Mathematical modelling of mutual electromagnetic influences of related power transmission lines in a transition process mode

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Abstract. A mathematical model is obtained using the variable state method, allowing the simulation of electromagnetic processes in transmission lines and communication lines (both homogeneous and heterogeneous) taking into account the reciprocal inductive and capacitive bonds in transition and established modes. The resulting mathematical model can solve a number of important scientific and practical problems related to the design and operation of electric power systems (EEC). The model also allows for a variety of situations and tasks related, for example, to the calculation of circuit breakage, short circuit (SC) or single-phase SC in isolated neural networks. In particular, it is possible to calculate the steered voltage of the line from high-voltage or high-precision power lines. Based on this methodology, it is also proposed to develop an algorithm for the machine formation of mathematical models for the study of transition processes in complex electrical networks, where the initial data will be their parameters and the structure of graphs of the studied networks.

1. Introduction
Currently, the problem of electromagnetic compatibility (EMC) in the power supply systems of enterprises where frequency-controlled electric drives based on power semiconductor transducers are widely used is becoming topical, since these electric drives distort the shape of the power and current curves of the network, which in turn is negative affects the operating modes of various types of electrical installations, leads to false triggering of relay protection and automation, being the main cause of violation of EMC electrical equipment [1–4].

In addition, power lines are constantly influenced by external electromagnetic fields of one origin or another. The third-party electromagnetic fields are induced in the lines of interference, which not only reduce the quality of the transmission, but sometimes cause high voltages and currents, which
lead to the destruction of the power transmission lines and equipment, as well as creating a danger to the life and health of operating personnel.

This problem is common to all systems and devices related to the generation, transmission, reception and processing of electrical signals, and is called the problem of electromagnetic compatibility. Its essence is that the design, construction and operation of the above-mentioned devices and systems must take into account: on the one hand, the impact on them of external electromagnetic fields of a predetermined nature and sufficient protection for their normal work against these effects, and, on the other hand, the measures to limit the levels of influence of electromagnetic fields of the designed devices and systems on other devices by acceptable values.

Thus, the presence of a large number of electromagnetic disturbances (EMF) of different types makes it difficult, and at worst impossible, for certain types of electrical equipment to function normally, which causes failures and emergencies in the networks of these enterprises.

The EMF can be virtually any electromagnetic phenomenon in a wide range of frequencies.

### 2. Compilation of a mathematical model of mutual electromagnetic influences of related power transmission lines in transition mode

In [5] the simulation of electromagnetic interactions of adjacent power transmission lines is considered for the case where one line is active, i.e. the source $e(t)$ first acts and the load is connected by a resistance at the end $Z_H = R_H + jX_L$. The second line, the ends of which are connected to the resistors $R_A$ and $R_B$, is arranged parallel to the first line at a certain distance.

The mathematical model of these lines is obtained by the method of state variables [6].

In this work a further synthesis of this model is made, with the aid of which it is possible to analyze different interaction variants for two lines. The equivalent circuit of the investigated network and the corresponding normal graph are shown in figures 1 and 2.

![Research network equivalent circuit](image)

**Figure 1.** Research network equivalent circuit.

In Figure 1, the switches $SD_1$ and $SD_2$ are shown only in order to be able to analyze the electromagnetic interactions of adjacent power lines for two variants. The first option is that the top line is active and the bottom passive line is active ($SD_1$ is closed, $SD_2$ is opened); the second option is that both lines are active ($SD_1$ is open, $SD_2$ is closed).

As variable states we choose voltages $u_2, u_3, \ldots, u_n, u_3', u_5', \ldots, u_n'$ and currents $i_1, i_2, \ldots, i_n, i_1', i_2', \ldots, i_n'$ along the 1st and 2nd lines. Distinguishing these values as variables characterizing the
energy state of the electrical circuit allows to form differential equations in normal form, since only in these elements currents and voltages are interconnected via derivatives.

The equations for independent sections and independent contours according to the first and second Kirchhoff laws for the first variant, when the branch of the tree (0-1') of a normal graph (Figure 2) contains a \( R_i \), i.e. the switching apparatus \( SD_i \) is closed and the \( SD_j \) is open.

\[
\begin{align*}
\frac{\partial i_{c1}}{\partial t} &= G_1 u_2; \quad i_{c12} = C_{12} \frac{\partial u_{23}}{\partial t} = C_{12} \frac{\partial u_2}{\partial t} - C_{12} \frac{\partial u_3}{\partial t};
\end{align*}
\]

then we’ll get

\[
(C_1 + C_{12}) \frac{\partial u_3}{\partial t} - C_{12} \frac{\partial u_{23}}{\partial t} = i_1 - i_2 - G_1 u_2. \tag{1}
\]

