Sea Quark or Anomalous Gluon Interpretation for $g^p_1(x)$?

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Abstract

Contrary to what has been often claimed in the literature, we clarify that the hard photon-gluon cross section $\Delta \sigma_{\gamma G}^{\text{hard}}(x)$ in polarized deep inelastic scattering calculated in the gauge-invariant factorization scheme does not involve any soft contributions and hence it is genuinely hard. We show that the polarized proton structure function $g^p_1(x)$ up to the next-to-leading order of $\alpha_s$ is independent of the factorization convention, e.g., the gauge-invariant or chiral-invariant scheme, chosen in defining $\Delta \sigma_{\gamma G}^{\text{hard}}(x)$ and the quark spin density. Thereby, it is not pertinent to keep disputing which factorization prescription is correct or superior. The hard-gluonic contribution to $\Gamma_1^p$, the first moment of $g^p_1(x)$, is purely factorization dependent. Nevertheless, we stress that even though hard gluons do not contribute to $\Gamma_1^p$ in the gauge-invariant scheme, the gluon spin component in a proton, which is factorization independent, should be large enough to perturbatively generate a negative sea polarization via the axial anomaly. We briefly comment on how to study the $Q^2$ evolution of parton spin distributions to the next-to-leading order of QCD in the chiral-invariant factorization scheme.
1. Recently we have analyzed the EMC [1] and SMC [2] data of the polarized proton structure function $g_1^p(x)$ to extract the polarized parton distributions to the next-to-leading order (NLO) of QCD by assuming that $Q^2 = \langle Q^2 \rangle = 10 \text{ GeV}^2$ for each $x$ bin of the $g_1^p(x)$ data [3]. Our analysis is performed in two extreme factorization schemes: gauge-invariant and chiral-invariant ones. In the former factorization, hard glueons do not contribute to $\Gamma_1^p$, the first moment of $g_1^p(x)$, and the quark net helicity is $Q^2$ dependent but has a local gauge-invariant operator expression, whereas glueons do make contributions to $\Gamma_1^p$ and the chiral-invariant quark spin does not evolve in the latter scheme. However, we have stressed that physics is independent of the choice of the factorization prescription; the size of the hard gluonic contribution to $\Gamma_1^p$ is purely a matter of convention, as first realized and strongly advocated by Bodwin and Qiu [4] sometime ago.

Because of the availability of the two-loop polarized splitting functions $\Delta P_{ij}^{(1)}(x)$ very recently [5], it becomes possible to embark on a full NLO analysis of the experimental data of polarized structure functions by taking into account the measured $x$ dependence of $Q^2$ at each $x$ bin. Two of such analyses are now available. A NLO QCD analysis is carried out in [6] in the conventional MS scheme, a gauge-invariant factorization, within the framework of the radiative parton model. While a sizeable and negative sea polarization is required to describe all presently available data, the gluon spin density is found to be rather weakly constrained by the data. On the contrary, a NLO fit presented in [7] in the chiral-invariant scheme shows that the gluon contribution is large and positive. Furthermore, the authors of [7] have criticized the gauge-invariant scheme that it is pathological and inappropriate because soft contributions are partly included in the hard coefficient function rather than being factorized into parton spin densities. More precisely, the hard gluon-photon cross section has the expression [4]

$$\Delta \sigma_{\text{hard}}^{\gamma G}(x)_{GI} = \frac{\alpha_s}{2\pi} \left[ (2x - 1) \left( \ln \frac{Q^2}{\mu^2_{\text{fact}}} + \ln \frac{1 - x}{x} - 1 \right) + 2(1 - x) \right], \quad (1)$$

in the gauge-invariant factorization scheme with $\mu_{\text{fact}}$ a factorization scale. An objection to this scheme has been that the last term in Eq.(1) proportional to $2(1 - x)$ appears to arise from the soft region $k_{\perp}^2 \sim m^2 << \Lambda_{\text{QCD}}^2$, and hence it should be absorbed into the polarized quark distribution [7-9].

