Asymmetric superradiant scattering and abnormal mode amplification induced by atomic density distortion

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The superradiant Rayleigh scattering using a pump laser incident along the short axis of a Bose-Einstein condensate with a density distortion is studied, where the distortion is formed by shocking the condensate utilizing the residual magnetic force after the switching-off of the trapping potential. We find that very small variation of the atomic density distribution would induce remarkable asymmetrically populated scattering modes by the matter-wave superradiance with long time pulse. The optical field in the diluter region of the atomic cloud is more greatly amplified, which is not an ordinary mode amplification with the previous cognition. Our numerical simulations with the density envelop distortion are consistent with the experimental results. This supplies a useful method to reflect the geometric symmetries of the atomic density profile by the superradiance scattering.

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I. INTRODUCTION

Matter-wave superradiance scattering is induced by the highly anisotropic geometry of Bose-Einstein Condensate (BEC). Ever since the first observation of superradiant scattering from the BEC, a series of experiments [1–9] have been explored to simulate the origin of this phenomenon and the dynamical properties of the light-atom coupling process. Experiments of different geometries, such as the Rayleigh and Raman superradiant scattering, in which the lights are pumped in the axial or radial direction [10, 11] were intensively studied. The fledged quantum theories describing the establishment of superradiant by mode competition [12–15], or semi-classical theories [16] describing the gain process are given and checked to be consistent with the relevant experiments in pretty accuracy. Because of the high controllability of matter-wave systems, the atomic superradiance has found its applications in areas such as coherent imaging, quantum memory and manipulation [6, 17, 18].

The spatial anisotropy is crucial for the generation of superradiance. In this process the photons are emitted into the end-fire modes. In previous works about superradiant radiation of BEC, the phases and densities of the condensates can be treated as uniformly distributed, if we neglect the boundary effects. Before switching off the trap, by tuning the incident intensity, detuning, and angle of the pumping beam, various kinds of superradiant processes of their own features have been realized [1, 2, 13, 19]. However, besides the experimental configuration, the local properties such as the local phase or density distribution are also very important factors in determining the dynamics of light-atom coupling systems, resulting in interesting phenomena in matter-wave superradiant processes. Unfortunately, the relevant research of superradiance in this area has not been paid enough attention as it deserved.

In this paper, we investigate the superradiant Rayleigh scattering by a pump laser incident along the short axis of a density distorted atomic condensate confined in an unbalanced magnetic trap. Even though, in our experiment, the density distortion of the condensate is tiny in a time-of-flight imaging, it induces rather prominent asymmetry in the atomic population for the first-order superradiant scattering modes. Unlike the ordinary optical mode amplification, the superradiance scattering optical field in the diluter region of the condensate is better amplified than that in the denser region. Our numerical simulation using the coupled Maxwell-Schrödinger equation with a density envelope modulation shows a good consistence with the experimental results. Thus the matter-wave superradiance supplies a useful tool for monitoring the spatial symmetry of the density distribution of BECs, without any too complicated optical laser techniques being involved.

The paper is arranged as follows. Section 2 gives our experimental description and results of asymmetrically populated scattering modes of the density distorted matter-wave superradiance. In Section 3, we present the theoretical model describing the superradiance of the distorted condensate. The effects of the atomic density distortion on the scattering process are analyzed and numerically simulated in Section 4. Finally, the conclusion is summarized in Section 5.

II. EXPERIMENTAL DESCRIPTION

For a BEC of highly anisotropic geometry as shown in Fig. (a), when it is exposed to a laser beam with wave vector $k_l$ (and frequency $\omega_l$) along its short axis (the $x$ direction), as the typical superradiance scatter-
ing demonstrated [1, 2], it will scatter photons to the
end-fire modes with wave vector \( \{ \mathbf{k}_{nm} \} \) along the lon-
gitudinal (the \( \hat{z} \) direction) axis of the condensate. At the
same time, the atoms within the condensate will get re-
coiled to the discrete side-modes \((n, m)\) with momentum
\( n\hbar \mathbf{k}_1 + m\hbar \mathbf{k}_{nm} \) \((n, m\) are integer). Here, the frequency
of the optical fields generated in each end-fire mode shall
be different with one another for energy-momentum con-
servation, depending on the status of the BEC and the
experimental configuration. However, one can approxi-
mate that \( k_{nm} \approx k_1 \approx k \) because of the dispersion re-
lation of light. After the superradiant process has been
initiated spontaneously, the recoiled atoms would inter-
fere with the condensate, forming a matter-wave grating,
and then be amplified by stimulated Rayleigh scattering.
At the initial stage, the population of atoms in each re-
coil side-mode \((n, m)\) grows exponentially following the
gain equation [1],

