Inconsistency between Thermodynamics and Probabilistic Quantum Processes

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We present a Gedankenexperiment that reveals an inconsistency between quantum theory and thermodynamics. It introduces a non-reciprocal waveguide coupled to a collection of two-level systems that absorb and emit radiation statistically. The experiment therefore combines coherent superposition of wave functions with the probabilistic description of absorption and emission processes. We show that this combination of coherence and collapse forces an isolated system to reduce its total entropy, starting from standard, thermodynamic equilibrium. The Gedankenexperiment demonstrates that the basic constituents of quantum physics, namely coherence and acausal (probabilistic) events, contradict the second law of thermodynamics. Several implications are briefly discussed.

The probabilistic nature of quantum physics is not related to the equations of motion which are fully deterministic and describe norm-preserving (unitary) transformations in the state space, but is instead related to a process whose correct interpretation or mathematically sound formulation is still under debate, the so-called collapse of the wave function [1]. The quantum mechanical collapse, which is here understood as a physical, not merely as an epistemic process, is of fundamental importance for our common-sense concept of reality. Using the collapse process as an integral part of their Gedankenexperiment, Einstein, Podolsky and Rosen showed that quantum mechanics is incomplete under the assumption of a local concept of reality [2]. Experiments in [3–5] that tested the associated Bell inequality [6] found the predictions of quantum mechanics to be correct, proving that reality must have a non-local character. Schrödinger’s famous Gedankenexperiment on a cat demonstrates drastically the consequences of the assumption that the collapse is not physical but rather concerns only the knowledge of the observer [7]. As emphasized by v. Neumann, this point of view leads inevitably to complete subjectivism and the abandonment of an objective world [8]. The “many-worlds” interpretation denies the collapse altogether and thereby forgoes the goal to describe macroscopic reality in terms of physics [9]. Therefore, we adopt in the following the hypothesis held at least implicitly by numerous physicists. These state that the collapse process is real in the sense that it occurs independently from the presence of an observer or a measurement apparatus, or argue that macroscopic ensembles of material objects have to be described as if the collapse were such a process [10].

The second law is one of the cornerstones of physics. Within classical physics and quantum physics, innumerable efforts have been undertaken to prove, see, e.g., [11–16] or to refute it [17–21]. In recent years, a new field of research called quantum thermodynamics has emerged, e.g., [22]. Quantum thermodynamics has achieved ongoing success in refining the second law and adapting it to quantum systems, see, e.g., [23]. Many of these achievements consider entangled many-body states at low temperatures, revealing that under such circumstances several formulations of the second law, such as the Clausius inequality or the Carnot efficiency, have to be modified with respect to their classical versions [24]. “Fluctuation theorems” describe the probability of irreversible processes violating the second law for small systems and on short time scales [25]. However, none of these investigations have challenged the simplest and most intuitive formulation of the second law: Spontaneous processes in isolated systems never reduce entropy on average, the overall temporal development always progresses from order to disorder and not vice versa [26, 27]. This entails that the state with the highest entropy which corresponds to thermal equilibrium, is the final steady state of any isolated system [28].

![Gedankenexperiment Diagram](image)

FIG. 1. Layout of the Gedankenexperiment. Two reservoirs $A$ and $B$ containing black-body radiation are coupled via a non-reciprocal, open waveguide to a collection of two-level systems (TLS) and to each other. The right-, respectively left-moving modes in channels 1 and 2 couple with different parameters $g_1$, $g_2$ to the two-level systems.

The collapse processes are usually considered to be one of the reasons for the validity of the second law [8]: They are irreversible and are supposed to always enhance the von Neumann entropy

$$S(\hat{\rho}(t)) = -k_B \text{tr}(\hat{\rho}(t) \ln \hat{\rho}(t)), \quad (1)$$

of a state $\hat{\rho}(t)$, because a collapse transforms the pure
state \( \hat{\rho}(0) \) with \( S(\hat{\rho}(0)) = 0 \) into a mixture \( \hat{\rho}_m(t > 0) \) with \( S(\hat{\rho}_m(t > 0)) > 0 \). This argument, however, rests on the ensemble interpretation of quantum mechanics. In contrast, the unitary Schrödinger dynamics of \( \hat{\rho}(t) \) does not alter \( S(\hat{\rho}(t)) \) and is therefore reversible in time, much as the time evolution in classical mechanics which is governed by time-reversal invariant laws. In classical mechanics, the fine-grained entropy never changes. A coarse-graining procedure or a statistical assumption (Boltzmann’s “Stosszahlansatz”) ensures a growing entropy in processes assumed to be irreversible if a sufficient number of particles are involved [29, 30]. In contrast, the quantum mechanical collapse enforces an intrinsically statistical formulation of the temporal development. If the collapse is attributed to the real dynamics of isolated systems, the dynamics acquires an irreducibly probabilistic character.

We shall now demonstrate that it is precisely this feature of the collapse process, its intrinsic randomness, that would violate the second law of thermodynamics in its basic sense [26, 27] if the interactions are not reciprocal. Due to the non-reciprocity, the coupling strength of localized degrees of freedom to itinerant particles depends on the propagation direction of the latter. The non-reciprocity can be achieved by a variety of techniques, which for example involve spin-orbit coupling in solid state devices [31] or spin-momentum locking in chiral quantum optics [32].

Experiments provide evidence [34, 35] that it is possible to statistically describe elementary inelastic processes associated with the absorption and emission of light quanta by atoms. This description utilizes differential equations based on the “golden rule” [33], which quantifies the transition rate between different states of a quantum system. The transition itself is considered to be a stochastic process happening in a finite time interval (see [35], p.419). Because such a process requires a continuum of eigenstates with different energies, the golden rule is only applicable if the involved modes of the radiation field form a continuum around the resonance frequency of the atoms [34, 35]. It is emphasized that the golden rule is not just an approximation to the full unitary time development as given by solving the Schrödinger equation. The concept of a transition rate implies that a decohering (collapse) process is involved which replaces the deterministic treatment with a probabilistic description.

In the following, we employ such a description, modeling the atoms by simple two-level systems (TLS). We do not try to derive the statistical description given by the golden rule from an underlying unitary dynamics of “system plus environment”, but – based on the microscopic Hamiltonian of our Gedankenexperiment – follow the line of thought used by A. Einstein in his derivation of Planck’s radiation law.

In the Gedankenexperiment, we consider two identical cavities \( A \) and \( B \) supporting a quasi-continuous mode spectrum described by bosonic annihilation operators \( a_j^\dagger \), \( a_j \) and frequencies \( \omega_j \). They are coupled bilinearly to the right- and left-moving modes \( a_{1k} \), \( a_{2k} \) of an open-ended waveguide which form also a quasi-continuum [36]. The modes \( a_{1k} \) and \( a_{2k} \) are coupled in turn to a collection of \( m \) two-level systems located at the center of the waveguide (see Fig. 1).

