Ring-tool profiling – graphical method in CATIA based on Generating trajectories theorem

G Frumușanu, V Teodor and N Oancea
“Dunărea de Jos” University, Manufacturing Engineering Department, Domnească Street 111, 800201 – Galați, Romania

E-mail: virgil.teodor@ugal.ro

Abstract. Machining of threads having high dimensions and multiple starts by turning is a challenging problem. An alternative possibility is to machine them by milling. The most productive milling solution is when using tools with inner active surface, namely ring tools. In the case of threads with multiple starts, the reciprocal enwrapped profile of the ring tool is considerably different to the shape of the thread axial (normal) section. In this paper, we suggest a methodology to profile the generator ring tool, based on a complementary theorem from enwrapped surfaces field. At the same time, a graphical algorithm aiming to find the ring tool profile, developed in CATIA graphical environment has been applied in the concrete case of a trapezoidal thread. The graphical profiling solution is presented in comparison to an analytical solution, in order to test the results precision. The graphical profiling method proves to be rigorous, easy to apply and highly intuitive.

1. Introduction
The helical surfaces can be generated with tools delimited either by an outer revolution surface (disc-tool, end mill) [1, 2], or by an inner revolution surface (ring-tool) [3]. The ring-tool for worms machining do have the big advantage of being much more productive, relative to the other above mentioned types of tools.

In all cases, the tool profiling methods currently used are lying on the fundamental theorems of the enwrapped surfaces (Olivier, Gohman, Nikolaev) [1, 2], or on complementary theorems (Minimum distance method, Family of substitutive circles method, Tangents method) [4].

The present paper proposes a modality for finding the ring-tool profile on the base of a new expression for the enwrapping theorems – the Plane generating trajectories method [5]. The novelty in using this method is the enabling of an easy interpretation in graphical form, under CATIA designing environment. Hereby, a dedicated algorithm to profile the ring-tool has been developed together with a graphical method in CATIA, having the same purpose. The method might be later extended for also profiling other types of tools – e.g. the tangential ring-tool.

2. Generating helical surfaces with a ring-tool
The helical surfaces generation by using a ring-tool is a highly productive process, currently used for machining long screws (e.g. the driving screws from machine tools construction) in large series.

The tools have relative high diameters, their active surface being an inner revolution surface. The ring-tool axis is disjoint to the helical surface axis, being placed on a position normal to the tangent at
the helix corresponding to exterior diameter of the helical surface.

The generating process kinematics involves the following motions:

- **I** – worked piece rotation around its own axis, \( \vec{V} \);
- **II** – worked piece translation along \( \vec{V} \) axis, in correlation with the rotation motion, the ensemble formed by the two motions giving a helical motion having the same axis and parameter as the helicoid to generate;
- **III** – cutting motion, executed by the ring-tool.

The formal expression of the above presented kinematics can be obtained after defining the following reference systems (see figure 1):

- \( XYZ \), meaning a system associated to the helical surface (the worked piece) and having \( Z \) axis overlaid to surface axis, and
- \( X_iY_iZ_i \) – system attached to the ring-tool axis, having \( Z_i \) and \( X_i \) axis overlaid to ring-tool axis \( \vec{A} \), respectively to \( X \) axis of the surface system.

The distance between \( Z \) and \( Z_i \) axis, measured along \( X \) (\( X_i \)), is denoted by \( a \) and represents a constructive parameter. The angle \( \alpha \) between \( Z \) and \( Z_i \) axis can be found by imposing to \( \vec{A} \) axis the condition of being normal to the helix of \( D_{ex} \) diameter, from the helical surface to be generated (see figure 2):

\[
\alpha = \arctan\left(\frac{2p}{D_{ex}}\right). \quad (1)
\]

The problem needing to be solved is that in which, by starting from the helical surface shape, one intends to find the profile of its generator ring-tool.

**3. Ring-tool profiling algorithm**

**3.1. Plane generating trajectories method**

The method, if applied in the addressed problem, starts from the equations of the helical surface to be generated, having the general form:

\[
\Sigma \left| X = X(u, \varphi); Y = Y(u, \varphi); Z = Z(u, \varphi) \right. \quad (2)
\]
with \( u \) and \( \varphi \) – independent variables.

The equations of \( \Sigma \) surface are then transferred from \( XYZ \) into \( X_1Y_1Z_1 \) system through the transform:

\[
X_1 = \alpha \left[ X - \delta \right],
\]

with

\[
\alpha = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha \\
0 & -\sin \alpha & \cos \alpha
\end{pmatrix},
\]

\[
\delta = \begin{pmatrix}
a \\
0 \\
0
\end{pmatrix},
\]

\( a \) being the above mentioned constructive parameter.

