Competing Species Dynamics: Qualitative Advantage versus Geography

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Abstract
A simple cellular automata model for a two-group war over the same “territory” is presented. It is shown that a qualitative advantage is not enough for a minority to win. A spatial organization as well as definite degree of aggressiveness are instrumental to overcome a less fitted majority. The model applies to a large spectrum of competing groups: smoker-non smoker war, epidemic spreading, opinion formation, competition for industrial standards and species evolution. In the last case, it provides a new explanation for punctuated equilibria.

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Physics has dealt with quite a success in describing and understanding collective behavior in matter. Very recently many physicists have used basic concepts and
techniques from the physics of collective disorder to study a large spectrum of
problems outside the usual field of physics such as social behavior\cite{1,2,3}, group
decision making \cite{4}, financial systems \cite{5} and multinational organizations \cite{6}.
See \cite{7} for a review of these applications.

A few years ago, Galam has developed a hierarchical voting model based on
the democratic use of majority rule \cite{8}. In the simplest case of two competing
parties $A$ and $B$ with respective support of $a_0$ and $b_0 = 1 - a_0$, it was shown
that, for the $B$, winning the elections at the top of the hierarchy (i.e. after
several tournaments) does not depend only on $b_0$ but also on the existence of
some local biases. In particular, in the case of voting cells of four persons, a
bias is introduced (usually in favor of the leading party, e.g. $B$) to solve the
$2A - 2B$ situations. Then, the critical threshold of support for the ruling party
to win can be as low as $b_c = 0.23$. The model showed how a majority up to 0.77
can self-eliminate while climbing up the hierarchy, using locally the democratic
majority voting rule. This self-elimination occurs within only few hierarchical
levels.

Following this previous study, we address here the universal and generic
problem of the competing fight between two different groups over a fixed area.
We present a “voter model” which describes the dynamical behavior of a popu-
lation with bimodal conflicting interests and study the conditions of extinction
of one of the initial groups.

This model can be thought of as describing the smoker - non smoker fight:
in a small group of persons, a majority of smokers will usually convince the few
others to smoke and vice versa. The point is really when an equal number of
smokers and non-smokers meet. In that case, it may be assumed that a social
trend will decide between the two attitudes. In the US, smoking is viewed as
a disadvantage whereas, in France, it is rather well accepted. In other words,
there is a bias that will select the winner party in an even situation. In our
example, whether one studies the French or US case, the bias will be in favor of
the smokers or the non-smokers, respectively.

The same mechanism can be associated with the problem of competing stan-
dards (for instance PC versus Macintosh for computer systems or VHS versus
Beta MAG for video systems). The choice of one or the other standard is often
driven by the opinion of the majority of people one meets. But, when the two
competing systems are equally represented, the intrinsic quality of the product
will be decisive. Price and technological advance then play the role of a bias.

Here we consider the case of four-person confrontations in a spatially ex-
tended system in which the actors (species $A$ or $B$) move randomly. The process
of spatial contamination of opinion plays a crucial role in this dynamics.

In the original Galam model \cite{8}, the density threshold for an invading emer-
gence of $B$ is $b_c = 0.23$ if the $B$ group has a qualititative bias over $A$. With
a spatial distribution of the species, even if $b_0 < b_c$, $B$ can still win over $A$
provided that it strives for confrontation. Therefore a qualitative advantage
is found not to be enough to win. A geographic as well a definite degree of
aggressiveness are instrumental to overcome the less fitted majority.

The model we use to describe the two populations $A$ and $B$ influencing each other or competing for some unique resources, is based on the reaction-diffusion automata proposed by Chopard and Droz [9]. However, here, we consider only one type of particle with two possible internal states ($\pm 1$), coding for the $A$ or $B$ species, respectively.

The individuals move on a two-dimensional square lattice. At each site, there are always four individuals (any combination of $A$’s and $B$’s is possible). These four individuals all travel in a different lattice direction (north, east, south and west).