Although the issue of whether or not gluons contribute to $\Gamma_1^p$ was resolved five years ago that it depends on the factorization convention chosen in defining the quark spin density and the hard cross section for the photon-gluon scattering [4,10], the fact that the interpretation of $\Gamma_1^p$ is still under dispute even today and that many recent articles and reviews are still biased towards or against one of the two popular implications of the measured $g_1^p(x)$, namely sea quark or anomalous gluon interpretation, demands a further clarification on this issue. In the present paper, we will point out that, irrespective of the soft cutoff, the $2(1 - x)$

\[ \text{See also a very nice roundtable summary at the Polarized Collider Workshop (University Park, PA, 1990) on the theoretical interpretation of the measured } g_1^p(x) \text{ [11].} \]
term in Eq.(1) actually arises from the axial anomaly, i.e., from the region where $k_{\perp}^2 \sim \mu_{\text{fact}}^2$. Consequently, many criticisms to the gauge-invariant factorization scheme are in vain. We will show that even though hard gluons do not contribute to $\Gamma_1^p$ in the gauge-invariant scheme, the bulk of negative sea polarization must be perturbatively generated by gluons via the axial anomaly. We would like to emphasize that none of the results presented in this paper (it reads like a status review) are new and they are scattered in the literature. Nevertheless, we believe that the present paper is useful for clarifying several confusing issues and for making a unified and consistent understanding of $g_1^p(x)$.

2. As far as the first moment of $g_1^p(x)$ is concerned, the parton-model and OPE approaches are equivalent. However, in order to consider QCD corrections to $g_1^p(x)$ itself, we shall consider the parton model. To NLO, the polarized proton structure function is given by

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_i^n e_i^2 \{ [\Delta q_i(x, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \Delta f_q(x) \otimes \Delta q_i(x, Q^2)]$$
$$+ \Delta \sigma_{\text{hard}}^{\gamma G}(x, Q^2) \otimes \Delta G(x, Q^2) \}, \quad (2)$$

where $n_f$ is the number of active quark flavors, $\Delta q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)$, $\Delta G(x) = G^\uparrow(x) - G^\downarrow(x)$, and $\otimes$ denotes the convolution

$$f(x) \otimes g(x) = \int_x^1 \frac{dy}{y} f \left( \frac{x}{y} \right) g(y).$$

The $\Delta f_q(x)$ term in Eq.(2) depends on the regularization scheme chosen, but its first moment is scheme independent at least to NLO: $\int_0^1 \Delta f_q(x) dx = -2$. Since the polarized photon-gluon cross section $\Delta \sigma^{\gamma G}(x)$ has infrared and collinear singularities at $m^2 = p^2 = 0$ and $k_{\perp}^2 = 0$, where $m$ is the quark mass, $p^2$ is the gluon momentum squared and $k_{\perp}$ is the quark’s transverse momentum perpendicular to the virtual photon direction, it is necessary to introduce a soft cutoff. Depending on the infrared regulators, one obtains

$$\Delta \sigma_{\text{CCM}}^{\gamma G}(x, Q^2) = \frac{\alpha_s}{2\pi} \left[ (2x - 1) \left( \frac{\ln \frac{Q^2}{-p^2x(1-x)}}{p^2x(1-x)} + \ln \frac{1-x}{x} - 1 \right) + 1 - 2x \right], \quad (4)$$

$$\Delta \sigma_{\text{AR}}^{\gamma G}(x, Q^2) = \frac{\alpha_s}{2\pi} \left[ (2x - 1) \left( \ln \frac{Q^2}{m^2} + \ln \frac{1-x}{x} - 1 \right) + 2(1-x) \right], \quad (5)$$

$$\Delta \sigma_{\text{R}}^{\gamma G}(x, Q^2) = \frac{\alpha_s}{2\pi} \left[ (2x - 1) \left( \ln \frac{Q^2}{\mu_{\text{MS}}^2} + \ln \frac{1-x}{x} - 1 \right) + 2(1-x) \right], \quad (6)$$

