Experimental studies of QCD using flavour tagged jets from DELPHI

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Identified $b\bar{b}q$ and $q\bar{q}\gamma$ events from DELPHI are used to measure the ratio of the mean charged particle multiplicity distribution between gluon and quark jets. The dependence of this ratio with the jet energy is established using about three million $Z^0$ decays. Results from all other detectors are discussed and compared. A nice agreement is found among all them. The ratio between the normalized total three-jet cross sections of $b\bar{b}g$ and $q\bar{q}g$, $q \equiv u, d, s$, events is also determined. The preliminary value obtained indicates that $b$ quarks are experimentaly seen to radiate less than light quarks due to their higher mass. The suggested experimental error is $\sim 300$ MeV for the $b$ mass determination at the $M_Z$ scale.

1. INTRODUCTION

In Quantum Chromodynamics (QCD), quarks ($q$) and gluons ($g$) are coloured objects that carry different colour charges. Quarks have a single colour index while gluons are tensor objects carrying two colour indices. Due to this fact, quarks and gluons differ in their relative coupling strength to emit additional gluons, and, in consequence, jets originating from the fragmentation of energetic quarks and gluons are expected to show differences in their final particle multiplicities, energies and angular distributions.

The masses of quarks are also fundamental parameters of the QCD lagrangian not predicted by the theory. The definition of the quark masses is however not unique because quarks are not free particles and various scenarios are possible. The perturbative pole mass, $M_q$, and the running mass, $m_q$, of the $\overline{MS}$ scheme are among the most currently used. At first order in $\alpha_s$ the predicted expression for an observable is not able to resolve the mass ambiguity and, only when second or higher order terms are included, the mass definition becomes known. At orders higher than one the renormalization scheme used as the baseline of the calculation has to be chosen and this contains the information about the mass definition. Earlier calculations of the three-jet cross section in $e^+e^-$ including mass terms already exist at $O(\alpha_s)$ [1,2] and have been used to evaluate mass effects for the $b$-quark when testing the universality of the strong coupling constant, $\alpha_s$. They could not however be used to evaluate the mass of the $b$-quark, $m_b$, because these calculations are ambiguous in this parameter. Recently, expressions at $O(\alpha_s^2)$, for the multi-jet production rate in $e^+e^-$ are available [3] and, thus, they enable measuring $m_b$ in case the flavour independence of $\alpha_s$ is assumed and enough experimental precision is achieved.

There are well known existing difficulties to measure all the above parameters in quantitative agreement with the predictions from perturbative QCD, since partons, quarks and gluons, are not directly observed in nature and only the stable particles, produced after the fragmentation process, are experimentally detected. However, the massive statistics and improved jet tagging techniques available at LEP presently allow overcoming these difficulties by applying restrictive selection criteria which lead to quark and gluon jet samples with high purities. The selected data samples are almost background free and small corrections to account for impurities are needed. A smaller model dependence than ever is now achieved, bringing the possibility to perform quantitative studies of quark and gluon fragmentation according to perturbative QCD.

The analyses reported in here include more
than 3 million \( Z^0 \) decays as collected by DELPHI at center-of-mass energies of \( \sqrt{s} \approx M_Z \). In the first analysis, the ratio between the gluon jet multiplicity and the quark jet multiplicity, \( r = \langle N_g \rangle / \langle N_q \rangle \), is presented and discussed in comparison with other detector results. In the second study, preliminary values of errors associated to the determination of \( m_b \) at the \( M_Z \) scale are given.

### 2. EVENT SELECTION

Gluon and quark jets were selected using hadronic three-jet events. Jets were mainly reconstructed using the DURHAM algorithm although the JADE algorithm was also used \(^4\), in particular, to observe the effects due to different angular particle acceptance of the various algorithms.

In the gluon splitting process \( (g \rightarrow q\bar{q}) \), the heavy quark production is strongly suppressed \(^8\). Gluon jets can thus be extracted from \( q\bar{q}g \) events by applying \( b \) tagging techniques. The two jets which satisfy the experimental signatures of being initiated by \( b \) quarks are associated to the quark jets and the remaining one, by definition, assigned to be the gluon jet without any further requirement. Algorithms for tagging \( b \) jets exploit the fact that the decay products of long lived \( B \) hadrons have large impact parameters and/or contain inclusive high momentum leptons coming from the semileptonic decays of the \( B \) hadrons. Gluon purities of 94% and 85% are achieved when using these techniques, respectively. Obviously, the quark jets belonging to these events cannot be used to represent an unbiased quark sample. Thus the quark jets whose properties are to be compared with the gluon jets must be selected from other sources which in any case should preserve the same kinematics. Two possibilities have been proposed in the current literature. One consists in selecting symmetric three-jet event configurations \(^3\) in which one \((Y)\) or the two \((\text{Mercedes})\) quark jets have similar energy to that of the gluon jet. The quark jet purities reached are \( \sim 52\% \) and \( \sim 66\% \), for \( Y \) and for \( \text{Mercedes} \) events, respectively. In a second solution \(^3\) radiative \( q\bar{q}\gamma \) events are selected, allowing a sample of quark jets with variable energy to be collected. In this latter case, misidentifications of \( \gamma \)'s due to the \( \pi^\pm \) background and radiative \( \tau^+\tau^-\gamma \) contamination give rise to quark jet purities of \( \sim 92\% \). This method gives a higher purity but unfortunately suffers from the lack of statistics.

