Abstract. We study the dating market decision problem in which men and women repeatedly go out on dates and learn about each other. We consider a model for the dating market that takes into account progressive mutual learning. This model consists of a repeated game in which agents gain an uncertain payoff from being matched with a particular person on the other side of the market in each time period. Players have a list of preferred partners on the other set. The players that reach higher rank levels on the other set preferences list have also higher probability to be accepted for dating. A question can be raised, as considered in this study: Can the less appreciated players do better? Two different kinds of dating game are combined "à la Parrondo" to foster the less attractive players. Optimism seems to be highly recommendable, especially for losers.

Keywords: Matching Model, Parrondo’s Paradox, Dating Market, Bandit Problem

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INTRODUCTION

Matching problems where the elements of two sets have to be matched by pairs have broad implications in economic and social contexts. As possible applications one could think of job seekers and employers, lodgers and landlords, or simply men and women who want to date. Standard models of matching in economics [1] almost always assume that each agent knows his or her preferences over the individuals on the other side of the market. This assumption, generally associated with neo-classical economics, is too restrictive for many markets, and some interesting work on matching problems with partial information has recently been published [2, 3]. Specifically, perfect information supposition is very far from being a good approximation for the dating market, in which men and women repeatedly go out on dates and learn about each other. Recently, a model for the dating market that takes into account progressive mutual learning was introduced by Das and Kamenica [4]. This model consists of a repeated game in which agents gain an uncertain payoff from being matched with a particular person on the other side of the market in each time period. The problem is related to bandit problems [5], in which an agent must choose which arm of an n-armed bandit to pull in order to maximize long-term expected reward, taking into account the tradeoff between exploring, that is learning more about the reward distribution for each arm, and exploiting, pulling the arm with the maximum expected reward. However, in Das and Kamenica model the arms themselves have agency - they can decide whether to be pulled or not, or whom to be pulled by, and they themselves receive rewards based on who the puller is. This motivates Das and Kamenica formulation of the problem as a "two-sided" bandit problem.

The matching problems describe systems where two sets of persons have to be matched pairwise. Players have a list of preferred partners on the other set. The players that reach higher rank levels on the other set preferences list have also higher probability to be accepted for dating. An interesting question is: Can the less appreciated players do better? In other words: Can the usual dating game losers achieve a better performance? A possible way to accomplish this goal may be the phenomenon known in the literature as Parrondo’s paradox [6], devised in 1996 by the Spanish physicist Juan M.R. Parrondo, where the alternation of two fair (or losing) games can result in a winning game. Although initially introduced as individual games, multiplayer versions of the paradox, played by a set of N players, have also been studied [7, 8]. In these collective games, a set of N players are arranged in a ring and each round a player is chosen randomly to play either game A or B. The game A is a simple coin tossing game, where the player wins or loses one unit of capital with probabilities $p_A$ and $1 - p_A$, respectively in [7], and a redistribution process where a player is chosen randomly to give away one coin of his capital to another player in [8]. Game A is combined with game B, for which the winning probability depends on the state (winner/loser) of the nearest neighbors of the selected player. A player is said to be a winner (loser) if he has won (lost) his last game. Recently [9] a new version of collective games was introduced, where besides obtaining the desired result of a winning game out of two fair games, another feature appears: the value of the $A/B$ mixing probability $\gamma$ determines whether you end up with a winning or a losing game.
In each round, a player is selected randomly for playing. Then, with probabilities $\gamma$ and $1 - \gamma$, respectively, game $A$ or $B$ is played. Game $A$ is the original coin tossing game. The winning probabilities in game $B$ depend on the collective state of all players. More precisely, the winning probability can have three possible values, determined by the actual number of winners $w$ within the total number of players $N$, in the following way:

$$p_B = \text{winning probability in game } B = \begin{cases} 
 p_B^1 & \text{if } w > \left\lceil \frac{2N}{3} \right\rceil, \\
 p_B^2 & \text{if } \left\lceil \frac{N}{2} \right\rceil < w \leq \left\lfloor \frac{2N}{3} \right\rfloor, \\
 p_B^3 & \text{if } w \leq \left\lfloor \frac{N}{2} \right\rfloor,
\end{cases}$$

where the brackets $[x]$ denote the nearest integer to the number $x$. The set of values $p_B^1, p_B^2, p_B^3$ are determined in order to give a fair game and depend on the total number of players $N$ [9].

