Drag force with different charges in STU background and $AdS/CFT$

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Abstract

In this paper we consider the drag force problem in the STU model. Already this problem and related topics were studied in $\mathcal{N} = 4$ SYM theory. Here we study the same problem in $\mathcal{N} = 2$ supergravity theory with an R-charge black hole (the STU model). This paper extends our previous work [1,2] by studying more general charge configurations and by taking into account the effect of a constant electromagnetic field and higher derivative corrections. We consider a single quark and a $q\bar{q}$ pair in a three charge non-extremal black hole background. First by using the AdS/CFT correspondence, we obtain the drag force, the diffusion coefficient, the total energy and momentum, quasi-normal modes, and the effect of higher derivative corrections for the single quark. Next we calculate the drag force for a $q\bar{q}$ pair which rotates around its center of mass. For the first time we study the motion of a rotating $q\bar{q}$ pair in this background. We show that our results in the near-extremal limit and finit chemical potential agree with the energy loss of the moving quark which was calculated in the $\mathcal{N} = 4$ SYM plasma.

Keywords: AdS/CFT correspondence; STU model; String theory; Drag Force

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1 Introduction

The formulation of QCD as a string theory plays an important role in theoretical physics. In the other hand the AdS/CFT correspondence [3-9] prepares useful methods for the studying QCD in the strong coupling. Recently, the problem of QCD from AdS/CFT correspondence is discussed by several papers. This problem is the motion of a heavy charged particle through a thermal medium. Already this problem considered in the $\mathcal{N}=4$ super Yang-Mills (SYM) thermal plasma and well studied in many papers such as [10-16], the basis of these papers was a toy model. In the CFT side there is a moving quark through the $\mathcal{N}=4$ SYM thermal plasma with the momentum $P$, mass $m$ and constant velocity $v$, which is influenced by an external force $F$. So, one can write the equation of motion as $\dot{P} = F - \mu P$, where in the non-relativistic motion $P = mv$ and in the relativistic motion $P = \frac{mv}{\sqrt{1 - v^2}}$, also $\mu$ is called friction coefficient. In that case, one can find the friction coefficient $\mu$ as a velocity independent [14] or velocity dependent [11]. In order to obtain drag force, one can consider two special cases. The first case is the constant momentum ($\dot{P} = 0$), so for the non-relativistic motion one can obtain $F = (\mu m)v$. In this case the drag force coefficient $(\mu m)$ will be obtained. In the second case, external force is zero, so one can find $P(t) = P(0)exp(-\mu t)$. In other words, by measuring the ratio $\frac{\dot{P}}{P}$ or $\frac{\dot{v}}{v}$ one can determine friction coefficient $\mu$ without any dependence to mass $m$. These methods lead us to obtain the drag force for a moving quark in the thermal plasma. In the other hand in AdS side there is type II string theory in AdS$_5 \times$ S$^5$ space. So, instead of the quark in the gauge theory, there is an open string in AdS space which stretched from D-brane to black hole horizon.

By using AdS/CFT correspondence the rate of energy loss of a heavy moving quark through the $\mathcal{N}=4$ SYM thermal plasma at large 't Hooft coupling are determined by [10-16]. Also there are many calculations to obtain the jet-quenching parameter [7-24]. The jet-quenching parameter of quarks is one of the interesting properties of the strongly-coupled plasma. The jet-quenching parameter controls the description of relativistic heavy quarks. Above considerations may be generalized to the case of a quark-antiquark pair which moves with the constant velocity $v$ through a strongly-coupled thermal $\mathcal{N}=4$ SYM plasma [25-30]. Ref. [27] found the $q\bar{q}$ pair feels no drag force. Actually the $q\bar{q}$ pair may have more degrees of freedom such as the rotational motion and oscillation along the connection axis. In the Refs. [28, 30] the description of $q\bar{q}$ system instead single quark well explained. Also the problem of spining open string (meson) in description of non-critical string/gauge duality considered [31]. In that paper the relation between the energy and angular momentum of spining open string for the Regge trajectory of mesons in a QCD-like theory is studied [32, 33]. In this paper we would like to add rotational motion to the $q\bar{q}$ pair and calculate drag force. In that case our method is different with Refs. [31, 34]. Then, it is possible to consider higher derivative corrections to the black hole solution. In that case the effect of curvature-squared corrections on the drag force of moving heavy quark in the $\mathcal{N}=4$ SYM plasma is considered [35] and found that curvature-squared corrections may increases or decreases the value of drag force on the quark.
The main subject of this work is to study moving quark and quark-antiquark pair through $\mathcal{N}=2$ supergravity plasma in the general form, i.e. five dimensional black hole with three $U(1)$ charges. We will begin with free string in the AdS space and obtain drag force. This force, in the absence of any external field, decreases the velocity of string, so, after long time, the motion of string will be small fluctuations around static string. But in the presence of appropriate external field (for example, constant electromagnetic field) string can remain in constant speed. In this paper, we consider a general STU model with non-zero charges which is the $\mathcal{N}=2$ supergravity theory and has non-extremal black hole. The solutions of $\mathcal{N}=2$ supergravity may be solutions of supergravity theory with more supersymmetry such as $\mathcal{N}=4$ and $\mathcal{N}=8$. Indeed, $\mathcal{N}=8$ supergravity correspond to the dual $\mathcal{N}=4$ SYM. Also $\mathcal{N}=4$ supergravity correspond to $\mathcal{N}=2$ SCFT [36, 37], and $\mathcal{N}=2$ supergravity correspond to $\mathcal{N}=1$ SCFT. Already the duality between gravity and $\mathcal{N}=2$ gauged theory investigated [38, 39]. The $\mathcal{N}=2$ supergravity theory in five dimensions can be obtained by compactifying eleven dimensional supergravity in a three-fold Calabi-Yau [40]. In order to consider effect of $R^2$-term in the curvature tensor and higher derivative corrections to the $\mathcal{N}=2$ supergravity theory we used from results of Refs. [41, 42], and obtain modified drag force. It is important to note that, above analysis might be invalid in the lower dimension with higher derivative effective action without consideration of proper Kaluza-Klein reductional dimension [43].

The aim of this paper is to study the drag force problem in STU model with three-charge non-extremal black hole (with different non-zero charges). Already we studied the drag force on moving quark at the $\mathcal{N}=2$ supergravity and STU model in two papers [1, 2]. In the first paper [1], we considered a moving quark in the thermal plasma at the $\mathcal{N}=2$ supergravity theory and obtained the drag force on the quark and quasi-normal modes of the string. Also, we calculated the effect of higher derivative terms and existence of external NSNS B-field on the drag force. In that paper we found the drag force problem in the $\mathcal{N}=2$ supergravity theory at the $\eta \to 0$ limit correspond to drag force problem in the $\mathcal{N}=4$ SYM plasma, where $\eta$ is the non-extremality parameter. Then in the second paper [2], we considered a moving quark in $D = 5$, $\mathcal{N}=2$ supergravity plasma and used three-charge non-extremal black hole solution (STU solution) with only one non-zero charge and calculated drag force on the quark and quasi-normal modes of the string.

Now, we generalize our previous papers to the case of three non-zero charges ($q_1 \neq q_2 \neq q_3 \neq 0$) and calculate the drag force on the quark-antiquark pair with linear motion with constant speed $v$ and circular motion with angular velocity $\omega$. In that case if we set $q_1 = q$ and $q_2 = q_3 = 0$ our problem reduces to the Ref. [2] and if we set $q_1 = q_2 = q_3 = q = \eta \sinh^2 \beta$ then our problem reduces to the Ref. [1].

The remaining part of the paper is organized as following. In the section 2 we briefly review the STU model and calculating method of the drag force in three-charge non-extremal black hole background. Then in the section 3 we calculate the drag force for single quark moving through $\mathcal{N}=2$ thermal plasma. The quasi-normal modes of the curved string and corresponding energy and momentum are obtained in the section 4. In the section 5 the effect of constant electromagnetic field, as the external field, on the drag force is discussed.
In section 6, first we consider the quark-antiquark pair moving with constant speed and then add a rotating motion to obtain drag force for these configurations. Finally, in the section 7 we find effect of higher derivative correction to the drag force for the single quark solution, and in the section 8 we summarize our results and giving several suggestions for the future works.