\[ \dot{N}_{n,m} = (G_{n,m} - \Gamma_{n,m})N_{n,m}, \quad (1) \]

where \( N_{n,m} \) is the atomic number of side-mode \((n,m)\),
\( G_{n,m} \) and \( \Gamma_{n,m} \) are the gain and loss coefficient, respec-
tively, describing the amplification and decoherence of
the process.

In our experiment, by laser cooling and rf-evaporation
cooling, a cigar-shaped BEC with \( N = 1.4 \times 10^7 \) \(^{87}\)Rb
atoms is prepared in the \(|F = 2, m_F = 2\)\) hyperfine
ground state with a long axis \( L = 80 \mu m \) and short axis
\( D = 8 \mu m \) in an Ioffe-quadrupole trap with trapping fre-
cuencies \( \omega_x = \omega_y = 2\pi \times 220 \text{Hz} \) and \( \omega_z = 2\pi \times 20 \text{Hz} \).
In order to manipulate the atomic density, we shock the
condensate by generating a magnetic force via abruptly
switching off the trapping potential by the fact that the
residual currents remained in the coils of the quadrupole
and Ioffe trap decay at different rates. The time control
sequence is shown in Fig.1(b). This results in a BEC
with a density profile like that in Fig.2(a) measured by
absorption imaging after free expansion. It can be seen
that the density of the BEC is a little higher on the left
side. Then after a delay \( \Delta t \approx 500 \mu s \), when the trapping
potential has vanished, a laser field with tunable pulse
length \( t \) and wave length \( \lambda_t = 780 \mu m \), which is
red detuned by \( \delta = -0.88 \text{GHz} \) to the \(|F = 2, m_F =
2\rangle \rightarrow |F' = 3, m_F' = 3\rangle \) transition, is incident on the con-
densate along the radial direction to initialize the matter-
wave superradiance. The diameter of the pump laser is
about 2.2 \( \mu m \), which is more than 20 times the longitudi-

\[ \lambda_{l} = 780 \mu m \]

n. The atomic populations
in two first-order forward scattering modes \((1, -1)\) and
\((1, 1)\) are measured with the pump beam incident to a

BEC before and after the switching-off of the trapping
potential, respectively, as shown in the TOF images of
the condensate in Figs.2(b) and 2(c). In the case that the
superradiance takes place before the magnetic trap is
shut down, the density of a dilute BEC (the interatomic
interaction can be neglected in the dilute BEC) is sym-
metrically distributed within the trap, because the size
of the BEC is small compared to the character length
scale of the spacial fluctuations of the trapping poten-
tial, resulting in a density profile of the Gaussian form
symmetric about its geometric center. It is also true that
envelope of the BEC should be symmetric even if the con-
densate is dense. In this situation, the wave-function of
the ground state shall take the Thomas-Fermi form

\[ \begin{align*}
\hat{N}_{n,m} & = (G_{n,m} - \Gamma_{n,m})N_{n,m}, \\
\end{align*} \quad (1) \]

which is in-

\[ \begin{align*}
\dot{N}_{n,m} & = (G_{n,m} - \Gamma_{n,m})N_{n,m}, \\
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tial, resulting in a density profile of the Gaussian form
symmetric about its geometric center. It is also true that
envelope of the BEC should be symmetric even if the con-
densate is dense. In this situation, the wave-function of
the ground state shall take the Thomas-Fermi form in
mean-field approximation because of the presence of the
atomic interactions. The superradiance of such symmet-
ric BEC has been studied 1–9; however, research of that

graphic patterns with our pump pulse intensity \( I = 80mW/cm^2 \)
and the pulse time \( t = 100 \mu s \). The atomic populations
in two first-order forward scattering modes \((1, -1)\) and
\((1, 1)\) are measured with the pump beam incident to a

