Global analysis of oscillation parameters

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Abstract. We present an analysis of oscillation neutrino data in the standard framework of three massive and mixed neutrinos. The status of the known and unknown three-neutrino mass-mixing parameters is discussed. The main results of this work are that normal ordering (NO) is favored over inverted ordering (IO) at 3σ level and that preferred values of the Dirac CP phase around nearly-maximal CP-violating cases. Finally, the octant of the atmospheric mixing angle remains undetermined.

1. Introduction
This work is mainly based on our latest analysis reported in [1], where a global analysis of oscillation and non-oscillation data was performed and where all included data are described. The three known flavor states \( \nu_\alpha = (\nu_e, \nu_\mu, \nu_\tau) \) are linear combinations of three states \( \nu_i = (\nu_1, \nu_2, \nu_3) \) with definite masses \( m_i = (m_1, m_2, m_3) \) through the Pontecorvo-Maki-Nakagawa-Sakata [2, 3] unitary matrix \( U \). The mixing matrix \( U \) is parameterized in terms of three mixing angles \( (\theta_{12}, \theta_{13}, \theta_{23}) \) and one so-called Dirac phase \( \delta \), associated to possible CP violation processes. To describe neutrino oscillations we define two independent squared mass differences, \( \delta m^2 = m_2^2 - m_1^2 > 0 \) and \( \Delta m^2 = m_3^2 - m_1^2 + m_2^2 \), with two possible options for the neutrino mass spectrum ordering: either \( \Delta m^2 > 0 \) (normal ordering, NO) or \( \Delta m^2 < 0 \) (inverted ordering, IO). A final remark: all results presented here will be expressed in terms of standard deviations \( N\sigma \) from a local or global \( \chi^2 \) minimum, \( N\sigma = \sqrt{\Delta \chi^2} \).

2. Methodology
To understand how bounds on mass-mixing oscillation parameters arise from the combination of data from many different experiments we group them as in [4], in order to show more clearly the progressive impact of different data sets on both known and unknown parameters. This methodology was adopted after the first indications in favour of a nonzero value of \( \theta_{13} \) [5] and its experimental discoveries [6]. We start by combining solar and long-baseline reactor data, that probe the \( \nu_e \rightarrow \nu_e \) flavor disappearance channel via oscillations driven by the \( (\delta m^2, \theta_{12}, \theta_{13}) \) parameters. These data provide precise measurements of \( (\delta m^2, \theta_{12}) \) and a rough measurement of \( \theta_{13} > 0 \) at the \( \sim 2\sigma \) level. Long-baseline accelerator data probe both the \( \nu_\mu \rightarrow \nu_\mu \) disappearance
and the $\nu_\mu \rightarrow \nu_e$ appearance channel via oscillations driven mainly by the $(\Delta m^2, \theta_{23}, \theta_{13})$ parameters, but they are also sensitive to subleading effects driven by $(\delta m^2, \theta_{12})$, as well as by $\delta$ and $\text{sign}(\Delta m^2)$. The combination of these data provides a good measurement of mixing angles and mass square differences, and also a hint in favor of $\sin \delta \neq 0$ and of normal ordering. These hints are enhanced by adding then short-baseline reactor data, mainly sensitive to $(\Delta m^2, \theta_{13})$, and then atmospheric neutrino data, sensitive in different ways to all the oscillation parameters via disappearance and appearance channels. This way of grouping different data sets allows to gauge how the current indications about neutrino CP violation and mass spectrum ordering are progressively enhanced, by using increasingly rich data sets in the global analysis.

3. Results of the global fit

The main results on oscillation parameters are summarized in Fig. 1. The squared mass differences $(\delta m^2, |\Delta m^2|)$ are determined within a couple of percent, while the mixing parameters $(\sin^2 \theta_{12}, \sin^2 \theta_{13}, \sin^2 \theta_{23})$ within a few percent. A preference for NO emerges at $3\sigma$ level from the global analysis, with coherent contributions from various data sets. If this $\Delta \chi^2$ difference is taken at face value, no allowed region survives for IO up to $3\sigma$. By considering NO and IO as separate cases, we also find that the Dirac CP phase $\delta$ is constrained within $\sim 15\%$ ($\sim 9\%$) uncertainty in NO (IO) around nearly-maximal CP-violating values, $\delta \sim 3\pi/2$. The CP-conserving value $\delta = 0$ (or $2\pi$) is disfavored at $3\sigma$ in both NO and IO: the value $\delta = \pi$ is also disfavored at $3\sigma$ in IO but not in NO (where it is still allowed at $2\sigma$). Finally, with respect to $\theta_{23}$, we find an overall preference for the second octant (more pronounced in IO), although both octants are allowed at $2\sigma$.

