THE ROLE OF BAG SURFACE TENSION IN COLOR CONFINEMENT

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We discuss here the novel view at the color confinement which, on the one hand, allows us to find out the surface tension coefficient of quark gluon bags and, under a plausible assumption, to determine the endpoint temperature of the QCD phase diagram, on the other hand. The present model considers the confining color tube as the cylindrical quark gluon bag with non-zero surface tension. A close inspection of the free energies of elongated cylindrical bag and the confining color tube that connects the static quark-antiquark pair allows us to find out the string tension in terms of the surface tension, thermal pressure and the bag radius. Using the derived relation it is possible to estimate the bag surface tension at zero temperature directly from the lattice QCD data and to estimate the (tri)critical endpoint temperature. In the present analysis the topological free energy of the cylindrical bag is accounted for the first time. The requirement of positive entropy density of such bags leads to negative values of the surface tension coefficient of quark gluon bags at the cross-over region, i.e. at the continuous transition to deconfined quarks and gluons. We argue that the cross-over existence at supercritical temperatures in ordinary liquids is also provided by the negative surface tension coefficient values. It is shown that the confining tube model naturally accounts for an existence of a very pronounced surprising maximum of the tube entropy observed in the lattice QCD simulations, which, as we argue, signals about the fractional surface formation of the confining tube. In addition, using the developed formalism we suggest the gas of free tubes model and demonstrate that it contains two phases.

1 Introduction

A new paradigm of heavy ion phenomenology that the quark gluon plasma (QGP) is a strongly interacting liquid \cite{1} proved to be very successful not only in describing some of its properties measured by lattice quantum chromodynamics (QCD), but also in explaining some experimental observables that cannot be reproduced otherwise. Probably, the two most striking conclusions obtained within the new paradigm are as follows: first, at the cross-over temperature, where the string tension of color tube is almost vanishing, the potential energy of color charge is of the order of a few GeV \cite{2}, i.e. it is 10 times larger than its kinetic energy, and, second, the QGP, so far, is the most perfect fluid since its shear viscosity in units of the entropy density is found to be the smallest one \cite{3, 4}. The first of these conclusions tells us that at the cross-over region there is no color charge separation \cite{5}, whereas the second one naturally explains the great success of ideal hydrodynamics when applied to relativistic heavy ion collisions.

Here we would like to discuss the recent progress achieved in our understanding of both the confinement phenomenon \cite{6, 7} and the physical origin of the cross-over \cite{8, 9, 10, 11, 12}. As we demonstrate below such a progress was made possible after realizing a principal role played by negative values of the surface tension coefficient of large QGP bags \cite{6, 8}. Also here we argue that the negative values of the surface tension, that are responsible for an existence of the cross-over transition to QGP at low baryonic densities, play the same role in ordinary liquids. Moreover, in this work we would like to draw an attention to the problem of the temperature dependence of surface tension coefficient in liquids by clearly showing that for many liquids the well known Guggenheim relation (see Eq. (10)) is not so well established experimentally as it is usually believed.

The work is organized as follows. Section 2 is devoted to the confining tube model, in which the Fisher topological term of the QGP bag free energy is accounted for. In section 3 we show that at the cross-over region the surface tension of QGP bags is necessarily negative and argue that this is the case for ordinary liquids as well. The maximum of the tube entropy observed in lattice QCD is explained in section 4, where the model of gas of free tubes is also developed. The conclusions are given in the last section.

2 Color confining tube and sQGP

A color confinement, i.e. an absence of free color charges, is usually described by the free energy of heavy (static) quark-antiquark pair $F_{Q\bar{Q}}(T, L) = \sigma_{str} \cdot L$. In the lattice QCD the functional dependence of $F_{Q\bar{Q}}(T, L)$ on the temperature $T$ and the separation distance $L$ can be extracted from the Polyakov line correlation in a color singlet channel. Then it is customary to define
The internal energy $U_\infty$ (left) and entropy $S_\infty$ (right) of confining tube connecting two static color charges found by lattice QCD simulations for infinite separation distance between charges [2]. The internal energy is shown for 2 quark flavors.

