Dipolar degree of freedom and dynamical correlations in Isospin equilibration processes

Papa M and Giuliani G
Istituto Nazionale Fisica Nucleare-Sezione di Catania, Via S. Sofia 64 95123 Catania Italy
E-mail: massimo.papa@ct.infn.it

Abstract. The asymptotic time derivative of the total dipole signal is proposed as an useful observable to investigate on Isospin equilibration phenomenon in multi-fragmentation processes. The study has been developed to describe charge/mass equilibration processes involving the gas and liquid "phases" of the total system formed during the early stage of a collision. General properties of this observable and the links with others isospin dependent phenomena are discussed. In particular, the $^{40}\text{Cl} + ^{28}\text{Si}$ system at 40 MeV/nucleon is investigated by means of semiclassical microscopic many-body calculations based on the CoMD-II model. The study of the dynamical many-body correlations produced by the model also shows how the proposed observable is rather sensitive to different parameterizations of the isospin dependent interaction.

1. Introduction
An interesting subject related to Heavy Ions Isospin physics is the process leading to the equilibration of the charge over mass ratio between the main partners of the reaction as described in Ref.[1]. The so-called "isospin diffusion" phenomenon has been indicated as the relevant mechanism acting between the reaction partners in essentially binary processes [2, 3, 4]. In particular, in the collision of the 124 and 112 Tin isotopes at 50 MeV/nucleon [2], evidence of partial equilibrium in the charge/mass ratios of the quasi-projectile and quasi-target has been deduced through the study of the iso-scaling parameters related to the isotopic distributions. In this case dynamical calculations, based on the Boltzmann-Uehling-Uhlenbeck model [5], show that the degree of equilibration depends on the behavior of the symmetry energy as a function of the density. In this work we want to extend the study of the isospin equilibration processes, looking at the system in a global way, by using the following quantity: $\vec{V}(t) = \sum_{i=1}^{Z_{\text{tot}}} \vec{v}_i$. At a microscopic level, the sum on the index $i$ is performed on all the $Z_{\text{tot}}$ protons of the system. $\vec{V}(t)$ corresponds, apart from the elementary charge, to the time derivative of the total dipole of the system. The velocities $\vec{v}_i$ are computed in the center of mass (c.m.) reference frame. We note that, as due to the total momentum conservation, the global effect related to the motion of the neutral particles (bound and free) is implicitly contained in $\vec{V}(t)$. Therefore we can expect, in a quite general way, a dependence of the behavior of $\vec{V}(t)$ on the isospin dependent interaction. We note also that several studies were based on the time dependence of this dynamical variable to describe pre-equilibrium Giant Dipole Resonance (GDR) $\gamma$-ray emission (see Ref.[6] and references therein ). Various reasons suggest us to use the same variable to also describe isospin equilibration in complex processes.
- (i) by taking into account only the effects associated to the strong interaction, after the pre-equilibrium stage, starting from the time \( t_{\text{pre}} \), when a second stage characterized by an average isotropic or symmetric emission of the secondary sources (statistical equilibrium) takes place, the ensemble average of \( \langle \vec{V}(t) \rangle \) satisfies the following relation: \( \langle \vec{V}(t_{\text{pre}}) \rangle = \langle \vec{V}(t > t_{\text{pre}}) \rangle \equiv \langle \vec{V} \rangle \) [6]. Therefore the average value of this dynamical variable at \( t_{\text{pre}} \) is invariant with respect to statistical processes and the value of \( \langle \vec{V} \rangle \) is determined only by the complex dynamics which characterizes the early stage of the collision, when fast changes of the average nuclear density are expected. \( \langle \vec{V} \rangle \) can be expressed as a function of the charge \( Z \), mass \( A \), average multiplicity \( \langle m_{Z,A} \rangle \) and the average value of the momentum \( \langle \vec{P}_{Z,A} \rangle \) of the detected particles in the generic event:

\[
\langle \vec{V} \rangle = \langle \sum_i Z_i v_{\text{cm}}^i \rangle = \frac{1}{m_0 c^2} \sum_i \frac{Z_i}{A} \langle m_{Z,A} \rangle \langle \vec{P}_{Z,A} \rangle C^{Z,A}_P
\]

