Adaptive Control of Four Motor Servo Systems Based on Characteristic Model and Gradient Projection Estimator

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ABSTRACT

For multi-motor servo system, its complex system model makes controller design difficult and hard to realize in practical situation. To solve this problem, an easily realized adaptive control scheme based on characteristic modeling method and gradient projection estimator is proposed in this article. Firstly, characteristic modeling method is applied into servo systems to obtain a simplified system model, which makes controller design much easier. Also, system’s uncertainties will be integrated into three time-varying characteristic parameters, which will be estimated online through gradient projection method. With this estimator, characteristic model can reflect real-time system status accurately but in a simple format. Then, based on characteristic model and parameter estimator, an adaptive controller is proposed to guarantee high track precision and enhance system robustness. The stability of the estimator’s updating law and adaptive control law is analyzed, and the effectiveness of the proposed control scheme is proved through simulation and experiments.

INDEX TERMS

Multi-motor servo systems, characteristic modeling, gradient projection estimator, adaptive control.

I. INTRODUCTION

Multi-motor servo system has been admitted as a promising technology in turntable fields owing to its reliability and high inertia-ratio [1], [2]. It is commonly used in systems like radar tracking or weapon aiming platforms, which all requires strong driving torque and high tracking precision [2]–[4]. Four-motor servo system has advantages in power supply and fault tolerant, which improves its reliability. However, its complex system model, high coupling and nonlinear features all present a huge challenge to controller design, which requires high precision, strong robustness, fast convergence rate and easy realization at the same time [5], [6]. Therefore, designing an easily realized adaptive controller becomes the key point to make multi-motor synchronization systems efficient and reliable.

Various control methods have been applied into multi-motor servo systems to acquire an ideal performance [7], [8]. Typically, like in paper [9], [10] and [11], [12], fuzzy systems or neural networks are chosen as an efficient way to enhance the adaptive capability and robustness of processor. Beyond that, in paper [13], researchers introduce a robust term into the tracking controller and the synchronization controller respectively to guarantee the H-infinity performances of tracking error and synchronization error. Besides, some researchers adopted the active disturbance compensation [14] and back-stepping control [15] to improve control precision. What need to be pointed is that most of these methods depend heavily on the complex system models, which brings inevitable complexity to controller. Meanwhile, the accurate parameters in dynamic models are hard to obtain in practical systems. In view of all these challenges, we develop the novel characteristic modeling theory into our multi-motor synchronization system to build a new system model with less structural or parametric information. By this way, the negative influence of uncertainties can be reduced and the controller could be simplified.

Characteristic modeling method is developed by Hongxin and Jun [16] to simplify the complex system models with all high-order system features remained. According to this...
theory, a high-order complex system could always be described by a low-order discrete time-varying system as long as system features and control objectives are confirmed. All the information contained in high-order system plants could be integrated into several time-varying characteristic parameters and finally make up the characteristic system model. Upon this method, many attempts have been made to deal with control problems of multi-motor systems [17]–[19]. In these researches, characteristic modeling offers a significantly simplified system model and helps avoid lots of measurement errors.

It is worth pointing out that the key issue in using characteristic modeling is to identify the characteristic parameters. In our case, three time-varying parameters contain main information and play a decisive role in controller design. Therefore, methods like genetic algorithm [20]–[22] and neural network based estimators [23]–[25] are applied to identify them online. However, the key parameters in these algorithms all need to be manually adjusted to fit different signals. Considering this factor, gradient projection method is finally chosen due to its full-automation and estimation performance.

The target of this article is to obtain an easily realized adaptive output feedback control scheme for multi-motor servo systems. Firstly, an accurate characteristic model of servo system is derived for the first time, which considers only input and output information. All the uncertainties and time-variations are integrated into three characteristic parameters. Secondly, based on the characteristic model, a gradient projection estimator is designed with estimation error as feedback input. This estimator quickly identifies time-varying characteristic parameters online, and the convergence is proved. Thirdly, an adaptive controller is designed, which is simple but easy to implement. Its effectiveness is also verified through simulation and experiment results.

Reminder of this article is organized as following: Section 2 presents the dynamic model and characteristic model of four-motor servo system. In section 3, a gradient projection method based estimator is designed to online estimate characteristic parameters. In section 4, an adaptive control scheme is finally built, and the stability is proved with Lyapunov theorem. Section 5 and section 6 show the simulation and experiment results respectively. Finally, conclusions are given in section 7.
several characteristic parameters in a two-order linear time-varying difference equation. By this way, system’s dynamic characters, environmental factors and control requirements are all concerned, rather than considering the dynamic model only. When given a same input, characteristic model could be treated equal to the subject within an acceptable error. Therefore, characteristic modeling method offers an opportunity to design a low-order controller for many complicated subjects and has been successfully applied into engineering programs.

