Predicting leptonic CP phase by considering deviations in charged lepton and neutrino sectors

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Abstract
Recently, the reactor mixing angle \( \theta_{13} \) has been measured precisely by Daya Bay, RENO, and T2K experiments with a moderately large value. However, the standard form of neutrino mixing patterns such as bimaximal, tri-bimaximal, golden ratio of types A and B, hexagonal, etc., which are based on certain flavor symmetries, predict vanishing \( \theta_{13} \). Using the fact that the neutrino mixing matrix can be represented as

\[
V_{\text{PMNS}} = U_{\text{PMNS}} P_{\nu},
\]

where \( U_{\text{PMNS}} \) result from the diagonalization of the charged lepton and neutrino mass matrices and \( P_{\nu} \) is a diagonal matrix containing Majorana phases, we explore the possibility of accounting for the large reactor mixing angle by considering deviations both in the charged lepton and neutrino sector. In the charged lepton sector we consider the deviation as an additional rotation in the (12) and (13) planes, whereas in the neutrino sector we consider deviations to various neutrino mixing patterns through (13) and (23) rotations. We find that with the inclusion of these deviations it is possible to accommodate the observed large reactor mixing angle \( \theta_{13} \), and one can also obtain limits on the charge-conjugation parity-violating Dirac phase \( \delta_{CP} \) and Jarlskog invariant \( J_{CP} \) for most of the cases. We then explore whether our findings can be tested in the currently running NuMI Off-axis \( \nu_e \) Appearance experiment with three years of data taking in neutrino mode followed by three years with the anti-neutrino mode.

1. Introduction

The phenomenon of neutrino oscillation is found to be the first substantial evidence for physics beyond the standard model. The results from various neutrino oscillation experiments \([1]\) established the fact that the three flavors of neutrinos mix with each other as they propagate and form mass eigenstates. The mixing is described by the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix \( V_{\text{PMNS}} \) \([2]\), analogous to the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix in the quark sector. The mixing matrix is unitary, and, hence, parameterized in terms of three rotation angles \( \theta_{12}, \theta_{23}, \theta_{13} \) and three CP-violating phases, one Dirac type \( (\delta_{CP}) \) and two Majorana types \( (\rho, \sigma) \), as

\[
V_{\text{PMNS}} \equiv U_{\text{PMNS}}, \quad P_{\nu} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\
-s_{12}c_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & -s_{13}c_{23} \\
s_{12}s_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}c_{23}
\end{pmatrix}
P_{\nu},
\]

where \( c_{ij} \equiv \cos \theta_{ij}, s_{ij} \equiv \sin \theta_{ij} \) and \( P_{\nu} \equiv |e^{\rho}, e^{\sigma}, 1 \rangle \) is a diagonal phase matrix, which is physically relevant if neutrinos are Majorana particles.

The solar and atmospheric neutrino oscillation parameters are precisely known from various neutrino oscillation experiments. Recently the reactor mixing angle \( \theta_{13} \) has also been measured by the Double Chooz \([3]\), Daya Bay \([4, 5]\), RENO \([6]\), and T2K \([7, 8]\) experiments with a moderately large value. After the discovery of sizable \( \theta_{13} \), much attention has been paid to determine the charge-conjugation parity (CP) violation effect in the lepton sector in the currently running as well as in future long-baseline neutrino oscillation experiments. As \( \theta_{13} \)
is non-zero, there could be CP violation in the lepton sector, analogous to the quark sector, provided the CP violating phase \( \delta_{CP} \) is not vanishingly small. Hence, it is of particular importance to determine the Dirac CP phase \( \delta_{CP} \) both theoretically and experimentally. The global analysis of various neutrino oscillation data has been performed by various groups [9–12], and the hint for non-zero \( \delta_{CP} \) was anticipated in [11, 12]. Including the data from T2K and Daya Bay, Forero et al [13] performed a global fit and found a hint for a non-zero value of \( \delta_{CP} \) and a deviation of \( \theta_{23} \) from \( \pi/4 \), with the best fit values as \( \delta_{CP} \approx 3\pi/2 \) and \( \sin^2 \theta_{23} \approx 0.57 \). The best fit values, along with their 3σ ranges of various oscillation parameters from [13], are presented in table 1.

Understanding the origin of the patterns of neutrino masses and mixing that emerge from the neutrino oscillation data is one of the most challenging problems in neutrino physics. In fact, it is part of the more fundamental problem of particle physics of understanding the origin of masses and the mixing pattern in the quark and lepton sector. As we know, the phenomenon of neutrino oscillation is characterized by two large mixing angles, the solar (\( \theta_{12} \)) and the atmospheric (\( \theta_{23} \)), and one not-so-large reactor mixing angle \( \theta_{13} \). Initially it was believed that the reactor mixing angle would be vanishingly small and motivated by such anticipation many models were proposed to explain the neutrino mixing pattern that are generally based on some kind of discrete flavor symmetries like \( S_3 \), \( S_4 \), \( A_4 \), etc [14–16]. For example, the tri-bimaximal (TBM) mixing pattern [17] is one such well-motivated model having \( \sin^2 \theta_{12} = \frac{1}{3} \) and \( \sin^2 \theta_{23} = \frac{1}{2} \), which plays a crucial role for model building. However, in the TBM mixing pattern the value of \( \theta_{13} \) is zero, and the CP phase \( \delta_{CP} \) is consequently undefined. After the experimental discovery of moderately large \( \theta_{13} \), various perturbation terms were added to the TBM mixing pattern, and it was found that it can still be used to describe the neutrino mixing pattern or model building with suitable modifications [18].