In this paper we consider a repeated mixing of two different dating games based on both Das and Kamenica learning dating model and on the last version [9] of collective Parrondo games to analyze the possibility that the less attractive players do better. Both dating games are assumed to be fair. For the sake of clarity let us imagine that the different games that we call $A$ and $B$ are played in different places with different rules. In game $A$, the probability that the man proposal be accepted is modelled by coin tossing, that is $p_A = 1/2$. This does not necessarily mean that every woman flips actually a coin, but it can be thought that all the variables not considered to construct the preference list, such as woman’s mood, man’s way, or the group size dependance of females selectivity [10] are contributing to the probability of acceptance $p_A$. On the other hand, the probability of acceptance for game $B$, $p_B$, depends on the number of previous winners within all players. This collective influence may seem not so clear at first sight, but it may be thought that in $p_B$ collective moods contribute, such as if all woman’s friends are dating, that particular woman is better disposed to consider a date to be must propose to the woman $i$.

**THE MODEL**

There are $N$ men and $N$ women, who interact for $T$ time periods. $v^m_j$ is the value of woman $j$ to every man, and $v^w_i$ is the value of man $i$ to every woman. These values are constant through time. In each period, a man $i$ is chosen randomly from the $N$ possible men. The expected $i$’s payoff of dating woman $j$:

$$payoff_{i,j}[t] = Q_{i,j}^m[t] \times p_{i,j}^m[t],$$

where $Q_{i,j}^m[t]$ is the man $i$’s estimate of the value of going out with woman $j$ at time $t$ and $p_{i,j}^m[t]$ is the man $i$’s estimate at time $t$ of the probability that woman $j$ will go out with him if he asks her out. In this way man’s decision is based on any prior beliefs and the number of rewards he has received. Both the expected value on a date with that particular woman and the probability that she will accept his offer are taken into account by (2). The expected woman $j$’s payoff of dating man $i$ is:

$$payoff_{i,j}^w[t] = Q_{i,j}^w[t],$$

where $Q_{i,j}^w[t]$ is the woman $j$’s estimate of the value of going out with man $i$ at time $t$. No probability is considered because man $i$ considered as a date to be must propose to the woman $j$. Since the underlying $v^m_j$ and $v^w_i$ are constant we define $Q_{i,j}^m[t]$ as man $i$’s sample mean at time $t$ of the payoff of going out with woman $j$:

$$Q_{i,j}^m[t] = \sum (v^m_j + \varepsilon),$$

where the sum is made on the effective dates between $i$ and $j$ and $\varepsilon$ is noise drawn from a normal distribution. In the same way, $Q_{i,j}^w[t]$ is woman $j$’s sample mean at time $t$ of the payoff of going out with man $i$:

$$Q_{i,j}^w[t] = \sum (v^w_i + \varepsilon).$$

In order to deal with the nonstationarity of $p_{i,j}^m[t]$’s, on the other hand, we use a fixed learning rate for updating the probabilities which allows agents to forget the past more quickly:

$$p_{i,j}^m[t] = (1 - \eta)p_{i,j}^m[t-1] + \eta \frac{offers_{i,j,accepted}[t-1]}{offers_{i,j,made}[t-1]},$$

where $\eta$ is a constant parameter.
The Man’s Decision Problem

The top ranked woman from the list of preferred partners of $i$ is selected to ask out for a date. The rank levels of the preference list are distributed according to the expected $i$’s payoff of dating woman $j$ (2). The man $i$ acts in a greedy way asking out woman $j$ at the top of his preference list.

The Woman’s Decision Problem

The rank levels of the women preference lists are distributed according to the expected woman $j$’s payoff of dating man $i$ (3). The woman’s decision problem depends on the game:

Game A

In both games women have to consider the exploration-exploitation tradeoff. Exploitation means maximizing expected reward (greedy choice). Exploration happens when the player selects an action with lower expected payoff in the present in order to learn and increase future rewards. One of the simplest techniques used for bandit problems is the so-called $\varepsilon$-greedy algorithm. This algorithm selects the arm with highest expected value with probability $1 - \varepsilon$ and otherwise selects a random arm. We will use slightly changed versions of the $\varepsilon$-greedy algorithm in both games.

In game A the exploration-exploitation tradeoff depends on coin tossing, that is the woman accepts the man’s $i$ offer to date with probability $p_A = 1/2$ (exploration) or she acts greedily and goes out with her best payoff $w$ choice with probability $1 - p_A = 1/2$.

Game B

In game B the choice of exploration or greedy behavior depends on the collective state of all men players. A man player is said to be a winner or a loser when he got his date or not, respectively, in his last game. More precisely, the winning or exploration probability can have three possible values, determined by the actual number of winners $w$ within the total number of players $N$, in the following way

$$p_B = \text{exploration probability in game B} = \begin{cases} p_B^1 & \text{if } w > \left\lceil \frac{N}{3} \right\rceil, \\ p_B^2 & \text{if } \left\lceil \frac{N}{3} \right\rceil < w \leq \left\lfloor \frac{2N}{3} \right\rfloor, \\ p_B^3 & \text{if } w \leq \left\lfloor \frac{N}{3} \right\rfloor, \end{cases}$$

where the brackets $[x]$ denote the nearest integer to the number $x$. The woman accepts the man’s $i$ offer to date with probability $p_B$ (exploration) or she goes out with her best payoff $w$ choice with probability $1 - p_B$. The set of values $p_B^1, p_B^2, p_B^3$ are determined in order to give a fair game and depend on the total number of players $N$ [9].