2 STU Model Solution and Drag Force

The STU model is just some $\mathcal{N}=2$ supergravity, which has generally 8-charge (4 electric and 4 magnetic) non-extremal black hole. But there are many situations with less than charges such as four-charge and three-charge black hole. In that case there is great difference between the three-charge and four-charge black hole. For example if there are only 3 charges the entropy vanishes (except in the non-BPS case). So, one really needs four charges to get a regular black hole. In 5 dimensions the situation is different and actually much simpler, there is no distinction between BPS and non-BPS branch. So, in 5 dimensions the three-charge configurations are the most interesting ones [44]. Therefore, we begin with the three-charge non-extremal black hole solution in $\mathcal{N}=2$ gauged supergravity which is called STU model and described by the following solution [39, 45],

\[ ds^2 = -\frac{f_k}{\mathcal{H}^3} dt^2 + \mathcal{H}^3 \left( \frac{dr^2}{f_k} + \frac{r^2}{L^2} dx^2 \right), \]

(1)

where,

\[ f_k = k - \frac{m_k}{r^2} + \frac{r^2}{L^2} \mathcal{H}, \]

\[ \mathcal{H} = \prod_{i=1}^{3} H_i, \]

\[ H_i = 1 + \frac{q_i}{r^2}, \quad i = 1, 2, 3, \]

(2)

where $L$ is the constant $AdS$ radius and $r$ is radial coordinate along the black hole, so the boundary of $AdS$ space is at $r \to \infty$ (on the D-brane). And the black hole horizon specified by $r = r_h$ which is obtained from $f_k = 0$. In STU model there are three real scalar fields as $X^i = \frac{H_i}{\mathcal{H}}$ which satisfy $\prod_{i=1}^{3} X^i = 1$. For the three R-charges $q_i$ in the equation (2) there is an overall factor such that $q_i = \eta \sinh^2 \beta_i$, where $\eta$ is non-extremality parameter and $\beta_i$ are related to the three independent electric charges of the black hole. Finally, the factor of $k$ indicates the space curvature, so the metric (1) includes a $S^3$ (three dimensional sphere) for $k = 1$, a pseudo-sphere for $k = -1$ and a flat space for $k = 0$. The metric (1) written with the assumption that the motion is only on the transverse axis which is specified by $x$. We would like to consider a moving charged particle through the thermal plasma in the above
background. In the CFT side there are complicated calculations to obtain drag force on charged particle. Therefore we are going to the AdS side. In that case string theory give us useful mathematical tools to calculate the drag force. According to the Maldacena dictionary we have stretched string from D-brane to the black hole horizon in the AdS space. The end point of string on D-brane represents the charged particle. Also from string theory it is known that the temperature in the supersymmetric gauge theory is equal to the existence of a black hole (black brane) with a flat horizon in the center of the AdS space.

In the dual picture of QCD, instead charged particle, we have an open string which is described by the Nambu-Goto action,

\[ S = -\sqrt{-g} = -T_0 \int d\tau d\sigma \sqrt{-g}, \]

where \( T_0 \) is the string tension and \( \tau \) and \( \sigma \) are the string world-sheet coordinates, also \( g \) is determinant of the world-sheet metric \( g_{\mu \nu} \). We use static gauge, where one fixes \( \tau = t \) and \( \sigma = r \). Therefore the string world-sheet is described by \( x(r, t) \), so in order to write lagrangian density one can find,

\[ -g = \frac{1}{\mathcal{H}^2} \left[ 1 - \frac{\mathcal{H} r^2}{f_k L^2} x'^2 + \frac{f_k r^2}{L^2} x'^2 \right], \]

where dot and prime denote \( t \) and \( r \) derivative respectively. Also the lagrangian density is given by \( \mathcal{L} = -T_0 \sqrt{-g} \). Then, by using Euler-Lagrange equation one can obtain the string equation of motion as the following expression,

\[ \frac{\partial}{\partial r} \left( \frac{f_k r^2}{\mathcal{H}^2} x' \right) = \frac{\mathcal{H} r^2}{f_k} \frac{\partial}{\partial t} \left( \frac{x'}{\sqrt{-g}} \right), \]

where \( \sqrt{-g} \) is given by square of the relation (4). In order to obtain the total energy and momentum, drag force or energy loss of particle in the thermal plasma, we have to calculate the canonical momentum densities. In that case one can obtain the following expressions,

\[ \left( \begin{array}{c} \pi^0_x \\ \pi^1_x \\ \pi^0_t \\ \pi^1_t \end{array} \right) = -\frac{T_0}{\mathcal{H}^2 \sqrt{-g}} \left( \begin{array}{cccc} -\frac{\mathcal{H} r^2}{f_k L^2} x' & \frac{f_k r^2}{L^2} x'^2 & 1 & \frac{f_k^2 r^2}{L^2} x'^2 \\ \frac{\mathcal{H} r^2}{f_k L^2} x' & \frac{f_k r^2}{L^2} x'^2 & -\frac{f_k^2 r^2}{L^2} x'^2 & 1 \end{array} \right). \]

After that, by using the relation \( \dot{P} \propto \pi^1_x \) we can obtain the drag force. Here, one can check that above results are agree with Herzog’s works (specially Ref. [10]) at near extremal limit.

In this paper we would like to consider two cases: first, a moving quark, as a charged particle, through the \( \mathcal{N}=2 \) supergravity thermal plasma which is main subject of sections 3-5, and second, a moving quark-antiquark pair in the same background which is considered in the section 6.

For the single quark in CFT side, we have an open string in AdS space which stretched from
$r = r_m$ on D-brane to $r = r_h$ at the horizon. In that case the total energy and momentum of string are obtained by the following integrals,

$$E = - \int_{r_h}^{r_m} \pi^0_t dr,$$

$$P = \int_{r_h}^{r_m} \pi^0_x dr.$$  \hspace{1cm} (7)

We will use above relations for single quark in the next section. Also the Hawking temperature of the black hole solution (1) will be as [45],

$$T_H = \frac{r_h^2 2 + \frac{1}{r_h^2} \sum_{i=1}^3 q_i - \frac{1}{r_h^2} \prod_{i=1}^3 q_i}{2\pi L^2 \sqrt{\prod_{i=1}^3 (1 + \frac{q_i}{r_h^2})}}.$$ \hspace{1cm} (8)

In the following section we will use equation (8) and obtain the diffusion coefficient for the quark and discuss about drag force of the single quark.

### 3 Single Quark Solution

As we know a quark in the $\mathcal{N}=2$ supergravity thermal plasma is equal to the stretched string from $r = r_h$ on D-brane to the black hole horizon. Indeed the D-brane covers an sphere $S^3$ in $S^5$ space which has minimum radius $r_m$, so the string lives on $AdS_5 \times S^5$ space. Therefore in solutions (1) and (2) we take $k = 1$ and interpret $m_1$ as the non-extremality parameter $\eta$ [2], so from equation (2) we have $f(r) = 1 - \frac{\eta}{r^2} + \frac{r^2}{L^2} \mathcal{H}$. Hence from now we set $k = 1$ to have $AdS_5 \times S^5$ space [2].

Here, there are special cases such as $q_2 = q_3 = 0$ and $q_1 = q$ which already discussed [2]. And the case of $q_1 = q_2 = q_3 = \eta \sinh^2 \beta$ and $L^2 = \frac{1}{\Lambda}$, where $\Lambda$ is the cosmological constant, reduce to result of the Ref. [1]. In this paper we take most general case with $q_1 \neq q_2 \neq q_3 \neq 0$.

