Global finite-time set stabilization of spacecraft attitude with disturbances using second-order sliding mode control

Zeyu Guo · Zuo Wang · Shihua Li

Received: 12 October 2021 / Accepted: 20 January 2022 / Published online: 3 February 2022
© The Author(s), under exclusive licence to Springer Nature B.V. 2022

Abstract The performance of attitude stabilization control algorithms for rigid spacecraft can be limited by disturbances. In this paper, the global finite-time attitude stabilization problem with disturbances is investigated and handled by constructing a second-order sliding mode controller. Firstly, a virtual controller based on set stabilization idea is constructed to globally finite-time stabilize the system. Then, a relay polynomial second-order sliding mode controller is constructed to guarantee that the tracking error toward the virtual controller will converge to zero in finite-time. Finite-time Lyapunov theory is applied to support the proof and stability analysis. The global finite-time stability holds even with bounded disturbances. The effectiveness and feasibility of the controller are illustrated by the numerical simulations.

Keywords Spacecraft · Attitude stabilization · Global finite-time stability · Second-order sliding mode · Bounded disturbances

1 Introduction

In recent years, the attitude stabilization control of the spacecraft has attracted extensive attentions due to its prominent role in space missions [1–3], such as spacecraft pointing, maneuvering and alignment. It is well known that the linear controllers, e.g., the PID controllers [4–6], are widely adopted in spacecraft due to their simple structure and easy implementation. However, PID controllers are hard to satisfy the requirements of high performance owing to the existence of couplings, nonlinearities and disturbances in spacecraft dynamics. Thus, the ensuing research efforts are devoted to the nonlinear approaches. These advanced control algorithms focus on improving the performance of spacecraft attitude control from different aspects.

Zeyu Guo and Shihua Li these authors contributed equally to this work.

Z. Guo · Z. Wang (✉) · S. Li
School of Automation, Southeast University, Nanjing 210096, People’s Republic of China
e-mail: z.wang@seu.edu.cn

Z. Guo · Z. Wang · S. Li
Key Laboratory of Measurement and Control of Complex Systems of Engineering, Ministry of Education, Nanjing 210096, People’s Republic of China
control approaches as compared with the corresponding asymptotically stable ones [17,18]. After years of research, significant progress has been made in finite-time control, among which numerous satisfactory approaches have been presented. These approaches can be divided into following categories: homogeneous control, adding a power integrator control (APIC), terminal sliding mode control (TSMC) and higher-order sliding mode control (HOSM). In [19], the actuator saturation is taken into account and the finite-time controller is constructed by using homogeneous method. The output feedback homogeneous controller is designed in [20], which considers the actuator saturation and rate as well. In [21,22], two different finite-time APIC controllers are constructed to guarantee the finite-time convergence of the system states. In [23], the disturbance observer-based APIC controller is constructed to handle the problem of finite-time attitude stabilization under mismatched disturbances. However, there exist some limitations when homogeneous control and APIC are applied in the attitude control problems. For the homogeneous controllers, disturbances in the spacecraft dynamics cannot be handled [24]. For the APIC approach, only the finite-time boundedness can be ensured when dealing with disturbances.

Among the algorithms investigated for finite-time attitude stabilization, finite-time SMC approaches have been proved to be effective owning to their robustness to disturbances. In [25], a TSMC method is proposed and the system states can be stabilized in finite time, while the unexpected singularity problem may occur in the implementation. In order to handle this problem, two different nonsingular TSMC (NTSMC) methods are proposed in [26,27]. To accelerate the convergence rates when the system states are far from the equilibrium, the fast NTSMC is proposed in [28]. Moreover, in [29], a fixed-time SMC is presented and the convergence time is independent of the initial states. Apart from the methods mentioned above, there exist various extensions of TSMC to improve the control performance from different aspects, e.g., observer-based output feedback TSMC [30–32], actuator saturated TSMC [33,34], adaptive TSMC [35,36], etc. In addition, the discontinuous control inputs lead to the chattering phenomenon. To this end, HOSM methods are applied in spacecraft to address chattering and guarantee the finite-time convergence as well. In [37], an integral second-order SMC (SOSM) is constructed, by which the finite-time stability of the system is guaranteed. On this basis, a third-order SMC method is proposed in [38]. By hiding the discontinuous switching in the derivative of the control inputs, the continuous control inputs are derived. However, the above two papers involve the inverse of a time-varying matrix, which may be irreversible in some specific states; thus, the singularity problem may occur.

To sum up, the aforementioned papers provide various finite-time control methods for the attitude stabilization of spacecraft. However, there still exists one main problem that the results are not global. It is well known that the quaternion is usually used to describe the arbitrary attitude motion in the three-dimensional space globally. The attitude systems under quaternion-based descriptions all have two equilibria. For these systems, the stability involved is called set stability and its definition can be found in [39,40]. For the literature using quaternion-based attitude descriptions, the global result means that the system is set stable, specifically, both of the two equilibria should be designed to be stable. If only one of the equilibria is designed to be stable while the other equilibrium is designed to be unstable, the attractive domain of the stable equilibrium will cover the global state space excluding one particular point. In this case, the entire system states will converge to the stable equilibrium although some states are closer to the unstable one, which is called ‘unwinding’ phenomenon [41]. Hence, the controllers that only consider one equilibrium fail to derive set stability and their results can only be considered as almost global results like that in [42].

The global attitude stabilization methods can be seen in [43–45] and the references therein. In [43], the idea of set stability is combined with the SOSM to obtain global stability. However, since the sliding mode surface selected in this paper is linear, the system is asymptotically stable. In [44], an adaptive tripod hybrid control scheme is raised to globally stabilize the system, but the system cannot ensure the finite-time convergence as well. In [45], by applying the set stabilization idea, the system states will converge to different equilibria depending on the initial states, however, only in the absence of disturbances. There are fewer controllers which can obtain global finite-time results with disturbances. Therefore, the motivation of this investigation is to design a controller which can derive global finite-time stability of the system when dealing with disturbances.
In this paper, inspired by the set stabilization idea, a relay polynomial SOSM (RPSOSM) controller is constructed by using a backstepping-like way to handle the global finite-time attitude stabilization problem. Firstly, a virtual controller is constructed to globally stabilize the system. Then the SOSM controller is designed. The finite-time tracking toward the virtual controller and the global finite-time stability of the system are proved based on Lyapunov theory. The major remarkable features are listed as follows:

1. Since the unit quaternion is adopted for the description of spacecraft attitude system, the set stability is introduced to design a global controller. In this control framework, the two equilibria of the spacecraft attitude closed-loop system can be stable, which avoiding the so-called unwinding phenomenon.

2. A discontinuous second-order sliding mode controller is designed to suppress the disturbances in the spacecraft dynamics. The controller guarantees the finite-time stability of the closed-loop system in the presence of disturbances. Meanwhile, to alleviate the chattering phenomenon, a modified continuous controller is also presented by using the saturation function.

