Improved Radiometric Calibration by Brightness Transfer Function Based Noise & Outlier Removal and Weighted Least Square Minimization

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SUMMARY An improved radiometric calibration algorithm by extending the Mitsunaga and Nayar least-square minimization based algorithm with two major ideas is presented. First, a noise & outlier removal procedure based on the analysis of brightness transfer function is included for improving the algorithm’s capability on handling noise and outlier in least-square estimation. Second, an alternative minimization formulation based on weighted least square is proposed to improve the weakness of least square minimization when dealing with biased distribution observations. The performance of the proposed algorithm with regards to two baseline algorithms is demonstrated, i.e. the classical least square based algorithm proposed by Mitsunaga and Nayar and the state-of-the-art rank minimization based algorithm proposed by Lee et al. From the results, the proposed algorithm outperforms both baseline algorithms on both the synthetic dataset and the dataset of real-world images.

key words: radiometric calibration, camera response function, brightness transfer function, noise & outlier rejection, weighted least square minimization

1. Introduction

Camera response function (CRF) is the function that maps the irradiance (the amounts of light falling on the image sensor) to the pixel intensity in the acquired image. If \( I \) denotes the irradiance and \( M \) denotes the pixel intensity, both terms are related by the CRF \( f \) by

\[
M = f(I)
\] (1)

The CRF can be inverted to a function \( g = f^{-1}(\cdot) \), referred to as inverse CRF. That is, \( I = g(M) \).

Radiometric calibration is a process used to estimate the camera response function from the image observation acquired from the camera. A popular approach for radiometric calibration is to use multiple images captured from a static scene with different exposure times. Generally, the least-square minimization is used as the core of estimation process. However, in [1], Lee et al point out a major shortcoming of least-square minimization. The shortcoming lies in its weak behaviour when applying to the biased observations, which leads to an overfitting problem, as well as when applying to the observations with outliers and noise. Furthermore, the authors also propose to use a rank minimization approach for the estimation process. Unlike the least square minimization approach, the overfitting problems can be avoided using the rank-minimization based method. This method can handle the presence of noise and outlier in data. As pointed out in [1], their proposed minimization is a simple but sufficient approach to obtain the right solution. However, the main limitation of Lee et al approach [1] is that, in many cases, their algorithm could converge to a local minimum or a trivial solution. Furthermore, the authors also mention that the Mitsunaga & Nayar’s least square minimization based method [2] work accurately when the data used in the calibration have small level of noise, have no outlier and are uniformly distributed. This inspires us to pursue a way that could improve the capability of least square approach.

In order to tackle the shortcoming of previous least square minimization based methods [2]–[6], an improved algorithm to the least square minimization based on Mitsunaga and Nayar’s [2] method is proposed. The resulting improvements lie in two major aspects. First, a noise & outlier removal procedure based on the analysis of brightness transfer function [7] of image observations is incorporated. This aims to tackle the weakness of least square minimization approach when noise and outlier is present in measurement data. Second, a variant of weighted least square minimization is proposed, in which the weights are derived from a set of irradiance distributions. The irradiance data used to compute the distributions are computed from the image intensity measured by using a set of representative CRFs. This is aimed to tackle the weakness of least square minimization approach when dealing with biased distributions of observations.

Specifically, the key contributions of the proposed algorithm can be stated as follows.

- To the best of the authors’ knowledge, this work is novel in that the analysis of brightness transfer function (BTF) on image observations is incorporated in order to decide which observation is noise & outlier.
• The introduction of irradiance distributions in the derivation of weights for the weighted least square minimization for radiometric calibration is made.

With regard to our first contribution, we adopt the BTF estimation method presented in [8] as the core of our noise & outlier module. In [8], the authors use BTF to detect incorrect pixel matches (between two images) that could result in holes in the warped output image. So they can properly deploy a repairing method to these pixels. In our work, the BTF is estimated from several pairs of image observations that could cover the entire range of intensity values. Then, we exploit the estimated BTF to select the data that should be further used in the CRF estimation.

Note that, recently Kim et al [9] propose a more complicated in-camera imaging pipeline in which, besides radiometric calibration, they additionally include color & white-balance transformation and gamut mapping into the pipeline. However, our work can be directly applied as a module in Kim et al work [9]. Furthermore, we also show an experiment where the white-balance transformation and the color-space transformation are included into the pipeline as suggested in [9]. The results of camera RAW image recovery with our proposed radiometric calibration seem promising.

The remainder of this paper are organized as follows. The related work is presented in Sect. 2. The details of the algorithm are presented in Sect. 3. The experimental result is reported in Sect. 4. Finally, the conclusion is drawn in Sect. 5.

2. Previous Work

A main approach for radiometric calibration uses a set of images acquired from a fixed camera with various exposures.

In [10], Mann and Picard proposed an idea that estimated the CRF by assuming the CRF was represented by a gamma-correcting function and that the exposure ratios between images were known. In [3], a non-parametric method was proposed to estimate the CRF.Debevec and Malik also imposed smoothness constraint into the estimation. In [2], Mitsunaga and Nayar proposed a method based on least square minimization to estimate the CRF. In this work, the CRF was represented with a polynomial function. In [11], Grossberg and Nayar proposed a representation for CRF, referred to as EMoR (empirical model of response function). This representation was derived from a database of real-world camera response functions (DoRF). In [4], Mann and Mann proposed a least square minimization method that iteratively estimated the CRF with a non-parametric model. In this work, the exposures were also simultaneously determined. In [5] and [6], Kim and Pollefeys proposed a method that estimated the CRF from the brightness transfer function (BTF). The BTF was computed from the comparison by dynamic programming technique. The method was also used in [12] for high resolution mosaic generation with Pan-Tilt-Zoom cameras. In [13], Kim et al proposed a method that jointly tracked features and estimated the CRF from a video sequence. The proposed method was applied for structure-from-motion and stereo reconstruction. In addition to using BTFs for analyzing the images acquired from the same camera, in [14], the authors proposed an idea to use BTFs to model the appearance relationships from the images acquired across non-overlapping cameras.

