Zero bias anomaly in the density of states of low-dimensional metals

Lorenz Bartosch and Peter Kopietz
Institut für Theoretische Physik, Universität Frankfurt, Robert-Mayer-Str. 8-10, D-60054 Frankfurt am Main, Germany
August 20, 2001

Abstract. We consider the effect of Coulomb interactions on the average density of states (DOS) of disordered low-dimensional metals for temperatures $T$ and frequencies $\omega$ smaller than the inverse elastic life-time $1/\tau_0$. Using the fact that long-range Coulomb interactions in two dimensions ($2d$) generate $\ln^2$-singularities in the DOS $\nu(\omega)$ but only $\ln$-singularities in the conductivity $\sigma(\omega)$, we can re-sum the most singular contributions to the average DOS via a simple gauge-transformation. If $\lim_{\omega \to 0} \sigma(\omega) > 0$, then a metallic Coulomb gap $\nu(\omega) \propto |\omega|/e^4$ appears in the DOS at $T=0$ for frequencies below a certain crossover frequency $\Omega_1$ which depends on the value of the DC conductivity $\sigma(0)$. Here, $-e$ is the charge of the electron. Naively adopting the same procedure to calculate the DOS in quasi 1d metals, we find $\nu(\omega) \propto (|\omega|/\Omega_1)^1/2 \exp(-\Omega_1/|\omega|)$ at $T=0$, where $\Omega_1$ is some interaction-dependent frequency scale. However, we argue that in quasi 1d the above gauge-transformation method is on less firm grounds than in 2d. We also discuss the behavior of the DOS at finite temperatures and give numerical results for the expected tunneling conductance that can be compared with experiments.

PACS. 71.10.Pm Fermions in reduced dimensions − 71.23.-k Electronic structure of disordered solids − 72.15.Rn Localization effects (Anderson or weak localization)

1 Introduction

In the early eighties, Altshuler and Aronov [1] perturbatively studied the effect of electron-electron interactions on the density of states (DOS) of low-dimensional weakly disordered interacting electronic systems. For temperatures $T$ and frequencies $\omega$ smaller than the inverse elastic life-time $1/\tau_0$, they found that in reduced dimensions the interplay between disorder and electron-electron interactions gives rise to singular corrections to the averaged DOS. In two dimensions ($2d$), the long-range Coulomb interaction gives rise to a $\ln^2$-correction to the average DOS [1],

$$\nu(\omega) \sim \nu_0 \left[ 1 - \frac{r_0}{4} \ln(|\omega|/\tau_0) \ln(|\omega|/\tau_1) \right], \quad 2d,$$

where

$$r_0 = \frac{(2\pi)^2 \nu_0 D_0}{2e^2} = \frac{2e^2}{(2\pi)^2 \sigma_0} = \frac{1}{\pi k_F \ell} \tag{2}$$

is a dimensionless measure for the resistance of the system at frequency scale $\omega \approx \tau_0^{-1}$ (where $\sigma_0$ is the Drude conductivity), and the interaction-dependent time $\tau_1$ is given by

$$\tau_1 = \frac{1}{D_0^2 \kappa^2 \tau_0} = \frac{4\tau_0}{(\kappa \ell)^2} \tag{3}$$

Here, $\kappa = 2me^2$ is the Thomas-Fermi screening wave-vector in two dimensions, $\nu_0 = m/2\pi$ is the DOS at the Fermi energy (per spin projection) of electrons with effective mass $m$ and charge $-e$, $D_0 = v_F \ell/2$ is the diffusion coefficient in 2d, and $\ell = v_F \tau_0$ is the elastic mean free path. We use units such that $\hbar = k_B = 1$. Note that for a good metal at high densities the Thomas-Fermi screening length is short compared with the elastic mean free path ($\kappa \ell \gg 1$) so that $\tau_1 \ll \tau_0$. In quasi 1d metallic wires (which consist of many transverse channels but permit diffusive motion only in one direction) the leading correction to the average DOS is [1]