for the momentum regulator ($p^2 \neq 0$) [12], the mass regulator ($m^2 \neq 0$) [13], and the modified dimensional regulator ($\mu_{\text{MS}}^2 \neq 0$) [14], respectively. Note that the term $(2x - 1)$ in Eqs.(4-6) is nothing but the spin splitting function $2\Delta P_{qG}(x)$ and that the term proportional to $2(1-x)$ in (5) and (6) is an effect of chiral symmetry breaking: It arises from the region.
where \( k_\perp^2 \sim m^2 \) in the mass-regulator scheme, and from \( k_\perp^2 \sim \mu_{\text{MS}}^2 \) in the \( n \neq 4 \) \((n > 4)\) dimensions in the dimensional regularization scheme.

The cross sections given in (4-6) are, however, not the desirable perturbative QCD results since they are sensitive to the choice of the regulator. Although the \( \ln(Q^2/ - p^2) \) and \( \ln(Q^2/m^2) \) terms, which depend logarithmically on the soft cutoff, make no contributions to the first moment of \( g_1^p(x) \), it is important to have a reliable perturbative QCD calculation for \( \Delta \sigma^{\gamma G}(x) \) since we are interested in QCD corrections to \( g_1^p(x) \). To do this, we need to introduce a factorization scale \( \mu_{\text{fact}} \), so that

\[
\Delta \sigma^{\gamma G}(x, Q^2) = \Delta \sigma^{\gamma G}_{\text{hard}}(x, Q^2, \mu_{\text{fact}}^2) + \Delta \sigma^{\gamma G}_{\text{soft}}(x, \mu_{\text{fact}}^2)
\]

and the polarized photon-proton cross section is decomposed into

\[
\Delta \sigma^{\gamma p}(x, Q^2) = \sum_{i} \left( \Delta \sigma^{\gamma q}(x) \otimes \Delta q_i(x, \mu_{\text{fact}}^2) + \Delta \sigma^{\gamma G}_{\text{hard}}(x, Q^2, \mu_{\text{fact}}^2) \otimes \Delta G(x, \mu_{\text{fact}}^2) \right),
\]

That is, the hard piece of \( \Delta \sigma^{\gamma G}(x) \) contributes to \( g_1^p(x) \), while the soft part is factorized into the nonperturbative quark spin densities \( \Delta q_i(x) \). Since \( \Delta \sigma^{\gamma p}(x) \) is a physical quantity, a different factorization scheme amounts to a different way of shifting the contributions between \( \Delta \sigma^{\gamma G}_{\text{hard}}(x) \) and \( \Delta q(x) \). An obvious partition of \( \Delta \sigma^{\gamma G}(x) \) is that the region where \( k_\perp^2 \gtrsim \mu_{\text{fact}}^2 \) contributes to the hard cross section, whereas the soft part receives contributions from \( k_\perp^2 \lesssim \mu_{\text{fact}}^2 \) and hence can be interpreted as the quark and antiquark spin densities in a gluon, i.e., \( \Delta \sigma^{\gamma G}_{\text{soft}}(x, \mu_{\text{fact}}^2) = \Delta q^G(x, \mu_{\text{fact}}^2) \). Physically, the quark and antiquark jets produced in deep inelastic scattering with \( k_\perp^2 \lesssim \mu_{\text{fact}}^2 \) are not hard enough to satisfy the jet criterion and thus should be considered as a part of one-jet cross section [12]. The choice of the “ultraviolet” cutoff for soft contributions specifies the factorization convention. In the present paper we only focus on two extremes: the chiral-invariant scheme in which the ultraviolet regulator respects chiral symmetry, and the gauge-invariant scheme in which gauge symmetry is respected but chiral symmetry is broken by the cutoff.

