Complexity Dynamic Analysis of Fractional-order Permanent Magnet Synchronous Motor in CNC Machine Tool

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Abstract. In this paper, permanent magnet synchronous motor system for fractional-order machine tools is studied. The nonlinear term of the system is decomposed by Adomian decomposition method, and the attractor and poincare cross-section of the system are drawn by Matalb. Meanwhile, the system is analyzed by numerical simulation including bifurcation diagram, SE complexity, C0 complexity and attractor phase diagram under parameter change, which further reveals the realizable dynamic characteristics of fractional-order chaotic system. The related research results lay a good foundation for the control of CNC (Computer Numerical Control) machine tools.

1. Introduction
At present, the cutting vibration of machine tools not only directly affects the quality and efficiency of machining, but also adversely affects the service life of machine tools and tools, and also produces noise that deteriorates the environment. Avoiding and suppressing chatter is an important problem in designing, manufacturing and using machine tools [1]. The vibration of traditional machine tools has been studied in many articles [2~5]. Permanent magnet variable speed motor has the characteristics of strong nonlinearity, strong coupling and partial switching, etc. When disturbed by external environment changes or internal parameters change, chaotic behaviors such as fast and slow rotating speed or heating will occur, which seriously affects the stability of numerical control system, so the study of chaotic characteristics in machine tools is of great significance [3,4].

In recent years, the study of fractional-order chaotic systems has become one of the main fields of nonlinear system research. Nowadays, there are many achievements in the research of fractional-order synchronous control, but few achievements in the analysis of complex oscillation behavior of fractional-order systems. At present, the numerical calculation based on Cpu fractional calculus mainly includes frequency domain method, prediction-correction method and Adomian decomposition method (ADM). In papers [8, 9], the fractional order model will be approximated as a high-order integer order model by using the complex frequency method, and the complex oscillation behavior and circuit design of fractional Lorenz and fractional super Lorenz chaotic systems are carried out respectively. In document [10], the dynamic behavior of fractional super Lü chaotic systems is numerically simulated by using the predictive correction method (ABM). Ref [7], the simplified fractional Lorenz chaotic system is numerically simulated by using ADM. Ref [11], the chaotic characteristics of simplified fractional Lorenz are performed by using improved Adomian. There are many achievements in the synchronization control of fractional order chaotic systems, such as
adaptive synchronization control [12, 13], backstepping control [14], generalized synchronization control [15] and so on.

2. Solution of Fractional Order Permanent Magnet Synchronous Motor (PMSG) Model

In this paper, the dynamic model equation of NC machine tool motor is [4].

\[
\begin{align*}
L_d \frac{di_d}{dt} &= -R_i i_d + \omega_e L_q i_q + u_d \\
L_q \frac{di_q}{dt} &= -R_i i_q - \omega_e L_d i_d - \Phi \omega_x - u_q \\
J_{eq} \frac{d\omega_x}{dt} &= T_e - T_x - B \omega_x
\end{align*}
\]

In the above formula: \(i_d, i_q, u_d, u_q, L_d, L_q\) is the current, voltage and inductance components of the direct axis (d) and the quadrature axis (q) of the stator respectively; \(R_s\) is stator resistance; \(\omega_e\) is the electrical angular frequency, and \(\omega_x\) is the generator speed; \(J_{eq}\) is the equivalent moment of inertia of the unit; \(\Phi\) is magnetic flux; \(T_e\) is the rotating magnetic torque; \(B\) is the rotational viscosity coefficient of the generator.

Assuming that the air gap of the generator is uniform, and the inductance of d-axis and q-axis are the same, considering the transient condition that the motor is started for a short time (\(\vec{u}_d=0, \vec{u}_q=0, \vec{T}_w=0\)), taking into account the system damping and the inductance fractional order factor in the system, the fractional order permanent magnet synchronous motor model is as follows [1]:

\[
\begin{align*}
\frac{d^\gamma x}{dt^\gamma} &= \sigma (y - x) \\
\frac{d^\gamma y}{dt^\gamma} &= -y - zx + \gamma x \\
\frac{d^\gamma z}{dt^\gamma} &= -z + xy
\end{align*}
\]