FIG. 1: (a) An elongated condensate is pumped by a single
off-resonant laser beam \((\omega_i, \mathbf{k}_i)\). The beam is collec-

tively scattered to two end-fire modes and the atoms get recoiled to

\[(2,-2) \quad (2,0)\]

the discrete side-modes \((m,n)\). (b) The time sequence of our
experiment. When the magnetic trap is turned off, there is a
residual magnetic fields generated, resulted from the different
decay rates of the quadrupole and Ioffe trap. Then after a
delay \( \Delta t \), a pump field with duration \( t \) and intensity \( I \) is in-
cident upon the BEC along the short axis of the condensate
to initiate the superradiant scattering. Finally, after a 22ms
time-of-flight, a probing beam is applied to the system to get
the absorption images of the scattered side-modes.
than the leftward end-fire mode, even though the density of the atomic cloud which functions as the gain medium for the light amplification is lower on the right half of the condensate. The rightward end-fire mode scatters the atoms into the $(1, -1)$ side-mode. This results in a higher atomic population in this mode than that in the $(1, 1)$ mode. To get insight in this unwonted mode amplification, we have to go into the details of the light-atom interaction.

### III. THEORETICAL MODEL OF THE DENSITY DISTORTED MATTER-WAVE SUPERRADIANCE

In the electric dipole approximation, the coupling between the superradiant matter-wave and light fields (the pump field and the end-fire modes) can be described by semiclassical Maxwell-Schrödinger equations \[16\]

\[
\begin{align*}
\text{i}\hbar \frac{\partial}{\partial t} \psi (\mathbf{r}, t) &= -\frac{\hbar^2}{2M} \Delta \psi (\mathbf{r}, t) \\
&\quad + \frac{(\mathbf{d} \cdot \mathbf{E}^{(\pm)}) (\mathbf{d} \cdot \mathbf{E}^{(\pm)})}{\hbar\delta} \psi (\mathbf{r}, t),
\end{align*}
\]

(2)

\[
\frac{\partial^2 \mathbf{E}^{(\pm)}}{\partial t^2} = c^2 \Delta \mathbf{E}^{(\pm)} - \frac{1}{\varepsilon_0} \frac{\partial^2 \mathbf{P}^{(\pm)}}{\partial t^2},
\]

(3)

where, $\psi (\mathbf{r}, t)$ is the atomic wave function. $\mathbf{E}^{(\pm)}$ are the positive and negative frequency parts of the electric field. $\mathbf{P}^{(\pm)} = -\mathbf{d} \psi (\mathbf{r}, t)^2 \frac{\mathbf{d} \mathbf{E}^{(\pm)}(\mathbf{x}, t)}{\hbar\delta}$ is the density distribution of the atomic electric dipoles. $M$ is the atomic mass, $\mathbf{d}$ the atomic dipole moment operator, $\varepsilon_0$ the vacuum dielectric constant, and $c$ the speed of light in vacuum. Here, the local density distribution of the condensate $|\psi(\mathbf{r}, t)|^2$ has to be taken into consideration in Eq. (3) via $\mathbf{P}^{(\pm)}$, because the modification of the light fields propagating through the matter-wave grating is prominent in the conditions of the present experimental parameters. In our experiment, the atomic gas is so dilute that the influence from the inter-particle interaction upon the atomic motion is very weak. Therefore, the nonlinear matter-wave self interacting term is not included in this equation, and the atomic loss term resulted from the nonlinear effects in the time scales of our experiment should also be negligible \[4, 6, 9\].

The Fresnel number $F = \pi D^2/(4\lambda L)$ of the optical pump field in our experiment is around 1, so that we can using the quasi-2D model to investigate the dynamics of the condensate in the superradiant processes. In our experiment, the atomic gas is dilute, thus one can decompose the atomic wave function in the discrete modes \[16\],

\[
\psi (\mathbf{r}, t) = \sum_{n,m} \frac{\psi_{n,m}(z,t)}{\sqrt{A}} e^{-i(\omega_{n,m}(t-nkz-mkz)},
\]