Thus, in a nutshell the experimental discovery of a moderately large value of the reactor mixing angle caused a profound change in the subject of flavor models that describe leptonic mixing. Some of the models are outdated, while others are suitably modified by including appropriate perturbations/corrections to accommodate the observed value of \( \theta_{13} \) [19]. In this paper, we would like to consider the effect of perturbations to a few such well-motivated models, which are based on certain discrete flavor symmetries like \( A_4 \), \( \mu - \tau \), etc. These models include TBM [17], bi-maximal mixing (BM) [20], golden ratio type A (GRA) [21, 22], golden ratio type B (GRB) [23, 24], and hexagonal (HG) [25] mixing patterns. However, as we know, these forms do not accommodate a non-zero value for the reactor mixing angle \( \theta_{13} \) and hence need to be modified suitably to provide the leptonic mixing angles compatible with the experimental data. In this paper, we are interested in looking for such a possibility. Although this aspect has been widely studied in the literature (see, for example, [18, 19, 24, 26–28]), the main difference between the previous studies and our work is that we have considered a very simple form of deviation matrix in terms of a minimal number of new independent parameters, which can provide corrections to both charged lepton and the standard neutrino mixing matrices. This in turn not only accommodates the observed mixing angles but also constrains the Dirac CP-violating phase \( \delta_{CP} \).

It is well known that the determination of the CP-violating phases and, in particular, the Dirac CP phase \( \delta_{CP} \) is an important issue in the study of neutrino physics. Many dedicated long-baseline experiments are planned to study CP violation in the neutrino sector. The theoretical prediction for the determination of the CP phase in the neutrino mixing matrix depends on the approach as well as the type of symmetries one uses to understand the pattern of neutrino mixing. Obviously a sufficiently precise measurement of \( \delta_{CP} \) will serve as a very useful constraint for identifying the approaches and symmetries, if any. In this work, we would also like to explore whether it is possible to constrain the CP phase \( \delta_{CP} \) by considering corrections to the leading-order charged lepton and neutrino mixing matrices and, if so, whether it is possible to verify such predictions with the data from the ongoing NuMI Off-axis \( \nu_e \) Appearance (NO\( \nu_e \)) experiment.

| Mixing Parameters | Best Fit value | 3σ Range |
|-------------------|---------------|---------|
| \( \sin^2 \theta_{12} \) | 0.323 | 0.278 \( \rightarrow \) 0.375 |
| \( \sin^2 \theta_{13} \) (NO) | 0.567 | 0.392 \( \rightarrow \) 0.643 |
| \( \sin^2 \theta_{13} \) (IO) | 0.573 | 0.403 \( \rightarrow \) 0.640 |
| \( \sin^2 \theta_{13} \) (NO) | 0.0234 | 0.0177 \( \rightarrow \) 0.0294 |
| \( \sin^2 \theta_{13} \) (IO) | 0.0240 | 0.0183 \( \rightarrow \) 0.0297 |
| \( \delta_{CP} \) (NO) | 1.34π | (0 \( \rightarrow \) 2π) |
| \( \delta_{CP} \) (IO) | 1.48π | (0 \( \rightarrow \) 2π) |
| \( \Delta m^2_{12}/10^{-3} \) eV\(^2\) | 7.60 | 7.11 \( \rightarrow \) 8.18 |
| \( \Delta m^2_{12}/10^{-3} \) eV\(^2\) (NO) | 2.48 | 2.3 \( \rightarrow \) 2.65 |
| \( \Delta m^2_{13}/10^{-3} \) eV\(^2\) (IO) | -2.38 | -2.54 \( \rightarrow \) -2.20 |
The paper is organized as follows. In section 2 we present the basic framework of our analysis. The deviations to the various mixing patterns due to neutrino and charged lepton sectors are discussed in sections 3 and 4, respectively. Section 5 contains a summary and conclusion.

2. Framework

It is well known that the lepton mixing matrix arises from the overlapping of the matrices that diagonalize charged lepton and neutrino mass matrices; i.e.,

$$U_{\text{PMNS}} = U_\nu^\dagger U_L.$$  \hspace{1cm} (2)

