CHARGINO PAIR PRODUCTION
AT $e^+e^−$ COLLIDERS WITH POLARIZED BEAMS

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Abstract

The chargino $\tilde{\chi}^{±}_{1,2}$ system can be reconstructed completely in $e^+e^−$ collisions. By measuring the total cross sections and the asymmetries with polarized beams in $e^+e^− \rightarrow \tilde{\chi}^{+}_i\tilde{\chi}^{-}_j [i,j = 1,2]$, the chargino masses and the gaugino–higgsino mixing angles of these states can be determined very accurately. If only the lightest charginos $\tilde{\chi}^{±}_1$ are kinematically accessible, transverse beam polarization is needed to determine the mixing angles unambiguously. From these observables the fundamental SUSY parameters can be derived: the SU(2) gaugino mass $M_2$, the modulus and the cosine of the CP-violating phase of the higgsino mass parameter $\mu$, and $\tan\beta = v_2/v_1$, the ratio of the vacuum expectation values of the two neutral Higgs doublet fields. [The remaining two-fold ambiguity of the phase can be resolved by measuring the normal polarization of the charginos.]
1. Introduction

Charginos $\chi_{1,2}^{\pm}$ in supersymmetric theories are generally mixtures of the spin–1/2 partners of the $W^\pm$ gauge bosons, $W^\pm$, and of the charged Higgs bosons, $H^\pm$. The chargino mass matrix \cite{1} is given in the $(\tilde{W}^-, \tilde{H}^-)$ basis by

$$M_C = \begin{pmatrix} M_2 & \sqrt{2}m_W \cos \beta \\ \sqrt{2}m_W \sin \beta & \mu \end{pmatrix}$$ (1)

The mixing matrix is built up by the fundamental supersymmetry (SUSY) parameters: the SU(2) gaugino mass $M_2$, the higgsino mass parameter $\mu$, and the ratio $\tan \beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields which break the electroweak symmetry. By reparametrization of the fields, the phase $\Phi_\mu$ in CP–noninvariant theories, may be attributed to $\mu$,

$$\mu = |\mu| e^{i\Phi_\mu} \text{ with } 0 \leq \Phi_\mu \leq 2\pi$$ (2)

while $M_2$ can be assumed real and positive; $\mu$ is real in CP–invariant theories.

Since the chargino mass matrix $M_C$ is not symmetric, two different unitary matrices acting on the left– and right–chiral $(\tilde{W}, \tilde{H})$ states are needed to diagonalize the matrix:

$$U_{L,R} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}^- \end{pmatrix}_{L,R} = \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix}_{L,R}$$ (3)

The unitary matrices $U_L$ and $U_R$ can be parametrized in the following way \cite{2}:

$$U_L = \begin{pmatrix} \cos \phi_L & e^{-i\beta_L} \sin \phi_L \\ -e^{i\beta_L} \sin \phi_L & \cos \phi_L \end{pmatrix}$$

$$U_R = \begin{pmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{pmatrix} \begin{pmatrix} \cos \phi_R & e^{-i\beta_R} \sin \phi_R \\ -e^{i\beta_R} \sin \phi_R & \cos \phi_R \end{pmatrix}$$ (4)

The four phase angles $\{\beta_L, \beta_R, \gamma_1, \gamma_2\}$ are not independent but can be expressed in terms of the invariant angle $\Phi_\mu$. All four phase angles vanish in CP–invariant theories for which $\Phi_\mu \to 0$ or $\pi$.

The mass eigenvalues $m_{\tilde{\chi}_{1,2}^\pm}^2$ and the rotation angles $\phi_L$ and $\phi_R$ are determined by the fundamental SUSY parameters $\{M_2, |\mu|, \cos \Phi_\mu, \tan \beta\}$;

$$m_{\tilde{\chi}_{1,2}^\pm}^2 = \frac{1}{2} \left[ M_2^2 + |\mu|^2 + 2m_W^2 \mp 4m_W^2 \Delta \right]$$ (5)
and
\[
\cos 2\phi_{L,R} = -\frac{M_2^2 - |\mu|^2 \pm 2m_W^2 \cos 2\beta}{4m_W^2 \Delta}
\]
\[
\sin 2\phi_{L,R} = -\frac{\sqrt{M_2^2 + |\mu|^2} \cos 2\beta + 2M_2|\mu| \sin 2\beta \cos \Phi_{\mu}}{2m_W \Delta}
\]
with \(\Delta\) involving the phase \(\Phi_{\mu}\)
\[
\Delta = \sqrt{(M_2^2 - |\mu|^2)^2 + 4m_W^2 (M_2^2 + |\mu|^2 + 2M_2|\mu| \sin 2\beta \cos \Phi_{\mu}) + 4m_W^4 \cos^2 2\beta/4m_W^2}
\]
(6)

Conversely, the fundamental SUSY parameters \(\{M_2, |\mu|, \tan \beta\}\) and the phase parameter \(\cos \Phi_{\mu}\) can be constructed from the chargino \(\tilde{\chi}_{1,2}\) parameters: the masses \(m_{\tilde{\chi}^{\pm}_{1,2}}\) and the two mixing angles \(\phi_{L,R}\) of the left– and right–chiral components of the wave function (see Sect.4).

