We propose a scheme of lepton mixing in which the unitary matrix that diagonalizes the neutrino mass matrix is bimaximal and the deviation from bimaximal of the lepton mixing matrix is due to the unitary matrix that diagonalizes the charged-lepton mass matrix. This matrix is assumed to be hierarchical, like the quark mixing matrix. It is shown that in general it is possible to have a sizable value for $|U_{e3}|$ together with an effective two-neutrino maximal mixing in solar neutrino experiments. If the effective mixing in solar neutrino experiments is less than maximal, as indicated by current data, $|U_{e3}|$ is bounded from below. Furthermore, in general the violation of CP could be relatively large.

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I. INTRODUCTION

Solar and atmospheric neutrino experiments found recently strong evidence in favor of neutrino oscillations with large mixing. The Super-Kamiokande experiment found model independent evidence of disappearance of atmospheric $\nu_\mu$’s [1] and the SNO solar neutrino experiment found model independent evidence of $\nu_e \rightarrow \nu_\mu, \nu_\tau$ solar neutrino transitions [2]. These model-independent evidences are further supported by the results of the K2K long-baseline experiment [3], of the Soudan 2 [4] and MACRO [5] atmospheric neutrino experiment and by the Homestake [6], GALLEX [7], SAGE [8], GNO [9] and Super-Kamiokande [10] solar neutrino experiments.

Three-neutrino mixing is the simplest and most natural known explanation of the results of solar and atmospheric neutrino experiments (see Ref. [11], and Ref. [12] for an extensive and updated list of references). Taking into account also the negative results of the CHOOZ long-baseline reactor neutrino experiment [13], it turns out that the lepton mixing matrix must have a bilarge form, i.e. close to bimaximal [14, 15, 16].

In this paper we extend the scheme proposed in Ref. [17], in which bilarge lepton mixing is obtained as a deviation from bimaximal mixing due to the unitary matrix that diagonalizes the charged lepton mass matrix. With respect to Ref. [17], in this paper we write the mixing matrix in a more general form, adding phases that were omitted in Ref. [17]. As a consequence, we will show that it is possible to have relatively large CP violation in the lepton sector, contrary to the results of Ref. [17], where CP violation was found to be very small.

In Section [1] we briefly review the relevant experimental results. In Section [II] we describe our scheme, with the help of Appendix [A]. In Section [IV] we discuss the phenomenological consequences of our scheme. Conclusions are drawn in Section [V].
II. EXPERIMENTAL RESULTS

The atmospheric neutrino data of the Super-Kamiokande experiment are well fitted by $\nu_\mu \rightarrow \nu_\tau$ transitions with large mixing [19]:

$$1.2 \times 10^{-3} \text{ eV}^2 < \Delta m_{\text{atm}}^2 < 5.0 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\vartheta_{\text{atm}} > 0.84 \quad (99\% \text{ C.L.}),$$

(2.1)

where $\Delta m_{\text{atm}}^2$ is the atmospheric neutrino squared-mass difference and $\vartheta_{\text{atm}}$ is the effective mixing angles in two-generation analyses of atmospheric neutrino data.

The global analysis of all solar neutrino data in terms of $\nu_e \rightarrow \nu_\mu, \nu_\tau$ performed in Ref. [20] yielded

LMA: \quad $2.3 \times 10^{-5} \text{ eV}^2 < \Delta m_{\text{sol}}^2 < 3.7 \times 10^{-4} \text{ eV}^2, \quad 0.24 < \tan^2 \vartheta_{\text{sol}} < 0.89 \quad (99.73\% \text{ C.L.}),$

(2.2)

LOW: \quad $3.5 \times 10^{-8} \text{ eV}^2 < \Delta m_{\text{sol}}^2 < 1.2 \times 10^{-7} \text{ eV}^2, \quad 0.43 < \tan^2 \vartheta_{\text{sol}} < 0.86 \quad (99.73\% \text{ C.L.}),$

(2.3)

where $\Delta m_{\text{sol}}^2$ is the atmospheric neutrino squared-mass difference and $\vartheta_{\text{sol}}$ is the effective mixing angles in two-generation analyses of solar neutrino data. In Eqs. (2.2) and (2.3) we reported only the boundaries of the so-called LMA and LOW regions, where matter effects contribute to neutrino transitions in the Sun and in the Earth (see Ref. [21]). The LMA region is currently favored, because it is much larger than the LOW region and it contains the minimum of the $\chi^2$ (a LOW region appears only at 99% C.L.). Additional small VAC regions, in which only neutrino oscillations in vacuum contribute, are marginally allowed (at 99.73% C.L.) [20].

The limits in Eqs. (2.2) and (2.3) show that also the mixing relevant for solar neutrino oscillations is large. However, maximal mixing seems strongly disfavored (neglecting the above-mentioned small and marginal VAC regions) from the analysis of solar neutrino data in Ref. [20]. This conclusion is supported by the results of some other authors [10, 22, 23, 24], whereas the authors of Refs. [25, 26, 27, 28] found slightly larger allowed regions, with marginally allowed maximal mixing. Therefore, it is not clear at present if maximal mixing in solar neutrino oscillations is excluded or not. Hopefully, this problem will be solved soon by the KamLAND [29] experiment, or by the BOREXINO [30] experiment.

