Flavor Violating Lepton Family U(1)$_\lambda$

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The Standard Model is extended minimally with a new flavor-violating family symmetry U(1)$_\lambda$, which acts only on leptons including the right-handed neutrinos. The model is anomaly free with family-dependent U(1)$_\lambda$ charges, and consistent with the observed neutrino mixing angles. It predicts charged lepton flavor-violating processes mediated by a new gauge boson. Under certain conditions, the smallness of $\theta_{13}$ of neutrino mixing can be justified in terms of the muon-to-tau mass ratio, at the same time explaining the electron-to-tau large mass hierarchy.
The observations of the neutrino flavor violation indicate the existence of new physics beyond the Standard Model (SM)[1]. This is usually interpreted in terms of the neutrino oscillations based on the mismatch between the flavor and mass eigenstates[2][3]. However, if neutrino masses are originated from some kind of Higgs mechanism, the theory must include tree-level neutrino flavor violating Yukawa interactions. Furthermore, this neutrino flavor violation would lead to charged lepton flavor violations (cLFV) at one-loop with a $W$-boson exchange. Although this induced cLFV is negligibly small, but it inevitably raises a question that if there is a Beyond-SM(BSM) with more significant and explicit lepton flavor violating interactions including charged leptons (see [4][5] for recent reviews).

In this Letter, we will explore the possibility how the right-handed neutrinos can play the role of going beyond the SM. One way is to introduce some gauge charge explicitly for them[6]. So we will consider an extra $U(1)_\lambda$ gauge interaction, which acts solely on leptons including the right-handed neutrinos. $U(1)_\lambda$ has different charges for different eigenstates similarly as in the Froggatt-Nielsen case[7], which leads to a flavor violating interaction in the flavor basis[6]. Then we can investigate the physics of cLFV, in particular, generated by a gauge interaction beyond Yukawa interactions. Note that our proposal is different from gauging the lepton number as a gauge symmetry, which was first suggested in [8]. Recent references include [9][10][11], but all in the flavor conserving context.

The Model

The model we consider includes the right-handed neutrinos and the gauge group is

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\lambda.$$  

(1)

$U(1)_\lambda$ acts only on leptons including $\nu_{\lambda R}$ with charges $q_\lambda$ depending on the flavor generation. This also allows $\nu_{\lambda R}$ to be genuine right-handed partners of left-handed neutrinos. Although we expect that $U(1)_\lambda$ is broken at a scale near or above the Electroweak (EW) scale, the coupling constant can be very small so that its gauge boson can couple to leptons very weakly. We assume that $U(1)_\lambda$ is left-right (LR) symmetric to minimize new anomalies, but this still is not enough to cancel all anomalies. For example, $(SU(2))^2 U(1)_\lambda$ anomaly does not cancel unless the sum of $U(1)_\lambda$ charges for all generations cancel. In fact, the condition

$$\sum q_\lambda = 0,$$  

(2)

where $q_\lambda$ are diagonalized $U(1)_\lambda$ charges for $U(1)_\lambda$ eigenstates, is the only extra we need to
cancel all anomalies as shown below:

\[(SU(2))^2U(1)_\lambda \propto \sum q_\lambda, \quad (3a)\]
\[(U(1)_\lambda)^3 \propto \sum_L (q_\lambda)^3 - \sum_R (q_\lambda)^3 = 0, \quad (3b)\]
\[(U(1)_\lambda)^2U(1)_Y \propto (2(-\frac{1}{2}) - (-1)) \sum (q_\lambda)^2 = 0, \quad (3c)\]
\[(U(1)_Y)^2U(1)_\lambda \propto (2(-\frac{1}{2})^2 - (-1)^2) \sum q_\lambda, \quad (3d)\]
\[(\text{Gravity})^2U(1)_\lambda \propto \sum_L q_\lambda - \sum_R q_\lambda = 0, \quad (3e)\]

where all anomalies are summed over all generations since they do not necessarily cancel separately in each generation. This structure clearly indicates the $U(1)_\lambda$ is not a gauged lepton number symmetry.