\[
C^{Z,A}_P = \frac{\langle m_{Z,A} \vec{P}_{Z,A} \rangle}{\langle \vec{P}_{Z,A} \rangle \langle m_{Z,A} \rangle}, \quad Z_i, \; v_{\text{cm}}^i \text{ are the generic charge and the center of mass velocity of} \; \text{the produced fragments respectively} \; [6]. \; \frac{C^{Z,A}_P}{P} \text{ are the correlation coefficients between the multiplicity and the mean momentum. These coefficients play a key role for the invariance property and therefore require for an event by event analysis in which many-body correlations cannot be neglected. For symmetry reasons,} \; \langle \vec{V} \rangle \text{ lies on the reaction plane. It is directly linked with a weighted mean of the charge/mass ratio, as Eq.(3) suggests. It also takes into account the average isospin flow direction through the momenta} \; \vec{P}_{Z,A}. \; \text{The long range Coulomb interaction can produce differences between the value of} \; \langle \vec{V}(t_{\text{pre}}) \rangle \; \text{and the observed asymptotic value. These changes, however, are rather small at the involved energies and can be evaluated with the necessary precision by taking into account the corrections due to the Coulomb repulsion up to the asymptotic stage.}

- (ii) in the general case, we find attractive the following decomposition: \( \langle \vec{V} \rangle = \langle \vec{V}_G \rangle + \langle \vec{V}_{GL} \rangle + \langle \vec{V}_L \rangle \) where \( \langle \vec{V}_G \rangle \) and \( \langle \vec{V}_L \rangle \) are the average dipolar signals associated to the gas "phase" (light charged particles) and to the "liquid" part corresponding to the motion of the produced heavy fragments. The signal \( \langle \vec{V}_{GL} \rangle \) is instead associated to the relative motion of the two "phases". By supposing, for simplicity, the gas "phase" formed by neutrons and protons, \( \langle \vec{V} \rangle \) can be further decomposed as:

\[
\langle \vec{V} \rangle = \frac{A_G (1 - \beta_G^2)}{4} \vec{v}_r^{PN} + \frac{\mu_{GL} (\beta_L - \beta_G)}{2} \vec{v}_{\text{cm},LG} + \langle \vec{V}_{r,L} \rangle
\]

In the above expression the first term represents the contribution related to the proton-neutron relative motion of the gas "phase" composed of \( A_G \) nucleons, expressed through the relative velocity \( \vec{v}_r^{PN} \); the second term concerns the relative velocity \( \vec{v}_{\text{cm},LG} \) between the centers of mass of the "liquid" complex and the "gas", \( \mu_{GL} \) is the reduced mass associated to the two sub-systems; the last term represents the contribution produced by the relative motion of the fragments. A similar expression can be obtained including others light particles in the gas "phase" as , for example, light Intermediate Mass Fragments (IMF). From this decomposition we can see how the equilibration condition \( \langle \vec{V} \rangle = 0 \), for the total system, requires a delicate balance which depends on the average neutron excess of the produced "liquid drops" (\( \beta_L \)), on the one associated to the "gas" pre-equilibrium emission (\( \beta_G \)), and on the relative velocities between the different parts. To enlighten the role played by some of the terms reported in Eq.(4), we can discuss the idealized decay of a charge/mass asymmetric source through neutrons and protons emission (or the case in which the liquid drops are produced through a statistical mechanism...
\((\nabla^2 r_{L}) = 0\). This example can also schematically describe the case of the complete stopping in heavy ion collisions. Moreover, for simplicity, we can consider uncorrelated fluctuations between the velocities, masses and neutron excesses. For non-identical colliding nuclei, if pre-equilibrium emission exists, then \( \langle \nabla_{\text{cm},L} \rangle \neq 0 \). In this case, if \( \langle \beta_{G} \rangle \neq \langle \beta_{L} \rangle \), due, for example, to the isospin “distillation” phenomenon, the first term has to be necessarily different from zero and it will contribute to the neutron-proton differential flow (see also Sect.3). Therefore, according to our description, the understanding of the isospin equilibration process for the total system requires the “gas” pre-equilibrium contribution to be taken into account. This term can be regarded as a kind of “dissipation” with respect to the system formed by the liquid part. In this work, as an example, we will discuss the results obtained through the Constrained Molecular Dynamics-II approach (CoMD-II) [9] applied to the charge/mass asymmetric \(^{40}\text{Cl} + ^{28}\text{Si}\) system at 40 MeV/nucleon. The study is performed by using different options for the isospin potential term.

