Testing MOG/Non-local/MOND gravity with rotation curve of dwarf galaxies

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ABSTRACT
The MOdified Gravity (MOG) and Non-local Gravity are two different alternative theories to General Relativity where in the limit of weak field approximation behave almost in similarly way and are able to play the role of dark matter and explain the rotation curve of spiral galaxies and cluster of galaxies (Moffat & Rahvar 2013, 2014; Rahvar & Mashhoon 2014). The effective gravitational potential in these theories compose of two terms, (i) Newtonian gravity with an enhanced gravitational constant and (ii) the second term with Yukawa type repulsive force which is defined with the length scale of \(1/\mu\). In this work we analysis the rotation curve of dwarf galaxies in the LITTLE THINGS catalog and compare them with MOG/Non-local gravity and Modified Newtonian Dynamics (MOND). We obtain almost the same \(\alpha\) factor as in our analysis of the spiral galaxy and cluster of galaxies, however we need a smaller length scale of \(\mu = 2.77 \text{kpc}^{-1}\) to describe the rotation curve of dwarf galaxies compare to \(\mu = 0.042 \text{kpc}^{-1}\) for the larger scales. This result guides us for the possibility of a running \(\mu\)-parameter as a function of scale of structures in these theories. For the case of MOND, the best value for the characteristic acceleration of this model is consistent with the Millgrom’s value (Milgrom 1983).

1 INTRODUCTION
In the galactic as well as cluster of galaxy scales, observations show that there are discrepancy between the observed dynamical mass and the mass inferred from the luminous matter in the structures (Zwicky 1937; Rubin et al. 1965, 1970). One of solutions for interpretation of this observation is using the concept of missing mass of Universe, so-called dark matter in the cosmology. The particles composing dark matter fluid interact with each other only gravitationally and they have possibly very weak interaction with the ordinary matter and themselves. No explicit signal of dark matter particle from the interaction with the ordinary matter is found yet (Moore et al. 2001; Gaitskell 2004; Angloher et.al 2012; Akerib et.al 2014).

The other approach for the interpretation of observation is using the idea of modification of gravity law in order to replace it with the dynamical effect of missing mass. One of well known models is the Modified Newtonian Dynamics (MOND) where the Newtonian dynamics changes in such a way that in the small accelerations produces a flat rotation curve for the spiral galaxies (Milgrom 1983). This theory also is developed to relativistic version by Bekenstein (2004). One of the challenging questions regarding MOND and other modified gravity theories is the interpretation of gravitational lensing of systems like the bullet cluster (Markevitch et al. 2004) and the large scale structure formation in the Universe without using the dark matter component of the structures.

Another attempt to solve the missing mass problem with modifying the gravity-law has been introduced by Hehl & Mashhoon (2009) with the Non-Local Gravity (NLG). This theory is the extension of non-local special relativity to the accelerating frames. In the Newtonian limit of non-local gravity, the effective gravitational potential is similar to a Newtonian potential, in addition to an extra repulsive term similar to the Yukawa force (Hehl & Mashhoon 2009). This theory has been compared with the rotation curve of spiral galaxies as well as the temperature profile of hot gas in Chandra database for the cluster of galaxies. With the fixed values for the parameters of this theory, the dynamics of spiral galaxies as well as the cluster of galaxies are consistence with the baryonic distribution of matter in these structures and there is no need to the dark matter (Rahvar & Mashhoon 2014).

The other alternative theory to the dark matter is so-called MOdified Gravity (MOG) theory which is a covariant extension of General Relativity. In this theory besides the tensor field as the generator of gravity there are additional scalar and vector fields (Moffat 2006). The important feature of this theory is the fifth force charge for each particle which is proportional to the inertial mass and it couples via a massive vector field. The geodesics of a test particle in this theory has an extra term similar to the Lorentz force in the electrodynamics, coupling with the fifth force charge and the result is a deviation from the standard geodesic equation in General Relativity. With this extra term in the weak field approximation, a modified Poisson equation is obtained where the solution of this equation for a point like mass results...
in a Newtonian potential and a Yukawa repulsive term. The comparison of the dynamics of spiral galaxies with THINGS catalog results in universal values for the parameters of this model with reasonable fit to the rotation curve of galaxies and cluster of galaxies (Moffat & Rahvar 2013, 2014). One of interesting similarities between the Non-local gravity and MOG theories is that while these theories have completely different physical axioms, in the weak field approximation they behave almost similar.

To test the universality of parameters of modified gravity models in the intermediate scales, we apply the effective potential of MOG, Non-local gravity as well as MOND for interpreting data in dwarf galaxies of LITTLE Things catalog. The observational data in this catalog are the density distribution of stars and gas of each galaxy as well as the dynamics in form of rotation curve of galaxy.

In Section (2), we review the alternative theories of gravity to compensate the dark matter as (a) MOdified Gravity (MOG) and the weak field approximation of this theory, (b) non-local gravity and the corresponding weak field approximation and Modified Newtonian Dynamics (MOND). In Section (3), we introduce the LITTLE THINGS catalog and apply the results of weak field approximation of alternative models of gravity to the dwarf galaxies. In this section we obtain the best values for the parameters of gravity model and compare them with what has been obtained from the Spiral galaxies and cluster of galaxies. In Section (4) for the conclusion, we discuss about the universality of parameters of modified gravity theories and possibility of having running parameters in these theories.

