Numerical simulation of flow on circular crested stepped spillway

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Abstract

The spillways are one of the most important hydraulic structures used in river engineering, dam construction, irrigation, and drainage engineering projects. Recently, a new type of such spillways with a circular crest has been proposed. In this paper, the hydraulic properties of the circular crested stepped spillway (CCSS) including flow pattern, distribution of velocity on the crest and pressure, turbulence intensity, discharge coefficient ($C_d$) and energy dissipation ratio (EDR) were investigated numerically. To model the free surface of flow the volume of fluid technique, and for modeling the turbulence of flow, $k-\varepsilon$ (RNG) was utilized. Results declared that there is a good agreement between the laboratory observations and numerical simulation. The $C_d$ of the CCSS changes between 0.9 and 1.4 considering the range of relative upstream head ($h_{up}/R$) between 0.33 m and 2.67. The observation of the flow streamlines showed that they are tangential to the curvature of the crest and there is no separation of the flow from the crest. Examination of the pressure distribution on the CCSS model shows that just downstream part of the crest, the pressure is partially negative. Of course, the same partial negative pressure is observed on the edge of the steps. The steps increase the maximum intensity turbulence by 50%. The CCSS can dissipate the energy of flow between 90 and 30%, and in the skimming flow regime, the portion of each step in the energy dissipation regardless of their position is almost identical.

Keywords CFD simulation · Discharge capacity · Energy dissipation · Skimming flow · Spillway

List of symbols

- $h_s$ : Height of steps
- $h_{up}$ : Head of flow over
- $C_d$ : Discharge coefficient
- $H_{down}$ : Total head of flow at upstream of spillway
- $H_{up}$ : Total head of flow downstream of the spillway
- $L_s$ : Length of steps
- $V_1$ : Flow velocity downstream of the spillway
- $V_{up}$ : Flow velocity at upstream of spillway
- $y_{up}$ : Depth of flow over
- CCSS : Circular crested stepped spillway
- CFD : Computational fluid dynamic
- EDR : Energy dissipation ratio
- $N$ : Number of steps
- $S$ : Slope of chute
- $V$ : $H$: Vertical; horizontal
- VOF : Volume of fluid
- $y_c$ : Critical depth of flow
- $P$ : Height of spillway
- $R$ : Radius of crest
- $g$ : Acceleration due to gravity
- $q$ : Discharge per width

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Introduction

Modeling the important hydraulic structures such as spillways always is one of the fundamental steps in hydraulic engineering projects such as dam construction and river diversion works. The design of spillways in each project according to their specific conditions can be a unique case. Although there are technical manuals for designing spillways, the final plan should be tested using a physical model (Hager and Pfister 2010; Bagheri and Kabiri-Samani 2020a; Haghiabi et al. 2018). The modeling aims to investigate the hydraulic properties including the flow pattern, discharge capacity (determine the rating curve), and distribution of velocity and pressure of flow through the spillway. Over the past decades, the hydraulic properties of various components of the spillway, including the guide walls (Wang and Chen 2010), approach channel (Zhang et al. 2015; Parsaie et al. 2018), crest (Mohammadzadeh-Habili et al. 2016; Shamsi et al. 2022; Parsaie and Haghiabi 2021), chute (Parsaie et al. 2016) and energy dissipator located at the toe of the chute (Valero et al. 2018), have been investigated using physical and numerical simulation. Today, the use of numerical simulation as a powerful tool in the design of spillways and evaluation of their alternative plans is common. In this regard, numerous studies have been published, most of which confirm the accuracy of numerical modeling results (Ghaderi et al. 2020a, 2020b; Rahimzadeh et al. 2012; Tadayon and Ramamurthy 2009; Bai et al. 2017; Zhan et al. 2016; Mohammad Rezapour Tabari and Tavakoli 2016; Bagheri and Kabiri-Samani 2020b). The velocity of the flow on the spillway’s chute is very high, which increases the risk of cavitation as well as scouring downstream of the spillway. Spreading downstream scour to the toe of the spillway may damage the dam body (Nou et al. 2021; Ghaderi et al. 2020b; Haghiabi 2017; Samadi et al. 2015). One of the effective ways to dissipate the energy of the flow passing through the spillway’s chute is to step its surface (Parsaie and Haghiabi 2019a), which both cause energy dissipation and increase the aeration of the flow (Parsaie and Haghiabi 2019c).

Recently, a circular crest stepped spillway (CCSS) due to having high $C_d$, suitable ability in energy dissipation and flow aeration has been proposed (Parsaie and Haghiabi 2019b). Hence, in this study, the hydraulic properties of the CCSS including flow pattern according to the flow streamlines, the distribution of flow velocity on the crest and pressure on the stepped chute, and energy dissipation are investigated using computational fluid dynamic (CFD) method.

