The mathematical model of an astrophysical jet simulation by the laboratory facility “plasma focus”

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Abstract. One of the most interesting properties of astrophysical jets is their propagation on the distances, which are many times greater than their diameters. Physical processes being in the basis of that behavior are not clearly understood by us. In addition, the majority of astrophysical parameters cannot be measured and stay unknown. It complicates a verification of any theoretical models. Using of Z-pinch facilities allows to carry out well controlled and well monitored experiments of laboratory jet investigation with the same scaling parameters as an astrophysical jets. Thereby we are able to observe processes, which are unobtainable for direct astronomical observations. Because of a large nonlinearity of the problem the creating of analytical theory, as a rule, is limited to some qualitative assessments. Therefore numerical simulations play a huge role for studying of dynamics of jets propagation. In this work results of numerical calculations of ideal magnetohydrodynamics equations, which describe jet propagation in PF-3 laboratory facility, are presented. We have made conclusions about parameters, influencing on collimated jet moving through ambient plasma. Differences of dynamics between single jet and continuous flow of matter have been discussed.

1. Introduction

According to the modern views the most part of observed matter in the universe consists of plasma: stars, galaxy cores, interstellar and intergalactic space, nebulae, accretion disks surround compact objects etc. Usually those objects have a very complicated nonlinear dynamic and it can be difficult to reveal some mechanisms leading to observed features. The situation is exacerbated with a low precision of observations of astrophysical parameters. Ordinarily it is able to get an accuracy about one or two orders. But in such dynamical systems qualitative changes of their structure may take place even with small changes of its parameters. Because of this we are not always able to say which regime does an astrophysical object work in. Therefore an carrying out of laboratory experiments with astrophysical parameters is very momentous for understanding of astrophysical objects dynamic because in such experiments it is possible to measure more precisely a lot of parameters.

Astrophysical parameters of propagation of nonrelativistic jets may be achieved in laboratory experiments using lasers [1] and Z-pinch [2]. Those astrophysical jets are streams of matter which are emitted along the axis of rotation of a compact object like young star object (YSO).
Table 1. Parameters of observed and laboratory jets.

| Parameter                        | Astrophysics observation | PF-3 facility |
|----------------------------------|--------------------------|---------------|
| Reynolds number, $Re$            | $> 10^8$                 | $10^3 - 10^4$ |
| Mach number, $Ma$                | $\gg 5$                  | 10            |
| Peclet number, $Pe$              | $> 10^7$                 | $10^8$        |
| Magnetic Reynolds number, $Rm$   | $> 10^{15}$              | $10 - 10^2$   |
| Density contrast, $n_{jet}/n_{ambient}$ | $> 1$               | 4             |

Matter appears from accretion disk with compact object collimation and propagates along the axis of rotation on the distance which is many times greater than jet diameter. The mechanism of that forming is not fully understood (see [3]) and, for this moment, cannot be modeled with a laboratory facility. But jet propagation can be reproduced with, for example, our laboratory installation PF-3 (Kurchatov Institute) [4].

We set ourselves the aim to investigate theoretically the dynamic of laboratory jet using our numerical code. Comparing calculated results with experimental data, we have been verifying that code and mathematical model which is being in the basis of the code. Modeling results could give us detail picture of the flow structure. This can be used for comparison between some features of astrophysical and laboratory jets.

2. The experimental facility

PF-3 is Filippov-type plasma focus facility and is known as a source of intensive plasma flows. Recently the wide cycle experiments were undertaken on studies of plasma flow parameters. To solve this problem the PF-3 facility was upgraded. A new diagnostic drift chamber with manifold set of diagnostic tools was designed to measure the jet and the ambient plasma parameters at the distances up to 100 cm from the place of jet generation (figure 1). These experiments show that PF-3 facility may be effectively used for laboratory modeling astrophysical jets.

![Figure 1. The scheme of the laboratory facility PF-3.](image-url)
The comparison between common dimensionless parameters of cosmic jets [5] and observed parameters of PF-3 [6] [7] is presented at table 1. These parameters do not have the same value but it is assumed that it is sufficient the same parameters would be simultaneously much more or much less than one.

3. The mathematical model

The ideal MHD is believed to be a good approximation of both laboratory and astrophysical jets. But there is an open question about magnetic viscosity of laboratory jet. The matter is viscosities depend on ionization degree of plasma and they could change widely. Changing of dynamic viscosity cannot make a big influence on the character of flow because for each changing Reynolds number does not fall lower than $10^3$. But if degree of ionization is low magnetic Reynolds number may be $\sim 10$. It may have an impact on the flow. This question is enough important and it requires addition investigations.

