Reflection of shear elastic waves from the interface of a ferromagnetic half–space

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Abstract. In this paper, the problems of reflection and refraction of a pure elastic wave incident from a nonmagnetic medium on the surface of contact between two semi-infinite media of an infinite nonmagnetic/magnetic structure are considered. The resonance character of the interaction between elastic and magnetic waves is shown, and the dependence of the magnetoelastic wave amplitudes on the incident elastic wave amplitude is also established.

1. Introduction

It is well known that there can be interconnected magneto-elastic (magneto-acoustic) waves in magnetically ordered media. The interrelated spin and elastic waves propagating through a ferromagnetic material are called magnetoelastic or elastic-spin waves. The theory of magnetoelastic coupling in ferromagnets was developed by Akhiezer A I et al. [1] and Kittel C [2]. The coupling effect is known as a magnetoacoustic effect. A number of monographs and almost infinitely many articles deal with this problem, and here are some of them [1–17].

In 1961, Damon R W and Eshbach J R [4] established the existence of a pure magnetostatic spin surface wave in ferromagnets, latter Maugin G A [5] and Hakmi A [6] proved the existence of shear magnetoelastic waves of Bleustein–Gulyaev-type in ferromagnets and obtained conditions under which the surface mode of these waves can be forced to propagate in a ferromagnetic medium.

Interest in these waves was caused by their great practical importance in engineering. Let us list some specific technical devices, where these waves find application: supersonic and hypersound generators, high-frequency magnetostrictive transducers, wave and frequency filters, delay lines, sensors and actuators for smart structures, radio transmitters, seismic devices, etc. Some of these devices operate due to conversion of the fluctuation energy of magnetic spin into the mechanical energy.

The coupled magnetoelastic problems are studied using equations of mechanical displacements, the Landau–Lifshits equations of motion of a magnetic moment, and Maxwell’s equations of a magnetic field.

This paper studies the reflection and refraction of elastic shear waves incident from a nonmagnetic elastic medium on the surface of a ferromagnetic half-space bordering on a nonmagnetic medium.
2. Statement of the problem

We consider a structure consisting of two adjacent half-spaces one of which is not electromagnetically active and the other is ferromagnetic (figure 1). The elastic properties of both media are taken into account.

In figure 1, $G_1$, $\rho_1$ and $G$, $\rho$ denote the shear moduli and the material densities of both media. It is assumed that an elastic shear wave is incident on the surface of the ferromagnetic half-space from the nonmagnetic medium ($y < 0$), where the displacement $w_1$ is directed along the axis $Oz$ (the axis $Oz$ is perpendicular to the plane of the figure).

The equation of mechanical motion in the nonmagnetic medium ($y < 0$) has the form

$$\frac{\partial^2 w_1}{\partial t^2} = c_{11}^2 \Delta w_1, \quad c_{11}^2 = \frac{G_1}{\rho_1}. \quad (1)$$

We assume that the magnetoelectric half-space ($y > 0$) is in a constant magnetic field $\vec{H}_0 = (0, 0, H_0)$. The saturation magnetization $\vec{M}_0$ is also directed along the axis $Oz$. The system comprising the equations of the mechanical displacement $w$ in a ferromagnetic medium, the Landau–Lifshits equations of motion of a magnetic moment $\vec{\mu}$ ($\mu, \nu, 0$), and Maxwell’s equations of a magnetic field becomes [1, 5, 7]

$$\frac{\partial^2 w}{\partial t^2} = c_t^2 \Delta w + M_0 f \left( \frac{\partial \mu}{\partial x} + \frac{\partial \nu}{\partial y} \right),$$

$$\frac{\partial \mu}{\partial t} = \omega_M \left( \frac{1}{\rho} \frac{\partial \varphi}{\partial y} + \tilde{b} \nu + \tilde{b}_\mu \frac{\partial w}{\partial y} - \lambda \Delta \nu \right),$$