In principle, the equations of the helical surface (2), after applying the transform (3), become:

\[
\Sigma \big| X_1 = X_1(u, \varphi); Y_1 = Y_1(u, \varphi); Z_1 = Z_1(u, \varphi).
\]

By sectioning \( \Sigma \) surface with \( Z_1 = H \) plane (with \( H \) – arbitrary variable), \( C_{\Sigma_n} \) curve results, having, in principle, the equations:

\[
C_{\Sigma_n} \big| X_1 = X_1(u); Y_1 = Y_1(u); Z_1 = H,
\]

see also figure 3.

3.2. The family of plane generating trajectories

It is defined as the family described by the curve belonging to the helical surface and placed in a plane section, normal to the future ring-tool axis, which has a rotation motion of \( \nu \) angular parameter around ring-tool axis [6]:

\[
X_1 = \omega_3^T \begin{pmatrix}
X_1(u) \\
Y_1(u) \\
H
\end{pmatrix},
\]

where \( \omega_3^T \) is the matrix of the co-ordinates transform corresponding to rotation around \( Z_1 \) axis. After developing, we obtain:

\[
\begin{pmatrix}
X_1 = X_1(u) \cdot \cos \nu - Y_1(u) \cdot \sin \nu; \\
Y_1 = X_1(u) \cdot \sin \nu + Y_1(u) \cdot \cos \nu; \\
Z_1 = H.
\end{pmatrix}
\]

3.3. The enveloping condition

The enveloping condition can be deducted by starting from the specific theorem of the Family of generating trajectories method. According to this, the inner revolution surface reciprocal enveloped to the family of generating trajectories is the locus of that points owning to \( C_{\Sigma_n} \) curves, from different plane sections \( H \), for which the normal to the curve does intersect the ring-tool axis.

The normal in the current point of \( C_{\Sigma_n} \) curve (6) is then defined and its direction parameters are calculated:
The equations of the normal can be now written as:

\[
\begin{align*}
\vec{N}_{C_{x,y}} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \vec{X}_{i} & \vec{Y}_{i} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{Y}_{i} \cdot \hat{i} - \vec{X}_{i} \cdot \hat{j}. \\
X_{i} &= X_{i}(u) + \lambda \cdot \vec{Y}_{i} \\
Y_{i} &= Y_{i}(u) - \lambda \cdot \vec{X}_{i} \\
Z_{i} &= H,
\end{align*}
\]

(9)

The equations of the normal can be now written as:

\[
\begin{align*}
\vec{N}_{C_{x,y}} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \vec{X}_{i} & \vec{Y}_{i} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{Y}_{i} \cdot \hat{i} - \vec{X}_{i} \cdot \hat{j}. \\
X_{i} &= X_{i}(u) + \lambda \cdot \vec{Y}_{i} \\
Y_{i} &= Y_{i}(u) - \lambda \cdot \vec{X}_{i} \\
Z_{i} &= H,
\end{align*}
\]

(10)

with \(\lambda\) – scalar variable parameter.

The equation of normals family (9) result from (10) and (7):

\[
\begin{pmatrix}
X_{i} \\
Y_{i} \\
Z_{i}
\end{pmatrix} = \begin{pmatrix}
\cos \nu & -\sin \nu & 0 \\
\sin \nu & \cos \nu & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
X_{i}(u) + \lambda \cdot \vec{Y}_{i} \\
Y_{i}(u) - \lambda \cdot \vec{X}_{i} \\
H
\end{pmatrix},
\]

(11)

or, after developing:

\[
\begin{align*}
X_{i} &= \left[ X_{i}(u) + \lambda \cdot \vec{Y}_{i} \right] \cos \nu - \left[ Y_{i}(u) - \lambda \cdot \vec{X}_{i} \right] \sin \nu; \\
Y_{i} &= \left[ X_{i}(u) + \lambda \cdot \vec{Y}_{i} \right] \sin \nu + \left[ Y_{i}(u) - \lambda \cdot \vec{X}_{i} \right] \cos \nu; \\
Z_{i} &= H,
\end{align*}
\]

(12)

with \(v\) being the family parameter.