$$\nu(\omega) \sim \nu_0 \left[ 1 - \frac{4\Omega_1}{\pi |\omega|} \right], \quad \text{quasi } 1d, \tag{4}$$

where $\nu_0 = 1/(\pi v_F)$ is the DOS per spin in 1d, and the frequency scale $\Omega_1$ depends on the effective electron-electron interaction constant $f_0$,

$$\Omega_1 = \frac{f_0^2}{32\pi D_0}. \tag{5}$$

Here, $D_0 = v_F \ell$ is the (bare) diffusion coefficient in quasi 1d.

Obviously, the correction terms in Eqs. (1) and (4) diverge for $\omega \to 0$, so that at low frequencies these perturbative expressions cease to be valid. What is the true low-frequency behavior of the DOS of disordered metals in
reduced dimensions? The answer to this question is relevant for a number of recent tunneling experiments [3,6,7], where a strong suppression of the tunneling conductance $G(V)$ as a function of the applied voltage has been observed (zero bias anomaly). The tunneling conductance is related to the DOS via $G(V) \propto \nu(\omega = eV)$, so that the experimentally observed zero bias anomaly in the tunneling conductance reflects the strong suppression of the average DOS at the Fermi energy.

In 2d, the low-temperature behavior of the DOS of a strongly correlated disordered metal has recently been measured by Bielejec et al. [3]. While at higher temperatures they found a logarithmic correction, at the lowest temperatures they found a stronger-than-linear suppression of the DOS, which has been interpreted in terms of a hard correlation gap. The knowledge of the low-energy behavior of the DOS of a 2d disordered metal with a finite conductivity might also be important to gain a better understanding of the physical mechanism that is responsible for the apparent metal-insulator transition in doped semiconductor devices [3,6]. An intensely studied quasi 1d system where under certain conditions the electrons propagate diffusively in only one direction are multi-wall Carbon nanotubes [3,4].

Let us briefly review previous calculations of the zero bias anomaly. In 2d, the first attempt to determine the true low-frequency asymptotics of $\nu(\omega)$ was apparently due to Finkelstein [3] who found that $\nu(\omega) \propto |\omega|^{1/4}$ for $\omega \to 0$. However, in the derivation of this result he assumed that the conductivity $\sigma(\omega)$ diverges logarithmically for $\omega \to 0$. The behavior of the DOS of 2d disordered electrons with a finite conductivity was not calculated by Finkelstein. Later this problem was reconsidered by Belitz and Kirkpatrick [3], who found that for frequencies exceeding the crossover frequency $\Omega_2 \equiv \tau_0^{-1} \exp[-1/r_0]$,

$$\Omega_2 \equiv \tau_0^{-1} \exp[-1/r_0] , \tag{6}$$

the perturbative expression given in Eq. (3) can actually be exponentiated, so that

$$\nu(\omega) \approx \nu_0 \exp \left[ -\frac{r_0}{4} \ln(|\omega|/\tau_0) \ln(|\omega|/\tau_1) \right] . \tag{7}$$

This expression has been re-derived in different ways by several authors [3,6,11,12]. Note, however, that Eq. (6) is not valid for frequencies smaller than the crossover frequency $\Omega_2$ defined in Eq. (6). A simple interpolation formula for the DOS, which yields a physically sensible result even for $\omega \to 0$, has been proposed by one of us in Ref. [6]. This formula is based on a re-summation of the leading $|\omega|^2$-singularities to all orders in perturbation theory, consistently neglecting sub-leading terms that involve only logarithmic corrections. In this approximation, one obtains

$$\nu(\omega) \approx \nu_0 \frac{2}{\pi} \int_{r_0}^{\infty} dt \frac{\sin(|\omega|t)}{t} \exp \left[ -\frac{r_0}{4} \ln(t/\tau_0) \ln(t/\tau_1) \right] . \tag{8}$$