Materials and methods

A CCSS consists of a circular crest that its curvature is an arc of a circle as well as a downstream stepped chute. The CCSS as shown in Fig. 1 is categorized as a short crested weir. In this figure, $h_s$ and $L_s$ are the height and length of step, respectively. $R$ is the radius of crest. $P$ is the height of spillway. $h_{up}$ and $y_{up}$ are the head and depth of flow over the crest upstream. Equation 1 can demonstrate the discharge capacity of the short crested weirs (Bos 1976). In this equation, $q$ is the discharge per width, $g$ is acceleration due to gravity, and $C_d$ is the discharge coefficient.

$$q = \frac{2}{3} C_d h_{up} \sqrt{\frac{2}{3} g h_{up}}$$  \hspace{1cm} (1)

To estimate the performance of CCSS regarding energy dissipation, the total head of flow downstream (Eq. $2: H_{down}$) minus the total head flow upstream (Eq. $2: H_{up}$). The ratio of their difference to the $H_{up}$ (Eq. 4) is defined as the performance of CCSS regarding energy dissipation (Haghiabi et al. 2022).
\[ H_{up} = P + y_{up} + \frac{V_{up}^2}{2g} = P + y_{up} + \frac{q^2}{2g(P + y_{up})^2} \] (2)

\[ H_{down} = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1} \] (3)

\[ \Delta H = \frac{H_{up} - H_{down}}{H_{up}} = \left(1 - \frac{H_{down}}{H_{up}}\right) \times 100 \] (4)

where the \( V_{up} \) and \( V_1 \) are the flow velocity at the upstream and downstream of the CCSS.

**Experimental setups**

The laboratory investigation on hydraulic models of the CCSS was carried out in a flume of 12 m long, 0.5 m wide and 0.45 m deep. The sidewalls of the flume were made of Plexiglas. The water surface profile in the flume was measured using a point gauge. The flow discharge was measured by a calibrated triangular weir located at the end of the flume. The height of the models was 0.3 m. The radius of the crest of the models was considered to be 0.06 m. The slope of the downstream step chute equal to (V: H) was 1:1.25. The models were made of concrete. The details of the smooth and CCSS models and the range of discharge of flow during the test are presented in Table 1.

**CFD modeling**

In this study, the numerical modeling of hydraulic properties of flow over the CCSS was performed using Flow 3D. The Flow 3D uses the finite volume method for the discretization of the governing equations of flow (Navier–Stokes equations). The strategy of this software to discretize the computational domain is using the structured mesh. The volume of fluid (VOF) method is utilized for modeling the free surface of the flow. The standard flow equations such as continuity equations (Eq. 5) and Navier–Stokes equations (Eq. 6) are solved numerically for all computational domains (Moukalled et al. 2015; Parsaie et al. 2016).

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_A) + \frac{\partial}{\partial y}(\rho v_A) = \frac{PSOR}{\rho} \] (5)

where \( u, v, z \) are the velocity components in the x, y and w directions. \( A_x, A_y \) are the cross-sectional area of the flow, \( \rho \) is fluid density, \( PSOR \) is the source term, \( v_j \) is the volume fraction of the fluid and three-dimensional momentum equations given in Eq. (6).

\[ \frac{\partial u}{\partial t} + \frac{1}{v_j} \left( uA_x \frac{\partial u}{\partial x} + vA_y \frac{\partial u}{\partial y} + wA_z \frac{\partial u}{\partial z} \right) = \frac{1}{\rho} \frac{\partial P}{\partial x} + G_x + f_x \]

\[ \frac{\partial v}{\partial t} + \frac{1}{v_j} \left( uA_x \frac{\partial v}{\partial x} + vA_y \frac{\partial v}{\partial y} + wA_z \frac{\partial v}{\partial z} \right) = \frac{1}{\rho} \frac{\partial P}{\partial y} + G_y + f_y \] (6)

where \( P \) is the fluid pressure, \( G_x, G_y \) the acceleration created by body fluids, \( f_x, f_y \) viscosity acceleration in three dimensions and \( v_j \) is related to the volume of fluid, defined by Eq. (7).

\[ \frac{\partial F}{\partial t} + \frac{1}{v_j} \left( \frac{\partial}{\partial x}(FA_xu) + \frac{\partial}{\partial y}(FA_yv) \right) = 0 \] (7)

To model turbulent flow, numbers of turbulence models including one-equation turbulent model (Prandtl mixing length), two types of two-equation k-\( \varepsilon \) models, renormalized group model (RNG- k-\( \varepsilon \)) and large-eddy simulation (LES) have been proposed. The RNG- k-\( \varepsilon \) model is a powerful turbulence model that has suitable performance for modeling the fine vortex; therefore, this model is very useful for modeling flow pattern problems. Most equations used in the RNG model are given in Eqs. (8 and 9).