Now, we are going to discuss about single quark solution and try to obtain drag force. There is the simplest solution for equation of motion (5), namely $x = x_0$, where $x_0$ is a constant and the string stretched straightforwardly from D-brane at $r = r_m$ to the horizon at $r = r_h$. It means that in the dual picture there is a static quark in the thermal plasma. For such configuration one can obtain $-g = (H_1 H_2 H_3)^{-\frac{1}{2}}$ and $\pi_x^0 = \pi_y^0 = \pi_z^1 = \pi_x^1 = 0$, so drag force is zero, as it expected for the static quark. Only non-zero components of momentum density are $\pi_r^1 = \pi_t^0 = -T_0 [\prod_{i=1}^3 (1 + \frac{q_i}{r_h^2})]^{-\frac{1}{2}}$, so total energy of the string is obtained as,

$$E = T_0 \left[ r + \frac{1}{6r} \sum_i q_i + \frac{1}{36r^3} \sum_{i\neq j} q_i q_j + \frac{1}{30r^5} \prod_i q_i \right]_{r_h}^{r_m},$$ \hspace{1cm} (9)
where we assume that the black hole charges \( q_i \) are small. In zero temperature one can interpret \( E \) as the rest mass of the quark, which is obtained by the following expression,

\[
M_{\text{rest}} = T_0 \left[ r_m - \frac{16}{15} r_h + \left( \frac{1}{6r_m} - \frac{1}{5r_h} \right) \sum q_i + \left( \frac{1}{r_m^3} - \frac{1}{r_h^3} \right) \sum_{i \neq j} q_i q_j + \frac{1}{30r_m^5} \prod_i q_i \right].
\]

In STU model there is non-extremal black hole which is described by non-extremality parameter, but in the \( \mathcal{N}=4 \) super Yang-Mills theory there is near-extremal black hole. So, if we take \( \eta \to 0 \) limit, we have near-extremal black hole, then the total energy of string obtained as \( E = T_0 (r_m - r_h) \) \[10\], so, the zero temperature is equal \( r_h = 0 \) and the physical mass of quark (rest mass) become \( M_{\text{rest}} = T_0 r_m \). In the other hand if we set \( q_1 = q_2 = q_3 = q \), the total energy of string is agree with equation (3.10) of Ref. [1] (under assumption of small charge with \( q = \eta \sinh^2 \beta \)).

As we told the total energy of string (9) obtained for static quark. Now we are going to consider most physical time-dependent solution of moving quark through plasma which in dual picture is a curved string described by \( x(r,t) = x(r) + vt \), where \( v \) is the constant velocity of the single quark \[1, 2, 10\]. In that case by using equation of motion (5) one can find,

\[
\frac{f(r)r^2}{L^2 v \mathcal{H}^{\frac{1}{3}} \sqrt{-g} } x' = C,
\]

where \( C \) is an integration constant and \( \sqrt{-g} \) is obtained in the following equation,

\[
-g = \frac{1}{\mathcal{H}^{\frac{1}{3}}} \left[ 1 - \frac{\mathcal{H}r^2}{f(r) L^2 v^2} + \frac{f(r)r^2}{L^2} x'^2 \right].
\]

Therefore it is easy to check that \( \pi^1_t = -\pi^1_x v = T_0 C v^2 \). These constant components of momentum density is obtained already for \( \mathcal{N}=4 \) SYM theory \[1, 10, 11\]. Now, by combining equations (11) and (12) one can obtain,

\[
-g = \frac{r^2 - \mathcal{H} r^2 v^2}{\mathcal{H}^{\frac{1}{3}} f(r)r^2 - C^2 v^2 \mathcal{H}^{\frac{1}{3}}}. \tag{13}
\]

Reality condition for \( \sqrt{-g} \) and therefore \( x' \) tell us that we should choose,

\[
C = \left[ \prod_{i=1}^3 (1 + \frac{q_i}{r_c^2}) \right]^\frac{1}{3} r_c^2, \tag{14}
\]

to have real energy and momentum. In the equation (14) critical radius \( r_c \) is the root of the following equation,

\[
r^6 + Ar^4 + Br^2 + \prod_{i=1}^3 q_i = 0, \tag{15}
\]
where \( A = \sum_{i=1}^{3} q_i + \frac{L^2}{1-L^2v^2} \) and \( B = \frac{1}{2} \sum_{i \neq j} q_i q_j - \frac{\eta L^2}{1-L^2v^2} \). Thus, one can find the drag force on single moving quark as,

\[
\dot{P} = -T_0 \pi_x^1 = -T_0 \left[ \prod_{i=1}^{3} (1 + \frac{q_i}{r_c^2}) \right]^\frac{1}{3} vr_c^2.
\]

(16)

Indeed the equation (16) is the momentum current into the horizon. Here, as already discussed [11], we have field theory interpretation of our system. One can image single quark moving in a constant external field with strength \( \varepsilon = T_0 \left[ \prod_{i=1}^{3} (1 + \frac{q_i}{r_c^2}) \right]^\frac{1}{3} vr_c^2 \). This external field keeps the curved string moving at the constant speed \( v \). We know that electromagnetic field lives on a D-brane on which this dragging string ends. The \( \varepsilon \) changes the boundary conditions for the string. Usually, the string should satisfy Dirichlet boundary conditions orthogonal to the D-brane and Neumann boundary conditions parallel (\( \pi_x = 0 \)). In the presence of \( \varepsilon \), the Neumann boundary conditions can be altered. The fact that \( \pi_x \neq 0 \) for the string solution indicates the presence of an electromagnetic field on the D-brane. We will study this statement in detail in the section 5. Also by using relations \( \dot{P} = -\mu mv, D = \frac{T_0}{\mu m} \), (8) and (16) one can obtain diffusion coefficient \( (D) \) of the quark.

Now, final expression for \( x'(r) \) is,

\[
x'(r)^2 = \alpha \frac{\mathcal{H}(r)^\frac{2}{3}}{f(r)^2 r^4},
\]

(17)

where \( \alpha = v^2 L^2 I^4 \mathcal{H}(r_h)^{\frac{1}{3}} \) is a constant. Then by using equations (6) and (17) we arrive to the following relations,

\[
\pi_t^0 = -\gamma \frac{1 + \frac{\alpha \mathcal{H}(r)^{\frac{2}{3}}}{f(r) r^2 L^2}}{\mathcal{H}(r)^{\frac{1}{3}}},
\]

\[
\pi_x^0 = \gamma v \frac{\mathcal{H}(r)^{\frac{2}{3}} r^2}{f(r)},
\]

(18)

where \( \gamma = T_0 \left( \frac{r_c}{r_h} \right)^2 \mathcal{H}(r_h)^{\frac{1}{3}} \) is another constant. Then by using equations (7) and (18) one can obtain total energy and momentum of the string.

It is interesting to consider the relativistic motion. In that case \( P = \frac{mv}{\sqrt{1-v^2}} \) and therefore friction coefficient \( \mu \) may be expressed as the following,

\[
\mu m = T_0 \left[ \prod_{i=1}^{3} (1 + \frac{q_i}{r_c^2}) \right]^\frac{1}{3} \sqrt{1-v^2 r_c^2}.
\]

(19)

Now in the case \( q_1 = q_2 = q \) and \( q_3 = 0 \) we have,

\[
\mu m = \frac{T_0}{2} \sqrt{1-v^2} (1 + \frac{q}{r_c^2})^3 \left[ 2q + \frac{1}{1-v^2} \pm \sqrt{(2q + \frac{1}{1-v^2})^2 - 4\left( \frac{q^2}{2} - \frac{\eta}{1-v^2} \right)} \right],
\]

(20)
for simplicity we set $L^2 = 1$. Relation (20) for small $q$ reduces to,

$$
\mu m = T_0 q \sqrt{1 - v^2} \left[ 1 + \frac{1}{3(1 - v^2)r_c^2} \right] \left[ 1 \pm 1 + \frac{2\eta(1 - v^2)}{1 + 4q(1 - v^2)} \right] + O(q^2),
$$

(21)

Also in the near-extremal limit $\eta \to 0$ we have $\mu m = \frac{T_0}{\sqrt{1 - v^2}}$ in the relativistic motion and $\mu m = \frac{T_0}{1 - v^2}$ for the non-relativistic motion. In that case we used positive sign in relation (20) because negative sign yield to zero friction coefficient in $\eta \to 0$ limit. Therefore we should take minus sign in relations (20) and (21). These relations show that increasing of $q$ increase the value of $\mu m$, and therefore value of drag force. The drag force coefficient $\mu m$ in the relation (21) has a maximum at the following value of $q$,

$$
q \simeq \frac{1}{4(1 - v^2)} \frac{1 - \eta(1 - v^2)}{2 + \eta(1 - v^2)(1 - \eta(1 - v^2))},
$$

(22)

which reduces to $q \simeq \frac{1}{8(1 - v^2)}$ at the near-extremal limit. In this limit the maximum of the drag force coefficient is,

$$
(\mu m)_{\text{max}} = \frac{T_0}{4\sqrt{1 - v^2}} \left[ 1 + \frac{1}{3(1 - v^2)r_c^2} \right].
$$

(23)

Here, it is interesting to compare our above results to the calculations of Ref. [11] (specially equation (20) in this paper with equation (5.9) of Ref. [11]), where SYM in Minkowski space ($k = 0$) is considered. It is expected that for YM on a sphere Lorentz invariance to be broken and one can see that our solutions should be identical to Ref. [11] at non-relativistic case ($v^2 \to 0$).

