Parametric amplification of vacuum fluctuations in a spinor condensate

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Parametric amplification of vacuum fluctuations is crucial in modern quantum optics, enabling the creation of squeezing and entanglement. We demonstrate the parametric amplification of vacuum fluctuations for matter waves using a spinor $F = 2^{87}\text{Rb}$ condensate. Interatomic interactions lead to correlated pair creation in the $m_F = \pm 1$ states from an initial unstable $m_F = 0$ condensate, which acts as a vacuum for $m_F \neq 0$. Although this pair creation from a pure $m_F = 0$ condensate is ideally triggered by vacuum fluctuations, unavoidable spurious initial $m_F = \pm 1$ atoms induce a classical seed which may become the dominant triggering mechanism. We show that pair creation is insensitive to a classical seed for sufficiently large magnetic fields, demonstrating the dominant role of vacuum fluctuations. The presented system thus provides a direct path towards the generation of non-classical states of matter on the basis of spinor condensates.

FIG. 1: (a) Fraction of atoms transferred into $|m_F = \pm 1\rangle$ within 21 ms as a function of the applied magnetic field. The error bars indicate statistical uncertainties. The blue line is a triple Gaussian fit to guide the eye. (b) Theoretical prediction for our experimental parameters (see text), with an initial BEC in $|0\rangle$ with $N = 50000$ and $N_s = 2$ seed atoms. The black line shows the GP result, whereas the blue line shows the result of our calculation, including both this classical seed and vacuum spin fluctuations.

Parametric amplification of vacuum fluctuations in nonlinear media plays a crucial role in the investigation and application of non-classical states of light. These states have revolutionized the field of quantum optics in the past decades. Since the first observation of squeezed light, these non-classical states of light have become a valuable tool in modern optics, e.g. for the enhancement of modern interferometers. Similarly, the production of entangled photon pairs has triggered an ongoing series of fundamental tests of modern quantum mechanics and has many possible applications for quantum computing. The tools developed for the production and manipulation of ultracold neutral atoms now bring many of these seminal investigations within the scope of experiments with matter waves. In this sense, the production of number-squeezed Bose-Einstein condensates (BECs) and spin squeezed thermal clouds has been recently demonstrated.

Spinor BECs, consisting of atoms with non-zero spin $F$, provide an optimal non-linear medium for the production of non-classical states of matter. In these systems, inter-particle interactions lead to a coherent population transfer between different Zeeman $m_F$ sublevels (spin dynamics). The case of a sample prepared in $m_F = 0$ (represented by $|0\rangle$) is particularly interesting. In that case the initial stages of the spin dynamics are characterized by the production of correlated atom pairs in $|\pm 1\rangle$, resembling the production of Einstein-Podolsky-Rosen (EPR) pairs in optical parametric down conversion.

Ideally a pure initial $|0\rangle$ BEC acts as a vacuum for atoms in $m_F \neq 0$. Hence pair creation into $|\pm 1\rangle$ can be understood as a parametric amplification of quantum vacuum fluctuations. However, parametric amplifiers are exponentially sensitive to any spurious initial seed in the amplified modes, which may easily dominate the effect of vacuum fluctuations. Despite careful purification procedures a tiny number of spurious atoms in $|\pm 1\rangle$ is experimentally unavoidable. Hence it is crucial to determine whether these spurious seed atoms play a significant role in the spin dynamics. Only in the case they do not, the system may safely be considered a parametric amplifier for vacuum fluctuations.

In this Letter we investigate the triggering mechanism for the parametric amplification of correlated pairs. Similar to the gain of the amplification, studied in Ref. [12], the triggering mechanism and its sensitivity depend crucially on the interplay between Zeeman energy, interac-
ations and external confinement. In particular, the relative importance of triggering by a classical seed and by vacuum fluctuations depends on the magnetic field $B$. At low $B$ the amplification is strikingly sensitive to any unavoidable spurious seed. On the contrary for sufficiently large $B$ the classical seed plays no significant role, and the system can indeed be characterized as a parametric amplifier of vacuum fluctuations. The amplification of vacuum fluctuations opens fascinating perspectives towards the analysis of two-mode squeezing and EPR entanglement of matter waves on the basis of unstable spinor BECs \[10,11\].

