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Varied approaches to loading assessment in fatigue studies

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**ABSTRACT**

In this paper, the authors want to draw readers’ attention to one long-standing question: which approach is preferable for estimation longevities in fatigue problem, the time domain approach (Rainflow) or the frequency domain one (Dirlik and others)? This question is important in engineering problems, particularly in problems of prolongation of the guaranteed service life. The discussion here is restricted by the longevity evaluation only at the post-processing stage of unidirectional loaded machine parts. It means the realizations might be recorded. Some experimental and speculative evidence of preferable use the Rainflow method is shown. Taking into account the huge computer’s power nowadays, the question of the irrelevance of appellation to the calculation accelerating using the spectral methods is specially discussed. There are areas, where the spectral methods are really necessary. There is only a need to recommend the restriction of their application scope to these special situations. It seems there is no need in inventing new spectral complicated algorithms only to stress out at the end, that their result coincides with the Rainflow outcome. That might be confusing for the practicing engineers. Currently, the main attention among supporters of spectral methods is focused on non-stationary and non-Gaussian random processes. (to estimate the spectral density is impossible for non-stationary processes, according to definition). These researchers seem to have forgotten, that even for these complicated situations the decision has already existed: that is the Rainflow and its analogues. The paper shows extensive laboratory experiment results of random fatigue testing of aluminum flat specimens under regular (to build the fatigue curve) and irregular (random) loading. The fatigue life curve (Gassner curve) has been built. These results allowed to compare the existing computation methods of longevity estimation. In the particular situation of narrow-band process, the methods seem to provide comparative results. Considered are methodological issues related to the assessment of the necessary and sufficient realization length, the influence of RMS, cycle counting methods and some possibilities of computing resources saving when using the Rainflow method. The stability of the Rainflow estimates is confirmed. Some problems with the choice of parameters during the longevity assessment by the Dirlik method were noted.

1. Introduction

This paper is an extended version of [1], which has been prepared for VAL4 Conference, canceled because of coronavirus. (See Table 1)

In situations where both component prototype as well its service load realization are available, the question arises as to whether the time domain or frequency domain approach preferred to assess component durability. In the time domain, the Rainflow method[2] is a proven and experimentally validated approach to consider random loading while studying machines’ longevities under fatigue. On the other hand, the spectral methods have their own sphere of applications, where they are the only way to estimate durability.

Contemporary approaches often recommend spectral methods at the post-prototype stage for reasons that may not be truly justifiable. In doing so, similarity of their results with Rainflow based estimates is often cited [3,4]. Today’s tendency is to adjust the applicability of the spectral method into non-Gaussian and non-stationary processes problems [5]. In that situation, spectral density cannot be estimated. However, Rainflow works well even in the situation with such processes. Also, the requirement of the linearity of the systems is not a must for Rainflow.

Reasons cited in preference of spectral methods include the calculation accelerating by applying the frequency domain estimation [6]; 2)
It can be argued that nowadays it is not quite reasonable to concentrate on selecting only the turning points (extremums) of random processes because they are supposed to be responsible for fatigue accumulation. Proper processing of the random processes with concentration exclusively on extremums accelerates the Rainflow procedure. The level crossing technique required for this treatment is described in App.1.

With regard to the second reason, one may argue, that any sample estimation bears “the curse of sample”, i.e. the precision of any estimate whether in frequency or time domain is restricted because of the finite length of registered loading realization.

There are several examples of successive application of Rainflow in engineering problems. There are 589 citations of the original paper written by Endo by now [2]. A short survey made on the base of the book of VAL4 Proceedings [10], concerning comparative use the Rainflow and the spectral approaches, has shown that among the authors of 74 papers in total in VAL4, more than 13% employ the Rainflow for their problems. Among the papers presented in the session D (Commercial Vehicles and Transport – which are more practical tasks) the percentage is even higher: 50%. This meta-analysis gives us a hint, that in practical applications the Rainflow plays a leading role.