2 ALTERNATIVE THEORIES TO DARK MATTER

One of the important questions in the cosmology is the missing mass of Universe where the baryonic matter is not enough to interpret the dynamics of structures such as cluster of galaxies and spiral galaxies (Zwicky 1937, Rubin et al. 1963, 1970). Beside using the concept of dark matter, the alternative approach is the modification of gravity law in such a way that one can explain the dynamics of structures with only baryonic matter. In this section we introduce three models for the modification of gravity law. These models are (a) MOdified Gravity (MOG) which is introduced by Moffat (2006) and the weak field approximation obtained in Moffat & Rahvar (2013), (b) Non-Local Gravity (NLG) where the Einstein equations in teleparallel form are similar to the Maxwell equations, hence these equations are written in a non-local way (Hehl & Mashhoon 2009) and (c) Modified Newtonian Dynamics (MOND) where the Newtonian dynamics is modified in the small accelerations (Milgrom 1983).

2.1 Field equations in MOG

We use the metric signature convention (−, +, +, +). The general form of action for MOG which also is called the Scalar-Tensor-Vector-Gravity (STVG), is given by (Moffat 2006, Moffat & Toth 2009),

\[ S = S_G + S_\phi + S_S + S_M, \]

where the first term is the standard Einstein action

\[ S_G = \frac{1}{16\pi} \int \frac{1}{G} (R + 2\Lambda) \sqrt{-g} \, d^4x, \]

and the second term (i.e. \( S_\phi \)) and third term (i.e. \( S_S \)) of action corresponds to the massive vector and scalar fields that are composed by the following terms:

\[ S_\phi = \frac{1}{4\pi} \int \omega \left[ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{2} \mu^2 \phi^2 \phi^\mu \phi^\mu \right. \]

\[ \left. + \ V_\phi (\phi^2 \phi^\mu) \right] \sqrt{-g} \, d^4x, \]

and

\[ S_S = \int \frac{1}{G} \left[ \frac{1}{2} g^{\alpha\beta} (G - 2 \nabla_\alpha G \nabla_\beta G + \mu^2 \nabla_\alpha \nabla_\beta \mu) \right. \]

\[ \left. - \ G^2 V_G(G) - \mu^{-2} V_\mu(\mu) \right] \sqrt{-g} \, d^4x, \]

where \( \nabla_\mu \) is the covariant derivative with respect to the metric \( g_{\mu\nu} \), the Faraday tensor of the vector field is defined by \( B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu \), \( \omega \) is a dimensionless coupling constant, \( G \) is a scalar field representing the gravitational coupling strength and \( \mu \) is a scalar field corresponding to the mass of the vector field. Also \( V_\phi (\phi^2 \phi^\mu) \), \( V_G(G) \) and \( V_\mu(\mu) \) are the self-interaction potentials associated with the vector field and the scalar fields, respectively. Here in our analysis we consider a simplified version of this action assuming that all the potentials set to zero and \( \omega \) and \( \mu \) fields as the constant parameters.

Also the action for pressureless dust can be written as

\[ S_M = \int (\rho \sqrt{-u^\mu u_\mu} - \omega Q_5 u^\mu \phi_\mu) \sqrt{-g} \, dx^4, \]

where assuming a point mass particle by replacing \( \rho(x) = m \delta^3(x) \) and varying this action results in the geodesic equation of

\[ \frac{du^\mu}{dt} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = \omega \kappa B^\alpha_{\alpha} u^\alpha, \]

where we assume that the fifth force charge is related to the inertial mass by \( Q_5 = \kappa m \) and the right hand side of this equation is similar to the Lorentz force in the electrodynamics. The only difference is that vector field in this action is massive, hence provides a short range interaction.

In the astrophysical scales we use the weak field approximation of this theory which has good agreement with the dynamics of spiral galaxies and cluster of galaxies (Moffat & Rahvar 2013). Here, we review the same procedure by expanding fields around the Minkowski space-time in the action as follows:

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \]

where \( \eta_{\mu\nu} \) is the Minkowski metric. For the vector field, we write

\[ \phi_\mu = \phi_\mu(0) + \phi_\mu(1), \]

where \( \phi_\mu(0) \) is the zeroth order and \( \phi_\mu(1) \) is the first order perturbation of the vector field. For the Minkowski space-time, we set \( \phi_\mu(0) \) equal to zero which means that in the absence of matter there is no gravity source for the vector field \( \phi_\mu \). Also we perturb the energy-momentum tensor about the Minkowski background:

\[ T_{\mu\nu} = T_{\mu\nu}(0) + T_{\mu\nu}(1). \]
where \( T_\mu\nu(0) \) is zero.