The total Hamiltonian of such a setup is given by

\[
H = H_A + H_B + H_{wg} + H_{TLS} + H_{\text{int}} + H_{\text{int}}^2.
\]

(2)

\( H_q \) denotes the Hamiltonian in cavity \( q = A, B \),

\[
H_q = h \sum_j \omega_j a_j^\dagger a_j.
\]

(3)

Similarly, the Hamiltonian of the waveguide reads

\[
H_{wg} = h \sum_k \omega_k \left( a_{1k}^\dagger a_{1k} + a_{2k}^\dagger a_{2k} \right),
\]

(4)

where the modes 1 and 2 belong to waves traveling to the right and to the left, respectively. For the TLS we have

\[
H_{\text{TLS}} = (\hbar \Omega/2) \sum_{l=1}^m \sigma^z_l,
\]

(5)

with Pauli matrix \( \sigma^z \). The coupling between the reservoirs and the modes 1 and 2 of the waveguide is bilinear,

\[
H_{\text{int}} = \sum_{q=A,B} \sum_{j,k} \hbar_{jk} \left( a_j^\dagger [a_{1k} + a_{2k}] + \text{h.c.} \right).
\]

(5)

Using the rotating wave approximation, the interaction with the TLS has the standard form [34, 35],

\[
H_{\text{int}}^2 = \sum_{l=1}^m \sum_k \left( g_{1k} a_{1k} + g_{2k} a_{2k} \right) \sigma^z_l + \text{h.c.},
\]

(6)

where \( \sigma^z_l \) denotes the raising operator of the \( l \)th TLS. We consider in the following the (time-dependent) average occupancy per mode \( j \), \( \langle n_j(t) \rangle \) for reservoir \( q = A, B \) in an energy interval around the TLS energy, \( \Omega - \Delta/2 < \omega_j < \Omega + \Delta/2 \) where \( \Delta \) is of the order of the natural linewidth of spontaneous emission from an excited TLS into the waveguide. The occupancy does not depend on \( j \) if the couplings \( \hbar_{jk} \), \( g_{1(2)k} \) are constant in the frequency interval of width \( \Delta \) around \( \Omega \).

It is crucial that \( g_{1k} \neq g_{2k} \) which specifies that the TLS couples with unequal strength to the right- and left-moving photons in the waveguide, a hallmark of chiral quantum optics [32].

At the initial time, \( t = 0 \), we assume separate thermal equilibria in \( A \), \( B \) and the ensemble of TLS at temperature \( T(0) \). For the radiation in cavity \( q \), the occupation number per mode at frequency \( \omega \) follows from the Bose distribution

\[
\langle n_q(\omega) \rangle(0) = \frac{1}{e^{\hbar \omega/k_B T(0)} - 1}.
\]

(7)
Because the temperature depends on \( \langle n_q \rangle \) as described by (7), we can define effective temperatures \( T_q(t) \) for each reservoir \( q \) under the assumption that the photons in each reservoir thermalize in the usual way quickly as a non-interacting Bose gas. Furthermore, we assume \( \langle n_{(1,2)k} \rangle(t) = 0 \) for the occupancy of the modes in the waveguide, i.e., the waveguide is populated through the sufficiently weak coupling to the reservoirs and its modes appear only as intermediate states [38]. The coherences of the TLS are neglected, because they decohere fast compared to the timescale set by the coupling to the waveguide [37]. This is assumed also in Einstein’s derivation of Planck’s law.

As the system is open and loses energy through the waveguide, its total entropy diminishes with time and the temperatures of the reservoirs and the TLS system fall to zero for \( t \rightarrow \infty \). The rate equations for \( \langle n_q \rangle(\Omega, t) \) and \( n_e(t) \), the number of excited TLS, are

\[
\begin{align*}
\frac{d\langle n_A \rangle}{dt} & = -\gamma_{\text{dec}}\langle n_A \rangle + \gamma_0 (-\langle n_A \rangle + \langle n_B \rangle) - \gamma_1 (m - n_e)\langle n_A \rangle + \gamma_2 m_e (\langle n_A \rangle + 1), \\
\frac{d\langle n_B \rangle}{dt} & = -\gamma_{\text{dec}}\langle n_B \rangle + \gamma_0 (-\langle n_B \rangle + \langle n_A \rangle) - \gamma_2 (m - n_e)\langle n_B \rangle + \gamma_1 m_e (\langle n_B \rangle + 1), \\
\frac{dm_e}{dt} & = -[\tilde{\gamma}_{11}(\Omega) + \tilde{\gamma}_{12}(\Omega)]m_e + \tilde{\gamma}_1(\Omega)\langle n_A \rangle(m - m_e) - (\langle n_B \rangle + 1)m_e \\ & \quad + \tilde{\gamma}_2(\Omega)(\langle n_B \rangle(m - m_e) - (\langle n_A \rangle + 1)m_e).
\end{align*}
\]

The first terms on the right side of Eqs. (8) – (10) describe the loss of photons through the open ends of the waveguide and are of first order in \( |h_{jk}|^2 \), resp. \( |g_{jk}|^2, |g_{2k}|^2 \). The following terms correspond to coherent processes of second order in the couplings. The effective rates \( \gamma_{\text{dec},0,1,2} \) and \( \tilde{\gamma}_{1r}, \tilde{\gamma}_{12} \) used for the numerical solution of (8) – (10) shown in Figs. 2 and 3 belong to the strong coupling regime of the TLS and the waveguide, with values accessible within a cavity QED framework [39]. The chiral nature of the waveguide entails \( g_{jk} \neq g_{kj} \) and therefore \( \gamma_1 \neq \gamma_2 \). In our example we have assumed \( \tilde{\gamma}_2 = \gamma_2 = \tilde{\gamma}_{12} = 0 \), i.e., channel 2 is not coupled to the TLS. One sees from (8) – (10) that this leads to a breakdown of the detailed balance condition in second-order processes because absorption is no longer balanced by stimulated and spontaneous emission. It generates an effective transfer of photons from reservoir \( A \) to \( B \) on time scales given by the strong coupling between channel 1 and the TLS, and a corresponding difference in the local temperatures calculated via Eq. (7). Fig. 2 shows the temporal behavior of the temperatures of reservoirs \( A \), \( B \) and the TLS for intermediate times. Although both reservoirs \( A \) and \( B \) lose photons through the open waveguide, the ratio between \( \langle n_A \rangle \) and \( \langle n_B \rangle \) attains a constant value for \( t \rightarrow \infty \). (Fig. 3). It shows clearly that the losses of reservoir \( B \) set in at a much later time than the population of \( B \) through \( A \) and the TLS.

