MODELING PLANETARY SYSTEM FORMATION WITH N-BODY SIMULATIONS: ROLE OF GAS DISK AND STATISTICS COMPARED TO OBSERVATIONS

HUIGEN LIU, JI-LIN ZHOU, AND SU WANG

Department of Astronomy & Key Laboratory of Modern Astronomy and Astrophysics in Ministry of Education, Nanjing University, Nanjing 210093, China; zhoujl@nju.edu.cn

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ABSTRACT

During the late stage of planet formation, when Mars-sized cores appear, interactions among planetary cores can excite their orbital eccentricities, accelerate their merging, and thus sculpt their final orbital architecture. This study contributes to the final assembling of planetary systems with N-body simulations, including the type I or II migration of planets and gas accretion of massive cores in a viscous disk. Statistics on the final distributions of planetary masses, semimajor axes, and eccentricities are derived and are comparable to those of the observed systems. Our simulations predict some new orbital signatures of planetary systems around solar mass stars: 36% of the surviving planets are giant planets (>10 M⊕). Most of the massive giant planets (>30 M⊕) are located at 1–10 AU. Terrestrial planets are distributed more or less evenly at <1–2 AU. Planets in inner orbits may accumulate at the inner edges of either the protostellar disk (3–5 days) or its magnetorotational instability dead zone (30–50 days). There is a planet desert in the mass–eccentricity diagram, i.e., a lack of planets with masses 0.005–0.08M J in highly eccentric orbits (e > 0.3–0.4). The average eccentricity (~0.15) of the giant planets (>10 M⊕) is greater than that (~0.05) of the terrestrial planets (<10 M⊕). A planetary system with more planets tends to have smaller planet masses and orbital eccentricities on average.

Key words: celestial mechanics – methods: numerical – planetary systems – planet–disk interactions – planets and satellites: formation

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1. INTRODUCTION

To date, around 490 exoplanets have been detected, mostly by Doppler radial velocity measurements (see, e.g., Udry & Santos 2007 for a review of their statistics). The masses of the planets range from the order of Earth mass (M⊕) to tens of Jupiter masses (MJ). Among them, the most recently observed HD 10180 system has the most number of planets among exoplanetary systems (Lovis et al. 2011). A study of multi-planet systems is helpful in understanding the formation history of planetary architecture due to planetary interactions (e.g., Wittenmyer et al. 2009). Recent statistics of single and multiple planetary systems have revealed several new possible signatures (Wright et al. 2009).

1. Including systems with long-term radial velocity trends, at least 28% of the known systems appear to contain multiple planets. Thus, multi-planet systems seem to be a common phenomenon.

2. The distribution of orbital distances of planets in multi-planet systems and single planets is inconsistent: single-planet systems show a pileup at a period of ~3 days and a jump near 1 AU, while multi-planet systems show a more uniform distribution in log period.

3. Planets in multi-planet systems have somewhat smaller eccentricities than single planets.

4. Exoplanets with minimum masses larger than Jupiter have eccentricities broadly distributed across 0 < e < 0.5, while lower-mass exoplanets exhibit a distribution peak near e = 0.

5. There may be a positive correlation between stellar masses and the occurrence rate of Jovian planets within 2.5 AU (Johnson et al. 2007). This might be a reflection of the correlation of stellar masses with their circumstellar disks.

Concurrently, our theoretical insights into the process of planet formation have improved greatly. According to the conventional core-accretion scenario, planets form in circumstellar disks. Through sedimentation of dust and cohesive collisions of planetesimals, Mars-sized embryos will form through runaway and oligarchic growth (Safronov 1969; Kokubo & Ida 1998). Angular momentum exchanges between the embryos and the gas disk will cause a fast inward migration (type I) of the embryos in an isothermal disk (Goldreich & Tremaine 1979; Ward 1997; Tanaka et al. 2002). In some circumstances, for example, in a region where magnetorotational instability (MRI) is active (Laughlin et al. 2004; Nelson & Papaloizou 2004), or in a radiative disk (Paardekooper & Mellema 2006; Kley et al. 2009), the speed of type I migration can be greatly reduced or its direction reversed. Thus, the embryos can be effectively sustained. As long as they grow beyond some critical masses (~10 M⊕), quasi-hydrostatic gas sedimentation begins in the Kelvin–Helmholtz timescale (Pollack et al. 1996; Ikoma et al. 2000), which may last for millions of years until the final runaway gas accretion sets in. Type I migration of planetary cores is helpful in shortening the timescale of giant planet formation, provided the migration speed is reduced by a factor of ~10 (Alibert et al. 2005). The newly formed giant planets then undergo type II migration before the gas disk is depleted.

According to the above scenario of single-planet formation, planetary population synthesis has successfully reproduced most of the statistical characteristics of the observed systems. In a series of works, Ida & Lin (2004a, 2004b, 2005, 2008) predicted a planet desert, with masses of 10–100 M⊕, due to the runaway gas accretion of giant planets, and confirmed the
observed correlation that the planet detection rate increases with the metallicity of their host star (Fischer & Valenti 2005). Other studies with a population synthesis of planets, such as Kornet & Wolf (2006) and Mordasini et al. (2009a, 2009b), give a more constrained view of the planet formation model based on observational data.

However, planets form in an environment with many siblings. Perturbations from neighbor protoplanets may excite their eccentricities, which is helpful in planet formation in some cases. For example, the inward type II migration of gas giants in outside orbits may trap the planetary cores in inner mean-motion resonances (mainly 2:1 resonance), thus resulting in their merging into hot super-Earths (Zhou et al. 2005; Raymond et al. 2006). Although planetary perturbations may inhibit core growth in other cases, the lack of planets in the asteroid belt may be due to Jupiter’s perturbations. Moreover, planet–planet scattering is thought to be the major cause of the eccentricities of the observed planetary systems (Rasio & Ford 1996; Zhou et al. 2007; Chatterjee et al. 2008; Juric & Tremaine 2008). The planet population synthesis method, without including the planet–planet scattering procedure, incompletely accounts for the observed planetary signatures.

By taking the planet–planet scattering effect into consideration, N-body integration of the full planetary equations is necessary. Based on N-body simulations and a self-consistent disk evolution, Thommes et al. (2008) investigated the formation of giant planets in the context of disk evolution and revealed some interesting tendencies that relate planetary systems to their parent disks. Ogihara & Ida (2009) studied the formation and distribution of terrestrial planets around M-dwarf stars and found that final configurations of planets depend on the speed of type I migration.

In this work, we employ the N-body method to study the final assembly of planetary systems. The aims of this study are twofold: (1) to explain the observed statistics of planet masses and orbit parameters, thus constraining the parameters that are most suitable for planet formation; and (2) to guide future observations by extending our study to the mass regime that is not yet detectable. We adopt the standard model of the core-accretion scenario, which includes the standard type I and II migrations of planets.

For N-body simulations of planetary formation and evolution, one of the major difficulties consists of the initial conditions of planetary embryos and the embedded protoplanetary disk. During the later stage of planet formation, most of the embryos are assumed to be formed with masses ranging from that of Mars to several Earths (or even bigger). However, their radial distributions are uncertain. Also, the parameters of viscous disks (their extensions, surface density distributions, depletion timescales, etc.) are uncertain. These parameters may be coupled, and their effects are different. As shown in Thommes et al. (2008), disk properties play a key role in determining planetary migration. They choose the mass and viscosity of the protostellar disk as two free parameters. To simplify the model used in this paper, through some tests, we fix most of the parameters to some fiducial values, except we let the gas disk depletion timescale ($\tau_{\text{disk}}$) and disk mass ($\Sigma_g$) be two free parameters. The effects of other parameters on the final architecture of planetary systems are also discussed.

The arrangement of this paper is as follows. First, we describe the model and method of the paper in Section 2. Then, some analytical estimation of the orbital configuration under type I and II migrations is given in Section 3. The effects of disk parameters on the evolution and final architecture of planetary systems are discussed in Section 4. In Section 5, the distributions of planet parameters (masses, semimajor axes, and eccentricities) from simulations are compared to observations. To check if our results are credible in a two-dimensional model, we review the differences in a three-dimensional model in Section 6. The conclusions are presented in Section 7, with discussion and some implications for future work.

2. MODEL AND INITIAL SETUP

Our investigation starts at the stage where most of the planetary embryos have cleared their feeding zones and obtained their isolation masses. This may correspond to an epoch of ~1 Myr after the birth of the protostar. Embryos in inner orbits might undergo type I migration, which is assumed to be stalled at some locations, such as the inner edge of the MRI’s dead zone of the gas disk (Kretke & Lin 2007; Kretke et al. 2009). Some of the embryos outside the snow line are large enough (several $M_{\oplus}$) for the onset of efficient gas accretion. They begin to accrete gas, open gaps, and undergo type II migration. The details of the model and parameters used in this paper are presented as follows.

