COORDINATING A SUPPLY CHAIN WITH DEMAND INFORMATION UPDATING

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Abstract. We investigate how to coordinate a two-echelon supply chain in which a supplier builds production capacity in advance and a manufacturer makes the ordering decision based on updated demand information. By combining European call option and buyback mechanisms, we propose a new hybrid option-buyback contract to coordinate such a supply chain with demand information updating. We construct a two-stage optimization model in that the supplier offers option price and the manufacturer decides initial ordering quantity in the first stage, then the supplier offers exercise price and buyback price and the manufacturer decides final ordering quantity in the second stage after demand information is updated. In both the centralized and decentralized settings, we analytically derive the optimal equilibrium solutions of two-stage ordering quantity. Particularly, we obtain closed-form formulae to describe the members' optimal behavior with a bivariate uniformly distribution. We prove that the proposed contracts can realize the perfect coordination of the supply chain and analyze how the proposed contracts affect the members' decisions. The theoretical results show that, by tuning the option price or buyback price, the supply chain profit can be arbitrarily split between the members, which is a desired property for supply chain coordination. Compared with the standard option and buyback contract, the proposed contract results in a greater supply chain profit and achieves Pareto improvement for the supply chain members. Furthermore, extending the baseline model focusing on price-independent demand to the case of price-dependent demand, we show that the proposed contract still can achieve supply chain coordination. Numerical examples are also conducted to complement the theoretical results.

1. Introduction. In real-world businesses, one of the key challenges faced by supply chains is how to effectively meet uncertain market demand (see [7, 39, 43]). For example, due to the long production cycle and short sales season, fashion apparel industries are suffering from the drastic impacts of market fluctuation and mass inventory (see [40]). A survey by Walmart on the apparel industry shows that if the manufacturers order 26 weeks before the sales season, the demand forecast has a margin of error of approximately 40%. However, if they order 16 weeks in advance,
the forecast error drops down to approximately 20%. And, if they place an order at the beginning of the sales season, the margin of error is only 10%. It is obvious that the information collected from market can help manufacturers improve their decision-making.

To minimize excessive inventory, manufacturers often postpone making their ordering decisions so that more accurate demand information can be collected in the form of market signals or observations (see [17, 19]). In practice, however, the production season is often long (see [42]). Thus, suppliers need to prepare production capacity in advance based on early demand information. This preparation comes with a risk to the supplier, as it must bear overproduction loss. To avoid potential loss, the supplier often acts conservatively to shed part of its excess capacity (see [13]). These self-interested actions unavoidably result in double marginal effects and poor supply chain performance.

To overcome the inherent shortcomings of such a supply chain, we propose a new hybrid option-buyback contract combining European call option and buyback mechanisms. The supplier charges an option price in a bid to reduce the financial loss due to overproduction and offers a buyback to induce a larger order from the manufacturer. With the proposed contract, not only can the manufacturer place an order flexibly based on updated demand information, but the members also voluntarily act in the best interest of the whole supply chain. We formulate a two-stage optimization model to study the decisions and coordination of the supply chain in which the supplier needs a long time to prepare capacity and the manufacturer may update demand information when making the ordering decision. In the first stage (i.e., the production season), the supplier first charges an option price, and then the manufacturer places an initial order based on early demand information (this initial order can be thought of the amount of reserved capacity). In the second stage (i.e., the sales season), the supplier first charges an exercise price and offers a buyback price, and then the manufacturer places final orders not exceeding the reserved capacity based on further demand information.

We analyze the coordination issue for a two-echelon supply chain with demand information updating. Our analysis suggests that the standard option and buyback contracts fail to coordinate the supply chain presented in this paper. However, a new hybrid option-buyback contract would be able to achieve perfect coordination of such a supply chain. The proposed contract possesses two desirable features. First, it is superior to the standard option and buyback contract. It not only allows the manufacturer to modify the initial order flexibly according to the latest demand information and current exercise price but can also effectively mitigate the double marginal effects. Second, the proposed contract form is flexible, i.e., the supply chain profit can be arbitrarily split between the members by tuning the option price or buyback price. We have also shown that the hybrid contract can coordinate the supply chain even in the case of price-dependent demand, demonstrating the robustness of the proposed contract.

This paper is distinct from the existing literature on demand information updating (see [4, 6, 7, 13, 39, 40, 43]). First, we combine European call option and buyback mechanisms to design a new hybrid option-buyback contract. In contrast to the standard option and buyback contract, the proposed contract includes a desirable feature of two-way compensation. As a result, supply chain members voluntarily act in the best interest of the supply chain. Second, the proposed contract is more advantageous than most of the existing contracts in coordinating the supply
chain with demand information updating, such as a bidirectional return policy (see [13]). In an extension with price-dependent demand, the proposed contract can also achieve the coordination of such a supply chain with demand information updating.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model setup and main assumptions. Section 4 analyzes the quantity decisions of the two-stage optimization model in centralized and decentralized supply chain. The coordination mechanism of the hybrid option-buyback contract and performance comparison among three contracts are also presented. Section 5 provides numerical examples and sensitivity analysis. In Section 6, the main model is extended to the scenario of price-dependent demand. Section 7 concludes and discusses the main management insights.

2. Literature review. This paper contributes to two streams of literature in supply chain management: coordination mechanism of the supply chain contracts, and inventory models with demand information updating.

2.1. Coordination mechanism using supply chain contracts. Many scholars have documented that a decentralized system consisting of independent, profit-maximizing firms often results in poor supply chain performance (see [13]). Spengler [33] attributes this poor performance to the “double marginal effects”. To mitigate double marginalization, numerous studies have designed various supply chain contracts to induce channel members to achieve system-optimal performance (see [5, 8, 22, 27]). These include wholesale price contract (see [9, 10]), revenue sharing contract (see [41]), return contract (see [13, 14]), quantity flexibility contract (see [35]), and option contract (see [4]). To address complex business situations, an increasing number of scholars have begun to concentrate on hybrid contracts with stronger flexibility (see [1]). For example, Li et al. [23] investigate the role of a forward commitment and option contract between a supplier and a retailer under price and demand uncertainty. They find that a combination of forward commitment and option contract can coordinate the supply chain even in the presence of asymmetric information. Jörnsten et al. [21] propose a hybrid wholesale-option contract under random discrete demand and demonstrate that the proposed contract is superior to the option contract if the manufacturer is risk-averse. Arani et al. [1] introduce a hybrid revenue-sharing and option contract to coordinate a retailer-manufacturer supply chain in which both the retailer-led and the manufacturer-led supply chains are considered. The results show that the proposed contract dominates a wholesale contract and a standard option contract. Our work is close in structure to that of [1] but is significantly distinct from their model. To characterize the features of a supply chain with demand information updating, we combine the European call option and buyback mechanisms to design a hybrid option-buyback contract. The proposed hybrid option-buyback contract is superior to the standard option and buyback contract in coordinating the supply chain with demand information updating.

2.2. Inventory models with demand information updating. Recently, the design of supply chain contracts has brought a broad appreciation for economic modeling in production research with information updating (see [34]). Prior research has examined how demand information updating influences supply chain coordination. There are two main research streams in these literatures. First, demand information updating occurs in the sales season in a two-period modeling framework. The
downstream member can use the realized demand in the first period to update the demand forecast for the second period (see [10, 12, 24, 25, 42]). For example, Barnes-Schuster et al. [4] investigate the role of options in a buyer-supplier system with two-stage correlated demand. They find that channel coordination can be achieved only when the exercise price is piecewise linear. Zhou and Wang [41] study supply chain coordination for newsvendor-type products with two ordering opportunities and demand information updating. The results show that the revenue-sharing contract fails to coordinate such a supply chain. They further propose an improved revenue-sharing contract to achieve perfect coordination of the supply chain in which the manufacturer requires the buyer to bear the production cost in the second period as compensation for its excess capacity.

In the second stream, demand information updating occurs in the production season for fashion products with short lifecycles and long lead times. In this situation, the market signals during the production season can be collected to improve the demand forecast (see [9, 10, 11, 16, 38, 44]). In particular, Chen et al. [13, 14] establish a coordination mechanism for the supply chain with lead-time considerations under exogenous price and price-dependent demand separately. The authors first show that the return contract fails to coordinate the supply chain and further propose a risk- and profit-sharing contract to achieve channel coordination. Yang et al. [40] address the optimal reservation pricing problem for a fashion supply chain with forecast updating and asymmetric cost information. They design a new menu of reservation contracts consisting of unit reservation fee, reservation quantity and final order quantity to ensure the manufacturer truthfully reveals its cost information. Cheaitou and Cheaytou [15] propose a two-stage capacity reservation contract in which the retailer has two ordering opportunities before and after demand information updating. Using the dynamic program approach, they characterize the structure of the retailer’s optimal policy.

The above models build on the assumption that the buyer’s order is fulfilled immediately with full production capacity. However, for products such as computers and fashion apparels, the supplier’s capacity is often limited, and it takes a long time to prepare production capacity (see [40]). Therefore, the buyer’s order quantity cannot exceed the supplier’s initial capacity. The another assumption used in this literature is that the supplier operates under voluntary compliance, even if the supplier knows the buyer’s demand updating behavior. In fact, demand information updating may hurt the supplier’s performance due to overproduction capacity. For example, Wu [36] considers a two-echelon supply chain where one manufacturer trades with one retailer under a quantity flexibility contract. The results demonstrate that quantity flexibility always benefits the retailer, whereas the supplier is benefited only when the level of flexibility is small. Therefore, without any commitment or compensation from the buyer, the supplier may act conservatively and produce less than the buyer’s optimal order. By relaxing the above assumptions, we consider a more general coordination mechanism for a supply chain with demand information updating. We investigate two possible cases of production capacity: (1) one in which the supplier can satisfy the final order quantity and (2) the other in which the supplier cannot satisfy the final order quantity. Further, we extend the setting of stochastic demand with a fixed retail price to a setting with retail price-dependent stochastic demand to examine the application of the new hybrid option-buyback contract.
This paper contributes to the existing literature on information updating in three main aspects. First, to the best of our knowledge, this paper is the first to propose a hybrid option-buyback contract involving bidirectional compensation to coordinate a supply chain with demand information updating. Comparatively, most existing literatures focus on improving certain standard contracts, such as the improved option contract (see [4]), the improved return contract (see [13, 14]) and the improved revenue sharing contract (see [41]). Through redesigning the contract parameters, those papers explore the coordination of the supply chain when the member can update demand information. Second, we extend the model setting of “fixed retail price stochastic demand” to “retail price-dependent stochastic demand”. We demonstrate that the proposed contract still achieves supply chain coordination. Compared with those improved standard contracts, the new hybrid option-buyback contract is more robust in practice. Third, with a hybrid option-buyback contract, we analytically derive the supply chain Pareto improvement region in which the supply chain not only achieves the system-optimal profit but also makes all members at least as well off as the standard option and buyback contract in Barnes-Schuster et al., Chen et al., Zhou and Wang (see [4, 13, 14, 41]).

3. Model setup. We consider a two-echelon supply chain consisting of a supplier and a manufacturer in which the manufacturer can utilize demand information updating to make the ordering decision. For convenience, the supplier is referred to as “she”, and the manufacturer is referred to as “he”. The manufacturer purchases a key component from the upstream supplier to assemble final products before the sales season. The supplier arranges the production capacity for the key component according to the initial order placed by the manufacturer. She needs a long lead time to prepare production capacity to satisfy the manufacturer’s order. To reduce the risk of overproduction capacity and overstock, a new hybrid option-buyback contract is employed.

The manufacturer faces uncertain market demand \( x \). At the beginning of the production season, the manufacturer reserves \( Q^I_k \) units of the production capacity from the supplier at an option price \( c_e \) based on the prior demand distribution \( f(x) \). The supplier prepares \( Q^I_k \) units of the production capacity at a unit cost \( c_1 \). During the production season, the manufacturer continuously observes demand information \( I \) to improve the demand forecast. At the end of the production season, the manufacturer acquires the posterior demand distribution \( h(x|i) \) when the value of \( I \) is realized as \( i \). The manufacturer finally purchases \( Q^F_i \) quantities of the component at an exercise price \( w_e \) based on \( h(x|i) \). At the same time, the supplier produces \( \min\{Q^I_k, Q^F_i\} \) units of the component and delivers the manufacturer at a unit cost \( c_2 \). The salvage values of excess capacity and leftover inventory are \( v_1 \) and \( v_2 \), respectively. The supplier compensates the manufacturer a buyback price \( b \) per unsold product at the end of the sales season. Figure 1 illustrates the sequence of events.