The two rotation angles \(\phi_{L,R}\) and the phase angles \(\{\beta_L, \beta_R, \gamma_1, \gamma_2\}\) define the couplings of the chargino–chargino–Z vertices:
\[
\langle \tilde{\chi}^-_{1L} | Z | \tilde{\chi}^-_{1L} \rangle = -\frac{g_W}{c_W} \left[ s_W^2 - \frac{3}{4} - \frac{1}{4} \cos 2\phi_L \right]
\]
\[
\langle \tilde{\chi}^-_{1L} | Z | \tilde{\chi}^-_{1R} \rangle = -\frac{g_W}{c_W} \left[ s_W^2 - \frac{3}{4} - \frac{1}{4} \cos 2\phi_R \right]
\]
\[
\langle \tilde{\chi}^-_{2L} | Z | \tilde{\chi}^-_{2L} \rangle = +\frac{g_W}{4c_W} e^{-i\beta_L} \sin 2\phi_L
\]
\[
\langle \tilde{\chi}^-_{2L} | Z | \tilde{\chi}^-_{2R} \rangle = +\frac{g_W}{4c_W} e^{-i(\beta_R - \gamma_1 + \gamma_2)} \sin 2\phi_R
\]
\[
\langle \tilde{\chi}^-_{2L} | Z | \tilde{\chi}^-_{2L} \rangle = -\frac{g_W}{c_W} \left[ s_W^2 - \frac{3}{4} + \frac{1}{4} \cos 2\phi_L \right]
\]
\[
\langle \tilde{\chi}^-_{2L} | Z | \tilde{\chi}^-_{2R} \rangle = -\frac{g_W}{c_W} \left[ s_W^2 - \frac{3}{4} + \frac{1}{4} \cos 2\phi_R \right]
\]
and the electron–neutrino–chargino vertices:
\[
\langle \tilde{\chi}^-_{1R} | \tilde{\nu} | e^+ \rangle = -g_Y e^{i\gamma_1} \cos \phi_R
\]
\[
\langle \tilde{\chi}^-_{2R} | \tilde{\nu} | e^+ \rangle = +g_Y e^{i(\beta_R + \gamma_2)} \sin \phi_R
\]
(8)

with \(s_W^2 = 1 - c_W^2 \equiv \sin^2 \theta_W\) denoting the electroweak mixing angle. \(g_W\) and \(g_Y\) are the \(e\nu W\) gauge coupling and the \(e\tilde{\nu} \tilde{W}\) Yukawa coupling, respectively. They are identical in supersymmetric theories:
\[
g_Y = g_W = e/s_W
\]
(9)

Since the coupling to the higgsino component, which is proportional to the electron mass, can be neglected in the sneutrino vertex, the sneutrino couples only to left–handed electrons.

Charginos are produced in \(e^+ e^-\) collisions, either in diagonal or in mixed pairs \([3]-[9]\):
\[
e^+ e^- \rightarrow \tilde{\chi}^+_i \tilde{\chi}^-_j \quad [i, j = 1, 2]
\]
In the analysis of the chargino system the polarization of the electron and positron beams plays a central role. In addition to standard longitudinal (L/R) polarization, it turns out that transverse (T) polarization of the beams will be quite useful in the measurement of the chargino wave functions. The analysis will be carried out in two steps.

(i) In the first step we assume that the collider energy will only be sufficient to generate the light $\tilde{\chi}_1^\pm$ pair. From the steep threshold behavior of the cross section, the mass of $\tilde{\chi}_1^\pm$ can be determined very accurately [10]. From the size of the polarized cross sections in the continuum, the mixing parameters can be extracted. The transverse beam polarization is required in order to obtain a unique solution in general. Moreover, the mass $m_{\tilde{\nu}_e}$ of the sneutrino exchanged in the $t$–channel can be measured.

(ii) In the second step we assume the collider energy to be large enough to produce the entire ensemble of diagonal and mixed chargino pairs $\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^+\tilde{\chi}_2^-$ and $\tilde{\chi}_2^+\tilde{\chi}_2^-$. In this case longitudinal beam polarization is sufficient to determine the mixing parameters unambiguously. Moreover, if the sneutrino mass is known from $\tilde{\nu}_e\tilde{\nu}_e$ pair production, the threshold behavior and the continuum values of the polarized cross sections can be exploited to carry out a high–precision analysis of the masses $m_{\tilde{\chi}_{1,2}}$, the mixing parameters $\phi_{L,R}$ of the wave functions and the $\tilde{e}\tilde{\nu}_e\tilde{W}$ Yukawa coupling.

From these observables the underlying fundamental SUSY parameters, $M_2, |\mu|$ and $\tan \beta$, can be extracted unambiguously; the phase $\Phi_\mu$ can be determined up to a twofold ambiguity $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$. This ambiguity can only be resolved by measuring manifestly CP–noninvariant observables related to the normal polarization of the charginos, cf. Ref. [2]. To clarify the analytical structure, the reconstruction of the basic SUSY parameters presented here is carried out at the tree level; the small higher–order corrections include parameters from other sectors of the MSSM demanding iterative higher–order expansions in global analyses at the very end.

The analysis of the chargino sector is independent of the structure of the neutralino sector which is potentially very complex in theories beyond the Minimal Supersymmetric Standard Model (MSSM). The structure of the chargino sector, by contrast, is isomorphic to the form of the MSSM for a large class of supersymmetric theories. Moreover, the analysis is based strictly on low–energy SUSY. Once these basic parameters are determined experimentally, they provide essential components in the reconstruction of the fundamental supersymmetric theory at the grand unification scale.

Many facets of the neutralino sector have been discussed in the literature; for recent results see Ref. [11] where mass relations are exploited, and Ref. [12] where spin correlations have been considered.
2. Chargino Production in $e^+e^-$ Collisions

The production of chargino pairs at $e^+e^-$ colliders is based on three mechanisms: $s$-channel $\gamma$ and $Z$ exchanges, and $t$-channel $\tilde{\nu}_e$ exchange, cf. Fig.1. The transition matrix element, after a Fierz transformation of the $\tilde{\nu}_e$ channel and $e^+$, can be expressed in terms of four bilinear charges, defined by the chiralities $Q_{\alpha\beta}$ for the diagonal $\chi_{\alpha\beta}$ of the associated lepton and chargino currents. After introducing the following notation,

$$T[e^+e^- \to \tilde{\chi}_i^\pm \tilde{\chi}_j^\pm] = \frac{e^2}{s} Q_{\alpha\beta} \left[ \tilde{\nu}(e^+) \gamma_{\mu} P_{\alpha} u(e^-) \right] \left[ \bar{u}(\tilde{\chi}_i^-) \gamma^{\mu} P_{\beta} v(\tilde{\chi}_j^+) \right]$$