(4)

where $\psi_{n,m}$ represents the wave function of the side-mode $(n, m)$, and $\hbar\omega_{nm} = (n^2 + m^2)\hbar\omega_r$ is the kinetic energy of one single atom in this mode with $\omega_r = \hbar k^2/2M$ the single photon recoil frequency. $A$ is the effective cross section area of the condensate. In the case of an symmetric BEC, $A$ should be constant. Atomic number in the $(n,m)$ mode can be calculated as

\[
N_{n,m} = \int |\psi_{n,m}|^2 dz.
\]

(5)

Similarly to the discrete expansion of the matter-wave function, the optical fields can be expressed in the form of running wave with few frequency components, because the band width of the laser beam is very narrow,

\[
\begin{align*}
E^{(\pm)} (\mathbf{r}, t) &= \varepsilon_\pm e_i e^{\pm i(\omega t - kz)} / 2 \\
&\quad + \varepsilon_+ (z, t) e_i e^{\pm i(\omega t - kz)} \\
&\quad + \varepsilon_- (z, t) e_i e^{\pm i(\omega t + kz)},
\end{align*}
\]

(6)

where $\varepsilon_\pm$ are the envelope functions of the pump laser and the end-fire modes, respectively ($+$ corresponds to the end-fire mode propagating along the positive $z$ direction, and $-$ represents that in the opposite direction). $e_y$ is the unit polarization vector of the electric field along the $y$ direction.

Under the slowly-varying-envelope approximation (SVEA) and using the transformation $\psi_{n,m} \rightarrow \frac{\psi_{n,m}\sqrt{E}}{\sqrt{A}}$, $\varepsilon_{\pm,t} \rightarrow e_{\pm,t} \sqrt{\frac{\hbar k}{2\varepsilon_0 A}}, \tau = 2\omega t$, and $\xi = kz$, Eq. (2) can be
recast to the following dimensionless form,

\[
\frac{i \partial \psi_{n,m}(\xi, \tau)}{\partial \tau} = -\frac{1}{2} \frac{\partial^2 \psi_{n,m}(\xi, \tau)}{\partial \xi^2} - i \kappa \frac{\partial \psi_{n,m}(\xi, \tau)}{\partial \xi}
\]

\[
+ \kappa e^+_{\pm}(\xi, \tau) \psi_{n-1,m+1}(\xi, \tau) e^{i(n-m-2)\tau}
\]

\[
+ e^-_{\pm}(\xi, \tau) \psi_{n-1,m-1}(\xi, \tau) e^{i(n+m-2)\tau}
\]

\[
+ e^+_{\pm}(\xi, \tau) \psi_{n+1,m-1}(\xi, \tau) e^{-i(n-m)\tau}
\]

\[
+ e^-_{\pm}(\xi, \tau) \psi_{n+1,m+1}(\xi, \tau) e^{-i(n+m)\tau},
\]

where \( \kappa = \frac{g}{2\hbar^2} \sqrt{\frac{\xi}{\delta}} \) is the dimensionless coupling constant with \( g = \frac{|d^2 \varepsilon_1|}{2\hbar^2} \sqrt{\frac{\hbar \omega}{2\alpha A L}} \).

Neglecting the retardation effects, the envelope functions \( e_\pm \) of the end-fire modes in the dimensionless form are given by

\[
e_{\pm}(\xi, \tau) = -\frac{\kappa}{\chi} \int d\xi' \sum_{n,m} e^{i(n\pm m)\tau}
\]

\[
\times \psi_{n,m}(\xi', \tau) \psi^*_{n+1,m+1}(\xi', \tau),
\]

with \( \chi = \frac{\hbar}{\delta} \). The two end-fire modes are propagating against each other, hence the spacial integration for \( e_+ \) is taken from \(-\infty \) to \( \xi \), while that for \( e_- \) is taken from \( \xi \) to \( \infty \).

There is one thing that is important we have to emphasize. Besides the fact that, during the interval between the turning off of the magnetic trap and switching-on of the pump pulse, the density distribution of the condensate is distorted by the residual magnetic field as shown in the experimental data in Fig. 2(a), the condensate is also accelerated by the unbalanced residual magnetic force. This has been confirmed by the measured center of mass motion of the condensate in our experiment. Then after long enough time, at which the magnetic force died out, the BEC acquires a constant velocity \( v_0 \). In a typical matter-wave interference experiment, the interference patterns of the scattered matter-wave is sensitively dependent on the local phase and density distributions of the condensate. However, we find that, in our situation, the local phase imprinted upon the condensate by the atomic acceleration has little influence upon the dynamics of the condensate (for making good consistency of the different parts of the paper, the detailed analysis of the influence upon the atomic motion from the acceleration is presented in the Appendix). In drastic contrast, even a tiny disturbance of atomic density distribution would generate prominent asymmetric interference patterns of the scattered matter-wave in the superradiant process. This makes our experiment a rather robust technique for measuring the geometric symmetry of BECs with high precision.