RESULTS

Our simulations involve a market of $N = 4$ men and $N = 4$ women. The learning rate of probability $p_{i,j}^m[t]$ is $\eta = 0.05$. The set of probabilities for game $B$, $p_B^1, p_B^2, p_B^3$ determined to give a fair game [9] for $N = 4$ are $p_B^1 = 0.79$, $p_B^2 = 0.65$ and $p_B^3 = 0.15$. The noise signal is drawn from a normal distribution of standard deviation 0.5. $v_m$’s and $v_w$’s are:

$$v_k^m = v_k^w = N - k + 6,$$

where $1 \leq k \leq 4$. Reported results are obtained with 1000 simulations averages on $10^4$ time steps.

Parrondo’s Paradox

All players benefit from Parrondo mixing of both games A and B as can be observed in Fig. 1. In this Figure the total matches, that is the number of accepted dating offers from all players, is shown as a function of time for game B.
FIGURE 1. Time evolution of total number of offers accepted between dating game $B$ played alone and games $AB$ switched with mixing probability $\gamma = 1/2$. Total matches vs. time (a) Game $B$; (b) Games $AB$.

FIGURE 2. Time evolution of number of accepted offers from the last of preferences list (loser) between dating game $B$ played alone and games $AB$ switched with mixing probability $\gamma = 1/2$. Loser matches vs. time (a) Game $B$; (b) Games $AB$.

played alone and games $A$ and $B$ switched with mixing probability $\gamma = 1/2$. In the first case, the fairness of game $B$ is verified when half of all attempts produce a match. When $A + B$ are played, the paradox produces that more than half of all attempts are successful, as can be seen in Fig. 1b.

On the other hand, less favored players, i.e. the lowest $v_w$ ones, have an evolution that is shown in Fig. 2, for game $B$ played alone and games $A$ and $B$ switched with mixing probability $\gamma = 1/2$. In this Figure the advantage of playing $A + B$ over $B$ for losers can be appreciated. The comparison with game $A$ is essentially the same for all the results studied.

Optimistic Results

Das and Kamenica propose as an alternative method to obtain asymptotic stability in their model for dating couples to suppose that players are initially optimistic and their level of optimism declines over time. As they say “This is another form of patience - a willingness to wait for the best - and it should lead to more stable outcomes” [4]. A systematic overestimate of the probability that the dating offer will be accepted is used to represent optimism.

Let us analyze optimistic players performance at our dating games model. At time $t$ optimistic players use the
FIGURE 3. Optimistic Players: Time evolution of number of accepted offers from the last of preferences list (loser) between dating game B played alone and games AB switched with mixing probability $\gamma = 1/2$. Loser matches vs. time (a) Game B; (b) Games AB

probability estimate:

$$p_{i,j}^m[t] = \alpha(t) + (1 - \alpha(t))p_{i,j}^m[t],$$

where $p_{i,j}^m[t]$ is updated as before by (6) and $\alpha(t) = (T - t)/T$, with $T$ the total number of time steps in simulations. Figure 3 shows the evolution of loser accepted offers corresponding to optimistic players. The order of loser acceptance increase by a factor 10 comparing optimistic (Fig. 3) and non-optimistic (Fig. 2) losers. On the other hand, the advantage of playing $A + B$ over $B$ for losers is conserved.

$N$ and $\gamma$ dependance

The results are highly dependent on both, the number $N$ of players and the mixing probability $\gamma$ of the games $A$ and $B$. As thoroughly explained on [9], the losing or winning character of the mixed $A + B$ games depends on $N$ and $\gamma$. We will present in a more extensive way the dependance on the number $N$ of players and the mixing probability $\gamma$ of the games $A$ and $B$ elsewhere.

CONCLUSIONS

We find a way for less qualified dating game players to improve their performance by means of a repeated mixing of two different dating games $A$ and $B$ based on a recent dating market model [4] and on a recent collective game version of Parrondo’s paradox [9]. In game $A$, the probability associated to exploration-exploitation tradeoff, that the man proposal be accepted is modelled by coin tossing, that is $p_A = 1/2$. On the other hand, the probability of acceptance for game $B$ depends on the collective state of the men set obtained through the number of previous winners within all players. We show that losers benefit from Parrondo mixing of both games $A$ and $B$. In the optimistic version of our model, when it is assumed that players are initially optimistic and their level of optimism declines over time, loser acceptance increase by a factor 10 and the paradoxical advantage of playing $A + B$ over $B$ or $A$ for losers is conserved. The results are highly dependant on both, the number $N$ of players and the mixing probability $\gamma$ of the games $A$ and $B$.

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