Without loss of generality, we assume that (1) \( 0 < c_e < c_1 - v_1 \) to avoid the supplier arbitraging with the option (see [1]); (2) \( v_1 < c_1 \) and \( v_2 < c_2 + v_1 \) to avoid unreasonable cases in which the supplier prepares infinite production capacity and the manufacturer produces the final product as much as possible (see [40]); and (3) \( c_1 + c_2 < w < p \) to make the supplier and manufacturer profitable (see [43]). In addition, the products are sold at an exogenous retail price \( p \).
Following Chen et al. [13] and Gurnani and Tang [19], the process of demand information updating can be described as follows: At the beginning of the production season, the manufacturer uses the prior demand distribution $f(x)$ to reserve the supplier’s production capacity. During the production season, more demand information $I$ is available, so the manufacturer continuously updates demand information. At the end of the production season, based on the latest demand information $I = i$, the manufacturer acquires the posterior demand distribution $h(x|i)$. Without loss of generality, we assume that (1) $i$ is expressed in the same unit as total demand with a lower support $i_{\text{min}} = 0$ (see [16, 13]); (2) the information makes the demand forecast more accurate than the original forecast (see [13]); (3) the random variable $x|i$ is increasing in $i$ (see [19]); (4) when $i_1 < i_2$, the cumulative distribution $H(x|i)$ for $x$ given $i$ satisfies $H(x|i_1) \geq H(x|i_2)$; (5) the demand information and the conditional distribution of the final demand is symmetric for all supply chain parties (see [13]); and (6) all distributions are continuous, differentiable, invertible and independent of the cost and price (see [13]). Many conjugate distributions can satisfy the above assumptions, such as uniform-uniform, uniform-Pareto, beta-negative binomial, gamma-Poisson and normal-normal distributions. To illustrate, suppose that the demand follows the uniform-uniform distribution. Then, the demand information $i$ and posterior demand $x|i$ are expressed as follows:

$g(i) = \frac{1}{\alpha}$ and $G(i) = \frac{1}{\alpha} (i - \gamma + \frac{\alpha}{2})$, where $i \in [\gamma - \frac{\alpha}{2}, \gamma + \frac{\alpha}{2}]$, and

$h(x|i) = \frac{1}{\beta}$ and $H(x|i) = \frac{1}{\beta} (x - i + \frac{\beta}{2})$, where $x \in [i - \frac{\beta}{2}, i + \frac{\beta}{2}]$.

For easy interpretation, the key notation is listed in Table 1.

The proposed model possesses three desirable features. First, the proposed model not only offers flexibility of ordering for the manufacturer by option, but also effectively mitigate double marginal effects through buyback in a decentralized supply chain. Thus, the hybrid model dominates the single option and single buyback model. Second, the proposed model characterizes the market demand distribution before and after updating by utilizing Bayesian approach. The manufacturer exercises option according to more accurate demand information, which is significantly distinguished from the previous studies in that the option is exercised after demand realization. Third, the proposed model is a dynamic game-theoretic decision-making model for the supply chain that incorporates logistic flow and information flow, which is more complex but closer to the real business environment.

In addition, some study limits in the proposed model are existed. For example, there is a risk of shortage for the manufacturer since his exercise quantity of option cannot exceed initial purchased option. To obtain more profit, a spot market can
Table 1. Notations and explanations

| Notation | Explanation |
|----------|-------------|
| \(x\)   | Random demand |
| \(p\)   | Retail price |
| \(c_{e}\) | Option price (decision variable) |
| \(w_{e}\) | Exercise price (decision variable) |
| \(\lambda\) | Revenue sharing ratio. |
| \(b\)   | Buyback price (decision variable) |
| \(Q_{k}^{j}\) | Capacity reservation quantity of the key component for the manufacturer, where \(j = c, d\) and \(k = sc, m, s\) (decision variable) |
| \(Q_{e}^{j}\) | Final order quantity of the manufacturer, where \(j = c, d\) (decision variable) |
| \(I\)   | Demand information |
| \(c_{1}\) | Capacity investment cost of the supplier |
| \(c_{2}\) | Production cost of the supplier |
| \(v_{1}\) | Salvage value of excess capacity |
| \(v_{2}\) | Salvage value of leftover inventory |
| \(f(\cdot), F(\cdot)\) | Prior probability density and distribution functions of \(x\) |
| \(g(\cdot), G(\cdot)\) | Probability density and distribution functions of \(I\) |
| \(h(x|i)\) | Probability density function of \(x\) given \(I = i\) |
| \(H(x|i)\) | Cumulative distribution functions for \(x\) given \(I = i\) |
| \(\Pi\) | Total profit |

Note: The superscripts “\(c\)” and “\(d\)” represent the centralized and decentralized supply chains, respectively. In addition, the subscripts “\(sc\)”, “\(m\)” and “\(s\)” represent the supply chain, manufacturer and supplier, respectively.

be considered as an emergency source of the key components. Besides, the supplier is assumed to use the “make-to-order” policy for production, i.e., her production quantity equals the manufacturer’s initially purchased option quantity. The manufacturer’s final exercise quantity of option may be lower than his initially purchased quantity after updating. Thus, the supplier faces overproduction risk. To reduce risk of overproduction, the supplier may not use the “make-to-order” policy but plan production for her own interest.

4. Quantity decision analysis.

4.1. The centralized supply chain. In the centralized system, the supply chain first determines the reservation quantity of the production capacity \(Q_{sc}^{c}\) at the beginning of the production season. Then, the supply chain utilizes the latest demand information \(I = i\) to decide the final order quantity \(Q_{e}^{c}\) at the beginning of the sales season. Using the backward induction method, we next solve this two-stage news vendor model with demand information updating.

4.1.1. Decision in the sales season. Given \(Q_{sc}^{c}\) and \(I = i\), the optimal final order quantity \(Q_{e}^{c}\) is obtained by solving the optimization problem (P1).

\[
\begin{align*}
\text{maximize} & \quad \Pi^{c}(Q_{sc}^{c}, i; Q_{e}^{c}) \\
\text{subject to} & \quad Q_{e}^{c} \leq Q_{sc}^{c},
\end{align*}
\]

(P1)
Corollary 1. Given Proposition 1 can be given as follows: 

where \( Q_e \) of the reservation quantity. If \( i > i_c \) and during the sales season is purchase the entire reservation quantity. The supply chain’s optimal expected profit \( \Pi^c(Q_{sc}, i; Q_e^c) \) increases in \( i \).

Let \( Q_e^c | \gamma = H^{-1} \left( \frac{p - c_2 - v_1}{p - v_2} \right) \), where \( H^{-1}(\cdot) \) is the inverse function of \( H(\cdot) \). Note that \( Q_e^c \leq Q_{sc}^c \). The following proposition establishes the optimal final order quantity.

Proposition 1. Given \( Q_{sc}^c \) and \( i \) in the sales season, \( \Pi(Q_{sc}, i; Q_e^c) \) is concave in \( Q_e^c \), and

\[
Q_{e}^c = \begin{cases} Q_{c}^c | i, & \text{if } i \leq i Q_{sc}^c, \\ Q_{sc}^c, & \text{if } i > i Q_{sc}^c, \end{cases}
\]

where \( i Q_{sc}^c \) satisfies \( Q_c^c | i Q_{sc}^c = Q_{sc}^c \).

When demand follows the uniform-uniform distribution, an equivalent form of Proposition 1 can be given as follows:

Corollary 1. Given \( Q_{sc}^c \) and \( i \) in the sales season, there exists a unique positive information threshold \( i Q_{sc}^c \) such that

\[
Q_e^c = \begin{cases} Q_e^c | i = i - \frac{\beta}{2} + \frac{p - c_2 - v_1}{p - v_2} \beta, & \text{if } \gamma - \frac{\beta}{2} \leq i \leq i Q_{sc}^c, \\ Q_{sc}^c, & \text{if } i Q_{sc}^c < i \gamma + \frac{\beta}{2}. \end{cases}
\]

From Proposition 1, \( Q_{e}^c \) depends on the reservation quantity of production capacity and the updated demand information. The variable \( x | i \) stochastically increases in \( i \). If \( i \leq i Q_{sc}^c \), then \( Q_e^c | i \leq Q_{sc}^c \). The reservation quantity can satisfy the manufacturer’s final order quantity. The manufacturer would only purchase part of the reservation quantity. If \( i > i Q_{sc}^c \), then \( Q_e^c | i > Q_{sc}^c \). The reservation quantity cannot satisfy the manufacturer’s final order quantity. The manufacturer would purchase the entire reservation quantity. The supply chain’s optimal expected profit during the sales season is

\[
\Pi^c(Q_{sc}, i; Q_e^c) = \begin{cases} \Pi^c(Q_{sc}, i; Q_e^c | i), & \text{if } i \leq i Q_{sc}^c, \\ \Pi^c(Q_{sc}, i; Q_{sc}^c), & \text{if } i > i Q_{sc}^c, \end{cases}
\]

where

\[
\Pi^c(Q_{sc}, i; Q_e^c | i) = v_1 Q_e^c + (p - c_2 - v_1)Q_e^c | i - (p - v_2) \int_0^{Q_e^c | i} H(x | i) dx,
\]

and

\[
\Pi^c(Q_{sc}, i; Q_{sc}^c) = (p - c_2)Q_{sc}^c - (p - v_2) \int_0^{Q_{sc}^c} H(x | i) dx.
\]

4.1.2. Decision in the production season. At the beginning of the production season, the supply chain needs to decide the optimal reservation quantity of production capacity \( Q_{sc}^c \). By solving the optimization problem (P2), we can obtain the optimal \( Q_{e}^c \) in the production season.

\[
\begin{align*}
\text{maximize} & \quad \Pi^c(Q_{sc}^c) \\
\text{subject to} & \quad Q_{sc}^c \geq 0,
\end{align*}
\]

where

\[
\Pi^c(Q_{sc}^c) = \int_0^{Q_{sc}^c} \Pi^c(Q_{sc}, i; Q_e^c | i) dG(I) + \int_{Q_{sc}^c}^{+\infty} \Pi^c(Q_{sc}, i; Q_{sc}^c) dG(I) - c_1 Q_{sc}^c.
\]

To obtain the optimal capacity reservation quantity of the supply chain \( Q_{sc}^c \), we first study the property of \( \Pi^c(Q_{sc}^c) \).
Lemma 1. \( \Pi^*(Q_{sc}^*) \) is concave in \( Q_{sc}^* \).

Lemma 1 ensures that there is a unique solution to (P2). Based on that, we establish the following proposition to obtain the optimal production quantity \( Q_{sc}^* \).

**Proposition 2.** In the production season, there is a unique solution for \( Q_{sc}^* \) that satisfies the equation of

\[
\int_{Q_{sc}^*}^{+\infty} [H(Q_e|i) - H(Q_{sc}^*|i)]dG(I) = \frac{c_1 - v_1}{p - v_2}. \tag{5}
\]

When the demand follows the uniform-uniform distribution, \( Q_{sc}^* = \gamma + \frac{\alpha - \beta}{2} + \frac{p - c_2 - v_1}{p - v_2} \beta - \sqrt{\frac{2(c_1 - v_1)\alpha\beta}{p - v_2}}. \]

Proposition 2 characterizes the sufficient condition of the optimal reservation quantity in a centralized system. The demand information is unknown in the production season, so the optimal reservation quantity of the manufacturer maximizes the expected supply chain profit. The demand uncertainty in our model involves two aspects: the range of possible demand scenarios in the production season and the inherent demand uncertainty in the sales season. The former can be solved by observing the market reaction to the product at the beginning of the sales season, while the latter will exist until the end of the sales season. Undoubtedly, demand information updating helps the manufacturer make a more accurate ordering decision relative to the noninformation updating order decision. However, note that demand information updating may lead to a greater mismatch between the supplier’s production capacity quantity and the manufacturer’s final order quantity. Since the supplier’s production capacity quantity is identical to the manufacturer’s initial order quantity, in the case of noninformation updating, only the manufacturer would face the risk of overordering due to demand uncertainty. On the contrary, in the case of demand information updating, the manufacturer’s initial order may be modified. Thus, the supplier’s production capacity quantity may be higher than the manufacturer’s final order, which leads to a risk of overproduction. To some extent, the resultant mismatch would hurt the supplier and further lead to more serious double marginal effects. To eliminate the potential negative effect of demand information updating, a novel hybrid option-buyback contract is proposed to coordinate the supply chain with demand information updating.

4.2. The decentralized supply chain. In this section, with a hybrid option-buyback contract, each member in the supply chain acts in the manner that maximizes their own expected profits. By the backward induction method, we first derive the manufacturer’s optimal final order quantity \( Q_{e}^* \) at an exercise price \( w_e \) based on the latest demand information \( I = i \) at the beginning of the sales season. We then solve the manufacturer’s optimal production capacity reservation quantity \( Q_{m}^* \) at an option price \( c_e \) at the beginning of the production season. In terms of the buyback mechanism, the supplier buys back all unused units at a unit buyback price \( b \) at the end of the sales season. Finally, compared with the centralized system, we characterize the sufficient condition under which a hybrid option-buyback contract \( \{w_e, c_e, b\} \) can coordinate the supply chain with demand information updating.

4.2.1. Decisions of the decentralized supply chain. Given the manufacturer’s reservation quantity \( Q_{m}^* \) and the latest demand information \( I = i \), the manufacturer’s optimal final order quantity \( Q_{e}^* \) at the beginning of the sales season can be
obtained by solving problem (P3).

$$\begin{align*}
\text{maximize} & \quad \Pi_m(Q^d_m, i; Q^d_e) \\
\text{subject to} & \quad Q^d_e \leq Q^d_m,
\end{align*}$$

(3)

where $$\Pi_m(Q^d_m, i; Q^d_e) = E_{[x|\beta]}\{p\min(x, Q^d_e) + b(Q^d_e - x)^+ - w_c Q^d_e\}$$. Let $$Q^d_e = H^{-1}\left(\frac{p-w_c}{p-b}\right)$$. By solving (P3), we obtain the manufacturer’s optimal final order quantity $$Q^d_e$$ in the sales season.