In [1], Lee et al proposed a method that estimated the CRF by using rank minimization on an irradiance observation matrix. The key idea of their work was based on the notion that the correct inverse CRF would transform a set of the same scene images (captured with different exposures) into the collection of irradiance observations that were different by the exposure ratio. By concatenating these irradiance observations into the columns of an observation matrix, the rank of this matrix must be equal to one as each column of irradiance observation matrix was linearly dependent to each other. Therefore, unlike other previous work presented above that are mainly based on the minimization of least square error, in [1] the CRF was estimated based on the minimization of the rank of irradiance observation matrix. The results of the rank minimization based approach showed one advantage over the least square minimization (i.e., Mitsunaga and Nayar [2]) in which the overfitting problem usually found in the least square approach could be alleviated. In [15], Badki et al proposed a method based on the rank minimization on the observation matrix constructed from the recovered BTFs of multiple exposure images of a dynamic scene. In [16], Litvinov and Schechner proposed a method to estimate the CRF from an image sequence using a nonparametric linear least square estimation. In [17], Akyuz and Genctav presented an evaluation of four CRF estimation methods, [2], [3], [11], [18], in the aspects of consistency, accuracy and robustness.

In addition to the multiple exposure image approach, there were other approaches that performed radiometric calibration using properties presented in images. In [19] and [20], only a single image was used. The intensity distributions of local edge regions were computed. Then, a search for the best CRF that could map the nonlinear intensity distributions into linear distributions was conducted. In [21], Matsushita and Lin proposed a method that estimated CRFs from noise observation. The method searched for the best CRF that transforms the asymmetric measured noise distribution into the symmetric irradiance noise distribution. In [22], Kim et al proposed a method that estimated the CRF and the imaging exposures from a set of acquired images of static outdoor scene. In [23], Tai et al proposed a method for estimating the CRF based on image deblurring.

Recently, some works [24]–[26], are proposed. In [24], the authors propose a photometric method that simultaneously estimates the surface normals and the CRFs. The consistency between the irradiance values obtained from the estimated inverse CRF and the ones obtained from the surface normal estimation are enforced in their optimization. In [25], the authors propose a radiometric calibration method for a set of images from Internet photo collection.
The method incorporates the scene reflectance consistency of corresponding pixels in their formulation and applies the rank minimization framework of Lee et al. [1] to estimation the CRFs. In [26], the authors present a radiometric calibration method from a single image consisting of a human face. The key idea of the method is based on the low-rank property of skin albedo gradients.

For the work presented in this paper, an algorithm based on the weighted least square minimization that yields to the better accuracy over the rank minimization method [1] is proposed. The purpose of the weights presented in the error function is to compensate the bias in the irradiance observations. In order for the weighted least square minimization to work accurately for the presence of noise and outlier, a noise and outlier removal method based on the analysis of brightness transfer function created from the image observations is proposed. From the perspective of the literature review, the proposed noise and outlier removal method is inspired by the previous work by [8] and [15]. In these work, the BTFs were directly used in the CRF estimation. In the proposed work, the recovered BTF for the purpose of noise and outlier filtering is used.

3. The Proposed Calibration Algorithm

The input is taken from the observation in the form of collections of pixel intensities from multiple images (of the same scene) taken at various exposures. In this algorithm, it is strictly assumed that the exposure ratios of the adjacent image pairs must be equal to one another. It is also assumed that the exposure ratio is known. That is, if there are \( N \) images, \( M_1, M_2, M_3, M_4, \ldots, M_N \), at different exposure times \( (e_1, e_2, \ldots, e_N) \), the exposure ratios of all adjacent image pairs must be equal to a constant. In other words, \( e_1/e_2 = e_2/e_3 = \ldots = e_{N-1}/e_N = \alpha \), where \( \alpha \) is the constant referred to as exposure ratio.

For a pair of adjacent images \( M_i \) and \( M_{i+1} \), the irradiance values at a pixel \( x \) of both images are related by

\[
g(M_i(x)) = \alpha g(M_{i+1}(x))
\]

where \( M_i(x) \) is the intensity value of the observation of image \( i \) at pixel \( x \). \( M_{i+1}(x) \) is the intensity value of the observation of image \( i + 1 \) at pixel \( x \). And \( \alpha \) is the exposure ratio used to capture the image \( i \) and the image \( i + 1 \).

By considering all pixels \( x \in X \) of all M images, \( M_1, M_2, M_3, M_4, \ldots, M_N \), the inverse CRF \( g \) can be estimated by solving the least square minimization expressed by

\[
\hat{g} = \arg \min_g \sum_{i=1}^{N} \sum_{x \in X} (g(M_i(x)) - \alpha g(M_{i+1}(x)))^2
\]

As mentioned earlier, the least square minimization as in Eq. (3) does not perform well when the data used in the calibration possesses the following three conditions. First, the data are noisy. Second, there are outliers in the data. Finally, the data are nonuniformly distributed. In this paper, improvement of the least square minimization is proposed so that the three aforementioned problems can be handled. The proposed algorithm consists of two major steps as shown in Fig. 1. The first step is to remove noisy data and outliers from the input observation. For this step, the idea proposed in [8] is adapted. The detail of the first step is presented in Sect. 3.1. In the second step, noise and outlier free observation will be used in a weighted least square minimization to estimate the camera response function. In particular, the weighted of each observation is set to compensate for non-uniformity in the data. The detail of the second step is presented in Sect. 3.2.