In the next section we are going to discuss about quasi-normal modes. In the section 3 we considered a moving string through thermal plasma without any external field, and obtained non-zero drag force. It means that the motion of string should be reduces to small fluctuations after the long time, which known as quasi-normal modes of string.

4 Quasi-Normal Modes of Curved String

In this section, we consider small perturbations of a curved string which stretched from $r = r_m$ to $r = r_h$ in STU background with three non-zero charges. This consideration allows us to obtain the friction coefficient $\mu$ in the non-relativistic regime of the quark. In that case we consider a quark moving in the $\cal N$=2 supergravity thermal plasma without any external field. Indeed, we want to study the behavior of the curved string at the $t \to \infty$ and low velocity limits. We use the same method with Ref. [11], where small perturbations of a straight string in $\cal N$=4 SYM thermal plasma considered. For more detail of such configuration see Refs. [1, 2, 45, 46, 47, 48, 49, 50]. The small fluctuations around the
curved string means that $\dot{x}^2$ and $x'^2$ are small, so one can neglect them in the expression (4). Hence the equation of motion (5) reduces to the following equation,

$$
\frac{\partial}{\partial r} \left( \frac{f(r)r^2}{\mathcal{H}^\frac{5}{6}} x' \right) - \frac{\mathcal{H}^\frac{5}{6} r^2}{f(r)} \ddot{x} = 0.
$$

(24)

Then under assumption of time-dependent solution of the form $x(r, t) = x(r)e^{-\mu t}$, equation of motion (24) reduces to the simple eigenvalue equation $Ox(r) = \mu^2 x(r)$, where we introduce operator $O$ as the following,

$$
O = \frac{f(r)}{\mathcal{H}^\frac{5}{6}} \int \frac{f(r)r^2}{\mathcal{H}^\frac{5}{6}} \frac{d}{dr} \left( f(r) \right) \mathcal{H}^\frac{5}{6} r^2 dr.
$$

(25)

In order to obtain friction coefficient, we assume that $\mu$ is small and one can use expansion $x(r) = x_0(r) + \mu x_1(r) + \mu^2 x_2(r) + \cdots$. Also by applying Neumann boundary condition we find $x'(r_m) = \mu x_1'(r_m) + \mu^2 x_2'(r_m) = 0$ and $x_0$ specified as a constant. Therefore under assumption of small $\mu$ one can find,

$$
\mu = -\frac{A}{x_0 L^4} \left[ \frac{(L^2(r^2 - \eta) + \mathcal{H}(r)r^4)^2}{\mathcal{H}(r)^\frac{5}{6}} \right] \int \frac{\mathcal{H}(r)^\frac{5}{6}}{L^2(r^2 - \eta) + \mathcal{H}(r)r^4} dr \mid_{r=r_m},
$$

(26)

where $A$ is an integration constant. Then, by using the relation (26) we can determine the value of the drag force. One can check that the drag force will obtained as a constant and at the near-extremal limit it is proportional to the $\tan^{-1}(r_m)$.

Now we can use these results to obtain the total energy and momentum of curved string. In that case we use equation of motion (24) and relations $\dot{x} = -\mu x, \quad P = \int_{r_{min}}^{r_m} \pi x^0 dr$ and also Neumann boundary condition ($x'(r_m) = 0$), so the total momentum of string will be as the following relation,

$$
P = \frac{T_0}{\mu L^2} \left[ \frac{r_{min}^6 + (L^2 + \sum_i q_i r_{min}^4 + (\frac{1}{2} \prod_{i \neq j} q_i q_j - \eta L^2)r_{min}^2 + \sum_i q_i x'(r_{min})}{r_{min}^{12} + r_{min}^6 \sum_i q_i + \frac{1}{2} r_{min}^3 \prod_{i \neq j} q_i q_j + r_{min}^2 \sum_i q_i} \right],
$$

(27)

where we insert $r_{min} > r_h$ as lower limit of integral to avoid divergency.

In order to obtain the total energy we keep second order of velocities and expand $\sqrt{-g}$ to find,

$$
E = -\frac{T_0}{2L^2} \left[ \frac{r_{min}^2 L^2 - \eta L^2 + r_{min}^4 \prod_i (1 + \frac{q_i}{r_{min}^2}) x(r_{min}) x_1'(r_{min})}{\prod_i (1 + \frac{q_i}{r_{min}^2})^{\frac{1}{6}}} \right] - T_0 \int_{r_{min}}^{r_m} dr \prod_i (1 + \frac{q_i}{r^2})^\frac{1}{6},
$$

(28)
where for small values of charge reduces to the following expression,

\[
E \simeq -\frac{T_0}{2L^2} \left[ r_{min}^4 + r_{min}^2 (L^2 + \frac{5}{6} \sum_i q_i) + \frac{1}{2} \prod_{i \neq j} q_i q_j - \frac{L^2}{6} \sum_i q_i - \eta L^2 \right] x(r_{min})x'(r_{min}) \\
- T_0 \left( r_m - r_{min} + \frac{1}{6} \left( \frac{1}{r_m} - \frac{1}{r_{min}} \right) \sum_i q_i \right),
\]

(29)

In order to check validity of equations (27) and (29) we take the near-extremal limit and find that,

\[
P = \frac{T_0}{\mu L^2} r_{min}^2 (r_{min}^2 + L^2) x'(r_{min}),
\]

\[
E = T_0 \left[ r_m - r_{min} - \frac{1}{2L^2} (r_{min}^2 + L^2) r_{min}^2 x(r_{min}) x'(r_{min}) \right],
\]

(30)

which is agree with [1, 10] under assumption \( L^2 = \frac{1}{\Lambda^2} \). Therefore one can write \( E = T_0 (r_m - r_{min}) - \frac{\mu P^2}{2m} \). The first term of right hand side interpreted as \( M_{rest} \) of the quark.

## 5 Effect of Constant Electromagnetic Field

In the previous sections the moving quark through plasma considered without any external field. In the present section we consider the single quark which moves with constant speed \( v \) through thermal plasma in STU background, and introduce a constant electromagnetic field as external field. We assume that the constant electromagnetic field is along \( x^1 \) and \( x^2 \) directions. Therefore we add a constant \( B \)-field in the form \( B = B_{01}dt \wedge dx_1 + B_{12}dx_1 \wedge dx_2 \) to the line element (1), where \( B_{01} \) is the constant electric field and \( B_{12} \) is the constant magnetic field. Also \( B_{01} \) and \( B_{12} \) are antisymmetric fields and other components of the \( B \)-field are zero. We must note that the same work was done originally for \( \mathcal{N}=4 \) SYM theory [15].

Because of introducing \( B_{01} \) and \( B_{12} \), the curved string dual to quark may be described by the \( x_1(r,t) = x_1(r) + v_1 t, x_2(r,t) = x_2(r) + v_2 t \) and \( x_3(r,t) = 0 \). Therefore the square root of lagrangian density (4) takes the following form,

\[
-g = \frac{1}{\mathcal{H}^2} \left[ 1 - \frac{\mathcal{H} r^2}{f(r)L^2} \tilde{v}^2 + \frac{f(r) r^2}{L^2} x'^2 - (B_{01} x'_1 + B_{12} (\tilde{v} \times \tilde{x}')^2) \right],
\]

(31)

where \( \tilde{v} = (v_1, v_2) \) is the vector of velocity and \( \tilde{x}' = (x'_1, x'_2) \) is the projected directions of string tail. We would like to consider three separated cases, first, we assume that only electric field is exist and \( B_{12} = 0 \) and second, we have non-zero magnetic field and there is \( B_{01} = 0 \), and finally we discuss about the case where \( \tilde{v} \perp B_{01} \).
In order to study effect of constant electric field, one may choose the moving direction of the quark to be in the $x_1$ direction, so we have $x_1(r,t) = x_1(r) + vt$ and $x_2(r,t) = x_3(r,t) = 0$. In that case the $x_1$-component of momentum density obtained as,

$$
\pi_{x_1} = \sqrt{\frac{r_c^2}{L^2} \prod_i (1 + \frac{q_i}{r_c^2}) + \frac{r_c^2 - \eta}{(\prod_i (1 + \frac{q_i}{r_c^2}))^\frac{3}{4}} - B_{01}^2},
$$

(32)

where $r_c$ is the root of the equation (15). It is easy to check that if $B_{01} = 0$, then the equation (32) reduces to the equation (16). On the other hand if we choose the value of electric field as $B_{01}^2 = \frac{r_c^2}{L^2} \prod_i (1 + \frac{q_i}{r_c^2}) + \frac{r_c^2 - \eta}{(\prod_i (1 + \frac{q_i}{r_c^2}))^\frac{3}{4}}$, then $\pi_{x_1} = 0$ and string feels no drag force, therefore the string can continue its motion at constant velocity. Indeed, it is what we mentioned after the equation (16).