IV. THE DENSITY MODULATION ANALYSIS AND NUMERICAL SIMULATION

Now we consider the influence upon the atomic motion and modes amplification from the distorted density distribution of the condensate. As shown in the experimental data in Fig. 2(a), the left half of the condensate has a higher density than that of the right half. Thus the superradiant scattering in the denser side would have a greater gain leading to a coordinate-dependent coupling constant \( \kappa \) in Eq. (7) via the \( z \)-dependence of the effective cross section area of the condensate \( A \). To describe such a mechanism that the atomic density distortion modifies the matter-wave superradiance, we multiply a factor \( S^{-2}(z) \) to the effective cross section area of the condensate \( A \rightarrow A/S^2(z) \). In the regime where the distortion is very weak, \( S(z) \) can be approximated up to the linear order of \( z \), \( S(z) = (1 + \frac{z}{\Lambda} + O(z^2)) \), with a parameter \( \Lambda \) that should be fitted from the experimental data. During the radial expansion, the atomic number \( N(z) = |\psi_{00}(z)|^2 \) will not change, thus the asymmetric cross section \( A/S^2(z) \) would lead to a transformation of the coupling constant \( \kappa \rightarrow \kappa(1 + \xi/\Lambda) \) with \( \Lambda = kL \). The envelope of the density distribution of the initial distorted condensate \( \psi \) used in our simulation is shown in Fig. 3(a). It is a modified function in Thomas-Fermi approximation \( \psi = \sqrt{N/\Lambda}(1 + \xi/\Lambda)\psi_0 \), where \( |\psi_0(z)|^2 = 3/4L^2[(L/2)^2 - z^2]\Theta(L/2 - |z|) \) with \( \Theta(z) \) the Heaviside step function. \( \psi_0 \) is a seed wave function indicating one atom in a side-mode. To measure the asymmetry of the initial BEC, we define a parameter \( dz \) in describing the displacement of the peak density point of the condensate from its geometric center. In our simulation, \( dz = 3.16\mu m \). This tiny shift is hardly measurable in ordinary imaging system. However, by monitoring the symmetry of the dynamics of the scattered matter-wave in the superradiant process, one can obtain information of the geometric symmetry of the condensate, which is exclusively available in using the \textit{in situ} imaging methods [20, 21].

A. Early stage

At the early stage of the superradiant process, depletion of the condensate is negligible and only the two first-order side-modes are dominantly excited with exponential amplification. The coupling between each of these modes to the ground state reads

\[
\frac{\partial \psi_{1, \pm 1}}{\partial \tau} = -\frac{1}{2} \frac{\partial^2 \psi_{1, \pm 1}}{\partial \xi^2} - i \frac{\partial \psi_{1, \pm 1}}{\partial \xi} + \kappa e_{\pm} S^2 \psi_{0,0},
\]

with the electric field in each end-fire mode taking the following form,

\[
e_{\pm}(\xi, \tau) = -\frac{i\kappa S(\xi)}{\chi} \int d\xi' \psi_{00}(\xi') \psi_{1, \pm 1}^*(\xi', \tau) S^2,
\]

where, \( \psi_{00} = \sqrt{N} \psi_0, \psi_{1, \pm 1} = \sqrt{N_{1, \pm 1}} \psi_{1, \pm 1} \). The first two terms on the right hand side of Eq. (10) represent the kinetic energy of each side-mode, while the last term describes the coupling between \( \psi_{00} \) and \( \psi_{1, \pm 1} \) by the electric field \( e_{\pm} \), respectively.
In the early stage $\tau \to 0$, the first two kinetic terms in Eq. (9) are negligible, leading to an exact expression of the atomic population in each side-mode as,

$$N_{1,\pm 1}(\tau) = N_{1,\pm 1}(0)e^{G_{1,\pm 1}\tau},$$

with

$$G_{1,1} = \frac{N\kappa^2}{\chi} \int_{-\infty}^{\infty} d\xi \psi_\pm^*\psi_\pm S^2 \int_{-\infty}^{\xi} d\xi' \psi_\pm^*\psi_\pm S^2,$$

$$G_{1,-1} = \frac{N\kappa^2}{\chi} \int_{-\infty}^{\infty} d\xi \psi_\pm^*\psi_\pm S^2 \int_{-\infty}^{\xi} d\xi' \psi_\pm^*\psi_\pm S^2.$$