Into \(X, Y, Z\) reference system, associated to ring-tool, \(A\) axis has the equations:

\[
A \begin{pmatrix}
X_{1} \\
Y_{1} \\
Z_{1}
\end{pmatrix} = 0; Y_{1} = 0; Z_{1} = H,
\]

(13)

with \(H\) variable. Hereby, the condition meaning that the family normals intersect the ring-tool axis is:

\[
\begin{align*}
X_{1} &= \left[ X_{1}(u) + \lambda \cdot \vec{Y}_{i} \right] \cos \nu - \left[ Y_{1}(u) - \lambda \cdot \vec{X}_{i} \right] \sin \nu = 0; \\
Y_{1} &= \left[ X_{1}(u) + \lambda \cdot \vec{Y}_{i} \right] \sin \nu + \left[ Y_{1}(u) - \lambda \cdot \vec{X}_{i} \right] \cos \nu = 0.
\end{align*}
\]

(14)

By eliminating the parameter \(\lambda\) from equations system (14), the specific enveloping condition is obtained as:

\[
X_{1}(u) \cdot \vec{X}_{i} + Y_{1}(u) \cdot \vec{Y}_{i} = 0.
\]

(15)

### 3.4. The profile of the ring-tool axial section

The equations ensemble (8) giving the plain trajectories family together with the condition (15), when \(H\) is arbitrarily varying, represent the enveloping surface, namely the inner revolution surface of the future ring-tool, which has, in principle, the equations:

\[
S \big| \begin{align*}
X_{1} &= X_{1}(v, H); Y_{1} = Y_{1}(v, H); Z_{1} = H.
\end{align*}
\]

(16)

The ring-tool axial section results in the form (see also figure 4):

\[
S_{R} \left[ R = \sqrt{\left[ X_{1}(v, H) \right]^{2} + \left[ Y_{1}(v, H) \right]^{2}} \right],
\]

(17)
4. Numerical application

A numerical application of the profiling algorithm from above is further presented. It addresses the profiling of the ring-tool for generating a trapezoidal thread (figure 5), such like the ones used at driving screws, into machine tools kinematical chains.

The screw axial profile is referred to a system XYZ, having its Z axis overlaid to the axis of the thread to be generated, while X is the symmetry axis of the axial profile gap, which is formed by three straight segments:

\[
\alpha_0 - u \cdot \cos \alpha_0; \ Z = b_v / 2 + \left( u_{1_{\text{max}}} - u_{1}\right) \sin \alpha_0, \tag{18}
\]

with \( u_1 \) – variable, \( u_{1_{\text{min}}} = 0, \ u_{1_{\text{max}}} = (R_e - R_i) / \cos \alpha_0, \)

\[
\alpha_0 - u \cdot \cos \alpha_0; \ Z = b_v / 2 - u_{2}, \tag{19}
\]

with \( u_2 \) – variable, \( u_{2_{\text{min}}} = -b_v/2, \ u_{2_{\text{max}}} = b_v/2, \)

\[
\alpha_0 - u \cdot \cos \alpha_0; \ Z = -b_v / 2 + \left( u_{3_{\text{max}}} - u_{3}\right) \sin \alpha_0, \tag{20}
\]

with \( u_3 \) – variable, \( u_{3_{\text{min}}} = 0, \ u_{3_{\text{max}}} = (R_e - R_i) / \cos \alpha_0. \)

The composite helical surface – right hand worm, of \( p = p_{ax}/2\pi \) helical parameter, is generated through the helical motion of \( \vec{V} \) axis and \( \varphi \) angular parameter:

\[
X = -(R_v - u_1 \cdot \cos \alpha_0) \sin \varphi; \]

\[
\Sigma_{AB} Y = (R_v - u_1 \cdot \cos \alpha_0) \cos \varphi; \tag{21}
\]

\[
Z = b_v / 2 + \left( u_{1_{\text{max}}} - u_{1}\right) \sin \alpha_0 + p \cdot \varphi;
\]
\[
\begin{align*}
X &= -R_c \cdot \sin \varphi; \\
Y &= R_c \cdot \cos \varphi; \\
Z &= b_v / 2 - u_2 + p \cdot \varphi;
\end{align*}
\] (22)

\[
\begin{align*}
X &= -(R_c - u_3 \cdot \cos \alpha_0) \sin \varphi; \\
Y &= (R_c - u_3 \cdot \cos \alpha_0) \cos \varphi; \\
Z &= -b_v / 2 + (u_3 - u_{3\max}) \sin \alpha_0 + p \cdot \varphi.
\end{align*}
\] (23)