It is clear that at the near-extremal limit the critical radius reduces to $r_c^* = \frac{L^2}{\Delta^2 - 1}$ and we have $\pi_{x_1} = \sqrt{(\frac{vL^2}{\Delta^2 - 1})^2 - B_{01}^2}$. So, if electric field is switched off, then the drag force is proportional to $\frac{vL^2}{\Delta^2 - 1}$ which is in agreement with Refs. [1, 10, 12] under assumption of $L^2 \Lambda^2 = 1$. In that case for infinitesimal electric field the drag force modified by a term in the form of $\frac{1}{2} \tau \cdot \frac{v^2 - 1}{v} B_{01}^2$. We see that there are four states where the drag force may be increase or decreases. In the case of $v > 0$, $v^2 > 1$ and $v < 0$, $v^2 < 1$ the effect of existence constant electric field is increasing of the drag force. But in the case of $v > 0$, $v^2 < 1$ and $v < 0$, $v^2 > 1$ the constant electric field decreases the drag force. However in the relativistic limit $v \to 1$ the constant electric field has no effect to the drag force.

In the second case we want to consider only constant magnetic field $B_{12}$. In that case we can choose $x_1(r,t) = x_1(r) + vt$, $x_2(r,t) = x_2(r)$ and $x_3(r,t) = 0$. Under this assumption one can find,

$$
x_1'(r) = \pi_{x_1} \left[ \frac{\beta \left( \frac{1}{\hat{\mathcal{H}}^3} - \frac{\hat{\mathcal{H}}^3 r_0^2}{f(r)} \right)}{f(r)r^2 \left( \left( \frac{\pi^2_{x_1} - f(r)r^2}{\hat{\mathcal{H}}^3} \right) \left( \frac{\pi^2_{x_2} - \beta}{\pi^2_{x_2}} \right) - \frac{\pi^2_{x_1} \pi^2_{x_2}}{\pi_{x_2}} \right)} \right]^\frac{3}{2},
$$

$$
x_2'(r) = \pi_{x_2} \left[ \frac{f(r)r^2 \left( \frac{1}{\hat{\mathcal{H}}^3} - \frac{\hat{\mathcal{H}}^3 r_0^2}{f(r)} \right)}{\beta \left( \left( \frac{\pi^2_{x_1} - f(r)r^2}{\hat{\mathcal{H}}^3} \right) \left( \frac{\pi^2_{x_2} - \beta}{\pi^2_{x_2}} \right) - \frac{\pi^2_{x_1} \pi^2_{x_2}}{\pi_{x_2}} \right)} \right]^\frac{3}{2},
$$

(33)

where $\beta = \frac{f(r)r^2}{\hat{\mathcal{H}}^3} - v^2 B_{12}^2$ and we set $\pi_{x_i} \equiv \pi_{x_i}$. Now reality condition implies that $\pi_{x_2} = 0$ and therefore we have,

$$
\pi_{x_1} = \left( 1 + \frac{q_1}{r_c^2} \right) \left( 1 + \frac{q_2}{r_c^2} \right) \left( 1 + \frac{q_3}{r_c^2} \right)^\frac{3}{4} \nu_{12}^2 v_c^2.
$$

(34)
where \( r_c \) is the root of equation (15). It tells us that there is no drag force in \( x^2 \) direction and \( B_{12} \) have no effect on the motion along \( x^1 \) direction since the equation (34) is coincide with equation (16) which obtained without any external field. Actually vanishing of \( \pi_{x_2} \) is consequence of vanishing of \( v_2 \).

With respect to these two cases, (electric and magnetic fields) we found that the constant magnetic field have no effect on the motion of string and it is appropriate electric field which keeps the string at constant speed \( v \), so this is agree with results of Refs. [10, 11].

Before end of this section we consider the case of \( \vec{v} \perp B_{01} \). It means that one may choose the solutions of equation of motion as, \( x_1(r, t) = x_1(r), x_2(r, t) = x_2(r) + vt \) and \( x_3(r, t) = 0 \). A possible drag force may be found as the following relation,

\[
\dot{P}_2 = - T_0 \sqrt{\frac{v^2}{\mathcal{H}} \prod_i (1 + \frac{q_i r}{r_c^2}) + r_c^2 - \eta}{\prod_i (1 + \frac{q_i r}{r_c^2})}^\frac{1}{2} - v^2 B_{12}^2, \tag{35}
\]

and \( \dot{P}_1 = 0 \). In this case the constant electric field has no effect on drag force. We should mention that, because of special direction of motion which we considered already, this situation is not happen for our string which studied in the previous sections.

6 Quark-Antiquark Solutions

In this section we consider a quark-antiquark pair, which may be interpreted as a meson, moving with the constant speed \( v \) in STU background. Already the energy of a moving quark-antiquark pair in \( \mathcal{N}=4 \) super Yang-Mills plasma calculated [51]. Now we would like to repeat same calculation STU background. To represent a quark-antiquark pair in the dual picture one may consider an open string in \( AdS_5 \) space which two endpoints of string lie on D-brane in the \( (X, Y) \) plan. Two end points of string on the D-brane represent quark and antiquark which separated from each other by a constant \( l \). We assume that at the \( t = 0 \) string is straight and two endpoints of string move with the constant velocity \( v \) along the \( X \) direction. The dynamics of such configuration discussed in detail in the Ref. [51]. Similar to the calculations of the previous sections one can obtain the square root quantity of the lagrangian density as a following expression,

\[
-g = \frac{1}{\mathcal{H}^\frac{1}{2}} \left[ 1 + \frac{f(r)r^2}{L^2}(x'^2 + y'^2) - \frac{\mathcal{H}r^2}{f(r)L^2}(x'^2 + y'^2) - \frac{r^4}{L^4} \mathcal{H}(x'^2 y'^2 + y'^2 x'^2 - 2x'y'y) \right], \tag{36}
\]
and the equations of motion with respect to $x$ and $y$ is given by the following equations, respectively,

$$
\frac{\partial}{\partial r} \left[ \frac{1}{\sqrt{-g}} \left( \frac{f(r)}{f(r)} \right)^{\frac{r^2}{L^2}} x' + \frac{r^4}{L^2} \mathcal{H} y^2 (x' - x y') \right] + r^2 \mathcal{H} \frac{\partial}{\partial t} \left[ \frac{1}{\sqrt{-g}} \left( \frac{\dot{x}}{f(r)} + \frac{r^2}{L^2} (y^2 x' - x y') \right) \right] = 0,
$$

$$
\frac{\partial}{\partial r} \left[ \frac{1}{\sqrt{-g}} \left( -\frac{f(r)}{f(r)} \right)^{\frac{r^2}{L^2}} y' \right] + r^2 \mathcal{H} \frac{\partial}{\partial t} \left[ \frac{1}{\sqrt{-g}} \left( \frac{yL^2}{f(r)} + r^2 (x^2 y' - x y') \right) \right] = 0,
$$

then momentum densities obtained by following equation,

$$
\begin{pmatrix}
\pi^0_x & \pi^1_x \\
\pi^0_y & \pi^1_y \\
\pi^0_r & \pi^1_r \\
\pi^0_t & \pi^1_t
\end{pmatrix} = -T_0 \frac{r^2 \mathcal{H}^2}{L^2 \sqrt{-g}} \times
\begin{pmatrix}
\frac{r^2}{L^2} \mathcal{H} y^2 x' y' - \left( \frac{\mathcal{H}^2}{f(r)} \right)^{\frac{r^2}{L^2}} x' \dot{y} & \frac{r^2}{L^2} \mathcal{H} y^2 y' \dot{x} - \left( \frac{\mathcal{H}^2}{f(r)} \right)^{\frac{r^2}{L^2}} y^2 x' \\
\frac{r^2}{L^2} \mathcal{H} y^2 y' \dot{x} & \frac{r^2}{L^2} \mathcal{H} y^2 y' \dot{y} - \left( \frac{\mathcal{H}^2}{f(r)} \right)^{\frac{r^2}{L^2}} y^2 x' \\
\frac{\mathcal{H}^2}{f(r)} (\dot{x} x' + \dot{y} y') & -\frac{L^2}{\mathcal{H}^2} - \frac{\mathcal{H}^2}{f(r)} (\dot{x}^2 + \dot{y}^2) \\
\frac{\mathcal{H}^2}{f(r)} (\dot{x} x' + \dot{y} y') & -\frac{L^2}{\mathcal{H}^2} - \frac{\mathcal{H}^2}{f(r)} (\dot{x}^2 + \dot{y}^2)
\end{pmatrix},
$$