The integrals in Eqs. (12) and (13) can be numerically carried out to give in the dimensionless notion $G_{1,1} = 16.7$ and $G_{1,-1} = 15.9$, implying that the atomic populations in these two side-modes increase exponentially at the beginning of the matter-wave superradiant process as shown in the experimental data and numerical simulation in Fig. 3(b). Even though such growing is unbalanced for the $(1,\pm 1)$ modes, the asymmetry of the matter-wave superradiance is hardly visible in experiment in this limit, because the atomic populations in these two modes are too small. However, the numerical simulation is consistent with the experimental data in predicting the exponential growing of the atomic numbers in the side-modes in this stage.

**B. Long pulse regime**

In the long pulse regime, things are quite different. Equation (11) is not valid in describing the dynamics of the matter-wave superradiance any more. By solving the semiclassical equations (7) and (8), we can calculate the atomic populations on the different side-modes. Our numerical stimulation has taken into account the coupling between the side-modes of high orders to a large cut-off. The result in this regime is also contained and shown in Fig. 3(b) with a fitted parameter $l = -500 \mu m$. To fit the experimental result with the quasi-two dimensional model, the numerical result is scaled by a factor of 2. This discrepancy is partly due to the collision between the side-modes and the condensate (12) (22). At the same time, in a real experiment, plenty of atoms in the condensate far away from the center with a much lower density are not included in our theoretical model. The superradiant gain is dependent on the density of the BEC, thus the collective gain does not occur below a critical density (22), then a big amount of these atoms will stay in their initial state, leading to a greater atomic population in the $(0,0)$ mode in experiment than that predicted by the numerical calculation.

It is known that, for BEC prepared in trap of perfect symmetry in the longitudinal direction, there is no asymmetry in the atomic occupations among the different side-modes in the time domain in the matter-wave superradiant process, provided that the pump field is incident on...
the condensate along the radial direction of BEC and the atomic distribution maintains unperturbed during the superradiant process. For example, for the superradiant process happens in the trap, there is no asymmetry between the two side-modes (1,±1) as shown in Fig. 2(b).

While, for BEC in the unbalanced trap with the time controlling sequence chosen such that ∆t = 500µs in our experiment, the condensate is shocked and distorted by the residual magnetic force to take the shape like that in Fig. 2(a). The matter-wave superradiance shows unconventional mode amplification behaviors. It is seen in Fig. 3(c) that, the ε₊ and ε₋ end-fire modes are mainly amplified and concentrated in the right part and left part of the condensate, respectively, and the condensate here functions as the gain medium for the optical mode amplification. However, for longer pulse durations, even though the density in the left side of the condensate is higher than that in the left side, the (1,−1) mode grows faster than the (1,1) mode, implying that ε₊ end-fire mode is more quickly amplified than that propagating in the opposite direction. This is in contradiction with our common cognition that the optical mode should be better amplified provided that the gain medium is denser.

The above abnormal side-mode amplification can be well explained by investigating the mode competition in the central region of the condensate. As the duration of the pump laser is prolonged, the atoms in the outer regime of the condensate have been greatly exhausted (as shown in Fig. 3(d)). At long pulse durations, there are more atoms being scattered to (1,−1) mode by the ε₊ light, leading to a higher atomic population in the (1,−1) side-mode. This phenomenon has been observed in the experiment as shown in Fig. 2(b).

As the duration of the pulse field further increases, high order side-modes such as the (2,0) mode begin to participate in the matter-wave superradiant process, as shown in Fig. 3(d). At t = 120µs, there has been a considerable number of atoms that are excited to the (2,0) mode. The pump laser, together with the light fields in all the end-fire modes, couples the different atomic side-modes with each other. This makes the atomic superradiance a process rather like the atomic Bragg diffraction. Besides the fact that the (1,−1) mode is stronger than the (1,1) mode, the atomic population in each side-mode begins to oscillate in the time domain.