Solar and atmospheric neutrino data can be well fitted in the framework of three-neutrino mixing, that allows solar $\nu_e \rightarrow \nu_\mu, \nu_\tau$ transitions with $\Delta m_{\text{sol}}^2 = \Delta m_{21}^2 = m_2^2 - m_1^2$ and atmospheric $\nu_\mu \rightarrow \nu_\tau$ transitions with $\Delta m_{\text{atm}}^2 \simeq \Delta m_{31}^2 = m_3^2 - m_1^2$ (see Refs. [11, 31]), where $m_1, m_2, m_3$ are the three neutrino masses.

From the results of the CHOOZ long-baseline reactor neutrino experiment [13], it is known that the element $U_{e3}$ of the three-generation neutrino mixing matrix is small [28]:

$$|U_{e3}|^2 < 5 \times 10^{-2} \quad (99.73\% \text{ C.L.}).$$

(2.4)

The results of the CHOOZ experiment have been confirmed by the Palo Verde experiment [32], and by the absence of $\nu_\tau$ transitions in the Super-Kamiokande atmospheric neutrino data [19].

An important consequence of the smallness of $U_{e3}$ is the practical decoupling of solar and atmospheric neutrino oscillations [33], which can be analyzed in terms of two-neutrino oscillations with the effective mixing angles $\vartheta_{\text{sol}}$ and $\vartheta_{\text{atm}}$ given by

$$\cos^2 \vartheta_{\text{sol}} = \frac{|U_{e1}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \vartheta_{\text{sol}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2},$$

$$\cos^2 \vartheta_{\text{atm}} = \frac{|U_{\tau3}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \vartheta_{\text{atm}} = \frac{|U_{\mu3}|^2}{1 - |U_{e3}|^2}.$$

(2.5)

(2.6)

From the limits in Eqs. (2.3)–(2.4) we get the following allowed intervals for the absolute values of the elements of the mixing matrix (the intervals are correlated, because of unitarity):

$$|U| \approx \begin{pmatrix}
0.71 & -0.90 & 0.43 - 0.69 & 0.00 - 0.22 \\
0.24 & 0.24 - 0.66 & 0.40 - 0.81 & 0.85 - 0.84 \\
0.24 & 0.24 - 0.66 & 0.40 - 0.81 & 0.85 - 0.84 \\
0.24 & 0.24 - 0.66 & 0.40 - 0.81 & 0.85 - 0.84
\end{pmatrix}.$$

(2.7)

Hence, the three-neutrino mixing is bilarge, not too far from bimaximal [14, 15, 16].

As explained in the following section, in this paper we extend the work presented in Ref. [17], in which we discussed the possibility that the deviation from bimaximal mixing is due to the unitary matrix that diagonalizes the charged lepton mass matrix.
III. LEPTON MIXING

Lepton mixing is due to the fact that in general the charged lepton and neutrino fields in the weak charged current

\[ j^{CC\dagger}_\rho = 2 \sum_{\alpha'=e',\mu',\tau'} \bar{\ell}_{\alpha'} L \gamma_\rho \nu_{\alpha'} L \]  

(3.1)
do not have a definite mass, but are unitary linear combinations of massive charged lepton and neutrino fields:

\[ \ell_{\alpha'} L = \sum_{\alpha=e,\mu,\tau} V^{(\ell)}_{\alpha'\alpha} \ell_\alpha L, \quad \nu_{\alpha'} L = \sum_{k=1}^{3} V^{(\nu)}_{\alpha'k} \nu_{kL}. \]  

(3.2)
The unitary matrices \( V^{(\ell)} \) and \( V^{(\nu)} \) diagonalize, respectively, the charged lepton and neutrino mass matrices. The weak charged current (3.1) is written in terms of the massive charged lepton and neutrino fields as

\[ j^{CC\dagger}_\rho = 2 \sum_{\alpha=e,\mu,\tau} \bar{\ell}_\alpha L \gamma_\rho U_{\alpha k} \nu_{kL}. \]  

(3.3)

with the unitary lepton mixing matrix

\[ U = V^{(\ell)\dagger} V^{(\nu)}. \]  

(3.4)

Since the charged leptons with definite mass are directly observable (through their electromagnetic interactions in detectors), it is convenient to assign them lepton numbers \( L_\alpha \) (\( \alpha = e, \mu, \tau \)) and define the corresponding flavor neutrino fields

\[ \nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \nu_{kL}. \]  

(3.5)

In this way, the weak charged current (3.4) can be written as

\[ j^{CC\dagger}_\rho = 2 \sum_{\alpha=e,\mu,\tau} \bar{\ell}_\alpha L \gamma_\rho \nu_{\alpha L}, \]  

(3.6)

showing that the destruction of a flavor neutrino \( \nu_{\alpha} \) (or the creation of a flavor antineutrino \( \bar{\nu}_{\alpha} \)) is associated with the creation of a charged lepton \( \ell^+_\alpha \) (or the destruction of a charged lepton \( \ell^-_\alpha \)). However, as shown in Eq. (3.3), a flavor neutrino \( \nu_{\alpha} \) is not an elementary particle, but the superposition of neutrinos \( \nu_{k} \) with masses \( m_k \) \((k = 1, 2, 3)\). This phenomenon is called “neutrino mixing” or “lepton mixing” and generates neutrino oscillations (see Ref. [21]). The name “lepton mixing” appropriately recalls that the mixing matrix \( U \) is given by the product (3.2) of the unitary matrices \( V^{(\ell)} \) and \( V^{(\nu)} \) that diagonalize the charged lepton and neutrino mass matrices.

