Research Article

A. Rushi Kesava and A. N. S. Srinivas*

Exploration of peristaltic pumping of Casson fluid flow through a porous peripheral layer in a channel

https://doi.org/10.1515/nleng-2022-0247
received December 11, 2021; accepted August 23, 2022

Abstract: This article is aimed to investigate the peristaltic pumping of a two-layered model in a two-dimensional channel. The core region occupies Casson fluid, while the porous medium occupies the peripheral region. The fluid flow in a porous medium was described with a suitable model using the Brinkman-extended Darcy equation. In the interface between fluid and porous medium, a shear stress jump boundary condition was applied. Closed-form solutions were obtained in both regions (core and peripheral). The physical quantities of peristaltic flow, such as axial velocity, pumping and change in the interface, were derived and explained. The fluid flow was analyzed by different physical parameters such as viscosity, permeability, porosity, Casson parameter and Darcy number. It is observed that the peristalsis mechanism has greater pressure in a two-layered model containing a non-Newtonian fluid in contact with a porous medium compared to a viscous fluid in the peripheral layer. It was observed that pumping decreased with the increase in Darcy number and an increase in shear stress jump constant resulted in increasing the pumping. The outcomes of the pumping phenomenon may be helpful for understanding the fluid flow aspects of blood flow in capillaries.

Keywords: Casson fluid, peristaltic transport, porous medium, interface

1 Introduction

The transportation of physiological fluids pumped from one place in the body to another place with a wavelike muscle contraction and relaxation is called peristalsis. Intensive research has been carried out on the peristaltic flow of biological fluids. The study of biological fluids is very important in understanding physiological systems such as esophagus, ureter, stomach, blood vessels and bile duct. Porous walls and deformable porous layers have been noticed in many biological applications. Some of the examples are capillary walls, gastrointestinal tract and intra-pleural membrane. The capillary walls are also surrounded by a layer of flattened endothelial cells, which is porous in nature [1]. The transport in and around the capillary walls plays a beneficial role in maintaining metabolism as well as fluid balance. Hou et al. [2] discussed a two-fluid model in a channel having viscous fluid as synovial fluid and a porous layer as articular cartilage. The gastrointestinal tract absorbs nutrients from food as well as fluids passing through it. Here, the epithelial cells are held responsible for absorbing water from the intestine. Furthermore, there are pores through the tight junction of them. It is quite significant to study the peristaltic behavior of two-layered systems with porous peripheral layers as well as the porous boundaries of a channel.

In order to explore the importance of peristalsis, various theoretical and experimental works were carried out by many researchers. The importance of peristaltic transport using theoretical fluid mechanics and experimental studies was reviewed by Jaffrin and Shapiro [3], Rath [4] and Srivastava and Srivastava [5]. The first attempt at experimental work of peristaltic transport is made by Latham [6]. The experiments made by Bugliarello and Sevilla [7], Cockelet [8] and Scott Blair [9] reveal that blood is a non-Newtonian fluid in many situations. To understand the rheological properties of blood, many models were developed, and among them, the Casson fluid model gave better results.

* Corresponding author: A. N. S. Srinivas, Department of Mathematics, School of Advanced Sciences, VIT, Vellore, 632014, Tamilnadu, India, e-mail: anssrinivas@vit.ac.in, tel: +91-8903312379