For $|\omega| \gtrsim \Omega_2 = \tau_0^{-1} \exp[-1/r_0]$, this expression reduces to Eq. (6). Note that Eq. (8) amounts to an exponentiation of the perturbative result in the time domain, whereas in Eq. (3) the perturbation series is exponentiated in frequency space. In the limit $\omega \to 0$, the leading term in the asymptotic expansion of Eq. (3) is

$$\nu(\omega) \sim \nu_0 \frac{4}{\pi^{1/2}} \left( \frac{\tau_1}{\tau_0 r_0} \right)^{1/2} \frac{|\omega|}{\Omega_2} . \tag{9}$$

Noting that $\sqrt{\tau_0 \tau_1} = r_0/(\nu_0 e^4)$, Eq. (3) can also be written as

$$\nu(\omega) \sim C \frac{|\omega|}{e^4} , \tag{10}$$

where the numerical constant $C$ is given by

$$C = 4(r_0/\pi)^{1/2} \exp[1/r_0] . \tag{11}$$

Surprisingly, Eq. (10) resembles the well-known classical Efros-Shklovskii Coulomb gap of two-dimensional electrons in the localized regime, where the DC conductivity $\sigma(0)$ vanishes [3]. Note, however, that in the derivation of Eq. (10) we have assumed that the DC conductivity $\sigma(0)$ remains finite, i.e. the electrons are assumed to be delocalized. The difference between the localized and the delocalized regime manifests itself in the dimensionless prefactor $C$: whereas in the case of the classical Coulomb gap of localized electrons the constant $C$ is a number of the order of unity that depends on the geometry of the underlying lattice, for the quantum Coulomb gap discussed here, $C$ depends on the dimensionless conductivity of the system. The existence of the Coulomb gap in the delocalized regime of a disordered interacting 2d metallic system was also found numerically by Efros and Pikus [13]. More recently, an intermediate delocalized phase in small clusters of disordered interacting electrons has been found numerically in Ref. [14].

In deriving Eq. (11), we have implicitly assumed that the DC conductivity $\sigma(0)$ does not deviate significantly from the conductivity $\sigma(\omega)$ at frequency $\omega \approx 1/\tau_0$. We have argued elsewhere [10] that a finite renormalization of the conductivity can be approximately taken into account by replacing $r_0 \to r_*$ in Eq. (11) where $r_* = e^2/(2\pi^2 \sigma(0))$ corresponds to the true DC conductivity of the system. The connection between low-frequency behavior of the conductivity and the DOS has also been emphasized by Nazarov [15] and by Levitov and Shytov [16].

The zero bias anomaly in 1d has so far received much less attention than the corresponding anomaly in 2d. Recent experiments on metallic Carbon nanotubes have motivated Mishchenko et al. [16] to study the fate of the perturbative Altshuler-Aronov correction in 1d at low frequencies. They found that long-range Coulomb interactions in a quasi 1d metal lead for $\omega \to 0$ to an exponentially small DOS,

$$\nu(\omega) \propto \exp \left[ -\frac{\varepsilon_0}{|\omega|} \ln \left( \frac{\varepsilon_1}{|\omega|} \right) \right] , \tag{12}$$

where $\varepsilon_0$ and $\varepsilon_1$ are some finite energy scales. We shall further comment on this result below. For frequencies exceeding a crossover scale $D_0/(2\pi R)^2$, where $R$ is the radius
of the nanotube, Egger and Gogolin [17] found a crossover to two-dimensionality, which results in a power-law dependence of the DOS. However, below this crossover scale, 1d behavior is expected.

