A combined mean-field and three-body model tested on the $^{26}$O-nucleus

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We combine few- and many-body degrees of freedom in a model applicable to both bound and continuum states and adaptable to different subfields of physics. We formulate a self-consistent three-body model for a core-nucleus surrounded by two valence nucleons. We treat the core in the mean-field approximation and use the same effective Skyrme interaction between both core and valence nucleons. We apply the model to $^{26}$O where we reproduce the known experimental data as well as phenomenological models with more parameters. The decay of the ground state is found to proceed directly into the continuum without effect of the virtual sequential decay through the well reproduced $d_{3/2}$-resonance of $^{23}$O.

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Introduction

Self-consistent mean-field calculations efficiently provide accurate average properties for $N$-body systems \cite{11,13}. Approximate methods have been developed to treat correlated systems \cite{4,14}. All these methods are first of all aimed at describing bound states. Resonances and their decay, and continuum states in general often are addressed, but only with much more difficulty. The challenges are often referred to as problems in connection with open quantum systems \cite{15}. This concept is defined as quantum systems in interaction with the environment via external fields or, more appropriate in the present context, by coupling to continuum degrees of freedom.

When $N$ is less than about 15, complete correlated solutions can be obtained with modern computers \cite{16,20}. The solutions are obtained with few-body techniques, that is as analytically or numerically accurate as necessary in the context. Thus, all necessary effects from couplings to the continuum are in principle fully included. These few-body techniques are designed to operate for distances larger than the radii of the constituent particles necessarily assumed to be inert. This is in contrast to the many-body methods designed to treat approximately bound or at least quasi stable states at small distances.

None of the existing methods treat both small-distance structure and large-distance decay properties equally well. The traditional few-body cluster methods assume small-distance boundary conditions at the surface of the constituents and provide corresponding large-distance behavior. The many-body methods provide detailed small distance structures and therefore correct boundary conditions for few-body calculations.

Combining these properties in one model would be extremely useful and of general interest in all subfields of physics. More specifically, a number of interdisciplinary topical problems can be better understood microscopically. This applies in particular to the concept of universality in connection with halo formation and decay \cite{21} and the extreme of Efimov Physics \cite{22}. Both phenomena appear in nuclei and nuclear astrophysics, as well as in cold atomic and molecular gases.

The purpose of this letter is to provide an overall framework to combine the few- and many-body treatments of relative and intrinsic motion of the constituent particles. We shall use the hyperspherical adiabatic Fadeev expansion method for the few-body part and the mean-field approximation for the many-body part. The effective interactions in this combined model design is a challenge, because the allowed many-body Hilbert space requires specially adjusted interactions, which must be renormalized for use in the few-body calculations. The applied many-body effective interaction is in this letter consistently incorporated in the few-body treatment, except for two terms of minor importance where we indicate possible improvements.

The practical implementation is crucial to test applicability, accuracy and efficiency. We choose $^{26}$O, where the nucleons in $^{24}$O-core and the additional two neutrons require different treatment \cite{23}, and where the traditional methods are inappropriate. The $^{26}$O-nucleus is an ideal test case on the neutron dripline with the double magic $^{24}$O and spherical $^{24}$O-core and the two valence neutrons $^{25,26}$. A detailed phenomenological investigation has appeared in the course of this work \cite{25}. We compare results and predict other observables.

Theoretical formulation. We consider an $A+2$ nucleon system divided into a core with mass number $A$ and two valence nucleons. We assume the same two- and three-body interactions, $V_{ij}$ and $V_{ijk}$, acting between all the nucleons in core and valence space. The general Hamiltonian can then be written

$$H = \sum_{i=1}^{A+2} T_i - T_{cm} + \sum_{i<j}^{A+2} V_{ij} + \sum_{i<j<k}^{A+2} V_{ijk}$$

