Comparison of Improved and Unimproved Quenched Hadron Spectroscopy

A. Cucchieri\textsuperscript{a}, T. Mendes\textsuperscript{a}, R. Petronzio\textsuperscript{a,b}

\textsuperscript{a}Dipartimento di Fisica, Universit\`a di Roma “Tor Vergata”
and INFN, Sezione di Roma II
Via della Ricerca Scientifica 1, 00133 Rome, Italy
\textsuperscript{b}CERN, Theory Division, CH-1211 Geneva 23, Switzerland

Abstract

We make a comparison between our quenched-hadron-spectroscopy results for the non-perturbatively-improved Wilson action and the corresponding unimproved case, at $\beta = 6.2$ on the same set of gauge configurations. Within our statistics, we find a sizeable improvement for the baryon spectrum and for the determination of the strange-quark mass.
The computation cost of the extrapolation to the continuum limit of lattice QCD simulations can be significantly reduced by using improved actions, where the leading cutoff effects are cancelled by suitable counterterms. It has been shown that on-shell improvement of $O(a)$ is achieved by adding to the usual Wilson action the clover term, with a coefficient that has been determined non-perturbatively by the ALPHA collaboration [1].

We have studied the influence of considering this non-perturbatively-improved Wilson action with respect to the usual, unimproved, action. A detailed analysis of our results for the improved case has appeared elsewhere [2], and here we concentrate on the comparison between the two cases (a preliminary study can be found in [3]). We refer to [1] for the description of the improvement programme.

We consider a lattice of volume $24^3 \times 48$ and coupling $\beta = 6.2$. We choose the following values for the hopping parameter $\kappa$ in the unimproved case: 0.1350, 0.1400, 0.1450, 0.1506, 0.1510, 0.1517, 0.1526 (for the improved case we used: 0.1240, 0.1275, 0.1310, 0.1340, 0.1345, 0.1350, 0.1352). We consider, for the improved as well as for the unimproved case, all nondegenerate flavor combinations from the different values of $\kappa$. Our simulations were carried out on the 512-processor computer of the APE100 series at the University of Rome “Tor Vergata”.

Our statistics come from 104 quenched gauge configurations, generated by a hybrid over-relaxation algorithm, with each update corresponding to a heat-bath sweep followed by three over-relaxation sweeps. The configurations are separated by 1000 updates. The numerical inversion of the propagator for the improved case is described in [2]. For the unimproved case we have performed the inversion in a similar manner, but for the last part of our configurations we have implemented the SSOR algorithm [4], which corresponds roughly to a gain of a factor 4 in the inversion time.

The analysis of the data for the improved case is described in [2]. For the unimproved case, we have followed the same procedure, namely hadron masses are obtained from single-mass fits to the large time behaviour of zero-momentum hadron correlators, and the errors are estimated through a single-elimination jack-knife procedure.

We remark that we determine $\kappa_c$ for both cases by using the so-called Ward identity mass $m_{WI}$, considered in [2] (in the unimproved case we set the coefficient $c_A$ to zero). We thus obtain for the unimproved case the value

$$\kappa_c = 0.153230(9) \quad (\text{from } m_{WI} = 0).$$

(1)

This may be compared with the value $\kappa_c = 0.153291(15)$, obtained from $M_{PS}^2 = 0$.

For the fits to the dependence of hadron masses upon quark masses we use in the unimproved case the bare quark mass, defined as $m_q(\kappa) \equiv (1/\kappa - 1/\kappa_c)/2$, while for the improved case we use an improved bare quark mass [1] defined by

$$\bar{m}_q(\kappa) \equiv m_q(\kappa) \left[ 1 + b_m \, m_q(\kappa) \right].$$

(2)

(Note that $\bar{m}_q$ is the renormalized mass with $Z_m = 1$.) The improvement coefficient $b_m$ has been determined non-perturbatively [5] to be $b_m = -0.62(3)$. For non-degenerate-flavour cases, we use symmetric averages of the masses defined above.
We use physical inputs at the strange quark mass for the lattice spacing. This avoids the inclusion of systematic uncertainties deriving from a chiral extrapolation. The value of the strange quark mass in lattice units is obtained by using a fit to the ratio $M_{PS}^2/M_V^2$ at the experimental value $(M_K/M_{K^*})^2$. We get for the unimproved case the value:

$$m_s = 0.0316(47),$$

(3)

to be compared with the improved case \[\text{[3]}\]: $m_s = 0.0315(45)$.