(10)

can be expressed in terms of four bilinear charges, defined by the chiralities $\alpha, \beta = L, R$ of the associated lepton and chargino currents. After introducing the following notation,

$$D_L = 1 + \frac{D_Z}{s_W^2 c_W^2} (s_W^2 - \frac{1}{2}) (s_W^2 - \frac{3}{4})$$

$$D_R = 1 + \frac{D_Z}{c_W^2} (s_W^2 - \frac{3}{4})$$

$$F_L = \frac{D_Z}{4 s_W^2 c_W^2} (s_W^2 - \frac{1}{2})$$

$$F_R = \frac{D_Z}{4 c_W^2}$$

(11)

and

$$D'_L = D_L + \left( \frac{g_Y}{g_W} \right)^2 \frac{D_\nu}{4 s_W^2}$$

$$F'_L = F_L - \left( \frac{g_Y}{g_W} \right)^2 \frac{D_\nu}{4 s_W^2}$$

(12)

the four bilinear charges $Q_{\alpha\beta}$ are linear in the mixing parameters $\cos 2\phi_{L,R}$ and $\sin 2\phi_{L,R}$; for the diagonal $\tilde{\chi}_1^- \tilde{\chi}_1^+$, $\tilde{\chi}_2^- \tilde{\chi}_2^+$ modes and the mixed mode $\tilde{\chi}_1^- \tilde{\chi}_2^+$:

$$\{11\}/\{22\} : \begin{align*}
Q_{LL} &= D_L + F_L \cos 2\phi_L \\
Q_{LR} &= D'_L + F'_L \cos 2\phi_R
\end{align*}$$

$$\begin{align*}
Q_{RL} &= D_R + F_R \cos 2\phi_L \\
Q_{RR} &= D'_R + F'_R \cos 2\phi_R
\end{align*}$$

(13)

$$\{12\}/\{21\} : \begin{align*}
Q_{LL} &= F_L e^{\mp i\beta_L} \sin 2\phi_L \\
Q_{LR} &= F'_L e^{\mp i(\beta_R - \gamma_1 + \gamma_2)} \sin 2\phi_R
\end{align*}$$

$$\begin{align*}
Q_{RL} &= F_R e^{\mp i\beta_L} \sin 2\phi_L \\
Q_{RR} &= F'_R e^{\mp i(\beta_R - \gamma_1 + \gamma_2)} \sin 2\phi_R
\end{align*}$$

(14)

The first index in $Q_{\alpha\beta}$ refers to the chirality of the $e^\pm$ current, the second index to the chirality of the $\tilde{\chi}_{i,j}^\pm$ current. The $\tilde{\nu}$ exchange affects only the $LR$ chirality charge $Q_{LR}$ while all other amplitudes are built up by $\gamma$ and $Z$ exchanges only. The first term in $D_{L,R}$ is generated by the $\gamma$ exchange; $D_Z = s/(s - m_Z^2 + i m_Z \Gamma_Z)$ denotes the $Z$ propagator and $D_\nu = s/(t - m_\nu^2)$ the $\tilde{\nu}$ propagator with momentum transfer $t$. The non-zero $Z$ width can in general be neglected for the energies considered in the present analysis so that the charges are rendered complex in the Born approximation only through the CP–noninvariant phase.

For the sake of convenience we introduce eight quartic charges for each of the production processes of the diagonal and mixed chargino pairs, respectively. These charges
correspond to independent helicity amplitudes which describe the chargino production processes for polarized electrons/positrons with negligible lepton masses. Expressed in terms of bilinear charges they are collected in Table 1, including the transformation properties under P and CP.

Table 1: The independent quartic charges of the chargino system, the measurement of which determines the chargino mass matrix.

| P   | CP   | Quartic charges                                                                 |
|-----|------|----------------------------------------------------------------------------------|
| even| even | $Q_1 = \frac{1}{4} [ |Q_{RR}|^2 + |Q_{LL}|^2 + |Q_{RL}|^2 + |Q_{LR}|^2 ]$               |
|     |      | $Q_2 = \frac{1}{2} \text{Re} [Q_{RR}Q_{RL}^* + Q_{LL}Q_{LR}^*]$                   |
|     |      | $Q_3 = \frac{1}{4} [ |Q_{RR}|^2 + |Q_{LL}|^2 - |Q_{RL}|^2 - |Q_{LR}|^2 ]$               |
|     |      | $Q_5 = \frac{1}{2} \text{Re} [Q_{LR}Q_{RR}^* + Q_{LL}Q_{RL}^*]$                   |
| odd |      | $Q_4 = \frac{1}{4} \text{Im} [Q_{RR}Q_{RL}^* + Q_{LL}Q_{LR}^*]$                   |
| odd | even | $Q_1' = \frac{1}{4} [ |Q_{RR}|^2 + |Q_{RL}|^2 - |Q_{LR}|^2 - |Q_{LL}|^2 ]$             |
|     |      | $Q_2' = \frac{1}{2} \text{Re} [Q_{RR}Q_{RL}^* - Q_{LL}Q_{LR}^*]$                   |
|     |      | $Q_3' = \frac{1}{4} [ |Q_{RR}|^2 + |Q_{LR}|^2 - |Q_{RL}|^2 - |Q_{LR}|^2 ]$             |

The charges $Q_1$ to $Q_3$, and $Q_5$, $Q_4$ are manifestly parity–even, $Q_1'$ to $Q_3'$ are parity–odd. The charges $Q_1$ to $Q_3$, $Q_5$, and $Q_1'$ to $Q_3'$ are CP invariant\footnote{When expressed in terms of the fundamental SUSY parameters, these charges do depend nevertheless on $\cos \Phi_\mu$ indirectly through $\cos 2\phi_L$, in the same way as the $\tilde{\chi}_{1,2}^\pm$ masses depend indirectly on this parameter.} while $Q_4$ changes sign under CP transformations\footnote{The P–odd and CP–even/CP-odd counterparts to $Q_5/Q_4$, which carry a negative sign between the corresponding $L$ and $R$ components, do not affect the observables under consideration.}. The CP invariance of $Q_2$ and $Q_2'$ can easily be proved by noting that

$$2m_{\tilde{\chi}_1^\pm}m_{\tilde{\chi}_2^\pm} \cos(\beta_L - \beta_R + \gamma_1 - \gamma_2) \sin 2\phi_L \sin 2\phi_R$$

$$= (m_{\tilde{\chi}_1^\pm}^2 + m_{\tilde{\chi}_2^\pm}^2) (1 - \cos 2\phi_L \cos 2\phi_R) - 4m_W^2$$

(15)

Therefore, all the production cross sections $\sigma[e^+e^- \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-]$ for any combination of pairs $\{ij\}$ depend only on $\cos 2\phi_L$ and $\cos 2\phi_R$ apart from the chargino masses, the sneutrino mass and the Yukawa couplings. For longitudinally–polarized electron beams the sums
and differences of the quartic charges are restricted to either $L$ or $R$ components (first index) of the $e^\pm$ currents.