Extra $U(1)$ is very common in extended SM[12][13]. (Also see [14].) Other models that accommodate family-dependent $U(1)$ family symmetry include [15], but flavor violation is from the Yukawa sector, not gauge sector. Also these models are not left-right symmetric so that the anomaly cancellation is more complicated, some even possible only by inheriting from the Green-Schwarz mechanism in string theory. Their extra $U(1)$ acts on both quarks and leptons. So none of these is similar to our model. The closest is [16], however, which did not consider the possibility of LFV. [17] and [18] consider the flavor changing neutral current (FCNC) due to $Z'$ by recognizing that the $Z'$ couplings to fermions can be non-diagonal, but the extra $U(1)$ acts on quarks as well. So, it is different from our model.

Generic Structure and Notations

Some comments on the notations are in order. We will use, generically, $\psi_\ell$ to denote the (Weak) flavor eigenstates, $\psi_j$ mass eigenstates, and $\psi_\lambda$ $U(1)_\lambda$ eigenstates. More specifically, $\ell$ ($\nu_\ell$) stands for flavor eigenstates for charged leptons (neutrinos, respectively), and $\lambda$ ($\nu_\lambda$) stands for $U(1)_\lambda$ eigenstates for charged leptons (neutrinos, respectively). For charged leptons, eventually we will identify the mass and flavor eigenstates such that $\ell$ stands for physical states $e, \mu, \tau$.

In terms of the (generic) flavor eigenstates $\psi_\ell$, the $U(1)_\lambda$ coupling to leptons are of the form

\[\mathcal{H}_{\text{int}}^{(\lambda)} \equiv \overline{\psi_\ell} \gamma^\mu q_{\ell\ell} Z_{\mu}^{(\lambda)} \psi_\ell, \quad (4)\]

where $Z_{\mu}^{(\lambda)}$ is the $U(1)_\lambda$ gauge boson. If the hermitian matrix

\[q \equiv (q_{\ell\ell}) \quad (5)\]
is not diagonal, we have LFV. The U(1)$_{\lambda}$ gauge invariance can be shown in terms of the U(1)$_{\lambda}$ eigenstates $\psi_{\lambda}$, for which the U(1)$_{\lambda}$ gauge coupling constant matrix $q_d$ becomes diagonal. Since $q$ is a matrix in the flavor space, the corresponding matrix $q_d$ in the U(1)$_{\lambda}$ eigenstate basis can be generated by a unitary transformation

$$q_{d\lambda\lambda} = (V^\dagger)_{\lambda\nu} q_{\nu\ell} V_{\ell\lambda},$$

where $V$ is given by

$$\psi_{\ell} = V_{\ell\lambda} \psi_{\lambda}, \quad V^\dagger V = 1$$

and parametrized\(^1\)

$$V(\tilde{\theta}_{12}, \tilde{\theta}_{23}, \tilde{\theta}_{13}) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \tilde{c}_{23} & \tilde{s}_{23} \\
0 & -\tilde{s}_{23} & \tilde{c}_{23}
\end{pmatrix} \begin{pmatrix}
\tilde{c}_{13} & 0 & \tilde{s}_{13} \\
0 & 1 & 0 \\
-\tilde{s}_{13} & 0 & \tilde{c}_{13}
\end{pmatrix} \begin{pmatrix}
\tilde{c}_{12} & \tilde{s}_{12} & 0 \\
-\tilde{s}_{12} & \tilde{c}_{12} & 0 \\
0 & 0 & 1
\end{pmatrix},$$

where $\tilde{c} \equiv \cos \tilde{\theta}$ and $\tilde{s} \equiv \sin \tilde{\theta}$. Note that this unitary transformation does not affect any of the SM gauge couplings since they are all family independent, i.e. proportional to identity in the flavor basis. The anomaly cancellation requires that the coupling constant matrix be traceless. So, in the U(1)$_{\lambda}$ basis,

$$q_d = \text{diag}(q_1, q_2, -q_1 - q_2).$$

There are three distinctively different cases: $q_1 = 0$, $q_2 = q_1$, or $q_2 \neq q_1 \neq 0$.

