U(1) masses in intersecting D-brane SM-like models

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Abstract

For recently constructed classes of D6-brane models, yielding the Standard Model fermion spectrum and gauge symmetry, we compute lower bounds on the masses of new $U(1)$ fields that such models predict in addition to the hypercharge $U(1)_Y$. In models with extra dimensions, generic uncertainties due to unknown values of the compactification radii of the extra dimensions affect the value of the string scale and thus the predictive power of such models. Using $\rho$ parameter and $Z - U(1)$ mixing-angle constraints we show how to avoid such uncertainties, to provide lower mass bounds for the additional $U(1)$ fields. These are in the region above 750 GeV for mixing angles less than $1.5 \times 10^{-3}$ (and as low as 550 GeV for mixing angles of $3 \times 10^{-3}$).

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1 Introduction.

The interest in physics of large extra dimensions triggered an intense research activity at both effective field theory and string theory level as well. Recent progress in the latter included specific (intersecting) D-branes string constructions which were particularly successful in yielding in the low energy limit a spectrum and symmetry close to that of the Standard Model (SM) \([1]\) (or supersymmetric extensions \([2]\)). Such constructions were extensively analysed at string level, and may provide us with consistent, possibly realistic models. However, addressing the phenomenological implications of such D-brane SM-like models is at an early stage, thus motivating this work as a step in this direction.

Chiral D-brane models of possible phenomenological interest correspond to D-branes at singularities \([3, 4, 5]\) and D-branes intersecting at non-trivial angles \([1, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]\). Such models share some common properties: a value for the string scale which is lower than in the heterotic case, the gauge symmetry includes direct products of groups \(U(N_\alpha) \times U(N_\beta)\) (each \(U(N_\alpha)\) emerging from the set/stack “\(\alpha\)” of \(N\) individual \(U(1)\) branes), the matter fields transform as bi-fundamental representations of such products (as \((N_\alpha, N_\beta)\) or \((N_\alpha, N_\beta)\)), \(U(1)\) factors originating each from initial \(U(N)\) groups are a generic presence beyond the SM gauge group, and three generation models close to the Standard Model or supersymmetric extensions may be obtained.

In the present work we consider for our phenomenological investigation the class of models of \([1]\) with the purpose of extending the analysis started in \([16]\). These models are non-supersymmetric string constructions which predict a fermion massless spectrum \textit{identical} to that of the Standard Model. The Standard Model gauge group emerges from four stacks of D6-branes wrapping a three cycle on a six-torus (orientifelded) type II A string theory, in the presence of a background NS B-field. Similar constructions may be possible with D5 branes in type II B orientifold compactifications on an orbifold \(T^2 \times T^2 \times T^2 / \mathbb{Z}_N\), and their analysis follows the pattern presented in this work \([15]\).

The resulting gauge group \(U(3) \times U(2) \times U(1) \times U(1)\) consequently contains four initial \(U(1)\) groups in addition to the non-Abelian part of the Standard Model gauge group. These may be identified with the baryon number, lepton number, Peccei-Quinn-like symmetry and hypercharge (or linear combinations thereof).

Upon dimensional reduction to four dimensions, three (linear combinations) of these additional \(U(1)\) fields become massive through couplings \(B_i \wedge F\) where \(B_i\) stands for four dimensional RR two-form fields (present in the models of \([4]\)) and \(F\) for the \(U(1)\) field strength tensor, as detailed in \([16]\). This mechanism applies to anomalous \(U(1)\)’s as well\(^3\) as to \(U(1)\) fields which are \textit{not} anomalous, and does not involve a Higgs mechanism/particle. Consequently, after \(U(1)\) fields become massive, the corresponding \(U(1)\) symmetries remain as perturbatively exact \textit{global} symmetries in the effective Lagrangian. For example baryon number remains a global symmetry which may be welcome to explaining the stability of the proton (troublesome in models with a low string scale). The need for a low string scale is even more important for the class of models considered is non-supersymmetric, requiring a value for \(M_S\) in the region of few TeV, to avoid a hierarchy problem. The situation is

\(^2\)For other models see also \([17]\).

\(^3\)The anomalies are cancelled by a generalised Green-Schwarz mechanism, see ref. \([1]\).
unlike that of heterotic counterpart (and supersymmetric) models, where baryon number violation interactions may instead be suppressed by the existence of a high UV (string) scale.

The class of models of [1] “accommodate” three generations of matter fields and this is briefly justified in the following way. The tadpole cancellation condition requires the number of fundamental $N_a$ and anti-fundamental $\overline{N}_a$ representations for any $U(N_a)$ group be equal, even if the gauge group is $U(2)$. This restricts the assignment of quarks and leptons as $U(2)$ doublets or anti-doublets. Indeed, if all left-handed quarks were $U(2)$ doublets, and leptons were $U(2)$ anti-doublets, one could not satisfy in this case $\# N_a = \# \overline{N}_a$. The only possibility is to have two left-handed quarks $Q^i_L$, $i=1,2$ as $U(2)$ doublets (anti-doublets) with the third one and with left-handed leptons as anti-doublets (doublets). For three families of quarks/leptons the total number of doublets and anti-doublets is in this way equal, ensuring tadpole cancellation and relating the number of colours to that of generations [1].

The class of models we address being non-supersymmetric require a low (TeV region) string scale to avoid a hierarchy problem. In [16] it was shown that such a low value of the string scale ($M_S \approx 1.5 \text{TeV}$) can indeed be obtained while still complying with precision electroweak measurements. It remains to explain why the string scale may take such low values. The usual explanation is that some transverse dimensions become very large. This can apply to D5 (toroidal-like) models [15], but in the case of models with D6 branes discussed in [1] there is no compact dimension which is simultaneously transverse to all the SM branes. Possible solutions were suggested in [16] and in fact the problem is elegantly solved in models with D6 branes wrapped on 3-cycles in the more general Calabi-Yau compactifications [18].

One potential drawback of the models (both D5 and D6 branes) we refer to may be that the models cannot comply with the successful unification of the gauge couplings of the supersymmetric version of the SM (MSSM). This is however expected since the models have SM-like spectrum and are also non-supersymmetric. The problem is actually more general and present even in supersymmetric models, for cases with a low (TeV range) string scale/large extra dimension(s) [19].

