Evaluation of error vector magnitude due to combined IQ imbalances and phase noise

Apostolos Georgiadis¹, Christos Kalialakis²,³

¹Department of Microwave Systems and Nanotechnology, Centre Tecnologic de Telecomunicacions de Catalunya – CTTC, Castelldefels, Barcelona 08860, Spain
²Radiocommunications Laboratory, Aristotle University of Thessaloniki, Thessaloniki, GR54124, Greece
³Spectrum Department, EETT, Thessaloniki State Airport, Thessaloniki, GR55103, Greece
E-mail: ckal@physics.auth.gr

Abstract: Novel closed form expressions for the error vector magnitude (EVM) are presented. The expressions combine the in-phase quadrature (IQ) amplitude and phase imbalances and the DC offsets along with the phase noise. Both the Gaussian and the Tikhonov probability density functions are utilised for the oscillator phase noise distribution. The explicit conditions when the EVM computations based on the Tikhonov distribution converge to a Gaussian based are investigated. Furthermore, the application of the proposed EVM expressions is demonstrated by including phase noise masks, providing a direct means to the EVM computations based on the Tikhonov distribution converge to a Gaussian based are investigated. Furthermore, the application of the proposed EVM expressions is demonstrated by including phase noise masks, providing a direct means to the EVM computations based on the Tikhonov distribution converge to a Gaussian based are investigated. Furthermore, the application of the proposed EVM expressions is demonstrated by including phase noise masks, providing a direct means to

1 Introduction

Error vector magnitude (EVM) is an important performance metric [1, 2] for the RF transceivers used in the high data rate wireless systems, such as WLAN and digital television. EVM can be derived numerically by system simulators [2, 3] and linked to the other system parameters of interest such as the bit error rate (BER) and the signal-to-noise ratio (SNR) [2, 4]. There is a great interest in the closed form solutions which provide fast results and an insight in terms of the sources of EVM degradation. EVM expressions that consider separately the effects of the IQ amplitude imbalances, the IQ phase imbalances and the phase noise have been presented in [5–13]. In this paper, the focus is on the expressions that combine the IQ amplitude, the phase, the DC offsets and the phase noise in one closed formula.

The phase noise and the IQ imbalances are the limiting factors in chipset performance. However, only the IQ amplitude and the phase imbalances can be adaptively adjusted to low values [13, 14]. The phase noise probability density function considered in the previously reported EVM expressions was Gaussian [2–13, 15] and suitable for the low-noise oscillator references. The phase noise in the phase-locked loop (PLL) systems has been shown to satisfy the Tikhonov distribution [16–18]. The Tikhonov distribution is expected to converge to a Gaussian distribution at the low noise limit [18, 19]. These convergence limits for the EVM have not yet been established. In this paper, these limits are determined in the context of the EVM by considering the analytical formulae, for both the Gaussian and the Tikhonov phase noise probability density distributions.

In Section 2, the general EVM formulation is outlined. Novel expressions based on the combined phase noise and IQ amplitude, phase imbalances and DC offsets are derived in Section 3 for a Gaussian distribution. In Section 4, the Tikhonov distribution is discussed and novel EVM expressions for the Tikhonov phase noise and the IQ amplitude and phase imbalances are derived. The comparison between the two expressions is discussed in Section 5 in order to establish concrete limits for the convergence. A validation of the proposed EVM expressions is given in Section 7 based on the measurement set-up described in Section 6 by taking advantage of the phase noise masks. Finally, the conclusions are drawn in Section 8.

2 EVM formulation

Consider a transmitter model using an IQ modulator and a local oscillator. A voltage controlled oscillator (VCO) is necessary for the multichannel operations such as Wifi systems and multiband cellular systems. A matrix system model is employed as in [5] (Fig. 1). An oscillator exhibiting a random phase noise $\phi_C$ can be represented as a matrix $R$

$$R = \begin{bmatrix} \cos(\phi_C) & -\sin(\phi_C) \\ \sin(\phi_C) & \cos(\phi_C) \end{bmatrix}$$ (1)

The modulator matrix $M$, for an amplitude imbalance $g$ and a phase imbalance $\phi$, respectively, is expressed as [5]

$$M = \sqrt{\frac{2}{1 + g^2}} \begin{bmatrix} g \cos \frac{\phi}{2} & \sin \frac{\phi}{2} \\ g \sin \frac{\phi}{2} & \cos \frac{\phi}{2} \end{bmatrix}$$ (2)
The value $1$ (in linear units) of the imbalance parameter $g$ corresponds to no amplitude imbalances. The values of the IQ amplitude imbalance up to $2$ dB and the phase imbalance of $5^\circ$ are the typical upper limits in practice [14].