In Ref. [17] we supposed that the neutrino unitary matrix \( V^{(\nu)} \) has the bimaximal form

\[ V^{(\nu)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]  

(3.7)

Although not natural in general, such bimaximal matrix could be due to an appropriate symmetry (as a \( L_e - L_\mu - L_\tau \) symmetry \([33, 34, 35, 36, 37]\)), maybe related to the special Majorana nature of neutrinos. On the other hand, since the masses of charged leptons are generated by the same Higgs mechanism that generates quark masses, we naturally supposed that the charged lepton unitary matrix \( V^{(\ell)} \) has the CKM form

\[ V^{(\ell)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}, \]  

(3.8)
where \( s_{ij} \equiv \sin \theta_{ij} \) and \( c_{ij} \equiv \cos \theta_{ij} \), with the hierarchy

\[
s_{12} \ll 1, \quad s_{23} \sim s_{12}^2, \quad s_{13} \sim s_{12}^3
\]

(3.9)
similar to the one in the quark sector (see Ref. [38]). Using the matrices (3.4) and (3.4) in Eq. (3.4), we found that the size of \( U_{e3} \) and the deviation of the effective solar mixing angle \( \theta_{sol} \) from its bimaximal value \( \pi/4 \) are related by their leading order proportionality to \( s_{12} \). On the other hand, the effective atmospheric mixing angle \( \theta_{atm} \) is insensitive to the contribution of the charged lepton matrix \( V^{(\ell)} \), keeping its bimaximal value \( \pi/4 \) up to negligible corrections of order \( s_{12}^2 \). This is in agreement with the indications of maximal mixing found in atmospheric neutrino experiments. Unfortunately, the suppositions in Ref. [17] lead to very small CP violation: the maximum possible value of the Jarlskog invariant is more than one order of magnitude smaller than its absolute maximum possible value.

In this paper we extend the lepton mixing scheme proposed in Ref. [17], noting that in general the unitary matrices \( V^{(\ell)} \) and \( V^{(\nu)} \) may depend on more phases than the single phase \( \phi_{13} \) in Eq. (3.8).

In the Appendix A we show that in general it is possible to choose the phases of the charged lepton fields in Eq. (2.3) in order to write the charged lepton matrix \( V^{(\ell)} \) in the form of Eq. (3.8), with only one phase \( \phi_{13} \) (see Eq. (A25)), but in this case the most general neutrino matrix \( V^{(\nu)} \) depends on three angles \( \theta_{12}^{(\nu)}, \theta_{13}^{(\nu)}, \theta_{23}^{(\nu)} \), three “Dirac type” phases \( \psi_{12}, \psi_{23}, \psi_{13} \), and two “Majorana type” phases \( \lambda_{21}, \lambda_{31} \) (see Eq. (A27)).

A bimaximal form for the neutrino matrix \( V^{(\nu)} \) is obtained by setting \( \theta_{12}^{(\nu)} = \theta_{23}^{(\nu)} = \pi/4 \) and \( \theta_{13}^{(\nu)} = 0 \), leading to

\[
V^{(\nu)} = W^{(23)}(\pi/4, \psi_{23}) W^{(12)}(\pi/4, \psi_{12}) D(\bar{\lambda}) ,
\]

(3.10)
in the notation of Appendix A. This matrix depends on two “Dirac type” phases \( \psi_{12}, \psi_{23} \). The diagonal matrix of phases \( D(\bar{\lambda}) \) on the right, with \( \bar{\lambda} = (1, \lambda_{21}, \lambda_{31}) \), is present only if neutrinos are Majorana particles. Explicitly, we have

\[
V^{(\nu)} = \begin{pmatrix}
e^{\psi_{12}} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & e^{-i\psi_{23}}
\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
e^{-i\psi_{12}} & 0 & 0 \\
0 & e^{i\lambda_{21}} & 0 \\
0 & 0 & e^{i(\lambda_{31}+\psi_{23})}
\end{pmatrix},
\]

(3.11)

In order to write the mixing matrix in this form we used the property in Eq. (A8) for the phase \( \psi_{12} \) and the property in Eq. (A9) for the phase \( \psi_{23} \). Obviously, the matrix \( D^{(1)}(\psi_{12}) \) commutes with \( W^{(23)}(\pi/4, \psi_{23}) = D^{(3)}(\psi_{23}) R^{(23)}(\pi/4) D^{(3)}(\psi_{23}) \) and the matrix \( D^{(3)}(\psi_{23}) \) commutes with \( W^{(12)}(\pi/4, \psi_{12}) = D^{(1)}(\psi_{12}) R^{(12)}(\pi/4) D^{(1)}(\psi_{12}) \).

