Simulation of the process of conduction of thermal energy transfer through the multilayer wall of the apparatus

A I Tuishchev, T P Tretyakova, T S Ozerova and Yu P Kulakova
Togliatti State University, 445020, Togliatti, ul. Belarusian 14
E-mail kaf_ttp@mail.ru

Abstract. We consider a multilayer wall of the apparatus, consisting of dissimilar materials, having heat loading during heat transfer in the form of conduction (thermal conductivity). Using the basic law of thermal conductivity, mathematical expressions are derived that makes it possible to calculate the basic indicators of thermal conductivity for various multilayer walls consisting of different materials. As a demonstration of the application of the above approach, an example of calculating the equivalent coefficient of thermal conductivity and specific heat flux of a printed circuit board is given.

Thermal conductivity (conduction) is the process of transfer of thermal energy that takes place in spacecraft, in equipment of the chemical, food, electronic industries, from areas with a higher temperature to areas with a lower temperature or when two solids come in contact with different temperatures.

It is known that the process of thermal conductivity is associated with concepts such as temperature field and temperature gradient. Mathematically, the temperature $\theta$ can be written as a function of the coordinates of the three-dimensional space $x$, $y$, $z$ and time $t$.

$$\theta = f(x, y, z, t)$$ (1)

When considering issues related to the thermal conductivity of materials used in various industries, as a rule, the temperature field is considered as a set of temperature values for all points of the space under study in a certain period of time [1]. Distinguish between three-dimensional, two-dimensional and one-dimensional space.

For a one-dimensional space, equation (1) can be represented as

$$\theta = f(x)$$ (2)

It is known that the locus of points having the same temperature forms an isothermal surface. Since at the same point in space there cannot be two different temperatures, it can be said that isothermal surfaces do not intersect with each other - all of them either end at the boundary of the body, or become closed in themselves [2]. Therefore, a change in body temperature can take place only in directions crossing isothermal surfaces (Figure 1)
The largest change in temperature $\theta$ takes place in the direction of the normal $n$ to the isothermal surface [3].

$$\lim_{\Delta n \to 0} \left( \frac{\Delta \theta}{\Delta n} \right) = \frac{\partial \theta}{\partial n} = \text{grad} \theta = \nabla \theta \quad (3)$$

where $\nabla$ is the mathematical symbol of the gradient.

The temperature gradient is a vector having a normal direction with respect to the isothermal surface in the direction of increasing temperature. Heat transfer is carried out only in the direction of decreasing temperature. We denote the amount of heat transmitting through an arbitrary surface per unit time, through $Q$, and the specific heat flux - $q$.

Specific heat flux is the amount of heat (heat flux) per unit surface. The heat flux density coincides with the direction of heat at a given point in the body and is opposite to the direction of the heat gradient vector (figure 2) [4].

The basic law of thermal conductivity is called the Fourier law

$$q = -\lambda \text{grad} \theta = -\lambda \Delta \theta \quad (4)$$

where $\lambda$ is the coefficient of thermal conductivity, W / m °C

Coefficient $\lambda$ characterizes the ability of a given material to conduct heat

---

Figure 1. Isothermal surface and temperature gradient.

Figure 2. Temperature gradient and specific heat flux.
\[ \lambda = - \frac{|q|}{\text{grad} \theta} = \frac{Q}{S \cdot \text{t} \cdot \Delta \theta / \Delta x} \]  

where \( Q \) is the heat flux; \( S \) is the area; \( t \) is the time; \( \Delta \theta \) is the temperature change; \( \Delta x \) is the distance between the considered isothermal surfaces.