A. Rushi Kesava: Department of Mathematics, School of Advanced Sciences, VIT, Vellore, 632014, Tamilnadu, India

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Two-fluid model considerations become very important in understanding biofluid flow analysis in physiological systems. The influence of the outer layer with different viscosities on the flow rate of Newtonian fluids was studied by Brasseur et al. [10]. Studies have been carried out by Mishra and Pandey [11] on peristaltic transport of blood flow in a small vessel by considering the Casson fluid in the central region but the Newtonian fluid in the outer region. The peristaltic transport of both viscous fluids in a tube was studied by Ramachandra Rao and Usha [12]. For analyzing the results, Ochaoa-Tapia and Whitaker [14] considered shear stress jump boundary conditions at porous as well as fluid boundary regions. El Shehawey and Husseney [15] analyzed the properties of porous boundaries on peristaltic pumping through a porous medium in a channel. Magnetohydrodynamic flow effects in peristaltic transport through a porous medium were studied by Mekheimer [16]. Furthermore, the peristaltic flow of fluids at the interface of a fluid layer and a porous medium was explained by Alazmi and Vafai [17] by considering two-fluid models. The peristaltic movement of Casson fluid with a symmetrical tube was described by Mernone et al. [18] and considered perturbation methods to explain the fluid flow in terms of amplitude ratio. Peristaltic flow in the gastrointestinal tract was explained by Mishra and Ramachandra Rao [19]. Vajravelu et al. [13,20,21] and Sreenadh et al. [22] explained some of the two-layered models by considering Newtonian/non-Newtonian fluids in core as well as peripheral regions.

Ponalagusamy and Tamil Selvi [23] investigated a two-fluid model in blood flow by arterial stenosis under the impact of magnetic field and heat transfer. As suction and internal heat generation (exponentially decaying) are significant, the behavior of Casson fluid flow over an exponentially stretching surface was examined. Anima-saun et al. [24] showed an increase in the variable plastic dynamic viscosity parameter of Casson fluid with an increase in the velocity profiles. However, there is a decrease in temperature throughout the boundary layer. Thumma et al. [25] recently studied generalized differential quadrature analysis of unsteady Casson fluid and discovered that increasing the Casson parameter decreases the axial velocities in both directions significantly. Recently, some researchers have worked on peristaltic transport of different biofluid flows in different geometries with permeable boundaries [26–31,36,37]. Kesava and Srinivas [32] analyzed the peristaltic pumping of viscous fluid through a porous peripheral layer in a channel with an external magnetic field.

Two-layered peristaltic pumping in a channel having non-Newtonian Casson fluid in the inner layer and a porous medium in the outer layer is proposed. In both regions, closed-form solutions were derived. Computational outcomes show the existence of porous medium in the peripheral layer that affects the flow field significantly.

2 Problem formulation by governing equations

Consider peristaltic transport in a two-dimensional channel filled with a non-Newtonian fluid in the core region and a porous medium in the peripheral region. The fluids are immiscible. An infinite sinusoidal wave of amplitude \( b \) and axial wavelength \( \lambda \) with a constant velocity \( c \) passes through the flexible walls of the channel. The walls taken are \( Y = \pm a \pm b \sin \frac{2\pi}{\lambda} (X - ct) \), having \( t \) as the time. Here, \( 2a \) is the mean width of the channel as well as the interface equation, which is denoted by \( Y = H(x) \) (Figure 1).

We assume that the length of the conduit is an integral multiple of the wavelength. The porous medium is assumed to be homogeneous and isotropic. Across the boundaries of the channel, a constant pressure difference was considered. The flow is unsteady in the lab frame taking the Cartesian coordinate system \( (X, Y) \) and the flow becomes steady in a wave frame of reference \( (x, y) \) by moving along with velocity \( c \) in the wave propagation direction.

The fixed and wave frames of reference were coupled together by using the transformation of equations as follows (Brasseur et al. [10]):

\[
v_1 = V_1; \quad \lambda = X - ct; \quad p_1(x) = P(X, t); \quad u_i = U_i - c; \quad y = Y,
\]

\[\text{(1)}\]

**Figure 1: Sketch of the flow pattern.**
where \((U_i, V_i)\) and \((u_i, v_i)\) are velocity coordinates along the transverse and axial directions. The pressure coordinates along transverse and fixed frames were considered as \(p_i\) and \(P_i\). The core and peripheral sections are denoted by the subscripts \(i = 1, 2\).

