Decoherence in non integrable systems.

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Self-induced decoherence formalism and the corresponding classical limit are extended from quantum integrable systems to non-integrable ones.

I. INTRODUCTION.

Decoherence was initially considered to be produced by destructive interference [1]. Later the strategy changed and decoherence was explained as caused by the interaction with an environment [2], but this approach is not conclusive because:

i.- The environment cannot always be defined, e.g. in closed system like the universe.
ii.- There is not a clear definition of the “cut” between the proper system and its environment.
iii.- The definition of the pointer basis is not simple.

So we need a new and complete theory: The self-induced approach [3], based in a new version of destructive interference, which will be explained in this talk in its version for non-integrable systems. The essential idea is that this interference is embodied in Riemann-Lebesgue theorem where it is proved that if $f(\nu) \in L^1$ then

$$\lim_{t \to \infty} \int_{-a}^{a} f(\nu)e^{-i\frac{\nu t}{\hbar}} dt = 0$$

If we use this formula in the case when $\nu = \omega - \omega'$, where $\omega, \omega'$ are the indices of the density operator $\hat{\rho}$, in such a way that $\nu = 0$ corresponds to the diagonal, we obtain a catastrophe, since all diagonal and not diagonal terms would disappear. But, if $f(\nu) = A\delta(\nu) + f_1(\nu)$, where now $f_1(\nu) \in L^1_{\nu}$, we have

$$\lim_{t \to \infty} \int_{-a}^{a} f(\nu)e^{-i\frac{\nu t}{\hbar}} dt = A$$

and the diagonal terms $\nu = 0$ remain while the off-diagonal ones vanish. This is the trick we will use below.

II. WEYL-WIGNER-MOYAL MAPPING.

Let $\mathcal{M} = M_{2(N+1)} \equiv \mathbb{R}^{2(N+1)}$ be the phase space. The functions over $\mathcal{M}$ will be called $f(\phi)$, where $\phi$ symbolizes the coordinates of $\mathcal{M}$

$$\phi^a = (q^1, ..., q^{N+1}, p^1, ..., p^N)$$

Then the Wigner transform reads

$$symb\hat{f} \doteq f(\phi) = \int \langle q + \Delta | \hat{f}(q - \Delta) e^{i\frac{\Delta q}{\hbar}} d^{N+1} \Delta$$

where $\hat{f} \in \hat{A}$ and $f(\phi) \in \mathcal{A}$ where $\hat{A}$ is the quantum algebra and the classical one is $\mathcal{A}$. We can also introduce the star product

$$symb(\hat{f} \cdot \hat{g}) = symb\hat{f} * symb\hat{g} = (f * g)(\phi), \quad (f * g)(\phi) = f(\phi) \exp \left( \frac{i\hbar}{2} \sum a \omega^{ab} \frac{\partial}{\partial b} \right) g(\phi)$$

and the Moyal bracket, which is the symbol corresponding to the commutator

$$\{f, g\}_{mb} = \frac{1}{i\hbar}(f * g - g * f) = symb\left( \frac{1}{i\hbar}[f, g] \right)$$
so we have

\[(f \ast g)(\phi) = f(\phi)g(\phi) + 0(h), \quad \{f, g\}_m = \{f, g\}_{pb} + 0(h^2)\]  \hspace{0.5cm} (1)

To obtain the inverse \(symb^{-1}\) we will use the symmetrical or Weyl ordering prescription, namely

\[symb^{-1}[q'(\phi)p'(\phi)] = \frac{1}{2}(q'p' + \hat{p}'\hat{q}')\]

Then we have an isomorphism between the quantum algebra \(\hat{\mathcal{A}}\) and the classical one \(\mathcal{A}\)

\[symb^{-1} : \mathcal{A} \rightarrow \hat{\mathcal{A}}, \quad symb : \hat{\mathcal{A}} \rightarrow \mathcal{A}\]

The mapping so defined is the \textit{Weyl-Wigner-Moyal symbol}. For the state we have

\[\rho(\phi) = symb^{\ast} = (2\pi\hbar)^{-N-1}symb\text{for operators}\]

and it turns out that

\[\langle \hat{\rho}| \hat{O} \rangle = (symb^{\ast}\text{symb}\hat{O}) = \int \text{d}\phi 2^{(N+1)} \rho(\phi)O(\phi)\]  \hspace{0.5cm} (2)

Namely the definition \(\hat{\rho} \in \hat{\mathcal{A}}\), as a functional on \(\hat{\mathcal{A}}\), is equal to the definition \(symb\rho \in \mathcal{A}'\), as a functional on \(\mathcal{A}\).