At first we have considered only ideal MHD equations having the form:

$$\frac{\partial \rho}{\partial t} = -\text{div} \rho \mathbf{v},$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v}, \nabla) \mathbf{v} = -\nabla p - \frac{1}{4\pi} [\mathbf{H} \times \text{rot} \mathbf{H}],$$

$$\frac{\partial \mathbf{H}}{\partial t} = \text{rot} [\mathbf{v} \times \mathbf{H}],$$

$$\text{div} \mathbf{H} = 0,$$

$$\frac{\partial e}{\partial t} = -\text{div} \left( \mathbf{v} \left( e + p + \frac{\mathbf{H}^2}{8\pi} \right) - \frac{\mathbf{H}(\mathbf{v} \cdot \mathbf{H})}{4\pi} \right) + S,$$

$$p = (\gamma - 1) \left( e - \frac{\rho v^2}{2} - \frac{\mathbf{H}^2}{8\pi} \right),$$

where $e = \rho e + \frac{\rho v^2}{2} + \frac{\mathbf{H}^2}{8\pi}$ is full internal energy.

For solving these equations we have used our own Godunov type scheme in two dimensions cylindrical coordinates with the well proven solver HLLD [8]. At first we have performed the calculations for hydrogen because for it a cooling function is well known. As initial conditions we have chosen ellipsoidal shape of the jet with the sizes as in PF-3 experiments (see figure 2a). Temperatures of the jet and its ambient have been taken as 1.1 eV and 1 eV respectively. In case of this choice of thermodynamic values all observed parameters are achieved after several steps of calculation. Magnetic field has been taken like measured: it grows from zero to jet border linearly and decrease from jet border like $r^{-1}$ [6].

Let’s consider jet modeling with PF-3 parameters: $n_{\text{jet}} = 3 \cdot 10^{17} \text{cm}^{-3}$, $V_{\text{z, jet}} = 5 \cdot 10^6 \text{cm s}^{-1}$, $B_{\phi, \text{jet}} = 4.5 \cdot 10^3 \text{Gs}$ and the achieved temperature $T_{\text{jet}} \sim 6 \text{eV}$. The concentration of ambient gas is $n_{\text{ambient}} = 10^{17} \text{cm}^{-3}$ [6] [7]. On the figures 2b and 2c we can see that the jet collimation occurs, but some matter is located far from the jet and it forms a mushroom. This is a classical picture of shock waves but because of presence of the magnetic field the most part of matter is located within the jet. If the magnetic field was absent all matter would be within the mushroom.

During jet flight sometimes small pieces of plasma move down relative to the jet (it is well seen on the figure 3a). This phenomenon takes place because of density of a shock front is more and its speed is low than ones behind the shock front. In this case matter from the down part of the jet pushes off from the shock front like peas from a wall. In astrophysical conditions such process may lead to the forming of nodosum structure of jet. Also the same structure may appearers because of a forming jet mechanism. The question about when and which does mechanism work is still open.
The most important for jet collimation fact which we can see on the figures 2 and 3 is after jet passing behind it an area with low plasma density remains. This area pulls ambient matter in. For continuous flow it could lead to collimation of a part of flow behind a shock front, because it will be pulled in that area. It means a collimation of continuous flow is possible even without magnetic field. See also [9].

Now let’s consider compares between jets with different parameters. On figure 3a two jets with lower (left) and higher (right) density contrast are shown. As we can see the mushroom of the high density contrast jet contains less matter. It happens because of this jet experiences less resistance of ambient plasma. In astrophysical conditions a density contrast may achieve...
very big values. Therefore it may be one of important factors leading to collimated flow. Figure 3b shows us difference between jets with different velocities. The fast jet (right) does not have a lot of collimated matter. Its mushroom has more plasma than slow one (left). This obvious result tells us collimation of faster jets requires higher values of control parameters.

4. Conclusions
The astrophysical jet parameters have been achieved with PF-3 and numerical simulations. The morphologies and the most part of parameters of YSO jets agree with the experimental and calculated ones as well. It has been found axial magnetic field prevents forming of a mushroom instability. Also it has been shown the observed nodosum structure may be produced with a jet own motion. The emptiness formation behind the jet passing has been observed. This may be very important factor for collimation of a continuous flow. The comparison between jets with different parameters show us the importance of a density contrast relation. Also from the comparison it can be concluded for collimation of fast jets it requires higher values of parameters which are charge for this. All these features must be investigated in the future.

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References
[1] Albertazzi B et al. 2014 Science 6207 325-8
[2] Suzuki-Vidal F et al. 2012 J. Phys.: Conf. Ser. 370 0120002
[3] Beskin V S 2010 Physics-Uspekhi 53 1199-235
[4] Krauz V et al. 2015 Proceedings of the 42nd EPS Conf. on Plasma Phys 39E 4.401
[5] Ciardi A 2010 Lect. Notes Phys. 793 3150
[6] Mitrofanov K N et al. 2014 JETP 119 910-23
[7] Ananyev S S et al. 2016 Plasma Phys Rep. 42 269-78
[8] Miyoshi T and Kusano K 2005 J. Comput. Phys. 208 315-44
[9] Frank A 1998 AIP Conf.Proc. 431 513-20