3.1 Brightness Transfer Function (BTF) Based Noise and Outlier Removal

The input of this step is the observation in the form of collection of pixel intensities from multiple images of the same scene taken at different exposures. The output of this step is the clean observation in which the noise and outlier pixels have been excluded from the estimation.

Given a pair of the same scene images, a 2D joint histogram of the pair can be computed. This histogram is referred to as a comparagram [27]. If a pair of intensity values is denoted by \( (B_1, B_2) \), then value of the bin \( J(B_1, B_2) \) in the comparagram \( J \) is the number of pixels with intensity value \( B_1 \) in the first image and the intensity value \( B_2 \) at the same pixel location in the second image. The function that describes the mapping between intensity values in the first image into the intensity values in the second image is referred to as Brightness Transfer Function (BTF) [7]. Given a comparagram \( J \), the BTF can be determined by comparagram regression [27].

From (2), the intensity values at pixel \( x \) of image \( i \) \( (M_i(x)) \) and image \( i + 1 \) \( (M_{i+1}(x)) \) can be related by the BTF, denoted as \( \tau \), by

\[
M_i(x) = \tau(M_{i+1}(x)) := g^{-1}(\alpha g(M_{i+1}(x)))
\]

where \( \tau \) is defined as \( g^{-1}(\alpha g(.)) \).

As it is assumed that the exposure ratios of all pairs of adjacent images (i.e., \( M_1 \) vs \( M_2 \), \( M_2 \) vs \( M_3 \), \ldots, \( M_i \) vs \( M_{i+1} \), \ldots) are the same and equal to \( \alpha \), then these pairs of adjacent images will have the same BTF. Therefore, in this work, a single comparagram can be computed from any pair of adjacent image observations. Figures 2 (a)-(d) show the comparagrams of 4 pairs of adjacent image observations. And, Fig. 2 (e) shows the comparagram combined from all four comparagrams as shown in Figs. 2(a)-(d). In the same manner as comparagram regression, the BTF from the comparagram can be estimated. The line plot in Fig. 2 (e) shows
Based on the estimated BTF, a pair of pixel intensity values (of adjacent images) is considered as clean data if it lies in the vicinity of BTF. Otherwise, it will be considered as noise or outlier. Figures 3 (a)-(d) show the examples of raw data represented as the scatter plots (alternative visualization of comparagram) for four different cases: clean data, data with noise, data with outlier, and data with noise and outlier, respectively. Remark that, the reason to show comparagrams as scatter-plots here is to make us be able to observe the intensity value pairs with small frequencies (e.g., intensity value pairs corresponding to outliers).

By applying this key notion, the noise & outlier removal procedure is proposed and can be briefly explained as follows. First, the comparagram from a set of adjacent pairs of images is computed. Then, the BTF is determined from the comparagram. Finally, the noise and outliers are filtered out from the observations with regard to the estimated BTF.

It is worth to mention that our proposed idea in this section is extended from [8] in which the BTF is used to detect incorrect pixel matches that could yield to holes in the output image. So a repairing method can be properly applied. In our work, the BTF is estimated from several pairs of images that cover the entire intensity ranges. Then, we use the estimated BTF to select the data used for the CRF estimation.

Furthermore, our work is different from [5] and [6] in the sense that we do not directly use the BTF in our CRF estimation. We only use BTF for the purpose of noise & outlier removal.

The BTF estimation method proposed in [5] and [6] is based on dynamic programming technique. Meanwhile, the BTF estimation method proposed in [8] is based on Random Sample Consensus (RANSAC) [28]. Therefore, for the purpose of noise & outlier removal as in our case, it is suitable to use the RANSAC based method [8].

The details of Noise and Outlier Removal step can be shown in Fig. 4, and can be explained as follows.

3.1.1 Pre-Selection Based on Comparagram Thresholding

In this step, data obviously considered as noise or outliers are rejected. First, the comparagram from a set of adjacent pairs of images is computed. Second, a constraint is imposed such that intensity of a pixel of image taken at higher exposure time must be higher than the one taken at lower exposure time. That is, any pair of intensity values that does not satisfy the constraint will be rejected.

Next, thresholding is applied on the comparagram. If the frequency of any bin in comparagram is less than a threshold value, the bin value (frequency) is set to zero. Otherwise, the bin value is still be the same. The threshold value is automatically set to one that the selected data satisfies the following two conditions. The first condition is that the frequencies of selected bins must lie above the k-Percentile of all non-zero bins. In this work, k is set to k \leq 40. The second condition is that the number of selected data points must be at least 50% of original data. In other words, if the amount of selected data points at 40-Percentile (i.e., k=40) is less than 50% of original data, another value at lower percentile of the data (e.g., k=35 or 30 or 25, etc.) is selected as the threshold value. These selected data points will be used as the seed points (reliable data) to be used in the next step.

Remark that, in this step, it is possible that some good data points corresponding to the bins that lie below the k-Percentile may have been removed. However, as the data selection step is explained in Sect. 3.1.3 at the end, these good data points can be recovered.
3.1.2 Hermite Spline Fitting with RANSAC

From the data obtained the pre-selection step, the BTF is determined. Hermite Spline fitting in conjunction with RANSAC as proposed in [8] is applied. The procedure can be explained as follows.