(1)

where $T_i$ and $T_{cm}$ are the kinetic energy operators for the $i$'th nucleon and for the total $A+2$ system, respectively. We reorganize $H$ into terms related to core, $H_c$, \[\text{etc.}\]
and valence, $H_v$, particles, i.e. explicitly

$$H = H_c(r_1, \cdots, r_A) + H_v(r_{v1}, r_{v2}) ,$$

$$H_c = \sum_{i=1}^{A} T_i - T_\text{cm}^\text{core} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} ,$$

$$H_v = T_\text{cm}^\text{core} + T_{A+1} + T_{A+2} - T_\text{cm} + V_{A+1,A+2} + \sum_{i=1}^{A} (V_{i,A+1} + V_{i,A+2}) + \sum_{i=1}^{A} V_{i,A+1,A+2} ,$$

where the spin and space coordinates of the $i$'th core or valence nucleons are $r_i$ and $r_{v_i}$, respectively.

The decisive approximation is now the choice of the Hilbert space allowed for the wave function, that is

$$\Psi = A(\Phi_c(r_1, \cdots, r_A)\Phi_v(r_{v1}, r_{v2})) ,$$

where $\Phi_c = \det(\{\phi_i\})$, is the Slater determinant, of single-particle wave functions, $\phi_i$, for the core nucleons, $\Phi_v$ is the three-body wave function, and $A$ symbolizes anti-symmetrization of all nucleons. The form of $\Psi$ in Eq. (5) clearly exhibits how we combine mean-field treatment of the core and ordinary treatment of the two three-body relative degrees of freedom. The total energy, $E$, is a sum of two terms corresponding to core, $E_c$, and valence, $E_v$, Hamiltonians in Eq. (2), that is

$$E = E_c + E_v = \langle \Psi | H_c | \Psi \rangle + \langle \Psi | H_v | \Psi \rangle .$$

We find the equations for the lowest energy solution by varying the wave functions over the allowed Hilbert space, that is

$$\frac{\delta E}{\delta \Phi_c} = \frac{\delta E}{\delta \Phi_v} = 0 ,$$

where both $\Phi_c$ and $\Phi_v$ must be normalized during the variation. The form of the two resulting equations are

$$H_{HF}(\Phi_c, \Phi_c)\phi_i = \epsilon_i \phi_i ; \quad H_v(\Phi_v, \Phi_v)\phi_v = E_v \phi_v ,$$

where the effective interactions in both Hartree-Fock single-particle, $H_{HF}$, and three-body, $H_v$, Hamiltonians depend on both $\Phi_c$ and $\Phi_v$. The coupled equations in Eqs. (8) must be solved simultaneously, which in practice means iteratively, to determine $\Phi_c$ and $\Phi_v$, and subsequently the energy, in a self-consistent procedure.

**Interactions.** We focus on the neutron dripline nucleus, $^{20}$O with the dominating configuration of two neutrons around a $^{24}$O-core. The choice of mean-field approximation requires a corresponding effective interaction. For the nucleon-nucleon interactions we choose the popular Skyrme form with SLy4 parameters. The three-body interaction in Eq. (3) is implemented as a density dependent two-body interaction.

The Skyrme interaction is of zero range and therefore not directly applicable in Hilbert spaces beyond Slater determinants. The necessary renormalization is possible but requiring additional investigations. Instead we use the finite-range nucleon-nucleon interaction in vacuum for $V_{A+1,A+2}$ in Eq. (4). We leave a more consistent adjustment to future refinements, because this interaction has very little influence on the small-distance structures, while more importantly the large-distance asymptotic properties are correct.

The phenomenological density dependence of the Skyrme interaction parametrizes all otherwise omitted influences, e.g. three-body effective forces. These effects are all accounted for by the two-body terms in Eqs. (3) and (4) except $\sum_{i} V_{i,A+1,A+2}$. Again we leave a more consistent derivation to future studies, because this term has very small structure influence, but it is necessary, if fine tuning of the global Skyrme energy is needed. We replace this term by $V_{c,A+1,A+2} = S_0 \exp(-\rho^2/\rho_0^2)$, where the hyperradius $\rho$ is defined as

$$(m_n + m_c)\rho^2 = m_c [(r_{v1} - R_c)^2 + (r_{v2} - R_c)^2] + m_n (r_{v1} - r_{v2})^2 ,$$

where $m_n$, $m_c$ and $R_c$ are neutron mass, core mass and core center-of-mass coordinate, respectively. The limit of zero at large-distances is correct by construction. The range and strength parameters are $\rho_0 = 6$ fm and $S_0 = -6.45$ MeV. The Skyrme interactions lead to density dependent effective masses, which also appear in the coupled Eqs. (8). This is a new feature in three-body equations and in few-body physics in general.

