Modern Michelson-Morley experiments
and gravitationally-induced anisotropy of $c$

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Abstract

The recent, precise Michelson-Morley experiment performed by Müller et al. suggests a tiny anisotropy of the speed of light. I propose a quantitative explanation of the observed effect based on the interpretation of gravity as a density fluctuation of the Higgs condensate.
1. The aim of this Letter is to propose a quantitative explanation of the tiny anisotropy of the speed of light suggested by the recent, precise Michelson-Morley experiment of Müller et al. [1]. Their result can be conveniently expressed in the form

$$B^{\text{exp}} = (-3.1 \pm 1.6) \cdot 10^{-9}$$  \hspace{1cm} (1)

where \(B\) enters the Robertson-Mansouri-Sexl [2] parametrization for the two-way speed of light \((c = 2.9979 \ldots 10^{10} \text{ cm/sec})\)

$$\frac{\bar{c}}{c} = 1 - (A + B \sin^2 \theta) \frac{v^2}{c^2}$$  \hspace{1cm} (2)

in a reference frame \(S'\) that moves with speed \(v\) with respect to a preferred frame \(\Sigma\), \(\theta\) denoting the angle between the direction of \(v\) and the direction of the light beam.

2. In order to explain the experimental result reported in Eq. (1), as a first step, I’ll adopt the tentative idea that light propagates in a medium with refractive index \(N_{\text{medium}} > 1\) so that there is a small Fresnel’s drag coefficient \(1 - \frac{1}{N^2_{\text{medium}}} \ll 1\). This provides a general framework to analyze any Michelson-Morley type of experiment (see refs. [4, 5]). In our case, where the medium is the vacuum itself, the physical interpretation of \(N_{\text{vacuum}}\) will represent a second step and provide a quantitative estimate to be compared with Eq. (1).

In this perspective, I’ll start introducing an isotropic speed of light

$$u = \frac{c}{N_{\text{medium}}}$$  \hspace{1cm} (3)

doing to the ideal case of a medium that extends to infinity in all spatial directions. Real experiments, however, are performed in finite portions of medium that might even reduce to just fill the arms of an interferometer. In this situation, an observer placed on the earth’s surface has no argument to think that light should propagate with the same velocity \(u\) in all directions.

However, one may adopt the point of view that any observed anisotropy is due to the earth’s motion with respect to a preferred frame \(\Sigma\) where light propagates isotropically. In this case, if \(\Sigma\) were identified with the cosmic background radiation, one expects the relevant value of the earth’s velocity to be \(v_{\text{earth}} \sim 365 \text{ km/sec}\). By adopting this point of view, and recalling that Lorentz transformations are valid both in Special and Lorentzian Relativity, for a frame \(S'\), moving with respect to \(\Sigma\) with velocity \(v\), light will be seen to propagate at a speed \((\gamma = 1/\sqrt{1 - v^2/c^2})\)

$$u' = \frac{u - \gamma v + \nu(\gamma - 1) \frac{\nu u}{c^2}}{\gamma(1 - \frac{\nu u}{c^2})}$$  \hspace{1cm} (4)
To second order in $v/u$, one obtains ($\theta$ denotes the angle between $v$ and $u$)

$$\frac{u'(\theta)}{u} = 1 - \alpha \frac{v}{u} - \beta \frac{v^2}{u^2}$$  \hspace{1cm} (5)

where

$$\alpha = (1 - \frac{1}{N_{\text{medium}}^2}) \cos \theta + O((N_{\text{medium}}^2 - 1)^2)$$  \hspace{1cm} (6)

$$\beta = (1 - \frac{1}{N_{\text{medium}}^2}) P_2(\cos \theta) + O((N_{\text{medium}}^2 - 1)^2)$$  \hspace{1cm} (7)

with $P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1)$.

Thus, the two-way speed of light is

$$\frac{\bar{u}'(\theta)}{u} = \frac{1}{u} \frac{2u'(\theta)u'(\pi - \theta)}{u'(\theta) + u'(\pi - \theta)} = 1 - \frac{v^2}{c^2}(A + B \sin^2 \theta)$$  \hspace{1cm} (8)

where

$$A = N_{\text{medium}}^2 - 1 + O((N_{\text{medium}}^2 - 1)^2)$$  \hspace{1cm} (9)

and

$$B = -\frac{3}{2}(N_{\text{medium}}^2 - 1) + O((N_{\text{medium}}^2 - 1)^2)$$  \hspace{1cm} (10)

In this way, using the experimental values $N_{\text{air}} \sim 1.00029$ or $N_{\text{helium}} \sim 1.000036$, one can re-analyze [4, 5] the classical ‘ether-drift’ experiments. For instance, by defining

$$v_{\text{earth}} \sqrt{N_{\text{medium}}^2 - 1} = v_{\text{obs}}$$  \hspace{1cm} (11)

(and an in-air-operating optical system) one predicts $v_{\text{obs}} \sim 9$ km/sec for $v_{\text{earth}} \sim 365$ km/sec, in good agreement with Miller’s results [6].