We note that the function of the proposed device is based on the phenomena taking place in the little-explored realm where the quantum world interfaces classical physics, as we use a statistical description together with a unique quantum feature which also makes the emission rate of the TLS dependent on the occupation of the final states. In section C of [38], we demonstrate that this counter-intuitive effect leads to the restoration of detailed balance in a cavity system without non-reciprocal elements. The non-unitary, probabilistic state development of the device can neither be achieved in classical Hamiltonian dynamics nor in the unitary quantum regime. The collapse processes that link the classical world and the quantum regime [40] are the cause of the thermal imbalance between the otherwise equivalent reservoirs \( A \) and \( B \).

Our Gedankenexperiment reveals a clear conflict between thermodynamics and quantum theory – as it is commonly understood and used – because, as shown,
FIG. 3. The occupations of reservoirs A and B for long times, plotted on a logarithmic scale. The ratio \( \langle n_A \rangle / \langle n_B \rangle \) is asymptotically time-independent. The losses through the open waveguide are much slower than the breakdown of local equilibrium caused by the non-reciprocal interaction with the TLS.

the probabilistic dynamics caused by collapse processes does not satisfy the condition for detailed balance. Instead it leads to a time-dependent state that violates local thermal equilibrium as mandated by the second law. According to quantum theory, the chirally coupled cavities are expected to develop unequal occupation numbers, thereby creating a temperature gradient between them, although no work is done on the system, coupled to the environment only through the open waveguide.

The rate equations (8) – (10) have been derived under the assumption that the two directional channels of the waveguide are fed by A and B through the emission of wavepackets which in turn interact with the TLS in a causal fashion. The emission and absorption of single photons by the TLS are considered thus as probabilistic processes taking place within a finite time span, due to the quasi-continuum of modes available in the waveguide and the reservoirs. They satisfy causality: It is not possible for a right-moving photon in channel 1 to be emitted by the TLS and subsequently absorbed by reservoir A. Pure scattering events at the TLS are neglected in this approximation because they are of higher order in the coupling constants. Their inclusion cannot restore the detailed balance broken in the chiral setup.

Our reasoning is based on the assumption that the interaction of the TLS with the radiation continuum leads to real events [41] which must be described statistically. The physical mechanism of this “real” collapse plays no role in these considerations. The argument is therefore obviously also in accord with dynamical collapse models, see, e.g., [42].

The “sorting” between the reservoirs A and B in our device resembles the action of a Maxwell demon [12, 13, 43]. Note that information is neither processed, stored or erased in the TLS nor in the waveguide [16]. In fact, our model does not contain any “hidden” degrees of freedom, able to receive entropy from either the photonic system or the TLS. The entropy loss towards the environment through the waveguide is independent from the interaction between the reservoirs and the TLS and cannot account for the local temperature gradient build up. The coupling to the environment just causes decoherence in the standard way. Therefore, the use of the golden rule is justified, even without the assumption of real collapse events within the system itself. If the system would be completely isolated, one could argue that it must be described by the full unitary dynamics, leading to a trivial reconciliation with the second law. However, also in this case a decohering “environment” is present which consists of the infinitely many degrees of freedom of the photon gas, the working substance of the device. Then the golden rule can be applied also in a closed system. A similar temperature difference between A and B appears and a new steady state develops from initial thermal equilibrium for \( t \to \infty \), having a lower entropy than the initial state. This is shown in Fig. 4 (see also section B of [38]).

It has been argued that it is not possible to discern experimentally an interpretation of quantum mechanics based on probabilistic dynamics and real collapse from the decoherence interpretation which replaces the physical collapse by an epistemic operation: the tracing over environmental degrees of freedom in the full density matrix at the final observation time \( t_{\text{fin}} \) [44–47]. It is not known whether the photon densities in the reservoirs at \( t_{\text{fin}} \), calculated with the tracing procedure, would differ from the results in [38] based on the golden rule. If so, our proposed experiment, if performed with an isolated system, would allow to decide between interpretations based on real and epistemic collapse, respectively. Only the latter do not contradict the second law, provided a solution of the full many-body problem would effectively restore detailed balance conditions in the statistical description. As such a computation appears out of reach at
present, the question can only be decided experimentally. An implementation of our model within a quantum optical platform appears feasible with current technology [32, 36, 39].

In case that the experiment reported unequal distributions in $A$ and $B$ for the closed system, one would be forced to conclude that statistical processes such as spontaneous emission and absorption are able to reduce the total entropy for arbitrary large systems and on average, not only for short times and small systems as expected from fluctuations. Then the state of classical thermal equilibrium with maximal entropy is unstable and the system moves to steady states with a lower entropy. As the processes presented occur in a closed system, they violate the second law of thermodynamics in its basic sense [26, 27].

Likewise, the temperature gradient in the open system cannot be attributed to a fluctuation phenomenon, as the imbalance between $A$ and $B$ is completely determined by the chiral waveguide and the two reservoirs are macroscopic objects.

Further, entropy-reducing processes would be expected to occur in nature, in structures differing greatly from the device presented in Fig. 1 [48], with ramifications for thermodynamics and various other fields of science as well as engineering, including information science, life sciences and astrophysics [49].

The authors gratefully acknowledge an outstanding interaction with T. Kopp, support by L. Pavka, and also very helpful discussions with and criticism from J. Annett, E. Benkiser, H. Boschker, A. Brataas, P. Bredol, C. Bruder, H.-P. Böchler, T. Chakraborty, I.L. Egusquiza, R. Frésard, A. Golubov, S. Hellberg, K.-H. Höck, G. Ingold, V. Kresin, L. Krten, G. Leuchs, F. Marquardt, A. Morpurgo, A. Parra-Rodríguez, H. Rogalla, A. Roulet, M. Sanz, P. Schneebeiß, A. Schnyder, C. Schn, E. Solano, R. Stamps, R. Valenti, J. Volz and R. Wanke. D.B. thanks Raymond Frésard for the warm hospitality during his stay at CRISMAT, ENSICAEN, which was supported by the French Agence Nationale de la Recherche, Grant No. ANR-10-LABX-09-01 and the Centre national de la recherche scientifique through Labex EMC3.