For an isotropic as well as incompressible transfer of Casson fluid, the rheological equation in accordance with Nakamura and Sawada [33], Mukopadhyay [34], Selvi and Srinivas [35] is given as follows:

\[
\tau_{ij} = \begin{cases} 
\frac{2}{\sqrt{2\pi}} \left( \frac{P_i}{\sqrt{2\pi}} + \mu_B \right) e_{ij}, & \pi > \pi_c, \\
\frac{2}{\sqrt{2\pi}} \left( \frac{P_i}{\sqrt{2\pi}} + \mu_B \right) e_{ij}, & \pi < \pi_c,
\end{cases}
\]

where \(P_i\), \(\mu_B\) and \(\pi\) are yield stress, plastic dynamic viscosity and product of the components, respectively. Furthermore, the deformation component product is represented as \(\pi = e_{ij}e_{ij}\), where \(e_{ij}\) is the \((i,j)\)th component, \(\pi_c\) is the critical value of the non-Newtonian model.

The governing equations of motions in the fluid (core) region and porous (peripheral) region that describes the flow in the moving frame are (Mishra and Ramachandra Rao [19]).

### 2.1 Core region

\[
\rho_i \left( \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} \right) = -\frac{\partial p_i}{\partial x} + \mu_i \left( \frac{\partial^2 u_i}{\partial y^2} + \frac{\partial^2 u_i}{\partial x^2} \right) + \mu \left( \frac{\partial^2 u_i}{\partial y^2} + \frac{\partial^2 u_i}{\partial x^2} \right),
\]

\(\rho_i \left( \frac{\partial v_i}{\partial x} + v_i \frac{\partial v_i}{\partial y} \right) = -\frac{\partial p_i}{\partial y} + \mu_i \left( \frac{\partial^2 v_i}{\partial y^2} + \frac{\partial^2 v_i}{\partial x^2} \right) + \mu \left( \frac{\partial^2 v_i}{\partial y^2} + \frac{\partial^2 v_i}{\partial x^2} \right),
\]

\[
\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0.
\]

### 2.2 Peripheral region

\[
\rho_2 \left( \frac{\partial u_2}{\partial x} + v_2 \frac{\partial u_2}{\partial y} \right) = -\frac{\partial p_2}{\partial x} + \mu_2 \left( \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_2}{\partial x^2} \right) - \frac{\mu_2}{k} u_2,
\]

\[
\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0.
\]

\[
\rho_2 \left( \frac{\partial v_2}{\partial x} + v_2 \frac{\partial v_2}{\partial y} \right) = -\frac{\partial p_2}{\partial y} + \mu_2 \left( \frac{\partial^2 v_2}{\partial y^2} + \frac{\partial^2 v_2}{\partial x^2} \right) - \frac{\mu_2}{k} v_2,
\]

\[
\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0.
\]

where \(\mu_1\) and \(\mu_2\) are viscosities in the inner and outer regions; \(\rho_1, \rho_2\) are densities in the core and peripheral regions and \(\varepsilon\) and \(k\) are the porosity and permeability in the porous medium as non-dimensional parameters.

By considering the following dimensionless variables,

\[
\mu = \frac{\mu_2}{\mu_1}, \quad a_i = u_i/c, \quad x = x/l, \quad \ell = ct/l, \\
\bar{p}_i = a^2 \rho_i/c, \quad \bar{v}_i = V_i/c, \quad y = y/a, \quad \bar{\psi}_i = \psi_i/ac, \\
Re = \rho_1 c a/\mu_1, \quad h = H/a, \quad \alpha = a/l, \\
\rho = \rho_2/\rho_1, \quad \phi = b/a, \quad Da = k/a^2, \quad h_1 = H_1/a,
\]

in the governing Eqs. (3)–(8) and in terms of the stream function, we obtain

\[
\delta \text{Re}(\psi_{y} \psi_{xx} - \psi_{x} \psi_{yy}) = -p_1 + (\delta^2 \psi_{xx} + \psi_{yy})(1 + \frac{1}{\beta_2}),
\]

\[
\delta^2 \text{Re}(\psi_{y} \psi_{xxx} - \psi_{x} \psi_{yyy}) = p_1 + (\delta^2 \psi_{xxx} + \psi_{yyy})(1 + \frac{1}{\beta_2}),
\]

\[
\delta_0 \text{Re}(\psi_{y} \psi_{y} - \psi_{x} \psi_{x}) = p_1 + (\delta^2 \psi_{xx} + \psi_{yy})(1 + \frac{1}{\beta_2}),
\]

\[
\delta^2 \text{Re}(\psi_{y} \psi_{y} \psi_{y} - \psi_{x} \psi_{x} \psi_{x}) = p_1 + (\delta^2 \psi_{xx} + \psi_{yy})(1 + \frac{1}{\beta_2}).
\]

Let us define \(u_i = \psi_{x} ; v_i = -\psi_{y} \).