The characteristic model theory is shown as below:

For a nonlinear system shown in equation (2):
\[
\dot{x} = f(x, u) = f(x, \dot{x}, \ldots, x^{(n)}, u, \dot{u}, \ldots, u^{(m)})
\]  
(2)

Define \(x_1 = x, x_2 = \dot{x}, \ldots, x_{n+1} = x^{(n)}\) and \(u_1 = u, u_2 = \dot{u}, \ldots, u_{n+1} = u^{(m)}\), then the nonlinear system could be transformed into:

\[
\dot{x}(t) = f(x_1, x_2, \ldots, x_{n+1}, u_1, u_2, \ldots, u_{n+1})
\]  
(3)

**Assumption 1:** [16] The nonlinear system (2) has the following qualities:

1. Single input single output;
2. \(u(t)\) is a first-order control input signal;
3. If all \(x_j\) and \(u_j\) equal to zero, then \(f(\bullet) = 0\);
4. \(f(\bullet)\) is continuously differentiable for all variables and the partial derivatives are bounded;
5. \(|f(x(t + t_s), u(t + t_s)) - f(x(t), u(t))| < M t_s\), where \(M\) is a positive constant and \(t_s\) is sample time;
6. All the variables \(x_j\) and \(u_j\) are bounded.

**Lemma 1:** [16] If a control system could be described with a same format as formula (3) and it meets the qualities (1-6) in Assumption 1, then the characteristic model of the system could be described by a second-order time-varying difference equation:

\[
x(k + 1) = f_1(k)x(k) + f_2(k)x(k - 1) + g_0(k)u(k) + g_1(k)u(k - 1)
\]  
(4)

And it has the following characters [16]:

1. With a same input, the output of characteristic model and practical objects are equivalent (maintained in an acceptable output error with a suitable sample time) in dynamic progress, and are equal in stable state.
2. \(f_1(k), f_2(k),\) and \(g_0(k)\) in equation (11) are slow time-varying and their ranges can be determined in advance.
3. Generally, in engineering application, If the controlled plant is a minimum-phase system or a weak non-minimum-phase system, the item \(g_1(k)u(k - 1)\) can be ignored.

**C. CHARACTERISTIC MODELING OF SYSTEM**

To simplify the complex dynamic model, the characteristic model is established in decoupled form. According to dynamic model (1), for each single motor:

\[
\begin{align*}
\dot{x}_j(t) &= w_j(t) \\
J_j \frac{d\dot{x}_j(t)}{dt} &= C_{Tj}i_{sagj}(t) - T_j(t) - T_cj/r_{cj} + d_{2j} \\
L_{sagj} \frac{di_{sagj}(t)}{dt} &= u_{sagj}(t) - R_{sagj}i_{sagj}(t) - C_{ej}w_j(t) + d_{1j}
\end{align*}
\]  
(5)

In order to obtain the characteristic model, we first differentiate \(\dot{\theta}_j\) to its second order. And combining with the dynamic form 5,

\[
\ddot{\theta}_j = \alpha_j\dot{\theta}_j + \beta_ju_{sajj} + \gamma_j
\]  
(6)

where

\[
\begin{align*}
\alpha_j &= \frac{C_{Tj}C_{ej}}{J_jR_{cj}} \\
\beta_j &= \frac{C_{Tj}}{J_jR_{cj}} \\
\gamma_j &= \frac{C_{Tj}d_{1j} - C_{Tj}L_{sagj}i_{sajj}'}{J_jR_{cj}} + d_{2j}r_{cj} - T_jr_{cj} - T_{cj}
\end{align*}
\]  
(7)

In the following, we derive the system model (6) in discrete-time form. Denoting the sampling period of the system as \(T\) and applying Taylor series expansion on \(\theta_j((k + 1)T)\) and \(\theta_j((k - 1)T)\) at the time of \(kT\) yield:

\[
\begin{align*}
\theta_j((k + 1)T) &= \theta_j(kT) + \dot{\theta}_j(kT)T + 0.5\ddot{\theta}_j(kT)T^2 \\
&+ \frac{\dddot{\theta}_j(kT + \tau_1)}{3!}T^3 \\
\theta_j((k - 1)T) &= \theta_j(kT) - \dot{\theta}_j(kT)T + 0.5\ddot{\theta}_j(kT)T^2 \\
&+ \frac{\dddot{\theta}_j(kT - \tau_2)}{3!}T^3
\end{align*}
\]  
(8)

where \(0 < \tau_1, \tau_2 < T\).

