A proof-of-principle experiment of eliminating photon-loss errors in cluster states

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Abstract. Quantum computation can be efficiently achieved by adopting the cluster-state model in principle, that is, first preparing multi-qubit cluster states and then performing single-qubit measurements. One problem encountered in this model is that qubits are easily lost, leaving the remaining qubits in a mixed state. Here, we experimentally demonstrate a simple way to eliminate such errors in one-dimensional cluster states using the polarizing beam splitter (PBS) gate, and we also perform the indirect-Z measurement to overcome qubit loss. The methods of quantum state tomography and entanglement witness are exploited to verify the performance of the schemes, and the experimental results indicate good agreement with the theoretical predictions.
1. Introduction

Since Knill, Laflamme and Milburn (KLM) revealed that measurement-induced nonlinearity suffices for efficient quantum computation [1], considerable attention has been paid to quantum computation using linear optical elements and single photons. However, the KLM scheme requires tens of thousands of optical elements to achieve a single entangling gate with high probability. Alternatively, the cluster-state model proposed by Raussendorf and Briegel [2] provides another way to universal quantum computation. This model also called ‘one-way quantum computation’ has attracted much attention because of its favorable properties such as requiring only single-qubit measurements on the cluster state [3, 4]. Remarkably, this helps to reduce the physical sources by several orders of magnitude [5, 6], and experimental demonstration in this direction has also been reported recently [7]–[9].

Many efforts have been devoted to the construction of cluster states. Type-I and Type-II fusion operations [6] are proposed to construct cluster states efficiently with maximally entangled photons, and experimental demonstration can be found in [8]. A simple C-phase gate is proposed and experimentally demonstrated by Kiesel et al [10, 11]. In addition, an entanglement operation (EO) gate is another efficient way of constructing cluster states in a sense that it does not waste the qubits [12, 13]. The EO gate is described in the context of matter qubits, but it can also be used in the context of optical qubits. As shown by Bodiya and Duan [14], polarizing beam splitter (PBS) can be used to generate a scalable tree-shaped cluster state and the scheme is applied in a recent experiment to construct a six-photon cluster state [9].

In the model of cluster states, one problem is that qubits may be easily lost owing to imperfect photon sources, detectors and noise outside the quantum system. It is very necessary to deal with such kinds of qubit-loss errors, because the density matrix of the quantum system will be left in a mixed state if some qubits are lost. Here, the problem we would like to solve is ‘Given a cluster state, how can we deal with qubit-loss errors?’ Recently, there have been several proposals of quantum loss-tolerant codes about tolerating qubit-loss errors, including ones based on cluster states [15] and parity states [16]. In our paper, we briefly review the methods of eliminating photon-loss errors by performing indirect-Z measurements [15] and using a PBS gate, where a different approach from Bodiya and Duan’s [14] is taken. Here, the meaning of deleting lost qubits and keeping the coherence refers to ‘eliminating photon-loss errors’, and we demonstrate a proof-of-principle experiment of how these methods can be used. The rest of the paper is organized as follows: we begin with section 2 showing the construction of the PBS gate, which can efficiently eliminate the lost qubit in the one-dimensional cluster state. In the same section, we review the novel scheme introduced by...
Varnava et al [15] to fault-tolerantly cope with qubit loss. In section 3, we demonstrate a proof-of-principle experiment to demonstrate the key operations of these methods.

2. The PBS gate operation and the indirect-Z measurement

First, we review the PBS gate [14] in the following paragraph. This operation, the same as parity check [17, 18], is implemented by mixing two photons at a PBS and accepting the case when each of the detectors receives one and only one photon. As shown in figure 1(a), the basic resource is two-photon Bell states which are relatively easy to obtain. This gate is conditioned on detecting one and only one photon in each output, a technique called post-selection [19], so it is naturally non-deterministic with a success probability of 50% and it has destructive characters. However, it can be useful for scalable quantum computation when exploiting ‘Divide and Conquer’ [14] or ‘percolation’ schemes [20]. Furthermore, the PBS gate can be easily converted to the well-known Type-I and Type-II fusion gates which are important for efficient one-way quantum computation [6, 21].