The problem of calculating the average DOS decouples from the problem of calculating the average conductivity. As a consequence, for sufficiently small frequencies, we may re-sum the most singular terms in the singularities [1,7]. As a consequence, for sufficiently small frequencies, we may re-sum the most singular terms in the singularities [1,7]. In 2d, the problem of calculating the DOS of disordered interacting electrons [9,10,11] is plagued by lnω-singularities, so that a naive application of the above gauge-transformation trick for short-range interactions in 2d is at least problematic. This is also the case for diffusive quasi 1d electrons, where the perturbative calculation of both ν(ω) and σ(ω) leads to |ω|−1/2-singularities [1]. Hence, also in this case it is problematic to calculate the average DOS using Eqs. (15–17) without considering simultaneously the low-frequency behavior of the conductivity. We shall come back to this point in Sec. 4.

3 Zero bias anomaly in 2d

In 2d the RPA-interaction is given by [1]

\[ f_{q_{\text{RPA}}} \approx (2D_0|q|^{-1}) |\omega_m|/q^2. \]  

This expression is valid in the frequency-momentum regime

\[ |\omega_m|/D_0 \ll |q| \ll (|\omega_m|/D_0)^{1/2}, \quad |\omega_m| \ll \tau_0^{-1}, \]  

shown in Ref. [1], under these conditions we may sum the most singular contributions to the average DOS via a simple gauge transformation, which at finite temperature T leads to the following expression for the imaginary time Green function,

\[ G(\tau) \approx G_0(\tau)e^{Q(\tau)}, \]  

where \( G_0(\tau) \) is the Green function of free fermions,

\[ G_0(\tau) = -\nu_0 \frac{\pi/\beta}{\sin(\pi\tau/\beta)}, \]  

and the Debye-Waller factor \( Q(\tau) \) is given by [18]

\[ Q(\tau) = -\frac{1}{\beta V} \sum_{q,\omega} f_{q_{\text{RPA}}}^{\text{RPA}} \frac{\omega + |q|^2 + |\omega_m|^2}{|q|^2 + |\omega_m|^2} \left[ 1 - \cos(\omega_m \tau) \right]. \]  

2 Summing the leading singularities via a gauge transformation

The average DOS of an interacting Fermi system at finite temperature \( T = 1/\beta \) can be written in terms of the disorder-averaged Green function at coinciding space points \( G(\omega) \equiv \tilde{G}(r,r,\omega) \) as

\[ \nu(\omega, T) = -\frac{1}{\pi} \coth \left( \frac{\beta \omega}{2} \right) \text{Im} G(\omega). \]  

To make contact with the disorder-averaged imaginary-time Green function at coinciding space points \( G(\tau) \), we notice that due to particle-hole symmetry near the Fermi energy we have

\[ G(\omega) = -2 \int_0^\infty dt \sin \omega t G(\tau \to it + 0^+). \]  

The problem of calculating the average DOS is now reduced to the problem of calculating the disorder-averaged imaginary-time Green function. An attempt to calculate \( \nu(\omega) \) within perturbation theory fails at low frequencies because the perturbative expansion of \( \nu(\omega) \) is plagued by singularities, see Eqs. (13–17). In 2d with long-range Coulomb interactions, these singularities diverge as ln^2 ω. At the same time, however, the perturbative expansion of the conductivity \( \sigma(\omega) \) contains only less severe lnω-singularities [1]. As a consequence, for sufficiently small frequencies, we may re-sum the most singular terms in the expansion of the average DOS without considering simultaneously a similar re-summation for the average conductivity, because we know a priori that ln^2-singularities do not appear in the calculation of \( \sigma(\omega) \). In this sense, the problem of calculating the average DOS decouples from the problem of calculating the average conductivity. As

2 Summing the leading singularities via a gauge transformation

The average DOS of an interacting Fermi system at finite temperature \( T = 1/\beta \) can be written in terms of the disorder-averaged Green function at coinciding space points \( G(\omega) \equiv \tilde{G}(r,r,\omega) \) as

\[ \nu(\omega, T) = -\frac{1}{\pi} \coth \left( \frac{\beta \omega}{2} \right) \text{Im} G(\omega). \]  

