Discovery of Large Scale Tensor Mode and
Chaotic Inflation in Supergravity

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Abstract

The BICEP2 collaboration has recently reported a large tensor fluctuation in the cosmic microwave background, which suggests chaotic inflation models. In this letter, we reconsider the chaotic inflation model in the supergravity. We introduce a non-holomorphic shift-symmetry breaking parameter, which we expect to exist in general, and discuss its effect on the inflaton dynamics. We show that the model predicts a sizable deviation from the original chaotic inflation model and the predicted tensor fluctuation can lie between the BICEP2 result and the upper bound given by the Planck experiment with a small shift-symmetry breaking parameter. The model is characterized by only two parameters, which yields predictability and testability in future experiments.
I. INTRODUCTION

Cosmic inflation \([1]\) is a natural scenario which not only solves the flatness and the horizon problem, but also explains the large scale structure of the universe and the fluctuation of the cosmic microwave background (CMB) radiation. Precise observations of the CMB \([2,3]\) begins to reveal nature of inflation. Recently, the BICEP2 collaboration has reported a large tensor fraction, \(r = \mathcal{O}(0.1)\) \([5]\), which favors chaotic inflation models \([6]\). Chaotic inflation models have been studied in the literature, especially in the context of the supergravity theory (SUGRA). In this letter, we reconsider chaotic inflation models in the SUGRA.

In SUGRA chaotic inflation models, the shift-symmetry proposed in Ref. \([7]\) is a crucial assumption. In order to obtain non-zero potential energy, the shift symmetry must be explicitly broken. In Ref. \([7]\) the shift-symmetry breaking is introduced in the superpotential. However, it would be natural to consider that the Kahler potential also has shift-symmetry breaking terms.

The shift-symmetry breaking in the Kahler potential is discussed in Refs. \([8,9]\), and it is shown that the prediction of the model is deviated from that of Ref. \([7]\) significantly. However, it is not clear whether the model possesses predictability. Higher dimensional terms in the Kahler potential may change inflaton dynamics due to large inflaton field value during inflation, once the shift-symmetry breaking is introduced.

In this letter, we propose to treat the shift-symmetry breaking in a systematic way by introducing a non-holomorphic shift-symmetry breaking spurion \(E\) and discussing its effect on the inflaton dynamics. We restrict our attention to the range of the breaking parameter where higher dimensional terms are negligible for the inflaton dynamics and the model possesses predictability and testability. We show that the prediction for the spectral index and the tensor fraction can lie between the results of the Planck and the BICEP2 experiments with a small shift-symmetry breaking parameter. We also show that future observations of the CMB can quantify the reheating temperature of the universe within a factor of \(\mathcal{O}(10)\).

This letter is organized as follows. In the next section, we review the SUGRA chaotic inflation model. In Sec. \([III]\), we introduce a non-holomorphic shift-symmetry breaking parameter \(E\) and discuss how the prediction on the spectral index and the tensor fraction is modified by the shift-symmetry breaking. We estimate the range of the shift symmetry breaking where higher dimensional terms in the Kahler potential are negligible, and show
the prediction of the model within the range. The last section is devoted to discussion and conclusions.

II. REVIEW ON SUGRA CHAOTIC INFLATION MODELS

In this section, we review SUGRA chaotic inflation models in Ref. [7]. For simplicity, we discuss a quadratic chaotic inflation model. In the SUGRA, the scalar potential is determined by the Kahler potential $K(\phi^i, \phi^{*\bar{i}})$ and the superpotential $W(\phi^i)$, where $\phi^i$ and $\phi^{*\bar{i}}$ are chiral multiplets and their conjugates, respectively. The scalar potential is given by

$$V = e^K \left[ K_{ii} D_i W D_i W^* - 3 |W|^2 \right],$$

$$D_i W \equiv W_i + K_{ii} W,$$  

where subscripts $i$ and $\bar{i}$ denote derivatives with respect to $\phi^i$ and $\phi^{*\bar{i}}$, respectively. $K_{ii}$ is the inverse of the matrix $K_{i\bar{i}}$. Throughout this letter, we use a unit with the reduced Planck mass $M_{\text{pl}} \simeq 2.4 \times 10^{18}$ GeV being unity.