In the next section we review the mechanism by which the $U(1)$ fields become massive and describe the procedure we use for setting explicit lower bounds on the masses of $U(1)$ bosons. Finding such lower bounds on the $U(1)$ masses is the main purpose of this work. The results are presented in Section 2.1. This completes the analysis of [16] which only addressed the bounds on the value of the string scale complying with constraints from electroweak scale measurements ($\rho$ parameter). Finally, the lower bounds on $U(1)$ masses are compared to those from other (string) models predicting additional Z’ bosons in the low energy regime.

2 $U(1)$ masses from string theory.

The class of D6 models that we investigate in this work are constructed from Type II A string theory compactified on a six-torus $T^2 \times T^2 \times T^2$. Apart from the Minkowski space, the remaining three dimensions of D6 brane models are wrapped each on a different torus $T^2$. One actually considers

\footnote{Localised in a small region of the Calabi-Yau manifold.}
four stacks of branes, $D6_\alpha$ branes, $\alpha = a, b, c, d$, each stack $\alpha$ bringing a $U(N_\alpha)$ group. Further, $n_{\alpha i}(m_{\alpha i}) \ i = 1, 2, 3$ stand for the wrapping numbers of each $D6_\alpha$ around the $x$ ($y$) coordinate of the $i$-th two-torus. The general set of allowed wrapping numbers yielding the Standard Model spectrum was presented in [1] with remaining independent parameters given in Table 1. In addition to the non-Abelian part of the SM gauge group, a $U(1)_\alpha$ factor emerges from each stack of branes. Therefore the final gauge group is $SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$ with the hypercharge group to arise as a linear combination of the four initial $U(1)_\alpha$. The charges of SM fields under $U(1)_\alpha$ are given in the Appendix, Table 4. For a full description of the models see [1] with applications in [16].

Although we do not investigate them in the following, similar constructions exist in models with D5 branes as well. D5-brane orientifold models are obtained from Type II B compactifications on an orbifold $T^2 \times T^2 \times T^2 / Z_N$. Four stacks ($a, b, c, d$) of D5 branes are wrapped on cycles of $T^2 \times T^2$ and are located at a $Z_N$ fixed point of $T^2 / Z_N$. The six dimensional world volume includes the Minkowski space with the remaining two extra dimensions wrapping each a different two-torus. Each stack of branes is then specified by $n_{\alpha i}(m_{\alpha i}) i=1, 2$. One obtains the SM fermionic spectrum and a gauge group with four additional $U(1)_\alpha$ factors, similar to D6 brane models. The gauge group is again $SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$ with hypercharge to emerge as a linear combination of the four Abelian groups. Details of D5 models may be found in [15].

For both D6- and D5-brane models there are four RR two-form fields $B_i$ which couple to the field strength tensor of the four $U(1)_\alpha$ fields, in the following way

$$\mathcal{L} \supset \sum_{i=1}^{4} \sum_{\alpha} c^\alpha_i B_i \wedge Tr F^\alpha, \quad \alpha = a, b, c, d. \quad (1)$$

Such couplings can provide masses to three linear combinations of $U(1)$ fields but not to the (linear combination yielding the) hypercharge $U(1)_Y$ field, which remains a local symmetry of the theory, before electroweak symmetry breaking. As a consequence not all $(4 \times 4 = 16)$ coefficients $c^\alpha_i$ are independent. It turns out that only seven of them are non-vanishing and five of them are actually independent. Their explicit expressions were presented in [16] for the D6 and D5 brane models considered and are also included in the Appendix.

The exact mechanism by which $U(1)_\alpha$ fields become massive, enabled by the term (1) and by kinetic terms for $B_i$ was presented in [16]. The idea is that the kinetic term of $B_{i,\mu \nu}$ in the action combines with the couplings (1) to give in the action a mass term for the $U(1)$ fields and a corresponding gauge kinetic term. This is just a re-arrangement of the degrees of freedom of 4D two-form fields $B_i$ whose scalar dual is each “eaten” by a $U(1)_\alpha$ field. Thus the mechanism does not require introducing additional scalar fields with vacuum expectation values, to provide mass terms for the $U(1)$ gauge bosons, nor does it bring the presence of massive Higgs-like fields in the end. Thus the mechanism is different from the usual Higgs mechanism. The following mass terms for $U(1)_\alpha$ fields emerge

$$\mathcal{L} \supset \frac{1}{2} \sum_{\alpha, \beta} (M^2)_{\alpha \beta} A_\alpha A_\beta \equiv \frac{1}{2} \sum_{\alpha, \beta} \left[ g_\alpha g_\beta M^2_3 \sum_{i=1}^{3} c^\alpha_i c^\beta_i \right] A_\alpha A_\beta, \quad \alpha, \beta = a, b, c, d. \quad (2)$$
The sum over $i$ runs over the three (massive) RR-fields present in the models and $g_\alpha$ is the coupling of $U(1)_\alpha$. $(U(1)_a$ arises from $U(3)$ thus $g_a^2 = g_{QCD}^2/6$ and $U(1)_b$ arises from $U(2)$ thus $g_b^2 = g_2^2/4$). Upon diagonalisation the mass matrix leads to positive (masses)$^2$, $M_i^2 > 0$ ($i=2,3,4$) ($M_1^2 = 0$ for hypercharge) for the three $U(1)$ bosons.

The coefficients $c_\alpha^i$ in (2) depend on the normalisation of the kinetic terms for the RR fields $B_i$. Such kinetic terms are in general radii dependent and once one redefines the fields to canonical kinetic terms, extra volume factors $\xi^i$ appear in (2) multiplying each $c_\alpha^i$. It is difficult to estimate the (numerical) value of such factors on string theory grounds, and this is reducing the predictive power of the models considered. Their expressions in terms of the radii $R^i_{1,2}$ of torus $i$ are for the D6-brane case$^5$
\[ \xi^i = \left[ \frac{R^i_1 R^i_2 R^k_1}{R^i_1 R^j_1 R^k_2} \right]^{1/2}, \quad i \neq j \neq j \neq k \neq i \]

As in [16] we assume that the factors $\xi^i$ are equal in magnitude and thus may be absorbed into the re-definition of the string scale $M_S$ in eq.(2). As a consequence the string scale prediction is subject to the uncertainty induced by such volume factors [16]. For the remaining of this work $M_S$ will thus stand for this re-scaled value.