For a massless quark, one will expect that, based on helicity conservation or chiral symmetry, the quark spin \( \Delta q = \int_0^1 \Delta q(x) dx \) is \( Q^2 \) independent and that there is no sea polarization perturbatively induced by hard gluons. One way of calculating \( \Delta \sigma^{\gamma G}_{\text{soft}}(x) \) is to make a direct cutoff on the \( k_\perp \) integration so that its integral expression is exactly the same as \( \Delta \sigma^{\gamma G}(x, Q^2) \) except that \( k_\perp^2 \) is integrated over from 0 to \( Q^2(1-x)/4x \) for the latter, but from 0 to \( \mu_{\text{fact}}^2 \) for the former. For \( \mu_{\text{fact}}^2 >> m^2, -p^2 \), the results are [4,15] (for a complete expression of \( \Delta \sigma^{\gamma G}_{\text{soft}}(x) \) to NLO using mass or momentum regulator, see [16])

\[
\Delta \sigma^{\gamma G}_{\text{soft}}(x, \mu_{\text{fact}}^2)_{\text{CI}} = \Delta q_{\text{CI}}^G(x, \mu_{\text{fact}}^2) = \frac{\alpha}{2\pi} \left[ (2x - 1) \ln \frac{\mu_{\text{fact}}^2}{m^2 - p^2 x (1-x)} + (1-x) \frac{2m^2 - p^2 x (1-2x)}{m^2 - p^2 x (1-x)} \right],
\]

for various soft cutoffs, and the subscript CI indicates that we are working in a chiral-invariant factorization scheme. Note that, as stressed in [9], the soft cross sections or quark
spin densities in a helicity + gluon given by (9) do not make sense in QCD as they are derived using perturbation theory in a region where it does not apply. Nevertheless, it is instructive to see that

$$
\Delta q_{CI}^G = \int_0^1 \Delta q_{CI}^G(x) dx
$$

vanishes when \(m^2 = 0\) or \(-p^2 >> m^2\), as expected. Hence, a sea polarization for massless quarks, if any, is produced nonperturbatively. Now it does make sense in QCD to subtract \(\Delta \sigma_{\gamma G}^{\text{soft}}\) from \(\Delta \sigma_{\gamma G}^{\text{hard}}\) [see Eqs.(4-6)] to obtain a reliable perturbative QCD result for \(\Delta \sigma_{\gamma G}^{\text{hard}}\):
comes from the region \( k_{\perp}^2 \sim \mu_{\text{fact}}^2 \). As noted in passing, the quark spin distribution in a gluon cannot be reliably calculated by perturbative QCD; however, the difference between \( \Delta q^{G}_{\text{GI}}(x) \) and \( \Delta q^{G}_{\text{CI}}(x) \) is trustworthy in QCD. It is interesting to see from Eq.(12) that
\[
\Delta q^{G}_{\text{GI}}(\mu_{\text{fact}}^2) = -\frac{\alpha_s(\mu_{\text{fact}}^2)}{2\pi} \quad \text{for massless quarks,}
\]
that is, the sea-quark polarization perturbatively generated by helicity + hard gluons via the anomaly is negative! It follows that the hard cross section has the form
\[
\Delta \sigma^{G}_{\text{hard}}(x, Q^2)_{\text{GI}} = \Delta \sigma^{G}_{\text{hard}}(x, Q^2)_{\text{CI}} + \frac{\alpha_s}{\pi} (1 - x) \left[ (2x - 1) \left( \ln \frac{Q^2}{\mu_{\text{fact}}^2} + \ln \frac{1 - x}{x} - 1 \right) + 2(1 - x) \right],
\]
and its first moment vanishes:
\[
\int_0^1 dx \Delta \sigma^{G}_{\text{hard}}(x, Q^2)_{\text{GI}} = 0
\]
in the gauge-invariant factorization scheme. Bodwin and Qiu [4] have shown generally that the gluonic contribution to \( \Gamma^p_1 \) vanishes so long as the ultraviolet regulator for the spin-dependent quark distributions respects gauge invariance and the analytic structure of the unregulated distributions. From Eqs.(9) and (12) we also see that, contrary to what has been often claimed in the literature [7-9], though the \( 2(1 - x) \) term in (5) and (6) drops out in \( \Delta \sigma^{G}_{\text{hard}}(x)_{\text{CI}} \) because it arises from the soft region \( k_{\perp}^2 \sim m^2, \mu_{\text{fact}}^2 \), it emerges again in the gauge invariant scheme due to the axial anomaly and this time reappears in the hard region \( k_{\perp}^2 \sim \mu_{\text{fact}}^2 \). Therefore, the hard gluonic coefficient function is genuinely hard!