In [11] it was pointed out, that for the stresses which occur during load oscillation processes, the basic principles are known from the theory of continuous random processes can be applied only under certain conditions. The values, which matter for random loading fatigue estimation are the maximum nominal stress for the spectrum \( \sigma_{\text{max}} \), the loading block length \( L_b \), and the shape of the spectrum \( v \). All the named values are the results of the Rainflow cycle counting. Further important quantities include the mean stresses and the cress factor. Some investigations show on the dependence of the fatigue on frequency [12].

The experiment was planned to provide a basis for proving and comparing varied methods for fatigue durability estimation under random loading at the first stage of fatigue. The crack propagation stage was not considered here.

The experimental study of the durability of aluminium AMg-61 specimens (Fig. 1) with thickness 2 mm under narrow-band loading was performed on the electro-vibrator device (Fig. 2) until complete failure [14]. Chemical composition of the AMg-61 alloy is 94.30% of Al and 5.70% of Mg; the tensile strength is \( R_n = 395 \) MPa. The total number of specimens tested under harmonic and random loading was 104.

The specimens were tested in cantilever bending, as the type of one of the most common types of structures loading. The equipment used makes it possible to eliminate tensile stresses when fixing the samples (Fig. 2).

To control the setting of the stress level in the specimens during vibration tests, the dependence of the maximum bending stresses in the specimen on the displacement of its edge was derived. The dependence was obtained by analyzing the design finite element model of the specimen. In Fig. 1a the distribution of equivalent stresses on the specimen surface is shown. To verify the model, an experimental assessment of the stress–strain state of the specimen under static loading was performed. Specimens were fixed in the tooling on the movable table of the
vibrator. Static loading was carried out by a fixed displacement of the movable table of the shaker using compressed air. Deformations in the specimens were recorded by strain gauges with a base of 3 mm, connected by a half-bridge circuit (Fig. 1b). The experimental data were compared with the calculated ones, which were averaged on the plate of the installed accelerometer sensor. The tests were carried out at the standard temperature of 20 °C and the air humidity of 40–60%.

The most detailed testing was performed to get the fatigue curve characteristics under symmetrical regular loading. The experimental points are shown in Fig. 3. Testing was carried out at seven stress amplitude levels \(\sigma_a\): 155, 145, 127, 105, 98, 70 and 61 MPa. One point in the graph corresponds to one broken specimen. Using the linear regression analysis, the coefficients of the fatigue curve were estimated:

\[
\lg(N) = 14.947 - 4.717\lg(\sigma_a)
\]  

Following the purpose of the study, the series of random loading were performed. To provide more stable experimental estimation, several specimens have been tested in each level of random loading. The stationary Gaussian loading was imitated to meet the spectral density of vibration acceleration (Fig. 4a). The frequency range of accelerations was set from 5 to 25 Hz. The hardware equipment guarantees the performing the random vibrations implemented stationary ergodic random processes with the normal law of distribution of ordinates of vibration accelerations on the vibration table (Fig. 2). On the base of registered stresses in Fig. 4b, the estimated stresses spectral density for the most loaded point is shown.

Although the random realization of acceleration shows the traits of a normal Gaussian random process, the resulting realization of stress is restricted in its emissions due to some physical reasons, that means, that the crest factor of stress is limited. It provided the opportunity to estimate the reasonable realization length (the explanations see below, also Fig. 10).

The part of the random process realization with the details of the process is shown in Fig. 5. The random loading process \(S(t)\) was normalized by dividing its ordinates by RMS(S) to \(x(t)\). That means, that RMS(\(x\)) = 1. The longer part of the same process is shown in Fig. 6.
Fig. 6, the time-scale is different to demonstrate the envelope curve of the process better. To estimate the errors of characteristics of a recorded random process, the analysis has been made to estimate the influence of the length of realization recorded (see below, in paragraph 5 and Fig. 10). Looking at Figs. 5 and 6, one can see, that although the process possesses a clear structure, it is not simple. The Irregularity factor, estimated numerically by the realization is \( I \approx 0.67 \). Estimation of \( I \) value on the base of Rice theory [14] provides almost the same value of \( I = 0.67 \). As one can see, those estimates coincide pretty well. Additional information concerning \( I \) factor estimation is in Appendix 2.