Before to show the results of our calculations, in the following section we briefly recall the way in which the isospin dependence of the nuclear interaction is introduced in the CoMD-II model and the related correlations.

2. Symmetry interaction and correlations

According to the results shown in Ref.[10], starting from a Skyrme type two-body microscopic interaction, the two-body effective potential in CoMD-II model can be expressed through the nucleon-nucleon overlap integral \(\rho_{ij} = \int d^{3}r_{i}d^{3}r_{j}\delta(\vec{r}_{i} - \vec{r}_{j})\rho(\vec{r}_{i})\rho(\vec{r}_{j})\). \(\vec{r}_{i}, \vec{r}_{j}\) represent the nucleon spatial coordinates, \(\rho\) is the Gaussian distribution in the coordinate space related to the generic nucleonic wave-packet. The microscopic isospin dependent interaction for the Stiff2 option is given by the following expression:

\[
V_{\tau} = \frac{a_{0}}{2\rho_{0}} \sum_{i<j}^{A} \left(2\delta_{\tau_{i},\tau_{j}} - 1\right)\delta(\vec{r}_{i} - \vec{r}_{j})-
\]

where \(A\) is the total mass number, \(\tau_{i}\) represent the generic third nucleonic isospin component and \(\rho_{0}\) is the one-body density at the saturation point. The coefficient \(a_{0} = 72\text{MeV}\) determines the strength of the isospin dependent interaction at the saturation density. As shown in Ref.[10] the structure of \(V_{\tau}\) reflects the fact that the two-body nuclear forces, in \(S\) wave, around the ground state (g.s.) density, are less attractive in isospin triplet states \((T = 1)\) with respect to the singlet states \((T = 0)\). From the above expressions it results that the associated effective interaction \(U_{\tau}\) (after the convolution with the nucleonic wave-packets) can be expressed as a function of the average overlap integrals per couple of neutrons \((nn)\), \(\rho_{nn}\), protons \((pp)\) \(\rho_{pp}\), and neutron-proton \((np)\) \(\rho_{np}\). As discussed in Ref.[10] for small asymmetries we can assume \(\rho_{nn} \approx \rho_{pp} \approx \rho_{nn} + \rho_{np} / 2 = \tilde{\rho}\). To characterize the differences associated to the nucleon-nucleon dynamics, at a two-body level, we can introduce the correlation coefficient \(\alpha\) in such a way \(\rho_{np} = (1 + \alpha)\tilde{\rho}\). \(\alpha\) depends on both \(\tilde{\rho}\) and the asymmetry parameter \(\beta\). Results on nuclear matter simulations [10] show that the behavior of \(\alpha\) as a function of \(\beta\) can be approximated for moderate asymmetries by a parabolic law. In this case, eqs.(6,7) of Ref.[10] give the following expression for the effective isospin potential in the Non Local (N.L.) approximation \(U_{\alpha,\beta}^{N.L.}\):

\[
U_{\alpha,\beta}^{N.L.} \approx \frac{a_{0}}{2\rho_{0}} \tilde{\rho}A^{2}F'(s)[(1 + 2\alpha_{0} - \alpha')\beta^{2} - 1/2\alpha_{0}]
\]

\(\alpha' = \frac{1}{4}\rho_{np} / 2\beta = 0\). \(\beta^4\) terms are neglected in the previous expression. \(\alpha_{0} \equiv \alpha(\tilde{\rho}, \beta = 0)\) represents the correlation coefficient related to the difference in the dynamics of the \(np\) couples with respect to the \(nn\) and \(pp\) ones for symmetric nuclear matter. \(\alpha_{0}\) depends on the average overlap integral per couple of nucleons \(\tilde{\rho}\) which reflects the degree of compression. \(s = \frac{1}{4\pi} \sum_{i<j} \rho_{ij}\) is associated to the total overlap integral per nucleon. \(F'\) is a form factor which modulates the changes of the iso-vectorial interaction as a function of the average overlap integral \(s\). For the Stiff1 option we use \(F' = \frac{2s}{s_{p,n} + s}\), for the Stiff2 case \(F' = 1\) and for the Soft option \(F' = (\frac{2s}{s_{p,n}})^{1/2}\). From
Eq.(3) we note that in our approach the iso-vectorial forces generate, beyond the $\beta$ dependent potential, also another density dependent term, independent on $\beta$ but proportional to the degree of correlation $\alpha$ for symmetric systems and to the strength of the iso-vectorial forces. As discussed in [10], at small asymmetries, this term determines the high sensitivity of the experimental observable to the different functional forms of $F'$. The finite value of $\alpha$, which is on the order of 15%, apart from the Pauli principle and the Coulomb interaction, is strongly affected by the isospin interaction itself. In fact the neutron-proton couples, at variance with the others ones, suffer the more attractive singlet interaction.