- **Step 1:** The overall range of intensity values on x-axis is divided into L non-overlapped intervals (here, \( L = 7 \) as suggested in [8]). These intervals can be illustrated in Fig. 5(a).
- **Step 2:** RANSAC iteration loop is performed. For each iteration, one sample in each divided interval is randomly selected. Then, Hermite spline fitting is applied to the selected L sample data that result into a Hermite spline fitted curve (Hermite spline model). We consider a datum to be an inlier if it falls within the range of \(+/−\delta\) of the fitted curve. Otherwise, it is an outlier. In this work, \( \delta \) is set to 2% of the maximum intensity value. That is, we accept intensity variation that deviates from the BTF within \(+/−5\) (from the scale 0–255). We consider the number of inliers as the quality of Hermite spline model. That is, the more number of inliers the better model. Therefore, the above steps are iterated to find the best Hermite spline model that yields to the maximum number of inliers. The number of iterations, i.e., \( N \), is calculated adaptively by (5):

\[
N = \frac{\log(1 − p)}{\log(1 − s^2)}
\]

where \( p \) is the probability that at least one of the randomly selected data is free from outliers. In this work, \( p \) is set to 0.95. And, \( s \) is an estimated ratio of the current number of inliers to the total number of data points.

Therefore, the output of this step is the BTF that is the Hermite spline curve that fits to the most input data (maximum number of inliers). An example of the best fitted Hermite spline curve is shown in Fig. 5(b).

3.1.3 Data Selection

It is possible that there may be some clean data in the original data that are not included in the pre-selected data. These data may not occur frequently in the observation (due to the nature of observation distribution). Therefore these data cannot pass the pre-selection criteria. However, these data are highly valuable in the CRF estimation. To recover these good but unselected data, we will bring the BTF obtained from the previous step to re-select the clean data from the original observation data. That is, we consider a data to be a clean if the data falls within the boundaries of \(+/−\delta\) of the BTF. Similar to Section 3.1.2, the value of \( \delta \) is set to 2% of the maximum intensity value. These boundaries are shown as the dash lines in relative to the BTF in Fig. 5(b).

The selected data in this step will be used in the CRF estimation explained in the next section.

3.2 Weighted Least Square Minimization for CRF Estimation

From the previous step, most of noise and outliers from the observations have been filtered out. The remaining data will be used in the CRF estimation. However, due to the nature of image acquisition, the distribution of acquired irradiance data may not be uniformly distributed. Specifically, as pointed out in Lee et al work [1], this effect can introduce biased observations that lead to an overfitting problem in the least square minimization based estimation process [2].

Specifically, if the minimization in (3) is solved, the values of irradiance data \( g(M(x)) \) that occur more frequently than others will have more influence on the error function. That is, a slightly change in the error of these frequent pixels can collectively make a huge impact on the error function. Therefore, the minimizer tends to find a solution that emphasizes on the ranges of irradiance that occur more frequently. Meanwhile, on the ranges of the irradiance data that are found less frequently, the solution may not reflect to the correct one.

To tackle the problem, a variant of weighted least square minimization for radiometric calibration by extending the Mitsunaga-Nayar minimization framework [2] expressed in (3) is proposed. The formulation for the weighted least square error function \( (E_{WLS}) \) is shown as (6)

\[
E_{WLS} = \sum_{i=1}^{n-1} \sum_{x \in X} (w_{i+1}(x) [g(M_i(x)) − αg(M_{i+1}(x))])^2
\]

where \( w_{i+1}(x) \) is the weight of pixel \( x \) in the least square based estimation. The weight is computed according to the intensity of image \( i + 1 \) at pixel \( x \).

For the definition of the weight, less weighted values is applied to the pixels that their corresponding irradiance values are found more frequently. That is, the weight for each pixel \( x \) is defined by

\[
w_{i+1}(x) = \frac{1}{1 + h(g(M_{i+1}(x)))}
\]

where \( h(g(M_{i+1}(x))) \) is the number of pixels (i.e., frequency)
in the image \( i+1 \) that their corresponding irradiance values are equal to \( g(M_{i+1}(x)) \).

The main purpose for the weight is to lessen the impacts of more frequent observations into the estimation results as well as to lift up the contributions of the less frequent observations. In some sense, it can be considered that the effect of weight in the minimization is to make the distribution of data used in the estimation to be closed to a uniform distribution. As shown in [1], when the distribution of irradiance observation is uniform, the least square minimization based CRF estimation [2] shows the superior performance. Note that, the less frequent observations are proper to be used in the CRF estimation because now all of the remaining data (after noise & outlier removal) are clean and reliable.

Typically, CRFs and inverse CRFs are monotonic increasing functions. Therefore, similar to [1], a monotonicity constraint is also imposed into the minimization by

\[
\hat{g} = \arg\min_g \left( E_{WLS} + \lambda \sum_{i} H\left(-\frac{dg(t)}{dt}\right) \right)
\]  

(8)

In (8), the monotonic increasing constraint is represented as \( \frac{dg(t)}{dt} > 0 \) for all ranges of intensity values. And, \( H(\cdot) \) is the Heaviside step function in which \( H(x) = 1 \) for \( x \geq 0 \). Otherwise, \( H(x) = 0 \). The derivatives \( \frac{dg(t)}{dt} \) are computed by numerical differentiation at several equally sampled points. And, \( \lambda \) is a large constant value to reflect the data that violate the monotonic constraint.

3.2.1 Representative CRFs for Weight Calculation

As expressed in (6), the minimization is performed on the irradiance domain. The irradiance data can be obtained by transforming the image intensity observations with the correct CRF. However, the CRF is the unknown that needs to be estimated. The correct CRF can be obtained after the minimization is finished. To deviate this dilemma, it is proposed to calculate the weights from the image intensity observations according to 4 different prior CRF representatives. These 4 representative CRFs, as shown in Fig. 6, are obtained from analyzing the CRFs in DoRF [11].