By substituting the perturbation in the action and varying the action with respect to metric and ignoring the higher orders of perturbation, we get
\[
R_\mu\nu(1) - \frac{1}{2} R(1) \eta_\mu\nu = 8\pi G_0 T_\mu\nu^{(M)}(1) + 8\pi G_0 T_\mu\nu^{(\gamma)}(1),
\]
(10) where the first term on the right-hand side of this equation represents the energy-momentum tensor of matter, and the second term corresponds to the energy-momentum tensor of the vector field given by
\[
T_\mu\nu^{(\gamma)} = \frac{\omega}{4\pi} (B_\mu^a B_\nu a - \frac{1}{8} g_{\mu\nu} B^\alpha_\beta B_{\alpha\beta}) + \frac{\mu^2 \omega}{4\pi} (\phi_\nu \phi_\mu - \frac{1}{2} \phi_\nu \phi_\mu g_{\mu\nu}).
\]
(11)

In the weak field approximation, ignoring the higher order terms from the vector field in the energy momentum tensor, for (0,0) component we have
\[
R_{00}(1) = -\frac{1}{2} \nabla^2 \phi_0(0),
\]
(12) and substituting in equation (10) results in
\[
-\frac{1}{2} \nabla^2 \phi_0(0) = 4\pi G_0 \rho.
\]
(13)

Varying action with respect to the vector fields, we have the following equation
\[
\nabla_\nu B^{\mu\nu} - \mu^2 \phi^{\mu} = -\frac{4\pi}{\omega} J^{\mu}.
\]
(14)

Let us assume that the current of matter, \( J^\mu \) is conserved, (i.e. \( \nabla_\mu J^\mu = 0 \)), which implies the constrain of \( \phi_\mu^{\mu,\rho} = 0 \) in the weak field approximation. For the static case, this equation simplifies to
\[ \nabla_\nu B^{0\nu} - \mu^2 \phi^0 = -\frac{4\pi}{\omega} j^0, \]
(15)

which has the solution of
\[ \phi^0(x) = \frac{1}{\omega} \int \frac{e^{-\mu|\vec{x} - \vec{x}'|}}{|\vec{x} - \vec{x}'|} J^0(\vec{x}') d^3 x'. \]
(16)

In order to obtain the field equation for an effective potential in the weak field approximation, we use the geodesic equation of a test mass particle in equation (13) and in the weak field of gravitation we take the divergence of the spatial component form this equation, i.e.
\[ \nabla \cdot \vec{a} - \frac{1}{2} \nabla^2 \phi_0(0) = -\omega \nabla \cdot \nabla \phi_0^0, \]
(17)

where \( \vec{a} \) represents the acceleration of the test particle. We substitute \( \nabla^2 \phi_0(0) \) from (13) into (17) and define the effective potential for the test particle by, \( \vec{a} = -\nabla \Phi_{eff} \), and relate it to the distribution of matter as
\[ \nabla \cdot (\nabla \Phi_{eff} - \kappa \omega \nabla \phi_0^0) = 4\pi G_0 \rho, \]
(18)

where by replacing the solution for \( \phi^0 \) from (16) and substituting \( J^0 \) with \( \kappa \omega \rho \), we derive the effective potential of
\[ \Phi_{eff}(\vec{x}) = -\int \frac{G_0 \rho(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3 x' + \kappa^2 \int \frac{e^{-\mu|\vec{x} - \vec{x}'|}}{|\vec{x} - \vec{x}'|} \rho(\vec{x}') d^3 x', \]
(19)

where with redefinition of parameters we get the conventional form that derived in [Moffat & Rahvar (2013)]
\[ \Phi_{eff}(\vec{x}) = -G_N \left[ \int \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} \left( 1 + \alpha - \alpha e^{-\mu|\vec{x} - \vec{x}'|} \right) d^3 x' \right], \]
(20)

and consequently the acceleration of a test particle is given by
\[ a(x) = -G \int \frac{\rho(x') (x - x')}{|x - x'|^3} \times \left[ 1 + \alpha - \alpha e^{-\mu|\vec{x} - \vec{x}'|} (1 + \mu |x - x'|) \right] d^3 x'. \]
(21)

The observational test of this theory with the spiral galaxies as well as the cluster of galaxies provides a unique values of \( \alpha = 8.89 \pm 0.34 \) and \( \mu = 0.04 \pm 0.004 \ kpc^{-1} \) [Moffat & Rahvar 2013, 2014] for these scales.

### 2.2 Non-local Gravity

The Lorentz covariance in the spatial relativity is valid for the inertial observers. The fundamental assumptions of transformation of physical laws between the non-accelerating observers and the accelerating observers is that, locally these two frames are equivalent. This is called the equivalence principle. However, for accelerating observers which are the real observers of the physical world, the Lorentz covariance might not be valid. This extension is modelled with the introduction of non-locality in the accelerating frames and consequently in gravity. It has been argued that one must in general go beyond the locality hypothesis of standard special relativity theory and include the past history of an accelerated observer [Mashhoon 1999].

The extension of non-locality to the general relativity is done by the averaging procedure where a kernel acts as the weight function for the gravitational memory of the past events. In the non-local gravity, it is assumed that deviation from the locality is proportional to \( \lambda / L \) where \( \lambda \) is the characteristic length of phenomenon and \( L \) is the characteristic length that can be defined by the acceleration of the observer. In the case that \( \lambda / L > 1 \), assuming that the locality is breakdown, we can not use the equivalence principle.

The Non-local gravity is formulated in a tetrad formalism. This field relates the space time metric to the Minkowski space with \( g_{\mu\nu}(x) = e^{\alpha}_\mu e^{\beta}_\nu \eta_{\alpha\beta} \). Here the field have sixteen degrees of freedom. It is necessary to extend the Riemannian structure of space-time by using the Weitzenböck connection where \( \nabla_\nu e^{\alpha}_\mu = 0 \) which simplifies this equation results in \( \Gamma^{\lambda}_{\mu\nu} = e^\alpha_\lambda \partial_\mu e^{\alpha}_\nu - e^\alpha_\nu \partial_\mu e^{\alpha}_\lambda \).