3. To compare with Eq. (11) I’ll now try to provide a quantitative estimate of $N_{\text{vacuum}}$, to be used in Eq. (10), starting from the idea of a ‘condensed’ vacuum, as generally accepted in modern elementary particle physics. Indeed, in the physically relevant case of the Standard Model, the situation can be summarized saying [7] that "What we experience as empty space is nothing but the configuration of the Higgs field that has the lowest possible energy. If we move from field jargon to particle jargon, this means that empty space is actually filled with Higgs particles. They have Bose condensed."

In this case, where the condensing quanta are just neutral spinless particles (the ‘phions’ [8]), the translation from ‘field jargon to particle jargon’, amounts to establish a well defined functional relation (see ref. [8]) $n = n(\phi^2)$ between the average particle density $n$ in the $k = 0$
mode and the average value of the scalar field $\langle \Phi \rangle = \phi$. Thus, Bose condensation is just a consequence of the minimization condition of the effective potential $V_{\text{eff}}(\phi)$. This has absolute minima at some values $\phi = \pm v \neq 0$ for which $n(v^2) = \tilde{n} \neq 0$.

The symmetric phase, where $\phi = 0$ and $n = 0$, will eventually be re-established at a phase transition temperature $T = T_c$. This, in the Standard Model, is so high that one can safely approximate the ordinary vacuum as a zero-temperature system. Thus, the vacuum might be compared to a quantum Bose liquid, a medium where bodies can flow without any apparent friction, as in superfluid $^4$He, in agreement with the experimental results.

On the other hand, the condensed particle-physics vacuum, while certainly different from the ether of classical physics, is also different from the ‘empty’ space-time of Special Relativity which is assumed at the base of axiomatic quantum field theory. Therefore, following this line of thought, the macroscopic occupation of the same quantum state ($k = 0$ in a given reference frame) can represent the operative construction of a ‘quantum ether’ whose existence might be detected through a precise ‘ether-drift’ experiment.

On a more formal ground we observe that the coexistence of exact Lorentz covariance and vacuum condensation in effective quantum field theories is not so trivial. In fact, as a consequence of the violations of locality at the energy scale fixed by the ultraviolet cutoff $\Lambda$, one may be faced with non-Lorentz-covariant infrared effects that depend on the vacuum structure.

To indicate this type of infrared-ultraviolet connection, originating from vacuum condensation in effective quantum field theories, Volovik has introduced a very appropriate name: reentrant violations of special relativity in the low-energy corner. In the simplest case of spontaneous symmetry breaking in a $\lambda \Phi^4$ theory, where the condensing quanta are just neutral spinless particles, the ‘reentrant’ effects reduce to a small shell of three-momenta, say $|k| < \delta$, where the energy spectrum deviates from a Lorentz-covariant form. Namely, by denoting $M_H$ as the typical energy scale associated with the Lorentz-covariant part of the energy spectrum, one finds $\frac{\delta M_H}{M_H} \rightarrow 0$ only when $\frac{M_H}{\Lambda} \rightarrow 0$.

The basic ingredient to detect such ‘reentrant’ effects in the broken phase consists in a purely quantum-field-theoretical result: the connected zero-four-momentum propagator $G^{-1}(k = 0)$ is a two-valued function. In fact, besides the well known solution $G^{-1}_a(k = 0) = M_H^2$, one also finds $G^{-1}_b(k = 0) = 0$.

The b-type of solution corresponds to processes where absorbing (or emitting) a very small 3-momentum $k \rightarrow 0$ does not cost any finite energy. This situation is well known in
a condensed medium, where a small 3-momentum can be coherently distributed among a large number of elementary constituents, and corresponds to the hydrodynamical regime of density fluctuations whose wavelengths $2\pi/|k|$ are \textit{larger} than $r_{\text{mfp}}$, the mean free path for the elementary constituents.

