Can a flavour-conserving treatment improve things?

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In this work I would like to present some ideas on how to improve on the gauge sector in our lattice simulations at finite baryon density. The long standing problem, that we obtain an onset in thermodynamic quantities at a much smaller chemical potential than expected, could be related to an unphysical proliferation of flavours due to hard gluons close to the Brillouin edges. These hard gluons produce flavour non-conserving vertices to the fermion sector. They also produce excessive number of small instantons due to lattice dislocations. Both unphysical effects could increase the propagation in (di)-quarks to give the early onset in $\mu$. Thus we will present here a modified action that avoids large fields close to the lattice cutoff. Some of these ideas have been tested for $SU(2)$ and are being implemented for $SU(3)$.

1. Introduction

There has been the long standing problem, since the introduction of a chemical potential $\mu$ coupled to the baryon density [1,2], that the onset of thermodynamic quantities happens in our lattice simulations [3,4] at a much smaller $\mu$ than at the expected $\mu = m_N/3$. The baryon density and the chiral condensate start changing from their vacuum values at $\mu$ of the order of or earlier than $m_\pi/2$, as if controlled by a Goldstone mode with net Baryon number, which is clearly unphysical in the low Temperature normal phase.

The community had the hope that including properly the fermion determinant, which for non-zero $\mu$ is complex and therefore very difficult to implement with lattice methods, will by itself solve this problem. The careful work by the Barbour group in ref.[6] and the follow up presented by I. Barbour and S. Morrison at this Workshop, seem to show a pinching at a higher $\mu$ but the onset still seems to be around $m_\pi/2$.

Several years ago I described another possible problem, on top of the fermion determinant one, that could be producing the early onset in our lattice simulations. As is well known, for Kogut Susskind fermions the flavour number is not conserved along their propagation due to hard gluons that can make jump the fermion propagator pole to other corners of the Brillouin zone, thus changing its flavour. This flavour changing would produce a proliferation of flavours even for the valence quarks, forming a much tighter bound nuclear matter than in nature[7,8]. As we heard on various talks by F. Wilczek, E. Shuryak and others at this workshop, diquark condensates could be produced easier the more flavour species, which could drive the observed early onset due to the flavour

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proliferation. Furthermore, these hard gluons, with momenta \( p \approx \pi/2a \) or higher, produce easily lattice dislocations that give many unphysical “small instantons”\(^8\). Again E. Shuryak in his model of quark propagating among instantons, gets increased propagation with a higher instanton density.

We see then, that from several points of view it seems important to find a way to suppress the gluons with momenta of the order of the lattice cutoff. We could regain flavour conservation and suppress unphysical mini instantons. Both unwanted effects tend to lower the \( \mu \) onset value from the physical one.

In order to achieve such a hard gluon suppression I have been experimenting, first for \( SU(2) \), with an “improved” gauge action which has the same naive continuum limit as the usual one in the field strengths, but diverges for plaquette phases of the order of the lattice cutoff. Even though this suppression method is gauge invariant, it is important to use a Metropolis updating scheme with small changes in the link phases, in order to guarantee that we stay locally without spurious dislocations. In this way we can hope to suppress small unphysical instantons. The physical gluon momentum transmitted to the fermions, given in the continuum by the Poynting vector, being essentially in the nonabelian case shaped as a “chair” formed with two contiguous plaquettes, could then also be kept low so as to conserve flavour.

With L. Polley we had also tried out a new kind of lattice fermions\(^9,10\), the t-asymmetric ones, where one can reduce to 2 flavours from the start (in contrast to the 4 staggered ones), but have several technical problems to surmount, related with regaining the \( O(4) \) symmetry, in order to calculate the hadron masses and compare them with the onset. This will be treated in the next section.

In sec. 3 we will describe the new gauge action, mainly for \( SU(2) \), and discuss its implementation for \( SU(3) \). The ultimative test will be to generate configurations generated with such an improved gauge action, to then compute the baryon density or chiral condensate for the full theory.