(37)

In this paper we consider two special cases, the first is the moving quark-antiquark pair with constant speed $v$ and in the second case we add rotational motion to the pair. The first case may be described by the $x(r, t) = vt + x(r)$ and $y(r, t) = y(r)$. These solutions satisfy boundary conditions as $x(\infty, t) = vt$ and $y(\infty) = \pm \frac{t}{2}$. In this case equation (38) reduces to the following expression,

$$
\begin{pmatrix}
\pi^0_x & \pi^1_x \\
\pi^0_y & \pi^1_y \\
\pi^0_r & \pi^1_r \\
\pi^0_t & \pi^1_t
\end{pmatrix} = -T_0 \frac{r^2 \mathcal{H}^2}{L^2 \sqrt{-g}} \times
\begin{pmatrix}
-v \left( \frac{1}{f(r)} \right)^{\frac{r^2}{L^2}} x' & \frac{f(r)}{\mathcal{H}^2} \frac{r^2}{L^2} y^2 x' \\
\frac{r^2}{L^2} y^2 x' & -\frac{\mathcal{H}^2}{f(r)} \left( \frac{r^2}{L^2} y^2 + \frac{f(r)}{v^2} \right) y'
\end{pmatrix},
$$

(39)

where,

$$
g = \frac{1}{\mathcal{H}^2} \left[ 1 + \frac{f(r)}{L^2} (x^2 + y^2) - \mathcal{H} r^2 v^2 - \frac{r^4}{L^4} \mathcal{H} v^2 \right].
$$

(40)
In order to obtain drag force, we take $\pi^1_x$ and $\pi^1_y$ components and solve them for $x'$ and $y'$ respectively and obtain,

\[
x'(r) = \pi^1_x \frac{L}{r} (1 - \frac{Hr^2v^2}{f(r)L^2}) \left[ \left( \frac{f(r)}{H} - \frac{r^2v^2}{L^2} \right) (T_0^2 + \frac{r^2}{L^2} f(r)H^2 - H\pi^1_x) - f(r)\pi^1_y \right]^{-\frac{1}{2}},
\]
\[
y'(r) = \pi^1_y \frac{L}{r} \left[ \left( \frac{f(r)}{H} - \frac{r^2v^2}{L^2} \right) (T_0^2 + \frac{r^2}{L^2} f(r)H^2 - H\pi^1_x) - f(r)\pi^1_y \right]^{-\frac{1}{2}}. \tag{41}
\]

As before, by using reality condition one can obtain,

\[
\pi^1_y^{12} = \left. \left[ \left( \frac{f(r)}{H} - \frac{r^2v^2}{L^2} \right) (T_0^2 + \frac{r^2}{L^2} f(r)H^2 - H\pi^1_x) - f(r)\pi^1_y \right] \right|_{r=r_{\text{min}}}^{-\frac{1}{2}}, \tag{42}
\]

where $r_{\text{min}}$ is turnaround point. One can check easily that $r_{\text{min}} \geq r_c$ ($r_c$ is critical radius which introduced in the section 3). If $\pi^1_y = 0$, then $r_{\text{min}} = r_c$ and above solutions are similar to single quark solution ($l = 0$). Here, in order the string have a chance of turning around smoothly, it requires that $\frac{\partial y}{\partial x} = \frac{y'}{x'} = \infty$ at $r_{\text{min}}$ [51]. So, it necessary to have $\pi^1_x = 0$.

Therefore one can find $\pi^1_y = \frac{T_0}{L} r_{\text{min}} H^\frac{1}{r_{\text{min}}} (r_{\text{min}})^{-\frac{1}{2}} \sqrt{\frac{f(r_{\text{min}})}{H(r_{\text{min}})} - \frac{r_{\text{min}}^2 v^2}{L^2}}$.

In the second case we add a rotational motion with angular velocity $\dot{\theta}$. Therefore the string may be described by the $x(r, t) = vt + x(r) \sin \theta$ and $y(r, t) = y(r) \cos \theta$. So Fig. 1 shows the configuration of rotating string. As we can see, $\theta(t)$ is an angle with $Y$ axis. These solutions satisfy boundary conditions $x(\infty, t) = vt \pm \frac{l}{2} \sin \theta$ and $y(\infty, t) = \pm \frac{l}{2} \cos \theta$, where for $\theta = 0$ reduce to the boundary condition without rotational motion. Also, from our conjecture we have another condition as $\frac{\dot{y}}{\dot{x}} = \cot \theta$, which reduces to $\frac{y'}{x'} \to \infty$ at the $\theta \to 0$ limit, which is agree with the first case.
Figure 1: A rotating ∩ - shape string dual to a q̅q pair that can be interpreted as a meson. A and B represent quark and antiquark with separating length \( l \). The radial coordinate \( r \) varies from \( r_h \) (black hole horizon radius) to \( r = r_0 \) on D-brane. \( r_c \) is a critical radius, obtained for single quark solution, which the string can't penetrate beyond it and \( r_{\text{min}} \geq r_c \). \( r_{\text{min}} = r_c \) is satisfied if points A and B located at origin \( (l = 0) \), in that case there is straight string which is dual picture of single static quark. \( \theta \) is assumed to be the angle with \( Y \) axis and the string center of mass moves along \( X \) axis with velocity \( v \).

These boundary conditions can also satisfy with two separated string which move at velocity \( v \) along \( X \) axis and simultaneously swing a circle with radius \( \frac{l}{2} \). Specifying these boundary conditions doesn’t lead to a unique solution for equation of motion, so we should specify additional conditions for this motion. Here we assume that the string is initially upright, move at velocity \( v \) and rotates around its center of mass. Now by using above solutions in the equation (38) and solving resulting equations with respect to \( x' \) and \( y' \) one can obtain following equations,

\[
A x'^2 + B y'^2 + C x'y' + D = 0,
\]

\[
A' x'^2 + B' y'^2 + C' x'y' + D' = 0,
\]

(43)
where,

\[ A = R^2 \sin^2 \theta \left[ \pi_x^{12} \left( \frac{f(r)}{H^2} - \mathcal{H}^2 y^2 \dot{\theta}^2 R^2 \sin^2 \theta \right) - T_0^2 \mathcal{H}^2 R^2 \left( \mathcal{H}^2 y^2 \dot{\theta}^2 R^2 \sin^2 \theta - \frac{f(r)}{H^2} \right)^2 \right], \]

\[ B = R^2 \cos^2 \theta \left[ \pi_x^{12} \left( \frac{f(r)}{H^2} - \mathcal{H}^2 (v + x \dot{\theta} \cos \theta)^2 R^2 \right) - T_0^2 \mathcal{H}^2 y^2 \dot{\theta}^2 R^2 \sin^2 \theta (v + x \dot{\theta} \cos \theta)^2 \right], \]

\[ C = -2y \dot{\theta} R^4 \sin^2 \theta \left[ \pi_x^{12} \mathcal{H}^2 \cos \theta + T_0^2 \mathcal{H}^2 R^2 \left( \mathcal{H}^2 y^2 \dot{\theta}^2 R^2 \sin^2 \theta - \frac{f(r)}{H^2} \right) \right] (v + x \dot{\theta} \cos \theta), \]

\[ D = R^2 n_x^{12} \frac{\mathcal{H}^2}{f(r)} \left[ \frac{f(r)}{R^2 \mathcal{H}} - y^2 \dot{\theta}^2 \sin^2 \theta - (v + x \dot{\theta} \cos \theta)^2 \right], \]