The \( b \)-quark purity in the \( bbq \) sample reached in the DELPHI analyses is \( \sim 93\% \) and for the light \( uds \)-quarks is \( \sim 80\% \). Table 1 summarizes the number of events selected and their corresponding energy intervals.

### 3. MULTIPLEITIES OF QUARK AND GLUON JETS

Results on the charged multiplicity of quark and gluon jets \(^3\) using symmetric \( Y \) configurations and reconstructed with DURHAM at 24 GeV gluon jet energy, give a ratio of \( r \approx 1.23 \pm 0.04 \text{(stat.} + \text{syst.)} \) which does not depend on the cut-off parameter \( (y_{\text{cut}}) \) selected to reconstruct jets \(^3\). It is significantly higher than one, which indicates that quark and gluons in fact fragment differently, but it remains far from the asymptotic lowest order expectation of \( C_F/C_A = 9/4 \), suggesting that higher order corrections and non-perturbative effects are very important to understand the measured value. A next-to-leading order correction \(^9\) in MLLA (Modified Leading Log Approximation) at \( O(\sqrt{\alpha_S}) \) already lowers the prediction towards \( r \) values slightly below two and exhibits a small energy dependence due to the running of \( \alpha_S \). However this is still insufficient to explain the value of \( r \) determined by the experiments. Solutions based on the Monte Carlo
method give a better approximation [5]. The parton shower option of the Jetset generator [10] which uses the Altarelli-Parisi splitting functions for the evolution of the parton shower reduces the theoretical prediction [4] for \( r \). At parton level, at 24 GeV jet energy, the expected value is \( \sim 1.4 \) and it is further reduced to \( \sim 1.3 \) if the value of \( r \) is computed after the fragmentation process. In both cases there is a clear dependence of \( r \) with the jet energy [4] which can be parametrized using straight lines with slopes of \( \Delta r/\Delta E = (+90 \pm 3 \text{ (stat.)}) \cdot 10^{-4} \text{ GeV}^{-1} \) at parton level and \( \Delta r/\Delta E = (+76 \pm 2 \text{ (stat.)}) \cdot 10^{-4} \text{ GeV}^{-1} \) after fragmentation. The absolute value of \( r \) predicted at parton level is however largely affected by the choice on the \( Q_0 \) parameter (cut-off at which the parton evolution stops) but has negligible influence on its relative variation with the energy, i.e., the slope. The DELPHI analysis uses symmetric and non-symmetric three-jet event configurations with quark and gluon jets of variable energy, allowing thus all these properties and predictions to be tested. A value of \( r = 1.23 \pm 0.03 \) (stat.+syst.) is measured corresponding to an average jet energy of \( \sim 27 \text{ GeV} \).

The energy dependence of \( r \) is also suggested at \( 4\sigma \) significance level, with a fitted slope of \( \Delta r/\Delta E = (+104 \pm 25 \text{ (stat.+syst.)}) \cdot 10^{-4} \text{ GeV}^{-1} \).

In a recent review [11] all published data from various experiments [4,5,6,7,13,14] were used to perform a general study of \( r \) as a function of the jet energy. At present, more data can be added to this comparison. These are the new analysed DELPHI data sample presented above and the most recent measurements of \( r \) performed by CLEO [12] and OPAL [16] at 4-7 GeV and 39 GeV average jet energies, respectively. The updated new DELPHI analysis incorporates two times more statistics than the previous analysis [4], therefore, significantly reduces the statistical errors. The analysis from CLEO compares the charged particle multiplicity in \( \Upsilon(1S) \rightarrow gg\gamma \) decays to that observed in \( e^+e^- \rightarrow q\bar{q}\gamma \) just in the continuum. This study does not rely on the Monte Carlo simulation to associate the final hadrons to the initial partons and can consequently be fairly considered as being model independent. The obtained value is \( r = 1.04 \pm 0.05 \) (stat.+syst.). The OPAL analysis uses a new technique [17] which selects gluon jets at \( \sim 39 \text{ GeV} \) by dividing the events into two hemispheres. While one of these hemispheres is required to contain two tagged quark jets, the other is left untouched being regarded as the gluon jet. The result from OPAL, expressed for only light \( uds \)-quarks, is \( r_{uds} = 1.55 \pm 0.07 \) (stat.+syst.). As it can be observed in figure [6,b] all these data agree with the predicted energy behaviour of [12,13] when the correction to the quark multiplicity to account for
the same flavour composition is applied. In our case it is: 56% \(uds\)'s, 33% \(c\)'s and 11% \(b\)'s. The OPAL number considering this quark mixture becomes \(r = 1.48 \pm 0.07\) (stat.+syst.).