Similar to the electromagnetic [Blagojević & Hehl 2012, Aldrovandi & Pereira 2013, Maluf 2013], we can define the field strength as follows
\[ C^{\lambda}_{\mu\nu} = \partial_\mu e^{\lambda}_\nu - \partial_\nu e^{\lambda}_\mu. \]
(22)

We define another auxiliary field strength, a modified torsion tensor of
\[ e^{\lambda}_{\mu\nu} = \frac{1}{2} \left( C^{\lambda}_{\mu\nu} - C^{\lambda}_{[\mu\nu]} + 2 \epsilon^{[\mu}_{\alpha} C_{\nu]}^{\lambda} \right) \]
(23)

where by defining a tensor density of
\[ H^{\mu\nu}_{\rho}(x) = \frac{\sqrt{-g(x)}}{\kappa} \epsilon^{\mu\nu}_{\rho}, \]
(24)
and $\kappa = 8\pi G$, the Einstein equation can be written in a simpler form of
\begin{equation}
\partial_{\mu}C_{\nu\mu}^{\alpha} = 0,
\end{equation}
where $\partial_{\mu}$ is the energy-momentum tensor of matter and $E_{\alpha}^{\mu}$ is the energy momentum of tetrad field and similar to the energy momentum tensor of electromagnetic field, it is traceless. The advantage of teleparallel formalism of general relativity is that we can write the Einstein equations in the form of Maxwell equations in a non-vacuum media. Here $\partial_{\mu}C_{\nu\mu}^{\alpha}$ plays the role of electromagnetic filed tensor of $F_{\mu\nu}$ (i.e. $C_{\mu\nu}^{\alpha} \rightarrow F_{\mu\nu}$), $\partial_{\mu}H_{\nu\sigma}^{\alpha}$ plays the role of displacement tensor of $H_{\mu\nu}$ (i.e. $H_{\nu\sigma}^{\alpha} \rightarrow H_{\mu\nu}$) and finally $\sqrt{-g}(T_{\alpha}^{\nu} + E_{\alpha}^{\nu})$ plays the role of current in the electromagnetism (i.e. $\sqrt{-g} E_{\mu\nu} \rightarrow j_{\mu}$).

It is well known that constitutive relation between $F_{\mu\nu}$ and $H_{\mu\nu}$ in the electrodynamics for a generic medium is non-local and is given by
\begin{equation}
C_{\mu\nu}^{\alpha} = \frac{1}{2}\xi^{\alpha \beta \gamma \delta}(\partial_{\gamma} H_{\beta\mu}^{\delta} + \partial_{\delta} H_{\gamma\mu}^{\beta} - \partial_{\mu} H_{\gamma\delta}^{\beta} - \partial_{\gamma} H_{\delta\mu}^{\beta}),
\end{equation}
where $q_{0}$ is the energy-momentum tensor of matter and $q_{f}$ is the energy momentum of tetrad field.

A more general situation can be supposed if we use one of general following kernels of:
\begin{equation}
q_{1} = \frac{1}{4\pi \lambda_{0}} \frac{1 + \mu (a_{0} + r)}{(a_{0} + r)^{2}} e^{-\mu r},
\end{equation}
\begin{equation}
q_{2} = \frac{1}{4\pi \lambda_{0}} \frac{1 + \mu (a_{0} + r)}{\mu (a_{0} + r)} e^{-\mu r},
\end{equation}

Which leads to the following acceleration law for non-local gravity:
\begin{equation}
a(\mathbf{x}) = -G \int \rho(\mathbf{x}'|(\mathbf{x} - \mathbf{x}') \times \left[ 1 - \mathcal{E}(r) + \alpha - \alpha e^{-\mu(\mathbf{x} - \mathbf{x}')}(1 + \frac{\mu}{2}(\mathbf{x} - \mathbf{x}')) \right] d^{3}x'.
\end{equation}

In which $\mathcal{E}(r)$ is either $E_{1}(r)$ or $E_{2}(r)$ associated to $q_{1}$ and $q_{2}$ consequently, and are given by
\begin{equation}
E_{1}(r) = \frac{a_{0}}{\lambda_{0}} \left\{ -\frac{r}{\mu a_{0}} e^{-\mu r} + 2e^{-\mu a_{0}} \left[ E_{1}(\mu a_{0}) - E_{1}(\mu a_{0} + \mu r) \right] \right\}
\end{equation}
and
\begin{equation}
E_{2}(r) = \frac{a_{0}}{\lambda_{0}} e^{-\mu a_{0}} \left[ E_{1}(\mu a_{0}) - E_{1}(\mu a_{0} + \mu r) \right].
\end{equation}

where $E_{1}(u)$ is the exponential integral function:
\begin{equation}
E_{1}(u) := \int_{u}^{\infty} \frac{e^{-t}}{t} dt.
\end{equation}

The numerical values of parameters for the simple kernel in equation (30) by comparing the dynamics of spiral galaxies with the theory has been obtained as $\alpha = 10.94 \pm 2.56$ and $\mu = 0.059 \pm 0.028$ $\text{km}^{-1} \cdot \text{s}$ [Rahvar & Mashhoon 2014].

