Dark Matter and Yukawa Unification with Massive Neutrinos

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Abstract

We revisit the WMAP dark matter constraints on Yukawa Unification in the presence of massive neutrinos. The large neutrino mixing indicated by the data modifies the predictions for the bottom quark mass, and enables Yukawa also for large \( \tan \beta \), and for positive \( \mu \) that were previously disfavoured. As a result, the allowed parameter space for neutralino dark matter also increases, particularly for areas with resonant enhancement of the neutralino relic density.

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1 Introduction

Reconciling the Cold Dark Matter (CDM) predictions of supersymmetric models with the stringent constraints from the Wilkinson Microwave Anisotropy Probe (WMAP) has been one of the challenges within the particle physics community in recent years. The amount of CDM deduced from WMAP data [1, 2] puts severe constraints on possible Dark Matter Candidates, including the lightest supersymmetric particle (LSP). Additional constraints on the model parameters are obtained by imposing Yukawa unification, and by taking into account the bounds from Flavour Changing Neutral Currents (FCNC).

In addition to the above, the neutrino data of the past years provided evidence for the existence of neutrino oscillations and masses, pointing for the first time to physics beyond the Standard Model. As expected, the additional interactions required to generate neutrino masses also affect the energy dependence of the couplings of the MSSM, and thus modify the Yukawa unification predictions. A first observation had been that the additional interactions of neutrinos, which affect the tau mass, may spoil bottom-tau Unification for small \( \tan \beta \) [3]. Subsequently, however, it has been realised that large lepton mixing naturally restores unification, and even allows Unification for intermediate values of \( \tan \beta \) that were previously disfavoured [4, 5]. This is done by making the simple observation that the \( b - \tau \) equality at the GUT scale refers to the \((3, 3)\) entries of the charged lepton and down quark mass matrices, while the detailed structure of the mass matrices is not predicted by the Grand Unified Group itself. It is then possible to assume mass textures, such that, after diagonalisation at the GUT scale, the \((m^{diag}_E)_{33}\) and \((m^{diag}_D)_{33}\) entries are no-longer equal.

In the current work, based on [6], we revisit the issues of Dark Matter and Yukawa Unification taking into account the effects of massive neutrinos and large lepton mixing in See-Saw models, and extending previous results to large \( \tan \beta \). We find that the effects on the allowed parameter space are significant and, in fact, it turns out that Yukawa Unification in the presence of neutrinos is also compatible with a negative \( \mu \), unlike what happens if the effects of neutrinos are ignored. Passing to the relic density of neutralinos, we study the consequences of large lepton mixing in the \( \chi - \tilde{\chi} \) coannihilation region and in resonances in the \( \chi - \chi \) annihilation channels, finding sizeable effects, particularly in the latter case. It is interesting to note that for the cosmologically favoured area, it is also possible to observe tau flavour violation at the LHC, in the framework of non-minimal supersymmetric Grand Unification [7].

2 Massive Neutrinos and Unification

In the presence of massive neutrinos, the predictions for \( m_b \) and unification clearly get modified. Radiative corrections from the neutrino Yukawa couplings have to be included in renormalisation group runs from \( M_{GUT} \) to \( M_N \) (scale of the heavy right-handed neutrinos). Below \( M_N \), right-handed neutrinos decouple from the spectrum and an effective see-saw mechanism is operative; the relevant equations are given in [8]. In addition, if the GUT scale lies significantly below a scale \( M_X \), at which gravitational effects can no longer be neglected, the renormalization of couplings at scales between \( M_X \) and \( M_{GUT} \) may induce additional effects to the running and the simplest
example is provided by minimal SU(5) \cite{9} (however, modifications to soft masses are in this simplest case proportional to the $V_{CKM}$ mixing \cite{9}, and thus are significantly suppressed). Nevertheless, it has been realized that the influence of the runs above the GUT scale on the Dark Matter abundance can be very sizeable \cite{10}, due to changes in the relation between $m_\tau$ and $m_\chi$, which is crucial in the coannihilation area. This we discuss in a subsequent section.