In this paragraph, we present a description of the PBS gate similar to Benjamin’s [13]. The initial state is prepared as

\[
|H\rangle + |V\rangle \sigma^Z_L \left( |H\rangle + |V\rangle \sigma^Z_R \right) |X\rangle,
\]

where \(H, V\) denotes the horizontal and vertical polarizations, \(|X\rangle\) represents the state of the entire cluster state minus the two marked nodes as shown in figure 1(b). The operator \(\sigma^Z_L \equiv \Pi_1 \sigma^Z_1 \sigma^Z_2 \cdots \sigma^Z_j\) is the product of \(\sigma^Z\) operators applied to each of those qubits \(1 \cdots j\) connected to one of the qubits which will go through the PBS. The \(\sigma^Z_R\) is defined the product of \(\sigma^Z\) operators applied to each of those qubits \(1 \cdots j\) connected to the other. For a PBS, the photons from the two input modes will go to different sides if and only if both photons have the same polarization, either \(HH\) or \(VV\). If we detect one photon in each side, the PBS will be
described by the projector \(|HH\rangle\langle HH| + |VV\rangle\langle VV|\). After the PBS and the post-selection, the initial state will become
\[
(|HH\rangle + |VV\rangle \sigma_z^L \sigma_z^R) |X\rangle.
\] (2)

Then, we make a Hardmard transformation of one of the qubits. The final state will be
\[
(|+H\rangle + |-V\rangle \sigma_z^L \sigma_z^R) |X\rangle,
\] (3)
where \(|\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)\). In this state, a ‘node qubit’ connected to a ‘leaf qubit’ inherits all the bonds of the previous qubits as shown in figure 1(b). Note that, here, we consider the PBS gate as a non-destructive gate. The PBS gate above is also described in the language of stabilizer operators of cluster states by Bodiya and Duan [14].

An interesting character of the PBS gate is that it can be used to eliminate photon-loss errors naturally. As shown in the formula (3), when two nodes join together, the remaining node will inherit all the previous bonds. If the two nodes bond to a common qubit and the common qubit does not bond to other qubits, the gate will result in the state
\[
(|+H\rangle + |-V\rangle \sigma_z^L' \sigma_z^R') |X - 1\rangle |+\rangle_c,
\] (4)
where \(|+\rangle_c\) is the common qubit, \(|X - 1\rangle\) is the state \(|X\rangle\) minus the common qubit and \(\sigma_z^L' = \Pi_i \sigma_i^Z \cdot \cdot \cdot \sigma_{(j-1)L}^Z\), \(\sigma_z^R' = \Pi_i \sigma_i^Z \cdot \cdot \cdot \sigma_{(j-1)R}^Z\), so the common qubit is not connected to the cluster state any more (see figure 2(a)). This was first introduced by Benjamin [13] using the EO gate.

Next, we review the indirect-Z measurement in the following paragraph. An \(n\)-qubit cluster state can be expressed in terms of a set of \(n\) stabilizers, one for each vertex in the graph. The stabilizers have the form
\[
\hat{S}_i = \hat{X}_i \bigotimes_{j \in v(i)} \hat{Z}_j,
\] (5)
where \(i\) denotes a qubit, \(v(i)\) denotes the neighborhood of \(i\), and \(\hat{X}_i\) and \(\hat{Z}_j\) denote the usual Pauli bit-flip and phase-flip operators, respectively, acting on qubits \(i\) and \(j\). As shown in figure 2(b), utilizing the property that cluster states are eigenstates of the stabilizers, Varnava et al [15] introduce the indirect-Z measurement to overcome qubit loss. More details about indirect-Z measurements can be found in [15].