We wish to stress that the quark spin density \( \Delta q^G(x) \) measures the polarized sea quark distribution in a helicity + gluon rather than in a polarized proton. Consequently, \( \Delta q^G(x) \) must convolute with \( \Delta G(x) \) in order to be identified as the sea quark spin distribution in a proton:
\[
\Delta q_{\text{GI}}^G(x, \mu_{\text{fact}}^2) - \Delta q_{\text{CI}}^G(x, \mu_{\text{fact}}^2) = -\frac{\alpha_s}{\pi} (1 - x) \otimes \Delta G(x, \mu_{\text{fact}}^2).
\]
This relation can be also derived from Eqs.(8) and (14) by noting that \( \Delta \sigma^{G}_{\text{hard}}(x) = e_q^2 \delta(x - Q^2/2p \cdot q) \) [19]. Now it becomes clear from Eqs.(2), (14) and (16) that despite of the factorization-scheme dependence of \( \Delta q(x) \) and \( \Delta \sigma^{G}_{\text{hard}}(x) \), the physical quantity \( g^G_1(x) \) remains to be scheme independent up to NLO, as it should be. The choice of factorization is thus a matter of convention. Since the valence quark spin distribution \( \Delta q_v(x) = \Delta q(x) - \Delta q_s(x) \) is factorization independent, it follows that \([19]\)
\[
\Delta q_{\text{GI}}(x, \mu_{\text{fact}}^2) - \Delta q_{\text{CI}}(x, \mu_{\text{fact}}^2) = -\frac{\alpha_s}{\pi} (1 - x) \otimes \Delta G(x, \mu_{\text{fact}}^2),
\]
which leads to
\[
\Delta q_{\text{GI}}(Q^2) - \Delta q_{\text{CI}}(Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2),
\]
where we have set $\mu_{\text{fact}}^2 = Q^2$. Eqs.(14) and (17) provide the necessary relations between the gauge-invariant and chiral-invariant factorization schemes. For a given $\Delta G(x, Q^2)$, the quark spin densities in these two different schemes are related to each other via (17). It should be remarked that in spite of a vanishing gluonic contribution to $\Gamma_{1}^{p}$ in the gauge-invariant scheme, it never means that $\Delta G$ vanishes in a polarized proton. Quite opposite to the naive expectation, if there is no sea polarization in the chiral-invariant scheme, then the size of the gluon spin component in a proton must numerically obey the relation $\Delta G(Q^2) = -(2\pi/\alpha_s(Q^2))\Delta q_{s}^{\text{GI}}(Q^2)$ in order to perturbatively generate a negative sea-quark polarization $\Delta q_{s}^{\text{CI}}(Q^2)$ via the QCD anomaly. In other words, even gluons do not contribute to $\Gamma_{1}^{p}$, the gluon spin can be as large as 2.5 for $\Delta q_{s}^{\text{GI}} = -0.10$ at $Q^2 = 10 \text{ GeV}^2$ provided that $\Delta q_{s}^{\text{CI}} = 0$. Recall that the gluon polarization induced from quark’s bremsstrahlung is positive (see the first moment of $\Delta P_{Gq}(x)$ in Eq.(25) below).

Phenomenologically, one has to specify the factorization scale $\mu_{\text{fact}}$ in order to extract the quark and gluon spin distributions from polarized DIS data as the hard photon-gluon cross section is dependent on $\mu_{\text{fact}}$. In practice, $\mu_{\text{fact}}^2$ can be fixed to be $\langle Q^2 \rangle$, the average $Q^2$ of the $g_{1}^{p}(x)$ data set. Of course, one should take into consideration the logarithmic term $\ln(Q^2/\mu_{\text{fact}}^2)$ in $\Delta \sigma_{\text{hard}}^{\gamma G}(x)$ to fully account for the measured $x$ dependence of $Q^2$ at each $x$ bin.