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for a certain configuration of occupations \{n_A\} in an open waveguide (see Fig. 1). In the coupling Hamiltonian, Eq. (5), the process is of first order. The initial state reads
\[ \rho = |\{n_A\}n_{A_j}\rangle \langle \{0\}1_{ik}, \{0\}_2, \{s\} \rangle, \]
for a certain configuration of occupations \{n_A\} for modes \( j' \) in \( A \) with \( j' \neq j \). Here, \( n_{A_j} \) is the occupation number of mode \( j \). Similarly, \( \{n_B\} \) denotes a configuration in reservoir \( B \). Further, \( \{s\} \) is the configuration of excited (\( s_i = e \)) or ground states (\( s_i = g \)) of the \( m \) TLS, \( l = 1, \ldots, m \). The waveguide channels 1 and 2 are not occupied in \( \psi_{in} \). The initial state is connected via the term \( \hbar_j^\pi a^\dagger_{ik} a_{A_j} \) to the final state
\[ \psi_{fin} = |\{n_A\}n_{A_j} - 1, \{n_B\}, \{0\}_1, \{0\}_2, \{s\} \rangle. \]
The \( S \)-matrix element reads then
\[ S^\text{in}_{t_0} = -2\pi i\delta^n(E_{\text{fin}} - E_{\text{in}})\langle \psi_{\text{fin}} | H^1_{\text{int}} | \psi_{\text{in}} \rangle \]
with \( E_{\text{fin}} - E_{\text{in}} = \hbar(\omega_k - \omega_j) \). The interaction is assumed to take place over a time interval \( t_0 \). The corresponding regularized \( \delta \)-function is (see ref.[35])
\[ \delta^n(E) = \frac{1}{\pi} \frac{\sin(Et_0/2\hbar)}{E}. \]
We have
\[ \langle \psi_{\text{fin}} | H^1_{\text{int}} | \psi_{\text{in}} \rangle = h_{jk}^\pi \sqrt{n_{A_j}}, \]
and, according to standard reasoning [35], the transition rate is given by
\[ \tau^{-1}_{\text{dec}(\omega_j)} = \frac{1}{t_0} \sum_k 4\pi^2|k_{kj}|^2 \frac{t_0}{2\pi \hbar} \delta^n(E_{\text{fin}} - E_{\text{in}}) = \frac{2\pi}{\hbar} h_{jk}^\pi n_{A_j} \rho_1(h\omega_j), \]
where \( \hbar \) is the value of \( \hbar_{jk} \) for \( \omega_j = \omega_k \), which is assumed to be constant in the interval \([\Omega - \Delta/2, \Omega + \Delta/2]\). Averaging over all modes \( j \) with frequency \( \omega \), we obtain \( \tau^{-1}_{\text{dec}(\omega)} = \gamma_{\text{dec}}(\omega)\langle n_A \rangle(\omega) \) with
\[ \gamma_{\text{dec}}(\omega) = \frac{2\pi}{\hbar} |h|^2 \rho_{\text{dec}}(h\omega), \]
assuming \( \rho_1 = \rho_2 = \rho_{\text{dec}} \). Adding the contribution of the decay into channel 2, the master equation for \( \langle n_A \rangle(\omega) \) describing the loss process reads
\[ \frac{d\langle n_A \rangle(\omega)}{dt} = -2\gamma_{\text{dec}}(\omega)\langle n_A \rangle(\omega) + \ldots \]
A similar expression holds for reservoir $B$ with the same rate constant $\gamma_{\text{dec}}$.

The TLS couple to the channels via spontaneous emission. The corresponding first-order process leads to

$$\frac{d\rho_{\text{eq}}}{dt} = -\gamma_1(\Omega) + \gamma_{\text{dec}}(\Omega)\rho_{\text{eq}} + \ldots,$$

with $\gamma_{\text{dec}}(\omega) = 2\pi g^2(\omega)\rho_{\text{eq}}(\omega)/h$ for $r = 1, 2$ (see below).

We compute now the coherent transfer of a photon from reservoir $A$ to $B$ through the chiral waveguide, which is of second order in the coupling $|\hbar|^2$. As this process concerns a wavepacket of finite width and takes place in a finite time interval, only the right-moving channel 1 is relevant, i.e., the coupling Hamiltonian is

$$H_{\text{int}}^1 = \sum_{q=A,B} \sum_{j,k} h_{jk} (a_{qj}^\dagger a_{1k} + \text{h.c.}).$$

We denote the initial state as

$$|\psi_{\text{in}}\rangle = |\{n_A\}n_{A_j}, \{n_B\}n_{B_l}, \{0_1\}0_{1k}, \{0_2\}, \{s\}\rangle.$$

The transition from $A$ to $B$ occurs via the intermediate states

$$|\psi_{\text{im}}\rangle = |\{n_A\}n_{A_j} - 1, \{n_B\}n_{B_l}, \{0_1\}1_{1k}, \{0_2\}, \{s\}\rangle,$$

towards the final state

$$|\psi_{\text{fin}}\rangle = |\{n_A\}n_{A_j} - 1, \{n_B\}n_{B_l} + 1, \{0_1\}0_{1k}, \{0_2\}, \{s\}\rangle,$$

involving the operators

$$h_{jk}^* a_{1k}^\dagger a_{A_j}, \ h_{lk} a_{B_l}^\dagger a_{A_j}.$$

The $S$-matrix $S_{fi}$ connecting initial and final states reads

$$S_{fi}^{\text{th}} = -2\pi i\delta_{n}(E_{\text{fin}} - E_{\text{in}}) \lim_{\eta \to 0^+} \sum_k \frac{V_{fk} V_{ki}}{E_{\text{in}} - E_{\text{im}} + i\eta},$$

and $E_{\text{in}} - E_{\text{im}} = h(\omega_l - \omega_k)$. The matrix elements $V_{fk}$ and $V_{ki}$ are

$$V_{fk} = \langle \psi_{\text{in}} | h_{lk} a_{B_l}^\dagger a_{1k} | \psi_{\text{im}} \rangle, \ V_{ki} = \langle \psi_{\text{im}} | h_{jk}^* a_{1k}^\dagger a_{A_j} | \psi_{\text{in}} \rangle.$$

We find

$$S_{fi}^{\text{th}} \approx -2\pi^2 \delta_{n}(h(\omega_l - \omega_k))|\hbar|^2\rho_{\text{eq}}(\omega_k)\sqrt{n_{A_j}(n_{BL} + 1)},$$

and obtain for the transition rate out of state $|\psi_{\text{in}}\rangle$,

$$\tau_{S_{fi}}^{-1} = \frac{1}{\hbar_0} \sum_{l} |S_{fi}^{\text{th}}|^2 = \frac{2\pi^3}{\hbar} \delta_{n}(h(\omega_l - \omega_k))|\hbar|^4\rho_{\text{eq}}^2(\omega_k)n_{A_j}(n_{BL} + 1).$$