- **confinement**: the case of non-zero string tension, i.e. $\sigma_{str} > 0$;
- **deconfinement**: the case of vanishing string tension $\sigma_{str} \to 0$ at $T \to T_{co}$, but one should remember that there is no color charge separation up to $T \geq 1.3 T_{co}$ values of the cross-over temperature $T_{co}$ [11,15].

The explanation of the latter is as follows: although at large distances $L \to \infty$ the potential energy of static $q\bar{q}$ pair is finite $U_{q\bar{q}}(T, L) = F_{q\bar{q}} - T^2 \frac{\partial F_{q\bar{q}}}{\partial T} = F_{q\bar{q}} + TS_{q\bar{q}}$ near $T_{co}$, the values of $U(T, \infty)$ are very large (see Fig. 1). From Fig. 1 one can conclude that near $T_{co}$ region QGP is a strongly interacting plasma (sQGP) which is similar to a liquid, since the ratio of the quark potential energy to its kinetic energy, the so called plasma parameter, $\frac{U(T, \infty)}{T^4} \in 1 - 10$ has the range of values that is typical for ordinary liquids [11].

The second striking feature of the confining tube can be seen in the right panel of Fig. 1 which clearly demonstrates that at $T = T_{co}$ the entropy of static $q\bar{q}$ pair is very large $S_{q\bar{q}}(T_{co}, \infty) \approx 20$. Such a value signals that really a huge number of degrees of freedom $\sim \exp(20)$ is involved, but the origin of large energy $U_{q\bar{q}}(T, \infty)$ and entropy $S_{q\bar{q}}(T, \infty)$ values near $T_{co}$ for a while remained mysterious [11] despite many attempts to explain it.

Another problem of principal importance for phenomenological models of deconfinement phase transition [8,9,10,11,12,13,14,15] is the value of the surface tension coefficient $\sigma_{surf}$ of QGP bags. There are several estimates for the surface tension coefficient $\sigma_{surf}$ of QGP bags [16], but the question is whether can we determine $\sigma_{surf}$ from lattice QCD? Therefore, in this section we consider an approach that allows us to determine the surface tension coefficient of QGP bags directly from the lattice QCD. As it will be shown in the section 4 such an approach naturally explains an existence of the ‘mysterious maximum’ [11] of the confining tube entropy.

In order to estimate the surface tension of QGP bags let us consider the static quark-antiquark pair connected by the unbreakable color tube of length $L$ and radius $R \ll L$. In the limit of large $L$ the free energy of the color tube is $F_{str} \to \sigma_{str} L$. Now we consider the same tube as an elongated cylinder of the same radius and length $L$. In this case we neglect the free energy of the regions around the color charges, but for our treatment of large separation distances $L \gg R$ this is sufficient. For the cylinder free energy we use the standard parameterization [8,9,10,11,12]

$$F_{cyl}(T, L, R) = -p_{v}(T)\pi R^2 L + \sigma_{surf}(T)2\pi RL + T\tau \ln \left( \frac{\pi R^2 L}{V_0} \right),$$

where $p_{v}(T)$ is the bulk pressure inside a bag, $\sigma_{surf}(T)$ is the temperature dependent surface tension coefficient, while the last term on the right hand side above is the Fisher topological term [17] which is proportional to the Fisher exponent $\tau = const > 1$ [8,9,10,11,12,15] and $V_0 \approx 1 \text{ fm}^3$ is a normalization constant. Since we consider the same object then its free energies calculated as the color tube and as the cylindrical bag should be equal to each other. Then for large separating distances $L \gg R$ one finds the following relation

$$\sigma_{str}(T) = \sigma_{surf}(T)2\pi R - p_{v}(T)\pi R^2 + \frac{T\tau}{L} \ln \left( \frac{\pi R^2 L}{V_0} \right).$$

In doing so, in fact, we match an ensemble of all string shapes of fixed $L$ to a mean elongated cylinder, which according to the original Fisher idea [17] and the results of the Hills and Dales Model (HDM) [18,19] represents a sum of all surface deformations of such a bag. The last equation allows one to determine the $T$-dependence of bag surface tension as

$$\sigma_{surf}(T) = \frac{\sigma_{str}(T)}{2\pi R} + \frac{1}{2} p_{v}(T)R - \frac{T\tau}{2\pi RL} \ln \left( \frac{\pi R^2 L}{V_0} \right),$$