So for dry air at \( \theta = 20 \, ^\circ\text{C} \) and normal pressure \( \lambda = 0.0276 \, \text{W/m} \, ^\circ\text{C} \); for water \( \lambda = 0.597 \, \text{W/m} \, ^\circ\text{C} \); get in ax \( \lambda = 0.17 \, \text{W/m} \, ^\circ\text{C} \); PCB \( \lambda = 0.27 \, \text{W/m} \, ^\circ\text{C} \); silver \( \lambda = 4.20 \, \text{W/m} \, ^\circ\text{C} \); copper \( \lambda = 3.90 \, \text{W/m} \, ^\circ\text{C} \); gold \( \lambda = 3.10 \, \text{W/m} \, ^\circ\text{C} \); aluminum \( \lambda = 2.30 \, \text{W/m} \, ^\circ\text{C} \); brass \( \lambda = 1.05 \, \text{W/m} \, ^\circ\text{C} \); steel \( \lambda = 0.5 \, \text{W/m} \, ^\circ\text{C} \); mica \( \lambda = 0.5 \, \text{W/m} \, ^\circ\text{C} \); duralumin \( \lambda = 1.80 \, \text{W/m} \, ^\circ\text{C} \) [5].

The value of \( \lambda \) depends on temperature. For metals with increasing temperature, \( \lambda \) decreases.

Heat distribution can occur in a stationary mode, in which the temperature field does not change in time, and in an unsteady mode, when the temperature field depends on time.  

![Figure 3. Temperature distribution on the surface of the apparatus wall.](image)

For the stationary mode, the heat flux \( Q \) passing through a flat homogeneous wall can be found from formula (5).

\[ Q = \lambda \cdot S \cdot \frac{\Delta \theta}{\delta} \]  

where \( \lambda \) is the coefficient of thermal conductivity of the wall material, \( \text{W/m} \, ^\circ\text{C} \); \( S \) - wall surface area, \( \text{m}^2 \); \( \Delta \theta \) is the temperature difference between the wall surfaces, \( ^\circ\text{C} \); \( \delta \) is the wall thickness, \( \text{m} \), \( \delta = h_{\text{A,P}} \).

Representing the wall with the temperatures of the sides \( \theta_1 \) and \( \theta_2 \) in the form of a flat plate, we show the temperature distribution in figure 3.

Often, the outer wall for obtaining the heat transfer process is covered with foil (a descent spacecraft, printed circuit board foil, etc.).
Considering the thickness of the foil in the form of an infinitely thin layer “cut out” perpendicular to the direction of the heat flux, the specific heat flux can be written as

\[ q = -\lambda \frac{d\theta}{dx} \]  \hspace{1cm} (7)

where \( q \) is the specific heat flux, \( W / m^2 \); \( d\theta / dx \) is the temperature gradient with a small increment of the x coordinate along the foil thickness [6].

In the stationary mode, \( \theta \) and \( S \) are the same in any layer of the wall, then at \( \lambda = \text{const} \) the temperature drop \( \theta_1 - \theta_4 \) is proportional to the thickness of the considered layer \( x \) of the wall.

So, for example, a double-sided printed circuit board represents a three-layer wall, in which the surface layers have a thickness \( h_1 = \delta_1 \), and the dielectric base has a layer \( h_{A,B} = \delta_2 \).

The temperature distribution in the cross section of the three-layer wall is presented in figure 4 [4].

![Figure 4](image)

**Figure 4.** Temperature distribution in the cross section of a three-layer wall.

For stationary thermal conditions we have [6]:

\[ q = \frac{\lambda_1}{\delta_1} (\theta_1 - \theta_2); \quad q = \frac{\lambda_2}{\delta_2} (\theta_2 - \theta_3); \quad q = \frac{\lambda_1}{\delta_1} (\theta_3 - \theta_4) \]  \hspace{1cm} (8)

Adding the left and right sides of the system of equations (8), we have

\[ \theta_1 - \theta_4 = q \cdot \left( 2 \frac{\delta_1}{\lambda_1} + \delta_2 \lambda_2 \right) \]  \hspace{1cm} (9)

Therefore, the specific heat flux is

\[ q = \frac{\theta_1 - \theta_4}{2 \frac{\delta_1}{\lambda_1} + \delta_2 \lambda_2} = \frac{\lambda_{eq}}{\sum \delta} \cdot (\theta_1 - \theta_{n+1}) \]  \hspace{1cm} (10)
For a wall with n layers, the concept of the equivalent thermal conductivity coefficient $\lambda_{eq}$ is introduced, then the specific flux can be written in the form of an expression

$$q = \frac{\theta_1 - \theta_{n+1}}{\delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n}$$