If it is assumed that these four representative CRFs are \( g_{R_1}, g_{R_2}, g_{R_3}, \) and \( g_{R_4} \), then the weights can be calculated as follows. Given image intensity observations, four different sets of irradiance data can be computed using the representative CRFs. Then, the four different set of weights, \( w_{R_1}, w_{R_2}, w_{R_3}, \) and \( w_{R_4} \) can be calculated from the computed irradiance data by using (7).

For each set of weight values, the minimization is solved as expressed in (8). Assuming that the resulting CRFs are \( g_1, g_2, g_3, \) and \( g_4 \) and the corresponding error values are \( e_1, e_2, e_3, \) and \( e_4 \), finally the CRF that yields to the minimum error value is selected.

The details of how the 4 representative CRFs are obtained can be explained as follows. First 201 CRFs in DoRF are divided into 4 different groups according to their curve characteristics. From our investigation of DoRF, four major groups of CRFs share the same 2\textsuperscript{nd} order characteristics of the CRFs as shown in Fig. 6. Specifically, the changes of gradients of CRFs are used as the grouping criteria. Accordingly, the changes of gradients of four representative CRFs in Fig. 6(a), (b), (c) and (d) are negative, positive & negative, small (almost zero) and positive, respectively. Then, the CRF that lies primarily in the gist of all curves in each group is arbitrarily selected as the representative CRFs. Note that, we try the option of using the averaged CRFs in each group as the representative CRFs. However, we found that some averaged curves are neither smoothed nor monotonic.

3.2.2 Minimization Algorithm

To perform the minimization in (8), the CRF is represented with the polynomial representation. The polynomial representation that is similar to [1] is used. That is, the CRF is expressed by

\[
g(M) = M + M(M-1) \sum_{i=1}^{n-1} c_i M^{n-i-1}
\]  

(9)

where \( n \) is the order of the polynomial function. \( c_i \) are the coefficients of polynomial function. These coefficients are the parameters to be estimated in the minimization.

Furthermore, the irradiance and intensity observation values are normalized into the range of \([0,1]\) with the boundary conditions \( g(0) = 0 \) and \( g(1) = 1 \). With the representation in (9), these boundary constraints are explicitly enforced by themselves in the minimization. That is, regardless of the values of \( c_i \) in (9), the values of \( g(0) \) and \( g(1) \) are always 0 and 1, respectively. For the minimization, Levenberg-Marquardt (LM) method is used. The overall algorithm can be explained as the pseudo-code in Algorithm 1. The value of \( \lambda \) is set to \( 1 \times 10^{10} \).

The explanation of Algorithm 1 can be briefly stated as follows. First, in Line 1, the noise and outlier removal procedure is applied as explained in Sect. 3.1 by using the procedure NoiseOutlierRemoval where the output of this step is the clean data \( (M_c) \). Next, in Line 2, by using the procedure ComputeWeight the weights are precomputed according to different representative CRFs as explained in Sect. 3.2.1. Then, in Line 4 to 14, the inverse CRF is calculated using...
Algorithm 1 Radiometric calibration process

Input: \( M \) = Image observation
Output: \( \hat{g} \) = Estimated inverse CRF

1: \( M_c \leftarrow \text{NoiseOutlierRemoval}(M) \)
2: \( W \leftarrow \text{ComputeWeight}(M_c) \)
3: where \( W = \{ w_{R_1}, w_{R_2}, w_{R_3}, w_{R_4} \} \)
4: for \( k = 1 \) to \( |W| \) do
5: Initialize \( \hat{g} \)
6: \( w_{R_k} \leftarrow w_{R_k} \)
7: while not converge do
8: \( l = g(M_c) \)
9: Calculate error using (8)
10: update \( \hat{g} \)
11: end while
12: \( \hat{g}_k \leftarrow \hat{g} \)
13: \( e_k \leftarrow \text{error} \)
14: end for
15: \( \hat{g} = \arg \min_{\hat{g}} e_k \)

weighted least square minimization according to the weights computed from different representative CRFs. For each set of weight, the resulting inverse CRF and its corresponding error as specified in Lines 12 and 13 are recorded. Finally, in Line 15, the inverse CRF \( \hat{g} \) that the corresponding error is minimum is selected.

4. Experiments

The proposed method is evaluated by performing experiments on both synthetic dataset and real-world images. The results are compared with those from two baselines methods, i.e., the classical least square based method proposed by Mitsunaga & Nayar [2] and the state-of-the-art rank minimization based method proposed by Lee et al’s method [1]. The Mitsunaga & Nayar method is referred to as MN and the Lee et al method as LEE. The minimizations of all CRF estimations are initialized with the linear response function.

For both baseline methods, the implementation that is kindly provided by Lee et al [1] is used. Particularly, the Levenberg-Marquard (LM) method is used to perform the minimization expressed in Algorithm 1. The value of \( \lambda \) in (8) is set to \( 1 \times 10^{10} \).

4.1 Results with Synthetic Data

We perform the experiment on synthetic data with two different datasets: (i) DoRF dataset [11] and (ii) NUS dataset [9]. In summary, the DoRF dataset consists of the set of 201 CRFs determined from several digital cameras. For the NUS dataset, it consists of 31 CRFs determined from more modern cameras.

For the CRF representation, the orders of polynomial functions in all methods (MN, LEE and our method) for DoRF dataset are set to 6. Meanwhile, for the NUS dataset, the polynomial functions with 6th degree are not enough to make all three methods to converge. We incrementally vary the degree of polynomial function. We found that the minimum order of polynomial function which makes all methods to converge is 9. Therefore, for the experiments with NUS dataset, the orders of polynomial functions in all methods (MN, LEE and our method) are set to 9.