### 2.3 Modified Newtonian Dynamics (MOND)

One of interesting approaches to answer the missing mass problem is the Modified Newtonian Dynamics that was proposed by Milgrom (1983). In order to interpret the flat rotation curve of spiral galaxies, Milgrom assumed that Newtonian dynamics should be modified in such a way that restore the Newtonian gravity at larger accelerations and is modified at low accelerations. In this theory, there is a universal acceleration of $a_{0} \approx 1.2 \times 10^{-10}$ $\text{m} \cdot \text{s}^{-2}$ at which the Newtonian dynamics is modified. An equivalent version of this theory is the modification of gravity law sourced by a single mass object as follows:
\begin{equation}
g = \begin{cases} 
\frac{GM}{r^{2}} & g \gg a_{0}, \\
\sqrt{GM a_{0}/r} & g \ll a_{0}. 
\end{cases}
\end{equation}

In order to ensure a smooth transition between the two regimes between the Newtonian gravity and Mondian gravity, Milgrom’s law is written in the following form:
\begin{equation}
\mu(\frac{g}{a_{0}})g = gn.
\end{equation}

where interpolating function satisfies the following constraints of $\mu(x) \rightarrow 1$ for $x \gg 1$ and $\mu(x) \rightarrow x$ for $x \ll 1$. A standard choice for the interpolating function is:
\begin{equation}
\mu(x) = x(1 + x^{2})^{-1/2}.
\end{equation}
From this interpolating function, it is straightforward to find the acceleration of test mass particle in terms of Newtonian gravity by using equations (38) and (39).

\[
g = \frac{g_N}{\sqrt{2}} \sqrt{1 + \sqrt{1 + \left(\frac{2g_m}{g_N}\right)^2}}. \tag{40}
\]

3 ROTATION CURVES OF GALAXIES

In this section we use a special type of galaxies so-called dwarf galaxies for testing the three alternative modifications of gravity law. The aim is to explain the dynamics of structures from the smallest galaxies to the cluster of galaxies without using the concept of dark matter. Dwarf galaxies are smaller than spiral galaxies, while in the context of dark matter scenario, some of them have more relative dark matter to the baryonic matter compared to the spiral galaxies (de Blok & McGaugh 1997).

In this section similar to the procedures used in (Mofat & Rahvar 2013 2014), we use the distribution of baryonic matter of dwarf galaxies including stars and interstellar baryonic gas to analyze the rotation curve of galaxies. For simplicity, we take a galaxy with cylindrical symmetry. The radial component of acceleration for the case of MOG and NLG can be calculated by discretizing space into small elements and adding up the acceleration of each element as follows

\[
a_r(r) = G_N \sum_{r' = 0}^{\infty} \sum_{\theta' = 0}^{\theta'} \frac{\Sigma(r')}{|r - r'|^3} (-r + r' \cos \theta')(1 + \alpha - \alpha e^{-\mu |r - r'|} - \mu \beta |r - r'| e^{-\mu |r - r'|}) |r'\Delta r'\Delta \theta', \tag{41}
\]

where \(\beta\) for MOG is 1 and for NLG is 1/2, \(\Sigma(r)\) represents the column density of a spiral galaxy. In the case of MOND, since this theory is nonlinear, we first find the Newtonian gravity (i.e. \(g_N\)) as we do in Newtonian case and then according to relation (40), obtain the overall acceleration.

From the observations, we have the column density of stars in 3.6 \(\mu\)m band as well as the column density of gas. Here we choose a sub-sample of nearby galaxies from the LITTLE THINGS catalog with high resolution measurements of velocity and density of stars and hydrogen profile (Hunter et.al 2015). Details about the observational parameters of these galaxies will be discussed in the next section part.

3.1 LITTLE THINGS catalog

Local Irregualrs That Trace Luminosity Extremes, The Hi Nearby Galaxy Survey (LITTLE THINGS) is a high-resolution (≈ 6” angular; < 2.6 km s\(^{-1}\) velocity resolution) observation with Very Large Array (VLA) that surveys nearby dwarf galaxies in the local volume within 10.3 Mpc. For this catalog, galaxies with distances >10 Mpc have been excluded in order to achieve a reasonably small spatial resolution in VLA Hi maps. The average distance to the galaxies in LITTLE THINGS sample is 3.7 Mpc. In addition galaxies with \(W_20 > 160\) km s\(^{-1}\) have been excluded in order to fit the galaxy emission comfortably in the bandwidth available for their desired velocity resolution (Oh et al. 2015). Here, parameter \(W_20\) is the full-width at 20% of the peak of an integrated Hi flux-velocity profile.

The high-resolution Hi observations enable us to derive reliable rotation curves of the sample galaxies in a homogeneous and consistent manner. The rotation curves are then combined with the Spitzer archival 3.6\(\mu\)m and ancillary optical U, B, and V images to construct mass models of the galaxies. Spitzer IRAC 3.6\(\mu\)m images is less affected by dust and less sensitive to the young stellar populations compared with optical images, so that they are better to be used for tracing old stellar populations that are overcoming in dwarf galaxies. This high quality multi-wavelength dataset significantly reduces observational uncertainties and thus allows us to examine the mass distribution in the galaxies in detail (Hunter et.al 2015). In high resolution HI data, observational systematic effects are properly reduced which provides us more accurate data.

The LITTLE THINGS catalog contains 37 dwarf galaxies, but the mass model of 26 dwarfs are available. We choose a sub-sample of 10 dwarf galaxies for our analysis, which have full coverage of rotation curves and baryonic matter distribution. Table (1) shows the list of galaxies in the sub-sample of the LITTLE THINGS catalog.