(11)

where

$$\lambda_{eq} = \frac{\delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n}{\delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n} = \frac{\sum \delta}{\sum \delta}$$

Example. Determine the equivalent coefficient of thermal conductivity and the specific heat flux of the two-sided wall of the apparatus, printed circuit board at temperatures of its boundary surfaces $\theta_1 = 70^{\circ}C$ and $\theta_2 = 20^{\circ}C$; the first layer is copper foil, $h_f = \delta_1 = 50 \mu m$, $\lambda_1 = 390 \text{ W/m}^{\circ}C$; the second layer is a dielectric base with a thickness of $h_{AP} = \delta_2 = 1 \text{ mm}$, the material is fiberglass - $\lambda_2 = 0.27 \text{ W/m}^{\circ}C$; the third layer is copper foil., $h_f = \delta_1 = 50 \mu m$, $\lambda_1 = 390 \text{ W/m}^{\circ}C$.

$$\lambda_{eq} = \frac{\delta_1 + \delta_2 + \delta_3}{\delta_1 + \delta_2 + \delta_3} = \frac{50 \cdot 10^{-6} + 1 \cdot 10^{-3} + 50 \cdot 10^{-6}}{\frac{50 \cdot 10^{-6}}{390} + \frac{1 \cdot 10^{-3}}{0.27} + \frac{50 \cdot 10^{-6}}{390}} = 0.297 \text{ W/m}^{\circ}C$$

Specific heat flux through a double-sided wall

$$q = \frac{\lambda_{eq}(\theta_1 - \theta_2)}{\sum \delta} = \frac{0.297 \cdot (70 - 20)}{1.1} = 13,51 \text{ W/m}^{2}$$

When determining the heat flux, the concept of the thermal resistance of the wall is introduced, then the Fourier law takes the form of Ohm's law [8].

$$Q = \frac{\theta_1 - \theta_2}{R_T}$$

or

$$R_T = \frac{\delta}{\lambda S}$$

(13)

The inverse of thermal resistance represents thermal conductivity.

$$G_T = \frac{1}{R_T} = \lambda \cdot S \cdot \delta^{-1}$$

(14)

In general, the thermal resistance of the wall of the structure

$$R_T = \int_{x_1}^{x_2} \frac{dx}{\lambda S(x)}$$

(15)

where $dx$ is the element of the heat flux path length; $S (x)$ is the isothermal surface area in the analytical expression; $x_1$ and $x_2$ are the distance from the reference point of isothermal surfaces.

Output. The presented method for calculating the heat transfer process in the form of conduction can be used in the design of thermal processes in the space, food, chemical, and electronic industries.

References

[1] Musteykis A I and Yunakov L P 2016 Numerical solution of heat and mass transfer problems. Part 1. Thermal conductivity (Moscow: SIC INFRA-M)
[2] Kudinov I V 2013 *Theoretical foundations of heat engineering. Part II Mathematical modeling of heat conduction processes in multilayer building envelopes* (Samara: Samara State University of Architecture and Civil Engineering)

[3] Tuishchev A I 1998 *Methods and means of computer control of radiation heating* (Moscow: GASBU) p 317

[4] Tuishchev A I 2002 *Design of household electronic equipment* (Moscow /MGUS) p 488

[5] Egorov V I 2015 *Exact methods for solving heat conduction problems: a training manual* (St. Petersburg: NRU ITMO)

[6] Vlasova E A 2016 *Mathematical models of heat conduction processes: guidelines* (Moscow: MSTU. N.E. Bauman)

[7] Gritsenko V V 2014 *Processes and apparatuses of food production: A manual for full-time and part-time students of the direction* (Rubtsov Industrial Institute / Rubtsovsk)

[8] Egorov V I 2018 *Analytical methods for solving heat conduction problems: a training manual* (St. Petersburg: NRU ITMO)