The squared rms value of the EVM is derived in detail in [5] and only the result is repeated here for brevity

$$ EVM_{\text{RMS}}^2 = \frac{1}{\text{SNR}} + c^T c + 2 - \text{Tr}(H) $$

(3)

where

$$ H = R \cdot M $$

(4)

and

$$ c = \sqrt{E_s} H \cdot \alpha_{\text{DC}} $$

(5)

with $\alpha_{\text{DC}}$ being a column vector representing the effect of the DC offsets in the modulator along the $I$ and the $Q$ paths, respectively. SNR $= E_s/N_0$ is the system signal-to-noise ratio because of the thermal noise with $E_s$ being the energy per symbol and $N_0$ being the noise spectral density, respectively. In (3), the symbol $\text{Tr}(\cdot)$ represents the trace of the matrix and the superscript $'T'$ represents the transpose of the matrix. Expression (3) is independent of the constellation format as long as linear memoryless signals are transmitted. This is unlike the BER which is in general different for each modulation scheme because of the use of the $Q$ function which is related to the complementary Gaussian error function. It should also be noted that this squared rms EVM expression does not depend on the correlation properties of the noise vector [5].

Using (1) and (2) in (4), yields the matrix $H$, whose trace is

$$ \text{Tr}(H) = \sqrt{\frac{2}{1 + g^2}} \left[ (1 + g) \cos\left(\frac{\varphi}{2}\right) \cos\left(\varphi_{\text{C}}\right) \right. $$

$$ + \left. (1 - g) \sin\left(\frac{\varphi}{2}\right) \sin\left(\varphi_{\text{C}}\right) \right] $$

(6)

The second term in (3), after some lengthy but straightforward matrix calculations, by using the definition of (5) and under the assumption of the identical DC offsets in the $I$ and the $Q$ branches, yields

$$ c^T c = E_s a_{\text{DC}}^2 \left[ 2 + \frac{4 g}{1 + g^2} \sin \varphi \right] $$

(7)

On using (6) and (7) in (3), the squared rms EVM because of the combined effect of the phase noise, the IQ amplitude, the phase imbalances and the DC offsets reads as (see (8)) Considering the fact that the phase noise is stochastic in nature, the squared rms value of the EVM in (8) must be averaged over the corresponding probability density function (pdf) $\rho(\varphi_{\text{C}})$ with a phase noise variance $\sigma$

$$ EVM_{\text{RMS,AVG}}^2 = \int EVM_{\text{RMS}}^2 \rho(\varphi_{\text{C}}) \ d\varphi_{\text{C}} $$

(9)

3 Gaussian phase noise and the IQ imbalances

The local oscillator reference is commonly provided by a PLL. The pdf of the PLL phase noise can be derived as the solution of a Fokker–Planck equation [18, 20]. In the limit of the low-noise variance $\sigma$, the pdf becomes Gaussian [18]

$$ \rho_g(\varphi_{\text{C}}) = \frac{1}{\sqrt{2 \pi \sigma^2}} \exp\left(-\frac{\varphi_{\text{C}}^2}{2 \sigma^2}\right) $$

(10)

The integration of (8) over the Gaussian pdf (10) requires the integration of the third term which contains the $\cos\varphi_{\text{C}}$ and the $\sin\varphi_{\text{C}}$ terms. The integral containing the cosine term in (8) can be computed in [5] by using the integral formulae from [21] whereas the sine term integral is zero because of the odd integrand. The average squared rms EVM eventually yields

$$ EVM_{\text{RMS,AVG}}^2 = \frac{1}{\text{SNR}} + a_{\text{DC}}^2 \left[ 2 + \frac{4 g}{1 + g^2} \sin \varphi \right] $$

$$ + \frac{2}{1 + g^2} \left[ (1 + g) \cos \varphi + (1 - g) \sin \varphi \right] $$

(11)

By successively letting $\varphi = 0^\circ$, $g = 1$ (linear units), $\sigma = 0^\circ$ and no DC offsets, the expression (11) folds back to the separate EVM expressions presented for the phase noise or the imbalances only in [5]. It should be noted that the combined expression (11) is not separable in the phase noise terms or the IQ imbalances (phase, amplitude and DC offset) only components.