**Proposition 3.** Given $$Q^d_m$$ and i in the sales season, the manufacturer’s expected profit in the sales season $$\Pi_m(Q^d_m, i; Q^d_e)$$ is concave in $$Q^d_e$$, and

$$Q^d_e = \begin{cases} 
Q^d_m(i), & \text{if } i \leq i_{Q^d_m}, \\
Q^d_m, & \text{if } i > i_{Q^d_m},
\end{cases}$$

where $$i_{Q^d_m}$$ satisfies $$Q^d_m|i_{Q^d_m} = Q^d_e$$.

Similarly, when the demand follows the uniform-uniform distribution, the manufacturer’s optimal final order quantity $$Q^d_e$$ can be derived in the following corollary.

**Corollary 2.** Given $$Q^d_m$$ and i in the sales season, there exists a unique positive information threshold $$i_{Q^d_m} = Q^d_m + \frac{\beta}{2} - \frac{p-w_c}{p-b} \beta$$ that satisfies $$Q^d_m|i_{Q^d_m} = Q^d_m$$ and further

$$Q^d_e = \begin{cases} 
Q^d_m(i) = i - \frac{\beta}{2} + \frac{p-w_c}{p-b} \beta, & \text{if } \gamma - \frac{\alpha}{2} \leq i \leq i_{Q^d_m}, \\
Q^d_m, & \text{if } i_{Q^d_m} < i < \gamma + \frac{\alpha}{2}.
\end{cases}$$

The manufacturer’s optimal final order quantity in the decentralized system depends on the updating demand information and the capacity reservation quantity. If the manufacturer’s final order quantity is below his capacity reservation quantity, the supplier would bear the risk of excessive capacity. From Proposition 3, the manufacturer’s optimal expected profit in the sales season is

$$\begin{align*}
\Pi_m(Q^d_m, i; Q^d_e) = \begin{cases} 
\Pi_m(Q^d_m, i; Q^d_e), & \text{if } i \leq i_{Q^d_m}, \\
\Pi_m(Q^d_m, i; Q^d_e), & \text{if } i > i_{Q^d_m},
\end{cases}
\end{align*}$$

(6)

where

$$\Pi_m(Q^d_m, i; Q^d_e) = (p - w_c)Q^d_e(i) - (p - b) \int_0^{Q^d_e(i)} H(x|i)dx,$$

(7)

and

$$\Pi_m(Q^d_m, i; Q^d_e) = (p - w_c)Q^d_m - (p - b) \int_0^{Q^d_e} H(x|i)dx.$$ 

(8)

Based on Equations (6)-(8), the manufacturer’s optimal expected profit in the production season is

$$\begin{align*}
\Pi_m(Q^d_m) &= \int_0^{i_{Q^d_m}} \Pi_m(Q^d_m, i; Q^d_e) dG(I) + \int_{i_{Q^d_m}}^{\infty} \Pi_m(Q^d_m, i; Q^d_m) dG(I) - ce Q^d_m.
\end{align*}$$

(9)

The following proposition establishes the optimal order quantity $$Q^d_e$$. 
Proposition 4. In the production season, $Q_{m}^{d_{s}}$ satisfies the equation of

$$
\int_{Q_{m}^{d}}^{+\infty} [H(Q_{m}^{d}|i) - H(Q_{m}^{d_{s}}|i)]dG(I) = \frac{c_{e}}{p-b}.
$$

(10)

When the demand follows the uniform-uniform distribution, $Q_{m}^{d_{s}} = \gamma + \frac{\alpha-\beta}{2} + \frac{p-w_{a}}{p-b} \beta - \sqrt{\frac{2\alpha-\beta}{p-b} \gamma \beta}.

Since the supplier prepares her production capacity quantity $Q_{s}^{d}$ according to the manufacturer’s reservation quantity $Q_{m}^{d}$ (i.e., $Q_{s}^{d} = Q_{m}^{d}$), the supplier’s expected profit in the sales season for any given $Q_{m}^{d}$ is

$$
\Pi^{d}_{s}(Q_{m}^{d}, i; Q_{s}^{d}) = \left\{ \begin{array}{ll}
\Pi^{d}_{s}(Q_{m}^{d}, i; Q_{s}^{d}), & \text{if } i \leq i_{Q_{m}^{d}}, \\
\Pi^{d}_{s}(Q_{m}^{d}, i; Q_{m}^{d}), & \text{if } i > i_{Q_{m}^{d}},
\end{array} \right.
$$

(11)

where

$$
\Pi^{d}_{s}(Q_{m}^{d}, i; Q_{s}^{d}) = (w_{e} - c_{2} - v_{1})Q_{s}^{d}|i - (b - v_{2}) \int_{0}^{Q_{s}^{d}|i} H(x|i)dx + v_{1}Q_{m}^{d}.
$$

(12)

and

$$
\Pi^{d}_{s}(Q_{m}^{d}, i; Q_{m}^{d}) = (w_{e} - c_{2})Q_{m}^{d} - (b - v_{2}) \int_{0}^{Q_{m}^{d}} H(x|i)dx.
$$

(13)

Based on Equations (11)-(13), given the manufacturer’s optimal reservation quantity $Q_{m}^{d_{s}}$ as given in Proposition 4, the supplier’s expected profit in the production season is

$$
\Pi^{d}_{s}(Q_{m}^{d_{s}}) = \int_{i_{Q_{m}^{d}}^{d}}^{i_{Q_{m}^{d}}^{d}} \Pi^{d}_{s}(Q_{m}^{d_{s}}, i; Q_{s}^{d})dG(I) + \int_{i_{Q_{m}^{d}}^{d}}^{+\infty} \Pi^{d}_{s}(Q_{m}^{d_{s}}, i; Q_{m}^{d})dG(I) + (c_{e} - c_{1})Q_{m}^{d}.
$$

4.2.2. Coordination of the supply chain. In this section, we consider a hybrid option-buyback contract to coordinate the supply chain with demand information updating. For convenience, we summarize the optimal quantity decisions of the centralized and decentralized system in Table 2.

| Supply chain | Final order quantity | Capacity reservation quantity |
|--------------|----------------------|-----------------------------|
| Centralized  | $Q_{e}^{*} = Q_{e}^{d}d_{s}$, if $i \leq i_{Q_{e}^{d_{s}}}$, $Q_{e}^{*} = Q_{e}^{d}$, if $i > i_{Q_{e}^{d_{s}}}$, \( H(Q_{e}^{d}|i) = \frac{c_{e}-c_{1}}{p-v_{2}} \int_{Q_{e}^{d}}^{+\infty} [H(Q_{m}^{d}|i) - H(Q_{m}^{d_{s}}|i)]dG(I) = \frac{c_{e}-c_{1}}{p-v_{2}} \) | \( H(Q_{e}^{d}|i) = \frac{c_{e}}{p-b} \) |
| Decentralized | $Q_{d}^{d_{s}} = Q_{d}^{d}$, if $i > i_{Q_{d}^{d_{s}}}$, \( H(Q_{d}^{d}|i) = \frac{c_{e}}{p-b} \int_{Q_{d}^{d}}^{+\infty} [H(Q_{m}^{d}|i) - H(Q_{m}^{d_{s}}|i)]dG(I) = \frac{c_{e}}{p-b} \) | \( H(Q_{d}^{d}|i) = \frac{c_{e}}{p-b} \) |

From Table 2, $Q_{d}^{d_{s}}$ may not be equal to $Q_{e}^{*}$. To coordinate the supply chain, the supply chain members need to negotiate contract terms \( \{c_{e}, w_{e}, b\} \) at the beginning of the production season to realize $Q_{d}^{d_{s}} = Q_{e}^{*}$. There exist two possible situations for the manufacturer’s reservation capacity quantity. When the reservation capacity quantity in the decentralized system is enough to satisfy the optimal final order quantity in the centralized system (i.e., $Q_{m}^{d} \geq Q_{e}^{*}$), the sufficient condition to
coordinate the supply chain can be derived from $H(Q^c_e|i) = H(Q^d_e|i)$. When the reservation capacity quantity is not enough to satisfy the optimal order quantity in the centralized system, i.e., $Q^d_m < Q^e_sc$, the manufacturer can only exercise the options up to the initial reservation capacity quantity, i.e., $Q^e_sc = Q^d_m$. The following proposition establishes the sufficient condition whereby a hybrid option-buyback contract coordinates the supply chain.

Proposition 5.

(i) The coordination of the supply chain with demand information updating is achieved with a hybrid option-buyback contract \( \{c_e, w_e, b\} \in M \), where
\[
M = \{(c_e, w_e, b) : c_e = \lambda(c_1 - v_1), w_e = p - \lambda(p - c_2 - v_1), b = p - \lambda(p - v_2), \lambda \in [0, 1]\};
\]

(ii) With \( \{c_e, w_e, b\} \in M \), $Q^e_sc = Q^d_m$ and $Q^e_sc = Q^d_m$;

(iii) With \( \{c_e, w_e, b\} \in M \), $\Pi^d_m(Q^d_m^*) = \lambda \Pi^c(Q^e_sc)$ and $\Pi^d_m(Q^d_m^*) = (1 - \lambda) \Pi^c(Q^e_sc)$.

Proposition 5 depicts the relationship among $c_e$, $w_e$ and $b$. Since $w_e = p - \frac{p - c_2 - v_1}{c_1 - v_1} c_e$ and $b = p - \frac{p - v_2}{c_2 + v_1} c_e$, we find that the exercise price and buyback price both decrease in the option price. With a hybrid option-buyback contract in the set $M$, the manufacturer realizes the same optimal final order quantity as the centralized system. Furthermore, the supply chain achieves the maximal system profit. At the same time, a hybrid option-buyback contract in the set $M$ permits an arbitrary allocation of the supply chain profit between the supplier and the manufacturer. Hence, a hybrid option-buyback contract in the set $M$ can effectively eliminate the double marginal effects in the decentralized system. It is also possible that a hybrid option-buyback contract in the set $M$ can improve each member’s profit through an appropriate profit allocation policy.

There are two types of risk in the above supply chain decisions: overproduction and overstock. With a hybrid option-buyback contract, the option price is essentially an allowance paid by the manufacturer to the supplier for the reserved capacity, and the buyback price is the compensation paid by the supplier to the manufacturer for unsold units. In so doing, the manufacturer bears part of the supplier’s overproduction risk, and the supplier bears part of the manufacturer’s overstock risk. By setting the option and buyback price, the supply chain realizes the risk transfer between the members.

In addition, note that the option-buyback contract permits an arbitrary allocation of the supply chain profit between the supplier and the manufacturer by adjusting $\lambda$. When $\lambda = 0$, then $c_e = 0$ and $w_e = b = p$. The hybrid option-buyback contract degenerates into a single buyback contract. The supplier bears all risks of overproduction and overstock, and obtains the entire supply chain profit. When $\lambda = 1$, then $c_e = c_1 - v_1$, $w_e = c_2 + v_1$ and $b = v_2$. The manufacturer bears all risks of overproduction and overstock, and extracts the entire profit of the supply chain. By setting the parameter of $\lambda \in (0, 1)$, we can adjust the option and buyback prices. Furthermore, the risks of overproduction and overstock can effectively be transferred between the supplier and manufacturer. The higher risk the member bears, the more profit the member would earn. Therefore, with a hybrid option-buyback contract in the set $M$, supply chain coordination can be achieved.

4.3. Performance comparison of the hybrid option-buyback contract. In practice, the standard option contract \( \{c_e, w_e\} \) (i.e., Policy A thereafter) may be applied to the high-tech, fashion, and automotive industries. It gives the holder of the option the right to buy an asset at a certain price (see [42, 43]). However, the
standard option contract fails to achieve supply chain coordination with demand information updating. To increase the supply chain profit, the standard buyback contract \{w, b\} (i.e., Policy B thereafter) is also introduced to partially mitigate the double marginal effects that cannot be avoided by the standard option contract (see [13]). Motivated by the advantages of two types of contracts, we combine the European call option and buyback mechanisms and propose a new hybrid option-buoyback contract \{\(c_e, w_c, b\)\} (i.e., Policy* thereafter) to coordinate the supply chain with demand information updating. The standard option contract and buyback contract are used as the benchmarks to evaluate the performance of the proposed hybrid option-buyback contract.

Case 1. With \{\(c_e, w_c\)\}, the supplier charges option price \(c_e\) and exercise price \(w_c\). The manufacturer requires \(Q_m^A\) options at \(c_e\) based on early demand information \(x\) at the beginning of the production season. The supplier then prepares production capacity. After demand information \(I = i\) is realized, the manufacturer purchases \(Q_m^A\) options at \(w_c\) based on later demand information \(x| i\). The expected profits of the manufacturer and supplier are given by

\[
\Pi_m^A(Q_m^A) = E[(i) \Pi_m^A(Q_m^A, i; Q_e^A)] - c_e Q_m^A, \text{ and}
\]

\[
\Pi_s^A(Q_m^A) = E[(i) \Pi_s^A(Q_m^A, i; Q_e^A)] + (c_e - c_1) Q_m^A,
\]

where \(\Pi_m^A(Q_m^A, i; Q_e^A) = p \min\{x, Q_e^A\} - w_c Q_e^A + v_2(Q_e^A - x)^+\) and \(\Pi_s^A(Q_m^A, i; Q_e^A) = (w_c - c_2 - v_1) Q_e^A + v_1 Q_m^A\).