Approximating very negligible inertial forces compared to viscous forces \(\text{Re} \to 0\) and long wavelength \(\delta \ll 1\), Eqs. (10) and (11) are reduced (bars removed) to

\[
\psi_{yy}(1 + \frac{1}{\beta_2}) = P, \quad 0 \leq y \leq h_1,
\]

\[
\psi_{yyy} = A^2 \psi_{yy}, \quad h_1 \leq y \leq h.
\]

The associated non-dimensional boundary conditions then become

\[
\text{at } y = h(x) : \frac{\partial \psi_{y}}{\partial y} = -1, \quad \psi_{z} = q,
\]

\[
\text{at } y = 0 : \quad \psi_{z} = 0, \quad \frac{\partial^2 \psi_{y}}{\partial y^2} = 0,
\]

\[
\text{at } y = h(x) : \quad \psi_{y} = 0,
\]

\[
\text{at } y = h(x) : \quad \frac{\partial \psi_{y}}{\partial y} = \frac{\beta_{1} \psi_{y}}{\beta_{2}},
\]

\[
\text{at } y = h(x) : \quad \frac{\partial^2 \psi_{y}}{\partial y^2} = \frac{\partial^2 \psi_{y}}{\partial y^2} - \beta_{1} \frac{\partial \psi_{y}}{\partial y},
\]

\[
\text{at } y = h(x) : \quad \frac{\partial^3 \psi_{y}}{\partial y^3} = \mu_{e} \left( \frac{\partial^2 \psi_{y}}{\partial y^2} - A \frac{\partial \psi_{y}}{\partial y} \right).
\]
where \( A^2 = \frac{\epsilon}{D_{a}} \), \( \beta_{2} = \frac{h_{c}}{\sqrt{D_{a}}} \) and \( \mu_{c} = \frac{\mu}{\epsilon} \) in Eq. (14), \( q \) represents the total flux. In Eq. (16), \( q_{1} \) represents the core flux. Moreover, it is observed that the limit \( Da \rightarrow \infty \) and \( \beta_{2} \rightarrow \infty \) in the governing equations and boundary conditions are reduced by Brasseur et al. [10].

### 3 Solution

The stream function for the flow within the two layers is determined by solving Eqs. (12) and (13) with boundary conditions (14)–(19) to obtain

\[
\psi_{1} = \frac{A^{2} \beta_{2} \mu_{c}}{6(\beta_{2} + 1)}(AC_{1} \cosh Ah + AC_{2} \sinh Ah + 1)y^{3}
+ \frac{\beta_{2}}{(\beta_{2} + 1)} By, \quad 0 \leq y \leq h_{1}(x),
\]

\[
\psi_{2} = q + h - y + C_{1}[\sinh Ay - \sinh Ah
+ A \cosh Ah(h - y)]
+ C_{2}[\cosh Ay - \cosh Ah
+ A \sinh Ah(h - y)]. \quad h_{1} \leq y \leq h.
\]

In Appendix I, the constants \( C_{1}, C_{2}, B_{1} \) are given. Substituting (20) and (21) in the momentum equations, we obtain the pressure gradient as follows:

\[
\frac{dp}{dx} = A^{2} \mu_{c}(1 + AC_{1} \cosh Ah + AC_{2} \sinh Ah).
\]

The dimensionless flux \( Q \) at any axial station in the fixed frame and the flux \( q \) in the wave frame of reference are connected by

\[
Q = \int_{0}^{h} (1 + u)dy = h + q.
\]

The time average yields to \((T = \lambda/c)\)

\[
Q = \frac{1}{T} \int_{0}^{T} q dt = \frac{1}{T} \int_{0}^{T} (q + h)dt = q + \int_{0}^{1} h dt = q + 1.
\]