4 Tikhonov phase noise distribution and the IQ imbalances

4.1 Tikhonov distribution

The Tikhonov distribution, also known as the von Mises distribution is a generalised form of the Gaussian distribution [16, 17]. A common form of the Tikhonov distribution is given by

$$ \rho_T(x; \ a, \ \xi) = \frac{1}{2\pi \nu(a)} \exp[a \cos(x - \xi)] $$

(12)

$$ EVM_{\text{RMS}}^2 = \frac{1}{\text{SNR}} + a_{\text{DC}}^2 \left[ 2 + \frac{4 g}{1 + g^2} \sin \varphi \right] + 2 - \frac{2}{1 + g^2} \left[ (1 + g) \cos \frac{\phi}{2} \cos \phi_{\text{C}} - (1 - g) \sin \frac{\phi}{2} \sin \phi_{\text{C}} \right] $$

(13)
where $\xi$ is a centrality parameter, $\alpha$ is called a concentration parameter and $I_n(\alpha)$ is the modified Bessel function of the order zero, respectively. Considering that the Tikhonov distribution function is also circular normal, the random variable $x$ has a support length of $2\pi$ around $\xi$ [17].

A form of the Tikhonov pdf (12) which is used in [18, 20] valid for the sinusoidal PLL systems is given as

$$
\rho_T(\phi_C) = \frac{\exp(a \cos \phi_C)}{2\pi I_0(a)}
$$

(13)

with $-\pi \leq \phi_C \leq \pi$. The concentration parameter $\alpha$ intuitively corresponds to a PLL signal-to-noise metric defined for a linearised PLL model [18]. In (8), it is assumed that the VCO is tuned to the frequency of the reference oscillator and the problem consists only of acquiring and tracking the phase.

### 4.2 Average EVM

As in the case of the Gaussian noise, the average RMS expressions require integrating (8) over the Tikhonov pdf (13) by using (9). Again, only the third term of (8), which contains the $\cos \phi_C$ and the $\sin \phi_C$ terms, requires averaging. The average value of the cosine of the phase noise under the Tikhonov distribution is given by

$$
\cos(\phi_C) = \frac{1}{2\pi I_0(a)} \int_{-\pi}^{+\pi} \cos(\phi_C) \rho_T(\phi_C) \, d\phi_C
$$

(14)

The integration limits stem from the fact that the Tikhonov has support in $[\pi, -\pi]$.

A convenient form of (13) is a modified Bessel function series form given by Viterbi [18]

$$
\rho_T(\phi_C) = \frac{1}{2\pi I_0(a)} \left[ I_0(a) + 2 \sum_{n=1}^{\infty} I_n(a) \cos n\phi_C \right]
$$

(15)

where the symbol $I_n(a)$ denotes the modified Bessel function of the $n$th order.

Using (15) in (14) and utilising the integral formulae from [21] yields

$$
\overline{\cos(\phi_C)} = \frac{I_1(a)}{I_0(a)}
$$

(16)

In a similar way, the average value of the sine of the phase noise is calculated as

$$
\overline{\sin(\phi_C)} = 0
$$

(17)

On taking into account (16) and (17), the averaged EVM for a combined Tikhonov phase noise and the IQ imbalances situation reads as

$$
EVM_{RMS, AVG}^2 = \frac{1}{SNR} + \frac{a^2_{DC}}{g^2} \left[ 2 + \frac{4g}{1 + g^2} \sin \varphi \right] + \left[ \frac{I_1(a)}{I_0(a)} \right] \frac{(1 + g)^2}{1 + g^2} \left( 1 + \cos \varphi \right)
$$

(18)

The phase noise variance $\sigma^2$ is included implicitly in (18), as follows in [18, 19]

$$
\sigma^2 = \frac{\pi^2}{3} + \frac{4}{I_0(a)} \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} I_k(a)
$$

(19)

It can be verified numerically that (19) is increasing strictly monotonically which leads to a one to one mapping between the phase noise variance $\sigma$ and the concentration parameter $\alpha$. Therefore, based on (18) and (19), $EVM_{RMS, AVG}$ is related to the phase noise variance by eliminating the parameter $\alpha$. In Fig. 2, the phase noise variance $\sigma^2$ is plotted against the concentration parameter $\alpha$.