Random testing was carried out at four levels of RMS: 126, 108, 83 and 73 MPa. The results of this experiment in semilogarithmic scale are shown in Fig. 7. The load was applied until breaking the specimens, and each point in Fig. 7 corresponds to one broken under random loading specimen. It is worth mentioning that the fatigue life curve presents the dependence of the number of cycles until failure on the value of random processes RMS. The number of cycles in this situation was estimated as the number of crossing mean process level by the centered random processes. Due to almost regular character of loading, there is no problem in estimating the cycle number (unlike in the strongly irregular loading case, see App. 2, Fig. 14). Similarly, the longevities \( N \) might have been expressed in time – the character of the graphs wouldn’t have changed much due to the stable effective frequency \( (f_{ef} = 17 \text{ Hz}) \).
experiment result shown in Fig. 7 was used for comparing varied methods of estimations (see below). The fatigue life curve (Gassner curve) obtained by linear regression analysis is shown in Fig. 7.

The clear structure of the random process under investigation (Fig. 5) also guarantees the simple way for recalculation the RMS coordinate into $\sigma_{\text{max}}$ as it was recommended in [15,16].

4. Methods for machines durability estimation

According to the time-domain method of durability estimation, the researchers deal with realization, recorded during representative enough time interval. To apply the Miner’s summation rule, the cycle counting procedure on this realization is required. This requirement follows from the Miner idea of linear summation of the damages, according to which the limiting case (fracture) happens when the sum of relative damages reaches unity:

$$\sum \frac{n_i}{N_i} = 1$$ (2)

In numerator of equation (2), which is further used for durability estimation, stand $n_i$ - the numbers of repetitions of the amplitudes $\sigma_{ai}$ until failure; in the denominator, stand $N_i$ - the limiting cycle numbers to failure taken from the fatigue curve equation (1).

Without diminishing the value of Miner hypothesis, which is the most widespread and works well especially in comparative studies, [15–17] one must note its biased nature. To diminish that bias in 1970-es the so-called Corrected linear hypothesis was introduced [15]. Later, in [18] it was transformed into the Simplified Corrected linear hypothesis:

$$\sum \frac{n_i}{N_i} = 0.25$$ (3)

The similar recommendations about the correction of Miner hypothesis may be found elsewhere [17,19]. Thus, on the base of the statistical study of experimental data, it was proposed that in case of the aluminium specimens under random loading:

$$\sum \frac{n_i}{N_i} = 0.37$$ (4)

This recommendation may be valid even in case of Haibach’s extension of the fatigue curve [20] for $N_i$ definition. It should be mentioned, that the experimental value of $\sum n_i/N_i$ until failure also demonstrates the large scatter [15,19].

The loading block of irregular loading is described by the $f(\sigma_{ai}, n_i)$ dependence. It is similar to the histogram and is the result of the cycle counting procedure. For durability estimation, it should be supplemented by the information about the length of the loading block $\text{lb}$. In

dependence $f(\sigma_{ai}, n_i)$ values $n_i$ show the number of repetition of the Rainflow cycles with amplitude $\sigma_{ai}$ in the realization $\text{lb}$. It might be also presented in the continuous form (see also the graphs in Fig. 9). The most widespread cycle counting procedure nowadays is the Rainflow [2,11,15]. In this particular situation of almost regular, narrow-band process, the other cycle-counting methods also work well. Peak Counting and Ranges Counting are the oldest ones and might serve for comparison of the calculating results: the methods provide the upper and the lower limiting estimation of durability.