\textbf{III. DECOHERENCE IN NON INTEGRABLE SYSTEMS.}

\textbf{A. Local CSCO.}

\textbf{a.-} When our quantum system is endowed with a CSCO of \(N+1\) observables, containing \(\hat{H}\), the underlying classical system is \textit{integrable}. In fact, let \(N+1\)-CSCO be \(\{\hat{H}, \hat{O}_1, ..., \hat{O}_N\}\) the Moyal brackets of these quantities are

\[\{O_I(\phi), O_J(\phi)\}_m = symb\left(\frac{1}{i\hbar}[\hat{O}_I, \hat{O}_J]\right) = 0\]

where \(I, J, ... = 0, 1, ..., N\) and \(\hat{H} = \hat{O}_0\). Then when \(\hbar \rightarrow 0\) from eq. (1) we know that

\[\{O_I(\phi), O_J(\phi)\}_{pb} = 0\]  \hspace{0.5cm} (3)

then as \(H(\phi) = O_0(\phi)\) the set \(\{O_I(\phi)\}\) is a complete set of \(N+1\) constants of the motion in involution, globally defined over all \(\mathcal{M}\), and therefore the system is integrable. q. e. d.

\textbf{b.-} If this is not the case \(N+1\) constants of the motion in involution \(\{H, O_1, ..., O_N\}\) \textit{always exist locally}, as can be shown integrating the system of equations (3). Then, if \(\phi_i \in \mathcal{M}\) there is \textit{maximal domain of integration} \(\mathcal{D}_{\phi_i}\) \textit{around} \(\phi_i \in \mathcal{M}\) where these constants are defined. In this case the system in \textit{non-integrable}. Moreover we can repeat the procedure with the system

\[\{O_I(\phi), O_J(\phi)\}_m = 0\]  \hspace{0.5cm} (4)

Then we can extend the definition of the constant \(\{H, O_1, ..., O_N\}\), defined in each \(\mathcal{D}_{\phi_i}\), outside \(\mathcal{D}_{\phi_i}\) as null functions. Their Weyl transforms \(\{\hat{H}, \hat{O}_1, ..., \hat{O}_N\}\) can be considered as a local \(N+1\)-CSCO\(s\) related each one with a domain \(\mathcal{D}_{\phi_i}\) that we will call \(\{\hat{H}, \hat{O}_{\phi_1}, ..., \hat{O}_{\phi_N}\}\) (we consider that \(\hat{H}\) is always globally defined).

\textbf{c.-} We also can define an \textit{ad hoc positive partition of the identity}

\[1 = I(\phi) = \sum_i I_{\phi_i}(\phi)\]

where \(I_{\phi_i}(\phi)\) is the \textit{characteristic function} or \textit{index function}, i. e.: \n
\[I_{\phi_i}(\phi) = \begin{cases} 1 & \text{if } \phi \in D_{\phi_i} \\ 0 & \text{if } \phi \notin D_{\phi_i} \end{cases}\]


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where the domains $D_{\phi_i} \subset D_{\phi}, D_{\phi_i} \cap D_{\phi_j} = \emptyset$. Then $\sum_i I_{\phi_i}(\phi) = 1$. Then we can define $A_{\phi_i}(\phi) = A(\phi)I_{\phi_i}(\phi)$ and

$$A(\phi) = \sum_i A_{\phi_i}(\phi)$$

and using $symb^{-1}$

$$\hat{A} = \sum_i \hat{A}_{\phi_i}$$

We can further decompose

$$\hat{A}_{\phi_i} = \sum_j A_{j,\phi_i} |j\rangle_{\phi_i} \langle j|_{\phi_i}$$

(5)

where the $|j\rangle_{\phi_i}$ are the corresponding eigenvectors of the local $N + 1$-CSO of $D_{\phi_i} \subset D_{\phi_i}$ where a local $N + 1$-CSO is defined. So

$$\hat{A} = \sum_{ij} A_{j,\phi_i} |j\rangle_{\phi_i} \langle j|_{\phi_i}$$

all over $\mathcal{M}$. It can be proved that for $i \neq k$ it is

$$\langle j|_{\phi_i} |j|_{\phi_k} = 0$$

so the last decomposition is orthonormal, thus decomposition (5) generalizes the usual eigen-decomposition of integrable system to the non-integrable case. We will use this decomposition below.