For the study of leptonic mixing it is generally assumed that the charged lepton mass matrix is diagonal, and, hence, the corresponding mixing matrix $U_L$ is an identity matrix. However, the neutrino mixing matrix $U_\nu$ has a specific form dictated by the symmetry which generally fixes the values of the three mixing angles in $U_\nu$. The small deviations of the predicted values of the mixing angles from their corresponding measured values are considered, in general, as perturbative corrections arising from symmetry breaking effects. A variety of symmetry forms of $U_\nu$ have been explored in the literature e.g., TBM, BM, GRA, GRB, HG, and so on. All these mixing patterns can be written in a generalized form, as shown in [24]. For the cases of TBM, BM, GRA, GRB, and HG forms for $U_\nu$ one can have $\theta_{12}^0 = -\pi/4$, while $\theta_{13}^0$ takes the values $\sin^{-1}(1/\sqrt{3})$, $\pi/4$, $\sin^{-1}(1/\sqrt{5} + \tau)$, $\sin^{-1}(\sqrt{3} - r/2)$ ($r$ being the golden ratio, i.e., $r = (1 + \sqrt{5})/2$), and $\pi/6$, respectively. Thus, the generalized neutrino matrix $U_\nu$ corresponding to these cases has the form [24]

$$U_\nu^0 = \begin{pmatrix} \cos \theta_{12}^0 & \sin \theta_{12}^0 & 0 \\ -\sin \theta_{12}^0 & \cos \theta_{12}^0 & -\sqrt{2}/2 \\ \sin \theta_{13}^0 & \cos \theta_{13}^0 & 1/\sqrt{2} \end{pmatrix}.$$ \hspace{1cm} (3)

The superscript ‘0’ is introduced to label the mixing matrix as the leading order matrix arising from certain discrete flavor symmetries. A common feature of these mixing matrices is that, they predict $\theta_{12} = \pm \pi/4$ and $\theta_{13} = 0$, if the charged lepton mixing matrix is considered to be a $3 \times 3$ identity matrix. However, they differ in their prediction for the solar mixing angle $\theta_{12}$, which has the value as $\sin^2 \theta_{12} = 0.5$ for the BM form, $\sin^2 \theta_{12} = 1/3$ for TBM, $\sin^2 \theta_{12} = 0.276$ and 0.345 for GRA and GRB mixing and $\sin^2 \theta_{12} = 0.25$ for HG mixing patterns. Thus, one possible way to generate corrections for the mixing angles such that all the mixing angles $\theta_{23}, \theta_{12},$ and $\theta_{13}$ should be compatible with the observed experimental data is to include suitable perturbative corrections to both the charged lepton and neutrino mixing matrices $U_L$ and $U_\nu$, respectively. In this paper we are interested in exploring such a possibility. While considering the corrections to the neutrino mixing matrix, we assume the charged lepton mixing matrix to be an identity matrix, and for correction to the charged lepton mass matrix we consider the neutrino mixing matrix to be either of TBM/BM/GRA/GRB/HG forms. Furthermore, we will neglect possible corrections to $U_\nu$ from higher dimensional operators and from renormalization group effects.

3. Deviation in Neutrino sector

In this section, we consider the corrections to the neutrino mixing matrix such that it can be written as

$$U_\nu = U_\nu^0 U_\nu^{\text{corr}},$$ \hspace{1cm} (4)

where $U_\nu^0$ is one of the symmetry forms of the mixing matrix, as described in equation (3), and $U_\nu^{\text{corr}}$ is a unitary matrix describing the correction to $U_\nu^0$. An important requirement is that the correction due to the matrix $U_\nu^{\text{corr}}$ should allow sizeable deviation of the angle $\theta_{13}$ from zero and also the required deviations to $\theta_{12}$ and $\theta_{23}$ so that all the mixing angles should be compatible with their measured values. As discussed in [29], $U_\nu^{\text{corr}}$ can be expressed as $V_s V_\nu V_\nu^\dagger$, where $V_{ij}$ are the rotation matrices in the \((ij)\) plane and, hence, can be parameterized by three mixing angles and one phase. In this work, we consider the simplest case of such a perturbation, which involves only a minimal set of new independent parameters; i.e., we consider the deviations involving only two new parameters (one rotation angle and one phase), which basically corresponds to the perturbation induced by a single rotation. There are several variants of this approach in the literature, generally for the TBM mixing pattern [18]. The main difference between the previous studies and our work is that, apart from predicting the values of the mixing angles compatible with their experimental range, we have also looked into the possibility of constraining the CP phase $\delta_{\text{CP}}$ not only for the TBM case but also for other varieties of mixing patterns.
3.1. Deviation due to 23 rotation

First, we would like to consider additional rotation in the 23 plane. Since the charged lepton mixing matrix is considered to be an identity in this case, the PMNS mixing matrix can be obtained by multiplying the neutrino mixing matrix \( U^0 \) with the 23 rotation matrix as follows

\[
U_{PMNS} = U^0 \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi & e^{i\alpha} \sin \phi \\
0 & -e^{i\alpha} \sin \phi & \cos \phi
\end{pmatrix},
\]

(5)

where \( \phi \) and \( \alpha \) are arbitrary free parameters. The mixing angles \( \sin^2 \theta_{12}, \sin^2 \theta_{23} \) and \( \sin \theta_{13} \), can be obtained using the relations

\[
\sin^2 \theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2},
\]

\[
\sin^2 \theta_{23} = \frac{|U_{\mu3}|^2}{1 - |U_{e3}|^2},
\]

(6)

\[
\sin \theta_{13} = |U_{e3}|.
\]

