Finite temperature phase transition in full QCD with $N_f = 2$ flavors of clover fermions at $N_t = 8$ and 10

Y. Nakamura, V. Bornyakov, M.N. Chernodub, Y. Koma, Y. Mori, M.I. Polikarpov, G. Schierholz, A. Slavnov, H. St"uben, T. Suzuki, P. Uvarov, and A.I. Veselov

aKanazawa University, Kanazawa 920-1192, Japan
bInstitute for High Energy Physics, RU-142284 Protvino, Russia
cITEP, B.Cheremushkinskaya 25, RU-117259 Moscow, Russia
dNIC/DESY Zeuthen, Platanenallee 6, D-15738 Zeuthen, Germany
eSteklov Mathematical Institute, Vavilova 42, RU-117333 Moscow, Russia
fKonrad-Zuse-Zentrum f"ur Informationstechnik Berlin, D-14195 Berlin, Germany

We present results for QCD with $N_f = 2$ flavors of dynamical quarks using nonperturbatively improved Wilson fermions at finite temperature on $16^3 \times 8$ and $24^3 \times 10$ lattices. We determine the transition temperature in the range of quark masses $0.6 \lesssim m_\pi/m_\rho \leq 0.8$. After fixing the Maximal Abelian gauge we investigate the contribution of Abelian monopoles to the Polyakov loop, Polyakov loop susceptibility and confirm Abelian and monopole dominance in full QCD.

1. INTRODUCTION

In order to obtain predictions for the real world from lattice QCD, we have to extrapolate the lattice data to the continuum and to the chiral limits. Recently the Bielefeld group and the CP-PACS collaboration using different fermion actions obtained consistent values for the critical temperature $T_c$ in the chiral limit, albeit on rather coarse lattices at $N_t = 4$ and 6. Edwards and Heller determined $T_c$ for $N_t = 4$, 6 using nonperturbatively improved Wilson fermions. We compute $T_c$ on finer lattices with $N_t = 8$ and 10 with high statistics. Our results for $N_t = 8$ were reported in Ref. [4].

2. SIMULATION

We use fermionic action for nonperturbatively $O(a)$ improved Wilson fermions:

$$S_F = S_W^c - \frac{i}{2} \kappa g c_{sw} a^5 \sum_x \bar{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu} \psi(x),$$

where $S_W^c$ is the original Wilson action, $c_{sw}$ was calculated in [5].

$*$Talk given by Y. N. at Lattice’03.

Configurations are generated on $16^3 \times 8$ ($\beta = 5.2$ and 5.25) and $24^3 \times 10$ ($\beta = 5.2$) lattices at various $\kappa$. The values of $\kappa$ and corresponding number of trajectories for $16^3 \times 8$ and $24^3 \times 10$ lattices can be found in Ref. [4] and Table 1, respectively. The number of configurations for $24^3 \times 10$ lattice is not large enough and results for this lattice are preliminary.

| $\kappa$ | 0.1352 | 0.1354 | 0.1356 | 0.1358 |
|---|---|---|---|---|
| # traj. | 5,400 | 7,400 | 3,130 | 1,650 |

Table 1: Simulation statistics on $24^3 \times 10$.

We use results obtained at $T=0$ to fix the scale. An updated version of the contour plot of lines of constant $r_0/a$ and $m_\pi/m_\rho$ is shown in Fig. [1].

3. CRITICAL TEMPERATURE

We use non–Abelian and Abelian Polyakov loops to determine the transition temperature. The Maximally Abelian gauge is fixed by maximizing the quantity

$$R = \sum_{s,\mu} \sum_{\mu=1}^{3} |\tilde{U}_{s,\mu}|^2,$$

with respect to gauge transformations $g$, $\tilde{U}(s,\mu) = g(s)U(s,\mu)g^\dagger(s + \mu)$. We use an
The lines of constant \( r_0/a \) and \( m_\pi/m_\rho \) at \( T = 0 \). Crosses correspond to parameters used in this work.

SU(3) version of the simulated annealing algorithm [9]. The Abelian link variables are defined as \( u_i(s, \mu) = \exp\{i \theta_i(s, \mu)\} \), where

\[
\theta_i(s, \mu) = \arg \tilde{U}_{ii}(s, \mu) - \frac{1}{3} \sum_{j=1}^{3} \arg \tilde{U}_{jj}(s, \mu) \mod 2\pi.
\]

We then define the Abelian, monopole and photon Polyakov loop operators as in [10].

The Polyakov loop susceptibility is used to determine the transition point. In Fig. 2 the non-Abelian Polyakov loop and its susceptibility are depicted. We get the following values for the critical temperatures:

- \( T_c \approx 196(8) \text{MeV}, \ m_\pi/m_\rho \approx 0.64 \) (Preliminary)
- \( T_c = 210(4) \text{MeV}, \ m_\pi/m_\rho = 0.77 \)
- \( T_c = 219(3) \text{MeV}, \ m_\pi/m_\rho = 0.81 \)

It is known [11] that in SU(2) gluodynamics the non-Abelian, Abelian and monopole Polyakov loops give the same \( T_c \) and critical indices. It follows from comparison of Fig. 2 and Fig. 3 that also in the full QCD these three Polyakov loops have similar behavior and their susceptibilities have maxima at the same temperature.