This interpretation \cite{13,14} of the gap-less branch, which is very natural on the base of general arguments, is unavoidable in a superfluid medium. In fact, ”Any quantum liquid consisting of particles with integral spin (such as the liquid isotope $^4\text{He}$) must certainly have a spectrum of this type...In a quantum Bose liquid, elementary excitations with small momenta $k$ (wavelengths large compared with distances between atoms) correspond to ordinary hydrodynamic sound waves, i.e. are phonons. This means that the energy of such quasi-particles is a linear function of their momentum” \cite{15}. In this sense, a superfluid vacuum provides for $k \to 0$ a universal picture. This result does not depend on the details of the short-distance interaction and even on the nature of the elementary constituents. For instance, the same coarse-grained description is found in superfluid fermionic vacua \cite{16} that, as compared to the Higgs vacuum, bear the same relation of superfluid $^3\text{He}$ to superfluid $^4\text{He}$.

Thus there are two possible types of excitations with the same quantum numbers but different energies when the 3-momentum $k \to 0$: a single-particle massive one, with $E_a(k) \to M_H$, and a collective gap-less one with $E_b(k) \to 0$. In this sense, the situation is very similar to superfluid $^4\text{He}$, where the observed energy spectrum is due to the peculiar transition from the ‘phonon branch’ to the ‘roton branch’ at a momentum scale $|k_o|$ where $E_{\text{phonon}}(k_o) \sim E_{\text{roton}}(k_o)$. The analog for the scalar condensate amounts to an energy spectrum with the following limiting behaviours:

\begin{enumerate}
  \item $E(k) \to E_b(k) \sim c_s |k|$ for $k \to 0$
  \item $E(k) \to E_a(k) \sim M_H + \frac{k^2}{2M_H}$ for $|k| \gtrsim \delta$
\end{enumerate}

where the characteristic momentum scale $\delta \ll M_H$, at which $E_a(\delta) \sim E_b(\delta)$, marks the transition from collective to single-particle excitations. This occurs for

$$\delta \sim 1/r_{\text{mfp}}$$

where \cite{17,18}

$$r_{\text{mfp}} \sim \frac{1}{\bar{n}a^2}$$

is the phion mean free path, for a given value of the phion density $n = \bar{n}$ and a given value of the phion-phion scattering length $a$. In terms of the same quantities, one also finds \cite{8}

$$M_H^2 \sim \bar{n}a$$

(14)
giving the trend of the dimensionless ratios \( \frac{\delta}{M_H} \sim \frac{M_H}{\Lambda} \sim \sqrt{\bar{n}a^3} \to 0 \) (15) in the continuum limit where \( a \to 0 \) and the mass scale \( \bar{n}a \) is held fixed.

By taking into account the above results, the physical decomposition of the scalar field in the broken phase can be conveniently expressed as (phys=`physical’) \[19\]

\[ \Phi_{\text{phys}}(x) = v_R + h(x) + H(x) \] (16)

with

\[ h(x) = \sum_{|k|<\delta} \frac{1}{\sqrt{2V}} \left[ \tilde{h}_k e^{i(k \cdot x - E_k t)} + (\tilde{h}_k)^\dagger e^{-i(k \cdot x - E_k t)} \right] \] (17)

and

\[ H(x) = \sum_{|k|>\delta} \frac{1}{\sqrt{2V}} \left[ \tilde{H}_k e^{i(k \cdot x - E_k t)} + (\tilde{H}_k)^\dagger e^{-i(k \cdot x - E_k t)} \right] \] (18)

where \( V \) is the quantization volume and \( E_k = c_s|k| \) for \( |k| < \delta \) while \( E_k = \sqrt{k^2 + M_H^2} \) for \( |k| > \delta \). Also, \( c_s\delta \sim M_H \).

Eqs. (16)-(18) replace the more conventional relations

\[ \Phi_{\text{phys}}(x) = v_R + H(x) \] (19)

where

\[ H(x) = \sum_k \frac{1}{\sqrt{2V}} \left[ \tilde{H}_k e^{i(k \cdot x - E_k t)} + (\tilde{H}_k)^\dagger e^{-i(k \cdot x - E_k t)} \right] \] (20)

with \( E_k = \sqrt{k^2 + M_H^2} \). Eqs. (19) and (20) are reobtained in the limit \( \frac{\delta}{M_H} \sim \frac{M_H}{\Lambda} \to 0 \) where the wavelengths associated to \( h(x) \) become infinitely large in units of the physical scale set by \( \xi_H = 1/M_H \). In this limit, where for any finite value of \( k \) the broken phase has only massive excitations, one recovers an exactly Lorentz-covariant theory.