The charged lepton masses in the U(1)$_{\lambda}$ basis are given by

$$M = (M_{\lambda\lambda}) = V^{-1} M_d V,$$

where $M_d$ is, with the identification of the mass and flavor eigenstates for charged leptons, the diagonalized mass matrix given in terms of the physical masses as

$$M_d = \text{diag}(M_\ell) = \text{diag}(M_e, M_\mu, M_\tau).$$

The Dirac neutrino mass matrix in the U(1)$_{\lambda}$ basis, $m$, is not necessarily diagonalized in the same way as $M$, but as

$$m_d = \text{diag}(m_{Di}) = U_D^{-1} m U_D.$$

The right-handed Majorana mass matrix $\tilde{m}_R$ in the U(1)$_{\lambda}$ basis is diagonalized as

$$\tilde{m}_{Rd} = \text{diag}(\tilde{m}_{Ri}) = U_R^{-1} \tilde{m}_R U_R.$$
Since $U(1)_{\lambda}$ gauge couplings to $\nu_L$ and $\nu_R$ are supposed to be the same in any basis,

$$U_D = U_R. \tag{14}$$

To satisfy this, neutrino mass matrices must commute with each other as

$$[m, \tilde{m}_R] = 0. \tag{15}$$

If $\tilde{m}_R \neq 0$, the physical neutrino masses are necessarily Majorana for both chiralities and given in eqs.(30c)(30d). Since we identify the flavor and mass eigenstates for charged leptons such that $U^\dagger_\ell = 1$, from eqs.(7)(12 or 13), the neutrino mixing matrix $U_\nu$ can be identified as

$$U_{\text{PMNS}} = U_\nu = VU_D = VU_R. \tag{16}$$

Note that the r.h.s has six angles, while the l.h.s has only three. For the best-fit values, we take the latest PDG numbers\[19\]: $\sin^2(2\theta_{12}) = 0.857^{+0.023}_{-0.025}$, $\sin^2(2\theta_{23}) > 0.95$, and $\sin^2(2\theta_{13}) = 0.095 \pm 0.01$. Then the best-fit values for the mixing matrix we can use are

$$\sin^2 \theta_{12} \simeq 0.311 \pm 0.016,$$

$$\sin^2 \theta_{23} \simeq (0.39 \sim 0.61),$$

$$\sin^2 \theta_{13} \simeq 0.024 \pm 0.003. \tag{17}$$

Without $U(1)_{\lambda}$ charged doublet $\Phi_\lambda$, we have twelve free parameters ($q_1$, $q_2$, momentum cut-off $\Lambda$, and nine diagonal Yukawa coupling constants) plus the number of vacuum expectation values $v_\lambda$ of the symmetry breaking scalar fields, but only eight constraints (three charged lepton masses, three neutrino mixing angles, and two neutrino mass relations). We can assume $\Lambda \sim v_\lambda$, but this still leaves at least four unconstrained parameters. The rest of details depend on the choices of these free parameters. With $\Phi_\lambda$, the number increases because of non-diagonal Yukawa coupling constants.

**Dirac Masses for Charged Leptons**

The Yukawa couplings for charged leptons are given by

$$Y_{\lambda\lambda'}L_{\lambda'\lambda}HL\ell_{\lambda'R} + \text{h.c.,} \tag{18}$$

where $H$ is the SM isospin doublet Higgs and $L$’s are the lepton doublets. Upon the EW symmetry breaking, this leads to Dirac mass terms for charged leptons as

$$\bar{\ell} M_{\lambda\lambda'}\lambda. \tag{19}$$
Since the SM Higgs does not carry \( U(1)_\lambda \) charge, the \( U(1)_\lambda \) gauge invariance requires that the tree-level SM Higgs Yukawa couplings should be diagonal in the \( U(1)_\lambda \) basis or some charges are degenerate\(^2\). Without any other off-diagonal contributions, this would imply that charged leptons have the same mass and \( U(1)_\lambda \) charge eigenstates so that \( U(1)_\lambda \) would not induce cLFV.