The onset of the spin dynamics is characterized by pair creation into $|±1\rangle$. In order to analyze this initial regime we consider the perturbed spinor $\Psi(\vec{r},t) = (\Psi_0(\vec{r}) + \delta\Psi(\vec{r},t))e^{-i\mu t}$, where $\Psi_0(\vec{r}) = (0,0,n_0(\vec{r})1/2,0,0)^T$ represents the initial BEC in $|0\rangle$, $\mu$ is the chemical potential, and $\delta\Psi(\vec{r},t) = (\delta\hat{\psi}_-1,\delta\hat{\psi}_0,\delta\hat{\psi}_1,\delta\hat{\psi}_2)^T$ describes fluctuations. Up to second order in $\delta\hat{\psi}_±\hat{\psi}_\mp = (\delta\hat{\psi}_+1 ± \delta\hat{\psi}_-1)/\sqrt{2}$, pair creation is described by the Hamiltonian $\hat{H} = \hat{H}_+ + \hat{H}_-$, where

$$\hat{H}_± = \int d^3r \left[ \delta\hat{\psi}_±^\dagger (\hat{H}_{eff} + q) \delta\hat{\psi}_± ± \frac{\Omega}{2} (\delta\hat{\psi}_2^2 + h.c) \right].$$

(1)

Here, $q \propto B^2$ characterizes the quadratic Zeeman energy (QZE), $\hat{H}_{eff} = -\hbar^2\nabla^2/2m + V_{\text{trap}}(\vec{r}) + (U_1 + U_0)n_0(\vec{r}) - \mu$, $V_{\text{trap}}(\vec{r})$ is the harmonic trap, $\Omega(\vec{r}) = U_1n_0(\vec{r})$, and $U_0 = (7g_0 + 5g_2 + 12g_4)/35$ and $U_1 = (7g_0 - 5g_2 + 12g_4)/35$ characterize the spin-preserving and spin-changing collisions. The coupling constant associated with the $s$-wave collisional channel with total spin $F$ is $g_F = 4\pi\hbar^2 F/m$. Note that $\hat{H}_±$ is identical to the Hamiltonian describing an optical parametric amplifier \[13\].

The problem is analyzed best in the basis of eigenstates $\hat{H}_±\varphi_n(\vec{r}) = \epsilon_n\varphi_n(\vec{r})$. By introducing $\delta\hat{\phi}_{±} = \sum_n \hat{b}_{n,±}\varphi_n$ and $\Omega_{n,n'} = \int d^3r\varphi_n\varphi_{n'}$ we may rewrite the Hamiltonian

$$\hat{H}_± = \sum_n (\epsilon_n + q)\hat{b}^\dagger_{n,±}\hat{b}_{n,±} ± \sum_{n,n'} \frac{\Omega_{n,n'}}{2} (\hat{b}_{n,±}\hat{b}^\dagger_{n',±} + h.c).$$

(2)

The Heisenberg equation $i\hbar\frac{d}{dt}\hat{O}_±^\dagger = [\hat{H}_±, \hat{O}_±^\dagger] = \xi_±^\dagger \hat{O}_±^\dagger$ then yields the eigenvalues $\xi_±$ and the corresponding eigenoperators $\hat{O}_±^\dagger = \sum_{n=1}^M (R_{n,±}^± b_{n,±} + R_{n,±}^± b^\dagger_{n,±})$, where $M$ indicates the maximal $\varphi_n$ level considered.

Imaginary eigenvalues with $\text{Im}(\xi_n) > 0$ result in an exponential amplification of spin fluctuations, which indicates the onset of pair creation. In that case, the amplification dynamics is dominated by the most unstable mode $\nu_0$ with the largest imaginary part $\text{Im}(\xi_{\nu_0}) = \Lambda(q)$. The instability rate $\Lambda(q)$ generally shows a non-monotonous multi-resonant $q$-dependence due to the interaction between QZE, interactions and external confinement \[12\].