\[ A' = R^2 \sin^2 \theta \left[ \pi_y^{12} \left( \frac{f(r)}{H^2} - \mathcal{H}^2 y^2 \dot{\theta}^2 R^2 \sin^2 \theta \right) - T_0^2 \mathcal{H}^2 y^2 \dot{\theta}^2 R^2 \sin^2 \theta (v + x \dot{\theta} \cos \theta)^2 \right], \]

\[ B' = R^2 \cos^2 \theta \left[ \pi_y^{12} \left( \frac{f(r)}{H^2} - \mathcal{H}^2 (v + x \dot{\theta} \cos \theta)^2 R^2 \right) - T_0^2 \mathcal{H}^2 R^2 \left( R^2 \mathcal{H}^4 (v + x \dot{\theta} \cos \theta)^2 - \frac{f(r)}{H^2} \right)^2 \right], \]

\[ C' = -2y \dot{\theta} R^4 \sin^2 \theta \cos \theta \left[ \pi_y^{12} \mathcal{H}^2 + T_0^2 \mathcal{H}^2 R^2 \left( R^2 \mathcal{H}^4 (v + x \dot{\theta} \cos \theta)^2 - \frac{f(r)}{H^2} \right) \right] (v + x \dot{\theta} \cos \theta), \]

\[ D' = R^2 n_y^{12} \frac{\mathcal{H}^2}{f(r)} \left[ \frac{f(r)}{R^2 \mathcal{H}} - y^2 \dot{\theta}^2 \sin^2 \theta - (v + x \dot{\theta} \cos \theta)^2 \right], \]

where we set \( \frac{\pi}{L} = R \). We must note that the variable \( C \) in equations (43) and (44) is different with integration constant in equation (11). Therefore from equations (43) one can obtain,

\[ x'(r) = 2 \left[ \frac{D(B - \pi_y^{12} A')}{C^2 - C'^2 - 4(AB - B'A')} \right]^{\frac{1}{2}}, \]

\[ y'(r) = 2 \left[ \frac{D(A - \pi_x^{12} B')}{C^2 - C'^2 - 4(AB - B'A')} \right]^{\frac{1}{2}}. \]

Here, if the rotational motion vanishes (\( \dot{\theta} = 0 \)), from equations (43) one can see that coefficients of \( x'y' \) vanish (\( C = C' = 0 \)) and our solutions recover the motion of quark-antiquark pair without rotation. In order to obtain drag force we use reality condition and find a relation between variable (44) as \( \frac{A}{A'} = \frac{B}{B'} = \frac{C}{C'} = \frac{D}{D'} = \frac{\pi_y^{12}}{\pi_x^{12}} \). Then one can find two equations as, \( C^2 - 4AB = 0 \) and \( C'^2 - 4A'B' = 0 \). These equations specify \( \pi_x^{12} \) and \( \pi_y^{12} \) respectively. After some calculations and simplifications we obtain,

\[ \pi_x^{12} = \frac{1}{2a} \left[ \pm \sqrt{b^2 - 4ac - b} \right], \]

\[ \pi_y^{12} = \frac{1}{2a'} \left[ \pm \sqrt{b^2' - 4a'c' - b'} \right]. \]
where

\[
\begin{align*}
a & = \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}} \cos^2 \theta (R_{\min}^2 \zeta + \xi \chi), \\
b & = T_0^2 \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}} \chi \left( 2R_{\min}^4 \zeta + \cos^2 \theta (R_{\min}^2 \xi \chi - \zeta) \right), \\
c & = T_0^4 \prod_i (1 + \frac{q_i}{r_{\min}^2}) R_{\min}^6 \sin^2 \theta \xi \chi^2, \\
a' & = \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{3}} (R_{\min}^2 \zeta + \xi \chi), \\
b' & = T_0^2 R_{\min}^2 \xi \left( R_{\min}^4 \zeta \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}} - 2R_{\min}^2 \zeta - \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}} \xi \chi \right), \\
c' & = T_0^2 \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}} R_{\min}^6 \xi^2 \left( \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}} - T_0^2 \right),
\end{align*}
\]

(47)

with,

\[
\begin{align*}
\zeta & = \prod_i (1 + \frac{q_i}{r_{\min}^2}) \frac{y^2 \dot{\theta}^2 R_{\min}^2 \sin^2 \theta (v + x \dot{\theta} \cos \theta)^2}, \\
\xi & = \frac{f(r_{\min})}{\prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}}} - \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}} (v + x \dot{\theta} \cos \theta)^2 R_{\min}^2, \\
\chi & = \prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}} \frac{y^2 \dot{\theta}^2 R_{\min}^2 \sin^2 \theta - \frac{f(r_{\min})}{\prod_i (1 + \frac{q_i}{r_{\min}^2})^{\frac{1}{2}}}}{y^2 \dot{\theta}^2 R_{\min}^2 \sin^2 \theta},
\end{align*}
\]

(48)

where \( R_{\min} = \frac{r_{\min}}{L} \) and \( r_{\min} \) is the turnaround point. The direct consequence of rotational motion is that drag force is no longer constant. From equation (46) one can see that the momentum densities of string vary with respect to \( x(r) \) and \( y(r) \).

But this is not appropriate description of a meson. Indeed, according to previous works [26, 27] the \( q\bar{q} \) pair should be close enough together and not moving too quickly. The presence of functions \( x(r) \) and \( y(r) \) in relations (48) is consequence of relativistic motion, which is not acceptable. On the other hand, because of non-vanishing drag forces, it is expected that the velocity of a \( q\bar{q} \) pair decreases. So, we consider a moving heavy \( q\bar{q} \) pair with non-relativistic speed, which rotates by angel \( \theta = \omega t \) around the center of mass. Indeed this situation is corresponding to motion of the heavy meson with large spin. Actually, in the very large angular momentum limit, a classical approximation is reliable. In this case, the angular velocity of the string is very small. Therefore we are going to the case of non-relativistic
motion \((\dot{\theta}^2 \rightarrow 0 \text{ and } \dot{\theta} v \rightarrow 0)\). In that case \(\zeta = c = c' = 0\) and we have,

\[
\begin{align*}
\pi_{x}^{12} &= \frac{r_{\text{min}}^2 T_0^2}{L^2} f(r_{\text{min}}) \mathcal{H}^{-\frac{1}{4}}(r_{\text{min}}), \\
\pi_{y}^{12} &= \frac{r_{\text{min}}^2 T_0^2}{L^2} \left( f(r_{\text{min}}) - \mathcal{H}(r_{\text{min}}) \frac{r_{\text{min}}^2 v^2}{L^2} \right) \mathcal{H}^{-\frac{1}{4}}(r_{\text{min}}).
\end{align*}
\]  

(49)

Now we assume that \(v^2 \rightarrow 0\) and angular velocity is infinitesimal constant \((\dot{\theta} = \omega \ll 1)\), and the quark-antiquark pair rotate around origin. In that case we neglect \(\omega^4\) terms and obtain values of momentum densities as,

\[
\pi_{x}^{1} = \pi_{y}^{1} = T_0 r_{c} \frac{r_{\text{min}}^2 \left[ 1 - \frac{\eta}{r_{c}^2} + \frac{r_{c}^2}{r_{\text{min}}^2} \Pi_i (1 + \frac{q_i}{r_{c}^2}) \right]^\frac{1}{2}}{\Pi_i (1 + \frac{q_i}{r_{c}^2})^\frac{1}{2}}.
\]  

(50)

In order to obtain non-zero components of momentum densities (49) and (50) we should use negative sign in relations (46), therefore correct sign in equations (46) is minus sign.

As we saw for the non-relativistic motion, we have constant drag forces.