The expression for the interface \( h_{1} \) is also determined from Eq. (16) by using any of the conditions \( \psi_{2} = q_{1} \) or \( \psi_{1} = q_{i} \) at \( y = h_{1}(x) \). The equation governing the interface is as follows:

\[
k(h_{1}) = -(q + h) + q_{1} + h_{1} - C_{1}A \cosh Ah(h - h_{1})
+ \sinh Ah_{1} - \sinh Ah - C_{2}[A \sinh Ah(h - h_{1})
+ \cosh Ah_{1} - \cosh Ah] = 0.
\]

The constants \( q \) and \( q_{1} \) do not depend on \( x \). By proposing the conditions \( h_{1} = y \) at \( x = 0 \) in Eq. (25), we obtain

\[
q_{1} = \frac{(Q - y)H_{3}H_{8} + H_{7}H_{8}H_{10} - H_{2}(H_{2}H_{5} + H_{4}H_{10})}{(H_{3} - H_{2})H_{5} - H_{4}H_{10}}. \quad (26)
\]

In Appendix II, the constants \( H_{j}, j = 1, \ldots, 10 \) are given.

Suppose we integrate Eq. (22) over one non-dimensional wavelength, then

\[
\Delta p = \int_{0}^{1} \frac{dp}{dx} dx = \int_{0}^{1} A^{2} \mu_{c} [AC_{1} \sin h(Ah) + AC_{2} \cos h(Ah) + 1] dx. \quad (27)
\]

### 4 Results and discussion

Peristaltic flow of Casson fluid in the core region and porous material in the peripheral region is explained by considering the two-layered system. The equations for pressure difference, stream functions, velocities and interface were derived in both core and peripheral regions. We have discussed about the important physical quantities in peristaltic transport like axial velocity, pumping and interface for different pertinent parameters governing the flow like Casson parameter, Darcy number, stress jump constant, porosity, viscosity and amplitude ratio. Ochoa-Tapia and Whitaker [14] used shear stress jump conditions between fluid and porous layers. The transcendental interface equation is calculated and solved. For assessing the quantitative effects of various parameters involved in the problem, the numerical computations can be carried out by using the software MATLAB, and graphs are plotted in origin. The governing equation and boundary conditions of this study are reduced to Brasseur et al. [10] with a few limitations.

The interface was considered as a streamline. Figure 2(a)–(d) shows the change in the shape of the interface for various parameters such as \( \mu, D_{a}, \beta_{2} \) and \( \phi \). Figure 2(b) shows the shape of the interface for different \( D_{a} \). It was found that an increase in Darcy number \( (Da) \) became a cause for the thinner peripheral layer in the dilated regime. Figure 2(c) shows the variation of interface with the Casson parameter \( \beta_{2} \), and it was found that increasing the Casson parameter increases the thickness of the layer in the crystal part of the channel, while the reverse performance is observed in the trough regions of the channel. Figure 2(a) and (d) shows the amplitude and
viscosity ratios at the interface. From the graphs, one can see that an increase in amplitude and viscosity ratios gives rise to thinner peripheral layers in the constricted region of the pump.

The variation of $\Delta p$ with $\bar{Q}$ is given by solving Eq. (27) using MATLAB. Figure 3(a)–(f) shows the graphical representation of pumping parameters for different parameters. The pumping features are drawn as graphs for different parameters such as amplitude and viscosity ratios, Darcy number, Casson parameter, stress jump constant and porosity by keeping other parameters constant. Figure 3(a) shows that the pumping speed decreased with the increase of the Casson factor for constant $\Delta p$. Figure 3(b) shows the effect of Darcy number $\text{Da}$ on the pumping factor. It can be observed from the figures that an increase in Darcy number increases permeability, which causes a decrease in pumping. These results indicate that the growing values of Darcy number decrease the pressure rise, which opposes the working of the pump. At a constant flux $\bar{Q}$, the growing values of Darcy number decrease the pressure rise.