The natural question that emerges is then whether one is indeed able to avoid the constraints induced by the (unknown) volume factors $\xi^i$ and make a prediction in the class of models under discussion. A low value of the string scale as found in [16] shows that one can construct string models with $M_S$ in the region of few TeV, while still complying with current constraints from $\rho$ parameter physics. In itself this finding is important and reassuring for the consistency of the model, since it does not require a string scale too large compared to TeV scale (which would re-introduce a hierarchy problem). However, for experimental searches it is less important the exact value of $M_S$ which still complies with current experimental constraints, or the aforementioned uncertainties affecting it, and more relevant the actual lower bounds on the masses of the additional $U(1)$ fields. It turns out that one can make a prediction for the latter independently of the volume factors mentioned above, by only assuming that they are equal for the three torii ($\xi^1 = \xi^2 = \xi^3$) for D6 case (or two torii in case of D5 branes). This can be respected if for example the ratio of the two radii of the $i$-th torus, $R^i_2/R^i_1$ is the same for any $i = 1, 2, 3$.

Using $\rho$ parameter constraints one can extract (lower) bounds on the value of $M_S$, as already done in [16] for D6 models. Further, using the mass eigenvalues $M_i^2$ of $(M^2)_{\alpha\beta}$ computed in terms of $M_S$ (to which we add their electroweak corrections) we are able to predict lower bounds on the total masses $M_t$ of the additional $U(1)$ fields, independent of the volume factors $\xi^i$. Therefore, while the string scale prediction is affected by such unknown factors, one can make predictions for the lower bounds on the masses of $U(1)$ fields. This motivated the present analysis and completes the

$^5$The hypercharge $U(1)_Y$ remains massless since there is no coupling $B_i \wedge F_y$ where $F_y$ is a linear combination of the initial four $U(1)_\alpha$ fields strengths. Due to this the sum in (3) runs only over $i=1,2,3$, unlike in [16]. This is possible because at string level there always exists one massless $U(1)$ which together with the model building constraint in the second-last column of Table 1 may be identified with $U(1)_Y$. The origin of the existence of a massless (gauged) $U(1)$ at string level is not clear, but it can be related to topological arguments.

$^6$Similar relations apply for D5-brane case.
| Higgs   | $\nu$ | $\beta_1$ | $\beta_2$ | $n_{a1}$ | $n_{b1}$ | $n_{c1}$ | $n_{d2}$ | $N_h$          |
|---------|------|----------|----------|---------|---------|---------|---------|----------------|
| $n_H = 1, n_h = 0$ | 1/3 | 1/2       | $\beta_2$ | $n_{a2}$ | -1      | 1       | $\frac{1}{\beta_2} - n_{a2}$ | $4\beta_2(1 - n_{a2})$ |
| $n_H = 1, n_h = 0$ | 1/3 | 1/2       | $\beta_2$ | $n_{a2}$ | 1       | -1      | $-\frac{1}{\beta_2} - n_{a2}$ | $4\beta_2(1 - n_{a2})$ |
| $n_H = 0, n_h = 1$ | 1/3 | 1/2       | $\beta_2$ | $n_{a2}$ | 1       | 1       | $\frac{1}{\beta_2} - n_{a2}$ | $4\beta_2(1 - n_{a2}) - 1$ |
| $n_H = 0, n_h = 1$ | 1/3 | 1/2       | $\beta_2$ | $n_{a2}$ | -1      | -1      | $-\frac{1}{\beta_2} - n_{a2}$ | $4\beta_2(1 - n_{a2}) + 1$ |
| $n_H = 1, n_h = 1$ | 1   | 1         | $\beta_2$ | $n_{a2}$ | 0       | 1       | $\frac{2}{\beta_2} - n_{a2}$ | $\beta_2(8 - \frac{4n_{a2}}{3}) - \frac{1}{3}$ |
| $n_H = 1, n_h = 1$ | 1   | 1         | $\beta_2$ | $n_{a2}$ | 0       | -1      | $\frac{1}{3}(-\frac{2}{\beta_2} - n_{a2})$ | $\beta_2(8 - \frac{4n_{a2}}{3}) + \frac{1}{3}$ |
| $n_H = 1, n_h = 1$ | 1/3 | 1         | $\beta_2$ | $n_{a2}$ | 0       | 1       | $\frac{2}{\beta_2} - n_{a2}$ | $\beta_2(8 - 4n_{a2}) - 1$ |
| $n_H = 1, n_h = 1$ | 1/3 | 1         | $\beta_2$ | $n_{a2}$ | 0       | -1      | $-\frac{2}{\beta_2} - n_{a2}$ | $\beta_2(8 - 4n_{a2}) + 1$ |

Table 1: Families of D6-brane models with minimal Higgs content as derived in [1]. The first four lines correspond to models of Class A of ref. [1], the remaining ones to models of Class B, which are distinguished by their different Higgs sector. The parameters of the models are $n_{a2}$ and $g_{d}/g_{c}$. The definition of $n_{a2}$ enables one to obtain the SM value for hypercharges, and thus to identify the massless (linear combination of) $U(1)$ with $U(1)_{Y}$. $\beta_i = 1 (1/2)$ is a parameter due (in a T-dual picture) to background NS B field (which modifies the complex structure of the $i^{th}$ two-torus) and corresponds to orthogonal (tilted) torus. The presence of $\beta_i$ enables an odd number of generations in these models [9] and also a shift in the effective wrapping numbers. Finally $N_h$ stands for the number of branes parallel to the orientifold plane, added for global RR tadpole cancellation [1].

A discussion of [16] which overlooked this observation.

### 2.1 Lower bounds on $U(1)$ masses.

For D6 models the parameters are presented in Table 1 while the Higgs sector quantum numbers are presented in Table 2. After diagonalisation of the mass matrix (2) one computes the masses $M_{i}^2$ ($i=2,3,4$) ($M_{1}^2 = 0$ for hypercharge) and eigenvectors for gauge bosons, according to the relations:

$$A_i^T = \sum_{a,b,c,d} F_{i\alpha} A_{\alpha}; \quad \delta_{ij} M_i^2 = F M^2 F^T, \quad i,j = 1,4.$$ (4)

where explicit entries for $F_{i\alpha}$ are given in the Appendix eqs. (A-13), (A-14) and $F F^T = F^T F = 1$.