In the following we assume the hierarchy (3.4) for the angles in the charged lepton matrix and we evaluate all quantities at the leading order in \( s_{12} \). As a first instance, the mixing matrix is given by

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & e^{-i\psi_{23}}
\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{2}} + \frac{1}{2} s_{12} e^{-i\psi_{12}} & \frac{1}{2} e^{i\psi_{12}} - \frac{1}{2} s_{12} & \frac{1}{\sqrt{2}} e^{i\psi_{12}} - \frac{1}{\sqrt{2}} s_{12} \\
-\frac{1}{2} e^{-i\psi_{12}} + \frac{1}{\sqrt{2}} s_{12} & \frac{1}{2} e^{i\psi_{12}} + \frac{1}{2} s_{12} e^{i\psi_{12}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} e^{-i\psi_{12}} - \frac{1}{\sqrt{2}} s_{12} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} e^{-i\psi_{12}} + \frac{1}{\sqrt{2}} s_{12}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\lambda_{21}} & 0 \\
0 & 0 & e^{i(\lambda_{31}+\psi_{23})}
\end{pmatrix} + O(s_{12}^2).
\]

(3.12)

Therefore, as in Ref. [17], the value of \( U_{e3} \),

\[
U_{e3} = -\frac{1}{\sqrt{2}} s_{12} e^{i(\lambda_{31}+\psi_{23})} + O(s_{12}^2),
\]

(3.13)
is proportional to \( s_{12} \), at leading order. From Eq. (3.12) one can see that at first order in \( s_{12} \) the phase \( \psi_{23} \) is factorized in the diagonal matrices on the left and right of the mixing matrix, because the matrix \( D^{(3)}(\psi_{23}) \) commutes with \( V^{(\ell)} \) at first order in \( s_{12} \) \( (V^{(\ell)} = R^{(12)}(\psi_{12}) + O(s_{12}^2)) \). This implies that, at first order in \( s_{12} \), the phase \( \psi_{23} \) is irrelevant for neutrino oscillations in vacuum as well as in matter (see Ref. [23]), which are invariant under the transformations \( U_{\alpha k} \rightarrow e^{-i\xi_k} U_{\alpha k} e^{-i\xi_k} \) with arbitrary phases \( \xi_\alpha (\alpha = e, \mu, \tau) \) and \( \xi_k (k = 1, 2, 3) \) that can eliminate the phase \( \psi_{23} \) in Eq. (3.13) (the Majorana phases \( \lambda_{21}, \lambda_{31} \) can be eliminated in any case and never contribute to neutrino oscillations).
IV. PHENOMENOLOGY

Since $U_{e3}$ in Eq. (3.13) is proportional to $s_{12}$, the value of $|s_{12}|$ is severely limited by the upper bound for $|U_{e3}|^2$ in Eq. (2.4):

$$|s_{12}| < 0.32.$$  \hspace{1cm} (4.1)

A. Solar Neutrinos

The effective solar mixing angle $\vartheta_{\text{sol}}$ is given by

$$\tan^2 \vartheta_{\text{sol}} = 1 - 2\sqrt{2}s_{12}\cos(\psi_{12}) + O(s_{12}^2).$$  \hspace{1cm} (4.2)

Hence, at first order in $s_{12}$ the deviation of $\tan^2 \vartheta_{\text{sol}}$ from unity, which corresponds to maximal mixing, is not only proportional to $s_{12}$ as in Ref. \[17\], but also to $\cos(\psi_{12})$. This means that in the scheme under consideration it is possible to have $U_{e3} \neq 0$ even with maximal solar mixing (with $\psi_{12} = \pi/2$). In general, the contribution of $\cos(\psi_{12})$ in Eq. (4.2) allows to have a solar mixing that is maximal or close to maximal. It is even possible to have an effective mixing angle $\vartheta_{\text{sol}}$ in the “dark side” ($\tan^2 \vartheta_{\text{sol}} > 1$) \[39\] with negative values of $s_{12}\cos(\psi_{12})$.

The upper limit on $\tan^2 \vartheta_{\text{sol}}$ in Eq. (2.2) implies that

$$s_{12}\cos(\psi_{12}) > 0.04.$$  \hspace{1cm} (4.3)

This lower limit is the same as that derived in Ref. \[17\] for $s_{12}$ alone, which follows trivially from Eq. (4.3):

$$s_{12} > 0.04.$$  \hspace{1cm} (4.4)

This lower limit for $s_{12}$ leads to the lower bound

$$|U_{e3}| > 0.03,$$  \hspace{1cm} (4.5)

already found in Ref. \[17\]. However, if $\psi_{12} \neq 0$, the lower limit for $s_{12}$ alone may be significantly larger than 0.04, leading to a lower bound for $|U_{e3}|$ larger than that in Eq. (4.3). Such values of $|U_{e3}|$ could be measured in the JHF-Kamioka long-baseline neutrino oscillation experiment, which has a planned sensitivity of $|U_{e3}| \approx 0.04$ at 90% CL in the first phase with the Super-Kamiokande detector and $|U_{e3}| < 10^{-2}$ in the second phase with the Hyper-Kamiokande detector \[40\].