2.1. Viscous Disks

Pre-main-sequence stars accrete gas through circumstellar disks. Although the origin of the disk viscosity is still controversial, the onset of MRI helps in the transportation of angular momentum through the disk (Balbus & Hawley 1991). In this paper, we adopt the ad hoc $\alpha$-prescription (Shakura & Sunyaev 1973) to model the effect of mass transportation in the disk around a classical T-Tauri star. In such a disk, the kinematic viscosity is expressed as $\nu = ac_s h$, where $c_s$ and $h$ are the sound speeds in the midplane disk and density scale height, respectively (Table 1). The surface density at stellar distance $a$ is given as (e.g., Pringle 1981)

$$\Sigma_g = \frac{\dot{M}}{3\pi \alpha c_s h} \left[ 1 - \left( \frac{R_*}{a} \right)^{1/2} \right],$$

where $\dot{M}$ is the gas accretion rate and $R_*$ is the stellar radius. For a T-Tauri disk with stellar mass of $0.2 M_\odot \lesssim M_* < 2.0 M_\odot$, the observed accretion rate is (Natta et al. 2006; Vorobyov & Basu 2009)

$$\dot{M} = 2.5 \times 10^{-8} M_\odot \text{ yr}^{-1} \left( \frac{M_*}{M_\odot} \right)^{1.3 \pm 0.3}.$$  (2)

In an optically thin disk, adopting the parameters in Table 1 and letting $a \gg R_*$, we get Equation (3) from Equation (1),

$$\Sigma_g = 2400 \text{ g cm}^{-2} \left( \frac{a}{10^3} \right)^{-1} \left( \frac{M_*}{M_\odot} \right)^{0.8} \left( \frac{a}{1 \text{AU}} \right)^{-1}.$$  (3)

For an $\alpha$-disk with constant $\alpha = 10^{-3}$, we derive the minimum mass solar nebular (MMSN), except with a different slope, $\beta = -\ln \Sigma_g / \ln a$ (cf., $\beta = 3/2$ in MMSN; Hayashi 1981; Ida & Lin 2004a). We also assume that the protoplanetary disk has a sandwich-layered structure: inside a location $a_{\text{crit}}$, the protostellar disk is partly thermally ionized, while outside $a_{\text{crit}}$ only the surface layer is ionized by stellar X-rays and diffuse cosmic rays, leaving in the central part of the disk a highly
neutral and inactive “dead zone” (Gammie 1996). The viscosity in the MRI’s active region could be one or two magnitudes larger than that of the dead zone (Sano et al. 2000). By assuming a constant accretion rate across the disk at a specific epoch in Equation (1), a positive density gradient is expected near \( \alpha_{\text{crit}} \), which helps to halt the embryos under type I migration (Kretke & Lin 2007; Kretke et al. 2009).

To model this effect, we let \( \alpha_{\text{MRI}} \) and \( \alpha_{\text{dead}} \) denote the \( \alpha \)-values of the MRI’s active and dead regions, respectively. The effective \( \alpha \) for the disk is modeled as (Kretke & Lin 2007)

\[
\alpha_{\text{eff}}(a) = \alpha_{\text{dead}} - \frac{\alpha_{\text{MRI}}}{2} \left[ \text{erf} \left( \frac{a - \alpha_{\text{crit}}}{0.1 a_{\text{crit}}} \right) + 1 \right] + \alpha_{\text{MRI}},
\]

where erf is the error function and 0.1\( a_{\text{crit}} \) is the width of the transition region. In this paper, we adopt \( \alpha_{\text{MRI}} = 0.02, \alpha_{\text{dead}} = 10^{-4} \) (Sano et al. 2000). The small value of \( \alpha_{\text{dead}} \) is adopted to obtain a reasonable timescale of type II migration (see Equation (16)). Alternative choices of \( \alpha_{\text{dead}} \) are also discussed in Section 4.1.

The location of the inner edge of the MRI’s dead zone, \( a_{\text{crit}} \), varies with disk temperature, kinematics, mass accretion rate, etc. Here, we adopt the expression from Kretke et al. (2009):

\[
a_{\text{crit}} = 0.16 \, \text{AU} \left( \frac{M}{10^{-8} M_\odot \, \text{yr}^{-1}} \right)^{4/9} \left( \frac{M_\star}{M_\odot} \right)^{1/3} \left( \frac{\alpha_{\text{MRI}}}{0.02} \right)^{-1/5}.
\]

During the evolution of a T-Tauri star and its disk, \( M \) decreases from \( 10^{-6} \) to \( 10^{-9} M_\odot \, \text{yr}^{-1} \), according to the infrared excess observation (e.g., Gullbring et al. 1998). So, the location of \( a_{\text{crit}} \) migrates inward as time proceeds. To simplify the procedure, we let the gas disk deplete uniformly in a timescale \( t_{\text{disk}} \), so \( M = M_0 \exp(-t/t_{\text{disk}}) \). Assuming a disk mass of 0.02 \( M_\odot \) (Andrews & Williams 2005), we obtain \( \tau_{\text{disk}} = 0.02 M_\odot / t_{\text{disk}} \).

Substituting Equations (4) and (5) into Equation (3), we obtain the following surface density for the circumstellar disk:

\[
\Sigma_g = \Sigma_0 f_g \left( \frac{a}{1 \, \text{AU}} \right)^{-1} \left( \frac{\alpha_{\text{eff}}}{10^{-4}} \right)^{-1} \left( \frac{M_\star}{M_\odot} \right)^{4/5} \exp \left( -\frac{t}{\tau_{\text{disk}}} \right),
\]

where \( f_g \) is the gas enhancement factor. \( \Sigma_0 = 280 \, \text{g cm}^{-2} \) is adopted in this work so that the total mass of the disk, up to 100 AU, is 0.02 \( M_\odot \). For \( f_g = 1 \), corresponding to the average disk mass in the Taurus– Auriga star formation region (Beckwith & Sargent 1996). For such a disk, Tommelle’s criterion for the onset of gravitational instability is expressed as \( Q = \frac{c_s^2}{\pi \alpha_{\text{eff}}} = 340(a/1 \, \text{AU})^{-3/2} f_g^{-1} \). So \( Q > 1 \) holds, and gravitational instability will not occur up to 100 AU as long as \( f_g \leq 10 \).

We truncate the inner disk at the disk cavity due to the stellar magnetic field. Around the corotation radius, the stellar magnetic torque acts to extract angular momentum from the disk and spins down disk material. At the location where the stellar magnetic field completely dominates internal disk stresses, sub-Keplerian rotation leads to a free fall of the disk material onto the surface of the star in a funnel flow along magnetic field lines, which results in an inner-disk truncation (Köningl 1991). The maximum distance of disk truncation is estimated at \( \sim 9 \) stellar radii. Considering that the radius of the protostar is generally 2–3 times larger than its counterparts in the main-sequence stage, inner disk truncation would occur at <0.1 AU. Thus, we set the inner edge as 0.05 AU in this paper.

### 2.2. Isolated Embryos and Type I Migration

According to the core-accretion scenario of planet formation, planetary embryos form through cohesive collisions of heavy elements near the midplane of the circumstellar disk. Within the solid disk, an embryo will grow until it accretes all the dust material around its feeding zone, so that an isolation body is achieved (Kokubo & Ida 1998, 2002). During the later stage of planet formation, inward migration of planetesimals under gas drag and planetary embryos under type I migration may change the distribution of embryos, which results in an uncertainty of the initial embryos’ masses. On the other hand, the MMSN model for the solid disk was derived according to the final solid masses of the present planets in the solar system (Hayashi 1981; Ida & Lin 2004a), which reflects the final distribution of the embryo, so we adopt the isolation masses with MMSN distributions, but with an enhancement factor \( f_d = f_r \). In a gas disk, the instability of the isolation masses is suppressed until the gas disk is depleted. Assuming that the dynamical instability timescale of a group of isolated masses is the same as the disk dispersal timescale, the isolation masses are correlated with their mutual separations and are given by (Kokubo & Ida 2002; Zhou et al. 2007)

\[
M_{\text{iso}} = 0.16 M_\oplus \left( f_d \eta_{\text{ice}} \right)^{3/2} \left( \frac{a}{1 \, \text{AU}} \right)^{3/4} \left( \frac{M_\star}{M_\odot} \right)^{-1/2} \left( \frac{k_{\text{iso}}}{10} \right)^{3/2},
\]

where \( f_d \) is the enhancement factor of heavy objects over the MMSN model and \( \eta_{\text{ice}} = 1 \) inside \( a_{\text{ice}} \) and 4.2 outside \( a_{\text{ice}} \), with \( a_{\text{ice}} \) being the location of the snow line, beyond which water is condensed as ice from disk gas (\( T \approx 170 \, \text{K} \)). Here, \( k_{\text{iso}} \) is the separation of embryos scaled by \( R_H = (2 M_{\text{iso}} / 3 M_\odot)^{1/3} \). For \( \tau_{\text{disk}} = 1–10 \, \text{Myr} \), \( k_{\text{iso}} = 8–10 \) at 1 AU and 7–9 at 10 AU for a disk with \( f_d = 1 \) (see Figure 11 of Zhou et al. 2007 and details therein).