Defining the $\tilde{\chi}_i^-$ production angles with respect to the electron flight–direction by the polar angle $\Theta$ and the azimuthal angle $\Phi$ with respect to the electron transverse polarization, the helicity amplitudes can be derived from eq. (10). While electron and positron helicities are opposite to each other in all amplitudes, the $\tilde{\chi}_i^-$ and $\tilde{\chi}_j^+$ helicities are in general not correlated due to the non-zero masses of the particles; amplitudes with equal $\tilde{\chi}_i^-$ and $\tilde{\chi}_j^+$ helicities are reduced only to order $\propto m_{\tilde{\chi}_i^\pm}/\sqrt{s}$ for asymptotic energies. The helicity amplitudes may be expressed as $T_{ij}(\sigma; \lambda_i, \lambda_j) = 2\pi\alpha\, e^{i\sigma\Phi} \langle \sigma; \lambda_i\lambda_j \rangle$, denoting the electron helicity by the first index, the $\tilde{\chi}_i^-$ and $\tilde{\chi}_j^+$ helicities by the remaining two indices, $\lambda_i$ and $\lambda_j$, respectively. The explicit form of the helicity amplitudes $\langle \sigma; \lambda_i\lambda_j \rangle$ can be found in Ref. [3].

In order to describe the electron and positron polarizations, the reference frame must be fixed. The electron momentum direction will define the $z$-axis and the electron transverse polarization vector the $x$-axis. The azimuthal angle of the transverse polarization vector of the positron is called $\eta$ with respect to the $x$–axis. In this notation, the polarized differential cross section is given in terms of the electron and positron polarization vectors $P=(P_T, 0, P_L)$ and $\bar{P}=(-P_T \cos \eta, P_T \sin \eta, -P_L)$ by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16s} \lambda^{1/2} \left[ (1 - P_L \bar{P}_L) \Sigma_{\text{unp}} + (P_L - \bar{P}_L) \Sigma_{LL} + P_T \bar{P}_T \cos(2\Phi - \eta) \Sigma_{TT} \right]$$

with the distributions

$$\Sigma_{\text{unp}} = \frac{1}{4} \sum_{\lambda_i\lambda_j} \left[ |\langle+; \lambda_i\lambda_j \rangle|^2 + |\langle-; \lambda_i\lambda_j \rangle|^2 \right]$$

$$\Sigma_{LL} = \frac{1}{4} \sum_{\lambda_i\lambda_j} \left[ |\langle+; \lambda_i\lambda_j \rangle|^2 - |\langle-; \lambda_i\lambda_j \rangle|^2 \right]$$

$$\Sigma_{TT} = \frac{1}{2} \sum_{\lambda_i\lambda_j} \text{Re} \left[ \langle-; \lambda_i\lambda_j \rangle \langle+; \lambda_i\lambda_j \rangle^* \right]$$

which depend only on the polar angle $\Theta$, but do not on the azimuthal angle $\Phi$ any more; $\lambda = [1 - (\mu_i + \mu_j)^2][1 - (\mu_i - \mu_j)^2]$ is the two–body phase space function and $\mu_i^2 = m_{\tilde{\chi}_i^\pm}/s$.

Carrying out the sum over the chargino helicities, the distributions $\Sigma_{\text{unp}}, \Sigma_{LL},$ and $\Sigma_{TT}$ can be expressed in terms of the quartic charges:

$$\Sigma_{\text{unp}} = 4 \left\{ \left[ 1 - (\mu_i^2 - \mu_j^2)^2 + \lambda \cos^2 \Theta \right] Q_1 + 4\mu_i\mu_jQ_2 + 2\lambda^{1/2}Q_3 \cos \Theta \right\}$$

$$\Sigma_{LL} = 4 \left\{ \left[ 1 - (\mu_i^2 - \mu_j^2)^2 + \lambda \cos^2 \Theta \right] Q_1' + 4\mu_i\mu_jQ_2' + 2\lambda^{1/2}Q_3' \cos \Theta \right\}$$

$$\Sigma_{TT} = -4\lambda \sin^2 \Theta \, Q_5$$

(18)
If the production angles could be measured unambiguously on an event–by–event basis, the quartic charges could be extracted directly from the angular dependence of the cross section at a single energy. However, since charginos decay mainly into the invisible lightest neutralinos and SM fermion pairs, the production angles cannot be determined completely on an event-by-event basis. The transverse distribution can be extracted by using an appropriate weight function for the azimuthal angle $\Phi$, cf. eq.(16). This leads us to the following integrated polarization–dependent cross sections as physical observables:

$$
\sigma_R = \int d\Omega \frac{d\sigma}{d\Omega} [P_L = -\bar{P}_L = +1] \\
\sigma_L = \int d\Omega \frac{d\sigma}{d\Omega} [P_L = -\bar{P}_L = -1] \\
\sigma_T = \int d\Omega \left( \frac{\cos 2\Phi}{\pi} \right) \frac{d\sigma}{d\Omega} [P_T = \bar{P}_T = 1, \eta = \pi]
$$

(19)

As a result, nine independent physical observables can be constructed at a given c.m. energy by means of beam polarization in the three production processes; three in each mode $\{ij\} = \{11\}, \{12\}$ and $\{22\}$.