The relationship between porosity and pumping was explained using Figure 3(c). We observe that the pumping decreases as the porosity ($\varepsilon$) increases. Thus, for large values of porosity ($\varepsilon$), more flux is observed in free pumping as well as co-pumping. The influence of shear stress jump constant ($\beta$) on pumping is represented in Figure 3(d). For large values of shear stress jump constant, the flow rate increases in the free pumping region and the co-pumping region. Figure 3(e) shows a change in flux with $\phi$, and it was found that at a constant $\Delta p$, an increase in flux increases $\phi$. Figure 3(f) shows the effect

Figure 2: Variation in the shape of the interface for $\mu$, $\text{Da}$, $\beta_z$ and $\phi$. (a) $\phi = 0.5$, $\text{Da} = 1$, $\beta_z = 0.9$, $\varepsilon = 0.7$, $\beta = 0.5$, $\gamma = 0.7$. (b) $\phi = 0.5$, $\beta_z = 0.9$, $\varepsilon = 0.7$, $\mu = 1$, $\beta = 0.5$, $\gamma = 0.7$. (c) $\phi = 0.5$, $\text{Da} = 1$, $\varepsilon = 0.7$, $\mu = 1$, $\beta = 0.5$, $\gamma = 0.7$. (d) $\text{Da} = 0.05$, $\varepsilon = 0.7$, $\mu = 1$, $\beta = 0.5$, $\gamma = 0.7$. (e) $\text{Da} = 0.1$, $\varepsilon = 0.7$, $\mu = 1$, $\beta = 0.5$, $\gamma = 0.7$. (f) $\text{Da} = 1$, $\varepsilon = 0.7$, $\mu = 1$, $\beta = 0.5$, $\gamma = 0.7$. 
Figure 3: Variation of $\Delta p$ with $\bar{Q}$ for $\beta_2$, $Da$, $\epsilon$, $\phi$ and $\mu$. (a) $\phi = 0.5$, $\epsilon = 0.7$, $\beta = 1$, $Da = 1$, $\mu = 2$. (b) $\phi = 0.5$, $\epsilon = 0.7$, $\beta = 1$, $\beta_2 = 0.5$, $\mu = 2$. (c) $\phi = 0.5$, $Da = 1$, $\beta = 1$, $\beta_2 = 0.5$, $\mu = 2$. (d) $\phi = 0.5$, $\epsilon = 0.7$, $Da = 1$, $\beta_2 = 0.5$, $\mu = 2$. (e) $\beta = 1$, $\epsilon = 0.7$, $Da = 1$, $\beta_2 = 0.5$, $\mu = 2$. (f) $\phi = 0.5$, $\epsilon = 0.7$, $\beta = 1$, $\beta_2 = 0.5$, $Da = 0.1$. 
of the viscosity ratio on flow rate. The flux increases as the viscosity ratio ($\mu$) increases in the pumping region. The pumping is better with $\mu > 1$ compared to $\mu < 1$, which is similar to the outcomes of Brasseur et al. [10]. Hence, it is concluded that for a given pressure rise, a better time-averaged flux with a porous peripheral layer having a large shear stress jump constant, high effective viscosity and small Darcy number is related to two Newtonian fluid layers.

Figure 4(a)–(d) shows the change in axial velocity at a constant axial station ($x = 0.25$) for various Darcy numbers ($Da$), shear stress jump constant ($\beta$), viscosity ratio ($\mu$) and Casson parameter ($\beta_2$). From Figure 4(a), it is observed that in the core region, an increase in the jump constant ($\beta$) decreases the axial velocity. But a reverse trend was observed in the peripheral region, as shown in Figure 4(a). Figure 4(b) shows the relationship between the axial velocity (along $y$) and the Casson parameter ($\beta_2$). From the figure, one can understand that fluid velocity increases with the increase in the Casson parameter ($\beta_2$).