Due to the variation of the disk accretion rate and stellar radiation at different epochs of the protostar, the location of the snow line varies (Garaud & Lin 2007). For a star with mass \( \lesssim 3 M_\odot \), we adopt the snow line location as (Kennedy & Kenyon 2008)

\[
a_{\text{ice}} = 2.7 \, \text{AU} \left( \frac{M_\star}{M_\odot} \right)^{4/9} \left( \frac{M}{10^{-8} M_\odot \, \text{yr}^{-1}} \right)^{2/9}.
\]
To model the type I migration of the embryos, we adopt the expression of the migration timescale from Cresswell & Nelson (2006), which is also valid for eccentric orbits:

\[
\tau_{\text{mig, I}} = -\frac{1}{C_1} \left[ 2.7 + 1.1\beta \right] \left( \frac{M_*}{M_p} \right) \left( \frac{h}{r} \right)^2 \times \left[ 1 + \left( \frac{er}{\Omega_M} \right)^5 \right] \Omega^{-1},
\]

where a negative (positive) value of \( \tau_{\text{mig, I}} \) corresponds to inward (outward) migration, respectively; and \( M_p, r, e, \) and \( \Omega \) are the mass, position, eccentricity, and angular velocity of the planet, respectively. Due to the MRI effect, \( \beta = -\partial \ln \Sigma / \partial \ln a \) can be negative near the location of maximum pressure, \( a_{\text{crit}} \).

Here, \( C_1 \) is a reduction factor. A large amount of literature has shown that to produce the observed planetary occurrence rate, \( C_1 \in [0.03, 0.3] \) is an appropriate range (e.g., Alibert et al. 2005; Ida & Lin 2008). To test its validity in our model, we set \( C_1 = 0.03, 0.1, 0.3 \) and execute some test runs. As shown in Figure 1(a), choosing both \( C_1 = 0.03 \) and 0.1 produces a very slow planet migration embedded in our disk model, which may render the formation of hot Jupiters very difficult. So, we set \( C_1 = 0.3 \) throughout this paper.

The presence of the gas disk will damp the eccentricities of embedded embryos (Goldreich & Tremaine 1980). The e-damping timescale can be described as (Cresswell & Nelson 2006)

\[
\tau_{\text{edap, I}} = \frac{Q_e}{0.78} \left( \frac{M_*}{M_p} \right) \left( \frac{M_*}{\Sigma g^2 \partial r / \partial a} \right)^{1/2} \left[ 1 + \frac{1}{4} \left( \frac{er}{h} \right)^3 \right] \Omega^{-1},
\]

where \( Q_e = 0.1 \) is a normalization factor to fit with the hydrodynamical simulations.

2.3. Giant Planet Formation: Gap Opening and Type II Migration

According to the conventional accretion scenario, giant planets form through three major stages (Perri & Cameron 1974; Mizuno 1980; Pollack et al. 1996): (1) Embryo growth stage: protoplanetary cores form and grow mainly via the bombardment of planetesimals before they attain isolation masses. (2) Quasi-hydrostatic sedimentation stage: the accretion of planetesimals tapers as their supply in the feeding zone is depleted. This induces quasi-hydrostatic sedimentation and the growth of the gaseous envelope due to the loss of entropy. (3) Runaway gas-accretion stage: when the mass of the gas envelope becomes comparable to that of the core, a runaway stage of gas accretion sets in continuously until the gas supply is exhausted by either the formation of a tidally induced gap near the protoplanet’s orbit or the depletion of the entire disk.

Through quasi-static evolutionary simulations, Ikoma et al. (2000) derived the critical mass where significant gas accretion occurs:

\[
M_{\text{crit}} \sim 7\ M_\oplus \left( \frac{M_\text{core}}{10^{-7}\ M_\oplus}\ yr \right)^{0.25} \left( \frac{\kappa}{1\ \text{cm}^2\ \text{g}^{-1}} \right)^{0.25},
\]

where \( M_\text{core} \) is the rate at which planetesimals are accreted onto the core and \( \kappa \) is the grain opacity (see also Rafikov 2006). Due to the uncertainty of \( M_\text{core} \) and \( \kappa \), we assume \( M_\text{core} \propto f_g \) and adopt \( M_{\text{crit}} = 4\ M_\oplus f_g^{0.25} \) in this paper. A planet core beyond this critical mass will begin to accrete gas in a Kelvin–Helmholtz timescale, \( \tau_{KH} \simeq 10^9\ yr (M_p/\ M_\oplus)^{-3} \), so that \( dM_p/\ dt \simeq \dot{M}_p / f_{KH} \) (Ikoma et al. 2000).

However, the accretion rate is also limited by the replenishing rate of the materials (\( M_{\text{disk}} \)), so the accretion rate of the planetary embryos is expressed as (Ida & Lin 2004)

\[
\frac{dM_p}{dt} = \min \left[ 10^{-9} \left( \frac{M_p}{M_\oplus} \right)^{4} \ M_\oplus \ yr^{-1}, M_{\text{disk}} \right].
\]

When the planet mass becomes sufficiently large, a tidally induced gap in the gas disk forms around its orbit (Lin & Papaloizou 1979). The critical mass for gap opening is determined by equating the timescale for type I torques to open a gap (in the absence of viscosity) with that for viscous diffusion to fill it. This gives (Armitage & Rice 2005)

\[
\frac{M_p}{M_*} \lesssim a_{\text{eff}}^{1/2} \left( \frac{h}{a} \right)^2.
\]

With \( a_{\text{eff}} \sim 10^{-4} \) in the MRI’s dead zone, the critical mass is given by

\[
M_{1,11} = 7.5 \left( \frac{a}{1\ \text{AU}} \right)^{1/2} \left( \frac{M_*}{M_\odot} \right) M_\oplus.
\]

After the giant planet opens a gap around the disk, it is embedded in the viscous disk and undergoes type II migration. As the mass of the planet grows and becomes comparable to the disk mass, migration slows and eventually stops. The migration speed with the slow-down effect can be expressed as (Alibert et al. 2005)

\[
\frac{da}{dt} = -\frac{3v}{2a} \times \min \left[ 1, \frac{2\Sigma_g a^2}{\dot{M}_p} \right].
\]

So, the migration timescale is given as

\[
\tau_{\text{mig, II}} = 0.6\ \text{Myr} \left( \frac{a_{\text{eff}}}{10^{-2}} \right)^{-1} \left( \frac{a}{1\ \text{AU}} \right) \left( \frac{M_\text{core}}{10^{-7}\ M_\oplus} \right)^{-1/2} \times \max \left[ 1, \frac{M_p}{2\Sigma_g a^2} \right].
\]
During the migration of the giant planets, their eccentricities will be damped by the disk tide (Goldreich & Tremaine 1979, 1980; Ward 1988), unless the planet is very massive (~20 $M_J$; Papaloizou et al. 2001). As the $e$-damping rate due to the gas disk is quite elusive for different mass regimes of the planets, we adopt an empirical formula for the $e$-damping timescale (Lee & Peale 2002):

$$\tau_{\text{edap},II} = \frac{\tau_{\text{mig},II}}{K},$$

(17)

where $K$ is a positive constant with values ranging from 10–100. To choose an appropriate value of $K$, we execute some test runs with $K = 10, 30, \text{and} 100$. As shown in Figure 1(b), the planet eccentricity is damped very quickly with $K = 100$. For $K = 30$, the eccentricity can be excited and effectively damped at the end of the simulation, when the gas disk has been nearly depleted. Therefore, we take $K = 10$ so that some planets in eccentric orbits can survive, in accordance with observations.

The gas accretion of a planet will stop when the gas material around the feeding zone ($R_H = h$) is exhausted; this gives the truncation mass of a gas giant:

$$M_{g,\text{iso}}(a) = 120 \left(\frac{a}{1\text{AU}}\right)^{3/4} \left(\frac{M_\odot}{M_g}\right) M_\oplus.$$  

(18)

This limits the mass of a giant planet by pure gas accretion. However, collision and coalescence merging may increase the masses of giant planets up to several Jupiter masses.

2.4. Equation of Motion

Based on the model described above, we investigate the late-stage formation of the planetary system in this paper. The stellar mass is taken as $1 M_\odot$. The gas disk is derived from Equation (6). We further assume that embryos have obtained their isolated masses (see Equation (7)) with $f_d = f_s$. We ignore embryos with mass $\leq 0.1 M_\oplus$ in the inner and outer orbits, so that there are in total 38–44 isolated embryos initially in circular and coplanar orbits in [0.5–13.5] AU for a system with a gas and a solid disk $f_d = f_s = 1$. The isolation masses and their radial extensions will be changed accordingly for different $f_d = f_s$.