7 Higher derivative corrections

In this section, we want to calculate the effect of higher derivative terms in the drag force on the single quark moving through \(\mathcal{N} = 2\) supergravity thermal plasma. If we consider the lowest order of string length then we can expand the effective action in power of \(T_0^{-1}\). So, the high order terms of \(T_0^{-1}\) is corresponding to the higher derivative terms. As we know, the higher order corrections are depend to the black hole physics [1, 41, 42, 52], and allow us to have better understanding of AdS/CFT correspondence. It is known that [1, 41], the full lagrangian including higher derivative terms is, \(\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{R^2} + \cdots\). Here just we consider the first order correction to the R-charged black hole solution. In presence of the first order correction the solution (2) should be modified as the following [42],

\[
\begin{align*}
f &= f_0 + c_1 f_1, \\
H_i &= h_{i0} + c_1 h_{i1}, \quad i = 1, 2, 3,
\end{align*}
\]  

(51)

where \(f_0 = 1 - \frac{\eta}{r_{c}^2} + \frac{r_{c}^2}{r_{\text{min}}^2} \mathcal{H}\) and \(h_{i0} = 1 + \frac{q_i}{r_{c}^2}\) and \(c_1\) is constant small parameter. The correction terms are obtained by [42],

\[
\begin{align*}
f_1 &= \frac{\eta^2}{96r^6 h_{i0}} - \frac{5q_i(q_i + \eta)}{72r^2 L^2 r^4}, \\
h_{i1} &= -\frac{q_i(q_i + \eta)}{72r^6 h_{i0}},
\end{align*}
\]  

(52)
and one can obtain,

\[ \mathcal{H} = \mathcal{H}_0 + c_1 \mathcal{H}_1 = \prod_{i=1}^{3} \left( 1 + \frac{q_i}{r^2} - \frac{c_1 q_i (q_i + \eta)}{72 r^2 (r^2 + q_i)^2} \right). \] (53)

By using above corrections to the case of moving single quark with constant speed \( v \), one can obtain lagrangian density as the following,

\[ \mathcal{L} = -\frac{1}{(\mathcal{H}_0 + c_1 \mathcal{H}_1)^{\frac{1}{2}}} \left[ 1 - \frac{(\mathcal{H}_0 + c_1 \mathcal{H}_1) r^2}{(f_0 + c_1 f_1)L^2 v^2} + \frac{(f_0 + c_1 f_1) r^2}{L^2} x^2 \right]^{\frac{1}{2}}. \] (54)

So, after some calculations similar to the section 3 one can find momentum density as the following relation,

\[ \pi^1_x = [(h_{10} + c_1 h_{11})(h_{20} + c_1 h_{21})(h_{30} + c_1 h_{31})]^{\frac{1}{2}} v r_c^2, \] (55)

where \( r_c \) is the critical point which is root of the following equation,

\[ f_0 + c_1 f_1 - r^2 v^2 (\mathcal{H}_0 + c_1 \mathcal{H}_1) = 0. \] (56)

So, we can generalize \( x^2 \) as a following,

\[ x^2 = v^2 L^2 r_h^4 [\mathcal{H}_0 + c_1 \mathcal{H}_1]^{\frac{1}{2}} \frac{(\mathcal{H}_0 + c_1 \mathcal{H}_1)^{\frac{3}{2}}}{(f_0 + c_1 f_1) r^4}, \] (57)

and the corrected horizon radius in presence of higher derivative terms is given by,

\[ r_h = r_{0h} \left[ 1 + \frac{c_1 \frac{\mathcal{H}_0 + c_1 \mathcal{H}_1}{r^4}}{24} \left( \sum q_i^2 - \frac{6r_{0h}^2}{3} \sum q_i + 3 r_{0h}^3 \right) - \frac{2\mathcal{H}_0 + c_1 \mathcal{H}_1}{24 r_{0h}^{\frac{2}{3} r_{0h}} (r_{0h}^2)^{\frac{1}{2} r_{0h}^2}} \left( \frac{3}{3} \sum q_i - 3 r_{0h}^2 \right) + 3 \right]. \] (58)

where \( r_{0h} \) is the horizon radius without higher derivative corrections which is obtained by the following equation, \( 1 - \frac{q_2}{r^2} + \frac{r^2}{L^2} \prod_{i=1}^{3} (1 + \frac{q_i}{r^2}) = 0 \). If we assume that \( q_2 = q_3 = 0 \) and \( q = \eta \sinh^2 \beta \), above equations reduce to [1]. Finally the energy and momentum densities along the string is modified (due to higher derivative terms) by the following equations,

\[ \pi^0_i = -T_0 \left[ \frac{\mathcal{H}_0 + c_1 \mathcal{H}_1}{r_h^4 (\mathcal{H}_0 + c_1 \mathcal{H}_1)} \right]^{\frac{1}{2}} \left( 1 + v^2 r_h^2 (\mathcal{H}_0 + c_1 \mathcal{H}_1) \right)^{\frac{1}{2}} (\mathcal{H}_0 + c_1 \mathcal{H}_1)^{\frac{1}{2}} \left( f_0 + c_1 f_1 \right)^{r = r_h}, \] (59)

and

\[ \pi^0_x = T_0 \frac{vr_c^2}{L^2 r_h^4 (\mathcal{H}_0 + c_1 \mathcal{H}_1)} \left( \mathcal{H}_0 + c_1 \mathcal{H}_1 \right)^{\frac{3}{2}} r^2 (f_0 + c_1 f_1). \] (60)
From the modified solution (52) one can see that, at the near-extremal limit of the first order corrections have not any effect to the lagrangian density and drag force, because at the \( \eta \to 0 \) limit, \( f_1 = h_{11} = 0 \) and \( H = 1 \). Actually if one consider higher order terms (second, third,...) at the near-extremal limit there are no effect on the drag force, because all of the correction terms are depend to the non-extremality parameter [41].

8 Conclusion

In this paper we developed our previous papers about drag force of moving quark and quark-antiquark pair through \( \mathcal{N}=2 \) supergravity thermal [1, 2]. We generalized our works to the case of three non-zero charged black hole. Indeed, we considered a moving quark and also a rotating quark-antiquark pair through \( \mathcal{N}=2 \) supergravity thermal plasma with three-charge non-extremal black hole background, where \( q_1 \neq q_2 \neq q_3 \neq 0 \). Furthermore, this paper has more discussions and calculations than Ref. [2], such as effect of constant NSNS B-field, effect of higher derivative corrections, and quark-antiquark solution. Also we found some new results which summarize here.

In the section 3 we concluded that the electromagnetic field strength on the D-brane is necessary to keep the motion of string at the constant velocity. Then in the section 5, we found that type of this constant NSNS B-field should be electric field. Also in the section 3, we discussed about special case of \( q_1 = q_2 = q \) and \( q_3 = 0 \), and found that for small velocity (non-relativistic case with \( v^2 \ll 1 \)) and near-extremal limit (\( \eta \to 1 \)) there is the maximum value of drag force if \( q = \frac{1}{8} \). In the case of relativistic motion at the near-extremal limit we obtained maximum value of drag force coefficient.

We used the method of Refs. [1, 10] in the case of STU model background with three non-zero charges and with different values For the quasi-normal modes of curved string. We have shown that all of our results are agree with Refs. [10, 11, 15, 53] at the near-extremal limit. In fact, we calculated the drag force in the \( \mathcal{N}=2 \) supergravity which has non-extremal black hole. But in the \( \mathcal{N}=4 \) SYM theory there are near-extremal black hole. So it is expected that the near-extremal limit of the \( \mathcal{N}=2 \) supergravity theory would be corresponding to the \( \mathcal{N}=4 \) SYM theory.

The section of the quark-antiquark solutions has two pieces. The first, was the simple extension of previous works such as [51] for the case of \( \mathcal{N}=2 \) supergravity thermal plasma with three non-zero black hole charges. The second, was consideration of rotating quark-antiquark pair. For the first time, we calculated the drag force of rotating quark-antiquark pair in the \( \mathcal{N}=4 \) SYM theory [29]. Now in STU model we have done the same calculations and found the drag force for rotating \( q \overline{q} \) pair with non-relativistic motion have two constant components (\( \pi_x \) and \( \pi_y \)), while in the case of just linear motion it is found that \( \pi_x \) should be vanishes to have real motion. In summary it should be noted that the \( \mathcal{N}=2 \) supergravity is an unusual interpretation, and the usual one is \( \mathcal{N}=4 \) SYM at \( \mu \to 0 \) limit, so we have shown that there is not difference at all between the two calculations.

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Finally in the last section by using results of the Ref. [42] we considered effect of higher order corrections (first order only). We represented modified horizon radius and generalized drag force of single quark due to higher derivative terms.

In here there are many interesting problems, which we introduce some of them as follows. There are other parameters more than the drag force such as shear viscosity [54, 55, 56] and jet quenching parameter [17, 18, 19, 20, 21, 22, 23, 24, 57] in the STU background with non-zero charges. Other interesting problem is consideration of more quark states such as four quarks in the baryon [58]. Also it is interesting to check the possibility of extension our work to the case of four-charge or 8-charge black hole in STU model [44, 59, 60].

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