3. Measuring Masses, Mixing Angles and Couplings

Before the strategies to measure the masses, mixing angles and the couplings are presented in detail, a few general remarks on the structure of the chargino system may render the techniques more transparent.

(i) The right–handed cross sections $\sigma_R$ do not involve the exchange of the sneutrino. They depend only, in symmetric form, on the mixing parameters $\cos 2\phi_L$ and $\cos 2\phi_R$.

(ii) The left–handed cross sections $\sigma_L$ and the transverse cross section $\sigma_T$ depend on $\cos 2\phi_{L,R}$, the sneutrino mass and the $e\bar{\nu}\bar{W}$ Yukawa coupling. Thus the sneutrino mass and the Yukawa coupling can be determined from the left-handed and transverse cross sections. [If the sneutrino mass is much larger than the collider energy, only the ratio of the Yukawa coupling over the sneutrino mass squared ($g_2^2/m_{\tilde{\nu}}^2$) can be measured by this method [12].]

The cross sections $\sigma_L$, $\sigma_R$ and $\sigma_T$ are binomials in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane. If the two–chargino model is realized in nature, any two $\sigma_L$ and $\sigma_R$ contours, for example, will at least cross at one point in the plane between $-1 \leq \cos 2\phi_L, \cos 2\phi_R \leq +1$. However, being ellipses or hyperbolae, they may cross up to four times. This ambiguity can be resolved by measuring the third physical quantity $\sigma_T$ for example. The measurement of $\sigma_T$ is particularly important if the sneutrino mass is still unknown. While the curve for $\sigma_R$ is fixed, the curve for $\sigma_L$ will move in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane with changing $m_{\bar{\nu}}$. 

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However, the third curve will intersect the other two in the same point only if the mixing angles as well as the sneutrino mass are chosen right.

The numerical analyses presented below have been worked out for the two parameter points introduced in Ref. [14]. They correspond to a small and a large $\tan \beta$ solution for universal gaugino and scalar masses at the GUT scale:

$$RR1 : (\tan \beta, m_0, M_2) = (3, 100 \text{ GeV}, 200 \text{ GeV})$$

$$RR2 : (\tan \beta, m_0, M_2) = (30, 160 \text{ GeV}, 200 \text{ GeV})$$

(20)

The CP-phase $\Phi_\mu$ is set zero. The induced chargino $\tilde{\chi}_i^\pm$, neutralino $\tilde{\chi}_1^0$ and sneutrino $\tilde{\nu}$ masses are given as follows:

$$m_{\tilde{\chi}_i^\pm} = 128/132 \text{ GeV} \quad m_{\tilde{\chi}_1^0} = 70/72 \text{ GeV}$$

$$m_{\tilde{\chi}_2^\pm} = 346/295 \text{ GeV} \quad m_{\tilde{\nu}} = 166/206 \text{ GeV}$$

(21)

for the two points $RR1/2$, respectively. The size of the unpolarized total cross sections $\sigma[e^+e^- \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-]$ as functions of the collider energy is shown for two reference points in Fig. 2. With the maximum of the cross sections in the range of 0.1 to 0.3 pb, about $10^5$ to $3 \times 10^5$ events can be generated for an integrated luminosity $\int \mathcal{L} \simeq 1 \text{ab}^{-1}$ as planned in three years of running at TESLA. The cross sections rise steeply at the threshold,

$$\sigma[e^+e^- \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-] \sim \sqrt{s - (m_{\tilde{\chi}_i^\pm} + m_{\tilde{\chi}_j^\pm})^2}$$

(22)

so that the masses $m_{\tilde{\chi}_i^\pm}$, $m_{\tilde{\chi}_2^\pm}$ can be measured very accurately in the production processes of the final–state pairs $\{11\}$, $\{12\}$ and $\{22\}$. Detailed experimental simulations have shown that accuracies $\Delta m_{\tilde{\chi}_1^\pm} = 40 \text{ MeV}$ and $\Delta m_{\tilde{\chi}_2^\pm} = 250 \text{ MeV}$ can be achieved in high-luminosity threshold scans [10].

### 3.1 Light Chargino Pair Production

At an early phase of the $e^+e^-$ linear collider the energy may only be sufficient to reach the threshold of the lightest chargino pair $\tilde{\chi}_1^+\tilde{\chi}_1^-$. Nevertheless, nearly the entire structure of the chargino system can be reconstructed even in this case.

By analyzing the $\{11\}$ mode in $\sigma_L\{11\}$, $\sigma_R\{11\}$, the mixing angles $\cos 2\phi_L$ and $\cos 2\phi_R$ can be determined up to at most a four fold ambiguity if the sneutrino mass is known and the Yukawa coupling is identified with the gauge coupling. The ambiguity can be resolved by adding the information from $\sigma_T\{11\}$. This is clearly demonstrated\[d\] in Fig. 3,

\[d\]With event numbers of order $10^5$, statistical errors are at the per–mille level.
for the reference point \textbf{RR1} at the energy $\sqrt{s} = 400$ GeV. Moreover, the additional measurement of the transverse cross section can also be exploited to determine the sneutrino mass. While the right–handed cross section $\sigma_R$ does not depend on $m_{\tilde{\nu}}$, the contours $\sigma_L$, $\sigma_T$ move uncorrelated in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane if not the correct sneutrino mass is used in the analysis. The three contour lines intersect exactly in one point of the plane only if all the parameters correspond to the correct physical values.

Thus, with transverse polarization available, the system of observables can be closed except for the mass value of the heavy chargino. It has been proved in Ref. \cite{6} that the SUSY parameters \{\text{\textit{M}}_2, \mu, \tan \beta\} can be derived from the observables $m_{\tilde{\chi}_1^{\pm}}$ and $\cos 2\phi_L, R$ up to at most a two–fold ambiguity.

