Spin Bottlenecks in the Quantum Hall Regime

A.H. MacDonald

Department of Physics, Indiana University, Bloomington, IN 47405
(October 15, 2018)

We present a theory of time-dependent tunneling between a metal and a partially spin-polarized two-dimensional electron system (2DES). We find that the leakage current which flows to screen an electric field between the metal and the 2DES is the sum of two exponential contributions whose relative weights depend on spin-dependent tunneling conductances, on quantum corrections to the electrostatic capacitance of the tunnel junction, and on the rate at which the 2DES spin-polarization approaches equilibrium. For high-mobility and homogeneous 2DES’s at Landau level filling factor \( \nu = 1 \), we predict a ratio of the fast and slow leakage rates equal to \((2K+1)^2\) where \( K \) is the number of reversed spins in the skyrmionic elementary charged excitations.

There has been renewed interest recently in nonequilibrium spin accumulation due to electronic transport in spin-polarized electron systems, in part because these accumulations are important in giant magnetoresistance. In this paper we address spin-accumulation in the tunneling current between a metal and a two-dimensional electron system (2DES) in the quantum Hall regime. Our work is motivated in part by recent experiments which have discovered unexplained two-rate leakage currents for such tunnel junctions, and also in part by the long spin-relaxation times of 2DES’s, especially long in the quantum Hall regime. We find that spin accumulation depends subtly on the interplay of spin-dependent tunneling conductances, thermodynamic densities-of-states, and spin relaxation rates. According to our theory, the double-rate current found in experiments which have discovered unexplained two-rate leakage currents for such tunnel junctions, and also in part by the long spin-relaxation times of 2DES’s, especially long in the quantum Hall regime. Gapped quantum Hall states lead to rapid variation of the chemical potential with density, and the presence of Skyrmion elementary charged excitations requires an unusual spin-relaxation process. We predict that, for homogeneous 2DES and thin tunneling barriers, the ratio of fast and slow relaxation rates at \( \nu = 1 \) will equal \((2K+1)^2\), where \( K \) is the number of reversed spins in the lowest energy Skyrmion quasiparticles.

We start from the following phenomenological linear response equations which we believe to be broadly applicable for tunneling between a metal and a 2DES.

\[
\begin{align*}
\dot{Q}_\uparrow &= -\mu_\uparrow G_\uparrow + (\mu_\downarrow - \mu_\uparrow)G_\downarrow, \\
\dot{Q}_\downarrow &= -\mu_\downarrow G_\downarrow + (\mu_\uparrow - \mu_\downarrow)G_\uparrow.
\end{align*}
\]

Here \( Q_\sigma \) and \( \mu_\sigma \) are the spin-\( \sigma \) particle-number and chemical potential in the 2DES. These equations express spin-partitioning of the tunneling current; the assumption of separate chemical potentials for the 2DES spin subsystems is valid when the spin-relaxation time is much longer than other characteristic scattering times in the 2DES. In these equations we have placed the zero of energy at the chemical potential of the metal. The two terms on the right hand side of Eqs. account respectively for tunneling across the junction and relaxation of the 2DES spin subsystems toward mutual equilibrium.

A closed description of electron transport in the system requires, in addition to the above conductance equations, which relate currents to chemical potential differences, a set of capacitance equations which relate these chemical potentials to accumulated charges:

\[
\begin{align*}
\mu_\uparrow &= -V_0 + (C^{-1})_{\uparrow\uparrow}Q_\uparrow + (C^{-1})_{\uparrow\downarrow}Q_\downarrow \\
\mu_\downarrow &= -V_0 + (C^{-1})_{\downarrow\uparrow}Q_\uparrow + (C^{-1})_{\downarrow\downarrow}Q_\downarrow
\end{align*}
\]