### 2. t-asymmetric fermions

An earlier attempt in order to reduce the number of flavours, was based on a new kind of fermions, also called t-asymmetric fermions, that don’t allow more than 2 flavours in contrast to the 4 flavours for staggered ones\(^9,10\). This fermions are obtained by just taking a one-sided time derivative, which still produces an hermitian hamiltonian and eventually a positive transfer matrix (for \( a_\tau < a_s/\sqrt{3 + m_q^2} \)). Consequently, the number of flavours gets reduced to just 2. These fermions are related to Susskind’s hamiltonian formalism ones, via a unitary transformation\(^11,12\) and a related form is also being investigated by the Heidelberg group\(^13\).

The details of this formulation can be found in\(^9,10\), with the result that after a diagonalization in spinor space like the usual Kawamoto-Smit one,

\[
\psi(x) = \alpha_1^x \alpha_2^x \alpha_3^x \chi(x) \quad \text{with} \quad \alpha_k = i \gamma_4 \gamma_k
\]

we get,

\[
K_{xy} = m \Gamma_4(x) \delta_{xy} + \frac{1}{a_\tau} \left( e^\mu U_{x,\dot{4}} \delta_{x,y-\dot{4}} - \delta_{x,y} \right) - \frac{i}{2a_\tau} \sum_k \Gamma_k(x) \left( U_{x,k} \delta_{x,y-k} - U_{x-k,k}^\dagger \delta_{x,y+k} \right)
\]
with the usual: \( \Gamma_\nu(x) = (-)^{x_1 + \cdots + x_\nu - 1} \), after having thinned out to the first diagonal component. This fermionic action \( \bar{\chi} K \chi \) has then just 2 flavours. Susskind showed that there is a \textit{discrete} version of chiral invariance left over, preventing mass counterterms in the interacting case, but not guaranteeing “Goldstone behaviour” for the pions. One could still get light pions, like in Aoki’s proposal for Wilson fermions [14].

We had already done first simulations with the baryon density for these fermions, \( \langle J_4 \rangle = \langle e^\mu \bar{\chi}_x U_{x,4} \chi_{x+4} \rangle \) and the chiral condensate, \( \langle \bar{\chi} \chi \rangle \), as function of \( \mu \). The operators have been calculated with the solved pseudofermion method [1]. The curves for \( \langle J \rangle \) are shown in ref. [1] (Fig. 1) and show a clear onset for two \( m_q = .01, .04 \) at the same \( \mu \approx 0.2 \). This result could be deceiving though as it is not clear how \( m_\pi \) scales with \( m_q \) for these fermions, as already mentioned above. In order to really check if we are getting better results, we have to compute for these fermions the \( \pi \) and \( N \) masses. For this, considering that we need a finer lattice in the \( \tau \) direction for positivity and a fine \( a_\tau \) for charge conjugation symmetry in the propagators, we have to scale the couplings to regain the \( O(4) \) symmetry. Once this is done, one can attempt to calculate the masses and the baryon density for this \( O(4) \) symmetric action. This has proven to be a formidable task and presently I consider more promissory to improve on the gauge sector.

3. Improvements on the gauge action

The other way out, in order to prevent flavour changing is to suppress the appearance of hard gluons with momenta \( p > \pi/2a \). Without these hard gluons the quarks cannot change their flavour by jumping to other corners of the Brillouin zone. As mentioned, these hard gluons can also cause unphysical mini-instantons that could enhance the quark propagation in Shuryak’s scheme, also affecting the \( \mu \) onset.

From my investigations in \( SU(2) \) with instantons, in order to try to suppress mini-instantons produced by lattice dislocations, one can do so by modifying the action for plaquettes with a large phase. Instead of the usual gauge action:

\[
S = \beta \sum_\square (1 - 1/N \Re \text{tr}(U_{\square})),
\]

which has a maximum plaquette action of \( 2\beta \) for a phase \( \theta_{\square} = \pi \), one can take an improved action of the form:

\[
S = \beta \sum_\square 2/\pi \tan(\pi/2 (1 - 1/N \Re \text{tr}(U_{\square}))),
\]