3. Compared with the existing controllers, the presented controller in this paper has the advantages of fewer parameters and simple structure, which can reduce the difficulties in parameter adjustments and controller implementations.

The rest of the paper is organized as follows. In Sect. 2, we present some lemmas, after which the dynamic and kinematic model of spacecraft attitude are provided. Also, the problem formulation is established at the end of the section. In Sect. 3, the design progress of the controller as well as the stability analysis of the closed-loop system are presented. In Sect. 4, simulation results are displayed and analyzed. The conclusion part is included in Sect. 5.

2 System model and problem formulation

2.1 Preliminaries

The expression and lemmas involved in subsequent derivations and proofs are listed here. For simplicity, we use the notation $|x|^r = \text{sign}(x)|x|^r$, $x \in \mathbb{R}$.

Definition 1 \[39,46\] Suppose there exists a compact set $M$ and a continuous function $V(x): \mathbb{R}^n \rightarrow [0, +\infty)$. Then the $V(x)$ is said to be a positive definite function with respect to $M$ if $\forall x \in \mathbb{R}^n \setminus M$, $V(x) > 0$ and $V(M) = 0$.

Lemma 1 \[17\] For the following system

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in \mathbb{R}^n,$$

suppose there exists a positive definite function $V(x): U \rightarrow \mathbb{R}$ such that the following condition holds

(i) There exist real numbers $c > 0$, $\alpha \in (0, 1)$ and an open neighborhood $U_0 \subset U$ of the origin such that

$$\dot{V}(x) + cV^\alpha(x) \leq 0, \quad x \in U_0 \setminus \{0\}.$$

Then the origin $0$ is a finite-time stable equilibrium of system (1). If $U = U_0 = \mathbb{R}^n$, the origin is a global finite-time stable equilibrium of system (1).

Lemma 2 \[47\] If $p_1 > 0$ and $0 < p_2 \leq 1$, then for $\forall x, y \in \mathbb{R}$, the following inequality holds

$$|x|^{p_1}p_2 - |y|^{p_1}p_2 \leq 2^{1-p_2} |x|^{p_1} - |y|^{p_1}p_2.$$

Lemma 3 \[48\] Let $c$ and $d$ be positive constants, then $\forall x, y \in \mathbb{R}$ satisfy the inequality

$$|x|^c|y|^d \leq \frac{c}{c + d}|x|^{c+d} + \frac{d}{c + d}|y|^{c+d}, \quad \forall x, y \in \mathbb{R}.$$

Lemma 4 \[49\] Let $p$ be a real number with $0 < p < 1$, then for $\forall x_i \in \mathbb{R}$, $i = 1, \ldots, n$, we have

$$(|x_1| + \cdots + |x_n|)^p \leq |x_1|^p + \cdots + |x_n|^p.$$

Lemma 5 \[50\] Let $V(x)$ be a continuous positive-definite function with respect to a compact set $M$ for system (1). If $V(x)$ satisfies $\dot{V}(x) \leq 0, \forall x \in \mathbb{R}^n \setminus M$, then system (1) is stable with respect to $M$.

2.2 Attitude model and problem formulation

For the attitude stabilization problems, the dynamic model of spacecraft attitude can be described as \[51\]

$$J \ddot{\omega} = a(\omega)J\omega + u(t) + M(t),$$

where $J = J^T$ is a positive-definite square inertia matrix whose dimension is 3, $\omega = [\omega_1, \omega_2, \omega_3]^T$ is the measured angular velocity, and $u(t) = [u_1, u_2, u_3]^T$ is the control torque. $M(t) = [M_1, M_2, M_3]^T$ is
the external disturbance. And \( a(\omega) \) is a 3 × 3 skew-symmetric matrix

\[
a(\omega) = \begin{bmatrix}
0 & \omega_1 & -\omega_2 \\
-\omega_1 & 0 & \omega_3 \\
\omega_2 & -\omega_3 & 0
\end{bmatrix}.
\]

The external disturbances are composed of solar radiation, magnetic effects or other uncertainties, which have the characteristics of small amplitudes and periodicity. Taking into account the above characteristics, sinusoidal disturbances can represent the disturbances in the attitude dynamics of the spacecraft well.

Due to the fuel consumption, motivations of the devices as well as other factors, the inertia matrix may be perturbed. Denote the inertia matrix \( J \) as \( J = J_0 + J_\delta \), where \( J_0 \) is the nominal value of the inertia matrix and \( J_\delta \) is the perturbed value. This fact leads to

\[
J_{\delta 0} = a(\omega)J_0\omega + u(t) + d_1(t),
\]

where \( d_1(t) = J_\delta \dot{\omega} + a(\omega)J_\delta \omega + M(t) \) is the lumped disturbances. Using the symbol \( || \cdot || \) to represent the Euclidean norm of a vector and the induced norm of a matrix, we have \( ||d_1(t)|| \leq ||J_\delta|| \omega + ||J_\delta|| \omega + ||M(t)|| \).

**Assumption 1** The lumped disturbances in (16) are bounded by a known constant, which is \( \sup|d_1(t)| \leq L, i = 1, 2, 3 \).

**Remark 1** The same assumptions on the lumped disturbances \( d_1 \) can be found in many papers such as [24, REMARK 3], [36, Assumption 2] and [52, Assumption 3], etc. If the lumped disturbances are to be regarded as the function-limited disturbances, then the subsequent constant gain of the designed controller should be substituted by the functional gain, which depends on the upper bound function of the disturbances. The finite-time stability of the closed-loop system under the function-limited disturbances can be guaranteed by the variable gain controller.

Using above dynamic model, with the quaternion being used to describe the spacecraft attitude, the kinematic model is [53]

\[
\dot{q} = \frac{1}{2} E(q)\omega.
\]

The quaternion \( q = [q_0, q_v^T]^T \), \( q_v = [q_1, q_2, q_3]^T \) in (4) is the unit quaternion. It satisfies

\[
q_0 = \cos\left(\frac{\theta}{2}\right), \quad q_i = e_i \sin\left(\frac{\theta}{2}\right), \quad i = 1, 2, 3.
\]

where \( \theta \) denotes the principal angle and \( e = [e_1, e_2, e_3]^T \) denotes the Euler axis with \( e_1^2 + e_2^2 + e_3^2 = 1 \). Thus, the unit quaternion satisfies

\[
q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1.
\]

\( E(q) \) is a 4 × 3 matrix

\[
E(q) = \begin{bmatrix}
-\omega_v^2 \\
-\omega_v^1 \\
-\omega_v^3
\end{bmatrix}
\]

where \( I_{3 \times 3} \) represents a 3 × 3 identity matrix. Based on (6), one can easily obtain that

\[
E^T(q)E(q) = I_{3 \times 3}.
\]

Using the nominal value \( J_0 \) to counteract the structural items in (3) and letting \( u(t) = [u_1, u_2, u_3]^T = -a(\omega)J_0\omega + J_0u_\delta(t) \), the system model can be simplified to

\[
\dot{q} = \frac{1}{2} E(q)\omega,
\]

\[
\dot{\omega} = u_\delta + d(t),
\]

where \( u_\delta(t) = [u_{\delta 1}, u_{\delta 2}, u_{\delta 3}]^T \) denotes the simplified control law. By Assumption 1, \( d(t) = J_0^{-1}d_1(t) = [d_1(t), d_2(t), d_3(t)]^T \) is also bounded by a known positive constant.