### 3. The experiment of repairing the cluster state

#### 3.1. Preparation of the state

In this section, we demonstrate a proof-of-principle experiment of how the methods in section 2 can be used. First, we prepare a four-photon cluster state with which we demonstrate the indirect-Z method. Applying a PBS gate to construct a four-qubit cluster state with two pairs of Bell states is the most simple but fundamental case as depicted in figure 1(b). A schematic drawing of our experimental setup is shown in figure 3. Two pairs of entangled photons in mode 1–2 and mode 3–4 are produced as the primary source by passing an ultraviolet laser pulse through two \(\beta\)-barium borate (BBO) crystals. The UV laser with a central wavelength of 394 nm has a pulse duration of 120 fs, a repetition rate of 76 MHz, and an average pump
Figure 2. The repair of the one-dimensional cluster state. Qubit 3 is assumed to be lost here. (a) Qubits 2 and 4 are passed through a PBS gate to eliminate qubit 3. (b) The indirect-Z measurement is performed to eliminate qubit 3.

Figure 3. Experimental setup of the generating four-photon cluster state as shown in figure 1(b). The sources are two pairs of maximally entangled photons produced by type-II spontaneous parametric down-conversion. The HWP before the PBS1 is used to transform the photon from H/V basis to +/− basis. Quarter-wave plates (QWPs), HWPs and polarizers before detectors are used for necessary polarization analysis. In the experiment, we managed to obtain an average two-fold coincidence of 25000 s−1.
power of 540 mW. After proper birefringence compensation and local unitary transformation with HWPs and nonlinear crystals, the initial state is presented as

\[ |\psi_0\rangle = |C_{12}\rangle \otimes |C_{34}\rangle = \frac{1}{2}(|+\rangle |H\rangle + |\rangle |V\rangle)_{12}(|+\rangle |H\rangle + |\rangle |V\rangle)_{34}. \]  

(6)

We then superpose the photon-2 and photon-3 at the PBS1. Their path lengths are adjusted so that they arrive simultaneously. To achieve good spatial and temporal overlaps, the outputs are spectrally filtered (\(\Delta U = 3.2\) nm) and monitored by fiber-coupled single-photon detectors. The filtering process stretches the coherence time to about 648 fs, substantially larger than the pump pulse duration [22]. This effectively erases any possibility to distinguish the two photons and subsequently leads to interference. After the post-selection, the final state will be

\[ |C_{1234}\rangle = \frac{1}{\sqrt{2}}(|+\rangle |+\rangle |H\rangle |+\rangle |V\rangle + |\rangle |\rangle |H\rangle |\rangle |V\rangle), \]  

(7)

which is the state in figure 1(b). It is equivalent to a four-photon Greenberger–Horne–Zeilinger (GHZ) state up to single-qubit unitary transformations.

We exploit the method of entanglement witness to prove its multipartite entanglement [23]. Entanglement witness is an observable which has a positive expectation value on all nine measurement settings are required. Figure 2(b) depicts the measurement results, yielding \(\text{Tr}(W_{C4}\rho_{\text{exp}}) = -0.24 \pm 0.01\), which is negative by 24 standard derivations and thus proves the presence of genuine multipartite entanglement. For \(|C_{1234}\rangle\), we use witness [24, 25]

\[ W_{C4} = 1/2 - |C_{1234}\rangle \langle C_{1234}|. \]  

(8)

\(|C_{1234}\rangle \langle C_{1234}|\) is decomposed into locally measurable observables as [26, 27]

\[ |C_{1234}\rangle \langle C_{1234}| = \frac{1}{2}(|+\rangle |+\rangle |H\rangle |+\rangle |H\rangle + |\rangle |\rangle |H\rangle |\rangle |H\rangle \rangle + \frac{1}{16}(ZZXX - YYYY + ZYXY + ZYYZ + YZYZ + YYZX - YXYZ)\],  