TABLE V: Simulation parameters and screening masses for the large lattice $32^2 \times 48 \times 6$. Lattice scales are estimated by

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on papers by Zahed, Liao and myself [mf] mg.] in which they are related to what happens for slow and fn...
if } R(T) \text{, } \sigma_{str}(T) \text{ and } p_v(T) \text{ are known}. \text{ This relation opens a principal possibility to determine the bag surface tension directly from the lattice QCD simulations for any } T. \text{ Also it allows us to estimate the surface tension at } T = 0. \text{ Thus, taking the typical value of the bag model pressure which is used in hadronic spectroscopy } p_v(T = 0) = -(0.25)^4 \text{ GeV}^4 \text{ and inserting into Eq. (3)} \text{, the lattice QCD values } R = 0.5 \text{ fm and } \sigma_{str}(T = 0) = (0.42)^2 \text{ GeV}^2 \text{ [20], one finds } \sigma_{surf}(T = 0) = (0.2229 \text{ GeV})^3 + 0.5p_vR \approx (0.183 \text{ GeV})^3 \approx 157.4 \text{ MeV fm}^{-2} \text{ [6]. The last term in (3) does not modify our above estimate at } T = 0, \text{ but, in contrast to [6, 7], we keep it in order to demonstrate its importance for the confining tube with free color charges.}

The found value of the bag surface tension at zero temperature is very important for the phenomenological equations of state of strongly interacting matter in two respects. Firstly, according to HDM the obtained value defines the temperature at which the bag surface tension coefficient changes the sign \([18, 19, 7]\) equations of state of strongly interacting matter in two respects. Firstly, according to HDM the obtained value demonstrates its importance for the confining tube with free color charges.

3 Surface tension coefficient at the cross-over temperature

The above results, indeed, allow us to tune the interrelation with the color tube model and to study the bag surface tension near the cross-over to QGP. Consider first the vanishing baryonic densities. The lattice QCD data indicate that at large \( T \) the string tension behaves as

\[
T_\sigma = \sigma_{surf}(T = 0)V_0^{\frac{2}{3}} \cdot \lambda^{-1} \in [148.4; 157.4] \text{ MeV},
\]

where the constant \( \lambda = 1 \) for the Fisher parameterization of the \( T \)-dependent surface tension coefficient \([17]\) or \( \lambda \approx 1.06009 \), if we use the parameterization derived within the HDM for surface deformations \([18, 19, 7]\).

Secondly, according to one of the most successful models of liquid-gas phase transition, i.e. the Fisher droplet model (FDM) \([17]\) the surface tension coefficient linearly depends on temperature. This conclusion is well supported by HDM and by microscopic models of vapor-liquid interfaces \([21]\). Therefore, the temperature \( T_\sigma \) in \([4]\), at which the surface tension coefficient vanishes, is also the temperature of the (tri)critical endpoint \( T_{cep} \) of the liquid-gas phase diagram. On the basis of these arguments in Ref. \([7]\) we concluded that the value of QCD critical endpoint temperature is \( T_{cep} = T_\sigma = 152.9 \pm 4.5 \text{ MeV} \). Hopefully, the latter can be verified by the lattice QCD simulations using Eq. (3).

Now the question is what is the surface tension coefficient above \( T_{cep} \), i.e. at supercritical temperatures. There are no experimental data on usual liquids in this region. In FDM and in the other well known model of liquid-gas phase transition, the statistical multifragmentation model (SMM) \([22, 23, 24]\), the surface tension at supercritical temperatures is assumed to be zero, while in other models such a question is usually not discussed. The only exceptions known to us are the exactly solvable statistical models of quark gluon bags with surface tension \([3, 9]\), their extension which includes the finite widths of large/heavy QGP bags \([10, 11, 12]\) and recently formulated generalization of the SMM \([25]\). For all these models it was demonstrated that the negative surface tension is the only physical reason of degeneration of the 1-st order phase transition into cross-over at supercritical temperatures. The question is whether the above suggested formalism can support such a conclusion.

\[
\sigma_{str} = \ln (L/L_0) - g_0, \tag{5}
\]

where \( L_0 > 0 \) and \( g_0 > 0 \) are some positive constants. Assuming the validity of Eq. (5) in the infinite available volume, one finds that for \( \sigma_{str}(T) \to +0 \) the string radius diverges, i.e. \( R \to \infty \).