Using equations (3), (5), and (6), one obtains the mixing angles as

\[
\sin \theta_{13} = \sin \theta_{13}^e \sin \phi,
\]

\[
\sin^2 \theta_{12} = \sin^2 \theta_{12}^e \cos^2 \phi + \sin^2 \theta_{12}^e \sin^2 \phi,
\]

\[
\sin^2 \theta_{23} = \frac{1}{2} \left[ 1 - \sin^2 \theta_{12}^e \sin^2 \phi - \cos \theta_{12}^e \sin 2\phi \cos \alpha \right].
\]

(9)

Thus, from equations (7)–(9), one can see that by including the 23 rotation matrix as a perturbation, it is possible to have nonzero \( \theta_{13} \), deviation of \( \sin^2 \theta_{23} \) from 1/2, and \( \sin^2 \theta_{12} \) from \( \sin^2 \theta_{12}^e \). With equations (7) and (8) one can obtain the relation between \( \sin^2 \theta_{12} \) and \( \sin^2 \theta_{13} \) as

\[
\sin^2 \theta_{12} = \frac{\sin^2 \theta_{12}^e - \sin^2 \theta_{13}}{1 - \sin^2 \theta_{13}}.
\]

(10)

Thus, it can be seen that in this case one can have \( \sin^2 \theta_{12} < \sin^2 \theta_{12}^e \), although the deviation is not significant. Therefore, the BM, GRA, and HG forms of neutrino mixing patterns cannot accommodate the observed value of \( \sin^2 \theta_{12} \) within its 3\( \sigma \) range.

Furthermore, as we have a non-vanishing and largish \( \theta_{13} \), this in turn implies that it could, in principle, be possible to observe CP-violation in the lepton sector analogous to the quark sector, provided the CP-violating phase is not vanishingly small, in the long-baseline neutrino oscillation experiments. The Jarlskog invariant, which is a measure of CP violation, has the expression in the standard parameterization as

\[
J_{CP} \equiv \text{Im} \left[ U_{e1} U_{\mu2} U_{\tau1}^* U_{\tau2}^* \right] = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta_{CP}
\]

(11)

and is sensitive to the Dirac CP-violating phase. With equation (5), one can obtain the value of the Jarlskog invariant as

\[
J_{CP} = -\frac{1}{4} \cos \theta_{12}^e \sin \theta_{12}^e \sin 2\phi \sin \alpha.
\]

(12)

Thus, comparing the two equations (11) and (12), one can obtain the expression for \( \delta_{CP} \) as

\[
\sin \delta_{CP} = \frac{(1 - \sin^2 \theta_{12}^e \sin^2 \phi) \sin \alpha}{\left[ (1 - \sin^2 \theta_{12}^e \sin^2 \phi)^2 - \cos^2 \theta_{12}^e \sin^2 2\phi \cos^2 \alpha \right]^{1/2}}.
\]

(13)

For numerical evaluation we constrain the parameter \( \phi \) from the measured value of \( \sin \theta_{13} \) and vary the phase parameter \( \alpha \) within its allowed range, i.e., \(-\pi \leq \alpha \leq \pi\). With equation (7) and using the 3\( \sigma \) range of \( \sin^2 \theta_{13} \) and the specified value of \( \sin^2 \theta_{12}^e \), we obtain the allowed range of \( \phi \) for various mixing patterns as: \((10.9 - 14.2)^\circ\) for BM, \((13.3 - 17.5)^\circ\) for TBM, \((14.8 - 19.2)^\circ\) for GRA, \((13.2 - 17.2)^\circ\) for GRB, and \((15.6 - 20.3)^\circ\) for the HG pattern. With these input parameters, we present our results in figure 1. The correlation plot between \( \sin^2 \theta_{12} \) and \( \sin^2 \theta_{13} \) is shown in the top panel, where the magenta, red, green, orange, and blue plots correspond to BM, TBM, GRA, GRB, and HG mixing patterns, respectively. The horizontal and vertical dashed black lines correspond to the best fit values for \( \sin^2 \theta_{12} \) and \( \sin^2 \theta_{13} \), whereas the vertical dashed magenta lines represent the 3\( \sigma \) allowed range of \( \sin^2 \theta_{12} \), and the horizontal dot-dashed lines correspond to the same for \( \sin^2 \theta_{13} \). As discussed before, one can see from the figure that the predicted values of the mixing angles \( \sin^2 \theta_{12} \) and \( \sin^2 \theta_{13} \) lie within their 3\( \sigma \) ranges only for TBM and GRB mixing patterns, whereas the predicted value of \( \sin^2 \theta_{12} \) lies outside its 3\( \sigma \) range for BM, GRA, and HG mixing patterns. With equation (13), we obtain the constraint on \( \delta_{CP} \), as shown in the middle panel of figure 1 for the TBM case, where we have used the 3\( \sigma \) allowed range of the mixing angles \( \theta_{12} \),
Using the predicted value of $\delta_{CP}$, the correlation between the Jarlskog invariant and $\sin^2 \theta_{13} / \sin^2 \theta_{23}$, is shown in the bottom panel of figure 1 for the TBM case. The corresponding results for the GRB mixing pattern are almost the same as the TBM case and, hence, are not shown explicitly in the figures. However, the allowed ranges of $\delta_{CP}$ and $J_{CP}$ are listed in table 2. Since BM, GRA, and HG mixing patterns cannot accommodate the observed mixing angles as discussed earlier in this section, the corresponding results are not listed.