To make contact with the disorder-averaged imaginary-time Green function at coinciding space points \( G(\tau) \), we notice that due to particle-hole symmetry near the Fermi energy we have

\[ G(\omega) = -2 \int_0^\infty dt \sin \omega t G(\tau \to it + 0^+). \]  

The problem of calculating the average DOS is now reduced to the problem of calculating the disorder-averaged imaginary-time Green function. An attempt to calculate \( \nu(\omega) \) within perturbation theory fails at low frequencies because the perturbative expansion of \( \nu(\omega) \) is plagued by singularities, see Eqs. (13–17). In 2d with long-range Coulomb interactions, these singularities diverge as ln^2 ω. At the same time, however, the perturbative expansion of the conductivity \( \sigma(\omega) \) contains only less severe lnω-singularities [1]. As a consequence, for sufficiently small frequencies, we may re-sum the most singular terms in the expansion of the average DOS without considering simultaneously a similar re-summation for the average conductivity, because we know a priori that ln^2-singularities do not appear in the calculation of \( \sigma(\omega) \). In this sense, the problem of calculating the average DOS decouples from the problem of calculating the average conductivity. As
which is responsible for the $\ln^2$-corrections to the DOS. Converting the sum over Matsubara frequencies into an integral, the Debye-Waller factor (analytically continued to real time) may be written as

$$Q(it) = \frac{r_0}{2} \int_0^{\tau_0-1} \frac{d\omega}{\omega} \ln \left( \frac{\omega}{D_0 e^{2}} \right)$$

\[ \times \left[ 1 - \cos(\omega t) \right] \tan(h \beta \omega/2) + i \sin(\omega t) \right], \quad (20)\]

where we have taken the thermodynamic limit $V \to \infty$. To be consistent with the approximation made in Eq. (15), we retain only $\ln^2$-singularities and ignore all terms involving only single logarithms: Within this approximation the imaginary part of $Q(it)$ vanishes, and for arbitrary temperatures we find $Q(it) \sim -\frac{1}{2} \ln(t/\tau_1) \ln(t/\tau_0)$. The DOS at finite temperature $T = 1/\beta$ is now given by

$$\nu(\omega, T) \approx \nu_0 \coth \left( \frac{\beta \omega}{2} \right) \frac{2}{\beta} \int_{\tau_0}^{\infty} dt \frac{\sin(\omega t)}{\sinh(\pi t/\beta)}$$

\[ \times \exp \left[ -\frac{\tau_0}{4} \ln(t/\tau_1) \ln(t/\tau_0) \right]. \quad (21)\]

In the limit $T \to 0$ this equation reduces to Eq. (8). On the other hand, if we first take the limit $\omega \to 0$ and then consider the leading behavior at low temperatures, we obtain for $T \ll \Omega_2$

$$\nu(0, T) \sim 2C T e^{-1}, \quad (22)\]

For example, taking the (sub-leading) imaginary part of the full Debye-Waller factor given in Eq. (22) into account would lead to a nonlinear suppression of the DOS. However, as discussed above, such an approximation would not be systematic.

**Fig. 1.** Graph of the average DOS $\nu(\omega)$ in 2d for various temperatures, see Eq. (21). We have chosen $r_0 \to r_1 = 1/\pi$ and $\tau_1 = \tau_0$. Curves from top to bottom are for $T/\Omega_2 = 0.8, 0.4, 0.2, 0.1, 0.05, 0.025, 0$.

**Fig. 2.** Temperature-dependence of the average DOS $\nu(T)$ at the Fermi energy in 2d. The parameters are again $r_0 \to r_1 = 1/\pi$ and $\tau_1 = \tau_0$.

### 4 Zero bias anomaly in 1d

As discussed in Sec. 3 the naive application of Eqs. (13, 17) in quasi 1d is problematic, because in this case the interactions corrections to the DOS and to the conductivity both involve the same type of singularities. Keeping this caveat in mind, let us nevertheless briefly discuss the
predictions of Eqs. (15-17) in quasi 1d. A similar calculation has recently been performed in Ref. [16].