Here \( V_0 \) represents the electrostatic contribution from charges external to the 2DES. Elements of the inverse capacitance \((C^{-1})_{\sigma\sigma'}\) matrix have an electrostatic contribution proportional to the width of the tunnel barrier and a quantum ‘chemical potential’ contribution due to the Fermi statistics and correlations of electrons in the 2DES:

\[
(C^{-1})_{\sigma\sigma'} = \frac{1}{C_g} + \frac{1}{A} \frac{d\tilde{\mu}_\sigma}{dn_{\sigma'}} = \frac{1}{C_g} + F_{\sigma,\sigma'}
\]

where \( C_g = Ae/(4\pi\epsilon^2d) \) is the electrostatic capacitance of the junction, \( A \) is the cross-sectional area of the two-dimensional electron system, \( \epsilon \) is the dielectric constant of the host semiconductor, \( d \) is the distance between the metallic electrode and the 2DES, and \( \tilde{\mu}_\sigma \) is the spin-\( \sigma \) chemical potential of the 2DES relative to its electric subband energy. The notation above is motivated by the relationship between \( F_{\sigma,\sigma'} \) and Fermi liquid theory interaction parameters. In the commonly employed Hartree mean-field approximation,

\[
F_{\sigma,\sigma'} = \frac{\delta_{\sigma,\sigma'}}{AD_{\sigma}^2}
\]
where $D_0^s$ is the non-interacting 2DES density-of-states per spin. In general $dP_{σ}/dt_{σ'}$ is non-zero for $σ ≠ σ'$ because of electronic correlations.

These equations can be used to describe various time-dependent or ac transport experiments: we apply them here to the situation studied recently by Chan et al. in which a chemical potential difference across the junction is created by external charges and the leakage current which reestablishes equilibrium is measured as a function of time. In an obvious matrix notation we rewrite the conductance and capacitance equations as $Q = Gμ$ and $G = −V_0 + C^{-1}Q$. Eliminating the chemical potentials using the capacitor equations yields a set of two coupled first-order inhomogeneous linear differential equations for the time-dependent spin-up and spin-down charges in the 2DES. Solving these with the boundary condition $Q_σ(t = 0) = Q_σ(t = 0) = 0$ yields the spin-dependent currents into the 2DES:

$$\dot{Q}_σ(t) = I_{σ, +} \exp(-t/τ_+) + I_{σ, -} \exp(-t/τ_-)$$

(6)

where $τ_+^{-1}$ and $τ_-^{-1}$, generalized RC relaxation rates, are the eigenvalues of $A = GC^{-1}$. We have obtained the following explicit expressions for $I_{σ, ±}$:

$$I_{σ, +} = \frac{G_+}{2} \left( \frac{A_{σ, τ} - A_{σ, τ}}{τ_+ - τ_-} + \frac{G_1 A_{σ, τ}}{τ_+ - τ_-} \right)$$

(7)

$$I_{σ, -} = \frac{G_+}{2} \left( 1 - \frac{A_{σ, τ} - A_{σ, τ}}{τ_+ - τ_-} \right) - \frac{G_1 A_{σ, τ}}{τ_+ - τ_-}.$$  

(8)

The corresponding expressions for $I_{±}$ are obtained by interchanging spin labels. In Fig. 1 we show results obtained for the dependence of leakage current on 2DES chemical potential near Landau level filling factor $ν = 1$ which follow from a non-interacting Skyrmion model of the 2DES described above. These replicate all major features found in experiment. The peak in both fast and slow relaxation rates is due to the sharp decrease in capacitance as the $ν = 1$ incompressible quantum Hall state is approached. The leakage current is dominated by the slow process, except in a narrow range very close to $ν = 1$ where the fast process takes over. The origin of this crossover is explained below.

Similar results can be obtained for the instantaneous chemical potentials of the spin-up and spin-down subsystems:

$$µ_σ(t) = −V_0 + ∑_σ μ_σ,s (1 - \exp(-t/τ_σ))$$

(9)

where $μ_σ, ± = ∑_σ C_{σ, ±}^{-1} I_{σ, ±}$. We note that $μ_σ, + + μ_σ, - = V_0$; current flows until the electrochemical potential change for each spin cancels the electric potential from external charges. The two spin subsystems are in equilibrium at both the beginning and the end of the relaxation process, but are, in general, out of equilibrium at intermediate times. The non-equilibrium spin accumulation $µ_σ(t) − μ_σ(0) = (µ_σ, + − µ_σ, -)(exp(-t/τ_+) − exp(-t/τ_-))$. What is readily separated in experiment are the fast and slow relaxation contributions to the current, not the spin subsystem contributions. Nevertheless, we see that non-equilibrium spin accumulations occur system between times $τ_+$ and $τ_-$ whenever both contributions are present. In Fig. 2 we plot, time-dependent chemical potentials calculated for the non-interacting Skrymion model at $μ = 0.05(e^2/ℓ)$.