However, this is not true due to the Majorana Yukawa couplings of \( \nu_R \) with the \( U(1)_\lambda \) breaking scalars (see eq.\((25)\)), which will generate non-diagonal Dirac mass terms for leptons at higher orders\(^3\). For example, the leading order of non-diagonal Dirac masses for charged leptons can be generated at one-loop, shown in Fig.1, as

\[
\Delta M_{\lambda'}^{(1)} \sim \sum_{\alpha,\beta} \frac{v_\alpha v_\beta v_{\text{EW}}}{\Lambda^2} Y_{\lambda' \lambda} y_{\lambda \lambda'} y_{\lambda' \lambda'} \bar{\nu}_{\lambda'} y_{\lambda' \lambda'} + (\lambda \leftrightarrow \lambda'),
\]

where Yukawa couplings at tree-levels are determined by

\[
Y_{\lambda \lambda} \frac{v_{\text{EW}}}{\sqrt{2}} = M_{\lambda \lambda}^{(0)}, \quad y_{\lambda' \lambda} \frac{v_{\text{EW}}}{\sqrt{2}} = m_{\lambda' \lambda}^{(0)}, \quad \bar{\nu}_{\lambda'} y_\alpha = \bar{m}_{\lambda' \lambda}^{(0)}.\]

### Dirac Masses for Neutrinos

The SM Higgs also has Yukawa couplings for neutrinos as

\[
y_{\lambda \lambda} \overline{L_{\lambda L}} i \sigma_2 H^* \nu_{\lambda R},
\]

and the most general (non-diagonal) Dirac masses for neutrinos are

\[
\bar{\nu}_{\lambda'} m_{\lambda' \lambda} \nu_{\lambda},
\]

with including non-diagonal Dirac neutrino masses generated at one-loop, shown in Fig.2, as

\[
\Delta m_{\lambda'}^{(1)} \sim \sum_{\alpha,\beta} \frac{v_\alpha v_\beta v_{\text{EW}}}{\Lambda^2} y_{\lambda \lambda} \bar{y}_{\lambda' \lambda} \bar{y}_{\lambda' \lambda'} \lambda_{\lambda'},
\]

\(^2\)Subsequently, we will assume charges are non-degenerate since, otherwise, it does not lead to cLFV.

\(^3\)Off-diagonal mass terms can also be generated if \( U(1)_\lambda \) charged isospin doublet scalar \( \Phi_\lambda \) is introduced.
which is larger than those from Fig.1 with $\ell \chi$ replaced by $\nu \chi$ if $\lambda_{H\phi}$ is not too small. This makes it possible that the off-diagonal masses of neutrino Dirac mass matrix are relatively larger even if those of charged lepton mass matrix are much smaller than the diagonal values.

**Majorana Masses for $\nu_R$**

For charged leptons, the Dirac masses are the entire masses, but it is not necessarily true for neutrinos. $U(1)_\lambda$ can be broken by SM singlet $\phi_\lambda$, which cannot couple to the left-handed leptons, but can only couple to the right-handed neutrinos, whose couplings are necessarily Majorana types. Then the Yukawa couplings are Majorana-type as

$$\overline{\nu}_\lambda \phi_\alpha \nu_{\lambda R},$$

and the Majorana masses for $\nu_R$’s are

$$\tilde{m}_{R\lambda} \overline{\nu}_{N R}(\nu_{\lambda R})^c,$$

which includes one-loop corrections according to Fig.3 such that

$$\Delta m^{(1)}_{R\lambda} \sim \sum_{\alpha, \beta, \gamma} \frac{v_\alpha v_\beta v_\gamma}{\Lambda^2} \overline{\nu}_{\lambda \alpha'} \tilde{y}_{\lambda \alpha' \lambda} \overline{\nu}_{\lambda' \alpha' \lambda} \lambda_\phi.$$  \hspace{1cm} (27)

Fig.4 also contributes at the same order, but does not generate any new nontrivial contributions. In other words, if Fig.3 leads to vanishing components, so does Fig.4.