The space allowed for the valence nucleons is only limited by the presence of the identical core nucleons. In the three-body calculation these core-occupied Pauli forbidden states are removed either by excluding the corresponding lowest adiabatic potentials or by constructing phase equivalent potentials with less bound states. The voluminous and tedious, but straightforward, derivation along with the subtleties and the space-requiring detailed formulae will be discussed in forthcoming publications.

Meaningful combination and parallel treatment of core and valence spaces require careful selection and perhaps adjustments of the interactions. Our choices are consistent but two less important links between the interactions still needs to be fine-tuned. Achieving rigorous consistency is probably difficult in general and the most obvious first application is on small- to large-distance dependence of two-nucleon correlations around a finite nucleus.

**Three-body energies.** With these interactions we find from Eq. (7) the self-consistent variational solution where the core particles are affected by the valence nucleons and vice versa. We use the hyperspherical adiabatic expansion method for the three-body part where the basic ingredients are the adiabatic potentials displayed in Fig. 1. The lowest potential is attractive at small
distorted

Hartree-Fock

285 keV

Hartree-Fock

18 keV (Exp.)

24O+n+n

26O

Figure 2: The distortion energy of 24O (left) within the 26O system, and the ground state energies of 26O (right) for the Hartree-Fock approximation and the present method. The zero-point is the energy of 24O in the Hartree-Fock ground state.

Three-body structure. The probability distribution is shown in Fig. 3 as a function of neutron-neutron distance, and core to neutron-neutron center of mass distance. Two sharp peaks are seen corresponding to a distance between the neutrons and their center of mass and the core of about (6, 2) fm and (3, 3) fm, respectively. These are approximately linear and equal sided triangular configurations as shown schematically in Fig. 3. A much fainter peak at distances (4,1.8) fm is also seen temptingly interpreted as a di-neutron signature. In Ref. [28] the same triple peak structure is obtained using phenomenological interactions, but the di-neutron configuration is concluded to be the dominating structure.

To compare properly we repeated the calculation using the neutron-core potential given in [28], and found very similar peak structures as in Fig. 3. We find that the “di-neutron” configuration in Ref. [28] is much smaller than, but separated from, the other two peaks. The differences from us are due to different adjustments of neutron-core Woods-Saxon potential to give the 25O properties, and the density dependent neutron-neutron pair potential to give the 26O energy with the use of the bare nucleon mass. Especially the last adjustment differs from our model.
We can also study the structure of the $^{25}\text{O}$ ground state resonance through the invariant mass spectra of two of the particles after knockout of the third one [34]. The results are shown in Fig. 4 where we first notice the expected structureless neutron-neutron spectrum. The neutron-core spectrum is more interesting with a peak at 0.85 MeV, which is only 0.1 MeV higher than the experimentally known $d_{3/2}$ resonance at 0.749(10) MeV [27].

This is in fact a remarkably good agreement for three reasons. First, this result of the neutron-core resonance energy is obtained without any free parameters, that is without any adjustment, and it is found directly from the same interaction as between the nucleons in the core. This is also in contrast to phenomenological models [28] where this two-body energy is used as input parameter. Second, a pure Hartree-Fock calculation of $^{25}\text{O}$, with the same Skyrme force, yields a $d_{3/2}$ energy of $-0.96$ MeV. This is a bound state and far from the experimental value, and as such revealing the inadequacy of the Hartree-Fock approximation. Third, the final state in $^{25}\text{O}$ is populated in two different reactions in experiment and theory, i.e. by high-energy proton and neutron knockout, respectively.