As presented in (5) and (6), to derive set stability, there should be two equilibria \( (1, 0, 0, 0)^T \) and \( (-1, 0, 0, 0)^T \). The difference in principal angle between them is 360°, and it seems to be exactly the same from a geometric point of view, based on which many papers claim themselves global stable though they only consider one equilibrium. Obviously, this is not entirely true for the reason that if the equilibrium \( (1, 0, 0, 0)^T \) is designed to be stable while \( (-1, 0, 0, 0)^T \) is unstable, the entire system states will converge to the equilibrium \( (1, 0, 0, 0)^T \) although some states are closer to \( (-1, 0, 0, 0)^T \), engendering the so-called unwinding phenomenon. To avoid this phenomenon, a SOSM control scheme based on set stability is introduced in subsequent sections.

### 3 Second-order sliding mode controller design

In this section, a SOSM controller is designed by using the idea of APIC and set stability. The design process and stability analysis are divided into two steps. At first, we select the sliding variables as follows:

\[
s_0 = q_0 - \text{sign}(q_0(0)), \quad s_i = q_i, \quad i = 1, 2, 3,
\]

where \( q_0(0) \) denotes the initial condition of \( q_0 \).
Remark 2 As the $q_0$ is always available throughout the control process, the value of $q_0(0)$ can be obtained directly. By introducing the sign ($q_0(0)$) term into the sliding variable, it will be proved later that whether the system states converge to $(1, 0, 0, 0)^T$ or $(-1, 0, 0, 0)^T$ depends on the initial value of $q_0$. In other words, if the above sliding variables can be driven to the origin in finite time, the global finite-time stability can be achieved.

Theorem 1 Considering the system \((8)\) with Assumption 1, there exists positive constants $k_1, k_2$ such that

\[
\dot{s}_i = -k_1 \text{sign}\left(|\omega_i| + \text{sign}(q_0(0)) k_2 s_i^2\right),
\]

\[
i = 1, 2, 3
\]

the SOSM $s_i = \dot{s}_i = 0, i = 0, 1, 2, 3$ will be established in finite time.

Proof Step 1 Choosing a $C^1$ Lyapunov function of the form:

\[
V_1(s) = s_1^2 + s_2^2 + s_3^2 + s_4^2 = \left[q_0 - \text{sign}(q_0(0))\right]^2 + q_v^T q_v.
\]

Then taking derivative of \((11)\) along system \((8)\), it provides

\[
\dot{V}_1(s) = 2 (q_0 - \text{sign}(q_0(0))) \dot{q}_0 + 2 q_v^T \dot{q}_v
\]

\[
= \text{sign}(q_0(0)) q_v^T \omega
\]

\[
+ \text{sign}(q_0(0)) q_v^T (\omega - \omega^*)
\]

\[
\omega^* = [\omega_1^*, \omega_2^*, \omega_3^*]^T
\]

is the virtual control law which is designed as

\[
\omega_i^* = -\text{sign}(q_0(0)) k_2 |q_i|^\frac{1}{2}, i = 1, 2, 3.
\]

Using Lemma 4, supposing that the tracking error toward the virtual controller has converged to zero in finite-time and considering the condition that $q_0(0) \geq 0$, then \((12)\) can be expressed as

\[
\dot{V}_1(s) = -k_2 \left(|q_1|^\frac{1}{2} + \right. \left.|q_2|^\frac{1}{2} + |q_3|^\frac{1}{2}\right)
\]

\[
\leq -k_2 \left[\left(|q_1|^\frac{1}{2} + |q_2|^\frac{1}{2} + |q_3|^\frac{1}{2}\right)\right]
\]

\[
\leq -k_2 \left(|q_0|^2\right)^{\frac{3}{4}}
\]

From system \((8)\), we have

\[
\dot{q}_0 = -\frac{1}{2} (q_1 \omega_1 + q_2 \omega_2 + q_3 \omega_3).
\]

This, together with virtual control law \((13)\), implies

\[
\dot{q}_0 = -k_2 \left(|q_1|^\frac{1}{2} + |q_2|^\frac{1}{2} + |q_3|^\frac{1}{2}\right).
\]

As $k_2$ is a positive constant, the condition $\dot{q}_0 > 0$ and $q_0 > 0$ will be reached in a finite time $t_1$. This fact leads to

\[
\dot{V}_1(s) = -k_2 \left(|q_1|^\frac{1}{2} + |q_2|^\frac{1}{2} + |q_3|^\frac{1}{2}\right)
\]

\[
\leq -k_2 \left(1 - q_0^2\right)^{\frac{3}{2}}
\]

\[
= -k_2 \left(1 - q_0^2\right)^{\frac{3}{2}} (1 + q_0^2)^{\frac{3}{2}}
\]

\[
\leq -k_2 \left(2 - 2q_0^2\right)^{\frac{3}{2}}.
\]

Another fact is $V_1(s) = (q_0 - 1)^2 + q_v^T q_v = 2 - 2q_0$. This, together with \((16)\), has

\[
\dot{V}_1(s) + k_2^2 V_1(s) \leq 0, \quad t \geq t_1.
\]

The same proof turns out to be correct for the case $q_0(0) < 0$. Using Lemma 1, it is not complicated to conclude that system \((8)\) under the designed virtual controller \((13)\) is globally finite-time stable.

One may consider that, based on backstepping control, a controller that drives the tracking error to zero in finite time can be constructed directly. However, the derivative operation in the design process will bring the control input with the negative fractional power, which may cause singularity problem. For this reason, we use a generalized ‘APIC’ \cite{54} to guarantee the finite-time tracking of the virtual controller and the establishment of SOSM.