The procedure of synthetic observation data generation is similar to [1] which can be explained as follows. First, the scene radiances in the range of \([0, 1]\) with four different distributions are generated. These four scene radiance distributions, referred to as D1, D2, D3, and D4, are shown in Fig. 7. For simulating the multi-exposure data of each test case, five irradiance observations of size 1,000 (1,000-pixel irradiance image) are generated from a scene radiance with exposure time of step 0.5 (0.0625, 0.125, 0.25, 0.5, 1).

To evaluate the effectiveness of the proposed algorithm against noise, the poisson noise distributions with four different noise levels of camera gains \( (C_g = 1, 3, 6, 9) \) are added into each irradiance observation. Finally, the image intensity observations are generated by applying the CRFs to the irradiance observations.

For evaluating the robustness to outliers of our algorithm, another dataset is also generated by adding outliers with 4 different levels (3%, 5%, 7%, 9%) to the irradiance observations. Outliers are added to the original dataset by randomly changing the intensity values of pixels. Similarly, the image intensity observations are obtained by applying the 201 CRFs.

To show the calibration accuracies of our algorithm and other two baseline algorithms, the root mean square errors (RMSEs) are calculated from the calibration results. An RMSE is calculated from the error that compares between the CRF obtained from an algorithm and the groundtruth CRF. The groundtruth CRFs mean the CRF data that are presented in the DoRF database. Then the cumulative histograms of number of successful results are computed. A successful result is defined as the corresponding RMSE being less than the threshold. Therefore, a better algorithm should yield to more number of successful results. Note that the maximum number of successful results (number of CRFs in dataset) are 201 and 31 for DoRF dataset and NUS dataset, respectively.

4.1.1 Results with Synthetic Data on DoRF Dataset

In this section, we present the results with synthetic data for DoRF dataset. The results can be shown in three dif-
different cases. First, the results for the clean dataset (without noise and outlier) for different radiance distributions (D1-D4) are shown in Figs. 8(a-1), 8(a-2), 8(a-3) and 8(a-4), respectively. Second, the results for the dataset with noise are shown in Figs. 8(b-1), 8(b-2), 8(b-3) and 8(b-4), respectively. Finally, the results for the dataset with outliers are shown in Figs. 8(c-1), 8(c-2), 8(c-3) and 8(c-4), respectively. From Figs. 8(a), Figs. 8(b), Figs. 8(c), it can be observed that the proposed method outperforms the MN method and the LEE method in all four data distributions with three differ-

**Fig. 8** Results on synthetic DoRF and NUS datasets. The curves show the cumulative histogram of the number of successful cases with regard to RMSE.
Therefore, a better algorithm should yield to the proposed method and the LEE method are compared. From the tables, three points can be observed. First, in the case of clean data, the proposed method outperforms MN method and LEE method in all four datasets. This shows the effectiveness of the proposed idea on weighted least square formulation as explained in Sect. 3.2. Second, for the uniform dataset (D1) at high levels of noise (i.e., $C_q = 6$ and $C_o = 9$) the proposed method and LEE method perform well and are significantly better than MN method. This shows the effectiveness of our proposed noise & outlier rejection method as explained in Sect. 3.1. Third, for the non-uniform datasets (D2, D3, and D4), the proposed method mostly outperforms the MN method and the LEE method. Furthermore, as can be seen in Table 2, the proposed method outperforms the LEE method in all four datasets with outliers.

### 4.1.2 Results with Synthetic Data on NUS Dataset

In this section, we present the results with synthetic data for NUS dataset. Similar to the results with DoRF dataset, the results on NUS dataset are shown in three different cases. First, the results for the clean dataset (without noise and outlier). Second, the results are shown for the dataset with noise. The last case, the results for the dataset with outliers.

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**Table 1**: Quantitative results on synthetic DoRF and NUS dataset with noise in RMSE and disparity

| Noise Level ($C_o$) | Mean | Standard Deviation | NUS | Mean | Standard Deviation |
|---------------------|------|--------------------|-----|------|--------------------|
|                      | D1   | D2     | D3 | D4   | D1   | D2     | D3 | D4   |
|                      | MN   | LEE    | Disparity | Ours | MN   | LEE    | Disparity | Ours | MN   | LEE    | Disparity | Ours | MN   | LEE    | Disparity | Ours | MN   | LEE    | Disparity | Ours | MN   | LEE    | Disparity | Ours | MN   | LEE    | Disparity | Ours | MN   | LEE    | Disparity | Ours |
|                      | RMSE |        |        |      | RMSE |        |        |      | RMSE |        |        |      | RMSE |        |        |      | RMSE |        |        |      | RMSE |        |        |      | RMSE |        |        |      | RMSE |        |        |      | RMSE |        |        |      |
|                      | 0    | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 |
|                      | 1    | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 | 0.0028 |

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**Table 2**: Quantitative results on synthetic DoRF and NUS dataset with outlier in RMSE and disparity

| Mean | Standard Deviation |
|-----|--------------------|
| D1  | D2     | D3 | D4   |
| MN  | LEE    | Disparity | Ours | MN  | LEE    | Disparity | Ours | MN  | LEE    | Disparity | Ours | MN  | LEE    | Disparity | Ours | MN  | LEE    | Disparity | Ours | MN  | LEE    | Disparity | Ours | MN  | LEE    | Disparity | Ours |
| RMSE | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 |

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ent cases (clean data, data with noise, and data with outlier). This can be observed that the red curves (our method) are on top of the blue curves (LEE method) and the black curves (MN method).

In Table 1 and 2, the quantitative results in terms of means and standard deviations of RMSEs and disparities over each data distribution (D1-D4) and each case (clean & noise in Table 1 and outlier in 2). Here, the disparity is defined as the maximum deviation from the groundtruth. Strictly speaking, these quantitative results show the quality of fit among the common set of successful calibration results. Note that in Figs. 8(a), Figs. 8(b), Figs. 8(c), the capability in term of the quantity of CRFs that an algorithm can successfully estimate (number of successfully estimated CRFs) is also shown. Clearly the proposed method outperforms the MN method and the LEE method.