The interaction takes place in the form of “fights” between the four individuals meeting on the same site. At each fight, the group nature ($A$ or $B$) is updated according to the majority rule, when possible, otherwise with a bias in favor of the best fitted group:

- The local majority species (if any) wins:

\[
\begin{align*}
nA + mB &\rightarrow \begin{cases} (n + m)A & \text{if } n > m \\ (n + m)B & \text{if } n < m \end{cases}
\end{align*}
\]

where $n + m = 4$.

- When there is an equal number of $A$ and $B$ on a site, $B$ wins the confrontation with probability $1/2 + \beta/2$. The quantity $\beta \in [0, 1]$ is the bias accounting for some advantage (or extra fitness) of species $B$.

The above rule is applied with probability $k$. Thus, with probability $1 - k$ the group composition does not change because no fight occurs.

Between fights both population agents perform a random walk on the lattice. This is achieved by shuffling randomly the directions of motion of the fours individuals present at each site and letting them move to the corresponding neighboring sites [9].

Initially, populations $A$ and $B$ are randomly distributed over the lattice, with respective concentrations $a_0$ and $b_0 = 1 - a_0$.

It is clear that the model richness comes from the even confrontations. If only odd fights would happen, the initial majority population would always win after some short time. The key parameters of this model are (i) $k$, the aggressiveness (probability of confrontation), (ii) $\beta$, the $B$’s bias of winning a tie and (iii) $b_0$, the initial density of $B$.

The strategy according to which a minority of $B$’s (with yet a technical, genetic, persuasive advantage) can win against a large population of $A$’s is not obvious. Should they fight very often, try to spread or accept a peace agreement? We study the parameter space by running cellular automata implementing the above system.

In the limit of low aggressiveness ($k \to 0$), the particles move a long time before fighting. Due to the diffusive motion, correlations between successive
Figure 1: Phase diagram for our socio-physical model with $\beta = 1$. The curve delineates the regions where either $A$ (on the left) or $B$ (on the right) wins depending on $b_0$, the initial density of $B$ and $k$, the probability of a confrontation.

Fights are destroyed and $B$ wins provided that $b_0 > 0.23$ and $\beta = 1$. This is the mean-field level of our dynamical model which corresponds to the theoretical calculations made by Galam in his election model [8].

More generally, and for $\beta = \text{const}$, we observe that $B$ can win even when $b_0 < 0.23$, provided it acts aggressively, i.e. by having a large enough $k$. Thus, there is a critical density $b_{\text{death}}(k) < 0.23$ such that, when $b_0 > b_{\text{death}}(k)$, all $A$ are eliminated in the final outcome. Below $b_{\text{death}}$, $B$ looses unless some specific spatial configurations of $B$’s are present.

This is a general and important feature of our model: the growth of species $B$ at the expense of $A$ is obtained by a spatial organization. Small clusters that may accidentally form act as nucleus from which the $B$’s can develop. In other words, above the mean-field threshold $b_c = 0.23$ there is no need to organize in order to win but, below this value only condensed regions will be able to grow. When $k$ is too small, such an organization is not possible (it is destroyed by diffusion) and the strength advantage of $B$ does not lead to success.

Figure 1 summarizes, as a function of $b_0$ and $k$, the regions where either $A$ or $B$ succeeds. It turns out that the separation curve satisfies the equation $(k + 1)^7(b_0 - 0.077) = 0.153$.

It is also interesting to study the time needed to annihilate completely the looser. Here, time is measured as the number of fights per site (i.e. $kt$ where $t$ is the iteration time of the automaton). We observed that, in this case, the dynamics is quite fast and a few units of time are sufficient to yield a collective change of opinion.