Based on this diagonalization of the Hamiltonian we can evaluate the experimentally relevant quantum evolution of the operators $a_{n,m_F}$ and $a^\dagger_{n,m_F}$, where $a_{n,±} = (b_{n,±} ± b^\dagger_{n,±})/\sqrt{2}$. The total population in $|±1\rangle$ is then $P_{m_F}(t) = \sum_n (a^\dagger_{m_F}(t)a_{n,m_F}(t))$, where the average is performed over the initial state $|\Psi_{m_F}(0)\rangle = |\Psi\rangle$.

In the presence of unstable spin excitation modes, the initial state $|\Psi\rangle$ triggers the subsequent amplification. Ideally $|\Psi\rangle$ should be a state with no particles in $m_F = ±1$ (defining our vacuum state $|\text{vac}\rangle$). In that case, pair creation is triggered by vacuum fluctuations (quantum triggering). However, unavoidable slight imperfections always lead to $N_s \ll N$ spurious atoms in $|±1\rangle$ at $t = 0$. Although small, these initial impurities serve as a classical seed of the spin dynamics (classical triggering), which can dominate the quantum triggering. It is hence crucial to determine the relative role of quantum and classical triggering.

Note that, in our experiments \[12\] the BEC in the $|0\rangle$ state is purified by quickly removing all atoms in the $|±1\rangle$ states. This results in a non-equilibrium state and we stress that the time after purification is too short to produce any thermal population of the $|±1\rangle$ states. Thus, thermal spin fluctuations do not play any role in our discussion and impurities in $|±1\rangle$ can only be created by radio-frequency noise and magnetic field jitter after the purification. Since these processes act equally on each single atom, any spurious $|±1\rangle$ atom is produced in the same spatial mode as the original BEC. These spurious single-atom processes may also transfer a tiny fraction of the thermal cloud into $|±1\rangle$. However these atoms lack significant spatial overlap with the most unstable excitations, and hence do not contribute to the spin dynamics.

To evaluate the triggering mechanism, the initial state $|\Psi\rangle$ including a classical seed can be represented by $|\Psi\rangle = (N_s!)^{-1}(\hat{\phi}_{±,1}^\dagger)^{N_s}|\text{vac}\rangle$, where $\hat{\phi}_{m_F}^\dagger (\hat{\phi}_{m_F})$ creates (annihilates) a $m_F$ particle in the mode of the initial BEC. In the basis of the effective Hamiltonian, they can be represented by a vector $\Phi$ where $\Phi_{m_F} = \sum_n \hat{\phi}_{m_F}^\dagger n |\phi_n\rangle$, with $\Phi_n = \int d^3r\sqrt{n_0}\varphi_n$. Now, one obtains the total population as a sum of a classically and a quantum triggered contribution $P_{m_F}(t) = P_C(t) + P_Q(t)$, where

$$P_C(t) = N_s\Phi^* \cdot \left(\hat{U}^\dagger(t) \cdot \hat{U}(t) + \hat{V}(t)^\dagger \cdot \hat{V}(t)\right) \cdot \Phi,$$

$$P_Q(t) = TR \left\{ \hat{V}^\dagger(t) \cdot \hat{V}(t) \right\}.$$
FIG. 2: Ratio between the classically-triggered \( P^c \) and quantum-triggered \( P^q \) populations for our experimental parameters, considering only the growth due to the most unstable mode. The quantum triggered dynamics is characterized by \( P^c \ll P^q \).

total number) during the typical investigation times of 20ms, they may significantly alter the spin dynamics, mainly due to the dynamical modification of the resonant conditions for the instability rate \( \Lambda(q) \). We have carefully included these losses by splitting the evolution time into sub-intervals. At the beginning of each sub-interval we introduce losses and recalculate the evolution operator (due to the moderate loss rate typically few sub-intervals already lead to convergence). Figure 1 (b) shows the numerical result for our experimental parameters. Note that the multi-resonant \( q \) dependence of the instability rate \( \Lambda(q) \) discussed in Ref. [12] directly maps into a multi-peaked pair creation efficiency, which is in very good agreement with our previous experimental results [12] shown in Fig. 1 (a). As we discuss in detail below, our experiments confirm this remarkable difference in sensitivity of the two resonances to any spurious classical seed. Note that this approach to determine the triggering mechanism avoids the difficulties arising in e.g. the investigation of fluctuations of the amplifier output [10], which is largely impeded by the significant statistical uncertainty of the atom number measurement (detection noise) and small shot-to-shot variations of the total atom number which lead to fluctuations in the pair creation efficiency (amplifier noise).