The inverse lattice spacing $a^{-1}$ in MeV is then obtained by normalizing the $K^*$ mass to its experimental value. We get:

$$a_{K^*}^{-1} = \frac{M_{K^*}}{M_V(m_s/2)} = 2943(100) \text{ MeV},$$

(4)

to be compared with the result obtained in the improved case with the same definition \[\text{[3]}\]: $a_{K^*}^{-1} = 2561(100)$. [The values for $a^{-1}$ coming from $M_\rho$ (the chiral value) and $M_\phi$ are obtained similarly, by linear extrapolation/interpolation.]

We show in Table 1 the values of the various ratios of different determinations of $a^{-1}$ and their jack-knife errors. The difference is between the unimproved value and the improved case. The jack-knife errors of the differences were computed as follows: we consider these differences for each cluster of configurations (obtained by single elimination), and we then evaluate their jack-knife errors. A distinctive difference between the two cases cannot be observed within our statistics, but for the ratio $r_1$, whose deviation from unity in the improved case is more significant than in the unimproved one.

|       | unimproved | improved | difference |
|-------|------------|----------|------------|
| $r_1 \equiv a_{K^*}^{-1}/a_\rho^{-1}$ | 1.037(23) | 1.030(33) | 0.007(27)  |
| $r_2 \equiv a_\phi^{-1}/a_{K^*}^{-1}$ | 1.030(31) | 1.029(27) | 0.0008(270) |
| $r_3 \equiv a_\phi^{-1}/a_\rho^{-1}$ | 1.067(56) | 1.060(62) | 0.008(54)  |

Table 1: Ratios of different determinations of the inverse lattice spacing $a^{-1}$

In Fig. 1 we show a plot of the vector-meson mass as a function of $M_{PS}^2$ for the two cases. The masses are normalized to the $K^*$-meson mass, taken at the strange-quark mass value given in eq. (3) above. The asterisks correspond to the experimental values, i.e. the vector-meson masses $M_\rho$, $M_{K^*}$, $M_\phi$, and appropriate combinations of pseudoscalar-meson masses. The errors in the plot are calculated from independent jack-knife procedures in the two cases. The difference between improved and unimproved simulations can be made statistically significant only if a jack-knife procedure is applied to the parameters of a fit. Indeed, the values of kappa where the inversions were taken did correspond to different quark masses and a direct jack-knife procedure for the hadron masses is not possible.
Figure 1: APE plot for $M_V$. The improved case (✸) is compared with the unimproved one (□).

As can be seen from Fig. 1, the experimental slope $dM_V/dM_{PS}^2$ is different from the one corresponding to our data (this happens for the improved as well as for the unimproved case). This slope is related to the behaviour of the dimensionless quantity $J$ defined by:

$$J \equiv M_V (dM_V/dM_{PS}^2)$$

obtained from $M_V$ as a function of $M_{PS}^2$. A comparison of the $J$ values for the two cases is made in Table 2. Again, the difference was computed for each cluster, and only then was its jack-knife error evaluated. We see that the improvement on $J$ is marginal, which seems to indicate that the discrepancy between the lattice data and the experimental values should come from the quenching approximation.

|       | exp. | unimproved | improved | difference  |
|-------|------|------------|----------|-------------|
| $J_{K^*}$ | 0.487 | 0.382(48) | 0.391(66) | -0.0081(525) |
| $J_{\phi}$ | 0.557 | 0.409(44) | 0.427(54) | -0.019(44)  |

Table 2: Comparison of values of the quantity $J$

We also consider the charm-quark mass $m_c$, which we obtain from a fit to the ratio $M_{PS}^2/M_{K^*}^2$ (the experimental input $M_{D}^2/M_{K^*}^2$ is used). We show in Table 3 a comparison
of the ratio \( m_c/m_s \) between the improved and the unimproved case. Note that this is also the ratio of renormalized quark masses, since in the ratio of charm over strange quark mass the renormalization factor \( Z_m \) drops out.

\[
\begin{array}{|c|c|c|}
\hline
\text{unimproved} & \text{improved} & \text{difference} \\
\hline
m_c/m_s & 12.58(190) & 10.11(140) & 2.47(140) \\
\hline
\end{array}
\]