Chaotic inflation is achieved by introducing two chiral multiplets $\Phi$ and $X$ and assuming the following Kahler and the superpotential,

$$W = m X \Phi,$$

$$K = K\left(XX^*, (\Phi + \Phi^*)^2\right) = \frac{1}{2}(\Phi + \Phi^*)^2 + XX^* + \cdots ,$$  

where $\cdots$ denote higher dimensional terms. This form of the potentials is realized by assuming an $R$ symmetry, a $Z_2$ symmetry and a shift-symmetry, which are listed in Table I. The breaking of the shift symmetry, which is necessary in order to obtain non-zero potential energy, is expressed by the holomorphic spurious field $m$. The $Z_2$ symmetry is crucial to prevent the over-production of gravitinos in the decay of the inflaton [11].

The inflaton field is identified with the imaginary part of $\Phi$, whose potential from the exponential factor in Eq. (1) is absent due to the shift symmetry [7], which solves the eta problem [10]. $X$ and the real part of $\Phi$ obtains masses as large as the Hubble scale during inflation by higher dimensional operators and hence are fixed to their origin during inflation. As a result, the potential of the imaginary part of $\Phi$, $\phi$, is given by

$$V(\phi) = \frac{1}{2} m^2 \phi^2,$$  

1 We neglect the D-term contribution, which is irrelevant for our purpose.
which is nothing but the potential of the quadratic chaotic inflation model [6]. The magnitude of the curvature perturbation, $P_\zeta \simeq 2.2 \times 10^{-9}$, determines the parameter $m$ as (see e.g. Ref. [12])

$$m \simeq 6.0 \times 10^{-6} = 1.5 \times 10^{13} \text{ GeV}.$$  \(4\)

The spectral index of the curvature perturbation $n_s$ and the tensor fraction $r$ are given by

$$n_s = 1 - \frac{2}{N_e} \simeq 0.967 \quad (N_e = 60),$$

$$r = \frac{8}{N_e} \simeq 0.13 \quad (N_e = 60),$$  \(5\)

where $N_e$ is the number of the e-fodings corresponding to the scale of the interest. Note that they are determined only by $N_e$, and the model has strong predictability.

### III. SHIFT-SYMMETRY BREAKING IN THE KAHLER POTENTIAL

In the previous section, we have reviewed the SUGRA chaotic inflation model. There, we have introduced the shift-symmetry breaking only to the superpotential. However, it would be more natural to consider that the Kahler potential also has shift-symmetry breaking terms. The shift-symmetry breaking in the Kahler potential is discussed in Refs. [8] [9]. However, it is not clear how higher dimensional terms change the prediction of the model once the shift-symmetry breaking is introduced, since the field value of the inflaton is far above the Planck scale during inflation.

In this section, we propose to treat the shift-symmetry breaking in a systematic way, such that the shift-symmetry breaking is expressed by a non-holomorphic spurious field $\mathcal{E}$. We estimate an upper bound on the magnitude of the shift symmetry breaking where higher dimensional terms in the Kahler potential are negligible and hence the model possesses

| $R$ | $Z_2$ | shift |
|-----|-------|-------|
| $X$ | 2 | $-1$ | $X \to X$ |
| $\Phi$ | 0 | $-1$ | $\Phi \to \Phi + ic$ |
| $m$ | 0 | $+1$ | $m \to m \frac{\Phi}{\Phi + ic}$ |

TABLE I. Charge assignment of (spurious) fields. $c$ is an arbitrary real number.
the predictability. We restrict our discussion to such a breaking parameter and show that the prediction of the model can lie between the results of the Planck [4] and the BICEP2 experiments [5] with a small shift-symmetry breaking parameter.