\subsection*{3.2 The Complete Chargino System}

From the analysis of the complete chargino system \{\text{\textit{\tilde{\chi}}}_1^{\pm}, \text{\textit{\tilde{\chi}}}_2^{\pm}, \text{\textit{\tilde{\chi}}}_1^{-}, \text{\textit{\tilde{\chi}}}_2^{-}\}, together with the knowledge of the sneutrino mass from sneutrino pair production, the maximal information can be extracted on the basic parameters of the electroweak SU(2) gaugino sector. Moreover, the identity of the $e\tilde{\nu}_\nu \tilde{W}_Y$ Yukawa coupling with the $e\nu \tilde{W}_g$ gauge coupling, which is of fundamental nature in supersymmetric theories, can be tested very accurately. This analysis is the final target of LC experiments which should provide a complete picture of the electroweak gaugino sector with resolution at least at the per-cent level.

The case will be exemplified for the scenario \textbf{RR1} with $\tan \beta = 3$. To simplify the picture, without loss of generality, we will not choose separate energies at the maximal values of the cross sections, but instead we will work at a single collider energy $\sqrt{s}=800$ GeV and an integrated luminosity $\int L = 1$ ab$^{-1}$. The polarized cross sections take the following values:

\begin{align}
\sigma_R\{11\} &= 1.8 \, \text{fb} & \sigma_L\{11\} &= 787.7 \, \text{fb} & \sigma_T\{11\} &= 0.53 \, \text{fb} \\
\sigma_R\{12\} &= 12.1 \, \text{fb} & \sigma_L\{12\} &= 106.2 \, \text{fb} & \sigma_T\{12\} &= 0.53 \, \text{fb} & \sigma_R\{22\} &= 67.1 \, \text{fb} & \sigma_L\{22\} &= 337.5 \, \text{fb} & \sigma_T\{22\} &= 1.07 \, \text{fb}
\end{align}

\begin{equation}
(23)
\end{equation}

Chargino pair production with right-handed electron beams provides us with the cross sections $\sigma_R_i (i = \{11\}, \{12\}, \{22\})$. Due to the absence of the sneutrino exchange diagram, the cross sections can be expressed symmetrically in the mixing parameters $c_{2L} = \cos 2\phi_L$ and $c_{2R} = \cos 2\phi_R$:

\begin{equation}
\sigma_{R_i} = A_{R_i} (c_{2L}^2 + c_{2R}^2) + B_{R_i} (c_{2L} + c_{2R}) + C_{R_i} c_{2L} c_{2R} + D_{R_i} \quad (i = \{11\}, \{12\}, \{22\})
\end{equation}

\begin{equation}
(24)
\end{equation}

The coefficients $A_{R_i}$, $B_{R_i}$, $C_{R_i}$ and $D_{R_i}$ involve only known parameters, the chargino masses and the energy. Depending on whether $A_{R_i}^2 \gtrsim C_{R_i}^2/4$, the contour lines in the $[c_{2L}, c_{2R}]$ plane (cf. Fig.4) are either closed ellipses or open hyperbolae. They intersect

\textit{The cross section $\sigma_R\{12\}$ is always represented by an ellipse.
in exactly two points in the plane which are symmetric under the interchange \( c_{2L} \leftrightarrow c_{2R} \); for \( RR1 \): \([c_{2L}, c_{2R}] = [0.645, 0.844] \) and interchanged.

While the right–handed cross sections do not involve sneutrino exchange, the cross sections for left-handed electron beams are dominated by the sneutrino contributions unless the sneutrino mass is very large. In general, the three observables \( \sigma_{Li} \ (i = \{11\}, \{12\}, \{22\}) \) exhibit quite a different dependence on \( c_{2L} \) and \( c_{2R} \). In particular, they are not symmetric with respect to \( c_{2L} \) and \( c_{2R} \) so that the correct solution for \([c_{2L}, c_{2R}] \) can be singled out of the two solutions obtained from the right-handed cross sections eq.(24). As before, the three observables can be expressed as

\[
\sigma_{Li} = A_{L_i} c_{2L}^2 + A'_{L_i} c_{2R}^2 + B_{L_i} c_{2L} + B'_{L_i} c_{2R} + C_{L_i} c_{2L} c_{2R} + D_{L_i} \ (i = \{11\}, \{12\}, \{22\}) \]

The coefficients of the linear and quadratic terms of \( c_{2L} \) and \( c_{2R} \) depend on known parameters only. The shape of the contour lines is given by the chargino masses and the sneutrino mass, being either elliptic or hyperbolic for \( A_{L_i} A'_{L_i} \gtrless C_{L_i}^2 / 4 \), respectively. These asymmetric equations are satisfied only by one solution, as shown in Fig. 4. Among the two solutions obtained above from \( \sigma_{R_i} \) only the set \([c_{2L}, c_{2R}] = [0.645, 0.844] \) satisfies eq.(25).

At the same time, the identity between the \( e\tilde{\nu}\tilde{W} \) Yukawa coupling and the \( e\nu W \) gauge coupling can be tested. Varying the Yukawa coupling freely, the contour lines \( \sigma_{Li} \) are shifted through the \([c_{2L}, c_{2R}] \) plane. Only for the supersymmetric solutions the curves \( \sigma_{Li} \) intersect each other and the curves \( \sigma_{R_i} \) in exactly one point. Combining the analyses of \( \sigma_{R_i} \) and \( \sigma_{Li} \), the masses, the mixing parameters and the Yukawa coupling can be determined to quite a high precision.

\[
m_{\tilde{\chi}_1^\pm} = 128 \pm 0.04 \text{ GeV} \quad \cos 2\phi_L = 0.645 \pm 0.02 \quad g[e\tilde{\nu}\tilde{W}] / g[e\nu W] = 1 \pm 0.001
\]

\[
m_{\tilde{\chi}_2^\pm} = 346 \pm 0.25 \text{ GeV} \quad \cos 2\phi_R = 0.844 \pm 0.005
\]

The 1σ level statistical errors have been derived for an integrated luminosity of \( \int \mathcal{L} = 1 \text{ ab}^{-1} \).