Table 3: Our lattice data for the ratio of charm-quark over strange-quark masses

The value of the lattice strange-quark mass given in eq. (3) and the ratio \( m_c/m_s \) in Table 3 can be used to obtain two independent determinations of the physical mass of the strange quark, as explained in [2]. The two resulting values are expected to agree, provided that they are evolved through the renormalization-group equation to the same scale. We use the conventional scale of 2 GeV. This gives, in the unimproved case:

\[
m_s^{(1)} = 115(13) \text{ MeV} \quad \text{from } m_s \\
m_s^{(2)} = 87(13) \text{ MeV} \quad \text{from } m_c/m_s
\]

while for the improved case (using the improved bare quark mass \( \tilde{m}_q \)) we obtained [2]:

\[
m_s^{(1)} = 111(15) \text{ MeV} \quad \text{and} \quad m_s^{(2)} = 111(16) \text{ MeV}.
\]

In order to compare the improved and unimproved cases, we consider the dimensionless ratio \( m_s^{(1)}/m_s^{(2)} \). Our results are shown in Table 4. We see a clear improvement here. This comes from two different sources: the variation of the lattice spacing (which influences the determination of \( m_s^{(1)} \)), and the use of the improved bare quark mass (which mainly affects the determination of \( m_s^{(2)} \)).

\[
\begin{array}{|c|c|c|}
\hline
\text{unimproved} & \text{improved} & \text{difference} \\
\hline
m_s^{(1)}/m_s^{(2)} & 1.315(56) & 1.005(49) & 0.310(48) \\
\hline
\end{array}
\]

Table 4: The ratio of the two strange-quark mass determinations

We now turn to the comparison of baryonic observables. We consider non-degenerate flavour combinations, and therefore we can draw APE plots with several values of the hadron mass. Plots are shown in Fig. 2 for the nucleon (or more precisely the \( \Sigma \)-like octet baryons) and for the \( \Delta \) (decuplet baryons). We refer to [2] for details on how our baryon masses are obtained. Again, masses are normalized to the \( K^* \) meson mass. Experimental points in these figures (asterisks) correspond to \( M_N, M_{\Sigma}, \) and \( M_{\Xi} \), and the use of the improved bare quark mass (which mainly affects the determination of \( m_s^{(2)} \)).
Figure 2: APE plot for (a) the nucleon mass (octet baryons) and (b) the Δ mass (decuplet baryons). The improved case (◇) is compared with the unimproved one (□).
In Table 5 we give some of our baryon mass values in MeV (for convenience, we also include the improved values already reported in [2]). The difference between unimproved and improved results can be taken for each cluster as was done before; we consider dimensionless ratios formed by the baryon mass divided by $M_{K^*}$. The results are shown in Table 5. The improved results are in much better agreement with the experimental points.

|       | exp  | improved | unimproved |
|-------|------|----------|------------|
| $M_N$ | 939  | 952(110) | 1054(90)   |
| $M_{\Sigma^{-}-\Lambda}$ | 73.7 | 70(30)   | 45(20)     |
| $M_{\Delta}$ | 1232 | 1265(110) | 1495(125) |
| $M_{\Delta^{-}-N}$ | 293  | 297(80)  | 336(55)    |

Table 5: Baryon masses in MeV (improved and unimproved) and comparison with experiment. The error in parentheses represents the statistical error, while the error due to the determination of the inverse lattice spacing is of about 4%.

|       | exp        | unimproved | improved | difference |
|-------|------------|------------|----------|------------|
| $M_N/M_{K^*}$ | 1.05       | 1.18(11)   | 1.07(13) | 0.11(13)   |
| $M_{\Delta}/M_{K^*}$ | 1.38       | 1.68(17)   | 1.35(14) | 0.34(13)   |

Table 6: Jack-knife comparison between improved and unimproved values for the nucleon and the $\Delta$ baryons.

The use of a non-perturbatively-improved action leads to a slightly smaller spread in the value of the lattice spacing extracted from light mesons with and without strange quarks. Within our statistics, it does not improve dramatically the meson spectrum, while it affects considerably the baryons quark-mass dependence, especially for the decuplet, and moves it into a better agreement with the experimental data. We have also found a remarkable agreement in the improved theory with respect to the unimproved case between two independent determinations of the strange-quark mass, one normalized through the lattice spacing and the other from the value for the charm mass extracted in the continuum from charmonium spectrum calculations.

References

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