The chemical potentials change quickly on a time scale $τ_+$ and $µ_σ$ overshoots $V_0$ on a much longer time scale the two chemical potentials approach $V_0$ from opposite sides. We note from Fig. 1 that the slow leakage current process actually dominates the capacitance at this value of $µ$; the naive view that the fast process is current flow to the 2DES while the slow process is spin-equilibration is incorrect.

Before turning to the quantum Hall regime, where non-equilibrium spin-accumulations are large, it is instructive to examine several limits for which spin-accumulation does not occur. For $F_{σ, σ'} C_g << 1$ we find that $τ_+^{-1} = (G_1 + G_σ) / C_g$, $τ_-^{-1} → 0$, $I_{σ, +} = G_σ V_0$ and $I_{σ, -} → 0$. In this limit, which usually holds for metallic electrodes, spin-independent electrostatic contributions dominate electrochemical potential changes; no non-equilibrium spin-accumulation occurs because the spin subsystems are not driven from equilibrium by electrostatic potentials. For strong tunnel barriers ($G_σ << G_g$), on the other hand, $τ_+^{-1} = G_σ(F_{τ, +} + F_{τ, -} - 2F_{τ, ±})$, $τ_-^{-1} = (G_τ + G_τ)(F_{τ, +} F_{τ, ±}) / (F_{τ, +} + F_{τ, ±} - 2F_{τ, ±})$ and the fast relaxation current $I_+ ≡ I_{σ, +} + I_{σ, -} = 0$. For this limit spin accumulation does not occur because the relaxation processes are fast enough to maintain instantaneous equilibrium. Unlike the electrostatic-dominance case discussed first, the leakage current flows at the slow rate $τ_-^{-1}$. A third more subtle limit in which spin-accumulation does not occur applies to Fermi gas 2DES’s in which we may ignore correlation contributions to the chemical potential and the commonly adopted forms $G_σ = c D_σ$, $G_σ = c_D D_1 D_2$ hold. These expressions result from golden rule estimates of quasiparticle tunneling and spin-flip transition rates respectively, $c$ is a constant which declines exponentially with the thickness of the tunneling barrier and $c_D$ is a constant dependent on spin-orbit scattering strength in the 2DES. For this model we find that $τ_+^{-1} = c/2(C_g + D_1 + D_2)$, $τ_+^{-1} = (c + c_D)(D_1 + D_2)/C_g$ and all the weight is in the fast leakage current. No spin-accumulation occurs because the spin subsystems are not coupled by interactions and the ratio of tunneling conductances equals the ratio of the rates at which the chemical potentials increase with density. Finally we mention the case in which the 2DES is paramagnetic to which we return below. For $G_τ = G_τ$ and $F_{τ, +} = F_{τ, -}$, symmetry forbids spin accumulations.
An explicit calculation finds no weight for the slow leakage current and the rate ratio

$$\frac{\tau_-}{\tau_+} = \frac{G_1(2/C_2 + F_{\uparrow,\uparrow} + F_{\downarrow,\downarrow})}{(G_1 + 2G_2)(F_{\uparrow,\downarrow} - F_{\downarrow,\uparrow})}. \quad (10)$$

None of these limits apply throughout the quantum Hall regime. Near integer Landau level filling factors, Fermi statistics and correlations in the 2DES, not electrostatics, dominate the electrochemical potential changes with density $\rho$. Equilibrium electronic states contain complex Skyrmion quasiparticles whose formation from the fully spin-polarized $\nu = 1$ ground state cannot be achieved by a single-particle process. Spin-equilibration will therefore be slow $\tau_+$. The two spin subsystems are intricately coupled so that the Fermi gas limit does not apply. Furthermore, the 2DES will generally be strongly spin-polarized.