2. For the 2nd section: \( i_2 - i_3 - i_{c1} - i_{G_1} - i_{c12} = 0 \),

where \( i_{c1} = C_1 \frac{\partial u_3}{\partial t}; \quad i_{G_1} = G_1 u_3; \quad i_{c12} = C_{12} \frac{\partial u_{23}}{\partial t} = C_{12} \frac{\partial u_2}{\partial t} - C_{12} \frac{\partial u_3}{\partial t}, \)

then we’ll get

\[
(C_1 + C_{12}) \frac{\partial u_3}{\partial t} - C_{12} \frac{\partial u_{23}}{\partial t} = i_2 - i_3 - G_1 u_3. \tag{2}
\]

Similar to other independent sections.

3. For the \( (n-1) \) section: \( i_{n-1} - i_n - i_{c1} - i_{G_1} - i_{c12} = 0 \),

where \( i_{c1} = C_1 \frac{\partial u_n}{\partial t}; \quad i_{G_1} = G_1 u_n; \quad i_{c12} = C_{12} \frac{\partial u_{n,n'}}{\partial t} = C_{12} \frac{\partial u_n}{\partial t} - C_{12} \frac{\partial u_{n'}}{\partial t}, \)

we’ll get

\[
(C_1 + C_{12}) \frac{\partial u_{n'}}{\partial t} = C_{12} \frac{\partial u_{n}}{\partial t} = i_{n-1} - i_n - G_3 u_n. \tag{3}
\]

4. For the 1' th section: \( i_{1'} - i_{2'} - i_{c2} - i_{G_2} + i_{c12} = 0 \),

where \( i_{c2} = C_2 \frac{\partial u_{23}}{\partial t}; \quad i_{G_2} = G_2 u_{n';} \quad i_{n'} = \frac{u_{n'}}{R_B} = G_B u_{n'}; \quad i_{c12} \) listed in paragraph 1 (see above),

then we’ll get

\[
(C_2 + C_{12}) \frac{\partial u_{23}}{\partial t} - C_{12} \frac{\partial u_2}{\partial t} = i_{1'} - i_{2'} - G_2 u_2. \tag{4}
\]

5. For the \( (n-1)' \) th section: \( i_{(n-1)'} - i_{n'} - i_{c2} - i_{G_2} + i_{c12} = 0 \),

\[
(C_2 + C_{12}) \frac{\partial u_{23}}{\partial t} - C_{12} \frac{\partial u_2}{\partial t} = i_{(n-1)'} - i_{2'} - G_2 u_2. \tag{5}
\]
where \( i_{c_2} = C_2 \frac{du_{n_1}}{dt}; i_{c_2} = G_2u_{n_2}; i_{c_2} = G_2u_{n_2}i_{c_{12}} \) listed in paragraph 3 (see above),

then we’ll get

\[
(C_2 + C_{12}) \frac{du_{n_1}}{dt} - C_{12} \frac{du_{n_2}}{dt} = -G_Bu_{n_1} - i_1(t) - G_2u_2.
\]

6. For the 1st circuit: \( u_{L_1} + u_{R_1} + u_{C_1} \pm u_{M_{12}} = e_1(t) \),

where \( u_{L_1} = L_1 \frac{di_1}{dt}, u_{R_1} = R_1i_1; u_{M_{12}} = M_{12} \frac{di_{12}}{dt} \).

we’ll get

\[
L_1 \frac{di_1}{dt} \pm M_{12} \frac{di_{12}}{dt} = e_1(t) - R_1i_1 - u_2.
\]

7. For the 2nd circuit: \( u_{L_1} + u_{R_1} + u_2 \pm u_{M_{12}} = 0 \),

where \( u_{L_1} = L_1 \frac{di_2}{dt}, u_{R_1} = R_1i_2; u_{M_{12}} = M_{12} \frac{di_{12}}{dt} \).

we’ll get

\[
L_1 \frac{di_2}{dt} \pm M_{12} \frac{di_{12}}{dt} = u_2 - u_3 - R_1i_2.
\]

Similar to other independent circuit.