B. Decoherence in the energy.

a.- Let us define in each $D_{\phi_i}$ a local $N + 1$-CSO $\{\hat{H}, \hat{O}_{\phi_i}\}$ (as we have said we consider that $\hat{H}$ is always globally defined) as

$$\hat{H} = \int_0^\infty \omega \sum_{im} |\omega, m\rangle_{\phi_i} \langle \omega, m|_{\phi_i} d\omega,$$

$$\hat{O}_{\phi_i} = \int_0^\infty \sum_m O_{m,\phi_i} |\omega, m\rangle_{\phi_i} \langle \omega, m|_{\phi_i} d\omega$$

where we have used decomposition (5). The energy spectrum is $0 \leq \omega < \infty$ and $m_{l_{\phi_i}} = \{m_{1_{\phi_i}}, ..., m_{N_{\phi_i}}\}, m_{l_{\phi_i}} \in \mathbb{N}$. Therefore

$$\hat{H}|\omega, m\rangle_{\phi_i} = \omega |\omega, m\rangle_{\phi_i}, \quad \hat{O}_{\phi_i}|\omega, m\rangle_{\phi_i} = O_{m,\phi_i} |\omega, m\rangle_{\phi_i}.$$ 

where, from the orthonormality of the eigenvector and eq. (5), we have

$$\langle \omega, m|_{\phi_i} |\omega', m'\rangle_{\phi_j} = \delta(\omega - \omega')\delta_{mm'}\delta_{ij}$$

b.- A generic observable, in the orthonormal basis just defined, reads:

$$\hat{O} = \sum_{imm'} \int_0^\infty \int_0^\infty d\omega d\omega' \tilde{O}(\omega, \omega')_{\phi_i, m, m'} |\omega, m\rangle_{\phi_i} \langle \omega', m'|_{\phi_i}$$

where $\tilde{O}(\omega, \omega')_{\phi_i, m, m'}$ is a generic kernel or distribution in $\omega, \omega'$. As explained in the introduction, the simplest choice to solve our problem is the van Hove choice [4].

$$\tilde{O}(\omega, \omega')_{\phi_i, m, m'} = O(\omega)_{\phi_i, mm'} \delta(\omega - \omega') + O(\omega, \omega')_{\phi_i, mm'}$$

(6)

where we have a singular and a regular term, so called because the first one contains a Dirac delta and in the second one the $O(\omega, \omega')_{\phi_i, m, m'}$ are ordinary functions of the real variables $\omega$ and $\omega'$. As we will see these two parts appear in every formulae below. So our operators belong to an algebra $\hat{A}$ and they read
The observables are the self adjoint \( O^\dagger = O \) operators. These observables belong to a space \( \mathcal{O} \subset \mathcal{A} \). This space has the basis \( \{|\omega, m, m'\rangle_{\phi_i}, \{\omega', m, m'\rangle_{\phi_i}\} \) defined as:

\[
|\omega, m, m'\rangle_{\phi_i} = |\omega, m\rangle_{\phi_i} (\omega, m')_{\phi_i}, \quad |\omega, \omega', m, m'\rangle_{\phi_i} = |\omega, m\rangle_{\phi_i} (\omega', m')_{\phi_i},
\]

where only the diagonal-singular terms remain showing that the system has decohered in \( \mathcal{A} \). This space has \( \mathcal{S} \subset \mathcal{O} \), where \( \mathcal{S} \) is a convex set. The basis of \( \mathcal{O} \) is \( \{\langle \omega, mm'\rangle_{\phi_i}, \langle \omega', mm'\rangle_{\phi_i}\} \) and its vectors are defined as functionals by the equations:

\[
|\omega, m, m'\rangle_{\phi_i}|\eta, n, n'\rangle_{\phi_j} = \delta(\omega - \eta)\delta_{m'n'}\delta_{ij}, \quad |\omega', m, m'\rangle_{\phi_i}|\eta, \eta', n, n'\rangle_{\phi_j} = \delta(\omega' - \eta)\delta_{m'n'}\delta_{ij}.
\]

and all others \( \langle ., . \rangle \) are zero. Then, a generic quantum state reads:

\[
\hat{\rho} = \sum_{im} \int_0^\infty d\omega \rho(\omega)_{\phi_i mm'} |\omega, mm'\rangle_{\phi_i} \langle \omega, mm'|_{\phi_i} + \sum_{imm'} \int_0^\infty d\omega \int_0^\infty d\omega' \rho(\omega, \omega')_{\phi_i mm'} |\omega, mm'\rangle_{\phi_i} \langle \omega', mm'|_{\phi_i}
\]