The outer boundary of embryos is set according to the standard core growth, beyond which the core masses that can grow within 5 Myr are less than 0.1 $M_\oplus$ (Kokubo & Ida 2002; Ida & Lin 2004a). Although this simplification may neglect their dynamical frictions with inner massive cores, this effect is similar to the damping effect by the gas-disk tide, which is more effective and already included in our simulations.

The angle elements (longitude of periastron and mean motion) of the embryos are randomly chosen. Figure 2 shows an example of the masses and initial locations of the embryos. According to Equation (7), a different $\tau_{\text{disk}}$ gives slightly different values of $k_{\text{iso}}$ and $M_{\text{iso}}$ (see Zhou et al. 2007).

The acceleration of an embryo with mass $M_i (i = 1, ...N)$ is given as

$$\frac{d}{dt} \mathbf{v}_i = -\frac{G(M_\star + M_i)\mathbf{r}_i}{r_i^3} + \sum_{j \neq i}^N \frac{GM_j}{|\mathbf{r}_j - \mathbf{r}_i|} \left[ \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|} - \frac{\mathbf{r}_j}{r_j} \right] - \frac{\mathbf{v}_i}{2\tau_{\text{mig}}} - \frac{(\mathbf{v}_i \cdot \mathbf{r}_j)\mathbf{r}_j}{r_j^2 \tau_{\text{edap}}},$$

(19)

where $\mathbf{r}_i$ and $\mathbf{v}_i$ are the position and velocity vectors of $M_i$ relative to the star, respectively; and the third and fourth terms on the right-hand side are the accelerations that cause the eccentricity damping and migration, respectively. For embryos with $M_i < M_{I,II}$ defined in Equation (14), $\tau_{\text{mig}} = \tau_{\text{mig},I}$ (Equation (9)) and $\tau_{\text{edap}} = \tau_{\text{edap},I}$ (Equation (10)) are used. For those with $M_i > M_{I,II}$, $\tau_{\text{mig}} = \tau_{\text{mig},II}$ (Equation (16)) and $\tau_{\text{edap}} = \tau_{\text{edap},II}$ (Equation (17)) are used instead. Note that, during the growth of an embryo, it may cross the critical mass $M_{I,II}$, thus passing from type I to type II migration, and the eccentricity-damping mode is switched.

In the absence of mutual planet perturbations ($M_i = 0$, $j \neq i$), i.e., without the second term on the right-hand side of Equation (19), the secular variations of orbital elements of the embryo $M_i$ under migration and eccentricity damping are derived from the classical perturbation theory:

$$\frac{da}{dt} = -\frac{a}{\tau_{\text{mig}}} - \frac{2e^2}{\tau_{\text{edap}}(1 + \sqrt{1 - e^2})},$$

$$\frac{de}{dt} = -\frac{e(1 - e^2)}{\tau_{\text{edap}}(1 + \sqrt{1 - e^2})},$$

$$\frac{d\omega}{dt} = 0,$$

(20)

where $\omega$ is the argument of the periastron of the embryo orbit.

3. PLANET CONFIGURATIONS UNDER MIGRATIONS: ANALYTICAL CONSIDERATIONS

Before we present the numerical results, it will be useful to investigate some ideal cases in which some cores undergo type I or II migration only, and to see the configuration of the system without considering their mutual perturbations. This will be helpful in understanding the onset of instability for the entire system.

3.1. Planetary Core Configurations under Type I Migration

Assume that there are N planet cores with masses below $M_{I,II}$ so that they undergo type I migration. Equation (9)
gives \( \dot{a} / a = \tau_{mig,I} = k M_p^{-1} \dot{a} \exp(t / \tau_{disk}) \) for \( a \gg a_{crit} \), where \( \dot{M}_p = M_p / M_\odot, \dot{a} = a / 1 \text{ AU}, \text{ and } k \approx 0.23 \text{ Myr if } C_1 = 0.3 \) with the expression of \( \Sigma_g \) from Equation (6). An embryo with an initial semimajor axis \( \dot{a}_0 \) will evolve with \( \dot{a} = \dot{a}_0 - \xi \dot{M}_p, \) where \( \xi = \tau_{disk}(1 - \exp(-t / \tau_{disk})) / k. \) So, the evolution of the relative separation between two neighboring embryos with the mass difference \( \Delta M_p \) is given as

\[
\frac{\Delta \dot{a}}{\dot{a}} = \frac{\Delta \dot{a}_0}{\dot{a}_0} \left[ 1 - \xi \Delta \dot{M}_p / \Delta \dot{a}_0 \right],
\]

where \( \Delta \dot{a}_0 \) is the initial separation. For a general case, \( \Delta M / \Delta \dot{a}_0 = \gamma \dot{M} / \dot{a}_0 \), and since \( \xi \dot{M}_p / \dot{a}_0 < 1 \) before \( a \) goes to \( a_{crit} \), Equation (21) approximates to

\[
\frac{\Delta \dot{a}}{\dot{a}} \approx \left( \frac{\Delta \dot{a}_0}{\dot{a}_0} \right) \left[ 1 + (1 - \gamma) \xi \ddot{M} / \dot{a}_0 \right].
\]

So, as long as \( \gamma < 1 \), the inward type I migration will increase the mutual distance scaled by its mutual Hill radii \( (R_H \propto \dot{a}) \). According to Zhou et al. (2007), such an increase will enhance the orbital crossing timescale of the system, for example, in the case of isolation masses in Equation (7) with \( \gamma = 3/4 \). However, due to the perturbation of giant planets and some other embryos, embryos in inner orbits will have their eccentricities excited, which may result in instability.

### 3.2. Giant Planet Configurations under Type II Migration

Similar to the previous analysis, giant planets undergoing type II migration with a timescale from Equation (16) may modify their mutual separations. As in the previous subsection, we assume that the planet mass is much smaller than the inner disk mass, and the disk has a constant \( \alpha_{eff} \). Thus, one has \( \dot{a} / a = \tau_{mig,II} = k^\prime \dot{a} \) for \( a \gg a_{crit} \), where \( k^\prime \approx 0.6 \text{ Myr} \) from Equation (16). So, \( \dot{a} = \dot{a}_0 - t / k^\prime \), and two neighboring planets will have constant separation \( \dot{a}_0 \). When it is scaled by \( R_H \propto \dot{a} \) at time \( t \),

\[
\frac{\Delta \ddot{a}}{\ddot{a}} = \left( \frac{\Delta \dot{a}_0}{\dot{a}_0} \right) \left[ 1 - \frac{t}{k^\prime \dot{a}_0} \right], \quad (t < k^\prime \dot{a}_0).
\]

This means that the mutual separation of the embryos scaled by Hill radii will increase during inward type II migration. However, secular perturbations among them will excite their eccentricities during their convergent migration, thus destabilizing the system if their eccentricities are high enough.

### 4. Numerical Results: Dependence on Disk Parameters

We numerically integrate the equations of planet motion (Equation (19)) with a time-symmetric Hermite scheme (Kokubo et al. 1998; Aarseth 2003). The regularization technique is used to handle the collisions between embryos; all embryos have their physical radii (a mean density of 3 g cm\(^{-3}\)) is assumed), and merging is expected when the mutual distance of two embryos is less than the sum of their physical radii. We assume a perfect inelastic collision between embryos. The inner boundary of the gas disk is set at 0.05 AU. If a planet migrates to <0.04 AU, we remove it from further integration. The external boundary of the gas disk is set at 100 AU. If a planet evolves to an orbit with \( a > 50 \text{ AU} \), we consider it as having escaped from this system. The tidal effects between the host star and the close-in planets are not included in our model because of their long, effective timescales.

Figure 2 shows an example of such an evolution. Initially, the masses and locations of 44 embryos are shown in Figure 2(a) (red dots). Finally, three planets are left, with masses 3, 6, and 10 and \( \alpha_{eff} \approx 10^{-6} \) migrating around 0.3 AU from the periastron alignment (with \( \sigma_2 - \sigma_3 \) librating around 0°) during the evolution.

As mentioned earlier, parameters of the protoplanetary disk are uncertain because of the present limited ability of observations. So, in this section, we mainly investigate the effect of disk mass, viscosity, and depletion timescale on planetary evolution.

#### 4.1. Influences of Disk Viscosity (\( \alpha_{dead} \)) and Disk Mass (\( f_g \))

The variations in disk mass and viscosity are modeled by two parameters, i.e., \( f_g \) in Equation (6) and \( \alpha_{dead} \) in Equation (4). We set \( f_g = 0.3, 1, 3, \) and 10, and \( \alpha_{dead} = 10^{-2}, 10^{-3}, \) and \( 10^{-4} \) to study their individual contribution to the architecture of planetary systems. Five runs for each parameter are executed. We fix the surface density profile as well as the disk depletion timescale \( (t_{disk} = 2 \text{ Myr}) \) for these test runs.