**Physical Majorana Neutrino Masses for $\nu_i^M$**

Since the Dirac neutrino mass matrix $m$ commutes with $\tilde{m}_R$, it is easy to diagonalize to obtain physical Majorana masses. The net neutrino mass terms are

$$(\nu_{\lambda R})^c \nu_{\lambda R} \overline{\nu}_{\lambda R}(\nu_{\lambda R})^c$$

and

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and

$$\nu_{\lambda R} m_{\alpha \lambda} (\nu_{\lambda R})^c + (\nu_{\lambda R})^c \nu_{\lambda R} m_{\alpha \lambda} (\nu_{\lambda L})^c,$$

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where
\[
\begin{pmatrix}
\nu_{iR}^M \\
(v_{iL}^M)^c
\end{pmatrix}
= 
\begin{pmatrix}
c_i & s_i \\
-s_i & c_i
\end{pmatrix}
\begin{pmatrix}
(v_{iL})^c \\
\nu_{iR}
\end{pmatrix}
\] (29)

with
\[
c_i = \frac{m_{Di}}{\sqrt{m_{Li}^2 + m_{Di}^2}} = \frac{m_{Li}}{\sqrt{m_{Di}^2 + m_{Di}^2}},
\] (30a)
\[
s_i = \frac{m_{Di}}{\sqrt{m_{Li}^2 + m_{Di}^2}} = \frac{m_{Di}}{\sqrt{m_{Di}^2 + m_{Di}^2}},
\] (30b)
\[
m_{Li} \equiv \frac{1}{2} \left( \frac{\tilde{m}_{Ri} - \sqrt{\tilde{m}_{Ri}^2 + 4m_{Di}^2}}{m_{Di}} \right),
\] (30c)
\[
m_{Ri} \equiv \frac{1}{2} \left( \frac{\tilde{m}_{Ri} + \sqrt{\tilde{m}_{Ri}^2 + 4m_{Di}^2}}{m_{Di}} \right).
\] (30d)

**Yukawa coupling constants**

The values of Yukawa couplings depend on the masses and symmetry breaking scales. If physical neutrino masses are Dirac types, \(y \ll 1\). If not, we can assume \(\tilde{y}_{\lambda} \sim O(1)\) for the maximum. Since we expect that the left-handed Majorana masses are of order \(m_{\ell} \sim 10^{11/2}\) eV \(\sim 100\) MeV so that we can obtain \(y_{\lambda} \sim 10^{-3}\). If we wish to get \(y_{\lambda} \sim 1\) such that \(m \sim 100\) GeV, then \(v_\lambda \sim \tilde{m}_{R} \sim 10^{11}\times 10^{-3} \sim 10^{14}\) GeV. So, for \(100\) MeV \(\lesssim m \lesssim 10^2\) GeV, \(10^{-3} \lesssim y_{\lambda} \lesssim 1\).

**Example I: \((q, 2q, -3q)\) with \(\phi_q\) and \(\phi_{2q}\)**

First, consider \(q_2 = 2q_1 = 2q\), and \(U(1)_\lambda\) is broken by two singlet scalars, \(\phi_q\) and \(\phi_{2q}\), then the \(U(1)_\lambda\) charge conservation on Yukawa couplings allows \(M\) and \(m\) at one-loop order have only (23)-components vanishing, while \(\tilde{m}_R\) has only the (22)-component vanishing.

For minimal mixing, i.e. \(\tilde{s} \ll 1\), we can assume a low \(v_\lambda \simeq 1\) TeV. Let \(Y_{\ell\ell} \sim M_{\ell}/v_{\text{EW}}, y \sim 10^{-3}, \tilde{y} \sim 1\), then we can obtain \(M_{12} \sim 10^{-4}\) MeV and \(M_{13} \sim 10^{-3}\) MeV and that \(\tilde{s}_{13} \sim 10^{-6} \sim \tilde{s}_{12}\) and \(\tilde{s}_{23} \sim 0\), which leads to