3. Calculation results

3.1. An example: the $^{40}$Cl+$^{28}$Si system at 40 MeV/nucleon

Now we discuss, as an example, the results concerning the isospin equilibration process for the $^{40}$Cl+$^{28}$Si system at 40 MeV/nucleon. In Fig. 1 we show the average total dipolar signals evaluated through CoMD-II calculations along the $\hat{z}$ beam direction $(V^z)$ and along the impact parameter direction $\hat{x}$, $(V^x)$, respectively. The reference frame is the c.m. one. The impact parameter $b$ is equal to 3 fm, in panels (a) and (b), and 1.5 fm in panels (c) and (d). In Fig. 1(a) and Fig. 1(c) the average dipolar signals are shown for the first 140 fm/c. Different lines refer to different iso-vectorial potentials, according to Ref. [9]. During the first 150 fm/c in all the cases wide oscillations exist. They are responsible for the pre-equilibrium $\gamma$-ray emission [6]. The damped oscillations converge towards smaller and almost constant values. This can be seen in Fig. 1(b) and Fig. 1(d) in which the dynamical evolution is followed from 110 fm/c up to 300 fm/c. The inclusion of corrections due to Coulomb interaction at longer time is on the order of some percent. The time interval in which the almost stationary behavior is reached is related to the lifetime of the coherent dipolar collective mode and it is strictly linked with the average time for the formation of the main fragments and pre-equilibrium emission.

![Figure 1](image.png)

Figure 1. Average dipolar signals $\langle \vec{V} \rangle$ (expressed in MeV/c) for $b=3$ fm along the $\hat{z}$ direction are plotted as a function of time in the intervals 0-140 fm/c (panel (a)) and 110-300 fm/c (panel (b)). Different lines indicate different options for the iso-vectorial interaction (see the text). Panels (c) and (d) display the average dipolar signals along the $\hat{x}$ direction for $b=1.5$ fm.

As we discussed in the introductory section, when the collision partners are not identical nuclei the isospin equilibration process produces a contribution to the neutron-proton differential flow $F_{np}$ [8] if the neutron and proton "gases" have different c.m. velocities. These pre-conditions are verified for the studied collision.

For $b = 3$ fm and for the Stiff1 and Soft options, in Fig. 2(a) we show $F_{np}$ as a function of the particles rapidity, $y$, normalized to the projectile one $y_{beam}$. The rapidity values are evaluated in the c.m. of the total system reference frame. The dashed vertical lines indicate the projectile and target reduced rapidity. From the figure we can see that, by averaging on the rapidity, the neutron-proton transversal velocity has a negative value. This means that, on average, the relative motion between the c.m. of the emitted neutrons and protons is deflected in the half-plane opposite to the impact parameter direction. The results shown in Fig. 2(a) can be compared with the calculations displayed in Fig. 2(b) obtained by subtracting, event
by event, the c.m. relative neutron-proton motion related to the "gas" phase. As can be seen, similarly to the case of identical nuclei, this correction restores (within the errors associated to the statistics of simulations) the almost specular behavior of $F_{np}$ with respect to the rapidity axes. The correction acts also along the beam direction.

Finally, in Fig. 2(c) we show the vectors associated to the average free proton-neutron relative motion $\vec{v}^{PN}_r$ for the two options. These quantities determine the correction on the differential flow due to the isospin equilibration process and are associated to the "gas" component of the total dipolar signal.