The Kahler potential is in general given by

\[ K = K \left( XX^*, \left( \Phi + \Phi^* \right)^2, \mathcal{E} \left( \Phi - \Phi^* \right)^2 \right), \]

with the transformation law of \( \mathcal{E} \),

\[ \mathcal{E} \rightarrow \mathcal{E} \frac{\left( \Phi - \Phi^* \right)^2}{\left( \Phi - \Phi^* + 2ic \right)^2}. \]

As we have mentioned, \( \mathcal{E} \) is non-holomorphic. In the following, we estimate the bound on \( \mathcal{E} \) so that \( O(\mathcal{E}^2) \) terms do not affect the inflaton dynamics.

The Kahler potential is expanded around the origin as

\[ K = XX^* + \frac{1}{2} (\Phi + \Phi^*)^2 - \mathcal{E} (\Phi - \Phi^*)^2 + \frac{\mathcal{E}^2}{4!} \kappa (\Phi - \Phi^*)^4 \cdots, \]

where \( \cdots \) denotes higher dimensional \( O(\mathcal{E}^3) \) terms and \( \kappa \) is an order one parameter. The normalization of \( \mathcal{E} \) is fixed by the third term in Eq. (8). The potential of the inflaton is given by

\[ V(\phi) = \exp \left( \mathcal{E} \phi^2 + \frac{\mathcal{E}^2}{6} \kappa \phi^4 + \cdots \right) \times \frac{1}{2} m^2 \phi^2, \]

where \( \cdots \) denotes higher dimensional \( O(\mathcal{E}^3) \) terms.

Let us discuss the dynamics of the inflaton. The first and the second slow-roll parameter \( \epsilon \) and \( \eta \) are given by

\[ \epsilon(\phi) = \frac{1}{2} \left( \frac{V_{\phi}}{V} \right)^2 \simeq \frac{2}{\phi^2} \left( 1 + 2\mathcal{E} \phi^2 + \frac{3 + 2\kappa}{3} \mathcal{E}^2 \phi^4 \right), \]
\[ \eta(\phi) = \frac{V_{\phi \phi}}{V} \simeq \frac{2}{\phi^2} \left( 1 + 5\mathcal{E} \phi^2 + \frac{6 + 7\kappa}{3} \mathcal{E}^2 \phi^4 \right), \]

where we have neglected \( O(\mathcal{E}^3) \) terms. The number of the e-folding \( N_e \) and the inflaton field values are related by

\[ N_e(\phi) = \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V_{\phi}} d\phi \simeq \frac{1}{4} (\phi^2 - \phi_{\text{end}}^2) - \frac{\mathcal{E}}{8} (\phi^4 - \phi_{\text{end}}^4) + \frac{3 - \kappa}{36} \mathcal{E}^2 (\phi^6 - \phi_{\text{end}}^6), \]

\[ ^2 \text{It is possible to introduce the shift symmetry breaking by a holomorphic parameter } m \text{ as } K \supset (m \Phi - m^* \Phi^*)^2 + \cdots. \text{ However, } m \text{ is too small to affect the inflaton dynamics.} \]

\[ ^3 O(\mathcal{E}^2) \text{ terms in the Kahler potential also contribute to field-dependent kinetic terms of the inflaton field.} \]

In order to simplify the analysis, we neglect them.
with \( \phi_{\text{end}} \simeq \sqrt{2} \). We have again neglected \( \mathcal{O}(\mathcal{E}^3) \) terms.

The spectral index \( n_s \) and the tensor fraction \( r \) are given by

\[
    n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon.
\]

(12)

In Figs. 1 and 2 we show \( n_s \) and \( r \) as functions of \( \mathcal{E} \) for \( \kappa = 0 \) and 1. It can be seen that \( n_s \) and \( r \) are significantly altered by the shift symmetry breaking expressed by \( \mathcal{E} \).

