Gluon-Exchange in Spin-Dependent Fragmentation

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Abstract

The fragmentation of an unpolarized quark into a transversely polarized spin-$\frac{1}{2}$ particle is studied in the framework of a simple model. Special attention is payed to the gluon exchange which is incorporated in the gauge link of the fragmentation function, and which we model by an abelian gauge field. The transverse single spin asymmetries in $e^+e^-$ annihilation and semi-inclusive deep-inelastic scattering are calculated in the one-loop approximation. For $e^+e^-$ annihilation one finds a cancellation between contributions from two on-shell intermediate states, which have no counterpart in deep-inelastic scattering. As a consequence of this cancellation, the model predicts the same spin asymmetry for both processes implying that, in the one-loop approximation, the corresponding fragmentation function is universal.

Recently, considering inclusive deep-inelastic scattering (DIS), the influence of (Coulomb) gluon exchange between the struck quark and target spectators has been studied in detail [1]. It has been emphasized that the rescattering of the struck quark causes (additional) on-shell intermediate states in the forward Compton amplitude, resulting in a shadowing contribution to the DIS cross section at leading twist. In Feynman gauge, this shadowing effect is described by the gauge link (path-ordered exponential) appearing in the definition of parton distributions [1].

Subsequently, the effect of rescattering has also been investigated in the case of semi-inclusive DIS [2]. Using a simple model, it has been shown that a transverse single target-spin asymmetry arises from the interference between the tree-level amplitude of the fragmentation process and the imaginary part of the one-loop amplitude, where the latter describes the gluon exchange between the struck quark and the target system. Afterwards, it has been demonstrated that the asymmetry calculated in Ref. [2] is nothing else but a model for the Sivers function including its gauge link [3]. The Sivers function, which belongs to the class of so-called time-reversal odd (T-odd) and transverse momentum dependent ($k_{\perp}$-dependent) parton densities, describes the distribution of unpolarized quarks in a transversely polarized target [4]. Its existence requires a relative transverse momentum between the target and

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the quark. Initiated by the recent studies on the Sivers asymmetry, the nontrivial question about the appropriate gauge link for $k_\perp$-dependent parton distributions in lightcone gauge has been addressed lately \cite{5, 6}.

By considering the behaviour of the path-ordered exponential under time-reversal a very interesting observation has been made in Ref. \cite{3}: the Sivers asymmetry in semi-inclusive DIS has the opposite sign compared to the one in Drell-Yan, i.e., the Sivers function is non-universal. This sign difference has been confirmed by an explicit model-calculation \cite{7}.

Comparing DIS and Drell-Yan, in the meantime also for unpolarized scattering a violation of universality has been pointed out, provided that the cross section is kept differential in target-related particles \cite{8}.

In the present paper we study the universality of $T$-odd spin-dependent fragmentation functions. At leading order in $1/Q$, where $Q$ denotes the hard scale of the process, two such objects exist for the fragmentation into a spin-$\frac{1}{2}$ hadron: the Collins function \cite{9} (fragmentation of a transversely polarized quark into an unpolarized hadron), and the Sivers-type fragmentation function \cite{10} (fragmentation of an unpolarized quark into a transversely polarized spin-$\frac{1}{2}$ hadron). It is important to notice that, in contrast to $T$-odd parton densities, these functions are non-vanishing in general, even if their gauge link is neglected \cite{9, 11}.

Rescattering of hadrons in the fragmentation process can provide the required imaginary part in the scattering amplitude.

In the following, we explore the effect due to gluon exchange as incorporated in the gauge link of the fragmentation functions. We focus on the Sivers-type single spin asymmetry, which is of particular interest for $\Lambda$ production (see e.g. \cite{12}). We employ the simple model used in Refs. \cite{2, 7} to calculate the transverse spin asymmetry for $e^+e^-$ annihilation and semi-inclusive DIS at the one-loop level. While for DIS only one on-shell intermediate state contributes to the asymmetry, there are three in $e^+e^-$ annihilation. However, the contributions of two of them cancel out each other, and the spin asymmetries for both processes are equal. Therefore, the Sivers fragmentation function is universal at one-loop. The same conclusion applies to the Collins function as well.