From the lower limit for $s_{12}\cos(\psi_{12})$ in Eq. (4.3) and the upper bound for $s_{12}$ in Eq. (4.4), for $\cos(\psi_{12})$ we get the lower bound

$$\cos(\psi_{12}) > 0.13.$$  \hspace{1cm} (4.6)

Figure \[1\] shows the allowed region in the positive $\sin(\psi_{12})$–$s_{12}$ plane, together with some curves with constant value of $\tan^2 \vartheta_{\text{sol}}$. These curves have been calculated using the exact expression of $\tan^2 \vartheta_{\text{sol}}$, because for large values of $s_{12}\cos(\psi_{12})$ higher-order terms are not negligible. In order to perform the calculation, we assumed, for illustration, the values $s_{23} = s_{12}^2$, $s_{13} = s_{12}^2$, $\phi_{13} = 0$, $\psi_{23} = 0$. Only the curves in the upper-left part of the figure are modified changing these values, whereas the thin solid lower-bound curve is insensitive to the values of $s_{23}$, $s_{13}$, $\phi_{13}$, $\psi_{23}$, because the leading order approximation in Eq. (4.2) is accurate.

B. Atmospheric Neutrinos

As in Ref. \[17\], the effective atmospheric mixing angle is insensitive to $s_{12}$, remaining practically maximal:

$$\sin^2 2\vartheta_{\text{atm}} = 1 + O(s_{12}^4).$$  \hspace{1cm} (4.7)

This is consistent with the experimental limit in Eq. (2.4).
C. Long-Baseline Oscillations and CP Violation

The value of $s_{12}$ could be measured in long-baseline oscillation experiments [1, 2, 3] sensitive to the largest squared-mass difference $|\Delta m^2_{31}|$. In these experiments the transition probabilities (neglecting matter effects, which in any case depend only on $|U_{e3}|^2$, $|U_{\mu 3}|^2$, $|U_{\tau 3}|^2$ and $|\Delta m^2_{31}|$) are well approximated by the standard two-generation formula with the effective oscillation amplitudes (see Ref. [21])

$$\sin^2 2\theta_{\nu_e \rightarrow \nu_e} = 4|U_{e3}|^2 (1 - |U_{e3}|^2) = 2 s_{12}^2 + O(s_{12}^4),$$

$$\sin^2 2\theta_{\nu_{\mu} \rightarrow \nu_e} = 4|U_{\mu 3}|^2 |U_{\nu 3}|^2 = s_{12}^2 + O(s_{12}^4),$$

for $\nu_e \rightarrow \nu_e$ and $\nu_{\mu} \rightarrow \nu_e$ or $\bar{\nu}_e \rightarrow \bar{\nu}_e$ transitions, respectively. In Eq. (4.9) we neglected possible CP violation effects which are measurable only by experiments sensitive to both the squared-mass differences $|\Delta m^2_{31}|$ and $|\Delta m^2_{21}|$. Indeed, for vacuum oscillations

$$P_{\nu_{\mu} \rightarrow \nu_e} - P_{\nu_e \rightarrow \nu_e} = 4J \left[ \sin \frac{\Delta m^2_{31} L}{2E} + \sin \frac{\Delta m^2_{21} L}{2E} - \sin \frac{\Delta m^2_{12} L}{2E} \right],$$

where $L$ is the source-detector distance and $E$ is the neutrino energy. The Jarlskog parameter [44],

$$J = \text{Im} \left[ U_{e2}^* U_{e3} U_{\mu 2}^* U_{\mu 3}^* \right]$$

is a measure of CP violation which is invariant under rephasing of the lepton fields. In contrast with Ref. [17], taking into account all the possible phases in the lepton mixing matrix leads to a linear contribution of $s_{12}$ to $J$:

$$J = \frac{1}{4\sqrt{2}} s_{12} \sin (\psi_{12}) + O(s_{12}^2).$$

(4.12)

Hence, if $\psi_{12}$ is not too small, we expect a sizable CP violation.

The maximum possible value for $|J|$ is obtained for $\psi_{12} = \pm \pi/2$. In this case Eq. (4.2) shows that the effective solar mixing is maximal and independent from the value of $s_{12}$. The upper limit for $s_{12}$ in Eq. (4.1)

$$|J|_{\text{max}} \simeq 5 \times 10^{-2},$$

(4.13)

which is not far from the absolute upper limit of the Jarlskog parameter (see Ref. [15])

$$|J|_{\text{absolute}} \simeq \frac{1}{6\sqrt{3}} = 9.6 \times 10^{-2}.$$

(4.14)

Although a maximal value of the effective solar mixing angle may be not completely excluded, as discussed in Section 2, it is certainly disfavored by current experimental data and out of the limits in Eqs. (2.2) and (2.3). Considering the allowed interval in Eq. (2.2) for $\tan^2 \vartheta_{\text{sol}}$ in the LMA region, which leads to the lower limit (4.3) for $s_{12} \cos (\psi_{12})$, we have

$$|s_{12} \sin (\psi_{12})| \leq \sqrt{(s_{12})_{\text{max}}^2 - (s_{12} \sin (\psi_{12}))_{\text{min}}^2} = 0.31,$$

(4.15)

leading to a maximal value of $|J|$ practically equal to that in Eq. (4.13). Let us notice that in this case the largeness of $|J|$ is due to a value of $|s_{12}|$ close to the upper bound in Eq. (4.1) and a large value of $|\sin (\psi_{12})|$, that is however sufficiently different from unity in order to satisfy the lower limit in Eq. (4.3).