Using Eqs. (1) and (3) we can write the total pressure \( p_{tot} \) of the cylinder as follows

\[
p_{tot}(L, R, T) = p_v(T) - \frac{\sigma_{surf}(T)}{R} - \frac{T\tau}{R^2 L} = \frac{\sigma_{surf}(T)}{R} - \frac{\sigma_{str}}{R^2} + \frac{T\tau}{R^2 L} \left[ \ln \left( \frac{\pi R^2 L}{V_0} \right) - 1 \right]. \tag{6}
\]

This equation shows that for fixed separation distance \( L \) in the limit \( \sigma_{str}(T) \to +0 \) the leading term is given by the surface tension contribution, while the next to leading term corresponds to the contribution of the Fisher topological term, whereas the second term on the right hand side of (6) is the smallest one. Therefore, it is evident that for small values of string tension (and large \( R \)) the main contribution to the total pressure and to its temperature derivative is given by the first term on the right hand side of (6).

To calculate the total entropy density \( s_{tot} \) of the cylinder let us parameterize the string tension as

\[
\sigma_{str} = \sigma_{str}^0 t^\nu \tag{7}
\]

where \( t \equiv \frac{T_{cep} - T}{T_{cep}} \to +0 \) and \( \nu = const > 0 \). From (7) it follows \( R = \left[ \frac{g_0 \ln (L/L_0)}{\sigma_{str}^0 \nu} \right]^\frac{1}{\nu} \) and then for \( t \to 0 \) the
entropy density $s_{\text{tot}}$ can be found from (6) and (7) as
\[
s_{\text{tot}} = \left( \frac{\partial p_{\text{tot}}}{\partial T} \right)_{\mu} \rightarrow -\frac{\nu}{2RT_{co}} \sigma_{\text{surf}} + \frac{1}{2R} \frac{\partial \sigma_{\text{surf}}}{\partial T} \rightarrow -\frac{\nu}{2T_{co}} \left[ \frac{\sigma_{\text{surf}}^0}{g_0 \ln (L/L_0)} \right]^{\frac{1}{2}} \sigma_{\text{surf}} > 0. \tag{8}
\]

This equation shows that at $T = T_{co}$ the entropy density diverges for $\nu < 2$ and also that at the cross-over region the surface tension coefficient must be negative otherwise the system would be thermodynamically unstable since its entropy density would be negative.

Figure 2. Surface tension of normal paraffins as a function of temperature from the triple point to the critical point. The filled circles indicate the experimental data [30]. The lines are the different theoretical parameterizations [28].

Clearly, the results [1]–[8] are valid for nonzero baryonic chemical potential $\mu$ up to the (tri)critical endpoint. The main modification in [1]–[6] is an appearance of $\mu$-dependences of $p_v(T, \mu)$ and $T_{co}(\mu)$ [6]. In the (tri)critical endpoint vicinity the behavior of $p_{\text{tot}}$ and $s_{\text{tot}}$ is defined by the $T$-dependence of the surface tension coefficient.

We stress that there is nothing wrong or unphysical with the negative values of surface tension coefficient, since $\sigma_{\text{surf}} 2\pi RL$ in [1] is the surface free energy and, hence, as any free energy, it contains the energy part $E_{\text{surf}}$ and the entropy part $S_{\text{surf}}$ multiplied by temperature $T$, i.e. $F_{\text{surf}} = E_{\text{surf}} - T S_{\text{surf}}$ [18, 19]. Therefore, at low temperatures the energy part dominates and the surface free energy is positive, whereas at high temperatures the number of configurations of a cylinder with large surface drastically increases and the surface free energy becomes negative since $S_{\text{surf}} > \frac{E_{\text{surf}}}{T}$. Moreover, the exactly solvable models with phase transition and cross-over [3, 9, 10] have region of negative surface tension coefficient and they clearly show that the only reason why the 1-st order deconfinement phase transition degenerates into a cross-over at low baryonic densities is the negative values of $\sigma_{\text{surf}}$ at this region and the above results independently prove this fact.