4. In conclusion, for finite values of \( \Lambda \) there are long-wavelength density fluctuations of the vacuum and Lorentz-covariance is not exact. Therefore, in the presence of such effects, one can try to detect the existence of the scalar condensate through a precise ‘ether-drift’ experiment. To this end, I observe that a simple physical interpretation of the long-wavelength density fluctuation field

\[ \varphi(x) \equiv \frac{h(x)}{v_R} \] (21)
has been proposed in refs. [13, 14]. Introducing \( G_F \equiv 1/v_R^2 \) and choosing the momentum scale \( \delta \) as

\[
\delta = \sqrt{\frac{G_N M^2 H}{G_F}} \quad (22)
\]

\((G_N \text{ being the Newton constant})\) one obtains the identification

\[
\varphi(x) = U_N(x) + \text{const.} \quad (23)
\]

\(U_N(x)\) being the Newton potential. Indeed, with the choice in Eq. (22), to first order in \( \varphi \) and in the limits of slow motions, the equations of motion for \( \varphi \) reduce to the Poisson equation for the Newton potential \( U_N \) [13, 14] so that the deviations from Lorentz covariance are of gravitational strength. If, as in the Standard Model, \( G_F \) is taken to be the Fermi constant one then finds \( \delta \sim 10^{-5} \text{ eV} \) and \( r_{\text{mfp}} \sim 1/\delta = O(1) \text{ cm} \). As anticipated, the variation of \( \varphi(x) \) takes place over distances that are larger than \( r_{\text{mfp}} \) and thus infinitely large on the elementary particle scale. Also, by introducing \( M_{\text{Planck}} = \sqrt{G_N M^2 H} \), and using Eqs. (15) and (22), one finds \( \Lambda = q_H M_{\text{Planck}} \) with \( q_H = \sqrt{G_F M^2 H} = O(1) \), or \( a \sim 1/\Lambda \sim 10^{-33} \text{ cm} \).

At the same time, to first order, the observable effects of \( \varphi \) can be re-absorbed [14] into an effective metric structure

\[
ds^2 = (1 + 2\varphi)dt^2 - (1 - 2\varphi)(dx^2 + dy^2 + dz^2) \quad (24)
\]

that agrees with the first approximation to the line element of General Relativity [20, 21]. In this perspective, the space-time curvature arises from two sources: i) a re-scaling of the length and time units associated with the modification of any particle mass and ii) a refractive index for light propagation

\[
N_{\text{vacuum}} \sim 1 - 2\varphi \quad (25)
\]

needed to preserve the basic particle-wave duality which is intrinsic in the nature of light.

Now, for a centrally symmetric field, and up to a constant, one has \( \varphi(R) = -\frac{G_N M}{e^2 R} \). Therefore, \( \varphi_{\text{earth}} \sim -0.7 \cdot 10^{-9} \) (for \( M = M_{\text{earth}} \) and \( R = R_{\text{earth}} \)) so that, using Eq. (10), I would estimate

\[
B^{\text{th}} \sim -4.2 \cdot 10^{-9} \quad (26)
\]

in good agreement with the experimental result in Eq. (1).

5. Summarizing: the vacuum is not ‘empty’ so that one should check the consistency between exact Lorentz covariance and vacuum condensation in effective quantum field theories.
For the specific case of the scalar condensate, the non-locality associated with the presence of the ultraviolet cutoff will also show up at long wavelengths in the form of non-Lorentz-covariant density fluctuations associated with a scalar function $\varphi(x)$.

If, on the base of refs. [13, 14], these long-wavelength effects are naturally interpreted in terms of the Newton potential $U_N$ (with the identification $\varphi = U_N + \text{const}$), one obtains the weak-field space-time curvature of General Relativity and a refractive index $N_{\text{vacuum}} \sim 1 - 2\varphi$. This value of $N_{\text{vacuum}}$, leading to the prediction in Eq. (20), can help to understand the experimental result Eq. (1) obtained by Müller et al. [1].

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[19] By ‘physical’ we mean that the normalization of the vacuum field $v_R$ is such that the quadratic shape of $V_{\text{eff}}(\phi_R)$ at $\phi_R = v_R$ is precisely given by the physical Higgs mass $M_H^2$.

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