\[
q \simeq q
\begin{pmatrix}
1 & -10^{-6} & 4 \times 10^{-6} \\
-10^{-6} & 2 & 0 \\
4 \times 10^{-6} & 0 & -3
\end{pmatrix}
\] (32a)
\[
V \simeq
\begin{pmatrix}
1 & -10^{-6} & -10^{-6} \\
-10^{-6} & 1 & 0 \\
-10^{-6} & 0 & 1
\end{pmatrix}
\] (32b)
such that $U_\nu = VU_R \simeq U_R$. With eq.(32a), the limit $\text{BR}(\mu \to 3e) < 10^{-12}$ can constrain the $U(1)_\lambda$ breaking scale $v_\lambda$ as $v_\lambda > v_{\text{EW}}$. So, our assumption $v_\lambda \sim 1 \text{ TeV}$ is consistently valid with any choice of $q$. The magnitude of $q$ cannot be directly constrained by any current data, but we can safely assume it is extra weak such that $q \ll g$, where $g$ is the Weak coupling constant.

As we increase $v_\lambda$ higher, we can generate larger off-diagonal masses for charged leptons, which allows more significant cLFV. For example, for $v_\lambda \sim 10^{14} \text{ GeV}$, if $\bar{s}_{13} \simeq -0.455$ and $\bar{s}_{12} \simeq -0.973$ and $\bar{s}_{23} \simeq 0.976$, we can obtain

$$q \simeq q \begin{pmatrix} 0.923 & 2 & -0.24 \\ 2 & -1.79 & 0.73 \\ -0.24 & 0.73 & 0.87 \end{pmatrix}$$ (33)

with significant $\mu \to e$ cLFV. However, we can even completely suppress $\mu \to e$ cLFV. For $v_\lambda \sim 10^{14} \text{ GeV}$, if $\bar{s}_{13} \simeq -0.5$ and $\bar{s}_{12} \simeq 0 \simeq \bar{s}_{23}$, we can obtain

$$q \simeq q \begin{pmatrix} 0 & 0 & \sqrt{3} \\ 0 & 2 & 0 \\ \sqrt{3} & 0 & -2 \end{pmatrix}.$$ (34)

So we need additional conditions to make predictions more specific.

With some specific assumptions, we can make it more interesting. Suppose $y_{22} = y_{33}$ and

$$m_3 = s_{R12}^2 m_1 + c_{R12}^2 m_2,$$ (35)

then

$$M \simeq \begin{pmatrix} 83.42 & 43.73 & 320.4 \\ 43.73 & 83.42 & 0 \\ 320.4 & 0 & 1716.15 \end{pmatrix} \text{ MeV},$$ (36a)

$$q \simeq q \begin{pmatrix} 1.11 & 0.467 & 0.638 \\ 0.467 & 1.76 & -0.357 \\ 0.638 & -0.357 & -2.86 \end{pmatrix}.$$ (36b)

This leads to $s_{R13} = 0$ for $U_R$. One notable fact in this case is that the largest ratio in eq.(36a) is about 40, but we can obtain $M_\tau/M_e \sim 4 \times 10^6$. In other words, we can obtain the large charged lepton mass hierarchy without a large mass hierarchy in the mass matrix. Furthermore, this is related to the smallness of $s_{13}$ in $U_\nu$. In fact, we can do even better in the next example.

Example II: $(q, 2q, -3q)$ with $\phi_{2q}$ and $\phi_{3q}$

Consider the previous example, but replace $\phi_q$ with $\phi_{3q}$ and assume $y_{22} \neq y_{33}$, $M_{11} = M_{22} = M_{1\mu}$, $\bar{m}_{R23} = 0$, and

$$\bar{m}_{R3} = s_{R12}^2 \bar{m}_{R1} + c_{R12}^2 \bar{m}_{R2},$$ (37)
then mass matrices are

\[
\mathbf{M} = \begin{pmatrix}
M_\mu & 0 & M_{13} \\
0 & M_\mu & 0 \\
M_{13} & 0 & M_{33}
\end{pmatrix},
\quad \mathbf{m} = \begin{pmatrix}
m_{11} & m_{12} & m_{13} \\
m_{12} & m_{23} & m_{24} \\
m_{13} & m_{23} & m_{33}
\end{pmatrix},
\quad \tilde{\mathbf{m}}_R = \begin{pmatrix}
\tilde{m}_{R11} & \tilde{m}_{R12} & \tilde{m}_{R13} \\
\tilde{m}_{R12} & \tilde{m}_{R22} & 0 \\
\tilde{m}_{R13} & 0 & \tilde{m}_{R22}
\end{pmatrix}.
\]