(9)

where \(X, Y\) and \(Z\) are short notations for the Pauli matrices. To implement this witness, nine measurement settings are required. Figure 4(a) depicts the measurement results, yielding \(\text{Tr}(W_{C4}\rho_{\text{exp}}) = -0.24 \pm 0.01\), which is negative by 24 standard derivations and thus proves the presence of genuine four-partite entanglement. From the expectation values of the witness, we can directly calculate the obtained fidelity as

\[ \langle C_{1234}| \rho_{\text{exp}} |C_{1234}\rangle = 0.74 \pm 0.01. \]  

(10)

After photon-1 is projected to \(|H\rangle\), the state remains a three-qubit linear cluster state

\[ |C_{234}\rangle = \frac{1}{\sqrt{2}}(|+\rangle |H\rangle |+\rangle |V\rangle + |\rangle |\rangle |V\rangle |\rangle |V\rangle), \]  

(11)

with which we demonstrate the PBS gate method against qubit-loss errors. We decompose its witness as

\[ W_{C3} = \frac{1}{2} - |C_{234}\rangle \langle C_{234}| \]  

\[ = \frac{1}{2} - \frac{1}{2}(|+\rangle |H\rangle \langle +\rangle |H\rangle + |\rangle |\rangle |H\rangle |\rangle |H\rangle \rangle + \frac{1}{4}(YYXY - YYYZ - ZZXY - ZYYX). \]  

(12)

From figure 5(b), the expectation value of \(W_{C3}\) results to be \(\text{Tr}(W_{C3}\rho_{\text{exp}}) = -0.31 \pm 0.01\) and the fidelity of the state is \(0.81 \pm 0.01\).
Figure 4. Experimental results of the state witness. (a) The expectation values of nine measurement settings of the witness in equation (9), where $M_1$–$M_9$ are separately $| + + H + \rangle \langle + + H + | + - V - \rangle \langle - - V - |$, $ZZXZ$, $ZZY$, $ZY$, $ZY$, $YYXZ$, $YYY$, $ZY$. (b) The expectation values of five measurement settings of the witness in equation (10), where $M_1$–$M_5$ are separately $| + H + \rangle \langle + H + | + - V - \rangle \langle - - V - |$, $ZY$, $ZZY$, $YY$, $XY$. Each of them is expected to be $+1$ or $-1$, and the error bars represent one standard deviation, deduced from propagated Poissonian statistics of the raw detection events.

3.2. Eliminating photon-loss errors in cluster states

After the preparation of the basic state, we next demonstrate experimentally the key operations of eliminating photon-loss errors. Because the post-selection technique is exploited in the preparation of multi-photon states in our experiment, an equivalence is assumed here, that is, we simulate the loss of a photon by detecting the photon without knowing its polarization information, which tells us that the photon is lost. Experimentally, this is done by placing no polarizer or PBS in front of the detector, and it does not prevent an in-principle verification of the schemes to eliminate photon-loss errors.

Suppose photon 3 of the cluster state $|C_{234}'\rangle$ is lost, which is experimentally performed by placing no polarizer in front of the detector 3, the damaged state remains to be a mixed state

$$|D_{24} \rangle \langle D_{24}'| = \frac{1}{2} (|+\rangle \langle +| + |+\rangle \langle +| - |+\rangle \langle -| - |+\rangle \langle -|). \quad (13)$$

As shown in figure 5(a), the state tomography is performed to depict the density matrix of $|D_{24} \rangle$. In the output modes, HWPs, QWPs and freely-rotatable polarizers are used to make projections onto the polarizations $\{H, V, +, R\}$. We perform each of these 16 correlation measurements for 105 s using all combinations of $\{H, V, +, R\}$. A maximum of 664 two-fold coincidence counts in 105 s are measured in the case of the setting ++. Instead of a direct linear combination of measurement results, which can lead to unphysical density matrices, we use a maximum-likelihood reconstruction technique. From figure 5(a), we can see that the state is very consistent with the highly mixed state.