One may ask how to accommodate the aforementioned two different factorization schemes in the approach of OPE? An examination of this issue also provides a clear picture on the differences between $\Delta q_{\text{GI}}$ and $\Delta q_{\text{CI}}$. Since the flavor-singlet axial-vector current $J_{\mu}^{5}$ has an anomalous dimension first appearing at the two-loop level [25], the quark spin

$$\Delta q_{\text{GI}} = \langle p|\bar{q}\gamma_{\mu}\gamma_{5}q|p\rangle s^{\mu},$$

with $s^{\mu}$ a spin 4-vector, is gauge-invariant but $Q^2$ dependent. The evaluation of the nucleon matrix element of $J_{\mu}^{5}$ involves connected and disconnected insertions (see e.g., [23]). The connected and disconnected insertions are related to valence quark and vacuum (i.e., sea quark) polarizations, respectively, and are separately gauge invariant. Thus we can make the identification:

$$\langle p|J_{\mu}^{5}|p\rangle = \langle p|J_{\mu}^{5}\text{ con}|p\rangle + \langle p|J_{\mu}^{5}\text{ dis}|p\rangle = \sum_{q}(\Delta q_{v}^{\text{GI}} + \Delta q_{s}^{\text{GI}})s^{\mu}.$$  (20)

Interestingly, lattice QCD calculations of $\Delta q_{v}^{\text{GI}}$ and $\Delta q_{s}^{\text{GI}}$ became available very recently [23,24]. It is found that $\Delta u_{s} = \Delta d_{s} = \Delta s = -0.12 \pm 0.01$ from the disconnected insertion [23].

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3A sea-quark interpretation of $\Gamma_{1}^{p}$ with $\Delta s = -0.10 \pm 0.03$ at $Q^2 = 10 \text{ GeV}^2$ [20] has been criticized on the ground that a bound $|\Delta s| \leq 0.052^{+0.052}_{-0.053}$ [21] can be derived based on the information of the behavior of $s(x)$ measured in deep inelastic neutrino experiments and on the positivity constraint. First of all, this argument is quite controversial [22]. Second, one can always find a polarized strange quark distribution with $\Delta s \sim -0.10$ which satisfies positivity and experimental constraints [3]. Third, a sea polarization of order $-0.11$ is also found by lattice calculations [23,24].
This empirical SU(3)-flavor symmetry within errors implies that the sea-quark polarization in the gauge-invariant scheme is indeed predominately generated by the axial anomaly. Recall that sea contributions in the unpolarized case are far from being SU(3) symmetric: $\bar{d} > \bar{s} > s$. In order to connect to the chiral-invariant scheme, one can write

$$J_5^\mu = J_5^\mu - K^\mu + K^\mu = \bar{J}_5^\mu + K^\mu,$$

with $K^\mu = (\alpha_s n_f/2\pi)\epsilon^{\mu\nu\rho\sigma} A^a_\nu(\partial_\rho A^a_\sigma - \frac{1}{2} g f_{abc} A^b_\rho A^c_\sigma)$ and $\epsilon_{0123} = 1$. Though neither $\bar{J}_5^\mu$ nor $K^\mu$ is gauge invariant, their matrix elements can be identified with quark and gluon spin components in the light-front gauge $A^+ = 0 [12]$ (it is not necessary to specify the coordinate)

$$s^+ \Delta q_{\text{CI}} = \langle p|J_5^+|p\rangle_{A^+=0}, \quad s^+ \Delta G = \langle p|(\vec{E} \times \vec{A})^+|p\rangle_{A^+=0} = -\frac{2\pi}{\alpha_s n_f}\langle p|K^+|p\rangle_{A^+=0}. \quad (22)$$