We believe that the same is true for many ordinary liquids otherwise one has to search for an alternative explanation for the disappearance of the 1-st order liquid-gas phase transition at the supercritical temperatures.
Of course, the experimental data in this region do not exists, but, nevertheless, there is indirect evidence for an existence of negative values of the surface tension coefficient at the supercritical temperatures. To demonstrate the validity of this statement we have to remind that the modern experimental data on the temperature dependence of the surface tension do not allow one to definitely conclude what is $T$-dependence at the vicinity of critical temperature $T_c$. In fact there are two alternative prescriptions \[26\] \[27\]

\[
\frac{\sigma_{\text{surf}}}{\rho_l^{2/3}} = a_E(T_c - T), \tag{9}
\]

\[
\frac{\sigma_{\text{surf}}}{\rho_l^{2/3}} = a_G(T_c - T)^n \quad \text{with} \quad n \approx \frac{11}{9}, \tag{10}
\]

where $\rho_l$ is the temperature dependent particle density of the liquid phase.

![Figure 3](image_url)

**Figure 3.** The surface tension $\Gamma$ (in N/m) in terms of cluster surfaces (first and third rows) and the surface tension $\Gamma/\rho_l^{2/3}$ (in (N/m)/(mole/l)$^{2/3}$) in terms of cluster number (second and fourth rows) as a function of $\epsilon = (T_c - T)/T_c$ for: quantum fluids (hydrogen and helium), noble gases (krypton and xenon) and more complex fluids (methane and water). The thin solid lines show data points \[23\] and the heavy dashed-dotted lines over the thin line show fits to $\Gamma = \Gamma_0 \rho_l^{2/3} \epsilon^{2n}$ and $\Gamma/\rho_l^{2/3} = \Gamma_0 \epsilon$ according to Eqs. \[10\] and \[9\], respectively.

After the Guggenheim work \[26\] the prescription \[10\] became a dominant one \[28\]. Sometimes there appeared even confusions. Thus, in \[29\] the authors determined the surface tension of water from the triple point to critical
of 9-th power \( \sigma_{surf}(T) = \sum_{i=1}^{9} a_i (T_c - T)^i \), but then the same authors refitted it to the prescription \( (10) \) \[30\]. Here in Fig. 2 which is taken from \[28\] we show the temperature dependence of some paraffins. As one can see for n-Pentane and n-Heptane the data on temperature dependence of surface tension near the critical point, indeed, may show the nonlinear behavior similar to \( (10) \), but for the n-Hexane and n-Octane one can see the linear \( T \)-dependence of Eq. \( (9) \).

Therefore, in order to clarify this issue a few years ago a thorough analysis \[32\] of the high quality NIST data \[33\] was performed. Some of the results are shown in Fig. 3 which is taken from Ref. \[32\]. As one can see from Fig. 3 for most of the analyzed liquids the linear prescription \( (9) \) provides an essentially better fit with the only exceptions of xenon and methane. Therefore, our first conclusion is that for many liquids the rule \( (9) \) better describes the data than the rule \( (10) \). The second conclusion one can draw from this discussion is that naive extrapolation of the linear \( T \)-dependence \( (9) \) of the surface tension coefficient \( \sigma_{surf} \) to supercritical temperatures \( T > T_c \) would lead to the negative values of the surface tension coefficient. Of course, one may think that \( \sigma_{surf} \approx 0 \) for \( T > T_c \) like in the FDM \[17\] and SMM \[22, 23, 24\], but in this case one has to explain the reason why the \( T \)-derivative of \( \sigma_{surf} \) has a discontinuity at \( T = T_c \) while the pressure and all its first and second derivatives are continuous functions of it arguments in this region.

4. The mysterious maximum of the lattice entropy and the gas of free tubes

The considered configuration of the unbroken confining tube is only one of many other configurations accounted by the lattice QCD thermodynamics. However, in order to explain a mysterious maximum of the lattice entropy (see Fig. 1) it is sufficient to assume that the probability of the unbroken confining tube among other configurations measured by lattice QCD is \( W(L) \sim [L g_0 \ln(L/L_0)]^{-1} \), i.e. in the limit \( L \rightarrow \infty \) it is negligible for any \( \nu \neq 0 \). Then the contribution of the unbroken confining tube into the lattice free energy is small, since \( W(L) F_{str}(L) \sim R^{-2} \) for \( t \rightarrow +0 \) and \( R \rightarrow R_{lat} - 0 \) \( (R_{lat} \) denotes the lattice size), but its contribution to the tube entropy