In supersymmetric models, unification is very sensitive to the model parameters \cite{11}, particularly the Higgs mixing parameter, $\mu$. To correctly obtain pole masses within this framework, the standard model and supersymmetric threshold corrections have to be included; for the bottom quark, these corrections result to a $\Delta m_b$ that can be very large, particularly for large values of $\tan \beta$ \cite{12}. Constraints from BR($b \rightarrow s\gamma$) are also included in the analysis. Before passing to the results, however, let us summarise a few facts on the possible range of the mass of the bottom quark: the 2-$\sigma$ range for the $\overline{MS}$ bottom running mass, $m_b(m_b)$, is from 4.1-4.4 GeV. Moreover, $\alpha_s(m_Z) = 0.1172 \pm 0.002$, and the central value of $\alpha_s$ corresponds to $m_b(m_Z)$ from 2.82 to 3.06 GeV.

In Fig. 2, we summarize the predictions for $m_b$ in mSUGRA and in the presence of massive neutrinos. In order to discuss the dependence of $m_b(M_Z)$ on $\tan \beta$, we consider the set of soft parameters: $m_{\tilde{t}_2} = 800$ GeV, $A_0 = 0$, $m_0 = 600$ GeV. The figure exhibits the well-known fact that in the absence of phases or large trilinear terms, $\Delta m_b$ is positive for $\mu$ positive, and therefore the theoretical prediction for the b quark pole mass is too high to be reconciled with $b-\tau$ unification. On the other hand, for $\mu < 0$, $\Delta m_b$ is negative and the theoretical prediction for the b quark mass can lie within the experimental range for values of $\tan \beta$ between roughly 25 and 45; clearly, for a large $\tan \beta$ it is mandatory to take into account the large supersymmetric corrections to $m_b$ \cite{13, 14}.

The analysis in the presence of massive neutrinos takes into account only the third generation couplings, from the $M_{GUT}$ to the scale of the right handed neutrino masses, and evolve the light neutrino mass operator from this scale down to $M_Z$. A large value of the Dirac-type neutrino Yukawa coupling, $\lambda_N$ at the GUT scale may arise naturally within the framework of Grand Unification, and its value is determined by demanding a third generation low energy neutrino mass of $m_{\nu_3} = 0.05$ eV. The predictions for $m_b(M_Z)$ using the lower and upper bounds of the 2-$\sigma$ experimental range of $\alpha_s$ and the corresponding range for $m_b(M_Z)$ after the evolution of the bounds on $m_b(m_b)$ are shown for a scale $M_N = 3 \times 10^{14}$ GeV.

We observe that for $\mu > 0$ the prediction for $m_b(M_Z)$ is always very large, despite its dependence on the soft terms through $\Delta m_b$. For the values of the soft terms considered in Fig 2 the allowed range of $\tan \beta$ shrinks from 27-44 to 30-45 when we introduce the effect of see-saw neutrinos. It is also shown that the influence of runs above $M_{GUT}$ is too small to have any significant effect.

The results are significantly modified once we consider the effects of lepton mixing in the diagonalisation and running of couplings from high to low energies. In order to show this, we focus on $b-\tau$ unification within the framework of SU(5) gauge unification and flavour symmetries that provide consistent patterns for mass and mixing hierarchies, and naturally reconcile a small $V_{CKM}$ mixing with a large charged lepton one. Taking into account the particle content of SU(5) representations (with symmetric up-type mass matrices, and down-type mass matrices that are transpose to the ones for charged
Figure 1: The value of $m_b(M_Z)$ versus $\tan \beta$ assuming $\lambda_b = \lambda_\tau$ at the high scale in the absence of lepton mixing. We work with the following set of parameters: $m_{1/2} = 800 \text{ GeV}$, $A_0 = 0 \text{ GeV}$, $m_0 = 600 \text{ GeV}$. The experimental range of $m_b$ (horizontal lines) is also shown, the black thick (blue thin) lines have $\alpha_s = 0.1212$ (0.1132), the dot-dash lines are obtained within the MSSM, the solid lines include neutrino Dirac Yukawa up to the scale $M_N = 3 \times 10^{14}$. The upper (lower) set of lines corresponds to $\mu > 0$ ($\mu < 0$).

leptons), one finds the approximate relations

$$M_\ell^0 \propto A \begin{pmatrix} 0 & 0 \\ x & 1 \end{pmatrix}, \quad M_\ell^0 \propto A \begin{pmatrix} 0 & x \\ 0 & 1 \end{pmatrix}$$

which, after diagonalization, lead to

$$\frac{m_b^0}{1 + x^2} = \frac{m_\tau^0}{1 - x^2} \rightarrow m_b^0 = m_\tau^0 \left( 1 - \frac{2 x^2}{\delta} + O(\delta^2) \right)$$

where $\delta$ parametrises the flavour mixing in the (2,3) sector.