Upon electroweak symmetry breaking, the mass matrix (2) receives corrections from the mixing of the Higgs state with some $U(1)_i$ fields, present if Higgs state is charged under $U(1)$. The fields $A_i^T$ receive an additional electroweak mass correction and $Z$ boson acquires a mass as well, a fraction of which is due to the string mechanism for mass, induced by the mixing of $Z$ boson with massive $U(1)$'s. The new mass matrix $M_{\gamma\gamma}'$ in the “extended” basis $a,b,c,d$ and $W_3^\mu$ (of $SU(2)_L$) contains a $4 \times 4$ sub-block $M_{\alpha\beta}^2$ ($\alpha, \beta = a,b,c,d$) to which we added electroweak corrections. Its explicit form was presented in [13]. The eigenvectors of $M_{\gamma\gamma}'$ (denoted by $F^*\gamma$) are computed in the Appendix eqs. (A-16) and satisfy

$$A_i^* = \sum_{\gamma=a,b,c,d,W_3} F_{i\gamma}^* A_\gamma; \quad \delta_{ij} M_i^2 = F^* M^2 F^{*T}, \quad i,j = 1,4.$$ (5)

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| Higgs $\sigma$ | $q_b$ | $q_c$ | $q'_b \equiv q_y$ | $T_3 = 1/2 \sigma_z$ | Higgs $\sigma$ | $q_b$ | $q_c$ | $q'_b \equiv q_y$ | $T_3 = 1/2 \sigma_z$ |
|---------------|------|------|------------------|-----------------|---------------|------|------|------------------|-----------------|
| $h_1$         | 1    | -1   | 1/2              | $+1/2$          | $H_1$         | -1   | -1   | 1/2              | $+1/2$          |
| $h_2$         | -1   | 1    | $-1/2$           | $-1/2$          | $H_2$         | 1    | 1    | $-1/2$           | $-1/2$          |

Table 2: Higgs fields, their U(1)$_{b,c}$ charges and weak isospin with $\sigma_z$ the diagonal Pauli matrix. Class A models contain either $H_{1,2}$ or $h_{1,2}$ while Class B contains both $H_i$ and $h_i$ with mixing angle $\theta$.

with $A_1^*$ to stand for the photon, $A_{2,3,4}^*$ for the massive $U(1)$ fields, $A_5^*$ for $Z$ boson and $F^*F^{*T} = F^{*T}F^* = 1$. The mass of $Z$ boson including the string corrections induced by the term $\xi$ is

$$M_Z^2 \equiv M_5^2 = M_5^2 \left[ 1 + \eta \xi_{21} + \eta^2 \xi_{31} + O(\eta^3) \right]$$

$$M_0^2 = \frac{1}{4}(4g_b^2 + g_c^2) < \phi >^2$$

where $M_0$ is the mass of $Z$ boson as given in the Standard Model, $< \phi >^2 \cos^2 \theta \equiv H_1^2 + H_2^2$, $< \phi >^2 \sin^2 \theta \equiv h_1^2 + h_2^2$ and $\theta$ is the mixing angle in the Higgs sector. $\theta = 0, (\pi/2)$ for Class A models with $n_H = 1, n_b = 0$ ($n_H = 0, n_b = 1$) respectively. For Class B models $\theta$ is not restricted, playing a role similar to $\tan \beta$ of the MSSM. Imposing $\rho$ parameter constraints on the mass of $Z$ boson, we derived the lower bound eq.(7) on the value of $M_S^2$ [16]. Note that $M_S^2$ includes possible volume factors effects $\xi^i$ (see eq.(3) and text thereafter).

$$M_S^2 = < \phi >^2 (\xi_{21}) \left[ 1 + \frac{\rho^0}{\Delta \rho} \right] \left[ 1 - 2 \frac{\xi_{31}}{\xi_{21}} + \frac{1}{\Delta \rho / \rho^0} \right]^{1/2}$$

The last bracket brings a correction to $M_S^2$ less than 0.1% relative to the case of a full numerical approach to computing $M_S^2$, and will thus be ignored hereafter. The value of $\xi_{21}$ is given by

$$\xi_{21} = - \left\{ \beta_1^2 \left[ 2 \beta_1 g_b^2 \nu n_{c1}(1 + R^2) - (36g_a^2 + g_c^2 R^2) \beta_2 n_{a2} \nu \right] \right\} + 4 \beta_1^2 \beta_2^2 \epsilon^2 (36g_a^2 + g_c^2 R^2)^2 + 9 \beta_1^2 \left[ g_b^2 n_{b1} \nu R^2 + 12g_a^2 (3n_{b1} \nu - n_{c1}(1 + R^2) \cos(2\theta))^2 \right] \right\} \left[ 5184 \beta_1^2 \beta_2^2 \epsilon^2 g_a^2 n_{c1}^2 (1 + R^2)^2 \right]^{-1}$$

The above two equations give the lower bounds on $M_S^2$ in terms of the chosen parameters, which are the ratio $R = g_d / g_c$ and $n_{a2}$ while $g_y$ is the hypercharge coupling, eq.(A-17).

One constraint in the case of additional $U(1)$ bosons is that on their mixing angle with the usual $Z$ boson, which may be read from the eigenvector of the latter. For the case when only one additional $Z'$ boson exists, the mixing is induced by the presence of an off-diagonal mass term $m_{ZZ'}^2 Z Z'$ in the action (in addition to $m_Z^2 Z^2$ and $m_{Z'}^2 Z'^2$). This mixing may be expressed in function of the mass eigenstates as (see for example [20])

$$\psi_0 = \arctan \left[ \frac{M_0^2 - M_Z^2}{M_{Z'}^2 - M_0^2} \right]^{1/2}$$

with $M_{Z'}$ the mass of the additional boson. Current experimental constraints provide bounds on the mixing angle $\theta$ in the region of $\psi_0 = k \times 10^{-3}$ with $k$ of order unity [20].
For more than one additional boson which is our case, relation (1) does not hold. In this case the mixing of $Z$ boson with the massive $U(1)$ fields can be computed using $\mathcal{F}$ and $\mathcal{F}^*$ of eqs. (A-13), (A-14), (A-16) to give

$$Z \equiv A^*_5 = \sum_{i=1}^{4} \psi_i A'_i + \mathcal{F}^*_{5W_3} W_3,$$

$$\psi_i \equiv \sum_{\alpha=a,b,c,d} \mathcal{F}^*_{5\alpha} \mathcal{F}_{i\alpha}$$