For the calculation of mean and standard deviation, only the CRFs that all methods commonly yield to the successful results, or with RMSEs less than 0.03, are considered. Therefore, a better algorithm should yield to the least values of mean and standard deviation. Note that, the columns in Table 1 that the noise level is zero ($C_q = 0$) means to the results for the cases of clean data (data without noise or outlier). Furthermore, the number of successful results of the MN method for the case of outliers are small in comparison with our method and the LEE method as can be seen in the third row of Fig. 8. Therefore, in Table 2, only the proposed method and the LEE method are compared.
are shown.

All three different cases are shown in Figs. 8(d), Figs. 8(e) and Figs. 8(f). The quantitative results in term of means and standard deviations of RMSEs and disparities over each data distribution (D1-D4) and each case (clear or noise or outlier) are shown in Table 1 and Table 2. From the figures and the tables, our proposed method outperforms the LEE method and the MN method in the similar way to the results on DoRF dataset. Note that, in Table 1, the cells that fill with the word “None” mean that there are no estimated CRF result that is jointly successful ($RMSE < 0.03$) for all three methods.

4.1.3 Accuracy When Varying Number of Input Images

In this section, we evaluate the effectiveness of all three algorithms when the number of input images used in the CRF estimation is varying. We extend the experiment with synthetic data on DoRF dataset presented in Sect. 4.1.1. We measure the number of successful CRF estimations when the number of images used in the estimation is decremented by one (5, 4, 3 and 2 images) on the DoRF clean data (no noise and no outlier). We keep the images with have the highest values of exposure time. For example, when the number of images used in the estimation is 4, we drop the image with lowest exposure time in the image set. The results on the number of successful estimations are shown in Fig. 9 (a) to (d) for different data distributions (D1-D4). The results show that our method and MN method which are based on least-square technique have barely no effect. On the other hand, the LEE method, which is based on rank minimization, seems to be variant to the effect when the number of images is dropped to 2. Although, it is not conclusive but it seems that our method requires less number of input images than the LEE method to produce the comparable results.

4.2 Results with Real-World Images

For the real-world experiment, we use the images from two different datasets. The first dataset, referred to as LEE dataset, consists of three different images of a color calibration plate from NUS dataset [9]. These images acquire from three different cameras, i.e., Canon EOS1Ds, Nikon D7000, and Nikon D40, respectively.

The images are obtained from each camera by capturing a static scene with different known exposure times. Specifically, each dataset of a camera consists of (i) a set of input images used for radiometric calibration, (ii) a test image, and (iii) the groundtruth RAW associated with the test image.

The first calibration results on real-world show for compare with the state of the art method by the same LEE dataset. And we add several experiments with images takes by a modern camera by use NUS dataset to indicate the validity of the proposed method.

Particularly, we use the same evaluation procedure as proposed in [1]. That is, we convert images to irradiance data (estimated RAW) as presented in [29]. Then we compare the estimated RAW to the groundtruth RAW for validating the accuracy of calibration.

The results on real-world images for the Nikon D40 camera (Image of color-chart object from NUS dataset), shown in Fig. 10, can be explained in details as follows. First, from a set of input images, we estimate the CRFs in R, G, B channels individually. The inverses of estimated CRFs in R, G, B channels are shown in Fig. 10(e). Then we apply the inverse CRF to the test image, as shown in Fig. 10(a), to obtain a calibrated sRGB image. The calibrated sRGB images for the MN method, the LEE method, and our method are shown in Fig. 10 (f), (i), and (l), respectively.

Based on the notion proposed in [29] and adopted in [1], we consider that there are white-balance transformation and color-space transformation for converting camera RAW data into sRGB image. And, both transformations can be represented with a $3 \times 3$ matrix. This matrix is estimated between the groundtruth RAW (shown in Fig. 10(b)) and the calibrated sRGB image using a least square method. We use the estimated matrix to convert the calibrated sRGB image into the estimated RAW image. The estimated RAW images for MN method, LEE method, and our method are shown in Fig. 10 (g), (j), and (m), respectively. Finally, the error map image that compares between the groundtruth RAW image and the estimated RAW image is calculated. The error maps
Fig. 10  Real-world experiment on Nikon D40. (a) Test image, (b) groundtruth RAW image, (c) estimated RAW without calibration, (d) error map between the groundtruth raw and the estimated raw without calibration, (e) estimated CRFs obtained from three methods in R, G, B channels (from top to bottom), (f), (i) and (l) calibrated sRGB images obtained from the MN method, the LEE method and our method respectively, (g), (j), and (m) estimated RAW images obtained from the MN method, the LEE method and our method respectively, (h), (k), and (n) error maps (R channel) computed from the groundtruth RAW image and the estimated RAW images from the MN method, the LEE method and our method respectively.

Table 3  RMSE of LEE and NUS real world experiments

| Camera          | LEE Dataset | NUS Dataset |
|-----------------|-------------|-------------|
| Canon 7D/5D    | MN          | 0.0027      | 0.0028      | 0.0026      | 0.0027      | 0.0028      |
|                 | LEE         | 0.0075      | 0.0110      | 0.0120      | 0.0111      | 0.0112      |
|                 | Ours        | 0.0075      | 0.0101      | 0.0129      | 0.0127      | 0.0129      |
| Canon 1Ds      | MN          | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | LEE         | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | Ours        | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
| Sony DSC-F828   | MN          | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | LEE         | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | Ours        | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
| Canon EOS1Ds    | MN          | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | LEE         | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | Ours        | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
| Nikon D7000    | MN          | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | LEE         | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | Ours        | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
| Nikon D40      | MN          | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | LEE         | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |
|                 | Ours        | 0.0009      | 0.0101      | 0.0102      | 0.0103      | 0.0103      |

on R channel for MN method, LEE method, and our method are shown in Fig. 10(h), (k), and (n), respectively. Note that, due to a gamut mapping and demosaicing/decompression artifacts around edges [1], [29] there still exists error in the estimated RAW images although the calibration is applied.