$$\frac{\partial \nu}{\partial t} = -\omega_M \left( \frac{1}{\rho} \frac{\partial \varphi}{\partial x} + \tilde{b} \mu + \tilde{b}_\mu \frac{\partial w}{\partial x} - \lambda \Delta \mu \right),$$

$$\Delta \varphi = \rho \left( \frac{\partial \mu}{\partial x} + \frac{\partial \nu}{\partial y} \right). \quad (2)$$

where $c_t^2 = G\rho$, $\tilde{b} = b + H_0/M_0$, $\tilde{b} = b + f$, $H_0$ is the intensity of the external magnetic field, $M_0$ is the bulk density of the saturation magnetization of the ferromagnetic medium, $b$ and $f$ are the coefficients of the piezomagnetic coupling, $\lambda$ is an exchange constant, $\omega_M = \gamma M_0$, $\gamma$ is the gyromagnetic ratio, $\mu_0 = \rho^{-1} M_0$ is the mass density of the saturation magnetization, $w$ is the displacement, $\mu$ and $\nu$ are the respective components of the magnetization vector $\vec{\mu}$ along the coordinate axes $Ox$ and $Oy$, and $\varphi$ is the magnetostatic field potential.

In what follows, we do not take the excitation of the magnetostatic field and the exchange
interaction into account; in this case, system (2) becomes

\[
\begin{align*}
\frac{\partial^2 w}{\partial t^2} &= c_t^2 \Delta w + M_0 f \left( \frac{\partial \mu}{\partial x} + \frac{\partial \nu}{\partial y} \right), \\
\frac{\partial \mu}{\partial t} &= \omega_M \left( \dot{b}_\nu + \dot{b}_\mu \frac{\partial w}{\partial y} \right), \\
\frac{\partial \nu}{\partial t} &= -\omega_M \left( \dot{b}_\mu + \dot{b}_\mu \frac{\partial w}{\partial x} \right).
\end{align*}
\]

(3)

The continuity conditions of the displacements and stresses on the boundary \(y = 0\) are written as

\[
w_1 \big|_{y=0} = w \big|_{y=0}, \quad \sigma^{(1)}_{yz} \big|_{y=0} = \sigma_{yz} \big|_{y=0}, \quad \text{(4)}
\]

where \(\sigma_{yz} = G_1 \partial w / \partial y\), \(\sigma_{yz} = G \partial w / \partial y + \dot{b} M_0 \nu\), \(w_1 = w_{1n} + w_{10}\); \(w_{1n}\) and \(w_{10}\) are the incident and reflected waves, respectively \([1, 7, 9]\).

The problem is to prove the existence of reflected and refracted waves, and to find the dependence of their parameters on the parameters of the incident wave.

3. The solution of the problem

It follows from equation (1) that the incident \(w_{1n}\) and reflected \(w_{10}\) waves are determined by the relations

\[
\begin{align*}
w_{1n} &= Ae^{i(\omega t - k_1 x - k_2 y)}, \\
w_{10} &= Be^{i(\omega t - k_1 x + k_2 y)}, \quad \text{(5)}
\end{align*}
\]

where \(k_1\) and \(k_2\) are components of the incident and reflected wave vectors and \(k_2^2 = \omega^2/c_t^2 - k_1^2\), \(A\), and \(B\) are constants.

We seek the refracted elastic-spin wave in a magnetic medium \(y > 0\) in the form

\[
\begin{align*}
w &= Ce^{i(\omega t - k_1 x - k_3 y)}, \\
\mu &= Me^{i(\omega t - k_1 x - k_3 y)}, \\
\nu &= Ne^{i(\omega t - k_1 x - k_3 y)}, \quad \text{(6)}
\end{align*}
\]

where \(C\), \(M\), and \(N\) are constants and \(k_3\) is a component of the wave vector of the refracted wave. In (6), we take into account the fact that the projections of the wave vectors of all three waves on the boundary must be the same, so that the boundary conditions are satisfied for all \(t\) and \(x\) \([8]\).