\[ \frac{\partial \rho k}{\partial t} + \frac{\partial}{\partial x}(\rho u_Ak) = \frac{\partial}{\partial x_i} \left( \alpha_k \mu_{eff} \frac{\partial k}{\partial x_i} \right) + G_k + G_b - \rho \varepsilon \] (8)

\[ \frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial}{\partial x}(\rho u_A \varepsilon) = \frac{\partial}{\partial x_i} \left( \alpha_k \mu_{eff} \frac{\partial \varepsilon}{\partial x_i} \right) + C_{1 \varepsilon} \frac{\varepsilon}{k} (G_k + C_{2 \varepsilon} G_b) \]

\[ - C_{2 \varepsilon} \rho \frac{\varepsilon^2}{k} - R \] (9)

in which \( G_k \) is the rate of kinetic energy creation, and \( R \) is the density of turbulence defined as below.
In these equations $\beta = 0.012, \eta_0 = 1.38$.

**CFD model Setup**

To simulate the flow over the CCSS, the number of steps including the definition of the two-dimensional models, definition and meshing of the computational domain, setting the boundary and initial conditions, and selecting the turbulence model should be followed. In this case, the upstream flow head and output were selected as the upstream and downstream boundary conditions, respectively. The symmetry was chosen as the boundary condition of the upper faces of the computational domain. For the lower face of the computational domain, the wall boundary condition was chosen. In this study, the $k - \varepsilon$ (RNG) model is used as the turbulence model. The size of the meshes was equal to 1.0 cm (horizontal) $\times$ 0.5 cm (vertical). These mesh dimensions are chosen to meet the criteria of mesh independence.

**Results and discussion**

In this section, the results of numerical simulation are presented. For Flow 3D validation, the water surface profile, flow head and flow discharge were used. Figure 2 shows the measured water surface profile and the results of numerical simulation in the validation stage.

Fig. 2 The results of the measured water surface profile and the numerical simulation
In this stage, the measured piezometric pressure and results of numerical simulation are plotted in Fig. 3. In these figures, the calibration results for the smooth and stepped model are shown. The examination of the calibration results shows that there is good agreement between the numerical results and the laboratory measurements.

In Fig. 4a and b, the measured discharge capacity, discharge coefficient ($C_d$) and the results of numerical simulations are shown, as well. In this figure, only the size of the steps is changed and it has been attempted to consider the slope of the stepped chute as constant. This figure shows that the geometric properties of the steps including size and slope of steps do not affect the discharge coefficient and the stage–discharge relation. These results support the theory of supercritical flow which defines that in the supercritical flow, the downstream cannot affect the upstream flow properties.

One of the most important reasons for the high value of $C_d$ of CCSS is the curvature of its crest. This curvature prevents the separation of the streamlines from the surface of the crest and results in a significant reduction in the energy loss of the flow passing over the crest part. Figure 5 shows the streamlines over the CCSS. As shown in this figure, all the streamlines are tangential to the curvature of the crest surface. Figure 5b shows the streamlines on the stepped chute. As can be seen from this figure, a vortex flow is formed between the stairs, which also reduces the flow energy. The examination of Fig. 5a shows that the flow regime at the upstream part of the crest is subcritical and the other part (downstream part) is supercritical. In other words, the critical point is formed on the crest. According to the subcritical flow theory, the crest geometry (crest radius) can affect the $C_d$. It should be noted that the critical depth increases with increasing flow discharge and scrolls upstream. This means that by increasing the discharge of flow the effect of the
Fig. 5 The streamline and Froude number of flow over the crest (a) and chute (b) of CCSS.

Fig. 6 The distribution of pressure over the smooth (a) and stepped chute (b).
upstream part of the crest on the $C_d$ is decreased. For this reason, as shown in Fig. 4, the $C_d$ remains relatively constant after the $\frac{y_u}{R} > 0.05$ m ($\frac{y_u}{R} \approx 1$).