In Fig. 1, the tree-level diagrams of the two fragmentation processes are displayed. For $e^+e^-$ annihilation we consider the decay of a timelike virtual photon into a $q\bar{q}$ pair, where the quark subsequently fragments into a spin-$\frac{1}{2}$ hadron (in the following we frequently talk
about a proton) and a scalar remnant, i.e.,
\[ \gamma^*(q) \rightarrow \bar{q}(p_1, \lambda^{'}) + p(p, \lambda) + s(p_2). \]  
(1)

The proton, including its polarization, is detected. The antiquark in the final state forms a jet. Just as well one might consider the fragmentation of the antiquark into an unpolarized hadron which, however, unnecessarily complicates the calculation even further. To make the transition from e^+e^- annihilation to semi-inclusive DIS one replaces the timelike photon by a spacelike one, and the outgoing antiquark by a quark in the initial state.

The one-loop corrections are shown in Fig. 2. For e^+e^- annihilation (semi-inclusive DIS) a single photon is exchanged between the remnant and the antiquark (initial quark). These diagrams provide a simple model for the lowest order contribution of the path-ordered exponential of the fragmentation function. (Actually, also the graph representing the one-loop correction to the vertex of the incoming photon in both processes is related to the path-ordered exponential.\(^1\) However, taking this diagram into account does not change any conclusion of the present work.) Obviously, two cuts (on-shell quark and antiquark, as well as on-shell antiquark and remnant) for e^+e^- annihilation have no counterpart in semi-inclusive DIS. Below we will demonstrate that these two on-shell states contribute to the transverse spin asymmetry. However, their contributions exactly cancel out each other. The quark-photon cut in e^+e^- annihilation corresponds to the cut in DIS. For the two processes the spin asymmetry due to the on-shell qγ state is equal. Particularly in the case of the qγ cut we will only quote the final result for the asymmetry and present details of the calculation, which is interesting in its own, elsewhere [13].

Before dealing with the model calculation, the kinematics is briefly discussed. We consider e^+e^- annihilation in the rest frame of the timelike photon. The proton in the final state has no transverse momentum, and its minus-momentum is given by zq^−, where q^- is the minus-momentum of the virtual photon. We fix the plus-momentum of the antiquark according to p^+_1 \approx q^+. The antiquark also carries a soft transverse momentum −Δ⊥, implying that the fragmenting quark and the outgoing proton have a relative transverse momentum, which is necessary for the Sivers asymmetry. These requirements specify the kinematics:

\[ q = \left( Q, Q, \vec{0}_\perp \right), \]

\(^1\)The author thanks John Collins for pointing this out.
\[ p_1 = \left( Q, \frac{\Delta^2_\perp + m_q^2}{Q}, -\Delta_\perp \right), \]
\[ p = \left( \frac{M^2}{z Q}, zQ, \bar{\Delta}_\perp \right), \]
\[ p_2 = \left( \frac{\Delta^2_\perp + m_s^2}{(1-z)Q}, (1-z)Q, \Delta_\perp \right). \]  

(2)

For simplicity, we listed in (2) always just the leading terms. Sometimes the \(1/Q^2\) corrections of \(p_1^+\) and \(p_2^-\) are needed, which can be obtained readily from 4-momentum conservation.

To calculate the Sivers-type fragmentation in \(e^+e^-\) annihilation the transverse component of the hadronic current has to be considered [14]. We define the various components of the current, depending on the helicities of the proton and the antiquark, via the invariant decay amplitude \(T\) according to

\[ T(\lambda, \lambda') = \epsilon_\mu J^\mu(\lambda, \lambda'), \]  

(3)

where \(\epsilon\) is the polarization vector of the virtual photon. In the following we focus on the \(x\)-component \(J^1\).

In the model we are using (see Ref. [2]) the proton carries no electromagnetic charge. Therefore, the charges of the fragmenting quark (denoted by \(e_1\)) and the one of the remnant are equal. The interaction between the quark, the proton, and the remnant is described by a scalar vertex with the coupling constant \(g\). This yields for the diagram on the \(lhs\) in Fig. 1 the current

\[ J_{(0)}^1(\lambda, \lambda') = e_1 g \frac{1}{s - m_q^2} \bar{u}(p, \lambda) (\not{q} - \not{p_1} + m_q) \not{\gamma}^1 v(p_1, \lambda') \]
\[ = e_1 g \frac{1 - z}{\sqrt{z}} \frac{Q}{\Delta^2_\perp + \tilde{m}^2} \left[ (\Delta^1 - i \lambda \Delta^2) \delta_{\lambda,-\lambda'} - \lambda \left( \frac{M}{z} + m_q \right) \delta_{\lambda,\lambda'} \right], \]  

