Behavior of liquid drop situated between two oscillating planes

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Abstract. The levitation drop technique is widely used for the measurement of the surface tension and viscosity of liquids. An experiment with a drop situated between two horizontal rigid planes gives the same possibilities. The dynamic problem is solved numerically in the following cases: (1) the free oscillations of the drop when the plates are motionless; (2) the forced oscillations when the upper plate makes a translational vibration in the normal direction. The possibility of viscosity determination in such experiments is shown.

1. Introduction

The theory for the surface tension driven oscillation have been elaborated in [1-3]. In the case of spherical, force-free, liquid drop the frequency and damping constant can be determined by the formula

$$\omega_l = \sqrt{\frac{\gamma}{\rho R^3}}\left(\frac{R^2}{R^2} - \frac{2\nu}{R^2}\left(l - \frac{l + 1}{2}\right)\right),$$

where $\gamma$ is the surface tension, $\rho$ is the density of the drop, $R$ is the radius of the drop in equilibrium and $l \geq 2$ is the mode’s number. The experiments in the Space Shuttle during its free fly or in a drop tower have shown good agreement with (1). On ground surface these experiments require the usage of complicated tools to establish and control the levitation force. As indicated in [4, 5], the resulting force is equal to zero only when integrated over the volume of the liquid drop, so the levitation force can produce sample rotation and disturb the drop’s spherical form by internal flows. The non-spherical form leads to a splitting of the Rayleigh mode just as the sample rotation. All these phenomena prevent the use of the analytical decision of the problem.

The liquid drop in contact with rigid plates oscillates when its equilibrium is disturbed. The advantage of this method lies in the simplicity of installation for such an experiment as there is no need for any complex experimental apparatus.

2. Assumptions and mathematical model

Our attention is restricted to a single liquid drop with volume $V$ bounded by two horizontal rigid planes with distance $H$ between them (figure 1). The material properties are: $\rho$ is the density of liquid, $\nu$ is the kinematic viscosity, $\gamma_{lg}$, $\gamma_{gs}$, $\gamma_{ls}$ are the surface tension on liquid-gas, gas-solid and
liquid-solid boundaries. Here and later the subscripts l, g and s denote the liquid, gas and solid phases respectively.

2.1. Assumptions
We make the following assumptions:
1) the liquid is Newtonian and incompressible; 2) there is no evaporation from the drop surface during the experiment; 3) the form of the drop has axial symmetry, the same is for the velocities and pressure on it; 4) it is possible to neglect the heat emission in the liquid, so the situation can be considered as isothermal; 5) the no-slip condition is correct on firm borders; 6) there is no contact angle hysteresis.

2.2. Fluid dynamics
The governing equations to describe the drop’s dynamics are the Navier-Stocks and continuity equations for liquid phase:

\[ \frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} + \nu \Delta \mathbf{u} - \frac{1}{\rho} \nabla p - \mathbf{g}, \]
\[ \nabla \cdot \mathbf{u} = 0, \]

where \( \mathbf{u} = (V_r, V_\theta, V_z) \) and \( p \) are the velocity and pressure in liquid.

The boundary conditions of the problem are:

\[ \mathbf{u} \big|_{S_{ls}} = 0, \quad \mathbf{n} \cdot \mathbf{T} \big|_{S_{lg}} = 0, \quad \mathbf{n} \cdot \mathbf{T} \big|_{S_{lg}} = \gamma_{lg} K, \]

where \( \mathbf{T}_{ij} = -p \delta_{ij} + \eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \) is a stress tensor, \( \delta_{ij} \) is the Kroneker delta and \( \eta = \nu \rho \) is the dynamic viscosity. The first of (6) means a tightness of firm borders and a no-slip condition on them, the second means that a tangential component of the stress vector is zero on a free surface and the third represents the Laplace formula for the normal stress vector on a free surface.

Equations (5, 6) are made dimensionless by normalizing the distances by the radius \( R \) of the equivalent free drop, the velocities by \( u_0 \), the pressure by \( \rho u_0^2 \) and the time by \( R/u_0 \), \( u_0 \) is the characteristic velocity.