All these results thus give evidence for an energy dependence of \(r\). The measured increase is

\[
\frac{\Delta r}{\Delta E} = (+110 \pm 13 \text{ (stat.+syst.)}) \cdot 10^{-4} \text{ GeV}^{-1},
\]

representing a \(\sim 8\sigma\) effect.

The measured value of \(r\) remains systematically lower than the JETSET prediction over the whole energy range, having an average value of

\[r = 1.23 \pm 0.01 \text{ (stat.)} \pm 0.03 \text{ (syst.),}\]

which corresponds to an average energy of \(\sim 23\) GeV. This ratio can be further expressed as

\[r_{uds} = 1.30 \pm 0.01 \text{ (stat.)} \pm 0.04 \text{ (syst.),}\]

if \(r\) is computed only for the light \(uds\)-quarks, extracting the \(b\) and \(c\) quark contribution to the quark jet multiplicity.

The absolute value of \(r\) depends on the reconstruction jet algorithm. For both the JADE and CONE schemes different results are obtained w.r.t. the DURHAM scheme. This is due to the combined effect of the different sensitivity of the various jet reconstruction algorithms to soft particles at large angles and of the expected different angular and energy spectra of the emitted soft gluons in the quark and gluon jets. A precise deconvolution of both effects is, at present, impossible. This jet algorithm dependence of \(r\) becomes however less apparent as the jet energy increases. The results from OPAL \([16]\), \(r = 1.48 \pm 0.07\) (stat.+syst.) and those from DELPHI \([6]\) at \(\sim 40\) GeV presented in this conference, \(r = 1.43 \pm 0.07\) (stat.+syst.) for DURHAM and \(r = 1.52 \pm 0.11\) (stat.+syst.) for JADE, agree within errors for the various methods and algorithms used. For the low energy interval, the JADE and DURHAM jet algorithms give a different description of the gluon jet properties \([4]\), although the DURHAM algorithm is in better agreement to those, model independent, results obtained by CLEO. Hence, the DURHAM jet algorithm seems to be better suited to describe the intermediate energy region than the JADE algorithm is.

The interpretation of these results in combination with those obtained by OPAL \([19]\) and ALEPH \([20]\) restrict the validity of the statement that gluon and \(b\)-quark jets have similar properties to the jet energy interval around 24 GeV and cannot be applied to the whole jet energy spectrum.

4. GLUON RADIATION IN \(b\)-QUARKS

For many observable quantities at LEP energies, \(\sqrt{s} \approx M_Z\), quark mass effects usually appear in terms proportional to \(m^2_{q}/M^2_Z\). This represents a \(\sim 3\%\) correction for \(m_q = m_b\) which in most of the cases can be safely neglected. This argument, for instance, is true for the total hadronic cross section \([2]\) but cannot be applied for the differential multi-jet cross sections that depend on the jet-resolution parameter, \(y_c\). The reason being the new scale, \(E_c = M_Z \sqrt{y_c}\), introduced in the analysis by the new variable which enhances the mass effects in the form \(m^2_b/E^2_c = (m^2_b/M^2_Z)/y_c\).

At \(\sqrt{s} \approx M_Z\) the three-jet production rate for \(b\)-quarks is in fact suppressed by a factor \(\sim 5-10\%\) w.r.t. that of light quarks \([1,2,21]\). This difference can then be expressed as a function of \(m_b\) and, therefore, is used to measure its value.

The experimental observation of such effects is however difficult and delicate because the effect is after all small and furthermore the correct theoretical framework to resolve the mass definition ambiguities is needed. This means that the observable has to be calculated including mass effects at \(O(\alpha^2_s)\). For this purpose a recent calculation \([3]\) of the ratio of the normalized three-jet cross sections between \(b\)-quarks and light \(uds\)-quarks

\[
R^{b\bar{d}}_3 = \frac{\Gamma^{Z^0 \rightarrow b\bar{b}g}(y_c)/\Gamma^{Z^0 \rightarrow \text{tot}b\bar{b}}}{\Gamma^{Z^0 \rightarrow d\bar{d}g}(y_c)/\Gamma^{Z^0 \rightarrow \text{tot}d\bar{d}}}
\]

has been performed.

The normalization in \(R^{b\bar{d}}_3\) to the total decay rates is introduced to cancel possible weak mass corrections depending on the top quark mass \([22]\) and the ratio of the three-jet cross sections be-
between $b$ and light $uds$-quarks minimizes uncertainties due to the hadronization process. In figure 2 the dependence of these uncertainties w.r.t the $\gamma_{\text{cut}}$ is shown and seen not to exceed $3\%$ for large enough values of $\gamma_{\text{cut}}$. The $R_3^{bd}$ distribution corrected for detector and fragmentation effects is also displayed in figure 3. The solid curves drawn in the figure are the theoretical $O(\alpha_s)$ prediction in steps of 1 GeV. The values of $m_b$ used to produce these curves are meaningless since they correspond to a calculation at $O(\alpha_s^2)$. They can nevertheless be used to evaluate the experimental precision assuming the difference between the theoretical curves remains similar to that at $O(\alpha_s^2)$. As can be observed the experimental error corresponds then to approximately 300 MeV for reasonably high values of $\gamma_{\text{cut}}$.

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