4. CONTINUUM LIMIT

At small enough lattice spacing and quark mass one can extrapolate the critical temperature \( T_c \) to the continuum and the chiral limits using formula:

\[
T_c r_0 = (T_c r_0)^{m_\pi/m_\rho} + C_a (a/r_0)^2 + C_q \left( \frac{1}{\kappa} - \frac{1}{\kappa_c} \right)^s,
\]

where \( r_0 = 0.5 \text{ fm} \) and \( (T_c r_0)^{m_\pi/m_\rho} \) corresponds to the extrapolated value.

We are brave enough to use four values for \( T_c r_0 \) (see Table 3), obtained at rather large quark
masses, to estimate the parameters in this extrapolation expression. A fit gives \((T_c)^{m_q,a \to 0}\) and \(\alpha\) with large errors: \((T_c)^{m_q,a \to 0} \sim 190\) MeV, \(\alpha \sim 0.8\).

| \(T_c r_0\) | \(a/r_0\) | \(N_t\) | \(\beta\) |
|------------|--------|-------|-------|
| 0.50(2)    | 0.20(1) | 10    | 5.2   |
| 0.53(1)    | 0.234(5)| 8     | 5.2   |
| 0.55(1)    | 0.225(5)| 8     | 5.25  |
| 0.57(2)    | 0.29(1) | 6     | 5.2   |

Table 3: Available data for \(T_c r_0\).

In \(N_f = 2\) QCD the critical indices are expected to belong to the universality class of the 3D \(O(4)\) spin model for which one expects \(\alpha = 0.55\). Fixing \(\alpha\) to this value we get the extrapolated temperature with a higher accuracy:

\[
T^{m_q,a \to 0} \sim 172.5(3.3)\text{ MeV}.
\]

This value agrees with values obtained in Refs. [1, 2].

5. CONCLUSIONS

We determined the critical temperature in full QCD on \(16^3 \times 8\) and \(24^3 \times 10\) lattices at \(\beta = 5.2\) and 5.25 with \(N_f = 2\) clover fermions using non-Abelian and Abelian Polyakov loop susceptibilities. Our results are in agreement with the results of other groups, as it is shown in Fig. 1. Assuming that the critical indices of the two flavor QCD belong to the universality class of the 3D \(O(4)\) spin model, we extrapolate the critical temperature \(T_c\) to the continuum and the chiral limits. The extrapolation result is given by [1]. We are continuing simulations on \(24^3 \times 10\) lattice to get better precision of \(T_c\) value on this lattice.

6. ACKNOWLEDGEMENTS

This work is supported by the SR8000 Supercomputer Project of High Energy Accelerator Research Organization (KEK). A part of numerical measurements has been done using NEC SX-5 at Research Center for Nuclear Physics (RCNP) of Osaka University. M.N.Ch. is supported by JSPS Fellowship P01023. T.S. is partially supported by JSPS Grant-in-Aid for Scientific Research on Priority Areas No.13135210 and (B) No.15340073. The Moscow group is partially supported by RFBR grants 02-02-17308, 01-02-17456, grants INTAS–00-00111, DFG-RFBR 436 RUS 113/739/0, and CRDF awards RPI-2364-MO-02 and MO-011-0. G.S. would like to thank Kanazawa University for its kind hospitality.

REFERENCES

1. F. Karsch, A. Peikert, E. Laermann, Nucl. Phys. B605 (2001) 579.
2. A.A. Khan et al., (CP-PACS), Phys. Rev. D63 (2001) 034502.
3. R. G. Edwards, U. M. Heller, Phys. Lett. B462 (1999) 132.
4. V. Bornyakov et al., [hep-lat/0301003] Nucl. Phys. B (Proc.Suppl.) 119 (2003) 703.
5. K. Jansen and R. Sommer [ALPHA collaboration], Nucl. Phys. B530 (1998) 185.
6. S. Booth et al., Phys. Lett. B519 (2001) 229.
7. A.S. Kronfeld, M.L. Laursen, G. Schierholz, U.-J. Wiese, Phys. Lett. B198 (1987) 516.
8. F. Brandstaeter, G. Schierholz, U.-J. Wiese, Phys. Lett. B272 (1991) 319.
9. G. Bali, V. Bornyakov, M. Müller-Preussker, F. Pahl, Nucl. Phys. B (Proc.Suppl.) 42 (1995) 852.
10. T. Suzuki et al., Phys. Lett. B347 (1995) 375; [Erratum-ibid. B351 (1995) 603].
11. S. Ejiri, S. Kitahara, T. Suzuki, K. Yasuta, Phys. Lett. B400 (1997) 163.