To estimate the sufficient for the stable durability estimation the realization length $\text{lb}$, the method from [21] was employed. The paper [21] provides a detailed explanation of that approach. The idea is based on the fact, that for many random loading processes after extending the definite realization length, the durability estimation based on it will not vary significantly. It follows from the fact of distribution $f(\sigma_{ai}, n_i)$ stabilization. The processes for which such stabilization exists form the so-called family of stable in specific sense processes. The word “specific” addresses the problem of durability estimation together with the Miner hypothesis. Of course, that method won’t work in the situation, when different exploitation modes are investigated. The stable in specific sense processes are not necessarily stable in the
classical theory of random possesses. The best value to characterize such stabilization in the tasks of durability estimation is the spectra fullness ratio \( v \) [11]:

\[
v = \frac{\sum n_i \left( \frac{a_{\text{max}}}{a_{\text{min}}} \right)^m}{\sum n_i}
\]

(5)

The value \( v \) characterizes the shape of the histogram \( f(\sigma_a, n_i) \). As one can see from the formula (4), \( v \) depends also on fatigue exponent \( m \). The value of non-dimensional \( v \) value is less than 1 and might serve for the equivalent amplitude estimation while estimating durability. The results of the stabilization of \( v \) together with stabilization of some other characteristics with the increase of the realization length \( l_b \) are shown in paragraph 5, Fig. 10.

It is not always possible to record actual random loading seen by a component, which is especially true for products at the design stage [8]. The way out of this situation is to use the spectral approaches. Their main idea is to represent the histogram of the distribution of Rainflow cycles by semi-empirical dependencies, using the spectral characteristic of the process as an input parameter. In this case, durability is usually also estimated according to the hypothesis of linear damage summation. The spectral approach to the assessment of fatigue life significantly reduces the amount of input data [6,8], but at the same time, it is at some extent controversial and has its own restrictions. For example, the applicability of spectral methods is limited to stationary Gaussian random processes. The choice among one spectral method or another strongly depends on the irregularity factor of the process [4-6]. The other limitation of the spectral methods is the linearity of the systems.

Considering the calculation methods in the time and frequency domains, it can be noted that the only fundamental difference is the method of calculating the loading cycle distribution \( f(\sigma_a, n_i) \). The further calculation is reduced to the application of one of the damage summation hypotheses, which implies comparing the accumulated number of cycles with its limiting value according to the material fatigue curve. However, it should be noted that the fatigue curve under constant amplitude loading is built with a certain fixed loading frequency, which introduces an additional error. Plotting the fatigue curve in RMS-N coordinates in some cases can make it possible to significantly refine the assessment of durability under random loading [14].

Another approach to assessing the durability under random loading is to obtain material fatigue diagrams for a fixed initial time series of a random loading process [14,22]. This eliminates serious errors that are actually associated with a comparison of a random loading process with a set of regular (harmonic) processes (it is under harmonic loading that the material fatigue curve is obtained) and the options for their summation. In other words, the assumptions associated with cycle counting and summation of damages are removed in that way. It should be noted that this approach is the most laborious and time-consuming and is recommended for use only for assessing the durability of particularly critical structures.

5. Results and discussion

While dealing with fatigue, one cannot avoid the question of the result scatter. The detailed analysis of the scatter by applying the model of normal distribution has been performed in [14]. In this study for judging about the experimental scatter of the sample data, the variation factor was chosen as a reliable estimate. The variation factor \( V_k \) of log-normal distribution in k-th level of loading is defined as:

\[
V_k = \frac{\text{RMS}_k (\lg(N))}{\text{MEAN}_k (\lg(N))}
\]

(6)

In Fig. 8, variation values \( V_k \) depending on durability \( \lg(N) \) are shown for both options: regular and random. One could expect that for random loading, the variation coefficient would be greater compared with the regular loading. The data, however, do not prove that suggestion. This might follow because of the different number of specimens tested.