8. For the \((n-1)\) th circuit: \( u_{L_1} + u_{R_1} + u_n - u_{n-1} \pm u_{M_{12}} = 0 \),

where \( u_{L_1} = L_1 \frac{di_{n-1}}{dt}, u_{R_1} = R_1i_{n-1}; u_{M_{12}} = M_{12} \frac{di_{(n-1),1}}{dt} \).

we’ll get

\[
L_1 \frac{di_{n-1}}{dt} \pm M_{12} \frac{di_{(n-1),1}}{dt} = u_{n-1} - u_n - R_1i_{n-1}.
\]

9. For the \(n\) th circuit: \( u_{L_H} + u_{R_H} - u_n = 0 \),

where \( u_{L_H} = L_H \frac{di_n}{dt}, u_{R_H} = R_Hi_n \),

we’ll get

\[
L_H \frac{di_n}{dt} = u_n - R_Hi_n.
\]

10. For the 1' th circuit: \( u_{R_A} + u_{L_2} + u_{R_2} + u_2 \pm u_{M_{12}} = 0 \),

where \( u_{L_2} = L_2 \frac{di_1}{dt}, u_{R_2} = R_2i_1; u_{R_A} = R_Ai_1; u_{M_{12}} = M_{12} \frac{di_1}{dt} \).

we’ll get

\[
L_2 \frac{di_1}{dt} \pm M_{12} \frac{di_1}{dt} = -R_Ai_1 - u_2 - R_2i_1.
\]

11. For the \((n-1)'\) th circuit: \( u_{L_2} + u_{R_2} + u_n - u_{n-1} \pm u_{M_{12}} = 0 \),

where \( u_{L_2} = L_2 \frac{di_{(n-1),1}}{dt}, u_{R_2} = R_2i_{(n-1),1}; u_{M_{12}} = M_{12} \frac{di_{n-1}}{dt} \).

we’ll get

\[
L_2 \frac{di_{(n-1),1}}{dt} \pm M_{12} \frac{di_{n-1}}{dt} = u_{n-1} - u_n - R_2i_{(n-1),1}.
\]

For the second variant, when the branch of the tree (0-1') of a normal graph (Figure 2) contains the source \( F_2(t) \), i.e. when the switching apparatus \( SD_1 \) is opened and the \( SD_2 \) is closed, the equations are drawn up in the same way (1)–(11). Then equations (1)–(11) are written in matrix form:

\[
H \frac{dx}{dt} = K \cdot x + S \cdot E \quad \text{or} \quad \begin{bmatrix} C & 0 \\ L & 0 \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} G & F_{12} \\ 0 & R \end{bmatrix} \cdot x + S \cdot E.
\]

where \( X = [u_2u_3 \ldots u_nu_2u_3 \cdots u_ni_1i_2 \cdots i_{n-1}i_{n-1}i_2 \cdots i_{(n-1)}]^T \).
The numerical integration of the obtained differential equations in the Cauchy form is carried out by MATLAB system are shown in graphs (Figure 3).

For numerical integration, write (12) in Cauchy form:

\[
\frac{dx}{dt} = A \cdot x + B \cdot E \quad \text{or} \quad \frac{dx}{dt} = f(x, t, e(t)), \quad x(0) = x_0, \quad (13)
\]

where \( A = H^{-1} \cdot K; \quad B = H^{-1} \cdot S; \quad f(x, t, e(t)) \) - \( m \)-dimensional vector function; \( x_0 - m \)-a dimensional vector of the initial values of the search values.

3. Results of the simulation of mutual electromagnetic influences of related power transmission lines in transition mode of the MATLAB system

The numerical integration of the obtained differential equations in the Cauchy form is carried out by the Runge-Kutta method of the fourth degree (fourth order of accuracy at one integration step) [6].

Results of line simulation in the MATLAB system are shown in graphs (Figure 3).
Consider a line with one $e_1(t)$ source with parameters: $U = 220$ kV; $R_1 = 0.0127$ Om/km; $R_2 = 0.0127$ Om/km; $L_1 = 0.9337$ mH/km; $L_2 = 0.9337$ mH/km; $G_1 = 0$ Cm/km; $G_2 = 0$ Cm/km; $C_1 = 12$ nF/km; $C_2 = 12$ nF/km; $C_{12} = 6$ nF/km; $f = 60$ Hz.

**Figure 3.** Current and active power graphs at the beginning and end of the 2nd line at $R_A = 0$, $R_B = 0$.

4. Conclusion

The resulting mathematical model makes it possible to model electromagnetic processes in the transmission (both homogeneous and heterogeneous) taking into account mutually inductive and capacitive couplings in transition and fixed modes. As a result, a number of important scientific and practical challenges related to the design and operation of EEC can be addressed.

The model also allows for a variety of situations and tasks related, for example, to the calculation of systems modes at breakdowns of lines, SC, or single-phase SC in isolated neutrals. In particular, it is possible to calculate the steered voltage of the line from high-voltage or high-precision power lines.

Despite the complication of power electrical equipment by the introduction of an additional converter, the effectiveness and feasibility of this solution were experimentally confirmed.

On the basis of this method, it is further proposed to develop an algorithm for the machine formation of mathematical models for the study of transition processes in complex electrical networks, where the initial data will be their parameters and the structure of graphs of the investigated networks.

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