If the lost-qubit is connected to a leaf qubit in the preparation step, we will be able to delete it with the method of indirect-Z measurement. Concretely, in the experiment, if qubit 3 has been connected to qubit 1 before it is lost, just as depicted in figure 1(b), we can eliminate qubit 3 under an X-measurement of qubit 1. The experimental result is shown in figure 5(b).
Figure 5. The experimental output density matrices. Each density matrix is represented as two bar charts, with the right bar chart depicting the real part of the matrix and the left chart depicting the imaginary part. The density matrices below are the ideal case. (a) After photon 3 is lost, the state remains a mixed state. (b) The method of indirect-Z operation is applied to eliminate qubit 3. When photon 1 is projected onto $|+\rangle$, the output state is $|+\rangle|+\rangle$, and the final state has a fidelity of $0.83 \pm 0.03$ with the ideal state. (c) The PBS gate operation is applied to eliminate qubit 3. The output state is an entanglement state $\frac{1}{\sqrt{2}}(|H\rangle|+\rangle + |V\rangle|\rangle)$, and the fidelity is $0.72 \pm 0.03$.

and it clearly agrees that what we obtained is $|+\rangle_2 \otimes |+\rangle_4$. The fidelity with the ideal state is $0.83 \pm 0.02$. Furthermore, we have performed the measurements of the polarization correlation between photon 2' and 4. Experimentally, the angle of polarizer 2 is set to $45^\circ$, $-45^\circ$ and $0^\circ$ with respect to the vertical direction, respectively, then the four-fold coincidence will oscillate with varying of the angle of polarizer 4. The results are depicted in figure 6(a), which clearly
Figure 6. Polarization correlation between photon $2'$ and 4 after elimination of the noise. If we fix the angle of polarizer 2 to $45^\circ$, $-45^\circ$ and $0^\circ$ with respect to the vertical direction, the four-fold coincidence will oscillate with varying of the angle of polarizer 4. (a) The experimental results clearly agree that the two qubits have no entanglement correlation. (b) The two sinusoidal curves with a visibility of $0.78 \pm 0.05$ demonstrate that photons $2'$ and 4 are in an entangled state $\frac{1}{\sqrt{2}}(|H\rangle |+\rangle + |V\rangle |-\rangle)$.

agree that photons 2 and 4 have no entanglement correlation. These results conclusively give the proof that qubit 3 has been eliminated and demonstrate the underlying principle of indirect-Z measurements against qubit-loss errors.

In the next step, we exploit the method of the PBS gate to remove the photon-loss errors. Now, we start from the cluster state $|C_{234'}\rangle$. Experimentally, as depicted in figure 3, photons $2'$ and 4 are passed through PBS2 to remove qubit 3 from the cluster state, leaving a smaller but pure state. After the post-selection, the final state is expected to be an entangled state

$$|R_{24}\rangle = \frac{1}{\sqrt{2}}(|H\rangle |+\rangle + |V\rangle |-\rangle). \quad (14)$$

To characterize the quality of the experimental process, we depict here the output density matrix of the final state in figure 5(c), where the maximum-likelihood reconstruction technique is also exploited. The agreement with the theory can be quantified from the fidelity $F = \langle R_{24}|\rho_{\text{exp}}|R_{24}\rangle = 0.72 \pm 0.03$. Again we have performed the measurements of the polarization correlation between photon $2'$ and photon 4. The experimental results can be found in figure 6(b), from which we can see that the final state is an entanglement state. This demonstrates the underlying principle of the method of the PBS gate against qubit-loss errors.

In conclusion, we experimentally demonstrate the methods of eliminating photon-loss errors in cluster states by using the PBS gate and performing indirect-Z measurements. The multi-photon states in this experiment are created probabilistically and a post-selection technique is exploited here. The photon-loss error is simulated by detecting it without knowing its polarization information. Although it does not defeat the proof-of-principle experiment here, on-demand entangled photon sources will be required for future application. The proposed schemes can be extended to other fields if we replace the PBS gate with general parity measurements. In addition, it remains an interesting question as to how the qubit-loss errors can be eliminated most efficiently, and our results can be seen as contributing to this effort.
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