Substituting \((13)\) into \((12)\), it yields

\[
\dot{V}_1(s) = -k_2 \left(|q_1|^\frac{1}{2} + |q_2|^\frac{1}{2} + |q_3|^\frac{1}{2}\right)
\]

\[
+ \text{sign}(q_0(0)) q_v^T (\omega - \omega^*)
\]

\[
= -k_2 \left(|q_1|^\frac{1}{2} + |q_2|^\frac{1}{2} + |q_3|^\frac{1}{2}\right)
\]

\[
+ \text{sign}(q_0(0)) q_v^T (\omega - \omega^*).
\]

We know sign \((q_0(0)) q_v^T (w - w^*) \leq |q_1||w_1 - w_1^*| + |q_2||w_2 - w_2^*| + |q_3||w_3 - w_3^*|.\]

Denoting \(|\omega_i - \omega_i^*|, i = 1, 2, 3\) as \(|\omega_i|^2 - |\omega_i^*|^2\) and using Lemma 2, it provides

\[
|q_i||w_i - w_i^*| \leq \sqrt{2}|q_i||\delta_i|^\frac{1}{2}, \quad i = 1, 2, 3,
\]

where \(\delta_i = |\omega_i|^2 - |\omega_i^*|^2\). Applying Lemma 3 to \((19)\), it yields

\[
|q_i||w_i - w_i^*| \leq \sqrt{2}|q_i||\delta_i|^\frac{1}{2}
\]

\[
\leq \sqrt{2}\left(\frac{2}{3}|q_i|^\frac{1}{2} + \frac{1}{3}|\delta_i|^\frac{1}{2}\right), \quad i = 1, 2, 3.
\]
Then combining (18) and (20) leads to
\[
\dot{V}_1(s) \leq \left( -k_2 + \frac{2\sqrt{2}}{3} \right) \left( |q_1|^2 + |q_2|^2 + |q_3|^2 \right) \\
+ \frac{\sqrt{2}}{3} \left( |\delta_1|^2 + |\delta_2|^2 + |\delta_3|^2 \right).
\] (21)

**Step 2** Considering the tracking error between \( \omega \) and \( \omega^* \), and defining
\[
V_i(s, \omega) = \frac{4}{3\sqrt{2}k_2} \int_{t_i}^{t_{i+1}} \left[ (|\delta_i|^2 - |\omega_i^*|^2)^{1/2} \right] dk, i = 1, 2, 3,
\]
based on which we can choose a Lyapunov function
\[
V(s, \omega) = V_1(s) + V_1(s, \omega) + V_2(s, \omega) + V_3(s, \omega).
\] (22)

Since \( V_1(s) \) and \( V_i(s, \omega) \) are positive definite, \( V(s, \omega) \) is also positive definite. According to Definition 1, \( V(s, \omega) \) is a Lyapunov function with respect to set M for system (8) where \( M = \{(q^T, \omega^T)|(-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)\}. \\
Taking the derivative of \( V_i(s, \omega) \) along the system we have
\[
\dot{V}_i(s, \omega) = -\frac{\sqrt{2}}{k_2} \left( \frac{\partial}{\partial q_i} |w_i|^2 \right) \dot{q}_i \int_{t_i}^{t_{i+1}} |\delta_i|^2 \, dk \\
- \frac{2}{3\sqrt{2}k_2} |\delta_i|^2 \, dk (u_{si} + d_i).
\] (23)

Now we analyze each of the terms on the right-hand side of (23). To begin with, The inequalities \( \dot{q}_i \leq |\dot{q}_i| \leq \frac{1}{2} (|q_1| + |q_2| + |q_3|) \) and \( \int_{t_i}^{t_{i+1}} |k|^2 - |\omega_i^*|^2 \, dk \leq |\omega_i - \omega_i^*| |\delta_i|^2 \) hold obviously. Thus, (23) can be rewritten as
\[
\dot{V}_i(s, \omega) \leq \frac{\sqrt{2}}{2k_2} \left( \frac{\partial}{\partial q_i} |w_i|^2 \right) (|w_1| + |w_2| + |w_3|) |\omega_i| \\
- \frac{2}{3\sqrt{2}k_2} |\delta_i|^2 (u_{si} + d_i). \] (24)

According to (13), one has
\[
\left| \frac{\partial}{\partial q_i} |w_i|^2 \right| = k_2^2.
\] (25)

This, together with (24), yields
\[
\dot{V}_i(s, \omega) \leq \frac{\sqrt{2}}{2k_2} (|w_1| + |w_2| + |w_3|) |\omega_i - \omega_i^*| |\delta_i|^2 \\
+ \frac{4}{3\sqrt{2}k_2} |\delta_i|^2 (u_{si} + d_i).
\] (26)

In (26), the term \( |\omega_i - \omega_i^*| |\delta_i|^2 \) can be enlarged further. Using Lemma 2, we have
\[
|\omega_i - \omega_i^*| |\delta_i|^2 |\omega_j| \leq \sqrt{2} |\delta_i| |\omega_j|.
\] (27)

Denoting \( |\omega_j| \) as \( |\omega_j + \omega_j - \omega_j^*| \) and applying the inequality \( \|x + y\| \leq \|x\| + \|y\| \), it provides
\[
|\omega_i - \omega_i^*| |\delta_i|^2 |\omega_j| \leq \sqrt{2} |\delta_i| |\omega_j^*| + \sqrt{2} |\delta_i| |\omega_j - \omega_j^*|.
\] (28)

Based on Lemma 2, we have \( |\omega_j - \omega_j^*| \leq \sqrt{2} |\delta_j|^2 \).

Substituting it into (28) and applying Lemma 3, it yields
\[
|\omega_i - \omega_i^*| |\delta_i|^2 |\omega_j| \leq \sqrt{2} |\delta_i| |\omega_j^*| \\
+ \sqrt{2} \left( \frac{2}{3} |\delta_i|^2 + \frac{1}{3} |\delta_j|^2 \right).
\] (29)

This, together with (13) and Lemma 3, implies
\[
|\omega_i - \omega_i^*| |\delta_i|^2 |\omega_j| \leq \sqrt{2} k_2 \left( \frac{2}{3} |\delta_i|^2 + \frac{1}{3} |q_j|^2 \right) \\
+ \frac{4}{3} |\delta_i|^2 + \frac{2}{3} |\delta_j|^2
\] (30)

Substituting (30) into (26) and simplifying it, we have
\[
\dot{V}_i(s, \omega) \leq \left[ \left( \frac{2\sqrt{2}}{k_2} + 2 \right) |\delta_i|^2 \\
+ \frac{2\sqrt{2}}{3k_2} \left( |\delta_1|^2 + |\delta_2|^2 + |\delta_3|^2 \right) \right] \\
+ \frac{1}{3} \left( |q_1|^2 + |q_2|^2 + |q_3|^2 \right) \\
+ \frac{4}{3\sqrt{2}k_2} |\delta_i|^2 (u_{si} + d_i).
\] (31)