The quark spin $\Delta q_{\text{CI}}$ does not evolve as the current $\bar{J}_5^\mu$ is conserved in the chiral limit. Of course, both $\Delta G(x)$ and $\Delta q_{\text{CI}}(x)$ (not just their first moments!) also can be recast as matrix elements of a gauge-invariant but nonlocal operator [4,15], as noted in passing. Applying (22) to the axial-current matrix element leads to

$$\langle p|J_5^\mu|p\rangle = \langle p|J_5^\mu|p\rangle_{\text{con}} + \langle p|\bar{J}_5^\mu|p\rangle_{\text{dis}} + \langle p|K^\mu|p\rangle_{\text{dis}} \rightarrow A^+=0 \sum_q (\Delta q_{v,\text{CI}} + \Delta q_{s,\text{CI}} - \frac{\alpha_s}{2\pi}\Delta G) s^+,$$

where use of $\Delta q_{v,\text{GI}} = \Delta q_{v,\text{CI}}$ has been made. It is clear that (20) and (23) are equivalent owing to the relation (18).

3. The $Q^2$ dependence of parton spin densities are governed by the Altarelli-Parisi equations:

$$\frac{d}{dt}\Delta q_{\text{NS}}(x, t) = \frac{\alpha_s(t)}{2\pi}\Delta P_{qq}^\text{NS}(x) \otimes \Delta q_{\text{NS}}(x, t),$$

$$\frac{d}{dt}\left( \begin{array}{c} \Delta q_s(x, t) \\ \Delta G(x, t) \end{array} \right) = \frac{\alpha_s(t)}{2\pi} \left( \begin{array}{cc} \Delta P_{qq}^s(x) & 2n_f \Delta P_{gq}(x) \\ \Delta P_{Gq}(x) & \Delta P_{GG}(x) \end{array} \right) \otimes \left( \begin{array}{c} \Delta q_s(x, t) \\ \Delta G(x, t) \end{array} \right), \quad (24)$$

where $\Delta q_{\text{NS}}(x) = \Delta q_i(x) - \Delta q_j(x)$, $\Delta q_s(x) = \sum_i \Delta q_i(x)$ and $t = \ln(Q^2/Q_0^2)$. The complete polarized splitting functions up to NLO, $\Delta P(x) = \Delta P^{(0)}(x) + \frac{\alpha_s}{2\pi}\Delta P^{(1)}(x)$, have been calculated in the $\overline{\text{MS}}$ scheme recently [5]. Hence, the NLO evolution of spin parton distributions in the gauge-invariant factorization scheme is completely determined. Explicitly, the AP equation for the first moment of flavor-singlet parton spin densities reads [5]

$$\frac{d}{dt}\left( \begin{array}{c} \Delta \Sigma_{\text{GI}}(t) \\ \Delta G(t) \end{array} \right) = \frac{\alpha_s(t)}{2\pi} \left( \begin{array}{cc} \frac{\alpha_s}{2\pi}(-2n_f) & 0 \\ 2 & \frac{\beta_0}{2} + \frac{\alpha_s \beta_1}{4} \end{array} \right) \left( \begin{array}{c} \Delta \Sigma_{\text{GI}}(t) \\ \Delta G(t) \end{array} \right), \quad (25)$$

where $\Delta \Sigma_{\text{GI}}(t) = \int_0^1 dx \Delta q_s(x, t)$, $\beta_0 = 11 - \frac{2n_f}{3}$ and $\beta_1 = 102 - \frac{38}{3} n_f$. It is clear that $\Delta \Sigma_{\text{GI}}$ to NLO is $Q^2$ dependent.
One may choose to work in the chiral-invariant factorization scheme, so that $\Delta \Sigma_{\text{CI}}$ does not evolve with $Q^2$. This requires that

$$\Delta \sigma^{\gamma G}_{\text{hard}}(Q^2)_{\text{CI}} = -\frac{\alpha_s}{2\pi}, \quad \gamma^{(1)S,1}_{qq} \equiv \int_0^1 \Delta P_{qq}^{(1)S}(x)dx = 0.$$  

(26)

Using the results obtained in the MS scheme, one may introduce a modification on NLO anomalous dimensions and hard coefficient functions to transfer from the GI scheme to the CI prescription \[26\]. However, this transformation cannot be unique since it is only subject to the constraints (26). Indeed, three different scheme changes have been constructed in \[7\]. As a consequence, the NLO evolution of polarized parton distributions in the CI scheme obtained in this manner \[7\] is ambiguous and not quite trustworthy as it depends on the scheme of transformation.