Figure 6 illustrates the pressure distribution over the circular crested spillway with a smooth (Fig. 6a) and stepped chute (Fig. 6b). Although the flow velocity at the downstream part of the crest increases, the main cause of the negative pressure zone is the sudden change in the direction of flow velocity and the formation of the wake region. Of course, this decrease in pressure even down to negative pressure values can be justified. It should be noted that the negative pressure in this section does not lead to cavitation and structural degradation. However, it leads to an increase in the discharge capacity due to a rise in the suction zone of flow. This is reminiscent of the criteria of the head design of the Ogee spillway, where the design head can be considered up to 1.5 times the head of the design flood discharge without any problems in the crest part. As the flow passes on the smooth chute, the flow velocity increases, and subsequently, the flow pressure decreases according to the energy equation. As the flow reaches the stilling basin or energy dissipator structure due to the collision of the flow with the surface of the concrete slab, the flow energy dissipation increases. It is noteworthy that the energy dissipation is reflected in the decrease in flow velocity and the amount of pressure at the toe of the chute increases. In the following, the distribution of pressure over the CCSS is investigated. As shown in Fig. 6, the value of pressure on the surface of
steps is positive and at their edges is negative. Of course, it should be checked whether this amount of negative pressure will damage the structure and how much of it (negative) is allowed. To do this, the cavitation index should be calculated. In other words, this will investigate the potential of the CCSS for cavitation to occur. According to Fig. 6, the inlet flow pressure to the stilling basin downstream of the stepped spillway is lower than that of the smooth chute spillway, which is due to the greater dissipation of the flow energy through the stepped chute. The results show that as the size of the steps increases, the energy dissipation of flow increases. This is because the intensity of the turbulence of the flow increases with the increase of the step size.

The performance of CCSS in terms of energy dissipation is shown in Fig. 7. As presented in this figure, the CCSS can dissipate the flow energy between 90 and 30%. The stepped chute can dissipate energy about 50% more than the smooth chute even in the skimming flow regime. As presented in this figure, by increasing the discharge, the performance of smooth and stepped chute decreases significantly. Of course, the intensity of the reduction of smooth chute performance is more than the stepped chute. In a smooth chute with an increasing flow rate, its performance decreases by about 91%, while in the same range of increasing flow, the performance of a stepped chute decreases by about 37%.

The effect of each part of CCSS including the crest and steps on the energy dissipation is shown in Fig. 8. As shown in this figure the portion of the crest in the energy dissipation is negligible. That reason is the least disturbance in the flow path (streamlines) due to its proper curvature of the crest. When the flow enters the first step, a considerable amount of its energy is dissipated compared to the crest part. At the low value of discharges, the energy loss is due to the collision of the flow with the surface of the steps. By increasing the flow discharge as shown in Fig. 5a, the vortex flow is generated between the steps. The structure of the vertex flows from a larger view is shown in Fig. 5b. After the formation of the vortex flow in the steps, a pseudo-boundary is formed between the steps and the main flow jet that reduces the effect of the steps' size on energy dissipation. This mode of flow is called the skimming flow regime. The $h_s/L_s$ of laboratory models was 0.8, 1.0 and 1.33; hence, when the ratio of $y_c/h_s$ is more than the 0.5, the skimming flow is formed. Theis finding confirms with criteria presented by Rajaratnam (1990), Chamani and Rajaratnam (1999), and James et al. (2001). After the full development of flow on the stepped chute and formation of the skimming flow on the steps, the
portion of each of the steps regardless of their position in the energy dissipation is almost identical. In other words, with the increase of the discharge, the contribution of each step to the energy dissipating decreases. Increasing the step size increases the intensity of flow turbulence, which further increases the rate of flow energy dissipation. It should be noted that the total head of flow must decrease during the spillway, but as seen, on each of the steps, the head of flow firstly decreases and then increases. This is because the direction of velocity in the vortex flow that forms on the steps is the opposite of the flow direction. Moving from the vortex flow to the tip of the steps, the direction of flow is synchronized with the overall direction of the flow passing through the stepped chute, As a result, the flow head is increased relative to the areas affected by the vortex flow.

The effect of the step size as the roughness of the chute on the turbulence intensity is shown in Fig. 9c and d. As shown in these figures, the maximum turbulence intensity of flow on the smooth chute is about 40, whereas this value is increased by up to 48 for the smallest steps, and as the steps become larger, this value is increased to about 78. As the size of the steps increases, the intensity of the flow turbulence increases, which in turn increases the amount of energy loss caused by them.

Conclusion

In this study, the hydraulic properties of circular crested stepped spillways including the pattern of streamlines, distribution of pressure, rating curve, discharge coefficient and mechanism of energy dissipation were investigated. Results declared that the length and curvature of the crest make it possible for the streamlines to adapt to the surface of the crest. The results also showed that crest curvature does not lead to dissipation of the flow energy. The downstream part of the crest performs the smooth transfer of flow from the crest to the stepped chute. By the formation of the skimming flow regime, the contribution of the steps to the energy dissipation decreases, and their portion in the energy dissipation is almost identical regardless of their position. As the step size increases, the intensity of the flow turbulence increases, resulting in a greater amount of energy dissipation.

Author contribution Dr. Haghiabi conceived of the presented idea; Dr. Parsaie developed the theory and performed the computations; Dr. Suleiman Shareef, Dr. Irzooki and Dr. Khalaf performed experimental tests.

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Data availability Some or all data, models or code that supports the findings of this study is available from the corresponding author upon reasonable request.

Declarations

Competing interests The authors declare that they have no competing interests.

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Applied Water Science (2022) 12: 215 1 3
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