\[
W(L) S_{str} = -W \frac{dF_{str}}{dT} = W L \frac{\sigma_{str}^0 \nu^\nu - 1}{T_{co}} \rightarrow W L \frac{\sigma_{str}^0}{T_{co}} \left[ \frac{g_0 \ln(L/L_0)}{1 - \nu} \right]^\nu R^{2(1 - \nu) / \nu} \sim R \frac{2(1 - \nu)}{\nu} (11)
\]

is an increasing function of the tube radius \( R \) for \( \nu < 1 \). Clearly, if the available size of the lattice \( R_{lat} \) would be infinite then the contribution of the unbroken tube would diverge, but for finite lattice size one should observe a fast increase at \( T \rightarrow T_{co} \).

The physical origin of a singular behavior of the tube entropy \( (11) \) encoded in \( \nu < 1 \) is rooted in the formation of fractal surfaces of the confining tube in the cross-over temperature vicinity \[6\]. This can be clearly seen from the power \( \frac{2(1 - \nu)}{\nu} \) of \( R \) on the right hand side of \( (11) \) which is fractal for any \( \nu \neq \frac{2n}{n+1} \) where \( n = 1, 2, 3, ... \) Moreover, the appearance of fractal structures at \( T = T_{co} \) can be easily understood within our model, if we recall that only at this temperature the fractal surfaces can emerge at almost no energy costs due to almost zero total pressure \( (7) \). An explanation of the tube entropy decrease for \( T < 0 \) is similar \[6, 7\]. It means that the fractal surfaces gradually disappear since for \( T > T_{co} \) the tube gradually occupies the whole available lattice volume.

Here we also would like to consider a toy model based on the total pressure \( (6) \) of the confining tube, but for the non-static (or free) quark-antiquark pair. In this case the parameter \( L \) should be considered as a free parameter which has to be determined from the maximum of the total pressure \( (6) \). Finding from this condition the radius dependent separation distance \( L_{w}(R) \) which corresponds to the most probable and the stable state of the free confining tube one has to substitute it into expression for the pressure \( (6) \) and find the corresponding radius of the tube from the equation \( p = p_{tot}(L_{w}(R), R, T) \) for the set of given external pressure \( p \) and temperature \( T \). Clearly the determination of an extremum of the total pressure \( (6) \) with respect to \( R \) with subsequent finding of the separation distance \( L \) should give the same result, but the first way is technically easier.

Instead of the quantitative analysis of the resulting pressure \( p_{tot}(L_{w}(R), R, T) \) which requires the knowledge of values of all constants, i.e. \( L_0, g_0, \tau \) etc., we prefer to discuss some qualitative properties of the model and show that it has two phases. Also we have to stress that such a model cannot be applied to high pressures (or high densities) directly because in this case the pressure of the system should account for the short range repulsion between the tubes. Therefore in what follows it is assumed that the gas of tubes has some low particle density \( \rho \) and, hence, one can consider this gas as an ideal gas with the pressure \( p = T \rho \).

To determine the density \( \rho \) one has to maximize the pressure \( p_{tot} \) first. From the vanishing derivative condition

\[
\frac{\delta p_{tot}(L, R, T)}{\delta L} = -\frac{g_0}{\pi R^4 L} - \frac{T \tau}{\pi R^2 L^2} \left[ \ln \left( \frac{\pi R^2 L}{V_0} \right) - 1 \right] + \frac{T \tau}{\pi R^2 L^2} = 0 (12)
\]
one can find the following equation for \( L_w(R) \)

\[
L_w = \frac{T \tau R^2}{g_0} \left[ 2 - \ln \left( \frac{\pi R^2 L_w}{V_0} \right) \right]
\]  

(13)

and show that it corresponds to a maximum of pressure. From Eq. (12) it is clearly seen that, in contrast to previous findings [6, 7], the role of the Fisher topological term is a decisive one for an establishing Eq. (13).