(4)

with \(\tilde{m}^2 = \frac{1}{z} \left( M^2 \frac{1-z}{z} + m_s^2 - m_q^2 (1-z) \right),\)

where the lightfront helicity spinors of Ref. [15] have been employed to evaluate the matrix element. We have also made use of the relation

\[ s - m_q^2 = (q - p_1)^2 - m_q^2 = \frac{z}{1-z} \left( \Delta^2_\perp + \tilde{m}^2 \right), \]  

(5)

which connects the total energy \(\sqrt{s}\) in the \(cm\)-frame of the outgoing proton and remnant with the variables \(z\) and \(\Delta^2_\perp\).

In the next step the one-loop correction on the \(lhs\) in Fig. 2 is included. To calculate the single spin asymmetry only the imaginary part of this diagram is important, where we focus here on the imaginary part caused by the on-shell \(q\bar{q}\) intermediate state. Generally, the imaginary part of a Feynman diagram is conveniently calculated by means of Cutkosky rules which determine the discontinuity of a diagram. Applying these rules the imaginary part of the one-loop graph is given by

\[ \text{Im}_{q\bar{q}} J^1_{(1)}(\lambda, \lambda') = \frac{1}{2i} \text{Disc}_{q\bar{q}} J^1_{(1)}(\lambda, \lambda') \]
\[ \frac{-1}{2i} i (e_1)^2 g \int \frac{d^4k}{(2\pi)^4} (-2\pi i)^2 \delta((p - k)^2 - m_q^2) \delta((p - q - k)^2 - m_q^2) \]
\[ \times \bar{u}(p, \lambda) \gamma^1 (\phi - \bar{k} + m_q) (\phi - \bar{q} - \bar{k} + m_q) \gamma^1 (\phi_2 - \bar{k}) v(p_1, \lambda') . \]  

Note that we have assigned a mass \( \mu \) to the gauge boson in order to avoid infrared singularities at intermediate steps of the calculation. The same recipe has been used in the calculation of the target-spin asymmetry [2, 7]. In the final result of the spin asymmetry the limit \( \mu \to 0 \) can be performed without encountering a divergence.

The \( \delta \)-functions in (6) are exploited to perform the integrations over \( k^+ \) and \( k^- \). We rewrite them according to
\[ \delta((p - k)^2 - m_q^2) = \frac{1}{|k^- - p^-|} \delta\left(k^- - p^- - \frac{\bar{k}^2 + m_q^2}{k^- - p^-}\right), \]  
\[ \delta((p - q - k)^2 - m_q^2) = \frac{1}{Q} \delta(k^- - (p^- + p^- - k^+ - Q)), \]  

and, hence, obtain
\[ \int dk^+ dk^- \delta((p - k)^2 - m_q^2) \delta((p - q - k)^2 - m_q^2) \ldots \]
\[ = \frac{1}{Q} \int dk^- \frac{1}{|k^- - p^-|} \delta\left(k^- - p^- + \frac{\bar{k}^2 + m_q^2}{k^- - p^-} + Q\right) \ldots \bigg|_{k^+ = p^+ + \frac{\bar{k}^2 + m_q^2}{k^- - p^-}} \]
\[ = \frac{1}{Q^2} \cdot \frac{1}{(1 - z)Q} \cdot \ldots \bigg|_{k_- = \frac{M^2}{zQ} - \frac{\bar{k}^2 + m_q^2}{Q}} \quad . \]  

From the second line in Eq. (9) one finds a quadratic equation for \( k^- \). Though this equation has two real solutions, only one of them provides a leading twist contribution in the end.

To obtain the result in (9) it also enters that, for the leading twist part of the current, the transverse loop-momentum satisfies the condition \( k_\perp \ll Q \).

Before performing the \( k_\perp \)-integration, the matrix element in the numerator of Eq. (6) is evaluated, keeping only those terms that can contribute at leading order in the hard scale,
\[ \text{Im}_{q\bar{q}} J^{(1)}(\lambda, \lambda') = \frac{(e_1)^2 g}{8\pi^2} \frac{1 - z}{\sqrt{z}} Q \int d^2 \vec{k}_\perp \frac{(k_1 - i\lambda k^2) \delta_{\lambda, -\lambda} + \lambda (M^2 + m_q) \delta_{\lambda, -\lambda}}{[\bar{k}_\perp^2 + m^2]() [(\vec{k}_\perp + \Delta_\perp)^2 + \mu^2]} . \]  