But surprisingly, quite the opposite results were found in the peripheral region, i.e., fluid velocity increased with a decrease in Casson parameter ($\beta_2$). Figure 4(c) shows the relation between the axial velocity and Darcy number ($Da$), and it was found that axial velocity is directly proportional to Darcy number, which is a clear indication of flow reversal in the core section reflecting a possible reflux. But the results are quite opposite in the peripheral region. Figure 4(d) indicates the relation between axial velocity and viscosity ratio, and from this figure, one can observe that axial velocity increases with the viscosity ratio parameter. But these results are quite the opposite in the core and peripheral regions.
5 Conclusions

The study on peristaltic transport of a two-layered system with a porous medium in the peripheral layer in a channel may represent the blood flow in capillaries and gastrointestinal tract. In this study, we investigated two-layered peristaltic pumping in a channel with non-Newtonian fluid in the inner layer and a porous medium in the outer layer. In the interface between the fluid and porous layers, a shear stress jump boundary condition is considered. The interface and pumping phenomena are observed for various physical parameters that govern the flow, such as Darcy number, porosity and shear stress jump constant. At a constant flux $Q$, an increase in Darcy number decreases the pressure rise. It is concluded that for a given pressure rise, the flux $Q$ is better with a small Darcy number, the values of shear stress jump constant are larger and the viscosity is higher compared to two Newtonian fluid layers.

Acknowledgments: The authors express their gratitude to the referees and the subject Editor for their valuable suggestions in the improvement of the paper.

Funding information: The authors state no funding involved.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

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Appendix I:

\[ C_1 = \frac{(2 + h - h_1)M_2 + M_2 M_6}{M_6 - M_4 M_6 - M_6 M_6}, \quad C_2 = \frac{C_1}{M_6}, \]

\[ B_1 = \frac{M_6 M_2 + M_6 M_6 + M_3 M_6 - M_6 M_6 + M_3 M_6}{K M_2}, \quad K = \frac{\beta_2}{\beta_2 + 1}, \]

\[ M_1 = \sinh A_h - h_2 A K \cosh A_h - \frac{h_1}{A} (\cosh A_h - \cosh A h), \]

\[ M_2 = \frac{\beta_1}{A} (\sinh A_h - \sinh A h) - \cosh A_h + h_2 A K \sinh A h, \]

\[ M_3 = \frac{\beta_1}{A} - K h_1, \quad M_4 = 1 + \frac{K A^2 \mu_y h_1^2}{2}, \]

\[ M_5 = A \cosh A_h - A M_4 \cosh A h, \]

\[ M_6 = A \sinh A_h - A M_4 \sinh A h, \]

\[ M_7 = \frac{K A^2 \mu_y^2}{6 M_2} (M_2 + M_5 A \sinh A h) + \frac{h_1}{M_2} (M_2 M_6 - M_2 M_6), \]

\[ M_8 = \frac{K A^2 \mu_y^2}{6 M_2} (M_2 A \cosh A h + M_5 A \sinh A h) \]

\[ + \frac{h_1}{M_2} (M_2 M_6 + M_2 M_6), \]

\[ M_9 = \sinh A h_2 \pm \sinh A h + (h - h_1) A \cosh A h, \]

\[ M_{10} = \cosh A h_1 - \cosh A h + (h - h_1) A \sinh A h. \]

Appendix II:

\[ H_1 = \sinh A Y - \gamma A K \cosh A - \frac{\beta_1}{A} (\cosh A Y - \cosh A), \]

\[ H_2 = \gamma A K \sinh A - \cosh A Y + \frac{\beta_1}{A} (\sinh A Y - \sinh A), \]

\[ H_3 = \frac{\beta_1}{A^2} - K \gamma, \quad H_4 = 1 + \frac{K A^2 \mu_y h_1^2}{2}, \]

\[ H_5 = A (\cosh A Y - H_4 \cosh A), \]

\[ H_6 = A (\sinh A Y - H_4 \sinh A), \]

\[ H_7 = \frac{K A^2 \mu_y^2}{6 H_2} (H_2 + H_5 A \sinh A) + \frac{Y}{H_2} (H_3 H_6 - H_2 H_6), \]

\[ H_8 = \frac{K A^2 \mu_y^2}{6 H_2} (H_5 A \cosh A + H_6 A \sinh A) \]

\[ + \frac{Y}{H_2} (H_3 H_6 + H_2 H_6), \]

\[ H_9 = \sinh A Y - \sinh A + (1 - \gamma) A \cosh A, \]

\[ H_{10} = \cosh A Y - \cosh A + (1 - \gamma) A \sinh A. \]