Substituting (21) and (31) into the derivative of (22), it yields
\[
\dot{V}(s, \omega) = \dot{V}_1(s) + \dot{V}_1(s, \omega) + \dot{V}_2(s, \omega) + \dot{V}_3(s, \omega) \\
\leq \left( -k_2 + \frac{2\sqrt{2}}{3} + 1 \right) \left( |q_1|^2 + |q_2|^2 + |q_3|^2 \right) \\
+ \left( \frac{\sqrt{2}}{3} + \frac{2\sqrt{2}}{k_2} \right) \left( |\delta_1|^2 + |\delta_2|^2 + |\delta_3|^2 \right) \\
+ \frac{4}{3\sqrt{2}k_2} \left( |\delta_1|^2 |u_{s1} + |\delta_1|^2 |d_1 + |\delta_2|^2 |u_{s2} \\
+ |\delta_2|^2 |d_2 + |\delta_3|^2 |u_{s3} + |\delta_3|^2 |d_3 \right).
\] (32)
On the basis of Lemma 5, it can be derived from (35) and (36) that system (8) under the controller (34) is stable with respect to set $\mathcal{M} = \{ (q^T, \omega^T) | (-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \}$, thus, the set stability of the system states to the equilibrium $(1, 0, 0, 0, 0, 0, 0, 0, 0)$ is finite. We know $V(s, \omega)$ satisfies

$$V(s, \omega) = [q_0 - \text{sign}(q_0(0))]^2 + q_1^2 + q_2^2 + q_3^2$$

$$+ V_1(s, \omega) + V_2(s, \omega) + V_3(s, \omega)$$

$$\leq [q_0 - \text{sign}(q_0(0))]^2 + q_1^2 + q_2^2 + q_3^2$$

$$+ \frac{4}{3\sqrt{2}k_2^3}|w_1 - w_1^*||\delta_1|^{\frac{3}{2}}$$

$$+ \frac{4}{3\sqrt{2}k_2^3}|w_2 - w_2^*||\delta_2|^{\frac{3}{2}}$$

$$+ \frac{4}{3\sqrt{2}k_2^3}|w_3 - w_3^*||\delta_3|^{\frac{3}{2}}.$$  

(36)

Substituting the fact $|\omega_j - \omega_j^*| \leq \sqrt{2}|\delta_j|^{\frac{1}{2}}$ into (36), it yields

$$V(s, \omega) \leq 2(1 - q_0 \text{sign}(q_0(0)))$$

$$+ \frac{4}{3k_2^3}\delta_1^2 + \frac{4}{3k_2^3}\delta_2^2 + \frac{4}{3k_2^3}\delta_3^2.$$  

(37)

Consider the condition where $q_0(0) \geq 0$, one has $\text{sign}(q_0(0)) = 1$ and $\text{sign}(q_0(0)) \geq q_0$. From (22) and (35), we can confirm that there exists a time $t^* < \infty$ such that $q_0 \geq 0$, $\forall t \geq t^*$, after which the following equation holds

$$1 - q_0 \text{sign}(q_0(0)) \leq 1 - q_0^2$$

$$= q_1^2 + q_2^2 + q_3^2, \quad t \geq t^*. \quad (38)$$

Combining (37) and (38), we have

$$V(s, \omega) \leq 2(q_1^2 + q_2^2 + q_3^2)$$

$$+ \frac{4}{3k_2^3}(\delta_1^2 + \delta_2^2 + \delta_3^2), \quad t \geq t^*.$$  

(39)

Letting $\beta = \max \{ 2, \frac{4}{3k_2^3} \}$, it provides

$$V(s, \omega) \leq \beta(q_1^2 + q_2^2 + q_3^2)$$

$$+ \beta(\delta_1^2 + \delta_2^2 + \delta_3^2), \quad t \geq t^*.$$  

(40)

Hence, using Lemma 4, we can obtain the following equation:

$$V(s, \omega) + \frac{k_3}{\beta^2}V^2(s, \omega) \leq 0,$$  

(41)

guaranteed by (35) and (40). Hence, it can be derived from Lemma 1 that the convergence time of the system states to the equilibrium $(1, 0, 0, 0, 0, 0, 0, 0, 0)$ is finite.

The same proof will not be repeated for the case $q_0(0) < 0$, because the theorems and methods used are similar as for $q_0(0) \geq 0$. Thus, we get straightly to the conclusion that when $q_0(0) < 0$, system (8) can converge to the equilibrium $(-1, 0, 0, 0, 0, 0, 0, 0, 0)$ in finite time.

Remark 3 As mentioned in the introduction, the controller designed in this paper owns a simple structure and only one parameter needs to be tuned. From (34), it can be concluded that only parameter $k_3$ needs to be selected because the parameter $k_2$ is totally decided by $k_3$, while $k_1$ is decided by $k_2$ and $k_3$. Actually, the parameter $k_3$ determines the convergence rates of the system states. When a faster convergence rate is required, $k_3$ needs to be adjusted to a larger value. If the over large control torques are to avoided, then $k_3$ needs to be adjusted smaller. In addition, owing to the conservative derivation process of APIC, the controller gain seems to be bigger than a overlarge constant to guarantee the stability of the system. For the possible problem
of excessive parameters, in practical, the controller gain can be adjusted appropriately small to avoid saturation of the actuator and energy waste without affecting the stability of the system.

**Remark 4** The formulas (13) and (41) indicate that the region 
\[ q_0 - \text{sign}(q_0(0)) = q_1 = q_2 = q_3 = 0, \omega_1 = \omega_2 = \omega_3 = 0 \] 
will be verified by simulations in Sect. 4. This, together with the fact that \( \dot{q} = \frac{1}{2} E(q) \omega \), the second-order sliding mode \( s_i = \dot{s}_i = 0, i = 0, 1, 2, 3 \) mentioned in Theorem 1 has been proved. Moreover, the accuracy of the second-order sliding mode \( s_i = \dot{s}_i = 0, i = 0, 1, 2, 3 \) and the influence of the discontinuous control torques on the second-order sliding mode will be relatively smaller after two times of integration.

**Remark 5** Since the attitude system is a second-order system, the discontinuous switching term of the second-order sliding mode controller still appears in the actual control torques. However, the ideas and derivation methods of this paper make it possible to extend the second-order sliding mode controller to the third-order, so as to completely address the problem of discontinuous control torques. In addition, our controller is of the form relay polynomial, it is simple to transform it into a continuous controller. This is also one of the methods to completely address the chattering problem.

It can be derived from the aforementioned analysis that the spacecraft attitude stabilization system (8) under the SOSM controller (34) is globally finite-time stable with respect to set \( M \).

**Remark 6** In practice, the discontinuous control inputs caused by the symbolic function in (34) may lead to chattering phenomenon. To avoid this, the continuous function \( \frac{|s|}{|s| + \phi} \) can be used as a substitute for the discontinuous symbolic function \( \text{sign}(s) \). Although the stability of the system under the continuous control inputs degrades from finite-time stable to finite-time boundedness, the boundary can be reduced to a tolerable range by choosing a appropriately small \( \phi \). The continuous controller is of the form

\[ u_{si} = -k_1 \frac{|\delta_i|}{|\delta_i| + \phi}, \quad i = 1, 2, 3, \tag{42} \]

where \( k_1, k_2, k_3 \) are the same as in (34), \( \phi \) is chosen as a small positive constant. The effectiveness of the controller will be verified by simulations in Sect. 4.

### 4 Simulation results

In this section, the simulation results are presented in the form of figures and tables. Choosing the nominal value of the inertia matrix as

\[ J_0 = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}. \]

The controllers involved in the following comparison section are designed as follows:

The integral SOSM (ISOSM) controller is designed as [37]

\[ u(t) = B^{-1}(q, \omega)[-A(q, \omega) - k_1|q_\nu|^\alpha_1 - k_2|\dot{q}_\nu|^\alpha_2 - \mu s - \lambda \text{sign}(s)], \tag{43} \]

where \( s(t) = \dot{q}_\nu(t) - \int_0^t (-k_1|q_\nu|^\alpha_1 - k_2|\dot{q}_\nu|^\alpha_2) d\tau \) is the integral sliding mode surface.