Figure 1 shows some curves with constant value of $J$ in the positive $\sin (\psi_{12}) - s_{12}$ plane. One can see that relatively large values of $J$, close to the upper limit in Eq. (4.13), can be realized if $s_{12}$ is not too far from the upper bound in Eq (4.1) and $\psi_{12}$ is not too small. In this case, CP violation may be measured in the JHF-Kamioka long-baseline neutrino oscillation experiment [40] or in a neutrino factory experiment [46].
V. CONCLUSIONS

We have proposed a scheme of lepton mixing in which the unitary matrix that diagonalizes the neutrino mass matrix is bimaximal and the deviation from bimaximal of the lepton mixing matrix is due to the unitary matrix that diagonalizes the charged-lepton mass matrix. This scheme generalizes the one proposed in Ref. [17] by taking into account the possible existence of additional phases.

The unitary matrix that diagonalizes the charged-lepton mass matrix is assumed to be hierarchical, like the quark mixing matrix, since presumably the charged-lepton and quark mass matrices are originated by the same standard Higgs mechanism. The neutrino mass matrix is generated by a different mechanism and the bimaximal form the unitary matrix that diagonalizes the neutrino mass matrix could be due to an appropriate symmetry and maybe related to the Majorana nature of neutrinos.

We have shown that in general it is possible to have a sizable value for $|U_{e3}|$ together with an effective two-neutrino maximal mixing in solar neutrino experiments. If the effective mixing in solar neutrino experiments is less than maximal, as indicated by current data, $|U_{e3}|$ is bounded from below (see Eq. (4.5)). Such values of $|U_{e3}|$ could be measured in the JHF-Kamioka long-baseline neutrino oscillation experiment [40].

In Ref. [17], it was found that CP violation is small. Here we have shown that the contribution of the additional possible phases allows the violation of CP to be relatively large (see Eq. (4.13)) and probably measurable in future experiments (JHF-Kamioka [40], neutrino factory [46]).

APPENDIX A: PARAMETERIZATION AND REPHASING OF THE MIXING MATRIX

A $3 \times 3$ unitary matrix $V$ can be written as (see [47, 48, 49] and the appendix of [50])

$$V = \left[ \prod_{a<b} W^{(ab)}(\theta_{ab}, \eta_{ab}) \right] D(\vec{\omega}) \quad (a, b = 1, 2, 3),$$

(A1)

with the unitary matrices

$$D(\vec{\omega}) = \text{diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3}),$$

(A2)

$$W^{(ab)}(\theta_{ab}, \eta_{ab})_{rs} = \delta_{rs} + (c_{ab} - 1) (\delta_{ra} \delta_{sa} + \delta_{rb} \delta_{sb}) + s_{ab} (\cos \eta_{ab} \delta_{ra} \delta_{sb} - e^{-i\eta_{ab}} \delta_{rb} \delta_{sa}),$$

(A3)

where $c_{ab} \equiv \cos \theta_{ab}$ and $s_{ab} \equiv \sin \theta_{ab}$. Here $D(\vec{\omega})$ is a diagonal matrix depending from the set of phases $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ and the matrices $W^{(ab)}(\theta_{ab}, \eta_{ab})$ are unitary and unimodular. For example, we have

$$W^{(12)}(\theta_{12}, \eta_{12}) = \begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} e^{i\eta_{12}} & 0 \\
-sin \theta_{12} e^{-i\eta_{12}} & \cos \theta_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}.$$  

(A4)

With an appropriate choice of the phases $\omega_k$ and $\eta_{ab}$, the angles $\theta_{ab}$ can be limited in the range

$$0 \leq \theta_{ab} \leq \frac{\pi}{2}.$$

(A5)

The order of the product of the matrices $W^{(ab)}$ in Eq. (A1) can be chosen in an arbitrary way. Different choices of order give different parameterizations.
The matrices \( W^{(ab)}(\theta_{ab}, \eta_{ab}) \) satisfy the useful identity\(^2\)

\[
D(\vec{\xi}) W^{(ab)}(\theta_{ab}, \eta_{ab}) D^\dagger(\vec{\xi}) = W^{(ab)}(\theta_{ab}, \eta_{ab} + \xi_a - \xi_b) ,
\]

(A7)

for any choice of the phases \( \vec{\xi} = (\xi_1, \xi_2, \xi_3) \).

The identity (A7) allows to write the matrix \( W^{(ab)}(\theta_{ab}, \eta_{ab}) \) as\(^3\)

\[
W^{(ab)}(\theta_{ab}, \eta_{ab}) = D^{(a)}(\eta_{ab}) R^{(ab)}(\theta_{ab}) D^{(a)\dagger}(\eta_{ab}) ,
\]

(A8)

or

\[
W^{(ab)}(\theta_{ab}, \eta_{ab}) = D^{(b)\dagger}(\eta_{ab}) R^{(ab)}(\theta_{ab}) D^{(b)}(\eta_{ab}) ,
\]

(A9)

with

\[
[D^{(a)}(\eta_{ab})]_{rs} = \delta_{rs} + (e^{i\eta_{ab}} - 1) \delta_{ra} \delta_{sa} ,
\]

(A10)

\[
[R^{(ab)}(\theta_{ab})]_{rs} = \delta_{rs} + (\cos \theta_{ab} - 1) (\delta_{ra} \delta_{sa} + \delta_{rb} \delta_{sb}) + \sin \theta_{ab} (\delta_{ra} \delta_{sb} - \delta_{rb} \delta_{sa}) .
\]