To compare the cycle counting results by varied methods, for the realization, shown in Figs. 5 and 6 the Kerner smoothing [23] is shown in Fig. 9. One can see that the distributions vary only slightly. It happens due to the fact, that the loading process possesses the simple structure (\( I = 0.68 \)). The most damaging is the result of Peak Counting. It shows more cycles with bigger amplitude. The lightest is for Range Counting. The Rainflow result is in the middle.

If in this particular example of random loading and estimated earlier fatigue exponent \( m = 4.73 \) the durability estimated by Rainflow would be taken as 100\%, then the durability estimated by Range counting is 108\% and the durability estimated by Peak Counting is 70\%.

In paragraph 4 the method of sufficient loading realization length estimation was mentioned [21]. The graphs of the main values responsible for durability estimation is shown in Fig. 10. Although the maximum amplitude \( a_{\text{max}} \) continues to grow with \( t \) increasing, the values of RMS and \( v \) stabilize after about one minute of recording. The random process in this study has the traits of the stationary in a specific sense process [21]. It means, that estimated durability stabilizes with \( l_b \) increasing. In this example increase \( l_b \) more than 60 s, will not change noticeably the durability estimation. This approach also serves as proof of the robustness of the Rainflow & linear summation approach.

For the construction of the Rainflow loading block \( f(\sigma_{ai}, n_i) \), which
was further used for the fatigue estimation, the realization with the length of \( t_b = 60 \) s might be taken, which guarantee the stabilization of estimated durability (see Fig. 10). Further, as it was recommended in [17] for aluminium materials, the correction of Miner hypothesis has been done with the constant 0.37, i.e. for fatigue life estimation the equation (4) has been used.

Fig. 11 presents information about experimental longevities (repeating at some extend the Fig. 7) with an addition of the calculated results. The estimation, based on Miner hypothesis (2), shows significant unconservative error. Calculations according to equation (4) with the correction “0.37” of Miner hypothesis, which was recommended in [17,19], seems to be consistent. On the other hand, the choice of the constant (0.37 or 0.25) or the choice among the equations (2 and 3) is rather arbitrary and is needed to be backed up with the detailed study. In fatigue, the scatter also plays an important role.

To assess the durability by spectral methods, the Dirlik method [24] was used as recommended by most researchers as the best. The results of comparison of the calculated according Dirlik method longevities (one of the options of initial data for calculation) with the experimental results are shown in Fig. 12.

The results coincide well— that is true in this particular situation. As it was known before [4–5], for a stationary narrowband process all approaches give the reasonably good estimate. On the other hand, more approbated Rainflow with clear physical meaning could be proposed as a more reliable instrument for durability estimation at the post-processing stage in uniaxial loading. As one of our correspondents said: “I am glad you insist on the time domain method as lots of people who use spectral methods do not fully understand the assumptions and risks behind these methods” [25].

6. Conclusions

This study was devoted to the comparison of time and frequency domain approaches to fatigue analysis.

The data of an extended experiment on the accumulation of fatigue damage in aluminium samples under random loading at a laboratory are presented. The metrology of the experiment made it possible to assess the durability using different methods for their subsequent comparison. The estimation of the durability in time domain based on Rainflow cycle counting and corrected linear hypothesis has shown fair, robust estimation for all levels of the stress of random loading in the experiment. It is shown also that in this particular case of loading processes close to narrow-band ones, different approaches give comparable results.

The final decision – Rainflow or spectral methods – is possible only thoroughly planned laboratory experiment. It would also be important to renew the previous attempts of registration of all extremums during the machine’s lifetime together with the Rainflow real-time processing. The modern processors will easily guarantee such record. Apart from service history monitoring, the obtained all service life Rainflow histogram will provide a unique chance to compare the estimated and service longevities, which means the service-life experiment.

As pointed out by Kogaev [15], Rice’s theory applies to a limited type of random processes. It might be employed on the design stage of a new machine when the prototype sample of the machine does not yet exist.