Let us estimate the bound on \( \mathcal{E} \) such that higher dimensional \( \mathcal{O}(\mathcal{E}^2) \) terms in the Kahler potential are negligible. Since we have expressed the magnitude of \( \mathcal{O}(\mathcal{E}^2) \) terms by the parameter \( \kappa \) (see Eq. (8)), we can estimate the bound on \( \mathcal{E} \) by investigating the dependence of the prediction for \( n_s \) and \( r \) on \( \kappa \). In Figs. 3 and 4 we show \( \Delta n_s \equiv |n_{s,\kappa=1} - n_{s,\kappa=0}| \) and \( \Delta r \equiv |r_{s,\kappa=1} - r_{s,\kappa=0}| \) as functions of \( \mathcal{E} \). We define the insensitivity to higher dimensional \( \mathcal{O}(\mathcal{E}^2) \) terms as \( \Delta n_s < 10^{-3} \) and \( \Delta r < 10^{-3} \), which is the typical resolution of future satellite experiments such as the CMBPol [13] and the LiteBIRD [14]. From Figs. 3 and 4 we put a bound on \( \mathcal{E} \) as

\[
    |\mathcal{E}| < 10^{-3.3}.
\]

(13)

In Fig. 5 we show the prediction on \( n_s \) and \( r \) for \( |\mathcal{E}| < 10^{-3.3} \). We also show constraints from the Planck experiment [4] and the BICEP2 experiment [5] for the pivot scale of 0.002 Mpc\(^{-1}\). It can be seen that the prediction of the model can lie between the results of the Planck and the BICEP2 experiments (see also Ref. [8]). We stress that the prediction is not affected by higher dimensional terms in the Kahler potential as long as the constraint given in Eq. (13) is satisfied.

Note that \( n_s \) and \( r \) also depend on \( N_e \). Difference of the reheating temperature by an order of magnitude changes \( N_e \) corresponding to the pivot scale by \( \mathcal{O}(1) \). The \( \mathcal{O}(1) \) change in \( N_e \) also modifies the prediction on \( n_s \) and \( r \) by \( \mathcal{O}(10^{-3}) \). Therefore, by measuring \( n_s \) and \( r \) with an accuracy of \( \mathcal{O}(10^{-3}) \), we can quantify the reheating temperature of the universe within a factor of \( \mathcal{O}(10) \). It should be noted that this is possible only within the parameter range given in Eq. (13).

In the above analysis, we have concentrated on the quadratic chaotic inflation model. This is because the constraint on the spectral index \( n_s \simeq 0.96 \) favors the quadratic model. In Ref. [15], however, it is pointed out that the central value of \( n_s \) is larger and amounts to \( \simeq 0.97 \). If that is the case, models with lower power potentials [16, 17] are favored.
IV. DISCUSSION AND CONCLUSIONS

In this letter, we have reconsidered chaotic inflation models in the SUGRA. We have introduced a non-holomorphic shift-symmetry breaking parameter $\mathcal{E}$ and discussed its effect on the inflaton dynamics. We have clarified the range of $\mathcal{E}$ where higher dimensional terms are negligible for the inflaton dynamics and the model possesses predictability and testability.
We have shown that the prediction for the spectral index $n_s$ and the tensor fraction $r$ are given by $n_s \sim 0.96$ and $r = 0.11 - 0.18$. The prediction can lie between the results of the Planck and the BICEP2 experiments. It is interesting that future experiments will measure $n_s$ and $r$ accurately and reveal the structure of the shift-symmetry breaking in the inflaton sector. We have also shown that future observations of the CMB can quantify the reheating temperature of the universe within a factor of $\mathcal{O}(10)$, as long as $\mathcal{E}$ is in the range we have clarified.

Let us comment on the magnitude of the shift symmetry breaking. We have introduced two shift-symmetry breaking parameters, $m$ and $\mathcal{E}$. The magnitude of the curvature pertur-
FIG. 5. The prediction on the spectral index $n_s$ and the tensor fraction $r$. We also show the constraint from the Planck and the BICEP2 experiments.

bation indicates that $m \sim 10^{-5}$ and the consistency with the observed spectral index and the tensor fraction suggests that $|\mathcal{E}| \sim 10^{-3}$. Therefore, the two shift-symmetry breaking parameters are different by order of magnitudes. Note that the $m$ is a holomorphic parameter while $\mathcal{E}$ is a non-holomorphic one, and hence they may have different origins. We hope that a more fundamental theory explains the origin of the shift-symmetry breaking.

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