The calculated width does not include effects of the unavailable experimental resolution and therefore understandably smaller than the observed value. The neutron-core spectrum is almost indistinguishable from the $d$-wave contribution also shown in Fig. 4. This reflects the structure of 90% neutron-core $d_{3/2}$-wave in the total three-body wave function. The rest is an equal distribution of $p_{3/2}$, $d_{5/2}$ and $f_{7/2}$ waves.

**Lifetime and decay properties** As already mentioned, a central aspect of the present method is how the presence of the valence nucleons affects the core. This is reflected in distortion of the core wave function and therefore also in the energy. By going from small to large values of hyperradius, $\rho$, the core should change from a distorted structure to the free solution. However, currently only the average effect is included and the core structure is maintained for all distances. This lack of a gradual relaxation means that while the bulk part of the calculated potential in Fig. 1 is correct, the large-distance asymptotic value of the potential is too high, in this case by 285 keV. However, by extending the adiabatic approach, and solving the coupled expressions from Eq. 3 for each step in $\rho$, a smooth transition could be obtained. We leave this more elaborate procedure for future improvements, because this would only marginally change any of the observables except possibly the width.

This width of the $^{25}\text{O}$ ground state resonance can still be fairly well estimated in the present implementation as the oscillator-approximated knocking rate multiplied by the WKB tunneling probability through the modified barrier in the lowest adiabatic potential [34]. First we shift the full adiabatic potential by 285 keV. This leads to an outer turning point of 66.3 fm and a lifetime of $10^{-16}$ s. Instead, when only lowering the potential by 285 keV in the tail outside 66.3 fm, the well-defined outer turning point at the energy of $18 - 285 = -267$ keV leads to a lifetime of about $10^{-15}$ s. Experimentally the half-life is found to be between $10^{-17}$ s and $10^{-15}$ s [27], and we conclude that even when employing a very crude transition mechanism this exponentially sensitive observable is predicted remarkably well by the model.

The resonance is decaying into two neutrons and the core. Following [38, 39], the single-particle energy distributions after decay are obtained in coordinate space from the stable spatial distribution of the particles for large values of $\rho$, where the hyperangles in coordinate and momentum space coincide. The results are shown in Fig. 5 for two $\rho$-values, 150 and 160 fm, to show convergence. The extremes, where one particle takes either maximum

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**Figure 4:** The invariant mass spectra of core-neutron (solid) and neutron-neutron (dotted) for the SLy4 Skyrme parameters. The dashed curve is the core-neutron contribution from $d$-waves. The black curve is the measurements from Ref. [27].

**Figure 5:** Single-particle energy distributions after decay of the ground state resonance [38][39]. Here $E_i$ is core (solid) or neutron (dashed) energy, and $E_{i,max}$ is the maximum energy available for particle $i$. The peak for the core energy is due to numerical inaccuracies.
or zero energy, leaves either zero or maximum energy in the relative motion between the other two particles.

We then see in Fig. 5 that the core energy is weighted towards its maximum with a corresponding decreasing fraction left for the relative motion of the neutrons (solid curves). Each neutron has largest probability for appearing with half of its maximum energy, again implying that the other half is in relative neutron-core motion (dashed curves). This shows that the decay mechanism is direct population of the continuum, consistent with the fact that the $d_{3/2}$ resonance in $^{25}$O is too high in energy to be even virtually populated during the decay.

Summary. The present study provides a consistent approach to including the intricacies of few-body formalisms into a many-body context. The model is practical and efficient as demonstrated in the application on the challenging nuclear neutron dripline nucleus, $^{26}$O. The data of both $^{25}$O and $^{26}$O are reproduced with fewer parameters than found in dedicated phenomenological models. Furthermore, the exponentially sensitive lifetime is obtained within measured uncertainties. The novel features of the quantum mechanical model are that few- and many-body properties respectively at large and small distances are self-consistently connected. The model is applicable to both bound and continuum states, and addresses challenges in open quantum systems. The universal character of halos and Efimov states suggests applications in other subfields of physics.

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