(38)

Then eq.(16) becomes

\[
U_{PMNS} = U_\nu(\theta_{23}, \theta_{13}, \theta_{12}) = V(\tilde{\theta}_{13}, \tilde{\theta}_{12} = 0 = \tilde{\theta}_{23}) U_R(\theta_{R23}, \theta_{R12}, \theta_{R13} = 0),
\]

(39)

where

\[
V(\tilde{\theta}_{13}) = \begin{pmatrix}
\tilde{c}_{13} & \tilde{s}_{13} \\
0 & 1 \\
-\tilde{s}_{13} & 0
\end{pmatrix},
\]

(40a)

\[
U_R(\theta_{R12}, \theta_{R23}) = \begin{pmatrix}
c_{R12} & s_{R12} & 0 \\
-c_{R23}s_{R12} & c_{R23}c_{R12} & s_{R23} \\
s_{R23}s_{R12} & -s_{R23}c_{R12} & c_{R23}
\end{pmatrix},
\]

(40b)

and

\[
\tilde{s}_{13} \equiv \sin \tilde{\theta}_{13} = \sqrt{\frac{M_\mu - M_e}{M_\tau - M_e}} \approx \sqrt{\frac{M_\mu}{M_\tau}}.
\]

(41)

With \(s_{13}^2\) and \(s_{12}^2\) given in eq.(17), eq.(39) leads to \(s_{23}^2 \simeq 0.61\), \(s_{R12}^2 \simeq 0.50\), and \(s_{R23}^2 \simeq 0.59\). So eq.(41) is perfectly consistent with the best-fit neutrino mixing angles. Due to \(c_{23} = t_{13}/\tilde{t}_{13}\), as \(s_{13}\) decreases or increases, other values vary to the opposite direction. For an interesting example, let us choose \(s_{13}^2 \simeq 0.0296\), \(s_{12}^2 \simeq 0.328\), then eq.(39) fixes \(s_{23}^2 \simeq 0.515\), \(s_{R12}^2 = 1/2 = s_{R23}^2\), i.e. \(U_R\) is bi-maximal[20] such that \(\tilde{m}_{R11} = \tilde{m}_{R22}\) and \(\tilde{m}_{R12} = \tilde{m}_{R13}\). This is within \(2\sigma\), so still plausible.

Note that

\[
s_{13} = \tilde{s}_{13}c_{R23} \approx \sqrt{\frac{M_\mu}{M_\tau}} c_{R23}
\]

(42)

justifies that the smallness of \(s_{13}\) in terms of the smallness of the muon-to-tau mass ratio. The cLFV is based on

\[
\mathbf{q} = \mathbf{q} \begin{pmatrix}
\tilde{s}_{13}^2 - 3 \tilde{s}_{13}^2 & 0 & -4 \tilde{s}_{13} \tilde{c}_{13} \\
0 & 2 & 0 \\
-4 \tilde{c}_{13} \tilde{c}_{13} & 0 & \tilde{s}_{13}^2 - 3 \tilde{c}_{13}^2
\end{pmatrix}.
\]

(43)

Example III: \((0, q, -q)\) with \(\Phi_q\)

In fact, we can get eq.(39) just for Dirac neutrinos, hence without singlet \(\phi_\lambda\). Let \(\mathbf{q}_d = \text{diag}(0, q, -q)\) and \(U(1)_\lambda\) be broken by a hypercharge \(1/2\) isospin doublet \(\Phi_q\) at the same time as the EWSB. This is allowed because \(q\) can be sufficiently smaller than the Weak coupling in
our model, and, above all, there is no experimental constraint against it. With assumptions \( Y_{12} = 0, Y_{11} = Y_{22}, y_{22} = y_{33} \), the tree-level mass matrices are

\[
M = \begin{pmatrix}
M_\mu & 0 & M_{13} \\
0 & M_\mu & 0 \\
M_{13} & 0 & M_{33}
\end{pmatrix},
\]

\[
m = \begin{pmatrix}
m_{11} & m_{12} & m_{13} \\
m_{12} & m_{22} & 0 \\
m_{13} & 0 & m_{22}
\end{pmatrix}.
\] (44)