After applying the transform (3), the helical surface equations are referred to \(X_1Y_1Z_1\) (see figure 1):

\[
\begin{align*}
X_1 &= -(R_c - u_1 \cdot \cos \alpha_0) \sin \varphi \cdot \cos \alpha - [b_v / 2 + (u_{1\max} - u_1) \sin \alpha_0 + p \cdot \varphi] \sin \alpha; \\
Y_1 &= (R_c - u_1 \cdot \cos \alpha_0) \cos \varphi + a; \\
Z_1 &= -(R_c - u_1 \cdot \cos \alpha_0) \sin \varphi \cdot \sin \alpha + [b_v / 2 + (u_{1\max} - u_1) \sin \alpha_0 + p \cdot \varphi] \cos \alpha;
\end{align*}
\] (24)

\[
\begin{align*}
X_1 &= -(R_c - u_3 \cdot \cos \alpha_0) \sin \varphi \cdot \cos \alpha - [-b_v / 2 + (u_3 - u_{3\max}) \sin \alpha_0 + p \cdot \varphi] \sin \alpha; \\
Y_1 &= (R_c - u_3 \cdot \cos \alpha_0) \cos \varphi + a; \\
Z_1 &= -(R_c - u_3 \cdot \cos \alpha_0) \sin \varphi \cdot \sin \alpha + [-b_v / 2 + (u_3 - u_{3\max}) \sin \alpha_0 + p \cdot \varphi] \cos \alpha.
\end{align*}
\] (25)

Regarding the condition (15) specific form for the addressed application, the partial derivatives expressions for each of the three profile segments result from relations (24) – (26):

\[
\begin{align*}
\frac{\partial}{\partial B} X_{1,i} &= \cos \alpha_0 \cdot \sin \varphi \cdot \cos \alpha + \sin \alpha_0 \cdot \sin \alpha; \\
\frac{\partial}{\partial B} Y_{1,i} &= -\cos \alpha_0 \cdot \cos \varphi;
\end{align*}
\] (27)

\[
\begin{align*}
\frac{\partial}{\partial C} X_{1,i} &= \cos \alpha_0 \cdot \sin \varphi \cdot \cos \alpha - \sin \alpha_0 \cdot \sin \alpha; \\
\frac{\partial}{\partial C} Y_{1,i} &= -\cos \alpha_0 \cdot \cos \varphi.
\end{align*}
\] (28)

The equations (24) – (26) together with condition (15) in its specific form, when \(H\) is varying, give the characteristic curves at the contact between the composite helical surface and the ring-tool one.

We further present a numerical example in the concrete case when the profile to be generated is defined by the following values of its geometric parameters: \(R_c = 45\) mm, \(R_i = 40\) mm, \(p_{av} = 16\) mm, \(a_v = 4\) mm, \(b_v = 4\) mm, \(\alpha = 20^{\circ}\), \(a = 225\) mm, thread number of starts \(k = 1\).

The characteristic curve has been found through 11 points, for the tool flank that generates CD flank of the trapezoidal thread. These points' co-ordinates have been determined with the help of a MatLab application, dedicated to this purpose and developed on the base of the suggested profiling algorithm, and they are presented in table 1. Table 2 shows the co-ordinates of the points that define the axial section of the ring-tool, corresponding to the same flank, in the form (17). In figure 6, one can see the results of the addressed numerical application, in graphical form.

Note In the points \(B\) and \(C\), onto the generated profile there are discontinuities that should be separately approached for tool profiling.
Table 1. The characteristic curve | Table 2. The axial section

| Point no. | X₁ [mm] | Y₁ [mm] | Z₁ [mm] | R [mm] | H [mm] |
|-----------|---------|---------|---------|-------|-------|
| 1         | 1.3417  | 269.9745| -3.80   | 269.9778| 3.80  |
| 2         | 1.2200  | 269.4775| -3.62   | 269.4802| 3.62  |
| 3         | 1.0998  | 268.9806| -3.44   | 268.9829| 3.44  |
| 4         | 0.9795  | 268.4839| -3.26   | 268.4857| 3.26  |
| 5         | 0.8592  | 267.9873| -3.08   | 267.9887| 3.08  |
| 6         | 0.7405  | 267.4909| -2.90   | 267.4919| 2.90  |
| 7         | 0.6219  | 266.9946| -2.72   | 266.9953| 2.72  |
| 8         | 0.5035  | 266.4985| -2.54   | 266.4989| 2.54  |
| 9         | 0.3861  | 266.0025| -2.36   | 266.0028| 2.36  |
| 10        | 0.2691  | 265.5067| -2.18   | 265.5068| 2.18  |
| 11        | 0.1525  | 265.0111| -2.00   | 265.0112| 2.00  |

5. Graphical method in CATIA

A graphical algorithm has also been developed in order to solve the same profiling problem, in the graphical design environment CATIA.