### 3.2. Sensitivity to the different options of the iso-vectorial interaction

In the following, we want to discuss the sensitivity of the dipolar signal to different options concerning the iso-vectorial interaction. For this aim we performed calculations at different reduced impact parameters $b_r = \frac{b}{b_{\text{max}}}$ ($b_{\text{max}} = 7.5 fm$)

![Figure 3](image)

**Figure 3.** - Relative changes $r$ for the ratio $R$ (see the text) evaluated for different couples of options are shown as a function of $b_r$. The lines which join the points are meant only to guide the eye through the shown trend.

We observe the greater sensitivity to the iso-vectorial interactions around $b = 3$ fm. This result is particularly evident by studying the ratios $R = \langle V^x \rangle / \langle V^z \rangle$. In Fig. 3(b) we in fact show the relative change $r = \frac{\Delta R}{R}$ between couples of different options. We can see that for $b_r$ less than about 0.5 relevant changes are predicted according to the different shapes of the form factor.
This impact parameter region produces a substantial stopping of the incident nuclei and is clearly dominated by large overlap between projectile and target which gives rise to processes changing from incomplete fusion reactions to IMF production. The region of intermediate impact parameters shows the higher sensitivity when the mechanism evolves with respect to the essentially binary processes which take place at the higher impact parameters. For $b_r$ greater than 0.6, in fact, the sensitivity is strongly reduced. In particular, according to what previously observed we have evaluated the partial contributions $\langle V_x^L \rangle$ and $\langle V_z^L \rangle$ related to the two main fragments. As an example, for $b = 3$ fm and for the Stiff2 option, the "liquid" asymptotic values are $\langle V_x^L \rangle = -111.8$ MeV/c and $\langle V_z^L \rangle = 150.9$ MeV/c while the total contributions are $\langle V_x \rangle = 44.9$ MeV/c and $\langle V_z \rangle = -25.15$ MeV/. Therefore, it results that the contributions carried by the two main fragments only partially contribute to the isospin equilibration process. The remaining part ("gas"), which in this case we have associated to particles and to the IMF, generates a term which contributes in a decisive way to the global equilibration process. For the same impact parameter, in Fig. 3(b) we show with the star symbol the sensitivity parameter $r$ evaluated by changing the option from Stiff1 to Stiff2 and only taking into account the contributions of the two main fragments. As we can see, the partial contribution shows a rather reduced sensitivity to the different options as compared to the case obtained by using the global information on the system.

4. Summary and conclusive remarks
In summary, in this work the isospin related equilibration process has been investigated by studying the ensemble average of the time derivative of the total dipole $\langle \vec{V} \rangle$ evaluated through CoMD-II calculations. Some general properties of this quantity have been discussed. In particular, it allows to generalize the definition of isospin equilibration also in complex reactions evolving through multi-fragmentation processes. As an example, calculations performed for the asymmetric charge/mass system $^{40}$Cl+$^{28}$Si at 40 MeV/nucleon show that the asymptotic values of $\langle \vec{V} \rangle$ and $R = \langle V_z^2 \rangle/\langle V_x \rangle$ for these processes are sensitive to different options for the iso-vectorial potential; moreover, in central and mid-peripheral collisions, the dipolar contribution associated to the pre-equilibrium emission of charged particles is relevant to determine the value of $\langle \vec{V} \rangle$ and the related sensitivity to different density dependent form factors.

References
[1] Shi L and Danielewicz P, Phys. Rev. C 68, (2003) 064604.
[2] Tsang M et al, Phys. Rev. Lett. 92, (2004) 062701.
[3] Steiner A W and Li Bao-an, Phys. Rev. C 72, (2005) 041601(R).
[4] Baran V, Colonna M, Di Toro M, Zielinska-Pfab M and Wolter H H, Phys. Rev. C 72, (2005) 064620.
[5] Bertsch G and Das Gupta S, Phys. Rep. 160, (1988) 189.
[6] Papa M et al, Phys. Rev. C 72, (2005) 064608. and references therein.
[7] Papa M, G.Giuliani, arXiv:0910,2923.
[8] Li Bao-an, Phys. Rep. 464, (2008) 113-281.
[9] Papa M, Maruyama T and Bonasera A, Phys. Rev. C 64, (2001) 024612; Papa M, Giuliani G and Bonasera A, J. Comput. Phys. 208, (2005) 403-415.
[10] Papa M and Giuliani G, Eur. Phys. J. 39, (2009) 117.