Analytic Performance Evaluation of Underlay Relay Cognitive Networks with Channel Estimation Errors

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Abstract—This paper evaluates the bit error rate (BER) performance of underlay relay cognitive networks with decode-and-forward (DF) relays in arbitrary number of hops over Rayleigh fading with channel estimation errors. In order to facilitate the performance evaluation analytically we derive a novel exact closed-form representation for the corresponding BER which is validated through extensive comparisons with results from Monte-Carlo simulations. The proposed expression involved well known elementary and special functions which render its computational realization rather simple and straightforward. As a result, the need for laborious, energy exhaustive and time-consuming computer simulations can be ultimately omitted. Numerous results illustrate that the performance of underlay relay cognitive networks is, as expected, significantly degraded by channel estimation errors and that is highly dependent upon both the network topology and the number of hops.

Index Terms—Multi-hop communication, channel estimation error, underlay cognitive radio.

I. INTRODUCTION

It was recently pointed out by a spectrum usage survey from the Federal Communications Commission (FCC), that the current licensed spectrum situation is significantly under-utilized \(^1\). Contrary to that, the current availability of spectrum resources for most emerging wireless applications such as video calling, online high-definition video streaming, high-speed Internet access through mobile devices, etc. are particularly scarce. In an attempt to improve the spectrum utilization in wireless communication systems, cognitive radio (CR) technology was proposed as a promising technology \(^2\)–\(^8\). In cognitive radio, secondary users-SUs (or unlicensed users) are generally allowed to use the licensed band primarily allotted to primary users-PUs (or licensed users), unless their operation interferes with the established communication of PUs. This operation can be realized in three distinctive modes: underlay, overlay and interweave \(^9\). In the underlay mode, SUs are allowed to use the spectrum when the interference caused by SUs on PUs is within a tolerated range by PUs. This mode is more preferable than its two counterparts thanks to its low implementation complexity \(^10\).

Due to the interference power constraint imposed on SUs operating in the underlay mode, their transmit power is limited and as such, their transmission range is reduced substantially. To overcome this constraint, SUs can apply relaying techniques, which take advantage of shorter range communication that results to lower path loss effects. Among various relaying techniques, decode-and-forward (DF) and amplify-and-forward (AF) deployments have been extensively investigated \(^11\). In DF, each relay decodes information from the source, re-encodes it, and forwards it to the destination. In AF, each relay simply amplifies the received signal and forwards it to the destination. Due to its capability of regenerating noise-free relayed signals, DF is employed in this paper.

This paper investigates underlay DF multi-hop cognitive networks with arbitrary number of hops. Most relevant works considering such network deployments focus in outage probability analysis \(^9\), \(^12\)–\(^17\), and BER analysis\(^1\)\(^13\)–\(^20\) assuming perfect channel estimation and two-hop communication. It is also recalled here that channel state information (CSI) is essential for coherent detection; nevertheless, existing channel estimators are unable to provide and guarantee perfect CSI. As a consequence, the impact of imperfect CSI on the system performance should be considered realistically.

In \(^21\), the BER analysis for single-hop cognitive networks is presented under the assumption of imperfect CSI only for SU-PU links. In \(^22\), an exact outage probability expression was proposed for AF dual-hop cognitive networks. However, to the best of our knowledge, the exact BER analysis for underlay DF N-hop cognitive networks, with N being arbitrary integer, and imperfect CSI on all wireless channels, has not been addressed in the open technical literature. Motivated by this, this paper is devoted to an analytic investigation of this topic by deriving a corresponding exact closed-form BER expression. The derived expression is validated by extensive computer simulations and is utilized in evaluating the corresponding system performance.

