A different way to read off the time –
A new idea for a binary clock

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Abstract

A new idea for a binary clock is presented. It displays the time using a triangular array of 15 bits. It is shown that such a geometric, triangular arrangement is only possible because our system of time divisions is based on a sexagesimal system in which the number of minutes in 12 hours equals the factorial of a natural number (720=6!).

1 Introduction

There are many ways to display the time. For example the familiar analog display with a dial and clock hands or digital displays using numerals. In addition there are binary displays which are a bit more difficult to read. A well known example is the so called Berlin clock (also called “set theory” clock), invented by Dieter Binninger and first installed in 1975 on the Kurfürstendamm in Berlin. The display of this clock consists of bits realized by lamps (or LEDs in a table version) which correspond to a certain amount of time. Lamps in the same row correspond to the same amount. For the Berlin clock, every lamp in the top row corresponds to 5 hours. The corresponding values in the second row is 1 h. In the third and fourth row the lamps represent 5 and 1 minute(s), respectively. For a better readability, every third lamp in the third row has a different color. The time is then just given by adding the amounts of all lamps lit. Fig. 1 shows the Berlin clock. The time displayed is 10:31.

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2 The new idea: A triangular clock

A new, more esthetic and geometric, form of display will be presented now. Here the lamps or LEDs are arranged in form of a triangle (see Fig. 2). Again, lamps in the same row correspond to the same amount of time. The corresponding values are listed in the following table.

| row | value       |
|-----|-------------|
| 1   | 6 hours     |
| 2   | 2 hours     |
| 3   | 30 minutes  |
| 4   | 6 minutes   |
| 5   | 1 minute    |

For the time displayed in Fig. 2 one finds:

\[
\begin{align*}
0 \times 6 \text{ h} & = 0h \\
+2 \times 2 \text{ h} & = 4h \\
+1 \times 30 \text{ min} & = 30 \text{ min} \\
+3 \times 6 \text{ min} & = 18 \text{ min} \\
+1 \times 1 \text{ min} & = 1 \text{ min}
\end{align*}
\]

\[
4h \quad 49 \text{ min}
\]
If all lamps are on, the time displayed is

\[6 \times 2 \times 2 \times 3 \times 30 \text{ min} + 4 \times 6 \text{ min} + 5 \times 1 \text{ min} = 11 \text{ h}59 \text{ min}\]

which means that this triangular form with 5 rows of lamps is perfectly suited for a 12 hour display. Using two different colors, one can achieve a 24h display; for example green for \textit{am} and red for \textit{pm}. Fig. 3 shows more examples. A Java applet of this clock can be found on [http://joerg.pretz.de](http://joerg.pretz.de).

### 3 Mathematical Background

We now come to the question why our system of time divisions allows such a triangular display? First let us compare the Berlin and the triangular clock. Common to both concepts is the fact that the amount of time a certain lamp corresponds to in the \((n - 1)\)th row equals \((m_n + 1)\) times the amount in the \(n\)th row where \(m_n\) is the number of lamps in the \(n\)th row. Here are two
examples to illustrate this:

- Berlin clock: 3rd row has $m_3 = 11$ lamps representing 5 minutes each. Thus for the 2nd row one finds $(11 + 1) \times 5 \text{ min} = 1 \text{ h}$.
- Triangular clock: 3rd row has $m_3 = 3$ lamps representing 30 minutes each. Thus for the 2nd row one finds $(3 + 1) \times 30 \text{ min} = 2 \text{ h}$.

The main difference is that for the triangular clock one has $m_n = n$, i.e. in the $n$th row there are also $n$ lamps. Only this allows the geometric, triangular arrangement of the lamps.

But what does $m_n = n$ mean? A row with $n$ lamps allows to display $n + 1$ states, from all lamps off to all lamps on. (Note that in the way the display is used here, not all of the $2^n$ possible states are used because if a lamp is on, also the lamps left to it are on in the same row.) For a triangular display with $n$ lamps in the bottom row and the number of lamps decreasing by one in every following row the total number of states is thus

$$(n + 1) \times ((n - 1) + 1) \times ((n - 2) + 1) \times \ldots \times (1 + 1) = (n + 1)!.$$

Now, note that our system of time measurement is based on divisions which have many factors, e.g. $12 = 3 \times 4$ and $60 = 1 \times 2 \times 5 \times 6$. For a 12 hour display with a precision of one minute the number of states one has to display is thus

$$12 \times 60 \text{ minutes} = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \text{ minutes} = 6! \text{ minutes}$$

which perfectly fits in a triangular display with five rows. Thus the whole concept works because our system of time divisions is based on a sexagesimal system dating back to the Babylonian [1] rather than a decimal system.

Fortunately an idea of the French Revolution to divide the day in 10 hours of 100 minutes of 100 seconds didn’t prevail [2]. With such a division this triangular display would not be possible.

References

[1] Otto Neugebauer, The Exact Sciences in Antiquity, Barnes & Noble, New York, 1993

[2] Richard A. Carrigan, Jr., Decimal Time, American Scientist, 66, 305 (78).