In order to carry out the \( k_\perp \)-integration it is convenient to combine the two factors in the denominator by means of the Feynman parameterization. For instance, this allows one to write in the case of the scalar integral:
\[ \int d^2 \vec{k}_\perp \frac{1}{[\bar{k}_\perp^2 + m^2]() [(\vec{k}_\perp + \Delta_\perp)^2 + \mu^2]} \]
\[ = \int_0^1 d\alpha \int d^2 \vec{k}_\perp \frac{1}{[\bar{k}_\perp^2 + \Delta_\perp^2 \alpha(1 - \alpha) + \mu^2 \alpha + m^2(1 - \alpha)]^2} . \]  

The \( k_\perp \)-integration on the rhs in Eq. (11) can be performed easily. The situation for the vector integral (containing a \( k_\perp \) in the numerator) is analogous.
Combining the tree level expression for the hadronic current in (4) with the imaginary part at one loop, one finds the result

\[ J^1(\lambda, \lambda') = e_1 g \frac{1 - z}{\sqrt{z}} Q \]

\[ \times \left[ (\Delta^1 - i \lambda \Delta^2) \left( h - i \frac{(e_1)^2}{8\pi} g_2 \right) \delta_{\lambda, \lambda'} - \lambda \left( \frac{M}{z} + m_q \right) \left( h - i \frac{(e_1)^2}{8\pi} g_1 \right) \delta_{\lambda, \lambda'} \right], \quad (12) \]

with

\[ h = \frac{1}{\Delta^2 + \tilde{m}^2}, \]

\[ g_1 = \int_0^1 d\alpha \frac{1}{\Delta^2 \alpha (1 - \alpha) + \mu^2 \alpha + \tilde{m}^2 (1 - \alpha)}, \]

\[ g_2 = \int_0^1 d\alpha \frac{\alpha}{\Delta^2 \alpha (1 - \alpha) + \mu^2 \alpha + \tilde{m}^2 (1 - \alpha)}. \]

Here the reason for introducing a finite mass of the gauge boson becomes very transparent. The functions \( g_1 \) and \( g_2 \) are divergent in the limit \( \mu \to 0 \), since in this case the integrands diverge for \( \alpha \to 1 \).

The last step is the calculation of the transverse spin asymmetry \( \sigma_{pol}/\sigma_{unp} \), where the unpolarized and polarized (polarization along \( x \)-axes) cross sections are given according to

\[ \sigma_{unp} \propto \frac{1}{2} \sum_{\lambda, \lambda'} J^1(\lambda, \lambda') \left( J^1(\lambda, \lambda') \right)^*, \]

\[ \sigma_{pol} \propto \frac{1}{2} \sum_{\lambda', \lambda} \left[ J^1(s_x = \uparrow, \lambda') \left( J^1(s_x = \uparrow, \lambda') \right)^* - J^1(s_x = \downarrow, \lambda') \left( J^1(s_x = \downarrow, \lambda') \right)^* \right]. \quad (13) \]

Eventually, one obtains from the on-shell \( q\bar{q} \) intermediate state the following contribution to the transverse single spin asymmetry:

\[ A_{x,q\bar{q}} = \frac{(e_1)^2}{8\pi} \frac{2 \left( \frac{M}{z} + m_q \right) \Delta^2}{(\frac{M}{z} + m_q)^2 + \Delta^2_1} \frac{g_1 - g_2}{h} \]

\[ = \frac{(e_1)^2}{8\pi} \frac{2 \left( \frac{M}{z} + m_q \right) \Delta^2}{(\frac{M}{z} + m_q)^2 + \Delta^2_1} \frac{\Delta^2_1 + \tilde{m}^2}{\Delta^2_1} \ln \frac{\Delta^2_1 + \tilde{m}^2}{\tilde{m}^2}. \quad (14) \]

As expected, the asymmetry is proportional to the \( y \)-component (component perpendicular to the proton spin) of the transverse momentum. Note that the transition \( \mu \to 0 \) has been performed. The difference \( g_1 - g_2 \) is finite in this limit.