\(^1\) The work in \(^20\) derives an approximate closed-form BER expression.
The structure of this paper is as follows: The next section presents the system model and the CSI imperfection model. The BER analysis is discussed in Section IV while simulated and analytical results are presented in Section V for derivation validity and performance evaluation. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

The underlay cognitive DF multi-hop network model under consideration is depicted in Fig. 1, where $N - 1$ secondary relays (SRs) numbered from 1 to $N - 1$ assist the transmission of the secondary source (SS) 0 to the secondary destination (SD) $N$. The SS and SRs use the same spectrum as a primary user $P$. The direct communication between SS and SD is bypassed, which is considered reasonable in scenarios where SS and SD are too far apart or their communication link is blocked due to severe shadowing and fading. We assume that the channel between any pair of transmitter and receiver experiences independent block frequency-flat Rayleigh fading i.e., frequency-flat fading is invariant during one phase but independently changed from one to another. Therefore, the channel coefficient between the transmitter $t \in \{0, 1, \ldots, N - 1\}$ and the receiver $r \in \{1, 2, \ldots, N, P\}$ is $h_{tr} \sim \mathcal{CN}(0, \eta_{tr} = d_{tr}^{-\alpha})$ where $d_{tr}$ is the distance between the two terminals and $\alpha$ is the path-loss exponent [23].

![System model](image)

An $N$-hop communication time interval consists of $N$ phases. In the first phase, SS 0 transmits a sequence of $K$ modulated symbols $x_0 = [x_0(1), x_0(2), \ldots, x_0(K)]$ with the symbol energy, $B_0$ i.e., $E[|x_0(k)|^2] = B_0$ where $E[\cdot]$ denotes the expectation and $k$ is the time index. SR 1 demodulates the received signal from SS 0 and re-modulates the demodulated symbol as $x_1 = [x_1(1), x_1(2), \ldots, x_1(K)]$ with the symbol energy, $B_1$, before forwarding to SR 2 in the second phase. The process continues until the signal reaches SD $N$. Without the notation confusion, the time index is omitted in the sequel and hence, the received signal through the hop $r$ can be expressed as

$$y_{tr} = h_{tr}x_t + n_{tr}, \quad (1)$$

where $y_{tr}$ denotes a signal received at the node $r$ from the node $t = r - 1$ and $n_{tr} \sim \mathcal{CN}(0, N_0)$ is additive white Gaussian noise at the node $r$.

In the underlay relay cognitive networks (e.g., [16], [24]), the SU $t$’s transmit power is limited such that the interference imposed on PU is under control. Without CSI errors, this interference constraint can be addressed as $B_t \leq I_T/|h_{tp}|^2$ where $I_T$ is the maximum interference level that PU still operates reliably. For the maximum transmission range, $B_t = I_T/|h_{tp}|^2$ is set. Following [25]–[28], we choose the CSI imperfection model as

$$h_{tr} = \hat{h}_{tr} + \varepsilon_{tr}, \quad (2)$$

where $\hat{h}_{tr}$ is the estimate of the $t-r$ channel and $\varepsilon_{tr}$ is the CSI error.

We assume that $h_{tr}$ and $\hat{h}_{tr}$ are jointly ergodic and stationary Gaussian processes. Therefore, $\varepsilon_{tr} \sim \mathcal{CN}(0, \sigma_{tr})$ and $\hat{h}_{tr} \sim \mathcal{CN}(0, \eta_{tr} = \eta_{t-r} - \sigma_{tr})$ with $\sigma_{tr}$ representing the quality of the channel estimator. For example [25], for the linear-minimum-mean-square-error (LMMSE) estimator,

$$\sigma_{tr} = E\left\{|h_{tr}|^2\right\} - E\left\{\hat{h}_{tr}^2\right\} = 1/(L_{p}\bar{\gamma}_{t,r,\text{training}} + 1)$$

where $L_{p}$ is the number of pilot symbols, $\bar{\gamma}_{t,r,\text{training}} = E\{/\gamma_{t,r,\text{training}}\gamma_{r/N_0}\}$ is the average SNR of pilot symbols for the $t-r$ channel, and $B_{t,\text{training}}$ is the pilot power.