Our next objective is to speculate the possible experimental indications, which could support or rule out our findings. As we know, neutrino physics has now entered the precision era as far as the measured parameters are concerned. The currently running experiments T2K and NO$\nu$A play a major role in this respect. These experiments will provide the precise measurement of atmospheric neutrino mass square difference and the mixing angle $\theta_{23}$ through the $\nu_{\mu}$ disappearance channel. They also intend to measure $\theta_{13}$, the CP violation phase $\delta_{CP}$ through $\nu_{\mu}$ to $\nu_{e}$ appearance. Furthermore, NuMI Off-axis $\nu_{e}$ Appearance (NO$\nu$A) can potentially resolve the mass-ordering through matter effects, as it has a long baseline. In this work, we would like to see whether the
Table 2. Predicted range of the CP phase $\delta_{\text{CP}}$ and the Jarlskog invariant $|\mathcal{J}_{\text{CP}}|$ due to possible deviations for various neutrino mixing patterns.

| Deviation type | Neutrino mixing matrix pattern | $\delta_{\text{CP}}$ Range (in radian) | $|\mathcal{J}_{\text{CP}}|$ Range |
|----------------|---------------------------------|----------------------------------------|----------------------------------|
| 23 rotation to $U^2_{13}$ | TBM and GRB | $\pm(0.7 - 1.5)$ | $(0 - 0.04)$ |
| 13 rotation to $U^2_{13}$ | TBM, GRA and GRB | $\pm(0 - 1.5)$ | $(0 - 0.04)$ |
| 12 and 13 rotation to $U_{1}$ | TBM and GRB | $\pm(1 - 1.5)$ | $(0.03 - 0.04)$ |
| | GRA | $\pm(0.6 - 1.5)$ | $(0.02 - 0.04)$ |
| | HG | $\pm(0 - 1.5)$ | $(0.03 - 0.035)$ |
| | BM | $\pm(0 - 0.8)$ | $(0 - 0.03)$ |

constraints obtained on $\delta_{\text{CP}}$ in our analysis could be probed in the NO$\nu$A experiment with three years of data taking with the neutrino mode followed by another three years with the antineutrino mode. For our study we do the simulations using GLoBES [30, 31].

3.2. Simulation details

NO$\nu$A (NuMI Off-axis Appearance) is an off-axis long-baseline experiment [32, 33], that uses Fermilab’s NuMI $\nu_\mu/\bar{\nu}_\mu$ beamline. Its detector is a 14 kton totally active scintillator detector placed at a distance of 810 km from Fermilab, near Ash River, which is 0.8° off-axis from the NuMI beam. It also has a 0.3 kton near detector located at the Fermilab site to monitor the unoscillated neutrino or anti-neutrino flux. It has already started data taking from late 2014. The experiment is scheduled to have a three-years run in neutrino mode, followed by a three-years run in anti-neutrino mode with a NuMI beam power of 0.7 MW and 120 GeV proton energy, corresponding to $6 \times 10^{20}$ proton on target per year. Apart from the precise measurement of $\theta_{13}$ and the atmospheric parameters, it aims to determine the unknowns such as neutrino mass ordering, leptonic CP violation, and the octant of $\theta_{23}$ by the measurement of $\nu_\mu/\bar{\nu}_\mu \rightarrow \nu_e/\bar{\nu}_e$ oscillations.

For the simulation for the NO$\nu$A experiment, the detector properties and other necessary details are taken from [34, 35]. We have used the following input true values of neutrino oscillation parameters in our simulations: $|\Delta m_{	ext{eff}}^2| = 2.4 \times 10^{-3}$ eV$^2$, $\Delta m_{12}^2 = 7.6 \times 10^{-5}$ eV$^2$, $\delta_{\text{CP}} = 0$, $\sin^2 \theta_{12} = 0.32$, $\sin^2 2\theta_{13} = 0.1$, and $\sin^2 \theta_{23} = 0.5$. The relation between the atmospheric parameter $\Delta m_{	ext{eff}}^2$ measured in main injector neutrino oscillation search and the standard oscillation parameter $\Delta m_{21}^2$ in nature is given as [36]

$$\Delta m_{12}^2 = \Delta m_{	ext{eff}}^2 + \Delta m_{21}^2 (\cos^2 \theta_{12} - \cos \delta_{\text{CP}} \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23}),$$

where $\Delta m_{	ext{eff}}^2$ is taken to be positive for normal ordering (NO) and negative for inverted ordering (IO).

In order to obtain the allowed region for $\sin^2 2\theta_{13}$ and $\delta_{\text{CP}}$, we generate the true event spectrum by keeping the above-mentioned neutrino oscillation parameters as true values and generate the test event spectrum by varying the test values of $\sin^2 2\theta_{13}$ in the range $[0.02:0.25]$ and that of $\delta_{\text{CP}}$ in its full range $[-\pi, \pi]$. Finally, we calculate $\Delta \chi^2$ by comparing the true and test event spectra. The obtained results in the $\sin^2 2\theta_{13} - \delta_{\text{CP}}$ plane are shown in figure 2, which are overlaid by our predicted value of $\delta_{\text{CP}}$. The top panel shows the 1σ contours for the running of $(3\nu + 0\bar{\nu})$ years, with NO as the true hierarchy. The bottom left (right) panel represents $(3\nu + 3\bar{\nu})$ years of data taking with NO (IO) as the true hierarchy. In these plots, the inner regions (bubbles) correspond to 1σ contours, whereas the outer curves represent 3σ contours. From these plots, one can see that our results are supported by NO$\nu$A data within 3σ confidence level (C.L.); however, with $(3\nu+3\bar{\nu})$ years of data taking, NO$\nu$A could marginally exclude these results at 1σ C.L.