A simple model of the 2DES valid at low temperature for $\nu$ near one, is obtained by ignoring interactions between Skyrmions. We obtain the following grand-canonical ensemble expressions for the occupation probabilities of the $N_\phi = A/(2\pi\ell^2)$ Skyrmion quasielectron and quasihole states with $K$ excess reversed spins:

$$n_{K\uparrow} = f(\epsilon_K + K\mu_\uparrow - (K + 1)\mu_\downarrow) \quad (11)$$

$$n_{K\downarrow} = f(\epsilon_K + (K + 1)\mu_\uparrow - K\mu_\downarrow). \quad (12)$$

Here $f(\epsilon) = (\exp(\epsilon/k_B T) + 1)^{-1}$ is a Fermi factor $\frac{1}{2}$, $\epsilon_K$ is the energy of a Skyrmion quasiparticle, $(2\pi\ell^2)^{-1}$ is the density of a full Landau level, and we have chosen the zero of energy so that quasielectron and quasihole Skyrmion states have the same energy $\frac{1}{2}$. When the spin subsystems are in equilibrium ($\mu_\uparrow = \mu_\downarrow = \mu$) we can use Eqs. (12) to calculate the chemical potential, given the Landau level filling factor. Eqs. (12) express the property that the $K$-th quasielectron Skyrmion is formed by adding $K + 1$ spin-down electrons and removing $K$ spin-up electrons from to the $\nu = 1$ ground state. For non-interacting electrons only $K = 0$ quasiparticles occur; for typical 2DES’s, on the other hand, the lowest energy quasiparticles have $K = 3$ $\frac{1}{1}$ and $F_{\uparrow,\downarrow}$ occur in the ratio $(K + 1)^2 : K^2 : K(K + 1) = 16 : 9 : 12$. This contrasts with the non-interacting electron case for which $F_{\uparrow,\uparrow}$ and $F_{\downarrow,\downarrow}$ vanish. (The same ratios apply for $\nu < 1$ with inverted spin-indices.) For $\nu = 1$ the low-temperatures equilibrium state charge is added to the $\nu = 1$ state in the form of $K = 3$ skyrmions, i.e. for each 4 up-spins added to the 2DES, three down spins are removed. It follows that the time-integrated spin-up and spin-down leakage currents are approximately equal in magnitude and opposite in sign.

In contrast to the partitioning of total leakage charge between spins, which is determined purely by thermodynamic considerations, the observable partitioning between fast and slow components is difficult to understand intuitively in the general case. Simplification occurs, however, when $\nu \approx 1$. It follows from particle-hole symmetry $\frac{2}{2}$, that the symmetries for leakage into a paramagnetic 2DES also hold at $\nu = 1$, explaining the vanishing weight in the slow leakage current channel seen in Fig. [1] as the filling factor approaches one. The ratio of fast to slow leakage rates, (Eq. [11]) depends only on thermodynamic quantities provided that the tunneling barrier is thin enough that $G_\uparrow >> G_\downarrow$. Then, provided that the temperature is sufficiently low that quantum terms dominate the inverse capacitance we find that $\tau_-/\tau_+ = (2K + 1)^2 = 49$, in rough agreement with the findings of Chan et al. $\frac{1}{1}$. We ascribe discrepancies to the inhomogeneity present at integer filling factors in all current samples.

I thank Ho Bun Chan and Ray Ashoori for stimulating discussions and the ITP at UC Santa Barbara for hospitality during a Quantum Hall Workshop. This work was supported by NSF Grants DMR-9714055 and PHY94-07194.

[1] M. Johnson and R.H. Silsbee, Phys. Rev. Lett. 55, 1790 (1985); M. Johnson, Phys. Rev. Lett. 70, 2142 (1993).

[2] A. G. Aronov, Pis'ma Zh. Eksp. Teor. Fiz. 24, 37 (1976) [JETP Lett. 24, 32 (1976)]; P.M. Tedrow and R. Merserey, Phys. Rep. 238, 174 (1994).

[3] T. Valet, and A. Fert, Phys. Rev. B 48, 7099 (1993); M.A.M. Gijs and G.E.W. Bauer, Adv. Phys. 46, 285 (1997); J-Ph Ansermet, J. Phys. C 10, 6027 (1998).

[4] H.B. Chan, R.C. Ashoori, L.N. Pfeiffer, and K.W. West, preprint (1999).