### 5 Comparison of the Tikhonov and the Gaussian-based EVM

By comparing the EVM expressions in (11) and (18), it can be deduced that the difference depends on the ratio of the Bessel functions term and the exponential variance term. The two terms are plotted in Fig. 3 by using (19) to provide the relationship between $\alpha$ and $\sigma^2$. The plot is made for the...
parameter \( \alpha \) in the range 0–700, which corresponds to the phase noise variance \( \sigma^2 \) in the range 5–350. The difference becomes notable when \( I_1(\alpha)/I_0(\alpha) < 0.45 \) which occurs when the parameter \( \alpha \) takes values less than one.

To establish a practical phase noise limit for the EVM calculations, a plot of the ratio against the phase noise rms is plotted in Fig. 4. From the plot, it is deduced that differences are generated for the rms values of 26° or more. Consequently, for all the practical cases, the EVM can be calculated with either distribution.

### 6 Measurement procedure

#### 6.1 Phase noise masks

Measured phase noise masks have usually the shape of a piecewise linear characteristic [15]. The mask is extended from a frequency \( f_{\text{min}} \) up to a frequency \( f_{\text{max}} \) (Fig. 5).

It requires a frequency \( f_{\text{OL}} \) along with two noise power density levels, a plateau level, PL (dBc/Hz) and an overshoot level, OL (dB) relative to the PL. The OL which occurs at \( f_{\text{OL}} \) is utilised to account for the humps that may appear in the PLL output spectra. The slope of the curve after \( f_{\text{OL}} \) is assumed to be \(-20 \text{ dB/dec (thermal noise)}\).

The corresponding phase noise rms value \( \sigma \) can be evaluated based on this phase noise mask model by integrating the piecewise linear curve over the frequency domain (see (20))

\[
\sigma = \sqrt{2 \times 10^{(\text{PL}/10)} \left[ f_{\text{min}}^{-m} \left( f_{\text{OL}}^{-m} - f_{\text{min}}^{-m} \right) \right] + 10^{(\text{OL}/10)} f_{\text{OL}}^{2} \left( \frac{1}{f_{\text{OL}}} - \frac{1}{f_{\text{max}}} \right)}
\]  

(20)

In (20), a factor ‘2’ was included to account for both the noise sidebands by assuming that the spectral plot of Fig. 5 corresponds to one sideband around the carrier.

In Fig. 6, the rms phase noise, as computed by (20), is plotted as a function of the PL (dBc/Hz) for the case of \( f_{\text{min}} = 1 \text{ kHz}, f_{\text{max}} = 1 \text{ MHz}, \text{ OL} = 0 \text{ dB} \) and \( f_{\text{OL}} = 100 \text{ kHz} \), respectively. Thus, a direct connection of the mask parameters with the EVM can be readily obtained.

#### 6.2 EVM measurements

A measurement test bench was set up (Fig. 7). The Agilent ESG 4438C Vector Signal Generator was used as a transmitter and the Agilent PSA E444A Spectrum Analyser with Vector Signal Analysis (VSA) software was used as the receiver.

The EVM evaluation method presented here is independent of the modulation format. This is demonstrated by the use of two different modulation schemes as signals namely QPSK and 16-QAM.

The ESG transmitted a carrier signal of 2.45 GHz modulated by using a 1MSPS signal and applying a root raised cosine (RRC) filter with a 0.35 roll-off factor. The PSA was used as a digital receiver and the VSA software was used to demodulate the digital radio signal and calculate the EVM in...
real time. It should be noted that the VSA references the EVM to the peak symbol energy whereas (11) and (18) use the average symbol energy. To compare the measured values for the 16-QAM, a $\sqrt{9/5}$ factor is required [2, 5].