Next we would like to briefly mention the implications of future-generation long-baseline experiments such as the Hyper–Kamiokande (T2HK) and the deep underground neutrino experiment (DUNE) experiments in our predicted results. All the details for simulation of the T2HK experiment are taken from [35] for $(3\nu+7\bar{\nu})$ years of running. The DUNE experiment, which is basically a slightly upgraded version of the Long baseline neutrino experiment, plans to use a 40 kton liquid argon detector. Except for the detector volume, other characteristics are taken from [37] for the simulation for $(5\nu+5\bar{\nu})$ years of data taking. We use the same true values of other input parameters as for the NO$\nu$A experiment. The correlation plots between $\delta_{\text{CP}}$ and $\sin^2 2\theta_{13}$ are shown in figure 3, overlaid by our predicted values for TBM. The plots on the top (bottom) panel are for the DUNE (T2HK) experiment with NO/IO as the true ordering, as labeled in the plots. It can be seen from these figures that as the $\delta_{\text{CP}}$–$\sin^2 2\theta_{13}$ parameter space is severely constrained, our predicted results are expected to be precisely verified by these experiments.
3.3. Deviation due to 13 rotation

Next we consider the corrections arising from an additional (13) rotation in the neutrino sector, for which the rotation matrix can be given as

$$U_{PMNS} = U_{
u}^0 \begin{pmatrix} \cos \phi & 0 & e^{-i\alpha} \sin \phi \\ 0 & 1 & 0 \\ -e^{i\alpha} \sin \phi & 0 & \cos \phi \end{pmatrix}$$ (15)

Proceeding in the similar way as in the previous case, we obtain the mixing angles, using equation (6), as

$$\sin \theta_{13} = \cos \theta_{12}^\nu \sin \phi,$$ (16)

$$\sin^2 \theta_{12} = \sin^2 \theta_{12}^\nu \frac{1}{1 - \cos^2 \theta_{12}^\nu \sin^2 \phi},$$ (17)

$$\sin^2 \theta_{23} = \frac{1}{2} \left[ \cos^2 \phi + \sin \theta_{12}^\nu \sin 2 \phi \cos \alpha + \sin^2 \theta_{12}^\nu \sin^2 \phi \right].$$ (18)

Analogously, the Jarlskog invariant and the CP-violating phase $\delta_{CP}$ are given as

$$J_{CP} = -\frac{1}{8} \cos \theta_{12}^\nu \sin 2 \theta_{12}^\nu \sin 2 \phi \sin \alpha,$$ (19)

and

$$\sin \delta_{CP} = -\frac{(1 - \cos^2 \theta_{12}^\nu \sin^2 \phi) \sin \alpha}{\left[ (1 - \cos^2 \theta_{12}^\nu \sin^2 \phi)^2 - \sin^2 \theta_{12}^\nu \sin^2 2 \phi \cos^2 \alpha \right]^{1/2}}.$$ (20)
In this case one obtains from equations (16) and (17)

$$\sin^2 \theta_{12} = \frac{\sin^2 \theta_{13}'}{1 - \sin^2 \theta_{13}},$$  \hspace{1cm} (21)

which implies that $\sin^2 \theta_{12} > \sin^2 \theta_{13}'$. This, in turn, implies that BM and HG mixing patterns cannot accommodate the observed value of $\theta_{12}$ within its $3\sigma$ range.

From equation (16) and using the $3\sigma$ allowed range of $\sin^2 \theta_{13}$, the allowed range of $\phi$ is found to be in the range $(9 - 15)\degree$ for various mixing patterns. Now using this value of $\phi$ and varying the free phase parameter $\alpha$ in the range $-\pi \leq \alpha \leq \pi$, we obtain the correlation plots between $\sin^2 \theta_{13}$ and $\sin^2 \theta_{13}$, as shown in the top left panel of figure 4, where red, blue, and green plots are for TBM, GRB, and GRA mixing patterns. The correlation plot for the HG and BM forms are not shown in the figure, as they lie outside the allowed $3\sigma$ region of $\sin^2 \theta_{12}$.

The $\delta_{CP}$ phase is very loosely constrained in this case as presented in figure 4. We also overlaid the predicted value of $\delta_{CP}$ for TBM over the NO$\nu$A simulated data. In this case also, the predicted result is consistent with expected NO$\nu$A data. The correlation plots between $\delta_{CP}$ and $\sin^2 \theta_{13}$, $I_{CP}$ and $\sin^2 \theta_{13}$, $\sin^2 \theta_{23}$, as well as between $I_{CP}$ and $\delta_{CP}$, are also shown in the figure. From the plots it can be seen that it could be possible to have a large CP violation $\mathcal{O}(10^{-3})$ in the lepton sector.