We require that:

\[
\frac{d}{dt} \rho(\omega, \omega')_{\phi_i mm'} = \rho(\omega', \omega)_{\phi_i mm'}, \quad \rho(\omega, \omega)_{\phi_i mm} \geq 0, \quad \langle \hat{\rho} | \hat{I} \rangle = \sum_{im} \int_0^\infty d\omega \rho(\omega)_{\phi_i} = 1,
\]

(7)

where \( \hat{I} = \int_0^\infty d\omega \sum_{im} |\omega, m\rangle_{\phi_i} \langle \omega, m|_{\phi_i} \) is the identity operator. Then, in fact, \( \hat{\rho} \in \mathcal{S} \), where \( \mathcal{S} \) is a convex set, and we have

\[
\langle \hat{O} \rangle_{\hat{\rho}(t)} = \langle \hat{\rho}(t) | \hat{O} \rangle = \sum_{im} \int_0^\infty d\omega \rho(\omega)_{\phi_i mm'} O(\omega)_{\phi_i mm'} + \sum_{imm'} \int_0^\infty d\omega \int_0^\infty d\omega' \rho(\omega, \omega')_{\phi_i mm'} e^{i(\omega - \omega')t/\hbar} O(\omega, \omega')_{\phi_i mm'}
\]

(8)

If we now take the limit \( t \to \infty \) and use the Riemann-Lebesgue theorem, being \( O(\omega, \omega') \) and \( \rho(\omega, \omega')_{\phi_i mm'} \) regular (namely \( \rho(\omega, \omega')_{\phi_i mm'} O(\omega, \omega') \in L_1 \) in the variable \( \nu = \omega - \omega' \)), we arrive to

\[
\lim_{t \to \infty} \langle \hat{O} \rangle_{\hat{\rho}(t)} = \langle \hat{\rho}_s | \hat{O} \rangle = \sum_{imm'} \int_0^\infty d\omega \rho(\omega)_{\phi_i mm'} O(\omega)_{\phi_i mm'}
\]

or to the weak limit

\[
W \lim_{t \to \infty} \hat{\rho}(t) = \hat{\rho}_s = \sum_{imm'} \int_0^\infty d\omega \rho(\omega)_{\phi_i mm'} (\omega, m, m')_{\phi_i}
\]

where only the diagonal-singular terms remain showing that the system has decohered in the energy.

Remarks

i.- It looks like that decoherence takes place without a coarse-graining, or an environment. It is not so, the van Hove choice (6) and the mean value (8) are a restriction of the information as effective as the coarse-graining is to produce decoherence.

ii.- Theoretically decoherence takes place at \( t \to \infty \). Nevertheless, for atomic interactions, the characteristic decoherence time is \( t_D = 10^{-15} \) sec [5]. For macroscopic systems this time is even smaller (e.g. \( 10^{-38} \) sec). Models with two characteristic times (decoherence and relaxation) can also be considered [6].

C. Decoherence in the other variables.

By a change of basis we can diagonalize the \( \rho(\omega)_{\phi_i mm'} \) in \( m \) and \( m' \):

\[
\rho(\omega)_{\phi_i mm'} \rightarrow \rho(\omega)_{\phi_i pp'} = \rho(\omega)_{\phi_i} \delta_{pp'}.
\]
in a new basis orthonormal \( \{ |\omega, p\rangle_{\phi} \} \). Therefore \( \rho_{\phi_i}(\omega) \delta_{pp'} \) is now diagonal in all its coordinates in a final local pointer basis in each \( D_{\phi_i} \), which, in the case of the observables is \( \{ |\omega, p, p'\rangle_{\phi_i}, |\omega', p, p'\rangle_{\phi_i} \} \) (i.e. essentially \( \{ |\omega', p'\rangle_{\phi_i}, \} \)), so in this pointer basis we have obtained a boolean quantum mechanic with no interference terms and we have the weak limit:

\[
W \lim_{t \to \infty} \hat{\rho}(t) = \hat{\rho}_* = \sum_{i} \int_{0}^{\infty} d\omega \rho_{\phi_i}(\omega) \rho_{\phi_i}(\omega, p, p|_{\phi_i})
\]

or in the case of \( \hat{P} \) with continuous spectra:

\[
W \lim_{t \to \infty} \hat{\rho}(t) = \hat{\rho}_* = \sum_{i} \int_{0}^{\infty} d\omega \int_{|p|_{\phi_i}}^{\infty} dp \rho_{\phi_i}(\omega, p, p|_{\phi_i})
\]

the only case that we will consider below.