We denote with $\langle n_B \rangle(\omega)$ the average occupation number per mode in reservoir $B$ at frequency $\omega$. It follows that

$$\tau_{j_{A\to B}}^{-1} = \frac{2\pi^3}{\hbar} |\hbar|^4\rho_{\text{eq}}^2(\omega_k)\rho_B(\omega_k)n_{A_j}(\langle n_B \rangle(\omega) + 1).$$

If we average over all modes $j$ in $A$ with frequency $\omega_l$, the transition rate from $A$ to $B$ at $\omega$ reads

$$\frac{1}{\tau_{A\to B}(\omega)} = \frac{2\pi^3}{\hbar} |\hbar|^4\rho_{\text{eq}}^2(\omega_k)\rho_B(\omega_k)|\langle n_A \rangle(\omega)(\langle n_B \rangle(\omega) + 1).$$

Similarly, the transition from $B$ to $A$, proceeding via channel 2, is

$$\frac{1}{\tau_{B\to A}(\omega)} = \frac{2\pi^3}{\hbar} |\hbar|^4\rho_{\text{eq}}^2(\omega_k)\rho_A(\omega_k)|\langle n_B \rangle(\omega)(\langle n_A \rangle(\omega) + 1).$$
Finally, the master equation for reservoir $A$ characterizing direct transitions between $A$ and $B$ through the waveguide is given by

$$\frac{d\langle n_A \rangle(\omega)}{dt} = -\frac{1}{\tau_{A\rightarrow B}(\omega)} + \frac{1}{\tau_{B\rightarrow A}(\omega)} = \gamma_0(\omega)(\langle n_B \rangle(\omega) - \langle n_A \rangle(\omega)), \tag{S-32}$$

where we have assumed identical densities of states in $A$ and $B$, $\rho_A = \rho_B = \rho$. The rate $\gamma_0(\omega)$ is defined as

$$\gamma_0(\omega) = \frac{2\pi^3}{\hbar}|\tilde{h}|^2 \rho_{w\gamma}(\hbar \omega)\rho(\hbar \omega). \tag{S-33}$$

The ratio of second and first order contributions follows as

$$\frac{\gamma_0(\omega)}{\gamma_{dec}(\omega)} = \pi^2|\tilde{h}|^2 \rho_{w\gamma}(\hbar \omega)\rho(\hbar \omega). \tag{S-34}$$

Next, we consider the absorption of radiation from reservoir $A$ by the $l$-th TLS. The initial state is

$$|\psi_{in}\rangle = |\{n_A\}n_{AJ}, \{n_B\}, \{0_1\}0_{1k}, \{0_2\}, \{s\}g_l\rangle. \tag{S-35}$$

The absorption of a wavepacket of finite spatial extension can only proceed via channel 1. The intermediate states read then

$$|\psi_{in}\rangle = |\{n_A\}n_{AJ} - 1, \{n_B\}, \{0_1\}1_{1k}, \{0_2\}, \{s\}g_l\rangle \tag{S-36}$$

and the final state is obtained by absorption of the photon in channel 1 by the TLS,

$$|\psi_{in}\rangle = |\{n_A\}n_{AJ} - 1, \{n_B\}, \{0_1\}0_{1k}, \{0_2\}, \{s\}e_l\rangle. \tag{S-37}$$

For the S-matrix element we have

$$S_{fi}^{\gamma_0} = -2\pi^2\hbar\tilde{g}_l|\tilde{\omega} - \omega_j\rangle \rho_{w\gamma}(\hbar \omega_j)\sqrt{n_{AJ}}, \tag{S-38}$$

where $\tilde{g}_l$ is the value of $g_{1k}$ for $\omega_k = \omega_j$. The corresponding term in the master equation for $m_e$, the average number of excited TLS, is obtained by summing over all initial states, which leads to the expression

$$\frac{dm_e}{dt} = -\frac{2\pi^3}{\hbar}(m - m_e)\hbar^2\tilde{g}_l^2 \rho_{w\gamma}(\hbar \omega)\rho(\hbar \omega)|\langle n_A \rangle(\Omega)| + \ldots \tag{S-39}$$

In an analogous manner, the radiation from reservoir $B$ is absorbed via channel 2 by the TLS,

$$\frac{dm_e}{dt} = -\frac{2\pi^3}{\hbar}(m - m_e)\hbar^2\tilde{g}_l^2 \rho_{w\gamma}(\hbar \omega)\rho(\hbar \omega)|\langle n_B \rangle(\Omega)| + \ldots \tag{S-40}$$

On the other hand, the emission from the TLS towards reservoir $A$ must proceed via the left-moving channel 2. The initial state is now

$$|\psi_{in}\rangle = |\{n_A\}n_{AJ}, \{n_B\}, \{0_1\}, \{0_2\}0_{2k}, \{s\}e_l\rangle, \tag{S-41}$$

the intermediate state

$$|\psi_{in}\rangle = |\{n_A\}n_{AJ}, \{n_B\}, \{0_1\}, \{0_2\}1_{2k}, \{s\}g_l\rangle, \tag{S-42}$$

and the final state

$$|\psi_{in}\rangle = |\{n_A\}n_{AJ} + 1, \{n_B\}, \{0_1\}, \{0_2\}0_{2k}, \{s\}g_l\rangle. \tag{S-43}$$

A calculation completely analogous to the one for absorption above leads to the following term in the master equation for $m_e$, this time summing over final modes in reservoir $A$,

$$\frac{dm_e}{dt} = \frac{2\pi^3}{\hbar}m_e\hbar^2\tilde{g}_l^2 \rho_{w\gamma}(\hbar \omega)\rho(\hbar \omega)|\langle n_A \rangle(\Omega) + 1| + \ldots \tag{S-44}$$
In this expression, the term proportional to \( \langle n_A \rangle \) is associated with stimulated emission from the TLS. It is noteworthy that the stimulated emission of the TLS is not caused by photons that have been emitted by \( A \) and then impinge on the TLS to create photons in the same mode. Here, in contrast, the stimulated emission of the TLS is induced by photons that reside in the receiving reservoir \( A \). This counterintuitive effect, which is solely due to the Bose statistics and therefore only possible in quantum physics, restores detailed balance for the case of a cavity embedded in another one, in accord with Kirchhoff's law on black body radiation (see section ).