Based on ADM, the equation solution of the system is obtained:

\[
\begin{align*}
x(t) &= c_0 + c_1 \left( t - t_0 \right)^y + c_2 \left( t - t_0 \right)^y \frac{y}{(q + 1)} + c_3 \left( t - t_0 \right)^q \frac{y}{(2q + 1)} + c_4 \left( t - t_0 \right)^q \frac{y}{(3q + 1)} + c_5 \left( t - t_0 \right)^q \frac{y}{(4q + 1)} + c_6 \left( t - t_0 \right)^q \frac{y}{(5q + 1)} \\
y(t) &= c_7 + c_8 \left( t - t_0 \right)^y + c_9 \left( t - t_0 \right)^{2q} \frac{y}{(2q + 1)} + c_{10} \left( t - t_0 \right)^q \frac{y}{(3q + 1)} + c_{11} \left( t - t_0 \right)^q \frac{y}{(4q + 1)} + c_{12} \left( t - t_0 \right)^q \frac{y}{(5q + 1)} \\
z(t) &= c_{13} + c_{14} \left( t - t_0 \right)^y + c_{15} \left( t - t_0 \right)^{2q} \frac{y}{(2q + 1)} + c_{16} \left( t - t_0 \right)^q \frac{y}{(3q + 1)} + c_{17} \left( t - t_0 \right)^q \frac{y}{(4q + 1)} + c_{18} \left( t - t_0 \right)^q \frac{y}{(5q + 1)}
\end{align*}
\]

Where \(x, y, z\) is system variables and \(\gamma, \sigma, q\) is parameter of system. When \(\gamma=20, \sigma=5.46, q=0.8\), according to formula (3), the analytical solution of the system can be obtained, and the phase diagram of the system (2), i.e., the existence of chaotic attractors, can be obtained by numerical simulation using Matlab, as shown in Figure 1.
3. Nonlinear dynamic analysis

In order to study the influence of parameters on fractional order permanent magnet motor, especially the influence of fractional order $Q$ on the system, this paper uses bifurcation diagram and complexity ($SE/C_0$) to describe it.

3.1 Influence of Order $q$ on System

Fixed parameters $\gamma=20, \sigma=5.46$, when $q \in [0.65, 1]$, the bifurcation diagram and complexity of the system are shown in Figure 2. When $q \in (0.7, 1]$, the system is in chaotic state, and the corresponding complexity $C_0/SE$ is higher. It can also be seen that in the chaotic region, as $q$ increases, the complexity of the system decreases, that is, as $q$ increases, the probability of chaos decreases. If the system is used in encryption system, the analysis and implementation of fractional system is very important. At the same time, it is concluded that the system is order 0.7, that is, this system is of order 2.1, which can produce chaos.

Fixed parameter $a = 3, c = 9$, changed parameter $b$, when $b \in [0, 3]$, the complexity of system (2) and the global bifurcation diagram of state variable $x(t)$ are shown in Figure 2.
3.2 The change of parameter $\sigma$

Fixed parameters $\gamma=20, q=0.8$, when $\sigma \in [1,8]$, the bifurcation diagram and complexity of the system are shown in Figure 3. It can be seen from Figure 3 that the system enters chaotic behavior with quasi-period. The system is in a periodic state, and the complexity of the system corresponding to this region is relatively small, while other regions are in a chaotic state, and the complexity of the self-system is relatively large, so it can be seen that the bifurcation diagram of the system is basically consistent with the complexity of the system.
3.3 The change of parameter \( \gamma \)

Fixed parameters \( \sigma=5.46, q=0.8 \), when \( \gamma \in [5,30] \), the bifurcation diagram and complexity of the system are shown in Figure 4. It can be seen from Figure 4 that the system enters chaos in the same way as the above two parameters, and there are three system periodic switching in the interval when \( \gamma \in [5,15] \), and the switching point is obvious, such as \( \gamma=10 \), and it can be seen that the system bifurcation diagram and system complexity can prove each other.

![Bifurcation diagram](image1.png) ![CO complexity](image2.png) ![SE complexity](image3.png)

Figure 4. Bifurcation diagram and complexity of system (1) when \( \gamma \) changes

4. Conclusion

This paper applies the basic theory of fractional calculus, based on the Adomian decomposition method, and uses numerical simulations such as bifurcation diagram, SE complexity and C0 complexity to analyze the basic dynamic behavior of a 0.8 order chaotic system. The following two conclusions are drawn:

(1) The dynamic behavior of permanent magnet synchronous motor in fractional order NC machine tools is obviously different from that of integer order system, and the complexity of fractional order system is higher.

(2) For the fractional-order CNC machine tool motor system, the probability that the periodic state is the convergence point is greater than the integer order.

Of course, the control of fractional-order permanent magnet motor system, especially the synchronous control of dual motors, needs further research, analysis and design.
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