We can avoid the aforementioned ambiguities and complications by working in the context of the gauge-invariant scheme where NLO polarized splitting functions are known. Once the evolution of the spin parton distributions $\Delta q_{\text{GI}}(x, Q^2)$ and $\Delta G(x, Q^2)$ is determined from the AP equations, $\Delta q_{\text{CI}}(x, Q^2)$ in the chiral-invariant prescription is simply related to $\Delta q_{\text{GI}}(x, Q^2)$ and $\Delta G(x, Q^2)$ by Eq.(17). One can check from Eq.(25) that $d(\Delta \Sigma_{\text{CI}}(t))/dt = 0$ to NLO, as it should be, where $\Delta \Sigma_{\text{CI}}(t) = \Delta \Sigma_{\text{GI}}(t) + (n_f\alpha_s/2\pi)\Delta G(t)$. In the absence of direct calculations of NLO polarized splitting functions in the chiral-invariant scheme, we believe that this is the right approach for studying the $Q^2$ evolution of parton spin distributions.

4. To conclude, contrary to what has been often claimed in the literature, we have clarified that the $2(1 - x)$ term in $\Delta \sigma^{\gamma G}_{\text{hard}}(x)$ in the gauge-invariant factorization scheme arises from the region $k^2_\perp \sim \mu^2_{\text{fact}}$ and hence is a genuinely hard contribution. We have shown explicitly that the physical quantity $g_{1p}^p(x)$ is independent of the choice of factorization convention. The sea-quark interpretation of $\Gamma_{1p}^p$ in the gauge-invariant scheme or the anomalous gluon interpretation in the chiral-invariant scheme is purely a matter of factorization convention chosen in defining $\Delta q(x)$ and $\Delta \sigma^{\gamma G}_{\text{hard}}(x)$. We have emphasized that even though hard gluons do not contribute to $\Gamma_{1p}^p$ in the gauge-invariant scheme, the gluon spin component in a proton should be large enough to perturbatively generate a negative sea polarization via the axial anomaly, recalling that $\Delta G(x, Q^2)$ is factorization independent.

As far as $g_{1p}^p(x)$ is concerned, both GI and CI factorization schemes are on the same footing. Thus it does not make sense to keep disputing which factorization prescription is correct or superior. Of course, once a set of $\Delta q_{\text{GI}}(x)$, $\Delta G(x)$, $\Delta \sigma^{\gamma G}_{\text{hard}}(x)$$_{\text{GI}}$ or of $\Delta q_{\text{CI}}(x)$, $\Delta G(x)$, $\Delta \sigma^{\gamma G}_{\text{hard}}(x)_{\text{CI}}$ is chosen, one has to stick to the same scheme in all processes.

In practice, it appears that the use of $\Delta q_{\text{GI}}(x)$ is more convenient than $\Delta q_{\text{CI}}(x)$. First of all, $\Delta q_{\text{GI}}$ corresponds to a nucleon matrix element of a local and gauge-invariant operator, and its calculation in lattice QCD became available recently. For $\Delta q_{\text{CI}}$, one has to compute the matrix element of $\tilde{J}_5^\mu$ in the light-front gauge, which will involve much more lattice con-
figurations. Second, NLO polarized splitting functions have been determined very recently in the gauge-invariant scheme, and it is straightforward to study the evolution of $\Delta q_{GI}(x, Q^2)$ through AP evolution equations. We have argued that a NLO analysis of polarized DIS data should be first carried out in the gauge-invariant factorization scheme and then related to the chiral-invariant prescription, if desired, via Eq.(17).

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