In our model, \( \alpha_{dead} \) mainly works on type I migration via Equation (9) and on type II migration via Equation (16). Since \( \Sigma_g \propto (\alpha_{eff})^{-1} \) (Equation (6)), \( \tau_{mig,I} \propto \alpha_{eff} \), i.e., planets in disks with smaller \( \alpha_{eff} \approx 10^{-6} \) migrate faster, which results in a smaller average semimajor axis (Figure 3(a)). For type II migration, if the planet mass is small \( (M_p \ll 2\Sigma_g a^2) \), which is about 90 \( M_\oplus \) at 5 AU and 170 \( M_\oplus \) at 10 AU), the migration speed decreases with \( \alpha_{eff} \), and thus larger \( \alpha_{eff} \approx 10^{-2} \) may also lead to a small average semimajor axis. Thus, disk viscosity may change the final location of planets. So, we take the most plausible value \( \alpha_{dead} = 10^{-4} \) in the following simulations (Sano et al. 2000). On the other hand, the average eccentricity is around 0.1, with large variations for all cases, which seems to be insensitive to \( \alpha_{dead} \).

The reason is that small \( \alpha_{dead} \) leads to a high density of the gas disk, resulting in two competing effects: (1) a fast type I migration so that a large frequency of embryo-scattering is expected, which enhances the eccentricities of embryos; and (2) a fast \( e^{-} \)-damping rate with a small \( e_{mean} \). As the timescales of these two effects are comparable, \( e_{mean} \) is nearly independent of \( \alpha_{dead} \).
There are three ways for \( f_g \) to influence orbital evolutions: the surface density of the gas disk (Equation (6)), initial isolation masses (Equation (7) with \( f_g = f_g \)), and the critical mass for the onset of runaway gas accretion (Equation (11)). Larger \( f_g \) may result in larger cores extended to an outer region. As we assume \( M_{\text{core}} \propto f_g \), we have \( M_{\text{crit}} \propto f_g^{0.25} \). Compared to the masses of the initial embryos (\( \propto f_g^{3/2} \)), it is much easier to form massive gas giants in a massive gas disk.

As shown in Figure 3(b), when \( f_g \geq 1 \), \( a_{\text{mean}} \) values are all around 3–4 AU. Although the migrations should be faster for planets in disks with larger \( f_g \), the initial embryos extended to an outer region in these cases, which resulted in similar average locations. For a less massive disk (\( f_g = 0.3 \)), these small embryos, initially in the inner region, never accreted gas, and thus they experienced type I migration throughout the evolution; therefore, they have smaller \( a_{\text{mean}} \) and \( e_{\text{mean}} \). More effective eccentricity damping results in a smaller \( e_{\text{mean}} \) for \( f_g = 3 \) than for \( f_g = 1 \). When \( f_g = 10 \), the largest embryos can reach 80 \( M_\oplus \) via collisions in 1 Myr. Finally, planets left in these systems are mainly gas giants, and their eccentricities are damped much less effectively than those of small planets. This is why the \( e_{\text{mean}} \) of \( f_g = 10 \) is much larger than that of \( f_g = 3 \).

4.2. Correlations with Disk Lifetime (\( \tau_{\text{disk}} \))

The role of the gas disk on planetary evolution is manifold: it causes the inward migration (type I or II) of protoplanets (Ward 1997; Lin & Papaloizou 1985) as well as the tidal damping of their orbital eccentricities. A long-survival gas disk may be helpful in the formation of giant planets, while planets in a short-lived disk may not have enough time to accrete gas, thus remaining either terrestrial or Neptunian planets. To investigate the effect of the disk depletion timescales (\( \tau_{\text{disk}} \)), we set 11 values of \( \tau_{\text{disk}} \), evenly ranging from 0.5 to 5 Myr in a logarithmic scale (Haisch et al. 2001). For each \( \tau_{\text{disk}} \), we performed 20 runs of simulations by randomly choosing the orbital phase angles of the embryos. So, in total, we performed 220 simulations for \( f_g = 1 \). All simulations were stopped at \( t = 10 \) Myr.

An interesting problem of planet formation is the growth epoch of different types of planets. We define (somewhat arbitrarily) the following two types of planets.

Gas giant planets (GPs), including massive GPs (\( M_p \geq 30 M_\oplus \)) and Neptune-sized GPs (10 \( M_\oplus \leq M_p < 30 M_\oplus \)) and terrestrial planets (TPs), including super-Earth TPs (1 \( M_\oplus \leq M_p < 10 M_\oplus \)) and sub-Earth TPs (\( M_p < 1 M_\oplus \)). Figure 4 shows the distribution of planets’ semimajor axes for the 220 runs of simulations at some epochs with \( f_g = 1 \). Only TPs are present at \( t = 10^5 \) yr (Figure 4(a)). When \( t = 1 \) Myr, runaway gas accretion occurs; thus, a few GPs appear (Figure 4(b)). The most efficient growth of GPs occurs at 1–3 Myr; thus, the number of GPs increases very quickly (Figure 4(c)), until they become dominant members at the end of the simulations (Figure 4(d)).

Figure 5 plots the correlations between the properties of the final systems and \( \tau_{\text{disk}} \). There are two regimes. For short-lived disks with \( \tau_{\text{disk}} \leq 1 \) Myr, the forming planets have small average masses (70–200 \( M_\oplus \); Figure 5(c)), and the average semimajor axes of these systems are large (5–6 AU; Figure 5(d)), indicating that most of the surviving planets were formed in distant orbits, while migrations did not strongly affect their orbital architectures. The average eccentricities (\( \sim 0.15 \)) are relatively large (Figure 5(a)), showing that the tidal damping effect is not very effective due to the short lifetime of the disk. However, planetary systems with a longer disk lifetime (\( \tau_{\text{disk}} > 1 \) Myr) tend to have larger average planet masses (\( \sim 1 M_\oplus \)), with lower average eccentricities (\( \sim 0.1 \)) due to the long period of disk damping. The semimajor axes of these systems decrease from 4 to 2 AU, indicating that inward migration indeed plays an important role in sculpting their orbital configurations.

Systems with \( \tau_{\text{disk}} < 1.5 \) Myr do not have many differences from the surviving planetary systems, while \( N_{\text{mean}} \) decreases slightly as \( \tau_{\text{disk}} \) surpasses 1.5 Myr (Figure 5(b)). The maximum of \( N_{\text{mean}} \) at \( \sim 1.5 \) Myr is due to the mechanism for halting small planets near \( a_{\text{crit}} \) (see Section 5.1). In disks with smaller \( \tau_{\text{disk}} \), the effects due to gas disks are not so dominant, and the surviving number of planetary systems mainly depends on the interactions between embryos. In disks with a longer lifetime, giant planets experience sufficient type II migration and will be closer to the boundary of the MRI’s region, where small planets were halted (see Section 5.1). Thus, small planets are more easily scattered out of the system or hit the host star, leaving fewer surviving planets.

One of the major effects that \( N \)-body simulations can describe is the eccentricity excitation due to mutual planetary perturbations. Such excitation may lead to some embryos being scattered out of the system, while others may be merged. Thus, the final number of planets should be greatly reduced from that of the initial embryos. Correlations between the number of surviving planets and the average mass, as well as the eccentricities of the planets in a planetary system, are shown in Figure 6. They are fitted by

\[
\begin{align*}
\langle e_{\text{mean}} \rangle &= 0.65 \times 0.67^{N_{\text{tot}}}, \\
M_{\text{mean}} &= 1.28 M_\oplus \times 0.85^{N_{\text{tot}}}. 
\end{align*}
\]

These correlations show that, in multi-planet systems like our solar system, planets have relatively lower average masses and eccentricities than those found in systems with one or only a few planets. Qualitatively, this law is easily understood. To achieve a longer orbital crossing time in a multi-planet system, either the eccentricities of the planets must be small or the planets must have large mutual separation scaled by their Hill radii; thus, smaller planetary masses are helpful in achieving a stable system (Zhou et al. 2007).
Figure 5. Correlations between the parameters of surviving planetary systems and the disk depletion timescale ($\tau_{\text{disk}}$) in 220 runs of simulations. Each point is averaged over 20 runs of systems with the same $\tau_{\text{disk}}$, with error bars indicating the standard deviations. Panels from (a) to (d) show the average eccentricities, numbers, masses, and semimajor axes of the surviving planets vs. $\tau_{\text{disk}}$.

(A color version of this figure is available in the online journal.)

Figure 6. Correlations between the surviving planet number $N_{\text{left}}$ and the average eccentricity $e_{\text{mean}}$ (a) as well as the average planet mass $M_{\text{mean}}$ (b) in a planetary system. Both $e_{\text{mean}}$ and $M_{\text{mean}}$ are inversely correlated with $N_{\text{left}}$. The error bars are the standard deviations.

(A color version of this figure is available in the online journal.)