which has the same naive continuum limit for small \( a \), but diverges for plaquettes with phases close to the cuttof. One can easily device functions which start the same and grow even faster for larger plaquette phases. The point being that at relatively moderate \( \beta \)’s one can suppress plaquette phases larger than \( \theta_{\square} \approx \pi/2 \) and therefore make sure that the effective \( \text{tr}(F_{\mu \nu}^2) \) stays away from the cutoff. ( For the usual gauge field action in \( SU(3) \), for example at \( \beta = 6.0 \), around 1/3 of the plaquettes still have a phase \( \theta_{\square} > \pi/2 \)). The actual gauge invariant definition for the gluon momentum, resembling the Poynting vector \( \text{tr}E \wedge B \) in the continuum, involves pairs of orthogonal plaquettes like a “chair”. If each of the plaquette phases is kept small also the combined one should be constrainable to get \( p < \pi/2a \).
In order to avoid large phases in the gauge links, which produce dislocations carrying a topological charge $|4|$, it is convenient to use the Metropolis algorithm for the link update, with small changes in the $SU(N)$ matrix on each iteration. In combination with an improved action as introduced above, one can obtain then fairly “smooth” gauge configurations, starting from the identity one. There is a problem though, related with a random walk like drifting of the links at a vertex corresponding to local gauge transformations, until we get large link phases close to $\pi$ that can produce dislocations. This can be reduced by gauge transformations that try to keep all the link phases low. The trick I used in $SU(2)$ is to check the phase $\phi$ of the new link $U = \exp(i \phi \hat{\phi} \cdot \sigma)$ and if it is larger than some phase, lets say $\pi/6$, make a local gauge transformation that shrinks that link’s phase to $\pi/8$ keeping its $SU(2)$ direction, while applying the inverse element to the other links at that vertex. These local gauge transformations succeed in keeping almost all $\phi$ smaller than $\pi/6$, thus suppressing almost all dislocations.

In this way I have been able to constraint the total topological charge to values $Q_v < |\pm 1|$ in periodic lattices (the field-theoretic definition does not give exactly integers), for fairly large $SU(2)$ lattices $(12^3 \times 24)$ with $\beta = 2.5$. For these lattices also the charge $Q_v$ with $v = V/2$ half the volume is also smaller than $|\pm 1|$ showing no $I - \bar{I}$ pairs.

One could also use more conventional action improvements in order to avoid hard gluons, by including terms in the action proportional to the Poynting operator (more generally the energy-momentum tensor $\Theta_{\mu\nu}$) that suppress large fields. In other words, include “chair” diagram terms in the action.

Another interesting alternative for an improved action in order to reduce the flavour breaking, was presented by Lagaë and Sinclair [15]. They smear the links by covariant displacements in all directions, in order to project the momenta to the central pole region in the Brillouin zone. In momentum space in the continuum they project the fields $A_\mu(k) \rightarrow 1/16 \prod_{i=1..4}(1 + \cos k_i) A_\mu(k)$, which in coordinate space corresponds to the smearing $A_\mu(x) \rightarrow 1/256 \prod_{i=1..4}(2 + D_i + D_{-i}) A_\mu(x)$. They report a large reduction in the flavour symmetry violations for the (non)-Goldstone $\pi$ splitting of masses. Up to now the only introduce this projected gauge fields in the minimal coupling terms in their action and it would be interesting to see what happens with the topology if they consider such improvement in the gauge action.
4. Conclusions

The inclusion of the fermion determinant could be not the only ingredient needed in order to shift the onset chemical potential to physical values close to the Nucleon mass. Our lattice simulations at finite $\mu$ have been done with an unimproved gauge sector. This produces several unwanted effects, like the proliferation of flavours even for valence quarks due to the hard gluons that can change the flavour along the quark propagation. Also the topological sector is distorted due to unphysical mini-instantons appearing due to non smooth gauge fields.

Both unwanted effects should be suppressed with an improvement in the gauge action. I introduced a simple to implement improvement that should help avoid hard gluons and dislocations. These ideas have been tested for $SU(2)$ with encouraging results and could also be implemented for $SU(3)$. In the Metropolis algorithm we cannot use anymore the trick to have the sum of staples precalculated as the action is not linear anymore, but the rest is very simple to implement.

Once we have the gauge sector under control, with no hard gluons close to the cutoff and no unphysical mini-instantons, we could again calculate the baryon density operator or the chiral condensate to find its $\mu$ dependence. This gauge updating could also be integrated in the unquenched code to redo Barbour’s group method to include the determinant.

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