Similar to the discussions in Section 5.1, with \{\(c_e, w_c\)\}, the manufacturer’s optimal order decisions are as follows: (1) In the production season, the optimal option quantity \(Q_m^{A*}\) satisfies the equation of \(\int_{Q_e^{A*}}^{+\infty} [H(Q_e^A | i) - H(Q_m^{A*} | i)]dG(I) = \frac{c_e}{p-v_2}\), where \(H(Q_e^A | i) = \frac{p-w_c}{p-v_2}\); and (2) in the sales season, the optimal exercise quantity

\[
Q_e^{A*} = \begin{cases} 
Q_e^A | i, & \text{if } i \leq i_{Q_m^{A*}}, \\
Q_m^{A*}, & \text{if } i > i_{Q_m^{A*}},
\end{cases}
\]

where \(i_{Q_m^{A*}}\) satisfies \(Q_m^{A*} | i_{Q_m^{A*}} = Q_m^{A*}\). When demand follows the uniform-uniform distribution, the following corollary presents the manufacturer’s optimal quantity decisions.

Corollary 3. With \{\(c_e, w_c\)\},

1. In the production season, \(Q_m^{A*} = \gamma + \frac{\alpha - \beta}{2} + \frac{p-w_c}{p-v_2} \beta - \sqrt{\frac{2c_e \alpha \beta}{p-v_2}}\).
2. In the sales season, there exists a unique positive information threshold \(i_{Q_m^{A*}} = Q_m^{A*} + \frac{\beta}{2} - \frac{p-w_c}{p-v_2} \beta\) that satisfies

\[
Q_e^{A*} = \begin{cases} 
Q_e^A | i = i - \frac{\beta}{2} + \frac{p-w_c}{p-v_2} \beta, & \text{if } \gamma - \frac{\beta}{2} \leq i \leq i_{Q_m^{A*}}, \\
Q_m^{A*}, & \text{if } i_{Q_m^{A*}} < i < \gamma + \frac{\beta}{2}.
\end{cases}
\]

Comparing Corollary 3 with Corollary 2, it is easy to find that \(Q_e^A | i < Q_e^d | i\) since \(b > v_2\). That is, the manufacturer’s optimal exercise quantity under \{\(c_e, w_c\)\} is less than that under \{\(c_e, w_c, b\)\}. The double marginal effects lead to a conservative order under \{\(c_e, w_c\)\} (see [33]). As is well known, only when \(c_e = c_1 - v_1\) and \(w_c = c_2 + v_1\) can the supply chain be coordinated by the standard option contract. However, the supplier gains a nonpositive profit and therefore does not set such option and exercise prices.
Following Zhao et al. [43], we define the profit gaps between two contracts of \{c_e, w_e, b\} and \{c_e, w_e\} as

\[
\text{Gap}_s^A = \frac{\Pi_s^d(Q_s^A) - \Pi_s^d(Q_s^A)}{\Pi_s^d(Q_s^A)} \quad \text{and} \quad \text{Gap}_m^A = \frac{\Pi_m^d(Q_m^A) - \Pi_m^d(Q_m^A)}{\Pi_m^d(Q_m^A)},
\]

where \text{Gap}_s^A and \text{Gap}_m^A represent the supplier’s and the manufacturer’s profit advantage with \{c_e, w_e, b\} ∈ M over \{c_e, w_e\}, respectively.

Both supply chain members would accept \{c_e, w_e, b\} ∈ M when \text{Gap}_s^A ≥ 0 and \text{Gap}_m^A ≥ 0. Compared with the benchmark of \{c_e, w_e\}, the following proposition establishes the conditions in which the coordination of the supply chain is achieved under \{c_e, w_e, b\} ∈ M.

**Proposition 6.** Compared with \{c_e, w_e\}, the coordination of the supply chain is achieved under \{c_e, w_e, b\} ∈ N^A ∈ M, where \(N^A = \{c_e, w_e, b : c_e = \lambda(c_1 - v_1), w_e = p - \lambda(p - c_2 - v_1), b = p - \lambda(p - v_2), \lambda \in [\lambda_{\min}, \lambda_{\max}], \lambda_{\min} = \frac{\Pi_s^d(Q_s^A)}{\Pi_m^d(Q_m^A)}\) and \(\lambda_{\max} = 1 - \frac{\Pi_s^d(Q_s^A)}{\Pi_m^d(Q_m^A)}\).

Proposition 6 shows that with \{c_e, w_e, b\} ∈ N^A ∈ M, both the supplier and manufacturer are beneficial compared with \{c_e, w_e\}. In there, \text{Gap}_s^A ≥ 0 and \text{Gap}_m^A ≥ 0. When \(\lambda = \lambda_{\min}\), we have \(\Delta \Pi_m^A = \Pi_m^d(Q_m^A) - \Pi_m^d(Q_m^A) = 0\), and \(\Delta \Pi_s^A = \Pi_s^d(Q_s^A) - \Pi_s^d(Q_s^A) = \Pi_m^d(Q_m^A) - \Pi_m^d(Q_m^A) := \Delta \Pi\). The supplier takes all the additional expected profit of the centralized supply chain while the manufacturer only reserves the initial expected profit of the decentralized supply chain under \{c_e, w_e\}. Comparatively, when \(\lambda = \lambda_{\max}\), then \(\Delta \Pi_m^A = \Delta \Pi\) and \(\Delta \Pi_s^A = 0\). The manufacturer gains the entire additional expected profit. Through tuning \(\lambda \in [\lambda_{\min}, \lambda_{\max}]\), the expected profit of the centralized supply chain is arbitrarily allocated between both members. Compared with \{c_e, w_e\}, the coordination of the supply chain is achieved under \{c_e, w_e, b\} ∈ N^A ∈ M.

**Case 2.** With \{w, b\}, the supplier first prepares capacity \(Q_s^B\) at a unit cost \(c_1\) based on early demand information \(x\) at the beginning of the production season. Then, the manufacturer orders \(Q_m^B\) at a unit wholesale price \(w\) based on later demand information \(x\) at the beginning of the sales season. At the end of the sales season, the supplier pays the manufacturer \(b\) per unit product unsold. The supply chain members’ profit functions are given by

\[
\Pi_m^B(Q_m^B) = E_{\{i\}}[E_{\{x\}}]\Pi_m^B(Q_s^A, i; Q_m^A)], \quad \text{and}
\]

\[
\Pi_s^B(Q_s^A) = E_{\{i\}}[E_{\{x\}}]\Pi_s^B(Q_s^A, i; Q_m^B)] - c_1 Q_s^B,
\]

where \(\Pi_m^B(Q_m^B, i; Q_m^B) = p \min\{x, Q_m^B\} - w Q_m^B + b(Q_m^B - x)^+\) and \(\Pi_s^B(Q_s^A, i; Q_m^B) = w Q_m^B + v_1(Q_s^A - Q_m^B) - (b - v_2)(Q_m^B - x)^+ - c_2 Q_s^B\).

Similar to the discussions in Section 5.1, with \{w, b\}, the supplier’s optimal capacity reservation quantity \(Q_s^B\) satisfies equation \(\int_{\substack{Q_s^B < x \leq \infty}} [(w - c_2 - v_1) - (b - v_2)H(Q_s^B | i)] dG(I) = c_1 - v_1\) in the production season. The manufacturer’s optimal order quantity in the sales season is

\[
Q_s^B = \begin{cases} 
Q_s^B | i, & \text{if } i \leq i_{Q_s^B}, \\
Q_m, & \text{if } i > i_{Q_s^B}, 
\end{cases}
\]
where \( Q^B_m | i = H^{-1} \left( \frac{p-w}{p-b} \right) \) and \( i_{Q^B_\ast} \) satisfies \( Q^B_m | i_{Q^B_\ast} = Q^B_\ast \). Specifically, the following corollary presents the optimal quantity decisions in which the demand follows the uniform distribution:

**Corollary 4.** With \( \{ w, b, \} \),

1. In the production season,
   \[
   Q^B_\ast = \gamma + \frac{\alpha - \beta}{2} + \frac{w-c_2-v_1}{b-v_2} \beta - \sqrt{\frac{2(c_1-v_1)\alpha \beta}{b-v_2} - \left( \frac{p-w}{p-b} \beta - \frac{w-c_2-v_1}{b-v_2} \beta \right)^2};
   \]
2. In the sales season, there exists a unique positive information threshold \( i_{Q^B_\ast} = Q^B_\ast \) that satisfies \( Q^B_m | i_{Q^B_\ast} = Q^B_\ast \) and further
   \[
   Q^B_\ast = \begin{cases} 
   Q^B_m | i = \gamma + \frac{\alpha - \beta}{2} + \frac{p-w}{p-b} \beta, & \text{if } \gamma - \frac{\alpha}{2} \leq i \leq i_{Q^B_\ast}, \\
   Q^B_\ast, & \text{if } i_{Q^B_\ast} < i < \gamma + \frac{\alpha}{2}.
   \end{cases}
   \]

Following Chen et al. [13], the standard buyback contract partially coordinates the supply chain with demand information updating. That is, with \( \{ w, b, \} \), the supply chain only reaches suboptimal quantity decisions of \( \{ Q^B_\ast, Q^B_m \} \). Comparatively, the hybrid option-buyback contract can achieve perfect coordination and reach the optimal quantity decisions of \( \{ Q^c_{sc}, Q^c_\ast \} \). Similarly, we define the profit gaps between two contracts of \( \{ c_e, w_e, b \} \) and \( \{ w, b \} \) as

\[
Gap^B_m = \frac{\Pi^d_s(Q_{sc}^d) - \Pi^d_s(Q^B_m)}{\Pi^d_s(Q_{sc}^d)} \quad \text{and} \quad Gap^B_m = \frac{\Pi^d_s(Q^d_m) - \Pi^d_s(Q^B_m)}{\Pi^d_s(Q^d_m)},
\]

where \( Gap^B_m \) and \( Gap^B_m \) represent the supplier’s and manufacturer’s advantage of \( \{ c_e, w_e, b \} \) over \( \{ w, b \} \), respectively. Compared with \( \{ w, b \} \), by solving \( Gap^B_m \geq 0 \) and \( Gap^B_m \geq 0 \), the following proposition establishes the conditions in which the coordination of the supply chain is achieved under \( \{ c_e, w_e, b \} \in M \).

**Proposition 7.** Compared with \( \{ w, b \} \), the coordination of the supply chain is achieved under \( \{ c_e, w_e, b \} \in N^B \in M \), where \( N^B = \{ \{ c_e, w_e, b \} : c_e = \lambda(c_1 - v_1), w_e = p - \lambda(p - c_2 - v_1), b = p - \lambda(p - v_2), \lambda \in [\lambda^B_{\min}, \lambda^B_{\max}] \} \), \( \lambda^B_{\min} = \frac{\Pi^d_s(Q^B_m)}{\Pi^d_s(Q_{sc}^d)} \) and \( \lambda^B_{\max} = 1 - \frac{\Pi^d_s(Q^B_m)}{\Pi^d_s(Q_{sc}^d)} \).

Proposition 7 shows that with \( \{ c_e, w_e, b \} \in M \), the manufacturer is hurt when \( \lambda < \lambda^B_{\min} \) and the supplier is hurt when \( \lambda > \lambda^B_{\max} \). Therefore, only when \( \{ c_e, w_e, b \} \in N^B \in M \) (i.e., \( \lambda \in [\lambda^B_{\min}, \lambda^B_{\max}] \)) are both members at least as well off as under \( \{ w, b \} \). Tuning \( \lambda \in [\lambda^B_{\min}, \lambda^B_{\max}] \) can arbitrarily allocate the supply chain profit between both members. Compared with \( \{ w, b \} \), the coordination of the supply chain is achieved under \( \{ c_e, w_e, b \} \in N^B \in M \).

5. **Numerical simulations.** Numerical examples are presented to illustrate the main theoretical results. For simplicity, the optimal quantity decisions with uniformly distribution is illustrated in table 3. Similar to (see [13, 14, 20, 37]), the parameters are set as \( p = 100, w = 95, c_1 = 30, c_2 = 50, v_1 = 15, v_2 = 30, \alpha = 1000, \beta = 800 \) and \( \gamma = 1500 \). Then, we obtain following numerical results.

5.1. **Supply chain coordination.** In the centralized supply chain, the supply chain optimal capacity reservation quantity is \( Q_{sc}^c = 1414 \). The critical demand information threshold is \( i_{Q_{sc}^c} = 1414 \). At the beginning of the sales season, there exist two cases: (1) when \( 1000 \leq i \leq 1414 \), then the supply-chain optimal final order
quantity is \( Q^*_c = i \); (2) when \( 1414 < i \leq 2000 \), then \( Q^*_c = 1414 \). The whole supply chain expected profit is \( \Pi^c = 2.1355 \times 10^4 \) (Propositions 1 and 2, and Corollary 1).