5. COMPARISON TO OBSERVATIONS

To compare our results to the observations, we set the mass of the disk ($f_d$) and its depletion timescale ($\tau_{\text{disk}}$) as two free parameters. We take 11 values of $f_d$ evenly ranging from 0.5 to 5 Myr in a logarithmic scale (Haisch et al. 2001). For each $\tau_{\text{disk}}$, we choose $f_d = 0.3$, 1, 3, and 10, and execute 6, 10, 5, and 3 simulations, respectively, to fit for the Gaussian distribution of log ($M_{\text{disk}}/M_\odot$) with a mean value of $\mu = -1.66$ and a standard deviation of $\sigma = 0.74$, according to the observations of Taurus–Auriga (Mordasini et al. 2009a). So, in total, we execute $11 \times 24 = 264$ runs of simulations.

We also make some statistical plots from the observed systems.\(^2\) To show planets that may not be observable yet, we distinguish planets in our simulations as detectables (with the induced stellar radial velocity $V_r \geq 3 \text{ m s}^{-1}$) and undetectables ($V_r < 3 \text{ m s}^{-1}$). Taking the mean value of $\sin i = 0.6$, among the 1437 surviving planets, 959 planets (66.7% of the total) from our simulations are undetectable, a large number of which are small planets (<1 $M_\oplus$).

5.1. Semimajor Axis Distributions

Figures 7(a) and (b) show the semimajor axis distribution from both the observations and simulations. The observed distribution shows two peaks (Figure 7(a)). (1) At 1–3 AU: this is roughly the snow line (where water is frozen) of the system, where planet cores may be stalled under type I migration (Kretke & Lin 2007; Ida & Lin 2008), and subsequent accretion of gas makes them giant planets. (2) At 0.04–0.06 AU (or 3–5 days): this is roughly the inner edge of the gas disk, where planets under type II migration will be stopped (Lin et al. 1996). From our simulations, there are a large number of planets beyond 3 AU; thus, the lack of planets at $>3$ AU is due to observational bias.

In addition, we show that there is an extra pileup of planets: (3) at around 0.2 AU (or around 30 days), which has not been revealed yet by observations. This location corresponds to the inner edge of the MRI’s dead zone ($a_{\text{crit}}$), where small planets under type I migration may be halted. However, as the location of $a_{\text{crit}}$ (see Equation (8)) moves inward, in the case of a short $\tau_{\text{disk}}$, its migration speed is faster than that of the type I migration; therefore, it is difficult to halt these planets near $a_{\text{crit}}$. When $\tau_{\text{disk}} \sim 1.5 \text{ Myr}$, $\frac{da_{\text{mig}}}{dt} \sim \frac{da_{\text{crit}}}{dt}$, so if $\tau_{\text{disk}} > 1.5 \text{ Myr}$, the mechanism for halting small planets near $a_{\text{crit}}$ works. After $\tau_{\text{disk}}$, the migration rate is reduced due to gas depletion. In fact, $a_{\text{crit}} \sim 0.2 AU$ at $t = 2 \tau_{\text{disk}}$. This explains the accumulation of planets near 0.2 AU. Some small planets were scattered into 0.04 AU.

Our simulations also show that the majority (70%) of giant planets are located in 1–10 AU. Most of them are massive GPs and experienced type II migration. Due to the depletion of the

\(^2\) http://exoplanet.eu
gas disk, they could not migrate to the proximity of the star; hence, they halted outside 1 AU. Due to observational bias of radial velocity measurements, massive giant planets are revealed only at \( < 4 - 5 \) AU.

5.2. Mass Distributions

Our simulations show that there are mainly two peaks for planet masses (Figure 7(c)).

1. At \( 1 - 2 \) \( M_J \). These are Jupiter-sized giant planets that have grown up with efficient gas accretion. Although our simulations reproduced this peak, there are not enough massive planets at \( M_p \sim 5 - 8 \) \( M_J \).

2. At \( 0.1 - 3 \) \( M_{\oplus} \). There are a large number of terrestrial planets, which have grown up mainly by mutual collisions from the perturbations of outside giant planets. This peak has not been revealed by observations yet, and can be checked by future observations of terrestrial planets.

One additional small peak is revealed by our simulations:

3. Around \( 10 \) \( M_J \). Massive embryos (\( M \sim 80 \) \( M_{\oplus} \)) formed after 1 Myr in disks with \( f_k = 10 \) and only experienced time-limited type II migration. More massive giant planets (\( > 10 \) \( M_J \)) may be born through some other mechanisms, e.g., the gravitational instability scenario.

The desert at \( 10 - 20 \) \( M_{\oplus} \) consists of Neptune-sized planets. According to our model in Section 2.3, the Kelvin–Helmholtz timescales are quite short (\( \sim 10^4 \) yr) for this mass regime, and runaway gas accretion quickly transforms them into giants. Only a few planets in the inner region with \( M_{\text{iso}} \sim 10 - 20 \) \( M_{\oplus} \) survive in this desert. Also, merging from lower planet embryos may form Neptune-sized planets.

5.3. Eccentricity Distributions

The distribution of eccentricities from simulations is similar to that from observations. In our simulations, the eccentricities of surviving planets vary from 0 to 0.84, while the observations show a maximum eccentricity of about 0.9 (Wright et al. 2009). We fit both distributions (simulations and observations) with an exponential decay function in the form of \( P(e)de = N(e)/N_{\text{tot}} \sim \exp(-Ae)de \). Since \( \int_0^1 P(e)de = 1 \), we have

\[
P(e) = \frac{A \exp(-Ae)}{1 - \exp(-A)},
\]

where \( A = 4.2 \) for observations and \( A = 7.8 \) for simulations of planets with \( V_r < 3 \) \( \text{m s}^{-1} \), as shown in Figures 7(e) and (f). A larger value of \( A \) from simulations indicates a steeper slope, indicating that more planets with small eccentricities are still not detected, as shown in Figure 8(d).

5.4. Correlation Graphs between \( a, e, \) and \( M_p \)

Figure 8 presents the correlation diagrams of \( a, e, \) and \( M_p \) from our simulations (right), with a comparison to the observational data (left). Our simulations reproduced all three correlation plots quite well. In particular, there is a gap in the \( M_p - e \) plot, indicating that a planet desert (0.005–0.08 \( M_J \)) depends on the eccentricity (Figures 8(c) and (d)). Figure 8(d) also shows that giant planets (\( M_p > 10 M_{\oplus} \)) tend to have larger eccentricities on average. To show this more clearly, we reproduce the eccentricity–semimajor axis correlation plots and the eccentricity distribution plots for giant planets (\( M_p > 10 M_{\oplus} \)) and terrestrial planets (\( M_p < 10 M_{\oplus} \)) in Figure 9. Giant planets have average eccentricities \( \sim 0.15 \) at all locations (except
Figure 8. Correlation graphs from observations (left column; data: http://exoplanet.eu) and our simulations (right column). Planets with induced stellar radial velocities \( V_r < 3 \, \text{m s}^{-1} \) are shown as red triangles. Solid lines in (e) and (f) show the induced stellar radial velocities. The boxes in (c) and (d) show the planet desert in the \( M_p - e \) diagram.

(A color version of this figure is available in the online journal.)

Figure 9. Variations of mean eccentricities with semimajor axes (a) and distributions of eccentricities (b) in giant planets \( (M_p \geq 10 \, M_\oplus) \) and terrestrial planets \( (M_p < 10 \, M_\oplus) \). Histograms in (b) are fitted by \( N(e)/N_{\text{total}} = 0.1 \exp(-Ae)/(1 - \exp(-4A)) \), with \( A = 7.78 \) for giant planets and \( A = 21.0 \) for terrestrial planets.

(A color version of this figure is available in the online journal.)

Figure 10. Variations of the eccentricity-damping timescale \( \tau_{\text{edap}} \) for different planet masses \( (M_p) \) at semimajor axes 0.3, 1, 3, and 10 AU. For planets with \( M_p > M_{1,11} \), \( \tau_{\text{edap}} \) is calculated by Equation (10) using a mean eccentricity of \( e = 0.05 \). Otherwise, \( \tau_{\text{edap}} \) is calculated by Equation (17). The jump of \( \tau_{\text{edap}} \) occurs at \( M_p = M_{1,11} \). Here, \( \tau_{\text{edap}} \) remains horizontal until the mass of the planet becomes comparable to the disk mass. The \( M_p - e \) desert in Figure 8(d) roughly corresponds to the mass regime with \( \tau_{\text{edap}} \) less than \( \sim 1 \, \text{Myr} \). Outside this region, eccentricities of planets can hardly be damped.

(A color version of this figure is available in the online journal.)