Declaration of Competing Interest

The authors declared that there is no conflict of interest.
Appendix 1

Simple tools for Rainflow computing optimization

Some authors (for instance, [6,8]) recommend using the spectral approach to speed-up the computation comparing with the Rainflow. Sometimes the complaints can be heard about the large data storages. Here, we could not miss an opportunity of presenting some simple instruments which help optimize the calculations while using the time-domain methods. Saving resources might be important also in the system of on-line monitoring of the technical state [26]. Here are some proposals.

1. Level-crossing technique. Time-consuming computation problems in time domain could be speeded-up by appropriate way of random process treatment. Because for the Rainflow as well as for other cycle counting methods, the crucial information is the information about extremums and their sequence, there is no need to perform detailed time discretization. The researcher can get the proper result much faster by performing the so-called level crossing discretization. The Table shows the time gain by using the level crossing method. Random sequences from [20] together with an original method for constructing the continuous process out of the sequence of extremums [27] were used for the model experiment. The Table shows a significant gain in time due to the use of the proper data processing (also, [11]).

2. Proper choice of the number of discretization levels. In comparison, realizations processing the proper choice of the number of levels by the ordinate is crucial. Typical analogue to digital conversion systems produce 6 to 16 bits, which corresponds to a range of 64 to 65,536 levels. Taking into consideration the accuracy of stresses registration, 64 levels seems to be sufficient. The employing of such level number simultaneously guarantees the primary filtration, which also speeds up the processing.

3. Markov’s matrix convolution. The well-known form of data storage is matrix representation [11,20]. In Fig. 13, the example of so-called Markov’s matrix is shown. For demonstration purpose, only 12-levels are presented. The grey areas in the Table show the indexes. This example corresponds to the filled-up matrix for the part of TWIST sequence adopted in testing in aviation. The matrix contains the number of repetitions of upward and downward ranges of the random TWIST process. That form of representation is compact enough and bears all the traits of the investigated process. The matrix might be as well be employed for random process modelling [28].

Appendix 2

Problems with estimating the irregularity factor $I$

Problem 1. This coefficient characterizes well the character of random loading. It also defines the distribution of the local maximums according to the Rice theory [13]. Nevertheless, some problems exist with $I$ estimation.

The most stick way is the numerical estimation through the representative enough loading realization. According to the definition

$$I = \frac{N_0}{N_e} \quad (7)$$

where $N_0$ is the number of mean value level crossing and $N_e$ is the number of extremums in realization. Rice theory proposes the estimation of $I$ through the integrals of moments of spectral density [13]. Basing on the integration of the spectral density function, which was shown in Fig. 4b integration provides the result: $I = 0.67$ which is very close to the value, estimated numerically by the formula (7) ($I = 0.68$).

Problem 2. It is also not so straightforward with the numerical estimation by the formula (7). While performing the digital processing of the signal, the number of digit levels $k$ for processing should be chosen. Due to specific of computer action $k = 2^n$, where $n = 5\ldots 8$. In problems connected with fatigue estimation, the division by levels serves also as the first filtration step to eliminate the small fluctuations (See also App.1).

Given as the example, the set of realizations of stationary loading random processes in some modes of exploitation of mountain bicycles [29] the analysis of the impact of $k$ on the estimated by formula (7) $I$ was performed. Those processes were of a rather complicated structure (wide-band processes). Depending on $k$ the estimated $I$ will vary (Fig. 14). The most drastically $I$ varies in the recommended interval of division ($k = 32\ldots 128$). $I$ decreases up to 25% ... 50% for different bike parts and different exploitation modes.