(10)

where $\psi_i$ accounts for the mixing $Z - A'_i$ in the basis (normalised) of the fields $A'_i, W_3, i = 1, 2, 3, 4$. In the limit of an infinite string scale, the massive $U(1)$ fields decouple ($\psi_{2,3,4} \to 0$) to leave the usual mixing of the Standard Model with $\mathcal{F}^*_{5W_3} \to \cos \theta_W$ and $\psi_1 \to \sin \theta_W$. Eq. (10) thus provides the eigenvector of the $Z$ boson for D6-brane SM-like models. Even in the case of more than one additional boson one may in principle define an effective angle $\psi'_i$ for the mixing of $Z$ boson with a massive $A'_i$ state (i=2,3,4) as in (1) with $M_{Z'} \to M_i$, $i = 2, 3, 4$. This may further be expressed as

$$\psi'_i = \arctan \left[ \frac{\Delta \rho/\rho_0 - 1}{\Delta \rho/\rho_0 + 1 \mathcal{M}_i^2/M_5^2 - 1} \right]^{1/2}, \quad i = 2, 3, 4.$$  

(11)

where we used that for the $\rho$ parameter $\rho_0 = M_W^2/(M_0^2 \cos \theta_W)$ and thus $\Delta \rho/\rho_0 = -1 + M_0^2/M_5^2$. Detailed comparison of this amount of mixing with that of (1) shows that $\psi'_i$ may provide an estimate for the amount of mixing for the cases we considered, but it is generally larger than that of (1). Also definition (11) does not contain information about the sign of the mixing and its dependence on the parameters of the model ($n_{a2}$ and $g_d/g_c$) is different from that of eq. (10) which will be used throughout this analysis.

From the mass eigenvalue equation after electroweak symmetry breaking $det(\mathcal{M}^2 - \mathcal{M}_i^2 I_5) = 0$, $i = 1, 5$ one finds $\mathcal{M}_i^2$ in terms of the string scale, $M_5$. Using the lower bounds (5) on the latter we can then make predictions for (lower bounds on) the masses of additional $U(1)$ fields $A^*_i$ (i=2,3,4). These bounds are therefore independent of the volume factors $\xi_i$, entering in the “re-scaled” value of $M_5$. The results are presented in Table 3 and discussed below in function of the associated mixing.

Model independent constraints on the value of the mixing lead to $|\psi| < 0.003$ [23], but there are cases when this may be smaller ($10^{-3}$) [24]. We thus presented in Table 3 our results for two (upper) values of $1.5 \times 10^{-3}$ and $3 \times 10^{-3}$ of the mixing angles of $Z$ boson with any massive $U(1)$ field. In Figure 2 lower bounds on the masses of $U(1)$ fields are also presented. For fixed $g_d/g_c$ the bounds on the mixing angle set bounds on $n_{a2}$ which is then used for finding the lowest allowed values for the $U(1)$ masses. For Class A models, Figure 1 (left column) one finds that generically $\mathcal{M}_2 > 25 \times 10^3$ GeV and $\mathcal{M}_4 > 3.5$ TeV. $\mathcal{M}_3$ can be as low as 1200 GeV for a mixing of order $\psi_3 \approx 1.5 \times 10^{-3}$ with $g_d/g_c = \mathcal{O}(1)$, see also Figure 3. For $\psi_3 \approx 3 \times 10^{-3}$ the bound on $\mathcal{M}_3$ decreases to $\mathcal{M}_3 > 750$ GeV. For Class A models with $\beta_2 = 1$ instead of $\beta_2 = 1/2$ the lowest bounds on $\mathcal{M}_3$ and $\mathcal{M}_4$ do not change significantly, while for $\mathcal{M}_2$ an increase of factor $\approx 4$ is present while still keeping the same amount of mixing. Changing the sign of $n_{c1}$ and $n_{b1}$ for fixed $\beta_{1,2}$ and $\nu$ does not affect significantly these bounds.

For Class B models, Figure 1 (right) the lower bounds on the masses have a similar dependence in function of $g_d/g_c$ giving $\mathcal{M}_2 > 4.5$ TeV, $\mathcal{M}_3 > 1.5$ TeV and $\mathcal{M}_4 > 1200$ GeV corresponding to a
**Figure 1:**

**Left column:** Masses of $U(1)$ fields (GeV) in Class A models with $\beta_1 = 1/2$, $\beta_2 = 1/2$, $\nu = 1/3$, $n_{c1} = 1$, $n_{b1} = -1$ in function of the parameter ratio $g_d/g_c$ for varying wrapping number $n_{a2}$. The lower bound on $M_4$ is saturated at large $n_{a2}$.

**Right column:** Masses of $U(1)$ fields (GeV) in Class B models with $\beta_1 = 1$, $\beta_2 = 1$, $\nu = 1$, $n_{c1} = 1$, $\phi = \pi/6$. in function of $g_d/g_c$ for varying wrapping number $n_{a2}$. The lower bound on $M_4$ is saturated at large $n_{a2}$, but corresponds to a mixing angle beyond $1.5 \times 10^{-3}$. $U(1)_b$ of mass $M_3$ does not mix with the rest of $U(1)$ fields (before electroweak symmetry breaking). These mass values comply with $\rho$ parameter constraints and include their electroweak corrections.
mixing of $1.5 \times 10^{-3}$ (or to $n_{a2} = 2$). $M_4$ decreases to 600 GeV for $\psi = 3 \times 10^{-3}$. $M_3$ is proportional to $M_S$ and is the analogue of the anomalous $U(1)$ in heterotic models (it does not mix with the remaining $U(1)$’s eq.([14-15]), unlike the case of Class A models [16]). Changing $\beta_2$, $n_{c1}$, $n_{b1}$ and $\rho$ brings in (small) changes on the bounds, as presented in Table 3.

To conclude, the lowest bounds for $U(1)$ masses with any of their associated mixing angles with Z boson in the region of $1.5 \times 10^{-3}$ or less, are 1100 GeV for Class A models and 750 GeV for Class B models. Note that for a fixed amount of mixing, when the two $U(1)$ couplings $g_c, g_d$ are comparable, one finds for generic cases lower bounds on masses than when the couplings are significantly different. These bounds should be compared to current experimental $Z'$ mass limit which is $> 690$ GeV and was obtained by CDF with the assumption that the $Z'$ boson has SM couplings strengths [22]. To help identify specific signatures of new $U(1)$ fields and distinguish from other models with additional $Z'$ bosons, the full Z boson eigenvector is presented in the Appendix. The mass bounds on the additional $U(1)$ bosons we found are somewhat larger than those of alternative models for similar amounts of mixing, which are [21] in the range of $545$ GeV (SO(10) GUT models), $564$ GeV (for left-right models with gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \subset SO(10)$ and $809$ GeV for the sequential $Z_{SM}$ boson defined to have the same couplings to fermions as the SM Z boson.