Letting \(\dot{\theta}_j(k)\) denote \(\dot{\theta}_j(kT)\), and substituting (6) into (9), we have

\[
\dot{\theta}_j(k) = \frac{1}{T - 0.5\alpha_jT^2}[\theta_j(k) - \theta_j(k - 1) + 0.5\beta_ju_{sajj}(k)T^2 + 0.5\gamma_j(k)T^2 + \frac{\dddot{\theta}_j(kT - \tau_2)}{3!}T^3]
\]  
(10)

Subtracting (9) from (8), and substituting (10) into the result, we can get

\[
\begin{align*}
\theta_j(k + 1) &= \frac{2}{1 - 0.5\alpha_jT}\theta_j(k) - \frac{1}{1 - 0.5\alpha_jT}\theta_j(k - 1) \\
+ \frac{\beta_jT^2}{1 - 0.5\alpha_jT}\theta_j(k - 1) \\
+ \frac{\beta_jT^2}{1 - 0.5\alpha_jT}u_{sajj}(k) + \Delta_{cj}
\end{align*}
\]  
(11)

where

\[
\begin{align*}
\Delta_{cj} &= \frac{\gamma_j(k)T^2}{1 - 0.5\alpha_jT} + \frac{\dddot{\theta}_j(kT + \tau_1)}{3!}T^3 \\
&+ \frac{0.5\alpha_jT\dddot{\theta}_j(kT - \tau_2)T^3}{3!(1 - 0.5\alpha_jT)}
\end{align*}
\]  
(12)

Thus, we can use the following characteristic model to describe each single motor:

\[
\theta_j(k + 1) = f_1(k)\theta_{cj}(k) + f_2(k)\theta_{cj}(k - 1) + g_0(k)u_{sajj}(k) + \Delta_{cj}
\]  
(13)

When it comes to four motor synchronization systems, four motors could be treated with same input and output but with small errors. These small errors could also be compensated.
by the coefficients of $\theta(k), \theta(k-1)$ and $u(k)$. Thus, the characteristic model could be obtained as:

$$\theta(k + 1) = f_1\theta(k) + f_2\theta(k - 1) + g_0u(k)$$  (14)

where three characteristic parameters $f_1, f_2$ and $g_0$ are related with the sampling time and the known range of system states, and will be estimated online by an improved gradient projection method estimator designed in section 3.

### III. DESIGN OF GRADIENT PROJECTION METHOD BASED ESTIMATOR

According to characteristic model (14), the error model of system can also be described as:

$$e(k + 1) = \Psi(k)\Phi(k)$$

$$= f_1e(k) + f_2e(k - 1) + g_0u(k)$$

$$\Psi(k) = [e(k) \quad e(k - 1) \quad u(k)]$$

$$\Phi(k) = [f_1(k) \quad f_2(k) \quad g_0(k)]^T$$  (15)

where $e(k) = \Psi(k) - \Psi^*(k), \Psi^*(k)$ is target position.

Considering formula (11)-(14), it is obvious that three parameters are bounded and they are also combination of $e(k)$ and will be estimated online by an improved gradient projection method designed in section 3.

Define $\hat{\Phi}_e(k) = \hat{\Phi}_e(k) - \Phi_e$. According to projection theorem, combining formula (18), we have

$$\|\hat{\Phi}_e(k)\|^2 \leq \|\hat{\Phi}_e(k)\|^2 + \frac{\lambda_1\Psi(k-1)}{\lambda_2 + \Psi^T(k-1)\Psi(k-1)} e_{ma}(k)$$

$$\|\hat{\Phi}_e(k)\|^2 \leq \|\hat{\Phi}_e(k)\|^2 + \frac{2\lambda_1 e_{ma}(k)\Psi(k-1)\hat{\Phi}_e(k-1)}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)}$$

$$\|\hat{\Phi}_e(k)\|^2 \leq \|\hat{\Phi}_e(k)\|^2 + \frac{\lambda_2^2 e_{ma}(k)\Psi(k-1)\Psi^T(k-1)}{(\lambda_2 + \Psi(k-1)\Psi^T(k-1))^2}$$