In the left panel of Fig.2 we show the change of $m_b$ as a function of $\tan \beta$, when the effects from large lepton mixing are appropriately considered. Comparing with the previous plots, we see how solutions with positive $\mu$ are now viable, for the whole range of $\tan \beta$. The appropriate size of the parameter $\delta$ in each case can be determined by imposing the relation $\lambda_\tau = \lambda_b(1 + \delta)$ at $M_{GUT}$ and investigating the values that are required in order to obtain a correct prediction for $m_b(M_Z)$. This is shown in the right pannel of Fig.2, where we demand a value of $m_b(M_Z)$ at the center of its experimental range, for the central value of $\alpha_s$.

3 Dark Matter constraints and Yukawa unification

In mSUGRA (or the CMSSM) for choices of soft terms below the TeV scale, the LSP is Bino like and the prediction for $\Omega_\chi h^2$ is typically too large for models that satisfy
Figure 2: In the left panel, we show $m_b$ as a function of $\tan\beta$, when including lepton mixing effects. In the right panel, we show the required values $\delta$ for consistent unification. We use the set of soft terms of Fig. 3, $\alpha_s = 0.1172$ and impose $m_b(M_Z) = 2.92$ GeV. The upper (lower) line corresponds to $\mu > 0$ ($\mu < 0$).

the experimental constraints on SUSY. In fact, the values of WMAP can essentially be obtained in two regions:

- $\chi - \tilde{\tau}$ coannihilation region that occurs for $m_\chi \sim m_{\tilde{\tau}}$.

- Resonances in the $\chi - \chi$ annihilation channel, which occur for $m_A \sim 2m_\chi$.

Since the above areas are “fine-tuned”, they will inevitably be sensitive to the changes induced by GUT unification and sizeable mixing in the charged lepton sector. The runs corresponding to $M_X > M_{\text{GUT}}$ have a big impact on the neutralino relic density. The large values of the gauge unified coupling $\alpha_{SU(5)}$ tend to increase the values of $m_{\tilde{\tau}}$, even if we start with small $m_0$ at $M_X$.

We see that the consideration of mixing effects, in combination with the inclusion of effects from the runs above $M_{\text{GUT}}$, significantly enhances the allowed parameter space (green area), an effect that is more visible for large $\tan\beta$. Areas with different colours are excluded [6]. The reduction of the allowed parameter space for smaller values of $\tan\beta$, is already evident for $\tan\beta = 35$. The $\tan\beta = 35$ picture has been discussed in [7], where it was shown that the WMAP allowed region can be compatible with observable flavour violation at the LHC.

The case with $\mu < 0$ and a more detailed discussion of lepton mixing effects are presented in [6].

4 Conclusions

We revisited the WMAP dark matter constraints on Yukawa Unification in the presence of massive neutrinos. Large neutrino mixing, as indicated by the data modifies the predictions for the bottom quark mass, and enables Yukawa also for large $\tan\beta$ and
Figure 3: WMAP allowed area (green) for the case of \( \tan \beta = 45, \mu > 0, A_0 = m_0, m_b(M_Z) = 2.92 \text{ GeV} \) and \( \delta \sim 0.42 \) for the same set of parameters as in Fig. 2, without (left) and with (right) the SU(5) running. The solid (dash) line corresponds to \( m_h = 114 \text{ GeV} \) \((BR(b \rightarrow s\gamma) = 2.8 \cdot 10^{-4})\). In the lower (red) area the LSP is a stau.

Figure 4: The same pair of plots as in Fig. 3 but for \( \tan \beta = 35 \). Here, the parameter \( \delta \sim 0.37 \) for positive \( \mu \) that were previously disfavoured. A direct outcome is that the allowed parameter space for neutralino dark matter also increases, particularly for areas with resonant enhancement of the neutralino relic density.

For completeness, we also note that for the cosmologically favoured parameter region, we found lepton flavour violating rates very close to the current experimental bounds [15]. Finally, interesting effects may arise in the case of non-universal soft terms. These are also discussed in detail in [15].

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