3 Conclusions

The analysis of phenomenological viability of consistent D-brane SM-like models is at an early stage, and this work was intended as a step in this direction. In such models, additional massive $U(1)$ fields are a generic presence. Previous analysis of the implications of these $U(1)$’s and of the value of the string scale revealed that $\rho$ parameter constraints can be respected for a string scale in the TeV

![Figure 2: Class A models: $(\beta_1 = 1/2, \beta_2 = 1/2, \nu = 1/3 \ n_{c1} = 1, n_{b1} = -1)$. The mixing $\psi_3$ of Z boson with $A'_3$ the lightest state among the additional $U(1)$’s. $n_{a2}$ varies as shown with step 2. The largest amount of mixing is usually manifest for $g_d \approx g_c$, increasing with $n_{a2}$. Upper bounds on the former translate into bounds on $n_{a2}$ and thus on the $U(1)$ masses, Figure 1.](image)
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| Higgs          | $\nu$ | $\beta_1$ | $\beta_2$ | $n_{c1}$ | $n_{b1}$ | $M_2$ (TeV) | $M_3$ (TeV) | $M_4$ (TeV) |
|----------------|------|-----------|-----------|---------|---------|------------|------------|------------|
| $n_H = 1, n_h = 0$ | $1/3$ | $1/2$    | $1/2$    | $1$     | $-1$    | $> 25$    | $1.2 (0.75)$ | $> 3.5$    |
| $n_H = 1, n_h = 0$ | $1/3$ | $1/2$    | $1/2$    | $1$     | $-1$    | $> 110$   | $1.7 (0.75)$ | $> 3.5$    |
| $n_H = 1, n_h = 0$ | $1/3$ | $1/2$    | $1/2$    | $-1$    | $1$     | $> 25$    | $1.2 (0.65)$ | $> 3.5$    |
| $n_H = 1, n_h = 0$ | $1/3$ | $1/2$    | $1$      | $-1$    | $1$     | $> 100$   | $1.2 (0.65)$ | $> 3.5$    |
| $n_H = 0, n_h = 1$ | $1/3$ | $1/2$    | $1/2$    | $1$     | $1$     | $> 25$    | $1.2 (0.6)$  | $> 3.5$    |
| $n_H = 0, n_h = 1$ | $1/3$ | $1/2$    | $1/2$    | $1$     | $1$     | $> 100$   | $1.2 (0.6)$  | $> 3.5$    |
| $n_H = 0, n_h = 1$ | $1/3$ | $1/2$    | $1$      | $-1$    | $-1$    | $> 25$    | $1.1 (0.65)$ | $> 3.5$    |
| $n_H = 0, n_h = 1$ | $1/3$ | $1/2$    | $1$      | $-1$    | $-1$    | $> 65$    | $1.2 (0.6)$  | $> 3.25$  |

Table 3: Lower bounds for Class A (Class B) models in the upper (lower) table. The values correspond to any of $Z$ bosons’ mixing with massive $U(1)$’s less than $1.5 \times 10^{-3}$. Values in brackets correspond to mixings of up to $3 \times 10^{-3}$. (For Class B models we chose $\theta = \pi/6$). These bounds are only reached in cases with $g_d/g_c = O(1)$, otherwise they may increase further.

This result was used in this work to set lower bounds on the masses of the additional $U(1)$’s, independent of the volume factors $\xi_1$ affecting the string scale prediction. The masses of the $U(1)$ fields are of string origin (with small electroweak corrections suppressed by the string scale), therefore no additional Higgs states (beyond the SM case) are required. The values of $U(1)$ masses were found to be somewhat larger than those of alternative models. The amount of mixing of the SM $Z$ boson with any of the new $U(1)$ fields and the eigenvectors of the $U(1)$ fields were computed. This information may be further used to improve our bounds on the $U(1)$ masses from (upper) bounds on the mixing derived from the dilepton decay modes of $Z'$ bosons.

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Appendix:

**D6 brane models:** The $U(1)_\alpha$ charges of quarks and leptons in D6-brane models of $[1]$ are:

| Intersection | Matter fields | $q_a$ | $q_b$ | $q_c$ | $q_d$ | $q_Y$ |
|--------------|---------------|-------|-------|-------|-------|-------|
| (ab)         | $Q_L$         | (3,2) | 1     | -1    | 0     | 0     | 1/6   |
| (ab*)        | $q_L$         | 2(3,2)| 1     | 1     | 0     | 0     | 1/6   |
| (ac)         | $U_R$         | 3(3,1)| -1    | 0     | 1     | 0     | -2/3  |
| (ac*)        | $D_R$         | 3(3,1)| -1    | 0     | -1    | 0     | 1/3   |
| (bd*)        | $L$           | 3(1,2)| 0     | -1    | 0     | -1    | -1/2  |
| (cd)         | $E_R$         | 3(1,1)| 0     | 0     | -1    | 1     | 1     |
| (cd*)        | $N_R$         | 3(1,1)| 0     | 0     | 1     | 1     | 0     |

Table 4: The hypercharge generator is defined by $q_Y = 1/6 q_a - 1/2 q_c + 1/2 q_d$. The asterisk denotes the “orientifold mirror” of each given brane. $U(1)_a$ and $U(1)_d$ can be identified with baryon number and (minus) lepton number respectively. $U(1)_c$ can be identified with the third component of right-handed weak isospin. $U(1)_b$ is an axial symmetry with QCD anomalies, much like a PQ-symmetry. $U(1)_b$ and $3U(1)_a - U(1)_d$ linear combination have triangle anomalies, cancelled by a generalised Green-Schwarz mechanism, whereas $U(1)_a + 3U(1)_d$ and $U(1)_c$ are anomaly-free.