(A11)

The matrix \( R^{(ab)}(\theta_{ab}) \) operates a rotation of an angle \( \theta_{ab} \) in the \( a-b \) plane. For example, we have

\[
R^{(12)}(\theta_{12}) = \begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} & 0 \\
-\sin \theta_{12} & \cos \theta_{12} & 0 \\
0 & 0 & 1
\end{pmatrix} ,
\]

\[
D^{(1)}(\eta_{12}) = \begin{pmatrix}
e^{i\eta_{12}} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} .
\]

(A12)

Let us consider now the mixing matrix (3.4). In general, we can write the unitary matrices \( V^{(\ell)} \) and \( V^{(\nu)} \) using Eq. (A11), leading to

\[
U = D^{\dagger}(\vec{\omega}^{(\ell)}) \left[ \prod_{a < b} W^{(ab)}(\theta_{ab}, \eta_{ab}) \right]^{\dagger} \left[ \prod_{a < b} W^{(ab)}(\theta_{ab}, \eta_{ab}) \right] D(\vec{\omega}^{(\nu)}) ,
\]

(A13)

in an obvious notation. Inserting pairs \( D^{\dagger}(\vec{\xi}) D(\vec{\xi}) \), Eq. (A13) can be written as

\[
U = D^{\dagger}(\vec{\omega}^{(\ell)} + \vec{\xi}) \left[ \prod_{a < b} D(\vec{\xi}) W^{(ab)}(\theta_{ab}, \eta_{ab}) D^{\dagger}(\vec{\xi}) \right]^{\dagger} \left[ \prod_{a < b} D(\vec{\xi}) W^{(ab)}(\theta_{ab}, \eta_{ab}) D^{\dagger}(\vec{\xi}) \right] D(\vec{\omega}^{(\nu)} + \vec{\xi}) ,
\]

(A14)

and using the identity (A7) we have

\[
U = D^{\dagger}(\vec{\omega}^{(\ell)} + \vec{\xi}) \left[ \prod_{a < b} W^{(ab)}(\theta_{ab}, \eta_{ab} + \xi_a - \xi_b) \right]^{\dagger} \left[ \prod_{a < b} W^{(ab)}(\theta_{ab}, \eta_{ab} + \xi_a - \xi_b) \right] D(\vec{\omega}^{(\nu)} + \vec{\xi}) .
\]

(A15)

---

\(^2\) Indeed,

\[
\left[ D(\vec{\xi}) W^{(ab)}(\theta_{ab}, \eta_{ab}) D^{\dagger}(\vec{\xi}) \right]_{rs} = \sum_{t,u} e^{i\xi_u} \delta_{rt} [\delta_{rs} + (c_{ab} - 1) (\delta_{ta} \delta_{ua} + \delta_{tb} \delta_{ub}) + s_{ab} (e^{i\eta_{ab}} \delta_{ta} \delta_{ub} - e^{-i\eta_{ab}} \delta_{tb} \delta_{ua})] e^{-i\xi_u} \delta_{us} = \delta_{rs} + (c_{ab} - 1) (\delta_{ra} \delta_{sa} + \delta_{rb} \delta_{sb}) + s_{ab} (e^{i(\eta_{ab} + \xi_a - \xi_b)} \delta_{ra} \delta_{sb} - e^{-i(\eta_{ab} + \xi_a - \xi_b)} \delta_{rb} \delta_{sa}) = [W^{(ab)}(\theta_{ab}, \eta_{ab} + \xi_a - \xi_b)]_{rs} .
\]

(A6)

\(^3\) Choosing \( \xi_a = -\eta_{ab}, \xi_b = 0 \) in Eq. (A8) and \( \xi_a = 0, \xi_b = \eta_{ab} \) in Eq. (A9). In both cases \( \xi_c = 0 \) for \( c \neq a, b \).
Since there are two independent differences \( \xi_a - \xi_b \), we can extract two phases from the product of \( W \)'s. Let us extract \( \eta_{12}^{(f)} \) and \( \eta_{23}^{(f)} \) with the choice
\[
\xi_1 - \xi_2 = -\eta_{12}^{(f)} , \quad \xi_2 - \xi_3 = -\eta_{23}^{(f)} .
\]
With this choice, Eq. (A15) can be written as
\[
U = e^{i(\omega_1^{(f)} + \xi_1)} D^\dagger (\vec{\omega}^{(f)} + \vec{\xi}) \left[ R^{(23)}(\vec{\theta}_{23}^{(f)} \theta_{13}^{(f)}, \phi_{13}^{(f)}) R^{(12)}(\vec{\theta}_{12}^{(f)}) \right]^\dagger \times \left[ W^{(23)}(\vec{\theta}_{23}^{(f)}, \psi_{23}) W^{(13)}(\vec{\theta}_{13}^{(f)}, \psi_{13}) W^{(12)}(\vec{\theta}_{12}^{(f)}, \psi_{12}) \right] D(\vec{\lambda}) ,
\]
with
\[
\phi_{13} = \eta_{13}^{(f)} + \xi_1 - \xi_2 = \eta_{13}^{(f)} - \eta_{12}^{(f)} - \eta_{23}^{(f)} , \quad \eta_{12}^{(f)} = \eta_{12}^{(f)} + \xi_2 - \xi_3 = \eta_{12}^{(f)} - \eta_{13}^{(f)} ,
\]
\[
\psi_{23} = \eta_{23}^{(f)} + \xi_2 - \xi_3 = \eta_{23}^{(f)} - \eta_{23}^{(f)} , \quad \psi_{13} = \eta_{13}^{(f)} + \xi_1 - \xi_3 = \eta_{13}^{(f)} - \eta_{12}^{(f)} - \eta_{23}^{(f)} ,
\]
and \( \vec{\lambda} = (1, \lambda_{21}, \lambda_{31}) \), where
\[
\lambda_{21} = \omega_2^{(f)} + \xi_2 - \omega_1^{(f)} + \xi_1 = \omega_2^{(f)} - \omega_1^{(f)} + \eta_{12}^{(f)} , \quad \lambda_{31} = \omega_3^{(f)} + \xi_3 - \omega_1^{(f)} + \xi_1 = \omega_3^{(f)} + \xi_1^{(f)} + \eta_{12}^{(f)} + \eta_{23}^{(f)} .
\]