It should be noted that deviation due to the 12 rotation matrix does not accommodate the observed value of $\theta_{13}$ as $U_{e3} = 0$ for such a case.

**4. Deviation in the charged lepton sector**

In this section we will consider the deviation arising in the charged lepton sector. For the study of lepton mixing it is generally assumed that the charged lepton mass matrix is diagonal, and, hence, the corresponding mixing matrix as an identity matrix. The deviation in the charged lepton sector and its possible consequences have been

![Figure 3. The correlation between $\delta_{CP}$ and $\sin^2 \theta_{13}$ for the TBM mixing pattern (red regions) superimposed on expected DUNE data (top panels), where the blue dashed lines represent the $3\sigma$ contours for (5+5 $\nu$) years of data taking, while the bottom panels represent the T2HK results for (3+7 $\nu$) years of running.](image-url)
Figure 4. Correlation plots between different oscillation parameters due to 13 deviation in the neutrino sector. In the top left panel the red, blue, and green plots correspond to TBM, GRB, and GRA mixing patterns. Other plots represent the correlation between different mixing parameters, as indicated in the plot labels for the TBM mixing pattern. The black solid lines (in the top right and second panels) represent the expected experimentally allowed parameter space (same as the dotted blue lines in figure 3).
studied by various authors [24, 26–28]. In [24, 26], the form for \( U_l \) is considered to be the product of two orthogonal matrices describing rotations in the 23 and 12 planes, which corresponds to two possible orderings, ‘standard’ with \( U_l \propto R_{23}(\theta_{13}^\nu)R_{12}(\theta_{12}^\nu) \) and ‘inverse’ with \( U_l \propto R_{12}(\theta_{12}^\nu)R_{23}(\theta_{13}^\nu) \). Using these forms for the lepton mixing matrix, the values of \( \delta_{\text{CP}} \) and the rephasing invariant \( J_{\text{CP}} \) have been predicted for the cases of TBM, BM, LG, GRA, GRB, and HG forms of neutrino mixing matrix \( U_\nu \). They have obtained the predictions for \( \delta_{\text{CP}} \) as \( \delta_{\text{CP}} \approx \pi \) for BM (LC) and \( \delta_{\text{CP}} \approx 3\pi/2 \) or \( \pi/2 \) for TBM, GRA, GRB, and HG. Here, we consider the simplest case, where the deviation matrix can be represented as a single rotation matrix in the \((ij)\) plane, as in the previous section for the neutrino sector.

Now considering the deviation to the charged lepton mixing matrix as a unitary rotation matrix either in the (12), (23), or (13) plane, one can write the PMNS matrix as

\[
U_{\text{PMNS}} = U_{ij} U_\nu^0,
\]

where \( U_{ij} \) is the rotation matrix in the \((ij)\) plane and \( U_\nu^0 \) is any one of the standard neutrino mixing matrix form TBM/BM/GRA/GRB/HG. However, corrections arising due to the \( U_{23} \) rotation matrix is ruled out, as it gives vanishing \( U_{e3} \).

### 4.1. Deviation due to rotation in 12 and 13 sector

Including the additional correction matrix \( U_{12} \) to the charged lepton sector, one can write the PMNS matrix as

\[
U_{\text{PMNS}} = \begin{pmatrix}
\cos \phi & -e^{-i\alpha} \sin \phi & 0 \\
e^{i\alpha} \sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{pmatrix} U_\nu^0.
\]

In this case we get the mixing angles as

\[
\sin \theta_{13} = \frac{\sin \phi}{\sqrt{2}},
\]

\[
\sin^2 \theta_{12} = \frac{2 \sin^2 \theta_{12}^\nu \cos^2 \theta + \cos^2 \theta_{12}^\nu \sin^2 \phi - \frac{1}{\sqrt{2}} \sin 2\theta_{12}^\nu \sin 2\phi \cos \alpha}{1 + \cos^2 \phi},
\]

\[
\sin^2 \theta_{23} = \frac{\cos^2 \phi}{1 + \cos^2 \phi}.
\]

With equations (24) and (26), we obtain the relation

\[
\sin^2 \theta_{23} = 1 - \frac{1}{2 \cos^2 \theta_{13}},
\]

which implies that \( \sin^2 \theta_{23} < 1/2 \). The Jarlskog invariant in this case is found to be

\[
J_{\text{CP}} = -\frac{1}{8\sqrt{2}} \sin 2\theta_{12}^\nu \sin 2\phi \sin \alpha,
\]

and the CP-violating phase as

\[
\sin \delta_{\text{CP}} = -\frac{(1 + \cos^2 \phi) \sin 2\theta_{12}^\nu \sin \alpha}{2\sqrt{Y}},
\]

where

\[
Y = \left(2 \sin^2 \theta_{12}^\nu \cos^2 \phi + \cos^2 \theta_{12}^\nu \sin^2 \phi - \frac{1}{\sqrt{2}} \sin 2\phi \sin 2\theta_{12}^\nu \cos \alpha \right) \times \left(1 + \cos 2\theta_{12}^\nu \cos^2 \phi - \cos^2 \theta_{12}^\nu \sin^2 \phi + \frac{1}{\sqrt{2}} \sin 2\theta_{12}^\nu \sin 2\phi \cos \alpha \right).
\]