Since screening is much less effective in one dimension than in higher dimensions, we simply approximate the one-dimensional RPA-interaction by a constant, \( f_{\text{RPA}} \approx f_0 \). This approximation should be correct up to logarithmic corrections in frequency. A calculation analogous to the one leading to Eq. (21) results in

\[
Q(it) = -\sqrt{2\Omega_1} \left[ \int_0^\infty \frac{d\omega}{2\pi} \frac{1 - \cos(\omega t)}{\omega^{3/2}\tanh(\omega/2)} + i\sqrt{t} \right],
\]

where \( \Omega_1 \) is given in Eq. (8). Note that in 1d, there is no need for an ultraviolet cutoff. At \( T = 0 \), we have \( \text{Re} \, Q(it) = \text{Im} \, Q(it) \), such that the imaginary part of \( Q(it) \) cannot be neglected. The DOS at finite temperature \( T = 1/\beta \) can now be written as

\[
\nu(\omega, T) \approx \nu_0 \coth \left( \frac{\beta \omega}{2} \right) \int_0^\infty dt \frac{\sin(\omega t) \cos(\sqrt{2\Omega_1 t})}{\sinh(\pi t/\beta)} \times \exp \left( -\sqrt{\frac{2\Omega_1}{\pi}} \int_0^\infty d\omega' \frac{1 - \cos(\omega' t)}{(\omega')^{3/2}\tanh(\omega'/2)} \right).
\]

As can be easily checked, for \( |\omega| \gg \max\{\Omega_1, T\} \), Eq. (24) reduces to the perturbative result given in Eq. (3). A graph of \( \nu(\omega, T) \) for different temperatures is shown in Fig. 2. The DOS at the Fermi energy as a function of temperature is shown in Fig. 4. The DOS is well approximated by \( \nu(T) \approx \nu_0 \exp\left(-\sqrt{2\Omega_1 T}/\pi\right) \) which is quite similar to a result found in Ref. [14]. At zero temperature, Eq. (24) may be simplified to

\[
\nu(\omega, T = 0) = \nu_0 \frac{\text{Re}}{\pi} \int_0^\infty dx \frac{\sin x^2}{x^2} \exp \left[ -2x \sqrt{\frac{i\Omega_1}{|\omega|}} \right].
\]

For \( |\omega| \ll \Omega_1 \), this integral is easily evaluated within a saddle-point approximation,

\[
\nu(\omega, T = 0) \sim \nu_0 \left[ \frac{\Omega_1}{|\omega| \pi\Omega_1} \right] \left( -\sqrt{\frac{\Omega_1}{|\omega|}} \right), \quad |\omega| \ll \Omega_1,
\]

which differs from the corresponding expression given by Mishchenko et al. [16] (see Eq. (12)) by a different prefactor. The exponential suppression of the DOS is quite surprising and is possibly an artifact of the inconsistent exponentiation of the \( |\omega|^{-1/2} \)-singularities inherent in Eqs. (21) and (12) for quasi 1d systems. Recall that for long-range Coulomb interactions in 2d it is possible to separate \( \ln^2 \)-terms from sub-leading terms already neglected in the approximation made in Eq. (13). In 1d, a similar separation is not possible. The ultimate low-frequency behavior of the DOS in 1d might therefore be altered by diagrams neglected in Eq. (13).