III. ERROR PROBABILITY ANALYSIS

Due to CSI errors, the transmit power of the node $t$ is modified as $B_{t}' = I_T/|\hat{h}_{tp}|^2$. Then, there are two possibilities: $|h_{tp}|^2 \leq |\hat{h}_{tp}|^2$ and $|h_{tp}|^2 > |\hat{h}_{tp}|^2$. Setting the transmit power as $B_{t}' = I_T/|\hat{h}_{tp}|^2$ meets the interference power constraint for $|h_{tp}|^2 \leq |\hat{h}_{tp}|^2$, since this case results in the interference power as $B_{t}'|h_{tp}|^2 = I_T|\hat{h}_{tp}|^2/|\hat{h}_{tp}|^2 \leq I_T$, but not for $|h_{tp}|^2 > |\hat{h}_{tp}|^2$, since this case results in the interference power as $B_{t}'|h_{tp}|^2 = I_T|\hat{h}_{tp}|^2/|\hat{h}_{tp}|^2 > I_T$. Given that $E\left\{|\hat{h}_{tr}|^2\right\} \leq E\left\{|h_{tr}|^2\right\}$, where the equality holds for no CSI errors, on average such transmit power setting may not meet the interference power constraint i.e., the interference at P is greater than $I_T$. Therefore, the primary system performance may be severely degraded if the channel estimator is not efficient. Consequently, in order to propose solutions to interference reduction on primary systems, statistics of interference at the PU receiver should be analyzed. The most important statistics is the probability that the interference exceeds $I_T$, namely the interference probability $P_I$ as used in [22]. It is noted that $P_I$ is derived for underlay AF dual-hop cognitive networks [24] and for underlay single-hop cognitive networks [21] with the CSI imperfection model slightly different. The CSI imperfection model in [21] and [24] is $h_{tr} = \rho_{tr}h_{tr} + \sqrt{1 - \rho_{tr}^2}\varepsilon_{tr}$, where $\rho_{tr}$ is the correlation coefficient between $h_{tr}$ and $\varepsilon_{tr}$.
to the space limitation, the interference probability analysis is deferred to the journal version of this paper. Instead, we focus on the BER analysis for underlay relay cognitive networks. To this effect, using the CSI imperfection model in 2, we rewrite (1) as,

\begin{equation}
\gamma_{tr} = \frac{2^{\gamma_{tr}}}{\sigma_{tr} + |h_{tr}|^2} \mu.
\end{equation}

where \( z_{tr} = |h_{tr}|^2, d_{tr} = \sigma_{tr} + |h_{tr}|^2/\mu, \) and \( \mu = I_T/N_0. \)

The average BER at the node \( r \) for square \( M-QAM \) with \( M = 2^q \) (q even) and rectangular \( M-QAM \) with \( M = 2^q \) (q odd) modulation schemes is expressed in (5) which is cited from [29, eq. (16)] and [29, eq. (22)], correspondingly. In (5), we define

\begin{align*}
g &= \frac{3}{(M-1)}, \\
u &= \frac{(I^2 + J^2 - 2)}{6}, \\
I &= 2\{q(1)/2\}, \\
J &= 2\{q(1)/2\},
\end{align*}

and \( \psi(s, v, M; \gamma) \) in (10) in which \( Q(.) \) is the Q-function eq. (1), [33] eq. (10).

Next, we derive \( f_\gamma(x) \) in order to enable the derivation of an explicit expression for (5). Since \( \hat{h}_{tr} \sim \mathcal{CN} \left( \frac{1}{\sqrt{\kappa_{tr}}}, 0 \right) \) and \( \hat{h}_{tr} \sim \mathcal{CN} \left( \frac{1}{\sqrt{\kappa_{tr}}}, 0 \right) \), the probability density functions (pdf’s) of \( z_{tr} \) and \( d_{tr} \) are \( f_{z_{tr}}(x) = \lambda_{tr}e^{-\lambda_{tr}x} \) and \( f_{d_{tr}}(x) = \lambda_{tr}e^{-\lambda_{tr}x+\mu_{tr}} \), respectively. As a result, the pdf of \( \gamma_{tr} \) is expressed in (5) which is cited from [32, eq. (6-60)]

\begin{align}
f_{\gamma_{tr}}(x) &= \int_{0}^{\infty} y f_{z_{tr}}(yx) f_{d_{tr}}(y) dy \\
&= \frac{\kappa_{tr} e^{\lambda_{d_{tr}}}}{(x + \kappa_{tr})^2}, \quad (11)
\end{align}

where \( \kappa_{tr} = \lambda_{tr}/\lambda_{tr}. \)