To initiate our experiments, we prepare BECs containing \( 7 \times 10^4 F = 2^{87} \text{Rb} \) atoms in the \( |0\rangle \) state in an optical dipole trap with trapping frequencies of \( 176, 132, 46 \) Hz. We carefully remove residual atoms in other spin components by briefly applying a strong magnetic field gradient of \( \approx 50 \text{ G/cm} \), which is ramped down within 10 ms. To investigate the spin dynamics, the applied homogeneous magnetic field is subsequently lowered from 7.9 G to a specific value between 0.12 and 2 G within 3 ms. The BEC is held at the chosen magnetic field for a variable time to allow for spin changing collisions. Finally, the number of atoms in all \( m_F \) components is measured independently by applying a strong magnetic field gradient during time-of-flight expansion. To evaluate the onset of the exponential amplification regime, we restrict our investigation to short spin evolution times and small populations in \( |\pm 1\rangle \).

The sensitivity of the system to a classical seed is investigated by deliberately producing a very small symmetric seed population in the \( |\pm 1\rangle \) states prior to the spin evolution. This is accomplished by using a radio frequency
pulse which transfers a variable number of atoms from the \( |0\rangle \) to \( |\pm 1\rangle \) states. This pulse is applied for 5 \( \mu s \) at a magnetic field of 7.9 G. A frequency of 5.6 MHz was identified, which couples the BEC in the \( |0\rangle \) state symmetrically to the \( |\pm 1\rangle \) states. The number of transferred atoms was calibrated at the smallest detectable numbers in the linear regime. Figure 3 shows a linear fit to the data which allows for an extrapolation to very small atom numbers. By reducing the radio-frequency power by a further 15 to 25 dB, it is thus possible to reproducibly prepare a very small number of seed atoms. Note that both spuriously and deliberately produced seeds result from similar single-atom processes, and hence have exactly the same spatial dependence as the original BEC. The sensitivity to the deliberately produced seed is therefore representative of the sensitivity to any spurious seed in the experiment.

![Fraction of atoms transferred into \( |m_F = \pm 1\rangle \) as a function of the deliberately prepared number of seed atoms.](image)

**FIG. 4:** Fraction of atoms transferred into \( |m_F = \pm 1\rangle \) as a function of the deliberately prepared number of seed atoms. (a) The fraction recorded at 0.65 G, corresponding to the low field resonance, shows a strong dependence on the classical seed after an evolution time of 15 ms. (b) The fraction recorded after \( t = 23 \) ms on the high field resonance at 1.29 G is independent of the number of seed atoms and we conclude that it is triggered by vacuum spin fluctuations. The error bars indicate statistical uncertainties. The solid lines are fits and the dotted line is a guide to the eye indicating saturation.

In conclusion, we have shown that a spinor \( F = 2 \) BEC initially prepared in an unstable \( |0\rangle \) state can provide a parametric amplifier for vacuum fluctuations, where the \( |0\rangle \) condensate acts as an effective vacuum for atoms in \( |\pm 1\rangle \). Similar to other parametric amplifiers the system is exponentially sensitive to a spurious classical seed. We have therefore carefully analyzed the dependence of the pair creation efficiency on a classical seed in \( |\pm 1\rangle \). For low magnetic fields the amplification is extremely sensitive to spurious classical seed atoms, whereas for large enough fields the classical seed is irrelevant, and the observed pair creation is due to a parametric amplification of vacuum fluctuations. This mechanism is identical to spontaneous optical parametric down conversion and paves the way for the development of non-classical atom optics on the basis of unstable spinor condensates. In particular, this system provides a direct path towards the observation of two-mode squeezing of matter waves, and a promising method for the creation of entangled atomic EPR pairs \[14] [11].

In addition magnetic dipole-dipole interactions may play a significant role \[17] [18] for the case of \( F = 1 \) BECs and will be the subject of future investigations.

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[20] An equivalent analysis shows that recent amplification experiments [16] could be dominantly quantum-triggered only for $q/h \lesssim 6$ Hz.