With the definition

\[
\tilde{\gamma}_r(\Omega) = \frac{2\pi^3}{\hbar} \hbar^2 g^2 \rho_{qg}^2 (\hbar \Omega) \rho(\hbar \Omega)
\]  

(S-45)

for \( r = 1, 2 \) we obtain the master equation for \( m_e \),

\[
\frac{dm_e}{dt} = -[\tilde{\gamma}_{11}(\Omega) + \tilde{\gamma}_{12}(\Omega)]m_e + \tilde{\gamma}_{11}(\Omega) [\langle n_A \rangle (m - m_e) - \langle n_B \rangle (m_e)]
\]

+ \[\tilde{\gamma}_{12}(\Omega) \langle n_B \rangle (m - m_e) - \langle n_A \rangle (m_e) \],

(S-46)

which is Eq.(10) in the main text. To compute the terms corresponding to the absorption and emission processes in the rate equations for \( \langle n_A \rangle \) and \( \langle n_B \rangle \), we note that the rates \( \tilde{\gamma}_r \) contain a summation over initial, respectively final modes in the reservoirs. The coefficients \( \gamma_r \) describing the temporal change in the average occupation number per mode, \( \langle n_q \rangle \), are therefore \( \tilde{\gamma}_r \) divided by the number of modes in the relevant frequency interval. Thus,

\[
N = \hbar \int_{\Omega - \Delta/2}^{\Omega + \Delta/2} d\omega \rho(\hbar \omega), \quad \gamma_r = \tilde{\gamma}_r / N.
\]  

(S-47)

Together with (S-18) and (S-32), it follows for \( \omega = \Omega \),

\[
\frac{d\langle n_A \rangle}{dt} = -\gamma_{\text{dec}}(-\langle n_A \rangle + \langle n_B \rangle) - \gamma_1(m - m_e)\langle n_A \rangle + \gamma_2 m_e(\langle n_A \rangle + 1),
\]  

(S-48)

\[
\frac{d\langle n_B \rangle}{dt} = -\gamma_{\text{dec}}(-\langle n_B \rangle + \langle n_A \rangle) - \gamma_2(m - m_e)\langle n_B \rangle + \gamma_1 m_e(\langle n_B \rangle + 1),
\]  

(S-49)

which are Eqs.(8) and (9).

**Derivation of the rate equations for the closed system**

This chapter discusses a variant of the open system characterized in section . In this variant, which is a closed system, the waveguide satisfies periodic boundary conditions, corresponding to a loop of length \( L \), where \( L \) is much larger than the distance between the reservoirs \( A \) and \( B \) see Fig. S5. We show that a description assuming real absorption and emission processes fulfills the detailed balance condition to first order in the coupling. Second order processes analogous to those described in Eqs. (S-35) – (S-44) break the detailed balance condition for the case that coherent absorption and emission is only possible along the short path between the reservoirs and the TLS, while dephasing occurs for wave packets emitted by a TLS and traveling the long way around the loop before reaching one of the reservoirs. The latter case is already accounted for by the first-order terms describing the equilibration between the reservoirs/TLS and the waveguide. Of course, if the dynamics of the closed system is considered to be unitary, corresponding to completely coherent evolution, the entropy does not change. This situation could be approximated by treating all second order processes as coherent, including those on the long path between the TLS and the cavities. Then the detailed balance condition would be satisfied, leading to stabilization of the state with maximal entropy. However, if the coherence is restricted to processes occurring along the short path, the ensuing steady state does not have maximum entropy. In the closed system, the occupancy of the waveguide can no longer be assumed to be zero, as the photons cannot escape towards infinity. We describe the occupancy in channel \( q \) by \( \langle n_q \rangle \) for \( q = 1, 2 \). The Hamiltonian is given in Eqs. (2) – (6).

The exchange of photons between cavity \( A \) and channel 1 of the waveguide proceeds via a first-order process analogous to that given in Eqs. (S-11) and (S-12), but the matrix element reads now (suppressing the frequency arguments),

\[
\langle \psi_{\text{in}} | H_{\text{int}}^1 | \psi_{\text{in}} \rangle = \hbar^*_{jk} \sqrt{n_{A_j} (n_1 + 1)}.
\]  

(S-50)
FIG. S5. The system obtained by closing the open waveguide of the system shown in Fig. 1 of the main text. The occupation numbers of the chiral channels 1 and 2 do not vanish due to equilibration with the reservoirs A and B.

This gives for the transition rate from mode $j$ in $A$ to channel 1 of the waveguide

$$\tau_{A\rightarrow 1}^{-1} = \gamma_{dec} n_{Aj} (\langle n_1 \rangle + 1), \quad (S-51)$$

and for the reverse process,

$$\tau_{1\rightarrow A}^{-1} = \gamma_{dec} \langle n_1 \rangle (n_{Aj} + 1), \quad (S-52)$$

where in this case a sum over initial states has to be performed to obtain the rate of emission into the fixed mode $j$ of $A$. The terms in the master equation for $\langle n_A \rangle$ are thus

$$\frac{d\langle n_A \rangle}{dt} = \gamma_{dec} (\langle n_1 \rangle - \langle n_A \rangle) + \ldots, \quad (S-53)$$

and for $\langle n_1 \rangle$

$$\frac{d\langle n_1 \rangle}{dt} = \gamma_{dec} (\langle n_1 \rangle - \langle n_1 \rangle) + \ldots, \quad (S-54)$$

with the rate constant $\gamma'_{dec} = (\rho/\rho_{wg})\gamma_{dec}$. Analogous expressions are obtained for $B$ and channel 2.

Another first-order process couples the TLS and the waveguide modes. We find for this contribution to the rate equation for $m_e$

$$\frac{d m_e}{dt} = \sum_{r=1,2} \tilde{\gamma}_{1r} [(m - m_e)\langle n_r \rangle - m_e(\langle n_r \rangle + 1)] + \ldots, \quad (S-55)$$

with $\tilde{\gamma}_{1r} = 2\pi g_{tr}^2 \rho_{wg}/\hbar$ (compare Eq.(S-19)). The corresponding terms in the rate equations for $\langle n_r \rangle$ are

$$\frac{d\langle n_r \rangle}{dt} = \gamma_{1r} [m_e(\langle n_r \rangle + 1) - (m - m_e)\langle n_r \rangle] + \ldots, \quad (S-56)$$
\[ \gamma_{1r} = \tilde{\gamma}_{1r}/N', \quad N' = h \int_{\Omega - \Delta/2}^{\Omega + \Delta/2} d\omega \rho_{wg}(\omega). \] (S-57)

All these first-order terms satisfy the detailed balance condition. They lead naturally to thermal equilibration between the reservoirs and the waveguide. For the second-order terms, we have first the process described by Eq. (S-46),

\[ \frac{dm_e}{dt} = \tilde{\gamma}_1(\langle n_1 \rangle + 1)[\langle n_A \rangle (m - m_e) - (\langle n_B \rangle + 1)m_e] \]
\[ + \tilde{\gamma}_2(\langle n_2 \rangle + 1)[\langle n_B \rangle (m - m_e) - (\langle n_A \rangle + 1)m_e] + \ldots, \] (S-58)

which depends also on the occupation numbers \( \langle n_r \rangle \) of the waveguide. This term is accompanied by corresponding terms in the rate equations for the reservoirs. It does not satisfy detailed balance because we have only considered the short path between the reservoirs and the TLS (the only available one in the open system). Including also the long path around the circular waveguide would again reinstate the detailed balance condition. We assume that this second process is not coherent due to dephasing of the photon while traveling along the loop. Such a dephasing may, for example, be caused by scattering processes induced in the long section of the loop.