Inserting (11) into (5) yields,

\begin{equation}
R_e(r) = \begin{cases}
\theta (I, u, W_{tr}) + \theta (J, u, W_{tr}), & q \text{ odd} \\
2\theta \left( \sqrt{M}, g, W_{tr} \right), & q \text{ even}
\end{cases}
\end{equation}

where \( W_{tr} = \{ M, \kappa_{tr}, \mu, \lambda_{tr}, \sigma_{tr} \} \) is a set of parameters and \( \theta(s, v, W_{tr}) \) is defined in (13). Also, \( \zeta(\beta, a) \) in (13) is defined as

\begin{equation}
\zeta(\beta, a) = \int_{0}^{\infty} Q \left( \sqrt{\beta x} \right) \left( a + x \right)^2 \, dx.
\end{equation}

Applying the integration by parts, we obtain the closed-form of \( \zeta(\beta, a) \) as follows,

\begin{align}
\zeta(\beta, a) &= \frac{1}{2a} - \frac{\sqrt{\beta}}{2\sqrt{\pi}} \int_{0}^{\infty} \frac{e^{-\frac{\beta}{a}}}{(x + a)^2} \, dx \\
&= \frac{1}{2a} - \frac{\sqrt{\beta}}{2\sqrt{\pi}} \int_{a}^{\infty} \frac{e^{-\frac{\beta}{a}}}{y\sqrt{y-a}} \, dy \\
&= \frac{1}{2a} - \sqrt{\frac{\beta \pi}{2a}} \left( 1 - erf \left( \frac{\beta a}{2} \right) \right),
\end{align}

where \( erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} \, dt \) is the error function eq. (8.250.1) and the closed-form expression of the integral in the second equality is deduced with the aid of [37] eq. (3.363.2).

Given the set of the average BERs of all hops \( \{ R_e(1), \cdots, R_e(N) \} \), the exact closed-form average BER of the underlay DF multi-hop cognitive networks is expressed as [35] eq. (9)

\begin{equation}
R_e = \sum_{n=1}^{N} \sum_{j=n+1}^{N} \left( 1 - 2R_e(j) \right).
\end{equation}

IV. NUMERICAL RESULTS

For illustration purpose, we arbitrarily select user coordinates as shown in Fig. 2 P at \((0.7, 0.5)\), SS 0 at \((0, 0)\), SR 1 at \((0.6, 0.2)\), SR 2 at \((0.8, 0.3)\), SD 3 at \((1, 0)\). SS 0, SD 3, and P are always fixed and thus, for 2-hop case only SR 1 is considered. Also, the number on the line is the distance between two corresponding terminals. The network topology in Fig. 2 is applied to all following results.

We consider the path-loss exponent of \( \alpha = 3 \) and the CSI error variance of \( \sigma_{er} = 1/\{(\sqrt{P})_{\text{training}}/N_0 + 1) \}. \) The value of \( B_{t,\text{training}} \) is selected such that the average received power at P does not exceed \( I_T \) (i.e., \( B_{t,\text{training}} \leq I_T \)). As a result, for illustration purposes we select \( B_{t,\text{training}} = I_T/\eta_{tr} \).

The study of channel estimators is outside the scope of this paper. Therefore, the selection of \( B_{t,\text{training}} \) in this paper is just an example to demonstrate the effect of CSI imperfection on the BER of underlay relay cognitive networks.