IQ amplitude and phase imbalances are introduced by the built in option of the Signal Generator ESG 4438C. Phase noise was introduced to the transmitted signal by additionally using an analogue phase modulation with a noise input at the ESG as the modulating signal. Effectively, this combination corresponds to a spectral profile similar to the one depicted in Fig. 5, with a flat plateau bandwidth of 100 kHz and no overshoot. Increasing the amount of phase deviation resulted in shifting the noise mask plateau to a higher value.

7 Validation of the combined EVM expressions

For a validation, the proposed EVM expressions are compared with the measurements. Based on (11), the effects of the phase noise on the EVM are plotted in Fig. 8. The combined effects of the phase noise along with the IQ amplitude and the phase imbalances are plotted in Figs. 9 and 10. The EVM measurement corresponding to the no amplitude, the phase imbalance and the phase noise was used to calculate an effective noise floor, which was evaluated to 47.6 dB. This SNR value was used in the theoretical calculations using (11).

An excellent agreement is observed in Figs. 8 and 9 regardless of the modulation format. The largest difference between the measurements and the proposed expressions is <1% in the EVM values.

Fig. 7 Set-up for the EVM measurements

Fig. 8 Measured and computed effects of the phase noise on the average rms value of the EVM without the IQ amplitude or the phase imbalances. Both the QPSK and the 16-QAM signals have been used in the measurements (SNR = 47.6 dB)

Fig. 9 Combined effects of the phase noise, the IQ amplitude imbalance of $g = 1$ dB and the phase imbalance, $\phi = 1.5^\circ$, respectively, on the EVM. QPSK signals have been used in the measurements (SNR = 47.6 dB)

Fig. 10 Combined effects of the phase noise and the large IQ amplitude and the phase imbalances. 16-QAM signals have been used in the measurements (SNR = 47.6 dB)

In Fig. 10, the typical upper limits of the amplitude (2 dB) and the phase imbalances (5°) are considered with the phase noise. The agreement is still very good for the values up to the 10° phase noise. For the phase noise values larger than 10°, the proposed theory seems to overestimate the EVM by 2%. This can be explained based on the way the VSA operates. The VSA computes the EVM by comparing each symbol with the nearest constellation symbol since the transmitted symbol information is not available [22], in contrast with the...
theoretical EVM calculation which is based on the comparison with a known transmitted symbol. This effectively results in the smaller measured EVM values in the case where the error vector because of the interferences and the noise is larger than the adjacent constellation symbol spacing [23].

8 Conclusions

Novel analytical expressions were presented to account for the combined effect of the phase noise, the IQ amplitude and the phase imbalances on the EVM. The expressions are not separable in the phase noise or the amplitude and the phase imbalances only contribute. The Tikhonov distribution was also utilised as a generalised phase noise model instead of the Gaussian distribution. It was shown analytically that for the EVM calculations using a Gaussian distribution as a phase noise model yields the same EVM results as the Tikhonov for all the practical VCO phase noise values. Furthermore, it was demonstrated how the proposed expressions can be linked to the phase noise masks thus allowing the designers to estimate directly the effect of the PLL parameters on the chipset EVM. The agreement with the measurements for the different digital modulation formats is very good.

9 Acknowledgments

The authors would like to acknowledge EU COST Action IC1301 Wireless Power Transmission for Sustainable Electronics. A. Georgiadis was supported by EU Marie Curie FP7-PEOPLE-2009-IAPP 251557 and the Spanish Ministry of Economy and Competitiveness project TEC 2012-39143.