[5] J.M. Kikkawa and D.D. Awschalom, Phys. Rev. Lett. 80, 4313 (1998).

[6] A. Berg, M. Dobers, R.R. Gerhardts, and K. v. Klitzing, Phys. Rev. Lett. 64, 2563 (1990); S. Kronmuller, W. Dietsche, K. v. Klitzing, G. Denninger, W. Wegscheider, and M. Bichler, Phys. Rev. Lett. 82, 4070 (1999).
[7] N.N. Kuzma, P. Khandelwal, S.E. Barrett, L.N. Pfeiffer, and K.W. West, Science 281, 686 (1998).
[8] I.D. Vagner and T. Maniv, Phys. Rev. Lett. 61, 1400 (1988); Dimitri Antoniou and A.H. MacDonald, Phys. Rev. B 43, 11686 (1991).
[9] T.P. Smith, W.I. Wang, and P.J. Stiles, Phys. Rev. B 34, 2995 (1986); V. Mosser, D. Weiss, K. v. Klitzing, K. Ploog, and G. Weimann, Solid State Commun. 58, 5 (1986); S.V. Kravchenko, D.A. Rabinberg, S.G. Semenichsky, and V.M. Pudalov, Phys. Rev. B 42, 3741 (1990); J.P. Eisenstein, L.N. Pfeifer, and K.W. West, Phys. Rev. Lett. 68, 674 (1992).
[10] S.L. Sondhi, A. Karlhede, S.A. Kivelson, and E.H. Rezayi, Phys. Rev. B 47, 16419 (1993); H.A. Fertig, L. Brey, R. Côté, and A.H. MacDonald, Phys. Rev. B 50, 11018 (1994); H.A. Fertig, L. Brey, R. Côté, and A.H. MacDonald, Phys. Rev. Lett. 77, 1572 (1996).
[11] We use Fermi statistics for the Skrymions to crudely represent their repulsive interactions. These calculations are all in the dilute Skyrmion limit where the actual particle statistics play at most a minor role.
[12] Kun Yang and A.H. MacDonald, Phys. Rev. B 51, 17247 (1995).
[13] J.J. Palacios, D. Yoshioka, and A.H. MacDonald, Phys. Rev. B 54, R2296 (1996).

FIG. 1. Leakage current between a metallic electrode and a 2DES as a function of chemical potential near \( \nu = 1 \). The chemical potential is in \( e^2/\ell \) units, and the zero of energy is chosen so that \( \mu = 0 \) at \( \nu = 1 \). The solid and dotted lines show leakage rates in fast and slow channels respectively, while the long and short dashed lines show the capacitance contributions, \( I_+ \tau_+ \) and \( I_- \tau_- \), from fast and slow rate channels respectively. These curves were calculated using the non-interacting Skyrmion model explained in the text to evaluate the quantum inverse capacitance contributions and a separation \( d = 5\ell \) between the metal and the 2DES. \( G_\sigma \) and \( G_{\sigma'} \) were assumed to have the form \( 1000/F_{\sigma,\sigma} + 10 \) and \( 1/(F_{\uparrow,\uparrow} F_{\downarrow,\downarrow}) + 0.01 \) in arbitrary units; the two terms representing uniform system golden rule and inhomogeneity contributions respectively. The leakage rates are in units of \( C_{\sigma}^{-1} \) times the conductance units and the capacitances per unit area are in units of \( \ell^{-1} \). This figure is for \( k_B T = 0.025 e^2/\ell \).

FIG. 2. \( \mu_\sigma + V_0 \) in units of \( V_0 \) as a function of time for the model of Fig. 1 at \( \mu = 0.05 e^2/\ell \). The maximum time illustrated is, \( 10^{-4} \) units so that \( t/\tau_+ \) large but \( t/\tau_- \) is still small at the largest times.
FIG. 3. Quantum contributions to the inverse capacitance for a 2DES near $\nu = 1$. The solid lines show the majority and minority spin results for $F_{\sigma,\sigma}$ for a non-interacting model which contains only $K = 0$ quasiparticles. For the Skrymion model, $F_{\uparrow,\uparrow}$, $F_{\downarrow,\downarrow}$ and $F_{\uparrow,\downarrow}$ are shown by dotted, short-dashed, and long-dashed lines respectively.