The second order term given in Eq. (S-32) is modified in the closed system as follows,

\[ \frac{d\langle n_A \rangle}{dt} = \gamma_0(\langle n_2 \rangle + 1)(\langle n_A \rangle + 1)\langle n_B \rangle - (\langle n_1 \rangle + 1)(\langle n_B \rangle + 1)\langle n_A \rangle) + \ldots, \] (S-59)

together with an equivalent term for \( \langle n_B \rangle \).

Another term of second order in \( |\hat{h}|^2 \) couples the two channels of the waveguide via a reservoir. For channel 1 it reads

\[ \frac{d\langle m_1 \rangle}{dt} = \gamma_3(\langle n_A \rangle + \langle n_B \rangle + 2)(\langle n_2 \rangle - \langle n_1 \rangle) + \ldots, \] (S-60)

with \( \gamma_3 = 2\pi^3|\hat{h}|^4\rho_{wg}/\hbar \).

Finally, there is a term connecting the channels via an intermediate excitation of the TLS, proportional to \( g_1^2g_2^2 \). This term can be neglected if one of the chiral couplings \( g_r \) is close to zero, as we have assumed in the numerical evaluation. Collecting all the terms, we obtain the rate equations for the photon occupation numbers,

\[ \frac{d\langle n_A \rangle}{dt} = \gamma_{dec}(\langle n_1 \rangle + \langle n_2 \rangle - 2\langle n_A \rangle) - \gamma_1(m - m_e)\langle n_A \rangle(\langle n_1 \rangle + 1) + \gamma_2m_e(\langle n_A \rangle + 1)\langle n_2 \rangle + 1) + \gamma_0(\langle n_2 \rangle + 1)(\langle n_A \rangle + 1)\langle n_B \rangle - (\langle n_1 \rangle + 1)(\langle n_B \rangle + 1)\langle n_A \rangle), \] (S-61)

\[ \frac{d\langle n_B \rangle}{dt} = \gamma_{dec}(\langle n_1 \rangle + \langle n_2 \rangle - 2\langle n_B \rangle) - \gamma_2(m - m_e)\langle n_B \rangle(\langle n_2 \rangle + 1) + \gamma_1m_e(\langle n_B \rangle + 1)(\langle n_1 \rangle + 1) + \gamma_0(\langle n_1 \rangle + 1)(\langle n_B \rangle + 1)\langle n_A \rangle - (\langle n_2 \rangle + 1)(\langle n_A \rangle + 1)\langle n_B \rangle), \] (S-62)

\[ \frac{d\langle n_1 \rangle}{dt} = \gamma_{dec}(\langle n_A \rangle + \langle n_B \rangle - 2\langle n_1 \rangle) - \gamma_{11}(m - m_e)\langle n_1 \rangle - m_e(\langle n_1 \rangle + 1) + \gamma_3(\langle n_A \rangle + \langle n_B \rangle + 2)(\langle n_2 \rangle - \langle n_1 \rangle), \] (S-63)

\[ \frac{d\langle n_2 \rangle}{dt} = \gamma_{dec}(\langle n_A \rangle + \langle n_B \rangle - 2\langle n_2 \rangle) - \gamma_{12}(m - m_e)\langle n_2 \rangle - m_e(\langle n_2 \rangle + 1) + \gamma_3(\langle n_A \rangle + \langle n_B \rangle + 2)(\langle n_1 \rangle - \langle n_2 \rangle). \] (S-64)

The average number of excited TLS is determined by

\[ \frac{dm_e}{dt} = \sum_{r=1,2} \tilde{\gamma}_{1r}[(m - m_e)\langle n_r \rangle - m_e(\langle n_r \rangle + 1)] \]
\[ - m_e(\tilde{\gamma}_1(\langle n_B \rangle + 1)(\langle n_1 \rangle + 1) + \tilde{\gamma}_2(\langle n_A \rangle + 1)(\langle n_2 \rangle + 1)) \]
\[ + (m - m_e)(\tilde{\gamma}_1(\langle n_A \rangle)(\langle n_1 \rangle + 1) + \tilde{\gamma}_2(\langle n_B \rangle)(\langle n_2 \rangle + 1)). \] (S-65)

The entropy \( S_{\text{rad}}(\omega) \) of the radiation per mode in the reservoirs and the waveguide depends only on \( \langle n_q(\omega) \rangle \) and
FIG. S6. Solutions of the rate equations (S-61)-(S-65) as function of time, starting from initial thermal equilibrium. Panel (a) displays \( \langle n_A \rangle(t) \), \( \langle n_B \rangle(t) \) and panel (b) \( \langle n_1 \rangle(t) \), \( \langle n_2 \rangle(t) \). The average occupations \( \langle n_q \rangle(t) \) per mode reach a novel steady state with \( \langle n_A \rangle \neq \langle n_B \rangle \). The photon densities in reservoir A and channel 1 fall to zero, whereas channel 2 stays occupied and reservoir B is populated. The displayed time interval corresponds to the time scale set by the coupling to the TLS. Parameters used are \( \gamma_{\text{dec}} = \gamma'_{\text{dec}} = \gamma_0 = \gamma_3 = 10 \text{ kHz} \), \( \tilde{\gamma}_1 = \tilde{\gamma}_{11} = 10 \text{ MHz} \), \( \gamma_1 = \gamma_{11} = 100 \text{ kHz} \), \( \tilde{\gamma}_2 = \tilde{\gamma}_{12} = \gamma_2 = \gamma_{12} = 0 \) and \( N = N'' = 100 \), \( \hbar\Omega/k_B T(0) = 1 \).

\[
\rho_{\text{rad}}(\omega) = \frac{\hbar\omega}{T} \langle n_l(\omega) \rangle + k_B \ln(1 + \langle n_l(\omega) \rangle), \tag{S-66}
\]

for \( l = A, B, 1, 2 \). Because the temperature depends on \( \langle n_l \rangle \) as described by Eq.(7) in the main text, the entropy is only a function of \( \langle n_l \rangle \). The effective entropies and temperatures in the reservoirs and the waveguide are computed under the assumption that the photons in each reservoir/channel thermalize in the usual way quickly as a non-interacting Bose gas. The probability \( m_e(0)/m \) for a TLS to be excited obeys the Boltzmann distribution

\[
\frac{m_e(0)}{m} = \frac{1}{e^{\hbar\Omega/k_B T} + 1}. \tag{S-67}
\]

