the contact between a helical surface, cylindrical and constant pitch and a revolution surface (the primary peripheral surface of disk cutter) is based on the specific theorem [3], as follows:

![Figure 7. Male rotor 3D solid](image)

The revolution surface (disk cutter), mutually envelope with one helical surface, cylindrical and constant pitch, is the locus made up of family circles, which have the common axis identical with the axis of the future disc cutter; those family circles are tangents to the intersection curves between the helical surface and the orthogonal planes to the disc cutter axis, in transverses planes with the future revolution surfaces axis (Figure 8).

![Figure 8. The circle parallel in plan $y_p=H$, belonging the family circles](image)
The coordinates ensemble \((H, R_H)\) defines the disk cutter axial section, mutually envelope with the helical groove belonging to the generated rotor, for different size of \(H\) parameter.

5. Profiling disk cutter generating helical groove of male rotor, using graphical procedure in AUTOCAD environment

Different sizes of \(H\) parameter and the coordinates ensemble \((H, R_H)\) determine the axial section of the future disk cutter, which is mutually envelope with the helical groove of male rotor (Figure 9). The centres of the circles radius \(R_H\) belong to the tool axis, \(Z_p\) and are in tangency with the curves \(\Sigma_H\) – planes curves of the surfaces to generate.

Consider the male rotor screw compressor component, having dimensions as follow: \(r_1=32\text{mm}; r_2=48\text{mm}; R_{e1}=50\text{mm}; R_{e2}=50\text{mm}; i=6/4; r=22\text{mm}; r_0=1.8\text{mm}\) [7], \(\alpha=45^0\), helical pitch \(P_E=188.67\text{mm}\); the axis \(\vec{A}\) of the future disk cutter will be determinate being in tangency to the counterclockwise helix.

5.1. AUTOCAD application description

It will be considere the 3D solid of the male rotor, shown in Figure 7.

The succession of commands it will be as follow:
-UCS command for rotating the coordinates system with the complement of twist angle of the helix;
-SLICE command for determination the transverse plan;
-SECTION command for definition the intersection surface by the transverse plan;
-UCS command for the new coordinates system in the transverse plan;
-EXPLODE the section defined previously;
-CIRCLE command for determination the circles radius minimal and tangent to the curve.
-Repeat the entire sequence above.

Table 1 describes the coordinates of the axial section of the disk tool which generates the helical groove of the male rotor.

Table 1. The axial section coordinates of the disk cutter

| \(R\)[mm] | 38.936 | 41.749 | 43.768 | 47.791 | 48.006 | .. | 39.993 | 39.252 | 38.560 | 38.207 |
|----------|-------|-------|-------|-------|-------|---|-------|-------|-------|-------|
| \(H\)[mm] | 3.300 | 4.300 | 5.300 | 6.300 | 8.000 | .. | 17.000 | 18.000 | 19.000 | 20.000 |
6. Conclusions

The paper consists in presentation in AUTOCAD environment, regarding a graphical algorithm to profiling the disk cutter which generates the male rotor screw compressor component.

The family circles method has been used, for both, the determination of male rotor transverse profile and for the disk cutter solid generating of helical groove.

The graphical method to express the substitute family circles allows an accurate and fast approach of the mutually envelope profiles, which are in rolling process; this method is also appropriate to shape the revolution surfaces who generate helical surfaces.

For the presented example, the solve of the problem was made up first manually, using AUTOCAD commands. In the future, it can be written a specific code AUTOLISP to allow the automation, after the parameterization of the tool and surfaces dimensions.

The graphical interpretation of points is made with o polyline edited with the option Spline. Once the increasing of number of points, the polyline is more accurate with the theoretical shape of the axial section of the tool.

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