Thus the parameters of the chargino system, masses \( m_{\tilde{\chi}_1^\pm} \) and \( m_{\tilde{\chi}_2^\pm} \), mixing parameters \( \cos 2\phi_L \) and \( \cos 2\phi_R \), as well as the Yukawa coupling can be used to extract the fundamental parameters of the underlying supersymmetric theory with high accuracy.

\[\text{In contrast to the restricted } \tilde{\chi}_1^+ \tilde{\chi}_1^- \text{ case, it is not necessary to use transversely polarized beams to determine this set of parameters unambiguously. If done so nevertheless, the analysis follows the same steps as discussed above. The additional information will reduce the errors on the fundamental parameters.}\]
4. Deriving the Fundamental SUSY Parameters

From the set (26) of measured observables the fundamental supersymmetric parameters \{M_2, \mu, \cos \phi_\mu, \tan \beta\} can be derived in the following way. To compactify the expressions, we introduce the abbreviations

\[
\Sigma = \frac{m^2_{\tilde{\chi}^\pm_1} + m^2_{\tilde{\chi}^\pm_2} - 2m^2_W}{2m^2_W}
\]
\[
\Delta = \frac{m^2_{\tilde{\chi}^\pm_2} - m^2_{\tilde{\chi}^\pm_1}}{4m^2_W}
\]

(27)

where \(\Delta\) is defined equivalently to eq.(7).

(i) \(M_2, |\mu|\) – Based on the definition \(M_2 > 0\), the gaugino mass parameter \(M_2\) and the modulus of the higgsino mass parameter read as follows:

\[
M_2 = m_W \sqrt{\Sigma - \Delta(c_2L + c_2R)}
\]
\[
|\mu| = m_W \sqrt{\Sigma + \Delta(c_2L + c_2R)}
\]

(28)

(ii) \(\cos \Phi_\mu\) – The sign of \(\mu\) in CP–invariant theories and, more generally, the cosine of the phase of \(\mu\) in CP–noninvariant theories is determined by the \(\tilde{\chi}^\pm_1, \tilde{\chi}^\pm_2\) masses and \(\cos 2\phi_{L,R}\):

\[
\cos \Phi_\mu = \frac{\Delta^2(2 - c^2_{2L} - c^2_{2R}) - \Sigma}{\sqrt{[1 - \Delta^2(c_{2L} - c_{2R})^2][\Sigma^2 - \Delta^2(c_{2L} + c_{2R})^2]}}
\]

(29)

(iii) \(\tan \beta\) – The value of \(\tan \beta\) is uniquely determined in terms of two chargino masses and two mixing angles

\[
\tan \beta = \sqrt{\frac{1 - \Delta(c_{2L} - c_{2R})}{1 + \Delta(c_{2L} - c_{2R})}}
\]

(30)

As a result, the fundamental SUSY parameters \(\{M_2, \mu, \tan \beta\}\) in CP–invariant theories, and \(\{M_2, |\mu|, \cos \Phi_\mu, \tan \beta\}\) in CP–noninvariant theories, can be extracted unambiguously from the observables \(m_{\tilde{\chi}^\pm_{1,2}}, \cos 2\phi_R, \cos 2\phi_L\). The final ambiguity in \(\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu\) in CP–noninvariant theories must be resolved by measuring observables related to the normal \(\tilde{\chi}^-_1\) or/and \(\tilde{\chi}^+_2\) polarization in non–diagonal \(\tilde{\chi}^-_1 \tilde{\chi}^+_2\) chargino–pair production [2].

For illustration, the accuracy which can be expected in such an analysis, is shown for both CP invariant reference points \(RR1\) and \(RR2\) in Table 2. If \(\tan \beta\) is large, this parameter is difficult to extract from the chargino sector. Since the chargino observables depend only on \(\cos 2\beta\), the dependence on \(\beta\) is flat for \(\beta \to \pi/2\) so that eq.(31) is not very useful to derive the value of \(\tan \beta\) due to error propagation. A significant lower bound can be derived nevertheless in any case.
Table 2: *Estimate of the accuracy with which the parameters \( M_2, \mu, \tan \beta \) can be determined, including sgn(\( \mu \)), from chargino masses and production cross sections; errors are statistical only at the 1\( \sigma \) level.*

|       | \( RR_1 \)          | \( RR_2 \)          |
|-------|----------------------|----------------------|
|       | theor. value         | fit value            | theor. value         | fit value            |
| \( M_2 \) | 152 GeV              | 152 \( \pm \) 1.75 GeV | 150 GeV              | 150 \( \pm \) 1.2 GeV |
| \( \mu \)  | 316 GeV              | 316 \( \pm \) 0.87 GeV | 263 GeV              | 263 \( \pm \) 0.7 GeV |
| \( \tan \beta \) | 3                   | 3 \( \pm \) 0.69     | 30                   | \( > \) 20.2         |

5. Sum Rules

The two–state mixing of charginos leads to sum rules for the chargino couplings. They can be formulated in terms of the squares of the bilinear charges, i.e. the elements of the quartic charges. This follows from the observation that the mixing matrix is built up by trigonometric functions among which many relations are valid. From evaluating these sum rules experimentally, it can be concluded whether \( \{\tilde{\chi}^{\pm}_1, \tilde{\chi}^{\pm}_2\} \) forms a closed system, or whether additional states, at high mass scales, mix in.

The following general sum rules can be derived for the two–state charginos system at tree level:

\[
\sum_{i,j=1,2} |Q_{\alpha\beta}|^2 \{ij\} = 2 \left( |D_{\alpha}|^2 + |F_{\alpha}|^2 \right) \quad (\alpha\beta) = (LL, RL, RR) \tag{31}
\]

The right–hand side is independent of any supersymmetric parameters, and it depends only on the electroweak parameters \( \sin^2 \theta_W, m_Z \) and on the energy, cf. eq. (11). Asymptotically, the initial energy dependence and the \( m_Z \) dependence drop out. The corresponding sum rule for the mixed left–right (LR) combination,

\[
\sum_{i,j=1,2} |Q_{LR}|^2 \{ij\} = 2 \left( |D_{\prime L}|^2 + |F_{\prime L}|^2 \right) \tag{32}
\]

involves the sneutrino mass and Yukawa coupling.