In the decentralized supply chain with \( \{c_c, w_e, b\} \in M \), given the values of \( \lambda \in [0.1, 0.9] \), the values of \( c_c \), \( w_e \) and \( b \) can be obtained by Proposition 5. Table 4 presents the optimal quantity and expected profits under the hybrid option-buyback contract. The feasible intervals of \( c_c \), \( w_e \) and \( b \) are \((1.5, 13.5), (96.5, 68.5)\) and \((93, 37)\), respectively. Specifically, when \( \lambda = 0.3 \), then \( c_c = 4.5 \), \( w_e = 89.5 \) and \( b = 79 \). The decisions of the supplier and manufacturer are as follows: in the first stage, the supplier determines the option price \( c_c = 4.5 \) and the manufacturer determines an initial quantity \( Q^d_m = 1414 \). In the second stage, the supplier determines the exercise price \( w_e = 89.5 \) and buyback price \( b = 79 \), and the manufacturer determines the final quantity \( Q^d_e = i \) when \( 1000 \leq i \leq 1414 \) or \( Q^d_e = 1414 \) when \( 1414 < i \leq 2000 \). Under such settings, \( Q^d_m = Q^*_c \) and \( Q^d_e = Q^*_c \). The optimal expected profits of the supplier and manufacturer are \( \Pi^d = 1.4948^4 \) and \( \Pi^d_m = 0.6406^4 \), respectively. Note that \( \Pi^d_e = \Pi^d \) and \( \Pi^d_m = 1.4948^4 = \Pi^d_c \). In addition, \( \Pi^d_e = 1 - \lambda \Pi^d_m = 0.3 \Pi^d_m = 0.3 \Pi^d = \lambda^2 \). The parameter \( \lambda \) is essentially the proportion to allocate the supply chain profit. According to Proposition 5, by tuning the option price or buyback price, the supply chain profit can be arbitrarily allocated between both members. With \( \{c_c, w_e, b\} \in M \), the coordination of the supply chain with demand information updating is achieved (Propositions 3 and 4, and Corollary 2).

![Table 3: Optimal quantity decisions with uniformly distribution](image)

| Optimal decisions | 2.1355 | 1.92919 | 0.2135 | 0.9 | 0.1 |
|-------------------|--------|----------|--------|-----|-----|
| Exercise quantity | \( i \) | \( 1.355 \) | \( 0.2135 \) | \( 0.9 \) | \( 0.1 \) |
| Option quantity   | \( Q^d_m = c_c \) | \( Q^d_m = c_c \) | \( Q^d_m = c_c \) | \( Q^d_m = c_c \) | \( Q^d_m = c_c \) |
| \( \lambda \)     | \( 0.1 \) | \( 0.2 \) | \( 0.3 \) | \( 0.4 \) | \( 0.5 \) |
| \( c_c \)         | \( 1.5 \) | \( 3.0 \) | \( 4.5 \) | \( 6.0 \) | \( 7.5 \) |
| \( w_e \)         | \( 96.5 \) | \( 93.0 \) | \( 89.5 \) | \( 86.0 \) | \( 82.5 \) |
| \( b \)           | \( 93 \) | \( 86 \) | \( 79 \) | \( 72 \) | \( 65 \) |
| \( Q^*_c \)       | \( 1414 \) | \( 1414 \) | \( 1414 \) | \( 1414 \) | \( 1414 \) |
| \( Q^d_e \)       | \( 1414 \) | \( 1414 \) | \( 1414 \) | \( 1414 \) | \( 1414 \) |
| \( \Pi^d \)       | \( 1.4948^4 \) | \( 1.4948^4 \) | \( 1.4948^4 \) | \( 1.4948^4 \) | \( 1.4948^4 \) |
| \( \Pi^d_m \)      | \( 0.6406^4 \) | \( 0.6406^4 \) | \( 0.6406^4 \) | \( 0.6406^4 \) | \( 0.6406^4 \) |
| \( \Pi^d_e / \Pi^d \) | \( 1.0677 \) | \( 1.0677 \) | \( 1.0677 \) | \( 1.0677 \) | \( 1.0677 \) |
| \( \Pi^d_m / \Pi^d \) | \( 1.0677 \) | \( 1.0677 \) | \( 1.0677 \) | \( 1.0677 \) | \( 1.0677 \) |

5.2. Performance comparison among three contracts. In the decentralized supply chain with \( \{c_c, w_e\} \), the values of \( c_c \) and \( w_e \) are given and kept the same as \( \{c_c, w_e, b\} \). Table 5 shows the optimal quantity and expected profits under \( \{c_c, w_e\} \). When \( \lambda = 0.3 \), the supply chain members’ decisions are as follows: in the first stage, the supplier determines option price \( c_c = 4.5 \) and the manufacturer determines the initial quantity \( Q^*_m = 1399 \). In the second stage, the supplier determines the exercise price \( w_e = 89.5 \), and the manufacturer determines the final quantity 1414.
$Q^*_e = i - 280$ when $1000 \leq i \leq 1679$ or $Q^*_e = 1399$ when $1679 < i \leq 2000$. With $\{c_e, w_e\}$, $\Pi^A < \Pi^c$. The decentralized supply chain optimal profit is less than the centralized supply chain. In addition, $\text{Gap}_{m}^A > 0$. The manufacturer can obtain greater expected profit from $c_e, w_e, b$ than from $c_e, w_e$ while the supplier may be hurt. Specifically, when $\lambda = 1$ (i.e., $c_e = 5$ and $w_e = 55$), then $Q^*_m = Q^*_c = 1414$, $\Pi^A = \Pi^c = 2.1355^A$ and $\Pi^A = 0$. Only when $c_e = v_1$ and $w_e = c_2 + v_1$, $\{c_e, w_e\}$ can reach the centralized supply chain optimal profit. The manufacturer, however, would occupy all profit (Corollary 3).

Table 5. Optimal quantity and expected profits with $\{c_e, w_e\}$

| $\lambda$ | $c_e$ | $w_e$ | $Q^*_m$ | $Q^*_c$ | $i$ | $\Pi^A$ | $\Pi^c$ | $\text{Gap}_{m}^A$ | $\text{Gap}_{m}^c$ |
|-----------|-------|-------|---------|---------|-----|---------|---------|----------------|----------------|
| 0.1       | 1.5   | 96.5  | 1454    | i-360   | 1.7374 | 1.5729  | 0.181   | 0.1645         | 0.025          |
| 0.2       | 3.0   | 93.0  | 1418    | i-320   | 1.8526 | 1.5062  | 0.105   | 0.3463         | 0.042          |
| 0.3       | 4.5   | 89.5  | 1399    | i-280   | 1.9339 | 1.3937  | 0.052   | 0.5402         | 0.052          |
| 0.4       | 6.0   | 86.0  | 1389    | i-240   | 1.9954 | 1.2512  | 0.015   | 0.7441         | 0.057          |
| 0.5       | 7.5   | 82.5  | 1386    | i-200   | 2.0425 | 1.0855  | -0.009  | 0.9570         | 0.058          |
| 0.6       | 9.0   | 79.0  | 1386    | i-160   | 2.0782 | 0.9001  | -0.023  | 1.1781         | 0.054          |
| 0.7       | 10.5  | 75.5  | 1390    | i-120   | 2.1043 | 0.6974  | -0.029  | 1.4069         | 0.046          |
| 0.8       | 12.0  | 72.0  | 1396    | i-80    | 2.1220 | 0.4791  | -0.027  | 1.6429         | 0.034          |
| 0.9       | 13.5  | 68.5  | 1404    | i-40    | 2.1322 | 0.2463  | -0.017  | 1.8859         | 0.019          |

In the decentralized supply chain with $\{w, b\}$, the values of $b$ are given and kept the same as $\{c_e, w_e, b\}$. Table 6 shows the optimal quantity and expected profits under $\{w, b\}$. When $\lambda = 0.3$, the supply-chain members' decisions are as follows: in the first stage, the supplier determines the initial quantity $Q^*_B = 1328$. In the second stage, the supplier determines the buyback price $b = 79$, and the manufacturer determines the final quantity $Q^*_m = i - 210$ when $1000 \leq i \leq 1538$ or $Q^*_e = 1328$ when $1538 < i \leq 2000$. With $\{w, b\}$, $\Pi^B < \Pi^c$. The standard buyback contract cannot reach the centralized supply chain optimal profit. There exists a Pareto improvement area under $\{c_e, w_e, b\}$ that makes both members better off than $\{w, b\}$ (Propositions 6 and 7).

Table 6. Optimal quantity and expected profits with $\{c_e, w_e\}$

| $\lambda$ | $w$ | $b$ | $Q^*_B$ | $Q^*_m$ | $i$ | $\Pi^B$ | $\Pi^m$ | $\text{Gap}_{m}^B$ | $\text{Gap}_{m}^c$ |
|-----------|-----|-----|---------|---------|-----|---------|---------|----------------|----------------|
| 0.1       | 95  | 93  | 1335    | i+171   | 1.8209 | 1.4827  | 0.228   | 0.3382         | -0.065         |
| 0.2       | 95  | 86  | 1358    | i-114   | 2.0587 | 1.5237  | 0.096   | 0.5350         | -0.056         |
| 0.3       | 95  | 79  | 1328    | i-210   | 1.9339 | 1.4712  | 0.012   | 0.5270         | 0.059          |
| 0.4       | 95  | 72  | 1302    | i-257   | 1.9386 | 1.4283  | -0.076  | 0.5102         | 0.179          |
| 0.5       | 95  | 65  | 1279    | i-285   | 1.8924 | 1.3988  | -0.172  | 0.4936         | 0.298          |
| 0.6       | 95  | 58  | 1258    | i-305   | 1.8560 | 1.3791  | -0.273  | 0.4769         | 0.418          |
| 0.7       | 95  | 51  | 1236    | i-318   | 1.8259 | 1.3668  | -0.378  | 0.4591         | 0.538          |
| 0.8       | 95  | 44  | 1213    | i-329   | 1.7990 | 1.3601  | -0.485  | 0.4388         | 0.661          |
| 0.9       | 95  | 37  | 1187    | i-337   | 1.7724 | 1.3581  | -0.595  | 0.4142         | 0.784          |

Figure 2 shows that with $\{c_e, w_e, b\}$, there exist $\lambda_m = 0.2476$ and $\lambda_s = 0.3205$, which make both members better off than $\{w, b\}$ and $\{c_e, w_e\}$. Therefore, when
$\lambda \in (0.2476, 0.3205)$, with $\{c_e, w_e, b\} \in M$, the supply chain realizes a Pareto improvement compared with $\{c_e, w_e\}$ and $\{w, b\}$ (Propositions 6 and 7).

Figure 2. Pareto improvement area

5.3. The impacts of the parameters. We investigate the impacts of the parameters $c_e$, $w_e$, $b$, $p$, $c_1$, $c_2$, $v_1$, and $v_2$ on the expected profits of the supply chain and its members. Set $c_e = 4.5$, $w_e = 89.5$, and $b = 79$ and other parameters are the same as the previous settings. When investigating the impacts of a certain parameter, we maintain the other parameters as unchanged.

Figure 3 shows the impacts of $c_e$ and $w_e$ on the expected profits of the supply chain and both members. Note that $c_e \in [1, 9]$ in Figure 3(1) and $w_e \in [82, 90]$ in Figure 3(2). From Figure 3, the manufacturer’s expected profit is decreasing in $c_e$ or $w_e$, in sharp contrast to the supplier’s tendency. As $c_e$ or $w_e$ increases, the supply chain expected profit increases. This increase suggests that although increasing $c_e$ or $w_e$ improves the performance of the supply chain and the supplier, it increases the manufacturer’s costs and further hurts the manufacturer.

Figure 3. The impacts of $c_e$ and $w_e$
Figure 4 shows the impacts of $b$ and $p$ on the expected profits of the supply chain and both members. Note that $b \in [71, 79]$ in Figure 4(1) and $w_e \in [96, 104]$ in Figure 4(2). From Figure 4, the optimal profits of the supply chain and manufacturer both increase in the buyback price or retail price. The supply chain and manufacturer would benefit from high $b$ or $p$ while the supplier is likely to be hurt.

**Figure 4.** The impacts of $b$ and $p$

Figures 5 and 6 illustrate the impacts of the costs and the salvage values on the expected profits of the supply chain and both members, respectively. Note that $c_1 \in [25, 34]$, $c_2 \in [45, 54]$ in Figure 5, and $v_1 \in [10, 19]$, $v_2 \in [25, 34]$ in Figure 6. The optimal profits of the supply chain and the supplier decrease in $c_1$ or $c_2$ and increase in $v_1$ or $v_2$ while the manufacturer remains unchanged. The low cost or high salvage value benefits the supplier and correspondingly reduces the risk of overproduction.