Figure 2 shows the regions allowed at $N\sigma$ in the plane $(\sin^2 \theta_{23}, \sin^2 \theta_{13})$, for both NO (upper panels) and IO (lower panels), for increasingly rich data sets. The leading LBL appearance amplitude is governed by a term proportional to $\sin^2 \theta_{23} \sin^2 \theta_{13}$ and it induces an anti-correlation...
between these two parameters, visible in the left panels. Subleading effects generate instead a difference in the allowed \( \theta_{13} \) ranges for NO and IO, the latter ones being generally higher. The middle panels show the combination with SBL reactor data. The comparison of the left and middle panels clearly shows that current accelerator and reactor constraints on \( \theta_{13} \) are more consistent in NO than in IO. Finally, atmospheric neutrino, in the right panels, provide an independent \( \Delta \chi^2 \) increment, so that the allowed regions are slightly reduced. Concerning \( \theta_{23} \), there is a slight \( \theta_{13} \) tension between accelerator and reactor data in IO, that is minimized for relatively large \( \theta_{23} \), hence the more pronounced preference for the second octant. However, the octant ambiguity remains unresolved at 2\( \sigma \) level in both NO and IO.

Figure 3 shows the covariance of the \( \Delta m^2, \theta_{13}, \sin^2 \theta_{23} \) parameters, in the same format as Fig. 2. There is a very good consistency of all the data on \( \Delta m^2 \), whose best-fit value remains practically constant in the three upper panels, while the value of \( \Delta m^2 \) slightly increases after the addition of SBL reactor data, as consequence of the slight increment in \( \sin^2 \theta_{23} \) discussed for Fig. 2. In general, at nearly maximal mixing one gets the lowest allowed values of \( \Delta m^2 \), while for nonmaximal mixing (in either octants) the preferred values of \( \Delta m^2 \) tend to increase. This correlation comes mainly from disappearance data in LBL accelerator experiments, where a decrease of the leading oscillation amplitude (governed by \( \sin^2 2\theta_{23} \)) can be partly traded for an increase of the leading oscillations phase (governed by \( \Delta m^2 \)), so as to keep the disappearance rate nearly constant.

Figure 4 shows the \( N\sigma \) regions allowed in the plane \( (\delta, \sin^2 \theta_{13}) \). The strong correlations between these two parameters (in the left panels) are mainly induced by the interplay between \( \delta \) and \( \theta_{13} \) arising in the subleading terms of the appearance probability of accelerator neutrinos. In NO, the best fit of \( \delta \) remains very close to \( \sim 1.3\pi \) by adding first SBL reactor and then atmospheric neutrino data. In NO, the consistency of all the data sets towards the same best-fit values of both the known \( (\Delta m^2, \theta_{23}, \theta_{13}) \) parameters and of the unknown \( \delta \) phase is striking. In IO there is a slight decrease of \( \delta \) from left to middle panels, correlated to the decrease of \( \theta_{13} \).

Figure 5 shows the \( (\delta, \sin^2 \theta_{23}) \) covariance. Only weak correlations (if any) emerge between

**Figure 2.** Covariance of the \( (\sin^2 \theta_{13}, \sin^2 \theta_{23}) \) parameters.
these two parameters at the current level of accuracy. In particular, in NO there is a slight anti-correlation, which implies that the best fit of $\delta$ might increase from $\sim 1.3$ to $\sim 1.4$ if the first (rather than the second) octant of $\theta_{23}$ were favored by upcoming data. In IO there is no significant correlation. These considerations about the interplay among three unknowns (the phase $\delta$, the $\theta_{23}$ octant and the mass ordering) are rather fragile and might change with future data. Conversely, the overall $3\sigma$ constraints on $\delta$ emerging in Fig. 4 and 5 appear to be relatively robust, with modest dependence on $(\theta_{13}, \theta_{23})$.

**Figure 3.** Covariance of the ($\Delta m^2$, $\sin^2 \theta_{23}$) parameters.

**Figure 4.** Covariance of the ($\delta$, $\sin^2 \theta_{13}$) parameters.
4. Conclusions

A global analysis of data coming from neutrino oscillation experiments is presented, within the standard framework including three massive and mixed neutrinos. We have discussed in detail the status of the three-neutrino mass-mixing parameters, both known and unknown. The main results of this work from the analysis of oscillation searches are summarized graphically in Fig. 1. A preference for NO emerges at $3\sigma$ level from the global analysis, with coherent contributions from various data sets. The Dirac CP phase $\delta$ is constrained within $\sim 15\%$ ($\sim 9\%$) uncertainty in NO (IO) around nearly-maximal CP-violating values. The octant of $\theta_{23}$ is still undetermined.

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