The overall factor \( e^{i(\omega_1^{(f)} + \xi_1)} \) and the diagonal matrix of phases \( D^\dagger (\vec{\omega}^{(f)} + \vec{\xi}) \) on the left of Eq. (A17) can be eliminated by appropriate rephasing of the charged lepton fields in Eq. (3.3). On the other hand, if neutrinos are Majorana particles the Lagrangian is not invariant under rephasing of the massive neutrino fields and the diagonal matrix of phases \( D(\vec{\lambda}) \) on the right of Eq. (A17) cannot be eliminated. Hence, the physical mixing matrix can be written as
\[
U = \left[ R^{(23)}(\vec{\theta}_{23}) W^{(13)}(\vec{\theta}_{13}, \phi_{13}) R^{(12)}(\vec{\theta}_{12}) \right]^\dagger \left[ W^{(23)}(\vec{\theta}_{23}^{(f)}, \psi_{23}) W^{(13)}(\vec{\theta}_{13}^{(f)}, \psi_{13}) W^{(12)}(\vec{\theta}_{12}^{(f)}, \psi_{12}) \right] D(\vec{\lambda}) ,
\]
with \( \vec{\theta}_{ab} = \vec{\theta}_{ab}^{(f)} \). In other words, in general the charged lepton and neutrino matrices in Eq. (3.4) can be written as
\[
V^{(f)} = R^{(23)}(\vec{\theta}_{23}) W^{(13)}(\vec{\theta}_{13}, \phi_{13}) R^{(12)}(\vec{\theta}_{12}) ,
\]
\[
V^{(\nu)} = \left[ W^{(23)}(\vec{\theta}_{23}^{(f)}, \psi_{23}) W^{(13)}(\vec{\theta}_{13}^{(f)}, \psi_{13}) W^{(12)}(\vec{\theta}_{12}^{(f)}, \psi_{12}) \right] D(\vec{\lambda}) .
\]
The charged lepton matrix (A25) has the standard explicit form given in Eq. (18), with three angles \( \vec{\theta}_{ab} \) and one phase \( \phi_{13} \). The neutrino matrix (A26) depends on three angles \( \vec{\theta}_{ab}^{(\nu)} \), three “Dirac type” phases \( \psi_{ab} \), and two “Majorana type” phases \( \lambda_{21}, \lambda_{31} \) (that can be eliminated in the case of Dirac neutrinos). Notice however, that Eqs. (A19)–(A23) show that the phases in the neutrino matrix (A26) may be due to the diagonalization of the neutrino mass matrix, or from the diagonalization of the charged lepton mass matrix, or both.

For the sake of clarity, let us finally remark that the mixing matrix \( U \) can be obviously written as
\[
U = \left[ R^{(23)}(\vec{\theta}_{23}) W^{(13)}(\vec{\theta}_{13}, \phi_{13}) R^{(12)}(\vec{\theta}_{12}) \right]^\dagger D(\vec{\lambda}) ,
\]
in terms of three mixing angles \( \vec{\theta}_{ab} \) and one physical phase \( \phi_{13} \). Our construction leading to Eq. (A24) shows that in general the mixing angles \( \vec{\theta}_{ab} \) and the phase \( \phi_{13} \) depend in a rather complicated way on the
angles and phases of both the matrices $V^{(\ell)}$ and $V^{(\nu)}$ that diagonalize, respectively, the charged lepton and neutrino mass matrices.

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FIG. 1: Allowed region in the positive $\sin(\psi_{12})$–$s_{12}$ plane. The thick solid line represents the upper bound in Eq. (4.1), $s_{12} < 0.32$. The thin solid line represent the limit in Eq. (4.3), corresponding to the upper limit on $\tan^2 \vartheta_{\text{sol}}$ in the LMA region (Eq. (2.2)). The dotted lines have the indicated constant values of $\tan^2 \vartheta_{\text{sol}}$. They have been calculated using the exact expression of $\tan^2 \vartheta_{\text{sol}}$ with $s_{23} = s_{12}^2$, $s_{13} = s_{32}^3$, $\phi_{13} = 0$, $\psi_{23} = 0$. The dashed lines have the indicated constant value of the CP-violation parameter $J$ in Eq. (4.11).