3.2 Observational test of Little Things galaxies with modified gravity models

In this section we find the best fit parameters of the three gravity models by comparing the dynamics of dwarf galaxies in the LITTLE THINGS catalog with our theoretical predictions. In the Newtonian gravity, one of crucial parameters in the estimation of dark matter content of a structure is the overall mass to the light ratio of a structure where we extract mass from the dynamics of structure and luminosity from the apparent luminosity of structure after correcting the absorptions in the intergalactic and galactic media. The luminosity of galaxies in our study is given in terms of magnitude in V-band (Hunter et al. 2012) and dynamics of galaxies are also obtained from the rotation curve of galaxies (Hunter et al. 2015). The overall mass to the light ratio of dwarf galaxies in our sample for the Newtonian gravity (i.e. \(\Upsilon = M_{dyn}/L\)) is given in Table (1).

3.2.1 Observational test of MOG

For this set of galaxies, we let \(\mu\) and \(\alpha\) as the free parameters and obtain the best values for these parameters from fitting the rotation curves of dwarf galaxies to the model. At the first step, we obtain the mass model of dwarf galaxies by means of linear interpolation method. According to this method by discretizing the pair of data points (\(\Sigma_i, x_i\)) where \(\Sigma_i\) represents the column density of a galaxy and \(x_i\) represents the distance from the centre of the galaxy, we obtain the density profile as follows

\[
\Sigma(x) = \Sigma_i + \frac{x_{i+1} - \Sigma_i}{x_{i+1} - x_i} (x - x_i), \tag{42}
\]

in which \(x\) is located between \(x_i\) and \(x_{i+1}\). After making a continuum density profile, we substitute the column density from the observation in equation (21) and calculate the acceleration of a test mass particle and consequently the rotational velocity of galaxy. Finally we compare this model
with the observed data by letting $\mu$ and $\alpha$ as the free parameter. We find the best value for the parameters of model by minimizing $\chi^2$

$$\chi^2 = \sum_{i=1}^{N} \frac{(V_i^{th} - V_i^{obs})^2}{\sigma_i^2},$$

(43)

where $V_i^{th}$ is the velocity predicted by the theory, $V_i^{obs}$ is the data from observation, $\sigma_i$ is the error bar of the observation and $N$ is the number of data points.

Once the best fit parameters are obtained, we find the region in the parameter space within $3\sigma$ confidence level, using the likelihood function as follows (Bevington & Robinson 2003):

$$L \propto \exp\left(-\frac{1}{2}(\chi^2 - \chi^2_{min})\right),$$

(44)

we plot the relative likelihood’s contours and investigate the two dimensional $\mu$ - $\alpha$ parameter space.

The contour plot of likelihood function for three $\sigma$ level of confidence with respect to the two dimensional parameter space is drawn in Figures 1. Moreover in Table 1 the chi-square, overall mass to light ratio, and best fit parameters of each galaxy are given. We find that the best value for $\alpha$ parameter is in the same range that was found for spiral galaxies ($\alpha = 8.89$) while for $\mu$ it is much larger than $\mu = 0.042$. Here the average value of this parameter for dwarf galaxies is $\mu = 3.56 \pm 0.49$.

In the next step we fix $\alpha = 8.89$ and let only the $\mu$ parameter changes in our sample galaxies. The results are given in Table 3. Here the average value of this parameter is $\mu = 2.77\text{kpc}^{-1}$ which is larger for the dwarf galaxies compare to the spiral galaxies. The rotation curves of each galaxy with the best value of $\mu$ is given in Figure 2. In this analysis the average value for $\chi^2$ per degree of freedom for our sample galaxies according to Table 3 is $\chi^2 = 1.32$. Comparing the best value of $\mu$ from the spiral galaxies and dwarf galaxies indicates the possibility of running of $\mu$ as a scale dependent parameter while $\alpha$ has a universal value.

3.2.3 Observational test of MOND

By using equations (40) and letting $a_0$ as the free parameter, we find the best fit parameter for MOND that is reported in Table 1. We infer from Table 1 that $a_0 \approx 1.17 \pm 0.75 \times 10^{-10}\text{ms}^{-2}$ is approximately equal to what is expected from Milgrom’s work ($a_0 \approx 1.2 \times 10^{-10}\text{ms}^{-2}$) from fitting this model to the Spiral galaxies (Milgrom 1983).

Swaters et al. (2010) examined for different set of dwarf galaxies and obtained the best value of $a_0 \approx 0.7 \times 10^{-10}\text{ms}^{-2}$ which is also consistent with our value. The rotation curve of our sample galaxies and theoretical rotation curves are drawn in Figure 2 and the mean $\chi^2$ per degree of freedom for our sample galaxies based on Milgrom’s theory is $\chi^2 = 0.87$.

4 CONCLUSIONS

To test the three modified gravity models of MOG, Non-local gravity and MOND with the dwarf galaxies, we used the LITTLE THINGS catalog of galaxies and fitted the theoretical rotation curves predicted by the modified gravity models to the observed data.

In the case of MOG and non-local gravity, we found that $\mu$ is larger compare to what Moffat & Rahvar (2013); Rahvar & Mashhoon (2014) reported for the spiral galaxies and cluster of galaxies, while we had almost consistent value for $\alpha$ parameter. The $\alpha$ parameter shows the strength of the gravitational constant at large distances where the effect of decaying Yukawa term is negligible. We adapted this parameter as in the spiral galaxy analysis and let the $\mu$ parameter which indicates the characteristic length of the Yukawa force as the free parameter. We have found that for the case of choosing a universal value for $\alpha$, the $\mu$-parameter depends on the size of structures. This results provides a hit on possibility of running of this parameter with the size of structure which has to be implemented in the theory. We also tested the dwarf galaxies with MOND and showed that our best value for $a_0$ is consistent with what has been obtained from fitting to the spiral galaxies.