The algorithm starts by creating the two reference systems mentioned in section 2. The helical surface’s generatrix is then drawn into \( XOY \) plane. The helical surface is virtually generated by applying \textit{SWEEP} command, after defining as director curve the helix of \( \vec{V} \) axis and \( p \) helical parameter and setting as translation direction the axis \( \vec{V} \). Into the same reference system, a line tangent to the helix is drawn, following to be used for defining the ring-tool axis.

Into the tool reference system – \( X₁ Y₁ Z₁ \), a plane is generated by the method “\textit{through point and line}”, the definition point and line being the origin of tool reference system and the above mentioned tangent-line, respectively. The axis \( \vec{A} \) is generated as a line normal to this plane, and which passes by the origin of \( X₁ Y₁ Z₁ \) system. For this line, we set the option “\textit{length type: infinite}”. The other two axis of the reference system have been defined as lines perpendicular to a curve, namely \( \vec{A} \), also passing by the origin of \( X₁ Y₁ Z₁ \) system, and having as support their corresponding planes (\( Z₁X₁ \) plane for \( X₁ \) axis and \( Z₁Y₁ \) plane for \( Y₁ \) axis).

A plane parallel with the plane \( X₁Y₁ \) is generated at offset at \( H \) value. It is obtained the intersection curve between the helical surface and this plane (“\textit{INTERSECTION}” command). The obtained curve is an in-plane curve and belongs to the helical surface.

On this curve it is generated a point, using the option “\textit{on curve}” and with “\textit{distance to reference}” on type “\textit{ratio of curve length}”. Initially, the value of this ratio is arbitrary (figure 7).

A point is generated as intersection between the tool’s axis and the plane \( Z₁ = H \). A line is drawn by this point and the one from the intersection curve (“\textit{LINE}” command, type “\textit{point-point}”, figure 8).

Consequently, from the point belonging to the intersection curve it is generated a line tangent to the curve and the angle between this line and the previously drawn line is measured.

Changing the value of “\textit{ratio of curve length}” parameter, the angle’s value is also changed. Tracking the angle’s value is possible to determine the ratio for which the angle is 90°. In this position, the tangent is perpendicular to the line which link a point from the curve with a point from the tool’s axis. This perpendicularly means that the line is normal to the curve and, from construction, passes through a point from the tool’s axis. So, the enveloping condition is accomplished and the point from the intersection curve belongs to the characteristic curve. Resuming this process for various values of \( H \) a set of points from the characteristic curve are determined.
The results such obtained are very close to the ones analytically determined. For example, instead of the point having the co-ordinates (0.6219; 266.9946), from $Z_1 = -2.72$ plane, analytically found, it was determined, by applying the graphical method, the point (0.6230; 266.9960), from the same plane. The distance between the two points is about $1.78 \cdot 10^{-3}$ mm, meaning a very good concordance between the results delivered by the two methods.

6. Conclusion

In this paper, an alternative method for profiling the ring-tools for machining by milling the helical surfaces is presented. The method lies on a complementary theorem of the enwrapped surfaces – the Plane generating trajectories theorem, which has been implemented in CATIA graphical design environment. Method application is sampled in a concrete case – profiling of a ring tool used for machining a screw trapezoidal thread.

In order to validate the method application, the same profiling problem has been solved analytically too. Between the results separately obtained in the two cases there is a very good concordance. Beyond its remarkable precision, the newly developed method proves to be easy to apply and has a pregnant intuitive character.

References

[1] Litvin F L 1984 Theory of gearing (Washington: NASA Scientific and Technical Information Division)
[2] Radzevich S P 2008 Kinematics geometry of surface machining (London: CRC Press)
[3] Berbinschi S, Teodor V and Oancea N 2012 A study on helical surface generated by the primary peripheral surfaces of ring tool Int. J. Adv. Manu. Tech. 61 p 15-24
[4] Oancea N 2004 Surface generation by enwrapping, vol. II. Complementary theorems (Galaţi: „Dunărea de Jos” Publishing House)
[5] Teodor V 2010 Contributions to the elaboration of a method for profiling tools – Tools which generate by enwrapping (Beau Bassin: Lambert Academic Publishing)
[6] Baroiu N, Teodor V and Oancea N 2015 A new form of the in-plane trajectories theorem. Generation with rotary cutters Bulletin of the Polytechnic Institute of Iaşi 3 p 29-36

Acknowledgement

This work was supported by a grant of the Romanian National Authority for Scientific Research and Innovation, CNCS – UEFISCDI, project number PN-II-RU-TE-2014-4-0031.