The results with calibration are compared to the results without calibration. To generate the results without calibration, the test image and the groundtruth RAW data are used to estimate the transformation $3 \times 3$ matrix. Then, the estimated RAW without calibration, as shown in Fig. 10(c), is generated by applying the $3 \times 3$ matrix to the test image. It can be observed that the error maps from the results with calibration (from all three methods) show significant lower values than the error map of the results without calibration.

Due to the lack of space, the error maps on G and B channels in the results of dataset are not shown. These errors are reflected in terms of RMSEs on each individual channel. The RMSEs computed from the error maps of R, G, B channels of different cameras with three calibration methods (MN, LEE, and ours) are shown in Table 3. From the results, it can be observed that our method is slightly better than the MN method and the LEE method. With the same rational, we show the other real world experiment on LEE and NUS database to indicate the validity of the proposed
method. The results show the inverses of estimated CRFs in R, G and B channels individually for each dataset. Finally, the RMSEs on each R, G, B channels of all real world experiment dataset are same shown in Table 3.

4.3 Discussions and Limitations

The proposed algorithm has limitations that are worth to be discussed as follows.

- First, currently the weights presented in Sect. 3.2 are computed from the representative CRFs that are derived from the DoRF. It is possible that the latent mapping in the CRFs of modern digital cameras could be significantly different from the representative CRFs. From our empirical experiment, if the latent mapping in CRF is a linear combination of 201 CRFs in the DoRF, we can get a good result by our technique. However, our proposed algorithm may fail in the calibration of a camera whose CRF does not lie in the space of DoRF.
- Second, the proposed algorithm is based on the assumption that the exposure ratios among image observation pairs must be equal and known. The main reason of relying on single known exposure ratio is due to our noise & outlier removal presented in Sect. 3.1. The single comparagram is established by combining all comparagrams of varying exposure pairs of image observations. Obviously, it is more likely that this combined comparagram could cover the entire range of intensity values. With the data spanning in entire range, we could yield to a very accurate BTF that can be effectively used to remove noise and outlier in the data. However, it is arguable that using only a single set of exposure ratio can lead to inaccurate estimation of the CRF. To get around the issue, our method can be extended to support multiple sets of exposure ratios, e.g. the data with exposure ratio 0.5 and 0.75. That is, our algorithm can still work with multiple sets of exposure ratios as long as the data in each set could cover the entire range of intensity value. This is to make the module of noise & outlier removal accurately infer the BTF. In addition to the aforementioned point, we need to slightly reformulate (6) to support several values of exposure ratios (\( \alpha \)). Regardless of using multiple sets of exposure ratios, we found from our empirical experiment that our proposed method can tolerate inaccuracy in exposure ratio of up to \( \pm 5\% \) for the case of using a single set of exposure ratio in the calibration.
- Third, the proposed method can fail in the case that the data in some intensity range is missing, i.e. incomplete data. This issue can be handled by incorporating a module that evaluates the completeness of data. If it is necessary, then the module can feedback to the user to ask for re-capturing the images so that a better set of data is obtained.
- Finally, we observe that from the plots of number of successful CRF estimation (\( \text{RMSE} < 0.03 \)) of DoRF datasets in column 1-3 of Figs. 11 the numbers of suc-
cessful estimations for D4 distribution with 3%, 5%, 7%, 9% outliers (171, 169, 162, 160) are slightly higher than the one for D4 with clean data (163) as shown in column 4. That is these results contradict to the intuition that the accuracy of algorithm when applying to the clean data should be better than the accuracies when applying to the data with outliers. To investigate this point, we set up a preliminary experiment to find the reason why the accuracy reversed like this. We still cannot exactly conclude the actual reason about this effect. To our best knowledge, we speculate the reason of this phenomenon that it may be due to the effect of outlier rejection module. As the outliers presented in the data could be rejected before passing to the CRF estimation module, therefore the distribution of filtered data could be changed from the original one in such a way that the distribution tends to be more uniform and then makes the algorithm performs slightly better. This issue could be as a future work for deeper investigation.

5. Conclusion

In this paper, an improved radiometric calibration algorithm by extending the least square minimization based method proposed by Mitsunaga and Nayar [2] is proposed. There are two major ideas in the proposed improvements. First, a noise & outlier removal procedure based on the analysis of brightness transfer function [7] of image observations is included. The main objective of this proposed improvement is to tackle one of the major drawback of least square minimization approach in which it poorly performs when there is noise and outlier in measurement data. Second, a variant of weighted least square minimization, is proposed, in which less weighted values are assigned to the pixels with higher frequency irradiance values. The main objective of resorting to the weighted least square is to tackle the drawback of ordinary least square minimization approach, which usually fails to converge to optimal solutions when the distribution of input data is biased or non-uniform. In comparison with two baseline algorithms, the Mitsunaga and Nayar algorithm [2] and the rank minimization based algorithm proposed by Lee et al [1], the evaluation using both the synthetic data and the real-world images has shown promising results by the proposed algorithm.

Some of possible future works can be listed as follows. First, a more generalized camera model (e.g. [9], [30]) that covers more modern cameras could be investigated in our framework. Second, tackling the issue of using multiple sets of exposure ratios in other perspectives can also be one of future work. Finally, a more rigorous treatment of weights in the estimation could also be investigated.

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