The validity of these sum rules is reflected in both the quartic charges and the production cross sections. However, due to mass effects and the \( t \)-channel sneutrino exchange, it is not straightforward to derive the sum rules for the quartic charges and the production cross sections in practice. Only asymptotically at high energies the sum rules (31) for the charges can be transformed directly into sum rules for the associated cross sections. Nevertheless, the fact that all the physical observables are bilinear in \( \cos 2\phi_L \)
and \( \cos 2\phi_R \), enables us to relate the cross sections with the set of the six variables \( \vec{z} = \{ 1, c_{2L}, c_{2R}, c_{2L}^2, c_{2R}^2, c_{2L}c_{2R} \} \). For the sake of simplicity we restrict ourselves to the left and right–handed cross section. We introduce the generic notation \( \vec{\sigma} \) for the six cross sections \( \sigma_R\{ij\} \) and \( \sigma_L\{ij\} \):

\[
\vec{\sigma} = \begin{bmatrix} \sigma_R\{11\}, \sigma_R\{12\}, \sigma_R\{22\}, \sigma_L\{11\}, \sigma_L\{12\}, \sigma_L\{22\} \end{bmatrix}
\]

(33)

Each cross section can be decomposed in terms of \( c_{2L} \) and \( c_{2R} \) as

\[
\sigma_i = \sum_{j=1}^{6} f_{ij} [m_{\tilde{\chi}^\pm_1}, m_{\tilde{\chi}^\pm_2}] z_j
\]

(34)

The matrix elements \( f_{ij} \) can easily be derived from Table 1 together with eqs. (11-14). Since the observables \( \sigma_R \) do not involve sneutrino contributions, the corresponding functions \( f_{ij} \) do not depend on the sneutrino mass. The 6×6 matrix \( f_{ij} \) relates the six left/right-handed cross section and the six variables \( z_i \). Inverting the matrix gives the expressions for the variables \( z_i \) in terms of the observables \( \sigma_i \), which are not independent. We therefore obtain several non–trivial relations among the observables of the chargino sector:

\[
z_1 = 1 : f_{1j}^{-1}\sigma_j = 1
\]

(35)

\[
z_4 = z_2^2 : f_{4j}^{-1}\sigma_j = [f_{2j}^{-1}\sigma_j]^2
\]

(36)

\[
z_5 = z_3^2 : f_{5j}^{-1}\sigma_j = [f_{3j}^{-1}\sigma_j]^2
\]

(37)

\[
z_6 = z_2 z_3 : f_{6j}^{-1}\sigma_j = f_{2j}^{-1} f_{3k}^{-1}\sigma_j \sigma_k
\]

(38)

where summing over repeated indices is understood. The failure of saturating any of these sum rules by the measured cross sections would signal that the chargino two–state \( \{\tilde{\chi}^\pm_1, \tilde{\chi}^\pm_2\} \) system is not complete and additional states mix in.

6. Conclusions

We have analyzed in this report how the parameters of the chargino system, the chargino masses \( m_{\tilde{\chi}^\pm_{1,2}} \) and the size of the wino and higgsino components in the chargino wave–functions, parameterized by the two mixing angles \( \phi_L \) and \( \phi_R \), can be extracted from pair production of the chargino states in \( e^+e^- \) annihilation. Three production cross sections \( \tilde{\chi}_1 \tilde{\chi}_1, \tilde{\chi}_1 \tilde{\chi}_2, \tilde{\chi}_2 \tilde{\chi}_2 \), for left– and right–handedly polarized electrons give rise to six independent observables. The method is independent of the chargino decay properties, i.e. the analysis is not affected by the structure of the neutralino sector which is very complex in supersymmetric theories while the chargino sector remains generally isomorphic to the
minimal form of the MSSM.

The measured chargino masses $m_{\tilde{\chi}^{\pm}_{1,2}}$ and the two mixing angles $\phi_L$ and $\phi_R$ allow us to extract the fundamental SUSY parameters $\{M_2, \mu, \tan \beta\}$ in CP–invariant theories unambiguously; in CP–noninvariant theories the modulus of $\mu$ and the cosine of the phase can be determined, leaving us with just a discrete two–fold ambiguity $\phi_\mu \leftrightarrow 2\pi - \phi_\mu$ which can be resolved by measuring the sign of observables related to the normal $\tilde{\chi}^{\pm}_{1,2}$ polarizations.

Sum rules for the production cross sections can be used at high energies to check whether the two–state chargino system is a closed system or whether additional states mix in from high scales.

To summarize, the measurement of the processes $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^- [i,j = 1,2]$ carried out with polarized beams, leads to a complete analysis of the basic SUSY parameters $\{M_2, \mu, \tan \beta\}$ in the chargino sector. Since the analysis can be performed with high precision, this set provides a solid platform for extrapolations to scales eventually near the Planck scale where the fundamental supersymmetric theory may be defined.

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Figure 1: The three exchange mechanisms contributing to the production of chargino pairs $\tilde{\chi}_i^{-}\tilde{\chi}_j^{+}$ in $e^+e^-$ annihilation.

Figure 2: The cross sections for the production of charginos as a function of the c.m. energy (a) with the $RR1$ set and (b) with the $RR2$ set of the fundamental SUSY parameters.
Figure 3: Contours of the cross sections $\sigma_L\{11\}$, $\sigma_R\{11\}$ and $\sigma_T\{11\}$ in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane for the set $RR1$ [$\tan \beta = 3$, $m_0 = 100$ GeV, $M_{1/2} = 200$ GeV] at the $e^+e^-$ c.m. energy of 400 GeV.
Figure 4: Contours of the cross sections (a) $(\sigma_R\{11\}, \sigma_L\{11\})$, (b) $(\sigma_R\{12\}, \sigma_L\{12\})$ and (c) $(\sigma_R\{22\}, \sigma_L\{22\})$ in the $[\cos 2\phi_L, \cos 2\phi_R]$ plane for the set $RR1$ [$\tan \beta = 3$, $m_0 = 100$ GeV, $M_{1/2} = 200$ GeV] at the c.m. energy of 800 GeV.