Choose Lyapunov function

$$V_1(k) = \hat{\Phi}_e^T(k)\hat{\Phi}_e(k)$$

Substituting (21) into the first difference of $V_1(k)$, we have

$$\Delta V_1(k) = V_1(k) - V_1(k - 1)$$

$$\leq - \frac{2\lambda_1 e_{ma}(k)\Psi(k-1)\hat{\Phi}_e(k-1)}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)}$$

$$+ \lambda_2^2 e_{ma}(k)\Psi(k-1)\hat{\Phi}_e(k-1)$$

$$= \frac{\lambda_1^2 e_{ma}(k)\Psi(k-1)\Psi^T(k-1)}{(\lambda_2 + \Psi(k-1)\Psi^T(k-1))^2}$$

Also, combining (17) and (18), we have

$$e_{ma}(k) = e_{ma}(k) - \lambda_3 \frac{e_{ma}(k)}{\lambda_3}$$

$$\text{sat}\left(\frac{e_{ma}(k)}{\lambda_3}\right) = \begin{cases} 1, & e_{ma}(k) > \lambda_3 \\ \frac{e_{ma}(k)}{\lambda_4}, & |e_{ma}(k)| \leq \lambda_3 \\ -1, & e_{ma}(k) < \lambda_3 \end{cases}$$

$$\lambda_1 > 0; \quad \lambda_3 > 0;$$

$$\lambda_1 > 0, \lambda_2, \lambda_3$$ are adjustable parameters need to be designed. $\pi_\mathcal{D}(\cdot)$ represents doing rectangular projection in compact set $\mathcal{D}$:

$$\mathcal{D} = \{(a_1, a_2, a_3)^T \in \mathbb{R}^3\}$$

$$a_1 \in [2 - TL - T^2L, \quad 2 + TL + T^2L];$$

$$a_2 \in [-1 - TL, \quad -1 + TL];$$

$$a_3 \in [g_1T^2, \quad g_2T^2]$$

(19)

where $T$ is sample time. $L, g_1, g_2$ are constants chosen according to system features [27].

Based on this estimator, three parameters will be identified online. And the estimation error is proved bounded in the following part:

**Theorem 1:** Considering parameter updating law (18), if choose parameters satisfy $\lambda_3 > |\Psi(k)\eta(k)|, 0 < \lambda_1 < 2, \lambda_2 > 0$, then the estimation error $e_{ma}(k)$ is bounded and it satisfies

$$\lim_{k \to \infty} \sup_{k} |e_{ma}(k)| \leq \lambda_3$$  (20)

**Proof:**

Firstly, define $\tilde{\Phi}_c(k) = \hat{\Phi}_c(k) - \Phi_c$. According to projection theorem, combining formula (18), we have

$$\|\tilde{\Phi}_c(k)\|^2 \leq \|\Phi_c(k - 1)\|^2 + \frac{\lambda_1\Psi(k-1)}{\lambda_2 + \Psi^T(k-1)\Psi(k-1)} e_{ma}(k)$$

$$\|\tilde{\Phi}_c(k)\|^2 \leq \|\Phi_c(k - 1)\|^2 + \frac{2\lambda_1 e_{ma}(k)\Psi(k-1)\Phi_c(k-1)}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)}$$

$$\|\tilde{\Phi}_c(k)\|^2 \leq \|\Phi_c(k - 1)\|^2 + \frac{\lambda_2^2 e_{ma}(k)\Psi(k-1)\Psi^T(k-1)}{(\lambda_2 + \Psi(k-1)\Psi^T(k-1))^2}$$

(21)