**IV. THE CLASSICAL STATISTICAL LIMIT.**

a.- Let us now take into account the Wigner transforms. *There is no problem for regular operators* which are considered in the standard theory. Moreover these operators are irrelevant since they disappear after decoherence.

b.- So we must only consider the singular ones as

\[
\hat{O}_S = \sum_{i} \int_{|p|_{\phi_i}}^{\infty} dp \int_{0}^{\infty} O_{\phi_i}(\omega, p|\omega, p, \phi_i) d\omega
\]

where now the \( \hat{P} \) have continuous spectra. So

\[
\hat{O}_S = \sum_{i} O_{\phi_i}(H, P_{\phi_i}) = \sum_{i} \hat{O}_{S\phi_i}
\]

But \( \hat{H}, P_{\phi_i} \) commute thus

\[
symb\hat{O}_S = O_S(\phi) = \sum_{i} O_{\phi_i}(H, P_{\phi_i}(\phi)) + 0(\hbar^2)
\]

and if \( O_{\phi_i}(\omega, p) = \delta(\omega - \omega')\delta(p - p') \) we have

\[
symb|\omega', p'|_{\phi_i}, \omega'\phi_i = \delta(H(\phi) - \omega')(P_{\phi_i}(\phi) - p)
\]

(really up to \( 0(\hbar^2) \), but for the sake of simplicity we will eliminate these symbols from now on).

Let us now consider the singular dual, the *symb\hat{O}_S* as the functional on \( \mathcal{M} \) that must satisfy eq. (2) that now reads

\[
(symb\hat{O}_S|symb\hat{O}_S) = (\hat{\rho}_S|\hat{O}_S)
\]

Then we define a density function \( \rho_S(\phi) = symb\hat{O}_S = \sum_{i} \rho_{S\phi_i}(\phi) \) such that

\[
\sum_{i} \int d\phi d^{2(N+1)} \rho_{S\phi_i}(\phi) O_{S\phi_i}(\phi) = \sum_{i} \int_{|p|_{\phi_i}}^{\infty} \rho_{\phi_i}(\omega, p) O_{\phi_i}(\omega, p) d\omega dp
\]

\( \hat{\rho}_S \) is constant of the motion, so \( \rho_{\phi_i}(\phi) = f(H(\phi), P_{\phi_i}(\phi)) \). Then we locally define at \( D_{\phi_i} \) the local action-angle variables \( (\theta^0, \theta^1, \ldots, \theta^N, J^0_{\phi_i}, J^1_{\phi_i}, \ldots, J^N_{\phi_i}) \), where \( J^0_{\phi_i}, J^1_{\phi_i}, \ldots, J^N_{\phi_i} \) would just be \( H, P_{\phi_i1}, \ldots, P_{\phi_iN} \) and we make the *canonical transformation* \( \phi^0 \rightarrow \theta^0_{\phi_i}, \theta^1_{\phi_i}, \ldots, \theta^N_{\phi_i}, H, P_{\phi_i1}, \ldots, P_{\phi_iN} \) so that

\[
d\phi^0 d^{2(N+1)} = dq^{(N+1)} dp^{(N+1)} = d\theta_{\phi_i} dH dP_{\phi_i}^N
\]

Now we will integrate of the functions \( f(H, P_{\phi_i}) = f(H, P_{\phi_i}, \ldots, P_{\phi_i}) \) using the new variables.

\[
\int_{D_{\phi_i}} d\phi^0 d^{2(N+1)} f(H, P_{\phi_i}) = \int_{D_{\phi_i}} d\phi^0 dH dP_{\phi_i}^N f(H, P_{\phi_i}) = \int_{D_{\phi_i}} dH dP_{\phi_i}^N f(H, P_{\phi_i})
\]
where we have integrated the angular variables $\theta_0^i, \theta_1^i, \ldots, \theta_N^i$, obtaining the configuration volume $C_{\phi_i}(H, P_{\phi_i})$ of the portion of the hypersurface defined by $(H = \text{const.}, P_{\phi_i} = \text{const.})$ and contained in $D_{\phi_i}$. So eq. (10) reads