3.2.2 Observational test for NLG

For the Non-local gravity, we follow the same procedure as in MOG for comparing the dynamics of dwarf galaxies with the gravity model. At first step, we find the best fit parameters of Non-local Gravity (i.e. $\mu$ and $\alpha$) by using equation (61) and calculate the dynamics of galaxies and compare them with the data. In Table 2 the best fit parameters and normalized $\chi^2$ for Non-local Gravity are derived. According to the table the average value of parameters are $\mu = 11.2 \pm 0.8$ and $\mu = 2.21 \pm 0.35\text{kpc}^{-1}$.

In the next step we fix the $\alpha$ and find the best value of $\mu$ for each galaxy. The best fit for $\alpha$ and $\mu$ of the theory are found as $\mu = 10.94 \pm 2.56$ and $\mu = 0.93 \pm 0.01\text{kpc}^{-1}$ and from the fitting the average value of $\chi^2$ is obtained as $\chi^2 = 1.22$. The rotation curve and the observed data of the dwarf galaxies for a fixed value of $\alpha = 10.92$ is plotted in Figure 2.
Testing MOG/Non-local/MOND gravity with rotation curve of dwarf galaxies

Figure 1. Contour plots of the relative likelihood function (i.e. \(\exp(-\frac{1}{2}[\chi^2 - \chi_{min}^2])\)) as a function of \(\alpha, \mu\). Table 2 provides the best \(\chi^2\) of these galaxies.

Table 1. Sub-set of dwarf galaxies from THINGS catalog. [Hunter et al. 2015]. Parameters in this table are calculated according to modified potential from MOG. The columns are as follows: (1) name of the galaxy, (2) distance of the galaxy from us, (3) The radius where the outermost part of the rotation curve is measured, (4) mass of gas, (5) mass of star in 3.6\(\mu\)m band, (6) the dynamical mass, (7) absolute V magnitude, (8) the normalized \(\chi^2\) for the best fit to the data in MOG theory, (9) \(\alpha\) parameter from the best fit in MOG, (10) \(\mu\) parameter from the best fit in MOG, (11) stellar mass to light ratio, (12) overall mass to light ratio assuming dark matter component ([Hunter et al. 2015]).
Table 2. Sub-set of dwarf galaxies from THINGS catalog. Parameters in this table are calculated according to modified potential from MOG. The columns are as follows: (1) name of the galaxy, (2) distance of the galaxy from us, (3) The radius where the outermost part of the rotation curve is measured, (4) mass of gas, (5) mass of star in 3.6µm band, (6) the dynamical mass, (7) absolute V magnitude, (8) the normalized $\chi^2$ for the best fit to the data in non-local gravity, (9) $\alpha$ parameter from the best fit in non-local gravity, (10) $\mu$ parameter from the best fit in non-local gravity, (11) stellar mass to light ratio, (12) overall mass to light ratio assuming dark matter component.

| Galaxy | Distance (Mpc) | $R_{\text{max}}$ (kpc) | $M_{\text{gas}}$ ($10^{10} M_\odot$) | $M_{\text{star}}$ ($10^{10} M_\odot$) | $\log(M_{\text{dyn}})$ ($M_\odot$) | $M_V$ (mag) | $\chi^2/N_{\text{d.o.f}}$ | $\alpha$ | $\mu$ (kpc$^{-1}$) | $\Upsilon_*$ $M_\odot/L_\odot$ | $\Upsilon$ $M_\odot/L_\odot$ |
|--------|----------------|------------------------|---------------------------------|---------------------------------|---------------------------------|-------------|------------------|--------|----------------|-------------------|------------------|
| DDO 52 | 10.3           | 5.43                   | 33.43                           | 7.20                            | 9.664                           | -15.4       | 1.19             | 10.5 ± 0.3 | 1.41 ± 0.2 | 0.36              | 36.31             |
| DDO 53 | 3.6            | 1.45                   | 7.00                            | 0.96                            | 8.567                           | -13.8       | 0.33             | 21 ± 2    | 0.53 ± 0.15 | 0.37              | 13.18             |
| DDO 87 | 7.7            | 7.39                   | 29.12                           | 6.18                            | 9.734                           | -15.0       | 1.34             | 10 ± 0.2  | 2.99 ± 0.7  | 0.39              | 63.09             |
| DDO 126| 4.9            | 3.99                   | 16.36                           | 2.27                            | 9.182                           | -14.9       | 1.04             | 4.5 ± 0.2 | 1.93 ± 0.2  | 0.3               | 19.95             |
| DDO 70 | 1.3            | 2.00                   | 3.80                            | 1.24                            | 8.768                           | -14.1       | 2.44             | 14.5 ± 0.5| 1.04 ± 0.04 | 0.34              | 15.85             |
| DDO 133| 3.5            | 3.48                   | 12.85                           | 2.62                            | 9.261                           | -14.8       | 3.08             | 6 ± 0.2   | 11.33 ± 1.8 | 0.35              | 25.7              |
| DDO 216| 1.1            | 1.12                   | 0.49                            | 1.60                            | 7.807                           | -13.7       | 0.92             | 6.5 ± 0.7 | 1.81 ± 0.3  | 0.56              | 2.5               |
| IC 1613| 0.7            | 2.71                   | 5.93                            | 1.94                            | 8.448                           | -14.6       | 1.15             | 19.5 ± 0.7| 0.4 ± 0.02  | 0.38              | 4.79              |
| IC 10  | 0.7            | 0.54                   | 1.65                            | 11.81                           | 8.213                           | -16.3       | 0.06             | 8 ± 1.4   | 0.5 ± 0.1   | 0.58              | 0.58              |
| NGC 1569| 3.4           | 3.05                   | 20.24                           | 20.69                           | 9.193                           | -18.2       | 0.14             | 11.5 ± 2  | 0.13 ± 0.02 | 0.49              | 0.95              |
Figure 2. The best fit to the rotation velocity curves of the LITTLE THINGS sample. We have plotted these curves with the universal value of $\alpha = 8.89$ for MOG and the best value of $\mu$ from Table (3) (dashed line), $\alpha = 10.94$ for Non-local gravity and the best value of $\mu$ from Table (3) (dotted line) and the rotation curve for MOND from the best value of $a_0$ from Table (4) (solid line).
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Table 3. The best values of $\mu$ with adapting universal value of $\alpha$ in MOG and non-local gravity models. The columns are as follows: (1) the name of the galaxy, (2) the universal parameter of $\alpha$ in MOG, (3) the best fit value for $\mu$ parameter in MOG for each galaxy, (4) Normalized $\chi^2$ for the best fit to the data in MOG, (5) the universal parameter of $\alpha$ in non-local gravity, (6) the best fit value for $\mu$ parameter in non-local gravity for each galaxy, (7) the normalized $\chi^2$ for the best fit to the data in the non-local gravity.