The coefficients $c_i^\alpha$ encountered in the text eq. (2) are given by $[1]$

$$c_i^\alpha = N_\alpha n_{\alpha j} n_{\alpha k} m_{\alpha i} ; \ i \neq j \neq k \neq i , \ i = 1,2,3$$

(A-12)

$N_\alpha$ is the number of parallel branes of type $\alpha$. The wrapping numbers $n_\alpha, m_\alpha$ as derived in $[1]$ are:

| $N_\alpha$ | $(n_{a1}, m_{a1})$ | $(n_{a2}, m_{a2})$ | $(n_{a3}, m_{a3})$ |
|------------|---------------------|---------------------|---------------------|
| $N_a = 3$  | $(1/\beta_1, 0)$    | $(n_{a2}, e \beta_2)$ | $(1/\nu, 1/2)$     |
| $N_b = 2$  | $(n_{b1}, -e \beta_1)$ | $(1/\beta_2, 0)$    | $(1, 3\nu/2)$      |
| $N_c = 1$  | $(n_{c1}, 3\nu e \beta_1)$ | $(1/\beta_2, 0)$    | $(0, 1)$           |
| $N_d = 1$  | $(1/\beta_1, 0)$    | $(n_{d2}, -e \beta_2/\nu)$ | $(1, 3\nu/2)$      |

Table 5: D6-brane wrapping numbers giving rise to a SM spectrum $[1]$. The general solutions yielding the SM spectrum are parametrized by a phase $\epsilon = \pm 1$, the NS background on the first two tori $\beta_1 = 1 - b_i = 1, 1/2$, four integers $n_{a2}, n_{b1}, n_{c1}, n_{d2}$ and a parameter $\nu = 1, 1/3$.

**Eigenvectors in Class A models.** For Class A models the matrix $\mathcal{F}_i^{\alpha}$ $(i = 1, 2, 3, 4, \ \alpha = a, b, c, d)$ is given below (with $\lambda_1 = 0$ (hypercharge) and $\lambda_i = M_i^2 / M_S^2, \ i = 2, 3, 4$ the roots of $\det(\lambda M_S^2 I_4 - M^2) = 0$, computed in $[14]$)

$$\mathcal{F}_i^{\alpha} = \frac{1}{|\mathcal{F}_i|} \{ g_d, 3g_a, 0, -g_d, g_c, 0 \}_{\alpha}, \ \ \alpha = a, b, c, d$$
\[ F_{ia} = \frac{1}{|F_i|} \frac{3\beta_2 g_a (2\beta_2 g_a^2 n_{c1} - n_{a2}\lambda_1 \beta_3 \nu^2)}{g_d [18\beta_2^2 g_a^3 n_{c1} + \beta_1 \nu \lambda_i (n_{a2} \beta_2 - 2\beta_1 n_{c1})]}, \quad i = 2, 3, 4. \]

\[ F_{ib} = \frac{1}{|F_i|} \left\{ -\frac{4\beta_1^2 \beta_2^2 \nu^2 \lambda_1^2 + 4\beta_2^2 [g_a^2 g_d + g_a^2 (g_c^2 + g_d^2)] n_{c1}}{6\nu n_{a1} g_b g_d [18\beta_2^2 g_a^3 n_{c1} + \beta_1 \nu \lambda_i (n_{a2} \beta_2 - 2\beta_1 n_{c1})]} \right. \\
\left. + \frac{\lambda_i \left( \beta_2^2 (9\beta_a^2 + \beta_d^2) (4\beta_2^2 + \nu^2 n_{a2}) - 4\beta_1 \nu \lambda_i (n_{a2} \beta_2 - 2\beta_1 n_{c1}) \right]}{6\nu n_{b1} g_b g_d [18\beta_2^2 g_a^3 n_{c1} + \beta_1 \nu \lambda_i (n_{a2} \beta_2 - 2\beta_1 n_{c1})]} \right\}, \quad F_{id} = \frac{1}{|F_i|} \quad (A-13) \]

with the notation \(|F_i| \equiv (\sum_a |F_{ia}|)^{1/2}.

**Eigenvectors in Class B models.** For Class B models the matrix \(F_{ia}\) is given by:

\[ F_{1\alpha} = \frac{1}{|F_1|} \left\{ \frac{g_d}{3g_a}, 0, -\frac{g_d}{g_c}, 1 \right\}_\alpha, \quad \alpha = a, b, c, d \]

\[ F_{2\alpha} = \frac{1}{|F_2|} \left\{ \frac{3g_d}{g_d}, \frac{2\omega_1}{(\omega_2 - 8\beta_1^2 \beta_2^2 \nu^2 y)}, 0, \frac{g_c}{g_d} \frac{1}{2\omega_1} (\omega_3 - 8\beta_1^2 \beta_2^2 \nu^2 y), 1 \right\}_\alpha \]

\[ F_{3\alpha} = \{0, 1, 0, 0\}_\alpha \]

\[ F_{4\alpha} = \frac{1}{|F_4|} \left\{ \frac{3g_d}{g_d}, \frac{2\omega_1}{(\omega_2 + 8\beta_1^2 \beta_2^2 \nu^2 y)}, 0, \frac{g_c}{g_d} \frac{1}{2\omega_1} (\omega_3 + 8\beta_1^2 \beta_2^2 \nu^2 y), 1 \right\}_\alpha \quad (A-14) \]

where

\[ \omega_1 = 36\beta_2^4 g_a^2 + \nu^2 (\beta_2 n_{a2} - 2\beta_1 n_{c1}) \left( 9\beta_2^2 g_a^2 n_{a2} + 2\beta_1 n_{c1} g_c^2 \right), \]

\[ \omega_2 = -\{\beta_2 (9\beta_a^2 - \beta_d^2) (4\beta_2^2 + \nu^2 n_{a2}) + 4\nu^2 \beta_1 n_{c1} (g_c^2 + g_d^2) (n_{a2} \beta_2 - 2\beta_1 n_{c1}) \}, \]

\[ \omega_3 = \{\beta_2 (9\beta_a^2 + \beta_d^2) (4\beta_2^2 + \nu^2 n_{a2}) - 4\nu^2 \beta_1 n_{c1} (g_c^2 + g_d^2) (n_{a2} \beta_2 - 2\beta_1 n_{c1}) \} \]

with the notation:

\[ y = \left\{ x^2 - \frac{n_{c1}^2}{\beta_1 \nu} \left[ g_c^2 g_d^2 + g_a^2 (g_c^2 + g_d^2) \right] \right\}^{1/2} \]

\[ x = \frac{1}{8\beta_1 \beta_2 \nu^2} \left\{ (9\beta_a^2 + \beta_d^2) [4\beta_2^4 + \nu^2 \beta_2^2 n_{a2}] + 4\beta_1 n_{c1} \nu^2 [\beta_1 (g_c^2 + g_d^2) n_{c1} - \beta_2 g_a n_{a2}] \right\} \quad (A-15) \]

The eigenvectors \(F_{ia}\) (i-fixed), correspond to mass eigenvalues \(M_i^2\) with \(M_1^2 = 0, M_2^2 = (x + y)M_3^2, M_3^2 = (2\beta_1 / \beta_2) g_a^2 M_2^2\) and \(M_4^2 = (x - y)M_3^2\) respectively.