Considering the \( m \) TLS as independent classical objects, their total entropy is given by

\[
S_M = -k_B m (p_e \ln p_e + (1 - p_e) \ln(1 - p_e)), \tag{S-68}
\]
with $p_e(t) = m_e(t)/m$. This approach is justified by the quick relaxation of the two-level systems by non-radiative processes, which decohere them on time scales much shorter than the time scale for spontaneous emission. These processes quickly quench finite coherences of the TLS, see ref.[37]. In any case, (S-68) provides an upper bound for the actual entropy of the TLS subsystem. We approximate the total entropy by the sum of the entropies of the subsystems, as is justified for large $\langle n_q \rangle$ and $m$. The total entropy is given by

$$S(t) = S_M(t) + \int_{\Omega - \Delta/2}^{\Omega + \Delta/2} d\omega [\rho(\omega)S^R(\omega, t) + \rho_{wg}(\omega)S^{wg}(\omega, t)],$$

with

$$S^R = S^A_{rad} + S^B_{rad}, \quad S^{wg} = S^1_{rad} + S^2_{rad}. \quad \quad (S-70)$$

For constant densities of states $\rho$ and $\rho_{wg}$, we may write

$$S(t) = S_M(t) + N S^R(\Omega, t) + N' S^{wg}(\Omega, t).$$

The temporal evolution of the closed system, starting with thermal equilibrium, is depicted in Fig. S6 for the same parameters as in the open system. The non-reciprocal interaction with the TLS empties reservoir $A$ and the active (coupled) channel 1, while the occupation of reservoir $B$ rises. The inert channel 2 is unaffected on this short timescales. It interacts via the weak couplings $\gamma_{12}$ and $\gamma_3$ with $B$, which manifests only on much longer timescales, as depicted in Fig. S7. This separation of timescales has the same origin in the closed and the open system.

**Detailed balance for a system with an embedded cavity**

We shall now demonstrate that the term in (S-44) corresponding to radiation stimulated by the receiving reservoir leads to the detailed balance condition in case a cavity is embedded into another one. Here, detailed balance is also obtained in the second-order terms, in contrast to the chiral system treated in the previous section. We consider a closed cavity $C$ with adiabatic walls. Inside of $C$ there is a smaller cavity $A$ which is coupled to $C$ through a small opening. Besides $A$, a collection of $m$ two-level systems is located in $C$ (Fig. S8). The Hamiltonian of this system is given as

$$H = H_A + H_C + H_{TLS} + H^1_{int} + H^2_{int}, \quad \quad (S-72)$$

where

$$H_q = \hbar \sum_j \omega_{qj} a_{qj}^\dagger a_{qj}, \quad H_{TLS} = \frac{\hbar \Omega}{2} \sum_l \sigma^z_l \quad (S-73)$$

for $q = A, C$. The interaction between $A$ and $C$ is given by

$$H^1_{int} = \sum_{j,k} h_{jk} a_{Aj}^\dagger a_{Ck} + \text{h.c.}, \quad \quad (S-74)$$

and $C$ interacts with $M$ as

$$H^2_{int} = \sum_{i=1}^{m} \sum_k g_k a_{Ck} \sigma^z_i + \text{h.c.} \quad (S-75)$$

The rate equations are computed as above, but now the exchange between $A$ and $C$ is given by terms of first order in the coupling $h_{jk}$,

$$\frac{d\langle n_A \rangle}{dt} = \gamma_4 [-\langle n_A \rangle(\langle n_C \rangle + 1) + \langle n_C \rangle(\langle n_A \rangle + 1)] + \ldots, \quad (S-76)$$

$$\frac{d\langle n_C \rangle}{dt} = \gamma_4 [-\langle n_C \rangle(\langle n_A \rangle + 1) + \langle n_A \rangle(\langle n_C \rangle + 1)] + \ldots \quad (S-77)$$
FIG. S7. The asymptotic temporal behavior of reservoir $B$ (panel (a)) and the inert channel 2 (panel (b)). The occupation in $A$ and channel 1 is almost zero. The novel steady state has unequal occupations in all photonic subsystems and therefore lower entropy than the initial state. The parameters used are the ones of Fig. S6.

with $\gamma_4' = (\rho_A/\rho_C)\gamma_4$, (compare Eq.(S-54)). The interaction between the TLS and $C$ leads to the terms

$$\frac{d\langle n_C \rangle}{dt} = -\gamma_5\langle n_C \rangle(m - m_e) + \gamma_5(\langle n_C \rangle + 1)m_e + \ldots,$$

(S-78)

which are of first order in $g_k$ and correspond to standard black-body radiation. Besides these first-order terms, there are also terms of second order in the couplings $h_{jk}$ and $g_k$, describing the interaction of the small cavity $A$ with the TLS via intermediate states belonging to $C$. However, in contrast to the second-order terms discussed in section , these second-order terms are compatible with detailed balance. The corresponding terms in the rate equation for $A$ read

$$\frac{d\langle n_A \rangle}{dt} = \gamma_6(\langle n_C \rangle + 1)\left[ -\langle n_A \rangle(m - m_e) + (\langle n_A \rangle + 1)m_e \right] + \ldots$$

(S-79)

As above, the term for stimulated emission, $\gamma_6(\langle n_C \rangle + 1)^2\langle n_A \rangle m_e$, is not related to radiation emerging from cavity $A$ into $C$ which would have the wrong direction (see Fig. S8), but comes from the occupation of $A$-modes in the final
FIG. S8. The cavity system. The small cavity $A$ exchanges radiation with the surrounding closed cavity $C$. The collection $M$ of two-level systems interacts with the radiation modes of $C$.

state. The rate equations for $A$ and $C$ are therefore

$$\frac{d\langle n_A \rangle}{dt} = \gamma_4 [-(\langle n_A \rangle + \langle n_C \rangle)] + \gamma_6 (\langle n_C \rangle + 1)^2 [-(\langle n_A \rangle)(m - m_e) + (\langle n_A \rangle + 1)m_e], \tag{S-80}$$

$$\frac{d\langle n_C \rangle}{dt} = \gamma'_4 [-(\langle n_C \rangle + \langle n_A \rangle)] - \gamma_5 \langle n_C \rangle(m - m_e) + \gamma_5 (\langle n_C \rangle + 1)m_e. \tag{S-81}$$

These equations fulfill the detailed balance condition and lead to thermal equilibrium between $A$, $C$ and $M$. The result is consistent with Kirchhoff’s law, which states that the interior of a thermally isolated hohlraum has no influence on the final steady state of the contained radiation. This radiation exhibits the black-body spectrum found by M. Planck.