To understand the \( M_p - e \) desert, i.e., very few planets with masses 0.005–0.08 \( M_j \) have eccentricities larger than 0.3–0.4, we plot the \( e \) damping timescales \( \tau_{\text{edap}} \) as a function of planet masses at 0.3, 1, 3, and 10 AU in Figure 10. For a planet, when \( M_p < M_{1,11} \), \( \tau_{\text{edap}} \) is calculated using a mean eccentricity, \( e = 0.05 \) and \( \Sigma_0 = 280 \, \text{g cm}^{-2} \). Otherwise, \( \tau_{\text{edap}} \) is calculated by Equation (17), with viscosity \( \alpha_{\text{dead}} = 10^{-4} \). According to Equation (10), small planets have longer \( \tau_{\text{edap}} \). The jumps of \( \tau_{\text{edap}} \) occur at \( M_p = M_{1,11} \). Note that \( \tau_{\text{edap}} \) stays horizontal until the mass of a planet becomes comparable to the disk mass \( (M_p > 2\Sigma_0\alpha_{\text{dead}}^2 = 21 \, M_\oplus/(a/1 \, \text{AU})) \). In the braking phase of type II migration, more massive planets have a longer
6. SIMULATIONS IN THE THREE-DIMENSIONAL MODEL

In all the above simulations, we adopted a coplanar planetary model (2D). When the orbital inclinations of the planets (3D) are included in the simulations, the final orbital characteristics of the planetary systems may be changed due to the different collision timescales between 2D and 3D simulations (Chambers 2001). In this section, we reveal the effects of including orbital inclinations via some more simulations. We executed 20 runs in the 3D model, setting the same initial conditions as those in the 2D model, except that the initial inclinations are set at $1^\circ$ for all randomly chosen embryos. The disk parameters are also set as standard values in our simulations: $f_g = 1$, $\tau_{\text{disk}} = 2$ Myr, and $\alpha_{\text{dead}} = 10^{-4}$.

6.1. Collisions with Different Masses

During the entire evolution period we simulated (10 Myr), there were 606 collisions in the 20 runs of 2D simulations, while 471 collisions occurred in the corresponding runs of the 3D model. In the 2D runs, the number of embryos colliding with the host star or being ejected out of the system was 120, compared to 231 in 3D runs. This is consistent with the result of Chambers (2001). As seen in Figure 11(a), the cumulative distribution functions (CDFs) of collisions in two models show that most collisions in 3D runs occurred in relatively later stages. Thus, earlier collisions in the 2D model produced embryos with larger masses. To show the effects of earlier collisions on the final planetary architectures, we divided the collisions into three regions, as shown in Figure 11(b): regions I ($M_0 < M_{\text{crit}}$, $M_1 < M_{\text{crit}}$), II ($M_0 < M_{\text{crit}}$, $M_1 > M_{\text{crit}}$), and III ($M_0 > M_{\text{crit}}$, $M_1 > M_{\text{crit}}$), where $M_0$ represents the larger mass of the embryo before a two-body collision, $M_1$ represents the mass after the collision, and $M_{\text{crit}} = 4 M_0$ is the critical mass beyond which efficient gas accretion begins (see discussion below Equation (11)).

Region I: Most collisions (2D: ~75%; 3D: ~85%) occurred in this region. These collisions influence only the Earth-like planets (TPs; see Section 4.2). Due to the shorter collision timescale in the 2D model, embryos have larger masses, on average, during earlier times and experience faster type I migrations and $e$ damping, according to Equations (9) and (10). Therefore, TPs have a smaller mean eccentricity (cf., $e_{\text{mean}} = 0.034$ for 2D and $e_{\text{mean}} = 0.047$ for 3D) and semimajor axis (see Figure 11(c)). As a result of more collisions occurring in 2D simulations, the final systems are expected to have fewer and larger masses of TPs than those from 3D simulations. In fact, we find 11 TPs left among the 74 surviving planets in the 20 runs of 2D simulations, while 24 of the 98 surviving planets in the corresponding 3D simulations have a small average mass (Figure 11(c)).

Region II: About 8.5% (2D) and 8.4% (3D) of collisions occurred in this region. These collisions make the embryos with masses $< M_{\text{crit}}$ grow beyond critical mass. However, this effect is limited by the small fraction of collisions and the slow gas accretion rate (Equation (12)) for embryos with masses slightly larger than $M_{\text{crit}} = 4 M_{\oplus}$. Thus, the differences between 2D and 3D models due to collisions in this region can be ignored.

Region III: Roughly ~16.5% (2D) and ~6.6% (3D) of collisions occurred in this region. These embryos can accrete gas before collisions. As the speed of type I migrations is much faster than that of type II, the final locations of planets are determined mainly by their cores. As planet cores in the 2D model undergo more collisions, they have bigger masses and a faster migration speed, so their final average semimajor axis is smaller than that in the 3D model (Figure 11(d)). As the inner region has less gas to accrete, $M_{\text{gas}} \propto \Sigma_0 R_H$, while $R_H \propto (M_p/3M_\star)^{1/3} a$ and $\Sigma \propto a^{-1}$, the giant planets in 2D simulations have less mass (Figure 11(d)). The type I eccentricity damping rate is much smaller than that of type II (Figure 10), and from Equation (10), $\tau_{\text{dead}} \sim a^{1/2} M_\star^{-1}$, so larger masses of planets in 3D simulations have shorter $\tau_{\text{dead}}$; thus, the mean eccentricity ($e_{\text{mean}} \approx 0.086$ in simulations) in the 3D model is smaller than that ($e_{\text{mean}} \approx 0.15$) in the 2D model.

6.2. Differences in Statistics

As most of our results in Section 5 are in statistics, we focus on the statistical differences between 2D and 3D models. The statistics of semimajor axes and masses of planets in the two models are presented in Figure 12. Some peaks and deserts in the 2D model are reproduced in the 3D model, such as the peaks at $a_{\text{crit}} \sim 2 A_U$, $M_{\text{crit}}$, and $1–2 M_\star$. However, some different characteristics still exist, especially the planet deserts at around $0.1 A_U$ and $10 M_\oplus < M_p < 0.1 M_\star$ in the 3D model. The lack of planets around $0.1 A_U$ in the 3D model may be due to the prolonged collision timescale, and a much longer time of integration may give results qualitatively similar to the 2D model. The deficit of planets with masses $10 M_\oplus < M_p < 0.1 M_\star$ in the 3D model makes the planet desert in Section 5.2 more obvious.

Figure 13 shows the orbital inclination distribution for the surviving planets in the 20 runs of 3D simulations. As one can...
Figure 12. Distributions of (a) semimajor axes, (b) planetary masses, (c) eccentricities, and (d) inclinations of the surviving planets in 20 runs of 2D simulations and 20 runs of 3D simulations.

(A color version of this figure is available in the online journal.)

Figure 13. Correlation graphs between inclinations and (a) semimajor axes and (b) masses in the 20 runs of 3D simulations.

see, most of the planets remain at $<2^{\circ}$; only a few planets with smaller masses have inclinations $\sim 10^{\circ}$.

To summarize, we point out that the differences between 2D and 3D simulations are small, and most of the statistical results in the 2D model are qualitatively credible.

7. CONCLUSIONS AND DISCUSSIONS

Based on the standard core accretion model of planet formation, we simulated the final assembly of planets by integrating the full N-body equations of motion. The stellar mass is fixed at 1 $M_\odot$. The circumstellar disk is assumed to have an effective viscosity parameter $\alpha_{\text{eff}}$ (Equation (4)) and undergo exponential decay in a timescale of $\tau_{\text{disk}} = 0.5-5$ Myr. Initially, embryos with isolation masses larger than 0.1 $M_\oplus$ (Equation (7)) are placed in each system. Embryos below (or above) the critical mass defined in Equation (14) will undergo type I (or type II) migration. The type I migration of embryos will be stalled near the inner edge of the MRI’s dead zone defined in Equation (5). The inner edge of the disk is set at 0.05 AU, where giant planets will be stalled under type II migration. The formulas governing the embryo’s motions are given in Equation (19).

We investigated the influences of different parameters (viscosities, disk masses, and disk depleted timescales) on the final architectures of planetary systems. Disk viscosity affects a planet’s location by determining its migration speed. In the regime of $\alpha_{\text{dead}} \in [10^{-4}, 10^{-2}]$, planets in disks with small viscosities migrate faster under type I planet–disk interactions, which results in a small average semimajor axis. For a moderate disk viscosity, the planetary system shows a moderate average location (Figure 3(a)). The average eccentricity does not show an obvious correlation with disk viscosity. Disk mass affects the system through the initial core masses. Large planet cores can be formed in the outer region of a massive disk, so giant planets are easier to form.

The disk depletion timescale ($\tau_{\text{disk}}$) plays an important role in final planetary masses and orbits. For short-lived disks with $\tau_{\text{disk}} \leq 1$ Myr, the forming planets have small masses (70–200 $M_\oplus$), with large average semimajor axes (5–6 AU). This indicates that most of the surviving planets are formed in distant orbits, while planet migrations do not yet affect their orbital architectures. Due to the short lifetime of the disk, their average eccentricities ($\sim 0.15$) are relatively large. Planetary systems with a long-lived disk ($\tau_{\text{disk}} > 1$ Myr) tend to have large planetary masses ($\sim 1 M_\oplus$), with low average eccentricities ($\sim 0.1$) due to extended disk damping. The average semimajor axes of these planets range from 4 to 2 AU, indicating that inward migration indeed plays an important role in the formation of planets in close-in orbits.