**Figure 5.** The impacts of $c_1$ and $c_2$

6. **An extension with price-dependent demand.** In the previous discussions, we assume that the market demand is independent of the retail price. However,
in some situations, manufacturers may influence market demand by adjusting retail prices. With a hybrid option-buyback contract, it is interesting to explore the coordination of the supply chain in which demand is price-dependent and the manufacturer updates demand information. To this end, we assume that demand has an additive linear form \( x(p) = a - p + \varepsilon = y(a, p) + \varepsilon \), where \( a \) is the extent to which the products are accepted by the market following the probability density \( \tilde{g}(a) \), and \( \varepsilon \) is the random noise following the probability density \( \tilde{f}(\varepsilon) \) and distribution \( \tilde{F}(\varepsilon) \) (see [14, 26, 29, 30]).

The sequence of events is as follows: At the beginning of the production season, the manufacturer reserves capacity \( Q^d_m \) based on the initial demand information \( x(p) \). At the end of the production season, the value of \( a \) is realized as \( \bar{a} \) by observing the reaction of the market to the product. The demand is updated as \( x(p|\bar{a}) = \bar{a} - p + \varepsilon = y(\bar{a}, p) + \varepsilon \). At the beginning of the sales season, the manufacturer decides his final order quantity \( Q^c \) and retail price \( p \) based on \( x(p|\bar{a}) \). Meanwhile, the supplier provides the manufacturer with \( \min\{Q^d_m, Q^d_e\} \) products. At the end of the sales season, the supplier buys back the unsold units. In such settings, we examine whether the new hybrid option-buyback contracts can coordinate the supply chain with demand information updating.

We first study the expected profit of the centralized supply chain. Given \( Q^c_{sc} \) and \( \bar{a} \) at the beginning of the sales season, there exist two possible quantity scenarios: \( Q^c_{sc} \geq Q^c_e \) and \( Q^c_{sc} < Q^c_e \). When \( Q^c_{sc} \geq Q^c_e \), the reservation capacity \( Q^c_{sc} \) is enough to satisfy the optimal order quantity \( Q^c_e \). The supply-chain profit is

\[
\Pi(Q^c_{sc}; \bar{a}; Q^c_e) = E_{[x|\bar{a}]} \left\{ \begin{array}{ll}
p^e x - c_2 Q^c_e + v_2(Q^c_e - x) + v_1(Q^c_{sc} - Q^c_e), & x \leq Q^c_e, \\
p^e Q^c_e - c_2 Q^c_e + v_1(Q^c_{sc} - Q^c_e), & x > Q^c_e. \end{array} \right.
\]

Let \( x(p^e|\bar{a}) = y(p^e|\bar{a}) + \varepsilon \) and \( z^c = Q^c_e - y(p^e|\bar{a}) \). Then, we have

\[
\Pi(Q^c_{sc}; \bar{a}; Q^c_e) = E_{[x|\bar{a}]} \left\{ \begin{array}{ll}
p^e[y(p^e|\bar{a}) + \varepsilon] - (c_2 + v_1)[y(p^e|\bar{a}) + z^c] + v_2[z^c - \varepsilon] + v_1 Q^c_{sc}, & \varepsilon \leq z^c, \\
(p^e - c_2 - v_1)[y(p^e|\bar{a}) + z^c] + v_1 Q^c_{sc}, & \varepsilon > z^c. \end{array} \right.
\]

Further, the supply chain expected profit is

\[
\Pi(Q^c_{sc}; \bar{a}; Q^c_e) = v_1 Q^c_{sc} + (p^e - c_2 - v_1)[y(p^e|\bar{a}) + \mu] - (c_2 + v_1 - v_2)I(z^c) - (p^e - c_2 - v_1)L(z^c). \quad (15)
\]
where \( I(z) = \int_{-\infty}^{z} (e^{z}-z-\varepsilon) d\varepsilon \) and \( L(z) = \int_{e^{-z}}^{\infty} (e^{z} - z - \varepsilon) d\varepsilon \).

When \( Q_{sc}^c < Q_{e}^c \), the reservation capacity \( Q_{sc}^c \) is not enough to satisfy the optimal order quantity \( Q_{e}^c \). The supply chain profit is

\[
\Pi(Q_{sc}, a; Q_{e}^c) = E_{\{x|\tilde{a}\}} \left\{ \begin{array}{ll}
p^e x - c_2Q_{sc}^c + v_2(Q_{sc}^c - x), & x \leq Q_{sc}^c, \\
p^d Q_{sc}^c - c_2Q_{sc}^c, & x > Q_{sc}^c.
\end{array} \right.
\]

Further, the supply chain expected profit is

\[
\Pi(Q_{sc}, a; Q_{e}^c) = (p^e - v_2)(y(p^e|\tilde{a})+\mu) - (p^e - v_2) \int_{Q_{sc}^c-y(p^e|\tilde{a})}^{+\infty} [y(p^e|\tilde{a})+\varepsilon - Q_{sc}^c] \tilde{f}(\varepsilon) d\varepsilon
\]

\[-(c_2 - v_2)Q_{sc}^c. \quad (16)\]

With \( \{c_e, w_e, b\} \), we study the supply chain members’ profit in the decentralized supply chain. Given \( Q_{m}^d \) and \( \tilde{a} \) at the beginning of the sales season, there also exist two possible scenarios: \( Q_{m}^d \geq Q_{e}^d \) and \( Q_{m}^d < Q_{e}^d \). Similarly, let \( x(p^d|\tilde{a}) = y(p^d|\tilde{a}) + \varepsilon \) and \( z^d = Q_{e}^d - y(p^d|\tilde{a}) \). When \( Q_{m}^d \geq Q_{e}^d \), the manufacturer’s profit is

\[
\Pi_{m}^d(Q_{m}^d, \tilde{a}; Q_{e}^d) = E_{\{x|\tilde{a}\}} \left\{ \begin{array}{ll}
p^d y(p^d|\tilde{a}) + \varepsilon + b(z^d - \varepsilon) - w_e[y(p^d|\tilde{a}) + z^d], & \varepsilon \leq z^d, \\
(p^d - w_e)[y(p^d|\tilde{a}) + z^d], & \varepsilon > z^d.
\end{array} \right.
\]

Further, the manufacturer’s expected profit is

\[
\Pi_{m}^d(Q_{m}^d, \tilde{a}; Q_{e}^d) = (p^d - w_e)y(p^d|\tilde{a}) + \mu) - (p^d - w_e)L(z^d). \quad (17)
\]

When \( Q_{m}^d < Q_{e}^d \), the manufacturer’s profit is

\[
\Pi_{m}^d(Q_{m}^d, \tilde{a}; Q_{e}^d) = E_{\{x|\tilde{a}\}} \left\{ \begin{array}{ll}
p^d y(p^d|\tilde{a}) + \varepsilon + b(Q_{m}^d - y(p^d|\tilde{a}) - \varepsilon) - w_eQ_{m}^d, & \varepsilon \leq z^d, \\
(p^d - w_e)Q_{m}^d, & \varepsilon > z^d.
\end{array} \right.
\]

Further, the manufacturer’s expected profit is

\[
\Pi_{m}^d(Q_{m}^d, \tilde{a}; Q_{e}^d) = (p^d - b)y(p^d|\tilde{a}) + \mu) - (p^d - b) \int_{Q_{m}^d-y(p^d|\tilde{a})}^{+\infty} [y(p^d|\tilde{a})+\varepsilon - Q_{m}^d] \tilde{f}(\varepsilon) d\varepsilon
\]

\[-(w_e - b)Q_{m}^d. \quad (18)\]

To verify whether \( \{c_e, w_e, b\} \in M \) can coordinate the supply chain with demand information updating, we investigate the manufacturer’s decisions in the sales season. Substituting Equation (14) into Equations (17) and (18) yields

\[
\Pi_{m}^d(Q_{m}^d, \tilde{a}; Q_{e}^d) = \lambda \{(p^d - c_2 - v_1)(y(p^d|\tilde{a})+\mu) - (c_2 + v_1 - v_2)I(z^d) - (p^d - c_2 - v_1)Q_{m}^d\}, \quad (19)
\]

\[
\Pi_{m}^d(Q_{m}^d, \tilde{a}; Q_{m}^d) = \lambda \{(p^d - v_2)(y(p^d|\tilde{a}) + \mu) - (c_2 - v_2 - v_1)Q_{m}^d \}
\]

\[-\lambda (p^d - v_2) \int_{Q_{m}^d-y(p^d|\tilde{a})}^{+\infty} [y(p^d|\tilde{a})+\varepsilon - Q_{m}^d] \tilde{f}(\varepsilon) d\varepsilon. \quad (20)\]

Comparing Equations (19) and (20) with Equations (15) and (16), we find that the supplier’s production decision and the manufacturer’s order decision coincide with the optimal decisions of the centralized supply chain. The hybrid option-buyback contract can coordinate the supply chain with demand information updating.
7. Conclusion and management insights. In this paper, we consider a two-echelon supply chain in which the supplier needs to prepare production capacity in advance according to an initial order at the beginning of the production season, and the manufacturer makes the final order by taking advantage of updated demand information at the beginning of the sales season. The existing literature shows that the conventional option contract and buyback contract both fail to coordinate the supply chain with demand information updating. We therefore propose a new hybrid option-buyback contract to investigate the coordination of such a supply chain with demand information updating. The analysis shows that demand information updating benefits the manufacturer and the supply chain but does not always benefit the supplier. Compared with the conventional option contract and buyback contract, the hybrid option-buyback contract creates greater supply chain profit and further realizes the supply chain members’ Pareto improvement. By adjusting the option price or buyback price, the hybrid option-buyback contract can arbitrarily allocate the whole supply chain profit between the supplier and the manufacturer and achieve the coordination of such a supply chain. When the supply chain is extended to situations with price-dependent demand, the coordination of such a supply chain can still be achieved with a hybrid option-buyback contract. In addition, the numerical simulations show that the parameters of salvage value, retail price and production cost significantly affect the efficiency of the hybrid option-buyback contract.

The paper reveals two main management insights. First, demand information updating benefits the manufacturer and the supply chain but does not always benefit the supplier. To improve the efficiency of the supply chain, the manufacturer that benefited from demand information updating should offer partial compensation to the supplier that was hurt due to overproduction. Second, to mitigate the individual loss, the supplier may take some measures to entice the manufacturer to order more. For example, the supplier may set a quantity discount to induce a larger order from the manufacturer.

Similar to the previous researches, we carry on the qualitative analysis to the demand uncertainty through Bayesian approach. It’s worth pointing out that the demand variance can be reduced by demand information updating but not completely eliminated. However, it’s difficult to provide more explicit and direct management advices. Thus, future research of uncertainty management should focus on quantitative analysis of uncertainty. That is, we need to study how build model to measure and control uncertainty. For example, some papers develop a robust-entropic optimal control technique to characterize uncertainty of model (see [2, 3]). Savku adn Weber [31] use a delayed Markov regime switching jump-diffusion approach to model a stochastic optimal control problem.

The contribution of this paper is to propose a new hybrid option-buyback contract to coordinate a supply chain with demand information updating, and characterize the conditions under which the supply chain coordination is achieved. Many researches also have considered the issue of inventory management and finance strategy (see [3, 28, 32] and references therein). Inspired by these classic literatures, this paper can be extended in the following directions. First, we assume that the manufacturer always updates demand information at the beginning of production season to observe more market signal. In reality, the ordering cost during production season always is uncertainty and may be time-dependent. There is a tradeoff between
update time and ordering cost. Incorporating the time of demand information updating as a decision variable will be an interesting topic in future research. In addition, the demand not only dependent on its retail price, but may also be affected by stock level. We can further extend our model to the case of price- and stock-sensitive demand. Second, current model implicitly assumes that the supply chain members are endowed with full capital. In practice, upstream and downstream enterprises of a supply chain, especially SMEs, often face funding shortfalls. A capital-constrained member must seek external financing from financial institutions or other supply chain members. For example, the manufacturer with capital constrained finances his inventory decision with bank credit or trade credit. Considering joint financing and ordering decisions for a capital-constrained manufacturer may yield interesting and meaningful results. Moreover, the manufacturer may also fund its business by portfolio financing scheme to satisfy uncertain demand.

Appendixes

Appendix (Proof of Proposition 1.)

Given $Q^*_e$ and $i$ in the sales season, the supply-chain expected profit is $\Pi^e(Q^*_e, i; Q^*_c) = l(Q^*_e - \int_0^{Q^*_c} H(x|i)dx) - c_2Q^*_e + v_2 \int_0^{Q^*_c} H(x|i)dx + v_1(Q^*_e - Q^*_c) = (p - c_2 - v_1)Q^*_e - (p - v_2) \int_0^{Q^*_c} H(x|i)dx + v_1Q^*_e$. Taking the first and second partial derivatives of $\Pi^e(Q^*_e, i; Q^*_c)$ with respect to $Q^*_e$ yields $\frac{\partial \Pi^e(Q^*_e, i; Q^*_c)}{\partial Q^*_e} = (p - c_2 - v_1) - (p - v_2)H(Q^*_c|i)$ and $\frac{\partial^2 \Pi^e(Q^*_e, i; Q^*_c)}{\partial Q^*_c \partial Q^*_e} = -(p - v_2)h(Q^*_c|i) < 0$. Therefore, $\Pi^e(Q^*_e, i; Q^*_c)$ is concave in $Q^*_e$, and the first-order condition can be written as $H(Q^*_c|i) = \frac{c_2 - v_1}{p - v_2}$. Particularly, let $Q^*_c|i = H^{-1}\left(\frac{c_2 - v_1}{p - v_2}\right)$, where $H^{-1}(\cdot)$ is the inverse function of $H(\cdot)$. Note that $Q^*_e \leq Q^*_e$, the optimal final order quantity $Q^*_e = Q^*_e(Q^*_e, i)$, can be written as $Q^*_e = Q^*_e(Q^*_e, i) = \min(Q^*_e, Q^*_e, i)$. In addition, if $x|i$ is stochastically increasing in $i$, then there exists an information threshold value $i_Q^e$ that satisfies $Q^*_e|i_Q^e = Q^*_e$. Further,

$$Q^*_e = \begin{cases} Q^*_e|i, & \text{if } i \leq i_Q^e, \\ Q^*_e, & \text{if } i > i_Q^e. \end{cases}$$

Appendix (Proof of Corollary 1.)