According to the expression of the matrix \( B \), we can get the determinant of \( B \), which is \( q_0 \). On this basis, we know that the matrix \( B \) is irreversible when the \( q_0 \) is equal to zero. By the definition of unit quaternion, the equation \( q_0 = 0 \) satisfies when the rotation angle around the axis is \( \pi \). Hence, the singularity problem of the ISOSM has been verified from a theoretical point of view.

The global finite time control (GFTC) controller is designed as [45]

\[ u_{si} = -k_1(\omega_{i0}^p + \text{sign}(q_0(0))k_3^p q_i)^2, \quad i = 1, 2, 3, \tag{44} \]

where \( k_1, k_2 \) are all positive constants.

#### 4.1 Validation of set stability

In order to verify the global finite-time stability mentioned above, the RPSOSM algorithm proposed in this paper is simulated separately. The initial conditions of \( q, \omega \) and \( d(t) \) are chosen as \( q(0)^T = (q_0(0), q_1(0), q_2(0), q_3(0))^T = (0, 0, 0, 0)^T, \omega(0)^T = (\omega_1(0), \omega_2(0), \omega_3(0))^T = (0, 0, 0)^T, d(t)^T = (d_1, d_2, d_3)^T = (0, 0, 0)^T \). As the initial condition of \( q_0 \) satisfies \( q_0(0) \geq 0 \) here, combining with the results in comparison section where \( q_0(0) \) is chosen as \( q_0(0) < 0 \), the global stability under the designed
controller can be verified. The simulation results are shown in Figs. 1, 2 and 3.

As is shown in Figs. 1, 2 and 3, when $q_0(0)$ satisfies $q_0(0) \geq 0$, $q^T$ converges to $(1, 0, 0, 0)^T$, which indicates that $(1, 0, 0, 0)^T$ is one of the equilibria of the system. Also, though there exist discontinuous items in actual control input, the accuracy of the controller remains at a satisfying level. Therefore, we can reasonably claim that the RPSOSM controller can provide a great steady-state attitude accuracy.

4.2 Comparisons of the control performance

In this section, the performance of the proposed RPSOSM algorithm (34), the ISOSM (43) algorithm and the GFTC (44) algorithm are compared in the absence/presence of disturbances.

The comparisons are divided into two cases: Case 1 considers the system without disturbances, while in Case 2 the external disturbances are chosen as sinusoidal disturbances. In Case 1, the initial conditions of $q$, $\omega$ and $d(t)$ are chosen as $q(0)^T = (q_0(0), q_1(0), q_2(0), q_3(0))^T = (-0.2, 0.6, -0.4, \sqrt{0.44})^T$, $\omega(0)^T = (\omega_1(0), \omega_2(0), \omega_3(0))^T = (0, 0, 0)^T$, $d(t)^T = (d_1, d_2, d_3)^T = (0, 0, 0)^T$. In Case 2, the disturbances $d(t)$ are selected as $d(t)^T = (d_1, d_2, d_3)^T = (\sin(10t), \sin(20t), \sin(30t))^T$, $J_\delta$ is selected as $J_\delta = 0.1 \times J_0$. Noting that in order to ensure the fairness of comparisons, suitable parameters are adjusted to ensure the performance of ISOSM and GFTC, and the control inputs are uniformly limited to $65N.m$. The parameters of the above-mentioned controllers are shown in Table 1. The simulation results of Case 1 and Case 2 are shown in Figs. 4, 5, 6, 7, 8 and 9, respectively.

When $q_0(0)$ is selected as $q_0 < 0$, $q^T$ converges to $(-1, 0, 0, 0)^T$. Therefore, $(-1, 0, 0, 0)^T$ is also the equilibrium of the system, thus verifying the global
Table 1 Controller parameters

| Controllers       | Parameters                  |
|-------------------|-----------------------------|
| RPSOSM (34)       | $k_1 = 30, k_2 = 1.5, k_3 = 0.01$ |
| ISOSM (43)        | $k_1 = 6, k_2 = 9, \alpha_1 = 0.5, \alpha_2 = 2/3, \lambda = 3.2, \mu = 0.5$ |
| GFTC (44)         | $k_1 = 35, k_2 = 1.7, k_3 = 0.01, p = 19/17$ |

Fig. 5 Response curves of angular velocity without disturbances.  
(a) $\omega_1$.  
(b) Details of $\omega_1$.  
(c) $\omega_2$.  
(d) Details of $\omega_2$.  
(e) $\omega_3$.  
(f) Details of $\omega_3$

Fig. 6 Response curves of control torque without disturbances.  
(a) RPSOSM (34).  
(b) ISOSM (43).  
(c) GFTC (44)

stability of the designed controller. In the absence of disturbance, RPSOSM has a faster convergence speed than ISOSM and GFTC, and the discontinuity existing in the control does not affect the accuracy of the controller.

Subsequently, when the system is subject to sinusoidal disturbances of different frequencies, it is obvious that RPSOSM still owns a faster convergence rate and higher stable precision than ISOSM and GFTC. The quantitative analysis of the simulation results is shown in Table 2.

4.3 Validation of the continuous controller

In this part, the proposed continuous controller (42) is simulated to verify its effectiveness. Since the main difference between the controller (42) and controller (34) is the disturbance rejection abilities, the simulation is carried out with disturbances. The parameters
Global finite-time set stabilization of spacecraft attitude

Fig. 9 Response curves of control torque with disturbances. a RPSOSM (34), b ISOSM (43), c GFTC (44).

Fig. 10 Response curves of quaternion under continuous controller (42).

of controller (42) are the same with controller (34) and the extra parameter $\phi$ is chosen as 0.05. With the same disturbances as that in the comparison section, the simulation results are shown in Figs. 10, 11 and 12.

5 Conclusion

The global finite-time attitude stabilization of the rigid spacecraft by using SOSM control has been investigated in this article. A RPSOSM controller has been

Table 2 Performance comparison

| Test cases             | Controllers | Performance indices |
|-----------------------|-------------|---------------------|
|                       |             | CT(s)$^a$ | QE$^b$ |
| Case 1 (without disturbances) | RPSOSM (34)  | 2.981   | 0      |
|                       | ISOSM (43)  | 4.551   | 0      |
|                       | GFTC (44)   | 4.023   | 0      |
| Case 2 (with disturbances) | RPSOSM (34)  | 2.979   | 0$^c$  |
|                       | ISOSM (43)  | 4.375   | 2.121e$^{-3}$ |
|                       | GFTC (44)   | 4.012   | 3.042e$^{-3}$ |
|                       | Controller (42) | 2.994   | 3.911e$^{-4}$ |

$^a$CT (convergence time) denotes the time after which $|q_i - q_i^*| < 5e^{-3}, i = 0, 1, 2, 3$ holds

$^b$QE (quaternion error) denotes the maximum error of the quaternion after the convergence time, which is obtained by $QE = \max |q_i - q_i^*|, i = 0, 1, 2, 3$

$^c$The QE is zero here because the actual error in the simulation is small enough, whose amplitude is less than 1e$^{-7}$. This error may be caused by the discretization of the continuous-domain sliding mode controller.
designed to obtain a finite-time result. The set stabilization idea has been applied to derive set stability for the system. The global finite-time stability has been proved based on a Lyapunov function. Simulation results reveal that the controller retain a satisfactory performance when dealing with bounded disturbances. Our future work is to further eliminate the discontinuity in the control input of the SOSM controller by using HOSMC.