| Galaxy   | $\alpha_{(MOG)}$ | $\mu_{(MOG)}$ | $\chi^2/N_{d.o.f_{(MOG)}}$ | $\alpha_{(Non-Local)}$ | $\mu_{(Non-Local)}$ | $\chi^2/N_{d.o.f_{(Non-Local)}}$ |
|----------|-----------------|---------------|-----------------------------|------------------------|---------------------|----------------------------------|
| DDO 52   | 8.89 ± 0.34     | 7. ± 2        | 1.07                        | 10.94 ± 2.56           | 1.35 ± 0.2           | 1.11                             |
| DDO 53   | 8.89 ± 0.34     | 0.97 ± 0.07   | 0.28                        | 10.94 ± 2.56           | 0.44 ± 0.04          | 0.39                             |
| DDO 87   | 8.89 ± 0.34     | 6.1 ± 1.1     | 1.53                        | 10.94 ± 2.56           | 1.26 ± 0.15          | 1.38                             |
| DDO 126  | 8.89 ± 0.34     | 0.75 ± 0.025  | 1.06                        | 10.94 ± 2.56           | 0.37 ± 0.015         | 1.06                             |
| DDO 70   | 8.89 ± 0.34     | 3.80 ± 0.4    | 2.58                        | 10.94 ± 2.56           | 1.67 ± 0.15          | 2.44                             |
| DDO 133  | 8.89 ± 0.34     | 2.17 ± 0.1    | 3.23                        | 10.94 ± 2.56           | 1. ± 0.05            | 3.69                             |
| DDO 216  | 8.89 ± 0.34     | 2.07 ± 0.2    | 0.73                        | 10.94 ± 2.56           | 0.98 ± 0.1           | 0.79                             |
| IC 1613  | 8.89 ± 0.34     | 3.37 ± 0.4    | 2.56                        | 10.94 ± 2.56           | 1.71 ± 0.2           | 1.24                             |
| IC 10    | 8.89 ± 0.34     | 1.09 ± 0.11   | 0.07                        | 10.94 ± 2.56           | 0.4 ± 0.07           | 0.07                             |
| NGC 1569 | 8.89 ± 0.34     | 0.41 ± 0.05   | 0.06                        | 10.94 ± 2.56           | 0.16 ± 0.02          | 0.07                             |
Table 4. The best parameter for $a_0$ in MOND for each galaxy. The columns are as follows: (1) the name of the galaxy, (2) the best value for $a_0$ parameter in MOND from the best fitting to the dwarf galaxies in LITTLE THINGS catalog, (3) the normalized $\chi^2$ for the best fit.

| Galaxy | $a_0$ ($10^{-10} \times \text{ms}^{-2}$) | $\chi^2$/N.d.o.f | MOND |
|--------|-------------------------------|----------------|-----|
| DDO 52 | 2.76 ± 0.16                   | 0.5            |     |
| DDO 53 | 0.52 ± 0.06                   | 1.06           |     |
| DDO 87 | 1.53 ± 0.06                   | 0.34           |     |
| DDO 126| 0.67 ± 0.03                   | 0.7            |     |
| DDO 70 | 1.63 ± 0.1                    | 1.77           |     |
| DDO 133| 1.9 ± 0.06                    | 2.23           |     |
| DDO 216| 0.38 ± 0.06                   | 0.5            |     |
| IC 1613| 0.53 ± 0.03                   | 1.38           |     |
| IC 10  | 1.25 ± 0.23                   | 0.1            |     |
| NGC 1569| 0.51 ± 0.08                  | 0.11           |     |