Then we reproduce eqs.(39)-(41) except now subscript \( R \) replaced by \( D \). In this case, the physical neutrinos are Dirac types and their masses satisfy

\[
m_1 + m_2 = 2m_3.
\] (45)

The cLFV is based on

\[
q = q \begin{pmatrix}
-\tilde{s}_{13}^2 & 0 & -\tilde{s}_{13}\tilde{c}_{13} \\
0 & 1 & 0 \\
-\tilde{s}_{13}\tilde{c}_{13} & 0 & -\tilde{c}_{13}^2
\end{pmatrix},
\] (46)

where \( e^+e^- \) coupling to \( Z^{(\lambda)} \) is relatively suppressed.

If \( Y_{12} \neq 0 \), but still \( Y_{11} = Y_{22} \), we can also recover eqs.(36a)(36b).

**Flavor Violating Higgs Decays**

Fig.5 shows further higher order contributions as

\[
\Delta M^{(2)}_{\lambda\lambda'} \sim \sum_{\alpha, \beta} v_\alpha v_\beta v_{EW}^2 Y_{\lambda\lambda'} y_{\lambda'\lambda} y_{\lambda'\lambda'} \tilde{y}_{\lambda'\lambda'} \lambda_H.
\] (47)

where \( \lambda_H v_{EW}^2 = m_H^2 \). This higher order may also generate flavor-violating Higgs decay with Yukawa couplings[21][22], provided it is not diagonal, of

\[
Y_{\ell'\ell} = Y_{\lambda'\lambda} v_{EW}^2 \frac{3\Delta M^{(2)}_{\lambda\lambda'}}{v_{EW}}.
\] (48)

**Discussions**

The model we have proposed in this Letter is a truly minimal extension of the SM because it only needs the right-handed neutrinos as additional matter accompanied by an extra abelian
gauge boson. The singlet scalar we need to break the extra $U(1)_\lambda$ only interacts with the right-handed neutrinos in addition to the $U(1)_\lambda$ gauge boson, so well hidden from the current detections. We can make the model more accessible with a SM doublet $\Phi_q$, which could be more easily testable.

This model makes interesting predictions on the LFV, which can be distinguished from other models. For example, since the cLFV process of production in our model is mainly caused by a gauge boson, compared with the cLFV based on Yukawa interactions, one should be able to distinguish our model by determining the spin of the boson in the charged lepton flavor violating processes whether the spin is one or zero.

We can construct models without $\sum q_\lambda = 0$ if the entire gauge symmetry is extended to that of the left-right symmetric models[23], as in [6]. In that case, $U(1)_\lambda$ breaking should be above the spontaneous parity breaking scale.

It also has an attractive theoretical aspect beyond cLFV as a benefit of having $U(1)_\lambda$. The charged lepton mass hierarchy is more manageable since large $M_\tau/M_\mu$ mass ratio can be explained in terms of more comparable masses. This is possible because non-diagonal charged lepton mass matrix can be constructed in the $U(1)_\lambda$ basis, which otherwise is hard to justify. This in turn links the charged lepton masses to the neutrino mixing matrix. In particular, the smallness of $\theta_{13}$ of the neutrino mixing can be related to the smallness of the muon-to-tau mass ratio, and we can reproduce the neutrino mixing angles perfectly consistent with the latest data. In this sense, our model is natural and we believe that this strongly suggests the worthiness of having $U(1)_\lambda$ family symmetry.

In fact, we can still obtain eqs.(39)-(41) (or eq.(36a) more generally) just in the SM with $\nu_R$'s, if we allow charged leptons have different flavor and mass eigenstates. All we need is to replace $V$ with $U^\dagger_\ell$ and $U_R$ with $U_\nu$, then demand eq.(38) (eq.(36a) respectively). So it deserves further investigations in this direction, both theoretically and experimentally.

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