Proceeding in a similar fashion as in the previous cases and considering the 3\( \sigma \) allowed range of \( \theta_{13} \), one can obtain the allowed range of \( \phi \) with equation (24) as \((10–15)\)°. Now varying the free parameters \( \phi \) and \( \alpha \) in their allowed ranges, we obtain the correlation plots between various mixing parameters, as depicted in figure 5. It should be noted that the correlation plots between \( \sin^2 \theta_{13} \) and \( \sin^2 \theta_{23} \) remain the same for all forms of neutrino mixing matrix \( U_\nu^0 \), as these mixing angles depend only on the free parameter \( \phi \) and are independent of \( \theta_{13}^\nu \) (which takes different values for different mixing patterns). For the correlation plots between \( \delta_{\text{CP}} = \sin^2 2\theta_{13}^\nu (\sin^2 \theta_{12}) \) and \( J_{\text{CP}} = \sin^2 \theta_{13} \), the red, green, blue, and magenta regions correspond to TBM,
GRA, HG, and BM mixing patterns. The GRB mixing pattern predicts the same constraints as the TBM pattern, and, hence, the corresponding results are not shown in the plots. Furthermore, the CP-violating phase is severely constrained in this scenario, and the Jarlskog invariant is found to be significantly large, as seen from the figure.

Next we consider deviation due to additional rotation in the 13 sector. In this case the PMNS matrix is given as

\[
U_{\text{PMNS}} = \begin{pmatrix}
\cos \phi & 0 & -e^{-i\alpha} \sin \phi \\
0 & 1 & 0 \\
e^{i\alpha} \sin \phi & 0 & \cos \phi
\end{pmatrix} U_{\nu}^0
\]  

(31)

Figure 5. Correlation plots between different observables due to 12 deviation in the charged lepton sector. The top left panel represents the correlation plot between \(\sin^2 \theta_{13}\) and \(\sin^2 \theta_{12}\). The descriptions of the other plots are indicated in the corresponding plot labels. In these plots the red, green, blue, and magenta regions correspond to TBM, GRA, HG, and BM mixing patterns. The black solid lines in the top (right panel) and the middle panel plots correspond to the experimentally allowed contours.
The mixing angles obtained are

\[ \sin \theta_{13} = \frac{\sin \phi}{\sqrt{2}}, \]
\[ \sin^2 \theta_{23} = \frac{1}{1 + \cos^2 \phi}, \]
\[ \sin^2 \theta_{12} = \frac{2 \sin^2 \theta_{13} \cos^2 \phi + \cos^2 \theta_{13} \sin^2 \phi - \frac{1}{\sqrt{2}} \sin 2 \theta_{13} \sin 2 \phi \cos \alpha}{1 + \cos^2 \phi}. \]  

(32)

In this case we obtain

\[ \sin^2 \theta_{23} = \frac{1}{2 \cos^2 \theta_{13}}, \]  

(33)

which implies \( \sin^2 \theta_{23} > 1/2 \). The Jarlskog invariant and the CP phase are found to be

\[ J_{CP} = \frac{1}{8 \sqrt{2}} \left( \sin 2 \theta_{12} \sin 2 \phi \sin \alpha \right), \]  

(34)
\[ \sin \delta_{CP} = \frac{\sin 2 \theta_{13} \sin \alpha (1 + \cos^2 \phi)}{2 \sqrt{2}}. \]  

(35)

Since the results for this deviation pattern are almost similar to the correction due to the 12 rotation case, one obtains the same constraints on \( \delta_{CP} \) as in the previous case, which are listed in table 2.

5. Summary and Conclusion

The recent observation of moderately large reactor mixing angle \( \theta_{13} \) has ignited a lot of interest to understand the mixing pattern in the lepton sector. It also opens up promising perspectives for the observation of CP violation in the lepton sector. The precise determination of \( \theta_{13} \), in addition to providing a complete picture of the neutrino mixing pattern, could be a signal of the underlying physics responsible for lepton mixing and for the physics beyond the standard model. In this context a number of neutrino mixing patterns like TBM/BM/GRA, etc., were proposed based on some discrete flavor symmetries like \( S_3, A_4, \mu - \tau, \) etc. However, these symmetry forms of the mixing matrices predict a vanishing reactor and maximal atmospheric mixing angles. To accommodate the observed value of relatively large \( \theta_{13} \), these mixing patterns should be modified by including appropriate perturbations. In this work, we have considered the simplest case of such perturbation, which involves only a minimal set of new independent parameters, i.e., one rotation angle and one phase, (which basically corresponds to perturbation induced by a single rotation), and found that it is possible to explain the observed neutrino oscillation data with such corrections. The predicted values of \( \delta_{CP} \) are expected to be supported by the data from the currently running NO\( \nu \)A experiment with (3+3 \( \bar{\nu} \)) years of data taking. We have also shown that it is possible to predict the value of the CP phase with such corrections. We have also found that a sizable leptonic CP violation characterized by the Jarlskog invariant \( J_{CP} \), i.e., \( |J_{CP}| \sim 10^{-2} \), could be possible in these scenarios.

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