Comparing our simulations to others, we find that the number of planets trapped in mean motion resonances (MMRs) is smaller than that obtained in other studies, such as in Terquem...
& Papaloizou (2007). Instead, we get a large number of planet pairs that have a history of passing periastron alignment (see Table 2, with the difference of their longitudinal periastron librating at 0°). The reason that we did not observe many surviving planets in MMRs may be that most of our surviving planets are giant planets. The choice of a small disk viscosity ($\alpha_{\text{dead}} = 10^{-4}$) makes type II migration too slow to cause significant convergent MMR trapping. For terrestrial planets, the speed of type I migration is faster than the type II migration of giant planets in outside orbits, so they are not easily trapped in the MMRs of giant planets.

Statistics of 264 simulated systems reproduced qualitatively the main features of the planet masses and orbital parameters for the observed exoplanetary systems. If we classify the planets into two major categories—GPs, including massive GPs ($M_p \geq 30 M_\oplus$) and Neptune-sized GPs ($10 M_\oplus < M_p < 30 M_\oplus$); and TPs, including super-Earth TPs ($1 M_\oplus < M_p < 10 M_\oplus$) and sub-Earth TPs ($M_p < 1 M_\oplus$)—then results from our simulations have the following implications for these planets.

Occurrence rate of planets. The ratio of GPs relative to TPs is low, i.e., 514 to 923 (or 36% to 64%) (1437 surviving planets; Table 2). This is mainly because only massive embryos can accrete sufficient gas to form GPs. Planets with smaller masses are more easily scattered by $N$-body interactions, which is the major difference from the population synthesis simulations of single-planet systems. Moreover, there exist some correlations between the surviving planets and the average eccentricity (or average planet mass) of a planetary system, i.e., a planetary system with more planets tends to have smaller planet masses and orbital eccentricities (Figure 6 and Equation (24)). These correlations are consistent with the stability of the system, i.e., a system with planets in less eccentric orbits and with larger mutual separation scaled by their Hill radii tends to have a longer crossing timescale and is thus more stable (Yoshinaga et al. 1999; Zhou et al. 2007).

Locations of GPs. Most (298, ∼58% of the total of 514) GPs are located in orbits with semimajor axes of 1–10 AU, with only ∼35% (182) in inner orbits < 1 AU. This may be linked with the snow line (∼2–3 AU) of the system. Because of the surface density enhancement of about 3–4 (Equation (7)), isolated masses beyond the snow line are larger; thus, they are more likely to accrete gas and become giant planets. Neptune-sized GPs are also formed in our simulations, but not as frequently (∼7.5%, 42 out of 1437), and they are mainly located in two regions of the systems: either at ∼10 AU, where they were scattered by giant planets formed first (like the formation of Uranus and Neptune); or in the inner edge of the gas disk (0.05 AU), where they seemed to be stalled under type II migration.

Location of TPs. TPs are almost evenly distributed, except for a group lying at 1–2 AU (Figure 4(d), for $f_g = 1$), just inside the snow line (∼3 AU). Also, there is a slight pileup of planets (both GPs and TPs) at 0.2–0.3 AU (30–50 days), which may correspond to the inner boundary of the MRI’s dead zone. According to Kretke & Lin (2007) and Morbidelli et al. (2008), super-Earths tend to be stalled there from type I migration and grow to be giant planets by gas accretion.

Eccentricities of planets. There are very few planets with masses 0.005–0.08 $M_\oplus$ that have eccentricities larger than 0.3–0.4. The average eccentricities of giant planets are larger (∼0.15) than those of the terrestrial planets (∼0.05; Figure 9). According to our simulations, the underlying mechanism is the relatively long $e$-damping timescale of massive planets due to the gas disk, especially when the planet mass is larger than the inner disk mass; see Equations (16) and (17).

We compared our results to some 3D simulations in Section 6. The qualitative differences between 2D and 3D models are not significant, indicating that our conclusions, based mainly on 2D simulations, are still reliable.

Now let us compare the above conclusions to the five new observational signatures that were stated in Section 1. According to our simulations, 260 of 264 runs result in multi-planet systems, which account for 98% of the total, compared to >28% from the observations. Planets in multiple systems have smaller eccentricities than single planets, which is clearly revealed in Figure 6. For signature 2, we did not classify our a, e, and $M_p$ distributions into single and multiple systems, as this classification has some uncertainty concerning undetected planets. While Figure 9(a) supports signature 3, massive planets seem to have larger eccentricities than those with smaller masses. The implication of this is interesting. As most of the observed exoplanets are Jupiter-sized planets in elliptical orbits, we can expect more planets with small masses in near-circular orbits of the multi-planet system, such as terrestrial planets in the solar system. Signature 4, i.e., massive planets tend to have larger eccentricities, is revealed by our simulations (Figure 9(b)). For signature 5, because we fix the stellar mass at 1 $M_\odot$, it could not be tested in the present work. We will investigate this correlation in future work.

However, due to some limits and uncertainties of the parameters we chose in this model, the conclusions obtained in the paper may be limited. For example, we investigated planet formation around 1 $M_\odot$ stars only, although with different sizes of the solid and gas disks. Not only the disk mass but also the accretion rate and metallicity have a correlation with the mass of the host star. For broader parameters, the occurrence ratio of TPs and GPs may need further investigation.

One of the major uncertainties arises from the initial masses and distributions of the embryos, which are the building blocks of planets. In this paper, we adopt the assumption that most of the embryos have already cleared their nearby heavy elements and achieved their isolation masses. This assumption is based on the linear growth of perturbations around a planetesimal disk, which is a reasonable approximation for a given planetesimal disk mass. However, this assumption may not be accurate in the case of high mass fractions, where the planetesimal disk is not fully evacuated. Therefore, we may need to consider a more complex model for the planetesimal disk mass fraction to improve the accuracy of our predictions.
on the full N-body simulation of planet growth (Kokubo & Ida 1998), and will be unrealistic when type I migration of embryos is taken into consideration. Also, initially there might be some smaller embryos between giant planets, so they may undergo planet scattering and become planets like Uranus and Neptune.

The second major uncertainty comes from our poor knowledge of the type I migration of the embryos. The speed (and even the direction) of type I migration will affect the number and location of the surviving terrestrial planets, especially planets in habitable zones (e.g., Ogihara & Ida 2009; Wang & Zhou 2010). We will further investigate this in our forthcoming work.

Third, our simulations mainly focus on the stage in which the gas disk is present and will be exponentially depleted. Further evolution due to the effect of a planetesimal disk was not included. During the later stage of planet formation, when the gas disk is totally depleted, planet–planetesimal interactions may damp the eccentricities of planets through dynamical friction and may induce migrations through angular momentum exchanges with embryos and scattered planetesimals (Fernandez & Ip 1984; Malhotra 1993). In the solar system, numerical simulations show that Jupiter will drift inward while Saturn, Uranus, and Neptune may migrate outward, resulting in a divergent migration. During the migration, the cross of 2:1 MMR between Jupiter and Saturn excites the eccentricities of four giant planets (Tsiganis et al. 2005). Such evolution may occur on the timescale of at least hundreds of millions of years, which is beyond the ability of our simulations.

Furthermore, we ignored the tidal effect between a host star and close-in planets. The tidal dissipation of the star–planet system may damp the eccentricity of the planets in close-in orbits in a gigayear timescale. Also, the gravitational potential of the gas disk was not included in our model. During disk depletion, the sweeping of secular resonance through the inner region may excite the eccentricities of inner orbits, thus inducing further merging of terrestrial planets (Nagasawa et al. 2003).

Although possessing many restrictions, the present paper, aimed at the statistics of the final assembly of planetary systems in the standard formalism (which includes type I and type II migrations, gas accretion, etc.), reproduces most of the observed orbit signatures. Thus, we think that the predictions from the simulations are helpful in guiding future planet detection.

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REFERENCES

Aarseth, S. J. 2003, Gravitational N-Body Simulations (Cambridge: Cambridge Univ. Press)
Alibert, Y., Mordasini, C., Benz, W., & Winn, S. J. 2005, A&A, 434, 343
Andrews, S. M., & Williams, J. P. 2005, ApJ, 631, 1134
Armitage, P. J., & Rice, W. K. M. 2005, in STScI Symp. Ser. 19, ed. M. Livio, K. Salu, & J. Valenti (Cambridge: Cambridge Univ. Press), 66
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Beckwith, S. V. W., & Sargent, A. I. 1996, Nature, 383, 139
Chambers, J. E. 2001, Icarus, 149, 262
Chatterjee, S., Ford, E. B., Matsumura, S., & Rasio, F. A. 2008, ApJ, 686, 580