Choose Lyapunov function

$$V_1(k) = \tilde{\Phi}_c^T(k)\tilde{\Phi}_c(k)$$

(22)
Then, substituting (24) into (23), we have
\[
\Delta V_1(k) \leq -\frac{1}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)}[(2\lambda_1 - \lambda_1^2)e_{ma}^2(k) + 2\lambda_1(\lambda_3 e_{ma}(k)) + \Psi(k)(\xi_1(k))e_{ma}(k))] \tag{25}
\]
Since \( \lambda_3 > |\Psi(k)(\xi_1)|, \lambda_1 > 0 \), we have
\[
\Delta V_1(k) \leq -\frac{(2\lambda_1 - \lambda_1^2)e_{ma}^2(k)}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)} \tag{26}
\]
Then, we can get
\[
V_1(k) = V_1(1) + \sum_{i=1}^{k} \Delta V_1(i) \leq V_1(1) - \sum_{i=1}^{k} \frac{(2\lambda_1 - \lambda_1^2)e_{ma}^2(k)}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)}
\]
\[
\Rightarrow \sum_{i=1}^{k} \frac{(2\lambda_1 - \lambda_1^2)}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)}e_{ma}^2(k) \leq V_1(1) - V_1(k) \leq \lim_{k \to \infty} \frac{(2\lambda_1 - \lambda_1^2)}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)}e_{ma}^2(k) = 0 \tag{27}
\]
Since \( 0 < \lambda_2 < 2 \), it is obvious that
\[
\frac{(2\lambda_1 - \lambda_1^2)}{\lambda_2 + \Psi(k-1)\Psi^T(k-1)} > 0 \tag{28}
\]
Also, in practical systems, \( |\Psi(k)| \) is bounded, we can get
\[
\lim_{k \to \infty} |e_{ma}(k)| = 0
\]
\[
\Rightarrow \lim_{k \to \infty} |e_m(k) - \lambda_3 \text{sat}(e_m(k)/\lambda_3)| = 0
\]
\[
\Rightarrow \lim_{k \to \infty} \text{sup} |e_m(k)| \leq \lambda_3 \tag{29}
\]
Thus, the estimation error is bounded. Proof of theorem 1 is completed.

IV. DESIGN OF ADAPTIVE CONTROL SCHEME BASED ON CHARACTERISTIC MODEL

According to sections above, the characteristic model can be rewritten as:
\[
e(k + 1) = \hat{f}_1(k)e(k) + \hat{f}_2(k)e(k - 1) + \hat{g}_0(k)u(k) + e_m(k) \tag{30}
\]
Based on characteristic model and control error feedback, the easily realized adaptive control law is designed as
\[
u(k) = \frac{1}{\hat{g}_0(k)}[\xi_1(k)e(k) - \hat{f}_1(k)e(k) - \hat{f}_2(k)e(k - 1)] \tag{31}
\]
To avoid singular problems, \( \hat{g}_0(k) \) is set not to be zero during estimation process.

Now we analysis the performance of the closed-loop system consisting of characteristic model (15) and adaptive control law (31):

**Theorem 2:** Considering adaptive control law (31), if choose parameters satisfy \( |\xi_1| < 1 \), then the tracking error \( e(k) \) is bounded and it satisfies:
\[
|e(k)| \leq \frac{\lambda_3}{1 - |\xi_1|} \tag{32}
\]

**Proof:** Substituting control law (31) into (30), we have
\[
e(k + 1) = \xi_1 e(k) + e_m(k) \tag{33}
\]
Choose Lyapunov function
\[
V_2(k) = e^2(k) \tag{34}
\]
The first difference of \( V_2(k) \) is given as:
\[
\Delta V_2(k + 1) = [\xi_1 e(k) + e_m(k)]^2 - e^2(k) = \xi_1^2 e^2(k) + 2\xi_1 e(k)e_m(k) + e_m^2(k) - e^2(k) = -(1 - \xi_1^2)e^2(k) + 2\xi_1 e_m(k)e(k) + e_m^2(k) \leq -(1 - \xi_1^2)e^2(k) + 2|\xi_1||e(k)||e_m(k)| + e_m^2(k) \leq -(1 - \xi_1^2)e^2(k) + 2\lambda_3|\xi_1||e(k)| + \lambda_3^2 \tag{35}
\]
Then, we can know that \( \Delta V_2(k) \) is guaranteed negative as long as
\[
|e(k)| > \frac{\lambda_3}{1 - |\xi_1|} \tag{36}
\]
Define
\[
\Upsilon = \left\{ e(k) \left| |e(k)| \leq \frac{\lambda_3}{1 - |\xi_1|} \right. \right\} \tag{37}
\]
It can be obtained that:
If tracking error \( e(k) \) is outside of \( \Upsilon, \) \( \Delta V_2(k) < 0, \) and \( e(k) \) will finally converge into \( \Upsilon. \) This completes proof of theorem 2.