V. CONCLUSION

In this paper, we have demonstrated experimentally and investigated asymmetric matter-wave superradiant scattering of a distorted BEC by measuring the dynamics of atoms in the two dominantly occupied first-order side-modes. It is found that, even a tiny atomic density disturbance would lead to prominent asymmetric atomic distributions among the different superradiant side-modes in the long pulse regime. The matter-wave side-modes are amplified in an unexpected way because of the abnormal magnification of the end-fire mode in diluter part of the BEC resulted from the mode competitions in the central part of the condensate. By means of superradiance, we have successfully signalized the symmetry of the atomic distribution of the condensate and finally measured it in the experiment. Our experiment is stable against the non-stationary disturbance, and presents a new way to detect the geometric symmetries of the density distribution of BEC.

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Appendix A: The influence from the atomic acceleration

In this appendix, we are going to prove that, the acceleration of the condensate will not add asymmetry to the matter-wave superradiant interference patterns, which is usually not true for the ordinary matter-wave interference experiment. This will make our experimental technique a rather stable method in detecting the geometric symmetry of BEC or in other potential applications against dynamical disturbance.

In the experiment, the condensate has been accelerated by the unbalanced magnetic trap to move at a velocity v₀ in the positive z direction. However, by changing the reference frame to the one moving with the condensate, it can be find that Eq. 7 is invariant under such a Galilean transformation. In fact, the pump laser together with each of the generated end-fire modes (with wave vector k± and frequency ω±) creates a pair of moving optical lattices with intensity \( I_±(r,t) \propto \cos(q_± \cdot r - \Delta \omega_± t) \) as shown in Fig. 4 in which \( q_± = k_± - k_±, \Delta \omega_± = \omega_1 - \omega_2 \).
moving with velocity $v_0$ along the $x$ and $y$ directions at a speed $2\omega_r/\hbar$ generated optical lattices move against each other in the $z$ direction. That is to say, these two generated optical lattices move against each other in the $z$ direction at a speed $2\omega_r/\hbar$.

When the condensate has an initial velocity $v_0 = 2\omega_r m'/k$, where $m'$ can be fractional, the frequency of each of the scattered light will have a Doppler shift, leading to the modification,

$$\Delta\omega_{\pm} \rightarrow \Delta\omega'_{\pm} = 2\omega_r(1 \pm m'),$$  \hspace{1cm} (A1) \nonumber

and resulting in a new sets of lattices with modified speed $v'_0 = 2\omega_r(\mp 1 + m')$ in the $z$ direction, i.e., compared to the stationary case, the speed of the generated optical lattices are both increased by $2\omega_r m'/\hbar$, which is exactly the same as $v_0$. In other words, the matter-wave superradiance of a moving condensate is equivalent with performing the superradiant experiment of a stationary BEC in a moving laboratory. Therefore, the atomic acceleration will induce no asymmetries in the atomic dynamics in the matter-wave superradiant processes.

Even though, the atomic acceleration appends a phase factor $e^{i(k'x - \omega't)}$ (where, $k' = \frac{Mv}{\hbar}$, $\omega' = \frac{Mv^2}{2\hbar}$) to the wave function of the condensate, this space and time dependent factor can be canceled by a unitary transformation. Therefore, it will not influence the symmetry of superradiant scattering in time domain, even though it seems that the directed acceleration of the condensate breaks the spacial inversion symmetry of the system and may lead to asymmetric dynamics in the matter-wave superradiant process. Thus any phase disturbance induced from the acceleration of the condensate by instability of the experimental setup will not influence the symmetry of the measured matter-wave superradiance interference patterns. This makes our experiment a rather stable technique for investigating the matter-wave superradiance against non-stationary disturbances.

FIG. 4: Schematic of matter-wave superradiance of a condensate moving at a velocity $v_0$. There are two moving optical lattices (with wave vector $q_{\pm}$) formed by the superposition the pump pulse and the light field in each of the end-fire modes, moving with velocity $v'_\pm$ along the $z$ direction, respectively.

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