5 Summary and conclusions

In summary, we have considered the fate of the singular perturbative corrections to the average DOS in quasi 1d and 2d disordered metals when the frequency and the temperature are reduced such that one leaves the perturbative regime. For a 2d metal with long-range Coulomb interactions, it is possible to re-sum the leading \( \ln^2 \)-singularities in the perturbative expansion of the average DOS to all orders in perturbation theory. The method relies on the fact that similar singularities do not appear in the perturbative expansion of the average conductivity. For a 2d system with short-range interactions or for a quasi 1d system a similar separation of energy scales does not exist, so that approximate expressions of the type given in Eq. (13), which involve the exponentiation of a certain subclass of Feynman diagrams, are problematic in this case. Thus, in practice the gauge-transformation trick described above, which has recently been employed by many authors [8].

\[\text{Fig. 3. Graph of the average DOS } \nu(\omega) \text{ in 1d for various temperatures, see Eq. (24). The curves from top to bottom are for } T/\Omega_1 = 32, 16, 8, 4, 2, 1, 0.5, 0.25, 0.\]

\[\text{Fig. 4. Solid line: graph of the average DOS } \nu(T) \text{ for } \omega = 0, \text{ see Eq. (24). Dotted dotted line: the approximation } \exp(-\sqrt{2\Omega_1 / T}).\]
is only controlled in the case of 2d disordered electrons interacting with long-range Coulomb forces in the regime where the conductivity has a finite DC-limit. In this case this method yields a simple interpolation formula (21) for the average DOS, which predicts a smooth crossover from the perturbative regime at high frequencies to a new low-frequency regime, where $\nu(\omega, T)$ vanishes linearly in $\omega$ or $T$. The average DOS of localized classical electrons is known to show a similar frequency- or temperature-dependence [12]. In contrast, the metallic Coulomb gap discussed here has a quantum mechanical origin and requires delocalized electrons with a finite DC conductivity. Numerical evidence for such a metallic Coulomb gap has been found in Ref. [13].

In principle, it should be possible to verify the existence of the metallic Coulomb gap experimentally via tunneling experiments in strongly correlated disordered systems with a finite DC conductivity. The expected shape of a typical trace of the tunneling conductance as a function of the applied voltage is shown in Fig. 1. Recent tunneling experiments by Bielejec et al. [2] in quench-condensed Beryllium films show a crossover from the perturbative regime with logarithmic corrections to an apparently linear Coulomb gap in the DOS. However, at the lowest temperatures the DOS exhibits a hard correlation gap, the origin of which remains open.

References

1. B. L. Altshuler and A. G. Aronov, in Electron-electron interactions in disordered systems, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
2. E. Bielejec, J. Ruan, and W. Wu, Phys. Rev. Lett. 87, 036801 (2001).
3. A. Bachtold, C. Strunk, J. P. Salvetat, J. M. Bonard, L. Forró, T. Nussbaumer, and C. Schönenberger, Nature 397, 673 (1999); C. Schönenberger, A. Bachtold, C. Strunk, J. P. Salvetat, and L. Forró, Appl. Phys. A 69, 283 (1999).
4. A. Bachtold, M. Fuhrer, S. Pylaunov, M. Forero, E. H. Anderson, A. Zettl, and P. L. McEuen, Phys. Rev. Lett. 84, 6082 (2000).
5. S. V. Kravchenko, W. E. Mason, G. E. Bowker, and J. E. Furneaux, Phys. Rev. B 51, 7038 (1995).
6. For a recent review see E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. 73, 251 (2001).
7. A. M. Finkelstein, Zh. Eksp. Teor. Fiz. 84, 168 (1983) [Sov. Phys. JETP 57, 97 (1983)].
8. D. Belitz and T. R. Kirkpatrick, Phys. Rev. B 48, 14072 (1993).
9. S. Levitov and A. V. Shytov, Prisma Zh. Eksp. Teor. Fiz. 66, 200 (1997) [JETP Lett. 66, 214 (1997)].
10. P. Kopietz, Phys. Rev. Lett. 81, 2120 (1998).
11. A. Kamenev and A. Andreev, Phys. Rev. B 60, 2218 (1999).
12. A. L. Efros and B. I. Shklovskii, in Electron-Electron Interactions in Disordered Systems, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
13. A. L. Efros and F. G. Pikus, Solid State Commun. 96, 183 (1995).