Consider a few limiting cases of Eq. (13). In the limit \( T \to 0 \) and finite \( R \) one gets

\[
L_w^0 \approx - \frac{T \tau R^2}{g_0} \ln \left( \frac{\pi R^4 \tau T}{V_0 g_0 e^2} \right) \to 0,
\]

which can be interpreted as a confinement of color charges. The same solution is true for the case of \( R \to 0 \) and finite \( T \). In the other extreme \( T \to \infty \) (or \( R \to \infty \)) one obtains a different solution of Eq. (13)

\[
L_w^\infty \approx \frac{V_0 e^2}{\pi R^2},
\]

(15)

which again shows that for large \( R \) values the separation distance is small. This situation resembles what is observed in lattice QCD for the non-static color charges: the long tubes that connect such charges simply break up at some separation distances \( L \).

From Eq. (13) one can show that the solution \( L_w \) is a monotonically increasing function of \( T \), while for \( T \neq 0 \) it always has a maximum as the function of \( R \). Searching for the maximum of \( L_w(R) \) \( (13) \) one can find the corresponding value of the radius \( R_{\text{max}} \) and \( L_w(R_{\text{max}}) \)

\[
R_{\text{max}} = \left[ \frac{g_0 V_0 e}{\pi \tau T} \right]^{\frac{1}{2}},
\]

(16)

\[
L_w(R_{\text{max}}) = \left[ \frac{V_0 e^2 \pi \tau T}{g_0} \right]^{\frac{1}{2}},
\]

(17)

which, evidently, obey the condition \( \pi R_{\text{max}}^2 L_w(R_{\text{max}}) = V_0 e \). The presence of the maximum of function \( L_w(R) \) leads to an existence of two different radii for the same value of separation distance \( L \), or in other words, there are two solutions of the equation \( L = L_w(R) \). Clearly, the parameters of the maximum \( R_{\text{max}} \) and \( L_w(R_{\text{max}}) \) given, respectively, by Eqs. (16) and (17), separate the regions of these solutions. Evidently, the latter correspond to two phases of the gas of tubes which have different tube radius for the same separation distance \( L \). The analysis of these solutions shows that there are many possibilities which strongly depend on the values of the involved constants \( L_0, g_0, \tau \) and \( V_0 \), whereas for low but non-vanishing temperatures one can show that the higher pressure corresponds to the phase of the tubes with smaller radius. This is clear because in case of low temperatures the leading contribution to the total pressure [6] is given by the surface tension term \( \frac{\sigma_{\text{surf}}}{R^2} \).

5 Conclusions

In this work we discuss the most general relation between the tension of the color tube connecting the static quark-antiquark pair and the surface tension of the corresponding cylindrical bag. Such a relation allows us to determine the surface tension of the QGP bags at zero temperature and, under the plausible assumptions that are typical for ordinary liquids, to estimate the temperature of vanishing surface tension coefficient of QGP bags at zero baryonic charge density as \( T_{\text{cep}} = 152.9 \pm 4.5 \text{ MeV} \). Using the Fisher conjecture [17] and the exact results found for the temperature dependence of surface tension coefficient from the partition of surface deformations [18, 19, 21], we conclude that the same temperature range corresponds to the value of QCD (tri)critical endpoint temperature, i.e. \( T_{\text{cep}} = T_c = 152.9 \pm 4.5 \text{ MeV} \). Then requiring the positive values for the confining tube entropy density we demonstrate that at the cross-over region the surface tension coefficient of the QGP bags is unavoidably negative. Furthermore, analyzing the data on the temperature dependence of the surface tension coefficient of some ordinary liquids in the vicinity of the critical endpoint we conclude that the negative values of the surface tension coefficient of QGP bags are not unique, but they also should exist at the supercritical temperatures of usual liquids. We believe such a conclusion is worth to verify experimentally for ordinary liquids.

Also we demonstrate that the long unbroken tube taken with a vanishing probability which generates a finite contribution into the lattice free energy may, under certain assumptions, provide a very fast increase of the lattice entropy of such configurations and, thus, it may explain the maximum of the tube entropy observed by lattice QCD. Additionally, we considered the non-static (free) tube and used the developed formalism to work out the model of the gas of free tubes. The performed analysis of such a model showed that there are two phases in this model which correspond to different radii of the tube for the same separation length \( L \). Since this
toy model resembles some important features of the confining tubes observed in the lattice QCD, we believe it is important for QCD phenomenology and can be used to build up more elaborate statistical model which accounts for a realistic interaction between tubes.

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