The previous results assume a constant bias. However, with the assumption...
that an individual surrounded by several of its congeners becomes more confident and thus less efficient in its fight, one may vary the bias $\beta$ as a function of the local density of $B$. For example, within a neighborhood of size $\ell^2$, the bias can decrease from 1 to 0 as follows: $\beta = 1 - b/(2\ell^2)$ if $0 \leq b \leq 2\ell^2$ (local minority of $B$'s) and $\beta = 0$ if $b > 2\ell^2$ (local majority of $B$'s), where $b$ designates the number of $B$'s in the neighborhood.

This rule produces an interesting and non-intuitive new behavior. Depending on the value of $\ell$, there is a region near $k = 1$ such that the $A$ species can win by preventing the $B$'s from spreading in the environment. This is achieved by a very aggressive attitude of the $A$'s. Note that this effect is already present in the previous case ($\ell = 1$ and $\beta = \text{const}$), but only on the line $k = 1$ and for $b_0 < 0.2$.

Figure 2 summarizes the regions where either $A$ or $B$ succeeds when $\ell = 7$. In addition to the separation line shown in light gray, the time needed to decimate the other opinion is indicated by the gray levels. We observe that this time may become large in the vicinity of the critical line. Depending on the time scale associated with the process, such a slow evolution may be interpreted as a coexistence of the two species (if a campaign lasts only a few days or a few weeks, the conflict will not be resolved within this period of time).

We have shown that the correlations that may exist between successive fights may strongly affect the global behavior of the system and that an organization

Figure 2: Same as figure 1 but for a bias computed according to the $B$ density on a local neighborhood of size $\ell = 7$. The gray levels indicate the time to eliminate the defeated species. The black dots on the left hand side of the separation curve show situation where the $B$ species wins due to an accidentally favorable initial configuration (dark for long time).
is the key feature to obtain a definite advantage over the other population. This observation is important. For instance, during a campaign against smoking or an attempt to impose a new system, it is much more efficient (and cheaper) to target the effort on small nuclei of persons rather than sending the information in an uncorrelated manner.

Also, according to figure 2, an hypothetical minority of smokers in France must harass non-smokers during social meetings (coffee break, lunch,...) rather often but not systematically, in order to reinforce their position. On the contrary, for an hypothetical majority of smokers in the US, either a smooth or a stiff harassment against the non-smokers is required to survive.

Aggressiveness is the key to preserve the spatial organization. Refusing a fight is an effective way for the $A$ species to use its numerical superiority by allowing the $B$ individuals to spread. With this respect, a minority should not accept a peace agreement (which would results in a lower $k$) with the leading majority unless the strength equilibrium is modified (i.e. $B$ is better represented).

Motion is also a crucial ingredient in the spreading process. There is a subtle tradeoff between moving and fighting. When little motion is allowed between fights ($k \rightarrow 1$), the advantage is in favor of $A$ again. In an epidemic system, our model shows that two solutions are possible to avoid infestation: either one let the virus die of isolation (dilute state due to a small $k$) or one decimates it before it spreads (large $k$).

Finally a simple variant of the above model provides a possible scenario to explain punctuated equilibria in the evolution of living organisms. It is well known that the transition between two forms of life may be quite abrupt. There is no trace of the intermediate evolutionary steps. To give some insights into this problem we modify our voter model by including a creation rate for the $B$ individuals ($A \rightarrow B$, with probability $p \ll 1$). In this context, the $B$ species is fitter than the $A$ species (the bias $\beta = 1$) but the numerical advantage of $A$ is too strong for $B$ to survive. However, if the simulation is run for a long enough time, nucleation in this metastable state will happen, which will produce locally a very favorable spatial arrangement of $B$’s. These $B$’s will then develop and, very rapidly, eliminate all $A$’s. In other words, a very numerous species may live for a considerable amount of time without endangering competitors and suddenly, be decimated by a latent, fitter species. This scenario needs a strong statistical fluctuation but no additional external, global event.

In conclusion, although the model we propose is very simple, it abstracts the complicated behavior of real life agents by capturing some essential ingredients. For this reason, the results we have presented may shed light on the generic mechanisms observed in a social system of opinion making.
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