Acknowledgements The work was supported in part by the National Natural Science Foundation of China under Grant (No. 62103102, 61973081, and 62173221), in part by Natural Science Foundation of Jiangsu under Grants (No. BK202210213), in part by China Postdoctoral Science Foundation (No. 2021M70077), in part by Jiangsu Postdoctoral Research Funding Program (No. 2021K009A).

Funding We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled Global Finite-Time Set Stabilization of Spacecraft Attitude with Disturbances using Second-Order Sliding Mode Control.

Data Availability The authors declare that all other data supporting the findings of this study are available within the article. Source data for figures are provided with the paper.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Joshi, S., Kelkar, A., Wen, J.: Robust attitude stabilization of spacecraft using nonlinear quaternion feedback. IEEE Trans. Autom. Control 40(10), 1800–1803 (1995). https://doi.org/10.1109/9.467669
2. Byrnes, C.I., Isidori, A.: On the attitude stabilization of rigid spacecraft. Automatica 27(1), 87–95 (1991). https://doi.org/10.1016/0005-1098(91)90008-P
3. Hu, Q., Li, B., Zhang, Y.: Robust attitude control design for spacecraft under assigned velocity and control constraints. ISA Trans. 52(4), 480–493 (2013). https://doi.org/10.1016/j.isatra.2013.03.003
4. Wen, T.Y., Kreutz-Delgado, K.: The attitude control problem. IEEE Trans. Autom. Control 36(10), 1148–1162 (1991). https://doi.org/10.1109/9.90228
5. Show, L.L., Jiang, J.C., Lin, C.T., Jan, Y.W.: Spacecraft robust attitude tracking design: PID control approach. In: American Control Conference, pp. 1360–1365 (2002)
6. Sari, N.N., Jahanshahi, H., Fakoor, M.: Adaptive fuzzy PID control strategy for spacecraft attitude control. Int. J. Fuzzy Syst. 21(3), 769–781 (2019). https://doi.org/10.1007/s40815-018-0576-2
7. Liu, Y., Zhang, T., Song, J., Liang, B.: Adaptive spacecraft attitude tracking controller design based on similar skew-symmetric structure. Chin. J. Aeronaut. 23(2), 227–234 (2010). https://doi.org/10.1016/S1000-9361(09)60209-0
8. Sun, L., Zheng, Z.: Disturbance-observer-based robust backstepping attitude stabilization of spacecraft under input saturation and measurement uncertainty. IEEE Trans. Ind. Electron. 64(10), 7994–8002 (2017). https://doi.org/10.1109/TIE.2017.2694349
9. Qiao, J., Li, X., Xu, J.: A composite disturbance observer and \( h_\infty \) control scheme for flexible spacecraft with measurement delay and input delay. Chin. J. Aeronaut. 32(6), 1472–1480 (2019). https://doi.org/10.1016/j.cja.2018.10.013
10. Wu, S., Wen, S.: Robust h-infinity output feedback control for attitude stabilization of a flexible spacecraft. Nonlinear Dyn. 84(1), 405–412 (2016). https://doi.org/10.1007/s11071-016-2624-5
11. Luo, W., Chu, Y.-C., Ling, K.-V.: Inverse optimal adaptive control for attitude tracking of spacecraft. IEEE Trans. Autom. Control 50(11), 1639–1654 (2005). https://doi.org/10.1109/TAC.2005.858694
12. Horri, N., Palmer, P., Roberts, M.: Gain-scheduled inverse optimal satellite attitude control. IEEE Trans. Aerosp. Electron. Syst. 48(3), 2437–2457 (2012). https://doi.org/10.1109/TAES.2012.6237602
13. Dwyer, T.A., III., Sira-Ramirez, H.: Variable-structure control of spacecraft attitude maneuvers. J. Guid. Control. Dyn. 11(3), 262–270 (1988). https://doi.org/10.2514/3.20303
14. Xia, Y., Zhu, Z., Fu, M., Wang, S.: Attitude tracking of rigid spacecraft with bounded disturbances. IEEE Trans. Ind. Electron. 58(2), 647–659 (2010). https://doi.org/10.1109/TIE.2010.2046611
15. Hu, Q., Xiao, L., Wang, C.: Adaptive fault-tolerant attitude tracking control for spacecraft with time varying inertia uncertainties. Chin. J. Aeronaut. 32(3), 674–687 (2019). https://doi.org/10.1007/s11071-018-0576-2
16. Seo, D., Akella, M.R.: High-performance spacecraft adaptive attitude-tracking control through attracting-manifold design. J. Guid. Control. Dyn. 31(4), 884–891 (2008). https://doi.org/10.2514/1.33308
17. Bhat, S.P., Bernstein, D.S.: Finite-time stability of homogeneous systems. In: Proceedings of the American Control Conference, vol. 4, pp. 2513–2514 (1997)
18. Bhat, S.P., Bernstein, D.S.: Finite-time stability of continuous autonomous systems. SIAM J. Control. Optim. 38(3), 751–766 (2000). https://doi.org/10.1137/S0363012997321358
19. Du, H., Li, S.: Finite-time attitude stabilization for a spacecraft using homogeneous method. J. Guid. Control. Dyn. 35(3), 740–748 (2012). https://doi.org/10.2514/1.56262
20. Zou, A., Kumar, K., de Ruiter, A.: Finite-time spacecraft attitude control under input magnitude and rate saturation. Nonlinear Dyn. 99(3), 2201–2217 (2019). https://doi.org/10.1007/s11071-019-05388-6
21. Jiang, B., Li, C., Ma, G.: Finite-time output feedback attitude control for spacecraft using “adding a power integra-
23. Yao, Q.: Robust finite-time control design for attitude stabilization of spacecraft under measurement uncertainties. Adv. Space Res. 68(8), 3159–3175 (2021). https://doi.org/10.1016/j.asr.2021.06.017

24. Hu, Q., Jiang, B.: Continuous finite-time attitude control for rigid spacecraft based on angular velocity observer. IEEE Trans. Aerosp. Electron. Syst. 53(3), 1082–1092 (2018). https://doi.org/10.1109/TAES.2017.2773340