The number of selected extremums also depends on $k$. In Fig. 15 for the same processes in bicycles [30] the dependences $N_{reg}(k)$ are shown. The graphs also show that the $N_{reg}$ might increase two-folds.

| k     | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|
| 1     | 0  | 0  | 0  | 0  | 0  | 2  | 0  | 0  | 0  | 0  | 0  | 0  |
| 2     | 0  | 0  | 0  | 0  | 0  | 17 | 1  | 1  | 0  | 0  | 0  | 0  |
| 3     | 0  | 0  | 0  | 0  | 0  | 191| 32 | 1  | 0  | 0  | 0  | 0  |
| 4     | 0  | 0  | 0  | 0  | 0  | 2277| 191| 17 | 0  | 2  | 0  | 0  |
| 5     | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 6     | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 7     | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 8     | 1  | 19 | 196| 2277|0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 9     | 1  | 0  | 26 | 197 |0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 10    | 0  | 0  | 2  | 17  |0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 11    | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 12    | 0  | 0  | 0  | 2  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |

Fig. 13. Markov’s matrix representation (simplified) for the TWIST sequence.
Appendix 3

**Influence of RMS on durability**

While speaking about the spectral density, the question of the connection of random process standard deviation RMS with durability will inevitably arise. It follows from the fact that the spectral density is the distribution of the process power RMS on frequency values. On the base of real stationary loading process [29] the connection of the estimations of two random non-independent values RMS (root mean square) and the maximum amplitude of the spectra $\sigma_{\text{amax}}$ was built (Fig. 16). According to the Rainflow method $\sigma_{\text{amax}} = 1/2 \text{ RANGE}$, where the range value of the realization $\text{RANGE} = \text{MAX} - \text{MIN}$. The random sample pairs correspond to the estimates for the selected parts of the long enough stationary realization of stresses in the parts of a mountain bike. It might be seen that the positive correlation exists, which is not surprising. The important fact here, that therefore the random value of the RANGE influents strongly upon the durability we have no serious reasons to deny the influence of RMS on it.

On the other hand, the data from the paper [16] prove, that elimination of small cycles from the loading process lead to the greater RMS, while the durability during this transformation did not change much. This fact contributes to the point of view of non-dependence of the durability on spectral...
densities. It might be said that the use the Rainflow is preferable.

Fig. 16. The connection of the sample RMS and $\sigma_{\text{max}}$ for 8 parts of loading realization [29].

References

[1] Gadolina IV, Erpalov AV. Using the experimental data in form of Gasser curve to compare the methods for durability estimation. Proceedings of the Fourth International Conference on Material and Component Performance under Variable Amplitude Loading (VAL4); 2020 Mar 30 – Apr 1; Darmstadt, Germany. Berlin: DVM; 2020. p. 83-9.

[2] Matsuishi M, Endo T. Fatigue of metals subject to varying stress. Fukuoka, Japan: Presented to the Japan Society of Mechanical Engineers; 1968.

[3] Mrsnik M, Slavic J, Boltzer M. Frequency-domain methods for a vibration-fatigue-life estimation - Application to real data. Int J Fatigue 2013;47:8-17. https://doi.org/10.1016/j.ijfatigue.2012.07.005.

[4] Larsen C, Irvin T. A review of spectral methods for variable amplitude fatigue prediction and new results. Procedia Eng. 2015;101:243-50. https://doi.org/10.1016/j.proeng.2015.

[5] Marques JM, Benasciutti D. More on variance of fatigue damage in non-Gaussian random loadings - effect of skewness and kurtosis. Procedia Struct Integrity 2020; 25:101–11. https://doi.org/10.1016/j.prostr.2020.04.014.

[6] Braccesi C, Cianetti F, Tomassini L. Random fatigue. A new frequency domain criterion for the damage evaluation of mechanical components. Int J Fatigue 2015; 70:417–27. https://doi.org/10.1016/j.ijfatigue.2014.07.005.

[7] Benasciutti D. Private correspondence.

[8] Wirsching PH, Paez TL, Ortiz K. Random vibration. Theory and Practice. New York: A Wiley-Interscience Publication; 1995.

[9] Bautin A. Monitoring of the elements of aviation structures using strain-gauge instrumentation. Vestnik mashinostroenija 2020;5:P.13-19.