V. SIMULATION RESULTS

In this section, the performance of the proposed control scheme is evaluated by simulations. Fig.2 shows the structure of adaptive control scheme. Considering practical situation and generality, two kinds of signal are chosen as target signal: slope signal \( y = 60 \pi t \) and sine signal \( y = 60 + 60\sin(t - \pi/2). \) To verify the accuracy of characteristic modeling, the outputs of characteristic model and dynamic model are monitored together. Also, the real-time value of three characteristic parameters is recorded to see if the gradient projection estimator meets the requirement of rapidity.
What’s more, to evaluate the control performance, a fuzzy PI controller is also simulated as comparison. The fuzzy control takes position tracking error and change rate as inputs. Then, the parameters of fuzzy controller ($k_p$ and $k_i$) are set online through fuzzification, fuzzy reasoning and defuzzification [28]–[30]. The final value is chosen as: $k_p = 15$, $k_i = 0.1$. The detailed information of motor parameters and control parameters are listed in Table 2.

| Parameter | Value       | Parameter       | Value       |
|-----------|-------------|-----------------|-------------|
| $C_T$     | 0.641N·m/krpm | $J_m$          | 250kg·m²   |
| $C_n$     | 0.6417N·m/krpm | $b_m$         | 0.005N·m/krpm |
| $L_{res}$ | 3.75mH      | $r_m$         | 8.5        |
| $R_{eq}$  | 1.31Ω       | $J_{eq}$       | 20         |
| $J_f$     | 0.000323kg·m² | $T_m$         | 10N·m      |
| $\lambda_1$ | 0.3       | $\lambda_2$   | 0.5        |
| $\lambda_3$ | 0.0005    | $\xi_1$       | 0.4        |

The simulation results are shown in Fig.3 and Fig.4. And Table 3 lists some details about simulation results.

**Simulation result analysis:**

1. From Table 3 and two figure (b)s, we can see that: the modeling error is no more than $0.6 \times 10^{-4}$, which indicates
the accuracy and effectiveness of characteristic modeling. Also, the time cost for estimator to identify three characteristic parameters are all less than 0.2s, which indicates the rapidity of characteristic modeling strategy.

(2) From Table 3 and two figure (c)s, we can see that: under two kinds of signals, there is no overshoot with adaptive controller while it is serious with Fuzzy PI controller. Besides, system reversing negatively affects Fuzzy PI seriously every $2\pi$ second in sine input, but the adaptive controller eliminates this phenomenon effectively. What’s more, the tracking error of adaptive controller is only half of Fuzzy PI in sine input. All these factors prove than the adaptive controller is simple but effective in controlling servo systems.

Considering the analysis, it can be obtained that the characteristic modeling method with a gradient projection estimator is proved useful in modeling four-motor servo systems, and the adaptive controller based on this model is obviously simplified but effective. This provides foundation for further research.

VI. EXPERIMENT RESULTS

In this section, the adaptive control scheme is applied into practical system to evaluate its effectiveness. The experiment platform is shown in Fig.5. In experiments, we choose the same input signals and do the same comparisons as in simulations. But the control parameters are adjusted according to platform parameters, which are all listed in Table 4.

The experiment results are shown in Fig.6 and Fig.7. And Table 5 lists some details about experiment results.

Experiment result analysis:

(1) From Table 5 and figure (b)s, we can see that: the modeling error is less than $1 \times 10^{-4}$, and the time cost of parameter identification is less than 1.2s. These data verify the conclusion of simulations: characteristic modeling is useful in servo systems and it can provide an accurate but simplified model for further control design.

(2) Considering figure (d)(e)(f) in Fig. 7, it can be obtained that a small value jump in three parameters will happen every $\pi$ second. The main reason is that: when system reverses, the backlash and other disturbances cause changes to system
status, thus the characteristic parameters adjust themselves to react to these changes. This phenomenon verifies the sensibility in real-time system status.

(3) From Table 5 and figure (c)s, it is obvious that the adaptive controller has no overshoot and has a better robustness for system disturbances which usually appear when system reversing. The tracking error also indicates the advantages of adaptive controller.

With all experiment results observed, we can conclude that: in practical servo systems, characteristic model-based control scheme has a good performance as expected. Compared to complex dynamic model, characteristic modeling provide a simplified option to control servo systems and avoids many measurement errors. Also, due to the real-time characteristic parameters, this control scheme has a good robustness naturally.

VII. CONCLUSION

In this article, we apply characteristic modeling method into four-motor servo systems to overcome the difficulty in controller design. To simplify the complex dynamic system model, characteristic modeling offers a simplified characteristic model with three time-vary parameters. Then, a gradient projection estimator is newly designed to identify three characteristic parameters online. With these two steps, a characteristic model is obtained, which could monitor system status accurately and rapidly. With characteristic model established, a simple adaptive controller is designed. These three parts make up the total control scheme. The stability of this control scheme is proved by Lyapunov theorem, and its effectiveness is verified through both simulation and experiment results.

What’s more, this control scheme has a good robustness due to time-varying characteristic parameters, and it can be easily applied into practical servo systems or other complex systems.

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