We substitute solutions (6) in system (3) to obtain

\[
\begin{align*}
[\omega^2 - c_t^2(k_1^2 + k_3^2)]C - ifM_0(k_1 M + k_3 N) &= 0, \\
i\omega M - \dot{b}_\omega M N + i k_3 \dot{b}_\mu \omega M C &= 0, \\
i\omega N - \dot{b}_\omega M M - i k_1 \dot{b}_\mu \omega M C &= 0.
\end{align*}
\]

(7)

Equating the determinant of system (7) to zero, we arrive at the characteristic equation of system (7)

\[
\begin{vmatrix}
-ifM_0 k_1 & -ifM_0 k_3 & \omega^2 - c_t^2(k_1^2 + k_3^2) \\
i\omega & -\dot{b}_\omega M & i k_3 \dot{b}_\mu \omega M \\
\dot{b}_\omega M & i\omega & -i k_1 \dot{b}_\mu \omega M \\
\end{vmatrix} = 0.
\]
After simple transformations, we obtain the relationship between the components of the refracted wave vector

$$k_1^2 + k_3^2 = \frac{\omega^2(\omega^2 - b^2\omega_M^2)}{f\tilde{b}\mu_0\omega_M^2M_0 + c_i(\omega^2 - b^2\omega_M^2)}.$$  \hspace{1cm} (8)

Further, we use system (7) to find the relationship between the amplitudes of the magnetization components and the amplitude of the elastic wave

$$M = \alpha C, \quad N = \beta C,$$  \hspace{1cm} (9)

where

$$\alpha = \frac{\omega M\tilde{b}\mu_0(\omega k_3 + ik_1\tilde{b}\omega_M)}{b^2\omega_M^2 - \omega^2}, \quad \beta = \frac{\omega M\tilde{b}\mu_0(\tilde{b}k_3\omega_M - k_1\omega)}{b^2\omega_M^2 - \omega^2}.$$

Formulas (9) show that the elastic oscillations in a ferromagnetic medium excite magnetic (spin) oscillations and vice versa, and the excitation of oscillations has a resonance character. In the literature, this phenomenon is called magnetoacoustic resonance. It follows from (9) that resonance occurs in our case at \(\omega = \tilde{b}\omega_M\).

We note that substituting (9) into the first equation of system (7), we again arrive at relation (8).

Now let us determine the dependence of the amplitudes of reflected and refracted waves on the amplitude of the incident wave. To this end, we use the boundary conditions, substitute solutions (5) and (6) in (3), and, after simplifying transformations, obtain

$$B = \frac{k_2G_1 - k_3G + \tilde{b}M_0\beta}{k_2G_1 - k_3G - bM_0\beta}A,$$

$$C = \frac{2k_2G_1}{k_3G + k_2G_1 - bM_0\beta}A.$$  \hspace{1cm} (10)

For the amplitudes of the magnetization components, we have

$$M = \frac{2\alpha k_2G_1}{k_3G + k_2G_1 - bM_0\beta}A,$$

$$N = \frac{2\beta k_2G_1}{k_3G + k_2G_1 - bM_0\beta}A.$$  \hspace{1cm} (11)

If we equate the amplitude of the reflected wave to zero, we get the transparency condition for the boundary of our structure

$$k_2G_1 - k_3G + \tilde{b}M_0\beta = 0.$$

In the particular case where the wave is incident perpendicularly to the surface \((k_1 = 0)\), from (8) we derive

$$k_3^2 = \frac{\omega^2(\omega^2 - b^2\omega_M^2)}{f\tilde{b}\mu_0\omega_M^2M_0 + c_i(\omega^2 - b^2\omega_M^2)}.$$  \hspace{1cm} (9)

Conclusions

Thus, a pure elastic shear wave incident from a nonmagnetic half-space on the interface between two media, one of which is magnetically ordered, is generally partially reflected as an elastic wave and partially refracted exciting an interrelated elastic-spin wave in the ferromagnetic medium. Relations (10) and (11) determine the relationship between the amplitudes of these waves.

Elastic oscillations generate magnetic (spin) oscillations in ferromagnetic media, and the oscillation excitation has a resonance character.
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