**Eigenvectors after Electroweak Symmetry Breaking.** The matrix \(F_{i\gamma}\) (with \(i = 1, 5\) and \(\gamma = a, b, c, d, W_3\)) after electroweak symmetry breaking has the same structure for both Class A and
Class B models and is given below. $\lambda_i^* = M_i^2/M_S^2$ is an eigenvalue of $M_{\gamma'}^2$, expressed in string units, $\lambda_1^* = 0$ (photon state) and $\delta \equiv \cos(\theta)$ with $\theta$ defined in the text as the mixing angle in the Higgs sector. For Class B models one should set in the eqs. below $n_{b1} = 0$.

\[
\mathcal{F}_{1\gamma}^* = \frac{1}{|\mathcal{F}_{1\gamma}|} \left\{ \frac{g_d}{3g_a}, 0, \frac{-g_d}{g_c}, 1, \frac{-g_d}{g_b} \right\}_\gamma, \quad \gamma = a, b, c, d, W_3.
\]  

(A-16)

\[
\mathcal{F}_{ia}^* = \frac{1}{|\mathcal{F}_{ia}|} \frac{3\beta_2 g_a (2\beta_2 g_a^2 n_{c1} - n_{a2} \lambda_1^* \beta_1 \nu_2)}{g_d [18 \beta_2^2 g_a^2 n_{c1} + \beta_1 \nu_2 \lambda_i^* (n_{a2} - 2\beta_1 n_{c1})]}, \quad i = 2, 3, 4, 5.
\]

\[
\mathcal{F}_{ib} = \left\{ (\eta g_b^2 + g_d^2 - \lambda_i^*) \left[ 4\beta_2 (9g_a^2 + g_d^2) \lambda_i^* - 4\beta_1 \beta_2 g_a^2 \lambda_i^* n_{a2} \nu_2 + \nu_2 \lambda_i^* \beta_2 ^2 ((9g_a^2 + g_d^2) n_{a2}^2 - 4\beta_1^2 \lambda_i^*) ight] + \beta_2^2 \left[ 4n_{c1}^2 (g_c^2 g_d^2 + g_d^2 g_a^2) (\lambda_i^* - g_b^2 \eta) + 9g_a^2 g_c^2 (\lambda_i^* - (g_b^2 + g_d^2) \eta) \right] + 4\beta_2^2 \lambda_i^* n_{c1}^2 \nu_2 \left[ (-\lambda_i^* + g_b^2 \eta) (g_c^2 + g_a^2) + g_d^2 \nu_2 \eta \right] \} \{ 2g_d [18 \beta_2^2 g_a^2 n_{c1} - \beta_1 \lambda_i^* (-\beta_2 n_{a2} + 2\beta_1 n_{c1}) \nu_2] \}^{-1} |\mathcal{F}_{ia}^*|^{-1}
\]

\[
\mathcal{F}_{ic}^* = \left\{ -g_c \left[ 4\beta_2 (9g_a^2 + g_d^2) \lambda_i^* \eta \delta - 4\beta_1 \beta_2 g_a^2 \lambda_i^* n_{a2} n_{c1} \eta \nu_2 \delta + 4\beta_2^2 \lambda_i^* n_{c1}^2 \nu_2 (9g_b^2 n_{b1} \nu_2 + g_d^2 n_{c1} \eta \delta) - 3\lambda_i^* n_{b1} \nu_2 \right] + \beta_2^2 \left[ -4\beta_1^2 \lambda_i^* \nu_2 \delta \left[ 12 n_{b1} \eta (g_b^2 n_{b1} \nu_2 + g_d^2 n_{c1} \eta \delta) + (g_b^2 + g_d^2) \lambda_i^* \nu_2 (12 n_{b1} \eta + n_{a2} \eta \delta) \right] \} \{ 2g_d [18 \beta_2^2 g_a^2 n_{c1} + \beta_1 \lambda_i^* \nu_2 (2\beta_2 n_{a2} - 2\beta_1 n_{c1})] \}^{-1} |\mathcal{F}_{ic}^*|^{-1}
\]

\[
\mathcal{F}_{id}^* = |\mathcal{F}_{ia}^*|^{-1}
\]

\[
\mathcal{F}_{iW_3} = \left\{ 2g_d g_b \left[ 12 n_{b1} n_{c1} \nu_2 (\beta_2 (9g_a^2 + g_d^2) - \beta_1^2 \lambda_i^* \nu_2) + [4\beta_2 (9g_a^2 + g_d^2) \lambda_i^* - 4\beta_2^2 n_{c1} (g_c^2 g_d^2 + g_d^2 (g_c^2 + g_a^2))] - \lambda_i^* [4\beta_1 \beta_2 g_a^2 n_{a2} n_{c1} - \beta_2^2 n_{c1}^2 (g_c^2 g_d^2 + g_d^2 g_a^2) + 4\beta_1^2 \lambda_i^* (g_c^2 + g_a^2) n_{c1}^2] \} \nu_2 \delta \} |\mathcal{F}_{ia}^*|^{-1}
\]

\[
\times \{ 2g_d [18 \beta_2^2 g_a^2 n_{c1} + \beta_1 \lambda_i^* \nu_2 (2\beta_2 n_{a2} - 2\beta_1 n_{c1})] \}^{-1} \{ -3\lambda_i^* n_{b1} \nu_2 + 3g_b^2 n_{b1} \eta \nu_2 + g_c^2 \eta (3n_{b1} \nu_2 - n_{c1} \delta) \} \}
\]

where we used the notation $|\mathcal{F}_{i\gamma}|^2 = \sum_{\gamma:a,b,c,d,W_3} |\mathcal{F}_{i\gamma}|^2$ and where one has that \[13\]

\[
\frac{1}{g_y^2} = \frac{1}{36 g_a^2} + \frac{1}{4 g_c^2} + \frac{1}{4 g_d^2}
\]

(A-17)

with $g_y$ the hypercharge coupling. Eq.\,(A-17) with fixed $g_y$ and $g_a^2 = g_{QCD}^2/6$ establishes a correlation for the allowed values of $g_c$ and $g_d$ and this is used in the text. To compute the mixing of Z boson with the massive $A'_i$ fields ($i = 2, 3, 4$) one replaces $\lambda_i^*$ with $M_i^2/M_S^2$ where $M_S^2$ and $M_S^2$ were given in the text.

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