For the on-shell antiquark and scalar remnant in the intermediate state the calculation is very similar to the previous case. In particular, it turns out that the asymmetry caused by this cut cancels out the one due to the \( q\bar{q} \) cut, i.e.

\[ A_{x,q\bar{q}} = -A_{x,q\bar{q}}. \quad (15) \]

The cancellation between the contributions of these two on-shell states is not a peculiarity of our specific model, but should rather hold in general [13]. Given that the two discontinuities
have different cut-thresholds the result (15) appears a bit surprising. However, this difference becomes unimportant in our calculation since in both cases one is above the threshold due to the (asymptotically) large $Q^2$ considered here. In fact, a closer inspection shows that $Q^2(1-z)$ needs to be large compared to typical soft scales of the process.

While our calculation of the two cuts discussed above is technically similar to the one of the target-spin asymmetry performed in Refs. [2, 7], we had to treat the $q\gamma$ cut along different lines. Here, merely the final result for the asymmetry due to the on-shell $q\gamma$ intermediate state is listed, and details will be presented elsewhere [13]. One finds

$$A_{x,q\gamma} = \left(-\frac{e_1^2}{8\pi} \frac{2 M^2}{(M^2)^2 + \Delta_1^2} \frac{\Delta_2}{m_s} \frac{\Delta_3}{m_s} \ln \frac{p_{20} - |\vec{p}_2| \cos \alpha}{m_s} + \cos \alpha \ln \frac{p_{20} + |\vec{p}_2|}{m_s} \right) \left[ 1 - \frac{\cos^2 \alpha}{4(1-z)} \left( 1 - \frac{p_{20}}{|\vec{p}_2|} \ln \frac{p_{20} + |\vec{p}_2|}{m_s} \right) \right],$$

(16)

where we restricted ourselves to the case $m_q = 0$ in order to simplify the expression. In Eq. (16) the energy and the three-momentum of the remnant, as well as the scattering angle (angle between the antiquark and the remnant) in the cm-frame of the proton and the remnant appear. In terms of the variables $z$ and $\Delta_1^2$ these quantities read

$$p_{20} = \frac{1}{2 \sqrt{s}} \left( s + m_s^2 - M^2 \right),$$
$$|\vec{p}_2| = \frac{1}{2 \sqrt{s}} \sqrt{ \left( s - (m_s + M)^2 \right) \left( s - (m_s - M)^2 \right) },$$
$$\cos \alpha = \frac{1}{2 \sqrt{s} |\vec{p}_2|} \left( (2z-1) s + m_s^2 - M^2 \right),$$

(17)

with $\sqrt{s}$ as given in Eq. (5). By explicit calculation we have shown that for semi-inclusive DIS the spin-asymmetry caused by the on-shell $q\gamma$ state coincides with the result in (16).\(^2\) Therefore, the total transverse spin asymmetries in $e^+e^-$ annihilation and in semi-inclusive DIS are equal, and the Sivers fragmentation function is universal in the one-loop model. The same conclusion holds for the Collins function as well since the $q\gamma$ cut leads, for both processes, to the same imaginary part for the four independent helicity amplitudes.

In summary, we have investigated the time-reversal odd fragmentation of an unpolarized quark into a transversely polarized spin-$\frac{1}{2}$ hadron. A one-loop approach with photon exchange has been used as a simple model for the gluon exchange incorporated in the gauge link of the fragmentation function. One finds essentially two results by comparing the transverse single spin asymmetries in $e^+e^-$ annihilation and semi-inclusive DIS: firstly, in $e^+e^-$ annihilation on-shell intermediate states exist which have no counterpart in DIS. However, the contributions of these on-shell states to the spin asymmetry cancel out each other. Secondly, the asymmetry caused by the one on-shell state, which $e^+e^-$ annihilation and semi-inclusive

\(^2\)The amplitudes for both processes are connected by a crossing relation. In order to obtain the correct crossing behaviour it is mandatory that the vertices of the gauge boson with the quark and the antiquark have the same sign. Using this rule we can confirm the reversed sign of the target-spin asymmetry for Drell-Yan in comparison to the one for DIS [3, 7].
DIS have in common, is equal for both processes. Therefore, the Sivers fragmentation function is universal. The same conclusion holds as well for the Collins function which plays an important role in measurements of the transversity distribution of the nucleon. It remains to be seen if the universality of time-reversal odd fragmentation functions survives once higher order gluon-exchange is taken into account.

Finally, we repeat that a one-loop calculation does not provide a sign change for the Sivers fragmentation function, although the gauge link of an incoming fermion is replaced by the one of the corresponding antifermion in the final state when making the transition from semi-inclusive DIS to $e^+e^-$ annihilation. This is in contrast to time-reversal odd parton distributions [3, 7].

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