10 References

1 Li, Y., Bakaloglu, B., Chakrabarti, C.: ‘A system level energy model and energy-quality evaluation for integrated transceiver front-ends’, IEEE Trans. VLSI Syst., 2007, 15, (1), pp. 90–103
2 Schmogrow, R., Nebendahl, B., Winter, M., et al.: ‘Error vector magnitude as a performance measure for advanced modulation formats’, IEEE Photonics Technol. Lett., 2012, 24, (1), pp. 61–63
3 Mckinley, M.D., Remley, K.A., Myśliński, M., et al.: ‘EVM calculation for broadband modulated signals’, 64th ARFTG Microwave Measurements Conf. Digest, Orlando, USA, December 2004, pp. 45–52
4 Mahmoud, H.A., Arslan, H.: ‘Error vector magnitude to SNR conversion for non-data aided receivers’, IEEE Trans. Wirel. Commun., 2005, 4, (2), pp. 673–680
5 Georgiadis, A.: ‘Gain, phase imbalance, and phase noise effects on EVM’, IEEE Trans. Veh. Technol., 2004, 53, (2), pp. 443–449
6 Guanbin, X., Manyuan, S., Hui, L.: ‘Frequency offset and I/Q imbalance compensation for direct-conversion receivers’, IEEE Trans. Wirel. Commun., 2005, 4, (2), pp. 744–748
7 Liu, R., Li, Y., Chen, H., Wang, Z.: ‘EVM estimation by analyzing transmitter imperfections mathematically and graphically’, Analog Integr. Circuits Signal Process., 2006, 4, (3), pp. 257–262
8 Liu, Q., Basley, R.J., Ma, X., Zhou, G.T.: ‘Error vector magnitude optimization for OFDM systems with a deterministic peak-to-average power ratio constraint’, J. Sel. Top. Signal Process., 2009, 3, (3), pp. 418–429
9 Chen, Z.Q., Dai, F.F.: ‘Effects of LO phase and amplitude imbalances and phase noise on M-QAM transceiver performance’, IEEE Trans. Ind. Electron., 2010, 57, pp. 1505–1517
10 Chen, M.-H., Han, K.-W., Yang, M.-H., Sun, X.-W.: ‘Effects of phase-locked loop bandwidth on error vector magnitude in transmitter’, J. Electromagn. Waves Appl., 2012, 26, (10), pp. 1315–1322
11 Jensen, T.L., Larsen, T.: ‘Robust computation of error vector magnitude for wireless standards’, IEEE Trans. Commun., 2013, 61, (2), pp. 648–657
12 Gregorio, F., Cousseau, J., Werner, S., Rihonen, T., Wichman, R.: ‘EVM analysis for broadband OFDM direct-conversion transmitters’, IEEE Trans. Veh. Technol., 2013, 62, (7), pp. 3443–3451
13 Valkama, M.,Renfors, M.,Kovunen, V.: ‘Advanced methods for I/Q imbalance compensation in communication receivers’, IEEE Trans. Signal Process., 2001, 49, (10), pp. 2335–2344
14 Tarighat, A., Bagheri, R., Sayed, A.H.: ‘Compensation schemes and performance analysis of IQ imbalances in OFDM receivers’, IEEE Trans. Signal Process., 2005, 53, (8), pp. 3257–3268
15 El Tanay, M.S., Wu, Y., Hazy, L.: ‘Analytical modeling and simulation of phase noise interference in OFDM-based DTT broadcasting systems’, IEEE Trans. Broadcast., 2001, 47, (1), pp. 20–31
16 Evans, M., Hastings, N., Peacock, B.: ‘Statistical distributions’ (Wiley-Interscience, 2000, 3rd edn.)
17 De Abreu, G.T.F.: ‘On the generation of Tikhonov variates’, IEEE Trans. Commun., 2007, 56, (7), pp. 1157–1168
18 Viterbi, A.: ‘PLL dynamics in the presence of noise by Fokker–Planck techniques’, Proc. IEEE, 1963, 51, (12), pp. 1737–1753
19 Leib, H., Pasupathy, S.: ‘The phase of a vector perturbed by Gaussian noise and differentially coherent receivers’, IEEE Trans. Inf. Theory, 1988, 34, (6), pp. 1491–1501
20 Lindsey, W.C.: ‘Nonlinear analysis of generalized tracking systems’, Proc. IEEE, 1969, 57, (10), pp. 1705–1722
21 Abramowitz, M., Stegun, I.: ‘Handbook of mathematical functions' (Dover, 1970)
22 Agilent Technologies: ‘Vector Signal Analysis Basics’, Application Note 150-15, Literature Number 5989-1121EN (Agilent 2004). Available at http://www.keysight.com担当作家/著者/論文/アイテム/リンク/ID/ID/index.html.pdf accessed January 2014
23 Georgiadis, A.: ‘AC-coupling and 1/f noise effects on baseband OFDM signals’, IEEE Trans. Commun., 2006, 54, (10), pp. 1806–1814