![Figure 1](image1.png)

**Figure 1.** The drop is situated between two rigid plates.

![Figure 2](image2.png)

**Figure 2.** The velocity distribution visualization in axial section.

3. Numerical solution procedure
A numerical solution is found by a finite-difference method. A uniform grid with the maximum partition of up to 30 steps in the z-direction is used. The time derivatives are discretized using a unilateral scheme with accuracy up to \( \Delta t \) and the spatial derivatives are discretized using a central difference implicit scheme with accuracy up to \( (\Delta x)^2 \), the tilde marks dimensionless variables.

The decision is carried out in the spirit of methods of splitting and can be divided into 4 stages at each time level. They are the following: 1) the shape of the equilibrium drop is defined by the procedure of energy minimization, so the equilibrium curvature \( K_0(z) \) is defined; 2) the pressure in
the drop is defined by the decision of Poisson equation. The shape of the drop and the velocity distribution in it do not vary during this stage; 3) the distribution of velocities is determined by the decision of Navier-Stocks equations; 4) the new shape of the free surface is defined by the formulae

\[ r_i^{t+\Delta t} = r_i^t + V_r \big|_{S_y} \cdot \Delta t; \quad z_i^{t+\Delta t} = z_i^t + V_z \big|_{S_y} \cdot \Delta t, \quad i = 2, m. \]

The grid is reconstructed on each time level according to the changes of the drop shape. The accuracy of the code developed was checked by comparing the results with various grid sizes and time-steps. Based on these comparisons, the mesh resolution required to obtain a size-independent solution are determined.

4. Results and discussion

4.1. Free oscillations
The instant visualization of velocity distribution in the axial section is shown in figure 2. The calculation is carried out for the drop of water with the following parameters: \( R = 1.5 \) mm, \( \mu_0 = 10^{-2} \) m s\(^{-1}\), \( H = 1.2R \), \( \gamma_{lg} = 0.073 \) N m\(^{-1}\), \( \gamma_{ls} - \gamma_{gs} = -0.01 \) N m\(^{-1}\). The lowest oscillation mode corresponds to mass transfer from the bottom part of the drop to its top part and back. The fluid is driven along the top and bottom plates in reverse directions and moved in the bulk along the central axis up or down, depending on oscillation phase. The dependences \( r_i(t) \) represent damping oscillations. To obtain the free oscillation frequency \( \omega_0 \) and the damping constant \( p \) we approximate \( r_i(t) \) by the function \( y(t) = A \cdot \exp(-p \cdot t) \sin(\omega_0 \cdot t + \varphi) + C \). The approximation was performed by the least squares method with minimization using the Rosenbrock algorithm.

![Figure 3. The initial phase’s dependence versus coordinate z](image1)

![Figure 4. The phase’s difference of the top and bottom wetting spots oscillations versus viscosity; ▲ - \( A = 2 \) \( \mu \)m; ■ - \( A = 3 \) \( \mu \)m; ● - \( A = 5 \) \( \mu \)m](image2)

4.2. Forced oscillations
Translational vibration of the top plane with frequency \( \omega_v \) in the normal direction causes changes of cross-section radii \( \{r_i\} \) in time. A momentum transfer from the top part of the drop to its bottom part is not a momentary process, therefore the oscillation of the top and bottom wetting spots have different initial phases. In figure 3 the typical dependence of the initial phases on coordinate \( z \) is represented. Obviously from the figure, the liquid layers adherents to plates oscillate with the same phases. As it is seen from the figure, the thickness of these layers is about 0.2\( H \). The difference of oscillation phases
of the top and bottom wetting spots should depend on the liquid’s viscosity. In figure 4 the dependence between the viscosity and the phase difference mentioned is represented, the vibration amplitudes were equal to $2\,\mu m$, $3\,\mu m$ or $5\,\mu m$. The phases are determined after a complete decay of free oscillations was achieved. Apparently from figure 4, for $A = 5\,\mu m$ the graph represents a direct line for all considered ranges of viscosity values. The dependences constructed at $A = 2\,\mu m$ and $A = 3\,\mu m$ are linear only at small values of viscosity and nonlinear effects appear at a smaller viscosity the less amplitude $A$ is. The less the vibration amplitude, the greater then sensitivity which can be determined as an angle of inclination of the curve $\Delta\psi(\nu)$.

Thus, in this work a drop bounded by two rigid horizontal plates has been considered, the problem of free and force oscillations dynamic was solved by numerical method and the possibility of viscosity determination in such experiments has been shown.

References

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