4The average BER of other modulation schemes such as M-PSK can be derived in the same approach.
\[ R_e (r) = \begin{cases} \int_0^\infty \left\{ \psi (I, u, M; \gamma) + \psi (J, u, M; \gamma) \right\} f_{\gamma_T} (\gamma) \, d\gamma, & q \text{ odd} \\ 2 \int_0^\infty \psi \left( \sqrt{M}, g, M; \gamma \right) f_{\gamma_T} (\gamma) \, d\gamma, & q \text{ even} \end{cases} \]  
\[ (5) \]

\[ \psi (s, v, M; \gamma) \triangleq \frac{2}{s \log_2 M} \sum_{k=1}^{\log_2 s} \sum_{i=0}^{(1-2^{-k})s-1} (-1)^{\frac{-k-1}{2}} Q \left( \frac{(2i+1)^2 v}{2k-1 - \left[ \frac{k-1}{s} + \frac{1}{s} \right]} \right), \]  
\[ (10) \]

\[ \theta (s, v, W_{tr}) \triangleq \frac{2}{s \log_2 M} \sum_{k=1}^{\log_2 s} \sum_{i=0}^{(1-2^{-k})s-1} (-1)^{\frac{-k-1}{2}} \kappa_{tr}^{\mu} e^{\lambda_{trP}^{\mu} \sigma_{trT}} \zeta \left( \frac{(2i+1)^2 v, \kappa_{trP}^{\mu}}{2k-1 - \left[ \frac{k-1}{s} + \frac{1}{s} \right]} \right)^{-1}, \]  
\[ (13) \]

Figs. 3 and 4 compare simulated and numerical results for two typical modulation levels, namely, 2-QAM for odd \( q \) and 4-QAM for even \( q \), \( N = \{2, 3\} \), and different degrees of CSI availability - perfect CSI and imperfect CSI with \( L_p = 1 \). It is seen that analytical results are well matched with simulated ones, validating the derived expression. Additionally, the BER performance is improved with respect to the increase in \( I_T \). This is obvious since \( I_T \) imposes a constraint on the transmit power and the higher \( I_T \), the higher the transmit power, eventually enhancing communication reliability. Moreover, the BER performance is deteriorated with the lack of CSI.

Fig. 5 investigates the impact of the quality of the channel estimator on the BER. The quality of the channel estimator can be enhanced by increasing the number of pilot symbols \( L_p \) at the cost of the bandwidth loss due to increased overhead. The results are reasonable since the BER performance is improved with the increased \( L_p \). Furthermore, for the selected channel estimator model, the performance is saturated at \( L_p = 4 \).

Given the specific network topology in Fig. 2 the results in Figs. 3 and 4 illustrate that 3-hop communication is worst than 2-hop communication for any set \( \{L_p, \alpha, I_T, M\} \). This means that in underlay DF multi-hop cognitive networks the advantage of the 3-hop communication over 2-hop communication in terms of the path loss reduction, e.g., the distance from the last relay to the destination in the 3-hop case (SR 2) is smaller than that in the 2-hop case (SR 1), can not sometimes turn into the performance improvement. This is because the last relay in the 3-hop case is closer to the primary user than in the 2-hop case, causing higher interference. Thus, the last relay in the 3-hop case should utilize lower transmit...
power than in the 2-hop case for reducing the interference level to the primary user, leading to higher performance degradation. These results recommend that the relay selection in underlay DF multi-hop cognitive networks is crucial in enhancing the network performance. A good relay not only provides reliable communication to the destination but also causes less interference to the primary user. The problem of the relay selection will be considered in a future work.

V. CONCLUSION

This paper investigated analytically the BER performance of underlay DF multi-hop cognitive networks over Rayleigh fading channel in consideration of imperfect CSI. The derived expression was shown to have a convenient algebraic form which allows straightforward to timely evaluation of the corresponding performance. The proposed analytical results were supported and validated with results from computer simulations while various results demonstrated that the imperfect CSI affects significantly the BER of underlay DF multi-hop cognitive networks. In addition, it was shown that the BER performance is dependent upon both the number of hops and the network topology.

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