If demand follows the uniform-uniform distribution, then we have $H(Q^*_c|i) = \frac{p - c_2 - v_1}{p - v_2} = \frac{1}{\beta}(Q^*_e - i + \frac{\beta}{2})$. Hence, $Q^*_e|i = i - \frac{\beta}{2} + \frac{p - c_2 - v_1}{p - v_2}\beta$. Let $Q^*_e|i_Q^e = i_Q^e = \frac{\beta}{2} + \frac{p - c_2 - v_1}{p - v_2}\beta = Q^*_e$. We have $i_Q^e = Q^*_e + \frac{\beta}{2} - \frac{p - c_2 - v_1}{p - v_2}\beta$. Therefore, the optimal final order quantity is

$$Q^*_e = \begin{cases} Q^*_e|i = i - \frac{\beta}{2} + \frac{p - c_2 - v_1}{p - v_2}\beta, & \text{if } \gamma - \frac{\gamma}{2} \leq i \leq i_Q^e, \\ Q^*_e, & \text{if } i_Q^e < i < \gamma + \frac{\gamma}{2}. \end{cases}$$

Appendix (Proof of Lemma 1.)

From Equations (2) and (3), we have $\frac{\partial \Pi^e(Q^*_e, i; Q^*_c)}{\partial Q^*_c} = v_1$ and $\frac{\partial^2 \Pi^e(Q^*_e, i; Q^*_c)}{\partial Q^*_c} = (p - c_2 - v_1) - (p - v_2)H(Q^*_c|i)$. Using the derivative formula of an integral with
variable upper and lower limits and $Q_{sc}^e|_{Q_{sc}^e} = Q_{sc}^e$ if $i = i_{Q_{sc}^e}$, we have
\[ \frac{\partial \Pi^c(Q_{sc}^e)}{\partial Q_{sc}^e} = \int_{0}^{i_{Q_{sc}^e}} \frac{\partial \Pi^c(Q_{sc}^e, i, Q_{sc}^e)}{\partial Q_{sc}^e} dG(I) + \int_{i_{Q_{sc}^e}}^{+\infty} \frac{\partial \Pi^c(Q_{sc}^e, i, Q_{sc}^e)}{\partial Q_{sc}^e} dG(I) - c_1 \]
\[ = \int_{0}^{i_{Q_{sc}^e}} v_1 dG(I) + \int_{i_{Q_{sc}^e}}^{+\infty} [(p - c_2 - v_1) - (p - v_2)H(Q_{sc}^e|i)]dG(I) - c_1 \]
\[ = \int_{i_{Q_{sc}^e}}^{+\infty} [(p - c_2 - v_1) - (p - v_2)H(Q_{sc}^e|i)]dG(I) - (c_1 - v_1), \]
and
\[ \frac{\partial^2 \Pi^c(Q_{sc}^e)}{\partial (Q_{sc}^e)^2} = -\int_{i_{Q_{sc}^e}}^{+\infty} (p - v_2)H(Q_{sc}^e|i)dG(I) < 0. \]
Therefore, $\Pi^c(Q_{sc}^e)$ is concave in $Q_{sc}^e$. 

**Appendix (Proof of Proposition 2.)**

From Lemma 1, the optimal option quantity $Q_{sc}^e$ satisfies the first order condition
\[ \int_{Q_{sc}^e}^{+\infty} [(p - c_2 - v_1) - (p - v_2)H(Q_{sc}^e|i)]dG(I) - (c_1 - v_1) = 0. \]
Since $H(Q_{sc}^e|i) = \frac{p - c_2 - v_1}{p - v_2}$, we have \[ \int_{Q_{sc}^e}^{+\infty} [H(Q_{sc}^e|i) - H(Q_{sc}^e*)|i]dG(I) = \frac{2(c_1 - v_1)}{p - v_2}. \] When demand follows a uniform-uniform distribution, the above first-order condition can be rewritten as
\[ \frac{1}{\alpha} \int_{Q_{sc}^e + \frac{\alpha - \beta}{\alpha}}^{+\frac{\alpha - \beta}{\alpha}} [p - c_2 - v_1 - \frac{1}{\beta} \left( Q_{sc}^e - i + \frac{\beta}{2} \right)] di = \frac{c_1 - v_1}{p - v_2}. \]
Then,
\[ \left( Q_{sc}^e - \gamma - \frac{\alpha - \beta}{2} \right)^2 - 2 \left( Q_{sc}^e - \gamma - \frac{\alpha - \beta}{2} \right) \frac{p - c_2 - v_1}{p - v_2} \beta + \left( \frac{p - c_2 - v_1}{p - v_2} \right)^2 \beta = \frac{2(c_1 - v_1)\alpha \beta}{p - v_2}. \]
That is,
\[ \left( Q_{sc}^e - \gamma - \frac{\alpha - \beta}{2} \right) - \frac{p - c_2 - v_1}{p - v_2} \beta = \frac{2(c_1 - v_1)\alpha \beta}{p - v_2}. \]
Hence,
\[ Q_{sc}^e = \gamma + \frac{\alpha - \beta}{2} + \frac{p - c_2 - v_1}{p - v_2} \beta - \sqrt{\frac{2(c_1 - v_1)\alpha \beta}{p - v_2}}. \]

**Appendix (Proof of Proposition 3.)**

Given $Q_{m}^d$ and $i$ in the sales season, the expected profit of the manufacturer is
\[ \Pi^d_m(Q_{m}^d, i; Q_{e}^d) = p \left( Q_{e}^d - \int_{0}^{Q_{e}^d} H(x|i)dx \right) + b \int_{0}^{Q_{e}^d} H(x|i)dx - w_e Q_{e}^d \]
\[ = (p - w_e) Q_{e}^d - (p - b) \int_{0}^{Q_{e}^d} H(x|i)dx. \]
Taking the first and second partial derivatives of $\Pi^d_m(Q_{m}^d, i; Q_{e}^d)$ with respect to $Q_{e}^d$ yields
\[ \frac{\partial \Pi^d_m(Q_{m}^d, i; Q_{e}^d)}{\partial Q_{e}^d} = (p - w_e) - (p - b)H(Q_{e}^d|i) \]
and
\[ \frac{\partial^2 \Pi^d_m(Q_{m}^d, i; Q_{e}^d)}{\partial (Q_{e}^d)^2} = -(p - b)h(Q_{e}^d|i) < 0. \]
Hence, $\Pi^d_{m}(Q^d_m, i; Q^d_e)$ is concave in $Q^d_e$, and the first-order condition can be written as $H(Q^d_e|i) = \frac{p-w_e}{p-b}$. Particularly, let $Q^d_e|i = H^{-1}\left(\frac{p-w_e}{p-b}\right)$. Since $Q^d_e \leq Q^d_m$, the optimal final order quantity $Q^d_{e*} = \min(Q^d_e | Q^d_m, i)$ can be written as $Q^d_{e*} = \min(Q^d_e | Q^d_m, i)$. In addition, if $x|i$ is stochastically increasing in $i$, then there exists an information threshold value $i_{Q^d_m}$ that satisfies $Q^d_e|i_{Q^d_m} = Q^d_m$. Further,

$$Q^d_{e*} = \begin{cases} Q^d_e|i, & i \leq i_{Q^d_m}, \\ Q^d_m, & i > i_{Q^d_m}. \end{cases}$$

\[\square\]

**Appendix (Proof of Corollary 2.)**

If demand follows the uniform-uniform distribution, then $H(Q^d_e|i) = \frac{p-w_e}{p-b} = \frac{1}{\beta} \left(Q^d_e - \frac{i + \beta}{2}\right)$. Further, $Q^d_e|i = i - \frac{\beta}{2} + \frac{p-w_e}{p-b} \cdot \beta$. Let $Q^d_e|i_{Q^d_m} = i_{Q^d_m} = \frac{\beta}{2} + \frac{p-w_e}{p-b} \cdot \beta = Q^d_m$. Then, we have $i_{Q^d_m} = Q^d_m + \frac{\beta}{2} - \frac{p-w_e}{p-b} \cdot \beta$. The optimal final order quantity is

$$Q^d_{e*} = \begin{cases} Q^d_e|i, & i \leq i_{Q^d_m}, \\ Q^d_m, & i > i_{Q^d_m}. \end{cases}$$

\[\square\]

**Appendix (Proof of Proposition 4.)**

From Equation (9), using the derivative formula of an integral with variable upper and lower limits and $Q^d_e|i_{Q^d_m} = Q^d_m$, if $i = i_{Q^d_m}$, then

$$\begin{align*}
\frac{\partial \Pi^d_{m}(Q^d_m)}{\partial Q^d_m} &= \int_{0}^{Q^d_m} \frac{\partial \Pi^d_{m}(Q^d_m, i; Q^d_m|i)}{\partial Q^d_m} dG(I) + \int_{Q^d_m}^{+\infty} \frac{\partial \Pi^d_{m}(Q^d_m, i; Q^d_m|i)}{\partial Q^d_m} dG(I) - c_e \\
&= \int_{Q^d_m}^{+\infty} [(p-w_e) - (p-b)H(Q^d_m|i)]dG(I) - c_e,
\end{align*}$$

and

$$\frac{\partial^2 \Pi^d_{m}(Q^d_m)}{\partial (Q^d_m)^2} = - \int_{Q^d_m}^{+\infty} (p-b)H(Q^d_m|i)dG(I) < 0.$$

Hence, $\Pi^d_{m}(Q^d_m)$ is concave in $Q^d_m$. Furthermore, the optimal option quantity $Q^d_{e*}$ satisfies the first-order condition $\int_{Q^d_m}^{+\infty} [(p-w_e) - (p-b)H(Q^d_{e*}|i)]dG(I) - c_e = 0$.

Since $H(Q^d_e|i) = \frac{p-w_e}{p-b}$, then $\int_{Q^d_m}^{+\infty} [H(Q^d_e|i) - H(Q^d_{e*}|i)]dG(I) = \frac{c_e}{p-b}$. When demand follows the uniform-uniform distribution, the above first-order condition can be written as

$$\frac{1}{\alpha} \int_{Q^d_m}^{Q^d_{e*}+\frac{\gamma}{2}} - \frac{p-w_e}{p-b} - \frac{1}{\beta} \left(Q^d_m - i + \frac{\gamma}{2}\right) \, di = \frac{c_e}{p-b}.$$

Further, we have

$$\left(Q^d_{e*} - \gamma - \frac{\alpha - \beta}{2}\right)^2 - 2 \left(Q^d_{e*} - \gamma - \frac{\alpha - \beta}{2}\right) \frac{p-w_e}{p-b} \beta + \left(\frac{p-w_e}{p-b} \beta\right)^2 = \frac{2c_e \alpha \beta}{p-b},$$

and

$$\left[\left(Q^d_{e*} - \gamma - \frac{\alpha - \beta}{2}\right) - \frac{p-w_e}{p-b} \beta\right]^2 = \frac{2c_e \alpha \beta}{p-b}.$$

Therefore, we have $Q^d_{e*} = \gamma + \frac{\alpha - \beta}{t} + \frac{p-w_e}{p-b} \beta - \sqrt{\frac{2c_e \alpha \beta}{p-b}}.$

\[\square\]
Appendix (Proof of Proposition 5.)

(i) Let \( H(Q^d_e|i) = H(Q^d_e|i) \). Then, \( w_e = p - \frac{p - b}{p - v_2} (p - c_2 - v_1) \). Introducing \( w_e \) into Equations (5) and (10), we find that when \( \frac{c_e - v_1}{p - v_2} = \frac{c_e}{p - v_2} \) (i.e., \( b = p - \frac{c_e}{c_1 - v_1} (p - v_2) \), Equations (5) and (10) are equivalent. The supplier acts in the manner to maximize the whole expected profit of the supply chain. Let \( \lambda = \frac{c_e}{c_1 - v_1} \), then \( w_e = p - \lambda (p - c_2 - v_1) \) and \( b = p - \lambda (p - v_2) \). The coordination of the supply chain is achieved.

(ii) With \( \{c_e, w_e, b\} \in M \), it is straightforward to verify \( Q^d_e = Q^d_m \) and \( Q^d_e = Q^d_m \).