[10] Kogaev VP, Gadolina IV, Zainetdinov RI, Grizlova TP, Petrova IM. Simulation of continuous switching random loads. Fatigue Fract Eng Mater Struct 2007;30(11):1016–29. https://doi.org/10.1111/j.1460-2975.2007.01171.x.

[11] Gadolina IV, Erpalov AV. Using the experimental data in form of Gasser curve to compare the methods for durability estimation. Proceedings of the Fourth International Conference on Material and Component Performance under Variable Amplitude Loading (VAL4); 2020 Mar 30 – Apr 1; Darmstadt, Germany. Berlin: DVM; 2020. p. 83-9.

[12] Matokhniuk L. Prediction of the characteristics of fatigue resistance of metals in the gigacycle region according to the results of high-frequency tests. Message 2. Application of the accumulation model of fatigue damage. Strength of Materials. 2012;6:67-80, in Russian.

[13] Rice SO. Mathematical analysis of random noise. Bell Syst Tech J 1944;23: 262-332. https://doi.org/10.1002/j.1538-7904.1944.tb00874.x.

[14] Erpalov AV, Shefer IA. Fatigue-based Classification of Loading Processes. Procedia Engin 2016;150:144-9. https://doi.org/10.1016/j.proeng.2016.06.737.

[15] Kogaev VP. Strength Calculations at stresses variable in time. Moscow: Mashinostr; 1993. in Russian.

[16] Sonsonsino CM. Limitation in the use of RMS values and equivalent stresses in variable amplitude loading. Int J Fatigue 1989;11(3):142-52. https://doi.org/10.1016/0142-1123(90)90023-7.

[17] Sonsonsino CM, Heim R, Melz T. Lightweight-Structural Durability Design by Consideration of Variable Amplitude Loading. Int J Fatigue 2016;92(Pt 2):328–36. https://doi.org/10.1016/j.ijfatigue.2015.07.030.

[18] Kogaev VP, Gadolina IV. Summation of fatigue damage in probability-based life calculations/ Vestnik mashinostroenija 1989;7:3-9.

[19] Eulitz KG, Kotte KL. Damage Accumulation – Limitations and Perspectives for Fatigue Life Assessment, Materials Week 2000 – Proceedings; 2000 September 25–28. Werkstoffwoche–Partnerschaft, Frankfurt.

[20] Hanel B, Hailbach E, Seeger T, Wirthen G, Zenner H. FKM-Richtlinie –Rechnerischer Festigkeitsnachweis für Maschinenbauteile (FKM-Guideline – Analytical Strength Assessment of Components in Mechanical Engineering), 5th extended ed.; 2007; VDM, Frankfurt, Germany.

[21] Gadolina IV, Dubin DA, Petrova IM, Serebriakova IL. Stabilization of the design loading characteristics in the problems of durability estimation of tracked vehicle parts. Journal of machinery manufacture and reliability. 2020;49(1):31–7. https://doi.org/10.3103/S1052618820010069.

[22] Poliakov BN. Some recommendations for increase of accuracy of defined rate fatigue durability of the equipment of heavy mechanical engineering. Tekhnika mashinostroenija. 2013(5):999-1008, in Russian.

[23] R Core Team. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria; 2020. Available from: https://www.R-project.org/.

[24] Dirlik T. Application of Computers in Fatigue Analysis [dissertation]. UK: University of Warwick; 1985.

[25] Wang L. Privat correspondence.

[26] Bautin A. Monitoring of the elements of aviation structures using strain-gauge measurement. Zavodskaya Laboratoriya. Diagnostika Materialov. 2019;8(1):57-6. in Russian. https://doi.org/10.26896/1028-6861-2019-85-1-1-57-63.

[27] Gadolina IV, Zainetdinov RI, Grizlova TP, Petrova IM. Simulation of continuous switching random loads. Fatigue Fract Eng Mater Struct 2007;30(11):1016–29. https://doi.org/10.1111/j.1460-2975.2007.01171.x.