$$
\sum_{i} \int_{prD_{\phi_i}} \int_{0}^{\infty} \rho_{\phi_i}(\omega, p)O_{\phi_i}(\omega, p)d\omega dp^N = \sum_{i} \int dHdP_{\phi_i}C_{\phi_i}(H, P_{\phi_i})\rho_{\phi_i}(S(H, P_{\phi_i})O_{\phi_i}(S(H, P_{\phi_i}))
$$

for any $O_{\phi_i}(\omega, p)$ so $\rho_{S\phi_i}(H, P) = \frac{1}{c_{\phi_i}}\rho_{\phi_i}(H, P)$ for $\phi \in D_{\phi_i}$, and

$$
\rho_S(\phi) = \rho_*(\phi) = \sum_{i} \frac{\rho_{\phi_i}(H(\phi), P_{\phi_i}(\phi))}{C_{\phi_i}(H, P_{\phi_i})}
$$

Putting $\rho_{\phi_i}(\omega, p) = \delta(\omega - \omega')\delta^N(p - p')$ for some $i$ and all other $\rho_{\phi_j}(\omega, p) = 0$ for $j \neq i$, we have

$$
symb(\omega', p', \phi)_{\phi_i} = \frac{\delta(H(\phi) - \omega') \delta^N(P(\phi) - p')}{C_{\phi_i}(H, P_{\phi_i})}
$$

c.- Moreover the symb of eq.(9) reads

$$
\rho_S(\phi) = \rho_*(\phi) = \sum_{i} \int_{prD_{\phi_i}} dp \int_{0}^{\infty} d\omega \rho_{\phi_i}(\omega, p) \frac{\delta(H(\phi) - \omega) \delta^N(P(\phi) - p_i)}{C_{\phi_i}(H, P_{\phi_i})}
$$

(11)

So we have obtained a decomposition of $\rho_*(\phi) = \rho_S(\phi)$ in classical hypersurfaces $(H = \omega, P_{\phi_i}(\phi) = p_{\phi_i})$, containing chaotic trajectories (since the system is not integrable), summed with different weight coefficients $\rho_{\phi_i}(\omega, p) / C_{\phi_i}(H, P_{\phi_i})$.

d.- Finally only after decoherence the positive definite diagonal-singular part remains and from eqs. (12) and (11) we see that

$$
\rho_{\phi_i}(\omega, p) \geq 0 \Rightarrow \rho_*(\phi) \geq 0
$$

so the classical statistical limit is obtained.

V. THE CLASSICAL LIMIT.

The classical limit can be decomposed in the following processes

Quantum Mechanics → (decoherence) → Boolean Quantum Mechanics → (symb and $\hbar \rightarrow 0$) →

Classical Statistical Mechanics → (choice of a trajectory) → Classical Mechanics

where the first two have been explained. It only remains the last one: For $\tau(\phi) = \theta_0^\phi(\phi)$ and at any fixed $t$ we have

$$
\sum_{i} \int_{D_{\phi_i}} \delta(\tau(\phi) - \tau_0 - \omega t)\delta(\theta_{\phi_i}(\phi) - \theta_{\phi_i,0} - p_{\phi_i}t)d\tau_0d\theta_{\phi_i,0} = 1
$$

then we can include this 1 in decomposition (11) and we obtain

$$
\rho_*(\phi) = \sum_{i} \int \frac{\rho_{\phi_i}(\omega, p_{\phi_i})}{C(\omega, p_{\phi_i})} \delta(H(\phi) - \omega)\delta(P_{\phi_i} - p_{\phi_i})\delta(\tau(\phi) - \tau_0 - \omega t)\delta(\theta_{\phi_i}(\phi) - \theta_{\phi_i,0} - p_{\phi_i}t)d\omega dp_{\phi_i}d\tau_0d\theta_{\phi_i,0}
$$

namely a sum of classical chaotic trajectories satisfying:

$$
H(\phi) = \omega, \tau(\phi) = \tau_0 + \omega t, \quad P_{\phi_i} = p_{\phi_i}, \theta_{\phi_i}(\phi) = \theta_{\phi_i,0} + p_{\phi_i}t
$$

weighted by $\frac{\rho_{\phi_i}(\omega, p_{\phi_i})}{C(\omega, p_{\phi_i})}$, where we can choose any one of them. In this way the classical limit is completed, in fact we have found the classical limit of a quantum system since we have obtained the classical trajectories, so the correspondence principle is also obtained as a theorem.
VI. CONCLUSION.

i.- We have defined the classical limit in the non-integrable case.

ii.- Essentially we have presented a minimal formalism for quantum chaos [7].

iii.- We have deduced the correspondence principle.

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