(iii) In the sales season, substituting Equation (14) into Equations (7) and (8), we have

\[
\Pi^d_m(Q^d_m, i; Q^d_e|i) = \lambda \left\{ (p - c_2 - v_1)Q^d_e|i - (p - v_2) \int_0^{Q^d_e|i} H(x|i)dx \right\},
\]

and

\[
\Pi^d_m(Q^d_m, i; Q^d_e|i) = \lambda \left\{ (p - c_2 - v_1)Q^d_m - (p - v_2) \int_0^{Q^d_m} H(x|i)dx \right\}.
\]

From Equations (2) and (3), the manufacturer orders the same final quantity between the decentralized and centralized systems. In the production season, the manufacturer’s profit function is

\[
\Pi^d_m(Q^d_m, i; Q^d_e|i) = \int_0^{Q^d_e} \Pi^d_m(Q^d_m, i; Q^d_e|i)dG_i(i) + \int^{Q^d_m} \Pi^d_m(Q^d_m, i; Q^d_e|i)dG_i(i) - c_e Q^d_m = (1 - \lambda) \Pi^d(Q^d_e|i).
\]

Further, \( \Pi^d(Q^d_m, i) = (1 - \lambda) \Pi^d(Q^d_e|i) \). The hybrid option-buyback contract permits an arbitrary allocation of the whole supply chain profit between both members through tuning \( \lambda \).

\[\Box\]

Appendix (Proof of Corollary 3.)

With \( \{c_e, w_e\} \), given \( Q^A_m \) and \( i \) in the sales season, the manufacturer’s expected profit is \( \Pi(Q^A_m; Q^A_e) = p \left( Q^e_A - \int_0^{Q^A_e} H(x|i)dx \right) - w_e Q^A_e + v_2 \int_0^{Q^A_e} H(x|i)dx = (p - w_e)Q^A_e - (p - v_2) \int_0^{Q^A_e} H(x|i)dx \). Taking the first and second derivatives of \( \Pi(Q^A_m, i; Q^A_e) \) with respect to \( Q^A_e \) yields \( \partial \Pi(Q^A_m, i; Q^A_e) \frac{\partial Q^A_e}{\partial Q^A_m, i} = (p - w_e) - (p - v_2) H(Q^A_e|i) \) and \( \frac{\partial^2 \Pi(Q^A_m, i; Q^A_e)}{\partial Q^A_m, i} = - (p - v_2) h(Q^A_e|i) < 0 \). Therefore, \( \Pi(Q^A_m, i; Q^A_e) \) is concave in \( Q^A_e \), and the first-order condition can be written as \( H(Q^A_e|i) = \frac{w - w_e}{p - v_2} \). Let \( Q^A_e|i = H^{-1} \left( \frac{w - w_e}{p - v_2} \right) \). Note that \( Q^A_e \leq Q^A_m \), and the optimal final order quantity in the sales season, \( Q^A_e* = Q^A_e(Q^A_m, i) \) can be written as \( Q^A_e* = Q^A_e*(Q^A_m, i) = \min\{Q^A_e, Q^A_e|i\} \). In addition, if \( x|i \) is stochastically increasing in \( I \), there exists an information threshold value \( i(Q^A_m) \) that satisfies \( Q^A_e|i(Q^A_m) = Q^A_m \). Further,

\[
Q^A_e* = \begin{cases} 
Q^A_e|i & \text{if } i \leq i(Q^A_m), \\
Q^A_m & \text{if } i > i(Q^A_m).
\end{cases}
\]

In the sales season, the manufacturer’s expected profit is

\[
\Pi(Q^A_m, i; Q^A_e*), \quad \Pi(Q^A_m, i; Q^A_m), \quad \text{if } i > i(Q^A_m).
\]

where \( \Pi(Q^A_m, i; Q^A_e|i) = (p - w_e)Q^A_e|i - (p - v_2) \int_0^{Q^A_e|i} H(x|i)dx \) and \( \Pi(Q^A_m, i; Q^A_m) = (p - w_e)Q^A_m - (p - v_2) \int_0^{Q^A_m} H(x|i)dx \). Therefore, in the production season, the manufacturer’s optimal expected profit is \( \Pi^A(Q^A_m) = \int_{Q^A_m}^{Q^A_e*} \Pi(Q^A_m, i; Q^A_e|i)dG(I) + \)
\[ \int_{Q_m}^{+\infty} \Pi(Q_m^i; i; Q_m^A) dG(I) - c_e Q_m^A. \]

Further, \( \frac{\partial \Pi^A_i(Q_m^A)}{\partial Q_m} = \int_0^\infty \frac{\partial \Pi^A_i(Q_m^A, q_m; i, Q_m^A)}{\partial q_m} dG(I) + \int_{Q_m^A}^{+\infty} \frac{\partial \Pi^A_i(Q_m^A, q_m; i, Q_m^A)}{\partial q_m} dG(I) - c_e = f_0 q_m, \quad \frac{\partial \Pi^A_i(Q_m^A, q_m; i, Q_m^A)}{\partial q_m} dG(I) - c_e \]

and

\[ \frac{\partial^2 \Pi^A_i(Q_m^A, q_m; i, Q_m^A)}{\partial q_m^2} = -f_0 q_m h(Q_m^A) dG(I) < 0. \]

Therefore, \( \Pi^A_i(Q_m^A) \) is concave in \( Q_m^A. \)

Further, the optimal option quantity \( Q_m^A \) satisfies the first-order condition

\[ \int_{Q_m^A}^{+\infty} \left[ H(Q_m^A) - H(Q_m^A; i, Q_m^A) \right] dG(I) = \frac{c_e}{p - v_2}. \]

Specifically, when demand follows the uniform-uniform distribution, then (i)

\[ H(Q_m^A; i, Q_m^A) = \frac{c_e}{p - v_2} = \frac{1}{\beta} \left( Q_m^A \right) \]

Hence, \( Q_m^A \) = \( i - \frac{\beta}{\gamma} + \frac{e}{p - v_2} \). Let \( Q_m^A = iQ_m - \frac{\beta}{\gamma} + \frac{e}{p - v_2} \). Therefore, we have

\[ Q_m^A = \left\{ \begin{array}{ll}
Q_m^A & \text{if } i \leq iQ_m^A,
\end{array} \right. \]

and (ii) the first order condition is equivalent to

\[ \frac{1}{\beta} \int_{Q_m^A}^{+\infty} \left[ \frac{p - w}{p - v_2} - \frac{1}{\beta} \left( Q_m^A \right) \right] dG(I) = \frac{c_e}{p - v_2}. \]

Furthermore, \( Q_m^A = \frac{\alpha - \beta}{2} - 3 \left( \frac{c_e}{p - v_2} \right) \frac{p - w}{p - v_2} \beta + \left( \frac{p - w}{p - v_2} \right)^2 = \frac{2c_e \beta}{p - v_2}. \]

Then,

\[ \left( \frac{Q_m^A - \gamma - \frac{\alpha}{2}}{2} - \frac{p - w}{p - v_2} \right)^2 = \frac{2c_e \alpha \beta}{p - v_2}. \]

Therefore, we have

\[ Q_m^A = \gamma + \frac{\alpha - \beta}{2} + \frac{p - w}{p - v_2} - \sqrt{\frac{2c_e \alpha \beta}{p - v_2}}. \]
value $i_{Q^B}$ that satisfies $Q^B_{m|i_{Q^B}} = Q^B_s$. Further,

$$Q^B_{m} = \left\{ \begin{array}{ll} Q^B_{m|i_{Q^B}}, & \text{if } i \leq i_{Q^B_*}, \\
Q^B_{s}, & \text{if } i > i_{Q^B_*}. \end{array} \right.$$ 

Then, in the sales season, we have

$$\Pi_s^B(Q^B_s, i; Q^B_m) = \left\{ \begin{array}{ll} \Pi_s^B(Q^B_s, i; Q^B_m|i), & \text{if } i \leq i_{Q^B_*}, \\
\Pi_s^B(Q^B_s, i; Q^B_m), & \text{if } i < i_{Q^B_*}. \end{array} \right.$$ 

Here $\Pi_s^B(Q^B_s, i; Q^B_m|i) = (w - c_2 - v_1)Q^B_m|i - (b - v_2) \int_{0}^{Q^B_m|i} H(x|i)dx + v_1 Q^B_s$ and $\Pi_s^B(Q^B_s, i; Q^B_m) = (w - c_2)Q^B_s - (b - v_2) \int_{0}^{Q^B_s} H(x|i)dx$. Based on the above equations, in the production season,

$$\Pi_s^B(Q^B_s) = \int_{0}^{Q^B_s} \Pi_s^B(Q^B_s, i; Q^m) dG(I) + \int_{Q^B_s}^{+\infty} \Pi_s^B(Q^B_s, i; Q^B_s) dG(I) - c_1 Q^B_s.$$ 

Further, $\frac{\partial \Pi_s^B(Q^B_s, i; Q^m)}{\partial Q^B_m|} = \int_{0}^{Q^B_s} \frac{\partial \Pi_s^B(Q^B_s, i; Q^m)}{\partial Q^B_s} dG(I) + \int_{Q^B_s}^{+\infty} \frac{\partial \Pi_s^B(Q^B_s, i; Q^m)}{\partial Q^B_s} dG(I) - c_1 = \int_{0}^{Q^B_s} v_1 dG(I) + \int_{Q^B_s}^{+\infty} (w - c_2) - (b - v_2)H(Q^B_m|i) dG(I) - c_1$ and $\frac{\partial \Pi_s^B(Q^B_s)}{\partial Q^B_s^2} = - \int_{Q^B_s}^{Q^B_s} (w - c_2) - (b - v_2)H(Q^B_m|i) dG(I) < 0$. Therefore, $\Pi_s^B(Q^B_s) i$ is concave in $Q^B_s$. Further, the optimal option quantity $Q^B_{s^*}$ satisfies the first-order condition $\int_{Q^B_s}^{Q^B_s} (w - c_2 - v_1) - (b - v_2)H(Q^B_m|i) dG(I) = c_1 - v_1$.

Specifically, when demand follows the Uniform-Uniform distribution, we have

(i) $H(Q^B_m|i) = \frac{p-w}{p-b} = \frac{1}{\beta} \left( Q^B_m - i + \frac{\alpha}{2} \right)$. Further, $Q^B_m|i = i - \frac{\beta}{2} + \frac{p-w}{p-b} \beta$. Let $Q^B_m|i_{Q^B} = i_{Q^B} - \frac{\beta}{2} + \frac{p-w}{p-b} \beta = Q^B_s$. Then, $i_{Q^B} = Q^B_s + \frac{\beta}{2} - \frac{p-w}{p-b} \beta$. Therefore,

$$Q_{m}^{s} = \left\{ \begin{array}{ll} Q^B_m|i_{Q^B} = i - \frac{\beta}{2} + \frac{p-w}{p-b} \beta, & \text{if } \frac{\alpha}{2} - i < i_{Q^B_*}, \\
Q^B_{s^*}, & \text{if } i_{Q^B_*} < i < \frac{\alpha}{2}. \end{array} \right.$$ 

and (ii) the first-order condition can be written as

$$\int_{Q^B_s}^{+\frac{\alpha}{2}} \left( (w - c_2 - v_1) - \frac{b - v_2}{\beta} \left( Q^B_s - i + \frac{\beta}{2} \right) \right) di = (c_1 - v_1) \alpha.$$ 

Further,

$$\left[ Q^B_s - \left( Q^B_s + \frac{\alpha - \beta}{2} \right) - \frac{w - c_2 - v_1}{b - v_2} \beta \right]^2 = \frac{2(c_1 - v_1)\alpha \beta}{b - v_2} + \left( \frac{p - w}{p - b} \beta - \frac{w - c_2 - v_1}{b - v_2} \beta \right)^2.$$ 

Therefore, we have

$$Q^B_{s^*} = \frac{\alpha - \beta}{2} + \frac{w - c_2 - v_1}{b - v_2} \beta - \sqrt{\frac{2(c_1 - v_1)\alpha \beta}{b - v_2} + \left( \frac{p - w}{p - b} \beta - \frac{w - c_2 - v_1}{b - v_2} \beta \right)^2}.$$ 

□

Appendix (Proof of Proposition 7.)

Compared with $\{c_e, w_e, b\} \in M$ with $\lambda^B_{\text{min}} = \frac{\Pi^B_s(Q^B_{s^*})}{\Pi^B_s(Q^B_{s^*})}$ makes the manufacturer earn the same expected profit since $\Pi^B_s(Q^B_{s^*})$ strictly increases in $\lambda$. Further, the contract of $\{c_e, w_e, b\} \in M$ with $\lambda^B_{\text{min}} < 1$ benefits the manufacturer. The contract of $\{c_e, w_e, b\} \in M$ with $\lambda^B_{\text{max}} = 1 - \frac{\Pi^B_s(Q^B_{s^*})}{\Pi^B_s(Q^B_{s^*})}$ makes the manufacturer earn the same expected profit since $(1 - \lambda)\Pi^B_s(Q^B_{s^*})$ strictly decreases in $\lambda$. Further, the contract of $\{c_e, w_e, b\} \in M$ with $0 \leq \lambda^B_{\text{max}}$ benefits the supplier. Therefore, the contract of $\{c_e, w_e, b\} \in M$ with $\lambda \in [\lambda^B_{\text{min}}, \lambda^B_{\text{max}}]$ makes both members at least as well off as the contract of $\{w, b\}$. □
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