25. Jin, E., Sun, Z.: Robust controllers design with finite time convergence for rigid spacecraft attitude tracking control. Aerosp. Sci. Technol. 12(4), 324–330 (2008). https://doi.org/10.1016/j.ast.2007.08.001

26. Hu, Q., Li, B., Zhang, A.: Robust finite-time control allocation in spacecraft attitude stabilization under actuator misalignment. Nonlinear Dyn. 73(1–2), 53–71 (2013). https://doi.org/10.1007/s11071-013-0766-2

27. Huo, X., Hu, Q., Xiao, B.: Finite-time fault tolerant attitude stabilization control for rigid spacecraft. ISA Trans. 53(2), 241–250 (2014). https://doi.org/10.1016/j.isatra.2013.11.017

28. Guo, B., Chen, Y.: Adaptive fast sliding mode fault tolerant control integrated with disturbance observer for spacecraft attitude stabilization system. ISA Trans. 94, 1–9 (2019). https://doi.org/10.1016/j.isatra.2019.04.014

29. Cao, L., Xiao, B., Golestani, M.: Robust fixed-time attitude stabilization control of flexible spacecraft with actuator uncertainty. Nonlinear Dyn. 100(3), 2020. https://doi.org/10.1007/s11071-020-05596-5

30. Hu, Q., Niu, G.: Attitude output feedback control for rigid spacecraft with finite-time convergence. ISA Trans. 70, 173–186 (2017). https://doi.org/10.1016/j.isatra.2017.07.023

31. Li, B., Qin, K., Xiao, B., Yang, Y.: Finite-time extended state observer based fault tolerant output feedback control for attitude stabilization. ISA Trans. 91, 11–20 (2018). https://doi.org/10.1016/j.isatra.2019.01.039

32. Shao, S., Zong, Q., Tian, B., Wang, F.: Finite-time sliding mode attitude control for rigid spacecraft without angular velocity measurement. J. Franklin Inst. 354(12), 4656–4674 (2017). https://doi.org/10.1016/j.jfranklin.2017.04.020

33. Jiang, B., Hu, Q., Friswell, M.: Fixed-time attitude control for rigid spacecraft with actuator saturation and faults. IET Control Theory Appl. 24(5), 1892–1898 (2016). https://doi.org/10.1049/iet-ctst.2016.2519838

34. Chen, Q., Xie, S., Sun, M., He, X.: Adaptive nonsingular finite time attitude stabilization of uncertain spacecraft. IEEE Trans. Aerosp. Electron. Syst. 54(6), 2937–2950 (2018). https://doi.org/10.1109/TAES.2018.2832998

35. Zhu, Z., Xia, Y., Fu, M.: Adaptive sliding mode control for attitude stabilization with actuator saturation. IEEE Trans. Ind. Electron. 58(10), 4898–4907 (2011). https://doi.org/10.1109/TIE.2011.2107719

36. Esmaeilzadeh, S., Golestani, M., Mobayen, S.: Chattering-free fault-tolerant attitude control with fast fixed-time convergence for flexible spacecraft. Int. J. Control Autom. Syst. 19(2), 767–776 (2020). https://doi.org/10.1007/s12555-020-0043-3

37. Tiwari, P.M., Janardhanan, S.U., Nabi, M.U.: Rigid spacecraft attitude control using adaptive integral second order sliding mode. Aerosp. Sci. Technol. 42, 50–57 (2015). https://doi.org/10.1016/j.ast.2014.11.017

38. Tiwari, P.M., Janardhanan, S.U., Nabi, M.U.: Attitude control using higher order sliding mode. Aerosp. Sci. Technol. 54, 108–113 (2016). https://doi.org/10.1016/j.ast.2016.04.012

39. Lin, Y., Sontag, E.D., Yuan, W.: Recent results on Lyapunov-theoretic techniques for nonlinear stability. In: Proceedings of the 1994 American Control Conference, vol. 2, pp. 1771–1775 (1994)

40. Rouche, N., Habets, P., Laloy, M.: Stability Theory by Liapunov’s Direct Method. Springer, New York (1977)

41. Bhat, S.P., Bernstein, D.S.: A topological obstruction to continuous global stabilization of rotational motion and the unwinding phenomenon. Syst. Control Lett. 39(1), 63–70 (2000). https://doi.org/10.1016/S0167-6911(99)00090-0

42. Angeli, D.: Almost global stabilization of the inverted pendulum via continuous state feedback. Automatica 37, 1103–1108 (2001). https://doi.org/10.1016/S0005-1098(01)00064-4

43. Tiwari, P., Janardhanan, S., un-Nabi, M.: Spacecraft anti-unwinding attitude control using second-order sliding mode. Asian J. Control 20(1), 455–468 (2018). https://doi.org/10.1002/asjc.1601

44. Hashemi, S., Pariz, N., Sani, S.: Observer-based adaptive hybrid feedback for robust global attitude stabilization of a rigid body. IEEE Trans. Aerosp. Electron. Syst. 57(3), 1919–1929 (2021). https://doi.org/10.1109/TAES.2021.3050665

45. Li, S., Ding, S., Li, Q.: Global set stabilisation of the spacecraft attitude using finite-time control technique. Int. J. Control 82(5), 822–836 (2009). https://doi.org/10.1080/00207170802342818

46. Liao, X.: Mathematical Theory and Application of Stability. Company of Center China Normal University, Wuhan, China (1988)

47. Ding, S., Li, S., Zheng, W.X.: Nonsmooth stabilization of a class of nonlinear cascaded systems. Automatica 48(10), 2597–2606 (2012). https://doi.org/10.1016/j.automatica.2012.06.060

48. Qian, C., Lin, W.: A continuous feedback approach to global strong stabilization of nonlinear systems. IEEE Trans. Autom. Control 46(7), 1061–1079 (2001). https://doi.org/10.1109/9.935058

49. Hardy, G.H., Littlewood, J.E., Polya, G.: Inequalities. Cambridge University Press, Cambridge (1952)

50. Fragopoulos, D., Innocenti, M.: Stability considerations in quaternion attitude control using discontinuous Lyapunov functions. IEE Proc Control Theory Appl. 151(3), 253–258 (2004). https://doi.org/10.1049/ip-cta:20040311
51. Kane, T.R., Likins, P.W., Levinson, D.A.: Spacecraft dynamics. AIAA J. **21**(6), 928–928 (1983). https://doi.org/10.1115/1.3167078

52. Cao, L., Xiao, B., Golestani, M., Ran, D.: Faster fixed-time control of flexible spacecraft attitude stabilization. IEEE Trans. Ind. Inf. **16**(2), 1281–1290 (2020). https://doi.org/10.1109/TII.2019.2949588

53. Pisacane, V.L.: Fundamentals of Space Systems. Oxford University Press, Pisacane and Robert C. Moore, New York (1994)

54. Ding, S., Li, S.: Second-order sliding mode controller design subject to mismatched term. Automatica **77**, 388–392 (2017). https://doi.org/10.1016/j.automatica.2016.07.038

**Publisher’s Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.